Relation of hard and total cross sections to centrality

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Abstract. We compare the fractions of the hard and geometric cross sections as a function of impact parameter. For a given definition of central collisions, we calculate the corresponding impact parameter and the fraction of the hard cross section contained within this cut. We use charm quark production as a definite example.

In this note, revised from Ref. [1], standard nuclear density distributions are described and the resulting geometrical overlap in nuclear collisions is calculated. We then compare the hard process cross section to the total geometric cross section as a function of impact parameter and discuss how they are related to the collision centrality.

A three–parameter Woods–Saxon shape is used to describe the nuclear density distribution,

$$\rho_A(r) = \rho_0 \frac{1 + \omega(r/R_A)^2}{1 + \exp((r - R_A)/z)}$$

(1)

where $R_A$ is the nuclear radius, $z$ is the surface thickness, and $\omega$ allows for central irregularities. The electron scattering data of Ref. [2] is used where available for $R_A$, $z$, and $\omega$. When data is unavailable, the parameters $\omega = 0$, $z = 0.54$ fm and $R_A = 1.19A^{1/3} - 1.61A^{-1/3}$ fm are used. The central density $\rho_0$ is found from the normalization $\int d^3r \rho_A(r) = A$. For results with other nuclear shape parameterizations, see the appendix of Ref. [3]. The nuclear shape parameters are given in Table 1. See also Ref. [4] for more detailed discussion of nuclear density distributions.

In minimum bias (impact parameter averaged) $AB$ collisions we expect the production cross section for hard processes to grow approximately as

$$\sigma_{AB}^{\text{hard}} = \sigma_{pp}^{\text{hard}} (AB)^{\alpha}$$

(2)
where $\alpha \equiv 1$ when no nuclear effects are included. However, central collisions are of the greatest interest since it is there that high energy density effects are most likely to appear. Central collisions contribute larger than average values of $E_T$ to the system, in the ‘tail’ of the $E_T$ distribution, $d\sigma/dE_T$. We would like to determine which impact parameters are important in the high $E_T$ tail, i.e. what range of $b$ may be considered central. We now define the central fraction of the hard cross section, Eq. (3), and the central fraction of the geometric cross section. We then discuss how the two are related.

Considering only geometry with no nuclear effects, $\alpha = 1$ in Eq. (2), the inclusive production cross section of hard probes increases as

$$d\sigma_{\text{hard}}^{\text{AB}} = \sigma_{\text{pp}}^{\text{hard}} T_{\text{AB}}(b) d^2b$$

and the average number of hard probes produced at impact parameter $b$ is $\bar{N}_{\text{AB}}(b) = \sigma_{\text{pp}}^{\text{hard}} T_{\text{AB}}(b)$ where $T_{\text{AB}}(b)$ is the nuclear overlap integral,

$$T_{\text{AB}}(\bar{b}) = \int d^2s T_A(\vec{s}) T_B(\bar{b} - \vec{s})$$

and $T_A = \int dz \rho_A(z, \vec{s})$ is the nuclear profile function. The nuclear overlap functions for Pb+Pb and Au+Au collisions are shown in Fig. 1 as a function of impact parameter. Integrating $T_{\text{AB}}$ over all impact parameters we find

$$\int d^2b T_{\text{AB}}(b) = AB$$

The central fraction $f_{\text{AB}}$, equivalent to the fraction of the total hard cross section, is defined as

$$f_{\text{AB}} = \frac{2\pi}{AB} \int_0^{b_c} b \, db \, T_{\text{AB}}(b)$$

Table 1. Nuclear shape parameters taken from Ref. [2].

| $A$ | $R_A$ (fm) | $z$ (fm) | $\omega$ | $\rho_0$ (fm$^{-3}$) |
|-----|------------|----------|---------|-----------------|
| 16  | 2.008      | 0.513    | -0.051  | 0.1654          |
| 27  | 3.07       | 0.519    | 0.0     | 0.1739          |
| 40  | 3.766      | 0.586    | -0.161  | 0.1699          |
| 63  | 4.214      | 0.586    | 0.0     | 0.1701          |
| 110 | 5.33       | 0.535    | 0.0     | 0.1577          |
| 197 | 6.38       | 0.535    | 0.0     | 0.1693          |
| 208 | 6.624      | 0.549    | 0.0     | 0.1600          |

[1]: https://example.com
[2]: https://example.com
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where \( b_c \) is the central impact parameter and \( b < b_c \) are central. To make a similar ‘centrality cut’ in \( pA \) collisions, the fraction

\[
f_A = \frac{2\pi}{A} \int_0^{b_c} b \, db \, T_A(b) \]

would be used. Fig. 2, taken from Ref. [5], shows the increase of \( f_{AB} \) with \( b_c \) for several symmetric, \( AA \), systems. Note that \( f_{AA} \approx 1 \) when \( b_c \approx 2R_A \). For example, if we choose \( \sigma_{\text{central}} = 0.1\sigma_{\text{hard}}^{AB} \), this corresponds to \( b_c = 2.05 \text{ fm} \) in \( Au+Au \) collisions and \( b_c = 1.05 \text{ fm} \) in \( O+O \) collisions. If we instead chose \( \sigma_{\text{central}} = 0.01\sigma_{\text{hard}}^{AB} \) then \( b_c = 0.52 \text{ fm} \) in \( Au+Au \) and \( b_c = 0.33 \text{ fm} \) in \( O+O \) collisions.

Note however that \( f_{AB} \) is not the fraction of the geometric cross section which includes both hard and soft contributions. The geometric cross section in central collisions is found by integrating the interaction probability over impact parameter up to \( b_c \),

\[
\sigma_{\text{geo}}(b_c) = 2\pi \int_0^{b_c} b \, db \, [1 - \exp(-T_{AB}\sigma_{NN})] .
\]

The nucleon-nucleon inelastic cross section, \( \sigma_{NN} \), is \( \approx 32 \text{ mb} \) at SPS energies and grows with energy. It is expected to be \( \approx 60 \text{ mb} \) at LHC energies. The fraction of

Fig. 1. The nuclear overlap function \( T_{AB}(b) \) as a function of impact parameter \( b \) for \( Pb+Pb \) and \( Au+Au \) collisions.
the geometric cross section is

$$f_{\text{geo}} = \frac{\sigma_{\text{geo}}(b_c)}{\sigma_{\text{geo}}}.$$  \hspace{1cm} (9)

In central collisions, where $T_{AB}$ is large, the impact parameter dependence is simple, $\sigma_{\text{geo}}(b_c) \propto b_c^2$. However, in peripheral collisions where the nuclear overlap becomes small, $\sigma_{\text{geo}}(b_c)$ deviates from the trivial $b_c^2$ scaling. Deviations from this scaling do not occur until $b_c \approx 2R_A$ in symmetric systems.

![Figure 2](image)

**Fig. 2.** The central fraction of the hard cross section as a function of impact parameter cut $b_c$ for several symmetric systems.

Figure 3 shows the numerical result, Eq. (9), relative to the integral where $b_c \to \infty$, for the same systems as in Fig. 2. We have used $\sigma_{NN} = 32$ mb in Eq. (9). A negligible difference in the most peripheral collisions can be expected if 60 mb were used instead. The growth of the fraction of the geometric cross section is slower than that of the hard fraction, $f_{AB}$. Indeed at $b_c \approx 2R_A$, $f_{\text{geo}} \approx 0.75$.

The total geometrical cross section for a variety of colliding nuclei is given in Table 2. We have also calculated the impact parameter $b_c$ corresponding to $f_{\text{geo}} = 0.05$, 0.1, and 0.2 or the central 5%, 10% and 20% of all collisions. The impact parameter corresponding to $f_{\text{geo}} = 0.2$ is $b_c \approx 1.04R_A$ when symmetric systems are considered. In asymmetric collisions, $b_c < R_A$ when $f_{\text{geo}} = 0.2$. If smaller centrality cuts are imposed, the impact parameters are reduced by factors of $\sqrt{2}$ and 2 for $f_{\text{geo}} = 0.1$ and 0.05 respectively. Note that when $B \gg A$, in the smaller nucleus is embedded in the larger with a 10% or smaller centrality cut.
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Table 2. Values of the geometric cross section and the impact parameter at which $f_{\text{geo}} = 0.05$, 0.1 and 0.2 respectively for several colliding systems.

| $A + B$ | $\sigma_{\text{geo}}$ (b) | $f_{\text{geo}} = 0.05$ (fm) | $f_{\text{geo}} = 0.1$ (fm) | $f_{\text{geo}} = 0.2$ (fm) |
|---------|-----------------|-----------------|-----------------|-----------------|
| 16+16   | 1.18            | 1.37            | 1.94            | 2.74            |
| 16+27   | 1.45            | 1.52            | 2.15            | 3.04            |
| 16+40   | 1.75            | 1.67            | 2.36            | 3.34            |
| 16+63   | 2.15            | 1.85            | 2.62            | 3.70            |
| 16+110  | 2.69            | 2.07            | 2.93            | 4.14            |
| 16+197  | 3.42            | 2.33            | 3.30            | 4.66            |
| 16+208  | 3.59            | 2.39            | 3.38            | 4.78            |
| 27+27   | 1.76            | 1.67            | 2.37            | 3.35            |
| 27+40   | 2.09            | 1.82            | 2.58            | 3.65            |
| 27+63   | 2.53            | 2.01            | 2.84            | 4.01            |
| 27+110  | 3.11            | 2.22            | 3.15            | 4.45            |
| 27+197  | 3.89            | 2.49            | 3.52            | 4.98            |
| 27+208  | 4.08            | 2.55            | 3.60            | 5.09            |
| 40+40   | 2.45            | 1.98            | 2.79            | 3.95            |
| 40+63   | 2.93            | 2.16            | 3.05            | 4.32            |
| 40+110  | 3.55            | 2.38            | 3.36            | 4.76            |
| 40+197  | 4.38            | 2.64            | 3.73            | 5.28            |
| 40+208  | 4.58            | 2.70            | 3.82            | 5.40            |
| 63+63   | 3.46            | 2.34            | 3.32            | 4.69            |
| 63+110  | 4.14            | 2.56            | 3.63            | 5.13            |
| 63+197  | 5.04            | 2.83            | 4.00            | 5.66            |
| 63+208  | 5.25            | 2.89            | 4.09            | 5.78            |
| 110+110 | 4.86            | 2.78            | 3.93            | 5.56            |
| 110+197 | 5.82            | 3.04            | 4.30            | 6.09            |
| 110+208 | 6.06            | 3.10            | 4.39            | 6.21            |
| 197+197 | 6.88            | 3.31            | 4.68            | 6.62            |
| 197+208 | 7.13            | 3.37            | 4.76            | 6.74            |
| 208+208 | 7.39            | 3.43            | 4.85            | 6.86            |
Fig. 3. The fraction of the total geometrical cross section as a function of impact parameter cut \( b_c \) for several symmetric systems. From left to right the curves are 16+16, 27+27, 63+63, 110+110, and 197+197.

For the same systems, Table 3 gives the value of the nuclear overlap at \( b = 0 \), \( T_{AB}(0) \), and the fraction of the hard cross section, \( f_{AB} \), corresponding to 5%, 10%, and 20% of the geometric cross section, calculated with the \( b_c \) values given in Table 2. For example, the central 10% of the total geometric cross section is obtained when \( b_c \sim 4.7 \) fm in Au+Au collisions, more than twice the corresponding impact parameter for the same percentage of the hard cross section. In this case, 10% of the geometric cross section encompasses \( \approx 40\% \) of the hard cross section. In fact, a 10% cut on the geometric cross section corresponds to 30-40% of the hard cross section for all systems considered. Even the central 5% of collisions encompasses 17-23% of the hard cross section while a less stringent cut of 20% garners 52-66% of all hard probes produced before nuclear effects are considered.

In Fig. 4, we show the ratio of \( f_{AB} \) relative to the geometric ratio for the same systems as in Figs. 2 and 3. The hard fraction grows more slowly relative to the geometric fraction in smaller systems, 16+16 and 27+27, but otherwise the results are similar. Figure 5 shows the relative ratios for the asymmetric systems 16+197, 27+197, 63+197, 110+197 and 197+197. In this case, the relative ratios cluster even closer together than for symmetric systems. Thus the larger nucleus sets the scale for both the hard and geometric cross sections in asymmetric systems.

The most appropriate way to obtain the number of hard probes produced in a central collision is to calculate \( b_c \) from the geometric cross section and then, with
Table 3. Values of $T_{AB}(0)$ and the fraction of the hard cross section for $f_{geo} = 0.05$, 0.1 and 0.2 respectively in several colliding systems.
Fig. 4. The fraction of the hard cross section as a function of the total geometrical cross section for the symmetric systems shown in Figs. 2 and 3. From left to right, the curves are $197+197$, $110+110$, $63+63$, $27+27$, and $16+16$.

As a specific example, the average number of $c\bar{c}$ pairs produced at $b = 0$ in Au+Au collisions is

$$\overline{N}_{AB}(0) = \sigma_{pp}^{\text{hard}} T_{AB}(0),$$

where $\sigma_{pp}^{\text{hard}}$ is the total hard process production cross section in $pp$ interactions. The rate in the impact parameter interval $0 < b < b_c$ is the ratio of the hard to geometric cross sections integrated over $b$,

$$\overline{N}_{AB}(b_c) = \frac{\int_0^{b_c} d\sigma_{AB}^{\text{hard}}}{\sigma_{\text{geo}}(b_c)} = \frac{\sigma_{pp}^{\text{hard}}}{\sigma_{\text{geo}} f_{\text{geo}}} AB f_{AB},$$

using Eqs. (8), (6), (9), and (8). In Fig. 3 the ratio

$$R_{AB}^{\text{hard}}(b_c) = \frac{\overline{N}_{AB}(b_c)}{\sigma_{pp}^{\text{hard}} AB f_{AB}} = \frac{1}{\sigma_{\text{geo}} f_{\text{geo}}} AB f_{AB}$$

is shown for the same set of symmetric systems as in Figs. 2–4 as a function of $b_c$.

As a specific example, the average number of $c\bar{c}$ pairs produced at $b = 0$ in Au+Au collisions is

$$\overline{N}_{AB}(0) = \sigma_{pp}^{\text{hard}} T_{AB}(0).$$
Fig. 5. The fraction of the hard cross section as a function of the total geometrical cross section for several asymmetric systems. From left to right, the curves are 197+197, 110+197, 63+197, 27+197, and 16+197.

At $\sqrt{s} = 200$ GeV, with MRS D′ parton distributions, $\sigma_{pp}^{\pi^0} = 0.344$ mb and $T_{AB}(0) = 29.3$ mb, resulting in $\approx 10 \pi$ pairs per Au+Au collision at $b = 0$. With a 10% centrality cut, the average number of $\pi$ pairs produced in the range $0 < b < b_c$ is

$$N_{AB}^{\pi}(b_c) = \frac{\sigma_{pp}^{\pi^0} AB f_{AB}}{\sigma_{geo}} 0.1.$$ 

(14)

Since the central 10% of the geometric cross section corresponds to 40% of the hard cross section, there are $\approx 7.7 \pi$ pairs in the 10% most central events.

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Fig. 6. The average number of hard probes produced within impact parameter $b_c$ for the symmetric systems shown in Figs. 2 and 3. From top to bottom, the curves are $197+197$, $110+110$, $63+63$, $27+27$, and $16+16$.

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