The Search for Supersymmetry Anomalies—Does Supersymmetry Break Itself?

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ABSTRACT

The established results concerning the BRS cohomology of supersymmetric theories in four space-time dimensions are briefly reviewed. The current status of knowledge concerning supersymmetry anomalies and the possibility that supersymmetry breaks itself through anomalies in local composite operators is then discussed.

It turns out that the simplest allowable supersymmetry anomalies occur only in conjunction with the spontaneous breaking of gauge symmetry. A simple example of such a possible supersymmetry anomaly is presented.

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1 Review and Discussion

Supersymmetry is currently one of the most popular ways to extend the standard model of strong, weak and electromagnetic interactions. It is also an essential ingredient of the superstring which is the only known candidate for a theory that might include gravity in a consistent way. However a major difficulty of supersymmetry is to explain why we do not observe it, assuming that it is really present. How and why is supersymmetry broken? It is possible that the considerations of the present paper may be relevant to that question.

It has been known for some time now that the BRS cohomology of $N = 1, D = 4$ rigid supersymmetry is very complicated \cite{5}\cite{6}\cite{7}. All the non-trivial structure occurs in Lorentz non-trivial sectors, and was consequently not found in some investigations, for example \cite{1}, but see also \cite{2}\cite{3}.

This means that in principle there may be anomalies in these theories that violate supersymmetry. We know that anomalies usually play an important role in theories, so any examples of such anomalies would be interesting.

In this paper, I briefly touch on some recent \cite{8}\cite{9} and also the older results concerning the BRS cohomology and then go on to some remarks concerning the current state of research on these questions \cite{10}\cite{11}.

The main problem now is that no supersymmetry anomalies have yet been found that correspond to this cohomology. I shall try to explain some of the fairly obvious reasons that such anomalies are not present in some
It is easy to write down the simplest examples where a supersymmetry anomaly could conceivably arise. The BRS cohomology of these theories indicates that there could be anomalies in the renormalization of composite operators (made from the elementary chiral superfields $S$ of the theory) which are antichiral spinor superfields. These composite operators take forms such as:

$$\Psi_{1\alpha} = D^2[S_1 D_\alpha S_2]$$  \hspace{1cm}(1)$$
$$\Psi_{2\alpha} = \overline{S}_1 D^2[S_2 D_\alpha S_3]$$  \hspace{1cm}(2)$$
$$\Psi_{3\alpha} = \overline{S}_1 D^2[\overline{D}^2 \overline{S}_1 D_\alpha S_3]$$  \hspace{1cm}(3)$$

One could add more chiral superfields $S$ or more supercovariant derivatives of course. The main things to keep in mind are:

1. The expression for $\Psi_{1\alpha}$ should not vanish

2. It is frequently necessary to use more than one flavour of superfield $S$ to prevent the expression from vanishing, because such expressions may be antisymmetric under interchange of flavour indices

To calculate the anomaly, one would couple such composite operators to the action with an elementary (i.e. not composite) antichiral spinor source
superfield $\Phi^\alpha$. This means that one simply adds the following term to the usual action of the theory:

$$S_\Phi = \int d^6z \, \Phi^\alpha \Psi_\alpha$$

where $\Psi_\alpha$ is some composite antichiral spinor superfield, examples of which are given above in (1-3).

Then the anomaly would appear in the form:

$$\delta \Gamma = m^k \int d^6z \, \Phi^\alpha c_\alpha S^n$$

where $\Gamma$ is the one-particle irreducible generating functional, $\delta$ is the nilpotent BRS operator, $\int d^6z$ is an integral over antichiral superspace, $c_\alpha$ is the constant ghost parameter of rigid supersymmetry, $m^k$ is the mass parameter $m$ to some power $k$ required by simple dimensional analysis, and $S^n$ is the $n^{th}$ power of the antichiral superfield (this might include a sum over indices which distinguish different $\mathcal{S}$ superfields from each other).

To count masses we use the following assignments for the variables and the superfields (Notation defined below):

$$m = 1; \partial_\mu = 1; \theta_\alpha = -\frac{1}{2}; S = 1; \Phi_\alpha = \frac{1}{2}; (6)$$

Now we define the component fields:

$$S = A + \theta^\alpha \psi_\alpha + \frac{1}{2} \theta^2 F$$

$$\Phi_\alpha = \phi_\alpha + W_{\alpha\beta} \theta^\beta + \frac{1}{2} \theta^2 \chi_\alpha$$
The dimensions of these coefficient fields are then:

\[ D_\alpha = \frac{-1}{2}; \quad A = 1; \quad \psi_\alpha = \frac{3}{2}; \quad F = 2; \]

\[ \chi_\alpha = \frac{3}{2}; \quad \phi_\alpha = \frac{1}{2}; \quad W_{\alpha\beta} = 1; \]  \quad (9)

An examination of examples shows that elementary dimensional counting prevents the powers of \( m \) from working correctly to yield (3) whenever the only vertices of the diagram are chiral vertices involving only chiral fields. It should be possible to show this by a dimensional argument, but I have not taken the time to try to do that yet--at any rate it certainly seems to hold for a wealth of examples, one of which can be found in [4].

However when there is at least one gauge propagator in the diagram, the powers of \( m \) easily work out correctly to yield (3). But then one has to confront another problem, which is that one has to analyze the cohomology again in the presence of the gauge fields. This unsolved problem has been partially finessed in [10].

Another problem that is also unsolved is the problem of solving the full BRS cohomology of any supersymmetric theory including the sources that are necessary to formulate the full BRS identity. Essentially, this brings in the complication of ensuring that the BRS cohomology space is orthogonal to the equations of motion of the fields. I will also make here some new comments on this question, which has been the subject of my work over the summer [10].
When one tries to compute the full BRS cohomology of chiral matter coupled to gauge fields, the BRS operator becomes quite formidable, and I will not try to give all the details here. However we can discuss a sub-problem of interest without solving the entire problem, and that is what we do below.

Whenever one formulates a BRS identity in the manner pioneered by Zinn-Justin, it is necessary to also include sources $\tilde{f}_i$ for the variation of the fields $f_i$, and in the resulting ‘full’ BRS operator, these give rise to terms that involve the equations of motion of the corresponding fields. This turns out to be more or less equivalent to the Batalin-Vilkovisky quantization method. The essential point is that this eliminates from the cohomology space anything which vanishes by the equation of motion, i.e. anything which vanishes ‘on-shell’. In our case this will eliminate all those objects in the cohomology space which involve superfields $\mathcal{S}$ which have mass terms in the action, as well as a number of higher order terms that are of no concern at present.

So we are now interested only in computing diagrams where the possible supersymmetry anomaly involves massless antichiral fields $\mathcal{S}$ in (5). But this raises another problem. The most promising simple case (see below) seems to involve a triangle diagram with the $\Phi^a$ superfield at one vertex, two chiral (or antichiral) superfields emerging from that vertex and the exchange of a vector superfield between these two lines. Now the mass counting implies that the anomaly has a higher power of mass than the composite operator from which it arises. The only way this can happen is if some of the interior
lines are massive. Is there any way that interior massive lines can give rise to exterior massless lines while exchanging a vector superfield? The answer to this question is of course well known—this will happen if and only if the gauge symmetry is spontaneously broken. We will therefore assume that gauge symmetry is spontaneously broken and that supersymmetry is not spontaneously broken. Since we are looking for supersymmetry breaking through anomalies, it is reasonable to assume that it is not otherwise broken.

This combination is in fact very easy to achieve—as is well known, gauge symmetry breaking is natural and very easy to achieve in rigid supersymmetry, but spontaneous supersymmetry breaking can only be achieved with very contrived models, particularly if the gauge group is semisimple.

So now, if we want to examine the question of supersymmetry anomalies, we are forced to consider a supersymmetric gauge theory with spontaneous breaking of the gauge symmetry. But there are more conditions, at least for the supersymmetry anomalies that involve matter superfields. In order for the relevant diagrams to exist, we must have matter multiplets which break under the gauge breaking into a combination of massive and massless fields, so that a massive vector superfield can have a vertex with a massless and a massive chiral superfield.

This happens of course for the Higgs multiplet itself, but then the massless Goldstone supermultiplets do not contribute to the BRS cohomology space, as will be shown in the forthcoming paper [10]. We must have additional
(non-Higgs) matter multiplets which break under the gauge breaking into a combination of massive and massless fields. There are many ways to do this, and an example is given below. Note that this happens also in the standard model, where the neutrino remains massless after spontaneous breaking of $SU(2) \times U(1)$ to $U(1)_{EM}$ simply because there is no right handed neutrino for it to form a mass with, (and because lepton conservation prevents the formation of a Majorana neutrino mass, in the minimal standard model at least). The relevant discussion of the standard model will also be the subject of a forthcoming paper [11].

So if we want to find a supersymmetry anomaly, we are driven to models with gauged supersymmetry and spontaneous breaking of the gauge symmetry through Higgs multiplets which develop a VEV in their ‘A’ components (but not their ‘F’ components–that would break supersymmetry). In addition these models must have matter which is massless at tree level, but which gets split into massive and massless components as a result of gauge breaking. These are the only models that have a chance of developing supersymmetry anomalies in some of their composite operators at the one loop level. Such models are of course very reminiscent of a supersymmetric version of the standard model of strong weak and electromagnetic interactions. It is just within the realm of possibility that these anomalies could account for the experimentally observed lack of supersymmetry in the world with no additional assumptions in the model at all–in which case we could say
that supersymmetry breaks itself. But there is plenty of work to do before we can determine whether this notion is right. Even if the supersymmetry anomalies exist, considerable work will be necessary to deduce the form of the supersymmetry breaking they give rise to.

A rather interesting and new feature is that we can see that the particular ‘soft’ mass-dependent supersymmetry anomalies we are examining here, if there coefficients are non-zero, would give rise to a kind of supersymmetry breaking that is a function of the VEV that breaks the gauge symmetries, and which vanishes in the gauge symmetric limit. In addition, it has been conjectured [4] that such anomalies might also provide a natural mechanism whereby ‘supersymmetry breaks itself’, while at the same time retaining the cosmological constant at the zero value it naturally has in many unbroken supersymmetric theories. Clearly the spontaneous breaking of the gauge symmetry would not interfere with this feature.

2 A Simple Example

We consider a supersymmetric gauge theory based on the gauge group $SU(2)$ with matter in two vector multiplets and a singlet: $L^a: I = 1; H^a: I = 0; R: I = 0$. These ‘a’ indices transform with $i\epsilon^{abc}$ and take the values $a=1,2,3$. Since the ‘a’ indices are real and since $\delta_{ab}$ is an invariant tensor of $SU(2)$, there is no difference when we raise and lower these indices.

Without any good reason, we shall assume that the superpotential does
not contain a mass term for the $L$ field.

Since the ‘Higgs field’ $H^a$ is in a real representation of the gauge group, it can have a mass term in the superpotential.

Now we assume the following form for the superpotential:

$$W = g_1 L^a H^a R + \frac{g_2 m}{2} H^a H^a + \frac{g_3}{4m} [H^a H^a]^2$$

(10)

Note that renormalizability is not a property of this superpotential.

If $g_2 g_3 < 0$, the Higgs field will develop a VEV in its ‘A’ term that breaks the gauge symmetry down to $U(1)$ while leaving supersymmetry unbroken. The $L$ and $R$ fields develop no VEV. Let us denote components as follows:

$$L^a = A^a + \theta^a \psi_\alpha^a + \frac{1}{2} \theta^2 F^a$$

(11)

$$H^a = B^a + m u^a + \theta^a \phi_\alpha^a + \frac{1}{2} \theta^2 G^a$$

(12)

$$R = A + \theta^a \psi_\alpha + \frac{1}{2} \theta^2 F$$

(13)

We distinguish $a = i, 3$ where $1 = 1, 2$. Here we have included a shift by the VEV:

$$< B^a > \text{ before shift} = \delta^{a3} m \sqrt{\frac{-g_2}{g_3}} \equiv \delta^{a3} m h \equiv m u^a$$

(14)

Then the ‘F’ term of the superpotential becomes:

$$W_F = [g_1 L^a (B^a + \delta^{a3} m h) R + \frac{g_2 m}{2} (B^a + \delta^{a3} m h)(B^a + \delta^{a3} m h)$$

$$+ \frac{g_3}{4m} [(B^a + \delta^{a3} m h)(B^a + \delta^{a3} m h)]^2]_F$$

(15)
In terms of components, this makes the following contribution to the action:

\[
S_{\text{Chiral}} = \int d^4x W_F = \int d^4x \left\{ g_1(mhA^3F + mh\psi^{3\alpha}\psi_\alpha + mhF^3A) \right. \\
+ g_1(A^aB^aF + \psi^a\phi^aA + F^aB^aA + A^a\phi^a\psi + A^aG^aA + \psi^aB^a\psi) \\
+ \text{terms involving } H \text{ superfield only} \} 
\]  
\tag{16}

The essential point to note here is that there is no mass term like

\[
m(A^iF + \psi^{i\alpha}\psi_\alpha + F^iA) \tag{17}
\]

for \(i = 1, 2\) which would give a mass to the \(L^1\) and \(L^2\) superfields. They are massless after spontaneous breaking of the gauge symmetry.

The simplest composite operator (together with a source \(\Phi_\alpha\) that could develop a supersymmetry anomaly seems to be:

\[
S_{\text{Composite}} = \int d^4x d^4\theta \left\{ \Phi^\alpha \overline{\psi}^\beta \left[ \overline{L^aH^a} \right] \overline{D^\beta D_\alpha R} \right\} \tag{18}
\]

After translation of the Higgs field, we find the terms

\[
S_{\text{Composite}} = mh \int d^4x \left\{ \chi^\alpha \left[ (\sigma^\mu)^{\gamma\beta} \partial_\mu \overline{A}^3 \sigma^\nu_{\alpha\beta} \partial_\nu \psi_{\gamma} \right. \\
+ \overline{\psi}^{3\beta} (\sigma^\mu)_{\alpha\beta} \partial_\mu F + \cdots \right\} + \cdots \tag{19}
\]

The form of the supersymmetry anomaly that we would like to calculate is

\[
\delta\Gamma = m^4 \int d^4xd^2\theta \Phi^\alpha c_\alpha \sum_{i=1,2} \overline{\mathcal{T}^i\mathcal{T}^i} = m^4 \int d^4x \chi^\alpha c_\alpha \sum_{i=1,2} \overline{A^iA^i} + \cdots \tag{20}
\]
Clearly at this point there are innumerable examples. It may be that even more structure is needed before non-zero examples of supersymmetry anomalies can be found, or this may be non-zero itself. I have not yet tried to calculate this example, but at least it does not appear to be obviously zero, since it passes all the tests mentioned above.

3 Higher Spins

Now we turn to another topic, which is a fuller analysis of the cohomology of the BRS operator defined by the supersymmetry invariance of chiral multiplets of rigid $N = 1$, $D = 4$ supersymmetry. The new result here [8][9] is that this cohomology space contains potential anomalies in the renormalization of fermionic superfields with all half-integer spins. Formerly it had been shown that there were potential anomalies for fermionic superfields with spin $\frac{1}{2}$ only [7].

This may be very important for superstring theories, since such higher spin multiplets necessarily occur in all such theories.

The result is that there is an infinite set of states of the form

$$\mathcal{X}_{\alpha_1...\alpha_k\beta_1...\beta_{n+g}} = \text{Sym}_{\beta_1...\beta_{n+g}} \int d^4x d^2\theta \left\{ \partial_{\alpha_1}\beta_1...\partial_{\alpha_k}\beta_k S_{a_1}...\right.$$  

$$...\partial_{\alpha_k(n-1)+1}\beta_{k(n-1)+1}...\partial_{\alpha_k}\beta_{kn} S_{a_n}\bar{\beta}_{kn+1}\bar{\beta}_{kn+2}...\bar{\beta}_{kn+g} \right\}.$$  \hspace{1cm} (21)

in the cohomology space of the Wess-Zumino model. The corresponding complex conjugate expressions are obtained by converting dotted into undotted indices and vice versa, and $S \rightarrow \bar{S}$, $\bar{c} \rightarrow c$. By contraction and
symmetrization of the undotted indices, we can decompose $X$ into operators of the form

$$A_{(a,b)} = A_{(\alpha_1\alpha_2...\alpha_a)(\dot{\beta}_1\dot{\beta}_2...\dot{\beta}_b)},$$

(22)

where $b = k_n + g$ and $k_n - a$ is even and greater than or equal to 0, since contractions always involve pairs of undotted indices. In particular, we are interested in polynomials with ghost charge $g = 1$, which correspond to anomalies. For these objects, we find $b - a$ is odd and positive. Therefore, the operators $A$ are spinors.

These objects could appear as anomalies in the renormalization of composite operators with the same spin structure as the anomaly. To compute the anomalies of a given such composite operator, a term of the form

$$S_\Psi = \int d^4x d^2\theta \left[ \Psi_{\alpha_1...\alpha_a\dot{\beta}_1...\dot{\beta}_b} \Phi^{\alpha_1...\alpha_a\dot{\beta}_1...\dot{\beta}_b} \right]$$

(23)

would be introduced into the action. Here $\Psi$ is a composite operator with ghost charge zero and $\Phi^{\alpha_1...\alpha_a\dot{\beta}_1...\dot{\beta}_b}$ is a chiral source superfield. There is a matching between the indices of the anomaly and those of the anomalous operator, because both must couple to the source $\Phi$. Now to compute the anomaly, some specific form for the composite operator $\Psi$ would be chosen and then the one particle irreducible generating functional $\Gamma$ including one vertex (23) would be calculated. If there is an anomaly, one would find that the supersymmetric variation of this part of $\Gamma$ would be of the form

$$\delta \Gamma = \kappa \int d^4x d^2\theta \left[ A_{\alpha_1...\alpha_a\dot{\beta}_1...\dot{\beta}_b} \Phi^{\alpha_1...\alpha_a\dot{\beta}_1...\dot{\beta}_b} \right]$$

(24)
where $\kappa$ is a calculable coefficient.

Because all possible anomalies have half-integer spin (see discussion of (22)), it follows that all operators which can be anomalous also have half-integer spin. Generally, the entire class of spinor operators in supersymmetric theories containing chiral matter can be anomalous.

We suspect that spontaneous breaking of the gauge symmetry will be necessary for non-zero computation of superanomalies in the higher spin cases just as in the spin one-half case.

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References

[1] Brandt, F., Dragon, N., Kreuzer, M.: Phys. Lett. B231, 263-270 (1989), Nucl. Phys. B332, 224-249 (1990), Nucl. Phys. B332, 250-260 (1990), Nucl. Phys. B340, 187-224 (1990)

[2] Brandt, F.: Lagrangedichten und Anomalien in vierdimensionalen supersymmetrischen Theorien. PhD-Thesis, Universität Hannover (1991)
[3] Brandt, F.: Lagrangians and anomaly candidates of $D = 1$, $N = 4$ rigid
supersymmetry. Nucl. Phys. B392, 928 (1993)

[4] Dixon, J. A.: A natural mechanism for supersymmetry breaking with
zero cosmological constant. CTP-TAMU-69/92, to be published in:
Duff, M. J., Khuri, R. R., Ferrara, S. (eds.) From superstrings to su-
pergravity. Workshop Proceedings, Erice 1992. Singapore, New Jersey,
Hong Kong: World Scientific

[5] Dixon, J. A.: BRS cohomology of the chiral superfield. Commun. Math.
Phys. 140, 169-201 (1991)

[6] Dixon, J. A.: Supersymmetry is full of holes. Class. Quant. Grav. 7,
1511-1521 (1990)

[7] Dixon, J. A.: Anomalies, Becchi-Rouet-Stora cohomology, and effective
theories. Phys. Rev. Lett. 67, 797-800 (1991)

[8] Dixon, J. A., Minasian, R.: BRS cohomology of the supertranslation in
$D = 4$. Preprint CPT-TAMU-13/93 (hep-th@xxx/9304035)

[9] Dixon, J. A., Minasian, R., Rahmfeld, J.: The Complete BRS Cohomol-
yogy of Supersymmetric Chiral Matter in $D = 4$. Preprint CPT-TAMU-
20/93

[10] Dixon, J. A.: BRS Cohomology of Supersymmetric Gauge Theories with
Spontaneous Breaking of Gauge Symmetry. Preprint CTP-TAMU-46/93

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[11] Dixon, J. A.: Supersymmetry Anomalies and the Standard Model.
    Preprint CTP-TAMU-47/93