Drag dynamics in one-dimensional Fermi systems

Jun’ichi Ozaki, Masaki Tezuka, and Norio Kawakami
Department of Physics, Kyoto University, Kyoto, Japan 606-8502
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We study drag dynamics of several fermions in a fermion cloud in one-dimensional continuous systems, with particular emphasis on the non-trivial quantum many-body effects in systems whose parameters change gradually in real time. We adopt the Fermi–Hubbard model and the time-dependent density matrix renormalization group method to calculate the drag force on a trapped fermion cluster in a cloud of another fermions species with contact interaction. A non-trivial peak in the resistance force is observed in the large cloud density region, and it is suggested that some of the collective modes have a crucial role in the excitation process. We propose a simplified model which explains the detail of the excitation process and also the origin of the resistance peak. This model emphasizes the difference between the full-quantum calculation and the semiclassical calculation, which is the quantum effects, in slow dynamics of many-body systems bound in a fermion cloud.

I. INTRODUCTION

Recently non-equilibrium dynamics of cold atom systems has been enthusiastically targeted, because cold atom systems are ideal as isolated quantum systems configured in laboratory, whose parameters can be modified dynamically [1]. In cold atom systems, strength and sign of interaction between atoms can be adapted by use of Feshbach resonance [2], and also the lattice potentials can be composed using optical lattices. The dynamics of quantum quench [3, 4] has been explored by suddenly changing the trap potential and the interaction, and also theoretically this dynamics has been studied [5, 26].

However, dynamics induced by a gradual change of parameters in real time has a lot more to be investigated. These dynamics are completely different from the quantum quench dynamics, because the constant change of the system parameters in time continuously causes the energy excitation and dissipation in the systems. In other words, these systems are not expected to relax to an equilibrium, but may reach a steady state in the sense of thermodynamics. Of these dynamics, especially we focus on drag dynamics of a fermion cluster trapped by a moving trap in a fermion cloud, interacting with cluster particles by contact interaction. Since the drag dynamics is one of the basic concepts of dynamics, a detailed study of drag dynamics is essential to the understanding of the non-equilibrium dynamics in quantum systems.

Thus in this study we simulate the drag dynamics in one-dimensional two-component Fermi systems with contact interaction. In this system, a cluster of fermions is forced to move by a species-dependent trap at a constant speed, interacting with a cloud of the other type of fermions by contact interaction. We calculate the excitation energy of the whole system and evaluate the energy increase per unit time by linear fit. Then we propose a simplified model of energy excitation process, which is based on a semiclassical viewpoint. This model explains intuitively the characteristic peak structure in the profile of the energy increase per unit time.

II. SIMULATION

A. System setup

We simulate drag dynamics, and calculate excitation energy of the whole system in one-dimensional two-component Fermi systems. Initially a cluster of \( n \) fermions is trapped by a harmonic potential within a cloud of the other type of free fermions (Fig. I(a)), where the average fermion density of the cloud is \( D \) (the Fermi momentum is \( \pi \hbar D \)). The mass of a cloud fermion is \( m_{\text{cloud}} = m_0 \), and the mass of a cluster fermion is \( m_{\text{cluster}} = m_0 r \); the mass ratio is \( m_{\text{cluster}}/m_{\text{cloud}} = r \). The harmonic trap potential is set as \( \frac{1}{2} m_0 \omega_0^2 x^2 \), where \( X \) is the location from the trap center. Therefore in the \( r = 1 \) case, the frequency of the harmonic trap is \( \omega_0 \) (and the oscillation cycle is \( T_0 = 2\pi/\omega_0 \)), and the typical width of the particle density distribution of a cluster fermion is \( \eta = \sqrt{\hbar/m_0 \omega_0} \). The interaction exists only between two fermions of the different types, and it is contact interaction which is expressed as \( u \delta(x_1 - x_2) \), in which \( x_1 \) and \( x_2 \) are the location of the two interacting fermions.

The trap moves within the region \( -5\eta < x < 5\eta \), and the whole system size is \( 40\eta \gg 5\eta \), so that the effect of the finite system size (e.g., the fluctuation of the cloud density) is negligible. The initial trap center is \( x = -5\eta \) and finally moves to \( x = 5\eta \). In this paper, we use \( \eta \), \( T_0 \) and \( \hbar \) as the units of the system, for example the unit of energy is \( \hbar T_0^{-1} \) and the unit of power is \( \hbar T_0^{-2} \). Therefore the independent variables in the system are \( n, D, r, u \) and \( v \), where \( v \) is a trap speed as mentioned below. We perform calculations for conditions of \( n \leq 0 \) and \( D \leq 1.5\eta^{-1} \) to obtain numerically exact results.

At \( t = 0 \) we suddenly move the trap potential by a constant speed \( v \), and simultaneously we give a speed \( v \) to the fermion cluster. Then the cluster pushes the cloud particles as shown in Fig.II(b), while it is forced to move by the moving trap potential. The moving trap increases the total energy of the system \( E(t) \), where we set \( E(0) = 0 \), as a linear function of the time approximately. Finally the trap reaches \( x = 5\eta \) in \( t = 10\eta/v \), and then we finish...
The discretized Hamiltonian is
\[ \hat{H}(t) = -\sum_{i,\sigma=\text{cluster,cloud}} \frac{\hbar^2}{2m_0} \frac{\partial}{\partial x^2} (\hat{a}_{i+1,\sigma} \hat{a}_{i,\sigma} + \hat{a}_{i,\sigma}^\dagger \hat{a}_{i+1,\sigma}^\dagger) + \sum_i V_i^{\text{cluster}}(t) \hat{n}_{i,\text{cluster}} + \frac{u}{\delta x} \sum_i \hat{n}_{i,\text{cluster}} \hat{n}_{i,\text{cloud}}, \]
where \( \hat{a}_{i,\text{cluster}} (\hat{a}_{i,\text{cloud}}) \) annihilates a fermion of the cluster (cloud) on site \( i \), and \( \hat{n}_{i,\sigma} = \hat{a}_{i,\sigma}^\dagger \hat{a}_{i,\sigma} \). We prepare the ground state of the system at \( t = 0 \), and then give the constant speed \( v \) to the cluster particles; starting from this state, we calculate the time evolution by that Hamiltonian with t-DMRG up to \( t = t_p = 10\eta/v \). The time step is \( 2 \times 10^{-4}T_0 \) and the maximum discarded eigenvalue of the reduced density matrix is below \( \varepsilon = 10^{-10} \). The simulation is conducted in the following range of parameters: the fermion number of the cluster \( n \leq 6 \), the fermion density of the cloud \( D \leq 1.5\eta^{-1} \), the trap speed \( v \sim 4\eta/T_0 \), the mass ratio \( 1 \leq r \leq 2 \), and the contact interaction strength \( u = 5\hbar T_0^{-1} \eta \) and \( 10\hbar T_0^{-1} \eta \) (this \( u \) is comparably large so that the reflection is dominant in typical collision cases).

III. RESULTS

Figure 2(a) shows the time dependence of the excitation energy obtained by the DMRG simulation for various values of the cloud density \( D \). The figure implies the linear increase in the system energy \( E \) with a damped oscillation. This fact suggests that the trapped fermion cluster goes to the steady state, and that the energy should increase linearly without the effect of the initial conditions. Therefore we evaluate \( P \), the energy increase per unit time, or the excitation power of the trap motion, by a linear fit of this plot neglecting the damped oscillation, so that we extract the asymptotic behavior in the longer period of time. \( P \) is expected to contain the information of the state going to the steady state, or the steady state itself. The resistance force \( F \) against the trap is calculated by the relation \( Fv = P \); \( P \) is proportional to \( F \) when the trap speed \( v \) is fixed. We focus on and discuss \( P \) in the following.

![FIG. 1. (Color online) (a) Initial particle density and trap potential at \( n = 6, D = 0.5\eta^{-1}, r = 1, u = 10\hbar T_0^{-1} \eta \). (b) Final particle density and trap potential at \( n = 6, D = 0.5\eta^{-1}, r = 1, u = 10\hbar T_0^{-1} \eta, v = 4\eta/T_0 \).](Image)

![FIG. 2. (Color online) (a) Time dependence of the excitation energy at \( n = 6, r = 1, u = 10\hbar T_0^{-1} \eta, v = 4\eta/T_0 \) for the 4 cases: \( D = 0.4\eta^{-1}, D = 0.6\eta^{-1}, D = 0.8\eta^{-1}, \) and \( D = 1.0\eta^{-1} \). (b) Trap speed dependence of energy increase per unit time at \( n = 1, D = 0.5\eta^{-1}, r = 1, u = 10\hbar T_0^{-1} \eta \) (log scale).](Image)
demonstrates the relation \( P \propto v^3 \), so that the reaction force to the trap is proportional to \( v^2 \); this means the existence of the inertial resistance. The inertial resistance is physically expected, because a cluster particle gives its momentum to a cloud particle in a single collision, whose rate is proportional to \( v \).

Next we investigate how \( P \) depends on the cloud density and the number of the cluster particles. Figures 3(a) and (b) show the energy increase per unit time \( P \) for \( 0.3\eta^{-1} \leq D \leq 1.5\eta^{-1} \) in the cases of \( u = 5hT_0^{-1}\eta \) and \( u = 10hT_0^{-1}\eta \), respectively. In the figures, \( D \) goes to zero exponentially in large \( D \); no resistivity is observed in this region. Also, the figures exhibit a single-peak structure for the cases of \( n = 1 \) or \( n = 2 \), and a double-peak structure for the cases of \( n \geq 3 \). In the large \( D \) region, the decay curves of \( 3 \leq n \leq 6 \) have very similar shapes at regular intervals, and the distance between the neighbors is about \( 0.10\eta^{-1} \) (i.e., the values of \( D\eta \) when \( P = 0.2hT_0^{-2} \) after the second peaks in Fig 3(a) are 0.93 at \( n = 3 \), 1.05 at \( n = 4 \), 1.15 at \( n = 5 \) and 1.24 at \( n = 6 \)). This suggests that the second peaks are at regular intervals of about \( 0.1\eta^{-1} \). These characteristics does not strongly depend on interaction strength \( u \). The important result from the plots is \( P < nP_{n=1} \) ( \( P > nP_{n=1} \) ) holds for small (large) \( D \), where \( P_{n=1} \) is \( P \) at \( n = 1 \). In other words, if one is to reduce the excitation in moving \( n \) fermions in a fermion cloud, one should trap the fermions in a single trap in the small \( D \) case, but in \( n \) traps independently in the large \( D \) case. In the following we mainly focus on the peak structure, especially the peak location, which is discussed later using our model.

Let us now explore the dependence of \( P \) on the mass ratio \( r \). Figures 4(a) and (b) show \( P \) in the same parameters as in Fig 3 except for \( r = 2 \). Except that the height of the second peak differs between different values of \( n \), these plots have the same features as in Fig 3. To extract the data close to the second peak as a quadratic curve, and obtain the average distance \( 0.1\eta \) (i.e., the estimated values of \( D\eta \) at the peak in Fig 4(a) are 0.86 at \( n = 3 \), 0.98 at \( n = 4 \), 1.09 at \( n = 5 \) and 1.20 at \( n = 6 \)); the distance between the neighboring second peaks has similar values despite the doubled mass ratio.

Here we focus on the energy of the cluster, which is the expectation value of the cluster Hamiltonian

\[
\hat{H}_\text{cluster} = - \sum_{i,\sigma=\text{cluster}} \frac{\hbar^2}{2m_\sigma} \delta x^2 (\hat{a}^\dagger_{i,\sigma} \hat{a}_{i+1,\sigma} + \hat{a}^\dagger_{i+1,\sigma} \hat{a}_{i,\sigma}) + \sum_i V_i(t) \hat{n}_i \text{cluster}.
\]

We plot the cluster-energy increase per unit time in Fig 4. In the figure, a peak (peaks) is observed in the case of \( n = 1, 2 \) (\( n > 2 \)). In the peak region, the energy of the
cluster increases over time. Therefore the system does not completely reach a steady state in the finite simulation time, since the cluster energy should not change in a steady state. The values of $D$ giving the peak-structure are a little larger than those in Fig. 3(b), so the excessive energy flux into the cluster corresponds to the decreasing $P$. When the cluster is overmuch excited, the cluster-energy release into the cloud becomes large, therefore the excitation energy of the whole system becomes small. Our model, which is described below, agrees with these results, indicating that the excessive energy release into the cloud decreases the excitation of the whole system.

Before we discuss our model, we compare naïve mean-field results with the DMRG results, in order to investigate the many-body effects in the system. Under a mean-field approximation, the cloud particles move in the potential created by the average interaction with the cluster, which is calculated by the density distribution of the cluster particles of the ground state (e.g., a gaussian function at $n = 1$). The potential moves by velocity $v$, pushes the cloud, and excites the system. Comparing the mean-field results with the DMRG results, we find that they agree in the case of $n = 1$, but the mean-field results have no peak in the large $D$ region. Thus, the many-body effects such as collective excitation have a crucial role in forming the peak structure in the large $D$ region; the model for this system has to contain its many-body effects.

Since the simple mean-field result does not agree with the DMRG result, we propose a simplified model to explain the DMRG simulation results, especially the appearance of the peak structure in Fig. 3 and Fig. 4. We start from the simplest case of $n = 1$, the one-body dynamics under the harmonic trap interacting with the fermion cloud. Now the classical picture is applied, where we set the trap center at $x_C(t)$. Initially the cluster particle has a speed $v_1$ at the location $x_1$ at a time $t_1$, then it moves around in the trap and interact with cloud fermions. To simplify the situation, it is fixed to the approximate mean value, $x_1 = x_C(t_1)$.

On the coordinates fixed to the trap, contributions from cloud particles with small velocity to the excitation process are canceled out, because there are the same number of cloud particles with the opposite velocity. Contributions from cloud particles near the Fermi surface, however, are not canceled, because there is no cloud particle moving in positive direction due to the moving coordinates. Therefore only the particles which have the momentum $\sim -\pi \hbar D$ have a role in the excitation process. Thus we presume that, in the excitation process, the cluster particle is accelerated by the cloud particles near the Fermi surface. Therefore the relation $v_1 = f(r)\pi \hbar D$ (in negative direction) is obtained, where $f(r)$ contains the momentum-transfer ratio by collisions and it is a decreasing function, because heavier cluster particles are given less velocity by the collision with the cloud particles. Since the velocity dependence of the resistance is $F \propto v^2$, we suppose that the resistance for a particle moving in the fermion cloud is proportional to square of its velocity, so the approximate classical equation of motion is

$$m_{\text{cluster}}\ddot{x} = -m_0\omega_0^2 x - \gamma |\dot{x}|,$$

where $\gamma$ is a function of $r$, $u$ and $D$. These are the basis of our model, and we use the model to explain the DMRG results in the following part.

Here we consider the $D$ dependence in the $n = 1$ case and explain this case by the model. We set the first peak at $D_{\text{peak}1}$. We start from $D = 0$, then (a) increase $D$ to $D_{\text{peak}1}$, and finally (b) increase $D$ over $D_{\text{peak}1}$ to the decay region. In the region (a), the larger $D$ is, the more frequently cloud particles collide with the cluster, therefore the work done by the trap potential increases. On the other hand, in the region (b), when $D$ is over $D_{\text{peak}1}$, the resistance from the cloud is much larger because both of $\gamma$ and $|\dot{x}|$ become large as $D$ increases. Then the particle releases its energy to the cloud so rapidly that it less interacts with the trap potential; the trap does not excite the system enough. That picture is supported by the result in Fig. 3 and it also explains the origin of the first peaks in Fig. 3 and Fig. 4. Thus our picture succeeds in explaining the $D$ dependence in the $n = 1$ case.

On the other hand, if our model is naïvely applied to the $n > 2$ case, it fails in the large $D$ region; the second peaks at $D_{\text{peak}2}$ arise in Fig. 3, Fig. 4 and Fig. 5. Nevertheless by introducing the quantum many-body effects properly into the initial speed $v_1$, our model can explain the emergence of these peaks. The underlying mechanism is as follows: the interaction within the cluster mediated by cloud particles distributes the given momentum to the whole cluster, or there are some collective excitation modes. Thus the particles have the distributed and smaller speed. Hence the particles can be slower even

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**FIG. 5.** (Color online) Increase of cluster energy per unit time $P_{\text{cluster}}$ for $1 \leq n \leq 5$ and $0.3 \eta^{-1} \leq D \leq 1.5 \eta^{-1}$ at $r = 1$, $u = 10\hbar T_0^{-1} \eta$, $v = 4\eta/T_0$.

**IV. DISCUSSION**

Before we discuss our model, we compare naïve mean-field results with the DMRG results, in order to investigate the many-body effects in the system. Under a mean-field approximation, the cloud particles move in the potential created by the average interaction with the cluster, which is calculated by the density distribution of the cluster particles of the ground state (e.g., a gaussian function at $n = 1$). The potential moves by velocity $v$, pushes the cloud, and excites the system. Comparing the mean-field results with the DMRG results, we find that they agree in the case of $n = 1$, but the mean-field results have no peak in the large $D$ region. Thus, the many-body effects such as collective excitation have a crucial role in forming the peak structure in the large $D$ region; the model for this system has to contain its many-body effects.

Since the simple mean-field result does not agree with the DMRG result, we propose a simplified model to explain the DMRG simulation results, especially the appearance of the peak structure in Fig. 3 and Fig. 4. We start from the simplest case of $n = 1$, the one-body dynamics under the harmonic trap interacting with the fermion cloud. Now the classical picture is applied, where
at large $D$, release less energy to the cloud, and interact with the trap potential more efficiently.

Here the momentum $mv_I$ is given to a cluster particle directly from the cloud, and finally the first particle has the momentum $mv_I$ and the other $n-1$ particles have $mv_{\text{dist}}$ as a result of the momentum transfer among the cluster particles. In the second peak, these single-particle velocities do not strongly depend on $n$, because the maximum work by the trap is achieved in almost the same velocities ($D$ at the second peak is not largely changed in $3 \leq n \leq 6$). To simplify the results, we neglect the dependence on $n$ and set $v_1 = v_1^{\text{peak2}}$ and $v_{\text{dist}} = v_{\text{dist}}^{\text{peak2}}$. Thus
\[
 f(r)\pi h D_{n}^{\text{peak2}} = v_1^{\text{peak2}} + (n-1)v_{\text{dist}}^{\text{peak2}}
\]
is obtained. That relation shows that $D_n^{\text{peak2}}$ is a linear function of $n$; this result agrees with the DMRG result in Fig.3 and Fig.4.

V. CONCLUSION

In summary, using the time-dependent density matrix renormalization group method and the Fermi–Hubbard model, we have calculated the drag dynamics of several fermions in a fermion cloud in one-dimensional continuous systems. We have obtained the steady energy excitation per unit time as a function of the particle number of the cluster $n$, the cloud density, the mass ratio between fermions, the interaction strength, and the trap speed. We have discovered the emergence of a double-peak structure; one is in the small cloud density region, and the other is in the large cloud density region. We have revealed that when one wants to reduce the system excitation in moving a fermion cluster in a fermion cloud, one should move the cluster packed together if the cloud density is low, but one should move the fermions in separate $n$ traps if the cloud density is higher. We have introduced a simplified model for the excitation process to explain the peak structure observed in the excitation energy per time as a function of the cloud density at $n = 1$. We have also elucidated the mechanism of the second peak at $n > 2$ using our model with a proper assumption about many-body effects, and have emphasized the quantum effects in the drag dynamics of a cluster bound in a fermion cloud.

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