Long term orbital dynamics of a retrograde Centaur 2006 RJ2

A D Pangestu\textsuperscript{1,2} and B Dermawan\textsuperscript{1}

\textsuperscript{1} Master Program in Astronomy, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia
\textsuperscript{2} National Institute of Aeronautics and Space, Indonesia

e-mail: ayu.dyah@lapan.go.id

Abstract. Centaurs are small bodies whose semi-major axes are located between Jupiter and Neptune. They have chaotic orbits and may be in orbital resonances with giant planets. An orbital resonance occurs when a simple commensurability presents between some fundamental frequencies of orbital elements. One of the resonance states is 1:1 (mean motion) resonance or co-orbital with the respected planet. This resonance can also occur in high inclination Centaurs. This study describes the results of long term orbital integration of a retrograde Centaur 2006 RJ2 (inclination of 165°) for 200,000 years. Resonance state of 2006 RJ2 was examined using the FAIR (Fast Identification of Mean Motion Resonance) method for 243 orbital clones with automation by developing Python codes. We report that 2006 RJ2 may be in retrograde 1:1 resonance with Saturn for about 160,000 years in the future. However, the clones exhibit chaotic orbits and show many temporally unstable resonances. Since the quality of its orbital elements is not quite precise, 2006 RJ2 shows a low possibility in the state of long retrograde 1:1 resonance with Saturn.

Keywords: centaurs, retrograde-polar resonance, co-orbital, temporally resonance, FAIR method automation.

1. Introduction
The presence of high inclination objects (more than 70°) in mean motion resonance with a giant planet is not well understood. Although mean motion resonance is a common feature in the Solar System, most of the objects known to be in resonance have small inclinations [4]. But recently, it was found an object in mean motion resonance capture with Jupiter at large inclination and retrograde orbit. The object is 2015 BZ509 that orbits Jupiter with an inclination of 163° and in 1:-1 retrograde mean motion resonance (co-orbital) with the planet [5]. This resonance can also occur on high inclination Centaurs, which are small bodies whose semi-major axes are between Jupiter and Neptune. Centaurs may be in orbital resonances with giant planets due to the gravitational perturbation from the planets itself, but the resonance may also just a temporal and unstable. In this paper, we report that Centaur 2006 RJ2 has a possibility in retrograde 1:1 resonance with Saturn for about 160,000 years in the future. This object has a retrograde orbit with inclination of 165°, so 2006 RJ2 will be in a retrograde co-orbital (1:-1) resonance. We generated 243 orbital clones of this object then integrated them for 200,000 years. The resonance state was examined using the FAIR method with Python codes automation as described in the following section.
2. Polar resonance, retrograde, and co-orbital motion

An orbital resonance occurs when a simple commensurability presents between some fundamental frequencies of orbital elements. It is a dynamical condition when the ratio of the orbital periods of two objects can be expressed as the ratio of two small integers \((j:k)\). If a high inclination small body is in mean motion resonance with a giant planet, that mean motion resonance is known as polar resonance. This resonance is not well understood and quite unstable due to the chaotic orbits of the high inclination objects. High inclination objects are orbiting the Sun in retrograde motion. From [6], retrograde orbital motion is a clockwise revolution of the objects around the Sun seen from the north ecliptic pole of the Solar System. Small bodies in this motion are rare in the Solar System. Among the 753,782 small bodies found so far, only 93 are in retrograde motion [2]. Because the majority of the objects are scattered or jump out of the system, the presence of high inclination objects in mean motion resonance is also rare.

One of the mean motion resonances is 1:1 to the respected planet. Based on this resonance, the orbital period of the object and the respected planet are the same. As mentioned before, 2015 BZ509 is found to be the first high inclination object and bounded in 1:1 resonance with a giant planet. This object has a stable 1:1 resonance orbit with Jupiter for a long time (million years) [6]. This is a unique phenomenon because the majority of high inclination retrograde objects are scattered. When a small body has the same period as the planet, this condition is also known as co-orbital. So, the 1:1 resonance is also a co-orbital motion. One example that has already well-known is Jupiter Trojan objects. They orbiting the Sun in Jupiter Lagrange triangular locations and have the same orbit as the planet. Other small bodies can also be co-orbital objects, when they have the same period as the respected planet or when they are in the state of 1:1 resonance with their host planets. This co-orbital motion has several types depends on the motion of the small body. The types of motion are tadpole, horseshoe, quasi-satellite, and trisectrix. The trisectrix motion type belongs to the geometry of the orbital motion of 2015 BZ509 (recently known).

A study about the resonance of a small body with its respected (giant) planet is not sufficient if it just considers the ratio of the small integers \((j:k)\) of the two orbital periods. It has to develop a disturbing function to identify the resonance precisely. Because this case is a polar retrograde resonance, the implemented disturbing function is for the nearly polar orbits. It is a series of expansion of the gravitational interaction of two bodies that revolve around the Sun. The polar disturbing function informs that the \((j:k)\) inclination resonant angle (or argument) is

\[
\theta = k\lambda - j\lambda_p + (j + k - 2l)\sigma + 2l\Omega, \tag{1}
\]

where \(\Omega\) is small body’s longitude of ascending node, \(\sigma\) is longitude of perihelion, \(\lambda\) and \(\lambda_p\) are the small body and planet’s mean longitudes, respectively. The integer \(l\) is even if \((j - k)\) is even, and odd if \((j - k)\) is odd. The integer \(l\) is basically a numerical value from that series of expansion depending on the inclination factor, so the integer is important for a high inclination resonant angle. But from [2], for a nearly co-planar retrograde object, the dominant value of \(l\) is zero because of \(\cos(i/2) \approx 0\). So, if the object is in state 1:1 retrograde resonance and nearly co-planar, the resonant angle is

\[
\theta = \lambda - \lambda_p - 2\sigma \tag{2}
\]

If an object is in resonance state with a giant planet, then the resonant angle will vary close to a particular value for some particular time. Since the object is not stable in its orbit due to the high inclination, then the resonance occurs in just a limited time (temporal resonance).

3. Data and methods

3.1. Data
The ephemeris data of 2006 RJ2 at epoch MJD 58600 was taken form JPL Small-Body Database (https://ssd.jpl.nasa.gov/sbdb_query.cgi). The orbital parameters are the semi-major axis, eccentricity, inclination, the longitude of ascending node, the argument of perihelion, and mean anomaly \((M)\). Data
quality of the provided orbital parameters of this object is not quite good (\( U \) parameter = 7), so we generate orbital clones of the object by varying its five among six orbital parameters (with an exception of the mean anomaly). This step reveals 243 orbital clones of 2006 RJ2. These orbital clones are then integrated by the SWIFT integrator for about 200,000 years to the future. The SWIFT integration scheme is a Regulated Mixed Variable Symplectic (RMVS) [3] and with assuming that the giant planets play as the main perturbers of 2006 RJ2.

3.2. Method
We identified the mean motion resonance of the object by the FAIR method [1]. This method is easy and efficient to identify the mean motion resonance by plotting the resonance variables to each axis. Generally, the resonance variables are defined for both inner and outer locations of the resonances relative to the respected planet (table 1).

Table 1. Resonance variables. The second column (Plot) shows the plot variables (subscript \( p \) denotes planet), the third and fourth columns show the number of intersecting stripes on the vertical (Ver) and horizontal (Hor) axes, respectively, when the object is in a state of resonance. The number of intersecting stripes also indicates the integer resonance of the object.

| Type   | Plot            | Ver | Hor |
|--------|-----------------|-----|-----|
| Inner  | \( M \) versus \( \lambda_p - \lambda \) (\( j - k \)) | \( j \) | \( j \) |
| Inner  | \( M_p \) versus \( \lambda - \lambda \) (\( j - k \)) | \( k \) | \( k \) |
| Outer  | \( M \) versus \( \lambda - \lambda_p \) (\( j - k \)) | \( j \) | \( k \) |
| Outer  | \( M_p \) versus \( \lambda - \lambda_p \) (\( j - k \)) | \( j \) | \( j \) |

Since 2006 RJ2 is probably in a state of 1:1 resonance, identification of the resonance by this method does not depend on the type because the object is in the same orbital period as the respected planet. Therefore, by plotting the mean anomaly versus the mean longitude difference, numbers of intersecting stripes with the horizontal and vertical axis can be identified. The numbers provide values of integer resonance (\( j \) and \( k \)). Accordingly, the resonant angle can then be obtained by using Equation (2). By plotting the resonant angle versus time, the duration along which the object is in a state of resonance can be examined.

We build automation of the FAIR method by developing Python code to efficiently identify the resonance. This automation makes the identification process is more efficient because it can derive the output plots for all the orbital clones simultaneously, not a manual identification for each orbital clone. This object has 243 orbital clones, so this automation is important. The Python code delivers several sets of 50 plots orbital clones (clone number 1 to 50, 51 to 100, and so on) in a single figure for 40,000 years segment time. Hence, for 243 orbital clones along 200,000 years of integration time, it reveals (5 \( \times \) 5) 25 figures. Another automation code was also built for the resonant angle presentation. This also gives another 25 figures for all 243 clones up to 200,000 years. Thus, using both codes we have finally 50 output figures.

Plots of the mean anomaly versus the mean longitude difference incorporate a histogram consisting of 9 classes for each plot. The frequency for each class is a number of numerical occurrences. If the object is in resonance 1:1, then there is one dark stripe intersecting horizontal axis. The dark stripe indicates high concentration occurrences in that area while the other area remains clear (low concentration). The histogram draws a curve (concave or convex) that depends on the location of the dark stripe. By this outlook, we approximate the curve with a simple quadratic fitting to specify the clarity of the resonance. The common equation for the quadratic fitting (variable \( x \) with parameters \( a_0 \), \( a_1 \), and \( a_2 \)) is
Because the curve that formed by 1:1 resonance is whether concave or convex, the main important parameter is $a_2$. If the absolute value of $a_2$ parameter is high ($a_2 \gg 0$), the curve will be more bent, but if the absolute value is nearly 0 ($a_2 \sim 0$), the curve will be flat. When the curve is more bent, then the occurrence of 1:1 resonance is high, and vice versa.

4. Results and discussions

After 200,000 years of integration, there are 3 orbital clones that discarded from the system. The limit of the outer system is set to 1000 au. Although the value of the $U$ parameter of 2006 RJ2 orbital element is 7, which is not quite good actually, surprisingly only 3 orbital clones are discarded away. This is indicative that the object is still in a stable orbit for about 200,000 years despite it has high inclination orbit and may be in a retrograde orbit around the Sun.

A sample plot of the mean anomaly versus the mean longitude difference for 50 orbital clones of 2006 RJ2 is presented in figure 1. Each plot in figure 1 is one periodic, so the dark stripes at about mean longitude 0 and 1 are the same. See the plot of orbital clone number 154 in figure 1. By using the FAIR method (table 1) to identify the mean motion resonance, the number of intersecting dark stripes with the horizontal and vertical axis on the plot is 1 and 0. Hence, the number of $k$ is 1 and the number of $(j - k)$ is 0. This brings the number of $j$ is 1. This can be concluded that this clone is in 1:1 $(j:k)$ co-orbital resonance with Saturn in the given segment time. The orbital clone also shows a concave curve of the fitting, meaning that the resonance occurs. Other orbital clones also show the resonance, such as clones number 160, 161, 193, and 195, but not quite clear.

Finally, we identify all the 243 orbital clones for all segment times up to 200,000 years. The result is as follows: 7.8% are clear resonance with $a_2$ parameter > 0.22. There are 14,77% belongs to not-quite-clear resonance with the value of $a_2$ parameter is in the range of 0.07–0.22. Although the orbital clones are in a stable orbit for about 200,000 years, their probability to be in stable resonance is small, just less than 10% and the value of $a_2$ parameter itself is not quite high.

Accordingly, plots of the resonance angle of the 50 orbital clones at the same segment time are presented in figure 2. Figure 2 presents the resonant angle versus time of the orbital clones in a state of resonance. Along the segment time, the resonant angles of the majority orbital clones vary around 0°. We also identify the orbital clones that exhibit the long resonance state. Based on this identification, we report that there are 8 orbital clones in state of 1:1 resonance that are stable up to 160,000 years, i.e. the clones number 68, 120, 132, 142, 154, 160, 193, and 223. Other clones that once be in 1:1 resonance are just in temporal resonance state for a limited time. Figure 3 shows the 2006 RJ2 trisectrix type of 1:1 co-orbital motion around the Sun with respect to the Saturn orbit (right panel). The top left panel is the object’s distance relative to Saturn ($\alpha$), and the bottom left panel is the relative distance versus the resonant angle.

There is also an indication that some of the clones are jumping into other resonances. We identify them with the help of the dark stripes features on the plot of mean anomaly versus the mean longitude difference. If the number of the intersecting dark stripes in horizontal and vertical axis are not 1 and 0, respectively, then the resonance occurrence on the orbital clones is not 1:1. For example, the plot of orbital clone number 185 in figure 1. The number of dark intersecting stripes in horizontal and vertical axis are 4 and 1, so the resonance state is 5:4. The phenomenon happens commonly in an unstable orbit object, and 2006 RJ2 is indeed not a stable object due to its high inclination orbit.
Figure 1. Plots of the mean anomaly versus the mean longitude difference (both axes are normalized to 1) for 50 orbital clones (orbital clones number 151 to 200, number of clones is at the upper part for each plot) for 40,000 up to 80,000 years segment time. The 9 yellow blocks on the bottom of each plot are the histogram and the red lines are the curves revealed by the quadratic fitting of the histogram.
Figure 2. Plots of the resonant angle of 50 orbital clones (orbital clones number 151 to 200) for 40,000 up to 80,000 years segment time. The vertical axis is normalized to 1.
Figure 3. The trisectrix motion type of 2006 RJ2 orbit for clone number 154. The left panel presents the relative distance and the resonant angle of the object along a short segment time. The right panel shows the Cartesian coordinate of the orbit. The Sun and Saturn are yellow and blue circles in locations of (0,0) and (10,0), respectively.

5. Conclusion
The probability of Centaur 2006 RJ2 in state of retrograde 1:1 resonance with Saturn is small. Among the orbital clones that have been analyzed, only 7.8% of the total 243 orbital clones show clear 1:1 resonance. Only 8 orbital clones are in the state of resonance for about 160,000 years. The others are merely temporal resonance within a very limited time, even just in some thousands of years. This small probability and the temporal resonance occur due to the high inclination and retrograde orbit. Also, the data quality of the available orbital elements is not quite good, so it makes the uncertainties remain large.

Acknowledgments
We express our gratitude to the Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung and National Institute of Aeronautics and Space, Indonesia for the support of this work and also to the reviewer for the helpful comments.

References
[1] Forgacs-Dajka E, Sandor Zs and Erdi B 2018 MNRAS 447 3383
[2] Li M, Huang Y and Gong S 2018 A & A 617 1
[3] Levison H F and Duncan M J 1994 Icarus 108 18
[4] Morais M H M and Namouni F 2017a MNRAS 472 L1
[5] Morais M H M and Namouni F 2017b Nature 543 635
[6] Wiegert P, Connors M and Veillet C 2017 Nature 543 687