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Numerical Modeling of the Sensitivity of X-Ray Driven Implosions to Low-Mode Flux Asymmetries

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The sensitivity of inertial confinement fusion implosions, of the type performed on the National Ignition Facility (NIF) [1], to low-mode flux asymmetries has been investigated numerically. It is shown that large-amplitude, low-mode shape shapes (Legendre polynomial $P_4$), resulting from low order flux asymmetries, cause spatial variations in capsule and fuel momentum that prevent the DT “ice” layer from being decelerated uniformly by the hot spot pressure. This reduces the transfer of implosion kinetic energy to internal energy of the central hot spot, thus reducing neutron yield. Furthermore, synthetic gated x-ray images of the hot spot self-emission indicate that $P_4$ shapes may be unquantifiable for DT layered capsules. Instead the positive $P_4$ asymmetry “aliases” itself as an oblate $P_2$ in the x-ray images. Correction of this apparent $P_2$ distortion can further distort the implosion while creating a round x-ray image. Long wavelength asymmetries may be playing a significant role in the observed yield reduction of NIF DT implosions relative to detailed post-shot 2D simulations.

Indirect-drive inertial confinement fusion (ICF) [1–3] uses lasers to heat the inside of a high-Z cavity (or hohlraum). The absorbed laser energy is re-emitted as x-rays. These x-rays heat the outer surface of a hollow, spherical, low-Z shell that contains a layer of frozen Deuterium and Tritium (DT) fuel. The heated outer shell ablates, creating a rocket-like reaction force, spherically imploding the shell at extremely high velocity ($\sim 350$ km/s). During the implosion, spherical convergence causes the pressure in the central gaseous void (or hot spot) within the shell to rise. This pressure decelerates the shell, both compressing the solid fuel, and converting the shell’s kinetic energy into hot spot internal energy, thus heating the hot spot, thereby initiating DT fusion reactions. Provided the hot spot areal density is sufficient, $\alpha$-particles will further heat the hot spot, causing bootstrap heating, ignition and thermonuclear burn propagation into the surrounding cold fuel. Numerical modeling indicates that the NIF can, for the first time, initiate inertial fusion ignition in the laboratory [4–6]. In comparison to detailed post-shot simulations [7], current NIF DT layered capsule implosions have neutron yields reduced by $\sim 3 - 10 \times$ and hotspot masses reduced by $2 - 3 \times$ [8, 9], while hot spot temperatures are similar. Low mode capsule shape distortions may explain some of this apparent discordancy [10], as simulations indicate they can reduce the conversion of implosion kinetic energy to hotspot internal energy, thereby bringing hot spot mass, energy, temperature and neutron yield more in line with experiments.

In this Letter, the effects of low-mode capsule shape asymmetries are examined numerically. The non-uniformity of the x-ray flux incident upon the shell and the resultant shell shapes can be described mathematically as a series of Legendre polynomials [11]. It is shown that a $P_4$ implosion asymmetry, that might result from low-order hohlraum generated flux asymmetries, causes spatial variations in the capsule & fuel momentum. This inhibits uniform deceleration of the capsule and fuel by the hot spot pressure, reducing the transfer of implosion kinetic energy to hot spot internal energy thus significantly reducing the capsule performance. Furthermore, simulated gated x-ray images of the hot spot self-emission show reduced sensitivity to the $P_4$ mode, instead the images appear to have a pronounced oblate $P_2$ shape. Reducing the amplitude of the oblate $P_2$ shape (as measured from the x-ray image) further reduces the sensitivity to the $P_4$ mode meaning the resulting x-ray images are round despite the capsule shape being highly distorted. Comparisons are made between key physical properties of the implosion, synthetically generated experimental observables, and NIF data.

The indirect-drive approach to ICF smooths high mode spatial non-uniformities in the x-ray flux incident upon the capsule, however the spatial distribution of the cones of laser beams which illuminate the hohlraum means that low mode x-ray flux non-uniformities can occur [1], these are considerably lower mode than those recently examined by Thomas et al [12]. Capsule-only, two-dimensional (2D), cylindrically-symmetric geometry simulations were performed with the radiative-magnetohydrodynamics code Hydra [13]. These were driven by an x-ray drive taken from an integrated hohlraum simulation which was adjusted to match the shock timing data from the VISAR diagnostic [14, 15] from NIF shot N110521, and the capsule implosion trajectory [16] measured on NIF shot N110625. QEOS [17] was used with tabular opacities and multi-group radia-
tion diffusion. The effects of hohlraum $P_4$ flux asymmetries were investigating by perturbing the applied flux with a $P_4$ distribution function of amplitude varying from $+10\%$ to $−10\%$. 2D Hydra modeling of the hohlraum & capsule [18] suggests the $P_4$ flux asymmetry incident on the capsule would be expected to be $<3\%$, except for in the first $\sim 2$ ns of the laser pulse where the flux asymmetry can be up to $10\%$. To date there is no direct measure of NIF hohlraum radiation asymmetry. The flux asymmetries were applied during the discrete time intervals $0 - 2$ ns (the ‘picket’ [19]), $2 - 11.5$ ns (the ‘trough’), $11.5 - 14$ ns ($2^{nd}$ shock), $14 - 16$ ns ($3^{rd}$ shock) and $16 - 18$ ns ($4^{th}$ rise) and $18 - 21.5$ ns (peak drive), creating $> 200$ 2D modeling runs of both DT layered capsules and DHe$^3$ gas filled capsules with a surrogate CH ‘fuel’ mass (symmetry capsules). In order to recreate images from the NIF gated x-ray diagnostic [20][GXD], time resolved, $11$ µm resolution, synthetic gated x-ray images of the hot spot self-emission $> 6$ keV, were created from polar and equatorial directions by post processing the Hydra runs. Hot spot and synthetic GXD shapes were characterised by a Legendre polynomial decomposition of the appropriate contour. The hot spot contour is defined as the minimum radius where $T_{ej} > \frac{1}{2} T_{ej_{max}}$ and $\rho_j < \frac{1}{2} \rho_{j_{max}}$ where $T_e$ is the electron temperature and $\rho$ is the mass density, ‘max’ denotes the maximum value within the $j^{th}$ angular ‘strip’ of cells. This is a robust definition of the hot spot even for highly distorted implosions. Based on previous studies the $17\%$ contour of the GXD is used both for the synthetic GXD and experimentally.

The applied Legendre $P_4$ flux asymmetries induce $P_4$ hot spot shapes at stagnation (see Figs. 1(a) & (c)), the sign of which is dependent on the timing of the applied flux asymmetry. If the asymmetry is present only during the shock compression phase (the first $\sim 18$ ns), shocks created in regions of the capsule exposed to higher flux propagate faster, these break out of the inner DT ice layer earlier, causing these regions to move ahead. This also causes ablator mass to flow laterally, away from the high flux region. Consequently during peak drive the regions initially exposed to high flux are at smaller radii, meaning they are accelerated less efficiently by the hohlraum flux and gain less total momentum. They can also have less aerial density ($\rho r$). The net effect is that the regions experiencing high flux during shock compression will protrude outwards at stagnation. Conversely if the flux asymmetry is applied during peak drive, the regions of the capsule exposed to more flux gain more momentum, and protrude inwards at stagnation. Regardless of the timing of the applied asymmetry, during the stagnation phase of the implosion, pressure within the lower density hot spot decelerates the higher density fuel from peak velocity, making any perturbation on this interface grow due to the Rayleigh-Taylor instability [21, 22], in addition to the Bell-Plesset growth due to convergence.

Figure 1. Axis of rotational symmetry is vertical at Radius $= 0$ µm. (a) DT layered capsule density plot at x-ray bangtime showing a positive Legendre polynomial $P_4$ shape. This simulation had a $10\%$ flux asymmetry applied from 11.5-14 ns. Black arrows indicate the mass flows which occur during stagnation. After bangtime ‘fingers’ of fuel continue to flow inwards (red arrows). White dots depict the hot spot contour. (b) Synthetic gated x-ray image of the hot spot self emission from 1(a), white dots show the $17\%$ contour, $a_4$ is greatly reduced compared to fig. 1(a). (c) The same implosion as fig. 1(a), but 100 ps later. Large $a_4$ brings the bangtime earlier, meaning this image is plotted at the neutron bangtime of an equivalent spherical implosion. (d) The synthetic GXD from 1(c), showing a large negative (oblate) $P_2$ and almost zero $a_4$ despite the obvious $P_4$ in 1(c).

Figure 2 summarizes the scalings of some important DT layered capsule implosion parameters as a function of hot spot $a_4$, all values are extracted from the simulations at x-ray bangtime. Fig. 2(a) depicts the ‘burn averaged’ $\rho r$ (the burn average of a quantity $Q_b = (\sum_{t=0}^{t=\infty} Q_t E_{proddt})/\int_{t=0}^{t=\infty} E_{proddt}$ where $Q_t$ is $Q$ at time $t$ and $E_{prodr}$ the thermonuclear energy production rate in time $dt$) as a function of hot spot $a_4$. Although the spatially averaged $\rho r$ is relatively constant, the lateral mass flows caused by the $P_4$ can create large spatial variations in $\rho r$. The regions with higher momentum continue to propagate radially inwards; fig. 2(b) depicts the remaining capsule kinetic energy (integrated from the hot spot surface to the ablation front) as a function of $a_4$, and the partition of that energy into hot spot internal
variations in a thermonuclear neutron yield as a function of hot spot
the corresponding maxima and minima in DT Layered Capsule: Hotspot Pressure

11.5 11.5 11.5 11.5 11.5 11.5 11.5 11.5

20 0 20 0 20 0 20 0

Figure 2. In (a), (c) and (d), colors depict timing of applied
flux asymmetry, see (c) for legend. (a) Burn averaged hot spot
+ fuel + ablator Pr vs hot spot a4 at x-ray bangtime: large
dots depict spatially averaged Pr, while the smaller points are
the corresponding maxima and minima in Pr. Large spatial
variations in Pr occur due to P4. (b) The burn averaged
energy partition as a function of hot spot a4; increasing P4
perturbations prevent the kinetic energy of the solid fuel +
remaining ablator (black) from being converted to both hot
spot internal energy (red) and solid fuel + remaining ablator
internal energy (blue) during stagnation. (c) Burn averaged
hot spot pressure as a function of hot spot a4. (d) Total
thermonuclear neutron yield as a function of hot spot a4; yield
varies by a factor of 15 over the asymmetry range examined.

tor material is rotationally symmetric about the vertical
axis, so the accumulated material absorbs the x-rays
emitted from the polar-lobes of the hot spot (top and bottom),
while allowing x-rays to more readily pass through
the equatorial regions (left and right). Consequently the
polar-lobes of the hot spot are almost completely invisible
in the synthetic GXD plots. This causes the x-ray image
in this context to have a negative (oblate) P2 shape. As the hot spot
a2 = 0 ± 1 µm (a4 is the amplitude of the P2 mode) for
all these pure P1 modelling runs, the P2 inferred from the
x-ray image is a “false” negative P2 mode. This suggests
that a negative P2 mode measured from the self-emission
x-ray image may in fact be a signature of a positive P1
mode, although it does not preclude the presence of a
true P2 mode. Fig. 3(b) quantifies this aliasing effect.
Symmetry capsules are qualitatively and quantitatively
very similar. This is potentially important for the
interpretation of GXD images from NIF DT implosions,
which often exhibit negative P2 modes [26].

In comparison to detailed 2D post-shot Hydra simulations [7],
DT implosions on the NIF currently have yields reduced by ∼ 3 − 10 ×,
while hot spot temperatures are similar. The inferred [8, 9] experimental hot spot volumes
are increased in comparison to the post-shot simulations,
while the hot spot mass is reduced, causing a 2 − 3 × reduction
in the hotspot density. P1 shape perturbations provide a mechanism which may explain these experimental observations, in particular bringing the yield and ion temperature relationship into better agreement. In the simulations discussed in this Letter, the DT fuel and hot spot do not mix; clear boundaries still exist (note these simulations use smooth capsules, but when nominal realistic capsule surface roughness [27] was employed
and modes up to 200 resolved, no significant implosion degradation occurred for the full range of a4). Consequently,
Unlike high mode ‘mix’ [1] (where the hot spot can be radiatively cooled by high Z impurities), the simulated ion temperature inferred from the neutron spectrum remains unaffected at 3.9 ± 0.05 keV for all $a_4$. The large $a_4$ does however truncate the thermonuclear burn, moving both the neutron and x-ray bangtimes earlier in time, so the capsule is still converging at bangtime. This, combined with the reduction in conversion of kinetic energy into internal energy, means the hot spot volume is increased. The hot spot mass decreases with positive $a_4$, bringing Hydra simulations approximately in line with experimental data, as shown in Table I. This compares NIF experimental data with two Hydra implosions; one is perfectly spherical ($a_4 = 0 \mu m$), and another with $a_4 = +20 \mu m$. Large positive $P_4$ brings the modeled implosion observables approximately in line with the experimental data. $P_0$ and $M_0$ are the amplitude of the 0th Legendre polynomial from the 17% contour of the equatorial and polar x-ray images respectively.

Table I. A comparison of NIF DT layered capsule experimental data from 4 shots N110608-N110908 with two Hydra implosions, one spherical ($a_4 = 0 \mu m$), and another with $a_4 = +20 \mu m$. Large positive $P_4$ brings the modeled implosion observables approximately in line with the experimental data. $P_0$ and $M_0$ are the amplitude of the 0th Legendre polynomial from the 17% contour of the equatorial and polar x-ray images respectively.

| Implosion Parameter | NIF exp. range[5] | Hydra ($a_4 = 0 \mu m$) | Hydra ($a_4 = +20 \mu m$) |
|---------------------|-------------------|-------------------------|---------------------------|
| Hot spot internal energy (kJ) | 0.7-1.4 | 3.1 | 1.3 |
| Hot spot mass (µg) | 2-6.4 | 8 | 5.5 |
| X-ray $P_0$ (µm) | 25-30 | 18.0 | 23.3 |
| X-ray $M_0$ (µm) | 25-35 | 17.0 | 27.1 |
| Ion Temperature (keV) | 3.3-4.4 | 3.9 | 3.9 |
| Fuel $pr$ (g cm$^{-2}$) | 0.77-0.98 | 0.7 | 0.72 |
| Yield (neutrons $\times 10^{14}$) | 1.9-6.0 | 74 | 5.3 |

In summary, numerical simulations have been used to examine the sensitivity of implosions similar to those currently taking place on NIF to low-mode flux asymmetries. It is shown that Legendre polynomial $P_4$ flux and empirically adjusted a $P_2$ flux asymmetry, in addition to the original $P_4$, in order to make the synthetic GXD image appear round. As the applied $P_2$ flux is increased in order to reduce the “false” GXD $a_2$ towards zero, there is a marked additional reduction in sensitivity to the $a_4$ measured from the x-ray image relative to that shown in Fig. 3(a) - in this simulation hot spot $a_4 = 25 \mu m$ and GXD $a_4 = 1 \mu m$. This suggests that attempts to tune the hohlraum to eliminate a “false” $P_2$ can have the unintended consequence of exacerbating overall asymmetry, while further reducing the diagnostic sensitivity to the asymmetry. A corollary of figure 4, is that it is possible to create imploded configurations which appear to be spherical based on both orthogonal GXD images but, in fact, are significantly asymmetric and have greatly reduced performance in comparison to equivalent spherical implosions because a large fraction of the imploding shell’s kinetic energy remains unstagnated.

The Science of Fusion Ignition Workshop [30] identified the understanding of the origin of the measured $pr$ asymmetries as a high priority. Experiments are currently being developed on the NIF to measure low mode asymmetry of the ablator in-flight using x-ray backlighting [16], and of the DT fuel at stagnation using Compton radiography [29]. These will eliminate the degeneracy in inferring implosion asymmetry form hot spot x-ray emission, as identified in this Letter. The $P_4$ x-ray drive asymmetry may be modified by repointing the laser beams within the hohlraum, moving the laser hot spots relative to the capsule. Large beam repointing may require changing the hohlraum length in order for the laser beams to pass cleanly through the laser entrance holes.

In summary, numerical simulations have been used to examine the sensitivity of implosions similar to those currently taking place on NIF to low-mode flux asymmetries. It is shown that Legendre polynomial $P_4$ flux
modes induce $P_4$ shape modes at the time of capsule stagnation. The largest $P_4$ amplitudes studied in this Letter can cause up to 50% of the capsule kinetic energy to remain unconverted to hot spot and DT ice internal energy, in turn reducing the neutron yield by up to 15×. Simulated x-ray images of the hot spot self-emission show reduced sensitivity to the positive $P_4$ mode, instead the images appear to have a pronounced oblate $P_2$ shape. Attempting to correct for this apparent $P_2$ distortion can further distort the implosion while creating x-ray images which appear round and self-consistent from both equatorial and polar directions. This also further reduces the sensitivity to the $P_4$ mode such that that no quantitative evaluation of the hot spot $a_4$ can be made. Long wavelength asymmetries may be playing a significant role in the observed yield reduction of NIF DT implosions relative to detailed post-shot 2D simulations.

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