Regularized Zero-Forcing for Multiantenna Broadcast Channels with User Selection

Zijian Wang and Wen Chen, Senior Member, IEEE

Abstract—A multiantenna multiuser broadcast channel with transmitter beamforming and user selection is considered. Different from the conventional works, we consider imperfect channel state information (CSI) which is a practical scenario for multiantenna broadcast channels. We propose a robust regularized zero-forcing (RRZF) beamforming at the base station. Then we show that the RRZF outperforms zero-forcing (ZF) and regularized ZF (RZF) beamforming even as the number of users grows to infinity. Simulation results validate the advantage of the proposed robust RZF beamforming.

Index Terms—Multiantenna multiuser, signal-to-interference-plus-noise ratio (SINR), beamforming, regularized zero-forcing (RZF).

I. INTRODUCTION

In recent years, multiple-input multiple-output (MIMO) has drawn considerable interest due to the advantages of increasing the data rate [1]. Several beamformings have been presented in the literature to provide the multiplexing gain. But for multiantenna broadcast channels, only precodings can be implemented at the transmitter because the receivers do not mutually cooperate. Linear transmit precodings for broadcast channels have been studied in [2], [3].

For broadcast channels with large number of users, user selection is necessary to provide multiuser diversity. In [4], [5], the authors propose zero-forcing (ZF) beamforming at the transmitter in conjunction with a semiothogonal user selection (SUS) algorithm. Performance analysis of ZF beamforming is studied in [6]. In [7], different beamforming and user selection schemes are compared and analyzed. To deal with the poor performance of ZF for small number of users, beamformings based on hybrid zero-forcing and orthogonal beamforming [8] and channel inversion regularization [9] are proposed. Methods to reduce the feedback needed for user selection have been studied in [10].

In this letter, we propose a robust regularized zero-forcing (RRZF) beamforming, where the user selection is based on the SUS algorithm as in [4]. While the ZF beamforming and regularized ZF (RZF) have degraded performance for imperfect channel state information (ICSI), the proposed RRZF significantly improves the performance. While the conventional optimal $\alpha$ in [9], [11] is $M/p$, where $M$ is the number of transmit antenna and $p$ is the signal-to-noise-ratio (SNR), we found that the optimal $\alpha$ grows with the number of users. Although the RRZF is optimized for small number of users, we show that in the extremal case when the number of users is infinity, the sum rate performance of RRZF still outperforms the ZF and RZF beamforming. Especially, we show that in this extremal case, the sum rate is monotonically increasing with the regularizing factor $\alpha$, and the optimal $\alpha$ is infinity.

In this letter, boldface lowercase letter and boldface uppercase letter represent vectors and matrices, respectively. Notations $||\mathbf{a}||$ stands for the Euclidean norm of a vector $\mathbf{a}$ and $|\alpha|$ stands for the modulus of a complex $\alpha$ respectively. $\text{tr}(\cdot)$ and $(\cdot)^H$ denote the trace and conjugate transpose operation of a matrix. Term $\mathbf{I}_N$ is an $N \times N$ identity matrix. $\overset{w.p.}{\rightarrow}$ represents convergence with probability one. Finally, we denote the expectation operation by $\mathbb{E}\{\cdot\}$.

II. SYSTEM MODEL

We consider a multiantenna multiuser broadcast network which consists of a base station equipped with $M$ antennas, and $K$ user terminals each with only a single antenna. It is assumed that $K > M$. So the base station needs to choose $M$ favorable users out of the $K$ users to transmit $M$ data simultaneously. Then the base station broadcasts $M$ precoded data streams after applying a linear precoder to the original data vector $\mathbf{s} \in \mathbb{C}^M$, where $\mathbb{E}\{\mathbf{ss}^H\} = \mathbf{I}_M$. We denote the precoding matrix at the base station as $\mathbf{W}$ and suppose that the base station transmit power is $P$. A power control factor can be derived as

$$\rho = \sqrt{\frac{P}{\mathbb{E}\{\mathbf{s}^H\mathbf{W}^H\mathbf{W}\mathbf{s}\}}} = \sqrt{\frac{P}{\text{tr}((\mathbf{W}\mathbf{W}^H))}}. \tag{1}$$

The received signal vector at the selected $M$ user terminals is

$$\mathbf{y} = \mathbf{HWs} + \mathbf{n}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{M \times M}$ is the Rayleigh broadcast channel matrix from the base station to the $M$ selected users, in which, all entries are i.i.d complex Gaussian distributed with zero mean and unit variance, and $\mathbf{n} \in \mathbb{C}^M$ is the noise vector, in which, all the entries are i.i.d complex Gaussian distributed with zero mean and variance $\sigma^2$.

From (2), the received signal at the $k$-th user can be rewritten as

$$y_k = \rho h_k^H \mathbf{W}_k \mathbf{s}_k + n_k = \rho h_k^H w_k s_k + \sum_{j=1, j\neq k}^{M} \rho h_j^H w_j s_j + n_k, \tag{3}$$

where $w_k$ is the $k$-th column of $\mathbf{W}$ and $h_k^H$ is the $k$-th row of $\mathbf{H}$ denoting the channel vector from the base station to the
k-th user. Therefore, the signal-to-interference-plus-noise ratio (SINR) of the k-th user is

$$\text{SINR}_k = \frac{\rho^2 |\mathbf{h}_k^H \mathbf{w}_k|^2}{\rho^2 \sum_{j=1, j \neq k}^M |\mathbf{h}_j^H \mathbf{w}_j|^2 + \sigma^2}. \tag{4}$$

For we aim to analyze the RZF for user selection instead of finding the optimal algorithm, we generalize a simplified SUS (semiorthogonal user selection) algorithm in [4] as follows. It will be stopped when |S| = M.

Step 1) Initialization:

$$X_1 = \{1, \ldots, K\}; \quad i = 1; \quad S = \emptyset. \tag{5}$$

Step 2) Select the i-th user as follows:

$$\pi(i) = \underset{k \in X_i}{\text{argmax}} \|\mathbf{h}_k\|; \quad S \leftarrow S \cup \pi(i); \tag{6}$$

Step 3) If |S| < M, then calculate $X_{i+1}$, and the set of users semiorthogonal to the selected user

$$X_{i+1} = \left\{ k \in X_i, k \neq \pi(i); \frac{|\mathbf{h}_k^H \mathbf{h}_{\pi(i)}|}{\|\mathbf{h}_{\pi(i)}\|} < \beta \right\}; \tag{7}$$

$$i \leftarrow i + 1. \tag{8}$$

In every step, the algorithm selects the best user among the user pool which are semiorthogonal to the selected users.

III. RRZF FOR ICSI AND PERFORMANCE ANALYSIS

In this section, we first propose an RRZF beamforming at the base station considering ICSI. The regularizing factor $\alpha$ in RRZF is larger than that in RZF since additional noise inherited from the CSI error is considered. Then we show that in the extremal case where the number of users is infinity, the sum rate is monotonically increasing with the $\alpha$, which implies that the proposed RRZF outperforms ZF and RZF. Since it is difficult to obtain the distribution of channel matrix for moderate user number, we give simulation results of optimal $\alpha$ in Fig. 1.

A. RRZF beamforming for ICSI

The power penalty problem exists in ZF because the beamforming vector does not match with the channel vector for each user. This can be solved by selecting users with nearly orthogonal channel vectors.

But it is still a severe problem for small user numbers because finding $M$ semiorthogonal users is not guaranteed. Adding an identity matrix multiplied by a regularizing factor $\alpha$ before the inversion manipulation is another efficient way to solve the power penalty problem [3]. Implementing RRZF beamforming, we have $W = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H + \alpha \mathbf{I})^{-1}$ in (3). Note that the channel inversion regularization brings interference among different users if $\alpha \neq 0$. The optimal tradeoff of $\alpha$ is obtained in [3] as $\alpha_{\text{opt}} = \sigma^2 / P$.

The CSI in the practical scenario is imperfect due to large delay caused by user selection. We propose a robust RZF (RRZF) by optimizing the $\alpha$. We model the imperfect CSI as [12]

$$\mathbf{H} = \hat{\mathbf{H}} + e \mathbf{\Omega}, \tag{9}$$

where $e \mathbf{\Omega}$ is the CSI error independent of $\hat{\mathbf{H}}$, and $\mathbf{\Omega}$ is unknown to the base station and the user terminals. The entries of $\mathbf{\Omega}$ are i.i.d complex Gaussian distributed with zero mean and unit variance, and $e^2$ denotes the power of the CSI error which is known to the base station. Then the received signal vector can be rewritten as

$$y = \hat{\mathbf{H}} \mathbf{W} s + n = \hat{\mathbf{H}} \mathbf{W} s + e \mathbf{\Omega} \mathbf{W} s + n, \tag{10}$$

where $\mathbf{W} = \mathbf{H}^H (\hat{\mathbf{H}} \mathbf{H}^H + \alpha \mathbf{I})^{-1}$ and $\hat{\rho}$ is derived by substituting $\mathbf{W}$ into (1). The covariance of the noise becomes

$$E \left\{ (e \hat{\mathbf{H}} \mathbf{W} s + n) (e \hat{\mathbf{H}} \mathbf{W} s + n)^H \right\} = e^2 \hat{\rho}^2 E \left\{ \mathbf{\Omega} \mathbf{W} \mathbf{s}^H \mathbf{W} \mathbf{\Omega}^H \right\} + E \{ nn^H \} = \left( e^2 P + \sigma^2 \right) \mathbf{I}_M, \tag{11}$$

where we used the fact $E \{ \mathbf{\Omega} \mathbf{A} \mathbf{\Omega}^H \} = \text{tr}(\mathbf{A}) \mathbf{I}_N$ for any $N \times N$ matrix $\mathbf{A}$ [13]. We use the diagonal decomposition

$$\hat{\mathbf{H}} \mathbf{H}^H = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^H \tag{12}$$

in the following analysis where $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \ldots, \lambda_M\}$ is a diagonal matrix. From (9), the imperfect CSI is a scaled version of Rayleigh channel matrix with eigenvalues scaled by $(1 - e^2)^{\frac{1}{2}}$. Since in the decomposition (12), $\mathbf{Q}$ and $\mathbf{\Lambda}$ are independent [14], the statistic distribution is the same as in the perfect channel matrix. Therefore, we can use the method as in [3] of taking expectations over $\mathbf{Q}$ to the desired signal and the interference to divide the desired signal and the interference in $\hat{\mathbf{H}} \mathbf{W} s$ and finally obtain the average SINR at each user terminal as a function of the eigenvalues of $\mathbf{H}$, that is

$$\text{SINR}(\lambda) = \frac{E \left\{ (\hat{\rho} \mathbf{H} s)_{k,k} \right\}}{\sum_{j=1, j \neq k}^M E \left\{ (\hat{\rho} \mathbf{H} s)_{j,k} \right\} + e^2 P + \sigma^2}.$$
We will use the optimal $\beta$ for each $K$ in the simulations. As $K$ grows to infinity, the optimal $\beta$ decreases to zero. For an extremal case $\beta = 0$, we obtain the following theorem which shows that, unlike the characteristic that RZF converges to ZF as $P/\sigma^2 \to +\infty$, the RZF does not converge to ZF as $K \to +\infty$, and the proposed RRZF outperforms RZF and ZF. Note that the MF beamforming is $\mathbf{W} = \mathbf{H}^H$. It can be viewed as an RZF beamforming with $\alpha = +\infty$, because in this case

$$
\rho \mathbf{W} = \sqrt{\frac{P}{\text{tr} \left( \mathbf{H}^H \left( \mathbf{H}^H + \alpha \mathbf{I}_M \right)^{-2} \right)}} \mathbf{H}^H \left( \mathbf{H}^H + \alpha \mathbf{I}_M \right)^{-1} = \sqrt{\frac{P}{\text{tr} \left( \mathbf{H}^H \right)}} \mathbf{H}^H.
$$

(15)

*Theorem 1*: If $\beta = 0$, then

$$
\text{SNR}_{ZF}^{\text{ZF}} < \text{SNR}_{RZF}^{\text{ZF}} < \text{SNR}_{RRZF}^{\text{ZF}} < \text{SNR}_{MF}^{\text{ZF}}.
$$

(16)

*Proof*: When $\beta = 0$, $\mathbf{h}_i^\dagger \mathbf{h}_j = 0$ for any $i \neq j$. Therefore,

$$
\mathbf{H} \mathbf{H}^H = [\mathbf{h}_1, \ldots, \mathbf{h}_M]^H \cdot [\mathbf{h}_1, \ldots, \mathbf{h}_M] = \text{diag} \left\{ ||\mathbf{h}_1||^2, \ldots, ||\mathbf{h}_M||^2 \right\} = \text{diag} \{ \lambda_1, \ldots, \lambda_M \}.
$$

(17)

Define the effective channel matrix $\mathbf{H}_{\text{eff}} = \mathbf{H} \mathbf{W}$. We have the average SNR of each user of the RZF beamforming as

$$
\text{SNR} = \frac{1}{M} \sum_{m=1}^{M} \frac{\rho^2}{\text{tr} (\mathbf{H}_{\text{eff}})} = \frac{\rho^2}{M (\epsilon^2 P + \sigma^2)} \frac{\text{tr} (\mathbf{H}_{\text{eff}})}{\text{tr} (\mathbf{H}^H)} = \frac{\rho^2}{M (\epsilon^2 P + \sigma^2)} \frac{\text{tr} \left( \mathbf{H}^H \left( \mathbf{H}^H + \alpha \mathbf{I}_M \right)^{-2} \right)}{\text{tr} \left( \mathbf{H}^H \right)} = \frac{\rho^2}{M (\epsilon^2 P + \sigma^2)} \sum_{m=1}^{M} \frac{\lambda_m^2}{(\lambda_m + \alpha)^3}.\tag{18}
$$

Taking derivative to (18) with respect to $\alpha$, we have

$$
\begin{align*}
\frac{d}{d\alpha} \sum_{m=1}^{M} \frac{\lambda_m^2}{(\lambda_m + \alpha)^3} &= \frac{2}{\left( \sum_{m=1}^{M} \frac{\lambda_m}{(\lambda_m + \alpha)^2} \right)^2} \left( \sum_{m=1}^{M} \frac{\lambda_m}{(\lambda_m + \alpha)^3} \right) \sum_{m=1}^{M} \lambda_m^2 - \sum_{m=1}^{M} \frac{\lambda_m^2}{(\lambda_m + \alpha)^3} \sum_{m=1}^{M} \lambda_m \right) .
\end{align*}
$$

(19)

IV. SIMULATION RESULTS

In this section, numerical results are carried out to show the advantage of the proposed RRZF beamforming with SUS algorithm. The performance is compared with ZF beamforming and the conventional RZF with SUS algorithm in terms of...
In this letter, we propose an RRZF beamforming for the multiantenna broadcast channel with the semiorthogonal user selection (SUS) algorithm for imperfect CSI. The RRZF has significant advantage to ZF and RZF for small number of users. We also show that RRZF outperforms ZF and RZF in the extremal case of $K = +\infty$. The optimal regularizing factor $\alpha$ in RZF is no more the conventional, but increases with $K$. Since it is difficult to derive the closed-form of $\alpha$ for moderate $K$, we obtain it by monte-carlo simulations.

V. CONCLUSION

In this letter, we propose an RRZF beamforming for the multiantenna broadcast channel with the semiorthogonal user selection (SUS) algorithm for imperfect CSI. The RRZF has significant advantage to ZF and RZF for small number of users. We also show that RRZF outperforms ZF and RZF in the extremal case of $K = +\infty$. The optimal regularizing factor $\alpha$ in RZF is no more the conventional, but increases with $K$. Since it is difficult to derive the closed-form of $\alpha$ for moderate $K$, we obtain it by monte-carlo simulations.

V. CONCLUSION

In this letter, we propose an RRZF beamforming for the multiantenna broadcast channel with the semiorthogonal user selection (SUS) algorithm for imperfect CSI. The RRZF has significant advantage to ZF and RZF for small number of users. We also show that RRZF outperforms ZF and RZF in the extremal case of $K = +\infty$. The optimal regularizing factor $\alpha$ in RZF is no more the conventional, but increases with $K$. Since it is difficult to derive the closed-form of $\alpha$ for moderate $K$, we obtain it by monte-carlo simulations.

REFERENCES

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels,” Euro. Trans. Telecommun., vol. 10, no. 6, pp. 585–596, Nov. 1999.
[2] G. Caire and S. Shamai, “On the achievable throughput of a multi-antenna gaussian broadcast channel,” IEEE Trans. Inf. Theory, vol. 43, pp. 1691–1706, July 2003.
[3] C. Peel, B. Hochwald, and A. Swindlehurst, “Vector-perturbation technique for near-capacity multiantenna multiuser communication—part I: channel inversion and regularization,” IEEE Trans. Commun., vol. 53, no. 1, pp. 195–202, Jan. 2005.
[4] T. Yoo and A. Goldsmith, “On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,” IEEE J. Sel. Areas Commun., vol. 24, no. 3, pp. 528–541, Mar. 2006.
[5] T. Yoo, N. Jindal, and A. Goldsmith, “Multi-antenna downlink channels with limited feedback and user selection,” IEEE J. Sel. Areas Commun., vol. 25, no. 7, pp. 1478–1491, Sep. 2007.
[6] P. Lu and H. Yang, “Sum-rate analysis of multiuser MIMO system with zero-forcing transmit beamforming,” IEEE Trans. Commun., vol. 57, no. 9, pp. 2585–2589, Sep. 2009.
[7] M. Sharif and B. Hassibi, “A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users,” IEEE Trans. Commun., vol. 55, no. 1, pp. 11–15, Jan. 2007.
[8] C. Zhang, W. Xu, and M. Chen, “Hybrid zero-forcing beamforming/orthogonal beamforming with user selection for MIMO broadcast channels,” IEEE Commun. Lett., vol. 13, no. 1, pp. 10–12, Jan. 2009.
[9] Y. Xu and T. L. Ngu, “A capacity-achieving precoding scheme based on channel inversion regularization with optimal power allocation for MIMO broadcast channels,” in Proc. 2007 IEEE GLOBECOM, pp. 3190–3194.
[10] E. Shin and D. Kim, “On the optimal sum-capacity growth of ZFBF with quality based channel reporting in multiuser downlink channels,” IEEE Trans. Commun., vol. 57, no. 6, pp. 1643–1647, June 2009.
[11] H. Wan, W. Chen, and J. Ji, “Efficient linear transmission strategy for MIMO relaying broadcast channels with direct links,” IEEE Wireless Commun. Lett., vol. 1, no. 1, pp. 14–17, 2012.
[12] A. D. Dabbagh and D. J. Love, “Multiple antenna MMSE based downlink precoding with quantized feedback or channel mismatch,” IEEE Trans. Commun., vol. 56, no. 11, pp. 1859–1868, Nov. 2008.
[13] C. Wang, E. K. S. Au, R. D. Murch, W. H. Mow, R. S. Cheng, and Y. Lan, “On the performance of the MIMO zero-forcing receiver in the presence of channel estimation error,” IEEE Trans. Wireless Commun., vol. 6, no. 3, pp. 805–810, Mar. 2007.
[14] A. Edelman, “Eigenvalues and condition numbers of random matrices,” Ph.D. dissertation, Dept. of Math., Mass. Inst. Technol., Cambridge, MA. 1989.