Neutrino oscillations and Leptogenesis

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Abstract:

The symmetry breaking of left right symmetric model around few TeV range permits the existence of massive right handed neutrinos or gauge bosons. In this work the decay of lightest right handed neutrinos in a class of minimal left right symmetric model is analysed for the generation of adequate lepton asymmetry. An analytical expression for the lepton asymmetry is developed. In an attempt to achieve the required baryogenesis, we have imposed certain constrains on the parameter space corresponding to low energy neutrino oscillation parameters (especially $\theta_{13}$) and the three phases (CP, majorana and higher energy phase).

Key words: Leptogenesis, Baryogenesis, Majorana Neutrino, Mass Matrix, CP Violation, Minimal left right Symmetric Model.

1 Introduction

Standard model fails to explain the origin of baryonic asymmetry present in the observable universe. The Big-Bang nucleosynthesis shows that the density of baryons compared to that of photons in the universe is very low; $\eta \equiv n_B/n_\gamma = (2.6 - 6.3) \times 10^{-10}$. This ratio $\eta$ can be related to the observed matter-antimatter or baryon-antibaryon asymmetry of the universe as, $Y_B \approx \eta / 7.04 = (3.7 - 8.9) \times 10^{-11}$ [1], where s denotes the entropy density. In order to produce a net baryon asymmetry in the standard Big-Bang model, three Sakharov necessary conditions must be satisfied: (a) Baryon number violation, (b) C and CP violation, and (c) a departure from thermal equilibrium [2] [3]. The first two conditions can be investigated only
after a particle physics model is specified, whereas the third condition can be discussed in a more general way. Among several interesting and viable baryogenesis scenarios proposed in the literature, Fukugita and Yanagida’s leptogenesis mechanism has attracted a lot of attention due to the fact that neutrino physics is entering a flourishing era [4][5].

Further, amongst numerous neutrino mass models under consideration, seesaw mechanism appears as the most attractive model [6] for generation of the tiny neutrino masses reported by the neutrino oscillation experiments [7][8]. The seesaw mechanism introduces a massive right handed neutrinos to the model. Since the right handed neutrinos are very heavy, usually in neutrino oscillation experiments, only the left handed neutrino mass or the low energy effective theory is considered for the study of neutrinos [9]. A more careful analysis of seesaw mechanism reveals that if the massive right handed neutrinos are completely ignored in the neutrino analysis or when seesaw mechanism is implemented in context of low energy effective theory, one can miss many essential ingredients of seesaw model required to explain the leptogenesis [10][11][12][13]. In general, without loss of generality one can work with both masses (tiny and massive), in a basis where the charged lepton mass matrix and right-handed neutrino Majorana mass matrix are diagonal with real eigenvalues [14]. In this case, there will be a total of 18 parameters in neutrino sector and the lepton asymmetry will depend on all of these 18 parameters, where as low energy observables will depend on 9 parameters only. In this paper, lepton asymmetry $\eta_B$ is estimated from light neutrino mass and mixing parameters by implementing the seesaw mechanism in the context of a class of supersymmetric left–right models. By considering a minimal Higgs sector, these models can predict the relation for the Dirac neutrino mass matrix, in a basis where the charged lepton mass matrix is diagonal; where $c \approx m_t/m_b$ is determined from the quark sector, and this assumption leaves only the Majorana mass matrix $M_R$ to be arbitrary. The three phases of $M_R$ can now be removed, leaving a total of 9 parameters which determine both the low energy neutrino masses and mixings as well as the baryon asymmetry. Various attempts have been made by different groups to establish a relationship between leptogenesis and low energy parameters that can be determined by the low energy neutrino experiments.
2 Numerical Approach:

In this section, we have discussed the Minimal Supersymmetric Model \[14\] consisting of majorana particles \((N_1, N_2, N_3)\) following the mass hierarchy pattern \(M_1 \ll M_2 < M_3\). The decay of particle \(N_1\) can be given as,

\[
\Gamma_{N_1 \to lH} = \frac{(M_D^\dagger M_D)_{11} M_1}{8\pi v^2}
\]  

(1)

Where \(M_D\) is dirac neutrino mass matrix and \(v = 174 \text{ GeV}\) is the electroweak VEV.

The asymmetry parameter arising from the decay of above heavy right handed majorana neutrino is given as \[9\],

\[
\varepsilon_1 = \frac{\Gamma_{N_1 \to lH} - \Gamma_{N_1 \to \bar{lH}}}{\Gamma_{N_1 \to lH} + \Gamma_{N_1 \to \bar{lH}}}
\]

(2)

In the basis, where right handed neutrino mass matrix is diagonal and real, the CP asymmetry parameter can be written as \[15\] \[16\].

\[
\varepsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \left[ \text{Im} (M_D^\dagger M_D)_{i1j} \left[ f_v \left( \frac{M_j}{M_1} \right)^2 \right] + \left[ f_s \left( \frac{M_j}{M_1} \right)^2 \right] \right]
\]

(3)

Where \(f_v(x)\) and \(f_s(x)\) are the contribution arising from vertex and self energy corrections respectively \[16\]. For the MSSM case,

\[
f_v(x) = \sqrt{x} \log(1 + \frac{1}{x}) \quad \text{and} \quad f_s(x) = \frac{2\sqrt{x}}{x-1}
\]

\[
f_v(x) + f_s(x) = -\frac{3}{2\sqrt{x}}
\]

In a model consisting of three heavy neutrino masses, the CP asymmetry parameter arising from the decay of \(N_1\) can be written as \[16\].

\[
\varepsilon_1 = \frac{-0.0000019 \left[ \text{Im} (M_D^\dagger M_D)_{12} \frac{M_1}{M_2} + \text{Im} (M_D^\dagger M_D)_{13} \frac{M_1}{M_3} \right]}{(M_D^\dagger M_D)_{11}}
\]

(4)
Here \( \epsilon \) depends on the (1,1), (1,2) and (1,3) entries of \((M_D^\dagger M_D)\). By the see-saw mechanism light neutrino mass matrix can be connected to heavy neutrino mass matrix by the expression [17],

\[
M_\nu = -M_D M_R^{-1} M_D^T
\] (5)

where \( M_D \) is,

\[
M_D = c M_l = c * \text{diag}(m_e, m_\mu, m_\tau)
\] (6)

here \( M_l \) is charged lepton mass matrix and \( c \) can be defined as \( c = m_t/m_b \).

In the case of Neutrino, flavor eigen states \((\nu_e, \nu_\mu, \nu_\tau)\) and mass eigen states \((\nu_1, \nu_2, \nu_3)\) can be connected as [18],

\[
M_\nu = U M_\nu^{\text{diag}} U^\dagger
\] (7)

Where \( M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) \) and \( U \) is 3 \( \times \) 3 mixing matrix consisting, majorana phases and dirac phase [19]. The solution for majorana mass matrix can be expressed as,

\[
M_R = c^2 M_l M_\nu^{-1} M_l^T
\] (8)

\[
= \frac{c^2}{m_1} \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & m_\tau \\ 0 & 0 & 1 \end{bmatrix} U_{PMNS} P^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & m_1/m_2 & 0 \\ 0 & 0 & m_1/m_3 \end{bmatrix} U^T_{PMNS} \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & 1 \end{bmatrix}
\] (9)

In an attempt to get proper form of \( M_R \), the values of different neutrino parameters are expressed in a small expansion parameter [15] [20] [21] \( \gamma \), which is defined as,

\[
\gamma = \frac{m_\mu}{m_\tau} \approx 0.059
\]
Different parameters in terms of small expansion parameter $\gamma$ are written as,

$$m_e = a_1 \gamma^3 m_2, \quad \frac{m_1}{m_3} = a_{13} \gamma, \quad \frac{m_1}{m_2} = \tan^2 \theta_{12} + a_{12} \gamma, \quad \theta_{23} = \frac{\pi}{4} + t_{23} \gamma, \quad \beta = \alpha + \pi/2 + b \gamma$$

and $\sin \theta_{13} = \theta_{13}$, since $\theta_{13}$ is very small.

Where $a, a_{13}, \tau_{13}, \tau_{23}$ are parameters of $O(1)$, with $a = 1.400, a_{13} = 1.3, a_{12} = 1$ and $b = 1$.

Now the right handed mass matrix can be written as,

$$M_R = \begin{pmatrix} P \gamma^3 & Q \gamma^3 & R \gamma^2 \\ S \gamma^3 & T \gamma^2 & U \gamma \\ V \gamma^2 & W \gamma & X \end{pmatrix}$$

(10)

The unitary matrices of $M_R$ can be defined as $(K U_3 U_2 U_1)$;

$$(K U_3 U_2 U_1) M_R (K U_3 U_2 U_1)^T = \begin{bmatrix} |M_1| & 0 & 0 \\ 0 & |M_2| & 0 \\ 0 & 0 & |M_3| \end{bmatrix}$$

(11)

Where $K = \text{diag}(k_1, k_2, k_3)$ with $k_1 = e^{-i \phi_1/2}$, where $\phi_1, \phi_2, \phi_3$ are phase factors which make each right handed neutrino masses real.

The values of masses $m_1$, $m_2$, and $m_3$ considered in this work are,

$$m_1 = 0.0027 \times 10^{-9} \text{GeV}$$

$$m_2 = 0.0068 \times 10^{-9} \text{GeV}$$

$$m_3 = 0.0380 \times 10^{-9} \text{GeV}$$
After the expansion in terms of small parameter, the values of $(M_D^†M_D)^{11}$, $(M_D^†M_D)^{12}$ and $(M_D^†M_D)^{13}$, which are connected to matrix $M_R$ takes the form,

$$(M_D^†M_D)^{11} = \frac{8a^2c^2m^2\gamma^2\cos^2\theta_{12}\sin^2\theta_{12}\tan^2\theta_{12}}{a_{13}\cos^4\theta_{12}[4a_{13}(a_{12}^2-b^2)\cos 2\theta_{12}+a_{13}(a_{12}^2+4b^2)(3+\cos 4\theta_{12})]}$$

(12)

$$(M_D^†M_D)^{12} = \frac{2a^2c^4m_4^2\gamma^2\tan^2\theta_{12}e^{-i(\phi_1-\phi_2)}e^{-2i(2\alpha+\delta)}[-4\theta_{12}^2\cos 2\theta_{12}+4\theta_{13}^2]}{3a_{12}a_{13}-2ia_{13}b+4\cos^2\theta_{12}a_{12}a_{13}+a_{13}(a_{12}+2ib)\cos 2\theta_{12}}$$

(13)

$$(M_D^†M_D)^{13} = \frac{2a^2c^4m_4^2\sin^2\theta_{12}e^{-i(\phi_1-\phi_2)}[a_{13}^2\cos^2\theta_{12}\gamma^2+e^{-2i(2\alpha+\delta)}\theta_{13}^2\sin^2\theta_{12}\gamma^2+2a_{13}\cos \theta_{12}e^{-i(2\alpha+\delta)}\theta_{13}\sin \theta_{12}\gamma^4]}{[a_{13}\gamma^4\cos^4\theta_{12}(a_{12}-2ib+(a_{12}+2ib)\cos 2\theta_{12})]^2}$$

(14)

Now the Eq. (4), representing the CP asymmetry can be expressed as,

$$\epsilon_1 = \frac{0.0000019}{(M_D^†M_D)^{11}}[2ABC+D(A^2-B^2)/(A^2+B^2)^2\times (M_1/M_2) + 2EFG+H(E^2-F^2)/(E^2+F^2)^2\times (M_1/M_3)]$$

(15)

Different terms used in the above expression are,

$$A = (3a_{12}a_{13}+4\cos^2\theta_{12}a_{12}a_{13}+a_{12}a_{13}\cos 4\theta_{12})$$

$$B = i(2ba_{13}\cos 4\theta_{12}-2a_{13}b)$$

$$C = 2a^2c^4m_4^2\gamma^2\tan^2\theta_{12}[\cos(\phi_1-\phi_2)\cos(4\alpha+2\delta)]$$
\[
\sin(\phi_1 - \phi_2) \sin(4\alpha + 2\delta) \left[ -4\theta_{13}^2 \cos 2\theta_{12} + 4\theta_{13}^2 \right]^2
\]

\[D = 2a^2 c^4 m_1^2 \gamma^2 \tan^2 \theta_{12} \sin(\phi_1 - \phi_2) \cos(4\alpha + 2\delta) + \cos(\phi_1 - \phi_2) \sin(4\alpha + 2\delta) \left[ -4\theta_{13}^2 \cos 2\theta_{12} + 4\theta_{13}^2 \right]^2\]

\[E = 2\sqrt{(a_{13})} \gamma^2 \cos \theta_{12} b (\cos 2\theta_{12} - 1)\]

\[F = i \sqrt{(a_{13})} \gamma^2 \cos^2 \theta_{12} a_{12} (1 + \cos 2\theta_{12})\]

\[G = 2a^2 c^4 m_1^2 \sin^2 \theta_{12} \cos(\phi_1 - \phi_3)\]

\[H = 2a^2 c^4 m_1^2 \sin^2 \theta_{12} \cos(\phi_1 - \phi_3)\]

The highest contributing factor in the CP asymmetry \(\varepsilon_1\) can be stated as,

\[\varepsilon_1 \propto \theta_{13}^2 \cos(\phi_1 - \phi_2) \cos(2\alpha + \delta) - \theta_{13}^2 \sin(\phi_1 - \phi_2) \sin(2\alpha + \delta) + [\cos(\phi_1 - \phi_3) + \sin(\phi_1 - \phi_3)] / \theta_{13}^2 + \ldots \]

(16)

For the generation of baryon asymmetry, the CP asymmetry parameter \(\varepsilon_1\) is related to leptonic asymmetric parameter \(Y_L\) by the expression,
\[ Y_L = \frac{(n_L - \bar{n}_L)}{s} = \sum_{i=1}^{3} \frac{\varepsilon_i k_i}{g_{s,i}} \]  \tag{17}

Where \( n_L \) and \( \bar{n}_L \) represents lepton and antilepton number density respectively, \( s \) is entropy, \( k_i \) is the efficiency factor for \( \varepsilon_i \) and \( g_{s,i} \) is number of degrees of freedom at \( T = M_i \).

In this work we are considering the decay of lightest right handed neutrino \( M_1 \) hence the above equation reduces to,

\[ Y_L = \frac{\varepsilon_1 k_1}{g_{s,1}} \]  \tag{18}

The baryon asymmetry \( Y_B \) produced through the sphaleron transition of \( Y_L \), while the quantum number B-L remains conserve, is given by,

\[ Y_B = \frac{n_B}{s} = CY_{B-L} = CY_L \]  \tag{19}

Where \( C = \frac{8N_f + 4N_H}{22N_f + 13N_H} \), \( N_f \) is the number of fermionic family and \( N_H \) is the number of Higgs doublets and \( s = 7.04 \, n_y \). Substituting the value of \( Y_L \) in Eq. (19) we have,

\[ Y_B = C \frac{\varepsilon_1 k_1}{g_{s,1}} \]  \tag{20}

The efficiency factor \( k_1 \) considered in our work can be expressed by an analytical equation for \( \tilde{m}_1 > m_* \) is taken from \cite{22} \cite{23},

\[ k_1(m_1, M_1) = (2 \pm 1) \times 10^{-2} (\frac{0.01eV}{\tilde{m}_1})^{1.1 \pm 0.1} \]  \tag{21}
Where $\tilde{m}_1$ is the effective neutrino mass and can be expressed as,

$$\tilde{m}_1 = \frac{(M_1^D M_D^†)_{11}}{M_1}$$  \hspace{1cm} (22)

The equilibrium neutrino mass $m_*$ can be expressed as,

$$m_* = \frac{16\pi^{5/2} \sqrt{g_*} v^2}{3\sqrt{5} M_{pl}}$$  \hspace{1cm} (23)

Where $g_* = g_{SM} = 106.75$ is the total number of degrees of freedom, and $M_{pl} = 1.22 \times 10^{19} \text{ GeV}/c^2$, is the Planck mass.

3 Results and Discussion

In developing the above formulation, we have assumed low energy supersymmetry, where the Dirac neutrino mass matrix has a determined structure. As a result, we have connected the lepton asymmetry with measurable low energy neutrino parameters. Here right handed neutrino masses are not independent of CP asymmetry parameter. In early universe, at temperature of order $N_1$, the main thermal process, which will enter in the production of lepton asymmetry is the decay of lightest right handed neutrino. In this work, we have restricted ourself to the case where asymmetry is generated only due to the decay of lightest right handed neutrino. The generated lepton asymmetry then gets converted to the baryon asymmetry in the presence of the sphaleron induced anomalous B-L violating processes before the electroweak phase transition. The Electroweak sphaleron processes will convert the induced lepton asymmetry to baryon asymmetry.

In order to estimate the baryon asymmetry arising due to the formulated analytical expression, the dilution factor, often referred as the efficiency factor $k$, that
takes into account the washout processes (inverse decays and lepton number violating scattering) has to be known a priori. In our work for the generation of baryon asymmetry \( Y_B = \frac{\epsilon_1}{\kappa_1} \) the efficiency factor \( \kappa_1 \) considered is of the form

\[
\kappa_1 = (2 \pm 1) \times 10^{-2}(\frac{0.01eV}{m_1})^{1.1\pm0.1},
\]

which is suggested in the work \[23\].

The parameter space corresponding to the parameters \( \theta_{13} \), the CP phase \( \delta \) and the Majorana phase \( \alpha \) are scanned. The value of \( \epsilon_1 \) is checked for a given set of parameters and the current best fit value of \( \theta_{13} \) \[24\] as expressed in table 1. In our analysis if \( \epsilon_1 < 1.3 \times 10^{-7} \) \[15\], the induced baryon asymmetry would be too small to explain the experimental observations. In fig.1 we observe that at current best fit value of oscillation angle \( \theta_{13} \), the value of \( \epsilon_1 \) is sufficient enough to generate observable the baryogengesis signals at low energy neutrino experiments. The value of leptogenisis change with the variation in input parameters.

| \( a \) | \( b \) | \( c \) | \( \theta_{12} \) | \( \theta_{13} \) | \( M_1 \) (GeV) | \( M_2 \) (GeV) | \( M_3 \) (GeV) | \( m_\tau \) (GeV) |
|---|---|---|---|---|---|---|---|---|
| 1.4 | 1.0 | 41.1 | 1.0 | 1.3 | 0.059 | \( 1 \times 10^9 \) | \( 8.7 \times 10^{14} \) | \( 2.6 \times 10^{14} \) | 1.77 | 34.5° | 8.45° |

From Eq.(16), we observe that the CP asymmetry factor depends strongly on parameters \( \theta_{13}, \alpha, \cos(\phi_1 - \phi_3) \) and \( \cos(2\alpha + \delta) \). In an attempt to observe the signatures of baryogenesis at neutrino oscillation experiments, we have imposed constraints on \( (\phi_1 - \phi_2), \alpha, \delta \) and \( (2\alpha + \delta) \) (dirac and majorana phases) parameters. In fig.2 baryon asymmetry \( Y_B \) is plotted as a function of \( \alpha \) and from this plot we can observe that the disfavoured range of \( \alpha \) is \( 100° - 170° \) and \( 280° - 340° \).

From fig. 3 we observe that the allowed range of dirac phase \( \delta \) lies in the range \( 114° - 220° \). The currently constrained value of \( \delta \) (or \( \delta_{CP} \)) by low energy neutrino oscillation experiment lies in the above mentioned range, which motivates the search of leptogenesis signatures at low energy neutrino oscillation experiments. From Eq. 16 we can observe that the dependent phase \( (2\alpha + \delta) \) contributes significantly in CP asymmetry. Fig. 4 illustrates, that the baryon asymmetry (originating from CP asymmetry) depends on \( (2\alpha + \delta) \), a low energy dependent phase. The allowed values of this dependent phase to generate observed baryon asymmetry are \( 0° - 57° \) and \( 292° - 360° \).
Fig 5. shows the dependance of baryogenesis on phases $\phi_1$ and $\phi_3$, which are the phases used for the diagonalization of the right handed mass matrix as shown in Eq. (11). The allowed range of ($\phi_1 - \phi_3$) is $30^\circ - 110^\circ$.

4 Conclusions:

In this work, we have performed a study of thermal leptogenesis which is considered as mechanics responsible for the generation of baryon asymmetry. The analytical expansion of $\varepsilon_{N1}$, neutrino CP asymmetry at low temperature is derived in terms of small expansion parameter $\gamma$. An approximate equation for the same is derived for non zero or the present value of oscillation angle $\theta_{13}$. For the final prediction, baryon asymmetry via CP asymmetry, CP asymmetry is connected with an efficiency factor. The results show that with the present value of $\theta_{13}$ and present bounds imposed on $\delta$ by NoVA and T2K [25], the neutrino experiments can be considered as one of potential source to compute the baryon asymmetry. However, a few relevant parameters are presently unknown. Meanwhile, one can try getting interesting constraints by imposing few assumptions on the high-energy parameters (the most relevant one being that right-handed neutrinos are hierarchical) [26].

It would be of great interest to perform similar calculations by including thermal corrections to CP asymmetry. This will be done in further work.

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Fig. 1: Evolution of CP asymmetry parameter $\varepsilon_1$ using analytical results as a function of neutrino oscillation angle $\theta_{13}$. 

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Fig. 2: Plot for Baryon asymmetry parameter $Y_B$ as a function of majorana phase angle $\alpha$.

Fig. 3: Evolution of Baryon asymmetry parameter $Y_B$ as a function of dirac phase angle $\delta$. 
Fig. 4: Evolution of Baryon asymmetry parameter $Y_B$ as a function of the dependent phase angle $(2\alpha + \delta)$.

Fig. 5: Evolution of Baryon asymmetry parameter $Y_B$ as a function of the dependent phase angle $(\phi_1 - \phi_3)$.