Higgs mass and unnatural supersymmetry

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Talk structure

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## The Minimal Supersymmetric Standard Model

### Chiral supermultiplets

| Name                  | Symbol | spin 0 | spin 1/2 | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|--------|--------|----------|-----------------------------|
| squarks, quarks       | Q      | $(\tilde{u}_L, \tilde{d}_L)$ | $(u_L, d_L)$ | $(3, 2, \frac{1}{6})$ |
| $(\times 3$ families) |        |        |          |                             |
| sleptons, leptons     | L      | $(\tilde{\nu}_L, \tilde{e}_L)$ | $(\nu, e_L)$ | $(1, 2, -\frac{1}{2})$ |
| $(\times 3$ families) |        |        |          |                             |
| Higgses, Higgsinos    | $H_u$  | $(H_u^+, H_u^0)$ | $(\tilde{H}_u^+, \tilde{H}_u^0)$ | $(1, 2, \frac{1}{2})$ |
|                       | $H_d$  | $(H_d^0, H_d^-)$ | $(\tilde{H}_d^0, \tilde{H}_d^-)$ | $(1, 2, -\frac{1}{2})$ |

### Gauge supermultiplets

| Name                  | spin 1/2 | spin 1 | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|----------|--------|-----------------------------|
| gluino, gluon         | $\tilde{g}$ | $g$   | $(8, 1, 0)$ |
| winos, W bosons       | $\tilde{W}^\pm$ | $W^\pm$ | $(1, 3, 0)$ |
| bino, B boson         | $\tilde{B}^0$ | $B^0$ | $(1, 1, 0)$ |
# Introduction

Where is SUSY now?

Unnatural SUSY EFT matching to the SM Phenomenological signatures THDM Conclusions

## ATLAS SUSY Searches

### Status: March 2016

| Model | $\ell$, $\mu$, $\tau$ | Jets | $E_{T}^{\text{miss}}$ | Mass limit | $\sqrt{s} = 7, 8 \text{ TeV}$ | $\sqrt{s} = 13 \text{ TeV}$ | Reference |
|-------|----------------------|------|------------------|------------|-----------------|-----------------|-----------|
| MSUGRA/CMSSM | 0-3 $c_{\mu}/1-2\tau$ | 2-10 jets | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ | 0 | 2-6 jets | Yes | 3.2 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (compressed) | mono-jet | 1-3 jets | Yes | 3.2 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-SP) | 0 | 2-3 jets | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-Z) | 0 | 7-10 jets | Yes | 3.2 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-W) | 1-2 $\tau + 1-2\ell$ | 0-2 jets | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-V) | 1 | 1-2 jets | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-N) | 2 | 2 jets | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-P) | 1 | 1 jet | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |
| $\mu$, $\tau$ (off-R) | 0 | 1 jet | Yes | 20.3 | $960 \text{ GeV}$ | $1.85 \text{ TeV}$ | ATLAS-CONF-2015-062 |

### EFT direct production

| Model | $\ell$, $\mu$, $\tau$ | Jets | $E_{T}^{\text{miss}}$ | Mass limit | $\sqrt{s} = 7, 8 \text{ TeV}$ | $\sqrt{s} = 13 \text{ TeV}$ | Reference |
|-------|----------------------|------|------------------|------------|-----------------|-----------------|-----------|
| 1H-1H | 1, 1, 0 | 0 | 2 | Yes | 3.2 | $840 \text{ GeV}$ | ATLAS-CONF-2015-066 |
| 1H-1H | 0 | 2 | Yes | 3.2 | $840 \text{ GeV}$ | ATLAS-CONF-2015-066 |
| 1H-1H | 0 | 2 | No | 3.2 | $840 \text{ GeV}$ | ATLAS-CONF-2015-066 |

### EW direct production

| Model | $\ell$, $\mu$, $\tau$ | Jets | $E_{T}^{\text{miss}}$ | Mass limit | $\sqrt{s} = 7, 8 \text{ TeV}$ | $\sqrt{s} = 13 \text{ TeV}$ | Reference |
|-------|----------------------|------|------------------|------------|-----------------|-----------------|-----------|
| $\ell$, $\mu$, $\tau$ | 0 | 2 | Yes | 20.3 | $90-335 \text{ GeV}$ | $1.78 \text{ TeV}$ | ATLAS-CONF-2015-067 |
| $\ell$, $\mu$, $\tau$ | 0 | 2 | Yes | 20.3 | $90-335 \text{ GeV}$ | $1.78 \text{ TeV}$ | ATLAS-CONF-2015-067 |

### Long-lived particles

| Model | $\ell$, $\mu$, $\tau$ | Jets | $E_{T}^{\text{miss}}$ | Mass limit | $\sqrt{s} = 7, 8 \text{ TeV}$ | $\sqrt{s} = 13 \text{ TeV}$ | Reference |
|-------|----------------------|------|------------------|------------|-----------------|-----------------|-----------|
| Direct | 1 | 0 | No | 20.3 | $270 \text{ GeV}$ | $1.15-370 \text{ GeV}$ | ATLAS-CONF-2015-067 |
| Stable | 0 | 1-5 jets | Yes | 27.9 | $495 \text{ GeV}$ | $850 \text{ GeV}$ | ATLAS-CONF-2015-067 |
| Metastable | 0 | 1 jet | Yes | 20.3 | $495 \text{ GeV}$ | $850 \text{ GeV}$ | ATLAS-CONF-2015-067 |
| GMIBS, stable $\ell$, $\mu$, $\tau$ | 1 | 1-2 jets | Yes | 20.3 | $495 \text{ GeV}$ | $850 \text{ GeV}$ | ATLAS-CONF-2015-067 |
| GMIBS, stable $\ell$, $\mu$, $\tau$ | 1 | 1-2 jets | Yes | 20.3 | $495 \text{ GeV}$ | $850 \text{ GeV}$ | ATLAS-CONF-2015-067 |

### RPV

| Model | $\ell$, $\mu$, $\tau$ | Jets | $E_{T}^{\text{miss}}$ | Mass limit | $\sqrt{s} = 7, 8 \text{ TeV}$ | $\sqrt{s} = 13 \text{ TeV}$ | Reference |
|-------|----------------------|------|------------------|------------|-----------------|-----------------|-----------|
| LFV | $\ell$, $\mu$, $\tau$ | 0 | 2 | Yes | 20.3 | $510 \text{ GeV}$ | $200 \text{ GeV}$ | ATLAS-CONF-2015-015 |

*Only a selection of the available mass limits on new states or phenomena is shown.*

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**Higgs mass and unnatural supersymmetry**

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Where is SUSY now?

Indirect measurements

- \( (g-2)_\mu \). 3.4σ discrepancy may be explained with \( \mathcal{O}(100) \) GeV smuons.
- \( M_W, M_Z, M_h \) and EWPO.
- Flavor observables \( (B_s \rightarrow \mu\mu, b \rightarrow s\gamma) \).

Dark matter

- Relic density and direct detection.

Collider

Higgs mass and unnatural supersymmetry

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Where is SUSY now?

Global fits

- In the unconstrained MSSM 105 new free parameters (masses, mixing angles and phases). Impossible/uninteresting to probe.
- Define a simplified model based on reasonable assumptions and a minor number of free parameters.
- Use of the available collider data, electro-weak precision observables and DM constraint to fit the best value and the likelihood profile of the model parameters.
- Effectively implement interplay between different searches (e.g. collider vs direct detection for DM).
The scenarios

**GUT Models**

| CMSSM       | NUHM1        | NUHM2        |
|-------------|--------------|--------------|
| $m_0, m_{1/2}, A_0, \tan \beta$ | $m_0, m_{1/2}, A_0, \tan \beta, m_H$ | $m_0, m_{1/2}, A_0, \tan \beta, m_{H_u}, m_{H_d}$ |

- Based on unifications assumptions for the soft-SUSY breaking mass terms.
- Introduce correlation between the colored and uncolored sectors.
- [1312.5250,1408.4060].

**pMSSM10**

| $M_1, M_2, M_3$ | $m_{\tilde{q}_{1,2}}, m_{\tilde{q}_3}, m_\tilde{t}$ | $A$ | $M_A, \tan \beta, \mu$ |

- Phenomenological model with 10 low-energy input parameters.
- We assume all left and right soft-SUSY mass breaking terms to be equal.
- We assume that the first two generations of squarks have the same soft-SUSY breaking term.
- All the trilinear coupling are the same.
- [1504.03260,1508.01173].
The framework

- Frequentist fitting framework written in Python/Cython and C++.
- We use SLHA standard as an interface between the external codes that are used to compute the spectrum and the observables.
- The Multinest algorithm is used to sample the parameter space.

| Parameter | Range     | Number of segments |
|-----------|-----------|--------------------|
| $M_1$     | (-1, 1) TeV | 2                  |
| $M_2$     | (0, 4) TeV  | 2                  |
| $M_3$     | (-4, 4) TeV | 4                  |
| $m_{\tilde{q}}$ | (0, 4) TeV  | 2                  |
| $m_{\tilde{q}_3}$ | (0, 4) TeV  | 2                  |
| $m_{\tilde{l}}$ | (0, 2) TeV  | 1                  |
| $M_A$     | (0, 4) TeV  | 2                  |
| $A$       | (-5, 5) TeV | 1                  |
| $\mu$     | (-5, 5) TeV | 1                  |
| $\tan \beta$ | (1, 60)   | 1                  |

Total number of boxes: 128

Codes

| Spectrum generation |
|--------------------|
| SoftSUSY |

| Higgs sector and $(g-2)_\mu$ |
|-------------------------------|
| FeynHiggs, Higgssignals, Higgsbounds |

| B-Physics |
|-----------|
| SuFla, SuperIso |

| EW precision observables |
|---------------------------|
| FeynWZ |

| Dark matter |
|-------------|
| MicroOMEGAs, SSARD |

Higgs mass and unnatural supersymmetry
Heavy Higgses, squarks, gluinos are relatively unconstrained.

Left-handed fermion decay chains evolve via $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$.

Sleptons are at less than 1 TeV.
Poor determination of the mass of colored sparticles (only lower bound from LHC searches).

Larger freedom allow to fulfill the $(g - 2)_\mu$ constraint without being in tension with the LHC searches.

Improved fit with respect to the GUT models.
The \((g-2)_\mu\) constraint

- 3.5\(\sigma\) discrepancy between the SM \((g-2)_\mu\) value and the measured one.
- In CMSSM, NUHM1 and NUHM2 there is a tension between the \((g-2)_\mu\) and LHC constraints from direct searches, due the universality relations.
- In the pMSSM10 we are able to fit perfectly the \((g-2)_\mu\).
- Impact of LHC8_{EWK} constraint limited.
pMSSM10 likelihood is very similar to the experimental value smeared by the theoretical uncertainty as given by FeynHiggs.

Lower value of $\tan\beta$ are disfavored at the 68% CL by LHC8$_{EWK}$, $(g-2)_{\mu}$ and DM constraints

The constraints interplay with the choice of a single soft SUSY-breaking mass-parameter for the sleptons.
Physical mass planes for the colored sparticles
Interplay between collider and direct detection

* CMSSM: best fit, 1σ, 2σ
* NUHM2: best fit, 1σ, 2σ
* NUHM1: best fit, 1σ, 2σ
* pMSSM10 w LHC8: best fit, 1σ, 2σ
* pMSSM10 w/o LHC8: best fit, 1σ, 2σ

- stau coann.
- A/H-funnel
- hybrid
- stop coann.
- focus point
- \(\tilde{\chi}_1^\pm\) coann.
- h-funnel
- Z-funnel

Higgs mass and unnatural supersymmetry

Emanuele A. Bagnaschi (DESY)
Most important results from LHC run 1 is the discovery a SM-like Higgs boson.

Mass measured with high-accuracy (e.g. [hep-ex/1503.07589]).
The hierarchy problem

- For a long time the hierarchy problem has been the guiding principle to try to understand what kind of Physics lies beyond the SM.
- It implies new physics at energy scales close to the TeV.
- From a theoretical point of view, the problem of the tuning of the Higgs mass is similar to the cosmological constant problem (it implies new dynamic at $10^{-3}$ eV).
- To justify this fine tuning an anthropic selection principle based on a large number of metastable vacua has been invoked.
- We could think of using an analogous explanation for the hierarchy problem.
The scenarios
Physical motivations

Various possible models:

1. Large masses for the scalar superpartners, much above the weak scale, violate naturalness, however supersymmetric fermions at a low scale are sufficient for DM and gauge coupling unification.

Split-SUSY

2. Both supersymmetric fermions and scalars are at the same scale, higher than the EW scale.

Quasi natural SUSY

High scale SUSY
**Single scale SUSY**

### Chiral supermultiplets

| Name                        | Symbol | spin 0      | spin 1/2    | (SU(3)_C, SU(2)_L, U(1)_Y) |
|-----------------------------|--------|-------------|-------------|----------------------------|
| squarks, quarks             | Q      | (\tilde{u}_L, \tilde{d}_L) | (u_L, d_L)  | (3, 2, \frac{1}{6})       |
| (×3 families)               | \tilde{u} | \tilde{u}^*_R | u^*_R       | (3, 1, \frac{-2}{3})     |
|                             | \tilde{d} | \tilde{d}^*_R | d^*_R       | (\bar{3}, 1, \frac{1}{3}) |
| sleptons, leptons           | L      | (\tilde{\nu}, \tilde{e}_L) | (\nu, e_L)  | (1, 2, \frac{-1}{2})     |
| (×3 families)               | \tilde{e} | \tilde{e}^*_R | e^*_R       | (1, 1, 1)                |
| Higgses, Higgsinos          | H_u    | (H_u^+, H_u^0) | (\tilde{H}_u^+, \tilde{H}_u^0) | (1, 2, \frac{1}{2})     |
|                             | H_d    | (H_d^0, H_d^-) | (\tilde{H}_d^0, \tilde{H}_d^-) | (1, 2, \frac{-1}{2})   |

### Gauge supermultiplets

| Name                        | spin 1/2 | spin 1     | (SU(3)_C, SU(2)_L, U(1)_Y) |
|-----------------------------|----------|------------|----------------------------|
| gluino, gluon               | \tilde{g} | g          | (8, 1, 0)                  |
| winos, W bosons             | \tilde{W}^\pm, \tilde{W}^0 | W^\pm, W^0 | (1, 3, 0)                  |
| bino, B boson               | \tilde{B}^0 | B^0       | (1, 1, 0)                  |
**At the EW scale**  
**Much above the EW scale**

### Split SUSY

#### Chiral supermultiplets

| Name                  | Symbol | spin 0 | spin 1/2 | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|--------|--------|----------|-----------------------------|
| squarks, quarks       | $Q$    | $(\tilde{u}_L, \tilde{d}_L)$ | $(\tilde{u}_L, \tilde{d}_L)$ | $(3, 2, \frac{1}{6})$ |
| $(\times 3$ families) | $	ilde{u}$ | $\tilde{u}^* _R$ | $u_R^+$ | $(3, 1, -\frac{2}{3})$ |
|                        | $	ilde{d}$ | $\tilde{d}^*_R$ | $d_R^+$ | $(3, 1, \frac{1}{3})$ |
| sleptons, leptons     | $L$    | $(\tilde{\nu}, \tilde{\nu}_L)$ | $(\nu, \nu_L)$ | $(1, 2, -\frac{1}{2})$ |
| $(\times 3$ families) | $\tilde{e}$ | $\tilde{e}^*_R$ | $e_R^+$ | $(1, 1, 1)$ |
| Higgses, Higgsinos    | $H_u$  | $(H_u^+, H_u^0)$ | $(\tilde{H}_u^+, \tilde{H}_u^0)$ | $(1, 2, \frac{1}{2})$ |
|                        | $H_d$  | $(H_d^0, H_d^-)$ | $(\tilde{H}_d^0, \tilde{H}_d^-)$ | $(1, 2, -\frac{1}{2})$ |

#### Gauge supermultiplets

| Name                  | spin 1/2 | spin 1 | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|----------|--------|-----------------------------|
| gluino, gluon         | $\tilde{g}$ | $g$ | $(8, 1, 0)$ |
| winos, W bosons       | $\tilde{W}^\pm, \tilde{W}^0$ | $W^\pm, W^0$ | $(1, 3, 0)$ |
| bino, B boson         | $\tilde{B}^0$ | $B^0$ | $(1, 1, 0)$ |
Higher order corrections to the Higgs mass in the MSSM

One can compute the Higgs mass by computing the complex zero of the inverse propagator matrix.

For the CP-even sector

$$M_{bH}^2(q^2) = \begin{pmatrix} q^2 - m^2_H + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{bH}(q^2) \\ \hat{\Sigma}_{bH}(q^2) & q^2 - m^2_b + \hat{\Sigma}_{bb}(q^2) \end{pmatrix}$$

where $\hat{\Sigma}_{ij}(q^2)$ ($i,j = b,H$) are the renormalized self-energies; $m^2_H$ and $m^2_b$ the tree level masses.

One then computes the complex roots of $\det(M_{bH}^2(q^2)) = 0$, $M_i$; and from those extracts the mass and width: $M_i^2 = M_i^2 - iM_i \Gamma_i$. 
Higher order corrections to the Higgs mass in the MSSM

Considering radiative corrections to the self-energies then all MSSM particles contributes.

\[ \hat{\Sigma}_{ij}(q^2) = \hat{\Sigma}_{ij}^1(q^2) + \hat{\Sigma}_{ij}^2(q^2) + \ldots \]

Structure of radiative corrections
Only stop-top sector for simplicity

1. At one loop:
   \[ \Delta(M_b^{(1)})^2 = m_t^4[L + C^{(1)}] \text{ with } L = \log\left(\frac{m_t}{m_{\tilde{t}}^2}\right) \]

2. At two loop:
   \[ \Delta(M_b^{(2)})^2 = m_t^2\left[ m_t^2\alpha_s(L^2 + L + C^{(2)}) + m_t^4(L^2 + L + D^{(2)}) \right] \]

In this case the problem of this approach is that, if directly applied to the High-scale SUSY/Split SUSY lead to large \( \log(M_S/Q_{EW}) \).
A tower of effective theories

► **Problem:** mass gap in the physical spectrum makes large logs of the ratio $m_{ew}/\tilde{m}$ appears in the perturbative expressions.

► **Solution:** For a proper computation these logs have to be resummed.

► **Method:** define a tower of effective field theories, where the heavy particles are integrated out, and match them at a proper scale. Use RGE to resum the large logarithms.
The Split-SUSY effective Lagrangian

▶ The Split-SUSY effective Lagrangian is obtained from the MSSM Lagrangian after the integration of the scalar supersymmetric partners and of an heavy Higgs doublet (A).

\[ \mathcal{L}_{\text{split}} \supset - \frac{M_3}{2} \tilde{g}^A \tilde{g}^A - \frac{M_2}{2} \tilde{W}^a \tilde{W}^a - \frac{M_1}{2} \tilde{B}\tilde{B} - \mu \tilde{H}_u^T \epsilon \tilde{H}_d + \]
\[ - \frac{H^\dagger}{\sqrt{2}} (\tilde{g}_{2u} \sigma^a \tilde{W}^a + \tilde{g}_{1u} \tilde{B}) \tilde{H}_u - \frac{H^T}{\sqrt{2}} (\tilde{g}_{2d} \sigma^a \tilde{W}^a + \tilde{g}_{1d} \tilde{B}) \tilde{H}_d + \text{h.c.} + \mathcal{O}(\frac{1}{m^2}) \]

▶ The other Higgs doublet (H) is finely tuned to be light.

\[
\begin{pmatrix}
  H \\
  A
\end{pmatrix} = \begin{pmatrix}
  \cos \beta & \sin \beta \\
  -\sin \beta & \cos \beta
\end{pmatrix} \begin{pmatrix}
  -\epsilon H_d^* \\
  H_u
\end{pmatrix}
\]

\[ H = -\cos \beta \epsilon H_d^* + \sin \beta H_u \]

▶ \( V(H) = m^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 \) as in the SM.
Tree level matching

- Matching conditions with the MSSM at the scale $\tilde{m}$.
  - Higgs quartic coupling:
    \[
    \lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta
    \]

- Higgs-higgsino-gaugino effective couplings
  \[
  \tilde{g}_{2u}(\tilde{m}) = g_2(\tilde{m}) \sin \beta, \quad \tilde{g}_{1u}(\tilde{m}) = \sqrt{\frac{3}{5}} g_1(\tilde{m}) \sin \beta, \\
  \tilde{g}_{2d}(\tilde{m}) = g_2(\tilde{m}) \cos \beta, \quad \tilde{g}_{1d}(\tilde{m}) = \sqrt{\frac{3}{5}} g_1(\tilde{m}) \cos \beta
  \]

- Note that $\tan \beta$ is not a parameter of the low-energy theory.
Threshold corrections

All corrections computed with the following assumptions:

1. Limit of unbroken EW symmetry ($v^2/\tilde{m}^2 \rightarrow 0$).
2. Limit of zero external momenta.
3. Limit of zero mass for the light particles.
4. Neglect all Yukawa couplings aside $g_t$.

For the couplings relevant for the Higgs boson mass computation we have:

- One loop threshold to the Higgs quartic coupling.

$$
\lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta + \Delta \lambda^{1\ell,\text{reg}} + \Delta \lambda^{1\ell,\phi} + \Delta \lambda^{1\ell,\chi^1} + \Delta \lambda^{1\ell,\chi^2} + \Delta \lambda^{2\ell}
$$

- One loop threshold to the Higgs-higgsino-gauge coupling.

$$
\tilde{g}_{2u}(\tilde{m}) = g_2(\tilde{m}) \sin \beta \left[ 1 + \Delta \tilde{g}_{2u} \right], \quad \tilde{g}_{1u}(\tilde{m}) = \sqrt{3/5} g_1(\tilde{m}) \sin \beta \left[ 1 + \Delta \tilde{g}_{1u} \right], \\
\tilde{g}_{2d}(\tilde{m}) = g_2(\tilde{m}) \cos \beta \left[ 1 + \Delta \tilde{g}_{2d} \right], \quad \tilde{g}_{1d}(\tilde{m}) = \sqrt{3/5} g_1(\tilde{m}) \cos \beta \left[ 1 + \Delta \tilde{g}_{1d} \right]
$$
Not needed for the computation of the Higgs mass but required to study the behavior at high energy (e.g. unification).

\[
\begin{align*}
\hat{g}_1(\tilde{m}) &= g_1(\tilde{m}) + \Delta g_1 \\
\hat{g}_2(\tilde{m}) &= g_2(\tilde{m}) + \Delta g_2 \\
\hat{g}_3(\tilde{m}) &= g_3(\tilde{m}) + \Delta g_3 \\
\hat{y} &= \frac{g_t(\tilde{m})}{\sin \beta} \left( 1 + \Delta g_\phi + \Delta g_\chi \right)
\end{align*}
\]
Computations of the spectrum and of the parameters/couplings

In Split SUSY two possibilities:

1. Bagnaschi et al [arXiv 1407.4081]

\[ \text{SM in } \overline{\text{MS}} \quad g_{1,2,3}, g_t, \lambda \quad \longleftrightarrow \quad \text{Split-SUSY in } \overline{\text{MS}} \quad g_{1,2,3}, g_t, \lambda, \tilde{g}_{1d}, \tilde{g}_{1u}, \tilde{g}_{2d}, \tilde{g}_{2u} \quad \longleftrightarrow \quad \text{MSSM in } \overline{\text{DR}} \quad g_{1,2,3}, y_t \]

2. Bernal et al [arXiv 0705.1496v3]

\[ \text{Split-SUSY in } \overline{\text{MS}} \quad g_{1,2,3}, g_t, \lambda, \tilde{g}_{1d}, \tilde{g}_{1u}, \tilde{g}_{2d}, \tilde{g}_{2u} \quad \longleftrightarrow \quad \text{MSSM in } \overline{\text{DR}} \quad g_{1,2,3}, y_t \]

- According to the scheme chosen, the corresponding threshold corrections have to be used. In theory one should define as many thresholds as needed by the mass spectrum.
Algorithm implementation

Matching conditions → High scale $\tilde{m}$ → EFT RGEs → Convergence? (yes) → Compute spectrum/observable

High scale $\tilde{m}$ → EFT RGEs → EW scale

SM input → EFT RGEs → EW scale → Convergence? (no)

EFT RGEs
Higgs mass prediction

- The Higgs mass is predicted in the low-energy theory, at the weak scale, using the relation between the $\lambda$ quartic coupling and the physical Higgs boson mass.
- At tree level

\[ m_H^2(Q) = 2\lambda(Q)v^2 = \frac{\lambda(Q)}{\sqrt{2}G_F} \]

- At one- and with two-loop $\mathcal{O}(g_t^4g_3^2)$ and $\mathcal{O}(g_t^6)$

\[ M_H^2 = m_H^2(Q) \left[ 1 + \delta_{1l}^{\text{SM}} + \delta_{1l}^{\text{Split}} \right] \]
\[ + \frac{g_t^4v^2}{128\pi^4} \left[ 16g_3^2(3l_t^2 + l_t) - 3g_t^2 \left( 9l_t^2 - 3l_t + 2 + \frac{\pi^2}{3} \right) \right] \]
Quasi-natural SUSY

- All superparticles have masses in the range between a few to tens TeV.

- Minimal stop mixing in the vicinity of $X_t = 0$.

- Maximal stop mixing close to $X_t = \sqrt{6\tilde{m}}$.

- Colored bands are due to the parametric uncertainty due to $M_t$ and $\alpha_s(M_Z)$.

- Two-loop corrections vanish for zero mixing and degenerate SUSY masses.

[Bagnaschi et al, 1407.4081]
SUSYHD uncertainty estimation

- Villadoro and Pardo-Vega: [1504.05200]
- Code available at http://users.ictp.it/~susyhd/.
Effect of dimension six operators

\[ \mathcal{O}_{GG} = g_s^4 |H|^2 C_{\mu\nu} C^{a\mu\nu} \]
\[ \mathcal{O}_{WW} = g^2 |H|^2 W^a_{\mu\nu} W^{a\mu\nu} \]
\[ \mathcal{O}_{BB} = g_s^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \]
\[ \mathcal{O}_{WB} = 2gg' t^a HW^a_{\mu\nu} B^{\mu\nu} \]
\[ \mathcal{O}_W = ig (H^\dagger t^a D^\mu H) D^\nu W^a_{\mu\nu} \]
\[ \mathcal{O}_B = ig' Y_H (H^\dagger D^\mu H) \partial^\nu B_{\mu\nu} \]

\[ \mathcal{O}_H = \frac{1}{2} (\partial_{\mu} |H|^2)^2 \]
\[ \mathcal{O}_T = \frac{1}{2} (H^\dagger D_{\mu} H)^2 \]
\[ \mathcal{O}_R = |H|^2 |D_{\mu} H|^2 \]
\[ \mathcal{O}_D = |D^2 H|^2 \]
\[ \mathcal{O}_6 = |H|^6 \]
\[ \mathcal{O}_t = |H|^2 \tilde{q}_L \tilde{H} t_R \]

contribution from the stops (at maximal mixing)

From Pardo Vega’s talk at KUTS2016-01.

Work in progress [Bagnaschi,Pardo-Vega,Slavich, 16xx.yyyyy].
Uncertainty in the Higgs mass prediction

- Different region of applicability for the two approaches (low SUSY vs large SUSY masses).
- Uncertainty estimation in the intermediate, phenomenologically interesting region, not trivial.

[SusyHD 1504.05200] [FlexibleSUSY Bagnaschi, Weiglein, Voigt 16xx.yyyy] [FeynHiggs 1312.4937]
High-scale SUSY

- All SUSY particles lie around the same scale $\tilde{m}$, which can be much higher than the weak scale.

- Thinner gray band due to $1\sigma$ variation of $\alpha_s(M_Z)$.

- Larger colored) due to $1\sigma$ variation of $M_t$. 

![Graph showing Higgs mass and stop mixing](image-url)
The darker (red) region denotes the effect of varying only $A_t$, in the range allowed by vacuum stability.

Larger (gray) band due to random scanning of each SUSY particle mass parameters ($M_1$, $M_2$, $M_3$, $m_{Q_i}$, $m_{U_i}$, $m_{D_i}$, $m_{E_i}$, $m_{L_i}$, distinguishing the third generation from the other two), up to a factor of 3 (1/3) above (below) the SUSY scale $\tilde{m}$.

Variation of $M_h$ around the value obtained with $\tan \beta$ and $A_t$ in such a way that $X_t = 0$ and $M_h = 125.15$ GeV, for a given mass scale $\tilde{m}$.
Unification in High-scale SUSY

- Use on the full one loop threshold corrections to the MSSM couplings $\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_t$.
- Two-loop MSSM RGEs.
- The gray band is obtained by scanning the SUSY mass parameters by up of a factor 3 (1/3) above (below) $\tilde{m}$.
- $\tan \beta$ in the scan is tuned to reproduce the observed Higgs mass.
- $A_t$ in the range allowed by vacuum stability.
Split-SUSY

- $M_1 = M_2 = M_3 = \mu = 1\ \text{TeV}$.
- All scalars degenerate at scale $\tilde{m}$.
- $A_t = 0$ (In Split-SUSY $A_t/\tilde{m} \ll 1$).

- Thinner gray band due to $1\sigma \alpha_s(M_Z)$.
- Larger colored band due to $M_t$ variation.
For $\tan \beta \lesssim 2$ dominant uncertainty from the top mass value.

For $\tan \beta \gtrsim 2$ dominant uncertainty from SUSY threshold effects.

Smallness of $A_t$ and $\mu$ implies small stop threshold corrections (smaller effect than in high-scale SUSY).

Less sensitivity to $M_t$ (in respect to high-scale SUSY).
Collider signatures: gluino decay

- Gluino lifetime (decay length) from the determination of $\tilde{m}$ due to the Higgs mass prediction.

$$c\tau_{\tilde{g}} = \left( \frac{2\text{TeV}}{M_{\tilde{g}}} \right)^2 \left( \frac{\tilde{m}}{10^7 \text{GeV}} \right)^4 \approx 0.4 \text{ m}$$

- $\tan \beta \approx 1 \rightarrow c\tau_{\tilde{g}} \gtrsim 10\text{ m}$ (out of detector decay).
- $1 < \tan \beta < 2 \rightarrow c\tau_{\tilde{g}} \gtrsim 50\mu\text{m}$ (displaced vertex).
- $\tan \beta > 2 \rightarrow$ prompt decay.

- Need EFT computation to resum large logs.

[hep-ph/0506214]
Heavy SUSY matched to a THDM

- Enrich the phenomenology by considering the possibility of a THDM for the scalar sector of low-energy EFT.
- First detailed in Haber & Hempfling ([hep-ph/9307301]).
- Matching performed recently by Lenz et al ([1203.0238]), however in the context of studying B meson mixing.
- First recent study focused on the Higgs mass by Lee & Wagner ([1508.00576]).
- Bagnaschi et al ([1512.07761]) studied vacuum stability with GUT study.
Matching to a THDM

\[ V = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - \left( m_{12}^2 H_1^\dagger H_2 + \text{h.c.} \right) + V_4, \]

\[ V_4 = \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 |H_1^\dagger H_2|^2 \]

\[ + \left( \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_2)(H_1^\dagger H_1) + \lambda_7 (H_1^\dagger H_2)(H_2^\dagger H_2) + \text{h.c.} \right). \]

- More complex scalar sector, requires computation of the thresholds for each \( \lambda_i \).

\[ \lambda_1 = \frac{1}{4} \left( g^2 + g'^2 \right), \quad \lambda_2 = \frac{1}{4} \left( g^2 + g'^2 \right), \]

\[ \lambda_3 = \frac{1}{4} \left( g^2 - g'^2 \right), \quad \lambda_4 = -\frac{1}{2} g^2, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0 \]
Results

- Lee and Wagner, ref. [1508.00576]
- Code will be available shortly.
Comparison with FeynHiggs in the low-\(\tan \beta\)-high scenario ([1508.00576]).
EW-vacuum stability in the THDM

- Published study on the EW stability of the THDM with GUT scale SUSY [1512.07761].

- Instability condition for the THDM

\[
\begin{align*}
\lambda_1 &> 0 \\
\lambda_2 &> 0 \\
\lambda_3 + (\lambda_1 \lambda_2)^{1/2} &> 0 \\
\lambda_3 + \lambda_4 + (\lambda_1 \lambda_2)^{1/2} &> 0
\end{align*}
\]

- Due to the matching with SUSY, only the fourth stability condition can be violated.
Metastability

- Derive a metastability bound from the $\lambda^4$ potential at tree level.
- Choose a gauge and field basis such that the problem become one-dimensional.

$$\lambda(\mu_r) \gtrsim -\frac{2.82}{41.1 + \log_{10} \frac{\mu_r}{\text{GeV}}}$$

with

$$\lambda = \frac{4(\lambda_1 \lambda_2)^{1/2} (\lambda_3 + \lambda_4 + (\lambda_1 \lambda_2)^{1/2})}{\lambda_1 + \lambda_2 + 2(\lambda_1 \lambda_2)^{1/2}}.$$
Comparison with Wagner et al

Preliminary

\[ M_A = 200 \text{ GeV}, \mu = M_S, X_t = 0 \text{ GeV} \]

- Good qualitative agreement for the THDM. Looking forward for a more thorough comparison of the implementations.
THDM with GUT-scale SUSY

$M_S = 2 \cdot 10^{14}$ GeV, $\mu = 2 \cdot 10^{13}$ GeV

Match the THDM instead of the SM. [Lee et al, Bagnaschi et al]
At low $\tan \beta$, the large top Yukawa at the low scale drives high $\lambda_2$ to high values in the IR.

At the high scale, gauge couplings approximately unify; $\lambda_4$ negative.

$\lambda_3 + \lambda_4 + (\lambda_1 \lambda_2)^{1/2} > 0$. 

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**RG running and vacuum stability**

**Renormalisation group running**

$M_A = 2000 \text{ GeV}, \tan \beta = 1.15$

**Vacuum stability conditions**

$M_A = 2000 \text{ GeV}, \tan \beta = 1.15$
If tan $\beta$ is large enough, the top Yukawa is unable to push $\lambda_2$
THDM+Higgsinos with GUT-scale SUSY

- Higgsinos have a minor effect on the Higgs mass since they couple only through gauge interactions (no gauginos in the spectrum).
THDM+Split with GUT-scale SUSY

Very large light Higgs mass, impossible agreement with the measured value.
Conclusions

- Due to the negative results of SUSY particle searches at the LHC, models with an unnatural spectrum are becoming more interesting.
- The Higgs mass represents the prime observable to probe these models.
- These models, like mini-split SUSY, have also interesting predictions for DM.

Future outlook

- Due to the importance of the Higgs mass prediction, it is important to improve the accuracy of the prediction by computing higher order corrections.
- Combination of the results with the Feynman diagrammatic approach.
- Extension to other spectrum-splitting configuration and/or other models.
Backup slides
The constraints

Indirect measurements

- $(g-2)_\mu$. $3.4\sigma$ discrepancy may be explained with $O(100)$ GeV smuons.
- $M_W, M_Z, M_h$ and EWPO.
- Flavor observables ($B_s \to \mu\mu, b \to s\gamma$).

Collider – GUT models

- Limits are independent of $A_0, \tan\beta, m_{H_u}^2$ and $m_{H_d}^2$.
- Due to unification, limits on squarks and gluinos are relevant also for sleptons and electroweakinos.

Dark matter

- Relic density and direct detection.

```
Higgs mass and unnatural supersymmetry
Emanuele A. Bagnaschi (DESY) 2 / 40
```
The constraints – collider pMSSM10

Three classes of constraints

Colored sparticle production
We have combined the following CMS searches:
- 0-lepton $M_{T2}$
- 1-lepton $M_{T2}^W$
- 2-lepton OS/SS
- $\geq 3$ leptons.

Compressed stop scenarios
This scenario is separately. The stop cross-section is set to zero.

Electroweakinos production
- Simplified ModelS (SMS) approach. Limited mass hierarchies.
- Slepton production.
- $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ via sleptons.
- $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ via WZ.
The MSSM Lagrangian

- Gauge part of the Lagrangian and fermion-scalar-gaugino interactions.
- Superpotential $W = h_e H_d L \tilde{e} + h_d H_d Q \tilde{d} + h_u Q H_u U^c - \mu H_u H_d$
- Soft SUSY-breaking mass and interaction terms for MSSM scalars

$$\mathcal{L}_{\text{soft-breaking}} = m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_Q^2 Q^\dagger Q + m_L^2 L^\dagger L$$
$$+ m_{\tilde{u}}^2 \tilde{u}_R^* \tilde{u}_R + m_{\tilde{d}}^2 \tilde{d}_R^* \tilde{d}_R + m_{\tilde{e}}^2 \tilde{e}_R^* \tilde{e}_R$$
$$+ \left( T_e H_d L \tilde{e}_R^* + T_d H_d Q \tilde{d}_R^* + T_u Q H_u \tilde{u}_R^* + B \mu H_u H_d + b.c. \right)$$

- SUSY-soft-breaking gauginos masses

$$\mathcal{L}_G = \frac{1}{2} \left( M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g} \right) + b.c.$$
Higgs mechanism in the MSSM

- Tree level Higgs scalar potential ($m_u^2 = m_{H_u}^2 + |\mu|^2$ and $m_d^2 = m_{H_d}^2 + |\mu|^2$)

\[
V_0 = m_u^2 \left| H_u^0 \right|^2 + m_d^2 \left| H_d^0 \right|^2 + B_\mu (H_d^0 H_u^0 + h.c.) + \frac{g^2 + g'^2}{8} \left( \left| H_d^0 \right|^2 - \left| H_u^0 \right|^2 \right)^2
\]

- The two Higgs doublet are supposed to acquire a v.e.v. different from zero
- Decomposition of the fields

\[
H_u^0 = \frac{1}{\sqrt{2}} (v_u + S_u + iP_u), \quad H_d^0 = \frac{1}{\sqrt{2}} (v_d + S_d + iP_d)
\]

- Diagonalization of the pseudoscalar mass matrix (rotation angle $\beta$) give a would-be Goldstone boson eaten by the Z and a pseudoscalar state with a mass

\[
m_A^2 = \frac{B_\mu}{\cos \beta \sin \beta}
\]

- Same diagonalization angle for the charged Higgs matrix
- Pseudoscalar couplings to quarks and leptons are given by

\[
g_{Auu} = \cot \beta \frac{m_u}{v}, \quad g_{Add,Aee} = \tan \beta \frac{m_{d,e}}{v}
\]
Higgs mechanism in the MSSM

- Mass matrix for the scalar sector ($m_u^2$ and $m_d^2$ replaced by a combination of $m_A^2$ and $\tan \beta$)

\[
\mathcal{M}_0 = \begin{pmatrix}
  m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \sin \beta \cos \beta \\
  -(m_A^2 + m_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta
\end{pmatrix}
\]

- Diagonalization angle $\alpha$. $m_h^2 \leq m_Z^2 \cos^2(2\beta)$ at tree level.

\[
\tan 2\alpha = \left( \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} \right) \tan 2\beta
\]

\[
m_{b,H} = \frac{1}{2} \left( m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2m_A^2 \sin^2(2\beta)} \right)
\]

- Scalar coupling to the gauge bosons: $g_{hVV} = \frac{2m_V^2}{v} \sin(\beta - \alpha)$, $g_{HVV} = \frac{2m_V^2}{v} \cos(\beta - \alpha)$

- Scalar couplings to the quarks and leptons are given by

\[
g_{h\nu\nu} = \frac{\cos \alpha}{\sin \beta} \frac{m_\nu}{v}, \quad g_{h\bar{d}d, h\bar{e}e} = \frac{-\sin \alpha}{\cos \beta} \frac{m_{d,e}}{v}
\]

\[
g_{H\nu\nu} = \frac{\sin \alpha}{\sin \beta} \frac{m_\nu}{v}, \quad g_{H\bar{d}d, H\bar{e}e} = \frac{\cos \alpha}{\cos \beta} \frac{m_{d,e}}{v}
\]
One loop corrections to the quartic coupling

\[
\lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta + \Delta \lambda^{1\ell, \text{reg}} + \Delta \lambda^{1\ell, \phi} + \Delta \lambda^{1\ell, \chi^1} + \Delta \lambda^{1\ell, \chi^2}
\]

- \( \Delta \lambda^{1\ell, \phi} \) contains the threshold corrections from diagrams involving scalars.
- Needed in all models.
One loop corrections to the quartic coupling

\[ \lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta + \Delta \lambda^{1\ell,\text{reg}} + \Delta \lambda^{1\ell,\phi} + \Delta \lambda^{1\ell,\chi^1} + \Delta \lambda^{1\ell,\chi^2} \]

- \( \Delta \lambda^{1\ell,\chi^1} \) contains the proper threshold corrections from SUSY fermions.

- Needed for single scale SUSY.

- In Split SUSY either introduced at the matching threshold with the SM or not present as a threshold (enters the relation between the quartic and the Higgs mass).
One loop corrections to the quartic coupling

\[ \lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta + \Delta \lambda^{1\ell,\text{reg}} + \Delta \lambda^{1\ell,\phi} + \Delta \lambda^{1\ell,\chi^1} + \Delta \lambda^{1\ell,\chi^2} \]

- \( \Delta \lambda^{1\ell,\text{reg}} \) contains term due to the fact that we are expressing the matching in terms of the low-energy effective theory in the \( \overline{\text{MS}} \) scheme.

- \( \Delta \lambda^{1\ell,\chi^2} \) contains the terms that are needed in single scale SUSY due to the fact that the tree level matching for \( \lambda \) is expressed in terms of the SM gauge couplings.
Two loop matching

- Two loop $O(g_s^2 g_t^4)$ corrections to $\lambda$ computed with EP techniques from the results of Slavich et al.

$$\Delta \lambda^{2\ell} = \frac{1}{2} \frac{\partial^4 \Delta V^{2\ell,\tilde{t}}}{\partial^2 H^\dagger \partial^2 H} + \Delta \lambda^{2\ell,\text{shift}}$$

- Two-loop diagrams involving strong gauge interaction of the stop squarks

$$\Delta V^{2\ell,\tilde{t}} = \frac{g_s^2}{64 \pi^4} \left( 2m_{\tilde{t}_1}^2 I(m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2, 0) + 2L(m_{\tilde{t}_1}^2, M_3^2, m_t^2) - 2m_t M_3 s_2^{} \theta I(m_{\tilde{t}_1}^2, M_3^2, m_t^2) 
+ \left( 1 - \frac{s_2^2}{2} \right) f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{s_2^2}{2} f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \left[ m_{\tilde{t}_1} \leftrightarrow m_{\tilde{t}_2}, s_2^{} \theta \rightarrow -s_2^{} \theta \right] \right)$$

- $\Delta \lambda^{2\ell,\text{shift}}$ contains one-loop renormalization term for the top Yukawa.
Algorithm implementation

- **SM input parameters**: $\alpha_s(M_Z)$, $\alpha(M_Z)$, $G_F$, $M_Z$, $m_b(m_b)$, $M_\tau$.

- **SUSY parameters**: $\mu(M_Z)$, $M_1(M_Z)$, $M_2(M_Z)$, $M_3(M_Z)$, $\tilde{m}$, $\tan \beta(\tilde{m})$, $A_t(\tilde{m})$, plus all the soft-susy breaking mass terms for the scalars.

- The running parameters are extracted with two loop precision.

- SM RGEs at three loop, Split SUSY RGEs at two loop.
FlexibleSUSY and EFT towers

- Framework developed by Athron, Park, Stöckinger and Voigt [1406.2319].
- Automatic generation of a SoftSUSY-like spectrum generator based for arbitrary models starting from a SARAH model file.
- SLHA input and output – easy interface with other codes and analysis pipelines.
- Native support for EFT towers. Boundary conditions can be specified in Mathematica code.
- Available at https://flexiblesusy.hepforge.org/

```plaintext
FSModelName = "THDM";
FSEigenstates = SARAH'EWSB;
AutomaticInputAtMSUSY = False;
FSDefaultSARAHModel = "THDM-II";
MINPAR = {
    {3, TanBeta}
};
EXTPAR = {
    {0, MSUSY}, {1, MEWSB},
    {2, MuInp}, {6, MAInp},
    {7, AtInp}, {8, AbInp},
    {9, AtauInp}, {100, LambdaLoopOrder}
};
EWSBOutputParameters = {M112, M222};
(High scale where we match to the MSSM)
HighScale = MSUSY;
HighScaleFirstGuess = MSUSY;
HighScaleInput = {
    {Lambda1, 1/2 (1/4 (GUTNormalization[g1] g1)^2 + g2^2) + UnitStep[THRESHOLD-1]
        UnitStep[LambdaLoopOrder-1]
        (deltaLambda1th1L + deltaLambda1Phi1L)
        + UnitStep[THRESHOLD-2]
        UnitStep[LambdaLoopOrder-2]
        deltaLambda1th2L),

    [...]}
```
Phenomenological predictions
Uncertainty of the Higgs mass prediction

There are different sources of uncertainty

- Missing higher order corrections in the translation from the physical measured data and the running parameters used in the iterative procedure.
- Missing higher terms in the RGEs.
- Power suppressed terms $1/(4\pi)^2 v^2 / \tilde{m}^2$ in the above two computations.
- Higher order corrections to the SUSY thresholds.
- $v^2 / \tilde{m}^2$ terms due to the fact that we neglect EWSB when matching the MSSM with the low-energy effective theory.

The procedure is valid only if there is a definite hierarchy between the particles.
Uncertainty of the Higgs mass prediction

- ±0.2 GeV are estimated to come from missing higher orders in the SM RGEs and in the relation between physical observables and running parameters in the SM.

- ±0.5 GeV from the missing higher orders in the SUSY threshold corrections (estimated by varying by a factor of two \( \tilde{m} \) – expected not to be much larger than the \( \mathcal{O}(g_3^2 g_t^4) \) corrections that are at most 0.4 GeV for large stop mixing and \( \tilde{m} = 10 \) TeV.

- The other uncertainties are suppressed by \( v^2/\tilde{m}^2 \), it should be already irrelevant for a scale \( \tilde{m} = 10 \) TeV.
At the EW scale

**THDM**

### Chiral supermultiplets

| Name              | Symbol | spin 0          | spin 1/2          | \((SU(3)_C, SU(2)_L, U(1)_Y)\) |
|-------------------|--------|-----------------|-------------------|---------------------------------|
| squarks, quarks   | Q      | \((\tilde{u}_L, \tilde{d}_L)\) | \((u_L, d_L)\)    | \((3, 2, \frac{1}{6})\)          |
|                   | \(\tilde{u}\) | \(\tilde{u}_R^*\) | \(u_R^+\) | \((3, 1, -\frac{2}{3})\)          |
|                   | \(\tilde{d}\) | \(\tilde{d}_R^*\) | \(d_R^+\) | \((\bar{3}, 1, \frac{1}{3})\)          |
| sleptons, leptons | L      | \((\tilde{\nu}, \tilde{e}_L)\) | \((\nu, e_L)\)   | \((1, 2, -\frac{1}{2})\)          |
|                   | \(\tilde{e}\) | \(\tilde{e}_R^*\) | \(e_R^+\) | \((1, 1, 1)\)          |
| Higgses, Higgsinos| \(H_u\) | \((H_u^+, H_u^0)\) | \((\tilde{H}_u^+, \tilde{H}_u^0)\) | \((1, 2, \frac{1}{2})\)          |
|                   | \(H_d\) | \((H_d^0, H_d^-)\) | \((\tilde{H}_d^0, \tilde{H}_d^-)\) | \((1, 2, -\frac{1}{2})\)          |

### Gauge supermultiplets

| Name             | spin 1/2          | spin 1          | \((SU(3)_C, SU(2)_L, U(1)_Y)\) |
|------------------|-------------------|-----------------|---------------------------------|
| gluino, gluon    | \(\tilde{g}\)    | \(g\)          | \((8, 1, 0)\)                  |
| winos, W bosons  | \(\tilde{W}^\pm\) | \(W^\pm\)      | \((1, 3, 0)\)                  |
|                  | \(\tilde{B}^0\)  | \(B^0\)        | \((1, 1, 0)\)                  |
High-scale SUSY

- For $\tan \beta \lesssim 2$ dominant uncertainty from the top mass value (dependence of $M_h$ on $\tilde{m}$ rather flat).

- For $\tan \beta \gtrsim 2$ larger sensibility to SUSY-threshold effects, with no strong dependence on $\tilde{m}$.
  This is due to two competing effects: flatness of the $M_h$ dependence on $\tilde{m}$ vs smallness of the SUSY thresholds at large $\tilde{m}$.

- Perturbativity of the top Yukawa satisfied ($\tilde{m} > 10^7$ GeV for $\tan \beta = 1$).
### At the EW scale

**Much above the EW scale**

**THDM + Higgsinos**

#### Chiral supermultiplets

| Name                  | Symbol | spin 0   | spin 1/2   | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|--------|----------|------------|------------------------------|
| squarks, quarks       | $Q$    | $(\tilde{u}_L, \tilde{d}_L)$ | $(u_L, d_L)$ | $(3, 2, \frac{1}{6})$ |
| ($\times 3$ families) | $\tilde{u}$ | $\tilde{u}_R^*$ | $u_R^*$ | $(\tilde{3}, 1, -\frac{2}{3})$ |
| ($\times 3$ families) | $\tilde{d}$ | $\tilde{d}_R^*$ | $d_R^*$ | $(\tilde{3}, 1, \frac{1}{3})$ |
| sleptons, leptons     | $L$    | $(\tilde{\nu}, \tilde{e}_L)$ | $(\nu, e_L)$ | $(1, 2, -\frac{1}{2})$ |
| ($\times 3$ families) | $\tilde{e}$ | $\tilde{e}_R^*$ | $e_R^*$ | $(1, 1, 1)$ |
| Higgses, Higgsinos    | $H_u$  | $(H_u^+, H_u^0)$ | $(\tilde{H}_u^+, \tilde{H}_u^0)$ | $(1, 2, \frac{1}{2})$ |
| ($H_d^0, H_d^-)$      | $H_d$  | $(H_d^0, H_d^-)$ | $(\tilde{H}_d^0, \tilde{H}_d^-)$ | $(1, 2, -\frac{1}{2})$ |

#### Gauge supermultiplets

| Name                  | spin 1/2 | spin 1   | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-----------------------|----------|----------|------------------------------|
| gluino, gluon         | $\tilde{g}$ | $g$     | $(8, 1, 0)$ |
| winos, $W$ bosons     | $\tilde{W}^\pm$ | $W^\pm$ | $(1, 3, 0)$ |
| bino, $B$ boson       | $\tilde{B}^0$ | $B^0$   | $(1, 1, 0)$ |
At the EW scale Much above the EW scale

THDM + Split SUSY

Chiral supermultiplets

| Name                  | Symbol | spin 0       | spin 1/2       | \((SU(3)_C, SU(2)_L, U(1)_Y)\) |
|-----------------------|--------|--------------|----------------|-------------------------------|
| squarks, quarks       | \(Q\)  | \((\tilde{u}_L, \tilde{d}_L)\) | \((u_L, d_L)\) | \((3, 2, \frac{1}{6})\)       |
| \((\times 3 \text{ families})\) | \(\tilde{u}\)  | \(\tilde{u}^*_R\) | \(u_R^\dagger\) | \((3, 1, -\frac{2}{3})\)       |
|                       | \(\tilde{d}\)  | \(\tilde{d}^*_R\) | \(d_R^\dagger\) | \((\bar{3}, 1, \frac{1}{3})\)       |
| sleptons, leptons     | \(L\)  | \((\tilde{\nu}, \tilde{e}_L)\) | \((\nu, e_L)\) | \((1, 2, -\frac{1}{2})\)       |
| \((\times 3 \text{ families})\) | \(\tilde{e}\)  | \(\tilde{e}^*_R\) | \(e_R^\dagger\) | \((1, 1, 1)\)       |
| Higgses, Higgsinos    | \(H_u\) | \((H^+_u, H^0_u)\) | \((\tilde{H}^+_u, \tilde{H}^0_u)\) | \((1, 2, \frac{1}{2})\)       |
|                       | \(H_d\)  | \((H^0_d, H^-_d)\) | \((\tilde{H}^0_d, \tilde{H}^-_d)\) | \((1, 2, -\frac{1}{2})\)       |

Gauge supermultiplets

| Name                  | spin 1/2 | spin 1 | \((SU(3)_C, SU(2)_L, U(1)_Y)\) |
|-----------------------|----------|--------|-------------------------------|
| gluino, gluon         | \(\tilde{g}\) | \(g\) | \((8, 1, 0)\) |
| winos, W bosons       | \(\tilde{W}^\pm, \tilde{W}^0\) | \(W^\pm, W^0\) | \((1, 3, 0)\) |
| bino, B boson         | \(\tilde{B}^0\) | \(B^0\) | \((1, 1, 0)\) |
Matching to a THDM

\[-\mathcal{L}_{Yuk} = \frac{\tilde{g}_d}{\sqrt{2}} H_1 \tilde{W} \tilde{h}_d + \frac{\tilde{g}'_d}{\sqrt{2}} H_1 \tilde{B} \tilde{h}_d + \frac{\tilde{g}_u}{\sqrt{2}} H_2^\dagger \tilde{W} \tilde{h}_u + \frac{\tilde{g}'_u}{\sqrt{2}} H_2^\dagger \tilde{B} \tilde{h}_u \]
\[+ \frac{\tilde{\gamma}_d}{\sqrt{2}} H_2 \tilde{W} \tilde{b}_d + \frac{\tilde{\gamma}'_d}{\sqrt{2}} H_2 \tilde{B} \tilde{b}_d + \frac{\tilde{\gamma}_u}{\sqrt{2}} H_1^\dagger \tilde{W} \tilde{h}_u + \frac{\tilde{\gamma}'_u}{\sqrt{2}} H_1^\dagger \tilde{B} \tilde{h}_u \]
\[+ \bar{b}_R H_1^* Q_L + b_R \bar{Q}_L H_2 + \tilde{\tilde{\gamma}}_b \bar{b}_R H_2^* Q_L + \tilde{\tilde{\gamma}}_t \bar{t}_R Q_L H_2 \]
\[+ \text{h.c.} \]

- “Wrong” yukawas $\tilde{\gamma}_i = \tilde{\eta}_j = 0$ at tree level.

- Ignored for now for simplicity, EFT is then a type II THDM. They will be included in a upcoming study.
The hMSSM approach

- Based on using the observed Higgs mass (and not the un-observed spectrum) as an input.

- Higgs sector of the MSSM described only in terms of $m_A, \tan \beta$ and $m_b$ (which is now an input).

- Predictive power in the heavy Higgs mass and the mixing angle $\alpha$.

\[
\mathcal{M}_\Phi^2 = \mathcal{M}_{\text{tree}}^2 + \begin{pmatrix}
\Delta \mathcal{M}_{11}^2 & \Delta \mathcal{M}_{12}^2 \\
\Delta \mathcal{M}_{12}^2 & \Delta \mathcal{M}_{22}^2
\end{pmatrix}
\]

\[
\tan \alpha = -\frac{(m_Z^2 + m_A^2) \cos \beta \sin \beta}{m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta - m_b^2}
\]

\[
m_h^2 = \frac{(m_A^2 + m_Z^2 - m_b^2)(m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta) - m_A^2 m_Z^2 \cos^2 2\beta}{m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta - m_b^2}
\]

It depends on the following assumptions:

- The neglected stop-top corrections in $\Delta \mathcal{M}_{11}^2, \Delta \mathcal{M}_{12}^2$ scale as $\mu X_t/m_{\text{SUSY}}^2$.

- SUSY sparticles do not affect the Higgs besides the effect in mass matrix (satisfied in the low-$\tan \beta$ scenarios).

- Proper approach is EFT (Lee and Wagner, Bagnaschi et al)

[1305.2172, 1307.5205, 1502.05653, LHCHXSWG-2015-002]
Results and comparison with the hMSSM

$m_H$ comparison with the hMSSM ([1508.00576]).
The renormalization of the mixing angle

- It is not useful to relate $\beta$ to the ratio of the vacuum expectation value of $H_u$ and $H_d$.
- $\beta$ should be interpreted as the fine-tuned mixing angle that rotates the two original doublets into one heavy doublet $A$ and a light one $H$.
- The divergent part of the CT for $\beta$ is required to cancel the divergence of the anti-symmetric part of the WFR matrix

$$\delta \beta^{\text{div}} = \frac{1}{2} \frac{\Pi^{\text{div}}_{HA}(m_H^2) - \Pi^{\text{div}}_{HA} hA(m_A^2)}{m_H^2 - m_A^2}$$

- Finite part of the CT is arbitrary and defines the renormalization scheme. In our case it cancels exactly the off-diagonal WFR contributions from the matching conditions of the effective couplings

$$\delta \beta^{\text{fin}} = \frac{\Pi^{\text{fin}}_{HA}(m_H^2)}{m_H^2 - m_A^2}$$

- It is the angle that diagonalized the radiatively corrected Higgs mass matrices at $p^2 = m_H^2$. 

Higgs mass and unnatural supersymmetry

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EW Tuning in High-scale SUSY

- **Tuning condition:**
  \[
  \tan^2 \beta = \left. \frac{m_{H_d}^2 + \mu^2}{m_{H_u}^2 + \mu^2} \right|_{\tilde{m}}
  \]

- **SUSY breaking pattern:**
  - common gaugino mass \( m_{1/2} \),
  - common scalar mass \( m_0 \),
  - Higgsino \( \mu \) and \( A_0 = 0 \).

- For any given value for \( m_{1/2}/\mu \) and \( m_0/\mu \), the measured Higgs mass and the EW tuning conditions determines \( \tan \beta \) and \( \tilde{m} \).

- New focus point for \( \tilde{m} \simeq 10^8 \) GeV and low \( \tan \beta \).
Vacuum stability in High-scale SUSY

- All the scans respect the vacuum stability constraints.
- Eliminates corrections that could reduce the Higgs mass when
  \[ \tilde{X}_t = \frac{(A_t - \mu \cot \beta)^2}{m_{Q_3} m_{U_3}} \geq 12. \]

Scalar potential for the stop-Higgs system

\[
V = m_{Q_3}^2 |\tilde{Q}_3|^2 + m_{U_3}^2 |\tilde{U}_3|^2 + \frac{g_t}{\sin \beta} (A_t H_u \tilde{Q}_3 \tilde{U}_3 + \mu H_d^* \tilde{Q}_3 \tilde{U}_3 + \text{h.c.}) \\
+ \frac{g_t^2}{\sin^2 \beta} (|H_u \tilde{Q}_3|^2 + |H_u \tilde{U}_3|^2 + |\tilde{Q}_3 \tilde{U}_3|^2) + \text{Higgs-mass terms + D-terms}
\]

- Requiring that the color-breaking minimum is not deeper than the EW one implies

\[
\tilde{X}_t = \frac{(A_t - \mu \cot \beta)^2}{m_{Q_3} m_{U_3}} < \left(4 - \frac{1}{\sin^2 \beta}\right) \left(\frac{m_{Q_3}}{m_{U_3}} + \frac{m_{U_3}}{m_{Q_3}}\right)
\]
Tuning conditions

- Less parameter at $\tilde{m}$ than high-scale SUSY.

- Tuning condition:
  \[
  \tan^2 \beta = \frac{m_{H_d}^2}{m_{H_u}^2} |_{\tilde{m}} .
  \]

- Assuming $SU(5)$ unification relations for the scalar, the tuning condition can be expressed in term of
  \[
  r_H = \frac{m_{H_d}^2}{m_{H_u}^2} |_{M_{GUT}} \quad \text{and} \quad r_Q = \frac{m_Q^2}{m_{H_u}^2} |_{M_{GUT}} .
  \]

- Solutions in a region close to universality ($r_H \sim r_Q \sim \mathcal{O}(1)$).
Mini-split SUSY

1. Simplest Split SUSY model emerging from anomaly mediation.
2. Theory characterized by four parameters: $\tilde{m}$, $m_{3/2}$, $\mu$ and $\tan \beta$.
3. One parameter fixed by the Higgs mass.
4. If gravity is the only mediator of SUSY breaking, $\tilde{m} \sim m_{3/2}$.
5. If $\mu$ and $B_\mu$ are generated from the same operator, then $\tan \beta$ is fixed.
Higgs mass in mini-split

- Soft scalar masses assumed to be all equal to $m_{3/2}$.
- $2 \lesssim \tan \beta \lesssim 3$ to get the correct Higgs mass.
- Mild dependence on the value of $\mu$. 

![Graph showing Higgs mass and gluino mass relationship]
Gaugino masses in mini-split SUSY

Anomaly mediation yields precise prediction for the gaugino masses.

\[
M_B = M_1(Q) \left[ 1 + \frac{C_\mu}{11} + \frac{g_1^2}{80 \pi^2} \left( \frac{-41}{2} \ln \frac{Q^2}{M_1^2} - \frac{1}{2} \ln \frac{\mu^2}{M_1^2} + \ln \frac{m_A^2}{M_1^2} + 11 \ln \frac{m_q^2}{M_1^2} + 9 \ln \frac{m_\ell^2}{M_1^2} \right) + \frac{g_3^2}{6 \pi^2} - \frac{13 g_t^2}{264 \pi^2 \sin^2 \beta} \right]
\]

\[
M_{\tilde{W}} = M_2(Q) \left[ 1 + C_\mu + \frac{g_2^2}{16 \pi^2} \left( \frac{19}{6} \ln \frac{Q^2}{M_2^2} - \frac{1}{6} \ln \frac{\mu^2}{M_2^2} + \frac{11}{3} \ln \frac{m_A^2}{M_2^2} + 3 \ln \frac{m_q^2}{M_2^2} + \ln \frac{m_\ell^2}{M_2^2} \right) + \frac{3 g_3^2}{2 \pi^2} - \frac{3 g_t^2}{8 \pi^2 \sin^2 \beta} \right]
\]

where

\[
M_1(Q) = \frac{33 g_1^2(Q)}{80 \pi^2} m_{3/2}, \quad M_2(Q) = \frac{g_2^2(Q)}{16 \pi^2} m_{3/2},
\]

\(g_i(Q)\) are the gauge couplings of the SM renormalized in the \(\overline{\text{MS}}\) scheme at a generic scale \(Q\), and
Dark matter in mini-split SUSY

- Rich phenomenology as $\mu$ is allowed to vary.
- Nature of LSP defined by $C_\mu = \frac{\mu m_A^2 \sin 2\beta}{m_A^2 - \mu^2} \ln \frac{m_A^2}{\mu^2}$.
| | $|C_\mu| \lesssim 4$ the LSP is the Wino (this is also the case of the usual Split SUSY with $\mu$ at the EW scale).

- $M_{\tilde{W}} = 2.7$ TeV if the Wino is a DM thermal relic.

- Most favorable case $M_{\tilde{g}} = 1.2 \times 2.7 = 3.24$ TeV $\to$ out of the LHC reach.
Dark matter in mini-split SUSY

- $C_\mu < 3.9$ and $4.1 < C_\mu < 7.8$ the LSP is the Bino.
- Thermal relic abundance would overclose the universe (need late entropy injection/low reheat temperature).
- In the range $4.1 < C_\mu < 7.8$ compressed gaugino mass spectrum.
- $C_\mu < 7.8$ the LSP is the gluino.
- Not acceptable for DM but interesting for collider phenomenology.
- Cosmological constraints evaded with R-violating effective interactions.
For $|C_\mu| \approx 4$, the LSP can be a well-tempered Bino-Wino.

For 10% mass splittings, the mass can be in the range of hundreds of GeV.

$M_\tilde{g}/M_\tilde{W} = 2.4$ (for $C_\mu \approx -4$) and $M_\tilde{g}/M_\tilde{W} = 1.2$ (for $C_\mu \approx 4$).

Case particularly favorable for the LHC.
For $C_\mu \approx 0$, the LSP can be a mixture of higgsino and wino.

- large annihilation cross section, not much gained in terms of relic abundance.

- Detection at DM experiments can be sizable due to Higgs boson exchange.
Dark matter in mini-split SUSY

- For $C_\mu \approx 7.8$, allows for an unusual co-annihilation between gluino and bino.
- For splittings in 100-150 GeV range, the bino can be a thermal relic DM and the gluino within reach of the LHC.
- However difficult detection at the LHC due to soft decay products.
For $C_\mu \approx 7.8$, allows for an unusual co-annihilation between gluino and bino.

For splittings in 100-150 GeV range, the bino can be a thermal relic DM and the gluino within reach of the LHC.

However difficult detection at the LHC due to soft decay products.
Dark matter in mini-split SUSY

\[ C_\mu = \frac{2 \mu \tan \beta}{m_{3/2}} \frac{\tilde{m}^2 + \mu^2}{(\tan^2 \beta + 1) \tilde{m}^2 + \mu^2} \ln \left[ (1 + \tan^{-2} \beta) \left( 1 + \frac{\tilde{m}^2}{\mu^2} \right) \right] \]

- We can express \( C_\mu \) in terms of the original parameter of the model.

- The 1-10 range can be easily obtained for natural choices of the parameters.
**Dark matter in mini-split SUSY**

- **Wino LSP** – **Bino LSP** – **Gluino/Bino LSP** (depends on the sign of $\mu$).

- Assumed exact universality of the scalar masses $\tilde{m} = m_{3/2}$.

- Contours of $\tan\beta$ as extracted from the Higgs mass measurements (mild dependence on $\mu$).

- Dashed blue line is where Wino DM abundance reproduces the observed DM density.

- Dash black line if $m_{H_d}^2 = \tilde{m}^2 = m_{3/2}^2$ is assumed.
Mini-split SUSY

- Spectrum characterized by:
  1. SUSY scalars at the mass scale $\tilde{m}$. Typical size related to the gravitino mass $\tilde{m} \approx (M_{PL}/M_*)m_{3/2}$ where $M_*$ is the mediation scale.
  2. Anomaly mediation gives precise predictions for the physical size of the gauginos in terms of $m_{3/2}$.
  3. $\mu$ is expected to be of $\mathcal{O}(m_{3/2})$ if there is no suppression related to PQ breaking. Otherwise $\mu$ is a free parameter between $m_{3/2}$ and the EW scale.
  4. $B_\mu$ is of order $\tilde{m}^2$
  5. $\tan \beta$ can assume any value, unless $\mu$ and $B_\mu$ are generated by the same operator.