Characterization of Water Dynamics and Modelling of an Open Channel Irrigation System

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Abstract. Management of open channel canals is very essential to avoid overflows and overcome water demand. Automatic control of delivery canals has been gaining significance in recent years with the purpose of improving the efficiency of water management. This paper presents the development of single reach irrigation canal with the objective of controlling the water level. Upstream gate opening of an irrigation canal reach is operated based upon a constant-level reservoir height. The canal reach between two gates is divided into N nodes, and the finite-difference forms of the continuity and the momentum equations (saint venant equations) are written for each node. The Taylor series expansion is applied to linearize the equations around the equilibrium conditions. The state space model of the canal system is developed and the same will further be used to design an efficient controller for maintaining the level of water in the canal.

1. Introduction

Incompetent usage of water networks is the major reason for the water scarcity faced by several developing countries. In South Asian countries, the infrastructure is on a constant decay whilst water resources are going dry besides the vast and complex water network system present. Reliability of water supply can be achieved through improved operation of partial and full automation in the main and branch canals [1].

Open channel flow is governed by steady and unsteady flow equations. Unsteady flow saint-venant equations are nonlinear hyperbolic partial differential equations which can be solved only by numerical methods. Published literature on unsteady flow modelling is numerous [3,4].

Mohan Reddy [10] linearized the saint-venant equation and local optimal control technique is applied for a single reach canal. Malaterre and Baume [11] addresses about all type of model that include: saint venant linearization, nonlinear features, strong unknown perturbations, and interactions among subsystems, neural network based models, fuzzy logic based models. Litrico and Fromion [6] developed frequency response approach for saint-venant equations. It is linearized around any stationary regime. Weyer [9] was performed the Model identification of the Haughton channel reaches in Australia. Ooi [7] was applied System identification in saint-venant equations. P.J.Van overloop [8] was used system identification method in open channel system. Klaudia Horvath, Eduard Galvis, Manuel Gomez, Jose Rodellar [5] linearized the saint venant equations and transformed in Laplace
domain. In this paper, linearization is explained elaborately by using Taylor series expansion [12,13, 14].

The objective of this study is to model the unsteady flow for a single reach system with rectangular cross section in an irrigation canal. The controller is designed to achieve the target depth at the downstream end of the canal so as to satisfy the downstream water demand.

2. Mathematical Modelling

An open channel is a conduit in which a liquid flows with a free surface which is actually an interface between the moving fluid and an overlying fluid medium. The schematic diagram of an open channel with a single reach is shown in Figure 1.

The unsteady flow Saint-Venant equations are given by

$$\frac{\partial y}{\partial x} + \frac{1}{T} + \frac{\partial Q}{\partial x} = 0$$

(1)

$$\frac{\partial Q}{\partial x} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \left( \frac{\partial y}{\partial x} - S_0 + S_f \right) = 0$$

(2)

In which \( y \) = depth of flow in the channel, m; \( Q \) = flow rate through the channel, m³/s; \( T \) = flow surface width, m; \( g \) = gravitational acceleration, m/s²; \( S_0 \) = channel bottom slope; \( x \) = longitudinal direction of channel, m; \( A \) = flow cross section, m²; \( t \) = time, s; and \( S_f \) = friction slope, which is expressed as

$$S_f = \frac{Q|Q|n^2}{A^2R^{4/3}}$$

(3)

Where \( n \) = Manning's roughness coefficient; and \( R \) = hydraulic radius, m. It is given by

$$R = \frac{A}{P}$$

(4)

Where \( P \) = wetted perimeter in m. Equation 1 and 2 are quasi linear, partial differential equations of the hyperbolic type and defy a closed-form solution.

In this study the hyperbolic equations are discretized using finite difference schemes and are linearized using Taylor's series approximations by neglecting higher order derivatives. The equilibrium condition is assumed to be the steady flow values and is governed by steady state gradually varied flow equation.
\[ \frac{\partial y}{\partial x} = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 B}{g A^3}} \]  

(5)

In which x = distance along the channel; \( S_0 \) = longitudinal slope of the channel bottom; \( S_f \) = slope of the energy line; \( B \) = top water surface width; \( g \) = acceleration due to gravity. A single reach canal is divided into ‘N’ nodes in each reach shown in Figure 2. It is divided into N-1 sub reaches of length \( \Delta x \) (distance between the nodes). The continuity and momentum equations are written for each of these N nodes.

**Figure 2**: Depth-time grid for finite difference scheme

2.1. **Upstream Node:**
Forward difference scheme is applied for the upstream node, Continuity equation:

\[ \frac{dy}{dt} = \frac{-1}{\Delta x T_{1,m}} (Q_2 - Q_g) - \frac{q_{1,m}}{\Delta x T_{1,m}} \]  

(7)

Where,

\[ Q_g = C_d U_m b \sqrt{2g} \sqrt{y_{N,m-1} - y_{1,m}} \]  

(8)

Where, \( C_d \) = gate coefficient; \( b \) = gate width, m; \( U_m \) = opening at gate, m; \( g \) = gravity acceleration, m/s\(^2\); \( y_{N,m-1} \) = depth of previous pool; \( \Delta x \) = length of segment.

2.2. **Interior Node:**
Central difference scheme for the interior nodes, Continuity equation:

\[ \frac{dy}{dt} = \frac{-1}{2\Delta x T_{j,m}} (Q_{j+1,m} - Q_{j-1,m}) - \frac{q_{j,m}}{2\Delta x T_{j,m}} \]  

(9)
Momentum equation:
\[
\frac{dQ}{dt} = -\frac{1}{\Delta x A_{j,m}} (Q_{j+1,m} - Q_{j-1,m}) + \frac{Q_{j,m}^2}{2\Delta x A_{j,m}^2} + (A_{j+1,m} - A_{j-1,m}) - \frac{gA_{j,m}}{2\Delta x} (y_{j+1,m} - y_{j-1,m}) - gA_{j,m}S_0 + \frac{gA_{j,m}Q_{\max}}{K_{j,m}^2}
\] (10)

2.3. Downstream Node:

Backward difference scheme for the downstream nodes

Continuity equation:
\[
\frac{dy}{dt} = -\frac{1}{\Delta x T_{N,m}} (Q_g - Q_{N,m}) - \frac{gQ_{N,m}}{\Delta x T_{N,m}}
\] (11)

Where,
\[
Q_g = C_d U_{m+1} b \sqrt{2g} \sqrt{y_{N,m} - y_{1,m+1}}
\] (12)

In which \(C_d\) = discharge coefficient of the downstream gate in reach; \(b\) = width of downstream gate in reach, \(m\); \(u_{m+1}\) = height of downstream gate opening in reach, \(m\). For the last pool, \(y_{1,m+1}\) is replaced with constant reservoir height of downstream end. Ordinary differential equations is still nonlinear in both \(y\), \(Q\), \(u\).

The state variables chosen are the depth of flow and the flow rate. The gates are the controls and the gate discharge equations in conjunction with continuity equations serve as boundary conditions in upstream node and downstream nodes. These equations are,
\[
Q_{N,m-1} = Q_{1,m} + Q_{\text{out},m-1}
\] (13)
\[
Q_{N,m-1} = C_d b U_m \sqrt{2g} \sqrt{\Delta h}
\] (14)

The depths at each node and the gate openings at the equilibrium conditions is obtained from the steady flow equations. In the linearization, we have linearize in the equilibrium states. The equilibrium variables are \(y_{e,j,m}, Q_{e,j,m}, u_{e,j,m}\). Actual variables are \(y_{j,m}, Q_{j,m}, u_m\). Difference between the equilibrium variables and actual variables are deviation variables \(\delta y_{j,m}, \delta Q_{j,m}, \delta u_m\).

\[
y_{j,m} = \delta y_{j,m} + y_{e,j,m}
\] (15)
\[
Q_{j,m} = \delta Q_{j,m} + Q_{e,j,m}
\] (16)
\[
u_m = \delta u_m + u_{e,j,m}
\] (17)

The linearization results in linear matrix state space equation.
\[
\delta x(t) = A\delta x(t) + B\delta u(t)
\] (18)
The objective is to maintain constant discharge at the gate. If there is a disturbance in the system, system should be bring back to the original equilibrium condition. In order to achieve this, a feedback controller is designed. For that problem, suitable controller is used to maintain the depth and constant discharge.

3. Results and Discussions
The cross sectional details and the parameters of the hypothetical canal system used in the model is given below Table 1.

|                      |                        |
|----------------------|------------------------|
| **Table 1:** Cross sectional parameters of the hypothetical canal |                        |
| Length of canal reach | 6250 m                 |
| Number of nodes      | 5                      |
| Number of sub reaches used | 4                  |
| Distance between the node | 1250 m              |
| Discharge required at end of canal | 47 m$^3$/s |
| Upstream reservoir height | 3.24 m               |
| Downstream reservoir height | 1.25 m           |
| Target depth at downstream end | 1.99 m          |
| Gate width            | 18.25 m               |
| Gate discharge coefficient | 0.9                  |

With the simulated model, the depth of water along the length of the canal system can be estimated for the input flow conditions. It should be noted that the depth of water along the length of the canal will not be uniform. Rather the depth of water gradually increase along the length due to the slope of the canal. The open loop response of the depth of the canal system for different nodes is given in Figure 3.

![Figure 3: Variation of depth of water in different nodes of the canal system](image)

The above results may be helpful to efficiently control the level of water in the canal system by manipulating the control gates placed at the upstream side of the canal.
4. Conclusion

In order to assure the availability of water for the final users and in order to prevent the canal overflows during the flood, water discharge, water depth level and stored water volume of the canal systems are the important control variables. Controlling the water level contributes additional advantages like, preventing canal overflows and increase in the stability of the system. Hence, the control of water level in the extraction zone is considered as the control objective.

The modelling of the irrigation canal is carried out by dividing the canal into reaches, and then characterizing the water dynamics at each reach separately. The boundary conditions between reaches also included. Steady flow condition is modelled by applying newton method. Open channel dynamics for unsteady flow are described using non-linear partial derivatives equations (Saint-Venant equations). The modelling of the channel using saint venant equations is developed and linearized for unsteady flow condition. The depth is obtained for steady flow and unsteady flow conditions of the canal. Controller will be designed to achieve water level control by manipulating the position of the control gates.

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Appendix

The following symbols are used in this paper:

- $A$ = flow cross section, m$^2$;
- $A_{i}$ = continuous-time feedback matrix;
- $B$ = continuous-time control matrix;
- $b_{i}$ = width of upstream gate in reach;
- $b_{i+1}$ = width of downstream gate in reach;
- $C$ = continuous-time disturbance matrix;
- $CD_i$ = discharge coefficient of upstream gate in reach $i$;
- $C_{di+1}$ = discharge coefficient of downstream gate in reach $i+1$;
- $D$ = hydraulic depth, m;
- $g$ = acceleration gravity, m/s$^2$;
- $N =$ number of nodes in each reach of canal;
- $N =$ manning coefficient;
- $P =$ wetted perimeter;
- $Q =$ flow rate through channel, m$^3$/s;
- $Q_g =$ discharge rate through gate, m$^3$/s;
- $q =$ turnout demand per unit length of channel, m$^2$/s;
- $R =$ hydraulic radius;
- $S_f =$ friction slope;
- $S_0 =$ channel bottom slope;
- $T =$ flow surface width;
- $u_i =$ height of upstream gate opening in reach $i$, m;
- $u_{i+1} =$ height of downstream gate opening in reach $i+1$, m;
- $x =$ longitudinal direction of channel, m;
- $\Delta x =$ length of subreaches;
- $Y =$ depth of flow in channel;
- $\delta Q =$ deviation in flow rate at node, m$^3$/s