Elastic solutions of an isothermal impervious cross-anisotropic half space due to hot fluid injection

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Abstract. The integral transform techniques are used to derive the closed-form solutions of a homogeneous cross-anisotropic half space with an isothermal impervious ground surface subjected to a hot fluid injection. The study developed a mathematical model for the distribution of temperature changes, excess pore fluid pressure and land deformation of the half space of strata. Analytic solutions are derived through the application of Hankel transform and Fourier transform with respect to the radial coordinate and axial coordinate, respectively. The results can provide better understanding of the hot fluid injection induced isothermal impervious half space responses of the cross-anisotropic porous strata.

1. Introduction

Responses of strata due to hot fluid injection into a formation below the ground surface are an important petroleum engineering issues. Considering impact on engineering safety, many studies were concentrated on mechanical, hydraulic, and thermal behavior due to hot fluid injection. Hydraulic and thermal disturbance usually result in a volumetric change of fluid and solid skeleton. The volumetric change can increase excess pore fluid pressure and lead to a decrease in effective stress. The loss of shear resistance of solid skeleton may result in a hydraulic or thermal failure in the strata. The simulation of these physical features is a complex task, and its validation is a major concern for the safety improvement of the hot fluid injection.

Alajmi et al. [1] presented the performance of hot water flooding compared to conventional water flooding in re-covering heavy oil from heterogeneous reservoirs through fine-mesh numerical simulations. In the study of Sasaki et al. [2], a system of gas production from methane hydrate layers involving hot water injection using dual horizontal wells was investigated. Rosenbrand et al. [3] addressed permeability change in sandstone due to heating from 20°C to 70~200°C. To simulate gas production from methane hydrate-bearing sand by hot water cyclic injection, a three-dimensional middle size reactor was used by Yang et al. [4].

In general, the strata are deposited through a geologic process of sedimentation over a long period of time. Under the accumulative overburden pressure, strata display significant anisotropy on mechanical, seepage and thermal properties. For this reason, theoretical and numerical models shall be able to simulate the layered soils and rocks as cross-anisotropic media [5-9].

The present investigation is focused on the closed-form solutions of a cross-anisotropic isothermal impervious half space due to hot fluid injection which still have not been derived in previous studies. The soil or rock mass is modelled as linearly elastic medium with cross-anisotropic properties in this
paper. The mechanical properties, hydraulic fluid flow and thermal conductivities are treated as cross-anisotropic. Using Hankel-Fourier integral transforms, the closed-form solutions of long-term excess pore fluid pressure, temperature changes and displacements of the strata due to a point of hot fluid injection into a half space are obtained. The results can provide better understanding of the hot fluid injection induced responses of the isothermal impervious half space.

2. Mathematical model

2.1. Basic equations

Figure 1 displays a point hot fluid injection into an isothermal impervious porous elastic half space. The cross-anisotropic soil or rock is modeled as a homogeneous elastic half space. For simplicity, the plane of symmetry of the stratum is positioned in the horizontal direction. Let \( r, \theta, z \) be a cylindrical coordinate system for this layer of stratum where the plane of isotropy coincides with the \( r-\theta \) horizontal plane. The constitutive law for an elastic medium with linear axially symmetric deformation can thus be expressed by

\[
\sigma_{rr} = A \varepsilon_{rr} + (A - 2N) \varepsilon_{\theta\theta} + F \varepsilon_{zz} - \beta_r \theta - p, \quad \text{(1a)}
\]

\[
\sigma_{\theta\theta} = (A - 2N) \varepsilon_{rr} + A \varepsilon_{\theta\theta} + F \varepsilon_{zz} - \beta_\theta \theta - p, \quad \text{(1b)}
\]

\[
\sigma_{zz} = F \varepsilon_{rr} + F \varepsilon_{\theta\theta} + C \varepsilon_{zz} - \beta_z \theta - p, \quad \text{(1c)}
\]

\[
\sigma_{rz} = 2L \varepsilon_{rz}, \quad \text{(1d)}
\]

The symbols \( \sigma_{ij}, \theta \) and \( p \) are the total stress components, temperature changes from the reference and excess pore fluid pressure of the half space, respectively. The material constants \( A, C, F, L, N \) of a cross-anisotropic stratum are defined by Love [10]. In equations (1a) to (1c), \( \beta_r \) and \( \beta_z \) represent the thermal expansion factors of the half space along and normal to the symmetric plane, respectively. Here, the relations between strain components \( \varepsilon_{ij} \) and displacement components \( u_i \) of the stratum are expressed by the linear law:

\[
\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{u_\theta}{r}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \frac{1}{2} \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right),
\]

(2)

where \( u_r \) and \( u_z \) are the displacements of the stratum in radial and vertical directions, respectively. For axially symmetric problem, it is noted that the shear stresses \( \sigma_{r\theta}, \sigma_{\theta z} \), and circumferential displacement \( u_\theta \) would vanish as the vertical \( z \)-axis is located through the hot fluid injection point. The expression for the thermal expansion factors \( \beta_r \) in radial direction and \( \beta_z \) in vertical direction are:

\[
\beta_r = 2(A - N) \alpha_{rr}, \quad \beta_z = 2F \alpha_{zz} + C \alpha_{zz},
\]

(3)

where \( \alpha_{rr} \) and \( \alpha_{zz} \) are the linear thermal expansion coefficients of the stratum in the horizontal and vertical directions, respectively. The material constants \( A, C, F, L, N \) employed in equations (1a) to (1d) are related through the following equations:

\[
A = \frac{E_z (1 - \nu_{rr} \nu_{zz})}{(1 + \nu_{rr})(1 - 2 \nu_{rr} \nu_{zz})}, \quad C = \frac{E_z (1 - \nu_{\theta\theta})}{1 - \nu_{\theta\theta} - 2 \nu_{rr} \nu_{zz}},
\]

where \( E_z \) is the vertical Young's modulus.
Here, the symbols $E_r$ and $E_z$ are defined as Young’s moduli with respect to directions lying on and perpendicular to the plane of isotropy, respectively; $\nu_{\theta\theta}$ is the Poisson’s ratio for strain in the angular direction due to a horizontal direct stress; $\nu_{\theta r}$ is the Poisson’s ratio for strain in the vertical direction due to a horizontal direct stress; $\nu_{rz}$ is the Poisson’s ratio for strain in the horizontal direction due to a vertical direct stress; and $G_{rz}$ is shear modulus for planes normal to the plane of isotropy.

For the cases of isotropic elastic medium, the material constants $A, C, F, L, N$ can be denoted as

$$A = C = \lambda + G, \quad F = \lambda, \quad L = N = G, \quad \beta_r = \beta_z = (2G + 3\lambda)\alpha_s.$$  \hspace{1cm} (5)

Here, $\lambda$ and $G$ are the Lame moduli of the homogeneous isotropic half space, and $\alpha_s$ is the linear thermal expansion coefficient of the isotropic solid skeleton.

In general, these axially symmetric total stresses $\sigma_{ij}$ must satisfy the equilibrium equations:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr}}{r} + \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta\theta}}{\partial z} + f_r = 0, \quad \frac{\partial \sigma_{\theta r}}{\partial r} + \frac{\sigma_{\theta r}}{r} + \frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{\partial \sigma_{\theta r}}{\partial z} + f_\theta = 0, \hspace{1cm} (6)$$

where $f_r$ and $f_\theta$ denote the body force components. For axisymmetric problems with effect of body forces neglected, the equilibrium equations can be expressed in terms of displacements, excess pore fluid pressure, and temperature change of the porous medium as follows:

$$A \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) + L \frac{\partial^2 u_r}{\partial z^2} + (F + L) \frac{\partial^2 u_\theta}{\partial r \partial z} - \beta_r \frac{\partial p}{\partial r} - \frac{\partial p}{\partial z} = 0, \hspace{1cm} (7a)$$

$$F + L \left( \frac{\partial^2 u_\theta}{\partial r \partial z} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} \right) + C \frac{\partial^2 u_\theta}{\partial z^2} - \beta_r \frac{\partial \theta}{\partial z} - \frac{\partial p}{\partial z} = 0. \hspace{1cm} (7b)$$

The hot fluid injection point is considered at a depth $h$ of point $(0, h)$ as shown in Figure 1. Using the laws of mass balance, energy conservation, continuity equation and heat conduction equation, the governing equations for fluid flow and thermal flow can be derived as shown in equations (8a) and (8b):
in which the permeability of the half space in the horizontal and vertical directions are expressed as \( k_h \) and \( k_v \), respectively; the constants \( \lambda_h \) and \( \lambda_v \) denote the horizontal thermal conductivity of heat flow in the plane of isotropy and the corresponding vertical thermal conductivity in the plane perpendicular to the isotropic plane, respectively; \( \gamma_f \) is the unit weight of injected fluid; the symbols \( \delta (r) \) and \( \delta (z) \) are the Dirac delta functions. The injected hot fluid is considered at a constant rate of thermal strength \( Q_t \) corresponding with fluid volume \( Q_f \) per unit time. Equations (7a), (7b), (8a) and (8b) govern the steady state responses of the porous medium subjected to axisymmetric disturbance of a point hot fluid injection.

### 2.2 Boundary conditions

The half space ground surface at \( z=0 \) is treated as a traction-free, isothermal and impervious boundary for all times \( t \geq 0 \). Therefore, its mathematical statements of the traction-free mechanical boundary conditions are:

\[
\sigma_{rr}(r,0) = 0, \quad \sigma_{zz}(r,0) = 0.
\]

The mathematical statements of the impervious isothermal condition at the boundary \( z=0 \) are given as the following equation (10):

\[
\frac{dp(r,0)}{dz} = 0, \quad \vartheta(r,0) = 0.
\]

The hot fluid injection point is assumed no effect at the remote boundary of \( z \rightarrow \infty \) for all times \( t \geq 0 \). Hence

\[
\lim_{z \rightarrow \infty} [u_r(r,z), u_z(r,z), p(r,z), \vartheta(r,z)] = \{0, 0, 0, 0\}.
\]

The responses can be derived from differential equations (7a), (7b), (8a) and (8b) corresponding with half space boundary conditions (10) and remote boundary conditions (11).

### 3. Elastic closed-form solution

The closed-form solutions of poroelastic deformation, excess pore fluid pressure and temperature increment of the strata due to a point of hot fluid injection into in a cross-anisotropic poroelastic half space can be obtained by using Hankel transform and Fourier transform [11-13] with respect to the radial coordinate variable \( r \) and axial coordinate variable \( z \), respectively, as below:

\[
u_r(r,z) = \frac{Q_f}{4\pi k_z} \sum_{i=1}^{10} a_i \frac{r}{R_i} + \frac{Q_f}{4\pi k_z} \sum_{i=1}^{10} a_i \frac{r}{R_i},
\]

\[
u_z(r,z) = \frac{Q_f}{4\pi k_z} \left[ h_j \sinh^{-1} \frac{\mu_{1f}(z-h)}{r} + b_{2f} \sinh^{-1} \frac{\mu_{2f}(z-h)}{r} + b_{3f} \sinh^{-1} \frac{\mu_{3f}(z-h)}{r} \right]
\]

\[
\vartheta(r,z) = \frac{Q_f}{4\pi k_z} \left[ h_j \sinh^{-1} \frac{\mu_{1f}(z-h)}{r} + b_{2f} \sinh^{-1} \frac{\mu_{2f}(z-h)}{r} + b_{3f} \sinh^{-1} \frac{\mu_{3f}(z-h)}{r} \right]
\]
Here, the hydraulic constants \( a_{if} (i = 1, \cdots, 10) \), \( b_{if} (i = 1, \cdots, 10) \), thermal constants \( a_{u} (i = 1, \cdots, 10) \) and \( b_{u} (i = 1, \cdots, 10) \) are defined by

\[
a_{if} = \frac{L + (F + L - C)\mu_{if}^2}{CL \mu_{if} (\mu_{if}^2 - \mu_{zf}^2)(\mu_{zf}^2 - \mu_{if}^2)},
\]

(13a)

\[
a_{zf} = \frac{L + (F + L - C)\mu_{zf}^2}{CL \mu_{zf} (\mu_{zf}^2 - \mu_{if}^2)(\mu_{if}^2 - \mu_{zf}^2)},
\]

(13b)

\[
a_{zf} = \frac{L + (F + L - C)\mu_{zf}^2}{CL \mu_{zf} (\mu_{zf}^2 - \mu_{if}^2)(\mu_{if}^2 - \mu_{zf}^2)}
\]

(13c)

\[
a_{zf} = \frac{\mu_{zf} + \mu_{if}}{\mu_{if} - \mu_{zf}},
\]

(13d)

\[
a_{zf} = \frac{\mu_{zf} + \mu_{if}}{\mu_{if} - \mu_{zf}}
\]

(13e)

\[
a_{zf} = \alpha_{zf},
\]

(13f)
\[
\begin{align*}
    a_{7f} &= \frac{2\mu_{2f}}{\mu_{1f} - \mu_{2f}} \frac{m_{2f}}{m_{1f}} a_{2f}, \\
    a_{8f} &= \frac{2\mu_{2f}}{\mu_{2f} - \mu_{1f}} \frac{F + C \mu_{3f} S_{3f}}{L_{m_{1f}}} a_{3f}, \\
    a_{9f} &= \frac{2\mu_{1f}}{\mu_{2f} - \mu_{1f}} \frac{m_{1f}}{m_{2f}} a_{1f}, \\
    a_{10f} &= \frac{2\mu_{1f}}{\mu_{1f} - \mu_{2f}} \frac{F + C \mu_{3f} S_{3f}}{L_{m_{2f}}} a_{3f}, \\
    a_{11} &= \frac{L \beta_{1} + [(F + L) \beta_{2} - C \beta_{1}] \mu_{i} \mu_{ii}}{C \mu_{i} (\mu_{i}^{2} - \mu_{2i}^{2}) (\mu_{i}^{2} - \mu_{3i}^{2})}, \\
    a_{21} &= \frac{L \beta_{2} + [(F + L) \beta_{2} - C \beta_{2}] \mu_{i}^{2}}{C \mu_{2i} (\mu_{2i}^{2} - \mu_{3i}^{2}) (\mu_{i}^{2} - \mu_{3i}^{2})}, \\
    a_{31} &= \frac{L \beta_{3} + [(F + L) \beta_{3} - C \beta_{3}] \mu_{i}^{2}}{C \mu_{3i} (\mu_{3i}^{2} - \mu_{2i}^{2}) (\mu_{i}^{2} - \mu_{2i}^{2})}, \\
    a_{41} &= \frac{L \mu_{i} a_{t}^{*} - y_{t}^{*} - \mu_{i} (F a_{t}^{*} + C \mu_{3} b_{3i})}{C L_{m_{i}} m_{i} (\mu_{i}^{2} - \mu_{2i}^{2}) (\mu_{i}^{2} - \mu_{3i}^{2})}, \\
    a_{51} &= \frac{L \mu_{i} a_{t}^{*} - y_{t}^{*} - \mu_{i} (F a_{t}^{*} + C \mu_{3} b_{3i})}{C L_{m_{2i}} m_{2i} (\mu_{2i}^{2} - \mu_{3i}^{2}) (\mu_{i}^{2} - \mu_{3i}^{2})}, \\
    a_{61} &= \frac{[A \alpha_{x} - F (F + L) \alpha_{x} b_{3i}^{*} - A L \alpha_{x} b_{3i}]}{C \mu_{3} [L \mu_{2i}^{4} + F (F + 2L) - A C] \mu_{2i}^{4} + A L}, \\
    a_{71} &= \frac{\mu_{2i} (F a_{t}^{*} + C \mu_{2} b_{2i}^{*}) - L (\mu_{2i} a_{t}^{*} - b_{2i}^{*})}{C L_{m_{2i}} m_{2i} (\mu_{2i}^{2} - \mu_{3i}^{2}) (\mu_{i}^{2} - \mu_{3i}^{2})}, \\
    a_{81} &= \frac{1}{L_{m_{2i}} m_{2i} (\mu_{i}^{2} - \mu_{2i}^{2})} \left[ C \mu_{2i} S_{3i} + L \frac{S_{3i}}{\mu_{2i}} + C \mu_{2i} \mu_{3i} b_{3i}^{*} - L (\mu_{2i} a_{t}^{*} - b_{3i}^{*}) \right], \\
    a_{91} &= \frac{1}{L_{m_{2i}} m_{2i} (\mu_{i}^{2} - \mu_{2i}^{2})} \left[ C \mu_{3i} S_{3i} + L \frac{S_{3i}}{\mu_{3i}} + C \mu_{3i} \mu_{5i} b_{5i}^{*} - L (\mu_{3i} a_{t}^{*} - b_{5i}^{*}) \right], \\
    a_{10} &= \frac{1}{L_{m_{2i}} m_{2i} (\mu_{i}^{2} - \mu_{2i}^{2})} \left[ C \mu_{i} S_{3i} + L \frac{S_{3i}}{\mu_{i}} + C \mu_{i} \mu_{5i} b_{5i}^{*} - L (\mu_{i} a_{t}^{*} - b_{5i}^{*}) \right], \\
    b_{a} &= a_{af} S_{af}, \\
    \beta_{f} &= a_{af} S_{f}.
\end{align*}
\]
\[ b_{2f} = a_{2f} S_{2f}, \]  
\[ b_{3f} = a_{3f} S_{3f}, \]  
\[ b_{4f} = \frac{\mu_{4f} + \mu_{2f}}{\mu_{4f} - \mu_{2f}} b_{2f}, \]  
\[ b_{5f} = \frac{\mu_{2f} + \mu_{1f}}{\mu_{2f} - \mu_{1f}} b_{2f}, \]  
\[ b_{b} = b_{2f}, \]  
\[ b_{7f} = \frac{2\mu_{2f} S_{1f} m_{2f}}{\mu_{1f} - \mu_{2f}} a_{2f}, \]  
\[ b_{8f} = \frac{2\mu_{2f} S_{1f}}{\mu_{1f} - \mu_{2f}} \frac{F + C_{5f} S_{5f}}{m_{2f} S_{4f} m_{1f}} a_{1f}, \]  
\[ b_{9f} = \frac{2\mu_{1f} S_{2f} m_{1f}}{\mu_{2f} - \mu_{1f}} a_{1f}, \]  
\[ b_{10f} = \frac{2\mu_{1f} S_{2f}}{\mu_{2f} - \mu_{1f}} \frac{F + C_{5f} S_{5f}}{m_{2f} S_{4f} m_{2f}} a_{1f}, \]  
\[ b_{1t} = \frac{L \beta \mu_{1i}^2 + (F + L) \beta_r - A \beta_z}{C L (\mu_{1i}^2 - \mu_{2i}^2) (\mu_{1i}^2 - \mu_{3i}^2)}, \]  
\[ b_{2t} = \frac{L \beta \mu_{2i}^2 + (F + L) \beta_r - A \beta_z}{C L (\mu_{2i}^2 - \mu_{3i}^2) (\mu_{2i}^2 - \mu_{3i}^2)}, \]  
\[ b_{3t} = \frac{L \beta \mu_{3i}^2 + (F + L) \beta_r - A \beta_z}{C L (\mu_{1i}^2 - \mu_{2i}^2) (\mu_{2i}^2 - \mu_{3i}^2)}, \]  
\[ b_{4t} = S_{1i} a_{4i}, \]  
\[ b_{5t} = S_{2i} a_{5i}, \]  
\[ b_{6t} = S_{3i} a_{6i}, \]  
\[ b_{7t} = S_{1i} a_{7i}, \]  
\[ b_{8t} = S_{1i} a_{8i}, \]  
\[ b_{9t} = S_{2i} a_{9i}, \]  
\[ b_{10t} = S_{2i} a_{10i}. \]
In addition, the characteristic roots $\mu_{ij} = \mu_{if}$, $\mu_{2j} = \mu_{2t}$, and $\mu_{ij}(i = 1, 2; j = f, t)$ must satisfy the following characteristic equation:

$$C L \mu_{ij}^{\Delta} - \left[ A C - F(F + 2L) \right] \mu_{ij} + AL = 0.$$  \hspace{1cm} (17)

The fluid and thermal characteristic roots $\mu_{3f}$ and $\mu_{3t}$ are defined by

$$\mu_{3f} = \sqrt{k_x / k_z}, \quad \mu_{3t} = \sqrt{\lambda_x / \lambda_z}. \hspace{1cm} (18)$$

In equation (12a), the distance symbols are shown below:

$$R_{1f} = \sqrt{r^2 + \mu_{1j}^2(z - h)^2 + \mu_{1j}(z - h)}, \hspace{1cm} (19a)$$

$$R_{2f} = \sqrt{r^2 + \mu_{2j}^2(z - h)^2 + \mu_{2j}(z - h)}, \hspace{1cm} (19b)$$

$$R_{3f} = \sqrt{r^2 + \mu_{3j}^2(z - h)^2 + \mu_{3j}(z - h)}, \hspace{1cm} (19c)$$

$$R_{4f} = \sqrt{r^2 + \mu_{4j}^2(z + h)^2 + \mu_{4j}(z + h)}, \hspace{1cm} (19d)$$

$$R_{5f} = \sqrt{r^2 + \mu_{5j}^2(z + h)^2 + \mu_{5j}(z + h)}, \hspace{1cm} (19e)$$

$$R_{6f} = \sqrt{r^2 + \mu_{6j}^2(z + h)^2 + \mu_{6j}(z + h)}, \hspace{1cm} (19f)$$

$$R_{7f} = \sqrt{r^2 + \left(\mu_{1j}z + \mu_{2j}h\right)^2 + \mu_{1j}z + \mu_{2j}h}, \hspace{1cm} (19g)$$

$$R_{8f} = \sqrt{r^2 + \left(\mu_{1j}z + \mu_{3j}h\right)^2 + \mu_{1j}z + \mu_{3j}h}, \hspace{1cm} (19h)$$

$$R_{9f} = \sqrt{r^2 + \left(\mu_{2j}z + \mu_{1j}h\right)^2 + \mu_{2j}z + \mu_{1j}h}, \hspace{1cm} (19i)$$

$$R_{10f} = \sqrt{r^2 + \left(\mu_{2j}z + \mu_{3j}h\right)^2 + \mu_{2j}z + \mu_{3j}h}, \hspace{1cm} (19j)$$

$$R_{1t} = \sqrt{r^2 + \mu_{1t}^2(z - h)^2 + \mu_{1t}(z - h)}, \hspace{1cm} (20a)$$

$$R_{2t} = \sqrt{r^2 + \mu_{2t}^2(z - h)^2 + \mu_{2t}(z - h)}, \hspace{1cm} (20b)$$

$$R_{3t} = \sqrt{r^2 + \mu_{3t}^2(z - h)^2 + \mu_{3t}(z - h)}, \hspace{1cm} (20c)$$

$$R_{4t} = \sqrt{r^2 + \mu_{4t}^2(z + h)^2 + \mu_{4t}(z + h)}, \hspace{1cm} (20d)$$

$$R_{5t} = \sqrt{r^2 + \mu_{5t}^2(z + h)^2 + \mu_{5t}(z + h)}, \hspace{1cm} (20e)$$

$$R_{6t} = \sqrt{r^2 + \mu_{6t}^2(z + h)^2 + \mu_{6t}(z + h)}, \hspace{1cm} (20f)$$

$$R_{7t} = \sqrt{r^2 + \left(\mu_{1t}z + \mu_{2t}h\right)^2 + \mu_{1t}z + \mu_{2t}h}, \hspace{1cm} (20g)$$
\[ R_{3t} = \sqrt{r^2 + (\mu_1 z + \mu_3 h)^2 + \mu_1 z + \mu_3 h}, \]  
(20h)

\[ R_{3s} = \sqrt{r^2 + (\mu_2 z + \mu_4 h)^2 + \mu_2 z + \mu_4 h}, \]  
(20i)

\[ R_{10t} = \sqrt{r^2 + (\mu_2 z + \mu_5 h)^2 + \mu_2 z + \mu_5 h}. \]  
(20j)

Besides, the symbols \( a_{it}^*(i=1,2,3) \), \( b_{ij}^*(i=1,2,3) \), \( m_{ij}(i=1,2) \), \( m_u(i=1,2) \), \( S_{ij}(i=1,\cdots,4) \) and \( S_a(i=1,\cdots,4) \) in equations (13) to (16) are defined in equations (21) to (26) as below:

\[ a_{it}^* = \frac{1}{\mu_{it}} \{ L \beta_i + [(F + L) \beta_z] - C \beta_i \mu_{it} \}, \]  
(21a)

\[ a_{2t}^* = \frac{1}{\mu_{2t}} \{ L \beta_i + [(F + L) \beta_z] - C \beta_i \mu_{2t} \}, \]  
(21b)

\[ a_{3t}^* = \frac{1}{\mu_{3t}} \{ L \beta_i + [(F + L) \beta_z] - C \beta_i \mu_{3t} \}, \]  
(21c)

\[ b_{it}^* = L \beta_i \mu_{it}^2 + [(F + L) \beta_r] - A \beta_i, \]  
(22a)

\[ b_{2t}^* = L \beta_i \mu_{2t}^2 + [(F + L) \beta_r] - A \beta_i, \]  
(22b)

\[ b_{3t}^* = L \beta_i \mu_{3t}^2 + [(F + L) \beta_r] - A \beta_i, \]  
(22c)

\[ m_{1f} = \frac{C \mu_{1f}^2 + F}{C \mu_{1f}^2 - L}, \]  
(23a)

\[ m_{2f} = \frac{C \mu_{2f}^2 + F}{C \mu_{2f}^2 - L}, \]  
(23b)

\[ m_u = \frac{C \mu_u^2 + F}{C \mu_u^2 - L}, \]  
(24a)

\[ m_{2t} = \frac{C \mu_{2t}^2 + F}{C \mu_{2t}^2 - L}, \]  
(24b)

\[ S_{1f} = \frac{L \mu_{1f}^2 - A}{(F + L) \mu_{1f}} = \frac{(F + L) \mu_{1f}}{L - C \mu_{1f}^2}, \]  
(25a)

\[ S_{2f} = \frac{L \mu_{2f}^2 - A}{(F + L) \mu_{2f}} = \frac{(F + L) \mu_{2f}}{L - C \mu_{2f}^2}, \]  
(25b)

\[ S_{3f} = \frac{L \mu_{3f}^2 + F + L - A}{L + (F + L - C) \mu_{3f}}. \]  
(25c)
The displacement expressions of the closed-form solutions (12a) and (12b) contain two terms, one for the constant thermal strength $Q_t$ and another describing the fluid volume $Q_f$ effect per unit time under the injection of hot fluid into an impervious isothermal half space. All of the derived field quantities are functions of the distance from the hot fluid injection source. Those quantities are inversely proportional to the hydraulic permeability or thermal conductivity. Besides, the mechanical moduli do not have influence on the long-term excess pore fluid and temperature increment of the strata.

4. Conclusions

Based on the theory of thermally poroelasticity, the long-term closed-form solutions of a homogeneous cross-anisotropic impervious isothermal half space for axially symmetric deformations, excess pore fluid pressure, and temperature increment subjected to a point injection of hot fluid are presented by equations (12a) to (12d). The displacement expressions of the closed-form solutions contain two terms, one regarding the constant thermal strength $Q_t$ and another describing the fluid volume $Q_f$ effect per unit time under the injection of hot fluid into the impervious isothermal half space. The results can provide better understanding of this kind of problem.

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