Circular polarization of cosmic photons due to their interactions with Sterile neutrino dark matter

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Abstract

In this paper, we explore the possibility of the polarization conversion of a wide energy range of cosmic photons to the circular polarization through their interaction with Sterile neutrino as a dark matter candidate. By considering the Sterile neutrino in the seesaw mechanism framework and right-handed current model, we estimate the Faraday conversion $\Delta \phi_{FC}$ of gamma ray burst (GRB) photons interacting with the Sterile neutrinos at both the prompt and afterglow emission levels. We show that for active-Sterile neutrino with mixing angle $\theta^2 \lesssim 10^{-2}$ motivated by models with a hidden sector coupled to the sterile neutrino, the Faraday conversion can be estimated as $\Delta \phi_{FC} \lesssim 10^{-2} - 10^{-17}$ rad. We also examine the V-mode power spectrum $C_{VL}$ of the cosmic microwave background (CMB) at the last scattering surface. We show that the circular polarization power spectrum at the leading order is proportional to the linear polarization power spectrum $C_{pl}$ and the mixing angle where for $\theta^2 \lesssim 10^{-2}$ leads to $C_{VL} \lesssim 0.01$ Nano-Kelvin squared.
1 Introduction

Over the past century, the existence of dark matter (DM), the non-baryonic substance of the universe, which accounts for 26% of the total energy density of the universe, has been discussed. The cosmological evidence like curves in the galactic halos \[1\] as well as astrophysical observations such as WMAP \[2\] and Planck \[3\], increase the DM existence probability. Beside cosmological and astrophysical evidence, it is crucial to attain information about interaction features of DM, if exist, with standard model (SM) particles. Such information can be obtained through direct detection experiments via scattering on target nuclei, such as XENON10 \[4\], XENON100 \[5\], XMASS \[6\], CoGEANT \[7\], DAMA \[8, 9\], PICASSO \[10, 11\] and indirect search using terrestrial experiments such as production signatures at colliders \[12, 13\] as well as searching for annihilation or decay signals \[14, 15\]. A different window into the nature of DM is to investigate the circular polarization effects arising from scattering of the cosmic photons, with various astrophysical sources, from DM particles. From the theoretical point of view, the circular polarization is generated from several mechanisms, mostly new physics interactions, which contribute to the Boltzmann equation. For instance, forward scattering of CMB photon from cosmic neutrino background leads to the circular polarization of CMB photon \[16\]. CMB photons scattered from electrons can acquire circular polarization in the presence of Lorentz violation \[19\], and also background fields as for example magnetic field \[19, 20\], non-commutative space-time \[19, 21\] and CP violation \[22\]. Conversion of linear polarization to circular one for GRB photons scattering from cosmic particles is also discussed in Ref. \[23\]. As another possibility, circular polarization for CMB can be raised from circularly polarized primordial gravitational waves \[24\].

Background radiation fields are spread over the universe at all wavelengths. CMB is the most intense radiation in which provides unique cosmological information at recombination epoch at the early universe. Beside background radiation, the cosmic rays originating in a different range of wavelengths are distinctive tools to study the properties of DM particle on the basis of observational effects. Non-uniform pulses of gamma-ray radiation, lasting commonly less than a minute, known as GRBs have detected at redshift less than ten \[25\]. It is believed that they are produced at the end of massive star evolution and forming black holes \[26, 27\] or combining of compact objects \[30\]. It can be seen at a random location on the sky and few times during a day. Generally, GRBs are followed-up by afterglow emissions including longer wavelength X-ray, optical, IR and radio frequencies.

In theoretical term, among the SM particles only neutrinos can fulfill properties of DM candidate. However, its small mass and large coupling to other particles of the SM keep neutrino relativistic at the epoch of freeze-out and it would only picture the hot DM \[31\]. There are many models beyond the SM which provide one or some unknown particles with different masses, interaction sorts, spin and strength to account for DM (for instance see refs. \[32, 38\] and the references therein). A large number of models beyond the SM, predict the existence of a spin 1/2 Majorana fermion which is singlet under gauge group of the SM with interactions weaker than the SM neutrinos known as right-handed Sterile neutrino, for a review see for example \[39\], and the right-handed current model see for example Refs. \[37, 40\]. Sterile neutrino is powerful enough to explain the baryon asymmetry \[11, 12\] and observed neutrino oscillations \[43\] if it shows up at triplet. With fewer mass \[44\], it can provide a viable DM through the seesaw mechanism \[45\]. The seesaw mechanism is implemented in three tree level ideas so-called as type-I \[46, 51\], type-II \[52, 53\] and type-III \[54\]. Meanwhile, there are some alternative extended models as well \[55, 56\].

Cosmological and astrophysical aspects of massive Sterile neutrino are studied broadly in literature \[57, 68\]. In this paper for the first time, we study the circular polarization phase shift caused by cosmic photons interacting with Sterile neutrinos, as the Warm DM (WDM).
provides a new tool to brighten the DM properties within a simple extension of the seesaw, so-called Type-I seesaw mechanism [46,49], as well as within the right-handed current model [37,40].

This paper is organized as the following: we first present a brief review of the seesaw type I model in section and the right-handed current model 2. In section 3 the time evolution of Stokes parameters for photon-Sterile neutrino interaction is calculated by using the scalar mode perturbation of metric and the generation of circular polarization. The circular polarization arising from GRB-Sterile neutrino and CMB-Sterile neutrino forward scatterings are estimated in section 4. Finally in section 5 we give a summary and conclusion.

2 Right Handed Neutrinos and Type-I seesaw

Right-handed Sterile neutrinos are elegantly embedded in the seesaw model. In type-I seesaw model the SM is extended by at least two heavy Sterile neutrino singlets $\nu^i_R$ ($i$ indicates the generation) with the following most general electroweak Lagrangian

$$L = L_{SM} + y_\nu^{ij} \bar{\ell}_L \tilde{H} \nu^j_R + \frac{1}{2} M^i_R \bar{\nu}^c_R \nu^i_R + h.c. \quad (1)$$

where $L_{SM}$ denotes the electeroweak Lagrangian of the SM and $y_\nu^{ij}$ is a matrix of Yukawa interactions, $H$ is the Higgs doublet and $\tilde{H} = \epsilon H^*$, with $\epsilon$ is the anti-symmetric SU(2)-invariant tensor, $\ell_L = (\nu_L, e_L)^T$ indicates the left handed lepton doublets, $\nu^c_R = C \nu^T_R$ with $C = i\gamma_2\gamma_0$. Furthermore, $\nu^i_R$’s are SM gauge singlets, hence the Majorana mass term $M^i_R$ is allowed in addition to the Dirac mass $m_D$. Consequently, after the electroweak symmetry breaking on e can obtain the Dirac mass as $m_D = y^\nu \langle H \rangle$ where by considering both Dirac and Majorana masses leads to the neutrino mass matrix as follows

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}, \quad (2)$$

where $M_R$ and $m_D$ are 3 × 3 matrices. However, the eigenvalues of $M_R$ can be chosen to be at a scale much higher than the electroweak scale suppressing the Dirac mass term. Meanwhile, to diagonalize the mass matrix, one needs a 6 × 6 mixing unitary matrix. In fact, the diagonalizing process occurs through two steps I) block diagonalizing and II) two unitary rotations. Therefore, there would be two sets of physical eigenstates related to the three light neutrinos of the SM particles. In the first set of eigenstates which are known as active neutrinos, the masses can be obtained as

$$m_\nu = -m_D^T M_M^{-1} m_D, \quad (3)$$

and the neutrinos belong to the $SU(2)$ doublets. In the second set, one has a set of heavy right handed Majorana neutrinos which are gauge singlets with mass $M_M$ the eigenvalues of $M_R$. The scale of $M_M$ is not determined by experiment and different constraints are available from particle physics, astrophysics and cosmology with different consequences [69]. As a result, Sterile and the SM neutrinos mix with $\theta \equiv m_D M_M^{-1}$ mixing angle. Therefore, all of the Majorana mass eigenstates can be represented by the flavor vector elements as:

$$N = V^\dagger_N \nu_R + \Theta^T \nu^c_L + h.c., \quad \text{and} \quad \nu = V^\dagger \nu_L - U^\dagger_\nu \theta \nu^c_R + h.c., \quad (4)$$

where $V_\nu$ is the usual neutrino mixing matrix connecting the observed light mass eigenstates $\nu_i$ to the active flavor eigenstates as follows:

$$V_\nu \equiv (1 - \frac{1}{2} \theta \theta^T) U_\nu, \quad (5)$$
and \( U_\nu \) is the unitary part of neutrino mixing matrix. Meanwhile, the corresponding parameters in the Sterile sector are \( V_N \) and \( U_N \) and the active-Sterile mixing angle is

\[
\Theta \equiv \theta U_N^\dagger.
\]

Thus the Sterile neutrinos interacts with the SM particles as follows

\[
\mathcal{L} \supset -\frac{g}{\sqrt{2}} \bar{N} \gamma'^\mu \Theta N W^-_\mu - \frac{g}{2 \cos \theta_W} \bar{N} \gamma'^\mu \nu_L Z_\mu - \frac{g}{2 \cos \theta_W} \bar{\nu}_L \gamma'^\mu \Theta h \bar{N} - \frac{g}{2 \cos \theta_W} \bar{\nu}_L \gamma'^\mu \Theta h \bar{N} \nu_L.
\]

where \( f_L = e_L, \mu_L, \tau_L \) is representing lepton in three generations and \( \nu_L \) denotes the SM neutrino in the flavor eigenstates, one can express it in terms of mass eigenstates as \( \nu_L = P_L (V_\nu \nu + \Theta N) \). Sterile neutrinos, with the properties of neutralness and being massive, have been largely proposed for DM candidate if their lifetime is longer than the age of the universe. Moreover, based on the available constraints on galaxy phase space density and also universal galaxy surface density and DM density, Sterile neutrinos can only stand for warm DM \[45\]. The production mechanism of Sterile neutrino in the case of WDM suggests that Sterile neutrinos have never been in thermal equilibrium in the early universe because they are produced at temperature much higher than active neutrino decoupling temperature, \( T \sim 1 \text{ MeV} \) (for a review see ref. \[35\] and the references therein) and their production can be resonantly enhanced in the presence of lepton asymmetries or non-resonant if the lepton asymmetries did not exist. Sterile neutrino can decay radiatively at loop level into SM neutrinos \( N \rightarrow \nu_L + \gamma \). There are also other dominant tree-level decay channel for Sterile neutrino which is \( N \rightarrow \nu_\alpha \nu_\beta \bar{\nu}_\beta \) with the following total decay width \[70, 71\]

\[
\Gamma = \frac{G_F^2 M_5^5}{96 \pi^3} \theta^2,
\]

where \( \theta^2 \equiv \sum_{i=e,\mu,\tau} |\theta_i|^2 \) and \( G_F \) is weak Fermi constant. By requiring the condition of Sterile neutrino lifetime being longer than the age of the Universe \( t_{\text{Universe}} = 4.4 \times 10^{17} \text{ sec} \) \[72\], the mixing angle \( \theta^2 \) should be constraint as

\[
\theta^2 < 1 \left( \frac{1 \text{ keV}}{M} \right)^5,
\]

where \( M \) denotes the mass of Sterile neutrino. Moreover, to prevent over production of keV Sterile neutrino through its mixing with SM active neutrinos in the early universe \[45, 73\] and also considering many astrophysical consideration constraints, the mixing angle is forced to be very small \( \theta^2 \ll 10^{-8} \) \[35\]. However, some models with additional hidden sector coupling to Sterile neutrino suggest that the production of Sterile neutrino in early universe can be suppressed if the oscillations of SM neutrinos to Sterile neutrinos instead of plasma temperature, \( T_{\text{max}} \sim 100 \text{ MeV}(M_1 \text{ keV})^{1/3} \), happens at much lower temperature \( T \ll T_{\text{max}} \) in order to prevent over-closure of the Universe. As a result, the large mixing angle \( \theta^2 \leq 10^{-1} \) are possible \[67\]. Besides, there are some direct laboratory loose bounds on mixing in keV mass range as \( \theta^2 \geq 10^{-4} \) \[74, 75\].

Before ending this section, we would like also to study that the right-handed neutrinos \( \nu_R \) of Dirac type, as WDM candidates, possibly couple to the SM particles via the right-handed current interactions with the SM intermediate gauge bosons \[37, 40, 76, 77\] such as

\[
\mathcal{L} \supset g_R (g/\sqrt{2}) \bar{\ell}_R \gamma'^\mu \nu_R W^-_\mu + \text{h.c.,}
\]

\[3\]
where \( \ell \) stands for a charged lepton. This was motivated by the parity symmetry reconstruction at high energies without extra gauge bosons. The counterpart of (10) in the quark sector has been also studied in Refs. [77, 78]. The effective coupling \( g_R^2 \ll 10^{-2} \) can be constrained by W-boson decay rate \( \Gamma(W \to \gamma + \nu_R) = (3g_R^2/32\pi)M_W^3 \) being smaller than the experimental W-boson decay width \( 4.2 \times 10^{-2} \text{GeV} \). Since \( \nu_R \) is supposed to be a WDM candidate, we also consider the constraint on \( \nu_R \) mass and coupling by studying the rate of decay \( \nu_R \to \nu_L + \gamma \) at the leading order with a W-boson exchange,

\[
\Gamma_R(\nu_R \to \nu_L + \gamma) = \frac{\alpha}{4} G_F^2 m_\nu^2 M^3 g_R^2.
\]

Using DM lifetime condition like (9), the obtained limit on coupling \( g_R \) is as follow

\[
g_R^2 \lesssim 10^{-4} \left( \frac{1.7 \text{ GeV}}{m_\nu} \right)^2 \left( \frac{1\text{eV}}{M} \right)^3,
\]

(12)

where \( m_\ell \) and \( M \) are the masses of charged lepton and right-handed neutrino, respectively. Rigorous astrophysical and cosmological constraints on the mass and effective coupling require further studies. In the following analysis, the sterile neutrino \( N \) or \( \nu_R \) plays the same role and parameters \( \theta^2 = g_R^2 \), unless otherwise stated.

### 3 Cosmic photons scattering from Sterile neutrino

The polarization of an ensemble of photons can be explained by the following density operator:

\[
\hat{\rho} = \frac{1}{\text{tr}(\hat{\rho})} \int \frac{d^3p}{(2\pi)^3} \rho_{ij}(\mathbf{p}) \hat{D}_{ij}(\mathbf{p})
\]

(13)

where \( \rho_{ij} \) shows the density matrix components in the phase space, \( \mathbf{p} \) represents the momentum of cosmic photons and \( \hat{D}_{ij}(\mathbf{p}) = a_i^\dagger(\mathbf{p})a_j(\mathbf{p}) \) is the number operator of photons. This can also be decomposed into well-known Stokes parameters in the polarization space as follows

\[
\hat{\rho} = \frac{1}{2} \begin{pmatrix}
I + Q & U - iV \\
U + iV & I - Q
\end{pmatrix},
\]

(14)

where \( I \) is radiation intensity, \( Q \) and \( U \) represent linear polarization and circular polarization is given by \( V \) parameter. The \( Q \) and \( U \) quantities are influenced by orientation of coordinate system while \( V \) and \( I \) are coordinate independent. Therefore, a coordinate independent combination \( Q \pm iU \) is considerable.

Stokes parameters for a propagating wave in the \( \hat{z} \) direction are defined as

\[
I \equiv \langle E_x^2 \rangle + \langle E_y^2 \rangle \\
U \equiv \langle 2E_x E_y \cos(\phi_x - \phi_y) \rangle \\
Q \equiv \langle E_x^2 \rangle - \langle E_y^2 \rangle, \\
V \equiv \langle 2E_x E_y \sin(\phi_x - \phi_y) \rangle.
\]

(15)

The amplitudes and phases of waves in the \( x \) and \( y \) directions are defined with \( (E_x, \phi_x) \) and \( (E_y, \phi_y) \), respectively. The \( \langle \cdot \cdot \cdot \rangle \) represents time averaging. In the standard model of cosmology, there is not any physical mechanism to generate \( V \) parameter for unpolarized cosmic photons. However, linear polarization can be converted to a circular polarization in the presence of a background field or by scattering off cosmic particles through Faraday conversion defined as

\[
\dot{V} = 2U \frac{d\Delta \phi_{\text{FC}}}{dt} \quad \text{and} \quad \dot{V} = 2Q \frac{d\Delta \phi_{\text{FC}}}{dt},
\]

(16)
Figure 1: The representative Feynman diagrams represent the photon-Sterile neutrino scattering. There are two more Feynman diagrams with antiparticle contributing in the loops.

where $\Delta \phi_{\text{FC}}$ is the Faraday Conversion (FC) phase shift [20].

The time evolution of V-Stokes parameter or equivalently the component of density matrix reads [79]

$$
(2\pi)^3 \delta^3(0)(2p_0) \frac{d}{dt} \rho_{ij}(p) = i\langle [H^0_I(t), D^0_{ij}(p)] \rangle - \frac{1}{2} \int dt \langle [H^0_I(t), [H^0_I(0), D^0_{ij}(p)]] \rangle,
$$

where the first order of the interaction Hamiltonian is given by $H^0_I$ and $p_0$ is the magnitude of photon momentum. The first on the right-hand side is the forward scattering and the second term represents higher-order collision terms.

In order to study the photon-Sterile neutrino forward scattering, we start with the seesaw Lagrangian given in (7). Within the seesaw model, the photon can scatter from Sterile neutrinos at one-loop level with a lepton and weak gauge bosons propagating in the loops. Representative relevant Feynman diagrams are shown in Fig1. There are t-channel Feynman diagrams involving $W$-boson and charged lepton loops (electron, muon and tau leptons can exchange in the loop). Two additional Feynman diagrams representing the contributions from antiparticles in the loops have been also taken into account. Contribution from a further s-channel diagram with the $W^+W^-\gamma\gamma$ vertex in which $W$-bosons exchange in a triangle loop as well as a t-channel diagram the same as Fig 1 where three $W$ bosons contribute in the box diagrams are negligible.

The electromagnetic free gauge field $A^\mu$ and Majorana fermion field $N(x)$, which are self-conjugate, can be indicated as creation $a_s^\dagger(p)$ ($b_r^\dagger(q)$) and annihilation $a_s(p)$ ($b_r(q)$) operators for photons (Majorana fermions) as

$$
A_\mu(x) = \int \frac{d^3p}{(2\pi)^3 2p^0} [a_s(p)\epsilon_{s\mu}(p)e^{-ip\cdot x} + a_s^\dagger(p)\epsilon^{*}_{s\mu}(p)e^{ip\cdot x}],
$$

$$
N(x) = \int \frac{d^3q}{(2\pi)^3 \sqrt{2q^0}} \left[b_r(q)u_r(q)e^{-iq\cdot x} + b_r^\dagger(q)v_r(q)e^{iq\cdot x}\right],
$$

where $\epsilon_{s\mu}(p)$ with $s = 1, 2$ are the photon polarization 4-vectors of two physical transverse polarization $u_r(q)$ and $v_r(q)$ are the Dirac spinors. The creation and annihilation operators respect the following canonical commutation (anti-commutation) relations

$$
[a_s(p), a_{s'}^\dagger(p')] = (2\pi)^3 2p^0 \delta_{ss'} \delta^3(p - p'),
$$

$$
\{b_r(q), b_{r'}^\dagger(q')\} = (2\pi)^3 \gamma^0 \delta_{rr'} \delta^3(q - q').
$$

(20)
The leading-order interacting Hamiltonian for this process can be expressed by the scattering amplitude as follows

$$
H^0_I(t) = \int dq' dq dp' (2\pi)^3 \delta^{(3)}(q' + p' - q - p) \exp(i[q^0 + p'^0 - q^0 - p^0])
\times \left[ b_{r'}^+(q') a_s^+(p') \right] M_{tot}(N\gamma \rightarrow N\gamma) a_s(p) b_r(q),
$$

with

$$
dq \equiv \frac{dq}{(2\pi)^3} e^{-i\frac{p^0}{\hbar}} dq', dp \equiv \frac{dp}{(2\pi)^3} e^{-i\frac{p'^0}{\hbar}} dp'$$

and the total amplitude $M_{tot}$ obtained from the sum of all Feynman diagrams in Fig. is as follows

$$
M_{tot}(q'r', p's', qr, ps) = M_1(q'r', p's', qr, ps) + M_2(q'r', p's', qr, ps)
- M_3(q'r', p's', qr, ps) - M_4(q'r', p's', qr, ps),
$$

where $M_{3,4}(q'r', p's', qr, ps)$ are the Hermitian conjugates of $M_{1,2}(q'r', p's', qr, ps)$, respectively, and have been contributed from antiparticles in the loops as follows

$$
M_1(q'r', p's', qr, ps) = \frac{1}{(2\pi)^4} \int d^4l \bar{u}_{r'}(q') \gamma^\alpha (1 - \gamma^5) S_F(l + p - p') \gamma^\beta (1 - \gamma^5) u_r(q) D_{F\alpha\beta}(q - l),
$$

$$
M_2(q'r', p's', qr, ps) = \frac{1}{(2\pi)^4} \int d^4l \bar{u}_{r'}(q') \gamma^\alpha (1 - \gamma^5) S_F(l - p - p') \gamma^\beta (1 - \gamma^5) u_r(q) D_{F\alpha\beta}(q - l),
$$

$$
M_3(q'r', p's', qr, ps) = \frac{1}{(2\pi)^4} \int d^4l \bar{u}_r(q) \gamma^\alpha (1 + \gamma^5) S_F(p' - p - l) \gamma^\beta (1 + \gamma^5) v_s(q') D_{F\alpha\beta}(l - q),
$$

and

$$
M_4(q'r', p's', qr, ps) = \frac{1}{(2\pi)^4} \int d^4l \bar{u}_r(q) \gamma^\alpha (1 + \gamma^5) S_F(p' - p - l) \gamma^\beta (1 + \gamma^5) v_s(q') D_{F\alpha\beta}(l - q),
$$

where $S_F$ indicates the fermionic propagator, the indices $r, r'$ and $s, s'$ in the above amplitudes, respectively, denote the Sterile neutrino and photon spin states. Moreover, we have considered the contribution of three generations of leptons in each diagram. To calculate the forward scattering term in (17), one should find the commutator $[H^0_I(t), D^0_{ij}(p)]$ then get the expectation value $\langle [H^0_I(t), D^0_{ij}(p)] \rangle$ according to the following operator expectation value

$$
\langle b_{s'}^+(q') b_r(q) \rangle = (2\pi)^3 \delta^{(3)}(q - q') \delta_{ss'} \delta_{ij} \frac{1}{2} f_N(x, q).
$$

To this end, we substitute (22,26) into (21) and then (17) to find the time evolution of the density matrix components as

$$
\frac{d}{dt} \rho_{ij}(p) = -\frac{\sqrt{2}}{12\pi p^0} \alpha \theta^2 G_F \int dq \left( \delta_{is} \rho_{s'j}(p) - \delta_{js} \rho_{is}(p) \right) f_N(x, q) \bar{u}_r(q) (1 - \gamma^5)
\times \left( q \cdot \epsilon_s \epsilon_{s'} + q \cdot \epsilon_s \epsilon_{s'} \right) u_r(q) + \frac{\sqrt{2}}{24\pi p^0} \alpha \theta^2 G_F \int dq \left( \delta_{is} \rho_{s'j}(p) - \delta_{js} \rho_{is}(p) \right)
\times \left( f_s \epsilon_{s'} \epsilon_{s'} - \epsilon_s \epsilon_{s'} \right) u_r(q),
$$

$$
\frac{d}{dt} \rho_{ij}(p) = -\frac{\sqrt{2}}{12\pi p^0} \alpha \theta^2 G_F \int dq \left( \delta_{is} \rho_{s'j}(p) - \delta_{js} \rho_{is}(p) \right) f_N(x, q) \bar{u}_r(q) (1 - \gamma^5)
\times \left( q \cdot \epsilon_s \epsilon_{s'} + q \cdot \epsilon_s \epsilon_{s'} \right) u_r(q) + \frac{\sqrt{2}}{24\pi p^0} \alpha \theta^2 G_F \int dq \left( \delta_{is} \rho_{s'j}(p) - \delta_{js} \rho_{is}(p) \right)
\times \left( f_s \epsilon_{s'} \epsilon_{s'} - \epsilon_s \epsilon_{s'} \right) u_r(q),
$$

(28)
where $\gamma^\mu$ and $\gamma^5$ are the usual gamma matrices. Consequently, reconstruction of the Stokes parameters through the density matrix elements leads to the Boltzmann equations as follows

$$\frac{dI}{dt} = C I_{e\gamma},$$

(29)

$$\frac{d}{dt} \Delta^\pm_P = C^\pm_{e\gamma} + i \dot{\eta}^P_{DM} \Delta^\pm_P + \mathcal{O}(V),$$

(30)

$$\frac{dV}{dt} = C^V_{e\gamma} + \dot{\eta}^{C^-}_{DM} \Delta^+_P + \dot{\eta}^{C^+}_{DM} \Delta^-_P,$$

(31)

in which $\Delta^\pm_P = Q \pm i U$ and $C^I_{e\gamma}, C^V_{e\gamma}$ and $C^\pm_{e\gamma}$ demonstrate the contributions from the usual Compton scattering to the time evolution of $I, V,$ and $\Delta^\pm_P$ parameters, respectively. Their explicit expressions are available in the literature for example see refs \[80, 81\]. Meanwhile, $\dot{\eta}^P_{DM}$ and $\dot{\eta}^{C^\pm}_{DM}$ which are considered for the photon-Sterile neutrino scattering can be obtained as

$$\dot{\eta}^P_{DM} = \frac{\sqrt{2}}{3\pi p^0} \alpha G_F \theta^2 \int dq f_N(x, q) \times (\epsilon^\mu_\nu \rho_\sigma \epsilon^\rho_\sigma^\nu \epsilon^\rho_\sigma^\nu q^\rho q^\sigma),$$

(32)

and

$$\dot{\eta}^{C^\pm}_{DM} = \frac{\sqrt{2}}{3\pi p^0} \alpha G_F \theta^2 \int dq f_N(x, q) \times \left[ (-q \cdot \epsilon_1 q \cdot \epsilon_2 - q \cdot \epsilon_2 q \cdot \epsilon_1) \right. \pm i(q \cdot \epsilon_1 q \cdot \epsilon_2 - q \cdot \epsilon_2 q \cdot \epsilon_1) \big],$$

(33)

where the scattered photons can be chosen from a wide range of energy from low to high energy cosmic photons. In this paper, we consider CMB as low energy photons and Gamma ray bursts (GRBs) as high energy photons and we calculate the amount of possible circular polarization for each case via their scattering from Sterile neutrino.

4 V-mode and circular polarization of cosmic photons

Astrophysical searches of WDM candidate in new physics are essential part of the experimental efforts to explore the nature of WDM. In this section, we propose an indirect method to search for WDM via studying WDM effects on cosmic photons polarization through cosmic photon-sterile neutrino forward scattering. The strategy is to search for WDM signals in regions of the sky with the highest expectation for WDM aggregation. Of these regions, the center of Galactic is one of the most promising locations for WDM searches and polarimetry of cosmic rays which come from these regions. We consider CMB and GRB photons as the source of cosmic ray photons which might open a new observational window to explore the nature of dark matter.

4.1 Circular polarization of GRB photons

The GRB photon-sterile neutrino forward scattering can generate the V polarization parameter. In this section, we estimate the Faraday conversion phase shift in two cases: (i) GRB photons at prompt emission interacting with Sterile neutrinos passing through internal and external shocks, (ii) GRB photons at afterglow intermediate emission interacting with sterile neutrinos on the way of their propagation.
Based on the time evolution of the V-Stokes parameter in [31] and considering [16], the Faraday conversion phase shift of the scattered photons from Sterile neutrinos evolve as follows

$$
\Delta \phi_{FC} = \frac{\sqrt{2}}{6\pi} \alpha G_F \theta^2 \int \frac{dt}{p_0} f_N(x) v_{DM}^2 (\hat{v}_1 \hat{q}_\beta e_1^\alpha e^\beta - \hat{v}_2 \hat{q}_\beta e_2^\alpha e^\beta),
$$

where we have supposed that the Sterile neutrino has the same global density of DM today $\rho_0 \approx 10^{-41}$ GeV$^4$ [82] and we have supposed that the Prompt $\gamma$-ray emission occurs at distance $\sim 10^{10}$ cm from the center.

However, in the second case, the afterglow radiation caused by the GRB photons can interact with the Sterile neutrinos in its way to the earth. Therefore, the GRB linear polarization is expected to be suppressed by the Faraday conversion phase shift. The integration over time in (34) can alternatively convert to integration over the redshift as $\int_0^z dt = \int_0^z \frac{dz}{H(z)}$ with $H(z) = H_0 [\Omega_\gamma (1+z)^4 + \Omega_M (1+z)^3 + \Omega_\Lambda]$ where $\Omega_\gamma \approx 10^{-4}$, $\Omega_M \approx 0.3$ and $\Omega_\Lambda \approx 0.7$ are the present densities of radiation, matter plus DM and dark energy, respectively, and $H_0 = 67.4$ km/s/Mpc is the value of Hobble constant at the present time [83]. Meanwhile, the bulk velocity of WDM, number density and energy of contributing particles depend on the redshift $\Delta \phi_{FC} = 10^{-31} \theta^2 (1+z)^2 (\frac{\text{GeV}}{k_0})(\frac{\rho_N}{10^{-41} \text{ GeV}^4}) (\frac{v_{DM}}{10^{-3}})^2 \int \frac{dl}{10^{10} \text{ cm}} (\hat{v}_1 \hat{q}_\beta e_1^\alpha e^\beta - \hat{v}_2 \hat{q}_\beta e_2^\alpha e^\beta),$

where the density of the Sterile neutrinos is assumed to be $\rho_N \approx 10^{-41}$ GeV$^4$ [82] and we have supposed that the Prompt $\gamma$-ray emission occurs at distance $\sim 10^{10}$ cm from the center.

Based on the time evolution of the V-Stokes parameter in [31] and considering [16], the Faraday conversion phase shift of the scattered photons from Sterile neutrinos evolve as follows

$$
\Delta \phi_{FC} = 10^{-31} \theta^2 (1+z)^2 (\frac{\text{GeV}}{k_0})(\frac{\rho_N}{10^{-41} \text{ GeV}^4}) (\frac{v_{DM}}{10^{-3}})^2 \int \frac{dl}{10^{10} \text{ cm}} (\hat{v}_1 \hat{q}_\beta e_1^\alpha e^\beta - \hat{v}_2 \hat{q}_\beta e_2^\alpha e^\beta),
$$

where we have supposed that the Sterile neutrino has the same global density of DM today $\rho_{DM} \approx \rho_N \approx 10^{-47}$ GeV$^4$ [83]. Conventionally, one can rewrite the above equation as a function of the decay rate and the mass of Sterile neutrino (see [3]). Therefore, the Faraday conversion phase shift due to the GRB photon - Sterile neutrino scattering depends on the mass of Sterile neutrino as follows

$$
\Delta \phi_{FC} = 10^{-13} \left(\frac{\text{keV}}{M}\right)^5 (\frac{\text{keV}}{p_0}) (\frac{\rho_{DM}}{10^{-47} \text{ GeV}^4}) (\frac{v_{DM}}{10^{-3}})^2,
$$

where the life time of Sterile neutrino $\tau_N \sim 10^x$ universe life time. However, the Faraday conversion phase shift for the GRB afterglow spectrum scattering from Sterile neutrinos can be estimated from (37) for different mixing angles in the range $\theta^2 \approx 10^{-2} - 10^{-6}$ which are shown in Tab.1.
Table 1: GRB Faraday conversion phase shift due to photon-Sterile neutrino dark matter interaction for the electromagnetic spectrum, regarding $z = 1$ and $\rho_{DM} = 10^{-47}$ GeV$^4$ [83]

| GRB types | $\lambda$ (cm) | $\Delta \phi_{FC}\big|_{\theta^2 \approx 10^{-2} - 10^{-6}}$ |
|-----------|----------------|----------------------------------|
| $\gamma$ ray | $10^{-10}$ | $\approx 10^{-17} - 10^{-21}$ |
| X ray | $10^{-8}$ | $\approx 10^{-15} - 10^{-19}$ |
| UV | $10^{-6}$ | $\approx 10^{-13} - 10^{-17}$ |
| visible | $10^{-4}$ | $\approx 10^{-11} - 10^{-15}$ |
| infrared | $10^{-3}$ | $\approx 10^{-10} - 10^{-14}$ |
| Microwave | $1$ | $\approx 10^{-7} - 10^{-11}$ |
| Radio | $10^{5}$ | $\approx 10^{-2} - 10^{-6}$ |

4.2 Circular polarization of the CMB photons

In this section, we discuss the circular polarization of the CMB photons in the conformal time $\eta$ due to photon-Sterile neutrino scattering. To do that we focus on the left-hand side of Boltzmann equation (31) including the information of photon propagation in the flat Friedman-Robertson-Walker (FRW) background space-time. As the circular polarization in presence of scalar perturbation is dominant comparing to vector and tensor perturbation, only the scalar perturbation is added to the metric and we neglect the vector and tensor perturbations. The DM distribution function is indicated as [86–88]:

$$f_N(\vec{x}, \vec{q}, \eta) = f_{N0}[1 + \Psi(\vec{x}, \vec{q}, \eta)],$$

where $f_{N0}(\vec{x}, \vec{q}, \eta)$ shows zeroth-order distribution, $\Psi(\vec{x}, \vec{q}, \eta)$ is perturbed part and $\vec{q} = q \hat{n}'$ where $\hat{n}'$ indicates the direction of dark matter velocity. Neglecting the collision term on the right hand side of the Boltzmann equation, the phase space distribution of Sterile neutrino can be obtain as follows

$$\frac{\partial f_N}{\partial \eta} + i \frac{q}{\varepsilon_N} (\vec{K} \cdot \hat{n}') \Psi + \frac{d \ln f_{N0}}{d \ln q} [\varphi - i \varepsilon_N \frac{q}{\varphi} (\vec{K} \cdot \hat{n}') \psi] = 0,$$

where $\varphi$ and $\psi$ indicate the scalar metric perturbation in the Newtonian gauge [89], $\vec{K}$ is wave number of the Fourier modes of scalar perturbations and $\varepsilon_N = (q^2 + a(\eta)^2 M^2)^{1/2}$ with the scale factor $a(\eta)$. The angular dependence of the perturbation can be expanded in a series of Legendre polynomials $P_l(\mu')$ as follows

$$\Psi(\vec{K}, q, \mu', \eta) = \sum_{l=0} (-i)^l (2l + 1) \Psi_l(\vec{K}, \eta) P_l(\mu'),$$
with $\mu' = \hat{K} \cdot \hat{n}'$. As a result, we expand (31) in terms of $\Psi_l$ and $\mu'$ as follows

$$
\dot{V} \simeq \frac{\sqrt{2}}{3\pi p_0} \alpha \theta^2 G_F \left[ (\eta_B - i \eta_A) \Delta_P^{+}(S) + (\eta_B + i \eta_A) \Delta_P^{-}(S) \right]
$$

$$
\times \frac{4\pi}{3} \left( \frac{1}{(2\pi)^3} \right) \int q^2 \, dq \, q^2 \, \varepsilon_N f_N[\Psi_0 - 2\Psi_2]
$$

$$
\simeq \dot{\eta}_{\text{DM}}^C \Delta_P^+ + \dot{\eta}_{\text{DM}}^C \Delta_P^-,
$$

where

$$
\dot{\eta}_{\text{DM}}^C = \dot{\eta}_{\text{DM}} (\eta_B \pm i \eta_A),
$$

with

$$
\eta_A = \left( \hat{K} \cdot \epsilon_1 \right)^2 - \left( \hat{K} \cdot \epsilon_2 \right)^2, \quad \eta_B = -2 \hat{K} \cdot \epsilon_1 \hat{K} \cdot \epsilon_2,
$$

and

$$
\dot{\eta}_{\text{DM}} = \frac{\sqrt{2}}{3\pi p_0} \alpha \theta^2 G_F \frac{1}{(2\pi)^3} \left[ \delta p_N - (\bar{\rho}_N + \bar{P}_N) \sigma_N \right],
$$

with $\sigma_N$ is shear stress and $\bar{\rho}_N$ and $\bar{P}_N$ are the unperturbed energy density defined as

$$
\bar{\rho}_N = a^{-4} \int q^2 \, dq \, \varepsilon_N f_N, \quad \bar{P}_N = \frac{1}{3} a^{-4} \int q^2 \, dq \, q^2 \, \varepsilon_N f_N,
$$

$$
(\bar{\rho}_N + \bar{P}_N) \sigma_N = \frac{8\pi}{3} a^{-4} \int q^2 \, dq \, q^2 \, \varepsilon_N f_N \Psi_2,
$$

and perturbation of pressure $\delta p_N$ is as follows

$$
\delta p_N = \frac{4\pi}{3} a^{-4} \int q^2 \, dq \, q^2 \, \varepsilon_N f_N \Psi_0,
$$

with

$$
\dot{\Psi}_0 = -\frac{q K}{\varepsilon} \Psi_1 - \phi \frac{d \ln f_N}{d \ln q},
$$

$$
\dot{\Psi}_1 = \frac{q K}{3\varepsilon} (\Psi_0 - 2\Psi_2) + \frac{\varepsilon N K}{3q} \frac{d \ln f_N}{d \ln q},
$$

$$
\dot{\Psi}_l = \frac{q K}{(2l + 1)\varepsilon} (l \Psi_{l-1} - (l + 1) \Psi_{l+1}), \quad l \geq 2.
$$

By inserting the initial condition, one can solve the above evolution equations numerically. Meanwhile, the time averaged value of perturbations $\beta_{\text{DM}} = \frac{\delta p_N}{\bar{\rho}_N} - (\bar{\rho}_N + \bar{P}_N) \sigma$ from last scattering up to today can be estimated at the order of matter anisotropy $\beta_{\text{DM}} \leq 10^{-4}$. In the presence of primordial scalar perturbations the CMB temperature and polarization anisotropy are given by the multi-pole moments as follows

$$
\Delta_{l,P,V}^S(\eta, K, \mu) = \sum_{l=0}^{\infty} (2l + 1)(-i)^l \Delta_{l,P,V}^I(\eta, K) P_l(\mu),
$$

(51)
where $P_l(\mu)$ is the Legendre polynomial of rank $l$, and $\mu$ indicates scalar product of the CMB propagating direction and the wave vector $K$. The time derivative in the left side can include the space-time structure and gravitational effects. Besides, the scattering of each plane wave can be described as the transport through a plane parallel medium \[92,93\]. Therefore, the Boltzmann equation for linear and circular polarization (42) casts into

$$
\frac{d}{d\eta} \Delta_V^{(S)} + iK\mu \Delta_V^{(S)} = C_{eg}^V + \frac{1}{2} a(\eta) (\eta_{DM}^{-} \Delta_P^{+} + \eta_{DM}^{+} \Delta_P^{-}),
$$

$$
\frac{d}{d\eta} \Delta_P^{\pm(S)} + iK\mu \Delta_P^{\pm(S)} = \frac{C_{eg}^{\pm} + i a(\eta) \eta^{P} \Delta_{P}^{\pm}}{\Delta p^{\pm}(S)}.
$$

(52)

The value of linear polarization $\Delta p^{\pm(S)}$ and $\Delta V^{(S)}$ in the direction $\hat{n}$ and at the present time $\eta_0$ is obtained by integrating above Boltzmann equation along the line of sight \[90\] and over all the Fourier modes $K$ as

$$
\Delta_P^{\pm(S)}(\hat{n}) = \int d^3K \xi(K) e^{\mp 2i \phi_{K,n}} \Delta_P^{\pm(S)}(K, p, \eta),
$$

$$
\Delta_V^{(S)}(\hat{n}) = \int d^3K \xi(K) \Delta_V^{(S)}(K, p, \eta),
$$

(53)

where $\xi(K)$ is a random variable utilized to characterize the initial amplitude of the mode, $\phi_{K,n}$ is the angle required to rotate the $K$ and $\hat{n}$ dependent basis to a fixed frame in the sky. Therefore we obtain

$$
\Delta_P^{\pm(S)}(K, \mu, \eta_0) = \int_{0}^{\eta_0} d\eta \eta_{e\gamma} e^{i x \mu - n_{e\gamma} + i \eta_{DM}^{P}} \left[ \frac{3}{4} (1 - \mu^2) \Pi(K, \eta) \right],
$$

(54)

and

$$
\Delta_V^{(S)}(K, \mu, \eta_0) = \frac{1}{2} \int_{0}^{\eta_0} d\eta \eta_{e\gamma} e^{i x \mu - n_{e\gamma} \left[ 3 \mu \Delta_V^{(S)} + (\eta_{DM}^{-} \Delta_P^{+} + \eta_{DM}^{+} \Delta_P^{-}) \right]},
$$

$$
\Delta_P^{(S)}(K, \mu, \eta_0) \approx \frac{1}{2} \int_{0}^{\eta_0} d\eta \eta_{e\gamma} e^{i x \mu - n_{e\gamma}} \left[ 3 \mu \Delta_V^{(S)} + 2 \eta \eta_{DM}^{- \epsilon} \Delta_P^{(S)} \right],
$$

(55)

where $x = K(\eta_0 - \eta)$, $\eta_{e\gamma} = n_e \sigma_T \chi_e$ and $\eta_{e\gamma} = \int_{\eta}^{\eta_0} \eta_{e\gamma} d\eta$ are the differential optical depth and total optical depth due to the Thomson scattering at time $\eta$ with $\chi_e$ being the ionization fraction, respectively. $\Delta P$ is defined as

$$
\Delta_P^{(S)}(K, \mu, \eta) = \frac{3}{4} (1 - \mu^2) \int_{0}^{\eta} d\eta \eta_{e\gamma} e^{i x \mu - n_{e\gamma} \eta_{e\gamma}} \Pi(K, \eta),
$$

(56)

with

$$
\Pi \equiv \Delta P^{S_2} + \Delta P^{S_3} - \Delta P^{S_1}.
$$

(57)

The value of $\eta_{DM}^{e\gamma}$ in \[55\] which obtained by the following ratio

$$
\tilde{\eta} = \frac{\eta_{DM}^{c\gamma}}{\eta_{e\gamma}} = \frac{\sqrt{2}}{8 \pi^2} \frac{m_e^2}{\alpha \chi_e} \frac{m_p}{m_p} \frac{\Omega_{DM}}{\Omega_{BM}} G_F \theta^2 \tilde{\beta}_{DM},
$$

(58)

with $\Omega_N \simeq \Omega_{DM}$, can determine the importance of CMB-Sterile neutrino interaction at CMB polarization. $\Omega_{BM} = \rho_{BM}/\rho_{cr}$ and $\Omega_{DM} = \rho_{DM}/\rho_{cr}$ are the baryonic matter density and the dark matter density parameters, respectively, and $\rho_{cr}$ is the critical density of the universe. In the above equation we supposed that the number density of electron or proton is approximately equal to the
Figure 2: $\tilde{\eta}$ as function of red-shift for mixing $\theta^2 = 10^{-2}$ and $\bar{\beta}_{\text{DM}} = 10^{-4}$.

barionic matter number density $n_e = n_\nu \simeq n_{\text{BM}}$. In Fig. (2), $\tilde{\eta}$ is plotted as function of red-shift for $\theta^2 = 10^{-2}$ and $\bar{\beta}_{\text{DM}} = 10^{-4}$, denoting the importance of CMB-Sterile neutrino interaction on the circular polarization. It can be seen that the maximum value of $\tilde{\eta}$ occurs at red-shift $z \simeq 20$.

In order to study this contribution more accurate, one needs to calculate the total value of the two-point correlation function of the $\Delta V$ mode. To this end, we consider the power spectrum as follows

$$C_{Vl} = \frac{1}{2l+1} \sum_m \langle a_{V,lm}^* a_{V,lm} \rangle$$

$$\simeq \frac{1}{2l+1} \int d^3K P_{\phi}^{(S)}(K) \left| \sum_m \int d\Omega^* Y_{lm}^*(\hat{n}) \int_0^{\eta_0} d\eta \eta e^{i2\mu - \eta e}\eta_B \eta \Delta^{(S)}_P \right|^2$$

(59)

where

$$a_{V,lm} = \int d\Omega Y_{lm}^* \Delta_V(\hat{n}),$$

(60)

and $P_{\phi}^{(S)}(K)$ is the power spectrum \[94\]

$$P_{\phi}^{(S)}(K) \delta(K' - K) = \langle \xi(K)\xi(K') \rangle.$$  

(61)

Therefore, the above relation leads to the following estimation of $C_{Vl}$ in terms of the linearly polarized power spectrum as follows

$$C_{Vl} \leq \tilde{\eta}_{\text{ave}}^2 C_{P_l},$$

(62)

where the $\tilde{\eta}_{\text{ave}}$ is average of the $\tilde{\eta}$ in (58) for $\theta^2 = 10^{-2}$ and $\bar{\beta}_{\text{DM}} = 10^{-4}$, calculated as follows

$$\tilde{\eta}_{\text{ave}} = \frac{1}{\eta_0 - \eta_{\text{ls}}^{\text{ss}}} \int_{\eta_0}^{\eta_{\text{ls}}^{\text{ss}}} d\eta \eta \tilde{\eta} \simeq 3 \times 10^{-4} \left( \frac{\theta^2}{10^{-2}} \right) \left( \frac{\bar{\beta}_{\text{DM}}}{10^{-4}} \right),$$

(63)

Then the value of circular power spectrum will be estimated as follow

$$C_{Vl} \leq 0.01 (nK)^2 \left( \frac{\theta^2}{10^{-2}} \right)^2 \left( \frac{\bar{\beta}_{\text{DM}}}{10^{-4}} \right)^2.$$  

(64)
5 Summary and conclusion

In this paper, we have introduced a new way to examine indirectly the DM signatures. We have considered the right-handed Sterile neutrino as a preferred WDM candidate which can be coupled to the SM particles within the context of the seesaw type I model and the right handed current model as well. We have shown that the polarization of cosmic photons which are naturally accelerated to high energy or even as a background can undergo a change via the forward scattering from the DM Sterile neutrino. For this purpose, we considered the GRB and the CMB as the sources of high and low energy cosmic photons through the formalism of Stockes parameters and Boltzmann equation. We have shown that the linear polarization of GRBs originated from a collapsing neutron star can be converted to the circular polarization by scattering from the DM surrounding the star. We have found that the Faraday conversion $\Delta \phi_{FC}$ of GRB-Sterile neutrino scattering at both the prompt emission and afterglow radiation are about $10^{-33}$ radian and $10^{-17} - 10^{-2}$ radian, respectively. To the leading order, the obtained results are in terms of the active-Sterile neutrino mixing angle $\theta^2$ or right-handed coupling $g_R^2$, and they are equivalent $\theta^2 \approx g_R^2$ and play the same role in the final results. We have summarized the Faraday conversion $\Delta \phi_{FC}$ for $\theta^2 \sim 10^{-2} - 10^{-6}$ in Tab 1. One should note that, here only the maximum value of the Faraday conversion phase shift for GRB-Sterile neutrino scattering using a simple model is estimated. Nevertheless, in order to calculate the exact value, one should consider a more complicated model using the distribution of WDM density in the galaxy and determines the direction of GRBs to the earth. We have also shown that the V-mode power spectrum of the polarized CMB as the low energy cosmic photons in the presence of the scalar perturbations can be expressed in terms of the linear polarization power spectrum. We have indicated that the V-mode power spectrum of the CMB photons $C_{Vl}$ caused by CMB-DM scattering is of the order of $0.01nK^2$, which is in the range of the accuracy of the experimental data [95–98] or will be available in the near future. Finally, we note that to fully interpret circular polarization for cosmic photons, one should take into account all the possible sources. Since there is not any source for producing the circular polarization of the cosmic photons in the SM, detecting any tiny circular polarization can provide an alternative investigation of WDM properties.

6 Acknowledgment

S.Tizchang would like to thank F. Elahi for fruitful discussions.

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