Dynamics and Infrared Spectroscopy of the Protonated Water Dimer**

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Catch phrase: Hydrated Proton

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Accurate infrared (IR) spectroscopy of protonated water clusters prepared in the gas phase has become possible in recent years,[1, 2, 3, 4, 5, 6] opening the door to a deeper understanding of the properties of aqueous systems and the hydrated proton, which are of main interest in central areas of chemistry and biology. Several computational studies have appeared in parallel, providing a necessary theoretical basis for the assignment and understanding of the different spectral features.[7, 8, 9, 10] It has been recently demonstrated that the $\text{H}_5\text{O}^+$ motif, also referred to as Zundel cation, plays an important role in protonated water clusters of 6 or more water molecules and as a limiting structure, together with the Eigen cation (H$_9$O$_4^+$), of the hydrated
proton in bulk water. The importance of the hydrated proton and the amount of work devoted to the problem contrast with the fact that the smallest system in which a proton is shared between water molecules, $\text{H}_5\text{O}_2^+$, is not yet completely understood, and an explanation of the most important spectral signatures and the associated dynamics of the cluster is lacking.

In this letter we report the simulation of the IR linear absorption spectrum of the $\text{H}_5\text{O}_2^+$ cation in the range between 0 to 4000 cm$^{-1}$ by state-of-the-art quantum-dynamical methods, we discuss the spectral signatures in terms of the underlying couplings and dynamics of the different degrees of freedom and compare our results to recent, accurate experiments on this system. For the first time the doublet-peak feature at around 1000 cm$^{-1}$ is fully reproduced, analyzed and assigned. The doublet is found to arise from the coupling between the proton-transfer mode and low frequency, large-amplitude displacements of both water molecules. Predictions are also made for the lowest frequency part of the spectrum, which has not yet experimentally been accessed. Several important features of the system are analyzed for the first time, namely the degeneracy of some of the vibrational levels and the extreme anharmonicity of the wagging motions and relative internal rotation of the two water molecules. In doing so, we do not resort to any low-dimensional model of the system, but we treat it in its full dimensionality, i.e., $3N - 6 = 15$ active internal coordinates (15D). The use of full dimensionality is found crucial in the reproduction of the complete absorption spectrum and dynamics. Our study provides a picture of the $\text{H}_5\text{O}_2^+$ system, extendable to larger aggregates, in which the clusters have to be viewed as highly anharmonic, flexible, multi-minima, coupled systems. From a methods perspective, we show that a full quantum-dynamical description of such a complex molecular system can still be achieved, providing explicative and predictive power and a very good agreement to available experimental data. In this respect, the reported simulations set a new state of the art in quantum dynamically describing an anharmonic, highly coupled molecular system of the size of the $\text{H}_5\text{O}_2^+$ cation. To account for the interatomic potential and the interaction with the radiation we make use of the potential energy surface (PES) and dipole-moment surfaces recently developed by Bowman and collaborators, which constitute the most accurate $ab\ initio$ surfaces available to date for this system.

The IR predissociation spectrum of the $\text{H}_5\text{O}_2^+$ cation has been recently measured in argon-solvate and neon- and argon-solvate conditions. It is expected that the photodissociation spectrum of the $\text{H}_5\text{O}_2^+-\text{Ne}_1$ complex is close to the linear absorption spectrum of the bare cation. This spectrum features a doublet structure in the region of 1000 cm$^{-1}$ made of two well-defined absorptions at 928 cm$^{-1}$ and 1047 cm$^{-1}$. This doublet structure was not fully understood, although the highest-energy component was assigned to the asymmetric proton-stretch fundamental based
on the quantal calculations on the IR spectrum of H$_5$O$_2^+$ of Bowman and coworkers.[6] Low-frequency modes may also play an important role in combination with the proton-transfer fundamental. Such a possibility has been already suggested,[2, 8, 10] but just which modes would participate in such combinations, and how, is still a matter of discussion.

The Hamiltonian used in the simulation of H$_5$O$_2^+$ is expressed in a set of polyspherical coordinates based on the Jacobi vectors in Figure 1.[12] It is found that only after the introduction of such a curvilinear set of coordinates an adequate treatment of the anharmonic large-amplitude vibrations and torsions of the molecule becomes possible. The kinetic energy operator is exact for $J = 0$, and the derivation of its lengthy formula (674 terms) will be discussed in a forthcoming publication. The correctness of the operator implemented was checked by comparison with data generated by the TNUM program.[13] The internal coordinates used are: the distance between the centers of mass of both water molecules ($R$), the position of the central proton with respect to the center of mass of the water dimer ($x, y, z$), the Euler angles defining the relative orientation between the two water molecules (wagging or pyramidalization: $\gamma_a, \gamma_b$; rocking: $\beta_a, \beta_b$; internal relative rotation: $\alpha$) and the Jacobi coordinates which account for the particular configuration of each water molecule ($r_1(a,b), r_2(a,b), \theta(a,b)$) where $r_{1x}$ is the distance between the oxygen atom and the center of mass of the corresponding H$_2$ fragment, $r_{2x}$ is the H–H distance and $\theta_x$ is the angle between these two vectors. These internal coordinates are body-fixed (BF) ones, where the water-water distance vector $\vec{R}$ points along the BF z-axis. These coordinates have the great advantage of leading to a much more decoupled representation of the PES than a normal-mode based Hamiltonian. The quantum-dynamical problem is solved in the time-dependent picture using the multiconfiguration time dependent Hartree method (MCTD H).[14, 15] The potential energy surface has been represented by a cut high-dimensional model-representation (cut-HDMR).[16, 17]

In Figure 2 probability-density projections on the wagging coordinates are shown for the ground vibrational state ($g_0$), as well as for one of the two fundamental states ($w_{1a}, w_{1b}$) of the wagging modes, which are degenerate vibrational states with an energy of 106 cm$^{-1}$. State $w_3$ is shown in Figure 2c and it will play a major role due to its coupling to the proton-transfer mode, as will be discussed later.

The probability-density of the wagging coordinates in $g_0$ (Figure 2a) presents four maxima in which the wagging angle is about 30 degrees with respect to the planar conformation for each water molecule. The probability for one or both water molecules to be found in a planar conformation is almost as high as the probability to be found pyramidal. This means that H$_5$O$_2^+$ interconverts already at $T = 0$, due to zero-point energy, between equivalent absolute minimum-energy structures in which both water molecules are found in a pyramidal conformation. Four equivalent minimum-
energy structures are accessible through wagging motions. The number of accessible equivalent minima at \( T = 0 \) doubles to eight since the relative rotation of both water molecules (\( \alpha \) coordinate) has also been found to be allowed through a low energy barrier.

The energies of the next three wagging-mode states (\( w_2, w_3, w_4 \)) are, respectively, 232, 374 and 422 cm\(^{-1}\). In a harmonic limit these states can be represented by kets \(|ab\rangle\) notation signifies the quanta of excitation in the wagging motions of waters \( a \) and \( b \). The degeneracy between \( w_2, w_3 \) and \( w_4 \) is broken due to anharmonicity. In harmonic approximation the energies of the two lowest wagging-fundamentals \( w_1a \) and \( w_1b \) are about 300 cm\(^{-1}\) larger than our result and do not account for their degeneracy, since harmonic modes are constructed taking as a reference only one of the equivalent absolute minima. The system, however, interconverts between eight equivalent \( C_2 \) structures and other stationary points through low-energy barriers (wagging motions and internal rotation), which leads to a highly symmetric ground-state wavefunction. Other vibrational states have been computed which are related to the internal rotation, rockings and water-water stretching modes. They will be reported and discussed in a forthcoming publication.

Figure 3 presents the IR predissociation spectrum of the \( \text{H}_5\text{O}^+ \cdot \text{Ne} \) complex [6] and the simulated spectrum of \( \text{H}_5\text{O}^+ \) in the range 700-1900 cm\(^{-1}\). The simulated spectrum is obtained in the time-dependent picture by Fourier transformation of the autocorrelation of a dipole-operated initial state:[18]

\[ \text{Eq. 1 HERE} \]

where \( E_0 \) is the ground-state energy and \(|\Psi_{\mu,0}\rangle \equiv \hat{\mu} |\Psi_0\rangle\). The simulated spectrum shows a good agreement with the experimental spectrum. The agreement on the doublet structure around 1000 cm\(^{-1}\) is very good, and the position of the doublet at 1700 - 1800 cm\(^{-1}\) is also in good agreement, despite the relative intensities being larger in the simulation.

The simulated spectrum in the range between 0 and 4000 cm\(^{-1}\) is depicted in Figure 4. The region below 700 cm\(^{-1}\) has not yet been accessed experimentally. Direct absorption of the wagging motions, excited by the perpendicular components of the field, appears in the range between 100 - 200 cm\(^{-1}\). The doublet starting at 1700 cm\(^{-1}\) is clearly related to bending motions of the water molecules, but its exact nature is still to be addressed. The simulated spectrum also shows the absorptions of the OH stretchings starting at 3600 cm\(^{-1}\).

The doublet absorption at around 1000 cm\(^{-1}\) and the related underlying dynamics deserve a deeper analysis. Due to the high density of states, it was not possible to obtain the fully converged states, but reasonably good approximations to the wavefunctions of the low-energy (\(|\Psi_{d1}^l\rangle, 930 \text{ cm}^{-1}\)) and high energy (\(|\Psi_{d1}^h\rangle, 1021 \text{ cm}^{-1}\)) eigenstates of the doublet were computed. Even though these wavefunctions contain all the possible informa-
tion on the two states, their direct analysis becomes complex due to the high dimensionality of such objects. In order to obtain a fundamental understanding of the observed bands, zeroth-order states \(|\Phi_z\rangle\) and \(|\Phi_{R,w3}\rangle\) were constructed, where \(|\Phi_z\rangle\) is characterized by one quantum of excitation in the proton-transfer coordinate whereas \(|\Phi_{R,w3}\rangle\) by one quantum in the water-water stretch and two quanta in the wagging motion. They were constructed by operating with \(\hat{z}\) on the ground state: \(|\Phi_z\rangle = \hat{z}|\Psi_0\rangle N\), where \(N\) is a normalization constant, and by operating with \((\hat{R} - R_0)\) on the third excited wagging state \(w_3\): \(|\Phi_{R,w3}\rangle = (\hat{R} - R_0)|\Psi_{w3}\rangle N\), respectively. The two eigenstates corresponding to the doublet were then projected onto these zeroth-order states. The corresponding overlaps read: \(|\langle \Phi_z | \Psi_{ld} \rangle|^2 = 0.20\), \(|\langle \Phi_{R,w3} | \Psi_{ld} \rangle|^2 = 0.53\) and \(|\langle \Phi_z | \Psi_{hd} \rangle|^2 = 0.48\), \(|\langle \Phi_{R,w3} | \Psi_{hd} \rangle|^2 = 0.12\). One should take into account that these numbers depend on the exact definition of the zeroth-order states, which is not unique. However, they provide a clear picture of the nature of the doublet: the low-energy band has the largest contribution from the combination of the water-water stretch and the third excited wagging (see Figure 2c), whereas the second largest is the proton-transfer motion. For the high-energy band the importance of these two contributions is reversed. Thus, the doublet may be regarded as a Fermi resonance between two zero-order states which are characterized by \((R, w_3)\) and \((z)\) excitations, respectively. The reason why the third wagging excitation plays an important role in the proton-transfer doublet is understood by inspecting Figure 2c and Figure 5. The probability density of state \(w_3\) has four maxima, each of which corresponds to a planar conformation of \(\text{H}_2\text{O-H}^+\) (\(\text{H}_3\text{O}^+\) character) for one of the waters, and a bend conformation (\(\text{H}_2\text{O}\) character) where a lone-pair \(\text{H}_2\text{O}\) orbital forms a hydrogen bond with the central proton. When the proton oscillates between the two waters, the two conformations exchange their characters accordingly. Thus, the asymmetric wagging mode \((w_3, 374 \text{ cm}^{-1})\) combines with the water-water stretch motion \((R, 550 \text{ cm}^{-1})\) to reach an energy close to the natural absorption-frequency of the proton transfer. As a consequence, the low-frequency wagging (or pyramidalization) motion of the water molecules becomes strongly coupled to the higher frequency, spectroscopically active proton-transfer motion and this coupling leads to the characteristic doublet feature of the IR spectrum.

In conclusion, we report a simulation of the dynamics and IR absorption spectrum of the \(\text{H}_5\text{O}_2^+\) cation by quantum-dynamical methodology in the full spectral range 0-4000 cm\(^{-1}\). The spectrum is directly comparable to available and future experiments on this system. We discuss some important features of the protonated water dimer which have remained until now elusive due to strongly anharmonic, large-amplitude motions. The floppy, multiminima nature of the cluster is presented and analyzed and the doublet-band absorption around 1000 cm\(^{-1}\) is fully reproduced and explained in terms of coupling of the proton-transfer motion to wagging torsions of the water
moieties. These calculations constitute an avenue for a detailed quantum-dynamical description of larger clusters and provide important fundamental information on the spectroscopy and dynamics of protonated aqueous systems and the hydrated proton.

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A brief discussion on the construction of the potential energy representation for the quantum-dynamical simulations is provided as supporting information. A more detailed discussion on the construction and accuracy of the potential energy representation will be given in a forthcoming publication.

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Figure Captions:

**Figure 1**: Set of Jacobi vectors in terms of which the kinetic energy of the system is expressed. The set of internal coordinates used corresponds to the length of these vectors and relative angles. The $z$ direction of the central proton is parallel to $\vec{R}$.

**Figure 2**: a) Probability density of the ground vibrational state, b) first and c) third wagging-mode states projected onto the wagging coordinates $\gamma_a$ and $\gamma_b$.

**Figure 3**: a) Predissociation spectrum of the $\text{H}_5\text{O}^+\cdot\text{Ne}$ complex; b) quantum-dynamical simulation. The resolution at which different peaks of the spectrum are resolved is given by the Fourier Transform. Due to the finite propagation time a finite resolution of about 30 cm$^{-1}$ is obtained.

**Figure 4**: Quantum-dynamics simulated spectrum in the range between 0 and 4000 cm$^{-1}$. Absorption is given in absolute scale in mega-barns (Mb).

**Figure 5**: Two most important coupled motions responsible for the doublet peak at 1000 cm$^{-1}$. 


The coupling shakes it: dynamics and infrared absorption spectrum of the protonated water dimer are reported by full quantum simulation. Strong couplings between the spectroscopically active proton-transfer motion and low-frequency, large-amplitude torsional modes are clearly identified and their role in the cluster dynamics is explained. These couplings are responsible for the characteristic doublet-peak around 1000 cm$^{-1}$, which was not understood and subject of debate. This spectral feature is reproduced, assigned and explained.

**keywords:** water clusters · proton transport · IR spectroscopy · quantum dynamics
\[ I(E) = \frac{E}{3e\epsilon_0 \hbar^2} \int_0^\infty \exp(\frac{i(E + E_0)t}{\hbar}) \times \langle \Psi_{\mu,0} | \exp(-i\hat{H}t/\hbar) | \Psi_{\mu,0} \rangle \, dt \] (1)
