Probing neutrino mixing angles with ultrahigh energy neutrino telescopes

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We point out that detecting $\nu_e$'s from distant astrophysical sources with the up-coming and future neutrino telescopes using the Glashow resonance channel $\bar{\nu}_e e^- \rightarrow W^- \rightarrow$ anything, which occurs over a small energy window around the $\bar{\nu}_e$ energy of $\sim 6.3$ PeV, offers a new way of measuring or setting limits on neutrino mixing angles, in particular the angle $\theta_{12}$, thereby providing an independent experimental probe of neutrino mixing angles. We also discuss how this exercise may throw light on the nature of the neutrino production mechanism in individual astrophysical sources.

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Experimental determinations of the parameters governing neutrino flavor oscillation phenomena, such as the mass-square differences and mixing angles amongst the three light neutrino species, have so far been done with solar- atmospheric-, reactor-, and accelerator neutrino experiments that involve neutrinos of few MeV – GeV energies. At a purely phenomenological level, one may ask if neutrinos of vastly different energies oscillate in the same way. In this context it is important to think of other possible independent ways of measuring the neutrino parameters that involve neutrinos of different (in particular higher) energies than those currently employed in neutrino oscillation experiments.

In this Letter we point out that up-coming and future neutrino telescopes capable of detecting neutrinos of very high (at least up to several PeV: $1 \text{ PeV} = 10^6 \text{ GeV}$) energies, would potentially also be able to measure one or more of the neutrino mixing angles at such energies. Specifically, we suggest that detecting $\nu_e$'s from distant astrophysical sources through the Glashow resonance (GR) channel, $\bar{\nu}_e e^- \rightarrow W^- \rightarrow$ anything, which occurs over a small energy window around the $\bar{\nu}_e$ energy $E_{\bar{\nu}_e}^{\text{GR}} = m_{1\text{W}}^2/2m_e = 6.3$ PeV, offers a new way of measuring the angle $\theta_{12}$ (see below).

Neutrino flavor oscillation phenomena are governed by six independent parameters: two mass-squared differences, $\Delta m^2_{12} \equiv |m_2^2 - m_1^2|$ and $\Delta m^2_{23} \equiv |m_3^2 - m_2^2|$, three mixing angles, $\theta_{12}, \theta_{23}$, and $\theta_{13}$, and a possible CP-violating phase $\delta$. Here $(m_1, m_2, m_3)$ are the masses corresponding to the three light neutrino mass eigenstates ($\nu_1, \nu_2, \nu_3$). The mixing angles parameterize the relation between the mass eigenstates and the flavor eigenstates ($\nu_e, \nu_\mu, \nu_\tau$); see equation 4 below. The current experimental situation, as summarized in Ref. 1, is the following: the solar neutrino data are consistent with flavor oscillations mainly between $\nu_e$ and $\nu_\mu$, driven by a mass-squared difference $\Delta m^2_{\text{sol}} = \Delta m^2_{12} \approx 7.1 \times 10^{-5} \text{ eV}^2$ and mixing angle $\theta_{\text{sol}} = \theta_{12} \approx 32^\circ.5$, while the atmospheric neutrino data are explained by oscillations mainly between $\nu_\mu$ and $\nu_\tau$ with $\Delta m^2_{\text{atm}} = \Delta m^2_{23} \approx 2.6 \times 10^{-3} \text{ eV}^2$ and $\theta_{\text{atm}} = \theta_{23} \approx 45^\circ$, the maximal value. The other mixing angle $\theta_{13}$ is constrained by reactor neutrino experiments to be very small, $\theta_{13} \lesssim 9^\circ$. The CP-violating phase $\delta$ remains undetermined by present experiments. The uncertainties (currently roughly 10–20\%) in the values of the above parameters may be expected to be significantly reduced by future experiments.

Very high energy neutrinos of energies well above several hundreds of TeV, and in some cases extending well into EeV ($10^6 \text{ GeV}$) energies, are predicted to be produced by astrophysical sources such as Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRB)'s. More “exotic” sources such as cosmic topological defects are predicted to produce neutrinos of energy even up to several hundred EeV. Detection of such high energy neutrinos from distant astrophysical sources is the primary goal of a host of proposed or already-under-construction large under-ice and underwater optical Cerenkov detectors such as ICE-CUBE, ANTARES, NESTOR, NEMO, as well as radio Cerenkov detectors such as RICE and ANITA. These detectors will be able to determine the energy as well the arrival directions of the neutrinos enabling point source neutrino astronomy.

High energy neutrinos are produced in astrophysical sources mainly from the decays of charged pions (and kaons) which are produced through interaction of high energy protons with either ambient photons (“$p\gamma$”) or protons (“$pp$”) within the source. In the $p\gamma$ process, charged pion (and hence neutrino) production occurs dominantly through production of the $\Delta$ resonance: $p\gamma \rightarrow \Delta^+ \rightarrow \pi^+ n$, with $\pi^+ \rightarrow \mu^+ \nu_\mu$, and $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. Thus each $p\gamma$ interaction eventually produces one each of $\nu_e, \nu_\mu$, and $\bar{\nu}_\mu$. Denoting by $\mathcal{F}_\nu = \{\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$ the fractional amounts of $\nu$'s and $\bar{\nu}$'s of various flavors, we thus have for the $p\gamma$ process the flavor fractions at the source, $\mathcal{F}_{p\gamma} = \{1/3, 0, 1/3, 1/3, 0, 0\}$. In the $pp$ process, on the other hand, each inelastic $pp$ collision produces a nearly equal mix of $\pi^+$'s and $\pi^-$'s whose decays together...
produce the flavor fractions, $F^{pp} = \{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0 \}$. Note that the $(\nu + \bar{\nu})$ fractions, $F_{(\nu + \bar{\nu})} \equiv \{(\nu_e + \bar{\nu}_e), (\nu_\mu + \bar{\nu}_\mu), (\nu_\tau + \bar{\nu}_\tau)\} = \{ \frac{1}{3}, \frac{2}{3}, 0 \}$ for both $p\gamma$ and $pp$ processes.

In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBs, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process \[14\]. However, for generality let us assume that a source is $\nu_\tau$ dominated. For high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBs, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBs, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBs, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process. In the tenuous (radiation dominated) environments within most potential astrophysical sources such as AGNs and GRBS, the dominant production mechanism for high energy neutrinos is expected to be the $p\gamma$ process.

Thus, $x = 0$ for a pure $p\gamma$ origin of the neutrinos. The flavor fractions at the flavor fractions at the source now work out to be

$$F_{\nu}^{\text{source}} = \left\{ \left(\frac{1}{3} - \frac{x}{6}\right), \frac{x}{6}, \frac{1}{3}, \frac{1}{3}, 0, 0 \right\}. \quad (1)$$

Note that $(\nu + \bar{\nu})$ fractions of different flavors at source are again $F_{\nu}^{\text{source}} = \{ \frac{1}{3}, \frac{2}{3}, 0 \}$, independent of the fraction $x$.

The flavor fractions of the $\nu'$s and $\bar{\nu}'$s arriving at Earth are different from the above fractions at the source due to neutrino flavor oscillations during the propagation from the source to the Earth. In the standard, 3-generation scenario of neutrino oscillations, the flavor eigenstates $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$) are related to the mass eigenstates $|\nu_i\rangle$ ($i = 1, 2, 3$) through $|\nu_{\alpha}\rangle = \sum U_{\alpha i} |\nu_i\rangle$. For values of $\Delta m^2$ as indicated by solar and atmospheric neutrino experiments, the vacuum oscillation length $L_{\text{osc}} = 4\pi E_{\nu}/|\Delta m^2| \approx 2.5 \times 10^{-12} (E_{\nu}/1 \text{TeV}) (10^{-3} \text{eV}^2/|\Delta m^2|) \text{Mpc}$ is always much smaller than the distances (several hundreds to thousands of Mpc) to possible astrophysical sources of neutrinos such as AGNs and GRBs, even for the highest energies of interest. The neutrinos thus oscillate many times before reaching Earth, and the oscillation (or survival) probabilities averaged over many oscillations take the simple form

$$P(\nu_{\alpha} \to \nu_{\beta}) = \sum_{i=1,2,3} |U_{\alpha i}|^2 |U_{\beta i}|^2. \quad (2)$$

A “standard” form \[1\] of the neutrino mixing matrix $U$ is

$$U = \left( \begin{array}{ccc} U_{e1} & U_{e2} & U_{e3} \\ \frac{U_{\mu 1}}{\sqrt{2}} & \frac{U_{\mu 2}}{\sqrt{2}} & \frac{U_{\mu 3}}{\sqrt{2}} \\ \frac{U_{\tau 1}}{\sqrt{2}} & \frac{U_{\tau 2}}{\sqrt{2}} & \frac{U_{\tau 3}}{\sqrt{2}} \end{array} \right) \left( \begin{array}{c} c_{12}c_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\epsilon^\delta \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\epsilon^\delta \end{array} \right) \left( \begin{array}{c} s_{12}c_{13} \\ c_{12}c_{23} - s_{12}s_{23}s_{13}\epsilon^\delta \\ -c_{12}s_{23} - s_{12}c_{23}s_{13}\epsilon^\delta \end{array} \right) \times \text{diag} \left( e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1 \right), \quad (3)$$

where $s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$, and $\delta, \phi_1, \phi_2$ are CP-violating phases. The phases $\phi_1$ and $\phi_2$ are present only for Majorana neutrinos; they do not enter into the expressions for oscillation probabilities \[2\], although the “Dirac” phase $\delta$, if it is non-zero, does.

In our numerical calculations below we use the above full form of the mixing matrix \[2\] in calculating the expressions for various oscillation probabilities \[2\] with $\delta$ set to zero for simplicity. However, simple analytical derivation of our main results are possible if we set $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$ (conditions which we hereafter refer to as “optimal”), in which case the mixing matrix $U$ takes the simple form

$$U_{\text{optimal}} = \left( \begin{array}{ccc} c_{12} & s_{12} & 0 \\ -\frac{\sqrt{2}}{\sqrt{3}}s_{12} & \frac{\sqrt{2}}{\sqrt{3}}c_{12} & \frac{\sqrt{2}}{\sqrt{3}} \end{array} \right), \quad (4)$$

involving only the mixing angle $\theta_{12}$.

Using Equations \[1\], \[2\], and \[4\] we can write the flavor fractions of $\nu'$s and $\bar{\nu}'$s arriving at Earth in the optimal case as

$$F_{\nu,\text{optimal}}^{\text{Earth}} = \left\{ \left(\frac{1}{3} - p\right), p, \frac{1 + q}{6}, \frac{1 - q}{6}, \frac{1 + q}{6}, \frac{1 - q}{6} \right\}. \quad (5)$$

where $p = \frac{1}{3} (x + 2\xi - 2x\xi)$, $q = (\xi - x\xi)$, with $\xi \equiv s_{12}c_{12} = \frac{1}{3}\sin^2 2\theta_{12}$.

Note that $(\nu + \bar{\nu})$ fractions of different flavors at Earth are again $F_{\nu}^{\text{source}} = \{ \frac{1}{3}, \frac{2}{3}, 0 \}$, independent of the fraction $x$ in the optimal case. Also, the $(\nu + \bar{\nu})$ fractions are independent of the neutrino mixing angles in the optimal case. Dependence on $x$ and mixing angles (only $\theta_{12}$ in the optimal case) is contained in the separate number fractions of $\nu'$s and $\bar{\nu}'$s of individual flavors. However, separate identification of the $\nu'$s and $\bar{\nu}'$s of individual flavors is not generally possible through the usual charged-current (CC) interactions of neutrinos in the currently operating or planned water or ice-based detectors, since the sign of the charge of the produced charged lepton cannot be determined. The only exception is the $\bar{\nu}_e$ which can be identified in water or ice based detectors through the Glashow Resonance (GR) channel already mentioned \[10\]. An experimentally measurable ratio such as $\bar{\nu}_e/(\nu_\mu + \bar{\nu}_\mu)$ giving the $\bar{\nu}_e$ fraction to $(\nu_\mu + \bar{\nu}_\mu)$ fraction measured over an energy interval centered at the GR energy can then yield a measurement of the angle $\theta_{12}$. Note that taking the ratio as above cancels out the unknown total neutrino flux coming from the source. Alternatively, if the spectrum of the neutrinos is the same across a broad range of ener-
energies, then one can also get the absolute normalization of the total neutrino flux from the total number of $(ν_μ + ̄ν_μ)$ events over a suitably chosen range of energies.

The cross section for the GR interaction of ̄νe can be written as

$$σ^{GR} \approx 0.675 \left( \frac{m_ν^2}{2m_e E_ν} \right) \delta \left( 2m_e E_ν - m_W^2 \right), \quad (6)$$

where $m_ν$ and $m_W$ are the $e^−$ and $W$ masses. The $ν_μ$ CC interaction cross section is $18 σ^{CC,ν_μ} ≈ 5.53 \times 10^{-30} \text{cm}^2/(E_ν/\text{GeV})^{0.563}$. Integrating the flux times cross section over an energy bin of say 5.01 PeV − 7.9 PeV (which spans 0.1 in log 10 on either side of the resonance energy 6.3 PeV), we can calculate the ratio, $R ≡ ̄ν_e^{GR}/(ν_μ + ̄ν_μ)CC$, of GR to $(ν_μ + ̄ν_μ)$-CC event rates in a water or ice based detector. For an assumed $E_ν^{-2}$ neutrino spectrum we get in the optimal case,

$$R_{\text{optimal}} = 30.5 \left[ ξ + \frac{x}{2} (1 - 2ξ) \right]. \quad (7)$$

In obtaining equation (7) we have considered muons with contained vertices only.

Equation (4), which is a linear function of $x$, is displayed in Figures 1 and 2 for various values of $θ_{12}$ (solid lines). In these Figures we also show the (dashed) lines obtained by allowing small deviations of the angles $θ_{23}$ (Fig. 1) and $θ_{13}$ (Fig. 2) from their “optimal” values; this we do by using the general form of the mixing matrix given by Equation (3).

Figures 1 and 2 show that the dependence of the ratio $R$ on the mixing angles progressively weakens as $x$ (i.e., fraction of $pp$ origin neutrinos) increases and, in the optimal case, disappears completely for $x = 1$ as expected (see equation (5)). Fortunately, for the dominant $pγ$ process ($x = 0$) the ratio $R$ has significant dependence on $θ_{12}$ thus allowing the possibility of measuring this angle. The flip side is that the expected number of ̄νe GR events is smaller in the case of $pγ$ as compared to $pp$ dominance.

Clearly, the accuracy with which the different mixing angles can be probed will be limited by the accuracy with which the ratio $R$ can be measured in a real detector. While the ratio $R$ always increases with increasing $θ_{12}$, its dependences on small deviations of the values of $θ_{23}$ and $θ_{13}$ from their optimal values are somewhat more complicated, though perhaps too small to be measurable. From the above Figures we see that for a given upper limit (say $≤ 10\%$) on the fraction $x$ for a given source from independent considerations (e.g., high energy $γ$-ray observations), a measurement of the ratio $R$ yields a lower limit on the angle $θ_{12}$. Conversely, for a given fixed value of $θ_{12}$, a measured value of $R$ gives a value of the fraction $x$, thus providing important clues to the nature of the neutrino production mechanism in the source under consideration.

At PeV energies neutrinos begin to get absorbed in passing through the Earth; this is especially true for ̄νe at the GR energy where the ̄νe interaction cross section is significantly enhanced. Thus only the downward-going to horizontal ̄νe’s will be detectable. Fortunately, the atmospheric neutrino flux at PeV energies is negligible and may not be of concern as a background. The detectability of ̄νe GR events has been discussed recently for the diffuse neutrino background (due to integrated contribution from all sources in the Universe) in Ref. 17 for a ICECUBE-type detector. The ̄νe GR events can be identified through the detection of the electromagnetic shower produced by the hadronic decay products of the $W$ in the detector. This corresponds to $≈ 70\%$ detection efficiency, reflecting a $\sim 70\%$ detection efficiency.
ergy window \cite{17}. Interestingly, since, at least in the optimal case, the \((\nu_e + \bar{\nu}_e)\) fraction is equal to the \((\nu_\mu + \bar{\nu}_\mu)\) fraction, and since the CC cross section is essentially the same \cite{18} for \(\nu_\mu\)'s and \(\nu_e\)'s, our ratio \(R\) in equation \cite{17} multiplied by a factor of \(\sim 0.7\) gives a good estimate of the expected ratio of signal-to-background number of events. At the same time, the accuracy with which the ratio \(R\) can be measured in a detector depends on the actual number of GR events detected in a reasonable time frame (say 6–7 years), which in turn depends on the flux from the source and the size of the detector.

In this context, we should mention that throughout the above discussion we have implicitly assumed individual point sources for which the meaning of the \(pp\) origin fraction \(x\) is well-defined. For the diffuse background, on the other hand, that fraction refers to some kind of weighted average over all sources. Our discussions above are then valid for diffuse background as well provided the above difference in the meaning of \(x\) is kept in mind. Clearly, from the point of view of signal-to-background ratio, individual point sources such as not too distant and yet sufficiently powerful GRBs, would be easiest to detect because then temporal as well as directional information could be used to increase the signal-to-background ratio. Also, individual sources are more easily subject to observations in other bands such as high energy \(\gamma\)-ray observations which may provide independent information on the fraction \(x\) (assuming hadronic origin of \(\gamma\)-rays through \(\pi^0\) decay). In any case, in view of important information regarding neutrino parameters as well as the nature of high energy astrophysical sources of neutrinos that may be obtained from detection of \(\nu_e\) GR events as discussed above, it would certainly be useful in designing future high energy neutrino telescopes to optimize them for detection of possible \(\nu_e\)'s from astrophysical sources through the GR channel.

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