Research on Airport Deicing Based on Queuing Theory
—— Take an airport in the west as an example

Yuming Zou and Liang Zhang*
Guangzhou Civil Aviation College, Guangzhou, 510000

*Corresponding author e-mail: 25267764@qq.com

Abstract. Each winter's snow and ice weather will cause a large number of flight delays, which will bring serious economic losses to airlines and airports. Among them, deicing work is an important factor influencing the punctuality of flights. It involves the coordination of airports, airlines, meteorological bureaus, de-icing companies, and many other uncontrollable factors such as the size of ice and snow, wind levels and emergencies. Based on the analysis of domestic and foreign expert deicing work, the theory of queuing theory was used to establish the airport deicing queuing system model. The example verified that the model could be used for the distribution of airport deicing machines, and was a problem for airport deicing queues. An effective quantitative analysis method is proposed.

1. Introduction
In recent years, China’s civil aviation industry has developed rapidly, air traffic has increased significantly, and the airport’s opening hours have increased. Therefore, the response time to the airport's deicing capacity has become more stringent. In order to ensure the punctuality of flights and improve competitiveness is the pursuit of each airport, airport management personnel have also gradually realized the importance of deicing, especially at the northern airports and high altitude airports, where low-temperature aircraft are more likely to freeze, and deicing work is even more important.

Many experts at home and abroad have studied the deicing work at the airport and put forward various theoretical and mathematical models. In 2006, MAO X et al proposed an agent-based deicing scheduling-FCFS scheduling rule [1]. The authors used the airlines as a selfish agent, and all airlines were seeking to maximize the use of airport deicing facilities. In 2007, Anna Norin et al. defined the problem of deicing vehicle scheduling during airport de-icing as a path optimization problem [2]. The author believes that deicing process optimization can be simplified into two goals: one is to find the shortest path between line deicing devices; the second is based on the status quo, according to the study of queuing theory to find a minimum delay path, and finally using GWAC (availability check greedy algorithm) to get the best results. In 2011, Xing Zhiwei used game theory to solve airline competition problems in the use of airport deicing resources [3]. In this model, the non-cooperative game is introduced to quantify the airport's ability to remove ice. The airline puts forward its own strategy based on the actual situation, improves the utility, simplifies the function of benefits, and makes the best choice.
It can be seen that the above studies have focused on the path optimization and interest issues in the de-icing process, and are rarely involved in the distribution of the number of deicing machines. Therefore, this paper first analyzes the deicing process, establishes the airport deicing model, and then combines the queue knowledge theory to finally solve the optimal number of open deicing machines. While reducing the queue length of aircraft deicing and improving the deicing efficiency, it has effectively reduced the operating costs of the airport.

2. Airport queuing issues

Queuing theory, or stochastic service system theory, is based on the statistical study of the arrival time of service objects and service time, and the statistical rules of these quantitative indicators (waiting time, queue length, busy duration, etc.) are obtained, and then according to these rules to improve the structure of the service system or re-organized by the service object. Simply put, it is a theory that solves the probabilistic characteristics of various service system waiting phenomena to solve the optimal design and optimal control of the service system. In queuing theory, the distribution of arrival interval and service time generally follows negative exponential distribution and Poisson distribution.

(1) Poisson distribution is a special random process used to describe the arrival of aircraft in queuing theory. Let $N(t)$ denote the aircraft arriving within $[0, t)$ and $P_n(t_1, t_2)$ denote the probability of $n$ aircraft arriving at $[t_1, t_2]$, which is

$$P_n(t_1, t_2) = P\{N(t_2) - N(t_1) = n\} \ (t_1 < t_2; n \geq 0) \quad (1)$$

When the $P_n(t_1, t_2)$ satisfies the following three conditions, the aircraft arrives to obey the Poisson distribution:

1) Stationary

   The probability that an aircraft arrives within $[t, t+\Delta t]$ is independent of the starting time $t$, but is only related to the interval length $\Delta t$ ($\Delta t$ is sufficiently small).

2) Independence

   It means that the number of aircraft arriving in the disjoint time zone is independent of each other, that is, the number of aircraft that arrive within $[t, t+\Delta t]$ has nothing to do with the number of aircraft that have arrived before time $t$. This property is also called after Effectiveness [7].

3) Generality

   It means that within a sufficiently small time interval $[t, t+\Delta t]$, there is a very small probability that two or more planes will arrive, which is

$$\sum_{n=2}^{\infty} P_n(t, t+\Delta t) = o(\Delta t) \quad (2)$$

So the probability of no one aircraft arriving in $[t, t+\Delta t]$ is

$$P_0(t, t+\Delta t) = 1 - \lambda \Delta t + o(\Delta t) \quad (3)$$

Obviously, the time interval $[0, t+\Delta t]$ can be decomposed into two intervals of $[0, t]$ and $[t, t+\Delta t]$. Then it can be seen that the time interval with time interval $t$ is exactly the probability that $n$ planes arrive.

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \quad (t > 0; n = 0, 1, 2, \ldots) \quad (4)$$

It can be seen that $N(t)$ obeys the Poisson distribution and its mathematical expectation and variance are
\[ E(N(t)) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda t}, \quad \text{Var}(N(t)) = \lambda t \] (5)

In particular, when \( t=1 \), there is \( E(N(1)) = \lambda \), which means the average number of aircraft arriving within a unit of time, also known as the arrival rate.

Since the Poisson flow is very similar to the actual flow and easy to process during analysis and calculation, the Poisson flow input is more studied in the queuing theory [4].

(2) Negative exponential distribution

If the random variable \( T \) has a probability distribution density of

\[ f(t) = \begin{cases} \lambda e^{-\lambda t} & (t \geq 0) \\ 0 & (t < 0) \end{cases} \quad (\lambda > 0) \] (6)

\( T \) is then said to have a negative exponential distribution with parameter \( \lambda \). Negative index distribution function is

\[ f(t) = \begin{cases} 1 - \lambda e^{-\lambda t} & (t \geq 0) \\ 0 & (t < 0) \end{cases} \quad (\lambda > 0) \] (7)

Obviously, \( E(T) = \frac{1}{\lambda} \), \( \text{Var}(T) = \frac{1}{\lambda^2} \), and \( \lambda \) is the average service rate of each deicer, that is, the average number of aircraft that leave the system after obtaining the service within the unit time.

3. Airport Deicing Model Construction and Application

3.1. Model representation

In the early 1950s, D.G.Kendall introduced the queuing system model notation. Usually six symbols were used and represented as \( X/Y/Z/A/B/C \). This token is called the Kendall notation. Each symbol represents a different meaning:

- \( X \) - The distribution of time differences between adjacent aircraft arriving at the queuing system;
- \( Y \) - Distribution of service time;
- \( Z \) - Number of deicing machines;
- \( A \) - How much capacity of the queuing system;
- \( B \) - The number of aircraft sources;
- \( C \) - Service rules.

If the last three items are omitted in the Kendall notation, it means \( X/Y/Z/\infty/\infty/FCF \). For example, \( M/M/s \) indicates that an aircraft arrival time interval follows a negative exponential distribution, service time is a negative exponential distribution, and \( s \) de-icing machine, the system capacity is infinite (waiting system), the aircraft source is infinite, and the queuing rule is the queuing model of first come first service [5].

3.2. Indicator Description

In order to make the service system's quality estimate more accurate, to know more about the working status of the system and to determine the best operating parameters, the following indicators are usually considered when analyzing and calculating.

(1) System status

The total number of aircraft in the system refers to the sum of the number of aircraft waiting for service and being serviced at any time. It is commonly referred to as \( N(t) \) and is also called transient. When the system runs smoothly, it is often referred to as \( N \), which is called steady state.
(2) System State Probability

The probability that the system has exactly $n$ aircraft at time $t$, called the transient probability, is denoted as $P_n(t)$ [11]. The probability of having $n$ aircraft when the system is stationary is called the steady-state probability, denoted as $T_n(t)$.

(3) Captains and Queue Leaders

Refers to the expected value of the number of aircraft in the captain system, which is the expected value $E(N)$ of the system steady state $N$, denoted as $L$. The queue length, also known as queue length, refers to the expected number of aircraft in the system waiting in line for service, denoted as $L_q$.

(4) Average aircraft arrival rate

Refers to the number of new arrivals arriving at the system per unit time when there are $n$ aircraft in the system, denoted as $\lambda_n$. If the average arrival rate is independent of the system state, the aircraft average arrival rate can be recorded as $\lambda$.

(5) System Average Service Rate

Refers to the average number of aircraft that have been serviced at the end of each unit of time when there are $n$ aircraft in the system, denoted as $\mu_n$. If the average service rate is independent of the system status, the system average service rate can be recorded as $\mu$.

(6) Time spent

Refers to the expected value of the aircraft staying at the system for the entire time, denoted as $W$.

(7) Waiting time

Refers to the expected value of the time the aircraft is waiting in the system for service, denoted as $W_q$. Obviously the waiting time is equal to the waiting time plus service time.

(8) Busy and Unoccupied

Busy period refers to the time that the aircraft starts when it arrives at an idle service agency, and it lasts until the service organization is free again. Idle period refers to the duration of the service agency from the beginning of the idle period to the time when it is busy again. It is often recorded as $I$.

In the above indicators, the main indicators that can be used to measure whether the system is normal are the captain, waiting time, period, and idle period. Captains and queue leaders, aircraft and service agencies are concerned about the captain and queue leader because it involves the size requirements of the space system. Waiting time is also an important indicator of the working conditions of the system, and each aircraft must be as short as possible. Busy periods and idle periods are mainly measured by their strength and efficiency. In services, they alternate.

3.3. Analysis of Airport Deicing Problems

The airport is located in the western part of China and is an international airport. It has 13 deicer machines and all of them are running side by side. In the day, 6 o'clock to 9 o'clock, 13 o'clock to 16 o'clock, 18 o'clock to 19 o'clock are the peak of the flight; other hours from 19 o'clock to 21 o'clock are stationary; from 21 o'clock to 6 o'clock the next day is a trough period. The arrival of the aircraft belongs to the waiting system, first-come-first-served, and later-to-later-service queuing rules.

According to the actual situation of the airport, it can be seen that the aircraft arrives in line with the Poisson flow, the time interval between successive arrivals of the aircraft obeys a negative exponential distribution, and the system space allows infinite queuing. Therefore, the airport aircraft deicing queue system belongs to the $M/M/s/\infty/\infty/FCFS$ model.

There are $s$ service systems, assuming $\lambda_n = \lambda$ and $\mu_n = \begin{cases} n\mu & (n = 1, 2, \ldots, s) \\ s\mu & (n = s, s + 1, \Lambda) \end{cases}$, so
If \( \rho_s = \frac{\rho}{s} = \frac{\lambda}{s \mu} \), then \( \rho_s < 1 \),

\[
p_n = \begin{cases} 
\frac{\rho^n}{n!} p_0 & (n=1,2, \ldots, s) \\
\frac{\rho^n}{s! s^{n-s}} p_0 & (n = s, s+1, \ldots) 
\end{cases}
\]

Equations 8 and 9 are the probability that the number of aircraft in the system is \( n \) under stable conditions. When \( n \geq s \), that is, the number of aircraft in the system is not less than the number of deicing machines, the probability that the returning aircraft will wait and must wait is

\[
\sum_{n=s}^{\infty} p_n = \sum_{n=s}^{\infty} \frac{\rho^n}{s! s^{n-s}} p_0 = \frac{1}{s!} \sum_{k=0}^{\infty} \left( \frac{\lambda}{s \mu} \right)^k p_0 = \frac{\rho^s}{s! (1-\rho_s) p_0}
\]

this formula is called Erlang wait equation.

Then other quantitative indicators can be expressed as

\[
L_q = \sum_{n=s}^{\infty} (n-s) p_n = p_0 \frac{\rho^s}{s!} \rho_s \frac{d}{d \rho_s} \left( \frac{1}{1-\rho_s} \right) = \frac{p_0 \rho^s \rho_s}{s! (1-\rho_s)^2}
\]

The average number of aircraft being serviced in the system is \( \bar{s} \). Obviously, \( \bar{s} \) is the average number of busy deicing machines.

\[
\bar{s} = \frac{s!}{n=0} n \rho_n + \frac{s!}{n=s} \rho_s = \rho p_0 \frac{\sum_{n=s}^{\infty} \rho^{n-1}}{n!(n-1)!} + \frac{\rho^s}{(s-1)! (1-\rho_s)} = \rho
\]

The above formula shows that the number of average busy deicing machines does not depend on the number \( s \) of deicing machines. So you can get the average captain

\[
L = L_q + \rho
\]

For the multiple deicing systems, the little formula is still established, so there is
\[ W = \frac{L}{\lambda}, W_q = \frac{L_q}{\lambda} = W - \frac{1}{\mu} \]  

(15)

3.4. Distribution of Airport Deicing Machines

As the airport has 13 de-icing machines, the number of service systems \( s = (1,2,3,\ldots,13) \), and the relationship between the number of aircraft racks deicing per unit time (also equivalent to the number of planes awaiting takeoff within the unit time) \( X \) and the number of aircraft racks arriving within unit time \( \lambda \) is \( \lambda = X / \text{(Unit time)} \), we can see that in the unit time the two values are equal. It can be seen that the values are equal in unit time. According to the aircraft deicing records at the airport, the average service rate of the deicing machines is \( \mu = 0.1 \), that is, the deicing work of one aircraft can be completed every 10 minutes. Finally, suppose the aircraft's waiting time at the deicing station is less than 30 minutes, so \( W_s \leq 30 \). Then, when the number of deicing machines \( c = (1,2,3,\ldots,13) \), the \( X \) value is the demand.

From 3.3 we can see that when \( c > 1 \), the Little formula of the M/M/c model is

\[ L_s = L_q + \frac{\lambda}{\mu} \]

\[ L_q = p_0(\rho)^c \frac{\rho}{c!} (1 - \rho)^2 \]

\[ W_s = L_s / \lambda \]

\[ W_q = L_q / \lambda \]

Among them, \( L_s \) is the number of aircraft in the deicing queuing system, \( L_q \) is the total number of aircraft waiting for service in the deicing queuing system, \( \rho \) is the service strength, \( W_q \) is the expected value of the aircraft waiting time in the queue, and \( p_0 \) is the entire deicing area free Probability [6], and

\[ p_0 = \left( \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} (1 - \rho)^2 \right)^{-1}, \rho = \frac{\lambda}{\mu} \]

Assuming that the expected value of the aircraft's waiting time at the deicing station is \( W_s = 1 / (\mu - \lambda) = 60 / (6 - X) = 30, X = 4 \) can be calculated.

When \( c = 2 \)

\[ p_0 = \frac{1}{1 + \frac{X}{6} + 0.5 \times \frac{1}{\frac{X}{6} - \frac{X^2}{30!}}} \]

\[ L_q = 2\rho^3 \times \left[ 1 + \frac{X}{6} + 0.5 \times \frac{1}{\frac{X}{6} - \frac{X^2}{30!}} \right] (1 - \rho)^2 \]

\[ L_s = 2\rho^3 \times \left[ 1 + \frac{X}{6} + 0.5 \times \frac{1}{\frac{X}{6} - \frac{X^2}{30!}} \right] (1 - \rho)^2 + \frac{X}{6} \]

\[ W_q = 2\rho^3 \times \left[ 1 + \frac{X}{6} + 0.5 \times \frac{1}{\frac{X}{6} - \frac{X^2}{30!}} \right] (1 - \rho)^2 \lambda \]
\[ W_s = 2\rho^3 \left\{ 1 + \frac{X}{6} + 0.5 \times \frac{1}{1 - \frac{X}{30!}} \times \left( \frac{X}{6} \right)^2 \right\} (1 - \rho^2 \lambda + \frac{X}{6}) \]

Solve \( X = 9 \) according to \( W_s = 30 \). Similarly, the X value when \( c = (3, 4, 5, \Lambda, 13) \) can be calculated separately. Then the comparison of the number of aircraft arriving at the statistical unit hour with the X value can determine the number of deicing stations that need to provide services at a certain moment, as shown in Table 1.

### Table 1. Arrangement mode of deicing machines at an airport in western China.

| Period          | Number of aircraft to be serviced per hour | The number of deicing machines should be open |
|-----------------|-------------------------------------------|---------------------------------------------|
| 6:00 to 9:00    | 43                                        | 11                                          |
| 9:00 to 13:00   | 28                                        | 7                                           |
| 13:00 to 16:00  | 36                                        | 9                                           |
| 16:00 to 18:00  | 23                                        | 6                                           |
| 18:00 to 19:00  | 35                                        | 9                                           |
| 19:00 to 21:00  | 27                                        | 7                                           |
| 21:00 to 6:00   | 6                                         | 2                                           |

It can be seen that when the number of aircraft to be served per hour reaches 13, the value of X when \( c = 3 \) can be calculated. If the value of X is close to 13, then three deicing machines can be opened. The de-icing of the airport provides a quantitative analysis method and data comparison, which can be directly used in the actual airport de-icing process. At the same time, it also provides a way of de-icing the other airports.

**4. Conclusion**

As the demand for air transportation at home and abroad has increased, the punctuality of flights and air services have increasingly attracted people's attention. Deicing and snow work during ice and snow has a great impact on the punctuality of flights and air services. The technical assurance and response time for removing ice and snow at airports have been gradually increased, and the ice and snow work has also received attention from all sides. Based on the theoretical knowledge of queuing theory, this paper analyzes the characteristics and operation flow of the airport deicing system, establishes the airport deicing queuing model, and uses the queuing theory to solve the problem that the number of airport deicing machines should be open. This not only solves the airport deicing process. The problem of long line-up time in the queue also saved operating costs and was worth the relevant machine.

**References**

[1] MAO X, TER M A, ROOS N, et al. Agent-based scheduling for aircraft deicing [C]. Proceedings of the 18th Belgium-Netherlands Conference on Artificial Intelligence, 2006.

[2] ANNA NORIN, ANDERSSON T, VARBRAND P, et al. A GRASP heuristic for Scheduling Deicing trucks at Stockholm Arlanda Airport [C]. Published in the proceedings of the 6th INO Workshop, 2007.

[3] Xing Zhiwei, Qiao Xiaohui. Non-cooperative game research on aircraft ground deicing operation [J]. Journal of System Simulation, 2011, 23(3): 433-437.

[4] Leo Marriott. ATW’s world Airline Report [J]. Airport Transport World. July, 2011.

[5] Zeng Yong, Dong Lihua, Ma Jianfeng. Modeling, Analysis and Simulation of Queuing [M]. Xi’an Xidian University Press, 2011.

[6] S Casado, M Laguna, J Pacheco. Heuristical labour scheduling to optimize airport passenger
flows [J]. Journal of the Operational Research Society. November 2008.