Structure and evolution of rotationally and tidally distorted stars

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ABSTRACT

\textbf{Aims.} This paper aims to study the configuration of two components caused by rotational and tidal distortions in the model of a binary system.

\textbf{Methods.} The potentials of the two distorted components can be approximated to 2nd-degree harmonics. Furthermore, both the accretion luminosity ($\sigma_r$) and the irradiative luminosity are included in stellar structure equations.

\textbf{Results.} The equilibrium structure of rotationally and tidally distorted star is exactly a triaxial ellipsoids. A formula describing the distortions to discuss the figures and dynamic parameters of synchronous orbiting satellites in the solar system. The equilibrium structure of the two components were treated as two non-symmetric rotational ellipsoids with two different semi-major axes ($a_1$ and $a_2$) by Huang (2004b). It is very important that Kippenhahn & Thomas (1970) introduced a method of simplifying the two-dimensional model with conservative rotation and allowed the structure equations for a one-dimensional star to incorporate the hydrostatic effect of rotation. This method has been adopted by Endal & Sofia (1976) and Meynet & Maeder (1997), who applied it to the case of shellular rotation law (Zahn 1992). In this case, the rotation rate takes the simplified form of $\Omega = \Omega(r)$. It was demonstrated that the shape of an isobar in the case of the shellular rotation law is identical to one of the equipotentials in the conservative case of Meynet & Maeder (1997).

\textbf{Key words.} star, stellar rotation, evolution

1. Introduction

In the conventional model of binary stars, there is no consideration of spin and tidal effects (Eggleton 1971, 1972, 1973; Hofmeister, Kippenhahn & Weigert, 1964; Kippenhahn et al. 1967; etc.); however, rotation and tide have been regarded as two important physical factors in recent years, so they need to be considered for a better understanding of the evolution of massive close binaries (e.g., Heger, Langer & Woosley 2000a; Meynet & Maeder 2000). The structure and evolution of rotating single stars has been studied by many investigators (Kippenhahn & Thomas 1970; Endal & Sofia 1976; Pinsonneau et al. 1989; Meynet & Maeder 1997; Langer 1998, 1999; Huang 2004a). However, it is also very important to study the evolution of rotating binary stars (Jackson 1970; Chan & Chau 1979; Langer 2003; Huang 2004b; Petrovic et al. 2005a,b; Yoon et al. 2006). The effect of spin on structure equations has been investigated (e.g., the present Eggleton’s stellar evolution code; Li et al. 2004a,b, 2005; Kühler 2002). They adopted the lowest-order approximate analysis in which two components were treated as spherical stars. In fact, with the joint effects of spin and tide, the structure of a star changes from spherically symmetric to non-spherically symmetric. Then, the stellar structure equations become three dimensional. Theory distinguishes two components in the tide, namely equilibrium tide (Zahn 1966) and the dynamical tide (Zahn 1975). Then, the dissipation mechanisms acting on those tides, namely the viscous friction for the equilibrium tide and the radiative damping for the dynamical tide, have been identified (Zahn 1966, 1975, 1977). The distortion throughout the outer regions of the two components is not small in short-period binary systems. The higher-order terms in the external gravitational field should not be ignored (Jackson 1970).

It is a very complex process to determine the equilibrium structure of the two components. Therefore, approximate methods have been widely adopted for studying these effects. In 1933, the theory of distorted polytropes was introduced by Chandrasekhar. Kopal (1972, 1974) developed the concept of Roche equipotential and of Roche coordinates to analyse the problem of rotationally and tidally distorted stars in a binary system. Burra (1989a, 1988) took advantage of the high-order perturbing potential to describe rotational and tidal deformations to discuss the figures and dynamic parameters of synchronously orbiting satellites in the solar system. The equilibrium structure of the two components were treated as two non-symmetric rotational ellipsoids with two different semi-major axes $a_1$ and $a_2$ ($a_1 > a_2$) by Huang (2004b). It is very important that Kippenhahn & Thomas (1970) introduced a method of simplifying the two-dimensional model with conservative rotation and allowed the structure equations for a one-dimensional star to incorporate the hydrostatic effect of rotation. This method has been adopted by Endal & Sofia (1976) and Meynet & Maeder (1997), who applied it to the case of shellular rotation law (Zahn 1992). In this case, the rotation rate takes the simplified form of $\Omega = \Omega(r)$. It was demonstrated that the shape of an isobar in the case of the shellular rotation law is identical to one of the equipotentials in the conservative case of Meynet & Maeder (1997).

At the semi-detached stage, both mass transfer between the components and luminosity change of a secondary exist due to the release of accretion energy which is correlated with the external potential of the two components. When the joint effect of rotation and tide are considered, the potential of the two components are different from those in non-rotational cases. Therefore, the luminosity due to the release of accretion energy, as well

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as irradiation energy, can significantly alter the structure and evolution of the secondary. In a rotating star, meridional circulation and shear turbulence exist, both of which can drive the transport of chemical elements. This effect is stronger and has already been studied by many scholars (Endal & Sofie 1978; Pinsonneault et al. 1989; Chaboyer & Zahn 1992; Zahn 1992; Meynet & Maeder 1997; Maeder 1997; Meader & Zahn 1998; Maeder & Meynet 2000; Denissenkov et al. 1999; Talon et al. 1997; Decressin et al. 2009). In this paper, the amplitude expression for the radial component of the meridional circulation velocity $U(r)$ considers the effect of tidal force, which may be important in a massive close binary system.

This paper is divided into four main sections. In section 2, the structure equations of rotating binary stars are presented. Material diffusion equations and boundary conditions are provided. Then, the accretion luminosity, including gravitational energy, heat energy, and radiation energy, is deduced. In section 3, the results of numerical calculation are described and discussed in detail. In section 4, conclusions are drawn.

2. Model for rotating binary stars

2.1. Potential of rotating binary stars

It is well known that the rotation of a component is synchronous with the orbital motion of a system thanks to a strong tidal effect. Such synchronous rotation also exists inside the component (Giuricin et al. 1984; Van Hamme & Wilson 1990); therefore, conventional theories usually assume that two components rotate synchronously and revolve in circular orbits (Kippenhahn & Weigert 1967; De Loore 1980; Huang & Taam 1990; Vanbeveren 1991; De Greve 1993). A coordinate system rotating with the orbital angular velocity of the stars is introduced. The mass centre of the primary is regarded as the origin, and it is presumed that the $z$-axis is perpendicular to the orbital plane, and the positive $x$-axis penetrates the mass centre of the secondary. The gravitational potential at any point $P(r, \theta, \varphi)$ of the surface of the primary can be approximately expressed as

$$\Psi = V + \frac{1}{3} \Omega^2 r^2 (1 - P_2 (\cos \theta)) + V_i,$$

where $V$ is the gravitational potential and given by Burša (1989a,1988),

$$V = \frac{GM_1}{r_p} + \frac{GM_2}{r_p} \left[ \frac{r_p}{r} f_2^{(0)} P_2^0 (\cos \theta) + \frac{r_p^3}{r^3} f_2^{(2)} P_2^2 (\cos \theta) \cos 2 \varphi \right].$$

Here, $V_i$ is the tidal potential (Burša 1989a)

$$V_i = \frac{GM_2}{D} \left[ \frac{r_p}{D} \right]^2 \left( \frac{1}{2} P_2^0 (\cos \theta) + \frac{1}{4} P_2^2 (\cos \theta) \cos 2 \varphi \right),$$

where it is assumed that the mean equatorial radius equals that of the equivalent sphere in the above equation for the convenience of calculation. Both $M_1$ and $M_2$ are the mass of the primary and the secondary, respectively, and $r_p$ represents each equivalent radius inside the star. $P_2^0 (\cos \theta)$ and $P_2^2 (\cos \theta)$ are the associated Legendre functions.

The orbital angular velocity of the system can be represented by

$$\Omega = \sqrt{G(M_1 + M_2)/D^3},$$

where $f_2^{(0)}$ and $f_2^{(2)}$ are dimensionless stokes parameters. If $M_1$ can generally be negligible compared to $M_2$, the stokes parameters can be expressed as (Burša 1989a,1988)

$$f_2^{(0)} = -\frac{1}{3} k_s + \frac{1}{2} k_t \left( \frac{r_p}{D} \right)^3 \frac{d}{d \ln P} = -J_2,$$

$$f_2^{(2)} = \frac{1}{4} k_s \left( \frac{r_p}{D} \right)^3,$$

where $k_s$ is the secular Love number, which is expressed as a measure of the body-yield-to-centrifugal deformation, and $k_t$ is an analogous parameter that is introduced to describe the secular tidal deformations. The response of the body to its centrifugal acceleration and to the tidal perturbing potential is different in the usual case. Therefore, the body-yield-to centrifugal deformation is not equal to the body-yield-to-tidal deformation. If the subject investigated is regarded as an ideal elastic body, the body-yield-to centrifugal deformation is equal to the body-yield-to-tidal deformation, $k_s = k_t$. In the ideal static equilibrium, $k_s = k_t = 1$ (Burša 1989a). We assume the ideal static equilibrium in this paper. $q$ is the mass ratio of the secondary to the primary $(q = \frac{M_2}{M_1})$. With Eqs. (2) and (3) being combined with Eq. (1), the potential of the primary can be obtained as

$$\Psi_P = \frac{GM_1}{D} \left( \frac{D}{r_1} + \frac{1}{2} \left( \frac{r_1}{r} \right)^2 \right) \left[ -J_2 (3 \cos^2 \theta - 1) \right.$$

$$\left. \left. + 6J_2 \sin^2 \theta \cos 2 \varphi \right] + \frac{1}{2} (1 + \sigma q) \left( \frac{r_1}{D} \right)^2 \sin^2 \theta \right.$$

$$\left. + \frac{1}{4} q \left( \frac{r_1}{D} \right)^2 (3 \sin^2 \theta (1 + \cos 2 \varphi) - 2) \right],$$

where $\sigma$ is the energy source per unit mass caused by mass overflow and irradiation. Because accretion luminosity is caused by the energy sources in the gainer’s outermost layer, there exists

$$\sigma = \Delta m = \Delta L_{acc},$$

where $\Delta m$ is the photosphere mass of the secondary. The surface temperature of the secondary may be approximated by the formula

$$L_2 + \Delta L_{acc} = 4\pi R^2 c T_{eff}^4,$$

where $L_2$ is the luminosity coming to the photosphere from the stellar interior, and $c$ is the Stefan-Boltzmann constant:

$$\frac{d \ln T}{d \ln P} = \left\{ \frac{\nabla_{\text{eq}} f_i / f_p}{\nabla_{\text{con}}} \right\}.$$
where \( < g_{eff} > \) and \( < g_{eff}^{-1} > \) are the mean values of effective gravity and its opposites over the isobar surface, and \( \nabla_R \) is the radiative temperature gradient. The factors \( f_P \) and \( f_T \) depend on the shape of the isobars.

2.3. Calculation of quantities \( f_P \) and \( f_T \)

2.3.1. Shape and gravitational acceleration of triaxial ellipsoid

To obtain the factors \( f_P \) and \( f_T \), the mean values \( < g_{eff} > \) and \( < g_{eff}^{-1} > \) over the isobar surface have to be calculated. Therefore, the shape of isobars must be given first. The functions for the semi-major axes \( a, b, \) and \( c \) to the radius of the equivalent sphere \( r_P \) can be obtained from Eq. (7) as

\[
\frac{4\pi abc}{3} = \frac{4\pi r_P^3}{3},
\]

\[
\frac{GM}{D} \left( \frac{1}{c} + \frac{1}{2} \frac{b^2}{c^2} (J_2 + 6J_2^{(2)}) \right) + \frac{1}{2}(1 + q)(\frac{b^2}{c^2} + q(\frac{b^2}{c^2})) = \frac{GM}{D} \left( \frac{1}{c} + \frac{1}{2} \frac{b^2}{c^2} (J_2 - 2J_2^{(2)} \right) - \frac{1}{2} q(\frac{b^2}{c^2}),
\]

\[
\frac{GM}{D} \left( \frac{1}{c} + \frac{1}{2} \frac{b^2}{c^2} (J_2 - 6J_2^{(2)}) + \frac{1}{2}(1 + q)(\frac{b^2}{c^2}) \right)
- \frac{1}{2} q(\frac{b^2}{c^2}) = \frac{GM}{D} \left( \frac{1}{c} + \frac{1}{2} \frac{b^2}{c^2} (J_2 - 2J_2^{(2)} \right) - \frac{1}{2} q(\frac{b^2}{c^2}).
\]

The left hand side of Eq. (17) corresponds to \( \theta = \frac{\pi}{2} \) and \( \varphi = 0 \), while the one of Eq. (18) corresponds to \( \theta = \frac{\pi}{2} \) and \( \varphi = \frac{\pi}{2} \). The three semi-major axes \( a, b, \) and \( c \) of a triaxial ellipsoid can be obtained numerically by solving (16), (17), and (18). From (7), the quantities \( g_r, g_\theta, \) and \( g_\phi \) at the surface of the two components take the forms of

\[
g_r = -\frac{\partial \Psi}{\partial r} = \frac{GM}{Dr} \left( \frac{D}{r} \right)^2 + \frac{1}{2} \frac{b^2}{c^2} (\frac{J_2}{r^2})^2 [\frac{1}{2}(1+q) \sin^2 \theta - \frac{1}{2} q \cos^2 \theta - 1],
\]

\[
g_\theta = -\frac{1}{r} \frac{\partial \Psi}{\partial \theta} = \frac{GM}{Dr} \left( \frac{D}{r} \right)^2 [3 J_2 + 6 J_2^{(2)} (2 \cos^2 \varphi - 1)] - 1, \]

\[
g_\phi = -\frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial \phi} = \frac{GM}{Dr} \left( \frac{D}{r} \right)^2 [3 J_2 + 6 J_2^{(2)} (2 \cos^2 \varphi - 1)] - \frac{1}{2} \frac{b^2}{c^2} \cos^2 \varphi \sin \theta \cos \theta ,
\]

\[
\sin \cos \varphi \sin \varphi.
\]

However, the total potential in the stellar interior (to first-order approximation) can be composed by four parts (Kopal 1959, 1960, 1974; Endal & Sofia 1976 and Landin 2009): \( \psi_s \), the spherical symmetric part of the gravitational potential: \( \psi_r \), the cylindrically symmetric potential due to rotation; \( \psi_i \) the non-symmetric potential due to tidal force, and \( \psi_d \), the non-symmetric part of the gravitational potential due to the distortion of the component considering the rotational and tidal effects. Therefore, the total potential at \( P(r, \theta, \varphi) \) is

\[
\Psi = \psi_s + \psi_i + \psi_r + \psi_d
\]

\[
= \frac{GM}{r} + \frac{1}{2} r^2 \sin^2 \theta \left[ \frac{GM_2}{r} [1 + \sum_{j=2}^{4} \frac{n_{ij}^2}{D}] P_j (\sin \theta \cos \varphi) \right]
- \frac{4\pi}{D} P_2 (\cos \theta) \int_{0}^{r_0} \frac{\rho_0 \Omega^2}{M_\phi} \frac{\sin^2 \theta + \eta_2}{2 + \eta_2} dr_0
+ 4\pi GM_2 \int_{j=2}^{4} \frac{P_j (\sin \theta \cos \varphi)}{(r_0)_{j+1}^4} \int_{0}^{\Omega_0} \frac{\rho_0^2}{M_\phi} \frac{j + 3 + \eta_1}{j + \eta_1} dr_0.
\]

The quantity \( \eta_j \) can be evaluated by numerically integrating the radial’s equation (cf. Kopal 1959)

\[
r_0 \frac{d}{dr} (\eta_j + 1) = \eta_j (\eta_j - 1) = j (j + 1),
\]

for \( j=2,3,4 \), and boundary condition \( \eta_j (0) = j - 2 \). The quantity \( \eta_0 \) is the mean radius of the corresponding isobar. The local effective gravity is given by differentiation of the total potential and is written as

\[
g_r = (\frac{\partial \Psi}{\partial r})^2 + (\frac{\partial \Psi}{\partial \theta})^2 + (\frac{\partial \Psi}{\partial \phi})^2 , \]

\[
S_r = \frac{4\pi}{3} (a^2 + b^2 + c^2).
\]

2.4. Element diffusion process

The effect of meridian circulation can drive the transport of chemical elements and angular momentum in rotating stars. For the components in solid-body rotation, no differential rotation exists that can cause shear turbulence. According to Endal & Sofia (1978) and Pinsoneault (1989), the transport of chemical composition is treated as a diffusion process. The equation takes the form of (Chaboyer & Zahn 1992)

\[
\frac{\partial \psi}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 D_{ij} \frac{\partial \psi}{\partial r} \right] + \left( \frac{\partial \psi}{\partial r} \right)_{nuc},
\]

\[
(27)
\]
where \( \left( \frac{d\mu}{d\theta} \right)_{nu} \) is a source term from nuclear reactions, and \( y_p \) is the relative abundance of \( \alpha \) – \( th \) nuclide. The diffusion coefficient \( D_{ff} \) given by Heger, Langer & Woosley (2000a) can be expressed as

\[
D_{ff} = \min \{d_{inst}, H_{x,ES} \} U(r),
\]

where \( d_{inst} \) and \( H_{x,ES} \) denote the extent of the instability and the velocity scale height, respectively. The expression for the amplitude of the radial component of the meridional circulation velocity \( U(r) \) (derived from Kippenhahn & Weigert 1990) has been modified to take the effects of radiation pressure and tidal force into account, which are important in a massive close binary system. It is noticed that

\[
U(r) = \left( \frac{\Omega^2}{GM} + \frac{\Omega_D r}{M D^2} \right) \frac{L}{8\pi \mu} \left( \frac{1}{2} \gamma_{\nu} \frac{1}{\nu_{\nu} - 1} \right) \left( 1 - \frac{\Delta r}{\Delta \rho_{\nu}} - \frac{\Delta \rho_{\nu}}{\rho_{\nu}} \right)
\]

where the term \( \frac{\Omega^2}{GM} + \frac{\Omega_D r}{M D^2} \) is the local ratio of centrifugal force and tidal force to gravity, \( \gamma \) is the ratio of the specific heats \( c_p/c_v \), \( L \) represents the luminosity at radius \( r \), \( M_f \) is the mass enclosed within a sphere of radius \( r \), \( \nu \) is the actual gradient and \( \nu_{\nu} \) is adiabatic temperature gradient, \( \epsilon_m = \frac{L}{D} \) gives the mean energy production rate, and \( \epsilon \) is local generation rate of nuclear-energy.

There is no source or sink at the inner and outer boundaries of the two components. Therefore, the boundary conditions are used as

\[
\left( \frac{\partial y_a}{\partial r} \right)_{r=1} = 0 = \left( \frac{\partial y_a}{\partial r} \right)_{r=0 M}
\]

where the subscript \( i \) denotes different layers inside stars. The initial abundance equals the one at the zero-age main sequence. Therefore, the initial condition is

\[
(y_a)_{r=0} = (y_a)_{\text{init}}.
\]

2.5. Luminosity accretion

In the case where the joint effect of rotation and tide is ignored, the two components are spherically symmetric. The star fills its Roche lobe and begins to transfer matter to the companion. However, in the case with the effects of rotation and tide being considered, the components are triaxial ellipsoids. The condition for the mass overflow through Roche lobe flow should be revised as \( \Delta t = r_{rhe} \) (Huang 2004b). It is assumed that the transferred mass is distributed within a thin shell at the surface of the primary before the transfer, and within a thin shell at the surface of the secondary after the transfer. Three forms of energy (including potential energy, heat energy, and radiative energy) are transferred to the secondary. The mass transfer rate is \( \dot{m} \). Two different cases are considered:

a) If the joint effect of rotation and tide is ignored, the accretion luminosity can be expressed directly in terms of the Roche lobe potential at the inner Lagrangian point, \( \Psi_{L1} \), and at the surface of the secondary \( \Psi_s \) (Han & Webbink 1999):

\[
\Delta L_P = \dot{m}(\Psi_{L1} - \Psi_s) = \frac{GmM_1}{D} \left( \frac{1}{X_{11}/D} + \frac{q}{1-X_{11}/D} + \frac{1+q}{2} \frac{X_{11}}{D} \right)
- \frac{q}{4} \frac{1}{r_{L1}^2} - \frac{1}{H_{L1}^2} \left( \frac{q}{r_{L1}^2} - \frac{1}{H_{L1}^2} \right),
\]

where \( X_{11} \) is the distance between the primary and \( L_1 \), and \( R_1 \) is the radius of the secondary.

b) If the joint effect of rotation and tide is considered, the equilibrium structure of the two components will be treated as triaxial ellipsoids. The release of potential energy because of the accretion of a mass rate \( \dot{m} \) to the secondary is given by

\[
\Delta L_P = \dot{m}(\Psi_{L1} - \Psi_s) = \frac{GmM_1}{D} \left( \frac{1}{X_{11}/D} + \frac{q}{1-X_{11}/D} + \frac{1+q}{2} \frac{X_{11}}{D} \right)
- \frac{q}{4} \frac{1}{r_{L1}^2} - \frac{1}{H_{L1}^2} \left( \frac{q}{r_{L1}^2} - \frac{1}{H_{L1}^2} \right),
\]

where \( \Psi_s \) is the potential of the secondary. Similarly, as the two components have different temperatures, the transmitted thermal energy will be

\[
\Delta L_T = \dot{m} \frac{3kT_{eff1}}{2\mu_1 m_p} - \frac{3kT_{eff2}}{2\mu_2 m_p},
\]

where \( T_{eff1} \) and \( T_{eff2} \) represent the effective temperature of the primary and the secondary, respectively, and \( \mu_1 \) and \( \mu_2 \) are the mean molecular weights of the primary and the secondary, respectively, \( m_p \) refers to proton mass. Because of the irradiation, energy accumulated by the primary and the secondary can be given by (Huang & Taam 1990)

\[
\Delta L_{acc} = \beta(\Delta L_P + \Delta L_T + \Delta L_r).
\]

Because a part of the total energy may be dissipated dynamically, \( \beta \) is assumed to range from 0.1 to 0.5 (Huang 1993). A value \( \beta = 0.3 \) is adopted.

3. Results of numerical calculation

The structure and evolution of binary system was traced with the modified version of a stellar structure program, which was developed by Kippenhahn et al., (1967) and has been updated to include mass and energy transfer processes. The calculation method is based on the technique of Kippenhahn and Thomas (1970) and takes advantage of the concept of isobar (Zahn 1992, Meynet and Maeder 1997). Both components of the binary are calculated simultaneously. The initial mass of the system components is set at 9\( M_0 \) and 6\( M_0 \). The initial chemical composition X equals X=0.70, and Z=0.02 is adopted for the two components. Similarly, the initial orbital separation between the two components for all sequences is defined as 20.771\( R_0 \), so mass transfer via Roche lobe occurs in case A (at the central hydrogen-burning phase of the primary). Two evolutionary sequences corresponding to the evolution with the joint effect of rotation and tide being considered or ignored are calculated. The sequence denoted by case 1 represents the evolution without the effects of rotation and tide being considered, while the sequence denoted by case 2 represents the evolution with the effects of rotation and tide being considered. The calculation of Roche lobe is taken from the study by Huang & Taam (1990). The non-conservative evolution in the two cases was considered. Because the local flux at colatitude \( \theta \) is proportional to the effective gravity \( g_e \), according to Von Zeipel theorem (Maeder 1999), the mass-loss rate due to the stellar winds intensified by tidal, rotational, and irradiative effects is obtained according to Huang & Taam (1990) (cf. Table 1). The angular velocity of the system and the orbital separation between the two components change due to a number of factors: changes in physical processes as
the binary system evolves, including the loss of mass and angular momentum via stellar winds, mass transfer via Roche lobe overflow, exchange of angular momentum between component rotation and the orbital motion of the system caused by tidal effect, and changes in moments of inertia of the components. The changes in the angular velocity of the system and the orbital separation between the two components can be calculated according to Huang & Taam (1990), and the results are listed in Table 1. Other parameters are treated in the same way for two sequences.

The evolution of the binary system proceeded as follows (cf. Table 1). Evolutionary time, orbital period, mass of two stars, luminosities and effective temperature of two stars, central and surface helium mass fraction of the primary, and mean equatorial rotational velocities of two stars are listed in Table 1. Points a, b, c, d, e, and f denote the zero-age main sequence, the beginning of H-shell burning, the end of central hydrogen-burning, the beginning of the mass transfer stage, the beginning of the central hydrogen-burning stage, and the end of calculation, respectively. At the beginning of mass exchange, the luminosity and effective temperature of the primary component decrease rapidly.

The secondary accretes 6.174M⊙ for case 1 and 5.502M⊙ for case 2 during the mass transfer in case A. Because of this mass gain, the luminosity and the temperature of the secondary go up. When the mass is transferred from the more massive star to the less massive one, the separation between the centres of the two components as well as the orbital period of the system decrease. Some orbital angular momentum is transformed into the spin angular momentum of both components, and this process is crucial to model the spin-up of the accretion star. With mass overflow, the mass of the primary will be less than that of the secondary. When the mass is transferred from the less massive star to the more massive one, the separation between the centres of the two components as well as the orbital period of the system increases. Some spin angular momenta in both of the components are transformed into orbital angular momentum. This physical process results in a longer epilogue after mass transfer.

The equilibrium configuration deviates from spherical symmetry because of the centrifugal forces and tidal forces. And the deviated region mainly lies in the outer layer of a star. In fact, the distorted stellar surface forms the shape of a triaxial ellipsoid. A distorted isobar surface can be expressed as

\[ r = r_p[1 + f(r)P_2(\cos \theta) + g(r)P_2(\cos \theta)\cos 2\varphi], \]  

(39)

which corresponds to the form of the disturbing potential (Zahn 1992). The coefficients \( f(r) \) and \( g(r) \) can be defined as

\[ f(r) = \frac{C_1D^2}{\pi \rho_0^2D^2} - \frac{C_M}{\pi \rho_0 D}, \]

and

\[ g(r) = \frac{C_M}{2\pi \rho_0 D}. \]

The quantity \( g \) is the mean density of a star with the mass of \( M_1 \). It was noticed that at the central hydrogen-burning phase, two parameters \( C_1 \) and \( C_2 \) in Eq. (39) gain the values of 0.703 ± 0.125 and 0.491 ± 0.102, respectively. This formula indicates that the shapes of the two components vary with the potentials of the centrifugal force and the tidal force. The radial deformation is inversely proportional to the mean density of the component. In order to describe the distortion, the distribution of the surface rotating velocities of the primary is illustrated in Fig. 1. The four panels (a), (b), (c), and (d) correspond to the evolutive time of 0, 2, 3, 5 years and corresponding periods of 2, 776, 2, 760, 2, 746, and 2, 628 days, respectively. The rotational velocity rates for the peaks of the semi-major axes \( b \) and \( a \) are

\[ \frac{\Delta v}{v} = \frac{\Delta v_1}{v_1} = 0.9867, 0.9401, 0.8814, \]

and 0.8664 in four panels, respectively. The results show that the surface deformation is intensified with the evolution and volume-expansion of the primary. The distortion throughout the outer region of the primary is considerable. The detailed theoretical models that focus on investigation of the outer regions have somewhat deviated from the Roche model. The high-order perturbed potential is required for studying the structure and evolution of short-period binary systems. Matthews & Mathieu (1992) examined 62 spectroscopic binaries with A-type primaries and orbital periods less than 100 days. They concluded that all systems with orbital periods less than or equal to three days have circular orbits or nearly circular orbits. Zahn (1977) and Rieutord & Zahn (1997) have shown how binary synchronization and circularization result from tidal dissipation. Based on smoothed particle hydrodynamics (SPH) simulation, Renvoizé et al. (2002) have quantified the geometrical distortion effect due to the tidal and rotational forces on the polytropic secondaries of semi-detached binaries. They suggest that the tidal and rotational distortion on the secondary may not be negligible, for it may reach observable levels of \( \sim 10\% \) on the radius in specific cases of polytropic index and mass ratio. Georgy et al. (2008) display that various effects of the rotation on the surface of a 20M⊙ star at a metallicity of 10^{-2} and at \( \sim 95\% \) of the critical rotation velocity. They point out that the star becomes oblate with an equatorial-to-polar radius ratio \( \frac{a}{b} \simeq 1.3 \). These results agree closely with ours.

The variation relative gravitational accelerations, the tidal force, and the ratio of \( f_{cen}/f_{tid} \) on the surface of the primary under the coordinate \( \theta \) and \( \varphi \) at the beginning of mass overflow are shown in Fig. 2. The quantities \( g_{cen}, \theta_{cen}, \) and \( g_{tid} \) are the three components of gravitational acceleration. The six panels (a), (b), (c), (d), (e), and (f) represent the distribution of \( g_{cen}/g, g_{tid}/g, g_{cen}/g_{tid}, g_{tid}/r_{tid}, \) and \( f_{cen}/f_{tid} \), respectively. The quantity \( g \) equals the gravitational acceleration of the corresponding equilibrium sphere \( g = \frac{GM}{R^2} \). When the joint effect of rotation and tide is considered, the gravitational accelerations are different from those in the conventional model. Gravitational acceleration generally has three components.

It is shown in panel (a) that the relative quantity \( \frac{g_{tid}}{g} \) reaches the maximum value of 1.048 at the two polar points and drops to the minimum value 0.6987 on the equatorial plane because the inward tidal force acts on the primary and causes the polar radius to become shorter. The tidal and centrifugal forces pull the primary outwards and change gravitational accelerations greatly on the equatorial plane. Furthermore, the maximum value is 0.9486 and the minimum value is 0.6987 on the equatorial plane. The lower values are at the peak of the longest axis \( a \) and the higher values are at the peak of the axis \( b \). The relative quantity \( \frac{g_{cen}}{g} \) reaches the maximum value of 0.20399 at the point of \( \theta = \frac{\pi}{2} + \frac{8\pi}{2}, \varphi = 2n \pi, k = 0, 1 \) and vanishes at the two polar points and on the equatorial plane in panel (b). It can be seen that a secondary maximal value of 0.10214 exists at point of \( \theta = \frac{8\pi}{2}, \varphi = 2n \pi + 2\pi, k = 0, 1 \). The relative quantity \( \frac{g_{cen}}{g} \) reaches the maximum value of 0.1050 at point of \( \theta = \frac{8\pi}{2}, \varphi = 2n \pi + 2\pi, k = 0, 1, 2, 3 \) and decreases to zero at the point of \( \varphi = \frac{8\pi}{2}, k = 0, 1, 2, 3 \) in panel (c). The total gravitational acceleration at the surface of the primary is shown in panel (d). Its distribution is similar to the one of \( \frac{g_{cen}}{g} \) because the radial component is the maximum value. It is noticed that, as expected, the average gravitational acceleration of the rotating model is less than for the non-rotating model. It can be observed that the tidal force reaches the highest value of 340.05 cm/s^{2} at the point of \( \varphi = \frac{8\pi}{2}, k = 0, 1 \) and decreases to the lowest of 136.24 cm/s^{2} at the two polar points. The quantity \( f_{cen}/f_{tid} \) reaches the maximum value of 2.4905 at the point of
Table 1. Parameters at different evolutionary points a, b, c, d, e, and f in sequences of cases 1 and 2.

| Sequence | Time $10^7$ yr | $P$ day | $M_1$ $M_0$ | $M_2$ $M_0$ | $\log L_1/L_0$ | $\log T_{1,eff}$ | $\log L_2/L_0$ | $\log T_{2,eff}$ | $Y_1 (c)$ | $Y_1$ | $V_{rot,1}$ km/sec | $V_{rot,2}$ km/sec |
|----------|----------------|---------|-------------|-------------|----------------|----------------|----------------|----------------|------------|------|-------------------|-------------------|
| a        |                |         |             |             |                |                |                |                |            |      |                   |                   |
| Case 1   | 0.0000         | 2.777   | 9.000       | 6.000       | 3.639          | 4.385          | 3.077          | 4.287          | 0.2800     | 0.2800 | 68.66             | 56.39             |
| Case 2   | 0.0000         | 2.776   | 9.000       | 6.000       | 3.629          | 4.381          | 3.055          | 4.280          | 0.2800     | 0.2800 |                   |                   |
| b        |                |         |             |             |                |                |                |                |            |      |                   |                   |
| Case 1   | 2.6725         | 2.737   | 8.929       | 5.994       | 3.959          | 4.285          | 3.110          | 4.267          | 0.8744     | 0.2800 | 143.69            | 63.39             |
| Case 2   | 2.6267         | 2.743   | 8.939       | 5.995       | 3.914          | 4.295          | 3.085          | 4.265          | 0.8240     | 0.2800 |                   |                   |
| c        |                |         |             |             |                |                |                |                |            |      |                   |                   |
| Case 1   | 2.6854         | 3.190   | 5.314       | 9.608       | 3.457          | 4.120          | 3.811          | 4.396          | 0.8799     | 0.2801 | 121.81            | 67.59             |
| Case 2   | 2.6329         | 3.192   | 5.318       | 9.616       | 3.264          | 4.135          | 3.834          | 4.406          | 0.8267     | 0.2801 |                   |                   |
| d        |                |         |             |             |                |                |                |                |            |      |                   |                   |
| Case 1   | 3.0784         | 11.743  | 2.727       | 12.168      | 3.776          | 4.128          | 4.145          | 4.461          | 0.9800     | 0.5892 | 58.41             | 41.79             |
| Case 2   | 3.3567         | 7.213   | 3.388       | 11.497      | 3.442          | 4.162          | 4.123          | 4.405          | 0.9800     | 0.3481 |                   |                   |
| e        |                |         |             |             |                |                |                |                |            |      |                   |                   |
| Case 1   | 3.0941         | 32.534  | 1.797       | 13.082      | 3.959          | 4.028          | 4.276          | 4.539          | 0.9799     | 0.8775 | 121.80            | 67.59             |
| Case 2   | 3.5470         | 32.531  | 1.714       | 13.037      | 3.280          | 4.686          | 4.308          | 4.419          | 0.9800     | 0.8261 |                   |                   |
| f        |                |         |             |             |                |                |                |                |            |      |                   |                   |

Fig. 1. Surface rotating velocity distribution of primary varying with time. Four panels (a), (b), (c), and (d) correspond to periods: 2.776, 2.760, 2.746, and 2.628 days, and corresponding evolutive time is 0, 2.3386×10$^7$, 2.6194×10$^7$, 2.6287×10$^7$ yrs, respectively.

$\theta = \frac{\pi}{2}; \psi = k\pi + \frac{\pi}{2}, k = 0, 1$ and reaches the secondary maximal value of 1.2445 at the point of $\theta = \frac{\pi}{2}; \psi = k\pi, k = 0, 1$. The results show that the effect produced by tidal distortion is lower in comparison with what is produced by rotational distortion on the equatorial plane. However, with the mass conversion, the opposite situation can emerge. It is concluded that tidal distortions are related to the mass ratio of the secondary to the primary. These results suggest that rotation and tide have strong influences on the stellar surface. They modify the gravity and change the spherically-symmetric shape into the triaxial ellipsoid shape.
Fig. 2. Variation of relative gravitational accelerations at the surface of primary under coordinate $\theta$ and $\varphi$ as mass overflow begins. The quantities $g_r$, $g_\theta$, and $g_\varphi$ are the three components of the gravitational acceleration $g_{\text{tot}}$. Quantity $g$ equals the gravitational acceleration of the corresponding equivalent sphere ($g = \frac{GM}{r^2}$).

Furthermore, the stellar structure equations are basically revised due to the distribution of the relative quantity in the outer region.

According to the Von Zeipel theorem, the mass loss due to stellar winds should be proportional to local effective gravity. Polar ejection is intensified by the tidal effect. The higher gravity at the peak of the axis $b$ makes it hotter. The ejection of an equatorial ring may be favoured by both the opacity effect and the higher temperature at the peak of the semi-axis $b$. This effect is called the $g_{\text{eff}}(\theta, \varphi)$-effect in this paper. It is predicted that the $g_{\text{eff}}(\theta, \varphi)$-effect is as important as the $g_{\text{eff}}$-effect suggested by Maeder (1999) and Maeder & Desjacques (2001). The shapes of planetary nebulae that deviate from spherical symmetry (axisymmetrical one in particular) are often ascribed to rotation or tidal interaction (Soker 1997). Frankowski and Tylenda (2001) suggest that a mass-losing star can be noticeably distorted by tidal forces, thus the wind will exhibit an intrinsic directivity and may be globally intensified. Interestingly enough, the group of the B[e] stars shows a two-component stellar wind with a hot, highly ionized, fast wind at the poles and a slow, dense, disk-like wind at the equator (Zickgraf 1999). Maeder and Desjacques (2001) have noticed that the polar lobes and skirt in $\eta$ Carinae and other LBV stars may naturally result from the $g_{\text{eff}}$ and $\kappa$-effects. Langer et al. (1999) have shown that giant LBV outbursts depend on the initial rotation rate. Tout and Eggleton (1988) pro-
posed a formula according to which the tidal torque would enhance the mass-loss rate by a factor of $\frac{1 + B \times (\frac{a}{R_{\odot}})^6}{1 - B \times (\frac{a}{R_{\odot}})^6}$, where $B$ is a parameter free to be adjusted (ranging from $5 \times 10^2$ to $10^4$). Mass loss and associated loss of angular momentum are anisotropic in rotating binary stars. The theories for describing the mass loss and angular momentum loss from stellar winds should be altered partly in future work.

The time variation of relative accretion luminosity at the semi-detached stage is shown in Fig. 3. The two panels (a) and (b) correspond to cases 1 and 2, respectively. The figure shows that the release of transferred thermal energy approaches zero, indicating that the mass transfer process is unstable.

The total H-burning energy-generation rates of the primary in the two cases are shown in Fig. 4. Panel (b) shows the H-burning energy-generation rate in case 2 after the main sequence. From the difference between curves in panel (a), it is noticed that the effect of rotation causes the total H-burning energy-generation rate lower. As a result, the evolutive time in the main-sequence stage gets longer (cf. Table 1). Moreover, the larger fuel supply and lower initial luminosity of the rotating stars help to prolong the time which they spend on the main sequence (Heger & Langer 2000b). The lifetime extension in rotating binary star at the main-sequence stage can also be illustrated according to Suchkov (2001). Their results show that the age-velocity relation (AVR) between F stars in the binary system is different from the one between “truly single” F stars. The discrepancy between the two AVRs indicates that the putative binaries are, on average, older than similar normal single F stars at the same effective temperature and luminosity. It is speculated that this peculiarity comes from the impact of the interaction of components in a tight pair on stellar evolution, which results in the prolonged main-sequence lifetime of the primary F star. Moreover, no central helium-burning stage exists for case 2 (cf. Table 1). From panel (b), it can be seen that the energy-generation rate of the primary vibrates at the H-shell burning stage in case 2. These facts suggest that the
burning of H-shell is unstable in case 2. The reason lies in the centrifugal force reducing the effective gravity at the stellar envelope. The luminosity and surface temperature there decrease (Kippenhahn 1977; Langer 1998; Meynet and Maeder 1997). Thus, the shell source becomes cooler, thinner, and more degenerated as the He core mass increases. As the hydrogen shell becomes unstable, the thickness $\rho$ and surface temperature are $\sim 0.203$ and $1.1885 \times 10^4 K$, respectively. This physical condition leads to thermal instability (Yoon et al., 2004), and the H-shell source experiences slight oscillation. It is well known that the energy-generation rate is proportional to temperature and density ($\dot{E} \propto \rho T^4$); therefore, the curve of the H-shell energy-generation rate fluctuates.

The time-dependent variation in the luminosity and the equivalent radius of the primary in the two cases are illustrated in Fig. 5. Because the rotating star has a lower energy-generation rate, the luminosity of the primary is lower, which is the consequence of decreased central temperature in rotating models due to decreased effective gravity (Meynet and Maeder 1997). Then, the primary expands slowly in case 2. It is observed that case 1 reaches point b at $t = 2.6725 \times 10^7 yr$, while case 2 reaches point b at $t = 2.6267 \times 10^7 yr$. The initiation time of mass transfer for case 2 is advanced by about $\sim 1.71\%$. Similarly, numerical calculation by Petrovic et al. (2005b) shows the radius of the rotating primary increases faster than that of the non-rotating primary due to the influence of centrifugal forces. Their results also show that mass transfer of Case A starts earlier in rotating binary system, which is consistent with ours. If the rotating star is still treated as a spherical star, the initiation time of mass overflow should be later than that in the non-rotational case. Actually, because of the distortion by rotation and tide, the time for mass overflow may be extended. Therefore, it is very important to investigate distortion in close binary systems.

The time-dependent variation in the helium compositions at the surface of the primary is illustrated in Fig. 6. The H-shell burning begins at $t = 2.6854 \times 10^7 yr$ in case 1 while at $t = 2.6329 \times 10^7 yr$ in case 2 (cf. Table 1). Therefore, the initiation time of H-shell burning is advanced by $1.71 \times 10^5 yr$. Moreover, the helium composition at the surface of the primary is $0.280051$ at point c, suggesting that the diffusion process progresses slowly in a rotating star. Cantiello et al. (2007) also indicate that rotationally induced mixing before the onset of mass transfer is negligible, in contrast to typical O stars evolving separately; hence, the alteration of surface compositions depends on both initial mass and rotation rates. The sample of the OB-type binaries with orbital periods ranging from one to five days by Hilditch et al. (2005) shows enhanced N abundance up to 0.4 dex. Langer et al. (2008) have discovered that for the same binary system, but with the initial period of six days instead of three days, its mass gainer is accelerated to a rotational velocity of nearly $500 km s^{-1}$, which produces an extra nitrogen enrichment from more than a factor two to about 1 dex in total.
Because there is no central helium-burning phase for case 2, the diffusion process can be neglected in the interior region of the primary after the main sequences.

4. Conclusions
The main achievements of this study may be summarised as follows.
(a) The distortion throughout the outer layer of the primary is considerable. The detailed theoretical models that investigate the outer regions of the two components have deviated somewhat from the lowest approximation of the Roche model. The high-order perturbing potential is required especially in the investigation of the evolution of short-period binary system.
(b) The equilibrium structures of distorted stars are actually triaxial ellipsoids. A formula describing rotationally and tidally distorted stars is presented. The shape of the ellipsoid is related to the mean density of the component and the potentials of centrifugal and tidal force.
(c) The radial components of the centrifugal force and the tidal force cause the variation in gravitation. The tangential components of the centrifugal force and the tidal force cannot be equalized and, instead, they change the shape of the components from perfect spheres to triaxial ellipsoids. Mass loss and associated angular momentum loss are anisotropic in rotating binary stars. Ejection is intensified by tidal effect. The ejection of an equatorial ring may be favoured by both the opacity effect and the higher temperature at the peak of semi-axis $b$. This effect is called the $g_\alpha(\theta, \varphi)$-effect in this paper.
(d) The rotating star has an unstable H-burning shell after the main sequence. The components expand slowly due to their lower luminosity. If the components are still treated as spherical stars, some important physical processes can be ignored.

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References
Baker, N. 1966, in Stellar Evolution, ed. R. F. Stein & A. G. W. Cameron (Plenum, New York) 333
Burša, M., 1988, Bull.Astron.Inst.czechosl.39,289
Burša, M., 1989a, Bull.Astron.Inst.czechosl.40,125B
Cantiello, M., Yoon, S.-C., Langer, N., & Livio, M. 2007, A&A, 465, L29
Chan, K.L., Chau, W.Y., 1979, APJ, 233,950
Chandrasekhar, S,; 1933, MNRAS,,93,390
Charbonnel, C., 1995, ApJ, 453, L41
Charbonnel, C., 1999, A&A, 328, 811
Charbonnel, C., 1999, A&A, 328, 811
Denissenkov, P.A., Ivanova,N.S., and Weiss,A, 1999, A&A, 341, 181
Decressin et al., 2009, A&A, 495, 271
Endal, A. S. & Sofia, S., 1976, ApJ, 210, 184
Endal, A. S. & Sofia, S., 1978, ApJ, 220, 279
Frankowski, A.& Tylenda,R., 2001, A&A, 367,513
Giuricin G., et al., 1984, A&A, 131, 152
Georgy, C., Meynet, G. and Maeder, A.,Proc.IAU-Symp., No. 255, 2008,L. K. Hunt, S. Madden @ R.Schneider.eds., in press [astroph08070561]
Han,Z., Webbink, R.F.,&A.A,1999, 236, 107
Hilditch, R. W., Howarth, I. D., & Harries, T. J. 2005, MNRAS, 357, 304
Heger, A.,Langer,N. and Woosley,S.E., 2000a,APJ, 528, 368
Heger, A.,Langer,N. 2000b,APJ, 544,1016
Hofmeister, E., Kippenhahn, R., Weigert, A.: Z.f.Aph.59, p.215-241,1964;