Reliability model of freight bogie damper based on real field data

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Abstract. In the present paper is estimated the reliability probabilistic model of the dry friction damper of the freight bogie Y25. The analysis is made on the basis of real field data, observed in actual operations of freight wagons. The theoretical probability distributions used to model the reliability are exponential and Weibull. The Weibull reliability models are estimated according to the Regression Method and to the Maximum Likelihood Method.

1. Introduction

Taking into account the nature of the service provided, reliability is an essential feature of railway vehicles. Many of the possible failure modes are potential causes of derailments, so it may have important social and / or economic implications and - in the worst case scenario – may lead to significant material damage or even to fatalities [1]. For this reason, the railway vehicles reliability is the subject of quite a lot of research. For example, in [2] are evaluated the failures and the reliability of three types of metro vehicles operating in Poland. The failures were classified and their occurrence in time was analysed. Also, in [3] a reliability analysis of subway vehicles subsystems was developed based on operational failures data. Failure statistics were made, and the optimal failure distribution model of each subsystem was determined. A reliability model of components of bogie is proposed in [4] based on the real data of the CRH2 high speed train.

Since the wheelset is a critical item in terms of safety, a significant number of studies address the issue of its reliability. Of these, can be mentioned the analysis of freight car wheelset failure modes based on field data, a reliability model being deduced using the regression method [5]. Also, in [1] are analysed the railway wheel failure modes and, for the major four failure modes, is carried out an additional analysis regarding the evolution in time of the failure rates, as well as the estimation of the failure probabilistic models, based on the Weibull distribution.

Besides the wheelset, the railway vehicle suspension is also a system with a high impact on safety. The suspension system has to isolate the car body from the vibrations determined by track irregularities, rail discontinuities, wheel defects, etc. It has also to ensure a stable dynamic behaviour of the vehicle in straight line and in curves and to reduce the vehicle-track mutual forces for reasons of traffic safety and passenger comfort or protection of the transported goods [6]. As shown in [7], the bogie suspension and structure failures are one of the main categories of rolling stock failures that have produced derailments in recent years in railway freight transport - both in European Union (second place) and in Romania (first place).
Among the concerns related to the reliability of the suspension, there is a growing interest in the field of condition monitoring and fault detection. For example, in [8,9] are proposed sensor configurations and are developed the methods (i.e. the associated mathematical models), while in [10] is proposed a failure detection method of the primary suspension damper, based on the analysis of cross-correlation of the bogie vertical accelerations.

As for reliability studies presenting systematically the failure modes of the suspension system or of its main components (dampers, springs) or in which is derived a reliability probabilistic model thereof, these are quite hard to find in the scientific literature. It can be mentioned a study investigating the freight vehicles suspension failures is carried out in [11], but the major emphasis was on the springs failure causes, the investigation consisting in failed spring material analysis and in stress analysis using analytical and finite element methods.

Under these circumstances, the present paper aim is to estimate the reliability probabilistic model of freight bogie dampers, the analysis being made based on real field data.

2. Estimation of the reliability probabilistic model
To determine a failure mathematical model, it is necessary to select a theoretical probability distribution and to estimate its parameters so that the obtained reliability model is adequate to the experimental data (i.e. accurately models, in terms of reliability, the real behaviour of the tested system). In this paper are analysed the cases of exponential and Weibull distributions, for the latter being used two distinct estimation methods.

2.1. Estimation of exponential reliability model
The exponential distribution is, chronologically speaking, the first distribution used extensively in reliability theory. Its fundamental feature is the constant failure rate (i.e. independent of time). Due to this peculiarity, the exponential distribution can be used especially when the effect of wear is not important, as is the case of the basic (useful) period of life of systems.

The exponential reliability equation is:

$$R(t) = \exp(-\lambda t),$$

where $t$ is the time and $\lambda$ is the failure rate. The exponential model is quite simple and its form leads to very simple mathematical expressions of the reliability indicators. On the other hand, this simplicity – given by the presumed constant failure rate - makes the exponential model less versatile, thus leads sometimes to a less accurate approximation of the empirical reliability.

It can be seen in equation (1) that the only parameter to determine is the failure rate $\lambda$, its estimator being given by:

$$\hat{\lambda} = \frac{r}{TTT(n,r)},$$

where $r$ is the number of failures registered during the observation period and $TTT(n,r)$ is the total time on test statistic, defined as the sum of the operating times of all units, regardless of whether they are still in service (units without failure) or not (units that have failed). Considering a sample of $n$ units and given their respective failure times arranged in ascending order, the empirical $TTT$ up to the $r$th order statistic is:

$$TTT(n,r) = \sum_{i=1}^{r} t_{i} + (n-r)t_{r},$$

Equation (3) gives thus the $TTT$ demonstrated by all $n$ units in the case of incomplete data, assuming that the test ends after the $r$th failure.
2.2. Estimation of Weibull reliability model using the Regression Method

The Weibull distribution is widely used in the reliability theory because of its versatility. Usually it is used the two-parameter version of Weibull distribution, according to which the evolution in time of the reliability $R(t)$ is given by:

$$R(t) = \exp \left[ -\left( \frac{t}{\eta} \right)^\beta \right], \quad (4)$$

where $t$ is the time, $\beta$ and $\eta$ are the shape and the scale parameters, respectively. The versatility mentioned above is given by the structure of the probabilistic law (i.e. by the two parameters), which makes it suitable to model the failure law of practically any industrial product.

To estimate the Weibull reliability model involves determining the values of the shape and the scale parameters so that the model is appropriate to the experimental data. The most used estimation method is the Regression (Least Squares) Method. It is based on the linearized form of the Weibull law and on the Least Squares principle in order to estimate the parameters of the Weibull distribution. The linear form is obtained by applying twice the natural logarithm in both members of equation (4), resulting:

$$\ln \ln \frac{1}{R(t)} = \beta \ln t - \beta \ln \eta, \quad (5)$$

thus the Weibull probability law can be written under a linear form:

$$Y = a_0 + a_1X \quad (6)$$

where

$$X = \ln t; \quad Y = \ln \ln \frac{1}{R(t)}. \quad (7)$$

To estimate the Weibull distribution parameters can then be used the linear regression and the Least Squares Method, which implies to minimize the sum of the squares of the deviations:

$$S = \sum_{i=1}^{n} (y_i - a_0 - a_1x_i)^2 \quad (8)$$

where

$$x_i = \ln t_i; \quad y_i = \ln \ln \frac{1}{1 - F(i)} \quad (9)$$

$n$ is the sample size, $t_i$ the failure time of damper $i$ and $F(i)$ an estimation of the failures cumulative distribution function.

The conditions for minimizing the expression in equation (8) are:

$$\frac{\partial S}{\partial a_0} = \frac{\partial S}{\partial a_1} = 0. \quad (10)$$

Solving the corresponding equations system, the expressions for coefficients $a_1$ and $a_0$ are:

$$a_1 = \left( \sum_{i=1}^{n} x_i y_i - \bar{y} \sum_{i=1}^{n} x_i \right) \left( \sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} x_i \right)^{-1}; \quad a_0 = \left( \bar{y} \sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} x_i y_i \right) \left( \sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} x_i \right)^{-1} \quad (11)$$

where
Consequently – see equations (5) and (6) - the estimations of parameters $\beta$ and $\eta$ are:

$$\hat{\beta} = a_1; \hat{\eta} = \exp\left( -\frac{a_0}{a_1} \right).$$  \hspace{1cm} (13)

The relationships above apply to the complete data sets (i.e. all $n$ failure times are known). In the case of a censored data set (i.e. only $r$ of the $n$ dampers have failed) the sums in equations (8) and (11) have the upper limit $r$ instead of $n$ and the means in equations (12) are calculated only for the corresponding $r$ terms.

The failures cumulative distribution function in equation (9) can be expressed using either the mean ranks or the median ranks. In most cases is preferred the Median Ranks Regression, situation in which the cumulative distribution function is estimated using Benard's approximation [12]:

$$F(i) = \frac{i - 0.3}{n + 0.4}. \hspace{1cm} (14)$$

2.3. Estimation of Weibull reliability model using the Maximum Likelihood Method.

The principle of the method consists in determining the most likely estimators (i.e. that maximize the likelihood of occurrence of the empirical data set) for the unknown parameters of a distribution. Considering a complete set of experimental data $x_1, x_2, \ldots, x_n$, the likelihood function is defined as [13]:

$$L(x_1, x_2, \ldots, x_n; \theta) = f(x_1; \theta) \cdot f(x_2; \theta) \cdot \ldots \cdot f(x_n; \theta) = \prod_{i=1}^{n} f(x_i; \theta), \hspace{1cm} (15)$$

where $\theta$ is the parameter to be estimated (there may be more than one) and $f$ is the probability density function of the assumed distribution.

The estimator that is most likely to have generated the observed sample is the one that maximizes the likelihood function, so it can be found by solving the equation [13]:

$$\frac{dL(x_1, x_2, \ldots, x_n; \theta)}{d\theta} = 0.$$

In the situation of a censored data set, when for the $n$ observed units only the first $r$ failure times are known ($r<n$), the likelihood function is defined as [13]:

$$L(x_1, x_2, \ldots, x_n; \theta) = \prod_{i=1}^{r} f(x_i; \theta) \prod_{j=r}^{n} R(x_j; \theta), \hspace{1cm} (17)$$

where $R$ is the reliability function.

Since the Weibull probability density function is given by:

$$f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp\left[-\left( \frac{t}{\eta} \right)^{\beta}\right],$$  \hspace{1cm} (18)

the corresponding likelihood function for a censored data set of $r$ failure times given by equation (17) is:
\[
L(t_1, t_2, \ldots, t_r; \beta, \eta) = \prod_{i=1}^{r} \left( \frac{\beta}{\eta} \right)^{t_i^{\beta-1}} \exp \left[ -\left( \frac{t_i}{\eta} \right)^{\beta} \right] \prod_{j=1}^{s-r} \exp \left[ -\left( \frac{t_j}{\eta} \right)^{\beta} \right].
\]

(19)

Applying the natural logarithm:

\[
\ln L(t_1, t_2, \ldots, t_r; \beta, \eta) = r \ln \beta - r \beta \ln \eta + (\beta - 1) \sum_{i=1}^{r} \ln t_i - \sum_{i=1}^{r} \left( \frac{t_i}{\eta} \right)^{\beta} - (n-r) \left( \frac{t_r}{\eta} \right)^{\beta},
\]

(20)

where \( t_r \) – the last observed failure time – is the censoring time, thus the time assigned to the \((n-r)\) units which have survived the time \( t_r \).

The maximum conditions are

\[
\frac{\partial \ln L(t_1, t_2, \ldots, t_r; \beta, \eta)}{\partial \beta} = 0 \quad \frac{\partial \ln L(t_1, t_2, \ldots, t_r; \beta, \eta)}{\partial \eta} = 0.
\]

(21)

By solving the equation system above are obtained the following expressions of the estimators for \( \beta \) and \( \eta \):

\[
\hat{\eta} = \left( \sum_{i=1}^{r} t_i^{\beta} + (n-r)t_r^{\beta} \right) / r
\]

(22)

\[
\frac{1}{\beta} - \frac{1}{r} \sum_{i=1}^{r} \ln t_i + \frac{(n-r)t_r^{\beta} \ln t_r}{\sum_{i=1}^{r} t_i^{\beta} + (n-r)t_r^{\beta}} = 0.
\]

(23)

3. Results and discussions

In this section is established the reliability mathematical model of freight bogie dampers, based on the real field data. The reliability model is estimated according to the previously presented probabilistic distributions and methods, so that more variants of this model are be obtained and a comparison is possible.

The analysis is based on empirical data, observed in actual operations of freight bogies in the period of their useful (normal) life. The sample size was \( n=1802 \) units, of which there have been observed failures in the case of \( r = 66 \) units over the testing period of one year. The studied dampers are of dry friction type, equipping the freight bogies type Y25. They are known as Lenoir dampers, and they have as an important feature the possibility to modify the damping force according to the axle load.

In the case of the exponential model, as the number of failures is known, it is only necessary to calculate the total time on test given by equation (3). Using the set of empirical data, it is obtained a \( TTT \) of 643214 days, which leads, through relationship (2), to an estimation of the failure rate of \( \hat{\lambda} = 1.0261 \times 10^{-3} \text{ (days}^{-1} \text{)} \).

For the Weibull distribution, the parameters of the reliability model are estimated using two distinct estimation methods. In the case of the model estimated using the Regression Method (RM) it is enough to simply apply the procedure described above. In the case of the Maximum Likelihood Method (MLM) the two estimates are determined using iterative procedures, by considering an initial value for the form parameter in equation (23) and by adjusting this value until the expression becomes null. As initial value it can be used the previously determined by the Regression Method. Once the form parameter estimation found, it is inserted in equation (22) thus the estimation of the scale parameter can be found.
In Table 1 are presented the calculated estimations of the parameters of the Weibull reliability model corresponding to the two methods.

**Table 1. Estimated parameters of Weibull distribution.**

| Method | \( \hat{\beta} \) | \( \hat{\eta} \) (days) |
|--------|----------------|------------------|
| RM     | 0.8799         | 12666            |
| MLM    | 0.7884         | 22905            |

It can be seen that there are significant differences between the results obtained for both parameters by using the two estimation methods. The scale parameter obtained through the Maximum Likelihood Method is significantly higher (almost double) than the one determined using Regression Method. On the other hand, the form parameter is about 10% lower. This confirms the widely accepted opinion according to which the use of the Maximum Likelihood Method leads to more optimistic models of the reliability.

Of the two Weibull parameters, the shape parameter \( \beta \) has a special significance, its value setting the monotony of the failure rate. For \( \beta = 1 \) the failure rate is constant, while for values higher and lower than 1, the failure rate is increasing and decreasing, respectively. A value close to 1 is an expected (usual) one in the case of a system during its normal (useful) life, as it is the case of the freight bogie dampers. In this regard, it can be seen in Table 1 that the \( \beta \) values are slightly below 1, this indicating a decreasing failure rate, i.e. a negative ageing (NWU – ‘new worse than used’) of the dampers.

![Figure 1. Theoretical and experimental damper reliability.](image)

In Figure 1 are shown the damper reliability laws according to the three estimated probabilistic models and the damper empirical reliability (dash-dot line).

It is obvious that the exponential model performs the poorest approximation of the empirical (real) damper reliability. This was somehow to be expected, as long as the exponential distribution, by its very structure, is suitable for modeling failure laws at a constant rate.

Better matching with experimental data is obtained in the case of the Weibull reliability models. Of them, as previously mentioned, the model resulting from the Maximum Likelihood estimation method indicates a higher reliability than the real one.

### 4. Conclusion

The aim of the present paper was to estimate the reliability probabilistic model of the dry friction dampers of the freight bogie Y25, the study being motivated by the fact that bogie suspension failures
are one of the main categories of rolling stock failures that have produced derailments in recent years in railway freight transport.

The analysis was based on real field empirical data, observed for one year in actual operations of freight wagons, in the period of their useful life.

The estimation of the reliability model was made using the exponential distribution and the Weibull distribution. Based on the latter, two models were obtained, following the estimation by two methods, the Regression Method and the Maximum Likelihood Method.

By comparing the obtained reliability models with the empirical reliability, it has been observed that the exponential model has the weakest match with the experimental data.

Regarding the Weibull-based models, although the two estimation methods produced significant differences in the two parameters of the Weibull distribution, the resulting models proved a good accuracy. Of the two models, it turned out that the Maximum Likelihood Method led to a more optimistic model of the damper reliability.

The shape parameter estimations of the two Weibull reliability models were slightly below 1, this indicating a slightly decreasing failure rate, i.e. a negative ageing (NWU – ‘new worse than used’) of the dampers. The fact that the values of parameter $\beta$ are close to 1 (situation which corresponds to a constant failure rate) is in line with the general theory regarding the behavior of systems during their normal/useful life - as it is the case of the analysed system.

References
[1] Spiroiu MA and Nicolescu M 2018 MATEC Web of Conferences 178 06005
[2] Melnik R, Koziaś S, Sowiński B and Chudzikiewicz A 2019 Transp. Res. Proc. 40 808–814
[3] Yin H, Wang K, Qin Y et al 2017 J. Wireless Com. Network 2017 212
[4] Yun T, Qin Y et al. 2016 Prognostics and System Health Management Conf. (Chengdu) (New York: IEEE) p 1
[5] Spiroiu MA 2015 Appl. Mechan. Mater. 809-810 1097-1102
[6] Spiroiu MA 2018 Mater. Plast. 55(1) 24-27
[7] Nicolescu M and Spiroiu MA 2019 U.P.B. Sci. Bull. Series D 81(3) 61-68
[8] Melnik R and Koziaś S 2017 J. Vibroeng. 19(1) 487-501
[9] Wei XK, Jia LM and Liu H 2013 Vehicle Syst. Dyn. 51(5) 700-720
[10] Dumitriu M 2019 Mech. Ind. 20(1) 102
[11] Kumbhalkara MA, Bhopec DV and Vanalkara AV 2015 Proc. Mat. Sci. 10 331-343
[12] McCool JI 2012 Using the Weibull Distribution: Reliability, Modeling and Inference (Hoboken: John Wiley & Sons)
[13] Rausand M and Hysland A 2004 System Reliability Theory Models, Statistical Methods and Applications (Hoboken: John Wiley & Sons)