The $X(2239)$ and $\eta(2225)$ as hidden-strange molecular states from $\Lambda\bar{\Lambda}$ interaction

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In this work, we propose a possibility to assign the newly observed $X(2239)$, as well as the $\eta(2225)$, as a molecular state from the interaction of a baryon $\Lambda$ and antibaryon $\bar{\Lambda}$. With the help of the effective Lagrangians, the $\Lambda\bar{\Lambda}$ interaction is described within the one-boson-exchange model with $\eta$, $\eta'$, $\omega$, $\phi$, and $\sigma$ exchanges considered. After inserting the interaction potential kernel into the quasipotential Bethe-Salpeter equation, the bound states from the $\Lambda\bar{\Lambda}$ interaction can be found by searching for the pole of scattering amplitude. Two loosely bound states appear near the threshold with spin parities $J^P = 0^-$ and $1^-$ with almost the same parameter. The $1^-$ state can be assigned into the $X(2239)$ observed at BESIII, which is very close to the $\Lambda\bar{\Lambda}$ threshold. The scalar meson $\eta(2225)$ can be interpreted as a $0^-$ state from the $\Lambda\bar{\Lambda}$ interaction. The results suggest further investigation is needed to understand the internal structure of $X(2239)$, $\eta(2225)$, and $X(2175)$.

I. INTRODUCTION

After the observation of $X(3872)$ at Belle, more and more $XYZ$ particles were reported at different experimental facilities, and attract great interest from the theoretical side [1]. Many $XYZ$ particles are suggested to be candidates of the exotic hadrons beyond the conventional $q\bar{q}/qqq$ picture. One of popular interpretation of the $XYZ$ particles are the molecular state, which is a loosely bound state composed of two hadrons. The possible molecular states are widely discussed in theory and applied to explain the observed exotic hadrons. The bound states from interaction of charmed/bottom and anticharmed/antibottomed mesons are often related to the $XYZ$ particles, such as the $Z_c(3900)$, $Z_c(4020)$, $Z_c(10610)$, and $Z_c(10650)$ [2–8]. The recent observed $P_c$ states near the $\Sigma^-\bar{D}^*$ threshold give people more confidence in the molecular state picture [9–18]. In the light sector, the $\Lambda(1405)$ is also proposed to be generated from the $KN$ interaction [19–22]. However, the study of a molecular state with a baryon and antibaryon is scarce in the literature, and the experimental hint about such state is also rarely reported. In charmed sector, the $Y(4630)$ was explained as a bound state from the $\Lambda\bar{\Lambda}$ interaction [23, 24]. Theoretically, the interaction between two baryons is analogous to that between two mesons. Moreover, generally speaking, a baryon-antibaryon pair is also not difficult to be produced in the experiment. Hence, it is interesting to study the molecular state composed of a baryon and an antibaryon.

In fact, even before the proposition of quark model, the possibility to interpret $\pi$ meson as a bound state of nucleon-antinucleon was discussed by Fermi and Yang [25]. However, such attempt is not so successful and not widely accepted. The $X(1835)$ was also be connected to a $N\bar{N}$ bound state [26, 27]. Recently, the BESIII collaboration reported a resonance structure by analyzing the cross section of the $e^+e^\to K^+K^-$ process at the center-of-mass energies ranging from 2 to 3.08 GeV. The structure is denoted as $X(2239)$ which has a mass of $2239 \pm 7.1 \pm 11.3$ MeV and a width of $139.8 \pm 12.3 \pm 20.6$ MeV [28]. A few investigations were conducted to interpret the $X(2239)$ [29, 30]. In Ref. [30], based on the mass estimated in a relativized quark model, the $X(2239)$ can be explained as a candidate for the P-wave $ss\bar{s}\bar{s}$ tetraquark state. An important observation about the $X(2239)$ is that it is almost on the threshold of the $\Lambda\bar{\Lambda}$ interaction after considering the experimental uncertainty of the mass. If we recall its spin parity and charge parity $J^{PC} = 1^{--}$ and observation of $X(2239)$ in the hidden-strange $K^+K^-$ channel, it is a good candidate of hidden-strange molecular state composed of $\Lambda$ and $\bar{\Lambda}$.

Before the observation of $X(2239)$, another state with the same quantum number, the $Y(2175)$, also named as $\phi(2170)$ in the literature, was observed by the Babar Collaboration in the initial-state-radiation process $e^+e^- \to \gamma\Sigma\phi(1020)f_0(980)$ with a mass of about $2175$ MeV [31]. Since the $Y(2175)$ was observed, it has been investigated within many theoretical interpretations, with include $qqg$ hybrid [32, 33], $ss\bar{s}\bar{s}$ tetraquark state [34–36], excited $1^{--}ss$ state [37], resonance state of $\varphi K\bar{K}$ [38, 39], and some other interesting speculations [40–42]. It is also possible that the $X(2239)$ and $X(2175)$ are the same state [1]. However, a $1^{--}$ state with a mass of $2135 \pm 8 \pm 9$ MeV and a width of $104 \pm 24 \pm 12$ MeV was also observed in the $\phi f_0(980)$ channel at BESIII [43]. It is more appropriate to take these two states as two separate states due to the large mass gap of these states. Besides, a state with a mass of about $2220$ MeV was observed by DM2 Collaboration and confirmed at MARK-III in the radiative decays $J/\psi \to \gamma\phi\phi$ [44, 45]. Later, the BES and BESIII Collaborations also observed a signal of $\eta(2225)$ [46, 47]. There exist also a few theoretical interpretation of the $\eta(2225)$, such as a $4^1S_0$ $ss$ state [48].

As indicated in Ref. [49], $Y(2175)$ and $\eta(2225)$ can be interpreted as $\Lambda\bar{\Lambda}(4^1S_1)$ and $\Lambda\bar{\Lambda}(4^1S_0)$ molecular states, respectively. However, it should be noticed that the $\Lambda\bar{\Lambda}$ threshold is about $2231.3$ MeV, while the mass of the $Y(2175)$ is about $60$ MeV lower than the threshold, which is too deep to be a molecular state. Moreover, recent measurement at BESIII indicate that the mass of $Y(2175)$ is about $2135$ MeV [43],

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which is about one hundred MeV below the $\Lambda\bar{\Lambda}$ threshold. As for the the newly observed state, $X(2239)$, its mass seems closer to the $\Lambda\bar{\Lambda}$ threshold. Hence, it is interesting to study the possibility of $X(2239)$ as $\Lambda\bar{\Lambda}(1^−)$ state rather than the $Y(2175)$.

Recalling the results in Ref. [49], one can find that the mass gap between the $Y(2175)$ and $\eta(2235)$ were claimed to be reproduced from S-wave $\Lambda\bar{\Lambda}$ interaction in the one-boson-exchange model by solving non-relativistic Schrödinger equation. However, from our experience, the relativistic effect and the $D$-wave component will vary the mass of the bound state, which is considerably large in the case considered system here. In this case, it is possible to produce a bound state with $1^-$ from the $\Lambda\bar{\Lambda}$ interaction with a mass very close to the threshold, which is more consistent with the molecular state as a loosely bound state of two hadron. To achieve this aim, we adopt a quasipotential Bethe-Salpeter (qBSE) approach where the theoretical frame is relativistic and $D$-wave contribution can be included naturally. For the $\Lambda\Lambda$ interaction, we still apply the one-boson-exchange potential model with pseudoscalar ($\eta$ and $\eta'$), scalar ($\sigma$), and vector ($\omega$ and $\phi$) exchanges. By inserting the exchange potential into the qBSE, the molecular states with spin parities $0^-$ and $1^-$ will be investigated.

The paper is organized as follows. After the introduction, we present the meson exchange potential and the qBSE approach in Section II. The numerical results are shown in Section III. Discussion and summary is given in Section IV.

II. THEORETICAL FRAME

First, we describe the interaction of the $\Lambda\Lambda$ interaction in the one-boson-exchange model. As in Ref. [49], to construct the potential, the Lagrangians for the couplings between the $\Lambda$ baryon and exchanged mesons can be written as,

$$\mathcal{L}_{\bar{\psi}\Lambda\Lambda} = -i g_{\bar{\psi}\Lambda\Lambda} \bar{\psi}_\Lambda \gamma_\mu \phi_\Lambda \psi_\Lambda,$$

where, the $\psi_\Lambda$, $\eta$, $\eta'$, $\sigma$, $\omega$, and $\phi$ are the field of $\Lambda$ baryon, $\eta$, $\eta'$, $\sigma$, $\omega$, and $\phi$ mesons. The coupling constants $g_{\bar{\psi}\Lambda\Lambda}$ can be derived by the SU(3) symmetry [49], and the mass of each exchange meson $m_i$ are cited from the Review of Particle Physics (PDG) [1], the explicit values are listed in the followings,

- $g_{\bar{\psi}\Lambda\Lambda}/4\pi = 4.473$, $m_\eta = 548.8$ MeV,
- $g_{\bar{\psi}\Lambda\Lambda}/4\pi = 9.831$, $m_\eta' = 957.7$ MeV,
- $g_{\bar{\psi}\Lambda\Lambda}/4\pi = 3.459$, $m_\sigma = 550.0$ MeV,
- $g_{\bar{\psi}\Lambda\Lambda}/4\pi = 8.889$, $m_\omega = 782.6$ MeV,
- $g_{\bar{\psi}\Lambda\Lambda}/4\pi = 222.2$, $m_\phi = 1019.5$ MeV.

In the current work, we consider the $\Lambda\bar{\Lambda}$ interaction instead of $\Lambda\Lambda$ interaction. Hence, the couplings between the light mesons and the antibaryon $\bar{\Lambda}$ are also required. As in the nucleon-antineucleon interaction, we adopt the well-known G-parity rule to write the $\Lambda\bar{\Lambda}$ interaction from the $\Lambda\Lambda$ interaction. By inserting the $G^{-1}G$ operator into the potential, the G-parity rule can be obtained easily as [50, 51]

$$V = \sum_i \zeta_i V_{i\Lambda\Lambda}.$$  

The G parity of the exchanged meson is left as a G factor for $i$ meson. Since $\omega$ and $\phi$ mesons own odd G parity, $\zeta_\omega$ and $\zeta_\phi$ should equal $-1$, and others still equal 1. Finally, we achieve a potential as

$$V_{\Lambda\bar{\Lambda}} = V_{\eta\Lambda\Lambda} + V_{\eta'\Lambda\Lambda} + V_{\sigma\Lambda\Lambda} - V_{\omega\Lambda\Lambda} - V_{\phi\Lambda\Lambda}.$$  

Now, we only need know the potential of $\Lambda\bar{\Lambda}$ interaction. With the lagrangians and the coupling constants given above, we can write the relevant meson exchange potential with the standard Feynman rule as,

$$iV_{\eta\Lambda\Lambda} = -g_{\bar{\psi}\Lambda\Lambda} \bar{u}_\Lambda \gamma_\mu u_\Lambda \frac{i}{q^2 + m_\eta^2} f_1(q^2) u_\Lambda \gamma_\mu u_\Lambda,$$

$$iV_{\eta'\Lambda\Lambda} = g_{\bar{\psi}\Lambda\Lambda} \bar{u}_\Lambda \gamma_\mu u_\Lambda \frac{i(-g^{\nu\sigma} + q^{\nu}q^{\sigma}/m_\eta^2)}{q^2 + m_\eta^2} f_1(q^2) u_\Lambda \gamma_\mu u_\Lambda,$$

$$iV_{\sigma\Lambda\Lambda} = g_{\bar{\psi}\Lambda\Lambda} \bar{u}_\Lambda \gamma_\mu u_\Lambda \frac{i}{q^2 + m_\sigma^2} f_1(q^2) u_\Lambda \gamma_\mu u_\Lambda,$$

where the $u_\Lambda$ is the spinor of $\Lambda$ baryon. The $q$, $m_\eta$, $m_\eta'$, and $m_\sigma$ are the exchanged momentum, and the masses of exchanged pseudoscalar $\bar{\Psi}$ ($\eta$ and $\eta'$), vector $\Psi$ ($\omega$ and $\phi$) and scalar $\sigma$ mesons. Usually, a from factor should be introduced at the vertexes because the exchanged mesons are not the point particle and have internal structure. Such form factors are also used to ensure the convergence of the integral. In the current work, we adopt three types of the form factors as,

$$f_1(q^2) = \frac{\Lambda_\epsilon^2 - m_\epsilon^2}{\Lambda_\epsilon^2 - q^2},$$

$$f_2(q^2) = \frac{\Lambda_\epsilon^4}{(m_\epsilon^2 - q^2)^2 + \Lambda_\epsilon^2},$$

$$f_3(q^2) = e^{-(m_\epsilon^2 - q^2)/\Lambda_\epsilon^2},$$

where we parameterize the cutoff in a form $\Lambda_\epsilon = m_\epsilon + \alpha_\epsilon 0.22$ GeV with $m_\epsilon$ being the mass of the exchange meson. Such parameterization of the cutoff can introduce the effect of the mass of exchanged meson, which is more reasonable than adoption of the same cutoff for different mesons. The $\alpha_\epsilon$ is introduced as a free parameter which is close to 1. Considering the explicit forms of the form factors, the $\alpha$ for $f_1$ should be larger than 0 to avoid an unphysical suppression near $\Lambda_\epsilon = m_\epsilon$. For other two types of the form factor, a value about zero can be chosen.

Different from Ref. [49], we will adopt the qBSE to explore the possible bound states from the $\Lambda\bar{\Lambda}$ interaction. The potential kernel obtained above is inserted into the the
Bethe-Salpeter equation to obtain the scattering amplitude, the poles of which correspond to the bound states. The Bethe-Salpeter equation is a 4-dimensional integral equation in the Minkowski space. Considering complexity and difficulty of directly solving such integral equation, we adopt a quasipotential approximation approach to reduce the 4-dimensional Bethe-Salpeter equation into a 3-dimensional integral equation. Then, using the partial-wave decomposition the 3-dimensional equation is further reduced into a 1-dimensional equation with fixed spin-parity \( J^P \) as [5, 55],

\[
iM_{J^P}(p', p) = \int \frac{d^2p''}{(2\pi)^3} \sum_{\lambda''} V^\alpha_{J^P}(p'', p') G_0(p'') iM_{J^P}(p'', p),
\]

where the sum extends only over nonnegative helicity \( \lambda'' \). In current case, we will consider \( J^P = 0^- \) and \( 1^- \), which can couple to \( \Lambda\Lambda \) in S wave, and be called S-wave states in the non-relativistic calculation [49]. Since we make the decomposition on spin parity directly, contributions from all possible orbital angular momenta \( L \) are included naturally. Hence, in the qBSE approach, no special treatment is needed to include the D-wave contribution.

Here, the reduced propagator with the spectator approximation can be written down in the center-of-mass frame with \( P = (W, 0) \) as

\[
G_0 = \frac{\delta^4(p_2'^2 - m^2)}{p_1'^2 - m^2} = \frac{\delta^4(p_2'^0 - E_2(p''))}{2E_2(p'')((W - E_2(p''))^2 - E_2^2(p''))},
\]

(13)

Here, as required by the spectator approximation, we put one of the particle, 2 here, on shell, which satisfies \( p_2'^0 = E_2(p'') = \sqrt{m^2 + p''^2} \). Here and hereafter, a definition \( p = |p| \) will be adopted.

The partial-wave potential is defined with the potential of the interaction obtained in the above as

\[
V^\alpha_{J^P}(p', p) = 2\pi \int d\cos \theta \left[ d_{J^P}(\theta) V_{X,\lambda}(p', p) + \eta d_{J^P}(\theta) V_{X,\lambda}(p', p) \right],
\]

(14)

where \( \eta = PP_1P_2(-1)^{J^P-J^P_1-M_1} \) with \( P \) and \( J \) being parity and spin for system, \( \Lambda \) meson or \( \bar{\Lambda} \) baryon. The initial and final relative momenta are chosen as \( p = (0, 0, p) \) and \( p' = (p' \sin \theta, 0, p' \cos \theta) \). The \( d_{J^P}(\theta) \) is the Wigner d-matrix. Since particle 1 is off shell in our qBSE approach, a form factor should be also introduced to reflect its internal structure. Here, we can adopt an exponential regularization by introducing a form factor into the propagator as

\[
G_0(p) \rightarrow G_0(p) e^{-\xi k_1^2 m_1^2/\Lambda^4},
\]

(15)

where the \( k_1 \) and the \( m_1 \) are the momentum and the mass of the off-shell particle. With such regularization, the convergence of the integral equation is guaranteed even without the form factor for the exchanged meson. The cutoff \( \Lambda \) is parameterized as in the \( \Lambda \) case, that is, \( \Lambda = m_{\pi} + \alpha r \), 0.22 GeV with \( m_{\pi} \) being the mass of the exchanged meson and \( \alpha \) having the same function as the parameter \( \alpha e \).

### III. NUMERICAL RESULTS

After obtaining the scattering amplitude by inserting the potential kernel into the qBSE, the bound state can be searched by finding the pole of scattering amplitude in the complex plane. The parameters in the qBSE approach is the cutoff \( \Lambda \) which has been parameterized into \( \alpha \). In the calculation, we choose \( \alpha \) equivalent to \( \alpha e \), and rename them as a parameter \( \alpha \), for simplification. We consider two spin parities \( 0^- \) and \( 1^- \), which can be obtained from S-wave coupling. The binding energy \( E_B = m_{\text{th}} - W \) with \( m_{\text{th}} \) and \( W \) being the threshold and the position of the pole obtained with different types of form factors are presented in Fig.1.

![FIG. 1: The binding energy \( E_B \) with the variation of the \( \alpha \). The upper and lower patterns are for bound states with spin parities \( 0^- \) and \( 1^- \), respectively. The black square, red circle, and blue triangle are for different types of form factors \( f_i(q^2) \) with \( i = 1, 2, 3 \) in Eqs. (9-11), respectively. The cyan line in the upper pattern is the suggested value of mass of the \( \eta(2225) \) in the PDG [1], and the cyan band in the lower pattern is for the uncertainty of the \( X(2239) \) [28]. More explanations are given in the text.](image-url)

The bounding energy of the bound state from the \( \Lambda\bar{\Lambda} \) interaction with quantum numbers \( J^{PC} = 0^{+-} \) is presented in the upper pattern of the Fig.1. The bound state can be produced from the interaction with reasonable \( \alpha \). For the monopole type of the form factor \( f_1(q^2) \), the bound state appears at an \( \alpha \) of about 1, which is standard value of the
In Fig. 1, the suggested value of the mass of $\eta(2225)$ in the PDG [1] is also shown as a cyan line, which can be reproduced at an $\alpha$ of about 1.2. Considering the uncertainty of mass of $\eta(2225)$ is about 10 MeV, which just fill the region we considered, we do not show it in the figure. The uncertainty corresponds to a range of $\alpha$ from 0.8 to 1.5. For other two types of form factors. The bound state is produced at $\alpha$ about zero, which corresponds to a standard cutoff about 1 GeV. The shapes of three curves for different form factors are analogous to each other. Considering the $\alpha$ is a free parameter in a reasonable range, we can say the different choices of the form factor does not effect on our conclusion. Hence, comparing the theoretical results with the experiment, the $0^-$ state from the $\Lambda\bar{\Lambda}$ interaction can be related to the $\eta(2225)$.

Now we turn to the $1^-$ case, which is shown in the lower pattern of Fig. 1. Different from the results in [49]. The binding energy of $1^-$ state are similar to these of $0^-$ state with the same parameters. For the monopole form factor $f_1(q^2)$, the bound state appears at an $\alpha$ of about 0.9, increases with the increase of $\alpha$, and reach a binding energy about 20 MeV at an $\alpha$ of about 1.5. For other two types of form factors, the bound state is produced at $\alpha$ of about zero. The experimental mass value of the $X(2239)$ reported by BESIII is $2239 \pm 7.1 \pm 11.3$ MeV. The central value is lightly higher than the $\Lambda\bar{\Lambda}$ interaction. After the uncertainty considered, the $X(2239)$ is just on the threshold. In Fig. 1, we also present the uncertainty of the mass of $X(2239)$ below the threshold as a cyan band. The experimental uncertainty of the $X(2239)$ corresponds to $\alpha$ from 0.8 to 1.2.

In our model, five exchanges including $\eta, \eta', \phi, \omega, \text{and } \sigma$ exchanges, are considered to construct the interaction potential. Usually, these exchanges play different roles in producing the bound states. In the qBSE approach, the potential can not be shown as the function of the range $r$ as in the non-relativistic calculation [49]. We check their contributions by turning on and off one or more exchanges and vary the parameter $\alpha$ from -1 to 3 to search for bound state. It is found that if we only keep one of five exchanges, no bound state can be produced from $\eta, \eta'$, or $\phi$ exchange while the interaction with only $\omega$ or only $\sigma$ exchange is still strong enough to produce bound states with a larger $\alpha$. Such result suggest that the $\omega$ and $\sigma$ exchanges play most important roles in producing the bound states. In the following we give more explicit results in Fig. 2 to show the role of exchanges.

We present the results after turning off $\eta, \eta'$ and $\phi$ exchanges and only keeping $\omega$ and $\sigma$ exchanges in patterns (a) and (b) of Fig. 2. As shown in the figure, the bound states with $0^-$ and $1^-$ can be produced from the $\omega$ and $\sigma$ exchanges with a lightly increase of the parameter $\alpha$ for all three types of form factors. We also check the case when removing both $\omega$ and $\sigma$ exchanges but keeping $\eta$ and $\eta'$ and $\phi$ exchanges. No bound state can be found with a reasonable parameter. Such results suggest that the $\omega$ and $\sigma$ exchanges are essential to produce the bound state with $0^-$ and $1^-$. In patterns (c) and (d), the results after removing $\sigma$ exchange is presented. Larger $\alpha$ is required for three types of form factors than the previous case. The dominate effect comes from the $\omega$ exchange as shown in patterns (e) and (f). To reproduce a binding energy in the full model, a parameter $\alpha$ should be increased by 0.5 or more.

IV. SUMMARY AND DISCUSSION

The molecular state composed of a baryon and an antibaryon is an interesting topic in the study of the exotic baryons. In the present work, we study the possibility to assign the newly observed $X(2239)$ as a $\Lambda\bar{\Lambda}$ molecular state in the qBSE approach. The potential kernel $\Lambda\bar{\Lambda}$ interaction is constructed in the one-boson-exchange model with the help of the Feynman rule, and inserted directly into the qBSE. After decomposition on spin parity, the bound state can be found by studying the pole of the scattering amplitude. Two bound states with spin parities $J^P = 0^-$ and $1^-$ are produced from the $\Lambda\bar{\Lambda}$ interaction. Our results suggest that these two bound states are both close to the $\Lambda\bar{\Lambda}$ threshold. Before observation of the $X(2239)$, there exists only one possible state $Y(2175)$
near the $\Lambda \bar{\Lambda}$ interaction, so it is often assigned as the $1^-$ molecular state. However, a binding energy is larger than 50 MeV for this assignment. Now, the $X(2239)$ was observed at $K^+K^-$ channel, and is just on the threshold if the experimental uncertainty is considered. In Ref. [56], an study of the strong decays of the $\Lambda\bar{\Lambda}$ bound state was performed. It was found that the dominant decay channel of the $1^-$ state is the $K\bar{K}$ channel, which is just the observation channel of the $X(2239)$ at BESIII. Hence, it is more suitable to assign these two states to $X(2239)$ and $\eta(2225)$, respectively.

We also discuss the effect of each exchange on producing the bound state. Among the five exchanges, including $\eta$, $\eta'$, $\phi$, $\omega$, and $\sigma$ exchanges, the $\omega$ and $\sigma$ exchanges, especially the former, are most important to produce two bound states. Such conclusion is consistent with the previous studies in Refs. [23, 49].

In the current work, we propose that the observation of the $X(2239)$ at BESIII provide a more suitable candidate of the $\Lambda\bar{\Lambda}$ molecular state with spin parity $1^-$. In the charmed sector, the $\Lambda_c \bar{\Lambda}_c$ molecular state are also studied in the theory, and was assigned as the $Y(4630)$ by some authors [23, 24]. As in the hidden-strange sector, many states are observed near $\Lambda_c \bar{\Lambda}_c$ threshold, including $Y(4630)$, $Y(4660)$ and a structure at 4625 MeV observed at Belle very recently [57]. More comprehensive investigation about the $\Lambda_c \bar{\Lambda}_c$ interaction in both theory and experiment is important to understand and these structures.

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