LIGHTCURVES OF THERMONUCLEAR SUPERNOVAE AS A PROBE OF THE EXPLOSION MECHANISM AND THEIR USE IN COSMOLOGY

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Thermonuclear supernovae are valuable for cosmology but their physics is not yet fully understood. Modeling the development and propagation of nuclear flame is complicated by numerous instabilities. The predictions of supernova light curves still involve some simplifying assumptions, but one can use the comparison of the computed fluxes with observations to constrain the explosion mechanism. In spite of great progress in recent years, a number of issues remains unsolved both in flame physics and light curve modeling.

1 Introduction

Supernovae of type Ia (SNe Ia) are important for cosmology (better to say, for cosmography) due to their brightness. They are not standard candles, but can be used for measuring distances with the help of the peak luminosity – decline rate correlation, established by Yu.P. Pskovskii \cite{21} and M.M. Phillips \cite{19} (see the review \cite{8}). To exclude systematic effects in linking the observed light of distant SNe Ia to the parameters of cosmological models, one has to understand the nature of supernova outbursts and to build accurate algorithms for predicting their emission.

This involves: identifying the progenitors of SNe Ia; the birth of thermonuclear flame and its accelerated propagation leading to explosion; light curve and spectra modeling.

When the full understanding will be achieved, one can try to evaluate the importance of evolution effects in using supernovae as distance indicators. In spite of great progress in recent years, a number of issues remains unsolved both in flame physics and light curve modeling. We point out some problems which seem most important to us.
2 Progenitors

Mechanical equilibrium and evolution of stars is easily understood from the virial theorem for a star: \( 3 \int P \, dV = -U \), where \( P \) is the pressure, \( V \) is volume, and \( U \) is the gravitational energy of a star. Crude estimates \( V \sim R^3 \), \( M \sim R^3 \rho \), \( U \sim -GM^2/R \) in the virial equation give \( P \sim GM^2/3 \rho^{4/3} \). For an ideal classical gas, the equation of state \( P = \mathcal{R} \rho T \) implies \( \mathcal{R} T \sim GM^2/3 \rho^{1/3} \), with \( \mathcal{R} \) the gas constant. This is already enough to understand the evolution of massive stars! While a star loses energy and contracts, its internal temperature \( T \) grows. If the losses are balanced by a nuclear energy release, then the contraction stops and a thermal equilibrium is established:

\[
\text{nuclear heating power } L^+ = \text{radiative cooling (luminosity) } L^-.
\]

The rate of thermonuclear heating scales as \( \langle \sigma v_0 \rangle \sim \exp[-(\alpha_G/T)^{1/3}] \) due to the Gamow’s peak: the chances to penetrate the Coulomb barrier for fast nuclei grow, but the tail of Maxwell distribution goes down. Here \( \alpha_G \) depends strongly on nuclei charges \( Z_i \): \( \alpha_G \propto Z_i^2 Z_2^2 \), thus high-\( Z \) ions can fuse only at high \( T \). Small perturbations of \( T \) produce huge variations in \( L^+ \) since, normally, \( T \ll \alpha_G \).

The cooling \( L^- \) depends on \( T \) moderately, and it seems that \( L^- \) cannot compensate an overheating perturbation. Why then do not all stars explode violently? The reason is the same as for the growth of \( T \) in stars losing the energy at contraction: they have negative heat capacity. For a star made of a classical plasma with \( \gamma = 5/3 \) the internal energy \( Q = \frac{3}{2} \mathcal{R}MT \) and the virial theorem implies \( U = -2Q \), so the total energy \( \mathcal{E} \) is negative: \( \mathcal{E} = U + Q = -Q < 0 \). Thus any growth in \( \mathcal{E} \) due to heating leads to the drop of \( T \) (because nuclear energy is used for expansion against gravity). The perturbations decay, and there is no hope to get a thermonuclear supernova from a normal star composed of a classical plasma.

The situation changes, if a star is made of a degenerate matter. Then the heat \( \frac{3}{2} \mathcal{R}MT \) resides in ions, but its absolute value is much less than \( Q \) which is now governed by Fermi energy of electrons. We have crudely for a mixture of non-relativistic electron Fermi gas and classical ions: \( P \sim K \rho^{5/3} + \mathcal{R} \rho T \sim GM^{2/3} \rho^{4/3} \), and \( \mathcal{R} T \sim GM^{2/3} \rho^{1/3} - K \rho^{2/3} \). So at high density, the equilibrium temperature \( T \) decreases with growing \( \rho \), i.e. goes the same way as \( \mathcal{E} \). The total heat capacity becomes positive, and runaway can set in as in terrestrial explosives. So, a progenitor of SN Ia must be a degenerate star - a white dwarf.

A single white dwarf is unable to explode, it cools down. But when it is in a
binary system the chances to produce a supernova do appear (we need only one in $\sim 300$ dying white dwarfs to explode in order to explain the rate of SNe Ia). Even if the binary has two dead white dwarfs, it can explode because they can merge due to emission of gravitation waves (double-degenerate, or DD scenario \[1\]). If one star in the binary is alive, the white dwarf can accrete its lost mass and reach an instability (single-degenerate, or SD scenario \[27, 5\]). It is unclear which scenario is most important, there are strong arguments\[14\] from chemical evolution that only SD is the viable one. On the other hand, it seems that DD can produce a richer variety of SN Ia events. Moreover, discoveries of intergalactic SNe Ia \[2, 7\] can be explained more naturally, because a DD system may evaporate from a galaxy. It is quite likely that both scenarios are being played, but their relative role may change in young and old galaxies. If so, a systematic trend may appear in SNe Ia properties with the age of Universe, and this may have important consequences for cosmology.

### 3 Thermonuclear flames

After merging in DD scenario, or after the white dwarf accretes large amount of material in SD case, the explosive instability develops. In principle, combustion can propagate either in the form of a supersonic detonation \[1\] wave, or as a subsonic deflagration \[12, 17\] (flame). In detonation, the unburned fuel is ignited by a shock front propagating ahead of the burning zone itself. In deflagration, the ignition is governed by heat and active reactant transport, i.e. by thermal conduction and diffusion.

Most likely, the runaway starts as a laminar flame propagating due to thermal conduction. In terrestrial flames, the ‘fusion’ of molecules goes with the rate: $\langle \sigma v_0 \rangle \sim \exp(-E_a/RT)$, the Arrhenius law of chemical burning. Here $E_a$ is activation energy. The parameter, showing the strong $T$-dependence of the heating $Z_e = \partial \log \langle \sigma v_0 \rangle / \partial \log T \approx E_a/RT$ is called the Zeldovich number in the theory of chemical flames. For them typically $Z_e \sim 10 \ldots 20$. The classical theory \[30\] predicts the flame speed $v_t \approx Z_e^{-1}[v_{Tl}/(\tau_{\text{reac}}(T_b))]^{1/2}$, with $\tau_{\text{reac}}(T) \propto \exp[E_a/(RT)]$. In SNe, for nuclear flames, $\tau_{\text{reac}}(T) \propto \exp[\alpha_1/(3T^{1/3})]$, and, $Z_e = \partial \log \langle \sigma v_0 \rangle / \partial \log T \approx \alpha_1^{1/3}/(3T^{1/3})$, which has values very similar to terrestrial chemical flames.

A big difference with chemical flames is the ratio of heat conduction and mass diffusion, the Lewis number, $Le = (v_T l_T)/(v_D l_D)$. One finds $Le \sim 1$ in laboratory gaseous flames, while $Le \sim 10^7$ in thermonuclear SNe, since heat is transported by relativistic electrons, $v_T \sim c$, and there is almost no diffusion,
Table 1: Flame speed $v_f$ and width $l_f$ in C+O \[26\]

| $\rho$ (gcc) | $v_f$ (km/s) | $l_f$ (cm) | $\Delta \rho/\rho$ | Ma |
|--------------|--------------|------------|-------------------|----|
| 6.0 $\times$ 10$^9$ | 214 | 1.8 $\times$ 10$^{-5}$ | 0.10 | 2 $\times$ 10$^{-2}$ |
| 1.0 $\times$ 10$^9$ | 36 | 2.9 $\times$ 10$^{-4}$ | 0.19 | 4 $\times$ 10$^{-3}$ |
| 0.1 $\times$ 10$^9$ | 2.3 | 2.7 $\times$ 10$^{-2}$ | 0.43 | 4 $\times$ 10$^{-4}$ |

$l_T \gg l_D$. Nevertheless, the modern computations \[26\] follow the old theory \[30\] closely. The conductive flame propagates in a presupernova with $v_f$ which is too slow to produce an energetic explosion: the ratio of $v_f$ to sound speed, i.e. the Mach number, Ma, is very small (see Table 1). The star has enough time to expand, to cool down, and the burning dies completely. So an acceleration of the flame is necessary in order to explain the SN phenomenon. This is the main problem in current research of SNe Ia hydrodynamics.

There is a rich variety of instabilities that can severely distort the shape of a laminar flame. The Rayleigh–Taylor (RT) instability governs the corrugation of the front on the largest scales. On the smallest scales the flame is controlled by the Landau-Darrieus (LD) instability. RT, LD instabilities and turbulence make computations difficult, but without them a star would not explode. All these instabilities were considered already by L.Landau \[15\] as a means to accelerate the flame.

Because of instabilities, the flame surface becomes wrinkled and its area grows as $S \propto \bar{R}^\alpha$, with average radius $\bar{R}$ and $\alpha > 2$, i.e. faster than $S \propto \bar{R}^2$. In other words the surface becomes ‘fractal’. The exponent $\alpha$ is actually the fractal dimension, $\alpha = D_F$. The effective flame speed is determined \[28\] by the ratio of the maximum scale of the instability to the minimum one: $v_{\text{eff}} = v_f(\lambda_{\text{max}}/\lambda_{\text{min}})^{D_F-2}$. When the vorticity is not important it is possible to study in detail the non-linear stage of LD instability and to find the fractal dimension \[3\]. A similar dependence of the flame fractal dimension on the density jump across the front was found in SPH simulations of the flame subject to RT instability \[4\].

The fractal description is good for LD while it remains mild, because it operates in a star on the scales from the flame thickness (a tiny fraction of a cm) up to $\sim$ 1 km. For the RT instability, $\lambda_{\text{max}}/\lambda_{\text{min}}$ is very uncertain and the fractal dimension is uncertain too. So a direct 3D numerical simulation is necessary. The same is true for a low density regime of LD when it is strongly
coupled to turbulence (generated on the front, or cascading from large RT vortices). A great progress is achieved here in several groups [2, 22, 23, 13]. When simulating 3D turbulent deflagrations one encounters two problems: the representation of the thin moving surface separating hot and cold material, and the prescription of the local velocity $v_f$ of this surface as a function of the large-scale flow with a crude numerical resolution $> 1$ km. One solution is sketched in [22]; for a different approach see [13]. In spite of the progress this problem cannot be treated as completely solved, and even 1D approach may give interesting results, especially for unusual SNe Ia [1].

4 Light curves of SNe Ia

Given a hydrodynamic structure of SN ejecta, one can compute a light curve which should be compared with observations. There are several effects in SNe physics which lead to difficulties in the light curve modeling of any type of SNe. For instance, an account should be taken correctly for deposition of gamma photons produced in decays of radioactive isotopes, mostly $^{56}$Ni and $^{56}$Co. To find this one has to solve the transfer equation for gamma photons together with hydrodynamical equations. Full system of equations should involve also radiative transfer equations in the range from soft X-rays to infrared for the expanding medium. There are millions of spectral lines that form SN spectra, and it is not a trivial problem to find a convenient way how to treat them even in the static case. The expansion makes the problem much more difficult to solve: hundreds or even thousands of lines give their input into emission and absorption at each frequency.

Currently, powerful codes appear aimed to attack a full 3D time-dependent problem of SN Ia light [10]. Yet there are some basic questions, like averaging the line opacity in expanding media, that remain controversial.

In our work we predict the broad-band UBVI and bolometric light curves of SNe Ia, using our 1D-hydro code which models multi-group time-dependent non-equilibrium radiative transfer inside SN ejecta. In our previous analysis we have studied two Chandrasekhar-mass models: the classical deflagration model W7 [18] and the delayed detonation one DD4 [29], as well as two sub-Chandrasekhar-mass models: helium detonation model LA4 [16] and low-mass detonation model WD065 with low $^{56}$Ni production [20], which was constructed for modelling subluminous SNe Ia events, such as SN 1991bg. All those models were simplified spherically-symmetrical (1D) ones.

The UBVI light curves of 1D models are shown in Fig. 1. The Chandrasekhar-
Figure 1: UBVI light curves for 4 1D models (W7 – solid line, DD4 – dots, LA4 – dashes, WD065 – dash-dots). Crosses, stars and triangles show the light curves for three observed SNe Ia.

mass models demonstrate almost identical light curves. The sub-Chandrasekhar-mass ones are more different. WD065 has almost similar element distribution as Chandrasekhar-mass models, and the shape of its light curve is in principle the same as that of W7 and DD4. It is just much dimmer due to a very low $^{56}$Ni abundance.

LA4 is very different from any other model, since the explosion there started on the surface of a white dwarf, not in the center, as for every other model, so there is a $^{56}$Ni layer near the surface in LA4. This feature explains why the model is essentially bluer than other ones.

Currently we employ our new corrected treatment for line opacity [24] in the expanding medium, which is important especially in UV and IR bands. It seems that 1D thermonuclear supernova models, e.g. the deflagration W7
Table 2: Parameters of SN Ia models

| Model | DD4  | W7   | LA4  | WD065 | MR  |
|-------|------|------|------|-------|-----|
| $M_{WD}^a$ | 1.3861 | 1.3775 | 0.8678 | 0.6500 | 1.4 |
| $M^{56}_{Ni}^a$ | 0.63 | 0.60 | 0.47 | 0.05 | 0.42 |
| $E_{51}^b$ | 1.23 | 1.20 | 1.15 | 0.56 | 0.46 |

$^a$in $M_\odot$

$^b$in $10^{51}$ ergs s$^{-1}$

[18] model and the delayed detonation DD4 [29] one, produce the light curves fitting the observations not so good as the recent 3D deflagration model MR computed at MPA [22]. We believe that the main feature of the latter model which allows us to get the correct flux during the first month, is strong mixing that moves the material enriched with radioactive $^{56}$Ni to the outermost layers of SN ejecta.

Fig. 2 demonstrates that in spite of quite different structure of the old W7 model and the new MR one their light curves are similar in many details. Moreover the new model behaves better in $U$ and $B$ bands. Unfortunately, the bolometric light curve for MR model is somewhat too slow. The ejecta must expand with a higher speed to let photons to diffuse out faster.

5 Conclusions

There are several points which require attention for applying SNe Ia in cosmology: progenitors may be different in younger galaxies; burning regimes may change with the age of Universe [23]. The physical understanding of the Pskovskii-Phillips is not yet achieved.

In the flame modeling the new 3D SN Ia model [22] is very appealing. Yet it is not a final one: a detailed post-processing of nucleosynthesis is not yet checked in the light curve calculation.

The SN light curve modeling still has a lot of physics to be added, such as a 3D time-dependent radiative transfer, including as much as possible of NLTE effects [10], which are especially essential for SNe Ia. All this will improve our understanding of thermonuclear supernovae and their role in cosmology.

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Figure 2: UBVI light curves for the 3D (MR; solid) and 1D (W7; dashed) models. Crosses, stars and triangles show the light curves for three observed SNe Ia.

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