Entanglement formation and violation of Bell’s inequality with a semiconductor single photon source.

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(Dated: October 28, 2018)

We report the generation of polarization-entangled photons, using a quantum dot single photon source, linear optics and photodetectors. Two photons created independently are observed to violate Bell’s inequality. The density matrix describing the polarization state of the postselected photon pairs is also reconstructed, and agrees well with a simple model predicting the quality of entanglement from the known parameters of the single photon source. Our scheme provides a method to generate no more than one entangled photon pair per cycle, a feature useful to enhance quantum cryptography protocols using entangled photons.

Entanglement, the counter-intuitive non-local correlations allowed by quantum mechanics between distinct systems, has recently drawn much attention due to its applications to the manipulation of quantum information [1]. These non-local correlations are often understood as the result of prior interactions between the quantum mechanical systems of interest. Following this idea, and as often quoted, entanglement would represent the memory of those interactions. But as suggested by the Innsbruck teleportation experiment [2], this is too limited a view. Entanglement can be induced between completely independent particles, due to the lack of which-path information, or in other words to the quantum indistinguishability of two identical particles. Pioneering work by Shih and Alley [3], followed by Ou and Mandel [4], already used the quantum indistinguishability to induce entanglement between two photons produced in a nonlinear crystal. More recently, entanglement swapping experiments [5, 6] used two independent entangled photon pairs to induce entanglement between photons of different pairs which never interacted. In this paper, we use a similar linear optics technique to induce polarization entanglement between single photons emitted independently in a semiconductor quantum dot source, at 2 ns time interval. We observed a clear violation of Bell’s inequality, which constitutes an experimental proof of non-local behavior for the first time with a semiconductor single photon source. The complete density matrix describing the polarization state of the two photons was also reconstructed, and satisfies the Peres criterion for entanglement [7]. We show that our results can be quantitatively explained in terms of basic parameters of the single photon source and derive a simple criterion for entanglement generation using those parameters. Eventually, we explain why our technique can be applied to quantum key distribution (QKD) in a straightforward and useful manner.

This experiment relies on two crucial features of our quantum dot single photon source, namely its ability to suppress multi-photon pulses [8], and its ability to generate consecutively two photons that are quantum mechanically indistinguishable [9]. The idea is to "collide" these photons with orthogonal polarizations at two conjugated input ports of a non-polarizing beam splitter (NPBS). A quantum interference effect ensures that photons simultaneously detected at different output ports of the NPBS should be entangled in polarization [4]. More precisely, when the two optical modes corresponding to the output ports ‘c’ and ‘d’ of the NPBS have a simultaneous single occupation, their joint polarization state is expected to be the EPR-Bell state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_c |V\rangle_d - |V\rangle_c |H\rangle_d)$$

Denoting ‘a’ and ‘b’ the input port modes of the NPBS, they are related to the output modes ‘c’ and ‘d’ by the 50-50% NPBS unitary matrix according to:

$$a_{H/V} = \frac{1}{\sqrt{2}}(c_{H/V} + d_{H/V})$$

$$b_{H/V} = \frac{1}{\sqrt{2}}(c_{H/V} - d_{H/V})$$

where subscripts ‘H’ and ‘V’ specify the polarization (horizontal or vertical) of a given spatial mode. The quantum state corresponding to single-mode photons with orthogonal polarizations at port ‘a’ and ‘b’ can be written as:

$$a_{H/V}^\dagger |\text{vac}\rangle = \frac{1}{2}(c_{H}^\dagger c_{V}^\dagger - d_{H}^\dagger d_{V}^\dagger - c_{H}^\dagger d_{V}^\dagger + c_{V}^\dagger d_{H}^\dagger) |\text{vac}\rangle$$

It is interesting to note that this state already features non-local correlations and violates Bell’s inequality, as pointed out in [10]. In this sense the post-selection is not an essential part of entanglement formation from two identical quantum particles. The experimental violation of Bell’s inequality with the global state would however require the discrimination between one-photon and two-photon pulses. Instead, if we discard the events when two photons occupy the same spatial mode ‘c’ or ‘d’ at the output (which is done naturally by detecting coincidence...
counts between 'c' and 'd'), we obtain the postselected state:

$$\frac{1}{\sqrt{2}}(c_H^d d_V^t - c_V^t d_H^t)|\text{vac}\rangle = |\Psi^-\rangle$$

as claimed. The post-selection can be done with regular single photon counter modules. Note that the generation of polarization entangled states via two-photon cascade emission [11] and parametric down converter [12] also rely upon a photon number post-selection.

The experimental setup is shown in fig 1. The single photon source consists of a self-assembled InAs quantum dot (QD) embedded in a GaAs/AlAs DBR microcavity [5]. It was placed in a Helium flow cryostat and cooled down to 4-10 K. The temperature was adjusted to tune the QD emission wavelength to cavity resonance. This increases the source brightness and reduces the effects of dephasing by increasing the radiative decay rate [13, 14]. Single photon emission was triggered by optical excitation of a single QD, isolated in a micropillar. We used 3 ps Ti:Sa laser pulses on resonance with an excited state of the QD, insuring fast creation of an electron-hole pair directly inside the QD. Pulses came by pairs separated by 2 ns, with a repetition rate of 1 pair/13 ns. The emitted photons were collected by a single mode fiber and sent to a Mach-Zender type setup with 2 ns delay on the longer arm. A quarter wave plate (QWP) followed by a half wave plate (HWP) were used to set the polarization of the photons after the input fiber to linear and horizontal. An extra half wave plate was inserted in the longer arm of the interferometer to rotate the polarization to vertical. One time out of four, the first emitted photon takes the long path while the second photon takes the short path, in which case their wavefunctions overlap at the second non-polarizing beam-splitter (NPBS 2). In all other cases, the single photon pulses "miss" each other by at least 2 ns which is greater than their width (100 - 200 ps). Two single photon counter modules (SPCMs) in a start-stop configuration were used to record coincidence counts between the two output ports of NPBS 2, effectively implementing the post-selection (if photons exit NPBS 2 by the same port, then no coincidence are recorded by the detectors). Single-mode fibers were used prior to detection to facilitate the spatial mode-matching requirements (they actually define the output modes). They were preceded by quarter wave and polarizer plates to allow the analysis of all possible polarizations.

The detectors were linked to a time-to-amplitude converter, which allowed to record histograms of coincidence events versus detection time delay $\tau$. A typical histogram is shown on fig 2. The integration time was two minutes. The number of coincidences for overlapping photons was measured as the area of the peak contained in the domain $-1\text{ns} < \tau < 1\text{ns}$. For given analyzer angle settings $(\alpha, \beta)$, we denote by $C(\alpha, \beta)$ this number normalized by the total number of coincidences in a time window of 100 ns. This normalization is independent of $(\alpha, \beta)$ since the input of NPBS 2 are two modes with orthogonal polarizations. $C(\alpha, \beta)$ measures the average rate of coincidences throughout the time of integration.

Two different QD microcavity devices were used to produce single photons. Both of them featured a high suppression of two-photon pulses and high overlap (indistinguishability) between consecutive photons. The overlap was measured by the Mandel dip [9], which was estimated by removing the HWP in the long arm, thus colliding completely identical particle at NPBS 2. A Mandel dip of zero means perfect indistinguishability between consecutively emitted single photons. Interestingly, the by-product of a Mandel test should be a photon-number entangled state $|0,2\rangle - |2,0\rangle$. However, the coincidence measurements alone presented in ref [9] do not rule out the possibility of a decohered mixture $|0,2\rangle|0,2\rangle + |2,0\rangle|2,0\rangle$. This decoherence issue will be fully addressed in the present work.

A Bell’s inequality test was performed for post-selected photon pairs from QD$_1$. Following ref [15], if we define

\[ \text{FIG. 1: Experimental setup. Single photons from the QD microcavity device are sent through a single mode fiber, and have their polarization rotated to H. They are split by a first NPBS. The polarization is changed to V in the longer arm of the Mach-Zender configuration. The two path of the interferometer merge at a second NPBS. The output modes are matched to single mode fibers for subsequent detection. The detectors are linked to a time-to-amplitude converter for a record of coincidence counts.} \]
Bell’s inequality is still violated by the short integration time used to insure high stability. The statistical error on the QD device. Two standard deviations, according to

The central window corresponds to photons that overlapped at the NPBS and took different output ports, i.e. the post-selected events.

The correlation function \( E(\alpha, \beta) \) for polarizer angle settings \( \alpha \) and \( \beta \) as:

\[
E(\alpha, \beta) = \frac{C(\alpha, \beta) + C(\alpha^\perp, \beta^\perp) - C(\alpha^\perp, \beta) - C(\alpha, \beta^\perp)}{C(\alpha, \beta) + C(\alpha^\perp, \beta^\perp) + C(\alpha^\perp, \beta) + C(\alpha, \beta^\perp)}
\]

then local realistic assumptions lead to the inequality:

\[
S = |E(\alpha, \beta) - E(\alpha, \beta)| + |E(\alpha, \beta) + E(\alpha, \beta)| \leq 2
\]

that can be violated by quantum mechanics.

Sixteen measurements were performed for all combination of polarizer settings among \( \alpha \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\} \) and \( \beta \in \{22.5^\circ, 67.5^\circ, 112.5^\circ, 157.5^\circ\} \). The corresponding values of normalized coincidence counts \( C(\alpha, \beta) \) are reported in table I up to an overall multiplicative constant. The statistical error on \( S \) is quite large, due to the short integration time used to insure high stability of the QD device. Bell’s inequality is still violated by two standard deviations, according to \( S \approx 2.38 \pm 0.18 \). This result constitutes the first observation of non-local correlations created between two single independent photons by linear-optics and photon number post-selection. Entanglement was created from a completely separable photon pair state.

To understand the degree of entanglement in detail, we reconstructed the postselected two-photon state, for comparison with a simple model. The two-photon polarization state can be completely characterized by a reduced density matrix, where only the polarization degrees of freedom are kept. This density matrix can be reconstructed from a set of 16 measurements with different polarizer settings, including circular. We performed this analysis, know as quantum state tomography, on photon pairs emitted by \( QD_2 \). The reconstructed density matrix is shown on fig 3. It can be shown to be non separable, i.e. entangled, using the Peres criterion (negativity \( \approx 0.43 \), where a value of 1 means maximum entanglement).

We next try to account for the observed degree of entanglement from the parameters of the QD single photon source. Due to residual two-photon pulses, giving a non-zero value to the equal time second-order correlation function \( g^{(2)}(0) \), a recorded coincidence count can originate from two photons of same polarization that would have entered NPBS 2 from the same port. A multi-mode analysis also reveals that an imperfect overlap \( V = |\int \psi_1(t) \psi_2(t)|^2 \) between consecutive photon wavefunctions washes out the quantum interference responsible for the entanglement generation. Including those imperfections, we could derive a simple model for the joint polarization state of the postselected photons. In the limit of low pump level, this model predicts the following density matrix in the \( (H/V) \otimes (H/V) \) basis:

\[
\rho_{\text{model}} = \frac{1}{4} \left( \begin{array}{cc} 2g^{(2)} & \frac{R}{T} - V \\ \frac{R}{T} - V & 2g^{(2)} \end{array} \right)
\]

R and T are the reflection and transmission coefficients of NPBS 2 (\( \frac{R}{T} \approx 1.1 \) in our case). Using the values for \( g^{(2)} \) and \( V \) measured independently, we obtain an excellent quantitative agreement of our model to the experimental data, with a fidelity \( Tr\left(\sqrt{\rho_{\text{Exp}}} \rho_{\text{model}} \sqrt{\rho_{\text{Exp}}}\right) \) as high as 0.997. The negativity of the state \( \rho_{\text{model}} \) is proportional to \( (V - 2g^{(2)}) \), which means that entanglement exists as long as \( V > 2g^{(2)} \). This simple criterion indicates whether a given single photon source will be able to generate entangled photons in such a scheme.
We now study the possible improvements and applications of our entanglement generation. If an optical switch is used to direct each photon on its proper path, our scheme will ideally succeed half of the time. Moreover, post-selection implies that the photons are destroyed when our scheme succeeds. This is a serious obstacle for some applications to quantum information systems, but not all. Indeed, the Ekert91 [17] or BBM92 [18] QKD protocols using entangled photons can directly be performed with our technique. The essence of these protocols is to establish a secure key upon local measurement of two distant photons from an entangled pair, which is exactly similar to our scheme. The bit error induced by uncorrelated photon pairs in those protocols is significantly suppressed [19] when single entangled pairs are used, a feature which only our source possesses among the currently demonstrated entangled photon sources. Therefore, those QKD protocols should actually benefit from our method to generate entanglement. When the scheme fails, one party, say Alice, does not receive any photon, so that Bob will discard his result. The intermittent failure of the scheme will effectively halve the communication rate, without compromising the secrecy of the key.

In summary, we demonstrated the violation of Bell’s inequality for the first time with a semiconductor single photon source. Polarization entanglement was induced between two independent but indistinguishable single photons, with linear-optics only. Our technique naturally produces no more than one entangled pair per cycle, which is a unique feature among previously demonstrated entangled photon sources. Our scheme can be straightforwardly applied to Ekert91/BBM92 QKD, and should perform better than current entangled photon sources for that purpose.

FIG. 3: Reconstructed polarization density matrix for the post-selected photon pairs emitted by QD2. The small diagonal HH and VV components are caused by finite two-photon pulses suppression ($g^{(2)} > 0$). Additional reduction of the off-diagonal elements originates from the imperfect distinguishability between consecutively emitted photons.

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