Latest results of Skyrme-Hartree-Fock-Bogoliubov mass formulas

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Abstract. The latest developments made in deriving accurate mass predictions within the microscopic Skyrme Hartree-Fock-Bogoliubov (HFB) approach are described. Using the HFB method, we have refitted our BSk20 and BSk21 models, on which were based the HFB-20 and HFB-21 mass tables, to the 2353 measured masses of nuclei with \(N\) and \(Z\) \(\geq 8\) given in the newly available 2012 Atomic Mass Evaluation. In addition, we have now investigated the sensitivity to the symmetry coefficient \(J\) and the stiffness of the neutron matter equation of state. We present here in all 5 new Skyrme interactions, along with their corresponding mass tables. The root-mean square deviation obtained with our preferred model (HFB-24) with respect to essentially all the 2353 available mass data falls to 0.549 MeV, the best value ever found within the mean-field framework. Since our Skyrme force is also constrained by the properties of pure neutron matter this new model is particularly well-suited for application to astrophysical problems involving a neutron-rich environment, such as the elucidation of the r-process of nucleosynthesis, and the description of supernova cores and neutron-star crusts.

1. Introduction

In Ref. [1] we presented a family of three Skyrme-type functionals, BSk19, BSk20 and BSk21, along with their corresponding mass tables, HFB-19, HFB-20 and HFB-21, respectively, that we had constructed with a view to providing a unified approach not only to the structure of nuclei as well as of all the different regions of neutron stars (outer crust, inner crust and core) but also to other phenomena associated with the birth and death of neutron stars, such as supernova-core collapse, the r-process of nucleosynthesis in the neutrino-driven wind or during the decompression of neutron-star matter. These three functionals are all based on effective forces with the 16-parameter generalized Skyrme form [1], which is characterized by unconventional terms that have a simultaneous density and momentum dependence.

The parameters of this form of force were determined primarily by fitting measured nuclear masses, which were calculated with the Hartree-Fock-Bogoliubov (HFB) method. For this it was necessary to supplement the Skyrme forces with a microscopic pairing force, phenomenological Wigner terms and correction terms for the spurious collective energy. However, in fitting the mass data we simultaneously constrained the Skyrme force to fit the zero-temperature equation of state (EOS) of homogeneous neutron matter (NeuM), as determined by many-body calculations with realistic two- and three-nucleon forces. Actually, several realistic calculations of the EOS of NeuM have been made, and while they all agree fairly closely at nuclear and subnuclear densities, at the much higher densities that can be encountered towards the centre of neutron
stars they differ greatly in their stiffness, and there are very few data, either observational or experimental, to discriminate between the different possibilities. We therefore considered three different constraining EOSs of NeuM, as follows. The softest is the one that we label FP [2] in Ref. [1], the one of intermediate stiffness is the “A18 + δv + UIX∗” EOS [3], which we label as APR, while our stiffest constraining EOS is the one labelled “V18” in Ref. [4], which we refer to as LS2 in Ref. [1].

After the publication of our HFB-19 to HFB-21 paper [1] the neutron stars PSR J1614−2230 and PSR J0348+0432 were shown to have masses of $1.97 \pm 0.04 \, M_\odot$ [5] and $2.01 \pm 0.04 \, M_\odot$ [6], respectively. This new observation confirmed the plausibility of the BSk20 and BSk21 neutron-star matter EOS, but not the one obtained with BSk19 (the corresponding maximum neutron-star mass being $1.86 M_\odot$ [1]) which is definitely too soft, and it must be discarded. This does not necessarily imply that HFB-19 mass model must also be discarded, nor that the NeuM EOS has to be much stiffer than that of FP, because the core of neutron stars may contain non-nucleonic particles (see, e.g., Ref. [7]).

In addition, at the end of 2012 a new Atomic Mass Evaluation (AME) [8] was published, with 2353 measured masses of nuclei having $N$ and $Z \geq 8$. When we calculate the root-mean square (rms) deviation for HFB-20 and HFB-21 with respect to this new data set, we find that it rises by 17 keV for the former and falls by 5 keV for the latter. The fact that the rms deviation in the case of HFB-21 is actually lower for the larger data base of the 2012 AME than for the 2003 AME [9], to which it was fitted, is an indicator of the reliability with which this model can be extrapolated into unknown regions of the nuclear chart.

For these two reasons, a new series of mass fits was performed. In refitting to the 2353 measured masses of nuclei having $N$ and $Z \geq 8$ in the 2012 AME, we took the opportunity of imposing different values of the symmetry coefficient $J$ on our fits. We generated in all five new parameter sets, BSk22 to BSk26, along with the corresponding mass tables, labelled HFB-22 to HFB-26, respectively [10]. BSk22 to BSk25 were fitted to $J = 32, 31, 30$ and 29 MeV, respectively, and were all constrained to LS2, while BSk26 was also fitted to $J = 30$ MeV but under the APR constraint. In Fig. 1 we show the NeuM EOSs of our five new models. These new mass models are described below.

2. The HFB-22 to HFB-26 mass models

The BSk22 to BSk26 interactions are all based on the 16-parameter generalized form of Skyrme force given in Eq. (1) of Ref. [1]. In addition to the above-described constraints on the symmetry energy, we also impose, as for our previous mass models, the following constraints: a) an optimal fit to the charge-radii data [11], b) a value of $0.8 M$ for the isoscalar effective mass $M^*_s$ in charge-symmetric infinite nuclear matter (INM) at the appropriate equilibrium density $n_0$, this being the value indicated by calculations with realistic forces (see, e. g., the discussion in Ref. [12]), c) an incompressibility $K_v$ of charge-symmetric INM falling in the experimental range $240 \pm 10$ MeV [13], d) the stability of NeuM and of $\beta$-equilibrated neutron-star matter (i.e., the homogeneous nucleon-lepton mixture of which neutron-star cores are comprised) against an unphysical polarization at any density relevant to neutron-star cores [14, 15], e) an EOS of charge-symmetric INM that is consistent with measurements in heavy-ion collisions of nuclear-matter flow over the density range $1.5 - 4.5 n_0$ [16, 17], f) a qualitatively acceptable distribution of potential energy among the four different spin-isospin channels in INM.

The form of our functionals is sufficiently flexible to allow all these constraints to be satisfied and at the same time for the 2353 measured masses of nuclei with $N$ and $Z \geq 8$ given in the 2012 AME to be fitted with an rms deviation as low as 0.55 MeV as shown in Table 1. The rms and mean (data - theory) values of the deviations between the measured masses and the predictions for the five new models are given in the first and second lines, respectively, of Table 1. The third line gives the the model error $\sigma_{mod}$, as defined by Eqns. (42) and (43) of Möller and Nix [18].
Table 1. Rms (σ) and mean (¯ε) deviations between data and predictions for the HFB-22 to HFB-26 models. The first pair of lines refers to all the 2353 measured masses M that were fitted [8], the third line to the model error on all the 2353 masses, and the last pair of lines to the masses Mn − r of the subset of 257 neutron-rich nuclei (Sn ≤ 5.0 MeV).

|                  | HFB-22 | HFB-23 | HFB-24 | HFB-25 | HFB-26 |
|------------------|--------|--------|--------|--------|--------|
| σ(M) [MeV]       | 0.629  | 0.569  | 0.549  | 0.544  | 0.564  |
| ¯ε(M) [MeV]      | -0.043 | -0.022 | -0.012 | 0.008  | 0.006  |
| σmod(M) [MeV]    | 0.619  | 0.561  | 0.542  | 0.537  | 0.556  |
| σ(Mnr) [MeV]     | 0.817  | 0.721  | 0.702  | 0.791  | 0.749  |
| ¯ε(Mnr) [MeV]    | 0.221  | 0.090  | 0.011  | 0.023  | 0.230  |

The last two lines of this table show the corresponding deviations for the subset consisting of the most neutron-rich measured nuclei, here taken as those with a neutron separation energy Sn ≤ 5.0 MeV (there are 257 nuclei in this subset). All five models display, not surprisingly, some deterioration as we move into the neutron-rich region. From the first line we see that the parameter sets BSk24 (J = 30 MeV) and BSk25 (J = 29 MeV) give the best global fits of all the new models, and in fact are better than any of our previous models. However, line 4 of Table 1 shows that the deterioration of BSk25 on moving into the neutron-rich region is much stronger than for BSk24: for all the other models the performance in the neutron-rich region correlates fairly well with the global performance. Thus the apparent high performance of this model in the global fit should be interpreted with caution; other defects of model BSk25 are discussed in Ref. [10]. Looking at BSk22 and BSk23, we see from both lines 1, 3 and 4 that J = 31 MeV works less well than either 29 or 30 MeV, while J = 32 MeV is still more strongly disfavoured.

Comparing BSk24 and BSk26 shows that for J = 30 MeV the high-density LS2 constraint (BSk24) gives slightly better fits than APR (BSk26). It must be stressed, however, that this discrimination in favour of LS2 as the constraining EOS of NeuM relates only to nuclear and subnuclear densities, so that we should not conclude that nuclear masses are telling us something about the EOS of NeuM at the higher densities found in neutron-star cores.

Overall, the clearest conclusion that can be drawn from Table 1 is that model BSk22 is the worst performing of all our models, ruling out J = 32 MeV. There are also very strong indications that J = 29 or 30 MeV (the latter in both its LS2 and APR forms) are to be preferred to J = 31 MeV, although we have expressed some concerns with regards to J = 29 MeV, i.e., to BSk25 [10]. In any case, it will be seen that the HFB models favour a value of J significantly different than the value of J ≃ 32 deduced from the finite-range droplet model [19] (see Ref. [10] for more details).

As was the case with our models BSk19, BSk20 and BSk21, the new models are well adapted to a unified treatment of all parts of neutron stars: the outer crust [20], the inner crust [21] and the core [1]. The relevance of the models to the core of neutron stars arises not only from their fit to a sufficiently stiff EOS of NeuM but also from their fit to nuclear masses, which implies that they take correct account of the presence of protons. For the same reason these models take account of inhomogeneities, and thus are appropriate for the calculation of the inner crust of neutron stars. As for the outer crust, its properties are determined entirely by the mass tables that we have generated for the appropriate interactions. This is in contrast to the very recent Ref. [22], which does not generate mass tables and thus cannot handle the outer crust.

Apart from the EOS of NeuM, other INM properties, such as the incompressibility coefficient, the isoscalar and isovector effective masses, the pressure in charge-symmetric INM (see Fig. 2)
are predicted to have values consistent with both experiments and calculations based on realistic nucleon-nucleon potentials. Moreover, INM is found to be stable against spin and spin-isospin fluctuations in agreement with realistic calculations.

Finally, we have found that all the neutron-star EOSs deduced from our new models support the massive neutron stars PSR J1614−2230 and PSR J0348+0432 (see Ref. [10] for more details).

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