Physical Fock Space of Tensionless Strings

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Abstract

We study the physical Fock space of the tensionless string theory with perimeter action which has pure massless spectrum. The states are classified by the Wigner’s little group for massless particles. The ground state contains infinite many massless fields of fixed helicity, the excitation levels realize CSR representations. We demonstrate that the first and the second excitation levels are physical null states.
1 Introduction

A string model which is based on the concept of surface perimeter was suggested in [1, 2, 17]. At the classical level the model is tensionless, because for the flat Wilson loop the action is equal to its perimeter [17]. In the recent articles one of the authors has found that it has pure massless spectrum of infinitely many integer spin fields [1, 2].

The solution of the corresponding two-dimensional world-sheet CFT for the canonically conjugate operators $X^\mu = (X_L^\mu + X_R^\mu)/2$ and $\Pi^\mu = (\Pi_L^\mu + \Pi_R^\mu)/2$ is [1]:

$$
X_L^\mu = x^\mu + \frac{1}{m} \pi^\mu \zeta - i \sum_{n \neq 0} \frac{1}{n} \beta_n e^{-in \zeta},
$$

$$
\Pi_L^\mu = me^\mu + k^\mu \zeta + i \sum_{n \neq 0} \frac{1}{n} \alpha_n e^{-in \zeta},
$$

where $k^\mu$ is momentum operator, $\alpha_n, \beta_n$ are oscillators satisfying the following commutator relations $[x^\mu, k^\nu] = i\eta^\mu\nu$, $[\alpha^\mu_n, \beta^\nu_l] = n \eta^\mu\nu \delta_{n+l,0}$ and $\zeta^\pm = \tau \pm \sigma$. A similar expansion also holds for the right moving modes $X_R^\mu, \Pi_R^\mu$ with the oscillators $\tilde{\beta}_n, \tilde{\alpha}_n$. The essential new feature of this theory is the appearance of the additional zero modes $[e^\mu, \pi^\nu] = i\eta^\mu\nu$, which means that the space-time wave function is a function of two continuous variables $k^\mu$ and $e^\mu$:

$$
\Psi_{phys} = \Psi(k, e).
$$

It was suggested in [1] that $e^\mu$ should be interpreted as a polarization vector. Its conjugate operator is defined as: $\pi^\mu = i\partial/\partial e_\mu$.

Our aim is to study spectral properties of the model on higher levels of physical Fock space in analogy with the standard string theory [10, 11, 12, 13, 14, 15]. This is quite possible using the mode expansion of the world-sheet fields and exploring corresponding new gauge symmetries. The tensionless string theory is invariant with respect to the standard conformal group with generators

$$
L_n = \sum_l : \alpha_{n-l} \cdot \beta_l :, \quad n = 0, \pm 1, \pm 2
$$

and with respect to the infinite-dimensional Abelian transformations generated by the operator $\Theta = \Pi^2 - 1$ with the following components:

$$
k^2, \quad k \cdot e, \quad k \cdot \alpha_n, \quad k \cdot \tilde{\alpha}_n, \quad \Theta_{nl} \quad n, l = 0, \pm 1, \pm 2, ...
$$

corresponding to the coefficients of $\tau^2, \tau, \bar{\tau}$ with exponentials and Fourier modes, respectively. Therefore in covariant quantization scheme the space-time equations, which define physical Fock space, have the following form [1, 2]:

$$
\begin{bmatrix}
k^2 \\
k \cdot e \\
k \cdot \alpha_n \\
k \cdot \tilde{\alpha}_n \\
\Theta_{nl} \\
L_n \\
\tilde{L}_n
\end{bmatrix}
\begin{bmatrix}
\Psi_{phys}
\end{bmatrix}
= 0, \quad n, l = 0, 1, 2, ...
$$

This model differs from the tensionless string models based on Schild’s work on ”null” string [3, 4, 5, 6] with its most likely continuous spectrum [7, 8, 9].
As one can clearly see from the first equation in (2), the spectrum of the model is purely massless: $k^2 = 0$.

The zero mode conformal operator is given by the expression

$$L_0 = (k \cdot \pi) + \hat{\Xi},$$

where $\hat{\Xi} = \sum_{n \neq 0} \alpha_{-n} \beta_n$ is the number operator with eigenvalues $\Xi = 0, 1, 2, \ldots$. This operator allows to represent the whole Fock space $\mathcal{F}$ as a sum of relativistically invariant subspaces

$$\mathcal{F} = \bigoplus_{\Xi=0}^{\infty} \mathcal{F}_\Xi$$

and to study each subspace $\mathcal{F}_\Xi$ separately. A natural question that arises is whether the physical Hilbert space on each subspace $\mathcal{F}_\Xi$ is positive-definite, i.e. ghost-free in analogy with the well known No-ghost theorem [10, 11, 12, 13, 14, 15] in the standard string theory.

As it was demonstrated in [2], the first two levels $\Xi = 0, 1$ corresponding to the ground state and the first excitation state, are well defined and have no negative norm waves. The vacuum state $\mathcal{F}_0$ is infinitely degenerate and contains massless particles of increasing tensor structure $A_{\mu_1 \ldots \mu_s}(k)$ for $s = 1, 2, \ldots$, while the first level $\mathcal{F}_1$ happens to be a physical null state.

We shall demonstrate below that the second level wave function, $\Xi = 2$, also represents a physical null state, therefore one can guess that all excitation levels $\Xi = 1, 2, 3, \ldots$ are physical null states. The general proof of this conjecture is still to be found despite the fact that some pattern already appears in the first two excitation levels, which explains why all excitations are physical zero norm states. The main reason is, that the polarization tensors must fulfill two constraints which follow from the main space-time equations (2): i) they have to be transverse $k_{\mu_1} \xi_{\mu_1 \ldots \mu_s} = 0$ and at the same time ii) to be longitudinal $e_{\mu_1} \xi_{\mu_1 \ldots \mu_s} = 0$. This leaves no options and the only solution has the form:

$$\xi_{\mu_1 \ldots \mu_s} = \xi(k, e) \ k_{\mu_1} \ldots k_{\mu_s}$$

and has zero norm!

In the next section we shall review the solution of the basic space-time equations (2) in Gupta-Bleuler quantization scheme for the ground state and the first excitation states [1, 2]. Then we shall turn to the study of the second level wave function.

2 $\Theta$-Algebra

The symmetries of the model are governed by the conformal operators $L_n$ and the new operators $\Theta_{n,l}$:

$$L_n = <e^{i n \zeta^+} : P_L^\mu \partial_\tau X_L^\mu :>, \quad \Theta_{n,l} = <e^{i n \zeta^+ + i l \zeta^-} : \Pi^\mu \Pi^\mu - 1 :>$$

(3)

where $P^\mu = \partial_\tau \Pi^\mu = (P_L^\mu + P_R^\mu)/2$ is the momentum operator defined by the mode expansion (1); thus

$$L_n = \sum_l : \alpha_{n-l} \cdot \beta_l : \quad \bar{L}_n = \sum_l : \bar{\alpha}_{n-l} \cdot \bar{\beta}_l :$$

4We shall use physical units in which $m = 1$. 

[3]
\[
\Theta_{0,0} = (e^2 - 1) + \sum_{n \neq 0} \frac{1}{4n^2} : (\alpha_{-n} \alpha_n + \tilde{\alpha}_{-n} \tilde{\alpha}_n) : \\
\Theta_{n,0} = \frac{i}{n} e \cdot \alpha_n - \frac{1}{4} \sum_{l \neq 0, n} \frac{1}{(n-l)} : \alpha_{n-l} \cdot \alpha_l : \quad n = \pm 1, \pm 2, \ldots \\
\Theta_{0,n} = \frac{i}{n} e \cdot \tilde{\alpha}_n - \frac{1}{4} \sum_{l \neq 0, n} \frac{1}{(n-l)} : \tilde{\alpha}_{n-l} \cdot \tilde{\alpha}_l : \quad n = \pm 1, \pm 2, \ldots \\
\Theta_{n,l} = -\frac{1}{2nl} : \alpha_n \cdot \tilde{\alpha}_l : \quad n, l = \pm 1, \pm 2, \ldots \tag{4}
\]

The conformal algebra has here its classical form, but with twice larger central charge

\[
[L_n, L_l] = (n-l)L_{n+l} + \frac{D}{6}(n^3 - n)\delta_{n+l,0} \tag{5}
\]

and with a similar expression for the right movers \(\tilde{L}_n\). The reason that the central charge is twice bigger than in the standard bosonic string theory \(2 \times \frac{D}{12} = \frac{D}{6}\) is simply because we have two left and two right moving fields \(^5\).

The full extended gauge symmetry algebra of constraints (3) takes the form

\[
\begin{align*}
&[L_n, \Theta_{0,0}] = -2n\Theta_{n,0} \\
&[L_n, \Theta_{l,0}] = -(n+l)\Theta_{n+l,0} \\
&[L_n, \Theta_{0,l}] = -2n\Theta_{n,l} \\
&[L_n, \Theta_{m,l}] = -(n+m)\Theta_{n+m,l} \\
&[L_n, k^2] = 0 \\
&[L_n, k \cdot e] = -i k \cdot \alpha_n \\
&[L_n, k \cdot \alpha_l] = -l k \cdot \alpha_{n+l}
\end{align*}
\]

where we have included commutators with the operators \(k^2, k \cdot e, k \cdot \alpha_l, k \cdot \tilde{\alpha}_l\), because they are constituents of the \(\tau\) dependent part of the operator \(\Theta = \Pi^2 - 1\), as one can see substituting the solution (1) into the definition of the operator \(\Pi_\mu = (\Pi_L^\mu + \Pi_R^\mu)/2\). One should stress that it is an essentially Abelian extension because

\[
[\Theta_{n,m}, \Theta_{l,p}] = 0, \quad n, m, l, p = 0, \pm 1, \pm 2, \ldots \tag{7}
\]

The relations (5), (6) and (7) define an Abelian extension of the conformal algebra and the equations (4) realize its oscillator representation.

### 2.1 Physical Fock space

The spectrum of this theory is pure massless and the subspaces \(\mathcal{F}_N\) defined by the operator \(L_0 = (k \cdot \pi) + \tilde{N}\) do not correspond to different mass levels, as it was in the standard string theory, where \(L_0 = k^2 + \tilde{N}\), but classify the states with respect to the eigenvalues of the operator \((k \cdot \pi)\) \[^2\]. This operator is equal to the length of the highest Casimir \(W = (k \cdot \pi)^2\) of the Poincaré group and defines fixed helicity states, when \(W = 0\) and continuous spin representations-CSR, when \(W \neq 0\) \(^{18, 19}\).

\(^5\)Such doubling of modes is reminiscent of the bosonic part of the \(N = 2\) superstring \([16]\). Here the coordinate field \(X\) has simply two sets of commuting oscillators and the conjugate oscillators are described by a separate field \(\Pi_\mu\).
2.2 Ground state, $\Xi = 0$

Let us first describe the ground state, $\Xi = 0$. The wave function $\Psi_0(k, e) \equiv |k, e, 0>$ is defined by $\alpha_n |0, k, e> = \beta_n |0, k, e> = 0$, $n = 1, 2, ..$ and the system (2) reduces to the following four equations [1]:

$$k^2 \Psi_0 = 0, \quad e \cdot k \Psi_0 = 0, \quad (e^2 - 1) \Psi_0 = 0, \quad (k \cdot \pi) \Psi_0 = 0. \quad (8)$$

It follows from the third equation that $\Psi_0$ is a function defined on a unit sphere $e^2 = 1$ with signature $\eta^{\mu\nu} = (- + + +)$ and can be expanded in the corresponding basis:

$$\Psi_0 = |0, k, e> = A(k) + A^\mu_1(k) e_\mu + A^\mu_1\nu_1(k) e_\mu e_\nu + ... |0, k>.$$  

The fields $A^\mu_1...\mu_s(k)$ describe massless particles of increasing tensor structure and the vacuum state in infinitely degenerate. They are symmetric traceless tensor fields because they are harmonic functions on a sphere $e^2 = 1$, and it follows from the last equation in (8), that $k_\mu A^\mu_1...\mu_s(k) = 0$. Thus the fields $A^\mu_1...\mu_s$ are: 1) symmetric traceless tensors of increasing rank $s = 0, 1, 2, ..$, 2) divergent free $k_\mu A^\mu_1...\mu_s = 0$ and 3) satisfying massless wave equations $k^2 A^\mu_1...\mu_s = 0$. All the above conditions are sufficient to describe integer spin fields [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. Thus the ground state is infinitely degenerate and contains gauge particles of arbitrary large fixed helicity, $W = (k \cdot \pi)^2 = 0$. Every tensor gauge field $A^\mu_1...\mu_s$, $s = 0, 1, 2, ..$ appears only once in the spectrum.

2.3 The First Excitation states, $\Xi = 1$

These states correspond to the continuous spin representations of massless little group, because for them $W = (k \cdot \pi)^2 = 1$. The wave function of the first excited state depends on four polarization tensors. The restriction on these tensors which follows from the space-time equations (2) gives the solution in the form [2]

$$\Psi_1 = [\xi_{\mu\nu}\alpha^\mu_1\alpha^\nu_1 + \omega_{\mu\nu}(\alpha^\mu_1\beta^\nu_1 - \beta^\mu_1\alpha^\nu_1)]|0, k, e>,$$  

where $k^\mu\omega_{\mu\nu} = \omega_{\mu\nu}k^\nu = 0$, $\omega_{\mu\nu} = \omega_{\mu\nu}e^\nu = 0$. Assuming that the second equation is valid for any vector $e^\nu$ the unique solution of the last equations is: $\omega_{\mu\nu} = \omega(k, e) k_\mu k_\nu$. The tensor $\xi_{\mu\nu}(k, e)$ remains arbitrary. The norm of this state is $<\Psi_1|\Psi_1> = -\omega_{\mu\nu}\omega_{\mu\nu} = -|\omega|^2 (k^2)^2$ and therefore is equal to zero! It is also normal to the ground state $<\Psi_0|\Psi_1> = 0$. Thus the first excitation is a physical null state. This consideration allows to make a conjecture that all excitations are physical null states and therefore define gauge parameters of the large gauge group.

3 The Second Excitation State, $\Xi = 2$

Let us consider the next level. The $L_0|\Psi_2> = 0$ equation will take the form

$$(k \cdot \pi)|\Psi_2> = -2 |\Psi_2>.$$  

The $\Xi = 2$ level contains twenty five tensors because we have now five relevant operators in the left sector

$$\alpha^\mu_1\alpha^\nu_1, \quad \alpha^\mu_1\beta^\nu_1, \quad \beta^\mu_1\beta^\nu_1, \quad \alpha^\mu_2, \quad \beta^\mu_2.$$
and the same amount in the right sector. They form three "families" of operators with tensor coefficients \( \chi, \eta, \xi \)

\[
|\Psi_2 > = \chi^{(1)}_{\mu \nu} \alpha_{-2}^\mu \tilde{\alpha}_{-2}^\nu + \chi^{(2)}_{\mu \nu} \alpha_{-2}^\mu \tilde{\beta}_{-2}^\nu + \chi^{(3)}_{\mu \nu} \beta_{-2}^\mu \tilde{\alpha}_{-2}^\nu + \chi^{(4)}_{\mu \nu} \beta_{-2}^\mu \tilde{\beta}_{-2}^\nu + \eta^{(1)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \eta^{(2)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \eta^{(3)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \eta^{(4)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \eta^{(5)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \eta^{(6)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \eta^{(7)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \eta^{(8)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \eta^{(9)}_{\mu \nu \lambda} \alpha_{-1}^\mu \tilde{\alpha}_{-2}^\nu + \eta^{(10)}_{\mu \nu \lambda} \alpha_{-1}^\mu \tilde{\beta}_{-2}^\nu + \eta^{(11)}_{\mu \nu \lambda} \beta_{-1}^\mu \tilde{\alpha}_{-2}^\nu + \eta^{(12)}_{\mu \nu \lambda} \beta_{-1}^\mu \tilde{\beta}_{-2}^\nu + \eta^{(13)}_{\mu \nu \lambda} \alpha_{-1}^\mu \tilde{\beta}_{-2}^\nu + \eta^{(14)}_{\mu \nu \lambda} \beta_{-1}^\mu \tilde{\alpha}_{-2}^\nu + \eta^{(15)}_{\mu \nu \lambda} \beta_{-1}^\mu \tilde{\beta}_{-2}^\nu + \eta^{(16)}_{\mu \nu \lambda} \beta_{-1}^\mu \tilde{\beta}_{-2}^\nu
\]

where we have taken into account that the operators involved commute \([\alpha^\mu_{-1}, \alpha^\nu_{-1}] = [\beta^\mu_{-1}, \beta^\nu_{-1}] = [\alpha^\mu_{-1}, \beta^\nu_{-1}] = 0\). The corresponding tensors \( \chi, \eta, \xi \) are subject to the constraints

\[
\begin{pmatrix}
  k^2 \\
  k \cdot e \\
  k \cdot \alpha_1 \\
  k \cdot \alpha_2 \\
  k \cdot \tilde{\alpha}_1 \\
  k \cdot \tilde{\alpha}_2 \\
  \Theta_{00} \\
  \Theta_{10} \\
  \Theta_{20} \\
  \Theta_{01} \\
  \Theta_{02} \\
  \Theta_{12} \\
  \Theta_{21} \\
  \Theta_{11} \\
  \Theta_{22} \\
  L_1 \\
  \bar{L}_1 \\
  L_2 \\
  \bar{L}_2
\end{pmatrix}
\begin{pmatrix}
  k^2 \\
  k \cdot e \\
  k \cdot \alpha_1 \\
  k \cdot \alpha_2 \\
  k \cdot \tilde{\alpha}_1 \\
  k \cdot \tilde{\alpha}_2 \\
  (e^2 - 1) + \frac{1}{2}(\alpha_{-1} \alpha_1 + \tilde{\alpha}_{-1} \tilde{\alpha}_1) + \frac{1}{8}(\alpha_{-2} \alpha_2 + \tilde{\alpha}_{-2} \tilde{\alpha}_2) \\
  i \cdot e \cdot \alpha_1 + \frac{1}{4}\alpha_{-1} \alpha_2 \\
  \frac{i}{2} \cdot e \cdot \alpha_2 - \frac{1}{4}\alpha_1 \alpha_1 \\
  i \cdot e \cdot \tilde{\alpha}_1 + \frac{1}{8}\tilde{\alpha}_{-1} \tilde{\alpha}_1 \\
  \frac{i}{2} \cdot e \cdot \tilde{\alpha}_2 - \frac{1}{4}\tilde{\alpha}_1 \tilde{\alpha}_1 \\
  -\frac{1}{2}\alpha_1 \alpha_2 \\
  -\frac{1}{2}\tilde{\alpha}_2 \tilde{\alpha}_1 \\
  -\frac{1}{4}\alpha_1 \tilde{\alpha}_1 \\
  -\frac{1}{8}\tilde{\alpha}_2 \tilde{\alpha}_2 \\
  k \beta_1 + \pi \alpha_1 + \alpha_{-1} \beta_2 + \beta_{-1} \alpha_2 \\
  k \beta_1 + \pi \tilde{\alpha}_1 + \tilde{\alpha}_{-1} \tilde{\beta}_2 + \tilde{\beta}_{-1} \tilde{\alpha}_2 \\
  k \beta_2 + \pi \alpha_2 + \alpha_1 \beta_1 \\
  k \beta_2 + \pi \tilde{\alpha}_2 + \tilde{\alpha}_1 \beta_1
\end{pmatrix}
\]

|\Psi_2 >= 0

It is a rather complicated system of space-time equations and our aim is to solve it. We shall start analyzing the equation \( \Theta_{00} |\Psi_1 > = 0 \), because it is the most restrictive on the wave function

\[
\Theta_{00} |\Psi_1 > = \frac{1}{4} \chi^{(2)}_{\mu \nu} \alpha_{-2}^\mu \tilde{\alpha}_{-2}^\nu + \frac{1}{4} \chi^{(3)}_{\mu \nu} \alpha_{-2}^\mu \tilde{\beta}_{-2}^\nu + \frac{1}{4} \chi^{(4)}_{\mu \nu} \alpha_{-2}^\mu \tilde{\alpha}_{-2}^\nu + \frac{1}{4} \chi^{(4)}_{\mu \nu} \beta_{-2}^\mu \tilde{\alpha}_{-2}^\nu
\]

plus the part which is cubic in the operators

\[
\frac{1}{2} \eta^{(2)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \alpha_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\alpha}_{-1}^\nu \tilde{\beta}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\alpha}_{-1}^\lambda + \frac{1}{2} \eta^{(4)}_{\mu \nu \lambda} \beta_{-2}^\mu \tilde{\beta}_{-1}^\nu \tilde{\beta}_{-1}^\lambda
\]
plus the part which is quartic in the operators with overall coefficient 1/2

\[
\begin{align*}
\xi^{(2)}_{\mu\nu\lambda\rho} & = \xi^{(4)}_{\mu\nu\lambda\rho} + \xi^{(5)}_{\mu\nu\lambda\rho} + \xi^{(6)}_{\mu\nu\lambda\rho} + \xi^{(8)}_{\mu\nu\lambda\rho} + \xi^{(10)}_{\mu\nu\lambda\rho} + \xi^{(12)}_{\mu\nu\lambda\rho} + \xi^{(13)}_{\mu\nu\lambda\rho} + \xi^{(14)}_{\mu\nu\lambda\rho} + \xi^{(16)}_{\mu\nu\lambda\rho} \\
\zeta^{(2)} + \zeta^{(5)} & = 0, \\
2\zeta^{(4)} + \zeta^{(6)} & = 0, \\
\zeta^{(8)} & = 0, \\
2\zeta^{(8)} + \zeta^{(14)} & = 0.
\end{align*}
\]

The sum of the coefficients in front of the identical operators should be equal to zero. Taking into account the symmetries of individual tensors in \(|\Psi_2| > 1\) one can get the following equations:

\[
\begin{align*}
\chi^{(4)} & = 0, \\
\chi^{(2)} + \chi^{(3)} & = 0, \\
\eta^{(8)} & = 0, \\
\eta^{(4)} + \frac{1}{2}\eta^{(6)} & = 0, \\
\eta^{(16)} & = 0, \\
2\eta^{(6)} & = 0, \\
\eta^{(12)} + \frac{1}{4}\eta^{(14)} & = 0, \\
\frac{1}{2}\eta^{(2)} + \frac{1}{2}\eta^{(5)} & = 0, \\
\frac{1}{2}\eta^{(14)} + \frac{1}{4}\eta^{(13)} & = 0.
\end{align*}
\]

This system of linear equations can be easily solved. As one can see, those tensors which are connected with the structures having three and four \(\beta\) operators, in the product, are equal to zero. This phenomenon already appeared when we analyzed the first level wave function: the tensor which is in front of the two \(\beta\) operators was equal to zero[2]. The solution can be expressed in the form

\[
\begin{align*}
\eta^{(4)} & = \eta^{(6)} = \eta^{(8)} = \eta^{(12)} = \eta^{(14)} = \eta^{(16)} = 0, \\
\eta^{(5)} & = -2\eta^{(2)}, \\
\eta^{(13)} & = -2\eta^{(10)}, \\
\xi^{(8)} & = \xi^{(14)} = \xi^{(16)} = 0, \\
\xi^{(5)} & = -\xi^{(2)}, \\
\xi^{(6)} & = -2\xi^{(4)} = -2\xi^{(13)}.
\end{align*}
\]
Therefore, after imposing the $\Theta_{00}$ constraint, the wave function has three sets of terms:

\[
|\Psi_2> = \chi^{(1)}_{\mu\nu} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-2} + \chi^{(2)}_{\mu\nu} (\alpha^{\mu}_{-2} \tilde{\beta}^{\nu}_{-2} - \beta^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-2}) + \\
\eta^{(1)}_{\mu\nu\lambda} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} + \eta^{(2)}_{\mu\nu\lambda} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} + \eta^{(5)}_{\mu\nu\lambda} \beta^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} + \\
\eta^{(9)}_{\mu\nu\lambda} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-2} + \eta^{(10)}_{\mu\nu\lambda} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-2} + \eta^{(13)}_{\mu\nu\lambda} \beta^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} + \\
\xi^{(1)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} + \xi^{(2)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} + \xi^{(5)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} + \\
\xi^{(4)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} + \beta^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} - \alpha^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} - \beta^{\mu}_{-1} \beta^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1}) + \left|k, e, 0\right>,
\]

where still we have the conditions $\eta^{(5)} = -2\eta^{(2)}$, $\eta^{(13)} = -2\eta^{(10)}$ and $\xi^{(2)} + \xi^{(5)} = 0$. Resolving these last equations we get nine independent tensors:

\[
|\Psi_2> = \chi^{(1)}_{\mu\nu} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-2} + \chi^{(2)}_{\mu\nu} (\alpha^{\mu}_{-2} \tilde{\beta}^{\nu}_{-2} - \beta^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-2}) + \\
\eta^{(1)}_{\mu\nu\lambda} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} + \eta^{(2)}_{\mu\nu\lambda} \alpha^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} + \eta^{(5)}_{\mu\nu\lambda} \beta^{\mu}_{-2} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} + \\
\eta^{(9)}_{\mu\nu\lambda} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-2} + \eta^{(10)}_{\mu\nu\lambda} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-2} + \eta^{(13)}_{\mu\nu\lambda} \beta^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} + \\
\xi^{(1)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} + \xi^{(2)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} + \xi^{(5)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} + \\
\xi^{(4)}_{\mu\nu\lambda \rho} \alpha^{\mu}_{-1} \tilde{\beta}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} + \beta^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1} - \alpha^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\beta}^{\lambda}_{-1} \tilde{\beta}^{\rho}_{-1} - \beta^{\mu}_{-1} \beta^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} \tilde{\alpha}^{\rho}_{-1})|k, e, 0>,
\]

where it was convenient to introduce the tensors

\[
\xi^{(4)}_{\mu\nu\lambda \rho} = \xi^{(2)}_{\mu\nu\lambda \rho}, \quad \eta^{(2)}_{\mu\nu\lambda} = \eta^{(10)}_{\mu\nu\lambda}, \quad \eta^{(13)}_{\mu\nu\lambda} = \xi^{(2)}_{\mu\nu\lambda}, \quad \chi^{(2)}_{\mu\nu} = \chi_{\mu\nu}
\]

in order to sum up the similar terms. The tensor $\xi_{\mu\nu\lambda \rho}$ is symmetric under simultaneous interchange of the indices $\mu \leftrightarrow \nu$ and $\lambda \leftrightarrow \rho$. One can simply check a posteriori that the wave function (12) fulfills the constrain $\Theta_{00}|\Psi_1> = 0$.

Our aim is now to impose the rest of the constraints on the wave function (12). The next constraint to be imposed is $\Theta_{10}|\Psi_2> = 0$:

\[
\Theta_{10}|\Psi_2> = - \frac{1}{4} \chi_{\mu\nu} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-2} - \frac{1}{2} \eta_{\mu\nu\lambda} \alpha^{\mu}_{-1} \tilde{\alpha}^{\nu}_{-1} \tilde{\alpha}^{\lambda}_{-1} + i (\xi_{\mu\nu\lambda \rho} e^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \xi^{(2)}_{\mu\nu\lambda \rho} e^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \xi^{(2)}_{\mu\nu\lambda \rho} e^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \beta^{\mu}_{-1} \alpha^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \beta^{\mu}_{-1} \beta^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \beta^{\mu}_{-1} \beta^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1} - \beta^{\mu}_{-1} \beta^{\nu}_{-1} \tilde{\alpha}^{\rho}_{-1})|k, e, 0> = 0
\]

and similar expressions that follow for the constraints $\Theta_{01}|\Psi_2> = \Theta_{11}|\Psi_2> = 0$. All equations which follow from these constraints can be summarized in the following form

\[
e^{\mu}_{\rho} \xi_{\mu\nu\lambda \rho} = e^{\lambda}_{\rho} \xi_{\mu\nu\lambda \rho} = 0, \\
\xi_{\mu\nu\lambda} = 0, \\
i e^{\mu}_{\rho} \xi_{\mu\nu\lambda} - \frac{1}{4} \chi_{\mu\nu} = 0, \quad i e^{\mu}_{\rho} \xi^{(2)}_{\mu\nu\lambda \rho} + \frac{1}{2} \eta_{\mu\nu\lambda \rho} = 0, \\
i e^{\lambda}_{\rho} \eta_{\mu\nu\lambda} + \frac{1}{4} \chi_{\mu\nu} = 0, \quad i e^{\rho}_{\mu} \xi^{(2)}_{\mu\nu\lambda \rho} - \frac{1}{2} \xi_{\mu\nu\lambda} = 0.
\]

(13)
Imposing the equations \( \Theta_{20}|\Psi_2 > = \Theta_{02}|\Psi_2 > = \Theta_{22}|\Psi_2 > = 0 \) we get
\[
e^\mu \chi_{\mu \nu} = \chi_{\mu \nu} e^\nu = 0, \\
i e^\mu \eta_{\mu \lambda \rho} - \frac{1}{4} \xi_{\mu \mu \lambda \rho} = 0, \quad i e^\lambda \zeta_{\nu \mu \lambda \lambda} - \frac{1}{4} \xi_{\nu \nu \lambda \lambda} = 0, \tag{14}\]
while \( \Theta_{12}|\Psi_2 > = \Theta_{21}|\Psi_2 > = 0 \) does not produce any new equations. Finally, imposing the constraints \( k \cdot \alpha_1 = k \cdot \bar{\alpha}_1 = k \cdot \alpha_2 = k \cdot \bar{\alpha}_2 = 0 \) we get
\[
k^{\mu} \xi_{\mu \nu \lambda \rho} = \xi_{\nu \mu \lambda \rho} k^{\rho} = 0, \\
k^{\mu} \chi_{\mu \nu} = \chi_{\mu \nu} k^{\nu} = 0, \\
k^{\lambda} \eta_{\mu \nu \lambda} = \eta_{\mu \nu \lambda} k^{\lambda} = 0, \\
k^{\lambda} \zeta_{\lambda \mu \nu \lambda} = \zeta_{\lambda \mu \nu \lambda} k^{\lambda} = 0, \\
k^{\lambda} \xi^{(2)}_{\lambda \mu \nu \rho} = \xi^{(2)}_{\mu \nu \lambda \rho} k^{\lambda} = 0. \tag{15}\]
Thus we have imposed all constraints associated with the \( \Theta \) operator, corresponding to the equations (13),(14) and (15).

Now we are in a position to consider the Virasoro constraints. The \( L_2|\Psi_2 > = \bar{L}_2|\Psi_2 > = 0 \) equations imply that
\[
k^{\mu} \chi^{(1)}_{\mu \nu \lambda} - \pi^{\mu} \chi_{\mu \nu \lambda} = 0, \quad k^{\nu} \chi^{(1)}_{\mu \nu \lambda} + \pi^{\nu} \chi_{\mu \nu \lambda} = 0, \\
k^{\mu} \eta^{(1)}_{\mu \nu \lambda} - 2 \pi^{\mu} \eta_{\mu \nu \lambda} = 0, \quad k^{\lambda} \zeta^{(1)}_{\mu \nu \lambda} - 2 \pi^{\lambda} \zeta_{\mu \nu \lambda} = 0, \\
\xi_{\lambda \mu \nu \lambda} = \xi_{\mu \nu \lambda \lambda} = 0. \tag{16}\]
Finally we came to the last constraints, \( L_1|\Psi_2 > = 0 \) which imply that
\[
k^{\mu} \zeta^{(1)}_{\nu \mu \lambda} = 0, \quad \chi_{\nu \lambda} = 0 \\
\eta^{(1)}_{\nu \lambda \rho} = 0, \quad k^{\mu} \xi^{(2)}_{\nu \mu \lambda \rho} + \pi^{\mu} \xi_{\mu \nu \lambda \rho} = 0 \\
k^{\mu} (\zeta^{(1)}_{\nu \lambda \rho} + \zeta^{(1)}_{\nu \mu \lambda}) + \pi^{\mu} \zeta_{\nu \mu \lambda} + 2 \chi^{(1)}_{\nu \lambda} = 0 \\
k^{\mu} (\xi^{(1)}_{\nu \lambda \rho} + \xi^{(1)}_{\nu \mu \lambda}) - \pi^{\mu} \xi_{\nu \lambda \rho} + 2 \eta^{(1)}_{\nu \lambda} = 0 \tag{17}\]
and \( \bar{L}_1|\Psi_2 > = 0 \), that gives:
\[
k^{\mu} \mathcal{N}_{\nu \lambda \rho} = 0, \quad \chi_{\nu \lambda} = 0 \\
\zeta^{(1)}_{\nu \lambda \rho} = 0, \quad k^{\mu} \xi^{(2)}_{\nu \mu \lambda \rho} + \pi^{\mu} \xi_{\nu \lambda \rho} = 0 \\
k^{\mu} (\eta^{(1)}_{\nu \lambda \rho} + \eta^{(1)}_{\nu \mu \lambda}) + \pi^{\mu} \eta_{\nu \lambda \mu} + 2 \chi^{(1)}_{\nu \lambda} = 0 \\
k^{\mu} (\xi^{(1)}_{\nu \lambda \rho} + \xi^{(1)}_{\nu \mu \lambda}) + \pi^{\mu} \xi^{(2)}_{\nu \lambda \rho} + 2 \xi^{(1)}_{\nu \lambda} = 0. \tag{18}\]

4 \hspace{1em} Solution of the Constraints

Our system of equations (13),(14), (15) and (16),(17),(18), which define the wave function (12), further reduces to the form:
\[
|\Psi_2 > = \chi^{(1)}_{\mu \nu} \alpha^{\mu}_2 \alpha^{\nu}_2 + \eta^{(1)}_{\mu \nu \lambda} \alpha^{\mu}_2 \alpha^{\nu}_2 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 + \zeta^{(1)}_{\nu \mu \lambda \rho} \alpha^{\mu}_1 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 + \xi^{(1)}_{\mu \nu \lambda \rho} \alpha^{\mu}_1 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 \\
+ \xi^{(2)}_{\mu \nu \lambda \rho} \alpha^{\mu}_1 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 + \zeta^{(2)}_{\nu \mu \lambda \rho} \alpha^{\mu}_1 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 + \xi^{(1)}_{\mu \nu \lambda \rho} (\alpha^{\mu}_2 \beta^{\nu}_2 \bar{\alpha}^{\lambda}_1 \bar{\alpha}^{\lambda}_1 \bar{\beta}^{\rho}_1 + \beta^{\mu}_2 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\beta}^{\rho}_1 - \beta^{\mu}_1 \alpha^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\beta}^{\rho}_1 + \alpha^{\mu}_1 \beta^{\nu}_1 \bar{\alpha}^{\lambda}_1 \bar{\beta}^{\rho}_1 |k, \epsilon, 0 >, \tag{19}\]
where we have used equations (17) and (18), $\chi_{\mu\nu} = \zeta_{\mu\nu\lambda} = \eta_{\mu\nu\lambda} = 0$. We shall summarize all remaining constraints in the form:

\begin{align*}
e^\mu \xi_{\mu\nu\lambda\rho} = e^\lambda \xi_{\mu\nu\lambda\rho} = 0, & \ 
 k^\mu \xi_{\mu\nu\lambda\rho} = e^\lambda \xi_{\mu\nu\lambda\rho} = 0, \\
e^\mu \xi_{(2)\mu\nu\lambda\rho} = e^\rho \xi_{(2)\mu\nu\lambda\rho} = 0, & \ 
 k^\mu \xi_{(2)\mu\nu\lambda\rho} = \rho^\mu \xi_{(2)\mu\nu\lambda\rho} = 0, \\
k^\mu \eta_{(1)\mu\nu\lambda\rho} + \xi_{(1)\mu\nu\lambda\rho} = 0, & \ 
 k^\mu \xi_{(1)\mu\nu\lambda\rho} + \chi_{(1)\mu\nu\lambda\rho} = 0, \\
k^\mu \chi_{(1)\mu\nu} = k^\nu \chi_{(1)\mu\nu} = 0.
\end{align*}

It is appropriate now to compute the norm of the wave function $|\Psi_2>$:

\[<\Psi_2|\Psi_2> = 12e^\mu \xi_{(2)\mu\nu\lambda\rho} \xi_{(2)\mu\nu\lambda\rho}.\]  

As one can see only one tensor $\xi_{\mu\nu\lambda\rho}(k, e)$ contributes into the norm of the second level wave function. The rest of the tensors $\chi_{(1)\mu\nu}$, $\eta_{(1)\mu\nu\lambda}$, $\zeta_{(1)\mu\nu\lambda\rho}$ and $\xi_{(1,2)\mu\nu\lambda\rho}$ do not contribute to the norm. We have the following equations on this basic tensor $\xi_{\mu\nu\lambda\rho}(k, e) = \eta_{\mu\nu\rho\lambda}(k, e)$

\begin{align*}
k^\mu \xi_{\mu\nu\lambda\rho} = \xi_{\mu\nu\lambda\rho} k^\lambda = 0, & \ 
 e^\mu \xi_{\mu\nu\lambda\rho} = \xi_{\mu\nu\lambda\rho} e^\lambda = 0, \\
\xi_{\mu\nu\lambda\rho} = 0, & \ 
 \xi_{\mu\nu\rho\lambda} = 0, \ 
 \chi_{\mu\nu\lambda\rho} = 0, \\
k^\mu \xi_{(1)\mu\nu\lambda\rho} = 0, & \ 
 k^\mu \xi_{(1)\mu\nu\lambda\rho} = 0, \\
k^\mu \chi_{(1)\mu\nu} = k^\nu \chi_{(1)\mu\nu} = 0. \quad (20)
\end{align*}

which generalize the corresponding equations on $\omega_{\mu\nu}$ for the first excited state $\Xi = 1$ where $k^\mu \omega_{\mu\nu} = \omega_{\mu\nu} k^\nu = 0$, $e^\mu \omega_{\mu\nu} = \omega_{\mu\nu} e^\nu = 0$. They tell us again that the tensor $\xi$ should be transverse and longitudinal at the same time. Unique solution of these equations therefore is:

\[\xi_{\mu\nu\lambda\rho} = \xi(k, e) \ k^\mu \xi(k, e) k^\nu \xi(k, e) k^\rho, \]

from which it follows that the norm of the second level wave function is equal to zero

\[<\Psi_2|\Psi_2> = 0.\]  

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