Michel parameters for $\tau$ decays $\tau \rightarrow l\nu \bar{\nu}$ ($l = e$, $\mu$) in a general two Higgs doublet model with $\mu - \tau$ flavor violation

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In a general two Higgs doublet model (2HDM), the anomaly of muon anomalous magnetic moment (muon g-2) can be explained by $\mu - \tau$ flavor violating Yukawa couplings, motivated by the recent CMS excess in Higgs boson decay $h \rightarrow \mu \tau$. We study Michel parameters for $\tau$ decays $\tau \rightarrow l\nu \bar{\nu}$ ($l = e$, $\mu$) in the 2HDM with the lepton flavor violation, and show that they can be sensitive to the flavor structure as well as the Lorentz and chiral structures of the model. We find that the correction to the Michel parameter $\xi_\mu$ in $\tau \rightarrow \mu \nu \bar{\nu}$ is correlated to the contribution to the muon g-2, and it can be as large as $10^{-4} - 10^{-2}$ in the parameter region where the $\mu - \tau$ flavor violating Yukawa couplings explain the muon g-2 anomaly. Therefore the precision measurement of the Michel parameter at the level of $10^{-4} - 10^{-2}$ would significantly probe the interesting parameter space for the solution to the muon g-2 anomaly.

I. INTRODUCTION

The discovery of a Higgs boson at the Large Hadron Collider (LHC) \cite{1,2} as well as the consistency of almost all low energy experimental results show the remarkable success of the standard model (SM) of elementary particles. On the other hand, the theoretical understanding of the Higgs sector is still poor. There are no apparent theoretical reason that the Higgs sector has to have the simplest structure (one Higgs doublet) in contrast to the matter sector which has three generation structure. Therefore, the extended Higgs sector would deserve to be studied to make a deep understanding of the nature of Higgs sector.

One of simple extensions of the SM Higgs sector is a two Higgs doublet model
(2HDM)\(^1\), where one more Higgs doublet is added into the SM. In a general 2HDM where both Higgs doublets couple to all fermions \(^2\), flavor violating phenomena mediated by the Higgs bosons are predicted \(^3\). Without any experimental supports, such a flavor violation beyond the SM has been considered to be problematic \(^{10–13}\).

However, the CMS collaboration has reported an excess in a flavor violating Higgs boson decay \(h \to \mu \tau\) at \(\sqrt{s} = 8\) TeV \(^{14}\), and the best fit value of the branching ratio is

\[
\text{BR}(h \to \mu \tau) = (0.84^{+0.39}_{-0.37})\%,
\]

(1)

and the deviation from the SM prediction is 2.4\(\sigma\). Recently, the CMS collaboration reported a result based on an integrated luminosity of 2.3 fb\(^{-1}\) at \(\sqrt{s} = 13\) TeV and no excess is observed \(^{15}\), but it is not sensitive enough to exclude the 8 TeV result. The ATLAS collaboration has also reported their results \(^{16,17}\) and the current best fit value of the branching ratio is

\[
\text{BR}(h \to \mu \tau) = (0.53 \pm 0.51)\%,
\]

(2)

which is consistent with the SM prediction as well as the CMS result shown above. Although the origin of the excess is not conclusive yet and more data are needed, the CMS excess in the flavor violating Higgs boson decay becomes a good motivation to study the flavor violating phenomena predicted in the 2HDM and multi-Higgs doublet model \(^3\).

In Refs. \(^{39,40}\), we have shown a possibility that a general 2HDM with \(\mu - \tau\) flavor violation can explain both the CMS excess in \(h \to \mu \tau\) and an anomaly of muon anomalous magnetic moment (muon g-2) as reported, for example, by \(^{41}\)\(^4\),

\[
a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}.
\]

(3)

\(^{1}\) See a recent review \(^3\).
\(^{2}\) This is sometimes called type-III 2HDM. (See, for example, Refs. \(^4\)–\(^7\).) However, sometimes the type-III 2HDM is referred to as a different type of 2HDM \(^8\). To avoid confusion, we call it a general 2HDM.
\(^{3}\) For earlier works, see, for example, Refs. \(^{18–23}\). The lepton flavor violation Higgs boson decays have been studied even before the CMS excess has been reported \(^{24,25}\).
\(^{4}\) Similar results have been obtained by \(^{42,44}\).
In the scenario where the 2HDM with $\mu - \tau$ flavor violation can resolve both anomalies, we have studied some predictions and constraints in $\mu$ and $\tau$-physics [40]. Especially we have found that the correction to the decay rate of $\tau \to \mu \nu \bar{\nu}$ is correlated to the correction to the muon g-2, and hence the precise measurement of the $\tau$ decay $\tau \to l \nu \bar{\nu}$ ($l = e, \mu$) is important to probe the scenario.

In this paper, we study Michel parameters for $\tau$ decays $\tau \to l \nu \bar{\nu}$ ($l = e, \mu$) in a general 2HDM with the lepton flavor violation. The Michel parameters in the leptonic decays $l \to l' \nu \bar{\nu}$ has been studied, for example, in [45–53], and within the framework of the 2HDM [11, 54–58]. However, the effect of the lepton flavor violation on the Michel parameters has not been well studied. Therefore, we analyze the corrections to the Michel parameters in $\tau$ decays $\tau \to l \nu \bar{\nu}$ ($l = e, \mu$) for the 2HDM in the presence of the lepton flavor violation. We stress that the precise measurement of the Michel parameters would have a sensitivity not only to the Lorentz and chiral structures but also to the flavor structure of the new physics models. Furthermore, we calculate the size of the corrections to the Michel parameters in the scenario where the muon g-2 anomaly is explained by the $\mu - \tau$ flavor violation in the 2HDM, and show that it can be as large as $10^{-4} - 10^{-2}$. We also find that there is an interesting correlation between the corrections to the Michel parameter $\xi_\mu$ in $\tau \to \mu \nu \bar{\nu}$ and the muon g-2, independent of the value of BR($h \to \mu \tau$). Therefore, the precise measurement of the Michel parameter at the level of $10^{-4} - 10^{-2}$ would significantly test the scenario.

This paper is organized as follows. In section II, we briefly review a general 2HDM. In section III, we study Michel parameters for $\tau$ decays $\tau^+ \to l^- \nu \bar{\nu}$ ($l = e, \mu$) in a general 2HDM with lepton flavor violation. Especially we show that the Michel parameters can be sensitive to the flavor structure as well as the Lorentz and chiral structures of the model. In section IV, we show the predicted values of the correction to the Michel parameter $\Delta \xi_\mu$ in the scenario where the muon g-2 anomaly can be explained by the $\mu - \tau$ flavor violation in the 2HDM. In section V, we summarize our results.
II. GENERAL TWO HIGGS DOUBLET MODEL

We briefly review a two Higgs doublet model. In a two Higgs doublet model, both neutral components of Higgs doublets get vacuum expectation values (vevs) in general. Taking a certain linear combination, we can always consider a basis (so called Georgi basis or Higgs basis [59, 60], and see also, for example, [61–65]) where only one of the Higgs doublets has the vev as follows:

\[
H_1 = \begin{pmatrix} G^+ \\ \frac{\nu+\phi_1+iG}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA}{\sqrt{2}} \end{pmatrix},
\]

(4)

where \(G^+\) and \(G\) are Nambu-Goldstone bosons, and \(H^+\) and \(A\) are a charged Higgs boson and a CP-odd Higgs boson, respectively. We have assumed that the CP is conserved in the Higgs potential for simplicity. CP-even neutral Higgs bosons \(\phi_1\) and \(\phi_2\) can mix and form mass eigenstates, \(h\) and \(H\) (\(m_H > m_h\)),

\[
\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\beta\alpha} & \sin \theta_{\beta\alpha} \\ -\sin \theta_{\beta\alpha} & \cos \theta_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}.
\]

(5)

Here \(\theta_{\beta\alpha}\) is the mixing angle.

Without imposing an extra symmetry, both Higgs doublets couple to all fermions. In mass eigenbasis for the fermions, the lepton Yukawa interactions are expressed by

\[
\mathcal{L} = -\bar{L}_i y^i e \phi_{ij} \bar{e}_L \phi e_{Rj} - \bar{L}_i H \rho \phi e_{Rj} - \bar{\nu}_L (V^\dagger_{\text{MNS}} \rho_e)_{ij} H^+ e_{Rj} + \text{h.c.,}
\]

(6)

where \(i, j\) represent flavor indices, \(L_L = (V_{\text{MNS}} \nu_L, e_L)^T\), and \(V_{\text{MNS}}\) is the Maki-Nakagawa-Sakata (MNS) matrix. Here all fermions \((f_L, f_R) (f = e, \nu)\) are mass eigenstates (i.e. \(e^1_{L,R} = e_{L,R}, \ e^2_{L,R} = \mu_{L,R}, \ e^3_{L,R} = \tau_{L,R}\)). We have assumed the seesaw mechanism with super-heavy right-handed neutrinos to explain the smallness of neutrino masses. The Yukawa coupling matrix \(\rho_{ij}\) is a general \(3 \times 3\) complex matrix and can be a source of the Higgs-mediated flavor violating processes. Although we only show Yukawa couplings in lepton sector, the Yukawa couplings in quark sector are understood similarly.

In mass eigenstates of Higgs bosons, the lepton Yukawa interactions are given by

\[
\mathcal{L} = -\sum_{\phi=h, H, A} \bar{y}^i_{\phi ij} \bar{e}_L \phi e_{Rj} - \bar{\nu}_L (V^\dagger_{\text{MNS}} \rho_e)_{ij} H^+ e_{Rj} + \text{h.c.,}
\]

(7)
where

\[ y_{eij}^e = \frac{m_e^i}{v} s_{\beta \alpha} \delta_{ij} + \frac{\rho_{ij}^e}{\sqrt{2}} c_{\beta \alpha}, \quad y_{Hij}^e = \frac{m_e^i}{v} c_{\beta \alpha} \delta_{ij} - \frac{\rho_{ij}^e}{\sqrt{2}} s_{\beta \alpha}, \quad y_{\lambda ij}^e = \frac{i \rho_{ij}^e}{\sqrt{2}}, \]  

(8)

where \( c_{\beta \alpha} \equiv \cos \theta_{\beta \alpha} \) and \( s_{\beta \alpha} \equiv \sin \theta_{\beta \alpha} \), and \( m_e^i = y_e^i v / \sqrt{2} \). Note that when \( c_{\beta \alpha} = 0 \) (\( s_{\beta \alpha} = 1 \)), the Yukawa interactions of \( h \) are equal to those of the SM Higgs boson. In general, however, there are flavor-violating interactions for \( h \) through the Higgs mixing \( c_{\beta \alpha} \). On the other hand, when \( c_{\beta \alpha} \) is small, the Yukawa interactions of heavy Higgs bosons (\( H, A, \) and \( H^+ \)) mainly come from the \( \rho_e \) couplings.

A scalar potential in the general 2HDM is given by

\[ V = M_{11}^2 H_1^\dagger H_1 + M_{22}^2 H_2^\dagger H_2 - \left( M_{12}^2 H_1^\dagger H_2 + \text{h.c.} \right) + \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2) (H_2^\dagger H_1) + \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + \left\{ \lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2) \right\} (H_1^\dagger H_2) + \text{h.c.}. \]

From this potential, one can calculate the relations among Higgs boson masses, and especially when \( c_{\beta \alpha} \) is close to zero (or \( \lambda_6 \sim 0 \)), the relations are simplified as

\[ m_h^2 \simeq \lambda_1 v^2, \quad m_H^2 \simeq m_A^2 + \lambda_5 v^2, \]
\[ m_{H^+}^2 = m_A^2 - \frac{\lambda_4 - \lambda_5}{2} v^2, \quad m_A^2 = M_{22}^2 + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2} v^2. \]

(9)

Fixing the couplings \( \lambda_i \), the heavy Higgs boson masses are parametrized by the CP-odd Higgs boson mass \( m_A \), which we treat as a free parameter of the model. We note that a dangerous contribution to Peskin-Takeuchi’s T-parameter (\( \rho \) parameter) \ref{66} are suppressed by the degeneracy between \( m_A \) and \( m_{H^+} \) as well as the small Higgs mixing parameter \( c_{\beta \alpha} \) \ref{67}. Therefore, we set \( \lambda_4 = \lambda_5 = 0.5 \) in our analysis, so that it guarantees the degeneracy between the CP-odd Higgs and charged Higgs bosons \( m_A = m_{H^+} \). \ref{5}

III. MICHEL PARAMETERS FOR \( \tau \) DECAYS \( \tau^- \rightarrow l^- \nu \bar{\nu} \) (\( l = e, \mu \))

In the 2HDM, charged Higgs boson interactions also induce \( \tau \) decays \( \tau^- \rightarrow l^- \nu \bar{\nu} \) at the tree level. Therefore, the detail study of the \( \tau \) decays is interesting to see the

\footnotetext{5}{In this Higgs boson mass spectrum, Peskin-Takeuchi’s S and U parameters are also small \cite{67}.
new physics effect. For an initial $\tau^-$ lepton polarization $P_\tau$, the final $l^-$ distribution ($l = e, \mu$) in the $\tau$ rest frame of $\tau^- \rightarrow l^- \nu \bar{\nu}$ decay is given in terms of Michel parameters $\rho_l$, $\eta_l$, $\xi_l$ and $\delta_l$ [45–53]:

$$
\frac{d\Gamma(\tau^- \rightarrow l^- \nu \bar{\nu})}{dx \cos \theta_l} = \frac{m_\tau w^3}{2\pi^3} \sqrt{x^2 - x_0^2} G_{F_l}^2 \left[ F_{1l}(x) - F_{2l}(x) P_\tau \cos \theta_l \right],
$$

where $G_{F_l}$ is an effective Fermi constant for $\tau^- \rightarrow l^- \nu \bar{\nu}$ process, and $\theta_l$ is the angle between the $\tau^-$ spin and the final $l^-$ momentum, $w$ is the maximum $l^-$ energy ($w = \frac{m_l^2 + m_\tau^2}{2m_\tau}$), and $x = E_l/w$ and $x_0 = m_l/w$ where $E_l$ and $m_l$ are energy and mass for the lepton $l$ ($l = e, \mu$), respectively. Here we have assumed neutrino masses are negligible. The decay rate for $\tau^- \rightarrow l^- \nu \bar{\nu}$ is expressed by

$$
\Gamma_l = \frac{G_{F_l}^2 m_\tau^5}{192\pi^3} \left\{ f(y_l) + 4\eta_l m_l g(y_l) \right\},
$$

where $y_l = m_l^2/m_\tau^2$, $f(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \log y$, and $g(y) = 1 + 9y - 9y^2 - y^3 + 6y(1 + y) \log y$.

In the 2HDM, the effective Fermi constant $G_{F_l}$ and the Michel parameters for $\tau^- \rightarrow l^- \nu \bar{\nu}$ are expressed by

$$
G_{F_l} = G_F \sqrt{1 + \Delta_l^1}, \quad \rho_l = \frac{3}{4}, \quad \delta_l = \frac{3}{4}, \quad \xi_l = \frac{1 - \Delta_l^1}{1 + \Delta_l^1}, \quad \eta_l = -\frac{\Delta_l^2}{1 + \Delta_l^1}, \quad (12)
$$

Here the corrections $\Delta_l^1$ and $\Delta_l^2$ are defined by

$$
\Delta_l^1 = \frac{(\rho_e^l \rho_e^\mu)(\rho_e^\mu \rho_e^\tau)_{\tau\tau}}{32G_F^2 m_{H^+}^4}, \quad \Delta_l^2 = \frac{\text{Re}(\rho_e^l \rho_e^\mu \rho_e^{\tau\tau})}{4\sqrt{2}G_F m_{H^+}^2}, \quad (13)
$$

where $m_{H^+}$ is the charged Higgs boson mass. Since the flavor of neutrinos and anti-neutrinos are not detected in the measurement, we have taken a sum of the flavor of neutrinos and anti-neutrinos in the final state. Thus we expect the deviation from the SM prediction in $\xi_l$ and $\eta_l$,

$$
\Delta \xi_l = \xi_l - \xi_{SM} = -\frac{2\Delta_l^1}{1 + \Delta_l^1} \approx -2\Delta_l^1, \quad (14)
$$

$$
\Delta \eta_l = \eta_l - \eta_{SM} = -\frac{\Delta_l^2}{1 + \Delta_l^1} \approx -\Delta_l^2, \quad (15)
$$
where $\xi_{SM} = 1$ and $\eta_{SM} = 0$ for the standard model values.

We note that if there are only flavor-conserving interactions assuming CP conservation for simplicity, the $\Delta_1'$ and $\Delta_2'$ are related:

$$\Delta_1' = \frac{(\rho^\mu e^\tau)^2}{32G_F^2m_H^4} = (\Delta_2')^2,$$

and hence we expect that $|\Delta_\eta| \gg |\Delta_\xi|$. On the other hand, if the flavor-violating interactions are dominant, the relation between $\Delta_\eta$ and $\Delta_\xi$ would be very different from the one in the flavor-conserving case. For example, if only $\rho^\mu_{e\tau}$ ($\tau\mu$) are non-zero and others are negligible,

$$\Delta_1^e \simeq 0, \quad \Delta_1^\mu \simeq \frac{|\rho^\tau e^\mu \rho^\mu e^\tau|^2}{32G_F^2m_H^4}, \quad \Delta_2^l \simeq 0 \text{ for } l = e \text{ and } \mu,$$

so that $|\Delta_\xi| |\gg |\Delta_\eta|$. Therefore we stress that the precise measurement of various Michel parameters are very important to understand not only the Lorentz and chiral structure but also the flavor structure of the new physics models.

Experiments have performed a test of lepton flavor universality by measuring the following quantity:

$$\left( \frac{g_\mu}{g_e} \right)_\tau^2 = \frac{\text{BR}(\tau^- \to \mu^-\bar{\nu}\nu)f(y_e)}{\text{BR}(\tau^- \to e^-\bar{\nu}\nu)f(y_\mu)},$$

where $f(y)$ is the same function shown in Eq. (11). The current world average \cite{6,8} is

$$\left( \frac{g_\mu}{g_e} \right)_\tau = 1.0018 \pm 0.0014.$$

In the 2HDM, this quantity is given by

$$\left( \frac{g_\mu}{g_e} \right)_\tau^2 = \frac{G_F^2}{G_F^2} \frac{1 + 4\eta_\mu \frac{m_\tau}{m_\mu} \gamma(y_\mu)}{1 + 4\eta_e \frac{m_\mu}{m_\tau} \gamma(y_e)} = \left( \frac{1 + \Delta_1^\mu}{1 + \Delta_1^e} \right) \left( \frac{1 + 4\eta_\mu \frac{m_\tau}{m_\mu} \gamma(y_\mu)}{1 + 4\eta_e \frac{m_\mu}{m_\tau} \gamma(y_e)} \right),$$

where $\gamma(y_i) = g(y_i)/f(y_i)$. Therefore, the measurement of the lepton flavor universality is sensitive to the non-universality of the effective Fermi constant $G_F$ (in other word, $\Delta_1^e$) as well as the parameter $\eta_l$ ($\Delta_1^l$). Especially, in the case with negligible $\eta_l$ ($\Delta_2^l$) as shown in Eq. (17), the correction to the lepton non-universality is

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\footnote{When the $\rho^\mu_{e\tau}$ ($\tau\mu$) flavor violating Yukawa couplings are dominant, the flavors of neutrino and anti-neutrino in the final state are different from those of the SM contribution. Therefore, there is no interference between the SM and charged Higgs contributions.}
sensitive to the lepton flavor violation \([40]\) and it is related to the correction to the Michel parameter \(\xi_\mu\):

\[
\left( \frac{g_\mu}{g_e} \right)_\tau \simeq 1 + \frac{\Delta_\mu^\tau}{2} \simeq 1 - \frac{\Delta_\xi_\mu}{4}.
\] (21)

Since \(\Delta_\xi_\mu < 0\), \((g_\mu/g_e)_\tau > 1\) in this scenario.

IV. CORRELATION BETWEEN CORRECTIONS TO MUON G-2 AND MICHEL PARAMETER \(\xi_\mu\)

In Refs. \([39, 40]\), we have found that the anomaly of muon g-2 can be explained by \(\mu - \tau\) flavor violating Yukawa interactions in a general 2HDM, which is motivated by the CMS excess in the Higgs boson decay \(h \rightarrow \mu \tau\) \([14]\). It will be interesting to see how large correction to the Michel parameters one can get in the parameter space where the muon g-2 anomaly is explained.

In an upper figure of Fig. 1 we show the absolute value of the correction to the Michel parameter \(|\Delta \xi_\mu|\) as a function of \(c_{\beta\alpha}\) and \(\text{BR}(h \rightarrow \mu \tau)\). Here we have assumed \(m_{H^+} = 350\) GeV. In a lower figure of Fig. 1 \(|\Delta \xi_\mu|\) is shown as a function of charged Higgs boson mass \(m_{H^+}\) and \(c_{\beta\alpha}\) fixing the branching ratio \(\text{BR}(h \rightarrow \mu \tau)\) \((\text{BR}(h \rightarrow \mu \tau) = 0.84\%)\). The dark (light) shaded region can explain the muon g-2 anomaly within \(\pm 1\sigma\) (\(\pm 2\sigma\)). In these figures, we have assumed that only flavor violating Yukawa couplings \(\rho_\mu^{\mu\tau} (\tau\mu)\) are non-zero, and others \(\rho_e\) Yukawa couplings are negligible as we have discussed in Eq. \((17)\) \(^7\). In order to explain the muon g-2 anomaly and to maximize its size, we have assumed \(\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}\), as discussed in Ref. \([40]\). As shown in Eq. \((17)\), \(\Delta^\mu_1\) is always positive and hence \(\Delta \xi_\mu\) is negative. As one can see from the upper figure of Fig. 1 there is an interesting correlation between the corrections to the muon g-2 and the Michel parameter \(\Delta \xi_\mu\) in \(\tau \rightarrow \mu \nu \bar{\nu}\) decay, that is almost independent of the value of \(\text{BR}(h \rightarrow \mu \tau)\). This is in contrast to the prediction of \(\tau \rightarrow \mu \gamma\) which depends on the value of \(\text{BR}(h \rightarrow \mu \tau)\) \([40]\). In the lower figure of Fig. 1 as the charged Higgs boson gets heavier, the predicted correction to the Michel parameter \(|\Delta \xi_\mu|\) becomes larger in the parameter region where the

\(^7\) As shown in Ref. \([40]\), many of \(\rho_e\) Yukawa couplings are strongly constrained in this scenario. Therefore, we simply neglect the others to focus on the effect of \(\rho_e^{\mu\tau} (\tau\mu)\) couplings.
FIG. 1: The correction to the Michel parameter $|\Delta \xi_{\mu}|$ is shown as a function of $c_{\beta\alpha}$ and $\text{BR}(h \rightarrow \mu \tau)$ (upper figure) and as a function of charged Higgs boson mass $m_{H^+}$ and $c_{\beta\alpha}$ (lower figure). We have assumed $m_{H^+} = 350$ GeV in the upper figure and $\text{BR}(h \rightarrow \mu \tau) = 0.84\%$ in the lower figure. The dark (light) shaded region can resolve the muon g-2 anomaly within $\pm 1\sigma$ ($\pm 2\sigma$).

muon g-2 anomaly is explained. The accuracy of the current Michel parameter measurements is at the $O(1)\%$ level [69], and hence the results are consistent with the current measurements on the Michel parameters. Since the correction $|\Delta \xi_{\mu}|$ and the lepton non-universality $(g_\mu/g_\tau - 1$ in the $\tau$ decays are related as shown in
Eq. (21), the current bound of the lepton non-universality Eq. (19) starts putting on the constraint in this scenario. Therefore, the future precise measurement of the Michel parameter $\xi_\mu$ at the level of $10^{-4} - 10^{-2}$ as well as that of the lepton non-universality would have a significant potential to probe this scenario.

V. SUMMARY

The theoretical understanding of the Higgs sector is still unsatisfactory. The more experimental data and theoretical studies will be needed to make a deeper understanding of the Higgs sector.

The CMS excess events of $h \rightarrow \mu\tau$ process might suggest the extension of the minimal structure of the SM Higgs sector. One of simple extensions of the SM Higgs sector is a two Higgs doublet extension of the SM. In a general 2HDM, the flavor violating phenomena mediated by Higgs bosons are predicted, and hence it is easy to explain the CMS excess in $h \rightarrow \mu\tau$ if this is due to new physics effect. In Refs. [39, 40], we have pointed out that the $\mu - \tau$ flavor violating Yukawa interactions can resolve the muon g-2 anomaly, and in Ref. [40], the correction to the decay rate of $\tau \rightarrow \mu\nu\bar{\nu}$ process is correlated to the contribution to the muon g-2 induced by the $\mu - \tau$ lepton flavor violating Yukawa interactions.

In this paper, we have studied the Michel parameters for the $\tau$ decays $\tau^- \rightarrow l^-\nu\bar{\nu}$ in a general 2HDM with lepton flavor violation, whose effect on the Michel parameters had not been well studied. We have shown that the precise measurement of the Michel parameters is sensitive to the flavor structure as well as the Lorentz and chiral structure of the model. Especially in the parameter region where the muon g-2 anomaly is explained by the $\mu - \tau$ flavor violating Yukawa couplings, the correction to the Michel parameter $|\Delta \xi_\mu|$ can be as large as $10^{-4} - 10^{-2}$ and it is correlated to the correction to the muon g-2, independent of the predicted value of $\text{BR}(h \rightarrow \mu\tau)$. Therefore, the precision measurement of the Michel parameters at the level of $10^{-4} - 10^{-2}$ would be crucial to probe the scenario where the $\mu - \tau$ flavor violating Yukawa couplings explain the anomaly of the muon g-2.
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[1] G. Aad et al. [ATLAS Collaboration], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716, 1 (2013) [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan et al. [CMS Collaboration], “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” Phys. Lett. B 716, 30 (2013) [arXiv:1207.7235 [hep-ex]].

[3] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, “Theory and phenomenology of two-Higgs-doublet models,” Phys. Rept. 516, 1 (2012) [arXiv:1106.0034 [hep-ph]].

[4] J. Liu and L. Wolfenstein, “Spontaneous CP Violation in the SU(2)$_L$ × U(1)$_Y$ Model with Two Higgs Doublets,” Nucl. Phys. B 289, 1 (1987).

[5] M. Aoki, S. Kanemura, K. Tsumura and K. Yagyu, “Models of Yukawa interaction in the two Higgs doublet model, and their collider phenomenology,” Phys. Rev. D 80, 015017 (2009) [arXiv:0902.4665 [hep-ph]].

[6] W. S. Hou, “Tree level $t \to ch$ or $h \to t\bar{c}$ decays,” Phys. Lett. B 296, 179 (1992).

[7] D. Atwood, L. Reina and A. Soni, “Phenomenology of two Higgs doublet models with flavor changing neutral currents,” Phys. Rev. D 55, 3156 (1997) [hep-ph/9609279].

[8] V. D. Barger, J. L. Hewett and R. J. N. Phillips, “New Constraints on the Charged Higgs Sector in Two Higgs Doublet Models,” Phys. Rev. D 41, 3421 (1990).

[9] J. D. Bjorken and S. Weinberg, “A Mechanism for Nonconservation of Muon Number,” Phys. Rev. Lett. 38, 622 (1977).

[10] S. L. Glashow and S. Weinberg, “Natural Conservation Laws for Neutral Currents,”
[11] B. McWilliams and L. F. Li, “Virtual Effects of Higgs Particles,” Nucl. Phys. B 179, 62 (1981).

[12] O. U. Shanker, “Flavor Violation, Scalar Particles and Leptoquarks,” Nucl. Phys. B 206, 253 (1982).

[13] T. P. Cheng and M. Sher, “Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets,” Phys. Rev. D 35, 3484 (1987).

[14] V. Khachatryan et al. [CMS Collaboration], “Search for Lepton-Flavour-Violating Decays of the Higgs Boson,” Phys. Lett. B 749, 337 (2015) [arXiv:1502.07400 [hep-ex]].

[15] CMS Collaboration [CMS Collaboration], “Search for Lepton Flavour Violating Decays of the Higgs Boson in the $\mu - \tau$ final state at 13 TeV,” CMS-PAS-HIG-16-005.

[16] G. Aad et al. [ATLAS Collaboration], “Search for lepton-flavour-violating $H \to \mu\tau$ decays of the Higgs boson with the ATLAS detector,” JHEP 1511, 211 (2015) [arXiv:1508.03372 [hep-ex]].

[17] G. Aad et al. [ATLAS Collaboration], “Search for lepton-flavour-violating decays of the Higgs and Z bosons with the ATLAS detector,” [arXiv:1604.07730 [hep-ex]].

[18] M. D. Campos, A. E. Crcamo Hernández, H. Ps and E. Schumacher, “Higgs $\to \mu\tau$ as an indication for $S_4$ flavor symmetry,” Phys. Rev. D 91, no. 11, 116011 (2015) [arXiv:1408.1652 [hep-ph]].

[19] D. Aristizabal Sierra and A. Vicente, “Explaining the CMS Higgs flavor violating decay excess,” Phys. Rev. D 90, no. 11, 115004 (2014) [arXiv:1409.7690 [hep-ph]].

[20] J. Heeck, M. Holthausen, W. Rodejohann and Y. Shimizu, “Higgs in Abelian and non-Abelian flavor symmetry models,” Nucl. Phys. B 896, 281 (2015) [arXiv:1412.3671 [hep-ph]].

[21] A. Crivellin, G. D’Ambrosio and J. Heeck, “Explaining $h \to \mu^{\pm}\tau^{\mp}$, $B \to K^{*}\mu^{+}\mu^{-}$ and $B \to K\mu^{+}\mu^{-}/B \to K e^{+}e^{-}$ in a two-Higgs-doublet model with gauged $L_{\mu} - L_{\tau}$,” Phys. Rev. Lett. 114, 151801 (2015) [arXiv:1501.00993 [hep-ph]].

[22] L. de Lima, C. S. Machado, R. D. Matheus and L. A. F. do Prado, “Higgs Flavor Violation as a Signal to Discriminate Models,” JHEP 1511, 074 (2015) [arXiv:1501.06923 [hep-ph]].
[23] I. Dorner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Konik and I. Niandic, “New Physics Models Facing Lepton Flavor Violating Higgs Decays at the Percent Level,” JHEP 1506, 108 (2015) [arXiv:1502.07784 [hep-ph]].

[24] A. Pilaftsis, “Lepton flavor nonconservation in \( H_0 \) decays,” Phys. Lett. B 285, 68 (1992).

[25] J. G. Korner, A. Pilaftsis and K. Schilcher, “Leptonic CP asymmetries in flavor changing \( H_0 \) decays,” Phys. Rev. D 47, 1080 (1993) [hep-ph/9301289].

[26] J. L. Diaz-Cruz and J. J. Toscano, “Lepton flavor violating decays of Higgs bosons beyond the standard model,” Phys. Rev. D 62, 116005 (2000) [hep-ph/9910233].

[27] K. A. Assamagan, A. Deandrea and P. A. Delsart, “Search for the lepton flavor violating decay \( A_0/H_0 \to \tau^\pm \mu^{\mp} \) at hadron colliders,” Phys. Rev. D 67, 035001 (2003) [hep-ph/0207302].

[28] A. Brignole and A. Rossi, “Lepton flavor violating decays of supersymmetric Higgs bosons,” Phys. Lett. B 566, 217 (2003) [hep-ph/0304081].

[29] S. Kanemura, K. Matsuda, T. Ota, T. Shindou, E. Takasugi and K. Tsumura, “Search for lepton flavor violation in the Higgs boson decay at a linear collider,” Phys. Lett. B 599, 83 (2004) [hep-ph/0406316].

[30] E. Arganda, A. M. Curiel, M. J. Herrero and D. Temes, “Lepton flavor violating Higgs boson decays from massive seesaw neutrinos,” Phys. Rev. D 71, 035011 (2005) [hep-ph/0407302].

[31] S. Kanemura, T. Ota and K. Tsumura, “Lepton flavor violation in Higgs boson decays under the rare tau decay results,” Phys. Rev. D 73, 016006 (2006) [hep-ph/0505191].

[32] S. Davidson and G. J. Grenier, “Lepton flavour violating Higgs and tau to mu gamma,” Phys. Rev. D 81, 095016 (2010) [arXiv:1001.0434 [hep-ph]].

[33] G. Blankenburg, J. Ellis and G. Isidori, “Flavour-Changing Decays of a 125 GeV Higgs-like Particle,” Phys. Lett. B 712, 386 (2014) [arXiv:1202.5704 [hep-ph]].

[34] R. Harnik, J. Kopp and J. Zupan, “Flavor Violating Higgs Decays,” JHEP 1303, 026 (2013) [arXiv:1209.1397 [hep-ph]].

[35] A. Arhrib, Y. Cheng and O. C. W. Kong, “Comprehensive analysis on lepton flavor violating Higgs boson to \( \mu^{\mp} \tau^{\pm} \) decay in supersymmetry without \( R \) parity,” Phys. Rev. D 87, no. 1, 015025 (2013) [arXiv:1210.8241 [hep-ph]].
[36] M. Arana-Catania, E. Arganda and M. J. Herrero, “Non-decoupling SUSY in LFV Higgs decays: a window to new physics at the LHC,” JHEP 1309, 160 (2013) Erratum: [JHEP 1510, 192 (2015)] [arXiv:1304.3371 [hep-ph]].

[37] E. Arganda, M. J. Herrero, X. Marcano and C. Weiland, “Imprints of massive inverse seesaw model neutrinos in lepton flavor violating Higgs boson decays,” Phys. Rev. D 91, no. 1, 015001 (2015) [arXiv:1405.4300 [hep-ph]].

[38] J. Kopp and M. Nardecchia, “Flavor and CP violation in Higgs decays,” JHEP 1410, 156 (2014) [arXiv:1406.5303 [hep-ph]].

[39] Y. Omura, E. Senaha and K. Tobe, “Lepton-flavor-violating Higgs decay $h \to \mu\tau$ and muon anomalous magnetic moment in a general two Higgs doublet model,” JHEP 1505, 028 (2015) [arXiv:1502.07824 [hep-ph]].

[40] Y. Omura, E. Senaha and K. Tobe, “$\tau$- and $\mu$-physics in a general two Higgs doublet model with $\mu - \tau$ flavor violation,” [arXiv:1511.08880 [hep-ph]].

[41] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, “$(g-2)_\mu$ and $\alpha(M_Z^2)$ re-evaluated using new precise data,” J. Phys. G 38, 085003 (2011) [arXiv:1105.3149 [hep-ph]].

[42] F. Jegerlehner and A. Nyffeler, “The Muon g-2,” Phys. Rept. 477, 1 (2009) [arXiv:0902.3360 [hep-ph]].

[43] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, “Reevaluation of the Hadronic Contributions to the Muon g-2 and to $\alpha(M_Z^2)$,” Eur. Phys. J. C 71, 1515 (2011) Erratum: [Eur. Phys. J. C 72, 1874 (2012)] [arXiv:1010.4180 [hep-ph]].

[44] F. Jegerlehner and R. Szafron, “$\rho^0 - \gamma$ mixing in the neutral channel pion form factor $F^0_\pi$ and its role in comparing $e^+e^-$ with $\tau$ spectral functions,” Eur. Phys. J. C 71, 1632 (2011) [arXiv:1101.2872 [hep-ph]].

[45] L. Michel, Proc. Phys. Soc. A63, 514 (1950).

[46] C. Bouchiat and L. Michel, “Theory of $\mu$-Meson Decay with the Hypothesis of Non-conservation of Parity,” Phys. Rev. 106, 170 (1957).

[47] T. Kinoshita and A. Sirlin, “Muon Decay with Parity Nonconserving Interactions and Radiative Corrections in the Two-Component Theory,” Phys. Rev. 107, 593 (1957).

[48] T. Kinoshita and A. Sirlin, “Polarization of Electrons in Muon Decay with General Parity-Nonconserving Interactions,” Phys. Rev. 108, 844 (1957).
[49] F. Scheck, “Muon Physics,” Phys. Rept. 44, 187 (1978).

[50] W. Fetscher, H. J. Gerber and K. F. Johnson, “Muon Decay: Complete Determination of the Interaction and Comparison with the Standard Model,” Phys. Lett. B 173, 102 (1986).

[51] A. Pich and J. P. Silva, “Constraining new interactions with leptonic $\tau$ decays,” Phys. Rev. D 52, 4006 (1995) [hep-ph/9505327].

[52] A. Pich, “Precision Tau Physics,” Prog. Part. Nucl. Phys. 75, 41 (2014) [arXiv:1310.7922 [hep-ph]].

[53] Y. Kuno and Y. Okada, “Muon decay and physics beyond the standard model,” Rev. Mod. Phys. 73, 151 (2001) [hep-ph/9909265].

[54] H. E. Haber, G. L. Kane and T. Sterling, “The Fermion Mass Scale and Possible Effects of Higgs Bosons on Experimental Observables,” Nucl. Phys. B 161, 493 (1979).

[55] A. Stahl, “The Michel parameter eta in tau decays,” Phys. Lett. B 324, 121 (1994).

[56] H. E. Logan and D. MacLennan, “Charged Higgs phenomenology in the lepton-specific two Higgs doublet model,” Phys. Rev. D 79, 115022 (2009) [arXiv:0903.2246 [hep-ph]].

[57] T. Abe, R. Sato and K. Yagyu, “Lepton-specific two Higgs doublet model as a solution of muon $g-2$ anomaly,” JHEP 1507, 064 (2015) [arXiv:1504.07059 [hep-ph]].

[58] E. J. Chun and J. Kim, “Leptonic Precision Test of Leptophilic Two-Higgs-Doublet Model,” arXiv:1605.06298 [hep-ph].

[59] H. Georgi and D. V. Nanopoulos, “Suppression of Flavor Changing Effects From Neutral Spinless Meson Exchange in Gauge Theories,” Phys. Lett. B 82, 95 (1979).

[60] J. F. Donoghue and L. F. Li, “Properties of Charged Higgs Bosons,” Phys. Rev. D 19, 945 (1979).

[61] L. Lavoura and J. P. Silva, “Fundamental CP violating quantities in a SU(2) $\times$ U(1) model with many Higgs doublets,” Phys. Rev. D 50, 4619 (1994) [hep-ph/9404276].

[62] L. Lavoura, “Signatures of discrete symmetries in the scalar sector,” Phys. Rev. D 50, 7089 (1994) [hep-ph/9405307].

[63] F. J. Botella and J. P. Silva, “Jarlskog - like invariants for theories with scalars and fermions,” Phys. Rev. D 51, 3870 (1995) [hep-ph/9411288].

[64] G. C. Branco, L. Lavoura and J. P. Silva, “CP Violation,” Int. Ser. Monogr. Phys.
[65] S. Davidson and H. E. Haber, “Basis-independent methods for the two-Higgs-doublet model,” Phys. Rev. D 72, 035004 (2005) Erratum: [Phys. Rev. D 72, 099902 (2005)] [hep-ph/0504050].

[66] M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys. Rev. D 46, 381 (1992).

[67] For example, see H. E. Haber and D. O’Neil, “Basis-independent methods for the two-Higgs-doublet model III: The CP-conserving limit, custodial symmetry, and the oblique parameters S, T, U,” Phys. Rev. D 83, 055017 (2011) [arXiv:1011.6188 [hep-ph]].

[68] B. Aubert et al. [BaBar Collaboration], “Measurements of Charged Current Lepton Universality and $|V_{us}|$ using Tau Lepton Decays to $e^- \bar{\nu}_e \nu_r$, $\mu^- \bar{\nu}_\mu \nu_r$, $\pi^- \nu_r$ and $K^- \nu_r$,” Phys. Rev. Lett. 105, 051602 (2010) [arXiv:0912.0242 [hep-ex]].

[69] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).