Goodness-of-fit analysis of the Cosmicflows-2 database of velocities

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ABSTRACT

The goodness-of-fit (GoF) of the Cosmicflows-2 (CF2) database of peculiar velocities with the ΛCDM standard model of cosmology is presented. Standard application of the χ² statistics of the full database, of its 4,838 data points, is hampered by the small scale non-linear dynamics which is not accounted for by the (linear regime) velocity power spectrum. The bulk velocity constitutes a highly compressed representation of the data which filters out the small scales non-linear modes. Hence the statistics of the bulk flow provides an efficient tool for assessing the GoF of the data given a model. The particular approach introduced here is to use the (spherical top-hat window) bulk velocity extracted from the Wiener filter reconstruction of the 3D velocity field as a linear low pass filtered highly compressed representation of the CF2 data. An ensemble 2250 random linear realizations of the WMAP/ΛCDM model has been used to calculate the bulk velocity auto-covariance matrix. We find that the CF2 data is consistent with the WMAP/ΛCDM model to better than the 2σ confidence limits. This provides a further validation that the CF2 database is consistent with the standard model of cosmology.

Key words: cosmology: large-scale structure of universe, peculiar velocities

1 INTRODUCTION

In the standard model of cosmology the large scale structure (LSS) of the universe grows out of a primordial perturbation field via gravitational instability. The continuity equation implies that the evolving density field is associated with a peculiar velocity field, both of which represent departures, or fluctuations, from a pure Hubble expansion. The Cosmic Microwave Background (CMB) dipole anisotropy, interpreted as the consequence of the peculiar motion of the Local group (LG) relative to the CMB frame of reference is the best evidence and example of that motion. First hints for the dipole anisotropy were given by Conklin (1969) and Henry (1971) and a more definitive determination by Smoot et al. (1977). In the standard model the density and the velocity fields are connected and one can opt for a full description of the LSS of the universe by means of either the density or the velocity (or both) field (Peebles 1980). Velocities of galaxies are taken here as the prime tracers of the LSS.

The quest for a mapping of the local universe has been a major driver of the study of the velocities of galaxies (Tonry & Davis 1981; Aaronson et al. 1982; Lynden-Bell et al. 1988; Bertschinger et al. 1990; Dekel et al. 1990; Shaya et al. 1995; Nusser & Davis 1995; Strauss & Willick 1995; Willick et al. 1997; Springob et al. 2007; Tully et al. 2008; Springob et al. 2014). The study of the bulk velocity dipole has been part of a major effort to do cosmology by velocities. The notion of a bulk velocity is that of the (weighted) mean velocity of a finite chunk of space, e.g. the mean velocity of a sphere of radius R centered on the LG. It follows that a bulk velocity is a moment taken over the full velocity field. Alternatively, a bulk velocity can be viewed as a highly compressed representation of a large database of velocities - namely a database of hundreds or thousands entries is compressed into three numbers, corresponding to the three Cartesian components of the bulk velocity vector. Indeed, velocity-based cosmology has been heavily dominated by attempts to estimate the bulk velocity on various scales (Watkins et al. 2009; Lavaux et al. 2010; Feldman et al. 2010; Colin et al. 2011; Nusser & Davis 2011; Nusser et al. 2011; Turnbull et al. 2012; Ma & Pan 2014; Watkins & Feldman 2015). (See Nusser 2016 for a thorough analysis of different methods of estimation of the bulk velocity.) The motivation for studies of the bulk velocity is twofold. One is the wish to understand the origin of the CMB dipole and its relation and possibly coherence with the flow field on larger scales. The other is to use it as a constraint on cosmological models and the values of various cosmological parameters (Jaﬀe & Kaiser 1995; Kaiser 1988; Watkins et al. 2009; Nusser & Davis 2011; Turnbull et al. 2012; Macaulay et al. 2012).

In the standard model structure has emerged from a primordial random Gaussian perturbations field. This implied that on scales...
larger than $\approx 10 \, h^{-1}\text{Mpc}$ (where $h$ is Hubble’s constant in units of $100\, \text{km}\, \text{s}^{-1}\text{Mpc}$) the velocity field constitutes a random Gaussian vector field. This enables a powerful and rigorous approach to the problem of the reconstruction of the LSS - both the density and velocity fields - from a given database of peculiar velocities. Given a database of peculiar velocities and assuming a prior cosmological model which postulates that the underlying primordial perturbation field is Gaussian of a given power spectrum, the LSS is readily reconstructed by means of the Wiener filter (WF) and constrained simulations (CRs) that sample the scatter around the mean (WF) field (Hoffman & Ribak [1999] Zaroubi et al. [1995] Hoffman [2001]). This WF/CRs Bayesian framework provides an appealing framework for the reconstruction of the LSS from velocities data within the realm of the standard model of cosmology (Zaroubi et al. [1999] 2001] [Courtois et al. 2012] Tully et al. [2014] Pomarède et al. [2015]). The WF/CRs reconstruction of the 3D velocity field was used to estimate the local bulk flow on scale of up to a few hundreds of Mpc (Hoffman et al. [2015]).

The Bayesian approach is indeed robust and optimal, within the context mentioned above. It focuses on the reconstruction of the LSS within the framework of the standard model of cosmology and for a given database. However, most previous studies have not addressed the question how consistent is the assumed cosmological model with the observed data. This is not a trivial issue - the model needs to agree with the data so as to provide a solid logical model with the observed data. This is not a trivial issue - the model needs to agree with the data so as to provide a solid logical model with the observed data. This is not a trivial issue - the model needs to agree with the data so as to provide a solid logical model with the observed data.

However, history does not always proceed in a linear fashion. The `history is circular’ approach was indeed applied to velocity databases (Zaroubi et al. 1997, 2001; Jaffe & Kaiser 1995 [Hoffman 2001] Press et al. 2007). The Bayesian approach was applied to various databases (Zaroubi et al. 1997, 2001, Jaffe & Kaiser 1995 [Hoffman 2001] Press et al. 2007). The application of the likelihood analysis to actual velocity databases suffers however from one major drawback. The gravitational dynamics of structure formation induces non-linear contributions to the velocities of galaxies. These non-linear corrections render the parameter estimation and GoF analysis to be rather uncertain. The remedy to the problem involves the filtering of small scales to give linearized data. The likelihood analysis can then be safely applied to the linearized data. Here we suggest such a small scales filtering procedure and study the extent by which Cosmicflows-2 (CF2) (Tully et al. 2013) is compatible with the $\Lambda$CDM standard model of cosmology.

The paper is structured as follows: The Bayesian and likelihood approach is derived in $\S2$. The issue of a possible circularity of the use of the WF to assess the GoF of the data by the model is discussed and refuted in $\S3$. The CF2 data is reviewed in $\S4$ and its application to the CF2 dataset is presented in $\S5$. The paper concludes with a discussion at $\S6$.

2 STATISTICAL ANALYSIS

2.1 Bayesian approach: The Wiener Filter

The problem addressed here is that of the estimation of the underlying 3D velocity field in general, and the bulk flow in particular, given the data. The Bayesian framework provides the means to do that (Zaroubi et al. 1995).

Modelling the data:

$$U_\mu = u_\mu + \epsilon_\mu = v(r_\mu) \cdot \hat{r}_\mu + \epsilon_\mu$$

(1)

Here $U_\mu$ is the $\mu$-th data point, corresponding to the radial velocity component at $r_\mu$ ($u_\mu$), and $\epsilon_\mu$ is the statistical observational error of that data point. (Here, $\mu$, $\nu = 1, \ldots, N_{\text{data}}$ where $N_{\text{data}}$ is the number of data points.)

The Wiener Filter (WF) estimator of the 3D velocity field on a cartesian grid:

$$u_{\mu}^{WF}(r) = \xi_{\mu}^{\ast}(r) \xi_{\mu}^{-1} U_\nu$$

(2)

Here $r_\mu$ is the coordinate of the i-th grid point, $\xi_{\mu}^{\ast}(r_\mu)$ is the cross-correlation function,

$$\xi_{\mu}^{\ast}(r_\mu) = \left\langle v_\mu(r_\mu) U_\nu \right\rangle = \left\langle v_\mu(r_\mu) v_\nu(r_\nu) \cdot \hat{r}_\nu \right\rangle ,$$

(3)

and the data-data auto-covariance matrix is:

$$\xi_{\mu\nu} = \left\langle U_\mu U_\nu \right\rangle = \left\langle u_\mu u_\nu + \sigma_\mu^2 + \sigma_\nu^2 \right\rangle$$

(4)

Given the WF reconstruction of the 3D flow field a Bayesian estimation of the bulk flow is easily obtained. The bulk flow is defined here as the mean velocity, in the sense of a top-hat window weighting, of a sphere of radius $R$, centred on the Local Group:

$$B_\alpha = \frac{1}{V_R} \int_{r < R} v_\alpha(r) d^3r$$

(5)

(Here $V_R = (4\pi/3)R^3$.) In the case of the WF reconstruction and for a series of spheres of radii $R_\mu$ ($\mu = 1, \ldots, N_{\text{spheres}}$) one finds:

$$B_{\mu,\alpha}^{WF} = \frac{1}{V_{R_\mu}} \int_{r < R_\mu} v_{\mu}^{WF}(r) d^3r = \frac{1}{V_{R_\mu}} \int_{r < R_\mu} \xi_{\mu\nu}(r) d^3r \xi_{\nu\nu}^{-1} U_\nu$$

(6)

It should be noted that $B_{\mu,\alpha}^{WF}$ is a random Gaussian variable, drawn from a random Gaussian field. It is a vector Gaussian variable obtained by convolving a set of Gaussian variable, $[U_\nu]$, with a given linear operator. This linear operator is determined by the assumed prior model, the distribution of the data points and the errors model.

2.2 Likelihood analysis

The problem addressed here is that of the degree of compatibility of the data with a given model, namely what is the likelihood of a given dataset to emerge from a given model.

A straightforward approach is to apply the likelihood analysis to the raw data (Zaroubi et al. 1995):

$$\mathcal{L}(U_\mu | \text{model}) = \frac{1}{\sqrt{\det(\xi_{\mu\nu})}} \exp \left\{ - \frac{U_\mu \xi_{\mu\nu}^{-1} U_\nu}{2} \right\}$$

(7)

For Gaussian variables, such as $[U_\nu]$, the value of the $\chi^2$ and the number of the degrees of freedom, determine the likelihood of the data given the model. In the present case $\chi^2 = U_\mu \xi_{\mu\nu}^{-1} U_\nu$, the number of degrees of freedom is the number of data points and the model is embodied by the data auto-covariance matrix, $\xi_{\mu\nu}$. This approach was applied to various databases (Zaroubi et al. 1997, 2001).
However, it turns out that this statistical analysis is sensitive to \( \sigma_s \), the small scale power induced by non-linear dynamics on small scales (Jaffe & Kaiser 1995; Zaroubi et al. 1997). This sensitivity to \( \sigma_s \) renders the likelihood analysis of the raw velocity data quite futile.

A different approach to the likelihood estimation is to apply it to a linear low pass filtered data, thereby removing or suppressing the small scale component that introduces non-linearities to the data. The bulk flow estimator is an ideal low pass filter - \( B_{WF}^{s} \) is affected predominantly by waves longer than \( \approx R_s \). It follows that a likelihood analysis that is shielded from small scales non-linear dynamics is provided by,

\[
L(B_{WF}^{s} | \text{model}) = \frac{1}{\sqrt{\text{det}(\Sigma_{WF}^{s})}} \exp \left[ -\frac{1}{2} \frac{B_{WF}^{s} \Sigma_{WF}^{-1} B_{WF}^{s}}{\text{variance}} \right],
\]

where the bulk velocity auto-covariance-matrix \( \Sigma_{WF}^{s} \) is

\[
\Sigma_{WF}^{s} = \left( B_{WF}^{s} B_{WF}^{s} \right) = \frac{1}{V_{RS} V_{RS}} \int_{r=r_S} \int_{r'=r_S} \epsilon_i(r) \epsilon_i(r') \, d^3r \, d^3r'.
\]

\[ 3 \text{ IS THE METHODOLOGY CIRCULAR?} \]

In the field of cosmology and LSS one often encounters the claim that the WF reconstruction in general, and of the bulk flow in particular cannot be used to assess cosmological parameters and GoF of peculiar velocity data by cosmological data. The scope and implications of the present section exceed and reach beyond the specific aim of the paper but it should be considered as one of our key results.

The methodology of Zaroubi et al. (1995) is the basis for extracting information from large scale structure observational data, where the underlying fields are assumed to be Gaussian. This methodology assumes a cosmological prior in terms of a power spectrum which allows an inference of velocity and density fields. The WF estimation of the bulk flow has appeared in several other, sometime implicit, forms. This is the case of the ‘all space constrained estimate’ (ASCE) of Nusser & Davis (2011) and the minimal variance (MV) estimator of Watkins & Feldman (2015). In fact, Nusser (2016) has demonstrated explicitly the tight relationship of the Watkins & Feldman (2015) MV estimation with the WF, with its explicit dependence of an assumed power spectrum.

The WF is the optimal linear (in the data) estimator of the underlying field in the sense of the minimal variance between the inferred and true underlying field (Rybicki & Press 1992; Zaroubi et al. 1995). To the extent that the power spectrum and the errors are correctly known then no other linear estimator can perform better than the WF. Yet, even if a wrong model is assumed the resulting WF provides an estimator, but of a lesser accuracy. The following simple example makes the point.

Let us pose the following question. How does a wrongly chosen prior model for the WF affects the \( \chi^2 \) of the reconstructed field with respect to the assumed (wrong) model?

We illustrate the point with the following straightforward example. Consider \( N \) data points \( d_i = s_i + e_i \) where the random error \( e_i \) is normally distributed with variance \( \sigma^2_e \). The underlying signal \( s_i \) obeys a Gaussian distribution of zero mean and variance \( \sigma^2_s \) and with no correlation between different points. Namely, both the signal and noise constitute a white noise. Take a prior which assumes that the signal has a (wrong) mean \( \bar{s} \neq 0 \) and a variance \( \sigma^2_{s_p} \), i.e. the correct value. Let \( f_i \) be the WF estimated signal for point \( i \), obtained by maximizing

\[
2 \ln P(f|d) \propto -(d_i - f_i)^2 / \sigma_s^2 - (f_i - \bar{s})^2 / \sigma_{s_p}^2
\]

which gives

\[
f_i = \frac{d_i \sigma_s^2 + \bar{s} \sigma_{s_p}^2}{\sigma_s^2 + \sigma_{s_p}^2}.
\]

Let us explore the conditions under which the \( f_i \) estimate is actually consistent with the assumed prior. Following the methodology presented in the current paper we first compute the variance \( \sigma^2_f = \chi^2 / \text{assuming the prior, i.e. the average is taken as an ensemble average with } d_i \) having mean \( \bar{s} \) and variance \( \sigma^2_s \). It is easily seen that

\[
\sigma^2_f = \sigma_s^2 \left( \frac{\sigma^2_s}{\sigma^2_s + \sigma^2_{s_p}} \right)^2.
\]

Note that, as expected (see Zaroubi et al. 1995), the variance of the WF estimate of \( s_i \) is biased. The \( \chi^2 \) of the estimated \( f_i \) with respect to its expected variance is

\[
\chi^2 = \sum \frac{f_i^2}{\sigma^2_f},
\]

where \( f_i \) is given by equation (11) with \( d_i \) being the actual data which has a mean of zero (in the ensemble average sense) and not \( \bar{s} \). We get for the ensemble average of \( \chi^2 \) the expression

\[
\chi^2 = N \left( 1 + \frac{\bar{s}^2}{\sigma^2_s} \right).
\]

Thus, \( \chi^2 \) for the prior can significantly be different from the mean value \( \chi^2 = N \) depending on the ratio \( \bar{s}/\sigma_s \). It is trivial to show that if the correct model has been assumed as a prior the \( \chi^2 \) per degree of freedom would have been unity. It follows that the WF does not yield by construction a \( \chi^2 = N \). This is obtained only for the correct model.

A further comment is due here. If the data is of good quality then the WF recovery of the underlying signal depends very weakly on the assumed prior. However in the cosmological case of peculiar velocities where the signal to noise ratio deteriorates strongly with the distance the opposite occurs. In fact, it has been shown that a significant enhancement of the large scale power in the assumed prior hardly affects the estimated bulk flow (Nusser & Davis 2011).

\[ 4 \text{ DATA} \]

Cosmflows-2 (CF2) is the second generation catalog of galaxy distances and velocities built by the Cosmflows collaboration (Tully et al. 2013). The CF2 database contains more than 8,000 galaxy peculiar velocities. Distance measurements come mostly from the Tully-Fisher relation and the Fundamental Plane methods (Colless et al. 2001). Cepheids (Freedman et al. 2001), Tip of the Red Giant Branch (Lee et al. 1993), Surface Brightness Fluctuation (Tonry et al. 2001), supernovae of type Ia (Jha et al. 2007) and other miscellaneous methods also contribute to this large dataset but to a minor extent (12%).

A partial remedy to the problem of non-linearities which affect the linear WF/CRs reconstruction is provided by the use of a grouped version of the database which contains 4,838 peculiar velocity values. The grouping is done by collapsing the velocities of all members of a given group or cluster of galaxies into one data point. The grouping is associated with the reduction of estimated
error by the square root of the number of group members. The mean statistical error for individual galaxies of \( \sim 20\% \) (of the distance) is thus reduced to a mean error for the groups of \( \sim 9\% \).

5 RESULTS

The standard model of cosmology, a flat \( \Lambda \) cold dark matter (ΛCDM) cosmology, is taken here as the prior model. The Wilkinson Microwave Anisotropy Probe 5 (WMAP5) cosmological parameters are assumed (Komatsu et al. 2009). The following parameters, in particular, have been used here: \( \Omega_m = 0.28 \) (the mass density parameter), \( h = 0.70 \) (the Hubble constant in units of 100km s\(^{-1}\)Mpc\(^{-1}\)) and \( \sigma_8 = 0.817 \) (the r.m.s. of the linear density fluctuations in a sphere of 8 h\(^{-1}\)Mpc). The model further assumes that the primordial fluctuations constitute a Gaussian random field.

The bulk flow derived by Hoffman et al. (2015) from the WF reconstruction of the CF2 database is used here as the low pass representation of the CF2 data.

The bulk velocity auto-covariance matrix \( \Xi_{\alpha \beta}^{WF} \) has been evaluated as an ensemble average taken over 2250 random realizations of the linear velocity field. These realizations have been evaluated on a Cartesian 256\(^3\) grid in a computational box of...
L = 1280 h^{-1} \text{Mpc}. Each realization has been sampled at the position of the CF2 data points. Normal errors have been added according to the nominal errors of the CF2 individual data points. Each realization is yielding a mock of CF2 data - much in the same way as mock data is calculated in the construction of CRs (Hoffman & Ribak 1991). The same WF operator that is applied to the actual CF2 database is applied to the mock catalog, yielding thereby an ensemble of mock WF bulk flows evaluated for spheres of R = 20, 30, ... 150 h^{-1} \text{Mpc}.

Our intention here is to calculate the GoF of the (WF bulk velocity) CF2 database with the WMAP/ΛCDM model. It follows that we do not change the model and therefore the pre-factor of the exponential term in the expression of the likelihood function (Eq. [8] is a constant (for a given set of data) and is irrelevant. The quantity of interest is the reduced χ² of all sphere of radius smaller or equal R,

\[ \chi^2(R, \sigma < R) = B_{a_\alpha}^{WF} Z_{a_\alpha b, b}^{-1} B_{b, b}^{WF}, \] (15)

and the χ² of a given sphere of radius R,

\[ \chi^2(R, R_0 = R) = B_{a_\alpha}^{WF} Z_{a_\alpha b, b}^{-1} B_{b, b}^{WF}, \] (16)

The upper panel of figure 3 shows the cumulative χ² for R = 20, 30, ... 150 h^{-1} \text{Mpc}. The confidence limits corresponding to the standard 1, 2 and 3σ normal distribution are shown as well, namely the upper dotted, dashed and dot-dashed curves indicate the reduced χ² for which the probability of not exceeding these values is 0.683, 0.945 and 0.997 (values corresponding to 1, 2 and 3σ of the normal distribution). The allowed deviation from the expected reduced χ² of unity decreases with radius as the number of d.o.f. increases. The plot shows very clearly that the CF2 data (manifested by the WF bulk velocity) is confined very comfortably within the 2σ envelope. The χ²/d.o.f. of the individual spheres is shown here as well for the sake of reference (figure 3 lower panel). The confidence level now assume constant (i.e. R independent) values as the number of d.o.f. is now fixed at 3 for all radii. Again the CF2 data lies comfortably within the 2σ confidence level.

6 DISCUSSION

We show here a new approach to the linearization of peculiar velocity data and apply it to the GoF analysis of the database with a cosmological model. The motivation here is to bypass the bias introduced by the small scale non-linear dynamics which affects the more traditional likelihood, or χ² analysis (cf. Zaroubi et al. 1995, 1997, Hoffman 2001). This is achieved by filtering the data of the small scale non-linear contributions. The particular approach that has been adopted here is to use the (spherical top-hat window) bulk velocity extracted from the Wiener filter reconstruction of the 3D velocity field. Our aim has been to assess by means of the χ² statistics the compatibility of the Cosmicflows-2 (CF2) database (Tully et al. 2013) with the WMAP/ΛCDM cosmological model. It is found here that the CF2 data is consistent with the WMAP/ΛCDM model to better than the 2σ confidence limits. This provides a further validation that the CF2 database is consistent with the standard model of cosmology and can be used for the reconstruction of the LSS and the setting up of constrained initial conditions for simulations that closely mimic the local universe.

The methodology presented here can be extended to perform a parameter estimation by performing a maximum likelihood analysis, using the likelihood function defined in Equation [8]. Limiting ourselves to the realm of GoF analysis of the data given a model it should be noted that the use of the WF bulk velocity estimator serves here primarily as a tool for the filtering of the small scales modes, and consequently the linearization of the data. The WF could have been evaluated with another cosmological model, not necessarily the ΛCDM one and the results would have not changed by much. The use of the WF with the ΛCDM model assures that the resulting estimated bulk velocity is the closest to the actual one - recalling that the WF is the minimal variance estimator - and thereby yielding the tightest possible GoF analysis among all bulk velocity estimators.

Bulk velocities are a linear representation of a velocity database, typically made of thousands data points. As such they play a double role. They provide an efficient filtering of the small scales nonlinear modes. At the same time they constitute a drastic data compression and reduction the number of the d.o.f. - in the CF2 case the number goes from close to 5000 down to a few tens. The current algorithm can be easily extended to include other moments of the velocity field - in particular the monopole and quadrupole terms. This will be provide a stricter GoF test on the data given a model.

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