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Extraction of optical solitons in birefringent fibers for Biswas-Arshed equation via extended trial equation method

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Abstract: This article obtains optical solitons to the Biswas-Arshedequation for birefringent fibers with higher order dispersions and in the absence of four-wave mixing terms, in a media with Kerr type nonlinearity. Optical dark, singular and bright soliton solutions are articulated by applying an imaginative integration technique, the extended trial equation scheme. Various additional traveling wave solutions are produced with this integration technique, which include rational solutions, Jacobi elliptic function solutions and periodic singular solutions. From the mathematical analysis some constraints are recognized that ensure the actuality of solitons.

Keywords: Extended trial equation method; Birefringent fibers; Biswas-Arshed equation; Optical solitons

1 Introduction

Solitons play a pivotal role in almost every field of life and are prominently domineering in optics by virtue of their development and control in optical fibers and transmission of data across trans-continental distances. In the context of optical fibers the investigation of solitons is the conspicuous area of research in electrical engineering, telecommunication industry and applied sciences [1–13]. There is an extensive range of models that commendably describe the dynamics of formation of solitons such as the nonlinear Schrödinger’s equation, Fokas-Lenells equation, Kaup-Newell equation, Lakshmanan-Porsezian-Daniel model, complex Ginzburg-Landau equation, Gerdjikov-Ivanov equation, Kundu-Eckhaus equation, Radhakrishnan-Kundu-Lakshmanan equation [14–25]. The propagation of solitons through optical fibers has been remain an engaging subject due its stunning applications in all optical communication systems. The area of telecommunication has experienced a remarkable development due to substantial progress in the industry of optical fibers. In recent few decades numerous results have been stated with polarization conserving fibers. Here we scrutinize the dynamics of solitons in the context of birefringent fibers where the pulses are polarized. Several investigations have been made for the extraction of optical solitons in birefringent fibers [26–34].

The Biswas-Arshed equation (BAE) reads [35]

\[ i \Omega_t + a_1 \Omega_{xx} + a_2 \Omega_{xt} + i(b_1 \Omega_{xxx} + b_2 \Omega_{xxt}) = i \left[ \lambda \left( |\Omega|^2 \Omega \right)_x + \mu \left( |\Omega|^2 \right)_x \Omega + \theta |\Omega|^2 \Omega_x \right], \]

where \( \Omega(x, t) \) represents the wave profile. In the recent works, this equation has engrossed a remarkable deliberation, see for Refs. [36–45], where the solitons are studied only along one component of this equation. Optical solitons for the BAE in birefringent fibers are investigated via extended \((G'/G)\)-expansion method, modified simple equation method, trial equation scheme, exp-function method, auxiliary equation method, Sine-Cosine method and Jacobi elliptic function expansion approached [46–52]. Additionally, the similar study on analytical solutions on different mathematical models involving fluid’s mechanics [53-58], heat and mass transfer [59-65], nanofluids and nanoparticles [66-69], electrical instruments [70] and few other multi-dimensional models are adhered therein [71-72]. However, in this article the BAE will be investigated for optical solitons in birefringent fibers without four-wave mixing (4WM) terms with the aid of extended trial equation method.
2 Governing equation

The coupled system obtained from (1) in birefringent fibers without 4WM reads [46]

\[
\begin{align*}
    ip_t + a_1 p_{xx} + b_1 p_{xt} + il(c_1 q_{xxx} + d_1 q_{xxt}) &= \frac{i[\lambda_1(|p|^2)_{xx} + y_1(|q|^2)_{xx} + i[\beta_1(|p|^2) + \Theta_1(|p|^2)|p_x] + i[\alpha_1(|q|^2)_{xx} + \mu_1(|p|^2)q_x]} \\
    iq_t + a_2 q_{xx} + b_2 q_{xt} + il(c_2 q_{xxx} + d_2 q_{xxt}) &= \frac{i[\alpha_2(|q|^2)_{xx} + y_2(|p|^2)_{xx} + i[\beta_2(|p|^2) + \Theta_2(|p|^2)|q_x] + i[\alpha_2(|q|^2)_{xx} + \mu_2(|p|^2)q_x]}.
\end{align*}
\]

(2)

Here \(b_{\ell}\) and \(a_{\ell}\) are the coefficients of spatio-temporal dispersion (STD) and group velocity dispersion respectively, while \(d_{\ell}\) and \(c_{\ell}\) are the coefficients of third order STD and third order dispersion respectively, for \(\ell = 1, 2\). Next, \(y_{\ell}\) and \(\lambda_{\ell}\) stand for self-steepening terms, while the nonlinear dispersions are confirmed by \(\beta_{\ell}\), \(\Theta_{\ell}\), \(a_{\ell}\) and \(\mu_{\ell}\).

2.1 Mathematical analysis

To retrieve optical solitons of the coupled system (2), we utilize the traveling wave transformations

\[
\begin{align*}
    p(x, t) &= \Omega_1(\zeta)e^{i\phi(x, t)}, \\
    q(x, t) &= \Omega_2(\zeta)e^{i\phi(x, t)},
\end{align*}
\]

where

\[
\phi(x, t) = -\kappa x + \omega t + \epsilon \quad \text{and} \quad \zeta = x - vt.
\]

(5)

Here \(\Omega_1(\zeta)\) is the amplitude, \(\nu\) gives the soliton velocity, \(\epsilon\) is the phase constant, \(\kappa\) and \(\omega\) are respectively the frequency and wave number of the soliton.

Implanting Eqs. (3)–(5) into (2) and dividing into imaginary and real parts, we attain

\[
\begin{align*}
    (c_{\ell} - d_{\ell}v)\Omega''_{\ell} + (\kappa b_{\ell} + \kappa^2 v d_{\ell} - \nu - 3\kappa^2 c_{\ell} + 2\kappa\omega d_{\ell} - 2\kappa a_{\ell} + b_{\ell}\omega)\Omega'_{\ell} \\
    -2(\mu_{\ell} + \Theta_{\ell} + 3\lambda_{\ell})\Omega'_{\ell}\Omega^2_{\ell} - 2a_{\ell}\Omega_{\ell}\Omega'_{\ell} - \beta_{\ell}\Omega^2_{\ell}\Omega'_{\ell} - 3y_{\ell}\Omega^2_{\ell}\Omega^2_{\ell} = 0,
\end{align*}
\]

(6)

and

\[
\begin{align*}
    (a_{\ell} - b_{\ell}v - 2\kappa v d_{\ell} + 3\kappa c_{\ell} + d_{\ell}\omega)\Omega''_{\ell} - (\omega + \kappa^2 a_{\ell} + \kappa^3 c_{\ell} - b_{\ell}\kappa - \kappa^2 d_{\ell}\omega)\Omega'_{\ell} \\
    -\kappa(\lambda_{\ell} + \Theta_{\ell})\Omega^3_{\ell} - \beta_{\ell}\Omega^2_{\ell}\Omega_{\ell} - y_{\ell}\kappa\Omega^2_{\ell} = 0,
\end{align*}
\]

(7)

where \(\ell' = 3 - \ell\) and \(\ell = 1, 2\). The balancing principle suggests that \(\Omega_{\ell'} = \Omega_{\ell}\).

Therefore, from Eq. (6) we have

\[
\begin{align*}
    (c_{\ell} - d_{\ell}v)\Omega''_{\ell} + (kb_{\ell} + \kappa^2 v d_{\ell} - \nu - 3\kappa^2 c_{\ell} + 2\kappa\omega d_{\ell} - 2\kappa a_{\ell} + b_{\ell}\omega)\Omega'_{\ell} \\
    -2(\mu_{\ell} + \Theta_{\ell} + 3\lambda_{\ell})\Omega'_{\ell}\Omega^2_{\ell} - 2a_{\ell}\Omega_{\ell}\Omega'_{\ell} - \beta_{\ell}\Omega^2_{\ell}\Omega'_{\ell} - 3y_{\ell}\Omega^2_{\ell}\Omega^2_{\ell} = 0.
\end{align*}
\]

(8)

From Eq. (8), we attain

\[
\nu = \frac{3\kappa^2 c_{\ell} - 2\kappa\omega d_{\ell} + 2\kappa a_{\ell} - b_{\ell}\omega}{b_{\ell}\kappa + d_{\ell}\kappa^2 - 1} \quad \text{and} \quad \nu = \frac{c_{\ell}}{d_{\ell}},
\]

(9)

along with the restraint conditions

\[
2\mu_{\ell} + \Theta_{\ell} + 3\lambda_{\ell} + \beta_{\ell} + 2a_{\ell} + 3y_{\ell} = 0, \quad \ell = 1, 2.
\]

(10)

Comparing the two values of \(\nu\) in (9), leads to the restraint

\[
3c_{\ell}d_{\ell}^2 - 2\kappa\omega d_{\ell}^2 + 2d_{\ell}a_{\ell} - \omega d_{\ell} - \kappa c_{\ell} - \kappa^2 d_{\ell}c_{\ell} + c_{\ell} = 0, \quad \ell = 1, 2.
\]

(11)

Now, from Eq. (7), we have

\[
\begin{align*}
    (a_{\ell} - b_{\ell}v - 2\kappa v d_{\ell} + 3\kappa c_{\ell} + d_{\ell}\omega)\Omega''_{\ell} - (\omega + \kappa^2 a_{\ell} + \kappa^3 c_{\ell} - b_{\ell}\kappa - \kappa^2 d_{\ell}\omega)\Omega'_{\ell} \\
    -\kappa(\lambda_{\ell} + \Theta_{\ell} + \beta_{\ell} + y_{\ell})\Omega^2_{\ell} = 0,
\end{align*}
\]

(12)

where \(\ell = 1, 2\). Eq. (12) will now be scrutinized by extended trial equation method.
2.2 Application of extended trial equation method

In this subsection, we employ the extended trial equation technique \([7,8]\) to Eq. (12) for constructing the exact solutions of the system (2).

**Case-1.** The solution of Eq. (12) can be expressed as

\[
\Omega_{\ell} = \sum_{i=0}^{\vartheta} \delta_i \Psi^i, \tag{13}
\]

where \(\delta_i\) are unknown constant to be determined such that \(\delta_0 \neq 0\) and

\[
(\Psi')^2 = \Lambda(\Psi) = \frac{\Gamma(\Psi)}{\Omega(\Psi)} = \frac{\eta_0 \Psi^\vartheta + \cdots + \eta_1 \Psi + \eta_0}{\chi_0 \Psi^\varrho + \cdots + \chi_1 \Psi + \chi_0}, \tag{14}
\]

where \(\eta_0, \ldots, \eta_\vartheta\) and \(\chi_0, \ldots, \chi_\varrho\) are arbitrary constants to be identified such that \(\eta_0 \neq 0\) and \(\chi_0 \neq 0\). Eq. (14) can be transformed into an integral form as:

\[
\pm (\zeta - \zeta_0) = \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}} = \int \sqrt{\frac{\Omega(\Psi)}{\Gamma(\Psi)}} d\Psi. \tag{15}
\]

The balancing process reveals that

\[
\sigma = \rho + 2Q + 2. \tag{16}
\]

By assuming \(\sigma = 4, \rho = 1\) and \(\rho = 0\) in (16), we arrive at

\[
\Omega_{\ell} = \delta_0 + \delta_1 \Psi. \tag{17}
\]

Implanting Eq. (17) along with Eq. (14) into (12) and evaluating the resultant system of equations, we attain

\[
\eta_0 = \eta_0, \quad \eta_1 = \eta_1, \quad \chi_0 = \chi_0, \quad \delta_0 = \delta_0, \quad \delta_1 = \delta_1,
\]

\[
\omega = \eta_1 \delta_1 \{a_{\ell} + 3c_{\ell}k - b_{\ell}v^2 - 2 \ell \alpha k - \ell v\} - 2 \ell \delta_0 \chi_0 \{a_{\ell}k + \ell \lambda + \ell \beta + \ell y\},
\]

\[
\eta_2 = 4 \ell \delta_0 \chi_0 \{b_{\ell}k + \ell \lambda + \ell \beta + \ell y\} \{a_{\ell}k + \ell \lambda + \ell \beta + \ell y\},
\]

\[
\eta_3 = \eta_1 \delta_1 \{a_{\ell} - a_{\ell} b_{\ell} k - \ell b_{\ell} v + \ell k_{2} k\},
\]

\[
\eta_4 = \frac{k_1 = b_{\ell}k + \ell \lambda + \ell \beta + \ell y\} \{a_{\ell} - a_{\ell} b_{\ell} k - \ell b_{\ell} v + \ell k_{2} k\},
\]

\[
\eta_5 = \frac{k_2 = b_{\ell}k + \ell \lambda + \ell \beta + \ell y\} \{a_{\ell} - a_{\ell} b_{\ell} k - \ell b_{\ell} v + \ell k_{2} k\},
\]

Substituting the values of parameters from (18) into (14) and using Eq. (15), we obtain

\[
\pm (\zeta - \zeta_0) = \chi \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}},
\]

where

\[
\chi = \sqrt{\frac{\chi_0}{\eta_4}} \tag{20}
\]

\[
\Lambda(\Psi) = \Psi^\vartheta + \eta_3 \Psi^3 + \eta_2 \Psi^2 + \eta_1 \Psi + \eta_0. \tag{21}
\]
As a consequence, the following exact solutions can now be written for the coupled system (2).

For $\Lambda(\Psi - u_1)^4$,

$$p(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_1 + 3 c_1 k - b_1 v - 2 d_1 k v] - 2 \delta_0 X_0 [a_1 k + c_1 k^2 + \delta_2 (\theta_1 + \lambda_1 + \beta_1 + \gamma_1)]}{\eta_1 \delta_1 d_1 - 2 \delta_0 X_0 (b_1 k + d_1 k^2 - 1)} \right] t + \epsilon} \times \left[ \delta_0 + \delta_1 v_1 \pm \frac{\delta_1 x}{x - vt - \zeta_0} \right],$$

(23)

$$q(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_2 + 3 c_1 k - b_2 v - 2 d_2 k v] - 2 \delta_0 X_0 [a_2 k + c_2 k^2 + \delta_2 (\theta_2 + \lambda_2 + \beta_2 + \gamma_2)]}{\eta_1 \delta_1 d_2 - 2 \delta_0 X_0 (b_2 k + d_2 k^2 - 1)} \right] t + \epsilon} \times \left[ \delta_0 + \delta_1 v_1 \pm \frac{\delta_1 x}{x - vt - \zeta_0} \right].$$

(24)

If $\Lambda(\Psi - u_1)^3(\Psi - u_2)$ and $u_2 > u_1$,

$$p(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_1 + 3 c_1 k - b_1 v - 2 d_1 k v] - 2 \delta_0 X_0 [a_1 k + c_1 k^2 + \delta_2 (\theta_1 + \lambda_1 + \beta_1 + \gamma_1)]}{\eta_1 \delta_1 d_1 - 2 \delta_0 X_0 (b_1 k + d_1 k^2 - 1)} \right] t + \epsilon} \times \left[ \delta_0 + \delta_1 v_1 + \frac{4 \delta_1 x^2 (u_2 - u_1)}{4 x^2 - [(u_1 - u_2)(x - vt - \zeta_0)]^2} \right],$$

(25)

$$q(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_2 + 3 c_2 k - b_2 v - 2 d_2 k v] - 2 \delta_0 X_0 [a_2 k + c_2 k^2 + \delta_2 (\theta_2 + \lambda_2 + \beta_2 + \gamma_2)]}{\eta_1 \delta_1 d_2 - 2 \delta_0 X_0 (b_2 k + d_2 k^2 - 1)} \right] t + \epsilon} \times \left[ \delta_0 + \delta_1 v_1 + \frac{4 \delta_1 x^2 (u_2 - u_1)}{4 x^2 - [(u_1 - u_2)(x - vt - \zeta_0)]^2} \right].$$

(26)

However, if $\Lambda(\Psi - u_1)^2(\Psi - u_2)^2$,

$$p(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_1 + 3 c_1 k - b_1 v - 2 d_1 k v] - 2 \delta_0 X_0 [a_1 k + c_1 k^2 + \delta_2 (\theta_1 + \lambda_1 + \beta_1 + \gamma_1)]}{\eta_1 \delta_1 d_1 - 2 \delta_0 X_0 (b_1 k + d_1 k^2 - 1)} \right] t + \epsilon} \times \left[ \frac{\delta_1 (u_2 - u_1)}{\exp \left\{ \frac{u_1 - u_2}{x} (x - vt - \zeta_0) \right\} - 1} \right],$$

(27)

$$q(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_2 + 3 c_2 k - b_2 v - 2 d_2 k v] - 2 \delta_0 X_0 [a_2 k + c_2 k^2 + \delta_2 (\theta_2 + \lambda_2 + \beta_2 + \gamma_2)]}{\eta_1 \delta_1 d_2 - 2 \delta_0 X_0 (b_2 k + d_2 k^2 - 1)} \right] t + \epsilon} \times \left[ \frac{\delta_1 (u_2 - u_1)}{\exp \left\{ \frac{u_1 - u_2}{x} (x - vt - \zeta_0) \right\} - 1} \right],$$

(28)

and

$$p(x, t) = e^{\left[ -\frac{\kappa x + \eta_1 \delta_1 [a_1 + 3 c_1 k - b_1 v - 2 d_1 k v] - 2 \delta_0 X_0 [a_1 k + c_1 k^2 + \delta_2 (\theta_1 + \lambda_1 + \beta_1 + \gamma_1)]}{\eta_1 \delta_1 d_1 - 2 \delta_0 X_0 (b_1 k + d_1 k^2 - 1)} \right] t + \epsilon} \times \left[ \frac{\delta_1 (u_1 - u_2)}{\exp \left\{ \frac{u_1 - u_2}{x} (x - vt - \zeta_0) \right\} - 1} \right],$$

(29)
By assuming $\psi_1$, the solutions given by (23)–(32) can be transformed to the plane wave solutions

$$q(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left[ \delta_0 + \delta_1 u_1 + \frac{\delta_1 (u_1 - u_2)}{\exp \left\{ \frac{u_1 - u_2}{\chi} (x - vt - \zeta_0) \right\} - 1} \right].$$

(30)

Whenever $\Lambda(\psi) = (\psi - u_1)^2(\psi - u_2)(\psi - u_3)$ and $u_1 > u_2 > u_3$,

$$p(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left[ \delta_0 + \delta_1 u_1 - \frac{2 \delta_1 (u_1 - u_2)(u_1 - u_3)}{2u_1 - u_2 - u_3 + (u_1 - u_2) \cosh \left( \frac{\sqrt{(u_1 - u_2)(u_1 - u_3)} (x - vt - \zeta_0)}{\chi} \right)} \right],$$

(31)

$$q(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left[ \delta_0 + \delta_1 u_1 - \frac{2 \delta_1 (u_1 - u_2)(u_1 - u_3)}{2u_1 - u_2 - u_3 + (u_1 - u_2) \cosh \left( \frac{\sqrt{(u_1 - u_2)(u_1 - u_3)} (x - vt - \zeta_0)}{\chi} \right)} \right].$$

(32)

Finally, if $\Lambda(\psi) = (\psi - u_1)(\psi - u_2)(\psi - u_3)(\psi - u_4)$ and $u_1 > u_2 > u_3 > u_4$,

$$p(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left[ \delta_0 + \delta_1 u_2 + \frac{\delta_1 (u_1 - u_2)(u_2 - u_4)}{u_4 - u_2 + (u_1 - u_4) s n^2 \left( \frac{\sqrt{(u_1 - u_2)(u_2 - u_4)} (x - vt - \zeta_0), m} {2 \chi} \right)} \right],$$

(33)

$$q(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left[ \delta_0 + \delta_1 u_2 + \frac{\delta_1 (u_1 - u_2)(u_2 - u_4)}{u_4 - u_2 + (u_1 - u_4) s n^2 \left( \frac{\sqrt{(u_1 - u_2)(u_2 - u_4)} (x - vt - \zeta_0), m} {2 \chi} \right)} \right].$$

(34)

where

$$m^2 = \frac{(u_2 - u_3)(u_1 - u_4)}{(u_1 - u_3)(u_2 - u_4)},$$

(35)

and $u_j, j = 1, \ldots, 4$ are the roots of $\Lambda(\psi) = 0$.

By assuming $\delta_0 = -\delta_1 v_1$ and $\zeta_0 = 0$, the solutions given by (23)–(32) can be transformed to the plane wave solutions

$$p(x, t) = e^{i \xi x + \eta \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 k v] - 2 \xi \delta_0 \chi_0 (a_1 k + c_1 k^2 + \delta_0^2 (\theta_1 + \lambda_1 + \beta_1 + y_3)] t + \epsilon}$$

$$\times \left( \frac{\delta_1 \chi}{x - vt} \right),$$

(36)
singular soliton solutions

\[
p(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_1 + 3c_2K - b_2v - 2d_1Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_2K + d_2K^2 - 1)} \right] t + \epsilon} \\
\times \left( \frac{\pm \delta_1 x}{x - vt} \right),
\]

(37)

\[
q(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_2 + 3c_2K - b_2v - 2d_2Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_2K + d_2K^2 - 1)} \right] t + \epsilon} \\
\times \left( \frac{4\delta_1 x^2 (u_2 - u_1)}{4\chi^2 - (u_1 - u_2)(x - vt)^2} \right),
\]

(38)

\[
q(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_2 + 3c_2K - b_2v - 2d_2Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_2K + d_2K^2 - 1)} \right] t + \epsilon} \\
\times \left( \frac{4\delta_1 x^2 (u_2 - u_1)}{4\chi^2 - (u_1 - u_2)(x - vt)^2} \right),
\]

(39)

bright soliton solutions

\[
p(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_1 + 3c_1K - b_1v - 2d_1Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_1K + d_1K^2 - 1)} \right] t + \epsilon} \\
\times \left[ \frac{\delta_1 (u_2 - u_1)}{2} \left\{ 1 + \coth \left( \frac{u_1 - u_2}{2\chi} (x - vt) \right) \right\} \right],
\]

(40)

\[
q(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_1 + 3c_1K - b_1v - 2d_1Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_1K + d_1K^2 - 1)} \right] t + \epsilon} \\
\times \left[ \frac{\delta_1 (u_2 - u_1)}{2} \left\{ 1 + \coth \left( \frac{u_1 - u_2}{2\chi} (x - vt) \right) \right\} \right],
\]

(41)

Moreover, when \( \delta_0 = -\delta_1 u_2 \) and \( \zeta_0 = 0 \), Jacobi elliptic function solutions (33) and (34) trimmed as

\[
p(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_1 + 3c_1K - b_1v - 2d_1Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_1K + d_1K^2 - 1)} \right] t + \epsilon} \\
\times \left[ \frac{-\delta_1 (u_1 - u_2)(u_1 - u_3)}{2u_1 - u_2 - u_3 + (u_3 - u_2) \cosh \left( \frac{\sqrt{(u_1 - u_2)(u_1 - u_3)}}{\chi} (x - vt) \right) \right],
\]

(42)

\[
q(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_2 + 3c_2K - b_2v - 2d_2Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_2K + d_2K^2 - 1)} \right] t + \epsilon} \\
\times \left[ \frac{-\delta_1 (u_1 - u_2)(u_1 - u_3)}{2u_1 - u_2 - u_3 + (u_3 - u_2) \cosh \left( \frac{\sqrt{(u_1 - u_2)(u_1 - u_3)}}{\chi} (x - vt) \right) \right],
\]

(43)

Moreover, when \( \delta_0 = -\delta_1 u_2 \) and \( \zeta_0 = 0 \), Jacobi elliptic function solutions (33) and (34) trimmed as

\[
p(x, t) = e^{i \left[ -\kappa x + \frac{\eta_1 \delta_1 [a_1 + 3c_1K - b_1v - 2d_1Kv]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_1K + d_1K^2 - 1)} \right] t + \epsilon} \\
\times \left[ \frac{\delta_1 (u_1 - u_2)(u_2 - u_3)}{u_4 - u_2 + (u_1 - u_4) \sn^2 \left( \frac{\sqrt{(u_1 - u_2)(u_1 - u_4)}}{2\chi} (x - vt) \right) \left( \frac{u_2 - u_3}{u_1 - u_3} \right) \left( \frac{u_2 - u_3}{u_1 - u_3} \right) \right],
\]

(44)
Through the transformation Case-2.

\[
p(x, t) = e^{i \left(-kx + \frac{\eta_1 \delta_1 [a_2 + 3c_2 \kappa - b_2 v - 2d_2 \kappa v]}{\eta_1 \delta_1 d_2 - 2\delta_0 \chi_0 (b_2 \kappa + d_2 \kappa^2 - 1)} + \epsilon \right)} 
\times \left[ \frac{\delta_1 (u_1 - u_2) (u_4 - u_2)}{u_4 - u_2 + (u_1 - u_4) \sin^2 \left( \frac{\sqrt{(u_1 - u_2)(u_2 - u_4)} (x - vt)}{2X} \right)} \right],
\]

(45)

Remark 1. When the modulus \( m \rightarrow 1 \), singular soliton solution emerge as

\[
p(x, t) = e^{i \left(-kx + \frac{\eta_1 \delta_1 [a_1 + 3c_1 \kappa - b_1 v - 2d_1 \kappa v]}{\eta_1 \delta_1 d_1 - 2\delta_0 \chi_0 (b_1 \kappa + d_1 \kappa^2 - 1)} + \epsilon \right)} 
\times \left[ \frac{\delta_1 (u_1 - u_2) (u_4 - u_2)}{u_4 - u_2 + (u_1 - u_4) \tanh^2 \left( \frac{\sqrt{(u_1 - u_2)(u_2 - u_4)} (x - vt)}{2X} \right)} \right],
\]

(46)

Remark 2. When the modulus \( m \rightarrow 0 \), periodic singular solution emerge as

\[
p(x, t) = e^{i \left(-kx + \frac{\eta_1 \delta_1 [a_2 + 3c_2 \kappa - b_2 v - 2d_2 \kappa v]}{\eta_1 \delta_1 d_2 - 2\delta_0 \chi_0 (b_2 \kappa + d_2 \kappa^2 - 1)} + \epsilon \right)} 
\times \left[ \frac{\delta_1 (u_1 - u_2) (u_4 - u_2)}{u_4 - u_2 + (u_1 - u_4) \sin^2 \left( \frac{\sqrt{(u_1 - u_2)(u_2 - u_4)} (x - vt)}{2X} \right)} \right],
\]

(48)

Case-2. Eq. (12) can be written as

\[
(a_\ell - vb_\ell - 2kd_\ell + 3k^2d_\ell - \omega d_\ell)(2P_\ell P_\ell^* - (P_\ell^\prime)^2) - 4\kappa(\theta_\ell + \lambda_\ell + \beta_\ell + y_\ell)P_\ell^2 \\
-4(\omega + a_\ell \kappa^2 + c_\ell \kappa^3 - b_\ell \omega \kappa - d_\ell \kappa^2 \omega)P_\ell^2 = 0,
\]

(50)

through the transformation \( \Omega_\ell = P_\ell^{1/2} \). Therefore, the solution of Eq. (50) can be expressed as

\[
P_\ell = \sum_{i=0}^{\theta} \delta_i \psi^i,
\]

(51)
where $\delta_{i}$ are unknown constant to be determined such that $\delta_{0} \neq 0$. The balancing process reveals that

$$\sigma = \rho + \rho + 2.$$  \hfill (52)

By assuming $\sigma = 3$, $\rho = 0$ and $\theta = 1$ in Eq. (52), we arrive at

$$P_{t} = \delta_{0} + \delta_{1}\Psi.$$  \hfill (53)

Implanting Eq. (53) along with Eq. (14) into Eq. (50) and evaluating the resultant system of equations, we attain

$$\chi_{0} = \chi_{0}, \quad \delta_{0} = \delta_{0}, \quad \delta_{1} = \delta_{1}, \quad \eta_{3} = \eta_{3},$$

$$\omega = \frac{2\eta_{3}(a_{\pm} + 3c_{/} \kappa - b_{/} \sqrt{2}d_{/} \nu - 2d_{/} d_{/} \nu)}{2\eta_{3}d_{/}} ,$$

$$\eta_{0} = \frac{\delta_{0}^{2} \left[ k_{1} - 2a_{/} \eta_{3}(b_{/} \kappa - 1) + 4\delta_{1}^{2} \chi_{0}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/}) \right]}{d_{/} \delta_{1}^{2}} ,$$

$$\eta_{1} = \frac{\delta_{0} \left[ k_{2} + k_{3} + 4b_{/} \nu \eta_{3}(b_{/} \kappa - 1) + 8\delta_{1} \chi_{0}(d_{/} + d_{/} \kappa - 1)(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/}) \right]}{d_{/} \delta_{1}},$$

$$\eta_{2} = \frac{k_{4} - 2a_{/} \eta_{3}(b_{/} \kappa - 1) + 4\delta_{1}^{2} \chi_{0}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/})}{d_{/} \delta_{1}} ,$$

where

$$k_{1} = 2\eta_{3} \left[ d_{/} \kappa \left( 3b_{/} \nu \kappa - 2(c_{/} \kappa + \nu) \right) + (b_{/} \kappa - 1)(b_{/} \nu - 3c_{/} \kappa) + 2d_{/}^{2} \nu \kappa \right] + \eta_{3} d_{/} \kappa \delta_{0}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/}),$$

$$k_{2} = 3d_{/} \delta_{0} \eta_{3}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/} + 4\kappa \delta_{1} \eta_{3} \left( 2d_{/}^{2} \nu \kappa - 2d_{/} (c_{/} \kappa + \nu) + 3c_{/} \kappa \eta_{3} \right),$$

$$k_{3} = -4b_{/} \nu \eta_{3} \left( 3\kappa^{2}(c_{/} \kappa - d_{/} \nu) + \nu \right) + 8b_{/} \nu \kappa \delta_{1} \chi_{0}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/}),$$

$$k_{4} = 2\eta_{3} \left[ d_{/} \kappa \left( 3b_{/} \nu \kappa - 2(c_{/} \kappa + \nu) \right) + (b_{/} \kappa - 1)(b_{/} \nu - 3c_{/} \kappa) + 2d_{/}^{2} \nu \kappa \right] + 3\eta_{3} d_{/} \kappa \delta_{0}(\theta_{/} + \lambda_{/} + \beta_{/} + \gamma_{/}).$$  \hfill (54)

Substituting all the values from (54) into (14) and using Eq. (15), we obtain

$$z(\zeta - \zeta_{0}) = X \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}},$$  \hfill (56)

where

$$X = \sqrt{\frac{\chi_{0}}{\eta_{3}}} ,$$

$$\Lambda(\Psi) = \Psi^{3} + \frac{\eta_{2} \Psi^{2} + \eta_{1} \Psi + \eta_{0}}{\eta_{3}} .$$  \hfill (58)

As a consequence, the following exact solutions can now be written for the coupled system (2).

For $\Lambda(\Psi) = (\Psi - \nu_{1})^{3}$, we obtain

$$p(x, t) = e^{i \left[ -kx + \frac{2\eta_{3}(a_{1} + 3c_{1} \kappa - b_{1} \nu - 2d_{1} \nu) - 4\delta_{1} \chi_{0}(\theta_{1} + \lambda_{1} + \beta_{1} + \gamma_{1})}{2\eta_{3}d_{1}} \right] t + e} \times \left[ \delta_{0} + \delta_{1} \nu_{1} + \frac{4\delta_{1} \chi}{(x - vt - \zeta_{0})^{2}} \right]^{1/2},$$

$$q(x, t) = e^{i \left[ -kx + \frac{2\eta_{3}(a_{2} + 3c_{2} \kappa - b_{2} \nu - 2d_{2} \nu) - 4\delta_{1} \chi_{0}(\theta_{2} + \lambda_{2} + \beta_{2} + \gamma_{2})}{2\eta_{3}d_{2}} \right] t + e} \times \left[ \delta_{0} + \delta_{1} \nu_{1} + \frac{4\delta_{1} \chi}{(x - vt - \zeta_{0})^{2}} \right]^{1/2} .$$  \hfill (59, 60)
If $A(\Psi) = (\Psi - u_1)^2(\Psi - u_2)\) and $u_2 > u_1$,

\[
p(x, t) = i \left[ -kx + \frac{2 \eta_1 (a_1 + 3 c_1 \kappa - b_1 v - 2 d_1 \kappa v) - 4 \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1)}{2 \eta_3 d_1} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_2 + \delta_1 (u_1 - u_2) \tanh \left( \frac{1}{2} \sqrt{\frac{v_1 - u_2}{X}} (x - vt - \zeta_0) \right) \right]^{1/2},
\]

(61)

\[
q(x, t) = i \left[ -kx + \frac{2 \eta_2 (a_2 + 3 c_2 \kappa - b_2 v - 2 d_2 \kappa v) - 4 \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2)}{2 \eta_3 d_2} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_2 + \delta_1 (u_1 - u_2) \tanh \left( \frac{1}{2} \sqrt{\frac{v_1 - u_2}{X}} (x - vt - \zeta_0) \right) \right]^{1/2}.
\]

(62)

However, if $A(\Psi) = (\Psi - u_1)(\Psi - u_2)^2$, and $u_1 > u_2$,

\[
p(x, t) = i \left[ -kx + \frac{2 \eta_1 (a_1 + 3 c_1 \kappa - b_1 v - 2 d_1 \kappa v) - 4 \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1)}{2 \eta_3 d_1} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_1 + \delta_1 (u_1 - u_2) \csch^2 \left( \frac{1}{2} \sqrt{\frac{v_1 - u_2}{X}} (x - vt - \zeta_0) \right) \right]^{1/2},
\]

(63)

\[
q(x, t) = i \left[ -kx + \frac{2 \eta_2 (a_2 + 3 c_2 \kappa - b_2 v - 2 d_2 \kappa v) - 4 \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2)}{2 \eta_3 d_2} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_1 + \delta_1 (u_1 - u_2) \csch^2 \left( \frac{1}{2} \sqrt{\frac{v_1 - u_2}{X}} (x - vt - \zeta_0) \right) \right]^{1/2}.
\]

(64)

Whenever $A(\Psi) = (\Psi - u_1)(\Psi - u_2)(\Psi - u_3)$ and $u_1 > u_2 > u_3$,

\[
p(x, t) = i \left[ -kx + \frac{2 \eta_1 (a_1 + 3 c_1 \kappa - b_1 v - 2 d_1 \kappa v) - 4 \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1)}{2 \eta_3 d_1} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_3 + \delta_1 (u_2 - u_3) \sn \left( \frac{1}{2} \sqrt{\frac{v_1 - u_3}{X}} (x - vt - \zeta_0) \right) \right]^{1/2},
\]

(65)

\[
q(x, t) = i \left[ -kx + \frac{2 \eta_2 (a_2 + 3 c_2 \kappa - b_2 v - 2 d_2 \kappa v) - 4 \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2)}{2 \eta_3 d_2} t + e \right]
\]

\[
\times \left[ \delta_0 + \delta_1 u_3 + \delta_1 (u_2 - u_3) \sn \left( \frac{1}{2} \sqrt{\frac{v_1 - u_3}{X}} (x - vt - \zeta_0) \right) \right]^{1/2},
\]

(66)

where

\[
m^2 = \frac{u_2 - u_1}{u_1 - u_3},
\]

(67)

and $u_j$, $j = 1, 2, 3$ are the roots of $A(\Psi) = 0$.

By assuming $\delta_0 = -\delta_1 u_1$ and $\zeta_0 = 0$, the solutions given by (59)–(64) can be transformed to rational solutions

\[
p(x, t) = \sqrt{4 \delta_1 \chi_0 \over x - vt} \cdot i \left[ -kx + \frac{2 \eta_1 (a_1 + 3 c_1 \kappa - b_1 v - 2 d_1 \kappa v) - 4 \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1)}{2 \eta_3 d_1} t + e \right],
\]

(68)

\[
q(x, t) = \sqrt{4 \delta_1 \chi_0 \over x - vt} \cdot i \left[ -kx + \frac{2 \eta_2 (a_2 + 3 c_2 \kappa - b_2 v - 2 d_2 \kappa v) - 4 \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2)}{2 \eta_3 d_2} t + e \right],
\]

(69)

bright soliton solutions

\[
p(x, t) = i \left[ -kx + \frac{2 \eta_1 (a_1 + 3 c_1 \kappa - b_1 v - 2 d_1 \kappa v) - 4 \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1)}{2 \eta_3 d_1} t + e \right]
\]

\[
\times \left[ \sqrt{\delta_1 (u_2 - u_1)} \sech \left( \frac{1}{2} \sqrt{\frac{v_1 - u_2}{X}} (x - vt) \right) \right],
\]

(70)
can be trimmed as

\[ q(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_2 + 3 c_2 \kappa - b_2 \nu - 2 d_2 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt) \right], \]  

(71)

and singular soliton solutions

\[ p(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_1 + 3 c_1 \kappa - b_1 \nu - 2 d_1 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{csch} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt) \right) \right], \]  

(72)

\[ q(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_2 + 3 c_2 \kappa - b_2 \nu - 2 d_2 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{csch} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt) \right) \right], \]  

(73)

where these solutions are valid for \( \delta_1 > 0 \). Moreover, if \( \delta_0 = -\delta_1 \nu_3 \) and \( \zeta_0 = 0 \), the Jacobi elliptic solutions (65) and (66) can be trimmed as

\[ p(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_1 + 3 c_1 \kappa - b_1 \nu - 2 d_1 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{sn} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt), \frac{(u_2 - u_3)}{(u_1 - u_3)} \right) \right], \]  

(74)

\[ q(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_2 + 3 c_2 \kappa - b_2 \nu - 2 d_2 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{sn} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt), \frac{(u_2 - u_3)}{(u_1 - u_3)} \right) \right]. \]  

(75)

**Remark 3.** When \( m \to 1 \), dark soliton solutions emerge as

\[ p(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_1 + 3 c_1 \kappa - b_1 \nu - 2 d_1 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_1 + \lambda_1 + \beta_1 + y_1) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{tanh} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt) \right) \right], \]  

(76)

\[ q(x, t) = e^{i \left[ -k^2 + 2 \eta^2 (a_2 + 3 c_2 \kappa - b_2 \nu - 2 d_2 \kappa y) - 4 \kappa \delta_1 \chi_0 (\theta_2 + \lambda_2 + \beta_2 + y_2) \right] t + \epsilon} \]

\[ \times \left[ \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} \mathrm{tanh} \left( \frac{1}{2} \sqrt{\frac{u_1 - u_2}{\kappa}} (x - vt) \right) \right], \]  

(77)

for \( u_1 = u_2 \).
3 Conclusion

The work expounded in this article successfully addresses optical solitons of Biswas-Arshed equation with Kerr-law nonlinearity in birefringent fibers with higher order dispersions and in the absence of four-wave mixing terms by the application of extended trial equation technique. With this integration scheme, we have commendably recovered dark, bright and singular optical solitons along with other traveling wave solutions, comprising rational solutions, periodic singular solutions and Jacobi elliptic function solutions in the presence of some constraints. It is concluded that our derived results for the Biswas-Arshed equation in birefringent fibers are exclusively new and have not been stated earlier. The outcomes of this paper are attention-grabbing and provide a stimulus to the audience of optical solitons. Later, this equation will be studied with the addition of four-wave mixing terms by the aid of appreciated integration schemes. These precious results will be presented as soon as possible.

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