Decoherence and recoherence from vacuum fluctuations near a conducting plate

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The interaction between particles and the electromagnetic field induces decoherence generating a small suppression of fringes in an interference experiment. We show that if a double slit-like experiment is performed in the vicinity of a conducting plane, the fringe visibility depends on the position (and orientation) of the experiment relative to the conductor’s plane. This phenomenon is due to the change in the structure of vacuum induced by the conductor and is closely related to the Casimir effect. We estimate the fringe visibility both for charged and for neutral particles with a permanent dipole moment. The presence of the conductor may tend to increase decoherence in some cases and to reduce it in others. A simple explanation for this peculiar behavior is presented.

The interaction of a quantum system with its environment is responsible for the process of decoherence, which is one of the main ingredients to understand the quantum–classical transition. In some cases, the interaction with the environment cannot be switched off. This is the case for charged particles that unavoidably interact with the electromagnetic field. As this interaction is fundamental, its effect is present in any interference experiment. In this letter we will analyze the influence of a conducting boundary in the decay of the visibility of interference fringes in a double slit experiment performed with charged particles (or neutral particles with a dipole moment). The reduction of fringe visibility is induced by the interaction between the particles and the electromagnetic field. Some aspects of this problem have been analyzed before. In fact, it is known that for charged particles, the interaction between the system (the particle) and the environment (the electromagnetic field) induces a rather small decoherence effect even if the initial state of the field is the vacuum. A particularly simple expression for the decay in the fringe visibility was obtained in: Assuming an electron in harmonic motion (with frequency \( \Omega \)) along the relevant trajectories of the double slit experiment, the fringe visibility decays by a factor \((1 - P)^2\) where \( P \) is the probability that a dipole \( p = eR \) oscillating at frequency \( \Omega \) emits a photon \((R \) is the characteristic size of the trajectory). This result is in accordance with the idea that decoherence becomes effective when a record of the state of the system is irreversibly imprinted in the environment. In this case, after photon emission, if the electron follows the trajectory \( \vec{X}_1(t) \) of the double slit experiment (see Figure 1) it becomes correlated with a state of the environment \( |E_1(t)\rangle \). This state is different from the one with which the electron correlates if it follows the trajectory \( \vec{X}_2(t) \). The absolute value of the overlap between these two different states is precisely given by \((1 - P)^2\).

In this letter we will analyze how the fringe visibility is modified when performing a double slit interference experiment in the vicinity of a conducting plane. Our analysis will serve not only to correct previous results but also to show that the effect of the conductor is quite remarkable and simple to understand. As we will see, the presence of the conducting plane may produce more decoherence in some cases and less decoherence in others. For example, we will show that if a conducting plate is placed perpendicular to the trajectories of the interfering charge, the fringe visibility decreases with respect to the vacuum case (absence of conducting plate). However, if the plate lies parallel to the electron’s trajectories, the contrast increases (the system recoheres!). We will show that this peculiar behavior can be understood in simple terms and the magnitude of the effect can be easily estimated. There are several interesting physical effects connected with the one we are analyzing here. Thus, it is well known that a conducting boundary modifies the properties of the zero point fluctuations, and therefore could affect the interference experiments of particles that interact with the electromagnetic field. Other consequences of the presence of nontrivial boundary conditions are the Casimir force between two conductors and the Casimir–Polder force affecting a probe particle in the vicinity of a conductor. These phenomena, that have been experimentally verified, are close relatives of the process we are studying here. In fact, the Casimir–Polder force can be thought as the dispersive counterpart of the decoherence effect we will discuss. The influence of boundaries on the electromagnetic vacuum is also responsible for changes in atomic lifetimes and interference phenomena for light emitted by atoms near conducting surfaces.

Let us first outline a simple method to compute the effect of electromagnetic interactions on the fringe contrast. We consider two electron wave packets that follow well defined trajectories \( \vec{X}_1(t) \) and \( \vec{X}_2(t) \) that coincide at initial \((t = 0)\) and final \((t = T)\) times as shown in Figure 1. In the absence of environment, the interference depends on the relative phase between both wave packets at \( t = T \). Because of the interaction with the quantum electromagnetic field, the interference pattern...
is the initial (vacuum) state of the field and \( |\psi(0)\rangle = (|\phi_1\rangle + |\phi_2\rangle) \otimes |E_0\rangle \). Here \( |E_0\rangle \) is the initial (vacuum) state of the field and \( |\phi_{1,2}\rangle \) are two states of the electron that are localized around the initial point and that in the absence of other interaction continue to be localized along the trajectories \( X_{1,2}(t) \) respectively. At later times, due to the particle-field interaction the state of the combined system becomes \( |\Psi(t)\rangle = (|\phi_1(t)\rangle \otimes |E_1(t)\rangle + |\phi_2(t)\rangle \otimes |E_2(t)\rangle) \). Thus, the two localized states \( |\phi_1(t)\rangle \) and \( |\phi_2(t)\rangle \) become correlated with two different states of the field. Therefore, the probability of finding a particle at a given position turns out to be

\[
\text{prob}(x, t) = |\phi_1(x, t)|^2 + |\phi_2(x, t)|^2 + 2\text{Re}\langle\phi_1(x, t)\phi_2^*(x, t)|E_2(t)|E_1(t)\rangle. 
\]

The overlap factor \( F = \langle E_2(t)|E_1(t)\rangle \) is responsible for two effects: Its phase produces a shift of the interference fringes. The absolute value \( |F| \) is responsible for the decay in the fringe contrast, which is the phenomenon we will analyze here. The calculation of the factor \( F \) is conceptually simple since it is nothing but the overlap between two states of the field that arise from the vacuum under the influence of two different sources (this factor is identical to the Feynman–Vernon influence functional \( \frac{1}{2}\eta \{A_\mu(x), A_\nu(y)\} \)). It is easy to show that the square of the overlap has a simple interpretation: \( |F|^2 \) is equal to the probability for vacuum persistence in the presence of a source \( j_\mu = (J_1 - J_2)_\mu \), which corresponds to a time dependent electric dipole \( e(\vec{X}_1(t) - \vec{X}_2(t)) \).

A conceptually similar and physically interesting problem can be analyzed along the same lines: the decoherence of neutral particles with a non-vanishing permanent dipole moment. In such case we can model the particle-field interaction using a Lagrangian \( L = L_{\text{int}} = P_{\mu\nu}(x)F_{\mu\nu}(x) \). Here \( F_{\mu\nu} \) is the field strength tensor and \( P_{\mu\nu} \) is a totally antisymmetric tensor whose non-vanishing components are given in terms of the electric and magnetic dipole densities. For particles with electric dipole \( \vec{p} \) and magnetic dipole \( \vec{m} \) moving along a trajectory \( \vec{X}(t) \), the dipolar tensor is such that \( P_{0i} = p_i(t)\delta^2(\vec{X} - \vec{X}(t))/2 \) and \( P_{ij} = \epsilon_{ijk}m_k(t)\delta^2(\vec{X} - \vec{X}(t))/2 \). In this case we can perform a calculation which is similar to the one above and show that the overlap \( F = \exp(-W_d) \) is

\[
W_d = \frac{1}{2} \int d\vec{x} d\vec{y} (P_1 - P_2)_{\mu\nu}(x)K_{\mu\nu\rho\sigma}(x, y) \times (P_1 - P_2)_{\rho\sigma}(y), 
\]

where the kernel is \( K_{\mu\nu\rho\sigma}(x, y) = \langle \{F_{\mu\nu}(x), F_{\rho\sigma}(y)\} \rangle \).

In what follows we will present results for the decoherence factors \( W_c \) and \( W_d \) (the subscripts stand for "charges" and "dipoles"). To compute \( W_c \) we need the two point function appearing in \( \frac{1}{2}\eta \{A_\mu(x), A_\nu(y)\} \). In the Feynman gauge and in the absence of conducting plates it is

\[
D_{\mu\nu}^{(0)}(x, y) = -\eta_{\mu\nu} \int \frac{d^3k}{(2\pi)^3} e^{|k(x - y)|} \cos(k(x_0 - y_0)),
\]

where the superscript \( (0) \) identifies this as the vacuum contribution. We will assume that the trajectories are such that \( \vec{X}_1(t) = -\vec{X}_2(t) = x(t)\hat{x} \). This is enough to describe a typical double slit experiment from the point of view of an observer moving at constant velocity from the source to the detector. In such case we obtain the following relatively simple expression for \( W_c \):

\[
W_c^{(0)} = e^2 \int \frac{d^3k}{8\pi^3} k \int_{-\infty}^{\infty} dt \hat{x}(t) \cos(kx(x(t)) e^{ikt} t^2. 
\]

This result, obtained first in [2], can be simplified further by assuming the validity of the dipole approximation (which is consistent in the nonrelativistic limit). Doing this, one can evaluate the decoherence factor for some special trajectories. In fact, for adiabatic trajectories, where \( x(t) = R \exp(-it^2/T^2) \), we find that \( W_c^{(0)} = 2e^2v^2/3\pi \), where \( v = R/T \) is a characteristic velocity. This result is finite and free of any cutoff dependence. However, for trajectories evolving over a finite time the situation is different. Thus, assuming that the motion starts at \( t = 0 \), ends at \( t = T \), and that is composed...
of periods of constant velocity \( v \), or constant acceleration \( v/\tau \), we obtain a result that diverges logarithmically when \( \tau \to 0 \): \( \Delta W_{c}^{(B)} = 2c^2v^2 \log(T/\tau)/\pi^2 \) (if \( \tau/T \ll 1 \)).

Previous results \( \Delta W_{c}^{(B)} \) were obtained for trajectories with discontinuous velocity using a natural UV cutoff arising from the finite size of the electron. The results of \( \Delta W_{c}^{(B)} \) agree with ours if the high frequency cutoff is identified with \( 1/\tau \). Thus, the cutoff dependence disappears in the adiabatic case and is a consequence of abrupt changes in velocity and the instantaneous preparation of the initial state. If the two wave packets are superposed after oscillating \( N \) times, it is possible to define a decoherence rate (the amount by which the decoherence factor grows in a single oscillation). Thus, if the time to complete one oscillation is much shorter than the period between oscillations we can show that, for large \( N \), the decoherence factor is proportional to \( N \): \( W_{c}^{(B)} = NW_{c}^{(0)}(1) \) where \( W_{c}^{(0)}(1) \) is the decoherence factor in a single oscillation.

We will now show how this result is modified by the presence of a perfect conductor located in the plane \( z = 0 \). To consider the effect of the conductor we only need to use the appropriate two point function \( D_{\mu\nu} \) that is the sum of two terms \( \Delta \): \( D_{\mu\nu} = D_{\mu\nu}^{(0)} + D_{\mu\nu}^{(B)} \). The vacuum term is the same as in \( \Delta \). The contribution of the boundary conditions (identified by the superscript (B)) can be obtained by the method of images and is:

\[
D_{\mu\nu}^{(B)}(x,y) = (\eta_{\mu\nu} + 2n_{\mu}n_{\nu}) \int \frac{d^{3}k}{(2\pi)^{3}2k} \times \exp(ik(\vec{x} - \vec{y})) \cos(k(x_0 - y_0)).
\]

Here \( n^\mu \) is the normal to the plane and \( \vec{y} \) is the position of the image point of \( \vec{y} \) (a prime denotes a vector reflected with respect to the plane, i.e. \( \vec{y}' = (y_x, y_y, -y_z) \)). Using \( \Delta \) we can derive a formula for the contribution of the boundary to the decay of the interference fringes. The complete equation is involved and will be given elsewhere \( \Delta \). Here we will restrict to the case where the trajectories are either perpendicular or parallel to the conductor’s plane. Thus, we will write \( \vec{X}_1 = z_0 \hat{z} \pm x(t) \hat{j} \) where \( \hat{j} \) defines a fixed vector aligned either along the \( \hat{z} \)-axis or along the plane perpendicular to it. In such case, the conductor’s contribution to decoherence is

\[
W_{c}^{(B)} = -\vec{y}'e^2 \int \frac{d^{3}k}{8\pi^{3}k} (1 - \frac{k_x^2}{k^2}) e^{2ikz_0} \times | \int_0^t dt' \hat{x}(t') \cos[k_j(x(t'))] e^{ikx(t')} |^2.
\]

The sign of \( W_{c}^{(B)} \) is determined by the orientation of \( \vec{y}' \) relative to \( \vec{j} \). \( W_{c}^{(0)} \) is negative when the trajectories are parallel to the conductor’s plane (since in that case \( \vec{y}' = \vec{j} \)). On the other hand, \( W_{c}^{(0)} \) is positive when the trajectories are perpendicular to the plane (where \( \vec{y}' = -\vec{j} \)). At small distances to the plane \( (z_0 \approx 0) \) we can see from \( \Delta \) that \( |W_{c}^{(B)}| \approx W_{c}^{(0)} \). Therefore, if the trajectories are perpendicular to the plane, in the limit of small distances the decoherence factor is \( W_{c} = W_{c}^{(0)} + W_{c}^{(B)} \approx 2W_{c}^{(0)} \): the effect of the conductor is to double the decoherence factor. However, if the trajectories are parallel to the conductor the effect is exactly the opposite: As \( W_{c}^{(B)} \) is negative, the conductor produces recoherence increasing the contrast of the fringes. In fact, for small distances the decoherence factor tends to vanish since \( W_{c} = W_{c}^{(0)} + W_{c}^{(B)} \approx 0 \). These results can be understood using the method of images taking into account that decoherence in empty space is related to the probability of photon emission for a varying dipole \( p = c\vec{x}(t) \). When the conducting plane is parallel to the dipole, the image dipole is \( \vec{p}_m = -\vec{p} \). Therefore the total dipole moment vanishes, and so does the probability to emit a photon. The image dipole cancels the effect of the real dipole and this produces the recovery of the fringe contrast. On the other hand, when the conductor is perpendicular to the trajectories, the image dipole is equal to the real dipole \( \vec{p}_m = +\vec{p} \). Therefore, the total dipole is twice the original one. This in principle would lead us to conclude that the total decoherence factor \( W_{c} = W_{c}^{(0)} + W_{c}^{(B)} \) should be four times larger than \( W_{c}^{(0)} \). However, one should take into account that in the presence of a perfect mirror photons can only be emitted in the \( z \geq 0 \) region. This introduces an additional factor of \( 1/2 \) that gives rise to the final result \( W_{c} \approx 2W_{c}^{(0)} \). The impact of conducting boundaries on the fringe visibility for interference experiments performed with charged particles was previously examined in \( \Delta \). However, results obtained in such papers are not correct due to inconsistent approximations that violate the conservation of the 4-current. Thus the expressions obtained there differ from ours in several ways: not only they are not proportional to \( v^2 \) but also they violate the positivity of the total decoherence factor \( W_{c} \).

Let us now describe the results for the case of neutral particles with permanent dipole moments. The calculation is tedious and details will be given elsewhere \( \Delta \). Here we will analyze it under somewhat simplified assumptions. Will assume that the dipole moments \( \vec{p} \) and \( \vec{m} \) remain constant along the trajectories. If \( \vec{X}_1 = \vec{X}_2 \equiv x(t) \hat{j} \) we find:

\[
W_{d}^{(0)} = \int \frac{d^{3}k}{8\pi^{3}k} k(p^2(1 - \frac{k_x^2}{k^2}) + \vec{m}^2(1 - \frac{k_x^2}{k^2})) \times | \int_0^t dt' \sin[k_j(\vec{x}(t'))] e^{ikx(t')} |^2.
\]
plane (i.e., $\vec{X}_{1,2} = z_0 \hat{z} \pm x(t) \hat{j}$) and assume that the dipole moments are either perpendicular or parallel to the conductor (the general case is more complex but the essential features can be seen here). Using this we obtain

$$W^{(B)}_d = - \int \frac{d^3 \vec{k}}{32\pi^3} k \{ \vec{m} \cdot e^{i \vec{k} \cdot \vec{p}} (1 - \frac{k^2}{k^2}) - \vec{m} \cdot e^{i \vec{k} \cdot \vec{p}'} (1 - \frac{k'^2}{k'^2}) \} \times \left| e^{i \vec{k} \cdot \vec{z}_0} \right|^2 \cdot \left( 1 - \frac{k^2}{k^2} \right) \cdot \left( 1 - \frac{k'^2}{k'^2} \right),$$

Thus, if the reflected dipole $\vec{p}'$ has the opposite direction than $\vec{p}$ (which is the case when $\vec{p}$ is parallel to the plate) the conductor tends to increase decoherence (since the contribution of the electric dipole to $W^{(B)}_d$ is positive). Likewise, when $\vec{p}$ is perpendicular to the plane, $\vec{p} = \vec{p}'$ and the contribution of the electric dipole to $W^{(B)}_d$ is negative. Therefore, in this case the conductor produces recoherence instead of decoherence. The opposite effect is found for the magnetic dipole. Indeed, when $\vec{m}' = -\vec{m}$ (magnetic dipole perpendicular to the plane) the conductor produces recoherence while more decoherence is produced if the magnetic dipole is parallel to the plane. This features can also be understood by thinking in terms of the image dipoles that are generated by the conductor. Thus, both when the $\vec{p}$ is perpendicular to the plane or when $\vec{m}$ is parallel, the direction of the image dipoles coincide with the source dipoles. In such case the decoherence increases. In the opposite situation ($\vec{p}$ parallel or $\vec{m}$ perpendicular to the plane) the effect of the conductor is to introduce recoherence. Again, in the limit of small distances the absolute value of $W^{(0)}_d$ and $W^{(B)}_d$ coincide and therefore the decoherence factor doubles with respect to the vacuum case.

For the two cases we considered (charges and dipoles) one can show that the boundary contribution to the decoherence factor decays algebraically with the distance to the conductor (in the limit of large distances). For small separations, explicit expressions for the decoherence factor can be obtained. For example, for charges moving close and parallel to the conductor, the lower order contribution of $W_d$ depends quadratically on $z_0$.

As expected, it exactly coincides with the decoherence factor produced by an electric dipole $p = 2e z_0$ in vacuum (with an additional factor of $1/2$ that takes into account that photons can only be emitted with $z \geq 0$).

In conclusion, our work shows that the effect of conducting boundaries on interference experiments has a rather simple interpretation: The way in which decoherence is affected is similar to the manner in which atomic emission properties are modified by the presence of conducting boundaries. Thus, the effect of the boundaries has not a well defined sign and may produce either more decoherence or complete recoherence (i.e. smaller or higher fringe visibility than in vacuum) depending on the orientation of the relevant trajectories with respect to the conductor’s plane. The effect discussed here is conceptually important due to its fundamental origin (i.e., it is always present) but its magnitude is too small to be under the reach of current experiments involving interference of neutral atoms in the vicinity of conducting planes.}

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