Lattice study of exotic $S = +1$ baryon

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We propose $S=+1$ baryon interpolating operators, which are based on an exotic description of the antidecuplet baryon like diquark-diquark-antiquark. By using one of the new operators, the mass spectrum of the spin-1/2 pentaquark states is calculated in quenched lattice QCD at $\beta = 6/ g^2 = 6.2$ on a $32^3 \times 48$ lattice. It is found that the $J^P$ assignment of the lowest $\Theta(udd\bar{s})$ state is most likely $(1/2)^-$. We also calculate the mass of the charm analog of the $\Theta$ and find that the $\Theta_c(udd\bar{c})$ state lies much higher than the $DN$ threshold, in contrast to several model predictions.

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Recently, LEPS collaboration at Spring-8 has observed a very narrow resonance $\Theta^{+}(1540)$ in the $K^{-}$ missing-mass spectrum of the $\gamma n \rightarrow nK^{+}K^{-}$ reaction on $^{12}C$. A remarkable observation is its strangeness is $S=+1$, which means that the observed resonance must contain a strange antiquark. Thus, the $\Theta^{+}(1540)$ should be an exotic baryon state with the minimal quark content $udd\bar{s}$. This discovery is subsequently confirmed in different reactions by several other collaborations. It should be noted, however, that the experimental evidence for the $\Theta^{+}(1540)$ is not very solid yet since there are a similar number of negative results to be reported.

Theoretically the existence of such a state was predicted long time ago by the Skyrme model. However, the prediction closest in mass and width with the experiments was made by Diakonov, Petrov and Polyakov using a chiral-soliton model. They predicted that it should be a narrow resonance and stressed that it can be detected by experiment because of its narrow width. In a general group theoretical argument with flavor $SU(3)$, $S=+1$ pentaquark state should be a member of antidecuplet or higher dimensional representation such as 27-plet or 35-plet. Both the Skyrme model and the chiral-soliton model predict that the lowest $S=+1$ state appears in the antidecuplet, $I=0$, and its spin and isospin are $(1/2)^+$. Experimentally, spin, parity and isospin of the $\Theta^{+}(1540)$ are not determined yet. After the discovery of the $\Theta^{+}(1540)$, many model studies for the pentaquark state are made with different spin, parity and isospin.

Lattice QCD in principle can determine these quantum numbers of the $\Theta^{+}(1540)$, independent of such arbitrary model assumptions or the experiments. We stress that there is substantial progress in lattice study of excited baryons recently. Especially, the negative parity nucleon $N^{*}(1535)$, which lies close to the $\Theta^{+}(1540)$, has become an established state in quenched lattice QCD. Here we report that quenched lattice QCD is capable of studying the $\Theta^{+}(1540)$ as well.

Indeed, it is not so easy to deal with the $qqq\bar{q}q\bar{q}$ state rather than usual baryons ($qqq$) and mesons ($qq$) in lattice QCD. The $qqq\bar{q}q\bar{q}$ state can be decomposed into a pair of color singlet states as $qq$ and $q\bar{q}$, in other words, it can decay into two-hadron states even in the quenched approximation. For instance, one can start a study with a simple minded local operator for the $\Theta^{+}(1540)$, which is constructed from the product of a neutron operator and a $K^{+}$ operator such as $\Theta = \varepsilon_{abc}(d^T_i C \gamma_5 u_b)d_c(\bar{s}_c \gamma_5 u_c)$. The two-point correlation function composed of this operator, in general, couples not only to the $\Theta$ state (single hadron) but also to the two-hadron states such as an interacting $KN$ system. Even worse, when the mass of the $qqq\bar{q}q\bar{q}$ state is higher than the threshold of the hadronic two-body system, the two-point function should be dominated by the two-hadron states. Thus, a specific operator with as little overlap with the hadronic two-body states as possible is desired in order to identify the signal of the pentaquark state in lattice QCD.

For this purpose, we propose some local interpolating operators of antidecuplet baryons based on an exotic description as diquark-diquark-antiquark. There are basically two choices as $3_c \otimes 3_c \otimes 3_c \otimes 6$ to construct a color triplet diquark-diquark cluster. We adopt the former for a rather simple construction of diquark-diquark-antiquark. We first introduce the flavor antitriplet ($\bar{3}_f$) and color antitriplet ($\bar{3}_c$) diquark field

$$\Phi_{1}^{\bar{3}_f}(x) = \frac{1}{2} \varepsilon_{ijk} \varepsilon_{abc} q_{i,b}^{T}(x) C T q_{k,c}(x)$$

where $C$ is the charge conjugation matrix, $abc$ the color indices, and $ijk$ the flavor indices. $\Gamma$ is any of the sixteen Dirac $\gamma$-matrices. Accounting for both color and flavor antisymmetries, possible $\Gamma$s are restricted within $1$, $\gamma_5$ and $\gamma_5 \gamma_\mu$ which satisfy the relation $(\Theta^T) = -\Theta$. Otherwise, the above defined diquark operator is identically zero. Hence, three types of flavor $3_f$ and color $3_c$ diquark; scalar ($\gamma_5$), pseudoscalar (1) and vector ($\gamma_5 \gamma_\mu$) diquarks are allowed. The color singlet state can be constructed by the color antisymmetric parts of diquark-diquark ($3_c \otimes 3_c \text{antisym} = 3_c$) with an antiquark ($\bar{3}_c$). In terms of flavor, $3_f \otimes 3_f \otimes 3_f = 1_f \otimes 8_f \otimes 8_f \oplus 10_f$. Manifestly, in this description, the $S=+1$ state belongs to the flavor antidecuplet. Automatically, the $S=+1$ state should have isospin zero. Then, the interpolating operator of the $\Theta(udd\bar{s})$ is obtained as

$$\Theta(x) = \varepsilon_{abc} \Phi_{1}^{\bar{3}_c}(x) \Phi_{1}^{\bar{3}_f}(x) C S_{c}^{T}(x)$$

(2)
for \( \Gamma \neq \Gamma' \). The form \( CS^T \) for the strange anti-quark field is responsible for giving the proper transformation properties of the resulting pentaquark operator under parity and Lorentz transformations [14]. Note that because of the color antisymmetry, the combination of the same types of diquark is not allowed. Consequently, we have three different types of exotic \( S=\pm 1 \) baryon operators through the combination of two different types of diquarks, which have different spin-parity [12]:

\[
\begin{align*}
\Theta^1_{\pm} &= \varepsilon_{abc} \varepsilon_{aef} \varepsilon_{bgh} (u_c u_T C_d f_j) (u_f g C_{\gamma_5} d_i h) C s^T_h, \\
\Theta^2_{\pm, \mu} &= \varepsilon_{abc} \varepsilon_{aef} \varepsilon_{bgh} (u_c u_T C_{\gamma_5} d_f j) (u_f g C_{\gamma_5} \gamma_\mu d_i h) C s^T_h, \\
\Theta^3_{\pm, \mu} &= \varepsilon_{abc} \varepsilon_{aef} \varepsilon_{bgh} (u_c u_T C_d f_j) (u_f g C_{\gamma_5} \gamma_\mu d_i h) C s^T_h
\end{align*}
\]

where the subscript " + (−)" refers to positive (negative) parity since these operators transform as \( P \Theta_{\pm} (\vec{x}, t) P^\dagger = \pm \gamma_4 \Theta_{\pm} (\vec{x}, t) \) (for \( \mu = 1, 2, 3 \)) under parity. The first operator of Eq. (3) is proposed for QCD sum rules in a recent paper [14] independently.

In this description, the operator of exotic \( \Xi_{5/2} \) (ssdd\( \bar{u} \)) or uuudd\( \bar{s} \) states, which are members of the antidecuplet, can be treated by interchanging \( u \) and \( s \) or \( d \) and \( s \) in the above operators. If a strange antiquark is simply replaced by a charm antiquark, the proposed pentaquark operators can be regarded as the anti-charmed analog of the isosinglet pentaquark state, \( \Theta_c(uudd\bar{c}) \).

Recall that any of local type baryon operators can couple to both positive- and negative-parity states since the parity assignment of an operator is switched by multiplying the left hand side of the operator by \( \gamma_5 \). The desired parity state is obtained by choosing the appropriate projection operator, \( 1 \pm \gamma_4 \), on the two-point function \( G(t) \) and direction of propagation in time. Details of the parity projection are described in Ref. [12]. We emphasize that the second and third operators, Eqs. (4) and (5), can couple to both spin-1/2 and spin-3/2 states. By using them, it is possible to study the spin-orbit partner of the spin-1/2 \( \Theta \) state, whose presence contradicts the Skyrme picture of the \( \Theta \) [11]. However, we will not pursue this direction in this article. We utilize only the first operator of Eq. (3), which couples only to a spin-1/2 state. Under the assumption of the highly correlated diquarks, we simply omit a quark-exchange diagram between diquark pairs contributing to the full two-point function in the following numerical simulations.

We generate quenched QCD configurations on a lattice \( L^3 \times T = 32^3 \times 48 \) with the standard single-plaquette Wilson action at \( \beta = 6/g^2 = 6.2 \) (\( a^{-1} = 2.9 \) GeV). The spatial lattice size corresponds to \( L a \approx 2.2 f m \), which may be marginal for treating the ground state of baryons without large finite volume effect. Our results are analyzed on 240 configurations. The light-quark propagators are computed using the Wilson fermions at four values of the hopping parameter \( \kappa = \{0.1520, 0.1506, 0.1497, 0.1489\} \), which cover the range \( M_\pi / M_p = \Phi_{\text{eff}} = 0.68 - 0.90 \). \( \kappa_s = 0.1515 \) and \( \kappa_c = 0.1360 \) are reserved for the strange and charm masses, which are determined by approximately reproducing masses of \( \phi(1020) \) and \( J/\Psi(3097) \). We calculate a simple point-point quark propagator with a source location at \( t_{\text{src}} = 6 \). To perform precise parity projection, we construct forward propagating quarks by taking the appropriate linear combination of propagators with periodic and anti-periodic boundary conditions in the time direction. This procedure yields a forward in time propagation in the time slice range \( 0 < t < T - t_{\text{src}} \).

In this calculation, the strange (charm) quark mass is fixed at \( \kappa_s (\kappa_c) \) and the up and down quark masses are varied from \( M_\pi \approx 1.0 \) GeV \( (\kappa_s = 0.1489) \) to \( M_\pi \approx 0.6 \) GeV \( (\kappa_s = 0.1520) \). Then, we perform the extrapolation to the chiral limit using five different \( \kappa \) values.

We first calculate the effective masses \( M_{\text{eff}}(t) = \ln\{G(t)/G(t + 1)\} \) for both parity states of the spin-1/2 \( \Theta(uudd\bar{s}) \). For example, Figs. show effective masses for the positive parity channel and the negative parity channel at \( \kappa = 0.1506 \) for up and down quarks with the fixed...
Therefore, the signals after mated. The correlators for the heavier mass state in this ≤ the two-point function in the plateau region 15

\[ \chi \]

where the respective KN threshold is defined as the total energy of the non-interacting KN state with zero momentum. However, all momenta are quantized as \( \vec{p}_{\text{min}} = 2\pi\vec{n}/L \) on a system of finite volume. The KN threshold is defined as the total energy of the non-interacting KN state with the smallest nonzero momentum \( |\vec{p}_{\text{min}}| = 2\pi/L \) in lattice units. Here, we stress the following two points. First, there is no clear signal for the KN state to be observed in the effective mass plot. It means that our proposed interpolating operator couples weakly to the KN scattering state. Secondly, our observed plateau in Fig. 1 (a) is considerably higher than the KN threshold. While the observed asymptotic state can be identified as a pentaquark (single hadron) state, our results seem to give no indication of the \( \Theta^+(1540) \) state in the positive parity channel.

In the negative parity channel, the gross feature is similar to the case of the positive parity. Fig. 1 (b) shows that a clear plateau appears in the range \( 15 \leq t \leq 20 \). The relatively noisy signals appear around the S-wave KN threshold after \( t = 21 \) and continue toward the maximum time slice \( t = T - t_{\text{src}} \) for the forwarding propagation. The errors after \( t = 21 \) are probably underestimated. The correlators for the heavier mass state in this euclidean time region usually have many orders of magnitude deviation and the distribution is non-Gaussian. Therefore, the signals after \( t = 21 \) are inconclusive.

We perform a covariant single exponential fit [22] to the two-point function in the plateau region \( 15 \leq t \leq 20 \), where the respective \( \chi^2 \) is indeed most favorable. The estimated mass is clearly higher than the KN threshold, which is evaluated as the total energy of the non-interacting KN state with zero momentum. The excitation energy of the observed asymptotic state from the KN threshold is roughly consistent with the experimental value. Although, without a finite volume analysis, it cannot be excluded that the observed plateau stems from only a mixture of the KN scattering states; we may regard it as a pentaquark state with a mass close to the experimental value of the \( \Theta^+(1540) \).

As a strange antiquark is simply replaced by a charm antiquark, we can explore the anti-charmed pentaquark \( \Theta_c(uddc) \) as well. A similar identification for the \( \Theta_c \) state can be made in the negative parity channel. The effective mass plot (Fig. 2) shows that a plateau, which terminates at \( t \approx 21 \), is much higher than the DN threshold. The relatively noisy signals appear around this threshold after \( t \approx 21 \). The observed asymptotic state is identified with a pentaquark (single hadron) state similarly.

In Fig. 2 we show the mass spectrum of the \( \Theta(udds) \) states with the positive parity (open squares) and the negative parity (open circles) as functions of the pion mass squared. Mass estimates are obtained from covariant single exponential fits in the appropriate fitting range. All fits have a confidence level larger than 0.3 and \( \chi^2/N_{DF} < 1.2 \). It is evident that the lowest state of the isosinglet \( S = +1 \) baryons has the negative parity. We evaluate the mass of the \( \Theta(udds) \) with both parities in the chiral limit. A simple linear fit for all five values in Fig. 2 yields \( M_{\Theta(1/2^-)} = 0.62 (3) \) and \( M_{\Theta(1/2^+)} = 1.00 (5) \) in lattice units. If we use the scale set by \( r_0 \) from Ref. [15], we obtain \( M_{\Theta(1/2^-)} = 1.84(8) \) GeV and \( M_{\Theta(1/2^+)} = 2.94(13) \) GeV. It is worth quoting other related hadron masses. The chiral extrapolated values for the kaon, the nucleon and the \( N^* \) state are \( M_K = 0.53(1) \) GeV, \( M_N = 1.06(2) \) GeV and \( M_{N^*} = 1.76(5) \) GeV in this calculation.

Our obtained \( \Theta(1/2^-) \) mass is slightly overestimated in comparison to the experimental value of the \( \Theta^+(1540) \), but comparable to our observed \( N^* \) mass, which is also overestimated. Needless to say, the evaluated values should not be taken too seriously since they do not include any systematic errors. Such a precise quantitative prediction of hadron masses is not the purpose of the present paper. Rather, we emphasize that, our results strongly indicate the \( J^P \) assignment of the \( \Theta^+(1540) \) is most likely \( (1/2)^- \). This conclusion is consistent with that of a recent lattice study [16] (if one corrects the parity assignment of their operator [17]) and that of QCD sum rules approach [14].

Results for the lowest-lying spin-1/2 \( \Theta_c \) state, which has the negative parity, are also included in Fig. 3. The \( \Theta_c \) state lies much higher than the DN threshold in contradiction with several model predictions [18, 19, 20]. The chiral extrapolated value of the \( \Theta_c \) mass is 3.45(7) GeV, which is about 500 MeV above the DN threshold (\( M_D = 1.89(1) \) GeV) in our calculation. This indicates
[54x327]pected as a bound state.

[54x303]otic baryon, Θ(c\bar{c}uudd) is not to be expected as a bound state.

[54x339]that the anti-charmed pentaquark Θc is not to be expected as a bound state.

[54x339]We have calculated the mass spectrum of the S=+1 exotic baryon, Θ(\text{uudd}), and the charm analog Θc(\text{uuddc}) in quenched lattice QCD. To circumvent the contamination from hadronic two-body states, we formulated the antidecuplet baryon interpolating operators using an exotic description like diquark-diquark-antiquark. Our lattice simulations seem to give no indication of a pentaquark with a mass close to the experimental value. Although more detailed lattice study would be desirable to clarify the significance of this observation, the present lattice study favors spin-parity (1/2−) for the Θc(\text{1540}). We have found that the lowest spin-1/2 Θc state, which has the negative parity, lies much higher than the DN threshold, in contrast to several model predictions [18, 19, 20].

To establish the parity of the Θc(\text{1540}), more extensive lattice study is required. Especially, a finite volume analysis is necessary to disentangle the pentaquark signal from a mixture of the KN scattering states. It is also important to explore the chiral limit. This calculation was performed using relatively heavy quark mass so that one may worry about a level switching between both parity states toward the chiral limit as observed in the case of excited baryons [4, 21]. We remark that a study for the non-diagonal correlation between our pentaquark operators and a standard two-hadron operator should shed light on the structure of the very narrow resonance Θ+(1540). The possible spin-orbit partner of the Θ state is also accessible by using two of our proposed operators. We plan to further develop the present calculation to involve more systematic analysis and more detailed discussion.

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[22] A simple exponential might not be an appropriate functional form for the decaying state. Recall that the pentaquark state can decay into two-hadron states even in the quenched approximation. Strictly speaking, we should take the decay width into account in the fitting form. However, nobody knows an appropriate analytic form for the two-point function of an unstable state in finite volume on the lattice.

FIG. 3: Masses of the spin-1/2 Θ(\text{uudd}) states with both positive parity (open squares) and negative parity (open circles) as functions of pion mass squared in lattice units. The charm analog Θc(\text{uuddc}) state (open diamonds) is also plotted. Horizontal short bar represents the KN(DN) threshold estimated by MN + MR(MN + MD) in the chiral limit.