Are the Sustainable Development Goals self-consistent and mutually achievable?

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Summary

• This paper uses the ICSU–ISSC Report on the Sustainable Development Goals (SDGs) to develop a network analysis of the SDGs.

• The analysis indicates that progress on Goals 1 (Poverty), 2 (Hunger) and 3 (Health) is highly positively reinforced; these Goals are the most likely to be achieved, and to be achieved earliest.

• The analysis also indicates that, due to trade-offs, Goals 14 (Oceans) and 15 (Terrestrial Ecosystems) are most at risk of not being achieved.

• A minimal mathematical model suggests that better outcomes on every Goal could be obtained through a combination of careful resource allocation and exploitation of the network linkages.

Abstract

On 18 September 2015 the United Nations General Assembly adopted a document with the title Transforming our world: the 2030 Agenda for Sustainable Development which sets out a plan of action to shape global development for the period 2015–2030 following on from the Millennium Development Goals.

In response to the 2030 Agenda, the International Council for Science (ICSU), in partnership with the International Social Science Council (ISSC) subsequently published a detailed commentary on the SDGs and the linkages between them. One issue raised by the ICSU-ISSC Report is the possibility that the SDG framework as a whole might not be internally self-consistent, and the report itself calls for a wider ‘systems perspective’.

In this paper we use the ICSU commentary as the basis for a quantitative theoretical analysis of the SDGs from a systems perspective. We provide a mathematical definition of self-consistency and show that the linkages we infer from the ICSU-ISSC report imply that the SDGs are not self-consistent.

However, using a simple dynamical model to investigate the combined outcome of direct efforts at tackling each Goal and the indirect effects on progress due to network effects, we show that network effects could be used to secure better outcomes on every Goal than would be possible if linkages between Goals did not exist at all. These better outcomes would be possible through an unequal, targeted re-allocation of direct efforts. Unequal distribution of direct effort can therefore make the SDGs mutually achievable.

Our conclusions contribute to the ongoing debate on the development of global strategies for achievement of the 2030 Agenda, the implementation of such strategies, and the monitoring of progress towards the Goals.
Keywords: Sustainable Development Goals; mathematical modelling; trade-offs; dynamical systems; network science; global perspective.

1 Introduction

The UN Sustainable Development Goals (SDGs) set out a very broad set of challenges for global development over the period 2015 - 2030. The SDGs propose a framework within which barriers to economic and societal progress should be addressed, for example inequality in opportunities and political representation within and between countries (Goal 7). Since their scope is significantly broader than that of the Millennium Development Goals, which preceded the SDGs, questions of coordination arise both within the SDGs and between the SDGs and other global initiatives, for example the UN Framework Convention on Climate Change [17] and the Sendai Framework for Disaster Risk Reduction 2015 - 2030 [20]. In this article we consider only the linkages within the SDGs themselves rather than these links to the UNFCCC and the SFDRR; even the restriction to the SDGs is a complex issue given that the set of 17 SDGs contains a collection of 169 separate targets.

There is a growing literature on interactions within the SDGs: recent contributions include the paper by Pradhan et al [12] which adopts a statistical viewpoint, computing correlations between the timeseries of the official SDG indicators, computed back to around 1990 in many cases, on a country-level basis. This provides assessments of the complementarities and trade-offs between SDGs through the computation of positive or negative correlations; it is unable to assess the direction of influences between Goals, and indeed might be subject to confounding variables. This is a valuable complementary direction to the present paper which takes a more theoretical approach, and focusses on questions of influence and directionality within the SDG network.

The large number of targets, and broad remit of the SDGs have led many authors to consider a subset of the Goals. For example the recent ICSU report [3] which focuses on four Goals: Goals 2 (Hunger), 3 (Health), 7 (Energy), and 14 (Oceans) and examines the interactions at the level of individual targets between those Goals and the other SDGs. [3] continues the science-based approach initiated in the earlier ICSU Report [2] on which we focus here. McCollum et al [8] discuss Goal 7 (Energy) and its links in detail, using the seven-point scale introduced by Nilsson et al [11]. Weitz et al [25] select two targets per Goal and analyse directed linkages between these, drawing directly on the context of Sweden. Le Blanc et al [5] discuss SDG 14 (Oceans) and consider directed links both between the seven targets associated with Goal 14, and between these targets and the other Goals. The directed nature of the interactions between SDGs is important, since it opens up a much wider understanding of how the SDG network is internally organised. Complementing the papers cited above, here we consider the linkages within the complete set of Goals 1-16, using some target-level information, but drawing conclusions only at the level of the Goals.

The analysis in this paper builds on the 2015 report [2] coordinated by the International Council for Science (ICSU), in partnership with the International Social Science Council (ISSC), and referred to below as ‘the ICSU report’. The ICSU report sets out a detailed and extensive qualitative commentary on the SDGs and the linkages between them and contains contributions from over 40 authors from 21 countries. It is one of the few documents that attempts consistently to treat the entire collection of Goals, rather than focussing on a subset (such as the Water–Energy–Food nexus). This makes it a distinctive, ambitious, and valuable assessment of the possible linkages between all SDGs, at the level of the overall Goals themselves.

The Executive Summary of the ICSU Report highlights in several places the need to consider linkages between Goals, and warns that the SDG framework does not in itself reflect those linkages. The authors comment that

[I]t is clear . . . that goal areas overlap, that many targets might contribute to several goals, and that some goals may conflict. The goals are also addressed without reference to possible links with other goals. Since the SDG framework does not reflect interlinkages . . . it is possible that the framework as a whole might not be internally consistent - and as a result not be sustainable.
In this paper we use the ICSU report as the basis for an attempt to make these linkages more quantitative and to explore the implications for the SDG network as a whole. It should be emphasised that the ICSU report itself emphasises a purely scientific perspective. Although the consequences of this are not described in detail, it suggests that social, political, and economic factors are not addressed explicitly. The report views progress towards the Goals in terms of scientific research and technological innovation, leaving unaddressed the questions of implementation, and political and economic influences. It also does not comment on the requirements of particular countries or regions; it takes an overarching global perspective. It would be of great interest to refine the analysis here either by attempting to combine scientific with political and economic arguments, and to focus attention on specific regions or individual countries.

Our conclusions are both about the system of SDGs as a whole and about the relative ease of progress on individual Goals. Since this paper is motivated, and underpinned, by expert judgements made in the ICSU report, our conclusions concern, more precisely, the systemic and relative progress possible on the SDGs as viewed through the lens of the ISCU assessments. As remarked on above, a particular bias is that a scientific perspective, rather than a political one, dominates: assessments of technological feasibility are more strongly represented than evidence of political will.

Because, as we show below, the network of SDGs does not satisfy the most obvious mathematical definition of self consistency, it is possible that misdirected efforts could lead, not just to a lack of progress on some Goals, but actually to make some Goals harder to achieve. In terms of specific Goals, the clearest implications are that Goals 14 and 15 (Oceans and Terrestrial Ecosystems respectively) are most at risk, while network effects significantly positively reinforce progress on Goals 1, 2 and 3 (Poverty, Hunger, and Health). The network of connections implies therefore that achieving Goals 1-3 should be relatively easy compared to achieving Goals 4-16. Moreover, this distinction between Goals 1-3 and Goals 4-16 is precisely because of the positive influences of the later Goals on Goals 1-3. This implies that more effort should be expended directly on Goals 4-16; this will in turn have a positive indirect impact on Goals 1-3.

Our results show that the targeting of direct effort can compensate for negative influences within the SDG network. Further, by combining network effects with carefully-targeted efforts that are distributed unequally across the Goals, it is possible to make significant progress on every Goal (including Goals 14 and 15) simultaneously and so avoid the kind of trade-offs that the ICSU report warns about.

Despite not satisfying a strict definition of self-consistency, we find that the directed network structure implied by the ICSU report is actually quite close to being self-consistent and certainly does not display some of the particular features than would make coherent progress on the SDGs significantly more difficult to attain: in this sense, although the SDGs are wide-ranging and potentially do contain many negative influences - trade-offs in the language of the ICSU report - the SDGs are by no means incoherent; a typical (in some sense) random network would be far less self-consistent.

More generally, we argue that the philosophy of attempting to describe and justify statements of the kind that ‘progress on Goal X is (positively or negatively) influenced by progress on Goal Y’ might be particularly useful when used in conjunction with data collected systematically country by country to monitor progress towards the SDGs over the coming decades. In this sense this paper attempts to complement data-driven approaches [16, 12], and the essential work of defining and collecting development statistics and progress indicators.

Ultimately this paper attempts to provide an accessible example of a network science-based approach to the SDGs, responding to the remark made in [19, section 2.1.8, page 43]: “The emerging disciplines of complexity science and network science provide an increasing body of knowledge which, however, has typically not been considered by policy makers to date, in large part because it is not readily accessible knowledge.”

2 Methodology and network properties

In this section we first describe the process by which we turn the ICSU report into a collection of quantitative assessments. Secondly, we propose a mathematical (but intuitively straightforward) definition of what it would
mean for the SDGs to be internally self-consistent, and discuss whether the network implicit in the ICSU report satisfies this definition or not; we find it does not, but that it is close to it.

We then introduce the additional influence of direct effort to support progress on each Goal. Through a simple dynamical model we discuss the balance between the necessary direct efforts and the progress due to the network effects. Robustness of the results to the modelling assumptions was tested through the addition of random weightings of the same magnitude as the initial couplings. This enables error bars at ±2 standard deviations to be estimated.

Throughout this article, as in the UN DESA Working Paper by David Le Blanc [4], we ignore Goal 17 ‘Strengthen the means of implementation and revitalise the Global Partnership for Sustainable Development’ since this Goal provides mechanisms that are intended to enable each of the more specific Goals 1 - 16. We also ignore the ‘means of implementation’ targets listed under each individual Goal, and keep only the targets that describe the specific goals themselves. After this pruning of the Goals and their targets we are left with 16 Goals which between them contain a total of 107 individual targets. Table 1 shows the number of targets associated with each Goal.

### 2.1 Network construction

For each SDG, the ICSU report provides a narrative table of linkages with each of the other Goals, and specifically the targets that each SDG is linked to within each of the other SDGs. For each entry in one of these tables, we determine the direction of the linkage given by the narrative, together with the sign of its influence (positive or negative). We estimate the strength of the linkage by the proportion of the total number of targets mentioned from the other Goal.

This estimate of strength appears reasonable since the ICSU report authors had freedom to cite linkages with as many targets in each linking Goal as they wished, and the number of targets cited varies from one to all that were possible. Since the ICSU report does not provide commentary on linkages between pairs of targets we have data that enables us to consider links only at the level of the entire Goals. The effect of this ‘normalisation’, dividing by the number of targets within each Goal, is explored in detail in section 2.4 below; results presented there indicate that it does not, in fact, significantly influence the results. A further point concerning the use of linkages at Goal-level rather than at the level of individual targets is that often a detailed analysis at the target level suggests that some links between targets in different Goals would be positive (reinforcing) and some would be negative (i.e. indicative of trade-offs). The expert opinion summarised by Singh et al [14] provides an example, for Goal 14 (Oceans) where target-based effects of different signs are proposed. Within the coarser-grained Goal level approach adopted here we effectively average over these individual effects; this may not be appropriate in all cases.

We now give an example. Consider the discussion of the linkages concerning Goal 1 (*End poverty in all its forms everywhere*) given on page 17 of [2]. In the row of the table for the linkage to Goal 3 (*Ensure healthy lives and promote well-being for all at all ages*) is the statement *Access to free health care is fundamental to poverty eradication* and target 3.8 (*Achieve universal health coverage . . .*) is explicitly listed. We interpret this as a directed link from Goal 3 to Goal 1, since the language implies that progress on Goal 3 will impact positively on progress on Goal 1, and we assign a weighting of 1/9 since, of the nine possible targets within Goal 3 that could have been referred to here, only one is explicitly listed.

In more formal mathematical language, the collection of linkages, made quantitative, are the entries of a $16 \times 16$
matrix $A$ which is the adjacency matrix of the corresponding weighted, directed graph. The entry $A_{ij}$ (which will also be written $A_{i,j}$ for clarity) describes the influence of Goal $j$ on Goal $i$. So the discussion in the example above would result in setting $A_{2,3} = 1/9$ and $A_{3,1} = 0$. Note that these matrix entries are also potentially affected by the discussion of linkages concerning Goal 3. Indeed, on page 25 of the ICSU Report we see that a link between Goal 1 and Goal 3 is also described: Poverty is a major cause of ill health and eradicating poverty will improve health and reduce health inequalities. Exactly three out of the five targets within Goal 1 are explicitly mentioned as having a positive impact on progress towards Goal 3, hence $A_{3,1} = 3/5$.

We carry out this procedure algorithmically, so that where no targets are listed, for example for Goal 1 on page 17 there is no direct link given with Goals 9, 12, 14 or 15, the corresponding entries $A_{1,9}$, $A_{1,12}$, $A_{1,14}$ and $A_{1,15}$, as well as $A_{9,1}$, $A_{12,1}$, $A_{14,1}$ and $A_{15,1}$, are zero. The direction of implication is usually clear from the text.

A small number of linkages are clearly indicated as negative influences. For example, on page 45 under Goal 8 (Growth), the linkage to Goal 6 (Water) states Increases in production (growth) can increase water pollution, …Protection of natural resources may inhibit production and growth. Since two of the six targets given under Goal 6 are listed here, and because the text implies negative linkages in both directions, we quantify this by setting both $A_{8,6} = -2/6$ and $A_{6,8} = -2/6$.

The above discussion shows that each entry $A_{ij}$ has usually only one, and at most two contributions, one from the section of the ICSU report commenting on Goal $i$ and one from the section on Goal $j$. Where there are two contributions they are aggregated. The entries are later scaled by a factor of $1/2$ to ensure that $A_{ij}$ always lies between $-1$ and $+1$; this helps in later interpretation. Figure 1 shows the sizes and signs of the entries in $A$ after this scaling has been applied and summarises the process of turning the ICSU report into a quantitative directed network structure. A complete description of the details of the process is given in the Appendix. Note that the data used in the construction of $A$ does not give rise to any diagonal entries in the matrix; there is no discussion in the report about the effect of any Goal on itself.

2.2 Self-reinforcing loops

A natural first question is to identify the SDGs that are connected by the edges with largest positive weights: these might be thought to provide some kind of ‘backbone’ to the interaction network. Given that the matrix $A$ has 162 non-zero entries, of which only 10 are negative, there are many closed directed loops formed from edges whose weights are all positive. The role of such self-reinforcing links has been discussed at length, but usually in the context of evolving networks, [6, 7]. Figure 2 shows the self-reinforcing loops with the highest edge weights.

Progress on Goal 13 (Climate) reinforces progress on Goal 6 (Water) which in turn reinforces progress on Goal 7 (Energy) which reinforces and is reinforced by Goal 13 again. Progress on Goals 13 and 15 (Terrestrial ecosystems) are similarly mutually reinforcing.

Clearly the view of the ICSU report is that progress on climate change is of fundamental importance. Another source of uncertainty, of particular relevance to Goals 7 and 13, occurs when a Goal has a significantly smaller number of targets compared to others. Goals 7 (Energy) and 13 (Climate) each contain only three targets. Hence, with the present methodology, the edge weights in the network identified by the authors of those sections of the ICSU Report can take only the values 0, 1/3, 2/3 or 1; this restriction in the set of possible values may lead the weights of these edges to be overstated, since it seems more likely that two out of three broad-based targets would be deemed relevant compared to six out of a set of nine or ten more specific targets, as would be the case for Goals 3, 8, 15 or 16. We comment further on this below in section 2.4.

2.3 A definition of self consistency for Agenda 2030

If all links in the network have positive edge weights, so that $A_{ij} > 0$ for all $1 \leq i, j \leq n$, then the matrix $A$ is called ‘positive’ (written $A > 0$). Positive matrices have an eigenvalue $\lambda_+$ that is real and positive and strictly larger in magnitude than any other eigenvalue. There exists an eigenvector $v_+$ corresponding to this largest eigenvalue that has all components positive. The eigenvector $v_+$ is often referred to as the Perron-Frobenius eigenvector (PFE) of $A$. In the case that all elements are known to be non-negative, i.e. $A_{ij} \geq 0$ for all $1 \leq i, j \leq n$ (written
Figure 1: Heat map showing the quantitative values $A_{ij}$ of links between the SDGs, constructed from a detailed analysis of the ICSU report. The colour of the matrix entry with coordinates $(i, j)$ indicates the support that progress on Goal $j$ lends to progress on Goal $i$. Entries left white indicate an absence of a link. For example, the dark (blue) squares indicating $A_{14,2}$ and $A_{14,11}$ show the significant negative impacts that progress on Goals 2 and 11 is indicated to have on Goal 14, corresponding to the red edges with negative weights in figure 2.

Figure 2: Short closed loops (of length 2 and 3) formed by directed edges of magnitude greater than $1/3$ in the SDG network, plus the links between Goals 5, 11, and 14 which join the loops. Positive values are indicated by arrows (blue edges), so that $j \rightarrow i$ indicates that Goal $j$ at the tail of the arrow has a positive influence on Goal $i$ at the arrowhead. Similarly, negative influences are indicated by (red) edges $j \rightarrow i$ ending in filled circles: Goal $i$ at the end with the circle is negatively influenced by Goal $j$.

For $A \geq 0$ a largest eigenvalue $\lambda_0$ that is real and positive still exists, with all other eigenvalues being less than or
equal in magnitude to $\lambda_0$. A Perron-Frobenius eigenvector $\mathbf{v}^{(0)}$ for $\lambda_0$ can similarly be shown to exist, for which all components are non-negative.

The idea of self-consistency can be expressed mathematically by considering it as applied to the linear dynamical system $\dot{\mathbf{x}} = A\mathbf{x}$. We could define the network of links between the state variables $x_1, \ldots, x_n$ to be self-consistent if $A\mathbf{x} \geq 0$ whenever $\mathbf{x} \geq 0$. Equivalently, this would mean that when all the state variables are positive or zero, then each state variable would be non-decreasing, so that state variables, starting from an initial condition in which $x_j \geq 0$ for all $j = 1, \ldots, n$ could never become negative. In fact, if state variables can never become negative then the possible kinds of behaviour at large times are limited: for each $j$, either $x_j(t) \to \infty$ as $t \to \infty$ or $x_j(t) = x_j(0)$ for all $t > 0$.

If we were to define self-consistent networks as those for which state variables could never become negative then a self-consistent network would be precisely one for which $A \geq 0$. However, it is more useful to make a weaker definition of self-consistency since we are more interested in the outcome of the network reinforcements, and the interplay between different links over long times, rather than the details of transients. Over long times, trajectories will (loosely speaking) become more aligned with the eigenvector corresponding to the largest positive eigenvalue of $A$; in the case that $A \geq 0$, and under the additional assumption that there is only a single eigenvector for the eigenvalue $\lambda_0$, this will be $\mathbf{v}^{(0)}$ and hence for every initial condition $\mathbf{x}(0) \geq 0$ there will exist a time $T \geq 0$ such that $\mathbf{x}(t) \geq 0$ for all $t > T$. Thus, self-consistency is guaranteed at long times but the time the network takes to become self-consistent might depend on the initial condition. This motivates the following definition.

**Definition:** A network of directed links between $n$ nodes with weights $A_{ij}$ is self-consistent if the matrix $A$ has an eigenvalue $\lambda_0$ that is (i) real, positive, and larger than the real parts of all other eigenvalues, and (ii) $\lambda_0$ has a unique non-negative eigenvector $\mathbf{v}^{(0)}$.

The Perron–Frobenius results imply that any network with $A \geq 0$ satisfies the definition of being self-consistent. But the definition of self-consistency is broader and it is possible that networks containing negative entries are still self-consistent.

If a network contains many entries of both signs then it is possible that the eigenvalues with the largest real part are a complex conjugate pair. This would indicate that the behaviour of the network effects over long times would drive oscillations: periodic rises and falls in the progress made on different Goals, with phase differences between the peaks in these cycles, as the result of the trade-offs identified by the network effects. In such a case it would be extremely difficult to guarantee simultaneous progress on every Goal.

The leading eigenvalue and eigenvector are therefore key properties of the network and indicate whether the network effects themselves would generate self-consistent progress on all Goals. For the SDGs we find that the leading eigenvalue is real, which is important, but that its eigenvector does not have entries that are all the same sign. So the SDG network is not self-consistent. Figure 3(a) plots the location of the eigenvalues of $A$ in the complex plane. We see that the largest eigenvalue $\lambda_0 \approx 1.467$ (3dp) is indeed real and positive, and it is significantly larger than the real part of the next eigenvalues to the left ($\lambda_{1,2} = 0.610 \pm 0.205i$, to 3dp) which form a complex conjugate pair.

The significance of complex eigenvalues is that they would drive oscillatory growth dynamics which would highlight trade-offs within the network. In this case, since their real part $Re(\lambda_1) = 0.610$ is much smaller than the leading eigenvalue $\lambda_0$, these effects are subdominant and in practice do not greatly contribute.

The eigenvector $\mathbf{v}^{(0)}$, shown in figure 3(b), contains both positive and negative entries. As a result, the network effects are not positively reinforcing for all Goals: we would expect negative progress on Goal 14 over long times.

### 2.4 Network robustness

In this section we discuss two issues in the construction of the network that are potentially unsatisfactory; first, that different authors contributed the assessments related to different Goals in the ICSU Report, and second, that there are potentially many variations on the protocol used for weighting the edges, from which we proposed dividing the number of targets mentioned by the total number available in section 2.1.
Chapters in the ICSU Report were drafted by between one and five authors each, and subject to review by a dozen other experts. This broader review should have ensured a balanced and consistent approach. There are clearly indications that the individual preferences of chapter authors persisted, most obviously in the section on Goal 11 (Cities) where no table of linkages was constructed.

These inconsistencies are, however, useful in that they show that the chapters clearly reflect different points of view. This suggests that it is important to ask whether there is a measure of the internal self-consistency of the ICSU Report available. An insight into this can be gained by examining the ways in which the chapter authors for each Goal draw out influences of that Goal on others, and also propose ways in which other Goals influence that Goal itself. In other words, the directed nature of the network also highlights that the two separate contributions to each edge, i.e. the influence $A_{ij}$ of Goal $j$ on Goal $i$ should, for consistency, be identified both by the authors of the chapter on Goal $i$ and by the authors of the chapter on Goal $j$.

Figure 4 provides a breakdown of the heat map shown in figure 1 into the separate contributions provided by the different chapter authors. Each chapter in the ICSU Report corresponds to one row in figure 4(a) and the corresponding column in figure 4(b). Matrix entries that are similar colours indicate agreement in the assessments of different chapter authors.

Overall there are 81 non-zero entries in $A^{in}$ (75 positive and six negative), and 138 non-zero entries in $A^{out}$ (132 positive and six negative), each out of 256 possible links. This indicates that the authors identified significantly more ways in which the Goal they were discussing influenced other Goals rather than the other way around. Out of these non-zero entries, there are 56 cases (i.e. 69% of the 81 possible cases) in which the two relevant chapters reported links between the same pair of Goals. This indicates a high degree of consistency in identifying the most important reinforcing links.

None of the six negative edges identified in each case coincided, perhaps indicating that, as well as there being far fewer trade-offs than positive reinforcements, the trade-offs are harder to identify. There are five cases in which the two relevant chapters identified links of different signs. These are $A_{2,7}$, $A_{8,6}$, $A_{14,8}$, $A_{14,15}$, and $A_{15,7}$. These disagreements might be useful in pointing to where more effort is particularly required in order to discern the linkages between these Goals: the effects of progress towards Goal 7 (Energy) on Goals 2 (Hunger) and 15 (Ecosystems); the effect of Goal 6 (Water) on Goal 8 (Growth); and the effects of Goal 8 and Goal 15 on Goal 14 (Oceans).

Out of these five cases of disagreement in the sign of the linkage, in one case the two contributions cancel
Figure 4: Heat maps showing the quantitative values of links between the SDGs, constructed from the ICSU report. As in figure 1, the colour of the matrix entry with coordinates \((i, j)\) indicates the support that progress on Goal \(j\) lends to progress on Goal \(i\). Entries left white indicate an absence of a link. (a) Heat map built up by rows, \(A_{ij}\) showing in row \(i\) the influences of other Goals on Goal \(i\); (b) Heat map built up by columns, \(A_{ij}^{\text{out}}\) showing in column \(j\) the influences of Goal \(j\) on the other Goals.

completely: \(A_{15,7}\). Hence the final matrix \(A\) contains one fewer non-zero entry than expected: \(A\) has 162 non-zero entries.

Turning to the second issue, we acknowledge that, although the procedure used in constructing the adjacency matrix \(A\) from the commentary in the ICSU Report is algorithmic, it is still in part subjective and of course open to biases of many kinds. Investigating the robustness of the approach is essential. To assess the robustness of the results presented in section 2.3, and in particular figure 3, to the weightings where the number of targets mentioned was divided by the total number of targets for the Goal in question, which varies in the range three to ten across different Goals, we considered a modified adjacency matrix \(\hat{A}\) in which the entries are replaced by the value +1 where the original edge weight in \(A\) is positive, −1 where the original edge weight is negative and zero otherwise. We refer to this as the Boolean version of the adjacency matrix. Figure 5(a) compares the eigenvalues of the Boolean version and the original, with the eigenvalues of the Boolean matrix rescaled so that the largest eigenvalues are equal; this rescaling adjusts for the fact that the mean of the entries in the matrix has increased. The Boolean matrix has a very similar characteristics to the original, indicating robustness of the main properties of the network described earlier. Notably, the eigenvalue of the Boolean adjacency matrix with the largest real part is also real and positive, and there is a very similar distance between this largest eigenvalue and the eigenvalues with the next largest real part. In the original case these next eigenvalues are a complex conjugate pair; in the Boolean case this is a real eigenvalue. This hints at the behaviour of the network as even less likely to show oscillatory transients than in the original case. Turning to the eigenvectors corresponding to the leading eigenvalues, as plotted in figure 5(b), we observe the their overall shape is extremely similar: both indicate a higher rate of progress on Goals 1, 2 and 3 compared to the remaining Goals, and the progress on Goal 14 (Oceans) implied by the network structure remains negative. The wide separation between the largest eigenvalue and the next-largest, in both cases, ensures that the eigenvector corresponding to the largest eigenvalue will in both cases dominate the dynamics of the network interactions over long times.

2.5 Discussion of Goal 1 (Poverty)

The UN Global Sustainable Development Report 2015 [19, pages 44-45] summarises an analysis of the ICSU report by stating that ‘When SDG 17 on ‘means of implementation’ … is excluded from the analysis, SDG 1 on
poverty is the most central node for the system. In other words, in the view of scientists, ... progress on poverty eradication is also central to all other goals.' [19, section 2.1.9]. Although this statement is immediately qualified by a cautionary note that the ICSU report did not define the precise nature of the linkages, this conclusion deserves to be treated with substantial caution. For example, figure 2-1 [19, page 45], which serves to illustrate the point made above, shows an undirected network, while, as we have set out above, the linkages between Goals indicated by the ICSU Report are clearly directed.

The broader discussion of Goal 1 in the ICSU Report reinforces this view: the elimination of poverty is a central goal, and relies on economic stability and growth, fair income distribution and governance and institutional relationships both within and between countries. These are all factors that drive the achievement of Goal 1. Less is said about the influence, in itself, of eliminating poverty on progress towards most of the other Goals. The ICSU report text is explicit about the effects of poverty reduction on progress on food security, healthcare, climate change and peaceful societies; hence there are directed links from Goal 1 to Goals 2, 3, 13 and 16.

Other outcomes of poverty elimination might well include, for example, education, economic growth and industrialisation which, as separate Goals within the SDG framework, we might expect to be represented by explicit links that are, in fact, harder to discern in the ICSU text. These links are therefore not present in figure 1, since the purpose of this paper is to understand the implications at the system scale of the individual assessments of linkages made Goal by Goal, and from a scientific viewpoint. It could be argued that the elimination of poverty leads to progress on education, economic growth, and industrialisation Goals only if other societal and political factors are supportive.

### 2.6 Discussion of Goal 14 (Oceans)

Linkages between Goal 14 and other Goals have been studied in particular by Le Blanc et al [5] and by the more recent ICSU report [3]. The interactions between Goal 14 and Goals 2 (Hunger) and 11 (Cities) are particularly of interest since in the ICSU Report (2015) they are identified as strongly negative influences of Goals 2 and 11 on Goal 14.

In the later ICSU report [3] these negative influences are clear. For Goal 2, two sources of negative influence are identified [3, page 191]: pollution from agricultural run-off of fertilizers into marine environments, reducing...
fish stocks, and that the creation of Marine Protected Areas, while overall enhancing food security, may serve to limit fishing access for coastal communities. Thus progress towards Goal 2 may negatively impact Goal 14 via pollution, and likewise Goal 14 may negatively impact Goal 14 via limiting access to food resources. Le Blanc et al [5] also identify similar possible negative linkages between Goal 14 and Goal 2, see [5, table 5].

The later ICSU report [3] comments also on negative interactions between Goals 2 and 11 [3, page 198]. In brief, the use of local materials in building construction will involve trade-offs with ecosystem management and marine conservation policies in coastal areas. Similar trade-offs between marine conservation and increased economic activity are noted in the discussion on linkages between Goal 14 and Goal 1 in [3].

In terms of the discussion of the ICSU Report [2], then, these remarks from the later ICSU report serve to reinforce the key trade-offs identifiable in the earlier report between Goal 14 and Goals 2 and 11.

3 A dynamical model

In this section we consider a combination of direct effort to achieve each of the Goals and network effects that reinforce, or hinder, progress over time.

Let the variable $x_i(t)$ describe the progress made towards Goal $i$ at time $t$ measured in years, so that $t = 0$ corresponds to 2015 and 2030 corresponds to $t = 15$, $t$ measured in years. We set $x_i(0) = 0$ indicating that initially there is no progress made towards Goal $i$ (i.e. we set zero as the reference point for measuring future progress) and we assume that, in the absence of any interactions between Goals, an amount of ‘direct effort’ (public expenditure, political will, private enterprise) is available so that constant annual progress $m_i$ is made in order to enable $x_i(15) = 1$, indicating that Goal $i$ has been fully achieved in 2030. This is, of course, a very great assumption. We make it in order to be able to address questions that compare progress on the different Goals, and questions that arise in the relative merits of direct effort, or indirect network effects, as mechanisms for achieving progress. We scale the values of the direct progress variables $m_i$ so that this scenario naturally corresponds to setting $m_i = 1$ for every $i$.

Now, since our focus is on the effect of the network structure described by the matrix $A$, compared to ignoring these positive and negative influences, we consider the evolution of progress on Goal $i$ to be a combination of the above constant progress at a rate $m_i$, together with feedback from the progress made on every other Goal, as described, in the simplest case, by the differential equation

$$\frac{dx_i}{dt} = \frac{1}{15} \left( m_i + \varepsilon \sum_{j=1}^{N} A_{ij}x_j \right),$$

so that $\varepsilon$ represents the proportional influence of progress on other Goals on Goal $i$: if $\varepsilon = 1.0$ and the entry $A_{ik} = 1$ then complete achievement of Goal $k$ (i.e. $x_k = 1$) drives the same increase in progress on Goal $i$ as the direct effort $m_i$ does. Similarly, if $\varepsilon = 0.1$ and $A_{ik} = 1$ then complete achievement of Goal $k$ produces a 10% increase in progress towards Goal $i$. We do not, in practice, expect that the implicit assumption of linearity here will hold precisely, but the network structure is an attempt to capture the largest of these interaction effects and this lack of independence could be considered a second-order correction.

On the other hand, the interpretation of $\varepsilon = 0.1$ as a 10% increase in the rate of progress is an overstatement since the values of the $x_k$ all start at zero and increase slowly over time, reaching values of order unity only towards the end of the time period under consideration. We note also that the solution of the system of differential equations (1) can be calculated explicitly.

With these caveats, we now turn to figure 6 which shows the progress on each Goal relative to the uncoupled case $\varepsilon = 0$ for which $x_i(t = 15) = 1.0$, indicating that direct efforts alone were used to drive complete progress towards every Goal over the 15 year time horizon. The uncoupled case is indicated by the solid horizontal line in each part of the figure.

For each Goal, the open circles within the error bars represent the progress made on that Goal when the network effects are introduced by setting $\varepsilon = 0.1$. The circles are at the same points in both figure 6(a) and 6(b). We see that
the positive reinforcements between Goals are particularly strong for Goals 1 (Poverty), 2 (Hunger) and 3 (Health), for which a coupling strength $\varepsilon = 10\%$ produces a 30-35% additional increase in overall progress towards these Goals, above that expected by the direct effort expended. Goals 13 (Climate Change) and 7 (Energy) also strongly reinforced, with increases in the 15-20% range. However, in the case of Goal 14 the network effects of progress on the other SDGs results in fact in lower progress than the amount of direct effort should yield on its own. This is a direct consequence of the large negative entries in row 14 in the matrix $A$, shown in figure 1 and the negative entry in the leading eigenvector $v^{(0)}$ shown in figure 3(b).

The error bars shown in both parts of figure 6 deserve careful definition. They have been added to attempt to assess the robustness of our results to fluctuations of up to 50% in the strengths of the interactions. For (a) we compute $10^4$ stochastically perturbed versions of the matrix $A$; in each perturbed version every entry $A_{ij}$ is perturbed by an iid random value drawn uniformly from the interval $[-\frac{1}{2}, \frac{1}{2}]$. Recall that the entries $A_{ij}$ are normalised to lie in the range from -1 to +1. We then compute the deterministic solution to the ODEs (1) using this perturbed version of the matrix $A$. Hence we randomly adjust the network links but we then hold them constant over the 15 year evolution of the model. The interval indicated is then $[\bar{x}_i - 2\sigma_i, \bar{x}_i + 2\sigma_i]$ where the mean $\bar{x}_i$ and standard deviation $\sigma_i$ are computed for Goal $i$ using the ensemble of $10^4$ results.

For (b) we proceed in a similar fashion to (a) but the stochastic perturbations of the matrix $A$ are constructed multiplicatively, thus preserving the non-existence of links where there are none indicated in the ICSU Report. We construct a perturbed version of the matrix $A$ by multiplying each entry $A_{ij}$ by a random factor drawn uniformly in the range $[\frac{1}{2}, \frac{3}{2}]$. As in (a), the interval indicated by the error bars is $[\bar{x}_i - 2\sigma_i, \bar{x}_i + 2\sigma_i]$ where the mean $\bar{x}_i$ and standard deviation $\sigma_i$ are computed for Goal $i$ using an ensemble of $10^4$ results.

The error bars in figure 6(b) therefore indicate the effects of fluctuations of at most 50% in the edge weightings computed from the Report text, but agreeing with the present or absence of linkages, as identified there. The error bars in figure 6(a) indicate the effect of fluctuations in the network as a whole, including linkages that were not identified in the ICSU Report.

At a coarse level, it is clear that the error bars in figure 6(b) and much smaller than those in figure 6(a) and one key reason for this is that in (a) all 256 entries of $A$ were available to be perturbed, while in (b) only the 162 non-zero entries were perturbed; in addition the multiplicative nature of the perturbations implies that the perturbations themselves are also on average smaller. We note that there is a positive correlation between means $\bar{x}_i$ and variances $\sigma_i^2$ in figure 6(b) as one would expect from multiplicative perturbations.

The effects of the perturbations allow us to conclude that the model results, in terms of which Goals are more easily achieved than others, are robust to the exact choices made for the weightings of the linkages.

Our last comment in this section is that the results above also give a prediction for the order in which the Goals are achieved. This provides a more qualitative indicator of the structure within the SDG system. As the network coupling parameter $\varepsilon$ increases the Goals will tend to be achieved earlier since the coupling provides a stronger overall driving. This argument holds for all Goals except for Goal 14 (Oceans): as we have seen above, the network effects provide a systematic suppression of progress on Goal 14. So, as we increase the network effects the absolute time at which progress on each Goal reaches the value 1 is not of as much interest as the order in which progress on each Goal reaches unity. Table 2 lists the Goals for the extreme values of the network coupling parameter, in order to highlight the differences caused by the network couplings. The pattern overall is very similar in the two cases, and hence for all values of $\varepsilon$ the prediction of the order for the first nine achieved does not change; also the final two Goals are always Goal 15 (Ecosystems) and Goal 14 (Oceans): in fact for sufficiently large $\varepsilon$ Goal 14 is never achieved: progress is made towards it but then decays as the influences of progress on the other Goals overcomes this and drives reductions in progress at longer times. Places 10 - 14 in the table contain the same set of Goals but in an order that is sensitive to the choice of $\varepsilon$. This ordering of the Goals may help in organising the selection and reporting of indicators towards progress on the SDGs overall: one might wish to focus more careful reporting on Goals further down the table.
Figure 6: Progress on each SDG when direct efforts are set to $m_i = 1$ for each $i$, and the network coupling parameter $\varepsilon = 0.1$, indicating indirect effects that are 10% of the strength of direct effects, normalised relative to the uncoupled case $\varepsilon = 0$. Circles (in both cases) show expected progress on each Goal, integrating the ODE model (1) from the initial state $x(0) = 1$ at $t = 0$ up to time $t = 15$. Error bars illustrate (a) the variation in results due to stochastic perturbations to all the network links in $A$; and (b) the variation in results due to stochastic perturbations only to the weightings for the non-zero edges identified in the ICSU Report.

Table 2: Predicted order in which the Goals are achieved when direct efforts are equal, i.e. setting $m_i = 1$ for each $i$. The left-hand column, labelled ‘weak network effects’ corresponds to $\varepsilon = 0.01$; the right-hand column, labelled ‘strong network effects’ corresponds to $\varepsilon \geq 2.0$ which is unlikely but serves to indicate the robustness of parts of the ordering to changes in $\varepsilon$. Goal 2 (Hunger) is predicted to be achieved first in both cases. In the case of strong network effects, Goal 14 is not achieved.

Table: Predicted order of Goal achievement

| Weak network effects | Strong network effects |
|----------------------|------------------------|
| 2. Hunger            | 2. Hunger              |
| 1. Poverty           | 1. Poverty             |
| 3. Health            | 3. Health              |
| 13. Climate          | 13. Climate            |
| 7. Energy            | 7. Energy              |
| 8. Growth            | 8. Growth              |
| 11. Cities           | 11. Cities             |
| 16. Peace            | 16. Peace              |
| 6. Water             | 6. Water               |
| 9. Industry          | 10. Inequality         |
| 10. Inequality       | 4. Education           |
| 4. Education         | 5. Gender              |
| 12. SCP              | 9. Industry            |
| 5. Gender            | 12. SCP                |
| 15. Ecosystems       | 15. Ecosystems         |
| 14. Oceans           | [14. Oceans]           |

4 Optimising the allocation of direct effort

The results presented in section 3 were obtained under the assumption of equal direct efforts $m_i = 1/15$ on every Goal $i$. In this section we depart from this, asking whether we can re-allocate the direct efforts on Goals, and,
through a combination of these direct efforts and the network effects, produce better overall results on all Goals. This is therefore an optimisation problem: to find the allocation of direct efforts \( m_1, \ldots, m_{16} \) that maximise the progress indicators \( x_1(15), \ldots, x_{16}(15) \). In order to make the problem tractable we demand that the progress made on every Goal must be equal, i.e. that \( x_1(15) = x_2(15) = \cdots = x_{16}(15) \).

Since the solution to the ODEs (1) is available in closed form:

\[
x(t) = \frac{1}{\varepsilon} A^{-1} \left[ \exp \left( \frac{\varepsilon t}{15} A \right) - I \right] m,
\]

where \( m \) is the vector of allocations \( m_1, \ldots, m_{16} \) and \( t \) is time. Setting \( t = 15 \) we use a quasi-Newton iteration routine in MATLAB to locate the optimal choices of the allocations. The results are shown in figure 7. Figure 7(a) shows the allocations \( m_i \) for each Goal, for three different values of \( \varepsilon \): \( \varepsilon = 0.01 \) (solid line, black), \( \varepsilon = 0.05 \) (dashed, blue), and \( \varepsilon = 0.1 \) (dot-dashed, red). As \( \varepsilon \) increases the allocations depart from equality of effort on every Goal, in a manner that is in some sense opposite to the progress achieved when equal direct efforts are made; compare with figure 6. In particular, as \( \varepsilon \) increases and the network effects become more important there is less requirement to directly invest effort on progress towards Goals 1 (Poverty), 2 (Hunger), 3 (Health) and 13 (Climate Change) and more need to invest direct effort on progress towards Goals 14 (Oceans) and 15 (Ecosystems). Other Goals that require more direct effort include 4 (Education), 5 (Gender), 9 (Industry), 10 (Inequality), and 12 (Sustainable Consumption and Production).

Figure 7(b) complements (a) by showing the overall improvement in progress towards all the Goals when the optimal direct resource allocation is made, as set out in (a). As \( \varepsilon \) increases, the improvements increase in a nonlinear fashion. This indicates that the network effects are potentially a route to more efficient resource allocation in order to achieve the SDGs. Moreover, it should be emphasised that the progress shown in figure 7(b) applies to every Goal, even those that are subject to trade-offs within the network such as 14 (Oceans): the uneven allocation of direct efforts can counterbalance the negative network effects and lead overall to better system-wide progress towards the SDGs.

A critical value of \( \varepsilon \approx 0.311 \) exists, beyond which it is no longer possible to find an optimal solution in which all the values \( m_i \) of the ‘relative direct effort’ are positive. This is indicated by the red square in figure 7(b). As
one expects from figure 7(a), this is caused by the direct effort $m_2$ for Goal 2 (Hunger) dropping to zero. The corresponding value of the overall relative improvement is around 1.55, on every Goal. We could conclude that, trusting everything to network effects, roughly a 50% additional benefit could be obtained; network effects are not necessarily small perturbations to the pursuit of each Goal independently.

5 Discussion

In this paper we have used the scientific evaluation of the Sustainable Development Goals, as summarised in the ICSU Report [2], to construct a directed, weighted network of the mutual influences between the SDGs. This approach emphasises the qualitative nature of linkages, and attempts to shape our understanding of the SDGs at a system-wide scale, in order to provoke discussion, even at this early stage in Agenda 2030, of which Goals are at higher risk of not being achieved.

The approach we take aims to understand the system-level nature of, and linkages between, the SDGs. In this sense it is similar to the use of causal loop models for understanding complex systems, as described in many books on system dynamics, for example [22] and [9]. Causal loop models describe positive and negative influences between variables with a view to enabling the visualisation of complexity and the identification of ‘reinforcing loops’ (those containing all positive edge weights) and ‘balancing loops’ (those containing both positive and negative edge weights that might tend to drive the relevant variables back towards equilibrium). A recent example of an analysis of a modern problem using causal loop models is the study of obesity published by the UK Government Office for Science [21]. The analysis presented here is more precise about the numerical values of the relevant positive and negative edges, pushing beyond the qualitative study of reports such as [21], towards (but stopping short of), more traditional economic systems dynamics ‘stock and flow’ models.

Specifically for the SDG system, as figure 1 illustrates, we find that there is a distinct asymmetry between Goals 1, 2 and 3 and the remaining Goals. Progress on Goals 4 – 16 generally promotes progress on Goals 1, 2 and 3 but there are far fewer links in the other direction. It therefore seems reasonable to conclude that a consequence of the ICSU Report is that we should consider the Goals as divided into two distinct subsets: Goals 1, 2 and 3 together form one subset, and Goals 4 - 16 form the other. Goals 4 - 16 interact strongly among themselves in many ways, and this subset, taken as a whole, in turn promotes Goals 1 - 3. But it is of interest to note that there is little feedback from Goals 1 –3 on Goals 4 – 16, although clearly the large negative entry $A_{14} 2$, noted above, is a specific exception to this general remark. In mathematical terms, the large number of zero entries indicated by the light blue in the lower left of the figure indicates that the matrix $A$ is not too far from being ‘block upper triangular’. The key practical conclusion of this decomposition into two subsets is that Goals 1, 2 and 3 are much more likely to be achieved, and that progress on Goals 4 - 16 will be more difficult. When we turn to the conceptual model that combines network effects with direct investment, a natural consequence is that achieving Goals 1, 2 and 3 requires a lower level of direct investment than for the other Goals.

For Goal 1 specifically, analysis of the ICSU Report, as summarised here in the present paper, shows that the linkages with Goal 1 (Poverty) are, in every case except one (the mutually reinforcing links between Goal 1 and Goal 2), that progress on other Goals will promote progress on Goal 1, rather than Goal 1 being a direct enabler of progress on the other Goals. This is a refinement of the statement in the UN Global Sustainable Development Report 2015 [19] which, as discussed in section 2.5 identified Goal 1 as the ‘most central node for the system’. The viewpoint of the expert scientists who contributed to the ICSU report should be more correctly represented as a set of influences that support Goal 1, but Goal 1 itself is, intriguingly, not described explicitly as enabling greater progress on other Goals (except for Goal 2). The network presented in this paper reflects the directions that influences operate in, motivated by the language and methodology used in the data source; our aim is to move away from the conclusion of mutual reinforcement, that ‘progress on every Goal leads to progress on every other Goal’.

In summary, according to the analysis presented in the ICSU report, Goals 1–3 emerge clearly as being much more heavily influenced by Goals 4–16 than influencing them in turn: Goals 1–3 are ‘downstream’ of Goals 4–16. This observation is at the root of the statement that Goals 1–3 benefit from the network of influences more than other Goals do. The systemic benefits to progress on Goals 1–3 of progress on Goals 4 (Education) and 5 (Gender
Equality) have been noted in data-driven studies, for example by Smith & Haddad [15]. It is also notable that, according to [10, page 16] within OECD countries there is a need for most progress to be made on Goals 4 and 5, looking at OECD averages.

Another interpretation of these results is in terms of the separate viewpoints of ‘science’ and ‘policy’, in the sense that the ‘science’ viewpoint emphasises the extent to which growth in scientific understanding, skills, and technological implementation generates linkages between Goals. Improved measurement or technological solutions in an area linked to one Goal (for example the provision of further low-cost generic drugs to developing countries) can drive improvements elsewhere in the SDG system. Thus the ‘science’ viewpoint is itself aligned with the intrinsic, and systemic, effects within the network. The ‘policy’ viewpoint accords more clearly with extrinsic effects: the direct investment that governments, and civil society more generally, are able to make in order to address local issues related to specific Goals. These are aligned to the ‘direct efforts’ $m_i$ described above. Hence the results shown in figure 7(a) may provide some kind of coarse-grained insight into the relative importance of assessing policy options related to each of the SDGs.

The ICSU report is, of course, the first (rather than the last) word on linkages between the SDGs. There are many issues left out, for example the influences of progress on poverty reduction (Goal 1) on many other Goals, as remarked on in section 2.5. The ICSU report takes a global viewpoint, and we attempt in this paper to capture this sense of which linkages were felt to be most important at the time the report was written: it is a snapshot of an expert community attempting, together, to understand the Agenda 2030 paradigm and how this complex system fits together. This paper attempts to continue this process of discernment.

The assumptions made in the modelling in section 3 include, most importantly, that the Goals are actually achievable by 2030. As a direction for future work it would be of interest to remove this assumption, together with attempting to estimate, perhaps, different values for the factors of $\varepsilon$ that appear in (1), for different Goals $i$, since progress on different Goals will be affected by network effects to an overall greater or lesser degree. A further refinement would be, when computing an optimal allocation of direct effort, as in section 4, to introduce weightings to capture the fact that direct ‘investment’ towards different Goals will have different marginal costs.

The Prototype Global Sustainable Development Report 2014 [18] provides another example of the kind of network analysis that the ICSU Report attempts, see table 18 on pp55-58 in [18]. In its section 3.2 ‘Reflection on synergies and trade-offs’ this table provides a qualitative discussion of the net effect of a number of global trends since 1950 on sustainable development progress described in terms similar to, but not precisely the same as, the SDGs. Entries in this table are either denoted as ‘positive’ (supporting sustainable development), ‘negative’ or as having no identifiable impact on sustainable development. This table is therefore similar to the Boolean version of the adjacency matrix described above, and it would be of interest in future work to investigate the properties of this matrix in itself, and to compare it to the matrix used here from the ICSU Report.

Further, the recent ICSU report [3] A Guide to SDG Interactions refines further the analysis of linkages initiated in the 2015 ICSU Report, for four of the SDGs: Goal 2 (Hunger), 3 (Health), 7 (Energy) and 14 (Oceans). In each case key interactions with a subset of the other Goals is discussed in extensive detail, using a seven point scale to describe the level of positive, neutral, or negative interactions between individual targets in these Goals, and those in other Goals. The overall conclusions of the 2017 ICSU Report are that there are no ‘fundamental incompatibilities’ between the Goals analysed, but some constraints were identified where coordinated policy interventions would be required in order to ensure that trade-offs between progress on different Goals did not arise.

It should also, of course, be pointed out that both the analysis presented here, and that presented in the matrix of linkages in [18] take a global viewpoint; a further direction for investigation would be the compilation of matrices for linkages at the level of regions, or indeed individual countries. This is clearly the ambition of data-driven approaches [27, 12], and so this more detailed level of analysis of the linkages would complement those studies.

Finally we briefly contrast the analysis here with work by Spaiser et al [16], see also [13]. The key difference is that these authors build a dynamical systems model directly from available historic data. The data series that they select are argued to be useful proxies for progress on the SDGs, and the models therefore describe trajectories of a low-dimensional dynamical system that indicates the linkages in a dynamic way, similar in philosophy to our model (1). That this modelling approach takes a very different set of starting points, but has similar ultimate aims to the work set out here, shows the wide range of possibilities for future work in this area, combining data with
models to provide a systems-level understanding of the SDG framework for global development.

References

[1] D. Griggs, M. Stafford Smith, J. Rockström, M. C. Öhman, O. Gaffney, G. Glaser, N. Kanie, I. Noble, W. Steffen, and P. Shyamsundar, An integrated framework for sustainable development goals. *Ecology and Society* **19**(4):49 (2014)

[2] International Council for Science (ICSU), International Social Sciences Council (ISSC). Review of the Sustainable Development Goals: The Science Perspective. Paris: International Council for Science (ICSU). ISBN: 978-0-930357-97-9 (2015)

[3] International Council for Science (ICSU). A Guide to SDG Interactions: from Science to Implementation. Eds: D.J. Griggs, M. Nilsson, A. Stevance, D. McCollum. International Council for Science, Paris. DOI: 10.24948/2017.01 (2017)

[4] D. Le Blanc, Towards integration at last? The sustainable development goals as a network of targets. UN Department of Economic & Social Affairs, DESA Working Paper No. 141 (2015)

[5] D. Le Blanc, C. Freire, M. Vierros, Mapping the linkages between oceans and other Sustainable Development Goals: A preliminary exploration UN Department of Economic & Social Affairs, DESA Working Paper No. 149 (2017)

[6] S. Jain and S. Krishna, Autocatalytic sets and the growth of complexity in an evolutionary model, *Phys. Rev. Lett.* **81**(25), 5684–5687 (1998)

[7] S. Jain and S. Krishna, Graph theory and the evolution of autocatalytic networks. In *Handbook of Graphs and Networks: From the Genome to the Internet* (eds S. Bornholdt and H.G. Schuster), Wiley-VCH Verlag. doi: 10.1002/3527602755.ch16 (2002)

[8] D.L McCollum, L. Gomez Echeverri, S. Busch, S. Pachauri, S. Parkinson, J. Rogelj, V. Krey, J.C. Minx., M. Nilsson, A.-S. Stevance, and K. Riahi, Connecting the sustainable development goals by their energy inter-linkages *Env. Res. Lett.* **13** 033006 (2018)

[9] D.H. Meadows, *Thinking in Systems: a Primer*. Editor: D. Wright. Chelsea Green Publishing Company. (2015)

[10] OECD, Measuring distance to the SDG targets: An assessment of where OECD countries stand. (2017)

[11] M. Nilsson, D. Griggs, and M. Visbeck, Map the interactions between sustainable development goals. *Nature* **534**, 320–322 (2016)

[12] P. Pradhan, L. Costa, D. Rybski, W. Lucht, and J.P. Kropp, A systematic study of Sustainable Development Goal (SDG) interactions. *Earths Future* **5**, 1169–1179 (2017)

[13] S. Ranganathan, S.C. Nicolis, R. Bali Swain and D.J.T. Sumpter, Setting development goals using stochastic dynamical system models. *PLoS ONE* **12**(2): e0171560. DOI:10.1371/journal.pone.0171560 (2017)

[14] G.G. Singh, A.M. Cisneros–Montemayor, W. Swartz, W. Cheung, J.A. Guy, T.-A. Kenny, C.J. McOwen, R. Asch, J.L. Geffert, C.C.C. Wabnitz, R. Sumaila, Q. Hanichj, and Y. Ota, A rapid assessment of co-benefits and trade-offs among Sustainable Development Goals. *Marine Policy*. In press DOI:10.1016/j.marpol.2017.05.030 (2018)
[15] L.C. Smith and L. Haddad, Reducing child undernutrition: past drivers and priorities for the post-MDG era. *World Development* **68**, 180-204 (2015)

[16] V. Spaiser, S. Ranganathan, R. Bali Swain and D.J.T. Sumpter, The sustainable development oxymoron: quantifying and modelling the incompatibility of sustainable development goals. *International Journal of Sustainable Development & World Ecology*, DOI: 10.1080/13504509.2016.1235624 (2016)

[17] United Nations, *United Nations Framework Convention on Climate Change*, New York (1992)

[18] United Nations. Prototype Global Sustainable Development Report. New York: United Nations Department of Economic and Social Affairs, Division for Sustainable Development, July 2014. [http://sustainabledevelopment.un.org/globalsdreport/](http://sustainabledevelopment.un.org/globalsdreport/) (2014)

[19] United Nations. Global Sustainable Development Report. New York: United Nations Department of Economic and Social Affairs, Division for Sustainable Development, 2015. [http://sustainabledevelopment.un.org/globalsdreport/](http://sustainabledevelopment.un.org/globalsdreport/) (2015)

[20] United Nations, *Sendai Framework for Disaster Risk Reduction 2015–2030*, Geneva (2015)

[21] P. Vandenbroek, J. Goossens, and M. Clements, Foresight Report. *Tackling Obesities: Future Choices – Building the Obesity System Map*. UK Government Office for Science. [www.foresight.gov.uk](http://www.foresight.gov.uk) (2007)

[22] J.A.M. Vennix, *Group Model Building* Chichester: Wiley (1996)

[23] K. Vladimirova and D. Le Blanc, How well are the links between education and other sustainable development goals covered in UN flagship reports? A contribution to the study of the science-policy interface on education in the UN system. UN Department of Economic & Social Affairs. DESA Working Paper No. 146 (2015)

[24] N. Weitz, M. Nilsson, and M. Davis, A nexus approach to the post-2015 agenda: formulating integrated water, energy, and food SDGs. *SAIS Review of International Affairs*, **34** (2), 37–50 (2014)

[25] N. Weitz, H. Carlsen, M. Nilsson, and K. Skånberg, Towards systemic and contextual priority setting for implementing the 2030 Agenda. *Sustain. Sci.*, **13**, 531–548 (2018)

[26] World DataBank: [databank.worldbank.org/data/](http://databank.worldbank.org/data/).

[27] World Bank. Atlas of Sustainable Development Goals 2017 : From World Development Indicators. *World Bank Atlas*; Washington, DC: World Bank. ©World Bank. [https://openknowledge.worldbank.org/handle/10986/26306](https://openknowledge.worldbank.org/handle/10986/26306). License: CC BY 3.0 IGO. (2017)
Appendix

This appendix provides a brief summary of the detailed analysis of the ICSU Report that was used to construct the network adjacency matrix $A$ illustrated in figure 1. $A$ is formed as the average of the two matrices $A^{\text{in}}$ and $A^{\text{out}}$ shown in figure 4, i.e. $A = (A^{\text{in}} + A^{\text{out}})/2$. The $A^{\text{in}}$ matrix is constructed row by row, with row $i$ containing the influences of the other Goals on Goal $i$. Similarly, the $A^{\text{out}}$ matrix is constructed column by column, with column $j$ containing the influences of Goal $j$ on each of the other Goals.

A.1 Goal 1. (Poverty): End poverty in all its forms everywhere

The report text (page 17) discusses the impacts of Goals 2 - 16 on Goal 1 and in only one case (Goal 2) does the text suggest a two-way link. Hence the contributions to the network weightings $A_{ij}$ are calculated to be

$$A^{\text{in}}_{1,j} = [0, 1/5, 1/9, 1/7, 1/6, 2/6, 1/3, 3/10, 0, 2/7, 2/7, 0, 2/3, 0, 0, 1/10]$$

$$A^{\text{out}}_{1,1} = [0, 1/5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

A.2 Goal 2. (Hunger): End hunger, achieve food security and improved nutrition and promote sustainable agriculture

The report text (pages 21-22) mainly discusses the impact of Goal 2 on Goal 1, and impacts of Goals 3 - 16 on Goal 2. The exceptions are Goals 13 and 15.

The linkage text for Goal 13 notes that enhanced resilience to climate change (Goal 13) will support more sustainable agriculture as well as food and nutrition security: this is a positive influence $13 \rightarrow 2$. But also the text notes the (harmful) influence of increased agriculture on climate change hence we note also the negative influence of Goal 2 on Goal 13 in entry $A_{13,2}$ in the weightings matrix.

For Goal 15 the text is similar: sustainable use and conservation of natural resources links directly with more sustainable agriculture as well as food and nutrition security; hence the positive influence $15 \rightarrow 2$ with weight $6/9$ since six out of the nine individual targets in Goal 15 are listed. But the text also notes the potential trade-offs between progress on Goal 2 and the environmental dimensions of Goal 15, mentioning biodiversity loss in particular. Since the word ‘biodiversity’ occurs in three out of the nine targets in Goal 15 we assign a weight of $-3/9$ to the influence of Goal 2 on Goal 15.

$$A^{\text{in}}_{2,j} = [0, 0, 4/9, 6/7, 3/6, 6/6, 3/3, 2/10, 2/5, 6/7, 2/7, 7/8, 3/3, 7/7, 6/9, 7/10]$$

$$A^{\text{out}}_{1,2} = [5/5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/5, -3/9, 0]$$

A.3 Goal 3. (Health): Ensure healthy lives and promote well-being for all at all ages

The report text (pages 25-26) discusses exclusively the effects on Goal 3 of progress on the other Goals. Positive weightings are assigned to every such link. The report therefore views progress on Goal 3 as an outcome of progress on the other Goals, rather than being an enabling factor that facilitates progress elsewhere. The report’s view of Goal 3 is similar in this respect to Goals 1 and 2.

$$A^{\text{in}}_{3,j} = [3/5, 2/5, 0, 3/7, 3/6, 3/6, 2/3, 5/10, 3/5, 3/7, 2/7, 2/8, 3/3, 1/7, 2/9, 3/10]$$

$$A^{\text{out}}_{4,3} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

A.4 Goal 4. (Education): Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all

The report text (page 29) discussing linkages between Goal 4 and the other SDGs takes the reverse view to Goal 3: Goal 4 is seen as an enabling factor, positively reinforcing progress on all other Goals, rather than being an
outcome of any others.

\[
A_{5,j}^{in} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
\]
\[
A_{5,4}^{out} = [3/5, 2/5, 2/9, 0, 1/6, 1/6, 1/3, 3/10, 2/5, 2/7, 2/7, 2/8, 2/3, 2/7, 2/9, 2/10]
\]

A.5 Goal 5. (Gender): Achieve gender equality and empower all women and girls

The report text (page 33) views Goal 5 similarly to Goal 4: as an enabling factor for progress on most other Goals. Interestingly, Goals 9 and 10 are clearly viewed differently, as factors that themselves enable progress on Goal 5: infrastructure provision (Goal 9) and reductions in inequality (Goal 10) are identified as both having positive influences on achieving gender equality.

\[
A_{5,j}^{in} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1/5, 4/7, 0, 0, 0, 0, 0, 0, 0]
\]
\[
A_{5,4}^{out} = [5/5, 4/5, 8/9, 7/7, 0, 2/6, 1/3, 4/10, 0, 0, 5/7, 3/8, 2/3, 0, 0, 5/10]
\]

A.6 Goal 6. (Water): Ensure availability and sustainable management of water and sanitation for all

The report text (page 27) clearly identifies water availability as an enabling factor for most other SDGs. There are three cases where the impact of progress on other Goals positively influences progress towards Goal 6, these are: Goal 9, where progress on infrastructure for flood and drought protection and water management is identified; Goal 12, where sustainable consumption and production practices will reduce water use and pollution emissions, hence enabling progress on Goal 6; and Goal 13, where progress towards climate change targets are identified as affecting water availability and sustainable water and sanitation development.

\[
A_{6,j}^{in} = [0, 0, 0, 0, 0, 0, 0, 0, 2/5, 0, 0, 3/8, 2/3, 0, 0, 0]
\]
\[
A_{6,4}^{out} = [3/5, 2/5, 3/9, 2/7, 2/6, 0, 1/3, 4/10, 0, 2/7, 3/7, 0, 0, 2/7, 3/9, 2/10]
\]

A.7 Goal 7. (Energy): Ensure access to affordable, reliable, sustainable and modern energy for all

The report text (page 41) identifies a complex set of linkages for Goal 7. The report describes energy as a vital resource that is required in order to meet other Goals. In four cases the narrative for linkages describes how progress on other Goals enables progress on Goal 7. These are: Goal 6, where water availability is identified as a requirement for the generation of power by conventional forms of generation; Goal 9, where infrastructure in the form of power grid and transportation networks are required in order to ensure access to energy for all; Goal 14 in which oceans are identified as a potential space for energy generation, citing offshore wind as an example; and Goal 16 in which transparent and corruption-free regimes are observed to be key to delivering energy services affordably.

Two negative linkages are described: Goal 7 negatively impacts on Goal 2 through competition for land, giving the example of biomass feedstock production, and Goal 7 negatively impacts on Goal 15 since energy projects can have a negative impact on ecosystems and biodiversity.

\[
A_{7,j}^{in} = [0, 0, 0, 0, 0, 0, 4/6, 0, 0, 0, 4/5, 0, 0, 0, 0, 3/7, 0, 4/10]
\]
\[
A_{7,5}^{out} = [2/5, -2/5, 1/9, 3/7, 1/6, 0, 0, 4/10, 0, 1/7, 3/7, 3/8, 2/3, 0, -3/9, 0]
\]

A.8 Goal 8. (Growth): Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all

The report text (pages 45–46) identifies a similarly complex set of linkages between Goal 8 and the other SDGs. In eight cases, growth is seen as a positive enabling factor supporting progress towards other Goals. In five cases (Goals 7, 9, 10, 12, and 16) the text indicates that this factor supports progress towards Goal 8 itself.
In two cases a negative influence is identified: Goals 6 and 15. In respect of Goal 6, the text comments that increases in production (growth) can increase water pollution, while water use efficiency can facilitate growth, but protection of natural resources may inhibit production and growth. Since two targets in Goal 6 are identified, the linkages between Goals 8 and 6 in both directions have been assigned a weighting of $-2/6$. The narrative for Goal 15 is even more complex: sustainable economic growth should minimise the degradation of terrestrial ecosystems. While the impact might be negative in the short term, synergies are expected over the long term. For the linkages between Goal 8 and both Goals 6 and 15 we have taken the worst case - the most pessimistic, negative, weighting that is compatible with the text.

$$A^{\text{in}}_{9,6} = [0, 0, 0, 0, 0, -2/6, 2/3, 0, 2/5, 2/7, 0, 2/8, 0, 0, 0, 0, 3/10]$$

$$A^{\text{out}}_{9,8} = [3/5, 2/5, 2/9, 3/7, 1/6, -2/6, 0, 0, 0, 3/7, 0, 1/3, 3/7, -2/9, 0]$$

A.9 Goal 9. (Industry): Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation

The report text (page 49) produces a reasonably straightforward set of positive linkages between Goal 9 and some, but not all, of the other Goals. In nine cases, progress on Goal 9 supports progress on other Goals. In one other case, as the narrative comments that inclusive sustainable industrialization requires access to education and skills for entrepreneurship, it is clear that this Goal, Goal 4, supports Goal 9 itself. In five cases (other than Goal 9 since self-links are not allowed) there are no direct links identified between other Goals and Goal 9.

$$A^{\text{in}}_{9,6} = [0, 0, 0, 2/7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$A^{\text{out}}_{9,8} = [2/5, 2/5, 0, 0, 0, 1/6, 2/3, 5/10, 0, 1/7, 4/7, 5/8, 1/3, 0, 0, 0]$$

A.10 Goal 10. (Inequality): Reduce inequality within and among countries

The report text (pages 53-54) indicates only zero or positive linkages between Goal 10 and the other Goals. The linkages with Goals 4 (Education), 8 (Growth) and 16 (Peace) are described as ‘both a consequence and a cause of’ Goal 10, interpreted as positive links in both directions between these Goals and Goal 10. Progress on Goals 9 (Industry) and 11 (Cities) are described as having the power to enable progress on Goal 10; in all other cases progress on other Goals is described as being enabled by progress on Goal 10.

$$A^{\text{in}}_{10,6} = [0, 0, 0, 1/7, 0, 0, 0, 0, 0, 2/10, 1/5, 0, 1/7, 0, 0, 0, 0, 5/10]$$

$$A^{\text{out}}_{10,8} = [2/5, 1/5, 2/9, 1/7, 1/6, 1/6, 1/3, 2/10, 0, 0, 0, 1/8, 1/3, 1/7, 0, 5/10]$$

A.11 Goal 11. (Cities): Make cities and human settlements inclusive, safe, resilient and sustainable

The report text (page 57) does not provide the same detailed analysis for the linkages to and from Goal 11 as for the other Goals. The summary text states that ‘Key goals that intersect with SDG 11 are 1, 3, 6, 7, 8, 9, 10, 13, and 16’. No detailed discussion is given of which targets would be enabled within these Goals. We have therefore allocated a weighting of 1/2 to each of these linkages, taken as being a linkage in which progress on Goal 11 enables progress on these other Goals. We take the linkages to be this way around since the report text continues by stating that ‘Progress on all other goals will have a positive impact in cities...’ which we interpret as a statement contrasting the general sense of linkages between all Goals, and in particular that progress on other Goals will enable Goal 11, with the subset of Goals and nature of linkage implied in the earlier statement.

$$A^{\text{in}}_{11,6} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$A^{\text{out}}_{11,8} = [1/2, 0, 1/2, 0, 0, 1/2, 1/2, 1/2, 1/2, 1/2, 0, 0, 0, 1/2, 0, 0, 1/2]$$
A.12 Goal 12. (SCP): Ensure sustainable consumption and production patterns

The report text (page 61) describes progress on Goal 12 (SCP) as enabling progress on other Goals. Although some positive linkage is vaguely indicated for each Goal, the targets listed against the linkage with Goals 5 (Gender) and 13 (Climate) are only described as ‘indirect links’ and means of implementation paragraphs are referred to, not specific targets. We have therefore assigned these a direct weighting of zero.

\[
A_{12,i,j}^{\text{in}} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
\]
\[
A_{12,i,j}^{\text{out}} = [3/5, 5/5, 3/9, 1/7, 0, 5/6, 3/3, 5/10, 4/5, 1/7, 4/7, 0, 0, 2/7, 5/9, 3/10]
\]

A.13 Goal 13. (Climate): Take urgent action to combat climate change and its impacts

The report text (page 65) highlights the many direct and indirect linkages between progress on Goal 13 and progress on other SDGs. These most take the form of impacts of climate change on other Goals, hence positive weightings in links from Goal 13 to other Goals, indicating that progress on Goal 13 has a positive influence on progress on other Goals. In two cases, Goal 7 (Energy) and Goal 12 (SCP) the text indicates an influence in the other direction, from these Goals to Goal 13. In the cases of Goal 1 (Poverty), Goal 6 (Water) and Goal 11 (Cities) there are positive links in both directions but we conclude that these are best described through unequal weightings in the two directions. For Goals 4 (Education) and 5 (Gender) the report text indicates only ‘indirect links’ and does not list any specific targets, so we set these weightings to zero.

\[
A_{13,i,j}^{\text{in}} = [1/5, 0, 0, 0, 0, 3/6, 3/3, 2/10, 1/5, 1/7, 1/7, 5/8, 0, 0, 0, 0]
\]
\[
A_{13,i,j}^{\text{out}} = [3/5, 2/5, 3/9, 0, 0, 1/6, 0, 1/10, 1/5, 1/7, 4/7, 0, 0, 2/7, 7/9, 1/10]
\]

A.14 Goal 14. (Oceans): Conserve and sustainable use the oceans, seas and marine resources for sustainable development

The report text (page 69) for Goal 14 strongly emphasises the existence of negative links between Goal 14 and other Goals, both in the use of the phrase ‘trade-offs’ and the explicit comment that “... some links are positive but there is also potential for goals to undermine each other (with action to achieve one goal resulting in other goals becoming harder to achieve).” As a result we have therefore taken a conservative approach to the weightings, resulting in a noticeably large number of negative weightings for edges between Goal 14 and other SDGs. The most negative weights are from Goals 2 (Hunger), 11 (Cities) and 15 (Ecosystems) to Goal 14, i.e. progress on these three Goals has the most potential to have a negative impact on progress towards Goal 14. It is also worth remarking that all the direct influences of progress on Goal 14 itself are positive.

\[
A_{14,i,j}^{\text{in}} = [0, -4/5, 0, 0, 0, 0, -1/3, -5/10, 0, 0, -5/7, 0, 0, 0, -7/9, 0]
\]
\[
A_{14,i,j}^{\text{out}} = [2/5, 0, 1/9, 0, 0, 2/6, 0, 0, 3/5, 0, 0, 5/8, 3/3, 0, 0, 4/10]
\]

A.15 Goal 15. (Ecosystems): Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss

The report text (pages 73–74) presents only positive links between Goal 15 and the other SDGs. Noticeably fewer individual targets in other Goals are listed, which leads to the conclusion that the overall strengths of links between Goal 15 and the rest of the SDGs are lower than for other Goals: Goal 15 is less clearly connected to the rest of the SDG network.

\[
A_{15,i,j}^{\text{in}} = [0, 0, 0, 1/7, 1/6, 0, 1/3, 0, 0, 0, 0, 0, 0, 0, 0, 0]
\]
\[
A_{15,i,j}^{\text{out}} = [2/5, 3/5, 1/9, 0, 0, 2/6, 0, 1/10, 1/5, 1/7, 4/7, 2/8, 2/3, 2/7, 0, 2/10]
\]
A.16 Goal 16. (Peace): Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels

The report text (pages 77–78) is noticeably sparse in indicating connections between Goal 16 and the other SDGs. There are no negative weightings. The highest weightings are with Goal 5 (Gender) and Goal 17 (Means of Implementation) but since Goal 17 is not considered within this network of linkages, this leaves only one significant link, with Goal 5. Because of the use of the term ‘synergies’ in the discussion related to Goals 4 (Education) and 5 (Gender), and the bidirectionality indicated in the general narrative on page 77, we assign the weights to links in both directions between Goal 16 and both of Goals 4 and 5.

\[
A_{16,j}^{\text{in}} = [2/5, 0, 0, 2/7, 4/6, 0, 0, 0, 1/5, 0, 0, 0, 0, 0, 0, 0] \\
A_{i,16}^{\text{out}} = [0, 0, 0, 2/7, 4/6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
\]
Table 3: A summary of the raw data from which the network adjacency matrices $A_{in}$ and $A_{out}$ are constructed. The overall linkage matrix is then computed as $A = (A_{in} + A_{out})/2$. Blanks indicate zero entries.