Spin-down of compact stars and energy release of a first-order phase transition

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ABSTRACT

The deconfinement phase transition from hadronic matter to quark matter can continuously occur during spins down of neutron stars. It will lead to the release of latent heat if the transition is the first-order one. We have investigated the energy release of such deconfinement phase transition for rotating hybrid stars model which include mixed phase of hadronic matter and quark matter. The release of latent heat per baryon is calculated through studying a randomly process of infinitesimal compressing. Finally, we can self-consistently get the heating luminosity of deconfinement phase transition by imputing the EOS of mixed phase, and based on the equation of rotation structure of stars.

1. Introduction

Neutron stars (NSs) provide us a unique playground to study the properties of super-dense matter in the most extreme physical conditions. NSs participation in various astrophysical phenomena usually presented challenges for us. The different equation of state (EOS) and their properties of matter at high densities would play an important role in understanding some astrophysical phenomena.

One of the most intriguing predictions of some theories of dense matter is a possibility of a phase transition into an ‘exotic’ state, including pion and kaon condensation, and deconfinement of quarks. Of them, the deconfinement phase transition represented the most profound effect on the structure and dynamics of NSs. Many investigations were interesting in the phase transition which is of the first-order type (Pisalski & Wilczek (1984) and Gavai et al. (1987)). In the simplest case, equilibrium phase transition from the normal, lower density phase to the pure exotic one, occurs at a constant pressure, and is accompanied by a density jump at the phase interface (Baym & Chin (1976)). However, as shown by Glendenning (Glendenning (1992), (1997)), the properties of phase transition with more than one conserved charge are quite different from the constant pressure transition. In such a case, a mixed phase (MP) can be made in the interior NSs. The NSs containing MP matter is so-called hybrid stars (HSs) by many investigators.

It is known that a NS will spin down due to braking (e.g. electric-magnetic radiation or gravitational wave radiation). Deconfinement transition can proceed to occur in HSs during the spin down. Such evolutionary processes induce not only the changes in stellar structure but also
continuous release of latent heat. In the present paper, we calculate the energy release due to a phase transition in a rotating HS. The heat arises only at the phase interface for a constant pressure transition. In our case, the heat release will be distributed over a MP region. Our calculations are based on the perturbation theory developed by Hartle (Hartle 1967). Regard EOS for hadronic matter and strange quark matter as the input, we will self-consistently acquire the deconfinement heat luminosity with respect to the characteristic age of the star.

In this work, we choose the simplest possible nuclear matter composition, namely neutrons, protons, electrons, and muons (npeµ matter) and ignore superfluidity and superconductivity.

The paper is organized as follows. In Sec.2, we introduce notation and describe general properties of deconfinement phase transition in stellar with particular emphasis on the existence of the MP and varying pressure in the MP. The rotating structure of star with deconfinement phase transition are presented in Sec.3. In Sec.4 we present the calculation of deconfinement phase transition heat luminosity associated with EOS of the MP and structure of rotating star. The conclusion and discussions are summarized in Sec. 5.

2.Deconfinement phase transition

Quark deconfinement phase transition is expected to occur in neutron matter at densities above the nuclear saturation density \( \rho_B = 0.16 \text{ fm}^{-3} \). Early works on the possible occurrence of quark matter in neutron stars (Baym & Chin (1976)) were based on the assumption that hadron matter and quark matter were both charge neutral (with only one independent chemical potential). As a consequence, the transition was described using Maxwell construction and the resulting picture of the star consisted of quark matter core surrounded by a mantle of hadron matter, the two phases being separated by a sharp interface.

In the 90s, Glendenning (Glendenning (1992), (1997)) pointed out that this assumption is too restrictive. More generally, the transition can through the formation of a MP of hadron matter and quark matter, total charge neutrality being achieved by a positively charged amount of hadron matter and a negatively charged amount of quark matter. Therefore, at present, most of the approaches to deconfinement matter in NS matter use a standard two-phase description of EOS where the hadron phase (HP) and the quark phase (QP) are modelled separately and resulting EOS of the MP is obtained by imposing Gibbs conditions for phase equilibrium with the constraint that baryon number as well as electric charge of the system are conserved (Glendenning (1997), schertler et al (2000)).

We have to deal with two independent chemical potentials \((\mu_\pi, \mu_e)\) if we impose the condition of weak equilibrium. The Gibbs condition for mechanical and chemical equilibrium at zero temperature between the HP and the QP reads

\[
p_{HP}(\mu_\pi, \mu_e) = p_{QP}(\mu_\pi, \mu_e).
\] (1)

where \(p_{HP}\) is the pressure of HP and \(p_{QP}\) is the pressure of QP. We use the EOS of the relativistic mean field model (Glendenning (1997)) for hadron matter and employ an effective mass bag-model
EOS for quark matter (Schertle et al. (1997)). Only two independent chemical potentials remain according to the corresponding two conserved charges of the $\beta$-equilibrium system. The volume fraction occupied by QP for every point on the MP curve

$$\chi = \frac{V_{QP}}{V_{HP} + V_{QP}}$$

(2)
can be obtained by imposing the condition of global charge neutrality in the MP

$$\chi q_{QP} + (1 - \chi) q_{HP} = 0.$$  
(3)

Where $q$ denotes the charge density. Finally, the total energy density $\epsilon$ and baryon number $\rho_B$ can be calculated using

$$\epsilon = \chi \epsilon_{QP} + (1 - \chi) \epsilon_{HP}$$

(4)

$$\rho_B = \chi \rho_{QP} + (1 - \chi) \rho_{HP}.$$  
(5)

Taking the charge neutral EOS of the HP, Eq.(1), (2) and (3) for MP and the charge neutral EOS of the QP, we can construct the full hybrid star EOS. The resulting of chemical potentials are shown in Fig.1. Below the charge neutral HP curve and above the charge neutral QP curve the HP is positively charged ($q_{HP} > 0$) and the QP is negatively charged($q_{HP} < 0$). Therefore, the charge of hadronic matter can be neutralized in the MP by an appropriate amount of quark matter.

In the MP the volume proportion of quark phase is monotonically increasing from $\mu[1]$ to $\mu[2]$. In Fig.2 we show the model EOS with deconfinement transition which is the typical scheme of a first order transition at finite density with MP. The phase transition construction in a two-component system leads to continuously increasing pressure of the MP with increasing density. We choose the parameters for hadronic matter EOS which have been given by Glendenning (1997) and quark matter EOS with s quark mass $m_s = 150$MeV, bag constant $B^{1/4} = 160$MeV, coupling constant $g = 3$.

### 3. Rotating evolution of hybrid stars

With the evaluated hybrid star EOS presented above we now turn to analyse the structure of the corresponding rotating HSs. Using the Hartle’s perturbation theory (1967), Chubarian et al (2000) have studied the change of the internal structure of the HSs due to rotation. In this paper, we also apply Hartle’s approach to investigate the structure of rotating HSs. Hartle’s formalism is based on treating a rotating star as a perturbation on a non-rotating star, expanding the metric of an axially symmetric rotating star in even powers of the angular velocity $\Omega$. The metric of a slowly rotating star to second order in the angular velocity $\Omega$, can be written as

$$ds^2 = -e^{\nu(r)}[1 + 2(h_0 + h_2 P_2)]dt^2 + e^{\lambda(r)}[1 + \frac{2(m_0 + m_2 P_2)}{(r - 2M(r))}]dr^2$$

$$+ r^2[1 + 2(v_2 - h_2)P_2]\{d\theta^2 + \sin^2 \theta [d\phi - w(r, \theta)dt]^2\} + O(\Omega^3)$$

(6)

Here $e^{\nu(r)}$, $e^{\lambda(r)}$ and $M(r)$ are functions of $r$ and describe the non-rotating star solution of the Tolman-Oppenheimer-Volkov (TOV) equations (Oppenheimer& Volkoff (1933)). $P_2 = P_2(\theta)$ is the
l = 2 Legendre polynomials. \( \omega \) is the angular velocity of the local inertial frame and is proportional to the star’s angular velocity \( \Omega \), whereas the perturbation functions \( h_0, h_2, m_0, m_2, v_2 \) are proportional to \( \Omega^2 \). we assume that matter in the star is described by a perfect fluid with energy momentum tensor

\[
T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}
\]

The energy density and pressure of the fluid are affected by the rotation because the rotation deforms the star. In the interior of the star at given \((r, \theta)\), in a reference frame that is momentarily moving with the fluid, the pressure and energy density variation is respectively

\[
\delta P(r, \theta) = [(\epsilon(r) + P(r))][p_0^* + p_2^* P_2(\theta)]
\]

\[
\delta \epsilon(r, \theta) = \frac{d\epsilon}{dP}[\epsilon(r) + P(r)][p_0^* + p_2^* P_2(\theta)]
\]

here, \( p_0^* \) and \( p_2^* \) are dimensionless functions of \( r \), proportional to \( \Omega^2 \), which describe the pressure perturbation. The rotational perturbations of the star’s structure are described by the functions \( h_0, m_0, h_2, m_2, v_2, p_2^* \). These functions are calculated from Einstein’s field equations. The effect of rotation described by the metric on the shape of the star can be divided into contributions: A spherical expansion which changes the radius of the star, and is described by the functions \( h_0 \) and \( m_0 \). The other part is a quadrupole deformation, described by functions \( h_2, v_2 \) and \( m_2 \). As a consequence of these contributions, the difference between the gravitational mass of the rotating star and the non-rotating star with the same central pressure is

\[
\delta M_{\text{grav}} = m_0(R) + \frac{J^2}{R^3}
\]

The change in the radius of the star is given by

\[
\delta R = \xi_0(R) + \xi_2(R) P_2(\theta)
\]

We wish to study sequences of the rotating stars with constant total baryon number at variable spin frequency \( \nu = \Omega/2\pi \). The expansion of total baryon numbers in powers of \( \Omega \) is

\[
N_B = N^0_B + \delta N_B + O(\Omega^4)
\]

where

\[
N^0_B = \int_0^R n_B(r)[1 - 2M(r)/r]^{-1/2}4\pi r^2 dr
\]

is the total baryons number of non-rotating star and

\[
\delta N_B = \frac{1}{m_N} \int_0^R (1 - \frac{2M(r)}{r})^{-1/2} \left( [1 + \frac{m_0(r)}{r - 2M(r)} + \frac{1}{3} \nu^2 [\Omega - \omega(r)]^2 e^{-\nu}] m_N n_B(r) + \frac{dn_N n_B(r)}{dP} (\epsilon + P)[p_0^*(r)] \right) 4\pi r^2 dr
\]

here \( m_N \) is the rest mass per baryon. To construct constant baryon number sequences, we first solve the TOV equations to find the non-rotating configuration for giving a central pressure \( P(r=0) \). And
then, for an assigned value of the angular velocity $\Omega$ the equations of star structure are solved to order $\Omega^2$, imposing that the correction to the pressure $p_0^*(r = 0)$ being not equal to zero. The value of $p_0^*(r = 0)$ is then changed until the same baryon number as that non-rotating star is obtained.

The results for the stability of rotating HSs configurations with possible deconfinement phase transition according to the EOS described above are shown in Fig.3, where the total gravitation mass is given as functions of the equatorial radius and the central baryon number density for static stars as well as for stars rotating with the maximum rotation frequency $\nu_k$. The dotted lines connect configuration with the same total baryon number and it becomes apparent that the rotating configurations are less compact than the static ones. In order to explore the increase in central density due to spin down, we create sequences of HSs models. Model in a particular sequence have the same constant baryon number, increasing central density and decreasing angular velocity. Fig.4 displays the central density of rotating HSs with different gravitational mass at zero spin, as a function of its rotational frequency. In the interior of these stars, the matter can be gradually converted from the relatively incompressible nuclear matter phase to more compressible quark matter phase.

4. Deconfinement heating rate

As the star spins-down, the centrifugal force decreases continuously, increasing its internal density. Fig.4 identifies the fact that quarks are accumulating in the interior of the star with decreasing rotation frequency $\nu$. Since the deconfined phase transition is first order phase transition, there are latent heat produced with the transformation of hadron matter into quark matter. The deconfinement phase transition heating play an important role in the process of compact star’s thermal evolution.

Corresponding to the EOS depicted in Fig.2, Fig.5 shows that energy per baryon of HS matter as a function of the baryon number density. Two intersectant solid lines denote the pure hadronic matter phase and pure quark matter phase respectively. The deconfinement transition just occurs at point 1 until point 2 after which a pure quark matter phase appears. Comparing HP curve to MP one, we find that the enthalpy increase in MP is slower than HP when baryon number density increases. It is a vivid representation of latent heat release when a phase transition occurs.

How to describe the latent heat is now a key issue. It is quite evident matter that the energy release due to the phase transition is direct proportion to the difference between the HP derivative and the MP one at point 1. Similarly, we can express the energy release per baryon as

$$\delta\tilde{e} - \delta e = (\frac{\delta\tilde{e}}{\delta \rho_B} - \frac{\delta e}{\delta \rho_B})\delta \rho_B$$ (15)

where $\frac{\delta\tilde{e}}{\delta \rho_B}$ denotes the enthalpy change per baryon for any density in MP curve, density increase assumed no phase transition proceeds to occur. For example, point A in Fig.5 has two possible enthalpy change ways, AC and AD, when the density increases. No transition occurs along AC while AD corresponds to the real situation. In order to derive $\frac{\delta\tilde{e}}{\delta \rho_B}$, We rewrite the expression of
HP and QP volume \( V_{QP} = \frac{N_{QP}}{\rho_{QP}} \), \( V_{HP} = \frac{N_{HP}}{\rho_{HP}} \) and define parameter \( \eta = N_{Q}/N_{B} \). Substitute these equation into Eq.(2), we can get
\[
\chi = \frac{\eta \rho_{HP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \quad (16)
\]
\[
1 - \chi = \frac{(1 - \eta) \rho_{QP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \quad (17)
\]
Replacing Eq.(16) and Eq.(17) in Eq.(4) and Eq.(5), we have energy density for a given \( \eta \),
\[
\tilde{\epsilon} = \frac{\eta \rho_{HP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \epsilon_{QP} + \frac{(1 - \eta) \rho_{QP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \epsilon_{HP} \quad (18)
\]
\[
\tilde{\rho} = \frac{\rho_{QP} \rho_{HP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \quad (19)
\]
In such a case, We can obtain the energy per baryon as
\[
\tilde{\epsilon} = \tilde{\epsilon} = \frac{\eta \rho_{HP}}{\eta \rho_{HP} + (1 - \eta) \rho_{QP}} \epsilon_{QP} + (1 - \eta) \epsilon_{HP} \quad (20)
\]
Furthermore we get
\[
\frac{\partial \tilde{\epsilon}}{\partial \rho_{B}} = \eta \frac{\partial \epsilon_{QP}}{\partial \rho_{B}} + (1 - \eta) \frac{\partial \epsilon_{HP}}{\partial \rho_{B}} \quad (21)
\]
Energy release per baryon at point A in the process of deconfinement phase transition can be given using above Eq.(15) and (21).
\[
\Delta e = \delta \tilde{\epsilon} - \delta e = (\eta \frac{\partial \epsilon_{QP}}{\partial \rho_{B}} + (1 - \eta) \frac{\partial \epsilon_{HP}}{\partial \rho_{B}}) \delta \rho_{B} \quad (22)
\]
In Eq. (22), we see \( \Delta e \) equals 0 when \( \eta \) takes 1 or 0, returning to pure phases, HP or QP.
Integrating for whole star, we get the total latent heat release unit time for a star
\[
H = \frac{d\epsilon}{dt} = \int \frac{d\epsilon}{dt} \rho_{B} dV \quad (23)
\]
Since the rotation frequency and its derivative can be observed, we can rewrite the Eq. (23) as
\[
H = \int \frac{d\epsilon}{d\nu} \dot{\nu}(t) \rho_{B} dV \quad (24)
\]
The heat release per baryon \( \frac{d\epsilon}{d\nu} \) due to reduction in rotating frequency is simulated for a given rotating sequence \( N = 1.66N_{\odot} \) (the static mass \( M = 1.5M_{\odot} \)), is plotted in Fig.6. We find that the energy release effectively is enhanced with increasing density and frequency. So the deconfinement transition in the star is stronger during the early ages, and the energy release rate becomes larger and larger from the outer region to the center of the star.

Of course, the heat luminosity \( H \) can be obtained only if the spin-down rate of a star given. The common case is one induced by magnetic dipole radiation. It reads
\[
\dot{\nu} = -\frac{16\pi^{2}}{3Ic^{3}}\mu^{2}\nu^{3}\sin^{2}\theta \quad (25)
\]
is induced by magnetic dipole radiation, where $I$ is the stellar moment of inertia, $\mu = \frac{1}{2}BR^3$ is the magnetic dipole moment, and $\theta$ is the inclination angle between magnetic and rotational axes.

The total heat luminosity as a function of the characteristic age for different magnetic fields, for the rotating sequence $N = 1.66N_\odot$ (the static mass $M = 1.5M_\odot$), is presented in Fig.7. Heating source inside the stars is an important factor effect on the cooling of NSs. NSs first cools (for $t < 10^6$ yrs) via various neutrino emission before the surface photon radiation take over. Several heating mechanisms, for example, rotochemical heating (Reisenegger et al. [1995], [2006]), compositional transitions in crust (Iida & Sato [1997]), crust cracking (Cheng et al. [1992]) and vortex pinning (Van Riper et al. [1993]), have been discussed in detail. It is generally expected that the heating sources significantly contribute to surface emission for the old NSs with low magnetic fields, characteristic of millisecond pulsars. Recently the thermal emission data showed to have high surface temperature for a few of millisecond pulsars. Especially for PSR J0437-4715, the surface temperature can be inferred as $1.2 \times 10^7 K$. In light of model for NS thermal evolution including rotochemical heating source, Reisenegger (Reisenegger et al. [2006]) found that the theoretical result is 20% lower than the inferred temperature. It is noteworthy in Fig.7 that our heat luminosity for magnetic field $\sim 10^8 G$ and $\sim 10^9 G$ lasts during a term far longer than $10^6$ yrs and is much higher than the other heat generation. We thus think that high temperature of some millisecond pulsars with low magnetic fields (Kargaltsev et al. [2004]) can be explained using predicted heating model of HSs. For PSR J0437-4715, we predict the surface emission $L_{bol} = H(\nu, \dot{\nu})(1 - \frac{2GM}{Rc^2})$ with rotation period and its derivative $P = 5.74, \dot{P} = 3.64 \times 10^{-12}$. The result is estimated as $L_{bol} \sim 6.6 \times 10^{29} $ergs$^{-1}$ considerably consistent with the observed thermal X-ray luminosity $L_X = 4\pi\sigma R^2 T^4_\infty \sim 2.5 \times 10^{29} $ergs$^{-1}$.

5. Conclusions and discussions

The nuclear matter can continuously be deconfined to quark matter during the spins-down of star. The deconfinement phase transition heating of rotating hybrid stars have been investigated in this work. The latent heat release of such a first order phase transition associated with the change of energy per baryon during transition and rotational structure evolution of star. Using Hartle’s perturbative approach, we have calculated the change of internal structure of rotating hybrid stars. For a process of infinitesimal compressing during the rotation of star, we can get the energy release per baryon by inputing the EOS of the MP. Furthermore, we self-consistently get the total energy release combining with the rotational structure of star. The results show that the latent heat release per baryon per unit frequency enhances with the increase of baryon number density and rotational frequency in HSs.

We have calculated the changes in the internal structure of a compact star during its spin-down. As shown in previous studies the deconfinement phase transition occurs if quark matter is stable state at high densities, since the nuclear matter is compressed in the interior of the star during spin-down. We consider a first-order transition presented by Glendenning (1992) to calculate the release of latent heat when the star is slow down. The heating rate arising from the processes has
been estimated. We find its significant effects on the thermal evolution and the effects is much more important than the past present heating mechanisms. For old neutron stars with low magnetic fields of $\sim 10^8 - 10^9 G$, especially, our predicted temperature is nearly identified with the values observed from millisecond pulsars. In future, we expect more observational examples in investigating effects of deconfinement heating mechanism on the NSs thermal evolution. Thermal evolution curves need to be compared to more data. In this paper, for simplicity we neglect Coulomb and surface effects on the MP. Many investigations (Endo et al. (2006)) have identified that both effects would restrict the region of the MP in the core of hybrid stars, which can effect the release of latent heat. Another problem which remains to be investigated is the unified description of middle-age and old pulsars. HS model may be no bad selection when our combining deconfinement heating, superfluidity effects in nuclear matter together.

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REFERENCES
Baym G., Chin, S. A., 1976, Phys.Lett.B, 62,241
Cheng K. S., Chau W. Y., Zhang J. L. & Chau H. F., 1992, ApJ, 396, 235
Chubarian E., Grigorian H., Poghosyan G., Blaschke D., 2000, A&A, 357, 968
Endo T., Maruyama T., Chiba S., Tatsumi T., 2006, Proceeding of 29th Johns Hopkins Workshop: "Strong Matter in the Heavens", to be published in Proceedings of Science, PoS(JHW)019
Gavai R. V., Potvin J., Sanielevici S., 1987 Phys.Rev.Lett, 58, 2519
Glendenning N. K., Compact Stars(Springer-verlag). 1997
Glendenning N. K., 1992, Phys. Rev. D, 46, 1274
Hartle J. B., 1967, ApJ, 150, 1005
Iida K., Sato K., 1997, ApJ, 477, 294
Kargaltsev O., Pavlov G. G., & Romani R., 2004, APJ, 602,327
Oppenheimer J.R, Volkoff G.M., 1939, Phys.Rev, 55,374
Pisalski R. D., Wilczek F., 1984, Phys. Rev. Lett, 29, 338
Reisenegger A., 1995, ApJ, 442, 749
Reisenegger A., Jofre P., Fernandez R., Kantor E., 2006, astro-ph/0606322
Schertler K., Greiner C., Thoma M.H., 1997, Nucl. Phys. A616,659
Schertler K., Greiner Schaffner-Bielich J. C., Thoma M.H., 2000, Nucl. Phys. A677, 463
Van R., Kenneth A., Link B., & Epstein R. I., 1995, ApJ, 448, 294
Fig. 1.— Electron chemical potential $\mu_e$ as a function of the neutron chemical potential $\mu_n$. The HP EOS is a relativistic mean-field model, the quark matter is effective mass MIT bag model with $m_s = 150MeV$, $B^{1/4} = 160MeV$, coupling constant $g = 3.0$.

Fig. 2.— Model EOS for the pressure of hybrid star matter as a function of the baryon number density. The HP EOS is a relativistic mean-field model, the quark matter is effective mass MIT bag model with $m_s = 150MeV$, $B^{1/4} = 160MeV$, coupling constant $g = 3.0$. 

Fig. 3.— Gravitational mass $M$ as a function of the equatorial radius (left figure) and the central density (right figure) for rotating hybrid stars configurations with a deconfinement phase transition. The solid curves correspond to static configurations, the dashed ones to those with maximum rotation frequency $\nu_k$. The lines between both extremal cases connect configurations with the same total baryon number.

Fig. 4.— Central density as a function of rotational frequency for rotating hybrid stars of different gravitational mass at zero spin. All sequences are with constant total baryon number. Dash horizontal lines indicate the density where quark matter is produced.
Fig. 5.— The energy of per baryon as a function of the baryon number density for hybrid star matter. Two intersectant solid lines denote the pure hadronic matter phase and pure quark matter phase respectively. Line AC represents the state of no deconfined phase transition during rotation of star.

Fig. 6.— For 1.5 $M_{\odot}$ hybrid star, the energy release of per baryon unit frequency as a function of the baryon number density in mixed phase region for various frequency.
Fig. 7.— For $1.5 M_{\odot}$ hybrid star, the deconfinement phase transition heating rate change with rotational evolution of star for various magnetic fields.