Heat transport scaling and transition in geostrophic rotating convection with varying aspect ratio

Hao-Yuan Lu1,*, Guang-Yu Ding2, Jun-Qiang Shi1, Ke-Qing Xia3,2,† and Jin-Qiang Zhong1
1School of Physics Science and Engineering, Tongji University, Shanghai 200092, China
2Department of Physics, The Chinese University of Hong Kong, Shatin, Hong Kong, China
3Center for Complex Flows and Soft Matter Research and Department of Mechanics and Aerospace Engineering, Southern University of Science and Technology, Shenzhen 518055, China

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We present high-precision experimental and numerical studies of the Nusselt number Nu as functions of the Rayleigh number Ra in geostrophic rotating convection with domain aspect ratio Γ varying from 0.4 to 3.8 and the Ekman number Ek from 2.0×10⁻⁷ to 2.7×10⁻⁵. The heat-transport data Nu(Ra) reveal a gradual transition from buoyancy-dominated to geostrophic convection at large Ek, whereas the transition becomes sharp with decreasing Ek. We determine the power-law scaling of Nu~Ra⁵, and show that the boundary flows give rise to pronounced enhancement of Nu in a broad range of the geostrophic regime, leading to reduction of the scaling exponent γ in small Γ cells. The present work provides new insight into the heat-transport scaling in geostrophic convection and may explain the discrepancies observed in previous studies.

Buoyancy-induced convection in the presence of rotation occurs widely in the Earth’s liquid core, the outer layer of the Sun, and the interior of gaseous planets. Heat transport by turbulent flows in rotating convection is an important process for many astro- and geo-physical systems, and is relevant to numerous industrial applications. Much of the previous studies has focused on the weak rotation regime in which Ra is far above the onset of convection. The Rayleigh number Ra and the Ekman number Ek characterize buoyancy and rotation, respectively. In this buoyancy-dominated flow regime, the thermal boundary layers (BL) remain the main throat to the heat transfer. When the rotation rate Ω increases and the ratio Ra/Ra_c falls below a transitional value, flow transition occurs from BL controlled convection to geostrophic convection where the local balance of Coriolis force and pressure gradient dominates the bulk flows. Although geostrophic convection possesses many important features of astro- and geo-physical flows, it has been a challenge to access this flow regime, particularly for high-resolution measurements of the heat transport when both the turbulent thermal forcing and strong rotations (Ek~10⁻⁷) are present.

To achieve a wide parameter range of low Ek, recent experimental and numerical studies have used convection cells with small aspect ratios Γ=D/H (D and H being the horizontal and vertical scale of the fluid domain), in the hope that measurements in these small-Γ domains still provide adequate sampling of the flow structures, since their horizontal scale (l~Ek⁻¹/³H) decreases with decreasing Ek. However, it remains an unanswered question whether the scaling of heat-transport determined in small Γ convection domains can be extrapolated to laterally extended and even unbounded systems.

The fluid dynamics of rotating, buoyancy-driven flows is often studied by a paradigmatic model, the rotating Rayleigh-Bénard convection (RBC), i.e., a fluid layer being heated from below and rotated about a vertical axis. In the geostrophic regime, the heat transport by rotating RBC, expressed by the Nusselt number Nu, exhibits a steep power-law scaling Nu~(Ra/Ra_c)γ. Asymptotic theory predicted that in this flow regime Nu=(Ra/Ra_c)⁵, based on the argument that the scaling exponent should be independent of the fluid dissipation properties. Experimental results suggested Nu~(Ra/Ra_c)³, which was interpreted through the BL crossing hypotheses. The γ=3 scaling was found in numerical simulations of the asymptotic theory when the effect of Ekman transport through non-slip boundaries was considered. However, recent experiment revealed 1.2<γ<1.6. Despite the large amount of experimental and numerical studies, there exists hitherto no generally accepted law of heat transport in the geostrophic convection regime. We note that these studies are conducted in domains with different aspect ratios under different boundary conditions.

In this Letter we demonstrate both experimentally and numerically that the scaling properties of heat transport in geostrophic rotating RBC depends sensitively on the aspect ratio Γ of the fluid domain. Remarkably, we report that the boundary flows in the sidewall region give rise to pronounced heat-transport enhancement, leading to a much slower scaling of Nu(Ra) in slender convection cells. Our convection apparatus was designed for high-precision heat transport measurements in rotating RBC. We used cylindrical cells that had copper top and bottom plates and Plexiglas sidewalls with an inner diameter D=240 mm and various heights (H=63, 120, 240, 480, 600 mm), yielding

*These authors contributed equally to this study.
FIG. 1: (a, b) The Nusselt number as functions of the Ra on logarithmic scales. (a) Nu(Ra) are measured in the Γ=0 cell with various Ω, and (b) with a fixed rotation rate Ω=3.14 rad/s but varying Γ. Results in (b) are for Ek=2.7×10^{-7} (red), 1.9×10^{-6} (gray), 4.6×10^{-7} (light blue) and 3.0×10^{-7} (blue). Open squares: DNS data for Γ=1 and Ek=1.9×10^{-6}. Open circles: data for Ω=0. Solid lines: the GL theory [29]. (c, d) The local exponent γ=d(lnNu)/d(lnRa) determined from a power-law fit of Nu(Ra) over various restricted ranges. Symbols are defined in (a) and (b), respectively. Solid curves are guides to the eye.

FIG. 2: (a) Nu as a function of Ra/Ra_c for various Ek with Γ=0.4. Symbols are defined in Fig. 1a. The vertical dashed lines denote the first transition values Ra_t/Ra_c for Ek=2.0×10^{-7} (purple), 3.0×10^{-7} (blue), 3.7×10^{-7} (green) and 5.9×10^{-7} (orange). The red dotted line represents the power-law fit to the data, Nu∼(Ra/Ra_c)^{1.39} for the range Ra≤Ra_t(Ek). The black dotted line indicates the non-rotating scaling Nu∼Ra^{0.317}. (b) The ratio Nu/Nu_0 as a function of RaEk^{3/2} with Nu_0=0.0921Ra_0^{0.317} and the exponent β=1.70 for Γ=0.4. The red dotted line represents the power-law fit to the data, Nu/Nu_0∼(RaEk^{3/2})^{1.19} for Ra≤Ra_t. The vertical dashed lines denote the first transition Ra_tEk^{3/2}=0.17±0.01. The arrow indicates approximately the second transition Ra_2Ek^{3/2}=3.4±0.4. Inset: an expanded view in the vicinity of the first transition Ra_tEk^{3/2}. For each Ek, Ra_t is determined by the interaction of the two locally fitted power-law lines.

Γ=3.8, 2.0, 1.0, 0.5 and 0.4, respectively. Deionized water at a mean temperature of 40.00°C was used as the working fluid. Measurements of Nu=qH/νΔT were taken with rotation rates Ω up to 4.71 rad/s and various applied temperature differences ΔT. The parameter range for Ra=qgΔTH^3/νκ and Ek=ν/2ΩH^2 was 1.4×10^7≤Ra≤2.9×10^{11} and 2.0×10^{-7}≤Ek≤2.7×10^{-5}. Here q is the heat-current density, g is the gravitational acceleration, ν, κ, λ are the fluid kinematic viscosity, thermal diffusivity and conductivity, respectively. Thus the reduced Rayleigh number spans 1.3≤Ra/Ra_c≤343. In the low-Ek regime (Ek<10^{-6}), the present study extended the measurement range, reducing Ra/Ra_c by about half a decade compared to earlier measurements in water [19, 23]. We refer to the rotation-dominated flow regime as geostrophic convection where Ra is above the convective onset but below a critical value Ra_t (defined below) for heat-transport scaling transition, as in this region the geostrophic balance holds [33]. We also made direct numerical simulations (DNS) that solved the Navier-Stokes equations in cylindrical cells with non-slip boundaries, using the multiple-resolution version of the CUPS code [34–36]. The numerical and experimental data covered different and overlapping parameter ranges and complemented each other; and where their parameter ranges overlapped, the corresponding data were in close agreement (see Supplemental Material [37] for detailed experimental and numerical methods).

Our measurements of Nu with Ω=0 suggest a heat-transport scaling Nu_0~Ra^{70} that agrees, within estimated systematic errors of about 2%, with previous stud-
ious coherent turbulent structures arise in the flow field under rotations \[ \Gamma=0 \], we speculate that the corresponding sharper transitions may be related to the different properties of turbulent structures that modify the heat-transport scaling, analogous to previous findings in weakly rotating RBC \[ \Gamma \leq 0 \]. Further studies, both experimental and numerical, are needed to substantiate this argument.

In Fig. 2 we examine the scaling properties of \( \Nu \) measured in the \( \Gamma=0.4 \) cell. Figure 2a shows \( \Nu \) as a function of \( \Ra/\Ra_c \) for various \( \Ek \). The data collapse approximately in the geostrophic convection regime where \( \Ra \) is below an \( \Ek \)-dependent transition value \( \Ra_c \). \( \Ra_c \) is determined as the upper bound of the steep power-law scaling of \( \Nu(\Ra) \) (shown in the inset of Fig. 2b). Linear regression of the data with \( \Ra \leq \Ra_c \) in the log-log plot suggests a power law \( \Nu \sim (\Ra/\Ra_c)^\gamma \), with the fitted exponent \( \gamma \) and its statistical error given in Table 1. The range of the power-law dependence expands as \( \Ek \) decreases, since \( \Ra_c/\Ra_c \) increases apparently with decreasing \( \Ek \). We note that for a higher \( \Ek \) fewer data points are available in the geostrophic convection regime. The power-law fitting here is thus in fact restricted in a relatively small range of \( \Ek \). For \( \Ra > \Ra_c \), \( \Nu(\Ra/\Ra_c) \) becomes dependent on \( \Ek \) with a greater value for a lower \( \Ek \). The spread of the data in this flow regime was reported and ascribed to the Ekman pumping effect \[ \Gamma \leq 0 \] that enhances \( \Nu \) with its strength depending on \( \Ek \). One expects that in the limit of large \( \Ra \) the heat-transport data approach the non-rotating scaling \( \Nu \sim \Ra^{\gamma_0} \), as shown in Fig. 2a.

| \( \Gamma \) | \( 10^6 \Ek \) | \( \gamma \) | \( \gamma_0 \) | \( \beta \) | \( \Ra \Ek^3 \) |
|---|---|---|---|---|---|
| 0.4 (EXP) | 0.30 | 1.48±0.06 | 0.317 | 1.70 | 0.17 |
| 0.5 (EXP) | 0.46 | 1.65±0.03 | 0.316 | 1.65 | 0.36 |
| 0.5 (DNS) | 1.9 | 1.42±0.06 | | | |
| 1.0 (EXP) | 1.9 | 1.82±0.06 | 0.303 | 1.60 | 1.01 |
| 1.0 (DNS) | 1.9 | 1.77±0.05 | | | |
| 2.0 (EXP) | 7.4 | 1.81±0.05 | 0.302 | 1.60 | 1.00 |
| 2.0 (DNS) | 1.9 | 2.04±0.05 | | | |

**TABLE 1:** Experimental (EXP) and numerical (DNS) results for the scaling exponents \( \gamma, \gamma_0, \beta \) and the transition value \( \Ra \Ek^3 \).

The heat-transport data from the \( \Gamma=0.4 \) cell, which cover a relatively small range of \( \Ek \), are incompatible with the power law scaling of \( \Nu \sim \Ra^{\gamma} \) in the geostrophic convection regime (Fig. 2). However, results of \( \Nu(\Ra) \) over a wider parameter range, depicted in Fig. 1b and 1d, suggest that the exponent \( \gamma \) is dependent on both of the control parameters \( \Gamma \) and \( \Ek \). Surprisingly, we find that \( \gamma \) decreases in slender cells with smaller \( \Gamma \) that span a parameter range of lower \( \Ek \) (Table 1), which appears to be contrary to what has been previously observed that the heat-transport scaling becomes steeper with decreasing \( \Ek \). To understand the \( \Ek \)- and \( \Gamma \)-dependence of \( \gamma \) in the geostrophic convection regime, we present in Fig. 3a both experimental and numerical results of \( \Nu(\Ra \Ek^{4/3}) \) for three values of \( \Gamma \) but with fixed \( \Ek=1.9 \times 10^{-6} \). One sees that in the geostrophic convection regime with \( \Ra \leq \Ra_c \), \( \Nu(\Ra) \) exhibits a steeper power-law for a larger \( \Gamma \). With increasing \( \Ra \) the three sets of data converge approximately at the same transition point towards the geostrophic turbulence regime, suggesting that the transitional value \( \Ra_t \) is independent of \( \Gamma \). Figure 3b plots the exponent \( \gamma \) as a function of \( \Gamma \) for various \( \Ek \). We find that for a given \( \Ek \) (e.g., \( 1.9 \times 10^{-6} \)), \( \gamma \) increases strongly with the aspect ratio; whereas for a fixed \( \Gamma \), \( \gamma \) increases with decreasing \( \Ek \). These results also suggest, for the parameter range studied, that the scaling exponent \( \gamma \) depends more sensitively
on Γ than on Ek for geostrophic convection.

Figure 3a shows that the different heat-transport scaling with varying Γ results in a higher Nusselt number for the slender cell (Γ=0.5), which exceeds the corresponding value for a wide cell (Γ=2.0) by over 150% for the same Ra near the convection onset. The enhancement of Nu for slender cells is observed in a wide range of geostrophic Ra near the convection onset. The enhancement of Nu [37]. Figures 4d-4f compare the distributions of \( j_z(r, z) \) for \( \text{Ra} \epsilon_k^{1/3}=19.8 \) in cells with Γ=0.5, 1.0 and 2.0. We find that although the lateral extend of the fluid layer increases with increasing Γ, the spatial structures of the boundary flow are similar. This is shown in the expanded view of the radial profiles \( j_z(r) \) in the inset of Fig. 4h, as the peaks of \( j_z(r) \) exhibit a similar structure with approximately the same magnitude and width. It is for this aspect-ratio invariant properties that the BF gives rise to a larger Nu in slender cells: with a smaller Γ the BF occupies a relatively larger volume of the cell (see the radially-scaled plot of \( j_z(r) \) in Fig. 4h), and makes a greater contribution to the overall enhancement in Nu. Near the rotation axis \((r=0)\) \( j_z(r, z) \) remains small, indicating that in this regime the centrifugal effect is insignificant for heat transport by the bulk flows, in line with previous studies [43, 44].

We have shown that lateral constraint of the flow domain impacts strongly the scaling properties of heat transport in the geostrophic rotating RBC. It is demonstrated that the local heat flux carried by the boundary flow (BF) makes up a significant portion of the global heat transport in a broad range of geostrophic convection, leading to the unexpected aspect-ratio-dependence of both the scaling exponents and the critical values for regime transitions. We conclude that the scaling relationship of Nu(Ra) measured in convection cells with finite Γ cannot be extrapolated to most large-scale, laterally unbounded geophysical and astrophysical flows, while theories of geostrophic convection that neglect the lateral boundary confinements provide an incomplete description of laboratory experiments. The present study brings new insight into understanding the diverse results of heat-transport scaling obtained from previous experiments and simulations [17, 19, 22, 26, 28].

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1 Electronic address: xiaqk@sustech.edu.cn
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