EVOLUTION AND NUCLEOSYNTHESIS OF ZERO-METAL INTERMEDIATE-MASS STARS

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ABSTRACT

New stellar models with masses ranging between 4 and 8 $M_\odot$, $Z = 0$, and $Y = 0.23$ are presented. The models have been evolved from the pre-main sequence up to the asymptotic giant branch (AGB). At variance with previous claims, we find that these updated stellar models do experience thermal pulses in the AGB phase. In particular, we show the following:

1. In models with a mass larger than 6 $M_\odot$, the second dredge-up is able to raise the CNO abundance in the envelope enough to allow a normal AGB evolution, in the sense that the thermal pulses and the third dredge-up take place.

2. In models of lower mass, the efficiency of the CNO cycle in the H-burning shell is controlled by the carbon produced locally via the $3\alpha$ reactions. Nevertheless, the He-burning shell becomes thermally unstable after the early AGB. The expansion of the overlying layers induced by these weak He-shell flashes is not sufficient by itself to allow a deep penetration of the convective envelope. However, soon after the development of the maximum luminosity of the He flash is attained, a convective shell systematically forms at the base of the H-rich envelope. The innermost part of this convective shell probably overlaps the underlying C-rich region left by the intershell convection during the thermal pulse so that fresh carbon is dredged up in a hot H-rich environment and an H flash occurs. This flash favors the expansion of the outermost layers already started by the weak thermal pulse, and a deeper penetration of the convective envelope takes place. Then the carbon abundance in the envelope rises to a level high enough that the further evolution of these models closely resembles that of more metal-rich AGB stars.

These stars provide an important source of primary carbon and nitrogen, so a major revision of the chemical evolution in the early Galaxy is required. We suggest that the chemical imprint of these Population III stars could be found in the old and metal-poor components of the Milky Way.

Subject headings: nuclear reactions, nucleosynthesis, abundances — stars: abundances — stars: evolution — stars: Population II

1. INTRODUCTION

Evidence for an expanding universe coupled to the observation of the fossil blackbody radiation leads to the natural conclusion that the primordial universe was hot and dense and that there was an epoch during which nuclear reactions among neutrons and protons took place. A few hours after the Big Bang, the primordial composition of the universe was defined. The standard homogeneous Big Bang nucleosynthesis (as reported by Alpher & Herman 1950, for a recent analysis, see Walker et al. 1991) predicts that the material that emerged from this epoch was mainly made up of $^1$H, $^4$He, and a small quantity of other light elements (D, $^3$He, and $^7$Li). The total mass fraction of the heavier elements was lower than $10^{-12}$, so the first generation of stars, the so-called Population III stars, was formed from a gas essentially lacking in metals. Such a peculiarity certainly influenced both the fragmentation of the primordial gas clouds and the evolutionary properties of this first stellar generation. In fact, in these extreme conditions, the cooling able to reduce the Jeans mass down to the stellar values was provided by the molecular hydrogen rather than dust or heavy molecules, and the progressive fragmentation was halted only when the gas became opaque because of the $^2$H$_2$ line absorption (Carlberg 1981; Silk 1983; Lepp & Shull 1983; Palla, Salpeter, & Stahler 1983; Yoshii & Saio 1986; Shapiro & Kang 1987; de Araújo & Opher 1989; Uehara, Susa, & Nishi 1996; Haiman, Thoul, & Loeb 1996; Omukai et al. 1998).

The initial mass function (IMF) emerging from this atypical star formation process has been the subject of a number of papers. Several groups have found that intermediate-mass stars (IMSs) could result from the fragmentation of primordial gas clouds. Yoshii & Saio (1986) found that the peak of the IMF for Population III stars ranges between 4 and 8 $M_\odot$, while Uehara et al. (1996) concluded that the minimum stellar mass is on the order of the Chandrasekhar mass (i.e., $\sim 1.4 M_\odot$). Nakamura & Umemura (1999) concluded that the typical stellar mass is around 3 $M_\odot$; this value may further increase by accretion of the environmental gas up to 16 $M_\odot$. On the other hand, three-dimensional simulations by Abel et al. (1998), Abel, Bryan, & Norman (2000), and Bromm, Coppi, & Larson (1999) obtained a Jeans mass on the order of $10^2$–$10^3 M_\odot$; their calculations, however, do not have enough resolution to...
explore further fragmentation into smaller masses. Recently, Nakamura & Umemura (2001) performed two-dimensional simulations and found that the primordial IMF is likely to be bimodal, with the first peak around $2\, M_\odot$ and the second peak between 10 and $10^2\, M_\odot$, consistent with the previously cited works. Wasserburg & Qian (2000a, 2000b), by comparing iron and r-element abundances in very metal-poor stars, have concluded that the first generation of Galactic stars was essentially composed of massive objects ($M \geq 100\, M_\odot$) capable of producing a prompt iron synthesis. Following their model, the average Galactic metallicity would increase up to $[\text{Fe}/\text{H}] \sim -3$ in a few $10^9\, \text{yr}$. In such a case, even low-mass stars could form after a very short time, but their iron content would be that of Population II stars. So the actual IMF of Population III stars is still an open question.

However, since the chemical composition of the matter ejected by a star largely depends on its mass, the study of the surface chemical composition of the most metal-poor stars could help shed light on the kind of stars that first populated our Galaxy (and probably the universe). Note, by the way, that the lifetime of the more massive IMSs is short enough so that they could have really polluted the interstellar medium during the very early phase of dynamical collapse of our Galaxy (masses larger than, say, $\approx 4\, M_\odot$ live less than $\approx 10^8\, \text{yr}$).

Great observational efforts have been made to identify very metal-deficient stellar populations. The first attempt (Bond 1981) failed to find stars with a metallicity of $[\text{Fe}/\text{H}] < -3$. Later on, a red giant with $[\text{Fe}/\text{H}] \sim -4.5$ was identified by Bessel & Norris (1984), and since then, the number of known low-metallicity stars has increased somewhat (see, e.g., Beers, Preston, & Shectman 1992; Ryan, Norris, & Bessel 1991), though only six of them have $[\text{Fe}/\text{H}] \leq -3.5$. Some authors suggest that four of these six stars have a metallicity of $[\text{Fe}/\text{H}] \sim -4$ (see Ryan, Norris, & Beers 1996, 1999; Sneden et al. 1994; Primas, Molaro, & Castelli 1994; Ryan et al. 1991; Carney & Peterson 1981; Molaro & Castelli 1990; Molaro & Bonifacio 1990). To date, however, no stars having the primordial chemical composition (i.e., $Z \sim 10^{-10}$ or $[\text{Fe}/\text{H}] \sim -8.3$) have been found. Such a lack of "observable zero-metal" stars actually supports the theoretical predictions that the primordial IMF did not favor the formation of low-mass, long-lived stars, but it does not rule out primordial intermediate-mass stars (PIMSs).

Though there is definite evidence that very metal-poor stars (and also just metal-poor stars) were formed by matter strongly polluted at least by the ejecta of massive stars (consider, for example, the overabundance of the $\alpha$ elements with respect to iron), there is some hint that a PIMS generation was formed: as an example, let us mention that Galactic nitrogen behaves as a primary element; i.e., it is solar scaled (although with an important dispersion around $[\text{N}/\text{Fe}] \sim 0.0$) in the lower metallicity stars observed in the Milky Way (Laird 1985; Carbon et al. 1987). While the nucleosynthesis occurring in massive stars, exploding as Type II supernovae, cannot account for this primary nitrogen component (see, for example, Timmes, Woosley, & Weaver 1995), the presence of a generation of PIMSs could robustly contribute to the production of primary nitrogen.

It goes without saying that the inclusion or otherwise of the ejecta of a PIMS generation in the general chemical evolution of a galaxy also alters drastically the interpretation of the heavy-element abundances measured in the very low metallicity Galactic stars and the intergalactic medium (IGM) at high redshifts (from $z \sim 2$ to 4; Cowie et al. 1995; Tytler et al. 1995; Songaila & Cowie 1996; Pettini et al. 1997; Ellison et al. 2000). For all these reasons, we decided to revise the evolutionary properties and the related nucleosynthesis of these primordial stars in a wide range of stellar masses. In this way, we hope to identify the possible footprints left by the primordial stellar population in the Galactic chemistry.

The necessity of calculating new PIMS models derives from the fact that most of the available theoretical studies of this kind of star stop at the end of the He burning (Ezer 1961, 1972; Ezer & Cameron 1971; Eryurt-Ezer 1981; Castellani, Chieffi, & Tornambè 1983; Tornambè & Chieffi 1986; Cassisi & Castellani 1993). As is well known, the most important contribution of the IMS to the Galactic chemical evolution comes from the nucleosynthesis occurring during the thermally pulsing phase of the asymptotic giant branch (AGB). These stars, in fact, produce in their interior carbon-nitrogen-, and neutron-rich isotopes that can enrich the interstellar medium by the combined efforts of the dredge-up and the mass loss. In spite of their potential importance in nucleosynthesis and chemical evolution, to date just one paper has addressed the computation of AGB models of this generation of PIMSs (Chieffi & Tornambè 1984), the main finding (based on the evolution of a $5\, M_\odot$ star) being that these stars do not experience the thermally pulsing phase (see below). In a companion paper, Fujimoto et al. (1984) addressed the general behavior of the helium-shell flashes in zero-metal AGB stars by means of an analytical model: their conclusion was that the thermally pulsing phase takes place in stars less massive than $4\, M_\odot$.

The scenario that emerged from those computations was that the overall impact of a PIMS generation on the chemical evolution of the light elements (mainly C and N) in the Galaxy was marginal. On the other hand, the lack of thermal pulses (TPs) coupled to a reduced mass-loss rate (due to the fact that these stars are significantly more compact and dimmer than the present ones) led the authors to suggest that these stars were more likely to increase the degenerate C-O core mass up to $1.4\, M_\odot$ and hence to explode as Type I 1/2 supernovae. The result would be a strong pollution of the interstellar medium by material formed by elements produced by complete explosive Si burning (Fe, Co, and Ni), incomplete explosive Si burning (Mn, Cr, and V), and explosive O burning (Si, S, Ar, Ca, and K).

In this paper and in a companion one (Limongi, Straniero, & Chieffi 2001), we present a full set of evolutionary models of zero-metal stars ranging in mass between 4 and $25\, M_\odot$ from the pre–main sequence (PMS) up to the AGB or up to the iron core collapse. In particular, this paper illustrates our results for models having $M \leq M_{up}$ (namely, the smallest mass that ignites carbon off-center in the degenerate core).

2. THE MODELS

The evolution of five PIMS models, with masses of 4, 5, 6, 7, and $8\, M_\odot$, $Z = 0$, and $Y = 0.23$, has been computed starting from the PMS up to either an advanced phase on the AGB or up to the carbon ignition for the more massive
model. These computations have been performed by means of the latest version of the FRANEC code, Release 4.8 (see Chieffi, Limongi, & Straniero 1998). Detailed references for the input physics adopted have been reported by Straniero, Chieffi, & Limongi (1997). The main characteristic of this code is that the set of equations used to describe both the chemical evolution (due to the nuclear burning) and the physical structure are fully coupled and solved simultaneously. This coupling is required to follow the final phases of the evolution of massive stars from the oxygen burning onward. In order to produce a homogeneous set of models, we have adopted the same computational scheme for IMSs. The nuclear network used in the present computations includes 48 isotopes (269 reactions) for the H burning, and 34 isotopes (147 reactions) for the He burning (see Fig. 1). In order to identify the lowest stellar mass for which a degenerate-carbon ignition occurs ($M_{\text{up}}$), a reduced set of nuclear species and related reactions has been added for the carbon burning, namely, nine isotopes and eight reactions. A revised version of the time-dependent mixing scheme first introduced by Sparks & Endal (1980) has been adopted. In particular, the mass fraction ($X_j$) of a certain isotope at the mesh point $j$, which is inside a convective region having total mass $M_{\text{conv}}$, is given by

$$X_j = X_j^0 + \frac{1}{M_{\text{conf}}} \sum_k (X_k^0 - X_k^*) f_{j,k} \Delta M_k,$$  \hspace{1cm} (1)

where the summation is extended over the whole convective region and the superscript “$0$” refers to unmixed abundances. The variable $\Delta M_k$ is the mass of the mesh point $k$, while the damping factor $f_{j,k}$ is

$$f_{j,k} = \frac{\Delta t}{\tau_{j,k}}$$  \hspace{1cm} (2)

if $\Delta t < \tau_{j,k}$, or

$$f_{j,k} = 1$$  \hspace{1cm} (3)

if $\Delta t \geq \tau_{j,k}$. Here $\Delta t$ is the time step and $\tau_{j,k}$ is the mixing turnover time between the mesh points $j$ and $k$, namely,

$$\tau_{j,k} = \int_{r(j)}^{r(k)} \frac{dr}{v(r)} = \sum_i \frac{\Delta r_i}{v_i}.$$  \hspace{1cm} (4)

The mixing velocity ($v_i$) is computed according to the mixing-length theory (Cox & Giuli 1968), and $\Delta r_i$ is the length of the mesh point $i$. This algorithm allows us to account for the partial mixing that occurs when the time step is reduced to or below the mixing timescale. This condition is sometimes fulfilled in the computation of advanced evolutionary phases. For example, thermally pulsing models require time steps of a few days (in some cases just a few hours), which may be comparable to or lower than the mixing timescale defined in equation (4). The borders of the convective regions are identified by means of the classical Schwarzschild criterion. Note that we have used for the H-rich material opacity tables computed by assuming $Z = 10^{-4}$ and various ratios for the H and He abundances (Alexander & Ferguson 1994; Iglesias, Rogers, & Wilson 1992). These tables allow a suitable description of the radiation transport in the stellar envelope as long as the metallicity does not exceed $Z = 10^{-4}$. For this reason, our evolutionary sequences become unreliable when the metallicity in the envelope, as modified by the third dredge-up, increases over this value.

The computation of stellar models is usually based on semiempirical (or phenomenological) descriptions of hydrodynamic phenomena occurring in real stars, namely, convection and mass loss. The widely used mixing-length theory of convection is a well-known example of this phenomenological approach to stellar hydrodynamics. This procedure does not provide, in general, a satisfactory physical description of these complex phenomena. Nevertheless, once the stellar models are properly calibrated, it may allow us to obtain adequate predictions for the stellar properties. This may be done by tuning the models (i.e., by changing the parameters left free in the phenomenological theory) until they reproduce certain selected and well-measured observable quantities. Unfortunately, because of the lack of an observational counterpart, such a procedure cannot be followed for $Z = 0$ models, so the question of the calibration becomes particularly critical for Population III stars. A possible alternative approach consists of adopting the values of the tuned free parameters obtained for larger metallicities. However, owing to the particular structure developed by metal-deficient stars, an extrapolation of the calibrated parameters down to $Z = 0$ may induce a substantial error. Thus, in order to estimate the uncertainties implicit in the above description of the stellar hydrodynamics, one should explore a range of values for these free parameters that is as wide as possible, but this would imply the computation of a great number of stellar models. A general discussion of this very important problem is beyond the goals of the present paper. In the following discussion, we will focus our attention on those phenomena that particularly affect the estimated nucleosynthesis of the PIMSs.

Mass loss has a negligible effect on IMSs up to the onset of the AGB. Strong mass-loss rates (between $10^{-8}$ and $10^{-4}$ $M_\odot$ yr$^{-1}$) are indeed observed in AGB stars. The duration of this phase, the mass of the resulting compact stellar remnant, and the chemical yields are then substantially affected by mass loss. Groenewegen & De Jong (1993, 1994a, 1994b), on the basis of synthetic AGB models, found that a Reimers (1975) mass-loss rate with a multiplicative factor $\eta \sim 4$ can reproduce the observed high-luminosity tail of the carbon stars’ luminosity function in our Galaxy and in the Magellanic Clouds as well as the observed abundances in LMC planetary nebulae. Dominguez et al. (1999) have shown that with such a mass-loss rate, the final masses of intermediate-mass stellar models are compatible with the initial-to-final mass relation (Weidemann 1987; Herwig

![Fig. 1.—Isotopes included in the nuclear network. Those identified by triangles have only been included during the H burning.](image-url)
More recently, other formulas have been proposed (Vassiliadis & Wood 1993; Blöcker 1995) that predict a huge mass-loss rate for AGB stars (see, for a recent revision of the initial-to-final mass relation, Weidemann 2000). We do not know if a similar strong mass-loss rate also characterizes Population III AGB stars. It has been suggested (see, for example, Vassiliadis & Wood 1993) that the increase of the mass-loss rate during the AGB may be caused by the onset of large-amplitude radial pulsation. Whether PIMS stars also suffered this kind of instability is a matter of opinion. The lower opacity of a metal-deficient atmosphere generally implies more compact stars. This could reduce the mass-loss rate. However, we find (see § 4) that, as a consequence of the second and third dredge-up, the PIMMs become C-rich. The carbon excess alters the structure of the atmosphere, which might favor the formation of grains and increase the molecular blanketing so that the opacity and, in turn, the mass-loss rate could grow. Then, without observational support, the actual mass-loss rate that characterizes the AGB evolution of PIMMs is practically unknown. Note that the overall characteristics of the internal structure of an AGB star are generally dependent on two quantities: the core mass and the envelope mass (see, for example, Iben & Renzini 1983). However, while the envelope mass is larger than 1.5–2 $M_\odot$, the main parameter is the core mass. Thus, since the initial envelope mass of an IMS is rather large, the internal structure remains weakly dependent on the mass loss for a large fraction of the AGB phase. The AGB stellar models presented here have been computed without mass loss. Our computations generally stopped before the onset of the mass loss-dominated AGB phase, even in the case of a particularly strong mass loss (except for the last computed TPs of the lowest mass model).

An even more complex situation concerns stellar convection in AGB stars. It has been widely discussed whether the classical treatment of convection (both physical and numerical) is sufficient to describe the second and third dredge-up occurring during the AGB phase or whether they require an extra mixing whose physical nature still has to be clarified, which is acting below the convective border (Iben 1975, 1981; Becker & Iben 1979; Iben & Renzini 1982, 1983; Hollowell & Iben 1989; Lattanzio 1989; Castellani, Chieffi, & Straniero 1990; Castellani, Marconi, & Straniero 1998; Herwig et al. 1997; Straniero et al. 1997; Frost & Lattanzio 1996; Langer et al. 1999; Mowlavi 1999; Herwig 2000). In summary, when the inner border of the convective envelope approaches the H-depleted (He-enriched) region, even a small perturbation, perhaps driven by a moderate mechanical overshoot, may induce a substantial dredge-up. In fact, as noted by Becker & Iben (1979; see also Castellani et al. 1990 and Frost & Lattanzio 1996), if the convective envelope (H-rich) penetrates a region that is progressively more He-enriched, a discontinuity of the radiative gradient forms at the interface between the stable and unstable layers. This occurs because the H-rich envelope has a significantly greater opacity than the He-rich layer immediately below it. In such a case, if some mixing occurs below the inner border of the convective envelope, the local hydrogen abundance rises, the opacity (and the radiative gradient) grows, and this layer becomes convectively unstable. A similar situation is encountered at the outer edge of the convective core during the central He burning (Paczyński 1970; Castellani, Giannone, & Renzini 1971a; 1971b); in this case, the discontinuity of the radiative gradient is caused by the conversion of helium (low-opacity) into carbon and oxygen (high-opacity) at the base of the convective core. Castellani et al. (1985) named this phenomenon induced overshoot (i.e., induced by the chemical discontinuity that forms at the boundary of a convective region) to be distinguished from the mechanical overshoot (i.e., the mixing caused by the convective elements that conserve a finite velocity beyond the unstable region). Note that in the conditions discussed above, the existence of a small (in most cases negligible) mechanical overshoot may trigger a substantial induced overshoot. Castellani et al. (1998) show that the overall impact on the second dredge-up of this phenomenon is generally small. On the contrary, Frost & Lattanzio (1996, see also Herwig et al. 1997; Herwig 2000; Mowlavi 1999) found that the inclusion of a moderate overshoot strongly increases the efficiency of the third dredge-up. Obviously, a change in the strength of the third dredge-up greatly affects the surface abundances of an AGB star. Thus, spectroscopic observation of the present-day AGB stars may be used to check the reliability of the various mixing algorithms and eventually to calibrate stellar models. Unfortunately, the same cannot be done with the missing Population III stars. Once again, because of the peculiarity of $Z = 0$ models, the descriptions obtained for more metal-rich stars could be unreliable. The basic set of models presented here has been obtained by limiting the mixing within the borders defined by the Schwarzschild criterion. However, we have computed some additional models to investigate the influence of a possible extra mixing below the convective envelope on the AGB evolution of $Z = 0$ stars. Then, following the hydrodynamic descriptions discussed by Herwig et al. (1997), we have tentatively assumed that the convective motions do not stop abruptly at the base of the convective unstable envelope but decrease exponentially below it. More precisely, we have used the following expression for the mixing velocity below the convective envelope:

$$v = v_{bce} \exp (-D/\beta H_p) ,$$

where $v_{bce}$ is the mixing velocity at the base of the convective unstable envelope (as obtained by means of the mixing-length theory), $D$ is the distance from the stability border, $H_p$ is the pressure scale height, and $\beta$ is a free parameter, which evidently determines the strength of the exponential decline. Herwig et al. (1997) suggest $\beta = 0.02$ (actually they use $f$ instead of $\beta$) for solar metallicity models. We have used various values for $\beta$ ranging between 0.005 and 0.04 (see §§ 4 and 5). Note that the extra mixing provided by this formula is, in most cases, negligible. In fact, since in the framework of the mixing-length theory the mixing velocity is proportional to the difference between the radiative and the adiabatic gradients, it falls to zero at the boundary of a convective region where the condition of marginal stability (i.e., $V_{rad} = V_{ad}$) is fulfilled. However, when the convective envelope moves inward, down to the region where the H has been converted into He, the discontinuity in the radiative gradient forms, and $v_{bce}$ grows. In such a case, the extra mixing provided by equation (5) smooths out the chemical profile and prevents the formation of the discontinuity in the radiative gradient. In order to reduce possible numerical noises and save computational time, we do not apply equation (5) to the borders of other convective regions.
3. THE CENTRAL H AND He BURNING

The overall characteristics of the central H- and He-burning phases of our models do not substantially differ from the ones already found in previous studies of very metal-poor stars (Ezer 1961, 1972; Ezer & Cameron 1971; Eryurt-Ezer 1981; Chieffi & Tornambè 1984; Tornambè & Chieffi 1986; Cassisi & Castellani 1993). A more quantitative comparison to the old models by Ezer and coworkers reveals major differences probably due to differences in the input physics. On the contrary, the models presented in the more recent papers are in good agreement with the present ones. This is no surprise because they were obtained by means of old versions of our code (the FRANEC). Small residual differences can easily be attributed to slightly different input physics.

In Table 1 we report the duration (in megayears) of the most relevant evolutionary phases, namely, the PMS and the central H and He burning. Central temperatures versus central densities are plotted in Figure 2, while the HR diagrams are shown in Figure 3. The temporal evolutions of the central convective region up to the end of the central He burning are illustrated for each star in Figure 4.

As has been clear since the pioneering papers, the approach to the main sequence and the central H-burning phase are influenced by the lack of CNO nuclei. In a more metal-rich IMS, in fact, the CNO cycle is the main energy supplier, and since it has a very strong dependence on the temperature, the burning remains confined within the innermost part of the star, and the convective core persists until the fuel is almost exhausted in the center. In the case of these PIMSs, on the contrary, the nuclear energy might be produced only via the PP chain (at least while the temperature remains below the threshold value for the activation of the $3\alpha$ reactions), and since it has a weaker dependence on the temperature, the whole structure is necessarily much hotter, and the active burning occurs in a more extended region around the center. The lack of a burning strongly concentrated in the center leads to the
Fig. 4.—Evolution with time of the central convective regions up to the end of the He burning for the computed models: H burning via the PP chains (first episode), H burning via the CNO cycle (second episode), and He burning (third episode).

following characteristics: (1) the convective core is much smaller than in more metal-rich stars of similar mass, and it even vanishes when the central mass fraction of H is still 0.5 and (2) in almost 80% of the total mass of the star, the He abundance increases. The lack of an extended convective core clearly influences the stellar track in the $T_C$-$o_C$ plane (see Fig. 2): in particular, since the PP chain never slows the gravitational contraction effectively, the central temperature and density continue to rise systematically until the $3\alpha$ reactions start up. The abundance of carbon produced in the center as a function of time is shown in Figure 5. When the $^{12}\text{C}$ mass fraction is about $10^{-10}$, the H burning switches to the CNO cycle. Then the local luminosity immediately increases, and a new convective core appears (see Fig. 4). The onset of the CNO burning induces the typical expansion of the central regions (see Fig. 2) and also clearly marks the evolutionary tracks (see Fig. 3).

At the end of the H burning, the central temperature is so large ($\sim 10^8$ K) that the He burning immediately follows. These stars spend their central He-burning lifetime at the blue side of the HR diagram, thus omitting the first dredge-up episode. For this reason, they enter the AGB phase with the original surface composition. As is usual at He ignition, a convective core develops, whose temporal evolution is shown in Figure 4. The H-burning shell, which forms immediately outside the H-exhausted core, is particularly hot.

The evolution of the temperature and that of the location (in mass coordinates) of the mesh where the energy generation rate of the H burning is maximum are shown in Figure 6 for the 7 $M_\odot$ model. Although the envelope is essentially lacking metals, the H-burning shell is mainly controlled by the CNO cycle. In fact, the temperature in the shell is large enough to allow for some carbon production via the $3\alpha$ reactions. As a consequence, carbon is partially converted into nitrogen within the shell. The resulting internal profiles of hydrogen, carbon, and nitrogen at the end of the central He burning are illustrated in Figure 7.

4. THE EARLY AGB PHASE

As is well known (see, for example, Becker & Iben 1979), during the early AGB phase, the H and the He burning are active in two separate shells. The energy generated by the He shell induces an expansion and a cooling of the outermost layers so that a convective envelope, which penetrates deeply within the star, develops. Though this is the first dredge-up episode for these stars, we will call it the second dredge-up because of the similarity with the one found in the more metal-rich models.
FIG. 6.—Evolution with time of the H-burning shell during the central He-burning phase for the $7 M_\odot$ model. Bottom: Position (in mass) where the nuclear energy generated by the H-burning shell is maximum. Top: Temperature at the same point.

The quantity (and the quality) of the matter dredged up to the surface differs significantly from that of more metal-rich models, and as we will see below, this causes the formation of two families of AGB stars. Let us note first that the amount of He dredged up by these PIMSs is much larger than that found in the more metal-rich AGB stars. This is due to the fact that since during most of the central hydrogen-burning phase the main energy source is the PP chain, He is produced in a wide region (in mass) of the stellar interior. The second important thing to note is that since the $3\alpha$ reactions are active up to the H-burning shell, primary carbon is dredged up. Some nitrogen also appears at the surface as a consequence of the mixing of material processed by the H-burning shell. Note that this is the only case in which the second dredge-up alters the total abundance, by number, of the CNO group. In the lower panel of Figure 8, we show the variations during the early AGB of the locations of the H- and He-burning shells (as identified by the points where the nuclear energy production is maximum) as well as the extension of the convective envelope for the $7 M_\odot$ (no extra mixing) model. The corresponding evolution of the surface abundances of C, N, O, and He are shown in the upper panel. In Table 2 we have summarized some properties of our models at the end of the early AGB phase, namely, in columns (1)–(7), the stellar mass, the assumed value of $\beta$ (see the definition of this parameter in § 2), the mass of the He core, and the surface mass fraction of $^4$He, $^{12}$C, $^{14}$N, and $^{16}$O, respectively. Note that the amount of CNO nuclei in the envelope of these stars strongly depends on the initial mass.

Chieffi & Tornambe (1984; see also Fujimoto et al. 1984) have shown that a minimum amount of CNO nuclei exists (roughly corresponding to a mass fraction of the order of $10^{-5}$) for which the H-burning shell is fully sustained by the CNO cycle. This means that the stars in which the CNO abundance in the envelope is raised above this threshold value by the second dredge-up will have a more or less standard H-burning shell, while the masses in which the amount of CNO catalysts remain below this threshold value are forced to raise the temperature in the H-burning region up to that typical of the He ignition. Also, in the latter case, the CNO cycle dominates the energy production in the H-burning shell, but its efficiency is controlled by the amount of carbon locally produced via the $3\alpha$ reactions. From the data reported in Table 2, it is evident that only for stellar masses larger than $6 M_\odot$ is this minimum amount of

FIG. 7.—C (solid line), N (dashed line), and H (dotted line) mass fraction profiles of the $7 M_\odot$ model at the end of the central He burning.

FIG. 8.—Early AGB phase of the $7 M_\odot$ model (no extra mixing). Top: evolution with time of the surface abundance (mass fraction) of C (long-dashed line), N (short-dashed line), O (dotted line), and He (solid line). Note that the He scale reads on the right y-axis. Bottom: evolution with time of the location (in mass coordinates) of the H- and He-burning shells. The dashed area shows the extension of the convective envelope.
CNO attained in the envelope at the end of the early AGB phase. In fact, in these models the expansion and the cooling induced by the He-burning shell promptly allow a deep penetration by the convective envelope throughout the intershell region (see the lower panel of Fig. 8). For models having lower masses, the inner edge of the convective envelope penetrates only slightly (or not at all) the H-He discontinuity, and the resulting amount of CNO nuclei is definitely lower than the threshold value.

The results shown in the first four rows of Table 2 refer to models obtained using the Schwarzschild criterion to fix the boundaries of the regions mixed by convection. As discussed in § 2, we have also performed some tests to investigate the effects of a possible extra mixing occurring below the border of the convective envelope. The results of these tests are given in the last three rows of Table 2. They indicate that such an extra mixing has only a negligible effect on the efficiency of the second dredge-up. A similar conclusion was previously obtained by Castellani et al. (1998) for more metal-rich stars.

Before closing this section, let us note that the 8 $M_\odot$ ($Z = 0$) model ignites carbon off-center during the early AGB. We followed part of this C burning up to the formation of an extended convective shell and found that the minimum mass that is able to ignite carbon in a mildly degenerate core (usually called $M_{\text{up}}$) is confined in the $7 \leq M_{\text{up}}/M_\odot \leq 8$ range for a generation of stars of zero initial metallicity, which confirms the previous result obtained by Tornambé & Chieffi (1986) and Cassisi & Castellani (1993).

### Table 2

| $M$ ($M_\odot$) | $\beta$ | $M_H$ ($M_\odot$) | $^{12}$C | $^{14}$N | $^{16}$O | Surface Mass Fraction |
|--------------|--------|-----------------|-------|-------|-------|---------------------|
| (1)          | (2)    | (3)             | (4)   | (5)   | (6)   | (7)                 |
| 4.0          | 0      | 0.8012          | 0.358 | 1.01 $\times 10^{-14}$ | 1.04 $\times 10^{-12}$ | 2.32 $\times 10^{-14}$ |
| 5.0          | 0      | 0.8861          | 0.365 | 9.47 $\times 10^{-10}$ | 4.29 $\times 10^{-10}$ | 4.99 $\times 10^{-11}$ |
| 6.0          | 0      | 0.9330          | 0.367 | 8.44 $\times 10^{-8}$ | 8.29 $\times 10^{-10}$ | 1.17 $\times 10^{-10}$ |
| 7.0          | 0.005  | 0.9875          | 0.369 | 2.08 $\times 10^{-6}$ | 1.59 $\times 10^{-9}$ | 2.88 $\times 10^{-9}$ |
| 7.0          | 0.01   | 0.9870          | 0.370 | 2.13 $\times 10^{-6}$ | 1.59 $\times 10^{-9}$ | 2.99 $\times 10^{-9}$ |
| 7.0          | 0.02   | 0.9867          | 0.370 | 2.16 $\times 10^{-6}$ | 1.59 $\times 10^{-9}$ | 3.04 $\times 10^{-9}$ |

5. THE ADVANCED AGB EVOLUTION

The early AGB ends when the He shell, progressively approaching the H-He discontinuity, loses its efficiency and allows the overlying layers to contract and reheat. As a consequence, the H-burning shell reactivates and begins to accumulate fresh He on the underlying He core. This is the beginning of the so-called thermally pulsing AGB (TPAGB) phase. When enough He is accumulated, the $3\alpha$ reactions start again at the base of the He-rich layer. For the sake of completeness (for a more complete description of the TPAGB stellar structures, see the review in Iben & Renzini 1983), let us briefly note that in stars with normal metallicity, the two shells (H and He) do not advance simultaneously in mass but, on the contrary, are active alternately. The H ignition is characterized by a rather strong thermonuclear runaway. The conditions of this He shell flash are somewhat different from those that lead to the He flash in a low-mass star at the tip of the red giant branch. The thermonuclear runaway, in the latter case, is induced by the strong degeneracy of the electron component of the stellar plasma, while in TPAGB stars, the He-rich layer is largely nondegenerate. If matter is nondegenerate, the tendency for temperature to increase because of the local release of thermonuclear energy normally is counterbalanced by an expansion caused by the temperature dependence of the pressure. In most cases, this pressure response rapidly quenches the heating, and a quiescent (self-regulated) burning takes place. However, in TPAGB models, the He ignition occurs in particular conditions (see Schwarzschild & Harm 1967), and the local temperature may rise for a certain time (how much mainly depends on the core mass) before the pressure response becomes effective. Since the $3\alpha$ reaction rate is strongly dependent on the temperature, the energy production rate of the He-burning shell rapidly increases up to $10^6$–$10^8 \ L_\odot$. The strength of the He flash depends on, among other things, the local values of temperature and density of the He-rich layer at the moment of the fuel ignition. For a given core mass, lower temperatures and larger densities imply higher He flash luminosities. It is important to note that these conditions are influenced by the rate at which the He-rich layer is accreted and, hence, by the properties of the H-burning shell. It is therefore clear that the amount of CNO catalysts in the envelope is a fundamental quantity that affects many properties of the TPAGB phase such as the duration of the interpulse ($\Delta t_{\text{ip}}$), the maximum $3\alpha$ luminosity, and the growth rate of the C-O core mass. For all of these reasons, one may expect a particular behavior of the $Z = 0$ TPAGB stars.

As already mentioned, Chieffi & Tornambé (1984) did not find thermal instabilities during the AGB phase of a 5 $M_\odot$ ($Z = 0$) stellar model. In fact, they showed that, owing to the lack of CNO nuclei, the temperature in the H-burning shell is so high that the $3\alpha$ reactions are active. Since the temperature at the base of the He-rich layer must be even higher, the two shells must be simultaneously active. In a companion paper, Fujimoto et al. (1984) showed that under this condition, the stability of the H-burning shell depends on the core mass: those stars having a core mass larger than a critical value ($\sim 0.73 \ M_\odot$) develop a stable He-burning shell, whereas for lower core masses, TPs occur. This prediction was confirmed by Chieffi & Tornambé: in their 5 $M_\odot$ model the two shells did advance (in mass) by consuming fuel at the same rate (at least within the phase that it was possible to compute at that time) so that a stable steady state was achieved during the AGB.

Concerning our set of PIMs, let us discuss separately, for the sake of clarity, the more massive models, in which the second dredge-up is able to raise the CNO mass fraction.
in the envelope above the threshold value for a self-sustained CNO burning (i.e., \( \sim 10^{-5} \)), and the less massive ones. It is important to note that all the present models develop a core mass at the end of the early AGB phase larger than the critical value obtained by Fujimoto et al. (1984).

5.1. The \( 6 \leq M/M_\odot < M_{\text{up}} \) Models

In these models, the CNO cycle in the H-burning shell reaches its full efficiency at a temperature lower than the value needed to activate the \( 3\beta \) reactions. In this case, the development of the thermally pulsing phase proceeds qualitatively as in the more metal-rich AGB stars. The luminosity contributions of both the He- and the He-burning shells in the case of the \( 7 \ M_\odot \) model and for two different values of the \( \beta \) parameter (see § 2 and below) are shown in Figures 9 and 10. In Figures 11 and 12, we show the evolution of the corresponding locations (in mass coordinates) of the two shells as well as the extension of the convective envelope. Moreover, some properties of these thermally pulsing models are listed in Tables 2 and 3, namely, in columns (1)-(8), the position of the H-burning shell at the onset of the TP (exactly when the He-burning luminosity becomes larger than the H-burning luminosity), the duration of the interpulse, the peak luminosity of the He flash, the temperature of the H-burning shell (exactly at the point where the nuclear energy production is maximum) at He rejoining, the density of the He-burning shell at He rejoining, and the surface mass fractions of \(^{12}\text{C}, \ ^{14}\text{N} \) and \(^{16}\text{O} \), respectively. First, we note that, owing to the lower amount of CNO nuclei in the envelope, the temperature of the H-burning shell is higher than that typically found in more metal-rich models (see, for example, Straniero et al. 2001). For example, in a \( 7 \ M_\odot \) model \((Z = 0.02)\), we found at the beginning of the tenth TP that the temperature of the H-burning shell is \( T_H = 7.93 \), while in the present \( Z = 0 \) model, even though the core mass is slightly lower, we find \( \log \frac{T}{T_\odot} = 8.10 \). As a consequence, the temperature of the He accumulated by the H-burning shell is higher, and in turn, a smaller density is required to start the \( 3\beta \) reactions. This occurrence becomes more evident if the properties of the He shell at the onset of a TP are compared to the corresponding ones in a model whose envelope has been enriched with a fresh CNO catalyst. Figure 13 shows the evolution of the temperature, the density, and the pressure of the He shell in the period between the ninth and the 12th TPs of the \( 7 \ M_\odot \) and \( \beta = 0.005 \) model. In this period, the surface CNO mass

\[ \log \frac{T}{T_\odot} = 8.10. \]

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| \( M_H \) \((M_\odot)\) | \( \Delta t_{\text{ip}} \) \((\times 10^3 \text{ yr})\) | \( \log \frac{L_{\text{He}}}{L_\odot} \) \((\log L_\odot)\) | \( \log T_H \) | \( \log \rho_{\text{He}} \) | \( \ ^{12}\text{C} \) | \( ^{14}\text{N} \) | \( ^{16}\text{O} \) |
|---|---|---|---|---|---|---|---|
| 0.9985 | ... | 4.84 | 8.070 | 3.530 | 2.229 \times 10^{-6} | 1.598 \times 10^{-9} | 3.173 \times 10^{-9} |
| 0.999 | ... | 5.02 | 8.076 | 3.563 | 2.229 \times 10^{-6} | 1.598 \times 10^{-9} | 3.173 \times 10^{-9} |
| 0.9903 | ... | 5.14 | 8.079 | 3.612 | 2.229 \times 10^{-6} | 1.602 \times 10^{-9} | 3.173 \times 10^{-9} |
| 0.9908 | ... | 5.25 | 8.085 | 3.616 | 2.229 \times 10^{-6} | 1.675 \times 10^{-9} | 3.174 \times 10^{-9} |
| 0.9913 | ... | 5.34 | 8.089 | 3.647 | 2.229 \times 10^{-6} | 1.771 \times 10^{-9} | 3.174 \times 10^{-9} |
| 0.9917 | ... | 5.43 | 8.092 | 3.672 | 2.229 \times 10^{-6} | 1.887 \times 10^{-9} | 3.175 \times 10^{-9} |
| 0.9922 | ... | 5.50 | 8.096 | 3.687 | 2.229 \times 10^{-6} | 2.043 \times 10^{-9} | 3.175 \times 10^{-9} |
| 0.9927 | ... | 5.56 | 8.098 | 3.715 | 2.229 \times 10^{-6} | 2.224 \times 10^{-9} | 3.176 \times 10^{-9} |
| 0.993 | ... | 5.62 | 8.101 | 3.715 | 2.229 \times 10^{-6} | 2.424 \times 10^{-9} | 3.177 \times 10^{-9} |
| 0.9936 | ... | 5.67 | 8.102 | 3.740 | 2.229 \times 10^{-6} | 2.642 \times 10^{-9} | 3.177 \times 10^{-9} |
| 0.994 | ... | 5.72 | 8.105 | 3.755 | 2.229 \times 10^{-6} | 2.884 \times 10^{-9} | 3.178 \times 10^{-9} |
| 0.9945 | ... | 5.76 | 8.106 | 3.764 | 2.227 \times 10^{-6} | 3.141 \times 10^{-9} | 3.179 \times 10^{-9} |
| 0.9949 | ... | 5.79 | 8.109 | 3.781 | 2.227 \times 10^{-6} | 3.414 \times 10^{-9} | 3.180 \times 10^{-9} |
| 0.9953 | ... | 6.29 | 8.074 | 3.885 | 4.156 \times 10^{-6} | 8.911 \times 10^{-6} | 2.801 \times 10^{-7} |
| 0.9957 | ... | 6.43 | 8.059 | 3.918 | 1.044 \times 10^{-5} | 1.643 \times 10^{-5} | 6.786 \times 10^{-7} |
| 0.996 | ... | 6.58 | 8.050 | 3.928 | 1.966 \times 10^{-5} | 2.534 \times 10^{-5} | 1.254 \times 10^{-6} |
| 0.9964 | ... | 6.67 | 8.044 | 3.953 | 3.150 \times 10^{-5} | 3.424 \times 10^{-5} | 1.958 \times 10^{-6} |
| 0.9967 | ... | 6.76 | 8.034 | 3.973 | 4.524 \times 10^{-5} | 4.328 \times 10^{-5} | 2.757 \times 10^{-6} |
| 0.997 | ... | 6.83 | 8.036 | 3.989 | 6.040 \times 10^{-5} | 5.222 \times 10^{-5} | 3.625 \times 10^{-6} |
| 0.9974 | ... | 6.88 | 8.034 | 4.004 | 7.672 \times 10^{-5} | 6.134 \times 10^{-5} | 4.560 \times 10^{-6} |
Fig. 10.—Evolution with time of the H- and He-burning luminosities during the TP-AGB phase in the case of the 7 M_☉ (β = 0.01) model. The initial time corresponds to the deepest penetration of the convective envelope after the second dredge-up.

fraction is approximately constant (∼2.2 × 10⁻⁶). The same quantities are given in Figure 14 but for the last computed TPs. In the latter case, the mass fraction of CNO in the envelope grows from about 6.4 × 10⁻⁵ to 1.33 × 10⁻⁴. Then the larger the CNO abundance in the envelope, the lower the temperature and the larger the density attained by the He-rich layer before the beginning of a TP. Note that in this case, the duration of the interpulse is longer because more time is needed to reach the conditions for the He ignition. The greater density of the He-rich material at the moment of the reignition clearly implies a stronger TP. This is evident from Figures 9 and 10 (see also col. [3] in Tables 3 and 4).

In spite of the lower intensity of the thermonuclear runaway, in the present Z = 0 models the He flashes also

Fig. 11.—Evolution with time of the location (in mass coordinates) of the H- and the He-burning shells of the same models as in Fig. 9. The dashed area shows the extension of the convective envelope.

Fig. 12.—Evolution with time of the location (in mass coordinates) of the H- and the He-burning shells of the same models as in Fig. 10. The dashed area shows the extension of the convective envelope.

Fig. 13.—Properties of the He-burning shell between the ninth and the 12th TPs of the 7 M_☉ (β = 0.005) model: density (top), temperature (middle), and pressure (bottom). The initial time corresponds to the deepest penetration of the convective envelope after the second dredge-up.
induce the formation of a convective region that extends after a few TPs over the whole intershell region. In such a way, the products of the He burning (essentially carbon) are distributed throughout the H-exhausted region. After each pulse, when the quiescent He burning takes place at the base of the He-rich layer, the inner border of the convective envelope moves inward and closely approaches the H-He discontinuity. For example, at the 10th TP of the 7 $M_\odot$ models, the inner border of the convective envelope penetrates the H-burning shell, reaching a layer that is only $\sim 10^{-5} M_\odot$ away from the C-enhanced region. Note that this situation also has been found in the case of a mixing strictly confined within the boundary defined by the Schwarzschild criterion. Becker & Iben (1979) were first to note that this is a very promising case in which an overshoot may be activated owing to the “finite and positive value of $\nabla_{rad} - \nabla_{ad}$ at the base of the formal convective zone.” They found that this happens when the amount of He-rich material that remains below the convective envelope is less than about 0.1 $M_\odot$. Such a condition is very soon fulfilled (after about 4 or 5 TPs) in our $Z = 0$ models that have a mass larger than 6 $M_\odot$. The resulting induced overshoot may be of great relevance for the possible dredge-up of carbon-rich material. This carbon dredge-up, which is commonly named the third dredge-up, has been invoked to explain the formation of the present generation of carbon stars (Iben 1975), and it is strongly supported by the observational evidence that there exists an evolutionary sequence of the various AGB components (from M to S and C stars) characterized by a progressive increase in the C/O ratio. As explained in § 2, we have activated this induced overshoot by artificially adding a small extra mixing below the base of the convective envelope. In order to check the dependence of the resulting dredge-up from the assumed strength of the extra mixing, we have computed various models by changing the free parameter $\beta$ in equation (5). In all the cases in which an extra mixing below the formal convective envelope has been included, a deep carbon dredge-up is obtained. Note the two different regimes that characterize the TPAGB evolution before and after the first C dredge-up episode (see Figs. 9–12 and Tables 3 and 4). For a few TPs, the C dredge-up induces a rapid H reignition (this is evident in the upper panels of Figs. 9 and 10). For $\beta \geq 0.02$, the first

\begin{table}[h]
\centering
\begin{tabular}{cccccccc}
\hline
&M$_I$ & $\Delta t_p$ & log $T_H^{max}$ & log $T_H$ & log $\rho_{H}$ & $^{12}$C & $^{14}$N & $^{16}$O \\
(M$_\odot$) & (10$^3$ yr) & (L$_\odot$) & & & & & & \\
\hline
0.9897 & ... & 5.04 & 8.076 & 3.572 & 2.229 $\times 10^{-6}$ & 1.598 $\times 10^{-9}$ & 3.173 $\times 10^{-9}$ & \\
0.9901 & 1.69 & 5.17 & 8.081 & 3.613 & 2.229 $\times 10^{-6}$ & 1.602 $\times 10^{-9}$ & 3.173 $\times 10^{-9}$ & \\
0.9906 & 1.71 & 5.28 & 8.086 & 3.628 & 2.229 $\times 10^{-6}$ & 1.674 $\times 10^{-9}$ & 3.174 $\times 10^{-9}$ & \\
0.9911 & 1.73 & 5.38 & 8.090 & 3.662 & 2.229 $\times 10^{-6}$ & 1.773 $\times 10^{-9}$ & 3.174 $\times 10^{-9}$ & \\
0.9916 & 1.75 & 5.46 & 8.092 & 3.692 & 2.229 $\times 10^{-6}$ & 1.896 $\times 10^{-9}$ & 3.175 $\times 10^{-9}$ & \\
0.9920 & 1.74 & 5.54 & 8.095 & 3.693 & 2.229 $\times 10^{-6}$ & 2.056 $\times 10^{-9}$ & 3.175 $\times 10^{-9}$ & \\
0.9925 & 1.74 & 5.60 & 8.098 & 3.716 & 2.228 $\times 10^{-6}$ & 2.250 $\times 10^{-9}$ & 3.176 $\times 10^{-9}$ & \\
0.9930 & 1.73 & 5.66 & 8.102 & 3.745 & 2.228 $\times 10^{-6}$ & 2.461 $\times 10^{-9}$ & 3.177 $\times 10^{-9}$ & \\
0.9934 & 1.73 & 5.71 & 8.104 & 3.753 & 2.228 $\times 10^{-6}$ & 2.689 $\times 10^{-9}$ & 3.177 $\times 10^{-9}$ & \\
0.9939 & 1.73 & 5.76 & 8.105 & 3.774 & 2.228 $\times 10^{-6}$ & 2.947 $\times 10^{-9}$ & 3.178 $\times 10^{-9}$ & \\
0.9943 & 1.72 & 5.81 & 8.107 & 3.778 & 2.227 $\times 10^{-6}$ & 3.222 $\times 10^{-9}$ & 3.179 $\times 10^{-9}$ & \\
0.9944 & 3.15 & 6.73 & 8.037 & 3.934 & 1.370 $\times 10^{-5}$ & 6.670 $\times 10^{-5}$ & 5.947 $\times 10^{-6}$ & \\
0.9947 & 2.92 & 6.81 & 8.035 & 3.968 & 2.495 $\times 10^{-5}$ & 9.167 $\times 10^{-5}$ & 6.502 $\times 10^{-6}$ & \\
0.9948 & 2.98 & 6.99 & 8.032 & 3.995 & 3.971 $\times 10^{-5}$ & 1.259 $\times 10^{-4}$ & 7.671 $\times 10^{-6}$ & \\
0.9951 & 2.84 & 7.02 & 8.029 & 4.008 & 5.542 $\times 10^{-5}$ & 1.525 $\times 10^{-4}$ & 8.448 $\times 10^{-6}$ & \\
0.9951 & 2.78 & 7.17 & 8.025 & 4.041 & 7.347 $\times 10^{-5}$ & 1.949 $\times 10^{-4}$ & 1.081 $\times 10^{-4}$ & \\
0.9954 & 2.48 & 7.20 & 8.026 & 4.019 & 8.965 $\times 10^{-5}$ & 2.145 $\times 10^{-4}$ & 1.143 $\times 10^{-4}$ & \\
0.9956 & 2.47 & 7.22 & 8.025 & 4.022 & 1.109 $\times 10^{-4}$ & 2.361 $\times 10^{-4}$ & 1.247 $\times 10^{-4}$ & \\
\hline
\end{tabular}
\caption{Selected properties of the 7 $M_\odot$ TPAGB models ($\beta = 0.01$)}
\end{table}
H thermonuclear runaway is particularly strong (the peak luminosity is larger than $5 \times 10^{6} L_{\odot}$). To follow this phase, our adaptive mesh algorithm puts more than 5000 mesh points in the convective envelope, with 3000 just in the innermost 0.1 $M_{\odot}$. In this case, the mixing algorithm described in § 2 consumes a huge amount of computer time (about 3 days to compute one dredge-up episode on a 700 MHz workstation). For this reason, we have followed only two C dredge-up episodes in the $\beta = 0.02$ case and just one in the $\beta = 0.04$ case. Note that the total amount of CNO dredged up in the first episodes of C ingestion in the $\beta = 0.02$ case (about $5 \times 10^{-4} M_{\odot}$) is equivalent to that cumulatively ingested during five dredge-up episodes in the $\beta = 0.005$ case. It is clear that, owing to the increase of the CNO in the envelope, the temperature of the H-burning shell and the strength of this H flash are progressively reduced (see col. [4] of Tables 3 and 4 and Figs. 9 and 10) until this peculiarity of the $Z = 0$ TPAGB models also disappears. Let us finally note that Herwig (2000) did not find a clear dependence of the dredge-up efficiency on the extra mixing efficiency. Perhaps this discrepancy may be due to the different chemical composition of the Herwig models (namely, $Z = 0.02$). Nevertheless, his tests were made by changing the parameter that controls the extra mixing for just one TP. We suspect that a more evident variation of the dredge-up should be found if the AGB evolution is followed for several TPs (as we actually do in the present paper) under different assumption for the extra mixing efficiency.

In any case, the great relevance of the dredge-up on our capability to predict the AGB properties demands a deeper analysis of this question, which is evidently beyond the scope of the present paper.

5.2. The $4 < M/M_{\odot} < 6$ Models

The scenario described above holds for the more massive PIMs, for which the CNO abundance in the envelope is raised by the second dredge-up above the threshold value that allows the full efficiency of the H-burning shell without the requirement of additional CNO catalysts coming from a local production of carbon. This condition implies the formation of fairly normal AGB stars. On the contrary, the behavior of the less massive $Z = 0$ stars is rather peculiar.

Figure 15 shows for a $4 M_{\odot}$ ($\beta = 0.01$) model the H and He luminosities as a function of time, while Figure 16 shows the locations (in mass coordinates) of the He- and the H-burning shells and the extension of the convective envelope as a function of time. Table 5 lists some properties of the same model.

The first thing worth noting is that, though the CNO mass fraction in the envelope is lower than $10^{-7}$ and the core mass is larger than the critical upper limit found by Fujimoto et al. (1984), these AGB models experience thermal instabilities. The H burning is indeed regulated by the carbon locally produced via the $3\alpha$ reactions, but the coupling of the H and He burning is so weak that they do not advance (in mass) at the same rate. As a consequence,

### Table 5

**SELECTED PROPERTIES OF THE 4 $M_{\odot}$ TPAGB MODELS ($\beta = 0.01$)**

| $M_{\odot}$ ($M_{\odot}$) | $\Delta t_{\text{d}}$ (10$^3$ yr) | $\log L_{\text{meq}}$ ($L_{\odot}$) | $\log T_{\text{H}}$ | $\log n_{\text{H}}$ | $^{12}\text{C}$ | $^{14}\text{N}$ | $^{16}\text{O}$ |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0.8337........ 0.8347...... 0.8352...... 0.8365...... 0.8379...... 0.8397...... 0.8415...... 0.8432...... 0.8450...... 0.8467...... 0.8484...... 0.8498...... 0.8508...... 0.8518...... 0.8527...... 0.8536...... 0.8545...... 0.8554...... 0.8562...... 0.8570...... 0.8578...... 0.8586...... 0.8594...... 0.8602...... 0.8610...... 0.8617...... 0.8625...... 0.8632...... 0.8639...... 0.8647...... | 7.73 7.65 7.29 8.78 9.18 9.01 8.80 8.58 8.36 8.15 9.00 9.09 9.10 9.08 9.05 9.00 8.95 8.90 8.86 8.79 8.73 8.65 8.58 8.49 8.40 8.31 8.23 8.12 8.04 | 4.80 5.48 6.09 8.63 6.41 6.23 6.59 6.65 6.71 6.75 7.19 7.37 7.49 7.59 7.67 7.74 7.80 7.86 7.90 7.94 7.98 8.01 8.04 | 8.104 8.056 8.097 8.000 8.005 8.009 8.015 8.016 8.021 8.023 7.977 7.965 7.957 7.952 7.949 7.947 7.943 7.944 7.942 7.941 7.939 7.938 7.936 7.937 7.935 7.936 | 3.637 3.744 3.668 3.909 3.939 3.963 3.979 3.994 4.004 4.011 4.127 4.166 4.194 4.216 4.235 4.251 4.266 4.278 4.290 4.299 4.306 4.314 4.322 4.326 4.333 4.337 4.342 4.344 4.349 | 1.008 $\times 10^{-14}$ 1.008 $\times 10^{-14}$ 2.838 $\times 10^{-13}$ 3.790 $\times 10^{-6}$ 3.790 $\times 10^{-6}$ 3.789 $\times 10^{-6}$ 3.788 $\times 10^{-6}$ 3.788 $\times 10^{-6}$ 3.788 $\times 10^{-6}$ 3.787 $\times 10^{-6}$ 3.787 $\times 10^{-6}$ 3.787 $\times 10^{-6}$ 3.787 $\times 10^{-6}$ 3.787 $\times 10^{-6}$ 3.786 $\times 10^{-6}$ 3.786 $\times 10^{-6}$ 3.786 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ 3.785 $\times 10^{-6}$ | 1.043 $\times 10^{-12}$ 1.043 $\times 10^{-12}$ 1.371 $\times 10^{-11}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$ 8.161 $\times 10^{-7}$
small-amplitude TPs take place. Owing to the weakness of these He-shell flashes, the inner border of the convective envelope does not penetrate the He-rich layer so that the induced overshoot is probably not activated.

The second, very important occurrence is that since the first TP, the H shell becomes unstable, and a convective zone develops (hydrogen convective episode: HCE) whose maximum extension is attained just after the usual He convective episode (HeCE) starts to shrink and just before the convective envelope reaches its maximum inward penetration. Let us stress that these new convective episodes are a systematic property of these $Z = 0$ AGB models with masses ranging between 4 and 6 $M_\odot$. The base of this HCE closely approaches the border of the carbon-enriched zone left by the HeCE in the intershell. This condition is similar to the one already encountered when the convective envelope penetrates the H shell (see previous subsection). In fact, owing to the greater opacity in the H-rich material, a discontinuity of the radiative gradient forms at the interface between the base of the HCE and the He-rich layer below. This condition is favorable to the induced overshoot that, in our opinion, would easily extend the base of the HCE down to the C-enhanced layer. In such a case, the carbon ingestion would induce a sudden thermonuclear runaway of the H-burning shell.

The consequences of this mixing event are illustrated in Figures 16 and 17. In this particular case, the two shells move simultaneously outward for some time, but when the innermost burning front gets closer to the H-He discontinuity, its rate relaxes. From now on, weak TPs take place, characterized by the two convective episodes described above: the standard HeCE and the unusual HCE. During the fifth TP, the HCE extends inward and overlaps the region previously mixed by the He convective shell. This situation is illustrated in Figure 17. Then carbon is dredged up, and immediately an H flash occurs. The energy released by this second (H) flash makes an important contribution to the expansion of the outermost layers started by the first (He) flash. Later on, during the postflash period, the convective envelope penetrates the H-burning region. Thus, the surface abundances of C and N are significantly enhanced (the CNO mass fraction in the envelope becomes $\sim 4 \times 10^{-6}$). This is an irreversible phenomenon since the amount of C dredged up is now so large that the star behaves, from now on, like the more massive ones. In par-
ticular, after a few more TPs, the standard third dredge-up takes place.

Note that in a model computed with a mixing strictly confined within the boundaries formally defined by the Schwarzschild criterion, the base of the HCE comes very close to the ex-convective shell generated by the TP, and the occurrence of the carbon dredge-up is arbitrarily dependent on the accuracy of the computation. Our experiments show that by changing the number of time steps and/or the spatial resolution, the HCE may or may not penetrate the C-rich region below it. However, once a small extra mixing is included (even just by putting $\beta = 0.001$ in eq. [5]), this uncertainty disappears, and the carbon dredge-up at the base of the HCE is systematically obtained after a few TPs. In the particular case illustrated in Figures 15, 16, and 17, a first close approach of the HCE to the C-rich interstellar region has been obtained after just three TPs (see col. [6] of Table 5), but only the fifth one is capable of turning off the AGB evolution of this model. In our opinion, the case in which the HCE does not enter the carbon-rich layers is unrealistic since it comes, in all cases, so close to the C-rich region that any small instability would lead to the irreversible and definitive mixing that brings the AGB properties of these models toward those of the more massive ones.

6. CONCLUSIONS

The occurrence of TPs in primordial AGB stars with masses ranging between 4 and 8 $M_\odot$ implies a major revision of the early nucleosynthesis scenario. We have shown that models with $M \geq 6 M_\odot$ develop a deep convective envelope that penetrates the H shell and approaches the region previously enriched with the carbon produced during the He shell flash. Moreover, we have demonstrated that in this condition, even a small extra mixing occurring below the base of the convective envelope induces a significant carbon dredge-up. We note that the abundances measured in the present generation of AGB stars demand the occurrence of the third dredge-up (see, for example, the review of Iben & Renzini 1983 and references therein). Then the fresh carbon engulfed in the envelope is partially converted into nitrogen by the CNO burning occurring at the base of the convective region. Thus, these AGB stars develop C- and N-rich envelopes.

Our models with masses ranging between 4 and 6 $M_\odot$ start the TP-AGB phase with weak He shell flashes so that the base of the convective envelope remains far from the H-burning shell. We show, however, that immediately after a TP, the H-burning shell becomes convectively unstable, and the inner border of this convective region (HCE) closely approaches the C-enriched zone. Also, in this case, we believe an induced overshoot is likely to occur. In our models, which include a small additional mixing at the base of the HCE, a thermonuclear runaway is activated by the carbon ingested by the HCE. The resulting expansion of the outermost layers leads to the deep penetration of the convective envelope that overlaps the whole region previously mixed by the HCE. Then the abundance of CNO in the envelope rises enough to allow a normal AGB evolution from now on. In particular, after a few TPs, the third dredge-up may further increase the carbon abundance in the envelope so that, once again, it is partially converted into nitrogen by the CNO burning occurring at the base of the external convective region.

The precise evaluation of the amount of nitrogen and carbon produced by these primordial AGB stars depends on the value of the parameter used to describe the strength of the extra mixing ($\beta$ in our eq. [5]). The uncertainty concerning the mass-loss rate is a further (important) limit to our comprehension of the nucleosynthesis contribution from these stars. Any attempt to calibrate these free parameters encounters the problem of the scarcity of observable constraints for Population III stars. Note, however, that though a quantitative derivation of the nucleosynthesis products requires a precise determination of the efficiency of mass loss and convective mixing, the new evolutionary scenario for PIMSs emerging from our models is not substantially affected by the above uncertainty.

The imprint of this nucleosynthesis could be sought in the more metal-deficient stars presently observed in our Galaxy and in the high-redshift Ly$\alpha$ systems. Observations show solar and relatively flat [C/Fe] and [N/Fe] ratios with metallicity that indicates a primary origin of both elements (see, for example, McWilliam 1997 and references therein). The nucleosynthesis of massive stars is sufficient to explain the primary carbon observed in metal-poor halo dwarfs, but it cannot account for the primary nitrogen (Timmes et al. 1995). In a companion paper (Abia et al. 2001), we investigate the implications of the new models for Population III stars on the chemical evolution of the early Galaxy. We show that the PIMS models presented here coupled with our new models for massive Z = 0 stars (Limongi, Straniero, & Chieffi 2001, in preparation) allow us to reproduce the primary behavior of both C and N down to the lowest observed metallicity. For example, by using the PIMSs computed assuming $\beta = 0.01$, we obtain $-0.3 \leq [\text{N/Fe}] \leq 1.0$, the precise value depending on the assumed IMF and on the iron amount ejected by $Z = 0$ Population III supernovae (see also Domínguez et al. 2001).

Recently, it has been pointed out that a considerable fraction (20%–25%) of the most metal-poor stars are carbon-rich, [C/Fe] $\geq 1$ (Rossy, Beers, & Sneden 1999). In most cases these extremely metal-poor carbon stars are also nitrogen-enhanced (Norris, Ryan, & Beers 1997; Bonifacio et al. 1998; Hill et al. 2000). Fujimoto, Ikeda, & Iben (2000) suggest that some of these carbon stars may be the product of mass transfer from an AGB companion of mass lower than 3 $M_\odot$ in close binary systems. Our computations show that in the case of an intermediate-mass companion, the surface composition of the secondary star after the mass transfer episode should also be CN enhanced.

Another interesting product of the nucleosynthesis occurring in these PIMS models is Li. When the temperature at the base of the convective envelope is higher than $(20-30) \times 10^6$ K, beryllium is efficiently produced through the $^3\text{He}(^4\text{He},\gamma)^7\text{Be}$ reaction. Then $^7\text{Li}$ is synthesized via the electron capture of $^7\text{Be}$, whose terrestrial half-life is about 53 days. If this electron capture occurs at the base of the envelope, the resulting Li would be immediately destroyed as a consequence of the $^7\text{Li}(p,\gamma)^8\text{Be}$ reaction. However, the convective motion might be so fast that most of the beryllium produced at the base of the envelope is redistributed in the outermost layers of the star, where the temperature is lower and Li is preserved. This is the well-known Cameron-Fowler mechanism (Cameron 1955). In our models of PIMSs, such a process is particularly efficient during the TP-AGB phase so that the mass fraction of Li in the envelope increases to a value comparable with the one
expected from the Big Bang nucleosynthesis (see, for example, Walker et al. 1991).

Let us finally mention an interesting nucleosynthesis channel that may be activated in zero-metal AGB stars by the production of free neutrons. As is well known (see, for example, Busso, Gallino, & Wasserburg 1999), the main component of the cosmic s-elements has been synthesized by neutron captures on iron seeds occurring in TP-AGB stars. Being deprived of iron, PIMMS cannot contribute to this nucleosynthesis. However, if the two major neutron sources, namely, the $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ and the $^{12}\text{C}(\alpha,n)^{16}\text{O}$ reactions, are activated, they might contribute to the synthesis of lighter elements. We found that the $^{14}\text{N}$ left by the advancing H-burning shell is fully converted into $^{22}\text{Ne}$ during the TP. Obviously, since this nitrogen depends on the amount of CNO in the envelope, the amount of $^{22}\text{Ne}$ in the convective region generated by the He flash is rather small at the beginning of the TP phase. However, after some dredge-up episodes, this value increases significantly. As an example, in the last computed TP of the 4 $M_\odot$ ($\beta = 0.01$) model, the mass fraction of $^{22}\text{Ne}$ in the internshell region is about $5 \times 10^{-3}$, which is about half the value typically found in AGB models of solar chemical composition. In our $Z = 0$ models, the maximum temperature at the base of the convective zone generated by the He shell flash ranges between $310 \times 10^6$ and $350 \times 10^6$ K so that the $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ is definitely involved. Since the amount of $^{22}\text{Ne}$ in the internshell increases with time and the temperature at the bottom of the convective shell also increases, the neutron production and the related nucleosynthesis would be particularly efficient in the final part of the TP-AGB phase of these PIMMS. For these reasons, many more models are needed to calculate this neutron capture synthesis. Nevertheless, the release of free neutrons (at least from the $^{22}\text{Ne}$ source) in an environment lacking iron seeds should activate a particular nucleosynthesis channel whose product could be used to trace the pollution caused by the missing Population III in the primordial Galactic material.

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