Comments on Marginal Deformations 
in Open String Field Theory

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Abstract

In this short letter we present a class of remarkably simple solutions to Witten’s open string field theory that describe marginal deformations of the underlying boundary conformal field theory. The solutions we consider correspond to dimension-one matter primary operators that have non-singular operator products with themselves. We briefly discuss application to rolling tachyons.
1 Introduction

One of the important features of string field theory is that it allows us to describe physics of different string theory backgrounds using data of only a single reference conformal field theory. This has been recently successfully applied in the context of Witten’s open bosonic string field theory [1]. It has been shown, in accordance with Sen’s conjectures [2], that the theory formulated on an arbitrary D-brane describes another vacuum [3] with no D-branes, and hence no conventional open string degrees of freedom [4].

In this letter we shall give a construction of string field theory solutions that correspond to less dramatic changes of the conformal field theory. Our exact solutions will describe conformal field theories deformed by exactly marginal operators. We shall construct such solutions perturbatively in a parameter $\lambda$, which to the first order can be identified with the coupling constant of a given exactly marginal operator [13, 14]. Following [3], we shall use the cylinder conformal frame parameterized by a coordinate $\tilde{z} = \arctan z$. The solution itself will be given by a series expansion in $\lambda$, each term will be given by a cylinder with simple insertions of the $c$ ghost, the exactly marginal operator called $J$, and vertical line insertions of the $b$ ghost. The mutual distances between $cJ(z)$ insertion points will be parameters that will be integrated over. Unfortunately, for the generic perturbation with singular operator product expansion our solution becomes ill-defined, so we shall restrict ourselves mostly to cases in which $J(x)J(y)$ is finite when $x$ approaches $y$.

One of the more interesting examples of this kind is the time-dependent rolling tachyon solution which is generated by exactly marginal operator $J(z) = e^{X'(z)}$ studied in [15]. We will look at the time-dependent behavior of the tachyon coefficient to get some clues on the tachyon matter problem [16]. Another example, in fact a simpler one, to which our results apply, are deformations generated by $\partial X^{\pm}$. Physically they correspond to turning on light-like Wilson lines, or in the T-dual picture, where the branes become localized both in space and time, they describe their separation in the light-like direction. We shall not however expand on this solution further.

In Section 4 we propose another type of solution, in what might be called a pseudo $B_0$ gauge. This seems easier to apply to situations with non-trivial self-contractions because of the absence of certain singularities.

Marginal deformation solutions in open string field theory have been studied previously in [17, 18, 19, 20, 21, 22], whereas [23] initiated their study in closed string field theory. In the course of this work we have learned that similar results to ours have been obtained independently by Kiermaier, Okawa, Rastelli and Zwiebach [24] and should appear in preprint at the same time as our work.

\footnote{For related recent development see [5, 6, 7, 8, 9, 10]. Nice reviews of string field theory include [11, 12].}
2 Marginal deformations in SFT

We shall start by solving the string field theory equation of motion

\[ Q_B \Psi + \Psi \ast \Psi = 0 \]  \hspace{1cm} (2.1)

perturbatively in a parameter \( \lambda \). Let us denote \( \phi_n \) the coefficient of order \( \lambda^n \), so that

\[ \Psi = \sum_{n=1}^{\infty} \lambda^n \phi_n. \]  \hspace{1cm} (2.2)

At order \( \lambda \) we find

\[ Q_B \phi_1 = 0. \]  \hspace{1cm} (2.3)

To obtain a non-trivial solution we shall take \( \phi_1 \) to be a non-trivial element of the cohomology.\(^2\) It is well known that for each cohomology class there is a representative of the form

\[ \phi_1 = cJ(0)|0\rangle, \]  \hspace{1cm} (2.4)

where \( J \) is a purely matter operator of conformal dimension one, so that indeed \( Q_B \phi_1 = 0 \). The solution to the equation of motion (2.1) can be determined recursively order by order using

\[ Q_B \phi_n = -[\phi_1 \phi_{n-1} + \phi_2 \phi_{n-2} + \cdots + \phi_{n-1} \phi_1]. \]  \hspace{1cm} (2.5)

The right hand side is manifestly \( Q_B \) closed, as one can convince themselves by induction, but it is a-priori not clear whether it is also \( Q_B \) exact. It turns out, that for operators which are exactly marginal in the conformal field theory, the right hand side is always exact.

To invert \( Q_B \) on an \( Q_B \)-exact state we have to fix a gauge. Popular option, which works well in the level truncation, is the Siegel gauge \([17]\); however, for analytic computations it is more convenient to use the \( B_0 \) gauge introduced in \([3]\), or some of its variants. In principle one could try using \( B_0 + \xi B_0^* \) gauge \([3]\) with \( \xi \) different at each order of \( \lambda \), but in this section we shall stick to the simplest \( B_0 \) gauge. We remind the reader that \( B_0 \) is the zero mode of the \( b \)-ghost in the cylinder coordinate.

Let us work out in detail the order \( \lambda^2 \) of the solution. Easy computation using the formalism

\(^2\)Note that similar construction can be used to construct the tachyon vacuum in the wedge state basis \([3]\). Therein one takes \( \phi_1 = Q_B(B_0^c|0\rangle) \). The solution appears to be pure gauge for all \( |\lambda| < 1 \), but becomes non-trivial at \( \lambda = 1 \).

\(^3\)The gauge with \( \xi = 1 \) was found very useful in computing scattering amplitudes \([25]\).
of \[3\] gives
\[\phi_2 = \frac{B_0}{L_0} (\phi_1 \ast \phi_1) = - \int_0^1 dr \, r^{L_0 - 1} B_0 (\phi_1 \ast \phi_1) \] (2.6)
\[= - \int_2^3 ds \hat{U}_s \left[ \frac{\pi}{4} (\bar{c}(x) + \bar{c}(-x)) - \frac{1}{2} \hat{B} \bar{c}(x) \bar{c}(-x) \right] J(x) J(-x) |0 \] (2.7)
where \(x\) in the second line stands for \((\pi/4)(s - 2)\). The operator \(\hat{U}_s\), in a notation borrowed from \[25\], is defined as
\[\hat{U}_s \equiv U_s^* U_s = e^{-\frac{4\pi^2}{L_0}}. \] (2.8)
The operator \(U_s\), in turn, is defined as the scaling operator \((2/s)L_0\) in the cylinder coordinate. The star denotes a BPZ conjugate and \(\hat{L}\) stands for \(L_0 + L_0^*\). For more properties the reader is referred to \[3\] as well as to older works \[26, 27\]. Note, that in the last two expressions, we have formally integrated over wedge states with \(r \in (0, 1)\). Such wedge states are ill-defined, have no meaning on their own, but in the present case they cause no problem. In fact, we use them only for notational convenience to denote a well defined operation of deleting part of the empty surface from the states \(\phi_1\). The real problem can only arise in the \(r \rightarrow 0\) limit, where the two insertions of \(cJ\) from the two \(\phi_1\)’s are approaching each other, squeezing in between a \(b\) line integral. For generic matter operators \(J\) there would be a singularity\[4\]. For simplicity we shall restrict our discussion in this section to operators with finite products at coincident points.

Before moving to higher orders in \(\lambda\), let us check that indeed
\[Q_B \phi_2 + \phi_1 \ast \phi_1 = 0. \] (2.9)
Acting with \(Q_B\) on \(\phi_2\) given by (2.7) is easy. It annihilates the two factors of \(\phi_1\), and acting on \((\pi/2)B_t^L |r\rangle\) produces \(-\partial_r |r\rangle\). The integral therefore localizes at the boundary, the \(r = 1\) contribution gives precisely \(-\phi_1 \ast \phi_1\) whereas the \(r = 0\) contribution vanishes thanks to the two \(c\)-ghost insertions approaching each other, again assuming absence of singularity from the matter currents. This is such a simple mechanism, that it is rather straightforward to guess the form of the \(n\)-th order term of the solution
\[\phi_n = \left( -\frac{\pi}{2} \right)^{n-1} \int_0^1 \prod_{i=1}^{n-1} dr_i \, \phi_1 \ast B_t^L |r_1\rangle \ast \phi_1 \ast \cdots \ast B_t^L |r_{n-1}\rangle \ast \phi_1. \] (2.10)

\[4\]The easiest way to deal with the singularity is to introduce a lower cut-off \(\varepsilon\) on the \(r\) integral and define \(\phi_2\) by the minimal subtraction. This amounts to defining \(\int_0^1 dr r^{-2} = -1\), which is the right definition for inverting \(L_0\) on weight \(-1\) state \(c_1\). Unfortunately, it turns out that in the \(B_0\) gauge there is also an additional \(1/r\) singularity associated with a state \(Q_B (Lc_1) |0\rangle\) on which \(Q_B\) cannot be inverted within the \(B_0\) gauge. The \(1/r\) singularity also appears for non-exactly marginal operators.
The proof that this solves (2.5) is easy and is left to the reader. Geometric picture of our solution is given in Fig. 1. The solution $\Psi_\lambda$ can be viewed as a functional on the Hilbert space of the

$$M$$

![Diagram](image)

Figure 1: Representation of the integrand (2.10) by a cylinder of circumference $\frac{\pi}{2} (2 + \sum r_i)$ with insertions of BRST nontrivial operator $cJ(z)$ and $b$-ghost line integral. The cylinder is formed by identifying the lines marked with an arrow. The BPZ contractions of the integrand are defined as a correlator on this surface with the contracting vertex operator being inserted at the puncture $P$.

open string, and as such, it can be represented as a surface with certain punctures. Instead of the conventional upper-half plane, we use a coordinate where the midpoint is sent to infinity so that the surface looks as a cylinder of canonical circumference $\pi$. But upon taking star product or acting with $1/L_0$, the natural circumference of the cylinder changes, and in our case, for the solution (2.10), it is given by $\frac{\pi}{2} (2 + \sum r_i)$. In addition, we get insertions of $cJ$ located at points

$$x_i = \frac{\pi}{4} \left( \sum_{k=1}^{n-1} r_k - 2 \sum_{k=1}^{i-1} r_k \right).$$

(2.11)

What about the $B_0$ gauge condition, does it remain true for our guess (2.10)? Using the identity

$$B_0 (X * Y) = -\frac{\pi}{2} (-1)^{g_h(X)} X * B_1^L Y,$$

(2.12)

valid for arbitrary $X$ and $Y$ satisfying $B_0 X = B_0 Y = 0$, we find successively that $\phi_1 * B_1^L |r_1\rangle$, $\phi_1 * B_1^L |r_1\rangle * \phi_1, \ldots$ are all in the $B_0$ gauge. To see that, one has to use also the identity $B_1^L |r\rangle * B_1^L \phi_1 = 0$.

The tachyon solution was originally found in a similar form [3], but later an elegant closed form was found by Okawa [5]. In the present case, just by simple inspection, we can formally sum up the whole series to obtain

$$\Psi = \frac{\lambda}{1 + \frac{\pi}{2} \lambda \int_0^1 \phi_1 * B_1^L |r\rangle * \phi_1}.$$  

(2.13)
Here again, the first factor by itself does not make much sense. However, as it acts on $\phi_1$, its action is well defined. Actually, it is possible to avoid using the negative wedge states. One can re-write the formula as

$$\Psi = \sqrt{|0\rangle} \star \frac{\lambda}{1 + \lambda \hat{\phi} \star A} \star \hat{\phi} \star \sqrt{|0\rangle},$$

(2.14)

where $\sqrt{|0\rangle}$ is a star square-root of the vacuum, or in other words the wedge state $|3/2\rangle$, $\hat{\phi} = \hat{U}_1 c J(0) |0\rangle$ and finally

$$A = \frac{\pi}{2} B_1^L \int_1^2 dr |r\rangle$$

(2.15)

is the homotopy operator used in \[4\] to prove that the cohomology around the tachyon vacuum is trivial.

Let us now ask what are the interesting properties of the marginal solution. For exactly marginal deformation one would expect that the energy of the configuration relative to the original brane is strictly zero. In the time independent setup, the energy is simply given by minus the action. Under the change of the parameter $\lambda$ the action changes as

$$\frac{\partial S}{\partial \lambda} = \langle \frac{\partial \Psi_\lambda}{\partial \lambda}, Q_B \Psi_\lambda + \Psi_\lambda \star \Psi_\lambda \rangle = 0,$$

(2.16)

where we used the fact that $\Psi_\lambda$ is a solution of the equations of motion for all values of $\lambda$. Integrating the equation we find that $S = 0$ for all finite values of $\lambda$.

At first sight, there is a little puzzle however. It seems that this proof works not only for exactly marginal deformations, but for all kinds of one-parameter families of solutions continuously connected to zero. One may think of a solution generated by a dimension zero operator

$$A_1^a c \partial X^1 \otimes \sigma_a + A_2^b c \partial X^2 \otimes \sigma_b$$

(2.17)

in a system of two D-branes, where the Chan-Paton factors are given by the Pauli matrices. This corresponds to turning on a constant non-abelian gauge potential $A = A_1 dx^1 + A_2 dx^2$ along two directions. As is well known, constant non-abelian fields have nonzero potential energy given by $\text{Tr} [A_\mu, A_\nu]^2$; this is true also in string field theory, as can be shown by integrating out infinite tower of massive fields [28]. It turns out, that for such deformations the recursive procedure for finding the solution breaks down. One has to go back and correct the initial starting point $\phi_1$ by higher order corrections. Typically what happens is that $e^{ikX}$ gets changed to $e^{ik(\lambda)X}$ which itself is a nice conformal operator, but its variation with respect to $\lambda$ is not. This is the point where our formal proof would break down. The problem does not arise in the fully compact Euclidean case, since there are no operators with continuous spectrum, and so the obstructions in the recursive procedure are unsurmountable. From the field theory perspective these obstructions manifest themselves as impossibility to turn on continuously a flux on a compact manifold.
Another general and interesting question to ask is how the cohomology of the theory changes under the marginal deformation. Had we worked out in detail, for instance, the solution corresponding to branes moving apart, we would have to be able to see how does the mass-spectrum of the stretched strings changes linearly with the brane separation. We do not have the solution yet, nevertheless, we can address the problem first from a formal viewpoint. Expanding the string field theory around the new vacuum $\Psi_\lambda$, we get the new BRST-like operator

$$Q_\lambda = Q_B + \{\Psi_\lambda, \cdot\}$$

and we want to find its cohomology. Formally, this is actually rather easy to determine. Start with a solution $\Psi_\lambda$ to the equation $Q_B \Psi + \Psi \ast \Psi = 0$ and perturb it in the direction of some operator $\varphi$. The variation of the solution solves

$$Q_\lambda \delta \Psi = Q_B \delta \Psi + \{\Psi_\lambda, \delta \Psi\} \ast = 0.$$

So the cohomology is given by perturbed solutions. These are very easy to construct. Deform the original theory by an operator $\lambda J(z) + \mu \varphi(z)$, pretending that it is still exactly marginal operator – in reality it is not, of course. The solution will be given by the formula (2.10), but only its first order term in $\mu$ will be relevant for the new cohomology representatives. Concretely the solution is

$$|O_\varphi\rangle = \sum_{n=1}^{\infty} (-\frac{\pi}{2})^{n-1} \int_0^1 \prod_{i=1}^{n-1} dr_i \left[ c\varphi \ast B_1^L |r_1\rangle \ast cJ \ast \cdots \ast B_1^L |r_{n-1}\rangle \ast cJ + (n-1 \text{ terms}) \right]$$

$$= c\varphi |0\rangle - \frac{\pi}{2} \lambda \int_0^1 dr \left[ c\varphi \ast B_1^L |r\rangle \ast cJ + cJ \ast B_1^L |r\rangle \ast c\varphi \right] + \cdots,$$

(2.20)

where the $n-1$ terms in the first line are obtained by exchanging the position of $\varphi$ with the remaining $J$’s. It is also possible to rewrite the formula in a closed form

$$|O_\varphi\rangle = (1 - \Psi_\lambda \ast B) \varphi (1 - B \ast \Psi_\lambda),$$

where

$$B = \frac{\pi}{2} \int_0^1 B_1^L |r\rangle$$

(2.22)

is a formal object, meaningful when sandwiched between two states containing half-strips of size $\pi/4$ without any insertions on the side adjacent to $B$. Apart of this purely notational formality, the solution (2.21) might be jeopardized when the OPE between $J$ and $\phi$ is singular (which is in fact the typical case). As we have been consistently ignoring these issues, we will do so once more. We shall postpone them to a future work. It is perhaps interesting that the straightforward formal proof of (2.21) does not require $\Psi$ to be a marginal deformation solution. It can be just any solution to the equations of motion. Of course, we do expect, that in the tachyon vacuum (2.21) will become singular.

5This question was touched upon in the context of string field theory in [29] and [30].
3 Rolling solutions

The most interesting application of the previous results is to the study of rolling tachyons [15,16]. Such solutions are generated by a primary field $J = e^{\pm X_0}$ of dimension one (we are using units in which $\alpha' = 1$). For definiteness, we shall take only the plus sign in the exponent – so that the tachyon field is in the perturbative vacuum in the far past. The important property of this vertex operator is that for positive powers its boundary OPE’s are non-singular

$$e^{mX_0(x)} : e^{nX_0(y)} : \sim |x - y|^{2mn} : e^{(m+n)X_0(y)} : .$$

(3.1)

To construct the solution we can simply use the results from the previous section, setting

$$\phi_1 = c_1e^{X_0}|0\rangle.$$  (3.2)

The solution itself is given by (2.10); in a form more suitable for level truncation analysis it reads

$$\Psi = \sum_{n=1}^{\infty} \lambda^n \left( -\frac{\pi}{2} \right) ^{n-1} \prod_{i=1}^{n-1} dr_i \sqrt{2 \sum_{i=1}^{n-1} r_i} \left[ -\frac{1}{\pi} \tilde{E}(x) + \frac{1}{2} (\tilde{c}(x) + \tilde{c}(x)) \right] \tilde{J}(x_1) \cdots \tilde{J}(x_n)|0\rangle,$$

(3.3)

where $x \equiv x_1$ and the $x_i$ are given by formula (2.11). Now let us extract the coefficients $c_1e^{nX_0}|0\rangle$. This will tell us the time dependence of the tachyon field. This has been previously studied in various approximation schemes in [15,16,31,32,33,34,35,36,37,38,39,40,41]. The puzzling feature encountered was that the solution, conjectured to be the tachyon matter, was oscillating with exponentially growing amplitude. The computed pressure was following the same pattern in stark contrast with Sen rolling tachyon conjectures [15,16]. Logically, there seem to be two possible explanations. Either the solution has a finite radius of convergence in $e^{X_0}$, so that beyond that one has to use proper re-summation formula. An example of such behavior is $(1 + x e^{X_0})^{-1}$, which in fact is quite reminiscent of the results from the boundary state analysis. Another possible explanation, perhaps the more likely one, is that the pressure is given by a more complicated formula, containing perhaps some improvement terms that are not given by the Noether procedure. In that case the oscillations in the tachyon field would not have any physical meaning.

The coefficient of the state $c_1e^{nX_0}|0\rangle$ in the rolling solution is given by

$$\lambda^n \left( -\frac{\pi}{2} \right) ^{n-1} \prod_{i=1}^{n-1} dr_i \left( \frac{2}{2 + \sum_{i=1}^{n-1} r_i} \right) ^{n^2+n-2} \cos^2 y \left[ 1 - \frac{2y}{\pi} - \frac{1}{\pi} \sin 2y \right] \times \left( I \circ e^{-nX_0(0)} \tilde{J}(y_1) \cdots \tilde{J}(y_n) \right),$$

(3.4)

where $y = y_1$, $\tilde{J}(y_k) = \cos^{-2} y_k e^{X_0(tan y_k)}$ and further

$$y_k^{(n)} = \frac{\pi}{2} - \frac{1 + \sum_{k=1}^{n-1} r_k}{2 + \sum_{k=1}^{n-1} r_k}.$$

(3.5)
The matter correlator can be computed using the OPE (3.1)

\[ \langle I \circ e^{-nX^0(0)} \tilde{J}(y_1) \cdots \tilde{J}(y_n) \rangle = \prod_{i=1}^{n} \frac{1}{\cos^{2n} y_i} \prod_{1 \leq i < j \leq n} \sin^2 (y_i - y_j) \]

\[ = \prod_{i=1}^{n} \frac{1}{\cos^2 y_i} \prod_{1 \leq i < j \leq n} (\tan y_i - \tan y_j)^2. \]  

(3.6)

To compute the coefficient (3.4) analytically, it is convenient to pass from \( r_i \) to the \( y_i \) variables. We were able to compute only the first three coefficients explicitly

\[ \Psi = \left[ \lambda e^{X^0} - \frac{\lambda^2}{243 \sqrt{3}} e^{2X^0} + \lambda^3 a_3 e^{3X^0} + \cdots \right] c_1 |0\rangle + \cdots, \]  

(3.7)

where \( a_3 \) is rather complicated expression which depends on polygamma function at special points. Equivalently, it can be expressed using the Hurwitz zeta functions \( \sum_{n=1}^{\infty} (n+\alpha)^{-s} \) for \( s = 2, 3, \ldots, 9 \). This is in fact quite natural, since the conformal dimension of \( e^{3X^0} \) is 9 and therefore the transcendentality pattern is similar to the one for the ghost number zero tachyon solution. The value of \( \alpha \) runs over the values 0, 1/12, 2/12, \ldots, 11/12.

Proceeding to higher orders in \( \lambda \) analytically seems an impossible task, so we have tried to obtain number of coefficients numerically by Monte Carlo integration.\(^6\) The first few values we got with \( 10^7 \) points are

\[ a_1 = 1, \quad a_2 = -0.152, \quad a_3 = 0.00215, \quad a_4 = -2.62 \times 10^{-6}, \]

\[ a_5 = 2.79 \times 10^{-10}, \quad a_6 = -2.80 \times 10^{-15}, \quad a_7 = 2.73 \times 10^{-21}, \quad a_8 = 2.59 \times 10^{-28}. \]  

(3.8)

With less accuracy, \( 10^5 \) points, we went up to values of \( a_{30} \). The results are plotted in the graph.\(^2\) They seem to be fitted remarkably well by a one-parameter fit \( n^{-0.38n^2} \). The behavior seems to be consistent with that of Moeller and Zwiebach,\(^{31}\) and Fujita and Hata,\(^{35, 37}\) who also found faster than exponential decay in the coefficients, which means that the sum over powers of \( e^{nX^0} \) has infinite radius of convergence and hence the infinitely growing oscillations will stay. To be completely honest, we have to point out, that our data are not entirely conclusive in this respect. Had the integrand been only moderately peaked around some cube, e.g. of volume \((1/10)^{10}\) for \( a_{10} \), there would be virtually no chance of detecting this region by the Monte Carlo method. The more reliable numerical or even analytic data for lower order coefficients do not however suggest this scenario. So although we are missing rigorous prove we have enough evidence to believe that \( a_n \) decay faster than exponentially, so that our series in \( e^{X^0} \) has infinite radius of convergence.

\(^6\)Actually we have used the built-in method QuasiMonteCarlo in Mathematica, so that our approximate values are exactly reconstructible.
Figure 2: Absolute value of a logarithm of the coefficients $a_n$ for $e^{n X^0} c_1 |0\rangle$. One parameter fit works remarkably well.

4 Discussion

We have presented a rather simple solution describing marginal deformation generated by dimension-one matter primary operator with finite $J(x)J(y)$ as $x \to y$. In order to be able to study really interesting examples, such as generic rolling tachyon process, or properties of unstable systems of branes and (anti)branes one has to understand well the case with singular $J(x)J(y)$. Our preliminary computations show that the otherwise successful $B_0$ gauge might not allow for existence of such solutions. There is one very simple alternative to the construction presented in section 2. When taking the $Q_B$ inverse of an $Q_B$ exact object, instead of demanding that the whole thing be in the $B_0$ gauge, we may as well simply demand that the argument behind $\hat{U}_r$ be in the $B_0$ gauge. For example for $\phi_2$ we find

$$\tilde{\phi}_2 = -\frac{1}{2!} \hat{U}_3 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} dz (\tilde{c}(z) + \tilde{c}(-z)) \tilde{J}(z)\tilde{J}(-z)|0\rangle. \quad (4.1)$$

Working out few more terms, it seems that the pattern is

$$|\Psi\rangle = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\lambda^n}{n!} \hat{U}_{n+1} \int \ldots \int \prod_{i=1}^{n} dz_i \delta \left( \sum z_i \right) \theta_M (z_i) \sum_{i=1}^{n} \tilde{c}(z_i) \prod_{i=1}^{n} \tilde{J}(z_i) |0\rangle, \quad (4.2)$$

where $\theta_M (z_1, \ldots, z_n)$ is the characteristic function of a domain specified by the set of inequalities

$$\sum_{j=1}^{k} z_{i,j} \leq \frac{\pi}{4} k(n - k) \quad (4.3)$$

This trick was invented, as far as we know, by Ian Ellwood in 2002 in the context of the Siegel gauge. He called this a pseudo-Siegel gauge.

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for $1 \leq k \leq n - 1$. We leave the proof of this proposal for the future.

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