The physics of articulated toys—a jumping and rotating kangaroo

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Abstract
We describe the physics of an articulated toy with an internal source of energy provided by a spiral spring. The toy is a funny low cost kangaroo which jumps and rotates. The study consists of mechanical and thermodynamical analyses that make use of the Newton and centre of mass equations, the rotational equations and the first law of thermodynamics. This amazing toy provides a nice demonstrative example of how new physics insights can be brought about when links with thermodynamics are established in the study of mechanical systems.

Keywords: classical mechanics, thermodynamics, physics of toys

(Some figures may appear in colour only in the online journal)

1. Introduction

Toys can be helpful in increasing student motivation in the classroom. In presentations for popularizing and communicating science to more general audiences, they can also help to increase the appreciation and interest in the physical science; sometimes in such a way that everyone (especially non-scientists) will probably grasp some fundamental concepts. However, they should be used with care: the physical description of some toys is not easy [1], even in the framework of simplified models, and their usefulness is sometimes limited. However, they are always valuable for motivation purposes at least [2].

In this paper we describe the motion of a toy that, due to an internal source of energy, jumps while rotating. It is a toy kangaroo but, as far as the physics description is concerned, being an object in the form of a kangaroo is just a detail, it could be something else (even a living being). Among the numerous possible objects suitable for illustration and demonstration purposes, a toy, performing on top of the instructor’s table during the lecture, is
definitely more likely to attract the student’s attention. The accurate description of all steps of the toy’s motion is intricate but some simplified assumptions are possible and meaningful. This allows us to transform the real complicated problem into a feasible one, which is useful, in this particular case, for establishing a correspondence between the descriptions of translations and rotations, on the one hand, and, on the other, to bridge mechanics and thermodynamics.

The jump of the kangaroo is funny, possibly even a bit mysterious, and our aim is to apply the pertinent physical laws to describe and understand the various phases of the motion. Though the mechanical description of rotations and translations is the result of Newton’s second law, students have a clear preference for translations. Since our toy performs a movement that is a combination of a translation and a rotation, it can be useful for underlining the parallelism between the mechanical treatment of each type of motion per se. We shall assume constant forces and constant torques; therefore the real problem reduces to an almost trivial one. Nevertheless, there are some subtle points that are easy to emphasize with a simple example. In previous papers [3, 4] we analysed, from the mechanical and thermodynamical point of view, quite a few systems, essentially either in translation or in rotation. Here we combine both types of motion and, again, we stress the thermodynamical aspects in each phase of the motion, their similarities and asymmetries.

The design and the manufacture of the toy enables the kangaroo to perform a full rotation (360°) in the air while it jumps. This is because of its mass, its shape (therefore of its moment of inertia), the articulations between the legs and the body, and also because of the power provided by the internal source of energy. The manufacturer should define and include an internal energy source suitable for the toy to perform a full turn around the centre of mass while its centre of mass raises sufficiently high and drops down in the air. If a rotation angle of ∼360° is not met, the toy does not work.

Our demonstration kangaroo is a plastic toy, bought from a street vendor for three euros, whose source of internal energy is a spiral spring (so, it is a low cost and very ecological item —no batteries inside).

After providing the necessary energy to the spiral spring, we put the toy on top of a table and release it. A back somersault by the toy immediately begins, as shown in figure 1, and it comprises three phases: (1) the kangaroo, initially with flexed legs, suddenly stretches them while rising its centre of mass, increases its speed and starts rotating (a)–(b); (2) in this phase, (c)–(h), the toy has no contact with the ground and it rotates while its centre of mass describes a parabola; (3) this is the ‘landing’ phase that starts when the feet first come in contact with the ground, and it lasts until the toy completely stops (i)–(j).

Mechanics and thermodynamics are two different branches of physics with many interrelations. However, interestingly enough, in most university physics curricula, as well as in the high school, thermodynamics and mechanics almost do not intersect. This is not the case in everyday life, where both are strongly connected: a most common example is the automobile [5], but there are many others [6]. We shall see that our funny kangaroo also helps to illustrate this kind of bridging. There is no ‘physics surprise’ in the interpretation of the motion, we just have to apply, in combination, basic laws of mechanics and thermodynamics. With reasonable simplifying assumptions, that do not spoil the essence of the physical description, we are able to reduce the real problem to a classroom example that students certainly enjoy while they learn how basic physics principles work.

In section 2, we briefly introduce the general formalism that will be applied in the analysis of the motion of the toy. The discussion of the dynamics is presented in section 3 and as it is essentially known. However, the subtle energetic issues related to the motion in phases
1 and 3, described in section 4, are probably less known or undervalued by instructors. In section 5 we present the conclusions.

2. Mechanics and thermodynamics

Let us briefly review the basic theoretical framework needed for the mechanical and thermodynamical analyses. A more detailed presentation can be found in [3, 4]. For a system of constant mass \( m \), Newton’s second law can be expressed by

\[
\Delta \vec{p} = \int_0^t \vec{F}_{\text{ext}} \, dt,
\]

where \( \vec{F}_{\text{ext}} = \sum \vec{F}_i^\text{ext} \) is the resultant external force acting upon the system and \( \Delta \vec{p} = m \Delta \vec{v}_{\text{cm}} \) is the variation of the system centre of mass linear momentum in the time interval \( \Delta t = t - t_0 \).

Equation (1) already incorporates the third law of mechanics, which implies that the resultant internal forces vanishes, \( \vec{F}_{\text{int}} = \sum \vec{F}_j^\text{int} = \vec{0} \). The above equation, expressed in a vector form, can also be equivalently given in a scalar form by means of

\[
\Delta K_{\text{cm}} = \int F_{\text{ext}} \cdot d\vec{r}_{\text{cm}}
\]

the so-called centre of mass equation. On the left-hand side, one has the variation of the centre of mass kinetic energy \( (K_{\text{cm}} = \frac{1}{2}m v_{\text{cm}}^2) \) and, on the right-hand side, the pseudo work [7–11] performed by the resultant external force. For the pseudo-work, the resultant force and the centre of mass displacement should be considered [see the integral in (2)], whereas for the work it is each force and its own displacement that matters. Equation (1), which is an integral form of Newton’s equation, and the centre of mass equation (2) are physically equivalent.

Figure 1. The real toy performing a back somersault. From a movie, we extracted pictures showing the different phases of the motion: the preparation of the jump (a)–(b); the jump when the toy has no contact with the ground (c)–(h); and the final phase (i)–(j) when the toy stops after an initial contact with the ground.
even though they use different physical magnitudes—they express the same fundamental law of mechanics, so they are general and apply to all systems undergoing whatever process. When the mechanical systems perform rotations, other forms of Newton’s fundamental law are better suited such as \[12, 13\]

\[
\Delta L = I \Delta \omega = \int_0^t \tau_{\text{ext}} \, dt \quad \Delta K_{\text{rot}} = \frac{1}{2} I \Delta \omega^2 = \int \tau_{\text{ext}} \, d\phi
\]  

(3)

for the rotation of a system of constant moment of inertia \(I\) around a principal axis of inertia containing the centre of mass. The system is acted upon by an external torque, of magnitude \(\tau_{\text{ext}}\), whose direction is along the rotation axis. These equations are simplified versions of the most general ones, and in (3) one does not need to consider the vector character of the angular momentum, \(\vec{L}\), of the angular velocity, \(\vec{\omega}\), or of the torque, \(\vec{\tau}_{\text{ext}}\). The two equations (3) for the rotation, together with (1) and (2), for the translation, are the pertinent ones for the mechanical description of the motion presented in the next section.

A different physical law, also applicable to any system and to any process, is the first law of thermodynamics which, incidentally, also involves typical mechanical quantities. That principle is a statement on energy conservation and it can be expressed by the equation

\[
\Delta K_{\text{cm}} + \Delta U = W_{\text{ext}} + Q.
\]  

(4)

On the left-hand side, in addition to the variation of the centre of mass kinetic energy, one has the variation of the internal energy and, on the right-hand side, the energy fluxes to/from the system are expressed. In other words, the left-hand side of equation (4) refers to the total energy variation of the system, whereas the right-hand side expresses the energy that crosses the system’s boundary, i.e. the energy that enters or leaves the system through its boundary. This energy transfer is the sum of two contributions: the external work—i.e. the sum of the works performed by each external force, \(W_{i,\text{ext}} = \int \vec{f}_{i,\text{ext}} \cdot d\vec{r}_{\text{ext}}\)—which is given by \(W_{\text{ext}} = \sum W_{i,\text{ext}}\), and \(Q\), that is the heat flow to/from the surroundings. If either \(W_{\text{ext}}\) or \(Q\) are positive, that means an energy transfer to the system, leading to an increase of the left-hand side of (4); if any of them is negative, that means an energy flow to the surroundings with a corresponding decrease of the total energy of the system. It is worth noticing that, whereas the pseudo-work of the resultant external force leads to the variation of the centre of mass kinetic energy, as stated by equation (2), the real work, together with the heat, may change both that kinetic energy and/or the internal energy of the system, as stated by equation (4). By expressing the first law of thermodynamics in terms of equation (4) one implicitly assumes that \(\Delta U\) includes all energy variations that may contribute to the total internal energy variation of the system. Those include the variations of rotational [such as \(\Delta K_{\text{rot}}\) as given by equation (3)] and translational kinetic energies with respect to the centre of mass, in addition to the variations due to temperature changes, variations of internal chemical energy (associated with chemical reactions), work performed by internal forces, etc. Of course, any process should also comply with the second law, besides the first law of thermodynamics.

We stress that equations (2) and (4) provide, in general, complementary information, since they correspond to two distinct fundamental laws of nature. The study of the movement of the toy described in the next sections illustrates that complementarity.

3. Back somersault by the ‘kangaroo’

In figure 1 we show the real toy performing the back somersault that comprises three phases. In figure 2 we illustrate pictorially these three phases of the kangaroo’s motion.
The forces exerted by the ground in phases 1 and 3 are surely time dependent [14]. As a simplifying assumption, we replace these variable forces by constant forces that produce, in each phase, exactly the same impulse as the real force [6]. In phase 1, for instance, the force $\vec{F}_t(t)$ exerted by the ground is replaced by the constant force $\vec{F}$ such that $\int_{t_0}^{\Delta t_0} \vec{F}_t(t) \, dt = \vec{F} \Delta t_0$, where $\Delta t_0$ is the duration of that phase, and similarly for phase 3. At the end of the initial phase, the centre of mass velocity is $\vec{v}_{cm}$ and, at the beginning of phase 3, the centre of mass velocity is $\vec{v}_{cm}'$. Regarding the rotation, the torque with respect to the centre of mass, in phase 1, is also a time dependent function. The torque acts during the time interval $\Delta t_0$, producing a certain variation of the angular momentum of the system (note that now the moment of inertia of the toy, $I$, also slightly varies because of the legs’ articulations. However, this variation is rather small—the legs are very light in comparison with the rest of the body—and we can assertively adopt a constant $I$). Here, our approximation consists in assuming a constant torque such that $\int_{t_0}^{\Delta t_0} \vec{\tau}(t) \, dt = \vec{\tau} \Delta t_0$, where $\vec{\tau}$ is a constant vector. The torque provides a clockwise angular acceleration, $\alpha_0$, being the angular velocity at the end of phase 1. In figure 3 we represent the constant forces and torques in phases 1 and 3. The constant torque $\vec{\tau}'$ in phase 3 leads to an counterclockwise angular acceleration that reduces the initial angular acceleration.
velocity \( \omega_0 \) to zero. Of course, in all phases, the weight, \( \vec{G} \), is always acting, but its torque always vanishes with respect to the centre of mass.

The fact that we are considering constant forces and torques considerably simplifies the integrals in the general expressions presented in section 2. These forces and torques are time dependent and they also depend on the position of the centre of mass (the forces) and of the rotation angle, \( \phi \) (the torques). The real forces and torques produce certain impulses and angular impulses (right-hand sides of equation (1) and of the first equation (3)). The simplifying assumption consists in replacing the real forces and torques by constant vectors that produce exactly the same impulses. In this way we are simplifying, but not oversimplifying, the problem, making it handy, even practicable in the classroom context. The constant forces and torques can be regarded as average ones, producing the same momentum (linear and angular) variations as the real ones. Of course we could use a probably more realistic force such as \( F(t) = F_0 \sin(\xi t) \), where \( \xi \) is a parameter. This would complicate the approach because (1) would no longer be a trivial integration. With a sophisticated force sensor (the toy is very light) it would be possible, in principle, to figure out \( \vec{F}(t) \) and \( \vec{F}'(t) \) and adjust them with analytic functions. With known time dependent analytic functions the integral in (1) would be straightforward but the integral in (2) still would require knowledge of \( \vec{F} = \vec{F}(x_{cm}) \) (the same for \( \vec{F}' \)) and the integral in (3) would require knowledge of \( \vec{\tau} = \vec{\tau}(\phi) \) (the same for \( \vec{\tau}' \)). A more quantitative analysis of the motion is out of the scope of the present study and this is why we are using constant forces (and torques) to keep the problem within manageable limits.

Phase 2 consists of a parabolic motion of the centre of mass (neglecting air resistance, of course) combined with an uniform rotation. In figure 4 we show the trajectory of the centre of mass of the toy. Assuming constant forces, the trajectory in phase 1 is exactly a straight line. In fact, for a constant force along \( x \) and \( y \), and for a body that starts from rest, the accelerations \( a_x \) and \( a_y \) are constants. Therefore, \( x = \frac{1}{2}a_x t^2 \) and \( y = \frac{1}{2}a_y t^2 \), hence, by eliminating \( t \) in both equations, this leads to \( y = cx \) \( (c \) is a constant), whose graph is a straight line. In phase 3 the function \( y = y(x) \) is more complicated because there are initial velocities along \( x \) and \( y \), and \( y(x) \) is, in general, not a linear function. However, since the centre of mass only moves very
little and for a very short time, the trajectory can be approximated by a straight line (the difference between the straight line—phase 3 in figure 4—and the actual trajectory is tiny, even indistinguishable within the precision of the drawing).

In summary, regarding the centre of mass motion, it is uniformly accelerated along both $x$ and $y$ in phase 1, it is a projectile motion in phase 2 and, finally, it is a uniformly retarded motion in phase 3 along $x$ as well as along $y$. Regarding the rotational motion, the angular acceleration is constant in phase 1, in phase 2 the angular velocity is constant, and in phase 3 the angular acceleration is again constant, producing an uniformly retarded angular motion.

Concerning the angular displacement, the toy performs a complete turn around its centre of mass. Most of the time, the toy is in the air, hence the $360^\circ$ turn is almost executed in phase 2. In phase 1, the supposedly constant torque produces a quadratic dependence with time of the angular displacement: $\phi(t) = \frac{1}{2}\alpha t^2$, where $\alpha$ is the angular acceleration; the function $\phi(t)$ varies linearly with time during phase 2: $\phi(t) = 2\pi + \omega_0 t$, where $\omega_0$ is the angular velocity at the end of phase 1 and during phase 2; and it again varies quadratically in phase 3: $\phi(t) = 2\pi - \Delta \phi_1 + \omega_0 t - \frac{1}{2}\alpha' t^2$, where $\alpha'$ is the magnitude of the angular acceleration in the third phase. The time, $t$, in the previous expressions starts at the beginning of each phase.

In figure 5 the function $\phi = \phi(t)$ is shown, assuming that the turn corresponds to exactly $2\pi$ (in practice, this is only an approximate value) and exaggerating, to make the figure more clear, the durations of phases 1 and 3.

In addition to our previous assumptions, we consider that the magnitude of the velocities $v_0$ and $v_0'$ is the same, i.e. the kangaroo touches the ground with its centre of mass exactly at the level it occupies when the parabolic motion starts (such an assumption is convenient to simplify the analysis but it could be relaxed). For the sake of a general discussion we let the displacement $\Delta x_0$ be different from $\Delta x_0'$ (the same for the initial and final $y$ displacements). In such a case, the magnitudes of the forces $\vec{F}$ and $\vec{F}'$ are different and so are the time intervals during which they act. Indeed, we can stay more general, since such generality has no drastic consequences on the formalism or on its clarity. So, we assume that the contact constant forces, $\vec{F}$ and $\vec{F}'$, do not necessarily have the same magnitude. In figure 6 we represent the vertical component of the resultant force, $R_y$, acting on the toy (part [a]) and the horizontal

![Figure 5. Angular displacement around the axis that passes through the kangaroo’s centre of mass versus time. The relative durations of the initial and final phases are exaggerated.](image-url)
component of the same resultant force, \( R_x \), (part \( b \)). Again the time intervals in the initial and final phases are exaggerated. The important point to notice is that, for each graph, the algebraic sum of the represented areas, i.e. of the impulses along \( y \) and \( x \) should add up to zero: the toy starts from rest and it comes to rest at the end of the jump; therefore the total variation of the linear momentum is zero in both the \( x \) and \( y \) directions. The same is true for the angular impulses: those in phases (1) and (3) cancel out and, in phase 2, the angular impulse is zero.

Each of the contact forces has a vertical component (normal reaction) and a horizontal component (static friction force). We may decompose the contact forces according to

\[ \vec{F} = f \, \vec{e}_z + N \, \vec{e}_z \quad \text{and} \quad \vec{F}' = -f' \, \vec{e}_z + N' \, \vec{e}_z. \]

To fix the notation we summarize in table 1 the displacements, velocity variations, accelerations and other magnitudes for each phase of the motion.

| Phase 1 | Phase 2 | Phase 3 |
|---------|---------|---------|
| Duration | \( \Delta t_0 \) | \( \Delta t \) | \( \Delta t'_0 \) |
| Force | \((f, N - G)\) | \((0, -G)\) | \((-f', N' - G)\) |
| Acceleration | \((\alpha_x, \alpha_y)\) | \((0, -g)\) | \((-\alpha'_x, \alpha'_y)\) |
| Velocity variation | \((v_{0x}, v_{0y})\) | \((0, -2v_{0y})\) | \((-v_{0x}, v_{0y})\) |
| Displacement | \((\Delta x, \Delta y)\) | \((D, 0)\) | \((-\Delta x', -\Delta y')\) |
| Torque | \(\tau\) | \(0\) | \(-\tau'\) |
| Angular acceleration | \(\alpha\) | \(0\) | \(-\alpha'\) |
| Angular velocity variation | \(\omega_{0z}\) | \(0\) | \(-\omega_{0z}\) |
| Angular displacement | \(\Delta \phi_0 \sim 0^\circ\) | \(\sim 360^\circ\) | \(\Delta \phi'_0 \sim 0^\circ\) |
Phase 1: \[
\begin{align*}
\frac{1}{2} m v_{0x}^2 &= f \Delta t_0 \\
\frac{1}{2} m v_{0y}^2 &= (N - G) \Delta t_0 \\
I o_{0} &= \tau \Delta t_0
\end{align*}
\] and
\[
\frac{1}{2} \omega_0^2 = f \Delta x_0 + (N - G) \Delta y_0
\]
(5)

where \( m \) is the mass of the toy and \( I \) its moment of inertia (that may slightly vary with time but we assume it is constant). Moreover, equation (4), leads to
\[
\text{Phase 1: } \frac{1}{2} m v_{0x}^2 + \frac{1}{2} I o_{0}^2 + \Delta U_x = -G \Delta y_0,
\]
(6)

where \( \Delta U_x \) is the part of the internal energy variation due to the spiral spring (this is the potential elastic energy delivered by the spring). The other part of the variation of the internal energy of the system is the kinetic rotational energy in (6).

The corresponding equations for phase 3 are
\[
\begin{align*}
\frac{1}{2} m v_{0x} &= f' \Delta t_0' \\
\frac{1}{2} m v_{0y} &= (N' - G) \Delta t_0' \\
I o_{0} &= \tau' \Delta t_0'
\end{align*}
\] and
\[
\frac{1}{2} \omega_0^2 = f' \Delta x_0 + (N' - G) \Delta y_0
\]
(7)

and
\[
\text{Phase 3: } - \frac{1}{2} m v_{0x}^2 - \frac{1}{2} I o_{0}^2 = G \Delta y_0' + Q,
\]
(8)

where \( Q \) is the heat transfer to the surroundings during the process (in this phase \( \Delta U_x = 0 \)).

We stress that neither \( \vec{F} \) nor \( \vec{F}' \) perform any work because (ideally) their application points do not move.

As already mentioned, phase 2 corresponds to a projectile motion combined with a uniform rotation. In this part only a conservative force is acting, hence the sum of the translational kinetic energy, of the rotational kinetic energy and of the potential gravitational energy remains constant (the rotational kinetic energy, \( \frac{1}{2} I o_{0}^2 \), is, itself, constant). For this phase, the centre of mass equation (2) and the first law of thermodynamics (4) provide exactly the same information, namely
\[
\text{Phase 2: } \frac{1}{2} m (v^2 - v_{0y}^2) = -G (y - \Delta y_0)
\]
(9)

(note that now, in equation (4), \( \Delta U = 0 \) and \( Q = 0 \), the process is a purely mechanical one, because we have neglected the air friction).

Regarding kinematical aspects, the maximum height reached by the centre of mass, the horizontal distance travelled by the centre of mass and the time of flight are given by
\[
\text{Phase 2: } h_{max} = \Delta y_0 + \frac{v_{0y}^2}{2g}, \quad D = \frac{2v_{0x} v_{0y}}{g}, \quad \Delta t = \frac{2v_{0y}}{g}
\]
(10)

For this phase, we may write \( \Delta \phi = o_0 \Delta t \). If we take \( \Delta \phi \sim 2\pi \), for the time of flight given in (10), one finds a relation between the vertical component of the velocity at the end of phase 1 and the angular velocity at that very same moment: \( o_0 = \frac{v_{0y}}{v_{0y}} \).
4. Energetic issues

In this section we explicitly show that there are asymmetric energetic issues related to the motion in phases 1 and 3, although the mechanical description of those phases is symmetric in the sense that in the first and in the third phases the impulses of the resultant forces are equal in magnitude with opposite directions.

Let us go back to phase 1. The required energy for the kangaroo to perform the back somersault is obviously provided by the spiral spring and ultimately by the person that winds it. When the spring is released, we assume the elastic energy decrease to be given by $\Delta U_1 = -\frac{1}{2} \kappa (\theta^2 - \theta_0^2)$, with $\frac{1}{2} \kappa \theta_0^2$ the initial stored energy and $\kappa$ the elastic constant characterizing the spring (the angle $\theta$, with initial value $\theta_0$, is a decreasing function of time). As this energy decreases, the articulated toy starts the jump.

Combining equations (5) and (6), we may express the internal energy variation by

$$\Delta U_1 = -\left(\frac{1}{2} m v^2 + \frac{1}{2} I_0 \omega^2 + G \Delta \theta^1\right)$$

$$=- \left[ f \Delta \chi_0 + (N - G) \Delta \theta^1 + \tau \Delta \phi^1 + G \Delta \theta^1 \right] < 0 . \quad (11)$$

The first line explicitly shows that the internal (elastic) energy is converted into another form of mechanical energy: translational and rotational kinetic energies plus potential gravitational energy. This phase is reversible—it evolves with no variation of the entropy of the Universe [15]. In the second line, the kinetic energy terms are expressed by the pseudo works related to the contact force and to its torque.

Regarding phase 3, equations (7) and (8) now lead to

$$Q = -\left(\frac{1}{2} m v^2 + \frac{1}{2} I_0 \omega^2 + G \Delta \theta^3\right)$$

$$=- \left[ f' \Delta \chi_0' + (N' - G) \Delta \theta^3 + \tau' \Delta \phi^3 + G \Delta \theta^3 \right] < 0 . \quad (12)$$

This is the heat released in phase 3 of the motion. This energy is simply lost in the sense that it flows from the system to the surroundings, where it is dissipated, and the entropy of the universe increases [16]. The relation between the original internal energy variation and this heat can be found by combining the previous equations, or simply by applying equation (4) directly to the whole process: on the left-hand side of that equation the variation of the centre of mass kinetic energy is zero and the variation of the internal energy is solely $\Delta U_1$ (to simplify the discussion we are assuming no temperature variations in the toy); on the right-hand side of equation (4), the work is only due to the weight (again we stress that neither the normal forces, nor the static friction forces perform any work); finally, there is the heat flow to the surroundings. Altogether, equation (4) yields for the overall process

$$Q = \Delta U_{\text{s}} + G \left( \Delta \theta_0 - \Delta \theta_0' \right) . \quad (13)$$

For $\Delta \theta_0 = \Delta \theta_0'$ the energy initially stored in the spring totally dissipates in the surroundings, which behaves as a heat reservoir at temperature $T$. If the final position of the centre of mass lies at a higher level than the initial one, part of the initial stored energy is used to raise the centre of mass of the kangaroo, as expressed in equation (13). In that case, the magnitude of the heat is smaller than the magnitude of the elastic potential energy initially stored in the toy. The overall process is clearly irreversible and, for $\Delta \theta_0 = \Delta \theta_0'$, the entropy variation of the
Universe is $\Delta S_0 = -\frac{q}{T} = -\frac{\Delta f}{T} > 0$ (the heat should now be considered from the point of view of the reservoir, and this is the reason for the minus sign).

When $\Delta x_0 = \Delta x'_0$ and $\Delta y_0 = \Delta y'_0$, even though the mechanical analysis becomes quite symmetric for the initial and final phases, there is a clear thermodynamical asymmetry: in phase 1, there is mechanical energy production at the expend of the internal energy of the system; in phase 3, the mechanical energy is ‘lost’ in the sense that it dissipates as heat in the surroundings and it cannot be recovered in an useful way. We may say that, at the end of phase 1, the system still keeps the energy in an ‘organized form’ as mechanical energy (hence, no entropy increase), whereas in the final phase there is no mechanism to recover the decrease of mechanical energy: in particular, energy cannot be stored back in the spiral spring. If that could be possible, the kangaroo would perform back somersaults continuously. We all know that this is not the case: after one jump one has to rewind the spiral spring for the next jump.

In a recent publication we discuss the same type of asymmetries in the context of human movements, like jumping and walking [17]. As in the example discussed in this paper, in one part of the process there is a direct conversion of internal energy into mechanical energy but, in the other part, the mechanical energy dissipates as heat in the surroundings.

5. Conclusions

The motion of the kangaroo studied in this paper can be used in the classroom to explicitly show the correspondence between translations and rotations. The example also serves to demonstrate that, for certain mechanical systems performing movements in which one part is symmetric with respect to the other, the thermodynamical behaviour of both parts may be different. More precisely, in one part there may be production of mechanical energy, due to an internal energy source, a process that does not increase the entropy; whereas in the other part of the motion there occurs dissipation of mechanical energy, which is transferred to the surroundings as heat, and there is an inescapable entropy increase.

We should emphasize that the first phase of the problem studied in this paper is somewhat similar to the springboard diver discussed in [18]—the main difference being that the internal (elastic) energy variation should then be replaced by the variation of the Gibbs free energy in the person’s muscles [17, 19]. There are of course many other examples, but the point we would like to emphasize here is the fact that energetic aspects are usually a bit underrated in textbooks [18].

The discussion presented in this paper comes in the sequence of a series of papers [3–6, 17] devoted to the link between mechanics and thermodynamics, a tendency that, fortunately, is already present in modern textbooks [20]. Our aim is to help fill the gap one encounters in most treatments of classical mechanics, and we do hope that our discussion, motivated by a ‘kangaroo’, is relevant for physics education.

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