A Survey of Knowledge Tracing

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Abstract—High-quality education is one of the keys to achieving a more sustainable world. In contrast to traditional face-to-face classroom education, online education enables us to record and research a large amount of learning data for offering intelligent educational services. Knowledge Tracing (KT), which aims to monitor students’ evolving knowledge state in learning, is the fundamental task to support these intelligent services. In recent years, an increasing amount of research is focused on this emerging field and considerable progress has been made. In this survey, we categorize existing KT models from a technical perspective and investigate these models in a systematic manner. Subsequently, we review abundant variants of KT models that consider more strict learning assumptions from three phases: before, during, and after learning. To better facilitate researchers and practitioners working on this field, we open source two algorithm libraries: EduData for downloading and preprocessing KT-related datasets, and EduKTM with extensible and unified implementation of existing mainstream KT models. Moreover, the development of KT cannot be separated from its applications, therefore we further present typical KT applications in different scenarios. Finally, we discuss some potential directions for future research in this fast-growing field.

Index Terms—Knowledge Tracing; Intelligent Education; Educational Data Mining; Adaptive Learning; User Modeling

1 INTRODUCTION

The world changes every day, education bears the responsibility of enabling individuals to keep learning so that they can adapt to the ever-changing society [1] [2] [3]. With the proliferation of the Internet and mobile communication technology, online education is developing on an unprecedented scale and gradually becoming a fashionable learning style [4] [5]. Compared with traditional education, online education breaks free of the limitations of physical classrooms and enables teaching and learning to occur flexibly, anytime and anywhere [6] [7] [8]. Moreover, online learning has the potential to bring high-quality and equitable education by providing each student with an optimal and adaptive learning experience. At the same time, online learning systems (such as Coursera, ASSISTment and Massive Online Open Courses) [9] [10] have proven to be even more effective than traditional learning styles, since they are able to offer more intelligent educational services, such as adaptive recommendations of individualized learning paths to students. In order to provide these intelligent services for each student, online learning systems continuously record a massive amount of available data about student-system interactions, which can be further mined to assess their knowledge levels and learning preferences. Specifically, Knowledge Tracing (KT) [11] is one of the most fundamental and critical research fields for online education, which utilizes a series of machine learning methods capable of exploiting educationally related data to monitor students’ dynamic knowledge states [12] [13].

Fig. 1 presents a simple schematic diagram of knowledge tracing. While studying, the learning system continuously records the student’s observed learning data, including exercises and the related knowledge concepts (e.g., equality, inequality, plane vector and probability, which are represented in different colors), and students’ answers (i.e., correct or incorrect responses). Benefiting from the development of intelligent education [14] and the methods of data anonymization [15], a large amount of side information is also recorded, such as response time, opportunity count and tutor intervention, which more completely reflect students’ learning process. In addition, new technologies and approaches, such as learning effective representations from limited educational data with crowdsourced labels, are also in constant development [16]. On the basis of these growing learning data, researchers are trying to maintain an estimate of students’ behind evolving knowledge states, which is exactly what KT is aiming for. Taking Fig. 1 as an example, the student’s prior knowledge states on the four knowledge concepts are 0.2, 0.4, 0.4 and 0.5 respectively. The student continues to absorb new knowledge in learning, which can be reflected by the gradually increased areas of the radar map that indicate the student’s knowledge mastery. After a period of learning, the student’s knowledge states reach 0.9, 0.8, 0.8 and 0.7 on corresponding knowledge concepts, suggesting good knowledge growth. In above learning process, KT models aim to monitor the changes in students’ knowledge states, which in turn provides intelligent learning services to students (e.g., recommending adaptive exercises that suit individual’s ability level). Indeed, KT is of great significance for both online learning systems and students. First, KT models enable the devel-
Development of personalized adaptive learning systems. Once grasping students’ knowledge states, the learning system can customize more suitable learning schemes for different students, making it possible to teach students in accordance with their proficiency. Second, students themselves can also better understand their learning process and gradually pay more attention to the skill with poor mastery [17, 18].

Knowledge tracing has been studied for decades. Early KT-related studies can be traced back to the late 1970s; these works focused primarily on confirming the effectiveness of mastery learning [19]. To the best of our knowledge, Corbett and Anderson [11] were the first to introduce the concept of knowledge tracing and they utilized Bayesian networks to model the student learning process, which was known as Bayesian Knowledge Tracing. Subsequently, the significance of KT was recognized by a broader spectrum of people, and increasing attention has been channeled into KT-related research. For example, many logistic models have been applied for KT, including Learning Factor Analysis [20] and Performance Factor Analysis [21]. In recent years, deep learning has boosted research into the KT task, due to its powerful capacity to extract and represent features and its ability to discover intricate structure [22]. For example, Deep Knowledge Tracing introduced recurrent neural networks (RNNs) [23] into the KT task and was found to significantly outperform previous methods [24]. After that, by considering various characteristics of the learning sequence, more methods have introduced various types of neural networks to the KT task [25, 26, 27]. Moreover, due to the requirements of practical applications, many variants of KT models have also been continuously developed, and KT has already been broadly applied in many educational scenarios.

While novel KT models and their large number of variants and applications continue to emerge, there are few survey papers on this young research field, especially on the emerging deep learning-based KT models. To this end, the present survey targets this research gap and aims to comprehensively review the current development and the state-of-the-art works on the KT task in a systematic manner. As shown in Fig. 2, we first categorize existing KT models from a technical perspective, which splits them into three categories: (1) probabilistic models, (2) logistic models and (3) deep learning-based models. Then, we comprehensively review existing KT models according the proposed taxonomy. Subsequently, we introduce a large number of variants of these basic KT models, which consider more strict assumptions about more complete learning process in different learning phases. Due to the complexity of different KT models, to better aid researchers and practitioners in implementing KT models, as well as to facilitate community development in this domain, we have open sourced two algorithm libraries, i.e., EduData [1] and EduKTM [2], which include most existing KT-related datasets, extensible and unified implementations of existing KT models and relevant resources. In addition, we present several typical applications of KT in real learning scenarios. Finally, we discuss some potential future research directions. In general, this paper presents an extensive survey of knowledge tracing that can serve as basic guidelines for both researchers and practitioners in future.

The remainder of this survey is organized as follows. Section 2 presents both the formal definition of the KT task and the taxonomy of KT models. Section 3 provides a review of the three categories of basic KT models. Section 4 describes the variants of basic KT models. Section 5 gives the summary of existing datasets for evaluating KT models and details of the algorithm libraries we have released. Section 6 introduces the extensive applications of KT in different scenarios. Section 7 discusses some potential future research directions. Finally, section 8 summarizes the paper.

2 OVERVIEW

2.1 Problem Definition

In an online learning system, supposing there exists a set of students $S$ and a set of exercises $E$, where students are asked to answer different exercises in order to achieve mastery of the related knowledge. Each exercise is related to specific Knowledge Concepts (KCs), which represent the basic units of knowledge. Generally, the name given to the knowledge related to exercises differs across online learning platforms. For instance, it is named skill in ASSISTments [28]. To promote better understanding, we refer to these uniformly as knowledge concepts throughout this paper, and denote the set of all KCs as $K$. Moreover, $M \in K$ and $K$ are respectively used to represent the total number of exercises and KCs. Subsequently, the learning sequence of a student can be represented as $X = \{(e_1, a_1, r_1), (e_2, a_2, r_2), ..., (e_i, a_i, r_i), ..., (e_N, a_N, r_N)\}$, where the tuple $(e_i, a_i, r_i)$ represents the learning data of the $i$-th exercise of student with KC $a_i$ achieving the score $r_i$.

1. https://github.com/bigdata-ustc/EduData
2. https://github.com/bigdata-ustc/EduKTM
interaction at the $t$–th time step, $e_t$ represents the exercise, $a_t$ represents the correctness label (i.e., with 1 for correct and 0 for incorrect answers), $r_t$ stands for the side information recorded in this learning interaction, and $N$ is the length of the learning sequence. The research problem of knowledge tracing can thus be defined as follows:

Given sequences of learning interactions in online learning systems, knowledge tracing aims to monitor students’ evolving knowledge states during the learning process and predict their performance on future exercises. The measured knowledge states can be further applied to individualize students’ learning schemes in order to maximize their learning efficiency.

It is worth noting that some works directly regarded the KT task as student performance prediction, without considering students’ knowledge states [29] [30] [31]. We agree that predicting student performance is of great significance. However, we have to point out that student performance prediction is now the most popular means to evaluate the quality of the knowledge state traced by KT models. KT focuses more on students’ knowledge states, especially their interpretability and rationality, which is related to the students’ acceptance of the conclusions given based on the KT model [11] [24] [32].

2.2 Categorization

As shown in Fig. 2, we categorize and summarize the existing KT models according to their technical differences. In more detail, the proposed taxonomy splits existing KT methods into three categories: (1) probabilistic models, (2) logistic models, and (3) deep learning-based models. In addition to these basic KT models, we also introduce a large number of their variants, which respectively consider a more complete learning process in three distinct learning phases: (1) modeling individualization before learning, (2) incorporating engagement and utilizing side information during learning, and (3) considering forgetting after learning. Moreover, we also summarize the extensive applications of KT in different educational scenarios, including learning resources recommendation, adaptive learning and educational gaming.

3 Basic Knowledge Tracing Models

In this section, as shown in Table 4, we present the basic KT models on the basis of our taxonomic framework. According to the timeline of development, we will first introduce the probabilistic models, followed by the logistic models and finally the deep learning-based ones.

3.1 Probabilistic Models

The basic paradigm for probabilistic models in KT assumes that the learning process follows a Markov process, where students’ latent knowledge states can be estimated by their observed performance [11]. In the following, we will present two basic probabilistic models in our taxonomy framework: the original Bayesian knowledge tracing (BKT) and the dynamic Bayesian knowledge tracing (DBKT).

3.1.1 Bayesian Knowledge Tracing

To the best of our knowledge, Bayesian Knowledge Tracing (BKT) is the first KT model to be proposed [11]. The topology of BKT’s structure is illustrated in Fig. 3; here, the unshaded nodes represent unobservable latent knowledge states, while the shaded nodes represent the observable answers of the student.

BKT is a special case of Hidden Markov Model (HMM). There are two types of parameters in HMM: transition probabilities and emission probabilities. In BKT, the transition probabilities are defined by two learning parameters: (1) $P(T)$, the probability of transition from the unlearned state to the learned state; (2) $P(F)$, the probability of forgetting previously mastered knowledge. Moreover, the emission probabilities are determined by two performance parameters: (1) $P(G)$, the probability that a student will guess correctly in spite of non-mastery; (2) $P(S)$, the probability that a student will make a mistake in spite of mastery. Furthermore, the parameter $P(L_0)$ represents the initial probability of mastery. BKT assumes a two-state student modeling framework: the knowledge is either learned or unlearned by the student, and there is no forgetting once a student has learned the knowledge. Given the observations of the student’s learning interactions, the following equation is used to estimate the knowledge state and the probability of correct answers:

$$
P(L_n) = P(L_n|\text{Answer}) + (1 - P(L_n|\text{Answer}))P(T),$$

$$P(C_{n+1}) = P(L_n)(1 - P(S)) + (1 - P(L_n))P(G),$$

(1)

where $P(L_n)$ is the probability that a KC is mastered at the n-th learning interaction, $P(C_{n+1})$ is the probability of correct answers at the next learning interaction. $P(L_n)$ is the sum of two probabilities: (1) the probability that the KC is already mastered; (2) the probability that the
A summary of different types of basic knowledge tracing models.

| Category | Typical approach | Technique | KC relationship | Knowledge state |
|----------|------------------|-----------|-----------------|-----------------|
| Probabilistic models | dynamic Bayesian knowledge tracing [33] | dynamic Bayesian networks | independent | unobservable node in HMM |
| Logistic models | learning factor analysis [20] | logistic regression | independent | the output of logistic regression function |
| Deep learning-based models | deep knowledge tracing [24] | RNN/LSTM | discover automatically | the hidden state |

Fig. 3. The topology of Bayesian Knowledge Tracing [11]. K are the unobserved knowledge nodes, A are the observed performance (answer) nodes, \( P(L_0) \) represents the initial probability, \( P(T) \) is the transition probability, \( P(G) \) is the guessing probability and \( P(S) \) is the slipping probability.

Knowledge state will convert to the mastered state. The posterior probability \( P(L_{n-1}|Answer) \) is estimated as follows:

\[
P(L_{n-1}|\text{correct}) = \frac{P(L_{n-1})P(\text{correct})}{P(L_{n-1})P(\text{correct}) + P(L_{n-1})P(\text{incorrect})} = \frac{P(L_{n-1})P(\text{correct})}{P(L_{n-1})P(\text{correct}) + P(L_{n-1})P(\text{incorrect})} \tag{2}
\]

3.2 Logistic Models

Logistic models are a large class of models based on logistic functions, where the probability of answering exercises correctly can be represented by a mathematical function of student and KC parameters. The logistic models first use different factors in students’ learning interactions to compute an estimation of the student and KC parameters, then utilize a logistic function to transform this estimation into the prediction of the probability of mastery [39]. In the following, we will introduce three logistic models: (1) Learning Factor Analysis (LFA), (2) Performance Factor Analysis (PFA) and (3) Knowledge Tracing Machines (KTM).

3.2.1 Learning Factor Analysis

The LFA model [20] considers the following learning factors:

- Initial knowledge state: parameter \( \alpha \) estimates the initial knowledge state of each student;
- Easiness of KCs: parameter \( \beta \) captures the easiness of different KCs;
- Learning rate of KCs: parameter \( \gamma \) denotes the learning rate of KCs.

The standard LFA model takes the following form:

\[
p(\theta) = \sigma \left( \sum_{i \in N} \alpha_i S_i + \sum_{j \in KCs} (\beta_j + \gamma_j T_j) K_j \right), \tag{4}
\]

where \( \Phi : A \times H \rightarrow \mathbb{R}^F \) denotes a mapping from the observed space \( A \) and the latent space \( H \) to an \( F \)-dimensional feature vector. \( Z \) is a normalizing constant, \( w \) denotes the weights.

3.2.2 Performance Factor Analysis

The PFA model [21] can be seen as an extension of the LFA model that is especially sensitive to the student performance. In contrast to the LFA model, PFA considers the following different factors:

- Previous failures: parameter \( f \) is the prior failures for the KC of the student;
- Previous successes: parameter \( s \) represents the prior successes for the KC of the student;
- Easiness of KCs: parameter \( \beta \) means the easiness of different KCs, which is the same as in the LFA model.

The standard PFA model takes the following form:
where \( \mu \) and \( \nu \) are the coefficients for \( s \) and \( f \), which denote the learning rates for successes and failures.

### 3.2.3 Knowledge Tracing Machines

The KTM model [34] utilizes factorization machines (FMs) [40, 41] to generalize previous logistic models to higher dimensions. FMs were originally proposed as a general predictor that works with any real valued feature vector, which can model all interactions between variables using factorized parameters [42]. FMs provide a means of encoding side information about exercises or students into the model. Therefore, KTM is able to model the knowledge mastery of the student based on a sparse set of weights for all features involved in the learning process. Let \( L \) be the number of features; here, the features can be related either to students, exercises, KCs or any other side information. The learning interaction is encoded by a sparse vector \( l \) of length \( L \), where \( l_i > 0 \) if feature \( i \) is involved in the interaction. The probability \( p(\theta) \) of the correct answer is determined by the following equations:

\[
p(\theta) = \sigma(\mu + \sum_{i=1}^{L} w_i l_i + \sum_{1 \leq i < j \leq L} l_i l_j \langle v_i, v_j \rangle),
\]

where \( \mu \) is the global bias, the feature \( i \) is modeled by the bias \( w_i \in \mathbb{R} \) and the embedding \( v_i \in \mathbb{R}^d \) (\( d \) is the dimension). Note that only features with \( l_i > 0 \) will have impacts on the predictions.

### 3.3 Deep Learning-based Models

The cognitive process can be influenced by many factors at both the macro and micro levels. It is difficult for probabilistic or logistic models to adequately capture a cognitive process of high complexity [24]. Deep learning has a powerful ability to achieve non-linearity and feature extraction, making it well suited to modeling the complex learning process, especially when there is a much larger amount of learning interaction data available [43]. In recent years, many research works on deep learning-based KT models have been proposed and achieved quite good performance. Nevertheless, deep learning-based models are poorly interpretable due to their end-to-end learning strategy, which limits their further applicability owing to the crucial significance of interpretability for KT. We will introduce deep learning-based models from the following five aspects: (1) deep knowledge tracing, (2) memory-aware knowledge tracing, (3) exercise-aware knowledge tracing, (4) attentive knowledge tracing, and (5) graph-based knowledge tracing.

#### 3.3.1 Deep Knowledge Tracing

Deep knowledge tracing (DKT) is the first approach to introduce deep learning into KT [24], which utilizes recurrent neural networks (RNNs) [23] to model the students’ learning process. DKT applies RNNs to process the input sequence of learning interactions over time, maintaining a hidden state that implicitly contains information about the history of all past elements of the sequence. The hidden state evolves based on both the previous knowledge state and the present input learning interaction [24]. DKT provides a high-dimensional and continuous representation of the knowledge state, making them better able to model the complex learning process. Generally, RNNs’ variant long short-term memory (LSTM) networks [44] are more commonly used in the implementation of DKT, which is made more powerful through considering forgetting.

Fig. 4 illustrates the process of deep knowledge tracing. In DKT, exercises are represented by their contained KCs. For datasets with different numbers of KCs, DKT applies two different methods to convert students’ learning interactions \( X = \{(e_1, a_1), (e_2, a_2), \ldots, (e_t, a_t), \ldots, (e_N, a_N)\} \) into a sequence of fixed-length input vectors. More specifically, for datasets with a small number \( K \) of unique KCs, \( x_t \in \{0, 1\}^{2K} \) is set as a one-hot embedding, where \( x_{tK} = 1 \) if the answer \( a_t \) of the exercise with KC \( k \) was correct or \( x_{tK+K} = 1 \) if the answer was incorrect. For datasets with a large number of unique KCs, one-hot embeddings are too sparse; therefore, DKT sets each input vector \( x_t \) to a corresponding random vector, then takes the embedded learning sequence as the input of RNNs and applies a linear mapping and activation function to the output hidden states to obtain the knowledge state of students:

\[
h_t = \tanh(W_{hh} x_t + W_{hh} h_{t-1} + b_h),
\]

\[
y_t = \sigma(W_{yh} h_t + b_y),
\]

where \( \tanh \) is the activation function, \( W_{hh} \) is the input weights, \( W_{hh} \) is the recurrent weights, \( W_{yh} \) is the readout weights, and \( b_h \) and \( b_y \) are the bias terms.

DKT has demonstrated superior performance relative to the probabilistic and logistic models. Nevertheless, DKT also has some unavoidable shortcomings. For example, the DKT model lacks interpretability: it is difficult to figure out how the hidden states can represent students’ knowledge states, and it cannot explicitly determine a student’s level of knowledge mastery from the hidden state [43]. Yeung and Yeung [45] further revealed that there are two unreasonable phenomena in DKT that violate common sense, i.e., (1) it fails to reconstruct the observed input, and (2) the predicted knowledge state is not consistent across time-steps. Overall, DKT remains a promising KT model [46].

#### 3.3.2 Memory-aware Knowledge Tracing

In order to enhance the interpretability of DKT, memory-aware knowledge tracing introduces an external memory module [47] to store the knowledge and update the corresponding knowledge mastery of the student. The most representative one is Dynamic Key-Value Memory Networks (DKVMN) for knowledge tracing [45], which points out students’ specific knowledge states on various KCs. DKVMN initializes a static matrix called a key matrix to store latent
KCs and a dynamic matrix called a value matrix to store and update the mastery of corresponding KCs through read and write operations over time.

As shown in Fig. 5, an embedding matrix is first defined to obtain the embedding vector $k_1$ of the exercises. A correlation weight $w_t$ is then obtained by taking the inner product between the exercise embedding $k_1$ and the key vectors $M^k$, followed by the softmax activation:

$$w_t = \text{Softmax}(k_1 M^k),$$

where the correlation weight $w_t$ represents the correlation between the exercises and all latent KCs.

In the read operation, DKVMN predicts student performance based on the student’s knowledge mastery. Specifically, DKVMN reads students’ mastery of the exercise $r_t$ with reference to the weighted sum of all memory vectors in the value matrix using the correlation weight. The read content and the input exercise embeddings are then concatenated together and passed to a fully connected layer to yield a summary vector $f_t$, which contains both the student’s knowledge mastery and the prior difficulty of the exercise. Furthermore, the student’s performance can be predicted by applying another fully connected layer with a sigmoid activation function to the summary vector:

$$r_t = \sum_{i=1}^{N} w_t(i) M_{t}^v(i),$$

$$f_t = \tanh(W_f r_t + b_f),$$

$$p_t = \sigma(W_p f_t + b_p),$$

where $W_f$ and $W_p$ are the weights, $b_f$ and $b_p$ are bias terms.

In the write operation, after an exercise has been answered, DKVMN updates students’ knowledge mastery (i.e., the value matrix) based on their performance. Specifically, the learning interaction $(e_t, a_t)$ is first embedded with an embedding matrix $B$ to obtain the student’s knowledge growth $v_t$. Then DKVMN calculates an erase vector $erase_t$ from $v_t$ and decides to erase the previous memory with reference to both the erase vector and the correlation weight $w_t$. Following erasure, the new memory vectors are updated by the new knowledge state and the add vector $add_t$, which forms an erase-followed-by-add mechanism that allows forgetting and strengthening knowledge mastery in the learning process:

$$e_t = \sigma(W_e v_t + b_e),$$

$$M_{t+1}^v(i) = M_{t}^v(i) [1 - w_t(i) erase_t],$$

$$add_t = \tanh(W_d v_t + b_d),$$

$$M_{t+1}^v(i) = \tilde{M}_{t}^v(i) + w_t(i) add_t,$$

where $W_e$ and $W_d$ are the weights, $b_e$ and $b_d$ are bias terms.

Abdelrahman and Wang [48] point out that DKVMN failed to capture long-term dependencies in the learning process. Therefore, they propose a Sequential Key-Value Memory Network (SKVMN) to combine the strengths of DKT’s recurrent modelling capacity and DKVMN’s memory capacity. In SKVMN, a modified LSTM called Hop-LSTM is used to hop across LSTM cells according to the relevance of the latent KCs, which directly captures the long-term dependencies. In the write process, when calculating the knowledge growth of a new exercise, SKVMN enables it to consider the current knowledge state in order to get more reasonable results.

### 3.3.3 Exercise-aware Knowledge Tracing

Generally, the text content is of great significance for students to understand the exercises (e.g., similarity and difficulty). Huang et al. [49] used text materials to automatically predict their difficulties and Liu et al. [50] utilized the text content to find similar exercises. Yin et al. [51] further proposed a pre-training model called QuesNet for comprehensively learning the unified representations of heterogeneous exercises. Huang et al. [52] also designed an unsupervised DisenQNet to distictively learn exercise representations. Exercises’ text contents also have significant impacts on the KT task, as understanding exercises is the first step towards answering them for students.

Therefore, Liu et al. [37] proposed the Exercise-aware Knowledge Tracing (EKT) to mine the potential value of exercises’ text contents for KT. Specifically, instead of using one-hot encoding of exercises, EKT automatically learns the
semantic representation of each exercise from its text contents. As shown in Fig. [3(a)], EKT first uses Word2vec [33] to pre-train the embedding vector for each word $w_i$ in exercise $e_i$. It then constructs a bidirectional LSTM, which captures the word sequence from both forward and backward directions to learn the semantic word representation. Finally, the element-wise max-pooling operation is utilized to merge $L$ words’ contextual representations into a global embedding as $x_i = \max(v_1, v_2, ..., v_L)$. After obtaining the semantic representation $x_i$ of each exercise, in order to distinguish the different influences of correct and incorrect answers, EKT extends the answer $a_t$ to a feature vector $0 = (0, 0, ..., 0)$ with the same dimensions as $x_i$ and represents the learning interaction $\tilde{x}_i$ as follows:

$$\tilde{x}_i = \begin{cases} [x_i \oplus 0], & \text{if } a_t = 1, \\ [0 \oplus x_i], & \text{if } a_t = 0, \end{cases}$$

where $\oplus$ denotes the concatenation operation.

In order to explicitly measure students’ mastery of specific KCs, EKT further incorporates the information of KCs associated with each exercise. As shown in Fig. [3(b)], a memory module consisting of a matrix $M$ is set up to represent the knowledge. The KCs of each exercise are converted into a one-hot encoding $KC_t \in \{0, 1\}^K$ with the dimension equal to the total number $K$ of all KCs. An embedding matrix $W_K \in \mathbb{R}^{K \times d_k}$ transfers $KC_t$ into a low-dimensional vector $v_t \in \mathbb{R}^{d_k}$ as follows: $v_t = W_K^T KC_t$. Another static memory network [42] is then utilized to calculate the knowledge impact $\beta_t^i$, which quantifies the correlation weights between the $i$-th KC of the exercise and each knowledge memory vector in $M$, as follows:

$$\beta_t^i = \text{Softmax}(v_t^T M_i) = \frac{\exp(v_t^T M_i)}{\sum_{j=1}^{K} \exp(v_t^T M_j)}. \quad (12)$$

With the knowledge impact $\beta_t^i$ of each exercise, the input $\tilde{x}_i$ is replaced by a new joint representation: $\tilde{x}_i^t = \beta_t^i \tilde{x}_i$. At each learning step $t$, EKT updates the student’s knowledge state $H_t^i \in \mathbb{R}^{K \times d_k}$ by the LSTM networks [44], as follows:

$$i_t = \sigma(W_{\tilde{x}\tilde{x}}^i \tilde{x}_t + W_r H_{t-1}^i + b_i),$$
$$f_t = \sigma(W_{\tilde{x}\tilde{x}}^i \tilde{x}_t + W_r H_{t-1}^i + b_f),$$
$$o_t = \sigma(W_{\tilde{x}\tilde{x}}^i \tilde{x}_t + W_r H_{t-1}^i + b_o),$$
$$c_t = f_t \cdot c_{t-1} + i_t \cdot \tanh(W_{\tilde{x}\tilde{x}}^i \tilde{x}_t + W_r H_{t-1}^i + b_c),$$
$$H_t^i = o_t \cdot \tanh(c_t),$$

where $i_t$ is the input gate, $f_t$ is the forget gate and $o_t$ is the output gate; moreover, $W_{\tilde{x}\tilde{x}}$ and $W_r$ are the weight parameters, while $b_i$, $b_f$, $b_o$, $b_c$ are the respective bias terms.

Next, EKT utilizes the attention mechanism to realize the insights that students have similar performance on similar exercises. The matrix $H_{att}$ represents the attentive hidden state, where each slot $H_{att}^i$ can be computed as follows:

$$H_{att}^i = \sum_{j=1}^{T} \alpha_j H_t^j, \quad \alpha_j = \cos(x_{t+1}, x_t). \quad (14)$$

Finally, EKT can predict the student’s performance on the next exercise $e_{t+1}$ as follows:

$$s_{t+1} = \sum_{i=1}^{K} \beta_t^i H_{att}^i,$$
$$y_{t+1} = \text{ReLU}(W_1 \cdot [s_{t+1} \oplus x_{t+1}] + b_1),$$
$$a_{t+1} = \sigma(W_2 \cdot y_{t+1} + b_2),$$

where $W_1$ and $W_2$ are weights, $b_1$ and $b_2$ are bias terms.

In addition to exploring the text content to capture meaningful exercise information, Liu et al. [54] present a method for obtaining pre-trained exercise embeddings to universally improve the effectiveness of existing KT models. In KT scenarios, the explicit exercise-KC relations and the implicit exercise similarity as well as KC similarity exist simultaneously. To capture all these relations in exercise embeddings, Liu et al. [54] first represent them together with the exercise difficulties as a bipartite graph. They then utilize a product layer to fuse these features in the defined bipartite graph to obtain the pre-trained exercise embeddings. The experiments indicate that the pre-trained exercise embeddings can uniformly improve the performance of some KT models, such as DKT.

### 3.3.4 Attentive Knowledge Tracing

In the development of deep learning, the Transformer is first proposed for neural machine translation [29], which abandons recurrence and relies entirely on the self-attention mechanism to capture global dependencies within a sequence. Transformer has demonstrated superior power in feature extraction and dependency capture while maintaining high computational efficiency, and some representative Transformer-based pre-training models, such as BERT [55] and GPT [56], have obtained state-of-the-art results on many natural language processing tasks. Pandey and Karypis [26] propose a self-attentive model for knowledge tracing (SAKT), which directly applied the Transformer to capture long-term dependencies between students’ learning interaction and achieved good performance. Moreover, Wang et al. [57] propose an adaptive sparse self-attention network to generate the missing features and simultaneously obtain fine-grained predictions of student performance. Zhu et al. [58] find there was a vibration problem in DKT and present an attention-based KT model to solve it, which also further use the Finite State Automaton (FSA) to provide deep analysis about the knowledge state transition.

However, the complexity of the KT task limits the performance of the above simple Transformer applications. Choi et al. [30] propose a novel method named separated self-attentive neural knowledge tracing (SAINT) to improve self-attentive computation for KT adaption. Specifically, SAINT has an encoder-decoder structure, where the exercise and answer embeddings are separately encoded and decoded by self-attention layers. The separation of the input allows SAINT to stack self-attention layers multiple times and capture complex relations in student interactions. Subsequently, Shin et al. [31] propose the SAINT+ model to incorporate two temporal features into SAINT: namely, the answering time for each exercise and the interval time between two continuous learning interactions. Both SAINT and SAINT+ have achieved superior performance relative to SAKT.

Besides, Ghosh et al. [27] observe that SAKT does not outperform DKT and DKVMN in their experiments. Unlike SAINT and SAINT+, they present a context-aware attentive knowledge tracing (AKT) model, incorporating the self-attention mechanism with psychometric models. AKT comprises four modules: Rasch model-based embeddings, exercise encoder, knowledge encoder and knowledge retriever. Specifically, in the embedding module, the classic
Rasch model in psychometrics [59] is utilized to construct embeddings for exercises and KCs, as follows:

$$x_t = c_{e_t} + \mu_{e_t} \cdot d_{e_t},$$

where $c_{e_t} \in \mathbb{R}^D$ is the embedding of the KC of this exercise, $d_{e_t} \in \mathbb{R}^D$ is a vector that summarizes the variations in exercises with the related KC, and $\mu_{e_t} \in \mathbb{R}^D$ is a scalar difficulty parameter that controls the extent to which this exercise deviates from the related KC. The exercise-answer tuple $(e_t, a_t)$ is similarly extended using the scalar difficulty parameter for each pair:

$$y_t = q_{(e_t,a_t)} + \mu_{e_t} \cdot f_{(e_t,a_t)},$$

where $q_{(e_t,a_t)} \in \mathbb{R}^D$ is the KC-answer embedding, $f_{(e_t,a_t)} \in \mathbb{R}^D$ is the variation vector. Through the above embedding, exercises labeled as the same KCs are determined to be closely related while retaining important individual characteristics. Then, in the exercise encoder, the input is the exercise embeddings $\{e_1, ..., e_t\}$ and the output is a sequence of context-aware exercise embeddings $\{\tilde{e}_1, ..., \tilde{e}_t\}$. AKT designs a monotonic attention mechanism to accomplish the above process, where the context-aware embedding of each exercise depends on both itself and the previous exercises, i.e., $\tilde{e}_t = \tilde{f}_{enc}(e_1, ..., e_t)$. Similarly, the knowledge encoder takes exercise-answer embeddings $\{y_1, ..., y_t\}$ as input and outputs a sequence of context-aware embeddings of the knowledge acquisitions $\{\tilde{y}_1, ..., \tilde{y}_t\}$ using the same monotonic attention mechanism; these are also determined by students’ answers to both the current exercise and prior exercises, i.e., $\tilde{y}_t = \tilde{f}_{enc}(y_1, ..., y_t)$. Finally, the knowledge retriever takes the context-aware exercise embedding $\tilde{e}_{1:t}$ and exercise-answer pair embeddings $\tilde{y}_{1:t}$ as input and outputs a retrieved knowledge state $h_t$ for the current exercise. Since the student’s current knowledge state depends on answering the related exercise, it is also context-aware in AKT. The novel monotonic attention mechanism proposed in AKT is based on the assumptions that the learning process is temporal and students’ knowledge will decay over time. Therefore, the scaled inner-product attention mechanism utilized in the original Transformer is not suitable for the KT task. AKT uses exponential decay and a context-aware relative distance measure to computes the attention weights. Finally, AKT achieves outstanding performance on predicting students’ future answers, as well as demonstrating interpretability due to the combination of the psychometric model.

Moreover, as the contextual information of exercises has been identified as highly significant in EKT, Pandey and Srivastava [60] propose a relation-aware self-attention model for knowledge tracing (RKT), which utilizes contextual information to enhance the self-attention mechanism. RKT defines a concept called the relation coefficient to capture the relations between exercises, which are obtained by modeling the textual content of the exercises and students’ forgetting behaviors respectively. The contextual exercise representation is then fed to the self-attention layer to trace students’ knowledge states.

It is worth noting that Pu and Becker [61] have recently proposed that the attentive knowledge tracing models significantly benefited from students’ continuous repeated interactions on the same exercises in the learning process. In their experiments, once the repeated interactions in the dataset were removed, AKT’s performance declined to be close to that of DKVMN.

### 3.3.5 Graph-based Knowledge Tracing

Graph neural networks (GNNs), which are designed to handle complex graph-related data, have developed rapidly in recent years [62]. The graph represents a kind of data structure that models a set of objects (nodes) and their relationships (edges). From a data structure perspective, there is a naturally existing graph structure within the KCs. Therefore, incorporating the graph structure of the KCs as additional information should be beneficial to the KT task. Nakagawa et al. [25] presented graph-based knowledge tracing (GKT), which conceptualizes the potential graph structure of the KCs as a graph $G = (V, E)$, where nodes $V = \{v_1, v_2, ..., v_N\}$ represent the set of KCs and the edges $E \subseteq V \times V$ represent relationships of these KCs; moreover, $h^t = \{h^t_i\}_{i \in V}$ represents the student’s temporal knowledge state after answering the exercise at time $t$. The architecture for graph-based knowledge tracing is composed of three parts: (1) aggregate, (2) update and (3) predict.

In the aggregate module, GKT aggregates the temporal knowledge state and the embedding for the answered KC $i$ and its neighboring KC $j$:

$$h_{k}^{t} = \begin{cases} h_{k}^{t}, a^{t}E_{c} & (k = i), \\ h_{k}^{t}, E_{c}(k) & (k \neq i), \end{cases}$$

where $a^{t}$ represents the exercises answered correctly or incorrectly at time step $t$, $E_{c}$ is the embedding matrix for the learning interactions, $E_{c}$ is the embedding matrix for the KC, and $k$ represents the $k$-th row of $E_{c}$.

In the update module, GKT updates the temporal knowledge state based on the aggregated features and the knowledge graph structure, as follows:

$$m_{k}^{t+1} = \begin{cases} f_{self}(h_{k}^{t}) & (k = i), \\ f_{neighbor}(h_{k}^{t}, h_{k}^{t}) & (k \neq i), \end{cases}$$

$$\bar{m}_{k}^{t+1} = G_{ea}(m_{k}^{t+1}),$$

$$h_{k}^{t+1} = G_{gru}(\bar{m}_{k}^{t+1}, h_{k}^{t}),$$

where $f_{self}$ is the multilayer perceptron, $G_{ea}$ is the same erase-followed-by-add mechanism used in DKVMN, and $G_{gru}$ is the gated recurrent unit (GRU) gate [53]. Moreover, $f_{neighbor}$ defines the information propagation to neighboring nodes based on the knowledge graph structure.

In the predict module, GKT predicts the student’s performance at the next time step according to the updated temporal knowledge state:

$$y_{k}^{t} = \sigma(W_{k}h_{k}^{t+1} + b_{k}),$$

where $W_{k}$ is the weight parameter and $b_{k}$ is the bias term. Recently, in the attempt to further explore knowledge structure, Tong et al. [63] propose structure-based knowledge tracing (SKT), which aims to capture the multiple relations in knowledge structure to model the influence propagation among concepts. SKT is mainly motivated by an education theory, transfer of knowledge [65], which claims that students’ knowledge states on some relevant KCs will also be changed when they are practicing on a specific KC due to the potential knowledge structure among KCs. Therefore, a student’s knowledge state is determined by
not only the temporal effect from the exercise sequence, but also the spatial effect from the knowledge structure. To concurrently model the latent spatial effects, SKT presents the synchronization and partial propagation methods to characterize the undirected and directed relations between KCs, respectively. In this way, SKT measures influence propagation in the knowledge structure with both temporal and spatial relations. To get rid of dependence on knowledge structure, Long et al. [65] propose the Automatical Graph-based Knowledge Tracing (AGKT), which utilizes the automatical graph to measure students’ knowledge states automatically without annotation manual annotations.

4 Variants of Knowledge Tracing Models

So far, we have presented all basic KT models. Generally, these basic models are proposed based on simplified assumptions of the learning process (i.e., the two-state assumption in BKT), and mainly leverage the learning interactions (i.e., exercises and responses) to estimate students’ knowledge states. However, the real learning process is not simply represented by exercises and responses, but is influenced by many factors, such as students’ learning behaviors. In summary, the above basic KT models are straightforward but have reduced performance in real-world learning scenarios. Therefore, many variants have been proposed under more strict assumptions that reflect more complete learning process in real scenarios. In the following, according to different learning phases, we classify and review the current variants of basic KT models from three categories: (1) modeling individualization before learning, (2) incorporating engagement and utilizing side information during learning, and (3) considering forgetting after learning.

4.1 Modeling Individualization before Learning

Everything and everyone has unique characteristics. For example, Liu et al. [67] considered several personalized factors (e.g., spatial and temporal preferences) of various tourists for personalized travel package recommendation. Similarly, individualization in the KT task refers to the fact that different students tend to have different learning characteristics (i.e., different learning rates or prior knowledge). Considering the student-specific variability in learning could promote KT Yudelson et al. [68]. In the following, we will introduce some variant KT models that consider the characteristics of individualization before learning.

4.1.1 Modeling Individualization in BKT

The original BKT paper has discussed individualization. Specifically, it uses all students’ learning interactions on a specific KC to learn the individual parameter. Similarly, for a specific student, all her learning interactions are utilized to fit her individual learning parameters [11]. In this way, BKT can learn different learning and performance parameters for different students and KCs. However, this approach achieves only a limited improvement compared with the original BKT model.

Subsequently, Pardos and Heffernan [69] propose two simple variants of BKT that respectively individualize students’ initial probability of mastery and the probability of transition from the unlearned state to the learned state. As shown in Fig. 7(a), a student node $S$ is added to individualize the initial probability of mastery $P(L_0|S)$ for each student. The student node assigns each student with a personalized initial probability of mastery. A conditional probability table is designed to determine the value of the student node. Similarly, if changing the connection of the student node from $P(L_0|S)$ to the subsequent knowledge nodes, the transition probability parameter can also be individualized. In this case, the student node gives individualized $P(T)$ parameters to each student, as shown in Fig. 7(b). Moreover, rather than individualizing only one kind of parameter in BKT, some other variants of BKT opt to individualize all four BKT parameters simultaneously [65]. Lee and Brunskill [70] suggest that when applied in an intelligent tutoring system, the individualized BKT model can yield good improvements to student learning efficacy, reducing by about half the amount of questions required for 20% of students to achieve mastery.

Another means of modeling individualization is clustering, which considers a wider range of students in different groups [71]. By clustering the students into $K$ groups, we can train $K$ different KT models and make predictions on the test data. The number of clusters $K$ is then varied from $K - 1$ to 1 and the predicting process is repeated iteratively. Finally, we can obtain a set of $K$ different predictions. Furthermore, there are two common methods used to combine these predictions [72]: (1) uniform averaging, which simply averages the $K$ predictions; (2) weighted averaging, which combines the models by means of a weighted average. To realize clustering, $K$-means is a basic clustering algorithm that randomly initializes a set of $K$ cluster centroids, which are identified using Euclidean distance. Another popular clustering algorithm is spectral clustering, which represents the data as an undirected graph and analyzes the spectrum of the graph Laplacian obtained from the pairwise similarities of data points. Recently, some novel clustering algorithms have been proposed, including discrete nonnegative spectral clustering [73] and clustering uncertain data [74].

4.1.2 Modeling Individualization in DKT

Minn et al. [75] propose a model named deep knowledge tracing with dynamic student classification (DKT-DSC), which introduces individualization to DKT by exploiting the idea of clustering. According to students’ previous performance, DKT-DSC assigns students with similar learning ability to the same group. The knowledge states of students in different groups are then traced by different DKT models. Moreover, considering the dynamic property of the learning ability, each student’s learning sequence is segmented into multiple time intervals. At the start of each time interval,
DKT-DSC will reassign students' learning ability and reassign their groups. In DKT-DSC, the K-means clustering algorithm is utilized to split students with similar ability levels into the same group at each time interval. After learning the centroids of all K clusters, each student is assigned to the nearest cluster. Through dynamic student clustering, DKT-DSC offers an effective approach to realizing individualization in DKT.

Shen et al. [75] propose a convolutional knowledge tracing model (CKT) to implicitly measure student individualization. Specifically, CKT considers two factors that influence students' individualization: individualized learning rates and individualized prior knowledge. Individualized learning rates represent students' differing capacities to absorb knowledge. The sequence of student learning interactions can reflect different learning rates in the sense that students with high learning rates can rapidly master knowledge, while others need to spend more time trying and failing. Therefore, it is reasonable to assess the differences in learning rate by simultaneously processing several continuous learning interactions within a sliding window of a convolutional neural network [72]. Besides, individualized prior knowledge refers to students' prior knowledge, which can be assessed via their historical learning interactions.

4.2 Incorporating Engagement during Learning

Student engagement is defined as "the quality of effort students themselves devote to educationally purposeful activities that contribute directly to desired outcomes" [77], which indicates a strong connection to the learning process. In general, higher engagement tends to result in more knowledge gains. Therefore, considering student engagement in learning can potentially improve KT results [78]. In this section, we will present some variant KT models that incorporate student engagement during learning into KT models.

4.2.1 Incorporating Engagement into BKT

Student engagement is difficult to be directly measured. In practice, some online learning systems have made use of sensor data to measure student engagement. For example, inexpensive portable electroencephalography (EEG) devices can help to detect a variety of student mental states in learning, which can be seen as reflections of student engagement [79]. Xu et al. [80] propose two methods that combine EEG-measured mental states to improve the performance of BKT. Concretely, the first one inserts a one-dimensional binary EEG measure into BKT, forming the EEG-BKT structure that extends BKT by adding a binary variable node $E$ between the knowledge node and the answer node. The second one, i.e., EEG-LRKT, utilizes logistic regression to combine an m-dimensional continuous variable $E$ extracted from the raw EEG signal in BKT.

However, in most cases, it is difficult to collect sensor data on every student. Therefore, Schultz and Arroyo [81] propose the knowledge and affect tracing (KAT) to model both knowledge and engagement in parallel. KAT is a sensorless model that does not rely on any sensor data. In this model, both knowledge and engagement are assumed to have direct influences on student performance. KAT considers three kinds of disengagement behaviors: quick guess (the student makes an attempt very quickly), bottom-out hint (all available hints are used) and many attempts (making more than three attempts at an exercise). These three behaviors are grouped as "gaming" behaviors in order to predict students' knowledge and engagement at each learning interaction. Rather than assuming equal influence of knowledge and engagement on students' knowledge state, one variation on the KAT model defines the connection between knowledge and engagement, and accordingly considers that students' knowledge states will influence their engagement. For example, students are more likely to disengage from knowledge they are not familiar with. Moreover, rather than explicitly modeling student engagement, Schultz and Arroyo [82] further propose the knowledge tracing with behavior (KTB) model, which has only one latent knowledge node that acts as a combination of both knowledge and engagement. KTB assumes that both engagement and performance are expressions of knowledge. The Bayesian estimation of the knowledge state needs to be inferred by both student engagement and performance.

4.2.2 Incorporating Engagement into DKT

To incorporate engagement into DKT, Mongkhonvanit et al. [83] propose to add five features in the process of watching videos on MOOCs to the input of DKT. These features reflect student engagement from various aspects, including playback speed, whether or not the video was paused, fast-forwarded or rewound, and whether or not the video was completed. For example, if a student watches a video at a much faster playback speed, it is likely that he/she is impatient and absent-minded. This model incorporates two further features: whether or not the exercise was submitted with an answer selected and whether or not the exercise was a part of an end-of-unit quiz, both of which are considered together. Experimental results indicate that DKT can achieve better performance through incorporating the above binarized engagement covariates.

4.3 Utilizing Side Information during Learning

Most KT models only leverage the performance data (i.e., exercises and student answers) to assess the students' knowledge states. Although these models have achieved quite good results and have been successfully applied in online learning systems, there are still many other types of side information collected during the learning process that can be utilized to obtain more precise knowledge states. In this part, we will introduce several variant KT models that attempt to take advantage of these different types of side information during learning.

4.3.1 BKT with Side Information

Many models that apply side information in BKT have been proposed. In this section, we first introduce several works that extend BKT to enable modeling only one kind of side information for specific purposes. Subsequently, we present a general model that can utilize all types of side information.

First, in terms of a student's first response time, a short initial response time could indicate either high proficiency or 'gaming' behavior, while a long initial response time
could indicate either careful thinking or lack of concentration. Since the connection between initial response time and knowledge state could be influenced by many complex and blended factors, Wang and Heffernan [84] propose to discretize the continuous first response time into four categories (i.e., extremely short, short, long, extremely long) in order to eliminate unnecessary information and simplify the latent complex possibilities. They then build a one-by-four parameter table for KT, in which each column represents the category of initial response time of the previous exercise, while the relevant values represent the probability of correct answers.

Second, regarding tutor intervention, Beck et al. [85] propose the Bayesian evaluation and assessment model, which simultaneously assesses students’ knowledge states and evaluates the lasting impact of tutor intervention. More specifically, it adds one observable binary intervention node to BKT: *True* means that the tutor intervention occurs in corresponding interactions while *False* indicates the opposite. The connection between the intervention node and knowledge node indicates the potential impact of the tutor intervention on students’ knowledge states. The intervention node is linked to all four BKT parameters. As a result, there are a total of eight parameters to learn in order to incorporate tutor intervention. One possible way to reduce the number of parameters is choosing to link only the intervention node to the learning rate parameter [86]. Similarly, Lin and Chi [87] develop the intervention-Bayesian knowledge tracing (Intervention-BKT), which incorporates various types of instructional interventions into BKT and distinguishes their different effects. More specifically, the intervention node in Intervention-BKT adds two types of interventions: *elicit and tell*. The relations between the intervention and performance nodes represent the impact of teaching interventions on student performance, while the relations between the intervention and knowledge nodes represent the impact of teaching interventions on students’ knowledge states. Therefore, at each learning interaction, while the present knowledge state is conditional on both the previous knowledge state and the current intervention, the student’s performance depends on both the present knowledge state and the current intervention.

Finally, rather than considering only one kind of side information, González-Brenes et al. [88] propose a feature-aware student knowledge tracing (FAST) model, which allows for the utilization of all kinds of side information. Traditional BKT uses conditional probability tables for the guessing, slipping, transition and learning probabilities, meaning that the number of features involved in inference grows exponentially. Therefore, as the number of features increases, the time and space complexity of the model also grow exponentially. To deal with this large number of features, FAST uses logistic regression parameters rather than conditional probability tables. The number of features and complexity increase linearly rather than exponentially. For parameter learning, FAST uses the Expectation Maximization with Features algorithm [89] and focuses on only emission features.

### 4.3.2 DKT with Side Information

With the goal of incorporating rich side information into DKT, Zhang et al. [90] propose an extension to DKT that explored the inclusion of additional features in DKT. More specifically, it incorporates an auto-encoder network layer to convert the higher-dimensional input data into smaller representative feature vectors, thereby reducing both the resource requirement and time needed for training. Students’ response time, opportunity count, and first action are selected as incorporated side information and all input features are converted into a fixed-length input vector. First, all input features are converted into categorical data and represented as a sparse vector by means of one-hot encodings. These encoded features are concatenated together to construct the higher-dimensional input vector as follows:

\[
C(e_t, a_t) = e_t + (max(e) + 1)a_t, \\
v_t = O(C(e_t, a_t) \oplus O(t_{t+1}) \oplus O(t_t)), \\
v_t' = \tanh(W_vv_t + b_v),
\]

where \( C \) is the cross feature, \( O \) is the one-hot encoder format, \( v_t \) represents the resulting input vector of each learning interaction, \( e_t \) is the exercise, \( a_t \) refers to the answer, \( t_t \) is the response time, \( W_v \) is the weight parameter and \( b_v \) is the bias term. Subsequently, an auto-encoder is introduced to reduce the dimensionality without incurring the loss of too much important information. Finally, the feature vectors extracted by auto-encoder will be the new input of DKT.

To achieve more feasible integration of side information, Loh [91] present a deep knowledge tracing method with decision trees (DKT-DT), which takes advantage of Classification And Regression Trees (CART) to preprocess the heterogeneous input features [92]. More specifically, CART is utilized to automatically partition the feature space and outputs whether or not a student can answer an exercise correctly. The predicted response and the true response are encoded into a four-bit binary code \( O(f_t, a_t) \); for example, \( O(f_t, a_t) = 1010 \) if the predicted response and the true response are both correct. \( O(f_t, a_t) \) is then concatenated with the original one-hot encoding of the exercise as the new input of DKT to train the corresponding model.

### 4.4 Considering Forgetting after Learning

In real-world scenarios, while learning, forgetting is also inevitable [93]. The *Ebbinghaus forgetting curve theory* indicates that students’ knowledge proficiency will decline due to forgetting [94]. Recently, Huang et al. [95] propose knowledge proficiency tracing (KPT) to model students’ knowledge proficiency with both learning and forgetting theories, which can dynamically capture the changes in students’ proficiency level on KCs over time and track them in an effective and interpretable manner. Therefore, the assumption that students’ knowledge states will not change over time is untenable. Nevertheless, basic KT models, such as BKT, usually do not take forgetting into consideration. In the following, we will introduce some variant KT models that have attempted to consider forgetting after learning for more precise knowledge states.

#### 4.4.1 Considering Forgetting in BKT

Qiu et al. [96] discover that BKT consistently overestimates the accuracy of students’ answers when a day or more had passed since the last attempt. In order to eliminate unnecessary information and simplify the latent complex possibilities, they then build a one-by-four parameter table for KT, in which each column represents the initial response category of the previous exercise, while the relevant values represent the probability of correct answers.

Finally, rather than considering only one kind of side information, González-Brenes et al. [88] propose a feature-aware student knowledge tracing (FAST) model, which allows for the utilization of all kinds of side information. Traditional BKT uses conditional probability tables for the guessing, slipping, transition, and learning probabilities, meaning that the number of features involved in inference grows exponentially. Therefore, as the number of features increases, the time and space complexity of the model also grow exponentially. To deal with this large number of features, FAST uses logistic regression parameters rather than conditional probability tables. The number of features and complexity increase linearly rather than exponentially. For parameter learning, FAST uses the Expectation Maximization with Features algorithm [89] and focuses on only emission features.
elapsedsincehershellastresponse.Thelandingreasonis
thatBKTassumesthatstudentperformancewillremainthesame
regardless of how much time has passed. To consider
how student performance declines with time, they propose
a BKT-Forget model, which hypothesizes that students may
forget information they have learned as days go by. In the
BKT-Forget model, a time node is added to specify which
parameters should be affected by a new day and the new
day node is fixed with a prior probability of 0.2. The param-
eter forget\_n is introduced to represent the forgetting rate
on a new day, while forget\_s denotes the forgetting rate on
the same day. However, although BKT-forget does consid-
ner the decline in student performance, it can only model for-
getting that occurs over the time scale of days. To model
the continuous decay of knowledge as time progresses,
Nedungadi and Remya [97] incorporate forgetting into BKT
based on the assumption that learned knowledge decays
exponentially over time [98]. An exponential decay function
is thus utilized to update the knowledge mastery level. They
further assumed that the chance of forgetting will increase
if a student does not practice the knowledge concepts within
30 days. Moreover, Khajah et al. [43] introduce an approach
that counts the number of intervening trials and treats each
as an independent opportunity for forgetting to occur.

4.4.2 Considering Forgetting in PFA
Recall the PFA model in Eq. 5, in which the probability
of students’ mastery is estimated using a logistic function:
\( p(\theta) = \sigma(\beta + \mu s + \nu f) \). The original PFA model ignores
the order of answers, in addition to the time between learning
interactions. It is therefore difficult to directly incorporate
time information into the original PFA model. Pelánek et al.
[29] propose PFAE (PFA Elo/Extended), a variant of the
PFA model that combines PFA with some aspects of the
Elo rating system [100]. The Elo rating system is originally
devised for chess rating (estimating players’ skills based on
match results). In PFAE, \( \theta \) is updated after each learning
interaction, as follows:
\[
\theta := \begin{cases} 
\theta + \mu \cdot (1 - p(\theta)) & \text{if the answer was correct,} \\
\theta + \nu \cdot p(\theta) & \text{if the answer was wrong.} 
\end{cases}
\]  

(22)

As the forgetting behavior of students is closely related
to time, in order to consider forgetting, Pelánek [101] add a
time effect function \( f \) to \( \theta \), i.e., using \( p(\theta + f(t)) \) instead of
\( p(\theta) \), where \( t \) is the time (in seconds) from the last learning
interaction, and \( f \) is the time effect function.

4.4.3 Considering Forgetting in DKT
To represent the complex forgetting behavior, the DKT-
forget model [102] introduces forgetting into DKT, which
considers three types of side information related to forget-
ting: (1) the repeated time gap that represents the interval
time between the present interaction and the previous in-
teraction with the same KC, (2) the sequence time gap that
represents the interval time between the present interaction
and the previous interaction, and (3) past trial counts that
represent the number of times a student has attempted on the
exercise with the same KC. All these three features are
discretized at \( \log_2 \) scale. Those side information is concate-
nated as additional information and represented as a multi-
hot vector \( c_t \), which is integrated with the embedding vector
\( v_t \) of the learning interaction, as follows:
\[
v'_t = \theta^{in}(v_t, c_t),
\]

(23)

where \( \theta^{in} \) is the input integration function. The integrated
input \( v'_t \) and the previous knowledge state \( h_{t-1} \) are passed
through the RNNs to update \( h_t \) in the same way as in Eq. 7.
The additional information at the next time step \( c_{t+1} \) is also
integrated with the updated \( h_t \):
\[
h'_t = \theta^{out}(h_t, c_{t+1}),
\]

(24)

where \( \theta^{out} \) is the output integration function.

Wang et al. [103] propose a novel HawkesKT model,
which introduces the Hawkes process to adaptively model
temporal cross-effects. The Hawkes process performs well at
modeling sequential events localized in time, as it controls
corresponding temporal trends by the intensity function.
The intensity function in HawkesKT is designed to character-
ize the accumulative effects of previous learning interac-
tions, along with their evolutions over time. In HawkesKT,
the temporal cross-effects and the ways in which they evolve
between historical learning interactions combine to form a
dynamic learning process.

5 Datasets and Baselines
After introducing above KT models and variants, to better
help researchers and practitioners who want to further
conduct related work and promote the application of KT,
we have open sourced two algorithm libraries, i.e., EduData
that for downloading and preprocessing most existing KT-
related datasets, and EduKTM that includes extensible and
unified implementations of existing popular KT models.
In the following, we will give detailed introduction of these
two algorithm libraries.

5.1 Datasets
As we have mentioned, KT emerges from the development
of online education, where a large number of students’ learn-
ing data is collected for analyzing their learning behav-
iors and knowledge states. In this section, we mainly
introduce existing public datasets available for evaluating
KT models. Table 2 lists all datasets, as well as their basic
information and statistics. In our released EduData, we pro-
vide the service of downloading and preprocessing all these
datasets, which is convenient to help beginners analyze and
utilize them quickly. In summary, these datasets are col-
collected in different learning scenarios, so that they exhibit
an extremely distinct difference in data scale, subject, and so
on, enabling them to meet a variety of research demands.

5.1.1 ASSISTments Datasets
ASSISTments [104], created in 2004, is an online tutoring
system in the United States, which provides students with
both assessment information and tutoring assistance. While
working on ASSISTments, students will be provided with
instructional assistance to help them solve the problem in
different substeps when they give wrong answers. After
obtaining correct answers, they will be given a new one.
Meanwhile, the system will learn about the students’ knowl-
edge states and predict how they will do in future tests.
Up to now, the organizers have released four public avail-
able datasets from ASSISTments, which are respectively
**TABLE 2**
Basic information and statistics of existing datasets available for evaluating KT models.

| Datasets            | Subjects     | Stages           | Side Information | Sources                          | Online platforms | Educational challenges | # of Students | # of Exercises | # of KCs | # of Learning records |
|---------------------|--------------|------------------|------------------|----------------------------------|-----------------|-----------------------|---------------|----------------|----------|----------------------|
| ASSISTments2009     | mathematics  | middle school    | Yes              | [104]                            |                |                       | 4,143         | 17,751         | 123     | 346,860              |
| ASSISTments2012     | mathematics  | middle school    | Yes              | [105]                            |                |                       | 46,674        | 179,999        | 265     | 6,123,270            |
| ASSISTments2015     | mathematics  | middle school    | No               | [106]                            |                |                       | 19,917        | /               | 100     | 708,631              |
| ASSISTments2017     | mathematics  | from middle school to college | Yes | [107] |                | 1,709          | 3,162         | 102     | 942,816              |
| Junyi               | mathematics  | from primary to high school | Yes | [108] |                | 247,606        | 722          | 131,441,538 |       | 29,018 students, 53,091 exercises, and 2,711,813 learning records. Therefore, the average learning record of students in ASSISTments2017 is much longer than other dataset, which is beyond the length of 1,000. ASSISTments2017 also has rich side information, including but not limited to AveKnow that indicates students’ average knowledge level based on BKT, timeTaken that represents the time spent on the current exercise, fristHelpRequest that represents whether the first response is a help request.

5.1.2 Junyi Dataset
The Junyi dataset [106] contains the problem log and exercise-related information on the Junyi Academy, a Chinese e-learning website established in 2012 on the basis of the open-source code released by Khan Academy. In contrast to the ASSISTments datasets, Junyi has less exercises and KCs, but includes an exercise hierarchy labeled by experts, the annotations of exercise relationship are also available. Therefore, many research works that focused on the knowledge structure in KT had utilized this dataset [64]. Junyi provides the prerequisite exercise of a specific exercise in the knowledge map, the topic and area of each exercise, as well as the coordinate position of the knowledge map.

5.1.3 Eedi2020 Dataset
The Eedi2020 dataset [109] is also released in an academic challenge, i.e., NeurIPS 2020 Education Challenge. This dataset contains students’ answers to mathematics questions from Eedi, an online educational platform which
millions of students interact with daily around the globe from school year 2018 to 2020. All exercises are multiple-choice problems with 4 possible answer choices, exactly one of which is correct. In Table 2, we give the statistics based on the training data in this competition, the total number of learning records in the full dataset exceeds 17 million. It is worth noting that Eedi2020 gives students’ exact answer choice so that we can also predict students options. Moreover, for the students, Eedi2020 records lots of valuable context information, including the Gender, DateOfBirth, PremiumPupil. For the learning records, Eedi2020 also presents their Confidence, GroupId, QuizId, and SchemeOfWorkId.

5.1.4 Statics2011 Dataset
Different from the above datasets that focus on mathematics exercises, the Statics2011 dataset is obtained from a college-level engineering statics course via online educational system developed by Carnegie Mellon University. The problems in college engineering course are totally different from traditional subjects, solutions can also contain many unique steps. We do not give the number of exercises and KCs as there is no explicit definition.

5.1.5 EdNet Dataset
The EdNet dataset is related to the English subject, which is consisted of students’ learning records in the multi-platform AI tutoring system Santa in South Korea. EdNet collected learning data of students over two years for their preparation of the e TOEIC (Test of English for International Communication) Listening and Reading Test. EdNet is now the largest public dataset in KT field with a total of 131,441,538 learning records from 784,309 students. Besides, it contains various features of students’ learning actions, such as the specific learning material they have interacted, how much time they have spent for answering a given exercise. There are four different versions of EdNet, respectively named EdNet-KT1, EdNet-KT2, EdNet-KT3, and EdNet-KT4 with different extents.

- **EdNet-KT1**. EdNet-KT1 contains students’ basic exercise-answering logs. This dataset has 784,309 students, 13,169 exercises, 188 KCs, and a total of 95,293,926 learning records. Exercises in EdNet-KT1 are organized by bundles, i.e., a collection of exercises sharing a common passage, picture or listening material. Therefore, exercises come up in bundles and students have to answer all contained exercises when a bundle is given.

- **EdNet-KT2**. EdNet-KT2 recorded students’ action sequences, which indicated their full learning behaviors. For example, a student who is not confident about the answer may alternately select among several answer choices before submitting. Such learning behaviors can reflect more fine-grained knowledge state of students. EdNet-KT2 contains three kinds of actions: enter when student first receives and views a bundle, respond when the student selects an answer choice to the exercise, and submit when the student submits his final answers to the given bundle. It is worth noting that EdNet-KT2 is a subset of EdNet-KT1.

- **EdNet-KT3**. On the basis of EdNet-KT2, EdNet-KT3 collected more students’ learning activities, such as reading explanations or watching lectures. These learning activities have potential impacts on students’ knowledge state so that they are valuable to be analyzed.

- **EdNet-KT4**. In EdNet-KT4, the very fine details of actions were provided. In particular, the following types of actions are added to EdNet-KT3: erase choice, undo erase choice, play audio, pause audio, play video, pause video, pay, refund, and enroll coupon.

5.1.6 CodeWorkout Dataset
The CodeWorkout dataset is utilized in the 2nd Computer Science Educational Data Mining Challenge (CSEDM). This dataset is collected from a CS1 course in the Spring and Fall 2019 semesters at a public university in United States. It contains the code submissions from students for 50 coding problems, each requiring 10–26 lines of code. In total, there are 329 and 490 students in the Spring and Fall semesters who completed the course. Each dataset contains more than 65,000 code submissions, the scores of the submissions (% of unit tests passes) are also available, as well as the compiler message if the compilation is not successful. The final grades of students are also provided for this dataset.

5.2 Baselines
The implementations of existing KT methods are not standardized, which may use different program languages (e.g., python, lua) and different deep learning frameworks (e.g., tensorflow, torch). Furthermore, some works did not well organize the codes systematically (e.g., the missing of running environments and dependencies), which brings difficulties in reproducing the models. To this end, we put forward the algorithm library for KT baselines, named EduKTM, which now has contained eight concurrent popular works. EduKTM will be always under development for including the latest KT models, more algorithms and features are going to be added. Besides, we provide detailed guidelines for everyone who is interested in contributing to EduKTM.

6 APPLICATIONS
Although knowledge tracing is an emerging research area, it has already been applied in a wide variety of scenarios. In the following, we survey the applications of KT models in three typical educational scenarios: learning resources recommendation, adaptive learning, and educational gaming.

6.1 Learning Resources Recommendation
Traditionally, learning resources for each student are selected in one of two ways. The first one requires teachers to manually select suitable resources that match students’ knowledge levels. However, this approach requires substantial time and effort, and different teachers may have different preferences. The second one allows students themselves to freely choose resources to learn. However, this may result in students choosing too easy or too difficult...
materials that will not benefit their learning [112], leading to low learning efficiency. In recent years, the prevalence of intelligent tutoring systems and the development of KT methods have made it possible to automatically recommend appropriate exercises to each student based on artificially designed intelligent algorithms.

Exercises are the most common learning resources in learning. Given the inferred knowledge states, one common strategy is selecting the next exercise that will best advance students’ knowledge acquisition. Desmarais and Baker [112] propose two extensions of the original BKT model, which respectively considered exercises’ difficulties and students’ multiple-attempt behaviors. These two extensions are integrated into a BKT-sequence algorithm to recommend exercises to students based on their knowledge states. Specifically, BKT-sequence first determines the predicted range of scores for each exercise. It then computes an expected score for each exercise that the student should get to achieve mastery, which is dependent on their current knowledge state (for instance, a lower knowledge state will result in higher expected scores). Finally, the algorithm returns the exercise with a predicted score that is closest to that of the expected score. Therefore, as the knowledge state of a particular KC grows, more difficult exercises will be recommended, as harder exercises are associated with a lower predictive score. Experimental results have shown that students using the BKT-sequence algorithm were able to solve more difficult exercises, obtained higher performance and spent more time in the system than students who used the traditional approach. Moreover, students also expressed that the BKT-sequence algorithm was more efficient.

In addition to exercises, there are also some other types of multi-modal learning resources, such as videos and figures. Machardy [113] utilizes an adaptation of BKT to improve student performance prediction by incorporating video observation. Experimental verification demonstrates the impact of both using and eschewing video data, as well as the learning rate associated with a particular video. In this way, they further developed a method to help people evaluate the quality of video resources. Concretely, they proposed the Template 1 Video model to incorporate video observations into BKT, which adds video activity as additional independent observation nodes to the BKT model. This model accordingly considers the probability that a given video resource will impart knowledge to a student. Moreover, the transition probability in BKT is conditional only on the presence of either a video or an exercise. Thus, the quality of the video can be determined by its promotion of learning, and this model can be leveraged as a tool to aid in evaluating and recommending video resources.

When recommending learning resources, the primary aim of existing solutions is to choose a simple strategy for assigning non-mastered exercises to students. While reasonable, it is also too broad to advance learning effectively. Huang et al. [114] accordingly propose three more beneficial and specific objectives: review and explore, smoothness of difficulty level and student engagement. In more detail, review and explore considers both enhancing students’ non-mastered concepts with timely reviews and reserving certain opportunities to explore new knowledge; smoothness of difficulty level indicates that the difficulty levels of several continuous exercises should vary within a small range as students gradually learn new knowledge; finally, student engagement considers that to promote students’ enthusiasm during learning, the recommended exercises should be in line with their preferences. In order to support online intelligent education with the above three domain-specific objectives, they developed a more reasonable multi-objective deep reinforcement learning (DRE) framework. DRE presented three corresponding novel reward functions to capture and quantify the effects of the above three objectives. This DRE framework is a unified platform designed to optimize multiple learning objectives, where more reasonable objectives also can be incorporated if necessary. Experimental results show that DRE can effectively learn from the students’ learning records to optimize multiple objectives and adaptively recommend suitable exercises.

### 6.2 Adaptive Learning

Adaptive learning broadly refers to "a learning process in which the content taught, or the way such content is presented, changes or ‘adapts’ based on individual student responses, and which dynamically adjusts the level or types of instruction based on individual student abilities or preferences" [115]. In contrast to learning resources recommendation, adaptive learning needs to design efficient learning schemes and dynamic learning paths to organize learning resources for students based on specific knowledge structures. However, the key problem associated with realizing adaptive learning is similar, i.e., dynamically measuring students’ evolving knowledge states, which is exactly what KT is pursuing.

The first few attempt made to apply KT to adaptive learning was the ACT Programming Tutor (APT) [11], where students were asked to write short programs and BKT was utilized to estimate their evolving knowledge state. This tutor can present an individualized sequence of exercises to each student based on their estimated knowledge states until the student has "mastered" each rule.

In recent years, Massive Open Online Courses (MOOCs) have become an emerging modality of learning, particularly in higher education. Pardos et al. [13] adapt BKT on the edX MOOC platform. The research object was a 14-week online course that included weekly video lectures and corresponding lecture problems. BKT was applied to study students’ learning phenomena and enhance their learning on this course. In order to better adapt BKT to the learning platform, the original BKT was modified in several respects. First, due to the lack of labeled KCs, the problems would be directly seen as the KCs, while the questions would be seen as the exercises belonging to the KC. Second, in order to capture the varying degrees of students’ knowledge acquisition at each attempt, the modified model assigned different guess and slip parameters to different attempt counts. Third, to deal with the problem of multiple pathways in the system, which reflected that the impacts on learning may come from various resources, they framed the influence of resources on learning as a credit/blame inference problem.

Generally, students’ cognitive structures include both students’ knowledge level and the knowledge structure of learning items (e.g., one-digit addition is the prerequisite knowledge of two-digit addition). Therefore, adaptive
learning should maintain consistency with both students’ knowledge level and the latent knowledge structure. Nevertheless, existing methods for adaptive learning often focus separately on either the knowledge levels of students (i.e., with the help of specific KT models) or the knowledge structure of learning items. To fully exploit the cognitive structure for adaptive learning, Liu et al. [18] propose a Cognitive Structure Enhanced framework for adaptive Learning (CSEAL). CSEAL conceptualized adaptive learning as a Markov Decision Process. It first utilized DKT to trace the evolving knowledge states of students at each learning step. Subsequently, the authors designed a navigation algorithm based on the knowledge structure to ensure that the learning paths in adaptive learning were logical and reasonable, which also reduced the search space in the decision process. Finally, CSEAL utilized the actor-critic algorithm to dynamically determine what should be learned next. In this way, CSEAL can sequentially identify the most suitable learning resources for different students.

6.3 Educational Gaming
The above two types of applications are most commonly used for KT in the traditional education field. Recently, the idea of KT has also been extended to be applied in more general scenarios, such as educational gaming. Long and Aleven [116] conduct a classroom experiment comparing a commercial game for equation solving, i.e., DragonBox, with a research-based intelligent tutoring system, i.e., Lynnette. The results indicated that students who used DragonBox enjoyed the experience more, while students who used Lynnette performed significantly better on the test. Therefore, it is possible to enable students to learn effectively and happily by designing suitable educational games on the online learning platform. In educational gaming, the paradigm of tracing students’ knowledge state can also work for player modeling. Here, player modeling, which is the study of computational models of players in games, aims to capture human players’ characteristics and cognitive features [117]. For instance, Fisch et al. [118] reveal that children engage in cycles of increasingly sophisticated mathematical thinking over the course of playing an online game. Kantharaju et al. [119] present an approach to trace player knowledge in a parallel programming educational game, which is capable of measuring the current players’ real-time state across the different skills required to play an educational game based only on in-game player activities.

7 Future Research Directions
This survey has reviewed the abundant current developments in the field of knowledge tracing, including their variants and typical applications, as completely as possible. Nonetheless, as KT is a young but promising research area, there are still a large number of research problems that need to be urgently solved. In this section, we discuss several potential future research directions.

7.1 Knowledge Tracing with Interpretability
It is difficult to obtain the true knowledge state of students, the performance of KT models is usually indirectly evaluated with reference to prediction tasks: the higher the prediction precision of students’ responses on future exercises, the better the performance of the KT model. Therefore, interpretability is typically not the major focus of existing KT models, especially for those deep-learning based models with the end-to-end learning characteristic. However, interpretability is of significant importance in the domain of education. For example, students usually care more about why a specific item is recommended rather than which/what item is recommended. More attention should therefore be paid to improving the interpretability of KT models. To this end, some educational theories can be considered, such as the Rasch model used in AKT [27] and the transfer of knowledge used in SKT [64]. Moreover, we could consider incorporating knowledge tracing along with some static cognitive diagnosis models. For instance, a neural cognitive diagnosis framework was recently proposed by Wang et al. [120] with the goal of obtaining more accurate and interpretable diagnostic results. By incorporating educational theories or static cognitive diagnosis, we expect to achieve both accurate and interpretable KT.

7.2 Knowledge Tracing with Continuous Responses
Most KT models assume that students’ answers are binary (i.e., either 0 or 1). The continuous value of students’ answers (e.g., those on subjective exercises) are usually omitted. In real-world scenarios, students may answer different types of exercises with either discrete or continuous responses, with the latter accounting for a large proportion that cannot be ignored [121]. Simple binarization of the continuous responses introduces inevitable systemic errors to the estimation of students’ knowledge states. It is necessary to develop KT models that can handle continuous responses and further measure students’ knowledge states from both discrete and continuous responses.

7.3 Knowledge Tracing with Students’ Feedback
Learning records are passive reflections of students’ knowledge states. By contrast, students’ feedback provides us with their proactive understanding about their knowledge states, which in turn yields direct and real indicators of their learning situation. However, there are few KT models that take advantage of training data related to students’ feedback, even though it can play an important role in fixing the KT results [122]. Wang et al. [123] have noted that feedback plays a positive role in learning, which may promote transfer and retention in learning from worked-out examples. Therefore, incorporating student feedback is a promising avenue that may yield improved results.

7.4 Knowledge Tracing with Less Learning Data
The learning of high-quality KT models inevitably requires a substantial amount of data to guarantee training stability. However, practical educational scenarios often suffer from the cold-start problem and the data isolation problem: e.g., students’ learning data tends to be distributed across different schools and is also highly proprietary, so that it is difficult to gather the data for training [124]. Therefore, potential methods of combining concepts such as federated learning or active learning to train novel KT models are also a promising research direction.
7.5 Knowledge Tracing for General User Modeling

Generally, user modeling refers to tools for characterizing users’ behaviors (e.g., frequent locations), personal information (e.g., age, gender, and occupation) and latent features (e.g., interests and abilities), which facilitate the provision of targeted services for different users. As a type of latent feature modeling, user ability modeling (including knowledge tracing) diagnoses the proficiency of users (not only individuals, but also groups of individuals, like user teams and companies) on specific skills/concepts. Therefore, in addition to education, knowledge tracing can be generally applied in a number of domains for user modeling, such as games, sports and recruitment.

8 Conclusions

In this survey, we conducted a comprehensive overview of knowledge tracing. Specifically, we first proposed a taxonomy from the technical perspective, which split the KT models into three categories: (1) probabilistic models, (2) logistic models and (3) deep learning-based models. Based on this taxonomy, we reviewed existing KT models comprehensively. Considering more strict learning assumptions, such as individualization, engagement, and forgetting, we also introduced rich variants of KT models involving three different learning phases. Moreover, we released two algorithm libraries for KT-related datasets and baselines, which helped to facilitate future research in this domain. Subsequently, we summarized some typical applications of KT in three common educational scenarios. Finally, we outlined some potential future directions for this young but promising research area. We hope that this extensive survey of knowledge tracing can help readers quickly understand the problem of modeling students’ dynamic knowledge states and serve as a basic framework for both researchers and practitioners in future research.

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