Dilepton decays of nucleon resonances and dilepton production cross sections in proton-proton collisions

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Abstract

Relativistic, kinematically complete phenomenological expressions for the dilepton decay rates of nucleon resonances with arbitrary spin and parity are derived in terms of the magnetic, electric, and Coulomb transition form factors. The dilepton decay rates of the nucleon resonances with masses below 2 GeV are estimated using the extended vector meson dominance (eVMD) model for the transition form factors. The model provides an unified description of the photo- and electroproduction data and of the vector meson and dilepton decays of the nucleon resonances. The constraints on the transition form factors from the quark counting rules are taken into account explicitly. The remaining parameters of the model are fixed by fitting the available photo- and electroproduction data and using results of the multichannel partial-wave analysis of the $\pi N$ scattering. The results are used to describe dilepton spectra measured at BEVALAC in proton-proton collisions.
1 Introduction

Dileptons are the clearest probe to study highly compressed nuclear matter. They provide a possibility to measure experimentally the in-medium widths and masses of vector mesons. The dilepton spectra measured by the CERES \[1\] and HELIOS-3 \[2\] Collaborations at CERN SPS found a significant enhancement of the low-energy dilepton yield below the $\rho$ and $\omega$ peaks in heavy systems ($Pb + Au$) compared to light systems ($S + W$) and proton induced reactions ($p + Be$). Theoretically, this enhancement can be explained in a hadronic picture assuming a dropping mass scenario for the $\rho$ meson or by the inclusion of in-medium spectral functions for the vector mesons. In both cases the enhanced low energetic dilepton yield is not simply due to a shift of the $\rho$ and $\omega$ peaks in the nuclear medium but it originates to most extent from an enhanced contribution of the $\pi^+\pi^-$ annihilation channel which, assuming vector dominance, runs over an intermediate in-medium $\rho$ mesons. An alternative scenario is the formation of a quark-gluon plasma in the heavy systems which leads to additional contributions to the dilepton spectrum from perturbative QCD ($pQCD$) such as quark-antiquark annihilation \[3, 4\].

Concerning the DLS experiment \[5\], the measured dilepton spectra do not match with the theoretical estimates, even when possible reduction of the $\rho$-meson mass and the $\rho$-meson broadening are taken into account \[6\]. This phenomenon is called 'DLS puzzle'. The HADES experiment at GSI will study the dilepton spectra in the same energy range in greater details \[7\].

The experimental data from heavy-ion collisions can only be compared to theoretical predictions from transport models which account for the complicated reaction dynamics. The elementary cross sections enter as an input into the transport simulations of heavy-ion collisions. A precise and rather complete knowledge of the decay channels of mesons and nucleon resonances, and their production, absorption, and reabsorption cross sections is therefore indispensable in order to draw reliable conclusions from studies of the dilepton production. The recent measurement of the dilepton production in proton-proton collisions by BEVALAC \[8\] provides a useful tool for testing the current theoretical schemes.

We give here relativistic, kinematically complete phenomenological expressions for the dilepton decay rates of nucleon resonances with arbitrary spin and parity and discuss these results in connection to the dilepton production in $pp$ collisions at BEVALAC energies.
2 Dilepton widths

The signs ± stand for the normal- and abnormal parity resonances, $J^P = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-$, ... (the upper sign) and $J^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+$, ... (the lower sign). In terms of the magnetic $(M)$, electric $(E)$, and Coulomb $(C)$ form factors, the decay width of a nucleon resonance with spin $J = \frac{l}{2} + 1 \geq \frac{3}{2}$ and mass $m_\ast$ into the nucleon with mass $m$ and a virtual photon with mass $M$ equals \cite{9}

$$
\Gamma(N_\ast^\dagger \to N \gamma^*) = \frac{9\alpha}{16} \frac{(l!)^2}{2^{(2l+1)!}} \frac{m_\ast^2 (m_\pm^2 - M^2)^{-1/2}}{m_\ast^{2l+1} m^2} \left( \frac{l+1}{l} \left| G_{M/E}^{(\pm)} \right|^2 + (l+1)(l+2) \left| G_{E/M}^{(\pm)} \right|^2 + \frac{M^2}{m_\ast^2} \left| G_{C}^{(\pm)} \right|^2 \right),$$

(1)

where $m_\pm = m_\ast \pm m$ and $G_{E/M}^{(\pm)}$ means $G_E^{(\pm)}$ or $G_M^{(\pm)}$. For $l = 1$, we recover the result of ref. \cite{10}.

The resonance decay widths of $J = \frac{1}{2}$ nucleon resonances can be found to be

$$
\Gamma(N_\ast^\dagger \to N \gamma^*) = \frac{\alpha}{8m_\ast} \left( m_\pm^2 - M^2 \right)^{3/2} \left( m_\pm^2 - M^2 \right)^{1/2} \left( 2 \left| G_{E/M}^{(\pm)} \right|^2 + \frac{M^2}{m_\ast^2} \left| G_{C}^{(\pm)} \right|^2 \right).$$

(2)

We use here the normalization for the monopole form factors identical to refs. \cite{9,11}. The $\Delta(1232)$-resonance form factors of refs. \cite{10,12} contain an additional factor of $\sqrt{\frac{2}{3}}$.

If the width $\Gamma(N^* \to N \gamma^*)$ is known, the factorization prescription (see e.g. \cite{13}) can be used to find the dilepton decay rate:

$$
d\Gamma(N^* \to Ne^+e^-) = \Gamma(N^* \to N \gamma^*) M\Gamma(\gamma^* \to e^+e^-) \frac{dM^2}{\pi M^4},$$

(3)

where

$$
M\Gamma(\gamma^* \to e^+e^-) = \frac{\alpha}{3} (M^2 + 2m_e^2) \sqrt{1 - \frac{4m_e^2}{M^2}},$$

(4)

is the decay width of a virtual photon $\gamma^*$ into the dilepton pair with invariant mass $M$. Eqs.(1)-(3) being combined give the $N^* \to Ne^+e^-$ decay rates.

The $\Delta(1232)$ dilepton decays are known as the dominant sources of the dilepton yield in nucleon-nucleon and heavy-ion collisions at low energies. As we discussed in ref. \cite{10}, previous calculations of the $\Delta(1232) \to Ne^+e^-$ decays, available in the literature, are incorrect.
3 Extended VMD & quark counting rules

To proceed further, one needs a specific model for transition form factors of nucleon resonances. We use extended VMD (eVMD) model. It is motivated by two observations:

(i) The naive VMD model should give, in principle, an unified description of the radiative \( R \to N\gamma \) and the mesonic \( R \to NV \) decays. However, a normalization to the radiative branchings \( RN\gamma \) strongly underestimates the mesonic branchings \( RNV \) as discussed in refs. [14, 15].

(ii) The electromagnetic nucleon form factors demonstrate experimentally a dipole behavior. The quark counting rules for the Sachs form factors predict \( G_E(q^2) \sim G_M(q^2) \sim 1/q^4 \) at \( q^2 \to \infty \). The naive VMD model with the ground-state \( \rho-, \omega-, \) and \( \phi \)-mesons cannot describe quantitatively the nucleon form factors and gives incorrect asymptotic behavior. It was proposed [16] to include in the electromagnetic current excited states of the vector mesons \( \rho', \rho'', \) etc.

The eVMD allows to solve naturally the problem of the \( RN\gamma \) to \( RNV \) ratios. The requirement of a stronger suppression of the transition form factors at high \( q^2 \) is equivalent to a destructive interference of the \( \rho \) and \( \omega \)-families away from the \( \rho \) and \( \omega \) poles. It reduces the \( RN\gamma \) to \( RNV \) ratios.

The monopole transition form factors, \( G_T^{(\pm)}(M^2) \) with \( T = M, E, C \), are expressed in terms of the covariant form factors, \( F_k^{(\pm)}(M^2) \) with \( k = 1, 2, 3 \), as follows

\[
G_T^{(\pm)}(M^2) = \sum_k M_{Tk}(M^2) F_k^{(\pm)}(M^2). \quad (5)
\]

The transformation matrices \( M_{Tk} \) can be found in ref. [12] for \( J^P = \frac{3}{2}^+ \) and in refs. [9, 11] for arbitrary \( J^P \).

The quark counting rules [17] predict the following asymptotics for the covariant form factors of \( J \geq \frac{3}{2} \) nucleon resonances:

\[
\begin{align*}
F_1^{(\pm)}(M^2) &= O\left( \frac{1}{(-M^2)^{l+2}} \right), \\
F_2^{(\pm)}(M^2) &= O\left( \frac{1}{(-M^2)^{l+3}} \right), \\
F_3^{(\pm)}(M^2) &= O\left( \frac{1}{(-M^2)^{l+3}} \right). \quad (6)
\end{align*}
\]
In the no-widths approximation for the vector mesons, these constraints can be resolved to give

\[
\begin{align*}
F_{1}^{(\pm)}(M^2) &= \frac{\sum_{j=0}^{n+1} C_{1j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)}, \\
F_{2}^{(\pm)}(M^2) &= \frac{\sum_{j=0}^{n} C_{2j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)}, \\
F_{3}^{(\pm)}(M^2) &= \frac{\sum_{j=0}^{n} C_{3j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)}.
\end{align*}
\tag{7}
\]

Here, \(C_{kj}^{(\pm)}\) are free parameters of the eVMD, \(l + 3 + n\) is the total number of the vector mesons with masses \(m_i\). The quark counting rules reduce the number of free parameters from \(l + 3 + n\) to \(n + 2\) for \(k = 1\) and to \(n + 1\) for \(k = 2, 3\). In the minimal case \(n = 0\), the knowledge of the four parameters \(C_{10}^{(\pm)}, C_{11}^{(\pm)}, C_{20}^{(\pm)}, \text{ and } C_{30}^{(\pm)}\) is sufficient to fix \(F_{k}^{(\pm)}(M^2)\). In the zero-width limit, the multiplicative representation (7) is completely equivalent to the usual additive representation.

The similar multiplicative representation motivated by the Regge theory is used in ref. [11]. The asymptotic dominance of the transverse covariant form factors, used in that work as an assumption, however, does not agree with the quark counting rules.

For spin \(J = \frac{1}{2}\) resonances, the constraints have the form

\[
F_{1,2}^{(\pm)}(M^2) = O\left(\frac{1}{(1 - M^2)^{3}}\right).
\tag{8}
\]

The general representation for the covariant form factors in the spin-\(\frac{1}{2}\) case becomes

\[
F_{k}^{(\pm)}(M^2) = \frac{\sum_{j=0}^{n} C_{kj}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)}.
\tag{9}
\]

The free parameters of the eVMD model are fixed by fitting the available photo- and electroproduction data, \(\gamma(\gamma^*)N \rightarrow N^*\), and the vector meson decays, \(N^* \rightarrow N\rho(\omega)\). When the experimental data are not available, the quark model predictions are used as an input.
4  Dilepton production in proton-proton collisions

A possibility to clarify the origin of the DLS puzzle has appeared since data from elementary \( pp (pd) \) collisions at \( T = 1 \div 5 \) GeV (\( T \) is the kinetic energy of the incident proton in the laboratory frame) became available from the DLS Collaboration [8].

For description of the dilepton production in proton-proton collisions at energies \( 1 \div 3 \) GeV, we use the nucleon resonance model. The mesons \( P (= \pi, \eta, ...) \) and \( V (= \rho, \omega, \phi) \) are produced through a two-step mechanism via the excitation of nuclear resonances, i.e. \( NN \rightarrow NR, R \rightarrow NV \). When energy increases, multiparticle final states become dominant. For such energies we used the experimental inclusive cross sections for the meson production.

The \( pp \rightarrow ppM \) cross section with \( M = P, V \) is given by

\[
\frac{d\sigma}{dM^2}(s, M)_{pp \rightarrow ppM} = \sum_R \int_{(m_p+M)^2} \left( \frac{\sqrt{s-m_p}}{m_p} \right)^2 \frac{d\sigma}{d\mu^2}(s, \mu)_{pp \rightarrow pR} dB(\mu, M)_{R \rightarrow pM}.
\]

(10)

The cross sections for the resonance production are given by

\[
d\sigma(s, \mu)_{pp \rightarrow pR} = \frac{|M_R|^2}{16\pi \sqrt{s} \pi^2} \Phi_2(\sqrt{s}, \mu, m_p) dW_R(\mu).
\]

(11)

with \( \Phi_2(\sqrt{s}, \mu, m_p) = \pi p^*(\sqrt{s}, \mu, m_p)/\sqrt{s} \) being the two-body phase space, \( p^*(\sqrt{s}, \mu, m_p) \) the final c.m. momentum, \( p_i \) the initial c.m. momentum, and \( \mu \) and \( m_R \) the running and pole masses of the resonances, respectively, and \( m_p \) is the proton mass. The mass distribution \( dW_R(\mu) \) of the resonances is described by the standard Breit-Wigner formula. The sum in (10) runs over the well established \((4*)\) resonances quoted by the PDG [18]. The branching to the \( V \) decay mode is given by

\[
dB(\mu, M)_{R \rightarrow pV} = \frac{d\Gamma_{NV}(\mu, M)}{\Gamma_R(\mu)}.
\]

(12)

In terms of the magnetic, electric, and Coulomb couplings \( g_M^{(\pm)}, g_E^{(\pm)}, \) and \( g_C^{(\pm)} \), the differential decay widths of nucleon resonances with spin \( J = l+1/2 \) into a vector meson, \( V \), with arbitrary mass \( M \) has the form [9]

\[
d\Gamma_{NV}(\mu, M) = \frac{9}{64\pi 2^l (2l + 1)!} \left( \frac{l!}{l!} \right) \frac{m_+^2 (m_+^2 - M^2)^{l+1/2} (m_-^2 - M^2)^{l-1/2}}{\mu^{2l+1} m^2}.
\]
\[
\left( \frac{l + 1}{l} \left| g^{(\pm)}_{M/E} \right|^2 + (l + 1)(l + 2) \left| g^{(\pm)}_{E/M} \right|^2 + \frac{M^2}{\mu^2} \left| g^{(\pm)}_C \right|^2 \right) dW_V(M),
\] (13)

with \(m_\pm = \mu \pm m_p\). The signs \(\pm\) refer to the natural parity and abnormal parity, \(g^{\pm}_{M/E}\) means \(g^+_M\) or \(g^-_E\). The above equation is valid for \(J \geq \frac{3}{2}\). For \(J = \frac{1}{2}\) one obtains

\[
d\Gamma^R_{NV}(\mu, M) = \frac{1}{32\pi\mu}(m^2_\pm - M^2)^{3/2}(m^2_\mp - M^2)^{1/2}
\left( 2 \left| g^{(\pm)}_{E/M} \right|^2 + \frac{M^2}{\mu^2} \left| g^{(\pm)}_C \right|^2 \right) dW_V(M).
\] (14)

The distribution \(dW_V(M)\) is also the Breit-Wigner distribution. The last two equations are similar to eqs.(1) and (2) for the virtual photon decays.

Due to the subthreshold character of the \(\omega\) production in decays of on-shell nucleon resonances, the \(M\)-dependence of the coupling constants \(g^{(\pm)}_M\), \(g^{(\pm)}_E\), and \(g^{(\pm)}_C\) can be important. At the \(\omega\) pole mass \(m_\omega\) these couplings are proportional to residues of the magnetic, electric, and Coulomb transition form factors. We assume that the coupling constants which enter into the covariant representation of the form factors are not mass dependent. The \(M\)-dependence of \(g^{(\pm)}_M\), \(g^{(\pm)}_E\), and \(g^{(\pm)}_C\) arises then exclusively from the \(M\)-dependent transformation from the covariant basis to the multipole basis according to

\[
g^{(\pm)}_T(M^2) = \sum_{kT'} M_{T'k}(M^2)M^{-1}_{kT}(m^2_\omega)g^{(\pm)}_{T'}(m^2_\omega),
\] (15)

with \(T, T' = M, E, C\). The dilepton spectra are shown in Fig. 1. More details on the calculations can be found in ref. [15].

## 5 Conclusions

We have considered the dilepton production in \(pp\) collisions at BEVALAC energies \(T = 1 \div 5\) GeV. The subthreshold production of vector mesons through the nucleon resonances is described within the eVMD model which allows to bring the transition form factors in agreement with the quark counting rules and provides an unified description of the photo- and electroproduction data,
Figure 1: The differential dilepton production cross sections as a function of the dilepton invariant mass, $M$, after applying the experimental filter and the smearing procedure. The solid curves are the total cross sections, the dashed curves correspond to the inclusive production, and the dotted curves correspond to the subthreshold production. The experimental data are from ref. [8].
γ(γ∗)N → N*, the vector meson decays, N* → Nρ(ω), and the dilepton decays, N* → Nℓ+ℓ−. The dilepton decay rates are described relativistically using kinematically complete phenomenological expressions.

The resulting dilepton spectra are reasonably well described at proton energies of \( T = 1.27 \div 1.85 \) GeV. At \( T = 1.04, T = 2.09, \) and \( T = 4.88 \) GeV the agreement is not perfect. The future experimental investigations at GSI will probably shed new light on these problems.

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