Terrestrial detector for low-frequency gravitational waves based on full tensor measurement

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Abstract. Two serious obstacles in constructing terrestrial gravitational wave (GW) detectors that can resolve low-frequency signals (\(\leq 10\) Hz) are seismic and Newtonian noises. Here we describe a new detector concept by adopting new measurement techniques and configurations to overcome the present low-frequency barrier due to these noises. Six magnetically levitated superconducting test masses, widely separated along three orthogonal axes, each with three degrees of freedom, constitute a tensor GW detector. The tensor outputs could be combined to better reject the Newtonian noise. Unlike current two-dimensional detectors, a single tensor detector is able to determine the polarization of GWs and the direction to sources on its own.

1. Introduction

Several astrophysical processes generate GWs below 10 Hz, and will not be observed by the advanced terrestrial laser interferometer detectors. Two serious obstacles in constructing terrestrial GW detectors that can resolve low-frequency signals are seismic and Newtonian noises (NN). Recently, the possibility of constructing three different types of low-frequency terrestrial GW detector has been discussed [1]. In this paper, we present an alternative design.

Due to the transverse nature of GWs, a detector that measures all the components of the curvature tensor could distinguish GWs from near-field Newtonian gravity. By combining six magnetically levitated superconducting test masses, one could construct a \textit{full-tensor} detector. We discuss a design concept that could reach sensitivity \(\leq 10^{-20}\) Hz\textsuperscript{-1/2} at 0.1-10 Hz and discuss the procedure of mitigating the seismic and Newtonian noise. We call this detector SOGRO (Superconducting Omni-directional Gravitational Radiation Observatory).

Because of the finite sizes of white dwarfs (WDs), WD-WD binary merger takes place at \(f < 1\) Hz. SOGRO will be able to detect any WD binaries within 1 Mpc. Within this horizon, there are two galaxies: Milky Way Galaxy and Andromeda (M31). The combined merger rate of WD-WD binaries within these two galaxies is estimated to be \(~3\) per 100 years [1]. If the signal is integrated over one year, the probability of finding a WD-WD binary during 1-year operation of SOGRO is \(~30\)% since each event is expected to last for \(~10\) years. Direct observation of WD-WD binaries via GW will provide valuable information on the evolution of binary stars composed of relatively low mass stars.

The existence of the intermediate mass black hole (IMBH) is still under vigorous debate, but there are several candidates of IMBH of up to a few tens of thousands of solar mass [2]. Binary mergers composed of IMBHs can be detected by SOGRO up to a few Gpc.
2. Full-tensor GW detector

According to general relativity, a gravitational field is characterized by a curvature tensor. A full-tensor detector could be constructed by measuring five degenerate quadrupole modes of a solid sphere [3]. A tensor detector is sensitive to GWs coming from any direction with any polarization and is thus capable of resolving the source direction and polarization.

One could construct a low-frequency (0.1 to 10 Hz) tensor GW detector by combining six almost free test masses. Figure 1 shows the test mass configuration of SOGRO. Six superconducting test masses, each with three linear degrees of freedom, are levitated over three orthogonal mounting tubes. The test masses are made of niobium (Nb) in the shape of a rectangular shell with circular flanges. Superconducting levitation/alignment coils and sensing capacitors (not shown) are located in the gap between the test masses and the mounting tubes, as well as on the outer surfaces of the test masses.

The along-axis motions of the two test masses on each coordinate axis are differenced to measure a diagonal component of the wave. The cross-axis (rotational) motions of the four test masses on each coordinate plane are differenced to measure an off-diagonal component of the wave. The detector also measures the three linear and three angular acceleration signals by summing the along-axis and cross-axis test mass motions. These common-mode (CM) signals are used to remove the residual sensitivity of the differential-mode (DM) channels of the detector to the platform accelerations [4].

Since test mass motion is measured with respect to the sensing circuit elements mounted on the platform, SOGRO requires a rigid platform with mode frequencies above the signal bandwidth, > 10 Hz. To alleviate excessive demand on cryogenics, the platform must not be too heavy yet sufficiently rigid. Our preliminary FEM analysis shows that, by building the 3D cross with aluminum (Al) square tubes triangulated by Al square tube struts (figure 1), a 30-m SOGRO platform can be constructed with all the mode frequencies > 19 Hz, except for one CM at 15 Hz, and a total mass of ~70 tons.

Magnetic levitation allows the test masses to have horizontal resonance frequencies \( f_h \leq 0.01 \) Hz. In the vertical direction, the stiffness due to the magnetic field can be made to cancel for the DM by connecting the levitation coils in series such that the total inductance remains constant [4]. In reality, the vertical DM frequencies \( f_v \) become \( \sim 1 \) Hz due to nonlinearities in the levitation coils. A "superconducting negative spring" has been demonstrated to reduce the resonance frequency of the SGG [4]. A similar technique could be applied to SOGRO to reduce \( f_v \) to 0.01 Hz.

An extremely low-noise transducer with high energy coupling is required for a sensitive GW detector. For SOGRO, we propose to employ a superconducting tuned capacitor bridge transducer [5]. Figure 2 shows a transducer for a diagonal-component channel, where \( C_i(t) \) is a Nb capacitor plate located near the \( j \)-th face of the \( i \)-th test mass. The bridge output is coupled to a nearly quantum-limited dc SQUID through a superconducting transformer. The sensing capacitances are modulated by GWs at \( \omega \) and the bridge is driven at the resonance frequency of the capacitor bridge coupled to the SQUID \( \omega_p \), well above the 1/f noise corner frequency of the SQUID. The electrical resonance increas-
es energy coupling constant $\beta$. The carrier signal at $\omega_p$ is precisely balanced by applying feedback forces to the test masses, eliminating the oscillator noise at the bridge output.

3. Detector sensitivity

The noise power spectral density of a GW detector with an active transducer is given [6] by

$$S_n(f) = \frac{16}{ML^2\omega^2} \left\{ \frac{k_B T \omega_p}{Q_D} + \frac{\omega^2 - \omega_p^2}{2\omega_p} \left( 1 + \frac{1}{\beta^2} \right)^{1/2} k_B T_N \right\}, \quad (1)$$

where $M$ and $L$ are the mass of each test mass and the arm-length of the detector; $T$ is the temperature; $\omega_p = 2\pi f_p$ and $Q_D$ are the DM frequency and Q; and $T_N$ is the noise temperature of the SQUID.

Table 1 shows the proposed parameter values and detector noises for two detector options: SOGRO with $L = 30$ m cooled to $T = 1.5$ K by pumping on liquid helium or using cryocoolers and advanced SOGRO (aSOGRO) with $L = 100$ m cooled to 0.1 K by using He$^3$-He$^4$ dilution refrigerators. We set $20\hbar$ and $2h$ as the SQUID noise goals for SOGRO and aSOGRO, respectively.

In figure 3, we plot the expected strain sensitivities of SOGRO and aSOGRO, as well as those of advanced LIGO (aLIGO) [7], advanced Virgo (AdV) [8] and eLISA [9]. SOGRO fills the sensitivity gap between advanced terrestrial interferometers and proposed space interferometer. The NN due the Rayleigh and infrasound waves before mitigation are also plotted [1]. SOGRO has a detector noise $\leq 10^{-20}$ Hz$^{-1/2}$ at 1-10 Hz. To reach the detector noise limit, the NN must be mitigated by up to 70 dB for SOGRO. The NN mitigation will be discussed in a separate paper in the same proceedings.

4. Seismic noise rejection

We propose to construct SOGRO underground at a depth $\geq 1$ km. The seismic noise level of an underground lab at 0.1-10 Hz is $\leq 3 \times 10^{-7}$ m s$^{-2}$ Hz$^{-1/2}$ [10]. This is 11 orders of magnitude above the target sensitivity of SOGRO. Vibration isolation at such low frequencies is very challenging.

Table 1. Detector parameters and expected sensitivities of SOGRO and aSOGRO.

| Parameter                  | SOGRO  | aSOGRO | Method employed (SOGRO/aSOGRO)                  |
|----------------------------|--------|--------|------------------------------------------------|
| Each test mass M           | 5 ton  | 10 ton | Nb square tube                                  |
| Arm-length L               | 30 m   | 100 m  | Over “rigid” platform                           |
| Antenna temperature T      | 1.5 K  | 0.1 K  | Superfluid He/dilution refrigerator             |
| Platform temperature $T_p$ | 1.5 K  | 1.5 K  | $Q_D = 5 \times 10^6/10^7$                      |
| DM frequency $f_D$         | 0.01 Hz| 0.01 Hz| Magnetic levitation                             |
| DM quality factor $Q_D$    | $5 \times 10^8$ | $10^9$ | Surface polished pure Nb                        |
| Signal frequency $f_s$     | 0.1-10 Hz | 0.1-10 Hz |                                                |
| Pump frequency $f_p$       | 50 kHz | 50 kHz | Tuned capacitor bridge transducer               |
| Amplifier noise number $n$| 20     | 2      | Nearly quantum-limited dc SQUID                 |
| Detector noise $S_n^{1/2}(f)$ | $2 \times 10^{-20}$ Hz$^{-1/2}$ | $2 \times 10^{-21}$ Hz$^{-1/2}$ | Computed at 1 Hz |

Figure 2. Superconducting tuned capacitor bridge transducer.
CM rejection techniques demonstrated with SGGs [11] can be applied to SOGRO and improved to one part in $10^{-9}$. By precisely matching the accelerometer scale factors and further compensating residual errors, the sensitivity to CM acceleration is reduced to $10^{-9}$, lightening the vibration isolation requirement to 20 dB.

The platform is suspended as a long ($\geq 15$ m) pendulum from its center. The platform will then have horizontal frequencies of $\leq 0.13$ Hz and angular frequencies ~1 mHz. This pendulum suspension will provide a passive isolation of 35 dB to horizontal accelerations and 120 dB to angular accelerations at 1 Hz, and much more at 10 Hz. For the vertical direction, 20-dB isolation must be provided by combining passive and active isolation.

5. Conclusions

A wideband tensor GW detector with sensitivity $\leq 10^{-20}$ Hz$^{-1/2}$ for the band 0.1-10 Hz could be constructed by using six widely separated, levitated superconducting test masses. Such a detector would be capable of determining the source direction and wave polarization. The tensor outputs could be combined to better reject the NN.

With SOGRO, one could set a new limit for GW flux at $f = 0.1$-10 Hz, not covered by the present terrestrial detectors. Binaries composed of two IMBHs of $10^4$ solar masses at the distance of a few billion light years or binaries composed of two WDs within the local group can be detected.

Major technical challenges to SOGRO are the construction of a large (30–100 m), rigid, platform that can be cooled to 0.1–1.5 K and obtaining the required high Q ($\sim 10^9$) in the levitated superconducting test masses. In spite of these challenges, SOGRO deserves serious consideration for a low-frequency terrestrial GW detector since it is a full-tensor detector with unique capabilities.

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