On computer modelling of operations, geometric characteristics and methods of pulsed optical tomography of nuclear fuel microobjects

E P Domoratskiy and T N Baybikova
School of Computer Engineering, National Research University Higher School of Economics, Myasnitskaya str., 20, Moscow, 101000, Russian Federation

E-mail: tbaibicova@hse.ru

Abstract. The paper refers to the area of morphological processing of projection images and its goal is to design some computer models of basic operations, geometric properties and methods of pulsed optical tomography which provide a high-speed production operational control and sorting of each microobjet (MO) of nuclear fuel in their flow according to their size and shape. Based on the proposed computer models, a precision laser method of industrial differential control and quality control of an MO flow of nuclear fuel was developed and experimentally tested. The method uses the statistical reconstruction of the size \( D \) and shape \( K \) of each microobjet and takes into account the overall dimensions of the outlines of three mutually orthogonal two-dimensional pulsed discrete projection images of a microobject. The processing speed of this method is 100 MO/s in the diameter range of 400–1500 \( \mu \)m. The relative error of an MO diameter control is no more than 0.25\% (at the reliability of \( P_D = 0.7 \) and \( K = 1.3 \) relative units), and the relative error of the non-sphericity coefficient control lies in the range of 2.3\% (\( P_K = 0.7 \) and \( K = 1.3 \)) to 0.6\% (\( P_K = 0.96 \) and \( K = 1.05 \)).

1. Introduction
One of the most promising and innovative ways of modern nuclear power development is the creation and application of High Temperature Gas-cooled Reactors (HTGR). The HTGR concept is being implemented in such countries as the USA (GA, ORNL), Japan (Fuji Electric), France (Framatom), Russia (National Research Center «Kurchatov Institute»), as well as in Germany, the UK, China, South Korea and other countries [1, 2]. There are two main HTGR reactor designs in the international development arena: pebble bed design and prismatic block design. The pebble-bed reactors use spherical fuel elements. The prismatic block reactors use cylindrical “fuel compacts”. The fuel compacts are massively loaded directly into the reactor active zone. The main part of fuel elements is granulated nuclear micro fuel particles which represent convex multi-layered three-dimensional microobjects (MO) of irregular spherical form with a size from 400 to 1500 \( \mu \)m, consisting of a solid kernel with a diameter of 500 \( \mu \)m and 4-5 layers of coatings with thickness from 50 to 90 \( \mu \)m which regulate the leakage of fission products. The technology of granulated nuclear materials (microobjects) manufacturing provides their mass (flow) production and consists of successive interrelated operations (steps): the formation of a dispersed jet of drops of original solvent based on UO\(_2\) (Sol-gel process); the fabrication of solid kernels from drops of solvent; the sequential covering of kernels with one, two, three, four coatings; the fabrication of micro fuel particles (kernels with five coatings).
The most important quality characteristics of an MO of granulated nuclear fuel and fuel elements fabricated on their basis which determine their reliability, physical, chemical and consumer properties are the geometric properties of their size and shape. To ensure mass (flowing) production and quality control of nuclear fuel in the on-line mode, the geometric control method must meet a unique set of requirements and be simultaneously automatic, contactless, high-speed, dynamic, differential, multidimensional, precision and reliable, as well as provide classification and sorting of moving (flying) MOs according to their shape and size in real time.

2. Problem statement
The paper relates to the area of experimental physics and consists in the development of some computer models of operations (procedures), geometric properties and methods of pulsed optical tomography of moving three-dimensional objects which provide production operational geometric control as well as quality control of nuclear fuel MO flow [3].

3. Description of modelling methods
In general terms, the controlled geometric parameter (CP) of a three-dimensional object is characterized by an objective function (1) with a set of image properties necessary and sufficient for its adequate (with a given accuracy, reliability and processing speed) description [4]:

\[ CP = f \left( Mo, Pi, Be, Md, Ai, Ni, Mr, T \right), \]  

(1)

where \( Mo \) is a method for obtaining an image; \( Pi \) are image parameters; \( Be \) are the basic elements (characteristics) of an image; \( Md \) is a method of determining the basic elements of an image; \( Ai \) is the viewing angle of an image (projection) relative to the object (Euler angles \( \alpha, \beta, \gamma \)); \( Ni \) is the number of images (projections); \( Mr \) is the method of reconstruction (recovery) of an image phantom with the usage of image features; \( T \) is the total time (duration) of CP control and is determined as the sum \( T = ti + tp + tr \), where \( ti \) is the time of image obtaining; \( tp \) is the time of image processing; \( tr \) is the reconstruction time.

In accordance with (1), the development of the method of geometric control of an MO involves the development of the following computer models of operations [4], which are understandable from the scheme (figure 1):

1. approximation (image) of a three-dimensional MO and spatial geometric properties of its size and shape;
2. spatial orientation of each MO in the flow;
3. determination of the optimal view, representative number and optimal spatial orientation (angles) of projection images of an MO;
4. method of producing (generating) the pulsed projection images of an MO;
5. determination of the numerical values of the optimal basic geometric properties of each projection image of an MO;
6. reconstruction methods of the size and shape of a three-dimensional MO on the basis of its discrete projection images.

Let us consider the operations of the modelling methods in more detail.

1. The ellipsoid of the General form is chosen as a mathematical model approximating the geometric properties of an MO (its three-dimensional image – phantom). The canonical equation of this ellipsoid is as follows:

\[ \left( x'^2/A_1^2 \right) + \left( y'^2/A_2^2 \right) + \left( z'^2/A_3^2 \right) = 1, \]  

(2)

where \( A_1, A_2, A_3 \) are the semi – axes of the ellipsoid; \( x', y', z' \) belong to the original coordinate system of the ellipsoid.
The geometric (overall) dimensions of an MO are determined by the numerical values of the mutually orthogonal axes of the ellipsoid $2A_1, 2A_2, 2A_3$, its average diameter ($D$) and shape factor ($K$) are determined respectively from the relations (3) [5]:

$$D = \sum_{i=1}^{3} 2A_i / 3; \quad K = \max \{A_1, A_2, A_3\} / \min \{A_1, A_2, A_3\}. \quad (3)$$

2. The model of spatial orientation of an MO in the flux is determined by the rotation matrix of the ellipsoid $R$ in a three-dimensional space to the Euler angles $\alpha, \beta, \gamma$:

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ -\sin \gamma & \cos \gamma & 0 \end{bmatrix} \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos \alpha \cdot \cos \beta & \sin \alpha \cdot \cos \beta & -\cos \alpha \cdot \sin \beta \\ -\sin \alpha \cdot \cos \beta & \cos \alpha \cdot \sin \beta & \sin \alpha \cdot \sin \beta \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos \alpha \cdot \cos \beta & \sin \alpha \cdot \cos \beta & -\cos \alpha \cdot \sin \beta \\ -\sin \alpha \cdot \cos \beta & \cos \alpha \cdot \sin \beta & \sin \alpha \cdot \sin \beta \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$
In this case, the size and shape of an MO model (ellipsoid) are determined deterministically, and its orientation (Euler angles) can be specified both deterministically and stochastically by even distribution.

3. The selection of the optimal type, number and angles of projection images of an MO, as well as their basic properties is determined on the basis of the optimal (best) combination of characteristics of accuracy, reliability and processing speed of the control method and is based on a numerical comparative analysis of their informativeness with the usage of simulation statistical modelling by the maximum entropy method. For this method, the optimal type of primary information is three discrete two-dimensional projection images. At the same time, their optimal spatial orientation is mutual orthogonality [6] (figure 1).

4. Pulsed projection images of an MO moving (flying) in the flow are formed by their exposure in parallel beams from three pulsed sources of optical radiation and their simultaneous synchronous registration by means of three corresponding optoelectronic sensors of discrete pulsed micro images [5]. The target exposure time of the video detector $T_e$ when obtaining the pulsed projection image of an MO is determined by the condition of the absence of the blurring of the image frame obtained from the moving MO in accordance with the equation:

$$T_e \leq R \cos \phi / (v + 0.5\omega D) K^*,$$

where $R$ is the resolution of a video detector on an image; $\phi$ is the angle between the trajectory of the movement of the MO and the plane of the video detector target; $v$ is the speed of the MO mass centre; $\omega$ is the maximum angular velocity of its rotation; $D$ is the MO diameter; $K^*$ is the image scale.

To reduce the information redundancy and improve the control speed, the spatial differentiation of pulsed electronic discrete images of an MO (that is, the creation of their outlines) is fulfilled. Thus, the computer model of each discrete outline is an ellipse (equation 6), the coordinates of the points of which are rounded to the nearest integers, which is adequate to the process of discretization of the primary information obtained from a real MO [5].

$$\sum_{i=1}^{3} A_i^2 R_{i1} x_i^2 - 2 \sum_{i=1}^{3} A_i^2 R_{i1} y_i x_i + \sum_{i=1}^{3} A_i^2 R_{i1}^2 y_i^2 = A_1^2 A_2^2 A_3^2 \sum_{i=1}^{3} \left(R_{i1}^2 / A_i^2\right).$$

5. The optimal basic properties of projection images are chosen on the basis of numerical analysis of their informativeness by the method of maximum entropy. These properties include: the image length of the one-dimensional projection of an MO to the selected direction $h_i$; the area of the image of a two-dimensional projection $s_i$; the maximum $l_{max}$, and minimum $l_{min}$ linear dimensions of an image of the two-dimensional projection of an MO ($i$ is a projection number). The numerical values of these basic properties are determined by the relations (7) [5]:
\[
\begin{align*}
    h_i &= 2 \left[ \sum_{j=1}^{3} A_j^2 R_{ji}^2 \right]^{\gamma/2}, \\
    s_i &= \pi A_i A_2 A_3 \left[ \sum_{j=1}^{3} \left( R_{ji}^2 / A_j^2 \right) \right]^{\gamma/2}, \\
    l_{\text{max}} &= 2 \left\{ I_i + \left[ I_i^2 - 4 A_i^2 A_2^2 A_3^2 \sum_{j=1}^{3} \left( R_{ji}^2 / A_j^2 \right) \right]^{\gamma/2} \right\}^{\gamma/2}, \\
    l_{\text{min}} &= 2 \left\{ I_i - \left[ I_i^2 - 4 A_i^2 A_2^2 A_3^2 \sum_{j=1}^{3} \left( R_{ji}^2 / A_j^2 \right) \right]^{\gamma/2} \right\}^{\gamma/2},
\end{align*}
\]

where: \( I_i = \sum_{j=1}^{3} A_j^2 \left( 1 - R_{ji}^2 \right) \); \( j, i \) are projection numbers and numbers of current orientation (angle);

6.1 The reconstruction method of a three-dimensional MO image based on the usage of basic characteristics \( h_i \) and \( s_i \) is defined by the simultaneous equations (8) [7]:

\[
\begin{align*}
    A_i^2 + A_2^2 + A_3^2 &= \frac{3}{4} h_i^2, \\
    A_i^2 A_2^2 + A_i^2 A_3^2 + A_2^2 A_3^2 &= \frac{3}{\pi^2} s_i^2, \\
    A_i^2 A_2^2 A_3^2 &= \left[ \sum_{j=1}^{3} h_j^2 s_j^2 / 4\pi^2 \right] - \left( h_i^2 h_j^2 h_k^2 / 32 \right) \left\{ 1 - \prod_{i=1}^{3} \left[ 1 - \left( 16/\pi^2 \right) h_i^2 s_i^2 / h_j^2 h_k^2 h_l^2 \right] \right\}^{\gamma/2}. 
\end{align*}
\]

6.2 The reconstruction method of a three-dimensional MO image based on the usage of basic characteristics \( l_{\text{max}}, l_{\text{min}} \) is defined by the simultaneous equations (9) [8]:

\[
\begin{align*}
    A_i^2 + A_2^2 + A_3^2 &= \sum_{i=1}^{3} m_i, \\
    A_i^2 A_2^2 + A_i^2 A_3^2 + A_2^2 A_3^2 &= \sum_{i=1}^{3} c_i, \\
    A_i^2 A_2^2 A_3^2 &= \sum_{i=1}^{3} m_i c_i - 2 m_1 m_2 m_3 \left\{ 1 - \prod_{i=1}^{3} \left[ 1 - \frac{m_i c_i}{m_1 m_2 m_3} \right] \right\}^{\gamma/2},
\end{align*}
\]

where \( m_i = \frac{1}{8} \sum_{j=1}^{3} \left( l_{ij}^2 + l_{ij}^2 \right) - \left[ l_{ij}^2 + l_{ij}^2 \right] / 4 \); \( c_i = l_{ij}^2 l_{ij}^2 / 16 \).

When solving the simultaneous equations (8) and (9) (for example, by the trigonometric method) with regard to the semi-axes of an ellipsoid \( A_1, A_2, A_3 \) and considering that only positive values of the
roots make sense, we obtain the linear dimensions of the semi-axes of an ellipsoid which approximates an MO. In this case, the geometric properties of the size and shape of an MO are determined from the relations (3).

On the basis of the computer models proposed in this paper, a laser precision high-speed method of geometric optic-electronic differential contactless production and quality control is developed and experimentally tested. The method allows to realize these types of control in real-time of an MO flow of nuclear fuels and is based on dynamic spatial-temporal statistical few-viewed reconstruction of the size and shape of each MO on the basis of the maximum and minimum overall dimensions of the discrete outlines of three mutually orthogonal two-dimensional pulsed projection images of an MO [4] (figure 1).

When describing the size of each MO, the spatial geometric properties are its overall dimensions and the average projection diameter ($D$) of its approximating ellipsoid of the General form. When describing the shape of an MO, the spatial geometric property is the coefficient of non-sphericity ($K$), determined by the ratio of the maximum and minimum dimensions (axes) of the approximating ellipsoid.

The processing speed of this method is not less than 100 MO/s in the diameter range of 400–1500 µm for exposure time 0.8–1.0 µs. The relative error of an MO diameter control is no more than 0.25% (at the reliability of $P_D = 0.7$ and $K = 1.3$ relative units), and the relative error of the non-sphericity coefficient control lies in the range of 2.3% ($P_K = 0.7$ and $K = 1.3$ relative units) to 0.6% (with $P_K = 0.96$ and $K = 1.05$ relative units).

4. Conclusion
The direct and inverse problems of computational pulsed optical tomography are formulated and solved by the developing of some methods of computer simulation of operations (processes) of obtaining (registration), morphological analysis and space-time few-viewed dynamic reconstruction of a three-dimensional MO of nuclear fuel on the basis of the basic properties of pulsed discrete projection images.

The developed methods of dynamic reconstruction and methods of production geometric control on their basis have an optimal combination of high characteristics of accuracy, reliability and processing speed, fully meet the necessary requirements and minimize their hardware and software implementation and control time. Therefore, they can be used in the measurement and control of various spherical three-dimensional objects of irregular shape (ball fuel elements, elements of thermonuclear fuel, aerosols of diamond granules, etc.)

References
[1] IAEA 2010 *High Temperature Gas Cooled Reactor Fuels and Materials* IAEA-TECDOC-1645 Vienna (IAEA Publ.) 182
[2] Grebennik V I, Kukharkin N E and Ponomarev-Stepnoy N N 2008 *Vysokotemperaturnye gazookhazhladaye reaktory — innovatsionnoe napravlenie razvitiya atomnoy energetiki* Moscow (Energoatomizdat Publ.) 136
[3] Moder J J and Elmaghraby S E 1981 *Isledovanie operatsiy. Metodologicheskie osnovy i matematicheskie metody* Moscow(Mir Publ.) 1 716
[4] Domoratskiy E P and Baybikova T N 2017 *Proc. XIV Int. Sci. Conf. on Optical methods of flow investigation (Electronic Materials)* (Moscow:MPEI) 275-81
[5] Domoratskiy E P 2016 Synthesis algorithm of the geometrical characteristics of the projection images of three-dimensional objects *J. Information Technologies* 22 8 605–09
[6] Domoratskiy E P and Baybikova T N 2015 *Proc. II Int. Sci. Conf. on Regional issues of Earth remote sounding* (Krasnoyarsk: SibGAU) 139–42
[7] Domoratskiy E P 2017 Method of statistical few view reconstruction of geometrical characteristics of three-dimensional objects by their discrete projection images *J. Information Technologies* 23 3 184–87
[8] Baybikova T N and Domoratskiy E P 2017 Technique for Dynamically Reconstructing Shape and Size of Three-Dimensional Objects from their Projected Dimensions *J. Herald of the Bauman Moscow State Tech. Univ., Nat. Sci.* **5** 109–17