$|\Delta B| = 1$ Weak Effective Lagrangian

in the Minimal Flavor Violation Supersymmetry

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Abstract
To evaluate the weak decays of b-hadrons, the $\Delta B = 1$ weak effective Lagrangian is the foundation. Any new physics beyond the standard model (SM) would contribute to the effective Lagrangian through the loop integration at the weak scale and evolution from the weak scale down to the hadronic scale. In this work we present a systematic analysis on the effective Lagrangian which mediates hadronic $|\Delta B| = 1$ processes in the framework of the minimal flavor violation supersymmetry as well as a numerical evaluation of the Wilson coefficients in the effective theory.

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1 Introduction
The forthcoming B-factories will make more precise measurements on the rare B-decay processes and those measurements would set more strict constraints on the new physics beyond the SM. The purpose to investigate B-decays, especially the rare decay modes is to search for traces of new physics and determine its parameter space.

The new physics effects on the rare B processes are intensively discussed in literature. If we believe that the SM is only an effective theory and the supersymmetry is more fundamental, measurements of rare B-processes will definitely enrich our knowledge in this field. But before we can really pin down any new physics effects, we need to carry out a thorough exploration in this area, not only in SM, but also in many plausible models, especially the supersymmetric model.
The calculation of the rate of inclusive decay $B \to X_s \gamma$ is presented by authors of [1, 2, 3] in the two-Higgs doublet model (2HDM). The supersymmetric effect on $B \to X_s \gamma$ is discussed in [4, 5, 6] and the next-to-leading order (NLO) QCD corrections are given in [7]. The transition $b \to s \gamma \gamma$ in the supersymmetric extension of the standard model is computed in [8]. The hadronic $B$ decays and CP-violation in those processes have been discussed also. The authors of [11] have discussed possibility of observing supersymmetric effects in rare decays $B \to X_s \gamma$ and $B \to X_s e^+e^-$ at the B-factory. Studies on decays $B \to (K, K^*) l^+l^-$ in SM and supersymmetric model have been carried out in [12]. The supersymmetric effects on these processes are very interesting and studies on them may shed some light on the general characteristics of the supersymmetric model. A relevant review can be found in [13]. For oscillations of $B_0 - \bar{B}_0$ ($K_0 - \bar{K}_0$), calculations have been done in the SM and 2HDM. As for the supersymmetric extension of SM, the calculation involving the gluino contributions should be re-studied carefully for gluino has a nonzero mass. At the NLO approximation, the QCD corrections to the $B_0 - \bar{B}_0$ mixing in the supersymmetry model have been discussed recently. The authors of [14, 15] applied the mass-insertion method to estimate QCD corrections to the $B_0 - \bar{B}_0$ mixing. The calculations including the gluon-mediated QCD were given in [16], and later we have re-derived the formulation by including the contribution of gluinos.

The supersymmetry effects influence the rare $B$ processes in two ways:

- the Wilson coefficients of the operators existing in the standard model case receive corrections from the supersymmetry sector. As far as the four-quark operators are concerned, the supersymmetric contribution begins at the order of $O(\alpha_s)$ along with the SM QCD corrections.

- when the supersymmetry effects are taken into account, the operator basis is enlarged, new operators emerge.

Generally speaking, theoretical predictions on the inclusive decay rates of $B$-mesons rest on solid grounds due to the fact that these rates can be systematically expanded in powers of $\frac{\Lambda_{QCD}}{m_b}$ [18, 19], where the leading term corresponds to the decay width of free $b$-quark. As the power corrections only start at $O(\frac{\Lambda_{QCD}^2}{m_b})$, they affect these rates by at most a few percent. Theoretically, the non-spectator effects of order $16\pi^2(\frac{\Lambda_{QCD}}{m_b})^3$ could be large [20, 21], especially for the charmless decay modes of $B$-mesons [22, 23, 24]. The main contributions to the lifetimes of $B$-mesons and $\Lambda_b$ are from the $b$-quark decays which are thoroughly studied in the framework of SM.

All the theoretical calculations are based on the weak effective Lagrangian which determines the effective vertices in the concerned Feynman diagrams[24]. The NLO calculation has been carried out in SM [25, 26], but is not complete for the supersymmetric extension. In order to study the supersymmetry effects in those low energy processes, one should obtain a complete effective Lagrangian which includes the supersymmetric contributions at the order $O(\alpha_s)$. Here we consider the supersymmetric model with minimal flavor violation (MFV), i.e. all flavor transitions occur only in the charged-current sector and are determined by the standard Cabibbo-Kobayashi-Maskawa (CKM) mechanism.

In this work, we perform a complete analysis on the $|\Delta B| = 1$ effective Lagrangian, including the current-current operators and penguin-induced operators within the framework of the MFV supersymmetry. After matching between the full MFV supersymmetric theory and the effective Lagrangian, the Wilson coefficients for concerned operators are obtained at the weak scale. Using the recently developed two-loop QCD anomalous dimension matrix of flavor changing four-quark operators [27], we discuss the evolution of the $|\Delta B| = 1$ non-leptonic effective Lagrangian from the weak scale down to the hadronic scale.

The paper is organized as follows. In section 2, we review the minimal flavor violation supersymmetric model and give the notations adopted in our analysis. In section 3, the detailed derivations of the effective Lagrangian at the weak and hadronic scales are made. Then in section 4, we present the numerical results which explicitly demonstrate the difference of the Wilson coefficients in MFV supersymmetry and SM. We discuss
the numerical results and make a short summary in section 5. Some complicated and tedious expressions are collected in the appendices.

2 The supersymmetry with minimal flavor violation

Throughout this paper we adopt the notation similar to Ref. [23]. the expressions of the concerned propagators and vertices can be found in the appendix of Ref.[23]. For convenience, we write down the superpotential which does not violate gauge invariance and the SM conservation laws is

\[ W = \mu \epsilon_{ij} \hat{H}_1^I \hat{H}_j^I + \epsilon_{ij} h_i^I \hat{H}_1^I \hat{L}_j^I \hat{R}_j^I - h_u^I (\hat{H}_1^I \hat{Q}_2^I - \hat{H}_2^I V^{IJ} \hat{Q}_1^J) \hat{D}^I - h_d^I (\hat{H}_2^I V^{*IJ} \hat{Q}_2^J - \hat{H}_3^I \hat{Q}_1^I) \hat{U}^I. \]  

(1)

Here, the weak SU(2) doublets of quark superfields have been written in the form

\[ \left( \begin{array}{c} V^{IJ} \hat{Q}_1^I \\ \hat{Q}_2^I \end{array} \right), \]

where \( I = 1, 2, 3 \) are the indices of generations. The Higgs and lepton doublets are denoted by \( \hat{H}_1, \hat{H}_2 \) and \( \hat{L}_I, \hat{R}_I \), respectively. The rest superfields \( \hat{U}_I, \hat{D}_I \) are quark superfields of \( u \)- and \( d \)-types and charged leptons in singlets of the weak SU(2). Indices \( i, j \) are contracted for the SU(2) group, and \( h_u, h_{u,d} \) are the Yukawa couplings.

In order to break the supersymmetry, the soft breaking terms are introduced as

\[ L_{soft} = -m_{H_1}^2 H_1^* H_1^I - m_{H_2}^2 H_2^* H_2^I - m_{L_1}^2 L_1^* L_1^I \\
- m_{R_1}^2 R_1^* R_1^I - m_{Q_1}^2 Q_1^* Q_1^I - m_{U_1}^2 U_1^* U_1^I \\
- m_{D_1}^2 D_1^* D_1^I + (m_1 \lambda_B \lambda_1 + m_2 \lambda_A \lambda_1^i + m_3 \lambda_A^3 \lambda_1^i + h.c.) + [B \epsilon_{ij} H_1^I H_2^J + \epsilon_{ij} A_1^I h_1^I H_1^I \hat{L}_j^I \hat{R}_j^I \\
- A_2^I h_{u,d}^I (H_1^I \hat{Q}_2^I - H_2^I V^{IJ} \hat{Q}_1^J) \hat{D}^I - A_3^I h_{u,d}^I (H_2^I V^{*IJ} \hat{Q}_2^J - H_3^I \hat{Q}_1^I) \hat{U}^I + h.c.] + \cdots \]  

(2)

where \( m_{H_1}, m_{H_2}, m_{L_1}, m_{R_1}, m_{Q_1}, m_{U_1}, m_{D_1} \) are the parameters of dimension two, while \( m_3, m_2, m_1 \) denote the masses of \( \lambda_3 \) (\( a = 1, 2, \cdots, 8 \)), \( \lambda_A \) (\( i = 1, 2, 3 \)), and \( \lambda_B \), the \( SU(3) \times SU(2) \times U(1) \) gauginos. \( B \) is a free parameter of dimension one. \( A_1^I, A_2^I, A_3^I \) (\( I = 1, 2, 3 \)) are the soft breaking trilinear couplings of scalars. The dots in Eq. [2] stand for flavor-off-diagonal terms (e.g. \( m_{Q_{IK}}^2 Q_1^I (\delta^{KJ} \delta^{KL} - V^{K1} V^{K1}) \)) that are assumed to be negligible. Such terms do occur in our numerical calculation in section 4, but are indeed very small.

Taking into account of the soft breaking terms Eq. [2], we can study the phenomenology within the supersymmetric extension of the standard model with minimal flavor violation (MFV MSSM). Once the favor-off-diagonal soft supersymmetry breaking terms are neglected, the squark mass matrices can be written as \( 2 \times 2 \) matrices for each flavor separately:

\[ m_{UL}^2 = \begin{pmatrix} m_{Q_{UL}}^2 + m_{u_1}^2 + (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) \cos 2\beta m_Z^2 & -m_{u_1} A_{u_1}^I + \mu \cot \beta \\
-m_{u_1} A_{u_1}^I + \mu \cot \beta & m_{u_1}^2 + m_{u_1}^2 + \frac{2}{3} \sin^2 \theta_W \cos 2\beta m_Z^2 \end{pmatrix}, \]

(3)

and

\[ m_{UD}^2 = \begin{pmatrix} m_{Q_{UD}}^2 + m_{d_1}^2 + (\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W) \cos 2\beta m_Z^2 & -m_{d_1} A_{d_1}^I + \mu \tan \beta \\
-m_{d_1} A_{d_1}^I + \mu \tan \beta & m_{d_1}^2 + m_{d_1}^2 - \frac{1}{3} \sin^2 \theta_W \cos 2\beta m_Z^2 \end{pmatrix}, \]

(4)
with \( m_{u_I}, m_{d_I} (I = 1, 2, 3) \) are the masses of the I-th generation quarks.

The SM and the MSSM differ in their Higgs sectors. There are four charged scalars, two of them are physical massive Higgs bosons and other are massless Goldstines. The mixing matrix can be written as:

\[
Z_H = \begin{pmatrix} \sin \beta & - \cos \beta \\ \cos \beta & \sin \beta \end{pmatrix}
\]

with \( \tan \beta = \frac{v_2}{v_1} \) and \( v_1, v_2 \) are the vacuum expectation values of the two Higgs scalars. Another matrix that we will use is the chargino mixing matrix. The supersymmetric partners of the charged Higgs and \( W^\pm \) combine to give two Dirac fermions: \( \chi_1^\pm, \chi_2^\pm \). The two mixing matrices \( Z^\pm \) appearing in the Lagrangian are defined as

\[
(Z^\pm)^T M_c Z^\pm = diag(m_{\chi_1}, m_{\chi_2}),
\]

where

\[
M_c = \begin{pmatrix} 2m_2 & \frac{\sqrt{2}g_2v_2}{\mu} \\ \frac{1}{2}g_2v_1 & \mu \end{pmatrix}
\]

is the mass matrix of charginos with \( g_2 \) denoting the gauge coupling of SU(2). In a similar way, \( Z_{U,D} \) diagonalize the mass matrices of the up- and down-type squarks respectively:

\[
Z_{U,I}^\dagger m_{U,I}^2 Z_{U,I} = diag(m_{\tilde{U}_1}, m_{\tilde{U}_2}),
\]

\[
Z_{D,I}^\dagger m_{D,I}^2 Z_{D,I} = diag(m_{\tilde{D}_1}, m_{\tilde{D}_2}).
\]

With those mixing matrices defined above, we can write the interaction vertices as in Ref.\[28\].

## 3 Matching the coefficients of operators

As in the SM case, we need to obtain the low-energy effective Lagrangian with five quarks, and while deriving it, the heavy supersymmetric degrees of freedom as well as that of SM, including top quark, W-bosons, charged Higgs bosons and the supersymmetric partners of the standard particles are integrated out. In this work we only retain the operators up to dimension six. In this approximation the effective Lagrangian for \( |\Delta B| = 1 \) reads

\[
\mathcal{L}_{\text{eff}} = - \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} \left( C_i^c(\mu) Q_i^c + \tilde{C}_i^c(\mu) \tilde{Q}_i^c \right) + \sum_{j=1}^{5} C_j^p(\mu) Q_j^p
\]

where \( G_F \) is the Fermi coupling constant, \( C_i^c(\mu), \tilde{C}_i^c(\mu), C_j^p(\mu) (i = 1, 2, \cdots, 10; j = 1, \cdots, 5) \) are the Wilson coefficients evaluated at the scale \( \mu \); \( V_{tb} \) and \( V_{ts} \) are the matrix elements of the CKM matrix. Making the effective Lagrangian close under the QCD renormalization, we include the penguin operator \( Q_5^p \) beside those four-quark operators.

As commonly adopted in literature, we classify the operators as the current-current operators which are originally induced by the tree level W-exchange interaction and one-loop 'box' diagrams, and the "penguin-"induced operators. Both of them would undergo QCD corrections and receive contributions from supersymmetric particles via loops.
The current-current operators are written as\textsuperscript{[27]}

\begin{align*}
Q_1^c &= \left( \bar{s}_\alpha \gamma_\mu \omega_- c_\beta \right) \left( \bar{c}_\beta \gamma^\mu \omega_- b_\alpha \right), \\
Q_2^c &= \left( \bar{s}_\alpha \gamma_\mu \omega_- c_\beta \right) \left( \bar{c}_\beta \gamma^\mu \omega_- b_\alpha \right), \\
Q_3^c &= \left( \bar{s}_\alpha \omega_- c_\beta \right) \left( \bar{c}_\beta \omega_- b_\alpha \right), \\
Q_4^c &= \left( \bar{s}_\alpha \omega_- c_\beta \right) \left( \bar{c}_\beta \omega_- b_\alpha \right), \\
Q_5^c &= \left( \bar{s}_\alpha \sigma_{\mu\nu} \omega_- c_\beta \right) \left( \bar{c}_\beta \sigma^{\mu\nu} \omega_- b_\alpha \right), \\
Q_6^c &= \left( \bar{s}_\alpha \sigma_{\mu\nu} \omega_- c_\beta \right) \left( \bar{c}_\beta \sigma^{\mu\nu} \omega_- b_\alpha \right), \\
Q_7^c &= \left( \bar{s}_\alpha \gamma_\mu \omega_- c_\beta \right) \left( \bar{c}_\beta \gamma^\mu \omega_- b_\alpha \right), \\
Q_8^c &= \left( \bar{s}_\alpha \gamma_\mu \omega_- c_\beta \right) \left( \bar{c}_\beta \gamma^\mu \omega_- b_\alpha \right), \\
Q_9^c &= \left( \bar{s}_\alpha \omega_- c_\beta \right) \left( \bar{c}_\beta \omega_- b_\alpha \right), \\
Q_{10}^c &= \left( \bar{s}_\alpha \omega_- c_\beta \right) \left( \bar{c}_\beta \omega_- b_\alpha \right),
\end{align*}

where \( \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \) and \( \omega_\pm = \frac{1 \pm i}{2} \omega_T \). In the standard model, there are only two such operators i.e. \( Q_1^c, Q_2^c \). 18 new operators are induced when supersymmetry takes part in the game. There are other ten current-current operators which are simply obtained by interchanging \( \omega_- \leftrightarrow \omega_T \) in \( Q_1^c \), i.e. \( \tilde{Q}_1^c = Q_1^c(\omega_- \leftrightarrow \omega_T) \). Due to the small CKM entries for the u-analog operators in the \(|\Delta B| = 1\) effective Lagrangian, \( V_{us}^* V_{ub} \ll V_{cb}^* V_{ub} \) the u-quark analogs of the effective Lagrangian in Eq.\textsuperscript{8} can be neglected. The basis of the penguin-induced operators consists of \textsuperscript{[29, 30, 31, 32, 33]}

\begin{align*}
Q_1^p &= \left( \bar{s}_\alpha \gamma_\mu \omega_- b_\alpha \right) \sum_q \left( \bar{q}_\beta \gamma^\mu \omega_- q_\beta \right), \\
Q_2^p &= \left( \bar{s}_\alpha \gamma_\mu \omega_- b_\alpha \right) \sum_q \left( \bar{q}_\beta \gamma^\mu \omega_- q_\alpha \right), \\
Q_3^p &= \left( \bar{s}_\alpha \gamma_\mu \omega_- b_\alpha \right) \sum_q \left( \bar{q}_\beta \gamma^\mu \omega_- q_\beta \right), \\
Q_4^p &= \left( \bar{s}_\alpha \gamma_\mu \omega_- b_\alpha \right) \sum_q \left( \bar{q}_\beta \gamma^\mu \omega_- q_\alpha \right), \\
Q_5^p &= \frac{1}{(4\pi)^2} s g_s G \cdot \sigma (m_s \omega_- + m_b \omega_+) b,
\end{align*}

with \( q = u, d, c, s, b, G_{\mu\nu} \equiv G_{\mu\nu}^a T^a \) denotes the gluon field strength tensor, \( G_{\mu\nu}^a = \partial_\mu G_{\nu}^a - \partial_\nu G_{\mu}^a + g_s f^{abc} G_{\mu}^b G_{\nu}^c \), and \( G \cdot \sigma \equiv G_{\mu\nu} \sigma^{\mu\nu} \).

In the following sections, we will derive the Wilson coefficients for the current-current operators in Eq.\textsuperscript{9} and penguin-induced operators in Eq.\textsuperscript{10}. 
3.1 The difference of quark field normalization in the full and effective theories while taking the $\overline{\text{MS}}$ scheme

In order to systematically investigate all corrections of the supersymmetry QCD to the vertex $\bar{d}uW$ including the self-energies of external quark legs, Ciuchini et. al used the on-shell scheme to subtract the divergence of the quark fields and the $\bar{d}uW$ vertex that originates from supersymmetric partners, whereas the divergence originating from the SM sector is still subtracted out in the $\overline{\text{MS}}$-scheme [7]. In this work, we will employ the $\overline{\text{MS}}$ scheme throughout and show that the results are qualitatively consistent with theirs.

It is noted that there is a difference of the normalization of the quark fields in the full supersymmetric theory and the effective theory while taking the $\overline{\text{MS}}$ scheme. As a matter of fact, for the self-energy which is in the vertex contributions and this normalization difference would cancel each other exactly and then the vertices, but also include this normalization difference. We find that the large logarithms which exist in effective theories. Namely, when we match the Lagrangians, we not only consider the contributions from the quark fields and the $\bar{q}q$ vertex that originates from supersymmetric partners, whereas the divergence originating from the SM sector is still subtracted out in the $\overline{\text{MS}}$-scheme [7]. In this work, we will employ the $\overline{\text{MS}}$ scheme throughout and show that the results are qualitatively consistent with theirs.

It is is understood that in both the full and effective theories, the SM quarks exist, but the supersymmetry particles (squarks and gluinos) only exist in the full theory, therefore there is not normalization ambiguity in the SM case. When the supersymmetry sector is included, there exists an extra term from the supersymmetry sector which cannot simply match the vertex-induced Lagrangian in the full and effective theories because the external quark fields have different normalizations. That is understood that in both the full and effective theories, the SM quarks exist, but the supersymmetry particles (squarks and gluinos) only exist in the full theory, but are integrated out to produce the effective Lagrangian. The difference $\Delta Z^{\text{full}} - \Delta Z^{\text{eff}} = \Delta Z^{\text{full}}$ is a finite renormalization effect which should be included when we match the Lagrangian in the full and effective theories. Namely, when we match the Lagrangians, we not only consider the contributions from the vertices, but also include this normalization difference. We find that the large logarithms which exist in the vertex contributions and this normalization difference would cancel each other exactly and then the decoupling theorem is obvious.

The difference manifests as a finite renormalization contribution to the $\Delta B = 1$ effective Lagrangian, which will be expressed explicitly in the following computation.

The supersymmetric contributions to the self-energy are

\[
\frac{1}{1 + \Delta Z_{\text{SM}}^{\text{full}} + \Delta Z_{\text{MSSM}}^{\text{full}}} \neq \frac{1}{1 + \Delta Z_{\text{SM}}^{\text{eff}}},
\]

where $\Delta Z_{\text{SM}}^{\text{full}} = \Delta Z_{\text{eff}}^{\text{full}}$ and the superscript ”eff” denotes the quantities in the effective theory. Thus one cannot simply match the vertex-induced Lagrangian in the full and effective theories because the external quark fields have different normalizations. That is understood that in both the full and effective theories, the SM quarks exist, but the supersymmetry particles (squarks and gluinos) only exist in the full theory, but are integrated out to produce the effective Lagrangian. The difference $\Delta Z_{\text{SM}}^{\text{full}} - \Delta Z_{\text{eff}}^{\text{full}} = \Delta Z_{\text{SM}}^{\text{full}}$ is a finite renormalization effect which should be included when we match the Lagrangian in the full and effective theories. Namely, when we match the Lagrangians, we not only consider the contributions from the vertices, but also include this normalization difference. We find that the large logarithms which exist in the vertex contributions and this normalization difference would cancel each other exactly and then the decoupling theorem is obvious.

The difference manifests as a finite renormalization contribution to the $\Delta B = 1$ effective Lagrangian, which will be expressed explicitly in the following computation.

The supersymmetric contributions to the self-energy are

\[
i \Sigma_{u}^{\text{susy}} (p) = -\frac{\alpha_s}{4\pi} C_F \left\{ \left( \Delta + \ln \frac{3}{2} + \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} + \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} - \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} \right) \left( \frac{Z_{\ell i}^{\text{1s}}}{Z_{\ell i}^{\text{1s}}} + \frac{Z_{\ell i}^{\text{2s}}}{Z_{\ell i}^{\text{2s}}} \right) \frac{Z_{\ell i}^{\text{1s}}}{Z_{\ell i}^{\text{1s}}} \frac{Z_{\ell i}^{\text{2s}}}{Z_{\ell i}^{\text{2s}}} \right\},
\]

\[
i \Sigma_{d}^{\text{susy}} (p) = -\frac{\alpha_s}{4\pi} C_F \left\{ \left( \Delta + \ln \frac{3}{2} + \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} + \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} - \frac{x_{\ell i}^{\ell i}}{x_{\ell i}^{\ell i} - x_{\ell i}^{\ell i}} \right) \left( \frac{Z_{\ell i}^{\text{1s}}}{Z_{\ell i}^{\text{1s}}} + \frac{Z_{\ell i}^{\text{2s}}}{Z_{\ell i}^{\text{2s}}} \right) \frac{Z_{\ell i}^{\text{1s}}}{Z_{\ell i}^{\text{1s}}} \frac{Z_{\ell i}^{\text{2s}}}{Z_{\ell i}^{\text{2s}}} \right\},
\]

where $x_{\ell i} = \frac{m_{\ell i}}{m_{\ell i}}$, $x_i = \frac{m_i^2}{m_N^2}$. In the expressions the first term is the correction to the wave function of the quarks whereas the second term corresponds to the supersymmetric contributions to the quark masses. In
Eq. [14], if we complete the renormalization for the quark fields in the $\overline{\text{MS}}$-scheme, the divergent part in the renormalization multiplier which should be subtracted is

$$\delta Z_{q}^+(\overline{\text{MS}}) = \frac{\alpha_s}{4\pi} C_F \Delta .$$

When matching the full and effective theories in $\overline{\text{MS}}$ scheme, there is a difference of the renormalized quark fields, so that we need to derive the extra contribution.

In literature [7], the on-mass-shell renormalization [44] is adopted, by which after renormalization the residue of the self-energy should be one at the physical mass, so that the normalization ambiguity does not exist. However, in the $\overline{\text{MS}}$ scheme, there is no such a requirement. From Eq. [11], we have the difference of the normalization of the quark fields in the full supersymmetric theory and the effective theory while taking the normalization difference into account, we obtain the Wilson coefficients of the current-current operators in the full and effective theories.

When matching the full and effective theories in $\overline{\text{MS}}$ scheme, we not only consider the vertex-induced contributions, but also need to include this finite renormalization contribution. Thus taking this normalization difference into account, we obtain the Wilson coefficients of the current-current operators in the full and effective theories.

**3.2 The Wilson coefficients of current-current operators**

The Wilson coefficients $C_i^c$, $\tilde{C}_i^c$ ($i = 1, 2, \cdots, 10$) in Eq. [8] can be determined by the requirement that the amplitude $A_{full}$ in the full theory is equal to the corresponding amplitude in the effective theory at the weak scale

$$A_{full} = A_{eff} = \frac{4G_F}{\sqrt{2}} V_{us}^* V_{cb} \sum_{i=1}^{10} \left\{ C_i^c Q_i^c + \tilde{C}_i^c \tilde{Q}_i^c - \left[ \Delta Z_c^- + \frac{1}{2} \Delta Z_b^- + \frac{1}{2} \Delta Z_s^- \right] Q_i^c \right\} .$$

The QCD induced one-loop Feynman diagrams responsible for $\Delta B = 1$ effective Lagrangian in the minimal flavor violation supersymmetric theory and effective theory are drawn in Fig. 3 and Fig. 4 respectively. The last term of Eq. [11] originates from the difference of quark field normalization in the full and effective theories. Considering the QCD corrections of current-current operators, we can extract the Wilson coefficients $C_i^c$, $\tilde{C}_i^c$ at the $\mu_W$ scale. The expressions for those coefficients are presented in appendix A and one can notice that all resultant $C_i^c$ and $\tilde{C}_i^c$ are free of infrared divergence. The first terms of $C_i^c(\mu_W)$ and $\tilde{C}_i^c(\mu_W)$ are the SM contribution whereas the other terms are due to the supersymmetry contributions. Other non-zero Wilson coefficients $C_i^c(\mu_W)$, $\tilde{C}_i^c(\mu_W)$ ($i = 3, 4, \ldots, 10, j = 1, 2, \ldots, 10$) all originate from contributions of scalar quarks and gluino. Provided $m_{\tilde{g}} = m_{\tilde{Q}} = m_{\text{SUSY}}$, we have

$$C_2^c(\mu_W) \sim C_{2_{\text{SM}}}^c(\mu_W) = 1 - \frac{\alpha_s}{4\pi} (\ln x_\mu + \frac{11}{6}),$$

$$C_1^c(\mu_W) \sim C_{1_{\text{SM}}}^c(\mu_W) = \frac{3\alpha_s}{4\pi} (\ln x_\mu + \frac{11}{6}),$$

as $m_{\text{SUSY}} \gg \mu_W$. 

$$\Delta Z_{a_i}^- = \frac{\alpha_s}{4\pi} C_F \left( \ln x_\mu + \frac{3}{2} + \sum_i Z_{U_i}^i Z_{D_i}^i \left( \frac{x_{\ell_i}^I}{x_{\ell_i}^H} \ln x_{\ell_i}^I - \frac{x_{\ell_i}^H \ln x_{\ell_i}^H - x_{\ell_i}^I \ln x_{\ell_i}^I}{(x_{\ell_i}^H - x_{\ell_i}^I)^2} \right) \right),$$

$$\Delta Z_{d_i}^- = \frac{\alpha_s}{4\pi} C_F \left( \ln x_\mu + \frac{3}{2} \sum_i Z_{U_i}^i Z_{D_i}^i \left( \frac{x_{\ell_i}^I}{x_{\ell_i}^H} \ln x_{\ell_i}^I - \frac{x_{\ell_i}^H \ln x_{\ell_i}^H - x_{\ell_i}^I \ln x_{\ell_i}^I}{(x_{\ell_i}^H - x_{\ell_i}^I)^2} \right) \right),$$

(13)
which recovers the SM result. When \( m_{\tilde{g}} \to \infty \), but \( m_{\tilde{\chi}} = m_{\tilde{\chi}^i} \) \((I = 1, 2, 3; i = 1, 2)\) and \( m_{\tilde{D}} = m_{\tilde{D}^I} \) \((I = 1, 2, 3; i = 1, 2)\) remain finite, we also have \( C_{i,2}^\mu(\mu_W) \sim C_{i,2}^{\mu,\text{SM}}(\mu_W) \). From the equations given in the appendix, one observes that in \( C_{i}^\mu(\mu_W) \) if all the mass parameters of the supersymmetric particles tend to infinity or they are the same, i.e. the supersymmetric particles are degenerate in mass (it is exactly the mSUGRA case) all the \( x \)-value related terms cancel each other. Here we take into account the fact \( \mathcal{Z}_{\tilde{U}1,2} \sim \mathcal{Z}_{\tilde{D}1,2,3} \sim \mathbf{I} \) in the mSUGRA scenario. In other coefficients \( C_{i}^\mu \), \( C_{i}^\mu \) etc. the logarithm-related terms are automatically suppressed to zero when the supersymmetric particles are very heavy (see appendix A for details). Those results indicate the decoupling of the supersymmetric sector as the supersymmetric partners turn to be very heavy.

In order to give a complete \( |\Delta B| = 1 \) non-leptonic effective Lagrangian with five quarks, we should include the contributions of penguin diagrams. In our present work, we only consider the gluon-penguin.

### 3.3 The Wilson coefficients of penguin-induced operators

In this section, we derive the Wilson coefficients for the penguin-induced dimension six operators. The basis for the penguin-induced operators is given in Eq.\([\text{III}]\). The one-loop Feynman diagrams for the penguin-induced operators are drawn in Fig.\([\text{IV}]\). The obtained coefficients \( C_5^\mu(\mu_W) \) read

\[
\begin{align*}
C_1^\mu(\mu_W) &= \frac{\alpha_s}{4\pi} \left[ -\frac{1}{9} \ln x_\mu - \frac{1}{6} E(x_t, x_H, x_{\tilde{\chi}^i}, x_{\chi}) + \frac{1}{9} \right], \\
C_2^\mu(\mu_W) &= \frac{\alpha_s}{4\pi} \left[ \frac{1}{3} \ln x_\mu + \frac{1}{2} E(x_t, x_H, x_{\tilde{\chi}^i}, x_{\chi}) - \frac{1}{3} \right], \\
C_3^\mu(\mu_W) &= \frac{\alpha_s}{4\pi} \left[ -\frac{1}{9} \ln x_\mu - \frac{1}{6} E(x_t, x_H, x_{\tilde{\chi}^i}, x_{\chi}) + \frac{1}{9} \right], \\
C_4^\mu(\mu_W) &= \frac{\alpha_s}{4\pi} \left[ \frac{1}{3} \ln x_\mu + \frac{1}{2} E(x_t, x_H, x_{\tilde{\chi}^i}, x_{\chi}) - \frac{1}{3} \right],
\end{align*}
\]

where

\[
E(x_t, x_H, x_{\tilde{\chi}^i}, x_{\chi}) = \left[ \frac{18 x_t - 11 x_H^2 - x_{\tilde{\chi}^i}^3}{12 (1 - x_t)^3} + \frac{(-4 + 16 x_t - 9 x_{\tilde{\chi}^i}^2) \ln x_t}{6 (1 - x_t)^4} \right] + \frac{1}{\tan^2 \beta} \left[ \frac{(2 x_t x_H^2 - 3 x^2_{\tilde{\chi}^i}^2 x_H^2)(\ln x_t - \ln x_H)}{6 (x_H - x_t)^4} + \frac{16 x_t x_H^2 - 29 x_{\tilde{\chi}^i}^2 x_H + 7 x_{\chi}^2}{36 (x_H - x_t)^3} \right] + \sum_{ij} |A_{ij}|^2 \left[ \frac{x_{\tilde{\chi}^i}^2 (\ln x_{\tilde{\chi}^i} - \ln x_{\chi})}{6 (x_{\chi} - x_{\tilde{\chi}^i})^4} + \frac{11 x_{\chi}^2 - 7 x_{\tilde{\chi}^i} x_H}{18 (x_{\chi} - x_{\tilde{\chi}^i})^3} \right].
\]

The Wilson coefficient of \( Q_5^\mu \) is

\[
\begin{align*}
C_5^\mu(\mu_W) &= x_t \left[ \frac{3 x_t \ln x_t}{4 (1 - x_t)^4} + \frac{2 + 5 x_t - x_{\tilde{\chi}^i}^2}{8 (1 - x_t)^3} \right] + \left\{ \frac{x_{\tilde{\chi}^i}^2 (\ln x_t - \ln x_H)}{2 (x_H - x_t)^3} + \frac{3 x_{\tilde{\chi}^i} x_t - x_{\tilde{\chi}^i}^2}{4 (x_H - x_t)^2} \right\} \\
&\quad + \frac{1}{\tan^2 \beta} \left[ -\frac{x_{\tilde{\chi}^i}^2 x_H^2 (\ln x_t - \ln x_H)}{4 (x_H - x_t)^4} - \frac{2 x_t x_{\tilde{\chi}^i}^2 - 5 x_{\tilde{\chi}^i} x_H - x_{\chi}^2}{24 (x_H - x_t)^3} \right] \\
&\quad + \left\{ \sum_{ij} |A_{ij}|^2 \left[ \frac{x_{\tilde{\chi}^i}^2 x_{\tilde{\chi}^j} (\ln x_{\tilde{\chi}^j} - \ln x_{\chi})}{2 (x_{\chi} - x_{\tilde{\chi}^i})^4} + \frac{2 x_{\chi}^2 + 5 x_{\chi} x_{\tilde{\chi}^i} - x_{\tilde{\chi}^i}^2}{12 (x_{\chi} - x_{\tilde{\chi}^i})^3} \right] \right\}
\end{align*}
\]
and the RGEs for the Wilson coefficients are terms in the perturbative expansion of the amplitudes are sufficient when the renormalization scale is close to \( \mu \approx \infty \). In the above expression, we only keep the contribution of the up-type scalar quarks of the third generation, and that from other squarks are ignored for their large masses. The matrix \( A, B \) are written as

\[
A^{ij} = -Z^{1i}_{\widetilde U^3} Z^{+s}_{1j} + \frac{m_t}{\sqrt{2} m_W \sin \beta} Z^{2i}_{\widetilde U^3} Z^{+s}_{2j},
\]

\[
B^{ij} = \left[ Z^{1i}_{\widetilde U^3} Z^{-s}_{1j} + \frac{m_t}{\sqrt{2} m_W \cos \beta} Z^{2i}_{\widetilde U^3} Z^{-s}_{2j} \right],
\]

with \( Z_{\widetilde U^3}, Z^\pm \) are the mixing matrices of scalar top quarks and charginos respectively. Because we are working in the framework of supersymmetric extension of SM, the effective vertex \( b \to sg \) must take in the contribution of supersymmetric particles, thus the penguin-induced Lagrangian is somewhat different from that within SM. It is noticed that the \( m_H \) related terms i.e. the last two terms in Eq.17 and Eq.18 tend to zero when \( m_H \to \infty \) and as well as \( m_{\text{SUSY}} \to \infty \).

When we only keep the Yukawa coupling of the top-quark to the Higgs boson and assuming that the masses of the scalar quarks except top scalar quarks are highly degenerate, and the weak eigenstates are the eigenstates of the masses, i.e. the Wilson coefficients of other operators vanish. This is exactly the operator basis existing in the standard model. In the \( \overline{\text{MS}} \)-scheme, the Wilson coefficients are simplified as

\[
C^c_\ell(\mu_W) = 3 \frac{\alpha_s}{4\pi} \left( \ln x_\mu + \frac{11}{6} \right) - 2 \frac{\alpha_s}{4\pi} \sum_k Z^{-1k}_{\widetilde U^3} Z^{+k}_{1k} \left[ \frac{x^2_{\bar{g}} \ln x_{\bar{g}}}{(x_{\bar{g}} - x_{\bar{g}})(x_{\bar{g}} - x_{\bar{g}})} + \frac{x^2_{\bar{g}} \ln x_{\bar{g}}}{(x_{\bar{g}} - x_{\bar{g}})(x_{\bar{g}} - x_{\bar{g}})} \right],
\]

\[
C^c_\ell(\mu_W) = \left( 1 - \frac{\alpha_s}{4\pi} \left( \ln x_\mu + \frac{11}{6} \right) \right) - \frac{\alpha_s}{2\pi} \left( \frac{x_{\bar{g}} - x_{\bar{g}} \ln x_{\bar{g}}}{x_{\bar{g}} - x_{\bar{g}}} + \frac{(x_{\bar{g}} - x_{\bar{g}})^2 \ln x_{\bar{g}}}{(x_{\bar{g}} - x_{\bar{g}})^2} \right),
\]

the Wilson coefficients of the penguin-induced operators remain unchanged and the other Wilson coefficients vanish.

### 3.4 The evolution of the Wilson coefficients

Many flavor-changing processes occur at the hadronic scale with \( \mu_{\text{hadron}} \ll \mu_W \). Generally, first a few terms in the perturbative expansion of the amplitudes are sufficient when the renormalization scale \( \mu \) is close to \( \mu_{\text{hadron}} \) rather than to \( \mu_W \). The Wilson coefficients \( C_1^c, \bar{C}_1^c, C_1^p, \bar{C}_1^p \) at \( \mu_{\text{hadron}} \) are obtained from \( C_1^c(\mu_W), \bar{C}_1^c(\mu_W), C_1^p(\mu_W) \) with help of the Renormalization Group Equations (RGEs) evolution. If we define a \( 1 \times 24 \) matrix as

\[
\bar{C} = \left( C_1^c, C_2^c, \ldots, C_{10}^c, \bar{C}_1^c, \bar{C}_2^c, \ldots, \bar{C}_{10}^c, C_1^p, C_2^p, \bar{C}_3^p, \bar{C}_4^p \right),
\]

and the RGEs for the Wilson coefficients are

\[
\mu \frac{d}{d\mu} \bar{C}(\mu) = \bar{C}(\mu) \bar{\gamma}(\mu),
\]
Here, $\hat{\gamma}$ is the anomalous dimension matrix which has the following form in the perturbative expansion

$$\hat{\gamma}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \hat{\gamma}^{(0)} + \left( \frac{\alpha_s(\mu)}{4\pi} \right)^2 \hat{\gamma}^{(1)}.$$  

(23)

Adopting the Naive Dimensional Regularization—MS (NDR-MS) scheme, Buras et al. have given the anomalous dimension matrix up to two-loop order in the basis of Eq. [4] and Eq.[10][27, 39].

With the RGEs (22) and the Wilson coefficients at the weak scale as the initial condition, we can derive the effective Lagrangian of five quarks at the hadronic scale.

4 Numerical results

Indeed, there are too many free parameters in the minimal supersymmetric extension of SM (MSSM). In order to reduce the number of free parameters, we assume that the MSSM is a low-energy effective theory of a more fundamental theory which exists at a higher scale, such as the grand unification scale or the Planck scale. A realization of this idea is the minimal supergravity (mSUGRA), which is fully specified by five parameters [40]

$$m_0, m_\perp, A_0, \tan \beta, \text{sgn}(\mu).$$

Here $m_0$, $m_\perp$ and $A_0$ are the universal scalar quark mass, gaugino mass and trilinear scalar coupling. They are assumed to arise through supersymmetry breaking in a hidden-sector at the GUT scale $\mu_GUT \simeq 2 \times 10^{16}$ GeV. In our numerical calculation, to maintain consistency of the theory and the up-to-date experimental observation, when we obtain the numerical value of the Higgs mass in the mSUGRA model with the five parameters, we include all one-loop effects in the Higgs potential [11]. Moreover we also employ the two-loop RGEs [2] with one-loop threshold corrections [11, 43] as the energy scale runs down from the mSUGRA scale to the lower weak scale. In the framework of minimal supergravity, the unification assumptions are expressed as

$$A_t^f = A_t^f = A_t^u = A_0, \quad (24)$$

$$B = A_0 - 1,$$

$$m_{H1}^2 = m_{H2}^2 = m_{Li}^2 = m_{Ri}^2 = m_{Qi}^2 = m_{\tilde{U}_i}^2 = m_{\tilde{D}_i}^2 = m_0^2,$$

$$m_1 = m_2 = m_3 = m_\perp.$$

For the SM parameters, we have $m_b = 5$ GeV, $m_t = 174$ GeV, $m_W = 80.23$ GeV, $\alpha_e(m_W) = \frac{1}{128}$, $\alpha_s(m_W) = 0.12$ at the weak scale. Taking above values, we find the SM prediction for Wilson Coefficients as $C^c_{iSM}(m_b) = -0.295, \ C^c_{2SM}(m_b) = 1.110, \ C^p_{1SM}(m_b) = 0.014$. In our numerical calculations of the supersymmetry corrections to those Wilson coefficients $C^c_i(m_b)$, $C^p_i(m_b)$, $\forall i = 1, ..., 10$, and their SM prediction values versus the parameter $m_\perp$ with $m_0 = 200$ GeV and $\tan \beta = 2$ or 20. The dependence of those ratios on $m_\perp$ ($m_\perp = 300$ GeV and $\tan \beta = 2$ or 20) is plotted in Fig.5.

5 Discussions and Conclusion

The $|\Delta B| = 1$ non-leptonic effective Lagrangian has been considered in the minimal flavor violating supersymmetry scenario. The supersymmetry contributions affect the effective Lagrangian via two aspects:
• new current-current operators emerge beside those 'old' operators in the SM case;

• for the 'old' operators, the Wilson coefficients at the weak scale are modified by the supersymmetric contributions.

Now let us briefly discuss our observation of the numerical results.

In Fig.4 (a) and (b), the two lines (solid and dash) corresponding to \( \tan \beta = 2 \) and \( \tan \beta = 20 \) differ from each other more obviously. The supersymmetry corrections to the Wilson coefficients \( C_{1,2}(m_b) \) are relatively large when the supersymmetry particles have masses of the same order of electroweak energy scale, for example, \( m_\chi = 300 \text{GeV}, \ m_0 = 200 \text{GeV} \), the supersymmetry corrections to \( C_{1,2}(m_b) \) can reach about 8%. When the masses of the supersymmetric particles become very large, the supersymmetry corrections turn to zero due to the decoupling theorem. In Fig.4(c), the two lines corresponding to \( \tan \beta = 2 \) and \( \tan \beta = 20 \) almost overlap on each other. It is noted that at the left part of Fig.5 (c) as \( m_\chi < 2 \text{ TeV} \), there is a sharp peak at the dependence of \( C_{1}^p(m_b) \) on \( m_\chi \). It drops very fast as \( m_\chi \) is away from this region, the resonance is due to an almost degeneracy of the mass of chargino and the mass of stop and it leads to an obvious deviation of the \( C_{1}^p(m_b) \) value from the prediction of SM (0.014). As \( m_\chi \) further increases, \( C_{1}^p \) tends to the predictive value of SM. This mass resonance only occurs for the coefficients of the penguin-induced operators, but not for the current-current-quark operators, so that the dependence of \( C_{1}^p(m_b), C_2^p(m_b) \) on \( m_\chi \) does not change drastically. All these are consistent with the common sense which is familiar to us even before the calculations are done.

In Fig.5, we plot the dependence of \( C_1^p(m_b), C_2^p(m_b) \) and \( C_1^b(m_b) \) on \( m_0 \), with \( A_0 = 0, \ sgn(\mu) = +1, \ m_\chi = 200 \text{ GeV} \). Similar to the case of Fig.4, the dependence of supersymmetry corrections to \( C_{1,2}(m_b) \) on \( m_0 \) is remarkable and the values obviously deviate from the SM prediction when \( m_0 \) take smaller values. When \( m_0 \sim 200 \text{ GeV}, \ C_{1,2}(m_b) \) deviate from the SM prediction by 8%, and when \( m_0 > 5 \text{ TeV} \), the deviation tends to zero. The dependence of \( C_1^p(m_b) \) on \( m_0 \) is in analog to its dependence on \( m_\chi \). When \( m_0 < 2 \text{ TeV} \), there is an obvious peak which damps steeply, the reason is still due to the mass degeneracy of heavy chargino and stop. When \( m_0 \) turns very large, \( C_1^p(m_b) \) approaches the SM prediction. By contrast, the dependence of \( C_1^p(m_b), C_2^p(m_b) \) on \( m_0 \) is more smooth, because there is no mass resonance effect in this situation.

In above discussions, we only list the Wilson coefficients of a few typical operators, the Wilson coefficients of other operators are in analogy. For simplicity, in all the numerical calculations, we always adopt \( A_0 = 0, \ sgn(\mu) = +1 \), but this convention is not necessary. If we dismiss this assumption and let \( A_0 \neq 0, \ sgn(\mu) = \pm 1 \), the parameter space is enlarged, but the qualitative conclusion remains the same.

Here we briefly discuss the decoupling of the supersymmetry particles as the concerned energy scale turns to infinity. When we match the effective theory to the full theory which is the standard model at the weak scale, the quark and gluon fields exist in both theories, thus the renormalized quark fields are the same in both theories. However, as the full theory is the supersymmetric extension of SM, scalar quarks and gluinos exist only in the full theory, but do not in the effective one. Thus when we match the two theories at a certain scale, disappearance of such supersymmetric partners in the effective theory would result in a difference of the normalization of the quark fields in the full and effective theories. Actually, this is a finite renormalization contribution of the self-energy. Ignoring this effect would lead to a situation that heavy supersymmetry particles do not decouple. This normalization difference would also affect evaluation of the vertex-induced contribution. When we match the full and effective theories we need to take this normalization difference into account, then the large logarithms emerging from the self-energy exactly cancel out that from the vertices, thus the decoupling is obvious. On other side, only in the \( \overline{\text{MS}} \) scheme, there exists the difference of the normalization of the renormalized quark fields, but it does not exist in the on-mass shell renormalization scheme. The reason is that requiring the external quarks to be on their physical mass shell and the residue for the self-energy to be one can serve as an additional condition by which the normalization of the quark fields are the same in the full and effective theories. A direct consequence of the correct renormalization
scheme is the decoupling of the supersymmetry sector as the supersymmetry particles are too heavy. If we include the normalization difference as we take the \( \overline{\text{MS}} \) scheme, the result coincides with that in the on-shell renormalization.

Because we have carefully considered the normalization difference, in our final expressions, one can immediately observe that the supersymmetry particles would decouple if their mass scale turns to infinity. That is consistent with the common sense.

From our discussion and numerical results, one can note that in general, the correction of supersymmetry to the Wilson coefficients can be as large as 8% when the supersymmetry particles are not very heavy. It is also noted that the \( \Delta B \) to the Wilson coefficients can be as large as 8% when the supersymmetry particles are not very heavy. It is consistent with the common sense.

With collection of large amount of data at the B-factory and elsewhere main laboratories in the world, the measurements on the rare processes \( b \rightarrow s \gamma \), \( b \rightarrow s \bar{u} u \) etc. would set more rigorous constraint on the parameter space of the supersymmetric model or we can, as expected, find some evidence of existence of supersymmetric particles.

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### A The Wilson coefficients of current-current operators at the weak scale

After matching at the weak scale, the Wilson coefficients for the current-current operators are written as

\[
C_{1}^{c}(\mu_{W}) = 3 \frac{\alpha_{s}}{4 \pi} \left( \ln x_{\mu} + \frac{11}{6} \right) - \frac{\alpha_{s}}{4 \pi} \sum_{\alpha = \tilde{g}, \tilde{\chi}_{k}^{\pm}} \left( \sum_{\beta \neq \alpha} \frac{x_{\alpha}^{2} \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_{1}}^{l_{j}^*} Z_{D_{1}}^{l_{i}} A_{i - k}^{-\alpha} A^{-j k} \right)
\]

\[
+ \sum_{\alpha = \tilde{g}, \tilde{\chi}_{k}^{\pm}} \frac{x_{\alpha}^{2} \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{U_{2}}^{l_{j}^*} Z_{U_{2}}^{l_{i}} B^{-i k} B^{-j k} \right],
\]

\[
C_{2}^{c}(\mu_{W}) = \left( 1 - \frac{\alpha_{s}}{4 \pi} \left( \ln x_{\mu} + \frac{11}{6} \right) \right) - \frac{\alpha_{s}}{8 \pi} \left\{ 2 \sum_{i} Z_{U_{2}}^{l_{i}} Z_{U_{2}}^{l_{i}^*} \left( \frac{x_{i}^{2}}{x_{g} - x_{i}^{2} x_{j}^{2}} + \frac{x_{i}^{2} \ln x_{i}^{2}}{x_{g} - x_{i}^{2}} \right) \right\} - \frac{(2 x_{g} x_{\beta_{i}^{2}} - x_{\beta_{i}^{2}}^{2}) \ln x_{g} - x_{g} x_{\beta_{i}^{2}} \ln x_{\beta_{i}^{2}}}{(x_{g} - x_{\beta_{i}^{2}}^{2})^{2}} + \sum_{i} Z_{D_{2}}^{l_{i}} Z_{D_{2}}^{l_{i}^*} \left( \frac{x_{i}^{2}}{x_{g} - x_{i}^{2}} + \frac{x_{i}^{2} \ln x_{i}^{2}}{x_{g} - x_{i}^{2}} \right)
\]

\[
- \frac{2 x_{g} x_{\beta_{i}^{2}}^{2} \ln x_{g} - x_{g} x_{\beta_{i}^{2}} \ln x_{\beta_{i}^{2}}}{(x_{g} - x_{\beta_{i}^{2}}^{2})^{2}} + 2 \sum_{i} Z_{D_{2}}^{l_{i}} Z_{D_{2}}^{l_{i}^*} Z_{U_{2}}^{l_{i}^*} Z_{U_{2}}^{l_{i}} \left( \frac{x_{i}^{2} \ln x_{i}^{2}}{x_{g} - x_{i}^{2}} + \frac{x_{i}^{2} \ln x_{i}^{2}}{x_{g} - x_{i}^{2}} \right) \right).\]
\[
\frac{x_{\ell}^2 \ln x_{\ell}}{(x_{g} - x_{\ell})(x_{\ell}^2 - x_{g}^2)} + \sum_i Z_{D_1}^{l_1} Z_{D_3}^{l_3} \left( \frac{x_{D_1}^3}{x_{g} - x_{D_1}^3} + \frac{x_{D_3}^3 \ln x_{D_3}^3}{x_{g} - x_{D_3}^3} \right) - \frac{(2x_{g} x_{\ell}^3 - x_{\ell}^2 x_{D_1}^3) \ln x_{g} - x_{g} x_{\ell} \ln x_{\ell}}{(x_{g} - x_{\ell})^2} + 2 \sum_{ij} Z_{D_1}^{l_i} Z_{D_3}^{l_j} Z_{U_2}^{l_j} Z_{U_2}^{l_j} \{ \\
\left( x_{g} (x_{D_1}^3 + x_{\ell}^2) - x_{\ell} x_{D_1}^3 \ln x_{g} \right) + \frac{x_{g}^2 \ln x_{g}}{(x_{g} - x_{\ell})(x_{\ell}^2 - x_{g})} + \frac{x_{\ell}^2 \ln x_{\ell}}{(x_{g} - x_{\ell})(x_{\ell}^2 - x_{g})} + \left( x_{g} - x_{\ell} \right) \left( x_{\ell}^2 - x_{g} \right) \} \\
C_{3}^{c} (\mu W) = -\frac{\alpha_s}{4\pi} \frac{2 m_{\chi}^2}{m_{W}^{2}} \left( \sum_{\alpha = \chi} \frac{x_{\alpha} \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_2}^{l} Z_{D_4}^{l} A_{bc}^{+} A_{sc}^{-} \right) \\
C_{4}^{c} (\mu W) = 0, \\
C_{5}^{c} (\mu W) = -\frac{1}{4} C_{3}^{c} (\mu W) , \\
C_{6}^{c} (\mu W) = 0, \\
C_{7}^{c} (\mu W) = -\frac{\alpha_s}{4\pi} \frac{2 m_{\chi}^2}{m_{W}^{2}} \left( \sum_{\alpha = \chi} \frac{x_{\alpha} \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_2}^{l} Z_{D_4}^{l} A_{bc}^{+} A_{sc}^{-} \right) \\
C_{8}^{c} (\mu W) = -\frac{\alpha_s}{4\pi} \sum_{\alpha = \chi} \frac{x_{\alpha}^2 \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_2}^{l} Z_{D_4}^{l} Z_{D_3}^{l} Z_{D_3}^{l}, \\
C_{9}^{c} (\mu W) = 2 \frac{\alpha_s}{4\pi} \left( \sum_{\alpha = \chi} \frac{x_{\alpha}^2 \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_2}^{l} Z_{D_4}^{l} A_{bc}^{+} A_{sc}^{-} \right) \\
C_{10}^{c} (\mu W) = 0, \\
C_{11}^{c} (\mu W) = -\frac{\alpha_s}{4\pi} \left( \sum_{\alpha = \chi} \frac{x_{\alpha}^2 \ln x_{\alpha}}{(x_{\beta} - x_{\alpha})} Z_{D_2}^{l} Z_{D_4}^{l} A_{bc}^{+} A_{sc}^{-} \right). 
\]
\[\sum_{\alpha=\tilde{g},\tilde{\chi}^+_1,\tilde{\mu}^2_{\nu},\tilde{\nu}_j} \prod_{\beta \neq \alpha} \frac{x_\alpha^2 \ln x_\alpha}{(x_\beta - x_\alpha)} \mathcal{Z}_{U_2}^{2j_2} \mathcal{Z}_{U_2}^{2i} B_{bc}^{ik} B_{sc}^{j^*k^*} \],
\]
\[
\bar{C}_2^{c}(\mu_W) = 0 ,
\]
\[
\bar{C}_3^{c}(\mu_W) = -\frac{\alpha_s}{4\pi} \frac{2 m_3 m_{\chi^+}}{m_W^2} \left( \sum_{\alpha=\tilde{g},\tilde{\chi}^+_1,\tilde{\mu}^2_{\nu},\tilde{\nu}_j} \prod_{\beta \neq \alpha} \frac{x_\alpha \ln x_\alpha}{(x_\beta - x_\alpha)} \mathcal{Z}_{D_2}^{2j_2} \mathcal{Z}_{D_3}^{2i} A_{bc}^{ik} A_{sc}^{j^*k^*} \right),
\]
\[
\bar{C}_4^{c}(\mu_W) = 0 ,
\]
\[
\bar{C}_5^{c}(\mu_W) = -\frac{1}{4} \bar{C}_3^{c}(\mu_W) ,
\]
\[
\bar{C}_6^{c}(\mu_W) = 0 ,
\]
\[
\bar{C}_7^{c}(\mu_W) = -\frac{\alpha_s}{4\pi} \frac{2 m_3 m_{\chi^+}}{m_W^2} \left( \sum_{\alpha=\tilde{g},\tilde{\chi}^+_1,\tilde{\mu}^2_{\nu},\tilde{\nu}_j} \prod_{\beta \neq \alpha} \frac{x_\alpha \ln x_\alpha}{(x_\beta - x_\alpha)} \mathcal{Z}_{D_2}^{2j_2} \mathcal{Z}_{D_3}^{2i} A_{bc}^{ik} A_{sc}^{j^*k^*} \right),
\]
\[
\bar{C}_8^{c}(\mu_W) = -\frac{\alpha_s}{4\pi} \sum_{\alpha=\tilde{g},\tilde{\chi}^+_1,\tilde{\mu}^2_{\nu},\tilde{\nu}_j} \prod_{\beta \neq \alpha} \frac{x_\alpha^2 \ln x_\alpha}{(x_\beta - x_\alpha)} \mathcal{Z}_{U_2}^{2i} \mathcal{Z}_{U_2}^{2j_2} \mathcal{Z}_{D_3}^{1j} \mathcal{Z}_{D_3}^{2j_2} ,
\]
\[
\bar{C}_9^{c}(\mu_W) = 2 \frac{\alpha_s}{4\pi} \left[ \sum_{\alpha=\tilde{g},\tilde{\chi}^+_1,\tilde{\mu}^2_{\nu},\tilde{\nu}_j} \prod_{\beta \neq \alpha} \frac{x_\alpha \ln x_\alpha}{(x_\beta - x_\alpha)} \mathcal{Z}_{B_2}^{1j} \mathcal{Z}_{B_3}^{1i} A_{bc}^{ik} A_{sc}^{j^*k^*} \right],
\]
\[
\bar{C}_{10}^{c}(\mu_W) = 0 .
\]
\[
(25)
\]

The notation $A_{IJ}^{+ij}$, $B_{IJ}^{+ij}$ ($I, J = 1, 2, 3$ are the indices of generations) are defined as
\[
A_{IJ}^{-ij} = \left( \mathcal{Z}_{B_j}^{1i} \mathcal{Z}_{1j}^{1} - \frac{m_d^f}{\sqrt{2m_W \cos \beta}} \mathcal{Z}_{D_3}^{2i} \mathcal{Z}_{2j}^{2j} \right),
\]
\[
A_{IJ}^{+ij} = \frac{m_d^f}{\sqrt{2m_W \sin \beta}} \mathcal{Z}_{B_j}^{1i} \mathcal{Z}_{1j}^{1} ,
\]
\[
B_{IJ}^{-ij} = \left( \mathcal{Z}_{U_j}^{1i} \mathcal{Z}_{1j}^{1} - \frac{m_d^f}{\sqrt{2m_W \sin \beta}} \mathcal{Z}_{U_3}^{2i} \mathcal{Z}_{2j}^{2j} \right),
\]
\[
B_{IJ}^{+ij} = \frac{m_d^f}{\sqrt{2m_W \cos \beta}} \mathcal{Z}_{U_j}^{1i} \mathcal{Z}_{1j}^{1} .
\]
\[
(26)
\]

Here, $\mathcal{Z}^{\pm}$ are the mixing matrices of charginos.
References

[1] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B527(1998)21.
[2] P. Ciafaloni, A. Romanino and A. Strumia, Nucl. Phys. B524(1998)361.
[3] F. Borzumati and C. Greub, Phys. Rev. D58(1998)074004.
[4] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B533(1991)591.
[5] R. Barbieri and G. F. Giudice, Phys. Lett. B309(1993)86.
[6] F. Borzumati and C. Greub, T. Hurth and D. Wyler, Phys. Rev. D62(2000)075005.
[7] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B534(1998)3; see also, C. Bobeth, M. Misiak, J. Urban, Nucl. Phys. B567(2000)153.
[8] S. Bertolini and J. Matias, Phys. Rev. D57(1998)4197.
[9] W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. D60(1999)114029.
[10] G. Barenboim and M. Raidal, Phys. Lett. B457(1999)109.
[11] J. L. Hewett and D. Wells, Phys. Rev. D55(1997)5549; Q. Yan, C. Huang, W. Liao and S. Zhu, Phys. Rev. D62(2000)094023.
[12] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D61(2000)074024.
[13] A. Masiero and L. Silvestrini,[hep-ph/9709244, [hep-ph/9711401].
[14] M. Ciuchini et.al, JHEP 9810(1998)008.
[15] R. Contion, I. Scimemi, Eur. Phys. J. C10(1999)347.
[16] F. Krauss, G. Soff, [hep-ph/9807238].
[17] Tai-Fu Feng, Xue-Qian Li, Wen-Gan Ma and Feng Zhang, Phys. Rev. D63(2001)015013.
[18] I. Bigi et al., Phys. Rev. Lett. 71(1993)496; A. Manohar and M. B. Wise, Phys. Rev. D49(1994)1310; B. Blok et al., Phys. Rev. D49(1994)3356; T. Mannel, Nucl. Phys. B413(1994)396; A. Falk, M. Luke and M. Savage, Phys. Rev. D49(1994)3367.
[19] I. Bigi et al., Phys. Lett. B293(1992)430, B297(1993)477(E).
[20] I. Bigi et al., in B decays, ed. S. Stone, 2nd edition, World Scientific, Singapore, 1994,132; I. Bigi, [hep-ph/9508408].
[21] M. Neubert and C. T. Sachrajda, Nucl. Phys. B483(1997)339.
[22] W.S. Dai, X.G. He, X.Q. Li and G. Zhao, Phys.Rev.D60 (1999) 034005.
[23] I. J. Kroll, [hep-ex/9602003], proceedings of the 17th Int. Symp. on Lepton Photon Interactions, Beijing, P. R. China,1995, 204.
[24] G. Altarelli and S. Petrarca, Phys. Lett. B261(1991)303.
[25] A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D\textbf{56}(1997)7228.
[26] A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D\textbf{59}(1999)034008.
[27] A. Buras, M. Misiak and J. Urban, Nucl. Phys. B\textbf{586}(2000)397.
[28] J. Rosiek, Phys. Rev. D\textbf{41}(1990)3464; Chao-Hsi Chang, Tai-Fu Feng, Eur. Phys. J. C\textbf{12}(2000)137.
[29] F. J. Gilman and M. B. Wise, Phys. Rev. D\textbf{20}(1979)2392.
[30] A. J. Buras, M. Jamin, M. Lautenbacher and P. Weisz, Nucl. Phys. B\textbf{370}(1992)69; B\textbf{375}(1992)501.
[31] A. J. Buras, M. Jamin, M. Lautenbacher and P. Weisz, Nucl. Phys. B\textbf{400}(1993)37.
[32] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Nucl. Phys. B\textbf{415}(1994)403.
[33] K. Chetyrkin, M. Misiak and M. Müinz, Nucl. Phys. B\textbf{520}(1998)279.
[34] T. Inami and C. S. Lim, Prog. Theor. Phys. 65(1981)297.
[35] R. Grigjanis, P. O'Donnell, M. Sutherland and H. Navelet, Phys. Rep. \textbf{228}(1993)93; Phys. Lett. B\textbf{213}(1988)355; B\textbf{286}(1992)413(E).
[36] B. Grinstein, R. Springer and M. Wise, Phys. Lett. B\textbf{202}(1988)138; Nucl. Phys. B\textbf{339}(1990)269.
[37] G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. B\textbf{248}(1990)181.
[38] M. Misiak, Phys. Lett. B\textbf{269}(1991)161.
[39] A. Buras, M. Jamin, M. Lautenbacher and P. Weisz, Nucl. Phys. B\textbf{400}(1993)37; Nucl. Phys. B\textbf{400}(1993)75; A. Buras, M. Jamin and P. Weisz, Nucl. Phys. B\textbf{347}(1990)491; H. Anlauf, Nucl. Phys. B\textbf{430}(1994)245.
[40] J. Feng, K. Matchev and F. Wilczek, Phys.Rev. D\textbf{63}(2001)045024; M. Hirsch, M. A. Diaz, W. Porod, J. C. Romao and J. W. F. Valle, Phys. Rev. D\textbf{62}(2000)113008.
[41] D. Pierce, J. Bagger, K. Matchev and R. Zhang, Nucl. Phys. B\textbf{491}(1997)3.
[42] S. Martin, M. Vaughn, Phys. Rev. D\textbf{50}(1994)2282; I. Jack and D. Jones, Phys. Lett. B\textbf{333}(1994)372; Y. Yamada, Phys. Rev. D\textbf{50}(1994)3537; I. Jack, D. Jones, S. Martin, M. Vaughn and Y. Yamada, Phys. Rev. D\textbf{50}(1994)5481.
[43] J. Bagger, K. Matchev and D. Pierce, Phys. Lett. B\textbf{348}(1995)443.
[44] A. Denner, Fortschr. Phys. \textbf{41}(1993)307; M. Böhm, H. Spiesberger and W. Hollik, Fortschr. Phys. \textbf{34}(1986)687.
Figure 1: The one-loop Feynman diagrams in the minimal flavor violation supersymmetry for the current-current operators in the full theory at the weak energy scale
Figure 2: The Feynman diagrams for QCD-corrections to the current-current operators in effective theory with five quarks
Figure 3: The one-loop diagrams for calculating the penguin-induced four-quark operators
Figure 4: The relative supersymmetry corrections (the supersymmetric corrections/ the SM predictions) to the Wilson coefficients at the $m_b$ scale versus $m_1$ with $\tan \beta = 2$ (Solid-lines), $\tan \beta = 20$ (Dash-Lines). The other parameters are set as $m_0 = 200\text{GeV}$, $A_0 = 0$, $\text{sgn}(\mu) = +$. 
Figure 5: The relative supersymmetry corrections to the Wilson Coefficients at the $m_h$ scale versus $m_0$ with $\tan \beta = 2$ (Solid-lines), $\tan \beta = 20$ (Dash-Lines). The other parameters are set as $m_{\tilde{g}} = 300$GeV, $A_0 = 0$, $sgn(\mu) = +$. 