Multiplicty Dependence of Hadron Spectra in Proton-Proton Collisions at LHC Energies and Super-Statistics

Karoly Urmossy

1 Institute for Particle and Nuclear Physics, Wigner RCP of the HAS
29-33 Konkoly–Thege Miklós Str., H-1121 Budapest, Hungary

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Abstract In this paper, transverse momentum spectra of $\pi^+$, $K^+$ and $p$ measured at fix event-multiplicities and $\sqrt{s} = 0.9$, 2.76 and 7 TeV collision energies by the CMS Collaboration are shown to fit the Tsallis-distribution. It is found that the power of the distribution shows a double-logarithmic dependence on the event-multiplicity $N$, while the $T$ parameter depends linearly on $N$. A similar double-logarithmic dependence of the $q$ parameter of $\pi^0$ spectra on the collision energy $\sqrt{s}$ is found too.

It is also shown that event-by-event fluctuations of the multiplicity $N$ and the total $E_T$ energy going into the transverse region can be the reason for the emergence of the Tsallis distribution in high-energy proton-proton collisions.

Keywords hadron spectra · Tsallis-statistics · super-statistics · KNO scaling · multiplicity fluctuations · transverse energy fluctuations

1 Introduction

Cut-power law hadron distributions (sometimes referred to as Tsallis distribution) are observed in various high-energy collisions: in electron-positron $e^+e^-$ [1] – [7], proton-(anti-)proton $pp$ [8] – [15], elastic $pp$ [19] and nucleus-nucleus $AA$ [20] – [33] collisions. While the dependence of the parameters of this distribution on the collision energy $\sqrt{s}$ [11,2,5,8,12,14,15,20,21,25,26,32], centrality [25,26,28,33], number of colliding nucleons $N_{coll}$ [23], hadron species [6,10,11,24,25,27,30,31] and momentum transfer [19] is thoroughly discussed in the literature, the dependence of hadron spectra on the event-multiplicity measured in [44] has not been analysed yet. In Sec. 3 this task is fulfilled.

One explanation for the emergence of cut-power laws may be found in thermal hadronisation models. These models are based on the conjecture that in high-energy collisions, small thermal droplets of matter (often referred to as “fireballs” or “clusters”) are created and these droplets fragment into hadrons. The calculations are carried through either in the canonical [13,14,37,38,42,43] or microcanonical [4,16,17,34,35,36,41] framework, and describe measured data on hadron spectra, total hadron multiplicities as well as multiplicity distributions. The latter can be approximated by either the negative binomial or by Euler’s gamma-distribution.

Since each formed fireball is different, they may carry different four-momenta, fragment into different number of hadrons and - if one works in the canonical framework - they may have different temperature. It has been shown that, if a single cluster or fireball is described by the Boltzmann-Gibbs distribution, however, the temperature [1,2,20,21,22,23,29,33] or cluster volume [23,34] or the multiplicity of hadrons stemming from a cluster [5,15] fluctuates from cluster to cluster, the average hadron spectrum may become a cut-power law (or Tsallis) distribution. The case of volume [34] and multiplicity [5,6,15] fluctuations has also been generalised to the microcanonical ensemble. The latter has been applied to the fragmentation of jets produced in $e^+e^-$ [5,6] and $pp$ collisions [15].

It is worth noting that if we approximate very narrow jets with one-dimensional bunches of massless hadrons, the conservation of four-momentum reduces to the conservation of energy, and the clusters (jets) become massless too (for a discussion, see [15]). Thus, jet-fragmentation models [5,6,15] may be considered as an approximation of microcanonical statistical hadronisa-
tion models \[1-14, 17, 31-39, 41\] for very high hadron energies and jet-like, very narrow, one-dimensional clusters. The latter models are sensitive to the species of hadrons and multiplicity distributions are obtained from them as a consequence. In the previous models, the multiplicity distribution is put in by hand but, because of their simplicity, analytic calculations are possible.

In this paper, I will focus on the effect of cluster energy fluctuations. Hadronic transverse momentum spectra in \(pp\) collisions are mixtures of particle yields coming from many clusters of different energy \(E_c\), four-velocity \(u_\mu\), and yielding different \(N\) number of hadrons. For the spectra published in \[14\] is soft, I will work in the canonical ensemble, supposing that each cluster produces hadrons according to an isotropic Boltzmann-Gibbs distribution in \(D = 3\) dimensions in the frame co-moving with the cluster:

\[
\frac{dN}{dp} \bigg|_{\text{cluster}} \sim \exp \left\{ -\beta(u_\mu p^\mu - m) \right\}, \tag{1}
\]

with \(\beta\) being the inverse temperature and \(p^\mu\) being the four-momentum of the hadron.

In Sec. 2 I show that an appropriate choice for fluctuations of the cluster energy result in cut-power law (or Tsallis) shaped hadron spectrum. As a first approximation, I will neglect the cluster velocities choosing \(u_\mu = (1, 0, 0, 0)\) and do calculations for massless \((m = 0)\) particles. This way, \(\beta = DN/E_c\) holds. Furthermore, I will assume that only one cluster is formed in one \(pp\) event.

In Sec. 3 I show that the Tsallis distribution obtained in Sec. 2 fits spectra of identified \(\pi^+\), \(K^+\) and \(p\) at fixed event multiplicity at LHC energies.

In Sec. 4 I show that multiplicity fluctuations of the Euler-gamma type superimposed over cluster energy fluctuations also result in an approximately Tsallis-shaped transverse \(\pi^+\) spectrum and fit measurements well. In addition, I also show that a double-logarithmic dependence of the \(q\) parameter of \(\pi^0\) spectra on the collision energy \(\sqrt{s}\) holds in the \(\sqrt{s} \in [0.2, 7]\) TeV range.

In Sec. 5 I make a prediction on the joint distribution of the event multiplicity \(N\) and total transverse energy \(E_T\) in an event, supposing that \(E_T \propto E_c\).

\section{Power-law Spectrum from Cluster Energy Fluctuations}

According to Eq. (1), let us conjecture that hadrons stemming from a cluster have an isotropic Boltzmann-Gibbs distribution in the frame co-moving with the cluster:

\[
\frac{dN}{dp} \bigg|_{\text{cluster}} = f_{N,E_c}(\epsilon) = A \exp \left\{ -\frac{DN}{E_c} \right\}, \tag{2}
\]

with \(E_c\) and \(N\) being the energy and multiplicity of the cluster and \(A = [DN/E_c]^D/\kappa_D \Gamma(D)\) comes from the normalisation condition

\[
\int d^D p f_{N,E_c}(\epsilon) = 1, \tag{3}
\]

where \(\kappa_D = \int d\Omega_D\) is the angular part of the \(D\) dimensional momentum space integral. The masses of the hadrons have been neglected, taking \(\epsilon = |p|\). Let us also conjecture cluster energy fluctuations of the form

\[
g_{N}(E_c) = \frac{1}{\Gamma(\alpha + 1)} \frac{\alpha E_0}{E_c^\alpha} \left( \frac{\alpha E_0}{E_c} \right)^\alpha e^{-\alpha E_0/E_c}. \tag{4}
\]

This way, the hadron distribution in a cluster of multiplicity \(N\), averaged over fluctuations of the cluster energy, becomes a cut-power law (or Tsallis) distribution:

\[
\frac{dN}{dp} \bigg|_{\text{cluster}} \bigg|_{E_c} = f_N(\epsilon) = \int dE_c g_{N}(E_c) f_{N,E_c}(\epsilon) = A_N \left( 1 + \frac{q - 1}{T} \epsilon \right)^{-1/(q-1)} \tag{5}
\]

with

\[
q = 1 + \frac{1}{\alpha + D + 1}, \quad T = \frac{\alpha E_0}{DN(\alpha + D + 1)}. \tag{6}
\]

The parameters \(\alpha\) and \(E_0\) (and thus \(q\) and \(T\)) may depend on the hadron multiplicity \(N\) in the cluster.

\section{3 Hadron Spectra at Fixed Multiplicity}

In this section, I show that Eq. (5) (in \(D = 3\) dimensions) fits transverse momentum spectra of various identified hadrons if the kinetic energy, \(\epsilon \rightarrow m_T - m\), is used as scaling variable:

\[
\frac{dN}{dp_T dy}_{|y=0}^{N=\text{fix}} = \frac{A p_T m_T}{\left[ 1 + \frac{q - 1}{T} (m_T - m) \right]^{1/(q-1)}}. \tag{7}
\]
The analysed data are: transverse momentum spectra of $\pi^+$ (Fig. 11) $K^+$ (Fig. 3) and $p$ (Fig. 4) stemming from $pp$ collision events of fixed multiplicities (indicated as $N_{\text{tracks}}$ in the figures) at $\sqrt{s} = 0.9$, 2.76 and 7 TeV collision energies and from the rapidity range $|y| \leq 1$. Figs. 2, 4, 6 and 7 show that a

$$q = 1 + \mu \ln(N/N_q),$$

$$T = T_0(1 + N/N_T)$$

dependence of the $q$ and $T$ parameters on the event-multiplicity $N$ is consistent with measurements. The parameters $\mu$, $N_q$, $T_0$, and $N_T$ take different values for each particle species, but do not seem to change significantly (within errors) as the collision energy $\sqrt{s}$ varies. It is interesting that while the $T$ parameters of $K^+$ and $p$ grow with $N$, the $T$ of $\pi^+$ is approximately independent of the event-multiplicity. Since the dominant part of the produced hadrons is pions, we may use Eq. (8) to estimate the dependence of the $q$ and $T$ parameters of pions on the pion-multiplicity $N_\pi$, by substituting $N \rightarrow N_\pi$. However, it is perhaps not true for kaons and protons.

4 The Effect of Multiplicity Fluctuations

It has been known for a time that multiplicity distributions of charged hadrons in high-energy collisions may be approximated by the negative binomial (NBD) or Euler’s Gamma distribution [4,5,16,18,34,35,38,39,40]. Fig. 8 shows fits of Euler’s Gamma distribution [4,5,16,18,34,35,38,39,40]. The hadron distribution in a cluster, averaged over the fluctuations of both the cluster energy $E_c$ and the cluster multiplicity $N$, is

$$p(N) = \frac{1}{\Gamma(a)} \frac{a}{N_0} \left( \frac{aN}{N_0} \right)^{a-1} e^{-aN/N_0}$$

(9)

to multiplicity distributions measured by the ALICE Collaboration [4,5,16,18,34,35,38,39,40]. Fig. 8 shows fits of Euler’s Gamma-distribution

Fig. 9 shows measured and calculated multiplicity-averaged $\pi^+$ spectra. In the evaluation of Eq. (10), the discrete sum was approximated by an integral with a lower bound $N_c = eN_q$ in order to ensure that $q \geq 1$. For the $a$ parameter of the multiplicity distribution Eq. (9), I used values obtained from fits to experimental data shown in Fig. 8. For the mean multiplicity parameter $N_0$, values higher then the ones obtained from fits to measured multiplicity distributions turned out to give best agreement with measured spectra (see the caption of Fig. 9).

It is also worth to note that the $q$ parameter of the transverse momentum spectra of neutral pions (which is also an $E_c$ and $N$ averaged observable) also show double-logarithmic dependence on the collision energy for $\sqrt{s} \in [0.2, 7]$ TeV

$$q(s) = 1 + q_1 \ln(\sqrt{s}/Q_0).$$

(11)

In the meanwhile, the $T$ parameter is not affected significantly (within errors) by the grows of $s$. Fig. 10 shows fit results. Similar functional forms for $q(s)$ have also been proposed in [13,14,21,22].

From the observation that in $pp$ collisions at $\sqrt{s} = 900$ GeV the $q$ parameter of various hadron species coincide within errors [10,39], we may infer that Eq. (11) might hold for other hadrons too.

5 Joint Distribution of the Multiplicity and the Total Transverse Energy in an Event

In the model presented above, the joint distribution of cluster energy $E_c$ and cluster multiplicity $N$ is

$$p(N, E_c) = p(N) g_N(E_c),$$

(12)

where $p(N)$ is the $E_c$ - averaged multiplicity distribution

$$p(N) = \int dE_c p(N, E_c)$$

(13)

shown in Fig. 8 and $g_N(E_c)$ is the normalised distribution of $E_c$ at fix multiplicity. From Eqs. (10) and (11), the parameters of $g_N(E_c)$ depend on $N$ as
\[ \alpha = \frac{1}{\mu \ln(N/N_q)} - D - 1, \]
\[ E_0 = \frac{D N T_0 (1 + N/N_T)}{1 - (D + 1) \mu \ln(N/N_q)}. \]

Since the hadron distribution in a cluster is assumed to be isotropic, the \( E_T \) energy going into the transverse region \((|y| \leq 1)\) in a single \( pp \) event is proportional to the energy of the cluster formed in the event: \( E_T \sim E_c \). Thus, the distributions of \( E_T \) and \( E_c \) have the same form. Fig. 11 shows the joint distribution \( p(N, E_T) \) with its projections, using parameters obtained from data on \( \pi^+ \) spectra measured at \( \sqrt{s} = 7 \) TeV and in the rapidity range \(|y| \leq 1\).

6 Conclusions

Cut-power law hadron distributions (sometimes referred to as Tsallis distribution) are observed in various high-energy collisions (from electron–positron to nucleus–nucleus reactions [1 - 33]). One possible explanation for this phenomena can be found in thermal hadronisation models. According to these models, in high-energy collisions, small thermal “fireballs” or “clusters” are formed and fragment into hadrons. Hadrons stemming from a single cluster inherit the thermal (canonical [37-38,12,43] or microcanonical [16,17,31,35,36,11]) distribution of the fireball. However, since the four-momentum (or four-velocity) and cluster mass [11,16,17,31,35,36,41], or the temperature [12,20,21,22,23,29,33] or cluster volume [23,34] or the hadron multiplicity [5,13] fluctuates from cluster to cluster, the average hadron spectrum may become a cut-power law (or Tsallis) distribution.

The measurement of transverse spectra of a few types of hadrons stemming from proton–proton collisions for some fixed event multiplicities [14] made it possible to get rid of the effect of event-by-event multiplicity fluctuations on hadron spectra. In Sec. 3 it is shown that transverse spectra of \( \pi^+, K^+ \) and \( p \) take a cut-power law shape (Eq. 7) even in fixed multiplicity proton–proton events. It is also found that the \( q \) parameter of the spectra follows a double-logarithmic dependence on the event-multiplicity \( N \), while the \( T \) parameter is independent of \( N \) (within errors) for pions, but grows linearly with \( N \) for kaons and protons (see Eq. 8 and Figs. 1 - 7).

In Sec. 2 it is shown that cluster energy fluctuations of the form of Eq. 3 result in cut-power law shaped hadron spectrum at fixed cluster multiplicity, if hadrons in a cluster are distributed according to the Boltzmann-Gibbs distribution. Averaging this spectrum over multiplicity fluctuations Eq. 9, using the weak multiplicity dependence of the \( q \) and \( T \) parameters of pion spectra found in Sec. 3 also results in an approximately cut-power law spectrum that describes transverse \( \pi^+ \) spectrum (see Sec. 4 and Fig. 9). This is consistent with the observation that multiplicity averaged transverse spectra (usually simply called “transverse spectra”) of hadrons stemming from \( pp \) collisions fit the Tsallis distribution. In addition, I have also shown that the \( q \) parameter of the transverse spectrum of \( \pi^0 \) shows a double-logarithmic dependence on the collision energy \( \sqrt{s} \) in the range \( \sqrt{s} \in [0, 7] \) TeV (see Fig. 10). Similar functional forms for \( q(s) \) have also been proposed in [13,14,12,22].

In order to be able to perform analytic calculations, I conjectured that only one cluster is formed in a single proton–proton event; hadrons stemming from the cluster have an isotropic Boltzmann-Gibbs distribution in the frame co-moving with the cluster; finally, I neglected particle masses and cluster velocities. In this “first approximation”, the \( E_c \) energy of the cluster formed in a single proton–proton event is proportional to the \( E_T \) energy that reaches the transverse region \((|y| \leq 1)\), \( E_c \sim E_T \). Thus, the joint distribution \( p(N, E_T) \) of the event multiplicity and the transverse energy in an event can be predicted (see Sec. 5 and Fig. 11).

Though, it is to be emphasized that there are rough approximations in the above presented calculations (the case of multiple cluster production in a proton–proton event as well as non-zero cluster velocities and finite hadron masses are to be taken into account in future works), it is clearly pointed out that event-by-event fluctuation of the hadron multiplicity as well as that of the energy reaching the transverse range may result in cut-power law (or Tsallis) shaped hadron spectra. Even if hadrons had Boltzmann-Gibbs distribution in a single event or cluster.

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Fig. 1 Transverse momentum spectra of $\pi^+$s stemming from $pp$ collisions of fixed multiplicities, measured at $\sqrt{s} = 0.9$ TeV (top), 2.76 TeV (middle) and 7 TeV (bottom) collision energies. Rapidity range: $|y| \leq 1$. Data of graphs were published in [44]. Curves are fits of Eq. (7).

Fig. 2 Dependence of the $q$ parameter of Eq. (7) on the event-multiplicity obtained from fits to $\pi^+$ spectra shown in Fig. 1. Top: $\sqrt{s} = 0.9$ TeV, middle: $\sqrt{s} = 2.76$ TeV bottom: $\sqrt{s} = 7$ TeV. Curves are fits of Eq. (8).
Fig. 3 Transverse momentum spectra of $K^+$-s stemming from $pp$ collisions of fix multiplicities, measured at $\sqrt{s} = 0.9$ TeV (top), 2.76 TeV (middle) and 7 TeV (bottom) collision energies. Rapidity range: $|y| \leq 1$. Data of graphs were published in [44]. Curves are fits of Eq. (7).

Fig. 4 Dependence of the $q$ parameter of Eq. (7) on the event-multiplicity obtained from fits to $K^+$ spectra shown in Fig. 3. Top: $\sqrt{s} = 0.9$ TeV, middle: $\sqrt{s} = 2.76$ TeV bottom: $\sqrt{s} = 7$ TeV. Curves are fits of Eq. (8).
Fig. 5 Transverse momentum spectra of p-s stemming from pp collisions of fixed multiplicities, measured at \( \sqrt{s} = 0.9 \) TeV (top), 2.76 TeV (middle) and 7 TeV (bottom) collision energies. Rapidity range: \( |y| \leq 1 \). Data of graphs were published in [44]. Curves are fits of Eq. (7).

Fig. 6 Dependence of the \( q \) parameter of Eq. (7) on the event-multiplicity obtained from fits to p spectra shown in Fig. 5. Top: \( \sqrt{s} = 0.9 \) TeV, middle: \( \sqrt{s} = 2.76 \) TeV bottom: \( \sqrt{s} = 7 \) TeV. Curves are fits of Eq. (8).
Fig. 7 Dependence of the $T$ parameter of Eq. (7) on the event-multiplicity obtained from fits to $\pi^+, K^+$ and $p$ spectra shown in Figs. 1, 3, and 5. Top: $\sqrt{s} = 0.9$ TeV, middle: $\sqrt{s} = 2.76$ TeV, bottom: $\sqrt{s} = 7$ TeV. Curves are fits of Eq. (8).

Fig. 8 Top: fits of Eq. (9) to multiplicity distributions of charged hadrons stemming from $pp$ collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV [45,49]. Middle and bottom: dependence of the distribution’s parameters on the collision energy.
Fig. 9 Transverse momentum spectra of $\pi^+\pi^-$ stemming from $pp$ collisions and averaged over multiplicity fluctuations. Measured data are published in [44]. Dashed lines show Eq. (10) with different parametrisations of the multiplicity distribution Eq. (9). For collision energies $\sqrt{s} = 0.9, 2.76$, and 7 TeV, $a = 1.38, 1.4, 1.12$ and $N_0 = 15.3, 24, 32$ were used respectively (see text below Eq. (10)).

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Fig. 10 Transverse momentum spectra of $\pi^0$-s stemming from $pp$ collisions at various collision energies and fits of Eq. (7) (top). $\sqrt{s}$ dependence of the $q$ parameter and fit of Eq. (11) (middle). $\sqrt{s}$ dependence of the $T$ parameter (bottom). Data of graphs were published in [15][10][17].
The plots of Eq. (12) show the joint distribution \( p(N, E_T) \) of the event-multiplicity \( N \) and total transverse energy \( E_T \) (bottom) and its projections (top and middle). The parameters used were obtained from fits to the multiplicity distribution of charged hadrons and transverse \( \pi^+ \) spectra measured in pp collisions at \( \sqrt{s} = 7 \) TeV collision energy. Used parameters are: 

- \( a = 1.11 \)
- \( N_0 = 12 \)
- \( \mu = 0.14 \)
- \( N_T = 1.2 \)
- \( T_0 = 70 \) MeV
- \( N_T = 5000 \)

Fig. 11

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