Fast wave heating in mirror traps

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Abstract. The magnetic beach, minority and second harmonic ion cyclotron heating schemes are addressed. Their specific features in application to mirror devices and the results of computations are discussed.

1. Introduction
In the Ion Cyclotron Range of Frequencies (ICRF) two fast wave branches could be separated: the fast Alfvén wave (FAW) and the fast magnetosonic wave (FMSW). The FAW is propagating if the frequency is lower than the ion cyclotron frequency while the FMSW occupies both frequency ranges. Both waves are weakly damped in plasma by electron Landau damping.

The ion cyclotron damping of the FMSW is weak because under condition of the fundamental cyclotron resonance its polarization is almost right-hand circular, which is opposite to the ion rotation. The polarization of the FAW is favorable for the cyclotron absorption. At higher frequencies the higher harmonic ion cyclotron damping of the FMSW occurs.

The cyclotron heating selectively increases the ion perpendicular energy. When the heating is strong enough this results in the formation of sloshing ions. Sloshing ions “slosh” between the magnetic mirrors forming density peaks near the reflecting mirror points. At the magnetic field minimum the sloshing ions velocity distribution is concentrated near a certain pitch-angle. The sloshing ions play an important role in mirror confinement [1].

A mirror trap plasma column is elongated along z, the direction of the steady magnetic field. This determines the character of the fast wave propagation in open trap plasma: along x and y, the directions perpendicular to the magnetic field, a fast wave normally forms a standing wave structure with a low number of nodes, while in the longitudinal direction the number of oscillations is quite high (see e.g. [2]). This perpendicular node structure of the wavefield normally remains as the wave propagates along the plasma column.

In this paper we address the major fast wave heating schemes for open traps: magnetic beach, minority and second harmonic heating.

2. Magnetic beach heating
Magnetic beach heating is the oldest ion cyclotron heating scheme [3]. It uses the FAW cyclotron absorption. The FAW is launched by the antenna from the high field side and propagates towards the

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cyclotron resonance zone. The major features of the fast wave propagation could, to a certain extent, be described by the dispersion relation

\[ k_z^2 = k_0^2 - k_\perp^2 / 2 \pm \sqrt{k_\perp^4 / 4 + k_0^2 g^2}, \quad (1) \]

where \( k_0 = \omega / c \), \( \varepsilon = \varepsilon_{11} = \varepsilon_{22} \), \( g = -i \varepsilon_{12} = i \varepsilon_{21} \) and \( \varepsilon_{ik} \) is the cold plasma dielectric tensor. If the frequency is lower than the ion cyclotron frequency the positive sign in front of the square root in equation (1) should be chosen for the FAW. A standing wave structure in the perpendicular direction is formed and, thus, \( k_\perp \) varies slowly with \( z \). The dielectric tensor components \( \varepsilon_\perp \) and \( g \) are positive and increase with approaching to the cyclotron zone. Equation (1) shows that \( k_z^2 > k_0^2 \varepsilon \) for the FAW. This means that the FAW propagates increasing its \( k_z^2 \) accompanied by growth of \( k_0^2 \varepsilon \). In cold plasma \( \varepsilon \) has a pole at the cyclotron resonance point and the FAW is damped residually [4]. If plasma is warm the Doppler shift widens the cyclotron zone, and the FAW could be damped even before the cyclotron layer is reached. The full damping of the FAW in this case provides high efficiency of the magnetic beach heating scheme.

In general, the antenna launches the waves travelling both to the high and low field sides. If there is a cut-off point for the wave traveling to the high magnetic field, the wave reflects from it and travels back to the cyclotron resonance layer making a deposit to plasma heating. For the FAW in single ion species plasma the cut-off condition does not depend on \( k_\perp \) and appears only if the plasma density nullifies. This follows from equation (1). So, the wave launched to high magnetic field may be reflected if mirror conducting end plates are present. If a mirror is instead ended with a plasma expander, cusp anchor or kinetic stabilizer, the FAW can penetrate through the magnetic mirror. This adverse fact should be accounted for in the antenna system design. A possible solution is to implement a travelling wave antenna system.

If the radial profile of the plasma density would be rectangular, the magnetic beach heating would only have a lower limit in the plasma density that could be estimated from the condition of the wavelength along the magnetic field to be short enough. If the radial plasma density profile is smoothly continuous, additional limiting factors come into play. The first is the appearance of the radial cut off zone at the plasma edge. To couple the wave with the antenna, tunneling of the wave through this zone should not be small. The condition of this could be written as \( |k_z \Delta_r| < 1 \). The width of the cut-off zone can be estimated from the equation

\[
\frac{d}{dr} \left[ D_\perp \frac{d}{dr} r E_\phi + \frac{m k_0^2 g}{r^2 D} r E_\phi + \frac{k_0^2 g^2}{D} E_\phi - D_\perp E_\phi \right] = 0
\]

(2)

(here \( D_\perp = k_z^2 - k_0^2 \varepsilon \), \( D = D_\perp + m^2 / r^2 \)) that describes the radial propagation in cylindrical geometry. Here \( m \) is the azimuthal Fourier mode number. The condition of cut-off zone transparency could then be estimated as

\[
|k_z \Delta_r| \sim k_0 \varepsilon \sqrt{\alpha} \left( 1 + m^2 - \frac{g \varepsilon}{m^2 + k_0^2 \alpha^2} \right) < 1.
\]

(3)

The condition (3) establishes the upper limit for plasma density and is barely restrictive. It is softer for modes with negative \( m \), especially for the mode with \( m = -1 \).

There is also a lower limit to the plasma density that comes from the requirement for the FAW to form an oscillating field structure in the radial direction [5]. It can be obtained analyzing the paraxial solution of the equation (3) for the marginal case \( k_z^2 = k_0^2 \varepsilon \). For single-species plasma it has the form

\[
- \frac{k_0^2 g^2}{\partial^2 \varepsilon / \partial r^2} \bigg|_{r=0} - m^2 - m \frac{\omega}{\omega_{ci}} > 0 \quad \text{for} \quad m \neq 0; \quad - \frac{2 k_0^2 g^2}{\partial^2 \varepsilon / \partial r^2} \bigg|_{r=0} > 1 \quad \text{for} \quad m = 0.
\]

(4)
The modes with negative $m$ and especially the mode $m = -1$ have an advantage to satisfy the condition (4) as well as for the antenna coupling condition (3).

In fact, the antenna for magnetic beach heating are compact and, therefore, have in principle a broad parallel wavenumber spectrum including small wavenumbers. The waves with low $k_z^2$ excite the Alfvén resonances. In the vicinity of the Alfvén resonance the slow wave is generated. In the radial direction it propagates to the plasma periphery in cold plasma. In hot plasma it propagates to the center when the Alfvén velocity is smaller than the electron thermal velocity. The ray-tracing calculations made in [6] show that in the last case the slow wave deposits more power to the ions with the cyclotron mechanism and less power to the electrons via Landau damping. Such kind of heating accompanies the magnetic beach heating.

### 3. Cyclotron heating of minority ions

Minority heating could be arranged for light ions having higher cyclotron frequency than the background ions. In its basic version the minority concentration is small enough to avoid a significant change in the wave pattern in the background plasma. At non-resonant frequencies, the FMSW has an elliptic polarization that includes both left-hand and right-hand circular parts. The left-hand part interacts with minority ions efficiently in their cyclotron resonance zone providing the cyclotron heating.

In [4] such a scheme was examined theoretically in a small mirror device of GDT (gas dynamic trap) type. There the global resonance of FMSW located near the midplane of the trap is excited and the cyclotron zone of the hydrogen minority is situated at the maximum of the global resonance field.

Computations show the validity of such a scheme. Power deposition to the minority ions is achievable. The computed antenna efficiency is high. However, the optimum condition for the heating corresponds to low minority concentration. For higher concentrations the eigenmode (global resonance) field becomes distorted, especially at the cyclotron zone and wave damping decreases. Even at the optimum condition the relative width of the global resonance is marginal.

Choosing the frequency lower than the cyclotron frequency at the center of the mirror and usage of the waves with high $|k_z|$ values improve the situation [7]. Because the magnetic field has a quadratic minimum, the cyclotron zone is widened. This increases wave damping. The maximum perturbation of the plasma dielectric response caused by the minority is decreased owing to high $|k_z|$, and the critical minority concentration increases with $|k_z|$. The computations show that global resonances are overlapped and the impedance curve is broad. The critical concentration $C_{H}$ of the minority depends on the parallel temperature of the minority ions and for $T_{H||} = 400$ eV, $C_{H} = 30\%$.

Such a minority heating scenario could be practical in small mirrors with moderate plasma density. It allows one to create a group of sloshing ions which are trapped near the magnetic field minimum. However, there is a necessity to have ions with smaller pitch angle in mirrors. They could be heated in the framework of the ion cyclotron minority scenario with conversion.

To explain this possibility we use figure 1 that shows the dependence of the normalized parallel wavenumber $k_{\parallel} = k_{\parallel}c / \omega_{pi}$ on the normalized magnetic field $\vec{B} = \omega_{H||} / \omega$ in a D–T (deuterium–tritium)
plasma according to equation (1). Here and below $\omega_{\text{pa}}$ and $\omega_{\text{ci}}$ are the plasma and cyclotron frequencies and $\alpha = H, D, T, e...$ is an index for the charged species.

When $\omega < \omega_{\text{ci}}$, i.e. $B > 3$, two propagating wave branches exist: the FAW with higher parallel wavenumber and the FMSW. As well as in single-ion component plasma the excitation of the FAW would result in magnetic beach heating.

When $\omega_{\text{iD}} > \omega_{\text{ci}}$, i.e. $3 > B > 2$, in the vicinity of $B = 3$, only the FMSW propagates. If $B$ approaches the value 2, the FAW becomes propagating. The two wave branches intersect resulting in wave conversion: The FMSW propagating from the high-field side converts to a FAW and the almost evanescent FAW converts to a FMSW that further passes through the deuterium cyclotron resonance. The non-evanescent FAW is fully absorbed in the deuterium cyclotron zone. Since the width of this conversion zone is narrow, a WKB approximation may break down in its vicinity, and thus reflections of the waves may appear. Also over-barrier FMSW–FMSW and FAW–FAW tunneling may occur.

Figure 1 prompts the following scenario for cyclotron heating of the deuterium minority: the FMSW is excited by the antenna and launched towards lower magnetic field at a position where the magnetic field is lower than the resonant value for tritium ions and the real part of $\epsilon$ is negative. This wave, traveling towards lower magnetic field, will convert to a FAW. The FAW will reach the vicinity of the cyclotron zone for deuterium ions, where it will be damped by the cyclotron mechanism.

As compared with the “magnetic beach” scheme, this scenario has some advantages. The first is that it is easier to excite by the antenna the FMSW than the FAW, since there is less restriction for the FMSW propagation in non-uniform plasma. In contrast to the FAW, the FMSW has a cut-off zone along the plasma column. This follows from equation (1) where the branch of waves, which describes FAW–FMSW conversion, has a positive sign in front of the square root. In the condition of cut-off $k_z^+ = k_0^+ \epsilon - k_0^+ g^2 / \epsilon$ in the dense plasma the right-hand side is proportional to the plasma density, and this condition could be met for finite density. The right-hand side also decreases on increase of the magnetic field. This factor also can cause cut-off. At the cut-off point $k_z = 0$. Under this condition the radial cut-off zone in equation (2) is minimal which means optimum conditions for antenna–plasma coupling.

Normally, at the cut-off region where the antenna could be placed $\epsilon$ is negative. The Alfvén continuum waves are evanescent there and, therefore, decoupling of the antenna from the Alfvén resonances is provided.

The calculations results for this ICRH scenario were presented in [8] for a reactor-scale straight field line mirror [9] with sloshing deuterium ions trapped in the magnetic field with the mirror ratio $R = 3$. A performed parameter scan has shown the basic properties of such a heating scheme. In all the calculations the single-path wave damping in the deuterium cyclotron resonance occurs. This results in a weak dependence of the antenna loading resistance on the plasma density variations. The almost full wave damping without a parallel ion temperature dependence indicates the magnetic beach character of the wave cyclotron absorption after the conversion of the FMSW to the FAW. Only a slight Alfvén resonance excitation is visible from the electromagnetic field patterns. The scheme is efficient in a wide range of deuterium concentrations $C_D = 5–50\%$. The fact that high minority concentrations may be used has practical importance. Calculations in which the antenna position along the axis of the mirror is varied show that the antenna excites those FMSW which have the longitudinal cut-off point at the antenna location. By varying the antenna position, the number of oscillations of the FMSW in the radial direction can be controlled.

4. Second harmonic ion cyclotron heating

The second harmonic ion cyclotron damping of the fast magnetosonic wave (see e.g. [10]) is caused by the electromagnetic field non-uniformity across the steady magnetic field and the ion thermal motion. The second harmonic ion heating has been experimentally tested in tokamaks. Attempts to use it have also been made in mirror experiments (see e.g. [11]). As compared with the fundamental
harmonic ion cyclotron heating, the second harmonic cyclotron damping is weaker, and the ratio of the damping rates is proportional to \( k_L^2 \alpha L_{\perp} k L_{\perp} \) which typically is a small number. Here, \( \rho_L \) is the ion Larmor radius. If the wave propagates along a non-uniform magnetic field, an estimated condition for single-path damping of the wave is:

\[
C_a (k_L \rho_L)^2 \frac{|E_+|}{|E|} |k_L \Delta L| > 1
\]

Here \( E_+ \) is the left-polarized component of the electric field, \( \Delta L \) is the resonant zone width that could be found from the condition \( |\beta_+| \leq 1 \) with \( \beta_+ = (\omega - \omega_{ci}, k_L v_{ti}) \), and \( v_{ti} \) is the parallel ion thermal velocity. The condition for single-path damping requires both components of the wave vector, \( \perp k \) and \( \parallel k \), not to be small. Thus \( k_L \sim k_\parallel \sim k_\perp \) where \( k_\perp = \omega / v_\perp \) and \( v_\perp \) is the Alfvén velocity. Using this, the inequality (5) could be rewritten as

\[
C_a \left( \frac{|E_+|}{|E|} \right) \frac{\alpha L_{\perp} k_{\perp}}{v_{ti} \perp} |k_\perp L_{Hi}| > 1,
\]

where \( L_{Hi} \) is the characteristic space scale of the magnetic field. In the left side of the inequality (6) all the terms are small except \( k_\perp L_{Hi} \) that has to compensate the smallness of the other terms. In small open traps this is possible only near the minimum of the magnetic field [2] where \( L_{Hi} \) reaches its maximum. In large reactor-scale devices the single-path damping condition is easier to meet.

The second order finite Larmor radius correction to the dielectric tensor accounting for the second harmonic effect

\[
\delta \varepsilon_{++} = -k_\perp ^2 \varepsilon_{++} / 4
\]

contributes only to the left-hand polarization component. The expression for \( \varepsilon_{++} \) could be obtained from the hot plasma dielectric tensor expressions [2]

\[
\varepsilon_{++} = \sum \frac{4 \omega_p^2 v_{ti}}{\omega} k_L v_{ti} \alpha L_{\perp} F(\beta_{++}) - \frac{k_\perp}{\sqrt{\pi}} \frac{\exp(-k_\perp^2)}{2} \right)
\]

where \( F \) is the Dawson integral. If the plasma density profile would be rectangular, \( k_\perp \) is constant along the radial coordinate and slowly varies along \( z \). In this case \( \delta \varepsilon_{++} \) is similar to the resonant contribution of the light minority, and the fast wave propagates as in the minority case. In the vicinity of the second harmonic cyclotron resonance the increase of \( \varepsilon_{++} \) takes place for the FAW.

If the plasma density profile is smoothly continuous, the physical pattern of the second harmonic cyclotron heating is different, and \( k_\perp \) varies now across the plasma column and nullifies at the radial cut-off point near the plasma edge. At the cut-off point the second harmonic correction to the dielectric tensor vanishes \( (\delta \varepsilon_{++} = 0) \). This means that the condition for cut-off is the same as in the cold plasma. If \( |k_\perp| \) increases, the cut-off point moves inside the plasma column and the region of the FMSW propagation shrinks. At a certain \( |k_\perp| \) value the propagation region ceases to exist. Since this value cannot be surpassed, no significant increase of \( |k_\perp| \) is expected.

The account of \( \delta \varepsilon_{++} \) in Maxwell’s equations increases the order of the problem. A new wave branch, the ion Bernstein wave (IBW), appears. The FMSW may convert to it. This would destroy the pattern of second harmonic heating and may result in heating of the plasma edge. The condition of conversion is difficult to evaluate analytically in two- or three-dimensionally non-uniform plasma. Qualitatively, the conversion may be expected if \( \delta \varepsilon_{++} \) has the order of the remaining cold dielectric tensor components. It may be avoided by decreasing \( k_\perp \) and increasing \( |k_\perp| \) for the launched FMSW.
The second harmonic heating arrangements are similar to the minority heating. In [12] computation results for minority and second harmonic heating in a large straight field line mirror are presented. The minority heating was studied for deuterium and second harmonic heating for tritium. For second harmonic heating the FMSW–IBW mode conversion is not observed. The power deposition is broader in this case, but other features are similar for the most part.

5. Conclusions
The fast wave heating schemes, i.e. magnetic beach, minority and second harmonic heating, are addressed and their similar and different properties are described.

The magnetic beach heating is efficient for plasma with a flat-top rectangular density profile. For more general profiles the plasma density value is restricted both by upper and lower limits. These limits do impede usage of this heating in large devices. In small devices with compact antennas the excitation of the Alfvén resonance plays an important role in this heating scheme.

The minority heating scenario is effective for mirror plasmas. In small mirrors single global resonance excitation can be used, but the global resonance overlapping scenario has an evident advantage. For large mirrors the minority heating is efficient in a wide range of minority concentrations and plasma densities; it allows one to place the antenna aside from the hot ion location; the calculations with a simple-design strap antenna have shown a good performance. However, this scenario is appropriate only for light minority ions.

The second harmonic heating can be applied for heavy minority. It could be realized in the regime of overlapping of the global resonances. The arrangements for it are similar to the minority heating. The conversion of the fast magnetosonic wave to the ion Bernstein wave may distort the heating pattern. However, the calculations show that this can be avoided. The efficiency of second harmonic heating is determined by weaker wave damping than for minority heating.

The numerical calculations show that in the reactor scale mirror the deuterium sloshing ions could be heated in the minority heating scheme, while the tritium ions could be sustained by the second harmonic heating.

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