Interpretable Detail-Fidelity Attention Network for Single Image Super-Resolution

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Abstract—Benefiting from the strong capabilities of deep CNNs for feature representation and nonlinear mapping, deep-learning-based methods have achieved excellent performance in single image super-resolution. However, most existing SR methods depend on the high capacity of networks which is initially designed for visual recognition, and rarely consider the initial intention of super-resolution for detail fidelity. Aiming at pursuing this intention, there are two challenging issues to be solved: (1) learning appropriate operators which is adaptive to the diverse characteristics of smoothes and details; (2) improving the ability of model to preserve the low-frequency smoothes and reconstruct the high-frequency details. To solve them, we propose a purposeful and interpretable detail-fidelity attention network to progressively process these smoothes and details in divide-and-conquer manner, which is a novel and specific prospect of image super-resolution for the purpose on improving the detail fidelity, instead of blindly designing or employing the deep CNNs architectures for merely feature representation in local receptive fields. Particularly, we propose a Hessian filtering for interpretable feature representation which is high-profile for detail inference, a dilated encoder-decoder and a distribution alignment cell to improve the inferred Hessian features in morphological manner and statistical manner respectively. Extensive experiments demonstrate that the proposed methods achieve superior performances over the state-of-the-art methods quantitatively and qualitatively. Code is available at github.com/YuanfeiHuang/DeFiAN.

Index Terms—Single image super-resolution, interpretable CNNs, Hessian matrix, detail fidelity.

I. INTRODUCTION

SINGLE image super-resolution (SISR), aiming at reconstructing a high-resolution (HR) image from a single low-resolution (LR) image obtained by limited imaging devices, is a representative branch of low-level vision and widely used in computer vision applications where high-frequency details are greatly desired, e.g., medical imaging, security and surveillance. For decades, numerous researches have been proposed to solve this ill-posed problem of SR, including interpolation-based [1], reconstruction-based [2], [3] and example-learning-based [4]–[7] methods. Recently, with the development of the high-profile deep convolutional neural network (CNN), deep-learning-based SR methods have achieved numerous attentions as its excellent performance and real-time processing.

Initially, Dong et al. [8] proposed a super-resolution convolutional neural network (SRCNN) by stacking a shallow CNN to learn the nonlinear LR-to-HR mappings, which outperforms most conventional example-learning-based SR methods. To facilitate the training phase and eliminate the vanishing/exploding gradients problem, Kim et al. further proposed the accurate VDSR [9] by stacking more than 20 convolutional layers with global residual learning. However, there also exists gradient vanishing/exploding by only using the global residual learning when the depth of network increases. Inspired by ResNet [10], the local residual learning was applied into SR task, e.g., Tai et al. proposed DRRN [11] by utilizing the multi-path local residual learning, Ledig et al. proposed EDSR [13] to obtain better performance by removing all the BN operations in SRResNet and achieved excellent performances. Furthermore, aiming at fully using the information flow of intermediate features in network, densely connection of DenseNet [14] was applied into SR task as SRDenseNet [15], MemNet [16], RDN [17] and etc. Besides,
the feature diversity and informativeness in each layer is also an important issue for improving performance, thus, attention mechanism was highlighted and widely applied for feature enhancement, such as channel-wise attention in SENet [18], spatial-wise attention in CBAM [19], non-local attention in [20], and further applied into the corresponding SR tasks: RCAN [21], CSFM [22] and RNAN [23].

However, higher computational complexities and numerous parameters are introduced as the networks become deeper and wider, then several SISR methods were proposed to reduce them. For example, aiming at reducing parameters, recursive learning was exploited by stacking deep convolution layers with weight sharing, which recursively calls a single block throughout the whole network [11], [24]. Meanwhile, to reduce the computational complexities in model training and inference, lightweight architectures was designed using information diffuence, e.g., IDN [25], MSRN [26] and CARN [27]. Moreover, Dong et al. [28] and Shi et al. [29] respectively utilized the transposed convolution and sub-pixel convolution module to upscale the inferred features in tail and limited the input of LR size to alleviate the computational loads. For larger upscaling factor, Lai et al. [30] proposed a Laplacian pyramid network for super-resolution (LapSRN) via reconstructing the sub-band residual HR images at multiple pyramid levels.

Nevertheless, the existing SR methods almost consider the information in local receptive fields where only 3 × 3 kernels of convolution are utilized to represent the local consistent details of feature. However, the collected LR images are full of low-frequency smoothes and high-frequency details, thus, it is natural to raise two issues: (1) It is difficult to learn a perfect convolutional operator, which is adaptive to the diverse characteristics of smoothes and details; (2) How to improve the ability to preserve the low-frequency smoothes and reconstruct the high-frequency details?

(1) For the first issue, since the low-frequency smoothes and high-frequency details have different characteristics of representation, and the ill-posed problem of SR is more sensitive to the fidelity of deficient details, it is better to solve it in a divide-and-conquer manner.

(2) For the second issue, following the first issue, we should preserve the low-frequency smoothes and reconstruct the high-frequency details as better as possible, which aims at reconstructing the residues (in architectures with global residual learning) using detail-fidelity features as in Fig. 1.

For these issues and to process the low-frequency smoothes and high-frequency details in a divide-and-conquer manner, we propose a purposeful and interpretable method to improve SR performance using Detail-Fidelity Attention in very deep Networks (DeFiAN), as Fig. 2 shows. The major contributions of the proposed method are:

- Introducing a detail-fidelity attention mechanism in each module of networks to adaptively improve the desired high-frequency details and preserve the low-frequency smoothes in a divide-and-conquer manner, which is purposeful for SISR task.
- Proposing a novel multi-scale Hessian filtering (MSHF) to extract the multi-scale textures and details with the maximum eigenvalue of scaled Hessian features implemented using high-profile CNNs. Unlike the conventional CNN features in most existing SR methods, the proposed MSHF is interpretable and specific to improve detail fidelity. Besides, the proposed multi-scale and generic Hessian filtering are the first attempts for interpretable detail inference in SISR task, and could be implemented using GPU-accelerate CNNs without any calculation of intricate inverse of matrix.

- Designing a dilated encoder-decoder (DiEnDec) for fusing the full-resolution contextual information of multi-scale Hessian features and inferring the detail-fidelity attention representations in a morphological erosion & dilation manner, which possesses characteristics of both full-resolution and progressively growing receptive fields.
- Proposing a learnable distribution alignment cell (DAC) for adaptively expanding and aligning the attention representation under the prior distribution of referenced features in a statistical manner, which is appropriate for residual attention architectures.

II. RELATED WORK

Benefiting from the ability of full-resolution deep CNNs to end-to-end non-linear mapping, deep-learning based SR methods have been developed as their excellent performance and real-time processing. Initially, Dong et al. [8] proposed a super-resolution convolutional neural network (SRCNN) to learn a nonlinear LR-to-HR mapping function, which outperforms most conventional example-learning based SR methods. Following this work, deep-learning-based SR methods have achieved excellent performances for decades and been divided into two branches of high-fidelity and lightweight applications:

A. Deep-learning based SR: High-fidelity Type

Aiming at stacking very deep networks to improve the capability of model to feature representation, Kim et al. proposed two deep networks for accurate SR: VDSR [9] and DRCN [24], both of which stack more than 20 convolutional layers with global residual learning. And following the residual block in ResNet [10], local residual learning has been applied in SISR task to stack very deep networks as EDSR [13] and DRRN [11]. Nevertheless, the residual learning still exists shortcomings and ineffectiveness, then MSRN [26] and OISR [31] were proposed to improve the ability of residual blocks for feature representation. Moreover, for building stronger relationships crossing information flows in each convolutional layers/blocks, the excellent densely skip connection has been utilized to build very deep model for more accurate super-resolution as MemNet [16], SRDenseNet [15], RDN [17] and etc. Inspired by the aforementioned trainable attention mechanism, several recent researches were proposed for effectively exploiting the diversity of features, e.g., RCAN [21] and CSFM [22]. Nevertheless, as higher computational complexities with increasing depth of model, these high-fidelity SR methods show difficult implement on mobile applications.

B. Deep-learning based SR: Lightweight Type

For mobile applications, Dong et al. [28] initially proposed a fast variant of SRCNN, which breaks the bottleneck of
computation by straightly using LR image as the input of network and applying deconvolutions into the tail of network for feature upsampling. For similar purpose, Shi et al. [29] proposed the sub-pixel convolutions to replace the deconvolutions, and achieves comparable performance with relatively less computations. Following them, the lightweight SR gets developed. Representatively, Lai et al. [30] proposed the LapSRN via progressively reconstructing the sub-band residual HR images at multiple pyramid levels. Besides, several researches were proposed focusing on the lightweight design of network using information diffluence, which splits the intermediate features and processes them separately in manner of connection or parallel manner, specifically (1), IDN [25] and CARN [27].

III. PROPOSED METHOD

Although stacking very deep networks with small kernel is a feasible way to represent the nonlinear mapping between LR and HR features, it is strongly limited by local consistence and holistically processing each pixel with single convolution in CNNs. Furthermore, in practice, the details and informative textures generally play the dominant roles on visual attention of human vision system, which pays more attentions to the impulse stimulations in scenes and images [32], [33]. Inspired by imitating human visual attentions, we suggest to deal with each components in divide-and-conquer manner, specifically for SR task, pay more attentions to detail fidelity.

Therefore, in this section, we firstly infer the detail-fidelity attention mechanism and how it works in deep CNNs. Next, we describe an interpretable hessian filtering and its multi-scale variant. Furthermore, we discuss and introduce the dilated encoder-decoder module and distribution alignment cell, which meet the condition of inferring better attention representation of Hessian features.

A. Detail-Fidelity Attention Network

It is known that, deep CNNs generally use groups of kernel to convolve with the input features in local receptive fields straightly, i.e., the input features \( \{ x_i, i = 1, 2, ..., c_{in} \} \) are convolved with the kernel weights \( w \) to get the output feature \( \{ y_j, j = 1, 2, ..., c_{out} \} \) as

\[
y_j = \sum_{i=1}^{c_{in}} x_i \otimes w_{j,i} + b_j
\]

where, \( c_{in} \) and \( c_{out} \) denote the number of input and output channel in a convolution layer. Specifically, in the receptive field \( \mathcal{R} \), for each location \( p_* \) on the output feature \( y_j \), we have

\[
y_j(p_*) = \sum_{i=1}^{c_{in}} x_i(p_* + p) \cdot w_{j,i}(p) + b_j
\]

Aiming at imitating the human visual attentions in divide-and-conquer manner, it is expected to convolve each patch of local receptive field \( \mathcal{R}_k \) with different kernels \( w^* \). However, it is inseparable to apply numerous parametric kernels for all the pixels of image. As a solution, under the assumption of fixed kernel weight \( w \), different transformation functions \( f^*(\cdot) \) are introduced into Eq.(2):

\[
y_j(p_*) = \sum_{i=1}^{c_{in}} \sum_{p \in \mathcal{R}_k} x_i(p_* + p) \cdot f^*(w)_{j,i}(p) + b_j
\]

specific to \( w \in \mathbb{R}^{K \times K} \) and \( f^*(w) \in \mathbb{R}^{K \times K} \), each element of a simple linear function \( f^*(w) \) could be represented as \( f^*(w)_{j,i} = a^*_j,i \cdot w_{j,i} \). Then,

\[
y_j(p_*) = \sum_{i=1}^{c_{in}} \sum_{p \in \mathcal{R}_k} (x_i(p_* + p) \cdot a^*_j,i) \cdot w_{j,i}(p) + b_j
\]

particularly, given \( a^*_j,i \) as a element of matrix \( a_j,i(p_* + p) \), then the divide-and-conquer attention mechanism could be formulated as

\[
y_j = \sum_{i=1}^{c_{in}} (x_i \cdot a_{j,i}) \otimes w_{j,i} + b_j
\]
Generally to an integrated system $\Phi$ with divide-and-conquer attention mechanism, the response output $y$ could be formulated as $y = \Phi(a \cdot x)$, where $x$ indicates the input stimulus signal. Furthermore, specific to SR task, the divide-and-conquer attention matrix $a$ are expected to represent detail fidelity of features $x$. Therefore, we introduce the detail-fidelity attention module $\{\text{DeFiAM}^{(n)}\}_{n=1}^{N}$ as the above divide-and-conquer attention system $\Phi$:

\[
y = \text{DeFiAM}^{(n)}(x) = x + \mathcal{F}^{(n)}(x) \cdot a = x + \mathcal{F}^{(n)}(x) \cdot \mathcal{S}(\mathcal{A}^{(n)}(\mathcal{D}^{(n)}(\mathcal{H}^{(n)}(\mathcal{F}^{(n)}(x)))))
\]

where $\mathcal{F}^{(n)}(\cdot)$, $\mathcal{H}^{(n)}(\cdot)$, $\mathcal{D}^{(n)}(\cdot)$ and $\mathcal{A}^{(n)}(\cdot)$ denote feature extraction module (FEM), Hessian Filtering, dilated encoder-decoder (DiEnDec) and distribution alignment cell (DAC) in $n$-th DeFiAM module, which are parametric and learnable. Besides, the non-linear gate cell $\mathcal{S}(\cdot)$ is implemented using Sigmoid function.

Specifically, $\{\text{FEM}^{(n)}\}_{n=1}^{N}$ stacks a chain of $M$ residual channel attention blocks (RCAB) $\{\mathcal{B}^{(n,m)}\}_{m=1}^{M}$ [21], and is formulated as

\[
\mathcal{E} = \mathcal{F}^{(n)}(x) = \mathcal{B}^{(n,M)}(\mathcal{B}^{(n,M)}(\cdots \mathcal{B}^{(n,M)}(x)))
\]

where $\mathcal{B}^{(n)}(x) = x + f_{FE}(x) \cdot f_{CA}(f_{FE}(x))$, specifically, each $f_{FE}$ stacks 2 convolution layers with $3 \times 3$ kernels for feature extraction, and $f_{CA}$ serves for channel attention by stacking global pooling operation, 2 full connection layers and a Sigmoid function successively as [21].

\section{B. Interpretable Hessian Filtering}

As a common criterion for describing the structural characteristics of images at the particular point, Hessian matrix is determined by the second partial derivative of the image in horizontal and vertical directions as

\[
\begin{bmatrix}
G_{hh}(x) & G_{hv}(x) \\
G_{hv}(x) & G_{vv}(x)
\end{bmatrix}
\]

where the $g_{hh}=G_{hh}(x)$, $g_{vv}=G_{vv}(x)$ and $g_{hv}=G_{hv}(x)$ represent the horizontal, vertical and mixed second partial derivative of $x$ respectively. Particulary, for edge detection [34], the maximum eigenvalue of this Hessian matrix is an excellent metric to draw the image structural edges with higher intensity and suitable for the residual images, and has positive effects on improving the detail fidelity of image.

1) Generic Hessian filtering: With the development of deep convolutional neural networks, almost gradient-based feature detection operators are attainable by using the convolutions with fixed weights on the efficient GPUs. From the Eq.(8), the Hessian matrix of $x$ depends on its second order derivations which are available by applying some specific filters:

\[
G_{hh} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix},
G_{vv} = G_{hh}^{T}, G_{hv} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}
\]

Furthermore, the maximum eigenvalue of Hessian matrix is necessary for representing the specific detail-fidelity features, which generally is calculated by applying SVD algorithm or eigenvalue decomposition and is hard to calculate with the acceleration of GPUs. Then, since the Hessian matrix $H(x)$ is real symmetric, we then utilize the linear algebra to infer the relationships between eigenvalues ($\lambda_1 > \lambda_2$), trace $\text{tr}(H(x))$ and determinant $|H(x)|$ of such real symmetric matrix:

\[
\text{tr}(H(x)) = G_{hh}(x) + G_{vv}(x) = \lambda_1 + \lambda_2
\]

\[
|H(x)| = G_{hh}(x)G_{vv}(x) - G_{hv}^2(x) = \lambda_1 \lambda_2
\]

By solving this linear formulation and being free of calculating the intricate inverse of matrix, the maximum eigenvalue $\lambda_1 = \lambda_1$ of $H(x)$ becomes a concise algebraic combination of the image gradients as

\[
\lambda = \mathcal{H}(x)
\]

\[
= (G_{hh}(x) + G_{vv}(x))/2 + \sqrt{(G_{hh}(x) - G_{vv}(x))^2 + 4G_{hv}^2(x)}/2
\]

since only the the pixel-wise gradients are required, it is accessible to utilize the specific convolutions as Eq.(9) to accelerate this Hessian Filtering (HF) procedure $\mathcal{H}(\cdot)$ and then obtain the definitive $\lambda \in \lambda$ for all the pixel-wise elements $x \in \mathcal{x}$.

More generically, $\lambda$ is an algebra combination of the second order gradients of $x$, and

\[
\lambda \propto \sum_{i=-1,0,1} \nabla x^i
\]

where $\nabla$ represents the gradient extractor via Eq.(9).

2) Multi-scale Hessian filtering: Furthermore, in deep residual networks for low-level vision, the residual features are becoming finer and thinner with the increasing depth [37]. That is, the features learned in shallower architecture would represent coarser residues such as geometric structures, as the architecture becomes deeper, more details such as textures and even noises could be generated.
Therefore, to extract various detail-fidelity components of image/feature, we introduce a scaled variant of the above generic Hessian filtering, which is named scaled Hessian Filtering and denoted as $\mathcal{H}_{k\text{ker}}(\cdot)$, where $k\text{ker}$ represents the kernel size of the specific filters in Eq.(9) by padding the zero-vectors. For example,

(1) for $\mathcal{H}_3(\cdot)$ filtering (generic Hessian filtering), the specific gradient extractors are

$$
G_3, hh = [1, -2, 1] \\
G_3, vv = [1, -2, 1]^T \\
G_3, hv = [1, 0, -1] [1, 0, -1]
$$

(14)

then its corresponding maximum eigenvalue of Hessian could be inferred as

$$
\lambda_3 \propto \sum_{i=-1,0,1} \nabla x^i
$$

(15)

(2) for $\mathcal{H}_5(\cdot)$ filtering, the specific gradient extractors are

$$
G_5, hh = [1, 0, -2, 0, 1] \\
G_5, vv = [1, 0, -2, 0, 1]^T \\
G_5, hv = [1, 0, -1] [1, 0, -1]
$$

(16)

where $\{0\}$ represents a 3-dimension zero vector. Then its corresponding maximum eigenvalue of Hessian could be inferred as

$$
\lambda_5 \propto \sum_{i=-2,0,2} \nabla x^i
$$

(17)

In this way, a smaller scaled Hessian filter (e.g., $\mathcal{H}_3(\cdot)$) pays more attention to infer the lower-order maximum eigenvalue of Hessian and exploits the gradients of closer neighboring pixels to generate finer information, e.g., the edges and textures. Instead, a larger one (e.g., $\mathcal{H}_5(\cdot)$) pays more attention to infer higher-order maximum eigenvalue of Hessian and generate coarser information, e.g., the structures. Therefore, we utilize the multi-scale Hessian filters to infer various informative details on each input feature $x$. As shown in Fig. 3, three scald Hessian filters $\mathcal{H}_3(\cdot), \mathcal{H}_5(\cdot)$, and $\mathcal{H}_7(\cdot)$ are exploited to implementing the Multi-Scale Hessian Filtering (MSHF) procedure to infer various interpretable detail-fidelity features, i.e., maximum eigenvalue of multi-scale Hessian matrix.

Particularly, since the size of the eigenvalue $\lambda_{k\text{ker}}$ is same as the corresponding inputs $x$, and under the assumption of Eq.(4) that the size of detail-fidelity attention $a$ should also be same as $x$. Therefore, a fusion unit is necessary to infer the detail-fidelity attentions from the multi-scale Hessian eigenvalues, and formulated as

$$
a = f_{FU}([\bar{\lambda}_3, \bar{\lambda}_5, \bar{\lambda}_7])
$$

(18)

specifically, the multi-scale Hessian eigenvalues are averaged in channel-wise as $\bar{\lambda}$, to alleviate computational complexity. Particularly, as Eqs.(15) and (17), the maximum eigenvalues of multi-scale Hessian matrix are proportional to the multi-order gradients of inputs, which only represent the thin residues but are incapable of illustrating attention maps. Therefore, aiming at both inferring the details of feature and capable of attention representation, the fusion unit $f_{FU}$ should meet several conditions:

(1) Full-resolution manner for the assumption of output $a$ and input $x$ with the same size;

(2) Channel extension manner for the dot product and summation operations of Eq.(4);

(3) Erosion & dilation manner for attention representation.

Under these predetermined conditions, we design a dilated encoder-decoder ($\text{DiEnDec}$) with full-resolution and erosion & dilation manners in morphology, and a distribution alignment cell (DAC) for channel extension manner in statistic.

**C. Dilated Encoder-Decoder**

Generally, an end-to-end architecture for low-level vision concerns more about the pixel intensities in full-resolution and aims at producing the expected details in a coarse-to-fine fashion which is contrary to the fashion in semantic segmentation or image classification. Therefore, full-convolution architecture is generally exploited for low-level vision tasks and meets the requested full-resolution manner.

Moreover, in order to learn some specific contextual information, namely implementation of erosion & dilation manner, encoder-decoder is designed to transform the input features into specific contextual styles. To utilize the global contextual information of images, hourglass-like encoder-decoder
(HGEnDec) [36], [38] are designed by combination of general convolutions and pooling operators. However, in the case of super-resolution, pooling is inappropriate to retain the full resolution. Then, Mao et al. [35] present a full-resolution convolutional encoder-decoder (ConvEnDec) for image restoration, in this way, the encoder cascaded by a group of $3 \times 3$ convolutions suppresses the noises and the decoder with deconvolution reconstructs the denoised features. However, this method only considers the information of small $3 \times 3$ local receptive field for each pixel and lacks contextual information. Therefore, a full-convolution and context-aware architecture is necessary.

Beyond the limitation of general convolution, the dilated convolution [39] considers a wider range of neighbors for each pixel with dilation in convolution and then aggregates more contextual information in full-resolution fashion. As Fig. 4(b) shows and differ from the existing encoder-decoders, we explore the ability of both full-resolution and extracting contextual information by introducing the dilated convolution, named Dilated Encoder-Decoder (DiEnDec) and parameterized as Fig. 4(a). In each DiEnDec, an exponential expansion of receptive field with no gridding effects [40] is available by using the exponential increasing dilation of convolution. For the $k \times k$ kernel of dilated convolution with dilation of $(k - 1)^i$ in the encoder with $d$ effective layers, the accumulative receptive field (ARF) could be formulated as

$$ARF = 1 + \sum_{i=1}^{M} (k - 1)^i$$ (19)

By reverse thinking, we use the dilated deconvolution to reproducing the information by decoding the contextual features. For better viewing, Fig. 4(a) shows the available ARF in each stage of encoding and decoding. Morphologically, by using these dilated convolutions, the encoder not only suppresses the noisy features and also obtain the neighboring relationships in larger receptive fields for feature erosion, then the decoder reconstruct these obscure eroded features into more representative dilated styles.

Specifically, under the assumption of $x \in \mathbb{R}^{c \times w \times h}$ and $\mathcal{F}(x) \in \mathbb{R}^{c \times w \times h}$, where $c$, $w$ and $h$ denote the channel size, width and height of features. As in Fig. 4(a), aiming at inferring the representative attention features with lower computational complexity, the output of DiEnDec is $v = \mathcal{D}(\mu_0, \sigma_0^2)$.

D. Distribution Alignment Cell

To meet the dot production and summation operations of Eq. (4) for $x \in \mathbb{R}^{c \times w \times h}$, it is necessary to expand the detail-fidelity attention $a \in \mathbb{R}^{c \times w \times h}$ from the above attention representation $v \in \mathbb{R}^{1 \times w \times h}$. Specifically, as Eq.(4), in each DeFiAM, the final output feature $y$ is a linear representation of input features $x$, residual features $\mathcal{F}(x)$ and attention representation $a$. Then it is necessary to align the attention representation and features for optimal convergency, and for this issue, the channel attention was proposed for feature recalibration in channel-wise [18], [21], but with the super-imposed effect that the observed features are enhanced with the channel-wise representation scalars of referenced features, and is generally helpless for alignment.

We then propose a distribution alignment cell (DAC) to expand the attention representation $v$ under the prior distribution of input features $x$, as illustrated in Fig. 5. Particularly, for a better understand of this subsection, we use $x_o$ (observed feature) and $x_r$ (referenced features) replace $v$ and $x$, respectively. Under the assumption that both of the observed and referenced features meet some prior distributions, e.g., assume $x_o \sim \mathcal{N}(\mu_o, \sigma_o^2)$ and $x_r \sim \mathcal{N}(\mu_r, \sigma_r^2)$, DAC firstly normalize the observed feature by

$$\tilde{x}_o = \frac{x_o - \mu_o}{\sqrt{\sigma_o^2 + \epsilon}}$$ (20)

and expand it into $\tilde{x} \in \mathbb{R}^{c \times w \times h}$ via channel-wise replication.

Besides, DAC also parameterize the distribution of referenced features using nonlinear neural networks as

$$\hat{\mu}_r = \text{FC}(\text{ReLU} (\text{FC}(\mu_r)))$$
$$\hat{\sigma}_r = \text{FC}(\text{ReLU} (\text{FC}(\sigma_r)))$$ (21)

where, $\text{FC}$ and $\text{ReLU}$ represent the full connection layer and Rectified Linear Unit (ReLU), respectively.

Then since the normalized observed feature meets the standard normal distribution, it is feasible to align them with the parameterized priors, as

$$\hat{x}_o = \tilde{x}_o \cdot \hat{\sigma}_r + \hat{\mu}_r$$ (22)

then, the aligned observed features meet similar distribution as the referenced features, namely, $\hat{x}_o \sim \mathcal{N}(\hat{\mu}_r, \hat{\sigma}_r^2)$.

To be emphasized, since the DAC procedure only works in channel-wise, the spatial-wise distribution of $x_o$ has been also retained for attention representation.

IV. EXPERIMENT

In this section, we first provide the benchmarks (including datasets and evaluation criterion) and implementation details (including hyper-parameters and optimization). Then we compare our DeFiA N model with other state-of-the-art methods on several benchmark datasets. Next we study the contributions of different components in the proposed DeFiA N by experimental demonstrations and finally illustrate discussion on limitation.
A. Benchmarks

1) Datasets: In testing phase, we implement our experiments on several benchmark datasets for evaluation, including Set5 [41], Set14 [5], BSD100 [42], Urban100 [43] and Manga109 [30] with general, structural and cartoon scenes. In training phase, 800 high-quality 2K-resolution images from DIV2K [44] dataset are considered as high-resolution groundtruth for training the proposed models, and downsampled using Bicubic algorithm to generate the corresponding low-resolution inputs. In detail, we use the 48 × 48 RGB patches from the low-resolution training set as input and the corresponding 48s × 48s high-resolution patches as groundtruth for ×s upscaling, and augment these LR-HR pairs with random horizontal and vertical flips and 90 rotations. Particularly, all the LR and HR images are pre-processed by subtracting the mean RGB value of the training sets.

2) Evaluation: For evaluating the SR performance, we apply two common full-reference image quality assessment criteria for evaluating discrepancies: Peak Signal-to-Noise Ratio (PSNR) and Structural SIMilarity (SSIM). Following the convention of super-resolution, luminance channel is selected for full-reference image quality assessment because the intensity of image is more sensitive to human-vision than the chroma. Moreover, we use two criteria to represent the computational complexities of methods, where #Params denotes the number of parameters in whole network, #FLOPs indicates the number of operations by Multi-Adds which is the number of composite multiply-accumulate operations for processing a 480 × 360 × 3 RGB image, which indicate the space complexity and time complexity respectively.

B. Implementation Details

1) Hyperparameters: As shown in Fig. 2, a full DeFiAN model depends on several hyperparameters, including the number of DeFiAM modules (N), the number of RCAB blocks in each FEM (M), the number of channels in each layer (C), and the specific settings of attention module. For the purposes on high-fidelity and lightweight applications, we propose two DeFiAN models:

i) High-fidelity DeFiAN_L has a chain of N = 10 DeFiAM modules, and in each module, M = 20 RCABs are stacked to reconstruct the primitive residues with C = 64 channels.

ii) Lightweight DeFiAN_S has a chain of N = 5 DeFiAM modules, and in each module, M = 10 RCABs are stacked to reconstruct the primitive residues with C = 32 channels.

Moreover, as described in Section III-A, for both DeFiAN_L and DeFiAN_S, aiming at inferring the detail-fidelity attention representations for feature enhancement, the primitive residues are delivered into the subsequent modules, including the interpretable MSHF which aggregates three Hessian eigenvalues \([\lambda_3, \lambda_2, \lambda_1]\) to infer the fine (edges and textures) and coarse (structures) details of feature, the DiEnDec with 3 dilated convolution layers for feature erosion and 3 dilated deconvolution layers for feature dilation as Fig. 4(a) shows, and the DaC with 2 full connection layers for parameterizing the distribution of referenced features.

2) Optimization: All of our models are optimized via minimizing the mean absolute error (MAE) between the super-resolved image \(DeFiAN_s(x)\) and the corresponding groundtruth image \(y\). Therefore, given a training dataset \(\{(x^k, y^k)\}_{k=1}^K\), where \(K\) is the batch size and \(\{x^k, y^k\}\) are the \(k\)-th LR and HR patch pairs. Then, the objective function is formulated as

\[
\mathcal{L}(\theta) = \sum_{k=1}^K \| y^k - DeFiAN(x^k) \|_1
\]

where \(\theta\) denotes the parameters of DeFiAN models, which are optimized using the Adam optimizer [46] with mini-batch of size \(K = 16\). The learning rate is initialized to \(10^{-4}\) and halved for every \(2 \times 10^5\) mini-batch updates. Each of the final models will get convergence after \(6 \times 10^5\) mini-batch updates on PyTorch framework and a 12GB NVIDIA Titan X Pascal GPU.

C. Comparison with State-of-the-art Methods

To illustrate the effectiveness of the proposed DeFiAN models, several state-of-the-art SISR methods are compared in terms of quantitative objective evaluation, qualitatively visual performance and computational complexity. As mentioned in Section IV-B1, for the purposes on high-fidelity and lightweight applications, two models DeFiAN_L and DeFiAN_S are proposed and compared with other corresponding state-of-the-art SISR methods.

1) High-fidelity models: Aiming at evaluating the high-fidelity super-resolved images from the low-resolution inputs, several state-of-the-art high-fidelity SR methods are selected to compare with our proposed DeFiAN_L, including MemNet [16], EDSR [13], D-DBPN [45], RDN [17], MSRN [26], RCAN [21], CSFM [22] and OISR-RK3 [31].

As posted in TABLE I, although compared with RCAN [21] and CSFM [22], both of which apply the attention mechanism into very deep networks, our DeFiAN_L achieves the highest performance with comparative spacial complexity and time complexity.

As described in Section III-A, since we stack a chain of residual channel attention blocks in the FEM modules, in a sense, RCAN [21] is considered as the baseline of our DeFiAN models. Particularly, by introducing the detail-fidelity attention mechanism, our DeFiAN is equipped with stronger ability of detail representation. As reported in TABLE I, especially on the challenging dataset Manga109, the proposed DeFiAN_L advances RCAN [21] with the improvement margins of 0.34 dB, 0.41 dB and 0.47 dB for ×2, ×3 and ×4 upscaling respectively. Besides, as shown in Fig. 6-7, our DeFiAN_L preserves the details better than other methods. Specifically, for structural images with regular buildings in Fig. 6, DeFiAN_L preserves the edges and repetitive structures well, and for synthesized comic scenes in Fig. 7, natural textures could be also preserved by using our DeFiAN_L.

1To be fair, all the comparative experiments are conducted on the super-resolved byte-format images generated using the released codes: SRCNN, FSRCNN, VDSR, LapSRN, DRRN, MemNet, IDN, EDSR, D-DBPN, RDN, RCAN, MSRN, CARN, OISR.
TABLE I: QUANTITATIVE COMPARISONS OF THE PROPOSED DeFiANL WITH THE STATE-OF-THE-ART HIGH-FIDELITY METHODS ON BENCHMARK DATASETS FOR $\times 2$, $\times 3$ AND $\times 4$ UPSCALING.

| Scale | Method     | #Params (K) | #FLOPs (G) | Set5 | Set14 | BSD100 | Urban100 | Manga109 |
|-------|------------|-------------|------------|------|-------|--------|----------|----------|
|       |            | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM |
| $\times 2$ | Bicubic   | -   | -   | 33.64 | 0.9292 | 30.22 | 0.8683 | 29.55 | 0.8425 |
|         | MemNet [16] | 2905.5 | 503.5 | 37.78 | 0.9597 | 33.28 | 0.9142 | 32.08 | 0.8978 |
|         | EDSR [13]  | 4071.7 | 1760.0 | 38.11 | 0.9600 | 33.82 | 0.9189 | 32.33 | 0.9011 |
|         | D-DBPN [45] | 5953.5 | 270.0 | 38.05 | 0.9599 | 33.70 | 0.9188 | 32.25 | 0.9001 |
|         | RDN [17]   | 5616.3 | 243.0 | 38.16 | 0.9603 | 33.88 | 0.9199 | 32.31 | 0.9009 |
|         | MSRNN [26] | 5930.1 | 256.5 | 37.98 | 0.9597 | 33.56 | 0.9171 | 32.20 | 0.8994 |
|         | RCAN [21]  | 15444.7 | 663.5 | 38.18 | 0.9604 | 34.00 | 0.9203 | 32.37 | 0.9016 |
|         | CSFM [22]  | 12071.9 | 519.9 | 38.17 | 0.9605 | 33.94 | 0.9200 | 32.34 | 0.9013 |
|         | OISR-RK3 [31] | 44270.0 | 1812.1 | 38.13 | 0.9602 | 33.81 | 0.9194 | 32.34 | 0.9011 |
|         | DeFiANL (Ours) | 15186.1 | 651.4 | 38.33 | 0.9618 | 34.28 | 0.9231 | 32.43 | 0.9029 |
| $\times 3$ | Bicubic   | -   | -   | 30.40 | 0.8686 | 27.34 | 0.7741 | 25.94 | 0.6672 |
|         | MemNet [16] | 2905.5 | 503.5 | 34.12 | 0.8923 | 30.42 | 0.8447 | 29.26 | 0.8089 |
|         | EDSR [13]  | 43660.8 | 839.6 | 34.64 | 0.9285 | 30.46 | 0.8451 | 29.24 | 0.8084 |
|         | RDN [17]   | 5800.6 | 111.7 | 34.67 | 0.9285 | 30.46 | 0.8451 | 29.24 | 0.8084 |
|         | MSRNN [26] | 6115.0 | 117.7 | 34.45 | 0.9265 | 30.29 | 0.8418 | 29.13 | 0.8054 |
|         | RCAN [21]  | 15592.4 | 290.8 | 34.72 | 0.9289 | 30.52 | 0.8458 | 29.29 | 0.8099 |
|         | CSFM [22]  | 12256.5 | 234.8 | 34.72 | 0.9290 | 30.52 | 0.8458 | 29.29 | 0.8099 |
|         | OISR-RK3 [31] | 44877.2 | 862.7 | 34.69 | 0.9287 | 30.46 | 0.8454 | 29.28 | 0.8096 |
|         | DeFiANL (Ours) | 15370.4 | 293.2 | 34.81 | 0.9305 | 30.75 | 0.8493 | 29.37 | 0.8120 |
| $\times 4$ | Bicubic   | -   | -   | 28.42 | 0.8101 | 25.99 | 0.6780 | 23.14 | 0.5673 |
|         | MemNet [16] | 2905.5 | 503.5 | 31.74 | 0.8900 | 28.23 | 0.7731 | 27.43 | 0.7291 |
|         | EDSR [13]  | 43071.0 | 542.9 | 34.12 | 0.8923 | 30.42 | 0.8447 | 29.26 | 0.8089 |
|         | RDN [17]   | 5763.7 | 67.4 | 34.67 | 0.9285 | 30.46 | 0.8451 | 29.24 | 0.8084 |
|         | MSRNN [26] | 6077.5 | 70.7 | 34.45 | 0.9265 | 30.29 | 0.8418 | 29.13 | 0.8054 |
|         | RCAN [21]  | 15197.0 | 168.1 | 34.22 | 0.9287 | 30.46 | 0.8454 | 29.28 | 0.8096 |
|         | CSFM [22]  | 12219.6 | 136.6 | 34.22 | 0.9287 | 30.46 | 0.8454 | 29.28 | 0.8096 |
|         | OISR-RK3 [31] | 44287.1 | 555.9 | 34.81 | 0.9305 | 30.75 | 0.8493 | 29.37 | 0.8120 |
|         | DeFiANL (Ours) | 15333.5 | 169.5 | 32.67 | 0.9009 | 28.99 | 0.7906 | 27.84 | 0.7448 |

Fig. 6: Subjective quality assessment for $\times 4$ upscaling on "img078" from Urban100 dataset. Structural textures (e.g., windows) in our DeFiANL are more distinct than other state-of-the-art high-fidelity SISR methods.

Fig. 7: Subjective quality assessment for $\times 4$ upscaling on "MomoyamaHaikagura" from Manga109 dataset. Our DeFiANL reconstruct the repetitive patterns better than other state-of-the-art high-fidelity SISR methods.
### TABLE II: Quantitative Comparisons of the Proposed DeFiAnS with the State-of-the-Art Lightweight Methods on Benchmark Datasets for ×2, ×3 and ×4 Upscaling.

| Scale | Method       | #Params (K) | #FLOPs (G) | Set5 | Set14 | BSD100 | Urban100 | Manga109 |
|-------|--------------|-------------|------------|------|-------|--------|----------|----------|
|       |              |             |            | PSNR | SSIM  | PSNR   | SSIM     | PSNR    | SSIM    |
| ×2    | Bicubic      | -           | -          | 33.64 | 0.9292 | 30.22  | 0.8683  | 29.55   | 0.8425  |
|       | SRCNN [8]    | 8.0         | 1.4        | 36.66 | 0.9542 | 32.47  | 0.8906  | 31.37   | 0.8879  |
|       | FSRCNN [28]  | 12.5        | 0.5        | 36.98 | 0.9556 | 32.62  | 0.9087  | 31.50   | 0.8904  |
|       | VDSR [9]     | 664.7       | 115.1      | 37.53 | 0.9587 | 33.05  | 0.9127  | 31.90   | 0.8960  |
|       | LapSRN [30]  | 435.3       | 18.9       | 37.52 | 0.9591 | 32.99  | 0.9124  | 31.80   | 0.8949  |
|       | DRRN [11]    | 297.2       | 1275.5     | 37.74 | 0.9591 | 33.25  | 0.9137  | 32.05   | 0.8973  |
|       | IDN [25]     | 715.3       | 31.0       | 37.83 | 0.9600 | 33.30  | 0.9148  | 32.08   | 0.8985  |
|       | CARN [27]    | 1592.4      | 41.9       | 37.80 | 0.9589 | 33.44  | 0.9161  | 32.10   | 0.8978  |
|       | DeFiAnS (Ours) | 1027.6     | 44.2       | 38.03 | 0.9605 | 33.63  | 0.9181  | 32.20   | 0.9099  |
| ×3    | Bicubic      | -           | -          | 30.40 | 0.8686 | 27.54  | 0.7414  | 27.21   | 0.7389  |
|       | SRCNN [8]    | 8.0         | 1.4        | 32.75 | 0.9090 | 29.29  | 0.8215  | 28.41   | 0.7863  |
|       | FSRCNN [28]  | 12.5        | 0.2        | 33.16 | 0.9140 | 29.42  | 0.8242  | 28.52   | 0.7893  |
|       | VDSR [9]     | 664.7       | 115.1      | 33.66 | 0.9213 | 29.78  | 0.8318  | 28.83   | 0.7976  |
|       | LapSRN [30]  | 435.3       | 8.5        | 33.82 | 0.9227 | 29.79  | 0.8320  | 28.82   | 0.7973  |
|       | DRRN [11]    | 297.2       | 1275.5     | 34.02 | 0.9244 | 29.98  | 0.8350  | 28.95   | 0.8004  |
|       | IDN [25]     | 715.3       | 13.8       | 34.12 | 0.9253 | 29.99  | 0.8356  | 28.95   | 0.8013  |
|       | CARN [27]    | 1592.4      | 22.3       | 34.28 | 0.9254 | 30.19  | 0.8397  | 29.06   | 0.8034  |
|       | DeFiAnS (Ours) | 1073.7     | 20.6       | 34.42 | 0.9273 | 30.34  | 0.8410  | 29.12   | 0.8053  |
| ×4    | Bicubic      | -           | -          | 28.42 | 0.8101 | 25.99  | 0.7023  | 25.96   | 0.6672  |
|       | SRCNN [8]    | 8.0         | 1.4        | 30.49 | 0.8629 | 27.51  | 0.7519  | 26.91   | 0.7104  |
|       | FSRCNN [28]  | 12.5        | 0.1        | 30.70 | 0.8657 | 27.59  | 0.7535  | 26.96   | 0.7120  |
|       | VDSR [9]     | 664.7       | 115.1      | 31.35 | 0.8838 | 28.02  | 0.7678  | 27.29   | 0.7252  |
|       | LapSRN [30]  | 435.3       | 4.8        | 31.52 | 0.8854 | 28.09  | 0.7687  | 27.31   | 0.7255  |
|       | DRRN [11]    | 297.2       | 1275.5     | 31.67 | 0.8888 | 28.22  | 0.7721  | 27.72   | 0.7284  |
|       | IDN [25]     | 715.3       | 7.7        | 31.81 | 0.8903 | 28.25  | 0.7731  | 27.41   | 0.7295  |
|       | CARN [27]    | 1592.4      | 17.1       | 32.12 | 0.8936 | 28.50  | 0.7791  | 27.58   | 0.7349  |
|       | DeFiAnS (Ours) | 1064.5     | 12.8       | 32.16 | 0.8942 | 28.63  | 0.7810  | 27.58   | 0.7363  |

Fig. 8: Subjective quality assessment for ×4 upscaling on "img092" from Urban100 dataset. Only our DeFiAnS can reconstruct the mixed edges correctly which are failed in other state-of-the-art lightweight SISR methods.

![Subjective quality assessment for ×4 upscaling on "GOOD_KISS_Ver2" from Manga109 dataset. Our DeFiAnS performs better than other state-of-the-art lightweight SISR methods on reconstructing the textures/characters.](image-url)
TABLE III: Ablation study on DeFiAN: ×3 upscaling evaluation on Set5 dataset.

|            | MSHF | DeEnDec | SAC | MSHF | DeEnDec | SAC | MSHF | DeEnDec | SAC | MSHF | DeEnDec | SAC | MSHF | DeEnDec | SAC |
|------------|------|---------|-----|------|---------|-----|------|---------|-----|------|---------|-----|------|---------|-----|
| #Params(K) | 15233.8 | 15291.4 | 15302.5 | 15244.1 | 15360.1 | 15301.7 | 15312.8 | 15370.4 |
| #FLOPs(G) | 290.8 | 291.9 | 292.1 | 290.8 | 293.2 | 291.9 | 292.1 | 293.2 |
| PSNR       | 33.91 | 34.12 | 34.09 | 33.96 | 34.17 | 34.15 | 34.13 | 34.22 |

Fig. 11: Comparison of DeFiAN with different model sizes: Average PSNR on Set5 dataset for ×3 upscaling.

D. Model Analysis

In this subsection, aiming at demonstrating the ability of detail-fidelity attention mechanism for interpretable detail inference in super-resolution task, the effects and contributions of each components in the proposed DeFiAN models are analyzed via experimental quantitatively verification, including the multi-scale Hessian filtering, dilated encoder-decoder, distribution alignment cell, and some implementation settings. Specifically, the models in this subsection is set as $N=10$, $M=20$, $C=64$, and are trained with only $5 \times 10^4$ mini-batch updates on DIV2K datasets, unless otherwise specified.

1) Ablation study: Firstly, we conduct the ablation study on each component of our DeFiAN as reported in TABLE III, including the multi-scale Hessian filtering (MSHF), dilated encoder-decoder (DiEnDec) and distribution alignment cell (SAC). Besides, since DeFiAN is designed to focus on high-frequency details of features in network, we also visualize the intermediate features in network. As shown in Fig. 10, we visualize the averaged attention features $\bar{a}$ of DeFiAM$_{10}$ by partly removing one or more modules, including MSHF, DiEnDec and SAC:

- if remove MSHF, the attention map would suffer more undesired discrepancies in region of details, such as subfigures (e)$\rightarrow$(c) and (h)$\rightarrow$(g).
- if remove DiEnDec, the attention map would focus on the holistic map but not the details, such as subfigures (e)$\rightarrow$(b) and (h)$\rightarrow$(f).
- if remove SAC, the gaps between distributions of $x$ and $a$ would be enlarged which is undesired in attention operation as Eq.(5), such as subfigures (g)$\rightarrow$(c) and (h)$\rightarrow$(e).

Note that, subfigure (a) represents DeFiAM with only FEM modules, so the averaged attention map $\bar{a}$=1 and the corresponding DeFiAN is same as RCAN.

2) Effect of model size: As illustrated in Section IV-B1, several hyperparameters play significant roles on designing a full model of DeFiAN, including the number of DeFiAM modules $N$, the number of RCABs $M$ in each DeFiAM and the number of channels $C$ in primitive residues. We then conduct the model analysis on different settings of these hyperparameters as in Fig. 11. Particularly, in this subsection, all the full models consist of FEM, MSHF, DiEnDec and SAC.

In Fig. 11, as the model size increases (i.e., larger $N$, $M$ and $C$), the performance of DeFiAN gets a large margin of...
improvement but reaches the bottleneck when the model size reaches a peak level (e.g., #FLOPs ≥ 200G) and emerges gradient exploding/E/vanishing/V when the depth reaches over 600 layers (the reason is discussed in Section IV-E). Therefore, we choose an appropriate setting of hyperparameter as described in Section IV-B1 for high-fidelity and lightweight applications, which achieves excellent performance with relatively lower computational complexities.

3) Effect of MSHF: As aforementioned in Section III-B, the proposed Hessian filtering is an interpretable architecture for detail representation and implemented using convolutional neural networks which is feasibly calculated in distributed GPUs. Therefore, in this subsection, we conduct some experiments to illustrate the efficiency and effect of the proposed Hessian filtering:

i) Efficiency of Hessian filtering. In a sense, our Hessian filtering is an accelerated CNN-based algorithm for calculating the maximum eigenvalues of Hessian matrix, and faster than the pixel-wise eigenvalue solving algorithm which is formulated as

\[ \lambda_{klij} = \max(Eig(H(x_{klij}))) \]  

(24)

where \(H(x_{klij})\) represents the Hessian matrix of \(klij\)-th pixel using Eq.(8) with 2-order gradients, \(Eig(\cdot)\) denotes the eigenvalue solver \(torch.eig\)1 in PyTorch.

To demonstrate the superiority of our Hessian filtering, we conduct two comparative experiments to illustrate the efficiency of conventional eigenvalue solver, the proposed CPU-based and GPU-based Hessian filtering. In particular, as Eq.(24), the conventional eigenvalue solver calculates the maximum eigenvalue in pixel-wise, then for fairness, we set the input tensor of small spatial size (1 × 1, 2 × 2, 4 × 4 and 8 × 8) and channel (1, 4, 16 and 64) to record the processing time. As shown in Fig. 12, to process the maximum eigenvalue of a single tensor, both of our proposed CPU-based and GPU-based Hessian filterings show superiority to the conventional solver in speed, especially with the increasing spatial sizes and channels of input, the GPU-based Hessian filtering is more robust to the size of input tensors than the others.

ii) Effectiveness of Hessian filtering on multi-order gradient representation. Furthermore, aiming at demonstrating the effectiveness and interpretability of (multi-scale) Hessian filtering for detail representation, we conduct the visualization experiments on the inferred Hessian features and attention representations in DeFiAM modules. As Fig. 1 shows, for the scaled Hessian filtering \(\mathcal{H}_{ker}(\cdot)\), the corresponding maximum eigenvalues of Hessian \(\lambda_{ker}\) focus on detail representation with different degrees. For example on image “img086” in Urban100, \(\lambda_3\) and \(\lambda_7\) focus on the fine edges and coarse structures respectively, which experimentally demonstrates the effectiveness of scaled Hessian filtering on representing multi-order gradients as the theoretical proof in Section III-B.

iii) Effect of multi-scale Hessian filtering. As illustrated in Fig. 3 and Section III-B2, to extract various detail-fidelity components of features, we introduce the multi-scale Hessian filtering (MSHF) to infer the multi-scale detail-fidelity attentions as Eq.(18). In order to demonstrate the effect of MSHF, we compare different settings of MSHF as reported in TABLE IV. By applying MSHF into DeFiAM modules, the ability of network to detail preservation and representation is largely improved with 0.21dB gains on Set5 dataset. For example, DeFiAM with single scaled Hessian filtering (e.g., \(ker=(3)\)) works better than the one without any Hessian filtering, but worse than the modules with double scaled Hessian filtering (e.g., \(ker=(3, 5)\)) and triple scaled Hessian filtering (e.g., \(ker=(3, 5, 7)\)) which performs best. Furthermore, as in Fig. 1, by applying MSHF with \(ker = (3, 5, 7)\), the detail-fidelity attention \(a\) captures both low-order and high-order details of feature and has higher ability to detail representation in field of residues.

4) Effect of DiEnDec: As described in Section III-C, DiEnDec is designed to meet the conditions on “full-resolution manner” and “erosion & dilation manner” for detail-fidelity attention representation. We then conduct some experiments to demonstrate the effect of DiEnDec in these two manners:

i) Full-resolution manner. As illustrated in Fig. 4(b), different from the existing encoder-decoder architectures (e.g., convolutional encoder-decoder (ConvEnDec) [35] which meets condition on full-resolution manner, and hourglass encoder-decoder (HGEnDec) [36] which is context-aware with growing receptive fields), our DiEnDec is a full-resolution and context-aware architecture with progressively growing accumulated receptive fields. Thus, we conduct some experimental comparisons by applying different encoder-decoder (with comparative #Params and #FLOPs) into each DeFiAM modules as in TABLE V. Particularly, “w/o MSHF” indicates the vanilla baseline model, “MSHF” indicates the full

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**TABLE IV:**

**EFFECT OF MULTI-SCALE HESSIAN FILTERING:**

**AVERAGE PSNR ON DATASETS FOR ×3 UPSCALING.**

| Settings | #Params (K) | #FLOPs (G) | Datasets |
|----------|-------------|------------|----------|
| w/o HF   | 15233.8     | 290.8      | Set5     |
| ker = (3)  | 15253.0     | 291.1      | 34.06    |
| ker = (5)  | 15253.0     | 291.1      | 34.05    |
| ker = (7)  | 15253.0     | 291.1      | 35.80    |
| ker = (3, 5)| 15272.2     | 291.5      | 34.09    |
| ker = (3, 7)| 15272.2     | 291.5      | 34.09    |
| ker = (5, 7)| 15272.2     | 291.5      | 34.04    |
| ker = (5, 7)| 15291.4     | 291.9      | 34.12    |

*ker = (a, b, c) indicates the alterable kernel for MSHF.*

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1. https://pytorch.org/docs/master/torch.html#torch.eig

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Fig. 12: Comparison of Hessian filtering (CPU-based and GPU-based) and eigenvalue solver \(torch.eig\) on number of channels (spatial size is 4 × 4) and spatial size of inputs (with 16 channels).
DeFiAN model, the hyperparameter $d$ here represents the half depth of encoder-decoder (when $d=0$ and $d=1$, HGEnDec=ConvEnDec=DiEnDec). Compared with ConvEnDec and HGEnDec, our DiEnDec achieves better performances. Furthermore, since DiEnDec is designed for further multi-scale Hessian features fusion, the DeFiAM module with MSHF gets improvements about 0.2dB. Besides, as the depth increases, ability of feature representation gets thrived and tends to be plateau as the balance between higher receptive field and consistency of local information.

ii) Erosion & dilation manner. As illustrated in Fig. 4, for $3 \times 3$ kernel of dilated convolutions in DiEnDec, the accumulative receptive field for encoding and decoding could be inferred as $ARF = 3 \rightarrow 7 \rightarrow 15$ and $ARF = 9 \rightarrow 13 \rightarrow 15$ respectively. Moreover, morphologically, the dilated convolutions of encoder work in pixel-aggregation manner which is similar to erosion, and the dilated deconvolutions of decoder work in pixel-divergency manner which is similar to dilation. To validate the ability of DiEnDec to erosion & dilation, we visualize the intermediate features in DiEnDec as Fig. 13 shows. We can find that, the features are progressively fining with pixel aggregation in encoding phase and coarsening with pixel divergency in decoding phase. Namely, on the view of morphology, the encoder works for feature erosion and the decoder works for feature dilation.

5) Effect of DAC: As aforementioned in Section III-D, to meet the dot production and summation operations of Eq.(4), we introduce the distribution alignment cell (DAC) to align the single attention representation $v$ on the distribution of prior features $x$, which represent the input of DeFiAM. As shown in Fig. 14, the distributions of detail-fidelity attention $a^{(n)}$ are aligned as the distributions of inputted features $x^{(n)}$ in channel-wise and retain the spatial-wise characteristics of the inferred attention representation $v^{(n)}$. Particularly, as in Eq.(21), in order to improve the capacity of feature representation, the distribution of referenced features are transformed into $x_r \sim N(\hat{\mu}_r, \sigma_r^2)$, that is why the distributions of aligned $a^{(n)}$ are uncompletely same as $x^{(n)}$.

E. Discussion on Limitation

In Fig. 11, unfortunately, as the depth of network increases over 600 layers, there exists gradient exploding/vanishing ($E/V$) in training phase. As illustrated in Section III-B, for example, taking only the scales Hessian features $\lambda_3$ into consideration, it can be inferred as an algebra combination of the second order gradient of $x$ in Eq.(15). Then in back propagation, the gradient of $x$ in MSHF can be represented as

$$grad = \frac{\partial L}{\partial x} \frac{\partial^3 x}{\partial \lambda_3} \sim \frac{\partial L}{\partial \lambda_3} \frac{\partial^2 x}{\partial x} \times \frac{\partial L}{\partial \lambda_3} \frac{\partial^2 x}{\partial x}$$

where $L$ represents the loss function in Eq.(23). We find that the gradient of $x$ in MSHF is proportional to the production of its 3-order partial gradients $\partial^3 x$ and $\partial L / \partial \lambda_3$. Since $\partial L / \partial \lambda_3$ is determined on the subsequent DiEnDec, we then set it aside. And $\partial^3 x$ is determined by only $x$, and raises two extreme problems: (1) if $\partial^3 x \gg 1$ (full of edges), the accumulated $grad$ would be exploding to $\pm \infty$; (2) if $\partial^3 x \rightarrow 0$ (full of smooths), the accumulated $grad$ would be vanishing to 0.

V. CONCLUSION

In this paper, we propose an interpretable detail-fidelity attention network for single image super-resolution, which is designed to pursue the initial intention of super-resolution for detail fidelity. Particularly, for two challenging issues on learning an adaptive operator to dividedly process low-frequency smoothes and high-frequency details, and improving the ability to detail fidelity, an interpretable multi-scale Hessian filtering is proposed, which is promising for detail fidelity representation in other computer vision applications. Besides,
to improve the attention representation of Hessian features, the dilated encoder-decoder and the distribution alignment cell are proposed in morphological and statistical manners respectively. Extensive experiments demonstrate the proposed methods achieve more excellent performance than the state-of-the-art methods quantitatively and qualitatively. Moreover, we suggest that divide-and-conquer for detail fidelity is expected to be a vital issue for super-resolution in the future.

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