Resilient Feedback Controller Design For Linear Model of Power Grids

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Abstract—In this paper, a resilient controller is designed for the linear time invariant (LTI) systems subject to attacks on the sensors and the actuators. A novel probabilistic attack model is proposed to capture vulnerabilities of the communication links from sensors to the controller and from the controller to actuators. The observer and the controller formulation under the attack is derived. Thereafter, by leveraging Lyapunov functional methods, it is shown that exponential mean square stability of the system under the output feedback controller is guaranteed if a certain LMI is feasible. The simulation results show the effectiveness and applicability of the proposed controller design approach.

I. INTRODUCTION

With the advent of digital network technology, the networked control systems (NCSs) have found successful application in industrial processes [1], power grids [2] and so forth because of their important advantages such as low cost, improved utilizing of resources, and simplicity of maintenance. But the vulnerability of communication links to the cyber attacks in NCS has increased. Some factors such as the adoption of open communication standards and protocols, novel energy storage technologies [3], [4] and connection to the internet help the attackers to launch a successful attack on different NCSs. To deal with security concerns in the modern control systems, it is essential to make the control systems resilient to the malicious attackers.

The topic of the resiliency of control systems to different types of cyber attacks is considered in several research on designing cyber-resilient systems [5]–[7]. In summary, a resilient control system is determined by its level of resiliency of the system under attacks condition [8]. For instance, the false data injection attacks and the data alteration attacks are examples of the unexpected threats which are used to deteriorate the normal operation of control systems [9]–[11]. Also, different approaches such as hybrid analysis [12] and machine learning methods [13] are utilized for the resilient detection and control in various control systems.

In recent years, many studies have discussed the definition, mathematical frameworks, and the application of resilient cyber systems in the modern NCS. [14] has proposed a game theoretic approach to solve the cascade failure problem in industrial control systems. Thereafter, the interdependency between cyber security and robust control design has been investigated, and some optimal criteria for the linear quadratic case have been obtained. In [15], a cyber-resilient controller is proposed for a class of nonlinear discrete systems under actuator attacks where the interaction of the attacker and the IDS is captured with a game in the detection layer. In [16], the condition for the existence of a robust resilient state feedback controller for a certain class of nonlinear discrete systems with norm bounded nonlinearity, is obtained. [17] proposes a robust estimator for a class of nonlinear time varying systems with randomly occurring uncertainties, and the specific $H_{\infty}$ performance is obtained for the estimator by leveraging stochastic stability analysis. In [18], a mathematical formulation for the examination of the security problems in the control system has been introduced, and the definition of the several key words in resilient control system design such as state awareness and operational normalcy in a mathematical framework is developed. [19] considers the problem of malicious behavior of the attackers in the control systems as an unknown input observer design problem. [20] has focused on effects of the specific type of attacks in industrial control systems. In [20], the impacts of the reply attacks on the performance of the control systems in the steady state conditions are analyzed. [20] shows that the attack is undetectable due to recording the sensors’ data and propagating the sensors’ data during false data injection attacks on the actuators of the system. [21] and [22] analyze the problem of the secure estimation and the secure control of a system when the sensors and the actuators are under attacks. [22] shows that it is impossible to recover the states of a system if more than half of the sensors are under the attacks.

In this paper, a probabilistic attack model is proposed to examine attacks on the sensors and the actuators of the LTI systems. The unreliability of the communication links between sensors to controller and controller to actuators have been modeled by Bernoulli random variables. The fraction of the time that each sensor/actuator is under the attack is modeled by a Bernoulli variable. Then, the observer and the controller equations are derived based on expected value of the random variable. Thereafter, the controller is designed based on feasibility of a certain LMI condition obtained by leveraging the stochastic Lyapunov function approach.

This paper is organized as follows: In section II the mathematical model of attacks and the closed-loop dynamics of the system under attacks are introduced. The controller design procedure and the stochastic stability analysis are examined in section III. The efficiency of the proposed resilient controller...
is shown by an example in section IV and conclusions are made at the end.

II. PROBLEM DEFINITION

Consider the following discrete time-invariant system:

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{align*}
\]

where \( x(k) \in \mathbb{R}^n \) is the system state, \( u(k) \in \mathbb{R}^m \) is the input control signal, \( y(k) \in \mathbb{R}^p \) is the measured output, \( A, B \) and \( C \) are the constant matrices with appropriate dimensions. Shown in figure 1, it is assumed the communication links between sensors to controller and controller to the actuators are not secure and the attackers are able to manipulate the sensor and actuator values.

The proposed attack model for the sensor measurements based on stochastic control theory is given as:

\[
\hat{y}(k) = \Pi_1 Cx(k) + (I - \Pi_1)\Pi_2 Cx(k) \tag{3}
\]

Where \( \hat{y}(k) \in \mathbb{R}^p \) is the measured output under attack, \( \Pi_1 = \text{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_p\} \) with \( \alpha_i (i = 1, 2, \ldots, p) \) are \( p \) uncorrelated stochastic variables coming from a Bernoulli distribution, \( \Pi_2 = \text{diag}\{\beta_1, \beta_2, \ldots, \beta_p\} \) with \( \beta_i (i = 1, 2, \ldots, p) \) are \( p \) uncorrelated stochastic variables coming from an unknown probability distribution. The mathematical expectation and variance of the random variables \( \alpha_i \) and \( \beta_i \) are defined as following:

\[
\begin{align*}
E\{\alpha_i\} &= \text{Prob}(\alpha_i = 1) = \bar{\alpha}_i \tag{4} \\
\text{Var}\{\alpha_i\} &= \bar{\alpha}_i^2 \tag{5} \\
E\{\beta_i\} &= \bar{\beta}_i \tag{6} \\
\text{Var}\{\beta_i\} &= \bar{\beta}_i^2 \tag{7}
\end{align*}
\]

where \( E(x) \) and \( \text{Var}(x) \) denote the mathematical expectation and variance of random variable \( x \) respectively. \( \bar{\beta}_i \) is the expected value of attack injected on the \( i^{th} \) sensor, and \( \bar{\alpha}_i \) models the deviation from the optimal strategy.

**Remark 1.** The random variable \( \alpha_i \) taking value on \( \{0, 1\} \) has been introduced to represent the attack on the \( i^{th} \) sensor. When \( \alpha_i \) is equal zero, the correct value of the \( i^{th} \) sensor is substituted by the injected attack value. The mathematical expectation of Bernoulli random variable \( \alpha_i \) determines the fraction of time that the \( i^{th} \) sensor is under attack.

Given (1) and (3), the observer dynamics and the controller attack model are:

\[
\begin{align*}
\hat{x}(k+1) &= Ax(k) + Bu(k) - L[\hat{y} - \Pi_1 C\hat{x}(k)] \\
u(k) &= \Pi_3 K \hat{x}(k) + (I - \Pi_3)\Pi_4 K \hat{x}(k) \tag{8}
\end{align*}
\]

where \( \hat{x}(k) \in \mathbb{R}^n \) denotes the system state estimation vector, \( L \in \mathbb{R}^{m \times p} \) and \( K \in \mathbb{R}^{m \times n} \) are observer gain and controller gain, \( \Pi_3 = \text{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_m\} \) with \( \gamma_i (i = 1, 2, \ldots, m) \) are \( m \) uncorrelated stochastic variables coming from a Bernoulli distribution, and \( \Pi_4 = \text{diag}\{\delta_1, \delta_2, \ldots, \delta_m\} \) with \( \delta_i (i = 1, 2, \ldots, m) \) are \( m \) uncorrelated stochastic variables coming from an unknown probability distribution. The mathematical expectation and variance of the random variables \( \gamma_i \) and \( \delta_i \) are defined as follows:

\[
\begin{align*}
E\{\gamma_i\} &= \text{Prob}(\gamma_i = 1) = \bar{\gamma}_i \tag{10} \\
\text{Var}\{\gamma_i\} &= \bar{\gamma}_i^2 \tag{11} \\
E\{\delta_i\} &= \bar{\delta}_i \tag{12} \\
\text{Var}\{\delta_i\} &= \bar{\delta}_i^2. \tag{13}
\end{align*}
\]

\( \bar{\gamma}_i \) is the expected value of attack injected on the \( i^{th} \) actuator, and \( \bar{\gamma}_i \) models the deviation from the optimal strategy. Since \( \Pi_1, \Pi_2, \Pi_3, \) and \( \Pi_4 \) are diagonal matrices with random independent elements, the following equations can be used to obtain expected value and variance of \( \Pi_i\Pi_j (i,j = 1, 2, 3, 4) \):

\[
\begin{align*}
E(XY) &= E(X)E(Y) \tag{14} \\
\text{Var}(XY) &= [E(X)]^2\text{Var}(Y) + [E(Y)]^2\text{Var}(X) \\
&\quad + \text{Var}(X)\text{Var}(Y) \tag{15}
\end{align*}
\]

where \( X \) and \( Y \) are two independent random variables.

The estimation error is defined as:

\[
e_k = x_k - \hat{x}_k. \tag{16}
\]

The closed-loop dynamics of the system is obtained by substituting (3) and (9) into (1) and (16):

\[
\begin{align*}
x(k+1) &= [A + B(\Pi_3 + (I - \Pi_3)\Pi_4)K]x(k) + B[(\Pi_3 - \Pi_3) + (\Pi_4 - \Pi_4) - (\Pi_3\Pi_4 - \Pi_3\Pi_4)]Kx(k) \\
&\quad - B[\Pi_3 + (I - \Pi_3)\Pi_4]Ke(k) - B[(\Pi_3 - \Pi_3) + (\Pi_4 - \Pi_4) - (\Pi_3\Pi_4 - \Pi_3\Pi_4)]Ke(k) \tag{17}
\end{align*}
\]

\[
\begin{align*}
e(k+1) &= (A - L(\Pi_1 + \Pi_2 - \Pi_1\Pi_2)Ce(k) \\
&\quad - L[(\Pi_1 - \Pi_1) + (\Pi_2 - \Pi_2) - (\Pi_1\Pi_2 - \Pi_1\Pi_2)]Ce(k) \tag{18}
\end{align*}
\]

where \( \Pi_1, \Pi_2, \Pi_3, \) and \( \Pi_4 \) are mathematical expectations of \( \Pi_1, \Pi_2, \Pi_3, \) and \( \Pi_4 \) respectively. The compact representation of the closed-loop system is:

\[
\zeta(k+1) = \Gamma_1\zeta(k) + \Gamma_2\zeta(k). \tag{19}
\]
where
\[ \zeta(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \]

\[ \Gamma_1 = \begin{bmatrix} A + B\Delta_2 K & -B\Delta_2 K \\ 0 & A - L\Delta_1 C \end{bmatrix} \]

\[ \Gamma_2 = \begin{bmatrix} B(\Delta_2 - \Delta_2) K & -B(\Delta_2 - \Delta_2) K \\ -L(\Delta_2 - \Delta_1) C & 0 \end{bmatrix} \]

\[ \Delta_1 = \Pi_1 + \Pi_2 - \Pi_1\Pi_2 \]

\[ \Delta_2 = \Pi_3 + \Pi_4 - \Pi_3\Pi_4 \]

### III. Controller Design

In this section, the sufficient conditions for the stability of LTI systems under the attack are derived. To deal with the stochastic variables defined in the attack models, it is necessary to use the concept of stochastic stability in the mean square sense. In the following, the notation of stochastic stability in the mean square sense is defined.

**Definition 1.** The solution \( \zeta = 0 \) for the closed-loop system given in (19) is said exponentially stable in mean square sense if there exists constants \( \rho \in [0, 1] \) and \( \sigma > 0 \) such that

\[ E\{\|\zeta(k)\|^2\} \leq \sigma \rho^k E\{\|\zeta(0)\|^2\} \]  

for any \( \zeta(0) \) and any \( k \geq 0 \).

Following Lemma is the base of deriving the sufficient conditions for mean square stability of the system given in (19).

**Lemma 1.** [24] If there exists real constant \( \tau \in [0, 1] \) such that

\[ E\{V(\zeta(k+1)|V(k))\} - V(\zeta(k)) \leq -\tau V(\zeta(k)) \]  

and \( V(k) \) is a quadratic Lyapunov function, then the sequence \( \zeta(k) \) will be exponential mean square stable.

**Theorem 1.** The system given in [19] is exponential mean square stable, if there exists matrices \( Q_1 > 0, Q_2 > 0, G, \) and \( H \) with appropriate dimensions such that satisfying the following LMI:

\[ \begin{bmatrix} -Q & * & * \\ \Sigma_1 & -Q & * \\ \Sigma_2 & 0 & -Q \end{bmatrix} < 0 \]  

where \( Q \) and \( Q_2 \) are the solution of (22). Utilizing the results of Lemma 1, we obtain:

\[ E\{V((k+1)|V(k))\} - V(k) = E[x^T(k+1)Q_1x(k+1) + e^T(k+1)Q_2e(k+1)] \]

where \( x(k), e(k), e(0) \) are i.i.d. \( T \) random variables such that satisfying the Q theorem.

Following Lemma is the base of deriving the sufficient conditions for mean square stability of the system given in (19).

**Lemma 2.** [25] For the full rank matrix \( B \in \mathbb{R}^{m \times n} \) in the singular value decomposition form, there exist a nonsingular matrix \( W \) such that \( BW = Q_1B \), if and only if there exists a symmetric matrix \( Q_1 \) in the following form:

\[ Q_1 = U \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} U^T \]

\[ B = U \begin{bmatrix} B_{10} \\ 0 \end{bmatrix} V^T \]
where \( Q_{11} \in \mathbb{R}^{m \times m} > 0 \), \( Q_{22} \in \mathbb{R}^{(n-m) \times (n-m)} > 0 \), \( U \in \mathbb{R}^{m \times m} \) and \( V \in \mathbb{R}^{n \times n} \) are unitary matrices, \( B_0 \in \mathbb{R}^{m \times m} \) is a diagonal matrix with positive diagonal elements.

It follows from Lemma \( (2) \) that we can substitute \( BW \) with \( Q_1 B \) in LMI \( (28) \). Thereafter, we can easily see that the proof of theorem \( (1) \) is complete, if \( G \) and \( H \) are defined as following in \( (29) \):

\[
G = W K \\
H = Q_2 L.
\]

Solution to the LMI given in theorem \( (1) \) is utilized to obtain controller gain \( (K) \) and observer gain \( (L) \) for mean square stability of the system defined in \( (19) \). Assume \( Q_1, Q_2, G, \) and \( H \) are the solution of \( (22) \), and \( Q_1 \) is in the form of \( (30) \). The matrix \( W \) in lemma \( (2) \) is computed as:

\[
U \begin{bmatrix} B_0 \\ 0 \end{bmatrix} V^T W = U \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} U^T U \begin{bmatrix} B_0 \\ 0 \end{bmatrix} V^T
\]

which implies:

\[
\begin{bmatrix} B_0 \\ 0 \end{bmatrix} V^T W = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} B_0 \\ 0 \end{bmatrix} V^T
\]

\[
W = (B_0 V^T)^{-1} Q_{11} B_0 V^T.
\]

It is concluded from \( (32), (33), \) and \( (36) \) that

\[
K = W^{-1} G \\
L = Q_2^{-1} H
\]

The \( K \) and \( L \) calculated by \( (37) \) and \( (38) \) guarantee the exponential mean square stability of the LTI system \( (1) \) when the sensors and actuators are under the attacks.

IV. Simulation Results

In this section, the following third order LTI system under attacks is considered.

\[
x(k + 1) = \begin{bmatrix} -1.7 & -0.5 & 0.1 \\ 1 & 0 & -0.7 \\ 0 & 0.8 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k)
\]

\[
y(k) = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x(k) + \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \right)
\times \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x(k)
\]

\[
(39)
\]

Given \( (39) \), the observer dynamics and the controller attack model are:

\[
\dot{x}(k + 1) = \begin{bmatrix} -1.7 & -0.5 & 0.1 \\ 1 & 0 & -0.7 \\ 0 & 0.8 & 0 \end{bmatrix} \dot{x}(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k)
\]

\[
- L[y(k)] - \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \dot{x}(k) - \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \right)
\times \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \dot{x}(k)
\]

\[
u(k) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} K \dot{x}(k) + \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \right)
\times \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} K x(k)
\]

(40)

where

\[
E\{\alpha_1\} = 0.7, \ E\{\alpha_2\} = 0.8 \quad E\{\gamma_1\} = 0.8, \ E\{\gamma_2\} = 0.9
\]

The expected values of \( \alpha_i (i = 1, 2) \) and \( \gamma_i (i = 1, 2) \) show the fraction of time that sensors and actuators work normally. Given \( (41) \), the sensors of the system are under attacks for 30 and 20 percent of the time in average respectively, and the actuators of the system are under attack for 20 and 10 percent of the time in average respectively.

\[
\begin{bmatrix}
-Q_1 & * & * & * & * & * \\
0 & -Q_2 & * & * & * & * \\
A + B \Delta_2 K & -B \Delta_2 K & -Q_1^{-1} & * & * & * \\
0 & A - L \Delta_1 C & 0 & -Q_2^{-1} & * & * \\
B \Delta_{22} K & B \Delta_{22} K & 0 & 0 & -Q_1^{-1} & * \\
L \Delta_{11} C & 0 & 0 & 0 & 0 & -Q_2^{-1}
\end{bmatrix} < 0
\]

(28)

\[
\begin{bmatrix}
-Q_1 & * & * & * & * \\
0 & -Q_2 & * & * & * \\
Q_1 (A + B \Delta_2 K) & -Q_1 B \Delta_2 K & -Q_1 & * & * \\
0 & Q_2 (A - L \Delta_1 C) & 0 & -Q_2 & * \\
Q_1 B \Delta_{22} K & Q_1 B \Delta_{22} K & 0 & 0 & -Q_1 \\
Q_2 L \Delta_{11} C & 0 & 0 & 0 & 0 & -Q_2
\end{bmatrix} < 0
\]

(29)
\[ \beta_i (i = 1, 2) \] and \( \delta_i (i = 1, 2) \) for the simulation purpose is considered as follows:

\[ \delta_1 = \beta_1 = 1.3, \delta_2 = \beta_2 = 1.1 \]

The solution to the LMI (22) provides the feedback gain and the observer gain for the unstable system defined in (39).

\[
K = \begin{bmatrix} 1.1475 & -0.1962 & -1.4460 \\ -0.7689 & 0.3120 & 1.3376 \end{bmatrix},
\]

\[
L = \begin{bmatrix} 0.0674 & 1.5850 \\ -0.0376 & -0.8844 \\ 0.0217 & 0.5095 \end{bmatrix}.
\]

Given the initial conditions \( x(0) = [1 \ 1.2 \ -0.8]^T \) and \( \dot{x}(0) = [0 \ 0 \ 0]^T \), the state responses is shown in figure 2. Figures 3 and 4 show the attack patterns on actuators and sensors of the system where \( \alpha_i = 0 \) and \( \gamma_1 = 0 \) denote the system is under the attack. The actuator signals and the sensor measurements under the attacks are shown in figures 5 and 6 respectively.

V. Conclusion

In this paper, a novel probabilistic model for the attack on sensors and actuators of LTI systems has been introduced.
Denial-of-Service (Dos) attack is considered as a subclass of the introduced stochastic attack models. The probabilistic attacks on sensors and actuators are allowed to happen simultaneously in the system and the mean square stability of the system under attacks on sensors and the actuators is proved by leveraging stochastic control theory. The sufficient conditions for the existence of resilient controller gain and the observer gain are obtained, and it is shown that the problem of designing an output feedback controller is solvable if a certain LMI condition is satisfied.

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