Spatially resolved NMR relaxation rate in a noncentrosymmetric superconductor

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Abstract

We numerically study the spatially-resolved NMR around a single vortex in a noncentrosymmetric superconductor such as CePt\textsubscript{3}Si. The nuclear spin-lattice relaxation rate $T^{-1}_1$ of the vortex core is calculated for an $s + p$-wave Cooper pairing state. The result is compared with that of an $s$-wave Cooper pairing state.

Key words: CePt\textsubscript{3}Si, unconventional superconductivity, vortex core, site-selective NMR, nuclear spin-lattice relaxation rate

The heavy fermion superconductor CePt\textsubscript{3}Si has a noncentrosymmetric crystal structure \cite{1}. The lack of the inversion symmetry leads to the mixture of Cooper pairing channels of different parity, resulting in unusual properties observed experimentally \cite{2,3,4}. For example, it is observed that while a Hebel-Slichter coherence peak appears in the nuclear spin-lattice relaxation rate $T^{-1}_1$ \cite{2,3,4} suggesting an $s$-wave like Cooper pairing, low-temperature experiments on the London penetration depth \cite{3,5} and the thermal conductivity \cite{6} indicate line nodes in the quasiparticle gap (i.e., unconventional superconductivity) \cite{8,14}. In this context, $T^{-1}_1$ was recently discussed in Refs.\cite{7,8,9} theoretically.

On the other hand, the spatially-resolved nuclear magnetic resonance (NMR) method has been revealed to be a powerful experimental technique \cite{10,11}. This technique as a probe of the electronic structure with spatial resolution is expected to reveal pairing symmetry of unconventional superconductors, because in spatially inhomogeneous systems there appear properties specific to the unconventional superconductivity.

In this paper, we study the spatially-resolved $T^{-1}_1$ around a single vortex for an $s + p$-wave pairing state. This pairing state is proposed for CePt\textsubscript{3}Si in Ref.\cite{12}. We consider a system described in Refs.\cite{12,13}. From now on, the notations are the same as those in Ref.\cite{8}.

To obtain $T^{-1}_1$ in the spatially inhomogeneous situation, we calculate the quasiclassical Green function $\hat{g}$ which is a $4 \times 4$ matrix in Nambu and spin spaces. $\hat{g}$ follows the Eilenberger equation for the noncentrosymmetric superconductivity \cite{8,14}:

\[
iv_F \cdot \nabla \hat{g} + \left[ i\omega_n \tau_3 - \alpha \hat{g}_k \cdot \hat{S} - \hat{\Delta}_k, \hat{g} \right] = 0,
\]

where $\alpha$ indicates the strength of the Rashba-type spin-orbit coupling, $\hat{g}_k$ is composed of the antisymmetric vector $\hat{g}_k = (-\hat{k}_y, \hat{k}_x, 0)$ \cite{13}, $\hat{S}$ is the electron spin operator, and $\hat{\Delta}_k$ is the superconducting order parameter. For a static magnetic field in the $z$ direction, the expression for $1/T_1 T$ is given as \cite{8,15}:

\[
\frac{T_1(T_c)T}{T_1(T)T} = \frac{1}{4T} \int_{-\infty}^{\infty} d\omega \frac{\cosh(\omega/2T)}{W(\omega)},
\]

where $\omega = \langle a_{i\uparrow}^2(\omega) \rangle a_{i\downarrow}^\dagger(\omega) - a_{i\uparrow}^\dagger(\omega) \langle a_{i\downarrow}^2(\omega) \rangle - \langle a_{i\uparrow}^2(\omega) \rangle a_{i\downarrow}^\dagger(-\omega) - \langle a_{i\uparrow}^\dagger(-\omega) \rangle a_{i\downarrow}(-\omega)$.

\[
\hat{a}(\omega) = \frac{i}{2\pi} \tau_1 \left[ \hat{g}(i\omega_n \rightarrow \omega + i\eta) - \hat{g}(i\omega_n \rightarrow \omega - i\eta) \right],
\]

where $\hat{a} = (\hat{a}^{1j}, \hat{a}^{2j})$, $\hat{a}^{1j}(\mu,\nu) = (a_{i\mu\nu}^{1j})$, and $\hat{a}^{2j}(\mu,\nu) = (a_{i\mu\nu}^{2j})$. The brackets $\langle \cdots \rangle$ denote the average over the Fermi surface. $\eta$ ($> 0$) is a small constant, which can roughly represent the impurity effect.
We consider two Fermi surfaces split by the spin-orbit coupling $\alpha k \cdot S$, and have the $s + p$-wave order parameter $\Psi + \Delta \sin \theta$ on one Fermi surface and $\Psi - \Delta \sin \theta$ on the other one [12] ($\Psi$: $s$-wave, $\Delta$: $p$-wave). We assume that the difference of the density of states and $v_F$ between the two Fermi surfaces is small, and neglect it here. We use the following empirical form of a single vortex. $\Phi/T_c = a F(t) \tan \left(r/\xi(t)\right) \exp(i \phi_v)$, $\xi(t) = \xi_0 \sqrt{t/bF(t)}$, $F(t) = \tan \left(1.74 \sqrt{1/t - 1}\right)$. Here, $\Phi = \{\Psi, \Delta\}$, $\xi_0 = v_F/T_c$, $t = T/T_c$ and the fitting parameters $a, b$. The flux line is along the $z$ axis, $r$ is the distance from the vortex center, and $\phi_v$ is the angle in the $x$-$y$ plane. We have performed a full numerical computation as in Ref. [16] and obtained a self-consistent vortex solution for the same parameters used in Ref. [8]. We then fitted it with the above expression, and obtained $(a, b) = (0.68, 2.2)$ for $\Psi$ and $(1.42, 2.2)$ for $\Delta$. We have also done the same fitting for the conventional $s$-wave pairing case, so that $(a, b) = (1.76, 2.2)$. We found that the fitting is good for $T > 0.1 T_c$, while for $T \leq 0.1 T_c$ it gives rather smaller core radius than actual one.

We calculate $1/T_1 T$ at several positions around the vortex, and show the results in Fig. 1 ($s + p$-wave state) and Fig. 2 (s-wave state). Away from the vortex ($r = 10 \xi_0$), $1/T_1 T$ exhibits the coherence peak just below $T_c$ in both pairing states, while it is rather smaller in the $s + p$-wave state [8]. With decreasing $r$ up to $\xi_0$, the height of the peak gradually decreases. With further decreasing $r$ below $\xi_0$ inside the vortex core, the height of the peak increases again and the value of $1/T_1 T$ at low temperatures increases also. The difference between $1/T_1 T$ at $r = 0.6 \xi_0$ and that at $r = 10 \xi_0$ in Fig. 1 ($s + p$-wave state) is similar to the difference between higher and lower frequency data of experimentally observed $1/T_1 T$ in Refs. [2,3,4]. That is, almost same height of the peak just below $T_c$ inside and outside the core, and larger values of $1/T_1 T$ at low temperatures inside the core than those outside the core. Finally, we note that $1/T_1 T$ increases further and the peak shifts to lower temperature side with approaching the vortex center (see plots for $r = 0.2 \xi_0$ in Figs. 1 and 2). The difference between $s + p$-wave state (Fig. 1) and $s$-wave state (Fig. 2) is conspicuous there in the low-temperature behavior. It would be due to a difference of the vortex core states between these pairing states.

In conclusion, we calculated spatially-resolved $1/T_1 T$ for $s + p$-wave and $s$-wave pairing states. The information on $1/T_1 T$ deeply inside the core might be valuable for experimentally detecting the pairing symmetry for a noncentrosymmetric superconductor.

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