The five accreting, millisecond X-ray pulsars in ultracompact binaries that were recently discovered with the Rossi X-ray Timing Explorer provide excellent candidates for constraining the deviations from general relativity described by the Brans-Dicke parameter $\omega_{BD}$. I calculate the expected rate of change of the orbital periods of these binaries and discuss the prospects of constraining $\omega_{BD}$ to values that are an order of magnitude larger than current constraints. Finally, I show how measurements of the orbital period derivative in ultracompact binaries can be used to place lower bounds on their orbital inclination.

Subject headings: gravitation–X-rays: binaries

1. INTRODUCTION

Neutron stars in compact binaries provide some of the best physical settings for testing the predictions of general relativity. The double neutron-star systems (the prototypical of which contains the Hulse-Taylor pulsar) have led to measurements of general relativistic predictions, such as the periastron precession and the Shapiro delay, and have given the first indirect evidence for the existence of gravitational waves (for a recent review see Will 2001).

When compared to solar system tests, however, double neutron star systems have provided only limited constraints, on alternative theories of gravity such as the Brans-Dicke theory (Will 2001). This is mostly due to the fact that the difference in the orbital period evolution of the binaries between general relativity and Brans-Dicke gravity is related to the mass difference of the two neutron stars in each system (Will 2001) and the two members of all double neutron-star systems have very similar masses (see Thorsett & Chakrabary 1999). Moreover, Brans-Dicke gravity can be constructed to be only slightly different from general relativity, through a single parameter $\omega_{BD}$, making the constraint on $\omega_{BD}$ rather weak.
Neutron stars in nature appear in various types of binaries with very small mass companions, which can in principle be used in placing stronger constraints on Brans-Dicke gravity. Tens of millisecond radio pulsars have been discovered in orbits around low mass white dwarfs, but with orbital separations too large for gravitational radiation to affect their orbital period evolution (Phinney & Kulkarni 1994). Neutron stars with low mass companions in close orbits often appear as bright X-ray sources but their orbital period evolution is dominated by mass transfer and mass loss from the companion stars in the form of a magnetic wind (Verbunt 1993). Moreover, most of these neutron stars show no periodic modulations of their X-ray flux at the stellar spin frequency, hampering measurements of the orbital periods and their evolution (Vaughan et al. 1994).

The most compact X-ray binary known to date, 4U 1820–30, which consists of a neutron star in a 11 min orbit around a $\sim 0.067 M_\odot$ companion (Rappaport et al. 1987), shows periodic X-ray eclipses, which were used by Morgan, Remillard, & Garcia (1988) to place an upper limit on its orbital period derivative of $\dot{P}/P < 3 \times 10^{-7}$ yr$^{-1}$. This limit was subsequently used by Will & Zaglauer (1989) to constrain the Brans-Dicke parameter to $\omega_{BD} \gtrsim 140$ or 600, depending on the stiffness of the neutron-star equation of state. However, 4U 1820–30 is near the center of the globular cluster NGC 6624 and hence an apparent change of its orbital period may be induced by gravitational acceleration in the potential of the cluster. This has been given as a possible reason for the orbital period decrease at a rate of $\dot{P}/P \simeq -1.1 \times 10^{-7}$ yr$^{-1}$ that was later inferred for this source by Tan et al. (1991). Furthermore, changes in the pattern of X-ray emission and the geometry of the X-ray eclipses may be responsible for the apparent changes of the orbital period, as suggested by van der Klis et al. (1993), who used a longer baseline than Tan et al. (1991) and measured a less statistically significant orbital period decrease at a rate of $\dot{P}/P \simeq -5.3 \times 10^{-8}$ yr$^{-1}$. Subsequent analysis of archival and more recent data from RXTE by Chou & Grindlay (2001) gave a marginal detection of orbital period decrease at a rate of $\dot{P}/P \simeq -3.47 \times 10^{-8}$ yr$^{-1}$. All these measurements render the constraint on the Brans-Dicke parameter $\omega_{BD}$ from 4U 1820–30 quite uncertain.

Recently, five accreting millisecond pulsars were discovered by RXTE in very compact binaries with orbital periods between 40 min and 4.3 hr (SAX J1808.4–3658: Chakrabarty & Morgan 1998; XTE J1751–305: Markwardt et al. 2002; XTE J0929–3314: Galloway et al. 2002; XTE J1807–294: Markward, Smith, & Swank 2003; XTE J1814–338: Markwardt & Swank 2003). Because the primary stars in these systems are millisecond pulsars, their orbital periods can be measured with high accuracy without significant systematic effects. Their orbital separations are very small and hence angular momentum losses via magnetic stellar winds are expected to be minimal. Moreover, all these binaries are in the galactic disk and thus are not subject to significant gravitational accelerations. Therefore, the five accreting millisecond pulsars provide prime candidates for constraining deviations from general relativity and, in particular, for constraining the parameter $\omega_{BD}$ of Brans-Dicke gravity.

In this article, I follow the analysis of Will & Zaglauer (1989) to calculate the constraints on the Brans-Dicke parameter $\omega_{BD}$ imposed by the measurement of an orbital period derivative for an
accreting millisecond pulsar. I argue that the expected constraints can be an order of magnitude larger than the constraints from solar system tests. I finally discuss how a limit on (or measurement of) the orbital period derivative can be used in placing a lower bound on the orbital inclinations of the binaries.

2. ORBITAL EVOLUTION IN BRANS-DICKE GRAVITY

In this section, I sketch the derivation of the rate of change of the orbital period of a binary in Brans-Dicke gravity. I follow closely the analysis of Will & Zaglauer (1989) but make use of more accurate relations that describe the evolution of the companion stars.

I consider the evolution of the nearly-circular orbit of a binary consisting of a neutron star, with mass $m_1$, and a low-mass companion, with mass $m_2$. The rate of change of the orbital angular momentum $J = \mu(Gma)$, where $m = m_1 + m_2$, $\mu = m_1m_2/m$, and $a$ is the semi-major axis, is

$$\frac{\dot{J}}{J} = \frac{1}{J} \frac{\partial J}{\partial m_1} \dot{m}_1 + \frac{1}{J} \frac{\partial J}{\partial m_2} \dot{m}_2 + \frac{1}{J} \frac{\partial J}{\partial a} \dot{a},$$

where $q \equiv m_1/m_2$ is the mass ratio in the binary and $\beta = -\dot{m}_1/\dot{m}_2$ is a parameter that describes the rate of mass loss from the system, if the mass-transfer is non-conservative.

For systems such as the newly discovered ultracompact binaries, orbital angular momentum may be lost because of gravitational-wave radiation or mass loss from the systems. Therefore, the left-hand-side of equation (1) is equal to

$$\frac{\dot{J}}{J} = \frac{\dot{J}_{\text{rad}}}{J} + j_w(1 - \beta) \frac{1}{q} \frac{\dot{m}_2}{m_2},$$

where $\dot{J}_{\text{rad}}$ is the rate of loss of angular momentum caused by gravitational-wave radiation and $j_w$ is the specific angular momentum carried away by the stellar wind, in units of $2\pi a^2/P$, where $P$ is the orbital period.

I am interested in comparing the predictions of different gravity theories to the observed rate of change of the orbital period, which I calculate using (e.g., Will & Zaglauer 1989)

$$\frac{P}{2\pi} = \frac{m}{m_1^3m_2^3} J^3 G^{-2}$$

so that

$$\frac{\dot{P}}{P} = \left( \frac{1 - \beta}{1 + q} - \frac{3}{2} \frac{1 - \beta}{1 + q} \right) \frac{\dot{m}_2}{m_2} + \frac{3}{2} \frac{\dot{a}}{a}.$$
In order to evaluate the rate of change of the semi-major axis, I will assume that the companion star always fills its Roche lobe, i.e., that its radius $R_2$ is equal to (Eggleton 1983)

$$R_2 = \frac{0.49q^{-2/3}}{0.6q^{-2/3} + \ln(1 + q^{-1/3})} a.$$ \hfill (5)

Using the fact that

$$\frac{\dot{q}}{q} = -\left(\frac{\beta + q}{q}\right) \frac{\dot{m}_2}{m_2},$$ \hfill (6)

I can evaluate the rate of change of the companion’s radius to be

$$\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{2}{3} \left(\frac{\beta + q}{q}\right) \left[1 - \frac{0.6 + 0.49q^{1/3}(1 + q^{-1/3})^{-1}}{0.6 + q^{2/3}\ln(1 + q^{-1/3})}\right] \frac{\dot{m}_2}{m_2}.$$ \hfill (7)

I can then turn the last equation into

$$\frac{\dot{a}}{a} = \left\{\xi_{ad} - \frac{2}{3} \left(\frac{\beta + q}{q}\right) \left[1 - \frac{0.6 + 0.49q^{1/3}(1 + q^{-1/3})^{-1}}{0.6 + q^{2/3}\ln(1 + q^{-1/3})}\right]\right\} \frac{\dot{m}_2}{m_2},$$ \hfill (8)

where I have introduced the adiabatic index $\xi_{ad} \equiv d\ln R_2/d\ln m_2$ for the companion star.

I now combine equations (1), (2), (4), and (8) to obtain

$$\frac{\dot{P}}{P} = 3 \left(\frac{n}{D}\right) \frac{\dot{J}_{rad}}{J},$$ \hfill (9)

where

$$n \equiv \frac{1}{2} \left(\xi_{ad} - \frac{1}{3} \frac{1 - \beta}{1 + q} - \frac{2}{3} \frac{\beta + q}{q} A\right),$$ \hfill (10)

$$D \equiv 1 + \frac{1}{2} \xi_{ad} - \frac{1}{2} \left(\frac{1 - \beta}{1 + q}\right) - \frac{1}{q} \left[\beta + j_w(1 - \beta)(1 + q)\right] - \frac{1}{3} \frac{\beta + q}{q} A,$$ \hfill (11)

and

$$A \equiv \left[1 - \frac{0.6 + 0.49q^{1/3}(1 + q^{-1/3})^{-1}}{0.6 + q^{2/3}\ln(1 + q^{-1/3})}\right].$$ \hfill (12)

Equation (9) is more general than the one derived by Will & Zaglauer (1989), as it is valid for a wider range of mass ratios of the binary. It provides the predicted rate of change of the orbital period of a binary, given a rate of angular momentum lost by the emission of gravitational waves and by mass loss from the binary. For a Brans-Dicke gravity, this rate is equal to (Will & Zaglauer 1989)

$$\frac{\dot{J}_{rad}}{J} = -\frac{1}{3} \left[\frac{96}{5} \frac{\mu m_2^2}{a^4} \left(\frac{k_1}{12}\right) + \frac{2\mu m}{a^3} G \xi_{s_1} s_1^2\right],$$ \hfill (13)

where

$$G \equiv 1 - \xi_{s_1},$$ \hfill (14)
Table 1: Observed Properties of Ultracompact Binaries

| Source            | $P_{\text{orb}}$ (min) | $a$ (lt-ms) | $f$ ($M_\odot$)       | Reference                                      |
|-------------------|------------------------|-------------|-----------------------|------------------------------------------------|
| SAX J1808.4−3658  | 120.9                  | 62.809      | $3.78 \times 10^{-5}$ | Chakrabarty & Morgan 1998                      |
| XTE J0929−3314    | 43.6                   | 6.290       | $2.7 \times 10^{-7}$  | Galloway et al. 2002                           |
| XTE J1751−305     | 42.4                   | 10.1134     | $1.278 \times 10^{-6}$| Markwardt et al. 2002                          |
| XTE J1807−294     | 40.1                   | 4.80        | $1.54 \times 10^{-7}$ | Markwardt et al. 2003; Markwardt priv. comm.  |
| XTE J1814−338     | 256.5                  | 390.3       | $2.016 \times 10^{-3}$| Markwardt & Swank 2003; Markwardt priv. comm. |

\[ k_1 = g^2 \left[ 12 \left( 1 - \frac{1}{2} \xi \right) + \xi \left( 1 - 2 \frac{m_1 s_1}{m_1 + m_2} \right) \right], \quad (15) \]

\[ s = -\left. \frac{\partial \ln m_1}{\partial G} \right|_0 \quad (16) \]

is the so-called neutron-star sensitivity, and

\[ \xi = \frac{1}{2 + \omega_{\text{BD}}} \quad (17) \]

is a parameter that describes the deviation of Brans-Dicke gravity from general relativity. Note that general relativity corresponds to $\omega_{\text{BD}} \to \infty$ and hence to $\xi = 0$. In writing the above equations, I assumed that the companion to the neutron star is a non-relativistic star and hence the sensitivity of the latter is negligible.

Given a rate of change of the orbital period of the binary ($\dot{P}/P$), the orbital parameters of the binary (i.e., $m_1$, $m_2$, $P$, and $a$), the properties of the neutron star (i.e., the value of $s$) and of the companion star (i.e., the value of $\xi_{\text{ad}}$), and the properties of mass loss in the binary (i.e., the values of $\beta$ and $j_w$), equations (9)-(17) can be used to place a constraint on the Brans-Dicke parameter $\omega_{\text{BD}}$.

### 3. APPLICATION TO ULTRACOMPACT BINARIES

In section §2, I sketched (following Will & Zaglauer 1989) the derivation of the rate of change of the orbital period of an ultracompact binary, when orbital angular momentum is lost due to emission of gravitational radiation, in Brans-Dicke gravity. In this section, I will discuss the observed properties of the recently discovered ultracompact binaries and their prospect for constraining deviations of this theory from general relativity.

**Properties of the binaries.**—Five accreting millisecond pulsars have been discovered to date, in ultracompact binaries with orbital periods between 40 minutes and 4.3 hours. Their orbital periods $P_{\text{orb}}$, projected semi-major axes $a$, eccentricities $e$, and mass functions $f$ are summarized in Table 1.

The constraint on the parameter $\omega_{\text{BD}}$ depends on the measured orbital period and semi-major axis of each orbit, as well as on the mass of the neutron star and of the companion star. A combination of observational and theoretical arguments strongly constrain the mass of the former...
Fig. 1.— The rate of orbital period evolution \( \tau_p^{-1} \equiv \dot{P}/P \) as a function of the Brans-Dicke parameter \( \omega_{BD} \), for different values of the neutron-star sensitivity \( s \). For this plot, I used the orbital parameters of XTE J1808.4–3658 and assumed a neutron star mass of 1.3\( M_\odot \), no mass loss, an edge-on orbit, and \( \xi_{\text{ad}} = 0 \) for the companion star.

The mass of the companion star is then calculated as a function of the unknown inclination \( i \) of the binary using the mass function, i.e.,

\[
f = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}.
\]  
(18)

For reasons related to the stability of mass transfer in X-ray binary systems, I will only consider cases in which the companion star is less massive than the neutron star.

For the parameters of the ultracompact binaries discussed here, the minimum absolute value of the rate of change of the orbital period increases with increasing mass of the neutron star or the companion star. This is caused by the fact that both the semi-major axis of a binary of given period and the rate of emission of gravitational waves increase with increasing total mass of the system. As equation (18) shows, the limiting absolute value for the rate of change of the orbital period corresponds to the maximum inclination (i.e., \( \sin i = 1 \)) and the minimum accepted value for the neutron-star mass, which we take to be 1.3 \( M_\odot \).

Properties of the neutron stars.—The separations of the ultracompact binaries under study are much larger than the radii of the neutron stars. Therefore, in general relativity, the specifics of the neutron-star structure, and hence its equation of state, do not enter in the calculation of the angular momentum lost due to gravitational radiation. However, this is not true in Brans-Dicke
Fig. 2.— The rate of orbital period evolution ($\tau_p^{-1} \equiv \dot{P}/P$) as a function of the Brans-Dicke parameter $\omega_{BD}$, for different values of the adiabatic index $\xi_{ad}$ of the companion star. For this plot, I assumed that the neutron star sensitivity is equal to 0.2 and used the same parameters as in Fig. 1.

gravity, in which the self-gravitational binding energies of the neutron stars (i.e., their sensitivities $s$) affect the result (Will & Zaglauer 1989).

A number of values for the sensitivity have been calculated by Will & Zaglauer (1989) and by Zaglauer (1990) using the relation

$$ s \equiv -\left( \frac{\partial \ln m_1}{\partial \ln G} \right)_N = \frac{3}{2} \left[ 1 - \left( \frac{\partial \ln m_1}{\partial \ln N} \right)_G \right], \quad (19) $$

where $N$ is the baryonic number in the star. According to these calculations, the sensitivity increases with increasing mass and with the stiffness of the equation of state, as both make the stars more compact and hence increase their self-gravitational binding energies.

The minimum absolute value of the rate of change of the orbital period also increases with increasing neutron star sensitivity, as shown for a typical case in Figure 1. The most stringent limit on the Brans-Dicke parameter $\omega_{BD}$ will, therefore, be obtained when using the smallest sensitivity for the lightest neutron star considered. For the rest of the paper, I will assume this to be equal to 0.2.

Properties of the companion stars.—The response of the companion star to mass loss is described by the adiabatic index $\xi_{ad}$ and depends on the nature of the star as well as on a number of factors such as the presence of external irradiation and the rate of mass loss. If all external effects
are neglected and the star is considered to be a polytrope, the parameter $\xi_{\text{ad}}$ takes the canonical value of $\xi_{\text{ad}} = -1/3$.

The companions to the neutron stars in the ultracompact binaries under consideration are believed to be white dwarfs (see Deloye & Bildsten 2003 and references therein), given their inferred very low masses and small sizes (both are of order a few hundredths of the solar mass and radius, respectively). The response of such stars to adiabatic mass loss was recently calculated in detail by Deloye & Bildsten (2003), who found that the corresponding parameter $\xi_{\text{ad}}$ decreases with increasing mass of the white dwarf and with decreasing atomic weight of the main compositional element, but depends rather weakly on its temperature. For the inferred values of the masses of the white dwarfs in the ultracompact binaries, they showed that $-0.3 \lesssim \xi_{\text{ad}} \lesssim 0$.

The dependence of the rate of orbital period evolution on the parameter $\xi_{\text{ad}}$ is shown in Figure 2. Clearly, for a weaker response of the companion star (i.e., for smaller values of $|\xi_{\text{ad}}|$), the rate of orbital period change is also smaller. This is expected, given our assumption that the companion star always fills its Roche lobe for mass transfer to occur and, therefore, a small change in the radius of the star caused by mass loss will be accompanied by a small change in the orbital period. Because the weakest companion-star response corresponds to the lowest mass stars (see Deloye & Bildsten 2003) and the latter also produce the lowest rate of orbital period change, using the lowest allowed value of $|\xi_{\text{ad}}|$ for the lowest companion mass in each binary will provide the most stringent limit.

Properties of the mass loss.—The last and hardest to constrain uncertainty in the calculation of the predicted rate of orbital period change arises from the properties of the mass loss from the binary, i.e., the parameters $\beta$ and $j_w$. Even though the companions to the neutron stars in the ultracompact binaries are white dwarfs and lie deep in the gravitational potential wells of the neutron stars, mass from their surfaces may be lost from the binary systems because of the ablation caused by the intense X-ray irradiation.

The dependence of the predicted rate of change of the orbital period on the amount ($\beta$) and strength ($j_w$) of mass loss is shown in Figure 3. Clearly, when there is very little mass loss, i.e., when $\beta \to 1$, the orbital period increases with time and the predicted rate depends very weakly on $j_w$. The situation is very different, however, when most of the mass is lost from the system, i.e., when $\beta \to 0$.

In order to understand this dependence, I rewrite equation (9) taking the limit $q \gg 1$, which is appropriate for the ultracompact binaries under study. In this case, $A \to 1/2$ and

$$\frac{\dot{P}}{P} \simeq 3 \left[ \frac{\xi_{\text{ad}} - \frac{1}{3}}{\frac{5}{3} + \xi_{\text{ad}} - j_w (1 - \beta)} \right] \frac{\dot{J}_{\text{rad}}}{J}. \quad (20)$$

In the absence of mass loss, equation (20) shows that, even though angular momentum is lost from the companion star due to the emission of gravitational waves, i.e., $\dot{J}_{\text{rad}} < 0$, the period of the binary increases, as long as $\xi_{\text{ad}} < 1/3$. This is a direct consequence of the assumption that the
The rate of orbital period evolution \( \tau_P^{-1} \equiv \dot{P}/P \) as a function of the mass-loss parameter \( \beta \), for different values of the angular momentum of the wind, \( j_w \). The binary parameters are the same as in Figure 1, the neutron-star sensitivity is set to \( s = 0.2 \), the adiabatic index for the companion star is set to \( \xi_{\text{ad}} = 0 \) and the Brans-Dicke parameter is set to \( \omega_{\text{BD}} = 1000 \). The lower half of the plot corresponds to an orbital period that decreases with time \( (\dot{P}/P < 0) \), whereas the upper half corresponds to an orbital period that increases with time \( (\dot{P}/P > 0) \).

Fig. 3.—

It is important to note, however, that, whether there is mass loss or not, there is always a lower limit on the absolute value of the rate of change of the orbital period, since

\[
\dot{P}/P \approx \frac{1 - 3\xi_{\text{ad}}}{\frac{3}{2} + \xi_{\text{ad}} + j_w \beta - j_w} \quad \text{and} \quad \left| \frac{J_{\text{rad}}}{J} \right| \geq \frac{1 - 3\xi_{\text{ad}}}{\frac{3}{2} + \xi_{\text{ad}} + j_w \beta + j_w} \left| \frac{J_{\text{rad}}}{J} \right|
\]

Equation (20) shows that the minimum positive rate of change of the orbital period corresponds to the limit \( \beta \rightarrow 1 \), whereas the maximum negative rate of change of the orbital period corresponds to the limit \( \beta \rightarrow 0 \) and \( j_w \rightarrow 1 \) (see also Fig. 3). Moreover, the absolute values of these two limits
Fig. 4.— The limiting rate for the evolution of the orbital period \( \tau_p^{-1} \equiv \dot{P}/P \) as a function of the Brans-Dicke parameter \( \omega_{BD} \), for the orbital parameters of the five known millisecond accreting pulsars. The lower half of the plot corresponds to an orbital period that decreases with time \( \dot{P}/P < 0 \), whereas the upper half corresponds to an orbital period that increases with time \( \dot{P}/P > 0 \).

are comparable in size. This fact allows for stringent constraints to be placed on the Brans-Dicke parameter \( \omega_{BD} \) by the measurement of an orbital period derivative, independent of whether the latter has a positive or negative value.

4. RESULTS AND DISCUSSION

In §3, I studied the rate of orbital period evolution in an ultracompact binary and its dependence on the various model parameters. When the binary period is increasing with time, I found that the rate of evolution is minimized for the lowest neutron-star mass (taken here to be \( \geq 1.3M_\odot \)), the largest inclination \( (\sin i = 1) \), the largest adiabatic index for the companion star (take here to be \( \xi_{ad} \leq 0 \)), and the case of no mass-loss \( (\beta = 1) \). On the other hand, when the binary period is decreasing with time, the absolute value of this rate is minimized for the same stellar parameters as before but at the limit of complete mass loss \( (\beta = 0) \) and with the wind carrying the orbital angular momentum \( (j_w = 1) \).

Figure 4 shows the limiting rate of orbital period evolution as a function of the Brans-Dicke
Fig. 5.— The limiting rate for the evolution of the orbital period \((\tau_p^{-1} \equiv \dot{P}/P)\) of SAX J1808.4–3658, in general relativity, as a function of the orbital inclination. The lower half of the plot corresponds to an orbital period that decreases with time \((\dot{P}/P < 0)\), whereas the upper half corresponds to an orbital period that increases with time \((\dot{P}/P > 0)\).

parameter \(\omega_{\text{BD}}\), for the five known accreting millisecond pulsars. For each source, the region between the two corresponding curves is excluded. As a result, a measurement of either a positive or a negative rate of orbital period evolution can be used in placing a lower bound on the Brans-Dicke parameter.

The limiting curves become insensitive to the deviation of Brans-Dicke gravity from general relativity for \(\omega_{\text{BD}} \gtrsim 10^4\). This number represents the tightest constraint that can be achieved with this method and is approximately an order of magnitude larger than the constraints imposed by solar-system and double-neutron-star tests (Will 2001). In deriving these constraining curves, I assumed no prior knowledge of any of the binary parameters other than the ones that can be inferred from X-ray timing. These constraints can be improved by measuring the masses of the neutron star and the companion star, by identifying the nature of the companions (and hence constraining their adiabatic indices \(\xi_{\text{ad}}\), and by placing constraints on the mass loss from the systems. The tightest limits can be achieved for an eclipsing ultracompact binary, with a relatively long orbital period, and limited mass loss.

Even when the binary parameters are not favorable, and hence the resulting constraints on
the Brans-Dicke parameter $\omega_{\text{BD}}$ are not stringent, the analysis presented here can be used in constraining the orbital inclination of the ultracompact binaries and the properties of mass loss. As an example, Figure 5 shows the limiting rate of evolution of the orbital period of the source SAX J1808.4–3658, in general relativity, as a function of the orbital inclination. The region between the two curves is not allowed for any neutron-star mass in the range $1.3 - 2.2M_\odot$ and for any mass-loss mechanism from the binary. As a result, a measurement of the rate of orbital period evolution can also be used in placing a lower bound on the inclination of the binary system.

The measurements required to place a stringent constraint on the Brans-Dicke parameter $\omega_{\text{BD}}$ can be achieved using the Rossi X-ray Timing Explorer by measuring the orbital periods of the binaries over a long period of time, i.e., between successive outbursts. Indeed, the source SAX J1808.4–3658 has shown four outbursts within six years and the accuracy of the measurement of the orbital period in the 1998 outburst alone is comparable to the value needed to achieve a useful constraint (see Chakrabarty & Morgan 1998). However, the properties of mass loss from the binary system may not remain constant over a long period of time. If the mass loss is driven by irradiation of the companion star, then during an outburst the mass loss will be significant and the orbital period may be decreasing with time, whereas in between outbursts the mass loss may be negligible and the orbital period will be increasing with time. The net result will be an artificially reduced rate of orbital period evolution and thus an artificially stringent constraint on Brans-Dicke gravity. The effects of a variable rate of mass loss can be minimized if an orbital period derivative can be measured in a single, long outburst. Given the short durations of the outbursts so far observed from the five known sources, such a measurement is unlikely with the capabilities of RXTE. It can be, however, one of the key scientific goals of the next X-ray timing mission (see, e.g., Markwardt 2004).

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Note added.—While this work was in its final stages, a new accreting millisecond pulsar, IGR J00291+5934, was discovered in an ultracompact binary (Markwardt et al. 2004, ATEL #353, #360). This new source has very similar orbital parameters with the five previously known accreting millisecond pulsars and can also be used in placing constraints on Brans-Dicke gravity.

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