\( \mathcal{N} = 1 \) Super Yang Mills renormalization schemes for Fractional Branes

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We investigate \( \mathcal{N} = 1 \) super Yang-Mills theory using fractional branes. We first define the \( \beta \)-function with respect to a supergravity coordinate. To provide the relation between the supergravity parameter and the renormalization group scale we use the UV known gauge theory \( \beta \)-function as a type of boundary condition. We show that there are no privileged renormalization schemes connected to a given supergravity solution while we investigate in some detail two schemes. The Wilsonian one where just one loop is manifest and the one containing multi-loops. A new functional relation between the gaugino condensate and the supergravity coordinates is finally determined.

I. INTRODUCTION

Recently using the gauge/gravity correspondence correspondence it was shown that many gauge theories can be investigated via their dual supergravity backgrounds. In the case of D3-branes in flat space this correspondence became even more stringent since an exact duality between \( \mathcal{N} = 4 \) super Yang-Mills in four dimensions and type IIB string theory on \( AdS_5 \times S^5 \) is conjectured to exist.

Clearly one would like to extend such a duality to less supersymmetric and non conformal gauge-theories. Fractional branes are a tool to construct non-conformal gauge theories with reduced supersymmetry \( \mathcal{N} \leq 4 \). More specifically by using fractional branes at orbifold singularities it has been possible to reproduce some gauge theory 1-loop perturbative results such as the \( \beta \)-function and the chiral anomaly. These results are not interpreted with a duality à la Maldacena, but are consistently interpreted as a open/closed string duality \( \mathcal{N} \leq 4 \). The classical supergravity solution, indeed, can be seen as an expansion for small \( \varepsilon \) 't Hooft coupling and as such it is not surprising that one is able to deduce the perturbative properties of the gauge theory. When this approach has been extended to describe fractional branes supporting in their world-volume \( \mathcal{N} = 1 \) super Yang-Mills theories, one seems able to deduce just the 1-loop Wilsonian \( \beta \)-function.

On the other hand for the \( \mathcal{N} = 1 \) Yang-Mills theory it has been shown that there exists a precise relation between the 1-loop Wilsonian \( \beta \)-function and the multi loop NSVZ one computed in a non holomorphic (in the coupling constant) scheme \( \mathcal{N} = 4 \). Having at hand such a relation it is reasonable to expect that the two schemes contain the same physical information.

Since it is believed valid the supergravity-gauge theory correspondence one should then be able to reproduce also the running of the coupling constant in this non-holomorphic renormalization scheme within the framework of the singular supergravity solutions. One of the main goal of this paper is to propose a solution to this problem.

In particular we will first observe that there is a good deal of arbitrariness when trying to identify the supergravity parameters (in this case the coordinates transverse to the brane) to the 4-dimensional renormalization scale of the theory. This arbitrariness is essentially due to the possibility of choosing in non-conformal backgrounds, different functional relations between distance in supergravity and energy scale in gauge theory \( \mathcal{N} \leq 4 \). We will then show how this arbitrariness can be used to go from one renormalization scheme to another.

We will also provide a new functional relation between a specific combination of the supergravity coordinates and the gaugino condensate. We will use again the UV boundary conditions to completely determine this function at large scales.

Since the specific supergravity solution we are considering is singular we will not attempt to understand the IR of the corresponding gauge theory. This has been explored in \( \mathcal{N} = 4 \) using non singular supergravity solutions \( \mathcal{N} \leq 4 \). In particular in Ref. \( \mathcal{N} \leq 4 \) it has been shown that the UV gauge theory boundary conditions are relevant to understand the non perturbative IR physics of super Yang-Mills.

In section II we will briefly review some of the relevant aspects of \( \mathcal{N} = 1 \) Super Yang-Mills theory. In section II we will introduce the fractional brane set up while in section III we define and impose the UV boundary conditions. The new relation between the gaugino condensate and the supergravity coordinates is introduced and studied in section IV. The conclusions are presented in V.

II. SUPER YANG MILLS PERTURBATIVE \( \beta \)-FUNCTIONS

In this section we summarize same general feature of pure \( SU(N) \) super Yang-Mills theory. At the classical level the theory possesses a \( U(1)_R \) symmetry which does not commute with the supersymmetric algebra. The ABJ anomaly breaks the \( U(1)_R \) symmetry to \( \mathbb{Z}_{2N} \). Non perturbative effects trigger the gaugino condensation lead-
ing to further breaking of $Z_{2N}$ symmetry to a left over $Z_2$ symmetry and we have $N$ equivalent vacua. The gaugino condensate is a relevant ingredient to constrain the theory. In literature it has been computed in different ways and we recall here its expression in two much studied schemes [14, 21]:

- The Holomorphic Scheme/Wilsonian

\[ \langle \lambda^2 \rangle = \text{Const.} \mu^3 e^{\frac{\tau_{YM}}{4\pi}} \]

where the gaugino condensate is holomorphic in $\tau_{YM} = \frac{\theta}{2\pi} + i\frac{\tau_{YM}}{g_Y^2}$. (1)

- The Non Holomorphic Scheme/Pauli Villars

\[ \langle \lambda^2 \rangle = \text{Const.} \mu^3 \text{Im} \left[ \frac{\tau_{YM}}{4\pi} \right] e^{\frac{\tau_{YM}}{4\pi}} \]

Here the condensate is non-holomorphic in $\tau_{YM}$.

The gaugino condensate is a universal physical constant independent on the scheme. This fact not only allows one to compute the perturbative $\beta$-functions in the two schemes since the respective coupling constant must depend on the scale in such a way to compensate the dependence on $\mu$ but it also enables us to establish a relation between the coupling constants in the two schemes.

The holomorphic scheme is very constrained leading to a pure one-loop type of running. This is a welcome feature from the point of view of the supercurrent chiral multiplet [4]. The second scheme is non holomorphic in the coupling constant. The theta dependence is constrained by the $U(1)_R$ anomalous symmetry. The two coupling constants are related in the following way:

\[ \tau_{YM}^H = \tau_{YM} - i\frac{N}{2\pi} \text{ln} \left[ \text{Im} \left( \frac{\tau_{YM}}{4\pi} \right) \right], \] (3)

where we added a superscript $H$ to distinguish the holomorphic coupling from the non holomorphic one. Note that the transition from one to the other scheme does not alter the theta dependence of the theory. This request manifestly breaks the holomorphicity in the coupling constant for the non Wilsonian scheme. The independence on the scale of the gaugino condensate leads to the following $\beta$-functions [4]:

\[ \beta(g_{YM}) = -\frac{3N}{16\pi^2} g_{YM}^3, \quad \text{Holomorphic} \] (4)

\[ \beta(g_{YM}) = -\frac{3N}{16\pi^2} g_{YM}^3 \left[ 1 - \frac{N^2 g_{YM}^2}{8\pi^2} \right]^{-1}. \] (5)

Where $\beta(g) = \frac{\partial g}{\partial L}$ and $L = \ln(\mu/\Lambda)$ while $\mu$ is the renormalization scale and $\Lambda$ a reference scale. Due to the relation $\beta$ the one-loop and the perturbative multi loop $\beta$-function carry the same perturbative physical information. Although these couplings may capture the all order perturbative terms the non perturbative contributions (if any) to the complete $\beta$-functions are still missing.

These two schemes display no universality of the two loop coefficient of the $\beta$-function. This is due to the non analytical relation between the coupling constants in the two schemes [4]. In fact the universality argument for the two loop $\beta$-function coefficient is valid for part of the possible choices of renormalization schemes [2].

If supergravity-gauge theory correspondence is valid it should be possible to adopt different renormalization schemes when connecting the supergravity solutions to the associated gauge theory. We will address this issue in the following sections.

### III. FRACTIONAL BRANES: THE SET UP

Let us now consider the pure SYM realized as the world-volume theory of a stack of N D3-fractional branes at the orbifold $\mathbb{R}^{1,3} \times \mathbb{C}^3 / (Z_2 \times Z_2)$. For definiteness we take the orbifold directions to be $x^1 \ldots x^9$ (labeled by $l, m \ldots$) introduce three complex coordinates:

\[ z_1 = x^4 + i x^5, \quad z_2 = x^6 + i x^7, \quad z_3 = x^8 + i x^9 \] (6)

and consider fractional branes that are completely transverse to the orbifold, and therefore extended along the directions $x^0 \ldots x^3$ (labeled by $\alpha, \beta \ldots$).

Within this orbifold one has four kinds of D3-fractional branes [4], corresponding to the four irreducible representations of the orbifold $Z_2 \times Z_2$, each of them supporting in its world-volume a $\mathcal{N} = 1$ SYM theory.

In the following we will consider the low energy dynamics of a bound state made of just one kind of D3-fractional brane. The gauge theory supported by a pile of $\mathcal{N} = 1$ SYM theory.

The supergravity solution determined by this source has been computed in the paper of Ref. [4]. In the following we specify that solution for the case that we are interested in; i.e. the case of only one type of fractional brane, obtaining for the metric and the R-R 4-form $C_4$:

\[ ds^2 = H^{-1/2} \eta_{\alpha\beta} dx^\alpha dx^\beta + H^{1/2} \delta_{lm} dx^l dx^m \] (7)

\[ F_5 = H^{-1} \wedge V_4 \ast (dH^{-1} \wedge V_4) \] (8)

while the classical profile of the twisted fields is:

\[ b_i = (2\pi^2 \alpha') \left( 1 + \frac{2g_s}{\pi} N \log \frac{\rho_i}{\epsilon} \right) \] (9)

\[ c_i = -(4\pi\alpha') g_s N \theta_i \] (10)

with $\rho_i = \sqrt{2 \theta_i}$, $\theta_i = \tan^{-1}(x^{2i+3}/x^{2i+2})$ ($i = 1, 2, 3$). The three pair of scalars $b_i$ and $c_i$ ($i = 1, 2, 3$) correspond to the components of the 2-forms $B_{(2)}$ and $C_{(2)}$, along
the anti-self dual 2-forms $\omega^i_{(2)}$ dual to the cycle $C_i$ that characterize this orbifold.

The gauge coupling and the theta angle of the dual gauge theory, are given in terms the twisted fields fluxes by the following relations \[13\]:

\[
\frac{1}{g_{YM}^2} = \frac{1}{16\pi g_s} + \frac{N}{8\pi^2} \sum_i \ln \frac{\rho_i}{\epsilon} \\
\theta_{YM} = -N \sum_i \theta_i. \tag{11, 12}
\]

Combining the previous equations we have that the complex coupling $\tau_{YM}$ is:

\[
\tau_{YM} = i \left[ \frac{1}{4g_s} + \frac{N}{2\pi} \sum_i \ln \frac{z_i}{\epsilon} \right] \equiv i \frac{N}{2\pi} \sum_i \ln \frac{z_i}{\langle z \rangle}, \tag{13}
\]

where $\langle z \rangle$ is nothing but $\langle \rho \rangle$ when restricting ourselves to the real part. $\langle z \rangle$ is real since we have chosen in eq.10 a null background value for the $c_i$ fields. However while still satisfying the equation of motion we can make a different constant choice for such background which promotes $\langle z \rangle$ to a complex variable.

It is always possible and natural to introduce the following dimensionless variable

\[
\rho^3 \equiv \prod_{i=1}^{3} \rho_i/\langle z \rangle, \tag{14}
\]

with respect to which the functional relation between the coupling constant reads:

\[
\frac{1}{g_{YM}^2} = \frac{3N}{8\pi^2} \ln \rho = F(\rho). \tag{15}
\]

Being the supergravity solution singular we cannot explore the IR of the theory which would correspond to small values of $\rho$. This is possible, as already discussed, in the case of the wrapped brane scenario. The hope is that, in a not too far future, by resolving the singularity one can check the super Yang-Mills IR physics using fractional branes.

It is worth mentioning that it has been very useful in the wrapped brane scenario to relate a known function of the supergravity coordinates with the gaugino condensate \[23\]. We shall suggest, in the last paragraph, how such a relation may emerge in the fractional brane scenario as well.

\section{UV Boundary Conditions: From One Loop to Multi Loops}

Supergravity provided the following functional relation:

\[
\frac{1}{g_{YM}^2} = F(\rho), \quad \rho \to \infty. \tag{16}
\]

In order to compute the standard gauge theory $\beta$-function we must differentiate the coupling constant with respect to $L = \ln (\mu/\Lambda)$ with $\mu$ the renormalization scale and $\Lambda$ a renormalization invariant scale. Supergravity is not able to predict such a relation between $\rho$ and $\mu$. Without loss of generality we can nevertheless define the following unambiguous supergravity $\beta$-function for fractional branes:

\[
\beta_\rho (g_{YM}) = \frac{\partial g_{YM}}{\partial \rho} = -\frac{3N}{16\pi^2} g_{YM}^2 \frac{1}{\rho}, \quad \rho \to \infty, \tag{17}
\]

which clearly captures asymptotic freedom. Once the relation between $\rho$ and $\mu$ is known the standard $\beta$-function is simply:

\[
\beta (g_{YM}) = \beta_\rho (g_{YM}) \frac{\partial \rho}{\partial L}. \tag{18}
\]

For non singular Jacobian $\partial \rho/\partial L$ most of the information carried in $\beta_\rho$ is transferred to the gauge theory $\beta$-function. If this were not the case the supergravity/gauge connection would be meaningless.

In order to fix the functional dependence between the supergravity parameter $\rho$ and $L$ we use the known form of the $\beta$-function in UV. This UV type of boundary conditions are similar to the ones introduced in the context of the wrapped brane scenario \[18\].

\subsection{Wilsonian UV Boundary Condition}

To reproduce the $\beta$-function in this scheme the UV boundary condition reads:

\[
\frac{\partial \ln \rho}{\partial L} = 1, \tag{19}
\]

yielding the following functional relations:

\[
\rho = \frac{\mu}{\Lambda}. \tag{20}
\]

This specific boundary condition reproduces the relation between $\rho$ and $\mu$ already adopted in literature.

\subsection{Pauli Villars UV Boundary Condition}

We have already stressed that the physical information encoded in the $\beta$-function expressed in this scheme is the same than the one carried in the Wilsonian scheme so there is no reason for the supergravity solution not to be linked to this scheme as well. Here the UV boundary condition is:

\[
\frac{\partial y(\rho)}{\partial L} = \left[ 1 - N g_{YM}^2 / 8\pi^2 \right]^{-1} = \left[ 1 - \frac{1}{3y} \right]^{-1}, \tag{21}
\]
with \( y(\rho) = \ln \rho \) and to write the last identity we have used eq. (15). After integrating we have:
\[
3L = 3 \ln \rho - \ln (\ln \rho) \tag{22}
\]
which can be solved (for large \( \mu \)) iteratively for \( \rho \) yielding:
\[
\rho \approx \mu \sqrt{\frac{\ln \mu}{\Lambda}} \quad \mu \to \infty. \tag{23}
\]

Note that in this scheme the two loop coefficient is not vanishing. Any other scheme in which the coupling constant relation can be written as a series expansion with respect to the new coupling constant will display the universality of the two loop coefficient of the \( \beta \)-function [23]. The Wilsonian coupling relation to the Pauli-Villars scheme is not analytical and as such does not display the universality of the loop coefficient of the respective \( \beta \)-functions. It is also interesting to observe that our functional relation eq. (23) does not change if we neglect in eq. (21) terms higher or of the order \( g_{\text{YM}}^4 \).

V. THE HIDDEN GAUGINO CONDENSATE

The gaugino condensate is a key ingredient when studying \( \mathcal{N} = 1 \) super Yang-Mills gauge theory. In particular, being a renormalization group invariant quantity is constant for any (non-zero) value of the coupling constant and in any scheme. In particular is present at weak coupling.

Here we suggest a possible correspondence between supergravity and the gaugino condensate within the fractional brane scenario. In order to find such a relation we recall that in supergravity the combination of twisted fields given by the eq. (13) is identified with the complex coupling of the gauge theory. Since under an R-transformation \( \tau_{\text{YM}} \to \tau_{\text{YM}} + \frac{N}{2} \alpha \), with \( \alpha \) the \( (1)_R \) transformation parameter for consistency
\[
\langle z \rangle \to e^{\frac{\alpha}{2} \langle z \rangle} \tag{24}
\]
So by construction the quantity
\[
\prod_{i=1}^{3} \rho_i \langle z \rangle, \tag{25}
\]
has \( (1)_R \) charge two and engineering mass dimension three. Eq. (23) is also precisely the combination appearing in eq. (13). Furthermore when choosing a particular supergravity scale (i.e. a value for the coordinate \( z_i \)) we break the R-symmetry to \( \mathbb{Z}_2 \). It is hence tempting to suggest that there exists a functional relation between eq. (23) and the inverse of the gaugino condensate of the form:
\[
\prod_{i=1}^{3} \left[ \rho_i \langle z \rangle \right] F(\rho) = \frac{\mu^3}{\langle \lambda^2 \rangle}, \tag{26}
\]
where the function \( F \) can be fixed using the UV boundary conditions used in the previous section or equivalently the expression for the gaugino condensate presented in eqs. (6), (8). Equation (26) shows the direct link between \( \langle z \rangle \) and \( \langle \lambda^2 \rangle \) and is consistent with the \( U(1)_R \) transformation. Indeed \( \langle z \rangle \) can be imagined to be the vacuum expectation value of \( z_i \) with R-charge 2/3 while \( \langle \lambda^2 \rangle \) is the condensate of the gluino which possesses, in the present conventions, a positive unit of R-charge while we hold fixed \( z_i \) and \( \mu \).

Using the gaugino expression in the Wilsonian scheme (see eq. (1)) one deduces \( F = \text{const.} \) while in the multi loop case [3] we have \( F = \text{const.}/\ln \rho \).

It is instructive to determine \( F \) again, within the non-holomorphic renormalization scheme, using the universality of the two loop coefficients of the \( \beta \)-function. This is equivalent to the boundary conditions just imposed but it shows the intimate relation between the gaugino condensate and the beta function.

By differentiating both sides of eq. (26), we get:
\[
\frac{\partial}{\partial \log \frac{\mu}{\Lambda}} \ln(\rho) = \frac{1}{1 + \frac{3}{4} \partial_\rho \ln F(\rho)} \tag{27}
\]
where \( \rho \) is defined in eq. (15) and we used the relation between the gaugino condensate and the dynamical scale \( \Lambda \) of the pure \( \mathcal{N} = 1 \) theory [21]; i.e. \( \langle \lambda^2 \rangle_{\theta=0} = \Lambda^3 \). Furthermore by using the relations (27) and the (11) we can compute from the classical solution, the \( \beta \)-function of the underlying gauge theory expressed in terms of the unknown function:
\[
\beta(\text{YM}) = -3 \frac{g_{\text{YM}}^3}{16 \pi^2} \left[ 1 + \frac{\rho}{3} \partial_\rho \ln F(\rho) \right]^{-1} - 3 \frac{g_{\text{YM}}^3}{16 \pi^2} \left[ 1 - \frac{\rho}{3} \partial_\rho \ln F(\rho) \cdots \right] \tag{28}
\]
The second expression follows because we are considering a perturbative expansion for the \( \beta \)-function. Using the universality of the first two orders of the \( \beta \)-function [23] we deduce (for large \( \rho \) i.e. small coupling constant) the following differential equation:
\[
\partial_\rho \ln F(\rho) = - \partial_\rho \ln \rho \tag{29}
\]
which fixes \( F = \frac{\text{const.}}{\ln \rho} \) in the UV regime. Substituting this expression in (28) yields:
\[
\beta(\text{YM}) = -3 \frac{g_{\text{YM}}^3}{16 \pi^2} \left( 1 - \frac{1}{3 \ln \rho} \right)^{-1} - 3 \frac{g_{\text{YM}}^3}{16 \pi^2} \left( 1 - \frac{N g_{\text{YM}}^2}{8 \pi^2} \right)^{-1} \tag{30}
\]
This is the NSVZ \( \beta \)-function [24], also found in Ref. [20].
VI. CONCLUSIONS

We investigated the UV properties of the $N = 1$ super Yang-Mills theory using singular supergravity solutions corresponding to fractional branes. We first defined the $\beta$-function with respect to the supergravity coordinate $\rho$ and found a supergravity type of asymptotic freedom. To provide the relation between the supergravity parameter and the renormalization group scale we used the UV known gauge theory $\beta$-function as a kind of boundary condition. The UV boundary conditions take the form of linear differential equations which can be easily solved. In particular we have seen that there are no privileged renormalization schemes connected to a given supergravity solution. We investigated in some detail two schemes. The Wilsonian one where, technically, just one loop is present and the one containing multi-loops. We recall that in the multi-loop $\beta$-function the first two coefficients are universal [22] while the others are scheme dependent.

Furthermore, by analyzing how the $U(1)_R$ symmetry is realized in supergravity, we determined a functional relation between the gaugino condensate and a particular combination of the coordinates transverse to the brane.

The IR physics cannot be uncovered with singular supergravity solutions. However the hope is that in the future, by resolving somehow the singularity, one can use fractional branes to get an independent check of the IR uncovered via wrapped branes [16] - [18]. Besides, at the moment, within the fractional brane scenario it is possible to investigate $N = 1$ supersymmetric gauge theories with matter fields.

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