A Data-driven Approach to Actuator and Sensor Fault Detection, Isolation and Estimation in Discrete-Time Linear Systems

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Abstract

We propose explicit state-space based fault detection, isolation and estimation filters that are data-driven and are directly identified from only the system input-output (I/O) measurements and through the system Markov parameters. The proposed procedures do not involve a reduction step and do not require identification of the system extended observability matrix or its left null space. The performance of the proposed filters is directly connected to and linearly dependent on the errors in the Markov parameters identification process. It is shown that the system observability will suffice for characterizing the fault detection, isolation and estimation filters for both sensor faults as well as fault detection observers for the actuator faults. However, characterizing the actuator fault isolation and estimation filters is feasible only if the system is minimum phase. We have also quantified the fault estimation error in terms of the Markov parameters identification errors. Finally, we have provided several illustrative case study simulations that demonstrate and confirm the merits of our proposed schemes as compared to methodologies that are available in the literature.

Key words: Data-driven; Fault diagnosis; Fault estimation; Linear systems.

1 Introduction

As engineering systems evolve, it is less likely that engineers have a detailed and accurate mathematical description of the dynamical systems they work with. On the other hand, advances in sensing and data acquisition systems can provide a large volume of raw data for most engineering applications. Consequently, one can find a trend towards data-driven approaches in many disciplines, including fault diagnosis. In addition to artificial intelligence based methods, some efforts exist in the literature that are aimed at extending the rich model-based fault diagnosis techniques to data-driven approaches. A trivial solution will be the one where one can first identify a mathematical dynamical model of the system from the available data, and using the resulting explicit model one then implements and designs conventional model-based fault detection and isolation (FDI) schemes. However, this approach suffers from the challenging errors that are introduced in the identification process and that may aggravate the FDI scheme design process errors and result in a totally unreliable fault diagnosis scheme.

In recent years, a new paradigm has emerged in the literature that aims at direct and explicit construction of the FDI schemes from the system input-output (I/O) data ([1,2,3,4]). Subspace-based data-driven fault detection and isolation methods ([5,6]) represent as one of the main approaches that are reviewed in [5]. These methods are developed based on identifying the left null space of the system extended observability matrix using the I/O data. An estimate of the system order and an orthogonal basis for the system extended observability matrix - or its left null space- are obtained via the SVD decomposition of a particular data matrix that is constructed from the system I/O data. This process is known as the reduction step. Basically, in the reduction step it is assumed that the number of the first set of significantly nonzero singular values and the associated directions provide an estimate of the system order and a basis of the extended observability matrix. However, in most cases, this process is erroneous due to the fact that the truncation point for neglecting the small singular values, as being non-significant, in the reduction step is not obvious and clear. Consequently, an erroneous system order and basis for extended observability matrix - or its left null space - are obtained. This error manifest itself into the fault diagnosis scheme performance in a nonlinear fashion. In other words, the performance representation of the FDI scheme is not a linear function of the gap between the estimated system order and the system extended observability matrix and the actual ones. Due to the above drawbacks, other works that have appeared in the literature are mainly concerned with the fault estimation in which the main objective is to eliminate and remove the above reduction step ([7,8,9]). In
these works, an FIR-filter is directly constructed for achieving the fault estimation task based on the system Markov parameters. Nevertheless, the conversion from the FIR-filter specifications to the state-space structure requires a reduction step if necessitated by the design requirements. Moreover, a complicated online optimization procedure, for implementing a corrective procedure is proposed in [7] for the FIR-filter tuning without which the estimation results will be biased. Consequently, the computational time of the method in [7] per sample is not reasonable for real life applications.

In this work, to overcome the above drawbacks and limitations we have proposed fault detection, isolation and estimation filters that are directly constructed in the state-space representation from only the available system I/O data. Our proposed schemes only require identification of the system Markov parameters that is achieved by using conventional methods, such as correlation analysis ([10]) or subspace methods ([11,12,13,14]) from the healthy I/O data. The advantages of our proposed fault detection and isolation scheme over the subspace-based detection and isolation methods are as follows: (a) Our method does not involve or require a reduction step, thus one is not forced to estimate a value for the system order and a basis for the system extended observability matrix. The order of the filters is determined by a parameter $i$ which can be selected based on the conditions of the system Markov parameters, and (b) It does not require constructing the extended observability matrix or its equivalent forms. Furthermore, our proposed scheme directly yields a state-space structure for the fault estimation filter by using only the system Markov parameters. The state-space filter structures are simply derived by following through a few processing steps starting from the available I/O data that allows a more general and comprehensive analysis if necessitated by the design requirements. Furthermore, we have provided explicit expressions for the actuator or sensor fault estimation performance errors in terms of the Markov parameters identification errors. We have also presented several simulation results that demonstrate the merits of our proposed scheme. Particularly, we have compared and evaluated our results with those provided in [7]. The results show that our scheme in its simplest form matches the performance of the most advanced algorithm that is proposed in [7], while requiring an online optimization scheme that imposes significant computational burden to the user. In contrast, our methodology does not suffer or require the extensive and costly computational resources that are necessary in other works in the literature.

The contributions of this paper can now be summarized as follows:

(1) A fault detection filter for both actuator and sensor faults is developed and directly constructed from the available system I/O data in the state-space form in a manner that does not involve a reduction step. Moreover, it does not require the exact a priori knowledge of the system order and only its upper bound is required.

(2) A fault isolation scheme for multiple actuator or multiple sensor faults is developed and directly constructed from the available system I/O data in the state-space form in a manner that does not involve a reduction step.

(3) A fault estimation scheme for actuator and sensor faults is developed and directly constructed from the available system I/O data in the state-space form in a manner that does not involve a reduction step, and

(4) It is shown that the error analysis and performance of the fault estimation scheme is a linear function of and dependent on the Markov parameters estimation errors.

The outline of the remainder of the paper is as follows. Following the preliminaries presented in Section 2, the principles behind our proposed approach are introduced in Section 3. We will show that a Luenberger observer can be directly identified and constructed from the available system I/O data without any prior knowledge of the system order and its matrices or through the identification of the extended observability matrix. In Section 4, we propose a fault detection filter that is directly constructed from the system Markov parameters in the state-space structure. However, the filter is not capable of isolating the faults. Consequently, fault isolation filters for multiple actuator or multiple sensor faults are then introduced in Section 5. Next, we propose a data-driven fault estimation filter for actuators and sensors identification in Section 6. The robustness of our proposed FDI schemes are then investigated in Section 7. Finally, we provide a number of illustrative simulation results in Section 8 to demonstrate and illustrate the advantages and benefits of our methodologies and comparisons with the available work in the literature are also provided.

The following notation is used throughout the paper. Specifically, $\hat{\cdot}$, $\mathcal{N}$ and $\mathbb{E}\{\cdot\}$ denote the Moore-Penrose pseudo-inverse, null space and the expectation operator. $A(p : q, r : s)$ denotes a matrix that is constructed from an original matrix $A$ by only containing the rows $p$ to $q$ and the columns $r$ to $s$. If $p$ and $q$ (or $r$ or $s$) are not specified, then it implies that we are dealing with all the rows (or columns) of $A$.

2 Preliminaries

Consider the following discrete-time linear system $S$,

$$
S : \begin{cases}
x(k+1) = Ax(k) + Bu(k) + w(k) \\
y(k) = Cz(k) + v(k)
\end{cases}
$$

(1)
where \( x \in \mathbb{R}^n, u \in \mathbb{R}^m, \) and \( y \in \mathbb{R}^l, \) \( w_k \) and \( v_k \) are white noise having zero mean and covariance matrices:

\[
E\left[ \begin{bmatrix} w_i^T \\ v_i^T \end{bmatrix} \right] = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{ij} \tag{2}
\]

We model a given actuator or a sensor fault through additive terms injected in the system \( S \) as follows,

\[
S_f : \begin{cases} x(k+1) = Ax(k) + Bu(k) + BF^a(k) + w(k) \\ y(k) = Cx(k) + f^s(k) + v(k) \end{cases} \tag{3}
\]

where \( f^a(k) \in \mathbb{R}^m \) and \( f^s(k) \in \mathbb{R}^l \) represent the actuator and sensor faults, respectively. These faults are commonly known as additive faults.

**Remark 1** The actuator and sensor faults are conventionally modeled in several manners in the literature. For instance, either as additive faults or multiplicative faults. The proper choice depends on the actual characteristics of the fault. Typically, sensor bias, actuator bias and actuator loss of effectiveness are considered as additive faults. Multiplicative fault models are more suitable for representing changes in the system dynamic parameters such as gains and time constants ([15]).

The basic challenge of this work is that the order of the system as well as the matrices \( A, B, C, Q, S \) and \( R \) are assumed not to be known. Instead, a sequence of healthy measured system I/O data, namely \( u(k) \) and \( y(k) \), for \( k = 1, \ldots, T \), are assumed to be available. Furthermore, it is assumed that \( u(k) \) is persistently exciting (PE) ([10]), which implies that the following matrix is full row rank,

\[
U_{ij}(k-i) = \begin{bmatrix} u(k-i) & u(k-i+1) & \ldots & u(k-i+j) \\ u(k-i+1) & u(k-i+2) & \ldots & u(k-i+j+1) \\ \vdots & \vdots & \ddots & \vdots \\ u(k) & u(k+1) & \ldots & u(k+j) \end{bmatrix} \tag{4}
\]

where \( i, j \in \mathbb{N} \). The parameter \( i \) should be selected according to Remark 2 stated subsequently and \( j \) is specified according to the size of the available I/O data. At this stage, let us state below the assumptions that we have made throughout this paper.

**Assumption 1:** The system \( S \) is stable and observable.

**Assumption 2:** The system matrices and system order are not known a priori and only measured I/O information is available.

**Assumption 3:** The Markov parameters are estimated by using the healthy data from the system \( S \) and the input \( u(k) \) that satisfies the persistently exciting (PE) condition.

**Assumption 4:** The faults in the system \( S_f \) are detectable and isolable as comprehensively discussed in [16].

We define the set \( \{ H_0, H_1, H_2, \ldots \} \), where \( H_i = CA^iB \) is known as the Markov parameter. If \( u(k) \) is persistently exciting, then several approaches are available in the literature to directly identify the Markov parameters from the I/O data \( u(k) \) and \( y(k) \) ([10], [9]).

Let us iteratively over the measurement equation of the system \( S \) to obtain,

\[
\begin{align*}
Y_{ij}(k) &= C_iA^sX_{ij}(k-s) + \mathcal{D}_{si}U_{(i+s-1)j}(k-s) + \\
& \hspace{1cm} \mathcal{E}_{si}W_{(i+s-1)j}(k-s) + V_{ij}(k-s)
\end{align*}
\tag{5}
\]

where \( Y_{ij}(k), W_{(i+s-1)j}(k-s) \) and \( V_{ij}(k-s) \) are constructed in a similar manner as in \( U_{ij}(k) \), and \( s \) is selected sufficiently large in order to ensure that \( A^s \approx \begin{bmatrix} 1 \end{bmatrix} \)

and

\[
C_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^i \end{bmatrix}; \mathcal{D}_{si} = \begin{bmatrix} H_{s-1} & H_{s-2} & \ldots & H_0 & 0 & \ldots & 0 \\ H_s & H_{s-1} & \ldots & H_1 & 1 & H_0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{i+s-1} & H_{i+s-2} & \ldots & H_{i-1} & H_{i-2} & \ldots & H_0 & 0 \end{bmatrix} \tag{6}
\]

\[
\mathcal{E}_{si} = \begin{bmatrix} CA^{s-1} & CA^{s-2} & \ldots & C & 0 & \ldots & 0 \\ CA^s & CA^{s-1} & \ldots & CA & C & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ CA^{i+s-1} & CA^{i+s-2} & \ldots & CA^{i-1} & CA^{i-2} & \ldots & C & 0 \end{bmatrix} \tag{7}
\]

\[
X_{ij}(k-s) = \begin{bmatrix} x(k-s) & \ldots & x(k-s+j-1) \end{bmatrix} \tag{8}
\]

\footnote{One can start with an initial \( s_0 \) and estimate the Markov parameters as shown in details subsequently. If the norm of the last estimated Markov parameter is less than a sufficiently small user defined bound, then \( s = s_0 \), otherwise, \( s \) should be increased and the procedure should be repeated until \( s \) with a desired Markov parameter norm is obtained.}
Therefore,
\[ Y_{ij}(k) \approx D_{si}U_{(i+s-1)j}(k-s) + \mathcal{E}_{si}W_{ij}(k-s) + V_{ij}(k-s) \]
(9)

An estimate of \( D_{si} \) - which contains the Markov parameters of the system - is given by the solution to the following problem,
\[ \hat{D}_{si} = \arg \min_{D_{si}} \| Y_{ij}(k) - D_{si}U_{(i+s-1)j}(k-s) \|_F^2 \]
(10)

where \( \| . \|_F \) denotes the Frobenius norm of a matrix. A practical solution to the optimization problem (10) that yields an estimate of the system Markov parameters, \( \hat{H}_l, l = 0, \ldots, i + s - 1 \), is provided in the Appendix A. Another approach for Markov parameters estimation can be pursued through the correlation analysis, which is comprehensively discussed in standard textbooks on linear systems identification ([10]).

We will subsequently use an equivalent form of the system \( S \) as follows,
\[
S : \begin{cases} 
  x(k-i+1) = Ax(k-i) + BI_iU_i(k-i) + w(k-i) \\
  Y_i(k-i) = C_i x(k-i) + D_i U_i(k-i) + V_i(k-i)
\end{cases}
\]
(11)

where,
\[
C_i = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{pmatrix}; \quad D_i = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{i-2} & H_{i-3} & \ldots & H_0 \end{pmatrix}
\]

\[
E_i = \begin{pmatrix} C_0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ C_{A^{-2}} & \ldots & C_0 \end{pmatrix}
\]
(12)

\[
U_i(k-i) = \begin{pmatrix} u(k-i) \\ u(k-i+1) \\ \vdots \\ u(k) \end{pmatrix}; \quad Y_i(k-i) = \begin{pmatrix} y(k-i) \\ y(k-i+1) \\ \vdots \\ y(k) \end{pmatrix}
\]
(13)

\[
I_i = \begin{bmatrix} I_{m \times m} & 0_{m \times (i-m)} \end{bmatrix}
\]
(14)

and \( V_i(k-i) \) is constructed similar to \( U_i(k-i) \) using \( v(k-i), \ldots, v(k) \). Moreover, we also define,
\[
D_{i+} = \begin{pmatrix} H_0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
H_{i-1} & H_{i-2} & \ldots & H_0 \end{pmatrix}; \quad D_{i+} = \begin{pmatrix} H_0 \\
\vdots \\
H_{i-1} \end{pmatrix}
\]
(15)

Using the above definitions, we now define \( D_{i-p}^+ \) and \( D_{i,p}^- \) that are obtained by deleting the columns \( p, 2p, \ldots, jmp \) and rows \( p, 2p, \ldots, jlp \) of \( D_i \), respectively, \( Y_i^+ (U_i^-) \) is obtained by deleting the rows \( p, 2p, \ldots, jmp \) and columns \( p, 2p, \ldots, jlp \) of \( Y_i^+ (U_i^-) \). Finally, \( D_{i+p}^+ \) and \( D_{i,p}^- \) are defined as matrices that only contain the columns \( p, 2p, \ldots, jmp \) and the rows \( p, 2p, \ldots, jlp \) of \( D_i \), respectively. The same property and structure holds for \( Y_i^+ (U_i^-) \). Also, we are now in a position to formally state the guideline for selection of parameter \( i \).

**Remark 2** The least value chosen for \( i \) should be such that \( D_{i+} \) is full column rank. Larger values for \( i \) have no impact on the performance of the scheme other than increasing the filters order, and consequently adversely increasing the computational costs. However, given that the Markov parameter estimation process is error prone, this may numerically affect the rank condition of \( D_{i+} \), therefore a sufficiently large value of \( i \) is desirable.

### 3 Problem Formulation

The problem that is envisaged in this work is a multifaceted challenge that involves residual generation, fault detection, isolation and estimation, and derivation of the residual filters parameters from only the available I/O data. We provide a description of the problem statement by outlining a sketch of the methodology and then discuss the details in subsequent sections.

Let us consider a signal \( \eta(k) \) that is governed by the following dynamics,
\[
\eta(k+1) = A_r \eta(k) + B_r U_r(k-i) + L_r Y_i(k-i)
\]
(16)

where \( \eta(k) \in \mathbb{R}^n \). Our goal is to determine the unknown matrices \( A_r, B_r \) and \( L_r \) given only the system I/O data \( U_r \) and \( Y_i \) such that for the healthy system \( S \),
\[
E(e(k)) = E(\eta(k) - T x(k-i)) \rightarrow 0 \text{ as } k \rightarrow \infty
\]
(17)

where \( T \in \mathbb{R}^{n \times n} \) is a full column rank matrix that is to be designed and specified subsequently. The error dynamics associated with \( e(k) \) is therefore given by,
\[
e(k+1) = e(k+1) - T x(k-i+1) = A_r e(k) + (A_r T - A_r C_i)x(k-i) + (B_r + L_r D_i - T B L_i) U_i(k-i) + E_i W_i(k-i) + V_i(k-i)
\]
(18)
which is obtained by substituting $\eta(k+1)$ from equation (16) and $x(k-i+1)$ from equation (11). Condition (17) is now satisfied if and only if (a) $A_r$ is a Hurwitz matrix, (b) $A_i T - T A_i + L_r C_i = 0$, and (c) $B_r + L_r D_i - T B I_i = 0$, which actually correspond to the Luenberger observer equations. The key concept that is introduced in this paper is that we specifically set

$$T = C_i$$  \hspace{1cm} (19)

In other words, we select $T$ to be equal to the extended observability matrix. In our subsequent data-driven derivations it is shown that our proposed filter design procedure does not actually require the computation of $C_i$. Typically in the literature [11], the extended observability matrix is obtained through a so-called reduction step from the I/O data or the Markov parameters. However, the estimate of the extended observability matrix that is obtained from this approach may have high or low dimensionality and lead to a completely different orthogonal basis as compared to the associated real system. Under both cases one will introduce and cause a nonlinear error effects into the estimation process. For this reason, in this work our goal is to eliminate the use of the reduction step.

Given (19), it follows readily that the previous conditions (b) and (c) can be expressed in terms of the extended observability matrix and the Markov parameters as (b') $A_i C_i - C_i A + L_r C_i = 0$, and (c') $B_r + L_r D_i - C_i B I_i = 0$, respectively. By examining the condition (c') above, it follows that $C_i B$ is in fact a column matrix associated with the second to $i+1$ Markov parameters of the system $S$. In other words, one can write,

$$C_i B = D_{i+}$$  \hspace{1cm} (20)

Therefore, the expression (c') only depends on the system Markov parameters $H_i$. Now, we examine the expression (b') above. The objective of the expression (b') is in fact to enforce,

$$(A_r C_i - C_i A + L_r C_i)x(k-i) \equiv 0$$  \hspace{1cm} (21)

On the other hand, from the measurement equation (11) it follows that,

$$C_i x(k-i) = Y_i(k-i) - D_i U_i(k-i) - V_i(k-i)$$  \hspace{1cm} (22)

By substituting equation (22) into equation (21) one gets,

$$(A_r C_i - C_i A + L_r C_i) x(k-i) = 0$$  \hspace{1cm} (23)

where,

$$M \triangleq A_r + L_r$$  \hspace{1cm} (24)

Iterating the equation (23) for the time steps from $k-i$ to $k-i+j$ gives us,

$$M (\Gamma_0 - E_i W_{ij}(k-i) - V_{ij}(k-i)) - (\Gamma_1 - E_i W_{(i+1)j}(k-i) - V_{(i+1)j}(k-i+1)) = 0$$  \hspace{1cm} (25)

where $\Gamma_0 = Y_{ij}(k-i) - D_i U_{ij}(k-i)$ and $\Gamma_1 = Y_{(i+1)j}(k-i+1) - D_{i+} U_{(i+1)j}(k-i)$. Once we estimate $M$ from the healthy I/O data (this is shown in detail subsequently) and $A_r$ is selected to be an arbitrary Hurwitz matrix, then $L_r$ and $B_r$ will be easily obtained from equation (24) and expression (c'), respectively.

Equation (25) forms the basis of our proposed data-driven solution for estimating $M$. Once the Markov parameters of the system are estimated by using the procedure of the Appendix A or the correlation analysis (10), we construct the estimated matrices $\hat{\Gamma}_0$ and $\hat{\Gamma}_1$ from the system I/O data (healthy data) as follows,

$$\hat{\Gamma}_0 = Y_{ij}(k-i) - D_i U_{ij}(k-i)$$  \hspace{1cm} (26)

$$\hat{\Gamma}_1 = Y_{(i+1)j}(k-i+1) - D_{i+} U_{(i+1)j}(k-i)$$  \hspace{1cm} (27)

where $D_i$ and $D_{i+}$ are constructed similar to $D_i$ and $D_{i+}$ but instead the estimated Markov parameters are utilized. Therefore, an estimate of $M$, namely $\hat{M}$, is given by,

$$\hat{M} = \hat{\Gamma}_1 \hat{\Gamma}_0^T$$  \hspace{1cm} (28)

where $K_1 \in \mathbb{R}^{l \times l}$ and $K_2 \in \mathbb{R}^{l \times (i-1)l}$ are nonzero matrices. The above analysis shows that the estimation filter (16) that satisfies the condition (17) can be directly synthesized from the system I/O data without requiring any reduction step. The problem that we are considering can therefore be stated as i) how one can use the estimation filter (16) to generate residual signals for accomplishing the fault detection goal? ii) how and under which conditions one can use the generated residuals for the purpose of accomplishing the fault isolation goal? and iii) how one can use the generated residuals for the purpose of accomplishing the fault estimation goal? We will discuss these three problems and propose below explicit data-driven fault detection, isolation and estimation schemes. Towards these end, we assume that Assumptions 1-4 which were given earlier in Section 2 hold.

Note that $M = \hat{A}_r + \hat{L}_r$, which in comparison with equation (21) implies that the true values of $A_r$ and $L_r$ are available only if $M$ is known. In some cases, as we will discuss in subsequent sections, $A_r$ can be an arbitrary Hurwitz matrix, which under this scenario implies that $A_r = A_r$. 

\[ \text{Equation 25} \]
Let us construct a residual generator filter that is governed by the following dynamics,

\[
\begin{align*}
\eta(k+1) &= \hat{A}_r \eta(k) + \hat{B}_r U_i(k-i) + \hat{L}_r Y_i(k-i) \\
\bar{r}(k) &= \eta(k) - Y_i(k-i) + \bar{D}_r U_i(k-i)
\end{align*}
\]

where \( \eta(k) \in \mathbb{R}^l \), \( \bar{A}_r \), \( \bar{B}_r \) and \( \bar{L}_r \) satisfy the following conditions, namely: \( (a') \) \( \bar{A}_r \) is Hurwitz, \( (b') \) \( \bar{A}_r + \bar{L}_r = M \) and \( (c') \) \( \bar{B}_r + L_r \bar{D}_1 - \bar{D}_1 L_1 = 0 \). Provided that \( A_r \) is selected to be a diagonal matrix, then it follows from the structure of \( M \) (equation (28)) that the first \( (i-1) \) rows of \( \bar{r}(k) \) that are generated by the filter (29) is trivially zero. However, the last \( l \) rows yield non-trivial values for \( \bar{r}(k) \). Therefore, the residual generator filter is now specified as follows,

\[
S_r : \quad \begin{cases}
\eta_r(k+1) = \hat{A}_r \eta_r(k) + \hat{B}_r U_i(k-i) + \hat{L}_r Y_i(k-i) \\
\bar{r}(k) = \eta_r(k) - \bar{D}_r U_i(k-i)
\end{cases}
\]

where \( \eta_r(k) \in \mathbb{R}^l \), \( \hat{A}_r = \hat{A}_r((i-1)l : il), (i-1)l : il) \), \( \bar{B}_r = \hat{B}_r((i-1)l : il), : il) \), \( \bar{L}_r = \hat{L}_r((i-1)l : il), : il) \) and \( \bar{D}_r = \hat{D}_1((i-1)l : il), : il) \).

Let us now consider a scenario where a sensor fault has occurred in the system \( S_f \). The residual signal dynamics is now given by \( (e_r(k) \) is the last \( l \) rows of \( e(k) \) where \( e(k) = \eta(k) - C_i x(k-i) \)

\[
\begin{align*}
e_r(k+1) &= \hat{A}_r e_r(k) + \Delta \hat{A}_r x(k-i) + \\
&\quad \Delta \hat{B}_r U_i(k-i) + \Delta \hat{L}_r Y_i(k-i) + \hat{D}_r \bar{F}_r^i(k-i) + \\
&\quad \bar{E}_r W_i(k-i) + \bar{L}_r V_i(k-i) \\
\bar{r}(k) &= e_r(k) + \Delta \hat{D}_1 U_i(k-i) + f^*(k)
\end{align*}
\]

where \( \bar{E}_r = E_r((i-1)l : il), : il) \) and \( \bar{F}_r^i(k-i) \) is defined similar to \( U_i(k-i) \). Moreover, \( \Delta \bar{D}_1 = \bar{D}_1 - D_1 \), \( \Delta \bar{D}_r = \Delta \hat{D}_1 ((i-1)l : il), : il) \), and \( \Delta \hat{A}_r, \Delta \hat{B}_r \) and \( \Delta \hat{L}_r \) are defined as follows. Assume that the conditions \( (a'') \), \( (b'') \) and \( (c'') \) are solved by using an exact mathematical model of the system to obtain \( A_r, B_r \) and \( L_r \). Then \( \Delta A_r = A_r - C_i A + L_r C_i, \Delta B_r = \hat{B}_r + L_r \bar{D}_1 - C_i B \), and \( \Delta L_r = L_r - L_r \). Therefore \( \Delta \hat{A}_r = \Delta A_r((i-1)l : il), (i-1)l : il) \), \( \Delta \hat{B}_r = \Delta B_r((i-1)l : il), : il) \), and \( \Delta \hat{L}_r = \Delta L_r((i-1)l : il), : il) \). Clearly, none of the \( \Delta \)'s can be evaluated since the system model is not available. This issue is addressed and resolved by the following assumption.

_Assumption 5_: It is assumed for the subsequent analysis that \( \Delta \bar{D}_1, \Delta \hat{A}_r, \Delta \hat{B}_r, \Delta \hat{L}_r \approx 0 \).

We will discuss in Section 7 the effects of the estimation errors on the performance of our proposed schemes by removing and relaxing this assumption. Furthermore, we use the notation \( \dot{e}(k) \) instead of \( e(k) \) to designate that Assumption 5 holds. Similarly, \( \dot{e}_r(k) \) denotes the last \( l \) rows of \( \dot{e}(k) \). When Assumption 5 is not imposed we use the notation \( e(k) \) to refer to the estimation error.

The error dynamics subject to Assumption 5 and subject to the presence of a sensor fault is now given by,

\[
\begin{align*}
\dot{e}_r(k+1) &= \hat{A}_r \dot{e}_r(k) + \hat{L}_r \bar{F}_r^i(k-i) + \\
\bar{E}_r W_i(k-i) + \bar{L}_r V_i(k-i) \\
\bar{r}(k) &= \dot{e}_r(k) + f^*(k)
\end{align*}
\]

Similarly, if an actuator fault has occurred in the system \( S_f \), the resulting residual error dynamics is governed by,

\[
\begin{align*}
\dot{e}_r(k+1) &= \hat{A}_r \dot{e}_r(k) - \hat{B}_r \bar{F}_r^i(k-i) + \\
\bar{E}_r W_i(k-i) + \bar{L}_r V_i(k-i) \\
\bar{r}(k) &= \dot{e}_r(k) - \bar{D}_r \bar{F}_r^i(k-i)
\end{align*}
\]

where \( \bar{F}_r^i(k-i) \) is defined in a similar manner as in \( U_i(k-i) \). We will discuss subsequently in Section 6 an important difference between the residual dynamics that is governed by the models (32) and (33) as far as the fault estimation objective is considered.

We are now in a position to state our proposed fault detection logic as follows. Note that \( \bar{r}(k) \) used below is generated by the filter (30),

\[
\begin{align*}
\text{If } \mathbb{E}\{\bar{r}(k)\} \neq 0 \Rightarrow & \quad \text{System is faulty} \\
\text{If } \mathbb{E}\{\bar{r}(k)\} = 0 \Rightarrow & \quad \text{System is healthy}
\end{align*}
\]

In practice, the fault detection logic (34) is made practical and is augmented by upper and lower bound thresholds. In other words, if the expected residual signal exceeds either some specified lower or upper thresholds then a fault is declared to be detected, otherwise the system is classified as healthy. The above completes the fault residual filter design that is realized by only the I/O data. It should be noted that \( M \) and the system Markov parameters (through which \( \bar{D}_1 \) and \( \bar{D}_1 \) are constructed) are estimated from the healthy available data. Once \( A_r, B_r \) and \( L_r \) are obtained by using the conditions \( (b'') \) and \( (c'') \), then the residual generator filter (30) is derived by using the I/O data of the system that is made available during its operation.

5 The Proposed Fault Isolation Scheme

The residual generator filter (30) generates a non-zero residual in presence of any fault in the sensor and/or actuator. However, this information is not sufficient for the purpose of performing the fault isolation task. Fault isolation is typically performed via structured residuals. In
other words, a bank of residual observers is constructed where each filter in the bank is insensitive to a particular fault but sensitive to all other faults. Therefore, in case of occurrence of a fault, all the filters generate non-zero residuals that exceed their thresholds except for the one filter that can be used for determining the isolated fault.

In this section, we propose a fault isolation scheme that can be realized from the identified system Markov parameters without requiring one to perform the reduction step. We develop the conditions for existence of this fault isolation filter. It should be pointed out that below we provide an approach for developing and constructing a set of structured residual filters under the presence of either multiple actuators or multiple sensors (but not simultaneously) faults. The case for isolating multiple actuators and sensors concurrent faults requires that one construct a bank of filters each of which is insensitive to a group of faults in the same manner. These details are not included in this paper and is left as a topic of future research.

5.1 Actuator Fault Isolation

Let us first consider the case corresponding to an actuator fault isolation problem. Assume that a fault has occurred in the actuator \( q \), where \( 1 \leq q \leq m \). Our goal is to design a residual generator filter that is insensitive to the fault in the actuator \( q \), but sensitive to faults in all the other actuators. This will be accomplished and possible if the columns \( \{ q, 2q, \ldots, mq \} \) of \( \hat{B}_r \) or \( \hat{B}_r \) are equal to zero. Consequently, the residual generator filter is clearly insensitive to the input channel \( q \). However, this condition cannot be satisfied at all times since it is closely related to the location of the transmission zeros of the subsystem that is governed in the transfer matrix from the input channel \( q \) to the outputs (and not the entire system).

Let us consider a filter that is governed according to the dynamics,

\[
\zeta(k + 1) = \hat{A}_r^q \zeta(k) + \hat{B}_r^q U_i^q(k - i) + \hat{L}_r^q Y_i(k - i) \quad (35)
\]

where \( \zeta(k) \in \mathbb{R}^{d_l} \), and the matrices \( \hat{A}_r^q, \hat{B}_r^q \) and \( \hat{L}_r^q \) are to be determined by the designer to satisfy the user specified design objectives. The filter (35) is known as an unknown input observer (UIO) and \( \{ \zeta(k) - C_r x(k - i) \} \to 0 \) as \( k \to \infty \) if and only if (i) \( \hat{A}_r^q \) is a Hurwitz matrix, (ii) \( \hat{A}_r^q C_r - C_r A + \hat{L}_r^q C_r = 0 \), (iii) \( \hat{B}_r^q + \hat{L}_r^q D + \hat{D}_r^q + \hat{L}_r^q I_i = 0 \), and (iv) \( \hat{L}_r^q = 0 \). It is well-known that conditions (i)-(iv) have a solution if and only if the transfer matrix from the input channel \( q \) to the outputs is minimum-phase. The condition (ii) is equivalent to \( \hat{A}_r^q + \hat{L}_r^q = \hat{M} \), where \( \hat{M} \) is estimated by using the healthy data according to equation (28). We also substitute condition (iv) into condition (iii). Consequently, the solution to conditions (i)-(iv) is given by,

\[
\hat{A}_r^q + \hat{L}_r^q = \hat{M} \quad (36) \quad \hat{L}_r^q = \hat{D}_r^q \hat{I}_i (\hat{D}_r^q)^\dagger \quad (37)
\]

In the data-driven problem formulation and context, one cannot determine the minimum-phassness of the subsystem transfer matrices in advance, given that the system matrices are unknown. Nevertheless, this property can be established if \( \hat{A}_r^q \) that is obtained from equations (36) and (37) is a Hurwitz matrix. Otherwise, the considered subsystem transfer matrix is non-minimum phase. Our procedure also provides a robust data-driven procedure for determining the minimum phaseness of the system \( S \).

The bank of residual filters that is to be employed for performing the actuator fault isolation task corresponds to an assembly of \( m \) unknown input observers (UIO) each of which operates with the information that is obtained from the \( m - 1 \) input channels and the \( l \) output channels. The governing dynamics of each filter bank is given by,

\[
SA_r^q = \left\{ \begin{array}{l}
\zeta(k + 1) = \hat{A}_r^q \zeta(k) + \hat{B}_r^q U_i^q(k - i) + \hat{L}_r^q Y_i(k - i) \\
\hat{r}_q^a(k) = C_r \zeta(k) - y(k), \quad q = 1, \ldots, l
\end{array} \right. \quad (38)
\]

where \( C_r = \begin{bmatrix} I_{1 \times l} & 0_{l \times (nl - i)} \end{bmatrix} \), \( \hat{B}_r^q = \hat{D}_r^q \hat{I}_i - \hat{L}_r^q \hat{D}_r^q \), and \( \hat{A}_r^q \) and \( \hat{L}_r^q \) are solutions to the equations (36) and (37). Our proposed fault isolation logic can now be presented as follows: A fault is detected and isolated in the actuator \( q \) if,

\[
\left\{ \begin{array}{l}
\mathbb{E}\{ \hat{r}_q^a(k) \} \neq 0 \quad : a \neq q, a \quad \text{and} \quad q = 1, \ldots, m, \quad \text{and} \\
\mathbb{E}\{ \hat{r}_q^a(k) \} = 0 \quad : a = q.
\end{array} \right.
\quad (39)
\]

As stated earlier for the fault detection problem, in practice, upper and lower threshold bounds should be established to robustly perform the isolation task. A simple comparison and observation reveals that the filters given by equations (30) and (38) are different in certain aspects. First, note that \( \eta(k) \in \mathbb{R}^d \) and \( \zeta(k) \in \mathbb{R}^{d_l} \). This is due to the fact that one could arbitrarily select \( A_r \) to be a diagonal matrix allowing a model order reduction (given the particular structure of \( M \)), whereas \( \hat{A}_r^q \) is to be determined and solved from equation (36), which does not necessarily yield a diagonal matrix. For the same reason, the first \( (i - 1)l \) rows of \( \hat{r}_q^a(k) \) are not trivially zero, as opposed to \( \hat{r}(k) \) in (29). For the observers that are given by equations (30) and (38), the residuals are defined according to \( \hat{r}(k) = \eta(k) - y(k) + \hat{D}_r U_i^q(k - i) \) and \( \hat{r}(k) = C_r \zeta(k) - y(k) \), respectively. The latter residual signal cannot be a function of the input, otherwise it fails to be insensitive to a particular actuator fault, which also explains the fact that we assumed the throughput term of the system \( S \) measurement equation to be zero.
Another significant difference arises due to the existence of a solution to conditions (a'), (b') and (c'), that are always guaranteed, whereas a solution to the conditions (i)-(iv) exists if and only if the subsystem transfer matrix from the input channel q to the outputs is minimum phase. Note that a non-minimum phase system may have minimum phase subsystem transfer matrices and vice versa, therefore a strict requirement of having an overall minimum phase system S is not necessary for design of the fault isolation filters given by (38).

5.2 Sensor Fault Isolation

In contrast to the actuator fault isolation problem, the accomplishment of the sensor fault isolation task is straightforward. We start by constructing a bank of isolation filters each of which is insensitive to a particular sensor fault and sensitive to all the other faults. In other words, each filter operates with \( l - 1 \) output measurements and \( m \) inputs. The design procedure for constructing the insensitive filter to the fault in the sensor \( p \) is similar to the residual filter design procedure that was provided in Section 4, with the difference that the output measurement \( p \) is not included. Formally speaking, we first construct a residual generator filter that is governed by

\[
\begin{align*}
\xi(k+1) &= \hat{A}_p^r \xi(k) + \hat{B}_p^r U_i(k-i) + \hat{L}_p^r Y_i^{p-}(k-i) \\
\hat{r}^p(k) &= \xi(k) - Y_i^{p-}(k-i) + \hat{D}_{i,p-} U_i(k-i)
\end{align*}
\]

(40)

where \( \xi(k) \in \mathbb{R}^d \), and \( \hat{A}_p^r, \hat{B}_p^r \) and \( \hat{L}_p^r \) satisfy (i'), \( \hat{A}_p^r \) is a Hurwitz matrix, (ii') \( \hat{A}_p^r + \hat{L}_p^r = \hat{M}^{p-} \), and (iii') \( \hat{B}_p^r + \hat{L}_p^r \hat{D}_{i,p-} - \hat{D}_{i+p-} I_l = 0 \). Note that we use \( \hat{M}^{p-} \) instead of \( \hat{M} \) that is estimated by using the healthy I/O data of the system according to \( \hat{M}^{p-} = \hat{G}_1^{-1} (\hat{G}_0^{p-})^{-1} \), where \( \hat{G}_0^{p-} = Y_{p-}^{i+1}(k-i+1) - \hat{D}_{i+p-} U_{i(k-i+1)} \) and \( \hat{G}_1^{-1} = Y_{1}^{p-} (k-i+1) - \hat{D}_{i+p-} U_{i(k-i+1)} \). Similar to the residual generator filter given by equation (29), the first \((l-1)(l-1)(l \geq 1)\) rows of \( \hat{r}^p(k) \) are trivially zero. Therefore, a reduced order model can be constructed as follows,

\[
\begin{align*}
\xi_r(k+1) &= \hat{A}_p^r \xi_r(k) + \hat{B}_p^r U_i(k-i) + \hat{L}_p^r Y_i^{p-}(k-i) \\
\hat{r}^p(k) &= \xi_r(k) - Y_i^{p-}(k-i) + \hat{D}_{i,p-} U_i(k-i)
\end{align*}
\]

(41)

where \( \xi_r(k) \in \mathbb{R}^1 \), \( \hat{A}_p^r = \hat{A}_p^r ((i-1)(l-1) : i(l-1), (i-1)(l-1) : i(l-1), (i-1)(l-1) : i(l-1), i(l-1) : i(l-1)), \hat{B}_p^r = \hat{B}_p^r ((i-1)(l-1) : i(l-1), (i-1)(l-1) : i(l-1), \hat{L}_p^r = \hat{L}_p^r ((i-1)(l-1) : i(l-1), i(l-1), \hat{D}_{i,p-} = \hat{D}_{i,p-} ((i-1)(l-1) : i(l-1), i(l-1), :).)

Two important conclusions can be drawn here. First, it is not possible to isolate a sensor fault for a single output system since \( l = 1 = 0 \) and all the equations will become singular. This fact also follows intuitively given that no other reference signal is available to perform comparison and isolation of multiple faults. Second, in contrast to the actuator fault isolation problem, no specific conditions are required for existence of a solution to the conditions (i')-(iii'). In fact, a solution always exists since a Hurwitz matrix \( \hat{A}_p^r \) can be arbitrarily selected, where then the matrices \( \hat{L}_p^r \) and \( \hat{B}_p^r \) can be specified and obtained.

6 The Proposed Fault Estimation Scheme

In many practical control problems, it is extremely crucial to estimate the faults once they are detected and isolated. In this section, we provide a simple data-driven based methodology for fault estimation filter design. First, we consider the case corresponding to the actuator fault estimation problem. Let us construct the following residual generator filter,

\[
\begin{align*}
\mathbf{SA}_{\text{est}} : \quad & \eta(k+1) = \hat{A}_r \eta(k) + \hat{L}_r Y_i(k-i) \\
& \hat{r}(k) = \eta(k) - Y_i(k-i) + \hat{D}_r U_i(k-i)
\end{align*}
\]

(42)

where \( \hat{A}_r \) and \( \hat{L}_r \) are obtained from the conditions: (i'') \( \hat{A}_r \) is a Hurwitz matrix, (ii'') \( \hat{A}_r + \hat{L}_r = \hat{M} \), and (iii'') \( \hat{L}_r \hat{D}_i - \hat{D}_{i+} I_l = 0 \). The above conditions have a solution if only if the system \( S \) is minimum phase. The error dynamics for the case of an actuator fault is given by,

\[
\begin{align*}
\mathbf{SS}_{\text{est}} : \quad & \hat{\eta}(k+1) = \hat{A}_r \hat{\eta}(k) + \hat{E}_r W_i(k-i) + \hat{L}_r V_i(k-i) \\
& \hat{r}(k) = \hat{\eta}(k) - \hat{D}_r F_i^{p}(k-i)
\end{align*}
\]

(43)

Since \( \hat{A}_r \) is Hurwitz, \( \mathbb{E}\{\hat{\eta}(k)\} \to 0 \) as \( k \to \infty \), which implies that \( \mathbb{E}\{\hat{r}(k)\} = -\hat{D}_r \mathbb{E}\{F_i^{p}(k-i)\} \). Therefore, the actuator fault can be estimated from the following expression,

\[
\hat{r}^a(k-i) = -I_l \hat{D}_r \mathbb{E}\{\hat{r}(k)\}
\]

(44)

Consequently, the fault estimator filter is now governed by equations (42) and (44).

The case associated with the sensor fault estimation is more straightforward. We first set up the following residual generator filter,

\[
\begin{align*}
\mathbf{SS}_{\text{est}} : \quad & \eta(k+1) = \hat{A}_r \eta(k) + \hat{B}_r U_i(k-i) \\
& \hat{r}(k) = \eta(k) - Y_i(k-i) + \hat{D}_r U_i(k-i)
\end{align*}
\]

(45)

where \( \hat{A}_r \) and \( \hat{B}_r \) are obtained from (i iii') \( \hat{A}_r = \hat{M} \), and (ii iii') \( \hat{B}_r = \hat{D}_{i+} I_l = 0 \). Note that \( \hat{M} \) is always a Hurwitz matrix, therefore \( \hat{A}_r \) that is obtained from the condition (i iii') is guaranteed to be Hurwitz. The resulting error
The process of Markov parameters estimation using the noisy data clearly leads to biases and errors. The fault detection and isolation schemes are less prone to these errors since these errors are handled through proper setting of the thresholds in most scenarios. However, the same cannot be said and is not valid for the fault estimation scheme. Any error that is present in the estimation of the Markov parameters will lead to biased results. It was stated earlier that the performance of our proposed schemes are linearly dependent on the Markov parameters estimation errors given the fact that our proposed algorithms are free of the reduction step commonly used in other techniques in the literature. In order to establish this claim, in this section we formally associate and show how the fault estimation errors are dependent on the Markov parameters estimation errors.

Towards this end, let us first consider the case of the sensor fault estimation problem. In this section, it is assumed that Assumption 5 is no longer valid for our subsequent analysis. Hence, the error dynamics is now governed by (recall that $e(k) = \eta(k) - C_i x(k-i)$),

$$
\begin{align*}
  e(k+1) &= \hat{A}_r e(k) + \Delta A_r x(k-i) + D_r U_i(k-i) \\
  + &C_i W_i(k-i) \\
  r(k) &= e(k) + \Delta D_i U_i(k-i) + F_i(k-i)
\end{align*}
$$

(48)

The matrix $A_r$ satisfies the condition $A_r C_i - C_i A = [\mathbb{I}]$.

If we replace $A_r$ with $\hat{A}_r$, then $A_r C_i - C_i A = \Delta A_r$, which is the reason for having the term $\Delta A_r x(k)$ in (48). By neglecting the noise terms, $\Delta A_r x(k)$ becomes,

$$
\Delta A_r x(k) = \hat{A}_r (Y_i(k-i) - D_i U_i(k-i)) \\
- (Y_{i+1}(k-i+1) - D_i U_{i+1}(k-i)) \\
= \hat{A}_r \Delta D_i U_i(k-i) - \Delta D_i U_{i+1}(k-i)
$$

(49)

The second equality of the above equation holds since $A_r = \mathbb{M}$. The above equation provides an explicit expression that directly and linearly associates the error dynamics to the Markov parameters estimation errors. If the reduction step was invoked, then $\Delta A_r x(k)$ would include a number of extra terms that would have been extremely challenging to evaluate. In contrast, it is straightforward to estimate $\Delta D_i$ from the Markov parameters estimation process. Also, note that $\Delta B_r = \Delta D_i I_i$. Therefore, the dynamics associated with $r(k) - \hat{r}(k)$ is governed by,

$$
\text{SS}^{\text{err}} : \begin{cases}
  e(k+1) - \hat{e}(k+1) = \hat{A}_r (e(k) - \hat{e}(k)) + \\
  \hat{M} \Delta D_i - \Delta D_i^{1+} + \Delta D_i I_i) U_i(k-i) \\
  r(k) - \hat{r}(k) = e(k) - \hat{e}(k) + \Delta D_i U_i(k-i)
\end{cases}
$$

(50)

where $\Delta D_i^{1+}$ is obtained by deleting the last $m$ columns of $\Delta D_i^{[4]}$. Both the input matrix $(M \Delta D_i - \Delta D_i^{1+} + \Delta D_i I_i)$ and the throughput matrix $(\Delta D_i)$ in equation (50) are linear functions of only the Markov parameters estimation errors, that clearly establish and confirm our earlier claim.

Following along the same procedure for the actuator fault estimation problem will yield exactly the same results, i.e., $\text{SA}^{\text{err}} = \text{SS}^{\text{err}}$, which not only shows that the actuator fault estimation filter bias is only a linear function of the Markov parameters estimation errors, but also it confirms that the dynamics associated with $r(k) - \hat{r}(k)$ are exactly the same for both cases. However, the fault estimation bias for the sensor case is given by $f^s(k-i) - \hat{f}^s(k-i) = I_i (r(k) - \hat{r}(k))$, whereas for the actuator fault case, it is given by $f^a(k-i) - \hat{f}^a(k-i) = I_i (\hat{D}_i^1 - D_i^1)(r(k) - \hat{r}(k))$. This completes the robustness analysis of our proposed scheme.

8 Simulation Results

In this section, we provide a number of numerical simulations to illustrate the merits and advantages of our proposed schemes. We consider a non-minimum phase...
function parameters are estimated by using a MATLAB built-in for the purpose of the fault estimation objective. In both cases, the healthy input is generated by a pseudo-random binary signal generator. The healthy output is generated by simulating the system subject to the healthy input in addition to state and measurement noise \( \mathcal{N}(0, 0.1) \) as governed by the system dynamics \( S \). The Markov parameters are estimated by using a MATLAB built-in function \texttt{impulseest}.

For the \textbf{first case} of simulations, we consider the following non-minimum phase system which includes the fault model for the actuator bias \( f_k^a \) and sensor bias \( f_k^s \) as additive terms,

\[
\begin{align*}
    x_{k+1} & = 
    \begin{bmatrix}
        0 & 0 & 0 & -0.01 \\
        1 & 0 & 0 & 0.08 \\
        0 & 1 & 0 & -0.27 \\
        0 & 0 & 1 & -0.54 \\
    \end{bmatrix}
    \begin{bmatrix}
        x_k \\
        u_k + f_k^a \\
    \end{bmatrix}
    \\
    y_k & = 
    \begin{bmatrix}
        1.58 & 0.725 & -0.60 & 0.31 \\
        2.4 & -0.08 & 0.42 & -0.05 \\
    \end{bmatrix}
    \begin{bmatrix}
        x_k + f_k^s \\
    \end{bmatrix} 
\end{align*}
\tag{51}
\]

The poles and zeros of the system are located at \( \{-0.39 \pm 53j, 0.11 \pm 0.09j\} \) and \( \{0.17, 1.49\} \), respectively. Figure 1a shows the output of the residual generator filter for performing the fault detection task (equation (30)) when a bias fault is injected in the actuator 1 at the instant \( k = 150 \). Figures 1b and 1c show the outputs of the fault isolation filters (equation (38)) for the actuators 1 and 2. The above results demonstrate that actuator faults are successfully detected and isolated by application of our proposed data-driven methodology. In the second set of simulations, let us consider the following minimum phase system,

\[
\begin{align*}
    x_{k+1} & = 
    \begin{bmatrix}
        0.32 & 0 & -0.3 & -0.18 \\
        -0.14 & 0.34 & 0 & -0.28 \\
        0.26 & 0.29 & -0.18 & 0.78 \\
        -0.17 & 0.13 & -0.82 & -0.14 \\
    \end{bmatrix}
    \begin{bmatrix}
        x_k \\
        u_k + f_k^a \\
    \end{bmatrix}
    \\
    y_k & = 
    \begin{bmatrix}
        0 & 0.96 & -1.05 & 0.98 \\
        -0.99 & -0.21 & -0.52 & 0 \\
    \end{bmatrix}
    \begin{bmatrix}
        x_k + f_k^s \\
    \end{bmatrix} 
\end{align*}
\tag{52}
\]

where \( f_k^a \in \mathbb{R}^2 \) and \( f_k^s \in \mathbb{R}^2 \). The poles and zeros of the system are located at \( \{-0.22 \pm 89j, 0.44, 0.34\} \) and \( \{-0.19, 0.549\} \), respectively. In the \textbf{first scenario} corresponding to this set, a bias fault equal to 2 is injected to the actuator 1 at the sample time \( k = 150 \) followed by an injection of another bias fault equal to 1 to the actuator 2 at the sample time of \( k = 200 \). Figure 2a shows that both actuator faults are accurately estimated by the filter given by the system (43). The average of the estimated fault severities over the last 10 time samples are given by \( \mathbb{E}(f_k^a(k)) = 1.87 \) and \( \mathbb{E}(f_k^s(k)) = 1.04 \).

In the \textbf{second scenario} corresponding to this set, a bias fault equal to 1 is injected to the sensor 1 followed by an injection of another bias fault equal to -1 to the sensor 2 at the sample time of \( k = 200 \). Figure 2b shows the sensor fault estimation results for the filter given by the system (46). The average of the estimated biases over the last 10 time samples are given by \( \mathbb{E}(\hat{f}_k^a(k)) = 0.92 \) and \( \mathbb{E}(\hat{f}_k^s(k)) = -0.96 \). These results again illustrate the capabilities and merits of our proposed scheme.

In order to quantify the effects of noise on the actuator fault estimation process, we have repeated the first scenario simulations for 500 Monte Carlo runs and plotted them in Figure 3a. In this figure, the difference between the estimated fault and the actual fault severity for the actuators are plotted versus each other. This figure shows that the estimation error distribution is close to the origin, namely, \( \mathbb{E}(f_k^a - \hat{f}_k^a, f_k^s - \hat{f}_k^s) = (-0.04, -0.01) \), \( \sigma(f_k^a - \hat{f}_k^a, f_k^s - \hat{f}_k^s) = (0.154, 0.056) \).

We have performed similar Monte Carlo simulation runs for the sensor fault estimation and the results are shown in Figure 3b. The average and standard deviation of the estimation errors over all the 500 simulations are \( \mathbb{E}(\hat{f}_k^a - f_k^a, \hat{f}_k^s - f_k^s) = (-0.006, -0.024) \) and \( \sigma(\hat{f}_k^a - f_k^a, \hat{f}_k^s - f_k^s) = (0.048, 0.051) \), which favorably suggests that our proposed scheme provides an almost unbiased estimation performance even if the estimated Markov parameters are erroneous. Table B.1 of Appendix B shows the average estimation errors corresponding to the first five Markov parameters associated with the Monte Carlo simulations.

Finally, in order to perform a comparative study, we consider the example that was provided in [7] and evaluate our results with those in this work. The system in [7] is continuous-time and represents a linearized model of a vertical take-off and landing (VTOL) aircraft that
The discrete-time model associated with the system (53) is obtained through a sampling rate of 0.5 seconds and was stabilized by invoking the following control law,

\[
\begin{align*}
\dot{x}(t) &= 
\begin{bmatrix}
-0.036 & 0.027 & 0.018 & -0.455 \\
0.048 & -1.01 & 0.002 & -4.020 \\
0.100 & 0.368 & -0.707 & 1.42 \\
0 & 0 & 1 & 0
\end{bmatrix} x(t) \\
&+ 
\begin{bmatrix}
0.44 & 0.17 \\
3.54 & -7.59 \\
-5.52 & 4.49 \\
0 & 0
\end{bmatrix} (u(t) + f^a(t)) \\
y(t) &= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1
\end{bmatrix} x(t) + f^s(t)
\end{align*}
\]

where \( f^s(t) \in \mathbb{R}^4 \), \( f^a(t) \in \mathbb{R}^2 \) and \( f^a(t) \) and \( f^s(t) \) represent the actuator faults and sensor bias faults, respectively. The discrete-time model associated with the system (53) is obtained through a sampling rate of 0.5 seconds and was stabilized by invoking the following control law,

\[
u(k) = 
\begin{bmatrix}
0 & 0 & -0.5 & 0 \\
0 & 0 & -0.1 & -0.1
\end{bmatrix} y(k) + \xi(k)
\]

where \( \xi(k) \) denotes the reference signal. The process and measurement noise are white having zero mean and covariances \( Q = 0.16I \) and \( R = 0.64I \), respectively. The injected fault signals to the actuators and sensors are given by,

\[
f^a(k) = \begin{cases} 
\begin{bmatrix} 0 \\ 0 \\ \sin(0.1\pi k) \end{bmatrix}^T & 0 \leq k \leq 50 \\
\begin{bmatrix} 0 & 0 & 0 \\ \sin(0.1\pi k) & 1 \end{bmatrix}^T & k > 50
\end{cases}
\]

\[
f^s(k) = \begin{cases} 
\begin{bmatrix} 0 \\ 0 \end{bmatrix}^T & 0 \leq k \leq 50 \\
\begin{bmatrix} \sin(0.1\pi k) & 1 & 0 \end{bmatrix}^T & k > 50
\end{cases}
\]

Figures 4a and 4b show the Monte Carlo simulation results for estimation of the actuator and sensor faults,
respectively. The average estimation errors are given by $\mathbb{E}(\hat{f}_1^1 - f_1^1, \hat{f}_2^1 - f_2^1) = (-0.09, -0.07)$ and $\mathbb{E}(\hat{f}_1^2, \hat{f}_2^2 - f_2^2) = (-0.06, 0.07)$. The standard deviations are given by $\sigma(\hat{f}_1^1 - f_1^1, \hat{f}_2^1 - f_2^1) = (0.109, 0.201)$ and $\sigma(\hat{f}_1^2 - f_1^2, \hat{f}_2^2 - f_2^2) = (3.153, 1.146)$.

The authors in [7] have investigated the performance of several algorithms that they proposed for this problem and concluded that the one with an online optimization procedure (based on an online solution of a mixed-norm problem using online I/O data) yielded their best performance in terms of the estimation errors. When we compare our results with the distribution plots of the fault estimation errors provided in [7] it can readily be observed and concluded that our proposed scheme match the performance of the most accurate algorithm that is proposed in [7]. However, our proposed methodology is superior to that developed in [7] in two important aspects, that are as follows: (I) First, our methodology directly yields the design solution to the residual generator filter in the state-space structure for performing the fault estimation objectives, whereas the proposed algorithm in [7] is in fact an FIR-filter that still involves a reduction step if the state-space structure of the filter is required, and (II) Second, the algorithm in [7] has a significantly higher computational complexity as compared to our proposed approach and indeed increases the computational burden to the user to the point where the average computational time per sample takes 1.70 seconds on a 3.4 GHz computer having 8 GB of RAM. On the other hand, the computational time of our proposed methodology per sample using the same computer takes only $4.9 \times 10^{-7}$ seconds, which represents a significant savings in the computational resources and constraints as far as any practical implementations are concerned.

9 Conclusion

We have proposed fault detection, isolation and estimation schemes that are all directly constructed in the state-space structure by utilizing only the system I/O data. We have shown that to design and develop our schemes it is only sufficient to estimate the system Markov parameters. Consequently, the reduction step that is commonly used in the literature and that also introduces nonlinear errors and requires an a priori knowledge of the system order is completely eliminated. We have shown that the performance of the estimation scheme is linearly dependent on the Markov parameters estimation process errors. Comparisons of our proposed schemes with those available in the literature have revealed that our methodology is mathematically simpler to develop and computationally more efficient while it maintains the same level of performance and requires a lower set of assumptions. Further research are required to investigate the robustness of our scheme to estimation errors and presence of concurrent sensors and actuators faults.

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In order to estimate the system Markov parameters, we neglect the noise terms and rearrange equation (9) as follows. Let us define \( \mathbf{V}_D = \begin{bmatrix} H^T_{t+s-1} & H^T_{t+s-2} & \cdots & H^T_0 \end{bmatrix}^T \), which is in fact a matrix of unknown Markov parameters of the system. Equation (9) is a system of linear equations that can be rearranged based on the definition of \( \mathbf{V}_D \) into the following format (by neglecting disturbances and noise),

\[
\mathbf{V}_Y = \mathbf{V}_U \mathbf{V}_D
\]  

(A.1)

Therefore, an estimate of the Markov parameters is now given by,

\[
\hat{\mathbf{V}}_D = \mathbf{V}_U^T \mathbf{V}_Y
\]  

(A.4)

Consequently, one can construct \( \hat{\mathbf{D}}_{si} \) (or any other matrix that is constructed from the system Markov parameters such as \( \mathbf{D}_s \) or \( \mathbf{D}_{t+} \)) from the elements of \( \hat{\mathbf{V}}_D = \begin{bmatrix} \hat{H}^T_{t+s-1} & \hat{H}^T_{t+s-2} & \cdots & \hat{H}^T_0 \end{bmatrix}^T \).

### B Markov parameters estimation errors for system (52)

The following Table shows the average estimation errors for the first 5 Markov parameters of the system, as an illustration, corresponding to the Monte Carlo simulation runs shown in Figure 3.

| Markov Parameter | \( \mathbb{E}\{H_i - \hat{H}_i\} \) |
|------------------|----------------------------------|
| \( H_0 \)       | \[0.259, 0.2834\]                 |
|                 | \[0.06, 0\]                       |
| \( H_1 \)       | \[-0.135, -0.47\]                 |
|                 | \[-0.15, -0.012\]                 |
| \( H_2 \)       | \[-0.20, -0.26\]                 |
|                 | \[0.08, 0.01\]                    |
| \( H_3 \)       | \[0.18, 0.15\]                   |
|                 | \[0.11, 0\]                      |
| \( H_4 \)       | \[0.69, -0.06\]                  |
|                 | \[0.69, 0.85\]                   |

Table B.1 Markov parameters estimation errors associated with the first five Markov parameters of the system (52)