Problems of Naturalness: Some Lessons from String Theory$^*$†

MICHAEL DINE

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

Abstract

We consider some questions of naturalness which arise when one considers conventional field theories in the presence of gravitation: the problem of global symmetries, the strong CP problem, and the cosmological constant problem. Using string theory as a model, we argue that it is reasonable to postulate weakly broken global discrete symmetries. We review the arguments that gravity is likely to spoil the Peccei-Quinn solution of the strong CP problem, and update earlier analyses showing that discrete symmetries can lead to axions with suitable properties. Even if there are not suitable axions, we note that string theory is a theory in which CP is spontaneously broken and $\theta$ in principle calculable. $\theta$ thus might turn out to be small along lines suggested some time ago by Nelson and by Barr.

$^*$ Invited Talk Presented at the Cincinnati Symposium in Honor of the Retirement of Louis Witten
† Work supported in part by the U.S. Department of Energy.
1. Introduction

Rather than deal here, as other speakers will, directly with the difficult questions raised by quantum gravity, I would like to focus on some questions of naturalness which Einstein’s theory raises. The three which will concern us here are: the cosmological constant; the problem of symmetries (both continuous and discrete) and, related to the second, the strong CP problem. In considering these questions, we will use string theory as a guide. In doing this, I am not assuming that string theory necessarily describes the real world, but rather that characteristics of string theory might plausibly be shared by any ultimate theory of nature.

The cosmological constant problem arises already if we consider (semi)-classical gravity coupled to quantum fields. At one loop, for example, in field theory one has a contribution to the vacuum energy

$$E_o = \Lambda = \sum_i \pm 1/2 \int \frac{d^3k}{(2\pi^3)} \sqrt{k^2 + m_i^2}$$

Here the sum runs over all physical helicity states in the theory; the $\pm$ refers to bosons and fermions, respectively. Generically, the result is

$$E_o \sim \mu^4$$

If $\mu \sim M_P$, this corresponds to a cosmological constant more than 120 orders of magnitude larger than the observational limit. In the presence of supersymmetry, the leading divergence cancels between bosons and fermions and one might hope to find $\mu$ of order the supersymmetry breaking scale, perhaps as small as $10^2$ GeV. This is a big improvement, but not nearly good enough. Of course, in field theory the cosmological constant is not calculable, and it is not clear we are asking a physically meaningful question. However, in string theory the cosmological constant is calculable; whenever supersymmetry is broken one finds it is large. So, with regards to this problem, string theory seems to offer no miracles: we will still need to search for some deeper explanation.

‡ Aficionados of hidden sector supergravity models will object that $\mu \sim 10^{11}$ GeV is more reasonable; I am just describing the best one can hope for.
Our second topic has to do with symmetries. The notion of an exact global symmetry is always a troubling one; it is particularly so in the presence of gravity. Global quantum numbers can disappear in black holes; wormholes, if they are relevant, will generate symmetry-violating operators. Gauge symmetries, on the other hand, enjoy a different status, and are expected to survive quantum gravity effects. Both these statements apply not only to continuous symmetries, but to discrete symmetries as well. Krauss and Wilczek have stressed that gauged discrete symmetries should survive quantum gravity effects. The simplest example of such a symmetry is provided by a spontaneously broken $U(1)$ gauge symmetry. Suppose, for example, one has two scalar fields, $\phi$, with $q = 2$, and $\chi$, with $q = 1$. An expectation value for $\phi$ leaves over the symmetry $\chi \rightarrow -\chi$.

String theory lends weight to these views. It is not difficult to prove that, even at tree level, the theory possesses no unbroken, continuous global symmetries. The strategy is to show that any such symmetry implies the existence of a conserved world sheet current, which in turn implies the presence of a massless vector particle. Discrete symmetries do frequently arise in compactifications of string theory. In many cases, these can be interpreted as relics of higher dimensional gauge and general coordinate invariance, i.e. as gauge symmetries. It has been widely speculated that all discrete symmetries in string theory of this kind; we will have more to say about this later.

The third problem we have mentioned is the strong CP problem. QCD possesses an additional parameter, $\theta$, which enters the lagrangian through the term

\[
\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} \int d^4x F\tilde{F}
\]

From the limits on the neutron electric dipole moment, one knows that $\theta < 10^{-9}$. Two solutions to this problem have been widely considered. Perhaps the most popular is the “axion.” Here one postulates that the classical lagrangian possesses a global $U(1)$ symmetry, the “Peccei-Quinn” symmetry, under which there is a massless field, the axion, which transforms non-linearly

\[
a(x) \rightarrow a(x) + \delta
\]
The axion is assumed to couple to $F \tilde{F}$ as

$$L_a = \frac{Ng^2}{16\pi^2} \int d^4x (\theta + \frac{a}{f_a}) F \tilde{F} \tag{1.5}$$

$f_a$ is the axion decay constant. QCD effects can then be shown to generate a potential for the axion,

$$V(a) \approx -m^2 f_a^2 \cos\left(\frac{a}{f_a} + \theta\right) \tag{1.6}$$

The minimum of this potential clearly occurs when $\theta_{eff} = \frac{a}{f_a} + \theta = 0$.

But the whole idea of the Peccei-Quinn symmetry is quite puzzling. Not only is one postulating a global symmetry, but a symmetry which is necessarily broken explicitly! String theory offers some insight into this question. Indeed, E. Witten pointed out early on that string theory exhibits symmetries of precisely this type.[7]

This can be understood in a number of ways. For example, if one compactifies the heterotic string to four dimensions, there is a two-index antisymmetric tensor, $B_{\mu\nu}$, $\mu, \nu = 0, \ldots 3$. The corresponding gauge-invariant field strength is $H_{\mu\nu\rho} = \partial_{\mu}B_{\nu\rho} + CS$, where $CS$ denotes the Chern-Simons term. Such an antisymmetric tensor is equivalent to a scalar field;

$$\partial_{\mu}a \propto \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \tag{1.7}$$

Because in perturbation theory the low energy effective lagrangian must be written in terms of $H$, no non-derivative couplings of $a$ appear, so in perturbation theory the lagrangian is symmetric under $a \rightarrow a + \delta^x$.

Thus one has a symmetry to all orders of perturbation theory, broken by effects of order $e^{-1/g^2}$ (perhaps $e^{-1/g^{[8]}}$). This sounds like precisely what one needs to solve the strong CP problem. So perhaps it is not so unreasonable, in general, to postulate such symmetries.

* Alternatively, this statement can be understood in terms of string vertex operators. Axion emission is described by $\int d^2\sigma \epsilon_{\mu
u}(k)e^{ik\cdot x}\partial_{\alpha}x^\mu\partial_{\beta}x^\nu e^{ik\cdot x}$. At zero momentum, this becomes the integral of a total divergence.
What about the possibility that CP is spontaneously violated, with vanishing bare $\theta$? Below we will argue that in the heterotic string theory, $CP$ is indeed conserved at a fundamental level; all observable $CP$ violation is necessarily spontaneous and, in principle, calculable.

We now take up each of the issues raised here in more detail.

2. Discrete Symmetries

We have argued that gauged discrete symmetries are safe, i.e. they are unbroken by gravity. We have also remarked that such symmetries arise in field theory and are quite common in string theory. We will now show that, unlike continuous symmetries, *approximate global* discrete symmetries also arise in string theory.

To motivate our treatment of this subject, consider the problem of anomalies. We are used to the notion that continuous gauge symmetries should be free of anomalies. What about discrete symmetries? That anomalies can arise in discrete symmetries can be understood by considering instantons in an effective low energy theory. Instantons generally give rise to effective operators which break symmetries; 't Hooft showed long ago, for example, that instantons of the electroweak theory generate an effective interaction which breaks both baryon and lepton numbers. The effective interactions generated by instantons can also violate discrete symmetries. For a gauge symmetry, such a breaking signals an inconsistency, and can be viewed as an anomaly.\cite{10,11,12} One can attempt to understand discrete anomalies by embedding discrete symmetries in continuous ones.\cite{10} However, this leads to constraints which depend on the quantum numbers of massive fields. The only constraints on discrete symmetries which involve exclusively properties of light fields can be understood in terms of instantons in the effective low energy theory.\cite{13,10}

What about string theory? If we assume that all discrete symmetries in string theory are gauge symmetries, it is natural to ask whether discrete anomalies ever arise for modular-invariant compactifications. If one found compactifications with such anomalies, they would be inconsistent. Such a situation would be reminiscent of global anomalies. It could be quite dramatic, representing a new, non-perturbative consistency condition on string compactifications. A priori, I don’t know of an argument that this can not occur; indeed, I know of no general
argument that other types of global anomalies (e.g. SU(2) anomalies) do not occur.

In fact, study of various compactifications quickly yields numerous examples of anomalies\(^{[14]}\). However, in all the cases which have been studied to date, one can cancel these anomalies in the following way.\(^*\) The axion, \(a\), couples to all of the low energy gauge groups:

\[
\frac{g^2}{16\pi^2 f_a} \sum a(x) F^{(i)} \tilde{F}^{(i)}
\]  

(2.1)

It turns out that one can always cancel all of the anomalies by assigning to the axion a non-linear transformation law of the form

\[
\frac{a}{f_a} \to \frac{a}{f_a} + \beta
\]  

(2.2)

for some number \(\beta\). To understand what is going on here, note that the instanton effective action is typically something of the form

\[
\psi \psi \ldots \psi e^{ia/f_a e^{-8\pi^2/g^2}}
\]  

(2.3)

So the phase rotation of the fermions is compensated by the shift in the axion field. This result is highly non-trivial (typically several anomalies are being taken care of by one such shift). It almost surely indicates that the symmetries are not, in fact, anomalous. So far only rather special classes of models have been examined, so that while I suspect that this is a general result, it is by no means certain. In any case, there is still no evidence for the existence of any new, (independent) consistency condition beyond those which hold in perturbation theory.

However, from these studies we learn something surprising: string theory possesses \textit{global} discrete symmetries which are valid to any order of perturbation theory and broken only non-perturbatively. For while the non-anomalous symmetry in all of these cases is \textit{spontaneously} broken by the non-linear transformation law of the axion, the original, anomalous symmetry is good to all orders, being broken only

\(^*\) The possibility that anomalies in discrete symmetries might be cancelled by a Green-Schwarz mechanism was noted in ref. 10.
non-perturbatively. If we adopt the view that phenomena which occur in string theory can plausibly occur in any ultimate theory, this means that it is reasonable to postulate approximate global symmetries in a low energy theory. Such symmetries have been suggested for many reasons, such as avoiding flavor changing neutral currents in multi-Higgs theories and proton decay in supersymmetric theories, for understanding the fermion mass matrix, and (see below) for understanding the strong CP problem.

3. Strong CP

3.1. Is CP Spontaneously Broken in String Theory?

In perturbation theory, CP is conserved in string theory.$^{[15]}$ One might ask whether this is true non-perturbatively. After all, in field theory, $\theta$ is a non-perturbative parameter which violates CP. It has been suggested that string theory might possess similar non-perturbative parameters.$^{[16]}$ If some of these are CP-violating, they might give rise to $\theta$ parameters in the low energy theory. However, it turns out that one can argue that CP is a gauge symmetry in string theory.$^{[17]}$ This means that there can be no such CP-violating parameters, since these would correspond to an explicit breaking of the symmetry.

As a result, if string theory describes nature, CP must be spontaneously broken and $\theta_{QCD}$ is calculable. This breaking might arise at $M_P$ (e.g. through expectation values for CP-odd moduli) or at lower scales (e.g. through vev’s for some matter fields). In either case, one expects that generically $\theta$ will be large, proportional to other CP-violating phases needed to explain the features of the $K$-meson system. However, in field theory, it is known that one can sometimes arrange things so that $\theta$ is small.$^{[18]}$ Preliminary investigation (to be described in ref. 19) indicates that certain “string inspired models” can accomplish this. In particular, in a class of models, discrete symmetries insure that CP is spontaneously violated at an “intermediate scale”, $M_{INT}$, of order $10^{11}$ GeV, with $\theta$ of order $\frac{M_{INT}}{M_P}$ (times coupling constants). Moreover, in these models, the low energy theory is supersymmetric, but the only CP violation lies in the KM phase and $\theta$.

$^{\dagger}$ When I presented this talk in Cincinnati, I was not sure of this statement, and only mentioned it as a possibility. I offered in addition some alternative arguments for absence of $\theta$ parameters.
3.2. ACCIDENTAL AXIONS IN STRING THEORY

Alternatively, one can explore axion solutions to the strong CP problem in string theory. There are, however, two potential problems with the stringy axion, $a$, which we have described above. First, in many compactifications of string theory, there is more than one strongly interacting gauge group; it is necessary to have at least one axion for each group. Second, even if $QCD$ is the only strong group, the decay constant, $f_a$, is a number of order $M_P$. This contradicts cosmological bounds, which give $f_a < 10^{12}$ GeV. However, one might choose to ignore these limits; there are a number of possible loopholes. For example, these analyses assume that there is no entropy generation after the QCD phase transition. However, plausible models exist in which there is such entropy generation, and yet an adequate baryon density is generated. These arguments also assume that the initial value of the axion field in the observable universe is simply a random number; in that case, for such a large $f_a$, only one universe in $10^3$ has a sufficiently small initial $\theta$. But Linde has pointed out that the size of the initial $\theta$ may be correlated with primordial density fluctuations. Only those regions with small enough $\theta$, in this view, might resemble ours. Thus a rather mild application of the anthropic principle (the “weak anthropic principle”) might solve the problem. You may not wish to take any of these possibilities too seriously; however, one should be aware that the cosmological axion limit rests on certain assumptions which may not be true.

For now, though, let us take the cosmological limit seriously, and ask how $f_a \sim 10^{11}$ might arise. We could, of course, simply postulate that there is another fundamental scale, and the axion arises in a manner similar to the string axion. Such an assumption is certainly troubling, however, and there is no reason to think such a scale should arise in string theory. Alternatively, the Peccei-Quinn symmetry might arise by accident, in the same way that baryon and lepton number arise in the standard model. Such an accident, however, would be quite startling if one simply assumes that gravity generates all operators consistent with the various local symmetries of the theory. The problem is that in order that the axion tune $\theta$ to the required precision, it is necessary that the leading operators which violate the symmetry be of very high dimension. This point was already raised in passing by Georgi, Glashow and Wise More recently, it has been discussed in a general and quantitative fashion by several authors. To gain some appreciation of the diffi-
culty, suppose that the lowest dimension, gauge-invariant operator which violates
the symmetry is $\mathcal{O}^{(4+n)}$, of dimension $4+n$. Then the leading symmetry-violating
term which can occur in a low-energy effective field theory is

$$\mathcal{L}_{SB} = \frac{\gamma}{M_P^n} \mathcal{O}^{(4+n)}$$

where \(\gamma\) is a dimensionless coupling constant. On dimensional grounds, this gives
rise to a linear term in the axion potential,

$$V_{SB} \propto \frac{f a^{n+3}}{M_P^n} a(x)$$

Since

$$m_a^2 \sim \frac{m_{\pi^2}^2 f_\pi^2}{f_a^2}$$

the resulting shift in $\theta$ is

$$\delta \theta = \frac{\delta a}{a} \sim \frac{\gamma}{m_{\pi^2} f_\pi^2 M_P^n} \frac{f a^{n+4}}{f_a} < 10^{-9}$$

For $f_a = 10^{\ 11}$, this gives $n > 6$ (i.e. the symmetry-violating operator must at least
be of dimension 12!) If $f_a = 10^{10}$, things are slightly better; one needs to suppress
all operators of dimension less than 9.

The lesson of all this is that if one wants a Peccei-Quinn symmetry to arise
by accident, one must forbid operators up to very high dimensions. How might
such a thing occur? The authors of refs. 25 noted that with a sufficiently com-
licated continuous gauge symmetry, one could indeed suppress operators of very
high dimension. However, by their own admission, the resulting models were not
particularly beautiful.

In light of our earlier discussion, it is natural to ask how easily discrete sym-
metries can accomplish the same objective. In fact, in the framework of string
theory, this question was asked some time ago by Lazarides et al[26] and by Ross
and Casas.[27] The latter authors also attempted to estimate how large a $\theta$ would

be induced by higher-dimension operators which violated the Peccei-Quinn symmetry, in precisely the spirit described above (we will see, however, that they failed to consider the most dangerous class of operators). Before reviewing these models, however, it is perhaps useful to illustrate just how powerful discrete symmetries are in this respect by considering theories in which the Peccei-Quinn symmetry is dynamically broken by fermion condensates. As an example, consider a theory with (unbroken) gauge group (in addition to the standard model gauge group) $SU(4)_{AC}$ ($AC$ is for “axi-color”), with scale $\Lambda_{AC} \sim f_a$. In addition to the usual quarks and leptons, we suppose that the theory contains additional fields $Q$ and $\bar{Q}$, transforming as $(4, 3)$ and $(\bar{4}, \bar{3})$ under $SU(4)_{AC} \times SU(3)_c$, and fields $\tilde{Q}$ and $\tilde{Q}$ transforming as a $(4, 1)$ and a $(\bar{4}, 1)$. Now suppose that the model possesses a discrete symmetry (gauged or global) under which

$$Q \rightarrow \alpha Q \quad \bar{Q} \rightarrow \alpha \bar{Q}$$

where $\alpha = e^{2\pi i N}$; all other fields are neutral. If, for example, $N = 3$, the lowest dimension chirality-violating operators one can write are of the form $(\bar{Q}Q)^3$, which is dimension 9; suppression of still higher dimension operators is achieved by choosing larger $N$. In this theory, the would-be PQ symmetry is

$$Q \rightarrow e^{i\omega Q} \quad \bar{Q} \rightarrow e^{-3i\omega \bar{Q}}$$

(3.4)

This symmetry has no $SU(4)$ anomaly, but it does have a QCD anomaly. One expects that this symmetry will be broken by the condensates

$$\langle \bar{Q}Q \rangle \sim f_a^3$$

(3.5)

This gives rise to an axion with decay constant $f_a$, which solves the strong CP problem.

Let us turn now to the ideas of Lazarides et al and of Casas and Ross. In particular, we will develop a variant of the model of the latter authors. Of course, ⋆ This has been noted independently, and much earlier, by A. Nelson (unpublished).
it is not presently clear how string theory might describe the real world, so we will view this model as “string inspired,” in that it shares features common to a class of compactifications. We will have to assume, also, some structure of soft supersymmetry breaking. Having said that, it should be stressed that models of this kind have a major virtue: the axion decay constant is naturally of order \( M_{INT} = \sqrt{M_W M_P} \), i.e. within the allowed axion window. Our only truly new point, beyond those made in ref. 27, will be that there are operators beyond those considered by these authors which one must eliminate if one is to insure sufficiently small \( \theta \).

Consider a theory with unification in the gauge group \( E_6 \), with \( E_6 \) broken to a rank 6 group at the unification scale; this is the structure which emerges from conventional Calabi-Yau compactification.\(^{[28]}\) Ordinary matter fields will be assumed to arise from 27’s of \( E_6 \). Casas and Ross assume that the theory possesses a \( Z_3 \times Z_2 \) symmetry. The 27 contains two standard model singlets, which we denote by \( S \) and \( N \). These authors suppose that there are two fields with the quantum numbers of \( S, S_i \), and two fields with the quantum numbers of \( \bar{S}, \bar{S}_i \). Under \( Z_3 \times Z_2 \), these fields transform as follows:

\[
S_1 \to -\alpha S_1 \quad \bar{S}_1 \to -\alpha \bar{S}_1 \quad \alpha = e^{2\pi i/3} \tag{3.6}
\]

while \( S_2 \) and \( \bar{S}_2 \) are invariant. The leading terms allowed in the superpotential are

\[
W = \frac{a}{M_p} S_2^2 \bar{S}_2^2 + \frac{b}{M_p^3} S_1^3 \bar{S}_1^3 + \frac{c}{M_p^9} S_1^6 \bar{S}_2^6 + \frac{d}{M_p^9} \bar{S}_1^6 S_2^6 \tag{3.7}
\]

This superpotential has an approximate \( U(1) \) symmetry, broken by the final two terms:

\[
S_2 \to e^{ib} S_2 \quad \bar{S}_2 \to e^{-ib} \bar{S}_2 \tag{3.8}
\]

This symmetry can play the role of a Peccei-Quinn symmetry. If \( S_2 \) and \( \bar{S}_2 \) have soft-breaking mass terms of the correct sign, these fields will acquire expectation values of order \( M_{INT} \), breaking the symmetry spontaneously (in this model, \( S_1 \) and \( \bar{S}_1 \) obtain larger expectation values).
The authors of ref. 27 estimated the $\theta$ which would arise in this model by considering the explicit breaking terms in the superpotential, above, as well as soft-breaking terms of the type $Am_{3/2}W$. It is easy to see that these lead to quite a small $\theta$. However, a generic supergravity model also leads to soft supersymmetry-breaking terms for scalar fields, $\phi$, of the type $m_{3/2}^2 \phi^* \phi^m$. In the present case, this allows the operator:

$$\frac{m_{3/2}^2}{M_p^2} S_1^* \bar{S}_1 S_2 \bar{S}_2^*$$  \hspace{1cm} (3.9)

This breaks the Peccei-Quinn symmetry. It is a huge term on the scale of axion physics; it gives, for example, a contribution to the axion mass of order $MeV$'s!

Clearly we can improve the situation if we consider a different symmetry. For example,

$$S_1 \rightarrow -\alpha S_1 \quad \bar{S}_1 \rightarrow \alpha^2 \bar{S}_1 \quad S_2 \rightarrow -S_2 \quad \bar{S}_2 \rightarrow \bar{S}_2$$  \hspace{1cm} (3.10)

again gives a lagrangian which admits a Peccei-Quinn symmetry. Now, however, the leading symmetry-violating operators are things like $\bar{S}_1^* S_1^2 S_2^2 \bar{S}_2^*$. This leads to a $\theta$ which is perhaps barely small enough.

We will not attempt, here, to consider all aspects of the phenomenology of these models; suffice it to say that it does appear to be possible to build realistic models along these lines. One can debate how reasonable – or contrived – this solution appears to be. As we will explain elsewhere, it is probably not much better or worse than is required to obtain a Nelson-Barr type solution of the problem.\[29\]

The main difference is that in the Nelson-Barr case, it is not necessary to suppress operators of such high dimension. However, as illustrated by the examples above, rather simple discrete symmetries can accomplish this.
4. Some Wild Speculations on Strong CP and the Cosmological Constant

I would like to conclude by describing some wilder ideas about the strong CP problem and the cosmological constant. These are associated with what T. Banks, N. Seiberg and I have dubbed “irrational axions.” Such axions do not arise in conventional theories. It is tempting to think that they might arise in string theory, but we have not found an example of this phenomenon. The basic ideas are very simple. Consider first the strong CP problem. Suppose that in addition to QCD, one has an additional strongly interacting gauge group; we will refer to this as axicolor, $QAD$. In addition, suppose one has a single Peccei-Quinn symmetry, and $a$ is the associated axion. The couplings of the axion to the two gauge groups are written

$$
\frac{1}{f_1} \frac{g_3^2}{16\pi^2} \int d^4x a F \tilde{F} + \frac{1}{f_2} \frac{g_2^2}{16\pi^2} \int d^4x a G \tilde{G} \tag{4.1}
$$

where $G$ refers to the axicolor gauge fields, and we assume $\Lambda_{QAD} >> \Lambda_{QCD}$. Then the axion acquires its mass principally from $ACD$ dynamics; one expects

$$
m_a^2 \sim \frac{\Lambda_{QAD}^4}{f_2^2} \tag{4.2}
$$

Ordinarily, such an axion would have nothing to do with the solution of the strong $CP$ problem. But suppose $\frac{f_1}{f_2}$ is irrational. The axion potential in this theory is something of the form

$$
V = \Lambda_{ACD}^4 \cos(a/f_1) + \Lambda_{QCD}^4 \cos(a/f_2 + \theta) \tag{4.3}
$$

Now since $f_1/f_2$ is irrational, one can always find integers $n_1$ and $n_2$ such that

$$
a \approx 2\pi n_2 f_2 \approx (2\pi n_1 - \theta) f_1 \tag{4.4}
$$

with arbitrary accuracy. Thus in this theory there exist ground states with arbitrarily small $\theta_{QCD}$. These states are stable on cosmological scales, but they are also rare; about 1 in $10^9$ local minima of the potential has such small $\theta$. Thus, in this model the $\theta$ problem is solved without a light axion; the price one pays is cosmological: why do we find ourselves in a suitable state?
This problem can (almost) be solved if we invoke the anthropic principle, in the “weak sense” discussed by Weinberg. Suppose that the cosmological constant of the theory is adjusted so that it vanishes as $\theta \to 0$. If this is the case, formation of galaxies requires that the cosmological constant not be larger than about 1000 times the present experimental limit. This gives $\theta$ far smaller than $10^{-9}$. We still have a factor of 1000 in cosmological constant to explain. Actually, it is not quite as bad as that; since the energy goes as $\theta^2$, we are really only out by a factor of 30.

We have already seen that there are many ways to solve the strong CP problem, so one more, which we don’t (yet) know how to get from an underlying microscopic theory might not seem that exciting. However, there is another fine tuning problem which we might like to solve without a light particle: the cosmological constant problem. This may also be possible with irrational axions. The idea we will describe here bears some resemblance to a suggestion of Abbott.

Suppose, in addition to QCD, the underlying theory has two strong gauge groups, with scales $\Lambda_1 \approx \Lambda_2$. Suppose the “irrational axion” couples to these, in the same irrational way as before. Suppose also that the bare cosmological constant, $\Lambda_4^0$, satisfies $\Lambda_4^0 < \Lambda_1^4$. With supersymmetry, one can show that these conditions can be natural. Now the axion potential is a sum of three terms:

$$V(a) = \Lambda_4^0 + \Lambda_4^1 \cos(a/f_1) + \Lambda_4^2 \cos(a/f_2 + \theta_2)$$

In this potential there exist vacua with arbitrarily small values of the cosmological constant. They are even more rare than in our previous example; for example, if $\Lambda_1 \sim 10^{10}$, then only one in about $10^{88}$ vacua are acceptable. Actually, the situation is even worse because in this case a typical local minimum with small cosmological constant is not even approximately stable. Only a small fraction are: for example, if several adjacent minima all have higher energy, the tunneling rate will be suppressed. Again, we can suppose that in an inflationary universe, some worlds like our own were created, and try and invoke the anthropic principle. Again, we are off by a factor of 1000.

However, I don’t believe this problem is so severe; for example, somewhat stronger versions of the anthropic principle might save the day. If we had examples of such irrational axions, they might well solve the cosmological constant problem.
Acknowledgements I wish to thank my collaborators T. Banks, R. Leigh, D. MacIntire and N. Seiberg for their insights, and Ann Nelson for several helpful discussions.

REFERENCES

1. R. Rohm, Nucl. Phys. B237 (1984) 553.
2. L. Krauss and F. Wilczek, Phys. Rev. Lett. 62, 1221 (1989).
3. T. Banks, L. Dixon, D. Friedan and E. Martinec, Nucl. Phys. B299 (1988) 613.
4. E. Witten, Nucl. Phys. B258 (1985) 75.
5. R.J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. 88B (1979) 123.
6. R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791; S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.
7. E. Witten, Phys. Lett. B149 (1984) 359.
8. S. Shenker, Rutgers preprint RU-90-47 (1990).
9. G. ’t Hooft, Phys. Rev. Lett. 37 (1976), Phys. Rev. D14 (1976) 3432.
10. L. Ibanez and G. Ross, Phys. Lett. 260B (1991) 291; Nucl. Phys. B368 (1992) 3.
11. J. Preskill, Sandip Trivedi, F. Wilczek and M. Wise, Nucl. Phys. B363 (1991) 207.
12. E. Witten, private communication c. 1983.
13. T. Banks and M. Dine, Phys. Rev. D45 (1992) 424.
14. Several examples of the phenomena which we now describe were presented in ref. 13. 1000’s more have been studied by D. MacIntire (SCIPP preprint in preparation).
15. A. Strominger and E. Witten, Comm. Math. Phys. 101 (341) 1985.
16. M. Douglas and S. Shenker, Nucl. Phys. B355 (1990) 635.
17. K. Choi, D. Kaplan and A. Nelson, UCSD preprint PTH 92-11 (1992); M. Dine, R. Leigh and D. MacIntire, SCIPP preprint SCIPP 92/16 (1992).
18. A. Nelson, Phys. Lett. 136B (1984) 387; S.M. Barr, Phys. Rev. Lett. 53 (1984) 329; Phys. Rev. D30 (1984) 1805; P.H. Frampton and T.W. Kephart,
Phys. Rev. Lett. **65** (1990) 1549.

19. M. Dine, SCIPP preprint to appear.

20. J. Preskill, M. Wise and F. Wilczek, Phys. Lett. **120B** (1983) 127; L. Abbott and P. Sikivie, Phys. Lett. **120B** (1983) 133; M. Dine and W. Fischler, Phys. Lett. **120B** (1983) 137.

21. J. Cline and S. Raby, Phys. Rev. **D43** (1991) 1381.

22. A.D. Linde, Phys. Lett. **259B** (1991) 38.

23. S. Weinberg, Phys. Rev. Lett. **59** (1987) 2607; Rev. Mod. Phys. **61** (1989) 1.

24. H. Georgi, S. Glashow and M. Wise, Phys. Rev. Lett. **47** (1981) 402.

25. M. Kamionkowski and J. March-Russell, Phys. Lett. **282B** (1992) 137; R. Holman et al., Phys. Lett. **282B** (1992) 132; S.M. Barr and D. Seckel, Bartol preprint BA-92-11.

26. G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. **56** (1986) 432.

27. J. Casas and G. Ross, Phys. Lett. **192B** (1987) 119.

28. M. Green, J. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press, New York, 1986.

29. M. Dine and R.G. Leigh, SCIPP preprint in preparation.

30. T. Banks, M. Dine, and N. Seiberg, Phys. Lett. **273B** (1991) 105.

31. L. Abbott, Phys. Lett. **150B** (1985) 427.