1. The theoretical background

The experience of mathematical structure may be supported by representing mathematical knowledge graphically in the form of networks. Knowledge maps are means to show ideas and concepts connected with a topic, in a well-structured form. Their special fitting as a pedagogical tool for mathematics education is pointed out, especially with regard to building of the mathematical structures. It turns out that knowledge maps, like mind maps and concept maps, may be efficient tools for building structure in mathematics.

The method of mind mapping takes into account that the two halves of the human brain are performing different tasks. While the left side is mainly responsible for logic, words, arithmetic, linearity, sequences, analysis, lists, the right side of the brain mainly performs tasks like multidimensionality, imagination, emotion, colour, rhythm, shapes, geometry, and synthesis. Mind mapping uses both sides of the brain, [1] letting them work together and thus increases productivity and memory retention. This is accomplished by representing logical structures using an artistic spatial image that the individual creates. Thus mind mapping connects imagination with structure and pictures with logic [2].

Mind mapping is a method of learning and knowledge testing and detecting whose essence lies in acquiring interconnections among terms. It is also a way how to formulate and regulate metacognitive learning strategies of students.

The mind mapping saves time, improves efficiency, presents information in an organized and easy to follow format on one page, which is simple for others to read and add their own ideas, develops organization skills. This flexible tool can be adapted to almost any task. The structure of a mind map improves memory and makes easier for anybody to remember more.

In spite of its well-structured and ordered contents a mind mapping has some limitations.

The essence of our pedagogical experiment realized in the academic year 2013/2014 (when teaching a subject Mathematics 1 on the Faculty of Civil Engineering, University of Zilina) was to include mind maps in the contents of lectures from the stated subject. The priority functions of the mind maps were the following ones:

The function of the “connecting bridge”; the aim of the mind map was to remind students of the facts and knowledge from the secondary school. This knowledge is necessary for the topic understanding (a phase reflect),

Cognitive maps mediated a recapitulation overview of the curriculum (a phase review).

With the help of the implemented mind maps students were pertinently “navigated and guided” in their own learning process.

2. Mind mapping in mathematical education

The beginnings of Mind mapping are connected with the name of Tony Buzan who in the late 70s of 20 century proposed mind maps as a technique of note taking. According to Fisher, it is an indication of “all procedures denoting thinking by the means of some projection”[3]. It is a visual denotation, which consists of words, concepts, ideas, symbols, pictures and essential junctions expressing interrelations between them. It is an effective tool for capturing ideas, notes and information, identification of key terms, projection of facts into the overall and meaningful
structure, an aid for creating associations which could otherwise get lost. Like a cartographic map it is a good way how to make thinking visible.

In fact the idea of the mind maps creation is much older. Also a great philosopher Rene Descartes in his *Discourse on the Method for Reasoning Well and for Seeking Truth in the Sciences* states the following rules of the so called Cartesian method: the rule of analytical procedure - to decompose complicated things into the simplest ones; the rule of synthesis - to proceed in the correct order from the simplest to the most difficult, to sum up relations and dependences from the simple ones up to the learning of the most complicated phenomena; and the rule of control - when solving the problem, pay attention to its different connections and aspects.

Mathematical knowledge has a character of the net. Mathematical terms, definitions, theorems, algorithms and rules are interconnected both between themselves and with the world outside. If we want our students to understand mathematics and make progress in it, we have to present it in relations (mathematical terms among themselves, mathematical terms ↔ real world).

From the stated facts it is evident that the visual depiction of cognitive structures can be very useful. Application of mind maps can significantly deepen individual understanding of the problem and make the whole learning process more valuable.

**Advantages of mind mapping:**
- **for a student** – it makes easier
  - curriculum understanding,
  - its recoding to a more memorable form,
  - distinguishing its nature and internal structures,
  - its remembering,
  - its restoring,
  - its reconstructing if new pieces of knowledge add up, and
  - creating “mental models” of the world.
- **for a teacher**
  - application when a teaching process is planned, when the curriculum is explained and summarised.
- it is a suitable tool for entrance diagnosis as well as diagnosis during or at the end of the educational process.

A mathematical world is a net of interconnected facts and terms and knowledge of all correlations among them is necessary for entering this wonderful world of mathematics. Mind maps will enable students to orientate themselves in the web of mathematical terms and they are also:
- an aid when identifying key terms, relations among them, creating a meaningful structure and making necessary links and relations understandable,
- they enable to implement new information in a broader context,
- combinations of words and a picture integrate both brain hemispheres in the learning process and make the learning of mathematics more effective,
- they help cognitive skills development, ability of analysis, classification and synthesis of terms,
- they enable and stimulate convergent, divergent, critical, strategic and complex mathematical thinking,
- they are an effective mnemotechnical aid (memory aid); the shape, colours, structure of a map will enable better remembering of information,
- they develop holistic and complex understanding of mathematical terms and characteristics, and
- they support the development of metacognitive skills - learn to learn and think about knowledge.

**Functions of mind maps:**
- **Auto diagnostic** (for a student) – a mind map enables a student to know explicitly his/her own realizing and cognitive arrangement of the discussed topic. It also offers a possibility to monitor his/her own learning procedure – it develops metacognitive abilities of a student.
- **Diagnostic** (for a teacher) – a mind map is a tool of identification the situations for making decisions about the character of pedagogic intervention. It also offers a diagnostic tool of identification on which level of understanding a student accepts new concepts.

| QUESTIONS FROM TEXTS, TEACHERS AND TESTS | THINKING PROCESSES | THINKING MAPS AS TOOLS |
|----------------------------------------|--------------------|-----------------------|
| How are you defining this thing or idea? What is the context? What is your frame of reference? | CIRCLE MAP DEFINING IN CONTEXT | Data Map |

**Basic types of the mind maps**

Table 1
| QUESTIONS FROM TEXTS, TEACHERS AND TESTS | THINKING PROCESSES | THINKING MAPS AS TOOLS |
|----------------------------------------|--------------------|------------------------|
| How are you describing this thing? Which adjectives would best describe this thing? | BUBBLE MAP DESCRIBING QUALITIES | Bubble Map |
| What are the similar and different qualities of these things? Which qualities do you value most? Why? | DOUBLE BUBBLE MAP COMPARING AND CONTRASTING | Double Bubble Map |
| What happened? What is the sequence of events? What are the sub stages? | FLOW MAP SEQUENCING | Flow Map |
| What are the causes and effects of these events? What might happen next? | MULTI- FLOW MAP CAUSE AND EFFECT | Multi-Flow Map |
| What is the analogy being used? What is the guiding metaphor? | BRIDGE MAP SEEING ANALOGIES | Bridge Map |
| What are the component parts and subparts of this whole physical object? | BRACE MAP PART - WHOLE | Brace Map |
| What are the main ideas, supporting ideas, and details in this information? | TREE MAP CLASSIFYING | Tree Map |
• **Intervention** – a mind map becomes a content-organised pillar of the learning procedure which guarantees a meaningful integration of new pieces of knowledge into already existing cognitive structures.

The stated functions of mind maps clearly indicate their important role in the formation of metacognitive learning strategies of students. Let us deal with some concrete examples of the mind maps that were used during our teaching experiment. In Table 1 we present some interesting types of the mind maps [4], which we used to create the specific models for teaching Mathematics 1.

3. Pedagogical experiment

The essence of our pedagogical experiment realized in the academic year 2013/2014 (when teaching a subject Mathematics 1 on the Faculty of Civil Engineering, University of Zilina in Zilina) was to include mind maps in the contents of lectures from the stated subject. The priority functions of the mind maps were the following ones:

1. The function of the “connecting bridge”, the aim of the mind map was to remind students of the facts and knowledge from the secondary school. This knowledge is necessary for the topic understanding (a phase reflect),

2. Cognitive maps mediated a recapitulation overview of the curriculum (a phase review).

With the help of the implemented mind maps students were pertinent “navigated and guided” in their own learning process.

We present in Figs. 1 and 2 two mind maps which were used during our experiment.

After creating and implementing several mathematical mind maps in the teaching process of the subject Mathematics 1 we approached carrying out a pedagogical experiment whose aim was to find out if this implementation would have a positive influence on the study results of students. On the basis of formulation of the pedagogical experiment’s aim the following hypothesis was set:

\[ H_1: \text{Students educated with the support of mind and conceptual maps will obtain at the end of an experimental teaching process at least an equal standard of knowledge in comparison with students educated without mind mapping use.} \]

When choosing experimental subjects we tried to find two groups which would be equivalent as much as possible. Finally we decided for a random sample of the first year students of the Faculty of Civil Engineering, University of Zilina in Zilina. During the winter term of the academic year 2013/2014 they passed the exam from the subject Mathematics 1 whose content is: basics of linear algebra, analytical geometry and differential calculus of real-valued functions of one variable.

When selecting control and experimental groups the agreement in the teacher competence was crucial (in both groups three-hour seminar and lectures were conducted by the same teacher), agreement in the content and range of curriculum and in teaching plans. The number of respondents in both groups was identical – 28. For an experimental group the work with mind maps was included in the teaching process every week. A control group attended traditional mathematics classes organised in the form of seminars.

After curriculum completion both groups solved an equivalent knowledge test which contained 10 tasks. The maximum number of the received points in the test was 30. Doing the experiment to verify the hypothesis \( H_1 \) was conducted according to an experimental plan without a pre-test. The following Table 2 presents the percentage success rate of a post-test in individual groups.

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**Fig. 1 Bubble map – Straight line**

**Fig. 2 Bubble map – Basics before Differentiation**
When using the stated teaching methods different study results were obtained. If we apply the one-sided hypothesis

\[ H_0: \mu_1 = \mu_2 \text{ versus } H_1: \mu_1 > \mu_2 \]

then \( H_0 \) is rejected on the significance level \( \alpha \) if \( T > t_{\alpha} (n + m - 2) \). This was confirmed in our case as it is true that

\[ 2.276 > t_{0.05} (54) = 1.676. \]

The one-sided hypothesis was rejected and the difference between mean values for the stated selective file was considered statistically significant. With the help of statistical methods it was confirmed that students educated by an innovative teaching method with the use of mind maps achieved better study results than the students educated by a traditional method.

4. Conclusion

The described pedagogical experiment convinced us about the positive influence of implementing mind maps in the process of teaching mathematics. Students perceived positively especially the fact that mind maps enabled them to bring a system into the amount of information, facts and terms and receive a detached view of the studied topic. Mind maps enabled them also to observe, revise, control and guide their learning process and thus develop their metacognitive learning strategies.

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