Physics Implications of a Perturbative Superstring Construction

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Abstract

We investigate the low energy physics implications of a prototype quasi-realistic superstring model with an anomalous $U(1)$. First, we present the techniques utilized to compute the mass spectrum and superpotential couplings at the string scale, and demonstrate the results for the effective theory along a particular flat direction/“restabilized vacuum” of the model. We then analyze the gauge symmetry breaking patterns and renormalization group equations to determine the mass spectrum at the electroweak scale for a particular numerical example with a realistic $Z - Z'$ hierarchy. Although the model considered is not fully realistic, the results demonstrate general features of quasi-realistic string models, such as extra matter (e.g. $Z'$ gauge bosons and an extended Higgs sector) at the electroweak/TeV scale, and non-canonical couplings (such as $R$-parity violating terms).

I. INTRODUCTION

Predictions from superstring theory provide natural possible extensions of the MSSM. However, there are several problems to be resolved in attempting to connect string theory to the observable, low energy world. First, many models can be derived from string theory, and there is no dynamical principle as yet to select among them. Furthermore, no fully realistic model, i.e., a model which contains just the particle content and couplings of the MSSM, has been constructed. In addition, there is no compelling scenario for how to break supersymmetry in string theory, and so soft supersymmetry breaking parameters must be introduced into the model by hand, just as in the MSSM.
We adopt a more modest strategy and consider a class of quasi-realistic models constructed within weakly coupled heterotic string theory. In addition to the necessary ingredients of the MSSM, such models generically contain an extended gauge structure that includes a number of $U(1)$ gauge groups and “hidden” sector non-Abelian groups, and additional matter fields (including a number of SM exotics and SM singlets). They predict gauge couplings unification (sometimes with non-standard Kač-Moody level) at the string scale $M_{\text{String}} \sim 5 \times 10^{17}$ GeV. The most desirable feature of models in this class is that the superpotential is explicitly calculable; in particular, the non-zero Yukawa couplings are $\mathcal{O}(1)$, and can naturally accommodate the radiative electroweak symmetry breaking scenario. In addition, string selection rules can forbid gauge-allowed terms, in contrast to the case in general field-theoretic models.

In addressing the phenomenology of these models, there are two complementary approaches. The first is the “bottom-up” approach, in which models with particle content and couplings motivated from quasi-realistic string models are studied to provide insight into the new physics that can emerge from string theory (such as additional $Z'$ gauge bosons) [1–5]. In this work we adopt the second (“top-down”) approach, and analyze a prototype string model (Model 5 of [10]) in detail. The analysis of this class of string models (done in collaboration with G. Cleaver and J. R. Espinosa) proceeds in several stages, which will be briefly summarized below and is documented in [6–9]. We then focus on the main results: the determination of the mass spectrum and trilinear couplings at the string scale, the renormalization group analysis, the low energy gauge symmetry breaking patterns and the mass spectrum of the model at the electroweak scale.

Our analysis shows that the prototype model is not fully realistic. In particular, many of the SM exotics remain massless in the low energy theory. However, we find there are other general features of the model which have interesting phenomenological implications, including an additional low-energy $U(1)'$ gauge group, $R$ parity violating couplings, “mixed” effective $\mu$ terms, and extended chargino, neutralino, and Higgs sectors (with patterns of mass spectra that differ substantially from the case of the MSSM).

In section II, we discuss the generation of the effective mass terms and the trilinear couplings associated with the flat direction. In section III, we present the effective couplings and the implications of the effective theory along a particular flat direction as an illustrative example. We conclude in section IV.

**II. FLAT DIRECTIONS AND EFFECTIVE COUPLINGS**

The model we have chosen as a prototype model to analyze is Model 5 of [10]. Prior to vacuum restabilization, the model has the gauge group

$$\{ SU(3)_C \times SU(2)_L \}_\text{obs} \times \{ SU(4)_2 \times SU(2)_2 \}_\text{hid} \times U(1)_A \times U(1)^6,$$

and a particle content that includes the following chiral superfields in addition to the MSSM fields:

$$6(1, 2, 1, 1) + (3, 1, 1, 1) + (\bar{3}, 1, 1, 1) +$$

$$4(1, 2, 1, 2) + 2(1, 1, 4, 1) + 10(1, 1, \bar{4}, 1) +$$
\[8(1, 1, 1, 2) + 5(1, 1, 4, 2) + (1, 1, \bar{4}, 2) +
8(1, 1, 6, 1) + 3(1, 1, 1, 3) + 42(1, 1, 1, 1),\]  
(2)

where the representation under \((SU(3)_C, SU(2)_L, SU(4)_2, SU(2)_2)\) is indicated. The SM hypercharge is determined as a linear combination of the six non-anomalous \(U(1)\)'s.

As the first step of the analysis, we address the presence of the anomalous \(U(1)_A\) generic to this class of models. The standard anomaly cancellation mechanism generates a nonzero Fayet-Iliopoulos (FI) term of \(O(M_{\text{String}})\) to the \(D^-\) term of \(U(1)_A\). The FI term would appear to break supersymmetry at the string scale, but certain scalar fields are triggered to acquire large VEV's along \(D^-\) and \(F^-\) flat directions, such that the new “restabilized” vacuum is supersymetric. The complete set of \(D^-\) and \(F^-\) flat directions involving the non-Abelian singlet fields for Model 5 was classified in [6].

In a given flat direction, the rank of the gauge group is reduced, and effective mass terms and trilinear couplings may be generated from higher order terms in the superpotential:

\[
W_M = \frac{\alpha K + 2}{M_{Pl}^2} \Psi_i \Psi_j \langle \Phi^K \rangle
\]

\[
W_3 = \frac{\alpha K + 3}{M_{Pl}^2} \Psi_i \Psi_j \Psi_k \langle \Phi^K \rangle,
\]

in which the fields which are in the flat direction are denoted by \(\Phi\), and those which are not by \(\{\Psi_i\}\). Hence, some fields acquire superheavy masses and decouple. The effective Yukawa couplings of the remaining light fields are typically suppressed compared with Yukawa couplings of the original superpotential (the \(\alpha_K\) coefficients are in principle calculable; for details, see [11]).

This procedure has been carried out for the prototype model in [6,8]. We carry out the analysis of the implications of the model for the flat directions that break the maximal number of \(U(1)'s\), leaving \(U(1)_Y\) and \(U(1)'\) unbroken. The list of matter superfields and their \(U(1)_Y\) and \(U(1)'\) charges are presented in Table 1.

III. EXAMPLE: LOW ENERGY IMPLICATIONS OF A REPRESENTATIVE FLAT DIRECTION

We choose to present the analysis of the model along a particular flat direction \[^2\]. The flat direction we consider is the \(P_1'P_2'P_3'\) direction (in the notation of [6,7]), which involves the set of fields \(\{\varphi_2, \varphi_5, \varphi_{10}, \varphi_{13}, \varphi_{27}, \varphi_{29}, \varphi_{30}\}\).

\(^1\)However, in the prototype model considered the effective trilinear couplings arising from fourth order terms are comparable in strength to the original Yukawas.

\(^2\)Other flat directions involve other interesting features, such as fermionic textures, baryon number violation, and the possibility of intermediate scale \(U(1)'\) breaking.
Along this flat direction, the effective mass terms which involve the observable sector fields and the non-Abelian singlets are given by

$$W_M = g h_f \bar{h}_b \langle \varphi_{27} \rangle + g h_g \bar{h}_d \langle \varphi_{29} \rangle + \frac{\alpha_4^{(1)}}{M_{Pl}} h_b \bar{h}_b \langle \varphi_5 \varphi_{10} \rangle + \frac{\alpha_4^{(2)}}{M_{Pl}} h_b \bar{h}_b \langle \varphi_2 \varphi_{13} \rangle$$

$$+ \frac{g}{\sqrt{2}} (e^c_a e_b + e^c_a e_a) \langle \varphi_{30} \rangle + \frac{g}{\sqrt{2}} (\varphi_1 \varphi_{15} + \varphi_4 \varphi_9) \langle \varphi_{10} \rangle + \frac{g}{\sqrt{2}} (\varphi_7 \varphi_{16} + \varphi_9 \varphi_{12}) \langle \varphi_2 \rangle$$

$$+ \frac{g}{\sqrt{2}} (\varphi_6 \varphi_{26} + \varphi_8 \varphi_{23} + \varphi_14 \varphi_{17}) \langle \varphi_{29} \rangle + \frac{\alpha_4^{(3)}}{M_{Pl}} \varphi_{21} \varphi_{25} \langle \varphi_{27} \varphi_{29} \rangle .$$

(5)

The effective trilinear couplings involving all fields which couple directly to the observable sector fields are given by:

$$W_3 = g Q_c u^c \bar{h}_c + g Q_c d^c \bar{h}_c + \frac{\alpha_4^{(4)}}{M_{Pl}} Q_c d^c \bar{h}_a \langle \varphi_{29} \rangle + \frac{g}{\sqrt{2}} e^c_a \bar{h}_a h_c + \frac{g}{\sqrt{2}} e^c_d \bar{h}_d h_c$$

$$+ \frac{\alpha_5^{(1)}}{M_{Pl}} e^c_h \bar{h}_c \langle \varphi_5 \varphi_{27} \rangle + \frac{\alpha_5^{(2)}}{M_{Pl}} e^c_h \bar{h}_a \langle \varphi_{13} \varphi_{27} \rangle + g h_b \bar{h}_c \langle \varphi_{20} \rangle .$$

(6)

In the observable sector, the fields which remain light include both the usual MSSM states and exotic states such as a fourth (SU(2)_L singlet) down-type quark, extra fields with the same quantum numbers as the lepton singlets, and extra Higgs doublets. There are other massless states with exotic quantum numbers (including fractional electric charge) that also remain in the low energy theory. The U(1)' charges of the light fields are family nonuniversal (and hence is problematic with respect to FCNC).

There are some generic features of the superpotential which are independent of the details of the soft supersymmetry breaking parameters. In addition to a large top-quark Yuakwa coupling (∼ O(1)) which is necessary for radiative electroweak symmetry breaking, the couplings indicate t – b and (unphysical) τ – μ Yukawa unification, with the identification of the fields h_c, h_c as the standard electroweak Higgs doublets. There is no elementary or effective canonical μ-term involving h_c and h_c, but rather non-canonical effective μ terms involving additional Higgs doublets. Finally, there is also a possibility of lepton-number violating couplings; thus this model violates R-parity, and has no stable LSP.

With the knowledge of the massless spectrum at the string scale, the gauge coupling beta-functions can be determined, and the gauge couplings can then be run from the string scale (where they are predicted to unify) to the electroweak scale. We determine the gauge coupling constant g = 0.80 at the string scale by assuming α_s = 0.12 (the experimental value) at the electroweak scale, and evolving g_3 to the string scale. We then use this value as an input to determine the electroweak scale values of the other gauge couplings by their (1-loop) renormalization group equations (RGE’s). The low energy values of the gauge couplings are not correct due to the exotic matter and non-standard k_γ = 11/3 for this

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3We refer the reader to [8,9] for further details of the model, such as the couplings involving the hidden sector fields.
FIG. 1. (a): Scale variation of the gauge couplings $\sqrt{k}$, with $t = (1/16\pi^2)\ln(\mu/M_{\text{String}})$, $M_{\text{String}} = 5 \times 10^{17}$ GeV, and $g(M_{\text{String}}) = 0.80$. (b): The running of the Yukawa couplings, in which the terms in (6) are denoted by $\Gamma_{Q1,2,3}$, $\Gamma_{l1,2,3,4}$, and $\Gamma_S$, respectively.

model; however, it is surprising that $\sin^2\theta_W \sim 0.16$ and $g_2 = 0.48$ are not too different from the experimental values 0.23 and 0.65, respectively.

The string-scale values of the Yukawa couplings of (6) are calculable (with the knowledge of the VEV’s of the singlet fields in the flat direction). Utilizing the RGE’s, we can also determine the low energy values of the Yukawa coupling constants. The running of the gauge couplings and the Yukawa couplings are shown in Figure 1.

To address the gauge symmetry breaking scenarios for this model, we introduce soft supersymmetry breaking mass parameters and run the RGE’s from the string scale to the electroweak scale. While the qualitative features of the analysis are independent of the details of the soft breaking, we choose to illustrate the analysis with a specific example with a realistic $Z - Z'$ hierarchy. General considerations [1,3–5] and an inspection of the $U(1)'$ charges of the light fields indicate that, in this example, the $U(1)'$ breaking is at the electroweak (TeV) scale. Due to the lack of a canonical effective $\mu$ term between $\bar{h}_c$, $h_c$ and a singlet, an extended Higgs sector is required, with an additional Higgs doublet and singlet ($\bar{h}_c$, $h_c$, $h_{\nu}$, and $s \equiv \varphi_{20}$). The symmetry breaking is characterized by a large ($O$(TeV)) value of the SM singlet VEV, with the electroweak symmetry breaking at a lower scale due to accidental cancellations.

We now present the mass spectrum for a concrete numerical example of this scenario, which requires mild tuning of the soft supersymmetry breaking mass parameters at the string scale. The initial and final values of the parameters for this example are listed in Table 2.
• **Fermion Masses**: The masses for the $t, b, \tau$, and $\mu$ are due to Yukawa couplings of the original superpotential, as shown in (3). With the identification of $Q_c$ as the quark doublet of the third family and $h_d, h_u$ as the lepton doublets of the third and second families, respectively, $m_t = 156$ GeV, $m_b = 83$ GeV, $m_\tau = 32$ GeV, and $m_\mu = 27$ GeV. The ratio $m_b/m_\tau$ is larger than in the usual $b-\tau$ unification because of the ratio $1 : 1/\sqrt{2}$ of the Yukawa couplings at the string scale, and is probably inconsistent with experiment (4) (of course, the high values for $m_b, m_\tau,$ and $m_\mu$ are unphysical). Finally, $u, d, c, s,$ and $e^-$ remain massless.

• **Squarks/Sleptons**: To ensure a large $M_{Z'}$ in this model, the squark and slepton masses have values in the several TeV range, with $m_{\tilde{t}} L = 2540$ GeV, $m_{\tilde{t}} R = 2900$ GeV; $m_{\tilde{b}} L = 2600$ GeV, $m_{\tilde{b}} R = 2780$ GeV; $m_{\tilde{\tau}} L = 2760$ GeV, $m_{\tilde{\tau}} R = 3650$ GeV; $m_{\tilde{\mu}} L = 2790$ GeV, and $m_{\tilde{\mu}} R = 3670$ GeV.

• **Charginos/Neutralinos**: The positively charged gauginos and higgsinos are $\tilde{W}^+, \tilde{h}_c, \tilde{h}_u$, and the negatively charged gauginos and higgsinos are $\tilde{W}^-, \tilde{h}_c, \tilde{h}_u'$. There is one massless chargino, and the other two have masses $m_{\tilde{\chi}^1_+} = 591$ GeV, and $m_{\tilde{\chi}^2_+} = 826$ GeV.

The neutralino sector consists of $\tilde{B}_0, \tilde{B}, \tilde{W}^0_3, \tilde{h}_c, \tilde{h}_u, \tilde{h}_u', \varphi_2^0,$ and $\tilde{h}_2^0$. The mass eigenvalues are $m_{\varphi_1}^0 = 963$ GeV, $m_{\varphi_2}^0 = 825$ GeV, $m_{\varphi_3}^0 = 801$ GeV, $m_{\varphi_4}^0 = 592$ GeV, $m_{\chi_2}^0 = 562$ GeV, $m_{\chi_3}^0 = 440$ GeV, $m_{\chi_4}^0 = 2$ GeV, and $m_{\chi_5}^0 = 0$.

In both cases, the absence of couplings of the Higgs field $\tilde{h}_u$ in the superpotential leads to a massless chargino and neutralino state. The absence of an effective $\mu$ term involving $h_c$ leads to an additional global $U(1)$ symmetry in the scalar potential, and an ultralight neutralino pair in the mass spectrum.

• **Exotics**: There are a number of exotic states, including the $SU(2)_L$ singlet down-type quark, four $SU(2)_L$ singlets with unit charge (the $\epsilon$ and extra $e^\epsilon$ states), and a number of SM singlet ($\varphi$) states, as well as exotics associated with the hidden sector. The scalar components of these exotics are expected to acquire TeV-scale masses by soft supersymmetry breaking. However, there is no mechanism within our assumptions to give the fermions significant masses.

• **Higgs Sector**: The non-minimal Higgs sector of three complex doublets and one complex singlet leads to additional Higgs bosons compared to the MSSM. Four of the fourteen degrees of freedom are eaten to become the longitudinal components of the $W^\pm$, $Z$, and $Z'$; and the global $U(1)$ symmetry is broken, leading to a massless Goldstone boson (which, however, acquires a small mass at the loop level) in the spectrum.

The spectrum of the physical Higgs bosons after symmetry breaking consists of two pairs of charged Higgs bosons $H_{1,2}^\pm$, four neutral CP even Higgs scalars ($h_i^0, i = 1, 2, 3, 4$), and one CP odd Higgs $A^0$, with masses $m_{H_1^\pm} = 10$ GeV, $m_{H_2^\pm} = 1650$ GeV, $m_{h_1^0} = 33$ GeV, $m_{h_2^0} = 47$ GeV, $m_{h_3^0} = 736$ GeV, $m_{h_4^0} = 1650$ GeV, and $m_{A^0} = 1650$ GeV.
In this model, the bound on the lightest Higgs scalar is different from the traditional bound in the MSSM. It is associated with the breaking scale of the additional global $U(1)$ symmetry; since this scale is comparable to the electroweak scale, not only one but two Higgs scalars will be light in the decoupling limit. In particular, the lightest Higgs mass satisfies the (tree-level) bound \[ m_{h_1}^2 \leq \frac{G^2}{4} v_1^2 + g_1^2 Q_1^2 v_1^2 = (35 \text{ GeV})^2. \] (7)

**IV. CONCLUSIONS**

The purpose of this work has been to explore the general features of this class of quasi-realistic superstring models through a systematic, “top-down” analysis of a prototype model. The results of the investigation of the low energy implications of the mass spectrum and couplings predicted in a subset of the restabilized vacua of this model demonstrate that in general, the TeV scale physics is more complicated than that of the MSSM.

In particular, we have found that noncanonical couplings, such as mixed effective $\mu$ terms and $R$-parity violating operators are typically present in the superpotential. In some other cases, there are possibilities for potentially interesting fermion textures. The model is also characterized by the presence of extra matter in the low energy theory such as SM exotics, extra $Z'$ gauge bosons with TeV scale masses, and additional charginos, neutralinos, and Higgs bosons with patterns of masses that differ substantially from the MSSM.

The particular model we studied is not realistic, in part due to the presence of (ultralight or massless) extra matter. However, due to the large number of possible models that can be derived from string theory, this result does not invalidate the potential viability of string models, or the motivation for investigating their phenomenological implications \[4\]. We stress that the features of this model are likely to be generic to this class of quasi-realistic models based on weakly coupled heterotic string theory, and thus warrant further consideration.

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\[4\] Progress has been made in exploring models in which the exotic matter decouples above the electroweak scale; e.g. see \[7\].
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| $(SU(3)_C, SU(2)_L, SU(4)_2, SU(2)_2)$ | $6Q_Y$ | $100Q'_{Y'}$ |
|--------------------------------------|-------|-------------|
| $(3,2,1,1)$:                         | $Q_a$ | 1           | 68          |
|                                      | $Q_b$ | 1           | 68          |
|                                      | $Q_c$ | 1           | −71         |
| $(3,1,1,1)$:                         | $u^c_a$ | −4          | 6           |
|                                      | $u^c_b$ | −4          | 6           |
|                                      | $u^c_c$ | −4          | −133        |
|                                      | $d^c_a$ | 2           | −3          |
|                                      | $d^c_b$ | 2           | 136         |
|                                      | $d^c_c$ | 2           | −3          |
|                                      | $d^c_d$ | 2           | −3          |
| $(1,2,1,1)$:                         | $h_a$  | 3           | −74         |
|                                      | $h_b$  | 3           | 65          |
|                                      | $h_c$  | 3           | 204         |
|                                      | $h_d$  | 3           | 65          |
|                                      | $h_a$  | −3          | 74          |
|                                      | $h_b$  | −3          | −65         |
|                                      | $h_c$  | −3          | −65         |
|                                      | $h_d$  | −3          | −65         |
|                                      | $h_e$  | −3          | −204        |
|                                      | $h_f$  | −3          | −65         |
|                                      | $h_g$  | −3          | −65         |
| $(3,1,1,1)$:                         | $D_a$  | −2          | −136        |

Table Ia: List of non-Abelian non-singlet observable sector fields in the model with their charges under hypercharge and $U(1)'$. 
|       | 6Q<sub>Y</sub> | 100Q<sub>Y</sub>′ |       | 6Q<sub>Y</sub> | 100Q<sub>Y</sub>′ |
|-------|----------------|-------------------|-------|----------------|-------------------|
| \(e_{a,c}^c\) | 6 | -9 | \(e_{b}^c\) | 6 | -9 |
| \(e_{d,g}^c\) | 6 | 130 | \(e_{e}^c\) | 6 | 130 |
| \(e_{f}^c\) | 6 | 130 | \(e_{h}^c\) | 6 | 130 |
| \(e_{i}^c\) | 6 | -9 | \(e_{a,b}^c\) | 6 | -130 |
| \(e_{c}^c\) | 6 | -130 | \(e_{d,c}^c\) | -6 | 9 |
| \(e_{f}^c\) | -6 | -269 | | | |

|       | 6Q<sub>Y</sub> | 100Q<sub>Y</sub>′ |       | 6Q<sub>Y</sub> | 100Q<sub>Y</sub>′ |
|-------|----------------|-------------------|-------|----------------|-------------------|
| \(\varphi_1\) | 0 | 0 | \(\varphi_{2,3}\) | 0 | 0 |
| \(\varphi_{4,5}\) | 0 | 0 | \(\varphi_{6,7}\) | 0 | 0 |
| \(\varphi_{8,9}\) | 0 | 0 | \(\varphi_{10,11}\) | 0 | 0 |
| \(\varphi_{12,13}\) | 0 | 0 | \(\varphi_{14,15}\) | 0 | 0 |
| \(\varphi_{16}\) | 0 | 0 | \(\varphi_{17}\) | 0 | 0 |
| \(\varphi_{18,19}\) | 0 | -139 | \(\varphi_{20,21}\) | 0 | -139 |
| \(\varphi_{22}\) | 0 | -139 | \(\varphi_{23}\) | 0 | 0 |
| \(\varphi_{24}\) | 0 | 0 | \(\varphi_{25}\) | 0 | 139 |
| \(\varphi_{26}\) | 0 | 0 | \(\varphi_{27}\) | 0 | 0 |
| \(\varphi_{28,29}\) | 0 | 0 | \(\varphi_{30}\) | 0 | 0 |

Table Ib: List of non-Abelian singlet fields in the model with their charges under hypercharge and \(U(1)′\).
|       | $M_Z$ | $M_{String}$ | $M_Z$ | $M_{String}$ |
|-------|-------|--------------|-------|--------------|
| $g_1$ | 0.41  | 0.80         | $M_1$ | 444          | 1695         |
| $g_2$ | 0.48  | 0.80         | $M_2$ | 619          | 1695         |
| $g_3$ | 1.23  | 0.80         | $M_3$ | 4040         | 1695         |
| $g'_1$| 0.43  | 0.80         | $M'_1$| 392          | 1695         |
| $\Gamma_{Q1}$ | 0.96  | 0.80         | $A_{Q1}$ | 3664 | 8682         |
| $\Gamma_{Q2}$ | 0.93  | 0.80         | $A_{Q2}$ | 4070 | 9000         |
| $\gamma_{Q3}$ | 0.27  | 0.08         | $A_{Q3}$ | 5018 | 1837         |
| $\Gamma_{11}$ | 0.30  | 0.56         | $A_{11}$ | −946 | 4703         |
| $\Gamma_{12}$ | 0.36  | 0.56         | $A_{12}$ | −707 | 4532         |
| $\Gamma_{13}$ | 0.06  | 0.05         | $A_{13}$ | 4613 | 4425         |
| $\Gamma_{14}$ | 0.11  | 0.13         | $A_{14}$ | 4590 | 4481         |
| $\Gamma_s$ | 0.22  | 0.80         | $A$   | 1695         | 12544        |
| $m^2_{Q_c}$ | (2706)$_2$ | (2450)$_2$ | $m^2_{d_d}$ | (4693)$_2$ | (2125)$_2$ |
| $m^2_{u_c}$ | (2649)$_2$ | (2418)$_2$ | $m^2_{d_e}$ | (2734)$_2$ | (2486)$_2$ |
| $m^2_{h_c}$ | (1008)$_2$ | (5622)$_2$ | $m^2_{h_b}$ | (826)$_2$  | (2595)$_2$ |
| $m^2_{e'_{W'}}$ | −(518)$_2$ | (5890)$_2$ | $m^2_{e_{W'}}$ | (3031)$_2$ | (11540)$_2$ |
| $m^2_{h_a}$ | (3626)$_2$ | (3982)$_2$ | $m^2_{h_c}$ | −(224)$_2$ | (5633)$_2$ |
| $m^2_{h_d}$ | (3666)$_2$ | (4100)$_2$ | $m^2_{h_b}$ | (4274)$_2$ | (4246)$_2$ |
| $m^2_{e_a}$ | (2770)$_2$ | (3564)$_2$ | $m^2_{e_f}$ | (2780)$_2$ | (3958)$_2$ |
| $m^2_{e_e}$ | (4195)$_2$ | (4254)$_2$ | $m^2_{e_h}$ | (4259)$_2$ | (4236)$_2$ |

Table II: $P_1 P_2 P_3'$ flat direction: values of the parameters at $M_{String}$ and $M_Z$, with $M_{Z'} = 735$ GeV and $\alpha_{Z-Z'} = 0.005$. All mass parameters are given in GeV.