Effect of many-body interaction on de Haas-van Alphen oscillations in insulators

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De Haas-van Alphen (dHvA) oscillations are oscillations in the magnetization as a function of the inverse magnetic field. These oscillations are usually considered to be a property of the Fermi surface and, hence, a metallic property. Recently, however, such oscillations have been shown to arise, both experimentally and theoretically, in certain insulators which have a narrow gap and an inverted band structure. In this work, we develop a theory to study the effect of many-body interaction on these unconventional oscillations. We consider weak interaction, focusing on the effect of renormalization of the quasiparticle spectrum on these oscillations. We find that interaction has an unusual effect: unlike in metals, in a certain regime the amplitude of oscillations may be enhanced substantially, both at zero and nonzero temperatures, even when the interaction is perturbatively weak.

One of the striking consequences of Landau quantization in a magnetic field in metals is the appearance of quantum oscillations. These are oscillations in physical observables of metals, both thermodynamic and transport, with a change in the magnetic field. The underlying mechanism is simple: as the magnetic field increases, the spacing between the Landau levels widens. This forces the highest occupied level to spill out of the Fermi level and become depopulated periodically, which manifests as oscillations in various observables. Evidently, these oscillations are expected only in metals and are measured routinely in experiments to map the Fermi surface of such systems [1, 2].

In recent years, however, quantum oscillations have been reported experimentally in various insulators. De Haas-van Alphen (dHvA) oscillations, which refer to oscillations in the magnetization, have been observed in the Kondo insulator SmB$_6$ [3, 4], while Shubnikov-de Haas (SdH) oscillations, which refer to oscillations in the resistivity, have been observed in the Kondo insulator YbB$_{12}$ [7], quantum well heterostructures [8, 9], WTe$_2$ [10], and moiré graphene [11]. Such unexpected findings prompted intensive theoretical investigations which have now revealed that contrary to the canonical picture, quantum oscillations can indeed arise in insulators, provided the insulators have a narrow gap and an inverted band structure [12, 13].

Although the phenomenon is now well-understood at the noninteracting level, the effect of interactions on these unconventional oscillations remains insufficiently explored. Research in this direction has predominantly concentrated on specific models of correlated insulators where interaction causes the opening of the gap [14–20] but not on generic band insulators with interaction. Exploring the latter is important since it offers a controlled approach to compare oscillations in insulators with those in metals. Moreover, from an experimental perspective, investigating this aspect is significant since some of the systems where unconventional oscillations have been observed are of this nature [8, 9], and the abundance of possibilities in this category suggests more explorations in the future.

In this work, we develop a theory of quantum oscillations in interacting band insulators, focusing specifically on dHvA oscillations. We consider weak interactions, incorporating it within the Hartree-Fock approximation. Notably, we find that in a certain parameter regime, even weak interactions can significantly modify the amplitude of oscillations unlike in metals. This arises because interactions now compete with a new energy scale in the form of a gap which is absent in a metal, thus influencing oscillations in a qualitatively different manner.

To put our results for the insulator in context, we first review the theory of dHvA oscillations in metals. The oscillating part of the magnetization is given by $M_{osc} = -\frac{\partial \Omega_{osc}}{\partial B}$, where $\Omega_{osc}$ is the oscillating part of the grand potential and $B$ is the external magnetic field. In a three-dimensional interacting metal with a parabolic spectrum of spinless electrons, considering only the renormalization of the energy spectrum due to a static interaction potential at the Hartree-Fock level, it is found that

$$\Omega_{osc} = \sum_{l=0}^{\infty} \tilde{A}_l(T) \cos \left[ 2\pi \frac{\tilde{\mu}}{\tilde{\omega}_c} \left( l + \frac{1}{4} \right) \right],$$

where $\tilde{\omega}_c = eB/\tilde{m}$ is the cyclotron frequency with $e$ as the absolute value of the charge and $\tilde{m}$ as the mass of an electron, respectively, $\tilde{\mu}$ is the chemical potential, and $\tilde{A}_l(T)$ is the temperature ($T$)-dependent amplitude of the $l$-th harmonic of oscillations given by ($k_B = 1$)

$$\tilde{A}_l(T) = \tilde{\omega}_c \frac{(eB)^{3/2}}{8\pi^4 \tilde{m}^{1/2}} \frac{2\pi^2 l^2 T/\tilde{\omega}_c}{\sinh(2\pi^2 l^2 T/\tilde{\omega}_c)}.$$  \hspace{1cm} (2)

Above, and henceforth, the presence (absence) of tilde denotes renormalized (bare) values of the respective quantities. The expressions (1) and (2) are, in fact, identical to the expressions for the appearance of renormalized parameters [22, 23]:

$$\mu \rightarrow \tilde{\mu} = \mu(1 + b)$$

$$\omega_c \rightarrow \tilde{\omega}_c = \frac{eB}{\tilde{m}} = \frac{eB}{m} (1 + a),$$

where $b$ and $a$ capture the degree of renormalization in $\mu$ and $m$, respectively. One can readily summarize the following salient features:
two bands are hybridized by an insulator. Consider the following Hamiltonian:

\[ H = \sum_{i,k} \varepsilon_{ik} c_{ik}^\dagger c_{ik} + \sum_{k} \left( \gamma_k c_{1k}^\dagger c_{2k} + \text{h.c.} \right) + H_{\text{int}}. \]  

(4)

Here, \( c_{ik}^\dagger (c_{ik}), i = 1, 2 \), denotes the creation (destruction) operators for particles with momentum \( \mathbf{k} \) in the \( i \)-th band with dispersion \( \varepsilon_{ik} \) in three dimensions. These two bands are hybridized by \( \gamma_k \). For simplicity, we choose \( \varepsilon_{1k} = \frac{k^2}{2m_1} - \Delta \) and \( \varepsilon_{2k} = -\frac{k^2}{2m_2} + \Delta \) with \( m_{1,2}, \Delta > 0 \), and assume \( \gamma_k = \gamma \) to be independent of \( \mathbf{k} \) with \( |\gamma| \ll \Delta \). Also, all particles are assumed to be spinless. The first two terms in Eq. (4) describe the noninteracting part and is easily diagonalized leading to an insulator with an inverted band structure and a narrow gap—see Fig. 1. The chemical potential \( \mu \) is chosen to lie inside the gap.

The last term in Eq. (4), \( H_{\text{int}} \), introduces interparticle interaction, whose exact form is not necessary for the results to be derived—we only assume that the interaction is static and weak so that its effect can be included perturbatively at the Hartree-Fock level which leads to a renormalization of the energy levels but no broadening.

In the presence of a magnetic field, discrete Landau levels are produced that are affected by the interaction. We calculate the grand potential using the standard formula \[ \Omega = -T \text{ Tr} \left( \sum_{\zeta_m} \ln \left\{ -[\tilde{G}^{-1}(\zeta_m)] \right\} \right). \]  

(5)

Here, \( \zeta_m = (2m + 1)\pi T \), with \( m \in \mathbb{Z} \), are the Matsubara frequencies, \( \text{Tr} \) stands for the trace over all energy states, and \( \tilde{G} \) is the Green’s function corresponding to \( H \) in a magnetic field given by \( \tilde{G}^{-1} = G^{-1} - \Sigma \), where \( G \) is the noninteracting Green’s function and \( \Sigma \) is the self-energy due to the interaction. In the band-basis, we have

\[ G_{11} = i\zeta_m - \omega_{1c} (n + \frac{1}{2}) - \frac{k^2}{2m_1} + \Delta + \mu, \quad G_{22} = i\zeta_m + \omega_{1c} (n + \frac{1}{2}) + \frac{k^2}{2m_2} + \mu, \]  

\[ G_{12} = G_{21} = -\gamma, \]  

where \( n \) is the Landau level index and \( \omega_{1c,2} = eB/m_{1,2} \). In general, \( \Sigma \) is a function of both \( B \) and \( T \) and requires a substantial effort to calculate. However, as far as dHvA oscillations in three dimensions are concerned, it suffices to consider \( \Sigma \) calculated at zero \( B \) and \( T \)—as in the case of metals, the effect of nonzero \( B \) and \( T \) leads to subleading corrections in orders of \( \omega_{1c,2}/\Delta \ll 1 \) and \( T/\Delta \ll 1 \), respectively \[20,25\]. Within this approximation, we evaluate the oscillating part of Eq. (14) and find,

\[ \tilde{\Omega}_{\text{osc}} = \sum_{l=1}^{\infty} \tilde{A}_l(T) \cos \left[ 2\pi l \frac{\Delta}{\omega_{1c} + \omega_{1c} - \pi \left( l + \frac{1}{4} \right)} \right], \]  

(6)

where

\[ \tilde{A}_l(T) = \frac{(eB)^{3/2}}{\pi^2 l^{3/2}} T \sum_{\zeta_m > 0} e^{-\frac{\pi l}{2\zeta_m}} \sqrt{\zeta_m^2 (\omega_{1c} + \omega_{1c})^2 + 4\omega_{1c} \omega_{1c} \gamma^2} \times \cosh \left[ \frac{\pi l \zeta_m (\omega_{1c} - \omega_{1c})}{\omega_{1c} \omega_{1c}} \right]. \]  

(7)

Details of the derivation are provided in Supplemental Material (SM) \[26\]. The above expressions are characterized by the following renormalized parameters:

\[ \omega_{1c,2} \rightarrow \omega_{1c,2} = \frac{eB}{m_{1,2}} = \frac{eB}{m_{1,2}} (1 + a_{1,2}), \]  

(8a)

\[ \Delta \rightarrow \tilde{\Delta} = \Delta (1 + b), \]  

(8b)

\[ \gamma \rightarrow \tilde{\gamma} = \gamma (1 + t), \]  

(8c)

\[ \mu \rightarrow \tilde{\mu} = \mu + \delta \mu. \]  

(8d)

Note that, while \( \omega_{1c,2}, \tilde{\Delta}, \tilde{\gamma}, \) and \( \tilde{\mu} \) appear explicitly in Eqs. \( 6 \) and \( 7 \), \( \mu \) enters implicitly through the Matsubara sum. Thus, it affects only the T-dependence of the oscillations, and has no effect on the \( T = 0 \) behavior, as long as \( \mu \) and \( \tilde{\mu} \) lie in the gap. For simplicity, we have assumed \( \tilde{\mu} = -\frac{\Delta \tilde{m}_1}{m_{1,2}} \) chosen such that it lies at the intersection of the two renormalized bands prior to hybridization \[27\]. Equation \( 6 \) along with Eqs. \( 7 \) and \( \tilde{\mu} \) generalize Eqs. \( 1 \) and \( 2 \) from an interacting metallic system to an interacting gapped system. Thus, for a given form of \( H_{\text{int}} \) in Eq. \( 1 \), one simply needs to calculate the renormalization parameters \( a_{1,2}, b, t \) to study the effect of interaction on dHvA oscillations. We will come back to this calculation later; for now, we discuss the qualitative features that arise from these expressions. It is seen that the phase remains unaltered and the frequency does not change appreciably since \( 1 \ll \frac{\tilde{\Delta}}{\omega_{1c,2} \omega_{1c} \omega_{1c}} \approx \frac{\Delta}{\omega_{1c,2} \omega_{1c} \omega_{1c}} \); thus, both these quantities remain qualitatively similar to those in metals.
FIG. 2. The effect of interaction on the amplitude at $T = 0$ according to Eqs. (9) and (19). $\tilde{A}_1(0)$ denotes the amplitude of the first harmonic at $T = 0$ in the presence of interaction. It is normalized by its noninteracting value $A_1(0)$. We show its variation with $a_1$ and $t$ keeping $a_2 = 0.2$ fixed in all the plots. The points $Q$, $R$, and $S$ are arbitrarily chosen in the interaction-parameter-space defined by $(a_1, a_2, t)$ which are referred to in Fig. 3 later.

contrast, the amplitude is significantly affected by interactions, in a way that is qualitatively different from that in metals.

First, we consider $\tilde{A}_1(0)$, the amplitude at $T = 0$. Changing the summation to an integral over the frequency in Eq. (7), we find,

$$\tilde{A}_1(0) = \frac{|\tilde{\gamma}| eB}{2\pi^{3/2}} K_1 \left( \frac{4\pi l |\tilde{\gamma}|}{\omega_{c1} + \omega_{c2}} \right),$$

where $K_\alpha$ is the modified Bessel function of the second kind. Equation (9), together with Eqs. (19), gives a quantitative description of how the amplitude is affected by interaction. It leads to an unusual feature that is unique to the insulating case: even a weak interaction can lead to a substantial change in the amplitude of oscillations at zero temperature. Indeed, the fate is decided by a delicate interplay between $\tilde{\gamma}$ and $\tilde{m}_1, \tilde{m}_2$ in Eq. (9). Using the parametrization of Eq. (19) in Eq. (9), we plot the first-harmonic-amplitude in Fig. 2 for different values of $m_1/m_2$ (determining the particle-hole asymmetry) and $\gamma/\omega_{c1}$ (determining the strength of the gap as compared to the Landau level spacing). It is seen that when $\gamma/\omega_{c1} \sim 1$, there is a pronounced enhancement in the zero-temperature-amplitude, which can amount to even an order of magnitude.

Next, we consider the dependence of the amplitude on $T$. This is calculated numerically from Eq. (7) and is presented in Fig. 3 for various choices of interaction parameters $m_1/m_2$ and $\gamma/\omega_{c1}$ as used in Fig. 2. In the limit $T >> \gamma$, as expected, the $T$-dependence follows the usual metallic behavior, contributed by the two participating bands. It only depends on $m_1, m_2$ and is independent of $\tilde{\gamma}$. In the other limit, $T \lesssim \gamma$, the $T$-dependence deviates from the metallic behavior, and depends on both $m_1, m_2$ and $\tilde{\gamma}$. The deviation is most striking when $m_1/m_2 \gg 1$ and $\gamma/\omega_{c1} \sim 1$. In this regime [Fig. 3(d)] there is a sizeable enhancement in the amplitude in the form of a nonmonotonic upturn driven by $T$ on top of the enhancement at $T = 0$ discussed earlier. An interesting observation is that since the behavior of the amplitude at low $T$ in the particle-hole asymmetric case is very sensitive to $\gamma/\omega_{c1}$, for a given material (with a fixed $\gamma$ and $\tilde{\gamma}$) the effect of temperature depends crucially on the field at which the oscillations are being studied to extract the amplitude. Indeed, Figs. 3(c) and (d) can be interpreted as the temperature dependence of the amplitude of the same dHvA
FIG. 3. The temperature-dependence of the amplitude in the presence of interaction. $\tilde{A}_1(T)$ denotes the amplitude of the first harmonic at temperature $T$ in the presence of interaction. It is normalized by its noninteracting zero-temperature value $A_1(0)$. We show its variation with $T$ for the various cases considered in Fig. 2. $Q, R, S$ denote three choices of $(a_1, a_2, t)$ defined in Fig. 2 earlier. $P$ denotes the noninteracting case. The curves are derived from numerical calculation of Eq. (7).

An expression for the $T$-dependence of the amplitude at low $T$ can be obtained by employing the Euler-Maclaurin formula to carry out the Matsubara sum in Eq. (7):

$$\tilde{A}_k(T) \approx \tilde{A}_k(0) - \alpha \left[ \frac{1}{2\pi} \int_0^{\pi T} f(x) dx - \frac{T}{2} f(\pi T) + \frac{T}{12} f'(\pi T) \right],$$

(10)

where $f(x)$ is the summand in Eq. (7) with $\zeta_m \rightarrow x$ and $\alpha = (\pi B)^{3/2}/\gamma$. Equation (10) reproduces the numerically obtained curve for $\tilde{A}_1(T)$ very well for $T \lesssim \gamma$. This can be further reduced to a closed analytical form by expanding in $T$; however, the resulting expression is accurate only when $T \ll \gamma$ and does not describe the features that arise at $T \gtrsim \gamma$. We discuss this in SM [26].

Having discussed the qualitative features, we now return to the calculation of the renormalization parameters $a_i, b, t$. These are related to the self-energy $\Sigma$ as follows (see SM [26]):

$$a_i = v_F^{-1} \delta k \Sigma_{ii} \big|_{k_F},$$

(11a)

$$b = \frac{1}{\Delta} [\Sigma_{22} - \Sigma_{11} + \mu (a_1 - a_2)] \big|_{k_F},$$

(11b)

$$t = \frac{1}{\gamma} \Sigma_{12} \big|_{k_F},$$

(11c)

$$\delta \mu = \mu a_2 - \Sigma_{22} \big|_{k_F},$$

(11d)

where $v_F$ and $k_F$ are the Fermi velocity and momentum, respectively. To calculate $\Sigma$, we need $H_{\text{int}}$. We consider the following form:

$$H_{\text{int}} = \sum_{i,j,k,k',q} V_{ijq} c_i^{\dagger} k c_j^{\dagger} q c_j^{\dagger} q c_i k.$$  

(12)

The above term describes an interaction of strength $V_{ijq}$ between particles belonging to either the same band ($i = j$) or different bands ($i \neq j$). For simplicity, we have only considered interband interaction which preserves the band index, but extending the calculation to a more general form of the interaction is straightforward. As remarked earlier, $\Sigma$ needs to be calculated at zero $B$ and $T$. At the Hartree-Fock level, we have $\Sigma_{ii}(k) =$ oscillation but at different field values.
This yields a normalized by its noninteracting value at $T = 0$. We have used $m_1 = m_2 = m$ (electron’s mass), $B = 1$ T, and $\frac{\Delta}{\omega_{1,2}} = 4.3 \times 10^7$. In (a) $\frac{\gamma}{\omega_{1,2}} = 0.9$ and in (b) $\gamma = 0 [T$ is scaled with the $\gamma$ used in (a) for comparison]. The interaction term is taken as $V_{ijq} = \frac{e^2}{\epsilon_0 (q + \kappa)^2}$, where $\kappa$ is the Thomas-Fermi wavevector. This yields $a_1 = a_2 = 0.022$ and $t = 0.329$ in Eqs. (20).

We plot the amplitude as a function of $T$ for a particle-hole symmetric case ($m_1 = m_2$) in Fig. 4, comparing the insulating case with its metallic counterpart. The change in amplitude due to interaction is considerably more pronounced in the insulating case.

In summary, we observe the following salient features of dHvA oscillations in interacting insulators:

1. The phase does not change with interaction.
2. The change in frequency is negligible: Because $\Delta \gg \omega_{1,2}, V$, where $V$ is the strength of interaction, $\Delta/(\omega_{1,2} + \omega_{1,2}) \approx \Delta/(\omega_{1,2} + \omega_{1,2})$.
3. The change in the amplitude can be substantial even in weak interaction, at both $T = 0$ and $T \neq 0$, in a certain regime. The amplitude may vary nonmonotonically with $T$ showing an upturn at low $T$, which is greatly amplified by interactions.

In comparing the above features with their counterpart for metals stated earlier, we find that, while the phase and the frequency behave similarly, the behavior of the amplitude is very different, both qualitatively and quantitatively.

In this work, we have considered weak interaction at the Hartree-Fock level. Going beyond, one can include the effects of a finite lifetime induced by interactions or disorder, which is expected to give rise to other features distinct from that in metals. We defer these questions for future investigation. Note, however, the assumption of weak interaction does not necessarily imply that our results do not apply to correlated insulators, such as Kondo and excitonic insulators [28, 29], where interaction is strong. In such systems, the strong interaction is primarily responsible for giving rise to the insulating gap via a phase transition. Once such a phase is reached, the effective interaction between the new quasiparticles may indeed be weak. In passing, we note that in the Kondo insulator SmB$_6$, the dHvA oscillations show an unusual enhancement of the amplitude at low $T$ [30, 31]. This material is strongly particle-hole asymmetric and is most likely in the regime of Fig. 3(d). The upturn observed in SmB$_6$ experimentally (see, for example, Fig. 5c in Ref. [31]) bears some resemblance to the upturn in Fig. 3(d).

In conclusion, we have presented a theory to study the effect of many-body interactions on dHvA oscillations in insulators at the Hartree-Fock level. We have shown that the amplitude of oscillations can change substantially even if the interaction is weak, unlike in metals. The difference originates from the interplay between interactions and the gap in the band structure, a feature absent in metals.

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SUPPLEMENTAL MATERIAL

A. THE OSCILLATORY GRAND POTENTIAL

We want to calculate the oscillatory part of the Grand potential given by

$$\tilde{\Omega} = -T \text{Tr} \left( \sum_{\zeta_m} \ln \{-[\tilde{G}^{-1}(\zeta_m)]\} \right),$$

(14)

where

$$\tilde{G}^{-1}(\zeta_m) = \left[ \begin{array}{cc} i\xi_m - \omega_{c1} \left( n + \frac{1}{2} \right) - \frac{k^2}{2m_1} + \Delta + \mu & -\gamma \\ -\gamma & i\xi_m + \omega_{c2} \left( n + \frac{1}{2} \right) + \frac{k^2}{2m_2} + \mu \end{array} \right] - \Sigma,$$

(15)

Here, \( \xi_m = (2m + 1)\pi T \), with \( m \in \mathbb{Z} \), \( n \) is the Landau level index, \( \omega_{c1,2} = eB/m_{1,2} \), and Tr stands for the trace over all energy states, i.e., \( \text{Tr} = \frac{eB}{2\pi} \sum_n \int_{-\infty}^{\infty} dk_z \). Using the well-known relation \( \text{Tr}([\ln M])=\text{Tr}([\ln |M|]) \), where \( |M| \) is the determinant of any matrix \( M \), we get

$$\tilde{\Omega} = -DT \int_{-\infty}^{\infty} dk_z \sum_n \left( \sum_{\zeta_m} \ln \{-[\tilde{G}^{-1}(\zeta_m, n, k_z)]\} \right),$$

(16)

where \( D = \frac{eB}{2\pi} \) and we have explicitly written all the variables on which \( \tilde{G}^{-1} \) depends for clarity. The summation over the Landau levels in Eq. (16) can be changed into an integral with the help of Poisson’s summation formula:

$$\sum_n F(n) = \int_{-\infty}^{\infty} F(x) dx + 2 \int_{\frac{2\pi}{x}}^{\infty} dx F(x) \cos(2\pi lx).$$

It is evident that only the second term induces oscillations. To evaluate this term, we use the method described in Refs. 24 and 25. Utilizing the method of integration by parts and retaining only the terms that contribute to oscillations, we obtain:

$$\tilde{\Omega}_{osc} = 2DT \int_{-\infty}^{\infty} dk_z \left( \sum_{l=1}^{\infty} \sum_{\zeta_m} \int_{0}^{\infty} \frac{1}{| -\tilde{G}^{-1}(\zeta_m, x, k_z)|} \frac{d}{dx} - \tilde{G}^{-1}(\zeta_m, x, k_z) \right) \sin 2\pi lx 2\pi dx \right).$$

(17)

It can be seen that the discrete variable \( n \) has been replaced by a continuous variable \( x \) above. To proceed further, we need to know how \( \Sigma \) depends on the three variables \( \zeta_m, x, k_z \) in Eq. (15). First, we assume a static interaction so that \( \Sigma \) has no dependence on \( \zeta_m \), i.e., \( \Sigma(\zeta_m, x, k_z) \rightarrow \Sigma(x, k_z) \). Next, for any generic weak interaction, one can expand the self-energy near \( x = x_F \) and \( k_z = 0 \) 24, 25. Writing explicitly in the band-basis,

$$\Sigma_{ii}(x, k_z) \approx \Sigma_{ii}(x_F, 0) + (x - x_F) \frac{\partial \Sigma_{ii}(x, k_z)}{\partial x} \bigg|_{x_F, 0} + \frac{k^2}{2} \frac{\partial^2 \Sigma_{ii}(x, k_z)}{\partial k_z^2} \bigg|_{x_F, 0},$$

(18a)

$$\Sigma_{12}(x, k_z) \approx \Sigma_{12}(x_F, 0).$$

(18b)

Above, \( x_F = (\mu + \Delta)/\omega_{c1} - 1/2 = -\mu/\omega_{c2} - 1/2 \). The expansions above have been carried out to the leading order which affects oscillations. For brevity, we write \( \frac{1}{\omega_{c1}} \frac{\partial \Sigma_{ii}(x, k_z)}{\partial x} \bigg|_{x_F, 0} = a_i \) and \( \frac{\partial^2 \Sigma_{ii}(x, k_z)}{\partial k_z^2} \bigg|_{x_F, 0} = \frac{1}{m_1} a_i \). A considerable simplification occurs by noting that it suffices to calculate \( \Sigma_{ij}(x_F, 0) \), \( a_i \), and \( \alpha_i \) at zero field since the effect of the field is to introduce higher order corrections in the field in oscillations which are negligible 24, 25. Further, it is also easily shown that \( a_i = \alpha_i \) at zero field. With this in mind, we define the following:

$$\omega_{c1,2} = \omega_{c1,2}(1 + a_{1,2}),$$

(19a)

$$\Delta = \Delta(1 + b),$$

(19b)

$$\gamma = \gamma(1 + t),$$

(19c)

$$\mu = \mu + \delta\mu,$$

(19d)
where

\[ a_i = v_F^{-1} \partial_i \Sigma_{ii} \big|_{k_F}, \quad (20a) \]

\[ b = \frac{1}{\Delta} \left[ \Sigma_{22} - \Sigma_{11} \right] \big|_{k_F}, \quad (20b) \]

\[ t = \frac{1}{\gamma} \Sigma_{12} \big|_{k_F}, \quad (20c) \]

\[ \delta \mu = \mu a_2 - \Sigma_{22} \big|_{k_F}, \quad (20d) \]

where \( v_F \) and \( k_F \) are the Fermi velocity and momentum, respectively. Using \( (18) \) in \( (15) \) along with the definitions \( (20) \) and \( (19) \), we have

\[
\tilde{G}(\zeta_m)^{-1} = \begin{bmatrix} i\zeta_m - \omega_c (x + \frac{1}{2}) - \frac{k_F^2}{2m_1} + \Delta + \tilde{\mu} & -\gamma \\ -\gamma & i\zeta_m + \omega_c (x + \frac{1}{2}) + \frac{k_F^2}{2m_2} + \tilde{\mu} \end{bmatrix}.
\] (21)

We now solve Eq. \( (17) \) using the above expression for \( \tilde{G}^{-1} \). The determinant of matrix \( -\tilde{G}^{-1} \) is the product of its eigenvalues. Setting \( \tilde{\mu} = -\frac{\omega_c \Delta}{\omega_c + \omega_c} \) and after solving, we get

\[
\tilde{\Omega}_{osc} = \frac{DT}{\pi} \int_{-\infty}^{\infty} dk_z \left( \sum_{l=1}^{\infty} \frac{1}{l} \sum_{\zeta_m} \left( -\frac{1}{\omega_c \tilde{\omega}_c} \right) \int_0^{\infty} \frac{2\pi l x}{\sin 2\pi l x} \left( i\zeta_m (\tilde{\omega}_c - \omega_c) + \frac{2\tilde{\omega}_c \omega_c}{\omega_c + \omega_c} \tilde{\Delta} - 2\tilde{\omega}_c \omega_c x - (1 + \frac{k_F^2}{2B}) (\tilde{\omega}_c) \right) dx \right),
\] (22)

where

\[
x_{1,2} = \left( \frac{\tilde{\Delta}}{\omega_c + \tilde{\omega}_c} - \frac{1}{2} - \frac{k_F^2}{2cB} \right) + i\zeta_m (\tilde{\omega}_c - \omega_c) + \frac{2\tilde{\omega}_c \omega_c}{\omega_c + \omega_c} \tilde{\Delta} - 2\tilde{\omega}_c \omega_c x - (1 + \frac{k_F^2}{2B}) (\tilde{\omega}_c) \pm i\sqrt{(\zeta_m (\tilde{\omega}_c - \omega_c))^2 + 4\tilde{\omega}_c \omega_c \gamma^2}.
\] (23)

Using the exponential form of \( \sin x \),

\[
\tilde{\Omega}_{osc} = \frac{DT}{\pi} \int_{-\infty}^{\infty} dk_z \left( \sum_{l=1}^{\infty} \frac{1}{l} \sum_{\zeta_m} \left( -\frac{1}{\omega_c \tilde{\omega}_c} \right) \int_0^{\infty} \frac{2\pi l x}{\sin 2\pi l x} \left( i\zeta_m (\tilde{\omega}_c - \omega_c) + \frac{2\tilde{\omega}_c \omega_c}{\omega_c + \omega_c} \tilde{\Delta} - 2\tilde{\omega}_c \omega_c x - (1 + \frac{k_F^2}{2B}) (\tilde{\omega}_c) \right) dx \right),
\] (24)

We now want to compute the integral over \( x \) using the residue theorem. Notice that the integrand contains the factors \( e^{i2\pi lx} \) and \( e^{-i2\pi lx} \), which have different convergence properties in the complex plane. Accordingly, we pick the appropriate contours: \( C_1 C_2 C_3 \) in the upper-half-plane for the former and \( C_4' C_2' C_3' \) in the lower-half-plane for the latter, as shown in Fig. \ref{fig:contour}. Integrals over paths \( C_2 \) and \( C_2' \) vanish according to Jordan’s Lemma. On the other hand, integrals over the paths \( C_3 \) and \( C_3' \) lead to nonoscillatory contributions which can be ignored. Thus, we eventually have

\[
\tilde{\Omega}_{osc} = \frac{DT}{\pi} \int_{-\infty}^{\infty} dk_z \sum_{l=1}^{\infty} \frac{1}{l} \sum_{\zeta_m} \left( -\frac{1}{\omega_c \tilde{\omega}_c} \right) \int_0^{\infty} \frac{2\pi l x}{\sin 2\pi l x} (\text{summation over residues}).
\] (25)

Because the location of the poles depends on the sign of \( \zeta_m \), it is convenient to split the sum over \( \zeta_m \) as \( \sum_{\zeta_m > 0} + \sum_{\zeta_m < 0} \) leading to \( \tilde{\Omega}_{osc} = \tilde{\Omega}_{osc}^+ + \tilde{\Omega}_{osc}^- \). Equation \( (25) \) then gives the following two contributions:

\[
\tilde{\Omega}_{osc}^+ = DT \int_{-\infty}^{\infty} dk_z \left[ \sum_{l=1}^{\infty} \frac{1}{l} \sum_{\zeta_m} \left( e^{i2\pi l x} \left( \frac{\Delta}{\omega_c + \tilde{\omega}_c} - \frac{1}{2} - \frac{k_F^2}{2\pi B} \right) e^{-2\pi l \left( \frac{\zeta_m (\tilde{\omega}_c - \omega_c)}{\omega_c \tilde{\omega}_c} + \frac{\sqrt{(\zeta_m (\tilde{\omega}_c - \omega_c))^2 + 4\tilde{\omega}_c \omega_c \gamma^2 + 4\omega_c \omega_c \gamma^2}}{\frac{\omega_c + \omega_c}{\omega_c \omega_c}} \right)} + e^{-i2\pi l \left( \frac{\Delta}{\omega_c + \tilde{\omega}_c} - \frac{1}{2} - \frac{k_F^2}{2\pi B} \right) \gamma} \right) \right]
\] (26)
and

\[ \tilde{\Omega}^-_{\text{osc}} = DT \int_{-\infty}^{\infty} dk_z \sum_{l=1}^{\infty} \sum_{\zeta_m} \left( e^{2\pi i \left( \frac{\Delta}{\omega_{c1}} - \frac{k_z^2}{\omega_{c1}^2} \right)} - 2\pi i \left( \frac{\zeta_m (\tilde{\omega}_{c2} - \tilde{\omega}_{c1}) \sqrt{\zeta_m (\tilde{\omega}_{c1} + \tilde{\omega}_{c2})^2 + 4\omega_{c1} \omega_{c2}^2}}{2\omega_{c1} \omega_{c2}} \right) \right) \]

To carry out the integration over \( k_z \), we use the method of steepest descents. We get the following expressions for \( \tilde{\Omega}^\pm_{\text{osc}} \).

\[ \tilde{\Omega}^+_{\text{osc}} = DT \sqrt{\epsilon B} \sum_{l=1}^{\infty} \frac{1}{l^{3/2}} \sum_{\zeta_m} \left( e^{2\pi i \left( \frac{\Delta}{\omega_{c1}} - \frac{\omega_{c2}}{\omega_{c1}} \right)} e^{-\frac{\omega_{c2}}{\omega_{c1}}} e^{-2\pi i \left( \frac{\zeta_m (\tilde{\omega}_{c2} - \tilde{\omega}_{c1}) \sqrt{\zeta_m (\tilde{\omega}_{c1} + \tilde{\omega}_{c2})^2 + 4\omega_{c1} \omega_{c2}^2}}{2\omega_{c1} \omega_{c2}} \right)} \right) \]

and

\[ \tilde{\Omega}^-_{\text{osc}} = DT \sqrt{\epsilon B} \sum_{l=1}^{\infty} \frac{1}{l^{3/2}} \sum_{\zeta_m} \left( e^{2\pi i \left( \frac{\Delta}{\omega_{c1}} - \frac{\omega_{c2}}{\omega_{c1}} \right)} e^{-\frac{\omega_{c2}}{\omega_{c1}}} e^{-2\pi i \left( \frac{\zeta_m (\tilde{\omega}_{c2} - \tilde{\omega}_{c1}) \sqrt{\zeta_m (\tilde{\omega}_{c1} + \tilde{\omega}_{c2})^2 + 4\omega_{c1} \omega_{c2}^2}}{2\omega_{c1} \omega_{c2}} \right)} \right) \]

Finally, adding \( \tilde{\Omega}^\pm_{\text{osc}} \), we get the oscillatory grand potential,

\[ \tilde{\Omega}_{\text{osc}} = \frac{(\epsilon B)^{3/2}}{\pi^2} T \sum_{l=1}^{\infty} \frac{1}{l^{3/2}} \sum_{\zeta_m > 0} e^{-\frac{\pi l}{\omega_{c1} \omega_{c2}} \sqrt{\zeta_m (\tilde{\omega}_{c1} + \tilde{\omega}_{c2})^2 + 4\omega_{c1} \omega_{c2} \tilde{\omega}_{c2}^2}} \left( \frac{\pi l \zeta_m (\tilde{\omega}_{c2} - \tilde{\omega}_{c1})}{\tilde{\omega}_{c1} \tilde{\omega}_{c2}} \right) \cos \left[ 2\pi l \left( \frac{\tilde{\Delta}}{\tilde{\omega}_{c1} + \tilde{\omega}_{c2}} - \frac{1}{2} - \frac{\pi}{4} \right) \right]. \]
FIG. 6. The plots are drawn for scenario R of Fig. 3 in the main text. The solid, red curve corresponds to Eq. (31), the thick, dashed, black curve corresponds to Eq. (34), and the thin dash-dotted, black curve corresponds to an expansion of Eq. (34) in $T$ to $O(T^3)$. In all the plots, $\gamma = 1$.

B. DEPENDENCE OF AMPLITUDE ON TEMPERATURE

The amplitude of $\tilde{\Omega}_{\text{osc}}$ can be read off from Eq. (30):

$$\tilde{\Omega}_{\text{osc}}(T) = \frac{eB}{\pi T^{3/2}} \sum_{m=0}^{\infty} e^{-\pi^2 l \frac{m^2}{\omega_{c1}\omega_{c2}}} \sqrt{\frac{2m+1}{\omega_{c1}\omega_{c2}} + 4\omega_{c1}\omega_{c2} \gamma^2} \cosh \left[ \frac{\pi l (2m+1) \pi T (\omega_{c2} - \omega_{c1})}{\omega_{c1} \omega_{c2}} \right].$$  \hspace{1cm} (31)

At low $T$, the summation over $m$ can be carried out using the Euler-Maclaurin formula:

$$\sum_{m=0}^{\infty} F(m) = \int_{0}^{\infty} F(r)dr + \frac{1}{2} [F(\infty) + F(0)] + \frac{1}{12} [F'(\infty) - F'(0)] + \cdots,$$  \hspace{1cm} (32)

where $F(m)$ is the summand in Eq. (31). Making the change of variable $(2m+1)\pi T \rightarrow x$, which changes $F(m) \rightarrow f(x) = e^{-\pi^2 l \frac{x^2}{\omega_{c1}\omega_{c2}}} \sqrt{\frac{2x + 1}{\omega_{c1}\omega_{c2}} + 4\omega_{c1}\omega_{c2} \gamma^2} \cosh \left[ \frac{\pi l x (\omega_{c2} - \omega_{c1})}{\omega_{c1} \omega_{c2}} \right]$, we have

$$\tilde{\Omega}_{\text{osc}}(T) \approx \alpha \left[ \frac{1}{2\pi} \int_{\pi T}^{\infty} f(x)dx + \frac{T}{2} f(\pi T) - \frac{T}{12} f'(\pi T) \right],$$  \hspace{1cm} (33)

where $\alpha = \frac{eB}{\pi T^{3/2}}$ and we have used the fact that $f(x \rightarrow \infty) = f'(x \rightarrow \infty) = 0$. Writing $\int_{\pi T}^{\infty} f(x)dx = \int_{0}^{\infty} f(x)dx - \int_{0}^{\pi T} f(x)dx$, and recognizing that the first integral is independent of $T$, we have

$$\tilde{\Omega}_{\text{osc}}(T) \approx \tilde{\Omega}_{\text{osc}}(0) - \alpha \left[ \frac{1}{2\pi} \int_{0}^{\pi T} f(x)dx - \frac{T}{2} f(\pi T) + \frac{T}{12} f'(\pi T) \right].$$  \hspace{1cm} (34)
Equation (34) reproduces the numerically obtained curve for $\tilde{A}_1(T)$ very well for $T \lesssim \gamma$ as seen in Fig. 6 implying that the truncation used in the Euler-Maclaurin formula is justified. But, it is not in a closed analytical form, thanks to the integral. A possible way out is to further expand the integral in $T$. For consistency, we also need to expand the other two terms to the same order in $T$ even though they are algebraic expressions. Such a procedure does yield a closed analytical form for $\tilde{A}_1(T)$, but it is not very useful. We demonstrate this in Fig. 6 where we consider an expansion of Eq. (34) up to $O(T^5)$. It is seen that the expression reproduces the numerically obtained curve only for $T << \gamma$ but not for $T \lesssim \gamma$.

C. Self-energy

In this section, we will compute the self-energy for the Hamiltonian given in Eqs. (4) and (12) in the main text:

$$H = \sum_{i,k} \varepsilon_{ik} c_{ik}^\dagger c_{ik} + \sum_k \left( \gamma_k^2 c_{1k}^\dagger c_{2k} + \text{h.c.} \right) + \sum_{i,j,k,k',q} V_{ijq} c_{iqk} c_{j(qk'-q)k'} c_{ik'} c_{jik}.$$  \hspace{1cm} (35)

The corresponding mean-field Hamiltonian can be written as follows:

$$H_{MF} = \sum_{i,k} \left[ \varepsilon_{ik} + \Sigma_{ii}(k) \right] c_{ik}^\dagger c_{ik} + \sum_k \left( (\gamma_k + \Sigma_{12}) c_{1k} c_{2k} + \text{h.c.} \right)$$

where

$$\Sigma_{ii}(k) = \sum_{k'} \left[ (V_{ii0} - V_{ikk'}^{-1} - k) \langle c_{ik'}^\dagger c_{ik} \rangle + V_{10} \langle c_{1k'}^\dagger c_{1k} \rangle \right], \quad (i \neq j),$$

$$\Sigma_{12}(k) = -\sum_{k'} V_{12k'}^{-1} - k \langle c_{1k'} c_{2k'} \rangle.$$ \hspace{1cm} (36)

The averages over the ground state at $T = 0$ can be calculated simply by going to the diagonal basis of the noninteracting part of the Hamiltonian:

$$H = \sum_{i,k} \varepsilon_{ik} c_{ik}^\dagger c_{ik} + \sum_k \left( \gamma_k^2 c_{1k}^\dagger c_{2k} + \text{h.c.} \right)$$

$$= \sum_k \left[ E_{-k} d_{-k}^\dagger d_{-k} + E_{+k} d_{+k}^\dagger d_{+k} \right],$$ \hspace{1cm} (38a)

where $E_{\pm k'} = \frac{1}{2} \left( \varepsilon_{1k'} + \varepsilon_{2k'} \right) \pm \sqrt{(\varepsilon_{1k'} - \varepsilon_{2k'})^2 + 4\gamma^2}$, with $-,$ referring to the hybridized valence and conduction bands, respectively. The basis transformation is given by the matrix:

$$\begin{bmatrix} c_{1k} \\ c_{2k} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} d_{-k} \\ d_{+k} \end{bmatrix},$$ \hspace{1cm} (39)

where $u_{ij}$ are derived from the eigenvectors as usual. To calculate $\langle c_{ik'}^\dagger c_{ik} \rangle$, we can simply ignore $\gamma$ and write $\langle c_{ik'}^\dagger c_{ik} \rangle = n_F(\xi_{ik'})$, where $\xi_{ik'} = \varepsilon_{ik'} - \mu$ and $n_F(x)$ is the Fermi function. To calculate $\langle c_{1k'}^\dagger c_{2k} \rangle$, we go to the new basis and write $\langle c_{1k'}^\dagger c_{2k} \rangle = \langle (u_{11} d_{-k'}^\dagger + u_{12} d_{+k'}^\dagger)(u_{21} d_{-k'} + u_{22} d_{+k'}) \rangle$. Noting that $\langle d_{-k}^\dagger d_{-k} \rangle = n_F(E_{-k})$ and $\langle d_{+k}^\dagger d_{+k} \rangle = 0$, together with $n_F(E_{+k}) = 0$ at $T = 0$, we finally arrive at the following expressions:

$$\Sigma_{ii}(k) \approx \sum_{k'} [(V_{ii0} - V_{ikk'}^{-1} - k) n_F(\xi_{ik'}) + V_{10} n_F(\xi_{jik'})],$$ \hspace{1cm} (40a)

$$\Sigma_{12}(k) = \gamma \sum_{k'} V_{12k'}^{-1} - k n_F(E_{-k'}) \frac{1}{(E_{+k'} - E_{-k'})},$$ \hspace{1cm} (40b)

[1] W. J. De Haas and P. M. Van Alphen, The dependence of the susceptibility of diamagnetic metals upon the field, Proc. Netherlands Roy. Acad. Sci 33, 1106 (1930).
[2] D. Shoenberg, Magnetic Oscillations in Metals (Cambridge University Press, Cambridge, England, 1984).

[3] G. Li, Z. Xiang, F. Yu, T. Asaba, B. Lawson, P. Cai, C. Tinsman, A. Berkley, S. Wolgast, Y. S. Eo, et al., Two-dimensional fermi surfaces in kondo insulator sb6, Science 346, 1208 (2014).

[4] B. Tan, Y.-T. Hsu, B. Zeng, M. C. Hatnean, N. Harrison, Z. Zhu, M. Hartstein, M. Kiourlappou, A. Srivastava, M. Johannes, et al., Unconventional fermi surface in an insulating state, Science 349, 287 (2015).

[5] M. Hartstein, W. Toews, Y.-T. Hsu, B. Zeng, X. Chen, M. C. Hatnean, Q. Zhang, S. Nakamura, A. Padgett, G. Rodway-Gant, et al., Fermi surface in the absence of a fermi liquid in the kondo insulator sb6, Nature Physics 14, 166 (2018).

[6] H. Liu, M. Hartstein, G. J. Wallace, A. J. Davies, M. C. Hatnean, M. D. Johannes, N. Shitsevalova, G. Balakrishnan, and S. E. Sebastian, Fermi surfaces in kondo insulators, Journal of Physics: Condensed Matter 30, 16LT01 (2018).

[7] Z. Xiang, Y. Kasahara, T. Asaba, B. Lawson, C. Tinsman, L. Chen, K. Sugimoto, S. Kawaguchi, Y. Sato, G. Li, et al., Quantum oscillations of electrical resistivity in an insulator, Science 362, 65 (2018).

[8] Z. Han, T. Li, L. Zhang, G. Sullivan, and R.-R. Du, Anomalous conductance oscillations in the hybridization gap of InAs/GaSb quantum wells, Phys. Rev. Lett. 123, 126803 (2019).

[9] D. Xiao, C.-X. Liu, N. Samarth, and L.-H. Hu, Anomalous quantum oscillations of interacting electron-hole gases in inverted type-II InAs/GaSb quantum wells, Phys. Rev. Lett. 122, 186802 (2019).

[10] P. Wang, G. Yu, Y. Jia, M. Onyszczak, F. A. Cevallos, S. Lei, S. Klemenz, K. Watanabe, T. Taniguchi, R. J. Cava, et al., Landau quantization and highly mobile fermions in an insulator, Nature 589, 225 (2021).

[11] L. Liu, Y. Chu, G. Yang, Y. Yuan, F. Wu, Y. Ji, J. Tian, R. Yang, K. Watanabe, T. Taniguchi, et al., Quantum oscillations in field-induced correlated insulators of a moiré superlattice, Science Bulletin (2023).

[12] J. Knolle and N. R. Cooper, Quantum oscillations without a fermi surface and the anomalous de haas–van alphen effect, Phys. Rev. Lett. 115, 146401 (2015).

[13] L. Zhang, X.-Y. Song, and F. Wang, Quantum oscillation in narrow-gap topological insulators, Phys. Rev. Lett. 116, 046404 (2016).

[14] H. K. Pal, F. Piechon, J.-N. Fuchs, M. Goerbig, and G. Montambaux, Chemical potential asymmetry and quantum oscillations in insulators, Phys. Rev. B 94, 125140 (2016).

[15] H. K. Pal, Quantum oscillations from inside the fermi sea, Phys. Rev. B 95, 085111 (2017).

[16] H. K. Pal, Unusual frequency of quantum oscillations in strongly particle-hole asymmetric insulators, Phys. Rev. B 96, 235121 (2017).

[17] O. Erten, P. Ghaemi, and P. Coleman, Kondo breakdown and quantum oscillations in sm6, Phys. Rev. Lett. 116, 046403 (2016).

[18] I. Sodemann, D. Chowdhury, and T. Senthil, Quantum oscillations in insulators with neutral fermi surfaces, Phys. Rev. B 97, 045152 (2018).

[19] P. Ram and B. Kumar, Theory of quantum oscillations of magnetization in kondo insulators, Phys. Rev. B 96, 075115 (2017).

[20] R. Peters, T. Yoshiida, and N. Kawakami, Quantum oscillations in strongly correlated topological kondo insulators, Phys. Rev. B 100, 085124 (2019).

[21] I. Lifshitz and A. Kosevich, Theory of magnetic susceptibility in metals at low temperatures, Sov. Phys. JETP 2, 636 (1956).

[22] J. M. Luttinger, Theory of the de haas-van alphen effect for a system of interacting fermions, Phys. Rev. 121, 1251 (1961).

[23] A. Wasserman and M. Springford, The influence of many-body interactions on the de haas-van alphen effect, Advances in Physics 45, 471 (1996).

[24] Y. Adamov, I. V. Gornyi, and A. D. Mirlin, Interaction effects on magneto-oscillations in a two-dimensional electron gas, Phys. Rev. B 73, 045126 (2006).

[25] C. Kämpersbusch and L. Fritz, Modifications of the lifshitz-kosevich formula in two-dimensional dirac systems, Phys. Rev. B 96, 205410 (2017).

[26] See supplemental material for the derivations of the main results.

[27] We assume that $\mu$ is a free parameter chosen such that it results in the $\tilde{\mu}$ assumed. This is not a unique choice—the chemical potential can lie anywhere inside the gap. Although this has no consequence at $T = 0$, new qualitative features can arise at $T \neq 0$. This was demonstrated in Ref. [13] for the noninteracting case; additional new features may be expected in the interacting case which we do not consider in this work.

[28] M. A. Continentino, G. M. Japiassú, and A. Troper, Excitonic phase transitions in electronic systems, Journal of Physics: Condensed Matter 7, L701 (1995).

[29] A. A. Allocca and N. R. Cooper, Quantum oscillations in interaction-driven insulators, SciPost Phys. 12, 123 (2022).