Nonequilibrium noise spectrum and Coulomb–blockade–assisted Rabi interference in a double-dot Aharonov-Bohm interferometer

Jinshuang Jin

Department of Physics, Hangzhou Normal University, Hangzhou, Zhejiang 311121, China

We investigate the charge–state coherence underlying the nonequilibrium transport through a spinless double-dot Aharonov-Bohm (AB) interferometer. Both the current noise spectrum and real-time dynamics are evaluated with the well–established dissipaton–equation–of–motion method. The resulted spectrums show the characteristic peaks and dips, arising from coherent Rabi oscillation dynamics, with the environment–assisted indirect inter-dot tunnel coupling mechanism. The observed spectroscopic features are in a quantitative agreement to the real-time dynamics of the reduced density matrix offdiagonal element between two charge states. As the aforementioned mechanism, these characteristics of coherence are very sensitive to the AB phase. While this is generally true for cross-correlation spectrum, the total circuit noise spectrum that is experimentally more accessible shows remarkably rich interplay between various mechanisms. The most important finding of this work is the existence of Coulomb–blockade–assisted Rabi interference, with very distinct signatures arising from the interplay between the AB interferometer and the interdot Coulomb interaction induced Fano resonance.

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I. INTRODUCTION

Quantum transport through a parallel double-dot embedded in Aharonov-Bohm (AB) interferometer has been attracted much attention, in both experiments [1,2] and theories [3,13]. This is a promising candidate for solid state qubit devices [2,16]. The observed transport current shows the AB oscillation as a function of the magnetic flux [2,3]. Physically, the observed features exhibit a rich interplay between Coulomb interaction [9,14], interdot tunneling [5,9], inelastic electron cotunneling processes [8,15], and so on. Recent interests include the dynamical effect of charge-state coherence on transient transport in double–dots AB interferometer [17–22]. It is found that in the absence of Coulomb interaction, the charge qubit states experience the phase localization at values of $m \pi$ [20,21]. On the other hand, the interdot Coulomb interaction opens Coulomb–assisted channels [22], which interfere with the single-electron impurity channels. The relative phase is no longer localized but continuously controllable with the magnetic flux [22]. Moreover, an almost decoherence–free charge qubit state emerges at the current–voltage turnover position [22]. The related Coulomb interaction opens Coulomb–assisted channel.

Comparing to the average current characteristics, the shot noise spectrum offers much detailed insights in nonequilibrium quantum transport [24,27]. The zero–frequency noise describes mainly the statistical behavior of the steady state. It has been used to measure the effective carrier charge that can be either fraction [28,30] or integer [31,32]. As anticipated, shot noise spectrum is more sensitive to current to quantum interference, which is particularly prominent in such as the double–dot AB interferometer in the Kondo regime [33,34]. The evaluated shot noise oscillation behavior in Ref. [33] well reproduced that of experiment [2].

Recent progress in on-chip detection allows the high–precision measurement of quantum current–current correlator spectrum in the full frequency range [30,42]. Nonequilibrium quantum noise spectrum contains rich correlated dynamics information [43,52]. It is neither asymmetric nor detailed–balance related, with respect to the sign of frequency. However, most of theoretical studies were carried out for symmetrized noise spectrum, based on the MacDonald’s formula [53]. This is a Markovian and quasiclassical treatment. Nevertheless, the evaluated spectrum does show the oscillation dynamics induced by interdot transfer couplings [54]. It had also identified a pronounced super/sub-Poissonian statistics at spin-spin interaction energies [55].

In this work, we investigate the nonequilibrium quantum current noise spectrum for the nondegenerate double-dot AB (DD-AB) interferometer systems. We do not include the Kondo effect that is important in the low temperature regime. Both lead-specified and total circuit current noise spectrums are accurately evaluated based on the well–established dissipaton–equation–of–motion (DEOM) theory [56,58]. It is well-known that the total noise spectrum is the consequence of the lead-specified current-current interference. Our main results but not limited are as follows. (i) The current noise spectrums show rich characteristics in relation to the coherent Rabi oscillation. This is in a good agreement to the real-time dynamics of the reduced density matrix off–diagonal element between two charge states; (ii) The observed Rabi characteristics arise from the environment-
assisted indirect interdot coupling; \((iii)\) In the absence of the interdot Coulomb interaction, the total circuit noise spectrum for pristine double-dot (AB phase \(\phi = 0\)) only shows the non-Markovian quasi-step features. There are no Rabi signal, which exists in the lead-specified auto-correlation and cross-correlation current noise spectra. The disappearance of the Rabi signal in the total circuit noise is due to the lead-specified current destructive interference; \((iv)\) However, the Rabi dips do appear when AB phase is nonzero and reach maximum at \(\phi = \pi\); \((v)\) Remarkably, in the presence of the strong interdot Coulomb interaction, the Rabi dips in the total circuit noise spectrum become Fano profiles. This is the so-called Coulomb blockade assisted (CB- assisted) Rabi interference phenomenon. Its signature in the circuit noise develops from the peak at AB phase \(\phi = 0\) to dip at \(\phi = \pi\), and Fano line shape in between.

The remainder of this paper is organized as follows. In Sec. II we present the model and the quantities of interest, with their evaluations via the DEOM approach being detailed in Appendix A. In Sec. III we demonstrate the numerically accurate results of the current noise spectra, including the auto-correlation, cross-correlation and the total circuit ones together with the real-time dynamics of reduced density matrix. First, we consider the non-interacting scenarios in Sec. IIIA. We then exploit the effect of interdot Coulomb interaction and CB- assisted Rabi interference in Sec. IIIB and Sec. IIIC respectively. In line with the experimental realization of AB interferometer \(\text{[6]}\), we discuss the effect of the indirect interdot tunneling strength, by introducing the indirect coupling parameter \(\text{[2]}\), on the noise spectra and the real-time dynamics of reduced density matrix in Appendix B. Finally, we conclude this work with Sec. IV.

II. METHODOLOGY

Consider a parallel double-dot embedded in an AB interferometer and coupled to the electron reservoirs. The total Hamiltonian is composed of the three parts, \(H_\text{T} = H_S + H_B + H_\text{SB}\). The double-dot system Hamiltonian is modeled by

\[
H_S = \sum_{u=1,2} \varepsilon_u \hat{a}_u^{\dagger} \hat{a}_u + U \hat{n}_1 \hat{n}_2. \tag{1}
\]

Here, \(\hat{a}_u\) (\(\hat{a}_u^{\dagger}\)) denotes the annihilation (creation) operator of the electron in dot-\(u\) with the energy level \(\varepsilon_u\), \(\hat{n}_u = \hat{a}_u^{\dagger} \hat{a}_u\) is the electron occupation, and \(U\) is the interdot, capacitive Coulomb interaction strength. The involved charge states in the double dots are \(|0\rangle = |00\rangle\), \(|1\rangle = |10\rangle\), \(|2\rangle = |01\rangle\), and \(|d\rangle \equiv |11\rangle\), i.e., the empty, the dot-1 occupied, the dot-2 occupied, and double–dots–occupancy states, respectively. The two single-electron charge states of \(|1\rangle = |10\rangle\) and \(|2\rangle = |01\rangle\) can be served as a charge qubit \(\text{[5, 6]}\). The quantum coherence properties of the double-dot states are described by the reduced system density matrix, \(\rho(t) \equiv \text{tr}_B \rho_{\text{tot}}(t)\), i.e., the partial trace of the total density operator \(\rho_{\text{tot}}\) over the electrode bath degrees of freedom.

The environment bath Hamiltonian of the two-electron reservoirs is given by

\[
H_B = \sum_{\alpha k} \left( \epsilon_{\alpha k} + \mu_{\alpha} \right) \hat{c}_{\alpha k}^{\dagger} \hat{c}_{\alpha k}. \tag{2}
\]

Here, \(\hat{c}_{\alpha k}^{\dagger} \hat{c}_{\alpha k}\) denotes the creation (annihilation) operator of the electron with momentum \(k\) and energy \(\epsilon_{\alpha k}\), in the left (\(\alpha = L\)) or right (\(\alpha = R\)) reservoir, under the applied bias voltage potential, \(eV = \mu_L - \mu_R\). The system–bath coupling assumes the standard tunneling form, which in the presence of an AB interferometer reads

\[
H_{SB} = \sum_{\alpha k} \left( e^{i \phi_{\alpha u}} \epsilon_{\alpha u} \hat{c}_{\alpha k}^{\dagger} \hat{c}_{\alpha k} + \text{H.c.} \right). \tag{3}
\]

The AB phases via threading a magnetic flux \(\Phi\) satisfy \(\phi_{L1} - \phi_{L2} + \phi_{R2} - \phi_{R1} = \phi = 2 \pi \Phi/\Phi_0\), where \(\Phi_0\) is the flux quantum. Without loss of generality, we adopt \(\phi_{L1} = -\phi_{R1} = \phi/4\), due to the gauge invariant \(\text{[13, 21]}\). To complete the description, one requires the reservoir hybridization spectral function,

\[
J_{\alpha uu}(\omega) = \pi e^{i \phi_{\alpha u} - \phi_{\alpha u}^*} \sum_k \Gamma_{\alpha k}^u \rho_{\alpha k}^u \delta(\omega - \epsilon_{\alpha k}).
\]

For simplicity, we set it a Lorentzian—type form \(\text{[62, 63]}\),

\[
J_{\alpha uu}(\omega) = \frac{\Gamma_{\alpha uu} \omega^2}{\omega^2 + W^2}, \tag{4}
\]

with \(\Gamma_{\alpha uu} = \Gamma_{\alpha}\) and

\[
\Gamma_{L12} = \Gamma_{L21}^* = \lambda_L \Gamma_L e^{-i \phi/2},
\]

\[
\Gamma_{R12} = \Gamma_{R21}^* = \lambda_R \Gamma_R e^{i \phi/2}. \tag{5}
\]

Here, \(0 \leq |\lambda_\alpha| \leq 1\), is the indirect interdot coupling parameter \(\text{[4, 6]}\). Moreover, the AB interferometer takes effect only when \(\lambda_\alpha \neq 0\). Throughout this work, we adopt the unit of \(e = \hbar = 1\), for the electron charge and the Planck constant.

The current operator for the electron transfer from \(\alpha\)-reservoir to impurity system is \(\hat{I}_\alpha \equiv \hat{N}_\alpha = -i[\hat{N}_\alpha, H_\text{T}]\). Here, \(\hat{N}_\alpha = \sum_k \hat{c}_{\alpha k}^{\dagger} \hat{c}_{\alpha k}\) is the electron number in the \(\alpha\)-reservoir. Let \(I_\alpha^\text{st}\) be the steady-state current \((I_\alpha^\text{st} \equiv \overline{I_\alpha})\) and \(\delta \hat{I}_\alpha(t) \equiv \hat{I}_\alpha(t) - I_\alpha^\text{st}\). The fluctuating current correlation function is then

\[
\langle \delta \hat{I}_\alpha(t) \delta \hat{I}_{\alpha'}(0) \rangle \equiv \langle \hat{I}_\alpha(t) \hat{I}_{\alpha'}(0) \rangle - I_\alpha^\text{st} I_{\alpha'}^\text{st}. \tag{6}
\]

The susceptibility of the shot noise fluctuation is related to the half-Fourier transform of

\[
C_{\alpha \alpha'}(\omega) \equiv \int_0^\infty dt e^{i \omega t} \langle \delta \hat{I}_\alpha(t) \delta \hat{I}_{\alpha'}(0) \rangle. \tag{7}
\]

The lead-specified current noise spectrum via the full Fourier transformation is then

\[
S_{\alpha \alpha'}(\omega) = C_{\alpha \alpha'}(\omega) + C_{\alpha' \alpha}^*(\omega). \tag{8}
\]
Its values at positive ($\omega > 0$) and negative ($\omega < 0$) frequencies correspond to energy absorption and emission processes, respectively. It is worth noticing that in the equilibrium case, the absorptive and emissive components are related each other via the detailed–balance relation or the equivalent fluctuation–dissipation theorem. However, they are generally independent in a nonequilibrium scenario. The widely studied symmetrized noise spectrum corresponds to $S^{sym}_{\alpha\alpha}(\omega) = S_{\alpha\alpha}^a(\omega) + S_{\alpha\alpha}^a(-\omega)$. This can not distinguish the nonequilibrium absorption versus emission processes.

Moreover, the net circuit current in experiments is given by $I(t) = aI_L(t) - bI_R(t)$, with the junction capacitance parameters, $a, b \geq 0$, satisfying $a + b = 1$. The circuit current noise spectrum is then $S(\omega) = a^2S_{LL}(\omega) + b^2S_{RR}(\omega) - 2abRe[S_{LR}(\omega)]$, and for the symmetric structure of $S$ this can not distinguish the nonequilibrium absorption versus emission processes.

$$S(\omega) = \frac{1}{4}\{S_{LL}(\omega) + S_{RR}(\omega) - 2Re[S_{LR}(\omega)]\}. \quad (9)$$

Numerical evaluations will be carried out via the fermionic DEOM method; see Appendix A for the details. This is a quasi-particle extension to the well–established hierarchical–equations-of-motion formalism. As an efficient and universal method for strongly correlated quantum impurity systems, DEOM converges rapidly and uniformly to the exact results, with increasing the truncated tier level, $L = n_{trun}$. The minimal truncation tier $L$ required to achieve convergence is closely dependent on the configurations of system as well as bath. In practice, the convergence with respect to $L$ is tested case by case. For the parameters exemplified in the following numerical calculations, the DEOM evaluations effectively converge at $L = 3$ tier level.

III. NUMERICAL RESULTS

To clarify the physical picture, we consider the symmetrical–leads situation, where $\mu_L = -\mu_R = V/2$, $\Gamma_L = \Gamma_R = \Gamma$ and $\lambda_L = \lambda_R = \lambda$.

Moreover, we focus on the sequential–dominated tunneling motion, with $\mu_L > \varepsilon_1, \varepsilon_2 > \mu_R$ and $k_BT \gtrsim \Gamma$, under the environment–assisted indirect interdot tunnel coupling with $\lambda = 1$. The effect of the indirect coupling parameter $\lambda$ on the current noise spectra and the corresponding coherent charge dynamics of the charge states will be given in the Appendix B. Set the wide bandwidth value of $W = 10\text{meV}$ for the electron reservoirs. The details of other parameters are given in the captions of the figures. It is worth noting that the characteristics on the auto–correlation noise spectra, $S_{LL}(\omega)$ and $S_{RR}(\omega)$, are quite similar; thus only the results of $S_{LL}(\omega)$, together with $Re[S_{LR}(\omega)]$, are explicitly reported below.

To elucidate the underlying mechanisms, the reduced system density matrix charge states dynamics will also be reported. Note that the degenerate DD–AB systems ($\Delta \varepsilon = 0$) were studied before. The resulted density matrix evolves smoothly without oscillations. For noninteracting case, the indirect interdot coupling would lead to a charge qubit phase localization. However, for strong interacting case, the phase is continuously tuned by AB flux. Moreover, the resulted charge qubit is an almost coherent pure state. The present paper focuses on the nondegenerate case ($\Delta \varepsilon \neq 0$), elucidating in particular the spectroscopic signature of the two charge states with finite energy–splitting.

A. Noninteracting scenarios

This subsection is concerned with the noninteracting scenario; i.e., $U = 0$ in the Hamiltonian Eq. 48. To single out different effects, we consider first the pristine double–dots system, in the absence of magnetic flux ($\phi = 0$). Figure 1 displays the numerically accurate results on the noise spectra and the real-time charge–states dynamics in the pristine system. As seen in Fig. 1(a) and (b), the noise spectra of both auto and cross correlations show dips at the energy splitting, i.e., $|\omega| = \Delta \varepsilon$ with $\Delta \varepsilon = \varepsilon_1 - \varepsilon_2$. This feature is similar to that of the intrinsic coherent Rabi oscillation induced by the direct interdot coherent coupling in both serial and parallel coherent coupled double-dot. Indeed, we find the coherent Rabi oscillation dynamics between the two nondegenerate charge states, $|1\rangle$ and $|2\rangle$ in the double-dot. This is concerned with the reduced density matrix ele.
as a function of the AB phase in the inset. (b) The cross-function of the AB phase in the inset. The other parameters spectrum, with the Rabi signal in the absorption part as a function of the AB phase in the inset. The cross-correlation noise spectrum, with the oscillation signal in the absorption part as a function of the AB phase in the inset. The other parameters are the same as in Fig. 1.

FIG. 2: (Color online) The accurate results for the current noise spectra (in 2f) and the real-time dynamics of the reduced density matrix off-diagonal element, with different AB phase $\phi$ for noninteracting $U = 0$ and energy splitting $\Delta \varepsilon = 0.7$ meV. (a) The auto-correlation noise spectrum of the left lead, with the oscillation signal in the absorption part as a function of the AB phase in the inset. (b) The cross-correlation noise spectrum, with the oscillation signal in the absorption part as a function of the AB phase in the inset. (c) The time evolution of $|\rho_{12}(t)|$. (d) The circuit noise spectrum, with the Rabi signal in the absorption part as a function of the AB phase in the inset. The other parameters are the same as in Fig. 1.

ment of $\rho_{12}(t)$, as plotted in Fig. 1(c) and (d). In contrast, the population dynamics of the two charge states, as reported in the inset in Fig. 1(d), show no oscillations at all. While the Rabi oscillation frequency occurs at the energy splitting, $\omega = \pm \Delta \varepsilon$, its manifestation as a quantum interference transport goes also by the environmental-assisted indirect interdot tunnel coupling ($\Gamma_{12} \neq 0$). It will be further demonstrated in the Appendix [3].

It is also noticed the Lorentzian-like (Markovian) dip at $\omega = 0$, in both the auto- and cross-correlation noise spectra. This feature arises from the Pauli exclusion principle and is dictated by long-time dynamics [58]. Moreover, the transport induced transitions show in $S_{\alpha \alpha}(\omega)$ the quasi-steps rising around $\omega = \pm |\varepsilon_u - \mu_\alpha|$, whereas in Re[$S_{LR}(\omega)$], they are the non-Markovian peaks at $\omega = -|\varepsilon_u - \mu_\alpha|$. These features are also consistent with our previous study on a single-impurity Anderson system [58]. They affect distinctly the aforesaid dips at the Rabi frequency, $\omega = \pm \Delta \varepsilon$, which appear more remarkably in the cross-correlation noise spectrum than the auto-correlation ones. The observations here also agree well with the Rabi dips nature of the indirect interdot tunnel coupling.

Explore now the effect of AB phase $\phi$ by treading the magnetic flux $\Phi$, exemplified with $U = 0$ and $\Delta \varepsilon = 0.7$ meV. Interestingly, the auto-correlation noise spectrum is weakly modified by the flux, as displayed in Fig. 2(a). The inset highlights that $S_{\Delta \varepsilon}(\omega = \Delta \varepsilon)$ is a periodic function of the AB phase, but of a small amplitude change.

In contrast, the cross-correlation noise spectrum, as shown in Fig. 2(b), is very sensitive to the AB phase. In particular, Re[$S_{LR}(\omega)$], at the characteristic oscillation frequency, $\omega = \pm \Delta \varepsilon$, changes from a valley at $\phi = 0$ to a peak feature at $\phi = \pm \pi$. It satisfies Re[$S_{LR}(\Delta \varepsilon)$] $\approx$ Re[$S_{LR}(-\Delta \varepsilon)$] $\propto -\cos \phi$; see the inset of Fig. 2(b). In parallel, we present $\rho_{12}(t)$ in Fig. 2(c), at different values of the AB phase. This reduced system density matrix element describes coherence between the two specified non-degenerate charge states, [1] and [2], in the double-dot. The dynamical phase, which could be represented with Re$\rho_{12}(t = 2\pi/\Delta \varepsilon)$, is tuned by the AB phase. The observed time-domain Rabi oscillation dynamics, which manifests the periodic behavior with the AB phase and does reflect in Re[$S_{LR}(\omega = \pm \Delta \varepsilon)$] $\propto -\cos \phi$. This is purely the quantum interference component and can be sensitively controlled by the AB phase $\phi$. In fact, the quantum interference occurs in electron transport between the two reservoirs. That is, the electron tunneling from the left (right) reservoir to the right (left) reservoir is via the interference pathes, i.e., dot 1 (channel 1) and dot 2 (channel 2). Apparently, as the aforementioned quantum interference characteristics are concerned, the cross-correlation noise spectrum [Fig. 2(b)] appears more prominent than the auto-correlation counterpart [Fig. 2(a)]. The latter shows the $\phi$-insensitive dips only. In this sense, one may consider $S_{LL}(\omega) + S_{RR}(\omega)$ as the background to the total circuit noise spectrum $S(\omega; \phi)$ of Eq. 2.

The details of $S(\omega; \phi)$ depicted in Fig. 2(d) are as follows. First of all, $S(\omega; \phi = 0)$, the black-curve in Fig. 2(d), shows only those non-Markovian quasi-step features, which rise around $\omega = \pm |\varepsilon_u - \mu_\alpha|$, as described earlier. Here, $\varepsilon_1 = -\varepsilon_2 = 0.35$ meV and $\mu_\alpha$ = $-\mu_\alpha$ = $0.7$ meV. There are no Lorentzian-like dips, which exist in the individual auto- and cross-correlation noise spectrums at $\omega = 0$ and $\omega = \pm \Delta \varepsilon$, but are now completely canceled out. The non-Markovian peaks at $\omega = -|\varepsilon_u - \mu_\alpha|$ in Re[$S_{LR}(\omega; \phi = 0)$], the black-curve in Fig. 2(b), are also smeared out in the total circuit noise spectrum. Interestingly, recovered in $S(\omega; \phi \neq 0)$, the colored curves in Fig. 2(d), are only those Lorentzian-like dips at $\omega = \pm \Delta \varepsilon$ that specifies the non-degeneracy of the double dots. Apparently, the depths of the characteristic dips reach maxima when the AB phase $\phi = \pm \pi$; see the inset of Fig. 2(d). It comes from the maximum peak feature of the cross-correlation noise spectrum Re[$S_{LR}(\omega; \phi = \pm \pi)$] as described above, see Fig. 2(b). Therefore, the total circuit noise spectrum is also sensitive to the AB phase.

B. Effect of interdot Coulomb interaction

Consider now the effect of interdot Coulomb interaction $U$ on the coherent Rabi oscillation dynamics, exem-
splitting $\Delta \varepsilon = 0.7$ meV. Coulomb-assisted transport channels ($\varepsilon_{1,2} + U$) modifies the characteristic quasi-step at $\omega = \pm (\varepsilon_{1,2} + U - \mu_0)$ and related features in the current noise spectra.

Figure 3 reports the results of the pristine double–dots system ($\phi = 0$), with different values of $U$. In the weak Coulomb interaction regime ($U = 0.15$ meV; red–curves), where $\mu_L > \varepsilon_{1,2} + U$, all transport channels are inside the sequential tunneling or diac window. For the lead-specified noise spectrum components and the density matrix coherence, Fig. 3(a)–(c), the Rabi oscillation signals differ significantly from the noninteracting counterparts (red–curves versus black–curves). However, the total circuit noise spectrum $S(\omega)$ as shown in Fig. 3(d) differs distinctly. It shows the peaks at $\omega = 0$ and $\omega = \pm \Delta \varepsilon$, which would be completely cancelled out if $U = 0$. In other words, the total circuit noise spectrum is sensitive to the Coulomb interaction, even when the Coulomb–assisted transport channels are inside the bias window.

In the strong Coulomb interaction regime ($U = 1.25$ meV; blue–curves), with $\varepsilon_{1,2} + U > \varepsilon > \mu_R$, where the Coulomb-assisted transport channels are outside of the bias window. Akin to the intrinsic Rabi oscillation via a direct interdot coupling [54, 60], the dips at $\omega = \pm \Delta \varepsilon$ in the auto-correlation noise spectrum, $S_{\alpha\alpha}(\omega)$, becomes inferences. Note that the strong capacitive coupling makes the double–dots system in the interdot Coulomb blockade (CB) regime. Thissuppresses the zero-frequency shot noise ($S_{LL}(0) = S_{RR}(0) = -\text{Re}[S_{LR}(0)]$). Counter-intuitively, as inferred from Fig. 3(b) and (c) the blue–curves, the strong capacitive coupling gives rise to the CB-assisted Rabi interference. It enhances the Rabi resonance at $\omega = \pm \Delta \varepsilon$ in the cross-correlation noise spectrum and the corresponding amplitude of the density matrix coherence oscillation away from the short-time scale, i.e., $t > 2/\Delta \varepsilon$ as inferred from Fig. 3(c). The CB–assisted Rabi interference arises from the Coulomb-assisted transport channels $\varepsilon_{1,2} + U$ which interfere with the single-electron impurity channels. Despite that $\varepsilon_{1,2} + U$ is above the bias window, it is accessible by finite-frequency current noise spectrum. The resultant total circuit noise spectrum, $S(\omega)$ in Fig. 3(d), the blue–curve in comparing to the red–curve, shows a stronger peak at $\omega = \pm \Delta \varepsilon$ but a weaker one at $\omega = 0$.

C. Coulomb–blockage–assisted Rabi interference with AB interferometer

1. Emergence of Fano interference

Figure 4 depicts the effects of AB phase $\phi$ on those CB–assisted Rabi interference relevant quantities. Here, we only plot the absorption noise with $\omega > 0$, since the Rabi oscillation signals in both the emission and absorption noise parts are quite similar (see the above figures). We also do not repeat the auto-correlation noise spectrum because it is AB phase $\phi$–insensitive and remains largely like Fig. 4(a). The fact that the cross-correlation noise is $\phi$–sensitive but the auto-correlation is not, is common for both noninteracting and interacting scenarios. The Rabi oscillation, the evolution of the reduced density matrix off-diagonal element as seen in Fig. 4(c), is
also \(\phi\)-sensitive.

There is a remarkably distinct feature of CB-assisted Rabi interference with AB interferometer: In the total circuit noise spectrum, the Rabi signatures are peak and dip at AB phase \(\phi = 0\) and \(\pi\), respectively, but Fano profiles in between. Evidently, this remarkable feature arises from the cross-correlation noise spectrum, as shown in Fig. 4(a) versus (b). This novel phenomenon could be understood below.

2. Understanding via canonical transformation

We restore to the canonical transformation on the electron operators in the dots which is given by \(22\),

\[
\hat{d}_1 = (\hat{a}_1 + \hat{a}_2)/\sqrt{2} \quad \text{and} \quad \hat{d}_2 = (\hat{a}_1 - \hat{a}_2)/\sqrt{2}.
\]

The two single-electron impurity states become \(|1\rangle\) and \(|2\rangle\), with \(|i\rangle = \hat{d}_i^\dagger|0\rangle\). The Hamiltonians described in Eqs. (1) and Eq. (3) can be rewritten, respectively, as

\[
\tilde{H}_s = \frac{\varepsilon_0}{2} \sum_{u=1,2} \hat{d}_u^\dagger \hat{d}_u + \frac{\Delta \varepsilon}{2} (\hat{d}_1^\dagger \hat{d}_2 + \hat{d}_2^\dagger \hat{d}_1) + U \hat{n}_1 \hat{n}_2,
\]

with \(\hat{n}_u = \hat{d}_u^\dagger \hat{d}_u\), and

\[
\tilde{H}_{SB} = \sum_{\alpha uk} (\tilde{t}_{\alpha uk} \hat{d}_u^\dagger \hat{c}_\alpha + H.c.),
\]

with the tunneling coefficients \(\tilde{t}_{\alpha uk} = \sqrt{2t_{\alpha k}} \cos(\phi/4)\) and \(\tilde{t}_{L2k} = \tilde{t}_{R2k} = i\sqrt{2t_{\alpha k}} \sin(\phi/4)\). They lead to the coupling strengths in the hybridization spectral function of Eq. (11) as \(22\),

\[
\tilde{\Gamma}_{a11} = 2\Gamma \cos^2(\phi/4), \quad \tilde{\Gamma}_{a22} = 2\Gamma \sin^2(\phi/4),
\]

\[
\tilde{\Gamma}_{L12} = \tilde{\Gamma}_{L21} = \tilde{\Gamma}_{R21} = \tilde{\Gamma}_{R12} = i\Gamma \sin(\phi/2).
\]

Note that here, we considered the symmetrical coupling strength and \(\lambda_L = \lambda_R = \lambda = 1\).

The observed CB-assisted interference with AB interferometer could be understood based on Eqs. (11). First of all, the auto-correlation noise describes the fluctuations of the electrons tunneling forth and back from the dots. Its spectrum is insensitive to the AB phase as we elaborated earlier, cf. Fig. 4(a). Therefore, as the Rabi characteristic in relation to the AB phase is concerned, the circuit noise spectrum [Fig. 4(b)] is effectively opposite to that of cross-correlation [Fig. 4(a)]. The latter will be the focus below. We identify the following three cases according to the value of AB phase.

(i) For \(\phi = 2m\pi\) [black-curve in Fig. 4(a)], we have either \(\tilde{\Gamma}_{a11} = 0\) or \(\tilde{\Gamma}_{a22} = 0\), with odd or even \(m\), respectively, and accordingly \(\tilde{\Gamma}_{a12} = 0\). The electron tunnels through only one of the transformed single-electron channels, either \(|1\rangle\) or \(|2\rangle\). However, the second term in Eq. (11) induces the transition between \(|1\rangle\) and \(|2\rangle\). The

IV. SUMMARY

In summary, we have thoroughly investigated the nonequilibrium quantum noise spectrum of the transport current through a nondegenerate double–dot embedded in an AB interferometer. Based on accurate DEOM evaluations, we demonstrate the rich spectroscopic signatures in relation to the Rabi oscillation between the two charge states in the double dots. For noninteracting double dots, the total circuit noise spectrum only shows non-Markovian quasi-steps at \(\phi = 0\), but Rabi dip at nonzero AB phase. For strong interdot Coulomb interaction, the spectrum signatures are peak and dip at \(\phi = 0\) and \(\phi = \pi\), respectively, but Fano profiles at other values of AB phase in between. Occurs there a remarkable Coulomb-blockade-assisted Rabi interference, arising from the interplay between various mechanisms. These include the environment-assisted indirect interdot coupling, coherent Rabi oscillation, lead-specified current-current interference, interdot Coulomb interaction, and further AB phase. All these mechanisms and the interplay between them are elaborated in the present work. The rich characteristics are inaccessible in the average current and zero-frequency shot noise. To the best of our knowledge, these results are the first uncovered in the present work and may have potential applications in the field of quantum computer and quantum information.

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Appendix A: The DEOM theory

In this appendix, we briefly outline the derivation of the two-time current-current correlation function based on the DEOM approach. The details see the references [50, 51]. Consider an electron transport setup, in which an impurity system ($H_\text{ss}$) is sandwiched by electrodes bath ($h_\alpha$), under electric bias potential ($eV = \mu_L - \mu_R$) applied across the leads, $\alpha = L$ and $R$. The total Hamiltonian reads $H_T = H_\alpha + h_\alpha + H_{\text{ss}}$. The system Hamiltonian $H_\text{ss}$ includes the electron-electron interaction and is given in terms of local electron creation $\hat{a}_\alpha^\dagger$ (annihilation $\hat{a}_\alpha$) operators of the specified system, for instance, see Eq. (10) in the present study. The electrodes bath is modeled as noninteracting electrons reservoir described by Eq. (2). The transport coupling between the reservoir and the system is given by Eq. (3) which is rewritten as

$$H_{\text{ss}} = \sum_{\alpha u} \left( \hat{a}_\alpha^\dagger \hat{F}_{\alpha u} + \text{H.c.} \right), \quad (A1)$$

with $\hat{F}_{\alpha u} = \sum_k e^{i\phi_{\alpha k}} t_{\alpha u k} c_k$. For the calculation of the current noise spectrum, we should restore to the technique of the dissipatons decomposition on the hybrid bath [50, 51], i.e., $\hat{F}_{\alpha u}^\sigma = \sum_{k=1}^K \sigma \hat{f}_{\alpha u k}^\sigma$ in Eq. (A1), with the dissipatons operator $\hat{f}_{\alpha u k}^\sigma$ satisfying $\langle \hat{f}_{\alpha u k}^\sigma(t) \hat{f}_{\alpha k'}^\sigma(0) \rangle_B = -\delta_{\sigma\sigma'} \delta_{\alpha\alpha'} \delta_{kk'} \eta_{\alpha u k} \delta_{-\gamma_{\alpha u k} t}$. This dissipatons decomposition arises from the nonequilibrium interacting reservoirs bath correlation functions [60, 61, 62, 71, 72], in an exponent expansion form of

$$\langle \hat{F}_{\alpha u}^\sigma(t) \hat{F}_{\alpha u}^\sigma(0) \rangle_B = \sum_{k=1}^K \eta_{\alpha u k}^\sigma e^{-\gamma_{\alpha u k} t}. \quad (A2)$$

Such an exponential expansion in Eq. (A2) is realized via a sum-over-poles decomposition for the Fourier integral of the relation which is the so-called fluctuation-dissipation theorem [61, 62, 72, 77]: $\langle \hat{F}_{\alpha u}^\sigma(t) \hat{F}_{\alpha u}^\sigma(0) \rangle_B = \frac{1}{2\pi} \int d\omega e^{i\omega t} \sum_{\alpha u k} \hat{f}_{\alpha u k}^\sigma \hat{J}_{\alpha u k}(\omega) \hat{J}_{\alpha u k}(\omega)^\dagger$, followed by Cauchy’s contour integration. Here, we adopt the Lorentz form of the hybridization function, see Eq. (10), with $J_{\alpha u k}(\omega) = J_{\alpha u k}(\omega) = J_{\alpha u k}(\omega)$. The exponents $\gamma_{\alpha u k}$ in Eq. (A2) arise from both the Fermi function and the hybridization function.

For bookkeeping, we adopt the abbreviations, $j \equiv (\sigma u k)$ and $\hat{f} \equiv (\sigma u k)$. For the collective indexes in fermionic dissipatons, such that $f_j = f_{\alpha u k}$ and so on. The superindex $\sigma = +, -$ (and $\bar{\sigma}$ is its opposite sign) is used to redefine the fermion creation and annihilation operators, e.g., $\hat{a}_\alpha^\dagger \equiv \hat{a}_\alpha^\dagger \bar{\sigma}$ and $\hat{a}_\alpha \equiv \hat{a}_\alpha \bar{\sigma}$ in Eq. (A6). The quantum coherence and/or decoherence dynamics of the impurity systems are described by the reduced density matrix, $\rho(t) \equiv \text{tr}_B \rho_{\alpha\alpha}(t)$, i.e., the partial trace of the total density operator $\rho_{\alpha\alpha}(t)$ over the electrode bath degrees of freedom. Dynamical variables in DEOM are the reduced dissipatons density operators (DDOs),

$$\rho_j^{(n)}(t) \equiv \rho_{j_1 \ldots j_n}(t) \equiv \text{tr}_B \left[ \left( \hat{f}_{j_n} \cdots \hat{f}_{j_1} \right)^n \rho_{\alpha\alpha}(t) \right]. \quad (A3)$$

Here, $\left( \hat{f}_{j_n} \cdots \hat{f}_{j_1} \right)^n$ specifies an ordered set of $n$ irreducible dissipatons. A swap of any two irreducible fermionic dissipatons causes a minus sign, such that

$$\left( \hat{f}_{j_n} \hat{f}_{j_1} \right)^n = -\left( \hat{f}_{j_1} \hat{f}_{j_n} \right)^n. \quad (A4)$$

From Eq. (A3), it is clear that the reduced system density operator is just $\rho(t) \equiv \rho^{(0)}(t)$. The DEOM formalism addresses also the hybridizing bath subspace dynamics. After some algorithm together with Wick’s theorem, the DEOM formalism has been obtained as [50, 51]

$$\rho_j^{(n)} = -\left( iL_\varepsilon + \sum_{r=1}^n \gamma_{j_r} \right) \rho_j^{(n)} - i \sum_{r=1}^n \gamma_{j_r} \rho_j^{(n+1)} \quad \text{for} \quad \rho_j^{(n)} = \rho_j^{(0)} \quad \text{and} \quad \rho_j^{(0)} = \rho_j^{(0)}(t). \quad (A5)$$

The Grassmannian superoperators, $A_j = A_j^{\sigma_{\alpha u k}} = A_u^{\sigma}$ and $C_j \equiv C_{\alpha u k}$ in Eq. (A5), are defined via $64, 66, 68$:

$$A_u^{\sigma} \hat{O}_\pm \equiv \alpha^\sigma \hat{O}_\pm \pm \hat{O}_\pm \hat{a}_u^\sigma \equiv \left[ \hat{a}_u^\sigma, \hat{O}_\pm \right] \pm C_{\alpha u k} \hat{O}_\pm \equiv \sum_{\nu} \left( \eta_{\alpha u k}^\sigma \hat{a}_\nu^\sigma \hat{O}_\pm + \eta_{\alpha u k}^\sigma \hat{O}_\pm \hat{a}_\nu^\sigma \right). \quad (A6)$$

Here, $\hat{O}_\pm$ is an arbitrary operator, with even ($+$) or odd ($-$) fermionic parity, such as $\rho^{(2m)}$ or $\rho^{(2m+1)}$, respectively.

The hierarchical structure in Eq. (A5) is formally identical to that derived originally via the calculus on path integral influential functions [64]. All $\{ \rho_j^{(n)} \}$ are now the physically well-defined functions with Eq. (A3) that goes by the mathematical irreducibility. The hierarchical coupling can be simply truncated by setting all $\rho_j^{(n>L)} = 0$, at a sufficiently large $L$. While all $L$-body dissipatons dynamics are treated exactly, the resulting closed DEOM for $\{ \rho_j^{(n)} ; n = 0, 1, \ldots, L \}$ represent also a dynamical mean-field scheme for higher-order DDOs. In this sense, DEOM is naturally a nonperturbative many-particle theory. The present algebraic construction, based on the quasi-particle (dissipaton) description of hybridizing bath, renders the DEOM, represented by Eq. (A5), a correlated system and bath dynamics theory. It can be used in the accurate evaluation on nonequilibrium properties of current noise spectrum below.

From the definition, $\hat{I}_\alpha = -\frac{1}{\mu} \left( \sum_k c_k^\dagger c_k \right)$, the current operator, for the electron transfer from $\alpha$-reservoir to the impurity system, reads

$$\hat{I}_\alpha = -i \sum_{\mu} \left( \hat{a}_\mu^\dagger \hat{F}_{\alpha u} - \hat{F}_{\alpha u} \hat{a}_\mu \right) = -i \sum_{j \in J} \hat{a}_j \hat{f}_j. \quad (A7)$$
The second identity in Eq. (A7) is expressed in the dissipatons decomposition, where \( \hat{a}_j \equiv \hat{a}^\sigma_{auk} = \hat{a}_u = \hat{a}_u \) and \( j_a \equiv \{ \sigma uk \} \in j \equiv \{ \sigma uk \} \). The mean current can then be evaluated in terms of the first-tier DDOs as

\[
I_\alpha(t) = \text{Tr}_T[\hat{I}_{\alpha} \rho_{\text{tot}}(t)] = -i \sum_{j_a \in j} \text{tr}_s[\hat{a}_{j} \rho_j^{(1)}(t)].
\] (A8)

The trace \( \text{tr}_s \) runs over the system degrees of freedom. It has been used in evaluating both steady-state and time-dependent transient current in various situations [70–78], including dynamical Kondo memory [67] and thermopower [79] responses.

Similarly, the two-time current-current correlation function can be evaluated as

\[
\langle \hat{I}_{\alpha}(t) \hat{I}_{\alpha'}(0) \rangle = \text{Tr}_T[\hat{I}_{\alpha} \rho_{\text{tot}}(t; \alpha')] = -i \sum_{j_a \in j} \text{tr}_s[\hat{a}_{j} \rho_j^{(1)}(t; \alpha')],
\] (A9)

where

\[
\rho_{\text{tot}}(t; \alpha') \equiv e^{-i\mathcal{H}_T t} \hat{I}_{\alpha} \rho_{\text{tot}}^{\text{st}},
\] (A10)

with \( \rho_{\text{tot}}(t = 0; \alpha') = \hat{I}_{\alpha} \rho_{\text{tot}}^{\text{st}} \) and \( \rho_{\text{tot}}^{\text{st}} \) denoting the steady-state total composite density operator, under the bias voltage of \( V = \mu_L - \mu_R \). The key step for the calculation of the correlation function based on the DEOM evaluation is to identify the initial DDOs, \{\rho_j^{(n)}(0; \alpha')\} that are associated with \( \rho_{\text{tot}}(t; \alpha') = \hat{I}_{\alpha} \rho_{\text{tot}}^{\text{st}} \). Based on the underlying dissipatons algebra and the generalized Wick’s theorem (for the details see Refs. [56–58]). The initial values of DDOs in Eq. (A9) are given by

\[
\rho_j^{(n)}(0; \alpha') = -i \sum_{j_a' \in j'} \bar{a}_{j'} \rho_{j'}^{(n+1);\text{st}} - i \sum_{r=1}^n (-)^{n-r} \bar{C}_{j'} \rho_{j'}^{(n-1);\text{st}},
\] (A11)

where, \{\rho_{(n\pm 1);\text{st}}\} are the steady-state solutions to DEOM (A5) by using the conditions, \{\rho_j^{(n);\text{st}} = 0; \forall n\} together with the normalization constraint, \( \text{tr}\rho^{(0)} = 1 \), under given constant bias potential. The resulting initial \{\rho_j^{(n)}(0; \alpha')\} are then propagated, by using the real-time dynamics of Eq. (A5) again, to obtain \{\rho_j^{(n)}(t; \alpha')\}. Finally, the lead-specified current correlation function, \( \langle \hat{I}_{\alpha}(t) \hat{I}_{\alpha'}(0) \rangle \), is evaluated according to Eq. (A9). Consequently, we can get the current noise spectrum Eq. (B3) together with Eq. (7).

**Appendix B: The effect of the parameter \( \lambda \)**

As expected, both the Rabi signal in the noise spectrum and the Rabi oscillation dynamics become weaker as the indirect coupling parameter decreases. Figure (B3) and (c) report this characteristic, in the auto-correlation noise spectrum and the reduced density matrix coherence evolution, respectively, as demonstrated with \( \phi = \pi \). The oscillatory quantum interference pattern disappears if there is no indirect coupling (\( \lambda = 0 \)). Meanwhile, the effect of AB phase \( \phi \) on the noise spectrum remain qualitatively the same as Fig. (2a) and (b). It affects little on the auto-correlation noise spectrum, but dramatically on the cross-correlation one. The latter dictates the effect of AB phase \( \phi \) on the total circuit noise spectrum, \( S(\omega) \) of Eq. (5).

Figure (B3) b) and (d) depict the characteristic peak/dip values of the cross-correlation and circuit noise spectra, respectively, at the Rabi frequency \( \omega = \Delta \epsilon \). The observations here can be understood via \( \Gamma_{\nu 12} \) of Eq. (B3). This cross-type of system–reservoirs coupling is responsible for the Rabi interference in transport. It is nonzero only when the indirect interdot tunnel coupling parameter \( \lambda \neq 0 \). Its absence leads to the electrons tunneling through channels 1 and 2 being independent. The indirect interdot tunnel coupling gives rise to the indistinguishability between the two channels. This leads further to the coherence between the two charge states \( |1\rangle \) and \( |2\rangle \) in the double-dot, and the Rabi oscillation occurs whenever \( |1\rangle \) and \( |2\rangle \) are also nondegenerate.
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