Structural Regularization*

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Abstract

We propose a novel method for modeling data by using structural models based on economic theory as regularizers for statistical models. We show that even if a structural model is misspecified, as long as it is informative about the data-generating mechanism, our method can outperform both the (misspecified) structural model and un-structural-regularized statistical models. Our method permits a Bayesian interpretation of theory as prior knowledge and can be used both for statistical prediction and causal inference. It contributes to transfer learning by showing how incorporating theory into statistical modeling can significantly improve out-of-domain predictions and offers a way to synthesize reduced-form and structural approaches for causal effect estimation. Simulation experiments demonstrate the potential of our method in various settings, including first-price auctions, dynamic models of entry and exit, and demand estimation with instrumental variables. Our method has potential applications not only in economics, but in other scientific disciplines whose theoretical models offer important insight but are subject to significant misspecification concerns.

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1 Introduction

Structural models are causal models based on economic theory. A complete structural model describes economic and social phenomena as the outcomes of individual behavior in specific economic and social environments. The structural approach to data analysis takes a structural model as a truthful representation of the data-generating mechanism and estimates the model parameters from observed data. The estimated model can then be used to make predictions, evaluate causal effects, and conduct welfare analyses\(^1\).

One of the main strengths of structural estimation lies in its ability to make claims of generalizability or external validity. Because a structural model is based on economic theory, its parameters – such as those governing preferences and technology – can be “deep,” or policy-invariant, so that the estimated model can be used to generate predictions in different environments. A key assumption involved, however, is that the model is correctly specified. In practice, there is no such guarantee and structural models are often criticized for relying on strong, unrealistic assumptions and identification by functional form. This has limited the usefulness of the structural approach and its empirical success\(^2\).

In this paper, we propose a new methodology for modeling data that both inherits the desirable property of structural estimation – the ability to make claims of external validity – and incorporates a robustness against model misspecification. The method, which we call the structural regularization estimator (SRE), treats a given structural model as the benchmark model and estimates a flexible statistical model with a penalty on deviance from the structural benchmark. Equivalently, we select the best statistical model to describe the data within a neighborhood of the structural model. We show that even if the structural model is misspecified, as long as it is informative about the true data-generating mechanism, our method can outperform both the (misspecified) structural model and un-structural-regularized statistical models.

\(^1\)See Reiss and Wolak (2007), Heckman and Vytlacil (2007) and Low and Meghir (2017) for surveys on structural estimation.

\(^2\)Heckman (2000): “The empirical track record of the structural approach is, at best, mixed. Economic data, both micro and macro, have not yielded many stable structural parameters. Parameter estimates from the structural research program are widely held not to be credible.” Rust (2014): “Looking back nearly four decades after the Lucas critique paper, it is fair to ask whether structural models really have succeeded and resulted in significantly more accurate and reliable policy forecasting and evaluation.”
Our method belongs to a class of regularized regression models. In contrast to popular methods such as ridge regression and the lasso, which shrink the parameters of a regression model toward zero to achieve a balance between bias and variance, the SRE shrinks the parameters of a statistical model toward those values implied by the structural model so as to achieve a balance between maximizing statistical fit and minimizing deviance from theory.

The SRE permits a Bayesian interpretation of using theory as prior knowledge. From a Bayesian perspective, regularization amounts to the use of informative priors that introduce our beliefs about the observed data (Li and Goel, 2006). Classic priors used for regularization in statistics and machine learning include sparsity and smoothness priors. In this paper, we argue that since theoretical models are formulated based on the results of previously observed information and conducted studies, they should naturally serve as priors for analyzing new evidence.

Our method can be used both for statistical prediction and causal inference. When used for statistical prediction, it contributes to the literature on transfer learning by showing how incorporating theory into statistical modeling can significantly improve out-of-domain prediction. Given a predictive task involving inputs $x$ and outcome $y$, a key limitation with most statistical methods is that they require the distributions governing the training and the test data to be the same in order to guarantee performance. In the machine learning literature, the problem of applying a model trained on a source domain with distribution $P_{xy}^S$ to a target domain with distribution $P_{xy}^T \neq P_{xy}^S$ is known as transfer learning. A majority of research on transfer learning so far has focused on domain adaptation, where the marginal distributions of the inputs differ across domains, i.e. $P_x^S \neq P_x^T$, but the conditional outcome distributions remain the same, i.e. $P_{y|x}^S = P_{y|x}^T$.

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3. This remains true for state-of-the-art deep learning models. See Donahue et al. (2014) and Yosinski et al. (2014) for discussions on how features extracted from deep convolutional neural networks trained on large image datasets are susceptible to various domain shifts.

4. Several definitions of domain exist in the transfer learning literature. In this paper, given $(x, y) \in O$ and a joint distribution $P_{xy}$ on $O$, we define domain as a pair $\langle O, P_{xy} \rangle$. Note that this notion of domain is different from that of the domain of a function.

5. See Pan and Yang (2010) for a survey on transfer learning. Ben-David et al. (2010) provides a theoretical treatment on learning from different domains.

6. Also known as covariate shift or transductive transfer learning (Pan and Yang, 2010).
Huang et al., 2007; Jiang and Zhai, 2007; Sugiyama et al., 2008) or by finding a domain-invariant transformation (Pan et al., 2010; Gopalan et al., 2011). Few studies, however, have dealt with the more difficult problem of when both $P_x$ and $P_{y|x}$ change across domains.

In this paper, we note that transfer learning can be viewed as a counterfactual prediction problem. If the source and the target domain are governed by the same data-generating mechanism, then a structural model that correctly describes this mechanism, when estimated on the source domain, will generalize naturally to the target domain, even if both the marginal and the conditional distributions have changed. In the context of transfer learning, the external validity of a structural model translates into domain-invariance. Fundamentally, this is because causal relationships are more stable than statistical relationships (Pearl, 2009). On the other hand, if a structural model is misspecified yet informative about the data-generating mechanism, then it may not compete with the best statistical models in-domain, but can still provide useful guidance for extrapolating out-of-domain prediction whether we are given a correctly specified or a misspecified but informative structural model.

Our method also contributes to the literature on causal effect estimation by offering a

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7 This includes the more recent deep domain adaptation literature that employs deep neural networks for domain adaptation. See Glorot et al. (2011); Chopra et al. (2013); Ganin and Lempitsky (2014); Tzeng et al. (2014); Long et al. (2015). Wang and Deng (2018) provides an overview of this literature in the context of computer vision.

8 The problem is known as inductive transfer learning (Pan and Yang, 2010). While a number of methods have been proposed to deal with this problem, they all require target domain data in training – we need to observe some $\{x_i, y_i\}$ in the target domain. See Schwaighofer et al. (2005); Dai et al. (2007); Gao et al. (2008); Wang and Schneider (2014). These methods mostly adapt multi-task learning algorithms and are not solutions to the problem of generalizing model predictions to different domains in a strict sense.

9 Motivated by the idea that causal relationships are more stable, Rojas-Carulla et al. (2018) propose “causal transfer learning.” Kuang et al. (2020) propose “stable prediction”. Both studies rely on the assumption that a subset of the input variables $v \subseteq x$ have a causal relation with the outcome $y$ and the conditional probability $P(y|v)$ is invariant across domains. However, it is not true that having a causal relationship implies $P(y|v)$ is domain-invariant. Let $w = x \setminus v$. The assumption only holds under very limited and untestable conditions, namely that $y \perp w|v$ and that the causal effect of $v$ on $y$ is homogeneous.

10 In this paper, we distinguish between the notion of out-of-domain and out-of-sample. Out-of-sample data are test data drawn from the same distribution as the training data.

11 Note that we do not claim superiority over un-structural-regularized statistical models in-domain, since one can always pick a statistical model flexible enough to generate good in-domain (out-of-sample) performance – performance on test data drawn from the same distribution on which the model is estimated. Hence the main contribution of the SRE to statistical prediction is in terms of its out-of-domain performance, i.e. the ability to extrapolate.
way to combine the nonstructural statistical approach to causal inference with the structural approach. The nonstructural approach, also known as the reduced-form approach\textsuperscript{12}, estimates causal effects from observational data using statistical models. Knowledge of the data-generating mechanism is used not to specify a complete causal model, but to inform research designs that can identify the causal effects of interest by exploiting exogenous variations in the data. Reduced-form methods, including selection on observables regression, instrumental variables regression, difference-in-differences estimation and so on, are widely used in applied economic analyses. At their best, these methods take advantage of credible sources of identifying information to deliver estimates that have high internal validity\textsuperscript{13}. On the other hand, they have also been criticized for learning effects that are local and lack justifications for external validity. Which approach should be preferred – the structural or the reduced-form – has been the subject of a long-standing debate within the economics profession\textsuperscript{14}. This debate has at times been framed as a disagreement over the role of theory in data analysis, with some authors emphasizing the limits to inference without theory (Wolpin, 2013) and others emphasizing the limits with theory (Rust, 2014).

We show that the SRE offers a way to reconcile and synthesize these two competing approaches and philosophies. Theory, in our approach, informs but not dictates data analysis. Technically, by using structural models to regularize the functional form of reduced-form models, we can effectively select models that sit “in the interior of the continuum between reduced-form and structural estimation” (Chetty, 2009). The resulting estimator has the ability to leverage the strengths of both approaches – the internal validity of reduced-form

\textsuperscript{12}As Chetty (2009) points out, the term “reduced-form” is largely a misnomer, whose meaning in the econometrics literature today has departed from its historical root. Historically, a reduced-form model is an alternative representation of a structural model. Given a structural model $M(x, y, \epsilon) = 0$, where $x$ is exogenous, $y$ is endogenous, and $\epsilon$ is unobserved, if we write $y$ as a function of $x$ and $\epsilon$, $y = f(x, \epsilon)$, then $f$ is the reduced-form of $M$ (Reiss and Wolak, 2007). Today, however, applied economists typically refer to nonstructural, statistical treatment effect models as “reduced-form” models. Perhaps reflecting the informal nature of the terminology today, Rust (2014) gives the following definitions of the two approaches: “At the risk of oversimplifying, empirical work that takes theory “seriously” is referred to as structural econometrics whereas empirical work that avoids a tight integration of theory and empirical work is referred to as reduced form econometrics.”

\textsuperscript{13}Angrist and Pischke (2010) offer an account of what they call “the credibility revolution” – the increasing popularity of quasi-experimental methods that seek natural experiments as sources of identifying information. Our definition of reduced-form methods include both quasi-experimental and more traditional, non-quasi-experimental methods that use prior information to locate exogenous sources of variation.

\textsuperscript{14}See Rosenzweig and Wolpin (2000); Angrist and Pischke (2010); Keane (2010a,b); Nevo and Whinston (2010); Deaton (2010); Heckman (2010) for different perspectives on the structural vs. reduced-form debate.
methods and the external validity of structural estimation – while defending against their weaknesses\(^{15}\).

We demonstrate the effectiveness of our approach using a set of simulation experiments designed to showcase its power under a variety of realistic settings in applied economic analyses, including first-price auctions, dynamic models of entry and exit, and demand estimation with instrumental variables. For each experiment, we compare the in-domain and out-of-domain performance of our estimator with that of structural and (reduced-form) statistical estimation. We consider a number of scenarios in which the benchmark structural model is misspecified. In particular, we consider cases in which individual agents deviate from perfect rationality and display various degrees of non-optimizing behavior or boundedly-rational expectations. These cases pose significant challenges to structural estimation due to a lack of identifiable, consensus models for non-rational behavior. Dynamic models in both macro- and microeconomics, for example, have long relied on the rational expectations assumption despite its well-known limitations. In all of these cases, we show that based on benchmark models that assume perfect rationality, the SRE is nevertheless able to obtain results that are much closer to the true non-rational models and, as a consequence, generates much more accurate out-of-domain predictions than (reduced-form) statistical models.

Several authors have proposed combining structural and reduced-form estimation (Chetty, 2009; Heckman, 2010). Their solution is to use structural models to derive sufficient statistics for the intended analysis and then use reduced-form methods to estimate them. In comparison, we offer a general algorithm rather than relying on ad hoc derivations\(^{16}\). In a paper concurrent with ours, Mao and Xu (2020) propose two novel ways for combining structural and reduced-form models, one with a doubly robust construction and the other a weighted ensemble. Their methods can be viewed as complementary to ours.

Our method is most closely related to Fessler and Kasy (2019) (FK) who also propose the idea of using theory to regularize statistical models. In their framework, theory is represented as a set of constraints on the parameters of a statistical model. They propose an empirical Bayes approach that first estimates the statistical model without constraints and

\(^{15}\)As a price to pay, the SRE largely loses its structural interpretation and cannot be used to conduct welfare analyses. We discuss this limitation in section on page 43.

\(^{16}\)However, our method cannot be used to conduct welfare analysis, which is the focus of Chetty (2009).
then project the estimated parameters, \( \hat{\beta} \), onto a subspace defined by theoretical restrictions. These projected values, \( \hat{\beta}_0 \), are then used as priors to obtain the parameters’ posterior means which shrink \( \hat{\beta} \) towards \( \hat{\beta}_0 \). Compared with their approach, our method is different in its construction and has arguably a number of key advantages. First, FK assumes a statistical model whose parameters are identified and are consistently and unbiasedly estimated in the absence of theoretical constraints. We do not impose such assumptions. The statistical model that we shrink toward our structural benchmark can be complex and high-dimensional. While FK also assumes their statistical model to be correctly specified, we regard ours as an approximation to an unknown target function, allowing the potential use of adaptive methods such as random forests and neural nets. Second, a key requirement for FK is that theory has to be expressed as a set of constraints on the statistical parameters. This puts significant limitations on the type of theoretical models that can be considered as well as requires an ad hoc approach to find a statistical model that nests the theoretical model for each application. In contrast, the structural model that we use as our benchmark can be highly complicated, whose assumptions have no obvious ways of being expressed as a set of constraints on a statistical model, and whose parameter space can have a higher dimension. This include models such as dynamic discrete choice models and dynamic games that are widely used in empirical applications. Our method is general and does not require ad hoc constructions. Third and perhaps most importantly, while FK focuses on improving the in-domain performance of statistical estimators with theory, our goal is to achieve both good in-domain and out-of-domain predictive performance and obtain estimates with both internal and external validity. Moreover, we note that when it comes to in-domain performance, compared to a purely statistical approach, FK’s method is mainly useful in a limited setting in which the sample size \( N \) is larger but not significantly larger than the number of parameters \( p \). This is because the empirical Bayes estimator can improve the precision of estimates when the sample size is small, but the improvement vanishes as the sample size grows large. In contrast, we show that the advantage of our SRE relative to a purely statistical approach is in its out-of-domain performance, i.e. its ability to extrapolate\(^{17}\). Such advantage does not

\(^{17}\)Fessler and Kasy (2019) prove that their estimator dominates (in terms of MSE) the unconstrained statistical model that they estimate in the first step. But this result holds for all James-Stein type shrinkage estimators. Conceptually, shrinking to anything has the effect of trampling down the variability of a statistical
disappear no matter how much data we observe in-domain.

Finally, this paper is related to the robustness literature in economics and statistics. Motivated by Hansen and Sargent (2001, 2010, 2020)’s work on robust decision making under model misspecification\footnote{See Watson and Holmes (2016) and Hansen and Marinacci (2016) for surveys of recent developments in statistical decision theory and robust estimation in the presence of model misspecification.}, Bonhomme and Weidner (2018) develop a locally robust minimax estimator that minimizes maximum expected loss over a statistical neighborhood of a benchmark model using local linearization techniques. In a Bayesian setting, Giacomini et al. (2019) analyzes partially identified models by constructing a class of priors in a neighborhood of a benchmark prior and obtaining the optimal posterior minimax decision over this class. Working on structural models, Christensen and Connault (2019) consider a class of models defined by equilibrium conditions and characterize the sensitivity of their counterfactuals to deviations from benchmark specifications of the distribution of unobservables. Like these studies, we are motivated by concerns over model misspecifications. However, our goal is not to achieve robustness in the sense of minimizing the worst-case impact of misspecification in a given neighborhood of the benchmark model or quantifying its local or global sensitivity. The shrinkage method we employ allows arbitrary deviation from the structural benchmark, so that when it is uninformative, the SRE is “reduced” to a (reduced-form) statistical model\footnote{Pun intended.}. Our method allows arbitrary misspecification of the structural model, unlike Christensen and Connault (2019) whose misspecification concerns are limited to the distribution of unobservables.

The rest of this paper is organized as follows. Section 2 provides a motivating example of how our method works in the context of a simple demand estimation problem. Section 3 lays out the details of our algorithm. In section 4 we apply our method to three sets of simulation experiments in the settings of first-price auctions, dynamic models of entry and exit, and demand estimation with instrumental variables and report their results. Section 5 concludes.

model when \( p \) is large relative to \( N \) and thereby helping to lower the MSE. In our simulations, therefore, we compare the performance of the SRE not against the statistical model that we regularize, but against the best statistical model we obtain using model selection. We argue that this is the more meaningful comparison.
2 Motivating Example

As a motivating example, consider a simple demand estimation problem. We observe the prices and quantities sold of a good $x$, as plotted in Figure 1a. Suppose the data are generated by the consumption decisions of $N$ consumers who purchased $x$ at different prices. Each consumer had fixed income $I$ and decided how much to purchase $x$ by solving the problem

$$\max_{q,q^o} u_i(q,q^o) \quad \text{subject to } pq + p^o q^o \leq I$$

Figure 1: Demand Estimation. Dots represent training data. Circles represent out-of-domain test data.
, where \((p, q, p^o, q^o)\) denote respectively the price and quantity of \(x\) and of an outside good \(o\), and

\[
u_i(q, q^o) = [\alpha_i q^o + (1 - \alpha_i) (q^o)\rho]^\frac{1}{\rho},
\]

with \(\rho = -\frac{1}{2}\), implying an elasticity of substitution equal to 0.67\(^{20}\).

We can fit the following statistical model to the data:

\[
q_i = \beta_0 + \beta_1 p_i + \beta_2 p_i^2 + \epsilon_i
\tag{2}
\]

The result is plotted in Figure 1b. If we further make the causal assumption that changes in \(p\) are exogenous, then (2) represents a reduced-form estimate of the individual demand curve.

The model fits the data well\(^{21}\) and would suffice if our goal is to make sales predictions \textit{in-domain} or obtain an \textit{internally valid} demand curve estimate. However, the fit becomes bad once we extrapolate outside the observed range of prices, as shown in Figure 1c. More sophisticated statistical and machine learning models wouldn’t help. In particular, domain adaptation methods do not apply since both the marginal distribution of \(p\) and the conditional distribution of \(q\) change once we extrapolate outside the observed domain.

On the other hand, structurally estimating model (1) would yield an estimated curve that has both internal and external validity (Figure 1c). This is not surprising since (1) describes the true data-generating mechanism. What happens if we estimate a structural model that is incorrectly specified? Figure 1d shows the result of estimating (1) but assuming \(\rho = 0.5\)^{22}. The structural fit is now poor both in-domain and out-of-domain, highlighting the fact that the validity of the structural approach hinges crucially on the model being correct.

Our structural regularization approach offers a way to combine statistical and structural models to address the shortcomings of each. Figure 1d also shows the result of structural

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\(^{20}\{\alpha_i\}\) are generated as follows:

\[
\alpha_i = \frac{\exp(\xi_i)}{1 + \exp(\xi_i)}, \quad \xi_i \sim \mathcal{N}(0, 0.5)
\]

\(^{21}\)In practice, (2) is selected from a set of nested polynomial models based on AIC.

\(^{22}\)That is, instead of estimating both \((\alpha_i, \rho)\) from the data, we estimate \(\alpha_i\) only while treating \(\rho = 0.5\) as an assumption of the model. The assumption, of course, is incorrect in this case.
regularization using the misspecified structural model as the benchmark. Compared to the structural fit, the SRE fit is closer to the true model both in-domain and out-of-domain. Compared to the statistical fit, the SRE performs slightly worse in-domain but significantly better out-of-domain\textsuperscript{23}. While we formally present the structural regularization method in the next section, this example helps illustrate why a misspecified structural model can be useful: although we misspecify the utility function, the assumption of consumer utility maximization subject to budget constraints provides important information on the relationship between price and demand that can be used to regulate the behavior of statistical models. The SRE is therefore able to achieve a balance between producing accurate descriptions of the data and incorporating theoretical (economic/behavioral) insight that allows it to better extrapolate beyond the observed domain.

3 Methodology

In this section, we first lay out our method in the context of conditional mean estimation. We then present it in the general framework of penalized extremum estimation and show how it can used to fit quantities identified via moment conditions. In each case, we discuss how our method can be used both for statistical prediction and causal inference.

3.1 Overview

We begin by considering the following statistical prediction problem: given variables \((x, y) \in \mathcal{O}\), let \(P_{xy}\) be a joint distribution defined on \(\mathcal{O}\) that governs \((x, y)\). Our goal is to learn a target function \(f(x)\) that minimizes the expected \(\ell_2\) loss \(E_{P_{xy}} [(y - f(x))^2]\). Equivalently, we are interested in estimating the conditional expectation function \(f(x) = E_{P_{y|x}} [y|x]\). We may not have access, however, to a random sample from \(P_{xy}\). Instead, we observe data \(D = \{(x_i, y_i)\}_{i=1}^N, (x_i, y_i) \in \mathcal{O}' \subseteq \mathcal{O}\), with data-generating probability distribution \(P'_{xy} = \mathbb{P}_{xy|(x,y)\in \mathcal{O}'}\). Classic statistical and machine learning algorithms built on the assumption that

\textsuperscript{23}To generate the results of this example, we shrink a 5-degree polynomial toward the structural benchmark. Note that if we compare the resulting SRE fit with this 5-degree polynomial in-domain, as in ?, the SRE fit will always perform no worse.
the training data is a random sample of the distribution of interest will thus have difficulty learning \( f \) from \( \mathcal{D} \).

We assume that we have at our disposal an identifiable structural model \( \mathcal{M} \) that we believe may describe the causal mechanism that generates \( \mathbb{P}_{xy} \). However, the model may also be misspecified. With this setup, our structural regularization estimator proceeds in two stages. In the first stage, we estimate the structural model \( \mathcal{M} \) on the data \( \mathcal{D} \) to obtain \( \hat{\mathcal{M}} \). We then use \( \hat{\mathcal{M}} \) to generate synthetic data \( \mathcal{D}_M = \{(x^M_i, y^M_i)\}_{i=1}^M \), \( (x^M_i, y^M_i) \in \mathcal{O} \). This is generally feasible since structural models are generative models capable of simulating new data and since \( \mathcal{M} \) is a causal model for \( \mathbb{P}_{xy} \), it can be used to simulate data on the entire domain \( \langle \mathcal{O}, \mathbb{P}_{xy} \rangle \) rather than on \( \langle \mathcal{O}', \mathbb{P}'_{xy} \rangle \) only. Based on the estimated model \( \hat{\mathcal{M}} \), we can also compute \( f^M(x) = \mathbb{E}^M[y|x] \) – the implied conditional expectation of \( y \) according to \( \hat{\mathcal{M}} \).

In the second stage, we estimate a flexible statistical model \( g(x; \theta) \) by seeking solution to the following problem:

\[
\min_{\theta \in \Theta} \left\{ \sum_{i=1}^N (y_i - g(x_i; \theta))^2 + \lambda \cdot \Omega(\theta, \hat{\theta}_M) \right\}
\]

(3)

where \( \Omega(.,.) \) is a distance function, \( \lambda \geq 0 \) is a penalty parameter, and \( \hat{\theta}_M \) is obtained by fitting \( g \) to \( \mathcal{D}_M \), i.e.

\[
\hat{\theta}_M = \arg \min_{\theta \in \Theta} \sum_{i=1}^M (y^M_i - g(x^M_i; \theta))^2
\]

(4)

\[
= \arg \min_{\theta \in \Theta} \sum_{i=1}^M (f^M(x^M_i) - g(x^M_i; \theta))^2
\]

(5)

\( g(x; \hat{\theta}_M) \) represents a statistical approximation to the model derived conditional mean,

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\( \text{24} \) In practice, this means that if we know, at the time of estimation, where we want to apply our model, i.e. the target domain input space, then we can use the estimated structural model to generate synthetic data on the target domain in addition to the source domain in this first stage.

\( \text{25} \) In practice, to avoid overfitting, one can either generate a very large synthetic set \( (x^M_i, y^M_i) \) or fit \( g \) directly to \( (x^M_i, f^M(x^M_i)) \), where \( x^M_i \) belongs to a grid of possible values of \( x \).
The method of indirect inference (Gourieroux et al., 1993), widely used for fitting structural models whose complexity makes direct likelihood evaluation difficult, also relies on the use of approximating statistical models generated by fitting to synthetic data.

Thus, in contrast to Fessler and Kasy (2019), our method does not require the structural model $\mathcal{M}$ to be expressed as a set of constraints on the parameters of a statistical model $g(x; \theta)$.
3.2 Bayesian Interpretation

The SRE permits a Bayesian interpretation. Specifically, in the case of $\ell_2$ regularization, $\hat{\theta}$ is the posterior mode of $\theta$ under a Gaussian prior centered around $\hat{\theta}$. This is illustrated in Figure 2 for a one-dimensional parameter. The standard deviation of the prior distribution is inversely proportional to regularization strength – the smaller the standard deviation, the larger the corresponding $\lambda$ is and the more confidence is placed on $\hat{\theta}$ being the “true value.”

As in Fessler and Kasy (2019), the use of informative priors centered around theoretically derived values gives the resulting estimator an appealing understanding of using theory as prior knowledge for analyzing new evidence.\footnote{Our method has a proper Bayesian interpretation due to our sample-splitting strategy (section 3.4) that separates the training data used for first and second stage estimation. Thus, from the perspective of second stage estimation, $\hat{\theta}$ is exogenously given, so that the prior distribution centered around it does not depend on the data. \footnote{See, e.g. James et al. (2013). For $\ell_1$ regularization, the corresponding prior is a double-exponential (Laplace) distribution (Tibshirani, 1996). Murphy (2012) provides more general discussions on the connection between regularization and MAP (maximum à posteriori) Bayesian inference. \footnote{Note that the estimator obtained by Fessler and Kasy (2019) is the posterior mean rather than the posterior mode.}}
3.3 Causal Inference

Causal Effect Estimation under Unconfoundedness In this section, we adapt the estimator introduced in section 3.1 to the problem of causal effect estimation under unconfoundedness. Let the observed variables be \((y, d, w) \in \mathbb{R} \times \mathbb{R} \times \mathcal{W}\), where \(y\) is the outcome variable, \(d\) is the treatment variable, and \(w\) is a set of control variables. We are interested in the causal effect of \(d\) on \(y\). Specifically, let our target be the average treatment effect (ATE) denoted by \(\tau\). We allow \(\tau\) to be fully nonlinear and heterogeneous, i.e. \(\tau = \tau (d, w)\). Under the unconfoundedness assumption of Rosenbaum and Rubin (1983)\(^{31}\),

\[
\tau (d, w) = \frac{\partial}{\partial d} \mathbb{E} [y|d, w] \tag{7}
\]

Let \(x = (d, w)\). The task of estimating \(\tau (d, w)\) is thus equivalent to the task of estimating \(\mathbb{E} [y|x]\). Suppose now that we have a structural model \(\mathcal{M}\) that describes the causal mechanism that generates these variables\(^{32}\), then we can use the SRE to produce \(g (x; \hat{\theta}) = \hat{\mathbb{E}} [y|x]\), from which we can derive \(\hat{\tau} (d, w)\)\(^{33}\).

As the preceding discussion shows, when the goal is to estimate the ATE under unconfoundedness, the difference between the reduced-form statistical approach and the structural approach boils down to a difference in the choice of the functional form of \(\mathbb{E} [y|x]\), with the former traditionally relying on simple linear models – although recent studies increasingly adopt more complex nonlinear and adaptive machine learning models, while the latter derive the functional form from theory. In a sense, one can argue that critics on either side

\(^{31}\)Using the notations of the Rubin causal model (Rubin, 1974), suppose the treatment variable \(d\) takes on a discrete set of values, \(d \in \{1, \ldots, D\}\), then the unconfoundedness – or conditional exchangeability – assumption can be stated as

\[
d \perp \perp (y(1), \ldots, y(D))|w
\]

, where \(y(d)\) is the potential outcome of \(y\) associated with treatment \(d\). This assumption is satisfied if \(d\) is not associated with any other causes of \(y\) conditional on \(w\). A more precise statement on the sufficient conditions for satisfying this assumption, made in the language of causal graphical models based on directed acyclic graphs (DAGs), is that \(w\) satisfies the back-door criterion (Pearl, 2009).

\(^{32}\)Importantly, \(\mathcal{M}\) does not have to support the unconfoundedness assumption, i.e. in the causal structure assumed by \(\mathcal{M}\), \(w\) does not have to satisfy the back-door criterion. This is because the structural model is used to aid the estimation of \(\mathbb{E} [y|d, w]\). The identifying assumption required for interpreting \(\partial \mathbb{E} [y|d, w]/\partial d\) as the (conditional) ATE remains that of unconfoundedness.

\(^{33}\)Technically, \(\tau (d, w)\) is the conditional ATE. With a slight abuse of notation, the population ATE \(\tau (d) = \mathbb{E}_w [\tau (d, w)]\).
of the methodological debate are motivated by a shared concern over model misspecification. Advocates for the reduced-form approach are concerned about the misspecification of \( E[y|x] \) due to the often strong and unrealistic assumptions – causal as well as parametric – made in structural models, while those advocating for the structural approach are concerned about misspecifications due to not incorporating theoretical insight – functional forms such as constant elasticity of substitution (CES) aggregation and the gravity equation of trade often encode important prior economic knowledge that sophisticated statistical and machine learning methods would not be able to capture based on training data alone\(^\text{34}\). The SRE addresses both of these concerns: our two-stage procedure effectively searches through a combined statistical and structural model space to arrive at an optimal functional form of \( E[y|x] \) that defends against both types of misspecifications.

**Instrumental Variables** When the unconfoundedness condition does not hold – when there is *unmeasured* confounding – one of the most widely used strategies in reduced-form inference is to rely on the use of instrumental variables, which are auxiliary sources of randomness that can be used to identify causal effects. Let our reduced-form statistical model be \( y = g(x; \theta) + \epsilon, x = (d, w) \), where \( \tau(d, w) = \partial g(x; \theta)/\partial d \) and \( \epsilon \) is a noise term that may be correlated with \( d \)\(^\text{35}\). If we have access to a variable \( z \) that is correlated with treatment \( d \) and is related to outcome \( y \) only through its association with \( d \), then \( z \) can serve as an instrument for \( d \)\(^\text{36}\). In general, given \( \theta \in \mathbb{R}^k \) and instrument \( z \in \mathbb{R}^l, l \geq k \), \( \theta \) can be identified via the following moment conditions:

\[
E[z(y - g(x; \theta))] = 0
\]

\(^{34}\)Rust (2014) makes a similar point: “Notice the huge difference in world views. The primary concern of Leamer, Manski, Pischke, and Angrist is that we rely too much on assumptions that could be wrong, and which could result in incorrect empirical conclusions and policy decisions. Wolpin argues that assumptions and models could be right, or at least they may provide reasonable first approximations to reality.”

\(^{35}\)In this reduced-form model, \( g(x; \theta) \) is a statistical model for \( E[y(d)|w] \) – the conditional expectation of the potential outcome of \( y \) under treatment \( d \) and \( \epsilon \) is defined as \( y - g(x; \theta) \). Thus by definition, the conditional ATE \( \tau(d, w) = \partial g(x; \theta)/\partial d \). When \( E[de] \neq 0 \), the received treatment \( d \) is related to unobserved factors that also affect \( y \), thus violating the unconfoundedness condition.

\(^{36}\)More precisely, the requirement is that \( E[ze] = 0 \) and \( \rho_{dzw} \neq 0 \), where \( \rho_{dzw} \) is the partial correlation of \( z \) and \( d \) given \( w \). On a causal graph, this translates into the requirement that \( z \) is correlated with \( d \) and that every open path connecting \( z \) and \( y \) has an arrow pointing into \( d \).
Algorithm 1 Structural Regularization with Sample-Splitting

Require: Observed data \( D \)

1: samples \( \{D_1, D_2\} \leftarrow \text{Partition}(D, K = 2) \)
2: output \( \hat{\theta} \leftarrow \text{StructuralRegularization}(D_1, D_2) \)

The function \( \text{Partition} \) randomly partitions a sample into \( K \) equal sized parts. The function \( \text{StructuralRegularization} \) takes in two data samples and uses them to produce the SRE estimates as follows:

1: \text{procedure:} \text{StructuralRegularization}(sample I, sample J) \n2: fit the structural model \( M \) on sample I to obtain \( \hat{M} \) \n3: use \( \hat{M} \) to generate a synthetic data set \( D^M \) \n4: solve problem (4) on \( D^M \) to obtain \( \hat{\theta}^M \) \n5: substitute \( \hat{\theta}^M \) into problem (3) \n6: solve problem (3) on sample J for a grid of \( \lambda \) values and find the optimal \( \lambda^* \) by cross-validation \n7: return \( \hat{\theta} \) as the solution to problem (3) on sample J at \( \lambda = \lambda^* \)

Assume again that we have a structural model \( M \) that describes the causal mechanism governing these variables\(^{37}\). Our SRE would proceed as before in the first stage and solve the following problem in the second stage:

\[
\min_{\theta \in \Theta} \left\{ \bar{m}(\theta)' W \bar{m}(\theta) + \lambda \cdot \Omega \left( \theta, \hat{\theta}^M \right) \right\}
\]

(9)

, where \( \bar{m}(\theta) = \frac{1}{N} \sum_{i=1}^{N} m_i(\theta) \), \( m_i(\theta) = z_i (y_i - g(x_i; \theta)) \) are the moment functions and \( W \) is a \( l \times l \) weight matrix. Once we obtain \( \hat{\theta} \) as a solution to (9), we can derive the conditional ATE as \( \hat{\tau}(d, w) = \partial g \left( x; \hat{\theta} \right) / \partial d \).

3.4 Implementation

In this section, we detail the implementation of our algorithm. We begin by showing that under the setup of section 3.1, our estimator has a closed form solution at any given \( \lambda \) in the special case of \( \Omega(., .) \) being an \( \ell_2 \) regularizer and \( g(x; \theta) \) being linear in \( \theta \).

Consider \( g(x; \theta) = \alpha + x' \beta \), where \( x \in \mathbb{R}^p \), \( \theta = (\alpha, \beta) \). In practice, the constant term \( \alpha \) should not be penalized. Let \( x \) be standardized into \( \tilde{x} \) with mean zero. Then we can

\(^{37}\)\( M \) does not have to contain \( z \). Once we have an estimated model \( \hat{M} \), we can use it to generate and fit \( g \) directly to a synthetic data set \( (x^M_i, g^M(x^M_i)) \) to obtain \( \hat{\theta}^M \), where \( g^M(x^M_i) = \mathbb{E}^M [y(d_i)|w_i] \) is the model derived conditional expectation of the potential outcome under treatment \( d \).
write our model as \( g(\tilde{x}; \tilde{\theta}) = \tilde{\alpha} + \tilde{x}' \tilde{\beta} \), \( \tilde{\theta} = (\tilde{\alpha}, \tilde{\beta}) \). We estimate \( g(\tilde{x}; \tilde{\theta}) \) as follows: in the first stage, after generating synthetic data \( D^M \) based on the estimated structural model \( \tilde{M} \), fitting \( g(\tilde{x}; \tilde{\theta}) \) to \( D^M \) gives \( (\tilde{\alpha}^M, \tilde{\beta}^M) \). In the second stage, because \( \tilde{x} \) is centered, we have \( \tilde{\alpha} = \bar{y} \). Let \( \tilde{y} = y - \bar{y} \). Let \( \Omega(\tilde{\beta}, \tilde{\beta}^M) = \| \tilde{\beta} - \tilde{\beta}^M \|_2^2 \). Then

\[
\tilde{\beta} = \arg\min_{\tilde{\beta}} \left\{ \| \tilde{y} - \tilde{X} \tilde{\beta} \|_2^2 + \lambda \| \tilde{\beta} - \tilde{\beta}^M \|_2^2 \right\} \\
= \left( \tilde{X}' \tilde{X} + \lambda I \right)^{-1} \left( \tilde{X}' \tilde{y} + \lambda \tilde{\beta}^M \right) 
\]

(10), where \( \tilde{X} = [\tilde{x}_1, \ldots, \tilde{x}_N]' \) and \( I \) is the \( p \times p \) identity matrix. In the case that \( \tilde{X} \) is orthonormal, (11) can be expressed as

\[
\tilde{\beta} = \frac{1}{1 + \lambda} \tilde{\beta}^{OLS} + \frac{\lambda}{1 + \lambda} \tilde{\beta}^M 
\]

(12), where \( \tilde{\beta}^{OLS} \) is the least squares estimate. In this case, the SRE can be viewed as a weighted average of statistical and structural estimation.

**Sample Splitting** We use the technique of *sample-splitting* (Angrist and Krueger, 1995) to avoid overfitting and ensure good statistical behavior specially when complex structural models are employed in the first stage\(^{38}\). The idea of sample-splitting is to split the training data into two parts to be used respectively for the two stages of estimation, so that \( \tilde{\theta}^M \) can be treated as exogenously given when \( g(\tilde{x}; \tilde{\theta}) \) is fit in the second stage. The details of our algorithm with sample-splitting are given in Algorithm 1.

The sample-splitting procedure reduces overfitting at a cost of wasting half of the data

\(^{38}\)Angrist and Krueger (1995) propose the use of sample-splitting in the context of instrumental variable estimation. Related ideas in the statistical literature goes back at least to Bickel (1982).
Algorithm 3 K-Fold Forward Cross-Validation

FowardCV is a subroutine for performing cross-validation in STRUCTURAL REGULARIZATION

1: procedure: FowardCV(sample $S$, $K$)
2: \hspace{1em} samples $\{S_1, S_2\} \leftarrow$ FowardSplit ($S$)
3: \hspace{1em} folds $\{I_1, \ldots, I_K\} \leftarrow$ Partition($S_1, K$)
4: \hspace{1em} for all $\lambda \in \Lambda$ do
5: \hspace{2em} for $k = 1 : K$ do
6: \hspace{3em} validation set $V_k \leftarrow I_k \cup S_2$
7: \hspace{3em} training set $T_k \leftarrow S_1 \setminus I_k$
8: \hspace{3em} solve problem (3) on $T_k$ to obtain $\hat{\theta}_k (\lambda)$
9: \hspace{3em} compute the prediction error of $g \left( x; \hat{\theta}_k (\lambda) \right)$ on $V_k$ to obtain validation error $e_k (\lambda)$
10: return optimal tuning parameter $\lambda^* \leftarrow \arg \min_{\lambda \in \Lambda} \frac{1}{K} \sum_{k=1}^{K} e_k (\lambda)$

$\Lambda$ is a grid of $\lambda$ values. The function FowardSplit randomly partitions $S$ into $\{S_1, S_2\}$, satisfying the following condition: let $X_1$, $X_2$, and $X_T$ be compact input spaces associated respectively with $S_1$, $S_2$ and the target domain, then $d_H (X_2, X_T) < d_H (X_1, X_T)$, where $d_H (\cdot, \cdot)$ denotes the Hausdorff distance.

In each stage of estimation. To improve efficiency, we can use the cross-fitting procedure of Chernozhukov et al. (2016, 2017). The idea is to similarly split the original sample into two parts, but alternately use each part for first and second stage estimation, so that each data point will participate in both stages albeit not at the same time. The details of our algorithm with cross-fitting are given in Algorithm 2.

**Forward Cross-Validation** We use cross-validation to choose the optimal penalty $\lambda$ in (3). If we know, at the time of estimation, the target domain on which we want to apply our model, then there are two ways to further improve the out-of-domain performance of our estimator. One is to use the estimated structural model to generate synthetic data on both the source and the target domain in the first stage, as discussed in section 3.1. In this section, we introduce a forward cross-validation procedure as another way to improve out-of-domain performance. The idea is to validate on subsets of the data that are “closer” to the target domain than the subsets on which the model is trained. More specifically, given a sample $S^{39}$, we partition $S$ into two parts, $S_1$ and $S_2$ with associated input spaces $X_1$ and $X_2$, such that $d_H (X_2, X_T) < d_H (X_1, X_T)$, where $X_T$ is the target domain input space and $d_H (\cdot, \cdot)$ is the Hausdorff distance. We then further partition $S_1$ randomly into $K - 1$ equal

\footnote{In practice, $S$ would be the subsample of $D$ on which the second stage estimation is conducted.}
Figure 3: Forward Cross-Validation. Illustrated is one iteration of the procedure, in which the folds in blue are used for training and the folds in brown are used for validation. While fold 1 – 5 are iteratively used for training and validation, fold 6 is always used for validation.

sized subsets and perform $K$-fold cross validation, each time using $K - 2$ subsets of $S_1$ for training and validating on a validation set that contains $S_2$ and the remaining subset of $S_1$. See Algorithm 3 for more details.

The idea of forward CV is perhaps best illustrated in the one-dimensional setting (Figure 3). Here we want to extrapolate the estimated model in the direction of increasing $x$. To this end, we perform cross validation by creating a six-fold partition of the sample data, where the sixth fold lies in the direction of increasing $x$ compared to the remaining five and is always in the validation set. Doing so helps produce tuning parameters whose corresponding models have superior extrapolation performance in the intended direction.

Adaptive Models The statistical model that we shrink toward the structural benchmark can be adaptive itself, allowing the potential use of machine learning methods like random forests and neural nets with structural regularization. Let $h(x; \theta, \gamma)$ be such a model with hyperparameter $\gamma$. To incorporate $h$ into our estimator, we can modify Algorithm 1 as follows. Split the initial training data into three parts: \{$D_1, D_2, D_3$\}. We first fit $h$ to $D_1$ to obtain the optimal $\lambda = \lambda^*$. Let $g(x; \theta) = h(x; \theta, \lambda^*)$. $g(x; \theta)$ then enters into the standard SRE algorithm, with $D_2$ and $D_3$ used respectively for structural estimation and

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\footnote{When the data has a time series structure, a rolling-window design can be used for cross-validation, as is commonly used for model selection in time series forecasting. See section B for an application.}
regularization. The modified algorithm thus becomes a three-stage procedure. Algorithm 2 can be adapted similarly with the three parts of data used alternately for the three stages of estimation.

3.5 Extension

In general, the SRE can be formulated as a penalized extremum estimator that seeks solution to the following problem in the second stage:

$$
\begin{equation}
\min_{\theta \in \Theta} \left\{ \mathcal{L}_g (S; \theta) + \lambda \cdot \Omega \left( \theta, \hat{\theta}^M \right) \right\}
\end{equation}
$$

(13)

, where $\mathcal{L}_g (S; \theta)$ is an objective function associated with $g (x; \theta)$ and evaluated on sample $S$.

This setup encompasses many possibilities. The statistical model $g$ can be discriminative or generative. The objective function can be based on any appropriate loss functions such as the quadratic loss and the cross-entropy (negative likelihood) loss. When $\theta$ is identified via moment functions $E [m (\theta)] = 0$, we obtain (9) as a special case of (13). In addition, in the special case that $g (x; \theta) = x' \theta$, $x \in \mathbb{R}^p$, $\Omega \left( \theta, \hat{\theta}^M \right) = \| \theta - \hat{\theta}^M \|^2_2$, and $m (\theta) = z (y - g (x; \theta))$, where $z \in \mathbb{R}^l$, $l \geq p$ is an instrument for $x$, we have the following analytical solution to (9) for a given $\lambda$:

$$
\hat{\theta} = (X' ZW Z' X + \lambda I)^{-1} \left( X' ZW Z' Y + \lambda \hat{\theta}^M \right)
$$

(14)

, where $X = [x_1, \ldots, x_N]'$, $Z = [z_1, \ldots, z_N]'$, and $I$ is the $p \times p$ identity matrix$^{41,42}$.

4 Applications

In this section, we demonstrate the effectiveness of our method and compare its finite-sample performance with that of statistical and structural estimation in three economic applications using Monte Carlo simulations. Taken together, these exercises cover prediction and causal inference (both under unconfoundedness and confounding) problems, static and dynamic

$^{41}$In practice, it is often desirable as in (10) not to penalize the constant term in $g$.

$^{42}$One can use $W = E [m (\theta_0) m (\theta_0)']^{-1}$, the efficient weight of Hansen (1982), and obtain $\hat{\theta}$ via a two-step procedure. This however may not be the optimal weight for our estimator. We leave the characterization of the asymptotic properties of the SRE as well as the optimal weighting matrix for (9) to future work.
settings, and individual behavior that deviates in various ways from perfect rationality.

A First-Price Auction

In our first application, we consider first-price sealed-bid auctions. Auctions are one of the most important market allocation mechanisms. Over the past twenty years, empirical analysis of auction data has been transformed by structural estimation of auction models based on games of incomplete information\textsuperscript{43}. Structural analysis of auction data views the observed bids as equilibrium outcomes and attempts to recover the distribution of bidders’ private values by estimating relationships derived directly from equilibrium bid functions. This approach, while offering a tight integration of theory and observations, relies on a set of strong assumptions on the information structure and rationality of bidders (Bajari and Hortacsu, 2005).

In this exercise, we conduct three experiments by simulating auction data with varying number of participants under three scenarios. The first scenario features rational bidders with independent private values drawn from a uniform distribution. The second scenario features rational bidders whose values are drawn from a beta distribution. The third scenario features boundedly-rational bidders whose bids deviate from optimal bidding strategies. Assume that we are interested in the relationship between the number of bidders \( n \) and the winning bid \( b^* \), \( \mathbb{E}[b^* | n] \). In each experiment, we estimate \( \mathbb{E}[b^* | n] \) using (a) a statistical model, (b) a structural model, and (c) the SRE. The structural model we use assumes rational bidders with uniform private value distribution and is thus correctly specified for Experiment 1 but

\textsuperscript{43}See Paarsch and Hong (2006); Athey and Haile (2007); Hickman et al. (2012); Perrigne and Vuong (2019) for surveys on econometric analysis of auction data.
misspecified in Experiment 2 and 3. Table 1 summarizes this setup. Below we detail the
data-generating models of the three experiments.

**Setup** Consider a first-price sealed-bid auction with \( n \) risk-neutral bidders with indepen-
dent private value \( v_i \sim \text{i.i.d.} F(v) \). Each bidder submits a bid \( b_i \) to maximize her expected
return

\[
\pi_i = (v_i - b_i) \times \Pr (b_i > \max \{b_{-i}\})
\]  

(15)

, where \( b_{-i} \) denotes the other submitted bids. In Bayesian-Nash equilibrium, each bidder’s
bidding strategy is given by

\[
b(v) = v - \frac{1}{F(v)^{n-1}} \int_0^{v_i} F(x)^{n-1}dx
\]  

(16)

For Experiment 1 and 3, we let \( F \) be \( U(0, 1) \). In this case, the equilibrium bid function
simplifies to \( b(v) = \frac{n-1}{n} v \). For Experiment 2, we let \( F \) be Beta \((2, 5)\). In each experiment, we
simulate repeated auctions with varying number of bidders\(^4\). For Experiment 1 and 2, the
observed bids \( b_i \) are the equilibrium outcomes, i.e. \( b_i = b(v_i) \). For experiment 3, we let \( b_i = \eta_i \cdot b(v_i) \), where \( \eta_i \) follows a normal distribution left-truncated at \( 0, \eta_i \sim \text{i.i.d.} \text{TN}(0, 0.25, 0, \infty) \).

Bidders in Experiment 3 thus “overbid” relative to the Bayesian-Nash equilibrium.

**Simulation** For each experiment, we simulate \( M = 100 \) auctions with number of bidders
\( n_m \) varying between 5 and 30. The observed data thus consist of \( \mathcal{D} = \{(n_m, b^*_m)^{n_m}_{m=1} \}^{M}_{m=1} \). In this
exercise, our goal is to learn \( \mathbb{E}[b^*|n]\), the relationship between the number of bidders and
the winning bid. To this end, three different types of estimators are used to estimate \( \mathbb{E}[b^*|n]\)
from the training data. To assess their performance, we use the true data-generating models
to compute \( \mathbb{E}[b^*|n]\) for \( n \in [5, 50] \), so that we can compare the predictions of the estimators
with the true value both in-domain and out-of-domain.

**Statistical Estimation** To estimate \( \mathbb{E}[b^*|n]\) using a statistical model\(^5\), the data we need
are \( \{(n_m, b^*_m)\}^{M}_{m=1} \), where \( b^*_m \) is the winning bid of auction \( m \). We fit the following \( p \)--degree

\(^{4}\)Assuming the same object is being repeatedly auctioned.

\(^{5}\)Since \( n \) is exogenous, \( \mathbb{E}[b^*|n]\) is also a causal relationship and (17) can also be thought of a reduced-form
model of the effect of the number of bidders on the winning bid.
polynomial to the data:
\[ b^*_m = \beta_0 + \sum_{j=1}^{p} \beta_j n_{m}^j + e_m \] (17)

, where the optimal degree \( p \) is determined based on information criteria.

**Structural Estimation** We structurally estimate the data from each experiment using a model \( \mathcal{M} \) that assumes bidders are rational, risk-neutral, and have independent private values drawn from a \( U(0,1) \) distribution. Under these assumptions, the bidders’ private values can be easily identified from the observed bids in each auction by \( v_i = \frac{n}{n-1} b_i \)\(^{46} \). The structural model makes it even easier to make predictions on the winning bid: the model implies that \( \mathbb{E}[b^* | n] = \frac{n}{n+1} \). No estimation is necessary.

**Structural Regularization** We use \( \mathcal{M} \) as the benchmark model for the SRE and specify a \( 5 \)-degree polynomial for the statistical model \( g(n; \theta) \) that we shrink toward the structural benchmark. Let \( g(\tilde{n}; \tilde{\theta}) \) be the model after \( n \) is standardized, as described in section 3.4. For regularizer, we use
\[ \Omega \left( \tilde{\beta}, \tilde{\beta}^\mathcal{M} \right) = \sum_{j=1}^{5} j \cdot (\tilde{\beta}_j - \tilde{\beta}_j^\mathcal{M})^2 \] (18)

, where \( \tilde{\beta} \) are the non-intercept coefficients of \( \tilde{\theta} \). (18) is commonly used for regularizing polynomial models. It puts more penalty on higher degrees of a polynomial and has the effect of making the resulting fit more stable\(^{47} \). The regularization procedure follows Algorithm 1 with sample-splitting and forward CV.

\(^{46}\)In general, if we do not impose the assumption that \( v_i \overset{i.i.d.}{\sim} U(0,1) \) and assume instead that \( v_i \overset{i.i.d.}{\sim} F(v) \), with \( F \) unknown, then we can identify and estimate \( v_i \) using the following strategy based on Guerre et al. (2000): let \( G(b) \) and \( g(b) \) be the distribution and density of the bids. (16) implies
\[ v_i = b_i + \frac{1}{n-1} G(b_i) \]

Thus, by nonparametrically estimating \( G(b) \) and \( g(b) \) from the observed bids, we can obtain an estimate of \( v_i \).

\(^{47}\)Assuming that the model parametrized by \( \tilde{\beta}^\mathcal{M} \) is more stable, which is typically the case, since it is obtained by fitting to a very large synthetic data set generated by the structural model.
Figure 4: First-price Auction - The relationship between the number of bidders and the winning bid. Dots represent training data. Circles represent out-of-domain test data. (a) - (b): Experiment 1; (c) - (d): Experiment 2; (e) - (f): Experiment 3.
Table 2: First-price Auction - Results

| Experiment | Estimator | In-Domain | | Out-of-Domain | |
|------------|-----------|-----------|-----------------|-----------------|
|            | Bias      | Var       | MSE             | Bias            | Var       | MSE             |
| 1          | Statistical | 0.0037   | 0.0003          | 0.0003          | 0.1256    | 118.8697        | 118.9157       |
|            | Structural | 0.0000   | 0.0000          | 0.0000          | 0.0000    | 0.0000          | 0.0000          |
|            | SRE       | 0.0035   | 0.0001          | 0.0001          | 0.0431    | 0.2690          | 0.2734          |
| 2          | Statistical | 0.1251   | 0.1129          | 0.1426          | 3.5506    | 572.1151        | 595.3736        |
|            | Structural | 9.8255   | 0.0000          | 98.5233         | 12.6631   | 0.0000          | 160.6319        |
|            | SRE       | 0.2772   | 0.0883          | 0.2399          | 0.3359    | 12.7092         | 12.8710         |
| 3          | Statistical | 0.1455   | 0.1987          | 0.2307          | 0.5285    | 18.2734         | 18.5692         |
|            | Structural | 7.7095   | 0.0000          | 60.4870         | 9.8956    | 0.0000          | 98.0982         |
|            | SRE       | 0.2787   | 0.1991          | 0.3215          | 0.4517    | 817.5363        | 817.8250        |

Notes: results are based on 100 simulation trials. Reported are the mean bias, variance, and MSE, averaged over the number of bidders $n$. Since the structural model predicts $\mathbb{E}[b^*|n] = (n - 1)/(n + 1)$, its predictions have zero variance.
Results  Figure 4a and 4b show the results of the first experiment. Figure 4a plots the number of participants \( n \) against the winning bid \( b^* \), the true \( \mathbb{E}[b^*|n] \), as well as predictions by the estimated statistical, structural, and SRE model. All three models fit very well in-domain. Since the structural model is the true model, it predicts the true expected winning bids. The statistical model – here a 2nd degree polynomial – also closely approximates the target function and could suffice if our goal is to obtain a good in-domain fit. Figure 4b plots the results of extrapolating the model predictions from \( n \in [5,30] \) to \( n \in [30,50] \). While the structural predictions still hold true, the statistical fit becomes very bad. On the other hand, the SRE fit remains close to the true relationship both in-domain and out-of-domain and can accurately predict winning bids well beyond the observed range of \( n \).

Figure 4c – 4f show the results of Experiment 2 and 3. In both experiments, the structural model is misspecified. In Experiment 2, it misspecifies the private value distribution. In Experiment 3, it assumes that bidders are rational and the observed bids are Bayesian-Nash equilibrium outcomes when they are not. As a consequence, in both cases, the structural fit deviates from the true model significantly. The statistical model, like in Experiment 1, is able to fit well in-domain but poorly out-of-domain. Remarkably, the SRE continues to perform well despite relying on a misspecified benchmark model. Its predictions are close to the true expected winning bids both in-domain and out-of-domain. Intuitively, the misspecified structural models still provide useful guidance on the functional form of \( \mathbb{E}[b^*|n] \) when we extrapolate beyond the observed domain, as evidenced in Figure 4d and 4f.

In Table 2, we report the mean bias, variance, and mean squared error of the three estimators for 100 simulation runs\(^{48}\). For all three experiments, the SRE has a low MSE comparable to those of the statistical model and of the true structural model in-domain, while achieving a significantly lower out-of-domain MSE than both the statistical model and the structural model when the latter is misspecified.

\(^{48}\)Reported are the mean bias, variance, and mean squared error, averaged over the number of bidder \( n \). Given an estimator \( f \), let \( f^{(r)}(n) \) denote the estimator’s prediction of the winning bid in simulation \( r \), then the empirical pointwise bias of \( f \) at \( n = i \) is \( \text{pwbias}(f,i) = \frac{1}{R} \sum_{r=1}^{R} |f^{(r)}(n = i) - \mathbb{E}[b^*|n = i]| \), where \( R \) is the total number of simulations. The empirical overall bias, or mean bias, of \( f \) is \( \text{bias}(f) = \frac{1}{|X|} \sum_{n \in X} \text{pwbias}(f,n) \), where \( X \) is the space associated with \( n \). The mean variance and the mean MSE are likewise defined.
B Dynamic Entry and Exit

Our second application concerns the modeling and estimation of firm entry and exit dynamics. Structural analysis of dynamic firm behavior based on dynamic discrete choice (DDC) and dynamic game models has been an important part of empirical industrial organization\textsuperscript{49}. These dynamic structural models capture the path dependence and forward-looking behavior of agents, but pays the price of imposing strong behavioral and parametric assumptions for tractability and computational convenience.

In this exercise, we focus our attention on the rational expectations assumption that has been a key building block of dynamic structural models in macro- and microeconomic analyses. The assumption and its variants state that agents have expectations that do not systematically differ from the realized outcomes\textsuperscript{50}. Despite having long been criticized as unrealistic, the rational expectations paradigm has remained dominant due to a lack of tractable alternatives and the fact that economists still know preciously little about belief formation.

We conduct three experiments in the context of the dynamic entry and exit of firms in competitive markets in non-stationary environments. Our data-generating models are DDC models of entry and exit with entry costs and exogenously evolving economic conditions. In our first experiment, agents have rational expectations about future economic conditions. In the second experiment, agents have a simple form of adaptive expectations that assume the future is always like the past. The third experiment features myopic agents who optimize only their current period returns. In all experiments, we are interested in predicting the number

\begin{table}[h]
\centering
\caption{Dynamic Entry and Exit - Setup}
\begin{array}{|c|c|c|}
\hline
\text{Experiment} & \text{True Mechanism} & \text{Structural Model} \\
\hline
1 & Rational Expectations & \\
2 & Adaptive Expectations & Rational Expectations \\
3 & Myopic & \\
\hline
\end{array}
\end{table}

\textsuperscript{49}See Aguirregabiria and Mira (2010); Bajari et al. (2013) for surveys on structural estimation of dynamic discrete choice and dynamic game models.

\textsuperscript{50}More precisely, rational expectations are mathematical expectations based on information and probabilities that are model-consistent (Muth, 1961).
of firms that are in each market each period. To this end, we estimate (a) a statistical model, (b) a structural model, and (c) the SRE. The structural model we estimate assumes rational expectations and is thus correctly specified only in Experiment 1. Table 3 summarizes this setup.

**Setup** Consider a market with \( N \) firms. In each period, the market structure consists of \( n_t \) incumbent firms and \( N - n_t \) potential entrants. The profit to operating in the market at time \( t \) is \( R_t \), which we assume to be exogenous and time-varying. At the beginning of each period, both incumbents and potential entrants observe the current period payoff \( R_t \) and each draws an idiosyncratic utility shock \( \epsilon_{it} \). Incumbent firms then decide whether to remain or exit the market by weighing the expected present values of each option, while potential incumbents decide whether or not to enter the market, which will incur a one-time entry cost \( c \). Specifically, let the entry status of a firm be represented by \((0,1)\). The time-\( t \) flow utility of a firm, who is in state \( j \in \{0,1\} \) in time \( t-1 \) and state \( k \in \{0,1\} \) in time \( t \), is given by

\[
 u_{it}^{jk} = \pi_{it}^{jk} + \epsilon_{it}^k
\]

(19)

where

\[
 \pi_{it}^{jk} = (\mu + \alpha \cdot R_t - c \cdot \mathbb{I}(j = 0)) \cdot \mathbb{I}(k = 1)
\]

(20)

is the deterministic payoff function and \( \epsilon_{it} = (\epsilon_{it}^0, \epsilon_{it}^1) \) are idiosyncratic shocks, which we assume are i.i.d. type-I extreme value distributed. The parameter \( \alpha \) measures the importance of operating profits to entry-exit decisions relative to the idiosyncratic utility shocks.

The *ex-ante* value function of a firm at the beginning of a period is given by

\[
 V_t^j(\epsilon_{it}) = \max_{k \in \{0,1\}} \left\{ \pi_{it}^{jk} + \epsilon_{it}^k + \beta \cdot E_t \left[ V_{t+1}^k \right] \right\}
\]

(21)

\[
 = \max_{k \in \{0,1\}} \left\{ \mathcal{V}_{it}^{jk} + \epsilon_{it}^k \right\}
\]

(22)

, where \( j \) is the firm’s state in \( t-1 \), \( \beta \) is the discount factor, \( \mathcal{V}_{it}^{jk} := E_t \left[ V_t^j(\epsilon_{it}) \right] \) is the expected value integrated over idiosyncratic shocks, and \( \mathcal{V}_{it}^{jk} := \pi_{it}^{jk} + \beta \cdot E_t \left[ V_{t+1}^k \right] \) is the choice-specific conditional value function.
At the beginning of each period, after idiosyncratic shocks are realized, each firm thus chooses its action, \( a_{it} \in \{0, 1\} \), by solving the following problem:

\[
a_{it} = \arg \max_{k \in \{0, 1\}} \left\{ V_{jk}^t + \epsilon_{it}^k \right\}
\]

(23)

, which gives rise to the conditional choice probability (CCP) function:

\[
p_t(k|j) := \Pr \left( a_{it} = k \mid a_{i,t-1} = j \right) = \frac{e^{V_{jk}^t}}{\sum_{\ell=0}^{1} e^{V_{j\ell}^t}}
\]

(24)

, which follows from the extreme value distribution assumption.

Since the value function involves the continuation values \( \mathbb{E}_t \left[ V_{k}^{t+1} \right] \), which requires expectations of the future profits \( (R_{t+1}, R_{t+2}, \ldots) \), its solution requires us to specify how such expectations are formed. In Experiment 1, we assume firms have perfect foresight on \( R_t \). This is a stronger form of rational expectations that assumes individuals knows the future realized values. Firms can then compute \( \mathbb{V}_t^j = \mathbb{E}_t \left[ V_{i}^{j} \left( \epsilon_{it} \right) \right], \ j \in \{0, 1\} \) in a model-consistent way, i.e. based on the distributional assumption of \( \epsilon_{it} \). In Experiment 2, we assume firms have a form of adaptive expectations, according to which beliefs about the future are formed based on past values. Here for simplicity, we assume that firms expect future profits to be always the same as in current period, i.e. \( R_t = R_{t+1} = R_{t+2} = \cdots \). Finally, in Experiment 3, we allow firms to be myopic, so that they do not care about the future and only maximize current payoffs.

**Simulation** For each experiment, we simulate \( N = 10,000 \) firms for \( T = 500 \) periods. The first \( T = 250 \) periods are used for training and the last \( T - T = 250 \) periods are used to assess the out-of-domain performance of our estimators. The training data thus consist of \( D = \left\{ \{a_{it}\}_{i=1}^N, R_t \right\}_{t=1}^T \). We simulate \( R_t \) to follow a rising time trend so that the environment is non-stationary. Figure 5 shows a realized path of \( R_t \). The model parameters for each experiment are chosen so that the entry and exit dynamics over the first \( T \) periods are significantly different from the last \( T - T \) periods, allowing us to better distinguish the performance of the estimators. Appendix A.2 reports the parameter values we use as well as other details of the simulation.
Our goal is to predict \( n_t \) – the number of firms operating in the market in each period. The data we need for statistical modeling are \((n_t, R_t)\). We fit the following nonlinear ARX model to the data:

\[
 n_t = \gamma_0 + \sum_{j=1}^{p} \gamma_j R_t^j + \sum_{\ell=1}^{q} \rho_{\ell} n_{t-\ell} + e_t
\]

, where \((p, q)\) are again determined based on information criteria.

To estimate the DDC model, we use a strategy that builds on Arcidiacono and Miller (2011) and estimates an Euler-type equation constructed out of CCPs. Here we sketch the strategy while presenting its details in Appendix A.2.\(^5\)

A key to our strategy is a rational expectations assumption: we assume that because agents have rational expectations, their expected continuation values do not deviate systematically from the realized values, i.e. \( V_{t+1}^j = \mathbb{E}_t \left[ V_{t+1}^j \right] + \xi_t^j \), where \( \xi_t^j \) is a time-\( t \) expectational error with \( \mathbb{E} (\xi_t^j) = 0 \). Given this assumption, and since our model has the finite dependence property of Arcidiacono and Miller (2011), solution to (21) can be written in the form of the

\(^5\)See Arcidiacono and Ellickson (2011) for a review of related CCP estimators. For empirical implementations, see, e.g. Artuc et al. (2010); Scott (2014).
following Euler equation:

\[ \ln \frac{p_t(k|j)}{p_t(j|j)} = \left( \pi_t^{ij} - \pi_t^{jj} + \beta \left( \pi_{t+1}^{kk} - \pi_{t+1}^{jk} \right) \right) - \beta \ln \frac{p_{t+1}(k|k)}{p_{t+1}(k|j)} + \epsilon_t^{jk} \]  

(26), where \( \epsilon_t^{jk} = \beta (\xi_t^k - \xi_t^j) \).

Replacing the CCPs with their sample analogues, i.e. let \( \hat{p}_t(k|j) = \) observed percentage of firms that are in state \( j \) in \( t = 1 \) and state \( k \) in time \( t \), we obtain the following estimating equations: for all \( j \neq k \),

\[ \ln \frac{\hat{p}_t(k|j)}{\hat{p}_t(j|j)} + \beta \ln \frac{\hat{p}_{t+1}(k|k)}{\hat{p}_{t+1}(k|j)} = \begin{cases} 
\mu + \alpha R_t - (1 - \beta) c + e_t^{01} & (j, k) = (0, 1) \\
-\mu - \alpha R_t + e_t^{10} & (j, k) = (1, 0) 
\end{cases} \]  

(27), where \( e_t = (e_t^{01}, e_t^{10}) \) is an error term that captures both the expectational errors in \( \epsilon_t^{jk} \) and the approximation errors in \( \hat{p}_t(k|j) \).

We assume that the value of the discount factor \( \beta \) is known. Estimating (27) gives us an estimate of the model parameters \( (\mu, \alpha, c) \). These estimates are consistent for a model that assumes rational expectations. Therefore, the DDC model estimated using this strategy is correctly specified for Experiment 1, but misspecified in Experiment 2 and 3.

**Structural Regularization** For structural regularization, we use the DDC model with rational expectations as the benchmark and use (25) with \( (p, q) = (2, 4) \) as the specification for the statistical model we regularize. Since the target variable \( n_t \) is serially correlated, we use Algorithm 1 with a cross-validation procedure based on a rolling-window design that is commonly used for time series modeling.

**Results** Figure 6 shows the results of the first experiment. Figure 6a plots the expected percentage of firms in the market, \( \mathbb{E} \left[ \frac{n_t}{N} \right] \), for the entire periods of \( t = 1 - 500 \), covering both the in-domain periods of \( t = 1 - 250 \) and the out-of-domain periods of \( t = 251 - 500 \), together with the predictions of the three estimators. The predictions are made using one-
Figure 6: Dynamic Entry and Exit - Experiment 1. Plotted are the true expected percentage of firms in the market (red) along with the predictions by the three estimators. Training data are not plotted for clarity. In (a), the entire periods of $t = 1 – 500$ are plotted, which covers both the in-domain periods of $t = 1 – 250$ and the out-of-domain periods of $t = 251 – 500$. (b) and (c) plot respectively the in-domain periods of $t = 11 – 110$ and the out-of-domain periods of $t = 301 – 400$ in order to show a more detailed picture (the plot in (b) starts at $t = 11$ due to $n_t$ during the initial periods being influenced by the initial states, where we randomly assign half of the firms as incumbents and the other half as potential entrants).
Figure 7: Dynamic Entry and Exit - Experiment 2. Plotted are the true expected percentage of firms in the market (red) along with the predictions by the three estimators. Training data are not plotted for clarity. In (a), the entire periods of $t = 1 - 500$ are plotted, which covers both the in-domain periods of $t = 1 - 250$ and the out-of-domain periods of $t = 251 - 500$. (b) and (c) plot respectively the in-domain periods of $t = 11 - 110$ and the out-of-domain periods of $t = 301 - 400$ in order to show a more detailed picture.
Figure 8: Dynamic Entry and Exit - Experiment 3. Plotted are the true expected percentage of firms in the market (red) along with the predictions by the three estimators. Training data are not plotted for clarity. In (a), the entire periods of $t = 1 - 500$ are plotted, which covers both the in-domain periods of $t = 1 - 250$ and the out-of-domain periods of $t = 251 - 500$. (b) and (c) plot respectively the in-domain periods of $t = 11 - 110$ and the out-of-domain periods of $t = 301 - 400$ in order to show a more detailed picture.
Table 4: Dynamic Entry and Exit - Results

| Experiment | Estimator | In-Domain | | | Out-of-Domain | | |
|------------|-----------|-----------|--------|--------|-----------|--------|
|            |           | Bias      | Var    | MSE    | Bias      | Var    | MSE    |
| 1          | Statistical | 0.0321    | 0.0018 | 0.0033 | 0.1347    | 0.0027 | 0.0400 |
|            | Structural | 0.0012    | 0.0009 | 0.0009 | 0.0180    | 0.0143 | 0.0148 |
|            | SRE       | 0.0014    | 0.0009 | 0.0009 | 0.0353    | 0.0310 | 0.0392 |
| 2          | Statistical | 0.0323    | 0.0046 | 0.0085 | 0.1258    | 0.6969 | 0.7371 |
|            | Structural | 0.0074    | 0.0002 | 0.0023 | 0.1621    | 0.0259 | 0.0731 |
|            | SRE       | 0.0037    | 0.0001 | 0.0004 | 0.0472    | 0.0506 | 0.0668 |
| 3          | Statistical | 0.0329    | 0.0015 | 0.0031 | 0.1558    | 0.0047 | 0.0583 |
|            | Structural | 0.0073    | 0.0025 | 0.0026 | 0.2111    | 0.0335 | 0.0986 |
|            | SRE       | 0.0010    | 0.0004 | 0.0004 | 0.0736    | 0.0661 | 0.1091 |

Notes: results are based on 100 simulation trials. Reported are the mean bias, variance, and MSE, averaged over time $t$. 
step ahead forecasting\textsuperscript{52}. To display the results more clearly, Figure 6b and 6c plot selected in-domain and out-of-domain periods to offer a more detailed picture. All three estimators fit relatively well in-domain. However, out-of-domain, the time series model is completely unable to capture the rising market entries as $R_t$ increases. This is partly by design: as we have discussed, we intentionally choose parameter values so that out-of-domain dynamics differ markedly from those in-domain. A statistical model that fits to the in-domain data is apparently unable to extrapolate well in this case. On the other hand, the structural model, which is correctly specified in this experiment, extrapolate very well, as expected. The SRE performs as well as the structural model in-domain. Out-of-domain, its predictions generally match the true values closely, except when the true percentages are close to 1. In those cases the SRE fit tends to overshoot, which is not surprising as the SRE model does not bind $n_t$ to be within $[0, N]$. Nonetheless, it is apparent that the SRE is able to capture the rising entries unlike the time series model.

Figure 7 shows the results of the second experiment. In Experiment 2, agents have adaptive expectations in the sense that they always assume $R_{t'} = R_t \forall t' > t$. Since in our simulations, $R_t$ follows a rising trend, this means that agents systematically underestimate future profits. The realized dynamics show that for most of the in-domain periods, there is little entry into the market. Entry increases significantly during the out-of-domain periods and indeed, for multiple periods of time, almost all firms are in the market. This marked difference between in-domain and out-of-domain dynamics pose significant challenges. Looking at the model fits, the time series model again fits relatively well in-domain but is completely unable to extrapolate out-of-domain. The structural model, being misspecified, is able to capture the rising entries, but tends to over-estimate the percentages of firms in the market. In particular, its predicted percentages tend to rise earlier and decline later than the real ones. The model that fits the best is the SRE, which is able to match the true dynamics closely both in-domain and out-of-domain, with the exception of periods in which the true percentages are close to 1, as the SRE fit is unbounded.

Figure 8 shows the results of the third experiment. In this experiment, agents are myopic

\textsuperscript{52}Given an estimated model, in each period $t$, we predict $n_t$ based on $\{(n_{t-1}, n_{t-2}, \ldots), (R_t, R_{t-1}, \ldots)\}$. To generate predictions for the structural model, we also assume agents have perfect foresight regarding $(R_{t+1}, R_{t+2}, \ldots)$.
Table 5: Demand Estimation - Setup

| Experiment | True Mechanism                        | Reduced-Form | Structural                      |
|------------|---------------------------------------|--------------|---------------------------------|
| 1          | linear demand, optimal monopoly pricing | linear demand |                                 |
| 2          | linear demand, non-optimal monopoly pricing | linear demand | linear demand, optimal monopoly pricing |
| 3          | linear demand, optimal monopoly pricing | log-log demand |                                 |
| 4          | linear demand, non-optimal monopoly pricing | log-log demand |                                 |

in that they only care about current period returns when making entry and exit decisions. The data-generating model is therefore static in nature. Looking at estimator performance, the story is broadly similar to that of Experiment 2, with the time series predictions biased toward 0 out-of-domain, the structural predictions biased toward 1, and the SRE offering the most accurate predictions both in-domain and out-of-domain.

Table 4 reports the mean bias, variance, and mean squared error of the estimators with respect to the true $E\left[\frac{a_t}{N}\right]$ over 100 trials. When correctly specified, the structural model performs the best, as can be expected. When misspecified, the structural model exhibits relatively large biases. The SRE consistently performs well both in-domain and out-of-domain throughout the experiments. In particular, it delivers significantly smaller biases, both in-domain and out-of-domain, than the statistical and the structural model when the latter is misspecified. Although it has a higher out-of-domain variance, presumably due to its predictions not being bounded within $[0, 1]$, its overall performance is clearly superior to that of the misspecified structural model in Experiment 2 and 3.

C Demand Estimation

In our final application, we revisit the demand estimation problem under a different setting. Suppose now that instead of observing consumer demand under exogenously varying prices, the prices we observe are set by a monopolist. In this case, changes in prices are endogenous and the relationship between price and quantity sold is confounded. As in the motivating
example of section 2, we are interested in learning the demand curve. To this end, if we have access to a variable that shifts the cost of production for the monopoly firm but does not affect demand directly, then it can be used as an instrumental variable to help identify the true demand curve. This is the reduced-form approach. Alternatively, we can estimate a structural model that fully specifies monopoly pricing behavior. This is the structural approach. Finally, we can combine the two using the SRE.

In this exercise, we conduct four experiments. In all four experiments, we assume that we do have access to a valid instrument so that the demand curve is nonparametrically identified. However, the functional form of the reduced-form statistical model may still be misspecified. On the other hand, using the structural approach, we estimate a model that assumes the observed prices are optimally set by a profit-maximizing monopoly firm. When this assumption is violated, as when for example the firm’s pricing is not optimal or it does not have monopoly power, the structural model will also be misspecified. The four experiments we conduct are thus arranged as follows: in the first experiment, both the reduced-form and the structural models are correctly specified. In Experiment 2 and 3, only one of the two is correctly specified. In Experiment 4, both are misspecified. Table 5 summarizes this setup.

This exercise differs from the previous two in two important aspects. First, our first two applications focus on the misspecification of structural models. The statistical models they fit are chosen using a model selection procedure so as to produce the best out-of-sample fit of the observed data. In practice, applied reduced-form research in economics often specifies simple linear models, so misspecification concerns are nontrivial. In this exercise, we highlight the functional form misspecifications of the reduced-form model as well as the structural. Second, this exercise focuses on comparisons of in-domain performance. We show that when either the reduced-form or the structural model is misspecified, the SRE will have better in-domain performance – more internal validity – than the misspecified model and has the ability to outperform both when both are misspecified.

Setup Consider $M$ geographical markets in which a product is sold. The equilibrium price and quantity sold in market $m$ are $(p_m, q_m)$. Assume that all markets share the same
aggregate demand function $Q^d(p)$:

$$q_m = Q^d(p_m) = \alpha - \beta \cdot p_m + \epsilon_m$$  \hspace{1cm} (28)

In Experiment 1 and 3, we assume the product is sold by a monopoly firm who sets the prices in each market to maximize its profit. The firm has different marginal costs $c_m$ for operating in different markets. Hence it sets

$$p_m = \arg \max_{p > 0} \{(p - c_m) Q^d(p)\}$$  \hspace{1cm} (29)

$$= c_m + \frac{1}{\beta} q_m$$  \hspace{1cm} (30)

Assume that we also observe a cost-shifter $z_m$, e.g. transportation costs, such that

$$c_m = a + b \cdot z_m$$  \hspace{1cm} (31)

, then $z_m$ can serve as an instrument for $p_m$ for identifying the demand curve.

In Experiment 2 and 4, we assume the monopoly firm fails to set optimal prices or does not have complete monopoly power. Its pricing decisions are given by

$$p_m = c_m + \frac{\lambda}{\beta} q_m$$  \hspace{1cm} (32)

, where $\lambda \in (0, 1)$. The firm thus earns a lower markup than a optimal price-setting monopoly.

Simulation For each experiment, we simulate $M = 1000$ markets and generate an observed data set of $D = \{(p_m, q_m, z_m)\}_{m=1}^M$. See Appendix A.3 for the parameter values we use in simulation.

Reduced-Form Estimation Because $p_m$ is now endogenous – $p_m$ and $\epsilon_m$ are correlated through (30) – the statistical relation between $p_m$ and $q_m$ is confounded and no longer represents the demand function. To estimate the demand curve using the reduced-form approach, we avail of the instrumental variable $z_m$ and estimate $Q^d(p)$ by two-stage least
squares (2SLS). In Experiment 1 and 2, our reduced-form model is correctly specified, i.e. we fit (28) to the data by 2SLS. In Experiment 3 and 4, however, we assume the demand function takes on a log-log form:

\[
\log q_m = \alpha - \beta \cdot \log p_m + \epsilon_m
\]  

(33)

, and is therefore misspecified in these two experiments.

**Structural Estimation**  We fit a structural model featuring linear demand function (28) and price-setting function (30). This structural model is correctly specified for Experiment 1 and 3, but misspecified for Experiment 2 and 4. The structural parameters are \((\alpha, \beta, a, b)\) and can be estimated as follows: from (28) and (30), we obtain

\[
p_m = a + b \cdot z_m + \frac{1}{\beta} q_m
\]  

(34)

If our model is correct, (34) is a deterministic linear equation system from which we can solve directly for \((\hat{a}, \hat{b}, \hat{\beta})\). Substituting \(\hat{\beta}\) into (28), we then obtain \(\hat{\alpha} = \frac{1}{M} \sum_{m=1}^{M} (q_m + \hat{\beta} p_m)\).

**Structural Regularization**  To estimate the demand curve using the SRE, we employ the structural model described above as the benchmark model and specify a 2nd degree polynomial \(g(p; \theta) = \theta_0 + \theta_1 p + \theta_2 p^2\) as the statistical model for \(Q^d(p)\). As in reduced-form estimation, we rely on the use of the instrumental variable \(z_m\) and identify \(\theta\) via the following moment conditions:

\[
\mathbb{E}[(q_m - g(p_m; \theta))|z_m] = 0
\]  

(35)

The SRE proceeds in two stages. In the first stage, we estimate the structural model and generate synthetic data \((p^M_m, Q^M(p^M_m))\), where \(Q^M(p)\) is the model derived demand function, i.e. the structural estimate of \(Q(p)\). We obtain \(\hat{\theta}^M\) by fitting \(g\) directly to the synthetic data. In the second stage, we minimize the objective function (9) with the following moment functions:

\[
m_m(\theta) = (q_m - g(p_m; \theta)) \phi(z_m)
\]  

(36)
Figure 9: Demand Estimation. Dots represent training data. The true demand curve is shown in red. (a) - (d) correspond respectively to Experiment 1 - 4.

where we let $\phi(z) = (1, z, z^2, \ldots, z^5)$.

Analytical solution to (9) for a given $\lambda$ is given by (14). For the weight matrix $W$, we use the 2SLS weight $W = (\phi(z)'\phi(z))^{-1}$. The regularization procedure follows Algorithm 1. Since no out-of-domain predictions are involved in this exercise, the standard cross-validation procedure for i.i.d. data is used to choose the optimal $\lambda$.

Results Figure 9a plots the results of the first experiment. As the figure shows, the observed data $(p_m, q_m)$ are significantly confounded – fitting a least squares model to the

\[^{53}\text{See footnote 42.}\]
Table 6: Demand Estimation - Results

| Experiment | Reduced-Form | | Structural | | | SRE |
|------------|--------------|----------------|-------------|----------------|-------------|----------------|----------------|
|            | Bias | Var  | MSE  | Bias | Var  | MSE  | Bias | Var  | MSE  |
| 1          | 0.2720 | 7.6780 | 7.7863 | 0.0770 | 0.9102 | 0.9161 | 0.9899 | 11.1375 | 13.3879 |
| 2          | 0.2884 | 5.1712 | 5.2821 | 12.3218 | 1.4233 | 203.8835 | 0.2783 | 11.4235 | 11.5223 |
| 3          | 25.9431 | 174.9081 | 2601.8750 | 0.1167 | 0.9648 | 0.9784 | 0.9703 | 13.0669 | 15.2066 |
| 4          | 11.8060 | 22.3152 | 423.3862 | 12.3212 | 1.4277 | 203.8519 | 0.3721 | 12.3088 | 12.5519 |

Notes: results are based on 100 simulation trials. Reported are the mean bias, variance, and MSE, averaged over p.

Data would produce an upward-sloping curve. Despite the significant confounding, reduced-form and structural estimation are both able to identify the true demand curve. This is because both use correctly specified models and z is a valid instrument. In this case, the SRE performs equally well. The three model fits and the true demand curve almost coincide.

Figure 9b plots the results of the second experiment. In this experiment, the reduced-form model is correctly specified, while structural model is not. The structural fit therefore deviates from the true demand curve, while the reduced-form model fits well. Figure 9c shows the other side of the coin. In Experiment 3, the structural model is correctly specified, but the reduced-form model is not. In this case, even though the reduced-form fit manages to capture the downward-sloping nature of the demand curve, it is badly “out of shape”. Finally, in Figure 9d, we show the results of Experiment 4 in which both models are misspecified and, as a result, produce fits that depart from the true relationship. In all of these experiments, however, the SRE fits the true demand curve well, regardless of which model – the reduced-form or the structural or even both – is misspecified.

Table 6 reports the mean bias, variance, and mean squared error of the estimators with respect to the true demand curve over 100 trials. When they are correctly specified, reduced-form and structural models exhibit low biases. The structural model, by virtue of imposing more structure on the data, attains a lower variance. When misspecified, both types of models exhibit large biases and MSEs. The SRE, in comparison, consistently attains a low bias. Although its variance is higher than that of structural estimation, its MSE remains relatively low and is significantly lower than the other two estimators when they are misspecified.
D Discussion

The tension between the goal of producing an accurate description of the data and the goal of estimating externally valid structural parameters that allow for counterfactual analysis and policy prediction is a lasting legacy of Cowles Commission research program (Heckman, 2000). Structural estimation, in its effort to achieve the second goal, often need to make strong and unrealistic assumptions, including both causal assumptions such as rational expectations, and parametric assumptions such as CES utility functions. Many efforts have been made to relax these assumptions. In the context of dynamic structural models, for example, these efforts include semiparametric estimation (Norets and Tang, 2014), robust estimation (Christensen and Connaught, 2019), and alternative specifications of expectations (Woodford, 2013). This paper offers an alternative: rather than seeking to minimize assumptions and estimate partially identified models, or specify more realistic models of behavior, which can be intractable and heterogeneous, we show the feasibility of adopting a tractable structural model with strong assumptions as an approximate model and estimate the data using structural regularization. A limitation with our approach is that by doing so, the SRE estimator no longer permits a structural interpretation and therefore cannot be used to conduct welfare analyses. We leave addressing this limitation to future work.

5 Conclusion

In this paper, we propose a general framework for incorporating theory into statistical modeling for statistical prediction and causal inference. We demonstrate the effectiveness of our method in a number of economic applications including first-price auctions, dynamic models of entry and exit, and demand estimation with instrumental variables. Many more potential applications are possible, such as forecasting long-run effects based on short-run observations or predicting effects at scale, which we leave for future work. Our method has potential applications not only in economics, but in other (social) scientific disciplines whose theoretical models offer important insight but are subject to significant misspecification concerns.
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