ADOMIAN DECOMPOSITION METHOD FOR STEADY FREE CONVECTIVE COUETTE FLOW IN A VERTICAL CHANNEL WITH NON-LINEAR THERMAL RADIATION, DYNAMIC VISCOSITY AND DYNAMIC THERMAL CONDUCTIVITY EFFECTS

Ajibade O. A., Jha B. K., Jibril H. M. and Yusuf A. B.

Department of Mathematics; Ahmadu Bello University Zaria, Kaduna State, Nigeria.

*Corresponding Author’s email: ayusuf@fudutsinma.edu.ng

ABSTRACT

In this paper, we investigate steady free convective Couette flow in a vertical channel with nonlinear thermal radiation, dynamic viscosity and dynamic thermal conductivity effects. The investigation is motivated by the studies of some researchers which assumed linear thermal radiation and constant fluid properties. However, this is uncalled for; as these assumptions do not reflect true behavior of the flow. For instance; increase in temperature affects fluid viscosity; thermal conductivity thereby changing the transport phenomenon. Here; the investigation considers both the fluid viscosity and thermal conductivity to be dependent on temperature with the thermal radiation adopting nonlinear form. Due to this reasons, the associated flow equations are highly nonlinear and exhibit no analytical solution and therefore require the use of Adomian decomposition method (ADM) of solution. The attained ADM solution is then coded into computer algebra package of mathematica where results under the parameters of interest are presented and discussed. Results of the investigation show that raising the thermal radiation leads to corresponding rise in both the velocity and temperature of the fluid in the channel. Furthermore; lessening the viscosity and thermal conduction of the fluid were identified to escalate both velocity and temperature of the fluid.

Keywords: Natural convection; Couette flow; Steady flow; Variable Fluid Properties; Nonlinear Thermal Radiation

INTRODUCTION

Flow of fluid induced by density difference occurring between the fluid particles due to temperature gradients is referred to as free convection flow. This type of flow has fundamental importance in many technological and industrial applications such as nuclear reactor, radiators, furnaces, rapid cooling process, sewage disposal and many more. Natural convection flows due to the movement of bounding surface surrounding the fluid is termed as “Couette flow”. This type of flow occurs in fluid machineries involving moving parts; especially in hydrodynamics lubrications. Couette flow has been used as a fundamental method for measurement of viscosity and as a means of estimating drag force in many wall driven applications (Yasutomi (1984)). A situation in flow formation which is not time dependent is called steady flow. This type of flow has applications in many engineering devices like boiler, turbine, condenser and water pump that run nonstop for many months before they are shut down for maintenance. Several scholars considered steady natural convection flow through channels due to its significance in engineering technology; especially; in cooling/heating applications. For example; it is used in computer engineering where electronic cabinets containing circuits are design in channel forms so as to enhance cooling of the computer system; in civil engineering, channels are used for irrigation purposes, measuring discharge of water in a river, studying the spread of pollutants and so on. In relation to this, Ostrach (1952) investigated steady laminar natural convection flow of viscous incompressible fluid between two vertical walls while Ostrach (1954) and Sparrow et al. (1952) studied combined effects of steady free and forced convective flow and heat transfer between vertical walls. The study of Miyatake and Fuzii (1972) presented results for steady natural convection between vertical walls on considering different physical situations of the flow process. Transient flow between two vertical walls heated/cooled asymmetrically was investigated by Singh and Paul (2006) and revealed that formation of upward flow occurs near the heated wall with down ward flow achieved near the cooled wall. Couette flow of heat generating/absorbing fluid was investigated by Jha and Ajibade (2010) and their result shows that reverse flow of the fluid is achieved with external heating of the moving plate. The study of Miyatake et al. (1973) pointed that the rate of heat transfer near the hotter wall is enhanced by the buoyancy force with the reversal flow attained near the cooler wall. Other connected studies can be witnessed in Mandal et al. (2014), Jha and Ajibade (2011). Nelson and Wood (1989a,b) and Jha et al. (2012).

Studies related to viscous fluid with temperature-dependent viscosity are of paramount importance; especially in petroleum industries for purification and filtration processes; it is also used in food processing and coating of metals. The ancient expression of temperature-dependent viscosity was first given by Reynold (1984). With the advent of this; several scholars have modeled the expression for temperature-dependent fluid viscosity in different forms; all of which revolved around the ancient Reynold’s expression. These can be viewed in Elbashbashy et al. (2000), Mukhapdya et al. (2009) and Vanden Berg et al. (2005); just a few to mention among others. In a related article, Carey and Mollendorf (1978) affirmed that when the viscosity of water is raised from
10^0C (\mu = 0.0033 g / cm/s) to 50^0C (\mu = 0.00548 g / cm/s).

Its viscosity is decreased by 240% while that of Grey et al. (1982) conveyed that; when the viscosity of a fluid is temperature-dependent, the flow mechanism of the fluid changes significantly compared to the assumption of constant viscosity. Mehta and Sood (1992) disclosed that; the usual assumption of constant viscosity of fluids evaluated at some reference temperature is not sufficient to describe a correct situation in the transport characteristics of viscous fluids. Temperature-dependent viscosity on free convective laminar boundary layer flow past a vertical isothermal flat plate was studied by Kafousius and Williams (1995) while the effect of temperature-dependent viscosity on mixed convection flow past a vertical flat plate in the region near a leading edge was investigated by Kafousius and Rees (1998). In the afore mentioned studies the latter researcher disclosed that when viscosity of fluid is sensitive to temperature change, the effect of temperature-dependent viscosity has to be taken into cognizance or else significant errors may occur in the flow mechanisms. Furthermore, Makinde and Ogulu (2011) concluded that a reduction in fluid viscosity amounts to the rise in its velocity. Interrelated scholarly articles can be seen in Costa and Macedonio (2003), Seddeck and Salem (2006), and Hossain et al. (2001).

Temperature-dependent thermal conductivity in the study of flow of viscous fluids has been considered by scholars due to its solicitations in technological innovations like in the extrusion of plastic sheets, polymer processing, spinning of fibers, cooling of elastic sheets etc. For instance, in heat sink/source applications; materials of high thermal conductivity are used while those of low conductivity are used in designing insulators. Similarly, metals in liquid form with small Prandtl number in the interval of 0.01 – 0.1 are commonly used for cooling purposes because of their high thermal conductivity. Numerous scholars investigated flow of viscous fluids on the assumption of constant thermal conductivity. However this is uncalled for; as variation in temperature affects the thermal conduction of the fluid. This is evidently observed in the study of Adrian et al. (1997) which disclosed that; when dry air is heated to 1000°C its thermal conductivity is 31.39×10^{-3} W / mK while at 2000°C the thermal conductivity is 37.95×10^{-3} W / mK. Similarly; Van den Berg et al. (2017) divulged that the use of variable thermal conductivity to study flow of molten magma can delay secular cooling of the mantle with constant viscosity model. Sharma and Aisha (2014) submitted that thermal conduction of fluid increases with decrease in Prandtl number. Other associated studies can be referred to the articles of Rihab et al. (2017), Dubuffet et al. (1999), Starlin (2000), Blas (2019) and Hofmeister (1999).

Release of energy in the form of electromagnetic waves by hot objects is termed thermal radiation. This has fundamental importance in cooling/heating processes; especially in the aspect of engineering applications for human survival on the earth. For instance thermal radiation is used in sterilization of medical instruments, toasting of bread, treatment of cancer and tumor, air conditioners and heaters. Due to this, Rosseland (1931), first gave the expression for thermal radiation and this expression was further simplified by Sparrow and Cess (1962). The simplified form is being used by scholars to study flow of fluids with thermal radiation; refer to Makinde et al. (2007), Makinde and Ibrahim (2017), Ganji et al. (2015), Sheikholeslami (2015), Makinde (2008) and Abel and Mashesha (2007). In view of this novelty, some researchers mentioned above have discussed the effect of thermal in their flow formation using linearized temperature in in their flow formation. The usage of this was queried by Magyari and Patokratoras (2011) arguing that the flow behavior is not accurately predicted via this procedure. They therefore proposed alternative method which adopted the use of nonlinear temperature in the expression for thermal radiation. In recognition to this, connected studies can be witnessed in Yusuf and Ajibade (2018a, 2020), Ajibade and Yusuf (2019), Yabo et al. (2016) and Jha et al. (2017).

There exists different method of solving differential equations arising from fluid flows. These include the method of undetermined coefficients, Runge-Kutta method, finite difference method, Laplace transform, Adomian decomposition method (ADM) and many others.

The present article investigates steady free convective Couette flow in a vertical channel on adopting nonlinear thermal radiation, dynamic viscosity, dynamic thermal conductivity and ADM method of solution (Adomian (1994)). This investigation is motivated by the works of some authors which failed to adopt the above parameters upon which the flow behaviors are either under-determined or over-determined. The choice of ADM is due to the following reasons: the technique avoids perturbation, it gives efficient, accurate and approximate solution, it does not require discretization of the solution, does not results to large equations. Furthermore; the method is not affected by computational round off errors, consumes less time and less amount of computer memory (Makinde et al. 2007).

MATHEMATICAL FORMULATION OF THE PROBLEM

Figure 1 consists of an infinite vertical channel formed by two parallel plates kept h distance apart. The channel is filled with an optically thick viscous incompressible fluid at the expense of radiative heat flux of intensity \( q_r \); which is absorbed by the plates and transferred to the fluid. Neglecting the effect of viscous dissipation and assuming all the fluid’s physical properties are constant except for its viscosity and thermal conduction which are assumed to be temperature-dependent. The \( x' \)-axis coordinate is taken along the channel in the vertically upward direction, being the direction of the flow while the \( y' \)-axis is taken normal to it. Also; assuming effect of radiative heat flux in the \( x' \)- direction to be negligible compared to that in the \( y' \)- direction with the temperature of
the plate kept at $y' = 0$ rise to $T_w$ and thereafter maintained constant while the other plate at $y' = h$ remains at $T_0$. Furthermore, the plate at $y' = 0$ moves on its own plate impulsively at uniform velocity $u' = Mu_0$ while the other plate remains at rest.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{Diagram of the problem}
\end{figure}

The appropriate governing equations under these assumptions together with the assumption of Boussinesq approximation are:

\begin{align}
\frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) + g \beta (T - T_0) &= 0 \\
\frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( k \frac{\partial T'}{\partial y} \right) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} &= 0
\end{align}

with the radiative heat flux of Sparrow and Cess (1962) given as:

\[ q_r = -4 \frac{\sigma}{3} \frac{\partial T^4}{\partial y} \]

Following Magyari and Pantokratoras (2011), the dynamic viscosity and dynamic thermal conductivity of the fluid are respectively expressed in the form:

\[ \mu = \mu_0 \left( 1 - \lambda \left( \frac{T' - T_0}{T_w - T_0} \right) \right) \]
\[ k = k_0 \left( 1 - \varepsilon \left( \frac{T' - T_0}{T_w - T_0} \right) \right) \]

with the boundary conditions for the velocity and temperature fields as:

\[ u = Mu_0 \]
\[ T' = T_w \quad \text{at} \quad y' = 0 \]

\[ u' = 0 \]
\[ T' = T_0 \quad \text{at} \quad y' = h \]

Non-dimensional of the Problem

The problem under consideration involves quantities in different dimensions and so equations (1-6) are therefore required to be transformed into non-dimensional form using the quantities:

\[ u = \frac{u'}{u_0}, \quad y = \frac{y'}{h}, \quad \theta(y) = \frac{T' - T_0}{T_w - T_0} \]

Using equation (4) and (7), the momentum equation (1) is transformed into dimensionless form and the following equation is obtained:

\[ u'(y) = \lambda \left( 1 + \lambda \theta(y) \right) \theta'(y) u'(y) - Gr \theta(y) \left( 1 + 2 \lambda \theta(y) \right) \]

The radiative heat flux in equation (3) is expanded nonlinearly on adopting Magyari and Pantokratoras (2011) and the following equation is grasped:
\[ \frac{\partial q_r}{\partial y} = -\frac{4\sigma}{3h^2\delta} \left( 12(T_w - T_0) + \theta(y) + \phi \right) \frac{\partial}{\partial y} \left( \frac{\partial^2}{\partial y^2} \theta(y) \right) \]

Now substituting equation (4), (7) and (9) into equation (2) gives the equation:

\[ \theta'(y) = \varepsilon \theta(y) \left[ 1 - \frac{4R_f}{3} \left( 1 + \varepsilon \theta(y) \right) \left[ \theta(y) + \phi \right]^3 \right] \]

\[ -4R_f \left( 1 + \varepsilon \theta(y) \right) \left[ \theta(y) + \phi \right]^2 \theta''(y) \left[ 1 - \frac{4R_f}{3} \left( 1 + \varepsilon \theta(y) \right) \left[ \theta(y) + \phi \right]^3 \right] \]

Again, using equation (7) in equation (5) and (6), the boundary conditions are:

\[ u = M, \quad \theta = 1 \quad \text{at} \quad y = 0 \]

\[ u = 0, \quad \theta = 0 \quad \text{at} \quad y = 1 \]

where \( R_f = \frac{4\sigma(T_w - T_0)^3}{3k_0\delta}, \quad \phi = \frac{T_0}{T_w - T_0}, \quad Gr = \frac{g \beta (T_w - T_0)}{4\nu U_0} \) (13)

Meaning of the parameters involved in equations (1-13) see the table of nomenclature.

Mathematical Description of ADM
Consider the differential equation in Adomian form:

\[ Lu + Su + Nu = g \]

where \( u \) is unknown function which is to be determined by a recursive relation, \( L \) is the highest order derivative which is also invertible, \( S \) is the remainder of the linear operator whose order is less than \( L \), \( Nu \) represents the nonlinear terms and \( g \) is the system input.

Operating \( L^{-1} \) to both sides of equation (14) and using the given initial boundary conditions, the following differential equation is obtained:

\[ u = w - L^{-1}(Su) - L^{-1}(Nu) \]

where \( w \) represents the term arising from integrating \( g \) and the auxiliary conditions.

According to ADM the solution \( u \) is defined by the series:

\[ u = \sum_{n=0}^{\infty} u_n \]

and \( Nu \) comprises the series of the Adomian polynomials:

\[ Nu = \sum_{n=0}^{\infty} A_n \]

where \( A_n \) are Adomian polynomials generated from the equation:

\[ A_n = \frac{1}{n!} \left[ \frac{d^n}{d\lambda^n} \left[ N \left( \sum_{i=0}^{\infty} \lambda_i u_i \right) \right] \right]_{\lambda=0} \]

The solution components \( u_0, u_1, u_2, \ldots \) are determined recursively as:

\[ u_0 = w \]

\[ u_{j+1} = -L^{-1}(Su_j) - L^{-1}(Nu_j), \quad j \geq 0 \]

where \( w \) is referred to as the zeroth-order component.

ADM Solution of the Problem
The differential equations (8) and (10) subject to equations (11) and (12) are solved using ADM as follow:

Let \( Lu(y) = u'(y) \), \( L\theta(y) = \theta'(y) \) where \( L^{-1} \chi(\bullet) = \int \chi(\bullet)dy \)
Putting equation (15); equations (8) and (11) can now be written as:
\begin{equation}
Lu(y) = \lambda \left(1 + \lambda \theta(y)\right) \theta(y) u(y) - Gr \theta(y)(1 + 2\lambda \theta(y))
\end{equation}
(22)
\begin{equation}
L \theta(y) = \epsilon \theta^2(y) \left[1 + 4 R_T^3 \frac{1}{3} \right] \left(1 + \epsilon \theta(y) \right) \left(1 + \phi \right) \theta(y) + \phi^3
\end{equation}
(23)
Operating \(L^{-1}\) to equations (22) and (23) we obtain:
\begin{equation}
L^{-1} Lu(y) = \lambda L^{-1} \left[(1 + \lambda \theta(y)) \theta(y) u(y)\right] - Gr L^{-1} \left[\theta(y)(1 + 2\lambda \theta(y))\right]
\end{equation}
(24)
\begin{equation}
L^{-1} L \theta(y) = \epsilon L^{-1} \left[\theta^2(y)(1 + \epsilon \theta(y)) \left[1 - 4 R_T^3 \frac{1}{3}\right] \left(1 + \epsilon \theta(y) \right) \left(\theta(y) + \phi\right)^3\right]
\end{equation}
(25)
But \(L^{-1} Lu(y) = u(y) - u(0) - yu(0)\) and \(L^{-1} L \theta(y) = \theta(y) - \theta(0) - y \theta(0)\)
(26)
Using equations (11) in equations (24) and (25) we have:
\begin{equation}
u(y) = M + yA + \epsilon L^{-1} \left[(1 + \lambda \theta(y)) \theta(y) u(y)\right] - Gr L^{-1} \left[\theta(y)(1 + 2\lambda \theta(y))\right]
\end{equation}
(27)
\begin{equation}
\theta(y) = 1 + By + \epsilon L^{-1} \left[\theta^2(y)(1 + \epsilon \theta(y)) \left[1 - 4 R_T^3 \frac{1}{3}\right] \left(1 + \epsilon \theta(y) \right) \left(\theta(y) + \phi\right)^3\right]
\end{equation}
(28)
Where \(A = f(0)\) and \(B = \theta(0)\) are values to be obtained using (12).
The ADM defines \(u(y)\) and \(\theta(y)\) in the forms:
\begin{equation}
u(y) = \sum_{n=0}^{\infty} u_n(y) \quad \text{and} \quad \theta(y) = \sum_{n=0}^{\infty} \theta_n(y)
\end{equation}
(29)
Substituting equation (29) into equation (27) and (28), we have:
\begin{equation}
\sum_{n=0}^{\infty} u_n(y) = M + yA + \lambda L^{-1} \left[(1 + \lambda \sum_{n=0}^{\infty} u_n(y)) \frac{d}{dy} \sum_{n=0}^{\infty} \theta_n(y) \frac{d}{dy} \sum_{n=0}^{\infty} u_n(y)\right]
\end{equation}
(30)
\begin{equation}
\sum_{n=0}^{\infty} \theta_n(y) = 1 + yB
\end{equation}
\begin{equation}
+ \epsilon L^{-1} \left[\frac{d}{dy} \sum_{n=0}^{\infty} \theta_n(y) \frac{d}{dy} \sum_{n=0}^{\infty} \theta_n(y) \left[1 + \epsilon \sum_{n=0}^{\infty} \theta_n(y) \left[1 - 4 R_T^3 \frac{1}{3}\right] \left(1 + \epsilon \sum_{n=0}^{\infty} \theta_n(y) \left(\sum_{n=0}^{\infty} \theta_n(y) + \phi\right)\right]\right]\right]
\end{equation}
(31)
\begin{equation}
- 4 R_T L^{-1} \left[\frac{d}{dy} \sum_{n=0}^{\infty} \theta_n(y) \left[1 + \epsilon \sum_{n=0}^{\infty} \theta_n(y) + \phi\right]\right]
\end{equation}
Setting \(\theta_0(y) = 1 + By\) and \(u_0(y) = M + yA - Gr L^{-1} \left[(1 + 2\lambda \theta(y)) \theta(y)\right]\)
(32)
\(u_{n+1}(y)\) and \(\theta_{n+1}(y)\) for \(n \geq 0\) are determined using the generating relations:
ADOMIAN DECOMPOSITION... Ajibade et al FJS

\[ u_{n+1}(y) = \lambda L^{-1}\left\{ (1 + \lambda \theta_n(y)) \frac{d}{dy}(\theta_n(y)) \frac{d}{dy}(u_n(y)) \right\} \quad \text{and} \quad (31) \]

\[ \theta_{n+1}(y) = \varepsilon L^{-1}\left\{ \frac{d}{dy}(\theta_n(y)) \frac{d}{dy}(\theta_n(y)) \left[ 1 + \frac{4R_T}{3} (1 + \varepsilon \theta_n(y)) \left[ \theta_n(y) + \phi \right]^3 \right] \right\} \]

\[ -4R_T L^{-1}\left\{ (1 + \varepsilon \theta_n(y)) \left[ \theta_n(y) + \phi \right]^2 \frac{d}{dy}(\theta_n(y)) \frac{d}{dy}(\theta_n(y)) \left[ 1 - \frac{4R_T}{3} (1 + \varepsilon \theta_n(y)) \left[ \theta_n(y) + \phi \right]^3 \right] \right\} \quad (33) \]

For more details on ADM, see Adomian (1994).

Convergence/Termination Criteria of the ADM Solution

It has been proven in Adomian (1994) and Cherruault (1990) that convergence of ADM solution always exists and is rapidly. Based on this the convergence of the solution is not tested here. For the termination criteria; the ADM solutions for \( u \) and \( \theta \) are all paused after the 3rd terms as subsequent terms after these contribute insignificantly to the final solution. The final solutions are not presented here due to their cumbersomeness but are used for discussing the results.

Nusselt Number and Skin Friction on the Channel Plates

The Nusselt number on the channel plates are evaluated on adopting Kay (2017) via:

\[ Nu_y = (1 - \varepsilon \theta) \frac{d\theta}{dy} \bigg|_{y=0} \quad \text{and} \quad Nu_1 = (1 - \varepsilon \theta) \frac{d\theta}{dy} \bigg|_{y=1} \quad (33) \]

with the skin friction \( \tau \) calculated using:

\[ \tau_0 = (1 - \lambda \theta) \frac{du}{dy} \bigg|_{y=0} \quad \text{and} \quad \tau_1 = (1 - \lambda \theta) \frac{du}{dy} \bigg|_{y=1} \quad (34) \]

RESULTS AND DISCUSSION

Steady free convective Couette flow in a vertical channel with dynamic viscosity, dynamic thermal conductivity and nonlinear thermal radiation has been investigated in this article. Influences of the physical parameters involved are examined with the results presented in figure 2 – 10 and on tables 1-3. For the aim of discussion, the values of \( R_T \) and \( \varepsilon \) taken in the range \( 0 \leq \varepsilon, R_T \leq 1 \) as terms associated with them behave as strong heat source/sink and their large values lead to finite time temperature blow up (Makinde and Chinyoka, 2010). Similarly, the values of \( \phi, \lambda, M \) are arbitrarily chosen between 0.1 – 3 while that of \( Gr \) are selected between 10, 12 and 14 which represent cooling of the plates by free convection current.

![Fig 2: Temperature profiles for different \( R_T \) \( (\phi = 0.1, \varepsilon = 0.1 \ldots R_T = 0.001, \_\_R_T = 0.1) \)](image)

![Fig 3: Temperature profiles for for different \( R_T \) \( (\phi = 0.1, \lambda = 0.1, M = 1, Gr = 10 \varepsilon = 0.1) \)](image)
Figure 2 demonstrates the effect of thermal radiation on the fluid temperature within the channel. The figure shows that an increase in $R_T$ contributes to the corresponding increase to the fluid temperature. The consequential effect of this on the fluid velocity is reflected on figure 3 where the fluid velocity is also seen to rise with increase in $R_T$, these behaviors are the attributes of the decrease in thermal conduction of the fluid.

Figure 4 and 5 illustrate the effect of changing thermal conductivity on the temperature and velocity of the fluid within the channel when other parameters are fixed. From these figures the temperature is seen to decrease with decrease in thermal conduction of the fluid. Similarly, on decreasing thermal conduction of the fluid the velocity in the channel is also witness to decrease. These fashions are the consequence of the decrease in thermal conduction of the fluid. This physically reveals the effect of decrease in thermal diffusivity with growing $\epsilon$ which act to diminish the influence of the applied boundary temperature, thus causing a decrease in the thermodynamics and consequent decrease of fluid velocity in the channel.
Figure 6 displayed the influence of temperature difference on the fluid temperature where the figure illustrates that the temperature in the channel increases with increase in $\phi$. This trend is attributed to the increase in the initial temperature of the fluid. The reflective effect of this on fluid velocity within the channel is graphed in figure 7 where the fluid velocity also increases with increase in $\phi$.

Effect of varying fluid viscosity on the fluid velocity is portrayed in figure 8. The figure demonstrates that the fluid velocity within the channel increase with decrease in viscosity (increase in $\lambda$) of the fluid. This behavior is accredited to the loosing of cohesive force between the fluid molecules.

The effect of varying Gr on the fluid velocity is depicted in figure 9 where the figure shows that the fluid velocity increases with increase in Gr. This is the consequence of the increase in the buoyancy force of the fluid molecules within the channel.
Figure 10 reflects the effect of varying the velocity of the moving boundary. The figure shows that the fluid velocity increases with increase in M. This is due to the physical fact that in a nonslip regime, the thin film of fluid adjacent to the moving plate moves with the velocity of the moving plate.

The effect of varying parameters on the rate of heat transfer (Nusselt number) between the working fluid and the channel plates is presented on Table 1. The table displays that with growing $R_T$; $Nu_0$ decreases while $Nu_1$ increases. This clearly reveals the increase in temperature difference between the channel plates. The table further shows that with small increase in $\epsilon$; $Nu_1$ increases whereas $Nu_0$ increases. Again; with small increase in $\phi$ and growing $R_T$. $Nu_0$ decreases while $Nu_1$ increases.
Effect of varying physical parameters on skin friction is presented on table 2. The table reflects that both $\tau_0$ and $\tau_1$ increases with initial increase in $\lambda$ and later they decreases with more increase in $\lambda$. As the thermal conduction of the fluid decreases; both $\tau_0$ and $\tau_1$ decreases with initial increase in $\lambda$ and later they decreases with further increase in $\lambda$. On escalating thermal radiation and temperature difference, $\tau_0$ and $\tau_1$ are view to increase with initial increase in $\lambda$ and they later decreases with further increase in $\lambda$.

VALIDATION OF THE RESULTS

This section validates the accuracy of the results realized in this investigation. In order to do this, the parameters $R_T$, $\lambda$ and $\varepsilon$ are suppressed and the parameter M is set to one (all in the present study). The resulting equations are then compared with those obtained in the published work of Jha and Ajibade (2010) on silencing heat generating/absorbing parameter (i.e. $S = 0$). The results of the comparison are displayed in table 3 below:

Table 3: Numerical values on table 3 shows that the two studies agree excellently with each other.

| Jha and Ajibade (2010) when $S = 0$ | Present study when $R_T = \varepsilon = 0$ and $M = 1$ |
|---|---|
| $y$ | $y$ | $u(y)$ | $u(y)$ |
| 0.1 | 0.9000 | 1.1850 | 0.9000 | 0.1850 |
| 0.3 | 0.7000 | 1.3000 | 0.7000 | 1.3000 |
| 0.5 | 0.5000 | 1.1250 | 0.5000 | 1.1250 |
| 0.7 | 0.3000 | 0.7600 | 0.3000 | 0.7600 |
| 0.9 | 0.1000 | 0.2650 | 0.1000 | 0.2650 |

CONCLUSION

In this paper, Adomian decomposition method has been successfully applied to study and investigate the steady free convective Couette flow through a vertical channel with nonlinear thermal radiation, dynamic viscosity and dynamic thermal conductivity. The following results are deduced:

i. The fluid velocity and temperature within the channel are both found to increase with increase in thermal conduction of the fluid.

ii. Decrease in viscosity of the fluid in the channel has been identified to increase the fluid velocity during flow.

iii. Increase in thermal radiation has been acknowledged to increase both the fluid velocity and temperature in the channel.

iv. Decreasing the fluid viscosity has been recognized to cause initial increase in the skin friction between the plates and the fluid.

v. With fixed parameters and with decrease in thermal conduction, the skin frictions on the plates are found to decrease with decrease in viscosity of the fluid.
vi. On decreasing the fluid thermal conduction, the Nusselt number on the heated plate is grasped to increase while it decrease on the cold plate.

Nomenclature and Greek symbols:

| Symbols | Interpretation                        | Unit               |
|---------|---------------------------------------|--------------------|
| \(y'\)  | Dimensional length                    | m                  |
| \(y\)   | Dimensionless length                  |                    |
| \(g\)   | Acceleration due to gravity           | ms\(^{-2}\)        |
| \(k\)   | Thermal conductivity                  | W/mK               |
| \(T\)   | Dimensional temperature               | K                  |
| \(h\)   | Dimensional channel width             | m                  |
| \(T_w\) | Wall temperature                      | K                  |
| \(T_0\) | Ambient temperature                   | K                  |
| \(u\)   | Dimensional velocity                  | m/s\(^{-1}\)       |
| \(U\)   | Dimensionless velocity                |                    |
| \(\alpha\) | Thermal diffusivity                  | m\(^2\)/s\(^{-1}\) |
| \(\delta\) | Absorption coefficient               |                    |
| \(\beta\) | Volumetric expansion coefficient      | K\(^{-1}\)         |
| \(\mu\) | Variable fluid viscosity              | kg/m\(^1\)s\(^{-1}\) |
| \(\mu_0\) | Dynamic fluid viscosity               | kg/m\(^1\)s\(^{-1}\) |
| \(Gr\)  | Grasshopper number                    |                    |
| \(R_f\) | Thermal radiation parameter           |                    |
| \(S\)   | Heat generating/absorbing parameter   |                    |
| \(q_r\) | Radiative heat flux                   | W/m\(^2\)          |
| \(\phi\) | Temperature difference parameter      | K                  |
| \(\theta\) | Dimensionless temperature            |                    |
| \(\sigma\) | Stefan-Boltzmann constant            | J/K\(^{-1}\)       |
| \(\varepsilon\) | Thermal conductivity variation parameter |                    |
| \(\lambda\) | Viscosity variation parameter        |                    |
| \(\mathbb{R}\) | Set of real numbers                 |                    |
| \(Nu_0\) | Nusselt number on the plate at \(y=0\) |                    |
| \(Nu_1\) | Nusselt number on the plate at \(y=1\) |                    |
| \(Nu\)  | Nusselt number                        |                    |
| \(\tau_0\) | Skin friction on the plate at \(y=0\) |                    |
| \(\tau_1\) | Skin friction on the plate at \(y=1\) |                    |
| \(\tau\) | Skin friction                         |                    |

REFERENCES

Abel, M.S. and Mashesha, N. (2007). Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. J. Applied Mathematical Modelling, 32: 1965-1983.

Andrian, N, Stefan, N., Martins, N. and Holger, V. (1997). Interpolation Correlation for properties of humid air in the temperature range 100 to 200. American Institute of Physics and American Chemical Society, 26(4):1111-1123.

Adomian, G. (1994). Solving Frontier Problems of Physics: The Decomposition Method, Boston, MA Kluwer.

Ajibade, A.O. and Yusuf, A.B. (2019). Variable Fluid Properties and Thermal Radiation Effects on Natural Convection Couette Flow through a Vertical Porous Channel. Journal of Advances in Mathematics and Computer Science, 3(1):1-17.

Blas, Z. Effects of thermophysical variable properties on liquid sodium convective flows in a square enclosure, J. Heat Trans., 141, 031301, 2019.

Carey, V.P. and Mollendorf, J. C. (1978). Natural convection in liquid with temperature dependent viscosity. In proceedings of the 6th International Heat Transfer Conference.Toronto, 2:211-217.

Dubuffet, F., Yuen, D.A. and Rabinowicz, M. Effects of a realistic mantle thermal conductivity on the patterns of 3D convection. Earth Planet Sci. Lett. 171 (3): 401 – 409, 1999.

Elbashbeshy, E. M. A. and Bazid, M.A. (2000). The
effect of temperature dependent viscosity on heat transfer over a continuous moving surface. Journal of Applied Physics, 33: 2716-2721.

Ganji, D.D., Mohsen, S., Younus, M.J. and Ellahi, R. (2015). Effect of thermal radiation on magneto hydrodynamics nanofluid flow and heat transfer by means of two phase model. Journal of Magnetism and Magnetic Materials, 374: 36-43.

Gray, J., Kassory, D.R., Tadjeran, H. and Zebib, A. (1982). Effect of significant viscosity variation on convective heat transport in water-saturated porous media. Journal of Fluid Mechanics, 117: 233-249.

Hofmeister, A.M. (1999). Mantle values of thermal conductivity and the geotherm from phonon life times Science. 283:1699-1706.

Jha, B K and Ajibade, A. O. (2010). Unsteady Free Convective Couette Flow of Heat Generating/Absorbing Fluid. International Journal of Energy and Technology. 2(12): 1-9.

Hossain, M. A., Khananer, K., and Vafai, K. (2001). The effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. Int. Therm. Sci., 40: 115-124.

Jha, B.K., Yabo, I.B. and Lin, J. (2017). Transient Natural Convection in an annulus with Thermal Radiation. Journal of Applied Mathematics, 8: 1351-1366.

Kafoussius, N.G. and Williams, E.W. (1995). The effect of temperature-dependent viscosity on the free convective laminar boundary layer flow past a vertical isothermal flat plate, Acta Mech., 110: 123-137.

Kafoussius, N.G. and Rees, D.A.S. (1998). Numerical study of the combined free and forced convective laminar boundary layer flow past a vertical isothermal flat plate with temperature-dependent viscosity, Acta Mech., 127: 39-50.

Costa, A. and Macedonio, G. (2003). Viscous heating in fluids with temperature dependent viscosity: Implication for magma flows. Non-linear Proceeding Geophysics., 10: 545-555.

Cherruault, Y. (1990). Convergence of Adomian’s method. J. of Mathl Comput. Modelling. 14: 83-86. Elbashbeshy, E. M. A. and Bazid, M.A. (2000). The effect of temperature dependent viscosity on heat transfer over a continuous moving surface. Journal of Applied Physics, 33: 2716- 2721.

Kay, A. (2017). Comments on ‘Combined effect of variable viscosity and thermal conductivity on free convection flow in a vertical channel using DTM’ by J.C. Imavathi and M. Shekar. Meccanica, 52 (6): 1493-1494.

Mandal, H.K., Das, S. and Jana, R.N. (2014). Transient Free Convection in a Vertical Channel with Variable Temperature and Mass Diffusion. IISTE, 23.

Magyari, E. and Pantokratoras, A. (2011). Note on the effect of thermal radiation in the linearized Rosseland approximation on the heat transfer characteristics of various boundary layer flows. International Communications in Heat and Mass Transfer.38: 554-556.

Mehta, K.N. and Sood, S. (1992). Transient free convection flow with temperature-dependent viscosity in a fluid saturated porous medium. International Journal of Engineering Science, 30: 1083-1087.

Makinde, O.D. and Ogulu, A. (2011). The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Journal of Chemical Engineering Communications. 195:1575-1584.

Makinde, O.D. (2008). Entropy generation analysis for variable viscosity channel flow with non-uniform wall temperature. Journal of Applied Energy. 85:384-393.

Makinde, O. D., Olajuwon, B. I. and Gbolagade A.W. (2007). Adomian Decomposition Approach to a Boundary Layer Flow with Thermal Radiation past a Moving Vertical Porous Plate. International Journal of Applied Mathematics and Mechanics, 3(3):62-70.

Makinde, O.D. and Ibrahim, S.Y. (2011). Radiation effect on chemically reacting magneto hydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate. International Journal of Physical Sciences, 6(6): 1508-1516.

Miyatake, O., Fuzii, T., and Tanaka, T. (1973). Natural Convection heat transfer between vertical plates, one plate with a uniform heat flux and the other thermally insulated, Heat Transfer-Jap.Res.4:25-33.

Miyatake, O. and Fuzii, T. (1972). Free convection heat transfer between vertical parallel plates-one plate isothermally heated and the other thermally insulated. Heat Transfer Jap. Res., 3:30-38.

Nelson D.J. and Wood, B.D. (1989a). Combined heat and mass transfer natural convection between vertical parallel plates. Int. J. Heat and Transfer, 32:1779-1787.

Nelson D.J. and Wood, B.D. (1989b). Fully developed combined heat and mass transfer natural convection between vertical parallel plates with asymmetric boundary conditions. Int. J. Heat and Transfer, 32:1789-1792.

Makinde, O.D. and Chinoyoka, T. (2010). Numerical Investigation of Transient Heat Transfer to Hydromagnetic Channel Flow with Radiative Heat and Convective Cooling. Communication in Nonlinear Science and Numerical Simulation, 15:3919-3930.

Mukhopadhyay, S. (2009). Effects of Radiation and Variable Fluid Viscosity on Flow and Heat Transfer along a Symmetric Wedge. Journal of Applied Fluid Mechanics, 2(2): 29-34.
Ostrach, S. (1952). Laminar Natural Convection flow and Heat Transfer of fluids with and without Heat sources in channels with constant wall Temperatures, NACA TN, pp 2863.

Ostrach, S. (1954). Combined natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channel with limberly varying temperature, NASA TN No.3141.

Reynold, O. (1883). An experimental investigation of the Circumference which determine whether the motion of water shall be director sinusoidal and of the law of resistance in parallel channels. Philosophical Transactions of the Royal Society. 174: 935-982.

Rihab, H. Raoudha, C., Faouzi, A., Abdelmajid, J. and Sassi, B.N. Lattice Boltzmann method for heat transfer with variable thermal conductivity. Int, J. Heat and Technology, 35(2):313-324, 2017.

Rosseland, S.E. (1931). Astrophysik and atom-theorische Grundlagen. Springer-Verlag, Berlin. 41-44

Singh, A.K. and Paul, T. (2006). Transient natural convection between two vertical walls heated/cooled asymmetrically, Int. J. of Applied Mechanics and Engineering, 1:143-154.

Seddeek, M. A. and Salem, A. M. (2006). Further results on the variable viscosity with magnetic field on flow and heat transfer to a continuous moving flat plate. Physics. Letter A., 353: 337-340.

Sheikholesmi, M., Ganji, D.D., Javed, Y. and Ellahi, R. (2015). Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, Journal of Magnetism and Magnetic Materials. 174:36-43.

Sharma, V. K. and Aisha, R. (2014). Effect of Variable Thermal Conductivity and Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM. Global Journal of Science Frontier Research: Mathematics and Decision Making. 14(2):56-63.

Sparrow, E. M. and Cess., R. D. (1962). Radiation Heat Transfer, augmented edition, Hemisphere,Washington,D.C. Sparrow, E.M., Eichhorn, T. and Gregg, J.L. (1959). Combined forced and free convection in boundary layer flow. Physics of Fluids. 2:319-328.

Starlin, I., Yuen, D.A. and Bergeron, S.Y. Thermal evolution of sedimentary basin formation with temperature-dependent conductivity, Geophys, Res, Lett., 27(02):265–268, 2000.

Van den Berg, A.P., Rainey, E.S.G. and Yuen, D.A. (2005). The combined influence of variable thermal conductivity, temperature- and pressure-dependent viscosity and core-mantle coupling on thermal evolution. Physics on the earth and Planetary Interior, 149:259-278.

Van den Berg, A.P., Yuen, D.A. and Steinbach, V. (2001). The Effects of Variable Thermal Conductivity on Mantle Heat-Transfer. Geophysical Research Letters, 28(5): 875-878.

Yabo, I.B., Jha, B.K. and Lin, J. (2016). Combined Effects of Thermal Diffusion and Diffusion-Thermo Effects on Transient MHD Natural Convection and Mass Transfer Flow in a Vertical Channel with Thermal Radiation. Journal of Applied Mathematics. 6: 2354-2373.

Yusuf, A.B. and Abiodun O.A. (2018a). Combined Effects of Variable Viscosity and Thermal Radiation on Unsteady Natural Convection Flow through a Vertical Porous Channel. FUDMA Journal of Sciences (FJS), 2(2): 273-287.

Yusuf, A. B. and Ajibade, A.O. (2020). Combined effects of variable viscosity, viscous dissipation and thermal radiation on unsteady natural convection Couette flow through a vertical porous channel. FUDMA Journal of Sciences (FJS), 2(2): 273-287, 4(2):135-150.