Predictive Power of Nuclear Cluster-Model Study

M. Kamimura
Department of Physics, Kyushu University, Fukuoka 812-8581, Japan and
RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan
E-mail: mkamimura@riken.jp

Abstract. Predictive power of theory needs good models and accurate calculation methods to solve the Schrödinger equations of the systems concerned. In this talk, I present some examples of successful predictions based on the nuclear cluster models of light nuclei and hypernuclei and on the calculation methods (CDCC and GEM) that have been developed by Kyushu group.

1. Introduction

Sometimes, theoreticians are requested to make suggestive, helpful predictions for the sake of experiments that are in a quickly developing stage, or in a preliminary stage. Sometimes, experimentalists enjoy to challenge theoreticians to make prediction before measurement which is to be done soon. Predictive power of theory needs good models and accurate calculation methods to solve the Schrödinger equations of the systems concerned.

In this talk, I shall present some examples of successful predictions by our nuclear cluster-model study of light nuclei and hypernuclei with the use of the calculation methods, CDCC (Continuum-Discretized Coupled Channels[1]) and GEM (Gaussian Expansion Method[2, 3] for few-body bound and scattering states) that have been developed by our Kyushu group.

The method of CDCC and its application to various breakup reactions up to 1986 were extensively summarized in a review volume of Ref. [1]. Nowadays, CDCC is used as one of standard methods to investigate breakup reactions with stable- and unstable-nucleus beams. Theoretical foundation of CDCC was presented in Ref. [4] in relation to the Faddeev method. In numerical calculations, the CDCC results for $^{12}\text{C}(d,d)^{12}\text{C}$ and $^{12}\text{C}(d,pn)^{12}\text{C}$ were precisely compared with that of the Faddeev calculation with a good agreement between them[5].

The nuclear cluster model has been extensively employed to perform a comprehensive study of the structure and reactions of light nuclei. The study in Japan in 1960’s and 70’s was performed as a long-term research project on ”Comprehensive Study of Structure of Light Nuclei—Based on the Viewpoint of the Alpha-Cluster Correlations and Molecular Structure—” that was supported by Yukawa Institute for Fundamental Physics (Kyoto). The study was summarized in three volumes of the Supplements of Theoretical Physics [6, 7, 8].

To the present author’s opinion, the most successful example of cluster-model wave function that has been used in many reaction calculations is our $3\alpha$-RGM (Resonating Group Method) wave function of $^{12}\text{C}$ [9, 10]. The $(e,e’)$ study of several states in $^{12}\text{C}$, especially of the Hoyle state (the second $0^+$ state at 7.65 MeV) will be discussed in §2. The calculated form factor (predicted in 1978) of the inelastic electron scattering from the Hoyle state was perfectly reproduced by an experiment [11] in 2006.
Using the wave functions of $^{12}$C, I shall present in §2, that the Kyushu group’s predictions on the differential cross sections (in absolute value) of $^6$Li scattering from $^{12}$C($0^+_1; 2^+_1, 3^+_1, 0^+_2$) were afterwards verified by experiments; and similarly for scattering of the unstable-nucleus $^7$Be from $^{12}$C with the use of CDCC for the projectile breakup channels.

Details of GEM are reviewed in Ref. [3] together with its applications to bound and scattering states of various few-body systems. An example of the most accurate 3-body GEM calculation is the determination of mass of antiproton as seen in the Particle Listings 2000 by the PDG; this is presented in §3. The cluster model has extensively been applied to the 4- and 5-body study of light hypernuclei using GEM. Some of successful examples of theoretical predictions on the structure of light hypernuclei will be shown in §4.

2. 3α RGM wave functions of $^{12}$C and application to reactions

The 3α RGM wave function is given by

$$\Psi_{JM}^{(12)C} = \mathcal{A} \left[ \Phi^{(\alpha_1)} \Phi^{(\alpha_2)} \Phi^{(\alpha_3)} \psi_{JM}(r, R) \right].$$

Here $\mathcal{A}$ is the total antisymmetrization operator, $\Phi^{(\alpha)}$ is the intrinsic wave function of the $\alpha$ cluster with the $(0s)^3$ configuration and $\psi_{JM}(r, R)$ describes the relative motion among three $\alpha$’s. How to determine $\psi_{JM}(r, R)$ is described in Refs.[9, 10]. The wave functions should first be severely examined by the $(e, e')$ experiments. The calculated form factors of the elastic scattering and transitions to the excited states with $J^\pi = 2^+, 4^+_1, 1^+_1, 3^-_1$ and $0^+_2$ are compared with the observed ones in Fig. 1. In all the cases, agreement is good between our calculation (solid lines) and the experimental data. Here, it is to be emphasized that, in the case of the second 0$^-$ state (the Hoyle state), our calculation was a prediction (1978) [9] before the measurement (1979). We see some differences in the second peak region. But, in 2006, the experimental data were much improved as shown in Fig. 2. I am glad to see that the latest experiment reproduced perfectly our prediction some 30 years ago (Recently, Funaki et al.[12] gave almost the same result as ours taking the alpha-condensation idea.)

We performed 3α RGM calculation for scattering states ($J^\pi = 0^+, 2^+$)[13] between $\alpha$ and $^8$Be allowing the virtual excitation of $^8$Be that is essentially important in this scattering process. We reproduced the position and width of the Hoyle-state resonance and predicted a broad 2$^+$ resonance at $E_X \sim 9.1$ MeV with $\Gamma \sim 1$ MeV. The prediction was supported by a recent experiment[14] giving $E_X(2^+) = 9.9 \pm 0.3$ MeV with $\Gamma = 1.0 \pm 0.3$ MeV.

Since the 3α RGM wave functions of $^{12}$C are so good, it happened that, in 1983, K. Katori et al. at RCNP (Osaka) challenged our Kyushu group as follows: They were going to measure the differential cross sections $\sigma(\theta)$ of the elastic and inelastic scattering $^{12}$C($^6$Li,$^6$Li)$^{12}$C($0^+_1; 2^+_1, 0^+_2, 3^-_1$) at $E(6$Li) = 170 MeV. They asked us to predict absolute values of $\sigma(\theta)$ before measurement which was planned to perform on 22 September, 1983. Our colleague Sakuragi accepted their challenge. On 16 September, he handed the experimentalists his prediction [15] of $\sigma(\theta)$ (solid lines in Fig. 2), in absolute values, which were calculated with the 4-state ($0^+_1, 2^+_1, 0^+_2, 3^-_1$) CC method. All the CC potentials were calculated by folding the M3Y NN force into the wave functions of $^{12}$C and $^6$Li and then multiplying them by a complex factor $(N_R + iN_I)$ with $N_R = 0.6$ and $N_I = 0.4$ (see Chapter VI of Ref. [1] for $N_R$ and $N_I$).

The observed differential cross sections [16] are shown in Fig. 2. At forward angles ($\leq 20^\circ$), the experiment reproduced very well the theoretical prediction except for the 3$^-_1$ excitation. It is to be emphasized that, the excitation of the second 0$^+$ state, which was considered to be difficult in calculation, is successfully predicted by the 3α cluster model. An extensive analysis of the reaction mechanism was performed in Ref.[17] with the use of CDCC which further includes the $\alpha + d$ d-state resonant breakup channel of $^6$Li and 3α breakup channel of $^{12}$C.
Figure 1. a)-g) Calculated and observed form factors of the elastic and inelastic electron scattering \(^{12}\text{C}(e,e')^{12}\text{C}(0_1^+,2_1^+,4_1^+,1_1^-,3_1^-,0_2^+)\). The figures are taken from [10]. h) The latest \((e,e')\) experimental data [11] for the second \(0^+\) state (the Hoyle state) reproduced perfectly our \(3\alpha\)-RGM prediction [9, 10] some 30 years ago.

Another example of the predictive power of the cluster-model study is as follows: In 1988 at RCNP, T. Yamagata \textit{et al.} performed the first experiment using unstable-nucleus beams in Japan, namely, the scattering \(^{12}\text{C}(\ ^7\text{Be},\ ^7\text{Be})^{12}\text{C}\) at 140 MeV. The secondary beams of \(^7\text{Be}\) were produced by the reaction \(^1\text{H}(\ ^7\text{Li},\ ^7\text{Be})n\). Since they had no experience in scattering of unstable nucleus, they requested Sakuragi to predict absolute values of \(\sigma(\theta)\) which would be a good guide to set up the detectors.

The calculated differential cross section predicted before measurement is shown in Fig. 3 by the solid line [18, 19] which is a sum of the elastic \((3/2_1^-)\) and inelastic \((1/2_1^-\) at 0.48 MeV) contributions since these two were not separated in the experiment. The CDCC calculation takes the \(\alpha-\ ^3\text{He}\) cluster-model wave functions of the bound states \((3/2_1^-\) and \(1/2_1^-\)) and the resonant states \((7/2^-\) and \(5/2^-\)) together with the non-resonant discretized continuum states. The experimental data [19] reproduced well the theoretical prediction.

Interestingly, during 1983-1988, Sakuragi accepted such types of challenge or request from experimentalists totally six times. His predictions were all successful, no failure. In that period, projectile-breakup experiments using light- and heavy-ion beams were in a quickly developing stage. I am proud of that Sakuragi was providing suggestive, helpful predictions to those experiments in such an important stage.
Figure 3. Calculated differential cross section (solid line) [18] predicted before measurement of $^{12}$C($^7$Be, $^7$Be)$^{12}$C at 140 MeV. See text for the calculation. The experimental data [19] reproduced well the theoretical prediction. This figure is taken from Ref.[19].

3. Determination of mass of antiproton

In order to demonstrate the accuracy of GEM, which precisely was reviewed in Ref.[3], I here present the best example, namely 3-body calculation to determine mass of anti-proton, $m_{\bar{p}}$.

Particle Listings 2000 by PDG gave the first recommended information on the antiproton mass that $|m_{\bar{p}} - m_p|/m_p < 10^{-7}$. This value was derived by spectroscopic study of highly-excited metastable states in antiprotonic helium atom composed of $\alpha + \bar{p} + e^-$. The sub-ppm laser spectroscopic data at CERN were analyzed by our colleague Kino making a 3-body calculation with the accuray of 10 significant figures. The laser experiment for the transition between the metastable states with $(J, v) = (34, 2)$ and $(J, v) = (33, 2)$ gave the wave length $\lambda_{\text{EXP}} = 470.7220(6)$ nm, which was precisely reproduced by Kino’s calculation $\lambda_{\text{CAL}} = 470.7220$ nm, assuming $m_{\bar{p}} = m_p$. Then Kino showed that the upper and lower bounds of $\lambda_{\text{EXP}}$ were reproduced by assuming $m_{\bar{p}} = (1 + \varepsilon)m_p$ with $\varepsilon = 5 \times 10^{-7}$. He then considered that even if $m_{\bar{p}}$ is changed, calculated wavelength should be within the experimental error. Then, we obtain $(1 - \varepsilon)m_p < m_{\bar{p}} < (1 + \varepsilon)m_p$, namely, $|m_{\bar{p}} - m_p|/m_p < \varepsilon = 5 \times 10^{-7}$, which was cited in Particle Listings 2000. Very recently, this bound becomes much smaller, $2 \times 10^{-9}$, in Particle Listings 2010. This can be a test of CPT invariance. I am happy to see that GEM is so accurate to determine, for the first time, mass of the elementary particle, antiproton.

A lot of transitions between excited states of antiprotonic helium atoms have been observed at the CERN. But, cost of the precise sub-ppm laser-scan search of the transition energy $\Delta E$ is very expensive. Kino was then requested to predict $\Delta E$ before measurement. A typical example[33] of the transition frequency by the Kino’s prediction ($\nu_{\text{CAL}}$) and the observed value
4. Application of cluster model to hypernuclei

Many of interesting light nuclei have two- or three-cluster structure. Therefore, if we inject one or two hyperons into them, we have interesting light hypernuclei with 3-, 4-, or 5-cluster structure. Thus, the light hypernucleus is a good place to apply our cluster model and make predictive suggestions to the hypernuclear experiment that is in a quickly developing stage. Strategy for studying hypernuclear physics is presented in Ref. [20].

Along this line, Hiyama has developed GEM so that it is applicable to precise 4- and 5-body calculations [3]. She has studied a lot of hypernuclei (reviewed in Refs. [21, 22]) such as

i) single-Λ hypernuclei (\(^7\)H, \(^6\)He, \(^6\)Li, \(^7\)He, \(^7\)Li, \(^7\)Be, \(^8\)Li, \(^8\)Be, \(^9\)Be, \(^10\)Be, \(^10\)B, \(^13\)C), ii) double-Λ hypernuclei (\(^5\)Li, \(^5\)He, \(^5\)Li, \(^5\)Be, \(^6\)Li, \(^6\)Be, \(^6\)Be, \(^7\)B, \(^7\)Be, \(^8\)Be, \(^9\)Be, \(^10\)Be, \(^11\)Be) and iii) single-Ξ\(^-\) hypernuclei (\(^5\)H, \(^7\)H, \(^9\)Li, \(^10\)Li, \(^12\)Be). Among those hypernuclei, I briefly introduce three types of her predictions compared with the experiment.

a) Shrinkage of hypernuclei

One of the interesting issues in hypernuclear physics is to investigate dynamical change of nuclear structure induced by a glue-like role of an added Λ hyperon. In 1999, Hiyama et al. [23] proposed to measure specifically the \(B(E2)\) value of the transition \(5/2^+_1 \rightarrow 1/2^+_1\) in \(^\Lambda\)Li and suggested how to derive the size of the ground state of \(^\Lambda\)Li from that \(B(E2)\) value. Taking \(^\Lambda\)He + N + N three-body model, they obtained \(B(E2; 5/2^+_1 \rightarrow 1/2^+_1) = 2.42e^2fm^4\) and therefore predicted that the size is shrunk by 25% due to the addition of Λ particle. Later, in Ref. [24] (2001), with the use of more precise \(\alpha + \Lambda + n + p\) four-body model, they recalculated the \(B(E2)\) value and obtained \(2.85e^2fm^4\) which means that the shrinkage is 22%.

The first observation of the hypernuclear \(B(E2)\) strength was made in the KEK-E419 experiment for \(B(E2; 5/2^+_1 \rightarrow 1/2^+_1)\) in \(^\Lambda\)Li. The strength was reported in 2001 to be \(3.6 \pm 0.5^{+0.5}_{-0.4}e^2fm^4\) [25]. From this, the shrinkage was estimated to be \(19 \pm 4\%\), which was consistent with our prediction. This confirmed, for the first time, the shrinkage of the nuclear size induced by the Λ particle. Our prediction about the shrinkage of \(^{12}\)C was given in [26, 27].

b) Spin-orbit splitting in hypernuclei

The strong \(NN\) spin-orbit force is one of the essential ingredients of nuclear physics. In hypernuclei, spin-orbit splitting energy was first precisely calculated in 2000 by Hiyama et al. [27] for the \(5/2^+_1 - 3/2^+_1\) doublet states in \(^9\)Be (= \(2\alpha + \Lambda\)) and the \(3/2^+_1 - 1/2^+_1\) states in \(^{13}\)C (= \(3\alpha + \Lambda\)); a very small splitting was predicted quite differently from the splitting in the ordinary nucleus when the quark-model based two-body \(NN\) spin-orbit force was taken. The calculated splitting was \(\Delta E_{CAL}(5/2^+_1 - 3/2^+_1) = 35 - 40\) keV in \(^9\)Be and \(\Delta E_{CAL}(3/2^+_1 - 1/2^+_1) = 150 - 200\) keV in \(^{13}\)C.

Afterwards, experimental values were reported as \(\Delta E_{EXP}(5/2^+_1 - 3/2^+_1) = 31.4^{+2.5}_{-3.6}\) keV in \(^9\)Be by BNL-E930 [28] in 2002 and \(\Delta E_{CAL}(3/2^+_1 - 1/2^+_1) = 150 \pm 54 \pm 36\) keV in \(^{13}\)C by BNL-E929 [29] in 2001, which is consistent with our prediction. The very weak spin-orbit component of the \(\Lambda N\) interaction compared with that of the \(NN\) interaction was confirmed.

c) Double Λ hypernuclei

The double strangeness \((S = -2)\) nucleus is an entry to multi-strangeness hadronic systems (including the core of a neutron star). Since the present experimental information on the \(S = -2\) system is very limited, study of the structure of double Λ hypernuclei is absolutely important.
In 2002 by Hiyama et al. [30], energy levels of $^7\Lambda\Lambda$He, $^7\Lambda\Lambda$Li, $^8\Lambda\Lambda$Li, $^9\Lambda\Lambda$Be, and $^{10}\Lambda\Lambda$Be were predicted with the framework of $\alpha\Lambda\Lambda$ four-body model, where $x = n, p, d, t, ^3$He, and $\alpha$, respectively. Also, energy levels of the double $\Lambda$ hypernucleus, $^{11}\Lambda\Lambda$Be were predicted within the framework of a $\alpha\alpha\Lambda\Lambda$ five-body model in Ref. [31] in 2010.

As for $^{10}\Lambda\Lambda$Be, we predicted $B_{\Lambda\Lambda}^{\text{cal}}(2^+_1) = 11.88$ MeV and $B_{\Lambda\Lambda}^{\text{exp}}(0^+_1) = 14.74$ MeV. The Demachi-Yanagi event, identified as $^{10}\Lambda\Lambda$Be and $B_{\Lambda\Lambda}^{\text{exp}} = 11.90 \pm 0.13$ MeV [32], was interpreted as an observation of its $2^+$ excited state. The Hida event, observed in KEK-E373 experiment as $B_{\Lambda\Lambda}^{\exp} = 20.83 \pm 1.27$ MeV for $^{11}\Lambda\Lambda$Be [32], was interpreted as an observation of the ground state of $^{11}\Lambda\Lambda$Be since our calculation gave $B_{\Lambda\Lambda}^{\text{cal}}(3/2^+_1, \text{g.s.}) = 18.23$ MeV and $B_{\Lambda\Lambda}^{\text{cal}}(5/2^+_1) = 15.48$ MeV (the experimental error is large in this case; further experiment at J-PARC is waited for).

Since the emulsion experiment itself cannot specify the spin-parity of the observed state and cannot tell whether the state is the ground state or an excited state, theoretical calculation is quite helpful in determining the property of the observed state.

5. Concluding remarks
Predictive power of theory requires good models and accurate calculation methods. It is desirable for theoreticians to make predictive suggestions to experiments in quickly developing fields. Here, we have shown some examples of the efforts, along this line, done by our Kyushu group taking the cluster model and calculation methods of CDCC and GEM.

References
[1] M. Kamimura et al., Prog. Theor. Phys. Suppl. No.89, 1 (1986).
[2] M. Kamimura, Phys. Rev. A38, 621 (1988).
[3] E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51 (2003) 223.
[4] N. Austern, Y. Yahiro and M. Kawai, Phys. Rev. Lett. 63, 2649(1989); Phys. Rev. C 53 (1996), 394.
[5] A. Deltuva et al., Phys. Rev. C 76, 064602(2007).
[6] K. Ikeda et al., Prog. Theor. Phys. Suppl. No.52 (1971) 1.
[7] S. Saito et al., Prog. Theor. Phys. Suppl. No.62 (1977) 1.
[8] K. Ikeda et al., Prog. Theor. Phys. Suppl. No.68 (1980) 1.
[9] M. Kamimura, Nucl. Phys. A 351 (1981) 456.
[10] A. Lenhardt et al., Nucl. Instrum. Method Phys. Res., Sect. A 562 (2006) 320.
[11] A. Funaki et al., Eur. Phys. J. A. 28 (2006) 259.
[12] M. Kamimura and Y. Fukushima, Proceedings of INS Interational Symposium on Nuclear Direct reaction Mechanism, Fukuoka, 1978, p.409.
[13] M. Kamimura et al., Nucl. Phys. A 738 (2004) 268.
[14] M. Kamimura, private communication (1983).
[15] K. Katori et al., Nucl. Phys. A480 (1988) 323.
[16] M. Kamimura, Phys. Lett. B 205 (1988) 204.
[17] Y. Sakuragi, private communication (1983).
[18] Y. Sakuragi, private communication (1988).
[19] Y. Yamagata, Y. Sakuragi, et al., Phys. Rev. C 39 (1989) 873.
[20] R. Hiyama et al., Prog. Theor. Phys. Suppl. No.185 (2010), 1.
[21] R. Hiyama et al., Prog. Theor. Phys. Suppl. No.185 (2010), 106.
[22] R. Hiyama et al., Prog. Theor. Phys. Suppl. No.185 (2010), 152.
[23] R. Hiyama et al., Phys. Rev. C 59, 2351 (1999).
[24] R. Hiyama et al., Nucl. Phys. A684 227c.
[25] K. Tanida et al., Phys. Rev. Lett. 86, 1982 (2001).
[26] R. Hiyama et al., Prog. Theor. Phys. 97, 881 (1997).
[27] R. Hiyama et al., Phys. Rev. Lett. 85, 270 (2000).
[28] K. Ikeda et al., Phys. Rev. Lett. 88, 82501 (2002).
[29] S. Ajimura et al., Phys. Rev. Lett. 86, 4225 (2001).
[30] E. Hiyama et al., Phys. Rev. C 66, 024007 (2002).
[31] E. Hiyama et al., Phys. Rev. Lett. 104, 212502 (2010).
[32] K. Nakazawa and H. Takahashi, Prog. Theor. Phys. Suppl. No.185 (2010), 335.
[33] M. Hori et al. Phys. Rev. Letters 87 (2001) 093401.