A Spalart–Allmaras local correlation–based transition model for Thermo–fluid dynamics

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Abstract. The study of innovative energy systems often involves complex fluid flows problems and the Computational Fluid-Dynamics (CFD) is one of the main tools of analysis. It is important to put in evidence that in several energy systems the flow field experiences the laminar-to-turbulent transition. Direct Numerical Simulations (DNS) or Large Eddy Simulation (LES) are able to predict the flow transition but they are still inapplicable to the study of real problems due to the significant computational resources requirements. Differently standard Reynolds Averaged Navier Stokes (RANS) approaches are not always reliable since they assume a fully turbulent regime. In order to overcome this drawback in the recent years some locally formulated transition RANS models have been developed. In this work, we present a local correlation-based transition approach adding two equations that control the laminar-to-turbulent transition process, \( \gamma \) and \( \tilde{\text{Re}}_{\theta,t} \), to the well-known Spalart–Allmaras (SA) turbulence model. The new model was implemented within OpenFOAM code. The energy equation is also implemented in order to evaluate the model performance in thermal-fluid dynamics applications. In all the considered cases a very good agreement between numerical and experimental data was observed.

1. Introduction

This paper presents the latest developments of a Spalart–Allmaras local correlation based transition model, introduced in [1], for Thermal–Fluid Dynamics applications.

The aim of this work was to contribute to the development of CFD techniques in the prediction of the flow’s laminar-to-turbulent transition. Although Direct Numerical Simulation (DNS) or Large Eddy Simulation (LES) are able to predict flow transition, the DNS approach is still inapplicable to the study of engineering interest problems due to the significant computational resources required. On the other hand, the applicability of the LES technique is very limited for the same reason. Moreover standard Reynolds Averaged Navier Stokes (RANS) models are not always reliable because they assume a fully turbulent regime. To overcome this drawback, some locally-formulated RANS transition models have been developed in recent years for general purpose unstructured CFD codes. They can be divided into two main classes: local correlation-based transition models (LCTM), [2, 3], and eddy viscosity phenomenological transition models, [4]. The first class includes the transitional effects by coupling standard RANS
models with further transport equations and empirical correlations. The second class of transition models, like the \( k - \omega \) introduced by Walters et al. [4], is based on laminar kinetic energy modelling, see [5]. These models have demonstrated an acceptable accuracy in the simulation of laminar-to-turbulent transition for an ample range of applications. The main drawback of LCTM methods, also named \( \gamma - \Re_\theta \) models, lies in the adoption of empirical correlations that are not typically applicable for a wide range of problems, whereas the \( k - \omega \) technique has not always produced satisfactory results in flow cases characterized by large pressure gradient, see [6].

It should be noted that \( \gamma - \Re_\theta \) models were initially coupled with the SST \( k - \omega \) turbulence model by its developers, but the \( \gamma - \Re_\theta \) model could be applied to other models too. In this work we present a coupling between the \( \gamma - \Re_\theta \) approach and the Spalart–Allmaras (SA) turbulence model, [7]. In particular we propose an implementation based on the open-source library OpenFOAM. Our source code has been also released with a GNU GPL license to make public all the computational details behind the \( \gamma - \Re_\theta \)-SA model; the model and a test case are freely downloadable on GitHub at the following address: https://github.com/vdalessa/gammaReThetatSA. The main reason behind the choice to investigate the \( \gamma - \Re_\theta \)-SA approach is that the SA equation has produced very good results in the computation of external flows. Moreover the model also has lower computational costs than the SST \( k - \omega \) model.

We have to remark that in this work we face the computation of the flow field past several wind turbine airfoil in transitional regime as well as conjugate heat transfer problems.

The paper is organized as follows. Sec. 2 presents the equations governing the transition model considered here. Sec. 3 briefly describes the discretization technique. Sec. 4 is devoted to the numerical results and Sec. 5 contains the conclusions.

### 2. Governing equations

The complete set of our flow governing equations can be written as

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0, \\
\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \nabla p - \nabla \cdot ((\nu + \nu_t) (\nabla \mathbf{u} + \nabla \mathbf{u}^T)) &= 0, \\
\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u} T) - \nabla \cdot \left( \left( \frac{\nu}{Pr} + \frac{\nu_t}{Pr_t} \right) \nabla T \right) &= 0, \\
\frac{\partial \nu_t}{\partial t} + \nabla \cdot (\mathbf{u} \nu_t) - P_{\nu} + D_{\nu} - \frac{c_{\nu 2}}{\sigma} \nabla \nu_t \cdot \nabla \nu_t - \frac{1}{\sigma} \nabla \cdot ((\nu + \nu_t) \nabla \nu_t) &= 0,
\end{align*}
\]

where \( \mathbf{u} \) is the velocity vector, \( p = P/\rho \) is the pressure divided by the density, \( T \) is temperature and \( d \) is the distance from the nearest wall; besides \( \nu \) is the kinematic viscosity and \( Pr \) the Prandtl number. The turbulent viscosity, \( \nu_t \), needed to take into account the turbulence, is computed according to the \( \nu_t \) variable as

\[
\nu_t = f_{\nu_1} \nu_t,
\]

on the other hand the turbulent Prandtl number, \( Pr_t \), is introduced to model the turbulent thermal diffusivity. The production and destruction terms appearing in the \( \nu_t \) transport equation are defined as follows:

\[
P_{\nu} = \gamma_{eff} c_{\nu 1} S \nu_t, \\
D_{\nu} = \max(\min(\gamma, 0.5), 1.0) \left[ c_{\nu 2} f_{\nu} \left( \frac{\nu_t}{d} \right)^2 \right].
\]
The term $\gamma_{\text{eff}}$ in eq. 3 is a term devoted to model the separation-induced transition and it is defined as follows:

$$\gamma_{\text{eff}} = \max (\gamma, \gamma_{\text{sep}})$$  \hspace{1cm} (4)

with

$$\gamma_{\text{sep}} = \min \left( 2.0 \cdot \max \left[ 0, \left( \frac{\text{Re}_\nu}{3.235 \text{Re}_{\theta,c}} \right) - 1 \right], \text{F}_{\text{reattach}}, 2.0 \right) \text{F}_{\theta,t}$$  \hspace{1cm} (5)

and

$$\text{F}_{\text{reattach}} = \exp \left( -\frac{R_T}{20} \right)^4.$$  \hspace{1cm} (6)

The following closure functions are now introduced to complete the definition of the SA equation given in eq. 1:

$$f_{v1} = \frac{\chi^3}{(\chi^3 + c_{b1}^3)^r},$$

$$f_{v2} = 1 - \frac{\chi}{(1 + \chi f_{v1})},$$

$$g = r + c_{w2} (r^6 - r),$$

$$f_w = g \left[ \frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{4/3},$$

$$\tilde{S} = |\Omega + \min (0, S - \Omega)| + \frac{\tilde{\nu}}{k^2d^2} f_{v2},$$

$$r = \begin{cases} r_{\max} & \text{if } \frac{\tilde{\nu}}{S k^2 d^2} < 0 \\ \min \left( \frac{\tilde{\nu}}{S k^2 d^2}, r_{\max} \right) & \text{otherwise} \end{cases},$$  \hspace{1cm} (7)

where $\chi = \tilde{\nu}/\nu$ is the dimensionless turbulent variable, $\Omega = \sqrt{2W} : W$ is the vorticity tensor module, $S = \sqrt{2D} : D$ is the strain rate tensor module and $\tilde{S}$ is a function of both the vorticity magnitude, $\Omega$, and $\tilde{\nu}$. The $r$ function is estimated as in [8] to increase the numerical stability, and $r_{\max}$ is a constant positive value typically set at 10. Finally, to fully define the SA model, the following standard closure constants are adopted

$$c_{b1} = 0.1355, \quad c_{b2} = 0.622, \quad c_v = 7.1, \quad \sigma = 2/3, \quad c_{w1} = \frac{c_{b1}}{k^2} + \frac{(1 + c_{b2})}{\sigma},$$

$$c_{w2} = 0.3, \quad c_{w3} = 2, \quad k = 0.41.$$  \hspace{1cm} (8)

The transport equations needed to model the transition are:

$$\frac{\partial \gamma}{\partial t} + \nabla \cdot (u \gamma) = P_\gamma - D_\gamma + \nabla \cdot \left[ \left( \nu + \frac{\nu_t}{\sigma_f} \right) \nabla \gamma \right],$$

$$\frac{\partial \tilde{\text{Re}}_{\theta,t}}{\partial t} + \nabla \cdot (u \tilde{\text{Re}}_{\theta,t}) = P_{\theta,t} + \nabla \cdot \left[ \sigma_{\theta,t} (\nu + \nu_t) \nabla \tilde{\text{Re}}_{\theta,t} \right].$$  \hspace{1cm} (11)

The source terms in the $\gamma$ equation are defined as:

$$P_\gamma = c_{a1} S \gamma F_{\text{onset}} \left[ 0.5 \left( 1 - c_{c1} \gamma \right) F_{\text{length}} \right],$$

$$D_\gamma = c_{a2} \Omega F_{\text{turb}} (c_{c2} \gamma - 1),$$

in $P_\gamma$ the term $F_{\text{onset}}$ is computed as:

$$F_{\text{onset}} = \max (F_{\text{onset},2} - F_{\text{onset},3}, 0)$$  \hspace{1cm} (12)
with
\[
F_{\text{onset},2} = \min \left( \max \left( F_{\text{onset},1}, F_{\text{onset},1}^4 \right), 4 \right),
\]
\[
F_{\text{onset},3} = \max \left( 2 - \left( \frac{R_T}{2.5} \right)^3, 0 \right),
\]
\[
F_{\text{onset},1} = \frac{\text{Re}_\nu}{2.193 \text{Re}_{\theta,c}}.
\]

In eq. 14 the terms \( \text{Re}_\nu \) and \( R_T \) are obtained as follows:
\[
\text{Re}_\nu = \frac{Sd^2}{\nu}, \quad R_T = \frac{\nu_t}{\nu}.
\]

Note that the parameter \( R_T \) is redefined because, for the \( k-\omega \) model, it requires the estimation of \( \omega \). The definition based on the viscosity ratio, \( \nu_t/\nu \), enables the SA equation to be adopted. The aspects concerning the terms: \( \text{F} \text{length} \) and \( \text{Re}_{\theta,c} \) are described in the next subsection. For the destruction term, \( D_\gamma \), on the other hand, the coefficient \( \text{F}_\text{turb} \) is defined as:
\[
\text{F}_\text{turb} = \exp \left( -\frac{R_T}{4} \right)^4,
\]

As for the source terms in the transport equation for \( \tilde{\text{Re}}_{\theta,t} \), \( P_{\theta,t} \), the following equation is adopted:
\[
P_{\theta,t} = \frac{c_{\theta,t}}{T} \left( \text{Re}_{\theta,t} - \tilde{\text{Re}}_{\theta,t} \right) \left( 1 - \text{F}_{\theta,t} \right).
\]

In eq. 17 the last term \( \text{F}_{\theta,t} \) is defined as:
\[
\text{F}_{\theta,t} = \min \left( \max \left( \exp \left( -\frac{\left| u \right|^2}{375 \Omega \nu \text{Re}_{\theta,t}} \right)^4, 1 - \left( \frac{\gamma - 1/c_{e2}}{1 - 1/c_{e2}} \right)^2 \right), 1.0 \right).
\]

The term \( T \) appearing in the source term of the \( \tilde{\text{Re}}_{\theta,t} \) equation is also defined as follows: \( 500\nu/\left| u \right|^2 \). Finally, the computation of \( \text{Re}_{\theta,t} \) in eq. 17 is discussed, together with the \( \text{F}_{\text{length}} \) coefficient, in the next subsection.

For the turbulence model, the following closure constants are adopted in order to close eq. 11
\[
c_{a1} = 2.0, \quad c_{a2} = 0.06, \quad c_{e1} = 1.0,
\]
\[
c_{e2} = 50, \quad c_{\theta,t} = 0.03, \quad \sigma_f = 1.0,
\]
\[
\sigma_{\theta,t} = 2.0.
\]

2.1. Empirical correlations in the model

Like other \( \gamma-\text{Re}_{\theta,t} \) approaches available in the literature, the present model contains three empirical correlations needed to compute \( \text{Re}_{\theta,t} \), \( \text{Re}_{\theta,c} \) and \( \text{F}_{\text{length}} \).

\( \text{Re}_{\theta,c} \), which appears in eq. 14, is the critical Reynolds number where the intermittency, \( \gamma \), starts to increase in the boundary layer. This typically occurs upstream from the transition Reynolds number, \( \text{Re}_{\theta,t} \). This element relates to the delay between when the turbulence starts and when appreciable turbulence levels are reached within the boundary layer. Note that this last feature is essential to obtain a significant change in the laminar velocity profile. \( \text{F}_{\text{length}} \), appearing in the production term of the \( \gamma \) transport equation, is an empirical correlation that controls the length.
of the transition region. In this paper for \( \text{Re}_\theta,t \) we adopt a correlation developed by Menter et al. [2]:

\[
\text{Re}_{\theta,t} = \begin{cases} 
(1173.51 - 589.428 \cdot Tu + 0.2196/Tu^2) F(\lambda_\theta) & Tu \leq 1.3 \\
331.5 (Tu - 0.5668)^{-0.671} F(\lambda_\theta) & Tu > 1.3 
\end{cases},
\]  

(22)

\[
F(\lambda_\theta) = \begin{cases} 
1 + [12.986\lambda_\theta + 123.66\lambda_\theta^2 + 405.689\lambda_\theta^3] \exp\left(-\frac{(Tu + 0.15)}{\lambda_\theta}\right) & \lambda_\theta \leq 0 \\
1 + 0.275 [1 - \exp(-35\lambda_\theta)] \exp\left(-\frac{Tu}{\lambda_\theta}\right) & \lambda_\theta > 0
\end{cases}.
\]  

(23)

It is important to note that the correlations in eq. 22, 23 contain the turbulence intensity \( Tu \). In the framework of the \( k-\omega \) model, \( Tu \) can be computed using the solution to \( k \) equation. In this work, we adopt the approach introduced in [9, 10]. Specifically, we establish that \( Tu = Tu_\infty \) for all the points of the flow field. Moreover \( \text{Re}_{\theta,t} \) is computed by iterating on the value of \( \theta_t \), since \( \text{Re}_{\theta,t} \) is a function of \( \theta_t \) itself because of the presence of \( \lambda_\theta \). Differently for \( \text{Re}_{\theta,c} \) and \( F_{\text{length}} \) we use the correlations introduced by Malan et al. [11]:

\[
\text{Re}_{\theta,c} = \min \left( 0.615\text{Re}_{\theta,t} + 61.5, \text{Re}_{\theta,t} \right),
\]

(24)

\[
F_{\text{length}} = \min \left( \exp\left(7.168 - 0.01173\text{Re}_{\theta,t}\right) + 0.5, 300 \right).
\]

(25)

### 2.2. Boundary conditions

Standard boundary conditions are adopted for \( \tilde{\nu} \): \( \tilde{\nu}_\infty = 3\nu \) at the free stream and \( \tilde{\nu} = 0 \) at the wall, while the boundary condition for \( \gamma \) at the wall is zero normal flux. At the inlet, the value of \( \gamma \) is 1. The boundary condition for \( \text{Re}_{\theta,t} \) at the wall is zero flux, while at the inlet \( \text{Re}_{\theta,t} \) is calculated from the specific empirical correlation based on the inlet turbulence intensity.

It is also very important to note that, in order to capture the laminar and transitional boundary layers correctly, the grid must have a viscous sub-layer scaled first cell height, \( y^+ \), of approximately 1 [3]. The value of \( y^+ \) is estimated as \( y^+ = \frac{u_*}{\nu} y_c \), where \( u_* = \sqrt{\tau_w/\rho} \) is the friction velocity, \( \tau_w \) is the viscous stress component measured at the wall, and \( y_c \) is the height of the cells next to the wall.

### 3. Numerical solution

The governing equations described in Sec. 2 are solved numerically by adopting a finite volume method (FVM). In particular, we used simpleFoam, which is the steady solver for incompressible flows available in the official releases of the OpenFOAM (Open-source Field Operation and Manipulation) code [12], which is unstructured finite volume solver released under the GNU Public License (GPL). The code’s object-oriented structure enables users to implement their own models and solvers in the baseline codes with relatively little effort (see for example [13, 14]). simpleFoam uses the well-established SIMPLE algorithm [15] for pressure–velocity decoupling and, like the other standard solvers, it is based on a colocated FVM approach. The Rhie–Chow correction is used to remove oscillations in the solutions [16]. For conjugate heat transfer (CHT) problems a fully coupled approach for energy equation is adopted.

For all the computations presented in this paper the diffusive terms and pressure gradients were approximated with second-order accurate central schemes. The convective terms for momentum, energy and turbulence equations were handled with a second order accurate linear–upwind scheme. For the linear solvers a preconditioned bi-conjugate gradient method (PBiCG) with the DILU preconditioner was used to solve the discretized momentum, \( \tilde{\nu} \), \( \gamma \) and \( \text{Re}_{\theta,t} \) equations. A preconditioned conjugate gradient method (PCG) with a diagonal incomplete-cholesky preconditioner was adopted instead for the pressure, while energy equation a Biconjugate Gradient
Stabilized (BiCGStab) method with Cholesky preconditioner was used. In particular, a local accuracy of $10^{-7}$ was established for the pressure, whereas other linear systems were considered as converged when the residuals reached the machine precision.

4. Results

4.1. Eppler 387 airfoil

In this sub-section we consider the flow over an Eppler 387 (E387) airfoil; see Fig. 1 for a representation of the solution. This case was chosen as a test case involving a separation-induced transition. The E387 airfoil is particularly appealing for this purpose because McGhee et al. [17] have published a large experimental dataset also involving the distribution of $c_p$ along the airfoil surface at several Re numbers. Indeed it was used in several paper for validation purposes, [18, 19].

The solutions reported here were computed using a structured computational grid having about $n_c = 2.2 \cdot 10^5$ and a C-shaped domain. The inflow and outflow boundaries were placed at about 10 times the airfoil chord, $c$, from the airfoil. The cells were clustered near the airfoil surface and the first cell height was arranged in order to obtain $O(y^+) \approx 1$. The effect of the angle of attack, $\alpha$, on the pressure distribution, for the case at $Re = 3 \cdot 10^5$, can be seen in Figure 2.

For all the reported angles of attack, the results obtained with the well-established XFOIL code [20] (which is a recognized method for use in airfoil design) have also been included. The data presented here were obtained using the default settings for the critical amplification factor, $N_{crit} = 9$. It is very easy to see that XFOIL tends to overestimate the $c_p$ value with respect to the experimental results, while the RANS transition model proposed here is globally very consistent with the experimental findings. The reported results reveal a good agreement between the experimental and numerical data in the context of laminar separation bubble (LSB) simulation.

The force coefficients plotted at $Re = 3 \cdot 10^5$ from Fig. 3, confirm the very good consistency between the numerical and experimental data. In all these figures, for further comparison, the results obtained with the XFOIL code have been included too. Prior to the stall region, the force
coefficients computed by the two completely different approaches are clearly always in very good agreement. Our $\gamma$–Re$_{\theta_{1}}$–SA model is able to capture airfoil performance in the stall region very well too.

4.2. SD7003 airfoil

The Selig–Donovan (SD) 7003 airfoil was also investigated at Re $= 6 \cdot 10^4$ and $\alpha = 4^\circ$, our numerical solutions are depicted in Fig. 4. The results obtained in this work were compared with LES and ILES computations performed by Catalano and Tognaccini, [21] and Galbraith and Visbal, [22] respectively. Very recently Islam et. al. [23] performed RANS computations based on SST $k$–$\omega$ and a local correlation based transition model. We have also compared our results with their ones.

The solutions reported in Fig. 5 were computed using a structured computational grid having

Figure 2. E387, Re $= 3 \cdot 10^5$. Pressure coefficients at several angles of attack.
Figure 3. E387, Re = 3 \cdot 10^5. Force coefficients.

Figure 4. SD7003, Re = 6 \cdot 10^4. \alpha = 4^\circ.

similar features to that employed for the E387 airfoil. Our pressure coefficient distribution is very close to Islam et. al. [23], however there are noticeable differences in the LSB zone with literature LES and ILES results. This result demonstrates objective evidence of LCTM based transition approaches in the prediction of separation-induced transition phenomena at very low Reynolds numbers. In Fig. 5(b) the skin friction coefficient, \( C_f \), distribution around the airfoil is showed. The LSB is situated between the separation point, \( x_s \), and the and reattachment point, \( x_r \); in Fig. 5(b) is clearly showed that between \( x_s \) and \( x_r \) we have \( C_f < 0 \), thus the flow experiences a recirculation region. We have to put in evidence that RANS solutions over-predict the reattachment point significantly when compared with the ILES/LES results. SA and \( k-\omega \)
models show similar $x_r$ values, however it is also evident that their $C_f$ results show sensible differences in the both LSB and turbulent attached region.

In Table 1, aerodynamic coefficients obtained from this research work with are compared with literature data. It can be seen that our approach exhibits a good agreement with $k-k_L-\omega$ data published by Islam et al. [23]. Differently DNS/LES based approach finds a better agreement with experimental data especially for what concerns the lift coefficient.

### 4.3. Conjugate Heat Transfer over a flat plate

In this subsection we present the CHT modeling of a case derived from the commonly used Schubauer and Klebanoff benchmark [26]. The flow problem is a zero pressure gradient flat plate with a very low free-stream turbulence intensity, i.e. $Tu = 0.03\%$, so it is used to test transition models in natural transition conditions.

The present calculations were performed on a computational mesh having two domains, one for a fluid and another for a solid. The fluid domain and the computational grid adopted for its geometric discretization were the same used in [1]. The solid one was represented by a flat plate with a thickness of 0.015 $m$; at the fluid interface the solid’s grid shares the same cells’ distribution used for the fluid domain, along its thickness the mesh elements were clustered.
near the fluid–solid interface. The bottom side of the plate was cooled, it was established by constant temperature $T = 300 \, K$; the incoming wind temperature was fixed to $293.15 \, K$. For the fluid domain we have adopted a Prandtl number equal to 0.71, on the other hand the solid was represented by a glass, i.e. $\rho = 2500 \, Kg/m^3$, $c = 789 \, J/kgK$ and $\lambda = 0.55 \, W/mK$.

Our results were obtained using an energy equation coupled CHT solver available within foam-extend code which is a fork of the OpenFOAM library (developed with the aim to integrate community contributions). Hence in this paper, in order to perform CHT computations, we have also ported the model described in Sec. 2 to foam-extend v4.0. For the sake of validation we have compared our results with $k-\omega-\gamma-\widetilde{Re}_t$ approach available within Ansys-Fluent v16.0 code. Since $k-\omega$ produces the decay of turbulence from the inlet to the flat plate leading edge the free-stream turbulence was increased up to $Tu = 0.2\%$. This element, in some sense, can be also considered a further advantage of the SA–$\gamma-\widetilde{Re}_t$ model. Indeed, our modeling technique fully transports $Tu$ avoiding the calibration process required by the turbulence decay.

In Fig. 6 we present the obtained the results in terms of skin-friction coefficient, $C_f = 2\tau_w/(\rho u_\infty^2)$, and Stanton number, $St = h/(\rho c_p u_\infty)$. The dimensionless parameters reported in Fig. 6(a) and Fig. 6(b) show very good agreement, thus we can retain the presented CHT approach sufficiently reliable.

5. Conclusions
In this work, we have presented a $\gamma-\widetilde{Re}_t$ RANS model for transitional flows based on the SA equation. The main reasons behind this approach are as follows: (i) the SA model offers very reliable results for external flow applications; and (ii) the SA model has lower computational costs than the SST $k-\omega$ equations.

We obtained a good feedback from our model’s implementation in predicting airfoil performance, as well as in the computation of a CHT problem on a zero pressure gradient flat plate. In all cases, our model implementation showed that the model is effective in predicting flow feature even in the airfoils stall region. Future research will focus on extending the $\gamma-\widetilde{Re}_t$–SA model to the Detached Eddy Simulation technique.
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