Global Monopole-BTZ black hole

S. Habib Mazharimousavi and M. Halilsoy
Department of Physics, Eastern Mediterranean University, Gazimağusa, Turkey.
(Dated: October 13, 2014)

In order to obtain the geometry of a global monopole we introduce the broken $O(2)$ symmetry in $2+1$-dimensions. Adding a negative cosmological constant yields the extension of BTZ black hole in a global monopole background. The global monopole effects the spacetime in the same way as the electric charge of a BTZ black hole.

Keywords: 2+1-dimensions; Global monopole; BTZ black hole;

I. INTRODUCTION

The analogue of Barriola-Vilenkin’s global monopole spacetime [1] in $3+1$-dimensions is constructed in $2+1$-dimensions. Considerable attention received by the lower dimensions during recent decades provides the main motivation for such a study. Not only it constitutes a test bed for higher dimensions but $2+1$-dimensions can also be considered as a brane in $3+1$-dimensions. Historically the idea was popularized first by the $2+1$-dimensional Banados-Teitelboim-Zanelli (BTZ) black hole solution [2,3] which was sourced by a negative cosmological constant. In other words, the absence of gravitational degrees of freedom in lower dimensions was filled by a cosmological constant. Now, the similar role will be played by both a cosmological constant and a global monopole together. We wish to dub such a spacetime as Global-Monopole-BTZ (GMBTZ) spacetime. The global monopole is localized in a core whose effect remains asymptotically much weaker relative to the cosmological monopole.

The global monopole in $3+1$-dimensions has the symmetry group $O(3)$ to be broken spontaneously to $U(1)$. The similar role is played in the $2+1$-dimensional case by the abelian group $O(2)$. Instead of a triplet of scalar fields we have now a doublet of scalar fields $\phi^a = \eta f(r) \frac{x^a}{r}$, (for $a = 1, 2$), with $\eta =$monopole charge constant, $f(r)$ a radial function to be determined and $(x^a)^2 = r^2$. The differential equation satisfied by $f(r)$ can’t be solved exactly, however, beyond the monopole’s core we can set $f(r) \simeq 1$. This makes a black hole whose horizon depends strongly on the monopole charge beside the cosmological constant. Interestingly, the geometry obtained for such a monopole is identical with the geometry of a charged-BTZ (CBTZ) black hole [4,6]. Thus, the geometry doesn’t differentiate whether the agent source is an external field of a global monopole or the electric charge of a central black hole. That is, global monopoles which are believed to emerge from spontaneous symmetry breaking during the big bang [2,4] plays pretty well suited the role of an electrostatic field.

II. GLOBAL MONOPOLE IN $2+1$-DIMENSIONS

We start with the general form of static, circularly symmetric line element in $2+1$-dimensions given by

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2d\theta^2$$

in which $A(r)$ and $B(r)$ are two functions only of $r$. Now, we introduce the action consisting of a doublet of real scalar fields of the form ($16\pi G = c = 1$)

$$S = \int d^3x\sqrt{-g} \left(R - 2\Lambda + L^{field}\right)$$

in which

$$L^{field} = -\frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{4}\lambda (\phi^a \phi^a - \eta^2)^2.$$  (3)

Here $a = 1, 2$, $R$ is the Ricci scalar, $\lambda$ is a coupling constant, $\Lambda$ is the cosmological constant, $\eta$ is the symmetry-breaking scale parameter and

$$\phi^a = \eta f(r) \frac{x^a}{r},$$

for $x^1 = r \cos \theta$ and $x^2 = r \sin \theta$. To find the field equation for $f(r)$ we express the field Lagrangian in terms of $f(r)$ only, i.e.,

$$L^{field} = -\frac{\eta^2}{2B}f'' + \frac{\eta^2}{2r^2}r^2f'' - \frac{1}{4}\lambda\eta^4 \left(f'' - 1\right)^2.$$  (5)

Now, variation of the action with respect to $f$ yields

$$f'' + \left(\frac{1}{r} + \frac{B}{2A} (\frac{A}{B})'\right)f' - \left(\frac{1}{r^2} + \lambda\eta^2 \left(f' - 1\right)\right)Bf = 0$$

in which a prime stands for the derivative with respect to $r$. Variation with respect to $g^{\mu\nu}$ yields the Einstein equations

$$G^\mu_\nu + \frac{1}{3}\Lambda\delta^\mu_\nu = T^\mu_\nu.$$  (7)
in which
\[ T^\nu_\mu = \frac{1}{2} \left( \partial_\mu \phi^a \partial^\nu \phi^a - \frac{1}{2} \partial_\mu \phi^b \partial^\nu \phi^a \delta^a_\mu \right) - \frac{1}{8} \lambda \left( \phi^a \phi^a - \eta^2 \right)^2 \delta^\nu_\mu. \] (8)

An explicit calculation gives
\[ T^t_t = -\frac{\eta^2}{4} \left( \frac{1}{B} f'^2 + \frac{1}{r^2} f'^2 + \frac{\lambda}{2} \eta^2 \left( f^2 - 1 \right)^2 \right), \] (9)
\[ T^\rho_\rho = \frac{\eta^2}{4} \left( \frac{1}{B} f'^2 - \frac{1}{r^2} f'^2 - \frac{\lambda}{2} \eta^2 \left( f^2 - 1 \right)^2 \right) \] (10)
and
\[ T^\theta_\theta = -\frac{\eta^2}{4} \left( \frac{1}{B} f'^2 + \frac{\lambda}{2} \eta^2 \left( f^2 - 1 \right)^2 \right). \] (11)

Nevertheless, the Einstein tensor’s components are given by
\[ G^t_t = -\frac{1}{r} B', \] (12)
\[ G^\rho_\rho = \frac{A'}{2} rAB \] (13)
and
\[ G^\theta_\theta = \frac{1}{4} \left( 2A'^2 AB - A'^2 B - A'B'^2 \right). \] (14)

We note that, as in 3+1–dimensional case, the size of the global monopole in flat spacetime is given by \( \delta = \frac{1}{\eta \lambda} \).

In the case of the flat spacetime \( B = 1 = A \) and the field equation for \( f \) becomes
\[ f'' + \frac{1}{r} f' - \left( \frac{1}{r^2} + \lambda \eta^2 \left( f^2 - 1 \right) \right) f = 0. \] (15)

It is not easy to find an exact solution for (15) but imposing the condition that \( \lim_{r \to \infty} f = finite = c_0 \) helps us to expand \( f(r) \) for large \( r \) as
\[ f = c_0 + \frac{c_1}{r} + \frac{c_2}{r^2} + \ldots. \] (16)

A direct substitution in (15) implies \( c_0 = 1, c_1 = 0 \) and \( c_2 = -\frac{1}{2 \lambda \eta^2} \) and therefore up to \( r^{-2} \) one finds
\[ f \simeq 1 - \frac{1}{2 \lambda \eta^2 r^2}. \] (17)

Let’s also add that a numerical solution for the field equation (15) with the boundary conditions \( f(0) = 0 \) and \( \lim_{r \to \infty} f(r) = 1 \) and \( \lambda = 1 = \eta \) is displayed in Fig. 1. We must add that from numerical analysis setting

\[ f'(0) = 0.583189029 \] is crucial which otherwise the solution would not approach to 1 asymptotically. Our numerical solution has similar counterpart in 3+1–dimensional flat spacetime.

Therefore for the flat spacetime one concludes that for large \( r \) which is outside the core of the monopole i.e. \( r \gg \delta \), a good estimation for \( f \) is \( f \simeq 1 \). As we shall see in the next section, for the case of curved spacetime due to a global monopole we also stick to the same assumption namely, for large \( r \) still \( f \simeq 1 \) and consequently the energy momentum tensor is given by
\[ T^{\nu}_\mu = diag \left[ -\frac{\eta^2}{r^2}, -\frac{\eta^2}{r^2}, 0 \right]. \] (18)

A. Global monopole-BTZ (GMBTZ) solution

To find solutions to the Einstein equations, let’s first combine the \( tt \) and \( rr \) components of the Einstein’s equations. This yields
\[ \frac{A'}{A} + \frac{B'}{B} = \eta^2 f'^2 \] (19)
whose integration implies
\[ B = \frac{1}{A} \exp \left( \eta^2 \int f'^2 dr + C \right). \] (20)
in which $C$ is an integration constant. Next, we consider the $tt$ component equation which admits

$$\frac{B'}{B^2} = \frac{2\Lambda r}{3} + \frac{\eta^2}{2r} + \frac{\eta^2 r}{2} \left( \frac{1}{B} f r^2 + \frac{1}{r^2} (f^2 - 1) + \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right).$$  \hspace{1cm} (21)

Although the exact solution to this differential equation is not available, in the limit of $f \simeq 1$ i.e. outside the core, one finds $B \simeq \frac{A}{r}$ (we set $C = 0$) and

$$\frac{B'}{B^2} = \frac{2\Lambda r}{3} + \frac{\eta^2}{2r}.$$  \hspace{1cm} (22)

Integration yields

$$A = \frac{1}{B} = -M + \frac{r^2}{r^2} - \frac{\eta^2}{2} \ln r$$  \hspace{1cm} (23)

where the constant $M$ can be interpreted as the mass of the possible black hole and $\frac{1}{r^2} = -\frac{1}{3} \Lambda$. The remained equations are the $\theta \theta$ component of the Einstein’s equation

$$\frac{1}{4} \frac{2A'' AB - A'^2 B - A'B'A}{B^2 A^2} - \frac{\Lambda}{3} = -\frac{\eta^2}{4} \left( \frac{1}{B} f r^2 + \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right)$$  \hspace{1cm} (24)

and the equation for the global monopole field (6). These equations, after the substitution from (23) and $f \simeq 1$ become

$$\frac{\eta^2}{4r^2} = 0$$  \hspace{1cm} (25)

respectively. Both equations up to our approximation order are satisfied.

The final solution given in (23) is isometric to the CBTZ \cite{6} black hole solution whose electric charge $q$ is replaced by the global monopole parameter (charge) $\eta$. This solution, therefore, can appropriately be called as the Global-Monopole-BTZ (GMBTZ) solution.

### III. CONCLUSION

We obtain the metric of a global, chargeless monopole in $2+1$-dimensions which is the analogue of the Barriola-Vilenkin’s monopole in $3+1$-dimensions \cite{11}. Outside the core of the global monopole we have an asymptotic black hole solution that may be attributed to the topological remnants of a $2+1$-dimensional big-bang. $O(2)$ is the spontaneously broken symmetry in this case but the results are much similar to its $3+1$-dimensional counterparts. The global monopole makes a black hole whose Hawking temperature is strongly dependent on the monopole parameter. When compared with the CBTZ black hole we observe that in $2+1$-dimensions the global monopole charge plays the similar role of electric charge in CBTZ black hole solution.

\[1\] M. Barriola and A. Vilenkin, Phys. Rev. Lett. 63, 341 (1989).
\[2\] M. Bañados, C. Teitelboim, J. Zanelli, Phys. Rev. Lett. 69, 1849 (1992).
\[3\] M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D 48, 1506 (1993).
\[4\] C. Martinez, C. Teitelboim and J. Zanelli, Phys. Rev. D 61, 104013 (2000).
\[5\] S. Carlip, Quantum Gravity in 2 + 1-Dimensions, Cambridge University Press, 1998.
\[6\] S. Carlip, Living Rev. Rel. 8, 1 (2005).
\[7\] A. Vilenkin, Phys. Rep. 121, 263 (1985).
\[8\] C. M. Chen, H. B. Cheng, X. Z. Li, X. H. Zhai, Class. Quantum Gravity 13, 701 (1996).
\[9\] X. Z. Li, Commun. Theor. Phys. 28, 101 (1997).
\[10\] D. Harari and C. Lousto, Phys. Rev. D 42, 2626 (1990).
\[11\] O. Dando and R. Gregory, Class. Quantum Grav. 15, 985 (1998).