THE MAD ERA:
A POSSIBLE NEW RESOLUTION TO
THE HORIZON, FLATNESS, AND MONOPOLE PROBLEMS

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submitted to Physical Review Letters
November 6, 1992

ABSTRACT

A cosmology with a dynamical Planck mass \(m_{\text{pl}}\) is shown to solve the horizon and monopole problems (and possibly flatness) if there is an early MAD (modified aging) era where the universe becomes older than in the standard model as a result of a large \(m_{\text{pl}}\): the causality condition is \(m_{\text{pl}}(T_c)/m_{\text{pl}}(T_o) \gtrsim T_c/T_o\) (\(T_c\) is some high temperature while \(T_o = 2.74 K\).) Unlike inflation, there is no period of vacuum domination nor any entropy violation. We study: a) bare scalar theories of gravity, b) self-interacting models, and c) bare theories with a phase transition in the matter sector.
The standard Hot Big Bang model is unable to explain the smoothness or flatness of the observed universe. The inflationary model\(^1\) solves the horizon, flatness, and monopole problems with an era of false vacuum domination during which the scale factor \(R\) grows superluminally. During inflation the temperature of the universe drops as \(T \propto R^{-1}\). Therefore, the next crucial ingredient for a successful inflationary model is a period of entropy violation which reheats the universe to a high \(T\).

We propose that a cosmology with a dynamical Planck mass can resolve the horizon and monopole problems without a period of vacuum domination. [We are still in the process of investigating how well our model addresses the flatness problem.] Further, entropy production is not required. We have considered here general scalar theories of gravity in which the Planck mass is some function of a scalar field, \(m_{pl} \propto f(\psi)\). However, we stress that the resolution we propose to these cosmological problems is more generally a feature of a dynamical Planck mass.

We consider a cosmology where the energy density of the universe begins radiation dominated and then goes over to a period of matter domination as in the standard model. In our model, in an early stage of the radiation dominated era, the Planck mass is very large. Thus, the universe is older at a given temperature than in a standard Hot Big Bang model. We call this epoch of ‘modified aging’ the MAD era. Larger regions of space come into causal contact at some high \(T\) and thereby become Smoot without violating causality.

The observable universe today (subscript \(o\)) can fit inside an early causally connected region (subscript \(c\)) if
\[
\frac{1}{R_c H_c} \geq \frac{1}{H_o R_o},
\]
where \(H\) is the Hubble parameter. [Inflation satisfies this condition by having a superluminal period of growth of the scale factor so that \(R_o / R_c\) is very large, followed by entropy violation]. In our model, this condition is satisfied by having \(H_c\) much smaller than in the standard model, i.e. \(t_c\) is very large. Extensions of Einstein gravity with a variable Planck mass \(m_{pl} = m_{pl}(t)\) can achieve this extra aging by having a large value of the Planck mass.
early on during the MAD era. For $H^2 = (8\pi \rho/3)m_{pl}^{-2}$ (as shown below, this is the correct equation of motion in the ‘slowly rolling’ limit where the time variation of the Planck mass is sufficiently small), the causality requirement becomes roughly

$$\frac{m_{pl}(T_c)}{M_o} \gtrsim \frac{T_c}{T_o},$$

where $M_o = m_{pl}(T_o) = 10^{19}$ GeV and $T_o = 2.74$K. For $T_c = 3 \times 10^{16}$ GeV, e.g., this requirement becomes $m_{pl}(T_c)/M_o \geq 10^{28}$. We discuss below three alternate theories of gravity which all satisfy causality in this way.

Subsequent to $T_c$, $m_{pl}$ must move down to the value $M_o$ by the time of nucleosynthesis. Case a) considers scalar theories of gravity without a potential for the scalar field. For pure Brans-Dicke gravity\(^2\), the Planck mass cannot drop to the required value in time; for models where the Planck mass is a more complicated function of a scalar field, our preliminary analysis indicates that $M_o$ may be obtained. We investigated various additional mechanisms to drive the Planck mass down. We considered case b), the addition of a potential for the Planck mass, and case c), scalar theories in the presence of a phase transition in the matter background. We found that all the constraints on a potential in case b) could not be met without inputing small parameters into the potential, creating a cosmological constant, or allowing the Planck mass to be away from the minimum of the potential today. Case c) seems to be a viable solution to causality which produces $M_o$ today; however, our analysis of this case is preliminary. Cases a) and b) have been worked out in detail and are discussed in two other papers\(^3,4\).

Extended\(^5\), hyperextended\(^6\), and induced gravity\(^7\) inflation use modified gravity as well. However, they differ from our work in that they require a vacuum dominated epoch and entropy violation, and extended models require an additional scalar field as inflaton. Since these models also need an additional mechanism, such as a potential for the dynamical $m_{pl}$, to drive it down to $M_o$ by today, the difficulties we illustrate in case b) will apply to these inflationary models as well.
**Action.** In scalar theories of gravity, such as those proposed by Brans and Dicke\(^2\) and studied by Bergmann\(^8\) and by Wagoner\(^9\), the Planck mass is determined dynamically by the expectation value of \(\Phi\). The action is

\[
A = \int d^4x \sqrt{-g} \left[ -\frac{\Phi(\psi)}{16\pi} R - \frac{\omega}{\Phi} \frac{\partial \mu \Phi \partial \rho \Phi}{16\pi} - V(\Phi) + \mathcal{L}_m \right],
\]

where \(\omega = 8\pi \frac{\Phi}{(\partial \Phi / \partial \psi)^2}\), we used the metric convention \((-, +, +, +)\), \(\mathcal{L}_m\) is the Lagrangian density for all the matter fields excluding the field \(\psi\), and \(V[\Phi(\psi)]\) is the potential for the field \(\psi\). Stationarizing this action in a Robertson-Walker metric gives the equations of motion for the scale factor of the universe \(R(t)\) and for \(\Phi(t)\),

\[
\ddot{\Phi} + 3H \dot{\Phi} = \frac{8\pi (\rho - 3p)}{3 + 2\omega} - \frac{\partial U}{\partial \Phi} - \frac{\partial \omega / \partial \Phi}{3 + 2\omega} \dot{\Phi}^2
\]

\[
H^2 + \frac{\kappa}{R^2} = \frac{8\pi (\rho + V)}{3\Phi} - \frac{\dot{\Phi}}{\Phi} H + \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\Phi} \right)^2
\]

where

\[
\frac{\partial U}{\partial \Phi} = \frac{16\pi}{3 + 2\omega} \left[ \Phi \frac{\partial V}{\partial \Phi} - 2V \right];
\]

\(U\) effectively acts as a potential term in the equation of motion for \(\Phi\). \(H = \dot{R}/R\) is the Hubble constant, while \(\rho\) is the energy density and \(p\) is the pressure in all fields excluding the \(\psi\) field. The entropy per comoving volume in ordinary matter, \(S = (\rho + p)V/T\), is conserved. We define \(\bar{S} = R^3T^3\) where \(S \simeq \bar{S}(4/3)(\pi^2/30)g_\ast\) and \(g_\ast(t)\) is the number of relativistic degrees of freedom in equilibrium at time \(t\).

**Case (a) Massless Scalar Theory:** \(V(\Phi) = 0\). Here we consider a scalar field \(\Phi = m_{pl}^2\) with \(V(\Phi) = 0\). During the radiation dominated phase, we take \(\rho - 3p = 0\). For a detailed presentation, see Ref. (3). There we considered two different forms of \(\Phi(\psi)\): i) the Brans-Dicke (BD) proposal of \(\Phi = (2\pi/\omega)\psi^2\) with \(\omega\) constant, and ii) general \(\Phi(\psi)\) with \(\omega\) not constant. We found that, no matter what the initial conditions for the BD field, it evolves quickly towards an asymptotic value which we call \(\Phi = \bar{m}_{pl}^2\). At this point the equations of motion reduce to those of an ordinary radiation dominated Einstein
cosmology with $M_o$, the usual Planck mass of $10^{19}$ GeV, replaced by $\tilde{m}_{pl}$. In particular, $R \propto \tilde{m}_{pl}^{-1/2}t^{1/2}$ and $H = 1/2t$.

We illustrate the derivation of these results briefly. The equation of motion (4) has solution $\dot{\Phi}R^3 = -C(1 + 2\omega/3)^{-1/2}$. $C$ is a constant of integration and can be positive, negative, or zero. Immediately we see that if $C \neq 0$, then $|\dot{\Phi}|$, which may initially have a large value, shrinks as $R$ grows. Eventually, $\Phi$ appears effectively constant and approaches $\tilde{\Phi}$. If $C > 0$, then $\dot{\Phi} < 0$; $\Phi$ starts out larger than $\tilde{\Phi}$ and approaches it from above. If $C < 0$, then $\dot{\Phi} > 0$; $\Phi$ starts out smaller than $\tilde{\Phi}$ and approaches it from below. If $C = 0$, then $\dot{\Phi} = 0$ and $\Phi = \tilde{\Phi}$ throughout radiation domination.

We found it easiest to parameterize $R$, $T$, and $H$ in terms of $\Phi = m_{pl}^2$, and then subsequently to solve for $\Phi(t)$. We present results here for the simplest case of pure BD gravity with spatial curvature $\kappa = 0$; more general results can be found in Ref. (3).

Integrating eqn. (5), we found $R(\Phi) = \frac{C}{2\bar{S}^{2/3}\gamma^{1/2}}\tilde{\Phi}^{-1/2}\exp(-\Theta/2\epsilon)\frac{1}{\sinh \Theta}$.

(7)

Here $\gamma(t) \equiv (8\pi^3/90)g_*(t)$, $\Theta \equiv \epsilon \ln(\tilde{\Phi}/\tilde{\Phi})$ and $\epsilon \equiv \pm(1 + 2\omega/3)^{1/2}/2$, where the +(-) in $\epsilon$ refers to $C > (<)0$. From adiabaticity, $T(\Phi) = \bar{S}^{1/3}/R$. We have $H(\Phi) = \dot{R}/R = (-C/R^4)(dR/d\Phi)$. Eqns. (4) and (5) and the conservation of matter equation determine $\Phi(t)$, $\rho(t)$, and $R(t)$ up to four constants of integration, which we can choose to be $\bar{S}$, $C$, $\Phi(t = 0)$, and $\tilde{\Phi}$. As $\Phi$ approaches $\tilde{\Phi}$, for $|\epsilon| > 1/2$ ($\omega > 0$), $R(\Phi)$ grows and thus $T(\Phi)$ drops adiabatically. In addition, the comoving horizon size grows, as does $H^{-1}R^{-1}$. The size of a causally connected region can grow large enough to resolve the horizon problem.

We can write $H_o = \alpha_o^{1/2}T_o^2/M_o$ with $\alpha_o = \gamma(t_o)\eta_o = 8\pi/3(\pi^2/30)g_*(t_o)\eta_o$ where $\eta_o \sim 10^4 - 10^5$. We can use adiabaticity, $RT = \bar{S}^{1/3} \propto (S/g_*)^{1/3}$, and our solutions to write the causality condition in eqn. (4) as

$$\frac{m_{pl}(T_c)}{T_c} \frac{2\epsilon}{\sinh \Theta_c + 2\epsilon \cosh \Theta_c} \geq \frac{\beta M_o}{T_o}$$

(8)
where \( \beta = \left( \gamma(t_c)/\alpha_o \right)^{1/2}(g_*(t_c)/g_*(t_o))^{-1/3} \). Although it is possible for the causality condition \( (8) \) to be satisfied while \( \Phi \) is still far from \( \tilde{\Phi} \), we find that the lowest possible value of \( \Phi^{1/2} \propto m_{pl}(t_c) \) that solves causality is given by \( \Phi_c \simeq \tilde{\Phi} \). For \( \Phi_c \approx \tilde{\Phi} \) \( (\Theta_c \approx 0) \), the causality condition becomes simply \( \frac{\tilde{m}_{pl}}{M_o} \gtrsim \beta \frac{T(\tilde{\Phi})}{T_o} \), where \( m_{pl}(t_c) \approx \tilde{m}_{pl} = \tilde{\Phi}^{1/2} \). We are free to specify the temperature at which we would like to resolve causality (the choice of \( T \) at which \( \Phi = \tilde{\Phi} \) is equivalent to making an appropriate choice for the ratio of arbitrary constants \( \tilde{S}/C \)). For \( T_c \simeq 250 \) GeV, e.g., condition \( (8) \) requires \( \tilde{m}_{pl} \geq 10^{13} M_o \).

We can verify that the resolution to the horizon problem is explained by an old universe. When \( \Phi \approx \tilde{\Phi} \), the universe evolves as an ordinary radiation dominated universe with \( M_o \) replaced by \( \tilde{m}_{pl} \). We can express the age of the universe in terms of \( t_{\text{einst}} \), the standard age in a cosmology described by Einstein gravity: \( t(\tilde{\Phi}) = t_{\text{einst}} (\tilde{m}_{pl}/M_o) \) at a given temperature. Since \( \tilde{m}_{pl} \gg M_o \), the universe is older than in the standard cosmology, e.g., at \( T_c = 3 \times 10^{16} \) GeV, \( t_{\text{einst}} \sim 10^{-40} \) sec while \( t(\tilde{\Phi}) \geq 10^{-12} \) sec.

As the universe cools below matter radiation equality, the nature of the solutions changes. Thus there is a built in off-switch to end the unusual radiation dominated behavior of \( R(\Phi), T(\Phi) \) and \( H(\Phi) \). The obvious difficulty with this resolution to the horizon problem is fixing the value of the Planck mass to be \( M_o \) by the time of nucleosynthesis. For pure BD, observations constrain the parameter \( \omega \geq 500 \). The rate at which \( \Phi \) changes is very suppressed for large \( \omega \). As an extreme example, for \( \omega = 500 \) and \( \tilde{\Phi}^{1/2} = 100 M_o \) at \( T_c \sim 1 \) eV, then today \( \Phi_{o}^{1/2} \geq 80 M_o \). For more general scalar-tensor theories with \( \omega \neq \text{constant} \), our preliminary analysis indicates that \( M_o \) may be obtained. Below, we discuss the possibilities of using a potential or an appropriate background matter (or vacuum) field to drive \( m_{pl} \) to \( M_o \) by nucleosynthesis.

**Case (b) Self Interacting Scalar Theory:** \( V(\Phi) \neq 0 \). A thorough treatment can be found in Ref. (4). We sketch here the difficulties encountered in simultaneously matching all the constraints on the model. We have only considered the slowly rolling limit where \( \dot{\Phi}/\Phi \ll H_R \), where \( H_R^2 = \left[ 8\pi (\rho + V)/3\Phi \right] \), so that \( H \simeq H_R \) and the causality
condition holds as in eqn. (2). In future work, it would be interesting to consider the opposite limit, where eqn. (2) would be modified.

There are two possible ways to satisfy the causality condition. First, for a scenario in which the potential is inconspicuous during the early radiation dominated era, the results of case a) are recovered. The sole purpose of the potential would be to push \( m_{pl} \) down to \( M_o \) after the causality condition was satisfied. Alternatively, one could imagine a potential with interactions large enough to thermalize a bath of \( \Phi \) particles and drive a phase transition. Then, ideally, the field could reside in the high-T minimum of the potential \( \Phi = \tilde{m}_{pl}^2 \) for \( T > T_c \) and quickly move to the low-T minimum of the potential \( \Phi = M_o^2 \) for \( T < T_c \). If \( \tilde{m}_{pl}/M_o \geq T_c/T_o \), then the causality condition would be satisfied.

In the cases we considered, the high-T minimum changes as \( T \) drops, and, unfortunately, the field does not stay in the minimum. However, in principle causality could still be solved as long as \( m_{pl}(T_c) = \tilde{m}_{pl} \) satisfies the above condition.

In either case, the model must satisfy the following list of constraints: 1) The cosmological constant today is below the observational limits, \( \Lambda_o = 8 \pi V(T_o, M_o^2)/M_o^2 < 10^{-122} M_o^2 \). 2) The Planck mass today is a minimum of \( U \), \( \frac{\partial U}{\partial \Phi}|_{M_o^2} = 0 \) and \( \frac{\partial^2 U}{\partial \Phi^2}|_{\Phi = M_o^2} \equiv m_{eff}^2 > 0 \). [Note that \( \Phi \) is driven to the minimum of \( U \) (not of \( V \)) in eqn. (3)]. 3) For Brans-Dicke like theories with \( \Phi = (2 \pi/\omega)\psi^2 \) and constant \( \omega \), time delay experiments require \( \omega \geq 500 \) if \( m_{eff} \leq 10^{-27} \text{GeV} \); these experiments place no bounds\(^{10} \) if \( m_{eff} \geq 10^{-27} \text{GeV} \). [Experimental bounds on the time variations of \( G \) require \( |\dot{G}/G| = |\dot{\Phi}/\Phi| \leq H_o \) and are automatically satisfied in the slow-roll limit]. 4) The universe is radiation dominated, not potential dominated, at high \( T \), \( V(\Phi) \lesssim \rho_{rad} \).

As an example, we consider the Brans-Dicke coupling \( \Phi = (2 \pi/\omega)\psi^2 \) with the potential \( V = \frac{\lambda}{4} \psi^4 - \frac{m^2}{2} \psi^2 + \delta \). To obtain numbers, as an example we take \( T_c = 3 \times 10^{16} \text{ GeV} \). Constraint 4) is most restrictive right at \( T = T_c \) and requires \( \lambda \omega^2 \leq 10^{-119} \). The only case that satisfies the combination of the first three of the constraints is \( V = \frac{\lambda \omega^2}{16 \pi^2} (\Phi - M_o^2)^2 \).

This potential satisfies \( \Lambda_o \propto V(\Phi = M_o^2) = 0 \). Using this potential, we can write constraint
4) in terms of $m_{\text{eff}}$ as 
\[(3 + 2\omega)(m_{\text{eff}}/M_o)^2 \leq 10^{-119}, \text{ or } m_{\text{eff}} \leq 10^{-41} \text{ GeV}; \] constraint 3) can then only be satisfied if $\omega \geq 500$. Constraint 4) then requires $\lambda \leq 10^{-122}$. With such small self-coupling the $\Phi$ field will not thermalize, there are no thermal corrections to the potential, and there is no phase transition. Satisfying time delay experiments as well as all the other constraints can only be accomplished with a potential that is very flat (and probably fine-tuned).

Alternatively, consider the case of $\omega \ll 1$. We find that, in general, all four constraints cannot be simultaneously satisfied (even if a linear term or a $\psi^6$ term is added to the potential). We cannot use the potential after $T_c$ to pin the field in the minimum in the presence of all four constraints. We must relax one of them. For example, the field could be moving slowly somewhere away from the minimum, i.e., constraint 2) does not hold. Then reasonable values of the other parameters ($\lambda = 1$) insist on a small value of $\omega \leq 10^{-60}$ to satisfy the constraints; i.e. gravity must deviate substantially from Einstein gravity. However, in this case, the potential is not really playing its intended role of pinning the Planck mass in its correct value today. In addition, compatibility of such a small value of $\omega$ with observational bounds on $\dot{G}/G$ depends on exactly which potential is chosen. Alternatively, there may be solutions to the cosmological constant problem which drive $\Lambda$ to zero without affecting the parameters in constraint 2); then one could relax constraint 1). Also, modifications to the model (e.g. $d\omega/d\Phi \neq 0$) change the constraints and may allow more freedom in the parameters. Generically, we expect the feature that survives will be substantial deviation from Einstein gravity, i.e., a small Brans-Dicke parameter.

If we allow the universe to become potential dominated at some point, we can relax constraint 4), and the parameters of the potential do not need to be small. It might be possible to construct a hybrid model combining some inflation with some MAD expansion.

Inflationary scenarios which use modified gravity, such as (hyper)-extended inflation, also need a potential to anchor $m_{pl}$ at $M_o$ today. There constraint 4) would be replaced by $V(\Phi) < V(\text{inflaton}) \sim \rho_{\text{rad}}(T_c)$. The problems outlined in this section would directly
apply to these models as well.

**Case (c) Phase Transitions in the matter sector.** If \( V(\Phi) = 0 \) then eqn. (4) becomes \( \ddot{\Phi} + f\dot{\Phi} = 8\pi(\rho - 3p)/(3 + 2\omega) \), where \( f \) is a friction term. All contributions to \( \rho - 3p \) will affect the dynamics of \( \Phi \). Any matter or vacuum energy with \( \rho - 3p < 0 \) on the right hand side of the equation will tend to drive \( \Phi \) to a smaller value; we will try to use this to drive \( \Phi \) to \( M_o \) after causality has been solved. Possibilities include i) a decaying negative vacuum energy and ii) a thermally corrected matter potential at temperatures above a phase transition. Case i) has a negative vacuum energy that decays in time, similar to the positive decaying vacuum energy considered previously\(^{11}\). Here we focus on case ii).

Consider the matter Lagrangian, \( \mathcal{L}_m = -(1/2)\partial_\mu \eta \partial^\mu \eta - V(\eta) \) where \( \eta \) is a scalar field and the bare, uncorrected potential is \( V(\eta) = (\lambda/4)(\eta^2 - \eta^2_{\text{min}})^2 \) with \( \eta_{\text{min}} = m/\sqrt{\lambda} \). With thermal corrections, for \( T > T_{\text{cr}} \), we have

\[
V(T, \eta) \simeq \frac{\lambda \eta^4}{4} + \frac{1}{2} \left( \frac{\lambda T^2}{4} - m^2 \right) \eta^2 + \frac{m^4}{4\lambda} - \frac{\pi^2}{90} T^4 - \frac{T^2 m^2}{24} .
\]  

(9)

At high \( T \) the minimum of the potential is at \( \eta_1 = 0 \). When \( T \) falls below \( T_{\text{cr}} = 2m/\sqrt{\lambda} \), then a new global minimum appears at \( \eta_2 = m/\sqrt{\lambda} \). We chose the potential so that \( V(\eta_2) = 0 \). Now, \( \rho - 3p = 4V - T(\partial V/\partial T) \). For \( T > T_{\text{cr}} \) (where \( \eta = 0 \)), we define \( -\alpha(T) \equiv \rho - 3p = -T^2 m^2/12 + m^4/\lambda \). Since this is negative, at high \( T \) the background matter potential will work to push \( m_{\text{pl}} \) to smaller values.

To illustrate we choose \( \Phi = \Phi_o + \xi \psi^2 \) so that \( \omega = \frac{4\pi \Phi_o}{\xi(\Phi_o - \Phi_o)} \). Eqn. (4) becomes

\[
\ddot{\Phi} + \left( 3H + \frac{\dot{\omega}}{2\omega + 3} \right) \dot{\Phi} = -\frac{8\pi}{(3 + 2\omega)} \alpha(T) .
\]

(10)

Imagine the scenario to proceed as follows: Very early on, the Planck mass evolves as described in case a) above. The high \( T \) contributions to the matter background will not strongly affect the \( \Phi \) evolution. Once \( \Phi \) approaches \( \Phi_o \) (and \( \dot{\Phi} \) becomes small), causality is solved. Since \( \dot{\Phi} \) becomes very small, subsequently the \( \alpha(T) \) term dominates and pushes \( \Phi \to \Phi_o \). Thus, the Planck mass is driven to \( m_{\text{pl}} = \Phi_o^{1/2} \equiv M_o \). At \( \Phi = \Phi_o \), \( \omega \to \infty \) and
the motion of $\Phi$ is effectively turned off. Further work on this proposed scenario is required to see if the Planck mass can indeed reach its present value prior to nucleosynthesis.

**The Flatness and Monopole Problems.** If there is a Grand Unified Epoch at temperature $T_G$, then magnetic monopoles are produced, typically one per horizon volume. In the standard cosmology, far too many are produced. If there is inflation at $T < T_G$, then the monopoles are inflated away. In our model of MAD Expansion, the monopole problem can be resolved as well. The number of monopoles in our observable universe today is given by the number of comoving horizon volumes at $T_G$ that would fit inside the comoving volume of our observable universe, \( N = \left( \frac{M_o}{m_{pl}(T_G)} \frac{T_G}{T_o} \right)^3 \). Thus, in our model, for \( m_{pl}(T_G) \gg M_o \) (a requirement similar to that for causality), the number of monopoles in our observable universe can be very small.

The universe can become flatter in a MAD cosmology. In the slow-roll limit where \( \dot{\Phi}/\Phi \) can be neglected, one can write \( \Omega = \frac{1}{1-x} \), where \( \Omega = \rho/\rho_c \), \( \rho_c = 1.88 \times 10^{-29} h_o^2 \) gm cm$^{-3}$, \( h_o = H_o/100 \) km s$^{-1}$ Mpc$^{-1}$, and \( x = \frac{\kappa}{8\pi G p/3} \). The observations that \( \Omega_o = O(1) \) would require \( \Omega(10^{-43} \text{ sec}) - 1 \simeq O(10^{-60}) \) in the standard Hot Big Band model. In our model, as the universe progresses from \( T_c \) to nucleosynthesis, \( m_{pl} \) changes from \( \tilde{m}_{pl} \) to \( M_o \). We can calculate the ratio \( \frac{x_{nuc}}{x_c} = \frac{\Phi_{nuc}}{\Phi_c} \frac{T_c^2}{T_{nuc}^2} \leq \frac{T_c^2}{\beta T_{nuc}^2} \simeq 10^{-17} \), where the second relation follows from eqn. (8). In the standard model, on the other hand, \( x \) would have grown by a factor \( (T_c/T_{nuc})^2 \), e.g., for \( T_c = 10^{16} \text{ GeV} \), by a factor \( 10^{55} \). Thus, our model assists the approach to flatness (\( \Omega \to 1 \)) by causing \( x \) to become many orders of magnitude smaller than in the standard model. However, because of the large early \( m_{pl} \) and thus small Planck time, \( \Omega \) would veer away from 1 very quickly; we are checking to see if this generates the same flatness problem as the standard model, only at higher temperatures.

**Conclusion.** In a cosmology with a large Planck mass, the universe grows older at a given high temperature than in a standard cosmology—old enough to explain how one end of our observable universe could have communicated with the other end if the Planck mass satisfies \( m_{pl}/M_o \gtrsim T_c/T_o \). This extra aging of the universe during the MAD era is
not in conflict with the observations of the age of the universe, which only place limits on the time elapsed since stars formed. We found that scalar theories coupled to gravity could slow the evolution of the universe so that the smoothness of our observable horizon volume is predicted. Additional mechanisms were proposed to anchor the Planck mass at today’s value by nucleosynthesis.

Acknowledgements. We thank F. Adams, A. Guth and H. Zaglauer for helpful conversations. We thank the ITP at U.C.S.B., the Max Planck Institut in Munich and the Aspen Center for Physics for hospitality. We acknowledge support from NSF Grant PHY-92-96020, a Sloan Foundation fellowship, and an NSF Presidential Young Investigator award.

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