CP violations in a predictive $A_4$ symmetry model

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Abstract

We study a seesaw model with $A_4$ flavor symmetry and the physics phenomenological consequences. After symmetry breaking, the model leads to the neutrino mixing matrix that satisfies the current data of neutrino oscillation experiments. We then study how the low energy CP violation parameter, $J_{CP}$, associates with the Dirac CP violation phase $\delta$. We also study the high energy CP violation associate with the decay of heavy right handed neutrino in leptogenesis process in order to explain the observed baryon asymmetry of the Universe, $\eta_B$. Numerically, we find a correlation between $J_{CP}$ and $\eta_B$ through the high energy phases. It is shown that our prediction for $J_{CP}$, and hence for the Dirac CP violating phase $\delta$, for some high energy fixed parameters can be constrained by the current data of $\eta_B$.

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**I. INTRODUCTION**

The data of neutrino oscillation experiments definitely affirmed that neutrinos have mass and they are mixing. Based on neutrino experimental data, in 2002, P. F. Harrison et al. [1] proposed the structure of neutrino mixing matrix which named Tri-Bimaximal (TB). According to this structure, the reactor mixing angle, $\theta_{13}$, is zero and the Dirac CP violating phase has no meaning. Subsequently, there have been a lot of efforts to build a simple model that leads to TB mixing pattern of leptons, and an interesting way seems to be the use of some discrete non-Abelian flavor groups added to the gauge group of the Standard Model (SM). There is a series of such models based on the symmetry groups $A_4$ [2, 3], $T'$ [4], and $S_4$ [5]... These models are usually realized at some high energy scale $\Lambda$ and the groups are spontaneously broken due to a set of scalar multiplets. Based on the most updated data of neutrino oscillation experiments, the values of neutrino mixing angles are given in [6] where the reactor mixing angle is relatively large, $\theta_{13} \sim 8^0$. As a result, the mentioned models are needed to re-examine for their consistence with recent experimental results. There are lots of efforts to generate the non-zero value of neutrino mixing angle $\theta_{13}$ as well as leptogenesis in the frame work of $A_4$ model, see for examples, Ref. [7]. However, according to these works, the only inclusion of higher order corrections would not produce such large value of $\theta_{13}$. Then the leading orders of the original $A_4$ models are required, and there are several attempts built in this direction, see for example in Refs. [8–10]. Besides two triplet flavons as usual, tree singlet flavons were used in Ref. [8], and two singlet flavons $\xi, \xi'$ transform as 1, 1' of the $A_4$ in Refs. [9] [10] in order to accommodate with the present neutrino data. However, leptogenesis have not studied in Ref. [9], whereas in Ref. [10], for having conventional leptogenesis, the authors considered the contribution of the next-to-leading order corrections to right handed neutrino (RHN) mass matrix in the suppersymmetry framework. In this work, we will study flavored leptogenesis with the help of renormalization group evolution of the Dirac Yukawa coupling matrix.

Besides the explanation of neutrino mixing structure, one has to find a way of generating neutrino tiny mass which is zero in the SM. And the seesaw mechanism [11] seems to be the most interesting and effective solution. The seesaw has another physics consequence which is called leptogenesis for the generation of the observed Baryon Asymmetry of the Universe (BAU) by the CP asymmetric decay of heavy RHNs [12]. If the BAU was generated
by leptogenesis, then CP violation in the lepton sector must be existed. For Majorana neutrinos, there are one Dirac and two Majorana CP violating phases. One of the phases (or a combination of them) in principle can be measured by neutrinoless double beta \((0\nu2\beta)\) decay \cite{13} experiments. Besides, the TB mixing structure forbids the low energy CP violation in neutrino oscillation, due to \(U_{e3} = 0\), and also forbids the high energy CP violation in leptogenesis. Therefore, any observation of the leptonic CP violation, for instance in \(0\nu2\beta\) decay and \(J_{CP}\), can strengthen our believe in leptogenesis by demonstrating that CP is not a symmetry of the leptons.

In this work, we consider an expansion of the SM by the seesaw realization of an \(A_4\) discrete symmetric model and it’s phenomenological relating with \(J_{CP}\) and leptogenesis. Apart from two SM scalar doublets taking responsibility for spontaneously breaking of the \(A_4\) and the SM gauge groups, this model contains additional \(SU(2)_L\) scalar singlets, namely two singlets \(\xi', \xi''\) transform as \(1', 1''\) and two triplets of the \(A_4\). If the RHN mass matrix’s components resulting from the contributions of VEVs of two scalar singlets (of both \(SU(2)_L\) and \(A_4\)) are exactly the same, then the model generates the TB pattern of lepton mixing matrix and hence leptogenesis does not work. We, therefore, study the case where those components are independent and we find the parameter space of the model that satisfies the low energy data and that the BAU is successful generated through flavored leptogenesis.

This work is organized as follows. In the next section, section II, we present the overview of the \(A_4\) model with seesaw mechanism. We also discuss the low energy phenomena of the lepton sector in this section. Section III is devoted to study the leptogenesis. Our discussions and the summary of our work are given in the last section, section IV.

II. THE \(A_4\) SYMMETRY MODEL WITH SEESAW MECHANISM

The non-Abelian \(A_4\) is a group of even permutations of 4 objects and has \(4! / 2 = 12\) elements. All properties of this group needed for model construction was given in \cite{2}. This paper will work in the \(A_4\) basis introduced by G. Altarelli and F. Feruglio, which is reviewed in the \cite{1}. In this work, we promote the \(A_4\) proposed in \cite{3} with two Higgs singlets to accompany with seesaw mechanism. The model contains several \(SU(2)_L \otimes U(1)_Y\) Higgs singlets, where two of them \((\xi', \xi'')\) are \(A_4\) singlets, while the remaining \((\phi_S, \phi_T)\) are triplets. The SM lepton doublets are assigned to be three components of one \(A_4\) triplet, while three
right handed charged leptons $e_R, \mu_R, \tau_R$ are assumed to transform as three different singlets $1, 1'', 1'$, respectively. The standard Higgs doublets $h_u$ and $h_d$ remain invariant under $A_4$. The particle content for leptons and scalars, their VEVs, and symmetry groups considered in the model are shown in Table I. Two more discrete symmetries $Z_3$ and $Z_4$ are included in order to get minimal and necessary Yukawa couplings. The Lagrangian for lepton sector

\begin{equation}
-L = \frac{y_e}{\Lambda} (\phi_T \bar{\psi}_L^d) e_R h_d + \frac{y_\mu}{\Lambda} (\phi_T \bar{\psi}_L^d)^\prime \mu_R h_d + \frac{y_\tau}{\Lambda} (\phi_T \bar{\psi}_L^d)^\prime \tau_R h_d + p \bar{\psi}_L^d N_R h_u + x_A^\prime \xi'(\bar{N}_L^c N_R)' + x_B^\prime \xi''(\bar{N}_L^c N_R) + H.c.,
\end{equation}

which is invariant under all symmetries given in Table I is:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Lepton & $SU(2)_L$ & $U(1)_Y$ & $A_4$ & $Z_3$ & $Z_4$ \\
\hline
$\psi^d$ & $2^*$ & 1 & $3^*$ & 1 & 1 \\
$e_R$ & 1 & -2 & $\frac{1}{2}$ & 1 & 1 \\
$\mu_R$ & 1 & -2 & $\frac{1}{2}'$ & 1 & 1 \\
$\tau_R$ & 1 & -2 & $\frac{1}{2}''$ & 1 & 1 \\
$N_{IR}$ & 1 & 0 & $\frac{3}{2}$ & $\omega$ & -i \\
\hline
Scalar & VEV & & & & \\
\hline
$h_u$ & 2 & -1 & $\frac{1}{2}$ & $\omega^2$ & i & $\langle h_u \rangle = v_u$ \\
h_d & 2 & 1 & $\frac{1}{2}$ & 1 & 1 & $\langle h_d \rangle = v_d$ \\
$\phi_S$ & 1 & 0 & $\frac{3}{2}$ & $\omega$ & -1 & $\langle \phi_S \rangle = (v_S, v_S, v_S)$ \\
$\phi_T$ & 1 & 0 & $\frac{3}{2}$ & 1 & 1 & $\langle \phi_T \rangle = (v_T, 0, 0)$ \\
$\xi'$ & 1 & 0 & $\frac{1}{2}'$ & $\omega$ & -1 & $\langle \xi' \rangle = u'$ \\
$\xi''$ & 1 & 0 & $\frac{1}{2}''$ & $\omega$ & -1 & $\langle \xi'' \rangle = u''$ \\
\hline
\end{tabular}
\caption{List of fermion and scalar fields, where $\psi^d = (\bar{\psi}_{La}, \bar{\psi}_{La})^T$ ($a = 1, 2, 3$) and $\omega = e^{2\pi i/3}$.}
\end{table}

The neutrino sector gives rise to the following Dirac and Majorana neutrino mass matrices

\begin{equation}
m_D = pv_u \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} = v_u Y_\nu, \quad Y_\nu = p \times 1,
\end{equation}

where $\Lambda$ is the cut-off scale of the model. After spontaneous symmetry breaking, the charged lepton mass matrix comes out diagonally with $m_e = \frac{y_e v_T}{\Lambda}$, $m_\mu = \frac{y_\mu v_T}{\Lambda}$, and $m_\tau = \frac{y_\tau v_T}{\Lambda}$.
where $X = 2x_B v_S, \bar{Y} = 2x_A' u', \bar{Z} = 2x_A'' u''$. $M_0 = 2X/3$ is the scale of RHN mass, $\kappa = Z/M_0, \omega = Y/M_0$. Hereafter, the complex parameters are distinguished by the tildes ($\bar{Z} = Z e^{i\phi_1}, \bar{X} = X e^{i\phi_2}$ and $X, Y, Z$ are real parameters). The active neutrino mass matrix is then obtained by the seesaw formula [11]:

$$m_\nu = -v_u^2 Y^T M_R^{-1} Y = -m_0 \begin{pmatrix} \tilde{a} & \tilde{b} & \tilde{c} \\ \tilde{b} & \tilde{d} & \tilde{h} \\ \tilde{c} & \tilde{h} & \tilde{f} \end{pmatrix}.$$  (4)

where $m_0 = (p u_v)^2/M_0$ is the scale of active neutrino mass, $v_u = v \sin \beta, v = 176$ GeV. And

$$\tilde{a} = -3 + 4\kappa e^{i\phi_1} + 4\omega e^{i\phi_2} (1 + \kappa e^{i\phi_1} \frac{DE}{\bar{DE}}); \quad \tilde{b} = -3 + 2\kappa e^{i\phi_1} + 4\kappa^2 e^{2i\phi_1} + 2\omega e^{i\phi_2} \frac{DE}{\bar{DE}};$$

$$\tilde{c} = -3 + 2\kappa e^{i\phi_1} + 2\omega e^{i\phi_2} + 4\omega^2 e^{2i\phi_2} \frac{DE}{\bar{DE}}; \quad \tilde{d} = -3 + 4\kappa e^{i\phi_1} + 4\omega e^{i\phi_2} \frac{DE}{\bar{DE}};$$

$$\tilde{h} = -3 + 2\kappa e^{i\phi_1} + 2\omega e^{i\phi_2} - 4\kappa \omega e^{(i\phi_1 + i\phi_2)} \frac{DE}{\bar{DE}}; \quad \tilde{f} = -3 + 4\kappa e^{i\phi_1} - 4\kappa^2 e^{2i\phi_1} + 4\omega e^{i\phi_2} \frac{DE}{\bar{DE}};$$

$$\bar{DE} = -9\kappa e^{i\phi_1} + 4\kappa^3 e^{3i\phi_1} - 9\omega e^{i\phi_2} + 4\omega^3 e^{3i\phi_2}.$$  (5)

The active neutrino mass matrix is diagonalized by $U_{PMNS}$ matrix as

$$U^T_{PMNS} m_\nu U_{PMNS} = \text{Diag.}(m_\nu) = \text{Diag.}(m_1, m_2, m_3),$$  (6)

where $U_{PMNS}$ is the neutrino mixing matrix parameterized as $U_{PMNS} = U_\nu K$ [6]. The precise forms of $U_\nu$ and $K$ are

$$U_\nu = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i\delta} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix},$$  (7)

$$K = \text{diag}(1, e^{i\beta_1/2}, e^{i\beta_2/2}).$$  (8)

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ $(ij = 12, 23, 13); \delta$ and $\beta_1, \beta_2$ are Dirac and two Majorana CP violating phases, respectively. However, instead of diagonalizing $m_\nu$, we diagonalize the Hermitian matrix $m_\nu^T m_\nu$ to examine the structure of $m_\nu$, so that the two Majorana phases
become irrelevant and we can easy obtain the mixing angles and phase appeared in \( U_\nu \) in terms of the parameters appeared in \( m_\nu \)\(^{[14]} \).

Then the Hermitian matrix \( m_\mu^\dagger m_\nu \) is diagonalized as

\[
m_\mu^\dagger m_\nu = U_{\text{PMNS}} \text{Diag}(m_1^2, m_2^2, m_3^2) U_{\text{PMNS}}^\dagger \equiv \begin{pmatrix} A & \tilde{B} & \tilde{C} \\ \tilde{B}^* & D & \tilde{H} \\ \tilde{C}^* & \tilde{H}^* & F \end{pmatrix} \quad (9)
\]

where

\[
A = m_0^2(\lvert \tilde{a} \rvert^2 + \lvert \tilde{b} \rvert^2 + \lvert \tilde{c} \rvert^2), \quad \tilde{B} = m_0^2(\tilde{a}^* \tilde{b} + \tilde{b}^* \tilde{d} + \tilde{c}^* \tilde{h}), \\
\tilde{C} = m_0^2(\tilde{a}^* \tilde{c} + \tilde{c}^* \tilde{f} + \tilde{b}^* \tilde{h}), \quad D = m_0^2(\lvert \tilde{b} \rvert^2 + \lvert \tilde{d} \rvert^2 + \lvert \tilde{h} \rvert^2), \\
\tilde{H} = m_0^2(\tilde{b}^* \tilde{c} + \tilde{d}^* \tilde{h} + \tilde{h}^* \tilde{f}), \quad F = m_0^2(\lvert \tilde{c} \rvert^2 + \lvert \tilde{f} \rvert^2 + \lvert \tilde{h} \rvert^2). \quad (10)
\]

Then, the straightforward calculation leads to the expressions for the masses and mixing parameters \(^{[15]}\)

\[
m_{1,2}^2 = \frac{\lambda_1 + \lambda_2}{2} + \frac{c_{23} \text{Re}(\tilde{B}) - s_{23} \text{Re}(\tilde{C})}{2 s_{12} c_{12} c_{13}}, \quad m_3^2 = \frac{c_{13}^2 \lambda_3 - s_{13}^2 A}{c_{13}^2 - s_{13}^2}, \quad (11)
\]
\[
\tan \theta_{23} = \frac{\text{Im}(\tilde{B})}{\text{Im}(\tilde{C})}, \quad \tan 2 \theta_{12} = \frac{2 c_{23} \text{Re}(\tilde{B}) - s_{23} \text{Re}(\tilde{C})}{c_{13}(\lambda_2 - \lambda_1)}, \quad (12)
\]
\[
\tan 2 \theta_{13} = \frac{2 \lvert s_{23} \tilde{B} + c_{23} \tilde{C} \rvert}{\lambda_3 - A}, \quad \tan \delta = -\frac{1}{s_{23} s_{23} \text{Re}(\tilde{B}) + c_{23} \text{Re}(\tilde{C})}, \quad (13)
\]

with

\[
\lambda_1 = c_{13}^2 A - 2 s_{13} c_{13} s_{23} \tilde{B} + c_{23} \tilde{C} \rvert + s_{13}^2 \lambda_3, \\
\lambda_2 = c_{23}^2 D + s_{23}^2 F - 2 s_{23} c_{23} \text{Re}(\tilde{H}), \quad \lambda_3 = s_{23}^2 D + c_{23}^2 F + 2 s_{23} c_{23} \text{Re}(\tilde{H}). \quad (14)
\]

It is worth to study the other low energy quantities such as effective neutrino mass in neutrinoless double beta decay \( (0\nu\beta\beta) \) |\langle m \rangle| and the Jarlskog invariant \( J_{CP} \) with the forms given in \( [6] \) as:

\[
|\langle m \rangle| = \left| m_1(U_{\text{PMNS}})^2 e_{i1} + m_2(U_{\text{PMNS}})^2 e_{i2} + m_3(U_{\text{PMNS}})^2 e_{i3} \right| = \left| \left( m_1 c_{12}^2 + m_2 s_{12}^2 e^{i \beta} \right) c_{13}^2 + m_3 s_{13}^2 e^{i (\beta_2 - 2 \delta)} \right|, \quad (15)
\]
\[
J_{CP} = \frac{1}{8} \sin 2 \theta_{12} \sin 2 \theta_{23} \sin 2 \theta_{13} \cos \theta_{13} \sin \delta. \quad (16)
\]

As can be seen from Eqs. \( [11] \quad [12] \quad [13] \quad [14] \), three neutrino masses, three mixing angles and a CP phase are presented in terms of five independent parameter \( p, \omega, \kappa, \phi_1, \phi_2 \). At
the present, we have five experimental results, which are taken as inputs in our numerical analysis given at $3\sigma$ by [6] for the normal hierarchy (NH) of active neutrino mass spectrum:

$$\Delta m^2_{21}(10^{-5}\text{eV}^2) = 6.93 - 7.97; \quad \Delta m^2(10^{-3}\text{eV}^2) = 2.37 - 2.63,$$

$$0.0185 \leq \sin^2 \theta_{13} \leq 0.0246, \quad 0.250 \leq \sin^2 \theta_{12} \leq 0.354,$$

$$0.379 \leq \sin^2 \theta_{23} \leq 0.616,$$

(17)

where $\Delta m^2_{21} = m_2^2 - m_1^2$, $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)$. Imposing the current experimental data on neutrino masses and mixing angles for the case NH of active neutrino masses into above relations and scanning all the parameter space $p, \kappa, \omega, \phi_1, \phi_2$, we investigate how those parameters are constrained and estimate possible prediction for leptogenesis. The allowed parameter spaces ($\kappa, \omega$) and ($\phi_1, \phi_2$) constrained by the experimental data given in Eq. (17) are respectively plotted in the figures 1 and 2. Whereas, the global parameter $p$ in the Dirac neutrino Yukawa coupling matrix $Y_{\nu}$ can be roughly estimated by $p^2 \simeq \frac{M_0 \sqrt{|\Delta m^2|}}{v^2}$ which is derived from the seesaw formula $m_0 = \frac{(\nu_{\nu})^2}{M_0} \simeq \sqrt{\Delta m^2}$. In the above numerical calculation, we have used the random value from zero to $2\pi$ for $\phi_1, \phi_2$, and $\tan \beta = 30$ as the inputs. The RHN mass scale is chosen as $M_0 = 5 \times 10^{13}$ GeV. Here we note that $M_0$ and $\tan \beta$ are absorbed into $m_0$ by the seesaw formula which is taken as 0.04 eV $\leq m_0 \leq 0.06$ eV in our numerical analysis. As a result, the parameter space plotted in figures 1, 2 are independent from the choice of the values of $\tan \beta$ and $M_0$.

The predicted values of the neutrino mixing angles are plotted in figures 3 and 4. The values of $\theta_{13}$ and $\theta_{12}$ are totally stayed in the $3\sigma$ range of the data given in Eq. (17), whereas the mixing angle $\theta_{23}$ is almost unchanged from it’s TB value ($\theta_{23} = 45^0$). It is worth to
mention that, the neutrino mixing angles are fixed by the model, namely, the original $A_4$ model predicts the TB structure of neutrino mixing matrix. In the current model, with an extra scalar singlet, only $\theta_{13}$ and $\theta_{12}$ are deviated from their TB values.

The prediction of the effective mass $|\langle m \rangle|$ is plotted in figure 5 as a function of the lightest active neutrino mass $m_1$. Numerically, our prediction of $|\langle m \rangle|$ is turned out to be $0.002$ eV $\leq |\langle m \rangle| \leq 0.023$ eV. Notice that, the results from $0\nu2\beta$ by KamLAND-Zen [16] and EXO-200 [17] indicate a limit on the effective neutrino mass parameter $|\langle m \rangle|$ as, $|\langle m \rangle| \leq (0.14 - 0.28)$ eV at $90\%$ CL. and $|\langle m \rangle| \leq (0.19 - 0.45)V$ at $90\%$ CL., respectively. Therefore, our result of $|\langle m \rangle|$ is expected to be measured by KamLAND-Zen and other $0\nu2\beta$ decay experiments in their new phase which is taking data since mid-2017 [18].

The prediction of $J_{CP}$ as a function of Dirac CP violation phase $\delta$ is plotted in figure 6. Once the exact value of $\delta$ is confirmed the exact value of $J_{CP}$ can be determined. As can be
seen from Eq. (16), within the constraints of $\theta_{13}$ and $\theta_{12}$ given in Eq. (17), $J_{CP}$ is strongly depends on $\sin \delta$ (or on the CP phase $\delta$). Notice that, from Eqs. (5, 10, 13) we find that $\delta$ directly depends on two high energy phases $\phi_1, \phi_2$ leading to the depend on $\phi_1, \phi_2$ of the $J_{CP}$ parameter. To be consistent, we would like to note that the CP asymmetry generated by the decay of RHN (Eq. 20) also directly depends on the phases $\phi_1, \phi_2$ (besides two Majorana phases). This makes the correlation between the BAU $\eta_B$ and $J_{CP}$.

III. LEPTOGENESIS

Now we consider how leptogenesis can work in our scenario. First of all, we have to diagonalize the RHN mass matrix $M_R$ given in Eq. (3) in order to go to the mass basis of the RHNs:

$$V^T_R M_R V_R = \text{Diag}(M_1, M_2, M_3) = (v_u p)^2 \text{Diag}(\frac{1}{m_1}, \frac{1}{m_2}, \frac{1}{m_3}),$$  \hspace{1cm} (18)

where $V_R = U^*_{PMNS}$. The parameters in this matrix are determined in the previous section with an exception that the Majorana CP phases will be taken as $0 \leq \beta_1, \beta_2 \leq 180^0$. And in the mass basis of the RHNs, the Dirac neutrino Yukawa coupling matrix is modified to be

$$Y'_\nu = V^T_R Y_\nu = U^*_{PMNS} Y_\nu \Rightarrow H = Y'_\nu Y'^\dagger_\nu = p^2 \times 1.$$  \hspace{1cm} (19)

We study the case of flavored leptogenesis, the CP asymmetry in the decay of RHN $N_i$ to lepton flavor $l_\alpha$ ($\alpha = e, \mu, \tau$) is defined as [19]:

$$\varepsilon_i^\alpha = \frac{\Gamma(N_i \rightarrow l_\alpha \phi) - \Gamma(N_i \rightarrow \bar{l}_\alpha \phi^\dagger)}{\sum_\alpha [\Gamma(N_i \rightarrow l_\alpha \phi) + \Gamma(N_i \rightarrow \bar{l}_\alpha \phi^\dagger)]}$$

$$= \frac{1}{8\pi H_{ii}} \sum_{j \neq i} \text{Im} \left[ H_{ij} (Y'_\nu)^* \alpha (Y'^\dagger_\nu)^* j \right] f \left( \frac{M_j^2}{M_i^2} \right),$$  \hspace{1cm} (20)

where $H = Y'_\nu Y'^\dagger_\nu$, and $M_i$ denotes the RHN masses. The loop function $f(x)$ containing vertex and self-energy corrections is given as:

$$f(x) = \sqrt{x} [(1 + x) \ln \frac{x}{1 + x} + \frac{2 - x}{1 - x}].$$  \hspace{1cm} (21)

As can be seen from Eq. (20), $\varepsilon_i^\alpha \sim (Y'_\nu)^2 \sim p^2 \sim \frac{M_0 \sqrt{|\Delta m^2|}}{v_d} \sim \frac{M_0}{\sin^2 \beta}$. Therefore, the CP asymmetry increases with the increasing of $M_0$, whereas it does not depend on $\tan \beta$ for a large range ($\tan \beta \geq 3$).
Notice from Eq. (20) that, in the studied model, a nonvanishing CP asymmetry requires $\text{Im}[H_{ij}(Y'\nu)_{i\alpha}(Y'\nu)_{j\beta}] \neq 0$ with $Y'\nu$ defined in (19). Therefore, to have leptogenesis we need to induce a nonvanishing $H_{ij}(i \neq j)$ at the leptogenesis scale. Indeed, this is possible by the RG (Renormalization Group) effects. The RG equation for the Dirac neutrino Yukawa coupling can be written as

$$\frac{dY'_\nu}{dt} = Y'_\nu[(T - \frac{3}{4}g_2^2 - \frac{9}{4}g_1^2) - \frac{3}{2}(Y'_{i\nu}Y'_{i\nu} - Y'_{\nu\nu}Y'_{\nu\nu})] ,$$

(22)

where $T = Tr(3Y'_{u\nu}Y_{u\nu} + 3Y'_{d\nu}Y_{d\nu} + Y'_{\nu\nu}Y_{\nu\nu}), Y_{u,d}$ and $Y_{i\nu}$ are the Yukawa couplings of up-type and down-type quarks and charged leptons, $g_{2,1}$ are the SU(2)$_L$ and U(1)$_Y$ gauge coupling constants, respectively. And $t = \frac{1}{16\pi} \ln(M/\Lambda')$, and $M$ is an arbitrary renormalization scale. The cutoff scale $\Lambda'$ can be regarded as the $G_f$ breaking scale $\Lambda' = \Lambda$ and assumed to be in order of the GUT scale, $\Lambda' \sim 10^{16}$ GeV.

As the structure of $M_R$ changes with the evolution of the energy scale, the $V_R$ depends on the scale $\Lambda'$ too. The RG evolution of $V_R(t)$ can be written as

$$\frac{d}{dt} V_R = V_R A,$$

(23)

where $A$ is an anti-Hermitian matrix $A^\dagger = -A$ due to the unitary of $V_R$. The RG equation for $Y'_{\nu}$ in the basis of diagonal $M_R$ is then obtained as

$$\frac{dY'_{\nu}}{dt} = Y'_{\nu}[(T - \frac{3}{4}g_2^2 - \frac{9}{4}g_1^2) - \frac{3}{2}(Y'_{i\nu}Y'_{i\nu} - Y'_{\nu\nu}Y'_{\nu\nu})] + A^T Y'_\nu.$$

(24)

Finally, we obtain the RG equation for the Hermitian matrix $H = Y'\nu Y'^\dagger\nu$ responsible for the leptogenesis as

$$\frac{dH}{dt} = 2(T - \frac{3}{2}g_2^2 - \frac{9}{4}g_1^2)H - 3Y'_\nu(Y'_{i\nu}Y_{i\nu})Y'^\dagger_{\nu\nu} + 3H^2 + A^T H + HA^*.$$

(25)

then, if we keep only the $\tau$-Yukawa coupling contribution as the leading order, we derive the off-diagonal terms of $H$ matrix as

$$H_{ij}(t) = -3g_1^2(Y'_{i\nu})_{i\alpha}(Y'^\dagger_{\nu\nu})_{j\beta} \times t.$$

(26)

The flavored CP asymmetries $\epsilon^\nu_i$ then can be obtained.

After the CP asymmetry in the decay of $N_i$, $\epsilon^\nu_i$, are calculated, the final value of $\eta_B$ can be calculated by solving the flavor dependent Boltzmann equations (BE). Those BEs describe the out-of-equilibrium processes such as the decay, inverse decay, and scattering
involving the RHNs, as well as the nonperturbative sphaleron interaction. Besides the CP asymmetries $\varepsilon_i^\alpha$, the final value of BAU also depends on the wash-out factors $K_i^\alpha$ which measure the effects of the inverse decay of Majorana neutrino $N_i$ into the lepton flavor $\alpha$ and scalars. The parameter $K_i^\alpha$ is defined as [21]:

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = (Y_\mu^\alpha)^\dagger (Y_{\mu}^\alpha)_{\alpha i} u_i^2 m_* M_i,$$

(27)

where $\Gamma_i^\alpha$ is the partial decay width of $N_i$ into the lepton flavors and Higgs scalars; $H(M_i)$ is the Hubble parameter at temperature $T = M_i$ defined as $H(M_i) \simeq (4\pi^2 g_*/45)^{1/2} M_i^2/M_{Pl}$, where $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, $g_* \simeq 116$ is the effective number of degrees of freedom of the SM with two Higgs doublets, and the equilibrium neutrino mass $m_* \simeq 10^{-3}$ eV.

Due to the flavor effects, each CP asymmetry $\varepsilon_i^\alpha$ contributes differently to the final formula for the baryon asymmetry as [21, 22],

$$\eta_B \simeq -2 \times 10^{-2} \sum_{N_i} \left[ \varepsilon_i^e \kappa_i^e \left( \frac{151}{179} K_i^e \right) + \varepsilon_i^\mu \kappa_i^\mu \left( \frac{344}{537} K_i^\mu \right) + \varepsilon_i^\tau \kappa_i^\tau \left( \frac{344}{537} K_i^\tau \right) \right],$$

(28)

if the RHN mass is about $M_i \leq (1 + \tan^2 \beta) \times 10^9$ GeV where the $\mu$ and $\tau$ Yukawa couplings are in equilibrium and all the flavors are to be treated separately. And if $(1 + \tan^2 \beta) \times 10^9$ GeV $\leq M_i \leq (1 + \tan^2 \beta) \times 10^{12}$ GeV where only the $\tau$ Yukawa coupling is in equilibrium and treated separately while the $e$ and $\mu$ flavors are indistinguishable, then baryon asymmetry is obtained as:

$$\eta_B \simeq -2 \times 10^{-2} \sum_{N_i} \left[ \varepsilon_i^e \kappa_i^e \left( \frac{417}{589} K_i^e \right) + \varepsilon_i^\tau \kappa_i^\tau \left( \frac{390}{589} K_i^\tau \right) \right],$$

(29)

here $\varepsilon_i^2 = \varepsilon_i^e + \varepsilon_i^\mu$, $K_i^2 = K_i^e + K_i^\mu$.

In the Eqs. (28, 29), the wash-out factors $\kappa_i^\alpha$ are defined as

$$\kappa_i^\alpha \simeq \left( \frac{8.25}{K_i^\alpha} + \left( \frac{K_i^\alpha}{0.2} \right)^{1.16} \right)^{-1}.$$

(30)

We study the case NH of active neutrino masses hence, as can be seen from Eq. (18), the lightest RHN mass is $M_3$, therefore the BAU is mainly generated by the decay of the 3rd generation of RHNs. The prediction of $\eta_B$ as a function of the lightest RHN mass, $M_3$ is shown in fig. 7. In this figure (and in figs. 8, 9), the central value of the experimental data of BAU is $\eta_B^{CMB} = 6.1 \times 10^{-10}$ [23], and $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$ is the phenomenologically
FIG. 7: The prediction of BAU, $\eta_B$, as a function of the lightest RHN mass, $M_3$.

FIG. 8: The prediction $\eta_B$ as a function of the Dirac CP phase, $\delta$, with $M_0 = 5 \times 10^{13}$ GeV.

FIG. 9: The prediction of $\eta_B$ as a function of $|J_{CP}|$ with $M_0 = 5 \times 10^{13}$ GeV.

allowed regions of $\eta_B$. The prediction of $\eta_B$ in flavored leptogenesis as a function of CP phase $\delta$ and of $J_{CP}$ are shown in figures 8 and 9 respectively where $M_0 = 5 \times 10^{13}$ GeV. As can be seen in figure 9, for successful leptogenesis, the scale of RHN mass, $M_0$, is required about $10^{13}$ GeV. However, as can be seen in the figure 9, the RHN mass scale for successful leptogenesis is also constrained by $J_{CP}$. Therefore, once the low energy CP violation $J_{CP}$ is precisely determined by future experiments then the value of $M_0$ for successful leptogenesis is well established. And vice versa, $J_{CP}$ is constrained by the current data of $\eta_B$, for some fixed value of $M_0$, we can pin down the value of $J_{CP}$ and hence the value of $\delta$.

IV. CONCLUSION

We have studied the seesaw version of a $A_4$ flavor symmetry model with two Higgs singlets beside other scalars as usual $A_4$ models. The neutrino mixing angles predicted by the model
come out satisfy the current experimental data at 3σ CL. We have also investigated how
effective neutrino mass $|\langle m \rangle|$ associated with $0\nu2\beta$ decay can be predicted as a function
of the lightest active neutrino mass $m_1$, and our prediction for $|\langle m \rangle|$ can be measured by
the in running $0\nu2\beta$ decay experiments. Besides, we have calculated the Jarlskog invariant
parameter $J_{CP}$ as a function of Dirac CP violation phase $\delta$. In the near future, if the value of
$\delta$ is precisely determined then we can point out the exact values of $J_{CP}$ and $\delta$. The flavored
leptogenesis is investigated in detail in this work. We find that the RHN mass about $10^{12}$
GeV is required to successfully generate BAU. We have found that there is a correlation
between low CP violation parameter $J_{CP}$ and high CP violation in the decay of RHN. Our
prediction for $J_{CP}$ and therefore for $\delta$ for some fixed parameters can be constrained by the
current observation of BAU.

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Appendix A: $A_4$ group: the AF (Altarelli-Feruglio) basis introduced by G. Altarelli and F. Feruglio

The non-Abelian $A_4$ is a group of even permutations of 4 objects and has $4!/2 = 12$ elements. The group is generated by two generators $S$ and $T$ satisfying the relations

$$S^2 = (ST)^3 = (T^3) = 1. \quad (A1)$$

There are three one-dimensional irreducible representations of the group denoted as

$$1 : \quad S = 1, \quad T = 1, \quad (A2)$$

$$1' : \quad S = 1, \quad T = e^{i4\pi/3} \equiv \omega^2, \quad (A3)$$

$$1'' : \quad S = 1, \quad T = e^{i2\pi/3} \equiv \omega. \quad (A4)$$

It is easy to check that there is no two-dimensional irreducible representation of this group. The three-dimensional unitary representations of $T$ and $S$ are given by

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad (A5)$$

where $T$ has been chosen to be diagonal. The multiplication rules for the singlet and triplet representations correspond to the above basis of two generators $T, S$ are given as below

$$1 \times 1 = 1, \quad 1' \times 1'' = 1, \quad 3 \times 3 = 3 + 3_A + 1 + 1' + 1''. \quad (A6)$$

For triplets

$$a = (a_1, a_2, a_3), \quad b = (b_1, b_2, b_3), \quad (A7)$$

one can write

$$1 \equiv (ab) = (a_1b_1 + a_2b_3 + a_3b_2), \quad (A8)$$

$$1' \equiv (ab)' = (a_3b_3 + a_1b_2 + a_2b_1), \quad (A9)$$

$$1'' \equiv (ab)'' = (a_2b_2 + a_1b_3 + a_3b_1). \quad (A10)$$

Note that while 1 remains invariant under the exchange of the second and the third elements of $a$ and $b$, $1'$ is symmetric under the exchange of the first and the second elements while $1''$
is symmetric under the exchange of the first and the third elements.

\[ 3 \equiv (ab)_S \]
\[ = \frac{1}{3} (2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1), \]  
(A11)  
\[ 3_A \equiv (ab)_A = \frac{1}{3} (a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_3b_1 - a_1b_3). \]  
(A12)

We will only focus only 3 since the 3\_A terms are antisymmetric and hence can not be used for neutrino mass matrix. In the triplet 3\_A, we can see that the first element has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry while the third element earns 1-3 interchange symmetry.

Moreover, if \( c, c', c'' \) are singlets of the type 1, 1', 1'', and \( a = (a_1, a_2, a_3) \) is a triplet, then the products \( ac, ac', ac'' \) are triplets explicitly given by \((a_1c, a_2c, a_3c), (a_3c', a_1c', a_2c'), (a_2c'', a_3c'', a_1c'')\), respectively.

Because the above basis, \( T \) is complex and \( T^\dagger \neq T \) in general so the complex conjugate representation \( r^* \) of a representation \( r (\text{r = 1', 1'', 3}) \) is not the same as \( r \). It is determined by the following rules \([24]\):

\[ c \sim 1 \rightarrow c^* \sim 1; \quad c' \sim 1' \rightarrow c''^* \sim 1'' = 1'', \quad c' \sim 1' \rightarrow c''^* \sim 1'' = 1', \]
\[ a = (a_1, a_2, a_3) \sim 3 \rightarrow a^* = (a_1^*, a_3^*, a_2^*). \]  
(A13)

For the one dimensional reps, it is easy to see these property because \((\omega^2)^* = \omega\). For the 3-reps we can find a transformation \( U \) that changes \( 3^* \) into 3 or \( 3^* \sim 3 \). This is similar to the case of \( SU(2) \) symmetry.

In considering model, the \( A_4 \) lepton triplet \( \overline{\psi}^d = (\overline{\psi}_1^d, \overline{\psi}_2^d, \overline{\psi}_3^d) \sim 3 \) has it complex conjugate of \( \psi^d = (\psi_1^d, \psi_3^d, \psi_2^d) \sim 3^* \). The \( 3 \times 3^* \) is used for constructing the kinetic term of lepton and Higgses, the Higgs potential,... For example some quadratic terms respecting \( A_4 \)-symmetry are:

\[ \overline{\psi}^d = (\psi_1^d, \psi_2^d, \psi_3^d) \sim 3, \]
\[ \rightarrow \left( \overline{\psi}^d \gamma^\mu D_\mu \psi^d \right)_1 = \overline{\psi}_1^d \gamma^\mu D_\mu \psi_1^d + \overline{\psi}_2^d \gamma^\mu D_\mu \psi_2^d + \overline{\psi}_3^d \gamma^\mu D_\mu \psi_3^d, \]
\[ \phi_S = (\phi_{S_1}, \phi_{S_2}, \phi_{S_3}) \sim 3, \quad (\phi_{S_1}^*, \phi_{S_2}^*, \phi_{S_3}^*) \sim 3^* \]
\[ \rightarrow \left( (D^\mu \phi_S)^\dagger D_\mu \phi_S \right)_1 = (D^\mu \phi_{S_1})^\dagger D_\mu \phi_{S_1} + (D^\mu \phi_{S_2})^\dagger D_\mu \phi_{S_2} + (D^\mu \phi_{S_3})^\dagger D_\mu \phi_{S_3}, \]
\[ \xi' \sim 1' \rightarrow \xi''^* \sim 1'' = 1'' \rightarrow (\xi''^* \xi')_1 = \xi'' \xi', \quad (\xi'''' \xi''')_1 = \xi'''' \xi''. \]  
(A14)
Note that the AF basis was used in ref. [25].

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