Killing the Straw Man: Does BICEP Prove Inflation?

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The surprisingly large value of $r$, the ratio of power in tensor to scalar density perturbations in the CMB reported by the BICEP2 Collaboration provides strong evidence for Inflation at the GUT scale. In order to provide compelling evidence, other possible sources of the signal need to be ruled out. While the Inflationary signal remains the best motivated source, the current measurement unfortunately still allows for the possibility that a comparable gravitational wave background might result from a self ordering scalar field transition that takes place later at somewhat lower energy. However even marginally improved limits on the possible isocurvature contribution to CMB anistropies could rule out this possibility, and essentially all other sources of the observed signal other than Inflation.

The recent claimed observation of primordial gravitational waves [1] provides a dramatic new empirical window on the early universe. In particular, it provides the opportunity, in principle, to definitively test the inflationary paradigm [2, 3], and to explore the specific physics of inflationary models. However, while there is little doubt that inflation at the Grand Unified Scale is the best motivated source of such primordial waves (e.g. [4–7], it is important to demonstrate that other possible sources cannot account for the current BICEP2 data before definitely claiming Inflation has been proved.

A surprisingly large value of $r$, the ratio of power in tensor modes to scalar density perturbations provides a challenge for other possible primordial sources, as such sources would have to generate gravitational waves efficiently without altering the observed adiabatic density fluctuations that are so consistent with inflationary predictions. Here we explore to what extent that challenge might rule out other possibilities.

We have previously explored a relatively generic possible competing source of a scale invariant spectrum of tensor modes [8–10], a simple self ordering scalar field (SOSF) in the early universe, and frankly had hoped that the BICEP2 observation would rule out this possibility, thus allowing a cleaner interpretation of the the existing data in terms of inflation. As we describe here unfortunately the measured value of $r$ falls just short of ruling out this other source as the dominant contribution of the observed effect. Nevertheless, as we also show, reducing the bound on any possible isocurvature component of the scalar power spectrum can rule out this possibility, and therefore any likely candidate source after inflation that produces gravitational waves. This would then imply the BICEP2 result definitely reflects gravitational waves from inflation, with all of the exciting concomitant implications (i.e. quantization of gravity [11]).

In the following we assume inflation occurs, and provides the measured adiabatic scalar density fluctuations inferred from CMB measurements (because that is strongly suggested by the data), but that a SOSF phase transition occurs after inflation, producing a gravitational wave signature that might overwhelm the inflationary signal.

Let $S_i$ and $T_i$ denote the scalar and tensor power generated by inflation and $S_\phi$ and $T_\phi$ the same quantities for the self-ordered scalar field. Out of these four quantities one can form several ratios of interest: (i) $r_{ad} = (T_i + T_\phi)/(S_i + S_\phi)$ is the tensor to scalar ratio incorporating both sources that has just been observed to have a central value of 0.2. (ii) The self-ordering scalar field produces isocurvature scalar fluctuations whereas inflation produces adiabatic ones. Measurements of the temperature anisotropies constrain the isocurvature fraction $x = S_\phi/(S_i + S_\phi)$ to lie in the range $0 < x < 0.09$ [12]. (iii) $r_\phi = T_\phi/S_\phi$, the tensor-to-scalar ratio for the SOSF case, can be calculated within the self-ordering scalar field model using the scalar power spectrum described in [13] along with the tensor power given in [9–10], and is found to be 2.34 (iv) $f = T_\phi/T_i$, the ratio of the tensor contributions from the SOSF mechanism to that produced by inflation, is given by $(140/N)(V_\phi/V_i)$ [9–10] where $N$ denotes the number of components of the self-ordering scalar field (presumed to be large and definitely greater than three), $V_\phi$ is the symmetry breaking scale for the self-ordering field and $V_i$ is the scale of inflation. We need $V_\phi < V_i$ to ensure that symmetry breaking occurs after inflation (otherwise evidence of it would be obliterated by inflation). This inequality constrains the ratio $f$. (v) The tensor to scalar ratio for inflation $r_i = T_i / S_i$ is the quantity of interest for inflationary models. In the
absence of the self-ordering scalar fields, \( r_i \) is equal to the measured quantity \( r_{\text{eff}} \), but the present measurement currently allows \( r_i \) to have a considerably lower value if self-ordering scalar fields dominate the observed signal.

A priori, this need not have been the case. Since only three of these ratios are independent, but there are now constraints on four of them, in principle, the data is capable of ruling out the existence of self-ordering scalar fields as a source. To explicitly determine the constraints we express \( f \) in terms of \( r_{\text{eff}}, x \) and \( r_\varphi \)

\[
 f = \frac{x r_\varphi}{r_{\text{eff}} - x r_\varphi}. \tag{1}
\]

Fig. 1 shows a plot of \( f \) as a function of \( x \) reveals that \( f \) grows monotonically with \( x \), diverging at \( x_\infty = r_{\text{eff}}/r_\varphi \approx 0.085 \). This corresponds to a situation where the SOSF contribution essentially accounts for all of the observed BICEP2 polarization, and therefore contributes a fraction 0.2/2.34 of the (isocurvature) power in scalar density perturbations.

Since \( x_\infty \) is less than the maximum iso-curvature ratio compatible with the temperature anisotropy data we arrive at the disappointing conclusion that the new measurement of \( r_{\text{eff}} \) does not additionally constraint self-ordering scalar fields. Had \( r_{\text{eff}} \) been larger, the isocurvature contribution of SOSF to scalar density perturbations would have to have been larger to account for the entire tensor signal, and existing constraints on this contribution would have therefore constrained \( f \), and thereby the symmetry breaking scale, \( V_\varphi \).

While this is disappointing, it is cause for hope. A small improvement on the iso-curvature fraction in CMB temperature fluctuations would imply that SOSF cannot give the full measured contribution to \( r_{\text{eff}} \) and therefore the signal from inflation is observable in the data. Alternatively a non-zero measured isocurvature fraction might be suggestive that an SOSF has occurred and contributes to the BICEP2 signal.

Note \( f \) must lie below a maximum value, \( f_{\text{max}} \), so \( x \) is actually constrained to lie in the range \( 0 < x < x_{\text{max}} \) rather than \( 0 < x < x_\infty \). Here

\[
x_{\text{max}} = \frac{f_{\text{max}}}{1 + f_{\text{max}}} x_\infty, \tag{2}
\]

obtained by inverting eq (1) and setting \( f \rightarrow f_{\text{max}} \). We estimate \( f_{\text{max}} \approx 35 \) by taking \( N = 4 \), and setting \( V_\varphi = V_i \) leading to \( x_{\text{max}} \approx 0.083 \).

Finally for completeness we display the inflationary tensor to scalar ratio \( r_i \) that may be inferred from the data as a function of the isocurvature fraction of scalar density perturbations induced by SOSF. This allows a quantitative estimate of how future constraints on this fraction can then allow one to infer the fraction of the BICEP2 signal that must result from Inflation.

Fig. 2 shows a plot of \( r_i \) as a function of \( x \) over its allowed range. As can be seen, if the current upper limit of 0.09 is reduced by a factor about 2, then the inflationary contribution must dominate. However, even a reduction by only 20% or so would imply a clear non-zero inflationary component to the observed BICEP2 signal.

While it is perhaps frustrating that the current observation cannot unambiguously rule out this toy model straw man as a source of gravitational waves that could polarize the CMB signal as observed by BICEP2. However, as we have described, we are at the threshold of being able to argue that Inflation unambiguously provides at the very least a non-zero component of the signal. Note that because the scale of inflation varies as the fourth root of \( r \), the scale of inflation will remain essentially identical to the Grand Unified Scale independent of whether it contributes all, or only a fraction of the observed polarization signal.

We also note that the current analysis has not included the possible contribution from vector modes due to SOSF. However since such modes are known to contribute roughly equally to scalar and tensor modes in the CMB it should not significantly affect ratios, although it would need to be calculated and included in a more complete future quantitative analysis.

Finally we note that while current data cannot definitively rule out a SOSF transition as the source of gravita-
tional waves, it nevertheless does imply that the source for such waves is at, or near the Grand Unified Scale. Thus, it allows an exploration of physics at a scale far larger than we can currently constrain at terrestrial experiments. This will be very important for constraining physics beyond the standard model, whether or not inflation is responsible for the entire BICEP2 signal, even though existing data from cosmology is strongly suggestive that it does.

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