Research on Robot Kinematic Calibration Based on Hand-eye Vision and Distance Error Model

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Abstract—Based on the MDH kinematic model of industrial robot, the homogeneous transformation error model between the coordinates of each joint is established; The distance error model is combined with the established homogeneous transformation error model to obtain the kinematic calibration model based on distance error, avoiding the complex calculation and coordinate conversion of the coordinate system; Finally, the ABB1410 robot is used as the experimental object. Through the compensation and correction of each joint parameter, the experimental results prove that the positioning accuracy and distance accuracy of the robot can be greatly improved, and it can be applied to the experimental industrial field.

Keywords—Kinematic Calibration, Distance error, Hand-eye vision, Modified DH model

I. INTRODUCTION

Due to the influence of the machining error of the shaft, there is a certain error between the actual kinematic parameters and the nominal kinematic parameters between the various links of the robot, which greatly affects the absolute positioning accuracy of the robot. Robot calibration is the process of obtaining the parameters of the robot model by using appropriate measurement methods and parameter solving methods to improve the accuracy of the robot [1].

A lot of research on robot kinematics calibration has been carried out by scholars. Hayati[2] introduced a modified DH model by introducing a rotational transformation to describe the deviation of adjacent parallel joint axes; Chen[3] used the distance error for DH kinematics model calibration and carried out the experimental verification; Tan[4] combined the screw theory with distance error model to calibrate robot parameters. In [5], monocular vision is used to perform kinematic calibration, but does not consider the problem of small view field of the camera. In [6-10], laser measuring instrument, three-coordinate measuring instrument and other measuring equipment are used to obtain the robot end position for calibration, however, the operation is complicated and the measuring speed is slow.

Considering the above problems comprehensively, a 6-DOF robot kinematic error calibration model is established based on the MDH kinematic model and distance error model in this paper. The vision measurement method is used to simplify the measurement process, and the equal distance calibration model is introduced to further improve the accuracy. The feasibility of the algorithm is verified by calibration experiments.

II. KINEMATIC CALIBRATION MODEL

A. Robot kinematic model

The D-H model is the most common model for describing the relationship between the rotation and position of each axis of the robot. It is generally described by \( a_{i-1}, \alpha_i, d_i, \) and \( \theta_i \), and Figure 1 is a schematic diagram of the link coordinate system of the ABB 1410 robot.

![Fig.1 Schematic diagram of link coordinates](image)

The D-H model established above is actually not complete. It can be seen from Figure 1 that, the absolute parallelism between rotation axis 2 and 3 of the robot does not exist in practice, and the minor assembly errors are inevitable, which will result in the error between their actual perpendicular line and ideal perpendicular lines. Therefore, when the rotation axes of two adjacent joints are approximately parallel, it is necessary to introduce the amount of rotation \( \beta \) around the y-axis to represent the modified DH (MDH) model. In MDH model, the transformation matrix of the adjacent joint coordinate system is given in Equation (1): the length of link is \( a \), the rotation angle of the link is \( \alpha \), the offset of the link is \( d \), the joint angle is \( \theta \), and the angle of rotation around the y-axis is \( \beta \).
\[ T_{i+1}^{*} = \text{Trans}(X_{i}, a_{i}) \cdot \text{Rot}(X_{i}, a_{i}) \cdot \text{Trans}(Z, d_{i}) \cdot \text{Rot}(Z, \theta_{i}) \cdot \text{Rot}(Y, \beta_{i}) \]  \hspace{1cm} (1)

**B. Distance error model**

The distance error refers to the deviation between the actual movement distance of the robot and the movement distance of the command. Figure 2 is a schematic diagram of the distance error model.

![Schematic diagram of distance error model](image)

Fig.2 Schematic diagram of distance error model

The coordinates of the measured point of the robot end effector in the basic coordinate system are \( P_{k}(i) \), and the coordinates in the measuring coordinate system are \( P_{\text{kw}}(i) \). The distance error of any two points can be expressed in Equation (2).

\[ \Delta d(i + 1) = \left| P_{k}(i + 1) \right| - \left| P_{\text{kw}}(i + 1) \right| \]  \hspace{1cm} (2)

Where, \( P_{k}(i + 1) \) represents the distance from \( P_{k}(i) \) to \( P_{k}(i + 1) \) on the actual trajectory of the robot, which is obtained by the measurement tool. \( P_{\text{kw}}(i + 1) \) represents the distance from \( P_{\text{kw}}(i) \) to \( P_{\text{kw}}(i + 1) \) on the instruction trajectory of the robot, which can be read directly from the robot teaching pendant. The relationship between the distance error and the position error between two adjacent points can be expressed in Equation (3).

\[ \Delta d[i + 1] = \frac{P_{k}(i + 1) - P_{\text{kw}}(i)}{P_{k}(i + 1) - P_{\text{kw}}(i)} \]  \hspace{1cm} (3)

Where, \( dp \) is the position deviation vector of a certain point in the basic coordinate system.

**C. Robot kinematic calibration model**

There are many factors that cause the error of the robot system, among which the geometric error of link has the greatest influence, these are \( \Delta a_{i-1} \), \( \Delta a_{i-1} \), \( \Delta d_{i} \), \( \Delta \theta_{i} \), \( \Delta \beta_{i} \). Under the influence of these five error quantities, the homogeneous transformation matrix \( T_{i}^{*} \) of the adjacent link coordinate system become \( T_{i+1}^{*} + dT_{i+1} \). \( dT_{i+1} \) is the differential perturbation matrix caused by the geometric error of the links. The matrix expression is shown in Equation (4).

\[ dT_{i+1} = \frac{\partial T_{i+1}}{\partial a_{i}} \Delta a_{i-1} + \frac{\partial T_{i+1}}{\partial a_{i}} \Delta a_{i-1} + \frac{\partial T_{i+1}}{\partial d_{i}} \Delta d_{i} + \frac{\partial T_{i+1}}{\partial \theta_{i}} \Delta \theta_{i} + \frac{\partial T_{i+1}}{\partial \beta_{i}} \Delta \beta_{i} \]  \hspace{1cm} (4)

Where,

\[ \frac{\partial T_{i+1}}{\partial a_{i}} = T_{i+1}^{*} \frac{\partial A_{i}}{\partial a_{i}} \], \[ \frac{\partial T_{i+1}}{\partial d_{i}} = T_{i+1}^{*} \frac{\partial d_{i}}{\partial d_{i}} \], \[ \frac{\partial T_{i+1}}{\partial \theta_{i}} = T_{i+1}^{*} \frac{\partial \theta_{i}}{\partial \theta_{i}} \], \[ \frac{\partial T_{i+1}}{\partial \beta_{i}} = T_{i+1}^{*} \frac{\partial \beta_{i}}{\partial \beta_{i}} \]

\[ \Delta a = A_{i} + A_{i} \Delta a_{i-1} + A_{i} \Delta d_{i} + A_{i} \Delta \theta_{i} + A_{i} \Delta \beta_{i} \]

Then, \( dT_{i+1} = T_{i+1}^{*} \Delta a_{i} \).

If every joint parameter error exists, then the adjacent joints have homogeneous disturbance matrix.

The simplification error matrix is shown in Equation (5).

\[ dT_{i} = \begin{bmatrix} 0 & -\delta_{i} & \delta_{i} & d_{i} \\ -\delta_{i} & 0 & -\delta_{i} & d_{i} \\ \delta_{i} & \delta_{i} & 0 & d_{i} \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (5)

The first three items in the fourth column of \( dT_{i} \) are robot positioning errors, which is \( dp = \begin{bmatrix} d_{x} & d_{y} & d_{z} \end{bmatrix}^{T} \), the error model is shown in Equation(6).

\[ dp = \sum_{i=1}^{N}(b_{x,i} \Delta a_{i-1} + b_{y,i} \Delta d_{i} + b_{z,i} \Delta \theta_{i} + b_{o,i} \Delta \beta_{i}) \]  \hspace{1cm} (6)

According to Equation (6), it can be converted to Equation (7).

\[ \Delta d(i + 1) = \frac{\Delta P_{kw}(i)}{\Delta P_{kw}(i)} (B_{i} - B_{\text{w}}) \Delta q \]  \hspace{1cm} (7)

In which, \( P_{kw}(i + 1) - P_{kw}(i) = \Delta P_{kw}(i) \).

**D. Simultaneous calibration of hand-eye relationship and kinematic parameters**

When using the CCD camera to obtain the actual distance, first calibrate the internal parameters of the camera, then find the external parameter matrix \( M \) of the captured image, and calculate the hand-eye calibration result \( X \), then the position and pose of the robot end-effector relative to the world coordinate system can be denoted as \( A = M^{-1} \cdot X^{-1} \), and the first three elements of the fourth column of \( A \) are the world coordinates of the robot end.

However, when the hand-eye calibration is performed to solve the Matrix \( X \) as camera coordinate system relative to the robot end-effector coordinate system, the nominal value
of the robot kinematics parameter is used, so there is a certain error \( dX \) in the calibrated hand-eye matrix \( X \). The matrix \( X^{-1} \) can be regarded as the rotation and translation relative to the coordinate axes of the camera [11], which denoted as Equation (8).

\[
X^{-1} = T_r(a) - R_r(a) - T_c(d) - R_c(\theta) - R_r(\beta) - T_c(l)
\]  

The relationship between \( X^{-1} \) and its error \( dX^{-1} \) is shown in Equation (9).

\[
A + dA = M^{-1}(X^{-1} + dX^{-1})
\]  

The actual distance between the points where the robot actually moves is denoted as \( d_w(i + 1) \). The distance between the points that calculated by the hand-eye calibration matrix \( X \) with error is denoted as \( d'_w(i + 1) \), the relationship between these two distances above is shown in Equation(10).

\[
d_w(i + 1) \cdot d'_w(i + 1) = \begin{bmatrix} x'_w(i + 1) - x'_w(i) \\ y'_w(i + 1) - y'_w(i) \\ z'_w(i + 1) - z'_w(i) \end{bmatrix} \cdot (B'_w - B_w) = K_w d_w
\]  

The nominal distance between the points along the robot trajectory according to the instructions is denoted as \( d_n(i + 1) \), the relationship between \( d_w(i + 1) \) and \( d_n(i + 1) \) is shown in Equation (11).

\[
d_n(i + 1) \cdot d_n(i + 1) = \begin{bmatrix} x_n(i + 1) - x_n(i) \\ y_n(i + 1) - y_n(i) \\ z_n(i + 1) - z_n(i) \end{bmatrix} \cdot (B_n(i) - B_n(i - 1)) = K_n d_n
\]  

Through Equation (10) and (11), the calibration for hand-eye relationship and kinematic parameters can be obtained.

\[
d'_w(i + 1) - d_n(i + 1) = [K_n - K_w] \begin{bmatrix} d'_w \\ d_n \end{bmatrix}
\]  

III. KINEMATIC ERROR CALIBRATION EXPERIMENT

The calibration experimental equipment is shown in Figure.3, and the schematic diagram of the experiment is shown in Figure.4. The CCD industrial camera is used to perform the kinematic parameters calibration of the ABB1410 robot. The error compensation amount includes 30 kinematic error items of each joint and 6 hand-eye relationship error items, so there must be at least 36 sets distance error equations. Two batches of data were measured in the experiment. The first batch of data including 37 points is used for the compensation calculation; the second batch of data including 36 points is used to test the corrected distance error. When using the first data batch for error compensation calculation, in principle, it is only needed to find 9 points in the space, so capture 9 images and then arrange the combination of these points, 36 sets of distance error equations can be obtained. However, the distance between the random points of space is an indefinite value, and the calculation rounding error due to the excessive difference of \( \Delta d(i + 1) \) may be produced. Therefore, the equidistant calibration model is adopted to make the equal distance between two adjacent points on the robot’s trajectory.

As shown in Figure 5, the position of the start point of the robot is determined firstly, and each subsequent point of motion is referenced to the previous point. The distance between two adjacent points is calculated by the latter point subtracted form the previous point, and the distance is denoted as \( d \). External camera parameter matrix \( A \) of each image is solved by using the MATLAB calibration toolbox. According to matrix \( A \), the actual coordinates denoted as \( P_w(i) \) of each position can be obtained, and the instruction coordinates denoted as \( P_{aw}(i) \) of each position can be read from the robot teach pendant. The error compensation value of kinematic parameters and hand-eye parameters can be obtained by solving the least squares solution.
TABLE 1 STATISTIC DATA OF DISTANCE ERRORS (ADE)

| Joint Num | Kinematics Parameters | Before Calibration | Parameter Errors | After Calibration |
|-----------|-----------------------|--------------------|------------------|------------------|
| Jnt 1     | $a_1$/mm              | 0                  | 0                | 0                |
|           | $a_2$/°               | 0                  | 0.004            | 0.004            |
|           | $d_1$/mm              | 475                | 0                | 475              |
|           | $\theta_1$/°          | 0                  | -0.032           | -0.032           |
| Jnt 2     | $a_2$/mm              | 150                | 0.075            | 150.075          |
|           | $a_2$/°               | -90                | 0                | -90              |
|           | $d_2$/mm              | 0                  | -0.3519          | -0.3519          |
|           | $\theta_2$/°          | 90                 | 0                | 90              |
| Jnt 3     | $a_3$/mm              | -600               | -0.075           | -600.075         |
|           | $a_3$/°               | 0                  | 0.01             | 0.01             |
|           | $d_3$/mm              | 0                  | -0.3519          | -0.3519          |
|           | $\theta_3$/°          | 0                  | -0.0002          | -0.002           |
| Jnt 4     | $a_4$/mm              | -120               | 1.88             | -118.12          |
|           | $a_4$/°               | 90                 | -0.16            | 89.84            |
|           | $d_4$/mm              | 720                | -0.30            | 719.7            |
|           | $\theta_4$/°          | 0                  | -0.31            | -0.31            |
| Jnt 5     | $a_5$/mm              | 0                  | 0.030            | 0.030            |
|           | $a_5$/°               | -90                | 0.002            | -89.998          |
|           | $d_5$/mm              | 0                  | 4.269            | 4.269            |
|           | $\theta_5$/°          | 0                  | 0.001            | 0.001            |
| Jnt 6     | $a_6$/mm              | 0                  | -0.042           | -0.042           |
|           | $a_6$/°               | 90                 | 0.006            | 90.006           |
|           | $d_6$/mm              | 85                 | -1.323           | 83.677           |
|           | $\theta_6$/°          | 0                  | 0                | 0                |

The geometric parameters of the robot links before and after calibration are shown in Table 1. It can be seen that after the parameter is solved, the error compensation amount of each link parameter can be well determined. After the offset is added to the robot model, the compensated link parameters can be obtained.

The distance error of the 36 points involved in the distance error check before and after correction is shown in Figure 6, and the distance error statistic is shown in Table 2. It can be seen from Table 2 that the accuracy of the distance error of 36 points is obviously improved, the mean value and the standard deviation of the absolute distance errors (ADE) are significantly reduced.

TABLE 2 STATISTIC DATA OF DISTANCE ERRORS (ADE)

| Distance Errors | Maximum of ADE | Mean of ADE | Standard of ADE |
|-----------------|----------------|-------------|-----------------|
| Before Calib    | 12.3006        | 4.8725      | 3.2907          |
| After Calib     | 6.2522         | 3.0336      | 1.4179          |

Fig.6 Distance errors before and after calibration

IV. CONCLUSIONS

(1) A kinematic calibration model based on MDH model and distance error model is constructed for robot ABB1410. The introduction of distance error avoids the conversion of measurement coordinate system to the basic coordinate system of the robot.

(2) The position and posture of the robot end-effector is obtained by vision measurement, which is convenient to perform. The method of equidistant calibration model for simultaneous calibration of hand-eye relationship and kinematic parameters is introduced, which further improves the calibration accuracy.

(3) The results of calibration experiments show that the robot kinematic calibration method based on visual and distance error can obviously improve the positioning accuracy and distance precision of the robot, which is of great significance for industrial practice.

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REFERENCES

[1] D.S Wang, J.N Chi. Survey on robot kinematics calibration [J]. Application Research of Computers, 2007, 24(9): 8-11.
[2] S.A Hayati. Robot arm geometric link parameter estimation[C]. 22nd IEEE Conference on Decision and Control. Piscataway, NJ, USA: IEEE, 1983: 1477-1483.
[3] G Chen, Q.X Jia, T Li. Calibration Method and Experiments of Robot Kinematics Parameters Based on Error Model [J]. ROBOT, 2012, 34(6): 680-688.
[4] Y.S Tan, H.X Sun, Q.X Jia. New manipulator calibration method based on screw theory and distance error [J]. Journal of Beijing University of Aeronautics and Astronautics, 2006, 32(9): 1104-1108.
[5] Z.X Xie, S.H Xin, X.Y Li. Method of Robot Calibration Based on Monocular Vision [J]. Journal of Mechanical Engineering, 2011, 47(5): 35-39.

[6] Y.J Ren, J.G Zhu. Measurement Robot Calibration Model and Algorithm Based on Distance Accuracy [J]. Acta Metrologica Sinica, 2008, 29(3): 198-202.

[7] W.B Gao, H.G Wang, Y Jiang, X.A Pan. Kinematic Calibration Method of Robots Based on Distance Error [J]. Robot, 2013, 35(5): 600-605.

[8] T Zhang, X.L Dai. Kinematic Calibration of Robot Based on Distance Error [J]. Journal of South China University of Technology (Natural Science Edition), 2011, 39(11): 98-103.

[9] C Li, Y.Q Wu, L Harald. POE-Based Robot Kinematic Calibration Using Axis Configuration Space and the Adjoint Error Model [J]. IEEE Transactions on Robotics, 2016, 32(5): 1264-1279.

[10] G Chen, L Wang, Q.X Jia. Calibration Method and Experiments of Kinematic Parameters for Robonaut Based on MCPC Model [J]. ROBOT, 2017, 39(2):151-159.

[11] H.W Zha. Calibration and Measuring Planning of Measuring Robot Based on Stereo Vision [D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2013.