On the Ternary Exponential Diophantine Equation Equating a Perfect Power and Sum of Products of Consecutive Integers

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Abstract: Consider the Diophantine equation \( y^n = x + x(x+1) + \cdots + x(x+1) \cdots (x+k), \) where \( x, y, n, \) and \( k \) are integers. In 2016, a research article, entitled – ‘power values of sums of products of consecutive integers’, primarily proved the inequality \( n = 19,736 \) to obtain all solutions \((x, y, n)\) of the equation for the fixed positive integers \( k \leq 10. \) In this paper, we improve the bound as \( n \leq 10,000 \) for the same case \( k \leq 10, \) and for any fixed general positive integer \( k, \) we give an upper bound depending only on \( k \) for \( n. \)

Keywords: Diophantine equation; Ternary Diophantine equation

MSC: 11D61; 11D45

1. Introduction

In 1976, Tijdeman proved that all integral solutions \((x, y, n), n > 0 \) and \(|y| > 1,\) of the equation

\[ y^n = f(x) \]

satisfy \( n < c_0, \) where \( c_0 \) is an effectively computable constant depending only on \( f \) if \( f(x) \) is an integer polynomial with at least two distinct roots (Shorey-Tijdeman [1], Tijdeman [2], Waldschmidt [3]). In 1987, Brindza in [4] obtained the unconditional form of the result for \( f(x) = f_1k_1(x) + f_2k_2(x) + \cdots + f_sk_s(x), \) where \( f_1, f_2, \ldots, f_s \) are integer polynomials and \( k_1, k_2, \ldots, k_s \) are positive integers such that \( \min\{k_i : 1 \leq i \leq s\} > s(s-1). \) In 2016, Hajdu, Laishram, and Tengely in [5] proved the above result for \( f(x) = x + x(x+1) + \cdots + x(x+1) \cdots (x+k). \) In 2018, Subburam [6] assured that, for each positive, real \( \epsilon < 1, \) there exists an effectively computable constant \( c(\epsilon) \) such that

\[ \max\{x, y, n\} \leq c(\epsilon)(\log \max\{a, b, c\})^{2+\epsilon}, \]

where \((x, y, n)\) is a positive integral solution of the ternary exponential Diophantine equation

\[ a^n = b^x + c^y \]
and \(a, b, c\) are fixed positive integers with \(\gcd(a, b, c) = 1\). In 2019, Subburam [7] provided the unconditional form of the first result for \(f(x) = (x + a_1)^r_1 + (x + a_2)^r_2 + \cdots + (x + a_m)^r_m\), where \(m \geq 2; a_1, a_2, \ldots, a_m; r = r_1, r_2, \ldots, r_m\) are integers such that \(r_1 \geq r_2 \geq \cdots \geq r_m > 0; \gcd(\eta, \cont(f(x))) = 1; \eta^{2\eta}\) is not an integer > 1; \(r_2 < r_1 - 1\) when \(r_2 < r_1\); \(\eta = |\{r_i : r_1 = r_i\}|\); and \(\cont(f(x))\) is the content of \(f(x)\). For further results related to this paper, see Bazsó [8]; Bazsó, Berczes, Hajdu, and Luca [9]; and Tengely and Ulas [10].

In this paper, we consider the Diophantine equation

\[
y^n = x + x(x+1) + \cdots + x(x+1) \cdots (x+k) =: f_k(x)
\]

in integral variables \(x, y,\) and \(n,\) with \(n > 0,\) where \(k\) is a fixed positive integer. In Theorem 2.1 of [5], Hajdu, Laishram, and Tengely proved that there exists an effectively computable constant \(c(k)\) depending only on \(k\) such that \((x, y, n)\) satisfy

\[
n \leq c(k)
\]

if \(y \neq 0, -1.\) For the case \(1 \leq k \leq 10,\) they explicitly calculated \(c(k)\) as

\[
n \leq 19,736.
\]

Here, we prove the following theorem. For any positive integers \(s, p_1, p_2, \ldots, p_m,\) we denote

\[
\lambda_s(p_1, \ldots, p_m) = \sum_{i_1, \ldots, i_s} p_{i_1}p_{i_2}\cdots p_{i_s}
\]

for \(1 \leq i_1 < \cdots < i_s \leq m\)

and \(\lambda_0(p_1, \ldots, p_m) = 1.\) This elementary symmetric polynomial and its upper bound have been studied in Subburam [11].

**Theorem 1.** Let \(k\) be any positive integer and

\[
b = 4\left|\sum_{i=0}^{k-1} (-1)^i A_{k-i-1} 2^i\right|
\]

where \(A_0 = 1, A_1 = 1 + a_1, A_{k-1} = 1 + a_{k-2},\) and \(A_j = a_{j-1} + a_j\) for \(j = 2, 3, \ldots, k - 2\) and where

\[
a_m = 1 + \sum_{i=0}^{m-1} \lambda_{i+1}(3, \ldots, k + i - m + 1)
\]

for \(m = 1, 2, \ldots, k - 2.\) Then, all integral solutions \((x, y, n),\) with \(y \neq 0, -1, x \neq 1, n \geq 1,\) of (1) satisfy

\[
n \leq c_2 \log b,
\]

where \(c_2\) can be bounded using the linear form of the logarithmic method in Laurent, Mignotte, and Nesterenko [12], and an immediate estimation is

\[
c_2 = \begin{cases} 
21,468 & \text{if } 21 > \log n \\
26,561(\log \log b)^2 & \text{if } 21 \leq \log n.
\end{cases}
\]

If

\[
b \leq 4 \times 9 \times 11 \times 467 \times 2,018,957,
\]
Theorem 2. Let \( k \) be odd. Then, we have the following:

\[
\begin{align*}
\text{Corollary 1. If } 1 \leq k \leq 10, \text{ then } n \leq 10,000.
\end{align*}
\]

Hajdu, Laishram, and Tengely studied each of the cases \( (n, k) \) where \( n = 2 \) and \( k \) is odd with \( 1 \leq k \leq 10^5 \) in the proof of Theorem 2.2 of [5]. Here, we prove the following theorem for any odd \( k \). This can be written as a suitable computer program by considering each step of the following theorem as a sub-program that can be separately and directly run.

**Theorem 2.** Let \( k \) be odd. Then, we have the following:

(i) There uniquely exist rational polynomials \( B(x) \) and \( C(x) \) with \( \deg(C(x)) \leq \frac{k - 1}{2} \) such that

\[
f_k(x) = B^2(x) + C(x).
\]

(ii) Let \( l \) be the least positive integer such that \( IB(x) \) and \( l^2C(x) \) have integer coefficients for any nonnegative integer \( i \) and \( \delta \in \{1, -1\} \)

\[
P_{i,\delta}(x) = \delta(IB(x) + \delta i)^2 - \delta(IB(x))^2 - \delta l^2C(x),
\]

\( r \) is any positive integer,

\[
H_1 = \{ \alpha \in \mathbb{Z} : P_{r,\delta}(\alpha) = 0, \delta \in \{1, -1\}, i = 0, 1, 2, \ldots, r - 1 \},
\]

and

\[
H_2 = \{ \alpha \in \mathbb{R} : P_{r,1}(\alpha) = 0 \text{ or } P_{r,-1}(\alpha) = 0 \},
\]

where \( \mathbb{R} \) and \( \mathbb{Z} \) are the sets of all real numbers and integers, respectively. If \( H_1 \) and \( H_2 \) are empty, then (1) has no integral solution \((x, y, 2)\). Otherwise, all integral solutions \((x, y, 2)\) of (1) satisfy \( x \in H_1 \) or

\[
\min H_2 \leq x \leq \max H_2.
\]

2. Proofs

**Lemma 1.** Let \( k \geq 3 \). Then, all integral solutions \((x, y, n)\), \( n > 0 \) and \( y \neq 0 \), of (1) satisfy the equation

\[
a_2b_1y^n_2 - b_2a_1y^n_1 = 2b_1a_1,
\]

where \( a_1, a_2, b_1, \) and \( b_2 \) are positive integers such that

\[
a_1a_2b_1b_2 \mid 4 \sum_{i=0}^{k-1} (-1)^i A_{k-1-i}2^i,
\]

\( A_i \) is the coefficient of \( x^{k-i-1} \) in the polynomial \( f_k(x) / x(x + 2) \),

\[
x = \left( \frac{b_2}{b_1} \right) y^n_1, \text{ and } x + 2 = \left( \frac{a_2}{a_1} \right) y^n_2.
\]
for some nonzero integers \(y_1\) and \(y_2\).

**Proof.** Let \(k \geq 3\). Let \((x, y, n)\), with \(n > 0\) and \(y \neq 0\), be any integral solution of the Diophantine equation

\[
y^n = x + x(x + 1) + \cdots + x(x + 1) \cdots (x + k).
\]

This can be written as

\[
y^n = x(x + 2)g_k(x)
\]

for some integer polynomial \(g_k(x)\), which is not divided by \(x\) and \(x + 2\), since \(k \geq 3\). Let \(d\) and \(q\) be positive integers such that

\[
gcd(x, (x + 2)g_k(x)) = d \quad \text{and} \quad \gcd((x + 2), xg_k(x)) = q.
\]

Let \(d_1, d_2, q_1\), and \(q_2\) be positive integers such that \(d_1d_2 = d\ \gcd(d_1, d_2) = 1\), \(\gcd(d_2^2, (x/d)) = \gcd(d_2^2, ((x + 2)g_k(x)/d)) = 1\), \(q_1q_2 = q\), and \(\gcd(q_1, q_2) = 1 = \gcd(q_2^2, ((x + 2)/q)) = \gcd(q_2, (xg_k(x)/q)) = 1\). Then,

\[
\left(\frac{d_1^2}{d}\right)x = y_1^q \quad \text{and} \quad \left(\frac{q_2^2}{q}\right)(x + 2) = y_2^q
\]

for some nonzero integers \(y_1\) and \(y_2\), since \(y \neq 0\) and \(n \geq 1\). From this, we have

\[
q_2d_1^2y_2^n - dq_1^2y_1^n = 2q_1d_1^2
\]

and so

\[
q_2d_1^2y_2^n - dq_1^2y_1^n = 2q_1d_1.
\]

Let

\[
g_k(x) = f_k(x)/(x(x + 2)) = x^{k-1} + A_1x^{k-2} + \cdots + A_{k-1}
\]

and

\[
g(x) = x^2 + 2x.
\]

Then, for each integer \(l\) with \(0 \leq l \leq k - 1\),

\[
h_l(x) = \left(\sum_{i=0}^{l}(-1)^iA_{l-i}2^i\right)x^{k-l-1} + A_{l+1}x^{k-l-2} + \cdots + A_{k-1}.
\]

In particular,

\[
h_{k-1}(x) = \sum_{i=0}^{k-1}(-1)^iA_{k-i-1}2^i.
\]

This implies that

\[
\gcd(g(x), g_k(x)) \mid \sum_{i=0}^{k-1}(-1)^iA_{k-i-1}2^i,
\]

where \(A_i\) is the coefficient of \(x^{k-i-1}\) in the polynomial \(g_k(x)\).

If \(x\) is odd, then \(d \mid x, d \mid g_k(x), q \mid (x + 2), q \mid g_k(x)\) and so \(dq \mid \gcd(g(x), g_k(x))\).

Suppose that \(x\) is even. Then,

\[
\frac{dq}{4} \mid \frac{x(x + 2)}{4} \quad \text{and} \quad \frac{dq}{4} \mid g_k(x).
\]

Hence, we have

\[
dq \mid 4\gcd(g(x), g_k(x)) \quad \text{and} \quad dq \mid 4\sum_{i=0}^{k-1}(-1)^iA_{k-i-1}2^i.
\]

This proves the lemma. \(\square\)
Lemma 2 (Hajdu, Laishram, and Tengely [5]). Let $a$, $b$, and $c$ be positive integers with $a < b \leq 4 \times 2,018,957 \times 99 \times 467$ and $c \leq 2ab$. Then, the Diophantine equation

$$au^n - bv^n = \pm c,$$

in integral variables $u > v > 1$, implies

$$n \leq \begin{cases} \max\{1000, 824.338 \log b + 0.258\} & \text{if } b \leq 100 \\ \max\{2000, 769.218 \log b + 0.258\} & \text{if } 100 < b \leq 10,000 \\ \max\{10,000, 740.683 \log b + 0.234\} & \text{if } b > 10,000 \end{cases}$$

Lemma 3 (Szalay [15]). Suppose that $p \geq 2$ and $r \geq 1$ are integers and that

$$F(x) = x^p + a_{rp-1} x^{p-1} + \cdots + a_0$$

is a polynomial with integer coefficients. Then, rational polynomials

$$B(x) = x^r + b_{r-1} x^{r-1} + \cdots + b_0$$

and $C(x)$ with $\deg(C(x)) \leq rp - r - 1$ uniquely exist for which

$$F(x) = B^p(x) + C(x).$$

Lemma 4 (Srikanth and Subburam [13]). Let $p$ be a prime number, $B(x)$ and $C(x)$ be nonzero rational polynomials with $\deg(C(x)) < (p-1) \deg(B(x))$, $l$ be a positive integer such that $l B(x)$ and $l^p C(x)$ have integer coefficients for any nonnegative integer $i$ and $\delta \in \{1, -1\}$:

$$P_{i,\delta}(x) = \delta (l B(x) + \delta i)^p - \delta (l B(x))^p - \delta l^p C(x),$$

$r$ be any positive integer,

$$H_1 = \{ \alpha \in \mathbb{Z} : P_{i,\delta}(\alpha) = 0, \delta \in \{1, -1\}, i = 0, 1, 2, \ldots, r - 1 \},$$

and

$$H_2 = \{ \alpha \in \mathbb{R} : P_{r,1}(\alpha) = 0 \text{ or } P_{r,-1}(\alpha) = 0 \}.$$

If $H_1$ and $H_2$ are empty, then the Diophantine equation

$$y^p = B(x)^p + C(x)$$

has no integral solution $(x, y)$. Otherwise, all integral solutions $(x, y)$ of the equation satisfy $x \in H_1$ or

$$\min H_2 \leq x \leq \max H_2.$$

In some other new way as per Note 2, using Laurent’s result leads to a better result. For our present purpose, the following lemma is enough.

Lemma 5 (Laurent, Mignotte, and Nesterenko [12]). Let $l$, $m$, $a_1$, $a_2$, $\beta_1$, and $\beta_2$ be positive integers such that $l \log(a_1/a_2) - m \log(\beta_1/\beta_2) \neq 0$. Let

$$\Gamma = \left| \left( \frac{a_1}{a_2} \right)^l \left( \frac{\beta_1}{\beta_2} \right)^m - 1 \right|.$$

Then, we have

$$|\Gamma| > 0.5 \exp\{-24.34 \log a \log \beta (\max\{\gamma + 0.14, 21\})^2\},$$
where $\kappa = \max\{3, \alpha_1, \alpha_2\}$, $\beta = \max\{3, \beta_1, \beta_2\}$ and $\gamma = \log\left(\frac{1}{\log \beta} + \frac{m}{\log \kappa}\right)$.

**Proof of Theorem 1.** Assume that $k \geq 3$. Then, by Lemma 1, all integral solutions $(x, y, n)$, $y \neq 0, -1$ and $n \geq 1$, of (1) satisfy the equation

$$ay^2 - by^n = c,$$

where $y_1$ and $y_2$ are nonzero integers, $a$ and $b$ are positive integers such that $c \leq 2ab$,

$$ab \mid 4 \sum_{i=0}^{k-1} (-1)^i A_{k-i-1}2^i,$$

and $A_i$ is the coefficient of $x^{k-i-1}$ in the polynomial $f_k(x)/x(x+2)$. Without loss of generality, we can take $y_1 > y_2$ to prove the result. From (2), we write

$$\left| 1 - \left(\frac{a}{b}\right) \left(\frac{y_2}{y_1}\right)^n \right| = \frac{c}{by_1^2}.$$

Next, take $\alpha_1 = a$, $\alpha_2 = b$, $\beta_1 = y_2$, $\beta_2 = y_1$, $l = 1$, and $m = n$ in Lemma 5. Then, by the lemma, we obtain

$$\frac{c}{by_1^2} \geq \exp\{-24.3414(\log \max\{3, a, b\})(\log \max\{3, y_1\}) \max\{21, (\log n)\}^2\}.$$

From this, we obtain the required bound. Next, assume that $1 \leq k \leq 2$. Then, we can write Equation (1) as

$$y_1^2 = c_1 x$$

and

$$y_2^2 = c_2 (x+2)^i,$$

where $c_1, c_2 \in \{1/4, 1/2, 1, 2, 4\}$ and $i \in \{1, 2\}$. In the same way, we can obtain the required bound. To find the exact values of $A_0, A_1, \ldots, A_{k-1}$, equate the coefficients of the polynomials

$$g_k(x) = 1 + (x+1)(1+(x+3)+\cdots+(x+3)(x+4)\cdots(x+k)).$$

and

$$g_k(x) = x^{k-1} + A_1 x^{k-2} + \cdots + A_{k-1}.$$

Then, we obtain $A_0 = 1$, $A_1 = 1 + \alpha_1$, $A_{k-1} = 1 + \alpha_{k-2}$, and $A_j = \alpha_{j-1} + \alpha_j$ for $j = 2, 3, \ldots, k-2$ and

$$\alpha_m = 1 + \sum_{i=0}^{m-1} \lambda_{i+1}(3, \ldots, k + i - m + 1)$$

for $m = 1, 2, \ldots, k-2$. \□

Next, we consider the case that

$$b \leq 4 \times 9 \times 11 \times 467 \times 2,018,957.$$

If $y_1 = 1, y_2 = 1$, or $y_1 = y_2$, then we have

$$x = \frac{d_2}{d_1} = 1, x = \frac{q_2}{q_1} - 2 = -1, x = \frac{2q_1 d_2}{d_1 q_2 - q_1 d_2}.$$
where \( d_1, d_2, q_1 \) and \( q_2 \) are positive integers such that \( d_1 d_2 q_1 q_2 = ab \). These three equations give the required upper bound. Hence, Lemma 2 completes the theorem.

**Proof of Corollary 1.** Take \( k = 10 \) in Theorem 1. Then, \( A_0 = 1, A_1 = 54, A_2 = 1258, A_3 = 16,541, A_4 = 134,716, A_5 = 700,776, A_6 = 2,309,303, A_7 = 4,589,458, A_8 = 4,880,507, A_9 = 2,018,957, \) and \( b/4 = 46,233 \) and so

\[
740.683 \log b \leq 8982.9.
\]

In a similar way, for the case \( k < 10 \), we have

\[
\max \{10,000, 740.683 \log b + 0.23\} \leq 10,000.
\]

Hence, Lemma 2 confirms the result. \( \square \)

**Proof of Theorem 2.** Take \( F(x) = x + x(x + 1) + \cdots + x(x + 1) \cdots (x + k) \) in Lemma 3. Since \( k \) is odd, so \( 2 \mid \deg(F(x)) \), \( p = 2 \), and \( r = \frac{k+1}{2} \). Then, by Lemma 3, there uniquely exist rational polynomials \( B(x) \) and \( C(x) \) with \( \deg(C(x)) \leq \frac{k-1}{2} \) such that

\[
F(x) = B^2(x) + C(x).
\]

Now, by Lemma 4, we have the theorem. \( \square \)

**Note 1.** First, find the values of the elementary symmetric forms \( \lambda_{i+1}(3, \ldots, k+i-m+1) \) for \( i = 0, \ldots, m-1 \) and \( m = 1, 2, \ldots, k-2 \). Next, obtain \( a_1, a_2, \ldots, a_{k-2} \) and so \( A_0, A_1, \ldots, A_{k-1} \). Using this, calculate \( |A_{k-i-1} - 2A_{k-i-2}| \) and so

\[
2^i |A_{k-i-1} - 2A_{k-i-2}| = |A_{k-i-1} 2^i - A_{k-i-2} 2^{i+1}|
\]

for \( i = 0, 2, 4, \ldots \). In this way, for any positive integer \( k \), we can find the exact value of \( b \) in Theorem 1. Therefore, it is not so hard to decide for which \( k \) is

\[
b \leq 4 \times 9 \times 11 \times 467 \times 2,018,957
\]

as in Theorem 1. For this work, we can use a suitable computer program.

**Note 2.** The result of Laurent [16] is an improvement on the result of Laurent, Mignotte, and Nesterenko [12]. From the proof, using the result of Laurent [16] and Proposition 4.1 in Hajdu, Laishram, and Tengely [5], we write the following:

Let \( A, B, \) and \( C \) be positive integers with \( C \leq 2AB, B > A \) and \( B \leq 4 \times 9 \times 11 \times 467 \times 2,018,957 \). Then, the equation

\[
Au^v - Bv^u = \pm C
\]

in integer variables \( u > v > 1, n > 3 \) implies

\[
n \leq C_m (\max \{m, h_n\})^2 (\log B) \left( 2\left(\frac{\tau - 1}{\log u_0} + \frac{1}{\log u_0}\right) + \frac{\log 4}{\log u_0}\right),
\]

where

\[
h_n = \log \left(\frac{n}{(\tau + 1) \log B} + \frac{1}{2 \log u + (\tau - 1) q_0}\right) + \epsilon_m,
\]

in which \( q_0, u_0, C_m, m, \tau, \) and \( \epsilon_m \) are positive real numbers such that \( u \geq u_0, \log(u/v) \leq q_0, C_m > 1, \epsilon_m > 1, \) and \( \tau > 1 \).

If we use the above observation in Lemma 1 of this paper, then we obtain the bound

\[
n \leq C'_b (\log n - \log \log b)^2 \log b
\]
and so an immediate estimation is

\[ n \leq c_2 \log b, \]

where \( c_2 \) is as in Theorem 1 and \( c'_2 \) is a positive real number depending on \( u_0, q_0, C_m, m, \tau, \) and \( \varepsilon_\eta. \) Though there are better bounds in the literature than what the linear form of the logarithmic method in Laurent, Mignotte, and Nesterenko [12] gives, it is sufficient to obtain an explicit bound only in terms of \( k \) using our method, which simplifies the arguments in Section 5 of [5] as well.

3. Conclusions

This article implied a method to obtain an upper bound for all \( n \) where \( (x, y, n) \) is an integral solution of (1) and to improve the method and algorithm of [4]. The same method can be applied to study the general Diophantine equation (see [8–10]),

\[ y^n = a_0 x + a_1 x(x + 1) + \cdots + a_k x(x + 1) \cdots (x + k), \]

where \( k, a_0, a_1, \ldots, a_k \) are fixed integers and \( x, y, n \) are integral variables in obtaining a better upper bound (depending only on \( k, a_0, a_1, \ldots, a_k \)) for all \( \max \{x, y, n\} \), where \( (x, y, n) \) is an integral solution of the general equation.

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