Cluster state entanglement of charge qubits

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Abstract. The cluster state is a special, highly entangled quantum state that provides a universal resource on which to perform measurement-based quantum computation. In this paper, we propose a scheme for the generation of such entanglement in a solid-state medium by the suppression of resonant tunneling of a ballistic electron by a nearby single-electron charge qubit. To investigate the viability of this scheme for the creation of cluster states, we performed numerical simulations and studied the entanglement interaction in detail. We show that high levels of entanglement can be achieved, with an average concurrence of 0.968.

1. Introduction

The cluster state model of quantum computation is a promising new approach to the design and formulation of quantum computers. In the "conventional" circuit model of quantum computation a register of qubits is acted upon by a series of successive logic gates in a manner analogous to that of its electronic counterpart. The cluster state model differs from this approach by instead using a system of highly entangled qubits as the universal computational resource [1]. Computation is then performed through a series of parallel single-qubit measurements that propagate quantum correlations through the system. While the results of these measurements are random, they can be classically combined to form a deterministic output and so universal computation achieved.

Through the choice of measurements, cluster state computation offers a unique way of designing quantum algorithms. More than this, it provides a robust scheme for implementing a practical computing device. Such a device does not suffer from many of the disadvantages associated with the quantum circuit implementation as it uses only single-qubit measurements to perform computation. Furthermore, it has been shown that a cluster state quantum computer would be more resilient to decoherence effects [2] and provide the ability for efficient error correction [3, 4]. Before cluster state computation can be practically performed, the controlled generation of the entanglement necessary to form the cluster state must be realized and this problem is the focus of this work.

2. Theoretical model

In our model the charge qubit is based on the single-electron lateral coupled quantum dots experimentally implemented by Petta et al [5]. These are constructed from a GaAs/AlGaAs heterostructure containing a two-dimensional electron gas (2DEG) 100nm below the surface and
controlled by Ti/Au top gates. The ballistic electron is produced by a single-electron source with an energy uncertainty of 1% and is confined to a GaAs quantum wire. This wire contains double symmetric resonant tunneling barriers positioned so that they lie over the center of one of the quantum dots. A schematic diagram of the proposed system is shown in figure 1. Currently, quantum wires have been produced down to diameters of a few nm [6] and so the ballistic electron is strongly radially confined and can be treated as one-dimensional. The barriers are formed of two InAs slices embedded in the quantum wire heterostructure. Such slices have been experimentally produced with widths of less than 50 nm and have a maximum barrier height of approximately 1.4 meV [7]. While the exact geometry of these potential barriers has yet to be established, similar experiments [8] with InP doped InAs wires have shown a good match to rectangular potential functions. Resonant tunneling occurs when the ballistic electron has an energy that closely matches one of the virtual energy eigenstates of the potential well formed by these barriers. Around these specific energies the transmission probability profile of the electron is sharply peaked and the barriers are effectively “transparent”.

Figure 1. A schematic diagram of the proposed entanglement system. The ballistic electron is pumped towards the barriers in the quantum wire by a single-electron source, entanglement occurs with the charge qubit through the Coulombic suppression of resonant tunneling through the double barriers.

To simulate this system, the electrons in the qubit and quantum wire are described by the three-dimensional joint wavefunction \( \psi(x_1, y_1, x_2, t) \), where \( x_1 \) and \( y_1 \) represent the position of the electron in the qubit, \( x_2 \) represents the position of the electron in the quantum wire, and \( t \) the time. The total Hamiltonian of the system is given by \( \hat{H} = \hat{H}_q + \hat{H}_w + \hat{H}_{\text{int}} \), where \( \hat{H}_q \) is the Hamiltonian of the isolated qubit, \( \hat{H}_w \) is the Hamiltonian of the isolated ballistic electron, and \( \hat{H}_{\text{int}} \) describes the Coulombic interaction that acts on the combined Hilbert space of the two electrons. The dynamics of the system are governed by the time-dependent Schrödinger equation \( i \frac{\partial \psi}{\partial t} = \hat{H} \psi \), which is solved using the numerical scheme described by Wang and Midgley [9] and Hines et al [10].

Entanglement occurs because the Coulombic interaction of the qubit distorts the potential experienced by the ballistic electron and so alters its resonant tunneling probability profile. By varying the separation and width of the barriers, as well as the ballistic electron’s incident energy, this transmission probability profile can be finely controlled. Through optimization the system is tuned so that resonant tunnelling will occur when the qubit electron is localized to the dot furthest from the wire and is suppressed when the qubit electron is localized to the dot closest to the wire, labelled by the \( |0\rangle \) and \( |1\rangle \) computational basis states respectively. The transmission coefficients of the ballistic electron with the charge qubit in initial states \( |0\rangle \) and \( |1\rangle \) states are plotted in figure 2 for a range of initial ballistic electron energies.

It has been shown that, while the particular mechanism of entanglement is unimportant, for a quantum system to be used as a cluster state then the localized entanglement between pairs of nearest neighbor
qubits must be at a maximum [11]. If this is not the case, measurements on the individual qubits will not result in the complete transfer of quantum information and computational errors will occur.

![Figure 2](image_url)

Figure 2. Transmission probabilities of the ballistic electron with 1% energy uncertainty for initial qubit states $|0\rangle$ and $|1\rangle$ over a range of initial ballistic electron energies using optimized barrier parameters.

To quantify the degree of bipartite entanglement present in the system we use the concurrence or “entanglement of formation” [12]. For an arbitrary $N$ qubit system, the concurrence is given by

$$C(\psi) = \max \left\{ 0, \| \psi^\dagger \sigma_1 \otimes \sigma_2 \otimes \ldots \otimes \sigma_N \psi \|_1 \right\},$$

where $\psi^\dagger$ is the complex conjugate of $\psi$ in a fixed basis, and $\sigma_i = \{\{0,-i\},\{i,0\}\}$ is the Pauli matrix acting on the Hilbert space of $i^{th}$ qubit in this basis. The concurrence of the system is then given in the range 0 to 1, where 1 means a maximally entangled state and 0 means a separable state. Shown in figure 3 is the average concurrence between the qubit and ballistic electrons ($N=2$) for a range of initial energies. As might be expected, the maximum average concurrence of 0.968 occurs when the difference in the transmission probability between the $|0\rangle$ and $|1\rangle$ states is at its maximum, namely at the ballistic electron energy of 1.585 meV.

![Figure 3](image_url)

Figure 3. The average concurrence for a range of initial ballistic electron energies with optimized barrier parameters.
3. Conclusions

In this work we have carried out a detailed simulation of a solid-state system for generating controlled quantum entanglement between a single-electron charge qubit and a ballistic electron by utilizing resonant tunneling in a quantum wire. The simulation was closely modeled on experimentally realizable components and treated the ballistic and qubit electrons and their interactions in detail. It was shown that the tunneling dynamics are highly sensitive to the system parameters. For symmetric rectangular potential barriers with an optimized height, width and separation of 1.4 meV, 50 nm and 90 nm respectively, the system produced entanglement with an average concurrence value of 0.968. This entanglement occurred due to the change in kinetic energy of the ballistic electron caused by the Coulombic repulsion from the qubit electron and is effectively elastic leaving the probability density of the qubit largely unchanged.

There are several key avenues of research that still need to be explored if this resonant tunneling scheme is to be shown to be suitable for cluster state generation. Further simulation needs to be performed to establish the exact nature of the phase shifts of the ballistic and qubit electrons to see whether it will be possible to compensate for these shifts during computation. It would also be desirable to try different potential barrier configuration in order to see if a more consistent concurrence could be produced across the whole range of energies contained within the ballistic electrons wave packet. If this can be achieved then it would facilitate entanglement pumping and so, with multiple tunneling events, the entanglement could be increased to an arbitrarily high value.

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