Error analysis of college students in calculus learning

Khairani1*, Mukhni1 and F Q Aini2
1 Mathematics Department, Universitas Negeri Padang, Prof. Dr. Hamka Street, Padang, Indonesia
2 Chemistry Department, Universitas Negeri Padang, Prof. Dr. Hamka Street, Padang, Indonesia

*Corresponding author email: khairani@fmipa.unp.ac.id

Abstract. Calculus is one of many courses in college that must be mastered by students. But, students faced difficulty in calculus learning. There were many students who failed in calculus course. They could not answer calculus problem correctly and made mistakes. Purpose of this study was to describe students’ error in calculus learning based on Newman Error Analysis. Type of research was descriptive qualitative. Sampling was done by purposive sampling. Sample was 29 students who took calculus course in 2018. Instrument was students’ answer worksheet of calculus test. Result shows that students have different type of error in solving calculus problem.

1. Introduction
Mathematics is always said as science language because mathematics can be used in many aspects in sciences. It can be used in chemistry, physics, biology, engineering and even in psychology and other social science. Application of mathematics theory and calculation can be found in physics such as velocity and acceleration. In chemistry, mathematics can be used to model the chemical phenomena, make the pattern of it and solve the problem. Biology use mathematics theory to analyses the biological phenomena and take conclusion from it. In psychology, there is psychometrics which uses mathematical statistics to measure individual’s mental ability and behavior style. Many use of mathematics in human life. That is to be the reason why mathematics must be learned in all education level, from kinder garden to college.

The subjects of mathematics that is learned in different education level have different standard according to the use of mathematics in that education level. In kinder garden, students learned to know the number and how to count simple arithmetic. So, the subject of mathematics that is learned by students is not many as primary students. And so on to college students who learned subjects of mathematics in many course, such as calculus, statistics, probability, real analysis and so on. Some subjects are learned by college students of mathematics major only and some of it is learned by college students in another major too. It happen based on the use of mathematics subjects in that major. It is because mathematics has large use in many aspects.

The large use of mathematics in different aspects makes it as one of must subjects which is to be learned in college. It is not only students of mathematics major who learned mathematics but also others students in different major and different faculty, learn mathematics concepts [1]. In college, the one of mathematics subject that is must be learned by students is calculus.
Calculus has concepts and theory which is to be foundation of many topics in other subjects especially in sciences. So, students of physic, chemistry and biology learn about calculus as mathematics’ students. Students are not only must understand the concept of calculus but also understand the application of calculus concepts in everyday life in their major. The problem in calculus is not always about counting and theorem but also application calculus in many aspects. The problem that involved application of calculus is always in mathematical word problem form.

There were problems in calculus learning that happen in college. Students seemed could understand calculus concepts. However, when students were given mathematical word problem in test, they could not answer the question correctly. It could be found many mistakes in their procedure to solve the problem. The mistakes were varies, it was different from one student to another. But, when students were given standard problem, they could solve it. It made students’ mark in calculus course was lower because there were many mathematical word problems in calculus exam. So, it could be found many students failed in calculus course and must take the course again a year later. Hence, students spent more time to study in university.

The reason behind this problem can be analysis from students’ answer sheet. It can show the mistakes that always done by students and can show how far students understand calculus concepts. Error analysis of students’ answer sheet can be done by using Newman analysis. Newman’s error analysis describes students’ error in five aspects of problem solving procedures: reading, comprehension, transformation, process skill and encoding [2]. This method has been used in other learning [3-8]. But, it has not been used in calculus learning. The finding of this analysis can be used to increase calculus learning quality.

2. Method
Type of research was descriptive research with qualitative approach. Samples of research were 29 students who took calculus course in semester January-June 2018. Samples were done by purposive sampling. Instrument of research were students’ answer sheet from calculus exam.

Test of calculus exam was in essay form. Students must answer the question on available answer sheet. The answer must show the procedure to solve the problem that is questioned in test. The questions consist of standard problems (mathematical routine problem) and mathematical word problems.

Students’ answer sheets were analysis used Newman’s Error Analysis that could be classified in to five categories below.

2.1. Error of reading
The first level to solve mathematical word problem is recognize word, symbol and notation that is used in problem. When students do not recognize it, they can be said that they have error of reading mathematical word problem. If they have error of reading, they cannot start the problem solving. So, they do not solve the problem. It can be said that students have error of reading.

2.2. Error of comprehension
The second level in problem solving the mathematical word is by understands the meaning of the problem. After students recognize word, symbol and notation that used mathematical word problem, they must understand what it is known from problem and what it is questioned by problem. If students can understand it, they can think what the best procedure to solve that problem. But, if they cannot understand it, they cannot solve it. It can be said that students have error of comprehension.

2.3. Error of transformation
The third level to solve word problem is by transforming the word and symbol in mathematical word problem to mathematics’ notation, equation or model. By transforming the problem to mathematical equation or model, it can be solved by using mathematical procedure. But, if students cannot transform it or they make mistake in transforming, it make them cannot solve the problem. It can be said that students have error of transformation.
2.4. Error of process skill
After students transforming mathematical word problem to mathematical model or equation, they can use mathematical procedure that is appropriated with the model to solve the problem. In this level, they must have good process skill. If they do not have it, they can make mistake or error in calculation. At last, they cannot solve it. It can be said that students have error of process skill.

2.5. Error of encoding
If students have good process skill and can solve the mathematical model of word problem, they can get the answer from mathematical model or equation. But, the answer is still in mathematical notation. So, it must be encode to appropriate the context of word problem. If they make mistakes in this level, it can be said that students have error of encoding.

3. Result and discussion
Data which had been collect from answer sheet were analyzed by Newman’s error analysis with qualitative approach. The data analyzing of instrument were presented below.

3.1. Analysis to error of reading
Mathematical word problem that was used in the test was about application of derivative in everyday life. The problem was “A farmer wishes to fence off three identical adjoining rectangular pens, each with 300 square feet of area. What should the width and length of each pen be so that the least amount of fence is required?”.

In this level, students should recognize word and symbol were used in the problem. But, from analysis students’ answer sheet, there were student who cannot recognize word and symbol in that problem. The mistake could be seen in the Figure 1 below.

![Figure 1](image1.png)

Figure 1. Student’s answer sheet that show error of reading.

In the problem, there was written that “What should the width and length of each pen be so that the least amount of fence is required “. But, student made mistake that student wrote the length and width. There was lack information in that statement. Student could not recognize word that was used in problem.

3.2. Analysis to error of comprehension
The second level in problem solving the mathematical word is by understands the meaning of the problem. Mathematical word problem that was used in the test was about derivative. The problem was “A 25 foot ladder is leaning against a building. If the bottom of ladder is sliding along the level pavement directly away from the building at 3 feet per second, how fast is the top of ladder moving down when the foot of ladder is 15 feet from the wall”.

In this level, students should understand the use of word and symbol in the problem. They should know what it is asked by the problem. But, from analysis students’ answer sheet, there were student who cannot understand the meaning of word and symbol that is used in problem. The mistake could be seen in the Figure 2 below.

![Figure 2](image2.png)
In the problem, there was written that distance between ladder and building is 15 feet. But, student made mistake that student wrote the distance from building is 15 feet. There was lack information in that statement. The information was really important to solve the problem. They did not understand the meaning of word that is used in problem. It is showed that student has error of comprehension.

3.3. Analysis to error of transformation

One of the procedures to solve mathematical word problem was transformation of word problem to mathematics’ notation. In this transformation, the word problem was transformed to appropriate mathematics’ symbol and notation. In the problem, “A 25 foot ladder is leaning against a building. If the bottom of ladder is sliding along the level pavement directly away from the building at 3 feet per second, how fast is the top of ladder moving down when the foot of ladder is 15 feet from the wall”, every information of the problem could be transformed to mathematics’ symbol. The use of transformation was to made problem solving easier. But, from analysis students’ answer sheet, there were student who cannot transformed word problem to mathematics symbol. The mistake could be seen in the Figure 3 below.

Figure 3. Student’s answer sheet that show error of transformation.

Figure 3 show that student can recognize the word in the problem. Student also understood what it is known from problem and what it is questioned by problem. But, student cannot transform the information to mathematics notation. It showed that student had error of transformation.

3.4. Analysis to error of process skill

After students transforming mathematical word problem to mathematical model or equation, they can use mathematical procedure that is appropriated with the model to solve the problem. From the problem, “A 25 foot ladder is leaning against a building. If the bottom of ladder is sliding along the level pavement directly away from the building at 3 feet per second, how fast is the top of ladder moving down when the foot of ladder is 15 feet from the wall”, the solution was used derivative concepts and Pythagoras theorem. The procedure used mathematics skill to solve it. But, from analysis students’ answer sheet, there were student who cannot use mathematics procedure correctly, so it had the mistakes. The mistake could be seen in the Figure 4 below.
Figure 4. Student’s answer sheet that show error of process skill.

Figure 4 showed that student used derivative to Pythagoras theorem. Student could derivative the variable but student made mistake in derivative the constant. The derivative of constant must be zero but student made mistake. It showed that student had error of process skill.

3.5. Analysis to error of encoding

The process of problem solving that use mathematics procedure will get the result. The result is still in mathematics notation. But, the mathematical word problem is problem that happen based on context in real life. So, in order the result appropriate to context of problem, it must encode to word that suit the context. But, there were student who did not encode mathematics notation to word. It could be seen in the figure below.

Figure 5. Student’s answer sheet that show error of encoding

Figure 5 showed that student got the result of problem by using mathematics procedure. But student did not encode the result to word that suit the context. If it left like that, the problem was not solved yet. So, student made mistake and it was error of encoding.

Analysis of students’ answer sheet shows that they have error of reading, comprehension, transformation, process skill and encoding. This results shows that students lack mathematical literacy
especially to solve mathematical word problems. This finding is coherent with [7] who found that students lacking in solving mathematical problem that need decoding mathematical representation such as tables, graph, number and symbolic notation.

4. Conclusion
Based on analysis of students’ answer sheet by using Newman Error Analysis with qualitative approach, it can be conclude that the error that is made by students are: (1) error of reading; (2) error of comprehension; (3) error of transformation; (4) error of process skill; and (5) error of encoding. That result shows that students still face difficulty in mathematical word problem. The error that is made by students is still high. The findings can be used as reference to decide exact solution for increasing quality in calculus learning.

5. References
[1] Vinsonhaler R 2016 Teaching calculus with infinimals *J. Humanistics Mathematics* 6 240-276
[2] White A L 2010 Numeracy literacy and newman’s analysis *J.Science and Mathematic Education in Southeast Asia* 2 129-148
[3] Riastuti N, Mardiyana M, Pramudia I 2017 Students’ error in geometry viewed from spatial intelligent *IOP Conf. Series: Journal of Physics: Conf. Series* 895 012029
[4] Junaedi I et all 2015 Disclose cause of students error in resolving discrete mathematics problem based on NEA as a means of enhancing creativity *International Journal of Education* 7 31-42
[5] Herold J F 2014 A cognitive analysis of students’ activity : an example in mathematics *Australian Journal of Teacher Education* 39 137-158
[6] Sumule et al 2018 Error analysis of Indonesia junior high school students in solving space and shape content PISA problem newman procedure *IOP Conf. Series: Journal of Physics: Conf. Series* 947 012053
[7] Vale P, Murray S & Brown B 2012 Mathematical literacy examination items and student errors: an analysis of english second language students’ response 28 2 65-83
[8] Wijaya A et al 2014 Difficulties in solving context-based PISA mathematics task: an analysis of students’ errors *The Mathematics Enthusiast* 1 555-584