Unparticle in (1+1) dimension with one loop correction

Anisur Rahaman
Hooghly Mohsin College, Chinsurah, Hooghly - 712101, West Bengal, India
(Dated: May 14, 2014)

The possible emergence of unparticle has been mooted recently including a mass like term for gauge field with the Schwinger model at the classical level. A one loop correction due to bosonization is taken into account and investigation is carried out to study its effect on the unparticle scenario. It is observed that the physical mass, viz., unparticle scale acquires a new definition, i.e., the effect of this correction enters into the unparticle scale in a significant manner. The fermionic propagator is calculated which also agrees with the new scale. It has also been noticed that a novel restoration of the lost gauge invariance reappears when the ambiguity parameter related to the current anomaly acquires a specific expression. We have also observed that a quantum effect can nullify the effect of violation of gauge symmetry caused by some classical terms.

PACS numbers:

I. INTRODUCTION

QED, viz., Schwinger model in (1+1) dimension [1], is an interesting and well known field theoretical model. It has been widely studied over the years in connection with the mass generation of photon, confinement aspect of fermion (quark), charge shielding etc. [2–5]. The description of this model in noncommutative space time [6, 7] has also attracted a considerable attention [8, 9]. The model in a dilatonic background has used to study the information loss problem too [10, 11]. Though this simple model describes the interaction of massless fermion with the Abelian gauge field the physics within it is much involved. Photon acquires mass via a kind of dynamical symmetry breaking and the fermions get disappeared from the physical spectra. The possibility of emergence of unparticle in (1+1) dimension [12] has added a new dimension to the fame of this model. The interest on this model concerning the possibility of presence of unparticle [12] has thus opened up further scope of investigation.

In general, unparticle is a scale invariant sector, that decouples at large scale and the spectrum of which might be detected in the missing energy momentum distribution [13]. Recently in [12], we have witnessed an exciting illustration where an attempt was made to establish the possibility of appearance of in (1+1) dimension and it has been made possible just by the inclusion of a masslike term for the gauge field into the action of the Schwinger model at the classical level [12]. It has been shown there that it approaches to a free field theory at high energies and a scale invariant theory at low energy. Despite the inclusion of the mass term at the classical level there lies more involved mechanism for a masslike term to get involved at the quantum mechanical level through a one loop correction [14]. An ambiguity parameter automatically enters into that loop correction. So the effective action of the Schwinger model in its generalized bosonized version contains a one loop correction along with an ambiguity parameter [14, 15]. For a specific choice, i.e., for the vanishing value of the ambiguity parameter the model reduces to the usual gauge invariant vector Schwinger model, but for the other admissible value of this parameter, the phase space structure as well as the the physical spectra acquire significant changes [14]. For instance, in this generic situation the confinement scenario of the fermion is found to get altered [14]. In fact, the fermion gets liberated in the similar fashion as it was found to happen in the Jackiw-Rajaraman version of Chiral Schwinger model [16]. We were familiar with this type of correction along with an ambiguity parameter from long back [16]. In [16, 20] we noticed, how amazingly ambiguity parameter removed the long suffering of the chiral generation of the Schwinger model [21]. The investigation related to unparticle in (1+1) dimension taking into account quantum correction would, therefore, be of interest in its own right. Inclusion of that a quantum correction in QED was attempted by us in [14]. Here we are embarked on a program to investigate what new light this one loop effect can shed in lower dimensional unparticle scenario.

In (1+1) dimension, the standard bosonization technique provides a room to study this newly conjectured unparticle scenario where both the classical and one loop corrected masslike term can simultaneously be taken into account in the same footing. What new signal this one loop corrected term can convey to this recently proposed lower dimensional unparticle physics, that is the main objective of this work. Other perspective of this work is to exploit arbitrariness of the ambiguity parameter which would be available here due to this very correction to restore the symmetry of this

*Electronic address: anisur.rahman@saha.ac.in
gauge variant version which was broken at the classical level to modify the standard QED model in order to establish the possible emergence of unpractice \[12\].

The model with this setting would, therefore, be the Sommerfield (Thirring-Wess) \[22, 23\] model along with a one loop correction term. An illustration with only loop correction was made by us for a different perspective in \[14\]. In this context, we would like to mention that the masslike term for gauge gauge field found in the Sommerfield(Thirring-Wess) model \[22, 23\] was included at the classical level. One may consider it as model that describe the interaction of fermionic field with Proca background. So there lies a sharp difference between the mass like term that entered in \[14, 15\] and \[12, 22, 23\] so far origin of this type of term is concerned. The masslike term considered in \[14, 15\] comes as a one loop correction in order to remove the divergence of the fermionic determinant during bosonization but in the illustration of unparticle in \[12\] the authors considered the masslike term for gauge field at the classical level. In fact, the old model available in \[22, 23\] has been considered in \[12\] with a different perspective.

The paper is organized in the following manner. Sec. II is devoted with the calculation of fermionic current where we will be able to see how the ambiguity parameter gets involved. In Sec. III, the bosonized model is quantized using the formalism developed by Dirac for constrained system to find out the physical spectrum. Fermionic propagator is calculated in Sec. IV. to see whether it agrees with the spectrum calculated in Sec. III. In Sec. V, it is shown that a novel gauge invariance can be restored exploiting the regularization ambiguity. Sec. VI contains the concluding remark.

II. CALCULATION OF FERMIONIC CURRENT

The vector Schwinger model is described by the following generating functional

\[ Z[A] = \int d\psi d\bar{\psi} e^{\int d^2 x \mathcal{L}_f}, \]  

with

\[ \mathcal{L}_f = \bar{\psi} \gamma^\mu \left[ i \partial_\mu + e A_\mu \right] \psi. \]  

The fermionic current for this model is \[ J_\mu = \bar{\psi} \gamma^\mu \psi \]. It is a composite operator build out of fermion fields. Since the products of local operators are often singular, it is instructive to calculate the current by point splitting method. Maintaining the Lorentz invariance the current can be defined in the following manner.

\[ J_\mu = \bar{\psi} (x + \epsilon) \gamma_\mu \left[ e^{i \epsilon f_s} e^{i \epsilon \tilde{A}} \mathcal{O}_\mu (z) - a \tilde{A} \right] : \psi (x), \]  

where \[ \tilde{A}_\mu = \epsilon_{\mu \nu} A^\nu \]. The choices of coefficients in front of \[ A_\mu \] and \[ \tilde{A}_\mu \] are very crucial. Note that \[ a = 0 \] correspond to the usual gauge invariant vector Schwinger model, but the non zero \[ a \]'s, correspond to the nonconfining Schwinger model proposed by us in \[14\]. In (1+1)dimension, the most general Ansatz for \[ A_\mu \] is

\[ A_\mu = - \frac{\sqrt{\pi}}{\epsilon} \left( \partial_\nu \sigma + \partial_\nu \tilde{\eta} \right), \]  

where \[ \tilde{\partial}_\mu = \epsilon_{\mu \nu} \partial^\nu \]. \[ \sigma \] and \[ \eta \] are two scalar field and the dual of \[ \eta \] is defined by \[ \tilde{\partial}_\mu \eta = \partial_\mu \tilde{\eta} \]. The definition \[ 4 \] of \[ A_\mu \] gives

\[ F_{\mu \nu} = - \frac{\sqrt{\pi}}{\epsilon} \left( \partial_\mu \tilde{\partial}_\nu - \partial_\nu \tilde{\partial}_\mu \right) \sigma = \frac{\sqrt{\pi}}{\epsilon} \epsilon_{\mu \nu} \Box \sigma, \]  

The equation of motion, that follows from \[ 2 \], for the field \[ \psi \] is

\[ [i \partial + \sqrt{\pi} \gamma_\mu (\sigma + \eta)] \psi = 0, \]  

and it provides the following solution for \[ \psi \]

\[ \psi (x) = : e^{i \sqrt{\pi} \gamma_\mu (\sigma + \eta)} : \psi (0) (x). \]  

If we substitute the expression for \[ A_\mu \] and \[ \psi \] in \[ 3 \], and use the identity

\[ < 0 | \tilde{\psi}^{(0)} (x + \epsilon) \psi^{(0)} (x) | 0 > = - \frac{i (\epsilon^\mu \gamma_\mu) \alpha \beta}{2 \pi \epsilon^2}, \]  

we arrive at

$$J^\text{reg}_\mu = J^f_\mu - \frac{a}{\sqrt{\pi}} \epsilon_\mu \epsilon_\nu - \tilde{\epsilon}_\mu \tilde{\epsilon}_\nu \frac{\tilde{\partial}}{\epsilon^2} \tilde{\partial}_\nu (\sigma + \eta)].$$

(9)

Taking the symmetric limit, i.e., averaging over the point splitting direction $\epsilon$, we find

$$J^\text{reg}_\mu = J^f_\mu - \frac{a}{\sqrt{\pi}} \tilde{\partial}_\mu (\sigma + \eta))$$

(10)

In terms of potential of the fermionic current, the above equation can be written down as

$$J^\text{reg}_\mu = -\tilde{\partial}_\mu \phi + e a e^2 A_\mu.$$  

(11)

Here $\phi$ stands for the potential of the free fermionic current and $J^f_\mu = \bar{\psi} \gamma_\mu \psi$. Note that the expression of current (11) contains the ambiguity parameter $a$. We were familiar with this parameter from long past [16]. In [14, 15] we have argued that this type of ambiguity may appear in the Schwinger model too however specific calculation was lacking there. An arguments illustration as given there like the argument made in [16]. For the the second case [16] it was calculated in [17] after few years. Rest of the work in this paper is devoted along with the ambiguity parameter $a$ to study the unpractice scenario in (1+1) dimension.

### III. STUDY OF PHYSICAL MASS THROUGH CONSTRAINT ANALYSIS

In (1+1) dimension exact Bosonization is possible and it interesting that one loop effect automatically enters through the process and that enable us to study the effect of one loop correction in the lower dimensional unparticle scenario. So we start our analysis with a generalized bosonized version where corrections from both the corners (classical as well as quantum) are taken into consideration. A standard way to get the bosonized lagrangian density from (2), is to be integrating out the fermions one by one [14], and that leads to the following bosonized lagrangian density

$$L_B = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - e \epsilon_\mu \epsilon_\nu \partial^\nu \phi A^\mu + \frac{1}{2} e^2 A^\mu A^\mu.$$  

(12)

Here $a$ is the regularization ambiguity parameter. It is needed to remove the divergence in the fermionic determinant. When it is treated through appropriate regularization, a masslike counter term enters in to the model through a one loop correction [14]. The current for this model is found out to be

$$J^B_\mu = -e \tilde{\partial}_\mu \phi + e^2 a A_\mu.$$  

(13)

Note that this expression of current agrees with the current computed in the previous section (11), and it is straightforward to see that $\partial_\mu J^B_\mu \neq 0$ that indicates the violation of gauge symmetry at the bosonized action due to the one loop effect. However at the fermionic level current was $J_\mu = \bar{\psi} \gamma_\mu \psi$ and gauge symmetry was intact there from classical point of view.

If Proca type electromagnetic background (along with the corresponding masslike term) for the gauge field is added with the fermionic version of the Schwinger model the model turns into

$$L_{fg} = \bar{\psi} \gamma^\mu [i \tilde{\partial}_\mu + e A_\mu] \psi + \frac{1}{2} m_0 A_\mu A^\mu.$$  

(14)

and gauge symmetry immediately breaks here at the classical level due to the presence masslike term of Proca background. As we have already mentioned, the standard bosonization technique allows us to take the advantage of including a one loop (quantum) effect here too. If we now include the one loop correction the bosonised lagrangian density reads

$$L_B = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - e \epsilon_\mu \epsilon_\nu \partial^\nu \phi A^\mu - \frac{1}{4} F^\mu_\nu F^\mu_\nu + \frac{1}{2} \epsilon^2 (a + m_0) A_\mu A^\mu.$$  

(15)

Here Lorentz indices runs over the two values 0 and 1 corresponding to the two space time dimension and the rest of the notations are standard. The antisymmetric tensor is defined with the convention $\epsilon_{01} = +1$. The coupling constant $e$ has one mass dimension in this situation. The parameter $m_0$ of course, has the dimension $e^2$. The terms containing $m_0$, has been introduced here to get the possible presence the unparticle is it was done in [12]. Of course
it was considered initially in the Sommerfield (Thirring-Wess) model. The term containing $a$ however help
to see the possible change of the unparticle scenario. Note that gauge symmetry has been broken by the combined
effect of the parameter $a$ and $m_0$.

Let us now proceed to study the phase space structure of the model in order to observe how physical mass term
of the model changes with the presence of one loop corrected masslike term. Though it can be seen in various
ways here we will follow the standard quantization of constrained system developed by Dirac. To this end, it is
necessary to calculate the the momenta corresponding to the field $A_0$, $A_1$ and $\phi$. From the standard definition of the
momentum, we obtain

$$\pi_0 = 0,$$
(16)
$$\pi_1 = F_{01},$$
(17)
$$\pi_\phi = \dot{\phi} - eA_1,$$
(18)

where $\pi_0$, $\pi_1$ and $\pi_\phi$ are the momenta corresponding to the field $A_0$, $A_1$ and $\phi$ respectively. Using the equations (16),
(17) and (18), the canonical Hamiltonian density is calculated:

$$\mathcal{H}_c = \frac{1}{2} (\pi_\phi + eA_1)^2 + \frac{1}{2} \pi_1^2 + \frac{1}{2} (A_0^2 - eA_0) - \frac{1}{2} e^2 (a + \frac{m_0}{e^2}) (A_0^2 - A_1^2).$$
(19)

Note that $\omega = \pi_0 = 0$, is the familiar primary constraint of the theory. The preservation of the constraint $\omega(x)$
requires $[\omega(x), H(y)] = 0$, which leads to the Gauss’ law as a secondary constraint:

$$\tilde{\omega} = \pi_1' + e\phi' + e^2 (a + \frac{m_0}{e^2}) A_1 \approx 0.$$
(20)

The constraints (16) and (20), form a second class set. Treating (16) and (20), as strong condition one can eliminate
$A_0$, and obtain the reduced Hamiltonian density.

$$\mathcal{H}_r = \frac{1}{2} (\pi_\phi + eA_1)^2 + \frac{1}{2m_0} (\pi_1' + e\phi')^2 + \frac{1}{2} (\pi_1'' + e\phi'')^2 + \frac{1}{2} e^2 (a + \frac{m_0}{e^2}) A_1^2.$$
(21)

According to the Dirac’s prescription of quantization, the Poisson brackets become inadequate for a theory
possessing second class constraint in its phase space. This type of system however, remains consistent with the
Dirac brackets. It is straightforward to show that the Dirac brackets between the fields describing the reduced
Hamiltonian (21) remain canonical. Using the Dirac brackets, the following first order equations of motion are found
out from the reduced Hamiltonian density (21).

$$\dot{A}_1 = \pi_1 - \frac{1}{e^2 (a + \frac{m_0}{e^2})} (\pi_1'' + e\phi''),$$
(22)
$$\dot{\phi} = \pi_\phi + eA_1,$$
(23)
$$\dot{\pi}_\phi = (1 + \frac{1}{(a + \frac{m_0}{e^2})}) \phi'' + \frac{1}{e (a + \frac{m_0}{e^2})} \pi_1'',$$
(24)
$$\dot{\pi}_1 = -e\pi_\phi - e^2 (a + \frac{m_0}{e^2} + 1) A_1.$$
(25)

A little algebra converts the above first order equations (22), (23), (24) and (25), into the following two second order
differential equations:

$$[\Box + e^2 (a + \frac{m_0}{e^2} + 1)] \pi_1 = 0,$$
(26)
$$\Box [\pi_1 + e (a + \frac{m_0}{e^2} + 1) \phi] = 0.$$
(27)
Equation (26), describes a massive boson field with square of the mass $m^2 = e^2(a + \frac{m_0}{e^2} + 1)$, whereas, equation (27), describes a massless scalar field. The equation (26), clearly shows that the physical mass acquires a generalized expression with the parameters involved in the masslike term at the classical level, as well as with the parameter entered through the one loop correction during bosonization. The welcomed entry of the one loop correction has thus brought a noticeable change in the physical mass and consequently, the definition of unparticle scale [12] acquires a novel expression with an adjustable parameter $a$.

Regarding the appearance of a dependent term, we should make some comment in order to make the fact transparent on one side, and to avoid confusion on the other. In two dimensional field theory, it is very crucial to define fermionic current, because lots of subtleties are involved in it. With the definition of current, the mathematical structure of the effective action changes significantly. In [25], it was pointed out that there was no arbitrariness in the chiral Schwinger model and that result corresponds to $a = 0$ of the Jackiw-Rajaraman version of chiral Schwinger model where it was maintained that for $a = 0$ the model suffered from non unitrity and that suffering could be removed only through non vanishing $a$. Few years later, through appropriate calculation of fermionic current it was shown in [17], that arbitrariness was certainly there and the $a$ dependent counter term resulted in. One can certainly calculate the fermionic current in various ways for the Schwinger model too. As one way calculation is exhibited in Sec. II. For Schwinger model, it is an usual practice to define the point splitting maintaining gauge symmetry and a generalized definition of current maintaining the gauge symmetry is available in [5]. However, one can certainly define it ignoring that symmetry. In the generalized gauge invariant definition [5] we found the emergence of a parameter dependent kinetic energy like term. The way Hagen defined the current, did not in any way include that possibility [26, 27]. So all the subtleties are not included even within the generalised definition of Hagen [26, 27]. The way we have defined the current in the present work, is also exclusive of the definition found in [26, 27], because the definition available in [26, 27] disagrees with non vanishing $a$.

**IV. CALCULATION OF FERMIONIC PROPAGATOR**

The nature of the theoretical spectrum becomes more transparent if we calculate the fermionic propagator to which we will now turn. To calculate fermion propagator one needs to work with the original fermionic model. The calculation of fermionic propagator is analogous to the Chiral Schwinger model [18, 19], and the so called nonconfining Schwinger model [14, 15]. The effective action obtained by integrating out $f$ from the bosonized action [15], is

$$S_{eff} = \int d^2x \frac{1}{2} [A_\mu(x)M^{\mu\nu}A_\nu(x)],$$

where

$$M^{\mu\nu} = e^2(a + \frac{m_0}{e^2})g^{\mu\nu} - \Box + e^2 \tilde{\partial}^{\mu} \tilde{\partial}^{\nu}.$$  \hfill (29)

The gauge field propagator is just the inverse of $M^{\mu\nu}$ and it is found to be

$$\Delta_{\mu\nu}(x - y) = \frac{1}{e^2(a + \frac{m_0}{e^2})} [g^{\mu\nu} + \frac{\Box + e^2}{\Box + e^2(a + \frac{m_0}{e^2} + 1)} \tilde{\partial}_\mu \tilde{\partial}_\nu] \delta(x - y).$$  \hfill (30)

Note that the two poles of propagator are found at the expected positions. One is at zero, and another is at $e^2(a + \frac{m_0}{e^2} + 1)$, indicating a massless and a massive excitations respectively and it agrees with physical spectrum obtained in the previous Sec.

We are now in a stage to calculate the Green function of the Dirac operator. To do that, let us consider the following Ansatz for the Green function of the Dirac operator $(i\partial + eA)$.

$$G(x, y; A) = e^{i\phi(x) - \Phi(y)} S_F(x - y),$$

where $S_F$ is the free massless fermion propagator and $\Phi$ will be determined when the Ansatz (31) will be plugged into the equation for the Green function. It will enable us to construct the propagator of the original fermion $\psi$. From the standard construction, the Green function is found out as

$$G(x, y; A) = e^{i\phi} \int d^2z A^{\nu}(z)J_\nu(z) S_F(x - y),$$

where the fermionic current $J_\mu$ is given by the following expression.

$$J_\mu = (\partial_\mu + \gamma_5 \tilde{\partial}_\mu)(D_F(z - x) - D_F(z - y)).$$  \hfill (33)
Here $D_F$ represents the propagator of a massless free scalar field. In order to avoid singularity such propagators have to be infra-red regularized in $(1+1)$ dimensions \[2\].

\[ D_F(x) = -\frac{i}{4\pi} \ln(-\mu^2 x^2 + i0). \]  

(34)

Here $\mu$ stands for the infra-red regulator mass. Finally, we obtain the fermion propagator by functionally integrating $G(x,y;A)$ over the gauge field:

\[ S_F = \int \mathcal{D}A e^{\frac{1}{2} \int d^2z \left( A_\mu(z)M^{\mu\nu}A_\nu(z)+2eA_\mu J^\mu \right) S_F(x-y)} \]

\[ = \mathcal{N} \exp \left[ \frac{D_F(m^2 = \epsilon^2(a + m_0^2 + 1))}{a + m_0^2 + 1} \right] S_F. \]  

(35)

Here $\Delta_F$ is the propagator of a massive free scalar field and $\mathcal{N}$ is a wave function renormalization factor.

The theoretical spectrum \[26\] and \[27\], as well as the fermionic propagator \[35\], therefore, make the fact confirm that the mass acquires a generalized expression with bare coupling constant and the parameters involved within the masslike terms for the gauge field. Thus, a new definition of unparticle scale emerges out with the introduction of one loop correction holding the hands of generalized physical mass term \[26\]. It is fascinating to get this new definition of unparticle scale when one loop correction is taken into consideration and it would be natural to see its importance in the area of physics where unparticle is expected to show its prominent role.

Note that, the mass, that generates via dynamical symmetry breaking in the vector Schwinger model, depend only on the bare coupling constant, and it does not contain any such parameters like $m_0$ or $a$. In fact, in the bosonized vector Schwinger model, no such parameter gets involved when bosonization is done there maintaining the gauge symmetry. It is true that setting the value of the parameter to $a = 0$, one can obtain vector Schwinger model \[1\] from the bosonized lagrangian of the so called nonconfining Schwinger model \[14\], where bosonization is done ignoring the gauge symmetry. But, this trivial choice $a = 0$, does not work to bring back the gauge symmetry in this generalized situation where masslike terms from both the corners classical as well as quantum are present. However, there lies some interesting possibility in connection with the retrieval of the gauge symmetry in the usual phase space, which we are going to uncover in the following Sec.

V. RESTORATION OF GAUGE SYMMETRY EXPLOITING THE AMBIGUITY PARAMETER

To exploit arbitrariness of the ambiguity parameter present in a model is more or less a standard practice in $(1+1)$ dimensional field theory. It has been used several times in different perspective. Sometimes even it has been used as a remedy in order to get out of the dangerous unphysical situation through which it is suffering \[16–20, 34–37\].

To get back the broken gauge symmetry of this modified model, if we exploit the available arbitrariness of the ambiguity parameter in this situation it will also stand as a sound example in its own right. Without violating any physical principal, the said arbitrariness allows us to set

\[ (a + m_0^2) = 0. \]  

(36)

This equation though look simple, it bears a deeper meaning. It reflects the competition between a classical and quantum mechanical term. During the competition if the effect one we succeeds to nullify the effect made by the other then only equation \[36\] will be satisfied. Now what it renders, is the opening up of a provision of getting an expression (definition) of $a$ in terms of the parameters involved in the theory, and that in turns, brings a remarkable change in the constraint structure of the theory. In fact, this definition of $a$ corresponds to a singularity in the phase space structure of the theory. The existing second class constraint structure gets converted into a first set. The following are those two first class constraints if \[36\] is maintained.

\[ \pi_0 \approx 0, \]  

(37)

\[ \pi_1' + e\phi' \approx 0. \]  

(38)

So, with this particular expression of the ambiguity parameter $a$, we get only the two first class constraints \[37\] and \[38\]. We now need two gauge fixing conditions to single out the physical degrees of freedom. The following two conditions would be the appropriate gauge fixing at this stage.

\[ A_0 \approx 0, \]  

(39)
\begin{equation}
A_1 \approx 0.
\end{equation}

When the gauge fixing conditions (39) and (40), are plugged as a strong condition into the hamiltonian (19), along with the two first class constraints (37) and (38), the hamiltonian (19) acquires the following simplified form.

\begin{equation}
H_{RF} = \int dx \left[ \frac{1}{2} \pi_\phi^2 + \frac{1}{2} e^2 \phi^2 + \frac{1}{2} \phi'^2 \right].
\end{equation}

But the price to pay at this point is to use the Dirac brackets [24] in place of canonical Poisson brackets. The Dirac brackets [24] for the fields describing the reduced hamiltonian (41), are found to remain canonical. We get the following equations of motion hamiltonian (41), when the canonical Dirac brackets are used for computation of equations of motion.

\begin{equation}
\dot{\phi} = \pi_\phi,
\end{equation}

\begin{equation}
\dot{\pi}_\phi = \phi'' - e^2 \phi.
\end{equation}

The above two equations give a single Klein-Gordon type second order differential equation

\begin{equation}
[\Box + e^2] \phi = 0.
\end{equation}

It describes a massive boson with square of the mass $e^2$. Note that, the mass in this situation is independent of the parameter $a$ or $m_0$. The presence of two first class constraint in the phase space indicates that the model with this has got back the gauge symmetry in its usual phase space when the mutual effect of the classical and quantum mechanical term cancels each other. The first class constraints will provide the generator of the gauge transformation. What can be more interesting than to see the reappearance of the lost symmetry of a model just by exploiting the arbitrariness of the ambiguity parameter? A careful look reveals that the expression of $a$ available by exploiting the arbitrariness in the theory [36], maps the present model onto the usual vector Schwinger model. A trivial choice, i.e., simultaneous setting of the parameters $a = 0$ and $m_0 = 0$, of course, convert this modified model into the usual Vector Schwinger model, however that choice does not bear any meaningful physical consequence as it is available for the present context exploiting the arbitrariness involved in the theory.

There are different ways to restore gauge invariance of a gauge nonsymmetric model. Mitra-Rajaraman prescription [28, 29] is a good example of that, where restoration of gauge symmetry of a given model is possible in its usual phase space. Some other prescriptions are also available, where restoration of gauge symmetry takes place in the extended phase space [30–33]. However, the restoration of gauge symmetry through a specific non vanishing expression of the ambiguity parameter $a$ resulted out of the contest of the two same type of terms coming from two different corner looks amazing. It is expected that it would be of considerable importance elsewhere, especially, in the physics at high energy regime, where unparticle is expected to show important role.

\section{VI. CONCLUSION}

Our work reveals that with this new setting the physical mass of the boson field acquires a generalized expression. Here physical mass term does not contain only the parameter involved in the classical masslike term like [13, 22, 23] but also it has acquired the effect of one loop correction. In fact, the physical mass term of the model described earlier in the work [12, 22, 23] has got modified with the parameter involved in one loop corrected term. So the quantum correction will now play a crucial role. Thus a novel definition in the unparticle scale and that too with an adjustable parameter $a$ has resulted in. So this new setting is expected to encompass both the theoretical and phenomenological aspects.

We have obtained an expression of the ambiguity parameter $a$ exploiting the arbitrariness available by virtue of exact bosonization without violating any physical principle which stands as a fascinating aspect of this model. The service, which this new definition of $a$ renders is remarkable: it has brought back the symmetry of the effective action in the usual phase space when classical masslike term for gauge fields destroyed that symmetry to start with at the fermionic version. So an opening has resulted in where a contest between a classical term and a quantum mechanically generated term can be examined in the context of symmetry restoration or violation. Some more studies that can be followed from the work are as follows. What would be the fate of that unparticle for that specific value of $a$ that brings back the symmetry? Will it die out when the system regain its original symmetry?

It would be worthwhile to mention that the term $\frac{1}{2} e^2(a + \frac{m_0}{2}) A^\mu A_\mu$ though looks like a gauge fixing term, it is not the case in true sense. With this term, the gauge redundancy of the vector Schwinger model though gets fixed
and it turns into a gauge non invariant model, nevertheless, it can not be considered as proper gauge fixing because a true gauge fixing term never brings any change in the theoretical spectrum. in contrast to that we noticed here that with the inclusions of the masslike terms, the mass of the massive boson has acquire a generalized expression. A massless boson has also been found in the theoretical spectrum. Instead of adding the terms $\frac{e^2}{2}\left(a + \frac{m_0}{2}\right)A^\mu A_\mu$, if the terms $\frac{\alpha}{2e^2}(\partial_\mu A^\mu)^2$, are added into the starting lagrangian of the vector Schwinger model, these do work as a true gauge fixing term, since in that situation one gets only a parameter free mass term for the photon, though the starting lagrangian in that situation contains the parameter $\alpha$.

One more point which we would like to emphasize is that the way the arbitraries of the ambiguity parameter has been exploited here to get back the symmetry of the model, has not in any, violated any physical principle and the technique is more or less standard in (1+1) dimensional QED and Chiral QED. We have witnessed several examples of the use of this mechanism in order to get rescued from different unfavorable as well as un physical situations [16–20, 34–37]. The most remarkable one in this context was the Jackiw and Rajaraman version of chiral Schwinger model [16], where they saved the long suffering of the chiral generation of the Schwinger model [21], from the non-unitary problem.

VII. ACKNOWLEDGMENTS

I would like to thank the Director, Saha Institute of Nuclear Physics for using computer facilities of the Institute.

[1] J. Schwinger, Phys. Rev., 128 2425 (1962)
[2] J. Lowestein and J. A. Swieca, Ann. Phys., 68 172 (1971)
[3] S. Coleman, Ann. Phys., 101 239 (1976)
[4] A. Casher, J. Kogut and L. Susskind, Phys. Rev. D10 732 (1974).
[5] G. Bhattacharyya, A. Ghosh, and P. Mitra, Phys. Rev., D50 4183 (1994)
[6] A. Saha, A. Rahaman and P. Mukherjee, Phys. Lett. B638 292 (2006), Phys. Lett B643 383 (2006)
[7] A. Saha, A. Rahaman and P. Mukherjee, Mod. Phys. Lett. A23 2947 (2008)
[8] F. Ardalan, M. Ghasemkhani and N. Sadoohhi, Eur. Phys. J. C71 1606 2011
[9] K. M. Keita, F. Wu, and M. Zhong, Phys.Lett. B681 367 (2009)
[10] M. Alford and A. Strominger, Phys. Rev. Lett. 69 563 (1992)
[11] A. Rahaman, Phys. Lett B697 260 (2011)
[12] H. Georgi and Y. Kats, Phys. Rev. Lett. 101 131603 (2008)
[13] H. Georgi, Phys. Rev. Lett. 98 221601 (2007)
[14] P. Mitra and A. Rahaman, Ann. Phys. (N.Y.) 249 34 (1996).
[15] A. Rahaman, Int. Jour. Mod. Phys. A19 3013 (2004).
[16] R. Jackiw and R. Rajaraman, Phys. Rev. Lett., 54 1219 (1985)
[17] R. Banerjee, Phys. Rev. Lett. 56 1889 (1986)
[18] H. O. Girotti, H. J. Rothe, and K. D. Rothe, Phys. Rev. D33 514 (1986)
[19] H. O. Girotti, H. J. Rothe, and K. D. Rothe, Phys. Rev. D34 592 (1986)
[20] H. O. Girotti and K. D. Rothe, Int. J. Mod. Phys. A4 5041 (1989)
[21] C. R. Hagen, Ann. Phys. (N.Y.) 81 67 (1973)
[22] C. M. Sommerfield Ann. Phys. 26 1 1963
[23] W. E. Thirring and J. E. Wess, Ann. Phys. 27 331 (1964).
[24] P. A. M. Dirac, Lectures on Quantum Mechanics(Yeshiva Univ. Press New York, 1964).
[25] A. Das, Phys. Rev. Lett. 55, 2126 (1985)
[26] C. R. Hagen, Phys. Rev. D55, 1021 (1997)
[27] C. R. Hagen, Nuov. Cim. B51, 169 (1967)
[28] P. Mitra and R. Rajaraman, Ann. Phys. (N. Y.) 203 137 (1990)
[29] P. Mitra and R. Rajaraman, Ann. Phys. (N. Y.) 203 157 (1990)
[30] E. S. Fradkin and G.A. Vilkovisky, Phys. Lett. B55 224 (1975)
[31] I. A. Batalin and E. S. Fradkin, Nucl. Phys. B279 514 (1987)
[32] T. Fujiiwara, I. Igarashi and J. Kubo, Nucl. Phys. B314 695 (1990)
[33] I. A. Batalin and V. Tyutin, Int. J. Mod. Phys. A6 3255 (1991)
[34] P. Mitra, Phys. Lett., 248 23 (1992)
[35] S. Ghosh and P. Mitra, Phys. Rev. D44 1332 (1991).
[36] K. Harada, Phys. Rev. lett., 64 134 (1990)
[37] E. Abdalla, M. C. B. Abdalla, and K. D. Rorhe, ‘Two Dimensional Quantum field theory’, (World Scientific, Singapore, 1991).