Electroweak Constraints on Minimal Higher-Dimensional Extensions of the Standard Model

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ABSTRACT

We derive electroweak constraints on the compactification scale of minimal 5-dimensional extensions of the Standard Model, in which all or only some of the SU(2)\textsubscript{L} and U(1)\textsubscript{Y} gauge fields and Higgs bosons feel the presence of the fifth dimension. In our analysis, we assume that the fermions are always localized on a 3-brane. In this context, we also present the consistent quantization procedure of the higher-dimensional models in the generalized $R_\xi$ gauge. We find that the usually derived lower bound of $\sim 4$ TeV on the compactification scale may be significantly lowered to $\sim 3$ TeV if the SU(2)\textsubscript{L} gauge boson is the only particle that propagates in all 5 dimensions.

1 Introduction

In the original formulations of string theory \cite{1}, the compactification radius $R$ of the extra dimensions and the string mass $M_s$ were considered to be set by the 4-dimensional Planck mass $M_P = 1.9 \times 10^{16}$ TeV. However, recent studies have shown \cite{2,3,4,5,6} that conceivable scenarios of stringy nature may exist for which $R$ and $M_s$ practically decouple from $M_P$. For example, in the model of Ref. \cite{5}, $M_s$ may become as low as of order TeV. In this case, $M_s$ constitutes the only fundamental scale in nature at which all forces including gravity unify. This low string-scale effective model could be embedded within e.g. type I string theories \cite{7}, where the Standard Model (SM) may be described as an intersection of higher-dimensional $D_p$ branes \cite{8,9,10}.

As such intersections may be higher dimensional as well, in addition to gravitons the SM gauge fields could also propagate independently within a higher-dimensional subspace with compact dimensions of order TeV$^{-1}$ for phenomenological reasons. Since such low string-scale constructions may result in different higher-dimensional extensions of the SM \cite{11,12}, the actual experimental limits on the compactification radius are, to some extent, model dependent. Nevertheless, most of the derived phenomenological limits in the literature were

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obtained by assuming that the SM gauge fields propagate all freely in a common higher-
dimensional space \[9, 10, 11, 12, 13, 14, 15\].

Here, we wish to lift the above restriction and focus on the phenomenological con-
sequences of models which minimally depart from the assumption of a universal higher-
dimensional scenario \[16\]. Specifically, we will consider 5-dimensional exten-
sions of the SM compactified on an \(S^1/Z_2\) orbifold, where the SU(2)\(_L\) and U(1)\(_Y\) gauge bosons may
not both live in the same higher-dimensional space, the so-called bulk. In all our models,
the SM fermions are localized on the 4-dimensional subspace, i.e. on a 3-brane or, as it
is often called, brane. For each higher-dimensional model, we calculate the effects of the
fifth dimension on the electroweak observables and analyze their impact on constrain-
ing the compactification scale.

The organization of this note is as follows: in Section 2 we introduce the basic concepts
of higher dimensional theories in simple Abelian models. After compactifying the extra
dimensions on \(S^1/Z_2\), we obtain an effective 4-dimensional theory, which in addition to
the usual SM states contains infinite towers of massive Kaluza–Klein (KK) states of the
higher-dimensional gauge fields. In particular, we consider the question how to consistently
quantize the higher-dimensional models under study in the so-called \(R_\xi\) gauge. Such a
quantization procedure can be successfully applied to theories that include both Higgs
bosons living in the bulk and/or on the brane. After briefly discussing how these concepts
can be applied to the SM in Section 3, we turn our attention to the phenomenological
aspects of the models of our interest in Section 4. Because of the limited space, technical
details are omitted in this note. A complete discussion, along with detailed analytic results
and references, is given in our paper in \[16\]. Section 5 summarizes our numerical results
and presents our conclusions.

\section{5-Dimensional Abelian Models}

As a starting point, let us consider the Lagrangian of 5-dimensional Quantum Electrodyn-
amics (5D-QED) compactified on an \(S^1/Z_2\) orbifold given by

\[\mathcal{L}(x, y) = -\frac{1}{4} F_{MN}(x, y) F^{MN}(x, y) + \mathcal{L}_{GF}(x, y),\]

where

\[F_{MN}(x, y) = \partial_M A_N(x, y) - \partial_N A_M(x, y)\]

denotes the 5-dimensional field strength tensor, and \(\mathcal{L}_{GF}(x, y)\) is the gauge-fixing term. The
Faddeev-Popov ghost terms have been neglected, because the ghosts are non-interacting
in the Abelian case. Our notation for the Lorentz indices and space-time coordinates is:
\(M, N = 0, 1, 2, 3, 5\); \(\mu, \nu = 0, 1, 2, 3\); \(x = (x^0, \vec{x});\) and \(y = x^5\).

In a 5-dimensional theory, the gauge-boson field \(A_M\) transforms as a vector under the
Lorentz group SO(1,4). In the absence of the gauge-fixing and ghost terms, the 5D-QED
Lagrangian is invariant under a U(1) gauge transformation:

\[A_M(x, y) \rightarrow A_M(x, y) + \partial_M \Theta(x, y).\]
To compactify the theory on an \( S^1/Z_2 \) orbifold and not to spoil the above property of gauge symmetry, we demand for the fields to satisfy the following equalities:

\[
\begin{align*}
A_M(x, y) &= A_M(x, y + 2\pi R), \\
A_\mu(x, y) &= A_\mu(x, -y), \\
A_5(x, y) &= -A_5(x, -y), \\
\Theta(x, y) &= \Theta(x, y + 2\pi R), \\
\Theta(x, y) &= \Theta(x, -y).
\end{align*}
\tag{2.4}
\]

The field \( A_\mu(x, y) \) is taken to be even under \( Z_2 \), so as to embed conventional QED with a massless photon into our 5D-QED. Notice that all other constraints on the field \( A_5(x, y) \) and the gauge parameter \( \Theta(x, y) \) in (2.4) follow automatically if the theory is to remain gauge invariant after compactification.

Given the periodicity and reflection properties of \( A_M \) and \( \Theta \) under \( y \) in (2.4), we can expand these quantities in Fourier series, e.g.

\[
A_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} A_\mu(0)(x) + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} A_\mu(n)(x) \cos \left( \frac{ny}{R} \right),
\tag{2.5}
\]

where the Fourier coefficients \( A_\mu(n)(x) \) are the so-called KK modes. Integrating out the \( y \) dimension we finally obtain the effective 4-dimensional Lagrangian

\[
\mathcal{L}(x) = -\frac{1}{4} F(0)_{\mu\nu} F(0)^{\mu\nu} + \sum_{n=1}^{\infty} \left[ -\frac{1}{4} F(n)_{\mu\nu} F(n)^{\mu\nu} + \frac{1}{2} \left( \frac{n}{R} A_{(n)\mu} + \partial_\mu A_{(n)5} \right) \right] + \mathcal{L}_{GF}(x),
\tag{2.6}
\]

where \( \mathcal{L}_{GF}(x) = \int_0^{2\pi R} dy \mathcal{L}_{GF}(x, y) \).

In addition to the usual QED terms involving the massless field \( A_{(0)}^{\mu} \), the other terms describe two infinite towers of massive vector excitations \( A_{(n)}^{\mu} \) and (pseudo)-scalar modes \( A_{(n)5} \) that mix with each other, for \( n \geq 1 \). The scalar modes \( A_{(n)5} \) play the rôle of the would-be Goldstone modes in a non-linear realization of an Abelian Higgs model, in which the corresponding Higgs fields are taken to be infinitely massive.

The above observation motivates us to seek for a higher-dimensional generalization of 't Hooft’s gauge-fixing condition, for which the mixing terms bilinear in \( A_{(n)}^{\mu} \) and \( A_{(n)5} \) are eliminated from the effective 4-dimensional Lagrangian (2.6). Taking advantage of the fact that orbifold compactification generally breaks SO(1,4) invariance \cite{17}, one can abandon the requirement of covariance of the gauge fixing condition with respect to the extra dimension and choose the following non-covariant generalized \( R_\xi \) gauge:

\[
\mathcal{L}_{GF}(x, y) = -\frac{1}{2\xi} \left( \partial^\mu A_\mu - \xi \partial_5 A_5 \right)^2.
\tag{2.7}
\]

\footnote{For a recently related suggestion, see \cite{18}.}
Nevertheless, the gauge-fixing term in (2.7) is still invariant under ordinary 4-dimensional Lorentz transformations. Upon integration over the extra dimension, all mixing terms in (2.6) drop out up to irrelevant total derivatives and the propagators for the fields $A_{(n)}^\mu$ and $A_5^{(n)}$ take on their usual forms that describe massive gauge fields and their respective would-be Goldstone bosons of an ordinary 4-dimensional Abelian-Higgs model in the $R_\xi$ gauge:

$$\begin{align*}
\mu \sim (n) & \quad \sim \nu \\
= \frac{i}{k^2 - (\frac{\pi}{\xi})^2} \left[ -g^{\mu \nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - (\frac{\pi}{\xi})^2} \right] \\
- \quad \sim -
= \frac{i}{k^2 - \xi(\frac{\pi}{\xi})^2}
\end{align*}$$

(2.8)

Hereafter, we shall refer to the $A_5^{(n)}$ fields as Goldstone modes.

Having defined the appropriate $R_\xi$ gauge through the gauge-fixing term in (2.7), we can recover the usual unitary gauge in the limit $\xi \to \infty$\cite{19, 20}. Thus, for the case at hand, we have seen how starting from a non-covariant higher-dimensional gauge-fixing condition, we can arrive at the known covariant 4-dimensional $R_\xi$ gauge after compactification.

The above quantization procedure can now be extended to more elaborate higher-dimensional models. Adding a Higgs scalar in the bulk, the 5D Lagrangian of the theory reads

$$\mathcal{L}(x, y) = -\frac{1}{4} F^{MN} F_{MN} + (D_M \Phi)^* (D^M \Phi) - V(\Phi) + \mathcal{L}_{\text{GF}}(x, y),$$

(2.9)

where $D_M = \partial_M + ie_5 A_M$ denotes the covariant derivative, $e_5$ the 5-dimensional gauge coupling, $\Phi(x, y) = (h(x, y) + i \chi(x, y))/\sqrt{2}$ a 5-dimensional complex scalar field, and $V(\Phi) = \mu_5^2 |\Phi|^2 + \lambda_5 |\Phi|^4$ (with $\lambda_5 > 0$) the 5-dimensional Higgs potential.

We consider $\Phi(x, y)$ to be even under $Z_2$, perform a corresponding Fourier decomposition, and integrate over $y$. For $\mu_5^2 < 0$, as in the usual 4-dimensional case, the zero KK Higgs mode acquires a non-vanishing vacuum expectation value (VEV) which breaks the U(1) symmetry. Moreover, it can be shown that as long as the phenomenologically relevant condition $v < 1/R$ is met, $h_{(0)}$ will be the only mode to receive a non-zero VEV $\langle h_{(0)} \rangle = v = \sqrt{2\pi R |\mu_5|^2}/\lambda_5$.

After spontaneous symmetry breaking, it is instructive to introduce the fields

$$G_{(n)} = \left( \frac{n^2}{R^2} + e^2 v^2 \right)^{-1/2} \left( \frac{n}{R} A_{(n)5} + ev \chi_{(n)} \right),$$

(2.10)

where $e = e_5/\sqrt{2\pi R}$, and the orthogonal linear combinations $a_{(n)}$. In the effective kinetic Lagrangian of the theory for the $n$-KK mode ($n > 0$), $G_{(n)}$ now plays the rôle of a Goldstone mode in an Abelian Higgs model, while the pseudoscalar field $a_{(n)}$ describes an additional physical KK excitation degenerate in mass with the KK gauge mode $A_{(n)\mu}$.
\( m_{a(n)}^2 = m_{A(n)}^2 = (n^2/R^2) + e^2 v^2 \). The spectrum of the zero KK modes is simply identical to that of a conventional Abelian Higgs model. It becomes clear that the appropriate gauge-fixing Lagrangian in (2.9) for a 5-dimensional generalized \( R_\xi \)-gauge should be

\[
\mathcal{L}_{GF}(x, y) = -\frac{1}{2\xi} \left[ \partial_\mu A^\mu - \xi \left( \partial_5 A_5 + e_5 v \frac{\chi}{\sqrt{2\pi R}} \right) \right]^2.
\]  

(2.11)

All the mixing terms are removed and we again arrive at the standard kinetic Lagrangian for massive gauge bosons and the corresponding would-be Goldstone modes. The CP-odd scalar modes \( a(n) \) and the Higgs KK-modes \( h(n) \) with mass \( m_{h(n)} = \sqrt{(n^2/R^2) + \lambda_5 v^2/\pi R} \) are not affected by the gauge fixing procedure. Observe finally that the limit \( \xi \to \infty \) consistently corresponds to the unitary gauge.

A qualitatively different way of implementing the Higgs sector in a higher-dimensional Abelian model is to localize the Higgs field at the \( y = 0 \) boundary of the \( S^1/Z_2 \) orbifold by introducing a \( \delta \)-function in the 5-dimensional Lagrangian

\[
\mathcal{L}(x, y) = -\frac{1}{4} F^{MN} F_{MN} + \delta(y) \left[ (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi) \right] + \mathcal{L}_{GF}(x, y),
\]  

(2.12)

where the covariant derivative and the Higgs potential have their familiar 4-dimensional forms. Because the Higgs potential is effectively four dimensional the Higgs field, not having KK excitations as a brane field, acquires the usual VEV. Notice that the bulk scalar field \( A_5(x, y) \) vanishes on the brane \( y = 0 \) as a result of its odd \( Z_2 \)-parity and does not couple to the Higgs sector.

After compactification and integration over the \( y \)-dimension, spontaneous symmetry breaking again generates masses for all the KK gauge modes \( A^\mu_{(n)} \). However, the Fourier modes are no longer mass eigenstates. By diagonalization of the mass matrix the mass eigenvalues \( m_{(n)} \) of the KK mass eigenstates are found to obey the transcendental equation

\[
m_{(n)} = \pi m^2 R \cot \left( \pi m_{(n)} R \right)
\]  

(2.13)

with \( m = ev \). Hence, the zero-mode mass eigenvalues are slightly shifted from what we expect in a 4D model. The respective KK mass eigenstates can also be calculated analytically \([10]\). To find the appropriate form of the gauge-fixing term \( \mathcal{L}_{GF}(x, y) \) in (2.12), we follow (2.11), but restrict the scalar field \( \chi \) to the brane \( y = 0 \), viz.

\[
\mathcal{L}_{GF}(x, y) = -\frac{1}{2\xi} \left[ \partial_\mu A^\mu - \xi \left( \partial_5 A_5 + e_5 v \chi \delta(y) \right) \right]^2.
\]  

(2.14)

As is expected from a generalized \( R_\xi \) gauge, all mixing terms of the gauge modes \( A^\mu_{(n)} \) with \( A_{(n)5} \) and \( \chi \) disappear up to total derivatives if \( \delta(0) \) is appropriately interpreted on \( S^1/Z_2 \). Determining the unphysical mass spectrum of the Goldstone modes, we find a one-to-one correspondence of each physical vector mode of mass \( m_{(n)} \) to an unphysical Goldstone mode with gauge-dependent mass \( \sqrt{\xi} m_{(n)} \). In the unitary gauge \( \xi \to \infty \), the would-be Goldstone modes are absent from the theory. The present brane-Higgs model does not predict other KK massive scalars apart from the physical Higgs boson \( h \).
3 5-Dimensional Extensions of the Standard Model

It is a straightforward exercise to generalize the ideas introduced in Section 2 for non-Abelian theories. Compactification, spontaneous symmetry breaking and gauge fixing are very analogous to the Abelian case and the non-decoupling ghost sector can be easily included [16]. Hence, in the effective 4D theory, we arrive at a particle spectrum similar to the Abelian case. In addition, the self-interaction of gauge-bosons in non-Abelian theories leads to self-interactions of the KK modes which are restricted by selection rules reflecting the $S^1/Z_2$ structure of the extra dimension.

Turning our attention to the electroweak sector of the Standard Model, its gauge structure $SU(2)_L \otimes U(1)_Y$ opens up several possibilities for 5-dimensional extensions, because the $SU(2)_L$ and $U(1)_Y$ gauge fields do not necessarily both propagate in the extra dimension. Such a realization may be encountered within specific stringy frameworks, where one of the gauge groups is effectively confined on the boundaries of the $S^1/Z_2$ orbifold [7, 8]. However, in the most frequently investigated scenario, $SU(2)_L$ and $U(1)_Y$ gauge fields live in the bulk of the extra dimension (bulk-bulk model). In this case, as has been presented in Section 2, both a localized (brane) and a 5-dimensional (bulk) Higgs doublet can be included in the theory. For generality, we will consider a 2-doublet Higgs model, where the one Higgs field $\Phi_1$ propagates in the fifth dimension, while the other one $\Phi_2$ is localized. The phenomenology of electroweak precision variables is not sensitive to details of the Higgs potential but only to their vacuum expectation values $v_1$ and $v_2$, or equivalently to $\tan \beta = v_2/v_1$ and $v^2 = v_1^2 + v_2^2$.

An even more minimal 5-dimensional extension of electroweak physics constitutes a model in which only the $SU(2)_L$-sector feels the extra dimension while the $U(1)_Y$ gauge field is localized at $y = 0$ (bulk-brane model). In this case, the Higgs field being charged with respect to both gauge groups has to be localized at $y = 0$ in order to preserve gauge invariance of the (classical) Lagrangian. For the same reason, a bulk Higgs is forbidden in the third possible model in which $SU(2)_L$ is localized while $U(1)_Y$ propagates in the fifth dimension (brane-bulk model).

In all these minimal 5-dimensional extensions of the SM we assume that the SM fermions are localized at the $y = 0$ fixed point of the $S^1/Z_2$ orbifold. The coupling of such a fermion to a gauge boson restricted to the same brane $y = 0$ has its SM value. On the other hand, the effective interaction Lagrangian describing the coupling of a fermion to the Fourier modes of a bulk gauge-boson has the generic form

$$\mathcal{L}_{\text{int}}(x) = \overline{\Psi}_\mu (g_V + g_A \gamma^5) \gamma^\mu \left( A_{(0)\mu} + \sqrt{2} \sum_{n=1}^{\infty} A_{(n)\mu} \right).$$

Again, the coupling parameters $g_V$ and $g_A$ are set by the quantum numbers of the fermions and receive their SM values. Because the KK mass eigenmodes generally differ from the Fourier modes, their couplings to fermions $g_{V(n)}$ and $g_{A(n)}$ have to be calculated for each model individually, after the appropriate basis transformations relating the weak to mass eigenstates have properly been taken into account.
4 Effects on Electroweak Observables

In this section, we will concentrate on the phenomenology and present bounds on the compactification scale \( M = 1/R \) of minimal higher-dimensional extensions of the SM calculated by analyzing a large number of high precision electroweak observables. To be specific, we proceed as follows. We relate the SM prediction \( \mathcal{O}^\text{SM} \) for an electroweak observable to the prediction \( \mathcal{O}^\text{HDSM} \) for the same observable obtained in the higher-dimensional SM under investigation through

\[
\mathcal{O}^\text{HDSM} = \mathcal{O}^\text{SM} \left(1 + \Delta^\text{HDSM} \mathcal{O}\right),
\]

(4.1)

Here, \( \Delta^\text{HDSM} \mathcal{O} \) is the tree-level modification of a given observable \( \mathcal{O} \) from its SM value due to the presence of one extra dimension. In five dimensions, all the tree-level modifications can be expanded in powers of the typical scale factor \( X = \frac{1}{3} \pi^2 m_Z^2 R^2 \). On the other hand, to enable a direct comparison of our predictions with the electroweak precision data \cite{22}, we include SM radiative corrections to \( \mathcal{O}^\text{SM} \). However, we neglect SM- as well as KK-loop contributions to \( \Delta^\text{HDSM} \mathcal{O} \) as higher order effects.

As input SM parameters for our theoretical predictions, we choose the most accurately measured ones, namely the Z-boson mass \( M_Z \), the electromagnetic fine structure constant \( \alpha \) and the Fermi constant \( G_F \). While \( \alpha \) is not affected in the models under study, \( M_Z \) and \( G_F \) generally deviate from their SM form. To first order in \( X \), \( M_Z \) and \( G_F \) may be parameterized as

\[
M_Z = M_{Z}^\text{SM} \left(1 + \Delta_Z X\right), \quad G_F = G_{F}^\text{SM} \left(1 + \Delta_G X\right),
\]

(4.2)

where \( \Delta_Z \) and \( \Delta_G \) are model-dependent parameters. For example, one finds

\[
\Delta_Z = \{ -\frac{1}{2} \sin^4 \beta, -\frac{1}{2} \sin^2 \hat{\theta}_w, -\frac{1}{2} \cos^2 \hat{\theta}_w \}.
\]

(4.3)

for the bulk-bulk, brane-bulk and bulk-brane models, with respect to the SU(2)\(_L\) and U(1)\(_Y\) gauge groups.

The relation between the weak mixing angle \( \theta_w \) and the input variables is also affected by the fifth dimension. Hence, it is useful to define an effective mixing angle \( \hat{\theta}_w \) by \( \sin^2 \hat{\theta}_w = \sin^2 \theta_w \left(1 + \Delta_\theta X\right) \), such that the effective angle still fulfills the tree-level relation

\[
G_F = \frac{\pi \alpha}{\sqrt{2} \sin^2 \hat{\theta}_w \cos^2 \hat{\theta}_w M_Z^2},
\]

(4.4)

of the Standard Model.

For the tree-level calculation of \( \Delta^\text{HDSM} \mathcal{O} \), it is necessary to consider the mixing effect of the Fourier modes on the masses of the Standard-Model gauge bosons as well as on their couplings to fermions. In addition, we have to keep in mind that the mass spectrum of the KK gauge bosons also depends on the model under consideration.

Within the framework outlined above, we compute \( \Delta^\text{HDSM} \mathcal{O} \) for the following high precision observables to first order in \( X \): the W-boson mass \( M_W \), the Z-boson invisible
width $\Gamma_Z(\nu\bar{\nu})$, $Z$-boson leptonic widths $\Gamma_Z(l^+l^-)$, the $Z$-boson hadronic width $\Gamma_Z(\text{had})$, the weak charge of cesium $Q_W$ measuring atomic parity violation, various ratios $R_l$ and $R_q$ involving partial $Z$-boson widths, fermionic asymmetries $A_f$ at the $Z$ pole, and various fermionic forward-backward asymmetries $A^{(0,f)}_{FB}$. For example, for the invisible $Z$ width $\Gamma_Z(\nu\bar{\nu})$ we obtain

$$\Delta_{\Gamma_Z(\nu\bar{\nu})}^{\text{HDSM}} = \begin{cases} 
\sin^2 \theta_w \left( \sin^2 \beta - 1 \right)^2 - 1 & \text{for the bulk-bulk model,} \\
-\sin^2 \theta_w & \text{for the brane-bulk model,} \\
-\cos^2 \theta_w & \text{for the bulk-brane model.}
\end{cases} \quad (4.5)$$

Employing the results of $\Delta_{\Delta O}^{\text{HDSM}}$ and calculating all the electroweak observables considered in our analysis by virtue of (4.4), we confront these predictions with the respective experimental values. We can either test each variable individually or perform a $\chi^2$ test to obtain bounds on the compactification scale $M = 1/R$, where

$$\chi^2(R) = \sum_i \frac{(O^{\text{exp}}_i - O^{\text{HDSM}}_i)^2}{(\Delta O_i)^2}, \quad (4.6)$$

$i$ runs over all the observables and $\Delta O_i$ is the combined experimental and theoretical error.

Figure 1 summarizes the lower bounds on the compactification scale $M = 1/R$ coming from different types of observables for the bulk-bulk model. In this model, we present the bounds as a function of $\sin^2 \beta$ parameterizing the Higgs sector. In Table 1, we summarize the bounds resulting for different confidence levels are given in Table 2.
Table 2: Lower bounds (in TeV) on the compactification scale $M = 1/R$ at $2\sigma$, $3\sigma$ and $5\sigma$ confidence levels.

5 Discussion and Conclusions

By performing $\chi^2$-tests, we obtain different sensitivities to the compactification radius $R$ for the three models under consideration: (i) the $SU(2)_L \otimes U(1)_Y$-bulk model, where all SM gauge bosons are bulk fields; (ii) the $SU(2)_L$-brane, $U(1)_Y$-bulk model, where only the $SU(2)_L$ fields are restricted to the brane, and (iii) the $SU(2)_L$-bulk, $U(1)_Y$-brane model, where only the $U(1)_Y$ gauge field is confined to the brane. The strongest bounds hold for the often-discussed bulk-bulk model no matter if the Higgs boson is living in the bulk or on the brane. For the bulk-brane models, we observe that the bounds on $1/R$ are significantly reduced by an amount even up to 1.4 TeV for $3\sigma$ if the $SU(2)$ bosons are the only fields that propagate in the bulk.

The lower limits on the compactification scale derived by the present global analysis indicate that resonant production of the first KK state may be accessed at the LHC, at which heavy KK masses up to 6–7 TeV $^4$ might be explored. In particular, if the $W^{\pm}$ bosons propagate in the bulk with a compactification radius $R \sim 3 \text{ TeV}^{-1}$, one may even be able to probe resonant effects originating from the second KK state, and so differentiate the model from other 4-dimensional new-physics scenarios.

In addition, we have paid special attention to consistently quantize the higher-dimensional models in the generalized $R_\xi$ gauges. Specifically, we have been able to identify the appropriate higher-dimensional gauge-fixing conditions which should be imposed on the theories so as to yield the known $R_\xi$ gauge after the fifth dimension has been integrated out $^4$.

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$^4$After $^1$ had been communicated, we became aware of $^2$, which also discusses the $R_\xi$ gauge before compactification in fermionless non-Abelian theories.
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