Emergence of space and cosmic evolution based on entropic force

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Abstract

From the perspective of the hologram, our universe is asymptotically holographic. We propose an additional pressure is added to the ideal fluid since the effects of the entropic force. Furthermore, we obtain the dynamic equation in FRW universe which contains the quantum gravitational effects based on the description of entropic force and emergence of space. Our model can well explain the age of the universe and current accelerating expansion. We give the relation between the luminosity distance and the redshift factor, and compare it with that of lambda cold dark matter model($\Lambda CDM$ model). It may cause inflation when the quantum effects are considered at very early universe. At last, we think the entropic force caused by the surface term may be the origin of the “dark energy”.

1. Introduction

In a variational approach to gravity, we usually choose the Einstein-Hilbert (EH) action as the fundamental action, which is an integral over the invariant four-volume of a scalar curvature. In order to obtain the Einstein equation, one usually add a surface term\cite{1}. However, it was pointed out that the true dynamical degrees of freedom of gravity which make gravity intrinsically holographic reside in the surface term rather than in the bulk term\cite{2}, and the EH action possesses a holographic relation between the surface and bulk terms\cite{3,4}. More significantly, the Einstein equation can be obtained just from the surface term of this action\cite{2} and it can be also obtained from the surface term of the more general actions in the case of pure gravity\cite{5}. Therefore, the surface term is so important that we can not disregard it.

A large number of cosmological observations have showed that our universe is in accelerating expansion. It is usually explained by $\Lambda CDM$ model. The
model is well in line with the data of astronomical observations and implies that 73% of the total energy of the universe is dark energy. However, since Verlinde[8] pointed out that gravity was explained as an entropic force caused by changes in the information associated with the positions of material bodies, it was proposed by Easson et al. that a driving term should be added to the Friedmann dynamic equation because of the existence of the surface term in the Einstein-Hilbert action[2]. In this entropic force scenario, called “entropic cosmology”, the expansion of the universe is caused by the entropy of the surface. Based on the idea of entropic cosmology, the current accelerating expansion [7] and inflation at early universe[8] were investigated. Cosmic expansion based on the entropic force scenario were also investigated in Refs. [9, 10].

Padmanabhan[11] suggested an idea that cosmic space is emergent as cosmic time progresses. He argued that the difference between the number of the surface degrees of freedom and that of the bulk degrees of freedom in a region of space drives the accelerating expansion of the universe through a simple equation \( \Delta V = \Delta t (N_{\text{sur}} - N_{\text{bulk}}) \), where \( V \) is the Hubble volume in Planck units and \( t \) is the cosmic time in Planck units. Further, he derived the standard Friedmann equation of FRW universe. The Gauss-Bonnet gravity and more general Lovelock gravity were also studied[12]. And emergent perspective of gravity has been further investigated[13, 14, 15].

In this paper, we are interested in how to obtain the dynamic equation without the cosmological constant. In other words, what may be the origin of the “dark energy”. The entropic force caused by the surface term can lead to a negative pressure which causes the expansion of the universe in the entropic force scenario. Hence we suggest that the additional pressure is added to the matter which is taken as ideal fluid since the effects of the entropic force. Furthermore, we obtain the number of modified degrees of freedom in the bulk and also get the corresponding dynamic equation in FRW universe by the idea that space is emergent as cosmic time progresses. This is a dynamic equation which contains the quantum gravity effects. Then we obtain the modified continuity equation. In order to more clearly see the properties of the evolution of the universe, we obtain the solutions at the early universe and the present epoch without considering the quantum effects. We can explain the current accelerating expansion without the cosmological constant. It may cause inflation at the early universe based on the quantum gravitational effects. Therefore, the entropic force caused by the surface term may be explained as the “dark energy”. In the paper the units are chosen with \( c = \hbar = k_B = 1 \) and we take the current Hubble constant \( H_0 = 70 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \) when we calculate the age of the universe.

2. Dynamic equation of Friedmann universe based on entropic force and emergence of space

The effective action is used as a starting point in any fundamental theories based on the viewpoint of modern physics. So does Einstein gravity theory. In
general, an extra surface term is added to the EH action in order to obtain the Einstein equation. But the surface term is so important that we can't cancel it as we point out in introduction.

In this paper, we use the EH action as our fundamental basis, this action is

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa} R + L_m \right), \]  

(1)

where \( \kappa = 8\pi G \). Varying this action with respect to the metric, we get

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} + J_{\mu\nu}. \]  

(2)

Generally, the last term is described by a delta function because of the locality of the theory. However, it should be a global effect from the holographic viewpoint [9]. In our paper, the global effect is described by the entropic force.

We consider the horizon of the universe. The horizon can be described by thermodynamics, and the thermodynamics of horizon can be described by the entropy. The corrected entropy with the quantum effects of the horizon is taken by [16, 17]

\[ S = \frac{A}{4L_P^2} + \alpha \ln \frac{A}{4L_P^2}, \]  

(3)

where \( A \) is the area of the horizon and \( \alpha \) is a parameter and \( L_P \) is the Planck length. The entropic force is given by [7]

\[ F_e = -\frac{dE}{dr} = -T \frac{dS}{dr}. \]  

(4)

Hence, we can obtain the corresponding entropic pressure by combining Eq.(3) with Eq.(4), it is

\[ P_e = \frac{F_e}{A} = -\frac{T}{2L_P^2} - \frac{\alpha T}{2\pi r^3}. \]  

(5)

In order to understand the emergence of space, a specific version of holographic principle was suggested by Padmanabhan[11]. He thought that our universe is being driven towards holographic equipartition (the number \( N_{\text{sur}} \) of degrees of freedom on the surface equals to the number \( N_{\text{bulk}} \) of degrees of freedom in the volume) and suggested which the law of evolution of the universe is given by

\[ \frac{dV}{dt} = L_p^2(N_{\text{sur}} - N_{\text{bulk}}). \]  

(6)

The effective number \( N_{\text{bulk}} \) of degrees of freedom in the volume is determined by the equipartition law of energy, so \( N_{\text{bulk}} = |E|/(1/2)T \). We take the ideal fluid as the matter of the universe, \( \rho \) and \( p \) represent respectively the energy density and the pressure of the ideal fluid. Further, we suggest that the additional pressure is added to the ideal fluid since the effects of the entropic force, so we can take the Komar energy \( E = |\rho + 3(p + P_e)|V \) contained inside the volume \( V = \frac{4\pi r^3}{3} \).
By taking the Hubble radius \( r = 1/H \) as the radius of the horizon and \( T = H/2\pi \) as the temperature of the horizon, we can obtain

\[
P_e = -\frac{H^2}{4\pi L_p^2} - \frac{\alpha H^4}{4\pi^2}.
\] (7)

Further, we can obtain

\[
N_{\text{bulk}} = -\frac{(4\pi)^2}{3H^4}(\rho + 3p) + \frac{4\pi}{H^2 L_p^2} + 4\alpha.
\] (8)

The number of degrees of freedom on the spherical surface is given by \[13\]

\[
N_{\text{sur}} = 4S,
\]

where \( S \) is the entropy of the horizon. Therefore, the number of the degrees of freedom on the surface is

\[
N_{\text{sur}} = \frac{4\pi}{H^2 L_p^2} + 4\alpha \ln \frac{\pi}{H^2 L_p^2}.
\] (9)

If the universe is holographic, it has \( N_{\text{sur}} = N_{\text{bulk}} \), then we can obtain

\[
\frac{3\alpha H^4}{4\pi^2}(1 - \ln \frac{\pi}{H^2 L_p^2}) = \rho + 3p.
\] (10)

From the above form, we can know that the energy density and the pressure respectively tend to zero when time tends to infinite, it is consistent with our physical picture.

Since our universe is asymptotically holographic equipartition, by using the law of evolution of the universe Eq.(6), we can obtain

\[
\frac{\dot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L_p^2}{3H^2}(\rho + 3p) + H^2 + \frac{\alpha H^4 L_p^2}{\pi}(1 - \ln \frac{\pi}{H^2 L_p^2}).
\] (11)

This equation describes globally the expansion of the universe, and it should be seen as the effective dynamic equation of FRW universe which contains the effective quantum gravitational effects. In \( \Lambda CDM \) model, the cosmological constant \( \Lambda \) satisfies \( \Lambda L_p^2 \approx 10^{-124} \) \[18\], that is \( \Lambda \approx 10^{-52}m^{-2} \). At present, the Hubble constant is \( H_0^2 \approx 10^{-53}m^{-2} \). Hence, the value of the cosmological constant is consistent with that of the dominated term \( H^2 \) at this epoch. The last term of Eq. (11) stands for the quantum gravitational effects, it decides the cosmic dynamic evolution at very early universe.

3. Properties of expansion of the universe

As usual, we think the continuity equation is kept, but its form is \( \dot{\rho} + 3H(\rho + p + P_e) = 0 \) because of the additional pressure caused by the entropic force. Substituting Eq.(7) to the continuity equation, we have

\[
\dot{\rho} + 3H(\rho + p) = \frac{3H^3}{4\pi L_p^2} + \frac{3\alpha H^5}{4\pi^2}.
\] (12)
In this section, in order to more clearly see some properties of expansion of the universe, we consider $\alpha = 0$, that is, we don’t consider the quantum effects. Then we obtain the dynamic equation of cosmic evolution and the continuity equation

$$\frac{\dot{a}}{a} = H^2 + \dot{H} = -\frac{4\pi L^2}{3}(\rho + 3p) + H^2 \quad (13)$$

and

$$\dot{\rho} + 3H(\rho + p) = \frac{3H^3}{4\pi L^2 \rho} \quad (14)$$

Using the equation of state of the ideal fluid

$$p = \omega \rho, \quad (15)$$

where $\omega$ is a non-negative parameter. By solving the above three equations, we can obtain the solutions $\dot{H} = -H^2$ and $\dot{H} = -\frac{1+3\omega}{2}H^2$.

### 3.1. Evolution of the universe under the solution $\dot{H} = -H^2$

As we have seen, the solution doesn’t have the parameter $\omega$, which implies that the rate of the expansion of the universe is same at the different epochs of the universe. We can obtain the solution of the scale factor

$$a(t) = t, \quad (16)$$

it varies linearly with time. The energy density is $\rho = \frac{3}{4\pi (1+3\omega)L^2}$, which is related with the parameter $\omega$. The age of the universe in this case is

$$t_0 = \frac{1}{H_0} = 9.8 \times 10^9 \text{ yr}. \quad (17)$$

The data of the supernova analyzed by the Supernova Cosmology Project indicate that the age of universe is $t_0 = 13.4^{+1.0}_{-1.3} \times 10^9 \text{ yr}$\cite{19}. The age obtained by us is greater than that of the standard model without the cosmological constant, but it is still less than the age which is obtained by the observation.

The general formula of the luminosity distance is given by\cite{20}

$$d_L = \frac{1+z}{H_0} \int_1^{1+z} \frac{dy}{H/H_0}, \quad (18)$$

where $z$ is the redshift factor defined by $z + 1 \equiv y = a_0/a$. Then we can obtain $\frac{H}{H_0} = \frac{a_0}{a} = y$ by Eq.(16). Substituting this result into Eq.(18), we can obtain the relation between luminosity distance and the redshift factor:

$$H_0 d_L = (1 + z) \ln(1 + z). \quad (19)$$

For $\Lambda CDM$ model, the luminosity distance of the flat universe is given by\cite{21}. Its form is

$$H_0 d_L = (1 + z) \int_0^z dz'[(1 + z')^2(1 + \Omega_m z' - z'(2 + z')\Omega_\Lambda)^{-1/2}, \quad (20)$$
where $\Omega_m = \frac{\rho_m}{\rho_c}$ and $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$. It has been find that the description of the universe for $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ is consistent with the data obtained by WMAP.

The deceleration parameter is defined by

$$q \equiv -\frac{\ddot{a}}{a^2}.$$  (21)

Substituting Eq. (16) into Eq. (21), we obtain the deceleration parameter $q = 0$.

### 3.2. Evolution of the universe under the solution $\dot{H} = -\frac{1 + 3\omega}{2}H^2$

In this case, we see the solution is related with the parameter $\omega$, it implies that the rate of the expansion of the universe is different at the different epoches of the universe. We will discuss separately the results at the early time and the late time of the universe.

(i). At the early time of the universe, the radiation dominates the universe at high energy scales, the parameter $\omega = 1/3$. Hence the evolution law of the universe is determined by $\dot{H} = -H^2$. We can get the solution $H(t) = \frac{1}{t}$, thus $a(t) = t$ and the energy density is $\rho = \frac{3}{8\pi G t^2}$.

(ii). At the late time of the universe, the matter dominates the universe, the parameter $\omega = 0$. Hence the evolution law of the universe is determined by $\dot{H} = -\frac{1}{2}H^2$. We can get the solution $H(t) = \frac{2}{t}$, so we have $a(t) = t^2$.  (22)
it varies with the square of time. The energy density is \( \rho = \frac{3}{2\pi L^2 z^2} \). The age of universe \( t_0 \) satisfies

\[
9.8 \times 10^9 < t_0 < 19.6 \times 10^9 \text{ yr},
\]

it is compatible with the observational result. Using the Eq.(18), we can obtain the relation between luminosity distance and the redshift factor

\[
H_0 d_L = z.
\]

By substituting Eq.(22) into Eq.(21), we obtain the deceleration parameter \( q = -\frac{1}{2} \). It shows that our universe is in accelerating expansion even if the quantum correction isn’t considered. The result is consistent with our observations.

### 3.3. Remarks of expansion of the universe

In this section, we analyze the solutions of the expansion of the universe without considering the quantum effects in our model. We obtain the relation between the luminosity distance and the redshift factor, and the age of the universe and the deceleration parameter. From the figures about the relationship between the luminosity distance and the redshift factor, we find that the solution \( \dot{H} = -\frac{1}{2} H^2 \) has a better description of the astronomical observational data. But the effect of accelerating expansion seems to feeble even it may be decelerating at the late-time universe and the age of the universe is less than astro-observation under this case. On the other hand, the relationship between the luminosity
distance and the redshift factor have some deviation from the results of $\Lambda CDM$ model, but the effect of accelerating expansion is evident under the solution $\dot{H} = -\frac{1}{2} H^2$.

In order to obtain the correct age of the universe, the law of the evolution of the universe has to have a transformation from $\dot{H} = -H^2$ to $\dot{H} = -\frac{1}{2} H^2$ at the epoch dominated by the matter. Thus we find that the transformation of the law of the evolution occurs at $t = 6.2 \times 10^9 \text{yr}$ if we take the age of the universe $t_0 = 13.4 \times 10^9 \text{yr}$. That is, the universe has an evident accelerating expansion effect from then on. This is compatible with the model of “big bang”, which says the universe started accelerating expansion several billion years ago. If it doesn’t have a transformation at that time, then it is hard to explain why we have two solutions at the late time in our model and we can obtain the correct age! Is this just an algebraic accident which doesn’t have a physical explanation?

As we know, it maybe cause the phase transition in the thermodynamic description. Our description is based on thermodynamics, hence it maybe cause the phase transition in the evolution of the universe at the epoch dominated by the matter. Does the phase transition cause that the solution $\dot{H} = -H^2$ transforms to the solution $\dot{H} = -\frac{1}{2} H^2$ at some time of the epoch dominated by the matter? If it is, what causes the phase transition? These similar questions may be interesting, but they are beyond this paper, so we will not discuss them in this paper.

However, it should be noted that we just discuss the classical evolution of the universe in the above description. That is, we do not consider the correction term in this section. If the quantum effects are considered, it may have an inflation at very early time of the universe because the quantum correction term dominates the evolution of the universe at that epoch (the letter[10] analyzed the similar equations at the early time in the different model). If it is, these puzzles of the horizon and flatness can be explained as long as we choose an appropriate parameter $\alpha$.

Although we don’t introduce the cosmological constant, it can also cause the current accelerating expansion of the universe which is well in line with the astro-observation. It may also cause inflation since the quantum correction term dominates the dynamic equation at very early universe. Therefore, the entropic force caused by the surface term may be the origin of the “dark energy”. In a word, from the perspective in which space is emergent and the entropy force causes the cosmic expansion may be a good attempt to discuss the problems of cosmology.

4. Conclusion

In this paper, our starting point is based on the EH action, it has a surface term by varying this action with respect to the metric. The surface term should cause a global effect from the holographic viewpoint [9]. The global effect is described
by the entropic force. We suggest that an additional pressure is added to the matter which is taken as ideal fluid since the effects of the entropic force in our model. Thus it leads to a change of the Komar energy of the matter. Further, we obtain the number of modified degrees of freedom in the bulk. Based on the idea that space is emergent as cosmic time progresses proposed by Padmanabhan, we obtain the corresponding dynamic equation in FRW universe. The dynamic equation describes globally the expansion of the universe. Moreover, it contains the quantum correction term.

We analyze the properties of the expansion of the universe. By using the modified continuity equation and the equation of state of the ideal fluid, we obtain these solutions of the expansion of the universe without the quantum correction. By analyzing the solution of the epoch dominated by the matter and the age of the universe, we find that it has a transformation from the solution $\dot{H} = -H^2$ to the solution $\dot{H} = -\frac{1}{2}H^2$ at $t = 6.2 \times 10^9$ yr. Thus we have a good description about the age of the universe and the current accelerating expanding expansion effect. The curve of the relation between the luminosity distance and the redshift factor in our model has some deviation from the result of $\Lambda CDM$ model at high redshift region, but it is understandable since the quantum correction term isn’t considered in our above discussions. We also find that the quantum correction term plays a leading role in the evolution of the universe at very early time, it may cause inflation at that epoch. If it is, these puzzles of flatness and horizon can be solved. Furthermore, we think that the entropic force caused by the surface term may be the origin of the “dark energy”. In the end of this paper, it should be stressed that the solutions we obtain don’t contain the quantum gravity effects. The properties of cosmic evolution with the quantum gravity effects are worthy further to investigate. Our results are useful for further understanding of the “dark energy” and the properties of the evolution of the universe.

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