THE LIFETIMES OF HEAVY FLAVOUR HADRONS - A CASE STUDY IN QUARK-HADRON DUALITY

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The status of heavy quark expansions for charm and beauty lifetime ratios is reviewed. Taking note of the surprising semiquantitative success of this description for charm hadrons I interpret the new data on $\tau(D_s)$ and re-iterate the call for more precise measurements of $\tau(\Xi_b^{0,+})$ and $\tau(\Omega_c)$. A slightly larger $B^-$ than $B_d$ lifetime is starting to emerge as predicted; the largest lifetime difference in the beauty sector, namely in $\tau(B_s)$ vs. $\tau(B)$ has correctly been predicted; the problem posed by the short $\Lambda_b$ lifetime remains. The need for more accurate data also on $\tau(B_s)$ and $\tau(\Xi_b^{0,+})$ is emphasized. I discuss quark-hadron duality as the central theoretical issue at stake here.

Contents

1 Introduction 2

2 Heavy Quark Expansions 3

3 Lifetimes of Charm Hadrons 4

4 Lifetimes of Beauty Hadrons 6

4.1 Orthodoxy .................................................. 6

4.2 Heresy ........................................................ 8

5 On Quark-Hadron Duality 8

5.1 General Remarks ........................................... 8

5.2 't Hooft Mode .............................................. 9

6 Summary and Outlook 10

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1 Introduction

The lifetime of a hadron represents an observable of fundamental as well as practical importance: (i) Its magnitude reveals whether the decay is driven by strong, electromagnetic or weak forces; (ii) it constitutes an essential engineering number for translating measured branching ratios into widths. Yet a strong motivation to measure a quantity does not necessarily imply a need for a precise theoretical description. Furthermore we all understand that nothing that is going to happen or not happen in the theory of weak lifetimes will make anybody abandon QCD as the theory of the strong interactions. After all, it is the only game in town after string theory has raised its ambition to become the theory of everything rather than merely the theory of the strong forces where it had first emerged.

The central theme of my talk will be that developing such a theory represents a forum for addressing the next frontier in QCD, namely quark-hadron duality or duality for short. The concept of duality constitutes an essential element in any QCD based description and it has been invoked since the early days of the quark model. For a long time little progress happened in this area; for a violation of duality can be discussed in a meaningful way only if one has a reliable theoretical treatment of nonperturbative effects.

Let me illustrate that through an example. A priori it would be quite reasonable to assume that relating the weak decay width of a heavy flavour hadron to the fifth power of its mass rather than the heavy quark mass – \( \Gamma(H_Q) \propto M^5(H_Q) \) – would incorporate boundstate effects as the leading non-perturbative corrections (and that is indeed what we originally did). Only after developing a consistent theory for the weak decays of such hadrons through the operator product expansion did we realize that such an ansatz would violate duality. For it leads to a large correction of order \( 1/m_Q - M^5(H_Q) \simeq (m_Q + \Lambda)^5 \simeq m_Q^5 (1 + 5\Lambda/m_Q) \) – which is anathema to the OPE.

This example already indicates that the study of heavy flavour decays had given new impetus to addressing duality: it has provided us with new theoretical tools, and it has re-emphasized the need to understand the limitations to duality since one aims at extracting fundamental KM parameters with high numerical accuracy from semileptonic decays.

Nonleptonic transitions provide a rich and multilayered lab to analyze duality and its limitations; they can act as a microscope exactly because they are thought of containing larger duality violations than semileptonic reactions.
2 Heavy Quark Expansions

The weak decay width for a heavy flavour hadron $H_Q$ into an inclusive final state $f$ can be expressed through an operator product expansion (OPE)\cite{1,2}.

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{CKM}|^2 \left[ c_3^{(f)} \langle H_Q|\bar{Q}Q|H_Q \rangle + c_5^{(f)} \frac{\mu^2_{Q}(H_Q)}{m_Q^2} + \sum_i c_{6,i}^{(f)} \frac{\langle H_Q|\bar{Q}\Gamma_i q|q\Gamma_i Q|H_Q \rangle}{m_Q^4} + \mathcal{O}(1/m_Q^4) \right], \tag{1}$$

where $\mu^2_{Q}(H_Q) \equiv \langle H_Q|\bar{Q}Q|H_Q \rangle$. Eq.\cite{1} exhibits the following important features:

- The expansion involves
  - $c$-number coefficients $c_i^{(f)}$ calculable within short-distance physics;
  - expectation values of local operators given by long distance physics;
  - their values can be inferred from symmetry arguments, other observables, QCD sum rules, lattice studies and quark models;
  - inverse powers of the heavy quark mass $m_Q$ scaling with the known dimensions of the various operators.

The nonperturbative effects on the decay width – a dynamical quantity – can thus be expressed through expectation values and quark masses. Those being static quantities can be calculated with decent reliability.

- A crucial element of Wilson’s prescription for this expansion is that it allows a selfconsistent separation of short-distance dynamics that is lumped into the coefficients $c_i^{(f)}$ and long-distance dynamics that enters through the expectation values of local operators. This is achieved through the introduction of the auxiliary scale $\mu$ that enters both in the coefficients and the matrix elements. As a matter of principle observables have to be independant of $\mu$ since Nature cannot be sensitive to how we arrange our computational tasks. In practise, however, $\mu$ has to be chosen judiciously for those very tasks: on one hand one would like to choose $\mu$ as high as possible to obtain a reliable perturbative expression in powers of $\alpha_S(\mu)$; on the other hand one likes to have it as low as possible to evaluate the expectation values in powers of $\mu/m_Q$. This ‘Scylla and Charybdis’ dilemma can be tackled by choosing $\mu \simeq 1$ GeV. For simplicity I will not state the dependance on $\mu$ explicitely although it is implied.
• The free quark model or spectator expression emerges asymptotically (for \( m_Q \to \infty \)) from the \( \bar{Q}Q \) operator since \( \langle H_Q | \bar{Q}Q | H_Q \rangle = 1 + \mathcal{O}(1/m_Q^2) \).

• No \( \mathcal{O}(1/m_Q) \) contribution can arise in the OPE since there is no independent dimension four operator (with colour described by a local gauge theory) \( \bar{Q}Q \). This has two important consequences:

  – With the leading nonperturbative corrections arising at order \( 1/m_Q^2 \), their size is typically around 5% in beauty decays. They had not been anticipated in the phenomenological descriptions of the 1980’s.
  – A \( 1/m_Q \) contribution can arise only due to a massive duality violation. Thus one should set a rather high threshold before accepting such a statement.

• Pauli Interference (PI)\( ^4 \), Weak Annihilation (WA)\( ^5 \) for mesons and W-scattering (WS) for baryons arise unambiguously and naturally in order \( 1/m_Q^3 \) with WA being helicity suppressed\( ^1 \). They mainly drive the differences in the lifetimes of the various hadrons of a given heavy flavour.

The expectation values of \( \bar{Q}Q \) and \( \bar{Q} \sigma \cdot GQ \) are known with sufficient accuracy for the present purposes from the hyperfine splittings and the charm and beauty hadron masses\( ^7 \).

The largest uncertainties enter in the expectation values for the dimension-six four-fermion operators in order \( 1/m_Q^3 \). For mesons I will invoke approximate factorization at a low scale of around 1 GeV. One should note that factorizable contributions at a low scale \( \sim 1 \text{ GeV} \) will be partially nonfactorizable at the high scale \( m_Q \)!

For baryons there is no concept of factorization, and we have to rely on quark model results.

Below I will discuss mainly hadron-specific duality violations affecting the ratios between different hadrons of a given heavy flavour.

3 Lifetimes of Charm Hadrons

One rough measure for the numerical stability of the \( 1/m_c \) expansion is provided by \( \sqrt{\mu_{D}^2(D)/m_c^2} \simeq 0.5 \) as an effective expansion parameter which is not small compared to one. Obviously one can expect – at best – a semiquantitative description. The mesonic four-quark matrix elements are calibrated by \( f_D \sim 200 \text{ MeV} \) and \( f_{D_s}/f_D \simeq 1.1 - 1.2. \)

\(^b\)The operator \( Qi \bar{D}Q \) can be reduced to the leading operator \( \bar{Q}Q \) through the equation of motion.
On general grounds one expects the following hierarchy in lifetimes:  
\[ \tau(D^+) > \tau(D^0) \sim \tau(D_s^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^+) > \tau(\Omega_c) \quad (2) \]

Table 1 shows the predictions and data. A few comments are in order here:

- You apply the $1/m_c$ expansion at your own risk. It is easy to list reasons why it should fail to reproduce even the qualitative pattern expressed in Eq.(2). However comparing the data with the expectations shows agreement even on the semiquantitative level. This could be accidental; yet I will explore the possibility that it is not. One should note that the longest and shortest lifetimes differ by a factor of about twenty!

- PI is the main engine driving the $D^+ - D^0$ lifetime difference as already anticipated in the ‘old’ analysis of Guberina et al.; the main impact of the HQE for this point was to show that WA cannot constitute the leading effect and that $BR_{SL}(D^0) \simeq 7\%$ is consistent with PI being the leading effect, see below. In quoting a lifetime ratio of $\sim 2$ I am well aware that the measured value is different from two. Yet that numerical difference is within the theoretical noise level: one could use $f_D = 220$ MeV rather than 200 MeV and WA, which has been ignored here, could account for 10 - 20 % of the $D^0$ width.

\footnote{\textit{\(\Gamma(D^+)\) is guaranteed to remain positive if the range in momentum over which PI can occur is properly evaluated. To put it differently: while one cannot count on obtaining a reliable value for \(\Gamma(D^+)\), a nonsensical result will arise only if one makes a mistake.}}
• Since \( \tau(D_s)/\tau(D^0) \approx 1.07 \) can be generated without WA, the ‘old’ data on \( \tau(D_s)/\tau(D^0) \) had provided an independent test for WA not being the leading source for \( \tau(D^0) \neq \tau(D^+) \); it actually allowed for it being quite irrelevant. The ‘new’ data reconfirm the first conclusion; at the same time they point to WA as a still significant process. A recent analysis of WA relying on QCD sum rules is not quite able to reproduce the observed lifetime ratio; further analysis along these lines is called for.

• The \( 1/m_b^2 \) contribution controlled by \( \mu_G^2(D) \) reduces the semileptonic width common to \( D^0 \) and \( D^+ \) mesons; this brings the absolute values observed for \( BR_{SL}(D^0) \) and \( BR_{SL}(D^+) \) in line with what is expected when it is mainly PI that generates the \( D^+ - D^0 \) lifetime difference.

• The description of the baryonic lifetimes is helped by the forgiving experimental errors. More accurate measurements of \( \tau(\Xi_{c}^+, 0_c, \Omega_c) \) are needed. They might well exhibit deficiencies in the theoretical description.

• Nonuniversal semileptonic widths – \( \Gamma_{SL}(D) \neq \Gamma_{SL}(\Lambda_c) \neq \Gamma_{SL}(\Xi_c) \) – are predicted with the main effect being constructive PI in \( \Xi_c \) and \( \Omega_c \) decays; the lifetime ratios among the baryons will then not get reflected in their semileptonic branching ratios; one estimates:

\[
BR_{SL}(\Xi_{c}^0) \sim BR_{SL}(\Lambda_c) \leftrightarrow \tau(\Xi_{c}^0) \sim 0.5 \cdot \tau(\Lambda_c)
\]

\[
BR_{SL}(\Xi_{c}^+) \sim 2.5 \cdot BR_{SL}(\Lambda_c) \leftrightarrow \tau(\Xi_{c}^+) \sim 1.3 \cdot \tau(\Lambda_c)
\]

\[
BR_{SL}(\Omega_c) < 15 \%
\]

• On general grounds one expects \( \Delta \Gamma(D^0)/\Gamma(D^0) \leq \theta_C^2 \cdot SU(3)_{FL} \) breaking \( \leq O(0.01) \). If the data show that the lifetime difference for the two neutral \( D \) mass eigenstates is significantly below this bound, one would have learned an intriguing lesson on duality.

4 Lifetimes of Beauty Hadrons

4.1 Orthodoxy

The numerical stability of the \( 1/m_b \) expansion is characterised by \( \sqrt{\mu_G^2(B)/m_b^2} \approx 0.13 < 1 \); i.e. such an expansion should yield rather reliable numerical results. Merely reproducing the qualitative pattern would be quite unsatisfactory. I will also use \( f_B \sim 200 \) MeV and \( f_B/f_{B_s} \approx 1.1 - 1.2 \). Table shows predictions and data. Again some comments to elucidate these findings:
• The original predictions for the meson lifetimes, which had encountered theoretical criticism, are on the mark. (i) A recent lattice study finds a result quite consistent with the original work based on factorization:

\[ \frac{\tau(B^+)}{\tau(B_d)} = 1.03 \pm 0.02 \pm 0.03 \]  \hspace{1cm} (6)

(ii) Sceptics will argue that predicting lifetime ratios close to unity is not overly impressive. In response one should point out that the largest lifetime difference by far – \( \frac{\tau(B_c)}{\tau(B_d)} \approx \frac{1}{3} \) – has been predicted correctly and that the absence of contributions \( \sim O(1/m_Q) \) had been crucial there!

• A serious challenge arises from the ‘short’ baryon lifetime. In terms of \( \Delta \equiv 1 - \tau(\Lambda_b)/\tau(B_d) \) the data can be expressed by

\[ \Delta_{\text{experim}} = 0.21 \pm 0.05 \]  \hspace{1cm} (7)

A detailed analysis of quark model calculations finds however

\[ \Delta_{\text{theor.}} = 0.03 - 0.12 \]  \hspace{1cm} (8)

Reanalyses by other authors agree with Eq. (8) – as does a pilot lattice study: \( \Delta_{\text{lattice}} = (0.07 - 0.09) \). A recent analysis based on QCD sum rules arrives at a significantly larger value: \( \Delta_{\text{QCD}} = 0.13 - 0.21 \). If true it would remove the problem. However, I would like to understand better how the sum rules analysis can differ so much from other studies given that it still uses the valence quark approximation.

• An essential question for future studies concerns the lifetimes of the beauty hyperons \( \Xi_b^{-0} \). On general grounds one expects \( \tau(\Xi_b^{-}) > \tau(\Lambda_b) \),

| \( \frac{\tau(B^+)}{\tau(B_d)} \) | \( \frac{\Delta \Gamma(B_s)}{\Gamma(B_s)} \) | \( \tau(B_s) \) | \( \tau(\Lambda_b) \) |
|-----------------|-----------------|--------|--------|
| 1.05 \( \frac{(f_{B_s} 200 \text{ MeV})^2}{2 \pi} \) | 1.066 ± 0.024 | 0.94 ± 0.04 | 0.46 ± 0.17 psec |

Table 2: Lifetime ratios in the beauty sector
More specifically, using the observed charm hyperon lifetimes and \(SU(3)\) symmetry a very sizeable effect has been predicted:\(^\text{16}\)

\[
\frac{\tau(\Xi^-_b) - \tau(\Lambda_b)}{\tau(\Lambda_b)} \sim 0.14 - 0.21
\]

4.2 Heresy

As said before, the ansatz \(\Gamma(H_Q) \propto M(H_Q)^5\) which would yield \(\tau(\Lambda_b)/\tau(B_d) \approx 0.75\) and therefore has been re-surrected\(^\text{19}\) is anathema to the OPE since it would imply the nonperturbative corrections to be of order \(1/m_Q^1\). The \(B^- - B_d\) lifetime difference is still an \(O(1/m_Q^3)\) effect.

Notwithstanding my employer I am willing to consider heresy, though, since it makes some further prediction that differ from the OPE findings:

\[
\frac{\bar{\tau}(B_s)}{\tau(B_d)} = \left(\frac{M(B_d)}{M(B_s)}\right)^5 \approx 0.94
\]

\[
\tau(\Lambda_b)/\tau(\Xi^0_b)/\tau(\Xi^-_b) \approx 1/0.85/0.85;
\]

the expectation \(M(\Xi_b) - M(\Lambda_b) \approx M(\Xi_c) - M(\Lambda_c)\) has been used in Eq.\(^{11}\).

On the down side I do not see how such an ansatz can yield a correct prediction for \(\tau(B_c)\) in a natural way.

One can also add that such an ansatz helps to understand neither the pattern nor the size of the lifetime differences in the charm sector. Agreement with the data can be enforced, though, by adjusting\(^\text{19}\) – in an ad-hoc fashion I would say – the contributions from PI, WA and WS.

5 On Quark-Hadron Duality

5.1 General Remarks

Duality has been an early and somewhat fuzzy element of quark model arguments. It can be expressed as follows:\(^\text{d}\) Rates evaluated on the parton level ‘approximate’ observable rates summed over a ‘sufficient number’ of hadronic channels.” It was never stated clearly how good an approximation it provided and what is meant by ‘sufficient number’; it was thought, though, that this number had to be larger than of order unity.

Heavy quark theory has opened up new theoretical tools as well as perspectives onto duality; it demonstrated that duality can hold even with one or...
two channels dominating – if an additional feature like heavy quark symmetry intervenes. This has been demonstrated for semileptonic $b \to c$ decays. The goal is to understand better the origins of limitations to duality and to develop some quantitative measures for it. The new tools that are being brought to bear on this problem are (a) the OPE; (b) the so-called small velocity sum rules and (c) the ’t Hooft model.

The results obtained so far show there are different categories of duality – local vs. global etc. duality – depending on the amount of averaging or ‘smearing’ that is involved and that duality can neither be universal nor exact.

Duality is typically based on dispersion relations expressing observable rates averaged over some kinematical variables through an OPE constructed in the Euclidean region. There are natural limitations to the accuracy of such an expansion; among other things it will have to be truncated. In any case, such a power expansion will exhibit no sensitivity to a term like $\exp(-m_Q/\Lambda_{QCD})$. Yet upon analytical continuation from the Euclidean to the Minkowskian domain this exponentially suppressed term turns into $\sin(m_Q/\Lambda_{QCD})$ – which is not surpressed at all! I.e., the OPE cannot account for such terms that could become quite relevant in Minkowski space and duality violations can thus enter through these ‘oscillating’ terms; the opening of thresholds provides a model for such a scenario. Actually they will be surpressed somewhat like $(1/m_Q^k)\sin(m_Q/\Lambda_{QCD})$ with the power $k$ depending on the dynamics in general and the reaction in particular. This could produce also a ‘heretical’ $1/m_Q$ contribution from a dimension-five operator:

$$\frac{1}{m_Q^2} \sin \left( \frac{m_Q}{\Lambda_{QCD}} \right) = \mathcal{O}(1/m_Q)$$

The colour flow in semileptonic as well as nonleptonic spectator decays and in WA is such that duality can arise naturally; i.e., nature had to be malicious to create sizeable duality violations. Yet in PI – because it is an interference phenomenon – the situation is much more complex leading to serious concerns about the accuracy with which duality can apply here.

5.2 ’t Hooft Model

The most relevant features of the ’t Hooft model are: (1) QCD in 1+1 dimensions obviously confines. (2) It is solvable for $N_C \to \infty$: its spectrum of narrow resonances can be calculated as can their wavefunctions.

Duality can then be probed directly by comparing the width evaluated through the OPE with a sum over the ‘hadronic’ resonances appropriately
smeared over the threshold region:

\[ \Gamma_{\text{OPE}}(H_Q) \leftrightarrow \sum_n \Gamma(H_Q \to f_n) \quad (13) \]

Such a program has been first pursued using numerical methods; it led to claims that duality violations arise in the total width through a $1/m_Q$ term and quantitatively more massively in WA. However, an analytical study has shown that neither of these claims is correct: perfect matching of the OPE expression with the sum over the hadronic resonances was found through high order in $1/m_Q$. The same result was obtained for the more intriguing case of PI.

### 6 Summary and Outlook

A mature formalism genuinely based on QCD has been developed for describing inclusive nonleptonic heavy flavour decays. It can tackle questions that could not be addressed before in a meaningful way; even failures can teach us valuable lessons on nonperturbative aspects of QCD, namely on limitations to duality.

A fairly successful semiquantitative picture has emerged for the lifetimes of charm hadrons considering the wide span characterised by $\tau(D^+)/\tau(\Omega_c) \sim 20$; while this might be a coincidence, it should be noted:

- PI provides the leading effect driving the $D^+ - D^0$ lifetime difference; this conclusion is fully consistent with the absolute value for $BR_{SL}(D^0)$.
- This year’s precise new experimental result
  \[ \frac{\tau(D_s)}{\tau(D^0)} = 1.211 \pm 0.017 \quad (14) \]
  confirms this picture: WA is nonleading, though still significant. It represents an interesting challenge to find the footprint of WA in some classes of exclusive final states.
- More precise data on the $\Xi_c^{0,+}$ and $\Omega_c$ lifetimes are very much needed. Those might reveal serious deficiencies in the predictions. It should be noted that the semileptonic widths for baryons are not universal!

The scorecard for beauty lifetimes looks as follows:

- The predictions for $\tau(B^-)/\tau(B_d)$ and even more impressively for $\tau(B_c)$ appear to be borne out by experiment.
• The jury is still out on $\tau(B_s)$.

• The $\Lambda_b$ lifetime provides a stiff challenge to theory. It should be noted that most authors have shown a remarkable lack of flexibility in accommodating $\tau(\Lambda_b)/\tau(B_d) < 0.9$, which is quite unusual in this line of work. Maybe experiment will show more flexibility.

• Accurate data on $\tau(\Xi_b^{-,0})$ will be essential to celebrate success or diagnose failure.

Having developed a theoretical framework for treating nonperturbative effects, we can address the issue of duality violations. While we have begun to understand better their origins, we have not (yet) found a model theory that could explain the short $\Lambda_b$ lifetime as a duality limitation.

There are, of course, different layers of failure conceivable and the lessons one would have to draw:

• A refusal by the data to move up the value of $\tau(\Lambda_b)$ could be interpreted as showing that the quark model provides a very inadequate tool to estimate baryonic expectation values.

• A low value of the average $B_s$ lifetime – say $\bar{\tau}(B_s) < 0.96\tau(B_d)$ – had to be seen as a very significant limitation to duality.

Clearly there is a lot we will learn from future data on lifetimes and other inclusive rates – one way or the other.

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