ON THE ULTRAVIOLET-BRIGHT PHASE OF METAL-RICH HORIZONTAL-BRANCH STARS

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ABSTRACT

We consider the origin of the UV-bright phase of metal-rich helium-burning stars, the slow blue phase (SBP) that was predicted by various earlier works. Based on improved physics including OPAL opacities, which is the same physics that was used in the construction of the new Yale Isochrones, we confirm the existence of the SBP. In addition to our grid of evolutionary tracks, we provide an analytical understanding of the main characteristics of the SBP phenomenon.

The SBP is slow because it is a slow-evolving, helium-shell-burning phase that is analogous to the early asymptotic giant branch phase. The SBP of a more metal-rich star is slower than a metal-poor counterpart if they have the same \( T_{\text{eff}} \) because a more metal-rich helium-burning star has a smaller mass than a metal-poor one and because lifetime increases as mass decreases.

Metal-rich helium-burning stars easily become hot because the luminosity from the hydrogen-burning shell is extremely sensitive to the mean molecular weight \( \mu \), whereas the luminosity from the helium-burning core is not. Under the assumption of a positive \( \Delta Y/\Delta Z \), helium abundance plays the most important role in governing \( \mu \), and thus Dorman et al. found that the SBP occurs only when \( Z < 0.4 \). When \( \Delta Y/\Delta Z \geq 0 \), we suggest that the SBP phenomenon is a major cause of the UV upturn phenomenon in giant elliptical galaxies, as will be shown in subsequent papers. The new HB tracks can be retrieved from S. Y.’s web site http://shemesh.gsfc.nasa.gov/astrology.html.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: evolution — galaxies: stellar content — stars: evolution — stars: horizontal-branch — ultraviolet: galaxies

1. INTRODUCTION

Early numerical studies of the advanced evolution of metal-rich stars by Demarque & Pinsonneault (1988) and by Horch, Demarque, & Pinsonneault (1992) suggested that, under the simplest assumptions about mass loss on the red giant branch (RGB) and galactic helium enrichment (\( \Delta Y/\Delta Z \approx 2–3 \)), stars in the core helium-burning (i.e., post-RGB) phase of evolution do not evolve into asymptotic giant branch (AGB) stars. Instead, before becoming white dwarfs (WDs), they evolve into UV-bright objects. Horch et al. (1992) named this evolutionary phase the slow blue phase (SBP). Horch et al. (1992) also found that the transition total mass, \( M_{\text{trans}} \), the total mass below which the UV-bright phase occurs, is highly sensitive to metallicity in the sense that it increases with increasing metallicity. In other words, a larger fraction of stars become UV bright instead of becoming post-AGB stars as metallicity increases.

These UV-bright SBP stars can be divided into two groups. Those in the first group are very low mass, helium-burning stars that never become red (i.e., AGB) but instead evolve directly into the WD phase (Sweigart, Mengel, & Demarque 1974). They spend all of their lifetimes on the hot horizontal branch and more luminous hot phases. Those in the second group are moderately low-mass helium-burning stars that first evolve to become AGB stars but then quickly return to the SBP without reaching the tip of the AGB. Both types are sensitive to metallicity in the sense that a larger fraction of helium-burning stars become SBP stars as metallicity increases.

Greggio & Renzini (1990) postulated the existence of the first group in metal-rich systems using a gedankenexperiment and named it the AGB-manqué stage. Meanwhile, Castellani & Tornambé (1991) independently noticed the second group in their calculations and called it the post-early AGB. The SBP phenomenon was later confirmed by more detailed numerical calculations (Horch et al. 1992; Dorman, Rood, & O’Connell 1993; Fagotto et al. 1994).

These discoveries are provocative because they are opposite to the traditional concept of the evolution of HB stars as a function of metallicity (more metal-rich HB stars are cooler). They suggest that old, metal-rich stellar systems may be, in fact, better UV light generators than metal-poor systems if they are sufficiently metal-rich (\( Z \geq Z_{\odot} \)) to experience the SBP phenomenon. Moreover, these discoveries are consistent with the hitherto mysterious UV flux-metallicity correlation (UV upturn phenomenon) in giant elliptical galaxies (Code & Welch 1979; Burstein et al. 1988). That is, if such metal-rich, UV-bright HB stars are indeed the major UV sources in giant elliptical galaxies, the observed positive UV upturn-metallicity relationship can be naturally understood.

In the same direction, another important step forward was made by Castellani & Castellani (1993), who pointed...
out that the inclusion of mass loss can have an effect on the evolution near the tip of the giant branch, particularly for stars with low envelope masses. Thus, if the mass loss takes place primarily near the giant branch tip, then the core contraction that triggers the helium flash can be initiated even if the helium core mass is below the critical mass for a no-mass-loss model. This result in the possibility of helium ignition as the star reaches the extended horizontal-branch and for substellar B core masses smaller than the core masses of ordinary horizontal branch stars. This work was recently followed up and extended by D'Cruc (1996), who explained the existence of substellar B stars by postulating extreme mass loss on the giant branch.

However, physics of such late stellar evolution is not well understood and is somewhat sensitive to the input physics, such as opacities. Moreover, because of difficulty in finding the actual stars, owing to their rarity in stellar samples, some doubt has been cast about this purely theoretical prediction (Lee 1994).

Motivated by these theoretical discoveries, we have carried out extensive calculations of advanced stellar evolution for a variety of chemical compositions and masses in order to investigate the validity of the theoretical predictions and better understand the physical basis of the SBP phenomenon.

2. CONSTRUCTION OF EVOLUTIONARY TRACKS

The same improved physics that has been used in the calculations of the new Yale Isochrones 96 (Demarque et al. 1996) has been used to construct the helium-burning phase evolutionary tracks. It includes OPAL opacities (Rogers & Iglesias 1992), Kurucz low temperature opacities (Kurucz 1991), and improved energy generation rates (Bahcall & Pinsonneault 1992).

Semiconvection is included, but overshooting, diffusion, and the Debye-Hückel correction have not been taken into account in order to be consistent with the MS through RGB calculations in the isochrones. The neutrino cooling rates used on the giant branch, which are important in determining the helium core mass at helium ignition at the tip of the RGB, are those of Itoh et al. (1989). These physical considerations have an influence particularly on the evolutionary timescale of the model, which is still uncertain by several gigayears (Chaboyer & Kim 1995; Chaboyer et al. 1996a).

For instance, the age scale adopted by Chaboyer et al. (1996a) in their Monte Carlo study is 17% lower than the one used in the study of the Galactic halo chronology recently published by Chaboyer, Demarque, & Sarajedini (1996b). This difference can be accounted for in the following way: a 7% decrease due to the improvements in the equation of state mentioned above, and another 7% decrease due to the inclusion of helium diffusion. The additional 3% decrease is due to an upward revision in the [$\alpha$/Fe] ratio from 0.4 to 0.55 in Chaboyer et al. (1996a).

Note that the sensitivity of globular cluster ages to [$\alpha$/Fe] is modest, a result in agreement with the earlier conclusion of Chieffi, Straniero, & Salaris (1991). Therefore, considering other unknown sources of uncertainty, only relative age is meaningful in this study. However, the general evolutionary patterns are not sensitive to these details in the input physics. Therefore all the conclusions that will be made in this paper remain the same except the absolute sense of the age.

Metal-rich ($Z \geq 0.01$) models have been constructed for $\Delta Y/\Delta Z = 2$ and 3. It is believed that this range embraces the true value, considering that $\Delta Y/\Delta Z = 2.75$ for the Sun, assuming primordial chemical composition ($Z, Y)_0 = (0, 0.23)$ and current composition ($Z, Y)_C = (0.0188, 0.2817)$, respectively (Anders & Grevesse 1989; Guenther et al. 1992). The galactic helium enrichment has not been taken into account for metal-poor ($Z < 0.01$) stellar evolution because the effect is negligible. Figures 1 and 2 show the evolutionary tracks, and detailed information on the model runs is given in Table 1.

Fewer models than actually constructed are displayed in Figures 1 and 2 for the sake of clarity. As generally accepted, more massive stars are cooler until the total mass $M_\text{tot}$ reaches a certain value (about 1 $M_\odot$), but above that mass the trend becomes reversed. For instance, the most massive model, $M_\text{tot} = 1.5 M_\odot$, is hotter than the next massive model, $M_\text{tot} = 0.9 M_\odot$, which is clearly seen in the $Z = 0.001$ panel (Demarque & Hirshfeld 1975). It is also seen that the SBP, the UV-bright phase, becomes much more prominent as metallicity increases.

To demonstrate the effect of the improved physics, some new models are compared in Figure 3 with models that were constructed using the old physics. As expected, the new metal-rich models with improved opacities are redder than the old models. On the other hand, the new metal-poor models, which are less sensitive to opacities, are slightly brighter than the old ones because of recent revisions to the nuclear energy generation rates.

While the evolutionary tracks are generally similar, some tracks show a notable difference. For example, the new 0.56 $M_\odot$ model in the left panel of Figure 3 becomes an AGB star, whereas the old model becomes a SBP star. Such a difference is present only for metal-rich models where the change in the opacities is significant. In addition, such an

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2 There may have been several empirical discoveries of such stars recently. The most promising examples are the hot stars in NGC 6791, an old, metal-rich open cluster (Kaluzny & Udalski 1992; Liebert, Saffer, & Green 1994; Kaluzny & Rucinski 1995).

3 There is still some uncertainty in the solar helium abundance that, when one allows for Coulomb interactions in the equation of state and for the effects of helium diffusion, would be closer to $\bar{Y} = 0.27$ (Proffitt 1992; Guenther, Kim, & Demarque 1996).

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| TABLE 1 | MODEL DATA USED IN FIGURES 1 AND 2 |
|---------|-----------------------------------|
| $Z$     | $Y$     | $M_{\text{core}}$ (M$_\odot$) | $M_{\text{env}}$ (M$_\odot$) |
|---------|---------|-------------------------------|------------------------------|
| 0.0001  | 0.236   | 0.501                         | 0.004, 0.02, 0.06, 0.10, 0.14, 0.22, 0.40, 1.00 |
| 0.0004  | 0.237   | 0.498                         | 0.007, 0.02, 0.06, 0.10, 0.14, 0.22, 0.40, 1.00 |
| 0.0100  | 0.242   | 0.488                         | 0.005, 0.02, 0.04, 0.07, 0.11, 0.15, 0.41, 1.01 |
| 0.0040  | 0.241   | 0.490                         | 0.007, 0.02, 0.04, 0.07, 0.11, 0.15, 0.41, 1.01 |
| 0.0100  | 0.250   | 0.470                         | 0.005, 0.02, 0.03, 0.05, 0.09, 0.43, 1.03 |
| 0.0100  | 0.260   | 0.470                         | 0.005, 0.02, 0.03, 0.05, 0.09, 0.43, 1.03 |
| 0.0200  | 0.270   | 0.470                         | 0.005, 0.02, 0.03, 0.05, 0.09, 0.43, 1.03 |
| 0.0200  | 0.290   | 0.460                         | 0.005, 0.02, 0.04, 0.06, 0.10, 0.44, 1.04 |
| 0.0400  | 0.310   | 0.460                         | 0.005, 0.02, 0.04, 0.06, 0.10, 0.44, 1.04 |
| 0.0400  | 0.350   | 0.450                         | 0.005, 0.02, 0.04, 0.07, 0.11, 0.45, 1.05 |
| 0.0600  | 0.350   | 0.450                         | 0.005, 0.02, 0.05, 0.07, 0.11, 0.45, 1.05 |
| 0.0600  | 0.410   | 0.440                         | 0.005, 0.02, 0.06, 0.08, 0.16, 0.46, 1.06 |
| 0.0100  | 0.430   | 0.440                         | 0.005, 0.02, 0.05, 0.12, 0.16, 0.20, 0.46, 1.06 |
| 0.0100  | 0.530   | 0.430                         | 0.005, 0.02, 0.05, 0.09, 0.17, 0.29, 0.47, 1.07 |

* Envelope mass: $M_{\text{env}} = M_{\text{tot}} - M_{\text{core}}$. 
The question of the origin of the SBP can be divided into two parts: (1) why is the SBP slow?, and (2) why is it blue? The first question is easier to answer. First, the SBP is a helium-shell-burning phase like the early AGB phase, in which stars evolve very slowly. Moreover, the lifetime of a star in this slow phase, from the zero-age HB (ZAHB) to the evolved HB phase, increases with metallicity. This can be understood easily based on the so-called mass-luminosity relation: a more massive star is brighter and, thus, dies more quickly. Similarly, a core helium-burning star with a smaller $M_{\text{core}}$ lives longer than one with a larger $M_{\text{core}}$ when the total masses are the same.

To begin with, a metal-rich red giant experiences the helium core flash before its core becomes as large as that of a metal-poor counterpart (Sweigart & Gross 1978). This is because in a more metal-rich red giant the higher opacity outside the degenerate core causes the temperature in the core to rise to that required to initiate the helium core flash more quickly. So, a more metal-rich core helium-burning star has a smaller $M_{\text{core}}$ than a less metal-rich one. Therefore, a more metal-rich model has a longer lifetime in the core helium-burning phase. For example, let us compare two core helium-burning stars with the same $M_{\text{env}}$ but with different metallicities. A model of $[M_{\text{tot}} = 0.445 M_\odot, Z = 0.06, Y = 0.41, M_{\text{env}} = 0.005 M_\odot, \log T_{\text{eff}} (\text{ZAHB}) = 4.38]$ has a lifetime of 270 Myr in the core helium-burning phase before it becomes a WD, which is 80% longer than the lifetime of a metal-poor model (150 Myr) of $(0.495, 0.004, 0.24, 0.005, 4.42)$. As a result, when we have two stellar systems that have similarly hot HBs but different metallicities as the examples given above, then the metal-rich system would have twice as many hot HB stars as the metal-poor system does.

The second question is more difficult to understand. The classical understanding about the SBP (Horch et al. 1992) is as follows: A core helium-burning star with a thin envelope burns up its hydrogen-burning shell quickly compared to the core helium-burning timescale, giving the star little time to expand the envelope slowly and become an AGB star. Instead, it becomes a hot, UV-bright star because it shrinks in radius (i.e., surface gravity increases) as its hydrogen...
Fig. 2.—Same as Fig. 1, but for $Z \geq 0.01$. Left and right panels are for $\Delta Y/\Delta Z = 2$ and 3, respectively. Note that more massive stars evolve into the UV-bright phase rather than into AGB as metallicity increases. This phenomenon is more conspicuous for $\Delta Y/\Delta Z = 3$. 
baking in the shell quickly decreases with time. In order to see this phenomenon, the envelope mass, $M_{env}$, should be as low as $M_{env} \lesssim 0.05 M_\odot$ for a star with $Z = Z_\odot$. It requires many billions of years for a stellar population to develop a substantial fraction of HB stars with such a low-mass envelope, assuming a moderate mass-loss efficiency. Horch et al. (1992) suggested that this UV-bright phase should occur even for stars with a larger total mass if their metallicity is higher. They suggested that the transition total mass, $M_{tot}^{tr}$, increases monotonically with increasing metallicity. Therefore even a star with a massive envelope may develop a UV-bright star. Dorman et al. (1993) also found the same phenomenon and emphasized that this phenomenon becomes efficient only when $Y \gtrsim 0.4$.

A more detailed and quantitative analysis has been carried out in this study in order to check the suggestions that were made by the previous theoretical predictions, and to understand the cause of the UV upturn phenomenon better. First, Figures 1 and 2 qualitatively but clearly show that the SBP occurs only in the very low mass, core helium-burning stars and that it happens to the more massive stars when the metallicity is higher under the assumption of a positive $Y/Z$. The UV-bright phenomenon is illustrated in Figure 4. The left panel is the relation between the transition total mass, $M_{tot}^{tr}$, and metallicity. It shows that $M_{tot}^{tr}$ is not sensitive to metallicity at all for $Z < Z_\odot$ and becomes very sensitive especially when $Y \gtrsim 0.4$ as Dorman et al. (1993) suggested. However, a more realistic comparison is the relationship between the transition envelope mass ($M_{env}^{tr}$) and metallicity, because the classical understanding of the SBP requires a small envelope mass, not a small total mass. The dependence of $M_{env}^{tr}$ on metallicity is shown in Figure 4b, in which $M_{env}^{tr}$ monotonically increases as a function of metallicity for the positive values of $Y/Z$.

It is important to understand why such a tight correlation between $M_{env}^{tr}$ and metallicity exists. To begin with, it is clear that chemical composition plays an important role in governing $M_{env}^{tr}$. We would like to know what aspect of metallicity is the main element of this behavior. Therefore the evolutionary tracks of the metal-rich models with the same mass, but with different chemical composition and/or core mass, have been compared with one another in order to disentangle the effects of one variable from those of another.

Figure 5 shows the evolutionary tracks of eight models with different sets of parameters. The range of variation in the input parameters is consistent with current expectations. It is apparent that only the models with higher $Y$ become UV bright. An increase in $Z$ from 0.06 to 0.1 does not play as important a role in making a star evolve to be a UV-bright star as an increase in $Y$ from 0.35 to 0.43 does. This agrees with Dorman et al.’s (1993) suggestion that the SBP occurs only when $Y \gtrsim 0.4$. Certainly the core mass plays little role within this range.

Figure 6 confirms Horch et al.’s (1992) suggestion; the UV-bright phase is caused by the fast exhaustion of the hydrogen-burning shell resulting from the high sensitivity of $L_H$ (luminosity from the hydrogen burning) to $\mu$ (mean molecular weight). Those models that become UV-bright stars burn up hydrogen in the CNO cycle near the ZAHB (at small ages) much more efficiently than the others, as shown in the top panel of Figure 6.

A simple analytic relationship between luminosity $L$ and $\mu$ in the standard model was well described in many early works including Clayton (1968, eq. [3-190]). This formalism was originally developed for the stars with a single energy source. Therefore, the exact validity for core helium-burning stars (with double energy sources) is debatable. We assume that such a simplified model should be reasonable if
it is used only to demonstrate the relation between $L$ and $\mu$ approximately. According to such a simplified model, luminosity in Clayton’s eq. (3-190) can also be expressed as follows:

$$L \propto M^3(\mu\beta)^4\langle\kappa\eta_n\rangle^{-1},$$  \hspace{1cm} (1)

where $\beta$ is the ratio of gas pressure to total pressure at the stellar center, $\kappa$ is opacity, $\eta_n$ is the ratio of the average rate of nuclear energy generation interior to a certain point to the average for the whole star, and $M$ is the total mass of the star.

Eq. (1) explicitly shows that the luminosity depends on the source of the opacities. Since the major opacity sources corresponding to the temperature and density of the helium-burning core and the hydrogen-burning shell are electron scattering ($\kappa \propto 1 + X$) and free-free absorption...
In the hydrogen-burning shell, the mean molecular weight, \( \mu \),
\[
\mu = (2X + 0.75Y + 0.5Z)^{-1}
\]
can be approximated using the surface chemical composition based on the assumption of a full ionization, because the hydrogen-burning region is very hot. On the other hand, the same expressions cannot be used for the \( \mu \) in the helium core, because the core is mostly (in general, \( Y \gtrsim 0.9 \)) composed of helium and a small amount of heavier elements (mostly carbon) regardless of the surface chemical composition. Therefore, the \( \mu \) in eq. (3) becomes more or less constant, causing \( L_{\text{He}} \) to be insensitive to the change in chemical compositions. Consequently, contrary to a simple conjecture, increase in helium abundance increases \( L_{\text{He}} \) significantly while it has little effect on \( L_{\text{H}} \), as shown in Figure 6. Figure 6 shows that, in the beginning of the core helium-burning phase, \( L_{\text{He}} \) is not affected by the change in chemical compositions much, whereas \( L_{\text{H}} \) is sensitive to helium abundance. Once evolution begins, even \( L_{\text{He}} \) increases as \( Y \) increases because of the inward effect from the higher \( L_{\text{H}} \) in the hydrogen-burning shell.

Indeed, Figure 7 shows that \( M_{\text{env}}^* \) is a sensitive function of \( \mu \). However, it should be noted that chemical composition has other complicated second-order effects on the evolution besides its effects on luminosity. Thus, the simple analytic formulae that are originally developed to work for MS stars are not always precisely true in the practical numerical calculation. This simplified analytical approach should be only demonstrating the relationship between luminosities and the mean molecular weight \( \mu \), which helps us understand the origin of the UV-bright phase of metal-rich stars.

4. SUMMARY

The earlier prediction of the UV-bright phase of helium-burning stars, the SBP, has been confirmed in this study based on improved physics. According to this calculation, the SBP is more easily achieved when helium abundance is higher because stars richer in helium burn up their hydrogen-rich envelopes faster. Under the assumption of a positive galactic helium enrichment (\( \Delta Y/\Delta Z \)), this means that more metal-rich stars become UV-bright SBP stars more easily. This confirms the results of both Horch et al. (1992) and Dorman et al. (1993).

The SBP, an intrinsically slow-evolving phase analogous to early AGB phase, becomes slower as metallicity increases because more metal-rich helium-burning stars are less massive than their less metal-rich counterparts and because lifetime increases as mass decreases. Metal-rich helium-burning stars easily become hot because of the different sensitivity of the luminosity from the hydrogen-burning shell and the helium-burning core to the mean molecular weight \( \mu \). Under the assumption of a positive \( \Delta Y/\Delta Z \), helium abundance, which plays the most important role in governing \( \mu \), has a dominant effect, and Dorman et al. (1993) found that the SBP occurs only when \( Y > 0.4 \) and \( \Delta Y/\Delta Z > 0 \). In principle, an extremely metal-rich hypothetical star may become a SBP star even if the helium abundance is not that high, since the SBP is not a direct function of \( Y \) but a function of \( \mu \). However, whether such hypothetical stars exist is questionable. On the contrary, if \( \Delta Y/\Delta Z \) is higher than we assumed in this study, namely \( \Delta Y/\Delta Z > 3 \), stars do not have to be very metal-rich in order to experience the SBP phenomenon. Existing empirical data...
make us believe that the true $\Delta Y/\Delta Z$ should be either within the bracket of 2–3 or at least close to it. A more accurate determination of $\Delta Y/\Delta Z$ is required.

The HB tracks were constructed using the same input physics that is used to construct the new Yale Isochrones. These new HB tracks are qualitatively consistent with the previous models (e.g., Horch et al. 1992; Dorman et al. 1993) but slightly different, mainly because of the improvement in opacities. When the mass of the star is near the transition mass, new models tend not to evolve into the SBP. Therefore, those who are using Yale Isochrones in their galaxy or star cluster modelings are strongly recommended to use the HB tracks listed in this study.4

Some elements in the stellar evolution theory are still uncertain, although general qualitative evolutionary phenomena would not be affected by such details. Thus only the relative ages should be taken seriously. In case one adopts the final result (e.g., galaxy models from population synthesis based on stellar evolution theory), the ages of the models should be renormalized with respect to the ages of the oldest stellar systems (e.g., Galactic globular clusters).

Whether the SBP phenomenon is mainly responsible for the UV upturn phenomenon in giant elliptical galaxies will be investigated in upcoming papers (Yi, Demarque, & Oemler 1996a, 1996b).

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4 Both the Yale Isochrones 1996 and the HB tracks are available at S. Y.'s web site http://shemesh.gsfc.nasa.gov/astronomy.html.