The Supersymmetric Two Boson Hierarchy*

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Abstract

We summarize all the known properties of the supersymmetric integrable Two Boson equation. We present its nonstandard Lax formulation and tri-Hamiltonian structure, its reduction to the supersymmetric nonlinear Schrödinger equation and the local as well as nonlocal conserved charges. We also present the algebra of the conserved charges and identify its second Hamiltonian structure with the twisted $N = 2$ superconformal algebra.

* Invited talk given by A.D. at the Workshop in Theoretical and Mathematical Physics, CAM’95, University of Laval, Quebec, Canada, June 11–16, 1995.
1. Introduction

The study of integrable models has provided for a long time a crossover arena between mathematical and theoretical physics [1]. Recently, these models have found a relevant role in the study of strings through the matrix models [2]. So, it is natural to study supersymmetric integrable models since they are likely to play an important role in the superstrings [3].

The most widely studied supersymmetric integrable system is the supersymmetric KdV (sKdV) equation [4]. But, there are other integrable systems, which can be obtained by supersymmetrization of other well known bosonic ones and may have an important role in physical applications in the study of the super matrix models. In this talk we give a description of some of our recent results [5-10] on the supersymmetric Two Boson (sTB) equation [6]. A detailed review of this system and other results not covered in this talk can be found in our review paper [10].

2. Two Boson Equation

The Two Boson system is a dispersive generalization of the long water wave equation [11] and has appeared in the literature in the study of bosonic matrix models [12]. The equations for this 1 + 1 dimensional integrable system are

\[
\begin{align*}
\frac{\partial J_0}{\partial t} &= (2J_1 + J_0^2 - J_0')' \\
\frac{\partial J_1}{\partial t} &= (2J_0J_1 + J_1')'
\end{align*}
\]

(1)

This equation can be obtained from the Lax operator

\[
L = \partial - J_0 + \partial^{-1}J_1
\]

(2)

through a nonstandard Lax equation [11,13]

\[
\frac{\partial L}{\partial t} = [L, (L^2)_{>1}]
\]

(3)

where \((\cdot)_{>1}\) refers to the differential part of the pseudo-differential operator. The conserved charges of the system are in involution and can be obtained in the conventional manner from

\[
H_n = \text{Tr} L^n = \int dx \text{ Res } L^n \quad n = 1, 2, 3, \ldots
\]

(4)
The TB equation has a tri-Hamiltonian structure [11] which can be obtained from modified Gelfand-Dikii brackets [13]. The second Hamiltonian structure is related to a Virasoro-Kac-Moody algebra for a $U(1)$ current (an affine algebra) [13]. Also, under an appropriate field redefinitions or reductions we can obtain from it the well known integrable systems such as the KdV, mKdV and the nonlinear Schrödinger (NLS) equations.

3. Supersymmetric Two Boson Equation

A supersymmetric generalization of (1) can be constructed [6] if we introduce the fermionic superfields

$$\Phi_0 = \psi_0 + \theta J_0$$
$$\Phi_1 = \psi_1 + \theta J_1$$

and the supercovariant derivative

$$D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial x}$$

We use the notation [6] where $z = (x, \theta)$ defines the coordinates of the superspace with $\theta$ representing the Grassmann coordinate.

The most general, local, dynamical equations in the superspace which are consistent with all the canonical dimensions and which reduce to (1) in the bosonic limit are

$$\frac{\partial \Phi_0}{\partial t} = -(D^4 \Phi_0) + 2(D \Phi_0)(D^2 \Phi_0) + 2(D^2 \Phi_1)$$
$$+ a_1 D(D^2 \Phi_0) + a_2 D(\Phi_0 \Phi_1)$$

$$\frac{\partial \Phi_1}{\partial t} = (D^4 \Phi_1) + b_1 D(D^2 \Phi_0) + 2(D^2 \Phi_1)(D\Phi_0) - b_2 D(\Phi_1 D^2 \Phi_0)$$
$$+ 2(D \Phi_1) D^2 \Phi_0 + b_3 \Phi_1 \Phi_0 (D^2 \Phi_0) + b_4 D(\Phi_1 \Phi_0)(D\Phi_0)$$
$$+ b_5 D(\Phi_0 (D^4 \Phi_0)) + b_6 D(\Phi_0 (D^2 \Phi_0))(D\Phi_0)$$

where $a_i$ and $b_i$ are arbitrary parameters. However, equations (7) are integrable only for specific choices of $a_i$ and $b_i$. In fact, a consistent Lax representation can be obtained for the system (7) with the Lax operator

$$L = D^2 - (D \Phi_0) + D^{-1} \Phi_1$$

and the nonstandard Lax equation

$$\frac{\partial L}{\partial t} = [L, (L^2)_{\geq 1}]$$
where $D^{-1} = \partial^{-1} D$. Here, (8) reduces in the bosonic limit to (2). So, the most general supersymmetric extension of the dynamical equations (1) which is integrable is given by

$$\frac{\partial \Phi_0}{\partial t} = -(D^4 \Phi_0) + (D(D\Phi_0)^2) + 2(D^2 \Phi_1)$$

$$\frac{\partial \Phi_1}{\partial t} = (D^4 \Phi_1) + 2(D^2((D\Phi_0)\Phi_1))$$

These are the supersymmetric Two Boson equations (sTB) [6] with the nonstandard Lax representation given by (8) and (9). In components the equations (10) read

$$\frac{\partial J_0}{\partial t} = (2J_1 + J_0^2 - J_0')'$$

$$\frac{\partial \psi_0}{\partial t} = 2\psi_1' + 2\psi_0' J_0 - \psi_0''$$

$$\frac{\partial J_1}{\partial t} = (2J_0 J_1 + J_1' + 2\psi_0' \psi_1)'$$

$$\frac{\partial \psi_1}{\partial t} = (2\psi_1 J_0 + \psi_1')'$$

and it is straightforward to check that these equations are invariant under the $N = 1$ supersymmetry transformation

$$\delta J_0 = \epsilon \psi_0'$$

$$\delta J_1 = \epsilon \psi_1'$$

$$\delta \psi_0 = \epsilon J_0$$

$$\delta \psi_1 = \epsilon J_1$$

In fact, as we will see, there is another supersymmetry transformation as well as additional symmetries present in our system (10).

Since equations (10) are integrable we have an infinite number of local conserved charges in involution which can be obtained from

$$Q_n = \text{sTr} L^n = \int dz \text{sRes} L^n \quad n = 1, 2, 3, \ldots$$

The first few charges are

$$Q_1 = -\int dz \Phi_1$$

$$Q_2 = 2\int dz (D\Phi_0)\Phi_1$$

$$Q_3 = 3\int dz \left[(D^3 \Phi_0) - (D\Phi_1) - (D\Phi_0)^2\right] \Phi_1$$

$$Q_4 = 2\int dz \left[2(D^5 \Phi_0) + 2(D\Phi_0)^3 + 6(D\Phi_0)(D\Phi_1) - 3(D^2(D\Phi_0)^2)\right] \Phi_1$$

(14)
They are bosonic and are invariant under the supersymmetry transformation (12).

Defining the Hamiltonians of the sTB system as

\[ H_n = \frac{(-1)^{n+1}}{n} Q_n \]  \hspace{1cm} (15)

we can write the sTB equations (10) as a Hamiltonian system [6]

\[
\partial_t \begin{pmatrix} \Phi_0 \\ \Phi_1 \end{pmatrix} = D_1 \begin{pmatrix} \frac{\delta H_3}{\delta \Phi_0} \\ \frac{\delta H_3}{\delta \Phi_1} \end{pmatrix} = D_2 \begin{pmatrix} \frac{\delta H_2}{\delta \Phi_0} \\ \frac{\delta H_2}{\delta \Phi_1} \end{pmatrix} = D_3 \begin{pmatrix} \frac{\delta H_1}{\delta \Phi_0} \\ \frac{\delta H_1}{\delta \Phi_1} \end{pmatrix} \]  \hspace{1cm} (16)

where the first structure has the local form

\[ D_1 = \begin{pmatrix} 0 & -D \\ -D & 0 \end{pmatrix} \]  \hspace{1cm} (17)

and the second has the nonlocal form [18]

\[ D_2 = \begin{pmatrix} -2D - 2D^{-1} \Phi_1 D^{-1} + D^{-1}(D^2 \Phi_0) D^{-1} & D^3 - D(D\Phi_0) + D^{-1} \Phi_1 D \\ -D^3 - (D\Phi_0) D - D\Phi_1 D^{-1} & -\Phi_1 D^2 - D^2 \Phi_1 \end{pmatrix} \]  \hspace{1cm} (18)

Defining

\[ R = D_2 D_1^{-1} \]  \hspace{1cm} (19)

the third structure, which is highly nonlocal, can also be written as

\[ D_3 = R D_2 = R^2 D_1 \]  \hspace{1cm} (20)

Using prolongation methods [14] and its generalization to the supersymmetric systems [15] the Jacobi identity as well the compatibility of the structures \( D_1, D_2 \) and \( D_3 \) can be checked. The second structure (18) corresponds to the twisted \( N = 2 \) superconformal algebra [16] after a linear change of basis of the fields, as we will discuss at the end of this talk.

Finally, the recursion operator defined in (19) has the form

\[ R = \begin{pmatrix} -D \left( D^2 - (D\Phi_0) \right) D^{-1} - D^{-1} \Phi_1 & 2 + D^{-1} \left( 2\Phi_1 + (D^2 \Phi_0) \right) D^{-2} \\ (D^2 \Phi_1 + \Phi_1 D^2) D^{-1} & D^2 + (D\Phi_0) + D\Phi_1 D^{-2} \end{pmatrix} \]  \hspace{1cm} (21)

and relates the conserved charges recursively as

\[
\begin{pmatrix} \frac{\delta H_{n+1}}{\delta \Phi_0} \\ \frac{\delta H_{n+1}}{\delta \Phi_1} \end{pmatrix} = R^\dagger \begin{pmatrix} \frac{\delta H_n}{\delta \Phi_0} \\ \frac{\delta H_n}{\delta \Phi_1} \end{pmatrix} \]  \hspace{1cm} (22)
4. Reductions of sTB

The sTB equation reduces to many other supersymmetric integrable models such as the sKdV and mKdV [6,10]. Let us show how the supersymmetric NLS equation [17,5] can be obtained from the sTB system [6,9].

Defining the fermionic superfields

\[
Q = \psi + \theta q \\
\bar{Q} = \bar{\psi} + \theta \bar{q}
\]

(23)

from the invertible transformation

\[
\Phi_0 = - (D \ln(DQ)) + (D^{-1}(\bar{Q}Q)) \\
\Phi_1 = - \bar{Q}(DQ)
\]

(24)

we obtain from (10), after a slightly involved derivation [6], the equations

\[
\frac{\partial Q}{\partial t} = -(D^4 Q) + 2 (D((DQ)\bar{Q})) Q
\]

\[
\frac{\partial \bar{Q}}{\partial t} = (D^4 \bar{Q}) - 2 (D((D\bar{Q})Q)) \bar{Q}
\]

(25)

These are the sNLS equations without free parameters obtained in [10] and shown to satisfy various tests of integrability [6].

In [12] we have shown that a scalar Lax equation operator for the sNLS equation (25) can be obtained from the Lax operator (8) of the sTB system. With the identifications (24) we can write (8) as

\[
L = G\tilde{L}G^{-1}
\]

(26)

where

\[
G = (DQ)^{-1} \\
\tilde{L} = D^2 - \bar{Q}Q - (DQ)D^{-1}\bar{Q}
\]

(27)

And this makes the Lax operators, \( L \) and \( \tilde{L} \), related by a gauge transformation in the superspace. However, different from the bosonic case [12], it is the formal adjoint of \( \tilde{L} \)

\[
\mathcal{L} = \tilde{L}^* \]

(28)

which gives the sNLS equations (25) through the nonstandard Lax representation

\[
\frac{\partial \mathcal{L}}{\partial t} = [\mathcal{L}, (\mathcal{L}^2)_{\geq 1}]
\]

(29)
It is possible, in principle, to obtain the bi-Hamiltonian structure for the sNLS system using the Gelfand-Dikii method in the superspace for the scalar Lax operator (28). However, a generalization of this method to superspace for nonstandard systems is not yet known. But, we can derive this structure directly from the sTB ones, through the field redefinition in (24). This can be found in reference [9].

If we rewrite the operator \( L \), given in (28), using the supersymmetric Leibnitz rule we get

\[
L = - \left( D^2 + \overline{Q}Q - \overline{Q}D^{-1}(DQ) \right)
= - \left( D^2 + \sum_{n=1}^{\infty} \Psi_n D^{-n} \right)
\]

where

\[
\Psi_{-1} = 0
\]
\[
\Psi_n = (-1)^{\frac{n+1}{2}} \overline{Q}(D^nQ), \quad n \geq 0
\]

and \( \Psi_{2n} \) (\( \Psi_{2n+1} \)) are bosonic (fermionic) superfields. In this way \( L \) above has the form of the Lax operator for the sKP hierarchy and, therefore, we can think of the sNLS system as a constrained sKP system but of the nonstandard kind (even the sTB can be viewed as a constrained sKP system). However, the Lax operator \( L \) in this case is an even parity operator [18] and not of the usual Manin-Radul form [4]. This is a new system, namely, a nonstandard supersymmetric KP hierarchy, and was studied in [7]. It gives a new supersymmetric KP equation and unifies all the KP and mKP flows.

Reduction of the sTB to the sKdV and msKdV are straightforward and details can be found in [6,9,10].

5. Nonlocal Charges

As we have already pointed out the sTB equation, given by the Lax operator (8), has conserved local charges \( Q_n \) obtained from the integer powers of the Lax operator as in (13). Also, the sTB has a supersymmetric charge \( Q \) which is local and conserved and implements the supersymmetric transformation in (12). However, supersymmetric integrable models also have nonlocal conserved charges. This was first discovered in [19] for the sKdV equation and explained in the Gelfand-Dikii formalism in [20]. For the sTB equation we also have the presence of nonlocal charges [8] and they can be obtained from

\[
F_{2n-1} = \text{str} L^{2n-1} \quad n = 1, 2, 3, \ldots
\]
and the first ones are

\[ F_{1/2} = - \int dz (D^{-1}\Phi_1) \]
\[ F_{3/2} = - \int dz \left[ \frac{3}{2} (D^{-1}\Phi_1)^2 - \Phi_0\Phi_1 - \left( D^{-1}((D\Phi_0)\Phi_1) \right) \right] \]
\[ F_{5/2} = - \int dz \left[ \frac{1}{6} (D^{-1}\Phi_1)^3 - (5(D^{-2}\Phi_1)\Phi_1 - 2\Phi_0\Phi_1 - 3(D\Phi_1) - (D^{-1}\Phi_1)^2)(D\Phi_0) 
+ \left( D^{-1}((D\Phi_1)\Phi_1 + \Phi_1(D\Phi_0)^2 - (D\Phi_1)(D^2\Phi_0)) \right) \right] \]  

(33)

These nonlocal charges are conserved and are fermionic. They reduce to the nonlocal charges of the sKdV [19,20] if we set \( \Phi_0 = 0 \). This is natural since we have already said that the sKdV equations is contained in the sTB system. Also, the nonlocal charges \( F_{2n-1}^{2n-1} \) are not supersymmetric since the integrand in (33) are not local functions of superfields. Even, the supersymmetric charge \( Q \) is not supersymmetric.

We can now ask about the algebra of these charges \( Q_n, Q \) and \( F_{2n-1}^{2n-1} \) [8]. First, we obtain

\[ \{Q_n, Q_m\}_1 = 0 \]
\[ \{Q_n, F_{2m-1}^{2m-1}\}_1 = 0 \]
\[ \{Q_n, Q\}_1 = 0 \]  

(34)

which simply implies that these charges are conserved under any flow of the hierarchy. The fact that \( Q \) and \( F_{2n-1}^{2n-1} \) are not supersymmetric is expressed by

\[ \{Q, Q\}_1 = - Q_2 \]
\[ \{Q, F_{1/2}\}_1 = Q_1 \]
\[ \{Q, F_{3/2}\}_1 = Q_2 \]
\[ \{Q, F_{5/2}\}_1 = \frac{1}{3} Q_3 + \frac{1}{24} Q_1^3 \]  

(35)

And the algebra for the lowest order nonlocal charges is

\[ \{F_{1/2}, F_{1/2}\}_1 = 0 \]
\[ \{F_{1/2}, F_{3/2}\}_1 = Q_1 \]
\[ \{F_{1/2}, F_{5/2}\}_1 = Q_2 \]
\[ \{F_{3/2}, F_{3/2}\}_1 = 2Q_2 \]
\[ \{F_{3/2}, F_{5/2}\}_1 = \frac{7}{3} Q_3 + \frac{7}{24} Q_1^3 \]
\[ \{F_{5/2}, F_{5/2}\}_1 = 3Q_4 - \frac{5}{8} Q_1^2 Q_2 \]  

(36)

So, the algebra of the conserved charges of the sTB system closes as a graded algebra. The Jacobi identity is trivially satisfied since the \( Q_n \)'s are in involution with all the charges. It
can also be seen that the algebra is closed with respect to the second Hamiltonian structure (18) and the resulting algebra [8] is a sorting of shifting of (34), (35) and (36).

The cubic terms in the algebra above arise from boundary contributions when nonlocal terms are involved. Using the following realization of the inverse operator

\[
(\partial^{-1} f(x)) = \frac{1}{2} \int dy \epsilon(x-y)f(y), \quad \epsilon(x) = \begin{cases} 
-1, & x < 0 \\
0, & x = 0 \\
+1, & x > 0 
\end{cases}
\]  
(37)

we can, for instance, obtain terms of the type

\[
\int dz (D^{-1} \Phi_1)^2 \Phi_1 = \frac{1}{3} \int dz D(D^{-1} \Phi_1)^3 = -\frac{1}{12} Q_3^3
\]  
(38)

showing the origin of the nonlinear character of the algebra. The algebra for the nonlocal charges of the sKdV equation also has cubic terms [8]. In fact, algebras of nonlocal charges showing nonlinearity of the cubic kind are present in various other systems and are related with Yangian structures [21,22]. The nonlinearity in (35) and (36) appear to be redefinable to cubic terms [8]. This is well known for the nonlinear sigma model [22]. With appropriate redefinitions, the right hand side of the algebra (35) and (36) appears to take the form

\[
a \hat{Q}_n + b \sum_{p+q+\ell=n} \hat{Q}_p \hat{Q}_q \hat{Q}_\ell
\]  
(39)

which is a cubic algebra. It suggests the presence of a Yangian structure, although the algebra for the local charges, in the present case, is involutive.

From (35) and (36) we see that \( F_{3/2} \) has the same algebra, with the other generators of the algebra, as the supersymmetry charge \( Q \). Thus, this can be identified with a second supersymmetry charge and this shows that the sTB equation has in fact an \( N = 2 \) extended supersymmetry [23]. The generators of the \( N = 2 \) supersymmetry can be identified with

\[
\frac{1}{3} (Q - F_{3/2}) \quad \text{and} \quad -\frac{1}{3} (2Q + F_{3/2})
\]  
(40)

Furthermore, with the linear change of variables

\[
\bar{\xi} = -\frac{1}{2} (\psi_0' - \psi_1) \\
\xi = \frac{1}{2} \psi_1
\]  
(41)
the second Hamiltonian structure (18), in terms of components, gives the following non-vanishing Poisson brackets [10]

\[
\begin{align*}
\{ J_0(x), J_0(y) \}_2 &= 2\delta'(x - y) \\
\{ J_0(x), J_1(y) \}_2 &= (J_0\delta(x - y))' - \delta''(x - y) \\
\{ J_0(x), \xi(y) \}_2 &= -\xi\delta(x - y) \\
\{ J_0(x), \xi(y) \}_2 &= \xi\delta(x - y) \\
\{ J_1(x), J_1(y) \}_2 &= J_1'\delta(x - y) + 2J_1\delta'(x - y) \\
\{ J_1(x), \xi(y) \}_2 &= \xi'\delta(x - y) + 2\xi\delta'(x - y) \\
\{ \xi(x), \xi(y) \}_2 &= -\frac{1}{4}J_1\delta(x - y) + \frac{1}{4}(J_0\delta(x - y))' - \frac{1}{4}\delta''(x - y)
\end{align*}
\]

This local algebra is nothing other than the twisted \( N = 2 \) superconformal algebra [16] whose bosonic limit is the Virasoro-Kac-Moody algebra for the sTB system [10].

Acknowledgements

This work was supported in part by the U.S. Department of Energy Grant No. DE-FG-02-91ER40685. J.C.B. would like to thank CNPq, Brazil, for financial support.
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