Avoidance of Big Rip In Phantom Cosmology
by Gravitational Back Reaction

Puxun Wu \(^b\) and Hongwei Yu \(^a,b\) \(*\)

\(^a\) CCAST(World Lab.), P. O. Box 8730, Beijing, 100080, P. R. China .
\(^b\) Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China\(^b\)

Abstract

The effects of the gravitational back reaction of cosmological perturbations are investigated in a cosmological model where the universe is dominated by phantom energy. We assume a COBE normalized spectrum of cosmological fluctuations at the present time and calculate the effective energy-momentum tensor of the gravitational back-reactions of cosmological perturbations whose wavelengths at the time when the back-reactions are evaluated are larger than the Hubble radius. Our results reveal that the effects of gravitational back-reactions will counteract that of phantom energy sooner or later and can become large enough to terminate the phantom dominated phase before the big rip as the universe evolves. This arises because the phase space of infrared modes grows very rapidly as we come close to the big rip.

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\(*\) To whom correspondence should be addressed
\(^b\) Mailing address .
I. INTRODUCTION

The analysis of data from supernovae[1], CMB[2] and WMAP[3] strongly indicates the existence of dark energy which dominates the present universe and drives the accelerating cosmic expansion. Many models, such as cosmological constant[4], quintessence[5], k-essence[6], braneworld[7], Chaplygin gas[8], quintom[9], and holography[10], are proposed to explain the dark energy. A phantom field[11], which has the super-negative equation of state($w \equiv p/\rho < -1$) in contrast to quintessence energy ($w > -1$) or cosmology constant($w = -1$), appears as another possible candidate of dark energy and has received increased attention recently. It has an unusual kinetic term in its Lagrangian which gives rise to some strange properties, such as the violation of the dominant energy condition[12, 13] and the increase of its energy density with time. As bizarre as it may appear, such terms could arise in a variety of theories[14].

It can be shown that once our universe enters the phantom energy dominated phase, the scalar factor will blow up in a finite proper time due to excessive expansion. This arises because[11, 15] the energy density of phantom fields increases with time instead of red-shifting away as the matter or radiation energy densities or as the energy density of ordinary quintessence. Thus the increasing phantom energy will ultimately strip apart gravitationally bound bodies and cause a cosmic doomsday or big rip[15]. The above conclusions are based upon a constant negative value of $w$. However, if the value of $w$ could change during the evolution of the universe and then in principle the big rip can be avoided[12]. Attempts have also been made toward avoiding the big rip by modifying the original Caldwell’s phantom model[16, 17, 18, 19, 20]. Note that it has been argued that a singularity can also develop at a finite future time even if $\rho + 3p$ is positive[21].

Here we would like to point out that the big rip may be avoided in phantom cosmology even with a constant negative value of $w$ without adding new physics. We will demonstrate that the gravitational back reaction of cosmological perturbations may terminate the phantom dominance before the big rip occurs.
II. GRAVITATIONAL BACK REACTION

Considering the fluctuations of metric and matter and expanding the Einstein equation to second order, due to non-linearity of Einstein equation, the second order equation can not be satisfied and the back reaction of these fluctuations to the background must be considered which is characterized by a gauge-invariant effective energy-momentum tensor $\tau_{\mu \nu}$

$$\tau_{\mu \nu} = \langle T^{(2)}_{\mu \nu} - \frac{1}{8\pi G} G^{(2)}_{\mu \nu} \rangle,$$ (1)

where $T^{(2)}_{\mu \nu}$ and $G^{(2)}_{\mu \nu}$ express the second order metric and matter perturbations and pointed brackets stand for spatial averaging. This formalism can be applied to both scalar and tensor perturbations and applies independent of the wavelength of the perturbations. The effects of gravitational back reactions have been studied in the context of cosmological models, where, for example, they have been used to address the issue of dynamical relaxation of the cosmological constant [25] and the possible termination of quintessence phase [26]. Here we will discuss the back reactions in phantom cosmology.

We will assume that $w$ is constant and $w < -1$, and the universe is not phantom dominated until the time $t_m$. That is, $t_m$ is the time of equal phantom energy density and matter energy densities. The universe is matter dominated if $t < t_m$ and is phantom dominated if otherwise. In the following we discuss the back reaction of cosmological perturbations during the phantom dominated phase ($t \geq t_m$). However, if phantom exists all the time, the back reaction should too before $t_m$. Here we assume that the back reaction is negligible or its effects are reinforcing before the phantom dominated era, since if otherwise the phantom phase could not occur. As the amplitude of each fluctuation mode is small, we need a very large phase space of modes in order to produce any interesting effects. During the phantom dominated phase, both the scalar factor $a$ and expansion rate $H$ increase with time, so Hubble distance decreases and the phase space of infrared modes grows. Hence we expect that the effects of the back reaction of infrared modes will grow and we therefore only focus on the back reaction of infrared modes on the evolution of the universe dominated by phantom energy.

A simple effective action of a phantom field can be expressed as

$$\mathcal{L}_{\text{phan}} = -\frac{1}{2} (\partial_{\mu} \varphi)^2 - V(\varphi),$$ (2)
where $V(\varphi)$ is the potential of the phantom field. Thus we can obtain the energy momentum tensor

$$T_{\mu \nu} = -\partial_\mu \varphi \partial_\nu \varphi + g_{\mu \nu} \left[ \frac{1}{2} \partial^\alpha \varphi \partial_\alpha \varphi + V(\varphi) \right]. \quad (3)$$

In longitudinal gauge the metric with scalar perturbations can be written as

$$ds^2 = a^2(\eta)[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\delta_{ij}dx^i dx^j] , \quad (4)$$

where $\Phi$ is the Bardeen potential. In addition to the geometrical perturbations, one must also consider the perturbations of the phantom field $\delta \varphi$ during the phantom dominated era.

Expanding the Einstein equation to the first order in $\Phi$ and $\delta \varphi$, we obtain the gauge-invariant equations of motion for small perturbations

$$\Phi' + h\Phi = -4\pi G \varphi_0' \delta \varphi , \quad (5)$$

$$\nabla^2 \Phi - 3h(h\Phi + \Phi') = 4\pi G[\varphi_0'^2 \Phi - \varphi_0' \delta \varphi' + V_{,\varphi} a^2 \delta \varphi] , \quad (6)$$

$$\Phi'' + 3h\Phi' + (2h' + h^2)\Phi = -4\pi G[-\varphi_0'^2 \Phi + \varphi_0' \delta \varphi' + V_{,\varphi} a^2 \delta \varphi] , \quad (7)$$

where $a'$ represents the derivative with respect to conformal time $\eta$ and $h = a'/a$. Eqs. (5,6,7) come from the $0i$, $00$ and $ij$ components of the Einstein equation respectively. Subtracting Eq. (6) from Eq. (7) and using Eq. (5) and the equation of motion for the phantom field, we get a second order partial differential equation for $\Phi$

$$\Phi'' - \nabla^2 \Phi + 2 \left( h - \frac{\varphi_0''}{\varphi_0'} \right) \Phi' + 2 \left( h' - h \frac{\varphi_0''}{\varphi_0'} \right) \Phi = 0 . \quad (8)$$

This equation is the same as that in which the matter perturbations are induced by a usual scalar field \[24\]. Thus, for long-wavelength perturbations we have

$$\Phi_k \simeq A_k \left[ 1 - \frac{H}{a} \int a dt \right] . \quad (9)$$

Here $A_k$ is an integration constant. For short-wavelength perturbations one gets

$$\Phi_k \propto \varphi_0' , \quad (10)$$

where a dot denotes the derivative with respect to coordinate time $t$. 

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Expansion of the energy-momentum tensor $T_{\mu\nu}$ and Einstein tensor $G_{\mu\nu}$ to the second order in $\Phi$ and $\delta\varphi$ yields the non-vanishing components of the effective back reaction energy-momentum tensor

$$\tau_{00} = \frac{1}{8\pi G} \left[ 12 H \langle \Phi \dot{\Phi} \rangle - 3 \langle \dot{\Phi}^2 \rangle + 9 a^{-2} \langle \nabla^2 \Phi^2 \rangle - \frac{1}{2} \langle \delta \dot{\varphi}^2 \rangle - \frac{1}{2} a^{-2} \langle (\nabla \delta \varphi)^2 \rangle + \frac{1}{2} V_{,\varphi \varphi} \langle \delta \varphi^2 \rangle + 2 V_{,\varphi} \langle \Phi \delta \varphi \rangle \right],$$

(11)

and

$$\tau_{ij} = a^2 \delta_{ij} \left\{ \frac{1}{8\pi G} \left[ (24 H^2 + 16 \dot{H}) \langle \Phi^2 \rangle + 24 H \langle \Phi \dot{\Phi} \rangle + \langle \dot{\Phi}^2 \rangle + 4 \langle \Phi \ddot{\Phi} \rangle - \frac{3}{4} a^{-2} \langle \nabla^2 \Phi^2 \rangle \right] - 4 \dot{\varphi}^2 \langle \Phi^2 \rangle + \frac{1}{2} \langle \delta \dot{\varphi}^2 \rangle - \frac{1}{2} a^{-2} \langle (\nabla \delta \varphi)^2 \rangle + 4 \dot{\varphi}_0 \langle \Phi \delta \varphi \rangle - \frac{1}{2} V_{,\varphi \varphi} \langle \delta \varphi^2 \rangle + 2 V_{,\varphi} \langle \Phi \delta \varphi \rangle \right\} ,$$

(12)

where $H = \dot{a}/a$.

Let us now apply these equations to the phantom dominated era. For $t > t_m$ the solution for the scale factor is given by

$$a(t) = a(t_m) \left[ -w + (1 + w) \frac{t}{t_m} \right]^{\frac{2}{3(1+w)}} .$$

(13)

Apparently $a$ diverges when $t = wt_m/(1 + w) \equiv t_{\text{brip}}$. During the phantom dominated era the energy density evolves as $\rho_{\text{phan}} \sim a^{-3(1+w)}$, thus $\rho_{\text{phan}}(t)$ is related to $\rho(t_m)$ by

$$\rho_{\text{phan}}(t) = \frac{\rho(t_m)}{[-w + (1 + w)t/t_m]^2} .$$

(14)

Here $\rho(t_m) \approx 1/(6\pi G t_m^2)$. The energy density $\rho_{\text{phan}}(t)$ increases with time and diverges when $t = t_{\text{brip}}$. Using $p = w\rho$, $p = -\frac{1}{2} \dot{\varphi}^2 - V(\varphi)$ and $\rho = -\frac{1}{2} \dot{\varphi}^2 + V(\varphi)$, we obtain

$$\dot{\varphi}^2 = \frac{-(w + 1)\rho(t_m)}{[-w + (1 + w)t/t_m]^2} ,$$

(15)

and

$$V(\varphi) = \frac{(-w + 1)\rho(t_m)}{2[-w + (1 + w)t/t_m]^2} = \frac{(-w + 1)\rho(t_m)}{2} \exp \left[ \frac{-2(1 + w)\varphi}{t_m \sqrt{-(w + 1)\rho(t_m)}} \right] .$$

(16)
Combining Eq. (9) and Eq. (13), we get in the long-wavelength limit

$$\Phi_k = \begin{cases} 
\beta A_k = \tilde{A}_k, & w \neq -5/3, \\
A_k[1 - \ln \chi(t)] \equiv A_k z(t), & w = -5/3,
\end{cases} \tag{17}$$

where $\beta$ is an integration constant and $\chi(t) \equiv \frac{5}{3} - \frac{2t}{t_m}$. As the universe evolves from $t_m$ to $t_{br}$, $\chi(t)$ varies from 1 to 0 and $z(t)$ from 1 to $\infty$. It then follows that $\dot{\Phi}_k = \ddot{\Phi}_k = 0$ for $w \neq -5/3$ and

$$\dot{\Phi}_k = \frac{2}{3t_m \chi(t) z(t)} \Phi_k, \tag{18}$$

and

$$\ddot{\Phi}_k = \frac{4}{9t_m^2 \chi(t)^2 z(t)} \Phi_k, \tag{19}$$

for $w = -5/3$. Defining $\rho_{br} \equiv \tau_0^0$ and $p_{br} \equiv -\frac{1}{3} \tau_i^i$ and using Eqs. (15, 16, 18, 19), the expression for $\tau_{\mu\nu}$ can be simplified to

$$\rho_{br} = \begin{cases} 
-\frac{(1-w)}{6\pi G t_m^2 [w+(1+w)t/t_m]^2} \langle \Phi^2 \rangle, & w \neq -5/3, \\
\frac{2}{9\pi G t_m^2 \chi(t)^2} \left( \frac{1}{z(t)^2} + \frac{3}{z(t)^2} - 2 \right) \langle \Phi^2 \rangle, & w = -5/3,
\end{cases} \tag{20}$$

$$p_{br} = \begin{cases} 
-\rho_{br}, & w \neq -5/3, \\
-\frac{2}{9\pi G t_m^2 \chi(t)^2} \left( \frac{2}{z(t)^2} + \frac{3}{z(t)^2} - 2 \right) \langle \Phi^2 \rangle, & w = -5/3.
\end{cases} \tag{21}$$

Here the subscript $br$ stands for back reaction. So the equation of state parameter for the dominant infrared contribution to the back reaction is given by

$$w_{br} = \begin{cases} 
-1, & w \neq -5/3, \\
-\frac{2+3z(t)-2z(t)^2}{1+3z(t)-2z(t)^2}, & w = -5/3.
\end{cases} \tag{22}$$

Hence, for $w \neq -5/3$, the contribution of infrared modes to the energy momentum tensor which describes the back-reaction takes the form of a negative cosmological constant, whose absolute value changes as a function of time. The result of this case is similar to that obtained in an inflationary background cosmology [23, 24], which has been used to address the issue of dynamical relaxation of the cosmological constant [25]. Meanwhile, for $w = -5/3$, $w_{br} = -3/2$ at $t = t_m$, and $w_{br}$ grows as time goes on and approaches to $-1$ at $t_{br}$. Using Eq. (14, 20), we have

$$\frac{\rho_{br}}{\rho_{phan}} = \begin{cases} 
-\frac{(1-w)}{3} \langle \Phi^2 \rangle, & w \neq -5/3, \\
\frac{2}{3} \left( \frac{1}{z(t)^2} + \frac{3}{z(t)^2} - 2 \right) \langle \Phi^2 \rangle, & w = -5/3.
\end{cases} \tag{23}$$
If the above ratio is negative, the phantom energy will be counteracted by the effects of the back reaction. When the ratio becomes negative unit, the phantom phase will end and the universe will re-enter the matter dominated era. In order to determine the value of this ratio, it is pivotal to evaluate the two-point function $\langle \Phi^2 \rangle$, which can be obtained by integrating over all Fourier modes of $\Phi$:

$$\langle \Phi(t)^2 \rangle = \int_{k_i}^{k_t} \frac{k^3}{2\pi^2} \langle \Phi_k(t)^2 \rangle \frac{dk}{k},$$

(24)

where $k_i = a_i H_i$ and $k_t = a(t) H(t)$ are infrared and ultraviolet cutoffs respectively. The infrared cutoff can be chosen as the length scale above which there are no significant fluctuations. If we assume that there was a period of inflation in the early history of our universe, we could take the infrared cutoff as the Hubble radius at the beginning of the inflation. The ultraviolet cutoff will be taken as the Hubble radius at time $t$ when the strength of the back reaction is to be evaluated.

### III. TERMINATION OF PHANTOM DOMINATED PHASE

We assume that our universe now is dominated by the phantom energy, and for simplicity, $t_m \sim t_0$, which means that the phantom just begins to dominate at today. As a matter of fact, one of the main features that distinguish the phantom energy from a cosmological constant or quintessence is that the onset of phantom energy dominance happens at the very last moment. So our assumption is fairly reasonable. Now we want to relate $\Phi_{pk}$ defined as the value of $\Phi_k$ during the phantom dominated phase, and $\Phi_{mk}$, which is the corresponding value just before the phantom energy begins to dominate. Let us consider separately modes whose wavelengths are outside the Hubble radius now and modes whose wavelengths are inside the Hubble radius today and exit the Hubble radius before time $t(t > t_0)$. For the former case, $\Phi_{pk}(t)$ and $\Phi_{mk}(t_0)$ can be related by the conservation of the Bardeen potential

$$\zeta = \frac{2}{3} \left( \frac{H^{-1} \dot{\Phi} + \Phi}{1 + w} \right) + \Phi.$$

(25)

In the matter dominated phase, since $w = 0$ and $\dot{\Phi}_{mk} = 0$, we obtain $\zeta(\Phi_{mk}) = 5\Phi_{mk}/3$. For $w \neq -5/3$, during the phantom dominated era, $\dot{\Phi}_{pk} = 0$. So we have $\zeta(\Phi_{pk}) = (5 + 3w)/(3+$
\[3w) \Phi_{pk}. \text{ It follows from the conservation of } \zeta \text{ that} \]
\[
\Phi_{pk} = \frac{5(1 + w)}{3w + 5} \Phi_{mk}. \quad (26)
\]

For modes whose their wavelengths are inside the Hubble radius today and exit the Hubble radius at time \(t_H\) given by \(a(t_H)H = k\). From Eqs. (10, 15, 17), we know how \(\Phi\) evolves with time on the wavelength scale smaller and larger than the Hubble radius. Thus we can express \(\Phi_{pk}(t > t_H)\) in terms of \(t_0, t_H\) and \(\Phi_{mk}(t_0)\)
\[
\Phi_{pk}(t) = \Phi_{mk}(t_0) \frac{t_0}{-wt_0 + (1 + w)t_H}. \quad (27)
\]

For the case of \(w = -5/3\), we have in the matter dominated era, from Eq. (9), that \(\Phi_{mk} = \frac{5}{a} A_k\) in the long-wavelength limit. For modes whose wavelengths are outside the Hubble radius now, using Eq. (17), we have
\[
\Phi_{pk}(t) = 5 \frac{3}{3} \Phi_{mk}(t_0) z(t). \quad (28)
\]

For those modes whose wavelengths are inside the Hubble radius today and exit the Hubble radius before time \(t(t > t_0)\), using Eqs. (10, 15, 17), we obtain
\[
\Phi_{pk}(t) = \Phi_{mk}(t_0) \frac{\chi(t_0)}{\chi(t_H)} \frac{z(t)}{z(t_H)}. \quad (29)
\]

We now assume that the spectrum of cosmological fluctuations at the present time is normalized by the recent observations of CMB anisotropies and the fluctuational spectrum has the form
\[
P(k) \equiv \frac{k^3}{2\pi^2} |\Phi_{mk}(t_0)|^2 = C \left( \frac{k}{k_{COBE}} \right)^\lambda, \quad (30)
\]
where \(k_{COBE} \equiv \alpha a_0 H_0\) and \(C^{1/2} \approx 10^{-5}\). By the joint analysis of the Maxima-1, Boomerang and COBE cosmic microwave anisotropy results and the estimate of the systematic errors, the blue spectrum tilt should be in the region of \(0 \leq \lambda < 0.27\) \(^1\). In this case \(\alpha \approx 7.5\).

Substituting Eqs. (26, 27, 30) into Eq. (24), we obtain, \(w \neq -5/3\)
\[
\langle \Phi(t)^2 \rangle \simeq \frac{25C}{(3w + 5)^2(\alpha a_0 H_0)^\lambda} \int_{k_0}^{k_{i}} k^{\lambda-1}dk + \frac{Ct_0^2}{(\alpha a_0 H_0)^\lambda} \int_{k_0}^{k_{i}} \frac{k^{\lambda-1}}{-wt_0 + (1 + w)t_H}^2 dk. \quad (31)
\]

\(^1\) According to the more recent WMAP data the upper bound could be slightly larger \(\underline{30, 31}\).
The first integration in the above can be evaluated as
\[ g_1 \equiv \frac{25C(w + 1)^2}{(3w + 5)^2(\alpha a_0 H_0)^\lambda} \int_{k_i}^{k_f} k^{\lambda - 1} dk \]
\[ = \frac{25C(w + 1)^2}{(3w + 5)^2} f_1 , \]
where we have defined
\[ f_1 = \left\{ \begin{array}{ll}
\ln \frac{a_0 H_0}{k_i}, & \lambda = 0 , \\
\lambda^{-1} a^{-\lambda} [1 - k_0^\lambda (a_0 H_0)^{-\lambda}], & \lambda \neq 0 .
\end{array} \right. \]

For an e-folding number as large as 60 ∼ 70, \( \ln(a_0 H_0/k_i) \sim O(10^1) \). Therefore, since \( C \sim 10^{-10} \), \( g_1 \ll 1 \). The second integration is given by
\[ g_2 \equiv \left( \frac{Ct^2}{\alpha a_0 H_0} \right)^\lambda \int_{k_0}^{k_f} \frac{k^{\lambda - 1}}{[-wt_0 + (1 + w)t_H]^2} dk \]
\[ = \frac{(1 + 3w)}{6(1 + w) + (1 + 3w)\lambda \alpha^x} \left( -w + (1 + w) \frac{t}{t_0} \right) \left( - \frac{1 + \lambda}{3(1 + w)^{\lambda - 2}} - 1 \right) . \]

Apparently \( g_2 \sim 0 \) when \( t \sim t_0 \) and it increases with time and approaches infinity at \( t = t_{brip} \). Thus hereafter, we will discard \( g_1 \) in discussing if and when the effects of back reaction can terminate the phantom dominated phase before everything is torn apart at the big rip.

For \( w = -5/3 \), the substitution of Eqs. (28,29,30) into Eq. (24) leads to
\[ \langle \Phi(t)^2 \rangle = \frac{25Cz(t)^2}{9k^\lambda_{COBE}} \int_{k_i}^{k_f} k^{\lambda - 1} dk + \frac{Cz(t)^2}{k^\lambda_{COBE}} \int_{k_0}^{k_f} \frac{k^{\lambda - 1}}{z(t_H)^2 \chi(t_H)^2} dk \]
\[ = 2Cz(t)^2 \left[ \frac{25}{18} f_1 + f_2(t) \right] , \]
where
\[ f_2(t) \equiv \left( \frac{1}{2k^\lambda_{COBE}} \right) \int_{k_0}^{k_f} \frac{k^{\lambda - 1}}{z(t_H)^2 \chi(t_H)^2} dk \]
\[ = 2a^{-\lambda} (1 + \lambda) e^{-\frac{2(1 + \lambda)}{\alpha^x}} \left[ 1 + \lambda \right] - \frac{\chi(t)^{-2(1 + \lambda)}}{\frac{2(1 + \lambda)}{\alpha^x}} . \]

Here \( Ei(x) \) is the exponential integral function. Appealing to the fact that when \( x \rightarrow \infty \), \( Ei(x) \) can be expanded as
\[ Ei(x) \sim e^x (x^{-1} + x^{-2} + \cdots) , \]
we have that
\[ f_2(t) \simeq \alpha^{-\lambda} \chi(t)^{-2(1 + \lambda)} \left[ \frac{z(t)^{-2}}{2(1 + \lambda)} + \cdots \right] . \]
when $t$ approaches $t_{\text{brip}}$. The above result blows up at the big rip, so we will discard $\frac{25}{18}f_1$ in Eq. (35). Using Eqs. (23, 31, 34, 35), we obtain

$$\frac{\rho_{\text{br}}}{\rho_{\text{phan}}} \simeq \begin{cases} \frac{8C}{3}[1 + 3z(t) - 2z(t)^2]f_2(t), & w = -5/3, \\ \frac{-w + (1 + w)^{1/2}}{6(1 + w)^{1/2}} - 1, & w \neq -5/3, \end{cases} \tag{39}$$

It is interesting to note that when $w \neq -5/3$, the ratio is always negative as long as $w < -1$ and becomes negative unity before $t \to t_{\text{brip}}$. While, for the case of $w = -5/3$, it can be shown that the ratio is positive at present and remains so for a period of time. The plot of this ratio vs $t/t_0$ in Fig. 1 shows that it turns negative approximately at $t \approx 1.81t_0$. This indicates that the back reaction reinforces the phantom energy in an early period of phantom dominated universe and then counteracts it. At the big rip this ratio becomes negative infinity. Therefore, the phantom dominated phase will be terminated sooner or later before the big rip by the back reaction effects. The behaviors of the energy density of the back reaction and that of the phantom background as a function of $t/t_0$ are plotted in Fig. 2 and Fig. 3. There we can see that the ratio of $|\rho_{\text{phan}}/\rho_{\text{br}}|$ reaches unity (i.e., $\log_{10}(|\rho_{\text{phan}}/\rho_{\text{br}}|)$ becomes zero) before the big rip and thus the phantom phase terminates. Note that the termination point appears very close to the big rip in the figures. However, since $t_0(\approx 15\text{Gyr})$ a is very large number, the actual time interval measured in terms of years is not small (see also the Table). Let us now discuss in more detail when this happens. For the case of $w \neq -5/3$ the time when the phantom phase is terminated by the effects of gravitational back reaction, i.e., when $\frac{\rho_{\text{br}}}{\rho_{\text{phan}}} \simeq -1$, can be explicitly given as

$$t \simeq \frac{t_0}{1 + w} \left( \frac{6(1 + w) + (1 + 3w)\lambda \alpha^w}{(1 + 3w)(1 - w)} + 1 \right)^{-\frac{3(1 + w)}{(1 + 3w)\lambda \alpha^w + (1 + w)}} + w. \tag{40}$$

Define $t'$ as the time of termination of the phantom phase before the big rip, we have

$$t' \equiv t_{\text{brip}} - t = -\frac{t_0}{1 + w} \left[ \frac{6(1 + w) + (1 + 3w)\lambda \alpha^w}{(1 + 3w)(1 - w)} + 1 \right]^{-\frac{3(1 + w)}{(1 + 3w)\lambda \alpha^w + (1 + w)}}. \tag{41}$$

For a fixed $w$, $t'$ is an increasing function of $\lambda$. This is similar to the case of quintessence in that the effect of back reaction is proportional to the blue tilt. For scale invariant spectra, i.e., $\lambda = 0$, the effect of back reaction for quintessence is negligible. In contrast, for the phantom case, it is easy to see from Eq. (41) that the effect of back reaction can still become large enough to terminate the phantom phase. When $w = -5/3$, it is almost impossible to
TABLE I: The time of termination of the phantom phase before the big rip: \( t_0 = 15Gyr, \alpha = 7.5, C = 10^{-10} \).

| \( w \) | \( t'(yr)(\lambda = 0) \) | \( t'(yr)(\lambda = 0.27) \) |
|---|---|---|
| -1.1 | \( 4.2 \times 10^6 \) | \( 6.4 \times 10^8 \) |
| -1.3 | \( 9.6 \times 10^5 \) | \( 1.9 \times 10^7 \) |
| -1.5 | \( 5.1 \times 10^5 \) | \( 5.2 \times 10^6 \) |
| -5/3 | \( 3.4 \times 10^5 \) | \( 2.7 \times 10^6 \) |
| -1.8 | \( 3 \times 10^5 \) | \( 2 \times 10^6 \) |
| -2 | \( 2.4 \times 10^5 \) | \( 1.3 \times 10^6 \) |

FIG. 1: \( \rho_{br}/\rho_{phan} \) vs \( t/t_0 \) is plotted for \( w = 1.5 \) and \( w = -5/3 \) at the early epoch of phantom dominated phase with \( \lambda = 0.27, \alpha = 7.5, C = 10^{-10} \). The vertical coordinate has been scaled by \( C = 10^{-10} \).

solve Eq. (39) to get an analytical expression for the time when \( \rho_{br}/\rho_{phan} \sim -1 \), but we can resort to numerical techniques. Listed in the Table are results of the calculations for the time of termination of the phantom dominated phase for both \( w \neq -5/3 \) and \( w = -5/3 \).

IV. DISCUSSIONS

The main conclusion we can draw from the Table is that the big rip can be avoided by the gravitational back reaction of cosmological perturbations. A comparison of our results with that of Ref. [15] for the case of \( w = -1.5 \) shows that our Solar system could be saved! We can also see from the table that for a fixed \( w \) the greater the blue tilt, \( \lambda \), the sooner the termination of the phantom phase. However, for a fixed \( \lambda \), the greater the \( w \), the earlier the
FIG. 2: Plotted is $\log_{10}(|\rho_{\text{phan}}/\rho_{\text{br}}|)$ vs $t/t_0$ for $w = -1.5$ with $\lambda = 0.27, \alpha = 7.5, C = 10^{-10}$. The step length of the horizontal axis is 0.0005. Note that $t_0 \approx 15\text{Gyr}$.

FIG. 3: $\log_{10}(|\rho_{\text{phan}}/\rho_{\text{br}}|)$ vs $t/t_0$ is plotted for $w = -5/3$ with $\lambda = 0.27, \alpha = 7.5, C = 10^{-10}$. The step length of the horizontal axis is 0.0002. Note that $t_0 \approx 15\text{Gyr}$.

determination of phantom phase before the big rip. The physical reason is that the greater the $w$, the farther away we are from the big rip and thus there is more time for the infrared modes to accumulate. Finally, let us consider a case of pure theoretical significance, i.e., the case in which $\lambda \to \infty$, then we find $t' \simeq 13\text{Gyr}$ for $w \neq -5/3$ and $t_0 = 15\text{Gyr}$. This means that it is not possible to kill the phantom dominated phase the time just when it kicks in.

In summary, We have, assuming a COBE normalized spectrum of cosmological fluctuations at the present time, calculated the gravitational back-reaction effects of cosmological perturbations whose wavelengths at the time when the back-reactions are evaluated are larger than the Hubble radius. Our results reveal that the gravitational back-reactions are growing with time and could become large enough to terminate the phantom dominated phase before the big rip occurs. An interesting feature to be noted of the gravitational
back reactions is that their effective energy momentum tensor is that of some form of "matter" which has negative energy density and positive pressure. This form of "matter" was postulated in Ref. [16]

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