A SUFFICIENT CONDITION FOR A COMPLEX POLYNOMIAL TO HAVE ONLY SIMPLE ZEROS AND AN ANALOG OF HUTCHINSON’S THEOREM FOR REAL POLYNOMIALS

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Abstract. We find the constant $b_\infty$ ($b_\infty \approx 4.81058280$) such that if a complex polynomial or entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $\omega \in \{2, 3, 4, \ldots\} \cup \{\infty\}$, with nonzero coefficients satisfy the conditions $\left| \frac{a_k^2}{a_{k-1} a_{k+1}} \right| > b_\infty$ for all $k = 1, 2, \ldots, \omega - 1$, then all the zeros of $f$ are simple. We show that the constant $b_\infty$ in the statement above is the smallest possible. We also obtain an analog of Hutchinson’s theorem for polynomials or entire functions with real nonzero coefficients.

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