The efficiency of gamma-ray emission from pulsars

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ABSTRACT

We present a modified scenario of gamma-ray emission from pulsars within the framework of polar cap models. Our model incorporates the possible acceleration of electron–positron pairs created in magnetospheres, and their subsequent contribution to the gamma-ray luminosity \( L_\gamma \). It also reproduces the empirical trend in \( L_\gamma \) for seven pulsars detected with Compton Gamma-Ray Observatory (CGRO) experiments. At the same time it avoids basic difficulties faced by theoretical models when confronted with observational constraints.

We show that the classical and millisecond pulsars form two distinct branches in the \( L_\gamma – L_{sd} \) diagram (where \( L_{sd} \) is the spin-down luminosity). In particular, we explain why the millisecond pulsar J0437–4715 has not been detected with any of the CGRO instruments despite its very high position in the ranking list of spin-down fluxes (i.e. \( L_{sd}/D^2 \), where \( D \) is a distance). The gamma-ray luminosity predicted for this particular object is about one order of magnitude below the upper limit set by EGRET.

Key words: pulsars: general – gamma-rays: observations – gamma-rays: theory.

1 INTRODUCTION

Numerous models of pulsars (see Michel 1991 for a review) proposed over the last three decades have tried to make specific predictions about emission of gamma-rays and X-rays. Gamma-rays are particularly important as a direct signature of basic non-thermal processes in pulsar magnetospheres, and potentially should help to discriminate among different models. Interpreting gamma-rays should also be less ambiguous compared with X-rays. In the latter case, especially for objects younger than 10^6 yr, contributions from initial cooling, internal friction, etc., of unknown magnitude may dominate the total X-ray emission.

There are seven positive detections of pulsars by CGRO (see Table 1), i.e. less than 1 per cent of all pulsars known to date. In all cases the sources had been identified by virtue of gamma-ray flux modulations with previously known period, \( P \). The Crab and Vela are the only pulsars seen by three of the CGRO detectors (EGRET, COMPTEL and OSSE).

The EGRET data have become so far the only testing ground for theoretical models. The latest critical review of three models of gamma-ray emission (polar cap models by Harding 1981 and Dermer & Sturner 1994, and the outer gap model of Yadigaroglu & Romani 1995) comes from Nel et al. (1996). They conclude that none of the models fits observations satisfactorily for two major reasons. First, the confrontation of polar cap models (Harding 1981; Dermer & Sturner 1994) with the observations leads to a relation

\[ L_\gamma \text{observed} \propto L_{sd}^\alpha \text{model}, \]

with \( \alpha = 0.6 \) and 0.5, respectively, instead of \( \alpha = 1 \). Secondly, there have always been some cases amongst the ~350 EGRET upper limits that apparently contradict predictions of \( L_\gamma \) made by the models. The troublesome limits come usually from the pulsars B1509−58, B1046−58, B0656+14, B1929+10 and B0950+08, as well as from the millisecond object J0437–4715.

Of these two problems raised by Nel et al. (1996), the latter is more severe in our opinion. The former problem may be solved to some degree by updating parameters used for two objects in the low-luminosity domain, i.e. B1055−52 and Geminga. After Thompson et al. (1994), Nel et al. used for B1055−52 the spectral index \( \gamma = 1.18 \) determined by Fierro et al. (1993) from the first three viewing periods of EGRET. However, a substantially higher value, \( \gamma = 1.59 \), based on the data from 10 viewing periods, recently became available (Fierro 1995). This value of the spectral slope reduces \( L_\gamma \) of B1055−52 by a factor of ~4. (We shall discuss other consequences of the steeper spectral slope for B1055−52 in Section 3.) Further reduction of \( L_\gamma \) in the case of B1055−52 is possible by lowering the distance \( D \) to the source. The argument for lowering the distance (the usually assumed value is 1.53 kpc, after the model of Taylor & Cordes 1993) may come from ROSAT PSPC observations. If the bulk of the (presumably) thermal X-ray emission from B1055−52 is due to initial cooling then the inferred radius for the neutron star at \( D = 1.53 \) kpc exceeds 30 km (Ogelman 1995) – a value hardly acceptable from the theoretical point of view. In the case of Geminga, its inferred luminosity drops by a factor of ~2.8 relative to the value used by Nel et al. owing to a newly determined HST parallax distance of 157 pc (Caraveo et al. 1996), instead of \( D = 250 \) pc. Fig. 1 presents the inferred EGRET luminosities versus model predictions of Harding (1981), with...
that offer a functional relation for $L_p$ visibly deviates from the trend predicted by Harding (1981). And observations looks quite satisfactory, even though the Crab $\gamma$-

The gamma-ray death line' on the $P$-$Q_g$ diagram (as originally presented by Arons). It is hard to reconcile such a line with the observed death line for radio emission, which corresponds to $\sim 10^{12}$ volts. The effect was already known earlier, when the polar cap models of Buccheri et al. (1978) and Harding (1981) were introduced. In the latter model, for a pulsar with $\dot{P} \approx 10^{-15}$ s$^{-1}$, $L_g$ would reach $L_{sd}$ at a characteristic age equal to $3 \times 10^7$ yr (Harding 1981).

Below we propose a polar cap model which is free of the two problems discussed above. This model reproduces gamma-ray luminosities inferred for seven observed pulsars. At the same time it avoids the problem of the empirical gamma-ray death line of Arons (1996), and it relaxes the upper limit constraints of Nel et al. (1996), especially for old classical pulsars and millisecond pulsars. In Section 2 we start by recalling the model for total power contained in outflowing particles, which refers directly to the relation analysed by Arons (1996). Then we present arguments for summing all available $CGRO$ data in order to get a better start for a modified polar cap model, rather than using the EGRET data alone. Section 3 contains a description of our model, and its reference to existing information on gamma-rays from pulsars. The summary and comments are in Section 4, which also contains the ranking of pulsars with the highest gamma-ray fluxes resulting from our model.

2 SIMPLE MODELS VERSUS $CGRO$ DATA

Although EGRET has been far more successful than other $CGRO$ instruments in detecting pulsars, this does not mean that pulsar gamma-ray emission above 100 MeV dominates energetically over gamma-ray emission from lower energy bands. In the case of the Crab pulsar most of the energy output occurs within the COMPTEL and OSSE energy ranges (e.g. Fierro 1995). In the extreme case of B1509$-$58 there is only OSSE detection. Early positive reports from the COMPTEL team (Carramiñana et al. 1995) have not been confirmed (Kuiper 1996), and EGRET gave only upper limits for the source (Thompson et al. 1994). Luminosities listed in Table 1 were inferred from phase-averaged fluxes assuming a beaming solid angle of gamma-ray emission $\Omega_r$ equal to 1 steradian. For three objects (the Crab, Vela and B1951+32) detected with more than one instrument, we can also construct a sum of inferred luminosities (ignoring possible changes of a beaming angle with energy range), to get an estimate of the 'total' gamma-ray luminosity. For the remaining four pulsars we will use the luminosities in the EGRET energy range as 'total'. The values of 'total' gamma-ray luminosities are shown in the last column of Table 1.

![Figure 1. The gamma-ray luminosities above 100 MeV for six EGRET sources are plotted against predictions of the polar cap model of Harding (1981). The normalization factor is arbitrary; $\tau = P/\dot{P}$ is a characteristic age expressed in yr. (Note: The Crab pulsar is the first dot from the right.) Bars indicate uncertainties in distance and flux. In the case of Vela (next to the Crab) the lower distance limit was extended down to 200 pc to conform to recent estimates based on X-ray and gamma-ray observations of the Vela SNR [400 $\pm$ 200 pc after Aschenbach, Egger & Trümper (1995) and $\approx 350$ pc after Oberlack et al. (1994), respectively).](image)
Let us now compare these ‘total’ gamma-ray luminosities with the simplest possible phenomenological polar cap model. According to this model, the gamma-ray luminosity $L_{\gamma}$ is proportional to the power contained in outflowing primary electrons $L_{\text{particles}}$, which in turn is proportional to the product of the primary electron energy $E_0$, the surface area of the canonical polar cap $A_{\text{pc}} \propto \pi R_{\text{pc}} \times 1/P$, and the Goldreich–Julian flux $\hat{n}_{\text{GJ}} \propto B/P$ (Goldreich & Julian 1969) of outflowing primary electrons:

$$L_{\gamma} \propto L_{\text{particles}} \propto E_0 \hat{n}_{\text{GJ}} A_{\text{pc}}.$$  \hspace{1cm} (2)

Assuming $E_0 = \text{constant}$ for all objects, one obtains

$$L_{\gamma} \propto L_{\text{particles}} \propto B/P^2 \propto L_{\text{sd}}^{1/2}. \hspace{1cm} (3)$$

[Note: The model for $L_{\text{particles}}$ from equation (3) was actually a starting point in the work of Harding (1981), who assumed $E_0 = 10^{15}$ eV and whose objective was to find a prescription for $L_{\gamma}(> 100\text{MeV})$. We shall return to this point in the next section.]

Fig. 2 shows how the relation of equation (3) compares with observations. The normalization factor ($C = 10^{16}$) has been chosen to obtain the best ‘by eye’ fit. The overall agreement looks quite impressive. There is a substantial improvement for the Crab compared with Fig. 1, owing to significant contributions from OSSE and COMPTEL data. Moreover, B1509−58 adds up smoothly to six EGRET pulsars. The same functional relation, but for EGRET points only, has been considered by Arons (1996) (see previous section).

The relation $L_{\gamma} = 10^{16} L_{\text{sd}}^{3/2} \text{ erg s}^{-1}$ presented in Fig. 2 cannot hold for all radio pulsars. It leads formally to $L_{\gamma}$ reaching $L_{\text{sd}}$ at $10^{32} \text{ erg s}^{-1}$ (which corresponds to the Arons empirical gamma-ray death line), whereas pulsars are observed down to $L_{\text{sd}} = 10^{30} \text{ erg s}^{-1}$. Clearly, pulsar models that predict $L_{\gamma}$ as a simple combination of $B$ and $P$ require some revision.

### 3 HOW DO ELECTRON–POSITRON PAIRS CONTRIBUTE TO GAMMA-RAYS?

According to the model of Daugherty & Harding (1982, DH82) primary electrons are accelerated along open magnetic field lines to high energies ($\sim 10^{19}$ eV) owing to rotation-induced electric field. The model assumes a dipolar structure of the magnetic field. Curvature photons emitted by primary electrons are absorbed by the magnetic field with subsequent creation of electron–positron pairs (Sturrock pairs). These pairs cool off instantly via synchrotron radiation. Synchrotron photons may lead to further pair creation. Electromagnetic cascades propagating in the pulsar’s magnetic field may be very rich, with several subsequent generations of pairs and photons.

Numerical treatment of electromagnetic cascades initiated by primary electrons above the polar cap, and propagating across the magnetosphere (DH82), does not include effects of possible acceleration of Sturrock pairs. The only contribution from pairs to gamma-rays taken into account is due to synchrotron emission of created pairs. The pairs themselves do not accelerate, and subsequently do not contribute to the curvature radiation. Such a simplification is usually justified by arguing that the appearance of conductive plasma above some height effectively leads to a screening of the electric field parallel to the local magnetic field lines. If, however, the density of created pairs is lower than the local corotation plasma density, the electrons from pairs will be subject to further acceleration (whereas positrons will be decelerated; eventually some of them will be stopped and reversed towards the stellar surface). In the context of polar cap models developed by Daugherty & Harding (1982, 1994, 1996) it became clear that a potential contribution to gamma-rays from pairs might be necessary to account for the observed gamma-ray fluxes.

If these secondary particles are indeed subject to effective acceleration at significant altitudes above the polar cap surface, e.g. at heights of several NS (neutron star) radii (Daugherty & Harding 1996), the resulting beaming angles of gamma-ray emission $\Omega_{\gamma}$ will be wider than those measured at the polar cap surface. The requirement for ‘nearly aligned rotators’ (Dermer & Sturmer 1994; Daugherty & Harding 1994), necessary to explain the large duty cycles of gamma-ray emission, might be then relaxed. [Note: Relaxing the assumption about small inclination angles between the rotation and magnetic axes may be inevitable on observational grounds. Inclination estimates carried out by Lyne & Manchester (1988) and Rankin (1990) show that in many pulsars inclination angles are large indeed. However, mutual comparison of these results shows that they are in agreement only for small inclination angles ($\leq 40^\circ$), and there is no correlation if either estimate is larger than this value (Miller & Hamilton 1993).] Hereafter we will assume that for all pulsars $\Omega_{\gamma} = 1\text{s}r$ (corresponding to opening angles of $\sim 30^\circ$).

Suppose that the secondary particles, which we assume to be created as described in the model of DH82, do participate in the gamma-ray production similarly to the primary electrons. In the spirit of equations (2) and (3), we propose the following prescription for $L_{\gamma}$:

$$L_{\gamma} = C n_+ E_{\pm} L_{\text{sd}}^{1/2}, \hspace{1cm} (4)$$

where $n_+$ is the number of created $e^\pm$ pairs per primary electron, and $E_\pm$ is the characteristic energy attained by particles due to acceleration. The normalization constant $C$ will be determined by fitting the observations. We will assume that $E_\pm$ achieved by the
secondary particles is similar to the energy attained by primary electrons $E_0$, i.e. $E_s = E_0$.

We begin with making a choice for the value of $E_0$, since it is this parameter which, along with $B$ and $P$, will determine the number of pairs $n_z$ created per primary electron. The analytical fit of Harding (1981) for $L_\gamma$ above 100 MeV, which gained so much popularity in testing her polar cap model against EGRET observations, was obtained from numerical simulations performed for a fixed value of the primary electron energy $E_0 = 10^{13}$ eV. Originally, this energy was chosen to make the best spectral fits above 100 MeV for COS-B data of the Crab and Vela. However, such an assumption about $E_0$ is not possible throughout the entire lifetime of the pulsar because the energy of outgoing electrons is subject to a twofold limitation (e.g. Sturrock 1971): there is an absolute upper limit $E_{\text{min}} = 1.2 \times 10^9 B_{12} P^{-2/3} \text{MeV}$ (equation 4) was determined by fitting numerical results to the seven detections (the last column of Table 1). Then we calculated two evolutionary tracks in the $L_{\gamma}$–$L_{\text{sd}}$ space for representatives of the classical pulsars (with typical magnetic field strength $B \sim 10^{12}$ G), as well as of the millisecond pulsars ($B \sim 10^9$ G). Both tracks are shown in Fig. 3 as solid curves. The upper curve ($B \sim 10^{12}$ G) starts at $L_{\gamma} = 10^{33} L_{\text{sd}}^2$ (see Section 2). The lower solid curve shows the evolutionary track for a millisecond pulsar with $B_{\text{pc}} = 10^9$ G. The evolutionary tracks end when the objects reach their ‘death points’ in the $L_{\text{sd}}$ space (marked with dotted lines).

An ultrarelativistic primary electron, sliding along a curved magnetic field line, emits curvature cooling in a purely dipolar magnetic field. The energy $E_0$ of primary electrons must conform to one of these two limits (whichever comes first) as the pulsar is slowing down, since both $E_W$ and $E_{\text{max}}$ decrease as $P$ increases. In consequence, the prescription for $L_\gamma ($> 100 MeV) of Harding (1981) should not be treated as accurate everywhere.

Moreover, accelerating electrons should cross the threshold energy $E_{\text{min}} = 1.2 \times 10^9 B_{12}^{-1/3} P^{1/3} \text{MeV}$ (5), required for pair creation. Whenever $E_W$ falls below $E_{\text{min}}$, a classical pulsar crosses the well-known death line, and enters ‘a graveyard for pulsars’. Another deathline occurs for millisecond pulsars even earlier, when $E_{\text{max}}$ falls below $E_{\text{min}}$ (Rudak & Ritter 1994).

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As the primary electron accelerates, its energy $E_0$ of primary electrons must conform to one of these two limits (whichever comes first) as the pulsar is slowing down, since both $E_W$ and $E_{\text{max}}$ decrease as $P$ increases. In consequence, the prescription for $L_\gamma ($> 100 MeV) of Harding (1981) should not be treated as accurate everywhere.

The development of cascades was followed by means of numerical simulations described in DH82. Calculations of number of pairs $n_z$ were performed with numerical simulations after choosing $P$ and $B$, and setting the primary electron energy $E_0$ according to equation (4). The normalization constant in our model of $L_\gamma$ (equation 4) was determined by fitting numerical results to the seven detections (the last column of Table 1). Then we calculated confidence intervals for $E_0$ according to equation (5).

where $\xi > 1$.

The best choice for the value of the parameter $\xi$ was made a posteriori, to reproduce the empirical trend of $L_{\gamma}$ for the seven CGRO pulsars with similar accuracy to equation (3) in Fig. 2. We found that the range $2 \leq \xi \leq 5$ fulfills this requirement. All results presented below are for $\xi = 2.5$. For $E_{\text{min}}$ we preferred to take the numerically obtained values whenever they differed from the analytical approximation (the analytical formulae for $E_W$, $E_{\text{max}}$ and $E_{\text{min}}$ are taken from Rudak & Ritter 1994). We found that the former are consistently smaller by a factor of $\sim 1.5$ (in most cases) than the latter.

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from EGRET observations (after Fierro et al. 1995). In addition, 350 EGRET upper limits (Nel et al. 1996), including seven limits for millisecond pulsars, are shown for comparison. Apart from J0437−4715, four objects with EGRET upper limits only are placed clearly below the upper evolutionary track. These are (from right to left): B1046−58, B0656+14, B1929+10 and B0950+08. If the proposed model is correct, these objects should be the best candidates for detection in gamma-rays (but not necessarily in the EGRET energy range). B1951+32 and B1509−58, present in the data of Nel et al. (1996), have been replaced with detections (filled dots) by EGRET and OSSE, respectively.

The comparison of how our model reproduces \( L_g \) for the seven \textit{CGRO} pulsars, along with combined upper limits from EGRET, COMPTEL and OSSE (from Thompson et al. 1994, Fierro et al. 1995, Schroeder et al. 1995 and Carramiñana et al. 1995), wherever available, is shown in Fig. 4. In the case of Geminga and B1509−58, the model overestimates the observed \( L_g \) by a factor of \( \sim 2 \). However, existing upper limits from COMPTEL and EGRET, respectively, improve the agreement. The upper limit for J0437−4715, based on EGRET only, is one order of magnitude above the predicted value of \( L_g \). For four other objects (B1046−58, B0656+14, B1929+10 and B0950+08) the stringent upper limits from EGRET differ from the model predictions by no more than a factor of \( \sim 3 \). Moreover, upper limits from COMPTEL on B1046−58 and B1929+10 place them on the safe side of the diagonal line of prefect agreement in Fig. 4. There is no information available about B0950+08 and B0656+14 from any COMPTEL observations.

4 SUMMARY

We have proposed a semi-phenomenological model of gamma-ray emission from pulsars, which is based on polar cap activity triggered by primary electrons. The energy of electrons is only a few times higher than the threshold energy required to induce pair creation in the presence of a dipolar magnetic field, with other restrictions applied when necessary. Electromagnetic cascades induced via curvature radiation were treated in the same way as described by DH82. The important ingredient of the model is the assumption that secondary particles, produced in cascades arising from one-photon absorption, contribute to the overall gamma-ray emission similarly to primary electrons.

The model was confronted with the gamma-ray luminosities for seven pulsars inferred from available data from OSSE, COMPTEL and EGRET experiments. We find that the model is consistent with the existing data. Moreover, the model does not lead to any violation of energetics for pulsars with low spin-down luminosity \( L_{sd} \) – the predicted gamma-ray luminosity \( L_g \) never reaches \( L_{sd} \). It avoids, therefore, the problem of ‘an empirical gamma-ray death line’ as raised by Arons (1996). We also used the EGRET archive of 350 upper limits, along with OSSE and COMPTEL upper limits (published for 15 and 18 pulsars, respectively), to find likely restrictions on the model. We have used some updates with respect to the data used by Nel et al. (1996) in their analysis. The EGRET upper limit for B1951+32 was replaced with its detections by EGRET (Ramanamurthy et al. 1995) and COMPTEL (Kuiper et al. 1996a). Similarly, the EGRET upper limit for B1509−58 was replaced with an OSSE detection (Schroeder et al. 1995). In the case of B1929+10 the model distance of 170 pc was replaced with 250 pc (see Yancopoulos, Hamilton & Helfand (1994) for detailed arguments).

For a fixed value of \( L_{sd} \), the predicted \( L_g \) depends rather weakly on the magnetic field strength \( B \) as long as \( 10^{11} \lesssim B \lesssim 10^{13} \) G (Dyks 1997), especially for pulsars with \( L_{sd} \approx 10^{34} \) erg s\(^{-1}\). That is why all evolutionary tracks calculated for high values of \( B \) converge roughly to the asymptotic relation of equation (3) for \( L_g \approx 10^{34} \) erg s\(^{-1}\) (dashed line in Fig. 3). Only as \( B \) enters the domain of millisecond pulsars \( (\sim 10^{18}–10^{19}) \) does \( L_g \) drop significantly. Therefore all millisecond pulsars, including J0437−4715, are expected to be very weak gamma-ray emitters regardless of their \( L_{sd} \), and their EGRET upper limits alone are still one order of magnitude above our predictions for their gamma-ray luminosities. A qualitatively quite similar behaviour of millisecond pulsars, although for different physical reasons, results from the model of Dermer & Sturmer (1994), which was used explicitly in the context of millisecond pulsars by Sturmer & Dermer (1994). For their luminosity \( L_{SD94} = 1.1 \times 10^{10} B^{1/2} P^{-3} \) erg s\(^{-1}\) of gamma-rays beamed into a solid angle \( \Omega_g = 1.5 \times 10^{-7} P^{-1} \) sr (equations 2 and 3 of Sturmer & Dermer 1994), the apparent gamma-ray luminosity for 1 sr can be expressed as

\[
L_g = \Omega_g^{-1} L_{SD94} = 1.2 \times 10^9 B^{1/2} P^{-1} \frac{L_{sd}}{10^{13} \text{erg s}^{-1}}, \tag{9}
\]

and accordingly for two pulsars with \( B = 10^9 \) and \( 10^{12} \) G but identical spin-down luminosity \( L_{sd} \) the former object will be placed below the latter one in a diagram like Fig. 3. The EGRET upper limit for J0437−4715 \( (\sim 10^{34} \text{erg s}^{-1}) \) is well above the 1.6 \( \times 10^{36} \) erg s\(^{-1}\) resulting from equation (9).

Out of several objects in the analysis of Nel et al. (1996) with uncomfortably low EGRET upper limits, and contradicting thus the models that they discuss, only two are left as a potential threat to our model: B0950+08 and B0656+14. The EGRET limits for these two sources are too low to be accommodated by the model. It is encouraging, however, that B0656+14 was reported as a possible EGRET source (Ramanamurthy et al. 1996). Both pulsars were on the priority list of COMPTEL but with low ranks, and no results are...
available for them so far. There are no OSSE limits available for B0656+14 either. B0656+14 definitely deserves more attention as a promising target for gamma-ray experiments below the energy range of EGRET. Its parameters are very similar to those of Geminga and B1055—a wide gap (of no gamma-ray detections) before the seventh gamma-ray pulsar, B1055—52, emerges as No. 33. The gap contains several millisecond pulsars, with their flagship J0437—4715, taking a very high overall position—No. 7.

Our ranking of the top 30 candidates for gamma-ray emission, arranged by a predicted flux resulting from equations (4) and (8),

\[ f_\gamma = \frac{C_n E_\text{peak} L_{\gamma,d}^2}{D^2}. \]  

is presented in Table 2. The pulsar data base of Taylor et al. (1993, 1995) ordered by \( f_\gamma \) starts now with Vela, then goes to the Crab and Geminga. B1055—52 advances by 13 positions to No. 20. The millisecond pulsars (from the gap), including J0437—4715, disappear from the list of the ‘top 30’.

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NOTE ADDED IN PRESS

After we had submitted our paper for publication, Lucien Kuiper pointed out to us that the COMPTEL group has found indications for a signal from B0656+14 in the energy interval of 10–30 MeV (Hermsen et al. 1997).

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\[ \gamma = 0.33 \log \tau - 3.08, \]  

where the characteristic age of pulsars \( \tau = P/2P \) is expressed in years. The trend is based essentially on the Crab pulsar \( \gamma = 2.16, \tau = 1.3 \times 10^5 \) on the one hand, and on B1055—52 \( \gamma = 1.18, \tau = 5.3 \times 10^5 \) on the other hand. With the new determination of the spectral slope for B1055—52, \( \gamma = 1.59 \) (Fierro 1995), the prescription for \( \gamma(\tau) \) looks questionable. As a consequence, upper limits for \( L_{\gamma} \) derived for old pulsars, especially for millisecond pulsars, might be somewhat tighter. That would put the models discussed by Nel et al. (1996) into even deeper trouble, whereas the model that we propose still remains intact.

Pulsars from the data base of Taylor, Manchester & Lyne (1993), extended by Taylor et al. (1995), plus Geminga, arranged in a traditional ranking based just on spin-down fluxes \( L_{\gamma,d}/D^2 \), start with the Crab and five other gamma-ray pulsars. Then, however, there is a wide gap (of no gamma-ray detections) before the seventh gamma-ray pulsar, B1055—52, emerges as No. 33. The gap contains several millisecond pulsars, with their flagship J0437—4715, taking a very high overall position—No. 7.
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