Semileptonic decays of $D$ and $D_s$ mesons in the relativistic quark model

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The form factors parameterizing the weak $D$ and $D_s$ transitions to light pseudoscalar and vector mesons are calculated in the framework of the relativistic quark model based on the quasipotential approach. The special attention is paid to the systematic account of the relativistic effects including transformation of the meson wave function from the rest to moving reference frame and contributions of the intermediate negative-energy states. The form factors are expressed through the overlap integrals of the meson wave functions, which are taken from previous studies of meson spectroscopy. They are calculated in the whole range of the transferred momentum $q^2$. Convenient parameterization of the form factors which accurately reproduces numerical results is given. The obtained values of the form factors and their ratios at $q^2 = 0$ agree well with the ones extracted from the experimental data. On the basis of these form factors and helicity formalism, differential and total semileptonic decay rates of $D$ and $D_s$ mesons as well as different asymmetries and polarization parameters are calculated. The detailed comparison of the obtained results with other theoretical calculations and experimental data is given.

I. INTRODUCTION

Semileptonic decays of heavy mesons provide an important information on the values of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements $V_{Qq}$ (with $Q$ denoting the heavy quark and $q$ the light one), which are essential ingredients of the standard model. Experimentally such decays can be measured more accurately than pure leptonic ones since there is no helicity suppression for them. Theoretically semileptonic decays are significantly less complicated than hadronic ones as they contain one meson and a lepton pair in the final state. The lepton part is easily calculated using standard methods, while the hadronic part factorizes thus reducing theoretical uncertainties. The hadronic matrix element is usually parameterized by the set of invariant form factors, which are calculated using nonperturbative approaches based on quantum chromodynamics (QCD), such as lattice QCD, QCD
sum rules, potential quark models.

Recently significant experimental progress has been achieved in studying semileptonic
decays of the open charm mesons \cite{1}. More precise and detailed measurements of the absolute
and differential branching fractions and form factors for $D$ and $D_s$ decays to pseudoscalar
and vector mesons became available due to high statistics accumulated at BES III \cite{2-10}.
Various CKM- favored and suppressed decay modes both with positron and muon were
investigated. This allows one to check the lepton universality in $D$ meson decays. Note
that possible hints of its violation were recently found in $B$ decays \cite{11}. More precise and
comprehensive data are expected form BES III and Belle II \cite{12} in near future.

In this paper we calculate the matrix elements of the flavor changing charged weak cur-
rent between initial $D$ or $D_s$ mesons and final light pseudoscalar or vector mesons in the
framework of the relativistic quark model based on the quasipotential approach. This model
was successfully applied for the calculations of the hadron spectroscopy \cite{13-16} and weak
decays \cite{17-21}. It was found that relativistic effects play very important role both for light
and heavy hadrons. Thus the form factors are calculated with the consistent account of the
relativistic quark dynamics. They are expressed through the overlap integrals of the meson
wave functions which are known from the study of their spectroscopy. The momentum trans-
fer $q^2$ dependence of form factors is explicitly determined in the whole kinematical range
without additional assumptions and extrapolations. Then we use these form factors and
the helicity formalism for the calculation of the differential and total branching fractions as
well as polarization and asymmetry parameters. We also compare our results with available
experimental data and previous predictions.

The paper is organized as follows. In Sec. II we briefly describe our relativistic quark
model with special emphasis on calculation of the weak decay matrix elements between
meson states with the account of relativistic effects. This model is applied in Sec. III to
the consideration of semileptonic decay form factors of open charm mesons. We give the
analytic expressions for the form factors which accurately reproduce the numerical results
for the momentum transfer $q^2$ dependence of the form factors in the whole accessible kinem-
atical range and compare them with available data. Then in Sec. IV we use these form
factors to calculate the differential and total $D$ and $D_s$ meson semileptonic decay rates and
different asymmetries and polarization parameters. Decays both with positrons and muons
are considered. This allows us to give predictions for the ratios of the corresponding de-
cay rates which can be used for the test of the lepton universality in charm meson decays.
Finally, Sec. VI contains our conclusions.

II. RELATIVISTIC QUARK MODEL

For the calculation of meson properties we employ the relativistic quark model based on
the quasipotential approach. In this model a meson with the mass $M$ is described by the
wave function $\Psi_M(p)$ of the quark-antiquark bound state which satisfies the Schrödinger-like
quasipotential equation \cite{13}

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R}\right)\Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M)\Psi_M(q),$$

(1)
where \( m_1, m_2 \) are the quark masses, \( \mathbf{p} \) is the relative quark momentum. The relative momentum squared in the center of mass system on the mass shell is given by

\[
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2},
\]

and the relativistic reduced mass is defined by

\[
\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}.
\]

The kernel of this equation \( V(\mathbf{p}, \mathbf{q}; M) \) is the QCD-motivated quark-antiquark potential which is constructed by the off-mass-shell scattering amplitude projected on the positive energy states. We assume [13] that it consists from the one-gluon exchange term which dominates at small distances and a mixture of the scalar and vector linear confining interactions which dominate at large distances. Moreover, we assume that the long-range vertex of the confining vector interaction contains additional Pauli term. Then the quasipotential is given by

\[
V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(\mathbf{p}) \gamma \cdot \mathbf{k} u_2(-\mathbf{p}) V(\mathbf{p}, \mathbf{q}; M) u_1(\mathbf{q}) \gamma \cdot \mathbf{k} u_2(-\mathbf{q}),
\]

with

\[
V(\mathbf{p}, \mathbf{q}; M) = \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma_1^\mu \gamma_2^\nu + V_{conf}(\mathbf{k}) \Gamma_1^\mu(\mathbf{k}) \Gamma_2^\mu(\mathbf{k}) + V_{conf}^S(\mathbf{k}), \quad \mathbf{k} = \mathbf{p} - \mathbf{q},
\]

where \( \alpha_s \) is the QCD coupling constant, \( D_{\mu\nu} \) is the gluon propagator in the Coulomb gauge, and \( \gamma_{\mu} \) and \( u(\mathbf{p}) \) are the Dirac matrices and spinors, respectively. The long-range vector vertex has the form

\[
\Gamma_{\mu}(\mathbf{k}) = \gamma_{\mu} + \frac{i\kappa}{2m} \sigma_{\mu\nu} \mathbf{k}^\nu,
\]

where \( \kappa \) is the long-range anomalous chromomagnetic quark moment. In the nonrelativistic limit confining vector and scalar potentials reduce to

\[
V_{conf}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{conf}^S(r) = \varepsilon(Ar + B),
\]

and in the sum they reproduce the linear rising potential

\[
V_{conf}(r) = V_{conf}^V(r) + V_{conf}^S(r) = Ar + B,
\]

where \( \varepsilon \) is the mixing coefficient. Thus this quasipotential can be viewed as the relativistic generalization of the nonrelativistic Cornell potential

\[
V_{NR}(r) = -\frac{4}{3} \alpha_s \frac{1}{r} + Ar + B.
\]

Our quasipotential contains both spin-independent and spin-dependent relativistic contributions.

All parameters of the model were fixed from the previous consideration of hadron spectroscopy and decays [13]. Thus the values of constituent quark masses are \( m_b = 4.88 \) GeV, \( m_c = 1.55 \) GeV, \( m_s = 0.5 \) GeV, \( m_{u,d} = 0.33 \) GeV; the parameters of the linear potential are \( A = 0.18 \) GeV\(^2\) and \( B = -0.30 \) GeV; the mixing parameter of the vector and scalar
confining potential is \( \varepsilon = -1 \), while the anomalous chromomagnetic quark moment \( \kappa = -1 \). We take the running QCD coupling constant with infrared freezing

\[
\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{A}},
\]

where \( \beta_0 = 11 - \frac{2}{3} n_f \), \( n_f \) is the number of flavors, \( \Lambda = 413 \) MeV, \( M_0 = 2.24\sqrt{A} = 0.95 \) GeV and the scale \( \mu \) is set to \( \frac{2m_1 m_2}{m_1 + m_2} \).

The spectroscopy of heavy-light and light mesons was discussed in detail in Refs. [14, 15]. The calculated masses for both of the ground and excited states were found in agreement with available experimental data and exhibit linear Regge trajectories. The meson wave functions were also calculated and can be used for the evaluation of the meson decays.

For the consideration of the \( D \) meson semileptonic decays it is necessary to calculate the hadronic matrix element of the local current governing the \( c \to f \) \( (f = s, d) \) weak transition. In the quasipotential approach the matrix element of this weak current \( J^W_\mu = \bar{f}\gamma_\mu(1 - \gamma_5)c \) between the initial \( D_{(s)} \) meson with four-momentum \( p_{D_{(s)}} \) and final meson \( F \) with four-momentum \( p_F \) is given by [17]

\[
\langle F(p_F)|J^W_\mu|D_{(s)}(p_{D_{(s)}})\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_F(p_F)\Gamma_\mu(p, q)\Psi_{D_{(s)}}(p_{D_{(s)}})(q),
\]

where \( \Psi_{M p_M} \) are the initial and final meson wave functions projected on the positive energy states and boosted to the moving reference frame with the three-momentum \( p_M \). The vertex function

\[
\Gamma = \Gamma^{(1)} + \Gamma^{(2)},
\]

where \( \Gamma^{(1)} \) is the leading-order vertex function which corresponds to the impulse approximation

\[
\Gamma^{(1)}_\mu(p, q) = \bar{u}_f(p_f)\gamma_\mu(1 - \gamma_5)u_c(q_c)(2\pi)^3\delta(p_q - q_q)
\]

and contains the \( \delta \) function responsible for the momentum conservation on the spectator \( q \) antiquark line. The vertex function \( \Gamma^{(2)} \) takes into account interaction of the active quarks \( (c, f) \) with the spectator antiquark \( (q) \) and includes the negative-energy part of the active quark propagator. It is the consequence of the projection on the positive energy states and has the form

\[
\Gamma^{(2)}_\mu(p, q) = \bar{u}_f(p_f)\bar{u}_q(p_q)\left\{\mathcal{V}(p_q - q_q)\frac{\Lambda^{(-)}_{f}(k')}{\epsilon_f(k') + \epsilon_f(q_c)}\gamma_1^0\gamma_1\mu(1 - \gamma_1^5) + \gamma_1\mu(1 - \gamma_1^5)\frac{\Lambda^{(-)}_{c}(k')}{\epsilon_c(k) + \epsilon_c(p_f)}\gamma_1^0\mathcal{V}(p_q - q_q)\right\}u_c(q_c)u_q(q_q),
\]

where \( k = p_F - \Delta; \ k' = q_c + \Delta; \ \Delta = p_F - p_D; \epsilon(p) = \sqrt{p^2 + m^2}; \) and the projection operator on the negative-energy states

\[
\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma_0 + \gamma_0(\gamma p))}{2\epsilon(p)}.
\]

Note that the \( \delta \) function in the vertex function \( \Gamma^{(1)} \) [Eq. (12)] allows us to take off one of the integrals in the expression for the matrix element Eq. (10). As the result the usual
expression for the matrix element as the overlap integral of the meson wave functions is obtained. The contribution $\Gamma^{(2)}$ [Eq. (13)] is significantly more complicated and contains the quasipotential of the quark-antiquark interaction $\mathcal{V}$ [Eq. (11)] which has nontrivial Lorentz-structure. However, it is possible to use the quasipotential equation (11) to get rid of one of the integrations in Eq. (10) and thus get again the usual structure of the matrix element as the overlap integral of meson wave functions (for details see Refs. [17, 18]).

Calculations of hadron decays are usually done in the rest frame of the decaying hadron, the $D_s$ meson in the considered case, where the decaying meson momentum $p_D = 0$. Then the final meson $F$ is moving with the recoil momentum $\Delta = p_F$ and its wave function should be boosted to the moving reference frame. The wave function of the moving meson $\Psi_F(\Delta)$ is connected with the wave function in the rest frame $\Psi_{F0}$ by the transformation [17]

$$\Psi_F(\Delta) = D^{1/2}(R^W_{\Delta})D^{1/2}_q(R^W_{\Delta})\Psi_{F0}(p),$$

(14)

where $R^W$ is the Wigner rotation, $L_{\Delta}$ is the Lorentz boost from the meson rest frame to a moving one and $D^{1/2}(R)$ is the spin rotation matrix.

### III. WEAK DECAY FORM FACTORS

In the standard model the semileptonic $D$ and $D_s$ meson decays to a pseudoscalar ($P$) or a vector ($V$) mesons are governed by the flavor-changing $c \to q \ell \nu_\ell$ ($q = s, d$) current. The corresponding matrix element $\mathcal{M}$ between meson states factorizes in the product of the leptonic ($L_\mu$) current and the matrix element of the hadronic ($H^\mu$) current with the corresponding CKM matrix element $V_{cq}$ and the Fermi constant $G_F$

$$\mathcal{M}(D_{(s)} \to P(V)\ell \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq} H^\mu L_\mu,$$

(15)

where $L_\mu = \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell$ and $H^\mu = \langle P(V) | \bar{q} \gamma_\mu (1 - \gamma_5) c | D_{(s)} \rangle$. The leptonic part is easily calculated using the lepton spinors and has a simple structure, while the hadronic part is significantly more complicated and requires nonperturbative treatment within QCD.

The hadronic matrix element of weak current $J^W$ between meson states is usually parameterized by the following set of the invariant form factors.

- For $D_{(s)}$ transitions to pseudoscalar $P$ ($\pi, K, \eta, \eta'$) mesons

$$\langle P(p_F) | \bar{q} \gamma^\mu c | D_{(s)}(p_{D_{(s)}}) \rangle = f_+(q^2) \left[ p^\mu_{D_{(s)}} + p^\mu_p - \frac{M_{D_{(s)}}^2 - M_P^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_{D_{(s)}}^2 - M_P^2}{q^2} q^\mu,$$

(16)

- For $D_{(s)}$ transitions to vector $V$ ($\rho, \omega, K^*, \phi$) mesons

$$\langle V(p_V) | \bar{q} \gamma^\mu c | D_{(s)}(p_{D_{(s)}}) \rangle = \frac{2iV(q^2)}{M_{D_{(s)}} + M_V} \epsilon^{\mu\nu\rho\sigma} \bar{c}_\nu p_{D_{(s)}} \rho p_V \sigma,$$

$$\langle V(p_V) | \bar{q} \gamma^\mu \gamma_5 c | D_{(s)}(p_{D_{(s)}}) \rangle = 2M_V A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M_{D_{(s)}} + M_V) A_1(q^2) \left( \epsilon^* - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right)$$

$$- A_2(q^2) \frac{\epsilon^* \cdot q}{M_{D_{(s)}} + M_V} \left[ p^\mu_{D_{(s)}} + p^\mu_p - \frac{M_{D_{(s)}}^2 - M_P^2}{q^2} q^\mu \right].$$

(17)
At the maximum recoil point \((q^2 = 0)\) these form factors satisfy the following conditions:

\[
f_+(0) = f_0(0), \quad A_0(0) = \frac{M_{D(s)} + M_V}{2M_V} A_1(0) - \frac{M_{D(s)} - M_V}{2M_V} A_2(0).
\]

We use the quasipotential approach and the relativistic quark model discussed in Sec. II for the calculation of the weak decay matrix elements and transition form factors. We substitute the leading \(\Gamma^{(1)}\) [Eq. (12)] and subleading \(\Gamma^{(2)}\) [Eq. (13)] vertex functions in the expression for the matrix element of the weak current between meson states (10). This matrix element is considered in the rest frame of the decaying \(D_s\) meson, then the boost of the final meson wave function \(\Psi_F\) from the rest to moving reference frame with the recoil momentum \(\Delta = p_F\) should be considered. It is given by Eq. (14). Thus we take into account all relativistic effects including the relativistic contributions of intermediate negative-energy states and relativistic transformations of the meson wave functions. The resulting expressions for the decay form factors have the form of the overlap integrals of initial and final meson wave functions. They are rather cumbersome and are given in Refs. [17, 18]. For the numerical evaluation of the decay form factors we use the meson wave functions obtained in calculating their mass spectra [14, 16]. This is a significant advantage of our approach since in most of the previous model calculations some phenomenological wave functions (such as Gaussian) were used. Moreover, our relativistic approach allows us to determine the form factor dependence on the transferred momentum \(q^2\) in the whole accessible kinematical range without additional approximations and extrapolations.

We find that the numerical results for these decay form factors can be approximated with high accuracy by the following expressions:

(a) \(f_+(q^2), V(q^2), A_0(q^2)\)

\[
F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M^2}\right) \left(1 - \sigma_1 \frac{q^2}{M^2_{D(s)}} + \sigma_2 \frac{q^4}{M^4_{D(s)}}\right)},
\]

(b) \(f_0(q^2), A_1(q^2), A_2(q^2)\)

\[
F(q^2) = \frac{F(0)}{\left(1 - \sigma_1 \frac{q^2}{M^2_{D(s)}} + \sigma_2 \frac{q^4}{M^4_{D(s)}}\right)},
\]

where for the decays governed by the CKM favored \(c \to s\) transitions masses of the intermediate \(D_s\) mesons are used: \(M = M_{D^*} = 2.112\) GeV for the form factors \(f_+(q^2), V(q^2)\) and \(M = M_{D_s} = 1.968\) GeV for the form factor \(A_0(q^2)\). While for the decays governed by the CKM suppressed \((c \to d)\) transitions masses of the intermediate \(D\) mesons are taken as follows: \(M = M_{D^*} = 2.010\) GeV for the form factors \(f_+(q^2), V(q^2)\) and \(M = M_D = 1.870\) GeV for the form factor \(A_0(q^2)\). The values of form factors \(F(0), F(q^2_{\text{max}})\) and fitted parameters \(\sigma_{1,2}\) are given in Tables [111]. We estimate the uncertainties of the calculated form factors to be less than 5%. The form factors are plotted in Figs. [12].

In Fig. [III] we compare our predictions for the product \(f_+(q^2)|V_{cq}|\) with experimental data from Belle [22] and BaBar [23] and lattice results [24, 25] for the weak \(D \to K\) and \(D \to \pi\)
TABLE I: Form factors of the weak $D$ meson transitions.

| Decay | Form factor | $F(0)$ | $F(q_{\text{max}}^2)$ | $\sigma_1$ | $\sigma_2$ |
|-------|-------------|--------|----------------------|----------|----------|
| $D \to K$ | $f_+$ | 0.716 | 1.538 | 0.902 | 1.07 |
|  | $f_0$ | 0.716 | 1.086 | 0.360 | 1.657 |
| $D \to K^*$ | $V$ | 0.927 | 1.305 | 0.356 | -0.490 |
|  | $A_0$ | 0.655 | 1.048 | 0.432 | -0.840 |
|  | $A_1$ | 0.608 | 0.660 | 0.410 | 0.166 |
|  | $A_2$ | 0.520 | 0.623 | 0.582 | -0.917 |
| $D \to \pi$ | $f_+$ | 0.640 | 2.336 | 0.332 | 0.557 |
|  | $f_0$ | 0.640 | 1.318 | -0.345 | 1.133 |
| $D \to \rho$ | $V$ | 0.979 | 1.884 | 0.264 | -2.001 |
|  | $A_0$ | 0.712 | 1.377 | 0.282 | -0.826 |
|  | $A_1$ | 0.682 | 0.782 | 0.567 | 0.352 |
|  | $A_2$ | 0.640 | 0.815 | 0.964 | 0.645 |
| $D \to \eta$ | $f_+$ | 0.547 | 1.228 | 1.153 | 1.519 |
|  | $f_0$ | 0.547 | 0.683 | 0.408 | 3.147 |
| $D \to \eta'$ | $f_+$ | 0.538 | 0.804 | -0.203 | -4.686 |
|  | $f_0$ | 0.538 | 0.547 | -0.950 | 1.038 |
| $D \to \omega$ | $V$ | 0.871 | 1.709 | 0.146 | -2.775 |
|  | $A_0$ | 0.647 | 1.178 | 0.224 | -0.759 |
|  | $A_1$ | 0.674 | 0.765 | 0.542 | 0.350 |
|  | $A_2$ | 0.713 | 0.802 | 0.997 | 2.176 |

TABLE II: Form factors of the weak $D_s$ meson transitions.

| Decay | Form factor | $F(0)$ | $F(q_{\text{max}}^2)$ | $\sigma_1$ | $\sigma_2$ |
|-------|-------------|--------|----------------------|----------|----------|
| $D_s \to \eta$ | $f_+$ | 0.443 | 1.554 | 0.675 | -0.856 |
|  | $f_0$ | 0.443 | 0.550 | -0.302 | 1.634 |
| $D_s \to \eta'$ | $f_+$ | 0.559 | 1.001 | 0.719 | -2.123 |
|  | $f_0$ | 0.559 | 0.654 | -0.499 | -0.124 |
| $D_s \to \phi$ | $V$ | 0.999 | 1.687 | 0.467 | -4.020 |
|  | $A_0$ | 0.713 | 0.988 | 0.412 | 0.903 |
|  | $A_1$ | 0.643 | 0.746 | 0.621 | -0.317 |
|  | $A_2$ | 0.492 | 0.645 | 0.447 | -3.622 |
| $D_s \to K$ | $f_+$ | 0.674 | 2.451 | 1.255 | -0.935 |
|  | $f_0$ | 0.674 | 1.174 | 0.216 | 1.241 |
| $D \to K^*$ | $V$ | 0.959 | 1.966 | 0.425 | -2.444 |
|  | $A_0$ | 0.629 | 1.103 | 0.281 | -0.435 |
|  | $A_1$ | 0.596 | 0.733 | 0.835 | 0.423 |
|  | $A_2$ | 0.540 | 0.702 | 1.266 | 1.425 |
FIG. 1: Form factors of the weak $D$ meson transitions.
FIG. 2: Form factors of the weak $D_s$ meson transitions. On the same plots we also show our results for $f_0(q^2)|V_{cq}|$ in comparison with lattice [24] data. We find the agreement within error bars with experimental values in the whole kinematical range for both transitions. There is also a nice accord with lattice results for the form factors of the $D \rightarrow \pi$ transition (there is a small difference only near $q^2_{\text{max}}$), while for the $D \rightarrow K$ transition our form factors have systematically somewhat larger values for $q^2 > 0.7$ GeV$^2$. Note that in general our form factors better agree with data in the accessible kinematical range than lattice ones.

In Table III we compare theoretical predictions for the form factors of the weak $D$ and $D_s$ meson transitions to pseudoscalar mesons at $q^2 = 0$ with available experimental data. The authors of Ref. [26] calculated form factors in the framework of the covariant confining quark model. The covariant light-front quark model was employed in Refs. [27, 28], while
FIG. 3: Comparison of our predictions for the product $f_+(q^2)|V_{cq}|$ (solid curve) and $f_0(q^2)|V_{cq}|$ (dashed curve) with experimental data for $f_+(q^2)|V_{cq}|$ form Belle [22] (orange dots with error bars) and BaBar [23] (red dots with error bars) and lattice results [24] for $f_+(q^2)|V_{cq}|$ (green dots with error bars) and $f_0(q^2)|V_{cq}|$ (magenta dots with error bars) of the weak $D \rightarrow K$ ($q = s$) and $D \rightarrow \pi$ ($q = d$) transitions.

TABLE III: Comparison of various theoretical predictions for the form factors $f_+(0)$ of the weak $D$ and $D_s$ meson transitions to pseudoscalar mesons with available experimental data.

| Decay | Our [26] | [27–29] | [30] | Lattice [24] | Experiment [7, 8, 31] |
|-------|-----------|---------|------|-------------|---------------------|
| $D \rightarrow K$ | 0.716 | 0.77 | 0.79(1) | 0.661(67) | 0.765(31) | 0.7361(34) |
| $D \rightarrow \pi$ | 0.640 | 0.63 | 0.66(1) | 0.635(66) | 0.612(35) | 0.6351(81) |
| $D \rightarrow \eta$ | 0.547 | 0.36 | 0.55(1) | 0.556(56) | 0.38(3) |
| $D \rightarrow \eta'$ | 0.538 | 0.36 | 0.45(1) |
| $D_s \rightarrow \eta$ | 0.443 | 0.49 | 0.48(3) | 0.611(62) | 0.4576(70) |
| $D_s \rightarrow \eta'$ | 0.559 | 0.59 | 0.59(3) | 0.490(51) |
| $D_s \rightarrow K$ | 0.674 | 0.60 | 0.66 | 0.820(80) | 0.720(85) |

in Ref. [30] calculations were done using light-cone sum rules in the framework of heavy quark effective field theory. Lattice QCD simulations in Ref. [24] were carried out with $N_f = 2 + 1 + 1$ dynamical quarks. Experimental values were taken from very recent report on world averages of measurements of hadron properties obtained by the Heavy Flavor Averaging Group [31]. Good agreement of our predictions with data is found. Only for the $D \rightarrow \eta$ transition $f_+(0)$ is somewhat larger than experimental value.

For the weak $D^+$ and $D_s$ meson transitions to vector mesons only the ratios of the form factors at maximum recoil of the final meson ($q^2 = 0$) are obtained experimentally

$$r_V = \frac{V(0)}{A_1(0)}, \quad r_2 = \frac{A_2(0)}{A_1(0)}.$$  \hspace{1cm} (20)

There have been many measurements and calculations of these ratios. In Table IV we compare some of theoretical predictions with averaged experimental data from PDG [1] and recent BES III [2–10] measurements. Once again our results agree well with data.
TABLE IV: Comparison of various theoretical predictions for the ratios of the form factors $r_V = V(0)/A_1(0)$ and $r_2 = A_2(0)/A_1(0)$ of the weak $D^+$ and $D_s$ meson transitions to vector mesons with available experimental data.

| Decay       | Ratio | Theory                          | Experiment                  |
|-------------|-------|---------------------------------|-----------------------------|
|             |       | Our [26] | [27–29] | [30] | PDG [1] | BES III [2–10] |
| $D \to K^*$ | $r_V$ | 1.53 | 1.22(24) | 1.36(2) | 1.39$^{(9)}_{(10)}$ | 1.49(5) | 1.406(62) |
|             | $r_2$ | 0.85 | 0.92(18) | 0.83(3) | 0.60$^{(8)}_{(9)}$ | 0.802(21) | 0.784(48) |
| $D \to \rho$ | $r_V$ | 1.44 | 1.26(25) | 1.46(3) | 1.34$^{(14)}_{(13)}$ | 1.48(16) | 1.695(98) |
|             | $r_2$ | 0.94 | 0.93(19) | 0.78(2) | 0.62$^{(8)}_{(9)}$ | 0.83(12) | 0.845(68) |
| $D \to \omega$ | $r_V$ | 1.29 | 1.24(25) | 1.47(4) | 1.33$^{(16)}_{(13)}$ | 1.24(11) |                |
|             | $r_2$ | 1.05 | 0.95(19) | 0.84(2) | 0.60$^{(9)}_{(9)}$ | 1.06(16) |                |
| $D_s \to \phi$ | $r_V$ | 1.56 | 1.34(27) | 1.42(2) | 1.37$^{(24)}_{(21)}$ | 1.80(8)  |                |
|             | $r_2$ | 0.77 | 0.99(20) | 0.86(1) | 0.53$^{(10)}_{(6)}$ | 0.84(11) |                |
| $D_s \to K^{*}$ | $r_V$ | 1.61 | 1.40(28) | 1.55(5) | 1.31$^{(16)}_{(19)}$ | 1.67(38) |                |
|             | $r_2$ | 0.90 | 0.99(20) | 0.82(2) | 0.53$^{(10)}_{(6)}$ | 0.77(29)  |                |

IV. SEMILEPTONIC DECAYS

The differential decay rate of the semileptonic $D_{(s)}$ decays can be expressed in the following form [26]

$$
\frac{d\Gamma(D_{(s)} \to F\ell^+\nu_\ell)}{dq^2d(\cos \theta)} = \frac{G_F^2}{(2\pi)^2} |V_{cq}|^2 \lambda^{1/2}(q^2 - m_\ell^2)^2 \lambda \left[ (1 + \cos^2 \theta) \mathcal{H}_U + 2 \sin^2 \theta \mathcal{H}_L + 2 \cos \theta \mathcal{H}_P \\
+ \frac{m_\ell^2}{2q^2} (2 \cos^2 \theta \mathcal{H}_U + 2 \sin^2 \theta \mathcal{H}_L + 4 \cos \theta \mathcal{H}_{SL}) \right],
$$

(21)

where $\lambda \equiv \lambda(M_{D_{(s)}}^2, M_F^2, q^2) = M_{D_{(s)}}^2 + M_F^2 + q^4 - 2(M_{D_{(s)}}^2 M_F^2 + M_F^2 q^2 + M_{D_{(s)}}^2 q^2)$, $m_\ell$ is the lepton mass, and the polar angle $\theta$ is the angle between the momentum of the charged lepton in the rest frame of the intermediate $W$-boson and the direction opposite to the final $F$ meson momentum in the rest frame of $D_{(s)}$. The bilinear combinations $\mathcal{H}_I$ of the helicity components of the hadronic tensor are defined by [26]

$$
\mathcal{H}_U = |H_+|^2 + |H_-|^2, \quad \mathcal{H}_L = |H_0|^2, \quad \mathcal{H}_P = |H_+|^2 - |H_-|^2, \quad \mathcal{H}_S = |H_1|^2, \quad \mathcal{H}_{SL} = \Re(H_0 H_1^*),
$$

(22)

and the helicity amplitudes are expressed through invariant form factors.

- For $D_{(s)} \to P$ transitions

$$
H_\pm = 0, \quad H_0 = \frac{\lambda^{1/2}}{q^2} f_+(q^2), \quad H_1 = \frac{1}{q^2} (M_{D_{(s)}}^2 - M_F^2) f_0(q^2).
$$

(23)

- For $D_{(s)} \to V$ transitions

$$
H_\pm(q^2) = \frac{\lambda^{1/2}}{M_{D_{(s)}} + M_V} \left[ V(q^2) \mp \frac{(M_{D_{(s)}} + M_V)^2}{\lambda^{1/2}} A_1(q^2) \right],
$$

where $V(q^2)$ and $A_1(q^2)$ are the form factors of the weak interaction.
\[ H_0(q^2) = \frac{1}{2 M V \sqrt{q^2}} \left[ (M_{D(s)} + M_V)(M_{D(s)}^2 - M_V^2 - q^2) A_1(q^2) - \frac{\lambda}{M_{D(s)} + M_V} A_2(q^2) \right], \]

\[ H_t = \frac{\lambda^{1/2}}{\sqrt{q^2}} A_0(q^2). \] (24)

The expression (21) normalized by the decay rate \( d\Gamma/dq^2 \), which is obtained by the integration of (21) over \( \cos\theta \), can be rewritten as

\[ \frac{1}{d\Gamma/dq^2} \frac{d\Gamma(D(s) \rightarrow F\ell^+\nu_\ell)}{dq^2d(\cos\theta)} = \frac{1}{2} \left[ 1 - \frac{1}{3} C_{FB}(q^2) \right] + A_{FB}(q^2) \cos\theta + \frac{1}{2} C_{FB}(q^2) \cos^2\theta, \] (25)

where the forward-backward asymmetry is defined by

\[ A_{FB}(q^2) = \frac{1}{\int_0^1 d(\cos\theta)d\Gamma(D(s) \rightarrow F\ell^+\nu_\ell)/d(\cos\theta)} \frac{d\Gamma(D(s) \rightarrow F\ell^+\nu_\ell)/d(\cos\theta)}{\int_0^1 d(\cos\theta)d\Gamma(D(s) \rightarrow F\ell^+\nu_\ell)/d(\cos\theta)} = 3 \frac{H_P - 2 m_{\ell}^2 H_{SL}}{4 H_{total}}, \] (26)

and lepton-side convexity parameter, which is the second derivative of the distribution (25) over \( \cos\theta \), is given by

\[ C_{FB}(q^2) = \frac{3}{4} \left( 1 - \frac{m_{\ell}^2}{q^2} \right) \frac{H_U - 2 H_L}{H_{total}}. \] (27)

Here the total helicity structure

\[ H_{total} = (H_U + H_L) \left( 1 + \frac{m_{\ell}^2}{2q^2} \right) + \frac{3m_{\ell}^2}{2q^2} H_S \] (28)

enters the differential decay distribution (21) integrated over \( \cos\theta \)

\[ \frac{d\Gamma(D(s) \rightarrow F\ell^+\nu_\ell)}{dq^2} = \frac{G_F^2}{(2\pi)^3 |V_{cq}|^2} \frac{\lambda^{1/2}(q^2 - m_{\ell}^2)^2}{24 M_{D(s)}^3 q^2} H_{total}. \] (29)

Other useful observables are the longitudinal polarization of the final charged lepton \( \ell \) defined by [26]

\[ P_{L}(q^2) = \frac{(H_U + H_L) \left( 1 - \frac{m_{\ell}^2}{2q^2} \right) - \frac{3m_{\ell}^2}{2q^2} H_S}{H_{total}}, \] (30)

and its transverse polarization [26]

\[ P_{T}(q^2) = -\frac{3\pi m_{\ell}^2 H_P + 2 H_{SL}}{8 \sqrt{q^2} H_{total}}. \] (31)

For the decays \( D(s) \rightarrow V \) the longitudinal polarization fraction of the final vector meson is given by [26]

\[ F_{L}(q^2) = \frac{H_L \left( 1 + \frac{m_{\ell}^2}{2q^2} \right) + \frac{3m_{\ell}^2}{2q^2} H_S}{H_{total}}, \] (32)

then its transverse polarization fraction \( F_T(q^2) = 1 - F_L(q^2) \).
V. RESULTS AND DISCUSSION

Now we substitute the form factors calculated in Sec. III in the expressions for helicity amplitudes, Eq. (23) and Eq. (24), and then evaluate differential and total decay rates of semileptonic $D_{(s)}$ decays. In Fig. 4 we confront our results with experimental data from BaBar [23, 32] (blue dots with error bars), CLEO [33] (orange dots with error bars) and BES III [2, 34] Collaborations for $D \rightarrow K\nu_e$ and $D \rightarrow \pi\nu_e$ differential decay rates. Good agreement in the whole accessible kinematical range is observed. In Tables V-VII we compare our and previous theoretical predictions [26, 27, 30] with experimental data from PDG [1] and recent data from BES III Collaboration. We roughly estimate the uncertainties of our calculations to be within 10%. For all decays we find agreement with experimental data within error bars. Only for the $D \rightarrow K^*\nu_\ell$ decay branching fractions we obtain somewhat lower central values than the data [1], while Refs. [26, 27] give significantly larger values. Thus the precise measurement of these branching fractions is the important test of the models.

Recently possible hints of the violation of the lepton universality were found in $B$ decays where deviations from the standard model predictions for the ratios of the semileptonic decay branching fractions involving muon and electron were observed. In Table VIII we give our results for the corresponding ratios of $D$ decays

$$R_F = \frac{\Gamma(D_{(s)} \rightarrow F\mu^+\nu_\mu)}{\Gamma(D_{(s)} \rightarrow Fe^+\nu_e)} \quad (33)$$

in comparison with previous predictions [26, 27], lattice [24] and experimental data form the BES III Collaboration [4, 6, 31]. We see that the standard model predictions are consistent with current experimental data.

We also calculate the forward-backward asymmetry $A_{FB}(q^2)$ [Eq. (26)], the lepton-side convexity parameter $C_F(q^2)$ [Eq. (27)], the longitudinal $P_L(q^2)$ [Eq. (30)] and transverse $P_T(q^2)$ [Eq. (31)] polarization of the final charged lepton, and longitudinal polarization $F_L(q^2)$ of the final vector meson, Eq. (32). As an example in Figs. 5 and 6 we plot these asymmetries and polarization parameters for $D^+ \rightarrow \pi^0\ell^+\nu_\ell$ and $D^+ \rightarrow K^{*0}\ell^+\nu_\ell$ decays.

In Table IX we present our predictions for the mean values of the helicity and asymmetry parameters for the semileptonic $D$ and $D_s$ decays. These values were obtained...
FIG. 5: Polarization and asymmetry parameters for the semileptonic $D^+ \to \pi^0 \ell^+ \nu_\ell$ decays.

FIG. 6: Polarization and asymmetry parameters for the semileptonic $D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$ decays.
by separately integrating corresponding partial differential decay rates in numerators and the total decay rates in denominators. Since we neglect the small positron mass, for all decays $D^+_s \to F e^+ \nu_e$, $\langle P_L \rangle = 1$ and $\langle P_T \rangle = 0$, while for decays $D^+_s \to P e^+ \nu_e$, $\langle A_{FB} \rangle = 0$ and $\langle C_F \rangle = -1.5$. Note that in Ref. [26] close values of these parameters were found. Experimentally only the ratios of the partial decay rates of the final vector meson states with longitudinal and transverse polarization $\Gamma_L/\Gamma_T = \langle F_L \rangle/(1 - \langle F_L \rangle)$ have been measured.

---

**TABLE V:** Comparison of various theoretical predictions for the branching ratios (in %) of the CKM-favoured $D \to K^{(*)} \ell \nu_\ell$ semileptonic decays with available experimental data.

| Decay | Theory | Epperiment |
|-------|--------|------------|
|       | Our [26] | [27] | [30] | PDG [1] | BES III [2, 6, 9] |
| $D^+ \to \bar{K}^0 e^+ \nu_e$ | 9.02 | 9.28 | 10.32(93) | 8.12(1.19) | 8.73(10) | 8.60(16) |
| $D^+ \to \bar{K}^0 \mu^+ \nu_\mu$ | 8.85 | 9.02 | 10.07(91) | 7.98(1.06) | 8.76(19) |
| $D^0 \to K^+ e^+ \nu_e$ | 3.56 | 3.63 | 4.1(4) | 3.20(47) | 3.542(35) | 3.505(36) |
| $D^0 \to K^- \mu^+ \nu_\mu$ | 3.49 | 3.53 | 4.2(4) | 3.10(46) | 3.41(4) | 3.413(39) |
| $D^+ \to \bar{K}^0 e^+ \nu_e$ | 4.87 | 7.61 | 7.5(7) | 5.37(24) | 5.40(10) |
| $D^+ \to K^0 e^+ \nu_e$ | 4.62 | 7.21 | 7.0(7) | 5.10(23) | 5.27(15) |
| $D^0 \to K^+ e^+ \nu_e$ | 1.92 | 2.96 | 3.0(3) | 2.12(9) | 2.15(16) | 2.033(66) |
| $D^0 \to K^- \mu^+ \nu_\mu$ | 1.82 | 2.80 | 2.8(3) | 2.01(9) | 1.89(24) |

**TABLE VI:** Comparison of various theoretical predictions for the branching ratios (in 10^{-3}) of the CKM-suppressed $D$ meson semileptonic decays with available experimental data.

| Decay | Theory | Epperiment |
|-------|--------|------------|
|       | Our [26] | [27] | [30] | PDG [1] | BES III [3, 5, 10] |
| $D^+ \to \pi^0 e^+ \nu_e$ | 3.53 | 2.9 | 4.1(3) | 3.52(45) | 3.72(17) | 3.63(9) |
| $D^+ \to \pi^0 \mu^+ \nu_\mu$ | 3.47 | 2.8 | 4.1(3) | 3.49(38) | 3.50(15) | 3.50(15) |
| $D^0 \to \pi^- e^+ \nu_e$ | 2.78 | 2.2 | 3.2(3) | 2.78(35) | 2.91(4) | 2.95(5) |
| $D^0 \to \pi^- \mu^+ \nu_\mu$ | 2.74 | 2.2 | 3.2(3) | 2.75(35) | 2.67(12) | 2.72(10) |
| $D^+ \to p^0 e^+ \nu_e$ | 2.49 | 2.09 | 2.3(2) | 2.29(23) | 2.18(17) | 1.860(93) |
| $D^+ \to p^0 \mu^+ \nu_\mu$ | 2.39 | 2.01 | 2.2(2) | 2.20(21) | 2.4(4) |
| $D^0 \to p^- e^+ \nu_e$ | 1.96 | 1.62 | 1.8(2) | 1.81(18) | 1.77(16) | 1.445(70) |
| $D^0 \to p^- \mu^+ \nu_\mu$ | 1.88 | 1.55 | 1.7(2) | 1.73(17) | 1.89(24) |
| $D^+ \to \eta e^+ \nu_e$ | 1.24 | 0.938 | 1.2(1) | 0.86(16) | 1.11(7) | 1.074(98) |
| $D^+ \to \eta \mu^+ \nu_\mu$ | 1.21 | 0.912 | 1.2(1) | 0.84(16) |
| $D^+ \to \eta^0 e^+ \nu_e$ | 0.225 | 0.200 | 0.18(2) | 0.20(4) | 0.191(53) |
| $D^+ \to \eta^0 \mu^+ \nu_\mu$ | 0.211 | 0.190 | 0.17(2) |
| $D^+ \to \omega e^+ \nu_e$ | 2.17 | 1.85 | 2.1(2) | 1.93(20) | 1.69(11) | 2.05(72) |
| $D^+ \to \omega \mu^+ \nu_\mu$ | 2.08 | 1.78 | 2.0(2) | 1.85(19) |
TABLE VII: Comparison of various theoretical predictions for the branching ratios (in %) of the $D_s$ meson semileptonic decays with available experimental data.

| Decay                  | Theory Our [26] | Theory [27] | Theory [30] | Experiment [1] | Experiment BES III [4, 7, 8] |
|------------------------|-----------------|-------------|-------------|----------------|------------------------------|
| $D_s \to \eta^+\nu_e$  | 2.37            | 2.24        | 2.26(21)    | 1.27(20)       | 2.29(19)                     |
| $D_s \to \eta^+\nu_\mu$| 2.32            | 2.18        | 2.22(20)    | 1.25(20)       | 2.4(5)                       |
| $D_s \to \eta^+\nu_e$  | 0.87            | 0.83        | 0.89(9)     | 0.74(14)       | 0.824(78)                    |
| $D_s \to \eta^+\nu_\mu$| 0.83            | 0.79        | 0.85(8)     | 1.1(5)         | 1.06(54)                     |
| $D_s \to \phi^+\nu_e$  | 2.69            | 3.01        | 3.1(3)      | 2.53(40)       | 2.39(16)                     |
| $D_s \to \eta^+\nu_\mu$| 2.54            | 2.85        | 2.9(3)      | 2.4(35)        | 1.9(5)                       |
| $D_s \to K^0\nu_e$     | 0.40            | 0.20        | 0.27(2)     | 0.390(74)      | 0.399(9)                     |
| $D_s \to K^0\nu_\mu$   | 0.39            | 0.20        | 0.26(2)     | 0.383(56)      | 0.325(4)                     |
| $D_s \to K^0\nu_e$     | 0.21            | 0.18        | 0.19(2)     | 0.233(29)      | 0.18(4)                      |
| $D_s \to K^0\nu_\mu$   | 0.20            | 0.17        | 0.19(2)     | 0.224(27)      | 0.237(33)                    |

TABLE VIII: Test of the $e - \mu$ lepton flavor universality. Comparison of theoretical predictions for the ratios $R$ of the weak $D$ and $D_s$ meson semileptonic decays with available experimental data.

| Decay                  | Theory Our [26] | Theory [27] | Lattice [25] | Experiment [3, 4, 6, 31] |
|------------------------|-----------------|-------------|--------------|--------------------------|
| $D \to K$              | 0.980           | 0.97        | 0.976        | 0.975(1)                 |
| $D \to \pi$            | 0.985           | 0.98        | 1.00         | 0.985(2)                 |
| $D \to K^*$            | 0.946           | 0.95        | 0.933        |                          |
| $D \to \rho$           | 0.959           | 0.96        | 0.957        |                          |
| $D \to \eta$           | 0.976           | 0.97        | 1.00         |                          |
| $D \to \eta'$          | 0.937           | 0.95        | 0.944        |                          |
| $D \to \omega$         | 0.959           | 0.96        | 0.952        |                          |
| $D_s \to \eta$         | 0.977           | 0.97        | 0.982        |                          |
| $D_s \to \eta'$        | 0.952           | 0.95        | 0.956        |                          |
| $D_s \to \phi$         | 0.944           | 0.95        | 0.936        |                          |
| $D_s \to K$            | 0.984           | 1.00        | 0.963        |                          |
| $D_s \to K^*$          | 0.958           | 0.95        | 1.00         |                          |

For $D^+ \to \bar{K}^*\ell^+\nu_\ell$ and $D_s \to \phi\ell^+\nu_\ell$ decays. The experimental values for these ratios are $1.13 \pm 0.08$ and $0.72 \pm 0.18$ [1], respectively. The first value is in agreement with our prediction $\Gamma_L/\Gamma_T = 1.16$, while the second one is somewhat smaller than predicted $\Gamma_L/\Gamma_T = 1.19$. Values of these ratios, close to ours, were obtained in Ref. [26].
TABLE IX: Predictions for the polarization and asymmetry parameters for the semileptonic $D$ and $D_s$ decays.

| Decay                  | $\langle A_{FB}\rangle$ | $\langle C_F^\ell \rangle$ | $\langle P_L^\ell \rangle$ | $\langle P_T^\ell \rangle$ | $\langle F_L \rangle$ |
|-----------------------|--------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------|
| $D_s^+ \rightarrow \bar{P}e^+\nu_e$ | 0                        | -1.5                        | 1                           | 0                           |                       |
| $D^+ \rightarrow \bar{K}\mu^+\nu_\mu$ | -0.053                   | -1.34                       | 0.85                        | -0.42                       |                       |
| $D^+ \rightarrow \pi^0\mu^+\nu_\mu$ | -0.040                   | -1.38                       | 0.89                        | -0.36                       |                       |
| $D^+ \rightarrow \eta\mu^+\nu_\mu$ | -0.052                   | -1.34                       | 0.85                        | -0.40                       |                       |
| $D^+ \rightarrow \eta'\mu^+\nu_\mu$ | -0.097                   | -1.20                       | 0.72                        | -0.56                       |                       |
| $D^+ \rightarrow \bar{K}^{*0}e^+\nu_e$ | -0.22                    | -0.47                       | 1                           | 0                           | 0.54                 |
| $D^+ \rightarrow \bar{K}^{*0}\mu^+\nu_\mu$ | -0.25                    | -0.37                       | 0.90                        | -0.15                       | 0.54                 |
| $D^+ \rightarrow \rho^0e^+\nu_e$ | -0.26                    | -0.42                       | 1                           | 0                           | 0.52                 |
| $D^+ \rightarrow \rho^0\mu^+\nu_\mu$ | -0.28                    | -0.34                       | 0.92                        | -0.12                       | 0.52                 |
| $D^+ \rightarrow \omega e^+\nu_e$ | -0.25                    | -0.39                       | 1                           | 0                           | 0.51                 |
| $D^+ \rightarrow \omega\mu^+\nu_\mu$ | -0.27                    | -0.32                       | 0.93                        | -0.11                       | 0.50                 |
| $D_s \rightarrow \bar{K}^0\mu^+\nu_\mu$ | -0.038                   | -1.38                       | 0.89                        | -0.34                       |                       |
| $D_s^+ \rightarrow \eta\mu^+\nu_\mu$ | -0.043                   | -1.37                       | 0.88                        | -0.35                       |                       |
| $D_s^+ \rightarrow \eta'\mu^+\nu_\mu$ | -0.080                   | -1.26                       | 0.77                        | -0.51                       |                       |
| $D_s \rightarrow \bar{K}^{*0}e^+\nu_e$ | -0.26                    | -0.41                       | 1                           | 0                           | 0.52                 |
| $D_s \rightarrow \bar{K}^{*0}\mu^+\nu_\mu$ | -0.29                    | -0.33                       | 0.92                        | -0.11                       | 0.51                 |
| $D_s \rightarrow \phi e^+\nu_e$ | -0.21                    | -0.49                       | 1                           | 0                           | 0.54                 |
| $D_s \rightarrow \phi\mu^+\nu_\mu$ | -0.24                    | -0.35                       | 0.90                        | -0.15                       | 0.54                 |

VI. CONCLUSIONS

In the framework of the relativistic quark model based on the quasipotential approach we calculated the form factors of the semileptonic $D$ and $D_s$ meson transitions. The relativistic effects including wave function transformations from the rest to moving reference frame and contributions of the intermediate negative energy states were consistently taken into account. This allowed us to reliably calculate the form factors in the whole accessible kinematical range without additional approximations and/or extrapolations. The form factors were expressed through the overlap integrals of the meson wave functions. These wave functions were obtained in our previous study of the heavy-light and light meson spectroscopy. This fact significantly increases self-consistency and reliability of our approach, since in most of the previous quark model studies of semileptonic decays some ad hoc form of the wave function (mostly Gaussian) had been used. It was found that our numerical results for the form factors and their $q^2$ dependence can be well approximated by Eqs. (18) and (19). The parameters of the fit are collected in Tables I II. The calculated values of the form factors $f_+(q^2)$ for the decays to pseudoscalar mesons and the ratios of the form factors $r_2$ and $r_V$ for the decays to vector mesons at $q^2 = 0$ agree within errors with the experimental data (see Tables III IV V). The form factors $f_+(q^2)$ of the weak $D$ transitions to pseudoscalar $K$ and $\pi$ mesons agree well with data form Belle 22 and BaBar 23 Collaborations in the whole $q^2$ range.

These form factors were applied for the calculation of differential and total decay rates
of semileptonic decays of $D$ and $D_s$ using the helicity formalism. The obtained differential decay distributions \cite{29} for $D$ decays to pseudoscalar mesons, plotted in Fig. 4, agree with experimental data from the BaBar \cite{23} and CLEO \cite{33} Collaborations. The calculated total branching fractions are also in good agreement with averaged experimental values from PDG \cite{1} and recent data from the BES III Collaboration \cite{2-10}. To test lepton universality in the semileptonic $D_s(\bar{s})$ meson decays we calculated the ratios of the branching fractions of decays involving muons to the ones involving positrons. These ratios are collected in Table VIII in comparison with other theoretical predictions and available experimental data. Within current experimental accuracy no deviations of data from the standard model predictions are found. We also calculated the forward-backward asymmetries, the lepton and vector meson longitudinal and transverse polarization parameters which can be measured in future experiments. Their mean values are collected in Table IX.

We presented the detailed comparison of our results with other theoretical predictions. In most cases better agreement of our values with data is found. The further increase of the experimental accuracy and new measurements can help to better understand quark dynamics in mesons. This could be realized in the future super tau-charm facility \cite{35-37} which is hotly discussed to construct.

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\begin{thebibliography}{10}
\bibitem{1} M. Tanabashi \textit{et al.} [Particle Data Group], “Review of Particle Physics,” Phys. Rev. D \textbf{98}, no. 3, 030001 (2018).
\bibitem{2} M. Ablikim \textit{et al.} [BESIII Collaboration], “Analysis of $D^+ \rightarrow \bar{K}^0 e^+ \nu_e$ and $D^+ \rightarrow \pi^0 e^+ \nu_e$ semileptonic decays,” Phys. Rev. D \textbf{96}, no. 1, 012002 (2017).
\bibitem{3} M. Ablikim \textit{et al.} [BESIII Collaboration], “Measurement of the branching fraction for the semi-leptonic decay $D_s^{0(+)\rightarrow \pi^-(0)\mu^+\nu_\mu$ and test of lepton universality,” Phys. Rev. Lett. \textbf{121}, no. 17, 171803 (2018).
\bibitem{4} M. Ablikim \textit{et al.}, “Measurements of the branching fractions for the semi-leptonic decays $D_s^+ \rightarrow \phi e^+ \nu_e, \phi \mu^+\nu_\mu, \eta \mu^+\nu_\mu$ and $\eta' \mu^+\nu_\mu$,” Phys. Rev. D \textbf{97}, no. 1, 012006 (2018).
\bibitem{5} M. Ablikim \textit{et al.} [BESIII Collaboration], “Study of the decays $D^+ \rightarrow \eta(0)^0e^+\nu_e$,” Phys. Rev. D \textbf{97}, no. 9, 092009 (2018)
\bibitem{6} M. Ablikim \textit{et al.} [BESIII Collaboration], “Study of the $D_s^0 \rightarrow K^-\mu^+\nu_\mu$ dynamics and test of lepton flavor universality with $D^0 \rightarrow K^-\ell^+\nu_\ell$ decays,” Phys. Rev. Lett. \textbf{122}, no. 1, 011804 (2019).
\bibitem{7} M. Ablikim \textit{et al.} [BESIII Collaboration], “Measurement of the Dynamics of the Decays $D_s^+ \rightarrow \eta(0)^0e^+\nu_e$,” Phys. Rev. Lett. \textbf{122}, no. 12, 121801 (2019).
\bibitem{8} M. Ablikim \textit{et al.} [BESIII Collaboration], “First Measurement of the Form Factors in $D_s^+ \rightarrow K^0e^+\nu_e$ and $D_s^+ \rightarrow K^{*0}e^+\nu_e$ Decays,” Phys. Rev. Lett. \textbf{122}, no. 6, 061801 (2019).
\bibitem{9} M. Ablikim \textit{et al.} [BESIII Collaboration], “Study of the decay $D^0 \rightarrow \bar{K}^0\pi^-e^+\nu_e$,” Phys. Rev. D \textbf{99}, no. 1, 011103 (2019).
\bibitem{10} M. Ablikim \textit{et al.} [BESIII Collaboration], “Observation of $D^+ \rightarrow f_0(500)e^+\nu_e$ and Improved Measurements of $D \rightarrow \rho e^+\nu_e$,” Phys. Rev. Lett. \textbf{122}, no. 6, 062001 (2019).
\bibitem{11} S. Bifani, S. Descotes-Genon, A. Romero Vidal and M. H. Schune, “Review of Lepton Uni-

\end{thebibliography}
versatility tests in $B$ decays," J. Phys. G 46, no. 2, 023001 (2019).

[12] A. J. Schwartz, “Semileptonic and leptonic charm meson decays at Belle II,” [arXiv:1902.07850 [hep-ex]].

[13] D. Ebert, R. N. Faustov and V. O. Galkin, “Properties of heavy quarkonia and $B_c$ mesons in the relativistic quark model,” Phys. Rev. D 67, 014027 (2003).

[14] D. Ebert, R. N. Faustov and V. O. Galkin, “Mass spectra and Regge trajectories of light mesons in the relativistic quark model,” Phys. Rev. D 79, 114029 (2009).

[15] D. Ebert, R. N. Faustov and V. O. Galkin, “Spectroscopy and Regge trajectories of heavy quarkonia and $B_c$ mesons,” Eur. Phys. J. C 71, 1825 (2011).

[16] D. Ebert, R. N. Faustov and V. O. Galkin, “Weak decays of the $B_c$ meson to charmonium and $D$ mesons in the relativistic quark model,” Phys. Rev. D 68, 094020 (2003).

[17] D. Ebert, R. N. Faustov and V. O. Galkin, “Weak decays of the $B_c$ meson to $B_s$ and $B$ mesons in the relativistic quark model,” Eur. Phys. J. C 32, 29 (2003).

[18] D. Ebert, R. N. Faustov and V. O. Galkin, “Spectroscopy and Regge trajectories of heavy quarkonia and $B_c$ mesons,” Eur. Phys. J. C 66, 197 (2010).

[19] D. Ebert, R. N. Faustov and V. O. Galkin, “Analysis of semileptonic $B$ decays in the relativistic quark model,” Phys. Rev. D 75, 074008 (2007).

[20] R. N. Faustov and V. O. Galkin, “Weak decays of $B_s$ mesons to $D_s$ mesons in the relativistic quark model,” Phys. Rev. D 87, no. 3, 034033 (2013).

[21] R. N. Faustov and V. O. Galkin, “Charmless weak $B_s$ decays in the relativistic quark model,” Phys. Rev. D 87, no. 9, 094028 (2013).

[22] V. Lubicz et al. [ETM Collaboration], “Scalar and vector form factors of $D \rightarrow \pi(K)\ell \nu$ decays with $N_f = 2 + 1 + 1$ twisted fermions,” Phys. Rev. D 96 (2017) no.5, 054514 Erratum: [Phys. Rev. D 99 (2019) no.9, 099902].

[23] L. Widhalm et al. [Belle Collaboration], “Measurement of $D^0 \rightarrow \pi\ell\nu(K\ell\nu)$ Form Factors and Absolute Branching Fractions,” Phys. Rev. Lett. 97, 061804 (2006).

[24] J. P. Lees et al. [BaBar Collaboration], “Measurement of the $D^0 \rightarrow \pi^-e^+\nu_e$ differential decay branching fraction as a function of $q^2$ and study of form factor parameterizations,” Phys. Rev. D 91, no. 5, 052022 (2015).

[25] Y. L. Wu, M. Zhong and Y. B. Zuo, “$B_s, D_s \rightarrow \pi, K, \eta, \rho, K^*, \omega, \phi$ Transition Form Factors and Decay Rates with Extraction of the CKM parameters $|V_{ub}|, |V_{cs}|, |V_{cd}|$,” Int. J. Mod. Phys. A 21, 6125 (2006).

[26] R. C. Verma, “Decay constants and form factors of s-wave and p-wave mesons in the covariant light-front quark model,” J. Phys. G 39, 025005 (2012).

[27] H. Y. Cheng and C. W. Hwang, “Covariant light front approach for s wave and p wave mesons: Its application to decay constants and form-factors,” Phys. Rev. D 69, 074025 (2004).

[28] C. W. Hwang and C. K. Chua, “Covariant light front approach for s wave and p wave mesons: Its application to decay constants and form-factors,” Phys. Rev. D 69, 074025 (2004).

[29] Y. L. Wu, M. Zhong and Y. B. Zuo, “$B_s, D_s \rightarrow \pi, K, \eta, \rho, K^*, \omega, \phi$ Transition Form Factors and Decay Rates with Extraction of the CKM parameters $|V_{ub}|, |V_{cs}|, |V_{cd}|$,” Int. J. Mod. Phys. A 21, 6125 (2006).

[30] Y. S. Amhis et al. [HFLAV Collaboration], “Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton
properties as of 2018,” arXiv:1909.12524 [hep-ex].

[32] B. Aubert et al. [BaBar Collaboration], “Measurement of the hadronic form-factor in $D^0 \to K^- e^+ \nu_e$,” Phys. Rev. D 76, 052005 (2007).

[33] D. Besson et al. [CLEO Collaboration], “Improved measurements of $D$ meson semileptonic decays to $\pi$ and $K$ mesons,” Phys. Rev. D 80, 032005 (2009).

[34] M. Ablikim et al. [BESIII Collaboration], “Study of Dynamics of $D^0 \to K^- e^+ \nu_e$ and $D^0 \to \pi^- e^+ \nu_e$ Decays,” Phys. Rev. D 92, no. 7, 072012 (2015).

[35] A. E. Bondar et al. [Charm-Tau Factory Collaboration], “Project of a Super Charm-Tau factory at the Budker Institute of Nuclear Physics in Novosibirsk,” Phys. Atom. Nucl. 76 (2013) 1072 [Yad. Fiz. 76, no. 9, 1132 (2013)].

[36] Q. Luo and D. Xu, “Progress on Preliminary Conceptual study of HIEPA, a super tau-charm factory in China”, talk at the 9th International Particle Accelerator Conference (IPAC 2018), held in Vancouver, British Columbia, Canada, April 29 - May 4, 2018.

[37] H.-p. Peng, “High Intensity Electron Positron Accelerator (HIEPA), Super Tau Charm Facility “STCF in China”, talk at Charm2018, Novosibirsk, Russia, May 21 - 25, 2018.