Holographic Cosmology from a System of M2-M5 Branes

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In this paper, we analyze the holographic cosmology using a M2-M5 brane configuration. In this configuration, a M2-brane will be placed in between a M5-brane and an anti-M5-brane. The M2-brane will act as a channel for energy to flow from an anti-M5-brane to a M5-brane, and this will increase the degrees of freedom on the M5-brane causing inflation. The inflation will end when the M5-brane and anti-M5-brane get separated. However, at a later stage the distance between the M5-brane and the anti-M5-brane can reduce and this will cause the formation of tachyonic states. These tachyonic states will be again open a bridge between the M5-branes and the anti-M5-branes, which will cause further acceleration of the universe.

I. INTRODUCTION

It is widely known that the entropy of the black holes is proportional to the area of the horizon, and its temperature is proportional to the surface gravity. Thus, a link between gravity and thermodynamics has been established. This link has become the basis of the Jacobson formalism [1]. In the Jacobson’s formalism, the Einstein’s equations are obtained from the first law of thermodynamics. It has been possible to derive the Friedmann equations from the Clausius relation using this Jacobson formalism [2]. In deriving the Friedmann equations, the entropy is assumed to be proportional to the area of the cosmological horizon. In fact, motivated by the Jacobson formalism, it has been proposed that the gravity is an entropic force [3]. In fact, Einstein’s equations have been obtained from this entropic force formalism. Furthermore, as the entropy of the bulk of the black hole is proportional to the area of the boundary, it has led to the development of the holographic principle. The holographic principle states that the number of degrees of freedom of a region in space is the same as the number of degrees of freedom on the boundary of that region.

The holographic principle have motivated the development of the holographic cosmology [4]–[8]. The holographic cosmology is based on the idea that the difference between the degrees of freedom in a region and the degrees of freedom on the boundary surrounding that region drives the expansion of the universe. The holographic cosmology has been studied in the context of Lovelock gravity, with special emphasis on the Gauss-Bonnet gravity [9]. The holographic cosmology for the brane world models [10], scalar-tensor gravity [43], and $f(R)$ [44], has also been discussed. It may be noted that this analysis was performed using a thermodynamic description of the brane world models, holographic scalar-tensor cosmology, and holographic $F(R)$ cosmology [13]. The holographic cosmology has also been generalized to the Friedmann-Robertson-Walker universe with an arbitrary spatial curvature [14]. This generalization has been performed for non-flat universes by using the aerial volume instead of the proper volume [15]. In this context, the Friedmann equation for the Lovelock gravity with an arbitrary spatial curvature have also been studied [16].

The holographic cosmology has been studied using the BIonic solution [17]. This BIonic solution is a configuration of a D3-brane and an anti-D3-branes with a wormhole in between them. This action for the D-branes is a non-linear action called the Dirac-Born-Infeld (DBI) action, and the non-linearity of this action is important in constructing this BIonic solution [18]–[21]. The F-string end on a point of a D-brane in this BIonic solution, and the F-string charge gets associated with the world-volume electric flux carried by the D-brane. The D3-branes of the BIonic has been identified with our universe and the BIonic solution has been used for analyzing the holographic cosmology [22]–[25]. In this solution, first a BIonic forms from black F-strings. Then the degrees of freedom of the D3-brane increase as energy flows from a anti-D3-brane into the D3-brane. However, as the D3-brane moves away from the anti-D3-brane, then the spike of the D3-brane gets separated from the spike of the anti-D3-brane spike, and the inflation ends at this stage. Finally, when the D3-brane comes close to the anti-D3-brane, a new wormhole forms due to the tachyonic
states. This wormhole also increases the degrees of freedom on the D3-brane, and this causes late time acceleration of the universe.

It may be noted there the F1-D3 intersection is U-dual to a system of M2-M5 branes \[26\]-\[27\]. The M2-branes intersecting with M5-branes have been analyzed in the supergravity regime \[28\]-\[29\]. This analysis has been done using the blackfold approach. Thus, it has been possible to recover the 1/4-BPS self-dual string solution \[30\]-\[31\], as a three-funnel solution of an effective five-brane world volume theory \[32\]-\[33\]. The finite temperature effects for non-extremal self-dual string solution solutions and wormhole solutions interpolating between stacks of M5-branes and anti-M5-branes have also been studied. These solutions define a BIon solution in M-theory \[36\]-\[37\]. It would be interesting to perform a similar analysis for this M-theory system. So, it is possible to study the holographic cosmology using a M2-M5 brane system. Thus, we will first study a M5-brane connected to an anti-M5-brane by a M2-brane. This will cause the degrees of freedom to flow into the M5-brane causing inflation. The inflation will end when the M5-brane and the anti-M5-brane get separated. However, at a later stage when the M5-brane approaches the anti-M5-brane, tachyonic states will be formed. These tachyonic states will form another bridge between the M5-brane and the anti-M5-brane. We will investigate the inflation in this model of M2-M5 branes, along with the consequences of the formation of these tachyonic states.

It is possible for the four dimensional universe to emerge from compactification of M5-branes. However, in this paper, we will not discuss such a compactification, and we will discuss the inflation using M5-brane geometry. In fact, inflation has already been studied using the M5-brane geometry \[38\]-\[41\]. However, in this paper, we study the inflation in the M-theory using the recently proposed proposal of holographic cosmology \[4\]-\[8\]. It may be noted that it is possible for the branes moving in the extra dimensional bulk to collide with each other. Such collisions have been studied in the context of Ekpyrotic universe \[42\]-\[47\]. In this paper, we will also use the formalism of holographic cosmology \[38\]-\[41\], to discuss the state of the universe before such a collision occurs. It may be noted that such a model has also been studied using BIon solution \[21\]-\[23\], and such a system is U-dual to a system of M2-M5 branes \[26\]-\[27\]. So, this system is actually U-dual to the BIon cosmology. In other words, this is the M-theory version of the holographic cosmology, which has so far only been studied in the string theory \[21\]-\[23\].

We would like to point out that the fact that F1-D3 intersection is U-dual to a system of M2-M5 branes is only mentioned as an observation at this stage. The analysis in this paper seems to be similar to the analysis performed for using BIon solution, so there might be a deeper relation between such a duality. However, at this stage we only mention this as a motivation to study inflation using a M2-M5-brane system. We would also like to point out that the bulk is populated by M5-branes and anti-M5-branes. So, a random collision between a M5-brane and an anti-M5-brane can occur. However, this will be a collision between a random M5-brane with an random anti-M5-brane in the bulk. Such collision between random branes occurs in Ekpyrotic universe \[42\]-\[47\]. However, in this model, we can calculate the state of the universe just before such an collision occurs.

The paper is organized as the follows. In section II, we discuss the holographic inflationary cosmology using the system of M2-M5 brane. In section III we analyze the tachyonic states the M2-M5 brane system. Finally, in the last section, we will summarize our main results. We will also discuss possible extension of the results obtained in this paper.

II. M2-M5 BIONIC SOLUTIONS

In this section, we analyze the holographic cosmology using M2-M5 Bionic solutions. This will be done by first analyzing the formation of a configuration of a M5-branes and an anti-M5-brane separated by a M2-brane. Now we can write the supergravity solution for black M2-brane lying along z and r directions as follows, \[48\]-\[49\],

\[
\begin{align*}
\text{ds}^2 &= H_2^{1/3}[2H_2^{-1}du dv + f du] + H_2^{-1}dz^2 + \Sigma_{i=1}^8 dx_i^2, \\
u &= -(t-r)/\sqrt{2}, \quad v = (t+r)/\sqrt{2}, \\
H_2 &= 1 + \frac{r_0^3 \sinh^2 \alpha}{r^3}, \\
f &= 1 - \frac{r_0^3}{r^3}
\end{align*}
\] (1)
Using this the definitions, we can write the following expression
\[
\cosh \alpha_\pm = \frac{k_\beta^3 \sqrt{1 \pm \sqrt{1 - \frac{4q_2^2}{k_\beta^6}(1 + \frac{k_\beta^2}{\sigma_r^2})}}}{\sqrt{2q_2}}
\]
\[
r_{0,\pm} = \frac{\sqrt{2q_2}}{k_\beta^3} \sqrt{1 \pm \sqrt{1 - \frac{4q_2^2}{k_\beta^6}(1 + \frac{k_\beta^2}{\sigma_r^2})}}
\]
\[
\beta = \frac{3}{4\pi T}, \quad q_2 = \sigma^2 \frac{r_0^3}{2} \sin \theta \sinh 2\alpha, \quad \tan \theta = \frac{k}{\sigma \beta^3}.
\]
In above equation, \(T\) is temperature of M2-brane and \(\sigma\) is the radius on the world-volume. The mass density along the \(z\) direction is given by
\[
\frac{M_{M2-brane}}{L_x \cdot L_z} = Q_2 \left(1 + \frac{\sqrt{2q_2}}{3\sqrt{2k_\beta^3}} + \frac{5q_2}{2k_\beta^6}\right)
\]
where \(Q_2 = \frac{N_c}{(2\pi)^2 L_p}\). Furthermore, the length along the \(x\) and \(z\) directions is denoted by \(L_x\) and \(L_z\), respectively. To describe the configuration of a M5-brane and an anti-M5-brane joined together by an M2-brane, we use embedding of the M5-brane in 11D Minkowski spacetime with metric [30]-[37].
\[
ds^2 = -dt^2 + (dx^1)^2 + (dx^2)^2 + r^2 d\Omega_3^2 + \sum_{i=6}^{10} dx_i^2.
\]
Here we have not considered the background fluxes. Using the standard angular coordinates \((\psi, \phi, \omega)\), it is possible to write the following expression,
\[
d\Omega_3^2 = -d\psi^2 + \sin^2 \psi (d\phi^2 + \sin^2 \phi d\omega^2).
\]
In the static gauge, we obtain
\[
t(\sigma^a) = \sigma^1, \quad x^1(\sigma^a) = \sigma^1, \quad r(\sigma^a) = \sigma^2 \equiv \sigma
\]
\[
\psi(\sigma^a) = \sigma^3, \quad \phi(\sigma^a) = \sigma^4, \quad \omega(\sigma^a) = \sigma^5, \quad x^6(\sigma^a) = z(\sigma),
\]
and the remaining coordinates are constant. The embedding function \(z(\sigma)\) describes the bending of the brane. As \(z\) is a transverse coordinate to the branes and \(\sigma\) is the radius on the world volume, we can write the induced metric on the effective five-brane world volume as
\[
\gamma_{ab} da^a da^b = -(da^0)^2 + (da^1)^2 + (1 + \zeta(\sigma)^2) da^2 + \sigma^2 (-d\psi^2 + \sin^2 \psi (d\phi^2 + \sin^2 \phi d\omega^2))
\]
Here we have imposed two boundary conditions in our system. Thus, we have imposed \(z(\sigma) \to 0\) for \(\sigma \to \infty\) and \(z'(\sigma) \to -\infty\) for \(\sigma \to \sigma_0\), where \(\sigma_0\) is the minimal two-sphere radius of the configuration. The mass density along the \(z\) direction at corresponding point can now be expressed as [30]-[37],
\[
\frac{dM_{Bion}}{L_x \cdot dz} = Q_2 \left(1 + \frac{\sigma^6}{k_\beta^6} + \frac{5q_2^2}{6\beta^6} \frac{1 + \sigma_0^6}{\sigma_0^6} \frac{1}{3/2} + \frac{11q_2^4}{8\beta^{12}} \frac{1 + \sigma_0^6}{\sigma_0^{12}} \frac{5/2}{3/2}\right)
\]
where we have used of this fact that \(q_2 = kq_5\). We now compare the mass densities for M2-M5 system to the mass density for black M2-brane and note that the thermal BIon at \(\sigma = \sigma_0\) behaves like black M2-brane. Furthermore, minimal two-sphere radius of the configuration \(\sigma_0\) depends on the temperature [30]-[37],
\[
\sigma_0 = \frac{q_2^{1/4}}{\beta^{1/2}} \left(1.234 - 0.068 \frac{q_2^{1/2}}{\beta^3}\right)
\]
The inflation ends when the the M5-brane gets separated from the anti-M5-brane, and this happens when the width of geometry between them reaches zero,

$$\sigma_0 = 0, \beta = \frac{3}{4\pi T} \rightarrow T_{\text{end}} = \frac{3}{4\pi} \left(\frac{0.05}{q_2^{1/2}}\right)^{2/7}. \quad (10)$$

Thus, the temperature was infinity at the beginning, and then it decreased to $T_{\text{end}}$ at the end of inflation. This corresponded to the decrease of the width of the geometry between M5-brane and the anti-M5-brane till it reached zero. This corresponded to the limiting value of the temperature was $T = T_{\text{end}}$.

This M2-brane geometry acted as a channel for the degrees of freedom to flow into the M5-brane, and this in turn led to inflation. It may be noted that by putting the black M2-brane charge along the radial direction and using equation (7), we obtain [36]-[37],

$$z_\pm(\sigma) = \int_\sigma^\infty ds \left(\frac{F_\pm(s)^2}{F_\pm(\sigma_0)^2} - 1\right)^{-\frac{1}{2}} \quad (11)$$

It is possible to express $F(\sigma)$ for the finite temperature M2-M5 brane system as

$$F_{\pm}(\sigma) = \sigma^3 \left(\frac{1 + \frac{k^2}{\sigma^2}}{1 + \sqrt{1 - \frac{4q_5^2}{\sigma^2}(1 + \frac{k^2}{\sigma^2})}}\right)^{3/2} \left(-2 + \frac{3\beta}{2} \frac{1 - \frac{1}{\sigma^2} \sqrt{1 - \frac{4q_5^2}{\sigma^2}(1 + \frac{k^2}{\sigma^2})}}{1 + \frac{k^2}{\sigma^2}}\right) \quad (12)$$

where

$$q_5 = \frac{\sqrt{3}}{2} \cos \theta \sinh 2\alpha$$
$$\tan \theta = \frac{k}{\sigma^3}, q_2 = kq_5 = -4\pi N_2 j_3 \frac{j_2}{N_4} \quad (13)$$

Here the number of M2-branes and M5-branes is denoted by $N_2$ and $N_5$, and the charges on these branes is denoted by $q_2$ and $q_5$, respectively. The temperature of this system is denoted by $T$. Attaching a mirror solution to Eq. (11), we construct thin shell wormhole configuration. Now we find an expression for the distance separating the $N$ M5-branes and $N$ anti-M5-branes. This is done by defining $\Delta = 2z(\sigma_0)$, where

$$\Delta = 2z(\sigma_0) = 2 \int_{\sigma_0}^\infty ds \left(\frac{F(s)^2}{F(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}. \quad (14)$$

We can now use these results for constructing the holographic cosmology in the M2-M5 Blon. In order to do that we need to compute the contribution of the M2-M5 system to the the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe. We mean by the bulk the region inside the cosmological horizon of a universe, and it should not be confused with the space in which the M2-M5 branes geometry. Now the relations between these degrees of freedom, the entropy of M2-M5, and the mass density along the transverse direction, can be written as

$$N_{\text{sur}} + N_{\text{bulk}} = N_{\text{M2-M5}} = N(\text{M5 - brane}) + N(\text{anti - M5 - brane}) + N(\text{M2 - brane})$$

$$\approx 4L_p^2 S_{\text{M2-M5}} = \frac{\Omega(3)\Omega(4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^3 \alpha} \quad (15)$$

where $\Omega_n$ denotes the volume of the round $N$-sphere. The solution to these equation, can be written as

$$N_{\text{sur}} \approx \frac{\Omega(3)\Omega(4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^3 \alpha} + \frac{\Omega(3)\Omega(4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^3 \sigma^3 \frac{3 \cosh^2 \alpha + 1}{\cosh^3 \alpha}$$
$$N_{\text{bulk}} \approx \frac{\Omega(3)\Omega(4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^4 \sigma^3 \frac{1}{\cosh^3 \alpha} - \frac{\Omega(3)\Omega(4)}{16\pi G} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \beta^3 \sigma^3 \frac{3 \cosh^2 \alpha + 1}{\cosh^3 \alpha} \quad (16)$$

The temperature of the Blonic system decreases with the increase in the degrees of freedom inside the universe.
It is also possible to express the Hubble parameter and energy density in terms of the quantities associated with the M2-M5 brane system. As the number of degrees of freedom on the apparent horizon is proportional to its area, we can write

\[ N_{\text{sur}} = \frac{4\pi r_A^2}{L_p^2} + \frac{8}{3} \alpha_0 \frac{\pi \mu}{L_p^2} r_A, \]

(17)

where the \( H \) is the Hubble parameter which can be expressed in terms of the scale factor \( a \) as \( H = \frac{\dot{a}}{a} \). The radius \( r_A = 1/\sqrt{H^2 + \frac{k}{a^2}} \) is the apparent horizon radius for the Universe. The Hubble parameter for flat universe can now be expressed as

\[ H_{\text{flat,inf}} \simeq \left( \frac{16\pi G k^4 q_0^4}{3\Omega(3)\Omega(4)q_0^6}\sigma_0^3 + \frac{8\pi G k^2 q_0^4}{\Omega(3)\Omega(4)q_0^6}\sigma_0^2 T^2 \right) \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0^2 \frac{\pi \mu}{L_p^2} + \frac{16}{3} \pi G k^4 q_0^4 \sigma_0^3 \right)^{-1}. \]

(18)

The universe energy density of the universe can be written as

\[ \rho_{\text{flat,inf}} = \frac{3}{8\pi L_p^2} H_{\text{flat,inf}}^2 \simeq \frac{3}{8\pi L_p^2} \left( \frac{16\pi G k^4 q_0^4}{3\Omega(3)\Omega(4)q_0^6}\sigma_0^3 + \frac{8\pi G k^2 q_0^4}{\Omega(3)\Omega(4)q_0^6}\sigma_0^2 T^2 \right)^2 \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0^2 \frac{\pi \mu}{L_p^2} + \frac{16}{3} \pi G k^4 q_0^4 \sigma_0^3 \right)^{-2}. \]

(19)

The energy density of the universe is related to the charges of M-branes. It is also related to the geometry of the M2-M5 system.

It is also possible to use Eq. (17), and the expression \( r_A = 1/\sqrt{H^2 + \frac{k}{a^2}} \), to obtain the Hubble parameter for non-flat universe as

\[ H_{o/c,inf} \simeq \left( \frac{16\pi G k^4 q_0^4}{3\Omega(3)\Omega(4)q_0^6}\sigma_0^3 + \frac{8\pi G k^2 q_0^4}{\Omega(3)\Omega(4)q_0^6}\sigma_0^2 T^2 \right)^2 \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0^2 \frac{\pi \mu}{L_p^2} + \frac{16}{3} \pi G k^4 q_0^4 \sigma_0^3 \right)^{-2} - \frac{k}{a^2}. \]

(20)

The scale factor for open \((k = -1)\) universe can be written as

\[ a_{o,inf}(t) \simeq \exp - \int dt [\mathcal{T} + \ln(t)]. \]

(21)

The scale factor for closed \((k = +1)\) universe can be written as

\[ a_{c,inf}(t) \simeq \exp - i \int dt [\mathcal{T} + \ln(t) + \frac{\pi}{2}]. \]

(22)

Here we have defined the quantity \( \mathcal{T} \) as

\[ \mathcal{T} = \left( \frac{16\pi G k^4 q_0^4}{3\Omega(3)\Omega(4)q_0^6}\sigma_0^3 + \frac{8\pi G k^2 q_0^4}{\Omega(3)\Omega(4)q_0^6}\sigma_0^2 T^2 \right)^2 \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0^2 \frac{\pi \mu}{L_p^2} + \frac{16}{3} \pi G k^4 q_0^4 \sigma_0^3 \right)^{-2} - \frac{k}{a^2}. \]

(23)

The scale factor for the open universe is very small at the the beginning \((T = \infty)\). However, as the temperature decreases the value for the scalar factor increases. Thus, at the end of the inflation, this scalar factor has a large value.
It may be noted that unlike a open universe, the scale factor for a closed universe oscillates. The energy density for open and closed universes, can now be expressed as follows,

\[
\rho_{o/c, inf} = \frac{3}{8\pi^2 l_p^2} \left( H_{o/c}^2 + k/a^2 \right) \\
\approx \frac{3}{8\pi^2 l_p^2} H_{flat}^2 \\
\approx \frac{3}{8\pi^2 l_p^2} \left( \frac{16\pi G k^4 q_5^2}{3\Omega(3)\Omega(4)q_5^2\sigma_0^2} T^3 + \frac{8\pi G k^2 q_5^4}{\Omega(3)\Omega(4)q_5^2\sigma_0^2} T^2 \right) \\
\times \left( 8 \sqrt{\frac{\pi\mu}{L_P^2}} \pm \sqrt{\frac{16}{3} \frac{3\mu}{L_P^2} + \frac{16\pi}{L_P^2} \left( \frac{16\pi G k^4 q_5^2}{3\Omega(3)\Omega(4)q_5^2\sigma_0^2} T^3 + \frac{8\pi G k^2 q_5^4}{\Omega(3)\Omega(4)q_5^2\sigma_0^2} T^2 \right)^{-1} \right)^2 = \rho_{flat, inf}. \quad (24)
\]

The energy density of the universes is related to the evolution of the M2-M5 brane system so it does not depend on type of universe. This can be noted from the fact that the energy density of the flat, open and closed universes is the same.

The inflation ends when the M5-brane is separated from the anti-M5-brane. At this stage the mass distribution along z-direction is absent. Now we can write the difference between the surface and bulk degrees of freedom as

\[
N_{sur} - N_{bulk} \simeq \int_{\sigma_0}^{\sigma_0} d\sigma \frac{dM_{Bion}}{dz} = 0, \quad (25)
\]

and so we have

\[
N_{sur} = N_{bulk}. \quad (26)
\]

The degrees of freedom on the cosmological horizon is equal to the degrees of freedom in the bulk, at the end of inflation.

### III. TACHYONIC STATES

In the previous section we analysed the inflation in the context of M2-M5 brane system. The inflation ended with the M5-brane getting separated from the anti-M5-brane. However, it is possible for the M5-brane to again come close to the anti-M5-brane at a later stage. This can result in a collision of the M5-brane with the anti-M5-brane. It may be noted that such a collision of branes has been studied in the context of Ekpyrotic universe [42]-[47]. In this section, we will analyse the state of the universe before such a collision using the formalism of holographic cosmology [4]-[8]. So, we will analyse phenomena of a M5-brane approaching an anti-M5-brane. It will be demonstrated that this will create tachyonic states, and these tachyonic states will in turn create a new bridge between the M5-brane and the anti-M5-brane. This may be responsible for the expansion at a later stage in the evolution of the universe. It may be noted here the universe starts with a non-phantom phase and then evolves to a phantom one.

The non-phantom phase can be constructed by using a M5-branes and an anti-M5-branes in the background [5]. Thus, in this model a M5-brane moving in the extra dimension approaches an anti-M5-brane, and tachyonic states form when the distance between the M5-brane and the anti-M5-brane reaches a critical distance \( l \). So, we will now analyse this critical system by placing one of the branes at \( z_1 = l/2 \), and the other brane at \( z_2 = -l/2 \). At this point the tachyonic states will form and the universe will enter a near collapse phase. We will use the formalism of holographic cosmology [4]-[8] for analysing this near collapse phase. So, the separation between the M5-brane and the anti-M5-branes becomes of the order one. We need a tachyonic action to analyse this state. Now we can write the open string tachyon for this system as [51]-[55],

\[
H_{DBI} = \frac{\Omega_3 \Omega_4 L_4 L_{ext}}{16\pi G} \frac{1}{\beta^6} \int d\sigma V(TA) \left( 1 + \frac{l'(\sigma)}{4} + TA^2 - TA'^2 \right) F_{DBI,M2-M5},
\]

\[
F_{DBI,M2-M5} = \sigma^3 \left( \frac{1 + \frac{\omega^2}{\beta^2}}{1 + \frac{\omega^2}{\beta^2}} \right)^{3/2} \left( -2 + \frac{3\beta^6}{2q_5^2} \frac{1 \pm \sqrt{1 - \frac{4q_5^2}{\beta^2}(1 + \frac{\omega^2}{\beta^2})}}{1 + \frac{\omega^2}{\beta^2}} \left( 1 - \frac{64q_5^2}{162\beta^3} \right). \quad (27)
\]
Here we have used
\[ V(TA) = \frac{\tau_3}{\cosh \sqrt{\tau TA}}. \] (28)

So, we can write the equation of motion for \( l(\sigma) \) and tachyon \( TA \), as
\[ \frac{l''V(TA)F_{DBI,M2-M5}}{4\sqrt{1 + \frac{\nu(\sigma)^2}{4} + TA^2 - TA^2}} + \]
\[ \frac{l'}{4\sqrt{1 + \frac{\nu(\sigma)^2}{4} + TA^2 - TA^2}} (V'(TA)F_{DBI,M2-M5} + V(TA)F'_{DBI,M2-M5}) + \]
\[ \frac{2l'(TAT'A' - TAT'A'^\prime)}{1 + \frac{\nu(\sigma)^2}{4} + TA^2 - TA^2} V(TA)F_{DBI,M2-M5} = 0. \] (29)

We also have
\[ \frac{1}{\sqrt{D_{TA,M2-M5}}} T'A'(\sigma)' = \]
\[ \frac{1}{\sqrt{D_{TA,M2-M5}}} \left[ \frac{(V(TA))'}{(V(TA))} (D_{TA,M2-M5} - (T'A'(\sigma))^2) + \frac{F'_{DBI,M2-M5}}{F_{DBI,M2-M5}} D_{TA,M2-M5} \right], \]
\[ D_{TA,M2-M5} = 1 + \frac{\nu(\sigma)^2}{4} + TA^2 - TA^2 \] (30)

The solution to these equations can be written as
\[ l(\sigma) = 2l_0 \left( \frac{1}{2} - \int_\sigma^\infty d\sigma \left( \frac{F_{DBI,M2-M5}(\sigma) + 4 \int d\sigma' (T'A^2 - TA^2)}{F_{DBI,M2-M5}(\sigma_0)} - 1 \right)^{1/2} \right), \] (31)

where
\[ TA \sim \left( \frac{\sigma_0^2}{\sigma^2 - \sigma_0^2} \right)^{1/3} \left( \frac{81\beta^3}{192 q_5} - 1 \right). \] (32)

The non-vanishing value of \( \sigma_0 \) represents a geometry that connects the M5-brane with the anti-M5-brane, and has a finite size width. The distance separating the M5-brane from the anti-M5-brane is \( l_0 \), when this geometry is formed. It may be noted that there are no tachyonic states present before this geometry forms, and as the temperature decreases the size of this geometry increases.

The entropy and mass density along \( z \)-direction in this tachyonic theory can be written as
\[ S_{tb} = \frac{2^{3/2} \eta \delta_0 \tau_4 \Omega_4}{16 \pi C_{\gamma}} \int d\sigma V(TA(\sigma)) \frac{F_{DBI,M2-M5}(\sigma)}{F_{DBI,M2-M5}(\sigma_0)} \beta^3 \sigma^3 \]
\[ \times \frac{1}{\cosh^\alpha} \frac{\sigma_0^{7/3}}{(\sigma^2 - \sigma_0^2)^{4/3}} \left( \frac{81\beta^3}{192 q_5} - 1 \right) \] (33)
\[ \frac{dM_{tb}}{dz} = \frac{2^{3/2} \eta \delta_0 \tau_4 \Omega_4}{4G} \int d\sigma V(TA(\sigma)) \frac{F_{DBI,M2-M5}(\sigma)}{F_{DBI,M2-M5}(\sigma_0)} \beta^3 \sigma^3 \frac{\cosh^2 \alpha + 1}{\cosh^\alpha} \]
\[ \times \frac{\sigma_0^{7/3}}{(\sigma^2 - \sigma_0^2)^{4/3}} \left( \frac{81\beta^3}{192 q_5} - 1 \right). \] (34)

These tachyonic states also effect the of degrees of freedom on the universe and cause further acceleration. Now we can again write the expression for the degrees of freedom in terms of the the entropy of this system. It is also possible
to write an expression for the mass density along the transverse direction for this system. Thus, we can write the following expression,

\[ N_{\text{surf}} + N_{\text{bulk}} = N_{\text{brane}} + N_{\text{anti-brane}} + N_{\text{wormhole}} \]

\[ \simeq \frac{2^{3/2} q_3^3 \Omega_4}{16 \pi G} \int d\sigma (T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta \sigma^3 \]

\[ \times \frac{1}{\cosh^3 \alpha (\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right) \]

\[ N_{\text{surf}} - N_{\text{bulk}} \simeq \int \frac{dM_{\text{M5-brane}}}{dz} \]

\[ = \frac{2^{3/2} q_3^3 \Omega_4}{4G} V(T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta^3 \sigma^3 \cosh \alpha + 1 \]

\[ \times \frac{\sigma^3}{(\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right). \] (35)

The solution of these equations can be written as

\[ N_{\text{surf}} \simeq \frac{2^{3/2} q_3^3 \Omega_4}{16 \pi G} \int d\sigma (T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta^4 \sigma^3 \]

\[ \times \frac{1}{\cosh^3 \alpha (\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right) \]

\[ + \frac{2^{3/2} q_3^3 \Omega_4}{4G} V(T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta^3 \sigma^3 \cosh \alpha + 1 \]

\[ \times \frac{\sigma^3}{(\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right). \]

\[ N_{\text{bulk}} \simeq \frac{2^{3/2} q_3^3 \Omega_4}{16 \pi G} \int d\sigma (T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta^4 \sigma^3 \]

\[ \times \frac{1}{\cosh^3 \alpha (\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right) \]

\[ - \frac{2^{3/2} q_3^3 \Omega_4}{4G} V(T A(\sigma)) \frac{F_{DBI, M2-M5}(\sigma)}{F_{DBI, M2-M5}(\sigma_0)} \beta^3 \sigma^3 \cosh \alpha + 1 \]

\[ \times \frac{\sigma^3}{(\sigma^2 - \sigma_0^2)^{1/3}} \left( \frac{81 \beta^3}{192 q_5} - 1 \right). \] (36)

The M5-brane approaches the anti-M5-brane, and this causes the production of tachyonic states. These states increase the number of degrees of freedom of the universe and this continues till the Big Rip singularity. The Hubble parameter for flat universe can be expressed as

\[ H_{\text{flat,ac}} \simeq \left( \frac{1}{V(T A)} \right)^{1/2} \left( \frac{1016 q_3^3 G \pi^5}{243 \Omega_3 \Omega_4 q_5^3 \sigma_0^{7/3} T^5} + \frac{64 q_2 G \pi^3}{27 \Omega_3 \Omega_4 q_5^{7/3} T^3} \right) \]

\[ \times \left( \frac{8}{3} \frac{\pi^2}{\alpha} \right) \left( \frac{\pi^2}{L_p^2} \right) \left( \frac{\pi^2}{L_p^2} \right) + \left( \frac{1016 q_3^3 G \pi^5}{243 \Omega_3 \Omega_4 q_5^3 \sigma_0^{7/3} T^5} + \frac{64 q_2 G \pi^3}{27 \Omega_3 \Omega_4 q_5^{7/3} T^3} \right)^{-1}. \] (37)

As the tachyonic potential increases, this Hubble parameter reduces to very small values.
The scale factor for closed universe can also be expressed as
\[ \rho_{\text{flat,ac}} = \frac{3}{8\pi L_p^2} H^2_{\text{flat,ac}} \]
\[ = \frac{3}{8\pi L_p^2} \left( \frac{1}{V(TA)} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^2 \]
\[ \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0 \frac{\mu}{L_p^2} + \frac{16\pi}{L_p^2} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^{-1} \right)^2 \]  
(38)

This energy density decreases with increasing tachyon potential. This occurs because the acceleration decreases the energy density. The acceleration occurs due to the tachyonic states forming a bridge between the M5-brane and the anti-M5-brane.

Now using the radius \( r_A = 1/\sqrt{H^2 + k/\pi} \), the Hubble parameter for non-flat universes can be written as
\[ H_{o/c,ac} \simeq \left[ \frac{1}{V(TA)} \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^2 \right] \]
\[ \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0 \frac{\mu}{L_p^2} + \frac{16\pi}{L_p^2} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^{-1} \right)^2 \]  
(39)

The scale factor for open universe can now be expressed as
\[ a_{o,ac}(t) \simeq \exp - \int dt [\mathcal{H}_t + \ln(t)], \]  
(40)

The scale factor for closed universe can also be expressed as
\[ a_{c,ac}(t) \simeq \exp - i \int dt [\mathcal{H}_t + \ln(t) + \frac{\pi}{2}], \]  
(41)

Here we have defined \( \mathcal{H}_t \) as
\[ \mathcal{H}_t = \left( \frac{1}{V(TA)} \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^2 \right] \]
\[ \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0 \frac{\mu}{L_p^2} + \frac{16\pi}{L_p^2} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^{-1} \right)^2 \]  
(42)

It may be noted that the scale factor are functions of the tachyonic potential. Thus, as the tachyonic potential increases, the open universe expands to infinity at \( TA = \infty \), and the closed universe oscillates.

We can now write an expression for the energy density of the open and closed universes as
\[ \rho_{o/c,ac} = \frac{3}{8\pi L_p^2} (H_{o/c,ac}^2 + k/a^2) \]
\[ \simeq \frac{3}{8\pi L_p^2} H^2_{\text{flat,ac}} \]
\[ \simeq \frac{3}{8\pi L_p^2} \left( \frac{1}{V(TA)} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^2 \]
\[ \times \left( \frac{8}{3} \alpha_0 \sqrt{\frac{\pi \mu}{L_p^2}} \pm \frac{16}{3} \alpha_0 \frac{\mu}{L_p^2} + \frac{16\pi}{L_p^2} \right) \left( \frac{1016q_2^3G\pi^5}{243\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^5 + \frac{64q_2G\pi^3}{27\Omega(3)\Omega(5)q_5^3\sigma_5^7/3} T^3 \right)^{-1} \right)^2 \]  
(43)

It may be noted that this energy density also depends only on the M2-M5 brane system, and not the type of the universe. Thus, the energy density for all the three types of the universe is the same.
IV. CONCLUSION

In this paper, we used the holographic cosmology conjecture to analyze the dynamics of a 6-D inflating spacetime. The inflation started with the formation of a system of M2-M5 branes. The M5-brane was connected to the anti-M5-brane by a M2-brane. This made it possible for the degrees of freedom to flow from the anti-M5-brane to the M5-brane causing inflation. The inflation was driven by the difference in the degrees of freedom between a region and the cosmological horizon surrounding that region. The inflation ended with the separation of the M5-brane from the anti-M5-brane. However, as the M5-brane approaches a second anti-M5-brane, at a later stage, tachyonic states were formed. These tachyonic states opened a new bridge between the M5-brane and the anti-M5-brane. This again caused the degrees of freedom to flow into the M5-brane, and hence the expansion of the universe. It may be noted that thermodynamic approach to gravity has also been studied using the generalized uncertainty principle \[56\]. In fact, holographic cosmology with the generalized uncertainty principle has also been studied \[57\]. It would thus be interesting to generalize the results of this paper using the generalized uncertainty principle.

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[1] J. Jacobson, Phys. Rev. Lett. 75, 1260 (1995)
[2] R. G. Cai and S. P. Kim, JHEP 0502, 050 (2005)
[3] E. P. Verlinde, JHEP 1104, 029 (2011)
[4] T. Padmanabhan, Class. Quant. Grav. 21, 4485 (2004)
[5] T. Padmanabhan, arXiv:1206.4916 (2012)
[6] T. Padmanabhan and H. Padmanabhan, Int. J. Mod. Phys. D 23, 1340011 (2014)
[7] T. Padmanabhan, arXiv:1210.4173 (2012)
[8] T. Padmanabhan, Res. Astron. Astrophys. 12, 891 (2012)
[9] K. Yang, Y. X. Liu and Y. Q. Wang, Phys. Rev. D 86, 104013 (2012)
[10] Y. Ling and J. P. Wu, JCAP, 1008, 017 (2010)
[11] M. Akbar and R. G. Cai, Phys. Lett. B 635, 7 (2006)
[12] S. Capozziello, V. F. Cardone, and A. Troisi, Phys. Rev. D 71, 043503 (2005)
[13] Y. Ling and W. J. Pan, Phys. Rev. D 88, 043518 (2013)
[14] A. Sheykhi, Phys. Rev. D 87, 061501 (2013)
[15] E. C. Young and D. Lee, JHEP 04, 125 (2014)
[16] M. Eune and W. Kim, Phys. Rev. D 88, 067303 (2013)
[17] A. Sepehri, F. Rahaman, A. Pradhan and I. H. Sardar, Phys. Lett. B 741, 92 (2014)
[18] C. G. Callan and J. M. Maldacena, Nucl. Phys. B 513, 198 (1998)
[19] L. Thorlacius, Phys. Rev. Lett. 80, 1588 (1998)
[20] R. Emparan, Phys. Lett. B 423, 71 (1998)
[21] G. W. Gibbons, Nucl. Phys. B 514, 603 (1998)
[22] M. R. Setare and A. Sepehri, JHEP 1503, 079 (2015)
[23] M. R. Setare and A. Sepehri, Phys. Rev. D91, 063523 (2015)
[24] A. Sepehri, A. Pradhan and S. Shoorevazi, Astrophys. Space Sci. 357, 18 (2015)
[25] M. R. Setare, M. Faizal, A. Sepehri, and A. F. Ali, arXiv:1502.03218 (2015)
[26] G. Grignani, T. Harmark, A. Marini, N. A. Obers and M. Orselli, JHEP 1106, 058 (2011)
[27] G. Grignani, T. Harmark, A. Marini, N. A. Obers and M. Orselli, Nucl. Phys. B 851, 462 (2011)
[28] R. Emparan, T. Harmark, V. Niarchos and N. A. Obers Phys. Rev. Lett. 102, 191301 (2009)
[29] R. Emparan, T. Harmark, V. Niarchos and N. A. Obers, JHEP 1003, 063 (2010)
[30] D. J. Smith, Class. Quant. Grav. 20, R233 (2003)
[31] D. Youm, Nucl. Phys. 556, 222 (1999)
[32] H. Nastase, C. Papageorgakis and S. Ramgoolam, JHEP 0905, 123 (2009)
[33] J. Armas and M. Blau, JHEP 1408, 140 (2014)
[34] J. Armas and T. Harmark, JHEP 1410, 63 (2014)
[35] R. Iengo and J. G. Russo, JHEP 0810, 030 (2008)
[36] V. Niarchos and K. Siampos, JHEP 1206, 175 (2012)
[37] V. Niarchos and K. Siampos, JHEP 1207, 134 (2012)
[38] K. Becker, M. Becker and A. Krause, Nucl. Phys. B715, 349 (2005)
[39] D. Battefeld, T. Battefeld and A. C. Davis, JCAP 0810, 032 (2008)
[40] A. Krause, JCAP 0807, 001 (2008)
[41] K. Becker, M. Becker and A. Krause, Phys. Rev. D74, 045023 (2006)
[42] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D 64, 123522 (2001)
[43] T. M. Sorensen, Phys. Rev. D 71, 107302 (2005)
[44] Y. Takamizu and K. Maeda, Phys. Rev. D 70, 123514 (2004)
[45] J. Lehners, P. McFadden and N. Turok, Phys. Rev. D 76, 023501 (2007)
[46] J. Lehners, P. McFadden, N. Turok and P. J. Steinhardt, Phys. Rev. D76, 103501 (2007)
[47] J. Lehners, Phys. Rept. 465, 223 (2008)
[48] T. Harmark and N. A. Obers, JHEP 05, 043 (2004)
[49] M. Tanabe and K. Maeda, Nucl. Phys. B738, 184 (2006)
[50] G. Grignani, T. Harmark, A. Marini and M. Orselli, JHEP 1403, 114 (2014)
[51] M. R. Garousi, JHEP 0501, 029 (2005)
[52] A. Dhar and P. Nag, JHEP 0801, 055 (2008)
[53] A. Dhar and P. Nag, Phys. Rev. D 78, 066021 (2008)
[54] M. R. Setare, A. Sepehri and V. Kamali, Phys. Lett. B 735, 84 (2014)
[55] A. Sen, Phys. Rev. D 68, 066008 (2003)
[56] A. Awad and A. F. Ali, JHEP 1406, 093 (2014)
[57] A. F. Ali, Phys. Lett. B 732, 335 (2014)