Characterization of arbitrary-order correlations in quantum baths by weak measurement

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The correlations of fluctuations are the driving forces behind the dynamics and thermodynamics in quantum many-body systems. For qubits embedded in a quantum bath, the correlations in the bath are the key to understanding and combating decoherence - a critical issue in quantum information technology. However, there is no systematic method for characterizing the many-body correlations in quantum baths beyond the second order or the Gaussian approximation. Here we present a scheme to characterize the correlations in a quantum bath to arbitrary order. The scheme employs weak measurement of the bath via projective measurement of a central system. The bath correlations, including both the “classical” and the “quantum” parts, can be reconstructed from the correlations of the measurement outputs. The possibility of full characterization of many-body correlations in a quantum bath forms the basis for optimizing quantum control against decoherence in realistic environments, for studying the quantum characteristics of baths, and for quantum sensing of correlated clusters in quantum baths.

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Introduction—The correlations of fluctuations are the driving forces underlying the quantum dynamics and thermodynamic processes (such as critical phenomena) of quantum many-body systems. Conventionally the correlations in many-body physics are considered at the second order or in the Gaussian approximation (which amounts to taking into consideration the quasi-particle excitations around the mean field), with the assumption that the higher order correlations are usually much smaller than the second order ones. Recent studies have revealed the importance of higher order correlations, especially in mesoscopic quantum systems [1–2]. The study of higher-order correlations, however, is challenging due to their many-body nature.

The quantum many-body correlations are particularly important to quantum information technology for their relevance in decoherence of central quantum systems embedded in quantum baths [3–7]. Recently, Gasbarri and Ferialdi [8] show that the dynamics of a central quantum system is determined by the correlations in the quantum bath and the effects of a quantum bath can be fully simulated by complex classical noises. This remarkable work paves the way of optimal quantum control for quantum gates and quantum memory in realistic environments. For classical noises, once the noise spectra are known, the quantum control of the central quantum systems can be designed to combat the decoherence [9–11]. Outstanding examples are dynamical decoupling [12–16] and dynamically optimized quantum gates [17–20]. For quantum baths, the back-action of the central system means that the bath correlations may need to be characterized each time for each new quantum operation and yet the optimization would be extremely time-consuming, if not impossible at all, due to the notoriously difficult quantum many-body problems. Now thanks to the progress in Ref. [8], the quantum control optimization can be applied to quantum baths as well, as long as the bath correlations can be characterized.

Thus, both for studying many-body physics and for applications in quantum information technology, highly desirable is a systematic method to measure the many-body correlations in a quantum bath. With the assumption of Gaussian noises, the noise spectroscopy (e.g., by dynamical decoupling or frequency combing) [21–26] can be employed to obtain the noise correlation spectra. The applications of the frequency comb approach, e.g., to higher-order correlations, however, are tricky due to the interference of nonlinear effects [21–23,24] and spurious signals [27] and yet are often limited to the case of pure dephasing [24].

In this paper, we present a general scheme for completely characterizing the correlations in a quantum bath. The scheme is based on weak measurement of the bath via projective measurement of the central system. By designing the measurement sequence, the bath correlations at arbitrary orders can be reconstructed from the correlations of the measurement outputs. Quantum weak measurement has been used to monitor quantum coherent oscillations [28,29], characterize spectral diffusion [30], and measure the non-symmetric oscillations [31,32]. Multitime correlations of continuous weak measurements have also been studied [33,35]. Recently, weak measurement was considered for improving spectral resolution in quantum sensing [36,37]. The application of weak measurement enabled the high-spectral-resolution magnetic resonance spectroscopy of single nuclear spins [32], which is possible due to the fact that the disturbance to the system caused by the weak measurement (i.e., measurement induced decoherence) is negligible. The weak disturbance feature of
weak measurement is exploited in our scheme of characterizing bath correlations.

Before proceeding to present our scheme, here we first summarize the main results of Ref. [8]. A general Hamiltonian $H = H_0 + V$ is considered, where $H_0 = H_S(t) + H_B$ contains the system Hamiltonian $H_S(t)$ (which may be time dependent due to external control) and the bath Hamiltonian $H_B$, and $V = \sum_n S_n B_n$ is the coupling between the bath operators $B_n$ (the noise fields) and the system operators $S_n$. In the interaction picture,

$$\hat{V}(t) = \sum_n \hat{S}_n(t) \hat{B}_n(t),$$

where the operator in the interaction picture is given by $\hat{A}(t) = U(t)\hat{A} U(t)^\dagger$ with $U(t) = \mathcal{T} e^{-i \int_{t_0}^t H_0(t') dt'}$ and $\mathcal{T}$ denoting time-ordering. The initial state of the system and the bath is assumed to be separable, described by the density operator $\rho(0) = \rho^S(0) \otimes \rho^B$. The density operator in the interaction picture, $\hat{\rho}(t) = U(t)^\dagger \hat{\rho}(0) U(t)$, evolves according to $\hat{\rho}(t) = \mathcal{T} e^{-i \int_{t_0}^t L(t') dt'} \hat{\rho}(t_0)$, with the Liouville superoperator $L$ defined by $L(t)\hat{\rho} = -i\mathcal{V}(t)\hat{\rho} - \hat{\rho}\mathcal{V}(t)$. Defining the superoperators $\hat{\mathcal{A}}\hat{\mathcal{B}} = -i(\hat{\mathcal{A}} \hat{\mathcal{B}} - \hat{\mathcal{B}} \hat{\mathcal{A}})/2$ (essentially a commutator) and $\hat{\mathcal{A}}^*\hat{\mathcal{B}} = (\hat{\mathcal{A}} \hat{\mathcal{B}} + \hat{\mathcal{B}} \hat{\mathcal{A}})/2$ (essentially an anti-commutator) and using the identity $-i\{\hat{\mathcal{A}},\hat{\mathcal{B}}\} = 2(\hat{\mathcal{A}}^*\hat{\mathcal{B}} + \hat{\mathcal{B}}^*\hat{\mathcal{A}})^t$, one obtains the reduced density operator of the central system $\hat{\rho}^S(t) \equiv \text{Tr}_B \hat{\rho}(t)$ determined by the bath field correlations

$$\mathcal{C}^{(N-\eta)\cdots(1-\eta)}_{\eta_1\cdots\eta_N} = \text{Tr} \left[ \mathcal{T} \mathcal{B}^{(N-\eta)\cdots(1-\eta)}_{\eta_1\cdots\eta_N} \mathcal{B}^{(1)\cdots(N)\eta} \mathcal{B}^{(1)\cdots(N)\eta} \right] \rho^B(1),$$

where $\eta_n = -\eta_n$. In terms of the irreducible bath correlations (cumulants) $\mathcal{C}^{(N-\eta)\cdots(1-\eta)}_{\eta_1\cdots\eta_N}$, the central system dynamics can be written as

$$\hat{\rho}^S(t) = \mathcal{T} e^{\frac{i}{\hbar} \int_{t_0}^t dt_1 \mathcal{L}(t_1) \hat{\rho}^S(t_1)} \rho^S(0)$$

Hereafter summation over the repeated indices $\eta_n$ and $\alpha_n$ is assumed. The effects of a quantum bath can be fully simulated by complex classical noises $b_n(t) = b_n^*(t) + ib_n(t)$ that have the correlations $\langle b_n(t) b_m(t') \rangle = \mathcal{C}^{(N-\eta)\cdots(1-\eta)}_{\eta_1\cdots\eta_N}$. The equivalence between a quantum bath and complex classical noises in their effects on central system dynamics offers an interesting venue for studying non-Hermitian quantum dynamics and thermodynamics in complex plane [33,40].

**Measurement of bath correlations** — We present a protocol for measuring the bath correlations to an arbitrary order. The scheme is based on weak measurement of the bath via projective measurement of the central system (see Fig. 1). To measure an $N$-th order correlation, a unit sequence of $N$ weak measurements [Fig. 1(a)] is applied on the quantum bath. In each unit, the quantum bath is prepared in the initial state $\rho^n_B$ at $t = 0$ and then evolves under the bath Hamiltonian $H_B$. At time $t_n$ (for $n = 1, 2, ..., N$), the central system is prepared in the state $\rho^n_S$, and then is coupled to the bath through the interaction $V = \sum_n S_n B_n$ for a small period $\delta t$ of evolution. The state of the central system and the bath, in the interaction picture, becomes $\hat{\rho}(t_n + \delta t) = e^{\mathcal{L}(t_n)\delta t} \hat{\rho}(t_n) \otimes \rho^n_S$, where $L(t) = 2 \sum_n [S_n^2 B_n^2(t) + S_n B_n(t)]$. A quantity $\Lambda_n$ of the central system is measured at $t_n + \delta t$. The output would be randomly an eigenvalue $\lambda_n$ of $\Lambda_n$ corresponding to the eigenstate $|\lambda_n\rangle$. The unit sequence of $N$ measurements is repeated many times. The outputs of the $N$ measurements, averaged over the repeated units, yield the measurement correlation $G^{(N)} = \langle \lambda_N \cdots \lambda_2 \lambda_1 \rangle$.

The projective measurement of the system operator $\Lambda_n$ constitutes a weak measurement of the bath (due to the weak entanglement during the interaction in the small period of time). See Fig. 1(b) for illustration. The weak measurement is characterized by the Kraus superoperator $M_{\lambda_n} = \text{Tr}_S [\lambda_n \langle \lambda_n \rangle e^{\mathcal{L}(t_n)\delta t} \rho^n_S]$, corresponding to the output $\lambda_n$. The probability of the output $\lambda_n$ is $p(\lambda_n) = \text{Tr}_B [M_{\lambda_n} \hat{\rho}^B(t_n)]$. The joint probability of a sequence of $N$ outputs is $p(\lambda_N, ..., \lambda_1) = \text{Tr}_B \left[ T M_{\lambda_N} \cdots \cdots M_{\lambda_1} \rho^B \right]$. The measurement correlation is $G^{(N)} = \sum_{\lambda_1, ..., \lambda_N} p(\lambda_N, ..., \lambda_1) \lambda_{\lambda_1} \cdots \lambda_{\lambda_N}$. For small $\delta t$, the evolution during the interaction $e^{\mathcal{L}(t_n)\delta t} \approx 1 + \hat{\mathcal{L}}(t_n)\delta t$. To pick up the signal proportional to the noise fields $B_n$ (hence proportional to the interaction $\mathcal{L}$), we choose the initial state $\rho^n_S$ and the observable $\Lambda_n$ such that the background term $\text{Tr}_n [\Lambda_n \rho^n_S] = 0$. Thus, the measurement correla-

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**FIG. 1.** Weak measurement for reconstruction of bath correlations. (a) A unit sequence (to be repeated many times) of $N$ weak measurements at different times for reconstructing the bath correlations at the $N$-th order. (b) Realization of a weak measurement on the bath via the projective measurement of the central system. (c) Reconstruction of the bath correlations by selecting a subset of outputs from a long measurement sequence, with the unused outputs in-between taken as “idle” (I).
up to the leading order of $\delta t$ is

$$G^{(N)} \approx \delta t^N \sum A^0_{\alpha_1}(t_\alpha) \cdots A^0_{\alpha_N}(t_N)A^0_{\eta_1}(t_1)C_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N},$$

(4)

where the coefficient

$$A^0_{\alpha}(t_{\alpha}) = 2\text{Tr}_{S} \left[ \Lambda_{\alpha} S^\alpha N\rho^S_n \right].$$

(5)

Equation (4) defines a linear equation for the bath correlations of the $N$-th order. Since the coefficients can be independently set by choosing the system state $\rho^S_n$ and the observable $\Lambda_{\alpha}$, a set of linearly independent equations can be established. By solving the set of linear equations, the bath correlations can be reconstructed.

With the cumulant expansion in Eq. (3), only the irreducible bath correlations are needed. Below we use the shorthand notation $C(N, \ldots, 1) \equiv C^0_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N}$. The irreducible bath correlations (cumulants) $C(N, \ldots, 1)$ can be recursively obtained by $C(1) = C(1), C(2,1) = C(2,1) - C(2)C(1), C(3,2,1) = C(3,2,1) - C(3,2)C(2,1) - C(3)C(2,1)C(1),\ldots$ and so on. The cumulant expansion can often be truncated at a rather low order. In particular, in the case of Gaussian baths (such as a quadratic boson bath [3]), the truncation at the second order irreducible correlations is exact. The truncation approximation would greatly reduce the number of measurements required to reconstruct the bath correlations.

We remark that both the “classical” and the “quantum” parts of the bath correlations can be extracted from the weak measurements. The “quantum” correlations refer to the terms that contain at least one bath superoperator $B^\alpha_{\eta}$ with $\eta_n = +$ (a commutator) and the “classical” correlations contain only bath superoperators with $\eta_n = -$ (anti-commutators). This classification of bath correlations is based on the observation that the commutator $[B^\alpha_{\eta}, H]$ would vanish if $B^\alpha_{\eta}$ is a classical noise field. As shown in Eq. (5), to extract a quantum correlation, one just need to choose the central system state $\rho^S_n$ and observable $\Lambda_{\alpha}$ such that $A^\alpha_{\alpha}(t_{\alpha}) = 2\text{Tr}_{S} \left[ \Lambda_{\alpha} S^\alpha N\rho^S_n \right] \neq 0$. It should be noted that for a bath at infinitely high temperature (such as a nuclear spin bath at room temperature [37]), $\rho^B \propto 1$ and $C^\alpha_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N} = 0$. In this case, one needs to examine at least the third order to extract the quantum correlations in a quantum bath. Such higher-order, “quantum” correlations are signatures of coherent clusters in baths [5]. For example, these signatures can be employed for quantum sensing of correlated nuclear spins in nuclear spin baths [5][41]. The higher-order quantum correlations may also be used to study the quantum characteristics (such as the Leggett-Garg inequality [42]) of quantum baths.

In practice, the protocol for reconstructing the bath correlations can be simplified by exploiting the facts that the perturbation of the weak measurement to the bath is small and the bath is usually in a thermal equilibrium state. One can perform an indefinitely long sequence of weak measurements on the bath at $t_k$ (with, e.g., $t_k = k\tau$ for $k = 1,2,\ldots$). In each shot of measurement, the central system is prepared in state $\rho^S_n$ at $t_k$, coupled to the bath through $V$ for time $\delta t$, and then measured on the observable $\Lambda_{\alpha}$ at $t_k + \delta t$. No preparation of the bath state is needed. See Fig. 1(c) for illustration. The measurement correlations at a given order $N$ and for a given timing $(t_1, t_2, \ldots, t_N)$ are obtained by selecting a subset of the measurements. The data from the other measurements (taken as idle) are discarded (but would be used for constructing correlations at other orders and/or for other timings). For the measurement whose output $\Lambda_{\alpha}$ is discarded, the evolution of the bath averaged over all possible outputs, is $\sum_{\eta} M_{\eta} \rho^B(t_k) \approx M_{\eta} \rho^B(t_k)$, which amounts to measurement-induced decoherence. If the measurement is weak ($|V\delta t| \ll 1$), the measurement-induced decoherence is negligible, i.e., $M_{\eta} \approx 1$ and $M_{\eta} \rho^B(t_k) \approx \rho^B(t_k)$. Furthermore, if the bath is in the thermal equilibrium state $\rho^B \propto e^{-H_B/(k_B T)}$ at temperature $T$, the bath Hamiltonian $H_B$ induces no evolution on it. Therefore, under the conditions that the bath is initially in a thermal equilibrium state and the measurement is sufficiently weak, the measurement correlations extracted from a subset of the measurements are the same, in the leading order of $\delta t$, as those obtained without the idle measurements [that is, the same as those in Eq. (3)]. In this simplified protocol, the sequential weak measurements can be carried out with a simple timing (e.g., equally spaced in time), there is no need to prepare the bath state in each unit sequence of measurement, and the output data can be reused for constructing correlations at different orders and for different timings [37].

Special case of central spin-1/2 — As an example, we present the explicit protocol for reconstructing the correlations in a quantum bath of a central spin-1/2 (qubit). The qubit-bath coupling can be written as $V = \frac{1}{2} \sum_{\alpha=x,y,z} \sigma^\alpha N \cdot B_{\alpha}$, where $\sigma^\alpha N$ is the Pauli matrix of the qubit along the $\alpha$-axis and $B_{\alpha}$ is the magnetic noise operator. Without loss of generality, we assume $t_N > t_{N-1} > \cdots > t_1$ in the correlation functions.

Let us consider the weak measurement of the bath at $t_1$ first. The central spin is polarized to be along, e.g., the $x$-axis, described by the density operator $\rho^S = (1 + \sigma_x)/2$ at $t = t_1$. After the interaction with the bath through $V$ for time $\delta t$, a spin operator $\Lambda_1$ is measured. To make the background term $\text{Tr} \left[ \Lambda_1 \rho^S \right]$ vanish, we choose the measurement axis to be along a direction perpendicular to the initial polarization, e.g., $\Lambda_1 = \sigma_y$. With the definition in Eq. (5), the coefficient in Eq. (4) becomes

$$A^+_1(t_1) = \text{Tr} \left[ \sigma_y \left( \sigma_x \rho^S + \rho^S \sigma_x \right) \right] = 1,$$

$$A^-_1(t_1) = -i \text{Tr} \left[ \sigma_y \left( \rho^S \sigma_x - \sigma_x \rho^S \right) \right] = 1,$$

and else $= 0$. Therefore the measurement correlation becomes

$$G^{(N)} = \delta t^N A^y_{\alpha_1}(t_N) \cdots A^y_{\alpha_N}(t_2) \left( C^{y \cdots y, \cdots y}_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N} + C^{\eta \cdots \eta, \cdots \eta}_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N} \right).$$

Or if we choose $\rho^S = (1 - \sigma_x)/2$ (central spin initially polarized along the $-x$ direction) and $\Lambda_1 = \sigma_y$, we have $A^+_1(t_1) = -A^-_1(t_1) = 1$ and else $= 0$. The measurement correlation would be

$$G^{(N)} = \delta t^N A^y_{\alpha_1}(t_N) \cdots A^y_{\alpha_N}(t_2) \left( C^{y \cdots y, \cdots y}_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N} - C^{\eta \cdots \eta, \cdots \eta}_{\eta_1 \cdots \eta_N, \alpha_1 \cdots \alpha_N} \right).$$
The summation and difference of $G^{(N)}$ and $G^{(N)}$ pick up the
bath correlations $C_{t_0\ldots t_{N-1}}^{a_N\ldots a_1}$ and $C_{t_0\ldots t_{N-1}}^{a_N\ldots a_1}$, respectively. That is
\begin{align}
G^{(N)} + G^{(N)} &= 2\delta t_N^N A_{t_N}^{a_N}(t_N) \cdots A_{t_1}^{a_1}(t_1) C_{t_0\ldots t_{N-1}}^{a_N\ldots a_1} \tag{6a} \\
G^{(N)} - G^{(N)} &= 2\delta t_N^N A_{t_N}^{a_N}(t_N) \cdots A_{t_1}^{a_1}(t_1) C_{t_0\ldots t_{N-1}}^{a_N\ldots a_1} \tag{6b}
\end{align}

The procedure can be similarly applied to the measurements at other times. For the latest time $t_N$, since the correlation function vanishes for $\eta_N = -\nu$, only one set of $(\rho_N^S, \Lambda_N)$ is needed to pick up the correlation function $C_{a_N\ldots a_1}^{\eta_N\ldots \eta_1}$. Thus, using measurement correlation functions for $2^{N-1}$ configurations of central spin initialization and measurement directions $\{\rho_N^S, \Lambda_N\}$, one can determine $2^{N-1}$ bath correlation functions $C_{a_N\ldots a_1}^{\eta_N\ldots \eta_1}$ with $a_n = y$ or $z$ corresponding to $\eta_n = -\nu$ or $+$ for each $n$. The correlations of noise fields along different directions can be similarly determined (e.g., correlations with $(a_n, \eta_n) = (x/2z, \pm z)$ can be extracted from measurements with $\rho_n = (1 \pm \sigma_z)/2$ and $\Lambda_n = \sigma_z$). For example, the third-order correlation (for $t_3 > t_2 > t_1$)
\begin{equation}
C_{x,y,z}^{+++} = \left(G_{x,y,z}^{+++} + G_{x,y,z}^{+++} - G_{x,y,z}^{+++} \right)/\left(4\delta t^3\right),
\end{equation}
where $G_{x,y,z}^{+++}$ denotes the measurement correlations for the central spin initialized along $x$ and measured along $y$ at $t_1$, initialized along $-x$ and measured along $y$ at $t_2$, and initialized along $y$ and measured along $z$ at $t_3$ (similarly for $G$ with other indices).

Among different types of decoherence, pure dephasing is often the most relevant to quantum information technology since it does not involve the slow energy dissipation process. For pure dephasing, the qubit-bath coupling assumes the form $V(t) = S_B(t)$. The qubit dynamics is determined by the bath correlations as
\begin{equation}
\rho^S(t) = \sum_{N=0}^{\infty} \frac{2^N}{N!} \int_0^t dt_N \cdots dt_1 C_{t_{-N}t_1}^{+++} S_z^{(t_N)} \cdots S_z^{(t_1)} \rho^S(0).
\end{equation}
Here we have used the fact that $C_{t_{-N}t_1}^{\eta_N\ldots \eta_1} = 0$ and $S_z^{(t_N)} = 0$. In the case of pure dephasing, the effects of quantum bath is fully determined by the correlation $C_{t_{-N}t_1}^{+++}$, which is directly related to the weak measurement correlations through, e.g.,
\begin{equation}
C_{x,y,z}^{+++} = C_{x,y,z}^{+++} /\delta t^3.
\end{equation}
Here we have used the fact that in the pure dephasing case, $C_{x,y,z}^{+++} = C_{x,y,z}^{+++} = C_{x,y,z}^{+++} = 0$ (for $B_x = 0$) and therefore $G_{x,y,z}^{+++} = -G_{x,y,z}^{+++} = -G_{x,y,z}^{+++}$ [according to Eq. (7)]. It should be noted that even though the pure dephasing is determined only by the “classical” bath correlations $C_{t_{-N}t_1}^{+++}$ the “quantum” correlations (those that contain at least one commutator) can still be measured by weak measurements. For example,
\begin{equation}
C_{x,y,z}^{+++} = C_{x,y,z}^{+++} /\delta t^3.
\end{equation}

**Conclusion** — We propose a general scheme for complete characterization of arbitrary order correlations in a quantum bath, based on weak measurement of the bath realized by projective measurement of a central system embedded in the bath. From the weak measurement correlations at the $N$-th order, one can reconstruct the $N$-th order bath correlations. The weak measurement has the advantage of negligible disturbance (i.e., measurement-induced decoherence) to the bath — this advantage allows the measurement data be collected at a simple timing and the correlations be extracted by selecting certain subsets of the data, which greatly reduces the time consumption for reconstructing the correlation functions [37]. Once the bath correlations are characterized, they can be used for optimizing quantum controls under all circumstances [9–11] [17–20]. Characterizing arbitrary-order correlations in quantum baths may provide an approach to studying the quantum characteristics (such as the Leggett-Garg inequality [42]) of many-body environments and enable quantum sensing of nuclear spin clusters of different types of correlations [5–41]. We expect that the experimental demonstration of the protocol is feasible in solid spin systems such as nitrogen-vacancy center spins [43], donor spins in silicon [44] and quantum dots [45].

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