CHARM HADROPRODUCTION IN $k_T$-FACTORIZATION APPROACH

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Abstract

We compare the theoretical status and the numerical predictions of two approaches for heavy quark production in the high energy hadron collisions, namely the conventional LO parton model with collinear approximation and $k_T$-factorization approach. The main assumptions used in the calculations are discussed. To extract the differences coming from the matrix elements we use very simple gluon structure function and fixed coupling. It is shown that the $k_T$-factorization approach calculated formally in LO and with Sudakov form factor accounts for many contributions related usually to NLO (and even NNLO) processes of the conventional parton model.

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1 Introduction

The investigation of the heavy quark production in high energy hadron collisions provides a method for studying the internal structure of hadrons. Realistic estimates of the cross section of the heavy quark production, as well as their correlations, are necessary in order to plan the experiments on existing and future accelerators.

QCD is the theory of quark and gluon interactions. The description of hard hadron collisions is possible only at the phenomenological level, reducing the hadron interactions to the parton-parton ones, treated in terms of the hadron structure functions. The hard process cross section is expressed as the convolution of the parton distributions in the colliding hadrons with the cross section of the elementary sub-process given by the square of the matrix element, calculated in the perturbative QCD. Such an approach can be justified only in the Leading Logarithmic Approximation (LLA) in the leading order (LO), or in the next to leading order (NLO) and so on. Strictly speaking, it is in disagreement with the quantum field theory, where the amplitudes, rather than the probabilities, should be added and/or multiplied. Thus all theoretical approaches to the high energy hadron interactions are in some sense phenomenological ones.

There are several phenomenological approaches to the calculations of hard processes cross sections in hadronic reactions.

The most popular and technically simplest approach is the so-called QCD collinear approximation, or parton model (PM). In this model all particles involved are assumed to be on mass shell, having only longitudinal components of the momenta, and the cross section is averaged over two transverse polarizations of the incident gluons. The virtualities \( q_2 \) of the initial partons are taken into account only through their densities (structure functions). The latter are calculated in LLA using DGLAP evolution equation and available experimental data. The probabilistic picture of "frozen" partons underlies this way of proceeding. The cross sections of QCD subprocess are calculated usually in the LO, as well as in the NLO \[1, 2, 3, 4, 5\]. However the transverse momenta of the incident partons are neglected in the QCD matrix elements in the direct analogy with the Weizsaecker-Williams approximation in QED. It allows to describe quite reasonably the experimental data on the total cross sections and one-particle distributions of produced heavy flavours, however it can not \[6, 7\] reproduce, say, the azimuthal correlations \[8\] of two heavy quarks, as well as the distributions over the total transverse momentum of heavy quarks pairs \[9\], which are just determined by the transverse momenta of incident partons.

The simplest way to incorporate the transverse momenta of the incident partons for the proper description of heavy quark correlations was suggested in \[9\], as the random shift of these momenta according to certain exponential distribution. However, this procedure has no serious theoretical background. While the shift of the order of \( \Lambda_{QCD} \) (\( \langle k_T \rangle \sim 300 \text{ MeV} \)) looks to be reasonable as having an origin in confinement forces at large distances, the value \( \langle k_T \rangle \sim 1 \text{ GeV} \) or more should be explained in terms of the perturbative QCD. Moreover, the averaged value of the shift seems to be dependent on the initial energy, kinematical region, etc. The simultaneous description of one-particle distributions over the transverse and longitudinal momenta seems to be impossible \[10\].
Another natural method to account for the incident parton transverse momenta is referred to as $k_T$-factorization [11, 12, 13, 14, 15], or the theory of semihard interactions [16, 17, 18, 19]. Here the Feynman diagrams are evaluated for the virtual incoming partons with all possible polarizations. Since there are no reason to neglect the gluon transverse momenta $q_1T$ and $q_2T$ in comparison with the quark masses and transverse momenta in the small $x$ domain, the QCD matrix elements of the sub-processes become more complicated. We obtain them in the LO, the NLO calculations being much more technically hard in this approach than in the conventional PM. On the other hand, the essential and, most probably, the major part of the NLO (and part of NNLO) corrections to the LO PM related to the finite values of parton transverse momenta are already included into LO contribution in case of $k_T$-factorization. Thus one can hope that NLO, NNLO... corrections will not be too large in $k_T$-factorization, making it to be more constructive.

The matrix element describing collision of the off-shell partons is not, of course, gauge invariant. This is the shortness of $k_T$-factorization. One can use it only in a "physical" (axial or planar) gauge, where the LO Feynman graphs responsible for the parton (DGLAP or BFKL) evolution take a ladder-like form, and the hard cross section can be factorized, that is written as a convolution of the initial parton wave functions with the hard sub-process matrix element squared. Recall, that an unintegrated parton distribution in this case is given not just by the derivative of the integrated parton density with respect to the scale, but includes the double logarithmic (Sudakov) form factor as well. This point has been discussed in more detail in [20] (see also Sect. 3 of the present paper). In Feynman gauge this double logarithmic contribution comes from the graphs where the soft gluons embrace the "hard" blob, as it is shown, for example, in Fig. 1b (contrary to the PM case shown in Fig. 1a).

Note, that when the initial energy grows, the momentum fraction $x$ carrying by the "active" (participating in hard interaction) parton decreases. As a rule the smaller $x$ the larger number of steps the parton passes in the evolution. Having emitted a lot of evolution gluons the parton gains a large transverse momentum $k_T$. It explains why a large phenomenological $k_T$ shift is needed to describe the high energy data in terms of the PM.

Today it is clear that the $k_T$-factorization approach is self-consistent and predicts the results in reasonable agreement both with the experimental data and with the collinear approximation when the last one can be applied. However, the detailed numerical analysis of hard processes in the framework of $k_T$-factorization is not provided yet.

The predictions of all phenomenological approaches depend significantly on the quark and gluon structure functions. The last ones are more or less known experimentally from the data of HERA, but unknown at very small values of Bjorken variable $x < 10^{-4}$. However it is just the region that dominates in the heavy quark production at high energies\footnote{For example, in the case of charm production, $m_c = 1.4\text{GeV}$, at LHC, $\sqrt{s} = 14\ \text{TeV}$, the product $x_1x_2$ of two gluons (both $x_1$ and $x_2$ are the integral variables) is equal to $4 \cdot 10^{-8}$ and the applicability of the existing structure functions at so small $x$ is not clear, see discussion in [21].}

The main goal of this paper is to compare the results of the conventional parton model and the $k_T$-factorization approach and to explain the source for the differences in their predictions. We present the main formalism of these approaches as well as the numerical
results. To clarify the results we consider very high energies, including the non-realistical one $\sqrt{s} = 1000$ TeV, and use the simple ”toy” gluon structure function of the nucleon.

2 Conventional parton model approach

The conventional parton model expression for the heavy quark hadroproduction cross sections has the factorized form [22]:

$$d\sigma(ab \rightarrow Q\overline{Q}) = \sum_{ij} dx_i dx_j G_{a/i}(x_i, \mu_F) G_{b/j}(x_j, \mu_F) d\hat{\sigma}(ij \rightarrow Q\overline{Q}), \quad (1)$$

where $G_{a/i}(x_i, \mu_F)$ and $G_{b/j}(x_j, \mu_F)$ are the structure functions of partons $i$ and $j$ in the colliding hadrons $a$ and $b$, $\mu_F$ is the factorization scale (i.e. virtualities of incident partons) and $d\hat{\sigma}(ij \rightarrow Q\overline{Q})$ is the cross section of the subprocess calculated in the perturbative QCD. It can be written as a sum of LO and NLO contributions,

$$d\hat{\sigma}(ij \rightarrow Q\overline{Q}) = \frac{\alpha_s^2(\mu_R)}{m_Q^2} \left[f^{(0)}_{ij} + 4\pi\alpha_s(\mu_R)\left[f^{(1)}_{ij} + \bar{f}^{(1)}_{ij} \ln(m_{R}^2/M_Q^2)\right]\right], \quad (2)$$

where $\mu_R$ is the renormalization scale, and the functions $f^{(0)}_{ij}$, $f^{(1)}_{ij}$ as well as $\bar{f}^{(1)}_{ij}$ actually depend only on a single variable

$$\rho = \frac{4m_Q^2}{\hat{s}}, \quad \hat{s} = x_i x_j s_{ab}. \quad (3)$$

The expression (1) corresponds to the process shown schematically in Fig. 1a. The main contribution to the heavy flavour production cross section at small $x$ is known to come from gluons, $i = j = g$.

The principal uncertainties of any numerical QCD calculation are the consequences of unknown scales $\mu_F$ and $\mu_R$ and the heavy quark mass, $m_Q$. Both scales (sometimes they are assumed to be equal) are to be of the order of hardness of the treated process, however which value is better to take, $m_Q$, $m_T = \sqrt{m_Q^2 + p_T^2}$ or $\hat{s}$, remains to be unknown. Principally, the uncertainties should not be essential because of the logarithmical dependence on these parameters. Unfortunately the masses of $c$ and $b$ quarks are not large enough and it leads to numerically large uncertainties (see e.g. [4]) at the existing energies for fixed target experiments.

Another principal problem of parton model is the collinear approximation. The transverse momenta of the incident partons, $q_i T$ and $q_j T$ are assumed to be zero, as is shown in Fig. 1a, and their virtualities enter only via structure functions; the cross section $d\hat{\sigma}(ij \rightarrow Q\overline{Q})$ is

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\[\text{These uncertainties have to disappear after all high order contributions are summed up. There is the opinion that the strong (weak) scale dependence in LO or NLO means the large (small) contribution of high order diagrams, but it is not the case. The strong or weak scale dependence in LO or NLO indicates only the same for the higher orders, which nevertheless can be numerically small or large at some particular fixed scale.}\]
assumed to be independent on the virtualities. This approximation considerably simplifies the calculations.

The conventional NLO parton model approach with collinear approximation works quite reasonably for one-particle distributions and the total cross sections; at the same time there is the serious disagreement with the data on azimuthal correlations and on the transverse momentum distributions of the heavy-quark pair. The reason is quite clear. Consider the processes of the fusion of two partons, having transverse momenta $q_1^T$ and $q_2^T$, into $Q\bar{Q}$ pair with transverse momenta $p_1^T$ and $p_2^T$. In LO parton model $q_1^T + q_2^T = p_1^T + p_2^T = 0$, and the distribution on the transverse momenta of the quark pair coincides with the distribution on the total transverse momentum of the initial partons. NLO gives a correction, numerically not very large because the shapes of $p_T$ distributions in LO and NLO are rather close to each other, see [3, 23]. In this case the qualitative features in NLO should be the same as LO ones, so the final transverse momentum distributions of the heavy-quark pair only slightly differ from $\delta$-functions, being in contradiction with the existing data [6, 9].

It was shown [9], that the conventional NLO parton model can describe the experimental data on azimuthal correlations and the momentum distributions of the heavy-quark pair, assuming the comparatively large intrinsic transverse momentum of incoming partons ($k_T$ kick) about 1 GeV/c. This procedure is, to a large extent, arbitrary. Let $p_T(Q\bar{Q})$ be the total transverse momentum of the pair. For each event the heavy quark pair, $Q\bar{Q}$, is boosted firstly to the rest in the longitudinal direction. Then a second transverse boost is performed, giving the pair a transverse momentum $p_T(Q\bar{Q}) = k_{1,T} + k_{2,T}$; $k_{1,T}$ and $k_{2,T}$ are the transverse momenta of the incoming partons, which are chosen randomly, with their moduli distributed according to

$$\frac{1}{N} \frac{dN}{d k_T^2} = \frac{1}{\langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right).$$

Alternatively, one can proceed as in the previous case, but giving the additional transverse momentum $k_{1,T} + k_{2,T}$ to the whole final-state system (at the NLO in QCD, this means the $Q\bar{Q}$ pair plus a light parton), and not to the $Q\bar{Q}$ pair only. These two methods lead to similar results [9].

However the $k_T$ kick method has no good theoretical background. The general phenomenological expression for the heavy quark production can be written as a convolution of the initial transverse momenta distributions, $I(q_{1T})$ and $I(q_{2T})$, with squared modulo of the matrix element,

$$\sigma_{QCD}(Q\bar{Q}) \propto \int d^2q_{1T} d^2q_{2T} I(q_{1T}) I(q_{2T}) |M(q_{1T}, q_{2T}, p_{1T}, p_{2T})|^2.$$  

Here there are two possibilities:

i) the typical gluon transverse momenta are much smaller than the transverse momenta of produced heavy quarks, $q_{iT} \ll p_{iT}$, or

ii) all transverse momenta are of the same order, $q_{iT} \sim p_{iT}$.

In the first case one can replace both initial distributions $I(q_{iT})$ by $\delta$-functions. It reduces

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\(^3\)We omit here for the simplicity all not important factors.
the expression (5) to the simple form (collinear approximation):
\[ \sigma_{\text{coll.}}(Q\bar{Q}) \propto |M(0,0,p_{1T},p_{2T})|^2 \]
(6)
in total agreement with Weizsaecker-Williams approximation.

In the second case we can not a priory expect good results from the Weizsaecker-Williams approximation. However it gives the reasonable numerical results in some cases.

The \( k_T \) kick discussed above effectively accounts for the transverse momenta of incident partons. It is based on the expression which symbolically reads
\[ \sigma_{\text{kick}}(Q\bar{Q}) \propto I(q_{1T})I(q_{2T}) \otimes |M(0,0,p_{1T},p_{2T})|^2 . \]
(7)
The main difference from the general QCD expression, Eq. (5), is that due to the absence of \( q_{iT} \) in the matrix element the value \( \langle k_{2T}^2 \rangle \) in Eq. (4) has to be differently taken for different processes (say, for heavy flavour production with comparatively small or large \( p_T \)). The reason is that the functions \( I(q_{iT}) \) in general QCD expression decrease for large \( q_{iT}^2 \) as a weak power (see next Sect.), i.e. comparatively slowly, and the \( q_{iT}^2 \) dependence of the matrix element is more important.

3 Heavy quark production in \( k_T \)-factorization approach

Consider another approach, when the transverse momenta of the incident gluons in the small-\( x \) region result from the diffusion in the parton (gluon) evolution. The diffusion is described by the function \( \varphi(x,q^2) \) giving the gluon distribution at the fixed fraction of the longitudinal momentum of the initial hadron, \( x \), and the gluon virtuality, \( q^2 \). It is approximately determined by the derivative of the usual structure function. Although generally it is the function of three variables, \( x, q_T \) and \( q^2 \), the transverse momentum dependence is comparatively weak since \( q_T^2 \approx -q^2 \) for small \( x \) in LLA in agreement with \( q^2 \)-dependences of structure function. Note that due to QCD scaling violation the value \( \varphi(x,q^2) \) for the realistic structure functions increases with decreasing of \( x \), and \( q_T \) becomes important in the numerical calculations.

The exact expression for \( q_T \) gluon distribution can be obtained as a solution of the evolution equation which, contrary to the parton model case, is nonlinear due to interactions between the partons in small \( x \) region. The calculations [24] of the \( q_T \) gluon distribution in leading order using the BFKL theory [25] result in difference from our \( \varphi(x,q^2) \) function only about 10-15%.

Here we deal with Eq. (3) with the matrix element accounting for the gluon virtualities and polarizations. Since it is much more complicated than in the parton model we consider only LO of the subprocess \( gg \rightarrow Q\bar{Q} \) which gives the main contribution to the heavy quark production cross section at small \( x \), see the diagrams in Fig. 2. The lower and upper ladder blocks present the two-dimensional gluon functions \( \varphi(x_1,q^2_1) \) and \( \varphi(x_2,q^2_2) \). In terms of the
conventional DGLAP parton distributions they can be determined as [10]

$$\varphi(x, q^2) = 4\sqrt{2}\pi^3 \frac{\partial[xG(x, q^2)]}{\partial q^2}.$$  \hspace{1cm} (8)

Such a definition of $\varphi(x, q^2)$ makes it possible to treat correctly the effects arising from the gluon virtualities.

Strictly speaking we have to include double logarithmic Sudakov-like form factor under the derivative in Eq. (8) [20]. It was shown in [20] that the probability to find a parton with a given longitudinal momentum fraction $x$ and transverse momentum $k_t$ within the LLA may be written as

$$f_a(x, k_t^2) = \frac{\partial}{\partial k_t^2} \left[xa(x, k_t^2)T_a(k_t^2, \mu^2)\right],$$  \hspace{1cm} (9)

where $a = g, q$; $T$ is the Double Logarithmic (DL) Sudakov form factor and

$$\varphi_a(x, q^2) = 4\sqrt{2}\pi^3 f_a(x, q^2).$$  \hspace{1cm} (10)

The first factor in (9) is evident. It corresponds to the real contribution/emission in the DGLAP evolution. Changing the scale from $\mu^2$ to $\mu^2 + \delta\mu^2$ one produces a new parton with $k_t^2$ between $\mu^2$ and $\mu^2 + \delta\mu^2$. So

$$f_a(x, k_t^2) = \frac{\partial}{\partial k_t^2} [xa(x, k_t^2)].$$  \hspace{1cm} (11)

Next factor

$$T = \exp \left(-C \int_{k_t^2}^{\mu^2} \frac{\alpha_s(q^2)}{2\pi} \ln \left(\frac{\mu^2}{q^2}\right) \frac{dq^2}{q^2}\right)$$  \hspace{1cm} (12)

($C = C_F = (N_c^2 - 1)/2N_c$ for the quarks and $C = C_A = N_c$ for the gluons) accounts for the virtual loop DGLAP contribution, which is needed to provide an appropriate normalization of the parton wave function and to satisfy the sum rules (i.e. the conservation of momentum, flavour, etc.). $T$ plays the role of survival probability, i.e. the probability not to emit an extra partons (gluons) with the transverse momenta $q'_t \subset [k_t, \mu]$.

Note that in DGLAP equation, written for the integrated (including all $k_t \leq \mu$) partons, there was a cancellation between the real and virtual soft gluon DL contributions. Emitting a soft gluon with the momentum fraction $(1 - z) \rightarrow 0$ one does not change the $x$-distribution of parent partons. Thus the virtual and real contributions originated by the $1/(1 - z)$ singularity of a splitting function $P(z)$ cancels each other.

Contrary, in unintegrated case the emission of soft gluon (with $q'_t > k_t$) alters the transverse momentum of parent ($t$-channel) parton. Eq. (9) includes for this effect through the derivative $\partial T(k_t^2, \mu^2)/\partial k_t^2$.

Thanks to this last derivative one obtains a positive unintegrated distribution $f_a(x, k_t^2)$ even at rather large $x$, where (due to the virtual term in DGLAP) the integrated parton density $a(x, \mu^2)$ decreases with $\mu^2$. 

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Unfortunately, the factor $T$ is known with the DLog accuracy only. When $k_t > \mu$ there are no double logarithms at all. So for $k_t > \mu$ we put $T = 1$.

In terms of Feynman diagrams in a physical (axial) gauge the factor $T$ comes from the self-energy graphs. In Feynman gauge the self-energy contribution has no double logarithms. Such a double logarithms come from the graphs with the gluons which embrace the hard blob (an example is shown in Fig.1b). Any diagram with a soft gluon emitted from one external line of hard blob (say, gluon $q_2$) and absorbed on another external line has the same double logarithm, except of the colour factor. The sum of colour factors, corresponding to all three graphs (soft gluon is absorbed by the heavy quark, antiquark or gluon $q_1$), is equal to the colour factor for self-energy diagram. This way one obtains in Feynman gauge the same expression (9).

Thus the true probability to find a parton with the momentum fraction $x$ and transverse momentum $k^2_T = k^2$ is given by the Eq. (9). (see [20] for more detail). To simplify the discussion we postpone the more exact Eq. (9) to the end of Section 4 and start with the Eq. (8) omitting the double logarithmic factor $T$.

A different way to introduce the gluon $q_T$ distributions is suggested in Ref. [26] and based on the Fourier transform of the structure functions.

In what follows we use Sudakov decomposition for quark momenta $p_{1,2}$ through the momenta of colliding hadrons $p_A$ and $p_B$ ($p_A^2 = p_B^2 \simeq 0$) and transverse ones $p_{1,2T}$:

$$p_{1,2} = x_{1,2}p_B + y_{1,2}p_A + p_{1,2T}. \quad (13)$$

The differential cross section of heavy quarks hadroproduction has the form:

$$\frac{d\sigma_{pp}}{dy_1^*dy_2^*d^2p_{1T}d^2p_{2T}} = \frac{1}{(2\pi)^8 (s)^2} \int d^2q_1d^2q_2\delta(q_1T + q_2T - p_{1T} - p_{2T}) \times \frac{\alpha_s(q_1^2)}{q_1^2} \frac{\alpha_s(q_2^2)}{q_2^2} \varphi(q_1^2, y) \varphi(q_2^2, x) |M_{QQ}|^2. \quad (14)$$

Here $s = 2p_Ap_B$, $q_{1,2T}$ are the gluon transverse momenta and $y_{1,2}$ are the quark rapidities in the hadron-hadron c.m.s. frame,

$$x_1 = \frac{m_1}{\sqrt{s}} e^{-y_1^*}, \quad x_2 = \frac{m_2}{\sqrt{s}} e^{-y_2^*}, \quad x = x_1 + x_2$$

$$y_1 = \frac{m_1}{\sqrt{s}} e^{y_1^*}, \quad y_2 = \frac{m_2}{\sqrt{s}} e^{y_2^*}, \quad y = y_1 + y_2 \quad (15)$$

$|M_{QQ}|^2$ is the square of the matrix element for the heavy quark pair hadroproduction.

In LLA kinematic

$$q_1 \simeq y_{PA} + q_{1T}, \quad q_2 \simeq x_{PB} + q_{2T}. \quad (16)$$

so

$$q_1^2 \simeq -q_{1T}^2, \quad q_2^2 \simeq -q_{2T}^2. \quad (17)$$

\footnote{We put the argument of $\alpha_S$ to be equal to gluon virtuality, which is very close to the BLM scheme [27]; (see also [19]).}
(The more accurate relations are $q_1^2 = -\frac{q_{1T}^2}{1-y}, q_2^2 = -\frac{q_{2T}^2}{1-x}$ but we are working in the kinematics where $x, y \sim 0$).

The matrix element $M$ is calculated in the Born order of QCD without standart simplifications of the parton model. In the axial gauge $p_B^\mu A_\mu = 0$ the gluon propagator takes the form $D_{\mu\nu}(q) = d_{\mu\nu}(q)/q^2$,

$$d_{\mu\nu}(q) = \delta_{\mu\nu} - (q^\mu p_B^\nu + q^\nu p_B^\mu)/(p_B q) .$$  \hspace{1cm} (18)

For the gluons in $t$-channel the main contribution comes from the so called "nonsense" polarization $g_{\mu\nu}^n$, which can be picked out by decomposing the numerator into longitudinal and transverse parts:

$$\delta_{\mu\nu}(q) = 2(p_B^\mu p_A^\nu + p_A^\mu p_B^\nu)/s + \delta_{\mu\nu}^T \approx 2p_B^\mu p_A^\nu/s \equiv g_{\mu\nu}^n .$$  \hspace{1cm} (19)

The other contributions are suppressed by the powers of $s$. It is easy to check that in axial gauge \((18)\) $p_B^\mu d_{\mu\nu} = 0$ and $p_A^\mu d_{\mu\nu} \approx -q_T^\nu/y$. Thus we effectively get

$$d_{\mu\nu}(q) \approx -2 \frac{p_B^\mu q_T^\nu}{y s} .$$  \hspace{1cm} (20)

Another way to obtain the same result is to use the transversality condition. Since the sum of the diagram Fig. 2a-c is gauge invariant the product $q_{1\mu} M_\mu = 0$, here $M_\mu$ denotes the amplitude of the gluon $q_1$ and hadron $p_B$ interaction described by the lower part of Fig. 2a–c. Using the expansion \((18)\) for the $q_1$ momentum we obtain

$$p_A^\mu M_\mu \approx -\frac{q_{1T}^\mu}{y} M_\mu ,$$

which leads to Eq. \((20)\) or

$$d_{\mu\nu}(q) \approx 2 \frac{q_{1T}^\mu q_{2T}^\nu}{x y s} ,$$  \hspace{1cm} (21)

if we do such a trick for the vector $p_B$ too.\footnote{Having in mind this trick one can say that the matrix element is gauge-invariant in $k_T$-factorization approach. However the polarization vectors of incoming gluons $q_1$ and $q_2$ are not arbitrary taken but fixed as $-q_{1T}^\mu/y$ and $-q_{2T}^\mu/x$, respectively (see \cite{14} for more detail).} Both these equations for $d_{\mu\nu}$ can be used but for the form \((20)\) one has to modify the gluon vertex slightly (to account for several ways of gluon emission – see ref. \cite{28}):

$$\Gamma_{eff}^\nu = \frac{2}{x y s} \left[(x y s - q_{1T}^2) q_{1T}^\nu - q_{1T}^2 q_{2T}^\nu + 2x (q_{1T} q_{2T}) p_B^\nu\right] .$$  \hspace{1cm} (22)

As a result the colliding gluons can be treated as aligned ones and their polarization vectors are directed along the transverse momenta. Ultimately, the nontrivial azimuthal correlations must arise between the transverse momenta $p_{1T}$ and $p_{2T}$ of the heavy quarks.

From the formal point of view there is a danger to loose the gauge invariance in dealing with the off mass shell gluons. Say, in the covariant Feynman gauge the new graphs (similar
to the "bremsstrahlung" from the initial or final quark line, as it is shown in Fig. 2d) may contribute in the central plateau rapidity region. However this is not the fact. Within the "semi-hard" accuracy, when the function $\varphi(x, q^2)$ collects the terms of the form $\alpha_s^k(\ln q^2)^n(\ln(1/x))^m$ with $n + m \geq k$, the triple gluon vertex \(\text{[22]}\) includes effectively all the leading logarithmic contributions of the Fig. 2d type \(\text{[25, 18]}\). For instance, the upper part of the graph shown in Fig. 2d corresponds in terms of the BFKL equation to the $t$-channel gluon reggeization. Thus the final expression is gauge invariant (except a small, non-logarithmic, $O(\alpha_s)$ corrections) \(\text{[1]}\).

Although the situation considered here seems to be quite opposite to the parton model there is a certain limit \(\text{[28]}\), in which our formulae can be transformed into parton model ones, namely when the quark transverse momenta, $p_{1,2T}$, are much larger than the gluon ones, $q_{1,2T}$.

### 4 Results of numerical calculations

Eq. (14) enables to calculate straightforwardly all distributions concerning one-particle or pair production. To illustrate the difference between our approach and the conventional parton model we present first of all the results of calculations of charm production ($m_c = 1.4$ GeV \(\text{[29, 30]}\)) with high $p_T$ at three energies, $\sqrt{s} = 1$ TeV, 10 TeV and $10^3$ TeV and with the same value of

$$x_T = \frac{2p_T}{\sqrt{s}} = 0.02,$$

i.e. $p_T = 10$ GeV, 100 GeV and $10^4$ GeV for the above energies.

We will use QCD scales $\mu_F = m_T = \sqrt{m_c^2 + p_T^2}$ and $\mu_R = m_c$, i.e. fixed coupling with $N_f = 3$ and $\Lambda = 248$ MeV.

However there exists a problem coming from the infrared region, because the functions $\varphi(x, q_1^2)$ and $\varphi(y, q_2^2)$ are unknown at the small values of $q_{1,2}^2$. Moreover, for the realistic gluon structure functions the value $\varphi(x, q^2)$ is positive in the small-$x$ region and negative in the large-$x$ region. The boundary between two regions, where $\varphi(x, q^2) = 0$, depends on the $q^2$, therefore the relative contributions of these regions is determined by the characteristic scale of the reaction, say, by the transverse momentum $p_T$.

To avoid this additional problem, we present the numerical calculations both in the $k_T$-factorization approach and in the LO parton model, using the "toy" gluon distribution

$$xG(x, q^2) = (1 - x)^5 \ln(q^2/Q_0^2),$$

for $q^2 > Q_0^2$, and $xG(x, q^2) = 0$ for $q^2 < Q_0^2$, with $Q_0^2 = 1$ GeV$^2$.

After the calculations of charm production cross sections using Eqs. (1) and (8), (14) one can see, that the values $d\sigma/dx_T$ at $x_T = 0.02$ are about 4–5 times larger in the $k_T$-factorization approach than in the LO parton model. Really this difference should not be considered as

6Taking the gluon polarization vector as $-q_{1T}/y$ we can completely eliminate the leading logarithm terms arising from Fig. 2d.
very large because, as it was discussed, an essential part of NLO and NNLO corrections is already included in the $k_T$-factorization, and it is known that the sum of LO and NLO contributions in the parton model is about 2-3 times larger than the LO contribution \[31\].

To show the most important values of the variables $q_{1,2T}$ in Eq. (14), as well as the kinematical region producing the main difference with the conventional parton model, we plot by dots in Fig. 3 the results of the $k_T$-factorization approach with the restrictions $|q_{1,2T}| \leq q_{max}$, as a function of $q_{max}$. The running coupling is fixed as $\alpha_S(m_c^2)$ instead of $\alpha_S(q_{1,2}^2)$ in Eq. (14). The dashed lines in Fig. 3 are the conventional parton model predictions with QCD scales $\mu_F = \sqrt{m_c^2 + p_T^2}$, and $\mu_R = m_c$. One can see that the $k_T$-factorization predictions exceed the parton model results when $q_{max} \geq p_T$.

Let us check now that the $k_T$-factorization predictions really coincide with the parton model ones, when the quark momenta, $p_{1,2T}$, are much larger than the gluon ones, $q_{1,2T}$ \[28\]. However we have to compare them at the same values of structure functions. When we submit Eq. (14) to the conditions $|q_{1,2T}| \leq q_{max}$ and neglect the $q_{1,2T}$ dependence in the matrix element, we recover the parton model approximation, and get the result proportional to $xG(x,q_{max}^2) \cdot yG(y,q_{max}^2)$ due to the direct consequence of Eq. (8) \[32\]

$$xG(x,q^2) = xG(x,Q_0^2) + \frac{1}{4\sqrt{2\pi}} \int_{Q_0^2}^{q^2} \varphi(x,q_1^2)dq_1^2.$$  \hspace{1cm} (25)

At the same time the original parton model yields the result proportional to $xG(x,\mu_T^2) \cdot yG(y,\mu_T^2)$ with $\mu_F = \sqrt{m_c^2 + p_T^2}$. Hence we expect the parton model to coincide with our calculations for the gluon distribution Eq. (24), $p_T \gg m_c$ and $|q_{1,2T}| \leq q_{max}$ after multiplying by an appropriate factor:

$$\left.\frac{d\sigma}{dx_T}\right|_{q_{1,2T} \ll p_T} = \left.\frac{d\sigma}{dx_T}\right|_{p_M} \left(\frac{\ln(q_{max}^2/Q_0^2)}{\ln(p_T^2/Q_0^2)}\right)^2.$$  \hspace{1cm} (26)

The right-hand side of Eq. (26) presented in Fig. 3 by solid curves shows a good agreement with $k_T$-factorization approach (open dots) even when $q_{max}$ is only slightly smaller (at the highest energy) than $p_T$. The same values $d\sigma/dx_T$, as in Fig. 3, but differentiated with respect to $ln q_{max}$ are presented in Fig. 4 for $k_T$-factorization approach. It exhibits, especially at the high energies, the logarithmic growth with $q_{max}$, until the value $q_{max} \sim p_T$. There is the narrow peak in this region, which provides about 70-80% of the $d\sigma/dx_T$ cross section integrated over the whole $q_{1,2T}$ phase space. Its physical nature seems to be quite clear. There are two kinematical regions for $t$-channel and $u$-channel diagrams, Fig. 2 a,b, giving comparatively large contributions to $d\sigma/dx_T$ for the high energies and high $p_T$ heavy quark production. One of them comes from the conventional parton model kinematics when both initial gluon transverse momenta, $q_{1,2T}$, are very small compared to $p_{1,2T}$, see Fig. 5a. Another large contribution appears from the region where, say, $q_{1T} \sim p_{1T}$, whereas $q_{2T}$ and $p_{2T}$ are comparatively small, as it is shown in Fig. 5b. In this case the quark propagator, \[1/(\hat{p}_1 - \hat{q}_1) - m_Q\] = \[1/(\hat{q}_2 - \hat{p}_2) - m_Q\], is close to the mass shell and gives rise to the narrow peak shown in Fig. 4. The general smallness of such processes comes from high-virtuality gluon propagator in Fig. 5b, and it is of the same order as in the case of Fig. 5a, even with
Table 1: The ratios of $c\bar{c}$ pair production in $k_T$-factorization approach and in LO parton model

| $\sqrt{s}$, TeV | 0.3 | 1  | 10  | 100 | 1000 |
|-----------------|-----|----|-----|-----|------|
| $R_{tot}$       | 4.0 | 4.0| 4.0 | 3.9 | 3.9  |
| $R(x_T = 0.02)$ | 3.4 | 4.5| 5.5 | 5.4 | 5.2  |

some suppression of the diagram Fig. 5a, where the contribution from the region of large rapidity difference of two produced heavy quark is suppressed by the fermion propagator.

The diagram shown in Fig. 5b can be considered as one of the most important contribution to the NLO parton model in the high energy limit, because it have spin-one gluon in the $t$-channel. Due to all these factors, combinatorics and the interference between diagrams, the summary contribution of the peak to the total $d\sigma/dx_T$ value is several times larger than the LO parton model contribution. In the Table 1 we present the calculated ratios of the total heavy quark pair production cross section, $R_{tot}$, and the ratio of $d\sigma/dx_T$, $R(x_T)$ at $x_T = 0.02$ (integrated over rapidity).

One can see that the relative contribution of the discussed peaks firstly increases due to increase the phase space. At the energy high enough ($\sqrt{s} \sim 10$ TeV this contribution is saturated. After that the relative contribution of the LO parton model increases logarithmically with $p_T$, and have to dominate at the extremely high energies and $p_T$, in academical asymptotic.

These results are confirmed by those presented in Fig. 6, where we reproduce the data from Fig. 3 for LO parton model and $k_T$-factorization approach with the condition $|q_{1,2T}| \leq q_{max}$. For comparison we show by stars the $k_T$-factorization predictions for all values $q_{1T}$ with the only condition $|q_{2T}| \leq q_{max}$, versus $q_{max}$. The last results are above the LO parton model even at not too large $q_{2T}$ because the peaks, discussed above, are included into the integral over $q_{1T}$.

Let us note that the calculation of $d\sigma/dx_T$ at $\sqrt{s} = 10$ TeV, $x_T = 0.02$, and with restriction $|q_{1,2T}| \geq p_T/2$ (see Fig. 5c) gives only about 2% of the total value of $d\sigma/dx_T$.

It is necessary to note that the essential values of $q_{1,2T}$ increase in our calculations when the transverse momentum, $p_T$ of the detected $c$-quark increases. At initial energy high enough, $q_{1,2T} \sim p_T$. In the $k_T$ kick language it means that the $\langle k_T^2 \rangle$ value also increases.

The values $d\sigma/dp_T$ are presented in Fig. 7. Here the lower dashed curves correspond to the LO parton model, solid curves — to the $k_T$-factorization approach with unintegrated gluon function $\varphi(x, q^2)$ given by the simplified expression (8) while the dotted and dash-dotted curves embody the double-logarithmic $T$-form factor (12) and use Eqs. (9) and (10) (see the discussion in section 3). Recall that from the diagram viewpoint the $T$-factor sums up the diagrams containing in the Feynman gauge the gluon lines embracing the hard blob as it is shown in Fig. 1b. In the axial gauge the corresponding Double Log contribution comes
from the parton (gluon) self-energy insertion, i.e. from the term proportional to $\delta (1 - z)$ in the DGLAP evolution equation. The $T$-factor reproduces a large part of the virtual NLO corrections with respect to the conventional PM and diminishes the cross section. It suppresses the contribution coming from small values of parton virtualities, $q_{1,2}^2 \ll m_T^2 = m_Q^2 + p_T^2$ in Eq. (14). Such a suppression becomes more important at very large $p_T$ as in this case the PM contribution, which is collected from the logarithmically large interval of $q_{1,2}^2 \ll p_T^2$ ($\int_{q_{0}^2}^{p_T^2} dq_{1,2}^2/q_{1,2}^2$), starts to dominate.

Recall that before we have no "scale" problem within the $k_T$-factorization approach since we deal with the explicit integral (14) over the parton virtualities $q_{1,2}^2$. The dependence of QCD matrix element $M(q_{1T},q_{2T},p_{1T},p_{2T})$ on the $q_{1,2T}$ values provides the convergency to the integral "automatically" fixing the "scale" $\mu_F$.

After $T$-factor is "switched on" one gets the problem of scale back. The scale $\mu^2$ is not fixed in (12) at LO accuracy. The reasonable $\mu^2$ values are expected to be somewhere between $m_T^2$ and $s = M_Q^2 = xys$. The numerical difference between these values is very small, as one can see from Fig.7.

The rapidity distributions of produced charm quark is presented in Fig. 8. Here again the dashed curves are the results of LO parton model calculations, the solid curves show the results of the $k_T$-factorization approach with unintegrated gluon function $\varphi(x,q^2)$ given by the simplified expression (8), while the dotted curves embody the double-logarithmic $T$-form factor (12) with $T(m_T^2)$ and use Eqs. (9) and (10). One can see that the last form factor decrease the cross section calculated in the $k_T$-factorization approach about 1.5 times.

5 Conclusion

We have compared the conventional LO Parton Model (PM) and the $k_T$-factorization approach for heavy quark hadroproduction. In order to concentrate on the specific features of the approach and not to deal with the peculiarities of the parton distributions a simple gluon density (24) has been chosen.

It has been shown that the contribution from the domain with strong $q_T$ ordering ($q_{1,2T} \ll m_T = \sqrt{m_Q^2 + p_T^2}$) coincides in $k_T$-factorization approach with the LO PM prediction. However a very numerically large contribution comes besides this in $k_T$-factorization from the region $q_{1,2T} \geq m_T$. It is kinematically related to the events where the transverse momentum of heavy quark $Q$ is balanced not by the momentum of antiquark $\overline{Q}$ but by the momentum of the nearest gluon.

This configuration is associated with NLO (or even NNLO, if both $q_{1,2T} \geq m_T$) corrections in terms of the PM with fixed number of flavours, i.e. without the heavy quarks in the evolution. Indeed, as it was mentioned in [1], up to 80% of the whole NLO cross section originates from the events where the heavy quark transverse momentum is balanced by the nearest gluon jet. Thus the large "NLO" contribution, especially at large $p_T$, is explained by the fact that the virtuality of the $t$-channel (or $u$-channel) quark becomes small in the region $q_T \simeq p_T$ and the singularity of the quark propagator $1/(\hat{p} - \hat{q} - m_Q)$ in the "hard"
QCD matrix element, \( M(q_1, q_2, p_1, p_2) \), reveals itself.

Including the double logarithmic Sudakov-type form factor \( T \) into the definition of unintegrated parton density one takes into account an important part of the virtual loop NLO (with respect to PM) corrections. Thus we demonstrate that \( k_T \)-factorization collects already at LO the major part of the contributions which play the role of the NLO (and even NNLO) corrections to the conventional PM. Therefore we hope that the higher order \( \alpha_S \) correction to the \( k_T \)-factorization would be rather small.

Another advantage of this approach is that a non-zero transverse momentum of \( Q\bar{Q} \)-system \( p_1 + p_2 = q_1 + q_2 \) is naturally achieved in \( k_T \)-factorization. The typical value of this momentum \( (k_T \text{-kick}) \) depends on the parton structure functions/densities. It increases with the initial energy \( (k_T \text{-kick}) \) increases with the decreasing of the momentum fractions \( x, y \) carried by the incoming partons) and with the transverse momenta of heavy quarks, \( p_T \). Thus one gets a possibility to describe a non-trivial azimuthal correlation without introducing a large "phenomenological" intrinsic transverse momentum of the partons.

A more detail study of the heavy quark production (including the correlations) in the \( k_T \)-factorization approach based on the modern realistic parton distributions will be published elsewhere.

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Figure captions

Fig. 1. Heavy quark production in the LO parton model (a) and in $k_T$-factorization approach with Sudakov T-factor (see discussions in Sect. 3).

Fig. 2. Low order QCD diagrams for heavy quark production in $pp$ ($p\bar{p}$) collisions via gluon-gluon fusion (a-c) and the diagram (d) formally violating the factorization, that is restored within logarithmic accuracy.

Fig. 3. The values of $d\sigma/dx_T$ for charm hadroproduction at $x_T = 0.02$ in $k_T$-factorization approach as a function of upper limits of integrals in Eq. (14) (points); the conventional parton model values (dashed lines) and the values of right-hand side of Eq. (26) (solid curves).

Fig. 4. The values of $d\sigma/dx_T/d\ln q_{\text{max}}$ for charm hadroproduction at $x_T = 0.02$ in $k_T$-factorization approach as a function of upper limits of integrals in Eq. (14) (points).

Fig. 5. The diagrams which are important in the case of one-particle distributions of heavy quark with large $p_T$. The situation similar to the LO parton model (a). The case, possible in NLO parton model, large $p_T$ of the quark is compensated by hard gluon and the fermion propagator is near to mass shell (b). The numerically small contribution, when large transverse momenta of heavy quarks are compensated by two hard gluons, whereas the fermion propagator is near to mass shell (c).

Fig. 6. The same as in Fig. 3 (points and dashed curves) together with the values of $d\sigma/dx_T$ (stars) for the case when only $q_{2T}$ upper limit integration is bounded by $q_{\text{max}}$.

Fig. 7. $p_T$-distributions of $c$-quarks produced at different energies. Dashed curves are the results of LO parton model. Solid curves are calculated with unintegrated gluon distribution $\varphi(x, q^2)$ given by Eq. (10), dash-dotted and dotted curves from Eqs. (11) and (12) for $\mu^2$ values in Eq. (12) equal to $\hat{s}/4$ and $m_T^2$, respectively.

Fig. 8. Rapidity distributions of $c$-quarks produced at different energies. Dashed curves are the results of LO parton model. Solid curves are calculated with unintegrated gluon distribution $\varphi(x, q^2)$ given by Eq. (10) and dotted curves from Eqs. (11) and (12) for $\mu^2$ values in Eq. (12) equal to $m_T^2$.  

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Fig. 1b
\[ \frac{d\sigma}{dx_T}, \text{ mb} \]

\( \sqrt{s} = 10^3 \text{ GeV} \)

\( \sqrt{s} = 10^4 \text{ GeV} \)

\( \sqrt{s} = 10^6 \text{ GeV} \)

\[ \ln|q_{\text{max}}|, \text{ GeV/c} \]

Fig. 3
Fig. 4
Fig. 5
\[ \sqrt{s} = 10^6 \text{ GeV} \]

Fig. 6
Fig. 7
