Experimental Status of Semileptonic $B$ Decays
Recent Results from LEP and Comparisons with $\Upsilon(4S)$ Experiments

M. Battaglia

$^a$Dept. of Physics, University of Helsinki, FIN-00014 Helsinki, Finland

Recent analyses of the LEP and $\Upsilon(4S)$ data have better outlined the picture of semileptonic $B$ decays. Results on inclusive and exclusive decay branching fractions and on the extraction of the $|V_{ub}|$ and $|V_{cb}|$ elements of the CKM mixing matrix are discussed, together with some of the still open questions and the sources of model systematics.

1. INTRODUCTION

Semileptonic (s.l.) $B$ decays represent a favorable laboratory to study the dynamics of heavy quark decays, to determine the size of the $|V_{ub}|$ and $|V_{cb}|$ CKM mixing matrix elements, crucial in the unitarity triangle test of the Standard Model, and to acquire informations on the structure of the $B$ meson itself.

The Aleph, Delphi, L3 and Opal experiments have analyzed together about 1.5 M s.l. $B$ decays recorded at LEP, at energies around the $Z^0$ pole from 1990 to 1995. This statistics is significantly lower than about 4 M s.l. decays recorded by CLEO and 7.5 M already logged by the BaBar and Belle experiments. However, the significant boost of the beauty hadrons, the confinement of their decay products in well separated jets and the production of almost all beauty hadron species make it possible to perform topological decay reconstruction with good efficiency on the LEP data sets, thus exploiting analysis techniques complementary to those used at lower energy $e^+e^-$ colliders. Since operator product expansion (OPE) predictions apply to sufficiently inclusive observables, it is advantageous to reconstruct s.l. decays inclusively, with only mild cuts on the lepton and hadron energies. Further, the LEP kinematics allow the two extreme kinematical regions at small and large momentum transfer to be accessed, since the $B$ decay products gain enough energy from the $B$ boost.

Section 2 summarizes recent results on inclusive and exclusive s.l. branching fractions obtained by the LEP experiments. These results have been averaged and compared with those from experiments running at the $\Upsilon(4S)$. Section 3 discusses the extraction of the $|V_{ub}|$ and $|V_{cb}|$ elements from inclusive and exclusive s.l. yields. Section 4 reviews some open questions in s.l. decays and model systematics presently limiting the precision in the interpretation of the data.

2. RECENT RESULTS

2.1. Inclusive s.l. Branching Fraction

The determination of the inclusive s.l. branching fraction $BR(b \to X$\(\ell\bar{\nu}\)) is important for the measurement of $|V_{cb}|$. Together with the measurement of the charm multiplicity and of exclusive decays discussed later in this Section, they provide with a test of the s.l. decay width. The main experimental issue here is the separation of the prompt $b \to \ell$ signal from the cascade $b \to c(c\bar{c}) \to \ell(\ell)$ and the $c \to \bar{\ell}$ backgrounds. This has been usually achieved by performing a fit to the lepton $p$ and $p_t$ distributions to profit from the harder spectra of the prompt signal. However, this method is limited by the model dependence in the description of the prompt and cascade spectra, and alternative techniques have recently been exploited to improve the signal sep-
aration in a more model-independent way. These analyses use the charge correlation between the $b$ and the lepton, the decay topology, and double tagged events where both beauty hadrons decay semileptonically. The dominant source of uncertainty remains, however, that due to the modeling of the $b \to \ell$ and $c \to \ell$ spectra. A global fit to the LEP and SLD electro-weak data, including also $R_b$, $R_c$ and asymmetry determinations by analyses using leptons and other independent methods, constrains the s.l. decay models, thus reducing the related systematics by about 20% \cite{1}. However, this depends on the relative size of the systematic uncertainties assigned to different asymmetry measurements, and a more conservative approach is to derive the LEP average from the direct determinations. This gives $\text{BR}(b \to X\ell\bar{\nu}) = 0.1056 \pm 0.0011 \text{ (stat)} \pm 0.0018 \text{ (syst)}$. After having rescaled this value by $1/2(\tau(b) + \tau(B_u)) = 1.021 \pm 0.013$, to account for the different beauty hadron species produced, this result can be compared with the most recent determination at the $\Upsilon(4S)$, obtained by CLEO using a lepton tagged method to separate the prompt lepton yield in $B$ decays from the backgrounds giving $\text{BR}(b \to X\ell\bar{\nu}) = 0.1049 \pm 0.0017 \text{ (stat)} \pm 0.0043 \text{ (syst)}$ \cite{2}.

It is useful to analyze these results in relation with the average number of charm particles in $B$ decays, $n_c$, since the semileptonic, double charmed and charmless yields are correlated once the total decay width is fixed. There are three main sources of data on $n_c$. The results and their average are summarized in Table 1. The first is based on the determination of the $B$ decay branching ratios into individual charm hadrons $\text{BR}(B \to D^+, D^0, D_s^+, \Lambda_c^+ + X)$ and their antiparticles.

| Method         | LEP     | CLEO     |
|----------------|---------|----------|
| Charm counting | 1.178 ± 0.074 | 1.132 ± 0.062 |
| Wrong sign $D$ | 1.293 ± 0.075 | 1.174 ± 0.053 |
| Topology       | 1.151 ± 0.047 | -        |
| Average        | 1.171 ± 0.040 | 1.159 ± 0.049 |

These rates have been determined at LEP and by CLEO and need to be further corrected to include the strange charm baryon and charmonium production in $B$ decays. A second approach is to express $n_c$ as $n_c = 1 + 2 \times \text{BR}(b \to c\bar{s}s) - \text{BR}(b \to charmless)$ and to determine $\text{BR}(b \to c\bar{s}s)$ more inclusively from $\text{BR}(b \to J/\Psi X)$ and $\text{BR}(b \to DDX)$. There have been several recent results on the decay branching fractions into open double charm $DDX$ or wrong sign $D(DX)$ at LEP, by CLEO and also by BABAR. Finally a third method, exploited by DELPHI and SLD, consists in extracting the double charm and charmless yield from the $B$ decay topology. The scaled LEP averages and the CLEO values can be independently compared with theoretical predictions, obtained using heavy quark expansion to order $1/m_c^2$. These are shown in Figure 1 for a range of values of the renormalization scale $\mu$ and of the ratio of the quark pole masses $m_c/m_b$ \cite{3}. The experimental results agree with these predictions at $\mu \simeq 0.25$ and $m_c/m_b \simeq 0.33$.

### 2.2. Charmless s.l. Branching Fraction

An accurate determination of the charmless s.l. $B$ branching fraction is important for the measurement of the $|V_{ub}|$ element and has represented a significant challenge to both theorists and experiments since CLEO first observed charmless s.l. decays as an excess of leptons at energies above...
the kinematical limit for decays with an accompanying charm hadron \( |V_{ub}| \). However, the limited fraction (\( \approx 10\% \)) of charmless decays populating this end-point region results in a significant model uncertainty for the extraction of \( |V_{ub}| \). An analysis technique based on the invariant mass \( M_X \) of the hadronic system recoiling against the lepton pair, peaked for \( b \rightarrow X_u \ell \bar{\nu} \) at a significantly lower value than for \( b \rightarrow X_c \ell \nu \), was proposed several years ago \(^1\) and it has been the subject of new theoretical calculations \(^{15–17}\). If \( b \rightarrow u \) transitions can be discriminated from the dominant \( b \rightarrow c \) background up to \( M_X \approx M(D) \), this method is sensitive to \( \approx 80\% \) of the charmless s.l. \( B \) decay rate. Further, if no preferential weight is given to low mass states in the event selection, the non-perturbative effects are expected to be small and the OPE description of the transition has been shown to be accurate away from the resonance region. The requirement to isolate the \( b \rightarrow u \) contribution to the s.l. yield from the \( \approx 60 \) times larger \( b \rightarrow c \) one, while ensuring an uniform sampling of the decay phase space to avoid biases towards the few exclusive low-mass, low-multiplicity states, make this analysis a major experimental challenge. \textsc{Aleph} \(^{18}\), \textsc{Delphi} \(^{19}\) and \textsc{L3} \(^{20}\) have developed new analysis techniques based on the observation that \( b \rightarrow X_u \ell \bar{\nu} \) decays can be inclusively selected from \( b \rightarrow X_c \ell \bar{\nu} \) by the difference in the invariant mass and kaon content of the secondary hadronic system and in the decay multiplicity and vertex topology. These features have been implemented differently in the analyses by the three experiments: \textsc{Aleph} used a neural net discriminant based on kinematical variables, \textsc{Delphi} preferred a classification of s.l. decays on the basis of their reconstructed hadronic mass \( M_X \), decay topology and presence of secondary kaons and \textsc{L3} adopted a sequential cut analysis based on the kinematics of the two leading hadrons in the same hemisphere as the tagged lepton. Starting from a natural signal-to-background ratio S/B of about 0.02, \textsc{Aleph} obtained S/B = 0.07 with an efficiency \( \epsilon = 11\% \), \textsc{Delphi} had S/B = 0.10 with \( \epsilon = 6.5\% \) and \textsc{L3} had S/B = 0.16 with \( \epsilon = 1.5\% \).

All three experiments observed a significant data excess over the estimated backgrounds corresponding to 303 \( \pm \) 88 events for \textsc{Aleph}, 214 \( \pm \) 56 for \textsc{Delphi} and 81 \( \pm \) 25 for \textsc{L3}. The characteristics of these events correspond to those expected for \( b \rightarrow X_u \ell \bar{\nu} \) decays (see Figure 2). The inclusive charmless s.l. branching ratios summarized in Table 2 were obtained. These results are compatible within their uncorrelated uncertainties. The differences in the analysis techniques resulted in systematic uncertainties that are only partially correlated. The \textsc{LeP} average value was therefore computed, resulting in \( \text{BR}(b \rightarrow X_u \ell \bar{\nu}) = (1.74 \pm 0.37 \text{ (stat.+exp.)} \pm 0.38 \text{ (b \rightarrow c)} \pm 0.21 \text{ (b \rightarrow u)} \times 10^{-3} = (1.74 \pm 0.57) \times 10^{-3} \) \(^{21}\).

The \textsc{LeP} experiments have shown the feasibility of these inclusive analyses, due to the favorable kinematics and the decay reconstruction capabilities of their detectors. The exact size of the systematic uncertainties attached to the inclusive analyses is still being debated within the community. While more precise data on \( B \) and \( D \) decays will decrease the dominant \( b \rightarrow c \) systematics of this measurement, future perspectives for inclusive charmless s.l. rate determinations are not yet clearly outlined. The hadronic mass analysis puts even further problems to symmetric and asym-
metric $B$ factories at the $\Upsilon(4S)$ peak, due to the confusion between the decay products of the $B$ and $\bar{B}$ and of the reduced $B$ decay length, compared with LEP. A feasibility study, performed by BABAR, requires full reconstruction of one $B$ meson through an exclusive decay mode to solve the first problem, and predicts a reconstructed signal sample with $M_X < 1.7$ GeV/$c^2$ of about 100 to 300 events for the full data statistics of 100 fb$^{-1}$ [22]. A new technique based on the reconstruction of the di-lepton $\ell\bar{\nu}$ invariant mass of the decay has been proposed recently [23]. While this method is sensitive to $\approx 20\%$ of the inclusive charmless rate, above the $b \to c$ threshold, it should provide a good control of theoretical uncertainties. At low-energy $B$ factories, the di-lepton mass measurement will profit from the $\nu$ reconstruction techniques that rely on the $B$ production at threshold, already successfully exploited by CLEO.

2.3. Exclusive s.l. $B$ Meson Decays

Exclusive s.l. $B$ decays have been studied at the $\Upsilon(4S)$ and at LEP, to establish the relative contribution of the individual channels to the s.l. decay width and to extract the relevant CKM elements. Charmless $B \to \pi \ell\nu$ and $\rho \ell\nu$ decays have been observed by CLEO and their rates measured [26]. Candidates for these exclusive decays have also been isolated by ALEPH [18] and DELPHI [19], from the inclusive analyses, but their statistics are insufficient for a rate determination with interesting accuracy.

Competitive accuracies have been obtained at LEP for charmed exclusive decay modes. The decay $B_d^0 \to D^{*+}\ell^-\bar{\nu}$ has received specific attention for the extraction of $|V_{cb}|$ from a study of its rate $\frac{d\Gamma}{dw} = K(w)F^2(w)|V_{cb}|^2$ as a function of the the $D^*$ and $B_d$ four-velocity product $w$. In this expression $K(w)$ is a phase space factor and $F(w)$ the hadronic form factor. The reconstruction of the decay can be performed fully exclusively, through the decay $D^{*+} \to D^0\pi^{+}$ followed by $D^0 \to K^-\pi^+$ and, at LEP, also by partial inclusive reconstruction of the $D$ meson (see Figure 3) resulting in a significant increase in the selection efficiency. At LEP, decays can be ef-

Table 2

Summary of the LEP $BR(b \to X_u\ell\bar{\nu})$ results with the sources of the statistical, experimental, uncorrelated and correlated systematic uncertainties.

| Experiment | $BR(b \to X_u\ell\bar{\nu})$ | (stat.) | (exp.) | (uncorrelated) | (correlated) |
|------------|-------------------------------|---------|--------|----------------|--------------|
| ALEPH [18] | (1.73 ± 0.48 ± 0.29 ± 0.29 (±0.29 $b \to c$) ± 0.47 (±0.43 $b \to u$)) $\times 10^{-3}$ |
| DELPHI [19] | (1.69 ± 0.37 ± 0.39 ± 0.18 (±0.13 $b \to c$) ± 0.42 (±0.34 $b \to c$)) $\times 10^{-3}$ |
| L3 [20] | (3.30 ± 1.00 ± 0.80 ± 0.68 (±0.68 $b \to c$) ± 1.40 (±1.29 $b \to c$)) $\times 10^{-3}$ |

Figure 3. The reconstructed $\Delta M = M(D\pi) - M(D)$ distribution for selected opposite (left) and same (right) sign lepton - pion combinations for the OPAL analysis. The $D^{*+}$ signal is visible in the opposite sign sample. In this inclusive analysis $M(D)$ is approximated by the invariant mass of the partially reconstructed $D$ decay.
efficiently reconstructed closer to zero $D^{*+}$ recoil energy than at the $\Upsilon(4S)$. But using exclusive reconstruction and at the $\Upsilon(4S)$, where the $B$ meson is almost at rest, better resolution on $w$ is achieved: CLEO obtains $\sigma(w) = 0.03$ compared with $\sigma(w) = 0.07$ to 0.12 from the inclusive LEP analyses (see Figure 4). The main background factor $F(w)$ is based on a dispersion-relation parameterization. By combining the measurements by ALEPH, DELPHI and OPAL, the LEP averages $F(1)|V_{cb}| = (34.5 \pm 0.7 \text{(stat)} \pm 1.5 \text{(syst}) \times 10^{-3}$ and $\rho^2 = 1.13 \pm 0.08 \text{(stat)} \pm 0.16 \text{(syst)}$ were obtained with a fit confidence level of 12% (see Figure 5). CLEO recently reported $F(1)|V_{cb}| = (42.4 \pm 1.8 \text{(stat)} \pm 1.9 \text{(syst}) \times 10^{-3}$ and $\rho^2 = 1.67 \pm 0.11 \text{(stat)} \pm 0.22 \text{(syst)}$ that gives 2.4 $\sigma$ higher extrapolated rate at $w = 1$ with a larger slope $\frac{\Delta w}{\Delta M}$. Since the intercept at $w = 1$ and the slope are highly correlated, this prompts further investigation of this decay through new analyses of the experimental data and of the underlying uncertainties.

![Figure 4](image_url)  
**Figure 4.** The differential rate $dN/dw$ for $B_d^0 \rightarrow D^{*+}\ell^{-}\bar{\nu}$ events for the DELPHI inclusive analysis (left) and the CLEO exclusive analysis (right).

![Figure 5](image_url)  
**Figure 5.** The $F(1)|V_{cb}|$ and $\rho^2$ determinations at LEP, corrected by the LEP $|V_{cb}|$ Working Group for a consistent set of input parameters, and the resulting LEP average. The ellipses indicate the 65% C.L. of each result.

![Figure 6](image_url)  
**Figure 6.** The invariant mass difference in the decays $\bar{B} \rightarrow D^{*+}\pi^{-}\ell^{-}X$, $D^0\pi^+\ell^{-}X$ and $D^+\pi^{-}\ell^{-}X$. The data are represented by the dots with error bars, the solid open histogram is the result of a fit to the data including narrow $D_1(2420)$ and $D_2^*(2460)$ states, broad $D^\ast\pi$ and $D\pi$ states (dashed histogram), fake $D$ (cross-hatched histogram) and fragmentation particle (hatched histogram) backgrounds.

The study of s.l. $B$ decays into orbitally and radially excited charmed mesons has established the production of the narrow orbital excitation $D_1(2420)$, while the $D_2^*(2460)$ and a broad state decaying into $D^{*+}\pi^-$ have been observed by CLEO in hadronic $B$ decays.
Table 3
Summary of the results on s.l. decays into excited charmed states.

| Experiment | BR($B \to D_1(2420)\bar{\ell}\bar{\nu}$) | BR($B \to D'_2(2460)\bar{\ell}\bar{\nu}$) | BR($B \to D^{*+}_{(0,1)}$ (broad)$\bar{\ell}\bar{\nu}$) |
|------------|--------------------------------------|--------------------------------------|--------------------------------------|
| ALEPH [3]  | $(7.0 \pm 1.1 \pm 1.2) \times 10^{-3}$ | $(2.4 \pm 1.0 \pm 0.5) \times 10^{-3}$ |                                                |
| DELPHI [3] | $(6.7 \pm 1.8 \pm 1.0) \times 10^{-3}$ | $(4.4 \pm 2.1 \pm 1.2) \times 10^{-3}$ | $(22.9 \pm 5.9 \pm 3.6) \times 10^{-3}$ |
| CLEO [3]  | $(5.6 \pm 1.3 \pm 0.9) \times 10^{-3}$ | $(3.0 \pm 3.3 \pm 0.8) \times 10^{-3}$ |                                                |

A recent analysis by DELPHI [3] has separately measured the narrow $D_1(2420)$ and broad, or non-resonant, $D^{(*)}\pi$ production (see Figure 3). Results are summarized in Table 3. The $D'_2(2460)$ production rate is less than that of $D_1(2420)$: $R^* = \frac{BR(B \to D_1^*(\bar{e}\bar{\nu}))}{BR(B \to D'_2(\bar{e}\bar{\nu}))} < 0.6$ in disagreement with the HQET prediction in the infinitely heavy quark mass limit $R^* \simeq 1.6$ [10], thus implying large $1/m_Q$ corrections [11]. Including these corrections restores agreement with the data for $R^*$, however the overall yield of the broad $D'_0 + D'_1$ states is not expected to exceed that for $D_1(2420)$. This apparent discrepancy with the preliminary DELPHI result can now be explained as an experimental effect, if the feed-through of non-resonant $D^*\pi$ production [12] is non-negligible.

A first estimate of the slope of the $\Lambda_b$ form factor has also been obtained at LEP. DELPHI performed a fit to the reconstructed $w$ distribution and event rate for a sample of candidate $\Lambda_b \to \Lambda_c \bar{\ell}\bar{\nu}$ decays obtaining $\rho^2 = 1.55 \pm 0.60$ (stat) $\pm 0.55$ (syst) [13], which is within the range of the current theoretical predictions.

3. EXTRACTION OF $|V_{ub}|$ AND $|V_{cb}|$

3.1. $|V_{ub}|$

The value of the $|V_{ub}|$ element can be extracted from the inclusive charmless s.l. branching fraction $BR(b \to X_u\bar{\ell}\bar{\nu})$ by using the following relationship derived in the context of Heavy Quark Expansion [24,25]:

$$|V_{ub}| = 0.00445 \left( \frac{BR(b \to X_u\bar{\ell}\bar{\nu})}{0.002} \right) \frac{1.55\text{ps}}{\tau_b} \times (1 \pm 0.20(\text{QCD}) \pm 0.035(m_b))$$

where the value $m_b = (4.58 \pm 0.06)$ GeV$/c^2$ has been assumed [26]. The theoretical uncertainties are small, due to the absence of $1/m_b$ term in the expansion and of $1/(m_b - m_c)$ dependence, and it is dominated by that on the $b$ mass. Inserting the LEP average $BR(b \to X_u\bar{\ell}\bar{\nu})$, $|V_{ub}|$ was determined to be:

$$|V_{ub}| = (4.13^{+0.42}_{-0.47} \text{(stat. + det.)}^{+0.43}_{-0.48} \text{(b \to c syst.)}^{+0.24}_{-0.25} \text{(b \to u syst.)} \pm 0.02(\tau_b) \pm 0.20(\text{HQE})) \times 10^{-3}.$$ 

The $b \to u$ and HQE model systematics is slightly below 10% and the large uncertainties from the modeling of $b \to c$ decays can be reduced in future by more precise data. This inclusive determination of $|V_{ub}|$ can be compared to that extracted from the determination of the exclusive rate for the decay $B \to \rho\ell\bar{\nu}$ by CLEO giving $|V_{ub}| = (3.25 \pm 0.14 \text{ (stat.)}^{+0.21}_{-0.29} \text{ (syst.)} \pm 0.55 \text{ (model)}) \times 10^{-3}$ [27]. The two measurements are consistent within their uncertainties, which are mostly uncorrelated. It is expected that the large model dependence of the exclusive method will be reduced by computing the hadronic form factor from lattice QCD.

3.2. $|V_{cb}|$

There are two methods, to extract the $|V_{cb}|$ element from s.l. decays: i) from the inclusive s.l. $B$ decay width, once the charmless contribution has been subtracted, and ii) by the extrapolation of the rate for exclusive decays, as $B \to D^*\bar{\ell}\bar{\nu}$, at zero recoil. The two results can be used as a check of the underlying theory and to improve the $|V_{cb}|$ accuracy by their averaging, the systematic
uncertainties being partially uncorrelated.

For the inclusive method the relationship:

\[ |V_{cb}| = 0.0411 \times \left( \frac{BR(b \rightarrow X_e \ell^\nu)}{1.55 \text{ps}} \times \frac{1.05}{\tau_b} \right)^{1/2} \times (1 - 0.024 \times (\mu_c^2 - 0.5/0.1)) \times (1 \pm 0.030(\text{pert.}) \pm 0.020(m_b) \pm 0.024(1/m_b^3)) \]

can be used to extract \(|V_{cb}|\) [27]. By taking the inclusive and charmless s.l. branching fractions presented above and the average b lifetime \(\tau_b = 1.564 \pm 0.014\) ps, the result is \(|V_{cb}| = (40.7 \pm 0.5 (\text{exp}) \pm 2.0 (\text{th})) \times 10^{-3}\). This inclusive method is limited by the lepton spectrum model systematics from the s.l. branching ratio and by the theory systematics due to the values of \(m_b\) and \(\mu_c^2\) (or \(-\lambda_1\)) and to the use of only the first terms of the operator product expansion as highlighted in Eq. (2).

In the exclusive method, the quantity \(|V_{cb}| \times F(1)\) is determined from the differential decay rate extrapolated to zero recoil. The extraction of \(|V_{cb}|\) relies on heavy quark symmetry for the form factor normalization \(F(1) = 1\) in the heavy quark limit and requires the computation of the finite b-quark mass effect and of non-perturbative QCD corrections. The value \(F(1) = 0.880 - 0.024 \pm 0.010\) (pert) \pm 0.025 (1/m_b^2) has been adopted by the LEP working group [27] giving \(|V_{cb}| = (39.8 \pm 1.8 (\text{exp}) \pm 2.2 (\text{th})) \times 10^{-3}\). This result is in agreement with that obtained with the inclusive method. There has been a recent lattice determination of \(F(1)\); unquenched computations will provide smaller and better understood systematics. However the larger value obtained by the recent CLEO analysis suggests a cautious attitude and the need for more experimental data.

4. OPEN QUESTIONS AND MODEL SYSTEMATICS

As more data on s.l. b decays have become available and the analysis techniques improved, the determinations of basic parameters of b decays, such as the \([V_{ub}]\) and \([V_{cb}]\) elements, are limited by theoretical uncertainties in their derivation from the experimental observables and by models in the description of the signal and backgrounds [28]. While this is promoting new phenomenological approaches, it also requires new measurements to better determine the input parameters and further constrain the models. This interplay between progresses in theory and new experimental insights addresses three main classes of questions: i) the definition of a coherent method to include bound state effects in the description of inclusive decays, ii) the accurate determination of the fundamental parameters, \(m_b\), \(m_b - m_c\) and \(p_b^2\) (or \(-\lambda_1\)), and iii) the estimate of the effects of the missing terms in the OPE and of violations of quark-hadron duality. The present experimental data and the theory predictions have already significantly progressed towards answers to these questions.

At present, inclusive spectra are predicted by a variety of specialized models ranging from fully inclusive models, such as the ACCMM model [30], to those saturating the inclusive decay width by the contribution of several exclusive final states, like the ISGW and ISGW-2 models [31].

![Figure 7. The lepton energy distribution, unfolded for detector effects, measured by L3 [33] with overlayed the spectra predicted by the ACCMM model and the ISGW model with 24% of D^{**} contribution](image-url)
tra are obtained from a free quark decaying into partons and the non-perturbative are corrections included by convoluting the parton spectra with a function encoding the kinematics of the b and spectator q quarks inside the heavy hadron [29]. The ACCMM model describes the B hadron as consisting of the b and spectator q quarks, moving back-to-back in the B rest frame, with a momentum p distributed according to a Gaussian distribution. The Fermi motion width pF represents a free parameter in the model. The value of pF is proportional to the average kinetic energy of the b quark in the hadron \( < p_b^2 > = \frac{3}{2} \bar{p}_F^2 \) and can be derived from the shapes of inclusive observables (see Figure 7).

![Figure 8. Fermi motion pF vs. charm mass mc obtained from fits to lepton spectra for the ACCMM model. The fit to CLEO data was repeated for different fixed values of mc [22], L3 used a single mc value [24] while OPAL performed a two-parameter fit [34].](image)

Figure 8 summarizes the results from fits to the energy spectrum of the lepton in s.l. \( b \to X\ell\bar{\nu} \) decays from LEP and CLEO data, showing a good consistency. The main issue here is to establish the validity of the ACCMM model, with these fitted values, to other decays, such as \( b \to X_u\ell\bar{\nu} \) and \( b \to s\gamma \).

This difficulty may be overcome by a description of the Fermi motion in the framework of QCD. This introduces a shape function, \( f(k_+ \rangle \), corresponding to the distribution of the light-cone residual momentum of the heavy quark inside the hadron [35]. At leading order and in the large \( m_b \) limit, the light-cone residual momentum \( k_+ \) can be expressed as the difference between the b quark mass and its effective mass \( m_b^* \) inside the hadron: \( m_b^* - m_b + k_+ \). However, the functional form of the function \( f(k_+) \) is not a priori known, except for its first three moments. The effect of different ansatz for the description of the Fermi motion of the b quark inside the heavy hadron has been studied in the case of \( b \to X_u\ell\bar{\nu} \) both at the parton level [36,41] and for the physical observables after full detector simulation [35,39]. It was found that the Fermi motion description contributes with an uncertainty on the branching fraction determination of \( \pm 5 - 15\% \), increasing as the hadronic mass \( M_X \) cut to distinguish \( b \to u \) from \( b \to c \) transitions is lowered and depending also on the other selection criteria.

Since \( m_b \) enters at the fifth power in the expression of the decay width, \( \Gamma \propto m_b^5 |V_{CKM}|^2 \), it is important to determine its value with an accuracy of better than \( \pm 100 \text{ MeV}/c^2 \) to guarantee a few percent error contribution in the extraction of \( |V_{ub}| \) and \( |V_{cb}| \) from inclusive decays. The dependence of the shape of inclusive spectra on the b quark mass represents a further source of systematics from \( m_b \). Recent estimates from \( \Upsilon \) spectroscopy, QCD sum rules and NNLO unquenched lattice computations have shown a remarkable agreement [38]. On the basis of the results [39], the LEP working groups have adopted \( m_b(1\text{GeV}) = (4.58 \pm 0.06) \text{ GeV}/c^2 \), where \( m_b(\mu) \) is defined such as \( \frac{dm_b(\mu)}{d\mu} = -\frac{16}{9} \frac{\alpha_s(\mu)}{\pi} + ... \) and the quoted uncertainty defines a 68% confidence region. The size of this error is still a topic of some controversy and its definition will have a significant impact on the accuracy of the determination of \( |V_{ub}| \) and \( |V_{cb}| \).

These uncertainties could be reduced by extracting information on the shape function and the quark masses from the s.l. decay spectra themselves. Due to its high statistics, the inclusive \( b \to X_u\ell\bar{\nu} \) decay is the obvious candidate. Since the shape function is expected to be universal in B decays, these data could in turn be...
compared with that extracted from other decays and used to constrain models and parameters for inclusive, rare decays such as $B \rightarrow X_u \ell \bar{\nu}$ and $b \rightarrow s \gamma$. This analysis appears to be most advantageous for experiments at the $\Upsilon(4S)$ since the small $B$ boost minimizes the finite resolution effects in converting from the lab frame to the $B$ rest frame. On the other hand the lower energy cut-off for lepton identification, at $E_\ell > 0.6$ GeV/$c$, introduces a model dependence when extrapolating to zero lepton energy. The first moments of the lepton energy and hadronic mass spectra can be used to determine $\Lambda = m_B - m_\ell + \ldots$ and $\lambda_3$ $^{[1]}$. An analysis of the CLEO lepton energy spectrum limited to the region $E_\ell > 1.5$ GeV gave $\Lambda = (0.39 \pm 0.11)$ GeV and $-\lambda_1 = (0.19 \pm 0.10)$ GeV$^2$ $^{[2]}$. CLEO reported the preliminary results from the first combined study of the first two moments of the hadronic mass and the lepton energy spectra $^{[3]}$. The hadronic mass moments were found to be in good agreement with the previous result, while the lepton energy spectrum gave unlikely and incompatible values. However, the model dependence in the extrapolation to the full spectrum and the unknown contributions of higher order terms $1/m^3$ prevent the derivation of any conclusions from these disagreements and highlight the importance of further data. A first lepton energy spectrum for $B \rightarrow X_u \ell \bar{\nu}$ has been extracted by the DELPHI analysis after unfolding of the detector effects. The available statistics is still too small for a significant test that may become possible after combining with the data from the $B$ factories.

The predictions for the inclusive analyses presented before and the extraction of the CKM matrix elements both rely on the basic assumption of duality, i.e. that the rates computed at the parton level correspond to those for the physical final states, after integrating enough hadronic channels. The validity of this assumption has been studied in a QCD model at the large $N_c$ and small velocity limit $^{[4]}$ and in the $(1+1)$ dimension ‘t Hooft model $^{[5]}$. It was found that the sum over exclusive channels corresponds to the inclusive OPE prediction to a very good accuracy soon after having integrated the first resonant states. It is interesting to comment here on the consistency of the determinations of $|V_{ub}|$ and $|V_{cb}|$ with inclusive and exclusive methods discussed above. The agreement between the measured values is a test of the accuracy of the theoretical methods and also of the validity of the quark-hadron duality assumption, up to the combined measurement uncertainties, i.e. $\approx 8\%$ for $b \rightarrow c$ and $25\%$ for $b \rightarrow u$. With improved analyses and computational techniques and using the larger data sets, already becoming available to CLEO III, BABAR and BELLE, these tests may reach a sensitivity of $\approx 10\%$ within a few years. Further tests of quark-hadron duality that have been proposed include the study of $D - \bar{D}$ mixing and the determination of $V_{ub}$ and $V_{cb}$ for $B^0_s$ and $B^0$ separately by an analysis of $B^0 \rightarrow X_u \ell \bar{\nu}$ and $B^0 \rightarrow D^* \ell \bar{\nu}$ decays. FOCUS $^{[6]}$ and CLEO $^{[7]}$ have already presented new results on neutral $D^0$ decays with enhanced sensitivity. The $B^0_s$ meson may further contribute to the investigation of quark-hadron duality at a future $e^+e^-$ linear collider able to deliver $10^9$ $Z^0$ events, i.e. about 7 M s.l. $B^0_s$ decays, continuing the complementary roles of low- and high-energy colliders in the understanding of $B$ decays.

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