Baryon helicity in B decay

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Abstract

The unexpectedly large transverse polarization measured in the decay $B \rightarrow \phi K^*$ poses the question whether it is accounted for as a strong interaction effect or possibly points to a hidden nonstandard weak interaction. We extend here the perturbative argument to the helicity structure of the two-body baryonic decay and discuss qualitatively on how the baryonic $B$ decay modes might help us in understanding the issue raised by $B \rightarrow \phi K^*$. We find among others that the helicity $+1/2$ amplitude dominates to the leading order in the $B (\bar{b}q)$ decay and that unlike the $B \rightarrow VV$ decay the dominant amplitude is sensitive to the right-handed $b \rightarrow s$ current, if any, in the penguin interaction.

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I. INTRODUCTION

In the two-body $B$ decay into light vector-mesons, the vector mesons should polarize longitudinally according to the simple $1/m_B$ power counting in the perturbative picture. The measured values of the longitudinal decay fraction $f_L$ are close to unity for the $\rho\rho$ modes [1–3] in good agreement with this theoretical prediction. Quantitatively more reliable calculation can be made for $D^*\rho$ with the heavy-quark symmetry and the experimental value of $f_L \approx 0.9$ [4] agrees with theory [5]. However, this simple prediction unexpectedly broke down for the decay $B \to \phi K^*$. The value of $f_L$ turned out to be approximately 0.5 for $B \to \phi K^{*0}$,

$$f_L = \begin{cases} 0.43 \pm 0.09 \pm 0.04 & [6], \\ 0.52 \pm 0.07 \pm 0.02 & [7]. \end{cases}$$  \hspace{1cm} (1)

The observed value for the charged mode $\phi K^{*+}$ is consistent with them; $f_L = 0.46 \pm 0.12 \pm 0.03$ [2]. As for the $\rho^+ K^{*0}$ modes, the situation is inconclusive at present since the numbers given by BaBar and Belle Collaborations are not perfectly consistent with each other; $0.79 \pm 0.08 \pm 0.04 \pm 0.02$ [8] vs $0.50 \pm 0.19^{+0.05}_{-0.07}$ [9].

Dominance of the longitudinal helicity is a direct consequence of the fact that the weak and strong forces are both mediated by gauge interactions, that is, chirality-conserving vector-axial-vector interactions. The longitudinal dominance should hold for all types of the decay interaction, either the tree or the penguin type, of the standard model. In the limit that the light quark ($u,d,s$) masses are zero and the valence $q\bar{q}$ are collinear inside fast mesons, the longitudinal fraction $f_L$ would be unity for all of $\rho\rho$, $\rho K^*$ and $\phi K^*$. The difference between the $s$-quark in $\phi K^*$ and the $u/d$-quark in $\rho\rho$ should not be important if the strong interaction is strictly perturbative except at hadron formation.

There are two conceivable origins of the large transverse polarization in $B \to \phi K^*$. The first one is breakdown of short-distance QCD dominance. That is, the strong interactions at long and/or intermediate distances may be somehow enhanced and cause helicity flip of quarks. For instance, if on-shell charm-anticharm meson intermediate states are important in the decay $B \to \phi K^*$, spins of slow charmed hadrons could flip with long-distance interactions and this effect would propagate into the light mesons in the final state [10]. But our limited knowledge of dynamical parameters of the charm hadron sector makes a reliable estimate difficult. Another proposal has been made from the perturbative side: It was argued [11] that soft collinear quarks and gluons can enhance the annihilation decay process, which would be otherwise subleading in $1/m_B$. Although the spin flip cost another $1/m_B$, the soft-collinear loop corrections in the annihilation decay might generate significant helicity flip in the case of $\phi K^*$ [12]. While one can parametrize such an effect, numerical estimate is subject to the uncertainty in the infrared and collinear cutoff. Yet another proposal is that the color-dipole decay operators may be nonperturbatively enhanced to generate a large transverse polarization [13]. Many different proposals are being made to point to possible sources and mechanisms of the long-distance interactions responsible for the large transverse polarization of $\phi K^*$. However, it is not clear at present whether any of these proposals will really explain it as a strong interaction effect.

If the origin is not in strong interaction, a nonstandard decay interaction must be responsible. Is there a new decay interaction whose chirality structure is different from the
standard gauge interaction? The case for such a new decay interaction is severely con-
strained. First of all, a new interaction must be of the scalar-pseudoscalar or the tensor
type. It should couple preferentially with the $s$-quark if the problem exists only in $\phi K^*$, not in $\rho K^*$. Furthermore the coupling should not have the quark-mass suppression $m_q/m_W$ unlike the standard Higgs coupling. While the possibility of the tensor weak coupling was pointed out [14], it is yet to find a way to incorporate such an ad hoc interaction in the context of the electroweak gauge theory.

The fundamental issue is whether the breakdown of the helicity rule is due to failure of the perturbative picture or to a new weak interaction. If nonperturbative strong interactions are responsible, how and where do they enter the decay processes? In addition to the pursuit from the theoretical side, more experimental information will help in reaching the root of the problem. Study of the decay $B \rightarrow V(1^-)T(2^+)$ such as $B \rightarrow K^*f_2$ and $\phi K_2$ will be useful for this purpose. Indeed the first crude measurement of polarization has been made for the latter [15]. We call attention here usefulness or relevance of the two-body baryon decays to the issue raised by the two-body meson modes. For instance, if large long-distance physics enters $B \rightarrow \phi K^*$ through the soft collinear corrections to the annihilation process, the violation of the helicity rule would smaller in the corresponding baryonic decay modes since the annihilation decay is suppressed more severely for the baryonic decays than for the mesonic decays. As for the exotic decay interaction, one advantage of the baryonic decay over the mesonic decay is that the dominant helicity amplitude is sensitive to the right-handed current. The helicity rule in the baryon-pair modes was discussed in an early paper by Körner [16]. He studied the baryonic decays with the tree interaction of $V-A$ using dynamical models for additional $q\bar{q}$ emission. Now the penguin interaction is of our primary concern because of the $\phi K^*$ puzzle. We distinguish between the two processes here and present the results in a way relevant to the current issue of the $\phi K^*$ polarization.

In the case of $B \rightarrow VV$, separation of the decay amplitudes into opposite-sign helicities $h = \pm 1$ requires measurement of the $s/p$-wave interference between the resonant $VV$ and the nonresonant $VPP$ background [17] or a three-body decay correlation for some modes [18] or else the angular correlation between decay products of different parents [19]. In contrast, the helicity amplitudes of $h = \pm \frac{1}{2}$ in the baryonic decay can be easily separated with the angular analysis of a single hyperon in the final state if the decay violates parity. It is done as part of hyperon identification. Although the branching fractions of the baryon-pair modes are small according to early indications [20], the simplicity in analysis will work to our advantage and allow us to accomplish the goal with much smaller samples of data on the baryonic modes.

The paper is organized as follows: After a brief review of the perturbative helicity selection rule for $B \rightarrow VV$ and its comparison with the data in Section II, we discuss the helicity rule for the baryon-antibaryon pair modes in Section III. In Section IV we discuss how to extract helicity information from measurement and then select the baryonic modes that are useful for our purpose. We will not attempt detailed dynamical computation of the baryonic decay amplitudes since despite numerous theoretical efforts over years [21,22] the results are

\footnote{The $S$-$P$ interaction arising from Fierz rearrangement of the penguin operators does not solve the problem.}
numerically less reliable for the baryonic modes than for $B \to VV$. Instead we quote only semiquantitative estimates which are based primarily on simple perturbative dynamics and symmetry, not on the specific form factors or the value of $\alpha_s$. Such crude estimates are in good agreement with experiment for $B \to VV$ other than $\phi K^*$ and $\rho K^*$. In Section V, we summarize our results and discuss prospects in theory and experiment.

II. PERTURBATIVE COUNTING RULE FOR MESON PAIRS

The perturbative helicity rule in B decay is based on two facts of the standard model. First, the weak and strong interactions are both gauge interactions so that, whenever a light quark pair is produced, its chirality is given by $q_Rq_R$ or $q_Lq_L$, not $q_Rq_L$ nor $q_Lq_R$.\(^2\) The energetic quarks may be produced either directly by the decay interaction or through the hard gluon interaction. The quark chirality does not change by emission nor absorption of hard gluons. Secondly, final hadron states are formed in the leading order by superposition of valence quarks with the light-cone wavefunctions. Therefore, helicity of a fast hadron is determined by helicities of its energetic constituents, $\bar{q}q$ for mesons and $qqq$ for baryons. The terms neglected in this approximation are of higher orders in $1/m_B$ or of higher-twist contributions in terms of the wavefunctions and effective operators. Breakdown of the helicity prediction therefore means that some long and/or intermediate distance strong interaction is enhanced to overcome the power suppression of $1/m_B$.

Under these conditions the chiral content of the energetic quarks produced in the final state of $B \to VV$ is:

$$\sim (\bar{q}_Lq_s)(\bar{q}_Lq_L) \text{ or } (\bar{q}_Lq_s)(\bar{q}_Rq_R),\quad (2)$$

where $q$ stands for the quark state of $u,d,s,c$, the subscript of $q_s$ stands for the “spectator”. It is understood that the colors are saturated appropriately. By parity invariance, $q_s$ has equal probabilities of spin up and down. The chiral content of Eq. (2) would not change in the limit of $m_q \to 0$ and $m_V \to 0$ even after any number of hard QCD interactions may take place. Eq. (2) gives the chiral content of the valence quarks/antiquarks of $VV$ not only for the spectator decay processes but also for the annihilation and exchange decay processes.

To derive the helicity rule, consider the decay,

$$B(\bar{b}q_s) \to V_1(\bar{q}q) + V_2(\bar{q}q_s).\quad (3)$$

If $q_L$ and $\bar{q}_L$ fly in parallel to form one vector meson, this meson $V_1(\bar{q}_Lq_L)$ is in the helicity state of $h = 0$. In the other meson $V_2$, the $\bar{q}_Lq_s$ pair alone can make $h = 0$ or $+1$ since the spin of $q_s$ can point to either direction. But requirement of the overall $J_z = 0$ forces the $V_2$ helicity to $h = 0$ in this case. (See the first figure in Fig.1.) The same argument holds in the case of $V_1(\bar{q}_Rq_R)$.

\(^2\)In the perturbative power counting, the light-quark-pair production through the color-magnetic decay operator $\propto \bar{b}_R \sigma_{\mu\nu} G^{\mu\nu} q_L$ picks up $O(1/m_B)$ through the light quark pair production $\bar{q}_L \sigma^{\kappa\lambda} G_{\kappa\lambda} q_R + (R \leftrightarrow L)$.
FIG. 1. The helicities of quarks and antiquarks in $B \rightarrow VV$ for $\bar{q}_L q_L + \bar{q}_L q_s$ (the upper figure) and for $\bar{q}_R q_L + \bar{q}_R q_s$ (the lower figure). The solid arrows indicate the quark-number directions, and the large open arrows stand for the dominant helicities. The two-end open arrow is for $q_s$.

Alternatively, if $q_R$ and $\bar{q}_R$ try to form $V_1$, helicity of $V_1 (\bar{q}_L q_R)$ is in $h = +1$, but helicity of $V_2 (\bar{q}_R q_s)$ can be only $h = 0$ or $-1$, not $+1$ (the second figure in Fig. 1). This conflicts with $J_z = 0$. Therefore one concludes that the only allowed helicity state is the longitudinal ($h = 0$) state for $V_1 V_2$. The kinematical corrections to this rule arise in $O(1/m_B)$ from the transverse motion of $qq$ inside a meson and the nonvanishing quark masses. Computation of these higher-twist terms can be carried out for $B \rightarrow VV$ by the QCD factorization method. In the case that one of the final mesons is a charmed meson, the form factor can be computed reliably with the heavy quark symmetry. In the case of light meson pairs, final results involve larger uncertainties due to the light-cone wavefunctions and the value of $\alpha_s$. Without going through this computation, however, a reasonable order-of-magnitude estimate can be made as we shall do below.

The $h = +1$ amplitude is realized by the small wrong helicity component of $q_L$ in $V_1$ when $\bar{q}_L q_L$ form $V_1$ as $\bar{q}_L (x p) (\gamma_1 - i \gamma_2) q_L ((1 - x) p)$ and the right-chiral component of $q_s$ combines $\bar{q}_L$ to form $V_2$ as $\bar{q}_L \sigma^{\mu \nu} q_s$. For the $h = -1$ amplitude, the wrong helicity component is needed for $\bar{q}_L$ in $V_1$ and also for $\bar{q}_L$ in $V_2$. According to the chiral projection of the plane Dirac wave, the wrong helicity component is suppressed by $m_q/(E_k + |k|)$. The transverse motion of $\bar{q}q$ inside a meson also acts as an effective quark mass under the longitudinal Lorentz transformation. Consequently the effect of the internal motion on the helicity can be incorporated by replacing the (current) quark mass $m_q$ with the transverse quark mass $m_T = \sqrt{m_q^2 + k_T^2}$. We are thus led to the well-known hierarchy of the helicity amplitudes $H_{hh}$ for $B \rightarrow V_1(p_h)V_2(-p_h)$ [5,25],

$$
\frac{|H_{++}|}{|H_{00}|} \approx \frac{|H_{--}|}{|H_{++}|} \approx \left\langle \frac{m_T}{E_k + |k|} \right\rangle, \quad (m_T \ll E_k),
$$

(4)

where the bracket $\langle \rangle$ denotes the average over the quark momentum with the light-cone wavefunction. It is a reasonable approximation to set $m_T \approx \frac{1}{2} m_V$ for the light mesons or a little more accurately $m_T \approx \sqrt{\frac{2}{3}} \times \frac{1}{2} m_V$ by taking account of $\langle k_T^2 \rangle = \frac{2}{3} \langle k^2 \rangle$. With $\langle E_k \rangle \approx \frac{1}{2} E_V \approx \frac{1}{4} m_B$, Eq. (4) is a counting rule in $1/m_B$ based on kinematics. It applies to

3Interchange as $H_{++} \leftrightarrow H_{--}$ for the $\bar{B}(b\bar{q})$ decay.
decay amplitudes of a given decay operator. A total amplitude may be sum of terms from different operators in general. Instead of going through detailed dynamical calculation, we proceed here with a semiquantitative estimate. Let us substitute $|k|$ with its peak value of distribution $\frac{1}{2}|p|$. Then we obtain with Eq. (4) the magnitude of the longitudinal fraction $f_L \equiv |H_{00}|^2 / \sum_h |H_{hh}|^2$ as

$$f_L \approx \begin{cases} 0.98 \sim 0.99 & (\rho \rho, \rho K^*) \\ 0.96 & (\phi K^*) \end{cases}.$$  

If the contribution of the end points of the wavefunctions is enhanced, these numbers can deviate more from unity. They are in line with measurement for the tree-dominated $\rho \rho$, off by two standard deviations or more on the larger side for the penguin-dominated $\rho K^*$, and clearly far too large for $\phi K^*$ which is expected to be almost purely a penguin decay. If a longitudinal helicity amplitude consists of more than one term and large cancellation occurs between different terms, the ratio of the transverse-to-longitudinal amplitude could be enhanced. We would need suppression of factor five for $H_{00}$ to explain $f_L \approx 0.5$ for $\phi K^*$ by such cancellation, which would translate to suppression of the $\phi K^*$ branching fraction by a factor of 25 relative to the case without cancellation. The observed values of the penguin decay branching fractions are not by order-of-magnitude off the conventional theoretical estimate. Therefore, it is not easy to attribute the observed large transverse polarization particularly in $\phi K^*$ to a strong suppression of the dominant longitudinal amplitude by cancellation:

It is tempting to attribute the large transverse polarization of $\phi K^*$ to a new interaction of an unconventional chiral structure hidden in the penguin loop. However, as long as the strong interaction dynamics is of short distances, the right-hand weak current would not solve the problem. Because the only difference of the right-handed weak interaction from the left-handed weak interaction is to interchange $H_{++} \leftrightarrow H_{--}$ in Eq. (4). To violate the helicity selection rule of Eq. (4), such a new interaction must emit a quark pair through the $S$-$T$-$P$ interaction, $\overline{q}_R q_L \pm \overline{q}_L q_R$, instead of $V$-$A$ interaction, $\overline{q}_R q_L \pm \overline{q}_L q_R$. The effective $S$-$T$-$P$ interaction from the Fierz-rearrangement of the left-right current interaction does not help since the helicity argument at the beginning of this Section can be made equally well for the interaction prior to the Fierz rearrangement. Because the coefficients of $S$-$T$-$P$ are fixed in such a case so that only $H_{00}$ survives after the $S$-$T$-$P$ contributions are summed over. Furthermore, if one has to explain that the transverse polarization is more pronounced in $\phi K^*$ than in $\rho K^*$, the new interaction should affect more strongly on $s$-quark than on $u/d$-quark. The Higgs interaction indeed follows such a coupling pattern, but the magnitude of the standard Higgs coupling is far too small to be relevant.

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4 The same estimate leads to $f_L \approx 0.92 - 0.93$ for $D^* \rho$. This number includes the $c$-quark mass effect (with the constituent mass $m_c \approx 1.7$ GeV) in the left-chiral $\pi$-quark state in $\mathcal{D}^0$. The value of $f_L$ does not deviate much from unity since it is the wrong helicity of $u_L/d_L$ in $\rho$ not of $\overline{c}_L$ in $\mathcal{D}^0$ that is needed to realize $h = +1$ for this tree-decay process. It is in reasonable agreement with experiment, $0.890 \pm 0.018 \pm 0.016(\mathcal{D}^{*0})$ and $0.885 \pm 0.016 \pm 0.012(\mathcal{D}^{*-})$ [23].
III. BARYON-ANTIBARYON MODES

When a baryon-antibaryon pair $\mathcal{B}\overline{\mathcal{B}}$ is produced in $B$ decay with the four-quark decay interaction, an additional pair of $q_Rq_R$ or $q_Lq_L$ must be produced through strong interaction. Since our interest is to test the perturbative picture of the helicity structure for the $\mathcal{B}\overline{\mathcal{B}}$ modes, we consider the case where the gluon producing the $q\bar{q}$ pair is hard and highly virtual as much as $\sqrt{s} = \frac{1}{2}m_B$ on average. This costs $\alpha_s$ suppression, which is unavoidable and common to all perturbative $\mathcal{B}\overline{\mathcal{B}}$ production. Körner noticed [16] that in the tree interaction of $V - A$, helicity mismatch occurs when the hard quark-antiquarks enter $\mathcal{B}\overline{\mathcal{B}}$ as valences directly from the primary decay interaction. He derived selection rules assuming no subsequent hard bend of the primary momenta. The helicity mismatch does not occur for the penguin interaction since it contains $(V - A)(V + A)$. Once a hard gluon emission is considered, however, the tree decay process can be saved from the helicity conflict under a certain circumstance: Imagine that the hard gluon emission reverses the primary quark momentum and takes away one unit of helicity. In this case the primary quark (or antiquark) momenta need not be parallel at the time of emission from the weak interaction. Then the selection rule of [16] is evaded. A closer inspection of the matrix element proves that the tree-decay amplitudes are indeed allowed even in the massless quark limit when a virtual gluon gives a hard backward kick to one of the primary quarks.

In our leading-order process all quark-antiquarks carry robust momenta except for the spectator, while in Körner’s picture the $q\bar{q}$ pair produced by a gluon is likely to be much softer. It is a dynamical question which process is more important to the actual baryonic decay modes through the tree process; the $\alpha_s$ suppression versus $(m/E \times$ the tail of the quark distribution in the baryon). The purpose of our paper is to derive and test the perturbative helicity rule for $\mathcal{B}\overline{\mathcal{B}}$ as an extension of the helicity rule in $B \rightarrow VV$. Therefore we assume the perturbative picture and proceed to study the $m/E$ expansion here.

Let us move to our argument. In the simple perturbative picture, three quarks fly in one direction and turn into valence quarks of a baryon while three antiquarks fly to the opposite direction and turn into valence antiquarks of an antibaryon. We derive the helicity selection rule on the same assumptions as in $B \rightarrow VV$. The chiral content of quarks and antiquarks is any one of the following three possibilities:

$$\begin{align*}
\left\{ \begin{array}{l}
(q_Lq_s)(\overline{q_L}q_L)(\overline{q_L}q_L) \cdots (A), \\
(q_Lq_s)(\overline{q_L}q_L)(\overline{q_R}q_R) \cdots (B), \\
(q_Lq_s)(\overline{q_R}q_R)(\overline{q_R}q_R) \cdots (C),
\end{array} \right.
\end{align*}$$

(6)

where colors are saturated separately among $qqq$ and among $\overline{qqq}$. In the standard model, the $(\overline{q_R}q_R)$ pair can come only from the gluon interaction. Therefore, the final quark state (A) and (B) are produced by either the tree or the penguin interaction, but the state (C) can be realized only by the penguin interaction. In these final states the antibaryon helicity can take the value of $h = +\frac{2}{3}$ [$(q_Lq_Lq_L)$ of (A)], $h = +\frac{1}{3}$ [$(q_Lq_Rq_L)$ of (B)] or $-\frac{1}{3}$ [$(q_Rq_Rq_R)$ of (C)]. The helicity of the baryon $(q_sqq)$ must match the antibaryon helicity to satisfy the overall $J_z = 0$ condition. The matching is possible only in the case of $h = +\frac{1}{2}$ from (B) since $h = -\frac{3}{2}$ or $-\frac{1}{2}$ for $(q_sqq)L$ of (A), $h = -\frac{1}{2}$ or $+\frac{1}{2}$ for $q_sq_Rq_L$ of (B), and $h = \frac{3}{2}$ or $-\frac{1}{2}$ for $q_sq_Rq_R$ of (C). This helicity matching is shown in Table I and depicted in Fig.2. It is easy to understand why neither the case (A) nor (C) can satisfy $J_z = 0$: When two pairs of quark-antiquark
TABLE I. Leading helicity states of the baryon and the antibaryon which are realized by three classes of quark contents (A, B, C). Only $+\frac{1}{2}$ (in boldface) of the column B is compatible with $J_z = 0$.

| Hadrons   | A                              | B                              | C                              |
|-----------|--------------------------------|--------------------------------|--------------------------------|
| Antibaryon| $+\frac{1}{2}(q_Lq_Lq_Lq_L)$  | $+\frac{1}{2}(q_Lq_Rq_L)$     | $-\frac{1}{2}(q_Lq_Rq_R)$     |
| Baryon    | $-\frac{1}{2}, -\frac{3}{2}(q_sq_Lq_L)$ | $+\frac{1}{2}, -\frac{1}{2}(q_sq_Rq_L)$ | $+\frac{3}{2}, +\frac{1}{2}(q_sq_Rq_R)$ |

$q_Lq_Lq_Lq_L$ (or $q_Rq_Rq_Rq_R$) fly back to back, they are in the state of $J_z = -2$ (or +2) along the baryon momentum. Then the remaining pair $q_Lq_s$ has no way to turn total $J_z$ to zero.

When the mass and transverse momentum corrections are included, the state of $h = -\frac{1}{2}$ is allowed for (B) with the small $h = -\frac{1}{2}$ component of $q_L$ and for (C) with that of $q_R$. The state of $h = +\frac{3}{2}$ requires two small components and the state of $h = -\frac{3}{2}$ needs three small components. In terms of the helicity amplitudes $H_{hh}$ for $B \to B(p)\overline{B}(-p)$, therefore, we expect most generally the hierarchy of

$$
\left| \frac{H_{-\frac{3}{2}, -\frac{3}{2}}}{H_{+\frac{3}{2}, +\frac{3}{2}}} \right| \approx \left| \frac{H_{+\frac{1}{2}, +\frac{1}{2}}}{H_{-\frac{1}{2}, -\frac{1}{2}}} \right| \approx \left| \frac{H_{-\frac{3}{2}, +\frac{3}{2}}}{H_{+\frac{1}{2}, -\frac{1}{2}}} \right| \approx \left\langle \frac{m_T}{E_k + |k|} \right\rangle,
$$

where $m_T/(E_k + |k|) \ll 1$. For the $\overline{B}$ ($bq_s$) decay, the dominant helicity amplitude is $H_{-\frac{3}{2}, -\frac{3}{2}}$ and the hierarchy similar to Eq. (7) holds with interchange of $H_{hh} \leftrightarrow H_{-h,-h}$. We have tabulated in Table II this helicity suppression for the amplitudes in each class (A $\sim$ C) of Eq. (6).

The amplitude of our primary interest is $H_{\pm\frac{3}{2}}$, since we can separate $H_{+\frac{3}{2}, +\frac{3}{2}}$ from $H_{-\frac{3}{2}, -\frac{3}{2}}$ only in $B \to \overline{B}B$. (See Section IV.) Since the $h = -\frac{1}{2}$ state can be realized by the small wrong helicity of either of two $\overline{q}_L$’s for (B), we may include this multiplicity factor in the ratio $H_{-\frac{1}{2}, -\frac{1}{2}}/H_{+\frac{1}{2}, +\frac{1}{2}}$;

$$
\left| \frac{H_{-\frac{1}{2}, -\frac{1}{2}}}{H_{+\frac{1}{2}, +\frac{1}{2}}} \right| \approx 2\left\langle \frac{m_T}{E_k + |k|} \right\rangle.
$$

The approximation of $m_T \ll E_p$ is a little less accurate for baryons than for vector-mesons since there are three valences instead of two and therefore the valences are slightly less energetic.

The fraction of the helicity content

$$
f_+ \equiv \frac{|H_{+\frac{1}{2}, +\frac{1}{2}}|^2}{|H_{+\frac{3}{2}, +\frac{3}{2}}|^2 + |H_{-\frac{1}{2}, -\frac{1}{2}}|^2}
$$

FIG. 2. The dominant helicities of quarks and antiquarks ($q_Lq_Rq_s + \overline{q}_Lq_Rq_R$) in $B \to B\overline{B}$ for the class (B). The open arrows indicate the helicity directions.
TABLE II. The helicity factors for the amplitudes $H_{hh}$ for a given decay operator in the three classes $A \sim C$ of the final quark states. Here $s \equiv (m_T/(E_k + |k|))$ is the helicity suppression factor. The $H_{+{1\over 2}+{1\over 2}}$ amplitude of $B$ is the dominant helicity amplitude. The relative normalization between different classes depends on dynamics as well as on the Wilson coefficients and the CKM-factors.

| $J^P$ of $B\overline{B}$ | $h$ of $H_{hh}$ | $A$ | $B$ | $C$ |
|------------------------|----------------|-----|-----|-----|
| $1^+ {1\over 2}+ {1\over 2}$ | $+{1\over 2}$ | $4s^2$ | 2 | 4s |
|                        | $-{1\over 2}$ | $2s^2$ | $-4s$ | 4s |
| $1^+ {3\over 2}+ {3\over 2}+{1\over 2}$ | $+{1\over 2}$ | $2\sqrt{2}s^2$ | $\sqrt{2}$ | $2\sqrt{2}s$ |
|                        | $-{1\over 2}$ | $-\sqrt{2}s^2$ | $2\sqrt{2}s$ | $-2\sqrt{2}s$ |
| $3^+ {3\over 2}+ {3\over 2}$ | $+{3\over 2}$ | $s^2$ | $3s^2$ | $-3s^2$ |
|                        | $+{1\over 2}$ | $2s^2$ | 1 | 2s |
|                        | $-{1\over 2}$ | $s^2$ | $-2s$ | 2s |
|                        | $-{3\over 2}$ | $-s^3$ | $-3s^3$ | $3s^3$ |

for $B \to B\overline{B}$ of $J^P = {1\over 2}^+$ can be estimated with Eq. (8) when one of the tree or the penguin operators dominates. Following the approximation made in $B \to VV$, we substitute a third of the baryon/antibaryon momentum for the valence momentum $k$ and set $m_T = \sqrt{2\over 3} \times {1\over 3} m_{\text{baryon}}$. For the kinematics of the mode $B \to \Lambda p$, for example, we find

$$f_+ \approx 0.89. \quad (\text{penguin})$$  \hspace{1cm} (10)

This is a ball-park figure for all $B\overline{B}$ modes. It should be reminded again that this is the number when a single decay operator dominates. If the dominant $H_{+{1\over 2}+{1\over 2}}$ amplitude consists of more than one term and large cancellation occurs among them, the value of $f_+$ can be smaller. However, if a large cancellation occurs within the $H_{+{1\over 2}+{1\over 2}}$ amplitude, its branching fraction would be abnormally small. Considering the small branching fractions of the $B\overline{B}$ modes in general, we will not be able to observe such abnormally suppressed $B\overline{B}$ modes in the near future.

If the $b$-quark should decay into $\overline{u}R$ through the right-handed current in either the penguin or the tree process, the $H_{+{1\over 2}+{1\over 2}}$ amplitude would dominate in such a process according to the argument above. In the baryonic decay, therefore, the chirality of the weak current manifests itself directly in the dominant helicity amplitude. In contrast, the chirality of the current affects only the subdominant helicity amplitudes in the two-body meson decays.

IV. BARYONIC DECAY MODES OF INTEREST

When the baryon (antibaryon) decays by strong interactions, the angular correlation of the decay products with the baryon momentum cannot distinguish between helicity $h$ and $-h$ since the correlation takes the same form for $h = \pm 1$ by parity conservation. This may look potentially a serious obstacle for carrying out the helicity test for the $B\overline{B}$ modes. Fortunately, however, hyperons decay through parity-violating weak interactions and the parity violation can separate between helicity $\pm h$ and allow us to determine $f_+$ with a relatively small number of events.
Let us take for concreteness the decay $B \to \bar{\Lambda}p$ again and choose $\bar{\Lambda}$ as the spin analyzer. The decay process is
\begin{equation}
B \to \bar{\Lambda}(p) + p(-p) \nonumber \rightarrow \bar{p}(q) + \pi^+(q), \tag{11}
\end{equation}
where $q$ is the decay momentum of $\bar{p}$ in the rest frame of $\bar{\Lambda}$. Then the decay angular distribution is given by
\begin{equation}
\frac{d^2\Gamma}{d\Omega_p d\cos\theta_q} = \frac{\Gamma_0}{8\pi} (1 + \tilde{\alpha}_\Lambda \cos\theta_q), \tag{12}
\end{equation}
where $\theta_q$ is the polar angle of $q$ with respect to $\bar{\Lambda}$ momentum $p$. It is easy to show that the asymmetry $\tilde{\alpha}_\Lambda$ is expressed with the nonleptonic decay parameter $\alpha_\Lambda$ and the helicity ratio $f_+$ in the form of
\begin{equation}
\tilde{\alpha}_\Lambda = (2f_+ - 1)\alpha_\Lambda. \tag{13}
\end{equation}
Note that according to approximate CP invariance in the hyperon decay, $\alpha_\Lambda = -\alpha_\bar{\Lambda}$ holds to accuracy of $O(10^{-4})$ or better.

Since we determine the helicity amplitudes with a parity-violating decay, we should choose $B$ or $\bar{B}$ from the hyperons or the antihyperons which decay nonleptonically with large parity asymmetry. Therefore $\Lambda$, $\Xi$, $\Sigma^*(\to p\pi^0)$ and their antiparticles are suitable for the spin analyzer. The baryon or the antibaryon that is not the spin analyzer may be a baryon resonance, though reconstruction with too many particles will degrade accuracy of $f_+$.

The observation of the large transverse polarization in $B \to \phi K^*$ points to the penguin process $b \to ss$ as a primary suspect. When an additional $ss$ pair is created by a gluon in $b \to ss$, the final quark state can end up in $\Omega \Xi$. Since the $ss$ from the additional pair fly back to back in $\Omega \Xi$, this must be a hard QCD process. Since this decay cascades down to six hadrons ($p\pi\pi\pi\pi K$), however, it will not be one of the easiest modes to reconstruct. In comparison the decay $\bar{\Xi} \Lambda$ can be more easily studied. This decay occurs through either "$\bar{b} \to ss$ (penguin) + $\bar{u}u$($d\bar{d}$)" or "$\bar{b} \to \bar{u}u$ (CKM-suppressed tree) + $\bar{ss}$". Since the tree process is strongly suppressed by the CKM-factors just as in $\rho K^*$, it is safe to assume that $B \to \bar{\Xi}Y$ ($Y = \Lambda, \Sigma$) is dominated by the penguin process $\bar{b} \to \bar{ss}$.

We thus expect that the mode $B \to \bar{\Xi}Y$ is the most suitable baryonic mode to study the issue raised by $\phi K^*$ in the penguin decay. When two nonstrange-quark pairs are emitted in the $\bar{b} \to \bar{ss}$ penguin process, the final baryon state is $\bar{\Xi}N$. This mode corresponds to $\rho K^*$ of $VV$. In contrast to $\phi K^*$ and $\rho K^*$, the $\rho\rho$ mode proceeds mainly through the tree decay $\bar{b} \to \bar{u}_{LL}u_{LL} + \bar{q}q$ since the penguin process $\bar{b} \to \bar{u}_{LL}(u\bar{u} + d\bar{d}) + \bar{q}q$ is down by the loop-suppression in the Wilson coefficients relative to the tree process. Therefore $\bar{\Xi}N$ is an $B\bar{B}$ counterpart of $\rho\rho$. However, this mode is not useful for the polarization study since we need a hyperon as a spin analyzer. If one goes after the tree decay, an alternative is the mode $\bar{\Sigma}Y$ which is dominated by the tree decay $\bar{b} \to \bar{u}_{LL}u_{LL} + \bar{s}s$. In short, the strangeness-changing modes ($\Delta S = 1$) are dominated by the penguin decay while the strangeness-conserving modes are dominantly through the tree process $\bar{b} \to \bar{u}_{LL}u_{LL}(s\bar{s})$. This observation is not our original, but rather a consensus among theorists in the recent
papers on the subject [22]. This simple approximation is in line with the limited numerical accuracy of our semiquantitative analysis.

With these remarks in mind, we have selected the promising baryonic modes and listed in Table III. They are the modes which require reconstruction of no more than five stable particles and do not contain a neutron. We have not listed the modes that contain $\Sigma^0$ since reconstruction of $\Lambda\gamma$ is often difficult. Although the helicity separation is impossible, we have included the $\bar{p}p$, $\bar{p}\Delta^{++}$ and $\bar{\Sigma}^0p$ modes in the Table since they give us an idea of how large the branching fractions of the interesting modes should be. The spin content of fast moving baryons is determined by the Lorentz-boosted valence quark spins, ignoring higher Fock states. We can relate the valence quark distributions of the octet and decuplet baryons with different $(I,Y)$ by using the constituent quark model, i.e., spin-flavor SU(6) symmetry. Then the baryon decay amplitudes are related to each other within each class ($A \sim C$) to the leading order of $\alpha_s/\pi$ for short-distance QCD. In Table III the relative magnitudes of the dominant helicity amplitudes $H_{1/2}^{+}$ are given within the penguin and the tree decay. Since long-distance QCD is included only in the baryon formation, they are more restrictive than the most general SU(6) symmetry prediction.

| Modes ($\Delta S = +1$) | Penguin ($\bar{b} \to \bar{s}_L q_L q_L$) |
|-------------------------|----------------------------------|
| $B^0 \to$              |                                   |
| $\Sigma^- p$           | $\sqrt{6}/9)P$                   |
| $\bar{\Lambda}\Delta^0$| 0                                |
| $\bar{\Sigma}^-\Delta^+$| $-2\sqrt{3}/9)P$               |
| $\Xi^0\Lambda$         | $\sqrt{2}/3)P$                  |
| $\bar{\Xi}^+\Sigma^-$  | $5\sqrt{6}/9)P$                 |
| $B^+ \to$              |                                   |
| $\bar{\Lambda}p$       | $P$                              |
| $\bar{\Xi}^+\Delta^0$  | 0                                |
| $\bar{\Xi}^+\Delta^+$  | $-2\sqrt{3}/9)P$               |
| $\Xi^+\Lambda$         | $\sqrt{2}/3)P$                  |
| $\Xi^0\Sigma^+$        | $5\sqrt{6}/9)P$                 |
| Modes ($\Delta S = 0$) |                                   |
| $B^0 \to$              |                                   |
| $\bar{p}p$              | $T$                              |
| $\Sigma^0\Lambda$      | $-\sqrt{6}T$                    |
| $\bar{\Xi}\Lambda$     | 0                                |
| $\bar{\Lambda}\Sigma^+, \Sigma^+\Lambda$ | $-\sqrt{6}T$ |
| $B^+ \to$              |                                   |
| $\bar{p}\Delta^{++}$   | $-\sqrt{6}T$                    |
| $\bar{\Delta}^0 p$     | $-\sqrt{2}T$                    |
| $\Sigma^0\Sigma^+$     | $T$                              |
| $\bar{\Lambda}\Sigma^+, \Sigma^+\Lambda$ | 0 |

Apart from the obviously forbidden modes ($B \not\to \bar{\Lambda}\Delta$ by isospin), several simple tree decay modes ($\Delta S = 0$) are absent in the leading order:

(i) In the tree process ($\Delta S = 0$), $su_Ld_L$ form the antihyperon. Therefore the antihyperon in the $\Delta S = 0$ process is chargeless, that is, $\Sigma^\pm$ cannot be produced.
(ii) The $u\bar{d}$ in $\Lambda$ is in the spin-zero state and the $\Lambda$ spin is carried by the $s$ spin. Since $u_Ld_L (h = +1)$ is in the spin-one state, $suLCD_L$ cannot form $\Lambda$. Therefore the modes $\Lambda\Lambda$ and $\Lambda\Sigma^+$ are forbidden.

The magnitudes of the tree and penguin amplitudes $T$ and $P$ are left as free parameters in Table III. Without breaking up into helicity states, a few brave attempts were made in the past to compute the decay rates $[21,22]$. While most agree in regard to the tree dominance for $\Delta S = 0$ and the penguin dominance for $\Delta |S| = 1$, the ratio $|P/T|^2$ spreads widely over two orders of magnitude depending on the methods and assumptions of calculation (the pole model, the diquark model, the sum rule, etc). This large theoretical uncertainty is not surprising since the baryonic decay rates depend on how the additional $q\bar{q}$ is created. The less-known quark distribution in the baryon compounds the uncertainty, not to mention the interference between the color-allowed and color-suppressed amplitudes. We give here only a simple order-of-magnitude estimate of $|P/T|$ based on the CKM factors $[26]$ and the dominant Wilson coefficients $[27]$. In the standard notation,

$$\frac{|P|}{|T|} \approx 3\sqrt{6} \times \frac{C_6}{C_2} \left| \frac{V_{ub}^* V_{td}}{V_{tb}^* V_{ud}} \right| \approx 2,$$

where $3\sqrt{6}$ comes from our normalization of $P$ and $T$ in Table III.

From the viewpoint of branching fractions and simplicity of analysis, the mode $B^0 \to \Lambda p$ appears to be the most promising among the penguin decays, while the modes $\Sigma^* \to Y$ are interesting for study of the tree decays. The expected branching fractions for the $\Xi\Sigma$ modes are as high as that of $\Lambda p$. It should be noted that they are the values in the absence of long-distance QCD corrections and are sensitive to dynamics too. Therefore they should not be taken as reliable predictions for the branching fractions.

Although the modes listed as zero in Table III are all forbidden in the leading order of the perturbative picture, they are allowed if long and/or intermediate distance strong interactions are enhanced or if the higher Fock configuration turns out to be important. That is, if substantial branching fractions are observed for them in future experiment, we may count them as an independent evidence against the perturbative argument. We have also listed the modes not useful for the helicity analysis, $\bar{p}p$, $\bar{p}\Delta^{++}$ and $\bar{\Lambda}^0 p$, since these easily identifiable modes may give some idea about magnitude of the tree modes of our interest.

One major difference from the $VV$ modes is that if the penguin decay contains $\bar{b} \to \bar{q}qq'$, this nonstandard interaction will manifest itself unambiguously in $f_+$. The ratio of $\bar{q}qq'$ to $\bar{q}qq'$ directly reflects on $f_+$ in the $B\bar{B}$ decays, while only a switch of $H_{++} \leftrightarrow H_{--}$ in the subdominant amplitudes occur in the $VV$ modes.

V. DISCUSSION

We have proposed to study the baryonic modes and collect more information about the source of the breakdown of the helicity rule. Since there have already been several proposals of possible sources, we comment on what impact the baryonic modes may possibly have on the issue.

If the large transverse polarization of $\phi K^*$ arises from enhancement of the annihilation process $[12]$, the same enhancement is unlikely in the baryonic modes for the following
reason: The annihilation decay amplitudes for $B \to VV$ are expressed with the vector-meson form factors in the time-like region in the leading order. They fall off like $1/q^2$ at large $q^2 = O(m_B^2)$ in perturbative QCD, but the author of Ref. [12] suspects that the soft-collinear loop corrections enhance the amplitudes numerically and upset the power suppression of the perturbative power counting. While the baryon form factors similarly describe the annihilation processes into a baryon pair, they fall off like $1/q^4$ in perturbative counting [24]. This difference in the asymptotic form factors can be traced back to the dimensions of the meson and baryon wavefunctions. Barring the possibility that the soft-collinear loops overcome one more factor of $1/m_B^2$ in rate, the annihilation process is far less competitive in the baryon-antibaryon decay modes. If so, our estimate of $f_+ \approx 0.9$ in Eq. (10) should hold for most baryonic modes. If experiment disagrees with it, we should look for other long-distance effects or an exotic decay interaction as the cause of breakdown of the perturbative helicity rule.

If the color-magnetic decay operator of $\bar{q}\sigma_{\mu\nu}qG^{\mu\nu}$ is responsible, as proposed in Ref. [13], conversion of the gluon $G^{\mu\nu}$ to $\bar{q}q$ must be enhanced to overcome the perturbative power suppression of $m_q/\sqrt{q^2}$ and the neutralization of the color in $\bar{q}q$. Since the mechanism of this soft enhancement has not been demonstrated quantitatively, it is hard to extend the argument to the baryonic modes. Nonetheless, we can argue that such a nonperturbative enhancement is highly unlikely in the baryon-antibaryon decays: Since the $\bar{q}q$ pair originating from $G_{\mu\nu}$ flies back to back to form $BB$, it is hard to avoid the short-distance chirality suppression of $m_q/\sqrt{q^2} \approx 2m_q/m_B$. We therefore suspect that enhancement of the color magnetic decay does not occur in the baryonic modes. We expect that $f_+$ should be around 0.9 even if this mechanism should be responsible for the large polarization of $VV$. If $f_+$ deviates largely from unity in $BB$, a more likely source would be a large mixture of the higher Fock configuration in the baryon composition.

A proposal [14] of the effective tensor four-quark interaction is ad hoc but similar to the enhanced color-gluon decay interaction in physical consequence. Such an effective four-quark interaction would be suppressed by $m_q/\sqrt{q^2}$ in perturbative QCD, if it arises as a short-distance-corrected gauge interaction. Unless one goes outside the framework of electroweak gauge theory, one cannot admit a large tensor interaction in any known way. If a short-distance tensor interaction of light quarks should be relevant (a long shot), it would generate the $H_{-\frac{1}{2}\frac{1}{2}}$ amplitude in the $BB$ decay without long-distance corrections.

If the large transverse polarization originates from the long-distance spin flip in the on-shell charmed hadron intermediate states [10], the observed effect would be net sum over many intermediate hadronic states. We have little reason to believe that a simple rule emerge for baryon helicity in this case. If the hadron-quark duality holds between the charmed-hadron-pairs and $c\bar{c}_L$, we may be able to make a crude estimate of $f_+$ with the $\langle m_T/(E_\parallel + |k|) \rangle$ factor of the $c$ and $\bar{c}$-quark. In this case the values of $f_+$ for different baryonic decays would be roughly equal to the value of $f_L(\approx 0.6)$ for $B \to J/\psi K^*$ [26].

If the quark-hadron duality is not applicable, our guess is that the values of $f_+$ would be

\[5\]In this scenario one has to make sure that the contribution of the on-shell charmed-hadron intermediate states of the Cabbibo-suppressed tree-process does not disturb $f_L \simeq 1$ for the $VV$ modes such as $\rho \rho$. 

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statistically random from one baryon mode to another over a wide range centered around 0.5.

To conclude, measurement of baryon helicity in any single $B$ decay mode will not decide on the source of the large transverse helicity observed in $B \to \phi K^*$. Nonetheless, the baryon helicity will be one useful additional piece of information not only to test the proposals so far made but also to search for a novel source yet unknown to us.

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