Beam profile sensitivity of the WMAP CMB power spectrum

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ABSTRACT

Using the published WMAP 5-year data, we first show how sensitive the WMAP power spectra are to the form of the WMAP beam. It is well known that the beam profile derived from observations of Jupiter is non-Gaussian and indeed extends, in the W band for example, well beyond its 12.1 FWHM core out to more than 1 degree in radius. This means that even though the core width corresponds to wavenumber \( l \approx 1500 \), the form of the beam still significantly affects the WMAP results even at \( l \approx 200 \) which is the scale of the first acoustic peak. The difference between the beam convolved \( C_l \) and the final \( C_l \) is \( \approx 70\% \) at the scale of the first peak, rising to \( \approx 400\% \) at the scale of the second.

New estimates of the Q, V and W-band beam profiles are then presented, based on a stacking analysis of the WMAP radio source catalogue and temperature maps. The radio sources show a significantly\( (3 - 4\sigma) \) broader beam profile on scales of \( 10' - 30' \) than that found by the WMAP team whose beam analysis is based on measurements of Jupiter. Beyond these scales the beam profiles from the radio sources are too noisy to give useful information. Furthermore, we find tentative evidence for a non-linear relation between WMAP and ATCA/IRAM 95 GHz source fluxes. We discuss whether the wide beam profiles could be caused either by radio source extension or clustering and find that neither explanation is likely. We also argue against the possibility that Eddington bias is affecting our results. The reasons for the difference between the radio source and the Jupiter beam profiles are therefore still unclear. If the radio source profiles were then used to define the WMAP beam, there could be a significant change in the amplitude and position of even the first acoustic peak. It is therefore important to identify the reasons for the differences between these two beam profile estimates.

Key words: cosmic microwave background – cosmology: observations-early universe–space vehicles: instruments

1 INTRODUCTION

The WMAP satellite has produced some of the best support for the standard \( \Lambda \)CDM cosmological model. By measuring the first two acoustic peaks it has shown that the Universe is spatially flat with \( \Omega_\Lambda = 0.74 \) and \( H_0 = 72 \text{ km s}^{-1} \text{Mpc}^{-1} \) (Hinshaw et al. 2009).

Of course, statistically precise measurements can also contain systematic errors which have to be guarded against. Such systematics include Galactic foregrounds which at the least cause mode coupling due to the incomplete sky (e.g. Hinshaw et al. 2003; Chon et al. 2004). There are also potentially more subtle systematics that arise from cosmological foregrounds. For example, Myers et al. (2004) and Bielby & Shanks (2007) have detected the SZ effect in the WMAP data by cross-correlating the CMB with rich cluster positions. Shanks (2007) has also discussed the effect of foreground lensing, prompted by QSO lensing results (Myers et al. 2003; 2005; Mountrichas & Shanks 2007). But SZ is unlikely to make a strong contribution to the first acoustic peak (Huffenberger et al. 2004). Also lensing requires a high anti-bias between galaxies and the mass distribution to have a significant effect at the first peak which needs to be reconciled with measures of bias from galaxy clustering dynamics (e.g. Ratcliffe et al. 1998; Hawkins et al. 2003).

However, there are also many potential systematics involved with the WMAP instrument, although the WMAP team have taken care that the effects of such systematics are minimised. One major potential systematic concerns the question of the WMAP radio telescope beam profile. We shall see that even at the wavenumber \( l \approx 220 \) of the first acoustic peak, the CMB power spectrum has significant dependence on the beam profile even in the highest resolution W band. Here the W-band resolution quoted by the WMAP team is 12.1 FWHM which is roughly equivalent to \( l \approx 1800 \). It is also noted that the beam is not Gaussian. Now the WMAP team have extensive papers devoted to the important question of measuring the beam (Page et al. 2003; Jarosik et al. 2007; Hill et al. 2009). The standard method is to use their observations of bright sources such as the planet Jupiter to measure the beam profile.

Here, after describing the WMAP5 data in §3, we re-derive in §4 the raw CMB power spectrum from the WMAP maps to show directly the effect of the beam. Then in §4 we use radio sources to make new estimates of the WMAP beam and discuss other tests of the beam profile. In §5 we then make fits to the radio source beam profiles and use these to de-beam the WMAP5 data and explore the

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range of power spectra that results. Possible reasons for the beam profile discrepancy and our conclusions are then presented in Section 4.

2 DATA

2.1 WMAP5 maps and point source catalogue

Here we use the five-year WMAP datasets which are available from the LAMBDA CMB website. The maps from the individual detectors in 5 frequency bands, K, Ka, Q, V and W are supplied. The FWHM of the 94 GHz W beam is 12.6′ compared to 19.8′ at V (61 GHz), 29.4′ at Q (41GHz), 37.2′ at Ka (33GHz) and 49.2′ at K (23GHz). There are 10 differencing assemblies (DAs), namely K1, K2, Q1, Q2, V1, V2, W1, W2, W3 and W4. The different DA maps can be cross-correlated to obtain power spectra free of uncorrelated detector noise bias (Hinshaw et al. 2003). The Jupiter beam profiles for each DA and the corresponding transfer functions are also given. The maps are in HEALPix (Górski et al. 2005) format with $N_{\text{side}} = 512$ and $N_{\text{side}} = 1024$. These give equal area pixels of dimension $\approx 7^\prime$ and $\approx 3^\prime$, respectively.

We use the radio sources drawn from the WMAP5 point source catalogue (Wright et al. 2009). These sources have to be detected to $>5\sigma$ in at least one WMAP band and their flux density is reported if they are detected at $>2\sigma$ in any of the other four WMAP bands. This gives a list of 390 sources to a limit of $\approx 0.5$ Jy in each band. The source positions are accurate to $\sim 4^\prime$ (Wright et al. 2008). 365 out of 390 sources are pre-detected at 4.85 GHz in the Greenbank (GB6) northern sky survey (Gregory et al. 1996) and the Parkes-MIT-NRAO (PMN) surveys (Griffith & Wright 1993). Here, we only use WMAP5 sources with $\sigma > 5$ and Jupiter counterparts and exclude sources (12 out of 365) that were found to be resolved at $\approx 4.7$ GHz resolution of GB6 and PMN. Table 1 shows the number of these sources in each band, also split into those brighter or fainter than 1.1 Jy.

From the optical identifications of Trushkin (2003) of the 208 WMAP 1st year sources the survey contains 77 percent QSOs or BL Lac with the remainder being radio galaxies/AGN. This is as expected given the dominance of flat-spectrum compact sources at the high WMAP frequencies.

Table 1. Summary of the WMAP sources listed as point sources in the Greenbank and PMN 5GHz catalogues.

| Band | $\geq 1.1$Jy | $< 1.1$Jy | Total ($>2\sigma$) |
|------|-------------|-----------|-------------------|
| Q    | 182         | 165       | 347               |
| V    | 164         | 153       | 317               |
| W    | 97          | 84        | 181               |

2.2 Ground-based 90-95GHz Radio Sources

We shall compare WMAP W-band fluxes with ground-based radio source fluxes from ATCA (Sadler et al. 2008) and IRAM (Steppe et al. 1988). The ATCA survey was made at 95 GHz and the IRAM survey at 90 GHz. The ATCA survey was based on sources selected at 20GHz. Of the 130 sources observed, 17 were detected at more than $2\sigma$ by WMAP5. The IRAM survey observed 294 sources at 90 and 230 GHz, targetting sources which are brighter than 1 Jy at 5 GHz. Here 66 sources were detected at more than $2\sigma$ by WMAP5. At these high frequencies the sources are mainly QSOs, BL Lacs or blazars. Many of the sources in the ATCA and IRAM surveys are variable and so where this is an issue we shall use the average source fluxes in our comparison with the WMAP fluxes.

3 DERIVING THE BEAM CONVOLVED $C_l$

We now analyse the WMAP W-band data to make an initial estimate of the power spectrum from the W band. To reduce the effect of correlated detector noise which would result from an autocorrelation of an individual CMB map, we make a cross-correlation of the maps from independent detectors. We derive the result by using the PolSpice code (Szapudi et al. 2001) to cross-correlate the W1 and W2 maps. The default WMAP5 temperature power spectrum mask (KQ85, Nolta et al. 2009) is used and the cut-sky corrected angular power spectrum, $C_l$, is obtained directly from the PolSpice code (see Chon et al. 2004). Hereafter, we shall call an angular power spectrum after a cut-sky and pixel transfer function (see Eq. 2) a ‘beam convolved $C_l$’. In Fig. 1 we immediately see that the beam convolved $C_l$ (green line) is not only drastically smoothed at the position of the second and third peaks but there is also a significant effect at the position of the first peak at $l \approx 220$ in that the amplitude of the standard CDM result (blue line) is $\approx 70$% higher. The reason for this is seen in Fig. 2 where the beam profile from the Jupiter observations using the W1 detector are shown. It can be seen that the beam is not Gaussian and has a $\theta^{-3}$ power-law tail out to $>1^\circ$.

Page et al. (2003) give the relation between the beam transfer function, $b_l$, and the radial beam profile, $b^S(\theta)$, as,

$$b_l = 2\pi \int b^S(\theta) P_l(\cos \theta) d\cos \theta / \Omega_B$$

(1)

The de-beamed cross-power spectrum measured from DA $i$ and $j$ is then given from the measured $C_l$ by

$$C_l = C'_l / b_i b_j p_i^2,$$

(2)

where $p_i$ is the pixel transfer function supplied with the HEALPix package. For $N_{\text{side}} = 512$, the pixelisation lowers the measured $C_l$ by $\approx 1$ and 10 per cent at $l \approx 200$ and 500, respectively.
If we use Eq. [2] with the Jupiter beam transfer function from the WMAP team, we find that we get back to the usual ΛCDM model (green and orange lines in Fig. 4). The black line shows the $C_l$ measured from a full-sky CMB simulation (WMAP5 best-fit ΛCDM model) after smoothing by a Gaussian beam, using synfast code (Górski et al. 2005). The red line shows the effect of the similar simulation now smoothed with W1W2 Jupiter beam profiles. The latter shows excellent agreement with our beam convolved $C_l$ measured from the W1 and W2 maps. The W1W2 $C_l$ is noisier than the simulation due to radiometer noise in the data. The effect of the Jupiter beam compared to the Gaussian beam is thus very significant in decreasing the height of the first peak. We also see that when the Jupiter beam is used, the ΛCDM model does give an accurate fit to the beam-convoluted $C_l$. Thus when we use the same parameters as the WMAP team, we reproduce the WMAP result.

4 TESTING THE WMAP BEAM PROFILE

4.1 Beam profiles via point sources

We then estimated a beam from the radio sources by making a stacking analysis of WMAP5 temperature maps around radio source positions. The extended foreground emission regions are excluded from the temperature maps using the ‘point source catalog’ mask (Wright et al. 2005). We calculated the average $\Delta T$ (per 49 arcmin$^2$ pixel) in annuli as a function of angular distance, $\theta$, between radio source position and the pixel centre. In the first instance we show the raw cross-correlation function for the Q, V and W bands in Fig. 2 split into bright ($> 1.1$ Jy) and faint ($< 1.1$ Jy) WMAP5 source sub-samples. The errors on the radio source profiles are jackknife errors. These are estimated using six equal-area sub-fields, given by $\sigma^2_{jk}(\theta) = (N_j - 1)(\langle \Delta T(\theta) - \bar{\Delta T}(\theta) \rangle^2)$, where $N_j = 6$ and $\Delta T$ is the stacked temperature measured from five out of the six sub-fields.

We see that the fainter source profiles appear to agree with the brighter source profiles at scales of $\theta \approx 30'$ but have significantly lower peak values. This is most clearly shown in the un-renormalised profiles shown for the bright and faint Q, V, W band sources in Fig. 2k, b and c. Although noise may be an issue for the faintest sources, this suggests that there may possibly be some form of non-linearity in the WMAP beam. We also note that the profiles from both bright and faint sources show a positive offset at the 0.01-0.02 mK level. The offset shows an increasing trend from $1^5 - 5^5$. The main uncertainty in estimating WMAP beam profiles from these data is in subtracting this offset at scales $> 1^5$.

Since WMAP has significant sidelobes stretching to $\approx 90^5$ (Barnes et al. 2003), there was a possibility that the offsets are also part of the beam. However, when we distributed points at random in the masked region and used these as our centres for our stacking analysis we also found a similar offset (dotted lines in Fig. 2h, b and c). This makes it look like the offset is not associated with the existence of sources and hence not associated with the WMAP beam. Our Monte Carlo simulations (see below) shows that these offsets are caused by the CMB fluctuations. We therefore employed a ‘photometry’ approach for the stacking analysis where we have subtracted the WMAP flux in an annulus at $1^5 < \theta < 2^5$ for the W band and proportionately bigger annuli in the V and Q bands. Using sky annuli close to the sources will clearly improve background subtraction in the presence of local background fluctuations.

The resulting WMAP radio source beam profiles for Q, V and W bands are shown in Fig. 2l, e and f. The profiles have been renormalised ($\approx 10\%$ statistical uncertainties in the normalising factors) to fit the peak in the WMAP Jupiter beam profile at $\theta < 4'$ and this profile is also shown. For each band we also compare the
profiles to a Gaussian beam with the FWHM as indicated in the plot. We see that on average the profiles from the radio sources are broader than the Jupiter profile in the W, V and Q bands. In the lower frequency, lower resolution K and Ka bands the radio source profiles fit the Jupiter beam better, indeed almost perfectly the lower frequency, lower resolution K and Ka bands the radio profile at $\theta > 30'$. Fig. 2 again shows the WMAP radio sources divided into faint and bright sources, split at 1.1 Jy. In the W and V bands particularly we again note that the fainter sources appear to be wider than the brighter sources. We also find similar results for W2, W3, W4, V2 and Q2 but choose not to include them here for clarity. These deviations from the WMAP Jupiter beam are puzzling and we now check to see if they could be caused by systematics.

At the referee’s request, we made 100 Monte Carlo simulations following Wright et al. (2009) to check our results. In summary, simulated maps are constructed to include point sources sampled from a power law $N(> S)$ distribution with spectral characteristics as seen in the data. The temperature map for each band is then smoothed with the WMAP Jupiter beam profile before being added to a simulated CMB map including radiometer noise. We then applied the five-band detection method following procedures described by Wright et al. (2009). Applying our stacking analysis described above, we found that even profiles as narrow as the W-band Jupiter profile can be accurately retrieved. The flux dependence of measured profiles were small with only a hint of possible Eddington bias in the faintest bin. The pixelisation effect is also too small to explain the wider profiles seen here. Furthermore, the estimated uncertainties using these simulations are consistent with the Jackknife error estimates. Further details are given by Sawangwit (2010), who, further finds similarly wide profiles for NVSS radio sources flux limited at 1.4 GHz, where any Eddington bias would be negligible.

We have checked the likely contribution of radio source clustering to the beam profiles, using the clustering analysis of the NVSS radio survey by Overzier et al. (2003). At $S \geq 200$ mJy where the sky density of NVSS sources is $n \approx 0.6$ deg$^{-2}$, $w(\theta) = 3 \times 10^{-5} \theta^{-3.4} + 6.6 \times 10^{-3} \theta^{-0.8}$. (3)

This is a 2-power-law form which changes slope at $\theta \approx 0.1'$. At smaller scales, doublelobed radio sources split into two components dominate while at larger scales source-source clustering dominates. We first calculate the excess number of sources in an annulus of area $\Delta A$ at radius $\theta$ from an average source, $N_{ex}(\theta) = w(\theta) n \Delta A$. The excess flux/temperature per unit area in the profile in the annulus is then given by $\Delta T_{ex} = N_{ex} \times f/\Delta A$ where $f$ is the average source flux. For a Gaussian point source of central intensity/temperature per unit area, $T_0$, and width, $\sigma$, the flux is $2\sigma^2 T_0$. Therefore in this case, $\Delta T_{ex}(\theta) = w(\theta) 2\pi \sigma^2 T_0$. We find $\Delta T_{ex} \approx 3 \times 10^{-4} T_0$ which is a negligible contribution in explaining the excess in Fig. 2 at this scale, if $T_0$ is taken to be the central profile value. Taking the parameters for 100 mJy from Overzier et al. (2003) makes the effect even smaller. These results are also likely to be upper limits for the WMAP sources which only have a density of $n \approx 0.01$ deg$^{-2}$ and an average 95 GHz flux of 500 mJy. We conclude that radio source clustering is not likely to be an issue for our radio source beam profiles.

We conclude that in the W and V bands and probably the Q band, the average radio source profile is wider than the Jupiter beam and the fainter sources may show a wider profile than the brighter sources. For W1 and $S > 1.1$ Jy sources, the beam profile measured here rejects that of Jupiter with 4.0, 3.0 and 3.5$\sigma$ significance for $\theta = 12.6', 20'$ and 31.6. These become 4.4, 3.2 and 2.8$\sigma$ when Monte Carlo errors are used instead. Note that the pixelisation has been taken into account when estimating these significances.

### 4.2 Comparison with ground-based fluxes

We now make a check of the WMAP5 W band fluxes as presented by Wright et al. (2009) in Table 1. We checked these against the ATCA and IRAM source flux densities. The comparison in Fig. 3 shows that for both surveys, the brighter sources with fluxes $> 3$ Jy are about a factor of 1.5 fainter in the WMAP source list than in the ATCA or IRAM lists. The agreement between the ATCA and IRAM fluxes appears better than for WMAP, if we use WMAP as an intermediary between these two surveys. If the scale error is due to WMAP, then this might suggest that there is a non-linearity in the WMAP flux scale. It could mean that a narrower WMAP beam at brighter fluxes is missing a significant amount of flux in the tail of the beam profile.

### 5 Impact on the De-beamed $C_l$

Finally, we use the information from our radio source beam profiles to judge what the effect might be on the de-beamed WMAP $C_l$. Unfortunately we will have to extrapolate our radio source fits in the regime beyond $\sim 1' \text{ out to } 5'$ because of the large errors on the radio source beam profile in this range. These results can therefore only be used to indicate the sensitivity of the $C_l$ to the beam profile and should not be regarded as alternative $C_l$ estimates. We first make an extrapolation where we fit the small-scale (total sample) beam profile points and then extrapolate continuing with the power-law as shown by the green dashed lines in Fig. 2i, e and f. We also made a more conservative extrapolation where we again fit the small-scale data but then extrapolate continuing parallel to the Jupiter beam profiles at large scales (orange dashed lines).

The range of the radio source de-beamed $C_l$ is shown by the two red lines and two cyan lines (for W and V bands) in Fig. 4. The most conservative profile model is $\approx 50\%$ higher than the Jupiter de-beamed $C_l$ (green and orange lines) at the scale of the first peak. But the most extreme model is now a factor of 2-3 higher even at $l = 220$ than the standard model power spectrum. We note that it has been possible to derive consistent $C_l$’s between the V, W (and Q) bands, although we accept that this is due to the freedom we have in extrapolating our radio source beam profiles beyond $\theta \approx 30'$. It seems that if the radio sources are indicating a wider
beam profile, then the systematic uncertainty in the beam at the largest scales will dominate the error budget of the $C_l$, even at the scale of the first acoustic peak. These larger errors would then allow a wider range of cosmological models to be fitted, including models where the first peak lies at $l$ as high as 330 (Shanks & Sawangwit 2010).

6 DISCUSSION AND CONCLUSIONS

Clearly it is important to understand why the radio source profiles are so wide in the Q, V and W bands. If there is a correlation between beam width and source flux then it will be wrong to use Jupiter to debeam the CMB power spectra because in the W band, for example, the $\approx 1$ Jy radio sources are much closer to the $\approx 0.5$ Jy rms flux of the CMB fluctuations than the 1200 Jy flux of Jupiter.

The non-linearity shown by the WMAP source fluxes compared to independently measured ATCA/IRAM fluxes is supporting evidence of non-linearity in the WMAP data calibration. It is possible that somehow the variability of the radio sources at W have caused problems that would not apply to the CMB. In passing, we note that the smaller than expected SZ decrements from WMAP observations of rich clusters (Myers et al. 2004; Bielby & Shanks 2007) may also be explained by a wider than expected WMAP beam at W. If so, this would argue that the non-linearity affects variable and non-variable WMAP sources alike.

In considering possible causes of WMAP non-linearity, we first note that detector saturation is unlikely to be the problem since this would lead to the brighter sources having a wider profile than the fainter sources, which is opposite to what is observed. However, Jupiter, being a moving source, has to be dealt with in a different way to the radio sources and the CMB fluctuations in the maps. This means that if there was a problem in the WMAP analysis, it would be necessary to check any filtering that is done to the maps. Otherwise, we do not understand the reason for the difference between the Jupiter and radio source beam profiles.

We conclude that;

- The WMAP power spectrum is heavily dependent on the beam profile. Indeed even the first acoustic peak at $l \approx 220$ is very dependent on the form of the profile at $1^\circ - 2^\circ$ where the profile is only $\approx 0.1$ per cent of its peak value.
- The radio point sources detected by WMAP in the Q, V and W bands generally show a broader beam profile than the Jupiter beam used by the WMAP team. For example, using bright point sources, our W1 beam profile rejects the Jupiter beam with $\gtrsim 99.5\%$ confidence.
- There may be evidence for a flux dependent effect within the WMAP data in that fainter radio sources appear to have systematically broader profiles than brighter sources, although the faint data are noisy.
- Non-linearity in the WMAP flux scale may also be indicated by comparisons of WMAP radio source fluxes with ATCA and IRAM fluxes which show 50 per cent reduced flux from WMAP.
- Further arguments against possible systematics such as Eddington bias affecting our results come from simulation checks and NVSS source samples selected at frequency where CMB fluctuations are subdominant (see Sawangwit 2010).
- The systematic errors on the WMAP $C_l$ due to the beam may be much larger than previously expected and in turn, this means that the systematic error on the best fit cosmological model may also be larger. It will be interesting to see if a revised estimate of the WMAP beam profile then allows a simpler cosmological model to be fitted than $\Lambda$CDM.

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