Measurement of $|V_{ub}|$ in semi-inclusive charmless $B \to \pi X$ decays

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We study semi-inclusive charmless decays $B \to \pi X$, where $X$ does not contain a charm (anti)quark. The mode $\bar{B}^0 \to \pi^- X$ turns out to be particularly useful for determination of the CKM matrix element $|V_{ub}|$. We present the branching ratio (BR) of $\bar{B}^0 \to \pi^- X$ as a function of $|V_{ub}|$, with an estimation of possible uncertainty. The BR is expected to be an order of $10^{-4}$.

1. Introduction

A precise measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements is one of the key issues in the study of $B$ mesons and $B$ factory experiments. In particular, the accurate determination of $V_{ub}$ is one of the most challenging problems in $B$ physics.

Theoretical and experimental studies for probing $V_{ub}$ have been mostly focused on the semileptonic $B$ meson decays. The CLEO result obtained using the exclusive semileptonic decay $B \to \rho l\nu$ is $(3.25 \pm 0.14^{+0.21}_{-0.26} \pm 0.55\text{(model)})$. (in $10^{-3}$)

The OPAL data obtained using the inclusive decay $B \to X_u l\nu$ is $(4.00 \pm 0.65^{+0.67}_{-0.74} \pm 0.19\text{(HQE)})$. (in $10^{-3}$)

In this work we study semi-inclusive charmless decays $B \to \pi X$ and investigate the possibility of extracting $|V_{ub}|$ from these processes. Compared to the exclusive decays, these semi-inclusive decays are generally expected to have less hadronic uncertainty and larger branching ratios. There are several possible processes in $B \to \pi X$ type decays, such as $\bar{B}^0 \to \pi^{\pm(0)} X$, $B^0 \to \pi^{\pm(0)} X$, $B_\pm \to \pi^{\pm(0)} X$, where $X$ does not contain a charm (anti)quark. Among these processes of the type $B \to \pi X$, we identify a certain mode, $\bar{B}^0 \to \pi^- X$, whose analysis is theoretically clean and which can be used for determining $|V_{ub}|$. Then, we calculate the branching ratio (BR) of $\bar{B}^0 \to \pi^- X$, and present the result as a function of $|V_{ub}|$ with an estimation of possible uncertainty. We also consider the $B^0 - \bar{B}^0$ mixing effect through $\bar{B}^0 \to B^0 \to \pi^- X$.

2. Classification of semi-inclusive charmless $B \to \pi X$ decays

Among the semi-inclusive charmless $B \to \pi X$ decays, let us first consider the mode $\bar{B}^0 \to \pi^- X$. Contributions for the decay amplitude of this mode arise from the color-favored tree ($b \to u d\bar{d}$) diagram and the $b \to d$ penguin diagram, so that the tree diagram contribution dominates. The charged pion $\pi^-$ in the final state can be produced via a $W$ boson emission at tree level and is expected to be energetic ($\sim m_B/2$). The decay amplitude can be written as

$$A(\bar{B}^0 \to \pi^- X) = A(b \to u^- d) \cdot h(u d \to X(u d)),$$

where $h$ denotes a hadronization function describing the combination of the $u d$ pair to make the final state $X$. To obtain the decay rate, $X(u d)$ should be summed over all the possible states, such as $\pi^+ \pi^0$, $\pi \pi \pi$ etc, so this process is effectively a two-body decay process of $b \to \pi^- u$. 

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Thus, in this mode, no hadronic form factors are involved, and as a result the model-dependence and uncertainty relevant to hadronic form factors do not appear. We note that the energetic charged pion $\pi^-$ in the final state can be a characteristic signal for this mode. (The net electric charge of $X$ should be positive so that $\pi^-$ cannot be produced in the case of $X = \pi\pi$.)

Now let us consider the mode $B^- \to \pi^- X$. Various contributions are responsible for this process: the color-favored tree diagram, the color-suppressed tree diagram, the $b \to d$ and $b \to s$ penguin diagrams. The color-favored tree contribution and one of $b \to d$ penguin contributions are similar to those in $B^0 \to \pi^- X$, which are effectively two-body type ($b \to \pi^- u$) processes. But, the color-suppressed tree and other penguins differ from those in $B^0 \to \pi^- X$. In fact, these diagrams correspond to a three-body decay process of $B^- \to \pi^- u\bar{u}$ in the parton model approximation. The charged pion $\pi^-$ in the final state contains the spectator antiquark $\bar{u}$. So the analysis involves the hadronic form factor for the $B \to \pi$ transition which is model-dependent. Furthermore, the $b \to s$ penguin contribution is not suppressed compared to the tree contributions, but dominant in this mode. Therefore, compared to the case of $B^0 \to \pi^- X$, the analysis of this mode is much more complicated and involves larger uncertainty.

Other modes of the type $B \to \pi X$ can be similarly classified. For instance, in the mode $B^0 \to \pi^- X$, the color-favored tree ($b \to \bar{u}ud$ and $b \to \bar{u}us\bar{s}$) diagrams and the $b \to d$ and $b \to s$ penguin diagrams are responsible for the decay process. In this case, the charged pion $\pi^-$ contains the spectator quark $d$ so that the process is effectively a three-body decay $B^0 \to \pi^- ud\bar{s}$ and the hadronic form factor for the $B \to \pi$ transition is involved. Other processes are effectively a combination of the two-body decay process ($b \to \pi q$) and the three-body decay process ($B^- \to \pi^- q\bar{q}'$).

3. Analysis of $\bar{B}^0 \to \pi^- X$ decay

We have seen that the process $\bar{B}^0 \to \pi^- X$ is particularly interesting, because it is effectively the two-body decay process $b \to \pi^- u$ and no uncertainty from hadronic form factors is involved. Thus, its theoretical analysis is expected to be quite clean.

We calculate the BR of the process $\bar{B}^0 \to \pi^- X$, where $X$ can contain an up quark and a down antiquark. We use the effective Hamiltonian and the effective Wilson coefficients given in Ref.\ [5]. The BR can be expressed as a polynomial of $|V_{ub}|$:

$$B(\bar{B}^0 \to \pi^- X) = \left|\frac{V_{ub}}{0.004}\right|^2 \cdot B_2 + \left|\frac{V_{ub}}{0.004}\right| \cdot B_1 + B_0 \ , \quad (4)$$

where for convenience we have scaled $|V_{ub}|$ by the factor 0.004 (the central value of the OPAL data).

![Figure 1](image-url)  
**Figure 1.** The branching ratio (in $10^{-4}$) versus $N_c$ for $\bar{B}^0 \to \pi^- X$ decay. $B_{tot}(\equiv B)$ has been calculated using $|V_{ub}| = 0.004$ and is denoted by the bold solid line. The solid, dotted, and dashed lines correspond to $\gamma = 65^\circ$, $85^\circ$, $110^\circ$, respectively.

In Figure 1, we present the BR of $\bar{B}^0 \to \pi^- X$ as a function of the effective number of color $N_c$ for three different values of the CP phase angle $\gamma(\equiv \phi_3) = 65^\circ$, $85^\circ$, $110^\circ$. $B_2$ and $B_0$ are independent of $\gamma(\equiv \phi_3)$, and only $B_1$ depends
on $\gamma(\equiv \phi_3)$. Three different lines for $B_1$ correspond to the relevant values of $\gamma(\equiv \phi_3)$, respectively. It is clearly shown that $B_2$ is dominant. An representative value of $B$ for $|V_{ub}| = 0.004$ and $\gamma(\equiv \phi_3) = 85^0$ is shown as the bold solid line in the figure. The value of $B$ does not vary much as $N_c$ varies.

![Figure 2](image-url)

Figure 2. The branching ratio (in $10^{-4}$) versus $|V_{ub}|$ for $\bar{B}^0 \rightarrow \pi^- X$ decay. The solid and the dotted line correspond to the smallest and the largest value of $B$ in the given parameter space, respectively.

In Figure 2, the BR of $\bar{B}^0 \rightarrow \pi^- X$ is presented as a function of $|V_{ub}|$. We vary the value of $N_c$ and $\gamma(\equiv \phi_3)$ in a reasonable range: from $N_c = 2$ to 10, and from $\gamma(\equiv \phi_3) = 60^0$ to $110^0$. The solid and the dotted line correspond to the smallest and the largest value of $B$ in the given parameter space, respectively. The BR is an order of $10^{-4}$. For the given $|V_{ub}|$, the BR $B$ is estimated with a relatively small error ($< 15\%$). Reversely, for the given (i.e., experimentally measured) BR $B$, the value of $|V_{ub}|$ can be determined with a small error ($< 10\%$). (Of course, since in a practical experiment the BR would be measured with some errors, $|V_{ub}|$ could be determined with larger error: e.g., for $B = (1.0 \pm 0.1) \times 10^{-4}$, our result expects $|V_{ub}| = (3.7 \pm 0.47) \times 10^{-3}$.)

Using the decay process $\bar{B}^0 \rightarrow \pi^- X$, one may need to consider the $B^0 - \bar{B}^0$ mixing effect: $\bar{B}^0 \rightarrow B^0 \rightarrow \pi^- X$. The neutral $\bar{B}^0$ has about 18% probability of decaying as the opposite flavor $B^0$ 

It turns out [4] that even considering the effect from the $B^0 - \bar{B}^0$ mixing, our result holds with resonable accuracy.

4. Conclusion

We have shown that among semi-inclusive charmless $B \rightarrow \pi X$ decays, the process $\bar{B}^0 \rightarrow \pi^- X$ is particularly interesting and one can determine $|V_{ub}|$ with reasonable accuracy, by measuring the BR of $\bar{B}^0 \rightarrow \pi^- X$.

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