Possibility of narrow resonances in nucleon-nucleon channels

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Compound states manifest themselves as bound states, resonances, or primitives, and their character is determined by their interaction with the continuum. If the interaction experiences a perturbation, a compound state can change its manifestation. Phase analysis of nucleon-nucleon scattering indicates the existence of primitives in the $^3S_1$, $^1S_0$, and $^3P_0$ channels. Electromagnetic interaction can shift primitives from the unitary cut, turning them into narrow resonances. We evaluate this effect on the $^1S_0$ proton-proton scattering channel in the framework of the Simonov-Dyson model. We show that electromagnetic interaction turns a primitive with a mass of 2000 MeV into a dibaryon resonance of approximately the same mass and a width of 260 keV. Narrow resonances of a similar nature may occur in other nucleon-nucleon channels. Experimental confirmation of the existence of narrow resonances would have important implications for the theory of nucleon-nucleon interaction.

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Analytical properties of scattering amplitudes were extensively studied in the 1950s, and for some time, the conditions of analyticity and unitarity were considered likely to be sufficient for the full recovery of scattering amplitudes. Low [1] found the dispersion relation for scattering amplitudes, which takes into account analyticity and unitarity. Solving this equation, Castillejo, Dalitz and Dyson [2] observed an ambiguity, which is now called the CDD poles. The physical meaning of these poles was clarified by Dyson [3]. Using a version of the Lee model [4], he showed that the CDD poles correspond to bound states and resonances (for a review, see [5]).

S-matrices with a finite number of poles, which are mainly used in the phenomenological parameterizations of scattering amplitudes, correspond to zero-range singular potentials. The character of the interaction in such systems is determined by analyzing the behavior of the scattering phase rather than the potential: if the phase increases with energy, the interaction is an attraction; if it decreases, the effect of interaction is a bound state or repulsion. According to the Breit-Wigner formula, isolated resonances drive the phase shift up. In the absence of bound states, the systems discussed in Refs. [1][2] correspond to phases that increase with energy.

In nucleon-nucleon interaction, the only bound state, the deuteron, arises in the $^3S_1$ channel, whereas all other two-nucleon channels are free of bound states.

Conversely, the nucleon-nucleon phase shifts decrease with increasing energy and provide evidence for repulsion. The Dyson model [3] was extended by Simonov [6] for the description of internucleon forces. In the generalized model, both attraction and repulsion may dominate, and the phase shifts behave bidirectionally. The repulsion has been included by weakening the condition of strict positivity of the imaginary part of the $D$ function, thereby allowing zeros of the $D$ function to appear on the unitary cut [7].

Numerous experimental searches for exotic multiquark states did not give decisive results. In the late ‘70s Jaffe and Low [8] proposed an experimental method to identify exotic hadrons using a special formalism, called the $P$-matrix. The application of the $P$-matrix formalism to meson-meson scattering revealed $P$-matrix poles that roughly correspond to the four-quark states predicted earlier by quark models [8]. States that show up as poles of the $P$ matrix, rather than the $S$ matrix, are called “primitives”. They correspond to zeros of the $D$ function on the unitary cut, modify the Low scattering equation, and generate the CDD poles [9].

The Simonov-Dyson model [6] provides a dynamical framework of the $P$-matrix formalism. It has been successfully applied to describe the nucleon-nucleon interaction dominated by repulsion at short distances [9][13]. In the Simonov-Dyson model, primitives are the objects that produce repulsion.

The state of the art that prevailed in the early ‘80s can be characterized as follows: The Dyson model describes systems dominated by attraction where bound states and resonances may exist. The Simonov-Dyson model describes systems with both attraction and repulsion where, in addition to bound states and resonances, primitives come into play.

The conventional approach to the description of nucleon-nucleon interaction is based on the Yukawa meson-exchange mechanism. As a result of the developments of the ‘70s and early ‘80s, a second approach has emerged, which ascribes nucleon-nucleon interaction to the $s$-channel exchange of $6q$-primitives (for a review, see [14]).

The mechanism of $t$-channel meson exchange is studied in detail and is widely used to analyze particular effects. The $s$-channel exchange mechanism [15] has solid theoretical background and is also largely self-sufficient in modeling nucleon-nucleon interaction.
The above two mechanisms may appear mutually exclusive. To decide which of them is more appropriate, it is necessary to find a prediction that could draw a clear distinction between the alternatives. The task is not simple, because they both successfully describe a wide range of experimental data.

However, these mechanisms can also be dual to each other in the sense in which the t-channel meson exchange is dual to the s-channel resonance exchange in the Veneziano model. In our case, this would mean that the t-channel meson exchange parameterizes the s-channel exchange of \( 6q \)-primitives, and vice versa. Simultaneous treatment of the t- and s-exchanges would represent in this case double counting.

There is, moreover, a third possibility. The hybrid Lee model can be reduced to the Simonov-Dyson model \([9]\), so the latter can be regarded as a phenomenological realization of the former. In the hybrid Lee model, the inter-nucleon forces are generated by the s-channel exchange of \( 6q \)-compound states.

We wish to clarify which of the aforementioned mechanisms is realized in nature, and whether there is a duality between the t-channel exchange of mesons and the s-channel exchange of primitives (\( 6q \)-compound states).

The existence of primitives imposes constraints on the interaction parameters. A characteristic feature of the Simonov-Dyson model is that small perturbations of the interaction parameters move primitives along the unitary cut. If there is a duality, the zeros of the \( D \) function do not leave the unitary cut, so primitives remain primitives. The duality condition thereby imposes constraints on the permissible perturbations of the parameters of the Simonov-Dyson model, which determine, in particular, compound state masses and their coupling with the continuum.

A similar situation occurs in the hybrid Lee model. The Simonov-Dyson model ensures the existence of stable primitives, if it arose as a reduction of the hybrid Lee model. In this case, arbitrary perturbations of the parameters of the hybrid Lee model are allowed, whereas the corresponding variations of parameters of the Simonov-Dyson model are those that keep primitives on the unitary cut.

Some discrimination between the aforementioned alternatives can be made, if one considers scattering in an external field. In the OBE models and in the case of duality, the scattering cross section will not be qualitatively changed. Conversely, if there is no duality and the s-channel exchange mechanism operates as assumed in the Simonov-Dyson model we can expect the appearance of a resonance in the place where we earlier observed a primitive. Because the mass of the resonance is approximately known (it corresponds to zero phase shift), the problem reduces to the estimation of the width of the resonance. In the hybrid model of Lee, primitives are rigidly connected with the unitary cut, so the resonance will not occur.

In fact, we can proceed without external fields. A meaningful example of the perturbation of strong-interaction parameters of the model is given by the electromagnetic interaction. In this case, the condition for the existence of primitives is violated as a result of electromagnetic mass shifts of the nucleons and the \( 6q \)-compound states, as well as modification of the imaginary part of the \( D \) function, which additionally receives the Gamow-Sommerfeld factor for the proton-proton channel. Inclusion of the electromagnetic effects allows the determination of the width of the resonance, as well as the (small) shift of the resonance mass relative to the primitive mass. We therefore consider the possibility that the displacement of primitives is associated with electromagnetic interaction. This takes place in all nucleon-nucleon channels because of the electromagnetic mass shifts and other electromagnetic corrections, including the initial- and final-state Coulomb interaction. For numerical estimates, we use the Simonov-Dyson model \([10]\).

The \( D \) function can be written as

\[
D(s) = \Lambda(s) - \Pi(s),
\]

where

\[
\Lambda^{-1}(s) = \sum_\beta \frac{g_\beta^2}{s - M_\beta^2} + f,
\]

\[
\Pi(s) = -\frac{1}{\pi} \int_{s_0}^{+\infty} \Phi_2(s') \frac{\mathcal{F}^2(s')}{s' - s} ds'.
\]

The indices \( \beta \) label \( 6q \)-compound states with masses \( M_\beta \), the coupling constants \( g_\beta \) parametrize the interaction with the continuum, and \( f \) describes the four-fermion contact interaction. \( \Phi_2(s) = \pi k/\sqrt{s} \) is the phase space volume of two nucleons, where \( k \) is the nucleon momentum in the center-of-mass frame. \( \mathcal{F}(s) \) is the form factor arising from the vertices of compound states and nucleons and the four-fermion interaction vertices:

\[
\mathcal{F}(s) = \left( \frac{s}{s_0} \right)^{1/4} \sin(kb) C_0^2(s).
\]

Here, \( s_0 \) is threshold value of \( s \) and \( b \) is the effective interaction radius. The Gamow-Sommerfeld factor \([17]\):

\[
C_0^2(k) = \frac{2\pi\eta}{\exp(2\pi\eta) - 1}.
\]
where \( \eta = \alpha \mu / k \) and \( \mu = m/2 \) is the reduced proton mass, accounts for the initial- and final-state Coulomb interaction of the \( S \)-wave protons. In the \( pn \) and \( nn \) channels, \( C_0(k) = 1 \).

In the proton-proton channel, the dispersion integral of Eq. (3) can be evaluated to give

\[
\kappa \Pi(s) = \frac{1}{2k^2 b} \left( 2 \text{Si}(2kb) \sin(2kb) + \left( \text{Ci}(2kb) + \text{Ci}(-2kb) \right) \cos(2kb) - 2C - \ln(-4k^2 b^2) \right) \]

\[
- \frac{\sin(kb)}{kb} e^{ikb} \left( C_0^2(k) + \frac{\pi}{k} \right) + \sum_{n=1}^{\infty} \frac{\sin(b/n)}{b/n} e^{-b/n} \frac{2}{1 + k^2 n^2},
\]

where \( \kappa = 2mb/\pi \) and \( C = 0.5772 \ldots \) is the Euler constant. Here units are chosen in which \( a_c \equiv 1/(\alpha \mu) = 1 \), later we will restore the dimension. In the expression \( \Pi \), a branch of the logarithm \( \ln(1) = 0 \) enters; in the complex \( k \)-plane, the cut extends from \( -\infty \) to \( 0 \), so \( \ln(-1 + i0) = \pm i\pi \). In the physical upper half of the complex \( k \)-plane, the self-energy operator \( \Pi(s) \) is an analytical function. On the real \( s \)-axis below \( s_0 \) or, equivalently, on the imaginary half-axis \( \Im k = 0 \), \( 3k > 0 \), \( \Pi(s) \) is a real function.

\( \Lambda^{-1}(s) \) is assumed to have a single pole at \( s = M^2 \), corresponding to a \( 6q \)-compound state. The contribution to \( \Lambda^{-1}(s) \) of the contact four-fermion interaction is described by a constant term. As a result,

\[
\kappa^{-1}\Lambda^{-1}(s) = c_p\left( \frac{r_p}{s-M^2} - \frac{r_p}{s_0-M^2} \right) - \frac{1}{\gamma},
\]

where \( \kappa c_p r_p = \eta_0^2 \) is the coupling constant entering Eq. (2) and \( r_p = 8\pi^2/b^2 \) is the residue of the free \( P \) matrix. The value of \( c_p \) measures the strength of interaction of the compound state with the continuum. The residue of \( \kappa^{-1}\Lambda^{-1}(s) \) is parameterized as follows

\[
c_p r_p = \frac{M^2 - s_0}{\gamma}.
\]

In the \( pn \) and \( nn \) channels, the \( D \) function has the correct analytical properties, provided \( \xi \in (0,1) \). The phase shifts are well reproduced for \( \xi = 0.9 \). Here we restricted ourselves to verification of the fact that, in the channel \( pp \) at \( \xi = 0.9 \), the analytical properties of the \( D \) function are correct.

The value of \( \gamma \) is determined by the scattering length. In the \( pp \) channel, we use the series expansion around \( k = 0 \) for \( \Re k > 0 \) [19]:

\[
kC_0^2(s) \cot \delta(k) + 2h(k) = \frac{1}{a} + \frac{r}{2} k^2 + ... \tag{8}
\]

This expansion allows one to draw a link between the low-energy parameters of the model and the phenomenological constants that characterize the low-energy proton-proton scattering. The following equation serves as the definition of an analytical function \( h(k) \):

\[
2\psi(1 + \frac{i}{k}) + ik + \ln(-k^2) = 2h(k) + ikC_0^2(s), \tag{9}
\]

with \( \psi(z) \) being the digamma function.

The strong-interaction phase shift can be found from the strong-interaction \( S \) matrix modified by the initial- and final-state Coulomb interaction

\[
S'(k) \equiv e^{2i\delta(k)} = \frac{D(s - i0)}{D(s + i0)}. \tag{10}
\]

We use the empirical value of the scattering length \( a \) to fix \( \gamma \):

\[
\gamma = 1 + \frac{b}{a} + \left( \ln(4b^2) + 4C - 3 \right) b
- \sum_{m=2}^{\infty} (-1)^m \frac{2m+1}{(m+1)!} b^m \zeta(m), \tag{11}
\]

where \( \zeta(m) \) is the Riemann zeta function. Once the model parameters are determined, we arrive at a prediction for the effective range

\[
r = \frac{2}{3} \left( 1 + \gamma \right) b - \frac{8\gamma \xi}{b (M^2 - 4m^2)} - \frac{7}{9} b^2
+ \sum_{m=2}^{\infty} (-1)^m \frac{2m+2}{3(m+3)!} b^{m+1} \zeta(m). \tag{12}
\]

In the \( ^3S_1 \) \( pn \) channel with \( a_c = \infty \), one has \( r = 2.1 \text{ fm} \) [19], which should be compared with \( r = 2.8 \text{ fm} \) [22] in the OBE models.

In bare strong interaction, the real and imaginary parts of the \( D \) function simultaneously vanish at some point on the real \( k \)-axis. The zeros of \( \text{Im} D(s) \), on the other hand, are the zeros of \( \gamma(s) \), the lowest-lying one is located at \( \gamma \) [20]

\[
k_0^2 = \pi/b_0. \tag{13}
\]

In the parameterization [7], \( \Re D(s) \) also vanishes at this point, whereas the compound state mass is equal to the total energy of protons in the center-of-mass frame:

\[
M_0 = 2\sqrt{\pi^2/b_0^2 + m_0^2}. \tag{14}
\]

In strong plus electromagnetic interaction (strong interaction + QED), the zero of the \( D \) function is shifted.
to the complex plane. The effect is related to the change of the model parameters $b_0 \rightarrow b = b_0 + \Delta b$, $m_0 \rightarrow m = m_0 + \Delta m$, etc., and to the Coulomb interaction of the protons. Let us discuss the change of parameters.

Electromagnetic mass splitting of hadrons is well studied. QED corrections to hadron masses are usually attributed to the Coulomb interaction between the quarks and their spin-spin interaction:

$$\Delta M = c < \sum_{i<j} e_i e_j > + h < \sum_{i<j} e_i e_j \sigma_i \sigma_j >.$$  \hfill (15)

Here, it is assumed that all the quarks are in $S$-wave states. The averaging is made over the color-spin-isospin wave function of the quarks. A reasonable description of the mass splitting in octet-baryon isomultiplets, accounting for the mass difference of nonstrange quarks, can be obtained for $c = 3.06$ MeV and $h = -1.35$ MeV [21].

The average values of the spin-isospin operators and the corresponding electromagnetic mass shifts of the proton, the neutron, and the $6q$-compound states with quantum numbers of the $^1S_0$ nucleon-nucleon channels are listed in Table I. The calculation of average values of the operators was carried out using fully antisymmetrized color-spin-isospin quark wave functions. The resonance is denoted by $d^*$. For the nonstrange quarks, the MIT bag model gives $b \sim R$ and $M \sim R^3$, where $R$ is the radius of the 6-quark bag, and thus the change of the parameter $b$ for the inclusion of electromagnetic interaction is simply given by

$$\frac{\Delta b}{b} = \frac{\Delta M}{3M}.$$  \hfill (16)

The values of the $^1S_0$ scattering lengths, derived from the experimental data, are as follows [22]: $7.83 \pm 0.01$ fm, $23.75 \pm 0.01$ fm, and $16.4 \pm 0.09$ fm in the $I_S = 1, 0, -1$ channels respectively. In our approach, the scattering length or, equivalently, the value of $\gamma$ is adjustable parameter. We obtain $a = 6.5$ fm in the $pp$ channel by fitting a maximum of the phase at $T \approx 10$ MeV.

To compare the physical phase with that of the bare strong interaction, we must have an estimate of $a_0$. We give some heuristic arguments in favor of the change in the scattering length being proportional to the electromagnetic mass shift of the $6q$-compound state with quantum numbers of the channel.

In the Born approximation, the scattering length is proportional to the averaged interaction potential. A part of this potential associated with the electromagnetic interaction leads to a difference of scattering lengths in different isospin channels. On the other hand, the electromagnetic mass splitting of hadrons is also determined by averaging the electromagnetic potential, although of quarks rather than nucleons. In any case, the equation $a = a_0 + C_2 \Delta M$, with $a_0 = 14.45$ fm and $C_2 = -1.45$ fm/MeV, gives scattering lengths $a = 7.3$ fm, $23.4$ fm, and $16.6$ fm in the $I_S = 1, 0, -1$ channels, respectively.

A slightly better fit of the proton-proton phase shift is obtained for $a = 6.5$ fm. Because $a/a_0 \sim 1/3$ (in dimensional units $a_c = 58$ fm), the electromagnetic corrections to low-energy parameters of the model are expected to be only moderately small. In the scattering lengths, the electromagnetic corrections $\sim 1$.

Parameter $\xi$ is also an adjustable parameter. We will accept $\Delta \xi = 0$.

We thus have estimates of the proton mass $m_0$ in the bare strong interaction, as well as the electromagnetic mass corrections for nucleons and $d^*$. The relative value of the correction to $b$ is known also.

The proton kinetic energy $T_3 = 244$ MeV in the laboratory frame corresponds to a vanishing phase shift. In the center-of-mass frame, one finds the momentum of the protons $k_3 = \sqrt{mT_3}/2$ and their total energy $M_3 = \sqrt{2mT_3} + 4m^2 = 1995$ MeV. The phase shift vanishes provided that the imaginary part of the $D$ function vanishes; that is, it is necessary to have $k_3 b = \pi$, so the interaction radius $b$ is fixed. The total mass of the compound state $M \approx M_3$ is known with an accuracy of $O(\Delta M)$, so we find from Eq. (10) the interaction radius $b_0$ with electromagnetic interaction switched off.

The condition (13) holds for bare strong interaction. Using Eq. (14) one may find a mass of the compound state with electromagnetic interaction switched off. Then, for $\alpha \neq 0$, the mass $M = M_0 + \Delta M$.

In Fig. 1 the proton-proton scattering phase shift is plotted in strong interaction + QED and in the bare strong interaction case. In Fig. 2 the $S$-wave proton-proton cross section is plotted. The narrow resonance peak is associated with the complex root of the equation

$$D(M^2 - iM \Gamma_\sigma) = 0.$$  \hfill (18)

If the electromagnetic interaction is switched off, Eq. (18) has a primitive root corresponding to $\Gamma_\sigma = 0$. When the electromagnetic interaction is switched on, a resonance

| Hadron | $I_S$ | $\sum_{i<j} e_i e_j$ | $\sum_{i<j} e_i e_j \sigma_i \sigma_j$ | $\Delta M$ |
|--------|--------|-----------------|-------------------------------|--------------|
| $p$    | $\frac{1}{2}$ | 0               | $\frac{2}{3}$               | -1.8         |
| $n$    | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$              | -2.4         |
| $d^{++}$ | 1       | 1               | $-\frac{1}{2}$           | 5.0          |
| $d^{+}$   | 0       | $-\frac{1}{3}$ | $\frac{10}{3}$            | -6.2         |
| $d^{0}$   | -1      | $\frac{2}{3}$  | $-\frac{2}{3}$             | -1.5         |

TABLE I: Electromagnetic mass shifts of the proton, the neutron, and the $6q$-compound states $d^*$ with isospin projections $I_S = \pm 1, 0$. The third and fourth columns show average values of the spin-isospin operators. Electromagnetic mass shifts of the hadrons are shown in the last column (in MeV).
of a width $\Gamma_\ast \neq 0$ appears in the neighborhood of the primitive.

The full set of the model parameters is shown in Table II. In the first row, the model parameters to which we previously attributed the subscript 0 are given.

The value of $M_\ast$ is defined to within a few MeV. If we take $M_\ast = 2000$ MeV and a width of $\Gamma_\ast = 250$ keV. Because the position of the peak is not very well defined, in an experimental search for the resonance, one needs to scan a region around the zero value of the scattering phase with a resolution in $\sqrt{s}$ better than 100 keV.

Phase analysis of nucleon-nucleon scattering did not reveal primitives in the $^3P_1$ and $^1P_1$ channels. In the $^3P_0$ channel, the data indicate the existence of a primitive with a mass of about 1970 MeV [13]. Arguments similar to those used for the $S$-wave scattering allow one to conclude that electromagnetic interaction in the $^3P_0$ proton-proton channel transforms the primitive to a resonance with a mass of about 1970 MeV and a width of a few hundred keV.

Astrophysical observations of neutron stars provide important constraints on the nuclear matter properties above saturation density and, indirectly, on the in-medium modifications of hadrons. The problem of the possible conversion of primitives to resonances, however, can most reliably be settled in the laboratory.

In the phase behavior of the nucleon-nucleon scattering, there are clear features that can be attributed to primitives. We therefore believe that in bare strong interaction some compound states remain on the unitary cut and manifest themselves as primitives, rather than resonances or bound states. When external conditions, such as non-zero density of matter or finite temperature, become different or new interactions come into play, compound states can leave the unitary cut, turning into resonances. These resonances can be observed in the laboratory as the usual Breit-Wigner peaks.

There is no conclusive evidence for the existence of exotic $6q$ resonances. However, in the phase analysis of $NN$ scattering $6q$ primitives are determined reliably. In the $P$-matrix formalism primitives are identified with exotic multiquark states. If primitives are shifted from the unitary cut under influence of perturbations, they can be observed as narrow resonances.

In the OBE models and the hybrid Lee model primitives remain on the unitary cut. In the Simonov-Dyson model, in general, the primitives, if exist, are shifted from the unitary cut. Some preferences for these models can be done after the search results of the narrow dibaryons will become available.

For an experimental search for narrow dibaryon resonances, one should have a beam of protons with a kinetic energy of $T \sim 250$ MeV and an energy spread below...
100 keV. The energy resolution of the detector is not important, so one can use scintillation detectors. In the CELSIUS accelerator at Uppsala, the beam momentum spread was a few times $10^{-3}$ before electron cooling and a few times $10^{-5}$ after electron cooling. Under such conditions, it is possible to measure $\sqrt{s}$ with an accuracy of 10 keV or better. The narrow width of $d^{++}$ (2000) is not an obstacle to its experimental search.

Resonances of the same nature should exist in the $pn$ and $nn$ channels. Experiments with neutron beams are, however, more complicated. Also, e.g., deuterium cannot be used as a fixed target, because the narrow resonances are smeared out by the Fermi motion. We thus estimated the effect in the proton-proton channel, which is of interest for experiments with protons bombarding a hydrogen target.

Suppose that, in proton-proton scattering, experimentalists scanned the energy region up to a few MeV around the value where the phase shift is zero and the resonance was not found. In this case, we would have to conclude that the primitives are stable under perturbations and are strictly adhered to the unitary cut.

Such stability is inherent in the OBE models and hybrid Lee model. Consequently, the Simonov-Dyson model would either be dual to the OBE models or be a phenomenological realization of the hybrid Lee model.

Assume instead that, in place of the primitive, experimentalists found the $d^{++}$ (2000) resonance with a width of about 260 keV. This would mean that the primitives are not strictly adhered to the unitary cut. This property holds only in the Simonov-Dyson model.

Observation of the resonance will have the following consequences: i) mutual transformation of primitives and resonances will be confirmed, ii) universality of the Yukawa meson-exchange mechanism will be questioned on the experimental basis, iii) the region of validity of the hybrid Lee model will be reduced, and iv) the absence of duality between the $t$-channel meson exchange and the $s$-channel exchange of $6q$-primitives will be proved.

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### Table II: Parameters of the Simonov-Dyson model for $M_N = 1995$ MeV with electromagnetic interaction switched off and on.

| $a$ | $a^*$ | $r$ | $b$ | $m$ | $M$ | $M^*$ | $\Gamma_*$ |
|-----|-------|-----|-----|-----|-----|-------|--------|
| 0   | 0.00  | 1.0 | 0.84| 6.50| 0.38| 1006  | 1006   |
| 0   | 0.00  | 1.0 | 0.84| 6.50| 0.38| 1006  | 1006   |
| 0   | 0.00  | 1.0 | 0.84| 6.50| 0.38| 1006  | 1006   |
| 0   | 0.00  | 1.0 | 0.84| 6.50| 0.38| 1006  | 1006   |

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