Microlensing by Stars in the Disk of M31

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Abstract

The optical depth to microlensing toward M31 due to known stars in the disk of M31 itself is \( \tau \sim 2 \times 10^{-7} e^{-r/d} \) where \( d \) is the disk scale length and \( r \) is the distance along the major axis. Thus, there can be significant lensing toward the M31 disk even if M31 contains no dark compact objects. The optical depth has a strong dependence on azimuthal angle: at fixed radius \( \tau \propto \left[ 1 + \left( h/d \right) \tan i \cos \phi \right]^{-2} \) where \( h \) is the scale height of the disk, \( i = 75^\circ \) is the inclination of M31, and \( \phi \) is the azimuthal angle relative to the near minor axis. By measuring the optical depth as a function of radial and azimuthal position, it is possible to estimate \( h \) and \( d \) for the mass of the M31 disk, and so determine whether the disk light traces disk mass. Ground-based observations in 0''.5 seeing of 0.8 deg\(^2\) once per week could yield \( \sim 3 \) events per year. With an ambitious space-based project, it would be possible to observe \( \sim 80 \) events per year. If lensing events were dominated by the M31 spheroid rather than the disk, the event rate would be higher and the spheroid’s parameters could be measured. The pattern of optical depths differs substantially for a disk and a spheroid.

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1. Introduction

Three groups have recently detected a total of at least 13 candidate microlensing events presumably caused by compact objects in the Galaxy which magnify the light of distant stars (Alcock et al. 1993, MACHO; Aubourg et al. 1993, EROS; Udalski et al. 1993, OGLE). The observations by MACHO and EROS of several million stars toward the Large Magellanic Cloud (LMC) were originally suggested by Paczyński (1986) as a way to detect massive compact halo objects (Machos) that may make up the dark matter in galactic halos. The observations by OGLE and MACHO toward the Galactic bulge were originally suggested by Paczyński (1991) and Griest et al. (1991) as a way to detect disk dark matter, to measure the low-mass end of the disk luminosity function, and better to constrain the distribution of Machos. However, even if some or all of the candidates prove to be real lensing events, this will not necessarily imply that the lenses make up (or even lie in) a dark halo. Gould (1994) suggested that the low event rate toward the LMC may indicate that the lenses lie in a dark thick disk. Gould, Miralda-Escudé, & Bahcall (1994) suggested that the lenses may lie in a thin disk and devised several methods to distinguish among the thin-disk, thick-disk, and halo models. Gould et al. argued that the high event rate toward the bulge compared to that toward the LMC tends to favor a disk origin of the events. Giudice, Mollerach, & Roulet (1994) suggested that the MACHO and EROS events may be due to a dark spheroid, that is a spherical structure that cuts off much more rapidly than an $r^{-2}$ halo.

Crotts (1992) pointed out that the optical depth toward the disk of M31 by Machos in an M31 halo would be far higher than the optical depth toward the LMC by Machos in a Galactic halo. He also noted that observations toward M31 would have several other advantages, most notably that the near side of the M31 disk would have a very much lower lensing rate and therefore would provide an excellent control field. The principal drawback of observations toward M31 is that the fields are too crowded to resolve the stars. However, Tomany & Crotts
(1994) have pioneered a ground-based method for detecting lensing events even for unresolved stars in $\gtrsim 1''$ seeing. Moreover, future space-based observations with $0.1''$ seeing would be able to resolve stars in M31 approximately as well as current ground-based observations do for stars in the LMC. Lensing searches toward M31 are therefore technically feasible.

Here I point out that even if there are no compact dark objects in M31, the optical depth toward the M31 disk (due to known stars within the M31 disk itself) is quite high, $\tau \sim 10^{-7}$. This evaluation holds at approximately one disk scale length along the major axis of the disk. The optical depth falls directly as the disk density (i.e. exponentially) as one proceeds radially outward. For fixed radius, the optical depth varies strongly as a function of azimuthal position. By measuring these variations, it should be possible to measure the scale height and scale length as well as the normalization of the mass of the M31 disk. These measurements would provide a direct check of the frequently used (but never tested) hypothesis that disk light traces disk mass.

Of course, it is possible that the dark, $r^{-2}$ halo of M31 is made of Machos. In this case, the microlensing due to low-mass stars in the M31 disk would be dwarfed by that due to Machos, and it is unlikely that one could recognize the characteristic signature of disk lenses. Similarly, it is possible that M31 has a dark spheroid (a roughly spherical structure with a steeper, e.g., $r^{-3.5}$ profile) which makes up some of the M31 dark matter, particularly in its inner portions. In this case also, it would be difficult to recognize the signature of a disk. However, in both these cases, it would be quite easy to recognize that the signal was due to a halo or to a spheroid and to map the characteristic features of either. Thus, in any event, lensing searches toward the M31 disk will yield valuable information about the mass distribution of M31.
2. Lensing By A Distant Disk

The Einstein radius, \( r_e \) about a lens of mass \( M \) has a physical size given by

\[
\frac{r_e^2}{\text{c}^2} = \frac{4GM D_{OL} D_{LS}}{D_{OS} c^2},
\]

(2.1)

where \( D_{OL}, D_{OS}, \) and \( D_{LS} \) are the distances between the observer, source, and lens. If both the lens and the source are in M31, then \( D_{OL} \sim D_{OS} \), so that

\[
\frac{r_e^2}{\text{c}^2} = \frac{4GM y}{c^2},
\]

(2.2)

where \( y \equiv D_{LS} \) is now the distance between the lens and source.

2.1. Lensing of a Source in the Disk Plane

I assume that the mass of the M31 disk is distributed in a double exponential profile,

\[
\rho(r, z) = \frac{\Sigma_0}{2h} \exp\left(-\frac{|z|}{h}\right) \exp\left(-\frac{r}{d}\right),
\]

(2.3)

where \( h \) and \( r \) are respectively the scale height and scale length of the disk and \( \Sigma_0 \) is the normalization. The density along the line of sight for a position \( y \equiv D_{LS} \) relative to a source at radius \( r \) in the plane is then

\[
\rho(y) = \frac{\Sigma_0}{2h} \exp\left(-\frac{y \cos i}{h} - \frac{y \sin i \cos \phi}{d}\right) \exp\left(-\frac{r}{d}\right)
\]

(2.4)

where \( i = 75^\circ \) is the angle of inclination of M31 and \( \phi \) is the azimuthal angle relative to the near minor axis. The optical depth to a source in the plane of the M31 disk is therefore

\[
\tau_0 = \frac{2\pi G \Sigma_0 h}{c^2} \sec^2 i \left(1 + \frac{h}{d} \tan i \cos \phi\right)^{-2} e^{-r/d}
\]

(2.5)
2.2. Lensing of Exponentially Distributed Sources

Equation (2.5) is appropriate if the majority of observed sources are very young (e.g. blue supergiants) and hence have a much lower scale height than the lenses. (The same result applies in the hypothetical case that the lenses were confined to a plane and the sources were distributed exponentially.) At the opposite extreme, one may assume that the sources are distributed like the lenses, with exponential scale height $h$. In this case I find that the optical depth, $\tau$, is higher $\tau = (3/2)\tau_0$, so that

$$\tau(r, \phi) = \frac{3\pi G \Sigma_0 h}{c^2} \sec^2 i \left( 1 + \frac{h}{d} \tan i \cos \phi \right)^{-2} e^{-r/d}. \quad (2.6)$$

For observations carried out at optical wavelengths, one may expect that sources behind the M31 plane will be essentially completely blocked by dust. I then find that if the sources and lenses have the same exponential scale height, the optical depth is lower, $\tau = (1/2)\tau_0$,

$$\tau(r, \phi) = \frac{\pi G \Sigma_0 h}{c^2} \sec^2 i \left( 1 + \frac{h}{d} \tan i \cos \phi \right)^{-2} e^{-r/d}. \quad (2.7)$$

For realistic optical observations, I expect that the actual optical depth will lie between $(1/2)\tau_0$ [eq. (2.7)] and $\tau_0$ [eq. (2.5)], and more likely closer to the former. For definiteness, and to be conservative, I henceforth compute optical depths according to equation (2.7).
3. Mapping the M31 Disk Mass Distribution

From equation (2.7) the lensing rate along the major axis is

$$
\tau = \frac{\pi G \Sigma_0 h}{c^2} \sec^2 i e^{-r/d}, \quad \text{(major axis).}
$$

(3.1)

I adopt a disk scale length, $d = 6.4$ kpc and total blue luminosity $L = 2.4 \times 10^{10} L_\odot$ from van der Kruit (1989). I somewhat arbitrarily adopt a mass-to-light ratio $M/L_B = 3$ for the M31 disk, and a disk scale height $h = 400$ pc. These values lead to an estimate of $\Sigma_0 = 280 M_\odot pc^{-2}$, and hence

$$
\tau \sim 2.5 \times 10^{-7} e^{-r/d},
$$

(3.2)

If the optical depth were measured at various positions along the major axis, then from equation (3.1) one could determine $d$ and $(\Sigma_0 h)$. The radial dependence should be quite strong. Then by measuring the azimuthal dependence at fixed radius, one could determine $(h/d)$. See equation (2.7). Note that for the adopted parameters, $[1 + (h/d) \tan i \cos \phi]^{-2}$ ranges between 0.65 and 1.7, so the azimuthal dependence should also be quite strong. For a dark thick disk of similar mass to the observed thin disk and of scale height 1400 pc (such as I have hypothesized to account for the Milky Way lensing events, Gould 1994), the optical depth would be $8 \times 10^{-7} e^{-r/d}$ along the major axis and would be a factor 30 higher along the far minor axis and a factor 0.30 lower along the near minor axis. Thus these two possible disk-like structures could be easily distinguished. As a practical matter, all three parameters would be fit simultaneously from the optical depths as measured over the whole M31 disk. The inner parts of the far minor axis would have to be excluded from the fit because of contamination from the M31 bulge.
4. Practical Requirements

The precision of the measurement described in the previous section rests fundamentally on the possibility of obtaining a statistically significant sample of lensing events, and hence on observing a large number of M31 stars. The MACHO group finds for both the LMC bar fields and for Galactic bulge fields, that it is possible to observe stars down to a crowding limit of $10^6$ stars deg$^{-2}$, in $\sim 2''$ seeing. In principle, it is possible to reach this crowding limit in any field by taking long enough exposures. That is, $n$, the number of observable stars per square degree should be related to the size of the seeing disk, $\theta_s$, by

$$n \sim 0.3 \theta_s^{-2} = 10^6 \left(\frac{2''}{\theta_s}\right)^2 \text{deg}^{-2}. \quad (4.1)$$

In practice, the coefficient of $\theta_s^{-2}$ will vary from field to field depending on the density of the partially resolved stars that are being monitored relative to the density of unresolved stars 1 or 2 mag below this limit. That is, if the crowding limit occurs at a flux level where there are many unresolved stars within 1 mag then it will be somewhat more difficult to follow lensing events than would be the case if the crowding limit occurred where there are few such stars. However, since Tomany & Crotts (1994) have demonstrated that there is no qualitative barrier to finding variables in extremely crowded fields, I will assume that equation (4.1) holds while recognizing that it will in general require some adjustment.

The total number of events observed is given by

$$N = \frac{2}{\pi} \omega T \int d\Omega n(\Omega) \tau(\Omega), \quad (4.2)$$

where $\omega^{-1}$ is the (harmonic mean) time scale of the events, $T$ is the length of the observations, and $\Omega$ is the position on the sky. Using this equation together with equation (2.7), I find a total lensing rate for the entire disk to be

$$N \sim 0.3 \omega T \left(\frac{d}{\theta_s D_{M31}}\right)^2 \frac{4\pi G \Sigma_0 h}{c^2} \text{csc} \ 2i \sim 0.5 \omega T \left(\frac{0''5}{\theta_s}\right)^2, \quad (4.3)$$

where $D_{M31}$ is the distance to M31, and where I have ignored a slight enhancement,
\[1 - (h/d)^2 \tan^2 i\]^{-1/2}, from the azimuthal integration. The fraction of events inside radius \(r\) would be \(1 - (1+r/d) \exp(-r/d)\), that is \(\sim 70\%\) inside the two scale lengths.

The expected time scale \(\omega^{-1}\) can be estimated as follows. Typically \(D_{LS} \sim h \sec i \sim 1500\) pc. For a typical disk star, \(M \sim 0.2 M_\odot\), the Einstein radius is \(r_e \sim 1.5\) A.U. Adopting 75 km s\(^{-1}\) for a typical relative transverse speed in the inner part of the disk, I find \(\omega \sim 9\) yr\(^{-1}\). Thus, from equation (4.3), observations in 0\(^{\prime}\)5 seeing would yield about 5 events per year.

The crowding limit can be reached in the inner part of M31 with short exposures (Tomany & Crotts 1994). The area within 13 kpc of the center of M31 covers only \(\sim 0.8\) square degrees (compared with 40 square degrees covered by MACHO). Since the events are expected to last a month or more, observations need be made only once every several days. Hence, nearly year-round observations are practical and do not require enormous amounts of telescope time. On the other hand, a substantial fraction of the events would be in regions with significant contamination by bulge lenses. Thus, while a pilot ground-based program could yield some information about the mass distribution of the M31 disk, better seeing would be required to obtain a detailed picture. For example, space-based observations with 0\(^{\prime}\)1 seeing would yield \(\sim 80\) events over the inner two scale lengths.

5. Hypothetical Observations With HST

As an example of a space-based program of observations, consider what could be done with the Wide Field Camera (WFC) on the repaired *Hubble Space Telescope* (HST). The WFC covers 4.4 square arcmin. Therefore \(\sim 2300\) square arcmin of the prime 2800 square arcmin field inside two scale lengths could be observed with \(\sim 500\) exposures. Although, the point spread function of HST is \(\lesssim 0^{\prime\prime}1\), the WFC is undersampled. I therefore adopt \(\theta_s = 0^{\prime\prime}1\), corresponding to \(\sim 7.5\) stars arcsec\(^{-2}\). I estimate the density of giant stars with \(M_I \lesssim 1.2\) by assuming there are 0.01 such stars per blue solar luminosity and that half of these stars lie behind the plane and are blocked by dust. I then find \(\sim 0.3\) pc\(^{-2}\) \(\exp(-r/d)\) or
\[ \sim 15 \text{arcsec}^{-2} \exp(-r/d). \] Thus, to achieve the crowding limit at 1 scale length, the observations must be sensitive to \( M_I \sim 1.2 \) or \( I \sim 25.8 \). While the new WFC camera has not yet been calibrated, I extrapolate from experience with the old camera to estimate that 30 minute exposures would yield 7\% relative photometric errors. Hence observations would require \( \sim 250 \) hours of telescope time, implying that the observations could not be carried out once per week even if the telescope were devoted to this project full time.

This calculation shows that HST is not suitable for this project. A specially designed telescope with a much larger field of view would be required.

6. Lensing By a Spheroid

As discussed by Giudice et al. (1994) it is possible that the MACHO and EROS events toward the LMC are generated by low-mass stars in the Galactic spheroid. If so, it would also be plausible to expect significant lensing by the M31 spheroid. Here, I present analytic results for lensing of sources in the M31 disk by a spherical spheroid having core radius \( a \) and asymptotic fall-off as \( r^{-n} \), \( n > 2 \). I then discuss the implications for lensing experiments toward M31.

6.1. Analytic Formulae

Suppose that lenses are distributed in the M31 spheroid with a density distribution

\[ \rho(r) = \frac{\rho_0}{[1 + (r/a)]^{n/2}}, \tag{6.1} \]

with \( n > 2 \). The Einstein ring radius for sources in the disk is again given by equation (2.1). As in the case of lensing by disk stars, \( D_{OL} \sim D_{OS} \), so that the Einstein ring reduces to equation (2.2). Hence the optical depth is given by

\[ \tau = \frac{4\pi G \rho_0}{c^2} \int_{-q}^{\infty} d\ell \frac{a^n(\ell + q)}{(a^2 + b^2 + \ell^2)^{n/2}}, \tag{6.2} \]

where \( b \) is the impact parameter, \( q \) is the distance from the disk star to the point
of nearest approach of the line of sight to the center of M31, and \( \ell \) parameterizes the distance along the line of sight. This becomes

\[
\tau = \frac{4\pi G \rho_0 a^2}{c^2} \left(1 + \frac{b^2}{a^2}\right)^{1-\frac{n}{2}} \left(\frac{\cos^{n-2} \theta_0}{n-2} + \tan \theta_0 \int_{-\theta_0}^{\pi/2} d\theta \cos^{n-2} \theta\right),
\]

(6.3)

where

\[
\tan \theta_0 \equiv \frac{q}{\sqrt{a^2 + b^2}}.
\]

(6.4)

Thus, along the major axis

\[
\tau = \frac{4\pi G \rho_0 a^2}{(n-2)c^2} \left(1 + \frac{b^2}{a^2}\right)^{1-\frac{n}{2}} \quad \text{(major axis)}
\]

(6.5)

For two special cases, the results are

\[
\tau = \frac{4\pi G \rho_0 a^3}{c^2(a^2 + b^2)} \left(q + \sqrt{a^2 + b^2 + q^2}\right), \quad (n = 3),
\]

(6.6)

and

\[
\tau = \frac{2\pi G \rho_0 a^4}{c^2(a^2 + b^2)} \left[1 + \frac{q}{\sqrt{a^2 + b^2}} \left(\frac{\pi}{2} + \tan^{-1} \left(\frac{q}{\sqrt{a^2 + b^2}}\right)\right)\right], \quad (n = 4).
\]

(6.7)

6.2. Implications For Observations of M31

The pattern of the optical depth for a power-law spheroid differs substantially from that of an exponential disk. The optical depth of a disk is exponential in radius, while the optical depth of a spheroid is a power law with an index two lower than the spheroid itself. This is immediately apparent for the major axis from equation (6.5), but in fact is true at any fixed azimuthal angle. The azimuthal dependence of the optical depth is more pronounced for a spheroid than for a disk. For \( a \ll b \), the ratio on the (far minor axis):(major axis):(near minor axis) at fixed radius is 0.51:1:29 for an \( n = 3 \) spheroid and 0.34:1:175 for \( n = 4 \). For a spheroid with a steep index, it is possible that at a given radius, the near minor axis will be dominated by the spheroid, while the major axes are dominated by the disk.
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