GRAVITATIONAL WAVES FROM GALAXY CLUSTERS: A NEW OBSERVABLE EFFECT

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ABSTRACT

A rich galaxy cluster showing strong resemblance to the observed ones is simulated. A cold dark matter spectrum, Gaussian statistics, a flat universe, and two components (baryonic gas plus dark matter particles) are considered. We have calculated the gravitational-wave output during the epoch of the fully nonlinear and nonsymmetric cluster evolution. The amplitudes and frequencies of the resulting gravitational waves are estimated. Since frequencies are very small—of the order of $10^{-17}$ Hz—a complete pulse cannot be observed during an admissible integration time; nevertheless, it is proved that these waves can produce an interesting secular effect, which appears to be observable with current technology.

Subject headings: cosmology: theory — gravitation — hydrodynamics — large-scale structure of universe — methods: numerical — radiation mechanisms: nonthermal

1. INTRODUCTION

Any dynamical and nonspherical self-gravitating astrophysical—or cosmological—system of mass $M$ and size $R$ is a potential source of gravitational waves. The most powerful systems are those combining a very asymmetric shape together with a violent nonstationary evolution.

In present Letter, we are interested in the gravitational waves radiated by cosmological objects. Flattened structures such as superclusters and walls show great departures from spherical symmetry, but they are evolving in the mildly nonlinear regime and, consequently, their evolution is slow. Gravitational waves from these structures will be considered elsewhere. Galaxy clusters evolve in the strongly nonlinear regime, and their dynamics is expected to be faster than in the case of larger flattened structures; however, departures from spherical symmetry seem to be small (even if their corresponding initial conditions were fully asymmetric). In this case, high-density contrasts—of the order of $10^3$—are reached and, consequently, fully nonlinear numerical simulations are necessary. This Letter is concerned with the characteristics of the gravitational radiation released during galaxy cluster evolution.

As is well known, clusters contain dark matter ($\sim 90\%$) and a subdominant baryonic component ($\sim 10\%$). Since the dynamics and spatial distribution of these components are different, the ratio between the gravitational luminosities coming from these two components is not expected to be constant. Numerical simulations including a baryonic component—gravitationally coupled to the dark one—are necessary to describe the evolution of that ratio.

Taking into account the above discussion, a two-component cluster mimicking the observed ones has been simulated numerically, paying particular attention to estimate its gravitational luminosity and the features of the radiated gravitational waves. To this aim, we have used the approach of considering that the sources satisfy the constraints of the so-called nearly Newtonian slow motion. As far as we know, similar calculations have not been performed up to now, probably owing to some pessimistic point of view justified by the combination of great numerical difficulties (three-dimensional calculations) together with very disappointing small values expected for the gravitational-wave luminosity, the amplitude, etc. However, our results have been more encouraging than we could foresee. A new observable effect has been found.

Hereafter, $t$ stands for the cosmological time, and $t_0$ is the age of the universe. $X$ stands for the derivative of the function $X$ with respect to the cosmological time. The present value of the Hubble constant is assumed to be $50$ km s$^{-1}$ Mpc$^{-1}$. The quantities $c$ and $G$ are the speed of light and the gravitational constant, respectively. Greek (Latin) indices run from 1 to 4 (1 to 3).

2. CLUSTER MODEL

A flat universe, cold dark matter (CDM), and adiabatic energy density fluctuations are assumed. The corresponding spectrum is normalized by the condition $\sigma_8 = 0.63$. Constrained Gaussian realizations of the density field—in the position space—can be obtained by using the method proposed by Hoffman & Ribak (1991) and improved by van den Weygaert & Bertschinger (1996). We have used this powerful tool to obtain an initial density field—at redshift $z = 100$—containing the cluster seed centered in a box. The constraints have been introduced in such a way that, after evolution, our initial overdensity leads to a feasible rich cluster (see Quilis et al. 1998 for details). It is assumed that the $90\%$ of the matter is CDM and the remaining, the baryonic one, is assumed to be a monatomic ideal gas. The initial values for the velocity field of both components are identical. No assumptions about the symmetry of the object have been made at all. The resulting departures from spherical symmetry correspond to an arbitrary statistical realization of the density field.

The cosmological hydrodynamic equations describing the evolution of the baryonic component (see Peebles 1980) are solved using a hydrocode based on modern high-resolution shock-capturing techniques. This code was described in Quilis et al. (1996). The motion of dark matter particles is studied by means of a standard particle mesh code (Hockney & Eastwood 1988). The total gravitational field is computed by solving the Poisson equation—which couples baryons and dark matter—with a multidimensional method based on fast Fourier transform (Press et al. 1987). Details about our galaxy cluster simulations are described in Quilis et al. (1998).

At the present time, the simulated cluster has (1) an X-ray luminosity of $\sim 10^{47}$ ergs s$^{-1}$, (2) a temperature of $\sim 3 \times 10^7$ K, and (3) a total mass inside the Abell radius (3 Mpc) of $5.4 \times 10^{14} M_{\odot}$; hence, we are considering a rich cluster having
features that are compatible with present observations (Böer-
ing 1991; Peebles 1994).

3. GRAVITATIONAL RADIATION

Our cluster evolves in a flat universe, and it is observed with
a detector of gravitational waves moving with the cosmological
expanding background. This is the most natural reference frame
in cosmology, and it is locally Minkowskian at any time. The
metric distance from the detector to the cluster \(D(t)\) is propor-
tional to the scale factor \(a(t) \propto (1 + z)^{-1}\). Its present value
is assumed to be \(D(t_0) = 100\) Mpc. Gravitons reaching the
detector at the present time were emitted by the cluster at time
\(t_0\) (emission event E). Could we perform our calculations in
the Minkowskian space tangent to the Robertson-Walker spa-
tetime at E? The particle point of view is appropriate for an-
swering this question. The emission and propagation of grav-
itons in the real and tangent spaces are now compared: (1) Since
graviton emission only depends on the internal dynamics of
the system and it is the same in both cases, the same amount of
gravitons with the same energies are emitted, (2) the back-
ground universe as well as the Minkowski empty spacetime are
transparent to graviton propagation, and (3) we have studied
the null geodesics in both spacetimes to calculate the tim-
es are transparent to graviton propagation, and (3) we have studied

observed from distances comparable to that of the horizon
\((D_c \geq 600\) Mpc), the Minkowskian point of view is not valid
and the approach used in this Letter must be improved.

Since our calculations can be carried out in the Minkowskian
tangent space and galaxy clusters are far from being relativistic
in both senses, special relativity, i.e., \(v/c \leq 10^{-3}\), and general
relativity \(r/R \leq 10^{-4}\) (\(r_g = 2GM/c^2\) is the Schwarzschild radius
of the object), these clusters can be described by using the so-
called slow-motion formalism. The spacetime metric can be
linearized in the usual way \((d_{\nu} = \eta_{\nu} + h_{\nu})\), the transverse
traceless (TT) gauge can be used, and, everywhere outside the
cluster (Misner, Thorne, & Wheeler 1973), the spatial com-
ponents of \(h^{TT}\) are

\[
I_{\nu} = \frac{c^2}{D} \int_{t_0}^{t} \frac{dD}{dt} \left[ 1 - \frac{D}{ct} \right] dt
\]

where \(I_{\nu}\) are the components of the traceless inertial tensor.
The contribution of the baryonic fluid \(I_{B}^{TT} = \int_{t_0}^{t} [\rho x, x, d^2x -
1/3\delta_{ij}] x dx \), where \(\delta_{ij}\) is Kronecker’s delta, \(\rho\) is the baryonic
density, and the \(x\)'s are the physical coordinates. The CDM
contribution \(I_{DM}^{TT}\) is given by the following summation extended
over all the CDM particles: \(I_{DM}^{TT} = \sum_{n} m_{n} (x_{n} - \frac{1}{3}\delta_{ij} x_{n}^{i} x_{n}^{j})\), \(n_{p}\)
being the number of CDM particles, and \(m_{n}\) being the mass of
each particle. Finally, we compute \(I_{TT} = I_{B}^{TT} + I_{DM}^{TT}\). Outside the
cluster, the relative motion of two neighboring particles \(A\) and
\(B\) moving with the cosmological background (ideal detector)
is fully described by the quantities \(h_{TT}^{TT}\). In TT gauge, there is
a system of coordinates attached to \(A\) in which the coordinate
variations of the particle \(B\) are

\[
X_{B}(t) - X_{B}(t_0) = \frac{1}{2} X_{B} [h_{TT}^{TT}(t)]_{1}\]

where \(X_{B}(t_0)\) stands for the initial coordinates of the particle \(B\),
and the quantities \(h_{TT}^{TT}\) are calculated at point \(A\). From this
formula, it follows that oscillations in the \(h_{TT}^{TT}\) quantities lead
to oscillations in the relative position of particles \(A\) and \(B\) with
related frequencies and amplitudes.

In the slow-motion approximation, the gravitational lumino-
sity \(L_{GW}\) is given by the well-known formula

\[
L_{GW} = \frac{G}{5c^3} \sum_{y} I_{y}^{TT}
\]

Since \(I_{y}\) is the addition of two terms, the gravitational lumino-
sity can be decomposed in the following evident manner:

\(L_{GW} = L_{B}^{GW} + L_{DM}^{GW} + L_{CM}^{GW} \). A direct calculation of \(L_{GW}\) based
on equation (3) would be very problematic (Finn & Evans
1990) as a result of the difficulties coming from the numerical
noise inherent to the numerical computation of third-order time
derivatives. However, as several authors have suggested (see,
e.g., Mönnchmeyer et al. 1991), the second-order time deriv-
atives involved in \(I_{y}\) can be written in terms of the quantities
\(\tilde{\rho}, \tilde{\varphi}, \tilde{x}, \) and \(\tilde{x}\), which, in its turn, can be expressed in terms of
related variables, taking advantage of the corresponding system
of equations governing the motion of baryonic and CDM

4. RESULTS

Figure 1 shows our numerical estimates of \(L_{DM}^{GW}\) (plus signs),
\(L_{B}^{GW}\) (triangles), and \(L_{GW}\) (solid line) versus the quantity
\((1 + z)^{-1}\). At redshifts between 30 and 25, CDM and baryons
have roughly the same velocity profiles and proportional (according to their relative abundances) density fields. As a consequence, the mass is the only relevant parameter, and the quantity $L_{GW}^0$ is smaller than $L_{GW}$. Since evolution leads to a CDM configuration that is more stationary and spherically symmetric than the baryonic one, the luminosity $L_{GW}^0$ becomes dominant at $z \sim 9$. The total luminosity $L_{GW}$ is a little greater than $10^{46}$ ergs s$^{-1}$ in the redshift intervals $(25, 30)$ and $(3, 7)$. The fact that luminosities appear to be similar in both periods can be easily understood taking into account that, in the first interval, deviations from spherical symmetry are greater than in the second one, while dynamics is more violent between redshifts 7 and 3. The luminosity, assumed constant, of a source radiating a total energy of during the age 2

In Figures 1 and 2 are compatible with rough estimates based in the second one, while dynamics is more violent between redshifts 7 and 3. The luminosity, assumed constant, of a source radiating a total energy of during the age 2

It is noticeable that the luminosities and amplitudes given in Figures 1 and 2 are compatible with rough estimates based on formulae from other astrophysical scenarios; in fact, according to Shapiro & Teukolsky (1983), the gravitational-wave luminosity of a self-gravitating system of mass $M$, size $R$, and typical velocity $v$ is

$$L_{GW} \approx L_0 \left( \frac{\rho_0}{M} \right)^{2/3} \left( \frac{R}{\rho_0} \right)^{1/3} \left( \frac{v}{c} \right)^{2/3},$$

being $L_0 = c^4 G \approx 3.63 \times 10^{59}$ ergs s$^{-1}$. For our simulated cluster, $(\rho_0/R) \approx 10^{-4}$, $(\rho_0/c) \approx 2 \times 10^{-3}$, and equation (4) leads to $L_{GW} \approx 2 \times 10^{45}$ ergs s$^{-1}$. Moreover, the relative strain or amplitude of the gravitational wave signal coming from a collapsing object at a distance $D$ of the terrestrial detector—in a form well-adapted to our cosmological scenario—is (Shapiro & Teukolsky 1983)

$$h \approx 3 \times 10^{-11} \left( \frac{\epsilon}{10^{-15}} \right)^{2/7} \left( \frac{M}{10^{15} M_\odot} \right) \left( \frac{D}{100 \text{ Mpc}} \right)^{-1}.$$  

For our simulated cluster ($\epsilon \approx 10^{-16}$), equation (5) leads to $h \approx 0.8 \times 10^{-11}$. Finally, Schutz (1997) gives the following alternative formula:

$$h \approx 5 \times 10^{-11} \left( \frac{E_{GW}}{10 M_\odot c^7} \right)^{1/2} \left( \frac{f_{GW}}{10^{-17} \text{ Hz}} \right)^{-1/2} \left( \frac{D}{150 \text{ Mpc}} \right)^{-1},$$

where $f_{GW}$ is the frequency of the wave. In the case of our cluster, the characteristic dynamical time is $\approx t_0^{-1}$. This gives $f_{GW} \approx 10^{-17}$ Hz. Furthermore, we have $E_{GW} \approx 0.15$, and $D = 100$ Mpc. Taking into account all these data, equation (6) leads to $h \approx 10^{-10}$.

After describing Figures 1 and 2 and analyzing their contents and consistency, let us focus our attention on an important consequence. The characteristic periods of the waves coming from the cluster are larger than $10^9$ yr, and, obviously, it is not possible to detect any complete pulse. However, we are going to show that the oscillatory fields $h_v$ and $h_s$ produce an effect that could be observable—during an admissible integration time—with a standard detector like the one described in equation (2). In fact, during the time interval $\Delta t$, the distance between two particles $A$ and $B$ aligned with the $x$- or $y$-axis undergoes a relative variation $1/2[\Delta h_0 / \Delta t]$, and when the particles are aligned with the directions forming angles of $45^\circ$ with these axes, their relative distances have changed in a quantity given by $1/2[\Delta h_0 / \Delta t] \Delta t$. The above derivatives—to be performed at present time—can be estimated from the data displayed in the curves of Figure 2. The resulting values are $[\Delta h / \Delta t]_0 = -1.26 \times 10^{-20}$ and $[\Delta h / \Delta t]_0 = -4.65 \times 10^{-21}$ yr$^{-1}$. In brief, our simulated cluster produces changes in the relative distance of the order of $10^{-20}$—detectable with current technology—in a short period of 4 days. This variation in the relative distance is an effect that would last—at the same rate—for many years; in this sense, we can speak about a secular effect. The relative separation distance would vary regularly, reaching the order of $10^{-19}$ after one decade.

5. CONCLUSIONS AND DISCUSSION

We have proved that gravitational radiation from clusters does not contribute significantly to the density parameter.

A new effect produced by the gravitational waves emitted by a cluster has been described. A cluster with the features of the observed ones has been simulated in the standard CDM model. In other cosmogonies, the effect produced by clusters having these features is expected to be comparable. This is because the clusters producing the secular effect are located...
near the observer and, consequently, their evolution—well inside the horizon—is dominated by internal interactions. Simulations support this idea; see, e.g., Huss, Jain, & Steinmetz (1998), who claim that the differences between individual cluster realizations of a given model are more important than the differences within differing scenarios for a given realization. However, the properties of the spatial distribution of clusters depend on the chosen scenario strongly. A few words about detection and future work are worthwhile.

We could take advantage of the fact that the secular effect can be observed for many years. In order to do that, this effect should be measured during periods greater than the characteristic time of any time-varying gravitational field acting on the detector. Thus, the pulses produced by these fields could be seen as local perturbations of the secular effect.

If the detector is pointing toward a certain direction, it is not receiving gravitational waves from a unique cluster; hence, the calculation of the total effect produced by a set of clusters is important. For the sake of brevity, we present a rough estimate based on idealized cluster distributions within a sphere of radius $R$. All of the clusters are identical to that of this Letter. The distance between any pair of clusters is constrained to be larger than a fixed distance $L$. No cluster correlations are considered. Given an observation direction, the contribution of each cluster to the relative variation of the distance between particles $A$ and $B$ (detector) is assumed to be $1/2\Psi\Delta t$ with $\Psi = 1.26 \times 10^{-20}(100/D)\kappa \cos \alpha \, \text{yr}^{-1}$, where $\alpha$ is the angle formed by the line of sight of the cluster and the observation direction. Since clusters do not radiate in a coherent way, their contributions to $\Psi$ are not correlated. This fact is simulated by using the random number $\kappa$ uniformly distributed in the interval $(-1, 1)$. The factor $(100/D)$ has been introduced because the effect of a given cluster decreases with distance as $D^{-1}$ (see eq. [1]). After superposing the contribution of all of the clusters, the mean $\Psi$ and the standard deviation $\sigma$ have been calculated from the $\Psi$ values corresponding to many observation directions. For each pair $(R, L)$, various simulations have been done. The mean $\Psi$ changes from one to another, but $\sigma$ is a very stable quantity. For the pairs $(R = 600, L = 50 \, \text{Mpc})$, $(R = 600, L = 100 \, \text{Mpc})$, and $(R = 500, L = 50 \, \text{Mpc})$, we have found $\sigma = 7.6 \times 10^{-20}$, $\sigma = 2.7 \times 10^{-20}$, and $\sigma = 7.0 \times 10^{-20}$, respectively. From these data, one concludes that, for $R \geq 500$, the quantity $\Psi$ depends on $R$, weakly. This fact suggests that the use of radius $R > 600$ is not necessary because the main part of the effect is produced by nearby clusters. Furthermore, the resulting $\sigma$ values are greater than the value $1.6 \times 10^{-20}$—the maximum value assumed for a single cluster and $D = 100$—even when large separations ($L = 100 \, \text{Mpc}$) are assumed. The large $\sigma$ values given by simulations strongly suggest the feasibility of anisotropy measurements. Improved simulations including cluster correlations should increase the anisotropy. The effect of realistic spatial distributions including correlations and clusters with different masses, sizes, etc., will be studied elsewhere. Various parameters—such as the proportions between baryonic and dark matter, the density parameter, etc.—would be involved in these simulations. The resulting anisotropy would depend on both cosmological and large-scale structure parameters. Comparisons between measurements and predictions of the secular effect and its anisotropies could play a crucial role in modern cosmology.

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REFERENCES

Boergering, H. 1991, New Insights into the Universe, ed. V. J. Martinez, M. Portilla, & D. Sáez (Berlin: Springer)

Finé, L. S., & Evans, C. R. 1990, ApJ, 351, 588

Hockney, R. W., & Eastwood, J. W. 1988, Computer Simulation Using Particles (Bristol: IOP Publishing)

Hoffman, Y., & Ribak, E. 1991, ApJ, 380, L5

Huss, A., Jain, B., & Steinmetz, M. 1998, MNRAS, in press

Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation (San Francisco: Freeman)

Möchmeyer, R., Schäfer, G., Müller, E., & Kates, R. E. 1991, A&A, 246, 417

Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton: Princeton Univ. Press)

Peebles, P. J. E. 1994, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)

Press, H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1987, Numerical Recipes: The Art of Scientific Computing (Cambridge: Cambridge Univ. Press)

Quilis, V., Ibáñez, J. M., & Sáez, D. 1996, ApJ, 469, 11

———. 1998, ApJ, in press

Schutz, B. F. 1997, in Les Houches School on Astrophysical Sources of Gravitational Radiation, ed. J.-A. Marck & J.-P. Lasota (Cambridge: Cambridge Univ. Press)

Shapiro, S. L., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs and Neutron Stars (New York: John Wiley)

van den Weygaert, R., & Bertschinger, E. 1996, MNRAS, 281, 84