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An FGM decomposition-based fuzzy MCDM method for selecting smart technology applications to support mobile health care during and after the COVID-19 pandemic

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Abstract

In a fuzzy multicriteria decision-making (MCDM) problem, a decision maker may have differing viewpoints on the relative priorities of criteria. However, traditional methods merge these viewpoints into a single one, which leads to an unrepresentative decision-making result. Several recent methods identify the multiple viewpoints of a decision maker by decomposing the decision maker’s fuzzy judgment matrix into several symmetric fuzzy subjudgment matrices, which is an inflexible strategy. To enhance flexibility, this study proposed a fuzzy geometric mean (FGM) decomposition-based fuzzy MCDM method in which FGM is applied to decompose a fuzzy judgment matrix into several fuzzy subjudgment matrices that can be asymmetric. These fuzzy subjudgment matrices are diverse and more consistent than the original fuzzy judgment matrix. The proposed methodology was applied to select the best choice from a group of smart technology applications for supporting mobile health care during and after the COVID-19 pandemic. According to the experimental results, the proposed methodology provided a novel approach to decomposing fuzzy judgment matrices and produced more diverse fuzzy subjudgment matrices.

1. Introduction

In multicriteria decision-making (MCDM), a decision maker sometimes select from a set of alternatives that are diverse but equally optimal (for his/her particular application), such as selecting professionals with distinct talents to enrich a department [1] and acquiring machines with unique strengths to support various purposes [2]. To address this requirement, some recent studies have adopted various fuzzy methods to provide high flexibility [3–6]. Other studies have formulated methods that recommend the top few alternatives, where only one among these similar alternatives is optimal. The present research was undertaken to address these problems.

The literature is reviewed as follows. Most previous studies have formulated methods that analyze (and aggregate) the diverse viewpoints of multiple decision makers [7–9], rather than identify all viewpoints held by a single decision maker. For example, Sun et al. [10] considered a fuzzy MCDM problem in which multiple decision makers are involved and these decision makers set different priorities for criteria. Instead of applying the prevalent fuzzy weighted average (FWA), fuzzy geometric mean (FGM), or fuzzy intersection (FI) aggregators [11–13], Sun et al. constructed a diversified binary fuzzy relation to aggregate the unequal priority sets of all decision makers to determine a joint decision. Several previous studies have identified the multiple viewpoints of one decision maker. For example, a decision maker exhibits bounded rationality and varying patterns of cognition, such as loss avoidance, sensitivity reduction, distorted probability judgment, and regret aversion [14,15]. Jin et al. [15] devised a regret–rejoice function in which the greater the deviation between the utilities of two alternatives is, the less a decision maker regrets selecting a particular alternative. By using the arithmetic mean, Lin and Chen [16] decomposed the judgment matrix of a decision maker into multiple submatrices that represented that decision maker’s numerous viewpoints. These subjudgment matrices were more consistent than the original judgment matrix and were designed to differ from each other as much as possible. On the basis of each subjudgment matrix, a set of priorities were derived for the given criteria, according to which the top performing alternative could be determined. Finally, multiple diverse alternatives could be chosen. Chen and Wu [17] applied a similar methodology to assess the suitability of a given eHealth application. Chen and Lin [2] transformed this approach into
a fuzzy one, applying the fuzzy arithmetic mean (FAM) to decompose a fuzzy judgment matrix. However, such methods apply FAM to decompose a fuzzy judgment matrix. Thus, if two fuzzy subjudgment matrices are generated, they are symmetric—the two fuzzy subjudgment matrices have the same distance from the original judgment matrix, which is inflexible. In addition, in theory, fuzzy judgment matrices can be decomposed by other suitable techniques.

Therefore, the objectives of this study are as follows.

- To generate fuzzy subjudgment matrices that can be asymmetric.
- To decompose a fuzzy judgment matrix by using other techniques: The existing FAM method can also be applied to generate fuzzy subjudgment matrices that are asymmetric by assigning different weights to these fuzzy subjudgment matrices, i.e., applying FWA instead. However, determining the weights of fuzzy subjudgment matrices is an extra task for the decision maker who may not know how to determine these weights. In contrast, this study does not require a decision maker to determine these weights.

To fulfill these objectives, a fuzzy geometric mean (FGM) decomposition-based fuzzy MCDM method was proposed. In the proposed methodology, FGM \cite{18,19} is applied to decompose a fuzzy judgment matrix into fuzzy subjudgment matrices that may be asymmetric (i.e., with unequal distances from the original fuzzy judgment matrix). However, this method faces the following challenges.

1. Each linguistic term is represented by a prespecified fuzzy number, but the FGM result may no longer belong to the set of prespecified fuzzy numbers, and a fuzzy subjudgment matrix containing prespecified fuzzy numbers only is preferable.
2. There are infinitely many ways to decompose a fuzzy judgment matrix by using FGM.

To cope with these challenges, the membership of the FGM result in the fuzzy judgment matrix is relaxed to be within (0, 1). Subsequently, a multiobjective fuzzy integer-nonlinear programming (FINLP) model is optimized to decompose a fuzzy judgment matrix into fuzzy subjudgment matrices. To easily solve the problem of decomposing the fuzzy judgment matrix into fuzzy subjudgment matrices (by using FGM), the multiobjective FINLP problem is converted into a crisp integer-linear programming (INLP) problem. The effectiveness of the proposed methodology was validated by applying it to a selection of diverse mobile ehealth applications during and after the COVID-19 pandemic.

The novelty of the proposed methodology is explained as follows. Although FGM has been widely used in MCDM to derive the fuzzy priorities of criteria or to aggregate the judgments of multiple decision makers, it has not been applied to decompose a fuzzy judgment matrix, which is much complicated than the previous two applications. The reason is that the elements of the decomposed sub-judgment matrices may not conform to the linguistic variables defined at the beginning, which needs to be resolved through a fuzzy mapping process.

The rest of this paper is organized as follows. Section 2 presents some preliminary arithmetic and geometric operations on triangular fuzzy numbers (TFNs). Section 3 introduces the proposed FGM decomposition-based fuzzy MCDM method. Section 4 details the application of the proposed methodology to the case of selecting the appropriate ehealth mobile technology (during and after the COVID-19 pandemic); it also reports the results of a comparison with some existing methods. Section 5 summarizes this study and outlines some topics for future research.

### 2. Preliminaries

Without loss of generality, all fuzzy parameters and variables in the proposed methodology are provided in (or approximated by using) TFNs \cite{20,21}. Thus, some crucial arithmetic and geometric operations on TFNs are outlined here.

**Theorem 1.** The arithmetic operations on two TFNs $\tilde{B} = (B_1, B_2, B_3)$ and $\tilde{C} = (C_1, C_2, C_3)$ are defined by the following equalities \cite{22}.

- **Fuzzy addition:** $\tilde{B}(+)\tilde{C} \equiv (B_1 + C_1, B_2 + C_2, B_3 + C_3)$.
- **Fuzzy subtraction:** $\tilde{B}(-)\tilde{C} \equiv (B_1 - C_1, B_2 - C_2, B_3 - C_3)$.
- **Fuzzy multiplication:** $\tilde{B}(\times)\tilde{C} \equiv (B_1C_1, B_2C_2, B_3C_3)$ if $B_1, C_1 \geq 0$.
- **Fuzzy division:** $\tilde{B}(\div)\tilde{C} \equiv (B_1/C_1, B_2/C_2, B_3/C_3)$ if $B_1 \geq 0, C_1 > 0$.

**Theorem 2.** The possible intersection points of two TFNs $\tilde{B} = (B_1, B_2, B_3)$ and $\tilde{C} = (C_1, C_2, C_3)$ include the following.

\[
\begin{align*}
(x_1, y_1) &= \left(\frac{B_1 - C_1}{C_2 - C_1 - B_2 + B_1}, \frac{B_1 - C_1}{C_2 - C_1 + B_1 + B_2}\right) & \text{if } 0 < y_1 \leq 1. \\
(x_2, y_2) &= \left(\frac{B_1 - C_1}{C_2 + C_3 - C_2 - B_2 + B_1}, \frac{B_1 - C_1}{C_2 + C_3 - B_2 + B_1}\right) & \text{if } 0 < y_2 \leq 1. \\
(x_3, y_3) &= \left(\frac{B_1 - C_1}{C_2 + C_2 - C_2 + B_1}, \frac{B_1 - C_1}{C_2 + C_2 - C_2 + B_1}\right) & \text{if } 0 < y_3 \leq 1. \\
(x_4, y_4) &= \left(\frac{B_1 - C_1}{C_2 + C_3 - C_3 - B_2 + B_3}, \frac{B_1 - C_1}{C_2 + C_3 - C_3 - B_2 + B_3}\right) & \text{if } 0 < y_4 \leq 1.
\end{align*}
\]

**Proof.** The required proof is trivial.

### 3. FGM decomposition-based fuzzy MCDM approach

The proposed FGM decomposition-based fuzzy MCDM approach involves the following steps.

- **Step 1**: Compare the relative priorities of criteria in pairs.
- **Step 2**: Construct the fuzzy judgment matrix.
- **Step 3**: Decompose the fuzzy judgment matrix into fuzzy subjudgment matrices by using FGM.
- **Step 4**: Formulate the FINLP model.
- **Step 5**: Convert the FINLP model.
- **Step 6**: Derive a fuzzy priority set from each fuzzy subjudgment matrix.
- **Step 7**: Assess alternatives on the basis of each priority set.
- **Step 8**: Combine the assessment results.

This procedure is illustrated in Fig. 1.

3.1. Steps 1 to 2: Pairwise criteria comparison and fuzzy matrix construction

In a fuzzy MCDM problem, decision makers are usually required to express their opinions on the relative priority of one criterion over another in linguistic (or semantic) terms. A common practice is to represent these linguistic terms by TFNs that are in the range $(1, 9)$ \cite{19}, as indicated in Table 1.

| TFNs for expressing linguistic terms. |
|--------------------------------------|
| **Linguistic Term**                   | **TFN** |
| As important as                       | $(1, 1, 3)$ |
| As important as or weakly more important than | $(1, 2, 4)$ |
| Weakly more important than            | $(1, 3, 5)$ |
| Weakly or strongly more important than| $(2, 4, 6)$ |
| Strongly more important than          | $(3, 5, 7)$ |
| Strongly or very strongly more important than | $(4, 6, 8)$ |
| Very strongly more important than     | $(5, 7, 9)$ |
| Very strongly or absolutely more important than | $(6, 8, 9)$ |
| Absolutely more important than        | $(7, 9, 9)$ |
The results are summarized with a fuzzy judgment matrix \( \tilde{A} = [\tilde{a}_{ij}] \); \( i,j = 1 \sim n \), where \( \tilde{a}_{ij} \) is the relative priority of criterion \( i \) over criterion \( j \) and \( n \) is the number of criteria. \( \tilde{A} \) meets the following requirements [23]:

\[
det(\tilde{A}(-\tilde{\lambda})I) = 0
\]

\[
(\tilde{A}(-\tilde{\lambda})I) \times \tilde{x} = 0,
\]

where \( \det() \) is the determinant function, \( \tilde{\lambda} \) is the fuzzy eigenvalue, and \( \tilde{x} \) is the fuzzy eigenvector. If \( \tilde{A} \) is sufficiently consistent, the fuzzy priorities of criteria \( \{\tilde{w}_i\} \) can be derived from \( \tilde{A} \) as follows [12]:

\[
w_{11} \tilde{\equiv} \frac{1}{1 + \sum_{m \neq i, n} \frac{n \prod_{j=1}^{n} a_{mj}}{\prod_{j=1}^{n} a_{ij}}}
\]

\[
w_{12} \tilde{\equiv} \frac{1}{1 + \sum_{m \neq i} \frac{n \prod_{j=1}^{n} a_{mj}}{\prod_{j=1}^{n} a_{ij}}}
\]

where \( \tilde{w}_i \) is the fuzzy priority of criterion \( j \). According to Eqs. (1)–(3), the fuzzy consistency ratio of \( \tilde{A} \), \( CR(\tilde{A}) \), can be calculated as follows [12]:

\[
CR_1(\tilde{A}) = \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} w_{ij}}{(n-1)RI}
\]

\[
CR_2(\tilde{A}) = \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} w_{ij}}{(n-1)RI}
\]

\[
CR_3(\tilde{A}) = \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} w_{ij}}{(n-1)RI}
\]

where \( RI \) denotes the random consistency index [23]. In basic terms, \( \tilde{A} \) is consistent if \( CR(\tilde{A}) \leq 0.1 \) [23]. However, for a large or complicated problem, the threshold can be slightly relaxed, e.g., to 0.3 [24,25]. Steps 1 and 2 involve only simple equations (i.e., Eqs. (1) to (6)) that can be readily solved. Therefore, there is no need to design an additional algorithm. The results of Steps 1 and 2 become the inputs to Step 3.

3.2. Step 3: Decomposing the fuzzy judgment matrix

A decision maker may have multiple viewpoints regarding the relative priorities of criteria. To investigate this possibility, the fuzzy judgment matrix of a decision maker can be decomposed into multiple fuzzy subjudgment matrices \( \tilde{A}(k); k = 1 \sim K \) to represent these viewpoints by using FGM as follows.

\[
\tilde{A}: = \sqrt[\times K]{\prod_{k=1}^{K} \tilde{A}(k)}
\]

or

\[
\tilde{a}_{ij} := \sqrt[\times K]{\prod_{k=1}^{K} a_{ij}(k)}\forall \tilde{a}_{ij} > 1
\]

where \( K \) is the number of fuzzy subjudgment matrices and \( : = \) means “is defined as”. All the fuzzy subjudgment matrices meet the following basic requirements [2]:

\[
det(\tilde{A}(k)(-\tilde{\lambda}(k))I) = 0
\]

\[
(\tilde{A}(k)(-\tilde{\lambda}(k))I) \times \tilde{x}(k) = 0.
\]

The FGM result can be precisely approximated by using TFNs [26]:

\[
\tilde{a}_{ij} \approx \sqrt[\times K]{\prod_{k=1}^{K} \tilde{a}_{ij}(k)}\forall \tilde{a}_{ij} > 1
\]

However, the methods of decomposing a fuzzy judgment matrix by using FGM are infinitely many. In addition, obtaining prespecified fuzzy numbers within \((1, 9)\) for all \( \tilde{a}_{ij}(k) \) values is difficult. To overcome these difficulties, the following principles are applied in the proposed methodology.

(1) \( \tilde{A}(k) \) is selected from the predefined TFNs. Consequently, for a \( n \times n \) fuzzy judgment matrix, the number of possible decompositions is at most \((9^K)^{n^2} \).
Table 2
Possible decomposition results with various levels of $\xi$.

| $\xi$ | Possible Decomposition Results |
|-------|--------------------------------|
| 0.1   | $\{(1, 1, 3), (1, 2, 4), \}, \{(6, 8, 9), (7, 9, 9)\}$, and all below |
| 0.2   | $\{(1, 1, 3), (1, 3, 5), \}, \{(1, 1, 3), (2, 4, 6)\}, \{(1, 2, 4), (2, 4, 6)\}, \{(5, 7, 9), (7, 9, 9)\}$, and all below |
| 0.3   | $\{(1, 1, 3), (3, 5, 7), \}, \{(1, 2, 4), (1, 3, 5)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 0.4   | $\{(1, 1, 3), (6, 8, 9), \}, \{(1, 2, 4), (2, 4, 6)\}, \{(5, 7, 9), (7, 9, 9)\}$, and all below |
| 0.5   | $\{(1, 1, 3), (6, 8, 9), \}, \{(1, 2, 4), (2, 4, 6)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 0.6   | $\{(1, 2, 4), (4, 6, 8), \}, \{(1, 3, 5), (2, 4, 6)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 0.7   | $\{(1, 2, 4), (4, 6, 8), \}, \{(1, 3, 5), (2, 4, 6)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 0.8   | $\{(1, 2, 4), (4, 6, 8), \}, \{(1, 3, 5), (2, 4, 6)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 0.9   | $\{(1, 2, 4), (4, 6, 8), \}, \{(1, 3, 5), (2, 4, 6)\}, \{(5, 7, 9), (5, 7, 9)\}$, and all below |
| 1.0   | $\{(3, 5, 7), (3, 5, 7)\}$ |

Table 3
Possible decomposition results obtained using FAM.

| $\xi$ | Possible Decomposition Results |
|-------|--------------------------------|
| 1     | $\{(1, 1, 3), (7, 9, 9), \}, \{(1, 2, 4), (6, 8, 9)\}, \{(3, 5, 7), (3, 5, 7)\}$ |

Fig. 2. Number of decomposition results obtained with distinct $\xi$ values.

Example 1. Assuming $\tilde{a}_{ij} = (3, 5, 7)$ and $K = 2$, the possible decomposition results with various levels of $\xi$ are summarized in Table 2. The number of possible decompositions increases rapidly as $\xi$ decreases (Fig. 2). Therefore, setting $\xi$ to a fairly high level is preferable. For comparison, the possible decomposition results obtained using FAM [2] are presented in Table 3. Notably,

1. The number of possible decompositions obtained using FGM is much greater than that of those obtained using FAM.
2. The two fuzzy subjudgment matrices generated using FAM are symmetric, whereas those generated using FGM may be asymmetric.

To select the optimal decomposition result, a multiobjective FINLP model is formulated, as described in the next section.

An algorithm for Step 3 is provided in Fig. 3.

3.3. Step 4: Formulating the FINLP model

To optimize the decomposition result, studies have focused on achieving the following objectives.

1. All fuzzy subjudgment matrices must be more consistent than the original fuzzy judgment matrix [2,16,17].
2. Fuzzy subjudgment matrices are diversified by maximizing their distances from each other [2,16,17].

In the proposed methodology, another objective is considered, namely the maximization of the membership of the FGM result in the fuzzy judgment matrix. Accordingly, the following multi-objective FINLP problem is solved to optimize the decomposition result.

(Multiobjective FINLP Model)

Min $\tilde{Z}_1 = \sum_{k=1}^{K} \tilde{\alpha}(k)$

Max $\tilde{Z}_2 = \sum_{k=1}^{K-1} \sum_{l=k+1}^{K} \tilde{d}(\tilde{A}(k), \tilde{A}(l))$

Max $Z_3 = \frac{2}{n(n-1)} \sum_{\tilde{a}_{ij} > 1} \max \mu_{\tilde{a}_{ij}}(\tilde{\psi}_{ij})$

These equations are subject to the following constraints.

$\tilde{\psi}_{ij} = \prod_{k=1}^{K} \tilde{a}_{ij}(k)$

$1 \leq \tilde{a}_{ij}(k) \leq 9 \forall i, j = 1 - n; k = 1 - K$

$\tilde{a}_{ij}(k) = 1/\tilde{a}_{ij}(k) \forall i, j = 1 - n; k = 1 - K$

$\tilde{a}_{ij}(k) \in \mathbb{Z}^+ \forall i, j = 1 - n; k = 1 - K$, $\xi \in (0, 1)$.
where $\tilde{CR}(k)$ is the fuzzy consistency ratio of $\tilde{A}(k)$, $\tilde{d}()$ is the fuzzy distance function, and $\tilde{\phi}_{ij}$ is the FGM result. Constraint (15) is derived directly from Eq. (7), and Constraints (16)–(18) are the basic requirements for a fuzzy judgment (or subjudgment) matrix [2]. Notably, $0 \leq Z_3 \leq 1$. The result of Step 4 is the FINLP model that becomes an input to Step 5 and will not be solved directly. Therefore, there is no need to design an algorithm for this step.

Fig. 3. Algorithm for Step 3.

The multiobjective FINLP problem must be converted into a more tractable form to be easily solved [27,28]. To this end, the fuzzy goal programming (or fuzzy satisfying) approach [29,30] is applied, as described in the next section.

3.4. Step 5: Converting the FINLP model into an equivalent INLP problem

First, the goals for the first two objective functions are established as follows:

$$\tilde{Z}_1 \leq \xi_1$$

$$\tilde{Z}_2 \geq \xi_2,$$

where $\xi_1$ and $\xi_2$ are positive goals. The satisfaction levels of achieving these goals are evaluated as follows:

$$s_1 = \xi_1 - Z_{11} - Z_{12}$$

$$s_2 = \frac{\xi_2 - Z_{23}}{Z_{22} - Z_{23}},$$

as illustrated in Fig. 4. $s_1$ and $s_2$ are satisfaction levels that are to be maximized, as is $Z_3$. Therefore, the three objective functions can be aggregated as follows.

$$\text{Max } Z_4 = s_1 + s_2 + Z_3$$

In Eq. (21), the values of $Z_{11}$ and $Z_{12}$ can be derived by applying Eqs. (4)–(6) to Eq. (12) as follows.

$$Z_{11} = \sum_{k=1}^{K} CR_i(k)$$

$$= \sum_{k=1}^{K} 1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j \neq i} a_{ij}(k)w_{ij}(k)$$

$$= \sum_{k=1}^{K} \frac{1}{(n-1)RI}$$

Fig. 4. Goals for the first two objective functions.
\[ Z_{12} = \sum_{k=1}^{K} C R_2(k) \]
\[ = \sum_{k=1}^{K} \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j \neq i} a_{ij}(k) w_{ij}(k)}{(n - 1) R I} \quad (25) \]

In Eq. (13), the fuzzy distance between two fuzzy subjudgment matrices can be measured using the fuzzy Frobenius distance as follows [2,31].

\[ \tilde{d}(\tilde{A}(k), \tilde{A}(l)) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} [\tilde{a}_{ij}(k) - \tilde{a}_{ij}(l)]^2}, \quad (26) \]

which can be decomposed into the following:

\[ d_1(\tilde{A}(k), \tilde{A}(l)) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \min(\max(a_{ij}(k) - a_{ij}(l), 0), \max(a_{ij}(l) - a_{ij}(k), 0))^2}, \quad (27) \]

\[ d_2(\tilde{A}(k), \tilde{A}(l)) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}(k) - a_{ij}(l))^2} \quad (28) \]

\[ d_3(\tilde{A}(k), \tilde{A}(l)) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \max(\max(a_{ij}(k) - a_{ij}(l), 0), \max(a_{ij}(l) - a_{ij}(k), 0))^2}. \quad (29) \]

Subsequently, from Eq. (14), max \( \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) \) is determined by the highest membership of the four intersection points of \( \tilde{q}_{ij} \) and \( \tilde{q}_{ij} \), which enables \( \gamma_{q_{ij}} \sim \gamma_{q_{ij}} \) to indicate the membership of the four intersection points. According to Theorem 2, the following equations can be derived.

\[ \gamma_{q_{11}} = \frac{a_{q_{11}} - a_{q_{11}}}{a_{q_{21}} - a_{q_{21}} - a_{q_{12}} + a_{q_{12}}} \quad (30) \]

\[ \gamma_{q_{12}} = \frac{a_{q_{12}} - a_{q_{12}} - a_{q_{12}} + a_{q_{12}}}{\tilde{a}_{q_{12}} - \tilde{a}_{q_{12}} - \tilde{a}_{q_{12}} + \tilde{a}_{q_{12}}} \quad (31) \]

\[ \gamma_{q_{13}} = \frac{a_{q_{13}} - a_{q_{13}}}{a_{q_{23}} - a_{q_{23}} - a_{q_{12}} + a_{q_{12}}} \quad (32) \]

\[ \gamma_{q_{14}} = \frac{a_{q_{14}} - a_{q_{14}}}{a_{q_{24}} - a_{q_{24}} - a_{q_{12}} + a_{q_{12}}} \quad (33) \]

where \( \varepsilon_{q_{ij}} \in \{0, 1\} \) is a state variable. Therefore,

\[ \max \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) \geq \gamma_{q_{ij}}; \, q = 1 \sim 4 \]

\[ \prod_{q=1}^{4} (\max \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) - \gamma_{q_{ij}}) = 0 \quad (35) \]

According to Eq. (10), Eq. (15) is decomposed into the following:

\[ \psi_{q_{11}} = \sqrt{\prod_{k=1}^{K} a_{q_{11}}(k)} \quad (36) \]

\[ \psi_{q_{12}} = \sqrt{\prod_{k=1}^{K} a_{q_{12}}(k)} \quad (37) \]

\[ \psi_{q_{13}} = \sqrt{\prod_{k=1}^{K} a_{q_{13}}(k)} \quad (38) \]

\[ Eq. (16) \text{ is equal to} \]

\[ 1 \leq a_{ij}(k) \leq 9; \, \gamma_{q_{ij}} = 1 \sim 3 \]

\[ a_{ij}(k) = \max(1, a_{ij}(k) - 2) \quad (40) \]

\[ a_{ij}(k) = \min(9, a_{ij}(k) + 2). \quad (41) \]

After removing the maximum function, Eq. (40) becomes the following:

\[ (a_{ij}(k) - 1)(a_{ij}(k) - a_{ij}(k) + 2) = 0. \quad (42) \]

Similarly, after removing the minimum function, Eq. (41) changes to the following:

\[ (a_{ij}(k) - 9)(a_{ij}(k) - a_{ij}(k) - 2) = 0. \quad (43) \]

Finally, the following INLP problem is solved.

(INLP Model)

\[ \text{Max } Z_4 = s_1 + s_2 + Z_3 \quad (44) \]

s.t.

\[ s_1 = \frac{s_1 - Z_{11}}{Z_{12} - Z_{11}} \quad (45) \]

\[ s_2 = \frac{s_2 - Z_{21}}{Z_{22} - Z_{23}} \quad (46) \]

\[ Z_{11} = \sum_{k=1}^{K} \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j \neq i} a_{ij}(k) w_{ij}(k)}{(n - 1) R I} \quad (47) \]

\[ Z_{12} = \sum_{k=1}^{K} \frac{1 - n + \frac{1}{n} \sum_{i=1}^{n} \sum_{j \neq i} a_{ij}(k) w_{ij}(k)}{(n - 1) R I} \quad (48) \]

\[ w_{i1}(k) = \frac{1}{1 + \sum_{m \neq i} \prod_{j=1}^{m} a_{ij}(k)}; \, i = 1 \sim n; \, k = 1 \sim K \quad (49) \]

\[ w_{i2}(k) = \frac{1}{1 + \sum_{m \neq i} \prod_{j=1}^{m} a_{ij}(k)}; \, i = 1 \sim n; \, k = 1 \sim K \quad (50) \]

\[ w_{i3}(k) = \frac{1}{1 + \sum_{m \neq i} \prod_{j=1}^{m} a_{ij}(k)}; \, i = 1 \sim n; \, k = 1 \sim K \quad (51) \]

\[ \prod_{q=1}^{4} (\max \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) - \gamma_{q_{ij}}) = 0 \quad (35) \]

According to Eq. (10), Eq. (15) is decomposed into the following:

\[ Z_{22} = \sum_{k=1}^{K-1} \sum_{h=1}^{K} \sum_{i=1}^{n} \sum_{j \neq i} (a_{ij}(k) - a_{ij}(l))^2 \quad (52) \]

\[ Z_{23} = \sum_{k=1}^{K-1} \sum_{h=1}^{K} \sum_{i=1}^{n} \sum_{j \neq i} \max(\max(a_{ij}(k) - a_{ij}(l), 0), \max(a_{ij}(l) - a_{ij}(k), 0))^2 \quad (53) \]

\[ Z_3 = \frac{2}{n(n - 1)} \sum_{q_{ij} \neq 1} \max \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) \quad (54) \]

\[ \max \mu_{\tilde{q}_{ij}}(\tilde{q}_{ij}) \geq \gamma_{q_{ij}}; \, i, j = 1 \sim n; \, i \neq j; \, q = 1 \sim 4 \]
\[
\prod_{q=1}^{4} \left( \max_{i,j} \mu_{q}(\varphi_{ij}) - \gamma_{q} \right) = 0; \; i, j = 1 \sim n; \; i \neq j
\]  
(55)
\[
\gamma_{q1} = \zeta_{q1} \left( \frac{\alpha_{q1} - \varphi_{ij} - \varphi_{ij} - \alpha_{q1}}{\varphi_{ij} - \varphi_{ij} - \alpha_{q1}} \right); \; i, j = 1 \sim n; \; i \neq j
\]  
(56)
\[
\gamma_{q2} = \zeta_{q2} \left( \frac{\alpha_{q2} - \varphi_{ij} - \varphi_{ij} - \alpha_{q2}}{\varphi_{ij} - \varphi_{ij} - \alpha_{q2}} \right); \; i, j = 1 \sim n; \; i \neq j
\]  
(57)
\[
\gamma_{q3} = \zeta_{q3} \left( \frac{\alpha_{q3} - \varphi_{ij} - \varphi_{ij} - \alpha_{q3}}{\varphi_{ij} - \varphi_{ij} - \alpha_{q3}} \right); \; i, j = 1 \sim n; \; i \neq j
\]  
(58)
\[
\gamma_{q4} = \zeta_{q4} \left( \frac{\alpha_{q4} - \varphi_{ij} - \varphi_{ij} - \alpha_{q4}}{\varphi_{ij} - \varphi_{ij} - \alpha_{q4}} \right); \; i, j = 1 \sim n; \; i \neq j
\]  
(59)
\[
\varphi_{ij} = \left\{ \sum_{k=1}^{k} \varphi_{ij}(k) \right\}; \; i, j = 1 \sim n; \; i \neq j
\]  
(60)
\[
\varphi_{ij} = \left\{ \sum_{k=1}^{k} \varphi_{ij}(k) \right\}; \; i, j = 1 \sim n; \; i \neq j
\]  
(61)
\[
(\alpha_{ij}(k) - 1)(\alpha_{ij}(k) - \alpha_{ij}(k) + 2) = 0; \; i, j = 1 \sim n; \; i \neq j; \; k = 1 \sim K
\]  
(62)
\[
(\alpha_{ij}(k) - 9)(\alpha_{ij}(k) - \alpha_{ij}(k) - 2) = 0; \; i, j = 1 \sim n; \; i \neq j; \; k = 1 \sim K
\]  
(63)
\[
1 \leq \alpha_{ij}(k) \leq 9; \; z = 1 \sim 3; \; i, j = 1 \sim n; \; i \neq j; \; k = 1 \sim K; \; t = 1 \sim 3
\]  
(64)
\[
0 \leq \gamma_{q} \leq 1; \; i, j = 1 \sim n; \; i \neq j; \; q = 1 \sim 4
\]  
(65)
\[
\zeta_{q} \in [0, 1]; \; i, j = 1 \sim n; \; i \neq j; \; q = 1 \sim 4
\]  
(66)
\[
\alpha_{ij}(k) \in \mathbb{Z}^+ \forall i, j = 1 \sim n; \; k = 1 \sim K; \; t = 1 \sim 3
\]  
(67)

The INLP problem can be solved using methods such as the hybrid outer approximation and generalized Bender’s decomposition [32] or branch-and-bound methods [33]. In the method devised in this study, the INLP problem is solved using a branch-and-bound algorithm, for which the pseudocode is provided in Fig. 5. In this figure, the computational complexity of each major step is discussed. The computational complexity of Step 4 is O(Kn), while those of Steps 5, 7, 22, and 24 are all O(n). With regard to the computational costs in these steps, the computational complexity of the algorithm is O(Kn).

The inputs and outputs of this algorithm are the fuzzy judgment matrix and fuzzy sub-judgment matrices, respectively. The equations used in the steps are shown in the pseudo code. In particular, the feasibility of the FGM decomposition result is validated according to Eqs. (54) and (55), as shown in the pseudo code. \(M_1\) is a positive integer representing the upper bound on the candidate queue length. A branch is formed as the average of two solutions. If all possible branches have been evaluated, the process stops and the best solution is returned.

4.5. Steps 6 to 8: Deriving fuzzy priority sets, assessing alternatives, and combining assessment results

From each fuzzy subjudgment matrix, a fuzzy priority set is derived, on the basis of which the optimal alternative is selected. Consequently, multiple fuzzy priority sets are generated and the decision maker can select multiple alternatives, each of which are optimal depending on the viewpoint. Steps 6 to 8 are standard fuzzy analytic hierarchy process (FAHP) calculations, and many methods have been proposed for this purpose, such as the FCM, FTOPSIS method. The required algorithm refers to Zhang and Xu [8], which is not repeated here.

4. Case study: Selecting a diverse set of mobile ehealth applications during and after the COVID-19 pandemic

4.1. Background

Smart technologies involve networked devices or systems that interact with each other [34–36]. Smart technologies have been widely applied in the medical and health care fields before the COVID-19 pandemic [37–40]. Since the start of the COVID-19 pandemic, numerous smart applications have emerged. The following are some examples.

- Smart robots (or smart drones) help to communicate with, or send medicine to, a quarantined patient to reduce the burden on health care professionals and contain local outbreaks [41]. Such smart technologies are also used to oversee security in public spaces, broadcast information to people in public spaces, and monitor foot or vehicle traffic more efficiently [42].
- In factories, to prevent the spread of COVID-19 through physical contact with machines, voice commands or gestures [43,44] or smartphones [45] are used to remotely control machines.
- Workers can wear smart wristbands or watches to detect their body temperature [46].
- In hotels, autonomous robots emit concentrated ultraviolet C light to disinfect room keys, guest rooms, and public areas such as lobbies and gyms [47].
- In museums, wearable sensors are used to measure the proximity of visitors and ensure social distancing is maintained [48].
- Location-based app services can monitor crowd sizes and be used to disperse large crowds [49].

However, policymakers must choose which of many smart technologies to implement due to resource constraints. Therefore, a mechanism for evaluation and comparison is required to determine the more suitable smart technology application. In addition, the wide-ranging implications of the COVID-19 pandemic has meant that a host of (preferably complementary) smart technologies, rather than a single one, should be applied [50]. The proposed methodology was thus formulated to solve this problem.

4.2. Key factors in mobile ehealth applications

4.2.1. Before the COVID-19 pandemic

According to the survey results of Wu et al. [51], perceived service availability and personal innovativeness in information technology are the most crucial factors in smart technology applications. Chen [20] contended that the key factors for selecting smart technology applications to support mobile health care include unobtrusiveness, support for online social networking, relaxation of related medical laws, size of the future ehealth market, and correct identification of users’ needs and context.

However, the aforementioned factors apply to normal life before the COVID-19 pandemic and may be inapplicable to the pandemic, which is the focus of this study.

4.2.2. During and after the COVID-19 pandemic

Although the estimated costs and effectiveness of a smart technology application are not primary considerations during the COVID-19 pandemic, these factors will become more rigorously examined after the pandemic [52]. In addition, in the
postpandemic era, the estimated costs and effectiveness of a smart technology application will undoubtedly be key, but they are difficult to estimate [13].

Some interventions initiated during the COVID-19 pandemic are obtrusive but necessary [13]. Nevertheless, as the pandemic progresses, people wish to be more free from the restrictions entailed by pandemic prevention measures, although many still fear the virus. Therefore, rather than being unobtrusive, it is more important for a smart technology application to be accepted by users [53].
Further, returning to normal life means resuming in-person interactions [54]. A smart technology application’s ease of implementation and maintenance is also crucial to its widespread adoption [55].

Therefore, the following factors must be considered in selecting a smart technology application for supporting mobile health care during and after the COVID-19 pandemic:

- Estimated cost [52,53,55]
- Effectiveness [13,52]
- Acceptability [13,53,54]
- Resumption of physical human interactions [54]
- Ease of implementation and maintenance [53,56]

This problem is illustrated with an analytic hierarchy process diagram [57] in Fig. 6.

The present 1-year project entitled “smart technology applications for supporting medical and health care after the COVID-19 pandemic” was undertaken by a team of three members: an industrial engineering professor, an information technology engineer, and a health care technology researcher. The project recommended (to the local government) suitable smart technology applications that would support mobile health care after the COVID-19 pandemic.

4.3. Application of the proposed methodology

At the outset, the team members jointly compared the relative priorities of key factors (in pairs). The following fuzzy judgment matrix summarizes the comparison results:

\[
\tilde{A} = \begin{bmatrix}
1 & - & - & - & (2, 4, 6) \\
(3, 5, 7) & 1 & - & (1, 3, 5) & (2, 4, 6) \\
(2, 4, 6) & (2, 4, 6) & 1 & (2, 4, 6) & (1, 3, 5) \\
(1, 3, 5) & - & - & 1 & (1, 3, 5) \\
- & - & - & - & 1
\end{bmatrix}
\]

The \(\tilde{CR}\) of \(\tilde{A}\) evaluated using Eqs. (38)–(40) was (0.00, 0.17, 9.05), which was somewhat inconsistent but still acceptable to the project team.

Subsequently, the INLP model of the problem was formulated and optimized to decompose the fuzzy judgment matrix into two fuzzy subjudgment matrices. The goals for the two fuzzy objective functions were set to the following:

\[\xi_1 = 2 \cdot 0.1 = 0.2\]

\[\xi_2 = \sqrt{10 \cdot \xi^2} = 12.65\]

Subsequently, a branch-and-bound algorithm was applied to solve the INLP problem by using Lingo on a PC with an Intel i7-7700 3.6 GHz CPU with 8 GB of RAM. Eventually, the following optimal decomposition result was obtained:

\[
\tilde{A}(1) = \begin{bmatrix}
1 & - & - & - & (1, 1, 3) \\
(1, 2, 4) & 1 & - & (1, 2, 4) & (1, 2, 4) \\
(6, 8, 9) & (2, 4, 6) & 1 & (1, 2, 4) & (1, 3, 5) \\
(1, 3, 5) & - & - & 1 & (2, 4, 6) \\
- & - & - & - & 1
\end{bmatrix}
\]

\[
\tilde{A}(2) = \begin{bmatrix}
1 & - & - & - & (6, 8, 9) \\
(5, 7, 9) & 1 & - & (1, 2, 4) & (6, 8, 9) \\
(3, 5, 7) & (2, 4, 6) & 1 & (6, 8, 9) & (6, 8, 9) \\
(1, 1, 3) & - & - & 1 & (5, 7, 9) \\
- & - & - & - & 1
\end{bmatrix}
\]

This gave the results of \(Z_1 = (0.00, 0.24, 11.04)\), \(Z_2 = (0.00, 13.89, 21.11)\), \(Z_1^* = 0.47\), and \(Z_2^* = 2.45\). The consistency ratios (CRs) of the two fuzzy subjudgment matrices were as follows:

\[\tilde{CR}(\tilde{A}(1)) = (0.00, 0.07, 2.96)\]

\[\tilde{CR}(\tilde{A}(2)) = (0.00, 0.16, 2.96)\]

Both were more consistent than the original fuzzy judgment matrix. The fuzzy priorities of the key factors derived from the two fuzzy subjudgment matrices were as follows:

- Viewpoint 1: \((0.038, 0.074, 0.208), (0.083, 0.184, 0.403), (0.223, 0.458, 0.667), (0.078, 0.199, 0.397), (0.034, 0.085, 0.206)\)
- Viewpoint 2: \((0.050, 0.094, 0.149), (0.154, 0.246, 0.393), (0.357, 0.528, 0.651), (0.070, 0.107, 0.213), (0.017, 0.025, 0.043)\)

According to the two viewpoints, the priorities of key factors were ranked as follows:

- Viewpoint 1: acceptability > resuming physical human interactions > effectiveness > ease of implementation and maintenance > estimated costs
- Viewpoint 2: acceptability > effectiveness > resuming physical human interactions > estimated costs > ease of implementation and maintenance

Thus, the two sets of results differed considerably.

The following smart technology applications to support mobile health care were considered:

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Fig. 6. Decision hierarchy of the mobile health application selection problem.
These are summarized in Table 5, in which the statistical results of the price quotes by related vendors to the project team and city government consultants. None of these critical points were defined (Table 9). Subsequently, the distances from each smart technology application to the two reference points were measured as 

\[ d_{q_1} \] and \[ d_{q_2} \], respectively. The results are summarized in Table 10.

Finally, the overall performance of each smart technology application, in terms of its fuzzy closeness (\( C_q \)), was evaluated. The results are listed in Table 11. \( C_q \) ranged from 0 (worst) to 1 (best) and could be defuzzified using the center-of-gravity method [62]. The smart technology applications were ranked according to the defuzzification results. The calculations in Tables 8 to 10 can be performed for viewpoint #2 in the same way, and therefore are not repeated here. The ranking results obtained based on viewpoint 2 are also presented in Table 11.

### 4.4. Discussion

An analysis of the experimental results prompts the following points for discussion.

1. The two fuzzy subjudgment matrices generated using the proposed methodology were diverse. For example, the following is a randomly generated decomposition result that meets the specified constraints:

\[ \tilde{A}(1) = \begin{bmatrix} 1 & - & - & - \\ (3, 5, 7) & 1 & - & (1, 2, 4) \\ (3, 5, 7) & - & - & 1 \\ (3, 5, 7) & - & - & 1 \end{bmatrix} \]

\[ \tilde{A}(2) = \begin{bmatrix} 1 & - & - & - \\ (3, 5, 7) & 1 & - & (2, 4, 6) \\ (2, 4, 6) & (1, 2, 4) & 1 & (3, 5, 7) \\ (1, 1, 3) & - & - & 1 \end{bmatrix} \]

The distance between the two fuzzy subjudgment matrices was (0.00, 12.59, 15.43). This was shorter than that achieved using the proposed methodology, wherein the two fuzzy subjudgment matrices were farther apart (more distinct) from each other.

2. The CRs of the two fuzzy subjudgment matrices were (0.00, 0.07, 8.08) and (0.00, 0.16, 2.96), respectively, whereas that of the original fuzzy judgment matrix was (0.00, 0.17, 9.05). Notably, the two fuzzy subjudgment matrices were more consistent than the original fuzzy judgment matrix.

3. The ranking results of the smart technology applications for supporting mobile health care during and after the COVID-19 pandemic were not equal.

4. According to the two distinct viewpoints, the optimal smart technology applications for the task of support mobile health care during and after the COVID-19 pandemic were determined to be the app indicating where individuals

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| Table 4 | Details of the four smart technology applications. |
| --- | --- |
| Smart Technology Application | Estimated costs (NT$) | Effectiveness | Acceptability | Resuming physical human interactions | Ease of implementation and maintenance |
| I | 1,500,000 | Very low | Low | Moderate | Very easy |
| II | 600,000 | Moderate | Very High | Low | Easy |
| III | 250,000 | High | High | High | Moderate |
| IV | 2,500,000 | High | Low | Moderate | Very difficult |

---

(1) Application I: Using smart robots (or smart drones) to oversee security in public spaces, observe crowds and broadcast information to them, and monitor traffic more efficiently. The cost of hiring such a smart robot was approximately NT$200 per hour.

(2) Application II: Providing high-risk groups with smart bracelets that detect body temperature and blood oxygen levels. The cost of each smart bracelet was approximately NT$600.

(3) Application III: Providing an app indicating the places that individuals with confirmed COVID-19 cases visit more frequently. In Taiwan, tracking the movement of confirmed individuals is the responsibility of city governments.

(4) Application IV: Establishing a wide-ranging surveillance system to detect the body temperature of all pedestrians regardless of whether they wear facemasks. The cost of each unit of such a surveillance system was approximately NT$200,000.

Collected data regarding these four smart technology applications are presented in Table 4. These data were real data based on the statistical results of the price quotes by related vendors to the project team and city government consultants. None of the smart technology applications were without defect.

Criteria corresponding to the key factors were established. These are summarized in Table 5, in which \( \tilde{p}_{qi} \) is the performance of the \( q \)th smart technology application in optimizing the \( i \)th criterion, \( q = 1 \sim 4 \), and \( i = 1 \sim 5 \). These criteria were then used to evaluate the performance of each smart technology application.

The evaluation results are summarized in Table 6.

The fuzzy priorities derived from a fuzzy subjudgment matrix were applied to evaluate the overall performance of each smart technology application in supporting mobile health care during and after the COVID-19 pandemic by using a fuzzy technique for ordering preferences by their similarity to the ideal solution (fuzzy TOPSIS) [58,59]. Other fuzzy evaluation methods, such as fuzzy preference ranking organization method for enriched evaluation (fuzzy PROMETHEE) [52], fuzzy elimination and choice expressing reality (fuzzy ELECTRE) [60], and fuzzy Vise Kriterijumska Optimizacija I Kompromisno Resenje (fuzzy VIKOR) [61] methods, are also applicable.

In the example of viewpoint 1, the performance of a smart technology application in optimizing each criterion was first normalized using fuzzy distributive normalization [58,59]:

\[ \tilde{\rho}_{qi} = \frac{\tilde{p}_{qi}}{\sqrt{\sum_{\phi=1}^{q} \tilde{p}_{\phi i}}} \]  

(69)

\( \tilde{\rho}_{qi} \) is the normalized performance. The results are summarized in Table 7.

After the derived fuzzy priorities were multiplied to determine the normalized performance, the fuzzy prioritized scores of smart technology applications, denoted by \( \tilde{\lambda} \), were obtained (Table 8).

On the basis of the fuzzy prioritized scores for all the smart technology applications, the fuzzy ideal (\( \tilde{\lambda}^+ \)) and anti-ideal (\( \tilde{\lambda}^- \))
Table 5
Criteria for evaluating the performance of a smart technology application.

| Critical Feature | Criterion |
|------------------|-----------|
|                  | \begin{align*}
|                  & \begin{cases}
  (0, 0, 1) & \text{if } 0.1 \cdot \min_{r} x_{1} + 0.9 \cdot \max_{r} x_{1} \leq x_{q1} \text{ or data not available} \\
  (0, 1, 2) & \text{if } 0.35 \cdot \min_{r} x_{1} + 0.65 \cdot \max_{r} x_{1} \leq x_{q1} < 0.1 \cdot \min_{r} x_{1} + 0.9 \cdot \max_{r} x_{1} \\
\end{cases}
\end{align*} |
| Estimated costs | \begin{align*}
| \tilde{p}_{q1}(x_{q1}) = & \begin{cases}
  (1.5, 2.5, 3.5) & \text{if } 0.65 \cdot \min_{r} x_{1} + 0.35 \cdot \max_{r} x_{1} \leq x_{q1} < 0.35 \cdot \min_{r} x_{1} + 0.65 \cdot \max_{r} x_{1} \\
  (3, 4, 5) & \text{if } 0.9 \cdot \min_{r} x_{1} + 0.1 \cdot \max_{r} x_{1} \leq x_{q1} < 0.65 \cdot \min_{r} x_{1} + 0.35 \cdot \max_{r} x_{1} \\
  (4, 5, 5) & \text{if } x_{q1} < 0.9 \cdot \min_{r} x_{1} + 0.1 \cdot \max_{r} x_{1} \\
\end{cases}
\end{align*} |
| Effectiveness | \begin{align*}
| \tilde{p}_{q2}(x_{q2}) = & \begin{cases}
  (1.5, 2.5, 3.5) & \text{if } x_{q2} = \text{"Moderate"} \\
  (3, 4, 5) & \text{if } x_{q2} = \text{"High"} \\
  (4, 5, 5) & \text{if } x_{q2} = \text{"Very High"} \\
\end{cases}
\end{align*} |
| Acceptability | \begin{align*}
| \tilde{p}_{q3}(x_{q3}) = & \begin{cases}
  (1.5, 2.5, 3.5) & \text{if } x_{q3} = \text{"Moderate"} \\
  (3, 4, 5) & \text{if } x_{q3} = \text{"High"} \\
  (4, 5, 5) & \text{if } x_{q3} = \text{"Very High"} \\
\end{cases}
\end{align*} |
| Resuming physical human interactions | \begin{align*}
| \tilde{p}_{q4}(x_{q4}) = & \begin{cases}
  (1.5, 2.5, 3.5) & \text{if } x_{q4} = \text{"Moderate"} \\
  (3, 4, 5) & \text{if } x_{q4} = \text{"High"} \\
  (4, 5, 5) & \text{if } x_{q4} = \text{"Very High"} \\
\end{cases}
\end{align*} |
| Ease of implementation and maintenance | \begin{align*}
| \tilde{p}_{q5}(x_{q5}) = & \begin{cases}
  (1.5, 2.5, 3.5) & \text{if } x_{q5} = \text{"Moderate"} \\
  (3, 4, 5) & \text{if } x_{q5} = \text{"Difficult"} \\
  (4, 5, 5) & \text{if } x_{q5} = \text{"Very Difficult"} \\
\end{cases}
\end{align*} |

with confirmed COVID-19 cases frequently visit and the smart wristbands allowing high-risk individuals to detect their body temperature and blood oxygen, respectively. These two smart technology applications both represented optimal solutions depending on the decision maker’s perspective.

(5) By contrast, when the fuzzy priorities of key factors were derived from the original fuzzy judgment matrix and used to evaluate the overall performances of smart technology applications, the ranking result was III > II > IV > I. Thus, the app indicating where individuals with confirmed COVID-19 cases frequently visit was the optimal smart technology application.

(6) The values of the two goals $\xi_1$ and $\xi_2$ were varied to carry out parametric analyzes: $\xi_1$: 0 to 0.4

$\xi_2$: 5 to 15

The optimal objective function values associated with various values of the two goals are summarized with a response surface plot in Fig. 7. Obviously, the optimal objective function value increased when $\xi_1$ increases and $\xi_2$ decreases. However, the larger the value
Table 7
Normalized performance of the smart technology applications.

| Q | Smart Technology Application | $\tilde{p}_{q1}$ | $\tilde{p}_{q2}$ | $\tilde{p}_{q3}$ | $\tilde{p}_{q4}$ | $\tilde{p}_{q5}$ |
|---|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | I (0.21, 0.36, 0.57)          | (0.00, 0.00, 0.22) | (0.00, 0.15, 0.37) | (0.23, 0.46, 0.72) | (0.54, 0.73, 0.83) |
| 2 | II (0.44, 0.58, 0.76)         | (0.21, 0.40, 0.64) | (0.57, 0.76, 0.86) | (0.00, 0.18, 0.48) | (0.44, 0.58, 0.76) |
| 3 | III (0.54, 0.73, 0.83)        | (0.44, 0.65, 0.83) | (0.46, 0.61, 0.78) | (0.49, 0.74, 0.92) | (0.21, 0.36, 0.57) |
| 4 | IV (0.00, 0.00, 0.19)         | (0.44, 0.65, 0.83) | (0.00, 0.15, 0.37) | (0.23, 0.46, 0.72) | (0.00, 0.00, 0.19) |

Table 8
Fuzzy prioritized scores of smart technology applications (viewpoint 1).

| q | Smart Technology Application | $\tilde{s}_{q1}$ | $\tilde{s}_{q2}$ | $\tilde{s}_{q3}$ | $\tilde{s}_{q4}$ | $\tilde{s}_{q5}$ |
|---|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | I (0.01, 0.03, 0.12)          | (0.00, 0.00, 0.09) | (0.00, 0.07, 0.25) | (0.02, 0.09, 0.29) | (0.02, 0.06, 0.17) |
| 2 | II (0.02, 0.04, 0.16)         | (0.02, 0.07, 0.26) | (0.13, 0.35, 0.57) | (0.00, 0.04, 0.19) | (0.01, 0.05, 0.16) |
| 3 | III (0.02, 0.05, 0.17)        | (0.04, 0.12, 0.33) | (0.10, 0.28, 0.52) | (0.04, 0.15, 0.37) | (0.01, 0.03, 0.12) |
| 4 | IV (0.00, 0.00, 0.04)         | (0.04, 0.12, 0.33) | (0.00, 0.07, 0.25) | (0.02, 0.05, 0.29) | (0.00, 0.00, 0.04) |

Table 9
Fuzzy ideal and anti-ideal points (viewpoint 1).

| Reference Point | $\tilde{\lambda}_1^+$ | $\tilde{\lambda}_1^-$ | $\tilde{\lambda}_1^{+/-}$ | $\tilde{\lambda}_4^+$ | $\tilde{\lambda}_4^-$ | $\tilde{\lambda}_4^{+/-}$ |
|-----------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Fuzzy ideal point | (0.02, 0.05, 0.17) | (0.04, 0.12, 0.33) | (0.13, 0.35, 0.57) | (0.04, 0.15, 0.37) | (0.02, 0.06, 0.17) |
| Fuzzy anti-ideal point | (0.00, 0.00, 0.04) | (0.00, 0.00, 0.09) | (0.00, 0.07, 0.25) | (0.00, 0.04, 0.19) | (0.00, 0.00, 0.04) |

Fig. 7. Results of parametric analyzes.

of $\xi_1$ is, the more inconsistent fuzzy subjudgment matrices will be. If the value of $\xi_2$ is too small, fuzzy subjudgment matrices cannot be diversified.

4.5. Comparison with existing methods

For comparison, the FAM-based decomposition method [2,63] was also applied to this case. Accordingly, to maximize the distance between the two fuzzy subjudgment matrices, the fuzzy judgment matrix was decomposed into two such subjudgment matrices by using FAM. The optimal solution was as follows.

The CRs of the two fuzzy subjudgment matrices were (0, 0.14, 7.17) and (0, 0.14, 5.70), respectively. Both subjudgment matrices were more consistent than the original fuzzy judgment matrix. However, both subjudgment matrices generated using FAM were (or were required to be) symmetric—for example, $d_2(\tilde{a}_{21}, \tilde{a}_{21}(1)) = d_2(\tilde{a}_{21}, \tilde{a}_{21}(2)) = 3$—which constituted the drawback of the FAM method. The distance between the two fuzzy subjudgment matrices was (0, 12.17, 20.27), which was 12% shorter than that achieved using the methodology proposed in this study. Therefore, the proposed methodology was more capable of differentiating the fuzzy subjudgment matrices. Subsequently, fuzzy TOPSIS was also applied to evaluate the overall performance of each smart technology application. The ranking
results of the smart technology applications from the two viewpoints, which differ from those generated using the proposed methodology, are listed in Table 12. Specifically the two methods yielded different results in the rankings of smart technology applications II and IV.

The ranking results obtained using the two methods were partly consistent: smart technology applications III and II were the optimal smart technology applications from viewpoints 1 and 2, respectively. Hence, the superiority of these two smart technology applications over their counterparts did not change by applying different methods.

When the FAM-based decomposition method is used, the fuzzy subjudgment matrices are symmetric. Consequently, a smart technology application that performs well from one viewpoint tends to perform poorly from another, which causes confusion for the decision maker in selecting the most suitable smart technology application. For example, in Table 12, smart technology application II performed well from viewpoint #2, but poor from viewpoint #1. By contrast, the proposed FGM decomposition-based approach is not limited by this difficulty. As shown in Table 11, smart technology application II performed approximately well from both viewpoints. In addition, in the present case, from distinct viewpoints, only the top two smart technology applications were different, as shown in Table 11. The rankings of the other smart technology applications (determined using the proposed methodology) were the same regardless of viewpoint. In addition, the number of possible decomposition results using the proposed methodology was much more than that using the existing FAM method. In fact, the decomposition result using the existing FAM method was also included in the decomposition result using the proposed methodology. In other words, the proposed methodology gave the decision maker more choices and flexibility.



| Application | Defuzzified Value | Rank | Application | Defuzzified Value | Rank |
|-------------|------------------|------|-------------|------------------|------|
| I           | (0.00, 0.31, 0.78) | 4    | II          | (0.00, 0.12, 0.69) | 2    |
| III         | (0.00, 0.08, 0.68) | 1    | IV          | (0.00, 0.30, 0.77) | 3    |

5. Conclusions

In a fuzzy MCDM problem, a decision maker may have different viewpoints regarding the relative priorities of criteria. This point has been largely overlooked in previous studies. Some recent studies have applied FAM to decompose a decision maker’s fuzzy judgment matrix into several fuzzy subjudgment matrices, each representing a single viewpoint. However, FAM possesses shortcomings and fuzzy judgment matrices can be decomposed using other methods. Therefore, this study proposed an FGM decomposition-based fuzzy MCDM methodology in which FGM is applied to decompose a fuzzy judgment matrix into several fuzzy subjudgment matrices that are diverse and more consistent than the original fuzzy judgment matrix. The method also resolves the difficulty of constructing fuzzy subjudgment matrices from pre-specified linguistic terms by formulating a multiobjective FINLP problem, which is solved after conversion into an equivalent INLP problem. To evaluate its effectiveness, the proposed methodology was applied to the case of selecting the best of four smart technology applications to support mobile health care during and after the COVID-19 pandemic.

According to the experimental results, the following conclusions were drawn:

1. The app indicating where individuals with confirmed COVID-19 cases often visit and the smart wristbands allowing high-risk individuals to detect their body temperature and blood oxygen levels was the optimal smart technology applications (depending on viewpoint).
2. On the basis of the original fuzzy judgment matrix, only one optimal smart technology application could be determined. By contrast, the decomposed fuzzy subjudgment matrices enabled the selection of multiple optimal smart technology applications.
3. The ranking results of smart technology applications obtained using the proposed methodology differed from those obtained using a previous FAM-based fuzzy decomposition method.

Over existing methods, the proposed methodology has the following advantages:

- In existing methods, determining the weights of fuzzy subjudgment matrices is an extra task for the decision maker who may not know how to determine these weights. In contrast, this study does not require a decision maker to determine these weights.
- The proposed methodology offers greater flexibility in decomposing a fuzzy judgment matrix.
- Using existing methods, the elements of the decomposed sub-judgment matrices may not conform to the linguistic variables defined at the beginning, which is resolved through a fuzzy mapping process in the proposed methodology.

However, the proposed methodology also has the following disadvantages:

- The decomposition results of the fuzzy judgment matrix are fine-tuned to conform to the previously defined linguistic variables. Doing so, while understandable, slightly degrades the precision of the decomposition results.
- The computational complexity involved with the INLP problem is an obvious limitation.

By default, all viewpoints are assigned equal importance in the proposed methodology, but this can be adjusted to emphasize certain decision maker viewpoints. In addition, other methods,
such as FI [64], partial consensus FI [13], linguistic ordered weighted average [65], and fuzzy weighted intersection [66], can be applied to decompose fuzzy judgment matrices. In future research, a method for aggregating the fuzzy judgment matrices of multiple decision makers can also be applied to decompose the matrix of a decision maker. These are some recommended directions for future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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