Symmetries of the Primordial Sky

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Essay written for the Gravity Research Foundation 2022 Awards for Essays on Gravitation

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(Dated: April 1, 2022)

Abstract

Quantum field theory, which is generally used to describe the origin of large-scale gravitational perturbations during cosmic inflation, has been shown to omit an important physical effect in curved space-time, the nonlocal entanglement among quantized modes from their gravitational effect on causal structure. It is argued here that in a different model of quantum gravity that coherently preserves nonlocal directional and causal relationships, primordial perturbations originate instead from coherent quantum distortions of emergent inflationary horizons; and moreover, that causal constraints account for approximate symmetries of cosmic microwave background correlations measured at large angular separations, which are highly anomalous in the standard picture. Thus, symmetries already apparent in the large-angle CMB pattern may be unique signatures of the emergence of locality and causal structure from quantum gravity.
The world works according to two all-encompassing theories: quantum theory, which
describes the behavior of matter and energy, and general relativity, which describes the
behavior of space and time, including gravity. Each one appears to be, in its own realm, a
practically flawless description of nature.

Although they describe the same physical world, the foundations of these theories seem
to be fundamentally incompatible. The exact nature of their conflict has been expressed in
many different ways, depending on different formulations of quantum mechanics. Locality
in space and time stops making any kind of sense on a very small scale; a quantum system
smaller than the Planck length, $l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35}$ meters, is more compact than
a black hole of the same mass. But the fundamental incompatibility is much broader and
deeper than that, and is not confined to small systems\[1, 2\]. The indeterminate consequences
of a quantum event spread everywhere at the speed of light, and its nonlocal gravitational
effect distorts space-time, including its causal structure, in all directions and on all scales. In
practice, context-dependent approximations must be used to reconcile quantum coherence
with geometrical locality and causality.

For most purposes, it is possible to set aside this knotty problem, since the active quantum
effects of gravity are very small in almost all physical systems we can actually measure. A
notable exception is cosmology: during cosmic inflation, the gravitational effects of quantum
fluctuations left permanent distortions in the large scale structure of space and time, which
ultimately led to the large scale structure we observe today. In this sense, quantum gravity is
the origin of all cosmic structure, and its effects are precisely measured. Maps of the cosmic
microwave background (CMB) on large angular scales preserve a largely intact image of the
pattern of gravitational potential on our cosmic horizon generated by quantum processes
during inflation.

The standard quantum theory of how structural perturbations form is a dynamical quan-
tum field theory that includes gravity, which I will call simply the “QFT model”. In this
picture, a classical (unquantized) cosmological space-time expands exponentially, due to the
gravitational effect of a bespoke scalar “inflaton” field that has a large expected vacuum
energy density. On this homogeneous classical background, which rapidly magnifies the
physical size of all structures, a quantum field theory is introduced to compute the quantum
fluctuations of the inflaton, and its gravitational effects on geometrical perturbations that
persist, after inflation is over, to create cosmic structure. QFT achieves localization of quan-
quantum states by borrowing it from the classical background: mode amplitudes and phases are quantized, each one like a harmonic oscillator, but their mapping onto events in space-time is classical.

The QFT model has been the basic framework adopted in the theory of cosmic inflation since it was invented. It is clear why this framework has been used for about 40 years: the powerful tools of effective field theory enable extensive and detailed calculations for many different models of matter fields during inflation, and lead to a cosmological model that can account for a great deal of precise data, especially on angular scales smaller than a few degrees.

Of course, the QFT model has not actually solved the basic problems of quantum gravity, it just hides them. An elegant essay by Hollands and Wald [1] explains its subtle flaw: the quantized distortion of causal structure from gravity on all scales is not correctly taken into account in the QFT model, since the fields are defined using a fixed classical background with a determinate causal structure. In the real system, differences in causal structure created by modes with wavenumber $k \to 0$, including events at distance $R \to \pm\infty$, entangle them with quantized modes on smaller scales. The renormalization of a field theory requires subtracting infinities to describe the physical modes of the vacuum; but with gravity included, as pointed out by Hollands and Wald [1], “An individual mode will have no way of knowing whether its own subtraction is correct unless it “knows” how the subtractions are being done for all the other modes.” The QFT mode decomposition models locality by acausally (and unphysically) constraining the system at infinity independently in all directions. During inflation, acausal phase correlations between null waves arriving from antipodal directions at $R = \pm\infty$ are built into the a priori classical definitions of the quantized stationary comoving modes. This assumption matters for locality, because the actual physical position of a particle along a line depends on phase information arriving from opposite directions.

Thus, we should not be surprised that the QFT framework, by ignoring important gravitational quantum entanglement with causal structure on large scales, leads to well-known gravitational paradoxes outside of inflation, including infrared inconsistencies [3], the apparent loss of information in black-hole evaporation, and a wildly incorrect estimate [4] for the value of the cosmological constant. Its great success in microscopic experiments, especially high energy particle collisions, apparently does not extrapolate to arbitrarily extended quantum systems where gravity is important. I argue here that the QFT approximation also
FIG. 1. A particle of mass $M$ decays into two photons $\gamma_A, \gamma_B$ that travel in opposite directions. Their gravitational shock wave distorts time as measured by observing clocks from the origin, by an amount $\delta \tau \sim GM/c^3$, independent of distance $R$, in a coherent pattern (Eq. 1) aligned with the particle axis, as shown. An $S$-wave decay isotropically superposes states with different particle axes; one other example, $\gamma_A', \gamma_B'$, is shown. The gravity of this state places the space-time into a Schrödinger-cat-like macroscopic superposition of different states, with causal structures that differ coherently on large scales by $\delta \tau \sim GM/c^3$. An observer at the center measures the same coherent large-scale angular pattern for a causal diamond of any size.

leads to incorrect predictions when applied to cosmic inflation, in particular, for large-angle correlations on horizons.

As an example of the gravitational effect of coherent causal entanglement on an angular pattern, consider the classical distortion of causal structure from the gravity of an idealized EPR-type system, consisting of a pointlike particle that decays into a pair of oppositely-propagating photons (Fig. 1). For a particle of mass $M$, a spherical null gravitational shock wave from the photons creates a coherent anisotropic displacement of time at radius $R$, at an angle $\theta$ from the decay axis, of magnitude

$$\delta \tau \simeq \frac{5GM}{24c^3}(3 \cos^2 \theta - 1).$$

This distortion is the time displacement of clocks read in all directions by a single observer in the rest frame of the original decaying particle. The gravitational memory of the decay recalls the whole history of a causal diamond: first the outgoing null shock, then the incoming
data from the clocks. Since $\delta \tau$ is independent of $R$, gravity generates the same coherent large-angle pattern of distortions on surfaces of causal diamonds of any radius $R$.

For a quantum decay where the axis is indeterminate, the correspondence principle demands that the space-time geometry is placed in a superposition of different states with causal structures that share the same macroscopic angular distortion, apart from the axis direction. The clock displacements represent a coherent measurement of the gravitational effect of a nonlocalized quantum state of matter— the angular structure of a quantum state of geometry measured “from inside”. The macroscopic coherent distortion of causal structure differs profoundly from naïve expectation of the QFT model that states couple mainly on the de Broglie scale, $\lambda \sim \hbar/Mc$.

The same coherent effects generalize to a gas of gravitational shocks from a system with many null particles, and to vacuum fluctuations. A sum of many distortions like Eq. (1) from more general quantum states of null propagating pointlike particles leads to a coherent large-angle pattern with a similar angular power spectrum, but a larger amplitude. Extrapolated to a cosmological horizon volume, where the particles are numerous enough to create a mean curvature radius of order $R$, the large-angle gravitational distortions $\Delta$ on causal surfaces of size $R$ from a gas of Planck mass particle particle states are of order\textsuperscript{5}

$$\langle \Delta^2 \rangle = \langle \delta \tau^2 \rangle / \tau^2 \simeq l_P/R,$$

much larger than the value $\langle \Delta^2 \rangle \sim (l_P/R)^2$ predicted by QFT (but in agreement with a conformal field theory of near-horizon vacuum states with a Planck cut-off\textsuperscript{6}). Such large distortions of causal surfaces from vacuum-state fluctuations could even be detectable in direct experiments\textsuperscript{7,8}. We are led to suspect that directionally coherent quantum gravity could dominate the production of perturbations, and possibly leave distinctive universal signatures in the pattern on large angular scales, where the curvature of cosmological horizons is important.

Standard inflation theory beautifully matches the angular power spectrum $C_{\ell}$ of CMB anisotropy\textsuperscript{9,10} on angular scales smaller than a few degrees, at angular wavenumbers $\ell \gtrsim 30$ where the difference between realizations, or “cosmic variance”, is relatively small. Indeed, apart from the amplitude and tilt of the initial spectrum, the famous spectrum of acoustic peaks at $\ell \gtrsim 100$ is shaped mostly by well-understood post-inflationary classical processes. However, realizations of the large-scale pattern in the QFT model vary signif-
icantly, depending on random variations in initial conditions. Thus, precise cosmological
tests and parameter estimates often omit data on angular scales larger than a few degrees,
\( \ell \lesssim 30 \), which are the main focus here.

At large angles, the measured pattern of the CMB temperature anisotropy, \( \delta T/T \), ap-
proximately preserves the primordial pattern of gravitational distortions \( \Delta \) on the spherical
surface of last scattering\[11\]. Since conformal invariance of the expanding universe preserves
angular relationships, and physical horizons have spherical boundaries localized in comoving
position, directional and causal constraints on inflationary horizons are more directly related
to angular separation \( \Theta \) than to angular wavenumbers \( \ell \). Thus, we choose to study the pat-
tern using the angular correlation function \( C(\Theta) \) instead of its more familiar transform \( C_{\ell} \).

Defined in terms of angular averages,
\[
C(\Theta) = \langle \Delta(\vec{\Omega}) \rangle \langle \Delta \rangle_{\Theta,\vec{\Omega}},
\]
where \( \langle \rangle_{\Theta,\vec{\Omega}} \) denotes an azimuthal mean on a circle at a polar angle \( \Theta \) about direction \( \vec{\Omega} \),
and \( \langle \rangle_{\vec{\Omega}} \) denotes a sky average.

It has been known for a long time that the measured large-angle CMB angular correlation
function does not appear to be at all typical of standard-model realizations\[14–16\]. In
particular, \( |C(\Theta)| \) is much smaller than expected from random cosmic variation at angular
separations larger than about 80 degrees (Fig. 2). Indeed, in the best maps it appears
to vanish near 90 degrees\[17\], and is consistent with zero over a range of larger angles,
after allowing for measurement uncertainties and for a small correction from an intrinsic
unmeasured dipole\[12\].

I will now argue that in a directionally coherent theory of quantum gravity, such a nearly-
vanishing large-angle correlation has a natural interpretation: it is a signature of an exact
null symmetry of the angular correlation function of primordial perturbations,
\[
C(90^\circ \leq \Theta \leq 135^\circ) = 0; \tag{4}
\]
and that moreover, this exact symmetry follows directly from constraints on causal relational
information, based on the hypothesis that primordial perturbations are directionally coherent
quantum distortions of causal diamonds.

Consider null trajectories in comoving conformal coordinates during inflation (Fig. 3).
A direction in the sky is associated with a null trajectory on the inflationary horizon that
FIG. 2. Comparisons of measured CMB maps with models, reproduced from ref. [12]. The colored lines in the left panel show the correlation function $C(\Theta)$ of the real CMB sky (Eq. 3), as measured by the WMAP and Planck satellites [9, 13], after subtracting various models of emission from our Galaxy. The “holographic model” (HM) plotted [12] is a simple analytic approximate model derived by integrating dipolar distortions of directionally-entangled horizons, normalized to agree with the standard expected inflationary power spectrum on angular scales less than a few degrees. All of these functions are shown with a $C_1 \simeq 80\mu K^2 \cos(\Theta)$ contribution added to include the effect of an intrinsic (but unobserved) dipole. With this, the data appear to be consistent with zero for $90^\circ < \Theta < 135^\circ$, as expected from causal symmetry; the spread among maps indicates the current measurement uncertainty due to contamination by the Galaxy. For comparison, on the right, 100 equally probable realizations in the standard QFT inflation model are shown on the same scale, none of which approximate the nearly-exact null symmetry that appears to describe the real sky. The substantial large-angle variation is mainly from wavelengths comparable to and larger than the horizon, which may be an artifact of unphysically independent IR modes in the model.

arrives at our world line at the end of inflation. Its 2D normal plane represents the surface of a causal diamond of an infinitely distant point. Correlated incoming data from that direction intersects our sky at a particular comoving location, a pole on the comoving 2D sphere (our “horizon footprint”) approximately where the CMB last scattering surface is. Information from the same infinitely distant point reaches the equator on the same sphere only at the end of inflation, and never reaches the antihemisphere. There is never a causal connection between incoming directional data at a polar point on our horizon and any point in its antihemisphere, so directionally coherent correlations at $\Theta > 90^\circ$ vanish.
FIG. 3. If cosmic perturbations originate from directionally coherent quantum distortions of horizons, boundaries of vanishing angular correlation of gravitational potential among world lines are bounded by intersections of their causal diamonds during inflation. The left panel shows a causal diagram of the inflationary era in comoving conformal coordinates [12]: the vertical direction represents conformal time, and vertical lines represent comoving world lines. Inflationary horizons \( \mathcal{H} \) are past null cones that arrive at the end of inflation. They intersect surfaces of constant conformal time on “horizon footprints”, comoving 2D spherical surfaces of causal diamonds. Suppose \( \mathcal{O}_A \) lies on a horizon footprint centered on \( \mathcal{O}_B \), to the left in this figure. Where \( A \) and \( B \) horizon footprints intersect, causal entanglement allows correlations. For \( R_B \to \infty \), the \( B \) footprint is a plane that intersects the \( A \) equator: no information from any point on \( \mathcal{O}_A \)’s horizon reaches the antihemisphere on \( A \) before inflation ends. The right panel shows two horizon footprints in 3D comoving space at the end of inflation. The polar angle \( \Theta \) refers to angular correlations on our sky, the \( A \) footprint. The equator at \( \Theta = 90^\circ \) defines the boundary of the “information shadow” cast from the north polar direction. The \( B' \) sphere shown, with \( R_{B'} = R_A/\sqrt{2} \), bounds the antihemispheric causal diamonds that can introduce nonzero correlation at \( \Theta > 135^\circ \).

This “information shadow” argument relies on directional coherence of quantum gravitational distortions on causal diamonds, similar to the directionally coherent gravitational memory in the decaying-particle system (Fig. [1]). It does not apply if the gravitational potential is quantized as a local scalar, as in QFT, where gravitational memory of a potential is “frozen in”, mode by mode, as a local scalar on world lines on surfaces of constant co-
moving time. This reasoning highlights why incorrect subtraction of the largest-scale modes in renormalized QFT matters: directional causal coherence leads to nonlocal, in-common displacement of $\Delta$ for whole causal diamonds relative to infinitely distant points.

The data also show a nonzero anticorrelation close to the antipode, that is, a tendency for antipodal patches of sky to have opposite signs. This appears to be impossible according to the classical causal reasoning just given, since no information from any direction on the horizon can directly reach its antihemisphere before the end of inflation. However, in a causally coherent theory of quantum gravity, spacelike correlations can arise in the usual quantum-mechanical way, through quantum nonlocality within a coherent causal diamond. As in the original EPR thought-experiment, such correlations appear to be classically acausal, but actually show the causal effect of a nonlocal coherent state; as discussed above, the same coherence extends to spacelike angular correlations of gravitational time distortions.

During inflation, nonzero correlation can arise near the antipode from some causal diamonds smaller than our horizon that partially overlap, but extend beyond it (see Fig. 3). As above, suppose there is a coherent in-common displacement of the potential of a whole causal diamond in relation to any direction, that is, to an infinitely distant point. Points in the antihemisphere of a horizon footprint share this mean displacement, a kind of correlation that is invisible to the observer in the center. A causal diamond centered on a point $O_{B'}$ in the antipodal direction, with $O_A$ on its boundary, also inherits a coherent polar displacement from $O_A$. But, if $O_A$ and $O_{B'}$ are neither too close nor too far apart, anisotropy on the $B'$ horizon also creates correlation visible from $O_A$. The reciprocal $AB'$ causal relationship in the antihemisphere mirrors that used to derive the $AB$ information shadow; thus, nonlocal spacelike correlations on $A$ can be introduced if the angular separation $\Theta_{B'} = 360^\circ - 2\Theta$ between the antipode and the $AB'$ horizon intersection circle, as viewed from $O_{B'}$, lies in the range $0 < \Theta_{B'} < 90^\circ$, or equivalently, if information on a null plane from $O_A$ reaches $O_{B'}$ before it reaches the $AB'$ horizon intersection. As viewed from $O_A$, these nonzero correlations appear for $180^\circ > \Theta > 135^\circ$, which approximately agrees with the real sky, after accounting for the unmeasured dipole. They reflect nonlocal correlations on $B'$ horizons over a limited range of sizes, $R_A/2 < R_{B'} < R_A/\sqrt{2}$. Moreover, we expect that the parity-inverted relationship of $A$ and $B'$ horizons maps positive small-angle correlation relative to
onto a negative large-angle correlation at the $A$ antipode,

$$C_{B'}(\Theta_{B'} \to 0) > 0 \rightarrow C(\Theta \to 180^\circ) < 0,$$

again as observed.

An approximate analytic "holographic" model of $C(\Theta)$ constrained by such causal relationships[12], based on integrating over dipolar distortions of directionally-entangled horizons with relative amplitudes set by causal symmetries, is shown along with data in Fig. (2). The overall amplitude and tilt of the 3D power spectrum are normalized to match the angular power spectrum of standard cosmology where it works well, at separations smaller than a few degrees, but the holographic model matches the real sky much better on large angular scales, with no additional adjustable parameters.

Model-independent comparisons[12] show that only a small fraction of standard realizations (between 1% and 0.1%, depending on the map) approximate the real sky as well as the large-angle null symmetry, Eq.(4). In addition, antipodal antisymmetry (Eq. 5) corresponds to another well-known measured anomaly of the large-scale angular spectrum, a systematic excess of odd-parity over even-parity fluctuations for $\ell \lesssim 30$ that occurs in fewer than 1% of realizations [16]. Other causal holographic symmetries can probe information beyond that in the correlation function; for example, previously-noted anomalies associated with alignments and shapes of low order multipoles may signify a universality of great circle variance, and reveal the orientation as well as the amplitude of the intrinsic dipole[18]. Right now, the main obstacle to more powerful statistical tests of primordial angular symmetries appears to be systematic uncertainty introduced by the foreground emission of our Galaxy, which can likely be improved.

According to the standard interpretation, these remarkable properties of the large-angle pattern are all statistical flukes. As pointed out many times in the literature, a small value of $C(\Theta)$ at large angles in the standard model is not impossible, because of the large variety of possible realizations; however, any exact null symmetry of angular perturbations is impossible. The reason is plain to see in Fig. (2): independent, unconstrained modes of QFT lead to random cosmic variation inconsistent with the universal behavior required by a directional symmetry. To achieve an exact symmetry requires a nonlocal omnidirectional conspiracy among amplitudes and phases of modes on all scales, of the kind that would arise from the causality-enforced entanglement described by Hollands and Wald. Although any
given outcome can be interpreted as a statistical fluke, this particular outcome looks like a clue to something deeper going on, since the measured shortcomings of standard inflationary predictions on large angular scales appear to match the theoretically known shortcomings of the quantum field theory used to compute them. Perhaps in the correct theory, the apparent anomalies are just symmetries from causal constraints.

In a causally-coherent model, the slow-roll behavior of the unperturbed expected inflationary background, as well as the successful match to the measured perturbation spectrum at $\ell > 30$, which depends only on the slightly tilted power spectrum produced by the slow roll, remain the same as in the standard picture\cite{19}. At the same time, a fundamental symmetry in $C(\Theta)$ at large angles provides concrete clues to the deep unsolved problem of how locality and coherence work in quantum gravity. If symmetries of the sky indeed represent physically fundamental constraints, the inflationary QFT approximation is unphysical. Primordial gravitational perturbations are not scalars attached to inflaton field mode fluctuations; instead, they are coherent distortions of the emergent metric arising from quantum uncertainty in localization of causal surfaces. Coherent quantum states of geometry are not localizable like quantum scalar particles; instead, they live on null cones of emergent comoving world lines, so that geometry retains quantum coherence on horizons.

A model of inflation with coherent horizons resembles a holistic model of a black hole horizon, in which an entire 2+1D horizon surface is viewed as a single quantum object that creates nonlocal spacelike entanglement with external states in all directions\cite{20}. A cosmology based on this idea represents a radical conceptual departure from the standard view of initial conditions, since its emergent definition of locality allows no physical definition of a pre-existing, unperturbed classical background. As exemplified by the decaying-particle thought-experiment, the geometry “becomes classical” (that is, emerges from an indeterminate superposition into a definite realization) only within each causal diamond. In this view, emergence of macroscopic space-time from a quantum state continues even to the present day, as each world line’s comoving horizon expands and new information enters from outside.

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