Nilpotent symmetries in the Jackiw-Pi model of 3D massive non-Abelian theory: superfield approach

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Abstract. In the available literature, only the Becchi-Rouet-Stora-Tyutin (BRST) symmetries are known for the Jackiw-Pi model of three (2 + 1)-dimensional (3D) massive non-Abelian gauge theory. We derive the full set of off-shell nilpotent \((s_{(a)b}^2 = 0)\) and absolutely anticommuting \((s_b s_{ab} + s_{ab}s_b = 0)\) (anti-)BRST transformations \(s_{(a)b}\) corresponding to the usual Yang-Mills gauge transformations of this model by exploiting the “augmented” superfield formalism where the horizontality condition and gauge invariant restrictions blend together in a meaningful manner. This superfield formalism leads to the derivation of (anti-)BRST invariant Curci-Ferrari restriction which plays a key role in the proof of absolute anticommutativity of \(s_{(a)b}\). The derivation of the proper anti-BRST symmetry transformations is important from the point of view of geometrical objects called gerbes. A novel feature of our present investigation is the derivation of (anti-)BRST transformations for the auxiliary field \(\rho\) from our superfield formalism which is neither generated by the (anti-)BRST charges nor obtained from the requirements of nilpotency and/or absolute anticommutativity of the (anti-)BRST symmetries for our 3D non-Abelian 1-form gauge theory.

PACS numbers: 11.15.-q, 03.70.+k, 11.10.Kk, 12.90.+b

Keywords: Jackiw-Pi model; 3D massive gauge theory; superfield formalism; (anti-)BRST symmetries; nilpotency and absolute anticommutativity; Curci-Ferrari condition
1. Introduction

The 4D (non-)Abelian 1-form gauge theories are at the heart of standard model (SM) of particle physics where there is a stunning degree of agreement between theory and experiment. One of the weak links of SM is connected with the very existence of the esoteric Higgs particle which is responsible for the mass generation of gauge bosons and fermions. In view of the fact that Higgs particle has not yet been observed experimentally with a hundred percent certainty, other theoretical tools for the mass generation of gauge bosons (in various dimensions of spacetime) have become important and they have generated a renewed interest in the realm of theoretical physics.

In the context of the above, it may be mentioned that the 4D topologically massive (non-)Abelian gauge theories have been studied in the past [1-4] where there is merging and mixing of 1-form and 2-form (non-)Abelian gauge fields through the celebrated topological $B \wedge F$ term. In such models, it has been shown that the (non-)Abelian 1-form gauge field acquires a mass in a very natural fashion without taking any recourse to the Higgs mechanism. However, these models suffer from problems connected with renormalizability, consistency and unitarity. We have studied [5-10] these models, within the framework of superfield and Becchi-Rouet-Stora-Tyutin (BRST) formalisms in the hope that we would be able to propose a model that would be free of the drawbacks of the earlier models [1-4]. However, it remains still an open problem to construct a 4D consistent, unitary and renormalizable non-Abelian 2-form gauge theory (where the 1-form and 2-form non-Abelian gauge fields are incorporated together).

In the above scenario, it is an interesting idea to propose and study some lower dimensional models which are free of the problems of 4D topologically massive theory and where mass and gauge-invariance co-exist together. One such massive model, that has been a topic of theoretical interest, is the Jackiw-Pi (JP) model in three $(2 + 1)$-dimensions of spacetime where the non-Abelian gauge invariance and parity are respected together due to the introduction of a 1-form vector field, endowed with a parity, that is opposite of the usual non-Abelian 1-form vector field [11]. In fact, the 3D gauge theories, in general, have been topic of theoretical research in the recent past because of the novel and attractive properties associated with them [12,13]. Furthermore, it has already been shown that, for sufficiently strong vector coupling, the gauge invariance does not necessarily imply the masslessness of gauge particles [14,15].

In the backdrop of the above statements, the JP model of 3D massive gauge theory has been studied from different theoretical angles. For instance, the Hamiltonian formulation and its constraint analysis have been carried out in [16]. The JP model is also endowed with some interesting continuous symmetries. In this context, mention can be made of the usual Yang-Mills (YM) symmetry transformation and a symmetry that is different (i.e. NYM) from the YM. The BRST symmetry and corresponding Slavnov-Taylor identity of this model have also been recently found [17]. However, the off-shell nilpotent and absolutely anticommuting anti-BRST symmetry transformations of this model have not been discussed [17] which are essential for the completeness of
the theory as their very existence is theoretically backed, guided and governed by the concept of mathematical objects called gerbes (see, e.g. [18,19] for details).

The local and continuous gauge symmetry, generated by the first-class constraints of a given gauge theory, is generalized to the nilpotent BRST and anti-BRST symmetry transformations in the BRST formalism. The anti-BRST symmetry is a new kind of symmetry transformation [20] that is satisfied by the Yang-Mills theory. It has also been shown [21] that the anti-BRST symmetry has not been a matter of choice rather it has real fundamental importance in providing necessary additional conditions for the ghost freedom that is essential for a consistent quantization. Both the nilpotent symmetries have been formulated in a completely model independent way in [22]. In our recent works [18,19], we have demonstrated the relevance of gerbes in the context of BRST formalism through the existence of CF-type restrictions. We have claimed that the latter is the hallmark of a gauge theory within the framework of BRST formalism. Thus, for the sake of completeness of the BRST analysis of the JP model, it is essential to derive a proper anti-BRST symmetry corresponding to the BRST symmetry of [17].

The main motivation behind our present investigation is to derive the full set of proper [i.e. off-shell nilpotent ($s^2_{(a)b} = 0$) and absolutely anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$)] (anti-)BRST symmetry transformations $s_{(a)b}$ corresponding to the usual YM gauge symmetry transformation for the JP model by exploiting the “augmented” superfield approach to BRST formalism [23-27]. This geometrical approach leads to the derivation of (anti-)BRST invariant Curci-Ferrari (CF) condition [28] which ensures the absolute anticommutativity of $s_{(a)b}$ and derivation of the coupled (but equivalent) Lagrangian densities that respect the above (anti-)BRST symmetry transformations.

In our present endeavor, we have purposely concentrated only on the usual YM gauge symmetries for the BRST analysis within the framework of superfield formalism. This is due to the fact that we plan to understand the JP model step-by-step so that we can gain deep insights into the key aspects of this model. This understanding, perhaps, would enable us to propose an accurate model for the 4D theory and would make us confident about the limiting cases of the general BRST analysis of this model where YM and NYM gauge symmetries would be combined together for their thorough discussions (within the framework of superfield formalism). Exactly similar kind of programme, we have followed for the BRST analysis of the 4D dynamical non-Abelian gauge theory [3,2] within the framework of superfield approach (see, e.g. [10] for details).

The prime factors that are responsible for our present investigations are as follows. First, the JP model is free of the problems encountered in the 4D topologically massive models with $B \wedge F$ term. Second, this 3D model does not invoke any higher form gauge field (like the 2-form $B$ field of 4D theory) for the mass generation. Third, the understanding of this 3D theory might provide some insights that would show the correct path to construct a renormalizable and consistent 4D massive theory. Fourth, we study JP model within the framework of BRST formalism where the renormalizability and unitarity could be proven with the help of Slavnov-Taylor identities and nilpotency of the conserved BRST charge. Finally, the 3D JP model is very special model because it
generates mass for the gauge field without violating the parity symmetry. This feature is drastically different from the mass generation by incorporating the Chern-Simons term in the Lagrangian density of 3D gauge theory where parity is violated.

The contents of our present investigation are organized as follows. In section 2, we discuss two sets of local gauge symmetry transformations associated with the JP model. Our section 3 incorporates the derivation of (anti-)BRST symmetry transformations for the gauge field \((A_\mu)\) and (anti-)ghost fields \([(\bar{C})C]\) with the help of Bonora-Tonin’s superfield formalism \([23,24]\). In section 4, we deal with the derivation of (anti-)BRST symmetry transformations for the vector field \(\phi_\mu\) and scalar field \(\rho\) within the framework of “augmented” superfield formalism \([25-27]\). Our section 5 is fully devoted to the derivation of coupled Lagrangian densities that respect the above (anti-)BRST transformations. We show the conservation of (anti-)BRST current (and corresponding charges) in section 6. Our section 7 contains the discussion of ghost symmetry transformations and the derivation of the algebra satisfied by all the symmetry generators. Finally, we made a few concluding remarks in section 8.

In our Appendix A, we capture the off-shell nilpotency and anticommutativity of the (anti-)BRST charges and the BRST invariance (as well as equivalence) of the coupled Lagrangian densities within the framework of “augmented” superfield formalism. We provide a geometrical interpretation for the existence of the (anti-)BRST invariant CF restriction \([28]\) in our Appendix B with a few clear and cogent physical arguments.

**Conventions and notations:** We adopt here the conventions and notations such that the background spacetime Minkowskian flat metric has the signature \((-,-,-)\), totally antisymmetric Levi-Civita tensor \(\varepsilon_{\mu\nu\eta}\) satisfies \(\varepsilon_{\mu\nu\eta}\varepsilon^{\mu\nu\kappa} = -3!, \varepsilon_{\mu\nu\eta}\varepsilon^{\mu\nu\kappa} = -2! \delta^\kappa_\eta, \text{etc.}, \) and \(\varepsilon_{012} = +1 = -\varepsilon_{012}.\) In the above, the Greek indices \(\mu, \nu, \eta, \ldots = 0, 1, 2\) correspond to the 3D time and space directions. We take the dot and cross products \(P \cdot Q = P^a Q^a, \ P \times Q = f^{abc} P^a Q^b T^c\) in the SU\((N)\) Lie algebraic space where the generators \(T^a\) of the SU\((N)\) Lie algebra satisfy the commutator \([T^a, T^b] = i f^{abc} T^c\) with \(a, b, c, \ldots = 1, 2, 3, \ldots N^2 - 1.\) The structure constants \(f^{abc}\) are chosen to be totally antisymmetric in \(a, b, c\) for the semi-simple SU\((N)\) Lie algebra \([29]\).

### 2. Preliminaries: continuous local gauge symmetries

We begin with the Lagrangian density of the three \((2 + 1)\)-dimensional \((3D)\) massive non-Abelian 1-form gauge theory proposed by JP where the SU\((N)\) Yang-Mills (YM) gauge invariance and parity are respected together. Furthermore, this theory respects a non-Yang-Mills gauge symmetry transformation as well. The Lagrangian density of the theory, in its full blaze of glory, is (see, e.g. \([11]\))

\[
\mathcal{L}_0 = -\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} - \frac{1}{4} (G^{\mu\nu} + g F^{\mu\nu} \times \rho) \cdot (G_{\mu\nu} + g F_{\mu\nu} \times \rho) + \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu} \cdot \phi_\eta \quad (1)
\]

where \(A_\mu\) and \(\phi_\mu\) are the vector fields with opposite parity, \(\rho\) is a scalar field, \(g\) is a coupling constant and \(m\) is the mass parameter. The 2-form \([F^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu) F_{\mu\nu} \cdot T]\) curvature tensor \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g \left( A_\mu \times A_\nu \right)\), corresponding to the 1-form
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\((A^{(1)} = dx^\mu A_\mu \cdot T)\) field \(A_\mu\), is derived from the Maurer-Cartan equation \(F^{(2)} = dA^{(1)} + ig (A^{(1)} \wedge A^{(1)})\). Similarly, the field strength tensor \(G_{\mu\nu} = D_\mu \phi_\nu - D_\nu \phi_\mu\), corresponding to the 1-form \((\phi^{(1)} = dx^\mu \phi_\mu \cdot T)\) field \(\phi_\mu\), is obtained from \(G^{(2)} = d\phi^{(1)} + ig (\phi^{(1)} \wedge A^{(1)} + A^{(1)} \wedge \phi^{(1)}) \equiv \frac{1}{2} (dx^\mu \wedge dx^\nu) G_{\mu\nu} \cdot T\) where the covariant derivative is defined as: \(D_\mu \phi_\nu = \partial_\mu \phi_\nu - g (A_\mu \times \phi_\nu)\).

The above Lagrangian density (1) respects the following usual local, continuous and infinitesimal Yang-Mills (YM) gauge transformations \(\delta_1\), as

\[
\begin{align*}
\delta_1 A_\mu &= D_\mu \Lambda \\
\delta_1 \phi_\mu &= -g (\phi_\mu \times \Lambda) \\
\delta_1 \rho &= -g (\rho \times \Lambda)
\end{align*}
\]

This theory also respects a non-Yang-Mills (NYM) gauge transformation \(\delta_2\). The infinitesimal version of this transformation is:

\[
\begin{align*}
\delta_2 A_\mu &= 0 \\
\delta_2 \phi_\mu &= D_\mu \Omega \\
\delta_2 \rho &= + \Omega \\
\delta_2 F_{\mu\nu} &= 0.
\end{align*}
\]

In the above, \(\Lambda = \Lambda \cdot T \equiv \Lambda^a T^a\) and \(\Omega = \Omega \cdot T \equiv \Omega^a T^a\) are the \(SU(N)\) valued infinitesimal (Lorentz-scalar) gauge parameters. It is straightforward to check that the Lagrangian density (1) transforms, under the local, continuous and infinitesimal transformations (2) and (3), as

\[
\delta_1 L_0 = 0 \quad \delta_2 L_0 = \partial_\mu \left[ \frac{m}{2} \epsilon^{\mu\nu\eta} F_{\nu\eta} \cdot \Omega \right].
\]

We note that the usual YM symmetry is a perfect symmetry for \(L_0\) because we have perfect invariance (i.e. \(\delta_1 L_0 = 0\)). However, the Lagrangian density \(L_0\) remains quasi-invariant under \(\delta_2\) because it transforms to a total spacetime derivative.

In our present investigation, we shall concentrate only on the usual YM gauge symmetry transformations \(\delta_1\) and derive the corresponding (anti-)BRST symmetry transformations that are off-shell nilpotent and absolutely anticommuting in nature.

3. (Anti-)BRST symmetries of gauge and (anti-)ghost fields: Bonora-Tonin’s superfield formalism

We apply the well-known Bonora-Tonin’s superfield approach [23,24] to derive the nilpotent (anti-)BRST symmetry transformations corresponding to the YM gauge transformations \(\delta_1\) for the 1-form gauge field \(A_\mu\) and (anti-)ghost fields \((\bar{C})C\). In this approach, first of all, we generalize the 3D bosonic vector field \((A_\mu = A_\mu \cdot T)\) and fermionic (anti-)ghost fields \((\bar{C} = \bar{C} \cdot T, C = C \cdot T)\) to their corresponding superfields. The latter are defined on the (3,2)-dimensional supermanifold parametrized by the superspace variables \(Z^M = (x^\mu, \theta, \bar{\theta})\) where \(x^\mu(\mu = 0,1,2)\) are the spacetime variables and \((\theta, \bar{\theta})\) are a pair of Grassmannian variables (with \(\theta^2 = \bar{\theta}^2 = 0, \theta \bar{\theta} + \bar{\theta} \theta = 0\)). These superfields can be expanded along the directions of the Grassmannian directions \(\theta\) and \(\bar{\theta}\) (of the (3,2)-dimensional supermanifold), as [23,24];

\[
\begin{align*}
\bar{B}_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{R}_\mu(x) + \bar{\theta} R_\mu(x) + i \theta \bar{\theta} S_\mu(x) \\
\bar{F}(x, \theta, \bar{\theta}) &= C(x) + i \theta \bar{B}_1(x) + i \bar{\theta} B_1(x) + i \theta \bar{\theta} s(x) \\
\tilde{F}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i \theta \bar{B}_2(x) + i \bar{\theta} B_2(x) + i \theta \bar{\theta} \bar{s}(x)
\end{align*}
\]
where the local secondary fields \([R_\mu(x), \bar{R}_\mu(x), s(x), \bar{s}(x)]\) are fermionic \((s^2 = 0, \bar{s}^2 = 0, R_\mu \bar{R}_\nu + \bar{R}_\nu R_\mu = 0, R_\mu R^\mu = 0, \text{etc.})\) and \([S_\mu(x), B_1(x), \bar{B}_1(x), B_2(x), \bar{B}_2(x)\] are bosonic in nature. These secondary fields can be determined in terms of the basic and auxiliary fields of the 3D local BRST invariant quantum field theory through the application of the celebrated horizontality condition (HC).

We note that the kinetic term \((-\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu}\), corresponding to the 1-form gauge field \(A_\mu\), remains invariant under the gauge transformations (2). The HC condition implies that the gauge invariant kinetic term remains invariant when we generalize the 3D ordinary non-Abelian gauge theory onto \((3,2)\)-dimensional supermanifold. The above statement of the gauge invariance can be, mathematically, expressed as:

\[
-\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu} = -\frac{1}{4} \tilde{F}^{MN} \cdot \tilde{F}_{MN}
\]

where the super curvature \(\tilde{F}^{MN}\), defined on the \((3,2)\)-dimensional supermanifold, is derived from the Maurer-Cartan equation: \(\tilde{F}^{(2)} = \tilde{d}\tilde{A}^{(1)} + i g (\tilde{A}^{(1)} \wedge \bar{A}^{(1)}) \equiv \frac{1}{2} (d d Z^M \wedge d Z^N) \tilde{F}_{MN}\). Here \(\tilde{d}\) is the super exterior derivative (with \(\tilde{d}^2 = 0\) and \(\tilde{A}^{(1)}\) is the super 1-form connection which are the generalizations of the ordinary exterior derivative \(d\) and 1-form connection \(A^{(1)}\) as

\[
d \longrightarrow \tilde{d} \equiv d Z^M \partial_M \equiv dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}
\]

\[
A^{(1)} \longrightarrow \tilde{A}^{(1)} = d Z^M \tilde{A}_M \equiv dx^\mu \tilde{B}_\mu(x, \theta, \bar{\theta}) + d\theta \tilde{F}(x, \theta, \bar{\theta}) + d\bar{\theta} \tilde{F}(x, \theta, \bar{\theta})
\]

where \(\tilde{B}_\mu(x, \theta, \bar{\theta}), \tilde{F}(x, \theta, \bar{\theta})\) and \(\tilde{F}(x, \theta, \bar{\theta})\) are the superfields on the \((3,2)\)-dimensional supermanifold and \(\partial_M = (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}})\). The celebrated HC condition (6) leads to the following relationships amongst the basic, auxiliary and secondary fields [23,24]

\[
R_\mu = D_\mu C \quad \bar{R}_\mu = D_\mu C \quad B_1 = -\frac{i}{2} g (C \times C) \quad \bar{B}_2 = -\frac{i}{2} g (\bar{C} \times \bar{C})
\]

\[
S_\mu = D_\mu B + i g (D_\mu C \times \bar{C}) \equiv -D_\mu \bar{B} - i g (D_\mu \bar{C} \times C)
\]

\[
s = -g (B \times C) \quad \bar{s} = +g (B \times \bar{C}) \quad B + \bar{B} = -ig (C \times \bar{C})
\]

where we have made the choices \(\bar{B}_1 = \bar{B}\) and \(B_2 = B\) which are, finally, identified with the Nakanishi-Lautrup type auxiliary fields of the 3D local quantum field theory.

It is to be noted that, to satisfy the HC (6), one sets equal to zero the Grassmannian components of the super tensor \(\tilde{F}_{MN}\) in super 2-form \(\tilde{F}^{(2)} = \frac{1}{2} (d d Z^M \wedge d Z^N) \tilde{F}_{MN}\). The equation \(\bar{B} + B = -ig (C \times \bar{C})\) [that is quoted in (8)] is the Curci-Ferrari (CF) restriction which is one of the key hallmarks of the non-Abelian 1-form gauge theory. This condition is derived from HC when one sets equal to zero the \(\tilde{F}_{\theta\bar{\theta}}\) component of the super curvature tensor \(\tilde{F}_{MN}\). The CF condition plays an important role in providing the proof for the absolute anticommutativity of the (anti-)BRST transformations. Furthermore, the CF condition is instrumental in obtaining a coupled set of Lagrangian densities [cf. (30), (31) below] that respect the (anti-)BRST transformations (see, also Appendix B).

Substituting the above relationships (8) into the super-expansions of the superfields [cf. (5)], we obtain the following explicit expansions:

\[
\tilde{B}_\mu^{(h)}(x, \theta, \bar{\theta}) = A_\mu(x) + \theta D_\mu \bar{C}(x) + \bar{\theta} D_\mu C(x) + \theta \bar{\theta} [i D_\mu B - g (D_\mu C \times \bar{C})](x)
\]
\[ \equiv A_\mu(x) + \theta (s_{ab} A_\mu(x)) + \bar{\theta} (s_b A_\mu(x)) + \theta \bar{\theta} (s_b s_{ab} A_\mu(x)) \]

\[ \tilde{F}^{(h)}(x, \theta, \bar{\theta}) = C(x) + \theta \left( \frac{g}{2} (C \times C)(x) \right) + \bar{\theta} \left( -i g (\bar{B} \times C)(x) \right) \]

\[ \equiv C(x) + \theta (s_{ab} C(x)) + \bar{\theta} (s_b C(x)) + \theta \bar{\theta} (s_b s_{ab} C(x)) \]

\[ \tilde{F}^{(h)}(x, \theta, \bar{\theta}) = \bar{C}(x) + \theta \left( \frac{g}{2} (\bar{C} \times \bar{C})(x) \right) + \bar{\theta} (i B(x)) + \theta \bar{\theta} \left( +i g (B \times \bar{C})(x) \right) \]

\[ \equiv \bar{C}(x) + \theta (s_{ab} \bar{C}(x)) + \bar{\theta} (s_b \bar{C}(x)) + \theta \bar{\theta} (s_b s_{ab} \bar{C}(x)) \] (9)

where \((h)\), as the superscript on the superfields, denotes the expansions of the superfields after the application of HC. The super 2-form curvature tensor can be expressed as

\[ \tilde{F}_{\mu\nu}^{(h)}(x, \theta, \bar{\theta}) = F_{\mu\nu}(x) - \theta \left[ g (F_{\mu\nu} \times \bar{C})(x) \right] - \bar{\theta} \left[ g (F_{\mu\nu} \times C)(x) \right] \]

\[ + \theta \bar{\theta} \left[ g^2 (F_{\mu\nu} \times C) \times \bar{C} - i g (F_{\mu\nu} \times B) \right](x) \]

\[ \equiv F_{\mu\nu}(x) + \theta (s_{ab} F_{\mu\nu}(x)) + \bar{\theta} (s_b F_{\mu\nu}(x)) + \theta \bar{\theta} (s_b s_{ab} F_{\mu\nu}(x)). \] (10)

It is clear, from the above expressions, that the kinetic term of \(A_\mu\) (i.e. \(-\frac{1}{4} F^{\mu\nu} \cdot F_{\mu\nu}\)) remains invariant (i.e. independent of the Grassmann variables \(\theta, \bar{\theta}\)) under the application of HC. In other words, we obtain \(-\frac{1}{4} F^{\mu\nu(h)}(x, \theta, \bar{\theta}) = F_{\mu\nu(h)}^{(h)}(x, \theta, \bar{\theta}) = -\frac{1}{4} F^{\mu\nu}(x) \cdot F_{\mu\nu}(x)\). Before we wrap up this section, we would like to state that the equations (9) and (10) imply that: \(s_b \leftrightarrow \lim_{\theta \to \bar{\theta}} (\partial / \partial \bar{\theta}), s_{ab} \leftrightarrow \lim_{\theta \to \bar{\theta}} (\partial / \partial \theta)\). This mapping establishes a relationship between the (anti-)BRST transformations and the translational generators along the Grassmannian directions of the \((3,2)\)-dimensional supermanifold. This key relationship entails upon the (anti-)BRST transformations, emerging from our superfield formalism, to be always nilpotent of order two \((s_{(a)b}^2 = 0)\) and absolutely anticommuting in nature because of the fact that \((\partial_\theta)^2 = (\partial_{\bar{\theta}})^2 = 0\) and \(\partial_\theta \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_\theta = 0\).

4. (Anti-)BRST symmetries for the vector field \((\phi_\mu)\) and scalar field \((\rho)\): augmented superfield formalism

It can be checked that the composite fields \((F_{\mu\nu} \cdot \phi_\eta)\) and \((F_{\mu\nu} \cdot \rho)\) remain invariant under the usual YM gauge transformations \(\delta_1\) because

\[ \delta_1 (F_{\mu\nu} \cdot \phi_\eta) = 0 \quad \delta_1 (F_{\mu\nu} \cdot \rho) = 0. \] (11)

Since \((F_{\mu\nu} \cdot \phi_\eta)\) and \((F_{\mu\nu} \cdot \rho)\) are the gauge-invariant quantities, therefore, these are physical quantities (in some sense). These quantities must remain unaffected by the presence of the Grassmannian variables when the former entities are generalized onto the \((3, 2)\)-dimensional supermanifold. Thus, we have the following gauge invariant restrictions (GIRs) on the (super)fields:

\[ \tilde{F}^{(h)}_{\mu\nu}(x, \theta, \bar{\theta}) \cdot \tilde{\phi}_\eta(x, \theta, \bar{\theta}) = F_{\mu\nu}(x) \cdot \phi_\eta(x) \]

\[ \tilde{F}^{(h)}_{\mu\nu}(x, \theta, \bar{\theta}) \cdot \bar{\rho}(x, \theta, \bar{\theta}) = F_{\mu\nu}(x) \cdot \rho(x). \] (12)

In the above, the expansion for \(\tilde{F}^{(h)}_{\mu\nu}(x, \theta, \bar{\theta})\) is quoted in equation (10). The bosonic superfields \(\tilde{\phi}_\mu(x, \theta, \bar{\theta})\) and \(\bar{\rho}(x, \theta, \bar{\theta})\) can be, in general, expanded on the \((3, 2)\)-dimensional supermanifold along the Grassmannian directions \((\theta, \bar{\theta})\), as

\[ \tilde{\phi}_\mu(x, \theta, \bar{\theta}) = \phi_\mu(x) + \theta \bar{P}_\mu(x) + \bar{\theta} P_\mu(x) + i \theta \bar{\theta} Q_\mu(x) \]
where the secondary fields \((P_\mu, \overline{P}_\mu, P, \overline{P})\) are fermionic and \((Q_\mu, Q)\) are bosonic in nature. These secondary fields can be determined with the help of GIRs [cf. (12)]. In fact, the equality (12), leads to the following relationships:

\[
P_\mu = -g (\phi_\mu \times C) \quad P = -g (\rho \times C) \quad Q_\mu = -i [g^2 (\phi_\mu \times C) \times \overline{C} - i g (\phi_\mu \times B)] \\
\overline{P}_\mu = -g (\phi_\mu \times C) \quad \overline{P} = -g (\rho \times C) \quad Q = -i [g^2 (\rho \times C) \times \overline{C} - i g (\rho \times B)].
\] (14)

Substituting the above values into (13), we obtain

\[
\tilde{g}^{(h,g)}(x, \theta, \overline{\theta}) = \phi_\mu (x) - \theta [g (\phi_\mu \times C)](x) - \overline{\theta} [g (\phi_\mu \times C)](x) \\
+ \theta \overline{\theta} [g^2 (\phi_\mu \times C) \times \overline{C} - i g (\phi_\mu \times B)](x) \\
\equiv \phi_\mu (x) + \theta (s_{ab} \phi_\mu (x)) + \overline{\theta} (s_{b} \phi_\mu (x)) + \theta \overline{\theta} (s_{ab} \phi_\mu (x))
\]

\[
\tilde{\rho}^{(h,g)}(x, \theta, \overline{\theta}) = \rho (x) - \theta [g (\rho \times C)](x) - \overline{\theta} [g (\rho \times C)](x) \\
+ \theta \overline{\theta} [g^2 (\rho \times C) \times \overline{C} - i g (\rho \times B)](x) \\
\equiv \rho (x) + \theta (s_{ab} \rho (x)) + \overline{\theta} (s_{b} \rho (x)) + \theta \overline{\theta} (s_{ab} \rho (x))
\] (15)

where the superscripts \((h, g)\) on the above superfields refers to the super-expansions of the superfields obtained after the application of celebrated HC condition and gauge invariant restrictions. Thus, we have already obtained the (anti-)BRST symmetry transformations in the above for the fields \(\phi_\mu\) and \(\rho\) in view of the mappings: \(s_b \longleftrightarrow \lim_{\theta \to 0} (\partial / \partial \theta)\), \(s_{ab} \longleftrightarrow \lim_{\theta \to 0} (\partial / \partial \theta)\).

Within the framework of superfield formalism, we can also calculate the nilpotent (anti-)BRST transformations for the field strength tensor \(G_{\mu \nu}\) and composite field \((F_{\mu \nu} \times \rho)\).

With the inputs from section 3 and section 4, we have the following:

\[
\tilde{G}_{\mu \nu}^{(h,g)} = \partial_\mu \tilde{g}_{\nu}^{(h,g)} - g (\tilde{B}_{\mu}^{(h)} \times \tilde{\phi}_{\nu}^{(h,g)}) - \partial_\nu \tilde{g}_{\mu}^{(h,g)} + g (\tilde{B}_{\nu}^{(h)} \times \tilde{\phi}_{\mu}^{(h,g)}).
\] (16)

The substitution of expansions from (9) and (15) leads to the following expansion

\[
\tilde{G}_{\mu \nu}^{(h,g)}(x, \theta, \overline{\theta}) = G_{\mu \nu}(x) + \theta [-g (G_{\mu \nu} \times \overline{C})] + \overline{\theta} [-g (G_{\mu \nu} \times C)](x) \\
+ \theta \overline{\theta} [g^2 (G_{\mu \nu} \times C) \times \overline{C} - i g (G_{\mu \nu} \times B)](x)
\] (17)

which results in the following off-shell nilpotent (anti-)BRST transformations for the tensor field \(G_{\mu \nu}\), namely;

\[
s_b G_{\mu \nu} = -g (G_{\mu \nu} \times C) \\
s_{ab} G_{\mu \nu} = -g (G_{\mu \nu} \times \overline{C}) \\
s_{b} s_{ab} G_{\mu \nu} = g^2 (G_{\mu \nu} \times C) \times \overline{C} - i g (G_{\mu \nu} \times B).
\] (18)

It is also interesting to check explicitly that,

\[
\tilde{G}_{\mu \nu}^{(h,g)}(x, \theta, \overline{\theta}) \cdot \tilde{G}_{\mu \nu}^{(h,g)}(x, \theta, \overline{\theta}) = G_{\mu \nu}(x) \cdot G_{\mu \nu}(x)
\] (19)

which establishes the Grassmannian independence of the l.h.s. As a consequence, we infer from this observation that \((G_{\mu \nu} \cdot G^{\mu \nu})\) is an (anti-)BRST invariant quantity.

In an exactly similar fashion, it is straightforward to note that [cf. (10) and (15)];

\[
(\tilde{F}_{\mu \nu}^{(h)} \times \tilde{\rho}^{(h,g)})(x, \theta, \overline{\theta}) = (F_{\mu \nu} \times \rho)(x) - \theta [g (F_{\mu \nu} \times \rho) \times \overline{C}] - \overline{\theta} [g (F_{\mu \nu} \times \rho) \times C](x) \\
+ \theta \overline{\theta} [g^2 ((F_{\mu \nu} \times \rho) \times C) \times \overline{C} - i g (F_{\mu \nu} \times \rho) \times B](x).
\] (20)
Nilpotent symmetries in the Jackiw-Pi model of 3D massive non-Abelian theory

The above expansion implies that the (anti-)BRST symmetry transformations of the composite field \( (F_{\mu\nu} \times \rho) \) are as given below:

\[
\begin{align*}
    s_b(F_{\mu\nu} \times \rho) &= -g \left( F_{\mu\nu} \times \rho \right) \times C \\
    s_{ab}(F_{\mu\nu} \times \rho) &= -g \left( F_{\mu\nu} \times \rho \right) \times \tilde{C} \\
    s_{ab} \left( F_{\mu\nu} \times \rho \right) &= g^2 \left\{ \left[ (F_{\mu\nu} \times \rho) \times C \right] \times \tilde{C} - ig \left( F_{\mu\nu} \times \rho \right) \times B. \right.
\end{align*}
\]

Furthermore, it is elementary to show that the following is correct, namely:

\[
\left[ \left( \tilde{F}^{(h)}_{\mu \nu} \times \tilde{\rho}^{(h,g)} \right)(x,\theta,\tilde{\theta}) \right] \cdot \left[ \left( \tilde{F}^{\mu \nu(h)}_{\mu \nu} \times \tilde{\rho}^{(h,g)} \right)(x,\theta,\tilde{\theta}) \right] = (F_{\mu\nu} \times \rho)(x) \cdot (F_{\mu\nu} \times \rho)(x) \tag{22}
\]

which establishes the Grassmannian independence of the l.h.s. As a consequence, we conclude that \( (F_{\mu\nu} \times \rho) \cdot (F_{\mu\nu} \times \rho) \) is an (anti-)BRST invariant quantity (i.e. \( s_{(a)b} \left[ (F_{\mu\nu} \times \rho) \cdot (F_{\mu\nu} \times \rho) \right] = 0 \)). Before we close this section, it is interesting to note that the following equation is correct, viz:

\[
\tilde{G}^{(h,g)}_{\mu \nu}(x,\theta,\tilde{\theta}) \cdot \left( \tilde{F}^{\mu \nu(h)}_{\mu \nu}(x,\theta,\tilde{\theta}) \right) = G_{\mu \nu}(x) \cdot (F_{\mu\nu} \times \rho)(x). \tag{23}
\]

It verifies the Grassmannian independence of the l.h.s. This observation, in turn, implies the (anti-)BRST invariance of \( G_{\mu \nu} \cdot (F_{\mu\nu} \times \rho) \) (i.e. \( s_{(a)b} \left[ G_{\mu \nu} \cdot (F_{\mu\nu} \times \rho) \right] = 0 \)). Finally, it is clear that \( \left( G_{\mu \nu} + g(F_{\mu\nu} \times \rho) \right) \cdot \left( G_{\mu \nu} + g(F_{\mu\nu} \times \rho) \right) \) of the Lagrangian density (1) is an (anti-)BRST invariant quantity. Furthermore, it is true to state that

\[
\frac{m}{2} \varepsilon^{\mu\nu\eta} \tilde{F}^{(h)}_{\mu \nu}(x,\theta,\tilde{\theta}) \cdot \tilde{A}^{(h,g)}_{\eta}(x,\theta,\tilde{\theta}) = \frac{m}{2} \varepsilon^{\mu\nu\eta} F_{\mu\nu}(x) \cdot \phi_{\eta}(x) \tag{24}
\]

which proves the (anti-)BRST invariance of the last term of \( \mathcal{L}_0 \) because the l.h.s. of (24) is actually independent of the Grassmannian variables.

5. Coupled Lagrangian densities: (anti-)BRST symmetries

In the previous section (cf. section 4), we have derived the (anti-)BRST symmetry transformations for the relevant fields of the theory. This can be seen from a close look at equations (9), (10) and (15). In explicit form, these (anti-)BRST transformations are

\[
\begin{align*}
    s_{ab} A_\mu &= D_\mu \tilde{C} \\
    s_{ab} \tilde{C} &= \frac{g}{2} \left( \tilde{C} \times \tilde{C} \right) \\
    s_{ab} B &= -g \left( B \times \tilde{C} \right) \\
    s_{ab} \rho &= -g \left( \rho \times \tilde{C} \right) \\
    s_{ab} \phi_\mu &= -g \left( \phi_\mu \times \tilde{C} \right) \\
    s_{ab} C &= i \tilde{B} \\
    s_{ab} \tilde{B} &= 0 \\
    s_{ab} F_{\mu\nu} &= -g \left( F_{\mu\nu} \times \tilde{C} \right) \tag{25}
\end{align*}
\]

\[
\begin{align*}
    s_b A_\mu &= D_\mu C \\
    s_b C &= \frac{g}{2} \left( C \times C \right) \\
    s_b \tilde{B} &= -g \left( \tilde{B} \times C \right) \\
    s_b \rho &= -g \left( \rho \times C \right) \\
    s_b \phi_\mu &= -g \left( \phi_\mu \times C \right) \\
    s_b \tilde{C} &= i B \\
    s_b B &= 0 \\
    s_b F_{\mu\nu} &= -g \left( F_{\mu\nu} \times C \right). \tag{26}
\end{align*}
\]

The above transformations are off-shell nilpotent of order two \( (s_{a(b)}^2 = 0) \) and absolutely anticommuting \( (s_b s_{ab} + s_{ab} s_b = 0) \) in nature in their operator form.

The (anti-)BRST symmetry transformations for the Nakanishi-Lautrup type auxiliary fields \( (B, \tilde{B}) \) have been obtained from the requirements of the nilpotency and absolute anticommutativity properties of \( s_{(a)b} \). In fact, the above requirements lead to the following proper (anti-)BRST symmetry transformations:

\[
\begin{align*}
    s_b B &= 0 \\
    s_b \tilde{B} &= -g \left( \tilde{B} \times C \right) \\
    s_{ab} \tilde{B} &= 0 \\
    s_{ab} B &= -g \left( B \times \tilde{C} \right). \tag{27}
\end{align*}
\]
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The absolute anticommutativity of the (anti-)BRST symmetry transformations \( s_{(a)b} \), applied onto the following fields, namely:

\[
\{ s_b, s_{ab} \} \ A_\mu = 0 \quad \{ s_b, s_{ab} \} \ \phi_\mu = 0 \quad \{ s_b, s_{ab} \} \ \rho = 0
\]

is true only when CF-condition \([ B + \bar{B} + ig \ (C \times \bar{C}) = 0 \] is satisfied. Thus, we note that our superfield formalism leads to (i) the derivation of the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations, and (ii) the CF-condition. The latter defines a hypersurface in the 3D Minkowski spacetime manifold on which the proper (i.e. nilpotent and absolutely anticommuting) (anti-)BRST transformations are defined. Furthermore, it can be checked that the CF-condition is (anti-)BRST invariant (i.e. \( s_{(a)b} \ [ B + \bar{B} + ig \ (C \times \bar{C}) ] = 0 \)). In other words, the key results of our superfield formalism are the derivation of the proper (anti-)BRST transformations and (anti-)BRST invariant CF-condition for the BRST invariant theory.

The expressions for the coupled (anti-)BRST-invariant Lagrangian densities of the 3D massive non-Abelian gauge theory can be written as

\[
L_B = L_0 + s_b \ s_{ab} \left[ \frac{i}{2} \ A_\mu \cdot A^\mu + C \cdot \bar{C} + \frac{1}{2} \ \phi_\mu \cdot \phi^\mu \right]
\]

\[
L_{\bar{B}} = L_0 - s_{ab} \ s_b \left[ \frac{i}{2} \ A_\mu \cdot A^\mu + C \cdot \bar{C} + \frac{1}{2} \ \phi_\mu \cdot \phi^\mu \right].
\]

We note that the terms in the square brackets are chosen in such a way that each term has a mass dimension one and ghost number equal to zero. Furthermore, all these terms (in the bracket) are Lorentz scalar. This is due to the fact that the proper nilpotent (anti-)BRST symmetries increase the mass dimension of fields by one on which they operate. As a consequence, we have the following expressions for the 3D (anti-)BRST invariant coupled (but equivalent) Lagrangian densities:

\[
L_B = -\frac{1}{4} \ F^{\mu \nu} \cdot F_{\mu \nu} - \frac{1}{4} \ (G^{\mu \nu} + g F^{\mu \nu} \times \rho) \cdot (G_{\mu \nu} + g F_{\mu \nu} \times \rho) + \frac{m}{2} \ \varepsilon^{\mu \nu \eta} \ F_{\mu \nu} \cdot \phi_\eta
\]

\[
+ B \cdot (\partial^\mu A_\mu) + \frac{1}{2} \ (B \cdot B + \bar{B} \cdot \bar{B}) - i \ \partial^\mu \bar{C} \cdot D_\mu C
\]

\[
L_{\bar{B}} = -\frac{1}{4} \ F^{\mu \nu} \cdot F_{\mu \nu} - \frac{1}{4} \ (G^{\mu \nu} + g F^{\mu \nu} \times \rho) \cdot (G_{\mu \nu} + g F_{\mu \nu} \times \rho) + \frac{m}{2} \ \varepsilon^{\mu \nu \eta} \ F_{\mu \nu} \cdot \phi_\eta
\]

\[- \bar{B} \cdot (\partial^\mu A_\mu) + \frac{1}{2} \ (B \cdot B + \bar{B} \cdot \bar{B}) - i \ D^\mu \bar{C} \cdot \partial_\mu C.
\]

We emphasize that there are no gauge-fixing and Faddeev-Popov ghost terms for the vector field \( \phi_\mu \) in the Lagrangian densities (30) and (31). The reason, behind this observation, is that the field \( \phi_\mu \) transforms covariantly [i.e. \( s_b \phi_\mu = -g \ (\phi_\mu \times C) \), \( s_{ab} \phi_\mu = -g \ (\phi_\mu \times \bar{C}) \)] under the (anti-)BRST transformations [cf. (25), (26)]. Thus, the term \( (\phi_\mu \cdot \phi^\mu) \) remains invariant under the (anti-)BRST transformations and, as a consequence, there is no contribution from this term to the (anti-)BRST invariant coupled (but equivalent) Lagrangian densities quoted in (30) and (31).

It can be checked that the above Lagrangian densities \( L_B \) and \( L_{\bar{B}} \) transform, under the off-shell nilpotent (anti-)BRST transformations, as

\[
s_b L_B = \partial_\mu [B \cdot (D^\mu C)] \quad s_{ab} L_B = \partial_\mu (B \cdot \partial^\mu \bar{C}) - D_\mu [B + \bar{B} + ig \ (C \times \bar{C})] \cdot \partial^\mu \bar{C}
\]
\( s_{ab} \mathcal{L}_B = - \partial_\mu \tilde{B} \cdot (D^\mu \tilde{C}) \quad s_B \mathcal{L}_B = - \partial_\mu (\tilde{B} \cdot \partial^\mu \mathcal{C}) + D_\mu [B + \tilde{B} + i g (C \times \tilde{C})] \cdot \partial^\mu \mathcal{C}. \) (32)

Thus, the corresponding actions (i.e. \( \int d^2x \, \mathcal{L}_B \) and \( \int d^2x \, \tilde{\mathcal{L}}_B \)) remain invariant on the constrained hypersurface in the 3D spacetime manifold where the CF-condition is satisfied. The explicit expressions in (32) explain that \( \mathcal{L}_B \) and \( \tilde{\mathcal{L}}_B \) are equivalent and both of them respect the nilpotent (anti-)BRST symmetry transformations.

6. Conserved charges: novel features

The invariance of the Lagrangian density (or action), under any arbitrary continuous symmetry transformation, leads to the derivation of the conserved current according to the Noether’s theorem. As a consequence, the local, continuous and off-shell nilpotent (anti-)BRST symmetry transformations \( (s_{(a)b}) \) lead to the derivation of the following Noether’s conserved currents \( J^\mu_{(a)b} \):

\[
J^\mu_a \equiv B \cdot (D^\mu C) - [F^\mu - g \left\{ (G^\mu + g F^\mu \times \rho) \times \rho \right\} - m \varepsilon^{\mu \nu \eta} \phi_\eta] \cdot (D_\nu C) \\
+ g \left[ G^\mu + g (F^\mu \times \rho) \right] \cdot (\phi_\nu \times C) + \frac{i}{2} g \partial^\mu \tilde{C} \cdot (C \times C)
\]

\[
J^\mu_{ab} \equiv -\tilde{B} \cdot (D^\mu \tilde{C}) - [F^\mu - g \left\{ (G^\mu + g F^\mu \times \rho) \times \rho \right\} - m \varepsilon^{\mu \nu \eta} \phi_\eta] \cdot (D_\nu \tilde{C}) \\
+ g \left[ G^\mu + g (F^\mu \times \rho) \right] \cdot (\phi_\nu \times \tilde{C}) - \frac{i}{2} g \partial^\mu C \cdot (\tilde{C} \times \tilde{C}).
\]

The above expressions for the Noether’s currents can be re-expressed (for the algebraic convenience), in the following form

\[
J^\mu_a \equiv B \cdot (D^\mu C) - \partial^\mu B \cdot C - \frac{i}{2} g \partial^\mu \tilde{C} \cdot (C \times C) \\
- \partial_\nu \left\{ (F^\mu - g \left\{ (G^\mu + g F^\mu \times \rho) \times \rho \right\} - m \varepsilon^{\mu \nu \eta} \phi_\eta) \cdot C \right\}
\]

\[
J^\mu_{ab} \equiv -\tilde{B} \cdot (D^\mu \tilde{C}) + \partial^\mu \tilde{B} \cdot \tilde{C} + \frac{i}{2} g \partial^\mu C \cdot (\tilde{C} \times \tilde{C}) \\
- \partial_\nu \left\{ (F^\mu - g \left\{ (G^\mu + g F^\mu \times \rho) \times \rho \right\} - m \varepsilon^{\mu \nu \eta} \phi_\eta) \cdot \tilde{C} \right\}.
\]

Now the proof of conservation law \( \left( \partial_\mu J^\mu_{(a)b} = 0 \right) \) becomes easier and it can be confirmed by exploiting the following Euler-Lagrange equations of motion, derived from the Lagrangian densities \( \mathcal{L}_B \) and \( \tilde{\mathcal{L}}_B \), respectively, namely;

\[
D_\mu F^\mu - g D_\mu [G^\mu + g F^\mu \times \rho] + m \varepsilon^{\mu \nu \eta} D_\mu \phi_\eta - \partial^\nu B \\
\quad + g \left\{ (G^\mu + g F^\mu \times \rho) \times \phi_\mu \right\} - i g (\partial^\nu \tilde{C} \times C) = 0
\]

\[
D_\mu [G^\mu + g (F^\mu \times \rho)] + \frac{m}{2} \varepsilon^{\mu \nu \eta} F^\mu_{\nu \eta} = 0
\]

\[
(G^\mu + g F^\mu \times \rho) \times F^\mu = 0 \quad \partial_\mu (D^\mu C) = 0 \quad D_\mu (\partial^\mu C) = 0
\]

\[
D_\mu F^\mu - g D_\mu [G^\mu + g F^\mu \times \rho] + m \varepsilon^{\mu \nu \eta} D_\mu \phi_\eta + \partial^\nu \tilde{B} \\
\quad + g \left\{ (G^\mu + g F^\mu \times \rho) \times \phi_\mu \right\} + i g (\tilde{C} \times \partial^\nu C) = 0
\]

\[
D_\mu [G^\mu + g (F^\mu \times \rho)] + \frac{m}{2} \varepsilon^{\mu \nu \eta} F^\mu_{\nu \eta} = 0
\]

\[
(G^\mu + g F^\mu \times \rho) \times F^\mu = 0 \quad \partial_\mu (\partial^\mu \tilde{C}) = 0 \quad D_\mu (\partial^\mu \tilde{C}) = 0.
\]
The above conserved currents [cf. (35), (36)] lead to the derivation of the following conserved (anti-)BRST charges \( Q_{(a)b} = f d^2 x \, J_{(a)b}^0 \), namely:

\[
Q_{ab} = - \int d^2 x \left[ \dot{B} \cdot D^0 \dot{C} - \dot{\dot{B}} \cdot \dot{C} - \frac{i}{2} g \, \dot{C} \cdot (\dot{C} \times \ddot{C}) \right]
\]

\[
Q_b = \int d^2 x \left[ B \cdot D^0 C - \dot{B} \cdot C - \frac{i}{2} g \, \dot{C} \cdot (C \times C) \right].
\]

The conserved \( \dot{Q}_{(a)b} = 0 \) and nilpotent (i.e. \( Q_{(a)b}^2 = 0 \)) (anti-)BRST charges \( Q_{(a)b} \) are the generators of the transformations (25) and (26), respectively. For instance, it can be checked that \( s_r \Phi = \pm i [\Phi, Q_r]_\pm \) \((r = b, ab)\). Here the field \( \Phi \) is the generic field of the theory and \((\pm)\) signs, on the square bracket, stand for the (anti)commutator for the generic field \( \Phi \) of the theory being (fermionic) bosonic.

It is interesting to point out that the nilpotent generators \( Q_{(a)b} \), even though produce the nilpotent (anti-)BRST transformations for the basic fields, they are unable to generate the (anti-)BRST transformations for the auxiliary field \( \rho \) of the theory. Even the requirements of the nilpotency and absolutely anticommuting properties of the proper (anti-)BRST symmetry transformations do not generate the (anti-)BRST transformations for the auxiliary field \( \rho \). This is a novel observation in this theory (within the framework of BRST formalism). For the usual 4D (non)-Abelian 1-form gauge theory, there are two inputs that lead to the derivation of all the (anti-)BRST symmetry transformations of all the relevant fields of the specific theory. These are (i) the (anti-)BRST charges \( Q_{(a)b} \) as the generators of the nilpotent (anti-)BRST symmetry transformations, and (ii) the requirements of nilpotency and absolute anticommutativity which lead to the derivation of the (anti-)BRST symmetry transformations for the auxiliary fields of the theory. Thus, the auxiliary field \( \rho \) is very special.

7. Ghost charge: BRST algebra

The Lagrangian densities (30) and (31) remain invariant under the following scale transformations for the (anti-)ghost and other basic as well as auxiliary fields, namely:

\[
C \rightarrow e^{+\Sigma} C \quad \bar{C} \rightarrow e^{-\Sigma} \bar{C} \quad (A_\mu, \phi_\mu, \rho, B, \bar{B}) \rightarrow e^0 (A_\mu, \phi_\mu, \rho, B, \bar{B})
\]

\( \Sigma \) is a global infinitesimal scale parameter. The \((+)\) signs, in the exponents, represent the ghost number of the fields \((\gamma)\gamma\) and the ghost number for the rest of the fields (i.e. \( A_\mu, \phi_\mu, \rho, B, \bar{B} \)) is equal to zero. As a consequence, the latter fields do not transform at all under the ghost-scale transformations. It is straightforward to check that the following infinitesimal transformations \( s_g \), obtained from (41), namely;

\[
s_g C = +\Sigma C \quad s_g \bar{C} = -\Sigma \bar{C} \quad s_g (A_\mu, \phi_\mu, \rho, B, \bar{B}) = 0
\]

are the symmetry transformations for the Lagrangian densities (30) and (31) because it is straightforward to check that \( s_g \mathcal{L}_B = 0 = s_g \mathcal{L}_B \).

The above ghost symmetry transformations \( s_g \) lead to the derivation of Noether’s conserved current and charge as given below:

\[
J^\mu_g = i \left[ \bar{C} \cdot D^\mu C - \partial^\mu \bar{C} \cdot C \right] \quad Q_g = i \int d^2 x \left[ \bar{C} \cdot D^0 C - \dot{\bar{C}} \cdot C \right].
\]
It can be proven that the above ghost charge $Q_g$ is the generator of (42). The nilpotent (anti-)BRST charge $(Q_{(a)b})$ and the ghost charge $Q_g$ satisfy the following standard BRST algebra, namely;

$$Q_b^2 = 0, \quad Q_{ab}^2 = 0 \quad \text{i} \ [Q_g, Q_b] = Q_b \quad \text{i} \ [Q_g, Q_{ab}] = -Q_{ab}$$

$$\{Q_b, Q_{ab}\} = Q_b Q_{ab} + Q_{ab} Q_b = 0 \quad Q_g^2 \neq 0$$

(44)

which shows that the ghost number of the BRST charge is $(+1)$ and that of the anti-BRST charge is $(-1)$. The above statements about the ghost numbers can be checked explicitly by starting with a state $|\psi\rangle_n$ that has the ghost number equal to $n$ (i.e. $i Q_g |\psi\rangle_n = n |\psi\rangle_n$). With this input and the above algebra (44), we can check that the ghost numbers of states $Q_b |\psi\rangle_n$ and $Q_{ab} |\psi\rangle_n$ are $(n + 1)$ and $(n - 1)$, respectively. Thus, the BRST charge increases the ghost number by one when it operates on a quantum state. On the other hand, the anti-BRST charge decreases the above number by one.

8. Conclusions

In our present investigation, we have exploited the usual classical Yang-Mills gauge symmetry of the JP model of 3D massive non-Abelian gauge theory and generalized it to the (anti-)BRST symmetry transformations at the quantum level that are off-shell nilpotent and absolutely anticommuting in nature. In this endeavor, the “augmented” superfield formalism (where the HC and GIRs blend together beautifully) [25-27] has played a decisive role in the derivation of the full set of appropriate transformations.

One of the important features of our superfield formulation is the derivation of the CF condition that enables us to obtain the absolutely anticommuting (anti-)BRST symmetry transformations. Thus, in addition to the BRST symmetry for the JP model [17], we have been able to derive the proper anti-BRST symmetry transformations for the sake of completeness. Furthermore, the celebrated CF condition has been able to help us in deducing the coupled Lagrangian densities [cf. (30), (31)] that respect the above proper (anti-)BRST symmetry transformations for our present theory.

In the context of 4D non-Abelian 1-form gauge theory, it is well-known that the existence of anti-BRST symmetry transformations $(s_{ab})$ is non-trivial. The off-shell nilpotent $(s_{a(b)}^2 = 0)$ (anti-)BRST symmetries anticommute $(s_b s_{ab} + s_{ab} s_b = 0)$ with each other only on a constrained hypersurface, described by the Curci-Ferrari field equations, on a 4D Minkowskian spacetime manifold [28]. It can be also checked, using the appropriate equations [cf. (25), (26), (39), (40)], that for our present theory:

$$s_b Q_{ab} = -i \{Q_b, Q_{ab}\} = -i \int d^2 x \left[ \tilde{B} \cdot \partial^0 \left( B + \tilde{B} + i g (C \times \bar{C}) \right) \right] = 0$$

$$s_{ab} Q_b = -i \{Q_{ab}, Q_b\} = i \int d^2 x \left[ B \cdot \partial^0 \left( B + \tilde{B} + i g (C \times \bar{C}) \right) \right] = 0$$

(45) (46)

are absolutely true on the 3D Minkowski spacetime manifold where the CF restriction $B + \tilde{B} + i g (C \times \bar{C}) = 0$ is satisfied. The absolute anticommutativity of the (anti-)BRST symmetries imply the linear independence of BRST and anti-BRST symmetry
transformations. The mathematical basis for the independence of the BRST and anti-BRST symmetries is encoded in the concept of gerbes that has been discussed in our earlier works [18,19] and illustrated geometrically in our Appendix B.

To obtain the full set of proper (anti-)BRST symmetry transformations, we are theoretically compelled to go beyond the HC and exploit the suitable GIRs to deduce the proper (anti-)BRST symmetries. This is a novel feature of this model. Furthermore, as it turns out, the auxiliary field $\rho$ is not like the other auxiliary (e.g. Nakanishi-Lautrup) fields of the theory because its (anti-)BRST symmetry transformations do not arise from the requirements of nilpotency and absolute anticommutativity of the (anti-)BRST symmetry transformations. This observation is also a novel feature of our present theory. The good thing about our present augmented superfield formalism [25-27] is that it leads to the precise derivation of the proper (anti-)BRST symmetry transformations associated with this special auxiliary field $\rho$, too.

It would be very nice endeavor to exploit the NYM gauge transformations (3) within the framework of our superfield formalism and obtain the novel results connected with the nilpotent (anti-)BRST symmetry transformations that emerge from it. In fact, two of us (RK and SG), have already made significant progress in obtaining the proper (anti-)BRST symmetry transformations corresponding to NYM symmetries by taking into account the following restrictions on the (super)fields [30]:

\[
\tilde{d}\tilde{\phi}^{(1)} + ig(\tilde{A}^{(1)} \wedge \tilde{\phi}^{(1)}) + ig(\tilde{\phi}^{(1)} \wedge \tilde{A}^{(1)}) - ig(\tilde{F}^{(2)} \wedge \tilde{\rho}) + ig(\tilde{\rho} \wedge \tilde{F}^{(2)})
\]

\[
= d\phi^{(1)} + ig(A^{(1)} \wedge \phi^{(1)}) + ig(\phi^{(1)} \wedge A^{(1)}) - ig(F^{(2)} \wedge \rho) + ig(\rho \wedge F^{(2)})
\]

(47)

where the symbols carry their standard meanings (as discussed in our present text). It would also be exciting to take the combination of the local YM and NYM gauge transformations together and obtain the full set of proper (anti-)BRST symmetry transformations, relevant coupled Lagrangian densities and exact (conserved, nilpotent and absolutely anticommuting) (anti-)BRST charges that generate the proper and full set of (anti-)BRST symmetry transformations for the JP model. Our present work and the forthcoming paper [30] would be the limiting cases of this general approach to the JP model (where the YM and NYM symmetries would blend together).

Acknowledgments

SG and RK would like to gratefully acknowledge the financial support from CSIR and UGC, New Delhi, Government of India, respectively.

Appendix A. Nilpotency and BRST invariance: superfield technique

The nilpotency and anticommutativity of the (anti-)BRST symmetry transformations and corresponding generators can be proven, in a simple and elegant manner, by exploiting the potential and power of superfield formalism. As has been pointed out
earlier, it can be checked, from the expansions in (9) and (15), that
\[ s_b \Psi(x) = \lim_{\bar{\theta} \to 0} \frac{\partial}{\partial \theta} \tilde{\Psi}^{(h,g)}(x, \theta, \bar{\theta}) \]
\[ s_{ab} \Psi(x) = \lim_{\bar{\theta} \to 0} \frac{\partial}{\partial \theta} \tilde{\Psi}^{(h,g)}(x, \theta, \bar{\theta}) \]
\[ s_b s_{ab} \Psi(x) = \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \tilde{\Psi}^{(h,g)}(x, \theta, \bar{\theta}) \] (A.1)
where \( \Psi(x) \) is the generic 3D field of the theory and \( \tilde{\Psi}^{(h,g)}(x, \theta, \bar{\theta}) \) are the corresponding superfields (defined on the \( (3,2) \)-dimensional supermanifold and expanded after the application of HC and/or GIRs). As a consequence, the nilpotency of the (anti-)BRST transformations is captured in the nilpotency (\( \partial_{\bar{\theta}}^2 = 0, \ \partial_{\theta}^2 = 0 \)) of the Grassmannian directions (\( \partial_{\theta}, \partial_{\bar{\theta}} \)).

In an exactly similar fashion, it can be checked that
\[ \left( \frac{\partial}{\partial \theta} \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \bar{\theta}} \frac{\partial}{\partial \bar{\theta}} \right) \tilde{\Psi}^{(h,g)}(x, \theta, \bar{\theta}) = 0 \] (A.2)
encodes the absolute anticommutativity (\( s_b s_{ab} + s_{ab} s_b = 0 \)) of the (anti-)BRST symmetry transformations in 3D spacetime for the JP model of massive theory.

That the (anti-)BRST charges are also nilpotent of order two, can be captured in the following expressions within the framework of superfield formalism, namely:
\[ Q_b = \lim_{\bar{\theta} \to 0} \frac{\partial}{\partial \theta} \int d^2x \left[ B(x) \cdot \tilde{B}_0^{(h)}(x, \theta, \bar{\theta}) + i \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \right] \]
\[ \equiv \int d^2x \int d\bar{\theta} \left[ B(x) \cdot \tilde{B}_0^{(h)}(x, \theta, \bar{\theta}) + i \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \right] \] (A.3)

Written in the ordinary 3D spacetime, the above expansions imply:
\[ Q_b = \int d^2x \ s_b \left[ B(x) \cdot A_0(x) + i \tilde{C}(x) \cdot C(x) \right] \] (A.4)
As a consequence, it is clear that \( s_b Q_b = -i\{Q_b, Q_b\} = 0 \) because of the nilpotency (\( s_b^2 = 0 \)) of the BRST transformations (\( s_b \)). In the language of superfield and superspace variables [with the inputs from the nilpotency (\( \partial_{\bar{\theta}}^2 = 0 \)) of derivative \( \partial_{\theta} \)]:
\[ \lim_{\bar{\theta} \to 0} \frac{\partial}{\partial \theta} Q_b = 0 \implies Q_b^2 = 0. \] (A.5)

There is an alternative way to express BRST charge (that is valid only on the constrained surface in 3D spacetime manifold where CF-condition \([B + B + i g (C \times C) = 0]\) is satisfied). This is given, within the framework of superfield formalism, as
\[ Q_b = i \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \int d^2x \left[ \tilde{B}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \right] \]
\[ \equiv \int d^2x \ s_b s_{ab} \left( i A_0(x) \cdot C(x) \right). \] (A.6)

The off-shell nilpotency of the BRST charge is, once again, proven by the nilpotency of Grassmannian derivative \( \partial \bar{\theta} \) (i.e. \( \partial_{\bar{\theta}}^2 = 0 \)) and the off-shell nilpotency (\( s_b^2 = 0 \)) of the BRST transformations \( s_b \). Thus, we note that the nilpotency of BRST charge is encoded in \( \partial_{\bar{\theta}}^2 = 0 \) and \( s_b^2 = 0 \) when it is expressed in terms of Grassmann derivative \( \partial \bar{\theta} \) on the \( (3,2) \)-dimensional supermanifold and in terms of the proper BRST transformations \( s_b \) existing in 3D ordinary spacetime.

As we have expressed the BRST charge \( Q_b \) in terms of the superfields (obtained after the application of HC and/or GIRs), we can also express the anti-BRST charge
Grassmannian independence of $\tilde{Q}_{ab}$ in the following two different ways, namely:

$$Q_{ab} = \lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \int d^2 x \left[ \tilde{B}(x) \cdot \tilde{B}_0^{(h)}(x, \theta, \bar{\theta}) + i \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}^*_0(x, \theta, \bar{\theta}) \right]$$

$$Q_{ab} = i \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \int d^2 x \left[ \tilde{B}_0^{(h)}(x, \theta, \bar{\theta}) \cdot \tilde{F}^{(h)}(x, \theta, \bar{\theta}) \right]$$  \hspace{1cm} (A.7)

where the second expression is true only on the 3D constrained surface where CF-condition $[B + \tilde{B} + i g (C \times \tilde{C}) = 0]$ is satisfied. The nilpotency ($\partial^2_{\bar{\theta}} = 0$) of the Grassmannian derivative $\partial^h_{\bar{\theta}}$ ensures that $Q_{ab} = 0$ because it can be seen that $\partial_{\bar{\theta}} Q_{ab} = 0$. The above expression can be expressed, in the 3D ordinary space, as given below

$$Q_{ab} = \int d^2 x \, s_{ab} \left[ \tilde{B}(x) \cdot A_0(x) + i \tilde{C}(x) \cdot \tilde{C}(x) \right]$$

$$\equiv i \int s_{ab} s_{b} \left( A_0(x) \cdot \tilde{C}(x) \right).$$ \hspace{1cm} (A.8)

The nilpotency of $Q_{ab}$ is captured in the equation $s_{ab} Q_{ab} = -i \{Q_{ab}, Q_{ab}\} = 0$ because we know, from the anti-BRST symmetry transformations (25), that $s_{ab}^2 = 0$.

The (anti-)BRST invariance and the equivalence of the coupled Lagrangian densities can also be captured in the language of superfield formalism. To this end in mind, it can be checked that the starting Lagrangian density $L_0$ can be generalized onto the $(3, 2)$-dimensional supermanifold as illustrated below

$$L_0 \rightarrow \tilde{L}_0 = -\frac{1}{4} \tilde{F}^{\mu \nu (h)} \cdot \tilde{F}^*_\mu \nu - \frac{1}{4} \left[ \left( G^{\mu \nu (h, g)} + g \tilde{F}^{\mu \nu (h)} \times \tilde{\partial}^{(h, g)} \right) \cdot \left( \tilde{G}^{\mu \nu (h, g)} + g \tilde{F}^{\mu \nu (h)} \times \tilde{\partial}^{(h, g)} \right) \right]$$

$$+ \frac{m}{2} \varepsilon^{\mu \nu \eta} \tilde{F}^{\mu (h)} \cdot \tilde{\partial}^{(h, g)}.$$ \hspace{1cm} (A.9)

The above expression for $\tilde{L}_0$ is, however, independent of the Grassmannian variables because of our clear discussion in section 3 and section 4. As a consequence, we have

$$\lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_0 = \lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_0 = 0 \iff s_{(a)b} L_0 = 0.$$ \hspace{1cm} (A.10)

Thus, we conclude that the above equation captures the (anti-)BRST invariance of the starting Lagrangian density $L_0$ in terms of the superfield and superspace variables.

The (anti-)BRST invariant coupled Lagrangian densities can be generalized onto the $(3, 2)$-dimensional supermanifold as:

$$L_B \rightarrow \tilde{L}_B = \tilde{L}_0 - \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \left[ \frac{i}{2} \tilde{B}^{(h)} \cdot \tilde{B}^{(h)} + \tilde{F}^{(h)} \cdot \tilde{F}^*_0 + \frac{1}{2} \tilde{\partial}^{(h, g)} \cdot \tilde{\partial}^{(h, g)} \right]$$

$$L_B \rightarrow \tilde{L}_B = \tilde{L}_0 + \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \left[ \frac{i}{2} \tilde{B}^{(h)} \cdot \tilde{B}^{(h)} + \tilde{F}^{(h)} \cdot \tilde{F}^*_0 + \frac{1}{2} \tilde{\partial}^{(h, g)} \cdot \tilde{\partial}^{(h, g)} \right].$$ \hspace{1cm} (A.11)

The nilpotency ($\partial_{\bar{\theta}}^2 = \partial^2_{\bar{\theta}} = 0$) of the Grassmannian derivatives $\partial_{\bar{\theta}}^h$ and Grassmann independence of $\tilde{L}_0$, lead to the following:

$$\lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_B = 0 \iff s_b L_B = 0 \iff \lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_B = 0 \iff s_{ab} L_B = 0.$$ \hspace{1cm} (A.12)

The equivalence of the coupled Lagrangian densities $L_B$ and $\tilde{L}_B$ is captured in the proof that $\lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_B = 0$, $\lim_{\theta \to 0} \frac{\partial}{\partial \bar{\theta}} \tilde{L}_B = 0$ because of the nilpotency ($\partial_{\bar{\theta}}^2 = 0, \partial_{\bar{\theta}}^2 = 0$) and anticommutativity of the Grassmannian derivatives (i.e. $\partial_{\bar{\theta}} \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_{\bar{\theta}} = 0$).
Appendix B. (Anti-)BRST symmetries, CF condition and gerbes: physical interpretation and geometrical meaning

In this Appendix, we provide a clear geometrical interpretation for the existence of CF condition in the language of (anti-)BRST symmetry transformations on the non-Abelian 1-form gauge field and associated (anti-)ghost fields. It is evident, from a close look at the off-shell nilpotent and absolutely anticommuting (anti-)BRST transformations (25), (26) and (27), that the following schematic diagram (Fig. B1) captures these transformations geometrically. We note that, in the whole diagram, there is a single point, at the ghost number zero, where there is a clustering of the auxiliary fields $B \equiv B \cdot T$ and $\bar{B} \equiv \bar{B} \cdot T$ which emerge from the (anti-)BRST symmetry transformations on the ghost fields $C \equiv C \cdot T$ and anti-ghost fields $\bar{C} \equiv \bar{C} \cdot T$, respectively. This is the place where the CF condition exists as it connects these auxiliary Nakanishi-Lautrup fields $B$ and $\bar{B}$ with the ghost number zero object constructed from the fermionic (anti-)ghost fields $(\bar{C})C$ of the theory [i.e. $B + \bar{B} + i g (C \times \bar{C}) = 0$].

In our earlier works on the 4D Abelian 2-form and 6D 3-form gauge theories [18,19], we have explained the existence and emergence of the (anti-)BRST invariant CF-type
restrictions by exploiting the fundamental notions of geometry and group theory from pure mathematics. As a warm-up exercise in [19], we have also considered the existence of CF condition for the 4D non-Abelian 1-form gauge theory within the framework of BRST formalism by taking the help of concepts from pure mathematics. In our present endeavor, we claim that, for any arbitrary $p$-form gauge theory within the framework of BRST formalism, it is the symmetry transformations that would provide us the clue for the existence of CF-type conditions in the theory. In fact, wherever, in the above type of diagram (cf. B1), there is clustering of different variety of fields at a particular ghost number, there will emerge a CF type condition which will be connected with the idea of gerbes [18,19]. Thus, physically, we interpret the mathematical object gerbes as some artifact that connects the fields which cluster at a particular ghost number (in the diagram like Fig. B1 for a given theory) and they basically prove the linear independence (i.e. the absolute anticommutativity) of BRST and anti-BRST symmetries.

The hallmark of a gauge theory, at the classical level, is the existence of first-class constraints on them in the language of Dirac’s prescription for classification scheme. On the other hand, one of the decisive features of a gauge theory, at the quantum level, is the existence of CF-type conditions which are mathematically backed by the idea of gerbes in the realm of BRST approach to gauge theories.

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