The $pp\rightarrow K^+\Sigma^+ n$ cross section from missing mass spectra

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Abstract. We utilize existing inclusive data on $K^+$-meson momentum spectra of the reaction $pp\rightarrow K^+ X$ at $T_p = 2.3 - 2.85$ GeV to deduce total cross sections for $pp\rightarrow K^+ \Sigma^+ n$. The method used to extract those cross sections is explained and discussed in detail. Our result for $T_p = 2.85$ GeV is consistent with the data point from a direct measurement at the same beam energy. The cross section obtained for $T_p = 2.3$ GeV is with $13.7 \pm 2.3 \, \mu b$ considerably smaller than the value found in a recent experiment by the COSY-11 Collaboration at a somewhat lower beam energy, indicating that the $pp\rightarrow K^+ \Sigma^+ n$ reaction cross section could exhibit a rather unusual energy dependence.

PACS. 13.75.Ev Hyperon-nucleon interactions – 13.75.Jz Kaon-baryon interactions – 14.20.Gk Baryon resonances

1 Introduction

Recently first near-threshold cross section data for the $pp\rightarrow K^+ \Sigma^+ n$ reaction were published by the COSY-11 Collaboration [1]. Surprisingly, it turned out that this cross section is larger than the one for $pp\rightarrow K^+ \Sigma^0 p$ by a factor of around 230 at the excess energy $\epsilon=13$ MeV and by a factor of around 90 at $\epsilon=60$ MeV. The excess energy $\epsilon$ is defined as $\epsilon=\sqrt{s}-m_K - m_{\Sigma^-} - m_N$, where $s$ is the squared invariant collision energy, while $m_K$, $m_{\Sigma}$ and $m_N$ are the masses of the kaon, the $\Sigma$ hyperon and the nucleon, respectively. It is also worth mentioning that none of the available model calculations [2,3,4,5,6] is able to describe those data. In fact, most of those models underestimate the cross section by an order of magnitude or even more.

Besides this rather large value for the production cross section as compared to the $\Sigma^0 p$ channel, the new results for the $pp\rightarrow K^+ \Sigma^+ n$ reaction are also somewhat startling when compared with the available high energy data. Indeed, one can find only five data points [7,8,9,10] for $pp\rightarrow K^+ \Sigma^+ n$ at higher energies in the literature. Moreover, those data show large fluctuations, even considering the large experimental uncertainties, and two of those points [9] were reported only in a preprint. But it is still obvious that the COSY-11 result at $\epsilon=60$ MeV [1] is as large as the $pp\rightarrow K^+ \Sigma^+ n$ cross section measured at higher energies [7,8,10], suggesting that there could be practically no energy dependence over the large energy region $60\leq \epsilon \leq 1000$ MeV with the mean cross section being $49 \pm 5 \, \mu b$. That is a rather unexpected result since the cross section of $pp\rightarrow K^+ \Sigma^0 p$, the only well investigated $\Sigma$ production channel, shows a significant energy dependence, as expected from the increasing phase space for the reaction. Indeed here the cross section changes by a factor of about 40 within the energy range indicated above.

The data points [7,8,9,10] at high energies are obtained from bubble chamber images where the identification of the $pp\rightarrow K^+ \Sigma^+ n$ as well as the $pp\rightarrow K^+ \Sigma^0 p$ reaction channel was done simultaneously and unambiguously. Therefore, these results at high energies might be fairly reliable. The situation with regard to the more recent counter experiments is different. Here the $pp\rightarrow K^+ \Sigma^+ n$ channel was often not considered because of the substantial difficulties in the final particle identification. The COSY-11 collaboration reconstructs the kaon and neutron four-momenta and identifies the $\Sigma^+$-hyperon by the missing mass. It was found [1] that the large background under the $\Sigma^+$-signal complicates the data analysis considerably and it introduces large uncertainties. A much better, i.e. direct, identification of the $\Sigma^+$ can be done by detecting the $\Sigma^+\rightarrow p\pi^0$ decay mode, though then a photon detector is required. Indeed, a corresponding experiment has already proposed [11] for the WASA detector [12] at the COSY facility.

With the present paper we want to supply some more values for the $pp\rightarrow K^+ \Sigma^+ n$ cross section to the data base. For that aim we utilize available data on inclusive $K^+$-meson momentum spectra measured at different angles in $pp$ collisions for the reaction $pp\rightarrow K^+ X$. Since the experimental $K^+$-meson momentum spectra [13,14,15] are available at energies that lie between the data of the COSY-11 Collaboration and the high energy data, the result of our analysis allows conclusions on the behavior of the $pp\rightarrow K^+ \Sigma^+ n$ cross section in this interesting energy region. Some of the spectra are available at energies that overlap with the bubble chamber results [7] and, therefore, we can also check whether the results based on our method are compatible with the high energy measurement. As a byproduct we also provide cross sections for the $pp\rightarrow K^+ Ap$ reaction and compare them with direct measurements, where the latter are based on the reconstruction of the final particles.
The paper is organized as follows: In Sec. 2 we describe the method. The analysis of the data is presented in Sec. 3. Our results are compared to other available data in Sec. 4. The paper ends with a short summary.

2 Method for the data evaluation

In this section we describe in detail the method for the data analysis. For completeness we include all relevant formulas, although some of them are given in Ref. [16]. Furthermore, since this method is not limited specifically to pp→K+Σ+n but applicable to any reaction with a three-body final state, we provide the formalism in a general form. The cross section for the a+b→1+2+3 reaction is given by

\[
\sigma = \frac{1}{2\pi^2 \lambda^{1/2}(s, m_a^2, m_b^2)} \int \frac{d^3p_1 \, d^3p_2 \, d^3p_3}{2E_1 \, 2E_2 \, 2E_3} \times \delta(P_1 + P_2 + P_3 - P_a - P_b) |A|^2 , \tag{1}
\]

where \( p_i \) and \( E_i \) are the 3-momentum and the energy of the \( i \)-th particle, respectively, while \( P_i \) stands for the 4-momentum. \( A \) denotes the reaction amplitude and the \( \lambda \)-function is defined by \( \lambda(x, y, z) = (x - y - z)^2 - 4yz \). We use the invariants

\[
s = p^2 = (P_a + P_b)^2, \quad s_Q = Q^2 = (P + P_3)^2 - (P - P_i)^2 \tag{2}
\]

where \( s_Q \) is the squared missing mass with respect to the first particle, which is identical to the squared invariant mass of the second and third particle. The Lorentz invariant differential cross section for the production of particle 1 is then written as

\[
\frac{E_1 \, d^3\sigma}{p_1^2 \, dp_1 \, d\Omega_1} = \frac{1}{2\pi^2 \lambda^{1/2}(s, m_a^2, m_b^2)} \int \frac{d^3p_2 \, d^3p_3}{2E_2 \, 2E_3} \times \delta(P_2 + P_3 - Q) |A|^2 \cdot \frac{1}{s_Q} \lambda^{1/2}(s, m_a^2, m_b^2) \times \frac{1}{s_Q} (s_Q, m_a^2, m_b^2) |A|^2. \tag{3}
\]

In the laboratory frame, i.e. for \( P_b = (0, m_b) \), \( s_Q \) can be expressed as

\[
s_Q = s + m_b^2 - 2(E_a + m_b)E_1 + 2p_{a1} \cos \theta_1 , \tag{4}
\]

where \( \Omega_1 \) and \( \theta_1 \) are the solid and polar production angle of the first particle. In Eq. (3), \( |A|^2 \) is the square of the reaction amplitude integrated over the kinematical variables related to the second and third particle. In general \( |A|^2 \) depends on \( p_1 \) (or \( s_Q \)), \( \Omega_1 = (\theta_1, \phi_1) \) and \( s \).

The relation between the differential momentum spectrum measured at the solid angle \( \Omega_1 \) and the missing mass \( (M_X) \) spectrum, where \( M_X^2 = s_Q \), is given by

\[
\frac{d^3\sigma}{dM_X \, d\Omega_1} = \sqrt{s_Q} \left[ \frac{E_a + m_b}{\sqrt{p_1^2 + m_b^2}} p_1 - p_a \cos \theta_1 \right]^{\frac{1}{3}} \frac{d^3\sigma}{dp_1 \, d\Omega_1}. \tag{5}
\]

In order to exemplify how we proceed in our analysis let us consider here two typical data samples for the pp→K+X reaction. One is from a measurement at the beam energy \( T_p = 2.3 \) GeV and the \( K^+ \)-production angle \( \theta_K = 10.3^\circ \) [14] and the other at \( T_p = 2.54 \) GeV and \( \theta_K = 30^\circ \) [13]. Both data sets are shown in Fig. 1 where the left panel illustrates the \( K^+ \)-meson momentum spectra, while the right panel shows the missing mass spectra obtained by Eq. (5). The arrows in Fig. 1 indicate the \( K\Sigma N \) and \( K\Lambda N \pi \) reaction thresholds, respectively. Below the \( K\Sigma N \) threshold, i.e. for \( M_X \leq m_{\Sigma^+} + m_N \), kaon production is primarily due to the pp→K+Ap reaction, though contributions from that channel with additional photons (K+Apγ, etc.) are also possible. The contributions to the missing mass spectrum for \( m_{\Sigma^+} + m_N \leq M_X \leq m_{\Lambda} + m_N + m_\pi \) come from the pp→K+Ap, pp→K+Σ0p and pp→K+Σ+n reaction channels and again channels with additional photons. Thus, by subtracting the contribution of pp→K+Ap in that invariant-mass region one can extract the sum of the pp→K+Σ0p and pp→K+Σ+n channels, under the assumption that the reactions with photons provide a negligible contribution. For a reliable estimation of the pp→K+Ap contribution it is crucial to know the \( K^+ \)-meson spectra below the \( K\Sigma N \) threshold. Then one can fix the pp→K+Ap channel directly from those data and use that result for the extrapolation to the invariant-mass region where the \( \Sigma \) channels are open. As is clear from Fig. 1, in some of the available experiments there are only a few data points below the \( K\Sigma N \) threshold. In such a case there is a sizable uncertainty in the data evaluation, which will be reflected, eventually, in the error bars of the corresponding results. This uncertainty is somewhat reduced if there is at least a clean signal of the opening of the \( K\Sigma N \) threshold in the spectrum, which is the case for most of the data.

The pp→K+Σ0p reaction can be well identified by the detection of the final particles and it has been intensively investigated [26,27]. This information can then be used to deduce the contribution of the pp→K+Σ+n channel from the missing mass spectra of inclusive \( K^+ \)-meson production in the region \( m_{\Sigma^+} + m_N \leq M_X \leq m_{\Lambda} + m_N + m_\pi \) for various beam energies. As is obvious from Fig. 1, while some experiments provide information on the \( K^+ \)-meson momentum spectrum over basically the whole available phase space, this is not the case with others. These limitations again introduce an uncertainty in the data analysis.

For fixing the contribution of the pp→K+Ap reaction channel to the missing mass spectrum we utilize Eqs. (3) and (5). Specifically, we determine the reaction amplitude \( |A| \) from the data for \( M_X \leq m_{\Sigma^+} + m_N \) and use that value for the extrapolation to \( M_X \geq m_{\Lambda} + m_N + m_\pi \). Since it is well known that the \( \Lambda \) final-state interaction (FSI) is sizable [17,18] we take it into account explicitly. This is done by assuming [19,20] that the reaction amplitude \( A \) can be factorized into a practically momentum and energy independent elementary production amplitude \( A_0 \) and an FSI factor, \( A_{\text{FSI}} \), where the latter is calculated within the Jost-function approach [21]. Details are summarized in Appendix A. As mentioned there, in our analysis we use Jost-function parameters (or equivalently, effective range parameters) from a global fit to the reaction pp→K+Ap. In principle, one could try to determine those parameters for each data set separately by using the corresponding \( M_X \)-spectra [22,23,24]. But then one would require \( M_X \)-spectra with rather good mass resolution and statistical accuracy, which is not always the case as seen in Fig. 1, for \( T_p = 2.54 \) GeV. Consequently, only
some of the data could be analyzed by including FSI effects that are determined by fitting directly to those data themselves.

Thus, we only fix the elementary production amplitude $A_0$ by a fit to the corresponding data for $M_X \leq m_\Sigma + m_N$. In practice we determine the constant $|A_0|$ for each angle and beam energy where experimental $K^+$-meson momentum and missing mass spectra are available. Then, by averaging the obtained values for $|A_0|$ (at a specific energy) over the angular dependence we deduce a result for $|A_0|^2$. The latter quantity can then be compared with the amplitudes deduced from directly measured $pp \to K^+\Lambda p$ cross sections, cf. the discussion in A and the results presented in Sect. 4. This allows us to examine whether the results we extracted from the measured invariant-mass spectra are consistent with the experimental information on the total $A$ production cross sections.

Let us now come back to the $pp \to K^+X$ invariant mass spectrum. The dotted lines in Fig. 1 show results of a calculation based on Eqs. (3) and (5) with the FSI included via Eq. (8) and the squared reaction amplitude $|A_0|$ appropriately adjusted to the spectra at $M_X \leq m_\Sigma + m_N$. The description of the $K^+$-meson momentum and missing mass spectra in terms of the contribution from the $pp \to K^+\Lambda p$ reaction looks reasonable. Note that so far we have neglected possible contributions from the reactions with photons in the final state, i.e. $pp \to K^+\Lambda p\gamma$, $pp \to K^+\Lambda p\gamma\gamma$ etc. However, judging from the measurement where a decent number of data points is available for $M_X < m_\Sigma + m_N$, there is not much room for such additional contributions anyway.

In order to estimate the uncertainty that could arise from our treatment of the $Ap$ FSI we consider also an alternative procedure. We perform a fit to the $K^+$ invariant mass spectrum without FSI, i.e. with pure phase space. But in this case we consider only data points that lie in an energy interval of about 30 MeV from the $K\Sigma N$ threshold downwards for the determination of the reaction amplitude $|A_0|$ at the various angles and energies. The data points closer to the $K\Lambda N\pi$ threshold exhibit, in general, such obvious FSI effects that it is meaningless to try to fit them with pure phase space. The dashed lines in Fig. 1 show those results obtained without inclusion of the $Ap$ FSI. We will use the predictions of those fits for the $K\Lambda N$ invariant mass spectrum in the region $m_\Sigma + m_N \leq M_X \leq m_\Lambda + m_N + m_\pi$.
for extracting the $\Sigma$ production cross section too. However, we want to emphasize already at this stage that we consider the extrapolation based on the fit that includes the $Ap$ FSI as much more reliable and, therefore, we consider the $\Sigma$ cross sections deduced from that fit as our definitive results.

Once the contribution from the $pp\to K^+Ap$ channel is established we subtract it from the $pp\to K^+X$ data in the region $m_{\Sigma^0}+m_{\eta} \leq M_X \leq m_A+m_N+m_\pi$ in order to obtain the sum of the contributions from the $pp\to K^+\Sigma^0p$ and $pp\to K^+\Sigma^+n$ reactions. Possible additional contribution from channels with photons in the final state are again neglected. Utilizing again Eqs. (3) and (5) we determine the corresponding (combined) $\Sigma$ amplitude $|A_0|$ for each angle and total energy where experimental $K^+$ mass spectra are available, etc. However, unlike in the $K^+Ap$ channel, now we do not include an FSI factor in the fitting procedure. Indeed, none of the available data sets exhibits a pronounced enhancement near the $K^+\Sigma N$ threshold that would warrant the inclusion of FSI effects. The solid lines in Fig. 1 show the final result, i.e. the contribution from the $pp\to K^+Ap$ reaction plus the fitted contribution from the $pp\to K^+\Sigma N$ channels.

### 3 Data analysis

In the present paper we analyze the measured $K^+$-meson momentum spectra published in Refs. \[13\] \[14\] \[15\]. The achieved results are summarized in Tables 1 and 2. In order to stay as close as possible to physical quantities we do not list the values obtained for the amplitudes $|A_0|$ but the corresponding cross sections. However, since those amplitudes correspond to data at different angles it is obvious that the given values are not totally real cross sections. Rather, they represent cross sections for specific angles, appropriately normalized to the full solid angle. In order to remind the reader on that we put the superscript $\theta$ on the corresponding symbols ($\sigma_A^\theta$ or $\sigma_0^\theta$).

Comparing the resulting values for $\sigma_A^\theta$ and $\sigma_0^\theta$ at different angles (at a specific energy) allows conclusions on the angular dependence of the reaction. For facilitating an easy general examination of that dependence we introduce the quantity $\xi$ which is the ratio of $\sigma_A^\theta$ and the corresponding (genuine) total cross section $\sigma_A$ obtained from the reference amplitude \[16\] in conjunction with Eq. \[11\]. Evidently, if there is full consistency between the latter parametrization of the experimental total cross section and the result stemming from our evaluation of the missing mass spectrum then the average of $\xi$ over the kaon angles would amount to unity.

Note that we have neglected the difference between the $p$ and $n$ masses and between the $\Sigma^0$ and $\Sigma^+$ masses in calculating the excess energies. These are inessential at the high reaction energies we are dealing with here.

As already mentioned, a major source for systematical uncertainties in the data analysis by the method described above is due to the extrapolation of the $K^+Ap$ mass spectrum to the region $m_{\Sigma^0}+m_{\eta} \leq M_X \leq m_A+m_N+m_\pi$. It is clear from Fig. 1 that the parametrization including the $Ap$ FSI still differs from the pure phase-space behavior in that region and, therefore, it affects the absolute value of the extracted $\Sigma$ production cross section. Thus, in order to estimate the uncertainty due to the extrapolation we determine $\sigma_Y^\theta$ (by a fit based on Eqs. (3) and (5)) for two scenarios: We subtract the contribution from the $pp\to K^+Ap$ (i) including FSI effects as shown by the dotted line in Fig. 1 and (ii) without FSI as given by the dashed line in Fig. 1. Both results for $\sigma_Y^\theta$ are given in Table 2.

We should mention that even within the Jost-function approach it is not always possible to reproduce the $M_X$-spectra around the $K^+Ap$ threshold in a perfect way. That might be a problem related to the use of the Jost function formalism or simply due to uncertainties of the parameters $\alpha$ and $\beta$ \[11\] used in Eq. \[9\]. In any case, such more subtle aspects of the $Ap$ FSI do not influence the shape of the $M_X$-spectra above the $K^+\Sigma N$ threshold significantly and are, therefore, not relevant for us.

Let us now discuss the different data sets for the $pp\to K^+X$ reaction one by one. Fig. 2 shows the missing mass spectra from Ref. \[14\] at the proton beam energy of $T_p=2.3$ GeV and kaon production angles of $\theta_K=8.3^\circ$, $10.3^\circ$ and $12^\circ$. The dashed lines are the results for the $pp\to K^+Ap$ reaction fitted with a constant reaction amplitude $|A_0|$ alone, while the dotted lines indicate corresponding results including the $Ap$ FSI.

The $M_X$-spectra exhibit a substantial enhancement close to the $K^+Ap$ threshold that originates from the $Ap$ FSI. Note that the shape of the near threshold spectra depends somewhat on the $K^+$-meson production angle and is not reproduced perfectly by using the Jost function (Eq. \[9\]), especially at the angles $\theta_K=8.3^\circ$ and $12^\circ$. As was shown in Ref. \[22\] the $M_X$-dependence generated by Eq. \[9\] can be varied by changing the parameters $\alpha$ and $\beta$ and, in principle, it is possible to achieve a better description of the spectra around the $K^+Ap$ threshold by allowing $\alpha$ and $\beta$ to depend on the $\theta_K$ angle. However, all those variations have only a marginal influence on the description of the missing mass spectra above the $K^+\Sigma N$ threshold, which is the region we are interested in in the present analysis.

The solid lines in Fig. 2 show the sum of the $pp\to K^+Ap$, $pp\to K^+\Sigma^0p$ and $pp\to K^+\Sigma^+n$ channels where the contribution of the latter two channels was determined by a fit to the difference between the experimental spectra and the contribution from the $pp\to K^+Ap$ reaction (including FSI effects) via Eqs. (3) and (5). In Tables 1 and 2 we list the corresponding values for $\sigma_A^\theta$ and $\sigma_0^\theta$ determined from the $M_X$-spectra for the cases with and without inclusion of the $Ap$ FSI. The quality of the least-square fit can be judged from the given reduced $\chi^2$. As can be seen from the table, there is practically no difference between the results obtained with and without $Ap$ FSI. This is primarily due to the fact that there are sufficient and accurate data on the mass spectrum below the $K^+\Sigma N$ threshold. Moreover, this threshold is clearly mapped out.

Fig. 3 shows the $M_X$-spectra from Ref. \[15\] at the proton beam energy of $T_p=2.4$ GeV and kaon production angles of $\theta_K=0^\circ$ and $17^\circ$. The notation for the lines are the same as in Fig. 2. Here the $K^+\Sigma N$ threshold is hardly visible in the data, especially at the larger angle, and accordingly there are huge differences in the extracted cross sections between the scenarios with and without $Ap$ FSI, cf. Tables 1 and 2. Ultimately, this is also reflected in the large error bars for the extracted value of $\sigma_0^\theta$. Please recall that the $\Sigma$ contributions are fitted to the invariant mass spectrum in the range $m_{\Sigma^0}+m_{\eta} \leq M_X \leq m_A+m_N+m_\pi$, i.e. between the arrows shown in the fig...
ures, which explains why the corresponding curves are in line with the data in that region but deviate from those at higher values.

Fig. 4 shows the missing mass spectra from Ref. [13] at the proton beam energy of $T_p=2.54$ GeV and kaon production angles of $\theta_K=20^\circ$, $30^\circ$ and $40^\circ$. There are only few points below the $K^+\Sigma N$ threshold and from those it is hard to see whether there is actually an enhancement due to the $\Lambda p$ FSI. Note that at this specific energy the description of the missing mass spectra for large $K^+$-meson production angles is very good, in particular, also for the data points above the $K^+\Lambda N\pi$ threshold. Thus, there seems to be not much room for contributions from the reaction channel with an additional pion.

The missing mass spectra from Ref. [14] at the proton beam energy of $T_p=2.7$ GeV and kaon production angles of $\theta_K = 12.6^\circ$, $16.1^\circ$, $20^\circ$ and $23.5^\circ$ are shown in Fig. 5. The data at

### Table 1. Analysis of available data on $K^+$-meson inclusive momentum spectra from the $pp\to K^+X$ reaction: Results for $pp\to K^+\Lambda p$.

| Reference | $T_p$ (GeV) | $\epsilon$ (MeV) | $\theta_K$ (degrees) | $\sigma^o_{\Lambda p}$ (µb) | $\chi^2$/ndf | $\sigma^o_{\Lambda p}$ (µb) | $\chi^2$/ndf |
|-----------|-------------|------------------|----------------------|-----------------|--------------|-----------------|--------------|
| [14]      | 2.3         | 252              | 8.3                  | 23.9±0.7        | 1.3          | 14.7±0.9        | 0.9          |
| [14]      | 2.3         | 252              | 10.3                 | 21.3±0.6        | 1.3          | 18.0±1.0        | 1.6          |
| [14]      | 2.3         | 252              | 12.0                 | 20.0±0.6        | 1.2          | 19.5±1.0        | 2.1          |
| [15]      | 2.4         | 285              | 0                    | 77.0±8.9        | 0.5          | 36.1±10.8       | 0.1          |
| [15]      | 2.4         | 285              | 17                   | 52.8±6.7        | 0.2          | 50.6±13.2       | 1.2          |
| [13]      | 2.54        | 331              | 20                   | 31.1±3.7        | 0.5          | 41.4±2.5        | 0.1          |
| [13]      | 2.54        | 331              | 30                   | 20.5±2.1        | 0.1          | 45.7±1.8        | 0.2          |
| [13]      | 2.54        | 331              | 40                   | 24.2±2.3        | 0.2          | 42.4±2.8        | 0.4          |
| [14]      | 2.7         | 383              | 12.6                 | 32.7±0.3        | 0.6          | 36.1±10.8       | 0.1          |
| [14]      | 2.7         | 383              | 16.1                 | 30.1±0.7        | 0.55         | 19.5±1.0        | 2.1          |
| [14]      | 2.7         | 383              | 20                   | 30.3±0.4        | 0.43         | 19.5±1.0        | 2.1          |
| [14]      | 2.7         | 383              | 23.5                 | 20.9±0.5        | 0.4          | 18.7±0.6        | 1.1          |
| [15]      | 2.85        | 431              | 0                    | 120.3±10.8      | 1.7          | 88.0±11.2       | 0.9          |
| [15]      | 2.85        | 431              | 17                   | 39.6±4.8        | 0.5          | 25.9±4.2        | 0.2          |
| [15]      | 2.85        | 431              | 32                   | 28.6±6.1        | 0.5          | 25.9±5.3        | 0.2          |

### Table 2. Analysis of available data on $K^+$-meson inclusive momentum spectra from the $pp\to K^+X$ reaction: Results for $pp\to K^+\Sigma N$.

| Reference | $T_p$ (GeV) | $\epsilon$ (MeV) | $\theta_K$ (degrees) | $\sigma^o_{\Sigma N}$ (µb) | $\chi^2$/ndf | $\sigma^o_{\Sigma N}$ (µb) | $\chi^2$/ndf |
|-----------|-------------|------------------|----------------------|-----------------|--------------|-----------------|--------------|
| [14]      | 2.3         | 178              | 8.3                  | 13.7±1.0        | 1.3          | 14.7±0.9        | 0.9          |
| [14]      | 2.3         | 178              | 10.3                 | 16.9±1.1        | 1.5          | 18.0±1.0        | 1.6          |
| [14]      | 2.3         | 178              | 12.0                 | 20.0±1.2        | 1.2          | 19.5±1.0        | 2.1          |
| [15]      | 2.4         | 211              | 0                    | 32.5±12.9       | 0.5          | 36.1±10.8       | 0.1          |
| [15]      | 2.4         | 211              | 17                   | 19.0±6.1        | 0.2          | 50.6±13.2       | 1.2          |
| [13]      | 2.54        | 257              | 20                   | 32.0±2.4        | 0.6          | 41.4±2.5        | 0.1          |
| [13]      | 2.54        | 257              | 30                   | 41.0±1.8        | 0.1          | 45.7±1.8        | 0.2          |
| [13]      | 2.54        | 257              | 40                   | 34.2±2.8        | 3.5          | 42.4±2.8        | 0.4          |
| [14]      | 2.7         | 309              | 12.6                 | 37.9±0.8        | 5.7          | 47.8±0.5        | 7.6          |
| [14]      | 2.7         | 309              | 16.1                 | 50.4±1.4        | 2.9          | 54.2±1.4        | 2.6          |
| [14]      | 2.7         | 309              | 20                   | 51.1±0.9        | 5.5          | 62.7±0.9        | 4.2          |
| [14]      | 2.7         | 309              | 23.5                 | 28.4±1.5        | 1.7          | 30.6±1.1        | 1.6          |
| [15]      | 2.85        | 357              | 0                    | 60.1±6.8        | 1.3          | 82.5±14.3       | 0.1          |
| [15]      | 2.85        | 357              | 17                   | 76.7±10.0       | 0.5          | 73.4±14.9       | 0.3          |
| [15]      | 2.85        | 357              | 32                   | 23.4±7.8        | 2.0          | 20.0±8.6        | 0.8          |
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Fig. 2. Experimental missing mass spectra obtained by Eq. (5) from the $K^+$-meson momentum spectra of the $pp\to K^+X$ reaction [13] at the proton beam energy of $T_p=2.3$ GeV at different kaon production angles $\theta_K$. The arrows indicate the $K\Sigma N$ and $K\Lambda N\pi$ reaction thresholds, respectively. The dotted lines show calculations based on Eqs. (3) and (5) for the $pp\to K^+\Lambda p$ reaction with $|A_0|$ fitted to the data for $M_X\leq m_\Sigma+m_N$ and $\Lambda p$ FSI effects included via Eq. (8). The dashed lines are result obtained without inclusion of the $\Lambda p$ FSI. The solid lines are the sum of the $pp\to K^+\Lambda p$, $pp\to K^+\Sigma^0 p$ and $pp\to K^+\Sigma^+ n$ cross sections where the sum of the cross sections for the two $\Sigma$-hyperon channels was fitted to the data for $m_\Sigma+m_N\leq M_X\leq m_\Lambda+m_N+m_\pi$.

Fig. 3. Experimental missing mass spectra from the $pp\to K^+X$ reaction [15] at the proton beam energy of $T_p=2.4$ GeV and at different kaon production angles $\theta_K$. Same description of curves as in Fig. 2.

$\theta_K=12.6^\circ$ and $\theta_K=20^\circ$ indicate an enhancement due to the $\Lambda p$ FSI and are well reproduced.

Fig. 6 shows the missing mass spectra from Ref. [15] at the proton beam energy of $T_p=2.85$ GeV and kaon production angles of $\theta_K=0^\circ$, $17^\circ$ and $32^\circ$. The $M_X$-spectrum at $\theta_K=0^\circ$ indicates a very strong enhancement due to the $\Lambda p$ FSI. On the other hand, it is somewhat disturbing that there are also data points below the $K^+\Lambda p$ threshold, i.e. outside of the kinematically allowed region. As stated in Ref. [15], this could be due to the momentum resolution of the measurement. In any case, those questionable events do not influence the extraction of the $\Sigma$ production cross section from the data.

4 Results and discussion

From the results collected in Tables 1 and 2 one can see that the fit to some of the data yielded a rather small $\chi^2$, reflecting the large statistical and systematical uncertainties of the experiments. The fit to the data at $T_p=2.7$ GeV, on the other hand, leads to a rather large $\chi^2$ because we assume that the missing mass spectra are smooth and, therefore, we cannot describe the large fluctuation of the data for which very small statistical errors are given, as can be seen in Fig. 5.

Let us first comment on the $\xi$ factor, the indicator for the $\theta_K$-angular dependence of the $pp\to K^+\Lambda p$ reaction amplitude. The spectra available at $\theta_K=0^\circ$ indicate a large $\xi$ and, therefore, a strong forward peaking of $|A_0|$. All other data show
Fig. 4. Experimental missing mass spectra from the \(pp \to K^+ X\) reaction [13] at the proton beam energy of \(T_p = 2.54\) GeV and at different kaon production angles \(\theta_K\). Same description of curves as in Fig. 2.

Fig. 5. Experimental missing mass spectra from the \(pp \to K^+ X\) reaction [14] at the proton beam energy of \(T_p = 2.7\) GeV and different kaon production angles \(\theta_K\). Same description of curves as in Fig. 2.

A smooth \(\theta_K\)-dependence within the range \(8.3^\circ \leq \theta_K \leq 32^\circ\). In general, for \(\theta_K \neq 0^\circ\) the factor \(\xi\) is about 0.4–0.8 so that the corresponding angle-averaged amplitude is smaller than the reference amplitude for the \(pp \to K^+ \Lambda p\) reaction computed from Eq. (16), which represents a fit to the measured total reaction cross section. Corresponding results are displayed in the left panel of Fig. 7. We want to point out, however, that for excess energies \(170 < \epsilon < 360\) MeV the uncertainty for \(|A_0|^2\) as determined directly from available data points from bubble chamber measurements is also in the order of a factor of \(\approx 2\), cf. the squares in Fig. 7 Therefore, we conclude that there is a reasonable consistency between the squared reaction amplitudes deduced from the missing mass and the values obtained from direct measurements.

Based on the values for \(\sigma^\theta\) at the same excess energies and for different \(\theta_K\) from Table 2 we can calculate the mean value and the standard deviation for the \(pp \to K^+ \Lambda p\) total reaction cross section, cf. Table 3 and the right panel of Fig. 7 (circles). Although for some energies the standard deviations are very large, there is reasonable overall agreement between the results.
extracted from the missing mass spectra and the cross section data from direct measurements.

The sum of the $pp \rightarrow K^+ \Sigma^0 p$ and $pp \rightarrow K^+ \Sigma^+ n$ cross sections ($\sigma^0_{\Sigma^0}$), extracted from the missing spectra, shows a more significant angular dependence only at the highest excess energy of $\epsilon=356.8$ MeV, cf. Table 2. For the other energies the dependence of $\sigma^0_{\Sigma^0}$ on the $K^+$-meson production angle is relatively smooth, at least within the uncertainties of the extracted values. From Table 2 one can also see that neglecting the $\Lambda p$ FSI in the extraction procedure yields results for $\sigma^0_{\Sigma^0}$ which are, in general, somewhat smaller. But it is reassuring to see that in most of the cases the error bars obtained for the $\sigma^0_{\Sigma^0}$’s with and without inclusion of the $\Lambda p$ FSI overlap, indicating that the results are not too sensitive to the specific subtraction prescription. There are only a few cases where there are indeed dramatic differences between the values for the two considered options. As already said, we consider the extrapolation based on the fit that includes the $\Lambda p$ as much more reliable and, therefore, we consider the $\Sigma$ cross sections deduced from that fit as our definitive results. Averaging over the $\sigma^0_{\Sigma^0}$ values extracted at different $K^+$-meson angles we can now evaluate the $\Sigma$ production cross section, $\sigma_{\Sigma}$, and determine the standard deviation for the results obtained from the fit to the $M_X$-spectra. The corresponding values, for the case where the $\Lambda p$ FSI was taken into account, are listed in Table 4.

In order to deduce the $pp \rightarrow K^+ \Sigma^+ n$ cross section one needs to subtract from $\sigma_{\Sigma}$ the $pp \rightarrow K^+ \Sigma^0 p$ cross section. In Fig. 8a we show the existing data for the reaction $pp \rightarrow K^+ \Sigma^0 p$ as a function of the excess energy. The circles are from the COSY-11 Collaboration [26,27], while the open squares are bubble
Table 3. Results for the total \(pp \rightarrow K^+\Lambda p\) cross section \(\sigma_A\). The excess energy \(\epsilon\) is given with respect to the \(\Lambda\)-hyperon production threshold. Our results, extracted from the missing mass spectra, averaged over the \(\theta_K\) angles, are listed in the upper part of the table. The error bars are computed from the corresponding standard deviation. Experimental results from direct measurements in a comparable energy region and the corresponding references are listed below.

| \(\epsilon\) (MeV) | \(\sigma_A\) (\(\mu b\)) | our evaluation | missing mass spectrum from Ref. |
|-------------------|--------------------------|----------------|---------------------------------|
| 252               | 22.2\(\pm\)1.5           |                | [14]                            |
| 285               | 52.1\(\pm\)20.8          |                | [15]                            |
| 331               | 18.2\(\pm\)2.9           |                | [12]                            |
| 383               | 23.1\(\pm\)3.8           |                | [14]                            |
| 431               | 46.6\(\pm\)29.3          |                | [15]                            |

The direct measurements are Ref.
| 138                   | 12.0\(\pm\)0.4          |                | [28]                            |
| 157                   | 18\(\pm\)5             |                | [39]                            |
| 431                   | 51\(\pm\)12            |                | [7]                             |

Table 4. Results for the \(\Sigma\) production cross sections. The excess energy \(\epsilon\) is given with respect to the \(\Sigma\)-hyperon production threshold. \(\sigma_\Sigma\) is the total \(\Sigma\) production cross section extracted from the missing mass spectra, averaged over the \(\theta_K\) angles. The error bars are computed from the corresponding standard deviation. The cross section for \(\sigma(K^+\Sigma^0p)\) was obtained via Eq. (11), utilizing the parametrization of the cross data for that channel from direct measurements as given in Eq. (6). The cross section for \(\sigma(K^+\Sigma^+n)\) is identified with the difference between \(\sigma_\Sigma\) and \(\sigma(K^+\Sigma^0p)\). Listed are also experimental results from direct measurements of the \(pp \rightarrow K^+\Sigma^0p\) and \(pp \rightarrow K^+\Sigma^+n\) channels and the corresponding references. Note that \(\sigma(K^+\Sigma^0p)\) at \(\epsilon = 727.6\) MeV is taken from Ref. [31] and those at 13 and 60 MeV are taken fromRefs. [25] (preprint) and [27], respectively.

| \(\epsilon\) (MeV) | \(\sigma_\Sigma\) (\(\mu b\)) | \(\sigma(K^+\Sigma^0p)\) (\(\mu b\)) | \(\sigma(K^+\Sigma^+n)\) (\(\mu b\)) | our evaluation | missing mass spectrum from Ref. |
|-------------------|--------------------------|----------------|----------------|----------------|---------------------------------|
| 178               | 17.7\(\pm\)2.3           | 4.0\(\pm\)0.3  | 13.7\(\pm\)2.3 |                | [14]                            |
| 212               | 43.4\(\pm\)7.3           | 5.2\(\pm\)0.5  | 38.2\(\pm\)7.3 |                | [15]                            |
| 258               | 43.2\(\pm\)1.8           | 6.9\(\pm\)0.7  | 36.3\(\pm\)1.9 |                | [13]                            |
| 309               | 48.8\(\pm\)11.8          | 8.9\(\pm\)1.0  | 39.9\(\pm\)11.8|                | [14]                            |
| 357               | 58.6\(\pm\)27.6          | 10.6\(\pm\)1.3 | 48.6\(\pm\)27.6|                | [15]                            |

The direct measurements are Ref.
| 13                   | 0.020\(\pm\)0.003        | 4.56\(\pm\)0.94\(\pm\)2.7  |                | [1]                             |
| 60                   | 0.482\(\pm\)0.144        | 44.8\(\pm\)10.7\(\pm\)15.2 |                | [1]                             |
| 357                 | 13\(\pm\)7              | 47\(\pm\)13                     |                | [7]                             |
| 728                 | 25\(\pm\)3              | 48.1\(\pm\)3.5                  |                | [8]                             |
| 849                 | 27\(\pm\)4              | 85\(\pm\)12                    |                | [9]                             |
| 1006                | 17\(\pm\)4              | 57\(\pm\)7                     |                | [10]                            |
| 1156                | 25\(\pm\)3              | 85\(\pm\)11                    |                | [9]                             |

chamber data [29]. Presently there are no experimental results available for 60\(\leq\)\(\epsilon\)\(\leq\)360 MeV, i.e. for the energy range of our analysis. The measurements from the TOF-Collaboration that cover the energy range of our interest are still at the stage of being analysed [30]. Therefore, to proceed further, we fit the \(pp \rightarrow K^+\Sigma^0p\) total reaction cross section by Eq. (11) with \(\kappa = 1\), i.e. by neglecting the \(\Sigma^0p\) FSI, and with the appropriate kinematics for the \(K^+\Sigma^0p\) channel. The resulting squared reaction amplitude is

\[
|A_{\Sigma^0}|^2 = (0.61 \pm 0.03) \cdot \exp[(1.34 \pm 0.2)\epsilon] \cdot 10^{-7} \, (\mu b),
\]

with the excess energy given in GeV. Note that the omission of possible \(\Sigma^0p\) FSI effects is in line with the experimental evidence for the \(pp \rightarrow K^+\Sigma^0p\) channel [26,27] – the available data do not show any visible indication for such a FSI [18,27]. It is also in line with the conclusions we draw from inspecting the experimental mass spectra analysed in the present paper, which likewise exhibit no sign for the presence of a \(\Sigma N\) FSI effects.

The parameterization (6) allows us to calculate the \(pp \rightarrow K^+\Sigma^0p\) cross section for each of the excess energies, where data on the \(pp \rightarrow K^+X\) reaction exist. The corresponding values are listed in Table 4. The last column in Table 4 is the difference between \(\sigma_\Sigma\) and \(pp \rightarrow K^+\Sigma^0p\) cross section, which we identify with the \(pp \rightarrow K^+\Sigma^+n\) cross section. The results are also shown in Fig. 8 (circles). Note that a linear scale is used for displaying the \(pp \rightarrow K^+\Sigma^+n\) cross section!
spectra are available, an UCLA preprint. For the lowest energy where missing mass with 85 $\mu b$ at 0.85 and 1.1 GeV represent the largest reported cross sections of fluctuations. We want to remark that the two points at pp → $K^+\Sigma^0p$ (a) and pp → $K^+\Sigma^+n$ (b) total reaction cross sections as a function of the excess energy. The triangles are data by the COSY-11 collaboration for the $K^+\Sigma^0p$ [25,27] and $K^+\Sigma^+n$ [11] channels, while the squares show bubble chamber data taken from Refs. [7,8,9,10,29,31]. The circles are results for pp → $K^+\Sigma^+n$ obtained from our analysis of the missing mass spectra. The solid lines in both panels show the result of Eq. (1) without $\Sigma^0p$ FSI (i.e. with $\kappa = 1$) and with $|\langle \mathcal{A}_{\Sigma^0} \rangle|^2$ given by Eq. (6). The dashed line in (b) is the same as the solid line, but multiplied by a factor 3.5.

First let us compare our results with the bubble chamber data [7,8,9,10] shown by the open squares in Fig. 8 and listed also in Table 4. The pp → $K^+\Sigma^+n$ cross section of 47±13 $\mu b$ at $\epsilon = 357$ MeV was measured by Louttit et al. [7] and it is in agreement with our result at the same energy, cf. Table 4 – though, unfortunately, at this highest energy of our analysis there is a large uncertainty due to the angular dependence of the extracted cross section as can be seen from Table 2. The pp → $K^+\Sigma^+n$ data [8,9,10] at higher energies indicate large fluctuations. We want to remark that the two points at $\epsilon \approx 0.85$ and 1.1 GeV represent the largest reported cross sections with 85 $\mu b$ (at both energies) and are available [9] only in an UCLA preprint. For the lowest energy where missing mass spectra are available, $\epsilon = 178$ MeV, we deduced a cross section of 13.7±2.3 $\mu b$ from the data. There are two so far unpublished measurements of the pp → $K^+\Sigma^+n$ cross section by the TOF collaboration at somewhat lower energy, i.e. at beam momenta of 2.06 GeV ($\epsilon = 98$ MeV) [32] and 2.157 GeV ($\epsilon = 128$ MeV) [33], respectively. It is worth mentioning that their results are roughly in line with the value we obtained. The results from the COSY-11 Collaboration at low excess energies have been available only recently [11]. Their pp → $K^+\Sigma^+n$ cross section of 44.8±10.7±15.2 at $\epsilon = 60$ MeV is as large as those at high energies, cf. Table 4 and Fig. 8.

For illustration purposes we include the result for the pp → $K^+\Sigma^0p$ cross section also in Fig. 8 (solid line). The dashed line shows the same cross section, but multiplied by a factor 3.5. Obviously its energy dependence is very different from that exhibited by the pp → $K^+\Sigma^+n$ cross section if one considers all available data. However, it is interesting to see that the curve would be roughly in line with the trend of the $K^+\Sigma^+n$ data, including the ones obtained from our analysis, if one disregards the COSY-11 events and the measurements from Refs. [8,10]. The data from the last two references are in clear contradiction to the results from Ref. [9] anyway. On the other hand, one has to keep in mind that the latter data were never officially published.

Provided that all data are indeed correct, it will be difficult to find plausible explanations for the drastically different behavior of the pp → $K^+\Sigma^+n$ cross section. The model calculations [2,3,4,5] available for this reaction channel indicate that the energy dependence of the cross section is similar to the one of pp → $K^+\Sigma^0p$. After all, the energy dependence is to a considerable part determined simply by phase-space factors. Evidently, the model predictions [2,3,4,5] disagree strongly with the new data [11] of the COSY-11 Collaboration. Since those calculations describe the pp → $K^+\Sigma^+n$, pp → $K^+\Sigma^0p$ and pp → $K^0\Sigma^+p$ reactions with the same dynamical input, additional mechanisms can be introduced only by assuming that they contribute to the pp → $K^+\Sigma^+n$ channel alone. But not even a recent study that focusses on the pp → $K^+\Sigma^+n$ reaction only and invokes contributions from the $\Delta(1620)$ resonance is able to describe the COSY-11 data satisfactorily [6]. Of course, possible additional contributions could arise from the excitation of crypto-exotic baryons, as was speculated in Ref. [11], that then decay into the $K\Sigma$ channel. Such crypto-exotic baryons were discussed [34] recently in the context of the new ANKE-COSY results [35] on $\phi$-meson production. An indication for a possible crypto-exotic baryon was also reported in Ref. [36], based on an analysis of the $\Sigma^0K^+$ invariant mass spectrum.

Anyway, instead of embarking on further speculations we believe that it would be more instructive to perform new measurements of the pp → $K^+\Sigma^+n$ reaction in the near-threshold region. The method used in the present paper can be also applied in the analysis of data that can be taken at ANKE [37] and HIRES [38] at COSY. These experimental facilities are perfectly suited for obtaining $K^+$-meson spectra with high statistics and high resolution. Such experiments would also allow to...
shed light on the angular dependence of the reaction amplitude, which we expect to be very weak at low excess energies.

5 Summary

In the present paper we determined the sum of the $pp \rightarrow K^+ \Sigma^+ n$ and $pp \rightarrow K^+ \Sigma^0 p$ cross sections from inclusive $K^+$-meson momentum spectra in the energy range $T_p = 2.3$ - 2.85 GeV, available in the literature. We showed that, after transformation of the momentum spectrum to the missing mass ($M_X$) spectrum, the contribution from the reaction channels with $A$ and $\Sigma$ hyperons can be isolated by inspecting the data between the $K^+ \Lambda p$, $K^+ \Sigma N$, and $K^+ \Lambda N \pi$ thresholds and we demonstrated that the $M_X$-spectra can be well described when taking into account the contributions from the $pp \rightarrow K^+ \Lambda p$, $pp \rightarrow K^+ \Sigma^+ n$ and $pp \rightarrow K^+ \Sigma^0 p$ reactions. The angular dependence of the reaction amplitude was accounted for by fitting the $K^+$-meson spectra at different angles. Total cross sections were deduced by averaging over the angles.

As a test we first determined the $pp \rightarrow K^+ \Lambda p$ cross sections at those excess energies where the invariant mass spectra are available. It turned out that the cross sections extracted by us are roughly in line with results from direct measurements in the same energy region.

Utilizing available information on the $pp \rightarrow K^+ \Sigma^0 p$ cross section, we then deduced total cross sections for the $pp \rightarrow K^+ \Sigma^+ n$ channel. The obtained results were discussed and compared with existing data from direct measurements. At the specific energy $T_p = 2.85$ GeV there is also a data point from a bubble chamber measurement [7] and it was reassuring to see that our result is compatible with that experiment. The cross section obtained for $T_p = 2.3$ GeV is with $13.7 \pm 2.3 \mu b$ considerably smaller than the value found in a recent experiment by the COSY-11 Collaboration at a somewhat lower beam energy. Thus, our new cross section values, together with the already available data, indicate that the energy dependence of the cross section from the reaction $pp \rightarrow K^+ \Sigma^+ n$ could differ drastically from that of the $pp \rightarrow K^+ \Sigma^0 p$ channel. This would be certainly rather surprising. Apparently, further experiments are necessary to confirm this unusual behaviour. If such experiments indeed corroborate the present findings then it is likely that peculiar and potentially exotic mechanisms play a role in the reaction $pp \rightarrow K^+ \Sigma^+ n$.

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A Treatment of the final-state interaction

In the present work we take into account effects of the final-state interaction in the $\Lambda p$ channel. Following standard arguments we assume that the reaction amplitude $A$ can be factorized into a practically momentum and energy independent elementary production amplitude $A_0$ and an FSI factor:

$$ A \approx A_0 \times A_{\Lambda p} \, . $$

The FSI effects are then taken into account within the Jost function approach

$$ \langle A_{\Lambda p} \rangle^2 \approx \frac{q^2 + \alpha^2}{q^2 + \beta^2} \ ,$$

where the momentum $q$ is given by

$$ q = \frac{\lambda^{1/2}(s_Q, m_{\Lambda}^2, m_p^2)}{2\sqrt{s_Q}} \ ,$$

and the parameters $\alpha$ and $\beta$ were taken as

$$ \alpha = -72.3 \text{ MeV} \ , \quad \beta = 212.7 \text{ MeV} \ .$$

These parameters are related to the scattering length $a$ and effective range $r$ of the $\Lambda p$ interaction [17]. To be specific, they correspond to the values $a = -1.8$ fm and $r = 2.8$ fm. The parameters of the $\Lambda p$ FSI that we use here were obtained in Refs. [17] [18] from a global phenomenological analysis of all available data on the reaction $pp \rightarrow K^+ \Lambda p$. But, one should keep in mind that the scattering parameters are not fixed uniquely. Actually, we have shown in Ref. [17] that a large set of different values for the scattering length $a$ and effective range $r$ allows to reproduce the energy dependence of the $pp \rightarrow K^+ \Lambda p$ cross section data. Some of these parameters coincide with results predicted by modern YN models [40-44].

Based on Eqs. (7,8) one can then write the total reaction cross section for the reaction $pp \rightarrow K^+ \Lambda p$ in the form

$$ \sigma_A(\epsilon) = \frac{\Phi_3}{2^{6} \pi^5 \lambda_{1/2}(s, m_{\Lambda}^2, m_p^2)} \langle A_0 \rangle^2 \kappa(\epsilon) \ .$$

Here $\langle A_0 \rangle^2$ is the (angle) averaged reaction amplitude squared while $\kappa$ is a factor that represents the FSI effects. The 3-body phase space is

$$ \Phi_3 = \frac{\pi^2}{4s} \left( \sqrt{s-m_K} \right)^2 \int_{(m_{\Lambda}+m_p)^2}^{(\sqrt{s-m_K})^2} \lambda_{1/2}(s, s_Q, m_K^2) \times \lambda_{1/2}(s_Q, m_{\Lambda}^2, m_p^2) \frac{ds_Q}{s_Q} \ .$$

In the nonrelativistic limit it reduces to

$$ \Phi_3 \rightarrow \frac{1}{2^{7} \pi^2} \left( \frac{\sqrt{m_Km_{\Lambda}m_p}}{m_K + m_{\Lambda} + m_p} \right)^{3/2} \epsilon^2 \ .$$

The non-relativistic form is a good approximation for the $pp \rightarrow K^+ \Lambda p$ reaction up to excess energies of $\epsilon \simeq 1$ GeV and we use it in the present investigation. Also, in this case the factor $\kappa$ can be computed analytically for the Jost function approach (8), and under the assumption that there is only a FSI in the $\Lambda p$ system [17]. It amounts to

$$ \kappa(\epsilon) = 1 + \frac{4 \beta^2 - 4 \alpha^2}{(-\alpha + \sqrt{\alpha^2 + 2 \mu \epsilon})^2} \ ,$$

where

$$ \mu = 2 \epsilon \ .$$
where $\mu$ stands for the reduced mass

$$\mu = \frac{m_{A}m_{p}}{m_{A} + m_{p}},$$

(15)

and the parameters $\alpha$ and $\beta$ for the $A\pi$ FSI given by Eq. (10). Note that $\kappa \equiv 1$ in case that FSI effects are neglected.

The $pp\to K^\pm A\pi$ reaction can be directly identified through the detection of the final particles and, therefore, there are already many precise cross section data available in the literature [21, 22, 23, 24]. These data, displayed in Fig. 4 (right side), allow a straightforward determination of the squared reaction amplitude $|A_0|^2$ by means of Eq. (11). Corresponding results are shown in Fig. 1 on the left hand side. For our purposes it is also convenient to parametrize the experimental cross section by means of a simple function. This is achieved with

$$|A_0|^2 = (1.89 \pm 0.04) \cdot \exp[(1.34 \pm 0.1) \epsilon \cdot 10^{-7} (\mu b)],$$

(16)

where the parameters were determined by a fit to the data in Fig. 1 for energies $\epsilon < 500$ MeV. Here $\epsilon$ is the excess energy in GeV. The corresponding curve, including also the $A\pi$ FSI via Eq. (14), is shown by the solid line in Fig. 1.

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