Study on Identification of SSI Damping System Based on Large-Scale Shaking Table Test

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Abstract. Damping system represents the energy dissipation characteristics of the overall structural system. The correct identification of damping system is not only the premise of selecting dynamical analysis method, but also the foundation of obtaining accurate results in seismic analysis. Traditionally, the identification of damping system under dynamic action is based on material properties. However, this method is lack of theoretical and experimental research. Based on fundamental identification method, the paper presents the study on the consistence of dynamic characteristics between different parts by shaking table test. The changing law of SSI damping system in different dynamic processes is investigated. The experimental results indicate that SSI system manifest obviously classical energy dissipation characteristics under certain conditions and can be considered as an approximate classical damping system.

1. Introduction

Damping system represents the energy dissipation characteristics of the overall structural system in dynamic analysis\textsuperscript{[1]}. The multi-degree of freedom system with equivalent viscous damping model can be divided into classical damping system and non-classical damping system. In classical damping system, motion states are coordinated, and motion equations can be decoupled in the modal space due to the classical energy dissipation characteristics. The real mode shapes can be obtained, so that modal superposition method is available for dynamic analysis. In non-classical damping system, motion equations cannot be decoupled because of significant sources of localized energy dissipation. Consequently, the mode shapes become complex valued numbers rather than real valued numbers. Therefore, to solve dynamic response, alternative methods are proposed, such as direct integration method, forced uncoupling method, equivalent damping method and complex mode method\textsuperscript{[2-5]}, which turns out to be complex and unpractical in engineering.

Soil-Structure Interaction System\textsuperscript{[6]} considers the coordination mechanism of soil and structure. SSI damping system represents the energy dissipation characteristics of SSI system in dynamic
analysis. Traditionally, a system composed of more than two materials, especially the SSI system that includes two materials with quite different damping behaviors, is often considered as a non-classical damping system \(^7\). However, the results of shaking table test of SSI system in Tongji University \(^8\) have illustrated that the non-classical damping characteristics disappeared gradually after initial small magnitude earthquake input, and the classical damping characteristics appeared progressively under certain conditions. The aforementioned study indicates that the traditional identification method based on material properties is lack of theoretical and experimental research, thus it is not accurate and it turns out to be inappropriate for the practical projects.

The soil-structure interaction effect is ubiquitous. However, study on the damping characteristics of SSI system and its identification has been rarely reported. For different damping systems, the methods could be totally different and the results can also vary greatly. The correct identification of SSI damping system is not only the premise of selecting dynamical analysis method, but also the foundation of obtaining accurate results in seismic analysis. Therefore, it is very necessary to conduct fundamental theoretical study and large-scaled experiment to explore the discipline of SSI damping system.

In order to discuss the damping characteristics of SSI system under earthquake and explore the existence of approximate classical damping system, the paper starts with fundamental structural dynamic analysis method \(^9\) and basic identification of damping system is discussed and verified by large-scale shaking table test. And to investigate the damping characteristics of SSI system, the consistency of modal frequencies are analyzed based on large-scale shaking table test. In the test, several earthquake waves are adopted, including bedrock wave, Jiangyou wave, El Centro wave and sine wave. The acceleration and displacement responses and dynamic characteristics of different parts of SSI system are measured under small, middle and large earthquakes. And the consistency of dynamic characteristics between different parts of the system are analyzed by acceleration self-power spectral density, Fast Fourier Transform, phase-plane analysis.

2. Identification of Damping System

For a system with viscous damping model \(^10\), its movement can be described by motion equation as shown below.

\[
[M][\ddot{u}(x,t)] + [C][\dot{u}(x,t)] + [K][u(x,t)] = \{p(t)\}
\]

The key difference between classical damping system and non-classical damping system is whether the motion equation above can be decoupled and whether the modal analysis method is available for dynamical analysis. If the motion equation can be decoupled, the movement can be expressed by linear combination of several decoupled modes. Any decouple mode is based on the assumption that the unique deflection curve exists. If the deflection curve is defined as the shape function and the movement of the base point is expressed by the generalized coordinate, then the modal displacement, velocity and acceleration of a decouple mode can be described as \(^11,12\):

\[
\begin{align*}
    u_n(x,t) &= \phi_n(x)q_n(t) \\
    \dot{u}_n(x,t) &= \phi_n(x)\dot{q}_n(t) \\
    \ddot{u}_n(x,t) &= \phi_n(x)\ddot{q}_n(t)
\end{align*}
\]

where \(\phi_n(x)\) is the shape function of the n-th mode, which is correlated with coordinate x, \(q_n(t)\), \(\dot{q}_n(t)\), \(\ddot{q}_n(t)\)is the displacement, velocity and acceleration of the n-th generalized coordinate, which is correlated with time t only.

According to dynamical analysis method, the necessary condition that the Eq. (2) is classical damping system is that shape function \(\phi_n(x)\) is continuous and the acceleration of the base point \(\ddot{q}_n(t)\) which is equivalent to
\{ \phi_n(x) \) continuous \} \quad \Rightarrow \quad \{ \hat{q}_n(t) \) exist \} \quad \Leftrightarrow \quad \{ \phi_n(x) \) continuous \} \quad \Leftrightarrow \quad \{ \hat{q}_n(t) \) derivable \} \quad \Leftrightarrow \quad \{ \phi_n(x) \) continuous \} \quad \Leftrightarrow \quad \{ \hat{q}_n(t) \) smooth and continuous \}

In other words, if the shape function \( \phi_n(x) \) is continuous and the velocity of base point \( \hat{q}_n(t) \) is smooth and continuous, the combined movement will be continuous, which means the system is classical damping system. Otherwise, if any condition above is not satisfied, the combined movement will not be continuous, and the system will be non-classical damping system.

Meanwhile, according to the modal superposition method, the displacement, velocity and acceleration response of the system can be expressed as the superposition of each modes:

\[
\{ u(t) \} = \sum_{n=1}^{N} \{ \phi_n \} q_n(t) \\
\{ \dot{u}(t) \} = \sum_{n=1}^{N} \{ \phi_n \} \dot{q}_n(t) \\
\{ \ddot{u}(t) \} = \sum_{n=1}^{N} \{ \phi_n \} \ddot{q}_n(t)
\]

Therefore, when the motion state of the system is continuous, each shape function is continuous with respect to coordinate \( x \) and each modal velocity function is continuous with respect to time \( t \). During the movement progress, there must be one or several order modal movement are discontinuous when the displacement, velocity or acceleration of adjacent gauging points of the system is discontinuous. In other words, whether the combined movement is continuous and smooth is related to the modal motion state. So that the damping system can be identified by investigating the state of adjacent gauging points of the system.

All in all, dynamic characteristics and motion state are the most fundamental method to discriminate damping system. To make a further understanding on the SSI damping system, a large-scale shaking table test is conduced and the consistency of dynamic characteristics between different parts is investigated.

3. Shaking Table Test of SSI Damping System

3.1. Test Model

The SSI damping system test model is an 8-storey frame-structure with pile and the soil underneath is typical loess in Xi’an. The similarity coefficient of modulus of elasticity \( SE=1/4 \), geometric similarity coefficient \( SL=1/10 \), and acceleration similarity coefficient \( Sa=2.5 \) \(^{[13,14]} \) are taken as controlling similarity constant. The geometric details of each component of the model are shown in Table 1.

| Category | Size (m) x Number |
|----------|------------------|
| Floor    | 0.6x0.6x0.02x8   |
| Beam     | 0.6x0.35x0.65x36 |
| Column   | 0.65x0.65x0.3x32 |
| Cap      | 0.3x0.9x0.9x2    |
| Pile     | 1x0.65x0.65x9    |
| Foundation | 3x1.5x1.5x1     |

Soil tank is one of the most crucial equipment in the shaking table test. To consider the effect of lateral soil pressure and shear deformation of soil during the earthquake. Considering the lateral soil...
pressure and shear deformation under earthquake input, a laminar shear box is used in SSI system shaking table test \cite{15,16} as shown in Fig.1. The size of the box is 3.0m in length, 1.5m in width and 1.392m in height. The laminar box consists of 13 horizontal steel-tube layers. The steel tube has a cross-section of 96 mm×96 mm, and a gap of 12mm each. Some ball bearings are arranged between steel tube layers so that the layers can move relatively to one another in accordance with the deformation of the soil inside. On the two side faces parallel to the vibration direction, four upright columns are installed to prevent the steel tube layers from torsion or slip. The soil box is bolted to the shaking table by a welded steel frame at the bottom. Antiskid strips are installed at the bottom of inner surface to satisfy the friction boundary condition, and 5mm thick rubber, 5cm thick polystyrene foam and 5mm thick waterproof rubber are laid on the side wall successively to satisfy the flexible boundary condition.

3.2. Arrangement of Gauging Points and Test Method

The main vibration direction is horizontal x direction. Various sensors are installed in the test, including accelerometers, displacement meters, strain gauges, soil pressure cells and pore water pressure transducers. As for the analysis object in this paper is SSI damping system, the arrangements of acceleration and displacement sensors are shown in Fig.1.

3.2.1. Investigation of Increasing Loading Process

In order to observe the mode characteristics of the SSI damping system and its change rule in dynamic process, the acceleration power spectral density curve of different material parts is a effective method.

To investigate the dynamic characteristics of different parts of SSI system, the dynamic characteristics of structure and soil were tested respectively at first. The first ordered and second ordered frequency of the structure with the additional mass are 5.14Hz and 30.26Hz respectively, the corresponding modal damping ratio are 2.79% and 3.73% respectively. The first ordered and second ordered frequency of the soil are 9.29Hz and 17.10Hz respectively, the corresponding modal damping ratio are 3.65% and 5.64% respectively. It is illustrated the dynamic characteristics of upper structure and that of soil is different in initial state.

According to Chinese Seismic Code, the frequently occurred earthquake, basic intensity earthquake and rarely occurred earthquake were simulated.

4. Research on the Consistency of Dynamic Characteristics of Different Parts of the System

4.1. Investigation of Increasing Loading Process

In order to observe the mode characteristics of the SSI damping system and its change rule in dynamic process, the acceleration power spectral density curve of different material parts is a effective method. The nature of distinguishing damping system is study on the overall energy dissipation distribution of the system. Auto-power spectral density function is the curve of power spectral density with frequency by Fourier transform of the auto correlative function. Reflecting the distribution of the response power in frequency domain, the auto-power spectral density curve reflects not only the
dynamic characteristics of the system, but also the distribution of energy in different frequencies compared with common frequency analysis methods\textsuperscript{[17,18]}.

Suppose $R_x(\tau) = E[X(t)X(t + \tau)]$ is the acceleration auto correlative function, according to Wiener-Khinchin theorem\textsuperscript{[17]}, the acceleration power spectral density $S_x(\omega)$ can be expressed as:

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau$$

The acceleration auto-power spectrum density curves of different measuring points of the SSI system under white noise condition before and after simulated earthquake with different intensity magnitude are shown in Figure 2.

![Acceleration Auto-power Spectrum Density under White Noise Conditions](image)

Figure 2 Acceleration Auto-power Spectrum Density under White Noise Conditions

It can be found that there exists large difference between structure and foundation in the peak vibrational energy distribution in frequency domain. The first two peak frequencies in Figure 2(a) are respectively corresponding to the fundamental frequencies of structure and foundation, which means the mode motion of the system is incongruous and the SSI system is non-classical at this time. However, Figure 2 (b) shows the interrelationship and driving effect between structure and foundation vibrational energy after small magnitude earthquake. In other words, the foundation vibration is driven at the frequency of superstructure peak energy and the superstructure vibration is also driven at the frequency of foundation peak energy. The frequencies of structure and foundation peak vibrational energy tend toward the same, and the mode motions start to be coordinated, which means the interaction of structure and foundation starts up, the motor coordination mechanism grows up gradually, and the classical damping characteristics of SSI system are emerging. Along with the strengthening of seismic action, the tendency of natural frequency getting closer is strengthened, and the unified modality of two parts of the SSI system is formed as shown in Figure 2(c), (d). Meanwhile, Along with the strengthening of seismic action, the system natural frequencies decrease, the power amplitude of lower order frequency reduces, and that of higher order frequency becomes obvious gradually. This indicates that though the rigidity of SSI system reduces and nonlinearity appears along with the strengthening of seismic action, the energy distribution characteristics of the two parts are getting further accordant in frequency. In other words, synthetic mode in transient sense of SSI system can still be formed in the case of dynamic nonlinear, and the approximate classical energy dissipation characteristics of the whole system will be performed so that the SSI system can be regarded as approximate classical damping system at this stage.
4.2. Investigation of Earthquake Loading Processes
In order to find out the distribution of vibrational energy in the frequency domain in SSI system and
the change trend of transient damping system in the process of earthquake loading, the measured
acceleration auto-power spectral density curves under different earthquake loading are shown in
Figure 3.

Though the mode can be hardly presented in the process of SSI system nonlinear development
under seismic dynamic forces, especially in the process of greater excitation, the change of auto-power
spectral density still reflects the distribution law of vibration energy and the development of damping
system. The main vibrational energy distribution in Figure 3(a) shows that energy is mainly
concentrated in lower vibration and the basic frequencies of structure and foundation are
uncoordinated in small earthquake loading process, thus the SSI system shows non-classical damping
characteristics obviously. Along with the increasing load magnitude, the higher order vibration energy
increases and gets dominant gradually, and the main frequencies of structure and foundation are
interlaced and closer to each other as shown in Figure 3(b)(c). The main frequency of the system
reduces greatly and the nonlinearity increases gradually with the further strengthening of the
vibrational energy, as shown in Figure 3(d), and the conformance trend of foundation and structure is
getting more obvious, which means the motor coordination mechanism between soil and structure
makes the motion state of the two parts tend to be coordinated under dynamic interaction. Meanwhile,
along with the increase of input energy and loss of high frequency stiffness, the redistribution of the
capacity of the system increases the low frequency energy load, but it does not affect the formation of
the classical damping characteristics of the SSI system.

In order to further prove the above-mentioned change regulation and get a clearer observation of
the changes in the motor coordination mechanism within the system, acceleration amplitude frequency
curve is adopted in every loading process. Taking the measured acceleration near the peak intensity as
object, the foundation and structure measured acceleration amplitude frequency curves of before peak,
peak and after peak in 2s duration in the process of incremental loading are shown in Figure 4-6.

Figure 4 shows acceleration amplitude frequency characteristics of foundation and structure in
small earthquake process, where the vibration frequencies of different parts of the system are
inconsistent and the non-classical damping characteristics are quite apparent, especially before the
peak. In middle earthquake process after small earthquake, however, motions of the upper and lower
parts of the system become to be coordinated, and the vibration frequencies tend to be consistent,
especially after the peak, as shown in Figure 5. When the vibration magnitude is larger, Figure 6
shows that the acceleration amplitude frequency curves of structure and foundation are getting further similar, the vibration frequencies of the two is basically the same, and the amplitudes are closer. The above indicates that the internal motor coordination mechanism of SSI system is strengthened in the incremental loading processes, and the classical damping characteristics of SSI system are also gradually becoming obvious.

Figure 4 Foundation and Structure Acceleration Amplitude Frequency Curves
Near Peak Intensity of Small Earthquake (0.125g Bedrock Wave)

Figure 5 Foundation and Structure Acceleration Amplitude Frequency Curves
Near Peak Intensity of Middle Earthquake (0.5g Bedrock Wave)

Figure 6 Foundation and Structure Acceleration Amplitude Frequency Curves
Near Peak Intensity of Large Earthquake (1.2g Bedrock Wave)

5. Conclusion
According to the basic judging method of the damping system, change rules of SSI damping system in certain range dynamic processes are studied through consistency of dynamic characteristics in SSI system shaking table model test in this paper. The results of acceleration auto-power spectrum density curves and amplitude frequency curves indicate that the motion states of different part are not harmonized before and at the beginning of earthquake, thus the non-classical damping characteristics are obvious. However, along with increasing loading processes after small earthquake, movement coordination mechanism of structure and foundation is formed gradually, and the frequency of different material parts tends to be consistent. The classical damping characteristics of SSI system are becoming apparent gradually.
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