Inclusive $CP$ Asymmetries in Semileptonic
Decays of $B$ Mesons\textsuperscript{1}

Hitoshi Yamamoto

Dept. of Physics, Harvard University, 42 Oxford St., Cambridge, MA 02138, U.S.A.

\textit{e-mail: yamamoto@huhepl.harvard.edu}

Abstract

We estimate the sensitivity of single lepton $CP$ violation measurements with respect to that of traditional di-lepton measurements. We find that the sensitivity of the single lepton method is better than that of the di-lepton method. The achievable sensitivity with the currently available data is already in the range relevant to standard model predictions. We also give general expressions for inclusive decay time distributions on $\Upsilon(4S)$ where the other $B$ is not measured, which will be used to obtain time dependent asymmetries. The expression is of general use whenever one deals with inclusive time-dependent as well as time-integrated measurements in $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ where the final state of the other $B$ is not reconstructed or when only the time difference is measured.

\textsuperscript{1}A talk given at the San Miniato conference, April 1997, and at the Hawaii CP violation conference, February 1997.
1 Introduction

The particle-antiparticle imbalance in the mass eigenstates of the neutral $B$ meson system,
\[ \delta \equiv \frac{|\langle B^0|B_{a,b}\rangle|^2 - |\langle B^0|\bar{B}^0|B_{a,b}\rangle|^2}{|\langle B^0|B_{a,b}\rangle|^2 + |\langle B^0|\bar{B}^0|B_{a,b}\rangle|^2}, \]
can be extracted from the same-sign di-lepton asymmetry in $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ \cite{1, 2}:
\[ A_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} \sim 2\delta. \]

The short distance calculation gives \cite{3, 4, 6}
\[ A_{\ell\ell} \sim -4\pi \frac{m_c^2}{m_t} \Re \left( \frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} \right) \sim 10^{-3}. \]

The long distance effects, however, are expected to dominate and one estimate gives $|A_{\ell\ell}| \sim 10^{-3}$ to $10^{-2}$ \cite{7}. Thus, a measurement of such asymmetry does not determine the $CP$ violating parameters in the standard model. However, an experimental value of $|\delta|$ above $\sim 10^{-2}$ would signal new physics \cite{8, 9}. The current published experimental number is not yet in the relevant range: $A_{\ell\ell} = 0.031 \pm 0.096 \pm 0.032 \pm 0.032 \pm 0.032$ \cite{10}. For $B_s$, relevant CKM factor is obtained by replacing $d$ by $s$ in the $B_d$ case, and it is expected to be small.

The $CP$ asymmetry in single lepton sample had been suggested as a possible observable to search for $CP$ violation in the case when the mixing is small \cite{5, 6, 11}. The logic was that if the mixing is small, then the statistics of the di-lepton events will decrease, making the di-lepton method impractical. After the observation of substantial mixing in the neutral $B$ meson system \cite{12}, however, the single lepton method has not received much attention. We point out that the advantage of the single lepton method over the di-lepton method actually increases for large mixings, and that, on $\Upsilon(4S)$, the single lepton method has a better sensitivity than the di-lepton method. This is so in spite of the fact that in the single lepton measurement, one usually cannot distinguish charged and neutral $B$ mesons. We first present general expressions the inclusive decay time distribution on $\Upsilon(4S)$.

2 Inclusive Time Distributions in $\Upsilon(4S) \rightarrow B^0\bar{B}^0$

The mass eigenstates can be written in terms of $B^0$ and $\bar{B}^0$ as
\[ \begin{cases} B_a = pB^0 + q\bar{B}^0 & \text{(mass:$m_a$, decay rate:$\gamma_a$)} \\ B_b = p'\bar{B}^0 - q'\bar{B}^0 & \text{(mass:$m_b$, decay rate:$\gamma_b$)} \end{cases} \]
where $|p|^2 + |q|^2 = |p'|^2 + |q'|^2 = 1$. We will adopt the Wigner-Weisskopf formalism\cite{13} which is valid when the oscillations caused by differences of masses and the decay rates are sufficiently slower than the time scale of decay transitions. If we have a pure $B^0$ or $\bar{B}^0$ at $t = 0$, the decay time distributions to a final state $f$ are given by \cite{14}

$$
\begin{align*}
\Gamma_{B^0 \to f}(t) &= |c|^2 \left[ |q'a_f|^2 e^{-\gamma_a t} + |qb_f|^2 e^{-\gamma_b t} + 2 \Re ((q'a_f) (qb_f) e^{-(\gamma_a + i \delta m) t}) \right], \\
\Gamma_{\bar{B}^0 \to f}(t) &= |c|^2 \left[ |p'a_f|^2 e^{-\gamma_a t} + |pb_f|^2 e^{-\gamma_b t} - 2 \Re ((p'a_f) (pb_f) e^{-(\gamma_a + i \delta m) t}) \right],
\end{align*}
$$

where

$$
e \equiv \frac{1}{p'q + pq'}, \ \delta m \equiv m_a - m_b, \ \gamma_\pm \equiv \frac{\gamma_a \pm \gamma_b}{2}
$$

and $a_f \equiv \text{Amp}(B_a \to f)$, $b_f \equiv \text{Amp}(B_b \to f)$ are normalized as $\sum_f |a_f|^2 = \gamma_a$, $\sum_f |b_f|^2 = \gamma_b$.

On $\Upsilon(4S)$, the $B^0 - \bar{B}^0$ pair is created in a $P$-wave state, and the quantum correlations of the two decays need to be carefully taken into account. In particular, when one side is detected as a certain final state $f$ that both $B^0$ and $\bar{B}^0$ can decay to, then at the same proper time the other side becomes a superposition of $B^0$ and $\bar{B}^0$ which depends on the decay amplitudes of $B^0$ and $\bar{B}^0$ to $f$. For the double-time decay distribution, the prescription is well known and the general expression for the decay probability where one side decays to $f_1$ at $t_1$ and the other side to $f_2$ at $t_2$ is \cite{14}

$$
\Gamma_{\Upsilon(4S) \to f_1 f_2}(t_1, t_2) = \frac{|c|^2}{2} \left[ e^{-\gamma_a t_1 - \gamma_b t_2} |a_{f_1} b_{f_2}|^2 + e^{-\gamma_b t_1 - \gamma_a t_2} |b_{f_1} a_{f_2}|^2 - 2 \Re (e^{-(\gamma_a - i \delta m) t_1} e^{-(\gamma_b + i \delta m) t_2} (a_{f_1} b_{f_2})^* (b_{f_1} a_{f_2})) \right],
$$

or in terms of $t_\pm \equiv t_1 \pm t_2$,

$$
\Gamma_{\Upsilon(4S) \to f_1 f_2}(t_+ , t_- ) = \frac{|c|^2}{4} e^{-\gamma_+ t_+} \left[ e^{-\gamma_- t_-} |a_{f_1} b_{f_2}|^2 + e^{\gamma_- t_-} |b_{f_1} a_{f_2}|^2 - 2 \Re ((a_{f_1} b_{f_2})^* (b_{f_1} a_{f_2}) e^{i \delta m t_-}) \right].
$$

In order to obtain the inclusive decay distribution where only one decay is detected, one has to integrate over time and sum over all possible final states on the other side. Performing the operation in \cite{14}, and using the Bell-Steinberger relation\cite{13}

$$
\sum_f a_{f_1}^* b_{f_2} = \langle B_a | B_{b} \rangle \ (= p' p^* - q' q^*) ,
$$

one obtains

$$
\Gamma_{\Upsilon(4S) \to f}(t) = 2 \sum_{f_1} \int_0^\infty \Gamma_{\Upsilon(4S) \to f_1 f}(t_1 , t) \ dt_1 = \Gamma_{B^0 \to f}(t) + \Gamma_{\bar{B}^0 \to f}(t) = |c|^2 \left[ |a_f|^2 e^{-\gamma_a t} + |b_f|^2 e^{-\gamma_b t} - 2 \Re ((p' p^* - q' q^*) a_{f_1} b_{f_2} e^{-(\gamma_a + i \delta m) t}) \right],
$$

3
where $\Gamma_{\Upsilon(4S)\rightarrow f}(t)$ is the probability density that one finds a given final state $f$ decaying at time $t$ in the process $\Upsilon(4S) \rightarrow B^0\overline{B}^0$, and the factor of two arises from the fact that the given final state can come from either side. This result may be expected since there is one $B^0$ and one $\overline{B}^0$ created and no correlation is measured, so the inclusive distribution can be expected to be simple sum of the two distributions. Such argument, however, is not in general true for general two-body states [16]. At $B$-factories, the absolute decay time is not measured well; instead the relevant time is the time difference $t_-$ of the two decays. This can be obtained by integrating (2) over $t_+$ and summing over $f_2$:

$$\Gamma_{\Upsilon(4S)\rightarrow f}(t_-) = \frac{|c|^2}{2\gamma_+} e^{-\gamma_+|t_-|} \times \left[ \gamma_b e^{-\gamma_-|t_-|} |a_f|^2 + \gamma_a e^{\gamma_-|t_-|} |b_f|^2 - 2\Re\left((\gamma_+ - i\delta m)(p'p^* - q'q^*)a_f b_f^* e^{-i\delta mt_-}\right) \right].$$

The sign of the time difference $t_-$ is defined to be the decay time of the final state $f$ minus the decay time of the other side. These formulae above are valid even when $CPT$ is violated.

### 3 Leptonic $CP$ Asymmetries and Experimental Sensitivities

The decay distributions for semileptonic decays are obtained by the substitution (assuming $CPT$ [14], and $\Delta B = \Delta Q$ [18])

$$a_{\ell+} = pA_0, \quad b_{\ell+} = pA_0,$$
$$a_{\ell-} = qA_0, \quad b_{\ell-} = -q\overline{A}_0,$$

where $A_0$ is the normalized semileptonic decay amplitude. The decay time distributions for a single $B^0$ or $\overline{B}^0$ created at $t = 0$ are then

$$\Gamma_{B^0\rightarrow \ell+}(t) = \frac{A_0^2}{2} e^{-\gamma_+ t} \left[ \cosh \gamma_- t + \cos \delta mt \right],$$

$$\Gamma_{\overline{B}^0\rightarrow \ell+}(t) = \frac{2}{|q|^2} e^{-\gamma_+ t} \left[ \cosh \gamma_- t - \cos \delta mt \right],$$

$$\Gamma_{B^0\rightarrow \ell-}(t) = \frac{2}{|p|^2} e^{-\gamma_+ t} \left[ \cosh \gamma_- t - \cos \delta mt \right].$$

We see that the ‘right-sign’ or ‘unmixed’ decay distributions for $B^0$ and $\overline{B}^0$ are identical, but the ‘wrong-sign’ or ‘mixed’ decay distribution can be different when $|p| \neq |q|$ namely when there is an imbalance of particle and antiparticle in the
Figure 1: Time-dependent asymmetries of flavor-untagged inclusive semileptonic decay shown in unit of $\delta$. The solid line is the asymmetry as a function of decay time difference, and the dashed line is that of absolute decay time. Used $\delta m / \gamma_+ = 0.73$ and $\gamma_- / \gamma_+ = 0.1$.

physical states. Note, however, that the shape is the same between $B^0$ and $\bar{B}^0$ even for the unmixed decay distributions.

Flavor-untagged inclusive single lepton decay distribution on $\Upsilon(4S)$ is then obtained from the distributions for $B^0$ and $\bar{B}^0$ by simple incoherent sum. Resulting time dependent lepton asymmetry is

$$A_\ell(t) = \frac{\Gamma_{B^0, \Upsilon^0 \to \ell^+}(t) - \Gamma_{B^0, \Upsilon^0 \to \ell^-}(t)}{\Gamma_{B^0, \Upsilon^0 \to \ell^+}(t) + \Gamma_{B^0, \Upsilon^0 \to \ell^-}(t)}$$

which is the asymmetry one would observe in hadronic machines or LEP where the absolute decay time can be measured. The asymmetry starts out as zero at $t = 0$ and reaches the first maximum at around $\delta m t \sim \pi$ (about 4 times the $b$ lifetime). At $B$-factories, the asymmetry as a function of $t_-$ is

$$A_\ell(t_-) = \delta \left( 1 - \frac{\gamma_+ \cos \delta m t_- - \delta m \sin \delta m t_-}{\gamma_+ \cosh \gamma_- t_- + \gamma_- \sinh \gamma_- t_-} \right),$$

which is a function similar to $A_\ell(t)$ above but the oscillation amplitude is slightly larger and it applies also to negative value of $t_-$. Figure 1 shows these asymmetries.

The time-integrated single lepton asymmetry is given by

$$A_\ell = 2D \chi \delta,$$
where \( D \) is the dilution factor due to charged \( B \) mesons, and is equal to the fraction of leptons coming from neutral \( B \) mesons. Other dilution effects such as those due to misidentified leptons or leptons from charmed hadrons could also be absorbed into \( D \). Assuming that there are the same number of leptons from charged \( B \)'s as from neutral \( B \)'s, we take \( D = 1/2 \).

We will now estimate the sensitivities to \( \delta \) of single and di-lepton asymmetry measurements. We assume that the lepton detection efficiency \( \epsilon_\ell \) for each lepton is the same in the single and di-lepton cases, and that they are uncorrelated in the latter. Also we assume \( \delta \ll 1 \) for the expressions of asymmetries below. In estimating statistics, we further assume \( \gamma_a \sim \gamma_h \) (or equivalently \( y \ll 1 \)). If we have \( N_0 \ U(4S) \rightarrow B^0\overline{B^0} \) decays, then the total number of same sign di-lepton events detected is \( N_0 b_{sl}^2 \chi \epsilon_\ell^2 \), where \( b_{sl} \) is the semileptonic branching fraction. The error in \( \delta \) is then

\[
\sigma_\delta(\ell\ell) = \frac{1}{2} \sqrt{\frac{1}{N_0 b_{sl}^2 \chi \epsilon_\ell^2}}.
\]

The total number of single lepton events detected is \( N_0 b_{sl} \epsilon_\ell \); thus the sensitivity to \( \delta \) of the single lepton measurement is

\[
\sigma_\delta(\ell) = \frac{1}{\chi} \sqrt{\frac{1}{N_0 b_{sl} \epsilon_\ell}}.
\]

The ratio of sensitivities of single to di-lepton measurements is then

\[
\frac{\sigma_\delta(\ell)}{\sigma_\delta(\ell\ell)} = \sqrt{\frac{b_{sl} \epsilon_\ell}{\chi}}.
\]

We see that the larger the mixing, the more advantageous the single lepton method becomes. This may be counter-intuitive, but can be understood as follows: as \( \chi \) increases, the statistics goes up linearly for the di-lepton sample while its asymmetry stays the same. For the single lepton sample, the statistics stays the same while the asymmetry goes up linearly, which is equivalent to statistics increasing quadratically for a fixed asymmetry.

A typical value for \( \epsilon_\ell \) is 0.5. Together with the experimental values for \( b_{sl} \) and \( \chi \), the ratio above is 0.78. Namely, the single lepton measurement has a sensitivity comparable to or better than that of the di-lepton measurement. Note also that the single and di-lepton datasets are largely statistically independent (only about 10% of the single lepton events are also in the di-lepton dataset). The two measurements can thus be combined to improve overall sensitivity. For example, the current CLEO data corresponds to \( N_0 \sim 2 \times 10^6 \). This gives \( \sigma_\delta(\ell) = 0.6\% \) and \( \sigma_\delta(\ell\ell) = 0.8\% \) with the combined sensitivity of 0.5% which is already in the range relevant to standard model predictions.
When $B^0$’s and $\bar{B}^0$’s are generated incoherently (e.g. on $Z^0$ or in $p\bar{p}$ collisions), one cannot perform the correlated di-lepton analysis. However, one can still perform single lepton asymmetry measurements. There, alternative method is to use flavor-tagging by lepton and/or jet charge to enhance the sensitivity.

Acknowledgments

This work was supported by the Department of Energy Grant DE-FG02-91ER40654.

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