Research Article

Resource Partitioning with Beamforming for the Decode-Forward Relay Networks

Duckdong Hwang, Junmo Kim, and Sungjin Kim

Communication and Network Laboratory, Samsung Advanced Institute of Technology, Mt. 14-1, Nongseo-Dong, Giheung-Gu, Yongin-Si, Gyeonggi-Do 446-712, South Korea

Correspondence should be addressed to Duckdong Hwang, duckdonh@yahoo.com

Received 22 May 2007; Revised 19 September 2007; Accepted 13 December 2007

Recommended by Sayande Mukherjee

A joint power and time slot partitioning scheme based on the channel status information (CSI) is proposed for networks of multiple relays using decode and forward (DF) protocol. A set of power constraints for the famous water pouring method is presented depending on the time slot partitioning and CSI. Optimizing the timing and the power distributions enhances the network throughput in addition to the diversity advantage well known for the open loop relay protocols. Beamforming techniques for the source or destination with multiple antennas are also proposed and utilized in the partitioning process.

Copyright © 2008 Duckdong Hwang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. INTRODUCTION

Due to the cost and size of mobile user terminals, the number of multiple antennas that can be mounted on them is limited in practice. Distributed antenna systems or user cooperation techniques (see [1–6]) have been proposed as an alternative or supplementary technology to enhance existing wireless links without multiple antennas on each terminal. The diversity enhancement capability of cooperative relaying has been the main concern of the research community for the last few years (see [2–6]). In support of this approach, the popular amplify-forward (AF) and DF protocols were introduced in [3].

Because of the half duplex constraint, it is anticipated that a loss in the throughput of a relay network is inevitable compared to the direct transmission. (The source uses only half of its transmission time.) This is demonstrated in the diversity-multiplexing tradeoff (DMT) analysis of relaying protocols [2], where the diversity is shown to be acquired at the expense of throughput loss. Azarian et al. propose the dynamic decode-forward (DDF) protocol to recover the throughput loss in DF protocol [7] by allowing the size of relay cooperation phase to vary adaptively depending on the channel status information (CSI) of the source-to-relay channel.

When the CSI is available at the transmitter in the closed loop systems, we can further optimize resources of the relaying networks to enhance not only the diversity but also the throughput. (In practice, only quantized CSI is available at the transmitter due to the bandwidth limitation of feedback channels. Thus, the full CSI assumption is the limiting case when the feedback channels expand their bandwidth to infinity. In [8], it is shown that the performance of AF relay power control with full CSI can be approached with small amount of feedback information. We leave the analysis of finite feedback effect in DF resource partitioning for future work.) In [8], a set of power allocation techniques for the AF relaying is considered for the full and finite feedback strategies. Optimization of power distribution in the symbol error sense is considered for the AF relay networks [9] and DF relay [5] networks, respectively. The authors in [10] present power allocation techniques for the relay protocols based on the long-term statistics of CSI. Instant CSI-based approach is taken in [11], where the power and the time slot are jointly optimized for the DF relays.

As shown in [7], the time slot partitioning based on CSI is crucial in targeting the throughput enhancement. In this paper, we try to show that appropriate partitioning of the resources (power and time slots) enhances the diversity and the throughput of DF relaying system at the same time. Contrary
to [11], where only the relay-to-destination link is considered, we consider the combined link of source-to-destination and relay-to-destination links. Consequently, the resulting optimization applies the famous water pouring method with different power constraints depending on the time slot division and CSI. The best time slot division with the maximum mutual information is searched along with the power allocation for that specific time division. As a way to quantify the throughput enhancement by the resource partitioning, the probability of choosing relay cooperation over the direct transmission is analyzed and compared to that of DDF protocol in [7]. To cover more general settings, we consider the multiple relay case and the multiple antenna case as well. For the multiple relay case, it is shown that the resource partitioning based on relay selection is enough to find the best relaying configuration. When multiple antennas are used at the source, we propose a way to combine beamforming with the resource partitioning proposed.

After introducing system model in Section 2, resource partitioning with multiple relays and analysis of the relaying probability are presented in Section 3. The method to combine beamforming with the resource partitioning is presented in Section 4. We analyze and present the simulation results in Section 5. Section 6 concludes this paper.

2. SYSTEM MODEL

The system model and channel gains \( (h_{ij}) \) of the relay network are shown in Figure 1. In the DF protocol, the source sends the information toward the destination with power \( P \) during the first time slot \( (T_1) \). The \( j \)th relay overhears this transmission. If it succeeds in decoding the message, it then re-encode the message with an independent code-book and transmits with power \( P_{r,j} \) during the second time slot \( (T_2) \). Otherwise, the relay remains silent. Note time slot \( T \) is divided such that \( T_1 + T_2 = T \) to support the source and relay transmissions. The destination leverages the observations from the two time slots to make the final decision of the RT bits of information sent.

Let us denote the distance of each link by \( d_{ij} \). The channel gains are assumed to be Rayleigh distributed with \( E[|y_{ij}|] = 1/d_{ij}^\alpha \). (The exponent \( \alpha \) denotes the path loss exponent. This is set to 2 in the simulations in Section 5.) The power of the additive noise at the relay and destination is assumed to be 1.

3. RESOURCE PARTITIONING AND RELAYING PROBABILITY

With full CSI at hand and \( M \) DF relays, we have an event space \( D \) of \( 2^M \) non overlapping elements, that is, \( i \)th event corresponds to direct transmission with no relay cooperation and \((2^M - 1)\)th event corresponds to full cooperation with \( M \) relays. Finding the optimum resource partitioning in each event and selecting the best choice among these event set gives the optimum resource allocation. We will see that this \( 2^M \) search space can be reduced to \( M + 1 \). The outage is defined to be the event when the mutual information from the source to the destination falls below the given rate. With the power constrain \( P \), we find the resource partitioning which minimizes the outage probability at the given rate \( R \).

3.1. Resource partitioning

Let \( D_i \) denotes the set of active relays in the \( i \)th event \((i = 0, 1, \ldots , 2^M - 1) \). If \( y_0 = |h_0|^2 \) and \( y_{k,j} = |h_{k,j}|^2 \), \( k = 1, 2, j = 1, 2, \ldots , M \), then the \( i \)th event is supported when all the links from the source to the relays in \( D_i \) have the mutual information greater than \( 2R \). Otherwise, this event is discarded from further consideration. This condition is described mathematically as

\[
T_1 \log_2(1 + y_{1,j}P_{s,j}) \geq RT \quad \forall j \in D_i \iff P_{s,j} \geq \frac{2^R - 1}{\gamma_{min}},
\]

where \( \mu = T_1/T \), \( \mu \in (0, 1) \); \( \gamma_{min} = \min \{y_{1,j}, j \in D_i \} \) and the source power \( P_{s,j} \) and \( \mu \) for the \( i \)th event will be determined later. Note that \( \gamma_{min} < (2^R - 1)/P_{s,j} \) is the condition when the \( i \)th event is discarded from the consideration.

Suppose the relays in \( D_i \) are not in outage, then the mutual information from the source to the destination is

\[
I(P_i, \mu \mid D_i)
= \mu \log_2(1 + y_{1,j}P_{s,j}) + (1 - \mu) \log_2 \left[ 1 + \sum_{j \in D_i} y_{2,j}P_{r,j} \right],
\]

where \( P_i = (P_{s,j}, P_{r,j}, j \in D_i) \). (Authors in [2] used orthogonal space-time block coding among the relays in the set \( D_i \) so that the multipaths from the relays can be coherently combined at the destination, which results in the mutual information of MISO channels as in the last logarithm expression of (2). Note the coherent combining of the MISO channel multipaths can also be done by precoding.) The last term in (2) can be maximized by allocation all the relay power

![Figure 1: The DF relay network model.](image)

\( R \)
with the third case, the interval \((0, 1]\) of \(\mu\) is divided into two sections \((0, \mu_1]\) and \([\mu_1, 1]\) with \(P = \mu_1 t(\mu_1)\); the first section, where the condition \(P \leq \mu t(\mu)\) is met, is discarded. The condition for the first case also divides the interval into two sections \((0, \mu_2]\) and \([\mu_2, \infty)\) with \(P = \mu_2 t(\mu_2)/(y_0 - y_{2,i})\); the second section is the region discarded this time. The condition \(\mu_2 \leq \mu_1\) makes all the values of \(\mu \in (0, 1]\) to be trapped in the first or the third case and the event \(E_i\) is dismissed. Otherwise, the condition for the second case divides the remaining interval into two sections \([\mu_1, \mu_2]\) and \([\mu_2, \infty)\) with \(t(\mu_2) = P + (1 - \mu_2)/(y_0 - y_{2,i})y_{0, 2} / y_0 y_{2, i}\); (7) is used for \(\mu \in [\mu_1, \mu_2]\) and (8) is used for \(\mu \in (\mu_2, \infty)\) to find \(\mu\) that maximizes the mutual information for the event \(E_i\). This process is repeated for all \(E_i, i = 0, 1, \ldots, M\) and the best resource partitioning among the \(M + 1\) events is found. If the maximum mutual information does not support the rate \(R\) with the power constraint \(P\), then the channel is in outage. As a baseline system, we consider the case where \(\mu\) is confined to be in the set \(\{1, 1/2\}\).

### 3.2. Relaying probability

While open loop DF protocol trades the throughput with the diversity gain (see [2]), the DMT analysis of DDF protocol in [7] shows that it achieves the diversity without much throughput loss compared to DF protocol. Hence in this subsection, we compare the proposed scheme with DDF in throughput aspect.

The DDF protocol tries to control the cooperation phase without power control, hence it seems that the DDF performs worse than the proposed scheme in this paper. But, the source in the DDF protocol continues the transmission during the time the relays cooperate, which is an advantage over the proposed system. Since the proposed system outperforms both the source-to-destination direct link and the conventional DF protocol where \(\mu\) is fixed to 1/2, the union of DMT curves of these protocols lower bound that of the proposed system. On the other hand, it is obvious that the proposed scheme performs worse than MISO or SIMO links with \(M + 1\) antennas since these correspond to perfect source-to-relay channels or relay-to-destination channels respectively. Thus, DMT curves of these links upper bound that of the proposed scheme. From this observation, we can conclude that the proposed joint time slot and power partitioning introduces the diversity advantage without sacrificing the throughput.

In another view, the probability that \(E_i, i \neq 0\) are selected quantifies how much the relaying contributes to this throughput enhancement. In Section 3, it is shown that a relay cooperates when two independent events \(\{y_0 \leq y_{2,i}/(1 - P y_{2,i}/\mu)\}, \{y_{1,i} \geq (2^R - 1)/P\}\) occur at the same time. The probability that at least a relay among \(M\) relays cooperates with the time slot partitioning \(\mu\) is

\[
P(\mu) = \sum_{i=1}^{M} Pr(y_0 \leq y_{2,i}/(1 - P y_{2,i}/\mu)) \cdot Pr(y_{1,i} \geq (2^R - 1)/P).
\]
Supposing that $\gamma_0$, $\gamma_{1,i}$ and $\gamma_{2,i}$ are Chi-square distributed with degree 2 and $\lambda_i$ being the statistical average of $\gamma$, then we have the following lower bound:

$$
Pr\left(\gamma_0 \leq \frac{\gamma_{2,i}}{1 - P\gamma_{2,i}/\mu}\right)
$$

$$
\geq \int_0^{\mu/2P} \int_0^{\gamma_0} f_\gamma(y)dy + \int_{\gamma_0}^{\mu/2P} \int_{\gamma_0 + 4\mu/2P}^{\gamma(y)} f_\gamma(y)dy
$$

$$
= \frac{\lambda_{2,i}}{\lambda_0 + \lambda_{2,i}} \left[ 1 - \exp\left( -\frac{\mu(\lambda_0 + \lambda_{2,i})}{2\lambda_0 \lambda_{2,i} P} \right) \right]
$$

$$
+ \frac{4\lambda_{2,i}}{\lambda_0 + 4\lambda_{2,i}} \exp\left( -\frac{\mu(33\lambda_0 + 4\lambda_{2,i})}{8\lambda_0 \lambda_{2,i} P} \right)
$$

$$
\approx \frac{\mu}{2P} + \frac{8\lambda_0 \lambda_{2,i} P - \mu(33\lambda_0 + 4\lambda_{2,i})}{2\lambda_0 (\lambda_0 + 4\lambda_{2,i} P)},
$$

(10)

where we used the two tangential lines of $\gamma_0 = \gamma_{2,i}/(1 - P\gamma_{2,i}/\mu)$ at $\gamma_{2,i} = 0$ and $\gamma_{2,i} = \mu/(2P)$ for the lower bounding. The approximation holds at high SNR. Sending $P \rightarrow \infty$ allows us to send $\mu$, the minimum value of $\mu$ in the saved section, to 0. Then, $P \rightarrow \infty$ and $\mu \rightarrow 0$ send (10) to

$$
\frac{4\lambda_{2,i}}{\lambda_0 + 4\lambda_{2,i}}.
$$

(11)

Consider the DDF protocol [7], where the source and relay power are fixed as $P_{s,i} = P_r = P$ and the system controls $\mu$ for the minimum outage transmission. In this case, the cooperation of the $i$th relay is selected if the conditions $\gamma_0 \leq \gamma_{2,i}$ and $\gamma_{1,i} \geq (2^R - 1)/P$ occur at the same time with $\mu = R/\log_2(1 + \gamma_{1,i} P)$. Then, we have

$$
Pr(\gamma_0 \leq \gamma_{2,i}) = \frac{\lambda_0}{\lambda_0 + \lambda_{2,i}}.
$$

(12)

Comparing this to (11) and assuming $\lambda_{2,i} \gg \lambda_0$, (11) is much closer to 1 than (12).

Compared to the fixed time slot case where $\mu$ is confined to be in the set $\{1, 1/2\}$, we can certainly find better $\mu$ with larger cooperation probability than $\mu = 1/2$. These analysis show that the joint partitioning of time slot and power has larger cooperation probability than the partitioning of individual resource only, thus contributes to the throughput enhancement.

4. BEAMFORMING

Recent developments show that multiple antenna technology is the key ingredient in enhancing the wireless communication performance. Therefore, we expect further enhancement of relaying networks by exploiting the beamforming gain from multiple antennas. In this section, we propose methods to combine beamforming and resource partitioning in Section 3 when multiple transmit antennas are used at the source or at the destination. First, we assume $N$ transmit antennas at the source and single antenna for the relays and the destination. The channel gains $h_0$, $h_{1,j}$, $j = 1, \ldots, M$ are $N$-dimensional vectors and $h_{2,j}$, $j = 1, \ldots, M$ are scalars.

Suppose $p$, $|p| = 1$ is the beamforming vector applied at the source and $E_i$ is the event being considered. We have $t(\mu) = (2^R - 1)/(\gamma_{1,i} |p^* h_{1,i}|^2)$, where $\gamma_{1,i} = |h_{1,i}|^2$ when $\tilde{a} = a/|a|$. The mutual information for this event is given as

$$
I(P, \mu|E_i) = \mu \log_2 \left[ 1 + \gamma_0 \left| P^* h_0 \right|^2 P_{s,i} \right]
$$

$$
+ (1 - \mu) \log_2 \left[ 1 + \gamma_{2,i} P_r \right].
$$

(13)

From the condition for the third case in Section 3, it is easy to see that $\mu_1$ is decreased if $|p^* h_{1,i}|$ increased. Since $|\mu_1, \mu_2|$ is the interval with a weakest constraint, we can, obviously, expect better outage performance with a wide second section. Also, increasing $|p^* h_0|$ contributes for the better mutual
information in (8). Thus, \( p \) should be jointly matched to \( \tilde{h}_{1,i} \) and \( \tilde{h}_0 \).

The vector \( \tilde{h}_{1,i} \) can be decomposed as

\[
\tilde{h}_{1,i} = \sqrt{1 - \lambda} \tilde{h}_0 + \lambda \tilde{h}_0 = \cos \theta \tilde{h}_0 + \sin \theta \tilde{h}_0^\perp,
\]

(14)

where \( \tilde{h}_0^\perp \) is perpendicular to \( \tilde{h}_0 \). If we set \( p = \cos \theta \tilde{h}_0 + \sin \theta \tilde{h}_0^\perp \), \( 0 \leq \theta \leq \theta_0 \), the vector \( p \) is positioned between \( \tilde{h}_0 \) and vector \( \tilde{h}_{1,i} \) as shown in Figure 2 and we have \( |p^* \tilde{h}_0| = \cos \theta \) and \( |p^* \tilde{h}_{1,i}| = \cos \theta - \theta \). This parametrization gives \( I(\mu) = (2\mu - 1)/[y_1, \cos^2(\theta - \theta)] \) and

\[
I(P, \mu|E_i) = \mu \log_2[1 + y_0 \cos^2 \theta P_{1,i}] + (1 - \mu) \log_2[1 + y_2, P_i].
\]

(15)

Optimum point in the parameter space determined by \( \mu \in (0, 1] \) and \( \theta \in [0, \theta] \) is to be searched with the four cases as in Section 3 depending on these parameter values.

When there are \( N \) receive antennas at the destination and single antenna for the relays and the source, the channel gains \( h_0, h_{2,j}, j = 1, \ldots, M \) are \( N \)-dimensional vectors and \( h_{1,j}, j = 1, \ldots, M \) are scalars. Since the source and relay transmissions use orthogonal channels in time, we can apply different receive beamforming vectors \( (p) \) for these transmissions. In the event \( E_i, p = h_0 \) is applied for the source transmission and \( p = h_{2,j} \) is applied for the relay transmission.

### 5. Simulation

In Figure 3, the cumulative distributions of mutual information corresponding to different resource partitioning schemes are plotted. The signal to noise ratio \( (P/N_0) \) is set to 10 dB. For the time slot partitioning, we quantize \( \mu \in (0, 1] \) into 10 uniform length regions, the quantized values of which are tested for the maximum mutual information with appropriate power allocation as in Section 3. For the beamforming, the angle \( \theta \in [0, \theta] \) is quantized into 4 uniform regions. Hence, \( 10 \times 4 \) quantized regions are tested for the set of \( \mu \) and \( \theta \). Note the power only control case corresponds to 2-level quantization, hence is different from open loop scheme because it relies on CSI and chooses \( \mu = 1 \), that is, the direct source-to-destination link, according to the conditions in Section 3. With power allocation only, more than three-fold increase in the rate is observed with 10\(^{-4}\) outage probability. Joint power and time slot partitioning gives additional 0.5 bps and the beamforming gives another 0.5 bps at the same outage probability.

The outage probabilities of different schemes against the SNR \( (P/N_0) \) are plotted in Figure 4. Besides that the number of relays is one, all the conditions are the same as in Figure 3. The slopes of the curves represent the diversity order enhancement from the relaying. As shown in Figure 4(a), the resource partitioning schemes give additional power gain over the transmission scheme without relay. As the rate \( (R) \)
increases from 1 bps/Hz to 3 bps/Hz, the benefit of partitioning time slot becomes prominent.

6. CONCLUSION

We present a joint time slot and power partitioning scheme along with a beamforming strategy for the network with multiple DF relays possibly with multiple antennas at the source or destination. Based on the CSI information, the proposed scheme further enhances the throughput as well as the diversity advantage known in open loop relay networks. The analysis of relaying probability indicates the enhancement from the resource partitioning. Supporting simulation results are presented.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. Part I. System description,” IEEE Transactions on Communications, vol. 51, no. 11, pp. 1927–1938, 2003.
[2] J. N. Laneman and G. W. Wornell, “Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks,” IEEE Transactions on Information Theory, vol. 49, no. 10, pp. 2415–2425, 2003.
[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Transactions on Information Theory, vol. 50, no. 12, pp. 3062–3080, 2004.
[4] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, “Coded cooperation in wireless communications: space-time transmission and iterative decoding,” IEEE Transactions on Signal Processing, vol. 52, no. 2, pp. 362–371, 2004.
[5] G. Scutari and S. Barbarossa, “Distributed space-time coding for regenerative relay networks,” IEEE Transactions on Wireless Communications, vol. 4, no. 5, pp. 2387–2399, 2005.
[6] A. Stefanov and E. Erkip, “Cooperative coding for wireless networks,” IEEE Transactions on Communications, vol. 52, no. 9, pp. 1470–1476, 2004.
[7] K. Azarian, H. El Gamal, and P. Schniter, “On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels,” IEEE Transactions on Information Theory, vol. 51, no. 12, pp. 4152–4172, 2005.
[8] N. Ahmed, M. A. Khojastepour, A. Sabharwal, and B. Aazhang, “Outage minimization with limited feedback for the fading relay channel,” IEEE Transactions on Communications, vol. 54, no. 4, pp. 659–669, 2006.
[9] P. A. Anghel and M. Kaveh, “On the performance of distributed space-time coding systems with one and two non-regenerative relays,” IEEE Transactions on Wireless Communications, vol. 5, no. 3, pp. 682–692, 2006.
[10] R. Annavajjala, P. C. Cosman, and L. B. Milstein, “Statistical channel knowledge-based optimum power allocation for relaying protocols in the high SNR regime,” IEEE Journal on Selected Areas in Communications, vol. 25, no. 2, pp. 292–305, 2007.
[11] E. G. Larsson and Y. Cao, “Collaborative transmit diversity with adaptive radio resource and power allocation,” IEEE Communications Letters, vol. 9, no. 6, pp. 511–513, 2005.