\[ D^+ \rightarrow K^- \pi^+ \pi^+ : \text{the low-energy sector} \]

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Abstract. An effective SU(3) × SU(3) chiral lagrangian, which includes scalar resonances, is used to describe the process \( D^+ \rightarrow K^- \pi^+ \pi^+ \) at low-energies. Our main result is a set of five S-wave amplitudes, suited to be used in analyses of production data.

Keywords: heavy meson, pion, kaon, chiral symmetry.

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INTRODUCTION

The pioneering E791 experiment[1] has shown that heavy-meson decays are reliable sources of information about scalar resonances. Data about these decays are encoded into Dalitz plots and have to be interpreted with the help of theoretical ansätze which, normally, rely on Breit-Wigner expressions. In the framework of field theory, one knows that expressions of this kind arise naturally when two-body interaction kernels are unitarized. However, usual Breit-Wigner functions are problematic, since they are based on kernels which do not describe well the meson-meson amplitude close to threshold. We propose to cure this problem by means of generalized Breit-Wigner expressions, based on interaction kernels which comply with low-energy chiral theorems. These alternative trial functions can be used as tools in analyses of the decay \( D^+ \rightarrow K^- \pi^+ \pi^+ \).

Our theoretical description is based on an effective SU(3) × SU(3) chiral lagrangian, which includes scalar resonances and also allows a consistent treatment of the primary weak vertex. Final expressions represent a compromise between reliability and simplicity, so that they could be employed directly in data analyses.

RESULTS

The decay \( D^+(P) \rightarrow \pi^+(q)\pi^+(q')K^-(k) \) involves both the weak quark transition \( c \rightarrow sW^+ \) and strong processes associated with final state interactions. We only consider hadronic degrees of freedom and need the following weak vertices: \( (D \rightarrow \pi KW)_{a} \), \( (D \rightarrow \bar{\kappa}W)_{a} \), \( (W \rightarrow \pi)_{a} \), \( (D \rightarrow KW)_{v} \), and \( (W \rightarrow \pi\pi)_{v} \), where \( \bar{\kappa} \) is the \( \kappa \)-resonance and the labels \( \nu \) and \( a \) refer to vector and axial currents involved in \( W \)-couplings. Strong interactions determine most structures observed in Dalitz plots and, in particular, the widths of resonances. The final state one is considering contains three mesons and a full treatment of their interactions is impossible. Therefore we work in the approximation in which one of the final mesons acts as a spectator. As the emerging
pions have isospin 2, their interactions can be neglected. Strong processes are then restricted to the $\pi K$ subsystem and described using the leading chiral $SU(3) \times SU(3)$ lagrangian given by Gasser and Leutwyler\cite{2}, complemented with resonance couplings from the work of Ecker, Gasser, Pich and De Rafael\cite{3}.

\[
\begin{align*}
\text{FIGURE 1.} & \quad \text{(Color online) Diagrams contributing to the decay } D^+ \rightarrow K^- \pi^+ \pi^+; \quad \text{(a) corresponds to a direct process, (b-e) involve the } \pi K \text{ scattering amplitude } T, \quad \text{and (f) depends on the production amplitude } \Pi. \\
\text{Our model is defined by fig.1. Diagram (a) represents a non-resonant background,} \\
\text{since the outgoing mesons reach the detectors without interacting. The other diagrams} \\
\text{involve the strong amplitudes } T \text{ and } \Pi. \text{ The former describes elastic scattering and is} \\
\text{unitary. The latter implements the propagation and decay of the } \kappa \text{ produced directly at} \\
\text{the weak vertex\cite{4} and has the dynamical structure shown in fig.2.}
\end{align*}
\]

\[
\text{FIGURE 2.} & \quad \text{(Color online) Resonance propagation (full line) and decay into } \pi^+ K^- \text{ (dashed lines); } T \text{ is} \\
\text{the unitary } I = 1/2 \text{ scattering amplitude.}
\]

Individual contributions from diagrams of fig.1 to the $D^+$ decay width read

\[
\delta a(\mu_{K\pi}^2, \mu_{K\pi'}^2) = \frac{1}{6\sqrt{2}} \left[ \delta a \right] G_F \cos^2 \theta_C \left[ (M_D^2 + 2M_{\pi}^2 + M_K^2 - \mu_{K\pi}^2) \right]
\]
\[
\mathcal{A}_b(\mu_{K\pi}^2) = -\frac{1}{9\sqrt{2}} [\delta_a] G_F \cos^2 \theta_C \left[ (P \cdot q + M_\pi^2) - \frac{(P \cdot q - M_\pi^2)(M_K^2 - M_\pi^2)}{M_D^2 + M_\pi^2 - 2P \cdot q} \right] \\
\times \left[ 2 \tilde{\Omega}_{1/2} T_{1/2}(\mu_{K\pi}^2) + \tilde{\Omega}_{3/2} T_{3/2}(\mu_{K\pi}^2) \right],
\]
\[
\mathcal{A}_{c+d}(\mu_{K\pi}^2) = -\frac{1}{3\sqrt{2}} [\delta_a] G_F \cos^2 \theta_C \left[ (P \cdot q' + M_\pi^2) - \frac{(P \cdot q' - M_\pi^2)(M_K^2 - M_\pi^2)}{M_D^2 + M_\pi^2 - 2P \cdot q'} \right] \\
\times \tilde{\Omega}_{1/2} T_{1/2}(\mu_{K\pi}^2),
\]
\[
\mathcal{A}_e(\mu_{K\pi}^2) = -\frac{\sqrt{2}}{3} [\delta_b] G_F \cos^2 \theta_C \left[ (M_D^2 - P \cdot q') - \frac{(M_D^2 - q K \pi')(M_K^2 - M_\pi^2)}{M_D^2 + M_\pi^2 - 2P \cdot q'} \right] \\
\times \left[ \tilde{\Omega}_{1/2} T_{1/2}(\mu_{K\pi}^2) - \tilde{\Omega}_{3/2} T_{3/2}(\mu_{K\pi}^2) \right],
\]
\[
\mathcal{A}_f(\mu_{K\pi}^2) = -4\sqrt{3} [\delta_c] G_F \cos^2 \theta_C \frac{P \cdot q'}{\mu_{K\pi}^2 - m_K^2} \left[ c_d/F^2 \right] \\
\times \left[ c_d (\mu_{K\pi}^2 - M_\pi^2 - M_K^2) + c_m (4 M_K^2 + 5 M_\pi^2)/6 \right] \left[ 1 - \tilde{\Omega}_{1/2} T_{1/2}(\mu_{K\pi}^2) \right],
\]

where \( G_F \) and \( \theta_C \) are the Fermi coupling constant and the Cabibbo angle, \( F \) is the pseudoscalar decay constant, \( c_d \) and \( c_m \) describe two possible \( \kappa \pi K \) couplings, and the [\( \delta_i \)] implement \( SU(4) \) symmetry breaking at the weak vertices. One has two possible \( \pi K \) subsystems and their invariant masses are \( \mu_{K\pi}^2 = (q + k)^2 \) and \( \mu_{K\pi'}^2 = (q' + k)^2 \). In the construction of Dalitz plots, one first evaluates \( \mathcal{A} = [\mathcal{A}_a + \cdots + \mathcal{A}_f] \) and then symmetrize with respect to \( \mu_{K\pi}^2 \leftrightarrow \mu_{K\pi'}^2 \). Allowed values for these invariant masses lie in the interval 0.40 GeV\(^2\) \( \leq \mu_{K\pi}^2, \mu_{K\pi'}^2 \leq 2.99 \) GeV\(^2\).

The functions \( \tilde{\Omega} \) and \( T \) are important substructures in our results and describe respectively two-meson propagation and \( \pi K \) elastic scattering. The labels 1/2 and 3/2 refer to \( \pi K \) isospin channels and we recall that only the former can couple with a s-channel \( \kappa \). The main features of these functions are summarized below.

**SCATTERING AMPLITUDE**

\[
T = \begin{array}{c}
|K| + \end{array} T K
\]

\[
T = \begin{array}{c}
|K| + \end{array} K K + \begin{array}{c}
|K| + \end{array} K K + \begin{array}{c}
|K| + \end{array} K K + \cdots
\]

**FIGURE 3.** (Color online) Bethe-Salpeter equation for the elastic \( \pi K \) amplitude: full equation (top) and perturbative solution (bottom).
The elastic $\pi K$ scattering amplitudes $T_I$ are derived by means of the Bethe-Salpeter equation, represented in figs.3, together with its perturbative solution. This geometrical series involves the two-body irreducible kernels $\mathcal{K}_I$ and the regularized two-meson propagators $\tilde{\Omega}_I$. As pointed out by Oller and Oset[5], at low-energies, the Bethe-Salpeter equation can be solved exactly, and one has

$$T_I = \frac{\mathcal{K}_I}{1 + \Omega_I \mathcal{K}_I}.$$ (6)

Propagators are complex above threshold and given by $\tilde{\Omega}_I = \tilde{R}_I + i I$. The scattering amplitudes are unitary and can be rewritten as

$$T_I = \frac{16\pi}{\rho} \sin \delta_I e^{i\delta_I} \leftrightarrow \tan \delta_I = -\frac{I \mathcal{K}_I}{1 + \tilde{R}_I \mathcal{K}_I}.\quad (7)$$

where $\delta_I$ are real phase shifts and $\rho = \sqrt{1 - 2(M_{\pi K}^2 + M_K^2)/s + (M_{\pi K}^2 - M_{\pi}^2)^2/s^2}$. Our unitarization of $T_I$ generalizes the $K$-matrix approximation, which amounts to neglecting the real parts of $\tilde{\Omega}_I$.

**FIGURE 4.** (Color online) Tree-level $\pi K$ amplitude; dashed and full lines represent respectively pseudoscalar mesons and scalar resonances.

The dynamical content of the scattering amplitudes is incorporated into the kernels $\mathcal{K}_I$, which are obtained by projecting out the $S$-wave content from the diagrams of fig.4. Contributions from $t$ and $u$ channels are very small and can be neglected. Our chiral lagrangians then yield

$$\mathcal{K}_{1/2} = \frac{1}{4F^2} \left[ 4s - 3\rho/2 - 4(M_{\pi}^2 + M_K^2) \right]$$
$$- \frac{3}{4} \frac{1}{s - m_K^2} \frac{4}{F^2} \left[ c_d (s - M_{\pi}^2 - M_K^2) + c_m (4M_K^2 + 5M_{\pi}^2)/6 \right]^2, \quad (8)$$

$$\mathcal{K}_{3/2} = -\frac{1}{2F^2} \left[ s - (M_{\pi}^2 + M_K^2) \right]. \quad (9)$$

**TWO-MESON PROPAGATOR**

The propagator for a system with total momentum $X = q + k$, is

$$\Omega = i \int \frac{d^4\ell}{(2\pi)^2} \frac{1}{[(\ell + X/2)^2 - M_{\pi}^2] [(\ell - X/2)^2 - M_K^2]}.$$ (10)

and can be written as $\Omega = -(1/16\pi^2) [L(s) + \Lambda_\infty]$, where the function $L(s)$ is given below and $\Lambda_\infty$ is an infinite constant that has to be removed by renormalization. Above
threshold, one has

\[
L(s) = \rho \log \left[ \frac{1 - \sigma}{1 + \sigma} \right] - 2 + \frac{(M_K^2 - M_{\pi}^2)}{s} \log \frac{M_K}{M_{\pi}} + i\pi \rho ,
\]

\[
\sigma = \sqrt{|s - (M_K + M_{\pi})^2|/|s - (M_K - M_{\pi})^2|} .
\]  

(11)

The renormalized propagators are given by \( \tilde{\Omega}_I = -(1/16\pi^2)[L(s) + c_I] \) and the constants \( c_I \) depend on the isospin channel. They are chosen by tuning the predicted phase shifts, eq.(7), to experimental results at a given point \( s = s_I \). For the channel \( I = 1/2 \), the constant \( c_{1/2} \) is chosen at the point \( s_{1/2} = m_K^2 \), where the experimental phase is \( \pi/2 \). One writes

\[
\tilde{R}_{1/2}(s) = -\frac{1}{16\pi^2} \Re[L(s) - L(m_K^2)]
\]  

(12)

and, by construction, \( \tilde{R}_{1/2}(m_K^2) = 0 \). In the \( I = 3/2 \) channel, fit to data requires \( c_{3/2} \sim 14.5 \). One notes that the functions \( \tilde{\Omega}_I \) introduce other phases into the problem, given by \( \tan \omega_I(s) \equiv 1/\tilde{R}_I \).

**SUMMARY**

![Graph](image_url)

**FIGURE 5.** (Color online) Full (black) and partial contributions to the real (continuous line) and imaginary (dashed line) components of the amplitudes \( A_I \); the vertical scale has to be multiplied by the weak factor \( G_F \cos^2 \theta_C \).

The purpose of this work is to map the relevant degrees of freedom of the amplitude \( D^+ \rightarrow K^- \pi^+ \pi^+ \) at low energies. This is achieved in eqs.(1-5), which represent individual contributions from the diagrams shown in fig.1. Masses and coupling constants
in these expressions are kept free, so that their values can be extracted from experiment. However, in order to discuss qualitative features of our results, we need to fix somehow these free parameters. In this case, we choose: \( F = 0.093 \text{GeV}, m_\kappa = 1.2 \text{GeV}, \) 
\((c_d; c_m) = (3.2; 4.2) \times 10^{-2} \text{GeV}\) and \(\delta_a = \delta_b = \delta_c = 1\). We show, in fig.5, the real and imaginary components of the amplitudes \(A_i\) and note that the magnitudes of all contributions are comparable. Nevertheless, it is also possible to see that diagrams (a) and (f), which describe the non-resonant background and the direct production of the resonance at the weak vertex, are especially important. The latter is very sensitive to the resonance coupling constant \(c_d\) and we take this conclusion with reserve.

In our calculation, the non-resonating background is given by a single and very simple diagram. It does not contain loops and hence \(A_a\) is necessarily a real function. As the inclusion of loops is associated with higher orders in the chiral expansion, we conclude that, at low-energies, there is no phase associated with the background. It is also worth noting that \(A_a\) is linear in the invariant masses and its distribution over a Dalitz plot will not be uniform, as sometimes assumed.

Other diagrams are complex and the presence of imaginary components can be unambiguously traced back to loops in the propagators \(\tilde{\Delta}_I\). By means of eq.(6), they blend with the real kernels \(\mathcal{K}_I\) and give rise to unitary \(\pi K\) scattering amplitudes. Direct inspection of eqs.(2-4) shows that their phases depend on combinations of \(\delta_I\) and \(\omega_I\). In diagram (f), representing the direct production of the \(\kappa\)-resonance at the weak vertex, the phase is just \(\delta_{1/2}\), as discussed in ref.[4].

We assume our results for the \(A_i\) to be reliable up to values of \(\mu_{K\pi}^2\) just above the point \(s = m_\kappa^2\), corresponding to \(\delta_{1/2} = \pi/2\). In the case of fig.5, this means invariant masses between \(0.4 \text{GeV}^2\) and \(1.6 \text{GeV}^2\). Constraints imposed by chiral symmetry are very important at the lower end of this interval. Full details of these calculations will be presented elsewhere.

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REFERENCES

1. E.M. Aitala et al. (E791), Phys. Rev. Lett. 86 770 (2001).
2. J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, 142 (1984); Nucl. Phys. B250, 465 (1985).
3. G. Ecker, J. Gasser, A. Pich and E. De Rafael, Nucl. Phys. B321, 311 (1989).
4. D.R. Boito and M.R. Robilotta, Phys. Rev. D 76, 094011 (2007).
5. J.A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997).