Bardeen-Stephen flux flow law disobeyed in the high-\(T_c\) superconductor

\(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\)

Á. Pallinger\(^1\), B. Sas\(^1\), I. Petthes\(^1\), K. Vad\(^2\), F. I. B. Williams\(^1,3\), and G. Kriza\(^1,4\)

\(^1\)Research Institute for Solid State Physics and Optics, PO Box 49, H-1525 Budapest, Hungary
\(^2\)Institute of Nuclear Research, PO Box 51, H-4001 Debrecen, Hungary
\(^3\)CEA-Saclay, Service de Physique de l’Etat Condensé, Commissariat à l’Énergie Atomique, Saclay, F-91191 Gif-sur-Yvette, France and
\(^4\)Institute of Physics, Budapest University of Technology and Economics, Budafoki út 8, H-1111 Budapest, Hungary

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Pulsed high current experiments in single crystals of the high-\(T_c\) superconductor \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) in c-axis directed magnetic field \(H\) reveal that the \(ab\)-face resistance in the free flux flow regime is a solely logarithmic function of \(H\), devoid of any power law component. Re-analysis of published data confirms this result and leads to empirical analytic forms for the \(ab\)-plane and c-axis resistivities: \(\rho_{ab} \propto H^{3/4}\), which does not obey the expected Bardeen-Stephen result for free flux flow, and \(\rho_c \propto H^{-3/4}\log^2H\).

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Free flux flow (FFF) resistivity describes how fast the vortices in a type II superconductor move in the direction of an applied force \([1]\). It is a measure of how the momentum of the superfluid is transferred to the host lattice via quasiparticle excitations. The velocity-force relation expressed by the FFF resistivity has to be taken into account in interpreting any vortex transport, be it global or local. Since the primary source of dissipation in a type II superconductor in magnetic field is vortex motion, FFF resistivity is also of great importance for technical applications.

A transport current exerts a Lorentz-Magnus force on a vortex and if other forces like vortex-defect interaction (pinning) are negligible (i.e., the vortex motion is “free”), the velocity-force relation can be inferred from the resistivity \(\rho_{\text{FFF}}\). The Bardeen-Stephen (BS) law \([2]\) states that \(\rho_{\text{FFF}}\) is proportional to the density of vortices and therefore to the magnetic field \(H\):

\[
\rho_{\text{FFF}} = \gamma \rho_n(H/H_{c2})^\beta, \quad \beta = 1, \quad (1)
\]

where \(\rho_n\) is the normal state resistivity, \(H_{c2}\) the upper critical field, and \(\gamma\) a constant \(\approx 1\). This law has been experimentally established \([3]\) for a number of conventional superconductors. In high-\(T_c\) materials the quasi-two-dimensional (2d) electronic structure, the nodes of the \(d\)-wave order parameter, and the structure of the vortex system may potentially influence FFF. The motion of 2d “pancake” vortices along their well conducting \(ab\) plane leads to dissipation by flux flow, whereas in the poorly conducting \(c\) direction dissipation is governed not by flux flow but by tunneling between weakly coupled \(ab\) planes. A quasiclassical calculation \([4]\) suggests that the BS law is valid also in the \(d\)-wave case.

Experiments to test the validity of Eq. (1) in high-\(T_c\) superconductors—and especially in \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) (BSCCO), the model system of this study—are contradictory. Data on low frequency transport in single crystals \([5]\) and thin films \([6, 7]\) as well as microwave and millimeter wave impedance \([8]\) are inconsistent with one another, agreeing only that the BS law is not obeyed. The resistivity often resembles a sublinear power law in field. The situation is similar in other high-\(T_c\) materials with the notable exception of the results of Kunchur et al. \([9]\) who find agreement with BS law in thin film resistivity measurements in \(\text{YBa}_2\text{Cu}_3\text{O}_7\). However, the resistance they measure does not saturate, i.e., it increases with current, up to the highest current they use, leading to an uncertainty in the value of \(\rho_{\text{FFF}}\). A re-analysis of the differential resistance indicates again a sublinear field dependence of \(\rho_{\text{FFF}}\).

The main difficulty in measuring the velocity-force relation is to take account of the pinning force about which one has little detail. One way is to model pinning to interpret the surface impedance arising from local vortex motion. Another approach is to create experimental conditions where pinning is irrelevant as occurs in a true (unpinned) vortex liquid. We extend our experiments to the non-ohmic regime by applying sufficiently high current that the pinning force is negligible compared with the Lorentz-Magnus force from the transport current.

To this end we have made pulsed high-current transport measurements on single crystal BSCCO with electrode contacts on the face parallel to the well conducting \(ab\) planes in a \(c\)-axis directed magnetic field. The global single crystal resistance measured on the \(ab\)-face in the ohmic regime and the asymptotic high-current differential resistance in the non-ohmic regime show the same logarithmic magnetic field dependence devoid of any power law. We combine this result with published data from other experiments \([10, 11]\) and set up empirical functional forms for the local resistivities \(\rho_{ab}\) and \(\rho_c\), valid over a broad range of temperature and field in the vortex liquid phase. The most striking and important of our conclusions is that for BSCCO \(\beta = 3/4\) in Eq. (1).
We selected for experiment three single crystals from 3 different batches of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ with typical dimensions $1 \times 0.5 \times 0.003 \text{ mm}^3$, the shortest corresponding to the poorly conducting $c$ axis. All were close to optimal doping with a resistance-determined critical temperature $T_c \approx 89 \text{ K}$ and transition width about 2 K in zero field; the diamagnetism in a 1 mT field set in progressively below $T_c$ to near 100% at low temperature.

Voltage-current ($V$-$I$) response was measured in the usual four-point configuration on an $ab$ face in perpendicular magnetic field with two current contacts across the width near the ends and two point voltage contacts near each edge of the same face. The contacts were made by bonding 25 $\mu\text{m}$ gold wires with silver epoxy fired at 900 K in an oxygen atmosphere resulting in current contact resistances of less than 3 $\Omega$. To avoid significant Joule heating, we employed short ($\lesssim 50$ $\mu$s) current pulses of isoceles triangular shape at 0.2 to 1 s intervals. Technical details and the issue of Joule heating are treated in Ref. [12] with the conclusion that the temperature change in the area between the voltage contacts is negligible for the duration of the pulse.

Typical $V$-$I$ characteristics at different temperatures in a field of 3 T are shown in Fig. 1. Above a temperature $T_{\text{lin}} < T_c$, the $V$-$I$ curves are linear (see the lower inset of Fig. 2 for the field dependence of the characteristic temperatures). Below $T_{\text{lin}}$ nonlinearity develops and the $I \to 0$ resistance decreases faster than exponentially until it becomes unmeasurably small even with the most sensitive technique. At low temperature dissipation sets in abruptly at a threshold current $I_{k}$; for higher temperatures a marked upturn in the $V$-$I$ curve (a “knee”) is seen at $I_k \lesssim I_{k0}$. Throughout the nonlinear range the differential resistance increases with increasing current; for currents several times $I_{k}$ or $I_{k0}$ it saturates (becomes current independent) at a value $dV/dI = R_{ab}$ as shown in the inset of Fig. 1. Since $I_{k}$ and $I_{k0}$ are hallmarks of depinning, the current-independent $R_{ab}$ observed at many times this current suggests that at these high currents pinning is irrelevant and $R_{ab}$ reflects FFF [13]. The focus of this article is the behavior of $R_{ab}$.

The upper inset of Fig. 2 shows the field dependence of $R_{ab}$ for the temperatures indicated in the phase diagram of the lower inset. The resistance is field and temperature independent at low temperature. With increasing temperature there is a crossover to a field-dependent behavior at $T_{\text{co}}$ (open circles in the phase diagram) situated between the magnetic irreversibility line and $T_{\text{lin}}$. Having in mind that in the BS law the characteristic field is $H_{c2}$, we interpolate the upper critical field using the form $H_{c2} = (120 \text{ tesla})[1 - (T/T_c)^2]$ constructed from $dH_{c2}/dT_{c2} = -2.7 \text{ T/K}$ [14] and use it to plot $R_{ab}$ against $H/H_{c2}$ on a logarithmic field scale in the main panel of Fig. 2. The high-temperature curves all collapse into a master curve representing a logarithmic field dependence:

$$R_{ab}(H, T) = R_{ab0}[1 + \alpha \log(H/H_{c2}(T))] ,$$

where $R_{ab0}$ is the zero field normal resistance at $T_c$ and $\alpha$ is a constant. This scaling only contains the temperature through $H_{c2}(T)$. Equation (2) provides an excellent description of all three samples; for the parameter $\alpha$ we find 0.16, 0.19 and 0.21, essentially the same values, in-
sensitive to the presumably different disorder between samples.

We emphasize that $R_{ab}$ cannot be compared directly to the BS law because the strong anisotropy of the electronic properties makes the current distribution very inhomogeneous and $R_{ab}$ reflects both $ab$-plane and $c$-axis properties. However, if the sample is thick in the $c$ direction, a simple scaling argument for a current independent local resistivity tensor yields $R_{ab} = A \rho_{ab} \rho_c$ where $A$ is a geometrical factor. This relation is valid in the linear region $T > T_{\text{lin}}(B)$ but also below $T_{\text{lin}}$ if the current is sufficiently high that the current density is well in the upper differentially linear portion of the response near the top surface of the crystal [13]. The analysis and experimental checks of Ref. [10] indicate that with the sample size, shape, and contact geometry used in their and our single crystal $ab$ plane studies in BSCCO, the thick sample limit provides a good description.

Independent confirmation of our results emerges from analysis of other experiments. Although high-current data are absent, the fact that the $V-I$ curves are linear for $T > T_{\text{lin}}(H)$ allows comparison with low-current data in this temperature and field range. In Fig. 3(a) we show $R_{ab}$ vs. $H$ curves extracted from the $R_{ab}$ vs. $T$ data taken by Busch et al. [10] in different magnetic fields. Excellent agreement with Eq. (2) is seen for $T > T_{\text{lin}}$ (full symbols in the figure) with $\alpha = 0.23$.

Busch et al. [10] were able to disentangle $\rho_{ab}$ and $\rho_c$ by using data from two additional contacts on the bottom of the crystal. $\rho_{ab}$ and $\rho_c$ results extracted from their work for a series of temperatures are shown as a function of $H/H_{c2}$ in Fig. 3(b) and (c). In the temperature and field range $T > T_{\text{lin}}(H)$ both quantities individually exhibit $H/H_{c2}$ scaling. The in-plane resistivity does not agree with the $\beta = 1$ BS law of Eq. (1), but is well described by a $\beta = 3/4$ exponent (best fit $\beta = 0.75 \pm 0.02$). Having definite analytic forms for both $R_{ab}$ and $\rho_{ab}$, we can use the relation $(R_{ab}/A)^2 = \rho_{ab} \rho_c$ to write an expression for the $c$-axis resistivity in summary:

$$
\rho_{ab} \equiv \rho_{\text{FF}} = \rho_{ab}^0 (H/H_{c2})^\beta, \quad (3)
$$

$$
\rho_c = \rho_c^0 (H/H_{c2})^{-\beta} [1 + \alpha \log(H/H_{c2})]^2, \quad (4)
$$

$$
\beta = 3/4, \quad \alpha = 0.2 .
$$

The prefactors $\rho_{ab}^0$ and $\rho_c^0$ are in good agreement with the respective normal resistivities at $T_c$.

Are these forms corroborated by other types of measurement? In principle $\rho_{ab}$ can be measured in thin films where the current density is expected to be homogeneous. In Fig. 3(b) we show the 77 K thin film resistivity obtained by digitizing the $V-I$ curves in Ref. 7. Data above about 1 T are reasonably well described by a $H^{3/4}$ dependence, but a closer look reveals that the log $\rho_{ab}$ vs. log $H$ curves are concave from below at every $H$, i.e., there is a systematic deviation from power law. This “logarithm like” (but not logarithmic) dependence is shared with other thin film results [6] but there are significant quantitative differences between data measured by different groups. A possible reason is that macroscopic defects like steps on the surface or mosaic boundaries force $c$-axis currents and the measured resistance is a sample-dependent combination of $\rho_{ab}$ and $\rho_c$.

The expression for $\rho_c$ reproduces well the maximum (at $H_{\text{max}} = H_{c2} \exp(8/3 - 1/\alpha) \sim 0.1 H_{c2}$) observed in the high field $c$-axis magnetoresistance [11] and the overall field dependence of these independently measured data is very well described by Eq. (4). We demonstrate this in Fig. 3(c) where we plot 70 K data for $\sigma_c(B) - \sigma_0$ from Fig. 5 of Ref. [11] where $\sigma_c = 1/\rho_c$ and the constant $\sigma_0$ is interpreted as the zero-field quasiparticle conductivity. Using our estimate of $H_{c2}(T) \approx 46$ T for $T = 70$ K, Eq. (4) fits the measured data with parameters $\sigma_c^0 = 1/\rho_c^0 = 6.6 \text{ (kfcem)}^{-1}$, $\sigma_0 = 3.6 \text{ (kfcem)}^{-1}$ and $\alpha = 0.20$ to obtain the curve indicated by the continuous line in the figure. The value of $\alpha$ is in excellent agreement with that inferred from $R_{ab}$ measurements. It should be pointed out, however, that $\sigma_0$ is significantly smaller than the

![Graph](image-url)
value $\sigma_0 \approx 8$ (kΩcm)$^{-1}$ inferred in Ref. [11]. In terms of resistivities, this means that $\rho_c$ decreases more slowly in high fields than described by Eq. (1) with $\beta = 3/4$ and is in fact best described with an exponent $\beta = 0.51$.

In the high-current limit the same form for $R_{ab}(H)$ also holds below $T_{\text{lin}}(H)$ where the $V$-$I$ curves are nonlinear. Since no change in the behavior of $R_{ab}$ is observed when the $T_{\text{lin}}(H)$ line is crossed, it is reasonable to assume the same for $\rho_{ab}$ and $\rho_c$. In the low field direction a lower limit of the validity of Eq. (2) is the zero of the equation at $H_0/H_{c2} = e^{-1/\alpha} \approx 10^{-3} \cdot 10^{-2}$, higher but in the order of the first order transition in the static vortex system. In the high-field direction Eq. (2) is valid up to the highest field $\approx 0.3 H_{c2}$ we investigated.

The temperature $T_{co}$ of the crossover from $R_{ab} = \text{const}$ to $R_{ab} \propto \log H$ is distinctly higher than the onset of magnetic irreversibility at $T_{\text{irr}}$, and also above the vortex glass transition $T_g \approx T_{\text{irr}}$ inferred from scaling analysis [16] of the $V$-$I$ curves. On the other hand, no change in the behavior of $R_{ab}$ is observed when the $T_{\text{irr}}(H)$ and $T_g(H)$ lines are crossed. This suggests that because the pinning potential is smoothed at high velocities, the phase diagram [17] of the far-from-equilibrium dynamic vortex system [18] is different from that of the unperturbed thermodynamic phases. Since $R_{ab}$ behaves the same in the pinned ($T < T_{\text{lin}}$) and unpinned ($T > T_{\text{lin}}$) liquid phases, we propose that the unpinned phase, otherwise observed only above $T_{\text{lin}}$, may be restored in the range $T_{co} < T < T_{\text{lin}}$. Then $T_{co}$ may approximate the melting transition in a hypothetical defect-free crystal.

Our most robust finding, invariably observed not only in our 3 batches but also in the data of Ref. [10], is the logarithmic field dependence of the high-current single crystal resistance $R_{ab}$. Although a power of $H$ factor is expected both in $\rho_{ab}$ [1] and $\rho_c$, no such factor is present in $R_{ab} \propto \sqrt{\rho_{ab}\rho_c}$. The most likely reason is that the power-law factors in $\rho_{ab}$ and $\rho_c$ cancel (exponents 3/4 and -3/4 in our analysis). The cancellation is very accurate; we estimate that a power law factor with exponent as low as 0.1 could be observed in our $R_{ab}$ data. Moreover, because we find no logarithmic correction to $\rho_{ab}$, the logarithmic dependence of $R_{ab}$ is carried by $\rho_c$.

Arguing that both $\rho_{ab}$ and $\sigma_c$ are proportional to the quasiparticle density of states at the Fermi level, $N(0)$, it cancels in the product $\rho_{ab}\rho_c$. In conventional superconductors $N(0)$ is proportional to the number of vortices therefore to $H$, leading to the $H$-linear resistivity of the BS law. In nodal gap superconductors near-nodal quasiparticles lead to a sublinear dependence; for line nodes $N(0) \propto H^{1/2}$ [20], as evidenced in recent low-temperature thermodynamic measurements [21]. Although delocalized near-nodal quasiparticles are not expected to contribute significantly to $\rho_{\text{FF}}$ because of the weak spectral flow force [4] they experience, the result may be different in the diffusive limit in the liquid phase. A possible reason for the $\beta = 3/4$ exponent is the different structure factors of the solid and liquid phases.

Nonlocal effects [22, 23] may influence the evaluation of the 6-contact measurements [11] and therefore the validity of Eqs. (3) and (4) (but not of Eq. (2)). This seems, however, unlikely in the light of the good agreement of $\rho_c$ inferred from independent $ab$-plane [10] and $c$-axis [11] measurements and of the broad temperature range of validity of Eq. (2).

In conclusion, we have set up empirical rules for the analytic form of single crystal resistance as well as for the $ab$ plane and $c$ axis resistivities in the high-current free flux flow limit in the vortex liquid state of BSCCO, valid over a broad range of temperature and field. Both the logarithmic field dependence of the single crystal resistance and the $3/4$-power law in the $ab$-plane free flux flow resistance are in disagreement with the current theoretical understanding of high-$T_c$ superconductors.

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