The Identification Difficulty of Quantitative Reasoning Process toward the Calculus Students’ Covariation Problem

Syarifuddin¹, T Nusantara², A Qohar², M Muksar²

¹Mathematics Education Program, STKIP Bima, Indonesia
²Faculty of Mathematics and Natural Sciences, State University of Malang, Indonesia

Abstract. This study aims to identify the calculus students’ difficulties who involve in the process of quantitative reasoning toward covariation problems. The students of mathematics education program who are taking the calculus involve as the subject in this research. The research process is done in some stages; namely: first, assigning covariation problem within 60 minutes. Second, correcting and selecting the results that show the student’s difficulties of the covariation problem, then it is followed by confirming students’ answers through task-based interview process. Further, the process analyzed qualitatively by describing the students’ difficulties in the reasoning process. The research results are the calculus students’ difficulties descriptions in quantitative reasoning problems of covariation process, including: 1) the difficulties in building quantitative structures to generate new quantities; 2) the difficulties in identifying the curved graph on the covariation problem due to the tendency of linear quantity construction; 3) there was 86.27% calculus students have difficulties in the quantitative reasoning toward covariation problem process.

1. Introduction

The studies of calculus students’ difficulties derived from their poor comprehension of function concepts. The initial study on function concepts revealed students’ misunderstanding of: interpreting a graph, in which they tended to see it as an event picture, instead of representation of how two quantities changed interrelated [14]; and of perceiving a function as an answer of mapping input to output process [1,3].

Carlson et al [5] studied the students’ readiness to learn calculus in terms of basic and reasoning competencies. In the study, Carlson et al utilized CCR (The Calculus Concept Readiness), which consists of 25 multiplied items. The result showed the students’ readiness of calculus concept was poor on basic and reasoning competencies. There were a few students of high CCR who were ready and passed calculus.

Stalvey & Vidakovic [19] and Yemen, Ulusoy, & Işıksa [20] conducted studies to observe the relationship of quantity and drawing Cartesian graph. Both studies revealed a few numbers of students who can reflect the relationship of two quantities in interpreting a dynamic event. In fact the high competence students found difficulties in modeling dynamic functional situation, including the covariational quantities [3,4,10,13]. Further study of Johnson [9], found the change level remained as the difficult concept for college and secondary students.
The students’ incompetence to relate the quantity in certain situation (covariation problem) is a significant matter of mathematics in term of quantitative reasoning. Elaborating quantitative reasoning is not simple and fast, it takes times for years to be proficient and skillful in providing solutions [18]. The students’ incapability of comprehending partial relationship as a whole is an including matter as well [7,12]. As a matter of fact, the quantitative reasoning needs to be expanded earlier to the primary and secondary students, in which it shapes second nature of using quantitative relationship among them.

2. Method
Recent study applies qualitative method, in which descriptive explorative approach is employed with following steps:

2.1. Subject selection
The students of mathematics education program who are taking the calculus involve as the subject in this research. There were 51 students joined in the research and did the test. As the study aimed to identify the difficulty of quantitative reasoning process toward the calculus students’ covariation problem, thus the subject is the students who provided the wrong answer of covariation problems. Of the conducted test, there were 44 students whose answers were incorrect. The 44 incorrect answers were categorized into similar groups among the similar criteria to be considered on the next step of analysis. The identification resulted 3 groups of difficulties. Among a difficulty of quantitative reasoning process, there were 2 students as the representative in the interview. So that, the students who involved in interview were 6 persons. Subject distribution can be seen as below:

| Subject                              | Total         |
|--------------------------------------|---------------|
| Whole subject                        | 51 students   |
| Subject with incorrect answers       | 44 students   |
| Research subject (interviewed)       | 6 students    |

2.2. Data collection and analysis
Data collection is conducted by giving the covariation problem to the students. Data used in the study are the students’ work and interview result. The covariation problem provided in the study as following:

A square metal sheet (2 meters by 2 meters) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank (see the figure). These edges will then be welded together.

a. How will the final volume of the tank depend upon the size of the squares cut from the corners?

b. Sketch anointer course graph of cutting size to the tank volume!

(The problem is adapted from Shell Centre for Mathematical Education, [16])

2.2.1. Data collection of students’ work
The data was collected from students’ worksheets after a 60 minutes test. After checking and grading the answer, the worksheets were selected due to purpose of the study that only utilized the incorrect
answer. The selected worksheets were used as the base to choose and prepare the subject in interview process.

2.2.2. Interview data collection
The audio and video data were obtained from the record of interview process toward the selected subject. Interview was conducted to confirm their answer of the test and to explore the students’ difficulties throughout the process of solving the covariation problem.

2.3. Data Analysis
Results of test and interview are analyzed interrelated. Result of the test is analyzed by comparing the representation flaws in the worksheets. The comparison output then paired with interview result to confirm the students’ reasoning process. Interview audio and video were able to playback over and over again to support the data analysis process.

3. Result and discussion
This part aimed to explore the research finding and result, in which representation and interview results, analysis and interpretation were discussed. 44 (86.27%) of 51 students provided wrong covariation’s answer. Generally, there were 3 categories of flaws identified from the subjects’ work. The categories can be identified as following. Of 6 interviewed subjects, there were 3 subjects which are explained in the research result. Those three subjects were the representatives of each previous three categorized flaws.

| No  | Flaws Categories                                      |
|-----|-------------------------------------------------------|
| 1.  | The flaws of connecting the quantities, as the focus on a quantity only |
| 2.  | The flaws in modeling the equation formed from the quantity |
| 3.  | The flaws of drawing and sketching the graph due to quantity’s misinterpretation and incorrect coordinating |

Following are several subjects’ work and interview quotation presenting the difficulties and flaws of quantitative reasoning process toward the covariation problem.

3.1. Work and interview results of Subject 1 (S1) and Subject 2 (S2)

3.1.1. S1’s work for part a
S1’s work on Figure 2:

![Figure 2](image)

Every cutting has the same size, a meters × a meters, so the length = 2 – a, width = 2 – a, and height = a. Volume = a(2 – a)^2

Figure 2. S1’s work for part a.

Answering process done by S1 was connecting quantities using equation form (model). S1 represented cutting size as a variable, so it resulted length = 2 – a, width = 2 – a, and height = a, and with the result as volume = a(2 – a)^2. The equation that S1 resulted was incorrect in term of an only one cutting side focus. It was confirmed as S1 is interviewed.
Interviewer: Please take a look back to the cut corner. Is only this corner to be cut? [pointing the corner of variable \( a \)]
S1: No, these ones too [pointing the other three corners]
Interviewer: Then, is this 2 meters reduced only by \( a \)?
S1: Oh that one. I was not aware of the cut on those two corners, so I only reduced it by \( a \).

The correct equation model of the covariation problem was below:

For instance the cardboard sized \( a \) meter \( \times \) \( a \) meter.

\[
l = 2 - 2a; \quad w = 2 - 2a; \quad h = a.
\]

So that the volume \( (v) = l \times w \times h \)
\[
= (2 - 2a)(2 - 2a)(a)
= 4a^3 - 8a^2 + 4a
\]

3.1.2. S2’s work for part b on Figure 3

![Figure 3. S2’s graph sketch](image)

S1 and S2’s answer for part a reached simply on the equation formation and, further, they did not decide coordinate points to draw the connection graph between cutting size and water tank volume (see figure 3). As confirmed in interview, S2 assumed the bigger cutting size the bigger water tank volume.

Interviewer: What does the graph shape in this sketch?
S2: Linear graph, Sir.
Interviewer: How does it become linear graph?
S2: It is because the bigger cutting of the corner the bigger water tank volume.

3.2. Work and interview results of Subject 3 (S3)
Answering process done by S3 was representing cutting size, so that S3 solely represented two cutting size, 0.1 meter and 0.2 meter. This representation shaped the graph (figure 5) into linear, as it connected only two coordinate points. The interview with S3 is presented as following:

Interviewer: Why did you represent only two points?
S3: As drawing graph needs only a few points, I felt it representative enough with two points.
Interviewer: Oh I see... Then, what did the graph you sketched shape?
S3: A graph of straight line.
Interviewer: Are you sure that the next additional cutting sizes will remain a straight line?
S3: Hmmm...

S3’s work is presented in figure 4:
For instance it is cut 0.1 meter, so the length \( l \) = 2 - (0.1 \times 2)
= 2 - 0.2
= 1.8 meter
\[ l = w \Rightarrow w = 1.8 \text{ meter} \]
\[ v = l \times w \times h \]
= 1.8 \times 1.8 \times 0.1
= 0.324 m³

- For instance it is cut 0.2 meter, so the length \( l \) = 2 - (0.2 \times 2)
= 2 - 0.4
= 1.6 meter
\[ l = w \Rightarrow w = 1.6 \text{ meter} \]
\[ v = l \times w \times h \]
= 1.6 \times 1.6 \times 0.2
= 0.512 m³

Figure 4. S3’s work for part a.

The question of possibility on the next additional cutting sizes made S3 hesitated and thought to do the other cutting size representation. S3 found it is hard to identify the graph model, as he simply comprehended the concept of drawing graph by only two coordinate points.

Figure 5. S3’s graph sketch

Research findings aforementioned show that S1, S2, and S3 tended to assume the increased cutting quantity only provides linear impact to the other quantity. They were not aware that the equation shaped is quadratic equation, so that it should result parabola graph. In a study of students’ quantification of ratio and rate for reasoning about change in covarying quantities, Johnson [8] used a square drawn from one of the vertices, and found that the change on the length of the square’s sides gives effect to the circumference and area. From the interview of Johnson’s study, the subject described the increase of area faster than the increase of circumference.

The difficulties of the subject in this study were confirmed and identified by the earlier research findings. Among the earlier studies, the difficulty of interpreting Cartesian graph as situation depiction was one of the findings [6,11]. The other difficulty involved the focus on the graph shape, not on the changing object [17] nor the interrelation among quantities [15]. This difficulty implicated interpretation of quantities interrelation, as it is visualized the interrelation from static perspectives, so that a quantity value can be paired to the other quantity value. On the other hand, when the students consider the relationship from a dynamic perspective, they can provide varied reasoning of graph interpretation [2,6].
4. Conclusion
The conclusion of the study is derived from the result of identification difficulty of quantitative reasoning process toward the calculus students’ covariation problem. Generally, 86.27% of calculus students found difficulties in quantitative reasoning process toward covariation problem. The difficulties vary from, the first, establishing quantitative structure generating new quantity, to, the second, identifying curved graph of covariation problem due to the tendency of established quantity linearity. Many pre-calculus students found difficulties on using and interpreting function notation. The others took a hard way to understand and express a quantity as a different function. Sometimes pre-calculus students were confused of function output and interpreted the same symbol as the equality statement, not a tool to define the connection between changing quantities simultaneously.

Acknowledgments
This research has been supported by Educational Fund Manager Institution of Indonesia (LPDP), No. 20161141091905.

References
[1] Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. Educational Studies in Mathematics, 23(3), 247–285.
[2] Cantoral, R., & Farfán, M. (2003). Mathematics education: A vision of its evolution. Educational Studies in Mathematics, 53(3), 255–270.
[3] Carlson, M. (1998). A cross-sectional investigation of the development of the function concept. In E. Dubinsky, A. H. Schoenfeld, & J. J. Kaput (Eds.), Research in collegiate mathematics education III (pp. 114–162). Providence: American Mathematical Society.
[4] Carlson, M. P., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. Journal for Research in Mathematics Education, 33(5), 352–378.
[5] Carlson, M. P., Madison, B., & West, R. D. (2015). A study of students’ readiness to learn calculus. International Journal of Research in Undergraduate Mathematics Education, 1(2), 209-233.
[6] Clement, J. (1989). The concept of variation and misconceptions in cartesian graphing. Focus on Learning Problems in Mathematics, 11(1–2), 77–87.
[7] Hackenberg, A. J., & Lee, M. Y. (2015). Relationships between students’ fractional knowledge and equation writing. Journal for Research in Mathematics Education, 46(2), 196-243.
[8] Johnson, H. L. (2012). Reasoning about variation in the intensity of change in covarying quantities involved in rate of change. The Journal of Mathematical Behavior, 31(3), 313-330.
[9] Johnson, H. L. (2015). Secondary students’ quantification of ratio and rate: A framework for reasoning about change in covarying quantities. Mathematical Thinking and Learning, 17(1), 64-90.
[10] Koklu, O. (2007). An investigation of college students’ covariational reasonings (Phd dissertation). Retrieved from Florida State University database.
[11] Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning and teaching. Review of Educational Research, 60(1), 1–64.
[12] Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades (pp. 93–118). Reston: Lawrence Erlbaum & National Council of Teachers of Mathematics.
[13] Monk, S. & Nemirovsky, R. (1994). The case of Dan: Student construction of a functional situation through visual attributes. In J. Dubinsky, E., Schoenfeld A. H, Kaput (Ed.), Research in Collegiate Mathematics Education I (Vol. 4, pp. 139–168). Washington, DC, American Mathematical Society.
[14] Monk, S. (1992). Students’ understanding of a function given by a physical model. In G. Harel & E. Dubinsky (Eds.), The concept of function: Aspects of epistemology and pedagogy. MAA Notes 25 (pp. 175–193). Washington, DC: Mathematical Association of America.
[15] Moore, K. C., Paoletti, T., & Musgrave, S. (2013). Covariational reasoning and invariance among coordinate systems. *The Journal of Mathematical Behavior, 32*(3), 461–473.

[16] Shell Centre for Mathematical Education (University of Nottingham). (1985). *The language of functions and graphs: An examination module for secondary schools*: Nottingham, UK: JMB/Shell Centre for Mathematical Education.

[17] Sierpinska, A. (1992). On understanding the notion of function. In G. Harel & E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 25–58). Washington, DC: Mathematical Association of America.

[18] Smith, J., & Thompson, P. W. (2007). Quantitative reasoning and the development of algebraic reasoning. In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 95-132). New York: Erlbaum.

[19] Stalvey, H. E., & Vidakovic, D. (2015). Students’ reasoning about relationships between variables in a real-world problem. *The Journal of Mathematical Behavior, 40*, 192-210.

[20] Yemen-Karpuzcu, S., Ulusoy, F., & Isiksal-Bostan, M. (2017). Prospective Middle School Mathematics Teachers’ Covariational Reasoning for Interpreting Dynamic Events During Peer Interactions. *International Journal of Science and Mathematics Education, 15*(1), 89-108.