Black holes with massive graviton hair

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No-hair theorems exclude the existence of nontrivial scalar and massive vector hair outside four-dimensional, static, asymptotically flat black-hole spacetimes. We show, by explicitly building nonlinear solutions, that black holes can support massive graviton hair in theories of massive gravity. These hairy solutions are, most likely, the generic end state of the recently discovered monopole instability of Schwarzschild black holes in massive graviton theories.

I. INTRODUCTION

Schwarzschild black holes (BHs) stand out among all possible solutions of general relativity as the only static, asymptotically flat, regular solution of vacuum Einstein equations. They are, in addition, stable solutions of the theory. Perhaps surprisingly, the Schwarzschild solution also solves many other field equations, such as generic scalar-tensor theories, f(R) theories and Chern-Simons gravity (see e.g. Refs. [1, 2]). In fact, it is possible to show that the Schwarzschild solution is the only static, asymptotically flat, regular solution also in the vacuum of these theories.

These uniqueness properties are in agreement with various “no-hair” proofs that Schwarzschild BHs cannot support regular scalar hair, nor fields mediating the weak or the strong interaction [3, 4].

The case of spin-2 hair is much less clear. It was shown by Bekenstein that BHs cannot support massive spin-2 fields in theories with generic nonminimal couplings to curvature, at least as long as the graviton mass is sufficiently large [5]. However, as proved by Aragone and Deser [6, 7], it is impossible to couple consistently a spin-2 field with a nonlinear gravitational theory. This result does not leave much room for BHs with spin-2 hair, unless the massive tensor field is itself the mediator of the gravitational interaction, i.e. in the case of massive theories of gravity [7, 8].

Even in the case of massive gravity, recent searches for nonlinear spherically symmetric solutions [9] seem to put a rest to the possibility of finding static, asymptotically flat BH solutions endowed with spin-2 hair.

On the other hand, the nonexistence of hairy BHs in massive gravities seems at odds with the recent finding that Schwarzschild BHs are unstable in generic theories with light massive spin-2 fields [10–13]. The instability is due to a propagating spherically symmetric degree of freedom and it is a long-wavelength instability. It only occurs for a nonvanishing mass coupling μMS ≲ 0.438, with μ being the graviton mass and MS the mass of the background BH (hereafter we use G = c = ̄h = 1 units).

Interestingly, for values of MS and μ that are phenomenologically relevant, the mass coupling μMS is always well within the instability region. Indeed, it is natural to consider the graviton mass of the order of the Hubble constant, μ ∼ H ∼ 10−33 eV, in order to account for an effective cosmological constant (see e.g. Ref. [14]). This tiny value implies that a graviton with mass μ ∼ H would trigger an instability for any Schwarzschild BH with mass smaller than 1022 MS! Even if the instability timescale τ can be extremely long (τ ∼ 1.43/μ in the small-mass limit [12]), as a matter of principle if Schwarzschild BHs are unstable in massive gravity, they must decay to something (or not even be formed in the first place) and, unless cosmic censorship is violated, the final state should be a spherically symmetric BH.

This apparent conundrum prompts the following question, which motivates the present study: do spherically symmetric, asymptotically flat BH solutions surrounded by a graviton cloud exist in theories with a massive graviton? Here, we show that such solutions do indeed exist and were not found in the thorough analysis of Ref. [10] simply because they were not searched for explicitly.

II. SETUP

Because our purpose is merely to show that hairy BHs do exist in theories of massive gravity, we focus on a specific example of such theories and consider the most general ghost-free Lagrangian of two interacting spin-2 fields, without matter couplings, given by [15]

\[ \mathcal{L} = \sqrt{|g|} \left[ m_g^2 R_g + m_f^2 \sqrt{1/g} R_f - 2 m_g^2 \sqrt{g} V(g, f) \right]. \] (1)

Here Rg and Rf are the Ricci scalars corresponding to \( g_{\mu\nu} \) and \( f_{\mu\nu} \), respectively; \( m_g^2 = 16 \pi G = 16 \pi, m_f^2 = 16 \pi G \) are the corresponding gravitational couplings, and \( m_v \) is written in terms of \( m_g, m_f \) and of the parameters of the potential term. The quantities \( f, g \) denote the determinant of the respective metric. The potential is
schematically written as
\[ V \equiv \sum_{n=0}^{4} \beta_n V_n(\gamma), \quad \gamma_{\mu \nu} = \left( \sqrt{g^{-1}} f \right)^{\mu \nu}, \] (2)
where \( \beta_n \) are real parameters and
\[ V_0 = 1, \quad V_1 = [\gamma], \quad V_2 = \frac{1}{2} (|\gamma|^2 - |\gamma|^2), \]
\[ V_3 = \frac{1}{6} (|\gamma|^3 - 3|\gamma||\gamma|^2 + 2|\gamma|^3), \quad V_4 = \det(\gamma), \] (3)
where the square brackets denote the matrix trace.

The parameters \( \beta_n \) are not all independent if flat space is to be a solution of the theory. They can be written in terms of two free parameters \( \alpha_3 \) and \( \alpha_4 \) defined as
\[ \beta_n = (-1)^{n+1} \left( \frac{1}{2} (3 - n)(4 - n) - (4 - n)\alpha_3 - \alpha_4 \right). \] (4)

The graviton mass \( \mu \) can be written in terms of the other parameters of the theory as
\[ \mu = \frac{m_{\gamma}^2}{m_f} \sqrt{1 + m_f^2/m_{\gamma}^2}. \] (5)

The Lagrangian (1) gives rise to two sets of modified Einstein equations for \( g_{\mu \nu} \) and \( f_{\mu \nu} \).
\[ R_{\mu \nu}(g) - \frac{1}{2} g_{\mu \nu} R(g) + \frac{m_f^4}{m_{\gamma}^2} T_{\mu \nu}^g(\gamma) = 0, \] (6)
\[ R_{\mu \nu}(f) - \frac{1}{2} f_{\mu \nu} R(f) + \frac{m_f^4}{m_{\gamma}^2} T_{\mu \nu}^f(\gamma) = 0, \] (7)
where the “graviton” stress-energy tensors \( T_{\mu \nu}^g \) and \( T_{\mu \nu}^f \) are explicitly given by
\[ T_{\mu \nu}^g = \sum_{n=0}^{3} (-1)^n \beta_n g_{\mu \nu} Y(\gamma), \] (8)
\[ T_{\mu \nu}^f = \sum_{n=0}^{3} (-1)^n \beta_4 - n f_{\mu \nu} Y(\gamma^{-1}), \] (9)
with \( Y(\gamma) = \sum_{n=0}^{3} (-1)^n \gamma^{n-2} V_n(\gamma) \). The Bianchi identity implies the conservation conditions
\[ \nabla_g^\mu T_{\mu \nu}^g(\gamma) = 0, \quad \nabla_f^\mu T_{\mu \nu}^f(\gamma) = 0, \] (10)
where \( \nabla_g \) and \( \nabla_f \) are the covariant derivatives with respect to \( g_{\mu \nu} \) and \( f_{\mu \nu} \) respectively. In fact, these two conditions are not independent due to the diffeomorphism invariance of the interaction term in (1), which is a general property of the “Fierz-Pauli like” interaction terms (16).

We consider static spherically symmetric solutions of Eqs. (6) and (7). The most general ansatz for the metrics is given by\( ^3 \)
\[ g_{\mu \nu} dx^{\mu} dx^{\nu} = -F^2 dt^2 + B^{-2} dr^2 + R^2 d\Omega^2, \] (11)
\[ f_{\mu \nu} dx^{\mu} dx^{\nu} = -p^2 dt^2 + b^2 dr^2 + U^2 d\Omega^2, \] (12)
where \( F, B, R, p, b \) and \( U \) are radial functions. The gauge freedom allow us to reparametrize the radial coordinate \( r \) such that \( R(r) = r \). To simplify the equations we also introduce the radial function \( Y(r) \) defined as \( b = U' / Y \), where \( ' = d/dr \).

Inserting (11) and (12) into the equations of motion (3) and (4), and using the conservation condition (10), we can reduce the problem to a system of three coupled first-order ordinary differential equations, which can be schematically written as (for a detailed derivation see 10)
\[ \begin{cases} B' = F_1(r, B, Y, U, \mu, m_f, m_g, \alpha_3, \alpha_4) \\ Y' = F_2(r, B, Y, U, \mu, m_f, m_g, \alpha_3, \alpha_4) \\ U' = F_3(r, B, Y, U, \mu, m_f, m_g, \alpha_3, \alpha_4) \end{cases} \] (13)

The remaining two functions \( F \) and \( p \) can then be evaluated
\[ F^{-1} F' = F_4(r, B, Y, U, \mu, m_f, m_g, \alpha_3, \alpha_4), \] (14)
\[ F^{-1} p = F_5(r, B, Y, U, \mu, m_f, m_g, \alpha_3, \alpha_4). \] (15)

The explicit form of the functions \( F_i \) is somewhat lengthy and not very instructive. The derivation of Eqs. (13)–(15) and their final form is publicly available online in a Mathematica notebook 17.

A. Boundary conditions at the horizon

Since we are interested in BH solutions, we assume the existence of an event horizon at \( r_H \), where \( F(r_H) = B(r_H) = 0 \). From the discussion in 18, 19 where it is shown that for the spacetime to be nonsingular the two metrics must share the same horizon, it follows that \( Y \) and \( p \) must also have a simple root at \( r = r_H \). On the other hand, the function \( U \) can have any finite value different from zero at the horizon. For numerical purposes we then assume a power-series expansion at the horizon,
\[ B^2 = \sum_{n=0}^{\infty} a_n (r - r_H)^n, \quad Y^2 = \sum_{n=0}^{\infty} b_n (r - r_H)^n, \] (16)
\[ U = u_H r_H + \sum_{n=1}^{\infty} c_n (r - r_H)^n. \] (17)

After inserting this into the system (13), \( a_n, b_n, c_n \) all can be expressed in terms of \( u_H \) and \( a_1 \) only, where the

\(^1\) Note that massive graviton theories might also allow for spherically symmetric solutions whose metrics are not both diagonal in the same coordinates 10.

Since we are interested in the end state of the monopole instability found in Refs. 11, 12, we focus here on the ansatz (11)–(12).
constant $u_H$ is arbitrary while $a_1$ is given by the solution of a quadratic equation
\[ Aa_1^2 + Ba_1 + C = 0, \tag{18} \]
where $A, B, C$ are functions of $u_H, r_H, \mu, m_f, m_g$ and $\alpha_3, \alpha_4$. Since there are two solutions for this equation for each choice of the parameters, there exist two different branches of solutions for the metric functions. Moreover, reality of $a_1$ requires $B^2 > 4AC$, and this condition restricts the parameter space.

Inserting (16)–(17) into Eqs. (14) and (15), we find
\[ F^2 = q^2(r - r_H) + q^2 \sum_{n \geq 2} d_n(r - r_H)^n, \tag{19} \]
\[ p^2 = q^2 \sum_{n \geq 1} e_n(r - r_H)^n, \tag{20} \]
where $d_n$ and $e_n$ can be expressed in terms of $u_H$ and of the other parameters and $q$ is an integration constant, which can be set arbitrarily and is related to the time-scaling symmetry.

Equations (13) are invariant under the following transformations:
\[ B(r) \to B(\lambda r), \quad U(r) \to U(\lambda r), \quad Y(r) \to Y(\lambda r), \]
\[ U(r) \to \frac{1}{\lambda} U(\lambda r), \quad \mu \to \frac{\mu}{\lambda}. \tag{21} \]
The parameter $u_H = U(r_H)/r_H$ remains invariant under the transformations above and the rescaling $r_H \to r_H/\lambda$. We use this rescaling to express all dimensionful quantities in terms of the mass of a Schwarzschild BH with horizon $r_H$, i.e., $M_S = r_H/2$. We also consider without loss of generality $m_f = m_g$.

Another important quantity that can be used to check the validity of the solutions is the temperature of each horizon, which can be evaluated as
\[ T = T_S = \frac{q \sqrt{\mu_1}}{4\pi} = T_F = \frac{q \sqrt{\mu_3}}{4\pi}. \tag{22} \]
These two temperatures can be shown to be the same for any value of the parameters, in agreement with the discussion of Ref. [10]. To evaluate the temperature we fix the constant $q$ by requiring that $F(r) \to 0$ (or, equivalently, $p(r) \to 0$) as $r \to \infty$.

B. Asymptotically flat solutions

We require the solutions to be asymptotically flat such that as $r \to \infty$, we have $B = 1 + \delta B$, $Y = 1 + \delta Y$, $U = r + \delta U$, where the variations are taken to be small. Inserting this in the field equations (13), we obtain to first order
\[ \delta B = \frac{C_1}{2r}, \quad \delta Y = \frac{C_1}{2r} - \frac{C_2(1 + r\mu)}{2r} e^{-\mu r}, \tag{23} \]
\[ \delta U = \frac{C_2(1 + r\mu + \mu^2 r^2)}{2r^2} e^{-\mu r}, \tag{24} \]
\[ \delta U = \frac{C_1}{2r} + \frac{C_2(1 + r\mu)}{2r} e^{-\mu r}, \tag{25} \]
where $C_1$ and $C_2$ are integration constants. Finally, we can find asymptotically flat solutions constants. Finally, we can find asymptotically flat solutions numerically using a shooting method.

III. RESULTS

For fixed values of $\mu, \alpha_3$ and $\alpha_4$, we integrate the system (13) starting from the horizon with the boundary conditions (16)–(17), towards large $r$ and find the values of the shooting parameter $u_H$ for which the solution matches the asymptotic behavior (23). For each choice of $\mu, \alpha_3$ and $\alpha_4$, there are two branches of solutions, corresponding to the two roots of the quadratic equation (18). In most cases only one of the branches will give an asymptotically flat solution.

As expected, a trivial solution for any value of $\mu, \alpha_3$ and $\alpha_4$ is obtained when $u_H = 1$, and it corresponds to the two metrics being equal and described by the Schwarzschild solution. However, we also find different solutions for which $u_H \neq 1$ and that correspond to regular, asymptotically flat BHs endowed with a nontrivial massive-graviton hair. We note that such solutions were not found in Ref. [10], most likely because the free parameter $u_H$ was not adjusted in order to enforce asymptotic flatness.

Our results are summarized in Figs. (14). The first important result is that hairy solutions exist near the threshold $\mu M_S \lesssim 0.438$ for any value of $\alpha_3, \alpha_4$. This number precisely corresponds to the critical threshold for the Gregory-Laflamme instability [20], which was found at the linear level [11, 12]. Solutions were expected to exist close to this threshold and, in fact, this expectation has prompted our search at the nonlinear level.

We also find that for any value of $\alpha_3$ and $\alpha_4, M/M_S, u_H$ and $(M_S T)^{-1}$ are monotonically increasing functions of $(\mu M_S)^{-1}$ as shown in Fig. (1). Here $M$ is the spacetime mass evaluated from the asymptotic behavior at infinity as $M = C_1/2$ [cf. Eqs. (23)].

Above the threshold $\mu M_S \gtrsim 0.438$, the Schwarzschild solution is linearly stable. Consistent with the linear analysis, the only asymptotically flat solution that we were able to find in this region is the Schwarzschild solution, labeled by $u_H = 1$ and $M = M_S$.

A. Parameter space

The behavior at smaller $\mu M_S$ is more convoluted as it depends strongly on higher curvature terms: the nonlinear terms of the potential [2] become important and the solution differs substantially from the eigenfunctions found in Ref. [12] at linearized level. Nevertheless, after an extensive analysis of the full two-dimensional parameter space $(\alpha_3, \alpha_4)$, we obtain the following classification:

(i) $\alpha_3 \neq -\alpha_4 \cup \alpha_3 = -\alpha_4 \lesssim 1$ — The solutions stop to exist below a cutoff $\mu_c M_S$, where the two branches of solutions near the horizon merge.
(ii) $1 \lesssim \alpha_3 = -\alpha_4 \lesssim 2$ – The solutions disappear only near $\mu M_S \sim 0.01$ and are singular at small $\mu M_S$, because some component of the metric $g_{\mu \nu}$ is vanishing where the metric $g_{\mu \nu}$ is regular (see Fig. 1). This causes the stress-energy tensor of Eq. (7) to become singular at these points.

(iii) $\alpha_3 = -\alpha_4 \gtrsim 2$ – The solutions exist for arbitrarily small $\mu M_S$ and are nonsingular. This schematic classification of the parameter space is shown in Fig. 2. Although the details of this division depend on the particular model we are considering (namely, the massive bimetric theories of Ref. [15]), we believe that qualitatively similar features are likely to occur in other possible nonlinear completions of massive gravity.

It is important to emphasize that an analysis of the full parameter space is an extraordinary task. As such, it is extremely difficult to guarantee that the parameter space is divided as depicted in Fig. 2 as we cannot rule out certain choices of $(\alpha_3, \alpha_4)$ not belonging to the above classes. Also, the numerical integration becomes increasingly more challenging in the small-$\mu$ limit. We were able to obtain solutions for mass coupling as small as $\mu M_S \sim 0.001$ and found no indication that, in the region $\alpha_3 = -\alpha_4 \gtrsim 2$, such solutions cease to exist. However, our numerical procedure cannot guarantee that hairy BHs exist for arbitrarily small $\mu$.

The change of behavior between different regions seems to be smooth, since near the boundaries the solutions do not change drastically. For example, in the vicinity of $\alpha_3 = -\alpha_4 = 1$ the solutions behave all in the same way. We compare the solutions for different choices of $\alpha_3$ and $\alpha_4$ in Figs. 3 and 4.

Nevertheless, the above classification seems very natural from the mathematical structure of the field equations. For instance, the choice $\alpha_3 = -\alpha_4$ corresponds to $\beta_3 = 0$, i.e., the higher-order term $V_3$ is absent in the potential (2). Furthermore, in this case,

$$\beta_0 = -6 + 3\alpha_3, \quad \beta_1 = 3 - 2\alpha_3, \quad \beta_2 = 1 + \alpha_3, \quad \beta_4 = \alpha_4.$$  \hspace{1cm} (26)

Thus, the boundaries where the behavior of the solutions change qualitatively correspond to a change of sign of the parameters $\beta_0$. It is also not surprising to find that $\alpha_3 = -\alpha_4 = 1, 2$ are special points of the parameter space, because they correspond to the cases where $V_2$ and $V_0$ are, respectively, absent in the potential (2).

Finally, the above picture does not hold in the limit where one of the metrics is taken to be the nondynamical Schwarzschild metric ($m_g \gg m_f$). In this case our numerical search suggests that, for any choice of $\alpha_3$ and $\alpha_4$, hairy BH solutions exist near the threshold but they do not exist for arbitrarily small $\mu M_S$. This could be explained by the fact that in the decoupling limit of massive gravity ($\mu \to 0, m_g \to \infty$, keeping $(\mu^2 m_g)^{1/3}$ fixed) the interactions of the helicity-0 coming from the potential (2) can be decomposed in Galileon-like terms [7], which cannot support nontrivial configurations around a spherically symmetric BH [21].
FIG. 3. The function $U'(r)$ for different values of the mass $\mu M_S$. The behavior is similar for any value $\alpha_3$ and $\alpha_4$ near the threshold $\mu M_S \sim 0.438$ but for small $\mu M_S$ it can be very different depending on the specific values of the parameters. Top panel: $\alpha_3 = 2, \alpha_4 = -2$. Middle panel: $\alpha_3 = 1, \alpha_4 = -1$. Bottom panel: $\alpha_3 = 1, \alpha_4 = 0$.

To summarize, although it is very challenging to infer the behavior of the solutions for all choices of the parameters $\alpha_n$, we have found convincing evidence that the term $V_3$ in the potential \[ V_3 \] plays an important role as it does not allow for hairy deformations of a Schwarzschild BH in the small graviton mass limit. This term is precisely the one that gives rise to a mixing between the helicity-0 and the helicity-2 components of the massive graviton in the decoupling limit \[ . \]

FIG. 4. Metric functions for small masses $\mu M_S$. Top panel: $\alpha_3 = 2, \alpha_4 = -2$. Bottom panel: $\alpha_3 = 1, \alpha_4 = -1$.

IV. DISCUSSION

As far as we are aware, the nonlinear solutions we have found are the first example of graviton-hairy BH solutions in asymptotically flat spacetime.

It is a matter of debate if the theory we considered can in fact be a viable alternative to general relativity (see e.g. \[ [22–25] \] and also Sec.VI of Ref. \[ [26] \] for a recent discussion on the status of massive gravity). Nevertheless, whatever the fate of the ghost-free massive and bimetric gravities, these solutions are interesting on their own as they provide the first example of an asymptotically flat graviton-hairy BH. Furthermore, we believe that several of the properties we have presented here are likely to be found in other hairy BH solutions of any putative nonlinear theory of massive gravity.

These solutions are also natural candidates for the final state of the monopole instability recently uncovered \[ [11–13] \]. The instability would presumably cause the Schwarzschild spacetime to evolve towards a hairy solution. Depending on the parameters of the theory, however, different types of solutions exist in the highly nonlinear regime. This suggests that hairy, static, asymptotically flat BH solutions exist only in certain regions of the parameter space. This in turn makes nonlinear time evolutions of Schwarzschild BHs highly desirable. It is of course possible that, in some regions of parameter space, Schwarzschild BHs are not the preferred outcome of gravitational collapse or even that these theories do not allow...
for stable static BH solutions. These issues can only be addressed by performing nonlinear collapse simulations.

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