Interacting Chaplygin gas

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We investigate a kind of interacting Chaplygin gas model in which the Chaplygin gas plays the role of dark energy and interacts with cold dark matter particles. We find that there exists a stable scaling solution at late times with the Universe evolving into a phase of steady state. Furthermore, the effective equation of state of Chaplygin gas may cross the so-called phantom divide \( w = -1 \). The above results are derived from continuity equations, which means that they are independent of any gravity theories. Assuming standard general relativity and a spatially flat FRW metric, we also find the deceleration parameter is well consistent with current observations.

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I. INTRODUCTION

The existence of dark energy is one of the most significant cosmological discoveries over the last century [1]. However, although fundamental for our understanding of the Universe, its nature remains a completely open question nowadays. Various models of dark energy have been proposed, such as a small positive cosmological constant, quintessence, k-essence, phantom, holographic dark energy, etc., see [2] for recent reviews with fairly complete lists of references of different dark energy models.

Recently the so-called Chaplygin gas, also dubbed quartessence, was suggested as a candidate of a unified model of dark energy and dark matter [3]. The Chaplygin gas is characterized by an exotic equation of state

\[
\rho_{ch} = \frac{-A}{\rho_{ch}},
\]

where \( A \) is a positive constant. The above equation of state leads to a density evolution in the form

\[
\rho_{ch} = \sqrt{A + \frac{B}{a^6}},
\]

where \( B \) is an integration constant. The attractive feature of the model is that it naturally unifies both dark energy and dark matter. The reason is that, from [2], the Chaplygin gas behaves as dust-like matter at early stage and as a cosmological constant at later stage. Some possible motivations for this model from the field theory points of view are investigated in [4]. The Chaplygin gas emerges as an effective fluid associated with \( d \)-branes [5] and can also be obtained from the Born-Infeld action [6]. Recently, the original Chaplygin gas model was generalized, with possible observational constraints on these generalized models presented in Ref. [7]. For example in the generalized Chaplygin gas (GCG) approach [8], the equation of state to describe the background fluid is generalized to

\[
p_{ch} = \frac{A}{\rho_{ch}^2},
\]

and the corresponding evolution of the scale factor is given by

\[
\rho_{ch} = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{-\frac{1}{1+\alpha}}.
\]

From the above equations, it is clear that when \( \alpha = 1 \) the GCG model recovers the original Chaplygin gas model. This approach has been thoroughly investigated for its impact on the 0th order cosmology, i.e., the cosmic expansion history (quantified by the Hubble parameter \( H[z] \)) and corresponding spacetime-geometric observables. An interesting range of models was found to be consistent with SN Ia data [9], CMB peak locations [10] and dimensionless coordinate distances to type Ia supernovae [11]. There seems to be, however, a flaw in unified dark matter (UDM) models that manifests itself only on small (Galactic) scales and that has not been revealed by the studies involving only background tests. In Ref. [11], it is found that GCG model produces oscillations or exponential blowup of the matter power spectrum inconsistent with observations. In fact, from this analysis, 99.999 % of previously allowed parameter of GCG model has been excluded (see, however, [12]).

Hence we may turn to a model with Chaplygin gas and dark matter. Although non-minimal coupling between the dark energy and ordinary matter fluids is strongly restricted by the experimental tests in the solar system [13], due to the unknown nature of the dark matter as part of the background, it is possible to have non-gravitational interactions between the dark energy and the dark matter components, without conflict with the experimental data. In this paper we investigate some physical properties of an interacting Chaplygin gas model. Here, by considering an interaction term between the Chaplygin gas fluid and dark matter particles similar to those studied in the context of quintessence scenarios [14], we investigate dynamical aspects of this interacting Chaplygin gas model.
Following the more accurate data a more dramatic result appears: the recent analysis of the type Ia supernovae data indicate that the time varying dark energy gives a better fit than a cosmological constant, and in particular, the equation of state parameter $w$ (defined as the ratio of pressure to energy density) crosses $-1$ at some low redshift region from above to below $16$, where $w = -1$ is the equation of state for the cosmological constant. It deserves to note that there are other independent fittings imply the probability that current $w < -1$ except for supernovae data $17$. The dark energy with $w < -1$ is called phantom dark energy $18$, for which all energy conditions are violated. To obtain $w < -1$, scalar field with a negative kinetic term, may be a simplest realization $19$. However, the equation of state of phantom scalar field is always less than $-1$ and can not cross $-1$. Also it has been shown that the equation of state cannot cross $-1$ in the k-essence model of dark energy under some reasonable assumptions $20$. Some dark energy models which contain a negative-kinetic scalar field and a normal scalar field have been considered in $21$; in these models crossing the border $w = -1$ can be realized. Some different suggestions on this crossing behavior are presented in $22$.

It has been pointed out that Chaplygin gas model can be described by a quintessence filed with well-connected potential $3$. So a model with mutually independent Chaplygin gas and dark matter is essentially a special quintessence model. The Chaplygin gas, here as dark energy, can not cross the phantom divide like quintessence. In this paper we shall see that an interaction term can realize this crossing naturally. At the same time we obtain a scaling solution: It may also shed light on the coincidence problem. Another interesting result is that the scaling solution inevitably leads to the steady state Universe $23$, which had been suggested many years ago but soon surpassed by expanding Universe, as the final state of our model.

We present our model in details in the next section and some observational predictions of this scenario and a comparison with recent observational data are also briefly discussed. Our conclusions and discussions appear in the last section.

II. THE MODEL

We consider the original Chaplygin gas, whose pressure and energy density satisfy the relation, $p_{ch} = -A/\rho_{ch}$. By assuming the cosmological principle the continuity equations are written as

$$\dot{\rho}_{ch} + 3H\gamma_{ch}\rho_{ch} = -\Gamma,$$

and

$$\dot{\rho}_{dm} + 3H\gamma_{dm}\rho_{dm} = \Gamma,$$

where the subscript $dm$ denotes dark matter, $H$ is the Hubble parameter and $\gamma$ is defined as

$$\gamma = 1 + \frac{p}{\rho} = 1 + w,$$

in which $w$ is the parameter of the state of equation, and $\gamma_{dm} = 1$ throughout the evolution of the Universe, whereas $\gamma_{ch}$ is a variable.

$\Gamma$ is the interaction term between Chaplygin gas and dark matter. Since there does not exist any microphysical hint on the possible nature of a coupling between dark matter and Chaplygin gas (as dark energy), the interaction terms between dark energy and dark matter are rather arbitrary in literatures $24$. Here we try to present a possible origin from fundamental field theory for $\Gamma$ (see $25$ for a thermodynamic discussion on $\Gamma$).

Whereas we are still lacking a complete formulation of unified theory of all interactions (including gravity, electromagnetic and strong), there at present is at least one very hopeful candidate, string/M theory. Although the recent developments in string theory, assisted by the discovery of the power of duality, have greatly improved our understanding of it, the theory is still not known in a way that would enable us to ask the questions about spacetime in a general manner, say nothing of the properties of realistic particles. Instead, we have to either resort to the effective action approach which takes into account stringy phenomena in perturbation theory, or we could study some special classes of string solutions which can be formulated in the non-perturbative regime. But the latter approach is available only for some special solutions, most notably the BPS states or nearly BPS states in the string spectrum: They seems to have no relation to our realistic Universe. Especially, there still does not exist a non-perturbative formulation of generic cosmological solutions in string theory. Hence nearly all the investigations of realistic string cosmologies have been carried out essentially in the effective action range $26$. Note that the departure of string-theoretic solutions away from general relativity is induced by the presence of additional degrees of freedom which emerge in the massless string spectrum. These fields, including the scalar dilaton field, the torsion tensor field, and others, couple to each other and to gravity non-minimally, and can influence the dynamics significantly. Thus such an effective low energy string theory deserve research to solve the dark energy problem. There a special class of scalar-tensor theories of gravity is considered to avoid singularities in cosmologies in $27$. The action is written below,

$$S_{st} = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi + \frac{1}{q(\phi)^2} L_m(\xi, \partial \xi, q^{-1} g_{\mu\nu}) \right],$$

where $G$ is the Newton gravitational constant, $\phi$ is a scalar field, $L_m$ denotes Lagrangian of matter, $\xi$ represents different matter degrees of matter fields, $q$ guarantees the coupling strength between the matter fields and
the dilaton. With action $\mathcal{S}$, the interaction term can be written as follow \cite{27},

$$
\Gamma = H \rho_m \frac{d \ln q'}{d \ln a}.
$$

Here we introduce new variable $q(a)' \equiv q(a)^{(3w_n-1)/2}$, where $a$ is the scale factor in standard FRW metric. By assuming

$$
q'(a) = q_0 e^{3 \int c(\rho_m + \rho_c)/\rho_m d \ln a},
$$

where $\rho_m$ and $\rho_c$ are the densities of matter and the scalar field respectively, one arrive at the interaction term,

$$
\Gamma = 3Hc(\rho_m + \rho_\xi),
$$

which is just the coupling form studied in contexts of quintessence and phantom dark energy models \cite{15}. Moreover the Chaplygin gas can be view as a scalar field with proper potential in cosmological models \cite{3}. So it may be reasonable to phenomenologically introduce such an interaction term between Chaplygin gas and dark matter.

Now return to the equation set $\mathcal{S}$ and $\mathcal{O}$. Set $x = -\ln(1+z)$, $\Gamma = 3Hc(\rho_\chi + \rho_d m)$, $u = (3H_0^2)^{-1} \kappa^2 \rho_d m$, $v = (3H_0^2)^{-1} \kappa^2 \rho_\chi$, $A' = A(3H_0^2)^{-2} \kappa^4$, where $H_0$ takes the value of present Hubble parameter, $\kappa$ is the Newton gravitational constant and $c$ is a constant without dimension. Eqs. $\mathcal{S}$ and $\mathcal{O}$ reduce to

$$
\frac{du}{dx} = -3u + 3c(u + v),
$$

$$
\frac{dv}{dx} = -3(v - A'/v) - 3c(u + v).
$$

We note that the variable time does not appear in the dynamical system $\mathcal{S}$ and $\mathcal{O}$ because time has been completely replaced by redshift $x = -\ln(1+z)$. The critical points of dynamical system $\mathcal{S}$ and $\mathcal{O}$ are given by

$$
\frac{du}{dx} = \frac{dv}{dx} = 0.
$$

The solution of the above equation is

$$
u_c = \frac{c}{1-c}, \quad v_c = (1-c)A'.
$$

We see the final state of the model contains both Chaplygin gas and dark matter of constant densities if the singularity is stationary. The final state contents perfect cosmological principle: the Universe is homogeneous and isotropic in space, as well as constant in time. Physically $\Gamma$ in $\mathcal{S}$ plays the role of matter creation term $C$ in the theory of steady state universe at the future time-like infinity. Recall that $c$ is the coupling constant, may be positive or negative, corresponds the energy to transfer from Chaplygin gas to dark matter or reversely. $A'$ must be a positive constant, which denotes the final energy density if $c$ is fixed. Also we can derive an interesting and simple relation between the static energy density ratio

$$
c = \frac{r_s}{1 + r_s},
$$

where

$$
r_s = \lim_{z \to -1} \frac{\rho_{dm}}{\rho_{ch}}.
$$

To investigate the properties of the dynamical system in the neighbourhood of the singularities, impose a perturbation to the critical points,

$$
\frac{d(\delta u)}{dx} = -3\delta u + 3c(\delta u + \delta v),
$$

$$
\frac{d(\delta v)}{dx} = -3(\delta v + A' v_c \delta v) - 3c(\delta u + \delta v).
$$

The eigen equation of the above linear dynamical system $(\delta u, \delta v)$ reads

$$
(\lambda/3)^2 + (2 + \frac{1}{1-c})\lambda/3 + 2 - 2c^2 = 0,
$$

whose discriminant is

$$
\Delta = [(1-c)^4 + (3/2-c)^2]/(1-c)^2 \geq 0.
$$

Therefore both of the two roots of eigen equation $\Delta$ are real, consequently centre and focus singularities can not appear. Furthermore only $r_s \in (0, \infty)$, such that $c \in (0, 1)$, makes physical sense. Under this condition it is easy to show that both the two roots of $\Delta$ are negative. Hence the two singularities are stationary. However it is only the property of the linearized system $\mathcal{S}$ and $\mathcal{O}$, or the property of orbits of the neighbourhoods of the singularities, while global Poincare-Hopf theorem requires that the total index of the singularities equals the Euler number of the phase space for the non-linear system $\mathcal{S}$ and $\mathcal{O}$. So there exists other singularity except for the two nodes. In fact it is a non-stationary saddle point at $u = 0$, $v = 0$ with index $-1$. This singularity has been omitted in solving equations $\mathcal{S}$ and $\mathcal{O}$. The total index of the three singularities is 1, which equals the Euler number of the phase space of this plane dynamical system. Hence there is no other singularities in this system. From these discussions we conclude that the global outline of the orbits of this non-linear dynamical system $\mathcal{S}$ and $\mathcal{O}$ is similar to the electric flux-lines of two negative point charges. Here we plot figures 1 and 2 to show the properties of evolution of the Universe controlled by the dynamical system $\mathcal{S}$ and $\mathcal{O}$. As an example we set $c = 0.2$, $A' = 0.9$ in all the figures except figure 4.

Further, to compare with observation data we need the explicit forms of $u(x)$ and $v(x)$, especially $v(x)$, since we have set $\gamma_{dm} = 1$ but $\gamma_{ch}$ is not a constant. We need
the properties of $\gamma_{ch}$ in our model, which is contained in $v(x)$, to compare with observations. Eliminate $u(x)$ by using (12) and (13) we derive

$$\frac{1}{3c} \frac{d^2v}{dx^2} + \frac{1 + (1 + A'/v^2)\frac{dv}{dx}}{v} + 3(1 - c) \left\{ v + \frac{dv}{dx} + 3(v - A'/v) \right\} \frac{1}{(3c)} = 0,$$  \hspace{1cm} (23)

which has no analytic solution. We show some numerical solutions in figure 3. We find that for proper region of parameter spaces, the effective equation of state of Chaplygin gas crosses the phantom divide successfully. We have no analytical result on this crossing phenomena yet in the present stage.

Up to now all of our results do not depend on Einstein field equation. They only depend on the most sound principle in physics, that is, the continuity principle, or the energy conservation law. Different gravity theories, such as standard general relativity, brane-induced gravity, 1/R gravity, and Lovelock gravity or cosmology of modified Friedmann equation such as Cardassian cosmology correspond to different constraints imposed on our previous discussions. Our improvements show how far we can reach without information of dynamical evolution of the Universe.

The most significant parameters from the viewpoint of observations is the deceleration parameter $q$, which carries the total effects of cosmic fluids. From now on we introduce the Friedmann equation of the standard general relativity. As a simple case we study the evolution of $q$ in a spatially flat frame. So $q$ reads

$$q = \frac{\ddot{a}a}{a^2} = \frac{1}{2} \left( \frac{u + v - 3A'/v^2}{u + v} \right),$$  \hspace{1cm} (24)

and density of Chaplygin gas $u$ and density of dark matter $v$ should satisfy

$$v(0) + v(0) = 1.$$  \hspace{1cm} (25)

And then Friedmann equation ensures the spatial flatness in the whole history of the Universe. Before analyzing the evolution of $q$ with redshift, we first study its asymptotic behaviors. When $z \to \infty$, $q$ must go to $1/2$ because both Chaplygin gas and dark matter behave like dust, while when $z \to -1$ $q$ is determined by

$$\lim_{z \to -1} q = \frac{1}{2} \left( \frac{u_c + v_c - 3A'/v_c^2}{u_c + v_c} \right).$$  \hspace{1cm} (26)

One can finds the parameters $c = 0.2$, $A' = 0.9$ are difficult to content the previous constraint Friedmann constraint (25). Here we carefully choose a new set of parameter which satisfies Friedmann constraint (25), say, $A' = 0.4$, $c = 0.06$. Therefore we obtain

$$\lim_{z \to -1} q = -1.95,$$  \hspace{1cm} (27)

by using (15) and (16). Then we plot figure 4 to clearly display the evolution of $q$. One can check $u(0) = 0.25$, $v(0) = 0.75$; $u(0) = 0.28$, $v(0) = 0.72$; $v(0) = 0.3$, $v(0) = 0.7$, respectively on the curves $v(-2) = 273$; $v(-2) = 250$; $v(-2) = 233$. One may find an interesting property of the deceleration parameter displayed in figure 4. The bigger the proportion of the dark energy, the smaller the absolute value of the deceleration parameter. The reason roots in the extraordinary state of Chaplygin gas, in which the pressure $p_{ch}$ is inversely proportional to the energy density $\rho_{ch}$.

Also we note that maybe an FRW Universe with non-zero spatial curvature fits deceleration parameter better than spatially flat FRW Universe. This point deserves to research further.

FIG. 1: The plane $v$ versus $u$. (a) We consider the evolution of the universe from redshift $z = e^2 - 1$. The initial condition is taken as $u = 0, v = 400$; $u = 50, v = 350$; $u = 100, v = 300$; $u = 120, v = 280$ on the four orbits, from the left to the right, respectively. It is clear that there is a stationary node, which attracts most orbits in the first quadrant. At the same time the orbits around the neighbourhood of the singularity is not shown clearly. (b) Orbit distributions around the node $u_e = u_c/(1 - c), v_c = (1 - c)A'$. 
We present a phase-space analysis of the evolution for a FRW universe driven by an interacting mixture of dark matter and Chaplygin gas. In the absence of interaction, there exist no scaling solutions because the state of equation of Chaplygin gas decreases with scale factor while state of equation of dark matter keeps a constant. Hence we study the existence and stability of the cosmological scaling solutions with interaction between Chaplygin gas and dark matter. For the interaction term \( \Gamma = 3cH(\rho_{ch} + \rho_{dm}) \), inspired by low energy effective string theory and scalar-tensor gravity theory, we find stationary scaling solutions for reasonable initial conditions. Further more this approach leads to an impressive result: The equation of state of Chaplygin gas traverses the phantom divide \( w = -1 \), which is favored by recent fittings with current type Ia supernovae data. At the same time, different from phantom model, there is no singularity in the future in our model. On the contrary the final state of our model is steady state Universe, in which Chaplygin gas ensures the continuous production of matter (dust) through the interaction term. The analytical researches on this crossing behavior deserve to investigate in the future work. We also calculate the deceleration parameter of this model in frame of spatially flat FRW cosmology. The result is consistent with the observation data.

At last, as a phenomenological model, we must constrain it by observational data. We see that this model is so simple that it is fully parameterized by two parameters: \( c \) determines the final ratio of Chaplygin gas and dark matter, and \( A \) governs the final total energy den-
sity of the Universe. We may consider the observational constraints on the parameter space arising from observation data from different observations, such as supernovae, CMB, X-rays, gravitational lensing effects or combination of these data in future work.

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