The Cost of Financial Frictions for Life Insurers*

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Abstract

During the financial crisis, life insurers sold long-term insurance policies at firesale prices. In January 2009, the average markup, relative to actuarial value, was −25 percent for 30-year term annuities as well as life annuities and −52 percent for universal life insurance. This extraordinary pricing behavior was a consequence of financial frictions and statutory reserve regulation that allowed life insurers to record far less than a dollar of reserve per dollar of future insurance liability. Using exogenous variation in required reserves across different types of policies, we identify the shadow cost of financial frictions for life insurers. The shadow cost of raising a dollar of excess reserve was nearly $5 for the average insurance company in January 2009.

JEL classification: G01, G22, G28

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1. Introduction

The traditional view of insurance markets is that insurance companies operate in an efficient capital market that allows them to supply insurance at nearly constant marginal cost. Consequently, the market equilibrium is primarily determined by the demand side, either by life-cycle demand (Yaari, 1965) or informational frictions (Rothschild and Stiglitz, 1976). Contrary to this traditional view, this paper shows that insurance companies are financial institutions whose pricing behavior can be profoundly affected by financial frictions and statutory reserve regulation.

Our key finding is that life insurers actively reduced the price of long-term insurance policies in January 2009 when historically low interest rates implied that they should have instead raised prices. The average markup, relative to actuarial value (i.e., the present discounted value of future policy claims), was $-25$ percent for 30-year term annuities as well as life annuities at age 50. Similarly, the average markup was $-52$ percent for universal life insurance at age 30. These deep discounts are in sharp contrast to the 6 to 10 percent markup that life insurers earn in ordinary times (Mitchell, Poterba, Warshawsky, and Brown, 1999).

In the cross section of insurance policies, the price reductions were larger for those policies with looser statutory reserve requirements. In the cross section of insurance companies, the price reductions were larger for those companies whose balance sheets were more adversely affected prior to January 2009.

This extraordinary pricing behavior was due to a remarkable coincidence of two circumstances. First, the financial crisis had an adverse impact on insurance companies’ balance sheets. Insurance companies had to quickly recapitalize in order to control their leverage ratio and to prevent a rating downgrade or regulatory action. Second, the regulation gov-

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erning statutory reserves in the United States allowed life insurers to record far less than a
dollar of reserve per dollar of future insurance liability in January 2009. Therefore, insurance
companies were able to lower their leverage ratio by selling insurance policies at a price far
below actuarial value, as long as that price was above the reserve value.

We formalize our hypothesis in a dynamic model of insurance pricing that is otherwise
standard, except for a leverage constraint that is familiar from macroeconomics and finance
(e.g., Kiyotaki and Moore, 1997; Brunnermeier and Pedersen, 2009). The insurance company
sets prices for various types of policies to maximize the present discounted value of profits,
subject to a leverage constraint that the ratio of statutory reserves to assets cannot exceed a
targeted value. When the leverage constraint binds, the insurance company optimally prices
a policy below its actuarial value if its sale has a negative marginal impact on leverage. The
Lagrange multiplier on the leverage constraint has a structural interpretation as the shadow
cost of raising a dollar of excess reserve.

We test our hypothesis on panel data of nearly 35,000 observations on insurance prices
from January 1989 through July 2011. Our data cover term annuities, life annuities, and
universal life insurance for both males and females as well as various age groups. Relative
to other industries, life insurance presents a unique opportunity to identify the shadow cost
of financial frictions for two reasons. First, life insurers sell relatively simple products whose
marginal cost can be accurately measured. Second, statutory reserve regulation specifies a
constant discount rate for reserve valuation, regardless of the maturity of the policy. This
mechanical rule generates exogenous variation in required reserves across policies of different
maturities, which acts as relative shifts in the supply curve that are plausibly exogenous.
We find that the shadow cost of financial frictions is essentially zero for most of the sample,
except around January 2001 and in January 2009. We find that the shadow cost of raising
a dollar of excess reserve was nearly $5 for the average insurance company in January 2009.
This cost varies from $1 to $13 per dollar of excess reserve for the cross section of insurance
companies in our sample.
From an investor’s perspective, January 2009 was an especially attractive opportunity to be in the market for insurance policies. For example, a 30-year term annuity could have been purchased for 25 percent less than a portfolio of Treasury bonds with identical cash flows. While solvency might have been a concern for some insurance companies, insurance policies are ultimately backed by the state guarantee fund (e.g., up to $250k for annuities and $300k for life insurance in California). Therefore, the only scenario in which an investor would not be repaid is if all insurance companies associated with the state guarantee fund were to systemically fail.

From an insurance company’s perspective, it is initially less obvious why the firesale of insurance policies was optimal in January 2009. A potential explanation is that insurance companies anticipated some chance of default, so that their expected liability was less than the full face value of insurance policies. We rule out this hypothesis based on several reasons. Perhaps the most compelling of these reasons is that insurance companies did not discount life annuities during the Great Depression, when the corporate default spread was even higher than the heights reached during the recent financial crisis. The absence of discounts during the Great Depression is consistent with the statutory reserve regulation that was in effect back then, which did not allow insurance companies to record liabilities at less than full reserve. Overall, the historical evidence is more consistent with our explanation based on financial frictions and statutory reserve regulation.

Our finding that the supply curve for life insurers shifts down in response to a balance sheet shock, causing insurance prices to fall, contrasts with the evidence that the supply curve for property and casualty insurers shifts up, causing insurance prices to rise (Froot and O’Connell, 1999). Although these findings may seem contradictory at first, they are both consistent with our theory of insurance pricing. The key difference between life insurers and property and casualty insurers is statutory reserve regulation. Life insurers were able relax their leverage constraint by selling new policies because their statutory reserve regulation allowed less than full reserve during the financial crisis. In contrast, property and
casualty insurers must tighten their leverage constraint when selling new policies because their statutory reserve regulation always requires more than full reserve (American Academy of Actuaries, 2000).

The remainder of the paper is organized as follows. Section 2 describes our data and documents key facts that motivate our study of insurance prices. Section 3 reviews key features of statutory reserve regulation that are relevant for our analysis. In Section 4, we develop a structural model of insurance pricing, which shows how financial frictions and statutory reserve regulation affect insurance prices. In Section 5, we estimate the structural model of insurance pricing, through which we identify the shadow cost of financial frictions. In Section 6, we calibrate the structural model of insurance pricing to show that it explains the observed magnitudes of the price reductions and the shadow cost of financial frictions in January 2009. Section 7 concludes with broader implications of our study for household finance and macroeconomics.

## 2. Annuity and Life Insurance Prices

### 2.1 Data Construction

#### 2.1.1 Annuity Prices

Our annuity prices are from the *Annuity Shopper* (Stern, 1989), which is a semianual publication (every January and July) of annuity price quotes from the leading life insurers. Following Mitchell, Poterba, Warshawsky, and Brown (1999), we focus on annuities that are single premium, immediate, and non-qualified. This means that the premium is paid upfront as a single lump sum, that the income payments start immediately after the premium payment, and that only the interest portion of the payments is taxable. Our data consist of three types of policies: term annuities, life annuities, and guaranteed annuities. For term annuities, we have quotes for 5- through 30-year maturities (every 5 years in between). For
life and guaranteed annuities, we have quotes for males and females between ages 50 and 90 (every 5 years in between).

A term annuity is a policy with annual income payments for a fixed term of $M$ years. Let $R_t(m)$ be the zero-coupon Treasury yield at maturity $m$ in month $t$. We define the actuarial value of an $M$-year term annuity per dollar of income as

$$V_t(M) = \sum_{m=1}^{M} \frac{1}{R_t(m)^m}. \tag{1}$$

A life annuity is a policy with annual income payments until the death of the insured. Let $p_n$ be the one-year survival probability at age $n$, and let $N$ be the maximum attainable age according to the appropriate mortality table. We define the actuarial value of a life annuity at age $n$ per dollar income as

$$V_t(n) = \sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-1} p_{n+l}}{R_t(m)^m}. \tag{2}$$

A guaranteed annuity is a variant of the life annuity whose income payments are guaranteed to continue for the first $M$ years, even if the insured dies during that period. We define the actuarial value of an $M$-year guaranteed annuity at age $n$ per dollar of income as

$$V_t(n, M) = \sum_{m=1}^{M} \frac{1}{R_t(m)^m} + \sum_{m=M+1}^{N-n} \frac{\prod_{l=0}^{m-1} p_{n+l}}{R_t(m)^m}. \tag{3}$$

We calculate the actuarial value for each type of policy at each date based on the appropriate mortality table from the Society of Actuaries and the zero-coupon Treasury yield curve (Gürkaynak, Sack, and Wright, 2007). We use the 1983 Annuity Mortality Basic Table prior to December 2000, and the 2000 Annuity Mortality Basic Table since December 2000. These mortality tables are derived from the actual mortality experience of insured pools, based on data provided by various insurance companies. Therefore, they account for adverse selection in annuity markets, that is, an insured pool of annuitants has higher life
expectancy than the overall population. We smooth the transition between the two vintages of the mortality tables by geometrically averaging.

2.1.2 Life Insurance Prices

Our life insurance prices are from COMPULIFE Software, which is a computer-based quotation system for insurance brokers. We focus on guaranteed universal life policies, which are quoted for the leading life insurers since January 2005. These policies have constant guaranteed premiums and accumulate no cash value, so they are essentially “permanent” term life policies.\(^1\) We pull quotes for the regular health category at the face amount of $250,000 in California. COMPULIFE recommended California for our study because it is the most populous state with a wide representation of insurance companies. We focus on males and females between ages 30 and 90 (every 10 years in between).

Universal life insurance is a policy that pays out a death benefit upon the death of the insured. The policy is in effect as long as the policyholder makes an annual premium payment while the insured is alive. We define the actuarial value of universal life insurance at age \(n\) per dollar of death benefit as

\[
V_t(n) = \left(1 + \sum_{m=1}^{N-n-1} \frac{\prod_{l=0}^{m-1} P_{n+l}}{R_t(m)^m}\right)^{-1} \left(\sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-2} P_{n+l}(1-P_{n+m-1})}{R_t(m)^m}\right). \tag{4}
\]

Note that this formula does not take into account the potential lapsation of policies, that is, the policyholder may drop coverage prior to the death of the insured. There is currently no agreed upon standard for lapsation pricing, partly because lapsations are difficult to model and predict. While some insurance companies price in low levels of lapsation, others take the conservative approach of assuming no lapsation in life insurance valuation.

We calculate the actuarial value for each type of policy at each date based on the ap-

\(^1\)While COMPULIFE has quotes for various types of policies from annual renewable to 30-year term life policies, they are not useful for our purposes. This is because a term life policy typically has a renewal option at the end of the guaranteed term. Because the premiums under the renewal option vary significantly across insurance companies, cross-sectional price comparisons are difficult and imprecise.
propriate mortality table from the Society of Actuaries and the zero-coupon Treasury yield curve. We use the 2001 Valuation Basic Table prior to December 2008, and the 2008 Valuation Basic Table since December 2008. These mortality tables are derived from the actual mortality experience of insured pools, based on data provided by various insurance companies. Therefore, they account for adverse selection in life insurance markets. We smooth the transition between the two vintages of the mortality tables by geometrically averaging.

2.1.3 Insurance Companies’ Balance Sheets

We obtain balance sheet data and A.M. Best ratings for insurance companies through the Best’s Insurance Reports CD-ROM for fiscal years 1992 through 2010. We merge annuity and life insurance prices to the A.M. Best data by company name. The insurance price observed in January and July of each calendar year is matched to the balance sheet data for the previous fiscal year (i.e., December of the previous calendar year).

2.2 Summary Statistics

We start with a broad overview of the industry that we study. Figure 1 reports the annual premiums collected for individual annuities and life insurance, summed across all insurance companies in the United States with an A.M. Best rating. In the early 1990’s, insurance companies collected nearly $100 billion in annual premiums for individual life insurance and about $50 billion for individual annuities. More recently, the annuity market expanded to $383 billion in 2008. The financial crisis had an adverse effect on annuity demand in 2009, which subsequently bounced back in 2010.

Table 1 summarizes our data on annuity and life insurance prices. We have 988 observations on 10-year term annuities across 98 insurance companies, covering January 1989 through July 2011. The average markup, defined as the percent deviation of the quoted price from actuarial value, is 6.9 percent. Since term annuities have a fixed income stream that is independent of survival, we can rule out adverse selection as a source of this markup.
Instead, the markup must be attributed to marketing and administrative costs as well as economic profits that may arise from imperfect competition. The fact that the average markup declines in the maturity of the term annuity is consistent with the presence of fixed costs. There is considerable cross-sectional variation in the pricing of 10-year term annuities across insurance companies, as indicated by a standard deviation of 5.9 percent (Mitchell, Poterba, Warshawsky, and Brown, 1999).

We have 11,879 observations on life annuities across 106 insurance companies, covering January 1989 through July 2011. The average markup is 9.8 percent with a standard deviation of 8.2 percent. Our data on guaranteed annuities start in July 1989. For 10-year guaranteed annuities, the average markup is 5.5 percent with a standard deviation of 6.1 percent. For 20-year guaranteed annuities, the average markup is 4.2 percent with a standard deviation of 4.8 percent.

We have 3,989 observations on universal life insurance across 52 insurance companies, covering January 2005 through July 2011. The average markup is −4.2 percent with a standard deviation of 17.9 percent. The negative average markup does not mean that insurance companies systematically lose money on these policies. With a constant premium and a rising mortality rate, policyholders are essentially prepaying for coverage later in life. When a universal life policy is lapsed, the insurance company earns a windfall profit because the present value of the remaining premium payments is typically less than the present value of the future death benefit. Since there is currently no agreed upon standard for lapsation pricing, our calculation of actuarial value does not take lapsation into account. We are not especially concerned that the average markup might be slightly mismeasured because the focus of our study is the variation in markups over time and across different types of polices.

2.3 Firesale of Insurance Policies

Figure 2 reports the time series of the average markup on term annuities at various maturities, averaged across insurance companies and reported with a 95 percent confidence interval.
The average markup varies between 0 and 10 percent, with the exception of a period of few months around January 2009. If insurance companies were to change annuity prices to perfectly offset interest rate movements, then the markup would be constant over time. Hence, the variation in average markup implies that insurance companies do not change annuity prices to fully offset interest rate movements (Charupat, Kamstra, and Milevsky, 2012).

For 30-year term annuities, the average markup fell to an extraordinary \(-25\) percent in January 2009. Much of this large negative markup can be explained by the fact that insurance companies aggressively reduced the of 30-year term annuities from July 2007 to January 2009. For example, Allianz Life Insurance Company reduced the price of 30-year term annuities from $18.56 (per dollar of annual income) in July 2007 to $13.75 in January 2009, then raised it back up to $18.23 by July 2009. Such price reductions cannot be explained by interest rate movements because relatively low Treasury yields implied relatively high actuarial value for 30-year term annuities in January 2009.

In January 2009, there is a monotonic relation between the maturity of the term annuity and the magnitude of the average markup. Average markup was \(-16\) percent for 20-year, \(-8\) percent for 10-year, and \(-3\) percent for 5-year term annuities. Excluding the extraordinary period around January 2009, average markup was negative for 20- and 30-year term annuities only twice before in our sample, in January 2001 and July 2002.

Figure 3 reports the time series of the average markup on life annuities at various ages. We find a similar phenomenon to that for term annuities. For life annuities at age 50, the average markup fell to an extraordinary \(-25\) percent in January 2009. There is a monotonic relation between age, which is negatively related to the effective maturity of the life annuity, and the magnitude of the average markup. Average markup was \(-19\) percent at age 60, \(-11\) percent at age 70, and \(-3\) percent at age 80.

Figure 4 reports the time series of the average markup on universal life insurance at various ages. We again find a similar phenomenon to that for term and life annuities. For
universal life insurance at age 30, the average markup fell to an extraordinary −52 in January 2009. There is a monotonic relation between age and the magnitude of the average markup. Average markup was −47 percent at age 40, −42 percent at age 50, and −29 percent at age 60.

2.4 Ruling Out Alternative Hypotheses

Our preferred explanation for the firesale of insurance policies in January 2009 is that insurance companies were financially constrained, and statutory reserve regulation allowed them to recapitalize by selling new policies. Before we turn to our preferred explanation, we rule out two alternative hypotheses.

2.4.1 Mispricing in Treasury Markets

The first alternative hypothesis is that Treasury yields were unnaturally low in January 2009, perhaps due to the Federal Reserve’s quantitative easing policy and the flight to liquidity in financial markets (Krishnamurthy and Vissing-Jørgensen, 2011). Consequently, our estimates of the actuarial value of insurance policies are potentially upward biased, which causes our estimates of the average markup to be downward biased.

We rule out this hypothesis based on three reasons. First, it does not explain why insurance companies actively reduced the price of their policies in January 2009. Insurance companies should have kept prices constant, if anything, if they believed that Treasury yields were temporarily lower than fundamental value. Second, standard economic theory (e.g., our model in Section 4) suggests that insurance companies should maximize profits, taking the Treasury yield curve as exogenously given. Therefore, the standard theory does not explain why it would ever be optimal for insurance companies to misprice their policies relative to the Treasury yield curve.

Third, this hypothesis cannot entirely explain the magnitude of the deviation of insurance prices from actuarial value. To illustrate this point, we recalculate the average markup in
January 2009 using the actuarial value of insurance policies in January 2008, long before any
evidence of potential mispricing in Treasury markets (Musto, Nini, and Schwarz, 2011). For
30-year term annuities, the average markup increases from −25 percent to −13 percent in
this counterfactual experiment. For life annuities at age 50, the average markup increases
from −25 percent to −11 percent. For universal life insurance at age 30, the average markup
increases from −52 percent to −15 percent. The implied discounts remain economically large
in this counterfactual experiment, which we view as a lower bound on the actual discounts
in January 2009.

2.4.2 Default Risk

The second alternative hypothesis is that insurance companies anticipated some chance of
default in January 2009, so that their expected liability was less than the full face value of
insurance policies. Therefore, their cost of capital was the Baa corporate bond yield, for
example, instead of the Treasury yield.

We rule out this hypothesis based on four reasons. First, if the Baa corporate bond yield
were used to calculate the actuarial value of insurance policies, it would imply that insurance
companies earn incredibly high markups in ordinary times (Mitchell, Poterba, Warshawsky,
and Brown, 1999). Second, the firesale of insurance policies was very short-lived around
January 2009, while the corporate default spread remained elevated for much longer. Third,
Appendix A shows that insurance companies did not discount life annuities during the Great
Depression, when the corporate default spread was even higher than the heights reached
during the recent financial crisis. The absence of discounts during the Great Depression is
consistent with the statutory reserve regulation that was in effect back then, which did not
allow insurance companies to record liabilities at less than full reserve. Fourth, Appendix B
shows that the appropriate cost of capital is the riskless interest rate in the presence of a
state guarantee fund that forces the surviving insurance companies to pay off the liabilities
of the defaulting insurance companies.
3. **Statutory Reserve Regulation for Life Insurers**

When an insurance company sells an annuity or life insurance policy, its assets increase by the purchase price of the policy. At the same time, the insurance company must record statutory reserves on the liability side of its balance sheet to cover future policy claims. In the United States, the amount of required reserves for each type of policy is governed by state law, but all states essentially follow recommended guidelines known as Standard Valuation Law (National Association of Insurance Commissioners, 2011, Appendix A-820). Standard Valuation Law establishes mortality tables and discount rates that are to be used for reserve valuation.

In this section, we review the reserve valuation rules for annuities and life insurance. Because these policies essentially have no exposure to market risk, finance theory implies that the economic value of these policies is determined by the term structure of riskless interest rates. However, Standard Valuation Law requires that the reserve value of these policies be calculated using a mechanical discount rate that is a function of the Moody’s composite yield on seasoned corporate bonds. Insurance companies care about the reserve value of insurance policies insofar as it is used by rating agencies and state regulators to determine the adequacy of statutory reserves. A rating agency may downgrade an insurance company whose asset value has fallen relative to its statutory reserves. In the extreme case, a state regulator may liquidate an insurance company whose assets are deficient relative to its statutory reserves.

### 3.1 Term Annuities

Let $y_t$ be the 12-month moving average of the Moody’s composite yield on seasoned corporate bonds, over the period ending on June 30 of the issuance year of the policy. Standard

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2In principle, rating agencies could calculate the economic value of liabilities and base their ratings on market leverage. However, their current practice is to take reserve valuation at face value, so that ratings are ultimately based on accounting leverage (A.M. Best Company, 2011, p. 31).
Valuation Law specifies the following discount rate for reserve valuation of annuities:

\[
\hat{R}_t - 1 = 0.03 + 0.8(y_t - 0.03),
\]  

which is rounded to the nearest 25 basis point. This a constant discount rate that is to be applied to all expected future policy claims, regardless of maturity. The exogenous variation in required reserves that this mechanical rule generates, both over time and across policies of different maturities, allows us to identify the shadow cost of financial frictions for life insurers.

Figure 5 reports the time series of the discount rate for annuities, together with the 10-year zero-coupon Treasury yield. The discount rate for annuities has generally declined over the last 20 years as nominal interest rates have fallen. However, the discount rate for annuities has declined more slowly than the 10-year Treasury yield. This means that statutory reserve requirements for annuities have become looser over time because a high discount rate implies low reserve valuation.

The reserve value of an \(M\)-year term annuity per dollar of income is

\[
\hat{V}_t(M) = \sum_{m=1}^{M} \frac{1}{\hat{R}_m}.
\]  

Figure 6 reports the ratio of reserve to actuarial value for term annuities (i.e., \(\hat{V}_t(M)/V_t(M)\)) at maturities of 5 to 30 years. Whenever this ratio is equal to one, the insurance company records a dollar of reserve per dollar of future policy claims in present value. Whenever this ratio is greater than one, the reserve valuation is conservative in the sense that the insurance company records reserves that are greater than the present value of future policy claims. Conversely, whenever this ratio is less than one, the reserve valuation is aggressive in the sense that the insurance company records reserves that are less than the present value of future policy claims.

For the 30-year term annuity, the ratio reaches a peak of 1.20 in November 1994 and a
trough of 0.73 in January 2009. If the insurance company were to sell a 30-year term annuity at actuarial value in November 1994, its reserves would increase by $1.20 per dollar of policies sold. This implies a loss of $0.20 in capital surplus funds (i.e., total admitted assets minus total liabilities) per dollar of policies sold. In contrast, if the insurance company were to sell a 30-year term annuity at actuarial value in January 2009, its reserves would only increase by $0.73 per dollar of policies sold. This implies a gain of $0.27 in capital surplus funds per dollar of policies sold.

### 3.2 Life Annuities

The reserve valuation of life annuities requires mortality tables. The Society of Actuaries produces two versions of mortality tables, which are called basic and loaded. The loaded tables, which are used for reserve valuation, are conservative versions of the basic tables that underestimate the mortality rates. The loaded tables ensure that insurance companies have adequate reserves, even if actual mortality rates turn out to be lower than those projected by the basic tables. For calculating the reserve value, we use the 1983 Annuity Mortality Table prior to December 2000, and the 2000 Annuity Mortality Table since December 2000.

Let \( \hat{p}_n \) be the one-year survival probability at age \( n \), and let \( N \) be the maximum attainable age according to the appropriate loaded mortality table. The reserve value of a life annuity at age \( n \) per dollar of income is

\[
\hat{V}_t(n) = \sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-1} \hat{p}_{n+l}}{\hat{R}_t^m},
\]

where the discount rate is given by equation (5). Similarly, the reserve value of an \( M \)-year guaranteed annuity at age \( n \) per dollar of income is

\[
\hat{V}_t(n, M) = \sum_{m=1}^{M} \frac{1}{\hat{R}_t^m} + \sum_{m=M+1}^{N-n} \frac{\prod_{l=0}^{m-1} \hat{p}_{n+l}}{\hat{R}_t^m}. 
\]
Figure 6 reports the ratio of reserve to actuarial value for life annuities, 10-year guaranteed annuities, and 20-year guaranteed annuities for males aged 50 to 80 (every 10 years in between). For these life annuities, the time-series variation in the ratio of reserve to actuarial value is quite similar to that for term annuities. In particular, the ratio reaches a peak in November 1994 and a trough in January 2009. Since the reserve valuation of term annuities depends only on the discount rates, the similarity with term annuities implies that discount rates, rather than mortality tables, have a predominant effect on the reserve valuation of life annuities.

### 3.3 Life Insurance

Let $y_t$ be the minimum of the 12-month and the 36-month moving average of the Moody’s composite yield on seasoned corporate bonds, over the period ending on June 30 of the year prior to issuance of the policy. Standard Valuation Law specifies the following discount rate for reserve valuation of life insurance:

$$
\hat{R}_t(M) - 1 = 0.03 + w(M)(\min\{y_t, 0.09\} - 0.03) + 0.5w(M)(\max\{y_t, 0.09\} - 0.09),
$$

which is rounded to the nearest 25 basis point. The weighting function for a policy with a term of $M$ years is

$$
w(M) = \begin{cases} 
0.50 & \text{if } M \leq 10 \\
0.45 & \text{if } 10 < M \leq 20 \\
0.35 & \text{if } M > 20 
\end{cases}
$$

As with life annuities, the American Society of Actuaries produces basic and loaded mortality tables for life insurance. The loaded tables, which are used for reserve valuation, are conservative versions of the basic tables that overestimate the mortality rates. The loaded tables ensure that insurance companies have adequate reserves, even if actual mortality rates
turn out to be higher than those projected by the basic tables. For calculating the reserve value, we use the 2001 Commissioners Standard Ordinary Mortality Table. The reserve value of life insurance at age \( n \) per dollar of death benefit is

\[
\tilde{V}_t(n) = \left( 1 + \sum_{m=1}^{N-n-1} \frac{\prod_{l=0}^{n-1} \hat{p}_{n+l}}{R_t(N - n)^m} \right)^{-1} \left( \sum_{m=1}^{N-n} \frac{\prod_{l=0}^{m-2} \hat{p}_{n+l}(1 - \hat{p}_{n+m-1})}{R_t(N - n)^m} \right). \tag{11}
\]

Figure 7 reports the ratio of reserve to actuarial value for universal life insurance for males aged 30 to 60 (every 10 years in between). In a period of few months around January 2009, the reserve value falls significantly relative to actuarial value. As shown in Figure 5, this is caused by the fact that the discount rate for life insurance stays constant during this period, while the 10-year Treasury yield falls significantly. If an insurance company were to sell universal life insurance to a 30-year old male in January 2009, its reserves would only increase by $0.87 per dollar of policies sold. This implies a gain of $0.13 in capital surplus funds per dollar of policies sold.

4. A Structural Model of Insurance Pricing

We now develop a model in which an insurance company sets prices for various types of policies to maximize the present discounted value of profits, subject to a leverage constraint that the ratio of statutory reserves to assets cannot exceed a targeted value. The model shows how financial frictions and statutory reserve regulation jointly determine insurance prices. We show that the model explains the magnitude of the price reductions in January 2009 through estimation in Section 5 and through calibration in Section 6.

4.1 An Insurance Company’s Maximization Problem

An insurance company sells \( I \) different types of annuity and life insurance policies, which we index as \( i = 1, \ldots, I \). These policies are differentiated not only by term, but also by sex and
age of the insured. The insurance company faces a downward-sloping demand curve $Q_{i,t}(P)$ for each policy $i$ in period $t$, where $Q'_{i,t}(P) < 0$. There are various micro-foundations that give rise to such a demand curve. For example, such a demand curve can be motivated as an industry equilibrium subject to search frictions (Hortaçsu and Syverson, 2004). We will simply take the demand curve as exogenously given because the precise micro-foundations are not essential for our purposes.

The insurance company incurs a fixed (marketing and administrative) cost $C_t$ in each period. Let $V_{i,t}$ be the actuarial value of policy $i$ in period $t$. The insurance company’s profit in each period is

$$\Pi_t = \sum_{i=1}^{I} (P_{i,t} - V_{i,t})Q_{i,t} - C_t. \quad (12)$$

A simple way to interpret this profit function is that for each type of policy that the insurance company sells for $P_{i,t}$, it can buy a portfolio of Treasury bonds that replicate its expected future policy claims for $V_{i,t}$. For term annuities, this interpretation is exact since future policy claims are deterministic. For life annuities and life insurance, we assume that the insured pools are sufficiently large for the law of large numbers to apply. Appendix B provides an alternative justification for why $V_{i,t}$ is the effective marginal cost of insurance policies in the presence of a state guarantee fund.

We now describe how the sale of new policies affects the insurance company’s balance sheet. Let $A_{t-1}$ be its assets at the beginning of period $t$, and let $R_{A,t}$ be an exogenous rate of return on its assets in period $t$. Its assets at the end of period $t$, after the sale of new policies, is

$$A_t = R_{A,t}A_{t-1} + \sum_{i=1}^{I} P_{i,t}Q_{i,t} - C_t. \quad (13)$$

As explained in Section 3, the insurance company must also record reserves on the liability side of its balance sheet. Let $L_{t-1}$ be its statutory reserves at the beginning of period $t$,
and let $R_{L,t}$ be the return on its statutory reserves in period $t$. Let $\hat{V}_{i,t}$ be the reserve value of policy $i$ in period $t$. Its statutory reserves at the end of period $t$, after the sale of new policies, is

$$L_t = R_{L,t}L_{t-1} + \sum_{i=1}^{I} \hat{V}_{i,t}Q_{i,t}. \quad (14)$$

The insurance company chooses the price $P_{i,t}$ for each type of policy to maximize firm value, or the present discounted value of its profits:

$$J_t = \Pi_t + E_t[M_{t+1}J_{t+1}], \quad (15)$$

where $M_{t+1}$ is the stochastic discount factor. The insurance company faces a leverage constraint on the value of its statutory reserves relative to its assets:

$$\frac{L_t}{A_t} \leq \phi, \quad (16)$$

where $\phi \leq 1$ is the maximum leverage ratio. The underlying assumption is that exceeding the maximum leverage ratio leads to bad consequences, such as a rating downgrade or forced liquidation by state regulators.\textsuperscript{3} At fiscal year-end 2008, many highly rated insurance companies were concerned that the upward pressure on their leverage ratio would trigger a rating downgrade, which would have an adverse impact on their business.\textsuperscript{4}

To simply notation, we define the insurance company’s excess reserves as

$$K_t = \phi A_t - L_t. \quad (17)$$

\textsuperscript{3}An alternative model, with similar implications to the leverage constraint, is that the insurance company faces a convex cost whenever the leverage ratio exceeds $\phi$.

\textsuperscript{4}For example, A.M. Best Company (2009) reports that MetLife’s “financial leverage is at the high end of its threshold for the current rating level. The company has projected that this will moderate down at year end 2009.”
The leverage constraint can then be rewritten as

\[ K_t \geq 0. \]  

(18)

The law of motion for excess reserves is

\[ K_t = \phi R_{A,t} A_{t-1} - R_{L,t} L_{t-1} + \sum_{i=1}^{I} \left( \phi P_{i,t} - \hat{V}_{i,t} \right) Q_{i,t} - C_t. \]  

(19)

4.2 Optimal Insurance Pricing

Let \( \lambda_t \geq 0 \) be the Lagrange multiplier on the leverage constraint (18). The Lagrangian for the insurance company’s maximization problem is

\[ \mathcal{L}_t = J_t + \lambda_t K_t. \]  

(20)

The first-order condition for the price of each type of policy is

\[
\frac{\partial \mathcal{L}_t}{\partial P_{i,t}} = \frac{\partial J_t}{\partial P_{i,t}} + \lambda_t \frac{\partial K_t}{\partial P_{i,t}} = \frac{\partial \Pi_t}{\partial P_{i,t}} + \lambda_t \frac{\partial K_t}{\partial P_{i,t}}
\]

\[ = Q_{i,t} + \left( P_{i,t} - \hat{V}_{i,t} \right) Q'_{i,t} + \lambda_t \left[ \phi Q_{i,t} + \left( \phi P_{i,t} - \hat{V}_{i,t} \right) Q'_{i,t} \right] = 0, \]  

(21)

where

\[ \lambda_t = \lambda_t + E_t \left[ M_{t+1} \frac{\partial J_{t+1}}{\partial K_t} \right]. \]  

(22)

Equation (21) implies that

\[ \lambda_t = \frac{\partial \Pi_t}{\partial K_t}. \]  

(23)
That is, \( \bar{\lambda}_t \) measures the marginal reduction in profits that the insurance company is willing to accept in order to increase its excess reserves by a dollar. Equation (22) implies that \( \bar{\lambda}_t = 0 \) if the leverage constraint does not bind today (i.e., \( \lambda_t = 0 \)), and increasing excess reserves does not relax future constraints (i.e., \( E_t[M_{t+1} \partial J_{t+1}/\partial K_t] = 0 \)). Therefore, we refer to \( \bar{\lambda}_t \) as the shadow cost of financial frictions because it measures the importance of the leverage constraint, either today or at some future state.

Rearranging equation (21), the price of policy \( i \) in period \( t \) is

\[
P_{i,t} = V_{i,t} \left( 1 - \frac{1}{\epsilon_{i,t}} \right)^{-1} \left( \frac{1 + \bar{\lambda}_t \hat{V}_{i,t}/V_{i,t}}{1 + \bar{\lambda}_t \phi} \right),
\]

where

\[
\epsilon_{i,t} = -\frac{P_{i,t} Q_{i,t}'}{Q_{i,t}} > 1
\]

is the elasticity of demand. If the shadow cost of financial frictions is zero (i.e., \( \bar{\lambda}_t = 0 \)), the price of policy \( i \) in period \( t \) is

\[
P_{i,t} = V_{i,t} \left( 1 - \frac{1}{\epsilon_{i,t}} \right)^{-1}.
\]

This is the standard Bertrand model of pricing, in which price is equal to marginal cost times a markup that is decreasing in the elasticity of demand.

If the shadow cost of financial frictions is positive (i.e., \( \bar{\lambda}_t > 0 \)), the price of policy \( i \) in period \( t \) satisfies the inequality

\[
P_{i,t} \geq V_{i,t} \left( 1 - \frac{1}{\epsilon_{i,t}} \right)^{-1} \text{ if } \frac{\hat{V}_{i,t}}{V_{i,t}} \geq \phi.
\]

That is, the price of the policy is higher than the Bertrand price if selling the policy tightens the leverage constraint on the margin. This is the case with property and casualty insurers,
whose statutory reserve regulation requires that $\hat{V}_{i,t}/V_{i,t} > 1$ (Froot and O’Connell, 1999). Conversely, the price of the policy is lower than the Bertrand price if selling the policy relaxes the leverage constraint on the margin. This was the case with life insurers in January 2009.

When the leverage constraint binds, equation (24) and the leverage constraint (i.e., $K_t = 0$) forms a system of $I + 1$ equations in $I + 1$ unknowns (i.e., $P_{i,t}$ for each policy $i = 1, \ldots, I$ and $\bar{\lambda}_t$). Solving this system of equations for the shadow cost of financial frictions,

$$\bar{\lambda}_t = \frac{1}{\phi} \left( \sum_{i=1}^{I} \left( \phi V_{i,t}(1 - 1/\epsilon_{i,t})^{-1} - \hat{V}_{i,t} \right) Q_{i,t} + K_{t-1} \right) \right).$$

(28)

To understand the intuition for this expression, consider the limiting case of perfectly elastic demand. The limit as $\epsilon_{i,t} \to \infty$ for all policies is

$$\bar{\lambda}_t \to \frac{1}{\phi} \left( \sum_{i=1}^{I} \left( \phi V_{i,t} - \hat{V}_{i,t} \right) Q_{i,t} \right) - K_{t-1} \right).$$

(29)

This expression shows that the shadow cost of financial frictions depends on the product of two terms. The first term says that the shadow cost is inversely related to the maximum leverage ratio. The second term says that the shadow cost is proportional to the marginal increase in excess reserves from selling new policies as a share of the initial shortfall in excess reserves.

5. Estimating the Structural Model of Insurance Pricing

In this section, we estimate the structural model of insurance pricing, through which we identify the shadow cost of financial frictions. Before doing so, we first present reduced-form evidence that is consistent with a key prediction of the model. Namely, the price reductions were larger for those insurance companies that experienced more adverse balance
sheet shocks just prior to January 2009, which are presumably the companies for which the leverage constraint was more costly.

5.1 Price Changes versus Balance Sheet Shocks

Figure 8 is an overview of how the balance sheet has evolved over time for the median insurance company in our sample. Assets grew by 3 to 14 percent annually from 1989 through 2010. The only exception to this growth is 2008 when assets shrank by 3 percent. The leverage ratio stays remarkably constant between 0.91 and 0.95 throughout this period, including 2008 when the leverage ratio was 0.93 for the median insurance company (Berry-Stölzle, Nini, and Wende, 2011).

Figure 9 is a scatter plot of the percent change in annuity prices from July 2007 to January 2009 versus asset growth from fiscal year-end 2007 to 2008. The four panels represent term annuities, life annuities, and 10- and 20-year guaranteed annuities. The dots in each panel represent the insurance companies in our sample in January 2009. The linear regression line shows that there is a strong positive relation between annuity price changes and asset growth. That is, the price reductions were larger for those insurance companies that experienced more adverse balance sheet shocks just prior to January 2009.

Our joint interpretation of Figures 8 and 9 is that insurance companies were able to maintain a low leverage ratio in 2008 and 2009 by taking advantage of statutory reserve regulation that allowed them to record far less than a dollar of reserve per dollar of future insurance liability. The incentive to reduce prices was stronger for those insurance companies that experienced more adverse balance sheet shocks and, therefore, had a higher need to recapitalize.
5.2 Empirical Specification

Let $i$ index the type of policy, $j$ index the insurance company, and $t$ index time. Based on pricing equation (24), we model the markup as a nonlinear regression model:

$$
\log \left( \frac{P_{i,j,t}}{V_{i,t}} \right) = -\log \left( 1 - \frac{1}{\epsilon_{i,j,t}} \right) + \log \left( \frac{1 + \lambda_{j,t} \hat{V}_{i,t}/V_{i,t}}{1 + \lambda_{j,t} L_{j,t}/A_{j,t}} \right) + e_{i,j,t},
$$

(30)

where $e_{i,j,t}$ is an error term with conditional mean zero.

We model the elasticity of demand as

$$
\epsilon_{i,j,t} = 1 + \exp\{-\beta' y_{i,j,t}\},
$$

(31)

where $y_{i,j,t}$ is a vector of policy and insurance company characteristics. In our baseline specification, the policy characteristics are sex and age. The insurance company characteristics are the A.M. Best rating, the leverage ratio, asset growth, and log assets. We also include a full set of time dummies to control for any variation in the elasticity of demand over the business cycle. We interact each of these variables, including the time dummies, with dummy variables that allow their impact on the elasticity of demand to differ across term annuities, life annuities, and life insurance.

In theory, the shadow cost of financial frictions depends only on insurance company characteristics that appear in equation (28). However, most of these characteristics do not have obvious counterparts in the data except for $\phi$, which is equal to the leverage ratio when the constraint binds (i.e., $\phi = L_t/A_t$). Therefore, we model the shadow cost of financial frictions as

$$
\lambda_{j,t} = \exp\{-\gamma' z_{j,t}\},
$$

(32)

where $z_{j,t}$ is a vector of insurance company characteristics. In our baseline specification, the insurance company characteristics are the leverage ratio and asset growth. Our use of asset
growth is motivated by the reduced-form evidence in Figure 8. We also include a full set of
time dummies and their interaction with insurance company characteristics to allow for the
fact that the leverage constraint may only bind at certain times.

5.3 Identifying Assumptions

If the elasticity of demand is correctly specified, regression model (30) is identified by the fact
that the markup has a nonnegative conditional mean in the absence of financial frictions:

$$- \log \left(1 - \frac{1}{\epsilon_{i,j,t}}\right) > 0.$$  \tag{33}

Therefore, a negative markup must be explained by a positive shadow cost of financial
frictions whenever the ratio of reserve to actuarial value is less than the leverage ratio (i.e.,
$\hat{V}_{i,t} / V_{i,t} < L_{j,t} / A_{j,t}$).

Even if the elasticity of demand is potentially misspecified, the shadow cost of financial
frictions is identified by exogenous variation in the ratio of reserve to actuarial value across
different types of policies. To illustrate this point, we approximate regression model (30)
through first-order Taylor approximation as

$$\log \left(\frac{P_{i,j,t}}{V_{i,t}}\right) \approx \alpha_{j,t} + \frac{1}{1/\lambda_{j,t} + L_{j,t}/A_{j,t}} \left(\frac{\hat{V}_{i,t}}{V_{i,t}} - \frac{L_{j,t}}{A_{j,t}}\right) + v_{i,j,t},$$  \tag{34}

where

$$v_{i,j,t} = -\alpha_{j,t} - \log \left(1 - \frac{1}{\epsilon_{i,j,t}}\right) + e_{i,j,t}.$$  \tag{35}

is an error term with conditional mean zero. For a given insurance company $j$ at a given
time $t$, the regression coefficient $\lambda_{j,t}$ is identified as long as $\hat{V}_{i,t} / V_{i,t}$ is orthogonal to $v_{i,j,t}$.

More intuitively, Standard Valuation Law generates relative shifts in the supply curve across
different types of policies that an insurance company sells, which we exploit to identify the

25
shadow cost of financial frictions.

5.4 Estimating the Shadow Cost of Financial Frictions

Since the data for most types of annuities are not available prior to July 1998, we estimate the structural model on the sub-sample from July 1998 through July 2011. Table 2 reports our estimates for the elasticity of demand in the nonlinear regression model (30). Instead of reporting the raw coefficients (i.e., $\beta$), we report the average marginal effect of the explanatory variables on the markup. The average markup on policies sold by A or A− rated insurance companies is 3.13 percentage points higher than that for policies sold by A++ or A+ rated companies. The leverage ratio and asset growth have a relatively small economic impact on the markup through the elasticity of demand. Every 1 percentage point increase in the leverage ratio is associated with a 6 basis point increase in the markup. Every 1 percentage point increase in asset growth is associated with a 4 basis point increase in the markup.

Figure 10 reports the time series of the shadow cost of financial frictions for the average insurance company (i.e., at the conditional mean of the leverage ratio and asset growth). The leverage constraint is not costly for most of the sample period. There is evidence that the leverage constraint was costly around January 2001 with a point estimate of $0.79 per dollar of excess reserve. The leverage constraint was clearly costly in January 2009 with a point estimate of $4.58 per dollar of excess reserve. That is, the average insurance company was willing to accept a marginal reduction of $4.58 in profits in order to increase its excess reserves by a dollar. The 95 percent confidence interval ranges from $2.78 to $6.39 per dollar of excess reserve.

In Table 3, we report the shadow cost of financial frictions for the cross section of insurance companies in our sample that sold annuities in January 2009. The table shows that there is considerable heterogeneity in the shadow cost of financial frictions. The shadow cost of financial frictions is positively related to the leverage ratio and negatively related to asset
growth. In January 2009, MetLife was the most constrained insurance company with a shadow cost of $13.38 per dollar of excess reserve. Metlife had a relatively high leverage ratio of 0.97 at fiscal year-end 2008 and suffered a balance sheet loss of 10 percent from fiscal year-end 2007 to 2008. American General was the least constrained insurance company with a shadow cost of $1.41 per dollar of excess reserve.

5.5 Inflow of Capital Surplus Funds

Insurance companies have two channels of raising capital surplus funds (i.e., accounting equity). The first, which we emphasize in this paper, is through the sale of new policies at a price above reserve value, which generates accounting profits. The second is direct inflow of capital surplus funds through issuance of surplus notes or reduction of stockholder dividends to the holding company. We now provide evidence that these two channels were complementary during the financial crisis.

For the same set of insurance companies as Table 3, Figure 11 reports the inflow of capital surplus funds for fiscal years 2008 and 2009 as a percentage of capital surplus funds at fiscal year-end 2007. The linear regression line shows that there is a strong positive relation between the inflow of capital surplus funds and the shadow cost of financial frictions in January 2009. In particular, MetLife had both the highest inflow of capital surplus funds (224 percent) and the highest shadow cost ($13.38 per dollar of excess reserve). American General is an outlier in Figure 11 with a relatively high inflow of capital surplus funds (158 percent), despite having the lowest shadow cost ($1.41 per dollar of excess reserves). This can be explained by the fact that its holding company received a government bailout in September 2008.

The picture that emerges from Figure 11 is that those insurance companies that were financially constrained received capital injections from the holding company, either through issuance of surplus notes or reduction of stockholder dividends. However, this direct inflow of capital surplus funds was insufficient at the height of the financial crisis and, therefore,
insurance companies had to raise additional capital by selling insurance policies at firesale prices.

6. Calibrating the Structural Model of Insurance Pricing

In this section, we calibrate the structural model of insurance pricing and solve explicitly for the insurance company’s policy and value functions. Relative to the estimation in the last section, the advantage of this approach is that we gain additional insight into how optimal insurance pricing is related to firm value and the shadow cost of financial frictions. The disadvantage, however, is that we must make additional parametric assumptions regarding asset returns and the demand function. We view the two approaches as providing complementary evidence that the model explains the magnitude of the price reductions in January 2009.

6.1 Additional Assumptions

Our goal is to calibrate and solve the simplest version of the model in Section 4 that captures the essence of our empirical findings. Therefore, we start with the version in which the insurance company sells only one type of policy. We assume that the return on assets and statutory reserves are constant and equal to the riskless interest rate (i.e., $R_{A,t} = R_{L,t} = R$). The stochastic discount factor is constant and equal to the inverse of the riskless interest rate (i.e., $M_t = 1/R$). We assume that both the reserve and the actuarial value of the policy are constant and denote them as $\hat{V}$ and $V$, respectively.

We parameterize the demand function as

$$Q_t = X_t P_t^{-\epsilon}.$$  (36)
where $\epsilon$ is a constant elasticity of demand. The demand shock follows a geometric random walk:

$$\Delta X_t = \frac{X_t}{X_{t-1}} = \exp\left\{ u_t - \frac{\sigma^2}{2} \right\}, \quad (37)$$

where $u_t \sim N(0, \sigma^2)$. Finally, we parameterize the fixed cost as

$$C_t = \overline{C} X_t V^{1-\epsilon}, \quad (38)$$

where $\overline{C}$ is a constant.

We calibrate the parameters of the model to explain the pricing of 30-year term annuities in January 2009. Therefore, we set the ratio of reserve to actuarial value to 0.73. We set the riskless interest rate to 0.5 percent, which is the 1-year nominal Treasury yield in January 2009. We set the elasticity of demand to 11, which generates a realistic markup of 10 percent when the leverage constraint does not bind. We set the standard deviation of demand shocks to 30 percent, which is the standard deviation of the growth rate for annual premiums on individual annuities in Figure 1. We set the fixed cost to 2%, which is MetLife's general expense ratio (excluding commissions) for individual annuities in fiscal year 2008. We assume a maximum leverage ratio of 0.97 to correspond to the highest leverage ratio for the cross section of insurance companies in Table 3. Table 4 summarizes the parameters of the calibrated model. Appendix C describes how we solve the model numerically using standard dynamic programming techniques.

### 6.2 Optimal Insurance Pricing and Firm Value

Given our simplifying assumptions, the insurance company’s maximization problem depends on only one state variable. Appendix C shows that the key state variable is its initial excess
reserves prior to the sale of new policies, appropriately scaled by market size:

\[ K_t = \frac{RK_{t-1}}{X_t V^{1-\epsilon}} - \overline{C}. \]  

(39)

Whenever \( K_t \) is negative, the insurance company has an initial shortfall in excess reserves that must corrected through the sale of new policies.

Figure 12 reports the optimal insurance price, firm value, and the shadow cost of financial frictions as functions of initial excess reserves. The leverage constraint does not bind when initial excess reserves are positive. In this region of the state space, the insurance company sells its policies at a markup of 10 percent. Its firm value is $100, and the shadow cost of financial frictions is zero. The leverage constraint binds when initial excess reserves are sufficiently negative. In this region of the state space, both the optimal insurance price and firm value are decreasing in the shortfall in excess reserves. In our calibration, the shadow cost of financial constraints is always equal to the Lagrange multiplier on the leverage constraint (i.e., \( \hat{\lambda}_t = \lambda_t \)). In other words, the insurance company does not have an incentive to increase excess reserves today in order to relax future constraints (i.e., \( E_t [M_{t+1} \partial J_{t+1} / \partial K_t] = 0 \)).

When initial excess reserves are \(-18\) percent of firm value, the insurance company sells its policies at a markup of \(-15\) percent. Its firm value is $71, and the shadow cost is $10 per dollar of excess reserve. Put differently, the insurance price falls by 25 percent, and firm value falls by 29 percent relative to when the leverage constraint does not bind. These magnitudes in the calibrated model are consistent with our empirical findings. Namely, 30-year term annuities sold at a markup of \(-25\) percent, and the shadow cost was nearly $5 per dollar of excess reserve for the average insurance company in January 2009.

7. Conclusion

This paper shows that financial frictions and statutory reserve regulation have a large and measurable impact on insurance prices. More broadly, we show that frictions on the supply
side have a large and measurable impact on consumer financial markets. The previous literature on household finance has mostly focused on frictions on the demand side of these markets, such as household borrowing constraints, asymmetric information, moral hazard, and near rationality. While these frictions on the demand side are undoubtedly important, we feel that financial and regulatory frictions on the supply side are equally important for our understanding of market equilibrium and consumer welfare.

Another broader implication of our study is that we provide micro evidence for a class of macro models based on financial frictions, which is a leading explanation for the Great Recession (see Gertler and Kiyotaki, 2010; Brunnermeier, Eisenbach, and Sannikov, 2012, for recent reviews of the literature). We feel that this literature would benefit from additional micro evidence on the cost of these frictions for other types of financial institutions, such as commercial banks and health insurance companies. In principle, the empirical approach in this paper can be used to estimate the shadow cost of financial frictions for other types of financial institutions.

Finally, we feel that further work is necessary on the optimal regulation of statutory reserves. The current regulation causes the statutory reserve requirement to vary arbitrarily, both over time and across different types of policies. While this exogenous variation is useful for identifying the shadow cost of financial frictions, it does not seem optimal from the perspective of insurance regulation. In the context of pricing equation (24), a simple reserve rule that achieves price stability is to set the reserve value equal to the targeted leverage ratio times the actuarial value (i.e., \( \hat{V}_{i,t} = \phi V_{i,t} \)). Under this reserve rule, the insurance price would always be the Bertrand price (26), even when the leverage constraint binds. Although this simple rule may not be the socially optimal policy in a fully specified model, it seems like a good starting point for thinking about optimal regulation.
A. Life Annuities during the Great Depression

Following Warshawsky (1988), our prices on life annuities from 1929 through 1938 are from annual editions of *The Handy Guide* (The Spectator Company, 1929). We focus on quotes for males between ages 50 and 80 (every 10 years in between). We match the quoted price for each year of *The Handy Guide* to the actuarial value in January of that year. We calculate the actuarial value at each date based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. We derive the zero-coupon yield curve from the constant-maturity yield curve reported in Cecchetti (1988).

Figure A1 reports the time series of the average markup on life annuities at various ages, averaged across insurance companies and reported with a 95 percent confidence interval. The key finding is that the markup remained positive throughout the Great Depression. In particular, the average markup for life annuities at age 50 was 28 percent in 1932, when the corporate default spread was much higher than the heights reached during the recent financial crisis.

Prior to the adoption of Standard Valuation Law in the mid-1940’s, individual states had their own standards for reserve valuation. However, many states used the annuitant mortality table from M’Clintock (1899) and a constant discount rate for reserve valuation (e.g., 3.5 percent in California). Figure A2 reports the ratio of reserve to actuarial value for life annuities for males aged 50 to 80 (every 10 years in between) at the discount rate of 3.5 percent. The ratio of reserve to actuarial value remained close to or above one throughout the Great Depression. This implies that insurance companies could not lower their leverage ratio by selling life annuities at a price below actuarial value, which is consistent with the absence of discounts in Figure A1.
B. Cost of Capital in the Presence of a Guarantee Fund

Insurance companies are ex ante identical, and we normalize their mass to one. The face value of future liabilities is \( Q \) for each insurance company, so that the aggregate quantity of liabilities is also \( Q \). There are \( S \) possible future states, and each state \( s = 1, \ldots, S \) realizes with probability \( \pi_s \). Let \( M_s \) be the stochastic discount factor associated with state \( s \), which satisfies the usual relation

\[
\sum_{s=1}^{S} \pi_s M_s R = 1,
\]

where \( R \) is the riskless interest rate. Each insurance company defaults with probability \( p_s \) in state \( s \).

There is a state guarantee fund such that the surviving insurance companies pay off the liabilities of the defaulting insurance companies. Conditional on survival, an insurance company must pay off its own liability plus \( p_s Q / (1 - p_s) \), which is the amount of defaulted liabilities \( p_s Q \) divided among the mass \( 1 - p_s \) of surviving insurance companies. Therefore, the present value of future liabilities for each insurance company is

\[
VQ = \sum_{s=1}^{S} \pi_s M_s (1 - p_s) \left( Q + \frac{p_s Q}{1 - p_s} \right) = \frac{Q}{R}.
\]

That is, the appropriate cost of capital for insurance liabilities is the riskless interest rate (i.e., \( V = 1/R \)) in the presence of a state guarantee fund.

C. Solving the Model by Dynamic Programming

Because demand follows a geometric random walk, we must scale both the value function and excess reserves by \( X_t V^{1-\epsilon} \) to make the model stationary. We rewrite the value function
as
\[ \bar{J}_t = \frac{J_t}{X_t V^{1-\epsilon}} = \bar{J}(K_t) = \left( \frac{P_t}{V} - 1 \right) \left( \frac{P_t}{V} \right)^{-\epsilon} - \bar{C} + \frac{1}{R} E_t [\Delta X_{t+1} \bar{J}_{t+1}] . \quad (C1) \]

We rewrite the law of motion for excess reserves as
\[ K_{t+1} = \frac{RK_t}{X_{t+1} V^{1-\epsilon}} - \bar{C} = \frac{R}{\Delta X_{t+1}} \left[ K_t + \left( \phi \frac{P_t}{V} - \hat{V} \right) \left( \frac{P_t}{V} \right)^{-\epsilon} \right] - \bar{C} . \quad (C2) \]

We rewrite the leverage constraint as
\[ K_t + \left( \phi \frac{P_t}{V} - \hat{V} \right) \left( \frac{P_t}{V} \right)^{-\epsilon} \geq 0. \quad (C3) \]

The insurance company chooses \( P_t \) to maximize firm value (C1) subject to the law of motion for excess reserves (C2) and the leverage constraint (C3).

The leverage constraint (C3) can be satisfied as long as initial excess reserves, prior to the sale of new policies, satisfies
\[ K_t \geq K = -\frac{\phi}{\epsilon} \left( \hat{V} \right)^{1-\epsilon} \left( 1 - \frac{1}{\epsilon} \right)^{\epsilon-1} . \quad (C4) \]

In the region of the state space \( \bar{K}_t < \bar{K} \), the maximization problem does not have a solution that satisfies the leverage constraint. Therefore, we impose an auxiliary assumption that \( \bar{J}(K_t) = \bar{J}(\bar{K}) \) for all \( \bar{K}_t < \bar{K} \). This assumption captures the fact that the insurance company may receive direct inflow of capital surplus funds when it cannot satisfy the leverage constraint through the sale of new policies alone.

We discretize the state space, which we denote as \( \{K_j\}_{j=1}^J \). We also discretize the demand shock into seven grid points using Gauss-Hermite quadrature. Starting with the initial guess
\[ P_1 (K_j) = V \left( 1 - \frac{1}{\epsilon} \right)^{-1} . \quad (C5) \]
for the policy function, we solve the model by value iteration.

1. Iterate on equation (C1) to compute the value function $J_i(K_j)$ corresponding to the current policy function $P_i(K_j)$.

2. For each point $K_j$ on the grid, find $P_{i+1}(K_j)$ that maximizes equation (C1) with $J_{t+1} = J_i(K_j)$.

3. If $\max_{K_j} |P_{i+1}(K_j) - P_i(K_j)|$ is less than the convergence criteria, stop. Otherwise, return to step 1.
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Table 1: Summary Statistics for Annuity and Life Insurance Prices

Markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is semiannual from January 1989 through July 2011.

| Type of policy       | Sample starts in | Number of | Markup |
|----------------------|------------------|-----------|--------|
|                      |                  | Observations | Insurance companies | Mean | Median | Standard deviation |
| Term annuities:      |                  |             |                    |      |        |                   |
| 5-year               | January 1993     | 732        | 83                 | 6.7  | 6.5    | 8.4               |
| 10-year              | January 1989     | 988        | 98                 | 6.9  | 7.0    | 5.9               |
| 15-year              | July 1998        | 418        | 62                 | 4.3  | 4.8    | 5.6               |
| 20-year              | July 1998        | 414        | 62                 | 3.8  | 4.4    | 6.6               |
| 25-year              | July 1998        | 339        | 53                 | 3.4  | 3.7    | 7.5               |
| 30-year              | July 1998        | 325        | 50                 | 2.9  | 2.8    | 8.8               |
| Life annuities:      |                  |             |                    |      |        |                   |
| Life only            | January 1989     | 11,879     | 106                | 9.8  | 9.8    | 8.2               |
| 10-year guaranteed   | July 1998        | 7,885      | 66                 | 5.5  | 6.1    | 7.0               |
| 20-year guaranteed   | July 1998        | 7,518      | 66                 | 4.2  | 4.8    | 7.5               |
| Universal life insurance | January 2005 | 3,989      | 52                 | -4.2 | -5.5  | 17.9              |
Table 2: Estimated Model of Insurance Pricing
This table reports the average marginal effect of the explanatory variables on the markup through the elasticity of demand in percentage points. The model for the elasticity of demand also includes time dummies and its interaction effects for life annuities and life insurance, which are omitted in this table for brevity. The omitted categories for the dummy variables are term annuities, A++ or A+ rated, male, and age 50. The $t$-statistics, reported in parentheses, are based on robust standard errors clustered by insurance company, type of policy, sex, and age. The sample is semiannual from July 1998 through July 2011.

| Explanatory variable | Average marginal effect |
|----------------------|------------------------|
| Rating: A to A−      | 3.13 (17.69)           |
| Rating: B++ to B−    | 9.16 (13.28)           |
| Leverage ratio       | 6.13 (23.55)           |
| Asset growth         | 3.91 (15.08)           |
| Log assets           | 2.31 (40.53)           |
| Interaction effects for life annuities: | |
| Rating: A to A−      | -2.26 (-17.94)         |
| Rating: B++ to B−    | -8.77 (-11.02)         |
| Leverage ratio       | 16.88 (26.27)          |
| Asset growth         | -5.58 (-19.58)         |
| Log assets           | -1.89 (-44.69)         |
| Female               | 0.27 (10.18)           |
| Age 55               | 0.25 (1.74)            |
| Age 60               | 0.60 (3.90)            |
| Age 65               | 0.83 (11.94)           |
| Age 70               | 1.14 (10.25)           |
| Age 75               | 1.45 (2.99)            |
| Age 80               | 1.80 (10.60)           |
| Age 85               | 2.36 (10.41)           |
| Age 90               | 3.28 (6.58)            |
| Interaction effects for life insurance: | |
| Rating: A to A−      | -23.21 (-5.12)         |
| Leverage ratio       | 21.78 (3.02)           |
| Asset growth         | -30.05 (-5.27)         |
| Log assets           | -13.21 (-7.36)         |
| Female               | 0.18 (1.03)            |
| Age 30               | 2.38 (0.21)            |
| Age 40               | 0.62 (0.03)            |
| Age 60               | 0.18 (0.00)            |
| Age 70               | 0.64 (0.31)            |
| Age 80               | 0.65 (0.27)            |
| Age 90               | 24.12 (4.74)           |
| $R^2$ (percent)      | 48.51                  |
| Observations         | 29,570                 |
Table 3: Shadow Cost of Financial Frictions in January 2009
This table reports the shadow cost of financial frictions for the cross section of insurance companies in our sample that sold annuities in January 2009, implied by our estimated model of insurance pricing.

| Insurance company                                      | A.M. Best rating | Leverage ratio | Asset growth | Shadow cost |
|--------------------------------------------------------|------------------|----------------|--------------|-------------|
| MetLife Investors USA Insurance Company                 | A+               | 0.97           | -0.10        | 13.38       |
| Allianz Life Insurance Company of North America         | A                | 0.97           | -0.03        | 10.47       |
| Lincoln Benefit Life Company                            | A+               | 0.87           | -0.45        | 8.76        |
| OM Financial Life Insurance Company                     | A-               | 0.95           | -0.04        | 8.31        |
| Aviva Life and Annuity Company                          | A                | 0.95           | 0.12         | 4.44        |
| Presidential Life Insurance Company                     | B+               | 0.91           | -0.06        | 4.33        |
| EquiTrust Life Insurance Company                        | B+               | 0.95           | 0.13         | 4.12        |
| Integrity Life Insurance Company                        | A+               | 0.92           | 0.03         | 3.85        |
| United of Omaha Life Insurance Company                  | A+               | 0.91           | -0.03        | 3.65        |
| Genworth Life Insurance Company                         | A                | 0.90           | 0.00         | 3.13        |
| North American Company for Life and Health Insurance    | A+               | 0.94           | 0.24         | 2.44        |
| American National Insurance Company                     | A                | 0.87           | -0.02        | 1.84        |
| American General Life Insurance Company                 | A                | 0.87           | 0.05         | 1.41        |
Table 4: Parameters in the Calibrated Model

| Parameter                              | Symbol | Value |
|----------------------------------------|--------|-------|
| Riskless interest rate                 | $R - 1$ | 0.5%  |
| Ratio of reserve to actuarial value    | $\tilde{V}/V$ | 0.73  |
| Elasticity of demand                   | $\epsilon$ | 11    |
| Standard deviation of demand shocks    | $\sigma$ | 30%   |
| Fixed cost                             | $\overline{C}$ | 2%    |
| Maximum leverage ratio                 | $\phi$ | 0.97  |
Figure 1: Annual Premiums for Individual Annuities and Life Insurance
This figure reports the total annual premiums collected for individual annuities and life insurance, summed across all insurance companies in the *Best’s Insurance Reports*. The sample is from fiscal year 1992 through 2010.
Figure 2: Average Markup of Term Annuities

Markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the zero-coupon Treasury yield curve. Average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating and time. The figure reports the conditional mean for policies sold by A++ and A+ rated companies. The confidence interval is based on robust standard errors clustered by insurance company. The sample is semianual from January 1989 through July 2011.
Figure 3: Average Markup of Life Annuities

Markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. Average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating, sex, and time. The figure reports the conditional mean for male policies sold by A++ and A+ rated companies. The confidence interval is based on robust standard errors clustered by insurance company, sex, and age. The sample is semiannual from January 1989 through July 2011.
Figure 4: Average Markup of Universal Life Insurance

Markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. Average markup is estimated from a regression of markups onto dummy variables for A.M. Best rating, sex, and time. The figure reports the conditional mean for male policies sold by A++ and A+ rated companies. The confidence interval is based on robust standard errors clustered by insurance company, sex, and age. The sample is semiannual from January 2004 through July 2011.
Figure 5: Discount Rates for Annuities and Life Insurance
This figure reports the discount rates used for statutory reserve valuation of annuities and life insurance (with term greater than 20 years), together with the 10-year zero-coupon Treasury yield. The sample is monthly from January 1989 through July 2011.
Figure 6: Reserve to Actuarial Value for Annuities

This figure reports the ratio of reserve to actuarial value for various types of annuities. The reserve value is based on the appropriate loaded mortality table from the American Society of Actuaries and the discount rate specified by Standard Valuation Law. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is monthly from January 1989 through July 2011.
Figure 7: Reserve to Actuarial Value for Universal Life Insurance
This figure reports the ratio of reserve to actuarial value for universal life insurance. The reserve value is based on the appropriate loaded mortality table from the American Society of Actuaries and the discount rate specified by Standard Valuation Law. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon Treasury yield curve. The sample is monthly from January 2005 through July 2011.
Figure 8: Asset Growth and the Leverage Ratio for Life Insurers
This figure reports the growth rate of total admitted assets and the leverage ratio for the median insurance company in our sample. The leverage ratio is the ratio of total liabilities to total admitted assets. The sample is from fiscal year 1989 through 2010.
Figure 9: Price Change versus Asset Growth in January 2009

The percent change in annuity prices is from July 2007 to January 2009. The percent change in total admitted assets is from fiscal year-end 2007 to 2008. For term annuities, the average price change is estimated from a regression of the price change onto dummy variables for insurance company and maturity. The figure reports the conditional mean for 30-year term policies. For life annuities, the average price change is estimated from a regression of the price change onto dummy variables for insurance company, sex, and age. The figure reports the conditional mean for male policies at age 50. The linear regression line weights the observations by annual premiums collected for individual annuities in fiscal year 2008.
Figure 10: Shadow Cost of Financial Frictions
This figure reports the shadow cost of financial frictions for the average insurance company, implied by our estimated model of insurance pricing. The confidence interval is based on robust standard errors clustered by insurance company, type of policy, sex, and age. The sample is semiannual from July 1998 through July 2011.
The inflow of capital surplus funds in fiscal years 2008 and 2009 is reported as a percentage of capital surplus funds at fiscal year-end 2007. An insurance company increases the inflow of capital surplus funds through issuance of surplus notes or reduction stockholder dividends. The shadow cost of financial frictions in January 2009 is reported for the same set of insurance companies as Table 3.
Figure 12: Optimal Insurance Price and Firm Value in the Calibrated Model
This figure reports the optimal insurance price ($P_t/V - 1$), firm value ($J_t$), and the shadow cost of financial frictions ($\lambda_t$) as functions of initial excess reserves ($K_{t-1}$). Firm value is normalized to $100 at the highest value of initial excess reserves. Initial excess reserves are normalized by the firm value corresponding to the highest value of initial excess reserves. Table 4 reports the parameters of the calibrated model.
Figure A1: Average Markup of Life Annuities: 1929–1938

 Markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. Average markup is estimated from a regression of markups onto dummy variables for time. The confidence interval is based on robust standard errors clustered by insurance company and age. The sample is annual from January 1929 through January 1938.
Figure A2: Reserve to Actuarial Value for Life Annuities: 1929–1938
This figure reports the ratio of reserve to actuarial value for life annuities. The reserve value is based on the annuitant mortality table from M’Clintock (1899) and a constant discount rate of 3.5 percent. The actuarial value is based on the annuitant mortality table from M’Clintock (1899) and the zero-coupon Treasury yield curve. The sample is monthly from January 1929 through January 1938.