Intrinsic alignments of dark matter halos in $f(R)$ gravity simulations

Yao-Tsung Chuang1,2*, Teppei Okumura1,3 † and Masato Shirasaki4,5

1 Institute of Astronomy and Astrophysics, Academia Sinica, No. 1, Section 4, Roosevelt Road, Taipei 10617, Taiwan
2 Department of Physics, National Taiwan University, No. 1, Section 4, Roosevelt Road, Taipei 10617, Taiwan
3 Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
4 National Astronomical Observatory of Japan (NAOJ), Mitaka, Tokyo 181-8588, Japan
5 The Institute of Statistical Mathematics, Tachikawa, Tokyo 190-8562, Japan

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ABSTRACT

There is a growing interest of utilizing intrinsic alignment (IA) of galaxy shapes as a geometric and dynamical probe of cosmology. In this paper, we present the first measurements of IA statistics of dark-matter halos using modified gravity models. We then show that the IA statistics are indeed useful to tighten the constraint on gravity models by combining with the conventional galaxy clustering statistics.

1 INTRODUCTION

The origin of the cosmic acceleration has been one of the most profound mysteries for decades (Weinberg et al. 2013). Many studies have explored it by considering dark energy as a source of the cosmic acceleration. Modifying the law of gravity at cosmological scales is an alternative way to explain the acceleration (Wang & Steinhardt 1998; Linder 2005). Conventionally, galaxy clustering observed in redshift surveys has been extensively exploited for this purpose (e.g., Guzzo et al. 2008; Reyes et al. 2010; Okumura et al. 2016).

Intrinsic alignment (IA) of galaxy shapes, originally focused as a contaminant to gravitational lensing signals (Croft & Metzler 2000; Heavens et al. 2000), and the gravitational shear-intrinsic ellipticity (GI) correlation functions, have been drawing attention as a new dynamical and geometric probe of cosmology (e.g., Chisari & Dvorkin 2013; Okumura et al. 2019; Hirata & Okumura 2020; Kurita et al. 2021; Reischke et al. 2021; Okumura & Taruya 2021). However, such a possibility has been explored merely by forecast studies or numerical simulations based on the ΛCDM model. Thus, we still do not know how the observed IA looks like in gravity models beyond general relativity (GR).

In this paper, we present the first measurements of IA statistics of dark-matter halos in modified gravity using N-body simulations of the $f(R)$ gravity model. We then show that the IA statistics are indeed useful to tighten the constraint on gravity models by combining with the conventional galaxy clustering statistics.

2 INTRINSIC ALIGNMENT STATISTICS

This section provides a brief description of the three-dimensional alignment statistics following Okumura & Taruya (2020). In this paper, we measure all the statistics in redshift space, and thus the halo overdensity field $δ_h$ below is sampled in redshift space and suffers from redshift-space distortions (RSD Kaiser 1987).

To begin with, orientations of halos or galaxies projected onto the sky are quantified by the two-component ellipticity, given as

$$γ(+, x)(x) = \frac{1 - (β/α)}{1 + (β/α)} \cos(2θ) \sin(2θ),$$

where $θ$, defined on the plane normal to the line-of-sight, is the angle between the major axis projected onto the celestial sphere and the projected separation vector pointing to another object, $β/α$ is the minor-to-major axis ratio and we set $β/α = 0$ for simplicity.

In this paper, together with the halo density correlation function, $ξ_{hh}$, abbreviated as the GG correlation, we study two types of IA statistics, the intrinsic ellipticity (II) correlation functions, $ξ_+$ and $ξ_-$ (Croft & Metzler 2000; Heavens et al. 2000), and the gravitational shear-intrinsic ellipticity (GI) correlation functions, $ξ_{hh}$ (Hirata & Seljak 2004). They are concisely defined as

$$1 + ξ_X(r) = \langle \{1 + δ_h(x_1)\} \{1 + δ_h(x_2)\} \rangle W_X(x_1, x_2),$$

where $X = \{hh, h+, -\}$ and $r = x_2 - x_1$. The GG, GI and II correlation functions are characterized by the function $W_X(x_1, x_2)$.
3 F(R) GRAVITY SIMULATIONS

In f(R) gravity theories, we replace the Ricci scalar R in the Einstein–Hilbert action by some general function of R, f(R), to mimic the effect of the cosmological constant Λ (Starobinsky 1980; De Felice & Tsujikawa 2010). We adopt the functional form of f(R) introduced by Hu & Sawicki (2007), which deviates from the law of gravity from GR is characterized by f_R0 \equiv d f(R)/dR|_{\Lambda = 0}

We perform N-body simulations under the ΛCDM model and f(R) gravity and study the difference of the alignment statistics measured from them. The cosmological N-body simulations have been run with ECOSMOCG code (Li et al. 2012). Every simulation in our paper consists of 512^3 particles in a cubic box with the side length being L = 316h^{-1}\text{Mpc}. The initial condition of the simulation is generated with the 2LPTic code using second-order Lagrangian perturbation theory (Crocce et al. 2006). We set the initial redshift to z = 50 and mainly work with the simulation outputs at z = 0 below. Note that the same initial condition has been used for both of ΛCDM model and f(R) gravity runs. Each simulation assumes the following cosmological parameters consistent with the measurement of the cosmic microwave background by Planck Collaboration et al. (2016): the total matter density Ω_m = 0.3156, the baryon density Ω_b = 0.0492, the cosmological constant Ω_Λ = 0.6844, the present-day Hubble parameter H_0 = 67.27\text{km/s/Mpc}, the spectral index of initial curvature perturbations n_s = 0.9645, and the amplitude of initial perturbations at k = 0.05\text{Mpc}^{-1} being A_s = 2.2065 \times 10^{-9}. For the f(R) gravity run, we choose the value of f_R0 as |f_R0| = 10^{-5} and set the functional form of f(R) \equiv R/(R+\text{const}). It is worth noting that the present-day linear mass variance smoothed at 8h^{-1}\text{Mpc}, referred to as σ_8, are set to 0.831 and 0.883 for the ΛCDM and f(R) gravity runs, respectively.

From the simulation outputs, we identify dark matter halos with a phase-space halo finder of ROCKSTAR (Behroozi et al. 2013). In this paper, we use the standard definition for the halo mass, M_{200}, defined by a sphere with a radius R_{200} within which the enclosed average density is 200 times the mean matter density (Navarro et al. 1996). We then create a halo catalog with the mass threshold of M_{200} > 10^{13}h^{-1}\text{M}_\odot. The lowest halo mass can be resolved with 485 particles in our simulation. Our halo sample does not include subhalos. The velocity of each halo is computed by the average particle velocity within the innermost 10% of the virial radius. We estimate the orientations of halos using the second moments of the distribution of member particles projected on the celestial plane.

4 CORRELATION FUNCTIONS IN F(R) GRAVITY

4.1 Correlation function measurements

Here we measure the GG, GI and II statistics in the N-body simulations. Since all these statistics have explicit angular dependences (Hamilton 1992; Okumura & Taruya 2020), we consider their multipoles in terms of the Legendre polynomials, P_l,

$$\xi_{X,\ell}(r) = \frac{2\ell + 1}{2} \int_{-1}^{1} \xi_X(r) P_{\ell}(\mu) d\mu,$$

where X = \{hh, h+, +, -\} and μ = \hat{x} \cdot \hat{r} with a hat denoting a unit vector. In linear perturbation theory all the four correlation functions, ξ_{X,\ell}, have non-zero values only for \ell = 0, 2 and 4. We thus consider only these three multipoles for each correlation function, and hence the number of the total statistics is 4 \times 3 = 12.

Our estimators for the multipoles, ξ_{X,\ell}, are expressed as (Okumura et al. 2020)

$$\xi_{X,\ell}(r) = \frac{2\ell + 1}{2} \frac{1}{RR(r)} \sum_{j,k=1} W_{X,jk} P_{\ell}(\mu_{jk}),$$

where RR is the pair counts from the random distribution, which can be analytically computed because we place the periodic boundary condition on the simulation box. For the GG, GI and II correlations, W_{hh, jk} = 1, W_{h+, jk} = γ_s(x_j), and W_{+, jk} = γ_s(x_j)γ_s(x_k) + γ_v(x_j)γ_v(x_k), respectively.

From the left to right of the first row in Fig. 1, we show the multipoles of the GG, GI, II− correlation functions measured from the f(R) gravity simulations at z = 0. This is the first measurements of intrinsic alignments from modified gravity simulations. We adopt 10 logarithmically spaced bins in r over 1 < r < 100 [h^{-1}\text{Mpc}]. The data at the scales below and above this range are severely affected by the halo exclusion effect and cosmic variance, respectively. The second, third and last rows of Fig. 1 are respectively the monopole, quadrupole and hexadecapole moments from the f(R) gravity simulation divided by those from the ΛCDM simulation, ξ_{X,\ell}/ξ_{X,\ell}_{GR}. Note that again, all the correlation functions are measured in redshift space which are a direct observable in real galaxy surveys, and thus they are affected by RSD.

Different gravity models predict different values of the growth factor D(z) and its derivative, f(z) = -d \ln D(z)/d \ln (1 + z). Furthermore, since the number density in the f(R) gravity model tends to be higher than that in the GR model, halos in f(R) gravity are less biased than those in ΛCDM with the same masses. As a result, the amplitude of the GG correlation in f(R) gravity becomes lower than that in GR (e.g., Alam et al. 2020), as seen in the lower panels of the leftmost column in Fig. 1.

Similarly to the GG correlation, the three multipoles of the GI correlation in the f(R) gravity model shows a negative deviation from those in ΛCDM. Note that the non-zero hexadecapole comes from the RSD effect (Okumura & Taruya 2020). Since the II+ and II− correlation functions are noisier than the GI correlation function, as is known for the ΛCDM case, it is harder to see the difference in the II correlation. Nevertheless, for the monopole and quadrupole one can see the same trend that the amplitude in f(R) gravity is smaller than that in ΛCDM, due to the fact that the amplitude of IA, often characterized by b_K (see equation 7), is positively correlated with the halo bias (Akitsu et al. 2021; Kurita et al. 2021); more massive halos tend to be more strongly aligned with the large-scale structure.

4.2 Covariance matrix

We use a bootstrap-resampling technique (Barrow et al. 1984) to estimate a covariance error matrix for the measured statistics in f(R) gravity simulations. With a given original halo catalog, we construct a new catalog by randomly choosing the same number of halos as the original catalog, allowing repetition. We repeat this process until we obtain the required number of bootstrap realizations, N_{bs}. For the
The kth realization (1 ≤ k ≤ N_{\text{bs}}), we measure the correlation function multipoles, $\xi_{X,\ell}(r)$, where $X = \{hh, h+, +,-\}$ and $\ell = 0, 2, 4$.

Given the measurements of the correlation functions, their covariance matrix, $C_{ij}^{Xk} = C[\bar{\xi}_{X,\ell}(r_i), \bar{\xi}_{X',\ell'}(r_j)]$ can be estimated as

$$C_{ij}^{Xk} = \frac{1}{N_{\text{bs}} - 1} \sum_{k=1}^{N_{\text{bs}}} \left[ \bar{\xi}_{X,\ell}(r_i) - \bar{\xi}_{X,\ell}(r_i) \right] \times \left[ \bar{\xi}_{X',\ell'}(r_j) - \bar{\xi}_{X',\ell'}(r_j) \right],$$

where $\bar{\xi}_{X,\ell}$ is the average of $\xi_{X,\ell}^k$ over $N_{\text{bs}}$ realizations, $\bar{\xi}_{X,\ell} = N_{\text{bs}}^{-1} \sum_{k=1}^{N_{\text{bs}}} \xi_{X,\ell}^k$. Since the number of bins of each statistic is 10, the size of the full covariance becomes $120 \times 120$. In this work, we choose $N_{\text{bs}} = 500$ to avoid the covariance matrix being singular. The obtained full covariance matrix normalized by the diagonal elements, $C_{ij}^{Xk} / (C_{ii} \cdot C_{jj})^{1/2}$, is shown in Fig. 2. The diagonal components, $C_{ii}^{Xk}$, are shown as the errorbars of our statistics in Fig. 1.

5 Distinguishing Gravity Models with IA

Here we investigate how well one can improve the distinguishability between the $\Lambda$CDM and $f(R)$ gravity models by considering the IA statistics. For this purpose, we introduce a parameter $A$ which characterizes the difference between the given statistics $\bar{\xi}_X$ of the two models, as $A = \bar{\xi}_X^{GR}(r) / \bar{\xi}_X^{GR}(r) = \bar{\xi}_X,\ell(r) / \bar{\xi}_X^{GR},\ell(r)$, where $X = \{hh, h+, +,-\}$ and $\ell = \{0, 2, 4\}$, and constrain the parameter. We add subscripts to $A$ depending on which statistics is used to be used for the constraints, e.g., $A_{GG}$, $A_{GI}$ and $A_{II}$ for the GG-, GI- and II-only analyses, respectively, and $A_{GG+GI+II}$ for their combination. We adopt a simple $\chi^2$ statistic to constrain the parameter $A$ which is given by

$$\chi^2(A) = \sum_{i,j,\ell,\ell'} C^{-1} X_i X_j \Delta_i \Delta_j,$$

where $\Delta_i = \bar{\xi}_{X,\ell}(r_i) - \bar{\xi}_X^{GR}(r_i)$. The covariance of the correlation functions in $f(R)$ gravity, $C_{ij}^{Xk}$, is the $120 \times 120$ matrix, and for the
single-statistics analysis of the GG or GI correlation, the covariance is reduced to a 30×30 submatrix, while the analysis of the II correlation, II(+ ) and II(− ), needs the 60 × 60 submatrix. Table 1 summarizes the degree of freedom for each choice of the statistics. Note that the constant model above is too simple and in reality the deviation of $A$ from unity is scale-dependent. To properly take into account the scale dependence, the detailed modeling of IA statistics in $f(R)$ gravity is required. Furthermore, the amount of the deviation from unity is not necessarily equivalent between different statistics. Thus, we do not focus on the best-fitting values of $A$ but are rather interested in how well we can exclude the possibility of the correlation functions under the two models being equal, namely $A = 1$, and whether the constraint gets tighter by combining the IA statistics with the clustering statistics.

Fig. 3 shows $\Delta \chi^2 = \chi^2 - \chi_{\text{min}}^2$ as a function of $A$, where $\chi_{\text{min}}^2$ is the minimized $\chi^2$ value with the best-fitting $A$. The black-dotted, yellow-dotted and blue-long-dashed curves are the constraints from the GG, GI and II correlations, respectively. While the GG correlation gives the tightest constraint as expected, the GI and II correlations also provide meaningful constraints. The best-fitting parameter $A_{\text{GG}}$ is shown as the horizontal lines in the second, third and fourth rows on the leftmost column of figure 1. Similarly, $A_{\text{GI}}$ is shown on the second leftmost column and $A_{\text{II}}$ is on the third and fourth columns.

As a demonstration, we calculate the model prediction of the GG, GI and II correlations in $f(R)$ gravity using linear perturbation theory. In Fourier space, the halo density field in redshift space, $\delta_h(k)$, is linearly related to the underlying matter density field in real space, $\delta_m^R(k)$, via the Kaiser factor (Kaiser 1987), $\delta_h(k) = (b_h + \mu_k^2)\delta_m^R(k)$, where $\mu_k = \hat{k} \cdot \hat{x}$ and $b_h$ is the linear halo bias. The superscript $R$ denotes a real-space quantity. For the IA statistics, we adopt the linear alignment (LA) model (Catelan et al. 2001; Hirata & Seljak 2004), which relates the ellipticity field linearly to the tidal gravitational field, described in Fourier space as

$$\gamma_+(k) = b_K(k_+^2 - k_0^2)\delta_m^R(k)/k^2,$$

where $b_K$ is the shape bias parameter. Following Okumura & Taruya (2020) we introduce the expression,

$$\Sigma_{\ell}(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 n dk P_m^R(k) f_{\ell}(kr),$$

where $P_m^R(k)$ is the matter power spectrum in real space, computed by the CAMB code (Lewis et al. 2000). We can then write all the IA statistics in the LA model in terms of $\Sigma_{\ell}$, similarly to the case of the GG correlation (Hamilton 1992). In the top row of Fig. 1, we show $\Sigma_{\ell}^{GR}$ multiplied by the best-fitting value of $A$. The bias parameter $b_h$ is determined as $b_h = 1.32$ by fitting the ratio of the GG correlation function to the matter correlation function $\xi_m^R = \xi_m^R(\ell_0)$, $b_h^2 = \xi_{hh,0}/\xi_m^R$, to the measurement on large scales. Similarly, $b_K$ is determined as $b_K = 0.40$ by fitting $b_K^2 = (15/8)\xi_{+\ell}/\xi_m^R$.

Finally, we study how well one can improve the distinguishability between the ΛCDM and $f(R)$ gravity models by combining the IA statistics with the conventional clustering statistics. The constraint using the combination of GI and II correlations is shown as the red curve in Fig. 3. It is interesting to see that though the constraint from the conventional GG-correlation analysis is stronger and excludes the case of $\xi_{hh,\ell} = \xi_{hh,\ell}^{GR}$ with 5.22σ C.L., one can achieve the meaningful constraint from GI+II (5.06σ). Once we combine all the statistics, namely 12 multipoles, the distinguishability reaches 6.31σ. Note that the analysis is based on various assumptions and simplifications, and thus these numbers do not have much importance. Rather, the amount of the improvement (~20%) matters. For higher redshifts, one can see the similar trend. All the numerical values obtained at $z = 0, 0.5$ and 1 are summarized in table 1.

6 CONCLUSIONS

In this paper, we have presented the first measurements of IA in a gravity model beyond GR using the two types of IA statistics, the GI and II correlation functions of halo shapes from $f(R)$ gravity simulations. By comparing them with the same statistics measured in ΛCDM simulations, we found that the IA statistics in different gravity models show distinguishable features, with a trend similar to the case of conventional galaxy clustering statistics. Quantitatively, IA
Table 1. Summary of constraints on the ratio of the statistics between $f(R)$ gravity and $\Lambda$CDM models, $A = \xi_X/\xi_X^{GR}$, at $z = 0, 0.5$ and 1. The second column represents the degree of freedom for each constraint.

| Statistics | d.o.f. | $\xi = 0$ | $\xi = 0.5$ | $\xi = 1$ |
|------------|--------|------------|------------|------------|
| GI         | 29     | 13.22      | 12.59      | 63.11      |
| GG         | 29     | 24.72      | 12.76      | 54.52      |
| II         | 59     | 48.44      | 36.37      | 54.52      |
| GI+II      | 89     | 78.45      | 54.52      | 63.11      |
| GG+GI+II   | 119    | 101.22     | 63.11      | 48.74      |

Intrinsic alignment in modified gravity simulations

Our constraints on different gravity models have been made assuming that the effect of the modified gravity on the clustering and IA statistics can be perfectly modeled. However, in the analysis of actual observations of IA, one needs to model the present statistics from linear to quasi non-linear scales. While there are several theoretical studies of IA beyond linear theory in GR (e.g., Blazek et al. 2019; Vlah et al. 2020), such predictions need to be carefully tested and extended to gravity models beyond GR. Furthermore, in real surveys, one observes shapes of galaxies, not of halos, thus misalignment between the major axes of galaxies and their host halos (Okumura et al. 2009) would degrade the detection significance of IA even in a modified gravity scenario. As a result, the distinguishability between different gravity models would be degraded compared to the results obtained in this paper. On the other hand, in this work we used only the amplitude of the multipole moments of the clustering and IA statistics, not the full shape of the underlying matter power spectrum which contains ample cosmological information but is more severely affected by the nonlinearities. Therefore, constraining power could eventually be either enhanced or suppressed. The more detailed and realistic analysis of clustering and IA statistics beyond a consistency test of GR will be performed in our future work.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding authors.

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