Core localized alpha-channeling via low frequency Alfvén mode generation in reversed shear scenarios

Z. Qiu\textsuperscript{1,2}, S. Wei\textsuperscript{1}, T. Wang\textsuperscript{1,2}, L. Chen\textsuperscript{1,3} and F. Zonca\textsuperscript{2,1}

\textsuperscript{1}Inst. Fusion Theory \& Simulation, School of Physics, Zhejiang Univ., Hangzhou, P.R.C.
\textsuperscript{2}Center for Nonlinear Plasma Science and C.R. ENEA Frascati, C.P. 65, 00044 Frascati, Italy
\textsuperscript{3}Dept. Physics \& Astronomy, University of California, Irvine CA 92697-4575, U.S.A.

Abstract

A novel channel for fuel ions heating in tokamak core plasma is proposed and analyzed using nonlinear gyrokinetic theory. The channel is achieved via spontaneous decay of reversed shear Alfvén eigenmode (RSAE) into low frequency Alfvén modes (LFAM), which then heat fuel ions via collisionless ion Landau damping. The conditions for RSAE spontaneous decay are investigated, and the saturation level and the consequent fuel ion heating rate are also derived. The channel is expected to be crucial for future reactors operating under reversed shear configurations, where fusion alpha particles are generated in the tokamak core where the magnetic shear is typically reversed, and there is a dense RSAE spectrum due to the small alpha particle characteristic dimensionless orbits.

Energetic particles (EPs) as well as fusion alpha particles related physics \cite{1,2} are key elements towards understanding the performance of future fusion reactors, among which two crucial topics are EPs transport loss by self-generated collective oscillations such as shear Alfvén wave (SAW) eigenmodes \cite{1,2} and searching for alternative/complementary routes to transfer EP power to fuel ions, i.e., alpha-channeling \cite{3}. Both processes are influenced by the saturation level and spectrum of SAWs. In this contribution, a channel for reversed shear Alfvén eigenmode (RSAE) \cite{4} nonlinear saturation is proposed and analysed, which is expected to play significant roles in future reactor-scale tokamaks with rich spectrum of core-localized RSAEs \cite{5} due to the reversed shear magnetic configuration and small dimensionless EP orbit size. In this proposed process, a RSAE spontaneously decays into another RSAE and a low frequency Alfvén mode (LFAM), which can be ion Landau damped, leading to effective heating of thermal ions in the reversed shear region, and consequently, enhanced fusion performance.

We consider for simplicity low-$\beta_i$ plasma such that the frequency separation between RSAE and LFAM required for resonant mode coupling can be well satisfied. The nonlinear coupling is dominated by thermal plasma contribution, while the RSAEs are excited by EPs, so the thermal plasma nonuniformity can be neglected, which is also consistent with the advanced scenario of reversed shear configuration. The governing equations describing nonlinear interactions among RSAEs and LFAM with all predominantly SAW polarization can be derived from nonlinear gyrokinetic vorticity equation \cite{6} and quasi-neutrality condition, with the particle response derived from nonlinear gyrokinetic equation \cite{7}.
The general equation for three SAWs nonlinear interaction, with the matching condition being \( \Omega_3(\omega_3, k_3) = \Omega_1(\omega_1, k_1) + \Omega_2(\omega_2, k_2) \), can be derived as

\[
\delta \phi_{k_3} = - \frac{i}{\omega_3} \Lambda_{k_3,k_1} \left[ (b_{k_3} - b_{k_1}) \left( 1 - \frac{k_{||}^2}{\omega_1 \omega_2} \right) + b_{k_1} V_A^2 k_{||}^3 \left( \frac{k_{||}^2}{\omega_1^2} - \frac{k_{||}^2}{\omega_2^2} \right) \right] \delta \phi_{k_1} \delta \phi_{k_2}, \tag{1}
\]

with \( \delta \phi_k \equiv -k_3^2 V_A^2 / \omega_3^2 + 1 - \omega_3^2 / \omega_k^2 \) being the SAW dielectric function in the WKB limit, \( \omega_G \equiv \sqrt{7/4 + T_e / T_{\text{liq}} / R_0} \) being the leading order geodesic acoustic mode frequency \([8]\), accounting for SAW continuum upshift and creation of beta-induced continuum gap, and \( \Lambda_{k'^n,k} \equiv (c / B_0) \hat{b} \cdot k'^n \times k' \) with \( \hat{b} \) being the unit vector along the equilibrium magnetic field \( B_0 \).

Equation (1) describes the nonlinear evolution of SAWs, with \( \Omega_3 \) modified by the beating of \( \Omega_1 \) and \( \Omega_2 \), the first term on the right hand side due to the competition of Reynolds and Maxwell stresses, and the second term from finite parallel electric field contribution to field line bending.

Note that, since \( (\omega_1 + \omega_2) \simeq (k_{||}^1 + k_{||}^2) V_A \), \( \Omega_3 \) naturally satisfies the SAW D.R. and can be strongly excited if it is a normal mode of the system, leading to significant spectral transfer of SAW turbulence. We note that, in the expression of \( \Lambda_{k} \), effects of wave-particle interactions are not included, consistent with the \( k_{||} v_i \ll \omega_k \) ordering for bulk non-resonant ions. However, finite Landau damping due to resonance with ions is crucial for alpha-channeling, and will be recovered formally in the later analysis by inclusion of the anti-Hermitian part of \( \delta \phi_k \) \([9]\).

**Parametric decay of RSAE**

Equation (1) will be applied to the nonlinear decay of a pump RSAE \( \Omega_0(\omega_0, k_0) \) into a RSAE sideband \( \Omega_1(\omega_1, k_1) \) and a LFAM \( \Omega_B(\omega_B, k_B) \), with the frequency/wavenumber matching condition \( \Omega_0 = \Omega_1 + \Omega_B \) assumed without loss of generality. For RSAE and LFAM being dominated by single-\( n \) and single-\( m \) mode structures, we take \( \delta \phi_k = A_k(t) \Phi_k(x) \exp \left( -i \omega_k t + i n \xi - i m \theta \right) \), with \( A_k(t) \) being the slowly varying mode amplitude, \( \Phi_k(x) \) the parallel mode structure localized about \( q_{\text{min}} \) with \( x \equiv nq - m \), and the normalization condition \( \int |\Phi_k|^2 dx = 1 \) is satisfied.

For the effective transfer of alpha particle energy to core ions, \( \omega_B \leq O(v_i / (q R_0)) \), and thus, \( |\omega_B| \ll |\omega_0|, |\omega_1| \) and \( k_B \approx 0 \). Thus, the \( q_{\text{min}} \) surface also corresponds to the rational surface of \( \Omega_B \), i.e., \( \Omega_B \) is the LFAM in the reversed shear configuration, as investigated theoretically \([10]\). We then have, \( \omega_0 \simeq \omega_1 \) and \( k_{||} 0 \simeq k_{||} 1 \). Effects of small frequency mismatch on the decay process will be discussed later.

The nonlinear RSAE sideband and LFAM equations can be derived from equation (1) as

\[
\mathbf{\hat{b}}_1 \cdot \Phi_1 A_1 = - \frac{i}{\omega_1} \left( A_{k_1}^* \Phi_0 \Phi_B \right)_x A_0 A_B^*, \tag{2}
\]

\[
\mathbf{\hat{b}}_B \cdot \Phi_B A_B = - \frac{i}{\omega_B} \left( A_{k_B}^* \Phi_0 \Phi_1 \right)_x A_0 A_1^*, \tag{3}
\]

with \( \alpha_1 \equiv (b_0 - b_B)(1 - k_{||} B k_{||} B V_A^2 / (\omega_0 \omega_B)) + b_B V_A^2 (k_{||} 1 / \omega_1) (k_{||} B / \omega_B - k_{||} 0 / \omega_0), \alpha_B \equiv (b_0 - b_B)(1 - k_{||} B k_{||} B V_A^2 / (\omega_0 \omega_B)) + b_B V_A^2 (k_{||} B / \omega_B) (k_{||} 1 / \omega_1 - k_{||} 0 / \omega_0), \ldots \right)_x \equiv \int \cdots dx \) denoting averaging over the fast radial scale, \( \mathbf{\hat{b}}_1 \cdot \Phi_1 b_1 \delta \phi_1 dx \) being the \( \Omega_1 \) eigenmode local dispersion function, and \( \mathbf{\hat{b}}_B \cdot \Phi_B \) being the local dispersion function for the LFAM eigenmode.
The parametric decay dispersion relation for RSAE decaying into another RSAE and LFAM can then be derived by combining equations (2) and (3)

$$\hat{\delta} B^* \simeq \left( \hat{\Lambda}_{k_0,k_B} \right) \frac{\alpha_N}{\hat{b}_B \omega_B \omega_0} \hat{C}^2 |A_0|^2,$$

with \( \hat{C} \equiv \langle \Phi_0 \Phi_0 \Phi_1 \rangle \), \( \hat{\Lambda}_{k_0,k_B} = \langle \hat{\Lambda}_{k_0,k_B} \rangle \), \( \alpha_N \equiv \alpha_1 \alpha_B \), and \( \hat{C} \simeq \sqrt{2 \Delta_B / (\sqrt{\pi} \Delta_0 \Delta_1) } \), with \( \Delta_0 \sim \Delta_1 \sim O(1) \) and \( \Delta_B \sim O(\beta^{1/2}) \) being the characteristic radial widths of the respective linear parallel mode structures. Expanding \( \hat{\delta} \simeq i \partial_{\omega_0} \hat{\delta}_1 (\partial_1 + \gamma_1) \simeq (2i/\omega_1)(\gamma + \gamma_1) \) and \( \hat{\delta} B^* \simeq (-2i/\omega_B)(\gamma + \gamma_B) \) in the local limit, with \( \gamma \) denoting the slow temporal variation of \( \Omega_1 \) and \( \Omega_B \) due to the parametric instability, and \( \gamma_1/\gamma_B \) being the linear damping rates of RSAE/LFAM accounted for by the anti-Hermitian part of \( \hat{\delta}_1/\hat{\delta}_B \). one obtains

$$\left( \gamma + \gamma_1 \right) \left( \gamma + \gamma_B \right) = \left( \hat{\Lambda}_{k_0,k_B} \right) \frac{\alpha_N}{4 \hat{b}_B \omega_0 \omega_1} \hat{C}^2 |A_0|^2.$$

The condition for the pump RSAE spontaneous decay can thus be obtained from equation (5) as \( \alpha_N > 0 \) and \( \left( \hat{\Lambda}_{k_0,k_B} \right)^2 \alpha_N \hat{C}^2 |A_0|^2 / (4 \hat{b}_B \hat{b}_1) > \gamma_B \gamma_1 \) for the nonlinear drive overcoming the threshold due to \( \Omega_1 \) and \( \Omega_B \) Landau damping.

The nonlinear dispersion relation is very complex, and depends on various conditions including the polarization and mode structure of the three modes involved. For further analytical progress, the WKB limit and the strong assumption of \( k_{\parallel B} \to 0 \) is adopted, and a parameter regime can be identified for the spontaneous decay process to strongly occur, which corresponds to \( k_{\perp 1} \gg k_{\perp 0} \), such that \( (b_0 - b_1)(b_0 - b_B - b_1) > 0 \); and \( \alpha_N > 0 \) can be satisfied with \( 1 - k_{\parallel 0} k_{\parallel 1} V_A^2 / (\omega_0 \omega_1) > 0 \), which generally requires \( \Omega_1 \) being excited above the local SAW continuum accumulation point with \( n_1 q_{\text{min}} < m_1 \).

The threshold condition for the RSAE spontaneous decay, for the proposed parameter region of RSAE “normal cascading” to \( |k_{\perp 1}| \gg |k_{\perp 0}| \), can be estimated as

$$\left| \frac{\delta B_{\perp 0}}{B_0} \right|^2 > \frac{4 \gamma_1 \gamma_B k_0^2}{\omega_0 \omega_1} \frac{k_{\perp 1}^2}{C^2} \frac{1}{1 - k_{\parallel 0} k_{\parallel 1} V_A^2 / (\omega_0 \omega_1)} \sim \mathcal{O}(10^{-7}),$$

and is comparable with or slightly higher than typical threshold condition for other dominant nonlinear mode coupling processes, e.g., ZS generation. This threshold amplitude, is also consistent with typical SAW instability intensity observed in experiments. Thus, this channel could be an important process in determining the nonlinear dynamics of RSAE.

**Nonlinear saturation and core-localized ion heating**

The RSAE saturation level can be estimated by considering the feedback of the two sidebands to the pump RSAE, which can be derived from equation (1) as

$$\hat{b}_0 \hat{b}_0 A_0 \simeq -\frac{i}{\omega_0} \hat{\Lambda}_{k_0,k_0} \hat{\alpha}_0 \hat{C} A_1 A_B,$$
with α₀ = (b₁ − b_B)(1 − k∥B∥k₁V₂/(ω₁ω₁)) + b₀V₂(k∥B∥ω₁)(k∥B∥ − k₁)/ω₁. The saturation level of LFAM, can be estimated from the fixed point solution of equations (2), (3) and (7), and one obtains, |A|² = y₁y₁b₁b₀V₂/(ω₁ω₁)(k∥B∥/ω₁)(k∥B∥/ω₁), and the ion heating rate due to LFAM Landau damping, can be estimated as

\[ P_i = 2γ_0b_B \frac{∂ \dot{E}_{B,\alpha}}{∂ \dot{E}_{B,\alpha}} \frac{n_{0}e^2}{T_i} |A|² \sim 10^{-3} γ_0 nT. \] (8)

The obtained core ion heating due to LFAM collisionless damping, can be comparable to Coulomb collisional heating estimated by nT/τE, with τE being the energy confinement time.

This channel, achieved via the Landau damping of secondary LFAM, noting that k∥B ≪ 1, is highly localized around the qmin surface (this conclusion can also be obtained, noting as the “secondary” LFAM structure will be determined by the primary RSAE, with a narrower extent than the primary RSAEs), will deposit fusion alpha particle power locally and heating core ions, leading to direct improvement of fusion performance in the tokamak center. The nonlinear dynamics of RSAE with multiple channels accounted for simultaneously [5, 11, 12] is crucial for the understanding of core plasma behaviour and fusion performance of future reactors.

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