THREE FLAVOR NEUTRINO OSCILLATION ANALYSIS OF THE KAMIOKANDE MULTI-GEV ATMOSPHERIC NEUTRINO DATA

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Abstract

Using the published Kamiokande data of the multi-GeV atmospheric neutrinos, we have searched the optimum set of the neutrino oscillation parameters among three flavors. It is found that $\chi^2$ is minimized for $(\Delta m^2_{21}, \Delta m^2_{31}) = (3.8 \times 10^{-2} \text{ eV}^2, 1.4 \times 10^{-2} \text{ eV}^2)$, $(\theta_{12}, \theta_{13}, \theta_{23}) = (19^\circ, 43^\circ, 41^\circ)$ with $\chi_{\text{min}} = 3.2$ (42%CL). The sets of parameters $(\Delta m^2_{21}, \Delta m^2_{31}) = (\mathcal{O}(10^{-11}\text{eV}^2) \text{ or } \mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(10^{-2}\text{eV}^2))$ which are suggested by the two flavor analysis fall within 0.7$\sigma$.

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There has been much interest in atmospheric neutrinos, which might give us an evidence for neutrino oscillations. While NUSEX and Frejus have reported consistency between the data and the predictions on atmospheric neutrino flux, Kamiokande, IMB and Soudan-2 have reported discrepancy. In particular, the Kamiokande group claimed that their multi-GeV data suggests the mass squared difference of neutrino is of order $10^{-2}\text{eV}^2$.

People have studied neutrino oscillations among three flavors, and it has been shown recently that the mass squared differences and the mixing angles have strong constraints from various experiments. The analysis of the multi-GeV atmospheric neutrino data by the Kamiokande group was based on the framework of neutrino oscillation between two flavors, and it is important to see what happens if we analyze the data in the three flavor framework. In this paper we will analyze the published multi-GeV data of the Kamiokande atmospheric neutrino experiment, taking into account mixings among three flavor neutrinos. Unlike other works in Ref. [10], we will take the matter effect into consideration, and evaluate the number of events by summing over the energy and the zenith angle of neutrinos, to reproduce the original analysis by the Kamiokande group as much as possible. Throughout this paper we will restrict our discussions only to the multi-GeV data by the Kamiokande group, not only because the Monte Carlo result for the neutrino energy spectrum is available only in Ref. [2], but also because this is the only data which gives both the upper and the lower bound on the mass squared difference of neutrinos.

We start with the Dirac equation for three flavors of neutrinos with mass

\footnote{A couple of works have discussed Kamiokande’s analysis from the viewpoints which are different from ours.}
in matter [13]:

\[
\frac{d}{dx} \Psi(x) = \left[ U \text{diag} \left( E_1, E_2, E_3 \right) U^{-1} + \text{diag} \left( A(x), 0, 0 \right) \right] \Psi(x) \tag{1}
\]

Here \( E_j = \sqrt{p^2 + m_j^2} \) is the energy of the neutrino, \( \Psi(x) \equiv (\nu_e(x), \nu_\mu(x), \nu_\tau(x))^T \) is the wave function of the neutrinos in the flavor basis, \( A(x) \equiv \sqrt{2} G_F N_\alpha(x) \) stands for the effect due to the charged current interactions between \( \nu_e \) and electrons in matter [13].

\[
U \equiv \begin{pmatrix}
  U_{e1} & U_{e2} & U_{e3} \\
  U_{\mu1} & U_{\mu2} & U_{\mu3} \\
  U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]

\[
\equiv \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix} \tag{2}
\]

with \( c_{ij} \equiv \cos \theta_{ij}, \ s_{ij} \equiv \sin \theta_{ij} \) is the orthogonal mixing matrix of neutrinos, and we will not discuss the CP violating phase of the mixing matrix here for simplicity [3].

The number of the expected charged leptons \( \ell_\alpha \) ( \( \ell_\alpha = \mu \) or \( \tau \) ) with energy \( q \) from a scattering \( \nu_\alpha e \rightarrow \nu_e \ell_\alpha \) is given by

\[
N(\ell_\alpha) = n_T \sum_{\beta=e,\mu} \int_0^\infty dE \int_0^\pi d\Theta \int_0^{q_{\text{max}}} dq \
\epsilon(q) F_\beta(E, \Theta) \frac{d\sigma_\alpha(E, q)}{dq} P(\nu_\beta \rightarrow \nu_\alpha; E, \Theta) \quad (\alpha = e, \mu) \tag{3}
\]

Here \( F_\beta(E, \Theta) \) is the flux of atmospheric neutrino \( \nu_\beta \) with energy \( E \) from the zenith angle \( \Theta \), \( n_T \) is the effective number of target nucleons, \( \epsilon(q) \) is the detection efficiency function for charged leptons \( \ell_\alpha \), \( d\sigma_\alpha(E, q)/dq \) is the differential cross section of the interaction \( \nu_\alpha e^- \rightarrow \nu_e \ell^-_\alpha \) (\( \alpha = e \) or \( \mu \)), \( P(\nu_\beta \rightarrow \)

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3Even if we include the CP violating phase \( \delta \) of the mixing matrix, the effect of \( \delta \) always appears in the combination of \( s_{13}e^{i\delta} \). \( s_{13} \) has to be small because of the constraints from the reactor experiments and from the solar neutrino observations, as we will discuss below.
$\nu_\alpha; E, \Theta \equiv |\langle \nu_\beta(0)|\nu_\alpha(L)\rangle|^2$ is the probability of $\nu_\beta \to \nu_\alpha$ transitions with energy $E$ after traveling a distance

$$L = \sqrt{(R + h)^2 - R^2 \sin^2 \Theta - R \cos \Theta},$$

(4)

where $R$ is the radius of the Earth, $h \sim 15$Km is the altitude at which atmospheric neutrinos are produced.

To reproduce the analysis of the multi-GeV data by the Kamiokande group, one needs the quantity

$$f_{\beta\alpha}(E, \Theta) \equiv n_T \int_{q^\text{max}}^0 dq \epsilon(q) F_{\beta}(E, \Theta) \frac{d\sigma_{\alpha}(E, q)}{dq} \quad (\alpha, \beta = e, \mu)$$

(5)

for each $E$ and $\Theta$, which is not given in [2]. However, the quantity

$$g_{\beta\alpha}(E) \equiv \int_0^\pi d\Theta f_{\beta\alpha}(E, \Theta),$$

(6)

which is obtained by integrating (5) over $\Theta$, is given in the Fig.2 (d)–(f) in Ref. [2]. The zenith angle dependence $n_\beta(E, \Theta)$ of the atmospheric neutrino flux for various neutrino energy $E$ has been given in Ref. [7] in detail. Here we multiply the quantity $g_{\beta\alpha}(E)$ by the zenith angle dependence in Ref. [7] with suitable normalization, and adopt the quantity

$$\tilde{f}_{\beta\alpha}(E, \Theta) \equiv \frac{g_{\beta\alpha}(E)n_\beta(E, \Theta)}{\int_0^\pi d\Theta n_\beta(E, \Theta)} \quad (\alpha, \beta = e, \mu)$$

(7)

instead of the original quantity $f_{\beta\alpha}(E, \Theta)$ used in [2]. (7) is the important assumption of the present analysis. (7) is not exactly the same as $f_{\beta\alpha}(E, \Theta)$ in the original analysis [2], but this is almost the best which can be done with the published data in [2].

We have solved (1) numerically for each $E \left(10^{-1/20} \text{ GeV} \leq E \leq 10^2 \text{ GeV}\right)$ and evaluated the number of events for a given range of the zenith
angle $\Theta_j \equiv \cos^{-1}\left(\frac{2j-7}{5}\right) < \Theta < \Theta_{j+1} \equiv \cos^{-1}\left(\frac{2j-5}{5}\right)$ \quad (1 \leq j \leq 5)

\[
X_j^\mu \equiv N(\mu, \Theta_j < \Theta < \Theta_{j+1}) \\
\equiv (1 + \alpha)(1 + \frac{\beta}{2}) \int_{\Theta_j}^{\Theta_{j+1}} d\Theta \int dE \\
\left[ \tilde{f}_{\mu\mu}(E, \Theta)P(\nu_\mu \rightarrow \nu_\mu; E, \Theta) + \tilde{f}_{e\mu}(E, \Theta)P(\nu_e \rightarrow \nu_\mu; E, \Theta) \right]
\]

\[
X_j^e \equiv N(e, \Theta_j < \Theta < \Theta_{j+1}) \\
\equiv (1 + \alpha)(1 - \frac{\beta}{2}) \int_{\Theta_j}^{\Theta_{j+1}} d\Theta \int dE \\
\left[ \tilde{f}_{\mu e}(E, \Theta)P(\nu_\mu \rightarrow \nu_e; E, \Theta) + \tilde{f}_{ee}(E, \Theta)P(\nu_e \rightarrow \nu_e; E, \Theta) \right].
\] (8)

Several groups [8] [7] have given predictions on the flux of atmospheric neutrinos but they differ from one another in the magnitudes, and the Kamiokande group assumed that the errors of the overall normalization $1 + \alpha$ and the relative normalization $1 + \beta/2$ are $\sigma_\alpha=30\%$ and $\sigma_\beta=12\%$, respectively. Here we regard these factors $\alpha$ and $\beta$ as free parameters of the theory, and adopt the following $\chi^2$ [14]:

\[
\chi^2 = 2 \sum_{\alpha=e,\mu} \sum_{j=1}^{5} \left( X_j^\alpha - N_j^\alpha - N_j^\alpha \ln \frac{X_j^\alpha}{N_j^\alpha} \right),
\] (9)

where $N_j^\alpha (\alpha = e, \mu; 1 \leq j \leq 5)$ is the data for each zenith angle $\Theta_j < \Theta < \Theta_{j+1}$. The theoretical prediction $X_j^\alpha (\alpha = e, \mu; 1 \leq j \leq 5)$ depends on seven free parameters ($\Delta m_{21}^2, \Delta m_{31}^2; \theta_{12}, \theta_{13}, \theta_{23}; \alpha, \beta$), where $\Delta m_{ij} \equiv m_i^2 - m_j^2$, so (8) is expected to obey a $\chi^2$ distribution with $10-7=3$ degrees of freedom. The number of degrees of freedom in the present analysis is smaller than the original one by the Kamiokande group ($5 \times 8 + 5 - 2 = 83$).

We could not reproduce exactly the zenith angle distributions in Fig. 3 in Ref. [2]. In particular our prediction for e-like events near $\cos \Theta \sim -1$ has a larger difference with the data than Kamiokande’s does, and this difference...
seems to be important to discuss the magnitude of $\chi^2$ later\footnote{If we try to fit the data with only two parameters $\alpha$ and $\beta$ as in Ref. \cite{12}, then the minimum value of $\chi^2$ is 21, which suggests that the $\Theta$ independent solution is excluded at the 99% confidence level in our analysis.}. Presumably this discrepancy arises not only because the data that we are using is different from the original one in Ref. \cite{2}, but also because the Kamiokande group has taken into account the smearing effect on the resolution of the angle ($15^\circ \sim 20^\circ$) and the effects of backgrounds \cite{2} \cite{13}. Throughout this paper we discuss the goodness of fit and the confidence level of set of the parameters etc. based on our calculation with (7).

The value of $\chi^2$ is affected to some extent by the presence of matter, and it is necessary to take into consideration the contribution of the second term in (1). Evaluation of $\chi^2$ requires a lot of CPU time of a computer since one has to solve (1) numerically for each $E$ and $\Theta$ and plug it into (9). We have meshed each parameter region into ten points ($\Delta m^2_{ij} = 10^{-5+\ell/2}$, $\theta_{ij} = \ell \pi/20$ ($0 \leq \ell \leq 10$)) and the evaluated the value of $\chi^2$. Furthermore, using the gradient-search method described in Ref. \cite{16}, we have found that $\chi^2$ has the minimum value for

\[
(\Delta m^2_{21}, \Delta m^2_{31}) = (3.8 \times 10^{-2} \text{eV}^2, 1.4 \times 10^{-2} \text{eV}^2)
\]

\[
(\theta_{12}, \theta_{13}, \theta_{23}) = (19^\circ, 43^\circ, 41^\circ)
\]

\[
(\alpha, \beta) = (2.8 \times 10^{-1}, -5.0 \times 10^{-2})
\]  

with $\chi^2_{\text{min}} = 3.2$. Note that the deviation of the two normalization factors $1 + \alpha$ and $1 + \beta/2$ from unity is within the errors $\sigma_\alpha = 30\%$, $\sigma_\beta = 12\%$ assumed in Ref. \cite{2}. We have also calculated the value of the modified chi square

$\tilde{\chi}^2 \equiv \chi^2 + \alpha^2/\sigma_\alpha^2 + \beta^2/\sigma_\beta^2$ with the weight for the errors $\sigma_\alpha = 30\%$, $\sigma_\beta = 12\%$, and we have found that the conclusions in the following discussions do not change very much with $\tilde{\chi}^2$ instead of $\chi^2$.\footnote{If we try to fit the data with only two parameters $\alpha$ and $\beta$ as in Ref. \cite{12}, then the minimum value of $\chi^2$ is 21, which suggests that the $\Theta$ independent solution is excluded at the 99% confidence level in our analysis.}
The zenith angle distributions of the e-like events, the \(\mu\)-like events and the double ratio \(R \equiv (\mu/e)_{\text{data}}/(\mu/e)_{\text{MC}}\) are given in Fig.1 and Fig.2 for the optimum set of parameters.

(Insert Fig.1 and Fig.2 here.)

The degrees of freedom of our analysis is 3, so the value of the reduced chi square is 1.1, which corresponds to 42 \% confidence level. This suggests that our fit in the present analysis is not particularly good, but as we mentioned earlier, this is probably due to the fact that the data from which we start is poorer than the original one by the Kamiokande group [2].

The region of the parameter space which is allowed at 90 \% CL is given by \(\chi^2 \leq \chi_{\text{min}}^2 + 12\) for seven free parameters. However, because it requires a lot of CPU time of a computer to solve (11) numerically, we could not give sets of contours in the parameter space. In fact, since the value \(\chi - \chi_{\text{min}}\) is rather large, if we project the allowed region onto the \(\Delta m_{21}^2 - \Delta m_{31}^2\) plane, it is conceivable that we have disjointed regions on this plane, and the calculation would be extremely tedious. So we restrict our analysis to a special case of particular interest here.

We have evaluated \(\chi^2\) for the sets of parameters, which are suggested by the solutions for the solar neutrino problem [13] [17]. In order not to spoil the success of these scenarios [13] [17] based on the two flavor framework, we consider only the case in which \(|U_{e3}|\) is small, since we have the formula which relates the probability \(P^{(3)}(\nu_e \rightarrow \nu_e)\) in the three flavor analysis to \(P^{(2)}(\nu_e \rightarrow \nu_e)\) in the two flavor one [18]:

\[
P^{(3)}(\nu_e \rightarrow \nu_e; A(x)) = (1 - |U_{e3}|^2)^2 P^{(2)}(\nu_e \rightarrow \nu_e; (1 - |U_{e3}|^2) A(x)) + |U_{e3}|^4.\tag{11}
\]

We note in passing that the constraint for \(|U_{e3}|\) also comes from the reactor experiments [13], which suggests \(|U_{e3}|^2 = s_{13}^2 \lesssim 10^{-1}\) or \(1 - |U_{e3}|^2 = c_{13}^2 \lesssim 10^{-1}\).
for $\Delta m^2_{31} \approx$ a few $10^{-2}$eV$^2$.

Irrespective of whether we consider the vacuum solution ($\Delta m^2_{21} \sim \mathcal{O}(10^{-11}$eV$^2)$) or the MSW solution ($\Delta m^2_{21} \sim \mathcal{O}(10^{-5}$eV$^2)$) for the solar neutrino, the mass squared difference $\Delta m^2_{21}$ is negligible compared to the contribution of the matter effect $A(x)$ in (1) and the other mass squared difference $\Delta m^2_{31}$, which should be at least of order $10^{-2}$eV$^2$ to account for the zenith angle dependence of the Kamiokande multi-GeV data. Thus we consider the case

$$\begin{align*}
(\Delta m^2_{21}, \sin^2 2\theta_{12}) &= (\Delta m^2, \sin^2 2\theta) \odot \\
\equiv & \begin{cases} 
(\mathcal{O}(10^{-11}$eV$^2), \mathcal{O}(1)), & \text{(vacuum solution)} \\
(\mathcal{O}(10^{-5}$eV$^2), \mathcal{O}(10^{-2})), & \text{(small angle MSW solution)} \\
(\mathcal{O}(10^{-5}$eV$^2), \mathcal{O}(1)), & \text{(large angle MSW solution)}
\end{cases} \\
0 \leq \theta_{13} \lesssim 5^\circ \\
\Delta m^2_{31}, \theta_{23} = \text{arbitrary}
\end{align*}$$

(12)

With this constraint we have found that $\chi^2$ has the minimum value for

$$\begin{align*}
\Delta m^2_{31} &= 3 \times 10^{-2}$eV$^2 \\
\theta_{13} &= 5^\circ \\
\theta_{23} &= 40^\circ \text{ or } 50^\circ \\
(\alpha, \beta) &= (3.6 \times 10^{-1}, -3.7 \times 10^{-2})
\end{align*}$$

(13)

with

$$\chi^2 = 9.6.$$  

(14)

Deviation of each parameter in this case is $(\chi^2 - \chi^2_{\text{min}})/7 = 0.9$, so we conclude that this set of parameters falls within $0.7\sigma$ for all three cases in (12). In fact we observe that any set of the parameters with the constraints (13) falls within $0.7\sigma$ as long as $\Delta m^2_{21} \ll \Delta m^2_{32} < \Delta m^2_{31}$ is satisfied (arbitrary $\theta_{12}$ is allowed in this case). In (13) the error of $\alpha$ is a little too large compared to
what the Kamiokande group assumed, but even if we take \( \alpha = 3.0 \times 10^{-1} \) with all other parameters the same as in (13), we find that the solution falls within 0.8\( \sigma \). The reason that we have weaker constraints in this analysis than in Ref. [2] is because we have larger numbers of free parameters, but this is inevitable as long as one assumes the general mixings among three flavors of neutrinos.

In this paper we have analyzed the multi-GeV atmospheric neutrino data by the Kamiokande group based on the framework of three flavor neutrino oscillations, and have shown that the best fit is obtained for the set of parameters \( \Delta m_{21}^2 \sim \Delta m_{31}^2 \sim \mathcal{O}(10^{-2} \text{eV}^2) \). We have also shown that the popular set of parameters \((\Delta m_{21}^2, \sin^2 2\theta_{12}) = (\Delta m^2, \sin^2 2\theta)\odot, (\Delta m_{31}^2, \sin^2 2\theta_{13}) = (\mathcal{O}(10^{-2} \text{eV}^2), \mathcal{O}(1)), \theta_{13} \simeq 0 \) fall within 0.7\( \sigma \). The minimum value of \( \chi^2 \) is 3.2 for 3 degrees of freedom, and the fit based on the hypothesis of neutrino oscillations is not particularly good due to that fact that we used only the information published in Ref. [2]. We hope that the situation will be improved much more when the SuperKamiokande experiment starts. If we combine the results here with other experimental data, then we get even stronger constraints, which will be reported somewhere [20].

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Figures

Fig.1 (a),(b) Zenith angle distributions for the e-like and μ-like multi-GeV events. The squares with error bars are data and the histograms stand for the predictions without neutrino oscillations (solid lines), and with neutrino oscillations (dashed lines for the optimum set of parameters $(\Delta m^2_{21}, \Delta m^2_{31}) = (3.8 \times 10^{-2} \text{ eV}^2, 1.4 \times 10^{-2} \text{ eV}^2)$, $(\theta_{12}, \theta_{13}, \theta_{23}) = (19^\circ, 43^\circ, 41^\circ)$), respectively. These quantities are obtained by multiplying the values in Fig.3(d) in Ref. [2] by the zenith angle dependence of the flux in Ref. [7].

Fig.2 Zenith angle distribution of the double ratio $R \equiv \frac{(\mu/e)_{\text{data}}}{(\mu/e)_{\text{MC}}}$. The solid lines stand for the prediction with neutrino oscillations for the optimum set of parameters. All the quantities are calculated based on the same assumption as in Fig.1.
Fig. 1(a)

# of e-like Events

$\cos \Theta$
Fig. 1(b)

Number of $\mu$-like Events vs. $\cos \Theta$
Fig. 2

$R$ vs. $\cos \Theta$