Charmed Baryon Weak Decays with Vector Mesons

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Abstract

We give a systematic study of $B_c \rightarrow B_n V$ decays, where $B_c$ and $B_n$ correspond to the anti-triplet charmed and octet baryons, respectively, while $V$ stand for the vector mesons. We calculate the color-symmetric contributions to the decays from the effective Hamiltonian with the factorization approach and extract the anti-symmetric ones based on the experimental measurements and $SU(3)_F$ flavor symmetry. We find that most of the existing experimental data for $B_c \rightarrow B_n V$ are consistent with our fitting results. We present all the branching ratios of the Cabbibo allowed, singly Cabbibo suppressed and doubly Cabbibo suppressed decays of $B_c \rightarrow B_n V$. The decay parameters for the daughter baryons and mesons in $B_c \rightarrow B_n V$ are also evaluated. In particular, we point out that the Cabbibo allowed decays of $\Lambda_c^+ \rightarrow \Lambda^0 \rho^+$ and $\Xi_c^0 \rightarrow \Xi^- \rho^+$ as well as the singly Cabbibo suppressed ones of $\Lambda_c^+ \rightarrow \Lambda^0 K^{*+}$, $\Xi_c^+ \rightarrow \Sigma^+ \phi$ and $\Xi_c^0 \rightarrow \Xi^- K^{*+}$ have large branching ratios and decay parameters with small uncertainties, which can be tested by the experimental searches at the charm facilities.
I. INTRODUCTION

Recently, the LHCb Collaboration has obtained the anti-triplet charmed baryon lifetimes with high precision, given by (1)

\[
(\tau_{\Lambda_c^+, \tau_{\Xi_c^0}, \tau_{\Xi_c^+}}) = (203.5 \pm 2.2, 456.8 \pm 5.5, 154.5 \pm 2.5) \text{ fs}. \tag{1}
\]

Note that the decay lifetime of \(\Xi_c^0\) is 3\(\sigma\) above the previous averaged value of \((112 \pm 12)\) fs in PDG [2]. Furthermore, BESIII [3] and Belle [4] Collaborations have precisely measured the absolute decay branching ratio for \(\Lambda_c^+ \rightarrow pK^-\pi^+\) with the world average of

\[
\mathcal{B}(\Lambda_c^+ \rightarrow pK^-\pi^+) = (6.28 \pm 0.32)\% \tag{2}
\]

in PDG [2]. Moreover, the Belle Collaboration has determined the absolute branching ratios in \(\Xi_c\), given by [5, 6]

\[
\mathcal{B}(\Xi_c^0 \rightarrow \Xi^-\pi^+) = (1.80 \pm 0.52)\% ,
\]
\[
\mathcal{B}(\Xi_c^+ \rightarrow \Xi^-\pi^+\pi^+) = (2.86 \pm 1.27)\% , \tag{3}
\]

from the decay chains of \(B\) mesons. These decay branching ratios are important as most of the other branching ratios of anti-triplet charmed baryons are measured relative to them.

It is known that there are some difficulties for the theoretical study in the non-leptonic decays of charmed baryons due to the failure of the factorization approach. On the other hand, one can use the \(SU(3)_F\) flavor symmetry to relate the amplitudes among different decays [7–9]. This becomes possible [10–27] as there have been recently many new experimental measurements for charmed baryon decays [3–6, 28–37]. In addition to the analysis of charmed baryon decays with \(SU(3)_F\), the theoretical calculations based on dynamical models have also been done in the literature [38–50]. However, the results are often not reliable and different among models. The main difficulties are due to the unknown baryon wave functions and nonfactorizable contributions.

In this work, we concentrate on the decays of \(B_c \rightarrow B_nV\) with the \(SU(3)_F\) flavor symmetry, where \(B_c\) and \(B_n\) correspond to the anti-triplet charmed and octet baryons, and \(V\) stand for the vector mesons, respectively. In fact, some of the decay branching ratios have been recently explored based on \(SU(3)_F\) in Ref. [25]. However, the approach in Ref. [25] has ignored the contributions from color-symmetric parts of the effective Hamiltonian and
correlations among the $SU(3)_F$ parameters. In addition, there should be four independent wave amplitudes [51], but only one is used in Ref. [25]. In this study, we shall include all the wave amplitudes and consider the full effective Hamiltonian. We shall also discuss the decay asymmetry parameters in $B_c \to B_n V$, such as the up-down and longitudinal polarization asymmetries of $B_n$ and asymmetry parameter of $V$.

This paper is organized as follow. In Sec. II, we present the formalism. In Sec. III, we extract the $SU(3)_F$ parameters from the experimental data. We conclude our study in Sec IV.

II. FORMALISM

The most general form of the amplitude for $B_c \to B_n V$ can be written as

$$M = \bar{u}_f(p_f) \epsilon^{\mu*} \left[ A_1 \gamma_\mu \gamma_5 + A_2 \frac{p_{f\mu}}{m_i} \gamma_5 + B_1 \gamma_\mu + B_2 \frac{p_{f\mu}}{m_i} \right] u_i(p_i),$$

where $\epsilon^\mu$ is the four vector polarization for the vector meson of $V$, $u_i(p_i)$ and $u_f(p_f)$ are the 4-component spinors (momenta) for the initial and final baryons, respectively, and $m_i$ represents the initial baryon mass. In general, the physical vector meson with its momentum in the $z$ direction has the vector polarizations of $\epsilon^\mu = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ for $\lambda_V = \pm 1$ and $\epsilon^\mu = (|\vec{p}_V|/m_V, 0, 0, E_V/m_V)$ for $\lambda_V = 0$, where $\lambda_V$ is the helicity and $m_V$, $\vec{p}_V$ and $E_V$ are the mass, 3-momentum and energy of the vector meson, respectively. In the center of the momentum frame (CMF), the kinematic factors of $A_2$ and $B_2$ in Eq. (4) can be further written as

$$\epsilon^{\mu*} p_{f\mu}/m_i = \epsilon^{\mu*} p_{i\mu}/m_i = \epsilon^0*.$$  

Here, we have used $p_i^\mu = p_f^\mu + p_V^\mu$ and $\epsilon_\mu p_V^\mu = 0$, where $p_V$ corresponds to the 4-momentum of the vector meson. It is clear that the terms associated with $A_2$ and $B_2$ will only contribute to the decay in the case of $\lambda_V = 0$, which are suppressed by the factor of $p_c/m_V$ with $p_c$ defined as the magnitude of the 3-momentum in the CMF, so that they can be ignored.

The decay width, up-down asymmetry and longitudinal polarization of $B_c \to B_n V$ are
given by
\[
\Gamma = \frac{p_c}{4\pi} \frac{E_f + m_f}{m_i} \left[ 2 \left( |S|^2 + |P_2|^2 \right) + \frac{E_V^2}{m_V^2} \left( |S + D|^2 + |P_1|^2 \right) \right],
\]
(6)
\[
\alpha = \frac{2E_V^2 \text{Re}(S + D)^* P_1 + 4m_e^2 \text{Re}(S^* P_2)}{2m_V^2 \left( |S|^2 + |P_2|^2 \right) + E_V^2 \left( |S + D|^2 + |P_1|^2 \right)},
\]
(7)
\[
P_L = \frac{2E_V^2 \text{Re}(S + D)^* P_1 - 4m_e^2 \text{Re}(S^* P_2)}{2m_V^2 \left( |S|^2 + |P_2|^2 \right) + E_V^2 \left( |S + D|^2 + |P_1|^2 \right)},
\]
(8)
where $S$, $P_{1,2}$ and $D$, corresponding to the orbital angular momenta of $l = 0, 1, 2$ in the non-relativistic limit, are given by [51]
\[
S = -A_1,
\]
(9)
\[
P_1 = -\frac{p_c E_V}{E_f + m_f} \left( \frac{m_i + m_p}{E_f + m_f} B_1 + B_2 \right),
\]
(10)
\[
P_2 = \frac{p_c}{E_f + m_f} B_1,
\]
(11)
\[
D = -\frac{p_c^2}{E_V(E_f + m_f)} (A_1 - A_2),
\]
(12)
respectively. Here, $\alpha$ and $P_L$ are defined by
\[
\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha \cos\theta,
\]
(13)
\[
P_L = \frac{\Gamma(\lambda_{B_n} = 1) - \Gamma(\lambda_{B_n} = -1)}{\Gamma(\lambda_{B_n} = 1) + \Gamma(\lambda_{B_n} = -1)},
\]
(14)
where $d\Gamma$ is the partial decay width, $\lambda_{B_n}$ is the helicity of $B_n$ and $\theta$ is the angle between the spin and momentum directions of $B_c$ and $B_n$, respectively.

Since the vector meson of $V$ subsequently decays into two pseudo-scalar mesons, its polarization can be determined. As a result, we can discuss the decay asymmetry parameter of $V$, defined by [38]
\[
\frac{d\Gamma_V}{d\cos\theta_V} \propto 1 + \alpha_V \cos^2\theta_V,
\]
(15)
with
\[
\alpha_V = \frac{E_V^2 \left( |S + D|^2 + |P_1|^2 \right) - m_e^2 \left( |S|^2 + |P_2|^2 \right)}{m_V^2 \left( |S|^2 + |P_2|^2 \right)},
\]
(16)
where $d\Gamma_V$ is the partial decay width for the $V$ decay and $\theta_V$ is the polar angle between $\vec{p}_V$ and the momentum directions of the pseudo-scalar mesons in the CMF of $V$.

The effective Hamiltonian responsible for the decay processes with $\Delta c = -1$ is
\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q, q' = d, s} \left[ c_1 V_{uq}^* V_{cq'}(\bar{u}q)(\bar{q}'c) + c_2 V_{uq}^* V_{cq'}(\bar{u}q)(\bar{q}'c) \right],
\]
(17)
where the quark operators are defined as \((\bar{q}_1 q_2) = (\bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2)\) with summing over the colors, the Wilson coefficient of \(c_1(c_2)\) is 1.246\((-0.636)\) at the scale of \(\mu = 1.25\) GeV \(52\) and \(G_F\) is the Fermi constant. Note that \((q, q') = (d, s), (d, d)\) or \((s, s)\) and \((s, d)\) correspond to the Cabbibo allowed, singly Cabbibo suppressed and doubly Cabbibo suppressed decays, respectively.

By using the CKM mixing parameters of \(V_{ca} = V_{ud} \approx 1\) and \(s_c \equiv V_{us} = -V_{cd} \approx 0.225\), the effective Hamiltonian in the flavor basis is given by

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} \left( \frac{c_2}{2} \epsilon^{kij} H(6)_{ik} + c_+ H(\overline{15})^i_{kj} \right) (\bar{q}_j q^k)(\bar{q}_i c) \tag{18}
\]

where \((q_1, q_2, q_3) = (u, d, s), c_\pm = c_1 \pm c_2\), and \(\epsilon^{kij}\) represents the total antisymmetric tensor with \(\epsilon^{123} = 1\). Here, the tensor components are given by

\[
H(6)_{ij} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 2 & 2s_c \\
0 & 2s_c & 2s_c^2
\end{pmatrix},
\]

\[
H(\overline{15})^i_{kj} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
(\epsilon^{kij} H(6)_{ik} + c_+ H(\overline{15})^i_{kj}) (\bar{q}_j q^k)(\bar{q}_i c) \tag{19}
\]

Two of the creation operators generated by \(H(\overline{15})\) are symmetric in color. As a result, \(H(\overline{15})\) does not contribute to the nonfactorizable amplitudes since the charmed baryons are total anti-symmetric in color \(53, 54\).

We separate \(A_1\) and \(B_1\) into 6 and \(\overline{15}\) parts under the \(SU(3)_F\) symmetry:

\[
A_1 = A_1^{(6)} + A_1^{(\overline{15})},
\]

\[
B_1 = B_1^{(6)} + B_1^{(\overline{15})}. \tag{20}
\]

In Eq. \(20\), \(A_1^{(6)}\) and \(B_1^{(6)}\) are parametrized as

\[
A_1^{(6)}(B_c \rightarrow B_n V) = a_0 H(6)_{ij}(B'_c)^{ik}(B_n)^j_k(V)^l_i + a_1 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i + a_2 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i + a_3 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i, \tag{21}
\]

\[
B_1^{(6)}(B_c \rightarrow B_n V) = b_0 H(6)_{ij}(B'_c)^{ik}(B_n)^j_k(V)^l_i + b_1 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i + b_2 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i + b_3 H_{ij}(6)(B'_c)^{ik}(B_n)^j_k(V)^l_i. \tag{22}
\]
where $a_i$ and $b_i$ are the $SU(3)_F$ parameters, while $B_{c,n}$ and $V$ can be written under the tensor components of the $SU(3)_F$ representations, given by

$$B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c)$$

$$B_n = \begin{pmatrix}
\frac{1}{2}\sum\Lambda & \sum^0 & p \\
\sum^- & -\frac{1}{2}\sum^0 & n \\
-\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda
\end{pmatrix},$$

and

$$V = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\omega + \rho^0) & \rho^+ & K^{*+} \\
\rho^- & \frac{1}{\sqrt{2}}(\omega - \rho^0) & K^{*0} \\
K^{-} & \bar{K}^{*0} & \phi
\end{pmatrix},$$

respectively.

On the other hand, the contribution from $H(15)$ to $c \to uq'\bar{q}$ is factorizable, given by

$$M(15) = \frac{G_F}{2\sqrt{2}} V_{uq} V_{cq'} c_+ \left(1 + \frac{1}{N_c}\right) \langle V|(\bar{u}q)|0\rangle \langle B_n|(\bar{q}'c)|B_c\rangle$$

for the vector mesons with positive charges, while the creation operators, $\bar{q}'$ and $\bar{u}$, are interchanged for the neutral vector mesons. Accordingly, $A_1^{(15)}$ and $B_1^{(15)}$ in Eq. (20) are given by [50]

$$A_1^{(15)} = -\frac{G_F}{2\sqrt{2}} V_{uq} V_{cq'} f_V m_V c_+ \left(1 + \frac{1}{N_c}\right) \left(g_1 - g_2 \frac{m_i - m_f}{m_i}\right),$$

$$B_1^{(15)} = \frac{G_F}{2\sqrt{2}} V_{uq} V_{cq'} f_V m_V c_+ \left(1 + \frac{1}{N_c}\right) \left(f_1 + f_2 \frac{m_i + m_f}{m_i}\right),$$

where $\langle V|(\bar{q}'q)|0\rangle = f_V m_V c^*_\mu$ and $N_c$ is the effective color number. In Eq. (26), we take that $f_V = 0.215$ GeV and the form factors of $f_i(g_i)$ are defined by

$$\langle B_n|(\bar{q}c)|B_c\rangle = \bar{u}_f(p_f) \left[f_1 \gamma_\mu - f_2 i\sigma_{\mu\nu} \frac{q^\nu}{m_i} + f_3 \frac{q_\mu}{m_i}\right] - \left(g_1 \gamma_\mu - g_2 i\sigma_{\mu\nu} \frac{q^\nu}{m_i} + g_3 \frac{q_\mu}{m_i}\right) \gamma_5 u_i(p_i).$$

In our calculation, we evaluate the form factors from the MIT bag model [55, 56]. The baryon wave functions and form factors are listed in Appendix A.

The factorizable parts in $A_2$ and $B_2$ are given by

$$A_2^{(fac)} = \frac{G_F}{\sqrt{2}} \left[c_+ \left(1 + \frac{1}{N_c}\right) \pm c_- \left(1 - \frac{1}{N_c}\right)\right] V_{uq} V_{cq'} f_V m_V g_2,$$

$$B_2^{(fac)} = -\frac{G_F}{\sqrt{2}} \left[c_+ \left(1 + \frac{1}{N_c}\right) \pm c_- \left(1 - \frac{1}{N_c}\right)\right] V_{uq} V_{cq'} f_V m_V f_2$$

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with the “±” signs for mesons with positive and neutral charges, respectively. In general, it is also possible to parametrize the nonfactorizable contributions in $A_2$ and $B_2$ according to the $SU(3)_F$ symmetry. However, since they are suppressed due to Eq. (5), we will neglect these parts.

To sum up, $A_1^{(1)}(B_1^{(6)})$ and $A_2^{(fac)}(B_2^{(fac)})$ can be calculated from the factorization approach, while $A_1^{(6)}(B_1^{(6)})$ are parametrized by the $SU(3)_F$ symmetry. The detail results are shown in Appendix B.

### III. NUMERICAL RESULTS

The effective color number can be extracted from the decay branching ratio of $\Lambda_c^+ \rightarrow p\phi$ since it only receives the factorizable contribution [43, 53]. The decay amplitude is given by

$$M(\Lambda_c^+ \rightarrow p\phi) = \frac{G_F}{\sqrt{2}} V_{us} V_{cs} \left( c_2 + \frac{c_1}{N_c} \right) \langle \phi | (\bar{s}s) | 0 \rangle \langle p | (\bar{u}c) | \Lambda_c \rangle .$$

(30)

With the form factors given in Appendix A, we obtain the decay parameters

$$\alpha(\Lambda_c^+ \rightarrow p\phi) = -0.08 , \quad P_L(\Lambda_c^+ \rightarrow p\phi) = -0.85 , \quad \alpha_V(\Lambda_c^+ \rightarrow p\phi) = 0.97 ,$$

(31)

which are independent of $N_c$. On the other hand, with the experimental data of $B(\Lambda_c^+ \rightarrow p\phi) = (1.06 \pm 0.14) \times 10^{-3}$ [2], we find that $(c_2 + c_1/N_c) = 0.49$, leading to $N_c = 9$, while the effective coupling strengths are found to be

$$A_1(\Lambda_c^+ \rightarrow p\phi) = 0.0110 \ G_F GeV^2 , \quad B_1(\Lambda_c^+ \rightarrow p\phi) = -0.0175 \ G_F GeV^2 ,$$

$$A_2(\Lambda_c^+ \rightarrow p\phi) = 0.0034 \ G_F GeV^2 , \quad B_2(\Lambda_c^+ \rightarrow p\phi) = 0.0109 \ G_F GeV^2 .$$

(32)

For the other decay modes of $B_c \rightarrow B_n V$, the nonfactorizable effects in $A_1^{(6)}$ and $B_1^{(6)}$ are sizable, which cannot be ignored. The calculations of the nonfactorizable amplitudes are non-perturbative, which are generally model dependent. To tackle with these effects, we determine the parameters in Eqs. (21) and (22) with the experimental data, which are listed in Table I. Here, we have used Eq. (C14) in Appendix C to exact the branching ratios in $\Xi_c^+$. In particular, we have that $B(\Xi_c^+ \rightarrow \Xi^0\rho^+) \approx B(\Xi_c^+ \rightarrow \Xi^0\pi^+\pi^0) = (8.3 \pm 3.6\%)$ as the experimental branching ratio as stated in Appendix C. In addition, the branching ratios of $\Xi_c^+ \rightarrow \Sigma^+\phi$ and $\Lambda_c^+ \rightarrow \Sigma^+\rho^0$ can be obtained by the event counting method in Refs. [57, 58]. For $\Lambda_c^+ \rightarrow p\phi$, we impose 10% error deviations for the effective coupling strengths in Eq. (32) to account for the errors in the form factors evaluated from the MIT bag model.
TABLE 1. Decay branching ratios of $B_c \to B_n V$ from the experimental data and our $SU(3)_F$ reconstructed values.

| channel | $10^2 B_{ex}$ | $10^2 B_{SU(3)}$ | channel | $10^3 B_{ex}$ | $10^3 B_{SU(3)}$ |
|---------|---------------|-------------------|---------|---------------|-------------------|
| $\Lambda_c^+ \to \Lambda^0 \rho^+$ | $< 6$ [2, 59] | $4.81 \pm 0.58$ | $\Lambda_c^+ \to p \omega$ | $0.94 \pm 0.39$ [2] | $0.63 \pm 0.34$ |
| $\Lambda_c^+ \to \Sigma^+ \omega$ | $1.70 \pm 0.21$ [2] | $1.81 \pm 0.19$ | $\Lambda_c^+ \to \Sigma^+ K^{*0}$ | $3.5 \pm 1.0$ [2] | $0.38 \pm 0.09$ |
| $\Lambda_c^+ \to p \bar{K}^{*0}$ | $1.96 \pm 0.27$ [2] | $2.03 \pm 0.25$ | $\Lambda_c^+ \to p \phi$ | $1.06 \pm 0.14$ [2] | $0.87 \pm 0.14$ |
| $\Lambda_c^+ \to \Sigma^+ \phi$ | $0.39 \pm 0.06$ [2] | $0.39 \pm 0.06$ | $\Xi_c^+ \to p \bar{K}^{*0}$ | $4.13 \pm 1.69$ [2, 6] | $4.71 \pm 1.22$ |
| $\Lambda_c^+ \to \Sigma^+ \rho^0$ | $1.0 \pm 0.5$ [2] | $1.43 \pm 0.42$ | $\Xi_c^+ \to \Sigma^+ \phi$ | $1.17 \pm 0.87$ [2, 6, 57] | $1.82 \pm 0.40$ |
| $\Xi^+ \to \Sigma^+ \bar{K}^{*0}$ | $2.88 \pm 1.06$ [2, 6] | $1.40 \pm 0.69$ | $\Xi_c^0 \to \Lambda^0 \phi$ | $0.49 \pm 0.15$ [2] | $0.44 \pm 0.08$ |
| $\Xi^+ \to \Xi^0 \rho^+$ | $8.2 \pm 3.6$ [2, 6] | $14.48 \pm 2.44$ |

In our numerical calculations, we adopt the minimal $\chi^2$ fitting. We find that the minimal value of $\chi^2/(\text{degree of freedom})$ is given by $18/4 = 4.5$, which is reasonable as $SU(3)_F$ is not an exact symmetry. The results of the effective coupling parameters are found to be

\[
(a_1, a_2, a_3, \bar{a}) = (-2.40 \pm 0.24, 0.82 \pm 0.44, -2.05 \pm 0.38, -1.59 \pm 0.10)G_F GeV^2, \quad (34)
\]

\[
(b_1, b_2, b_3, \bar{b}) = (6.91 \pm 0.28, -0.82 \pm 0.99, 2.82 \pm 0.52, 0.75 \pm 0.42)G_F GeV^2, \quad (35)
\]

with the correlation in the sequences $(a_1, a_2, a_3, \bar{a}, b_1, b_2, b_3, \bar{b})$, given by

\[
R = \begin{pmatrix}
1 & -0.087 & 0.085 & -0.043 & 0.423 & 0.161 & -0.091 & 0.083 \\
-0.087 & 1 & 0.599 & 0.325 & 0.043 & 0.540 & 0.105 & 0.363 \\
0.085 & 0.599 & 1 & -0.094 & 0.126 & 0.257 & 0.346 & -0.096 \\
-0.043 & 0.325 & -0.094 & 1 & -0.011 & 0.473 & -0.308 & 0.640 \\
0.423 & 0.043 & 0.126 & -0.011 & 1 & 0.314 & 0.135 & -0.150 \\
0.161 & 0.540 & 0.257 & 0.473 & 0.314 & 1 & -0.112 & 0.472 \\
-0.091 & 0.105 & 0.346 & -0.308 & 0.135 & -0.112 & 1 & -0.355 \\
0.083 & 0.363 & -0.096 & 0.640 & -0.150 & 0.472 & -0.355 & 1
\end{pmatrix}, \quad (36)
\]

where $\bar{a} = a_0 + (a_1 + a_2 - a_3)/3$ and $\bar{b} = b_0 + (b_1 + b_2 - b_3)/3$. Accordingly, the branching ratios, up-down asymmetries and longitudinal polarizations for the Cabbibo allowed, singly suppressed and doubly suppressed decays of $B_c \to B_n V$ are shown in Tables 2, 3 and 4.
respectively. The reconstructed branching ratios are also listed in Table 1. Most of the results are consistent with the experimental data except $\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K^{*0})$.

### TABLE 2. Cabbibo allowed decays of $B_c \to B_n V$.

| channel | $10^2\mathcal{B}_{SU(3)}$ | $\alpha$ | $P_L$ | $\alpha_V$ |
|---------|-----------------------------|---------|------|-----------|
| $\Lambda_c^+ \to \Lambda^0 \rho^+$ | $4.81 \pm 0.58^a$ | $-0.27 \pm 0.04$ | $-0.93 \pm 0.05$ | $2.01 \pm 0.39$ |
| $\Lambda_c^+ \to p\bar{K}^{*0}$ | $2.03 \pm 0.25^a$ | $-0.18 \pm 0.05$ | $-0.62 \pm 0.16$ | $4.96 \pm 0.76$ |
| $\Lambda_c^+ \to \Sigma^0 \rho^+$ | $1.43 \pm 0.42$ | $-0.34 \pm 0.18$ | $-0.66 \pm 0.34$ | $9.82 \pm 7.19$ |
| $\Lambda_c^+ \to \Sigma^+ \rho^0$ | $1.43 \pm 0.42^a$ | $-0.34 \pm 0.18$ | $-0.66 \pm 0.34$ | $9.82 \pm 7.19$ |
| $\Lambda_c^+ \to \Sigma^+ \omega$ | $1.81 \pm 0.19^a$ | $-0.34 \pm 0.11$ | $-0.67 \pm 0.22$ | $1.60 \pm 0.62$ |
| $\Lambda_c^+ \to \Sigma^+ \phi$ | $0.39 \pm 0.06^a$ | $0.02 \pm 0.03$ | $0.08 \pm 0.10$ | $0.16 \pm 0.01$ |
| $\Lambda_c^+ \to \Xi^0 K^{*+}$ | $0.10 \pm 0.10$ | $-0.15 \pm 0.20$ | $-0.40 \pm 0.55$ | $0.35 \pm 0.52$ |
| $\Xi_c^+ \to \Sigma^+ K^{*0}$ | $1.40 \pm 0.69^a$ | $0.32 \pm 0.30$ | $0.37 \pm 0.35$ | $40.30^{+68.54}_{-41.30}$ |
| $\Xi_c^+ \to \Xi^0 \rho^+$ | $14.48 \pm 2.44^a$ | $0.00 \pm 0.07$ | $-0.62 \pm 0.13$ | $1.07 \pm 0.09$ |
| $\Xi^0 \to \Lambda^0 K^{*0}$ | $1.37 \pm 0.26$ | $-0.28 \pm 0.10$ | $-0.67 \pm 0.24$ | $6.94 \pm 2.28$ |
| $\Xi^0 \to \Sigma^0 K^{*0}$ | $0.42 \pm 0.23$ | $-0.33 \pm 0.50$ | $-0.42 \pm 0.62$ | $38.99^{+82.32}_{-39.99}$ |
| $\Xi^0 \to \Sigma^+ K^{*-}$ | $0.24 \pm 0.17$ | $-0.37 \pm 0.31$ | $-0.76^{+0.64}_{-0.24}$ | $1.94 \pm 2.63$ |
| $\Xi^0 \to \Xi^0 \rho^0$ | $0.88 \pm 0.22$ | $-0.15 \pm 0.18$ | $-0.26 \pm 0.32$ | $20.55 \pm 5.91$ |
| $\Xi^0 \to \Xi^0 \omega$ | $2.78 \pm 0.45$ | $-0.40 \pm 0.07$ | $-0.71 \pm 0.12$ | $2.03 \pm 0.47$ |
| $\Xi^0 \to \Xi^0 \phi$ | $0.14 \pm 0.13$ | $0.22 \pm 0.10$ | $0.61 \pm 0.27$ | $0.71 \pm 0.51$ |
| $\Xi^0 \to \Xi^- \rho^+$ | $8.98 \pm 0.55$ | $-0.32 \pm 0.01$ | $-0.94 \pm 0.01$ | $2.45 \pm 0.21$ |

$^a$ reconstructed values

Note that our result of $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \rho^+) = (4.81 \pm 0.58) \times 10^{-2}$ agrees with the experimental upper limit of $6 \times 10^{-2}$ (90% C.L.) obtained from $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \rho^+, \rho^+ \to \pi^+ \pi^0)$ by CLEO, which is obtained from $\mathcal{B}(\Lambda_c^+ \to pK^- \pi^+) < 0.95$ by CLEO, where the resonant contribution of $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \rho^+, \rho^+ \to \pi^+ \pi^0)$ is included in $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \pi^+ \pi^0)$ along with $\mathcal{B}(\Lambda_c^+ \to pK^- \pi^+) = (6.28 \pm 0.32)\%$. However, it is inconsistent with the latest experimental measurement of $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \pi^+ \pi^0)/\mathcal{B}(\Lambda_c^+ \to pK^- \pi^+) = 1.20 \pm 0.11$, making the experimental upper limit for $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \rho^+)$ questionable.
TABLE 3. Singly Cabbibo suppressed decays of $B_c \to B_n V$.

| channel          | $10^3 B_{SU(3)}$ | $\alpha$   | $P_L$    | $\alpha_V$ |
|------------------|------------------|------------|----------|------------|
| $\Lambda^+_c \to \Lambda^0 K^{**}$ | $3.35 \pm 0.37$ | $-0.13 \pm 0.05$ | $-0.81 \pm 0.09$ | $1.03 \pm 0.22$ |
| $\Lambda^+_c \to p\rho^0$           | $0.02^{+0.07}_{-0.02}$ | $-0.27^{+1.27}_{-0.73}$ | $-0.28^{+1.28}_{-0.72}$ | $-$ |
| $\Lambda^+_c \to p\omega$            | $0.63 \pm 0.34^a$ | $0.36 \pm 0.17$ | $0.88^{+0.12}_{-0.21}$ | $2.95 \pm 1.01$ |
| $\Lambda^+_c \to p\phi$              | $0.87 \pm 0.14^a$ | $-0.06 \pm 0.04$ | $-0.83 \pm 0.08$ | $0.88 \pm 0.14$ |
| $\Lambda^+_c \to n\rho^+$            | $1.76 \pm 0.72$ | $-0.09 \pm 0.22$ | $-0.84^{+0.35}_{-0.16}$ | $1.48 \pm 0.47$ |
| $\Lambda^+_c \to \Sigma^0 K^{**}$    | $0.18 \pm 0.04$ | $-0.14 \pm 0.17$ | $-0.35 \pm 0.41$ | $11.71 \pm 5.00$ |
| $\Lambda^+_c \to \Sigma^+ K^{**0}$   | $0.38 \pm 0.09^a$ | $-0.14 \pm 0.17$ | $-0.34 \pm 0.41$ | $11.86 \pm 4.99$ |
| $\Xi^+_c \to \Lambda^0 \rho^+$       | $1.52 \pm 0.57$ | $0.49 \pm 0.22$ | $0.28 \pm 0.46$ | $2.05 \pm 0.82$ |
| $\Xi^+_c \to pK^{**0}$               | $4.71 \pm 1.22^a$ | $-0.12 \pm 0.15$ | $-0.23 \pm 0.29$ | $13.01 \pm 1.39$ |
| $\Xi^+_c \to \Sigma^0 \rho^+$        | $11.45 \pm 1.52$ | $-0.39 \pm 0.02$ | $-0.96 \pm 0.00$ | $3.32 \pm 0.67$ |
| $\Xi^+_c \to \Sigma^+ \rho^0$       | $2.85 \pm 0.81$ | $-0.42 \pm 0.04$ | $-0.91^{+0.12}_{-0.09}$ | $4.99 \pm 2.14$ |
| $\Xi^+_c \to \Sigma^+ \omega$       | $4.11 \pm 0.77$ | $-0.13 \pm 0.17$ | $-0.48 \pm 0.28$ | $1.68 \pm 0.23$ |
| $\Xi^+_c \to \Sigma^+ \phi$         | $1.82 \pm 0.40^a$ | $-0.56 \pm 0.02$ | $-0.75 \pm 0.06$ | $3.19 \pm 1.56$ |
| $\Xi^+_c \to \Xi^0 K^{**}$           | $4.28 \pm 1.64$ | $0.28 \pm 0.10$ | $-0.45 \pm 0.27$ | $0.40 \pm 0.07$ |
| $\Xi^0_c \to \Lambda^0 \rho^0$       | $0.13 \pm 0.11$ | $0.51 \pm 0.17$ | $0.72 \pm 0.21$ | $13.22 \pm 7.92$ |
| $\Xi^0_c \to \Lambda^0 \omega$       | $1.51 \pm 0.20$ | $-0.16 \pm 0.19$ | $-0.19 \pm 0.31$ | $2.12 \pm 0.19$ |
| $\Xi^0_c \to \Lambda^0 \phi$         | $0.44 \pm 0.08^a$ | $-0.10 \pm 0.13$ | $-0.63 \pm 0.32$ | $0.90 \pm 0.36$ |
| $\Xi^0_c \to pK^{**-}$                | $0.19 \pm 0.14$ | $-0.47 \pm 0.26$ | $-0.88^{+0.49}_{-0.12}$ | $3.36 \pm 3.92$ |
| $\Xi^0_c \to nK^{**0}$               | $2.52 \pm 0.79$ | $-0.31 \pm 0.19$ | $-0.58 \pm 0.36$ | $10.29 \pm 3.73$ |
| $\Xi^0_c \to \Sigma^0 \rho^0$        | $0.11 \pm 0.10$ | $-0.08 \pm 0.25$ | $-0.28 \pm 0.69$ | $6.42 \pm 4.14$ |
| $\Xi^0_c \to \Sigma^0 \omega$        | $0.70 \pm 0.13$ | $-0.13 \pm 0.17$ | $-0.48 \pm 0.28$ | $1.70 \pm 0.24$ |
| $\Xi^0_c \to \Sigma^0 \phi$          | $0.30 \pm 0.07$ | $-0.57 \pm 0.02$ | $-0.75 \pm 0.06$ | $3.21 \pm 1.57$ |
| $\Xi^0_c \to \Sigma^{+} \rho^{-}$    | $0.19 \pm 0.13$ | $-0.50 \pm 0.32$ | $-0.83^{+0.53}_{-0.17}$ | $3.24 \pm 4.27$ |
| $\Xi^0_c \to \Sigma^{-} \rho^{+}$    | $5.56 \pm 0.34$ | $-0.37 \pm 0.01$ | $-0.97 \pm 0.01$ | $3.32 \pm 0.27$ |
| $\Xi^0_c \to \Xi^0 K^{**0}$          | $0.79 \pm 0.23$ | $-0.33 \pm 0.15$ | $-0.71^{+0.32}_{-0.28}$ | $7.39 \pm 6.19$ |
| $\Xi^0_c \to \Xi^{-} K^{**+}$        | $3.36 \pm 0.23$ | $-0.12 \pm 0.01$ | $-0.87 \pm 0.03$ | $1.07 \pm 0.09$ |

$^a$ reconstructed values
The other possible resonant dominated contribution in \( \Lambda^+_c \to \Lambda^0 \pi^+ \pi^0 \) is given by

\[
B(\Lambda^+_c \to \Sigma(1385)^{++}(0) \pi^0(+) , \Sigma^{++}(0) \to \Lambda^0 \pi^+(0)) = (1.9 \pm 0.4) \times 10^{-3},
\]

where we have taken \( B(\Lambda^+_c \to \Sigma(1385)\pi) = (2.2 \pm 0.4) \times 10^{-3} \) from our previous work \([20]\) and \( B(\Sigma(1385) \to \Lambda \pi) = 0.87 \pm 0.01 \) in PDG \([2]\). By subtracting these resonant contributions in \( \Lambda^+_c \to \Lambda \pi^+ \pi^0 \), we find that

\[
B(\Lambda^+_c \to \Lambda^0 \pi^0 \pi^+)_{\text{non}} / B(\Lambda^+_c \to \Lambda \pi^+ \pi^0) < 44\%
\]

with 90% C.L. and \( B(\Lambda^+_c \to \Lambda^0 \pi^0 \pi^+)_{\text{non}} = (1.9 \pm 0.7)\% \) by neglecting other resonant channels, where the subscript of “non” represents the non-resonant contribution only. This result shows that \( \Lambda^+_c \to \Lambda^0 \pi^+ \pi^0 \) is dominated by the resonances, which is one of the important predictions in Ref. \([21]\).
The decays of $\Lambda_c^+ \to \Sigma^+ K^{*0}$ and $\Xi_c^+ \to p K^{*0}$ share the same coupling strengths in terms of the $U-$spin symmetry $[27]$ as they are related through interchanging $d$ and $s$ quarks. Naively, one expects that they should have the same decay widths. However, our results indicate that
\[
\frac{\Gamma(\Lambda_c^+ \to \Sigma^+ K^{*0})}{\Gamma(\Xi_c^+ \to p K^{*0})}_{SU(3)_F} = 0.18 \pm 0.01.
\] (39)
This hierarchy can be understood by the released energies, given by
\[
m_{\Lambda_c^+} - m_{\Sigma^+} - m_{K^{*0}} = 0.20\text{GeV}
\]
\[
m_{\Xi_c^+} - m_p - m_{K^{*0}} = 0.64\text{GeV}.
\] (40)
With a smaller kinematic phase space, the decay of $\Lambda_c^+ \to \Sigma^+ K^{*0}$ is suppressed compared to $\Xi_c^+ \to p K^{*0}$. It can be interpreted as the $SU(3)_F$ breaking effect, caused by the mass differences. Meanwhile, the experimental data lead to
\[
\frac{\Gamma(\Lambda_c^+ \to \Sigma^+ K^{*0})}{\Gamma(\Xi_c^+ \to p K^{*0})}_{ex} = 1.9 \pm 0.8,
\] (41)
which is much larger than the value in Eq. (39). Despite this inconsistence, we are still confident that our result in Eq. (39) due to the phase space suppression is correct. We view this result as one of our predictions and suggest the future experiments to revisit the channels.

It is interesting to note that the Cabbibo allowed decays of $\Lambda_c^+ \to \Lambda^0 \rho^+$ and $\Xi_c^0 \to \Xi^- \rho^+$ have large branching ratios and decay parameters with small uncertainties as shown in Table 2, so that they can be viewed as the golden modes for the experimental searches. Similarly, the singly Cabbibo suppressed decays of $\Lambda_c^+ \to \Lambda^0 K^{*+}$, $\Xi_c^+ \to \Sigma^+ \phi$ and $\Xi_c^0 \to \Xi^- K^{*+}$ are also recommended to future experiments for the same reasons. In addition, we point out that the decay parameters in $\Xi_c^{+(0)} \to \Sigma^{+(0)} \phi$ are almost the same in terms of the isospin symmetry. However, the decay branching ratio for the neutral $\Xi_c^0$ mode is suppressed due to the shorter lifetime compared to the $\Xi_c^+$ one and the factor 2 from the CG coefficient.

In Table 5, we compare our results of the Cabbibo allowed decays with those in the literature, where Körner and Krämer (KK) [38], Żenczykowski (Zen) [45] and Hsiao, Yu and Zhao (HYZ) [25] are the studies based on the covariant quark model, pole model and $SU(3)_F$, respectively. In Ref. [38], only the decay widths are provided instead of the branching ratios. To obtain the branching ratios, we have used the lifetimes in Eq. (11). As seen from Table 5, our results are consistent with those in Ref. [38]. However, the branching ratios of $\rho^+$
modes of $\Lambda_c^+ \to \Lambda^0 \rho^+$, $\Lambda_c^+ \to \Sigma^0 \rho^+$, $\Xi_c^+ \to \Xi^0 \rho^+$ and $\Xi_c^0 \to \Xi^- \rho^+$ in Ref. [38] are too large compared to our predictions as well as the existing data. Furthermore, most of our results are compatible with those in Ref. [45], whereas differ largely in $\Lambda_c^+ \to \Lambda^0 \rho^+$, $\Xi_c^0 \to (\Xi^- \rho^+, \Xi^0 \omega)$ and $\Xi_c^0 \to \Xi^- \rho^+$ in Ref. [38]. Note that in Ref. [25], the contributions from $H(15)$ and the correlations between the parameters are not included in their calculations, resulting in larger errors than ours. Except the decays with the existing experimental data, which are also the inputs for the fitting, the predictions in Ref. [25] are quite different from ours even though both of us take the $SU(3)_F$ approach. In particular, due to the different treatments of the wave amplitude, the predicted decay branching ratio of $\Lambda_c^+ \to \Xi^0 K^{*+}$ in Ref. [25] is about 8 times larger than ours and the one in the literature [38, 45].

IV. CONCLUSIONS

We have explored the charmed baryon decays of $B_c \to B_n V$ based on the $SU(3)_F$ flavor symmetry. In these processes, we have calculated the color-symmetric parts of the effective Hamiltonian by the factorization approach assisted with the MIT bag model, while the anti-symmetric ones are extracted from the experimental data. We have systematically obtained all decay branching ratios and parameters in $B_c \to B_n V$. We have found that our results are consistent with the experimental data except $\Lambda_c^+ \to \Sigma^0 K^{*+}$, for which our fitted value of $B(\Lambda_c^+ \to \Sigma^0 K^{*+}) = (0.38 \pm 0.09) \times 10^{-3}$ is much smaller than the data of $(3.5 \pm 1.0) \times 10^{-3}$. As our result contains a very small error, whereas the experimental one is large, we are eager to see the precision measurement of this mode in the future experiments. We have demonstrated that the branching ratios of $\Lambda_c^+ \to \Lambda^0 \pi^+ \pi^0$ and $\Xi_c^+ \to \Xi^0 \pi^+ \pi^0$ are dominated by the resonances with the decay chains of $\Lambda_c^+ \to \Lambda^0 \rho^+$, $\rho^+ \to \pi^+ \pi^0$ and $\Xi_c^+ \to \Xi^0 \rho^+$, $\rho^+ \to \pi^+ \pi^0$, respectively. We have shown that most of our results with $SU(3)_F$ are consistent with those calculated from the dynamical models in Refs. [38] and [45]. However, the predictions for the $\rho^+$ modes of $\Lambda_c^+ \to \Lambda^0 \rho^+$, $\Lambda_c^+ \to \Sigma^0 \rho^+$, $\Xi_c^+ \to \Xi^0 \rho^+$ and $\Xi_c^0 \to \Xi^- \rho^+$ in Ref. [38] are too large, whereas those of $\Lambda_c^+ \to \Lambda^0 \rho^+$ and $\Xi_c^0 \to (\Xi^- \rho^+, \Xi^0 \omega)$ in Ref. [45] are found too small, compared to our values. On the other hand, our results are very different from those in Ref. [25], in which the $SU(3)_F$ approach is also used but the contributions from color-symmetric parts of the effective Hamiltonian are ignored.
TABLE 5. Decay branching ratios (%) of the Cabbibo favored channels in our $SU(3)_F$ approach and those in Körner and Krämerl (KK) [38], Ženczykowski (Zen) [45] and Hsiao, Yu and Zhao (HYZ) [25] along with the data in Ref. [2].

| channel               | Our results | KK [38] | Zen [45] | HYZ [25] | Data [2] |
|-----------------------|-------------|---------|----------|----------|---------|
| $\Lambda_c^+ \rightarrow \Lambda^0\rho^+$ | 4.81 ± 0.58 | 19.4    | 1.80     | 0.74 ± 0.34 | < 6     |
| $\Lambda_c^+ \rightarrow p\bar{K}^{*0}$  | 2.03 ± 0.25 | 3.13    | 5.03     | 1.9 ± 0.3 | 1.96 ± 0.27 |
| $\Lambda_c^+ \rightarrow \Sigma^0\rho^+$  | 1.43 ± 0.42 | 3.19    | 1.56     | 0.61 ± 0.46 |
| $\Lambda_c^+ \rightarrow \Sigma^+\rho^0$  | 1.43 ± 0.42 | 3.17    | 1.56     | 0.61 ± 0.46 |
| $\Lambda_c^+ \rightarrow \Sigma^+\omega$  | 1.81 ± 0.19 | 4.09    | 1.10     | 1.6 ± 0.7 | 1.70 ± 0.21 |
| $\Lambda_c^+ \rightarrow \Sigma^+\phi$   | 0.39 ± 0.06 | 0.26    | 0.11     | 0.39 ± 0.06 | 0.39 ± 0.06 |
| $\Lambda_c^+ \rightarrow \Xi^0K^{*+}$    | 0.10 ± 0.10 | 0.12    | 0.11     | 0.87 ± 0.27 |
| $\Xi^+_c \rightarrow \Sigma^+\bar{K}^{*0}$ | 1.40 ± 0.69 | 2.42    | 7.38     | 10.1 ± 2.9 | 2.88 ± 1.06 |
| $\Xi^+_c \rightarrow \Xi^0\rho^+$        | 14.48 ± 2.44 | 99.0    | 5.48     | 9.9 ± 2.9 | 8.2 ± 3.6  |
| $\Xi^0_c \rightarrow \Lambda^0\bar{K}^{*0}$ | 1.37 ± 0.26 | 1.55    | 1.15     | 0.46 ± 0.21 |
| $\Xi^0_c \rightarrow \Sigma^0\bar{K}^{*0}$ | 0.42 ± 0.23 | 0.85    | 0.77     | 0.27 ± 0.22 |
| $\Xi^0_c \rightarrow \Sigma^+K^{*-}$      | 0.24 ± 0.17 | 0.54    | 0.37     | 0.93 ± 0.29 |
| $\Xi^0_c \rightarrow \Xi^0\rho^0$        | 0.88 ± 0.22 | 2.36    | 1.22     | 1.4 ± 0.4  |
| $\Xi^0_c \rightarrow \Xi^0\omega$        | 2.78 ± 0.45 | 3.21    | 0.15     | 0.10$^{+0.86}_{-0.10}$ |
| $\Xi^0_c \rightarrow \Xi^0\phi$          | 0.14 ± 0.13 | 0.25    | 0.10     | 0.015$^{+0.071}_{-0.015}$ |
| $\Xi^0_c \rightarrow \Xi^-\rho^+$         | 8.98 ± 0.55 | 16.9    | 1.50     | 0.86 ± 0.12 |
Appendix A: Dynamics

To get a consistent results with the $SU(3)_F$ representation in Sec. II, we adopt the baryon wave functions as

$$B_c = \frac{1}{\sqrt{6}} \left[ (B_c)_k \epsilon^{ijk} q_i q_j c \otimes \chi_A + (23) + (13) \right]$$ (A1)

and

$$B_n = \frac{1}{3} \left[ (B_n)_l \epsilon^{ijk} q_j q_k q_i \otimes \chi_A + (23) + (13) \right]$$ (A2)

for the anti-triplet charmed and octet baryons, respectively, where (23) stands for interchanging second and third quarks in the first term, while (13) for first and third ones. Here, the spin structure is defined as $\chi_A = (\uparrow \downarrow \uparrow - \downarrow \uparrow \uparrow)/\sqrt{2}$.

The definitions in Eqs. (A1) and (A2) have different signs for $\Xi^-$ and $\Lambda^+$ compared to those in Refs. [43, 44], while they differ in sign for $\Sigma^+$, $\Xi^0$ and $\Lambda^0$ in Ref. [56].

In this work, the form factors are evaluated from the MIT bag model [55, 56]. We follow the calculations in Ref. [56]. For completeness, the input parameters are given by

$$m_{u,d} = 0.005 \, \text{GeV}, \quad m_s = 0.28 \, \text{GeV}, \quad m_c = 1.5 \, \text{GeV}, \quad R = 5 \, \text{GeV}^{-1}, \quad (A3)$$

where $R$ is the radius of the quark bag. After correcting a typo in the original derivation, Eq. (19f) in Ref. [56] shall be read as

$$\mathcal{A}_T = (A - B)N^i N^j R^3 W_i W_j J_{11}(-2R^2/15), \quad (A4)$$

where $\mathcal{A}_T$ is one of the components in the axial vector, $A(B)$ is the quark overlapping factor for the spin up (down), $N^i(N^j)$ is the normalized factor for the initial (final) baryon, $W^i(W^j)$ is associated with the normalized factor for quarks and $J_{11}$ is the overlapping between two Bessel functions. The details can be found in Ref. [56].

Our results with $q^2 = 0$ are provided in Table 6 where we have assumed the dipole momentum dependences, given by

$$f_i = \frac{f(0)}{1 - \frac{q^2}{M_V^2}}, \quad g_i = \frac{g(0)}{1 - \frac{q^2}{M_A^2}}, \quad (A5)$$

with $(M_V, M_A) = (2.01, 2.42) \, \text{GeV}$ for $c \rightarrow s$ and $(M_V, M_A) = (2.11, 2.51) \, \text{GeV}$ for $c \rightarrow u/d$. The sign differences in the form factors compared to Ref. [56] are due to the baryon wave functions.
TABLE 6. Form factors for charmed baryons decaying to octet baryons with $q^2 = 0$.

| channel          | $f_1$  | $f_2$  | $f_3$  | $g_1$  | $g_2$  | $g_3$  |
|------------------|--------|--------|--------|--------|--------|--------|
| $\Lambda_c^+ \rightarrow \Lambda^0$ | -0.455 | -0.189 | -0.001 | -0.497 | 0.055  | 0.438  |
| $\Lambda_c^+ \rightarrow p$          | 0.328  | 0.181  | 0.000  | 0.407  | -0.070 | -0.501 |
| $\Lambda_c^+ \rightarrow n$          | 0.330  | 0.182  | -0.000 | 0.408  | -0.070 | -0.502 |
| $\Xi_c^+ \rightarrow \Lambda^0$      | -0.138 | -0.093 | 0.009  | -0.168 | 0.026  | 0.271  |
| $\Xi_c^+ \rightarrow \Sigma^0$       | 0.290  | 0.201  | -0.031 | 0.332  | -0.031 | -0.550 |
| $\Xi_c^+ \rightarrow \Sigma^+$        | -0.410 | -0.285 | 0.044  | -0.469 | 0.044  | 0.778  |
| $\Xi_c^+ \rightarrow \Xi^0$          | -0.587 | -0.309 | 0.029  | -0.630 | 0.053  | 0.732  |
| $\Xi_c^0 \rightarrow \Lambda^0$      | 0.137  | 0.093  | -0.009 | 0.167  | -0.026 | -0.271 |
| $\Xi_c^0 \rightarrow \Sigma^0$       | 0.288  | 0.201  | -0.031 | 0.330  | -0.031 | -0.549 |
| $\Xi_c^0 \rightarrow \Sigma^-$        | 0.408  | 0.284  | -0.044 | 0.467  | -0.045 | -0.777 |
| $\Xi_c^0 \rightarrow \Xi^-$          | 0.590  | 0.312  | -0.030 | 0.632  | -0.052 | -0.738 |

Appendix B: Amplitudes with $SU(3)_F$ representations

In this Appendix, we provide the effective coupling strengths in Tables 7, 8 and 9. We distinguish $A_1(B_1)$ in two different parts according to the effective Hamiltonian. $A_1^{(15)}$ and $B_1^{(15)}$ are purely factorizable, which are calculated through the form factors. On the other hand, $A_1^{(6)}$ is parametrized by the $SU(3)_F$ symmetry, while $B_1^{(6)}$ is obtained by substituting $b_i$ for $a_i$. The contributions from $A_2$ and $B_2$ are suppressed as implied by Eq. (5). We only consider the factorizable contributions in $A_2$ and $B_2$ for consistency with the calculation in $\Lambda_c^+ \rightarrow p\phi$.

Appendix C: Experimental branching ratios

In Refs. [5, 6], the experimental decay widths are given by

\[ \Gamma (B^- \rightarrow \bar{\Lambda}_c^- \Xi_c^0) = (5.81 \pm 1.39) \times 10^8 s^{-1}, \]  
\[ \Gamma (B^0 \rightarrow \bar{\Lambda}_c^- \Xi_c^+) = (7.64 \pm 2.94) \times 10^8 s^{-1}. \]  

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TABLE 7. Effective coupling strengths for Cabbibo allow decays with units $10^{-1}$ $G_F$ GeV$^2$.

| channel | $A_1^{(15)}$ | $A_2^{(fac)}$ | $B_1^{(15)}$ | $B_2^{(fac)}$ | $A_1^{(6)}$ |
|---------|-------------|--------------|-------------|--------------|-------------|
| $\Lambda_c^+ \rightarrow \Lambda^0 \rho^+$ | 0.281 | 0.084 | -0.430 | 0.312 | $-\sqrt{\frac{6}{3}}a_1 - \sqrt{\frac{6}{3}}a_2 - \sqrt{\frac{6}{3}}a_3$ |
| $\Lambda_c^+ \rightarrow p\bar{K}^*0$ | -0.305 | 0.052 | 0.460 | 0.154 | $-2a_1$ |
| $\Lambda_c^+ \rightarrow \Sigma^0 \rho^+$ | 0 | 0 | 0 | 0 | $-\sqrt{2}a_1 + \sqrt{2}a_2 + \sqrt{2}a_3$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \rho^0$ | 0 | 0 | 0 | 0 | $\sqrt{2}a_1 - \sqrt{2}a_2 - \sqrt{2}a_3$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \omega$ | 0 | 0 | 0 | 0 | $-2\sqrt{2}\bar{a} - \sqrt{\frac{2}{3}}a_1 - \sqrt{\frac{2}{3}}a_2 + \sqrt{\frac{2}{3}}a_3$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ \phi$ | 0 | 0 | 0 | 0 | $-2\bar{a} + \frac{2a_1}{3} + \frac{2a_2}{3} - \frac{2a_3}{3}$ |
| $\Lambda_c^+ \rightarrow \Xi^0 K^{*+}$ | 0 | 0 | 0 | 0 | $-2\bar{a}_2$ |
| $\Xi^+_c \rightarrow \Sigma^+ K^{*0}$ | 0.335 | -0.030 | -0.656 | -0.224 | $2a_3$ |
| $\Xi^+_c \rightarrow \Xi^0 \rho^+$ | 0.350 | 0.074 | -0.619 | 0.472 | $-2a_3$ |
| $\Xi^0_c \rightarrow \Lambda^0 K^{*0}$ | -0.123 | 0.018 | 0.213 | 0.073 | $-\frac{2\sqrt{6}}{3}a_1 + \frac{\sqrt{6}}{3}a_2 + \frac{\sqrt{6}}{3}a_3$ |
| $\Xi^0_c \rightarrow \Sigma^0 K^{*0}$ | -0.236 | 0.021 | 0.461 | 0.158 | $-\sqrt{2}a_2 - \sqrt{2}a_3$ |
| $\Xi^0_c \rightarrow \Sigma^+ K^{*-}$ | 0 | 0 | 0 | 0 | $2a_2$ |
| $\Xi^0_c \rightarrow \Xi^0 \rho^0$ | 0 | 0 | 0 | 0 | $-\sqrt{2}a_1 + \sqrt{2}a_3$ |
| $\Xi^0_c \rightarrow \Xi^0 \omega$ | 0 | 0 | 0 | 0 | $2\sqrt{2}\bar{a} + \frac{\sqrt{6}}{3}a_1 - \frac{2\sqrt{6}}{3}a_2 - \frac{\sqrt{6}}{3}a_3$ |
| $\Xi^0_c \rightarrow \Xi^0 \phi$ | 0 | 0 | 0 | 0 | $2\bar{a} - \frac{2a_1}{3} + \frac{4a_2}{3} + \frac{2a_3}{3}$ |
| $\Xi^0_c \rightarrow \Xi^- \rho^+$ | -0.351 | -0.073 | 0.624 | -0.477 | $2a_1$ |
TABLE 8. Effective coupling strengths for the singly Cabbibo suppressed decays with units $10^{-2} \, G_F GeV^2$.

| channel          | $A_1^{(15)}$ | $A_2^{(fac)}$ | $B_1^{(15)}$ | $B_2^{(fac)}$ | $s_c^{-1} A_1^{(6)}$ |
|------------------|--------------|--------------|--------------|--------------|----------------------|
| $\Lambda_c^+ \rightarrow \Lambda^0 K^{*+}$ | 0.812        | 0.242        | -1.291       | 0.937        | $-\sqrt{6}a_1 + 2\sqrt{6}a_2 - \sqrt{6}a_3$ |
| $\Lambda_c^+ \rightarrow p\rho^0$         | -0.397       | 0.067        | 0.574        | 0.192        | $-\sqrt{2}a_2 - \sqrt{2}a_3$ |
| $\Lambda_c^+ \rightarrow p\omega$         | 0.406        | -0.069       | -0.589       | -0.197       | $-2\sqrt{2}\tilde{a} + 2\sqrt{2}a_1 - \sqrt{2}a_2 + \sqrt{2}a_3$ |
| $\Lambda_c^+ \rightarrow p\phi$           | -0.889       | 0.150        | 1.420        | 0.476        | $-2\tilde{a} - \frac{4a_1}{3} + \frac{2a_2}{3} - \frac{2a_3}{3}$ |
| $\Lambda_c^+ \rightarrow n\rho^+$          | 0.563        | 0.248        | -0.815       | 0.716        | $-2a_2 - 2a_3$ |
| $\Lambda_c^+ \rightarrow \Sigma^0 K^{*+}$  | 0            | 0            | 0            | 0            | $-\sqrt{2}a_1 + \sqrt{2}a_3$ |
| $\Lambda_c^+ \rightarrow \Sigma^+ K^{*0}$  | 0            | 0            | 0            | 0            | $-2a_1 + 2a_3$ |
| $\Xi_c^+ \rightarrow \Lambda^0 \rho^+$     | -0.228       | -0.085       | 0.379        | -0.339       | $-\sqrt{6}a_1 - \sqrt{6}a_2 + 2\sqrt{6}a_3$ |
| $\Xi_c^+ \rightarrow pK^{*0}$               | 0            | 0            | 0            | 0            | $-2a_1 + 2a_3$ |
| $\Xi_c^+ \rightarrow \Sigma^0 \rho^+$      | 0.436        | 0.102        | -0.818       | 0.733        | $-\sqrt{2}a_1 + \sqrt{2}a_2$ |
| $\Xi_c^+ \rightarrow \Sigma^+ \rho^0$      | 0.436        | -0.039       | -0.818       | -0.280       | $\sqrt{2}a_1 - \sqrt{2}a_2$ |
| $\Xi_c^+ \rightarrow \Sigma^+ \omega$      | -0.446       | 0.040        | 0.840        | 0.287        | $-2\sqrt{2}\tilde{a} - \frac{\sqrt{2}}{3}a_1 - \frac{\sqrt{2}}{3}a_2 - \frac{2\sqrt{2}}{3}a_3$ |
| $\Xi_c^+ \rightarrow \Sigma^+ \phi$        | 0.975        | -0.087       | -2.025       | -0.692       | $-2\tilde{a} + \frac{2a_1}{3} + \frac{2a_2}{3} + \frac{4a_3}{3}$ |
| $\Xi_c^+ \rightarrow \Xi^0 K^{*+}$          | 1.011        | 0.215        | -1.858       | 1.418        | $-2a_2 - 2a_3$ |
| $\Xi c^0 \rightarrow \Lambda^0 \rho^0$     | -0.161       | 0.023        | 0.266        | 0.091        | $\frac{\sqrt{3}}{3}a_1 + \frac{\sqrt{3}}{3}a_2 - 2\sqrt{3}a_3$ |
| $\Xi c^0 \rightarrow \Lambda^0 \omega$      | 0.164        | -0.024       | -0.273       | -0.093       | $-2\sqrt{3}\tilde{a} + \frac{\sqrt{3}}{3}a_1 + \frac{\sqrt{3}}{3}a_2$ |
| $\Xi c^0 \rightarrow \Lambda^0 \phi$       | -0.359       | 0.051        | 0.658        | 0.225        | $-\sqrt{6}\tilde{a} - \sqrt{6}a_1 - \sqrt{6}a_3$ |
| $\Xi c^0 \rightarrow nK^{*+}$               | 0            | 0            | 0            | 0            | $2a_2$ |
| $\Xi c^0 \rightarrow n\bar{K}^{*0}$        | 0            | 0            | 0            | 0            | $-2a_1 + 2a_2 + 2a_3$ |
| $\Xi c^0 \rightarrow \Sigma^0 \rho^0$      | -0.307       | 0.028        | 0.575        | 0.197        | $-a_1 - a_2$ |
| $\Xi c^0 \rightarrow \Sigma^0 \omega$      | 0.314        | -0.028       | -0.591       | -0.202       | $2\tilde{a} + \frac{a_1}{3} + \frac{a_2}{3} + 2a_3$ |
| $\Xi c^0 \rightarrow \Sigma^0 \phi$        | -0.687       | 0.062        | 1.423        | 0.486        | $\sqrt{2}\tilde{a} - \frac{\sqrt{2}}{3}a_1 - \frac{\sqrt{2}}{3}a_2 - \frac{2\sqrt{2}}{3}a_3$ |
| $\Xi c^0 \rightarrow \Sigma^+ \rho^-$       | 0            | 0            | 0            | 0            | $-2a_2$ |
| $\Xi c^0 \rightarrow \Sigma^- \rho^+$       | 0.614        | 0.146        | -1.151       | 1.031        | $-2a_1$ |
| $\Xi c^0 \rightarrow \Xi^0 K^{*0}$          | 0            | 0            | 0            | 0            | $2a_1 - 2a_2 - 2a_3$ |
| $\Xi c^0 \rightarrow \Xi^- K^{*+}$          | -1.014       | -0.210       | 1.873        | -1.431       | $2a_1$ |
TABLE 9. Effective coupling strengths for the doubly Cabbibo suppressed decays with units $10^{-3} \, G_F \text{GeV}^2$.

| channel                  | $A_1^{(15)}$ | $A_2^{(fac)}$ | $B_1^{(15)}$ | $B_2^{(fac)}$ | $s_0^2 A_1^{(6)}$ |
|--------------------------|-------------|--------------|-------------|--------------|-----------------|
| $\Lambda_c^+ \to pK^{*0}$ | 1.623       | -0.275       | -2.449      | -0.820       | 2$a_3$          |
| $\Lambda_c^+ \to nK^{*+}$ | 1.638       | 0.723        | -2.481      | 2.179        | $-2a_3$         |
| $\Xi_c^+ \to \Lambda^0 K^{*+}$ | -0.664     | -0.246       | 1.154       | -1.031       | $-\frac{\sqrt{6}a_1}{3} + \frac{2\sqrt{6}a_2}{3} + \frac{2\sqrt{6}a_3}{3}$ |
| $\Xi_c^+ \to p\rho^0$   | 0           | 0            | 0           | 0            | $-\sqrt{2}a_2$  |
| $\Xi_c^+ \to p\omega$   | 0           | 0            | 0           | 0            | $-2\sqrt{2}a + \frac{2\sqrt{2}a_1}{3} - \frac{\sqrt{2}a_2}{3} - \frac{2\sqrt{2}a_3}{3}$ |
| $\Xi_c^+ \to p\phi$     | 0           | 0            | 0           | 0            | $-2\bar{a} - \frac{4a_1}{3} + \frac{2a_2}{3} + \frac{4a_3}{3}$ |
| $\Xi_c^+ \to n\rho^+$   | 0           | 0            | 0           | 0            | $-2a_2$         |
| $\Xi_c^+ \to \Sigma^0 K^{*+}$ | 1.268     | 0.296        | -2.491      | 2.232        | $-\sqrt{2}a_1$  |
| $\Xi_c^+ \to \Sigma^+ K^{*0}$ | -1.781     | 0.159        | 3.492       | 1.194        | $-2a_1$         |
| $\Xi_c^0 \to \Lambda^0 K^{*+}$ | 0.657      | -0.094       | -1.136      | -0.387       | $-\frac{\sqrt{6}a_1}{3} + \frac{2\sqrt{6}a_2}{3} + \frac{2\sqrt{6}a_3}{3}$ |
| $\Xi_c^0 \to p\rho^-$   | 0           | 0            | 0           | 0            | $-2a_2$         |
| $\Xi_c^0 \to n\rho^0$   | 0           | 0            | 0           | 0            | $\sqrt{2}a_2$   |
| $\Xi_c^0 \to n\omega$   | 0           | 0            | 0           | 0            | $-2\sqrt{2}a + \frac{2\sqrt{2}a_1}{3} - \frac{\sqrt{2}a_2}{3} - \frac{2\sqrt{2}a_3}{3}$ |
| $\Xi_c^0 \to n\phi$     | 0           | 0            | 0           | 0            | $-2\bar{a} - \frac{4a_1}{3} + \frac{2a_2}{3} + \frac{4a_3}{3}$ |
| $\Xi_c^0 \to \Sigma^0 K^{*0}$ | 1.255      | -0.114       | -2.454      | -0.839       | $\sqrt{2}a_1$   |
| $\Xi_c^0 \to \Sigma^- K^{*+}$ | 1.788      | 0.425        | -3.502      | 3.138        | $-2a_1$         |
The effective Hamiltonian responsible for the processes is \[52\]
\[
\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left\{ \xi_c [C_1(\mu)Q^c_1(\mu) + C_2(\mu)Q^c_2(\mu)] + \xi_u [C_2(\mu)Q^u_2(\mu) + C_2(\mu)Q^u_1(\mu)] \right\}
\]
\[
- \xi_t \sum_{i=6}^{10} C_i(\mu)Q_i(\mu) \right\},
\]
where \(\xi_i \equiv V^*_{ib}V_{is}\), \(C_i\) are the Wilson coefficients, and \(O_i\) are given as

\[
Q^i_1 = (\bar{b}_i q_j)_{V-A} (\bar{q}_j s_i)_{V-A},
\]
\[
Q^i_2 = (\bar{b} q)_{V-A} (\bar{q} s)_{V-A},
\]
\[
Q^i_3 = (\bar{b} s)_{V-A} \sum_q (\bar{q} q)_{V-A},
\]
\[
Q^i_4 = (\bar{b} s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A},
\]
\[
Q^i_5 = (\bar{b} s)_{V-A} \sum_q (\bar{q} q)_{V+A},
\]
\[
Q^i_6 = (\bar{b} s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A},
\]
with \(q = u, d, s, c, b\) in the summations.

The tree order operators, \(O^i_1\) and \(O^i_2\), are clearly isospin singlet since they do not contain either up or down quark. The penguin operators, \(O_3 \sim O_6\), are also isospin singlet since they treat \(u\) and \(d\) on equal footing. By neglecting \(O^u_1\) and \(O^u_2\) due to \(\xi_u < 0.001\), we find that the effective Hamiltonian is an isospin scalar. By using the identity

\[
\langle \frac{1}{2}, \frac{1}{2} | B \left( |0, 0\rangle \otimes |\frac{1}{2}, \frac{1}{2}\rangle \right)_{B_c} = \langle \frac{1}{2}, -\frac{1}{2} | B \left( |0, 0\rangle \otimes |\frac{1}{2}, -\frac{1}{2}\rangle \right)_{B_c} \right\rangle
\]

with

\[
\langle \frac{1}{2}, \frac{1}{2} | B = -|\bar{B}^0\rangle, \quad |\frac{1}{2}, -\frac{1}{2} | B = |B^-\rangle,
\]

\[
\langle \frac{1}{2}, \frac{1}{2} | B_c = |\Xi^+_c\rangle, \quad |\frac{1}{2}, -\frac{1}{2} | B_c = |\Xi^-c\rangle, \quad |0, 0\rangle | B_c = |\Lambda^+_c\rangle_B_c,
\]

we find that the two processes have the same decay widths as stated in Ref. [6].

We average the decay widths in Eqs. (C2) and (C1), given by

\[
\Gamma(B \to \Lambda^- \Xi^-c) = (6.14 \pm 1.26) \times 10^8 \text{ s}^{-1},
\]

which has a small uncertainty. With \(\mathcal{B}(\bar{B}^0 \to \bar{\Lambda}^-c \Xi^+_c) \mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (3.32 \pm 0.81) \times 10^{-5}\) in Ref. [6], we get

\[
\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (3.56 \pm 1.13)\%.
\]
From Eq. (C14) and the CLEO data [60], we have
\[ \mathcal{B}_{ex}(\Xi_c^+ \to \Xi^0 \pi^+ \pi^0) = (8.2 \pm 3.6)\% , \] (C15)
which contains both resonant and non-resonant contributions. On the other hand, the latest \( SU(3)_F \) analysis with the non-resonance shows that [21]
\[ \mathcal{B}_{non}(\Xi_c^+ \to \Xi^0 \pi^+ \pi^0) = (1.5 \pm 0.3)\% . \] (C16)
As a result, the experimental branching ratio is clearly dominated by the resonance. There are two dominating candidates in the resonances, \( \Xi^+ \to \Xi^0(1530) \pi^+ , \Xi^0 \rho^+ \). However, the first process is forbidden by the the color symmetry [20, 53, 54], which is supported by the experimental data [60].
\[ \mathcal{B}(\Xi_c^+ \to \Xi^0(1530) \pi^+) / \mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+ \pi^0) < 0.3 . \] (C17)
Consequently, we could safely treat the experimental value of \( \mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+ \pi^0) \) as \( \mathcal{B}(\Xi_c^+ \to \Xi^0 \rho^+) \).

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