Open Set Domain Adaptation: Theoretical Bound and Algorithm

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Abstract—Unsupervised domain adaptation for classification tasks has achieved great progress in leveraging the knowledge in a labeled (target) domain to improve the task performance in an unlabeled (source) domain by mitigating the effect of distribution discrepancy. However, most existing methods can only handle unsupervised closed set domain adaptation (UCSDA), where the source and target domains share the same label set. In this paper, we target a more challenging but realistic setting: unsupervised open set domain adaptation (UOSDA), where the target domain has unknown classes that the source domain does not have. This study is the first to give the generalization bound of open set domain adaptation through theoretically investigating the risk of the target classifier on the unknown classes. The proposed generalization bound for open set domain adaptation has a special term, namely open set difference, which reflects the risk of the target classifier on unknown classes. According to this generalization bound, we propose a novel and theoretically guided unsupervised open set domain adaptation method: Distribution Alignment with Open Difference (DAOD), which is based on the structural risk minimization principle and open set difference regularization. The experiments on several benchmark datasets show the superior performance of the proposed UOSDA method compared with the state-of-the-art methods in the literature.

Index Terms—Transfer Learning, Domain Adaptation, Machine Learning.

I. INTRODUCTION

STANDARD supervised learning relies on the assumption that both training and test samples are drawn from the same distribution. Unfortunately, this assumption does not hold in many applications, since the process of collecting samples is prone to dataset bias [1]. In object recognition, for example, there can be a discrepancy in the distributions between training and testing as a result of specific conditions, device type, position, orientation, and so on. To address this problem, unsupervised domain adaptation (UDA) [2], [3] was proposed to transfer the related knowledge from the source domain, which has abundant labeled samples, to an unlabeled domain (the target domain).

The aim of UDA is to minimize the distribution difference in learning the related knowledge between domains. The existing work on UDA falls into two main categories: (1) feature matching, which seeks a new feature space where the marginal distributions or conditional distributions from two domains are similar [4]–[6], and (2) instance reweighting, which estimates the weights of the source domain so that the distribution discrepancy is minimized [7], [8]. There is an implicit assumption in most existing UDA methods [9]–[11] that the source and target domains share the same label set. UDA under this assumption is also known as Unsupervised Closed Domain Adaptation (UCSDA) [12].

However, the assumption in UCSDA methods is not realistic in an unsupervised setting (i.e., when there are no labels in the target domain), since it is not known whether the classes of target samples are from the label set of the source domain. It is possible that the target domain contains additional classes (unknown classes) which are not found in the label set of the source domain [13]. For example, in the Syn2Real task [14], real-world objects (target domain) may have more classes than synthetic objects (source domain). If existing UCSDA methods are used to solve the UDA problem without the assumption, negative transfer [15] may occur, due to the mismatch between unknown and known classes (see Fig. 1(b)).

To address UDA problem without the assumption, Busto et al. [12] and Saito et al. [13] recently proposed a new problem setting, Unsupervised Open Set Domain Adaptation (UOSDA), in which the unlabeled target domain contains unknown classes that do not belong to the label set of the source domain (see Fig. 1). There are two key challenges [13] in addressing the UOSDA problem. The first challenge is how to classify unknown target samples, since there is insufficient knowledge to support learning which samples are from unknown classes. To address this challenge, it is necessary to mine deeper domain information to delineate a boundary

Fig. 1: Unsupervised open set domain adaptation problem (UOSDA), where the target domain contains “unknown” classes that are not contained in the label set of the source domain.
### Before Adaptation

| Source | Target for known | Target for unknown |
|--------|------------------|-------------------|
|        |                  |                   |

### After UCSDA Methods

| Aim of UOSDA | After UCSDA Methods |
|--------------|---------------------|
| Negative Transfer |                    |

(a) | (b) | (c)

| Source | Target for known | Target for unknown |
|--------|------------------|-------------------|
|        |                  |                   |

#### open set difference

The second challenge in UOSDA is **distribution difference**. When distributions are matched, unknown target samples should not be matched, otherwise negative transfer may occur.

#### Target for unknown classes

The first proposed UOSDA method is **Assign-and-Transform-Iteratively (ATI-λ)** [12], which recognizes unknown target samples by using a constraint integer programming then learns a linear map to match source domain with target domain by excluding predicted unknown target samples. However, ATI-λ has an additional assumption that the source domain also contains unknown classes which do not belong to the target classes. The first proposed deep UOSDA method is **Open Set Back Propagation (OSBP)** [13]. OSBP addresses the UOSDA problem without the assumption required by ATI-λ. It rejects unknown target samples by training a binary cross entropy loss.

It is clear that ATI-λ and OSBP mainly focus on UOSDA algorithms, however they have not analyzed UOSDA theoretically. Moreover, there is no work to give a generalization bound for the open set domain adaptation problem. To fill this gap, we research UOSDA from the theoretical aspect. We first study the risk of target classifier on unknown classes. We discover the risk of target classifier on unknown classes is closely related to a special term called **open set difference** which can be estimated by unlabeled samples. Minimizing open set difference help us to classify unknown target samples and address the first challenge.

Following our theory, we design a principle-guided UOSDA method referred to as **Distribution Alignment with Open Difference (DAOD)**. This method can accurately classify unknown target samples while minimizing the discrepancy between two domains for known classes. DAOD learns the target classifier by simultaneously optimizing the structural risk functional [18], the joint distribution alignment, the manifold regulariza-

### Related Work

#### Closed Set Domain Adaptation

Ben-David et al. [20] proposed generalization bounds for closed set domain adaptation. The bound represents that the performance of the target classifier depends on the performance of the source classifier and the discrepancy between the source and target domains. Many UCSDA methods [6], [10], [21] have been proposed according to the theoretical bound and attempt to minimize the discrepancy between domains. We roughly separate these methods into two categories: feature matching and instance reweighting.

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**Fig. 2:** 1) UCSDA methods match the source samples with target samples, however as Fig. (b) shows, the unknown target samples interfere with distribution matching. This may lead to negative transfer. 2) UOSDA classifies known target samples into correct known classes and recognizes unknown target samples as unknown.
Feature matching aims to reduce the distribution discrepancy by learning a new feature space to match distributions by employing the Maximum Mean Discrepancy (MMD) [22]. Joint distribution adaptation (JDA) [5] improves TCA by jointly matching marginal distributions and conditional distributions. Adaptation Regularization Transfer Learning (ARTL) [23] considers a manifold regularization term [19] to learn the geometric relations between domains, while matching distributions. Joint Geometrical and Statistical Alignment (JGSA) [24] not only considers the distribution discrepancy but also matches the geometric shift. Recent advances show that deep networks can be successfully applied to closed set domain adaptation tasks. Deep Adaptation Networks (DAN) [25] considers three adaptation layers for matching distributions and applies multiple kernels (MK-MMD) [26] for adapting deep representations. Wasserstein Distance Guided Representation Learning (WDGRL) [27] minimizes the distribution discrepancy by employing Wasserstein Distance in neural networks.

The instance reweighting method reduces distribution discrepancy by weighting the source samples. Kernel mean matching (KMM) [7] defines the weights as the density ratio between the source domain and the target domain. Yu et al. [8] provided a theoretical analysis for important instance reweighting methods. However, when the domain discrepancy is substantially large, a large number of effective source samples will be down-weighted, resulting in the loss of effective information.

Unfortunately, the methods mentioned above cannot be applied to open set domain adaptation, because unknown target samples in the closed set domain adaptation scenario are used to match distributions, which leads to negative transfer.

**Open Set Recognition.** When the source domain and target domain for known classes share the same distribution, the open set domain adaptation becomes Open Set Recognition. A common method for handling open set recognition relies on the use of threshold-based classification strategies [28]. Establishing a threshold on the similarity score means rejecting distant samples from the training samples. Open set Nearest Neighbor (OSNN) [29] recognizes whether a sample is from unknown classes by comparing the threshold with the ratio of similarity scores to the two most similar classes of the sample. Another trend relies on modifying Support Vector Machines (SVM) [30]–[32]. Multi-class open set SVM (OSVM) [32] uses a multi-class SVM as a basis to learn the unnormalized posterior probability which is used to reject unknown samples.

**Open Set Domain Adaptation.** The open set domain adaptation problem was proposed by Assign-and-Transform-Iteratively (ATI-λ) [12]. Using $\ell_2$ distance between each target sample and the center of each source class, ATI-λ constructs a constraint integer programming to recognize unknown target samples $S_u$, then learns a linear transformation to match the source domain and target domain excluding $S_u$. However, ATI-λ requires the help of unknown source samples, which are unavailable in our setting. Recently, a deep learning method, Open Set Back Propagation (OSBP) [13], has been proposed. OSBP relies on adversarial neural network and a binary cross entropy loss to learn the probability of target samples, then uses the estimated probability to separate unknown target classes. However, we have not found any paper that considers the generalization bound for open set domain adaptation. In this paper, we complete the blank in open set domain adaptation theory.

### III. PROPOSED METHOD

In this section, we first establish the basic definitions of domains, closed set domain adaptation (CSDA), and open set domain adaptation (OSDA), then introduce the problems which will be solved in this paper. Second, we present our main theoretical results. Lastly, we propose our UOSDA method based on our theoretical work.

#### A. Notation and Problem Setting

A domain is a joint probability distribution $P = \mathcal{X} \times \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ are the feature and label spaces respectively. Let $P_X$ and $P_Y$ be the marginal distributions corresponding to spaces $\mathcal{X}$ and $\mathcal{Y}$ respectively. We define closed set domain adaptation as follows.

**Definition 1** (Closed Set Domain Adaptation). Let $P^s$ and $P^t$ be the source domain and target domain respectively, where $P^s \neq P^t$, $\mathcal{X}_s = \mathcal{X}_t$ and $\mathcal{Y}_s = \mathcal{Y}_t$, $D_s = \{(x_{si}, y_{si})\}_{i=1}^{n_s} \sim P^s$, $D_t^u = \{(x_{ti}, y_{ti})\}_{i=1}^{n_t} \sim P^t$ and $D_t^l = \{(x_{tl}, y_{tl})\}_{i=n_t+1}^{n_t+m_t} \sim Q^t$ are samples drawn from domains i.i.d. The task of closed set domain adaptation is to learn a good target classifier $f_t : \mathcal{X}_t \rightarrow \mathcal{Y}_t$ given $D_s, D_t^u, D_t^l$ as the training examples.

When there are no labeled target samples $D_t^l$ ($m_t = 0$), the scenario is called unsupervised closed set domain adaptation. It is noteworthy that the assumption $\mathcal{Y}_s = \mathcal{Y}_t$ is crucial in the definition of closed set domain adaptation. However, the assumption does not hold in the open set setting. In open set domain adaptation, the target classes have two types: known classes and unknown classes. The unknown classes gather all additional classes which are not contained by the label set $\mathcal{Y}_s$. Let $\mathcal{O}_t$ be the conditional distribution $Q^t(x, y \mid y \in \mathcal{Y}_s)$. We define open set domain adaptation as follows.

**Definition 2** (Open Set Domain Adaptation). Let $P^s$ and $Q^t$ be the source domain and target domain respectively, where $P^s \neq Q^s_{\mathcal{X} \times \mathcal{Y}}$ and $\mathcal{X}_s = \mathcal{X}_t$, $D_s = \{(x_{si}, y_{si})\}_{i=1}^{n_s} \sim P^s$, $D_t^u = \{(x_{ti}, y_{ti})\}_{i=1}^{n_t} \sim Q^t_{\mathcal{X}}$ and $D_t^l = \{(x_{tl}, y_{tl})\}_{i=n_t+1}^{n_t+m_t} \sim Q^t_{\mathcal{X} \times \mathcal{Y} \mid y \in \mathcal{O}_s}$ are samples drawn from domains i.i.d. Given $D_s, D_t^u, D_t^l$ as the training examples, the tasks of open set domain adaptation is to learn a good target classifier $f_t : \mathcal{X}_t \rightarrow \mathcal{Y}_t$ such that:

1. $f_t$ classifies known target samples into correct known classes;
2. $f_t$ classifies unknown target samples as unknown.

When there are no labeled target samples $D_t^l$ ($m_t = 0$), the setting is called unsupervised open set domain adaptation.
Problem 1 (Unsupervised Open Set Domain Adaptation). Let $\mathbb{P}^s$ and $\mathbb{Q}^t$ be the source domain and target domain respectively, where $\mathbb{P}^s \neq \mathbb{Q}^t_{\mathcal{X} | \mathcal{Y}}$ and $\mathcal{X}_s = \mathcal{X}_t$. $\mathcal{D}_s = \{ (x_s, y_s) \}_{1 \leq i \leq n_s} \sim \mathbb{P}^s$ and $\mathcal{D}_t = \{ (x_t, y_t) \}_{1 \leq i \leq n_t} \sim \mathbb{Q}^t_s$ are samples drawn from domains i.i.d. How can we learn a good target classifier $f_t : \mathcal{X}_t \rightarrow \mathcal{Y}_t$ by using $\mathcal{D}_s, \mathcal{D}_t$ as the training samples?

Notions and their descriptions are summarized in Table I.

### Table I: Notations and their descriptions.

| Notation | Description |
|----------|-------------|
| $\mathcal{X}$ | feature space |
| $n_s, n_t$ | number of source/target samples |
| $n$ | the feature dimension |
| $C$ | the number of known classes |
| $P^s, Q^t$ | source/target joint distribution |
| $P^s_{\mathcal{X}}, Q^t_{\mathcal{X}}$ | source/target marginal distribution |
| $P^s_{\mathcal{X} | \mathcal{Y}}, Q^t_{\mathcal{X} | \mathcal{Y}}$ | source/target conditional distribution for class $t$ |
| $Q^t_{\mathcal{X} | Y \leq C}$ | target marginal distribution for known classes |
| $X_s \ell$ | data matrix $[x_{s,1}, ..., x_{s,n_s}] \in \mathbb{R}^{d \times n_s}$, source samples |
| $X_t \ell$ | data matrix $[x_{t,1}, ..., x_{t,n_t}] \in \mathbb{R}^{d \times n_t}$, target samples |
| $X_s^l$ | data matrix $[x_{s,1}^l, ..., x_{s,n_s}^l]$, source samples with label $l$ |
| $X_t^l$ | data matrix $[x_{t,1}^l, ..., x_{t,n_t}^l]$, target samples with pseudo label $l$ |
| $X_s^k$ | data matrix $[x_{s,1}^k, ..., x_{s,n_s}^k]$, samples predicted as known |
| $n_s^l$ | number of samples in $X_s^l$ |
| $n_t^l$ | number of samples in $X_t^l$ |
| $n_s^k$ | number of samples in $X_s^k$ |
| $\phi(\cdot), K(\cdot, \cdot)$ | kernel feature map and kernel function induced by $\phi(\cdot)$ |

B. Main Theoretical Results and Open Set Difference

We theoretically analyze the OSDA problem.

We consider multiclass classification with hypothesis space $H$ of classifiers

$$h : \mathcal{X} \rightarrow \mathcal{Y}_t(O) = \{1, ..., C, C+1\},$$

where $\mathcal{X} = \mathcal{X}_s = \mathcal{X}_t$, the classes $1, ..., C \in \mathcal{Y}_s$ and the class $C+1$ represents the unknown target classes. Denoted by

$$R_{s,i}(h) = \mathbb{E}_{x \sim \mathbb{P}^s(x | i)} \ell(h(x), i),$$

partial risks, where $\ell : \mathcal{Y}_t(O) \times \mathcal{Y}_t(O) \rightarrow \mathbb{R}$ is the symmetric loss function satisfying the triangle inequality. We note that when $i = C+1$, $R_{t,C+1}(h)$ is the risk of classifier $h$ on unknown target classes.

The risks of $h$ w.r.t. $\ell$ under $\mathbb{P}^s(x, y)$, $\mathbb{Q}^t(x, y)$ and $\mathbb{Q}^t(x, y | y \leq C)$ are given by

$$R_s(h) = \mathbb{E}_{P^s}(\ell(h(x), y)) = \sum_{i=1}^{C+1} \pi^s_i R_{s,i}(h),$$

$$R_t(h) = \mathbb{E}_{Q^t}(\ell(h(x), y)) = \sum_{i=1}^{C+1} \pi^t_i R_{t,i}(h),$$

$$R^s_t(h) = \mathbb{E}_{Q^t_{\mathcal{X} | \mathcal{Y}}}(\ell(h(x), y)) = \sum_{i=1}^{C} \pi^t_i R_{t,i}(h),$$

where $\pi^s_i = \mathbb{P}^s(y = i)$ and $\pi^t_i = \mathbb{Q}^t(y = i)$ are class-prior probabilities. Specifically, let

$$R_{t,C+1}(h) = \mathbb{E}_{Q^t_{\mathcal{X} | \mathcal{Y}}}(\ell(h,C+1)),$$

$$R^u_{s,C+1}(h) = \mathbb{E}_{P^s}(\ell(h,C+1)),$$

be risks that unlabeled samples are regarded as unknown samples. For stating the main theoretical result of the paper, we need to introduce discrepancy distance, $d_H^t(\mathbb{P}, \mathbb{Q})$, which measures the difference between two distributions $\mathbb{P}, \mathbb{Q}$.

Definition 3 (Discrepancy Distance [33]). Let $H$ be a set of functions from $\mathcal{X}$ to $\mathcal{Y}$, and $\ell$ be a loss function. The discrepancy distance between distributions $\mathbb{P}$ and $\mathbb{Q}$ over $\mathcal{X}$ is

$$d_H^t(\mathbb{P}, \mathbb{Q}) = \sup_{h \in \mathcal{H}} \left| \mathbb{E}_{\mathbb{P}}(\ell(h, h^*)) - \mathbb{E}_{\mathbb{Q}}(\ell(h, h^*)) \right|.$$

The following theorem provides an open set domain adaptation bound according to discrepancy distance.

**Theorem 1.** Given a hypothesis $H$ with a mild condition that constant function $C+1 \in H$, then for any $h \in H$, we have

$$R_s(h) = \frac{R_s(h)}{1 - \pi^s_{C+1}} \leq R_s^t(h) + d_H^t(Q^t_{\mathcal{X} | Y \leq C}, P^s) + \lambda,$$

$$R_{t,C+1}(h) = \frac{R_{t,C+1}(h)}{1 - \pi^t_{C+1}} \leq R_{t,C+1}^u(h) - \pi^t_{C+1} R_{t,C+1}(h),$$

where $\lambda = \min_{h \in H} R_s(h) + R_{t,C+1}(h), Q^t_{\mathcal{X} | Y \leq C} := \mathbb{Q}^t(x | y \neq C+1)$.

The proof can be found in Section IV. It is noteworthy that the open set difference $\Delta_o$ is the crucial term to bound the risk of $h$ on unknown target classes, since

$$R_{t,C+1}(h) \leq \frac{1 - \pi^t_{C+1}}{\pi^t_{C+1}} \left( \Delta_o + \frac{1}{2} d_H^t(Q^t_{\mathcal{X} | Y \leq C}, P^s) \right).$$

The risk of $h$ on unknown target classes is intimately bound up with the open set difference $\Delta_o$.

$$|\pi^t_{C+1} R_{t,C+1}(h) - (1 - \pi^t_{C+1}) \Delta_o| \leq \frac{1}{2} d_H^t(Q^t_{\mathcal{X} | Y \leq C}, P^s).$$

When $\pi^t_{C+1} = 0$, Theorem 1 degenerates the closed set scenario with the theoretical bound

$$R_t(h) \leq R_s(h) + \frac{3}{4} d_H^t(Q^t_{\mathcal{X}}, P^s) + \lambda.$$
source samples from being classified as unknown. According to Eq. [5], the negative term and distance discrepancy jointly prevent all target samples from being recognized as unknown classes. In addition, Corollary [11] also tells us that the positive term and negative term can be estimated just by unlabeled samples. Using Natarajan Dimension Theory [34] to bound the source risk $R_s(h)$, risks $R^{u}_{t,C+1}(h)$ and $R^{u}_{s,C+1}(h)$ by empirical estimates $\hat{R}_s(h)$, $\hat{R}^{u}_{t,C+1}(h)$ and $\hat{R}^{u}_{s,C+1}(h)$ respectively, we obtain the following result.

**Corollary 1.1.** Given a symmetric loss function $\ell$ satisfying the triangle inequality and bounded by $M$, and a hypothesis $H$ with conditions: 1) $C + 1 \in H$ and 2) the Natarajan dimension of $H$ is $d$, if a random labeled sample of size $n_s$ is generated by $P_s^X$-i.i.d and a random unlabeled sample of size $n_t$ is generated by $Q_{X}^U$-i.i.d, then for any $h \in H$ and $\delta \in (0, 1)$ with probability at least $1 - 2\delta$, we have

\[
\begin{align*}
\frac{R_t(h)}{1 - \pi_{C+1}} &\leq \hat{R}_s(h) + d^t_H(Q_{X|Y \leq C}^t, P_s^X) + \Delta_o + \lambda \\
&+ 4M \sqrt{2d \log n_s + 4d \log(C + 1) + 8 \log 4/\delta}.
\end{align*}
\]

where $\lambda = \min_{h \in H} R_s(h) + R_t^{u}(h) \text{ and empirical open set difference } \Delta_o = \frac{R^{u}_{t,C+1}(h)}{1 - \pi_{C+1}} - \hat{R}^{u}_{s,C+1}(h)$.

Next, we employ the open set difference $\Delta_o$ to construct our model, Distribution Alignment with Open Difference (DAO). C. Method

In this section, we propose our open set domain adaptation method. In Theorem [1] we derive the bound for open set domain adaptation which shows: 1) the first term (Source Risk) bounds the performance of the source domain; 2) the second term (Distribution Discrepancy) is a measure of the discrepancy between the source marginal distribution $P_s^X$ and the target marginal distribution for known classes $Q_{X|Y \leq C}^t$; 3) the third term is the open set difference $\Delta_o$, which is the difference between $R^{u}_{t,C+1}(h)$ and $R^{u}_{s,C+1}(h)$. In this paper, we utilize the term $\alpha R_{t,C+1}(h) - \gamma R^{u}_{s,C+1}(h)$ to simulate the open set difference $\Delta_o$, where $\alpha, \gamma$ ($\alpha, \gamma \geq 0$) are free parameters.

Let $X_s = [x_{s1}, ..., x_{sn_s}] \in \mathbb{R}^{d \times n_s}$, $X_t = [x_{t1}, ..., x_{tn_t}] \in \mathbb{R}^{d \times n_t}$ be the source and target data matrix respectively, and $Y_s = [y_{1}, ..., y_{n_s}] \in \mathbb{R}^{1 \times n_s}$ be the source label matrix. We can then write the bound as follows.

\[
\begin{align*}
\text{empirical source risk:} & \quad \frac{1}{n_s} \sum_{i=1}^{n_s} \ell(h(x_{si}), y_{si}) + \lambda D(P_s^X, Q_{X|Y \leq C}^t) \\
\text{empirical open set difference:} & \quad \frac{\alpha}{n_t} \sum_{i=1}^{n_t} \ell(h(x_{ti}), C + 1) - \frac{\gamma}{n_s} \sum_{i=1}^{n_s} \ell(h(x_{si}), C + 1),
\end{align*}
\]

where $D(P_s^X, Q_{X|Y \leq C}^t)$ is the distribution discrepancy for known classes.

**Structural Risk Minimization.** From a statistical machine learning perspective, we solve the UOSDA problem by the structural risk minimization (SRM) principle [18]. In SRM, the predicted function $h$ can be formulated as

\[
h^* = \arg \min_{h \in H} \hat{R}_s(h) + R(h),
\]

where $R(h)$ is the regularization term, and the hypothesis $H$ is defined as a subset of functional space

\[
H^{C+1} = \{h = [f_1, ..., f_{C+1}]^T : f_i \in \mathcal{H} \},
\]

here $\mathcal{H}$ is a reproducing kernel Hilbert space (RKHS) related to a kernel $K(\cdot, \cdot)$. Then, the classifier is

\[
f(x) = \arg \max_{s \in \{1, ..., C+1\}} h(x) = \arg \max_{s \in \{1, ..., C+1\}} \left[ f_1(x), ..., f_{C+1}(x) \right]^T,
\]

for any $x \in X$. Here the vector-value function $h$ is called the scoring function.

To effectively handle the different source domain and target domain for known samples, we can further divide the regularization term $R(h)$ as

\[
R(h) = \rho M(X_s, X_t) + \lambda D(P_s^X, Q_{X|Y \leq C}^t),
\]

where $M(X_s, X_t)$ is the manifold regularization [19], and the term $D(P_s^X, Q_{X|Y \leq C}^t)$ means the joint distribution alignment for known classes, defined as follows.

\[
D(P_s^X, Q_{X|Y \leq C}^t) = (1 - \mu) D_0(P_s^X, Q_{X|Y \leq C}^t) + \mu \sum_{i=1}^{C} D_i(P_s^X, Q_{X|Y \leq C}^t).
\]

Here $D_0(P_s^X, Q_{X|Y \leq C}^t)$ is the empirical marginal distribution alignment for known samples, $D_i(P_s^X, Q_{X|Y \leq C}^t)$ is the empirical conditional distribution alignment ($i = 1, ..., C$), and $\mu \in [0, 1]$ is the adaptive factor [35] to represent the importance between the empirical marginal distribution alignment and the empirical conditional distribution alignment.

As formula [7] shows, we also add the open set difference to learn the unknown samples. Lastly, we formula our optimization problem as follows.

\[
h^* = \arg \min_{h \in H} \frac{1}{n_s} \sum_{i=1}^{n_s} \ell(h(x_{si}), y_{si}) + \sigma \|h\|_K \\
+ \frac{\alpha}{n_t} \sum_{i=1}^{n_t} \ell(h(x_{ti}), C + 1) - \frac{\gamma}{n_s} \sum_{i=1}^{n_s} \ell(h(x_{si}), C + 1) + \rho M(X_s, X_t) + \lambda D(P_s^X, Q_{X|Y \leq C}^t),
\]

where $n = \max\{n_s, n_t\}$ and $\|h\|_K$ is the regulation term for avoiding over-fitting.

**Remark 1.** In this paper, we employ Maximum Mean Discrepancy (MMD) [22] to match distributions. However, this results in a gap with discrepancy distance $d^t_H$ which is used to measure the distribution difference in Theorem 1. Inspired by
Lemma 3 we also give a similar theoretical bound by using MMD distance. The details of the theoretical bound based on MMD are shown in Theorem 5. However, for proving Theorem 5 we also need an additional condition that the loss is squared loss $\ell(y, y') = \|y - y'\|_2^2$. Thus, we use the squared loss to design our method. In addition, we use scoring functions to represent classifiers, and one-hot vectors to represent labels. Related theoretical analysis about scoring functions can be found in Section IV.

Using the representer theorem, if the optimization problem has a minimizer $h^*$, then $h^*$ can be written as

$$ h^*(x) = \sum_{i=1}^{n_s + n_t} \beta_i K(x_i, x), \quad \forall x \in \mathcal{X}, $$

where $\beta_i \in \mathbb{R}^{(C+1) \times (n_s + n_t)}$ is the parameter and $x_i \in X_s \cup X_t$.

Distribution Alignment. We first introduce the definition of MMD distance and use MMD distance to match joint distributions $P_X$ and $Q_X$.

Given two distributions $P_X$ and $Q_X$, the MMD distance between $P_X$ and $Q_X$ is defined as:

$$ \text{MMD}_{H}(P_X, Q_X) = \left\| \int \phi dP_X - \int \phi dQ_X \right\|_{H}, $$

where $H$ is the reproducing kernel Hilbert space (RKHS) and $\phi$ is the kernel feature map.

Let $X'_t = \{x_{t1}, \ldots, x_{t(n_t)}\}$ be the target samples with label $t$, $X'_c = \{x_{c1}, \ldots, x_{c(n_c)}\}$ be the target samples with pseudo label $c$. DAOD minimizes the MMD distances between empirical marginal distributions $\hat{P}_{X'_t}$, $\hat{Q}_{X'_c}$, and conditional distributions $\hat{P}_{X'_t|Y \leq C}$, and $\hat{Q}_{X'_c|Y \leq C}$. To make MMD a proper regularization for the scoring function $h$, we adopt the projected MMD [23], [35], which is computed as

$$ D_0(\hat{P}_{X'_t}, \hat{Q}_{X'_c|Y \leq C}) = \left\| \frac{1}{n_t} \sum_{i=1}^{n_t} h(x_{t_i}) - \frac{1}{n_c} \sum_{i=1}^{n_c} h(x_{c_i}) \right\|_{H}, $$

where $n_t$ is the number of predicted known target samples. $x_{t_i}$ is the predicted known target sample.

Then using the representer theorem and kernel trick, we can write Eq. (10) as

$$ \text{tr}(\beta^T K M \beta), $$

where $\beta = [\beta_1, \ldots, \beta_{n_s+n_t}]^T$, $K$ is the $(n_s + n_t) \times (n_s + n_t)$ kernel matrix $[K(x_i, x_j)]$, and $M = \mu M_0 + (1 - \mu) \sum_{t=1}^{C} M_t$ is the MMD matrix:

$$ (M_t)_{ij} = \begin{cases} \frac{1}{n^2} & x_i, x_j \in X_s, \\ \frac{1}{n^2} & x_i, x_j \in X'_t, \\ -\frac{1}{n_s n_t} & \text{otherwise}; \end{cases} $$

Open Set Loss Function. Here we use a matrix to rewrite the loss function and open set difference. Let the label matrix be $Y = [y_1, \ldots, y_{n_s+n_t}] \in \mathbb{R}^{(C+1) \times (n_s+n_t)}$, where $y_i \in \mathbb{R}^{(C+1) \times 1}$ is a one-hot vector such that $y_i[l] = 1$ if the sample $x_i$ is from $X_s$ with label $l$, $y_i[C+1] = 1$ if the sample $x_i$ is from $X_t$ and $y_i[l] = 0$, otherwise. $\tilde{Y} = [\tilde{y}_1, \ldots, \tilde{y}_{n_s+n_t}] \in \mathbb{R}^{(C+1) \times (n_s+n_t)}$, where $\tilde{y}_i \in \mathbb{R}^{(C+1) \times 1}$ is a one-hot vector such that $\tilde{y}_i[C+1] = 1$ if the sample $x_i$ is from $X_s$ and $y_i[l] = 0$, otherwise.

Then

$$ \frac{n}{n_s} \sum_{i=1}^{n_s} \ell(h(x_i), y_i) + \alpha \|h\|_2^2 + \gamma \| \tilde{Y} - \beta^T K \|_2^2 + \sigma \text{tr}(\beta^T K \beta), $$

where $l = 1, \ldots, C$.
Proof.

D. Training

- Theorem 2. If the parameter $\gamma$ is smaller than 1 and the kernel function $K$ is universal, then Eq. (21) has a unique optimizer which can be written as: 

$$\beta = \left(\mathbf{A}^2 - \bar{\mathbf{A}}^2 + \lambda \mathbf{M} + \rho \mathbf{L} \mathbf{K} + \sigma \mathbf{I}\right)^{-1} \left(\mathbf{A}^2 \mathbf{Y}^T - \bar{\mathbf{A}}^2 \bar{\mathbf{Y}}^T\right).$$  

(22)

Proof. See Appendix A.

To compute a true value of Eq. (22), it was best for us to use the groundtruth labels of the target domain. However, the setting of our problem is unsupervised, which implies that it is impossible to obtain any true target labels. Inspired by methods JDA [5], ARTL [23] and MEDA [35], we use pseudo labels instead of the groundtruth labels. Pseudo labels are generated by applying an open set classifier $h$ trained on the source data to the target data.

In this paper, we use Open Set Nearest Neighbor for Class Verification-t (OSNNcv-t) [29] to help us learn pseudo labels. We select the two nearest neighbors $v, u$ from the test sample $s$. If both nearest neighbors have the same label $l$, $s$ is classified with the label $l$. Otherwise, we calculate the ratio

$$R = \frac{\|v - s\|_2}{\|u - s\|_2},$$

here we assume that $\|v - s\|_2 \leq \|u - s\|_2$. If $R$ is smaller than or equal to a pre-defined threshold $t$, $0 < t < 1$, $s$ is classified with the same label of $v$. Otherwise, $s$ is recognized as the unknown sample.

To make the pseudo labels more accurate, we use the iterative pseudo label refinement strategy, proposed by JDA [5]. The implementation details are demonstrated in Algorithm 1.

IV. GENERALIZATION BOUNDS FOR OPEN SET DOMAIN ADAPTATION

Since our method DAOD is based on MMD distance, but not the discrepancy distance used in Theorem 1, we also give a theoretical bound for OSDA that shows how MMD controls generalization performance in the case of the squared loss $\ell(y, y') = \|y - y'\|_2^2$.

We first prove Theorem 1 as follows.

Proof of Theorem 1. Given Eq. (1), we have

$$R_t(h) = \sum_{i=1}^{C} \pi_t^i R_{t,i}(h) + R_{t,C+1}^h(h) - (1 - \pi_{C+1}^h) E_{Q^t_{X|Y \leq C}} \ell(h, C+1).$$

(23)

Let $\Delta = R_{t,C+1}^h(h) - (1 - \pi_{C+1}^h) E_{Q^t_{X|Y \leq C}} \ell(h, C+1)$, and $h^* = \arg\min_{h \in H} R_s(h) + R_t^h(h)$, then

$$R_t(h) \leq \sum_{i=1}^{C} \pi_t^i R_{t,i}(h) + \Delta$$

$$\leq \sum_{i=1}^{C} \pi_t^i R_{t,i}(h^*) + (1 - \pi_{C+1}^h) E_{Q^t_{X|Y \leq C}} \ell(h, h^*) + \Delta$$

$$\leq \sum_{i=1}^{C} \pi_t^i R_{t,i}(h^*) + (1 - \pi_{C+1}^h) E_{p_X^t} \ell(h, h^*) + \Delta$$

$$+ \frac{1}{2} (1 - \pi_{C+1}^h) d_H^t(Q^t_{X|Y \leq C}, p_X^t)$$

$$\leq \sum_{i=1}^{C} \pi_t^i R_{t,i}(h^*) + (1 - \pi_{C+1}^h)(R_s(h^*) + R_s(h)) + \Delta$$

$$+ \frac{1}{2} (1 - \pi_{C+1}^h) d_H^t(Q^t_{X|Y \leq C}, p_X^t).$$

(24)

Consider $-(1 - \pi_{C+1}^h) E_{Q^t_{X|Y \leq C}} \ell(h, C+1)$,

$- (1 - \pi_{C+1}^h) E_{Q^t_{X|Y \leq C}} \ell(h, C+1)$

$$\leq (1 - \pi_{C+1}^h) \left(-E_{p_X^t} \ell(h, C+1) + \frac{1}{2} d_H^t(Q^t_{X|Y \leq C}, p_X^t)\right).$$

(25)

We obtain the result from inequalities (23), (24) and (25).
bound the discrepancy distance $d_H^*$, according to Lemma 3 which is given by Ghifary et al. [36].

**Lemma 3 (Domain Scatter Bounds Discrepancy [36]).** Let $\mathcal{H}$ be an RKHS with a universal kernel. Suppose that $\ell = \|y - y'\|_2^2$ is the squared loss, and consider the hypothesis set

$$H = \{f \in \mathcal{H}, \|f\|_H \leq R \text{ and } \|f\|_\infty \leq r\},$$

where $R, r > 0$ is a constant. Let $\mathbb{P}$ and $\mathbb{Q}$ be two distributions over $X$. Then the inequality holds:

$$d_H^*(\mathbb{P}, \mathbb{Q}) \leq 8R\text{MMD}_H(\mathbb{P}, \mathbb{Q}).$$

The proof of Lemma 3 can be found in Ghifary et al. [36].

However, in our method DAOD, the hypothesis set $H$ is a subset of

$$H^{C+1} = \{(f_1, ..., f_{C+1})^T : f_i \in \mathcal{H}\}.$$

Thus, we restate Lemma 3 with a slight modification.

**Lemma 4.** Let $\mathcal{H}$ be an RKHS with an universal kernel. Suppose that $\ell(y, y') = \|y - y'\|_2^2$ is the squared loss, and consider the hypothesis set $H^* = H^{C+1} \cup F$, where

$$H^{C+1} = \{(f_1, ..., f_{C+1})^T \in \mathcal{H} : \|f_i\|_H \leq R, \|f_i\|_\infty \leq r\},$$

and $F$ is a constant vector-value function $[0, ..., 0, ..., 1]^T \in \mathbb{R}^{(C+1)}$, here $R, r > 0$ is a constant. Let $\mathbb{P}$ and $\mathbb{Q}$ be two distributions over $X$. Then the inequality holds:

$$d_H^*(\mathbb{P}, \mathbb{Q}) \leq \text{LMMMD}_H(\mathbb{P}, \mathbb{Q}),$$

where $L = \max(2(C + 1)Rr + 4r, 8(C + 1)Rr)$.

Using the triangle inequality of $L^2$ norm and Lemma 4, we give a theoretical bound based on MMD as follows.

**Theorem 5.** Given a hypothesis $H^*$ defined in Lemma 4 and the loss function $\ell(y, y') = \|y - y'\|_2^2$, then for any $h \in H^*$, we have

$$\sqrt{\frac{R_t(h)}{1 - \pi^C_{C+1}}} \leq \sqrt{R_s(h)} + \sqrt{\frac{L}{2}\text{MMD}_H(Q^t_{X|Y \leq C}, P^t_X)} + \sqrt{\frac{\Delta_o + \frac{L}{2}\text{MMD}_H(Q^t_{X|Y \leq C}, P^t_X)}{1 - \pi^C_{C+1}}} + \lambda,$$

where $\lambda = \min_{h \in H^*} \sqrt{R_s(h)} + \sqrt{R_t(h)}$ and $L$ is a constant defined in Lemma 4.

**Proof.** See Appendix A.

Before stating the generalization bound similar to the bound in Corollary 1.1, we introduce Rademacher complexity, which measures the richness of a class of real-valued functions with respect to a distribution.

**Definition 4 (Rademacher Complexity).** Let $\mathcal{F}$ be a class of real-valued functions defined in a space $Z$. Given sample $S = \{z_1, ..., z_n\} \in Z$, then the Empirical Rademacher Complexity of $\mathcal{F}$ with respect to the sample $S$ is

$$\mathcal{R}_S(\mathcal{F}) = \mathbb{E}_\sigma \left[ \sup_{f \in \mathcal{F}} \frac{1}{n} \sum_{i=1}^{n} \sigma_i f(z_i) \right],$$

where $\sigma = (\sigma_1, ..., \sigma_n)$ are Rademacher variables, with $\sigma_i$'s independent uniform random variables taking values in $-1, +1$.

Using Rademacher Complexity, we obtain the following result.

**Theorem 6.** Given the hypothesis $H^*$ defined in Theorem 5 and the loss function $\ell(y, y') = \|y - y'\|_2^2$, if a random labeled sample $S$ of size $n_s$ is generated by $\mathbb{P}^s$-i.i.d and a random unlabeled sample $T$ of size $n_t$ is generated by $\mathbb{Q}^t_X$-i.i.d, then for any $h \in H^*$, $\delta \in (0, 1)$ with probability at least $1 - 3\delta$, we have

$$\sqrt{\frac{R_t(h)}{1 - \pi^C_{C+1}}} \leq \sqrt{R_s(h)} + \sqrt{\frac{L}{2}\text{MMD}_H(Q^t_{X|Y \leq C}, P^t_X)} + \sqrt{\frac{\Delta_o + \frac{L}{2}\text{MMD}_H(Q^t_{X|Y \leq C}, P^t_X)}{1 - \pi^C_{C+1}}} + \lambda,$$

where $\lambda = \min_{h \in H^*} \sqrt{R_s(h)} + \sqrt{R_t(h)}$, empirical open set difference $\Delta_o = \hat{R}_o^{u,C+1} - \hat{R}_o^u$, $H$ is the hypothesis set defined in Lemma 3 and $L$ is a constant defined in Lemma 4.

**Proof.** See Appendix A.

Theorem 5 and Theorem 6 highlight that the risk of the source domain, open set difference and MMD distance, $\text{MMD}_H(Q^t_{X|Y \leq C}, P^t_X)$, control the generalization performance in open set domain adaptation.

In summary, our theory is a bold attempt at filling the blank in open set domain adaptation theory. First, we provide an analysis of the theoretical bound for OSDA based on discrepancy distance, and discover that open set difference is the crucial element for assisting the classification of unknown target samples. Second, we develop a bound based on scoring functions, MMD distance and $L^2$ loss to reduce the gap between our method DAOD and our theoretical work.

**V. EXPERIMENTS AND EVALUATIONS**

In this section, we first utilize real world datasets to verify the performance of DAOD. Experiments are then designed to understand the behavior of the parameters.

**A. Real World Datasets**

We evaluated our method on three cross-domain recognition tasks: object recognition (Office-31, Office-Home), and face recognition (PIE). Table II lists the statistics of these datasets.
Office-31 [37] consists of three real-world object domains: AMAZON (A), DSLR (D) and WEBCAM (W). It has 4,652 images with 31 common categories. This means that there are 6 domain adaptation tasks: A → D, A → W, D → A, W → A, D → W, W → D. Following the standard protocol and for a fair comparison with the other methods, we extracted feature vectors from the fully connected layer-7 (fc7) of the AlexNet [38]. We introduced an open set protocol for this dataset by taking classes 1-10 as shared classes in alphabetical order. The classes 21-31 were used as the unknown classes in the target domain.

Office-Home [39] consists of 4 different domains: Artistic (Ar), Clipart (Cl), Product (Pr) and Real-World (Rw). Each domain contains images from 65 object classes. We constructed 12 OSDA tasks: Ar → Cl, Ar → Pr, ..., Rw → Ar. In alphabetical order, we used the first 25 classes as known classes and classes 26-65 as the unknown classes. Following the standard protocol and for fair comparison with the other methods, we extracted feature vectors from ResNet-50.

PIE [40] includes 41,368 facial images of 68 people with various pose, illumination, and expression changes. The face images are captured by 13 synchronized cameras (different poses) and 21 flashes (different illuminations and/or expressions). We focused on 5 out of 13 poses, i.e., PIE1 (C05, left pose), PIE2 (C07, upward pose), PIE3 (C09, downward pose), PIE4 (C27, frontal pose) and PIE5 (C29, right pose). These facial images were cropped to a size of 32 × 32. We took classes 1-20 as shared classes and classes 21-68 as unknown classes in the target domain. We constructed 20 tasks: PIE1→PIE2, PIE1→PIE3,..., PIE5→PIE4.

B. Baseline Methods

DAOD was verified and compared with several baseline methods as follows.

1) No Transfer:
   - OSNN [29]: OSNN recognizes an sample as unknown by computing the ratio of similarity scores to the two most similar classes of the sample and comparing the ratio with a pre-defined threshold.

2) Closed Set:
   - TCA [4] + OSNN. The aim in implementing TCA is to show that if the UCSDA method is used to solve the UOSDA problem, negative transfer will occur, leading to poor performance.

3) Open Set:
   - JDA [5] + OSNN. We extended JDA into the open set setting. Joint distribution matching is the main step for JDA, thus we simply matched the known samples predicted by OSNN when the JDA method was implemented.
   - JGSA [24] + OSNN. We extended JGSA into the open set setting. First, for learning new features, we implemented JGSA by using the source samples and known target samples predicted by OSNN. Then, we used OSNN to predict the pseudo labels. We repeated the process until convergence.
     - ATI-λ [12] + OSNN. ATI-λ is the first UOSDA method, but we also need the unknown source samples for implementation. To implement ATI-λ under our setting, we used ATI-λ to select the outliers, then learned the new features for matching the source domain and target domain excluding selected outliers. Lastly, OSNN was used to predict the labels.
     - OSBP [13]. OSBP utilizes adversarial neural network and a binary cross entropy loss to learn the probability for target samples, then uses the estimated probability to recognize unknown samples.

Two new UOSDA methods, Factorized Representations for Open Set Domain Adaptation (FRODA) [12] and Separate to Adapt (STA) [13], have recently been proposed. We compare the performance of DAOD with that of both methods reported in their papers (see Appendix B).

C. Hyper-parameter Settings

Before reporting the detailed evaluation results, it is important to explain how DAOD hyper-parameters are tuned. There are several hyper-parameters for DAOD: 1) the choice of the kernel K; 2) adaptation parameters λ, σ, ρ, p, μ; 3) open set parameters α, γ; and 4) #iterations T and threshold t ∈ (0, 1). Generally, it is impossible to tune the optimal parameters using cross validation because the labeled and unlabeled samples are from different distributions. Therefore, most domain adaption methods [5], [24] use a standard strategy grid-search that tunes all parameters in the parameter space and reports the best results. However, tuning all the hyper-parameters using the grid-search strategy might be impractical for two reasons [36]. The first is of the computational complexity. The second is that cross-validating a large number of hyper-parameters may worsen the generalization on the target domain. Our strategy for dealing with the issue is to reduce the number of tunable hyper-parameters.

For the kernel function, we choose the Gaussian kernel:

$$K_C(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right),$$

where the kernel bandwidth r is median(∥a − b∥), ∀a ∈ Xs, b ∈ Xt. The adaptive factor μ presents the relative importance of MMD distance for marginal distributions and MMD distance for conditional distributions. Wang et al. [35] made the first attempt to compute μ by employing A-distance [20], which is the special case d_{H,1} of discrepancy distance d_{H,1}. According to paper [20], the A-distance can also be defined as the error of building a binary classifier from hypothesis set H to discriminate between two domains. Wang et al. [35] used the linear hypothesis set to estimate A-distance. Let ε(h) be the error of a linear classifier h discriminating source samples Xs and target samples Xt. Then

$$d_A(X_s, X_t) = 2(1 - \epsilon(h)).$$

We adopt the same method as MEDA [35] to estimate μ:

$$\mu = 1 - \frac{d_M}{d_M + \sum_{l=1}^{C} d_l},$$

where
where $d_M = d_A(X_s, X'_t)$, $d_l = d_A(X'_s, X'_t)$ ($l = 1, ..., C$). Here $X'_t$ is the target samples predicted as known samples. This estimation has to be computed at every iteration of DAOD, since the predicted conditional distributions for the target may vary each time. In this paper, we fix $\rho = 10$, $0 \leq \rho < 1$ and $\sigma = 1$. We also set $T = 20$ and $t = 0.5$. Distribution alignment is inevitable, so we choose a slightly larger $\lambda = 50$.

Thus, only two hyper-parameters remain tuneable: $\alpha$ and $\gamma$. We interpret how to further tune those parameters. We choose parameters according to the following rules: 1) the positive term $R_{u,C}^{t+1}$ and negative term $R_{s,C}^{t+1}$ in open set difference inference with each other. A larger positive term means that more samples are recognized as belonging to the unknown classes. A larger negative term implies that more samples are classified as unknown samples. To ensure that the positive term and negative term balance, the difference $|\alpha - \gamma|$ should not be too large. Furthermore, the parameter $\alpha$ should be larger than $\gamma$, since the positive term’s coefficient $1/(1 - \pi_{C+1})$ is larger than 1. In this paper, we set the difference of the open set parameters as $\delta = \alpha - \gamma \in \{0.05, 0.1, 0.2\}$. 2) If $\alpha \rightarrow +\infty$, all target samples could be recognized as unknown samples, thus, we choose a slightly smaller $\alpha \in \{0.3, 0.4\}$. Lastly, we evaluate DAOD by empirically searching the parameter spaces $\{\alpha, \alpha - \gamma\}$, and report the best results. Additionally,
Fig. 4: The horizontal axis is the difference in the open set parameters $\delta = \alpha - \gamma$. In the figures, the difference $\delta$ is not larger than $\alpha$, since the parameter $\gamma$ is required to be larger than or equal to 0. If $\delta > 0$, $\alpha$ is larger than $\gamma$. If $\delta < 0$, $\gamma$ is larger.

we provide parameter sensitivity analysis and convergence for DAOD, which will verify that DAOD can achieve stable performance for a wide range of hyper-parameter settings. We use two types of accuracy [12], [13] to evaluate DAOD:

$$\text{Acc}(OS) = \frac{1}{C+1} \sum_{l=1}^{C+1} \frac{|x : x \in D^l_t \land h(x) = l|}{|x : x \in D^l_t|},$$

(31)

and

$$\text{Acc}(OS^*) = \frac{1}{C} \sum_{l=1}^{C} \frac{|x : x \in D^l_t \land h(x) = l|}{|x : x \in D^l_t|},$$

(32)

where $D^l_t$ ($l = 1, ..., C+1$) is the set of the target samples from class $l$ and $h$ is the predicted classifier. Acc(OS) is the main index for estimating the performance of the UOSDA methods [12].

D. Experimental Results

The classification accuracy of the UOSDA tasks is shown in Table III. The following facts can be observed from this table. 1) The closed set method TCA performs poorly on most tasks, even worse than the standard OSNN method, indicating that negative transfer occurs. 2) We observe that all open set methods achieve better classification accuracy than OSNN on most tasks. This is because the source samples and known target samples are from different distributions. 3) DAOD achieves much better performance Acc(OS) than the six baseline methods on most (26 out of 38) tasks. The average classification accuracy (Acc(OS), Acc(OS*)) of DAOD on the 38 tasks is 68.5%, 69.1% respectively, gaining a performance improvement of 2.6%, 2.3% compared to the best baseline OSBP. 4) The performance of the JDA+OSNN, JGSA+OSNN and ATI-\(\lambda\)+OSNN methods are generally worse than that of DAOD. A major limitation of these methods may be that they omit the selected unknown target samples when they construct a latent space to match the distributions for known classes. This may result in unknown samples being mixed with the known samples in the latent space. In DAOD, the negative term $R_{u,s, C+1}$ helps DAOD to avoid the problem suffered by JDA, JGSA and ATI-\(\lambda\). 5) Performance of the OSBP method is generally worse than that of DAOD. The main reasons may be that 1) OSBP only matches marginal distributions but not joint distributions; 2) OSBP does not keep the unknown target samples away from known source samples, with the result that many unknown target samples are recognized as known samples. However, DAOD uses the negative term $R_{u,s, C+1}$ to separate the source samples and unknown target samples.

E. Openness

Similar to open set recognition [41] and separate to adapt [17], we define openness as

$$\varnothing = 1 - \frac{|Y_s^u|}{|Y_t^u|}.$$  

(33)

The above Eq. (33) estimates the level of openness. $\varnothing = 0$ represents a completely closed problem and larger values denote more open problems.

In our experiments, we only tested special cases $\varnothing \approx 0.5, 0.6$ and 0.7. To verify that DAOD is robust to different levels of openness, we conducted experiments on the Office-Home dataset with openness ranging from 0.10 to nearly 0.85. We took classes from 1 to 10 as known classes and classes
from 11 to 11 + i (i = 0, 1, 2, 3, 4) as unknown classes. The openness \( \bigcirc \) therefore ranges from 0.10 to 0.30. We also took classes from 11 to 65 − 10 * i or 20 (i = 0, 1, 2, 3, 4) as unknown classes. In this setting, the openness \( \bigcirc \) changes from 0.50 to 0.85.

To show that DAOD is robust to openness change, we used the same parameters for all openness values. Due to space limitation, we report the average results and plot classification accuracy in Fig. 3. Compared with the best baseline method OSBP, DAOD performs steadily and achieves the best performance for almost all values of openness.

### F. Analysis for Open Set Parameters

In this section, we analyze the open set parameters \( \alpha \) and \( \gamma \). We find the relationship between \( \alpha \) and \( \gamma \) is related to another parameter, the difference \( \delta = \alpha - \gamma \). We conducted experiments on the Office-31 dataset with \( \alpha \) ranging from 0.2 to 1.2 and \( \delta \) ranging from −0.2 to \( \alpha \). Due to space limitation, we report the average results on Office-31 in Fig. 4. According to Fig. 4, we obtain the following results.

1) When \( \delta \) increases, the accuracy of unknown classes will also increase, since the larger positive term \( R_{t,C+1}^\delta(h) \) means that more samples are recognized as unknown.

2) When \( \delta < 0 \) (\( \alpha < \beta \)), for almost all \( \alpha \in [0.2, 1.2] \), the performance Acc(OS) is poorer than the best baseline method (dashed line). This is because when \( \delta < 0 \), more samples are recognized as known classes. This observation is the same as our theoretical results (Theorem 1), since in open set difference \( \Delta_o \), the positive term’s coefficient \( 1/(1 - p_{t,C+1}^\delta) \) is larger than the negative term’s coefficient 1. Thus, \( \delta \) should be larger than 0 (\( \alpha > \gamma \)).

3) All figures in Fig. 4 are similar for almost all \( \alpha \) from 0.4 to 1.2, which implies that \( \alpha \) may be not the most important factor influencing the performance of DAOD. Thus, the difference \( \delta \) is a more important factor for DAOD.

4) When \( \delta \) is larger than 0.2, the performance Acc(OS) begins to decrease. This is because for larger \( \delta \), more known samples are classified as unknown.

5) For all \( \alpha \) from 0.2 to 1.2, if \( \delta \) are chosen from \([0.05, 0.2]\), the performance Acc(OS) of DAOD achieves better performance than the best baseline method.

6) Though \( \alpha \) is not the main factor influencing the performance of DAOD, we compare figures (\( \alpha < 1.0 \)) with figures \((\alpha \geq 1.0)\) and find that a smaller \( \alpha \) achieves slightly better performance than a larger \( \alpha \). Thus, \( \alpha \in [0.2, 0.8] \) and \( \delta \in [0.05, 0.2] \) are good choices for DAOD.

### G. Parameter Sensitivity and Convergence Analysis

We analyze the parameter sensitivity of DAOD on different types of datasets to demonstrate that a wide range of parameter values can be chosen to obtain satisfactory performance. We evaluate important parameters \( \lambda, \sigma, \rho, p, t \) and \( T \). We report the average results for datasets Office-31, Office-Home and PIE respectively, and discuss the results. The dashed line denotes the results of the best baseline method on each dataset.

**Distribution Alignment \( \lambda \)** We run DAOD with varying values of \( \lambda \). We plot classification accuracy w.r.t. different values of \( \lambda \) in Fig. 5(a). We find larger values of \( \lambda \) make distribution alignment more effective. If we choose \( \lambda \) from \([40,300]\), we obtain better results than the best baseline method.

**Regularization \( \sigma \)** We run DAOD with varying values of \( \sigma \). We plot classification accuracy w.r.t. different values of \( \sigma \) in
In the future, we will mainly focus on universal domain adaptation [22], which is a unified domain adaptation framework that includes closed set domain adaptation, open set domain adaptation and partial domain adaptation [43].

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