Numerical study of turbulent mixed convection in a rotating inter-disk cavity with axial throughflow of cooling air

Sergei I Smirnov, Alexey G Abramov, Ekaterina E Kitanina and Evgeni M Smirnov

Higher School of Applied Mathematics and Computational Physics, Peter the Great St. Petersburg Polytechnic University, Saint-Petersburg, 195251, Russian Federation

E-mail: sergeysmirnov92@mail.ru, smirnov_em@spbstu.ru

Abstract. The paper presents the results of eddy-resolving numerical simulation of mixed convection developing in a rapidly rotating inter-disk cavity with axial throughflow of cooling air. Under conditions of the experiments available in the literature for the case of the cavity heating from the side of the disk surfaces, the computations were performed using two methods: Implicit LES and zonal RANS/ILES, with and without taking into account the gravity effects. In case of the zonal RANS/ILES approach, the axial pipe flow and the zone of turbulent mixing of throughflow with particles of heated gas were simulated on the base of the RANS equations added by a turbulence model, whereas the major part of the buoyancy-controlled cavity flow was computed with the ILES method. The ratio of the cavity width to its outer radius was 0.13. The computations were carried out at the rotational Reynolds number equal to $2 \times 10^5$, if evaluated with the outer radius; the axial flow Reynolds number was $2 \times 10^4$. A moderate size grid used in the computations (of about $5 \times 10^5$ cells) provided a good resolution of quasi-laminar Ekman layers. Data on dynamics of unsteady vortex structures forming in the cavity, on characteristics of local heat transfer, as well as on effects of the gravity force action are reported. The results of computations of the time-averaged local Nusselt number on the disks surfaces are compared with measurement data; an acceptable agreement has been obtained in case of the zonal RANS/ILES approach.

1. Introduction

Rapidly rotating annular cavities are typical components of the drum-type rotor of a compressor of a gas turbine engine. In annular cavities related to the last compressor stages, the conditions for development of turbulent mixed convection arise, where the driving reason for occurrence of natural convection in the centrifugal-force field is heating the drum surface by the compressed gas. The forced convection effects are caused by an axial throughflow of cooling air going to the turbine part of the engine.

Mixed convection in rapidly rotating annular cavities with axial throughflow is a challenging problem [1, 2]. Interest in research in this area has emerged since the early 1990s, when first systematic experiments were carried out to study the effect of throughflow on the integral and local time-averaged...
heat transfer in single cavities under various heating conditions [3-7]. Further experimental studies [8-12] were also focused on local heat transfer measurements, providing limited information on three-dimensional unsteady velocity fields, despite this information are extremely important for a deeper understanding of complex dynamic and heat transfer phenomena peculiar to this type of mixed convection.

Some attempts of numerical simulation of the rotating-cavity mixed convection based on the RANS (Reynolds-averaged Navier-Stokes) approach [13-15] have demonstrated principal limitations of this approach: typically, it leads to a dramatic discrepancy between the computed and measured radial distributions of the local heat flux on disks surfaces.

At present, a significant progress towards a deeper penetration into unsteady dynamics and heat transfer under mixed convection conditions in rapidly rotating annular cavities might be achieved with the use of eddy-resolving approaches. However, conceptually, applications of the models introducing the subgrid turbulent viscosity explicitly (Large Eddy Simulation, LES) may conflict with the quasi-laminar nature of thin Ekman-type layers forming on disk surfaces. The same issue occurs in case of using hybrid RANS/LES methods [16].

The highest expectations for progress in numerical studying the problem under consideration are associated with using the high-accuracy DNS (Direct Numerical Simulation) method. However, in case of flow conditions, which are typical for engineering applications, this method is extremely resource consuming. As a productive alternative, the Implicit LES (ILES) method can be used, in which the subgrid turbulent viscosity is not introduced, and the role of physical viscosity on subgrid scales is replaced by dissipative properties of a proper numerical scheme. An initial experience of using the ILES method is presented in [9, 17, 18].

When applying the ILES method for simulation of the considered mixed convection with grids of moderate dimensions, there is a doubt about sufficiency of the grid resolution for adequate reproducing turbulent exchange between the throughflow and the heated air circulating in the cavity. This gives a motivation to the use of the hybrid zonal RANS/ILES approach, in which the pipe flow and the region of turbulent mixing of throughflow with oncoming particles of heated gas are simulated using the RANS equations added by a semi-empirical turbulence model, whereas the major part of the convective flow in the cavity is computed with the ILES method.

In this work, the authors present the results of comparative computations performed with the ILES and RANS/ILES methods for mixed convection in a rapidly rotating inter-disk cavity in the presence of axial cooling airflow. The dynamics of unsteady vortex structures developing in the cavity, peculiarities of local heat transfer as well as the gravity effects, which are usually neglected in related studies, are analyzed.

2. Problem definition

The problem of airflow dynamics and heat transfer in a rapidly rotating symmetrically heated inter-disk cavity with throughflow of cooling air is considered (Fig. 1a). The problem setup corresponds to one of the flow regimes studied experimentally in [3-5]. The computational domain includes the inter-disk cavity and a central axial tube consisting of the inlet and outlet parts. The determining linear dimensions of the configuration are as follows: the inner radius of the tube $a = 0.045$ m, the outer radius of the cavity $b = 0.4845$ m, the cavity width $s = 0.065$ m. The lengths of the inlet and outlet sections of the tube are $l_1 = 0.598$ m and $l_2 = 0.293$ m, respectively. The inter-disk cavity under consideration can be conventionally considered as an annular one, in the sense that its central part is occupied by axial throughflow, the inlet and outlet radii of which are equal to $a$. With this interpretation, the radius $a$ can be treated as the inner radius of the annular cavity.
The computations presented below were fulfilled with the following values of the determining parameters: the Prandtl number $Pr = 0.71$, the axial flow Reynolds number $Re_z = \frac{\rho_\infty a W}{\mu_\infty} = 2 \cdot 10^4$, the rotational Reynolds number $Re_\Omega = \frac{\rho_\infty \Omega b^2}{\mu_\infty} = 2 \cdot 10^5$, the Rossby number $Ro = \frac{W}{\Omega (a)} = 5.74$. This corresponds to the cavity rotation speed of $\Omega = 12.8 \text{ rad/s}$. The centripetal-to-gravity acceleration ratio can be estimated as $\Omega^2 b/g = 8.1$ and $\Omega^2 a/g = 0.75$ at the periphery and in the axial region, respectively. The computations were carried out with and without taking into account the gravity effects.

3. Computational aspects

The computations were performed with the ANSYS Fluent 19.3 software package [19] using the method of coupled solving the continuity and the momentum equations with pressure correction (pressure-based coupled solver). The problem was considered in a reference frame rotating with a constant angular velocity, $\Omega$, on the basis of the perfect gas model with constant values of the dynamic viscosity, $\mu$, and thermal conductivity, $\lambda$, estimated at the cooling air inlet temperature, $T_\infty = 38 ^\circ C$.

Two eddy-resolving approaches were applied: the ILES method and the zonal hybrid RANS/ILES technique. In case of the hybrid approach, the RANS-based computations, using the $k-\omega$ SST turbulence model, covered the inlet and outlet tubes and the central part of the cavity ($-0.2 < r/b < 0.2$), which included also the mixing zone. At the tube inlet, the ratio of turbulent-to-molecular viscosity was set equal to 40, with a turbulence intensity of 10%. The Vortex Method [19] was used to create turbulent content at the RANS/ILES interface and to transfer the generated turbulent pulsations to the ILES zone.

In case of the ILES approach, the upwind scheme of second-order accuracy was used for spatial discretization of the convective terms of the transport equations. When using the RANS/ILES approach, the bounded central difference scheme was applied in ILES zone, which is a combination of the central difference and the upwind schemes [19]. Physical time advancement was performed with an implicit second-order scheme.

The used computational grid included $5.3 \cdot 10^5$ hexagonal cells; it was refined near the walls. In the RANS zone, the conventional requirement of $y^+ < 1$ for the first near-wall computational point was
satisfied. Each of the Ekman layers forming at the disks was covered by a grid layer containing about 15 cells in the normal-to-disk direction, and the thickness of the near-disk grid cells was of $1.5 \times 10^{-4} \text{s}$.

After a transient period, the flow and heat transfer statistics were gathered for a time corresponding to about 200 revolutions of the cavity. Sufficiency of this time for getting representative statistics was justified by monitoring of radial distributions of the disk-surface Nusselt numbers averaged over both time and circumferential coordinate.

4. Results of computations and discussion

4.1. Flow Structure

Figure 2 shows instantaneous temperature fields and streamline patterns in the middle axial section of the cavity. The fields presented for three instants were computed by the RANS/ILES approach with and without taking into account the gravity force action; the time interval between the chosen instants is 10.2 time units, if the physical time is normalized with the period of one revolution of the cavity. The figures show time evolution of cyclonic and anticyclonic vortex structures between which intensive centrifugal and centripetal radial flows break through. Due to low-frequency instabilities of the cavity flow, the number and local dynamics of vortex structures change in time. As well, a relatively slow precession of the flow core is observed. The arrows in the patterns given in Figure 2 indicate the position and direction of the cold gas flow from the throughflow region to the cavity periphery, in the form of radial “arms”. The predicted flow peculiarities are in a good accordance with experimental observations of the flow structure in heated rapidly rotating cavities with axial throughflow [3, 5].

Considering the maps given in Figure 2, one can conclude also that, for the adopted set of the determining parameters, the gravity action has a pronounced influence on the mixed convection under study. In particular, the gravity effects lead to an increase in the speed of global precession and to thickening of the centrifugal “arms”, in other words, to an increase in the distance between the vortex structures.

Figure 2. Instantaneous temperature distributions and streamlines in the middle axial section of the cavity computed (a-c) without and (d-f) with taking into account the gravity force action.
4.2. Heat Transfer
As it is seen in Figure 2, the air temperature in the core of the convection developing in the cavity is, as a whole, notably higher in the case of no gravity force action. Figure 3a illustrates the gravity influence on predicted instant temperature values at the monitoring point located in the middle axial section at a radius of $r = 0.83b$ (the data obtained with the RANS/ILES method are shown). Pronounced low-frequency temperature fluctuations seen in the figure give additional evidence of the presence of global precession of the flow core.

![Figure 3. Time variations of (a) temperature at a point located in the middle axial plane, $r = 0.83b$, and (b) heat flux averaged over the Disk 1 surface; the data shown were obtained using the RANS/ILES approach without (lines 1) and with (lines 2) gravity force action](image)

Time variations of the heat flux averaged over the Disk 1 surface are shown in Figure 3b. It is noteworthy that in the case of no gravity action, a significantly lower level of the integral heat transfer is predicted, as well as a lower saturation of the high-frequency part of the fluctuation spectrum.

The Nusselt number characterizing the intensity of local heat transfer was evaluated at a current radius $r$ as $Nu = qr/\Delta T\lambda$, where $q$ is the disk-surface heat flux averaged over time and over angular coordinate, $\Delta T$ is the difference between the local (prescribed) disk-surface temperature and the inlet temperature $T_{in}$.

Figure 4a shows radial distributions of the local Nusselt number obtained in the computations performed with the ILES method, taking into account the gravity force action, and with the RANS/ILES approach, both with and without including the gravity effects (negative values of $Nu$ seen in the plot correspond to heat fluxes directed from the gas to the disk).
In the ILES calculations, a significantly lower level of heat transfer rate was obtained. This is an expected consequence of insufficiency of the used computational grid for resolution of important exchange processes in the zone of cooling throughflow. The strongest difference between the data obtained with the ILES and the RANS/ILES is observed at the periphery of the cavity, where, in contradiction with the measurement data and with the hybrid method results, the ILES method predicts a non-monotonic behavior of the Nusselt number. At smaller radii, \( r/b < 0.4 \), the heat transfer rate evaluated for two disks differs significantly due to the reference asymmetry of the problem. Here, the heat transfer rate on Disk 1 increases with increasing the radial coordinate, whereas it decreases on Disk 2. Relatively high values of \( \text{Nu} \) on the Disk 2 surface at smaller radii are obviously due to the impingement of the peripheral part of throughflow onto the disk surface.

In Figure 4b, the local Nusselt numbers predicted with the RANS/ILES method (at \( g = 9.81 \text{ m/s}^2 \)) are compared with the experimental results [5]. Taking into account a rather significant discrepancy in the measurement data, the degree of agreement between the simulation results and the measurement data can be considered satisfactory.

5. Conclusion

Using vortex-resolving approaches, ILES and RANS/ILES, numerical simulations of turbulent mixed convection in a rapidly rotating inter-disk cavity with axial throughflow of cooling air were performed.

The computations have reproduced all principal peculiarities of the mixed convection known from the literature, including: formation of evolving large-scale cyclonic and anticyclonic vortices of the Rayleigh-Bénard type, which are separated by radial “arms” of the cold gas moving towards the periphery of the cavity; presence of a relatively slow global precession of the flow core; formation of thin quasi-laminar Ekman layers on the surfaces of the disks and a mixing zone of the axial throughflow with the flow in the cavity.

It was found that for the considered combination of determining parameters, which was included in the program of the extended experimental study [3-5], the dynamics of unsteady large-scale vortex structures and their intensity, as well as the heat transfer rate in the cavity, were very sensitive to take into account the gravity force action.
Additional methodological information on practical application of the eddy-resolving approaches, which can be useful for further studies of problems of this class, was obtained. Thus, the computations carried out with the hybrid zonal RANS/ILES approach using a grid of a relatively modest dimension, gave meaningful results, which were in accordance with previous experimental observations of the mixed convection structure and with measured characteristics of local heat transfer. In order to obtain reliable results using the ILES method, much more refined computational grids are required.

Acknowledgments
The study was supported by the Russian Foundation for Basic Research under grant no. 20-08-01090.

References
[1] Owen J M and Rogers R H 1995 Flow and heat transfer in rotating-disc systems. Volume 2: Rotating cavities (John Wiley & Sons, New York) p 296
[2] Owen J M, Tang H and Lock G D 2018 Aerospace 5 1–22
[3] Farthing P R, Long C A, Owen J M, and Pincombe J R 1992 J. Turbomach 114 237–246
[4] Farthing P R, Long C A, Owen J M and Pincombe J R 1992 J. Turbomach 114 229–236
[5] Long C A 1994 Int. J. Heat Fluid Flow 15 307–316
[6] Long C A and Tucker P G 1994 J. Turbomach. 11 525–534
[7] Kim S Y, Han J C and Morrison G L 1993 Proceedings of ASME Turbo Expo 1993 (Cincinnati, Ohio, USA) 93-GT-258
[8] Bohn B, Deutsch G, Simon B, and Burkhardt C 2000 Proceedings of ASME Turbo Expo 2000 (Munich, Germany) GT2000-280
[9] Bohn D E, Bouzidi F, Burkhardt C, Kitanina E E, Ris V V, Smirnov E M and Wolff M W 2002 Proceedings of 9th Int. Symp. on Transport Phenomena and Dynamics of Rotating Machinery (Honolulu, Hawaii, USA) 2002.02.10–14
[10] Owen J M and Powell J 2006 J. Eng. Gas Turb. Power 128 128–134
[11] Long C A and Childs P R N 2007 Int. J. Heat Fluid Flow 28 1405–1417
[12] Gunther A, Uffrecht W and Odenbach S 2014 Proceedings of ASME Turbo Expo 2014 (Düsseldorf, Germany) GT2014-26228
[13] Long C A, Morse A P and Tucker P G 1997 J. Turbomach. 119 51–60
[14] Tian S, Tao Z, Ding S and Xu G 2008 Int. J. Heat Mass Transfer 51 960–968
[15] Wróblewski W and Fraczek D 2014 Archives of Mechanics 66 343–364
[16] Atkins N R and Kanjirakkad V 2014 Proceedings of ASME Turbo Expo 2014 (Düsseldorf, Germany) GT2014-27174
[17] Smirnov E M 2002 Proceedings of 12th International Heat Transfer Conference (Grenoble, France)
[18] Bohn D, Ren J and Tuemmers C 2006 Proceedings of ASME Turbo Expo 2006 (Barcelona, Spain) GT2006-90494
[19] ANSYS Fluent Theory Guide. Release 2019 R3, ANSYS, Inc. (Canonsburg, PA, USA)