Phenomenological Analysis of Hybrid Textures of Neutrinos

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Abstract

We present a comprehensive phenomenological analysis of the allowed hybrid textures of neutrinos. Out of a total of sixty hybrid textures with one equality between the elements of neutrino mass matrix and one texture zero only twenty three are found to be viable at 99% C.L. whereas the earlier analysis found fifty four to be viable. We examine the phenomenological implications of the allowed hybrid textures including Majorana type CP-violating phases, 1-3 mixing angle and Dirac type CP-violating phase, δ. We, also, obtain lower bound on effective Majorana mass for all the allowed hybrid textures.

1 Introduction

In the Standard Model (SM), fermions acquire masses via spontaneous breakdown of SU(2) gauge symmetry. However, the values of fermion masses and the observed hierarchical fermion spectra are not understood within the SM. This results in thirteen free parameters in the SM which includes three charged lepton masses, six quark masses and the four parameters of the CKM matrix. The symmetries of the SM do not allow non-zero neutrino masses through renormalizable Yukawa couplings. However, non-zero neutrino masses can be introduced via non-renormalizable higher dimensional operators presumably having their origin in physics beyond the SM. Texture zeros, flavor symmetries, radiative mechanisms and the see-saw mechanisms are some of the widely discussed mechanisms for fermion mass generation. These mechanisms, most often, complement and reinforce each other. In the ongoing decade, significant advances have been made in understanding these mechanisms. In particular texture zeros and flavor symmetries have provided quantitative relationships between flavor mixing angles and the quark/lepton mass ratios. It has, now been realized that the “See-Saw GUT” scenario, on its own, cannot provide a complete understanding of the flavor structure of the quark and lepton mass matrices and new physics seems to be essential perhaps in the form of new symmetries mainly in the lepton sector.

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Moreover, a unified description of flavor physics and CP-violation in the quark and lepton sectors is absolutely necessary. This can be achieved by constructing a low energy effective theory with the SM and some discrete non-Abelian family symmetry and, subsequently, embedding this theory into Grand Unified Theory (GUT) models like $SO(10)^1$. For this reason, the discrete symmetry will have to be a subgroup of $SO(3)$ or $SU(3)^2$. The search for an adequate discrete symmetry has mainly focused on the minimal subgroups of these groups with at least one singlet and one doublet irreducible representation to accommodate the fermions belonging to each generation. One such subgroup is the quaternion group $Q_8^3$ which not only accommodates the three generations of fermions but also explains the rather large difference between values of 2-3 mixings in the quark and lepton sectors. Quaternion symmetry like some other discrete symmetries leads to nontrivial relationships amongst the non-zero mass matrix elements which underscores the inadequacy of texture zero analyses$^3$ in isolation. Such textures which have equalities between different elements alongwith the vanishing of some elements of the mass matrix have been referred to as hybrid textures in the literature. Frigerio and Smirnov$^12$ presented a comprehensive analysis of the hybrid textures alongwith other possibilities for the neutrino mass matrix. The detailed numerical analysis of sixty possible hybrid textures with one equality amongst mass matrix elements and one texture zero was presented by Kaneko et al.$^13$ who found fifty four out of sixty to be phenomenologically viable. Hybrid textures have, also, been discussed more recently$^14$ though in a somewhat different context.

In the present work, we present a detailed and improved numerical analysis of the hybrid textures found to be viable in the earlier numerical analysis by Kaneko et al.$^13$. Our analysis shows that only twenty three out of a total of sixty possible hybrid textures are phenomenologically viable. We examine the phenomenological implications of the viable hybrid textures. These hybrid textures have different hierarchical spectra and have different phenomenological consequences. We find the values of Majorana type CP-violating phases for which these textures are realized. We, also, calculate the effective Majorana mass and Jarlskog rephasing invariant $J_{CP}$ for all viable hybrid textures of neutrinos. In addition, we present correlation plots between different parameters of the hybrid textures of neutrinos for $3\sigma$ allowed ranges of the known parameters. It is found that positive results from neutrinoless double beta decay experiments will rule out three more hybrid textures with one equality between the matrix elements and one zero. Thus, the investigation of Majorana type CP-violating phases will be significant.

### 2 Hybrid Textures

The neutrino mass matrix, $M_\nu$, can be parameterized in terms of three neutrino mass eigenvalues $(m_1, m_2, m_3)$, three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and one Dirac type CP-violating phase, $\delta$. If neutrinos are Majorana particles then there are two additional CP-violating phases $\alpha, \beta$ in the neutrino mixing matrix. Thus, massive Majorana neutrinos increase the number of free parameters of the SM from thirteen to twenty two. In the charged lepton basis the complex symmetric mass matrix $M_\nu$ can be diagonalized by a complex unitary matrix $V$:

$$M_\nu = V M^{diag} V^T$$

(1)
where $M^\text{diag}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ is the diagonal neutrino mass matrix. The neutrino mixing matrix $V$ can be written as

$$V = U P \equiv U \text{PMNS} = \text{PMNS},$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The matrix $V$ is called the neutrino mixing matrix or PMNS matrix. The matrix $U$ is the lepton analogue of the CKM quark mixing matrix and the phase matrix $P$ contains the two Majorana phases. Therefore the neutrino mass matrix can be written as

$$M_\nu = U P M^\text{diag}_\nu P^T U^T. \quad (3)$$

The elements of the neutrino mass matrix can be calculated from Eq. (3). In hybrid textures of neutrinos, we have one equality of matrix elements and one zero, i.e.

$$M_{ab} = 0, \quad M_{pq} = M_{rs}. \quad (4)$$

These two conditions yield two complex equations as follows

$$m_1 U_{a1} U_{b1} + m_2 U_{a2} U_{b2} e^{2i\alpha} + m_3 U_{a3} U_{b3} e^{2i(\beta+\delta)} = 0 \quad (5)$$

and

$$m_1 (U_{p1} U_{q1} - U_{r1} U_{s1}) + m_2 (U_{p2} U_{q2} - U_{r2} U_{s2}) e^{2i\alpha} + m_3 (U_{p3} U_{q3} - U_{r3} U_{s3}) e^{2i(\beta+\delta)} = 0 \quad (6)$$

where $U$ has been defined in Eq. (2). These two complex equations involve nine physical parameters $m_1, m_2, m_3, \theta_{12}, \theta_{23}, \theta_{13}$ and three CP-violating phases $\alpha, \beta$ and $\delta$. The masses $m_2$ and $m_3$ can be calculated from the mass-squared differences $\Delta m^2_{12}$ and $\Delta m^2_{23}$ using the relations

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}} \quad (7)$$

and

$$m_3 = \sqrt{m_2^2 + \Delta m^2_{23}}. \quad (8)$$

Thus, we have two complex equations relating five unknown parameters viz. $m_1, \theta_{13}, \alpha, \beta$ and $\delta$. Therefore, if one out of these five parameters is assumed, other four parameters can be predicted. Solving Eqs. (5) and (6) simultaneously, we obtain

$$\frac{m_1 e^{-2i\beta}}{m_3} = \frac{U_{a2} U_{b2} (U_{p3} U_{q3} - U_{r3} U_{s3}) - U_{a3} U_{b3} (U_{p2} U_{q2} - U_{r2} U_{s2})}{U_{a1} U_{b1} (U_{p2} U_{q2} - U_{r2} U_{s2}) - U_{a2} U_{b2} (U_{p1} U_{q1} - U_{r1} U_{s1})} e^{2i\delta} \quad (9)$$

and

$$\frac{m_1 e^{-2i\alpha}}{m_2} = \frac{U_{a3} U_{b3} (U_{p2} U_{q2} - U_{r2} U_{s2}) - U_{a2} U_{b2} (U_{p3} U_{q3} - U_{r3} U_{s3})}{U_{a1} U_{b1} (U_{p3} U_{q3} - U_{r3} U_{s3}) - U_{a3} U_{b3} (U_{p1} U_{q1} - U_{r1} U_{s1})}. \quad (10)$$

Using Eqs. (9) and (10), the two mass ratios $(\frac{m_1}{m_2}, \frac{m_1}{m_3})$ and the two Majorana phases $(\alpha, \beta)$ can be written as

$$\frac{m_1}{m_2} = \left| \frac{U_{a2} U_{b2} (U_{p3} U_{q3} - U_{r3} U_{s3}) - U_{a3} U_{b3} (U_{p2} U_{q2} - U_{r2} U_{s2})}{U_{a2} U_{b2} (U_{p3} U_{q3} - U_{r3} U_{s3}) - U_{a3} U_{b3} (U_{p2} U_{q2} - U_{r2} U_{s2})} \right| \quad (11)$$
\[
\frac{m_1}{m_2} = \left| \frac{U_{a3}U_{b3}(U_{p2}U_{q2} - U_{r2}U_{s2}) - U_{a2}U_{b2}(U_{p3}U_{q3} - U_{r3}U_{s3})}{U_{a1}U_{b1}(U_{p3}U_{q3} - U_{r3}U_{s3}) - U_{a3}U_{b3}(U_{p1}U_{q1} - U_{r1}U_{s1})} \right|, \tag{12}
\]

\[
\alpha = -\frac{1}{2} \arg \left( \frac{U_{a3}U_{b3}(U_{p2}U_{q2} - U_{r2}U_{s2}) - U_{a2}U_{b2}(U_{p3}U_{q3} - U_{r3}U_{s3})}{U_{a1}U_{b1}(U_{p3}U_{q3} - U_{r3}U_{s3}) - U_{a3}U_{b3}(U_{p1}U_{q1} - U_{r1}U_{s1})} \right) \tag{13}
\]

and

\[
\beta = -\frac{1}{2} \arg \left( \frac{U_{a2}U_{b2}(U_{p3}U_{q3} - U_{r3}U_{s3}) - U_{a3}U_{b3}(U_{p2}U_{q2} - U_{r2}U_{s2})}{U_{a1}U_{b1}(U_{p2}U_{q2} - U_{r2}U_{s2}) - U_{a2}U_{b2}(U_{p1}U_{q1} - U_{r1}U_{s1})} \right) - \delta. \tag{14}
\]

Both the Majorana phases \(\alpha\) and \(\beta\) in Eqs. (13) and (14) are functions of \(\theta_{13}\) and \(\delta\) whereas the mass ratios \(\left(\frac{m_1}{m_2}, \frac{m_1}{m_3}\right)\) are functions of \(\theta_{12}, \theta_{23}, \Delta m_{12}^2\), and \(\Delta m_{23}^2\) are known experimentally. The values of mass ratios \(\left(\frac{m_1}{m_2}, \frac{m_1}{m_3}\right)\), from Eqs. (11) and (12) can be used to calculate \(m_1\). The mass eigenvalue \(m_1\) can be written in terms of two mass squared differences which are known from experiments.

\[
m_1 = \frac{m_1}{m_2} \sqrt{\frac{\Delta m_{12}^2}{1 - \left(\frac{m_1}{m_2}\right)^2}} \tag{15}
\]

and

\[
m_1 = \frac{m_1}{m_3} \sqrt{\frac{\Delta m_{12}^2 + \Delta m_{23}^2}{1 - \left(\frac{m_1}{m_3}\right)^2}} \tag{16}
\]

The two values of \(m_1\) obtained above from the two mass ratios must be equal to within the errors of the mass squared differences. Using the experimental inputs of the two mass squared differences and the two mixing angles we can constrain the \((\theta_{13}, \delta)\) plane. The experimental constraints on neutrino parameters with \(1\sigma\) errors are given below:

\[
\Delta m_{12}^2 = 7.67^{+0.22}_{-0.21} \times 10^{-5} eV^2, \\
\Delta m_{23}^2 = \pm 2.37^{+0.15}_{-0.10} \times 10^{-3} eV^2, \\
\theta_{12} = 34.5^{+1.4}_{-1.4}, \\
\theta_{23} = 42.3^{+5.1}_{-3.3}.
\]

Only an upper bound is known on the mixing angle \(\theta_{13}\) from the CHOOZ experiment. We vary the oscillation parameters \(\delta\) and \(\theta_{13}\) within their full physical ranges with uniform distributions. We, also, calculate the effective Majorana mass, which is given as

\[
M_{ee} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}| \tag{17}
\]

and Jarlskog rephasing invariant quantity \([17]\)

\[
J_{CP} = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13} \sin \delta \tag{18}
\]

within the allowed parameter space for all allowed hybrid textures. In our numerical analysis we generate as many as \(10^8\) data points (\(10^5\) data points were generated in the previous analysis\([13]\)) thus, making our numerical analysis more reliable. Earlier analysis of hybrid textures\([13]\) used the ratio of two known mass-squared differences

\[
R_\nu \equiv \frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \frac{1 - \left(\frac{m_1}{m_2}\right)^2}{\left(\frac{m_1}{m_3}\right)^2} \tag{19}
\]
to constrain the other neutrino parameters. However, our analysis makes direct use of the two mass-squared differences and is more constraining. Earlier analysis does not use the full experimental information currently available with us in the form of two mass-squared differences. Moreover, since $R_\nu$ is a function of mass ratios, it does not depend upon the absolute neutrino mass scale. It is obvious that the two mass ratios $\frac{m_1}{m_2}$ and $\frac{m_1}{m_3}$ may yield the experimentally allowed values of $R_\nu$ for mutually inconsistent values of $m_1$. However, our analysis selects only those mass ratios for which the values of $m_1$ obtained from Eqs. (15) and (16) are identical to within the errors of the mass squared differences. Moreover, the definition of $R_\nu$ used in many earlier analysis does not make use of the knowledge of solar mass hierarchy to constrain the neutrino parameter space. Our numerical analysis restricts the number of allowed hybrid textures to twenty three at 99% C.L. as enumerated in Table 1.

3 Results And Discussions

All the allowed hybrid textures of neutrinos give hierarchical spectrum of neutrino masses. There are ten textures with clear normal hierarchy, two with inverted hierarchy, and one with all hierarchies. The remaining ten textures give normal and quasi-degenerate spectrum as given in Table 2.

Table 3. depicts the lower bound on $\theta_{13}$ for viable hybrid textures. The measurement of $\theta_{13}$ is the main goal of future experiments. The proposed experiments such as Double CHOOZ plan to explore $\sin^2 2\theta_{13}$ down to 0.06 in phase I (0.03 in phase II)\[18\]. Daya Bay has a higher sensitivity and plans to observe $\sin^2 2\theta_{13}$ down to 0.01\[19\]. Thus some of these textures will face stringent experimental scrutiny in the immediate future. We have, also, calculated the effective Majorana mass which is the absolute value of $M_{ee}$ element of neutrino mass matrix. The neutrinoless double beta decay which is controlled by effective Majorana mass is forbidden for texture $C_1$, $C_2$ and $C_3$ as $M_{ee} = 0$ for these textures. Table 4. gives the lower bound on $M_{ee}$ for the remaining hybrid textures of neutrinos.

The analysis of $M_{ee}$ will be significant as many neutrinoless double beta decay experiments will give the predictions on this parameter. A most stringent constraint on the value of $M_{ee}$ was obtained in the $^{76}{Ge}$ Heidelberg-Moscow experiment\[20\] $|M_{ee}| < 0.35\text{eV}$. There are large number of projects such as SuperNEMO\[21\], CUORE\[22\], CUORICINO\[22\] and GERDA\[23\] which aim to achieve a sensitivity below 0.01eV to $M_{ee}$. Forthcoming experiment SuperNEMO, in particular, will explore $M_{ee} < 0.05\text{eV}$\[24\]. The Jarlskog rephasing invariant quantity $J_{CP}$ varies in the range (-0.05-0.05) for most of the allowed hybrid textures. We get a clear normal hierarchical mass spectrum for ten hybrid texture structures (Table 2). The Dirac type CP-violating phase $\delta$ is constrained to the range $90^\circ \text{ - } 270^\circ$ for $A1$ as seen in Fig.1(a). It can be seen from Fig.1(a) and Fig.1(c) that there exists a clear lower bound on $\theta_{13}$ and effective Majorana mass $M_{ee}$ However, this range of $\delta$ is disallowed for $B2$. For $B1$ the Majorana type CP-violating phase $\alpha$ is constrained to a value of $0^\circ$ or $180^\circ$ as seen in Fig.1(d). However, its range varies from $50^\circ$ to $130^\circ$ for $C1$, $C2$ and $C3$ and this range of $\alpha$ is disallowed for $A1$ and $B2$. The Majorana phase $\beta$ is unconstrained i.e. whole range of $\beta$ is allowed for all textures with normal hierarchy. Maximal value of $\theta_{23}$ is disallowed for $A3$, $A5$, $B4$, $B6$, $C1$, $C2$ and $C3$. Fig.1(b) depicts Jarlskog rephasing invariant quantity $J_{CP}$ as a function of $\delta$ for $B1$. The allowed range of $J_{CP}$
for $B_1$ is constrained to be (-0.018-0.018).

The other ten textures give quasidegenerate spectrum in addition to normal hierarchy. We get the lower bound on effective Majorana mass (Fig.2(a)) and the reactor mixing angle. Many forthcoming neutrino oscillation experiments aim to measure/constrain these two parameters. The Dirac type CP-violating phase $\delta$ is constrained for most of these hybrid texture structures as seen in Fig.2(a) for $B_3$. Fig.2(b) shows the mass spectrum for $B_3$. The correlation plot shows both normal and quasidegenerate behaviour. Dirac type CP-violating $\delta$ lies in the range $90^o$ or $270^o$ for $B_3, B_5$ while this range is disallowed for $A_2, A_4, D_1, D_3, D_4, D_5, E_3$ and $E_4$. The Majorana-type CP-violating phase, $\alpha$, takes a single value of $0^o$ or $180^o$ for $B_3, B_5, D_1, D_3, D_4$ and $E_4$. We find some hybrid textures having identical predictions for all parameters except the 2-3 mixing angle i.e. $\theta_{23}$ is above maximal for one and below maximal for the other set. Such textures are given in Table.5. There are some projects like Tokio- to Kamioka- Korea (T2KK) which intend to resolve the octant degeneracy of $\theta_{23}$ (i.e. $\theta_{23} < 45^o$ or $\theta_{23} > 45^o$)[25]. The results from future experiments will decide the fate of such texture structures. A comparative study of some predictions of the present work and the earlier analysis[13] is presented in Table.6.

Two hybrid texture structures viz. $D_2, E_2$ have inverted hierarchical mass spectrum. We find identical predictions for all the neutrino parameters for these two textures except for the Majorana type CP-violating phase $\alpha$ as given in Fig.3., which is restricted to a range of $82^o$ to $96^o$.

For the remaining texture $E_1$ all mass hierarchies are possible as shown in correlation plot 4. The Majorana phase $\alpha$ is constrained to a value of $0^o$ or $180^o$, while no constraint is obtained for the other Majorana phase $\beta$ in this hybrid texture.

4 Conclusions

In conclusion, we find that only twenty three hybrid textures with one equality between mass matrix elements and one texture zero are allowed at 99% C.L. by our analysis. We present systematic and detailed numerical analysis for twenty three allowed hybrid textures of neutrinos. These hybrid textures have different hierarchical spectra and, thus, have different phenomenological implications. Ten out of twenty three allowed hybrid textures give normal hierarchical mass spectra, two imply inverted hierarchy and one texture has all hierarchies. The remaining ten hybrid textures give quasidegenerate spectra in addition to normal hierarchy. Predictions for 1-3 mixing angle and Dirac type CP-violating phase, $\delta$, are given for these textures. These two parameters are expected to be measured in the forthcoming neutrino oscillation experiments. We, also, obtained the lower bound on effective Majorana mass for all the allowed hybrid textures. The possible measurement of effective Majorana mass in neutrinoless double $\beta$ decay experiments will provide an additional constraint on the remaining three neutrino parameters i.e. the neutrino mass scale and two Majorana type CP-violating phases. The observation of correlations between various neutrino parameters like $\theta_{13}, \theta_{23}, \delta, M_{ee}$ etc will confirm/reject hybrid textures with a texture zero and an additional equality among the matrix elements.
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References

[1] H. C. Goh, R. N. Mohapatra and Siew-Phang Ng, Phys. Rev. D 68, 115008 (2003).
[2] I. de Medeiros Varzielas, S. F. King and G. G. Ross Phys. Letts. B 644, 1532 (2007).
[3] M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto Phys.Rev. D 71, 011901 (2005).
[4] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, Phys. Letts. B 536, 79 (2002); Bipin R. Desai, D. P. Roy and Alexander R. Vaucher, Mod. Phys. Lett A 18, 1355 (2003).
[5] Zhi-zhong Xing, Phys. Lett. B 530 159 (2002).
[6] Wanlei Guo and Zhi-zhong Xing, Phys. Rev. D 67, 053002 (2003).
[7] Alexander Merle and Werner Rodejohann, Phys. Rev. D 73, 073012 (2006); S. Dev and Sanjeev Kumar, Mod. Phys. Lett. A 22, 1401(2007), arXiv:0607048 [hep-ph].
[8] S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, Nucl. Phys. B 784, 103-117 (2007).
[9] S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, Phys. Rev. D 76, 013002 (2007).
[10] M. Honda, S. Kaneko and M. Tanimoto, JHEP 0309, 028(2003).
[11] S. Kaneko, M. Tanimoto, Phys. Lett. B 551, 127 (2003).
[12] M. Frigerio, A. Y. Smirnov, Phys. Rev. D 67, 013007 (2003).
[13] S. Kaneko, H.Sawanaka and M. Tanimoto, JHEP 0508, 073(2005).
[14] Srubabati Goswami, Subrata Khan, Atsushi Watanabe, arXiv:0811.4744v1 [hep-ph], Amol Dighe, Narendra Sahu, arXiv:0812.0695v2 [hep-ph]
[15] G. L. Fogli et al, arXiv:0506083 [hep-ph].
[16] M. C. Gonzalez-Garcia, Michele Maltoni, Phys. Rept. 460 (2008) 1-129, arXiv:0704.1800v2 [hep-ph].
[17] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[18] http://doublechooz.in2p3.fr/ F. Ardellier et al. (Double Chooz Collaboration),
[19] [http://dayabay.ihep.ac.in/](http://dayabay.ihep.ac.in/) Daya Bay Collaboration, arXiv:0701029 [hep-ex].

[20] H. V. Klapdor- Kleingrothaus, *Nucl. Phys.Proc.Suppl.* **145**, 219 (2005).

[21] Arnaboldi C *et al.* 2004a *Nucl. Instrum. Meth.* A **518**, 775

[22] Arnaboldi C *et al.* (CUORICINO collaboration) *Phys. Lett. B* **584**, 20 (2004)

[23] I. Abt *et al.* (GERDA collaboration) arXiv:0404039 [hep-ex].

[24] Sarazin X *et al.* 2000 Preprint arXiv:0006031 [hep-ex].

[25] Hisakazu Minakata, arXiv:0701070 [hep-ph].
Table 1: Twenty three allowed hybrid textures. $X$ denote two equal elements.

|   | A                  | B                  | C                  | D                  | E                  |
|---|--------------------|--------------------|--------------------|--------------------|--------------------|
| 1 | $(X \ 0 \ e)\ b \ X$ | $(X \ X \ 0)\ b \ f$ | $(0 \ d \ e)\ X \ f$ | $(a \ X \ X)\ 0 \ f$ | $(a \ X \ X)\ b \ f$ |
| 2 | $(a \ 0 \ X)\ X \ f$ | $(a \ d \ 0)\ b \ X$ | $(0 \ d \ e)\ X \ X$ | $(a \ X \ e)\ 0 \ f$ | $(a \ X \ e)\ X \ f$ |
| 3 | $(a \ 0 \ e)\ X \ X$ | $(0 \ d \ e)\ b \ X$ | $(a \ X \ e)\ 0 \ f$ | $(a \ X \ e)\ X \ f$ |                       |
| 4 | $(a \ 0 \ X)\ X \ f$ | $(a \ d \ 0)\ X \ X$ | $(a \ d \ X)\ 0 \ f$ | $(a \ X \ e)\ X \ f$ |                       |
| 5 | $(a \ 0 \ e)\ X \ X$ | $(a \ d \ 0)\ X \ X$ | $(a \ d \ e)\ 0 \ X$ |                       |                       |
| 6 | -                  | $(a \ d \ 0)\ b \ X$ | -                  | -                  | -                  |

Table 2: Mass patterns for the allowed hybrid textures.
Table 3: Predicted Lower Bounds on $\theta_{13}$.

| $\theta_{13}$ | Hybrid Textures                  |
|---------------|----------------------------------|
| $> 0^o$       | $A2, A4, B3, B5, B6, C1, D3, D4, E3, E4$ |
| $\geq 1^o$   | $A1, A3, A5, B2, E1$             |
| $\geq 2^o$   | $B1, B4, D1, D5$                 |

Table 4: Predicted Lower Bounds on $M_{ee}$.

| $M_{ee}(eV)$ | Hybrid Textures                  |
|--------------|----------------------------------|
| $> 0$        | $A2, A3, A4, A5, B3, B4, B6$     |
| $\geq 0.003eV$ | $B1$              |
| $\geq 0.01eV$ | $A1, B2, D1, D2, D5, E1, E2$     |
| $\geq 0.02eV$ | $D3, D4, E3$            |
| $\geq 0.03eV$ | $E4$                        |
| $\geq 0.05eV$ | $B5$                        |

Table 5: The hybrid textures in the same block give identical predictions except $\theta_{23}$.

| Hybrid Textures | $\theta_{23}$ |
|-----------------|---------------|
| $A2$            | $< 45^o$      |
| $A4$            | $> 45^o$      |
| $B3$            | $< 45^o$      |
| $B5$            | $> 45^o$      |
| $D3$            | $< 45^o$      |
| $E3$            | $> 45^o$      |
| $D4$            | $< 45^o$      |
| $E4$            | $> 45^o$      |
| Hybrid Texture | Hierarchy | $\theta_{13}$ | $M_{ee}$ (eV) (Lower Bound) | $\theta_{23}$ |
|----------------|-----------|---------------|-----------------------------|-------------|
| A1 I I  | NH | $> 1^\circ$ | $\geq 0.01$ | - |
| A2 K I  | NH + QD | $> 0^\circ$ | $> 0$ | $< 45^\circ$ |
| L I  | NH | $> 1^\circ$ | $> 0$ | - |
| A4 N I  | NH + QD | $> 0^\circ$ | $> 0$ | $> 45^\circ$ |
| A5 O I  | NH | $> 1^\circ$ | $> 0$ | - |
| B1 G II | NH | $> 2^\circ$ | $\geq 0.003$ | - |
| B2 I II  | NH + QD | $> 1^\circ$ | $\geq 0.01$ | - |
| B3 J II  | NH + QD | $> 0^\circ$ | $> 0$ | $< 45^\circ$ |
| B4 L II  | NH | $> 2^\circ$ | $> 0$ | - |
| B5 M II  | NH + QD | $> 0^\circ$ | $\geq 0.05$ | $> 45^\circ$ |
| B6 O II  | NH | $> 0^\circ$ | $> 0$ | - |
| C1 C IV  | NH | $\geq 0^\circ$ | 0 | - |
| C2 L IV  | NH | $\geq 0^\circ$ | 0 | - |
| C3 O IV  | NH | $\geq 0^\circ$ | 0 | - |
| D1 D V  | NH + QD | $> 2^\circ$ | $\geq 0.01$ | - |
| D2 I V  | IH | $\geq 0^\circ$ | $\geq 0.01$ | - |
| D3 M V  | NH + QD | $> 0^\circ$ | $\geq 0.02$ | $< 45^\circ$ |
| D4 N V  | NH + QD | $> 0^\circ$ | $\geq 0.02$ | $< 45^\circ$ |
| D5 O V  | NH + QD | $> 2^\circ$ | $\geq 0.01$ | - |
| E1 E VI  | All | $> 1^\circ$ | $\geq 0.01$ | - |
| E2 I VI  | IH | $\geq 0^\circ$ | $\geq 0.01$ | - |
| E3 J VI  | NH + QD | $> 0^\circ$ | $\geq 0.02$ | $> 45^\circ$ |
| E4 K VI  | NH + QD | $> 0^\circ$ | $\geq 0.03$ | $> 45^\circ$ |

Table 6: A comparative study of some predictions obtained in the present work with earlier analysis[13]. Upper (lower) entry in each block corresponds to the value obtained in the present (earlier) analysis.
Figure 1: Correlation plots for $A_1$ and $B_1$ hybrid texture structures.

Figure 2: Correlation plots for $B_3$ hybrid texture structure.

Figure 3: Correlation plots for $D_2$ and $E_2$ hybrid textures.
Figure 4: Mass spectrum for hybrid texture $E1$. 
