International Conference on Computational Heat and Mass Transfer-2015

A Peclet Number Based Analysis of Mixed Convection for Lid-Driven Porous Trapezoidal Enclosure

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Abstract

Mixed convection flows in a lid-driven trapezoidal enclosure filled with porous medium are studied numerically using our recently proposed higher order compact (HOC) scheme presented in [1]. The top wall of the enclosure is allowed to move in its own plane at a constant speed, the bottom wall is heated uniformly while all other walls maintained at constant cold temperature. The relevant parameters in the present study are Darcy number ($Da$) ($10^{-5} \leq Da \leq 10^{-3}$), Grashof number ($Gr$) = $10^5$, Prandtl number ($Pr$) ($0.026 \leq Pr \leq 10$) and Reynolds number ($Re$) ($1 \leq Re \leq 100$). The influence of convection is analyzed with Peclet number ($Pe = Re.Pr$). The flow distribution affects significantly temperature distributions at high $Pe$ irrespective of $Da$. Effect of Peclet numbers has been further investigated for both natural convection and forced convection dominant regimes at high $Da$. Strong coupling between flow fields and temperature are observed at high $Pe$.

Keywords: Mixed convection, Porous medium, HOC scheme, Peclet number.

1. Introduction

The lid-driven cavity problem is recognized as the most popular benchmark problem in computational fluid dynamics (CFD) literature. This is not only due to the presence of analytical and numerical benchmark results, but also for its engineering applications where both the momentum and energy transport occur in closed enclosures.

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Among the various cavity geometries, rectangular and square lid-driven cavity geometry with various boundary conditions have received considerable attention in the literature due to its simple geometrical settings as well as various practical applications such as thermal design of buildings, cooling of electronic devices, furnace and nuclear reactors, lubrication and drying technologies, and chemical and food processing. The majority of works dealing with convection in enclosures is restricted to the cases of simple geometry like rectangular, square cavities. But the configurations of actual enclosures occurring in practice are often far from being simple. Ilyican and Bayazitoglu [2] investigated natural convective flow and heat transfer within a trapezoidal enclosure with parallel cylindrical top and bottom walls at different temperatures and plane adiabatic side walls. Karyakin [3] reported two-dimensional laminar natural convection in enclosures of arbitrary cross-section. Thermosolutal heat transfer within trapezoidal cavity heated at the bottom and cooled at the inclined top part was investigated by Boussaid et al. [4]. Very recent Ismael et al. [5] investigated the mixed convection heat transfer in a trapezoidal cavity. They noticed that the behavior of Nusselt number is different from Richardson number depending on the direction of the lid. Battachharya et al. [6] analyzed the flow structure and temperature patterns during mixed convection in a trapezoidal cavity with a cold top moving wall and the bottom wall heated by two different modes – isothermally and non-isothermally. A few other applications of mixed convection were also carried out by other investigators ([7]-[11]). The present study deals with a mixed convection flow within a trapezoidal enclosure where the bottom wall is heated uniformly and all other walls are cooled by means of a constant temperature whereas the top wall is allowed to move in its own plane at a constant speed. We have computed the results using our recently proposed fourth order accurate compact scheme [1] on nonuniform grids. From literature survey, it is noted that the role of flow fields on temperature distributions during mixed convection is poorly understood. The objective of this work is to present a generalized framework on understanding the flow and thermal coupling based on Peclet number.

![Figure 1. (a) Trapezoidal flow configuration with boundary conditions (b) Physical Plane](image)

**1.1 Mathematical formulation and numerical method:**

The physical domain under investigation is a two-dimensional trapezoidal cavity filled with fluid saturated porous medium. In Fig. 1, we have generated a grid system consisting of horizontal lines parallel to the bottom and top boundaries and inclined lines of gradually varying slope provides accurate representation of the boundaries. The transformed coordinates from physical \((x, y)\) plane to computational \((\xi, \eta)\) plane are given by the relations [12],

\[
x = \frac{W_2 - (W_2 - W_1)\eta}{\xi} - (W_1 - W_2)\frac{\eta}{2},
\]

\[
y = \eta H,
\]
where $H$ is the height of the cavity and $W_1$ and $W_2$ are the dimensions of the horizontal walls.

The velocity boundary conditions are assumed to be no-slip on solid boundaries. The fluid is assumed to be incompressible, Newtonian and laminar. For the treatment of the buoyancy term in the momentum equation, Boussinesq approximation is adopted to account for the variations of temperature as a function of density, and to couple in this way the temperature field to the flow field. The governing equations for unsteady mixed convection flow using conservation of mass, momentum and energy can be written in terms of non-dimensional streamfunction-vorticity ($\psi, \zeta$) formulation as follows:

\[
-\left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) = \zeta \tag{1}
\]

\[
\frac{\partial \zeta}{\partial t} - \frac{1}{Re} \frac{\partial^2 \zeta}{\partial x^2} - \frac{1}{Re} \frac{\partial^2 \zeta}{\partial y^2} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} + \frac{1}{Re Da} \zeta = \frac{Gr}{Re^2} \frac{\partial T}{\partial x} \tag{2}
\]

\[
\frac{\partial T}{\partial t} - \frac{1}{Re Pr} \frac{\partial^2 T}{\partial x^2} - \frac{1}{Re Pr} \frac{\partial^2 T}{\partial y^2} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = 0 \tag{3}
\]

Where, $Gr$, $Re$ and $Pr$ are respectively the Grassh of number, Reynolds number and Prandtl number. The dimensionless boundary conditions are as follows:

$u = 0, v = 0$ and $T = 0$ for $x = 0$ and $0 \leq y \leq 1$

$u = 0, v = 0$ and $T = 0$ for $x = 1$ and $0 \leq y \leq 1$

$u = 0, v = 0$, and $T = 1$ for $y = 0$ and $0 \leq x \leq 1$;

$u = 1, v = 0$, and $T = 0$ for $y = 1$ and $0 \leq x \leq 1$.

These governing equations (1) - (3) can be written in the transformed plane in general as

\[
\frac{\partial \phi}{\partial t} + l \frac{\partial^2 \phi}{\partial \xi^2} + a \frac{\partial^2 \phi}{\partial \eta^2} + g \frac{\partial^2 \phi}{\partial \xi \partial \eta} + b \frac{\partial^2 \phi}{\partial \xi^2} + c \frac{\partial \phi}{\partial \xi} + d \frac{\partial \phi}{\partial \eta} + p \phi = f \tag{4}
\]

Where $l$ is a constant and the coefficients $a, b, c, d, g, p$ are the transformed coefficients and $f$ is the transformed part of the source function in the computational plane. Here, $\phi$ stands for $\psi, \zeta, T$. The differential operators are used for the transformation as follows:

\[
\begin{pmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{pmatrix} =
\begin{pmatrix}
x_\xi & y_\xi \\
x_\eta & y_\eta
\end{pmatrix}^{-1}
\begin{pmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta}
\end{pmatrix}
\]

Now, we discretize the transformed equations of (1)- (3) spatially using a fourth-order accurate compact scheme as proposed in [1], on nonuniform grids. Let the approximate value of a function $\phi$ at a mesh point $(\xi_i, \eta_j)$ is denoted by $\phi_{i,j}$. The compact finite difference formulae for a reference point $(\xi_i, \eta_j)$ involves the nearest eight neighboring mesh points with the mesh spacing $h$ lying in the computational stencil of the nine compact grid points

\[
\begin{pmatrix}
\phi_{i-1,j+1} & \phi_{i,j+1} & \phi_{i+1,j+1} \\
\phi_{i-1,j} & \phi_{i,j} & \phi_{i+1,j} \\
\phi_{i-1,j-1} & \phi_{i,j-1} & \phi_{i+1,j-1}
\end{pmatrix}
\]

Assuming the transformed domain to be rectangular and constructing on it a uniform rectangular mesh of steps $h$ in the $\xi$- and $\eta$-directions respectively, the standard central difference approximation to equation (4) (i, j)-th node is given by
\[ l \delta_i^+ \phi_{i,j}^n + a_{i,j} \delta_i^2 \phi_{i,j}^n + g_{i,j} \delta_i \eta \delta_i \phi_{i,j}^n + b_{i,j} \delta_i \eta^2 \phi_{i,j}^n + c_{i,j} \delta_i \eta \phi_{i,j}^n + d_{i,j} \delta_i \phi_{i,j}^n + p_{i,j} \phi_{i,j}^n - T_{i,j}^n = f_{i,j}, \]

where \( \phi_{i,j}^n \) denotes \( \phi(\xi_i, \eta_j) \); \( \delta_i \), \( \delta_\eta \) and \( \delta_i^2 \), \( \delta_\eta^2 \) are the first and second order central difference operators along \( \xi \)- and \( \eta \)-directions respectively, and \( \delta_i \delta_\eta \) is the mixed second order central difference operator. The truncation error \( T_{i,j}^n \) is given by

\[ T_{i,j}^n = h^2 \left[ \frac{\partial^4 \phi}{\partial \xi^4} + 2g \frac{\partial^4 \phi}{\partial \xi^2 \partial \eta} + 2c \frac{\partial^4 \phi}{\partial \xi^3 \partial \eta} + b \frac{\partial^4 \phi}{\partial \eta^4} + 2d \frac{\partial^4 \phi}{\partial \xi \partial \eta^3} \right]_{i,j}. \]

These higher order derivatives can be approximated as second order accurate as in [13]. For temporal integration, the Crank-Nicolson iterative method is adopted. For more detailed derivations of the fourth-order compact scheme of the unsteady N-S equations the reader is referred to the works of ([1], [13]).

The governing equations are highly non-linear and the use of non-uniform grid invariably leads to non-symmetric matrices which attract us to use hybrid BiCGStab (Van Der Vorst [14]) algorithm for solving the resultant linear algebraic equations. This improves the convergence behavior of the algorithm. The steady-state results have been produced with a time step \( \Delta t = 0.001 \). Because of compactness and higher-order accuracy, this treatment may be taken as the model for similar computations. It is mentioned here that the steady-state solution is assumed to reach when the following condition is satisfied:

\[ \max |X_{i,j}^{(n+1)} - X_{i,j}^{(n)}| < 0.5 \times 10^{-6} \]

where \( X_{i,j}^{(n)} \) denotes numerical value of \( \xi_{i,j} \) or \( T_{i,j} \) at \( n \)-th time level.

2. Results and Discussions:

To assess the numerical accuracy of our computer code, we have compared the results of the problem described in Mcquain et al.[15]. In table 1 we compare our data for location of the center of the primary eddy and the corresponding vorticity with Mcquain et al. [15]. The results are in good agreement with previous results [15].

Results are shown for various parameters such as Darcy number (Da), Prandtl number (Pr), Grashof number (Gr) and Reynolds number (Re). The ranges of Da are within \( 3 \times 10^1 \leq Da \leq 10^3 \) for the simulations. Three representative values of Pr such as 0.026 (molten metal), 0.7 (air) and 10 (water) are considered for the simulation. Intensity of mixed convection is governed by Gr and Re. Role of convection on thermal characteristics is governed by Peclet number (Pe=Re. Pr) which is the ratio between heat transfer by convection and heat transfer by conduction. The flow and temperature fields are strongly coupled or the convective heat transport is dominant if Peclet number (Pe=Re. Pr) is high.

The effect of Darcy number for Pr=0.7, Re=100 i.e. Pe=70 and Gr=\( 10^5 \) is shown in Fig. 2 and Fig. 3. It is noted here that the primary circulation occupies most of the cavity and strength of circulation is weak as maximum value of streamfunction is 0.0001 at Da=\( 10^{-5} \). It is interesting to observe that, flow characteristics couple with temperature distributions at higher Pr limit. Based on energy balance equation (3), it may be inferred that Peclet number (Pe=RePr) is high (Pe=70) to induce convective effect on temperature distributions. However, intensity of flow and temperature are not strongly coupled at a smaller Da to increase heating rate at the top portion of the cavity (see Fig. 3). In addition, flow intensity is quite less at low Da (Da=\( 10^{-5} \)). Thus, a large thermal gradient near the bottom wall is observed. As the Darcy number increases to \( 10^{-4} \), and to \( 10^{-3} \) the flow intensity for both primary and secondary circulations become gradually stronger.

Analysis has also been carried out to study streamlines and isotherms for various Pr (Pe) with Da=\( 10^{-3} \) with Reynolds numbers (Re=10) at Gr=\( 10^5 \). The results are shown at a representative high Da (Da=\( 10^{-3} \)) and Gr=\( 10^5 \) to study the role of flow fields on thermal characteristics. Fig. 4 and Fig. 5 illustrate distributions on streamlines and isotherms at Re=10 for Gr=\( 10^5 \) with various Pr. It is found that two sets of oppositely rotating circulation cells
occur in each half of the cavity and the right circulation cells expand further near the top wall due to dominance of lid velocity. The effect of convection for various Pr on the temperature field may be explained based on Peclet numbers for each Pr. Flow field and temperature profiles are found to be decoupled and conduction dominant heat transport is observed for Pr=0.026 (Pe=0.26). Therefore, temperature profiles are found to be symmetric. The effect of flow on the temperature distribution can be viewed as the result at higher Pe (Pe=100) with Pr=10 and enhanced thermal mixing due to enhanced convection occurs near the central zone where a large portion is maintained at T \leq 0.3. It is interesting to note that the thermal boundary layer thickness near the top portion of side walls is much smaller than those cases with smaller Pr values. These phenomena in turn affect the flow field. Due to enhanced thermal mixing, at the central zone, buoyancy effect is negligible along the central zone. As a result, the flow induced by natural convection is lesser due to less thermal gradient induced by thermal mixing. Thus the flow due to natural convection would not add a significant contribution to the flow field induced by lid velocity.

Fig. 6 shows the heat transfer rates in terms of local Nusselt number on the bottom wall for (A) different Darcy numbers and (B) different Prandtl numbers. It is found that Nusselt numbers are quite small and less variation are found with distance for low Peclet numbers and Darcy numbers. It is also observed that local heat transfer rates increase with high Peclet number for mixed convection dominant regimes.

3. Conclusions:
Mixed convection flow within a trapezoidal enclosure filled with porous medium has been studied in the present investigation. Role of convection on temperature distributions are assessed via the concept of Peclet number (Pe). It is observed that at low Darcy number (Da=10^{-5}), the isotherms are smooth and monotonic for all the Gr, Pr and Re. It is also interesting to observe that the isotherms are found to be symmetric signifying negligible effect of lid velocity while processing low Pr fluid, corresponding to smaller values of Pe, irrespective of Da even at high Gr.

The natural and forced convection effects are prominent at higher Da (Da=10^{-3}). The effect of mixed convection on the temperature distributions is further quantified by Peclet number (Pe). The significant effect of lid velocity is observed at Re=10 and Pr=10 (Pe=100). It is interesting to observe that the flow intensity is reduced for Pr=10 due to less intensity of buoyancy caused by large isothermal cold fluid in the cavity. It is also found that Nusselt numbers are quite small and having less variations with distance for low Peclet numbers and Darcy numbers. Local heat transfer rates increase with high Peclet number for mixed convection dominant regimes.

Table 1. Comparison of steady-state primary vortex data for the lid-driven cavity flow for $1 \leq \text{Re} \leq 500$ with $\delta = 2/\sqrt{3}$ among the present solutions and those with Mcquain et al. [15].

| Re | Variables | Mcquain et al. [15] | Present Study |
|----|-----------|---------------------|--------------|
| 1  | $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.267, 1.200 (1.747, 2.382) | 0.267, 1.197 (1.769, 2.362) |
| 50 | $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.276, 1.247 (2.110, 2.319) | 0.276, 1.242 (2.102, 2.325) |
| 100| $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.290, 1.105 (2.023, 2.193) | 0.290, 1.208 (2.014, 2.175) |
| 200| $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.305, 1.019 (1.907, 2.105) | 0.305, 1.014 (1.905, 2.100) |
| 400| $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.315, 0.970 (1.848, 2.067) | 0.315, 0.966 (1.869, 2.062) |
| 500| $\psi_{\min} \times \xi_{v,c}$, (x, y) | 0.317, 0.964 (1.848, 2.055) | 0.316, 0.959 (1.868, 2.062) |
Figure 2. Streamline contours for various Darcy numbers with \( Pr=0.7, \ Re=100, \ Pe=70, \ Gr=100000 \)

Figure 3. Temperature contours for various Darcy numbers with \( Pr=0.7, \ Re=100, \ Pe=70, \ Gr=10^5 \).

Figure 4. Streamline contours for various Prandtl numbers with \( Da=0.001, \ Re=10, \ Gr=10^5 \).
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Figure 5. Temperature contours for various Prandtl numbers with Da=0.001, Re=10, Gr=10^5.

Figure 6. Variations of local Nusselt number at bottom wall with Re=10, Gr=10^5.

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