Is Poker a Skill Game? New Insights from Statistical Physics

Marco Alberto Javarone\textsuperscript{1,2}

\textsuperscript{1}Department of Mathematics and Computer Science, University of Cagliari, Cagliari (Italy)
\textsuperscript{2}DUMAS - Department of Humanities and Social Sciences, University of Sassari, Sassari (Italy)

(Dated: March 31, 2015)

During last years poker has gained a lot of prestige in several countries and, beyond to be one of the most famous card games, it represents a modern challenge for scientists belonging to different communities, spanning from artificial intelligence to physics and from psychology to mathematics. Unlike games like chess, the task of classifying the nature of poker (i.e., as 'skill game' or gambling) seems really hard and it also constitutes a current problem, whose solution has several implications. In general, gambling offers equal winning probabilities both to rational players (i.e., those that use a strategy) and to irrational ones (i.e., those without a strategy). Therefore, in order to uncover the nature of poker, a viable way is comparing performances of rational versus irrational players during a series of challenges. Recently, a work on this topic revealed that rationality is a fundamental ingredient to succeed in poker tournaments. In this study we analyze a simple model of poker challenges by a statistical physics approach, with the aim to uncover the nature of this game. As main result we found that, under particular conditions, few irrational players can turn poker into gambling. Therefore, although rationality is a key ingredient to succeed in poker, also the format of challenges has an important role in these dynamics, as it can strongly influence the underlying nature of the game. The importance of our results lies on related implications, as for instance in identifying the limits poker can be considered as a 'skill game' and, as a consequence, which kind of format must be chosen to devise algorithms able to face humans.

Nowadays, social dynamics and modeling human behavior represent challenging topics for scientists belonging to different communities, e.g., artificial intelligence, physics, mathematics and social psychology. Notably, the modern field of sociophysics\textsuperscript{11} aims to investigate social and economic phenomena by a strongly interdisciplinary approach, mainly based on analytical and computational tools, coming from the framework of the statistical physics\textsuperscript{2-4}. Moreover, several social issues as opinion formation, information spreading and social behaviors, can be represented and studied by using agent-based models\textsuperscript{5,6} often combined with the theory of network\textsuperscript{7-10}. In this work, we analyze poker games (hereinafter simply poker) by the framework of statistical physics (see also\textsuperscript{11}). Poker represents one of the major challenges for artificial intelligence and mathematics\textsuperscript{12-15}, and it is a topic of interest also for psychologists, economists and sociologists\textsuperscript{16} due to its wide diffusion over several countries. One of the most controversial aspects of poker, caused by the utilization of money, is related to its nature, i.e., 'skill game' or gambling. The related answer has not yet been solved\textsuperscript{17}, although it has a long list of social implications\textsuperscript{18,19}. Furthermore, all efforts made to define algorithms and strategies in the context of artificial intelligence are obviously based on the confident belief that computing skills are relevant to succeed in poker. Therefore, our investigations aim to shed some light on the nature of poker, by an approach based on statistical physics. There are several variants of poker, e.g., Texas Hold’em, Omaha, Draw, etc., each having its own rules. However, they all follow a similar logic: a number of cards is distributed among players, who in turn decide if to play or not, evaluating the possible combinations of their cards (called hand) with those on the table. Since players cannot see the cards of their opponents, when they have to take an action (e.g., to bet money), poker is an imperfect information game, unlike others like chess where all players get all the system information simultaneously\textsuperscript{20}. It is worth to observe that the utilization of money makes the challenge meaningful, just because the underlying dynamics of poker are constituted by a series of bets. Hence, without money players would have no reasons to fold their hands. In general, there are two main formats for playing poker, i.e., tournament and ‘cash game’. The former entails players pay an entry fee that goes into the prize pool plus a fee to play, receiving an amount of chips. Then, top players share the prize pool. Instead, playing poker in the ‘cash game’ format entails to use real money during the challenge. Therefore, in this last case, players can play until they have money and, although there are no entry fees to pay, a fraction of each pot is taxed (i.e., a small ‘rake’ is applied). In the work\textsuperscript{21}, the author defined a model for representing poker challenges, focusing his attention on tournaments, in order to study the role of rationality. Remarkably, his main result was that the nature of poker does not depend on its rules but on the players’ behavior, then identifying rationality as a key ingredient to

\textsuperscript{*}marcojavarone@gmail.com
succeed. Therefore, since the human behavior has such important role in poker, we perform further investigations on this direction, but considering the ‘cash game’ format.

Let us now briefly recall the model described in [21] and summarize the main achievements. This model represents ‘heads-up’ challenges, i.e., challenges that involve two players at a time. Players can be rational or irrational. The former move (e.g., bet and fold) by using the Sklansky table [22] as reference, whereas the latter play randomly. It is worth to note that, for the sake of simplicity, each round is composed of only one betting phase (instead, in real scenarios, usually there are more phases [22]). Numerical simulations showed that, under these conditions, rational players win a challenge against irrational players with probability \( P_r \approx 0.8 \). Hence, a rational player is supposed to win about three consecutive coin flip with winning probabilities \( x \) against an irrational one. As a consequence, since ‘heads-up’ tournaments have a tree-like structure, the final winner is a rational player when the number of total participants \( N \), regardless of their behavior, is \( N \leq 2^W \). After analyzing poker tournaments by different conditions (e.g., also allowing rationals to change behavior), the author [21] states that the nature of poker depends on the players’ behavior, but not on its rules.

Here, we focus our attention on the ‘cash game’ format. It is important observing that now each ‘heads-up’ challenge can last from 1 to \( n \) rounds, where the value of \( n \) depends on the amount of money opponents have available. Moreover, even after a single round one player can leave the table (i.e., ending the challenge) with her/his remaining money. In order to study this scenario, we consider a population of agents that interact by the dynamics of the classical voter model [23]. In so doing, each agent has a state that represents its behavior (i.e., an interaction corresponds to a full challenge). Then, regardless of their behavior, is assessed. After each interaction the state, characterized by the presence of a prevailing state \( \sigma = +1 \) or \( \sigma = -1 \). Figure 1 shows the average magnetization over time, achieved in the two considered cases (i.e., rational and irrational ones). Notably, both varying the density \( \rho_r \) and considering the two cases, the agent population always converges to the same state. At this point, it is worth to investigate the final population state \( \Sigma \), in order to know whether, after challenges, all agents play rationally (i.e., \( \Sigma = +1 \)) or not (i.e., \( \Sigma = -1 \)). Therefore, we analyze the amount of rational agents over time \( S(t) \), for different initial densities \( \rho_r \) — see figure 2. Remarkably, since values of \( S(t) \) are averaged over different simulation runs, and by knowing that at each attempt the population reaches an ordered phase, we may derive the probability that rational agents prevail on irrational ones on varying \( \rho_r \). Notably, these winning probabilities (indicated as \( P^r \)) have been computed for different values of \( \rho_r \) from 0 to 1, focusing on small values close to 0 (e.g., 0.0033, 0.01, 0.05) for the case a and on high values close to 1 (e.g., 0.97, 0.98, 0.99) for the case b. The main reason to explore in particular low \( \rho_r \) for the case a and high \( \rho_r \) for the case b.
FIG. 1. Evolution of the magnetization on varying the initial density of rational agents $\rho_r$. a Results achieved by implementing the case $a$: agents play full challenges. b Results achieved by implementing the case $b$: agents play single rounds. Results have been averaged over 100 different simulation runs.

FIG. 2. Amount of rational agents over time. a Results achieved by implementing the case $a$: agents play full challenges. b Results achieved by implementing the case $b$: agents play single rounds.

for the case $b$ lies in the fact that, observing figure 2, we found that rational agents easily prevail playing full challenges (i.e., $a$) against irrational agents that, in turn, prevail many times playing single rounds (i.e., $b$). A fast inspection of all achieved results allows to appreciate a strong difference between challenges performed in the two cases. Notably, for low values of $\rho_r$, in the case $a$, the magnetization initially decreases, and later it increases up to 1. This phenomenon is caused by the transition of several irrational agents to the rational behavior, as confirmed in panel $a$ of figure 2. On the other hand, when agents play single rounds, the value of $M$ only increases up to 1, revealing that there are no sensible transitions between the two states, when $\rho_r$ has both low and high initial values (see panel $a$ of figure 1).

Furthermore, even the summation $S(t)$ strongly differs in the two cases: when playing full challenges the population seems to reach always the same ordered ‘rational’ phase; instead, when playing single rounds, sometimes even few irrational agents can turn into irrational the whole population. Eventually, figure 3 further highlights the already detected differences between cases $a$ and $b$. Notably, we computed fitness functions for both cases, identifying a simple step function for $a$, and the function $P_{w}^{\rho_r}(\rho_r) = \rho_r^{3/2}$ for $b$. On one hand, it is interesting to note that in full challenges even the presence of only one rational agent can entails the transition to an ordered ‘rational’ phase. On the other hand,
FIG. 3. Winning probability of rational agents ($P_{wr}$) on varying $\rho_r$. In the legend, $F$ refers to the case $a$ and $S$ to the case $b$. The two black lines (i.e., the dotted and the continuous one) refer to the computed fitting functions.

when playing single rounds, rational agents prevail with a probability greater than 50% only if $\rho_r > 0.7$.

All these results confirm that classifying the nature of poker is a tricky task, as a lot of conditions must be considered in real scenarios. In particular, although in tournaments it seems rationality be a key ingredient to succeed [23], in the cash game format it may be sometimes appropriate to associate poker to gambling. Moreover, considering all risks of poker in the cash game format (see [23]), we think both players and scientists working on poker be aware of our results. A further important point to discuss, before to conclude, is related to the validity of our model in real scenarios. Notably, although it would be extremely interesting to compare outcomes of the proposed model with real data, this is not possible as no similar datasets exist. Anyway it may be possible to evaluate if a player is adopting mainly a random strategy or a rational one as, according to the rules [22], often players have to show their hands after the round to discover who is the winner. Finally, we deem the importance of our results lies on related implications. Notably, we found that not only the player’s behavior but also the format of poker must be considered when classifying the nature of this game, showing that there are well defined limits poker can be considered as a ‘skill game’.

ACKNOWLEDGMENTS

MAJ would like to thank Fondazione Banco di Sardegna for supporting his work.

[1] Galam, S.: Sociophysics: a personal testimony. *Physica A: Statistical Mechanics and its Applications* **336** 1-2 (2004)
[2] Castellano, C. and Fortunato, S. and Loreto, V.: Statistical physics of social dynamics. *Rev. Mod. Phys.* **81**-2 591–646 (2009)
[3] Barra, A., Contucci, P., Sandell, R., Vernia, C.: An analysis of a large dataset on immigrant integration in Spain. The Statistical Mechanics perspective on Social Action. *Scientific Reports* **4** 4174 (2014)
[4] Agliari, E., Barra, A., Galluzzi, A., Javarone, M.A., Pizzoferato, A., Tantari, D.: Emerging heterogeneities in Italian customs. arXiv:1503.00659 (2015)
[5] Biondo, A.E., Pluchino, A., Rapisarda, A.: Reducing financial avalanches by random investments *Physical Review E* **88-6** (2013)
[6] Biondo, A.E., Pluchino, A., Rapisarda, A.: The Beneficial Role of Random Strategies in Social and Financial Systems *Journal of Statistical Physics* **88-6** (2013)
[7] Javarone, M.A.: Social Influences in Opinion Dynamics: the Role of Conformity. *Physica A: Statistical Mechanics and its Applications* **414** 19–30 (2014)
[8] Tomassini, M., Pestelacci, E., Luthi, L.: Social Dilemmas and Cooperation in Complex Networks *International Journal of Modern Physics C* **18-7** (2007)
[9] Javarone, M.A., and Armano, G.: Perception of similarity: a model for social network dynamics. *J. Phys. A: Math. Theor.* **46** 455102 (2013)
[10] Nyczka, P., Sznajd-Weron, K.: Anticonformity or Independence? - Insights from Statistical Physics. *Journal of Statistical Physics* **151** 174–202 (2013)
[11] Sire, C.: Universal statistical properties of poker tournaments. *Journal of Statistical Mechanics: Theory and Experiment* P08013 (2007)
[12] Bowling, M., Burch, N., Johanson, M., Tammelin, O.: Heads-up limit holdem poker is solved. *Science* **347-6218** 145–149 (2015)
[13] Dahl, F.A.: A Reinforcement Learning Algorithm Applied to Simplified Two-Player Texas Holdem Poker. *Machine Learning: ECML 2001 - LNCS 2167* 85–96 (2001)
[14] Teofilo, L.F., Reis, L.P., Lopes Cardoso, H.: Computing card probabilities in Texas Hold’em. *Information Systems and Technologies (CISTI), 2013 8th Iberian Conference on* 1–6 (2013)
[15] Seale, D.A., Phelan, S.E.: Bluffing and betting behavior in a simplified poker game. *Journal of Behavioral Decision Making* **23**-4 335–352 (2010)
[16] Poker: A big deal. *Economist* (22 December 2007), p. 31.
[17] Hannum, R.C., Cabot, A.N.: Toward Legalization of Poker: The Skill vs. Chance Debate *UNLV Gam-
[18] Kelly, J.M., Dhar, Z., Verbiest, T.: Poker and the Law: Is It a Game of Skill or Chance and Legally Does It Matter?. *Gaming Law Review* 3-11 (2007)

[19] Cabot, A., Hannum, R.: Poker: Public Policy, Law, Mathematics, and the Future of an American Tradition. *TM Cooley L. Rev.* 22-443 (2005)

[20] Colman, A.M., Game Theory and Its Applications Digital Printing, 2008.

[21] Javarone, M.A.: Poker as a Skill Game: Rational versus Irrational Behaviors. *Journal of Statistical Mechanics* P03018 (2015)

[22] Sklansky, D., and Malmuth, M., Hold ‘em Poker for Advanced Players. *Two Plus Two Publications* (1999)

[23] Liggett T.M. Interacting Particle Systems. *Springer-Verlag, New York* (1985).

[24] Mobilia, M. and Redner, S.: Majority versus minority dynamics: Phase transition in an interacting two-state spin system. *Phys. Rev. E* 68-4 046106 (2003)

[25] Javarone, M.A.: Poker Cash Game: a Thermodynamic Description. *arxiv:1503.01418* (2015)