Fuzzy topological digital space and their properties of flat electroencephalography in epilepsy disease

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Abstract. There are an abnormal electric activities or irregular interference in brain of epilepsy patient. Then a sensor will be put in patient’s scalp to measure and records all electric activities in brain. The result of the records known as Electroencephalography (EEG). The EEG has been transfer to flat EEG because it’s easier to analyze. In this study, the uncertainty in flat EEG data will be considered as fuzzy digital space. The purpose of this research is to show that the flat EEG is fuzzy topological digital space. Therefore, the main focus for this research is to introduce fuzzy topological digital space concepts with their properties such as neighbourhood, interior and closure by using fuzzy set digital concept and Chang’s fuzzy topology approach. The product fuzzy topology digital also will be shown. By introduce this concept, the data in flat EEG can considering having fuzzy topology digital properties and can identify the area in fuzzy digital space that has been affected by epilepsy seizure in epileptic patient’s brain.

1. Introduction
Epilepsy disease is a disease where an irregular interference that found in seizure patient’s brain. For the initial stage, this disease can be controlled by some medicines. But in critical stage, the damaged tissue should be removed and the patient has to go through a surgery. A detector will be placed in the patient’s scalp to measure and record any abnormal activity. The result of the sensors is known as Electroencephalography (EEG). The purpose of EEG is to detect the area of damaged tissue. This EEG actually is high dimensional signals. So, [1] developed a method to mapping the EEG signal to a low dimensional signal that will be known as flat of EEG (fEEG). The benefit of fEEG is can be compressed and analyzed. The fEEG then will be considered as digital space. The fEEG that uncertainty has not been considered yet.

Topological digital space concept has been started by [2]. After that, many researchers began to extend to this field. Among them is [3] that has introduced Alexandroff’s topology and [4] studied the Khalimsky’s topology. Then [5] started to extend it into fuzzy topology field known as fuzzy topological digital space. The study is actually extension from the previous research [6]. For fuzzy topology’s field is actually an extended from crisp topology where the uncertainty element will be considered in general topology. In [7] has introduced fuzzy topology in his research by saying that fuzzy topology is a collection of fuzzy subset in universal set that must be fulfill three condition. Then from Chang’s fuzzy topology, many researchers began to produce new concepts in Chang’s fuzzy topology. For example [8] that introduced the fuzzy neighbourhood and fuzzy closure and interior
concept by [9] and [10]. For product of Cartesian, we will refer from [11], fuzzy relation from [12-13] and fuzzy product space from [14-18]

In this paper, we will introduce fuzzy topological digital space and some basic properties by using Chang’s fuzzy topology and fuzzy set digital concept. The properties of fuzzy topological digital space is such as openness, closedness, fuzzy neighbourhood digital, closure, interior and fuzzy product digital space. Then for our main case study, flat EEG need to be consider as fuzzy digital space for next usage. As a result for this paper, the EEG will be concluding as a fuzzy topological digital space.

2. Materials and methods
In this section, we will need to reintroduce the theory of fuzzy set digital and extend to some other new theory such as fuzzy point digital and fuzzy relation digital that will be used to achieve the main paper’s objectives which is fuzzy topological digital space.

2.1. Fuzzy set digital theory
In [22], the element of the fuzzy set has a grade of membership in real number of the closed interval [0,1]. Let $U$ be universal set. Fuzzy set $X$ will be characterized by $\mu_X : U \rightarrow [0,1]$ with $\mu_X$ is membership function or grade of membership. Consider fuzzy set is continuous. If $\mu_X$ is membership function for continuous universal set, then the fuzzy set $X$ known as analog fuzzy set. [23-27]. For fuzzy set digital theory has been discussed in generally in [28]. We need to redefine this theory for the reviewer easy to understand and more suitable for this research.

Definition 1 Let $U$ be universal set that consist discrete universal set $X_{dig} = \{x_i\}_{i=1}^n$ and $A_{dig} \subseteq X_{dig}$. Then $A_{dig}$ is said to be fuzzy subset digital of fuzzy set digital $X_{dig}$, where the membership function $\mu_{A_{dig}} : A_{dig} \subseteq X \rightarrow [0,1]$. Fuzzy subset, $A$ can be defined by $A = \{(x, \mu_A(x)) : x \in U, \mu_A(x) \in [0,1]\}$.

$$\mu_{A_{dig}}(x) = \begin{cases} 1 & \text{if } x \in A_{dig} \\ 0 & \text{if } x \notin A_{dig} \\ 0 < c < 1 & \text{if } x \in X_{dig} \end{cases} \quad (1)$$

Figure 1. Fuzzy set digital with discrete-universal set.

Same as fuzzy set operations, fuzzy set digital also consider the membership function or grade of membership. In digital implementation, an analog fuzzy set’s is discretized along both the universal set and membership function. Below is the operation of fuzzy set digital that will be used later.
**Definition 2** Let $U$ discrete-universal set and fuzzy set digital $\tilde{X}_{dig} \subseteq U$. Then,

i. $\tilde{A}_{dig} = \tilde{B}_{dig} \iff \mu_{\tilde{A}_{dig}}(x_i) = \mu_{\tilde{B}_{dig}}(x_i), \forall x_i \in U$

ii. $\tilde{A}_{dig} \subseteq \tilde{B}_{dig} \iff \mu_{\tilde{A}_{dig}}(x_i) \leq \mu_{\tilde{B}_{dig}}(x_i), \forall x_i \in U$

iii. $\tilde{C}_{dig} = \tilde{A}_{dig} \cup \tilde{B}_{dig} \iff \mu_{\tilde{C}_{dig}}(x_i) = \max\{\mu_{\tilde{A}_{dig}}(x_i), \mu_{\tilde{B}_{dig}}(x_i)\}, \forall x_i \in U$

iv. $\tilde{D}_{dig} = \tilde{A}_{dig} \cap \tilde{B}_{dig} \iff \mu_{\tilde{D}_{dig}}(x_i) = \min\{\mu_{\tilde{A}_{dig}}(x_i), \mu_{\tilde{B}_{dig}}(x_i)\}, \forall x_i \in U$

v. $\tilde{E}_{dig} = \tilde{A}_{dig} ^ \prime \iff \mu_{\tilde{E}_{dig}}(x_i) = \min\{\mu_{\tilde{A}_{dig}}(x_i), 1 - \mu_{\tilde{A}_{dig}}(x_i)\}, \forall x_i \in U$

**Definition 3** If $\{\tilde{A}_{dig,i}\}_{i \in I}$ collection of fuzzy subset digital in fuzzy set digital, $\tilde{X}_{dig}$, then $\tilde{C}_{dig} = \bigcup_{i \in I} \tilde{A}_{dig,i}$ and $\tilde{D}_{dig} = \bigcap_{i \in I} \tilde{A}_{dig,i}$ defined by

$$\mu_{\tilde{C}_{dig}}(x) = \sup_{i \in I} \{\mu_{\tilde{A}_{dig,i}}(x)\}, x \in U \text{ and } \mu_{\tilde{D}_{dig}}(x) = \inf_{i \in I} \{\mu_{\tilde{A}_{dig,i}}(x)\}, x \in U$$

(2)

2.2. Fuzzy point digital

Fuzzy point theory started from [22] that introduced a single fuzzy element. Then [8] and [29] also expand this theory to many fields. We extend from equation (1) to introduce fuzzy point digital.

**Definition 4** Fuzzy point digital $x_i \subseteq \tilde{X}_{dig}$ is fuzzy set digital with membership function

$$\mu_{x_i}(y) = \begin{cases} 0 < p \leq 1 & \text{if } y = x_i \\ 0 & \text{if } y \neq x_i \end{cases}$$

$\forall y \in U$ where $U$ is discrete-universal. Fuzzy point digital, $x_i$ is an element for fuzzy subset digital $\tilde{A}_{dig}$ in $\tilde{X}_{dig}$, $x_i \in \tilde{A}_{dig} \iff p \leq \mu_{\tilde{A}_{dig}}(x_i)$.

2.3. Fuzzy relation digital

Generally in mathematics, product of Cartesian means a product of two set. Then from the definition and its properties [11], there is fuzzy relations concept that has been studied by some of authors such as [12] and [13]. Fuzzy relations are fuzzy subsets of $X \times Y$ that is mappings from $X \rightarrow Y$. Therefore, for fuzzy relation digital is as follows:

**Definition 5** Let $\tilde{X}_{dig}, \tilde{Y}_{dig} \subseteq U$ where $U$ is discrete-universal sets, then

$$\tilde{X}_{dig} \times \tilde{Y}_{dig} = \{(x, y), \mu_{\tilde{X}_{dig} \times \tilde{Y}_{dig}}(x, y) : (x, y) \in \tilde{X}_{dig} \times \tilde{Y}_{dig}\}$$

(4)

is called a fuzzy relation digital on $\tilde{X}_{dig} \times \tilde{Y}_{dig}$.

2.4. Fuzzy topological digital space (FTDS) and their properties.

The concept of fuzzy topological space need to consider three conditions [7]. In digital perspective, [5] has introduced fuzzy topological digital space (FTDS) where the intersection of any fuzzy open set in $X$ is open. By using Chang’s fuzzy topology, we will generalize FTDS and discuss some of the properties.
Definition 6 Let $\tilde{T}_{dig} = \{ \tilde{A}_{dig} \}_{i \in I}$ be the collection of fuzzy subset digital for fuzzy set digital $\tilde{X}_{dig}$ satisfying three conditions: i. $\phi$, $\tilde{X}_{dig} \in \tilde{T}_{dig}$ . ii. If $\tilde{A}_{dig}, \tilde{B}_{dig} \in \tilde{T}_{dig}$ , then $\tilde{A}_{dig} \cap \tilde{B}_{dig} \in \tilde{T}_{dig}$ . iii. If $\{\tilde{A}_{dig}\}_{i \in I}$ , then $\bigcup_{i \in I}\tilde{A}_{dig} \in \tilde{T}_{dig}$.

Then $\tilde{T}_{dig}$ is fuzzy topology digital on fuzzy set digital $\tilde{X}_{dig}$. The pair $(\tilde{X}_{dig}, \tilde{T}_{dig})$ will be known as fuzzy topological digital space (FTDS).

Let $\tilde{T}_{dig_{1}}$ and $\tilde{T}_{dig_{2}}$ fuzzy topology digital on fuzzy set digital $\tilde{X}_{dig}$. If the inclusion relation $\tilde{T}_{dig_{1}} \subseteq \tilde{T}_{dig_{2}}$ holds, then $\tilde{T}_{dig_{2}}$ is finer than $\tilde{T}_{dig_{1}}$, and $\tilde{T}_{dig_{1}}$ is coarser than $\tilde{T}_{dig_{2}}$.

Generally in [7], every element in Chang's fuzzy topology is fuzzy open subset. Same as [5] for FTDS.

Definition 7 Let $(\tilde{X}_{dig}, \tilde{T}_{dig})$ be FTDS. Fuzzy subset digital of $\tilde{X}_{dig}$ is said to be open digital $\iff$ it belong to $\tilde{T}_{dig}$. i. $\phi$ and $\tilde{X}_{dig}$ are open digital. ii. The intersection for any fuzzy open subset digital is open digital. iii. The union for any fuzzy open subset digital is open digital.

The elements of $\tilde{T}_{dig}$ are called fuzzy open subset digital. A fuzzy subset digital $\tilde{C}_{dig}$ is called fuzzy closed subset digital if $\tilde{C}_{dig} \in \tilde{T}_{dig}$. We denote by $\tilde{T}_{o}$ the collection of all fuzzy open subset digital in this FTDS.

In [7] has introduced fuzzy neighbourhood theory by using fuzzy open subset in fuzzy topological space. After that, [8] using fuzzy point to build fuzzy neighbourhood. In [27] also studied about fuzzy neighbourhood. In [29] neighbourhood concept in topology digital had been extended into fuzzy topology. Below is the concept of fuzzy neighbourhood digital.

Definition 8 Let $(\tilde{X}_{dig}, \tilde{T}_{dig})$ are FTDS and $x_i \in \tilde{X}_{dig}$. Fuzzy subset digital $\tilde{N}_{dig}$ on $\tilde{X}_{dig}$ known as fuzzy neighbourhood digital for $x_i \iff \exists$ fuzzy open subset digital $\tilde{O}_{dig}$ where $x_i \in \tilde{O}_{dig} \subseteq \tilde{N}_{dig}$.

Theorem 1 Let $(\tilde{X}_{dig}, \tilde{T}_{dig})$ are FTDS. Then fuzzy subset digital $\tilde{A}_{dig}$ of $\tilde{X}_{dig}$ is open digital $\iff \tilde{A}_{dig}$ is fuzzy neighbourhood digital $\forall x_i \in \tilde{A}_{dig}$.

Proof. Suppose $\tilde{A}_{dig}$ is open digital. We need to prove $\tilde{A}_{dig}$ is fuzzy neighbourhood digital $\forall x_i \in \tilde{A}_{dig}$.

Let $x_i \in \tilde{A}_{dig}$. Because $\tilde{A}_{dig}$ is fuzzy open subset digital, then $\tilde{A}_{dig}$ can be fuzzy neighbourhood digital of $x_i$ where $x_i \in \tilde{A}_{dig} \subseteq \tilde{A}_{dig}$.

Conversely if $\forall x_i \in \tilde{A}_{dig}, x_i \in \tilde{A}_{dig} \subseteq \tilde{A}_{dig}$, then $\exists$ fuzzy open subset digital $\tilde{O}_{dig_{x_i}}$ where $x_i \in \tilde{O}_{dig_{x_i}} \subseteq \tilde{A}_{dig}$. Then,

$\tilde{A}_{dig} = \bigcup \{ x_i : x_i \in \tilde{A}_{dig}, x_i \in \tilde{O}_{dig_{x_i}} \} \subseteq \bigcup \{ \tilde{O}_{dig_{x_i}} : x_i \in \tilde{A}_{dig} \} \subseteq \tilde{A}_{dig}$

Therefore, $\tilde{A}_{dig} = \bigcup \tilde{O}_{dig_{x_i}} : x_i \in \tilde{A}_{dig}$.
Closure and interior theory in fuzzy topology can refer from [7], [9], [28] and [29]. The closure and interior for fuzzy topology digital are as followed.

**Definition 9** Let \((\tilde{X}_{dig}, \tilde{T}_{dig})\) is FTDS and fuzzy subset digital \(\tilde{A}_{dig}\) in \(\tilde{X}_{dig}\). The closure \(Cl(\tilde{A}_{dig})\) and the interior \(Int(\tilde{A}_{dig})\) are defined as

\[
Cl(\tilde{A}_{dig}) = \inf \{\tilde{C}_{dig} : \tilde{A}_{dig} \subseteq \tilde{C}_{dig}, \tilde{C}_{dig}' \in \tilde{T}_{dig}\}, \quad Int(\tilde{A}_{dig}) = \sup \{\tilde{O}_{dig} : \tilde{O}_{dig} \subseteq \tilde{A}_{dig}, \tilde{O}_{dig} \in \tilde{T}_{dig}\}
\]  

(5)

**Theorem 2** Let \(\tilde{A}_{dig}\) and \(\tilde{B}_{dig}\) fuzzy subset digital of \(\tilde{X}_{dig}\) with \((\tilde{X}_{dig}, \tilde{T}_{dig})\) is FTDS. Then,

i. \(Cl(\phi) = \phi\)
ii. \(\tilde{A}_{dig} \subseteq Cl(\tilde{A}_{dig}), \forall \tilde{A}_{dig} \in \tilde{T}_{dig}\)
iii. \(Cl(Cl(\tilde{A}_{dig})) = Cl(\tilde{A}_{dig}), \forall \tilde{A}_{dig} \in \tilde{T}_{dig}\)
iv. \(Cl(\tilde{A}_{dig}) \cap Cl(\tilde{B}_{dig}) = Cl(\tilde{A}_{dig} \cup \tilde{B}_{dig}), \forall \tilde{A}_{dig}, \tilde{B}_{dig} \in \tilde{T}_{dig}\)

**Theorem 3** Let \(\tilde{A}_{dig}\) and \(\tilde{B}_{dig}\) fuzzy subset digital of \(\tilde{X}_{dig}\) with \((\tilde{X}_{dig}, \tilde{T}_{dig})\) is FTDS. Then,

i. \(Int(\phi) = \phi\)
ii. \(Int(\tilde{A}_{dig}) \subseteq \tilde{A}_{dig}, \forall \tilde{A}_{dig} \in \tilde{T}_{dig}\)
iii. \(Int(Int(\tilde{A}_{dig})) = Int(\tilde{A}_{dig}), \forall \tilde{A}_{dig} \in \tilde{T}_{dig}\)
iv. \(Int(\tilde{A}_{dig}) \cap Int(\tilde{B}_{dig}) = Int(\tilde{A}_{dig} \cap \tilde{B}_{dig}), \forall \tilde{A}_{dig}, \tilde{B}_{dig} \in \tilde{T}_{dig}\)

The concept of fuzzy relation [12-13] has been extended to fuzzy topology. For more detail can refer from [14]. We need to introduce fuzzy product digital space for next usage later.

**Definition 10** Let \((\tilde{X}_{dig}, \tilde{T}_{X_{dig}})\) and \((\tilde{Y}_{dig}, \tilde{T}_{Y_{dig}})\) be FTDS. The fuzzy product digital space of \(\tilde{X}_{dig}\) and \(\tilde{Y}_{dig}\) is the cartesian product \(\tilde{X}_{dig} \times \tilde{Y}_{dig}\) of sets \(\tilde{X}_{dig}\) and \(\tilde{Y}_{dig}\) together with the fuzzy topology digital generated by the family \(\{P_{1}^{-1}(\tilde{A}_{dig})_{i_{1}} \times P_{2}^{-1}(\tilde{B}_{dig})_{i_{2}} : \tilde{A}_{dig} \subseteq \tilde{T}_{X_{dig}}, \tilde{B}_{dig} \subseteq \tilde{T}_{Y_{dig}}\}\), where \(P_{1}\) and \(P_{2}\) are projection of \(\tilde{X}_{dig} \times \tilde{Y}_{dig}\) onto \(\tilde{X}_{dig}\) and \(\tilde{Y}_{dig}\) respectively. Because \(P_{1}^{-1}(\tilde{A}_{dig})_{i_{1}} \times P_{2}^{-1}(\tilde{B}_{dig})_{i_{2}} = (\tilde{A}_{dig})_{i_{1}} \times (\tilde{B}_{dig})_{i_{2}}\)

forms a base for the fuzzy product topology digital on \(\tilde{X}_{dig} \times \tilde{Y}_{dig}\).

**Theorem 4** Let two different fuzzy topology digital \(\tilde{T}_{X_{dig}}\) and \(\tilde{T}_{Y_{dig}}\) in \((\tilde{X}_{dig}, \tilde{T}_{X_{dig}})\) and \((\tilde{Y}_{dig}, \tilde{T}_{Y_{dig}})\) respectively. The product of Cartesian, \(\tilde{T}_{X_{dig}} \times \tilde{T}_{Y_{dig}}\) also fuzzy topology digital.

**Proof.** Let \(\tilde{T}_{X_{dig}}\) and \(\tilde{T}_{Y_{dig}}\) fuzzy topology digital on fuzzy set digital \(\tilde{X}_{dig}\) and \(\tilde{Y}_{dig}\) respectively.

i. By definition 7 (i), \(\phi, \tilde{X}_{dig} \in \tilde{T}_{X_{dig}}\) and \(\phi, \tilde{Y}_{dig} \in \tilde{T}_{Y_{dig}}\). Thus \(\phi \in \tilde{T}_{X_{dig}}\) and \(\phi \in \tilde{T}_{Y_{dig}}\) \(\Leftrightarrow \) \(\phi \in \tilde{T}_{X_{dig}} \times \tilde{T}_{Y_{dig}}\). For \(\tilde{X}_{dig} \in \tilde{T}_{X_{dig}}\) and \(\tilde{Y}_{dig} \in \tilde{T}_{Y_{dig}}\) \(\Leftrightarrow \) \(\tilde{X}_{dig} \times \tilde{Y}_{dig} \in \tilde{T}_{X_{dig}} \times \tilde{T}_{Y_{dig}}\). Thus condition from definition 7 (i) satisfied.
ii. Let $\mathcal{A}_{\text{dig}}$ and $\mathcal{B}_{\text{dig}}$ be any collection of fuzzy subset digital for $\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$. Then $\mathcal{A}_{\text{dig}} \in \mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\mathcal{B}_{\text{dig}} \in \mathcal{T}_{\tilde{y}_{\text{dig}}}$. Because $\mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\mathcal{T}_{\tilde{y}_{\text{dig}}}$ is fuzzy topology digital satisfying definition 7(ii), then $\mathcal{A}_{\text{dig}} \cap \mathcal{B}_{\text{dig}} \in \mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\mathcal{A}_{\text{dig}} \cap \mathcal{B}_{\text{dig}} \in \mathcal{T}_{\tilde{y}_{\text{dig}}}$. So, $\mathcal{A}_{\text{dig}} \cap \mathcal{B}_{\text{dig}} \in \mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$. With that $\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$ satisfying definition 7(ii)

iii. Let any subcollection for $\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$. So $\{\mathcal{A}_{\text{dig}}\}_{i \in I} \in \mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\{\mathcal{A}_{\text{dig}}\}_{i \in I} \in \mathcal{T}_{\tilde{y}_{\text{dig}}}$. Then $\bigcup_{i \in I} \mathcal{A}_{\text{dig}} \in \mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\bigcup_{i \in I} \mathcal{A}_{\text{dig}} \in \mathcal{T}_{\tilde{y}_{\text{dig}}}$ because $\mathcal{T}_{\tilde{x}_{\text{dig}}}$ and $\mathcal{T}_{\tilde{y}_{\text{dig}}}$ is fuzzy topology digital. Then, $\bigcup_{i \in I} \mathcal{A}_{\text{dig}} \in \mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$. So, $\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$ satisfying definition 7(iii).

Thus, this conclude $\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}}$ fuzzy topology digital on $\tilde{X}_{\text{dig}}$ and $\tilde{Y}_{\text{dig}}$. □

2.5. Flat Electroencephalography (fEEG) as a fuzzy digital space.

Flat Electroencephalography (fEEG) is some transformation of EEG signals which in high dimensional into two-dimensional plane. This process involves three main parts. The readers can refer from [1] for more detail. In [30], fEEG has been considering in digital space by using Voronoi digitization [29]. In this section we will now need to consider fEEG in fuzzy digital space.

Let fEEG is Cartesian product of the x-axis and y-axis. Then the digitization of fEEG means the Cartesian product of the digitization of the x-axis and y-axis. The equation is

$$\text{dig}_{\text{EEG}}(x, y) = (p_x, p_y) \in Z^2 : x \in V_o(p_x), y \in V_o(p_y)$$

(6)

where Voronoi cell with nucleus $p$ at x-axis, $V_o(p_x) = \{x \in R: p_x - \frac{1}{2} < x \leq p_x + \frac{1}{2}, p_x \in Z \}$ and Voronoi cell with nucleus $p$ at y-axis, $V_o(p_y) = \{y \in R: p_y - \frac{1}{2} < y \leq p_y + \frac{1}{2}, p_y \in Z \}$. Hence, the fEEG in digital form is

$$\text{fEEG} = \{(p, \text{Volt}): p = (p_x, p_y) \in Z^2; \text{Volt} \in R^+ \}$$

$$\text{fEEG} = \{(x, y) \in Z; \text{Volt} \in R^+ \}$$

(7)

with Volt is electric potential.

By using fuzzy set digital (equation 1), we will transform the fEEG as fuzzy digital space. Then fuzzy digital space can be considered to have fuzzy topology digital characteristic. This will be shown in next section (in result and discussion section). Suppose fEEG is Cartesian product of the fuzzy point digital of the x-axis and y-axis. The fEEG (equation 7) will be redefine in fuzzy digital space

$$\tilde{\text{fEEG}} = \{(x, \mu_{\text{EEG}}(x)), (y, \mu_{\text{EEG}}(y)\} : x, y \in Z; \mu_{\text{EEG}}(x), \mu_{\text{EEG}}(y) \in [0, 1]$$

(8)

with $x_i$ and $y_i$ are element in discrete universal set of Z.

3. Result and discussion

In this section, we will take $\tilde{\text{fEEG}}$ from equation (8) will be shown as a FTDS. The data point in $\tilde{\text{fEEG}}$ has been considered as digital fuzzy point.

Theorem 5 Flat EEG, $\tilde{\text{fEEG}}$ is fuzzy topology digital.

Proof. Suppose $\tilde{\text{fEEG}}$ in equation (8). However $\tilde{\text{fEEG}}$ can be redefined as follows

$$\tilde{\text{fEEG}} = \{\mathcal{T}_{\tilde{x}_{\text{dig}}} \times \mathcal{T}_{\tilde{y}_{\text{dig}}} \}$$

(9)
where \( Z \) is product of two fuzzy set digital \( X_{\text{dig}} \times Y_{\text{dig}} \). If \( I_{\overrightarrow{X_{\text{dig}}}} \times I_{\overrightarrow{Y_{\text{dig}}}} \) has been considered as fuzzy topology digital by theorem 4, then \((\overrightarrow{Z_{\text{dig}}}, I_{\overrightarrow{Z_{\text{dig}}}})\) is FTDS. This concludes that \( \overrightarrow{EEG} \) is fuzzy topology digital.

**Theorem 6** \((\overrightarrow{EEG}, I_{\overrightarrow{Z_{\text{dig}}}})\) is fuzzy topological digital space.

**Proof.** By using definition 6 and theorem 5

**Theorem 7** Let \((\overrightarrow{EEG}, I_{\overrightarrow{Z_{\text{dig}}}})\) is FTDS and \( x_i, y_i \in \overrightarrow{EEG} \). Then there exist the fuzzy neighbourhood digital for every \( x_i, y_i \in \overrightarrow{EEG} \).

**Proof.** By theorem 1, \( X_{\text{dig}} \) and \( Y_{\text{dig}} \) is fuzzy open set digital on \( \overrightarrow{EEG} \). Therefore, \( X_{\text{dig}} \) and \( Y_{\text{dig}} \) is fuzzy neighbourhood digital for every \( x_i \in X_{\text{dig}} \) and \( y_i \in Y_{\text{dig}} \) respectively. In other word, \( \forall x_i, y_i \in \overrightarrow{EEG}, \exists X_{\text{dig}}, Y_{\text{dig}} \) where \( x_i \in X_{\text{dig}} \subseteq X_{\text{dig}} \) and \( y_i \in Y_{\text{dig}} \subseteq Y_{\text{dig}} \). From equation 9, it can be conclude that \( x_i, y_i \in X_{\text{dig}} \times Y_{\text{dig}} \subseteq X_{\text{dig}} \times Y_{\text{dig}} \).

**Theorem 8** Let \((\overrightarrow{EEG}, I_{\overrightarrow{Z_{\text{dig}}}})\) is FTDS and \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \in I_{\overrightarrow{Z_{\text{dig}}}} \). Then \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \) open digital and the largest fuzzy open subset digital that contain in \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \). Fuzzy subset digital \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \) are open digital if and only if \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} = \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \).

**Proof.** From definition 9 (equation 5), it’s clear \( \text{Int}(\overrightarrow{A_{\text{dig}}}), \text{Int}(\overrightarrow{B_{\text{dig}}}) \) are interior for fuzzy subset digital for \( \overrightarrow{A_{\text{dig}}} \in I_{\overrightarrow{X_{\text{dig}}}} \) and \( \overrightarrow{B_{\text{dig}}} \in I_{\overrightarrow{Y_{\text{dig}}}} \) respectively. Then there exist fuzzy open subset digital \( \overrightarrow{O_{\text{dig}}} \in I_{\overrightarrow{X_{\text{dig}}}} \) and \( \overrightarrow{V_{\text{dig}}} \in I_{\overrightarrow{Y_{\text{dig}}}} \). From equation 9 and theorem 4, it can be \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}} \subseteq \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \subseteq \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}}) \). But \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \) is interior for fuzzy subset digital \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \) where \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \subseteq \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \). Therefore \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) = \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \). So \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \) is open digital and the largest fuzzy open subset digital in \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \). If \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \) is open digital, then \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \subseteq \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \) where \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \) is interior for fuzzy subset digital for \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \). Then, \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} = \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \).

Conversely, if \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} = \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \), then \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \in I_{\overrightarrow{Z_{\text{dig}}}} \) is open digital because \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \) also open digital. By definition 9 (equation 5), \( \overrightarrow{O_{\text{dig}}} \subseteq \overrightarrow{A_{\text{dig}}} \) and \( \overrightarrow{V_{\text{dig}}} \subseteq \overrightarrow{B_{\text{dig}}} \).

Therefore, \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \) is open digital. Because \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \) is interior for fuzzy subset digital \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \), then \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} \subseteq \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \). Therefore, \( \overrightarrow{O_{\text{dig}}} \times \overrightarrow{V_{\text{dig}}} = \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \). So \( \text{Int}(\overrightarrow{A_{\text{dig}}} \times \text{Int}(\overrightarrow{B_{\text{dig}}}) \) is the largest fuzzy open subset digital in \( \overrightarrow{A_{\text{dig}}} \times \overrightarrow{B_{\text{dig}}} \).
4. Conclusion
In this study, we have show that flat Electroencephalography (fEEG) during epileptic seizure is fuzzy topological digital space (FTDS) by considering three conditions. Then this fEEG will have FTDS properties such as neighbourhood, interior and closure. Because of fEEG means Cartesian product of the digitization of the x-axis and y-axis, then we will need the fuzzy product digital space to show that fEEG is a fuzzy topological digital space. The area in fEEG that has been affected by epilepsy seizure will have fuzzy topology digital properties.

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