OPERA Superluminal Neutrinos per Quantum Trajectories

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Abstract
Quantum trajectories are used to study OPERA findings regarding superluminal neutrinos. As the applicable stationary quantum Klein-Gordon equation is real, real quantum reduced actions and subsequent real quantum trajectories follow. The requirements for superluminal neutrinos are examined. A neutrino that is self-entangled by its own backscatter is shown to have a nonlocal quantum trajectory that may generate a superluminal transit time. Various cases are shown to produce theoretical superluminal neutrinos consistent with OPERA neutrinos. Quantum trajectories are also shown to provide insight into neutrino oscillations.

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1 Introduction
The OPERA collaboration has initially reported, with due caution, that superluminal neutrinos have been observed [1]. Due to the profoundness of this event, the initial OPERA report has suggested that this experiment should be replicated for confirmation. The OPERA collaboration also intentionally offered neither theoretical nor phenomenological explanations. The physics community has responded with a plethora of papers addressing the causes for superluminal neutrinos including three papers by Matone [2,3] and Faraggi [4], who have couched their work in the quantum equivalence principle (QEP) [5–7]. They have developed real quantum trajectories for neutrinos [2–4]. QEP and the quantum trajectory representation are mutually compatible and are embedded in a common quantum Hamilton-Jacobi formulation.

Herein, I develop real quantum trajectories for a neutrino that is self-entangled by its own backscatter and discuss its implications bearing upon the OPERA experiment. “Backscatter” herein means that each neutrino contains an internal degree of backscatter, which is discussed further in §2. Two non-independent quantum effects are shown to induce superluminal propagation: self entanglement, which is the entanglement among the spectral components (one of which is backscatter) within an individual neutrino; and nonlocality. Possible physical causes of backscatter are discussed. Backscatter is shown to be a possible cause of neutrino oscillations. This relativistic quantum-trajectory investigation is embedded in a quantum Hamilton-Jacobi formulation of the associated stationary Klein-Gordon equation (SKGE).

Some in the physics community have investigated OPERA superluminal neutrinos as a group velocity phenomenon [8–11]. The study of group velocities is in the \(\psi\)-representation in Hilbert space and has much...
in common with the quantum trajectories representation. But quantum trajectories are presented in configuration space plus time consistent with the underlying quantum Hamilton-Jacobi formulation. Quantum trajectories are not burdened with issues of wave-packet integrity including the case of strongly peaked spectral components in the wave-number domain of a particle that, if treated as distinct entities, would spatially separate widely into distinct entities whose overlap and entanglement with each other would rapidly decrease \cite{12}. A quantum trajectory is derived from the quantum reduced action (a generator of quantum motion) for the particle as a whole. The quantum reduced action contains the internal backscatter so that all entanglement is retained. This entanglement induces nonlocal motion in the subsequent quantum trajectory.

Section 2 develops the formulation for relativistic quantum trajectories for neutrinos with internal backscatter. The subsequent quantum trajectories give insight into neutrino oscillation. Backscatter is identified as a spectral component of a dichromatic $\psi$ that travels in the counter direction of $\psi$’s other spectral component. In \S 3, quantum trajectories show that while a set of the constants of quantum motion for superluminal transit times for the OPERA experiment always exist, other sets of constants of the quantum motion result in subluminal transit times. For superluminal neutrinos, it is shown that the OPERA experiment must favorably bias the selection of constants of quantum motion. Some favorable systematic biases are examined. Section 4 presents the conclusion.

2 Formulation

Matone \cite{2} and Faraggi \cite{4} have shown that the applicable relativistic quantum stationary Hamilton-Jacobi equation (RQSHJE) for the OPERA neutrinos has the same form as the nonrelativistic quantum stationary Hamilton-Jacobi equation (NQSHJE). The RQSHJE for the OPERA experiment may be given in one-dimension, $q$, by \cite{13}

$$\left(\frac{\partial W}{\partial q}\right)^2 + m^2c^2 - \frac{E^2}{c^2} + \frac{\hbar^2}{2}\langle W; q \rangle = 0$$ \hfill (1)

where $W$ is the relativistic quantum reduced action (Hamilton’s quantum characteristic function), which is a generator of the quantum motion. And $\langle W; q \rangle$ is the Schwarzian derivative of $W$ with respect to $q$ and given explicitly by

$$\langle W; q \rangle = \frac{\partial^3 W}{\partial q^3} - \frac{3}{2} \left(\frac{\partial^2 W}{\partial q^2}\right)^2.$$

The Schwarzian derivative contains the quantum effects and makes the RQSHJE a third-order non-linear differential equation that requires more constants of the quantum motion than the relativistic classical stationary Hamilton-Jacobi equation, which is a first-order nonlinear differential equation \cite{14}.

A general solution for $W$ of Eq. (1) is given within an integration constant by \cite{2,4,15,17}

$$W(q) = \hbar \arctan \left(\frac{A\theta(q) + B\hat{\theta}(q)}{C\theta(q) + D\hat{\theta}(q)}\right)$$ \hfill (2)

where $\{\theta, \hat{\theta}\}$ is a set of independent, real solutions of the associated one-dimensional SKGE,

$$-\hbar^2c^2\frac{\partial^2 \psi}{\partial q^2} + (m^2c^4 - E^2)\psi = 0.$$

The coefficients $(A, B, C, D)$ are real constants here. The Wronskian $W$ is normalized so that

$$W^2(A\theta + B\hat{\theta}, C\theta + D\hat{\theta}) = (AD - BC)W^2(\theta, \hat{\theta}) = (\hbar c)^{-2}.$$ \hfill (3)
The formula $(\hbar c)^{-2}$ of the Wronskian normalization for the relativistic case replaces the analogous formula $2m/\hbar^2$ of the Wronskian normalization for nonrelativistic case \cite{15,16,18}. (The quantum trajectory representation of quantum mechanics uses a Wronskian normalization while the $\psi$-representation uses a Born probability normalization.) The coefficients $(A, B, C, D)$ are specified by the normalization of the Wronskian and the initial conditions for $W$ analogous to those for the NQSHJE, \cite{5,6,14,15}. The coefficients also must obey $AD - BC \neq 0$, otherwise by the principle of superposition $A\theta(q) + B\vartheta(q)$ and $C\theta(q) + D\vartheta(q)$ would be redundant and consequently $W(q)$ would be constant which is forbidden \cite{5}. The general solutions of the RQSHJE and the SKGE imply each other as has been shown elsewhere for the analogous NQSHJE and Schrödinger equation \cite{5,19}. One can always find a set $\{\vartheta, \vartheta\}$ of independent solutions for the SKGE for which the coefficients $B,C \neq 0$ by setting $\vartheta = A\theta + B\vartheta$ and $\vartheta = C\theta + D\vartheta$. Thus, by superposition the quantum reduced action, Eq. (2), may alternatively be expressed as $W = h \arctan[\theta(q)/\vartheta(q)]$.

In the course of this exposition, a redundant set of $\theta$s is nevertheless used to facilitate insight into the OPERA quantum trajectories. Let us consider that the particular quantum reduced action for OPERA neutrinos be specified by

$$W = h \arctan \left( \frac{\alpha \sin(+kq) + \beta \sin(-kq - \varphi)}{\alpha \cos(+kq) + \beta \cos(-kq - \varphi)} \right) $$

(4)

where $k = (E^2 - m^2c^4)^{1/2}/(\hbar c)$ and $\alpha^2 + \beta^2 = 1$ where unless explicitly stated otherwise $\alpha \geq \beta \geq 0$. The set of redundant solutions to the SKGE is \{ $\Gamma \sin(+kq), \Gamma \cos(+kq), \Gamma \sin(-kq - \varphi), \Gamma \cos(-kq - \varphi)$ \} where the $\Gamma$ is the Wronskian normalization factor given by $\Gamma = [(E^2 - m^2c^4)/(\alpha^2 - \beta^2)]^{1/2} = [\hbar kc(\alpha^2 - \beta^2)]^{-1/2}$ consistent with Eq. (3). The factor $\Gamma$ cancels out in Eq. (7). The phase shift $\varphi$ is restricted to $-\pi < \varphi \leq \pi$.

The set $\{E, \beta, \varphi\}$ is a sufficient set of constants of the quantum motion to describe the quantum reduced action $W(E, \beta, \varphi; q)$. As $W$ does not explicitly appear in the RQSHJE while its derivatives of first three orders do, the conjugate momentum, $\partial W/\partial q$ is also a solution of the RQSHJE. The conjugate momentum is given by

$$\frac{\partial W}{\partial q} = \frac{h k (\alpha^2 - \beta^2)}{\alpha^2 + \beta^2 + 2\alpha \beta \cos(2kq + \varphi)}.$$ 

Note that $\partial W/\partial q \neq 0$ for $\alpha > \beta$ consistent with $W$ never being constant \cite{5,6}. As developed elsewhere by a nonrelativistic analogy \cite{12}, this particular form for $W$ and $\partial W/\partial q$ may be linked with an eigenfunction of the associated SKGE that is a Wronskian-normalized dichromatic wave function, $\psi$ whose spectral representation may be given by

$$\psi = \begin{cases} 
\left[ \Gamma [\alpha^2 + \beta^2 + 2\alpha \beta \cos(2kq + \varphi)]^{1/2} \exp \left[ i \arctan \left( \frac{\alpha \sin(+kq) + \beta \sin(-kq - \varphi)}{\alpha \cos(+kq) + \beta \cos(-kq - \varphi)} \right) \right] \right]_{\psi_{\text{forward}}} \\
\left[ \Gamma \exp(+ikq) + \beta \exp(-i\varphi) \exp(-ikq) \right]_{\psi_{\text{backscatter}}}, 
\end{cases}$$

(5)

The intermediate formula in Eq. (5) for $\psi$ labeled “eigenfunction of SKGE” has an exponential argument that could be expressed as $iW/\hbar$ in the one-dimensional case. The amplitude for the intermediate form for $\psi$ in Eq. (5) is proportional to $(\partial W/\partial q)^{-1/2}$. The final formula of Eq. (6) exhibits the dichromatic resolution of $\psi$ into spectral components at $\pm k$. The very existence of a dichromatic spectral resolution for $\psi$ is sufficient for entanglement as $\psi \neq \psi_{\text{forward}} \times \psi_{\text{backscatter}}$, which in turn induces nonlocality \cite{19}. This self-entanglement is also manifested in $W$, Eq. (4), as $W$ is not separable into two terms: one containing only forward propagation and the other, only backscatter, that is $W \neq W_{\text{forward}} + W_{\text{backscatter}}$ \cite{19}. This particular selection of a redundant set of $\theta$s to describe $W$ lets one introduce a degree, $\beta$, of backscatter, \{ $\Gamma \sin(-kq - \varphi), \Gamma \cos(-kq - \varphi)$ \} that coherently interferes in a self-entangled manner with the forward...
motion \{\Gamma \sin(+kq), \Gamma \cos(+kq)\} of amplitude \alpha in the generator of quantum motion. The quantum reduced action for the neutrino, Eq. (1), retains the self-entanglement due to backscatter no matter how large q becomes. The phase shift, \(\varphi\) generalizes W and \(\psi\). If \(\beta = 0\) (i.e., no backscatter), then \(\alpha = 1\), \(\Gamma = k^{-1/2}\), \(W = +\hbar kq\) and \(\psi = (\hbar kc)^{-1/2} \exp(\pm ikq)\), which would be rectilinear motion.

Jacobi’s theorem is used to determine the relativistic equation of quantum motion [2–4,6,15,20]. As Jacobi’s theorem also determines the equation of motion in classical mechanics, Jacobi’s theorem does transcend across the division between classical and quantum mechanics to give a universal equation of motion. Jacobi’s theorem renders time generation as

\[
t - \tau = \frac{\partial W}{\partial E} = \frac{\tau_c}{c} \times \frac{E}{(E^2 - m^2c^4)^{1/2}} \times \frac{(\alpha^2 - \beta^2)}{1 + 2\alpha\beta \cos(2kq + \varphi)}
\]

where \(\tau\) is the Hamilton-Jacobi constant coordinate (a nontrivial constant of integration) that specifies the epoch and where \(\tau_c\) is the time needed to transit the distance \(q\) by light in a vacuum. Let us set \(\tau = 0\) herein. Interference between the spectral components (self-entanglement) of the associated \(\psi\), Eq. (5), is manifested in the quantum trajectory by the cosine term in the denominator of Eq. (6). This self-entanglement is sustained in Eq. (6) no matter how large \(q\) becomes.

The quantum factor, \(H_Q(q)\), has extrema at

\[
d(H_Q) = \frac{d}{dq} = \frac{2k(\alpha^2 - \beta^2)\alpha\beta \sin(2kq + \varphi)}{[1 + 2\alpha\beta \cos(2kq + \varphi)]^2} = 0.
\]

or

\[
q = \frac{n\pi}{2k} - \frac{\varphi}{2k}, \quad n = 0, \pm1, \pm2, \cdots.
\]

The interference due to backscatter (\(\beta \neq 0\)) induces the quantum trajectories to oscillate between maxima and minima times of propagation [12]. For every \(q\), there exist two \(\varphi\)s within the range \(-\pi, +\pi\) that satisfy Eqs. (7) and (8). One of which, \(\varphi_{\text{max}}\), renders \(\cos(2kq + \varphi_{\text{max}}) = -1\); the other, \(\varphi_{\text{min}}\), renders \(\cos(2kq + \varphi_{\text{min}}) = +1\). The different phase shifts, \(\varphi_{\text{max}}\) and \(\varphi_{\text{min}}\), as constants of the quantum motion, specify different quantum trajectories on the \(q, t\)-plane that arrive at the designated \(q\) at different times.

The local maxima propagation times, \(t\), occur in the vicinity of the local maximum of \(H_Q\) given by

\[
q = \left(\frac{(2n - 1)\pi}{2k} - \frac{\varphi_{\text{max}}}{2k}\right), \quad n = 0, \pm1, \pm2, \pm3, \cdots
\]

minima times at \(q = n\pi/k - \varphi_{\text{min}}/(2k), \quad n = 0, \pm1, \pm2, \pm3, \cdots\). The quantum factor, \(H_Q(q)\), has a maximum given by

\[
H_{Q,\text{max}} = \frac{\alpha + \beta}{\alpha - \beta}, \quad q = \frac{(2n - 1)\pi}{2k} - \frac{\varphi_{\text{max}}}{2k}, \quad n = 0, \pm1, \pm2, \pm3, \cdots.
\]

and a minimum value given by

\[
H_{Q,\text{min}} = \frac{\alpha - \beta}{\alpha + \beta}, \quad q = \frac{n\pi}{k} - \frac{\varphi_{\text{min}}}{2k}, \quad n = 0, \pm1, \pm2, \pm3, \cdots.
\]

The extrema, \(H_{Q,\text{max}}\) and \(H_{Q,\text{min}}\), form the envelope (caustics) for a wedge in the \(q, t\)-plane with its apex at the origin. This wedge is analogous to the light cone. Note that \(H_{Q,\text{max}}\) and \(H_{Q,\text{min}}\) are functions of only the constant of the quantum motion \(\beta\) for \(\alpha = +(1 - \beta^2)^{1/2}\).

The relativistic factor \(H_R\) is a function of \(E\) and \(m\), but not of the variable \(q\). The relativistic factor for the OPERA experiment for 17 GeV neutrinos may be approximated by Eq. (6) as

\[
H_R = 1 + \frac{m^2c^4}{E^2} + O(m^4c^8/E^4)
\]

4
Figure 1: Heuristic quantum trajectories in natural units ($\hbar, c, m = 1, k = \pi/2$) with constants of the quantum motion $E = (\pi^2/4 + 1)^{1/2}$ (or alternatively $k = \pi/2$), $\beta = 0.28$, and $\varphi = 0, \pi$. The quantum trajectory for $\varphi = 0$ is solid, while the quantum trajectory for $\varphi = \pi$ is dashed.

where $(m^4e^8/E^4) \ll 1$. For neutrinos, it is expected that $|1 - H_R| < 10^{-19}$ even in their most massive eigenstate [1]. If $\beta$ is sufficiently large so that $H_{Q,\text{min}} \times H_R^{-1} < 1$, then the OPERA neutrino may be superluminal. Herein, $\beta$ is assumed to be sufficiently large while $H_R$ is sufficiently close to unity to make the quantum trajectories superluminal consistent with the apparent superluminal OPERA neutrinos [1]. As $q$ increases along the OPERA neutrino’s quantum trajectory, the quantum factor $H_Q$ fluctuates within the quantum factor’s wedge between its maximum and minimum values. For any $q \geq 0$, there exists a phase shift $-\pi < \varphi_{\text{min}} \leq \pi$ for which $H_Q(E, \beta, \varphi; q) = H_{Q,\text{min}}$. Hence, all values of $q$ are on some superluminal quantum trajectory for the OPERA neutrino. For any $q$, there also exists another phase shift $-\pi < \varphi_{\text{max}} = \varphi_{\text{min}} \pm \pi \leq \pi$ such that the $H_Q(E, \beta, \varphi; q) = H_{Q,\text{max}}$, which implies the existence of also a subluminal quantum trajectory with a different phase shift. And for an interior $q, t$-point within the wedge, there exist two crossing quantum trajectories.

For completeness, the non-relativistic dichromatic particle also exhibits theoretical superluminal behavior [12]. The non-relativistic dichromatic particle differs with the neutrino investigated herein by its non-relativistic wave number. And elsewhere the relativistic particle deep into the forbidden zone has been shown to be superluminal [13]. Also, if $\beta = \alpha = 2^{-1/2}$, then $\psi$ manifests a standing wave.

Figure 1 exhibits the quantum trajectories for an example where $\hbar, c, m = 1$ (natural units) and where the constants of the quantum motion are given by $E = (\pi^2/4 + 1)^{1/2}$ (or alternatively $E$’s proxy $k = \pi/2$), $\beta = 0.28$, and $\varphi = 0, \pi$. While this $\beta$ is orders of magnitude too large to be representative of the OPERA anomaly, it does render a good heuristic example of quantum trajectories for dichromatic particles. The quantum trajectories have temporal turning points where the quantum trajectories alternately become temporally retrograde and forward. The self-entanglement between the two spectral components, Eq. [3], induces this retrograde motion. In turn, retrograde motion induces nonlocality as exhibited on Fig. 1 where the quantum trajectory permits multiple particle positions, $q_s$, for selected times, $t_s$, until a threshold time, $t_{th}$, after which all times permit multiple particle positions. As $q$ increases, the turning points asymptotically approach the bounds prescribed for $H_Q$, Eqs. [9] and [10]. The entwined quantum trajectories for $\varphi = 0, \pi$ on Fig. 1 demonstrate the existence of concurrent superluminal and subluminal propagation. Note that there
are two quasi-periodic meanderings for every wavelength as characteristic of nonlinear differential equations for the interference effect due to the cosine term of Eq. [6] has its frequency doubled, that is the $2kq$ component in the argument of the cosine term has a span of $4\pi$ radians per wavelength. For completeness, these quasi-periodic meanderings asymptotically approach periodicity as $q \to \infty$ [12].

While early latent, temporally retrograde segments may be suppressed as shown by Fig. 1, retrograde segments are realized as $q$ increases. Any finite $\beta$ will induce nonlocality for sufficiently large $q$ [12]. This may be shown by investigating the equation of quantum motion, Eq. [6]. Temporal turning points are smooth and exist where the reciprocal velocity, $dt/dq$, goes to zero. The behavior of reciprocal velocity may be derived from Eq. [6] and given by

$$\frac{dt}{dq} = \frac{d[q(c^{-1}H_R)]}{dq} \times H_Q + qc^{-1}H_R \times \frac{d(H_Q)}{dq}. \tag{11}$$

The conditions for a temporal turning point, $dt/dq = 0$, may be simplified to

$$\frac{1}{q} + \frac{4\alpha\beta k \sin(2kq + \varphi)}{1 + \alpha\beta \cos(2kq + \varphi)} = 0. \tag{12}$$

As $q$ may increase without bound in Eqs. (11) and (12), the $1/q$ term in Eq. (12) will decrease sufficiently to ensure the existence temporal turning points. The existence of temporally retrograde motion follows.

There is an alternative interpretation to segments of temporal retrograde motion: such retrograde motion is an antiparticle, here an antineutrino, $\bar{\nu}$, that moves forward in time. Under this interpretation, $\nu, \bar{\nu}$-pair creations occur at the temporal turning point (local temporal minimum) associated with $H_{Q,\text{min}}$ while $\nu, \bar{\nu}$-pair annihilations occur at the temporal turning point (local temporal maximum) associated with $H_{Q,\text{max}}$. For nonrelativistic quantum trajectories, it has been shown for temporal turning points of nonrelativistic quantum trajectories that neither pair creations need be an endoergic process nor pair annihilation need be an exoergic process [12]. Upon $\nu, \bar{\nu}$-pair creation at the local temporal minimum associated with $H_{Q,\text{min}}$, the $\bar{\nu}$ travels forward in time but spatially retrograde in the $-q$-direction while the $\nu$ travels forward in time and in the $+q$-direction. This gives the quantum trajectory a bi-directional character manifesting internal backscatter. The $\bar{\nu}$ travels in the $-q$-direction to the local temporal maximum associated with $H_{Q,\text{max}}$. There the $\bar{\nu}$ joins with another $\nu$, which has been traveling on the preceding forward segment toward the local temporal maximum, to form a $\nu, \bar{\nu}$-pair that is annihilated at that local temporal maximum.

The $\nu, \bar{\nu}$-pair creations may facilitate neutrino oscillations by possibly producing pairs of different flavors, $e, \mu, \tau$, sterile, or, perhaps a mixed pair thereof. The bidirectional character of the quantum trajectory, discussed in the previous paragraph, also manifests neutrino oscillation. On the other hand, one using the $\psi$ representation would conclude that the wave packet would lose integrity at a temporal turning point due to interference from backscatter. If a quantum trajectory has retrograde segments, then there exists by Fig. 1, as previously discussed, a threshold time, $t_{\text{th}}$, for which the quantum trajectory is multi-valued with regard to $q$ for all $t > t_{\text{th}}$. These $q$s for a particular time $t > t_{\text{th}}$ form the set $\{q_i\}$. The smallest $q_{1,i} \in \{q_i\}$ is on a temporally forward segment of the quantum trajectory exhibiting a $\nu$ particle of some particular flavor. The next greater $q_{2,i} \in \{q_i\}$ is on a temporal retrograde segment of the quantum trajectory exhibiting a $\bar{\nu}$ antiparticle of some flavor that may differ from the flavor of the $\nu$ particle associated with $q_{1,i}$. This alternating pattern repeats itself for the remainder of the quantum trajectory with the caveat that any rotation through the different flavors (neutrino oscillations) may be more arcane. While the quantum trajectory for any $i$ may determine the entire $\{q_i\}$, any measurement of some subset of $\{q_i\}$ would depend upon how the measuring experiment would be executed. As the primary objective here is to investigate the OPERA superluminal anomaly, it suffices that quantum trajectories for this investigation assume that no more than two types of neutrinos, $\nu$ and $\bar{\nu}$, with the same flavor would participate.
3 Application to OPERA experiment

The OPERA collaboration observed \[ \frac{v-c}{c} = \left( 2.37 \pm 0.32 \text{ (stat.)} ^{+0.34}_{-0.24} \text{ (sys.)} \right) \times 10^{-5} \] for 17 GeV neutrinos. From Eqs. (6) and (10), a selection of \( \beta \approx 1.185 \times 10^{-5} \) is chosen to mimic the OPERA collaboration’s neutrino superluminal anomaly. For 17 GeV neutrinos, the mass of a particular flavor is considered to have a negligible effect smaller than \( 10^{-19} \) [1]. For 17 GeV and a neutrino rest mass of 2 eV, the neutrino would have a wavelength given by \( \lambda = \frac{hc}{E^2 - m^2c^4} \approx 72.932 \) am. The \( 730534.61 \pm 0.2 \text{ m} \) between the CNGS target focal point at CERN and the origin of the OPERA detector at LNGS equates to approximately \( 1.0016670729207 \times 10^{22} \) wavelengths. The quantum trajectory for a simulated OPERA neutrino with rest mass 2 eV and constants of the quantum motion \( E = 17 \text{GeV}, \beta = 1.185 \times 10^{-5} \text{ and } \varphi = 0 \), is investigated herein. Segments of its computed quantum trajectory are presented in Figs. 2(a), 2(b) and 2(c).

Figure 2(a) shows the quantum trajectory for the first 2 wavelengths. The duration of one wavelength (two meanderings) is 0.2433 ys as exhibited on Fig. 2(b). The time coordinate (vertical axis) of Fig. 2(b) is \( \tilde{t}_b = t - 900.4 \text{ ys} \) while the distance coordinate (horizontal axis) is \( \tilde{q}_b = q - 269.8 \text{ fm} \). Figure 2(c) shows the quantum trajectory in the approximate vicinity of the OPERA detector at LNGS, which is \( 1.00166707 \times 10^{22} \text{ wavelengths} \) from the CNGS target focal point at CERN. The computed time of flight, \( \text{TOF}_\nu \), for a neutrino to reach the the lower (minimum) caustic at the OPERA detector at LNGS is 2.437 ms. Segments of the quantum trajectory alternate between forward and retrograde motion with regard to time. Between segments, there exist smooth turning points near the upper and lower caustics where neutrino speed becomes instantaneously infinite. [Note that this instantaneously infinite speed is consistent with \( dt/dq \to 0 \), cf. Eqs. (11) and (12).] Nevertheless, the velocity remains integrable as substantiated by Fig. 2(c). The coordinates for 2(c) are \( \tilde{q}_c = q - 730534.61 \text{ m} \text{ and } \tilde{t}_c = t - 2.437 \text{ ms} \). Note that the time scale of Fig. 2(c) is much coarser than those of Figs. 2(a) and 2(b) to accommodate...
the size of the range of the temporally forward and retrograde segments. These durations increase as \( q \) increases. While not apparent in Fig. 2(c), corresponding points on successive wavelengths advance in time about 0.2433 \( \tau \) consistent with Figs. 2(a) and 2(b). The \( TOF_\nu \) computation predicts an early arrival time with respect to that assuming the speed of light in a vacuum, \( TOFc \). The time difference is given by

\[
\delta t = TOF_\nu - TOFc = 57.75 \text{ ns}
\]  

while OPERA observed \( \delta t = (57.8 \pm 7.8(\text{stat.})^{+8.3}_{-5.9}(\text{sys.})) \text{ ns} \). The relative difference in the computed neutrino velocity, \( v \), and \( c \) is by Eq. (13)

\[
(v - c)/c = 2.37 \times 10^{-5}
\]

while OPERA observed \((v - c)/c = (2.37 \pm 0.32(\text{stat.})^{+0.34}_{-0.24}(\text{sys.})) \times 10^{-5}\). But the constants of quantum motion, \( \beta = 1.185 \times 10^{-5} \) and \( \varphi = 0 \), where chosen to match the OPERA anomaly for a 17 GeV neutrino at \( q = 730534.61 \text{ m} \). Note that value of \( \beta \) used here is well within the reported value \([1]\) for \( \bar{\nu}_\mu \) contamination at the OPERA detector of less than 2.1%. Again, there exists for other points, \( q \), on the quantum trajectory for which some value of the constant of the quantum motion \( \varphi \) in the range \((-\pi, +\pi) \) would also render Eq. (14) true.

We note in Fig. 2(c) that the time duration of the retrograde segments has increased to about 115.5 ns. These durations will continue to increase with increasing \( q \) as the quantum trajectory proceeds out the wedge of \( H_Q \). As discussed earlier, a temporally retrograde segment may be considered to manifest an antineutrino, \( \bar{\nu} \), traveling forward in time but in a negative \( q \)-direction. While all segments of the quantum trajectories between \( \nu, \bar{\nu} \)-pair creations in the vicinity of \( H_{Q,\text{min}} \) and \( \nu, \bar{\nu} \)-pair annihilations in the vicinity of \( H_{Q,\text{max}} \) are spatially one-quarter wavelength long (18.233 am) and temporally forward with an approximate duration of 115.5 ns as previously noted. The average speed over these quarter-wavelength segments is quite subluminal, not even pedestrian, at approximately 0.1579 nm/s. Hence, neither \( \nu \) particle advancing in the positive \( q \)-direction nor the \( \bar{\nu} \) antiparticle receding in the negative \( q \)-direction (spatial retrograde motion) are tachyons in neighborhoods that do not include temporal turning points. As such, the subluminal portions of neutrino’s quantum trajectory are not subject to the restriction of Cohen and Glashow \([21]\) that tachyons could not sustain superluminal quantum motion due to a loss of energy to spontaneous \( e^-, e^- \)-pair creations. On the other hand, the quantum trajectories are superluminal at isolated temporal points where either \( \nu, \bar{\nu} \)-pair creations or \( \nu, \bar{\nu} \)-pair annihilations exist. These \( \nu, \bar{\nu} \)-pair creations or \( \nu, \bar{\nu} \)-pair annihilations are respectively neither endoergic nor exoergic as energy \( E \) is explicitly one of the constants of quantum motion for the quantum trajectory and so used in Jacobi’s theorem to develop the relativistic equation of quantum motion, Eq. (6). Thus, the restrictions of Cohen and Glashow \([21]\) are also not applicable to the superluminal neighborhood associated with temporal turning points. The findings of this paragraph may be generalized for the entire quantum trajectory. And the full quantum trajectory through its nonlocality may have superluminal transit time even if all of its individual segments may have subluminal average absolute values of speed over their respective segments.

Had the 2007 MINOS neutrino velocity anomaly \([22]\),

\[
\frac{(v_{\text{MINOS}} - c)}{c} = [5.1 \pm 2.9(\text{stat.} + \text{sys.})] \times 10^{-5} \text{ 68\% confidence limits,}
\]

been chosen for investigation, then the constant of quantum motion \( \beta_{\text{MINOS}} = 2.55 \times 10^{-5} \) would have been selected. It is noted that the MINOS collaboration never asserted that MINOS neutrinos were superluminal as the MINOS confidence limits were insufficient.

As \( H_{Q,\text{max}} \) and \( H_{Q,\text{min}} \) are just functions of \( \beta \) as already noted and as \( H_Q \) is the more dominant than \( H_R \) for neutrinos at the energies of the OPERA experiment, the relative difference \((v - c)/c\) is more a function of the constant of the quantum motion \( \beta \) than a function of the constant of the quantum motion \( E \). This is consistent with the OPERA experiment where any energy dependence of \( v \) is less than one \( \sigma \) over the energy range of the experiment \([1]\).
However, OPERA collaboration obtained $\delta t$ by comparing the temporal distributions of neutrino interactions at LNGS with the temporal distribution of protons hitting the CNGS target \(^1\). This implies that many interactions over the range $(-\pi, +\pi)$ for the constant of the quantum motion $\varphi$ must be considered. The average for the quantum factor $H_Q$ of Eq. (6) is given by averaging over all quantum trajectories, each specified by its $\varphi$, that may intercept a given $q$. Averaging over $\varphi$ is given by \(^23\)

$$
< H_Q \varphi > = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H_Q d\varphi = \frac{(\alpha^2 - \beta^2)}{2\pi} \int_{-\pi}^{+\pi} \frac{d\varphi}{1 + 2\alpha\beta \cos(2kq + \varphi)} = \frac{(\alpha^2 - \beta^2)}{2\pi} \frac{2\pi}{[(\alpha^2 - \beta^2)^2]^{1/2}} = 1.
$$

So, uniform $\varphi$-averaging over all quantum trajectories that may intercept $q$ does not render an aggregate superluminal motion. This result is consistent with the results of Matone \(^2\) and Faraggi \(^4\) that while particular quantum trajectories may have superluminal transit times, the quantum correction (here $H_Q$) does not necessarily support the existence of superluminal motion for the general case.

Possible sources of backscatter in the quantum reduced action or SKGE's $\psi$ include, but not limited to, the CNGS neutrino beam assembly, neutrino oscillations, transmission effects through the earth including the Mikheyev-Smirnov-Wolfenstein (MSW) effect \(^24\), thermal effects \(^3\) and interference due to a quantum analogy to reverberation, and the OPERA detector at LNGS. If any single cause or some combination thereof systematically introduces a biased $\varphi$-distribution, perhaps in combination with a biased $\beta$ distribution, then this could permit superluminal quantum motion. Then the observed OPERA superluminal quantum motion \(^1\) for could be attributed to an internal interference effects described by particular values of the constants of quantum motion $\beta$ and $\varphi$. We shall briefly study three cases that lead to superluminal neutrinos.

In the first case, we consider $\nu, \bar{\nu}$-pair creations. Neutrino oscillations that create $\nu, \bar{\nu}$-pairs are adduced as a candidate source for backscatter with a favorable biased $\varphi$-distribution. These $\nu, \bar{\nu}$-pair creations occur at local minima temporal turning points as shown on Figs. 1 and 2(c). Pair creation phenomenon at maximum reinforcement by coherent interference is theoretically substantiated by Refs. \(^12\) and \(^25\). If the OPERA experiment systematically only detected these pair creations, then a $\beta = 1.185 \times 10^{-5}$, as used for Eq. \(^{14}\), would explain OPERA superluminal propagation. The OPERA experiment would systematically select $\varphi_{\min}$ to detect $\nu, \bar{\nu}$-pair creation.

On the other hand, let us suppose for a second case that the OPERA experiment could detect not only $\nu, \bar{\nu}$-pair creations but also $\nu, \bar{\nu}$-pair annihilations as well. The forward and backscatter spectral components of $\psi$ are in coherent reinforcement at the local minimum temporal turning points \(^{12}25\) where $\nu, \bar{\nu}$-pairs are created. For coherent reinforcement, the relative amplitude of the eigenfunction, $\psi_+$, would be proportional to $\alpha + \beta$ by Eq. \(^5\) where the spectral components would be in phase. For $\nu, \bar{\nu}$-pair annihilations at local maximum temporal turning points, the relative amplitude of the eigenfunction, $\psi_-$, would be $\alpha - \beta$ where the spectral components would be out of phase \(^{12}25\). The predicted relative difference of the neutrino velocity, analogous to Eq. \(^{14}\), for detecting just both $\nu, \bar{\nu}$-pair creations and annihilations would have the form

$$
1 - \frac{\alpha - \beta}{\alpha + \beta} \times \frac{\alpha + \beta}{\alpha - \beta} = 2 + \frac{\alpha + \beta}{\alpha - \beta} \times \frac{\alpha - \beta}{2} = 2 \beta^2 = 2.37 \times 10^{-5}
$$

(15)

where $\psi^\dagger \psi$ is the Born weighting function. The divisor “2” in Eq. \(^{15}\) is the normalization $\psi^\dagger \psi_+ + \psi^\dagger \psi_- = 2$ in Eq. \(^3\). As the internal interference between spectral components of $\psi$ are in reinforcement at local time minima while in opposition at local time maxima, the Born weighting function $\psi^\dagger \psi$ renders preponderant weighting to minimum transit times — hence, superluminal neutrinos. The constant of the motion $\beta$ that would be consistent with OPERA results for detecting just both $\nu, \bar{\nu}$-pair creations and annihilations is given by $\beta = 3.442 \times 10^{-3}$. This value of $\beta$ is also well within the reported value \(^1\) for $\bar{\nu}_\mu$ contamination at the OPERA detector of less than 2.1%. Note that $\beta$ for this second case must be greater than the $\beta$ for the first case as $\nu, \bar{\nu}$-pair annihilations by themselves would create subluminal neutrinos.
In the third case, the neutrino may be detected at any point $q$ within the OPERA detector at LNGS along the neutrino’s trajectory for any value of the constant of the motion $\varphi$. The cross section of neutrino detection at $q$ for each trajectory with a constant of the motion $\varphi$ has a Born weighting function given by $\psi^\dagger(q)\psi(q) = 1 + 2\alpha\beta\cos(2kq + \varphi)$ from Eq. (5). As the OPERA detector is more than $10^{22}$ wavelengths downstream from the CNGS target focal point at CERN, evaluation of the relative difference of neutrino velocity, Eq. (14), may be simplified by considering the contributions of only $H_q$. We average $H_q$ weighted by $\psi^\dagger(q)\psi(q)$ over $\varphi$ to account for all trajectories that intercept $q$. The predicted relative difference of the neutrino velocity, analogous to Eq. (14), for detecting a neutrino weighted by $\psi^\dagger(q)\psi(q)$ has the form

$$1 - \frac{\int_{-\pi}^{\pi} H_q \psi^\dagger \psi \, d\varphi}{\int_{-\pi}^{\pi} \psi^\dagger \psi \, d\varphi} = 2\beta^2 = 2.37 \times 10^{-5}.$$  

Analogous to the second case, the interference between the spectral components of $\psi$ are in reinforcement if $H_Q < 1$ and in opposition if $H_Q > 1$. This correlation between the Born weighting function $\psi^\dagger \psi$ and $H_Q$ is due to their common dependence upon the term $\cos(2kq + \varphi)$ in Eqs. (5) and (6). As $\int_{-\pi}^{\pi} \cos(kq + \varphi) \, d\varphi = 0$, superluminal times follow for neutrinos with internal backscatter. The value of the constant of the motion $\beta$ that would be consistent with OPERA results is $\beta = 3.442 \times 10^{-3}$. This value for $\beta$ is the same as the value for the second case. This sameness may be explained by

$$\psi^\dagger\bigg|_{\varphi=\hat{\varphi}} \psi\bigg|_{\varphi=\hat{\varphi}} + \psi^\dagger\bigg|_{\varphi=\hat{\varphi}+\pi} \psi\bigg|_{\varphi=\hat{\varphi}+\pi} = 2$$

where the difference of $\pi$ between the values for the constant of motion $\varphi$ in the two weighting functions causes their interference contributions to cancel each other.

4 Conclusion

In conclusion, quantum trajectories with backscatter can explain superluminal propagation for quantum trajectories of neutrinos. Backscatter can be incorporated into a general solution for the quantum reduced action so that self-entanglement is maintained in the generator of quantum motion of neutrinos. Backscatter may also be incorporated into an eigensolution of the associated Schrödinger equation so that self-entanglement maintains coherent interference. Entanglement, Eq. (6), begets nonlocality that in turn begets overall superluminal transit times for the full trajectory even if transit times for all segments of the trajectory (the absolute value of the time to transit from a temporal extremum to the next temporal extremum) are subluminal. These segments alternate between being forward or retrograde in time. For any point on the quantum trajectory with constants of quantum motion energy, $E$, and finite backscatter, $\beta > 0$, there exists a third constant of quantum motion phase shift, $\varphi$, for which the quantum trajectory is superluminal. When the interference due to backscatter is averaged for a uniform distribution of phase shift, $\varphi$, then no superluminal propagation in general is expected. If the OPERA experiment in a systematic way favorably biases the interference due to backscatter, then superluminal propagation may be expected. Different physically inspired processes that systematically favorably bias the interference are examined. Processes that include the favorable correlation with regard to phase shift, $\varphi$, between the Born weighting function $\psi^\dagger \psi$ and quantum factor $H_Q$ in the equation of motion, Eq. (6), lead to theoretical superluminal neutrinos. These superluminal neutrinos can replicate the apparent superluminal OPERA observations by adjusting the degree of backscatter, $\beta$, a constant of the motion. The degree of superluminality beyond a threshold $\beta$ increases as $\beta$ increases.

The restrictions of Cohen and Glashow are applicable to neither subluminal nor superluminal portions of the quantum trajectories for which energy is a constant of the quantum motion. At the temporal turning points of a quantum trajectory where the velocity of the quantum trajectory becomes instantaneously infinite, either $\nu, \bar{\nu}$-pair creations or annihilations occur instead of Cohen and Glashow $e^+, e^-$-pair creations.

A byproduct of these investigation is insight into neutrino oscillation. The $\nu, \bar{\nu}$-pair creations occur at the local time minima of the quantum trajectory while $\nu, \bar{\nu}$-pair annihilations occur at local time maxima.
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