String Creation and Heterotic–Type I’
Duality

Oren Bergman a, Matthias R. Gaberdiel b and Gilad Lifschytz c

a Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

b Department of Applied Mathematics and Theoretical Physics
Cambridge University
Silver Street
Cambridge, CB3 9EW
U. K.

c Joseph Henry Laboratories
Princeton University
Princeton, NJ 08544

November 1997

Abstract

The BPS spectrum of type I’ string theory in a generic background is derived using
the duality with the nine-dimensional heterotic string theory with Wilson lines. It is
shown that the corresponding mass formula has a natural interpretation in terms of
type I’, and it is demonstrated that the relevant states in type I’ preserve supersym-
metry. By considering certain BPS states for different Wilson lines an independent
confirmation of the string creation phenomenon in the D0-D8 system is found. We
also comment on the non-perturbative realization of gauge enhancement in type I’,
and on the predictions for the quantum mechanics of type I’ D0-branes.

∗E-mail address: bergman@string.harvard.edu
†E-mail address: M.R.Gaberdiel@damtp.cam.ac.uk
‡E-mail address: gilad@puhep1.princeton.edu
1 Introduction

An interesting phenomenon involving crossing D-branes has recently been discovered [1, 2, 3]. When a $D_p$-brane and a $D(8 - p)$-brane that are mutually transverse cross, a fundamental string is created between them. This effect is related by U-duality to the creation of a D3-brane when appropriately oriented Neveu-Schwarz and Dirichlet 5-branes cross [4]. These, and all other brane-creation phenomena can be obtained from M-theory, where crossing M5-branes lead to the creation of an M2-brane [4, 5]. A M(atrix) theory origin of this phenomenon has also been suggested recently [6].

An example of a system exhibiting the string creation effect is the D0-D8 system in type IIA string theory. Unlike the other $D_p \perp D(8 - p)$ systems, this system is further constrained by charge conservation to include vacuum strings between the branes [7, 3]. The resulting force between the D0- and the D8-brane vanishes, and the system is in a BPS state [2, 3]. This system is closely related to the 9-dimensional type I' string theory [8] which is T-dual to type I on $\mathbb{R}^9 \times S^1$. As type I string theory is unoriented, type I' string theory contains two orientifold fixed planes, and the theory lives on $\mathbb{R}^9 \times S^1 / \mathbb{Z}_2$, where the interval $S^1 / \mathbb{Z}_2$ is bounded by the two orientifold planes. In addition, the $SO(32)$ Chan-Paton factors of type I (32 D9-branes) become sixteen parallel D8-branes in type I' (together with their sixteen images on the other side of the fixed planes), and their positions in the interval are determined by a Wilson line in the type I theory. The conserved charges of type I string theory are Kaluza-Klein (KK) momentum and D-string winding number, and those of type I' are winding number and D-particle number. The system contains D0- and D8-branes as well as fundamental strings, and the above effect should therefore play an important role in the non-perturbative regime of the theory.

Type I string theory has been conjectured to be S-dual to the $Spin(32)/\mathbb{Z}_2$ heterotic string theory [9]. The map relating the two theories compactified on $\mathbb{R}^9 \times S^1$ is given by [10]

$$\lambda_I = \frac{1}{\lambda_h}, \quad G^{I}_{MN} = \frac{G^{h}_{MN}}{\lambda_h}, \quad R_I = \frac{R_h}{\sqrt{\lambda_h}},$$

(1.1)

where $G^{MN}$ is the ten-dimensional metric $(M, N = 0, \ldots, 9)$, $\lambda_h$ and $\lambda_I$ are the heterotic and type I coupling constants, and the radius of the circle is $R_h$ and $R_I$ when measured in the heterotic and type I metrics, respectively. It is easy to see from these relations that heterotic KK momentum is mapped to type I KK momentum, and that heterotic winding number is mapped to D-string winding number in type I. On the other hand, the winding number of the fundamental string in type I is not a conserved quantity, and it does not have a dual description in the heterotic string theory. Combining the above S-duality map with the T-duality map relating type I and type I',

$$\lambda_{I'} = \frac{\lambda_I}{R_I}, \quad G^{I'}_{MN} = G^{I}_{MN}, \quad R_{I'} = \frac{1}{R_I},$$

(1.2)
we find that the heterotic and type I' parameters are related as

$$\lambda' = \frac{1}{\sqrt{\lambda h R_h}}, \quad G'_{MN} = \frac{G^h_{MN}}{\lambda h}, \quad R' = \frac{\sqrt{\lambda h}}{R_h}. \quad (1.3)$$

In particular, the heterotic KK momentum is mapped to type I' winding, and heterotic winding is mapped to the D0-brane number in type I'. The KK momentum is not a conserved quantity in type I' (it is related to the winding of the fundamental string in type I), and it therefore has no heterotic dual.

The simple relations in (1.2) and (1.3) hold only when eight D8-branes are fixed at the location of each orientifold plane, which corresponds to the Wilson line $A = (0^8, (1/2)^8)$, breaking $SO(32)$ to $SO(16) \times SO(16)$. In this case, the type I' background metric is flat and the dilaton is constant, and it is therefore possible to define a (global) string coupling constant as $\lambda' \equiv \exp(\phi_I')$. This case is also special in the sense that the 9-dimensional $Spin(32)/Z_2$ heterotic theory for which the gauge group has been broken to $SO(16) \times SO(16)$ is T-dual to a suitable compactification of the $E_8 \times E_8$ heterotic string theory (where again the gauge group has been broken to $SO(16) \times SO(16)$) [11]. This in turn implies a duality between the $E_8 \times E_8$ heterotic theory and type I', which has been explored in [12, 13]. From the point of view of M-theory, this duality is simply the exchange of two compact directions, one of which is $S^1$ and the other $S^1/Z_2$. This has in turn been exploited to construct the heterotic matrix theory [14].

For Wilson lines other than $(0^8, (1/2)^8)$ the situation is more complicated. In particular, the dilaton of type I' is then not constant along the interval, the metric is curved, and the simple duality relations (1.2) and (1.3) no longer hold. Furthermore, since the two heterotic theories are not related by T-duality in the general case, type I' is then not related to the $E_8 \times E_8$ theory, and therefore has no simple M-theory interpretation.

The Wilson lines $(0^{8-N}, (1/2)^{8+N})$ (which in type I' correspond to backgrounds with $(8 - N)$ D8-branes at one fixed plane and $(8 + N)$ at the other) were partially explored in [10]. The heterotic theories in this class exhibit, at particular radii (which depend on $N$), gauge enhancement to exceptional groups, and it was shown that this occurs whenever the type I' theory develops a region where the coupling diverges. It is then possible that additional states become massless, thereby avoiding a potential contradiction of the duality with the fact that type I' (or type I) does not possess a perturbative gauge enhancement mechanism.

On the other hand, the precise form of the duality map was not analyzed in [10], and it is clear that it cannot be as simple as in the $N = 0$ case. In particular, type I' winding is quantized in multiples of 1/2, as a single open string which stretches from a D8-brane at one end to a D8-brane at the other end has winding number 1/2, and a single closed string which winds around the whole interval has winding number 1. The heterotic KK momentum, on the other hand, is shifted due to the presence of the Wilson line, and it is not always
half-integral. In fact, for $N = 0 \mod 4$ the heterotic KK momentum is half-integral, but for $N = 2 \mod 4$, it is of the form $n/4$, where $n \in \mathbb{Z}$, and for odd $N$, it is of the form $n/8$. If the simple duality map were true, this would give unphysical values for the type I’ winding number. It is one of the aims of this paper to derive the correct duality map in these cases, and to demonstrate that it gives rise to type I’ winding which is always half-integral. The critical observation is that the heterotic KK momentum is mapped to a state which has, in addition to the D0-branes, a certain amount of type I’ winding.

We will also show that the actual value of the type I’ winding number is somewhat ambiguous as it depends in general on the positions of the D0-branes along the interval. In particular, the difference in mass of a D0-brane as it changes its position can be absorbed into a shift of the winding number, and this can be understood as a consequence of the macroscopic D0-D8 strings, which were shown to be present in [7, 8]. Furthermore, the creation of D0-D8 strings can be demonstrated by analyzing the system as a function of the positions of the D8-branes; our analysis of the duality map between BPS states in the heterotic and type I’ string theories therefore gives further (and independent) evidence for this phenomenon.

It will also become apparent that the relevant BPS states in type I’ preserve supersymmetry. In the presence of D0- and D8-branes, this amounts to the condition that the string has a definite orientation, and we demonstrate that this follows from the BPS condition of the heterotic theory [15]. This latter condition requires that the right-moving oscillators are in their ground states, and for non-zero winding number (i.e. in the presence of D0-branes) this roughly implies that KK momentum can only flow in one direction, and therefore that type I’ strings can only wind in one direction.

The paper is organized as follows. In section 2 we review nine-dimensional heterotic string theory and gauge enhancement. In section 3 we derive the precise heterotic-type I’ duality map, including both heterotic momentum and winding, and show that the type I’ winding is always physical and supersymmetric, i.e. half-integer and non-positive. In section 4 we use the duality to exhibit the creation of D0-D8 strings by following the behavior of certain states as the Wilson line is varied, and discuss the role these strings play in gauge enhancement. In section 5 we present some conclusions, and comment on the predictions of the present analysis with regards to the D0-brane quantum mechanics. We have included two short appendices: in the first we give a table of the lightest type I’ BPS states for a small number of D0-branes, and in the second we generalize the results of section 3 to Wilson lines corresponding to bulk D8-branes.

While this paper was being finalized, a paper [19] appeared where some results which overlap with section 3 are obtained.
2 Review of $D=9$ heterotic string theory

Let us consider the $\text{Spin}(32)/\mathbb{Z}_2$ heterotic string theory compactified on $S^1$ with radius $R$ and a background gauge field $A$. The left- and right-moving momenta are

\[
\begin{align*}
    p_L &= \left(\sqrt{\frac{2}{\alpha'_h}}(P + w h A), \frac{n_h - A \cdot P - w h A^2/2}{R_h} + \frac{w h R_h}{\alpha'_h}\right), \\
    p_R &= \frac{n_h - A \cdot P - w h A^2/2}{R_h} - \frac{w h R_h}{\alpha'_h},
\end{align*}
\]

(2.1)

where $P$ is an element of the $\text{Spin}(32)/\mathbb{Z}_2$ lattice $\Gamma^{16}$, and $n_h$ and $w_h$ are the KK momentum and winding numbers, respectively. The mass spectrum is given by

\[
M^2_h = \frac{1}{2} p_L^2 + \frac{2}{\alpha'_h} (N_L - 1) + \frac{1}{2} p_R^2 + \frac{2}{\alpha'_h} (N_R - c),
\]

(2.2)

where $N_R, N_L$ are the right and left-moving oscillator numbers, and $c = 0$ or $1/2$ depending on whether the right-moving fermions are periodic (R) or anti-periodic (NS). Physical states satisfy the level matching condition

\[
p_L^2 + \frac{4}{\alpha'_h} (N_L - 1) = p_R^2 + \frac{4}{\alpha'_h} (N_R - c),
\]

(2.3)

and BPS states are given by the further requirement that $N_R = c$. The condition for states to be physical and BPS-saturated is thus

\[
p_L^2 - p_R^2 = \frac{4}{\alpha'_h} (1 - N_L),
\]

(2.4)

which can be simplified to

\[
\frac{1}{2} P^2 + n_h w_h = 1 - N_L.
\]

(2.5)

In this case, the mass formula becomes

\[
M^2_{h,BPS} = p_R^2 = \left(\frac{n_h - A \cdot P - w h A^2/2}{R_h} + \frac{w h R_h}{\alpha'_h}\right)^2.
\]

(2.6)

For example, states with $N_L = 1$ and $P^2 = 0$ include the gravity multiplet and the vector multiplets associated with the Cartan generators of the gauge group. Additional massless vectors, associated to the roots of the underlying gauge group, have $N_L = 0$, and therefore satisfy

\[
p_R^2 = 0, \quad p_L^2 = \frac{4}{\alpha'_h}.
\]

(2.7)
At zero winding number this gives the roots of the subgroup of $SO(32)$ which is left unbroken by the Wilson line, and for a generic radius $R_h$ there are no further massless gauge bosons. However, if the radius satisfies

$$R_h^2 = \alpha_h^\prime (1 - A^2/2) ,$$

(2.8)

where $A^2$ is the length of the shortest vector of the form $A + \lambda$ where $\lambda \in \Gamma^{16}$, there exist additional massless vectors for non-zero winding number which give rise to an ‘enhancement’ of the gauge symmetry.

In this paper we shall be mostly interested in the case where the Wilson line is of the form $A_N = (0^{8-N}, (1/2)^{8+N})$ where $N = 0, \ldots, 8$. This class of theories exhibits an interesting pattern of gauge enhancements (table 1). At a generic radius, the unbroken gauge group is $SO(16 - 2N) \times SO(16 + 2N) \times U(1)^2$. For $N > 0$, the smaller $SO$ group, together with one of the $U(1)$’s is enhanced at the critical radius

$$R_h(N) = \sqrt{\frac{\alpha_h^\prime N}{8}}$$

(2.9)

to $E_{9-N}$. On the other hand, for $N = 0$, the critical radius is $R_h(0) = 0$, and both $SO(16)$ groups are enhanced to $E_8$. In this case all winding numbers contribute to the gauge enhancement. This reflects the fact that a new continuum degree of freedom has appeared; indeed, by T-duality, the $N = 0$ theory at $R_h = 0$ is really the ten-dimensional $E_8 \times E_8$ heterotic string theory [11].

| $N$ | $G$ | $R_h^2/\alpha_h^\prime$ | $G_{\text{enhanced}}$ | $w_h$ | $p_h$ | $G$ charges |
|-----|-----|--------------------------|------------------------|-------|------|-------------|
| 0   | $SO(16) \times SO(16)$ | 1/8                      | $E_8 \times E_8$      | $\mathbb{Z}$ | $0$  | $(128,1), (1,128)$ |
| 1   | $SO(14) \times U(1)$   | $1/4$                    | $E_7$                  | $\pm1, \pm2$ | $\pm1/4, \pm1/2$ | $64_+, 64'_-, 14_+, 14_-$ |
| 2   | $SO(12) \times U(1)$   | $3/8$                    | $E_6$                  | $\pm1$ | $\pm3/8$ | $16_+, 16'_-$ |
| 3   | $SO(10) \times U(1)$   | $1/2$                    | $E_5$                  | $\pm1$ | $\pm1/2$ | $8_+, 8_-$ |
| 4   | $SO(8) \times U(1)$    | $5/8$                    | $E_4$                  | $\pm1$ | $\pm5/8$ | $4_+, 4'_-$ |
| 5   | $SO(6) \times U(1)$    | $3/4$                    | $E_3$                  | $\pm1$ | $\pm3/4$ | $2_+, 2_-$ |
| 6   | $SO(4) \times U(1)$    | $7/8$                    | $E_2$                  | $\pm1$ | $\pm7/8$ | $1_+, 1_-$ |
| 7   | $SO(2) \times U(1)$    | $1$                      | $E_1$                  | $\pm1$ | $\pm1$ | $+, -$ |
| 8   | $U(1)$                  | $1$                      |                        |       |      |             |

Table 1: Gauge enhancement in $D = 9$ heterotic string theory. Only the relevant part of the gauge group is presented, and $p_h = n_h - A \cdot P - w_h A^2/2$.

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There are other enhanced groups as well, such as $SO(34)$ when $A = (1,0^{15})$ (free-fermionic point), and $SU(18)$ when $A = (r^{16})$ with $0 < r < 1/2$.  

6
The cases \( N = 1, 2 \) are different from \( 3 \leq N \leq 8 \), in that the \( w_h = \pm 2 \) states contribute to the gauge enhancement as well as the \( w_h = \pm 1 \) states. Indeed, for \( N \geq 3 \) the enhancement from \( SO(16-2N) \times U(1) \) to \( E_{9-N} \) only requires the spinor representation to become massless, whereas for \( N = 1, 2 \) the vector and singlet representation, respectively, has to become massless as well.

### 3. Duality map and type I’ BPS states

Let us now analyze the duality between the above heterotic theories and the type I’ theory where \((8-N)\) D8-branes are at \( x^9 = 0 \) and \((8+N)\) D8-branes are at \( x^9 = 2\pi \). First, we need to fix some conventions.

We assume (as in [10]) that the D8-brane R-R charge is \(-8\mu_8\) just to the right of the orientifold plane at \( x^9 = 0 \), and that it jumps by \(+\mu_8\) each time we cross a D8-brane to the right. The charge is then \(-N\mu_8\) in the bulk of the interval.

The D0-D8 system preserves 1/4 of the supersymmetry. Let us denote by \( Q_L \) and \( Q_R \) the two sixteen-component supercharges of type IIA string theory. A D8-brane that extends along the \( x^0, \ldots, x^8 \) directions (and is a point in \( x^9 \)) is invariant under the linear combination \( \epsilon_L Q_L + \epsilon_R Q_R \), where

\[
\epsilon_L = \Gamma_0 \cdots \Gamma_8 \epsilon_R \quad \text{(3.1)}
\]

and a D0-brane is invariant under the linear combination with

\[
\epsilon_L = \Gamma_0 \epsilon_R \quad \text{(3.2)}
\]

Using \( \Gamma_0^2 = 1 \), the two equations (3.1) and (3.2) are solved by \( \epsilon_L = \Gamma_0 \epsilon_R \) provided \( \epsilon_R \) satisfies

\[
\epsilon_R = \Gamma_1 \cdots \Gamma_8 \epsilon_R \quad \text{(3.3)}
\]

and it is easy to check that such \( \epsilon_R \) exist.

On the other hand, an elementary string that is stretched along the \( x^9 \) axis is invariant under the supersymmetry generator provided that

\[
\epsilon_L = \Gamma_0 \Gamma_9 \epsilon_L \quad \epsilon_R = \pm \Gamma_0 \Gamma_9 \epsilon_R \quad \text{(3.4)}
\]

where the sign depends on the orientation of the string. It is then easy to check that the three conditions (3.1), (3.2) and (3.4) are compatible only for the upper sign in (3.4); this means that in a D0-D8 system, supersymmetry is only unbroken if any strings that are present are oriented in a specific direction. For definiteness we shall assume that configurations with unbroken supersymmetry correspond to strings stretching from right to left for which the winding number is negative.
It should be stressed that this constraint is a consequence of both the D0- and the D8-brane being present. In particular, for states without any D0-branes, there is no restriction on the direction of the string.

The winding number of type I' is quantized in units of $1/2$, which corresponds to a string that stretches once across the interval. The D0-brane number is also quantized in units of $1/2$; the minimal value corresponds to a single D0-brane that is stuck at one of the orientifold planes.

3.1 Type I' BPS spectrum

Our calculation follows the analysis of [10], but we find it useful to re-introduce all the dimensionful parameters (other than $\bar{h}$ and $c$ of course). The low energy effective actions of the two theories are given (up to terms that we shall not need) by

$$S_h = \int d^{10}x \sqrt{-G_h} e^{-2\phi_h} \left\{ \frac{1}{2\kappa^2} \left[ R_h + \cdots \right] - \frac{1}{8g_{YM}^2} \text{Tr}_V F^2 \right\},$$

(3.5)

where the Yang-Mills and gravitational couplings are related by $2\kappa^2 = \alpha'_I g_{YM}^2$ [13], and

$$S_{I'} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G_{I'}} e^{-2\phi_{I'}} \left[ R_{I'} + \cdots \right]$$

$$- T_8 \sum_i \int_{x^9=x_i^9} d^9x e^{-\phi_{I'}} \text{Tr}_f \left[ \sqrt{\det(\eta_{\mu\nu} - 2\pi\alpha'_{I'} F_{\mu\nu})} - \sqrt{-g_{I'}} \right].$$

(3.6)

Here the second term is the sum of the DBI actions on the world-volumes of the D8-branes, $g''_{I\mu\nu}$ is the nine-dimensional metric, and the D8-brane tension is given by $T_8 = \mu_8/(\sqrt{2}\kappa)$ [17].

The constants $\kappa$ and $\mu_8$ are fixed by the Dirac quantization condition for D-branes, and by the normalization of the D-string tension [18]

$$2\kappa^2 = (2\pi)^7 (\alpha'_I)^4, \quad \mu_8^2 = (2\pi)^{\gamma}(\alpha'_{I'})^{-5}.$$

(3.7)

Expanding the DBI action to quadratic order in $\alpha'_{I'}$ gives

$$S_{I'} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G_{I'}} e^{-2\phi_{I'}} \left[ R_{I'} + \cdots \right] - \frac{\pi(\alpha'_{I'})^2 \mu_8}{2\kappa^2} \sum_i \int_{x^9=x_i^9} d^9x \sqrt{-g_{I'}} e^{-\phi_{I'}} \text{Tr}_f F^2.$$

(3.8)

For $\kappa = 1$ this reproduces precisely the actions considered in [10]. The type I' background dilaton and metric are given as [10]

$$e^{\phi_{I'}(x^9)} = \left[ \kappa \Omega(x^9)/C \right]^5$$

$$G'_{MN}(x^9) = \Omega^2(x^9) \eta_{MN},$$

(3.9)
where
\[ \Omega(x^9) = \kappa^{-1} C [3\mu_8 C (B + N x^9) / \sqrt{2}]^{-1/6}, \]  
(3.10)

and \( B \) and \( C \) are constants \( (B \geq 0) \). The relation between these parameters and the heterotic parameters \( R_h \) and \( \phi_h \) is deduced by replacing the non-compact components of \( \eta_{MN} (M, N = 0, \ldots, 9) \) above with a slowly varying function of the non-compact dimensions \( \gamma_{\mu\nu} (\mu, \nu = 0, \ldots, 8) \), and comparing the dimensional reductions of the low energy effective actions. The duality relation involves a Weyl rescaling of the nine-dimensional heterotic metric, \( \gamma_{\mu\nu} = D_2 g_{h\mu\nu} \), where \( D \) inevitably depends on \( B \) and \( C \). The comparison of the two gravitational actions then leads to
\[ 2\pi R_h e^{-2\phi_h} = \kappa^{-10} D_7 C^{10} \int_0^{2\pi} dx^9 \Omega^{-2}(x^9), \]  
(3.11)

and comparing the Yang-Mills actions gives
\[ 2\pi R_h e^{-2\phi_h} = 2^{5/2} \pi^2 \mu_8 \kappa^{-4} \alpha_h^{-1/2} D^5 C^5. \]  
(3.12)

Combining (3.11) and (3.12) we then find
\[ DC^{5/3} = 3^{-2/3} 2^{7/3} \pi \mu_8 \kappa^{-1/2} \alpha_h N^{1/2} (B + 2\pi N)^{4/3} - B^{4/3}]^{-1/2}. \]  
(3.13)

The duality maps pure momentum states of the heterotic string \((w_h = 0)\) to pure winding states of type I', and so
\[ 1/R_h = D M_{winding}’, \]  
(3.14)

where \( M_{winding}’ \) is the mass of a type I' string that stretches twice along the interval as measured in the metric \( \gamma_{\mu\nu} \),
\[ M_{winding}’ = \frac{1}{\pi \alpha’} \int_0^{2\pi} dx^9 \Omega^2(x^9) = \frac{1}{D} \sqrt{\frac{8}{\alpha_h^2 N}} \left[ (B + 2\pi N)^{2/3} - B^{2/3} \right]^{-1/2}. \]  
(3.15)

Here we have used (3.7) to eliminate \( C \). The mass of a D0-brane which is located at \( x^9 \), as measured in the same metric, is given by
\[ M_{D0}’(x^9) = \frac{1}{\sqrt{\alpha’}} \Omega(x^9) e^{-\phi’(x^9)} = \frac{1}{D} \sqrt{\frac{2N}{\alpha_h}} \left( B + N x^9 \right)^{2/3} \left[ (B + 2\pi N)^{4/3} - B^{4/3} \right]^{1/2}, \]  
(3.16)

where we have used both (3.7) and (3.13). It is then easy to check that
\[ M_{D0}’(2\pi) - M_{D0}’(0) = \frac{N}{2} M_{winding}’, \]  
(3.17)
or more generally that the masses of D0-branes which are located at different values of $x^9$ differ by the mass of $N$ strings that are stretched between the two points in $x^9$,

$$M'_{D0}(x_1^9) - M'_{D0}(x_2^9) = NM'_{\text{string}}(x_1^9, x_2^9). \tag{3.18}$$

This is consistent with the fact that D0-branes feel a linear gravity-dilaton potential in the presence of D8-branes.

In remains to analyze the heterotic winding states whose KK momentum vanishes. From the situation for $N = 0$ one may expect that these states correspond in type I' to states that consist of D0-branes, but do not have any winding. However, inverting (3.14) and using (3.15) and (3.16), we find

$$R_h/\alpha'_h = D\left[1/2 \, M'_{D0}(0) + \frac{N}{8} M'_{\text{winding}} \right]. \tag{3.19}$$

This suggests that the heterotic winding states map to states that consist of D0-branes at $x^9 = 0$, together with some winding. The factor of $1/2$ corresponds to the fact that a single D0-brane is stuck at the orientifold plane, and therefore contributes only half of its mass to the bulk.

As it stands, the winding contribution in (3.19) is in $\mathbb{Z}/8$, but the winding number in type I' must be quantized in units of $1/2$. If we combine the heterotic momentum states (3.14) and the heterotic winding states (3.19) into the BPS mass formula (2.6), we find

$$M'_{BPS} = \left| w' M'_{\text{winding}} - n'I' M'_{D0}(0) \right|, \tag{3.20}$$

where

$$w' = n_h - A_N \cdot P - w_h(1 + N/4), \quad n'I' = w_h/2, \tag{3.21}$$

are the winding number and D0-brane number, respectively. Here we have assumed that all D0-branes are located at $x^9 = 0$.

It is easy to see that $w'$ is now in $\mathbb{Z}/2$, unless $N$ and $w_h$ are odd (in which case it can be of the form $1/4 + \mathbb{Z}/2$). On the other hand, because of (3.18), the winding number is not unambiguous, and it depends in fact on the precise locations of the D0-branes. In order to obtain a physical value for the winding number, we therefore have to assume that, when $w'$ is of the form $1/4 + \mathbb{Z}/2$, half a D0-brane is at $x^9 = 2\pi$ rather than at $x^9 = 0$; this leads

Actually, since the metric $\gamma$ is flat, the potential is due only to the dilaton in this metric. In curved metrics there would also be a gravitational contribution.

This result could have been anticipated as follows. There exist heterotic states with winding number 1 which become massless at the critical radius. These must correspond to a type I' configuration of half a D0-brane at $x^9 = 0$ without any winding. Using (3.14), the heterotic mass formula for this state at a generic radius then implies (3.19).
to an additional contribution to the winding number of $N/4$, and the resulting value for $w_I'$ is then in $\mathbb{Z}/2$.

More generally, it is clear that the analysis of the mass formula alone does not predict the actual locations of the D0-branes, and therefore the actual value of the winding number. In particular, from the point of view of the mass formula, it is always possible to ‘move’ whole D0-branes from $x^9 = 0$ to $x^9 = 2\pi$ without modifying the condition $w_I' \in \mathbb{Z}/2$. There exist, however, additional constraints that restrict these possibilities. In particular, as will be discussed in the following subsection, the condition of unbroken supersymmetry gives a constraint on the winding number (and therefore on the possible locations of the D0-branes). Furthermore, for a certain class of heterotic BPS states, the nature of the gauge charges fixes the locations of the D0-branes; this will be discussed in section 4.

### 3.2 A condition on the winding number

As we have explained before, the configuration of a D0-brane, a D8-brane and a fundamental string is only supersymmetric if the string is oriented in a particular way. We have chosen the convention that (in the presence of a D0-brane) the winding number must be non-positive.

It is clear that the BPS states of type I’ must satisfy this requirement. This is not apparent from (3.21), since it seems one could get positive winding by choosing $n_h$ arbitrarily large, and negative winding by choosing $w_h$ arbitrarily large. However, the values of $n_h$ and $w_h$ are not arbitrary since they must satisfy the heterotic physical-BPS condition (2.3). We can express this condition in terms of the type I’ variables, and we find that

$$2n_I' w_I' = 1 - \frac{N}{2}n_I'^2 - \frac{1}{2}(P + 2n_I'A_N)^{2} - N_L. \quad (3.22)$$

For $n_I' > 0$, it follows from (3.22) that

$$w_I' \leq \frac{1}{2n_I'} - \frac{Nn_I'}{4}, \quad (3.23)$$

as $N_L \geq 0$. Since $w_I'$ is in $\mathbb{Z}/4$, the only positive values for $w_I'$ can arise if

$$\frac{1}{2n_I'} - \frac{Nn_I'}{4} \geq 1/4. \quad (3.24)$$

This can only happen for $n_I' = 1/2$ if $N = 0, \ldots, 6$, $n_I' = 1$ if $N = 0, 1$, and $n_I' = 3/2, 2$ if $N = 0$. These cases can be checked directly, and it turns out that in each case (3.22) implies that $w_I' \leq 0$. For the benefit of the reader we have worked out the maximal value of $w_I'$ for a certain class of BPS states in appendix A.

We have thus shown that the winding number is always non-positive, provided that all D0-branes are located at $x^9 = 0$. It is also easy to check that for those states where the
condition \( w_I \in \mathbb{Z}/2 \) requires that at least half a D0-brane is at \( x^9 = 2\pi \), the modified winding number (which takes into account that half a D0-brane has been moved) also satisfies this condition. We can then turn the argument around, and regard the condition \( w_I \leq 0 \) as a constraint on the possible locations of the D0-branes.

Using the fact that \( w_I \leq 0 \) for BPS states, the BPS mass formula for type I’ then becomes

\[
M_{\text{BPS}}^{I'} = |w_I|h_{\text{winding}} + |n_I|h_{\text{D0}}(0) \tag{3.25}
\]

if \( w_I \in \mathbb{Z}/2 \), and

\[
M_{\text{BPS}}^{I'} = (|w_I| - N/4)h_{\text{winding}} + (|n_I| - 1/2)h_{\text{D0}}(0) + 1/2h_{\text{D0}}(2\pi) \tag{3.26}
\]

if \( w_I \in \mathbb{Z}/2 + 1/4 \). This means that the mass of a type I’ BPS state is simply the sum of the mass associated to the strings and the mass associated to the D0-branes, as may have been expected. It should be stressed that this is in marked contrast to the heterotic BPS mass formula (2.6).

4 String creation and gauge enhancement

In this section we examine how some of the type I’ BPS states evolve as the D8-branes are moved from one end of the interval to the other, \textit{i.e.} as the Wilson line of the heterotic theory is changed. As we shall see, the winding number of the states in type I’ changes as the D8-branes are moved, and we shall demonstrate that this can be accounted for in terms of D8-D8 strings that become longer or shorter, and fundamental strings that are created as D8-branes cross D0-branes.

4.1 D0-D8 strings

Let us begin with the \( N = 0 \) case, for which eight D8-branes are located at each orientifold plane. Let us further consider the BPS states that have zero winding number in type I’. These states were discussed in [12] from a slightly different point of view. There are three kinds of states: those charged under the first \( \text{SO}(16) \) (“left-charged”), those charged under the second \( \text{SO}(16) \) (“right-charged”), and neutral states. The charged states can be further subdivided according to whether the heterotic winding and momentum numbers are even or odd (table 2).

The number of D0-branes in these states is equal to \( w_h/2 \), and the winding number is zero. We shall only consider those states for which \( w_h > 0 \); analogous statements hold for the BPS states with \( w_h < 0 \), which correspond to anti-D0-branes. The duality with the heterotic theory suggests that the D0-branes form bound states with the orientifold planes.
Table 2: $N = 0$ heterotic BPS states with $w_{I'} = 0$. The first two rows are left-charged states, the next three rows are right-charged states, and the last row corresponds to neutral states. The various 1’s and $-1$’s can be located anywhere in the corresponding eight entries, and $l$ in the second row is a positive integer $\leq 4$.

| $N_L$ | $w_h$ | $n_h$ | $P$ | $P + w_hA$ |
|-------|-------|-------|-----|------------|
| 0     | $2k$  | $-2k$ | $k(0^8; (1)^8) \pm (1, \pm 1, 0^6; 0^8)$ | $(\pm 1, \pm 1, 0^6; 0^8)$ |
|       | $2k + 1$ | $-2k - 1$ | $k(0^8; (1)^8) + (-\frac{1}{2})^8 + (1^2, 0^8-2^2; 0^8)$ | $(\frac{1}{2})^2, (-\frac{1}{2})^8-2^2; 0^8)$ |
| 0     | $2k$  | $-2k \pm 1$ | $k(0^8; (1)^8) \pm (0^8; 1^2, 0^6)$ | $\pm (0^8; 1^2, 0^6)$ |
|       | $2k$  | $-2k$ | $k(0^8; (1)^8) \pm (0^8; 1, -1, 0^6)$ | $\pm (0^8; 1, -1, 0^6)$ |
|       | $2k + 1$ | $-2k + 1 - l$ | $k(0^8; (1)^8) - (0^8; 1^2, 0^8-2^2)$ | $(0^8; (-\frac{1}{2})^2, (\frac{1}{2})^8-2^2)$ |
| 1     | $2k$  | $-2k$ | $-k(0^8, 1^8)$ | $(0^{16})$ |

and D8-branes at $x^9 = 0$ and $x^9 = 2\pi$ for the left- and right-charged states, respectively, and that they form bound states in the bulk for the neutral states.

There exist two mechanisms by means of which gauge charges can arise in type I': the state can contain open strings that begin or end on D8-branes, and it may contain one or more D0-branes that are stuck at an orientifold plane. The former states account for weights in the conjugacy classes of the identity and vector representations. For example, open strings that begin and end on D8-branes which are located at the same orientifold plane correspond to the roots of the underlying gauge group, and D8-D8 strings which stretch across the interval correspond to states in the bivector representation. On the other hand, D0-D8 strings give rise to states in the vector representation.

A half D0-brane that is stuck on an orientifold plane generates, through its fermionic zero modes, states that transform under the corresponding spinor representation. If there are more such D0-branes on one of the orientifold planes, this gives rise to states that transform in the symmetric product of the spinor representation.

We shall assume that in any of the above BPS states at most half a D0-brane is stuck at an orientifold plane. In particular, this implies that weights in the conjugacy classes of the identity and the vector representations arise solely from open strings, whereas weights in the conjugacy classes of the spinors arise from a combination of open strings and half a D0-brane.

Consider for example the states with weight vectors $\pm(1, \pm 1, 0^6; 0^8)$. In the type I’ picture these charges arise from strings between the corresponding D8-branes. The overall sign corresponds to the orientation of the string, and the relative sign is $-$ for a string
between two D8-branes in the interval $0 \leq x^9 \leq 2\pi$, and + for a string between a D8-brane
and the image of another. From the point of view of the interval $0 \leq x^9 \leq 2\pi$, the string in
the second case begins on a D8-brane, is reflected by the orientifold, and ends on another
D8-brane (figure 1). It should be mentioned, that even in the presence of D0-branes, both
orientations of the string preserve supersymmetry as the string has vanishing length.

We want to vary the Wilson line in such a way as to move one of the D8-branes from
$x^9 = 0$ to $x^9(r)$, so

$$A = (0^8; (1/2)^8) + \Delta A, \quad \Delta A = (0^7, r; 0^8),$$

(4.1)

where $r$ can be in any of the first eight entries. The precise $r$ dependence of $x^9(r)$ is rather
involved, and the duality map in this case is worked out in appendix B. When the D0-branes
are near $x^9 = 0$, the map is given by

$$w'_I = n_h - A \cdot P - w_h(1 + r/2), \quad n'_I = w_h/2,$$

(4.2)

and when the D0-branes are near $x^9 = 2\pi$ the map changes to

$$w'_I = n_h - A \cdot P - w_h, \quad n'_I = w_h/2.$$

(4.3)

Let us describe the change in $w'_I$ for the different types of states in turn.

**Left-charged states, $w_h$ even**

For a given value of $k$ and two directions in the root lattice of the (left) $SO(16)$, there
are four states that correspond to the four different choices of sign. In type I’, the two
directions correspond to the choice of two D8-branes, and the four states correspond
to the various open strings between them. If $\Delta A$ is in one of the other six directions,
$A \cdot P$ does not change as $A \leftrightarrow A + \Delta A$; in type I’ this corresponds to the situation
where one of the other D8-branes is moved. If $\Delta A$ is in the direction of one of the two given lattice directions then $-A \cdot P$ changes by $\pm r$. In type I' this corresponds to moving one of the two chosen D8-branes, which increases the length of the string and therefore contributes $\pm r$ to the winding number. Here the sign depends on the orientation, and there are therefore two states for which the sign is plus, and two for which it is minus.

This appears to be in contradiction with the requirement that the string have a negative orientation to preserve supersymmetry in the presence of D0-branes. In the initial $N = 0$ background the contradiction is avoided since the strings have vanishing length, and in fact the two orientations are related by a gauge transformation. On the other hand, as the string becomes longer, the states with positive orientation would appear to break supersymmetry.

However, there is an additional contribution to the change in $w_I'$ given by $-w_h r/2$. This contribution is independent of the direction of $\Delta A$. In the type I' picture it corresponds to $w_h/2$ D0-D8 strings, which are created when the D8-brane crosses the D0-branes. The orientation of these strings is fixed by the sign of the background R-R 8-brane charge [3], and it is negative in our conventions.

As a consequence, if $\Delta A$ corresponds to moving one of the two selected D8-branes, both effects will contribute. In the two states for which the D8-D8 string is positively oriented, this string will combine with a negatively oriented D0-D8 string to give a positively oriented D0-D8 string of vanishing length (figure 2(a-c)), thereby avoiding the possible contradiction with supersymmetry, which is manifest in the heterotic picture.[4] This cancellation has important consequences for gauge enhancement, as we shall soon see. For the other two states the orientations are both negative, and the winding numbers add to a net negative number (figure 2(d)).

**Left charged states, $w_h$ odd**

For these states, half a D0-brane is stuck at the orientifold plane at $x^9 = 0$ and contributes a weight in the spinor representation. In addition there are $2l$ D8-branes at $x^9 = 0$ that support D8-D8 strings. The correspondence between the two terms in (4.2) and the number of D8-D8 and D0-D8 strings is not as simple as in the previous case. In fact $-A \cdot P$ changes by $\mp r/2$, depending on whether $\Delta A$ is in one of the $2l$ special directions or not. By itself, this change cannot correspond to a growing D8-D8 string, which would contribute $\mp r$ to the winding. However, the additional change in $w_I'$ from the term $-w_h r/2$ can be broken into two parts

$$-w_h r/2 = -kr - r/2.$$  \hspace{1cm} (4.4)

4In the final configuration there is a non-vanishing force on the D0-brane towards $x^9 = 0$, and therefore classically the length of the string is indeed zero. In quantum mechanics the picture is less clear.
Figure 2: Moving a D8-brane for left-charged states. (a) represents a state with weight \((+1, -1, 0^6; 0^8)\) and one D0-brane in the background of eight D8-branes on each side. (b) As the D8-brane associated with the \(-1\) entry moves to the right, the D8-D8 string becomes longer, and a D0-D8 string of opposite orientation is created. (c) The two strings combine into a D0-D8 string of vanishing length, resulting in a net constant force towards \(x^9 = 0\). (d) As the D8-brane associated with the \(+1\) entry moves to the right, the D8-D8 string becomes longer, and a D0-D8 string of the same orientation is created; there is no force in this case.

The first part corresponds to the D0-D8 strings that are created when the D8-brane crosses the \(k\) D0-branes which live just off the orientifold plane. The second part does not correspond to a D0-D8 string, since the D8-brane never crosses the additional \(1/2\) D0-brane\(^5\) but rather combines with the \(-A \cdot P\) contribution to give a D8-D8 string for the case when one of the \(2l\) special D8-branes is moved (figure 3), and to cancel the above winding when one of the other D8-branes is moved.

Right-charged states

In this case the type I’ winding number is given by (4.3), where only the term \(-A \cdot P\) can change. In fact this term does not change, since the first eight entries of \(P\) are zero. In the type I’ picture the winding remains unchanged since there are no D8-D8 strings at \(x^9 = 0\), and D0-D8 strings are not created until the D8-brane reaches \(2\pi\), which

\(^5\)In fact, there are no D0-D8 strings associated with this \(1/2\) D0-brane, as it is confined to precisely \(x^9 = 0\), where the value of the R-R 8-brane charge is 0.
is where the D0-branes are (see figure 4). These strings will therefore have vanishing length, and will not contribute to the winding. They will however change the weight vector, consistent with the heterotic picture.

Neutral states

In this case there are no D8-D8 strings in the type I’ picture. Any change in the type I’ winding should therefore arise from D0-D8 strings. The winding number is somewhat ambiguous in this case, as the D0-branes are not localized at either end. We can however define $w_I$ as if the D0-branes were all at $x^9 = 0$. Eq. (4.2) then shows that the change in winding corresponds precisely to the creation of $w_h/2$ D0-D8 strings.
To conclude, we see that the creation of D0-D8 strings is necessary in order to explain the evolution of type I' BPS states, given the duality with heterotic string theory. We have assumed that, except for a half D0-brane in the case when there are $\mathbb{Z} + 1/2$ D0-branes, D0-branes are not stuck at the orientifold, but rather are bound (together with their images) to the orientifold plane and D8-branes, and this has allowed us to explain the evolution of the above states satisfactorily.

4.2 Gauge enhancement

We are now in a position to understand how the gauge enhancement pattern of section 2 is realized non-perturbatively in type I'. For every $N$ there is a point in the moduli space of type I', corresponding to $B = 0$, where the dilaton (3.9) blows up at $x^9 = 0$, and D0-branes located at $x^9 = 0$ become massless (3.16). This point corresponds to the appropriate critical radius (2.9) in the heterotic picture [10]. The case $N = 0$ has already been explained in [12]. In all the other cases, the states in table 1 with $w_h = 1$ (or $-1$) correspond to type I' states consisting of $1/2$ D0-brane (or $1/2$ anti-D0-brane) at $x^9 = 0$, and no winding (e.g. figure 5(a)). The mass of these states is thus entirely due to the $1/2$ D0-brane at $x^9 = 0$, and it therefore vanishes for $B = 0$. These states transform in the spinor representation of the gauge group $SO(16 - 2N)$, and are of course spacetime vector multiplets.

For $N = 1, 2$ there are additional massless vectors with $w_h = \pm 2$. In type I' these are states that consist of a whole D0-brane without any net winding, and are therefore massless when $B = 0$. It follows from the analysis of the heterotic theory that these states are only BPS if $N = 1, 2$, but not for $N \geq 3$. This can be understood from the point of view of type I' as follows. Let us consider the $w_h = 2$ state in the $N = 0$ background which is charged under the left $SO(16)$, and which becomes massless at the critical (vanishing) radius. This state transforms in the adjoint of $SO(16)$. In type I' it consists of one whole D0-brane near $x^9 = 0$ without any winding, and therefore belongs to the first class of states discussed above. In fact figure 2(a) depicts one of its components.

As we move one of the two D8-branes to $2\pi$, leaving seven D8-branes at 0, the gauge group at $x^9 = 0$ becomes $SO(14)$. As discussed above, for the two components of the original state for which the D8-D8 string is positively oriented, the net winding number will vanish as a consequence of the cancellation between the D8-D8 string and the D0-D8 string. These states will therefore become massless at the new ($N = 1$) critical point. Both components will contain a short D0-D8 string (figure 5(b,c)), and will therefore give rise to a state in the vector (14) representation of $SO(14)$. Together with the $1/2$ D0-brane state which gives rise to the spinor (64) of $SO(14)$, and with the $1/2$ and 1 anti-D0-brane states, we get an enhancement of $SO(14) \times U(1)$ to $E_8$.

Next let us move the other D8-brane, i.e. the D8-brane in figure 5(b,c) on which the string ends (or begins); the result is shown in figure 6(b,c). The state in figure 6(b) has
Figure 5: $SO(14) \times U(1) \rightarrow E_8$. (a) spinor $64_+$ (b) and (c) give rise to the vector representation $14_+$. 

winding and is therefore always massive, but the state in figure 6(c) has no winding, and thus becomes massless at the new ($N = 2$) critical point. This state contains two short strings which are understood to stretch between the D0-brane and its image. (If we restrict attention to the interval $0 \leq x^9 \leq 2\pi$ then this is represented as in figure 6(c)). The state is therefore neutral with respect to the $SO(12)$ gauge group at $x^9 = 0$, but it is of course charged under the R-R $U(1)$. Together with the spinor (32) from the 1/2 D0-brane, and the anti-D0-brane states, we then get an enhancement to $E_7$.

Figure 6: $SO(12) \times U(1) \rightarrow E_7$ (a) spinor $32_+$. (b) is massive, and (c) is a singlet $1_+$. 

If we move any further D8-branes we will not lengthen any D8-D8 strings, but D0-D8-strings will still be created. As a consequence, the above states will acquire non-vanishing winding, and therefore will never be massless. We can therefore conclude that states with $n_{\nu} = 1$ only contribute to gauge enhancement for $N = 1, 2$; this is in agreement with the dual heterotic picture.
5 Conclusions

In this paper we have analyzed the duality between the nine-dimensional heterotic and type I' string theories for a large class of Wilson lines. We have given the duality map between the relevant BPS states in detail, and we have shown that the mass formula for type I' states has a simple interpretation in terms of D0-branes and winding strings. We have also shown that the heterotic BPS condition implies that, in the presence of D0-branes, the winding number of type I' has a definite sign; this is in accord with the fact that in the presence of D8- and D0-branes, supersymmetry is only unbroken if any fundamental string has a definite orientation.

A crucial element in our discussion was the realization that a heterotic winding state corresponds to a certain number of D0-branes together with some winding. We have also seen that the difference in mass between D0-branes at different positions along the interval corresponds to the mass of a string that stretches between these positions. We have analyzed a class of BPS states as one of the D8-branes is moved from one side of the interval to the other, and we have found independent evidence for the proposal that a fundamental string is created whenever a D8-brane crosses a D0-brane. Our analysis also suggests that, at least for the states in section 4, at most half a D0-brane is stuck at the orientifold plane, and that the remaining D0-branes are bound to the orientifold planes and D8-branes without being constrained to move on the orientifold plane.

The analysis also predicts that certain bound states should exist in the quantum mechanics of D0-branes in an $N \neq 0$ type I' background, and it would be interesting to check this directly. In particular, bound states of D0-branes without any winding are absent, with the only exception of two D0-branes for $N = 1$ and $N = 2$. (In these cases there exist however short strings which provide a constant attractive force). This is in marked contrast to the $N = 0$ background, where bound states of any number of D0-branes are predicted to exist. Since these bound states are necessary for an M-theory interpretation (or matrix theory formulation), we expect neither to be applicable to the $N \neq 0$ backgrounds. This is also consistent with the fact that D8-branes, by themselves, have no M-theory interpretation; only the combination of eight D8-branes and an orientifold plane is known to arise in M-theory.

Acknowledgments

We would like to thank Michael Green, Igor Klebanov and Juan Maldecena for useful discussions. O.B. is supported in part by the NSF under grants PHY-93-15811 and PHY-92-18167, M.R.G. is supported by a Research Fellowship of Jesus College, Cambridge, and G.L. is supported in part by the NSF under grant PHY-91-57482.

20
A Lightest BPS states

In this appendix we describe the lightest BPS states of type I’ for the first few values of the D0-brane number, and in the backgrounds with $N = 0, \ldots, 8$ (table 3). These states were found by determining the largest winding number for a physical BPS state with a given number of D0-branes, i.e. the largest winding number that is consistent with $(3.22)$ and $(3.27)$. We use the convention that all D0-branes are at $x^9 = 0$.

| $N \backslash n_I$ | $1/2$ | 1 | $3/2$ | 2 | $5/2$ |
|-----------------|-------|---|-------|---|------|
| 0               | 0_{(S,I)+(I,S)} | 0_{(A,I)+(I,A)} | 0_{(S,I)+(I,S)} | 0_{(A,I)+(I,A)} | 0_{(S,I)+(I,S)} |
| 1               | 0_{(S,I)} | 0_{(V,I)} | $-1/2(3,V)$ | $-1/2(3,V)$ | $-3/4(3,V)$ |
| 2               | 0_{(S,I)} | 0_{(I,I)} | $-1(V,S')+(S'I,I)$ | $-1(A,I)+(I,A)$ | $-3/2(A,S)+(S',I,V')$ |
| 3               | 0_{(S,I)} | $-1/2(I,V)$ | $-1(S',I)$ | $-3/2(V,V)$ | $-2(S,A)$ |
| 4               | 0_{(S,I)} | $-1(A,I)+(I,A)$ | $-3/2(S,V)$ | $-2(A,I)+(I,A)$ | $-5/2(S,V)$ |
| 5               | 0_{(S,I)} | $-1(V,I)$ | $-2(S',A)+(S',I)$ | $-5/2(V,V)$ | $-3(S,I)$ |
| 6               | 0_{(S,I)} | $-1(I,I)$ | $-2(S,I)$ | $-3(A,I)+(I,A)$ | $-4(S,V)+(S',I)$ |
| 7               | 0_{(S,I)} | $-3/2(I,V)$ | $-5/2(S,V)$ | $-7/2(V,V)$ | $-9/2(S',V)$ |
| 8               | 0_{I} | $-2_A$ | $-3_A$ | $-4_A$ | $-5_A$ |

Table 3: Winding number (assuming all D0-branes are at $x^9 = 0$) and $SO(16 - 2N) \times SO(16 + 2N)$ charges of lightest few charged type I’ BPS states with $n_I$ D0-branes.

For $N = 0$ and integer $n_I$, there exist in addition massless BPS-states with $N_L = 1$ and $(P + 2n_I,A) = 0$; these are neutral states (gravitons and photons). The representation $S$ corresponds to the weight $(\pm 1/2)^L$ with an even number of $+$ signs, and the representation $S'$ corresponds to an odd number of $+$ signs. For $N = 7$, the representations of $SO(2) \simeq U(1)$ are labeled by their $U(1)$-charge $q \in \mathbb{Z}/2$ modulo 2, where $q_I = 0$, $q_{S'} = 1/2$, $q_V = 1$, and $q_S = 3/2$. Brackets denote higher dimensional representations in the same conjugacy class of the given representation.

Some of the states in table 3 can be obtained from the states with $N = 1$ by moving a single D8-brane from $x^9 = 0$ to $x^9 = 2\pi$ as in section 4. For example, the states $0_{(A,I)}$, $0_{(V,I)}$, $0_{(I,I)}$, $-1/2(I,V)$ and $-1(A,I)$ at $n_I = 1$ and $N = 0, 1, 2, 3$ and 4, respectively, are connected in this way. On the other hand the other state at $n_I = 1$ and $N = 4$ ($-1(A,I)$ does not arise in this way.
B  D8-branes in the bulk

In this appendix we will derive the map between \((n_h, w_h)\) and \((n'_I, w'_I)\), in the case where some D8-branes are in the bulk. We shall consider specifically the situation where the Wilson line of the heterotic theory is

\[
A = (0^{8-N_1}, r^{N_1-N_2}, (1/2)^{8+N_2}) ,
\]

and \(0 \leq r \leq 1/2\).

On the type I’ side this corresponds to the configuration where \((8-N_1)\) D8-branes are at \(x^9 = 0\), \((N_1-N_2)\) D8-branes are at \(x^9 = x^9(r)\), and \((8-N_2)\) D8-branes are at \(x^9 = 2\pi\). (The relation between \(x^9(r)\) and \(r\) will given below in \((B.2)\).) The value of the background R-R 8-brane charge is then \(-N_1\mu_8\) in the region \(0 < x^9 < x^9(r)\), and \(-N_2\mu_8\) in the region \(x^9(r) < x^9 < 2\pi\).

Following \([10]\), we can solve the equation of motion for the type I’ background, and we find that

\[
\frac{1}{R_h} = DM'_{\text{winding}} = \frac{\sqrt{8}}{\alpha'_h} \left[ \frac{(a-b)/N_1 + (c-a)/N_2}{\alpha'_h [(a^2-b^2)/N_1 + (c^2-a^2)/N_2]^{1/2}} \right],
\]

\[
DM'_{D0}(0) = \frac{2}{\alpha'_h} \left[ \frac{b}{\alpha'_h [(a^2-b^2)/N_1 + (c^2-a^2)/N_2]^{1/2}} \right],
\]

\[
r = \frac{1}{2} \frac{(a-b)/N_1}{(a-b)/N_1 + (c-a)/N_2},
\]

where we have defined

\[
a = (B + x^9(r)N_1)^{2/3}, \quad b = B^{2/3}, \quad c = (B + (N_1-N_2)x^9(r) + 2\pi x^9(r))^{2/3}.
\]

As in section 3, we expect that a winding state in the heterotic theory is mapped to a D0-brane together with some winding. However the calculation is somewhat more elaborate as the connection between \(r\) and \(x^9(r)\) is rather complicated, as can be seen from equation \((B.2)\).

We make the ansatz that

\[
R = \frac{1}{\alpha'_h} \frac{1}{2} DM'_{D0} + DM'_{\text{winding}} \beta ,
\]

and solve for \(\beta\), using the equations \((B.2)\). After some algebra we find that

\[
\beta = \frac{N_1}{8} + \frac{(N_2-N_1)}{8N_2^2} (c-a)^2 ,
\]

(B.5)
which, in terms of the heterotic variables becomes

\[ \beta = \frac{N_1}{8} \left(1 + \frac{r - 1/2}{2} \right) (N_2 - N_1) . \]  \hspace{1cm} (B.6)

Using equations (B.6, 2.6) the map between \((n_h, w_h)\) and \((n_{\nu}, w_{\nu})\) is then given as

\[ w_{\nu} = n_h - A \cdot P - w_h \left(1 + \frac{N_2}{4} + \frac{(N_1 - N_2)r}{2}\right) , \]
\[ n_{\nu} = \frac{w_h}{2} . \]  \hspace{1cm} (B.7)

This formula is used in section 4 with \(N_1 = 1\) and \(N_2 = 0\). It should be noted that these transformations can also be found simply by assuming that massless \(w_h = 1\) states correspond to half a D0-brane at \(x^9 = 0\) without any winding.

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