Applying an overstress principle in accelerated testing of absorbing mechanisms

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Abstract. The relevance of using overstress test as a forced one to determine the pneumatic absorber lifespan was studied. The obtained results demonstrated that at low load overstress the relative error for the absorber lifespan evaluation is no more than 3%. This means that the test results spread has almost no effect on the lifespan evaluation, and this effect is several times less than that at high load overstress tests. Accelerated testing of absorbers with low load overstress is more acceptable since the relative error for the lifespan evaluation is negligible.

1. Introduction
Forced mode testing acceleration is achieved by intensifying the degradation processes by forcing the loading loads (stress, deformation, displacement amplitude, temperature, frequency, etc.). In this regard single loading mode change tests are of the greatest interest, when an overstressed (high) mode follows a low one or precedes it. Depending on the time of the "loaded" mode, such tests can be called high or low load accelerated overstress tests (figure 1). If an overstressed mode follows a low one, such forced testing is called high load overstress; if this mode precedes the low one, then it is low load overstress.

![Figure 1](image-url)

**Figure 1.** Forcing the overstress testing with high (a) and low (b) loads.
Overstress principle is as follows. No matter what conditions the product was in or was operated under, some fatigue changes accumulate in it. These changes do not disappear, and then they affect the fault-free operation time. Therefore, the testing should be constructed as a step loading, where for a number of cycles $n_{n}$ stress $\sigma_{n}$ is applied, and then for $n_{k}$ cycles the product is overstressed with even higher stress $\sigma_{k}$.

2. Problem statement
The objective is to determine the absorbing mechanisms lifespan $N_{n}$ at loading amplitude $F_{n}$, when firstly the sample is tested for $n_{n}$ cycles at amplitude $F_{n}$, and then it is overstressed at a relatively high amplitude $F_{k}$.

The duration of the preloading $n_{n}$ is known. The residual lifespan $n_{k}$ at the overstress is observed during the experiment. The lifespan $N_{k}$ at the amplitude $F_{k}$ is easy to determine by testing several samples, as $F_{k}$ is profound and the testing time is short. If the functional relation between $n_{k}$, $n_{n}$, $N_{n}$ and unknown value $N_{k}$ is set, the latter can be calculated by the step testing results. Such functional relations based on various suppositions were proposed by different authors [1, 2, 3].

3. Theory
Functional relation for the dependence $n_{k}=n_{k}(n_{n},N_{n},N_{k})$ can be identified by analysing physical nature of damages accumulation. Let the value of the damages accumulation be determined by the equation

$$D_{p}(n) = \int_{0}^{n} \dot{D}_{p}(x)dx,$$

where $\dot{D}_{p}(x)$ is the rate of the damages accumulation; $D_{p}(n)$ is the value of the damages accumulation.

At the sector $(D, n_{n})$ stress $\sigma_{n}$ acts, and at the sector $(n_{n}, n_{n}+n_{k})$ stress $\sigma_{k}$ does, and at the time moment $(n_{n}+n_{k})$ the product failure occurs. Then, the value of the damages accumulated by the time moment $n_{n}$ is represented by the dependence

$$D_{p}(n_{n}) = \int_{0}^{n_{n}} \dot{D}_{n}(x)dx.$$

At the sector $(n_{n}, n_{n}+n_{k})$ the damages accumulation rate depends on the stress $\sigma_{k}$. Let us denote it by $\dot{D}_{k}(n)$. Taking into account that the changes, accumulated in the product for the time $(0, n_{n})$, affect the subsequent damages accumulation, the value of the damages accumulated at the second sector is represented by

$$D_{p}(n_{n}, n_{n}+n_{k}) = \int_{n_{n}}^{n_{n}+n_{k}} \dot{D}_{k}(x)dx,$$

where $n_{n}^{*}$ is some number of cycles representing the impact of the stress $\sigma_{n}$ on the lifespan at the stress $\sigma_{k}$.

Equations (2) and (3) enable us to write the failure occurrence condition for the time $(0, n_{n}+n_{k})$ under the step load as follows

$$\int_{0}^{n_{n}} \dot{D}_{n}(x)dx + \int_{n_{n}}^{n_{n}+n_{k}} \dot{D}_{k}(x)dx = 1$$

or

$$\int_{0}^{n_{n}} \dot{D}_{n}(x)dx + \int_{0}^{n_{n}^{*}} \dot{D}_{k}(x)dx = 1.$$
\[
\int_{0}^{n_u} D_u(n)dn + \int_{0}^{n_k} D_k(n)dn = 1,
\]

(5)

where \(N_u\) and \(N_k\) are destroying cycles number under the stresses \(\sigma_u\) and \(\sigma_k\).

At constant damages accumulation rate, i.e. at the adherence of equation \(\dot{D}(n) = \text{const}\), it follows from formula (5) that

\[
D_u + D_k = 1,
\]

(6)

where \(D_u = \frac{n_u}{N_u}\) is the damage measure at the stress \(\sigma_u\);

\(D_k = \frac{n_k}{N_k}\) is the damage measure at the stress \(\sigma_k\);

\(n_u\) is the value of the residual lifespan at the stress \(\sigma_u\);

\(n_k\) is the number of loading cycles at the stress \(\sigma_k\);

\(\bar{N}_k\) is the mathematical expectation of the lifespan at the stress \(\sigma_k\);

\(\bar{N}_u\) is the mathematical expectation of the lifespan at the stress \(\sigma_u\).

Ratio (6) connects values \(n_u\), \(N_k\), and \(N_u\) and provides an opportunity to plan accelerated testing. From formula (6) one derives unknown value \(\bar{N}_u\) (figure 1).

\[
\bar{N}_u = \frac{n_k}{1 - D_u}.
\]

(7)

Knowing the lifespan value at the high load \(\bar{N}_u\), accelerated testing is planned as step loading. In this regard the loading is performed for \(n_u\) cycles at the stress \(\sigma_u\), and then the transition to the stress \(\sigma_k\) is carried out (figure 1). Several tests are done at constant \(n_u\) and \(\sigma_u\) with the observed values \(n_k^{(i)}\) noted and the mathematical expectation calculated for the value \(\bar{n}_u = \frac{1}{m} \sum_{i=1}^{m} n_k^{(i)}\). The lifespan \(\bar{N}_u\) estimation is found correspondingly as

\[
\bar{N}_u = \frac{n_k}{1 - D_k}.
\]

(8)

Accelerated testing with high load overstress is based on the fact that if the lifespan \(\bar{N}_u\) is to be determined for the low mode (amplitude \(S_u\)) with the known lifespan \(\bar{N}_k\) for the high mode (amplitude \(S_k\)), the product is preloaded for a few cycles \(n_u\) at the low mode, and then it is overstressed at the high one. The reduced testing time is achieved, when the value \(n_u\) is a small portion of the lifespan \(\bar{N}_u\), and the time to determine the values \(\bar{n}_k\) and \(\bar{N}_u\) is short as the overstress mode is high enough.

During accelerated testing with low load overstress, the product under test is loaded for a large number of cycles \(n_u\) at the high mode (amplitude \(S_u\)) when the lifespan \(\bar{N}_u\) is known, and then it is overstressed at the low mode (amplitude \(S_k\)). The reduced testing time is achieved, when at large \(n_u\) the lifespan \(\bar{n}_k\) is a small portion of the lifespan \(\bar{N}_k\), and the duration of the preloading and the lifespan \(\bar{N}_u\) determination is negligible. Compared to ordinary tests, the time advantage is more if the value \(n_u\) is more and the residual lifespan is less, consequently.
The biggest challenge is to find the means of fatigue damages accumulation, taking into consideration all real processes going on in the material at step loading with a single loading mode change. The following hypotheses can be applied for this purpose [5].

1. The Palmgren-Miner hypothesis

\[
\bar{n} = N \left( 1 - \frac{n_u}{N_u} \right). \tag{9}
\]

2. The Manson-Nachtigall-Frisch hypothesis

\[
\bar{n} = N \left( 1 - \frac{n_u}{N_u} \right)^\alpha. \tag{10}
\]

3. The hypothesis of Bolotina V. V.

\[
\bar{n} = N \left[ 1 - \left( \frac{n_u}{N_u} \right)^\alpha \right]. \tag{11}
\]

4. The Henry hypothesis

\[
\bar{n} = \frac{l - n_u}{l + \alpha n_u}. \tag{12}
\]

5. The hypothesis of Kordonskyi Kh. B.

\[
\bar{n} = a n_u \left( N^{\frac{l}{l_n}} - l \right). \tag{13}
\]

6. Parabolic relation

\[
\bar{n} = N \left( 1 - \frac{n_u}{N_u} \right) \left( l + \alpha \frac{n_u}{N_u} \right). \tag{14}
\]

The comparison of predicted and experimental results showed the following:

- The Palmgren-Miner hypothesis has an error depending on material hardening or weakening degree, and it can be significant.
- The Manson-Nachtigall-Frisch hypothesis provides a sufficiently accurate agreement with practice at material hardening, but the error becomes considerable at weakening.
- At the absence of material ageing, the hypothesis of Bolotin V.V. provides an accuracy sufficient for practice, if \( n_u > (20...30)\% \) of \( N_u \).
- The Henry hypothesis can also be used at the absence of material ageing.
- The hypothesis of Kordonskyi Kh. B. provides a reliable estimation of the lifespan in most cases if \( n_u > (15...20)\% \) of \( N_u \).
- Parabolic relation also has a good agreement with the practice at the absence of material weakening.

All these models of recalculating experimental results, obtained when applying overstress test to normal mode, are based on the so-called physical reliability concept [4]. The concept is that product reliability relies on the value of the used lifespan and not on the way it was used. One of the physical reliability concept sequences in case of linear models is the ratio validity [6].

\[
\sum_{i,j} \frac{M_{ij}}{M_{ij}(e_j)} = 1, \tag{15}
\]
where \( k \) is the number of loading mode steps;
\( M_{\varepsilon_j}(\varepsilon_j) \) is the mathematical expectation of fault-free operation time at single step testing in the mode \( \varepsilon_j \);
\( M_{\varepsilon_j}(\xi_j) \) is the mathematical expectation of operation time at \( j \)-th step in the mode \( \varepsilon_j \).

Using ratio (15), it is easy to determine the dependence for mathematical expectation estimation of fault-free operation time when using overstress test.

### 4. Experimental results

To evaluate the relevance of applying overstress as a forced method, experimental tests on pneumatic absorber lifespan were conducted.

The results of calculations on the basis of formulae (7) and (8) are given in table 1 for low load overstress and in table 2 for high load overstress. These results are compared with experimental values for the absorber lifespans. Tables 1 and 2 show relative error of the absorber lifespan estimation for each test.

#### Table 1. Relative error of the absorber lifespan estimation at the low load overstress

| Testing mode | \( F_n \), mm | \( F_k \), mm | \( n_n \cdot 10^3 \) | \( n_k \cdot 10^3 \) | \( \overline{N}_{\varepsilon_n} \cdot 10^3 \) | \( \overline{N}_{\varepsilon_k}^{\text{om}} \cdot 10^3 \) | \( \overline{N}_{\varepsilon_k}^{\text{p}} \cdot 10^3 \) | \( D_{\varepsilon} \) | \( \Delta = \frac{\overline{N}_{\varepsilon_n}^{\text{om}} - \overline{N}_{\varepsilon_k}^{\text{p}}}{\overline{N}_{\varepsilon_k}^{\text{p}}} \cdot 100\% \) |
|--------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------|----------------|
| 16→13        | 16          | 13          | 200            | 142            | 258            | 633.9          | 630            | 0.9995   | 0.62          |
| 15→12        | 15          | 12          | 250            | 177.5          | 331            | 704.4          | 724.4          | 1.0072   | -2.76         |

#### Table 2. Relative error of the absorber lifespan estimation at the high load overstress

| Testing mode | \( F_n \), mm | \( F_k \), mm | \( n_n \cdot 10^3 \) | \( n_k \cdot 10^3 \) | \( \overline{N}_{\varepsilon_n} \cdot 10^3 \) | \( \overline{N}_{\varepsilon_k}^{\text{om}} \cdot 10^3 \) | \( \overline{N}_{\varepsilon_k}^{\text{p}} \cdot 10^3 \) | \( D_{\varepsilon} \) | \( \Delta = \frac{\overline{N}_{\varepsilon_n}^{\text{om}} - \overline{N}_{\varepsilon_k}^{\text{p}}}{\overline{N}_{\varepsilon_k}^{\text{p}}} \cdot 100\% \) |
|--------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------|----------------|
| 13→16        | 13          | 16          | 100            | 269.2          | 400.9          | 401.5          | 302            | 1.1224   | -32.87        |

#### 5. Results and discussion

The analysis of the obtained results shows that at the low load overstress the value of relative error \( \Delta \) for the lifespan estimation is more than 30%. This can be explained by a profound impact of the fatigue strength values spread during testing and material hardening at changing deformation mode from low amplitudes to high ones.

The damage measure value in the testing equals \( D_{\varepsilon} = 1.1224 \), signifying a non-linear nature of the fatigue damages summation at the high load overstress.

At the low load overstress, as seen from table 1, the relative error \( \Delta \) for the lifespan estimation is not more than 3%. This demonstrates that the lifespan testing results spread has a minor effect on the jacket lifespan estimation and this effect is several times less than that at accelerated tests with the high load overstress. In this case, the damage measure value is close to one. The deviation from the fatigue damages linear summation in this case can be explained by the material weakening, when the deformation mode changes from higher amplitude to a lower one.

#### 6. Conclusion

The obtained experimental results demonstrate that high (low) loading amplitudes overstress provides lifespan values close to those of accelerated tests.

Accelerated testing with low loading amplitude overstress is more preferable since the relative error value of lifespan estimation is inconsiderable, and this method allows preloading time \( n_n \) to be set.
Relative total damage measure at high (low) loading amplitudes overstress lies in the ranges $D_{\Sigma} = 0.999\ldots1.122$, and its mean value is close to one. This points out the possibility of applying fatigue damages linear summation hypothesis to evaluate real lifespan of absorbing mechanisms.

References
[1] Krinetskiy L N et al. 1989 *The Foundations of Flying Vehicles Testing*. (Moscow, Mashinostroenie). Russian.
[2] Sudakov R S 1988 *System Testing: Choosing Volumes and Duration*. (Moscow, Mashinostroenie). Russian.
[3] Kubarev A I et al. 1987 *Machinery and Equipment Reliability*. (Moscow, Izdatelstvo standartov). Russian.
[4] Zarenin Yu G 1970 *Reliability Compliance Tests*. (Moscow, Izdatelstvo standartov). Russian.
[5] Nelson W and Hahn G 1972 *Technometrics*. 14(2) 235-244.
[6] Sudo H and Nakano Y 1985 *Microelectron. Reliab*. 25(3) 525-540.