Strings, unification and dilaton/moduli- induced
SUSY-breaking

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Abstract

I discuss several issues concerning the use of string models as unified theories of all interactions. After a short review of gauge coupling unification in the string context, I discuss possible motivations for the construction of $SU(5)$ and $SO(10)$ String-GUTs. I describe the construction of such String-GUTs using different orbifold techniques and emphasize those properties which could be general. Although $SO(10)$ and $SU(5)$ String-GUTs are relatively easy to build, the spectrum below the GUT scale is in general bigger than that of the MSSM and includes colour octets and $SU(2)$ triplets. The phenomenological prospects of these theories are discussed. I then turn to discuss soft SUSY-breaking terms obtained under the assumption of dilaton/moduli dominance in SUSY-breaking string schemes. I underline the unique finiteness properties of the soft terms induced by the dilaton sector. These improved finiteness properties seem to be related to the underlying $SU(1,1)$ structure of the dilaton couplings. I conclude with an outlook and some speculations regarding $N = 1$ duality.

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1 Introduction

Heterotic 4-D strings are considered today our best candidates for the construction of unified theories of all interactions including gravity. In spite of that, most of the effort in string theory has been devoted to understand the theory itself rather than to explore whether indeed one can really unify the standard model and gravity into a unique consistent framework. I discuss in this talk some of the ideas considered recently in this direction, using strings as unified theories. This is not supposed to be a general review and I will only discuss topics in which I have been more or less involved. These include gauge coupling unification, string-GUTs and soft SUSY-breaking terms from dilaton/moduli-induced SUSY-breaking. The las two sections include some discussion about the special properties of the the soft terms implied by dilaton-induced SUSY-breaking and an outlook including some speculations.

2 Some thoughts about gauge coupling unification

In 4-D heterotic strings the strength of both gauge and gravitational coupling constants is governed by the vev of the dilaton, $< Re S > = \frac{4 \pi}{g^2}$. Consider for example an hypothetical 4-D string whose gauge group contains the SM group. The corresponding SM couplings would be related as

\[ g_2^2 k_2 = g_3^2 k_3 = g_1^2 k_1 = G_{\text{Newton}} M_{\text{String}}^2 = \frac{4 \pi}{Re S}. \]  

(1)

at the string scale $M_s \approx 5.39 \times 10^{17} \text{GeV}$. Here $k_2, k_3$ are positive integers (the "levels" of the $SU(2)$ and $SU(3)$ algebras) and $k_1$ is a rational number which gives the normalization of the $U(1)_Y$ weak hypercharge. If the corresponding 4-D string was constructed by a simple compactification from either of the two supersymmetric 10-D heterotic strings, one always has $k_2 = k_3 = 1$ whereas $k_1$ is a model-dependent rational number. In this way one has at the string scale $\sin^2 \theta_W = 1/(1 + k_1)$ and $\alpha_s = (1 + k_1) \alpha_e$, where $\alpha_{s,e}$ are the strong and electromagnetic fine-structure constants. The standard $SU(5)$ GUT predictions are recovered for the choice $k_1 = 5/3$. As a first try one can assume this value for $k_1$ and compute $\sin^2 \theta_W$ and $\alpha_s$ at the weak scale $M_Z$ by running the renormalization group equations (r.g.e.) down to low energies. Assuming that the only massless particles charged under the SM are those of the minimal supersymmetric SM (MSSM) one finds $\sin^2 \theta_W = 0.218$ and $\alpha_s = 0.20$, several standard deviations away from the experimental values $\sin^2 \theta_W(M_Z) = 0.233 \pm 0.003$ and $\alpha_s(M_Z) = 0.11 \pm 0.01$. Is this a serious problem for the idea of a direct string unification of the SUSY standard model?

One may argue that, for an $SU(3) \times SU(2) \times U(1)$ string the value of $k_1$ should be considered as a free parameter. If we indeed take $k_1$ as a free parameter one can find much more successfull results for the coupling constants. In particular, for $k_1 = 1.4$ one gets $\sin^2 \theta_W(M_Z) = 0.235$ and $\alpha_s(M_Z) = 0.13$, in not unreasonable agreement with experiment. In fact it is amusing to note that, if the historical order of theoretical ideas would have been slightly different, the joining of coupling constants could have been considered an outstanding success of string theory! Indeed, imagine the stringers of the early seventies would have discovered the supersymmetric heterotic strings before GUTs (and SUSY-GUTs) would have been
introduced. Georgi, Quinn and Weinberg would have told them how to extrapolate the couplings down to low energies. However they would not have had any prejudice concerning the value of \( k_1 \) and they would have taken it as a free parameter. Then they would have found that for \( k_1 = 1.4 \) one gets the above successful results. This could have been then interpreted as a great success of string theory! (In fact even the results stated above for \( k_1 = 5/3 \) would have been considered rather successful given the experimental precision in the late seventies).

Let us forget now about virtual history and come back to the question whether direct string unification of the supersymmetric SM is or not a problem. Leaving \( k_1 \) as a free parameter is certainly a possibility. It is true however that up to now nobody has constructed any string model with \( k_1/k_2 < 5/3 \), which is what it seems to be required. For example, in the \( k = 1 \) orbifold models constructed up to now one has \( k_1 \geq 5/3 \). \( E_6 \)-like models (\( (2,2) E_8 \times E_8 \) compactifications) have \( k_1 = 5/3 \) and often bigger results are obtained for \( (0,2) \) orbifold models. In fact, straightforward compactifications (e.g., level=1) of the heterotic strings are likely to yield always \( k_1 \geq 5/3 \), but this is not necessarily the case in all generality. In particular, in higher KM level models one can think of a value \( k_1/k_2 \leq 5/3 \) even though \( k_1 > 5/3 \).

It would be very interesting to find model-independent constraints on \( k_1 \) or else find examples with \( k_1/k_2 \leq 5/3 \).

There are of course, other alternatives to understand the disagreement found for the joining of coupling constants. The infinite massive string states can give rise to substantial one-loop corrections [3, 8] to the gauge coupling constants (string threshold corrections). Indeed this possibility has been studied and the potential exists for such corrections to explain the discrepancy [4, 9]. Each coupling gets one-loop corrections as

\[
\frac{4\pi}{g_i^2(M_s)} = k_i ReS + \Delta_{Ih}^i, \quad i = 1, 2, 3
\]

where \( ReS = 4\pi/g_i^2 \) and \( \Delta_{Ih}^i \) are the threshold corrections. There are two type of threshold corrections: field-independent and field-dependent. The first of these are expected to be small and indeed this smallness has been confirmed in explicit computations for some \((0,2)\) orbifold examples [3, 11]. The field-dependent threshold corrections may on the contrary be large depending on the values of the fields. The fields relevant in this case are those related to marginal deformations of the underlying CFT like the moduli \( T_a, U_a \) fields. These fields parametrize the size (R) and shape of the six-dimensional compactification variety (orbifold, Calabi-Yau manifold). The corresponding threshold functions \( \Delta_{ih}(T_a, U_a, U_a^*) \) have been computed in a variety of 4-D strings [3, 8, 10, 11]. It seems to be a common feature that for large compactification radius \( R^2 \) one gets for all the examples studied a leading correction of the form [12]

\[
\Delta^i = \frac{b'_i}{12} R^2
\]

where the \( b'_i \) are model-dependent constant coefficients [8]. Now one can see that for \( k_1 = 5/3 \) and not too large values of \( R \) (e.g. \( R \simeq 2 - 4 \)) one can obtain good results for \( \sin^2 \theta_W \) and \( \alpha_s \) by appropriately choosing the \( b'_i \) coefficients [11, 13]. In fact this result is less trivial than it sounds since these \( b'_i \) coefficients are numbers which may be computed in specific (orbifold) models in terms of the quantum numbers and "modular weights" of the massless field of the theories. It was found in particular
that only a quite limited class of orbifold models \cite{9} could possibly have adequate threshold corrections. A random distribution of modular weights for the SM particles will in general lead to threshold corrections in the wrong direction. So even though indeed threshold corrections may be large there is no reason for them to conspire precisely in the way we want.

Another obvious alternative is to give up the idea of a direct string unification of the SM involving just the content of the MSSM, i.e., to consider the presence of extra massless particles in addition to those of the SM \cite{5,13}. After all, the explicit 4-D string models constructed up to now have always plenty of additional stuff! The problem with this is that we open a Pandora box of virtually unlimited possibilities in which we, rather than predicting the weak angle, are just adjusting it. A second problem with this is that explicit computations of the running couplings for different possible choices of extra matter fields have shown that there is no possible choice of extra matter fields which yields direct unification at the string scale \textit{new intermediate scale thresholds are necessary} \cite{5,13,2}. Thus we lose the beauty of direct string unification altogether. But, if we have to deal with extra intermediate scales, why not considering the possibility of GUTs themselves which naturally require a scale of order $10^{16}$ GeV?

3 Are there motivations for constructing GUTs from strings?

Standard SUSY-GUTs like $SU(5)$ and $SO(10)$ predict the unification of coupling constants at a scale $M_X$ which is a free parameter. This allows for the computation of one coupling constant as a function of the others. One finds $\alpha_s(M_Z) = 0.12$ for $\sin^2\theta_W(M_Z) = 0.233$ by chosing $M_X = 2 \times 10^{16}$ GeV, in good agreement with experiment. Given this success, in principle much better than the one obtained from direct string unification of the MSSM, it is natural to try and embed standard SUSY-GUTs into string theory. This is obviously an important motivation. However, I must remark that in order for a SUSY-GUT to yield the above nice prediction for $\alpha_s$, it is crucial the assumption that bellow the string scale the only particles present in the spectrum are those of the MSSM. So unification into a simple group like $SU(5)$ or $SO(10)$ is not enough, the breaking of those groups has to be such that the remaining low energy theory is the MSSM.

One could also consider as an argument in favour of GUTs the nice way in which the observed SM generations fit into representations of the $SU(5)$ or $SO(10)$ groups. A random 4-D string with $SU(3) \times SU(2) \times U(1) \times G$ group typically contains extra vectorlike heavy leptons or quarks. These extra particles are in fact chiral with respect to the extended group $G$ and remain light as long as the extended symmetries are unbroken. Furthermore, these extra leptons and quarks are not guaranteed to have integer charges or to obey the usual charge quantization of the SM particles. In order to obtain (level=1) $SU(3) \times SU(2) \times U(1) \times G$ string models in which the light fermions are just ordinary SM generations with standard charge asignements the simplest way is to start with heterotic strings with some underlying (level=1) $SU(5)$, $SO(10)$ or $E_6$ symmetry which is broken down to some smaller group \textit{at the string scale} through the Hosotani-Witten flux-breaking mechanism. Notice that these models are not GUTs, because the symmetry breaking is not carried out through a Higgs mechanism (i.e., through an adjoint vev) and also because the GUT-
like symmetries are never realized as GUT symmetries at any scale, they constitute just an intermediate step in the construction technique. The actual gauge symmetry is the one of the SM (or some simple extension) and hence they are just SM strings of a particular class. Anyway, if we need to have at some level some GUT-like structure (although not realized as a complete gauge symmetry) it is reasonable to try and study whether symmetries like \( SU(5) \) and \( SO(10) \) may be promoted to a complete symmetry of the massive spectrum. This would be the essence of string GUTs.

There are other features of SUSY-GUTs which their practitioners love, like the prediction for the \( m_b/m_\tau \) ratio and other fermion mass relationships; predictions for proton-decay and lepton-number violating processes etc. These are more model dependent and may also be present in string models so they would not constitute by themselves motivations to construct string GUTs. On the other hand it would be useful to construct GUTs from strings to check whether the dynamical assumptions that the GUT practitioners assume are or not natural within the context of strings. So one can try to extract some selection rules to constrain the rules of SUSY-GUT model-building. I will briefly discuss some of these in the next section.

4 SUSY-GUTs from strings

It is essential for SUSY-GUTs the existence in the spectrum of chiral fields (e.g., adjoints) appropriate to induce the breaking of the gauge symmetry down to the standard model. In the context of N=1, 4-D strings this is only possible if the affine Lie algebra associated to the GUT symmetry is realized at level \( k \geq 2 \). Straightforward compactifications of the supersymmetric heterotic strings have always \( k = 1 \) algebras which they inherit from the \( k = 1 \, E_8 \times E_8 \) or \( Spin(32) \, D = 10 \) heterotic strings. To obtain 4-D strings with higher level one has to go beyond simple compactifications of the heterotic strings. At the beginning it was thought that such higher level models would be very complicated to construct. This is why in the early days of string model-building there were no attempts at the construction of string-GUTs. Only a few papers dealt with the explicit construction of 4-D strings with affine Lie algebras at higher levels \([14, 15]\).

In the last year there have been renewed attempts for the construction of string GUTs at \( k = 2 \) using orbifold \([16]\) and free fermion techniques \([17]\). The first of these methods is relatively easy and is the one I am more familiar with. Furthermore world-sheet supersymmetry (which is quite a technical difficulty in fermionic models \([18]\)) is guaranteed by construction. Here I will thus discuss mostly results obtained using the (symmetric) orbifold methods of ref. \([16]\), although many of the conclusions may be easily extended to other 4-D string constructions.

The general idea is the following \([16]\). One starts with a \((0,2)\) orbifold compactification of the 10-D heterotic string. It turns out that it is convenient to start with the \( Spin(32) \) heterotic (instead of \( E_8 \times E_8 \)). Models in which the gauge group has the structure \( G_{GUT} \times G' \), where \( G' \) in turn contains as a subgroup a copy of \( G_{GUT} \) (i.e. \( G_{GUT} \in G' \)) are searched for. We would just have at this point a usual level \( k = 1 \) \((0,2)\) orbifold model with a particular gauge structure. Now we do some kind of modding or projection (to be specified below) in such a way that only gauge bosons corresponding to the diagonal \( G_{GUT}' \times G_{GUT}' \) subgroup of \( G_{GUT} \times G_{GUT} \) survive in the massless spectrum. We are thus left with a structure of type \( G_{GUT}' \times G'' \) where
is realized at $k = 2$.

In fact this scheme is quite general and may be implemented in other classes of (0, 2) 4-D string constructions. For example, it may be used starting with the class of (0, 2) models obtained by adding gauge backgrounds and/or discrete torsion to Gepner and Kazama-Suzuki models. The final step leading to the $k = 2$ group may be achieved by embedding an order-two symmetry ($\gamma$ in the notation of [19]) by a permutation of $G_{GUT}$ with $G'_{GUT}$.

In the case of orbifolds, three methods in order to do the final step $G_{GUT} \times G'_{GUT} \rightarrow G^D_{GUT}$ were discussed in refs. [15]. In the first method (I) the underlying $k = 1$ model with the $G_{GUT} \times G'$ structure is obtained by embedding the twist action of the orbifold into the gauge degrees of freedom by means of an automorphism of the gauge lattice (instead of a shift). In $k = 1$ models of this type one can have "continuous Wilson-line" backgrounds [20, 21] which can be added in such a way that the symmetry is broken continuously to the diagonal subgroup $G^D_{GUT}$. In the second method (II) one does the final step by modding the original model by a $Z_2$ twist under which the two groups $G_{GUT}$ and $G'_{GUT} \in G'$ are explicitly permuted. The third method (III) is field-theoretical. One explicitly breaks the original symmetry down to the diagonal subgroup by means of an ordinary Higgs mechanism. Although these three methods look in principle different, there are many $k = 2$ models which can be built equivalently from more than one of the above methods.

Giving all the details of these constructions here would be pointless. Let me just explain a few features. There are a few of them which are quite general due to their, in some way, kinematical origin. Consider the mass formula for the left-moving (bosonic) string states from any 4-D toroidal orbifold model:

$$\frac{1}{8}M_L^2 = N_L + h_{KM} + E_0 - 1.$$  (4)

Here $N_L$ is the left-moving oscillator number, $h_{KM}$ is the contribution of the KM gauge sector to the conformal weight of the particle and $E_0$ is the contribution of the internal (compactified) sector to the conformal weight.

Let us consider the case of symmetric (0, 2) Abelian orbifolds. All Abelian $Z_N$ and $Z_N \times Z_M$ orbifolds may be obtained by toroidal compactifications in which the 6 (left and right) compactified dimensions are twisted. There are just 13 possible orbifold twists [22] which can be characterized by a shift $v = (v_1, v_2, v_3)$, where $e^{2i\pi v_i}$ are the three twist eigenvalues in a complex basis. A consistent symmetric orbifold model is obtained by combining different twisted sectors in a modular invariant way. This procedure is well explained in the literature [22, 23]. To each possible twisted sector there corresponds a value for $E_0$ given by the general formula:

$$E_0 = \sum_{i=1}^{3} \frac{1}{2}|v_i|(1 - |v_i|).$$  (5)

Notice also that $E_0 = 0$ for the untwisted sector which is always part of any orbifold model. In the case of asymmetric orbifolds [24], obtaining $N = 1$ unbroken SUSY allows the freedom of twisting the right-movers while leaving untouched the (compactified) left-movers. In this case one can then have $E_0 = 0$ even in twisted sectors.

Let us go now to the other relevant piece in eq. (4), namely the contribution $h_{KM}$ of the KM sector to the conformal weight of the particle. A state in a representation
\((R_1, R_2, \cdots)\) will have a general weight

\[ h_{KM} = \sum_i C(R_i) \frac{k_i}{k_i + \rho_i} \tag{6} \]

Here \(C(R)\) is the quadratic Casimir of the representation \(R\). \(C(R)\) may be computed using \(C(R) \text{dim}(R) = T(R) \text{dim}G\), where \(T(R)\) is the index of \(R\). Unless otherwise explicitly stated, we use the standard normalization in which \(T = 1/2\) for the \(N\)-dimensional representation of \(SU(N)\) and \(T = 1\) for the vector representation of \(SO(2N)\). With this normalization, for simply-laced groups the Casimir of the adjoint satisfies \(C(A) = \rho\). The contribution of a \(U(1)\) factor to the total \(h_{KM}\) is instead given by \(Q^2/k\), where \(Q\) is the \(U(1)\) charge of the particle and \(k\) is the normalization of the \(U(1)\) generator, abusing a bit it could be called the level of the \(U(1)\) factor. Formula (6) is very powerful because the \(h_{KM}\) of particles can be computed without any detailed knowledge of the given 4-D string. This information is a practical guide in the search for models with some specific particle content.

Using eq. (4) and the values for \(E_0\), \(h_{KM}\) computed through eq. (5), (6) we can learn, for instance, what \(SU(5)\) or \(SO(10)\) representations may appear in the massless spectrum of any possible twisted sector of any given Abelian orbifold. In the case of these groups we are interested in knowing which twisted sectors may contain 24-plets or 45 and 54-plets respectively.

For a 24-plet one has \(h_{KM} = 5/7\); for \(SO(10)\) 45-plets one has \(h_{KM} = 4/5\) and, finally, for \(SO(10)\) 54-plets one has \(h_{KM} = 1\). From the condition \(h_{KM} + E_0 \leq 1\) one draws the following conclusions:

i. All those representations may be present in the untwisted sector of any orbifold.

ii. 54s of \(SO(10)\) \((k = 2)\) can only be present in the untwisted sector of symmetric orbifolds.

iii. 45s of \(SO(10)\) \((k = 2)\) may only appear either in the untwisted sector or else in twisted sectors of the type \(v = 1/4(0, 1, 1)\) or \(v = 1/6(0, 1, 1)\). This is a very restrictive result since Abelian orbifolds containing these shifts are limited.

iv. 24-plets of \(SU(5)\) can never appear in the twisted sectors of the \(Z_3, Z_4, Z_6'\) and \(Z_8\) orbifolds.

From the above conclusions it transpires that looking for models with GUT-Higgs fields in the untwisted sector should be the simplest option, since they can always appear in any orbifold. This option has another positive aspect in that the multiplicity of a given representation in the untwisted sector is never very large, it is always less or equal than three in practically all orbifolds and is normally equal to one in the case of \((0, 2)\) models. Proliferation of too many GUT-Higgs multiplets will then be avoided.

To be specific let us present a string \(SO(10)\)-GUT constructed from the first of the three orbifold methods mentioned above, i.e., the “continuous Wilson-line” method. In this method the orbifold twist \(\theta\) is realized in the gauge degrees of freedom in terms of automorphisms \(\Theta\). In the absence of Wilson line backgrounds \(L_i\), the action of \(\Theta\) can be described by an equivalent shift \(V\). In the presence of \(L_i\), the embedding is non-Abelian when \(\Theta L_i\) does not give back \(L_i\) up to lattice vectors. When embedding by automorphisms, not all Cartan gauge currents are given by combinations of derivatives of the 16 bosonic coordinates \(\partial F_I\) since the lattice coordinates \(F_I\) are generically rotated by \(\Theta\) and the unbroken gauge currents
must be invariant under $\Theta$. The Cartan sub-algebra, as well as the step currents, now arise from $\Theta$ invariant orbits of the $e^{iP\cdot F}$ operators of the form

$$|P\rangle + |\Theta P\rangle + \cdots + |\Theta^{N-1}P\rangle$$

where $|P\rangle \equiv e^{iP \cdot F}$ and $P^2 = 2$. After the continuous Wilson lines are turned on, states not satisfying $P \cdot L = \text{int}$ drop out from the spectrum. This projection kills some Cartan generators thus forcing a reduction of the rank of the gauge group. This is a necessary condition to get a residual affine Lie algebra realized at higher level.

Consider [16] the simplest symmetric orbifold with order 2 symmetries, namely, $Z_2 \times Z_2$. The internal six-dimensional twists $\theta$ and $\omega$ may be embedded into the gauge degrees of freedom by the order two automorphisms $\Theta$ and $\Omega$ defined by:

$$\Theta(F_1, F_2, \cdots, F_{16}) = (-F_1, -F_2, \cdots, -F_{10}, -F_{11}, -F_{12}, F_{13}, F_{14}, F_{15}, F_{16})$$

$$\Omega(F_1, F_2, \cdots, F_{16}) = (-F_1, -F_2, \cdots, -F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16})$$

(8)

The unbroken gauge currents correspond to states $|P\rangle$ with $P$ invariant plus the oscillators $\partial F_{15}, \partial F_{16}$. Also, from non-invariant $P$’s we can form orbits invariant under both $\Theta$ and $\Omega$. Altogether we find 200 currents that can be organized into an $SO(10) \times SO(18) \times U(1)^2$ algebra realized at level $k = 1$.

Next we turn on a Wilson line background $L$ along, say, the compactified direction $e_6$. $L$ has the form

$$L = (\lambda, \lambda, \lambda, \cdots, \lambda, 0, 0, 0, 0, 0, 0)$$

(9)

The parameter $\lambda$ can take any real value since $L$ is completely rotated by both $\Theta$ and $\Omega$. The gauge group is broken to $SO(10) \times SO(8) \times U(1)^2$. The currents associated to $SO(10)$ are given by

$$|+1, -1, 0, 0, \cdots, 0, 0, 0, 0, 0, 0, 0\rangle + |-1, +1, 0, 0, \cdots, 0, 0, 0, 0, 0, 0\rangle$$

(10)

where underlining means that all possible permutations must be properly considered. One can check that $SO(10)$ is realized at level $k = 2$ whereas $SO(8)$ has $k = 1$.

In the untwisted matter sectors $U_1, U_2$ and $U_3$, the corresponding left-moving vertices transform under $(\Theta, \Omega)$ with eigenvalues $(-1, 1), (1, -1)$ and $(-1, -1)$ respectively. The momenta involved must also satisfy $P \cdot L = \text{int}$. In sectors $U_1$ and $U_2$ there are matter fields transforming as $(1, 8)$ and with different $U(1)$ charges. In the $U_3$ sector we find the states

$$\partial F_I, \ I = 1, \cdots, 10$$

$$|+1, -1, 0, 0, \cdots, 0, 0, 0, 0, 0, 0, 0\rangle - |-1, +1, 0, 0, \cdots, 0, 0, 0, 0, 0, 0\rangle$$

(11)

These states have no $U(1)^2$ charges and belong to a $(54, 1) + (1, 1)$ representation of $SO(10) \times SO(8)$. In $U_3$ we also find four extra singlets, charged under the $U(1)$'s only. Altogether the spectrum of this GUT model is given in Table [1]. The charge $Q$ is non-anomalous whereas $Q_A$ is anomalous. The gravitational, cubic and mixed gauge anomalies of $Q_A$ are in the correct ratios in order to be cancelled by the 4-D version of the Green-Schwarz mechanism [23]. The degeneracies of the twisted sectors $\theta, \omega$ and $\theta \omega$ depend on the way one realizes the $Z_2 \times Z_2$ twist in the compactifying cubic
Table 1: Particle content and charges of the string-GUT example discussed in the text. The three rightmost columns display three examples of consistent soft masses from dilaton/moduli SUSY breaking.
lattice (see ref. [16] for details). This model has 4 $SO(10)$ generations and two pairs of $16 + \overline{16}$ Higgs fields plus additional 10-plets.

There is an interesting feature which turns out to be quite generic in $SO(10)$ string GUTs obtained from this method (I). In the 0-picture the full emission vertex operator for the singlet in $U_3$ has the form

$$
\partial X_3 \otimes \sum_{I=1}^{10} \partial F_I
$$

A Vev for this field precisely corresponds to the Wilson line background $L$ in eq. (9). The fact that this background may be varied continuously means that this singlet is a string modulus, a chiral field whose scalar potential is flat to all orders. Indeed, using the discrete $Z_2$ R-symmetries of the right-handed sector, it can be proven that its self-interactions vanish identically. The GUT Higgs contains the other 9 linear combinations of $\partial F_I$. These give the diagonal elements of the symmetric traceless matrix chosen to represent the 54-plet. The associated vertex operator is

$$
\partial X_3 \otimes \sum_{I=1}^{10} c_I \partial F_I ; \quad c_I \in \mathbb{R}, \quad \sum_I c_I = 0 .
$$

Vevs for these nine components of the 54 would correspond to the presence of more general Wilson backgrounds of the form $L = (\lambda_1, \lambda_2, \cdots, \lambda_{10}, 0, 0, 0, 0, 0, 0)$ with $\sum_{I=1}^{10} \lambda_I = 0$. These more general backgrounds break the symmetry further to some $SO(10)$ subgroup like $SU(4) \times SU(2)_L \times SU(2)_R$. The fact that these other nine modes may be continuously varied means that they are also string moduli or, more generally, that the 54-plet of $SO(10)$ in this model is itself a string modulus! We find that this property of the GUT-Higgs behaving as a string modulus, on equal footing with the compactifying moduli $T_i$, is very remarkable.

This example belongs to a whole class of models obtained through continuous Wilson lines. A general characteristic is that they are $SO(10)$ models in which the GUT Higgs is a 54 multiplet. Moreover, there is only one such GUT Higgs coming from the untwisted sector and behaving like a string modulus. On the other hand, the rest of the particle content is model dependent. This includes the number of generations, existence of Higgses 10s, $(16 + \overline{16})$s, hidden gauge group, etc. For instance, the number of generations can be changed by adding discrete Wilson lines to the original orbifold.

The second orbifold method (II), which is implemented by permutations, is more versatile [26]. In this case one may obtain models similar to the previous one both with either one 54 or one 45 in the untwisted sector. One can also find $SU(5)$ models with adjoints 24s in the untwisted sector (sometimes also in some twisted sectors). Instead of showing more examples it is perhaps more interesting to display some general properties and selection rules [26] which one can derive for this kind of string-GUTs. As will be clear, some of those will be more general and apply to any string-GUT constructed through any method.

**General selection rules for any $k=2$ string-GUT**

i) All superpotential terms have $\text{dim} \geq 4$ (i.e., no mass terms).

ii) At $k = 2$ the only reps. which may be present in the massless spectrum are: 5, 10, 15 and 24-plets for $SU(5)$; 10, 16, 45 and 54-plets for $SO(10)$.
iii) $SO(10)$: The rep. $5_4$ for $k = 2$ is special. All its left-handed conformal weight comes from the KM sector. As a consequence: a) A $5_4$ cannot be charged under any $U(1)$ nor any other gauge group. b) Couplings of the type $X(5_4)(5_4)$ or $X(5_4)(5_4')$ (where $X$ is some singlet) are not allowed.

In addition to these general rules one can prove several other ones for string-GUTs obtained from symmetric orbifolds. Some of these are as follows:

*Selection rules for $k=2$ string-GUTs from symmetric orbifolds*

i) There are no selfcouplings $5_4^n$ for any $n$.

ii) There cannot be couplings of type $(54)(45)(45)$.

iii) There cannot be $SU(5)$ self-couplings of type $24^3$.

In practice, when constructing explicit models in symmetric orbifolds, the constraints are even tighter. As we mentioned above, there is normally just one GUT-Higgs in $SO(10)$, either a $5_4$ or a $4_5$ in the untwisted sector. Being in the untwisted sectors, selfcouplings of the type $4_5^n$ or couplings of type $X(45)(45)$ are also forbidden (see ref. [26] for a more detailed explanation of selection rules). Many of these couplings have been used in the past in SUSY-GUT model building in order to trigger GUT-symmetry breaking while obtaining as the low energy sector the MSSM. With the above type of constraints it seems it is very difficult (if not impossible) to construct models whose low energy sector is indeed the MSSM. The absence of some relevant GUT-Higgs selfcouplings cause extra chiral multiplets to remain massless. That will be the case of the GUT-partners of the Goldstone bosons of GUT-symmetry breaking. For example, upon symmetry breaking by an adjoint $24$, twelve out of the 24 fields remain massless. They transform as

$$(8, 1, 0) + (1, 3, 0) + (1, 1, 0)$$

under $SU(3) \times SU(2) \times U(1)$. This seems quite a generic situation which I would expect to be present in more general 4-D string constructions like asymmetric orbifolds or models based on the fermionic construction. In these more general cases some of the above strict selection rules are relaxed in principle but not very much in practice. For example, if the $k = 2$ model comes from the diagonal sum of two $SO(10)$ $k = 1$ factors, the $45$s or $54$s obtained originate from a $(10, 10)$ rep. of the original theory. Such reps. do not admit cubic selfcouplings and the same is expected for the $54$ (due to its antisymmetry there are no cubic couplings for the $45$ anyhow). Since the extra particles above will have masses only of the order of the weak scale, they will sizably contribute to the running of the gauge coupling constants. One can perform a one-loop analysis of the running of the gauge coupling constants and check that, with the particle content of the minimal SUSY-SM plus the additional fields above, there is no appropriate gauge coupling unification in the vicinity of $10^{15} - 10^{17}$ GeV. In the case of $SO(10)$ an intermediate scale of symmetry breaking could improve the results. We thus see that gauge coupling unification is not particularly better in string-GUTs than in direct SM string unification, if the above analysis is correct.

The most severe problem of SUSY-GUTs is the infamous doublet-triplet splitting problem of finding a mechanism to understand why, for example, in the 5-plet Higgs of $SU(5)$ the Weinberg-Salam doublets remain light while their coloured triplet partners become heavy enough to avoid fast proton decay. The most simple, but clearly unacceptable, way to achieve the splitting is to write a term in the $SU(5)$ superpotential

$$W_H = \lambda H \Phi_{24} \bar{H} + M H \bar{H}$$
and fine-tune $\lambda$ and $M$ so that the doublets turn light and the triplets heavy. Since there are no explicit mass terms in string theory this inelegant possibility is not even present. Another alternative suggested long time ago is the “missing partner” mechanism [27]. Formulated in $SU(5)$ it requires the presence of 50-plets in the massless sector which is only possible for level $k \geq 5$, a very unlikely possibility [13, 28]. A third mechanism, put forward in the early days of SUSY-GUTs, is the “sliding singlet” mechanism [29, 30]. This requires the existence of a singlet field $X$, with no self-interactions, entering in the mass term in eq. (15). $W_H$ is then replaced by

$$W_X = \lambda H\Phi_{24}\bar{H} + XH\bar{H}.$$  

The idea is that the vev of the 24 is fixed by other pieces in the potential but the vev of $X$ is undetermined to start with, i.e. the vev “slides”. Now, once the electroweak symmetry is broken by the vevs of $H, \bar{H}$, the minimization conditions give $\lambda(-\frac{3}{2}v) + \langle X \rangle = 0$ where $\text{diag}(\langle \Phi_{24} \rangle) = v(1,1,1, -3/2, -3/2)$. In this way $X$ precisely acquires the vev needed for massless doublets. This is in principle a nice dynamical mechanism but it was soon realized that it is easily spoiled by quantum corrections [31, 32].

Interestingly enough, one finds that in string GUTs, couplings of the “sliding singlet” type are frequent, the main difference now being that the GUT-Higgs field also “slides”. In particular, this happens in models in which the GUT-Higgs is a modulus, as in the examples discussed above. Take for example the $SO(10)$ model discussed above whose massless spectrum is displayed in the table. The singlets in the $U_3$ sector $S^0 = (1,1)_{0,0}, S^+ = (1,1)_{0,1}, S^- = (1,1)_{0,-1}$ do also behave as moduli. Both these singlets and the 54 couple to the decuplets $H^+ = (10, 1)_{0,1}$ and $H^- = (10, 1)_{0,-1}$. The sub-indices in all these fields refer to their $Q$ and $Q_A$ charges. It is easy to check that there are flat directions in this scalar moduli space in which the gauge symmetry is broken down to $SU(4) \times SU(2) \times SU(2)$ and some of the doublets remain light whereas the colour triplets remain heavy (the symmetry is broken down to the SM through the vevs of the $16 + \bar{16}$ pairs). If the sliding-singlet argument were stable under quantum corrections, the regions in moduli-space in which there are light doublets would be energetically favoured.

As the above example shows, the appropriate language to describe the doublet-triplet splitting problem within the context of the above string-GUTs is in terms of the scalar moduli space of the model. At generic points in the moduli space there are no massless Higgs doublets at all, they are all massive. At some “multicritical” points in moduli space some Higgs fields become massless. This is very reminiscent of the behaviour of the moduli spaces of other well studied string moduli, those associated to the size and shape of the compact manifold usually denoted by $T_i$. It is well known that generically there are points in the $T_i$ moduli space in which extra massless fields appear. This is also apparently the case of the moduli space associated to the dilaton complex field $S$. The problem of understanding the doublet-triplet splitting within this context would be equivalent to finding out why we are sitting on a region of moduli space in which massless doublets are obtained. It could well be that an appropriately modified version of the sliding-singlet mechanism is at work and that region of moduli space is energetically favoured.

To summarize this section, I believe that the doublet-triplet splitting problem is a crucial issue and should be addressed in any model before trying to extract any further phenomenological consequences such as fermion masses. It is also important
to understand whether it is possible to build string GUTs in which the massless sector is just the MSSM, or else whether the existence of extra massless chiral fields is really generic. This would dictate the necessity of intermediate scales to attain coupling constant unification.

5 Soft SUSY-breaking terms from dilaton /moduli sectors

Let us turn now to a different subject. The idea is trying to extract some information about the structure of effective SUSY-breaking soft terms which are left out once supersymmetry is spontaneously broken. This is a very important objective since by the year 2005 the spectrum of supersymmetric particles might be tested at LHC and this could be one of the few experimental windows we could have to test the theory. Since very little is known about the origin of supersymmetry-breaking in string theory, this aim looks impossible. This is not as hopeless as it seems. We do not need to know all the details of a symmetry-breaking process in order to get important physical information. A well known example of this is the standard model itself. We do not know yet how SU(2) × U(1) breaking takes place. But, assuming that somehow a (composite or elementary) operator with the quantum numbers of a doublet gets a vev, we get a lot of information.

The idea is to apply a similar philosophy for SUSY-breaking in string theory [33,9]. We have to try and identify possible chiral fields φ_i such that their auxiliary fields F_i could get non-vanishing vev and break SUSY. In string models there are some natural candidates to do the job: the complex dilaton S = 4π/g^2 + iθ and the moduli fields T_i whose vevs determine the size and shape of the compact space. The field S is present in any 4-D strings and the T_i fields at least in any model obtained from compactification. Thus these singlet fields are generic in large classes of string models. An additional advantage is that these fields couple to matter only with non-renormalizable couplings supressed by powers of 1/M_{Planck}. This is a condition which is required in supergravity models with supersymmetry breaking in a” hidden sector”. The scalar potential for S and T_i is flat order by order in perturbation theory and it is expected that non-perturbative effects will i) induce a non-trivial scalar potential for those fields yielding < S >≠ 0, < T_i >≠ 0 and ii) break supersymmetry spontaneously.

The crucial assumption here is to locate the origin of SUSY-breaking in the dilaton/moduli sector [34,35]. It is perfectly conceivable that other fields in the theory, like charged matter fields, could contribute in a leading manner to supersymmetry breaking. If that is the case the structure of soft SUSY-breaking terms will be totally model-dependent and we would be able to make no model-independent statements at all about soft terms. On the contrary, assuming the seed of SUSY-breaking originates in the dilaton-moduli sectors will enable us to make some predictions which might be testable. We will thus make that assumption without further justification. Let us take the following parametrization [35,36] for the dilaton/moduli auxiliary fields F^S and F^i vevs:

\[ G_S^{3/2}F^S = \sqrt{3}m_{3/2}sin\theta ; \quad G_i^{3/2}F^i = \sqrt{3}m_{3/2}cos\theta \Theta_i \]  

where \( \sum_i \Theta_i^2 = 1 \) and \( m_{3/2} \) is the gravitino mass. The angle \( \theta \) and the \( \Theta_i \) just parametrize the direction of the goldstino in the \( S, T_i \) field space. This parametriza-
tion has the virtue that when we plugg it in the general form of the supergravity scalar potential, its vev (the cosmological constant) vanishes by construction. We now need some information about the couplings of the dilaton/moduli fields. Those are given by the Kahler potential $G$ and the gauge kinetic functions $f^a$. The latter are given at the tree level by $f^a = k^a S$, where $k^a$ are the Kac-Moody levels, for any 4-D string. The tree-level kahler potential for $S$ is $-\log(S + S^*)$, also for any model.

The kahler potential for the moduli are more complicated and model-dependent and we need to specify the class of models and moduli that we are considering. We will concentrate here on the three untwisted moduli $T_1, T_2, T_3$ always present in (0,2) toroidal symmetric orbifold constructions (for particular examples there may be additional untwisted moduli and also complex structure structure fields $U_1, U_2, U_3$). For this large class of models one can write for the kahler potential

$$\sum_{i} \log(T_i + T_i^*) + \sum_{i} |C_i|^2 \Pi_i (T_i + T_i^*)^n_i \quad (18)$$

where $C_r$ are charged chiral fields and the $n_r$ are fractional numbers called "modular weights" which depend on the given field $C_r$. The sum in $i$ may be extended to all the three moduli $T$-fields and also to the complex-structure $U$-fields. Plugging this information into the supergravity lagrangian one finds the following results for the scalar masses, gaugino masses and soft trilinear coupling $A_{rst}$ associated to a Yukawa coupling $Y_{rst}$:

$$m_r^2 = m_{3/2}^2 (1 + 3 \cos^2 \theta \, n_r, \bar{\Theta}^2)$$

$$A_{rst} = -\sqrt{3} m_{3/2} [\sin \theta + \cos \theta (\bar{\Theta} (\bar{u} + \bar{n}_r + \bar{n}_s + \bar{n}_t))] \quad (19)$$

where $\bar{u} = (1,1,...)$. (An additional term should be added to $A_{rst}$ for Yukawa couplings $Y_{rst}$ depending on the moduli). Several observations are in order. First of all, in the case of dilaton dominance in the SUSY-breaking process ($\sin \theta = 1$) one gets the remarkably simple universal result:

$$-A_{rst} = M^a = \sqrt{3} m_{3/2} \quad (20)$$

$$m_r^2 = m_{3/2}^2 \quad (21)$$

This result in fact applies for any 4-D string (not only orbifolds) whenever the dilaton dominates. A second observation is that a similar structure of soft terms is obtained for orbifold models in which $\bar{n}_r, \bar{\Theta}^2 = -1/3$ and $\bar{n}_r + \bar{n}_s + \bar{n}_t = -\bar{u}$. This happens for any value of $\theta$ (i.e., not necessarily dilaton dominance) if one assumes $F^1 = F^2 = F^3$ (equal SUSY-breaking contribution from the three untwisted $T$-fields, $\bar{\Theta}^2 = (1/3, 1/3, 1/3)$) and that the charged fields $C_r$ have overall modular weight $n_r = \bar{u}, \bar{n}_r = -1$ (this happens for all particles in the untwisted sector and particles in some types of twisted sectors). A third important observation is that the mass $m_r^2$ of the scalar fields is not positive definite and hence there are choices of $\sin \theta$ and the $\bar{\Theta}^2$ for which tachyons may appear.

A simplified case in which only the dilaton $S$ and the "overall modulus" $T$ contribute to SUSY-breaking (i.e.$\bar{\Theta}^2 = (1/3, 1/3, 1/3)$) was studied in ref. In this case one finds that 1) the particles in both untwisted and twisted sectors with overall modular weights $n = -1$ never become tachyonic; 2) In order to avoid
particles with smaller modular weights (i.e. $n = -2, -3$..) to become tachyonic one has to confine oneself to goldstino angles $\cos^2 \theta \leq 1/|n|$. 3) Due to these constraints one always has bigger gaugino than scalar masses ($M^2 \geq m_i^2$). There is only one situation in which the gauginos may become lighter than the scalars. This happens \[35\] when the chiral fields have all modular weights $n = -1$ and $\sin \theta \to 0$, in which case both $M, m \to 0$ and including loop corrections to $G$ and $f^a$ can reverse the situation and yield gaugino masses smaller than scalar masses.

Considering the more general case [36, 37] in eq. with several moduli $T_i$ modifies somewhat the general conclusions. One finds that 1) particles with overall modular weight $n = -1$ can also become tachyonic for some choices of the angles and 2) The gaugino masses may become lighter than scalar masses even at the tree level. As an example of the possibilities offered let me consider some consistent choices of soft mass terms for the string $SO(10)$ GUT discussed above. For simplicity I only consider the possibility of the $S$ and the $T_i i = 1, 2, 3$ , (and not the $U_i$) contributing to SUSY-breaking. Since it is a $Z_2 \times Z_2$ orbifold, the twisted modular weights are $\vec{n}_T = (\frac{-1}{2}, \frac{-1}{2}, 0)$, where the underlyning means permutations. The untwisted modular weights are as usual $\vec{n}_U = (-1, 0, 0)$. The three rightmost columns in the table show three consistent choices of soft masses: A) Dilaton dominated case ($\sin \theta = 1$). All the scalars have the same mass, $\sqrt{3}$ times lighter than the gauginos. B) Case with $\sin^2 \theta = 1/3$ and $\vec{\Theta}^2 = (0, 0, 1)$. In this case the gauginos and Higgs 10-plets have equal masses, the 16-plets have zero mass and the GUT-Higgs have negative mass$^2$. The latter property is interesting since it show us that SUSY-breaking may automatically trigger GUT-symmetry breaking; C) Case with $\sin \theta = 0$ and $\vec{\Theta}^2 = (0, 0, 1)$. In this case the dilaton does not contribute to SUSY-breaking and the gauginos are massless at the tree-level. The Higgs 10-plets have positive mass$^2$ but the GUT-Higgs and the 16-plets get negative mass$^2$. The latter may also enforce that there are $16 + \bar{16}$ pairs getting a vev. Thus we see that a variety of phenomenological possibilities open up depending on what is the role of the different moduli in the process of SUSY-breaking.

It must be emphasized that, given a specific string model, there is only certain type of soft terms which can be added consistently with the assumptions of dilaton/moduli dominance in SUSY-breaking. Taking a random choice of soft terms would lead to inconsistencies. For example there is always a rule [38] which connects the soft masses of particles in the three untwisted sectors $i = 1, 2, 3$ with that of the gauginos, $m_1^2 + m_2^2 + m_3^2 = M^2$ (see the discussion below). The reader may check this constraint in the three examples in the table. In fact a similar sum-rule is still correct for twisted particles with overall modular weight $n = -1$, and not only for untwisted fields. More details and examples can be found in a forthcoming publication [39].

6 Dilaton-induced SUSY-breaking is special

Indeed the soft terms relationships obtained under the assumption of dilaton-dominance SUSY-breaking is special in several respects. First, these boundary conditions for soft terms are obtained for any 4-D $N = 1$ string and not only for orbifolds. Second, these conditions are universal, gauge group independent and flavour independent. Thirdly, the soft masses obtained for scalars are positive definite, lead to no tachyons. This is to be compared with situations in which other fields like the mod-
uli contribute to SUSY-breaking. We have seen how easy is to get negative mass when the moduli contribute to SUSY breaking [33, 35]. We all hope that, whatever the string theory describing the spontaneously broken SUSY phase could be, it will be a tachyon-free modular invariant theory. Dilaton-dominance SUSY-breaking is not necessary for the absence of tachyons, there are also situations in which e.g., the moduli dominate and still there are no tachyons. But dilaton dominance guarantees the absence of tachyons. So also in this sense dilaton induced SUSY-breaking is special.

Dilaton SUSY-breaking is special in yet another aspect, which has past mostly unnoticed in the literature. It has been recently realized [40, 41] that the boundary conditions $-A = M = \sqrt{3}m$ of dilaton dominance coincide with some boundary conditions considered by Jones, Mezincescu and Yao in 1984 [42] in a completely different context. It is well known that one can obtain two-loop finite $N = 1$ field theories by considering appropriate combinations of matter fields (so that the one-loop $\beta$-function vanishes) and Yukawa couplings (so that the matter field anomalous dimensions vanish). It has also been argued in favour of the complete finiteness of this type of theories to all orders. What Jones, Mezincescu and Yao did is to look for SUSY-breaking soft terms which do not spoil one-loop finiteness when added to these finite theories. They came out with universal soft terms with $-A = M = \sqrt{3}m$. It was also shown in 1994 by Jack and Jones that two-loop finiteness was also preserved by this choice [43].

This coincidence is at first sight quite surprising since we did not bother about the loop corrections when extracting these boundary conditions from the dilaton dominance assumption. Also, effective $N = 1$ field theories from strings do not in general fulfill the finiteness requirements (in fact I do not know of any which does). Why dilaton-dominance bothers to yield soft terms with such improved ultraviolet behaviour?

A heuristic motivation goes as follows. The dilaton sector in a 4-D string is completely model independent (at least at the tree-level). Hence the gauge kinetic function $f_a = k_a S$ and $G(S, S^*) = -\log(S + S^*)$ for any 4-D string, independently of e.g., what compactification we used to obtain it. This means that, if the assumption of dilaton-dominance makes sense at all, it has to lead to soft terms which are consistent with any possible compactification and also has to be independent of the particular choice of compactification. In particular, the obtained soft terms have to be consistent with the simplest of all kinds of compactifications, a toroidal compactification preserving $N = 4$ supersymmetry. What do I mean by soft terms consistent with $N = 4$ supersymmetry? By that I mean that the soft terms should be in the list of terms which maintain the finiteness properties of $N = 4$ supersymmetry. The reason for that is that, if there is indeed some mechanism by which SUSY is spontaneously broken in the dilaton sector one does not expect the induced soft terms below the SUSY-breaking scale to produce new logarithmic divergences in a theory ($N = 4$ SUSY) which was originally finite.

The types of SUSY-breaking soft terms which may be added to $N = 4$ SUSY without spoiling finiteness is well known [14]. First, one can add $N = 1$ preserving masses for the $N = 1$ chiral multiplets contained in $N = 4$. These are not very interesting since we are interested in soft terms leaving no unbroken supersymmetry. A second more interesting possibility is to add soft masses $M$ for the gauginos along with masses $m_1^2, m_2^2, m_3^2$ for the three multiplets of adjoint scalars present in the
theory verifying:
\[ m_1^2 + m_2^2 + m_3^2 = M^2. \] (22)

This constraint may be interpreted just as Supertrace(Mass)\(^2\) = 0 for \(N = 4\). In addition, the presence of gaugino masses generates logarithmic divergences involving a holomorphic trilinear coupling (an \(A\)-term) which is only cancelled if
\[ M = -A. \] (23)

I already mentioned how a sum-rule like (22) is indeed verified by the dilaton/moduli induced soft terms involving the un twisted sector particles in orbifolds. In the case of a model-independent source of SUSY-breaking like the one we are discussing one expects \(m_1^2 = m_2^2 = m_3^2 = m^2\). So, one thus arrives for consistency with \(N = 4\) at the universal conditions \(-A = M = \sqrt{3}m\). One thus can understand the dilaton-induced boundary conditions as a consistency condition due to the fact that i) dilaton couplings are compactification-independent and thus 2) should obey consistency constraints from the most constrained compactifications, \(N = 4\) preserving compactifications. Notice that in an \(N = 1\) theory the sum rule (22) will not be in general preserved, it is the boundary conditions \(-A = M = \sqrt{3}m\) which generalize to the \(N = 1\) case, not the \(N = 4\) expressions themselves.

Coming back to the finiteness properties of this type of soft terms, it is clear that if we had an \(N = 1\) two-loop finite theory as the effective low energy theory from some 4-D string model, dilaton SUSY-breaking would respect these finiteness properties. On the other hand there is no reason for an \(N = 1\) theory from strings to be finite as a field theory, it is already finite anyhow due to the string cut-off (modular invariance). If we add these soft terms to a non-finite \(N = 1\) theory the ultraviolet properties do not specially improve, but at the scale at which those relationships hold \((M_{\text{string}})\) one finds e.g. that the \(\beta\) functions associated to the soft terms are proportional to the \(\beta\)-function of the Yukawa couplings \(h\), i.e.,
\[
\begin{align*}
\beta_A(h, g^2, M, m, A)|_{M_{\text{string}}} &\propto M\beta_h(h, g^2) \\
\beta_{m^2}(h, g^2, M, m, A)|_{M_{\text{string}}} &\propto M^2/h\beta_h(h, g^2)
\end{align*}
\] (24)

So the theory becomes finite if the underlying unbroken-SUSY theory was finite. On the other hand, if we start from a finite \(N = 1\) theory and we add random soft terms one finds that \(\beta_A \propto h^2(A + M)\) and \(\beta_{m^2} \propto h^2(3m^2 - M^2)\) so that the dilaton-dominated boundary conditions would constitute a fixed point of the renormalization group equations. This again shows us the special properties of this choice of soft terms.

The above discussion shows that the assumption that the auxiliary field associated to the dilaton breaks supersymmetry leads to soft terms with remarkable finiteness properties. This fact seems to be related to the \(S - duality\) structure of the dilaton couplings. For example, one can check that the result \(M = -A\) is obtained in the dilaton dominated scheme due to the fact that the following functional expression is verified:
\[
f(S)^S = 2Re f(S)G(S, S^*)^{S^{1/2}}
\] (25)

where \(f\) and \(G\) are the gauge kinetic function and \(S\)-field Kahler potential. This relationship is related to the \(SU(1, 1)\) structure of the field \(S\) in the \(N = 4\) supergravity.
Lagrangian \[15\]. So the dilaton-dominated soft terms are intimately connected to the $S - duality$ symmetry \[16, 17\] underlying these theories.

Recently Seiberg and others \[18\] have discussed the existence of certain duality properties between different classes of $N = 1$ theories with different particle content and with or without Yukawa couplings. It would be desirable to see to what extent their results could be extended to the $N = 0$ case. An obvious first step in that direction would be the addition of SUSY breaking soft terms. In view of the above discussion it is reasonable to think that soft terms of the type $-A = M = \sqrt{3} m$ could have a special status in this respect. Notice also that the proportionality between soft terms and Yukawa $\beta$-functions at $M_{\text{string}}$ shown in eq. (24) shows the presence of a non supersymmetric marginal operator structure analogous to the $N = 1$ examples discussed in ref. \[19\].

7 Outlook and speculations

The above lines discussed several directions recently explored in trying to establish contact between the physics at the string scale and the physics at the weak scale, which is the one amenable to experimental test. It is important to realize that by the year 2005 the LHC should provide us with important experimental information about the origin of the weak scale. If low energy supersymmetry is correct, the spectrum of SUSY particles should be tested. We do not have at the moment a theory of supersymmetry breaking but we still have ten years ahead to find one! I certainly believe that it should be easier to find a theory of soft terms rather than a theory of fermion masses. At least, it seems that the former could have a more model-independent origin than the second.

In the previous lines I parametrized SUSY-breaking in terms of the vacuum expectation values of the auxiliary fields of the dilaton and moduli but I never discussed what could be the dynamical origin of supersymmetry breaking. I did not discussed either what is the dynamics which fixes the vevs $<S>$ and $<T_i>$. The most popular scenarios assume that the same dynamics which break SUSY at the same time fix those vevs. It is not clear to me that this is necessarily the case. It is conceivable that some string dynamics could fix $<S>$ and/or $<T_i>$ to be of order the string scale and then some low energy field-theoretical effect (e.g., gaugino condensation) could break supersymmetry \[50\]. It is not clear what string effects could fix the $S, T$ vevs without breaking supersymmetry at the string scale, but one can use the duality symmetries associated to those fields to restrict the possibilities. Indeed, the well known $T$-duality symmetries would suggest that the most natural values for $<T>$ should be around the selfdual point, $<T> \approx 1$ and that kind of result is obtained in $T - duality$-invariant versions of gaugino condensation \[51\]. If some sort of $S - duality$ \[16\] is correct in $N = 1$ theories, one should also expect $<S> \approx 1$. I would argue that this is not necessarily unreasonable if the massless sector of the theory contains particles beyond the ones in the MSSM, which is in fact the generic case in explicit string models. In this case the gauge couplings will not be asymptotically free and may become quite large at the string scale.

Things may be more complicated than the tacit assumption hidden in the previous sections, that we are in a perturbative regime of the string. It could well be that the non-perturbative string effects modify in a substantial manner all the perturbative 4-D backgrounds that we are using at the moment in explicit constructions. In
this case one can still hope that these corrections do not modify substantially the
$N = 1$ superpotentials but only the D-terms (see talk by M. Dine in these proceedings). If we are less lucky even the superpotentials could be affected. This would make rather difficult to extract predictions from any 4-D string construction unless we know all the relevant non-perturbative dynamics, something which looks rather remote. On the other hand this possibility would have in my opinion one interesting aspect (probably the only one!). Standard 4-D strings are always excessively rigid in providing Yukawa couplings. They tend to have so many continuous and discrete symmetries that many (too many) couplings (including non-renormalizable ones) are forbidden. A typical example of this is the absence of selfcouplings of the GUT-Higgs discussed above. Perhaps string non-perturbative effects could generate new superpotential terms (e.g., like $24^3$ in $SU(5)$) which could be absent in the perturbative vacuum one started from.

The other tacit assumption is that we are identifying correctly the short distance elementary degrees of freedom of the standard model in trying to embed it into a 4-D string. The recent results by Seiberg and others [48] show how two different $N = 1$ theories with different gauge group could be dual to each other and describe the same physics in the infrared. An example of this is the equivalence of the physics of a $N = 1$ $SU(N)$ theory with $N_f$ flavours at weak coupling to the physics of an $SU(N_f - N)$ also with $N_f$ flavours at strong coupling. Thus, as suggested in the first article in [48], perhaps all or part of the known elementary particles of the standard model are in fact dual to the truly elementary particles at short distances. This is a very intriguing possibility. It could well be that e.g., the $SU(3)$ colour interactions and the quarks were not elementary but dual to the true elementary states. It would be the latter states which we should unify along with the $SU(2) \times U(1)$ interactions and the leptons into a string theory. The whole hypothesis of the ”desert” should then be reconsidered, including its emblematic prediction, gauge coupling unification, since it would not be the observed $\alpha_s$ coupling which should unify with the other two, but the dual. Although one cannot directly apply the arguments of ref. (48) to a non-semisimple chiral theory like the SM, it is amusing to note that for SUSY-QCD with the observed 6 flavours $N = N_f - N = 3$ and hence the gauge group would be the same in the dual theory.

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