IMPROVED RESUMMATIONS FOR THE THERMODYNAMICS OF THE QUARK-GLUON PLASMA

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Two recent attempts for overcoming the poor convergence of the perturbation expansion of the thermodynamic potentials of QCD are discussed: an HTL-adaption of “screened perturbation theory” and approximately self-consistent HTL resum-mations in the two-loop entropy.

At leading order, perturbation theory in the deconfined phase of QCD gives a reasonable estimate of the interaction pressure for temperatures a few times the critical one. But as soon as the beautiful machinery of resummed thermal perturbation theory comes into its own, its poor convergence properties seem to forbid its exploitation except at ridiculously high temperatures (or densities). This breakdown becomes apparent already at order $g^3$, which is entirely produced by the collective phenomenon of Debye screening (somewhat misleadingly dubbed “plasmon effect”), and already occurs in the simplest models such as scalar $\phi^4$ theory for rather small coupling.

At least in scalar theory, it has been shown that this impasse can be breached by Padé resummation and, more promisingly, by judiciously optimized perturbation theory such as “screened perturbation theory” (SPT). In SPT a coupling expansion is performed only with respect to couplings in explicit interactions, while any coupling constants buried in thermal (quasi-particle) masses are not expanded out, leading to nonpolynomial, i.e., nonperturbative, expressions in $g$. This has recently been adapted for QCD under the trademark “HTL perturbation theory”. There, in place of a simple mass term, the hard-thermal-loop (HTL) effective action is added, and subtracted again as a formally higher-order counterterm, from the ordinary action.

This approach differs from standard (HTL-)resummed perturbation theory in that resummed quantities are not only used in the soft momentum regime, but throughout. However, there is a price to be paid. At any finite loop order, the UV structure of the theory is modified—new (eventually temperature-dependent) divergences occur and must be subtracted, introducing a new source of renormalization scheme dependence.

An alternative approach for a more extensive resummation of the physics of HTL’s has been worked out by J.-P. Blaizot, E. Iancu, and myself, which is based on a self-consistent (“Φ-derivable”) two-loop approximation to

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the thermodynamic potentials. A central observation regarding the latter is that the entropy has a remarkably simple form,

\[ S = - \text{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial n}{\partial T} \text{Im} \log D^{-1} + \text{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial n}{\partial T} \text{Im} \Pi \text{Re} D \tag{1} \]

up to terms that are of loop-order 3 or higher, provided \( D \) and \( \Pi \) are the self-consistent one-loop propagator and self-energy. Thus, any explicit two-loop interaction contribution to the entropy has been absorbed by the spectral properties of quasiparticles. Remarkably, this holds true for fermionic as well as gluonic interactions.

Now, except for simple scalar models, such a self-consistent calculation is usually prohibitively difficult. In gauge theories it is moreover of questionable value because it is gauge-fixing dependent. However, these gauge dependences occur at an order which is beyond the (perturbative) accuracy of the above 2-loop approximation. If only the relevant leading and next-to-leading order contributions to the self-energies are considered, gauge invariance remains intact. We have therefore proposed approximately self-consistent (ASC) resum-mations based on Eq. (1) with, in a first approximation, the HTL self-energies, and, in a next-to-leading approximation (NLA), ones that are augmented by contributions given by NLO HTL perturbation theory for hard quasiparticles.

Employing HTL propagators, one obtains an expression, \( S^{\text{HTL}} \), which is no more complicated than the HTL-resummed one-loop pressure of Ref. (in one respect it is even simpler as it is manifestly UV finite and does not need artificial subtractions\(^a\)). And in contrast to the latter, when expanded to order \( g^2 \), it contains the correct leading-order interaction term, given by

\[ S^{(2)} = -2\pi N_g \int \frac{d^4k}{(2\pi)^4} \frac{\partial n}{\partial T} \epsilon(\omega)\delta(\omega^2 - k^2) \text{Re} \Pi^{\text{HTL}}_T(\omega, k) = -N_g \frac{m^{2}_{\infty} T}{6} \tag{2} \]

(in pure-glue QCD). Remarkably, this is directly related to the asymptotic thermal mass \( m^{2}_{\infty} = N g^2 T^2/6 \) of hard transverse gluons.

On the other hand, \( S^{\text{HTL}} \) contains only part of the plasmon effect \( \sim g^3 \); the main contribution \( \sim g^3 \) comes, rather surprisingly, from corrections to the dispersion laws of hard quasiparticles, determined by \( \delta \Pi^{\text{HTL}}_T \) as evaluated by standard HTL perturbation theory\(^b\). Both, \( \Pi^{\text{HTL}}_T \) and \( \delta \Pi^{\text{HTL}}_T \) turn out to be needed only for approximately light-like momenta\(^b\) which is gratifying as this is the only region where the HTL’s remain accurate for hard momenta.

\(^a\)As a matter of fact, the evaluation of the HTL pressure in Ref. has recently been found to suffer from an incomplete dimensional regularization that led to a larger-than-necessary over-inclusion of the leading-order interaction term.

\(^b\)The 2-loop entropy assumes its simple form of Eq. (1) only after the sum over Matsubara frequencies is carried out and an inherently real-time formula is obtained.
Figure 1. Pure-glue SU(3) gauge theory: Comparison of the HTL entropy (full lines) and the NLA estimates (dash-dotted lines) for $\overline{\mu} = \pi T \ldots 4\pi T$ with the lattice result of Ref. 12 (dark-gray band).

In Fig. 1, $S_{HTL}$ has been evaluated with $g(\overline{\mu})$ and $\overline{\mu} = \pi T \ldots 4\pi T$ and is found to compare favorably with available lattice data. Also included are estimates for our proposed next-to-leading approximation (NLA) which corrects the asymptotic thermal mass $\delta m_\infty$ by the (averaged) NLO contribution as given by standard HTL perturbation theory, and incorporated through an approximate gap equation (cf. Appendix).

This approach has also been applied successfully to QCD with fermions at zero and non-zero chemical potentials. Further elaborations and refinements are work in progress.

Appendix

Appendix

In Fig. 2, our approximately self-consistent entropy is considered for the “solvable” toy model of massless $O(N \to \infty)$ scalar field theory and compared to the results of screened perturbation theory at one- and two-loop order. In this model the unrenormalized Lagrangian is $\mathcal{L}(x) = \frac{1}{2} (\partial \phi)^2 - \frac{3N}{N+2} g_0^2 (\phi^2)^2$, to be taken in the limit $N \to \infty$, where the pressure per scalar degree of freedom coincides with that of $N = 1$ when keeping only “super-daisies” or “foam” diagrams. As is well known, this leads to

$$P(T) - P(0) = J_T(m) + \frac{1}{2} m^2 I_T(m) + \frac{1}{128 \pi^2} m^4$$

(3)

with the thermal mass $m$ given by the solution of the “gap equation”

$$m^2 = 4! g^2(\overline{\mu}) [I_T(m) + I_0^f (m, \overline{\mu})] = g^2 T^2 - \frac{3}{\pi} g^3 T^2 + \ldots$$

(4)
Figure 2. Large-$N$ scalar $O(N)$-model: (a) Comparison of (2nd- and 3rd-order) perturbative and HTL-improved approximations to the entropy. The shaded areas denote the variation under changes of the renormalization scale from $\bar{\mu}' = \pi T$ to $4 \pi T$. The band marked “HTL” refers to using the leading-order (HTL) mass in the 2-loop $\Phi$-derivable entropy, “NLA” to using the approximately self-consistent mass $m^2 = g^2 T^2 - 3 g T^2 / \pi$ or $m = g T - 3 g^2 T / 2 \pi$, respectively.

(b) The analogous comparison for the HTL-resummed one-loop pressure and the two-loop pressure in screened perturbation theory. The light-gray area marked (1) corresponds to the HTL-resummed one-loop pressure with in addition $\bar{\mu}_3$ varied from $\frac{1}{2} m$ to $2 m$. Full dark-gray lines refer to the “minimally subtracted” 2-loop pressure in SPT, Eq. (7), where $m$ is chosen by extremalization. Displayed are the results for $\bar{\mu}_3 = 2 m$ (upper three curves corresponding to $\bar{\mu} / (2 \pi T) = \frac{1}{2}, 1, 2$) and $\bar{\mu}_3 = \frac{1}{2} m$ (lower three curves, which have finite end-points beyond which there is no solution to the extremalization condition). With $\bar{\mu}_3 = \bar{\mu}$, the result coincides with the exact one. The a priori equally plausible prescription of subtracting $P(0)$ instead, Eq. (8), together with $\bar{\mu}_3 = \frac{1}{2} m$ or $2 m$ leads to the various dashed lines, the choice $\bar{\mu}_3 = \bar{\mu}$ to the dotted ones.

where $g(\bar{\mu})$ has been minimally renormalized and where we have introduced

$$I_0(m) = -\frac{m^2}{32 \pi^4} \left( \frac{2}{\epsilon} + |\log \frac{\bar{\mu}^2}{m^2} + 1| \right) \equiv I_0^{\text{div}}(m) + I_0^f(m, \bar{\mu}),$$

$$I_T(m) = \int \frac{d^3 k}{(2 \pi)^3} n(\varepsilon_k) \varepsilon_k, \quad J_T(m) = -T \int \frac{d^3 k}{(2 \pi)^3} \log(1 - e^{-\varepsilon_k / T}),$$

with $\varepsilon_k = \sqrt{k^2 + m^2}$.

In the present context, SPT amounts to replacing $\mathcal{L} \rightarrow \mathcal{L} - \frac{1}{2} m_0^2 \phi^2 + \delta \frac{1}{2} m_0^2 \phi^2$ where $\delta$ is treated as a one-loop quantity prior to putting $\delta = 1$, and $m$ is in the end some approximation to the thermal mass, e.g. as given by some (approximate) gap equation or by the HTL value $m = g T$.

Now this introduces new UV divergences, requiring also a mass counterterm, which however must be subtracted again in the $\delta$-counterterms, for
the original theory is massless and does not have mass counterterms in dimensional regularization. In our simple model, a renormalized mass $m$ can be introduced by $m_0^2 = m^2 - 4!g_0^2 I_0^{\text{div}}(m)$. The divergences in the two-loop pressure are then formally $T$-independent (before $m$ gets identified with some thermal mass), and their minimal subtraction yields

$$P_{\text{SPT, min}}^{(1)+2}(T) = m^2 I_0^f(m, \bar{\mu}_3) + \frac{m^4}{128\pi^2},$$

where the first three terms represent the one-loop contribution. Here $\bar{\mu}_3$ is the renormalization scale associated with the additional divergences of SPT.

Alternatively, one could, with equal plausibility, define a finite pressure by considering $P(T) - P(0)$. This explicitly thermal part reads

$$P_{\text{SPT, th}}^{(1)+2}(T) = m^2 I_0^f(m, \bar{\mu}_3) - 12g^2\{I_T(m) + [I_T^f(m, \bar{\mu}_3)]^2, \quad (7)$$

In Fig. 2, the (HTL)-resummed 1-loop pressure and the 2-loop pressure of SPT with $m$ fixed by extremalization, $\partial P/\partial m = 0$, are evaluated for various subtraction schemes. It turns out that SPT works well only beginning at 2-loop order and only in version (7), provided $\partial P/\partial m = 0$ has solutions.

In the ASC approach, already the HTL approximation is a significant improvement over standard perturbation theory. The NLA works extremely well provided the NLO corrections to the thermal mass are included by the ASC gap equation $m^2 = g^2T^2 - 3g^2Tm/\pi$.

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