Study of $B \to K_0^*(1430) K^{(*)}$ decays in QCD Factorization Approach

Ying Li*, Hong-Yan Zhang, Ye Xing, Zuo-Hong Li
Department of Physics, Yantai University, Yantai 264005, China

Cai-Dian Lü
Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China
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Within the QCD factorization approach, we calculate the branching fractions and $CP$ asymmetry parameters of 12 $B \to K_0^*(1430) K^{(*)}$ decay modes under the assumption that the scalar meson $K_0^*(1430)$ is the first excited state or the lowest lying ground state in the quark model. We find that the decay modes with the scalar meson emitted, have large branching fractions due to the enhancement of large chiral factor $r_8^{K_S}$. The branching fractions of decays with the vector meson emitted, become much smaller owing to the smaller factor $r_8^{K^*}$. Moreover, the annihilation type diagram will induce large uncertainties because of the extra free parameters dealing with the endpoint singularity. For the pure annihilation type decays, our predictions are smaller than that from PQCD approach by 2-3 orders of magnitudes. These results will be tested by the ongoing LHCb experiment, forthcoming Belle-II experiment and the proposing circular electron-positron collider.

I. INTRODUCTION

Although the quark model has made great success in describing most of the hadronic states, the lowest lying scalar mesons are too light to fit in the quark model. Two possible scenarios have been proposed on the basis of whether the scalars lower than 1 GeV belong to the four-quark states or the classical two scalar mesons. They are controversial for decades [1]. In scenario-1 (S1), the scalars such as $\kappa(800)$, $a_0(980)$ and $f_0(980)$ are naively seen as the lowest lying $q \bar{q}$ states, and $K_0^*(1430)$, $a_0(1450)$, and $f_0(1500)$ are the first excited states, correspondingly. In contrast, $K_0^*(1430)$, $a_0(1450)$, $f_0(1500)$ are treated as the the $q \bar{q}$ ground states and their first excited states are about (2.0 $\sim$ 2.3) GeV in scenario-2 (S2), in which the lightest scalar mesons are the four-quark bound states.

In the past decade, many $B$ decay modes involving scalar mesons have been reported by BaBar, Belle and large hadron collider-b (LHCb) experiments. This may provide a unique feature to distinguish these two scenarios by the $B$ meson tag [2]. It is thus hoped that the combination of the precise experimental measurements and the accurate theoretical predictions might provide us valuable information on the nature of scalar mesons.

To achieve this goal, some hadronic $B_q(q = u, d, s)$ decays to scalar mesons have been studied in detail in the framework of the QCD factorization (QCDF) [1, 3, 4] and the perturbative QCD approach (PQCD) [5, 6]. Using the QCDF approach, H-Y Cheng et.al had calculated the branching fractions and direct $CP$ asymmetries of most decay modes in refs. [3, 4], such as $B \to f_0 K$, $B \to K_0^*(1430)\phi(\rho)$ and $B \to K_0^*(1430)\pi$, where most results accommodate the data with large uncertainties.

Very recently, the decays $B \to K_0^*(1430) K^{(*)}$ dominated by $b \to d$ penguin operators, have been calculated within the PQCD approach [6], however they have not been touched in QCDF frame. To complete, we therefore shall calculate the branching fractions of $B \to K_0^*(1430) K^{(*)}$ decays in QCDF, as well as their $CP$ asymmetry parameters. In the experimental side, these observables are too small to be measured at current experiments, but they are hopeful to be measured in the future experiments such as the updated LHCb experiment, the high luminosity Belle-II experiment and the proposing high energy circular electron-positron collider.

The paper is organized as follows. In Section 2, we give the analytic formula including the effective Hamiltonian, the form factor and all corrections to the amplitudes. Presentation of results and discussions are given in Section 3. At last, we summarize this work in Section 4.

II. ANALYTIC FORMULA

In this section, we shall start from the weak effective Hamiltonian responsible for $B \to K_0^*(1430) K^{(*)}$ decays. In the standard model, it could be written as [7]

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* (C_1O_1 + C_2O_2) \right. \\
- V_{ub} V_{td}^* \left( \sum_{i=3}^{10} C_i O_i + C_{7\gamma}O_{7\gamma} + C_{8g}O_{8g} \right) \left] + \text{h.c.} \right. 
$$

(1)

The explicit form of the operators $O_i$ and the corresponding Wilson Coefficients $C_i$ at different scale $\mu$ could be found in ref.[7]. $V_{ut+jb}$ and $V_{ut+jd}$ are the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements. Note that $O_{1,2}$ are tree operators and others $O_{3-10,7\gamma,8g}$ are penguin ones.
In dealing with the nonleptonic charmless $B$ decays, the decay amplitude is usually separated into the emission part and the annihilation part in terms of the structure of the topological diagrams. According to the factorization approximation based on the heavy quark limit, the former part could be written as the product of decay constant and form factor. For the latter one, it is always regarded as being power suppressed. In QCDF $^8$ based on collinear factorization, the contribution of the non-perturbative sector of QCD, lack a precise solution. For the $B \rightarrow P$ and $B \rightarrow V$ transition form factors, we will employ the results of the QCD sum rule method $^9$, since they have been used in calculating $B \rightarrow PP, PV$ and $VV$ modes widely. The form factor of $F^{B \rightarrow S}$ as concerned, to the best of our knowledge, a number of approaches had been advocated to calculate them, such as QCD sum rule $^{10}$, light-cone QCD sum rule $^{11}$, PQCD $^{12}$ and covariant light front quark model (cLFQM) $^{13}$. In this work, we use the results obtained by cLFQM $^{13}$ for keeping consistent with the results of other $B \rightarrow SP(V)$ decay modes $^3$.

Following the standard procedure of QCDF approach, the emission part of decay amplitude could be written as

$$ A_s(B^0 \rightarrow M_1M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_i V_{pd}V_{^*d}^a a_i^p(\mu)(M_1M_2|O_i|B)_F, $$

where $(M_1M_2|O_i|B)_F$ is the factorizable matrix element, which can be factorized into a form factor times a decay constant, as stated before. The effective parameters $a_i^p$ can be calculated perturbatively, whose expressions are given by

$$ a_i^p(M_1M_2) \quad (C_l \equiv \frac{C_{l+1}}{N_c}) \quad N_i(M_2) $$

$$ + \frac{C_{l+1}}{N_c} \frac{CF_{\alpha s}}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2) \right] $$

$$ + P_{i1}^p(M_2), $$

with $i = 1, \ldots , 10$. The upper (lower) signs apply when $i$ is odd (even), $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$. The quantities $\lambda_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted quark and the spectator quark of the $B$ meson and $P_i(M_2)$ for penguin contractions. Similarly, the annihilation contributions are described by the terms $b_1$, and $b_{i,EW}$, which have the expressions

$$ b_1 = \frac{C_F}{N_c^2} C_l A_1^l, $$

$$ b_2 = \frac{C_F}{N_c^2} C_2 A_2^l, $$

$$ b_3 = \frac{C_F}{N_c^2} \left[ C_3 A_1^l + C_5 (A_3^l + A_4^l) + N_c C_6 A_5^l \right], $$

$$ b_4 = \frac{C_F}{N_c^2} \left[ C_4 A_1^l + C_6 A_2^l \right], $$

$$ b_{3,EW} = \frac{C_F}{N_c^2} \left[ C_9 A_1^l + C_7 (A_3^l + A_4^l) + N_c C_8 A_5^l \right], $$

$$ b_{4,EW} = \frac{C_F}{N_c^2} \left[ C_10 A_1^l + C_8 A_2^l \right], $$

with the subscripts 1,2,3 of $A_i^{l,f}$ stand for the annihilation amplitudes induced from $(V - A)(V - A)$, $(V - A)(V + A)$ and $(S - P)(S + P)$ operators, respectively. The superscripts $l$ and $f$ refer to gluon emission from the initial and final-state quarks, respectively. It should be stressed that the decays $B^0 \rightarrow K_0^{(*)+}K_0^{(*)-}$ are only induced by the annihilations type diagrams. Hereafter, the order of the $a_i^p(M_1M_2)$ coefficients is dictated by the subscript $M_1M_2$, where $M_2$ is the emitted meson and $M_1$ shares the same spectator quark with the $B$ meson. For the $b_i(M_1M_2)$ of the annihilation, $M_1$ means the one containing an anti-quark from the weak vertex, while $M_2$ contains a quark from the weak vertex. The explicit expressions of $V_i, H_i, P_i, b_i, b_{i,EW}$ and their inner functions could be found in refs.$^{[1,3]}$.

When dealing with the hard-scattering spectator and the weak annihilation contributions, we suffer from infrared endpoint singularities $X = \int_0^1 dx/(1 - x)$, that cannot be calculated from the first principle in the QCDF approach and only be estimated phenomenologically with a large uncertainty. Following the arguments of ref.$^8$, we also parameterize these kinds of contributions by the complex quantities, $X_H$ and $X_A$ namely,

$$ X_H = \left( 1 + \rho_H e^{i\phi_H} \right) \ln \frac{m_B}{\Lambda}, $$

$$ X_A = \left( 1 + \rho_A e^{i\phi_A} \right) \ln \frac{m_B}{\Lambda}, $$

where $\Lambda = 0.5$GeV. $\rho_H, \rho_A$ are real parameters, and $\phi_H$ and $\phi_A$ are free strong phases in the range $[-180^\circ, 180^\circ]$. All the above four parameters should be fixed by the experimental data, such as branching fractions and $CP$
asymmetries. In the so-called PQCD approach [14], one can eliminate these end-point singularities by keeping all small transverse momenta of gluons and inner quarks.

Including the emission and annihilation contributions, the decay amplitude can be finally given as

\[
A(B^- \to K^- K_{0}^{*0}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ \left( a_2^p - \frac{1}{2} a_1^p - (a_6^p - \frac{1}{2} a_8^p) r_K^* \right) _{K^- K_{0}^{*0}} f_{K^0} F_0^{B \to K}(m_K^2)(m_B^2 - m_K^2) + f_B f_{K_0^*} f_{K}(b_2 \delta_{u}^p + b_3 + b_{3,EW})_{K^- K_{0}^{*0}} \right\},
\]

(8)

\[
A(B^- \to K_{0}^* K^0) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - \left( a_2^p - \frac{1}{2} a_1^p - (a_6^p - \frac{1}{2} a_8^p) r_K^* \right) _{K_{0}^{*} K^0} f_{K} F_0^{B \to K^*}(m_{K^*}^2)(m_B^2 - m_{K^*}^2) + f_B f_{K_0^*} f_{K}(b_2 \delta_{u}^p + b_3 + b_{3,EW})_{K_{0}^{*} K^0} \right\},
\]

(9)

\[
A(B^0 \to K_{0}^{*0} K^0) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ \left( a_2^p - \frac{1}{2} a_1^p - (a_6^p - \frac{1}{2} a_8^p) r_K^* \right) _{K_{0}^{*0} K^0} f_{K^0} F_0^{B \to K^*}(m_{K^*}^2)(m_B^2 - m_{K^*}^2) + f_B f_{K_0^*} f_{K}(b_3 + b_4 - \frac{1}{2} (b_{3,EW} + b_{4,EW}))_{K_{0}^{*0} K^0} \right\} + \left( b_4 - \frac{1}{2} b_{4,EW} \right)_{K_{0}^{*0} K^0},
\]

(10)

\[
A(B^0 \to K_{0}^{*+} K^-) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ f_B f_{K_0^*} f_{K}(b_1 \delta_{u}^p + b_4 + b_{4,EW})_{K_{0}^{*+} K^-} + \left( b_4 - \frac{1}{2} b_{4,EW} \right)_{K_{0}^{*+} K^-} \right\},
\]

(12)

\[
A(B^0 \to K^{+} K_{0}^{-}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ f_B f_{K_0^*} f_{K}(b_1 \delta_{u}^p + b_4 + b_{4,EW})_{K^+ K_{0}^{-}} + \left( b_4 - \frac{1}{2} b_{4,EW} \right)_{K^+ K_{0}^{-}} \right\},
\]

(13)

\[
A(B^- \to K^{*-} K_{0}^{*0}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - \left( a_2^p - \frac{1}{2} a_1^p - (a_6^p - \frac{1}{2} a_8^p) r_K^* \right) _{K^{*-} K_{0}^{*0}} f_{K_{0}^{*}} A_0^{B \to K^*}(m_{K^*}^2) m_B p_e + f_B f_{K_0^*} f_{K}(b_2 \delta_{u}^p + b_3 + b_{3,EW})_{K^{*-} K_{0}^{*0}} \right\},
\]

(14)

\[
A(B^- \to K_{0}^{*-} K_{0}^{*}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ \left( a_2^p - \frac{1}{2} a_1^p - (a_6^p - \frac{1}{2} a_8^p) r_K^* \right) _{K_{0}^{*-} K_{0}^{*}} f_{K_{0}^{*}} F_1^{B \to K_{0}^{*}}(m_{K^*}^2) m_B p_e - f_B f_{K_0^*} f_{K}(b_2 \delta_{u}^p + b_3 + b_{3,EW})_{K_{0}^{*-} K_{0}^{*}} \right\},
\]

(15)
\begin{equation}
A(B^0 \to K^{0} K_0^{*0}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - \left( a_p^0 - \frac{1}{2} a^0_{10} + (a^p_6 - \frac{1}{2} a^p_8) r^K_{10} \right) \frac{2 f_{K^0} A_{B^0K^*0}(m^2_{K^0}) m_{B^0}}{r^K_{10} K_0^{*0}} + \left( b_3 + b_4 - \frac{1}{2} (b_{3,EW} + b_{4,EW}) \right) \right\},
\end{equation}

\begin{equation}
A(B^0 \to K_0^{*0} K^0) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - \left( a_p^0 - \frac{1}{2} a^0_{10} + (a^p_6 - \frac{1}{2} a^p_8) r^K_{10} \right) \frac{2 f_{K^0} A_{B^0K^*0}(m^2_{K^0}) m_{B^0}}{r^K_{10} K_0^{*0}} + \left( b_3 + b_4 - \frac{1}{2} (b_{3,EW} + b_{4,EW}) \right) \right\},
\end{equation}

\begin{equation}
A(B^0 \to K_0^{*+} K^{*-}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - f_B f_{K^0} f_K \left[ (b_1 \delta_p^0 + b_4 + b_{4,EW}) K_0^{*+} + \left( b_4 - \frac{1}{2} b_{4,EW} \right) K^{*-} K_0^{*+} \right] \right\},
\end{equation}

\begin{equation}
A(B^0 \to K^{*-} K_0^{*+}) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ - f_B f_{K^0} f_K \left[ (b_1 \delta_p^0 + b_4 + b_{4,EW}) K_0^{*+} + \left( b_4 - \frac{1}{2} b_{4,EW} \right) K^{*-} K_0^{*+} \right] \right\},
\end{equation}

where

\begin{align*}
r^K_0(\mu) &= \frac{2m^2_K}{m_b(\mu)[m_u(\mu) + m_d(\mu)]}, \\
r^K_0^{*}(\mu) &= \frac{2m^2_{K^0}}{m_b(\mu)[m_u(\mu) + m_d(\mu)]}, \\
r^{K^*}(\mu) &= \frac{2m^{K^*}_0}{m_b(\mu)} f^{K^*}_K(\mu) / f_K.
\end{align*}

III. NUMERICAL RESULTS AND DISCUSSION

To proceed, we shall present numerical results obtained from the formulas given in the previous section. Firstly, we introduce the adopted parameters in our calculation. Secondly, we give the numerical results and show the theoretical errors due to uncertainty of some parameters. At last, some discussions and comparisons will be added.

The scalar meson, unlike the pseudoscalar one, has two kinds of decay constants, namely the vector decay constant $f_3$ and the scale-dependent scalar decay constant $f_S$, the definitions of which are given by:

\begin{equation}
\langle K_0^{*0}(p)|\bar{s}\gamma_\mu d(0)\rangle = f_{K_0^{*0}} p_\mu,
\end{equation}

\begin{equation}
\langle K_0^{*0}(p)|\bar{s}d(0)\rangle = m_{K_0^{*0}} f_{K_0^{*0}}.
\end{equation}

The two decay constants satisfy

\begin{equation}
f_{K_0^{*0}} = \frac{m_u(\mu) - m_d(\mu)}{m_{K_0^{*0}}} f_{K_0^{*0}},
\end{equation}

where $m_u(\mu)$ and $m_d(\mu)$ are the running current quark masses. It should be stressed that the decay constants of $K_0^{*0}(1430)$ have the signs flipped from S1 to S2. The definition of the form factors for the $B \to S$ transitions are given by [13]

\begin{equation}
\langle S(p')|A_\mu|B(p)\rangle = -i \left[ \left( P_\mu - \frac{m_B^2 - m^2_{S}}{q^2} q_\mu \right) F_{1S}(q^2) + \frac{m_B^2 - m^2_{S}}{q^2} q_\mu F_{2S}(q^2) \right],
\end{equation}

where $P_\mu = (p + p')_\mu$ and $q_\mu = (p - p')_\mu$. In cLFQM, the momentum dependence of the form factor could be parameterized in a di-pole model form [13],

\begin{equation}
F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)}.
\end{equation}

Together with the decay constants, the parameters $F(0)$, $a$ and $b$ for $B \to S$ transitions in the different scenarios are summarized in Table I.

The twist-2 light-cone distribution amplitude (LCDA) $\Phi_S(x)$ and twist-3 $\Phi_S^2(x)$ and $\Phi_S^3(x)$ for the scalar meson $K_0^{*0}$ made of the quarks $s\bar{d}$ are given by

\begin{equation}
\langle K_0^{*0}(p)|\bar{s}(z_2) \gamma_\mu d(z_1)|0\rangle = p_\mu \int_0^1 dx e^{i(xp\cdot z_2 + \bar{z}_p \cdot z_1)} \Phi_S(x),
\end{equation}

\begin{equation}
\langle K_0^{*0}(p)|\bar{s}(z_2) d(z_1)|0\rangle = m_{K_0^{*0}} \int_0^1 dx e^{i(xp\cdot z_2 + \bar{z}_p \cdot z_1)} \Phi_S^2(x),
\end{equation}

\begin{equation}
\langle K_0^{*0}(p)|\bar{s}(z_2) \sigma_{\mu\nu} d(z_1)|0\rangle = -m_{K_0^{*0}}(p_\mu z_\nu - p_\nu z_\mu) \int_0^1 dx e^{i(xp\cdot z_2 + \bar{z}_p \cdot z_1)} \Phi_S^3(x) / 6.
\end{equation}
with \( z = z_2 - z_1, \bar{x} = 1 - x \), and their normalizations are

\[
\int_0^1 dx \Phi_S(x) = f_S, \quad \int_0^1 dx \Phi_S^a(x) = \int_0^1 dx \Phi_S^b(x) = \bar{f}_S. \tag{26}
\]

The above definitions of LCDAs can be combined into a single matrix element as

\[
\langle \bar{K}_0(p)|s_{2g}(z_2)d_{1a}(z_1)|0 \rangle = \frac{1}{4} \int_0^1 dx e^{i(xp_{z_2} + \bar{x}p_{z_1})} \times \left\{ p\Phi_S(x) + m_S \left( \Phi_S^a(x) - \sigma_{\mu\nu}p^\mu z^\nu \Phi_S^b(x) \right) \right\} \alpha_{\beta}. \tag{27}
\]

In general, the twist-2 light-cone distribution amplitude \( \Phi_S \) has the form

\[
\Phi_S(x, \mu) = \bar{f}_S(\mu) 6x(1 - x) \times \left[ B_0(\mu) + \sum_{m=1}^{\infty} B_m(\mu) C_{m/2}(2x - 1) \right], \tag{29}
\]

where \( B_m(\mu) \) are scale-dependence Gegenbauer moments and \( C_{m/2}(u) \) are the Gegenbauer polynomials. The \( B_m \) of different scenarios are also presented in Table I. As for the twist-3 distribution amplitudes, we shall adopt the asymptotic form for simplicity, shown as

\[
\Phi_S^a(x) = \bar{f}_S, \quad \Phi_S^b(x) = \bar{f}_S 6x(1 - x). \tag{30}
\]

For the LCDAs of \( K^{(*)} \), we will employ the formulae obtained from the QCD sum rules \([15, 16]\). The other used parameters, such as the CKM elements, the decay constants of the pseudo-scalar and the vector, and the form factors of \( B \to K^{(*)} \), are also listed in Table II for convenience.

Now, we turn to discuss the numerical results of the concerned decay modes. As we had stated in previous section, when calculating the hard spectator and the annihilation contributions, two endpoint singularities are parameterized phenomenologically by four free parameters, namely \( \rho_H, \phi_H \) and \( \rho_A, \phi_A \), which should be determined from the experimental data. In ref.[3], a global fit of \( \rho_A \) and \( \phi_A \) to the \( B \to S P \) data indicates \( \rho_A = 0.15 \) and \( \phi_A = 82^\circ \) with \( \chi^2 = 8.3 \), so in this work for the central values (or “default” results), \( \rho_A H = 0.15 \) and \( \phi_A H = 82^\circ \) are adopted. In Table III, we list the calculated branching fractions of \( B \to K_0^*(1430)K^{(*)} \) decays, where the first uncertainty is due to the variations of \( B_{1,3} \) and \( f_S \), the second comes from the form factors and the strange quark mass, and the last one is induced by weak annihilation and hard spectator interactions in ranges \( \rho_{A,H} \in [0, 0.3] \) and \( \phi_{A,H} \in [0, 180^\circ] \).

Here, we shall take \( B^- \to K^- K_0^*(1430) \) and \( B^- \to K^- K_0^*(1430)K^0 \) as examples to illustrate the relative size of each contribution. The decay amplitude formula of each mode has been presented in eqs.(8) and (9), respectively. The first decay mode is characterized by \( B \to P \) transition with scalar meson emitted; while the second decay mode is characterized by \( B \to S \) transition with pseudoscalar meson emitted. Because of the small vector decay constant of scalar meson \( K_0^*(1430) \), \( f_{K_0^*} F_B^{B \to K} (m_{K_0^*}^2) \) is suppressed relative to \( f_{K_0^*} F_B^{B \to K} (m_K^2) \). Therefore, the decay width of the first channel should be suppressed comparing with the second one. The numerical results of these two decay modes are given as below:

\[
A(B^- \to K^- K_0^*) \propto V_{ub} V_{ud}^*(0.53 + 0.14i) + V_{cb} V_{cd}^*(0.58 + 0.08i) + V_{ub} V_{ud}^*(-0.03 + 0.01i) + V_{cb} V_{cd}^*(-0.03 + 0.01i), \tag{31}
\]

\textit{emission diagrams}

\[
A(B^- \to K^- K_0^*) \propto V_{ub} V_{ud}^*(0.53 + 0.14i) + V_{cb} V_{cd}^*(0.58 + 0.08i) + V_{ub} V_{ud}^*(-0.03 + 0.01i) + V_{cb} V_{cd}^*(-0.03 + 0.01i), \tag{31}
\]

\textit{annihilation diagrams}
From these two equations, it is apparent that the emission diagrams are dominant. However, there is a large enhancement from $O_{6,8}$ operators due to the fact that the chiral factor $r_K^{K'} = 12.3$ at $\mu = 4.2$ GeV is much larger than $r_K = 1.5$ owing to the larger mass of $K_0(1430)$.

It follows that $(a_6^p - \frac{1}{4}a_8^p)r_{K0}^{K'}$ is much greater than $(a_6^p - \frac{1}{4}a_8^p)r_{K0}^{K'}$ and $(a_6^p - \frac{1}{4}a_8^p)r_{K0}^{K'}$. This compensates with the suppression of scalar meson decay constant to result in a larger branching ratio of $B^-\to K^-K_0^{*0}(1430)$ with scalar meson emitted.

In the $B\to PP(V)$ and $VV$ decay modes, the weak annihilation contribution is usually expected to be very small because it belongs to the next leading power correction. However, one can see from Table III that the uncertainties induced by the weak annihilation are very large, even much larger than the central values in some decay modes, such as $\mathcal{B}^{0}\to K_0^{*0}(1430)K^0$ and pure annihilation mode $\mathcal{B}^{0}\to K_0^{*0}(1430)K^*$. This phenomenon can be understood as follow, when discussing the penguin-induced annihilation diagram of $B\to PP$ mode, the decay amplitude is helicity suppressed heavily because the helicity of one of the final-state mesons cannot match with that of its own quarks. On the contrary, this kind of helicity suppression can be alleviated when the scalar meson involved due to the nonzero orbital angular momentum $L_z$ of the scalar meson.

From Table III, one could notice that our predicted central values for the branching ratios of $K_0^{*0}(1430)K^0$ and $K_0^{*0}(1430)K^0$ based on S2 are smaller than the results based on S1 by a factor of 4 $\sim$ 5. In contrast, the central values of of $K_0^{*0}(1430)$ and $K_0^{*0}(1430)$ based on S1 are a bit smaller than those of S2. For $K_0^{*0}(1430)$ and $K_0^{*0}(1430)$, the predicted central values of S2 is larger than those of S1 by a factor 2. Although there is difference between the central values of different scenarios, we cannot distinguish two scenarios due to large uncertainties taken by annihilation diagrams, unless one approach were proposed to deal with annihilation effectively in QCD. In previous studies [3], by comparing the calculated branching ratios of $B\to K_0^{*0}$ and $B\to K_0^{*0}$ with experimental data, one concluded that the S2 is preferred, i.e. $K_0^{*0}$ is very likely the lowest lying state. If so, the branching fractions of $K_0^{*0}(1430)$ and $K_0^{*0}(1430)$ could be measured by analyzing the data of $K_0^{*0}(1430)$ and $K_0^{*0}(1430)\pi^-$. Unfortunately, the predicted results of $K_0^{*0}(1430)K^0$ and $K_0^{*0}(1430)K^0$ is too small to be measured with the current data of Belle, but they are hopeful to be measured in the forthcoming Belle-II by analyzing the four-body final states of $K^+K^-\pi^+\pi^-$. Within the framework of PQCD, X. Liu et.al had calculated these decays [6], and both their results and ours are below the experimental upper limits. Comparing our predictions and theirs, we find that for pure annihilations they obtained rather larger branching fractions by keeping the transverse momenta, which are larger than ours by more than two order of magnitude. For decays with emission diagrams, they also got the larger branching ratios with rather large nonfactorizaion diagrams based on both two different scenarios. We hope the future measurements could distinguish two different frameworks. The $CP$ asymmetries have not been observed in any $B$ decays involving a scalar meson. For the charged decay
Then, we can write down the modes, according the definition,

$$A_{CP}^{dir} = \frac{A(B^- \rightarrow f) - A(B^+ \rightarrow \bar{f})}{A(B^- \rightarrow f) + A(B^+ \rightarrow f)}$$

the predictions of the direct CP asymmetries based on two different scenarios of scalar mesons are summarized in Table IV. The resource of this asymmetry is the interference between the emission diagrams and annihilations. The large uncertainties in the latter lead to the large errors in the direct CP asymmetries, as shown in the table.

For the neutral decay modes, the situation becomes more complicated due to the fact that both $B^0$ and $\bar{B}^0$ could decay to the same final states. For illustration, we will take $B^0(\bar{B}^0) \rightarrow \overline{K}^0(1430)K^0$ as an examples. Four decay amplitudes, $A_f$, $A_f$, $\bar{A}_f$ and $\bar{A}_f$ are denoted as

$$A_f = \langle K^0_0 \overline{K}^0 \rangle B^0), \quad A_f = \langle \overline{K}^0_0 K^0 \rangle B^0);$$
$$\bar{A}_f = \langle K^0_0 \overline{K}^0 \rangle \bar{B}^0), \quad \bar{A}_f = \langle K^0_0 K^0 \rangle \bar{B}^0).$$

Then, we can write down the CP asymmetry as

$$A_{CP}^{dir} = \frac{|A_f|^2 + |\bar{A}_f|^2 - |A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2 + |A_f|^2 + |\bar{A}_f|^2}. \quad (35)$$

Due to $B^0 - \bar{B}^0$ mixing, the four time-dependent decay widths are given by ($f = \overline{K}^0_0 (1430)K^0$)

$$\Gamma(B^0(t) \rightarrow f) = e^{-\gamma_1\frac{1}{2}}(|A_f|^2 + |\bar{A}_f|^2)$$

$$\Gamma(\bar{B}^0(t) \rightarrow \bar{f}) = e^{-\gamma_1\frac{1}{2}}(|A_f|^2 + |\bar{A}_f|^2)$$

$$\Gamma(B^0(t) \rightarrow \bar{f}) = e^{-\gamma_1\frac{1}{2}}(|A_f|^2 + |\bar{A}_f|^2)$$

$$\Gamma(\bar{B}^0(t) \rightarrow f) = e^{-\gamma_1\frac{1}{2}}(|A_f|^2 + |\bar{A}_f|^2)$$

where $\Delta m$ stands for the mass difference of two mass eigenstate of $B^0/\bar{B}^0$ meson, and $\Gamma$ for the average decay width of the $B$ meson. The auxiliary parameters $C_f$ and $S_f$ appearing in above equations are defined by

$$C_f = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2}, \quad (37)$$

$$S_f = \frac{2\text{Im}(\lambda_f)}{1 + |A_f/\bar{A}_f|^2}, \quad (38)$$

$$\lambda_f = \frac{V_{tb}V^*_{td}}{V^*_{tb}V_{td}} \frac{\bar{A}_f}{A_f}. \quad (39)$$

Replacing $f$ by $\bar{f}$, we could obtain the formulae for $C_f$ and $S_f$, corresponding. If the experiment could find the values of $C_f, S_f, C_f$ and $S_f$, we thus can obtain four new parameters:

$$C = \frac{1}{2}(C_f + C_f), \quad \Delta C = \frac{1}{2}(C_f - C_f), \quad (40)$$

$$S = \frac{1}{2}(S_f + S_f), \quad \Delta S = \frac{1}{2}(S_f - S_f). \quad (41)$$

Physically, $S$ is the mixing-induced CP asymmetry and $C$ is from the direct CP asymmetry, while $\Delta C$ and $\Delta S$ are CP-even under CP transformation $\lambda_f \rightarrow 1/\lambda_f$. In Table V, we present our estimations of $A_{CP}, C, \Delta C, S$ and $\Delta S$ for the final states $\overline{K}^0_0 K^0, K^0_0 K^0, K^0_0 K^0$ and $K^0_0 K^0$, under two different scenarios. It should be stressed that in PQCD [6], there is no CP asymmetries because of the absence of tree operators. However, in QCDF, by including the penguin contractions ($P_i$), the charming penguin namely, an extra strong phase together with another one from annihilations might lead to the large CP asymmetry, as shown in the table. Again, the large uncertainties of annihilations result in the large error for the pure annihilation decay modes, especially under S2. We hope these parameters can be measured in future colliders, such as high luminosity Belle-II, LHC-b and even higher energy $e^+e^-$ collider.

### IV. SUMMARY

In this work, we studied the $B \rightarrow K^0_0(1430)K^{(*)}$ decays under two different scalar meson scenarios by using the QCD factorization approach. We calculated the branching fractions and the CP asymmetry parameters. It is found that the decay modes with the scalar meson emitted, have large branching fractions due to the enhancement of large chiral factor $r_{K}^{K}$, with some of the branching fractions around the corner of Belle II. In contrast,
the branching fractions of decay modes with the vector meson emitted, are much smaller. Moreover, the annihilation contributions take large uncertainties because of the free parameter from endpoint singularity. For the pure annihilation type decays, we predicted very small branching fractions, which are 2-3 orders of magnitudes smaller than the results from PQCD. Some of the decay channels are hopeful to be measured in future colliders, such as Belle-II, LHC-b and even high energy e⁺e⁻ collider.

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1. H. Y. Cheng, C. K. Chua and C. C. Yang, Phys. Rev. D 73, 014017 (2006) [hep-ph/0508164], [13]
2. Wei Wang, Cal-Dian Lu, Phys. Rev. D82 (2010) 034016
3. H. Y. Cheng, C. K. Chua, K. C. Yang and Z. Q. Zhang, Phys. Rev. D 87, no. 11, 114001 (2013) [arXiv:1303.4403 [hep-ph]]; H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D 77, 014034 (2008) [arXiv:0705.3079 [hep-ph]]; H. Y. Cheng and C. K. Chua, Phys. Rev. D 82, 034014 (2010) [arXiv:1005.1968 [hep-ph]].
4. Y. Li, J. X. Fan, J. Hua and E. L. Wang, Phys. Rev. D 85, 074010 (2012) [arXiv:1111.7153 [hep-ph]]; Y. Li, E. L. Wang and H. Y. Zhang, Adv. High Energy Phys. 2013, 175287 (2013)[arXiv:1206.4106 [hep-ph]].
5. W. Wang, Y. L. Shen, Y. Li and C. D. Lu, Phys. Rev. D 74, 114010 (2006) [hep-ph/0609082]; Y. L. Shen, W. Wang, J. Zhu and C. D. Lu, Eur. Phys. J. C 50, 877 (2007) [hep-ph/0610380]; Z. Q. Zhang and Z. J. Xiao, Chin. Phys. C 33, 508 (2009) [arXiv:0812.2314 [hep-ph]]; X. Liu, Z. Q. Zhang and Z. J. Xiao, Phys. Rev. C 34, 157 (2010) [arXiv:0904.1955v2 [hep-ph]]; Z. Q. Zhang, Phys. Rev. D 82, 034036 (2010) [arXiv:1006.5772 [hep-ph]]; Z. Q. Zhang and J. D. Zhang, Eur. Phys. J. C 67, 163 (2010) [arXiv:1004.4426 [hep-ph]]; Z. Q. Zhang and Z. J. Xiao, Chin. Phys. C 34, 528 (2010) [arXiv:0904.3375 [hep-ph]]; C. S. Kim, Y. L. Li and W. Wang, Phys. Rev. D 81, 074014 (2010) [arXiv:0912.1718 [hep-ph]]; Z. Q. Zhang, Commun. Theor. Phys. 56, 1063 (2011); Z. Q. Zhang, Phys. Rev. D 83, 054001 (2011) [arXiv:1106.0386 [hep-ph]].
6. X. Liu and Z. J. Xiao, Commun. Theor. Phys. 53, 540 (2010) [arXiv:1004.0749 [hep-ph]]; X. Liu, Z. J. Xiao and Z. T. Zou, Phys. Rev. D 88, no. 9, 094003 (2013) [arXiv:1309.7256 [hep-ph]].
7. G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [hep-ph/9512380].
8. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) [arXiv:hep-ph/9905312]; M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000) [arXiv:hep-ph/0006124]; M. Beneke and M. Neubert, Nucl. Phys. B 765, 333 (2003) [arXiv:hep-ph/0308039]; M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B 774, 654 (2007) [hep-ph/0612260].
9. P. Ball and G. W. Jones, JHEP 0703, 069 (2007) [hep-ph/0702100 [hep-ph]].
10. M. -Z. Yang, Phys. Rev. D 73, 034026 (2006) [Erratum-ibid. D 73, 079901 (2006) [hep-ph/0509103]; T. M. Aliev, K. Azizi and M. Savić, Phys. Rev. D 76, 074017 (2007) [arXiv:0710.1508 [hep-ph]].
11. Y. -M. Wang, M. J. Aslam and C. -D. Lu, Phys. Rev. D 78, 014006 (2008) [arXiv:0804.2204 [hep-ph]]; Y. -J. Sun, Z. -H. Li and T. Huang, Phys. Rev. D 83, 025024 (2011) [arXiv:1011.3901 [hep-ph]].
12. R. -H. Li, C. -D. Lu, W. Wang and X. -Z. Wang, Phys. Rev. D 79, 014013 (2009) [arXiv:0811.2648 [hep-ph]].
13. H. -Y. Cheng, C. -K. Chuah and C. -W. Huang, Phys. Rev. D 69, 074025 (2004) [hep-ph/0310359].
14. Y. -Y. Keum, H. -n Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001) [hep-ph/0004004]; Y. -Y. Keum, H. -N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001) [hep-ph/0004173]; A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang and Y. M. Wang, Phys. Rev. D 88, 034005 (2013) [arXiv:1303.7256 [hep-ph]].
[15] P. Ball and R. Zwicky, Phys. Rev. D \textbf{71}, 014029 (2005) [hep-ph/0412079].

[16] P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D \textbf{75}, 054004 (2007) [hep-ph/0612081].