Angular dihadron correlations as interplay of elliptic and triangular flows

G. Eyyubova\textsuperscript{1,2}, V.L. Korotkikh\textsuperscript{2}, I.P. Lokhtin\textsuperscript{2}, S.V. Petrushanko\textsuperscript{2}, A.M. Snigirev\textsuperscript{2}, L.V. Bravina\textsuperscript{3}, E.E. Zabrodin\textsuperscript{2,3}

\textsuperscript{1} Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, CR-11519 Prague, Czech Republic
\textsuperscript{2} Skobeltsyn Institute of Nuclear Physics, Moscow State University, RU-119991 Moscow, Russian Federation
\textsuperscript{3} Department of Physics, University of Oslo, PB 1048 Blindern, N-0316 Oslo, Norway

Abstract

The hybrid model HYDJET++, which combines soft and hard components, is employed for the analysis of dihadron angular correlations measured in PbPb collisions at center-of-mass energy $\sqrt{s_{NN}} = 2.76$ TeV. The model allows for study both individual contributions and mutual influence of lower flow harmonics, $v_2$ and $v_3$, on higher harmonics and dihadron angular correlations. It is shown that the typical structure called ridge in dihadron angular correlations in a broad pseudorapidity range could appear just as interplay of $v_2$ and $v_3$. Central, semi-central and semi-peripheral collisions were investigated. Comparison of model results with the experimental data on dihadron angular correlations is presented for different centralities and transverse momenta $p_T$.

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1. Introduction

The measurement of azimuthal anisotropy and angular correlations of particles is an important tool for exploring properties of matter produced in nucleus-nucleus collisions. For non-central collisions of nuclei the initial azimuthal anisotropic overlap region leads to anisotropies in final particle distribution over azimuth $dN/d\phi$, which is characterized by the coefficients $v_n$ in the Fourier decomposition:

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n(p_T, \eta) \cos n(\phi - \Psi_{RP}),$$

where $\Psi_{RP}$ is the reaction plane of a collision and coefficients $v_n$ depend on transverse momentum $p_T$ and pseudorapidity $\eta$. The two-particle angular correlation function, $C(\Delta\eta, \Delta\phi)$, in relative pseudorapidity $\Delta\eta = \eta^1 - \eta^a$ and azimuth $\Delta\phi = \phi^1 - \phi^a$ is sensitive to collective flow of particles as well as to any other particle correlations in azimuthal angle and pseudorapidity. In the flow dominated regime the pair distribution can be expanded in Fourier series:

$$\frac{dN_{\text{pairs}}}{d\Delta\phi} \propto 1 + 2 \sum_{n=1}^{\infty} V_n(p_T^1, p_T^a) \cos n(\Delta\phi),$$

where superscript indices refer to the two particles in a pair, usually called "trigger" and "associated" ones. The study of dihadron angular correlations in heavy ion collisions has revealed the new phenomena in collision dynamics, the so-called ridge and double-hump structure \cite{1,3}. In order to explain this correlation structure many mechanisms were proposed, such as conical emission from either Mach-cone shock waves \cite{4,5} or Cerenkov gluon radiation \cite{6}, large-angle gluon radiation, jets deflected by radial flow and path-length dependent energy loss (see talks at Quark Matter'08 conference \cite{7}).

In paper \cite{8} the authors suggested that triangular flow might play an important role in the understanding of ridge. Triangular flow, as well as higher flow harmonics, should arise due to initial state fluctuations in a collision geometry. Then, the experiments at the LHC provided us with a new set of amazing results. Particularly, the ridge structure in two-dimensional correlation function was also observed in proton-lead \cite{9,13} and in high multiplicity proton-proton collisions \cite{13}. The origin of the ridge-like structure in $pp$ interactions and its similarity to that in PbPb collisions is still an open question. In $pPb$ reactions triangular flow was measured to be compatible with $v_3$ in PbPb collisions provided the multiplicity was the same. Traditionally proton-nucleus collisions are considered as cold nuclear matter effects, hence, the question is can the azimuthal anisotropy in cold nuclear matter have the the same strength as in hot nuclear matter.
In heavy ion collisions the long-range, i.e. $|\Delta \eta| > 2$, features of the angular dihadron correlations at low and intermediate transverse momenta in central and mid-central collisions were shown to be described with the sum of the Fourier harmonics $v_2 \approx v_3$, found from independent flow analysis methods [3, 14, 15]. This implies that the $V_n$ coefficients in Eq. (2) factorize into two single-particle flow coefficients

$$\frac{dN^{pairs}}{d\Delta \varphi} \propto 1 + 2 \sum_{n=2}^{\infty} v_n(p_T^0)^n (p_T^n) \cos n(\Delta \varphi). \quad (3)$$

The factorization was found to break at higher $p_T$ and also for the first coefficient $V_1$ for all $p_T$ range [3, 15]. What is the origin of this breakage? Also, are all of the six harmonics equally important for the description of long-range correlations?

To answer these questions we employ the HYDJET++ model [16], which merges parametrized hydrodynamics with jets. In addition to hard processes, the unique feature of the model is the possibility to switch on and off the elliptic and triangular harmonics in order to investigate both their individual contributions and the result of mutual interplay to the considered phenomena. In the present work we study the role of $v_2$ and $v_3$ in description of the dihadron correlation function $C(\Delta \eta, \Delta \varphi)$ in lead-lead collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The appearance of higher order harmonics $V_n$, $n > 3$ in correlation function is checked, and obtained results are confronted to the available experimental data.

2. HYDJET++ model

The basic principals of HYDJET++ model are described in a manual [16]. The model combines two components corresponding to soft and hard processes, respectively. The parameter which regulates the contribution of each component to the total event is minimal transverse momentum $p_T^{\text{min}}$ of hard scattering. The partons either produced at or quenched down to the momenta below $p_T^{\text{min}}$ are considered to be thermal ones. Such partons do not contribute to the hard part.

The hard part of the model is based on PYTHIA [17] and PYQUEN [18] generators, which simulate parton-parton collisions, parton radiative energy loss and hadronisation. The soft part of the model has no evolution stage from the initial state until the hadronisation, but rather represents a thermal hadron production already at the freeze-out hypersurface in accordance with the prescriptions of ideal hydrodynamics adapted from the event generator FAST MC [19].

Strength and direction of the elliptic flow $v_2$ are regulated in the HYDJET++ by two parameters. Spatial anisotropy $\epsilon(b)$ represents the elliptic modulation of the final freeze-out hypersurface at a given impact parameter $b$, whereas momentum anisotropy $\delta(b)$ deals with the modulation of flow velocity profile. Additional triangular modulation of freeze-out hypersurface,

$$R(\varphi, b) \propto \sqrt{1 - \epsilon(b)} \left[ 1 + \epsilon_3(b) \cos [3(\varphi + \Psi_3)] \right],$$

produces triangular flow $v_3$ [20, 21]. Here $\epsilon_3$ is the new anisotropy parameter. The reaction plane $\Psi_2$ is fixed to zero and $\Psi_3$ plane is generated randomly on event-by-event basis. Thus, the two planes are uncorrelated in accordance with the experimental data. The recent version of HYDJET++ is tuned to describe data on lead-lead collisions at the LHC energies [20, 21].

3. HYDJET++ and dihadron correlations

The two-particle correlation function is defined as the ratio of pair distribution in the same event (signal) to the combinatorial pair distribution (background), where pairs are not correlated. In experiment the background function is usually constructed with pairs from mixed events. The ATLAS and ALICE collaborations use the following definition [3, 15]:

$$C(\Delta \eta, \Delta \varphi) = \frac{d^2 N^{\text{pair}}}{d\Delta \eta d\Delta \varphi} = \frac{N^{\text{mixed}}}{N^{\text{same}}} \times \frac{d^2 N^{\text{same}} / d\Delta \eta d\Delta \varphi}{d^2 N^{\text{mixed}} / d\Delta \eta d\Delta \varphi}, \quad (4)$$

where $N^{\text{mixed}}$, $N^{\text{same}}$ are the number of pairs in the same event and mixed events, respectively. 1D correlation function $C(\Delta \varphi)$ is obtained by integrating $C(\Delta \eta, \Delta \varphi)$ over pseudorapidity range $\Delta \eta$. Another definition of correlation function is used by CMS collaboration [22]:

$$\frac{1}{N^{tr}} \frac{d^2 N^{\text{pair}}}{d\Delta \eta d\Delta \varphi} = B(0, 0) \times \frac{S(\Delta \eta, \Delta \varphi)}{B(\Delta \eta, \Delta \varphi)}, \quad (5)$$

where $N^{tr}$ is a number of trigger particles, and the signal and background are:

$$S(\Delta \eta, \Delta \varphi) = \frac{1}{N^{tr}} \frac{d^2 N^{\text{same}}}{d\Delta \eta d\Delta \varphi}, \quad B(\Delta \eta, \Delta \varphi) = \frac{1}{N^{tr}} \frac{d^2 N^{\text{mixed}}}{d\Delta \eta d\Delta \varphi}.$$

This definition depends on event multiplicity, since it involves the number of associated particles where the pair of particles comes with approximately the same $\eta$ and $\varphi$ angles, $B(0, 0)$.

The background can be constructed from two single particle spectra, $d^2 N^{tr}/d\eta d\varphi$ and $d^2 N^{\text{same}}/d\eta d\varphi$. Instead of
correlating every two particles in mixed events, one correlates the yields in given two bins. The yield represents the average over many events, therefore, the EbE correlations are washed out and the yield of pairs for background function would be:

\[ B(\Delta \eta, \Delta \varphi) = \int d^2 N_{a}^{\text{tr}} \frac{d^2 N_{a}^{\text{tot}}}{d\eta d\varphi} \delta_{a}^{\text{tr}} d\eta d\varphi d\varphi', \]

where \( \delta_{a}^{\text{tr}} = \delta(\eta^{\text{tr}} - \eta - \Delta \eta)\delta(\varphi^{\text{tr}} - \varphi - \Delta \varphi). \) Due to the absence of any detector effects in the model, spectra \( dN/d\varphi \) as well as a background function \( B(\Delta \varphi) \) should be flat. Thus, for function \( B(\Delta \varphi, \Delta \eta) \) we use only \( dN/d\eta \) distribution and assume flat distribution over \( \Delta \varphi. \)

Fourier harmonics \( V_{n} \) from Eq. (2) are defined directly from the correlation function \( C(\Delta \varphi) \):

\[ V_{n} = \langle \cos(n\Delta \varphi) \rangle = \frac{\sum_{i} C(\Delta \varphi_{i}) \cdot \cos(n\Delta \varphi_{i})}{\sum_{i} C(\Delta \varphi_{i})}. \] (6)

If collective azimuthal anisotropy is the dominant mechanism of the correlation at large \( |\Delta \eta| \), then \( V_{n} \) coefficients would depend on single-particle anisotropies \( v_{n} \) similar to Eq. (5):

\[ V_{n}(p_{T}^{\text{low}}, p_{T}^{\text{low}}) = v_{n}(p_{T}^{\text{low}}) \times v_{n}(p_{T}^{\text{low}}) + \delta_{n}. \] (7)

At low \( p_{T} \) region the non-flow contribution \( \delta_{n} \) is negligible.

In experiment one usually defines the single-particle flow \( v_{n}(2PC) \) via two-particle correlation function using \( v_{n} \) at low \( p_{T} \) as a reference,

\[ v_{n}(2PC)(p_{T}) = \frac{V_{n}(p_{T}, p_{T}^{\text{low}})}{v_{n}(p_{T}^{\text{low}})}, \] (8)

which effectively corresponds to two-particle cumulant method.

Angular dihadron correlations contains all possible types of two-particle correlations. Many sources of two- or many-particle correlations, such as femtoscopic correlations, resonance decays, jets, collective flow, are presented in the model. The long-range correlations over \( \eta \) appear in the model only due to collective flow. The correlation function \( C(\Delta \eta, \Delta \varphi) \) calculated in NYDIE++ in PbPb collision at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) for \( 2 < p_{T} < 4 \text{ GeV}/c \) and \( 1 < p_{T}^{\text{tr}} < 2 \text{ GeV}/c \) is presented in Fig.1 for the cases of (a) absence of collective flow at zero impact parameter, (b) centrality 0-5%, only elliptic flow \( v_{2} \) is turned on and (c) centrality 0-5%, both elliptic and triangular flow are present. Figure [1a] shows that the jet peak is highly suppressed at away-side \( \Delta \varphi \approx \pi \) due to jet quenching. Although remnants of it can be seen over a broad \( \Delta \eta \) range at away-side, no long-range azimuthal correlations are seen at the near-side. The long-range azimuthal correlations start to appear at the near-side in presence of elliptic flow with the characteristic \( \cos(2\Delta \varphi) \) pattern. They are flat in relative pseudorapidity up to \( \Delta \eta \approx 4 \), which corresponds to flat pseudorapidity shape of collective flow in the model. Finally, triangular flow enhances these near-side correlations, often referred to as ridge, and also modifies the away-side structure, producing a double-hump distinctly seen in Fig.[1c].

In HYDIE++ model \( v_{2} \) and \( v_{3} \) anisotropies are introduced at the stage of thermal freeze-out, by means of space modulation of freeze-out volume and additional modulation of flow velocity profile for the elliptic flow only. Thus, the model is insensitive to different origins of anisotropy and to the evolution dynamics from the initial state to the freeze-out stage. It is tuned, however, to describe the coefficients \( v_{2} \) and \( v_{3} \) pretty well at low transverse momenta. The interplay between \( v_{2} \) and \( v_{3} \) in the final state leads to appearance of higher order flow harmonics, which reasonably describe data at mid-central collisions [20].

The results for long-range azimuthal correlations obtained with Eq. (5) for \( 1 < p_{T} < 1.5 \text{ GeV}/c \) and \( 3 < p_{T}^{\text{tr}} < 3.5 \text{ GeV}/c \) in HYDIE++ calculations are plotted in Fig. 2 onto the CMS data [22] for different centralities. Since the correlation function given by Eq. (5) depends on multiplicity of associated particles, it does not always exactly coincide with the model. Therefore, HYDIE++ calculations are shifted on the constant value in such a way that the minima of \( C(\Delta \varphi) \) in data and in the model coincide. In central collisions the model underestimates the data while in peripheral collisions the tendency is opposite. The semi-central collisions are described quite well. To see the role of each of the Fourier coefficients \( V_{n} \) more distinctly, we plot in Figure 3 the values of first five \( V_{n} \) coefficients, calculated for distributions shown in Fig. 2. At very central collisions all the coefficients \( V_{n} \) in the model are lower than that extracted from the data. At semi-central, semi-peripheral and even peripheral collisions all but \( V_{1}, V_{2} \) describe data rather well. At peripheral collisions \( V_{2} \) in the model is much higher than in data. This circumstance reflects the fact that the model does not describe data well on single-particle elliptic flow in the region of intermediate transverse momenta \( 3 < p_{T} < 3.5 \text{ GeV}/c \), see [23].

Note that there is no directed flow \( v_{1} \) in the model, neither pseudorapidity odd \( v_{1} \) nor even \( v_{1} \), which is supposed to come from the initial state fluctuations as discussed in the literature [23]. Nevertheless, the \( V_{1} \) component appears here due to violation of the momentum conserva-
Figure 1: 2D correlation function in hydjet++ in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for $2 < p_T < 4$ GeV/c and $1 < p_T < 2$ GeV/c for (a) central collisions with impact parameter $b = 0$, (b) centrality 0-5% with only elliptic flow, and (c) centrality 0-5% with both elliptic and triangular flow present.

Figure 2: 1D correlation function at $2 < |\Delta \eta| < 4$ in hydjet++ in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for $3 < p_T < 3.5$ GeV/c and $1 < p_T < 1.5$ GeV/c for different centralities in comparison with CMS data [22].

Figure 3: Fourier coefficients $V_n$ extracted for 1D correlation function, presented in Fig. 2 in comparison with CMS data [22].

Figure 4 shows the coefficients $v_2\{2PC\}$, $v_3\{2PC\}$ and $v_4\{2PC\}$ extracted from $V_n$ at $|\Delta \eta| > 2$ by Eq. (7) with $p_T^n = p_T$ in comparison with $v_2$, $v_3$, $v_4$ calculated w.r.t. known reaction plane at generator level. In case of negative $V_n$, the coefficients $v_n\{2PC\}$ are taken as $v_n\{2PC\} = -\sqrt{|V_n|}$. Comparison is presented for two centralities, 0-5% and 30-35%. One can see that in the range of $p_T < 3.5$ GeV/c the 2PC-method describes the $v_n$ coefficients pretty well. This means that $V_n$ coefficients factorize in this region into a product of two singular flow coefficients, and the collective flow is the dominant source of correlation. At higher transverse momenta, $p_T > 3.5$ GeV/c, the non-flow contri-
at low to reaction plane. It shows that the substantial contribution of the jet component dominates. This contribution is negative for odd coefficients \( v_n \{2PC\} \).

It is worth noting that higher order coefficients \( V_n \), \( n > 4 \) also appear in the model in \( C(\Delta \varphi) \) decomposition, though they decrease rapidly with \( n \), as shown in Fig. 4. These coefficients at low \( p_T \) can only originate from the lower order flow harmonics, \( v_2 \) and \( v_3 \). Figure 6 depicts pentagonal flow, \( v_5 \{2PC\} \), obtained by Eq. (7) at different centralities. The result is compared to the product \( v_2(p_T) \times v_3(p_T) \), obtained at generator level with known reaction plane. It shows that the substantial contribution to \( V_5 \) comes from \( v_2 \) and \( v_3 \) harmonics at all centralities at low \( p_T \), in full accord with the theoretical estimates [26, 27].

4. Conclusions

In contrast to more sophisticated hydrodynamic models, in which the initial-state fluctuations and the subsequent hydrodynamic expansion give rise to infinite spectrum of Fourier components in the azimuthal particle correlations, the HYDJet++ model allows us to study the influence of a single harmonic, such as \( v_2 \) or \( v_3 \), as well as their interplay, on the final particle azimuthal distributions. This is the ideal situation, where all genuine higher-order initial fluctuations, which can distort the signal, are simply switched off. Elliptic flow component contributes to all even harmonics of higher order, whereas the interplay of \( v_2 \) and \( v_3 \) leads to appearance of odd harmonics in the model. In the present paper we got the clear evidence that this mechanism allows one to describe also the dihadron correlations, including the double-hump structure, at mid-central collisions, where collective flow dominates over fluctuations. The measured amplitude of the ridge at mid-central collisions is well described by a superposition of elliptic and triangular flows. This is the main result of the paper.

Also, for pairs of particles with a large pseudorapidity gap (|\( \Delta \eta \)| > 2) in a range of transverse momenta \( p_T < 3.5 \) GeV, the coefficients \( V_2 \), \( V_3 \) and \( V_4 \) are found to factorize into the product of corresponding collective flow coefficients \( v_n \) calculated in the model with known reaction plane. Pentagonal coefficient, \( v_5 \{2PC\} \), extracted from the dihadron correlation function follows approximately the scaling condition \( v_5 \{2PC\} \propto v_2 \times v_3 \) at low \( p_T \) only.

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References

[1] A. Adare et al. (PHENIX Collaboration), Phys. Rev. C 78 (2008) 014901.
[2] M.M. Aggarwal et al. (STAR Collaboration), Phys. Rev. C 82 (2010) 024912.
[3] K. Aamodt et al. (ALICE Collaboration), Phys. Rev. Lett. 107 (2011) 032301.
[4] H. Stoecker, Nucl. Phys. A 750 (2005) 121.
[5] J. Casalderrey-Solana, E. Shuryak, D. Teaney, J. Phys. Conf. Ser. 27 (2005) 22.
[6] I.M. Dremin, Nucl. Phys. A 767 (2006) 233.
[7] Proceedings of Quark Matter 2008, J. Phys. G 35 (2008) 104001-104167.
[8] B. Alver, G. Roland, Phys. Rev. C 81 (2010) 054905 [Erratum-ibid. C 82 (2010) 039903].
[9] S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B 718 (2013) 795.
[10] G. Aad et al. (ATLAS Collaboration), Phys. Rev. Lett. 110 (2013) 182302.
[11] B. Abelev et al. (ALICE Collaboration), Phys. Lett. B 719 (2013) 29.
[12] S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B 724 (2013) 213.
[13] V. Khachatryan et al. (CMS Collaboration), JHEP 1009 (2010) 091.
[14] K. Aamodt et al. (ALICE Collaboration), Phys. Let. B 708 249 (2012).
[15] G. Aad et al. (ATLAS Collaboration), Phys. Rev. C 86 (2012) 014907.
[16] I.P. Lokhtin et al., Comput. Phys. Commun. 180 (2009) 779.
[17] T. Sjostrand, S. Mrenna, P. Skands, JHEP 0605 (2006) 026.
[18] I.P. Lokhtin, A.M. Snigirev, Eur. Phys. J. C 45 (2006) 211.
[19] N.S. Amelin et al., Phys. Rev. C 77 (2008) 014903.
[20] L.V. Bravina et al., Eur. Phys. J. C 74 (2014) 2807.
[21] L.V. Bravina et al., Phys. Rev. C 89 (2014) 024909.
[22] S. Chatrchyan et al. (CMS Collaboration), Eur. Phys. J. C 72 (2012) 2012.
[23] D. Teaney, L. Yan, Phys. Rev. C 83 (2011) 064904.
[24] N. Borghini, P.M. Dinh, J.-Y. Ollitrault, Phys. Rev. C 62 (2000) 034902.
[25] S. Chatrchyan et al. (CMS Collaboration), JHEP 1107 (2011) 076.
[26] F.G. Gardim, F. Grassi, M. Luzum, J.-Y. Ollitrault, Phys. Rev. C 85 (2012) 024908.
[27] D. Teaney, L. Yan, Phys. Rev. C 86 (2012) 044908.