From Solar to Stellar Brightness Variations: The Effect of Metallicity

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ABSTRACT

Context. Comparison studies of Sun-like stars with the Sun suggest an anomalously low photometric variability of the Sun compared to Sun-like stars with similar magnetic activity. Comprehensive understanding of stellar variability is needed, to find a physical reasoning for this observation.

Aims. We investigate the effect of metallicity and effective temperature on the photometric brightness change of Sun-like stars seen at different inclinations. The considered range of fundamental stellar parameters is sufficiently small so the stars, investigated here, still count as Sun-like or even as solar twins.

Methods. To model the brightness change of stars with solar magnetic activity, we extend a well established model of solar brightness variations, SATIRE (which stands for Spectral And Total Irradiance Reconstruction), which is based on solar spectra, to stars with different fundamental parameters. For that we calculate stellar spectra for different metallicities and effective temperature using the radiative transfer code ATLAS9.

Results. We show that even a small change (e.g. within the observational error range) of metallicity or effective temperature significantly affects the photometric brightness change compared to the Sun. We find that for Sun-like stars, the amplitude of the brightness variations obtained for Strömgren (b + y)/2 reaches a local minimum for fundamental stellar parameters close to the solar metallicity and effective temperature. Moreover, our results show that the effect of inclination decreases for metallicity values greater than the solar metallicity. Overall, we find that an exact determination of fundamental stellar parameters is crucially important for understanding stellar brightness changes.

Key words. Stars: variables: general – Stars: activity – Stars: atmospheres – Stars: fundamental parameters – Radiative transfer

1. Introduction

Stellar activity is caused by magnetic fields emerging from below the stellar surface and evolving due to the complex interaction of gas dynamics and magnetic flux. The resulting concentrations of magnetic fields produce a variety of phenomena, including spots and faculae that lead to darkening and brightening on the stellar surface. Such emerging magnetic features can be directly observed on the Sun, due to its exclusive location, and thus have been studied in great detail (see, e.g., Solanki et al. 2006). In contrast, stellar activity can be accessed only by indirect manifestations of the surface structures, i.e. spectroscopic and brightness variations, as well as proxies of magnetic heating of the stellar atmosphere, chromospheric Ca II and coronal X-ray emission.

Stellar long-term variability investigations were launched in the 50’s by several observatories, such as the Lowell Observatory, to monitor variations of photometric brightness and chromospheric activity (see Wilson1978 Balunas et al.1995 Lockwood et al.2013 and references therein). These studies provide an observational survey of the relation between photometric variations and chromospheric activity among lower main-sequence stars (Radick et al.1998 Lockwood et al.2007 Hall et al.2009 Radick et al.2018), and led to several important findings. Firstly, the photometric variability is stronger for stars with higher magnetic activity. Secondly, it was found that among stars with a certain chromospheric activity level, a transition from faculae-dominated to spot-dominated stellar photometric variations occurs (Lockwood et al.2007 Hall et al.2009). So that the correlation between Ca II and photometry displayed by the Sun becomes an anticorrelation for more active stars. Thirdly, investigations comparing the Sun with other main-sequence stars (Lockwood et al.2007 2013) showed that solar brightness variability over the 11-year activity cycle appears to be anomalously low compared with stars of near-solar magnetic activity. The latter observation was used to suggest that historical solar variability and consequently the solar role in pre-industrial climate change might have been significantly larger than thought before (see, e.g. Lean et al.1992 Shapiro et al.2011 Judge et al.2012 Solanki et al.2013). Another recently proposed explanation (Shapiro et al.2016) is based on the fact that solar brightness variability is caused by a delicate balance between dark and bright magnetic features. This balance is sensitive to the combination of stellar fundamental parameters, which define properties of magnetic features, i.e. effective temperature,
metallicity and surface gravity. Consequently, stars with slightly different fundamental parameters can show significantly altered, e.g., higher, brightness variations. So far this hypothesis could not be tested due to the absence of reliable stellar atmospheric models for magnetic features, which hindered the development of quantitative models of stellar brightness variability for a long time.

In contrast to other stars, accurate model atmospheres of quiet regions and magnetic features exist for the Sun. The variability of Sun-like stars can reasonably be assumed to be based on the same mechanisms as on the Sun, where the processes that are responsible can be observed in detail. Therefore, it is possible to extend solar irradiance models (see Ermolli et al. 2013, Solanki et al. 2013 and references therein) to investigate Sun-like stars. Previous studies suggested several approaches depending on the issue of interest. For example, the effect of inclination for stars observed out of their equatorial plane on the photometric variability (Schatten 1993, Knaack et al. 2001, Vieira et al. 2012) Shapiro et al. (2014) was studied and a potential increase of variability was found for large inclinations. Lanza et al. (2009) developed different brightness variability models that account for active regions, but which have at least eleven free parameters, to analyse and fit observed light curves. Moreover, the correlation between the faculae and spot-dominated stellar variability and magnetic activity was investigated by modelling a hypothetical Sun at different activity levels and extrapolating the surface coverage by solar magnetic features to another mean chromospheric activity level (Shapiro et al. 2014). Whereas these investigations can explain the activity dependence of variability as well as the transition from faculae- to spot-dominated stars, they did not address the anomalously low brightness variability of the Sun, which remains a long-standing puzzle.

Generally, stars are characterised as Sun-like or even solar twins if their fundamental stellar parameters are close to the solar parameters. While such stars are close to the solar case, no star has the identical set of fundamental stellar parameters. Here, the goal is to shed light on the role of several stellar fundamental parameters for stellar brightness change, in particular different metallicities and effective temperatures. For that, we extend the successful brightness variations model SATIRE (Pigge et al. 2000, Krivova et al. 2003) to Sun-like stars with different fundamental stellar parameters. Thus we take advantage of building on the successful existing solar models that have been developed for decades and agree accurately with the solar observations. Whereas most stellar brightness variation studies assumed solar contrast of magnetic features, we take a different path. Our novel approach is based on calculating the spectral contrasts of spot umbrae, spot penumbrae, and faculae to the quiet stellar region depending on stellar fundamental parameters. This allows us to investigate the influence of stellar fundamental parameter, e.g., metallicity and effective temperature, on stellar brightness variations. Here, we limit our study to stars with the same surface distribution of magnetic features as the solar case. The main goal is to investigate stellar brightness change, in particular in the Strömgren $b$ and $y$ filters and the Kepler passband, on the timescale of magnetic activity cycles.

This paper is structured as follows. In Section 2, our theoretical approach is described. In Section 3, we present our results of the effect of the metallicity, inclination, and effective temperature on stellar brightness change. Finally, we provide a summary and draw conclusions in Section 4.

### 2. Model: From the Sun to Stars

In this study we focus on stellar photometric brightness change on the magnetic activity cycle time-scale, which for the Sun is approximately 11 years. Note, another important time-scale is the rotational time-scale, which will be investigated separately in a follow-up paper. To obtain a time dependent spectrum the observed magnetic feature distributions of solar cycle 23 are used, which was a cycle of intermediate strength. The effect of the cycle strength on the brightness variations was investigated in Shapiro et al. (2014), and here we choose a representative cycle for which we have good measurements of solar surface magnetic field. The typical distributions of faculae and spots on the solar surface during the maxima and minima of the solar cycle are displayed in Fig. 1 together with the time-evolution of the solar irradiance in Strömgren filter $b$. The difference between the year 2000 and 2008, which is indicated by a red horizontal line in the bottom panel of Fig. 1 is taken here to represent the brightness change on the activity cycle time-scale.

#### 2.1. Photometric Brightness Change

The SATIRE model that we describe here is used to compute the photometric brightness change. The SATIRE model separately accounts for the quiet stellar regions, star spot umbra, star spot penumbral and facular components (Krivova et al. 2003). Following the detailed description in Shapiro et al. (2014) the spectral flux can be decomposed into two main contributions:

\[ F(\lambda) = F_Q(\lambda) + F_m(\lambda), \]

where $Q$ denotes the quiet part of the stellar surface, $m$ is associated with different magnetic features and $\lambda$ is the wavelength. Then, the disk integrated flux $F_Q(\lambda)$ is obtained by integrating

\[ F_Q(\lambda) = \int_0^1 I_Q(\lambda, \mu) \omega(\mu) d\mu, \]

where $\omega(\mu) = 2\pi r_{star}/d_{star}$ is a weighting function with the stellar radius, $r_{star}$, and the distance between the star and observer, $d_{star}$. The considered emergent intensity, $I_Q(\lambda, \mu)$, depends also on $\mu$, which is the cosine of the angle between the observer’s direction and the local stellar radius. The magnetic features contribute through their contrast $I_m(\lambda, \mu) - I_Q(\lambda, \mu)$ to the stellar brightness

\[ F_m(\lambda) = \int_0^1 \sum_m \alpha_m(\mu) \left[ I_m(\lambda, \mu) - I_Q(\lambda, \mu) \right] \omega(\mu) d\mu, \]

where the fractional coverage of the ring at the corresponding $\mu$ on the stellar disk is given by the functions $\alpha_m(\mu)$. In this formulation the coverage at the stellar disk center is associated with $\alpha_m(\mu = 1)$ and at the limb with $\alpha_m(\mu = 0)$. In order to gain a more detailed understanding of separate contributions from faculae and spots, $F_m$ can be further decomposed. The facular spectral flux is defined as

\[ F_{Fac}(\lambda) = \int_0^1 \alpha_{Fac}(\mu) \left[ I_{Fac}(\lambda, \mu) - I_Q(\lambda, \mu) \right] \omega(\mu) d\mu, \]

whereas the spectral flux from spots consists of both the spot umbra and spot penumbra components:

\[ F_{spot}(\lambda) = \int_0^1 \alpha_{Pen}(\mu) \left[ I_{Pen}(\lambda, \mu) - I_Q(\lambda, \mu) \right] \omega(\mu) d\mu + \int_0^1 \alpha_{Umb}(\mu) \left[ I_{Umb}(\lambda, \mu) - I_Q(\lambda, \mu) \right] \omega(\mu) d\mu. \]
The solar intensities, $I(\lambda, \mu)$, and solar surface coverages, $\alpha_d(\mu)$, fully determine the solar Sun. In previous investigations, the magnetic feature coverages, $\alpha_m(\mu)$, were varied to model different stars [Knaack et al. 2001; Lanza et al. 2009; Shapiro et al. 2014], whereas the emergent intensities, $I_m(\lambda, \mu)$ and $I_Q(\lambda, \mu)$, and thus the contrasts of the magnetic features were fixed to be as on the Sun. Since the aim of this investigation is to understand potential differences between brightness changes on a hypothetical Sun with slightly different fundamental parameters, we utilize a complementary approach and use the same coverages of stellar features as for the solar case, but contrasts of the magnetic features and their center-to-limb variations (CLVs) will be adopted by recalculating the emergent intensities from the quiet and magnetic stellar regions.

2.2. Radiative Transfer Model

In this study we are interested in the amplitude of solar cycle brightness change, defined here as the difference between annually averaged solar brightness in 2000 (cycle max) and 2008 (cycle min). To obtain the brightness change using the Equations (1) - (3), the emergent spectra for the different stellar components are needed. Those can be calculated by solving the radiative transport equation for the corresponding atmospheric models. For the photospheric layers of the quiet Sun, spot umbra, and spot penumbra such atmospheric models are often computed assuming radiative equilibrium while convection is included through mixing length theory [Bohm-Vitense 1958]. The faculae models are semi-empirical [Vernazza et al. 1981; Lemaire et al. 1981; Fontenla et al. 1993, 2006], i.e. they are constructed to match their output to solar observations recorded at intermediate spatial resolution [Vernazza et al. 1981]. Thus, faculae models are not in radiative equilibrium. Despite the fact that these models do not account for the geometric properties of magnetic features such as hot walls in magnetic flux tubes [Solanki 1993; Steiner 2005], ensembles of which form network and faculae, they have been successfully used for many applications, in particular, for modelling solar brightness variations [Solanki et al. 2013].

In order to obtain the emergent intensities across a wide range of wavelengths from the atmospheric models, we use the LTE spectral synthesis code ATLAS9 [Kurucz 1992; Castelli & Kurucz 1994]. For the calculation of the stellar continuum opacities, the following contributors are taken into account: Free-free (ff) and bound-free (bf) transitions in H, H+, H2, He I, He II, He III. In addition, ff and bf transitions for low to high temperature absorbers such as C, N, O, Ne, Mg, Al and Si.
Moreover, electron scattering and Rayleigh scattering on H I, He I and H$_2$ are considered. ATLAS9 further exploits Opacity Distribution functions (ODFs) to account for the opacity of millions of atomic and molecular spectral lines. For that the ODFs are generated by the code DFSYNTHE (Castelli 2005) for two microturbulence velocities of 1.5 km s$^{-1}$ for the quiet stellar component and facular component, and of 2.0 km s$^{-1}$ for the spot components, where a higher velocity is chosen to partly account for the large Zeeman splitting in the spots (Fontenla et al. 2006; Anderson, R. I. et al. 2010). To account for the center-to-limb variation, the emergent intensities, $I(\lambda, \mu)$, are calculated for 11 different $\mu$ values. So far the SATIRE model made mainly use of pre-calculated emergent spectra by Unruh et al. (1999) to successfully reconstruct the solar irradiance variations. For our purpose it is necessary to calculate these emergent spectra for different fundamental stellar parameters. The ATLAS9 code has evolved since Unruh et al. (1999) used it to compute the emergent spectra generally needed for the SATIRE model. Therefore, we recomputed the change in the solar spectrum between 2000 & 2008, as well as the facular and spot contributions to this change (shown in Fig. 2) by exploiting the newest ATLAS9 version (together with recalculated ODFs for solar elemental composition taken from Anders & Grevesse (1989)). For that we use the same atmospheric structures as Unruh et al. (1999) for the quiet sun, sun spots, and faculae as our reference. The so obtained brightness changes agree very well with previous calculations by the SATIRE model based on the Unruh et al. (1999) spectra (see dotted lines in Fig. 2). The small deviations are due to up-dates incorporated over the last decades in the ATLAS9 and DFSYNTHE codes.

Now that we have validated the approach for the solar case, let us consider different metallicity values. These affect the spectrum in two ways. Firstly, they change the strengths of atomic and molecular lines. Secondly, for a cool star, changed metallicity values influence the continuum opacity, due to the change in the concentration of electron donors. We consider these effects separately, to judge the importance of each. In a first step, only the direct effect of the metallicity on the atomic and molecular lines is considered on its own. For this, we use the corresponding recalculated ODFs for the new metallicity, while keeping the atmosphere models used by Unruh et al. (1999) for the quiet Sun and magnetic features, thus neglecting the effect of a different metallicity on the atmosphere’s structure and the electron concentration. The results of this approach are presented in Subsection 3.1. In the next step, both, the effect of lines as well as the change of the electron concentration along with the back-reaction of the changed radiation field on the atmospheric structure is considered. For that the ATLAS9 code provides a tool, which self-consistently calculates 1D radiative equilibrium atmosphere models for different fundamental stellar parameters, i.e. effective temperature and metallicity. However, since the facular model was obtained semi-empirically, and no three-dimensional magnetohydrodynamic calculations (3D MHD) based calculations of magnetic features for different metallicities are currently available, the only option is to modify the reference facular model. The empirical modification of the atmospheric model for the faculae and the results of this approach are discussed in Subsection 3.2.

3. Results

To understand how fundamental stellar parameters affect the brightness change of a hypothetical Sun, we vary the metallicity, the inclination, and the effective temperature separately. We begin our study by investigating the effect of metallicity on stellar brightness change. For that we first consider only the effect of Fraunhofer lines on the opacity (Subsection 3.1), where at this first stage we neglect the feedback on the atmospheric structure. We then analyse the combined effect due to Fraunhofer lines, changed electron number density and changed stellar atmospheric structure (Subsection 3.2). The effect of inclination and effective temperature is considered in Subsections 3.3 and 3.4, respectively.

3.1. Direct Effect of Metallicity on Fraunhofer Lines

On time-scales greater than a day, brightness variations are determined by changes in the surface-area coverage by magnetic features (e.g. Solanki et al. 2013; Yeo et al. 2017). Therefore, the sum of the facular and star spot contributions is decisive for the amplitude of the photometric brightness change. Furthermore, Shapiro et al. (2015) showed that for the Sun, which is faculae-dominated, the main contribution to the brightness change on solar cycle time-scales in the UV and visible spectral domains comes from the molecular and atomic lines that are present in the emergent spectra of faculae (see also Mitchell & Livingston 1991; Unruh et al. 2000). This has to do with the particular temperature structure of faculae. While their temperature is that of the quiet Sun near the Rosseland mean optical depth, $\tau_{Ross}$, equal to unity, they are considerably hotter in the higher layers. Hence, at the disc centre their continuum contrast is relatively low, while in the cores of most spectral lines of neutral elements they are bright (Yeo et al. 2013). This is partly because, the spectral lines of neutral atoms and molecules are weakened in faculae due to enhanced ionization and dissociation, respectively.

![Fig. 2. The amplitude of solar cycle brightness change, here the difference between annually averaged solar brightness in 2000 (cycle max) and 2008 (cycle min), calculated with the SATIRE model (black). In addition, we plot the facular and spot components of the brightness change (red and blue). The gray shaded area indicates the spectral response function of the Kepler telescope measurements and the yellow shaded areas the Strömgren filters b and y (centered at 467 nm and 547 nm, respectively). Solid lines: Solar cycle brightness change calculated with the newest ATLAS9 code and ODFs. Dotted lines: Same but using emergent spectra calculated by Unruh et al. (1999).](image-url)
where \( \frac{N_{\text{metals}}}{N_{\text{H}}} \) is the number density of hydrogen. We start with the solar metallicity, \( M/H = 0.0 \), and the element composition by \cite{AndersGrevesse1989}. To obtain metallicities in the range \( -0.4 \leq M/H \leq 0.4 \) in steps of 0.1, we scale the ratio of all metals to hydrogen accordingly. The ODFs are then used together with the reference atmospheric models to calculate the emergent intensities for Equations (1) - (5) (SATIRE model). At this step, the approach is, however, not yet self-consistent. While the ratio of all metals to hydrogen is changed and thus the formation height of different lines is shifted, the \( H^{-} \) opacity, which is the main contributor to the continuum in the visible, remains unchanged because neither the electron density nor the temperature structure is recalculated. Note that in this approach, when only the metallicity for the line opacities is changed the effective temperature, \( T_{\text{eff}} \), is still somewhat affected, e.g. for higher metallicities \( T_{\text{eff}} \) becomes lower due to stronger lines.

The brightness change computed in this way is shown in Fig. 3. Figure 3 a) shows the effect of an increased metallicity \( M/H = 0.2 \) for the facular and spot contributions separately, while Fig. 3 b) shows the effect on the overall brightness change. The solar case as in Fig. 2 is also plotted for comparison. As explained earlier in this section, the main reason why the Sun is brighter during activity cycle maximum are the weaker spectral lines in the faculae \cite{Shapiro2015}. If we enhance metallicity, we increase the strength of spectral lines, so that to first order more lines get weakened in faculae and their contrast increases. Because the line density is higher in the UV, the increase in contrast is also larger there, as confirmed by Fig. 3 b). In contrast, the spot contribution changes little (Fig 3 a). This can be understood by looking at the spectral profiles of the faculae and spot contributions to the brightness change: While the facular profile contains a lot of spectral features brought about by the Fraunhofer lines, the spot profile is pretty smooth. This indicates that the contrast due to Fraunhofer lines is predominant for the faculae component, while continuum plays a more important role for the spot component. This is supported by the temperature profile of sunspot umbrae and penumbra, which have similar (or flatter) gradients than the quiet Sun, but a significantly lower effective temperature.

Stellar photometric brightness variation measurements spanning a decade or more have predominantly been made in the Strömgren \( b \) and \( y \) filters. Thus the dependence of brightness change on metallicity in these filters, shown in Fig. 4, is of particular interest. The brightness change in these filters is also particularly sensitive to metallicity. This is partly because facular and sunspot variations nearly balance each other for solar metallicity, especially in Strömgren \( b \). For \( M/H = 0.2 \), for example, the brightness change in the Strömgren filter \( b \) is increased by a factor of 10.6 relative to the Sun, whereas in the Strömgren filter \( y \) it is increased by a smaller, but still sizeable factor of 1.86.

Interestingly, for metallicities smaller than in the Sun, the brightness change becomes negative in the Strömgren \( b \) filter, which indicates that for low metallicities the brightness changes over the activity cycle are spot-dominated rather than faculae-dominated. Both regimes have been observed in main-sequence stars \cite{Radick1998,Lockwood2007,Monteiro2000}. The brightness change integrated over Strömgren \( b \) and \( y \) filters as a function of metallicity is shown in Fig. 4.
In order to quantify the full effect of metallicity on the brightness changes in the facular model leads to a greater brightness change and consequently the continuum forming layer has to be increased. The stronger continuum offsets the deeper spectral lines, so that $T_{\text{eff}}$ remains unchanged. On the other hand, increased opacity due to higher metallicity leads to a slight shift of the $\tau_{\text{Ross}} = 1$ on the column mass scale, so that the continuum is formed at somewhat higher layers compared to the solar case.

As the facular model is not in radiative equilibrium, it cannot be directly re-calculated by the ATLAS9 code. Therefore we assumed that a changed metallicity value has the same effect on the temperature structures of the faculae as on the quiet stellar regions, applying the $\Delta T$ shown in Fig. 5(b) to the solar facular model. In other words, we assumed that the change of the metallicity preserves the temperature contrast between the faculae and the quiet regions as a function of the column mass. We plan to test this assumption in future with a realistic 3D MHD calculation, which, however, is beyond the scope of the present paper. Taking all effects combined into account has a large impact on stellar photometric brightness change as shown by the difference between the solid black line and the dashed black line in Fig. 6.

Finally, the effect of metallicity on the brightness changes in the Strömgren $b$ and $y$ filters, and the Kepler pass-band is shown in Fig. 7. In this set of calculations the combined effect of lines, recalculated atmospheric model and changed electron concentration is captured. Note, that the solar case is not the same reference as in the previous section. This is because Unruh et al. (1999) adjusted the temperature structure of the initial facular model for the solar case to match the observed emergent spectra without changing the electron concentration. Recalculating the electron concentration for the adjusted temperatures in the facular model leads to a greater brightness change and consequently a shifted reference point. The combined effect leads to an even stronger increase in brightness change with greater metallicity than was found in Subsection 3.1 for the isolated effect of metallicity on Fraunhofer lines. A transition to a spot-dominated regime is found for the Strömgren $b$ filter close to $\text{M/H} = -0.1$, where a similar result was found in Subsection 3.1.

These findings support the hypothesis that the balance between radiation from dark and bright magnetic features that determines the brightness change on the magnetic activity cycle timescale is sensitive to stellar fundamental parameters. In particular, the solar metallicity value is close to a complete compensation regime, which is one possible explanation for the low brightness change of the Sun compared to other Sun-like stars of similar chromospheric activity (Lockwood et al. 2007, 2013). Previous analysis of the relationship between chromospheric activ-
Fig. 6. The amplitude of solar cycle brightness change as defined in Fig. 2. Orange solid line: the solar case (M/H = 0.0). Black solid line: A hypothetical Sun calculated with M/H = 0.3 using solar atmosphere models. Black dotted line: same as before, but using atmospheric models consistently recalculated with M/H = 0.3, i.e. the back-reaction of the radiation field on the atmospheric structure, and continuum opacity change due to a different electron concentration are considered. The yellow and grey shaded areas are the same as in Fig. 2.

Fig. 7. Change in radiative flux integrated over Strömgren b and y filters and Kepler pass-band as a function of metallicity for cases with recalculated atmosphere models.

Fig. 8. Brightness change in the Strömgren b and Kepler filters for different inclination angles (90° corresponds to an equator-on view) and different metallicity values. a) Brightness change vs. metallicity for four different inclination angles. b) Brightness change vs. inclination angle, for three different metallicities and two different filters (Strömgren b and Kepler).

3.3. Inclination Effects

The Sun is always observed from a close to an equator-on vantage point. This is different for other stars, which are observed at random inclination angles, defined as the angle between the observer’s viewing direction and the rotational axis of the star. We define the inclination angle such that a pole-on view corresponds to 0° and an equator-on view to 90°. Several investigations studied the effect of inclination on the solar brightness variation on the magnetic activity cycle time-scale (e.g., Schatten [1993], Knaack et al. [2001], Shapiro et al. 2014). All previous studies found that an inclined Sun would show a greater brightness variation, although they differed in the magnitude of the effect of inclination. However, prior studies considered only the solar case, and did not consider other stellar fundamental parameters. Here, we investigate the effect of the inclination for stars with different metallicities, with the focus on the brightness change in the Strömgren b and Kepler filters. Using the calculated spectra for different metallicities with the recalculated model (see Section 3.2), the brightness changes are obtained for different view angles.

Figure 8a shows the brightness change with the metallicity for four inclination angles \( \alpha = [0°, 30°, 60°, 90°] \). It is evident that for metallicity values between M/H = −0.1 and M/H = 0.2 the brightness change increases with decreasing inclination. With increasing M/H values the effect of inclination diminishes. The dependence of the inclination is more complex for metallicities...
greater than M/H = 0.2. Note, that our result for different inclinations of the solar case is in agreement with previous investigations (Radick et al. 1998; Knaack et al. 2001; Shapiro et al. 2014). Therefore, we conclude that the brightness change of stars with different fundamental parameters can show different dependences on inclination. This is illustrated in Fig. 8b), where the brightness change with inclination is shown for three metallicity values. For M/H = 0.3, starting from the equator-on view, the brightness change in the Strömgren b filter, (Fig. 8b)), decreases first with decreasing inclination angle. Then, at lower inclination angles the brightness change increases again towards a pole-on view. This is explained in more details in the Appendix A where we discuss the center-to-limb variations (CLVs) of faculae.

3.4. Effective Temperature

So far we only studied the effect of metallicity. Another important fundamental stellar parameter that likely affects stellar brightness change is the effective temperature. To understand the role of the effective temperature, the brightness change is calculated for effective stellar temperatures 100 K less and 100 K greater than the solar value. Such small deviations from the solar value are of special interest, as the measurement accuracy of this parameter is approximately 100 K (Pinsonneault et al. 2012). The atmospheric models for different effective temperatures were recalculated following the same procedure as described in Section 2.1.2. Subsequently, the brightness change was calculated with SATIRE. Figure 2 shows the total brightness change for the three considered T_eff values. Already a 100 K decrease in T_eff causes the brightness change to increase remarkably, while a similar increase in T_eff lowers the brightness change and leads to spot-dominated cycle in the Strömgren b filter (for solar M/H). Such a sensitive response to a slight change in the effective temperature emphasises the special combination of stellar parameters of our Sun.

Fig. 9. The amplitude of solar cycle brightness change as defined in Fig. 2. Black line: The solar case with solar effective temperatures for all features. Green line: A hypothetical Sun, but with T_eff reduced by 100K for all components. Orange line: Same as before, but with T_eff increased by 100K for all components.

Fig. 10. Estimated rms of the radiative flux change in the Strömgren (b + y)/2 filters for four different cases vs. metallicity. Black: Only the effect of Fraunhofer line on the opacity is considered. Blue: Recalculated atmosphere models are used.

3.5. Towards Observational Quantities

Having investigated the effect of metallicity on the spectral brightness change, together with the effect of the inclination, we now link our comprehensive theoretical results to observed stellar photometric brightness changes. Observed stellar photometric brightness variations is usually analysed in the Strömgren b and y filters separately or as one averaged quantity over Strömgren (b + y)/2 (Lockwood et al. 2007; 2013; Radick et al. 2018). Furthermore, observational studies also quantify the brightness change as the root-mean square (rms) variation of the annual mean magnitudes that are obtained from long-term observations. Since, however, for our modelling we consider the brightness difference between the magnetic cycle maximum and minimum of only one cycle, a one-to-one comparison cannot be made in a straightforward manner.

We approximate the rms variation for Strömgren (b + y)/2 by using the relation \( \text{rms} = A/\sqrt{2} \), where A is the amplitude of a sine function. Figure 10 shows the rms variations obtained in this way versus metallicity. Note, that the absolute value of the brightness change was considered, i.e. the phase at which maximum brightness is reached was neglected. Overall, Fig. 10 shows that photometric brightness variation for Strömgren (b + y)/2 is close to a local minimum for solar metallicity and inclination. The same holds for solar metallicity and inclination angles up to roughly 60°, as well as for the brightness change in the Strömgren b filter (see Figs. 7 and 8). The smallest rms variations among the computed cases is found for metallicity of M/H = −0.1. Combining this with the results of the effective temperature study reveals that the photometric brightness variations in the Strömgren b, and Strömgren (b + y)/2 filter for the solar case is close to a local minimum for the parameter space of the metallicity and effective temperature.

4. Conclusions

Physics-based models are of importance for a comprehensive understanding of long-term stellar and solar brightness variations, but have so far been rarely applied to stars other than the Sun. Of special interest is the long-standing puzzle that solar brightness variability on the time-scale of the 11-year activity...
cycle appears anomalously low in comparison to variability of Sun-like stars with a near-solar level of magnetic activity. Such models are also of importance for the detection of extra solar planets (see for example Borgniet et al. 2015). Thus, there has been a drive towards understanding the solar-stellar connection, especially after the launch of the Corot (Bordé, P. et al. 2003; Baglin et al. 2006) and Kepler (Borucki et al. 2010) space missions that provide broadband stellar photometry of unprecedented precision. In addition, detailed models of stellar variability are of interest for the upcoming TESS (Ricker et al. 2015; Lund et al. 2017) and PLATO (Catala 2009; Rauer et al. 2016) missions.

In this study we have extended the well-established solar variability model (SATIRE) to stars with different fundamental parameters. For that we kept the distribution of magnetic features fixed, but we calculated the emergent intensities for different values of metallicity and effective temperatures. In a first step, we demonstrated that changing metallicity affects the Fraunhofer lines in quiet stellar regions in a different way than in faculae. In particular, we find that higher metallicity values result in a significant increase in contrasts of faculae, i.e. bright magnetic features. The enhanced contrasts lead to a greater amplitude in the photometric brightness change over a magnetic activity cycle. While isolating the effect of Fraunhofer lines on the brightness change confirmed their important contribution, which was first established for the solar case (Shapiro et al. 2015), it is crucial to account for the back-reaction of the changed radiation field on the atmospheric structure. In a self-consistent approach, the brightness change is affected by metallicity, such that even small changes in metallicity values have a significant impact on stellar brightness change for pass-bands used in space- and ground-based stellar observations. Furthermore, examining the brightness changes for a hypothetical Sun with slightly changed effective temperature in both directions reveals an increase in brightness change for the Strömgren $b$ filter. All in all we conclude that the combination of the solar fundamental parameters corresponds to a local minimum in the brightness change in the Strömgren filters on the solar cycle time-scale. This finding thus explains the anomalously low solar brightness change by the incidental combination of fundamental solar parameters (Shapiro et al. 2015). This is a plausible explanation for low solar brightness change. In addition, a possible observational bias hinders the identification of stars with low brightness change, and thus potential solar twins.

We also find that the inclination does not have a strong impact on the brightness change for stars with metallicities somewhat higher than solar. Due to the dependence of center-to-limb variations on metallicity, a star with double the metallicity of the Sun would show almost no difference in the brightness changes when it is observed equator-on or pole-on. In contrast, inclination angles play an important role for stars with low metallicity, i.e. approximately $M/H = 0.2$ or lower. For stars with less metals than in the Sun, different inclination angles can even lead to a transition from faculae-dominated to spot-dominated brightness changes. While this result confirms previous investigations on the importance of the inclination for solar metallicity, it additionally reveals that the inclination effect becomes weaker for stars with greater metallicities.

In the future, our theoretical findings will be tested against observational data. Recently, sufficient observational data were obtained for one Sun-like star to perform an extensive analysis: The star HD 173701, whose metallicity is twice as large as the solar value, shows higher chromospheric variation, but an even higher photometric brightness variation. Both effects can be explained by the difference in the metallicity (Karoff et al. 2018). Another curious case is the Sun-like star HD 143761, which has a near-solar effective temperature but whose metallicity is half of the solar one (von Braun et al. 2014). This star shows a photometric variability that is spot-dominated, despite being less active than the Sun (Radick et al. 2018). Such a ‘rule-breaking’ behaviour can be explained by a lower facular contrast due to its low metallicity, which is in line with our results. Note, however, that this example should be treated with caution since observational data with two different instruments detected a change from correlated to anticorrelated behaviour (Radick et al. 2018). Unfortunately, for more detailed comparison we lack a complete set of measurements to determine fundamental parameters for many observed main-sequence stars. Therefore, an effort is currently underway to obtain a more complete set of observations, including accurate measurements of stellar fundamental parameters and long-term variability for an extended number of Kepler stars (Peigura et al. 2017; Kong et al. in prep.), for comparison between modelling and measurements.

We conclude that the complex interaction between radiation and matter is crucial to obtain correct brightness variation calculations. However, since the model adjustment uses simplified assumptions and cannot account for three-dimensional effects, 3D MHD calculations are needed for a more realistic approach. Such simulations can provide a more realistic modelling for their brightness changes using a 1.5-dimensional approach (Norris et al. 2017). Unfortunately, current 3D MHD calculations are only available for different effective temperatures on a coarse grid (Beeck et al. 2015). Consequently, we aim to obtain 3D MHD simulations on a finer effective temperature grid together with different metallicity cases and to compute the entire spectrum in order to study the dependence of stellar brightness changes on the fundamental stellar parameters.

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**References**

Anders, E. & Grevesse, N. 1989, Geochem. Cosmochim. Acta, 53, 197

Anderson, R. I., Reiners, A., & Solanki, S. K. 2010, A&A, 522, A81

Baglin, A., Auvergne, M., Boismard, L., et al. 2006, in COSPAR Meeting, Vol. 36, 36th COSPAR Scientific Assembly

Balintas, S. L., Donahue, R. A., Soon, W. H., et al. 1995, ApJ, 438, 269

Beeck, B., Schüssler, M., Cameron, R. H., & Reiners, A. 2015, A&A, 581, A42

Bohn, K.-H., & Veneva, E. 1995, ZAp, 46, 108

Bordé, P., Rouan, D., & Léger, A. 2003, A&A, 405, 1137

Borqui, S., Meunier, N., & Lagrange, A.-M. 2015, A&A, 581, A133

Borucki, W. J., Koch, D., Basri, G., et al. 2010, Science, 327, 977

Castelli, F. 2005, Memorie della Societa Astronomica Italiana Supplementi, 8, 34

Castelli, F. & Kurucz, R. L. 1994, A&A, 281, 817

Catala, C. 2009, Experimental Astronomy, 23, 329

Ermolli, I., Matthes, K., Dudok de Wit, T., et al. 2013, Atmospheric Chemistry and Physics, 13, 3945

Fligge, M., Solanki, S. K., & Unruh, Y. C. 2000, A&A, 353, 380

Fontenla, J. M., Avrett, E., Thuillier, G., & Harder, J. 2006, ApJ, 639, 441

Fontenla, J. M., Avrett, E., Thuillier, G., & Harder, J. 2008, The Astrophysical Journal, 639, 441

Fontenla, J. M., Avrett, E. H., & Loeser, R. 1993, ApJ, 406, 319

Kong et al. in prep., for comparison between modelling and measurements.

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Appendix A: Dependence of Brightness Change on Inclination and CLV

To understand why the dependence of brightness change on the inclination itself depends on metallicity, we investigate the center-to-limb variation of facular contrast. Figure A.1 shows the CLV of the facular contrast in the Strömgren \(b\) filter and the Kepler pass-band. These two cases represent the two different types of behaviour discussed in Subsection 3.3. The facular contrast is multiplied by the corresponding \(a\) to account for the foreshortening effect. For the solar case, the facular contrast increases from the disc center outwards. Since this increase continues almost to the edge of the disc, a relocation of the faculae towards the center will result in only a slight decrease of their contrast with inclination. Thus the effect of the brightness change is weak (see Fig. A.2 (a)).

For greater metallicity, the facular contrast in the middle of the disc is almost constant, while a steep drop starts at \(r/R \approx 0.7\). The difference between the facular contrast around the disc center and the limb is significantly greater than for the solar case. Therefore, two contributions compete in the brightness change when the star with higher metallicity is inclined towards the pole-on view. With inclination some part of the faculae are seen closer to the disc center, where the facular contrast does not change, another part is shifted to the limb where the facular contrast drops significantly. Therefore, the facular brightness change decreases greatly with inclination for a metallicity value of 0.3 (see Fig. A.2 (b)).

To explain the different behaviour with inclination for metallicity values in the range \(-0.1 < M/H \leq 0.2\) and metallicity values greater than 0.2, we plot the faculae and spot brightness changes with different inclinations for the solar value (\(M/H = 0.0\)) and the metallicity value \(M/H = 0.3\) in Fig. A.2. For the solar case, the facular brightness change decreases somewhat with inclination, at the same time spot brightness change decreases by a much larger amount, so that total brightness change goes up.

For the \(M/H = 0.3\) the brightness change due to spots drops with inclination by a similar amount as for \(M/H=0\), but at the same time facular brightness change drops by a similar amount, or even slightly more, so that the balance between faculae and spots remains either the same or is only slightly shifted towards the spot contribution. Consequently, the total brightness change either remains the same or decreases.

While the total surface area covered by magnetic features remains constant when the star is inclined, due to the equatorial symmetry of the distributions, the location of magnetic features on the disc changes. Thus the contrasts of the faculae and spots are modified. Consequently, the total brightness change is altered due to a changed balance of facular and spot contributions. For the solar metallicity value the faculae contribution decreases slightly with decreasing inclination, i.e. towards a pole-on view, but at the same time the negative contribution of the spots increases significantly. This leads to an increase of the total brightness change. We find an opposite behaviour for metallicity values greater than \(M/H = 0.2\). For such cases the facular contrast decreases significantly with inclination, while the spot contrast behaves almost as for the solar metallicity value. This results in a decrease of the overall brightness change with inclination.

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