Nonuniqueness and Structural Stability of Self-Consistent Models of Elliptical Galaxies

Christos Siopis
Department of Astronomy, University of Florida, P.O. Box 112055, Gainesville, FL 32611-2055

In recent years, high-resolution ground-based and HST observations have shown that the space density profiles $\rho(r)$ near the central regions of most elliptical galaxies exhibit power-law cusps, i.e. they scale as $\rho \sim r^{-\gamma}$ with $0 \leq \gamma \leq 2$. This finding contradicts the previously widely held view that ellipticals have extended central “cores” of near-constant density. Furthermore, evidence has accumulated over the past two decades that the constant-density surfaces of at least some ellipticals may actually be genuinely triaxial ellipsoids rather than spheroids. This has spurred some interest in the galactic community because theoretical and numerical work suggests that a central density cusp or mass concentration embedded in a generic nonaxisymmetric potential can lead to a significant expansion of the phase-space regions where motion is chaotic.

In view of this evidence, Merritt and Fridman (1996) used Schwarzschild’s (1979) method to construct self-consistent models of a triaxial generalization of Dehnen’s spherical potential, which contains a central density cusp. They found that self-consistency could be achieved only for models with a weak ($\gamma = 1$) cusp, and then only when the chaotic orbits populating the outermost regions of the model were allowed to be not fully mixed (i.e., they were not required to sample the invariant measure [Lichtenberg & Lieberman, 1992]). Based on this and later work, Merritt and collaborators have suggested that triaxiality may not in fact be compatible with central density cusps, and that, as a consequence, most elliptical galaxies may be (or may be evolving towards) axisymmetric configurations, at least in the central regions.

The construction of a self-consistent equilibrium $\rho(r)$ via Schwarzschild’s numerical method is effected by assigning appropriate weights to a large number of orbital templates, each evolved under the influence of the gravitational potential generated by $\rho(r)$, so that the weighted superposition of all the templates reproduces, at some level of coarse graining, the initial $\rho(r)$. This is usually implemented via some constrained optimization algorithm, such as linear or quadratic programming. Owing to its conceptual simplicity and relative ease of implementation, Schwarzschild’s method has been used quite extensively for the construction of stellar equilibria, especially when no known analytical solutions to the collisionless Boltzmann equation exist or they are difficult to compute.

In the present work (see also Siopis, 1999; Siopis & Kandrup, 1999) Schwarzschild’s method was first used to construct self-consistent models of a Plummer sphere and to calculate a number of velocity moments, which were then compared with known analytical solutions (Dejonghe, 1986) to assess the reliability of the numerical method. Subsequently, the method was applied to the construc-
tion of triaxial Dehnen models with weak ($\gamma = 1$) cusps. The principal moral derived from the extensive use of Schwarzschild’s method is that the importance of a good library of orbital templates cannot be overemphasized. The initial conditions must be selected carefully so as to provide a comprehensive coverage of phase space. This usually means that one requires a good understanding of the orbital structure of the system to be modeled. Failure to include enough orbits, or an injudicious choice of initial conditions that misses important families of orbits or violates the symmetries of the system, can lead to unphysical results. Furthermore, special care should be taken to ensure that each orbital template constitutes a truly time-independent building block. This means that orbits should be integrated until they uniformly cover their resonant tori (in the case of regular orbits) or until they uniformly sample the invariant measure (in the case of chaotic orbits).

Schwarzschild Plummer-sphere models were constructed both maximizing and minimizing the number of near-radial (low angular momentum) orbits. The resulting equilibria reproduce most of the structure that is present in the velocity distributions computed analytically (Dejonghe, 1986). Where agreement was less than satisfactory, the discrepancy could be traced to inadequacies in the library of orbits. Since velocities were not constrained explicitly in the construction of the models, these results suggest that Schwarzschild’s method can be used successfully to study the degeneracy of the solutions.

Self-consistent models of the triaxial Dehnen potential could not be constructed when the chaotic orbits at all energy levels were forced to be completely mixed so as to yield time-independent building-blocks: only the innermost 65% of the mass could be mixed. In these inner regions, it was possible to obtain alternative solutions that contain considerably different numbers of chaotic orbits, yet yield (at least approximately) the same mass density distribution. However, these solutions are not truly time-independent, since the unmixed chaotic orbits in the outer regions, which do not sample an invariant measure, will cause secular evolution.

Finally, some of the numerical equilibria were sampled to generate initial conditions for N-body simulations to test the stability of the models. Preliminary work showed that no catastrophic evolution takes place, but there is a weak tendency for the configuration to become more nearly axisymmetric over several dynamical times (Siopis et al., 1999). It is not yet clear whether this tendency is real or a numerical artifact.

References

Dejonghe, H. 1986. MNRAS, 224, 13
Lichtenberg, A.J. & Lieberman, M.A. 1992. Regular and Chaotic Motion, Springer-Verlag, New York
Merritt, D. & Fridman, T. 1996. ApJ, 460, 136
Schwarzschild, M. 1979. ApJ, 232, 236
Siopis, C. 1999. Ph.D. thesis, University of Florida
Siopis, C., Athanassoula, E. & Kandrup, H. E. 1999. In preparation
Siopis, C. & Kandrup, H. E. 1999. MNRAS, in preparation