Inclusive semileptonic $B$ decays: $|V_{cb}|$ and $|V_{ub}|$

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The present status of the measurement of the inclusive semileptonic $B$ decays is reviewed. In particular the determination of the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{cb}|$ and $|V_{ub}|$ is discussed and some future prospects are given.

1. Introduction

With the discovery of CP violation in the $B^0$ mesons, and the precise measurement of the angle $\beta$ [1] of the Unitarity Triangle (UT), the experimental effort focus on the measurements of other parameters to over-constrain the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. In particular the information on the side $R_b$, opposite to the angle $\beta$ is crucial to test the standard model prediction for CP violation. In the Wolfenstein parametrization $|R_b| = \lambda^{-1}(1 - \lambda^2/2)|V_{ub}/V_{cb}|$ where $\lambda = |V_{us}|/\sqrt{|V_{us}|^2 + |V_{ud}|^2} \approx 0.226$, so a precision measurement of $R_b$ require the measurement of $|V_{ub}|$ and $|V_{cb}|$ with high accuracy. Moreover the parameters $|V_{ub}|$ and $|V_{cb}|$ play a special role in the CKM matrix, because they can be extracted from pure tree level decays, so their values are to high accuracy, independent of any new physics contributions.

The semileptonic transitions $b \rightarrow c\ell\nu$ and $b \rightarrow u\ell\nu$ are characterized by an electroweak current that probes the $B$ dynamics allowing to determine the CKM matrix element $|V_{cb}|$ in a clear environment. Their simplicity is only apparent, because we are interested in precision measurements, the complexity specific of QCD dynamics have to be addressed and taken in consideration. Both inclusive and exclusive final states can be exploited to extract the CKM matrix elements. The experimental and theoretical techniques underlying these two approaches are different and provide crucial cross-check on our understanding of the QCD frameworks. At present the inclusive determinations for both $|V_{cb}|$ and $|V_{ub}|$ are more precise than the exclusive determinations. However improvements of the exclusive determinations, whose uncertainties are dominated by the lattice QCD calculations, are an important goal for the next future.

A comprehensive review of the semileptonic measurements is beyond the scope of this paper. We will present only the most recent measurements of both $|V_{ub}|$ and $|V_{cb}|$.

2. Inclusive $B \rightarrow X_c \ell\nu$ decays

The theoretical approaches used to extract both $|V_{cb}|$ and $|V_{ub}|$ from inclusive $B \rightarrow X\ell\nu$ decays use the fact that the $b$-quark mass is large compared to the $\Lambda_{QCD}$ scale, that characterize the low energy hadronic physics. This allows to use an effective-field-theory, the Heavy Quark Expansion (HQE), to separate the non-perturbative from perturbative contributions, and write a double expansion in powers of $\Lambda_{QCD}/m_b$ and $\alpha_s(\mu)$, with $\mu >> \Lambda_{QCD}$. In this framework the total semileptonic $B \rightarrow X_c\ell\nu$ decay width is given by

$$\Gamma_{sl} = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} (1 + A_{ew}) A^{pert} F(r, \frac{\mu^2}{m_b}, \frac{\mu^2_{\ell}}{m_b^2}, ...),$$

(1)

where $r = m_\ell/m_b$, $A_{ew}$ are the electroweak corrections, $A^{pert}$ summarize the QCD radiative corrections, and the term $F$ is written as an expansion in powers of $1/m_b$ and depends on non-perturbative parameters. At each order in $1/m_b$ new non-perturbative parameters come out. At present this expansion is computer till the $1/m_b^2$ term. A crucial feature of the $F$ expansion is that the first order term is zero, making the leading term in this expansion precise at the $\%$ level, and the higher order just small corrections to the leading term.

Expressions similar to Eq.1 can be computed for the moments of the lepton momentum $p_\ell$ spectra, and the moments of the squared hadronic mass spectra $m_X^2$ in $B \rightarrow X_c\ell\nu$ decays, together with the moments of the energy of the $\gamma$ emitted in the inclusive radiative $B \rightarrow X_\gamma$ decays. This allows to combine the measured $\Gamma_{sl}$ with the measurements of the moments of other kinematic variables to determine $|V_{cb}|$ with high precision, togheter with non-perturbative HQE parameters and $b$- and $c$-quark masses.

Measurements of hadronic mass distribution and leptonic spectrum have been made by many experiments CLEO [4], BABAR [5], Belle [6], DELPHI [7] and CDF [8] (the latter provides only the measuremant of the hadronic moments).

At the modern $B$-factories, the large samples of $Y(4S) \rightarrow BB$, can be exploited to provide a clean sample of tagged $B$ meson, by reconstructing one of the $B$ meson ($B_{\text{reco}}$) via fully hadronic modes, $B_{\text{reco}} \rightarrow D^{(*)}Y^{\pm}$, where $Y^{\pm}$ corresponds to a combination of $\pi^{0/+}$’s and $K^{0/+}$’s. The kinematic consistency of the $B_{\text{reco}}$ with a $B$ meson is tested using the variables $m_{ES} = \sqrt{s}/4 - p_B^2$ and $\Delta E = E_B - \sqrt{s}/2$, where $\sqrt{s}$ is the center of mass energy, and $E_B$ and $p_B$
are the energy and the momentum of the $B_{\text{reco}}$ candidate in the $\Upsilon(4S)$ frame. By fully reconstructing one of the $B$ mesons in the events, the charge, flavor and momentum of the second $B$ can be inferred. By knowing event by event the momentum of the signal $B$ meson, the kinematics of the decay product of the signal $B$ meson be computed in the center of mass of the $B$ meson, reducing the uncertainty due to the motion of the $B$ meson in the $\Upsilon(4S)$ frame.

Both BABAR and Belle have used the $B_{\text{reco}}$ sample for many studies on the $B$ semileptonic decays. The efficiency to reconstruct a $B_{\text{reco}}$ candidate is quite low, 0.3% for $B^0\overline{B}^0$ and 0.5% for $B^+B^-$ events, but the purity of the sample is higher than 80% allowing to reduce systematic uncertainties due to backgrounds knowledge.

BABAR recently presents a measurement of the moments of the hadronic mass ($m_X^k$), with $k = 1...6$, using a larger data-set (220-10^6 $B\overline{B}$) than the previous measurement (based on 88 - 10^6 $B\overline{B}$) [3]. Moreover, BABAR present for the first time the measurement of the mixed hadronic mass-energy moments $\langle n_X^k \rangle$, with $k = 2, 4, 6$, where $n_X$ is defined by $n_X^2 = m_X^2 - 2\Delta E_X + \Lambda^2$, $m_X$ and $E_X$ are respectively the mass and the energy of the $X_c$ system in the $B$ rest frame, and $\Lambda = 0.65$ GeV. The $n_X$ moments have been proposed in Ref. [10] due to their high sensitivity to higher order non-perturbative parameters.

The analysis uses the $B_{\text{reco}}$ sample, and proceed reconstructing a $B_{\text{reco}}$, and identifying a lepton in the event (both electron and muons are used). All particles, both neutral and charged, that are not used in the reconstruction of the $B_{\text{reco}}$ are assigned to the signal $B$. A kinematical fit imposes 4-momentum conservation, the equality of the masses of the two $B$ mesons, and constrains the mass of the neutrino, inferred from the missing momentum $P_\nu$, to zero with $P_\nu^2 = 0$. This allows to reconstruct with a good resolution the distribution of the mass of the hadronic system, $m_X$. An example of the hadronic mass spectra is shown in Fig.1. To correct for the effect of the lost particles, BABAR measures the hadronic moments using calibration curves obtained with the Monte Carlo that relate the measured moments to the true moments.

The main sources of systematics errors on the moments are due to the uncertainties related to the detector efficiency for tracks and photons, and to the $X_c$ signal modelling, mainly due to the poor knowledge of the $B \rightarrow D^{*+}\ell\nu$ composition, and particular care is also devoted to take out the radiative correction.

BABAR measures the hadronic and mixed moments as a function of the lower cut on the lepton momentum $p_{\ell, \text{min}}$ between $p_\ell > 0.8$ to $p_\ell > 1.9$ GeV/c in the $B$ rest frame. The results are compatible with the previous measurement [3], a comparison is shown in Fig.2 together with the results of the HQE fit described in the next section. The very high lepton momentum cuts, even if not used to constrain the HQE parameters, is important to test the Operator Product Expansion (OPE) in a region dominated only by the $D$ and the $D^*$ exclusive states.

### 2.1. Determination of HQE parameters and $|V_{cb}|$

BABAR performs a combined fit to the hadronic mass moments measurement presented above, the lepton energy moments in $B \rightarrow X_c\ell\nu$ decays [3] and photon energy moments in $B \rightarrow X_c\gamma$ decays [11], in the framework of the kinetic scheme [12]. The results (we do not report the higher order HQE parameters that are left free in the fit) are

$$|V_{cb}| \cdot 10^3 = 41.88 \pm 0.44_{\text{exp}} \pm 0.35_{\text{theo}} \pm 0.59_{\text{SL}}$$

$$m_b = (4.552 \pm 0.038_{\text{exp}} \pm 0.040_{\text{theo}}) \text{GeV}$$

$$\rho^2 = (0.471 \pm 0.034_{\text{exp}} \pm 0.062_{\text{theo}}) \text{GeV}^2,$$

where the additional uncertainty of 1.4% for the $|V_{cb}|$ is a normalization uncertainty due to non calculated terms in the total rate. The quality of the fit is very good, with a $\chi^2 = 8$ for 20 degree of freedom. In Fig.2 the fit results, using only the BABAR data, is superimposed to the measured moments. The moments $\langle m_X \rangle$ and $\langle m_X^2 \rangle$ are not included in the fit but provide an unbiased comparison with the fitted HQE prediction. The measured moments $\langle n_X^2 \rangle$ and $\langle (n_X^2 - \langle n_X \rangle^2 \rangle^2 \rangle$, also are not included in the fit, but are in good agreement with the prediction.

Recently Belle re-analyze the $B \rightarrow X_c\ell\nu$ data published in Ref.[13] and measures the first and the second moment of the photon energy spectrum with various cuts on the minimum energy threshold, $E_{\text{min}} = 1.9...2.3$ GeV. The new measured moments $\langle m_X \rangle$ are in agreement with the BABAR[11] and CLEO[15] measurements. Using these new moments, Belle performs a combined fit with the recent Bella data on the lepton energy and hadronic mass moments [3] in $B \rightarrow X_c\ell\nu$.
The fit has been performed in both the kinetic scheme and the 1S scheme [16]. The results are reported in Table I together with the result using BABAR only moments, and the results from the global HQE fit including all the moment measurements from BABAR, Belle, CDF, CLEO and DELPHI (see HFAG web page for more details [17]). It can be seen that including the photon energy moments reduces substantially the uncertainty on the photon energy moments reduces substantially the predictions that are available for $p_{\ell,\text{min}} < 1.5\text{ GeV/c}$.

Table I  Fitted values for $|V_{cb}|$, $m_b$ and the $\mu_x^2$ ($\lambda_1$) parameters for the new Belle and BABAR fits, described in the text. For comparison the results of the global fit provided by HFAG are also reported. BABAR reported results in the kinetic scheme, Belle reported results also in the 1S scheme.

| Scheme          | $|V_{cb}| \times 10^3$ | $m_b^{\text{kin}}$ | $\mu_x^2$ GeV$^2$ |
|-----------------|------------------------|--------------------|-------------------|
| Belle           | 41.52\(\pm\)0.90      | 4.543\(\pm\)0.075 | 0.539\(\pm\)0.079 |
| Belle only $X_\ell\nu$ | 41.46\(\pm\)0.99      | 4.573\(\pm\)0.134 | 0.523\(\pm\)0.106 |
| Babar           | 41.88\(\pm\)0.81      | 4.552\(\pm\)0.055 | 0.471\(\pm\)0.070 |
| Global          | 41.91\(\pm\)0.68      | 4.573\(\pm\)0.034 | 0.408\(\pm\)0.035 |
| Global only $X_\ell\nu$ | 41.68\(\pm\)0.70      | 4.677\(\pm\)0.053 | 0.387\(\pm\)0.039 |

| Scheme          | $|V_{cb}| \times 10^3$ | $m_b^{1S}$ | $\lambda_1$ GeV$^2$ |
|-----------------|------------------------|-----------|-------------------|
| Belle           | 41.56\(\pm\)0.68      | 4.723\(\pm\)0.055 | -0.303\(\pm\)0.046 |
| Belle only $X_\ell\nu$ | 41.55\(\pm\)0.80      | 4.718\(\pm\)0.119 | -0.308\(\pm\)0.092 |
| Global          | 41.78\(\pm\)0.31      | 4.701\(\pm\)0.030 | -0.313\(\pm\)0.025 |
| Global only $X_\ell\nu$ | 41.56\(\pm\)0.40      | 4.718\(\pm\)0.058 | -0.274\(\pm\)0.047 |

It should here be reported, that the exclusive determination of $|V_{cb}|$ using the recent $B \rightarrow D^*\ell\nu$ form factor computation [18], differs by the inclusive determination by more than 2$\sigma$. Further studies are needed on both theoretical and experimental side.
3. Inclusive $B \to X_u \ell \nu$ decays

As for the $B \to X_c \ell \nu$ decays, see Eq[1] the full rate for $B \to X_u \ell \nu$ decays is proportional to $|V_{ub}| \cdot m_b^5$, times a function that accounts for QCD and electroweak corrections. The theoretical framework also in this case, is the HQE, which predict the total $\Gamma(B \to X_u \ell \nu)$ decay rate with uncertainty of about 5% [20], dominated by the uncertainty on $m_b$.

In practice the measurable rate is strongly reduced, since the background from $B \to X_c \ell \nu$ decays (that dominates the signal by a factor 50) must be suppressed by requiring stringent kinematic cuts. The reduced accessible rate break the convergence of the HQE and this increase considerably the theoretical uncertainty. The cuts usually relies on the fact that the $u$-quark mass is much lighter than the $c$-quark, as a consequence the distribution of some kinematics variables differs between $b \to u$ and $b \to c$ transitions. For example the the lepton momentum $p_{\ell}$, extends to higher values for the signal, and analogously the distribution of the hadronic mass $m_X$ of the hadronic jet produced by the fragmentation of the quark $u$, extends toward lower values compared to the $m_X$ distribution for the quark $c$, that cannot be lower than the mass of the $D$ meson. It is so possible to select regions of the phase space where the signal over background is reasonable, but usually the acceptances tend to be small (from 6% requiring $p_{\ell}$ above the kinematic endpoint of the leptons from $B \to X_c \ell \nu$, to 30% requiring $m_X$ lower than the mass of the $D$ meson) and finite experimental resolution have to be accounted.

In the reduced phase space, the leading term of the non-perturbative correction to the expected rate, becomes of the order $\Lambda_{QCD}/m_b$ instead of $\Lambda^2_{QCD}/m_b^2$, and is described by the distribution function (called shape function, SF) of the momentum of the $b$ quark inside the $B$ meson. The SF cannot be computed perturbatively and must be determined experimentally. It is a function of $m_b$ and of other heavy quark parameters that describe the internal structure of the $B$ meson, which can be determined from the fit to the moments of the inclusive semileptonic $B \to X_c \ell \nu$ decays described above, and also directly from the $B \to X_c \gamma$ decays.

Because the uncertainty on the SF parameters (including also $m_b$) is one of the biggest source of uncertainty in the present $|V_{ub}|$ determinations, better understanding of the $B \to X_c \ell \nu$, $B \to X_c \gamma$ are important to reduce the uncertainty on $|V_{ub}|$. Additional uncertainty that require further theoretical studies are due to the sub-leading shape functions that affects in a different way the $b \to u$ transitions from the radiative or $b \to c$ transitions.

Moreover in the limited region of the $B \to X_u \ell \nu$ phase space used to extract the signal yields, further complications arise due to possible Weak Annihilation (WA) contributions, that affect differently $B^0$ from $B^+$ and other non-perturbative effects that contribute to the tail of the $B \to X_u \ell \nu$ phase space. Experimental constraints to the WA will be discussed in the Sec[3.3]

The measurement of the partial branching ratio $\Delta B(B \to X_u \ell \nu)$ can be translated into $|V_{ub}|$ by $|V_{ub}| = \sqrt{\Delta B/(\tau_B \cdot \Gamma_{th})}$, where $\tau_B$ is the $B$ average meson lifetime, and $\Gamma_{th}$ is the reduced decay rate defined as $\Gamma_{th} = \Delta \Gamma_{th}/|V_{ub}|^2$, where $\Delta \Gamma_{th}$ is the partial width into the phase space defined by the kinematic cuts, predicted by the theory. There are various calculations of the $\Delta \Gamma_{th}$ available, documented in Refs[21] (BLNP), [22] (GGOU), [23] (DGE), [24] (ADFR), and [25] (BLL).

BaBar recently published [26], using a sample of 382$M$ of $B\overline{B}$ mesons, analyses on various kinematic variables, studying semileptonic decays with $p_{\ell} > 1$ GeV, recoiling against a fully reconstructed $B$ mesons. The measured partial branching ratio have been obtained with cuts on the hadronic mass $m_X$ of the hadronic system $X_u$, the hadronic light-cone momentum $P_a = E_X - |p_{\ell}|$, and a combination of a cut on $m_X$ with a cut on the the invariant mass of the leptonic system $q^2 = (p_{\ell} + p_{\nu})^2$. In Fig[3] as an example, is shown the distribution of the measured $m_X$ variables. Within the signal region used to extract the partial branching ratio, $m_X < 1.55$ GeV, an yield of $803 \pm 60 X u \ell \nu$ events has been extracted. The experimental systematics have large contributions from the modeling of the signal and $B \to X_c \ell \nu$ decays, and also from the detector efficiency to charged, photons and neutral hadrons. The BABAR results are reported in Table II together with the analogous partial branching ratio measured by Belle using a similar technique [28]. The BABAR and Belle partial branching ratios, are in good agreements. But taking into account the large correlations between the analysis with the various kinematics cuts, both BABAR and Belle observe a discrepancy at the level of $\approx 2.0-2.5\sigma$ between the ratio of the measured $P_+ < 0.66$ GeV partial rate and the measured $m_X - q^2$ partial rate, compared with the corresponding ratio predicted by various theoretical

| cut | BABAR [26] | Belle [28] |
|-----|------------|------------|
| $M_\ell < 1.55$ GeV | $1.18 \pm 0.09 \pm 0.08$ | - |
| $M_\ell < 1.70$ GeV | - | $1.21 \pm 0.11 \pm 0.09$ |
| $P_a < 0.66$ GeV | $0.95 \pm 0.10 \pm 0.08$ | $1.08 \pm 0.10 \pm 0.10$ |
| $M_\ell < 1.7$ GeV | $0.81 \pm 0.08 \pm 0.07$ | $0.82 \pm 0.08 \pm 0.07$ |
| $|q^2| > 8$ GeV$^2$ | | |

Table II Summary of the measured partial branching ratio $\Delta B$ in units of $10^{-3}$. The uncertainty are statistical and systematics (this include the signal model systematics). For Belle, the published $\Delta \Gamma(X_u \ell \nu)$ has been translated into a partial branching ratio using the $B^{+}/0$ average lifetime from Ref[27].
Figure 3: Upper: Measured \( M_\pi \) spectra. The result of the fit is superimposed: \( B \to X_u \ell \nu \) signal generated inside (outside) the selected kinematic region \( M_\pi < 1.55 \) GeV, in white histogram (grey histogram), and the \( B \to X_e \ell \nu \) background (cyan). Lower: background subtracted plot (no unfolded), with finer binning.

This observation requires more investigations from both experimental and theoretical side, because can be an hint of the contributions of WA, or of other effects not accounted in the present theoretical calculations.

### 3.1. Determination of \( |V_{ub}| \)

As we have said, the determination of \( |V_{ub}| \) from the partial rates, require inputs from theory. HFAG extract the averages between the various experiments and kinematical variables, using all available calculations. In Tab. III we report only results using the BLNP, GGOU and BLL calculations.

For the analysis in Ref. 26 and Ref. 28, only the results for \( m_\chi \) has been used, due to the large correlations with the \( P_e \) and \( q^2 \) kinematic cuts. BLNP and GGOU agree pretty well, but the \( m_\chi \) value used in input are different. If the same \( m_\chi \) is used, than BLNP is more than 5% higher than the GGOU average. The BLL which is applied only for some cuts, give a value quite higher if compared with the other methods. For a reliable comparison between different calculations, consistent set of input parameters should be used, we hope in the near future, these inconsistencies will be fixed.

The sensitivity to the SF and to the WA can be strongly reduced if larger portion of the \( B \to X_u \ell \nu \) phase space are integrated. An analysis performed by BABAR on a sample of 80 fb\(^{-1}\) [37], integrates about 96% of the total rate is interesting for the smallness of the theoretical error (less than 3%), but the statistical uncertainty of 18% reduce the impact of this measurement on the global averages.

At CKM2008, very recently, Belle presented [38] a preliminary analysis based on a multivariate technique to reduce the large \( B \to X_e \ell \nu \) background, integrates about 90% of the phase space. This measurement extracts \( |V_{ub}| \) with a total uncertainty of 7%, where the contribution due to the theory and \( m_\chi \) is only 4%

### 3.2. \( M_X \) hadronic moments in \( B \to X_u \ell \nu \) decays

The determination of the OPE parameters from the measurements of the hadronic moments in charmless \( B \to X_u \ell \nu \) decays, is important to test the theoretical framework used to extract \( |V_{ub}| \). The measurement of the \( X_u \) hadronic moments has been performed for the first time by BABAR [40].

The analysis is based on the same technique of the analysis used to extract \( |V_{ub}| \). The \( B \to X_\ell \ell \nu \) background is subtracted by a fit to the hadronic mass spectrum, the result of the fit is reported in Fig. III. The \( m_X^2 \) spectrum is unfolded for the detector acceptance, efficiency and resolution effects and the first, second and third central moments are extracted from the unfolded spectrum.
An HQE fit of these moments, in the kinetic scheme \[12\] yields

\[
\begin{align*}
\mu_0 &= (4.604 \pm 0.125 \pm 0.193 \pm 0.097) \text{GeV} \\
\mu_2 &= (0.398 \pm 0.135 \pm 0.195 \pm 0.036) \text{GeV}^2
\end{align*}
\]

where the first error is statistical, the second is systematic, and the thirds come from the theory. The results for \(\mu_0\) and \(\mu_2\) are consistent with the determinations with \(B \rightarrow X_{u,\ell}\nu\) and \(B \rightarrow X_{c}\gamma\) events. Despite the large uncertainty, this studies deserve more studies in future.

### 3.3. Weak annihilation

One of the effects that is not included in current calculations of the partial decay rate, is the WA \[11\], which is expected to contribute at the level of a few percent \[12, 13, 14, 15\]. Simply speaking, WA refers to the annihilation of the \(b - \bar{u}\) pair to a virtual \(W\) boson, and results in an enhancement of the decay rate near the endpoint of the \(q^2\) spectrum. Here \(q^2\) refers to the mass squared of the virtual \(W\).

Experimentally, WA should be observable as a violation of isospin invariance, i.e. difference in the partial decay rates of \(B^0 \rightarrow X_{u}\ell^+\nu\) and \(B^+ \rightarrow X_c^0\ell^+\nu\), at high \(q^2\), since it occurs only for charged \(B\) mesons.

\(\text{Babar}\) performed a first measurement of the partial branching fraction for inclusive \(B^0 \rightarrow X_{u}\ell^+\nu\) decays above 2.3 GeV/c of the charged lepton momentum \[40\]. \(B^0\bar{B}^0\) events produced at the \(\Upsilon(4S)\) resonance are tagged by the partially reconstructed \(B^0 \rightarrow D^{*+}\ell\nu\) decays via the reconstruction of the soft pion \(p_{\text{soft}}\) emitted by the \(D^{*+}\) decays. The signal is extracted fitting the distribution of the neutrino mass of the tag side \(M_2^2\), that is peaked at 0 for true \(B \rightarrow \ell\) events. The large \(B \rightarrow X_{u}\ell\nu\) background is suppressed using the kaon veto and the soft pion veto on the signal side. The remaining charm background is fixed to the Monte Carlo estimation, and the large combinatoric background, is modelled using the wrong

charge sample in which the soft pion and the lepton on the tag side, have the same charge.

We identify the charmless semileptonic decay of the second \(B\) meson in the event and compare its partial decay rate with the partial rate for the sum of charged and neutral \(B\) mesons previously published \[47\], and extract the difference in these partial decay rates between \(B^+\) and \(B^0\) mesons. The result for the partial branching fraction for the interval \(2.3 < p_t < 2.6\) GeV is \(\Delta B(B^0 \rightarrow X_u\ell\nu) = (1.30\pm0.21\pm0.07)\times10^{-4}\). Combining this result with the inclusive lepton spectra allows to extract a ratio of the \(\Gamma(B^0)\) and \(\Gamma(B^+)\) decays widths for charmless semileptonic decays, \(R^{+/0} = \frac{\Delta\Gamma^{+}}{\Delta\Gamma^{0}} = 1.18 \pm 0.35 \pm 0.17\). Thus with the presently available data sample, there is no evidence for a difference in partial decay rates in \(B^0\) and \(B^+\) at the high end of the lepton momentum spectrum, where we would expect the impact of WA in \(B^+\) decays. Defining \(\Delta\Gamma_{WA} = \Delta\Gamma^{+} - \Delta\Gamma^{0}\) the contribution of the WA, the following limit ca be set

\[
\frac{|\Delta\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3 - 2.6)}, \quad \text{at 90\% C.L. ,} \tag{2}
\]

where \(f_{WA}(2.3 - 2.6)\) refers to the fraction of the weak annihilation rate contributing in the momentum interval \(2.3 - 2.6\) GeV/c. This limit is also consistent with a model dependent limit set by CLEO \[49\] studying the \(q^2\) spectra.

These limits can be translated in an uncertainty on \(|V_{ub}|\) of the order of 2 – 3% according to the acceptance of the kinematic cut. The flavor dependent partial decay rate could also be determination, using other \(B - flavor\) tagging techniques, like the fully reconstructed hadronic sample described above. These
studies should be performed in the future.

4. Conclusions

The CKM matrix elements $V_{cb}$ and $V_{ub}$ are fundamental parameters of the Standard Model. $|V_{cb}|$ with the inclusive decays already reached an error well below the \( \approx 2\% \). Future progress from the experimental side are important, in particular it is crucial to understand the model of the $X_c$ state which uncertainty is now dominated by the scarce knowledge of the $B \rightarrow D^{**}\ell\nu$ rates. At present the inclusive determination is only marginally compatible with the exclusive determination. The two determinations differs by \( \approx 2\sigma \). These discrepancy have to be understood in the next future. The total uncertainty on $V_{ub}$ is below 10\%. It is crucial to reduce this uncertainty for precise testing of the CKM picture of CP violation and for indirect searches of New Physics. There are many theoretical framework available on the market. The $B$–factories have enough statistics that detailed studies of the $B \rightarrow X_u\ell\nu$ kinematic can be exploited to prune the different available calculations. BABAR moved forward in this direction starting to study the moments of the hadronic mass in semileptonic $B \rightarrow X_u$ transitions, and the rate of $B \rightarrow X_u\ell\nu$ decays, in $B^0$ tagged events.

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