A Causal Linear Model to Quantify Edge Unfairness for Unfair Edge Prioritization and Discrimination Removal

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Abstract

The dataset can be generated by an unfair mechanism in numerous settings. For instance, a judicial system is unfair if it rejects the bail plea of an accused based on the race. To mitigate the unfairness in the procedure generating the dataset, we need to know and quantify where the unfairness is originating from, how it affects the overall unfairness, and how to prioritize these sources of unfairness to address the real-world issues underlying these sources. Prior work of (Zhang et al., 2017) identifies and removes discrimination after data is generated but does not suggest a methodology to mitigate unfairness in the data generation phase. We use the notion of an unfair edge, same as (Chiappa and Isaac, 2018), to be a source of discrimination and quantify unfairness along an unfair edge. We also quantify overall unfairness in a particular decision towards a subset of sensitive attributes in terms of edge unfairness and measure the sensitivity of the former when the latter is varied. Using the formulation of cumulative unfairness in terms of edge unfairness, we alter the discrimination removal methodology discussed in (Zhang et al., 2017) by not formulating it as an optimization problem. This helps in getting rid of constraints that grow exponentially in the number of sensitive attributes and values taken by them. Finally, we discuss a priority algorithm for policymakers to address the real-world issues underlying the edges that result in unfairness. The experimental section validates the linear model assumption made to quantify edge unfairness.

1. INTRODUCTION

The underlying procedure generating a dataset can be subjected to unfairness in many scenarios. For instance, the criminal recidivism dataset generated by a judicial system that rejects the bail plea of an accused based on their race (e.g. African American) (Mehrabi et al., 2019) is unfair. Similarly, Stop, Question, and Frisk (SQF) dataset generated by the police who search for the contraband based on the race of the pedestrian (Evans and Williams, 2017) is unfair. (Chiappa and Isaac, 2018) subjectively evaluates whether an edge is unfair or not but it does not help in mitigating the unfairness. Multiple statistical criteria have been proposed to identify discrimination such as demographic parity, calibration, etc., but none attempt to analyze the causes of discrimination in the data generation phase - where is the discrimination arising from? how does the cause of the discrimination impact the decision made? etc. Also, it is mathematically incompatible to satisfy multiple statistical criteria (Berk et al., 2018); therefore there is an additional task of selecting which criterion has to be achieved.

For instance, we are assigned a task to reduce the cumulative unfairness in a bail granting decision towards African Americans. Suppose we are given the causal graph (see Figure 1) that is representative of the procedure generating the bail dataset with each node generated using its parents. Also, we are given the conditional probability distributions (CPT) of every node in the causal graph. If resources are limited, it becomes essential to prioritize the unfair edges to decide where should the unfairness be mitigated. To prioritize, we need a metric that measures the amount of unfairness in an edge and overall unfairness, a metric to measure how unfairness in an edge translates to overall unfairness and an algorithm to prioritize amongst the set of unfair edges whose underlying unfairness needs to be mitigated. Edge $R \rightarrow E$ in Figure 1 is unfair as the admission ($E$) is dependent on the race ($R$). If one is aware that increasing unfairness along $R \rightarrow E$ increases cumulative unfairness in bail decision for African Americans, one can mitigate the underlying unfairness present in edge $R \rightarrow E$ by raising awareness across the African American community to apply to educational institutions, by providing financial aid to African American groups, etc. Also, if increasing unfairness in edges $R \rightarrow E$ and $R \rightarrow T$, increases the cumulative unfairness in the

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bail decision \((J)\) by say, 0.7 fold and 0.3 fold respectively, then the government can prioritize to attend to the issues related to \(R \rightarrow E\) over \(R \rightarrow T\). Similar to (Chiappa and Isaac, 2018), we use a Causal Bayesian network (Pearl, 2009) to represent the procedure generating the dataset and use the criterion that the presence of unfair edges results in unfairness in the decisions made.

The novelty in our work is the design of a linear model for conditional probability \(P(X|Pa(X))\) using \(d\)-connected concept in graphical models (Pearl, 2009; Koller and Friedman, 2009) that aids in quantifying edge unfairness. The model decomposes the conditional probability distribution \(P(X|Pa(X))\) of the destination node \(X\) of an unfair edge into beliefs along fair and unfair edges from the parents of \(X\) \((Pa(X))\) to \(X\). By using a formulation of cumulative unfairness similar to the measure of discrimination used in Zhang et al, 2017 (Zhang et al., 2017), we mathematically justify that when there is no edge unfairness across all edges, there is no cumulative unfairness towards any subset of sensitive attributes in any decision made. Our work also evaluates the impact of edge unfairness on cumulative unfairness that helps in deciding which set of unfair edges has to potential to amplify cumulative unfairness. Using the measure of impact and edge unfairness we assign priorities to unfair edges and propose a priority algorithm which the policymakers can use and decide where the underlying unfairness needs to be mitigated. We also propose a discrimination removal algorithm that sets edge unfairness in all unfair edges to zero and generates a fair distribution that is free from any discrimination. Not framing the discrimination removal procedure as an optimization problem, contrary to (Zhang et al., 2017), eliminates the problem of exponentially growing constraints in the number of sensitive attributes and their values.

This paper is organized as follows. In section 1, we introduce to the need to quantify edge unfairness, cumulative unfairness, and prioritization of the unfair edges. In section 2, we discuss the related work. In section 3, we introduce to the preliminaries from the fairness and causal inference literature. In section 4, we model the conditional probability \(P(X|Pa(X))\) that aids in quantifying edge unfairness. In section 5, we define and quantify edge and cumulative unfairness. We prove a theorem that eliminating edge unfairness across all unfair edges approximately eliminates the cumulative unfairness. In section 6, we discuss an algorithm to remove discrimination towards any subset of sensitive attributes in any decision without framing it as an optimization problem. We also discuss an algorithm that prioritizes unfair edges based on their edge unfairness and their potential to amplify cumulative unfairness. In section 7, we discuss experiments to validate the model assumption of the conditional probability \(P(X|Pa(X))\). In section 8, we discuss conclusions and future work.

2. RELATED WORK

(Gebu et al., 2018) suggests documenting the dataset by recording the motivation and creation procedure to mitigate the unfairness in the data generation phase. However, it does not discuss how to identify and remove discrimination in the existing dataset. (Zhang et al., 2017) uses a causal approach of path-specific effects (Pearl, 2013; Avin et al., 2005; Maathuis et al., 2018) to identify direct and indirect discrimination and modify the data distribution to eliminate the discrimination in the new distribution. (Nabi and Shpitser, 2018) addresses the limitations in (Zhang et al., 2017) of not solving the problem in a continuous variable setting, not providing concrete techniques to handle non-identifiability and not being able to decide the outcome of future samples that come from an unfair distribution as the discrimination removal procedure in (Zhang et al., 2017) modifies the original distribution and not future samples.

Both (Zhang et al., 2017) and (Nabi and Shpitser, 2018) do not discuss how to mitigate the unfairness in the data generation phase but instead propose methods to identify and remove discrimination in the dataset after it has been generated. (Zhang et al., 2017) discrimination discovery procedure does not have much utility in removing discrimination in the data generating phase, because it does not help in identifying the unfair edges that cause discrimination. A claim that the bail rejection decision \(J = \tilde{g}\) is unfair towards the African Americans \(R = \tilde{a}\) due to unfairness underlying the admission process \(E\) made based on gender \(G\) is a much stronger statement as compared to the claim that the bail rejection \(J = \tilde{g}\) is discriminative towards the African Americans \(R = a\). The former statement points to a particular cause that results in discrimination, whereas the latter just claims that discrimination exists. Using the former statement, one can address the practical problems associated with the cause, while the latter only raises an alarm that the discrimination exists without aiding in taking tangible steps that mitigate the discrimination in the data generation phase. In our work, we do not intend to make cautionary claims that the discrimination exists, be it...
direct and indirect discrimination as in Zhang et al., 2017 (Zhang et al., 2017), but aim to prioritize the unfair edges based on how discriminatory it is towards a subset of sensitive attributes concerning a particular decision. (Zhang et al., 2017) discrimination removal method enforces constraints of no direct and indirect discrimination towards all the subsets of sensitive variables and their specific values in every decision, thereby resulting in exponentially growing constraints in the number of sensitive attributes and their values.

Unlike (Zhang et al., 2017), in which every direct edge from the sensitive attribute and every indirect path from the sensitive attribute containing redlining attributes is unfair, we consider the presence of an unfair edge as the determining factor for the path to be unfair same as (Chiappa and Isaac, 2018). It is reasonable to consider the presence of an unfair edge as an indicator of whether a path is unfair as opposed to the presence of a redlining attribute. For instance, if Education $E$ is a redlining attribute, all paths from $G$ via $E$ is unfair. Yet, denying admissions based on a particular gender can be fair, if, only gender-specific institutions exist in the locality. Hence, a subjectively specified unfair edge truly captures the underlying unfairness than the presence of a redlining attribute. (Srinivas, 1993; Kim and Pearl, 1983; Henrion, 2013) separate the independent contributions from each of the parents onto the child by using unobserved variables in the explicit representation of causal independence (see Figure 2 in (Heckerman, 1993)). Although this representation has the advantage of only requiring linear probability assessments, it leads to intractable inference and difficulty in probability assessments due to unobserved variables. To overcome the issues of intractability and unobserved variables, (Heckerman, 1993) proposed a temporal definition of causal independence that if cause $c$ makes a transition from one instance to another between $t$ to $t + 1$ with other causes not making a transition, then the distribution of the effect at time $t + 1$ depends only on the effect at effect time $t$, cause at time $t$ and cause at time $t + 1$. Based on this definition, (Heckerman, 1993) gives a belief network representation of causal independence with only observed variables. This representation makes the probability assessment and inference tractable. (Heckerman and Breese, 1994) proposes an atemporal equivalent to the temporal definition of (Heckerman, 1993) in which a set of causes is independent of the effect if and only if all the temporal orderings are decomposable using the same nested function. However, the focus in any of the aforementioned works is to make inference and probability assessments tractable and not to quantify the individual dependency from each of the parents onto their child as in our work.

### 3. PRELIMINARIES

Throughout the paper, we use a bold uppercase letter $X$ to denote a set of attributes; an uppercase letter $X$ to denote a single attribute; a bold lowercase letter $x$ to denote a specific value taken by $X$; a lowercase letter $x$ to denote a specific value taken by its corresponding attribute $X$. $x_a$ restricts the values of $x$ to variable set $A$. For a node $X$, its parents are denoted by $Pa(X)$ and the specific values taken by them are denoted by $pa(X)$. Each node is associated with a conditional probability table (CPT), i.e., $\mathbb{P}(X|Pa(X))$.

#### 3.1. Node Intervention: Do-Operator and Potential Outcome (PO) Framework

Node intervention refers to forcibly setting $X$ to a constant $x$. Pearl, 2009 (Pearl, 2009) denoted node intervention using $do(X = x)$. Neyman and Rubin (Hernan and Robins, 2010; Imbens and Rubin, 2015) denoted node intervention using a Potential Outcome (PO) $Y_{xM}$ that denotes the response of each variable $Y$ in $Y$ to node intervention $X = x$. Potential outcomes (PO) can be single world or cross world (Steen and Vansteelandt, 2018). In a single world $PO$, value of the intervened variable is same across different random variables. For instance, $Y(x, M(x))$ is single world PO read as “outcome $Y$ if $X$ was set to $x$ and $M$ was set to whatever value it would have obtained had $X$ been set to the same value $x$”. In a cross world $PO$, the value of the intervened variable can vary across different random variables. For instance, $Y(x, M(x'))$ is cross world PO read as “outcome $Y$ if $X$ was set to $x$ and $M$ was set to whatever value it would have obtained had $X$ been set to a different value $x'"$. $PO$ framework is widely used in path specific analysis which is discussed later in Section 3.5.

**Definition 1** Node interventional distribution denoted by $\mathbb{P}(Y^{xM}) = \mathbb{P}(Y^{do(X = x)})$ is the distribution of $Y$ after forcibly setting $X$ to $x$. $\mathbb{P}_+ \text{ is the set of all node interventional distributions } \mathbb{P}(Y^{xM})$.

**Definition 2** A directed acyclic graph $G(E, V)$ (DAG) with observed variables $V$ is a Causal Bayesian Network (CBN) compatible with $\mathbb{P}_+$ [Definition 3.1.1 (Pearl, 2009), (Tian, 2003)] iff.

1. $\forall \mathbb{P}(V^{do(X=x)}) \in \mathbb{P}_+, \mathbb{P}(V^{do(X=x)})$ is Markov relative to $G_X$ which means it factorizes over $G_X$ when $V$ is consistent with $X$ i.e., $\mathbb{P}(V^{do(X=x)}) = \prod_{v \in V \setminus X} \mathbb{P}(v|pa(V)).\delta_{xv}$ \hspace{1cm} (1)
2. $\mathbb{P}(v|pa(V), do(X=x)) = 1, \forall V \notin X$ [when $v$ is consistent with $x$]
3. $\mathbb{P}(v|pa(V), do(X=x)) = \mathbb{P}(v|pa(V)), \forall V \notin X$ [when $pa(V)$ is consistent with $x$]

$DAG \ G$ represents the procedure for generating the dataset comprising of observed variables $V$ with each $V$ generated
using \(Pa(V)\). We assume that \(P(V) > 0\) throughout the paper to avoid zero probability of node interventional distribution, path-specific nested counterfactual distribution (introduced later), etc. helping in comparing the effect of different interventions.

**Remark 1** \(G_X\) is \(G\) with incoming edges of \(X\) removed.

### 3.2. Identification

A node interventional distribution \(P(Y|do(X = x))\) is said to be **identifiable** if it can be expressed using observational probability \(P(V)\). Although, a more formal definition for identification is in (Pearl, 2009), the aforementioned notion of identifiability is widely used in causal inference literature. For instance, when CBN comprises of only observed variables, node interventional distribution of a set of attributes \(Y\) i.e., \(P(Y = y|do(X = x))\) is identified as,

\[
P(Y = y|do(X = x)) = \sum_{\forall v \in V \setminus (X,Y)} \prod_{v \in \{X,Y\}} P(v|pa(V)), \delta_{x=y}
\]

(2)

(Tian, 2003) discusses techniques for identifying interventional distributions when unobserved variables are present. We do not elaborate it here, because our work assumes that all variables are observed.

### 3.3. D-Separation

**Definition 3** A trail \(V_1 \rightarrow \ldots \rightarrow V_n\) is said to be an **active trail** given a set of variables \(X\) in \(G\) if for every \(v\)-structure \(V_i \rightarrow V_j \leftarrow V_k\) along the trail, \(V_j\) or any descendent of \(V_j\) is in \(X\) and no other node in the trail belongs to \(X\).

**Definition 4** \(A\) is said to be **\(d\)-separated** from \(B\) given \(C\) in a graph \(G\) (\(d-sep_G(A; B|C)\)) if there is no active trail from any \(A \in A\) to any \(B \in B\) (Pearl, 2009), (Koller and Friedman, 2009). If there is at least one active trail from any \(A \in A\) to any \(B \in B\), then \(A\) is said to be **\(d\)-connected** from \(B\) given \(C\) in a graph \(G\) (\(d-con_G(A; B|C)\)) (see Fig. 1.3 (Pearl, 2009)).

### 3.4. Probabilistic Implications of D-separation

**Theorem 1** If sets \(X\) and \(Y\) are \(d\)-separated by \(Z\) in a DAG \(G(E,V)\), then \(X\) is independent of \(Y\) conditional on \(Z\) in every distribution \(P\) that factorizes over \(G\). Conversely, if \(X\) and \(Y\) are not \(d\)-separated by \(Z\) in a DAG \(G\), then \(X\) and \(Y\) are dependent conditional on \(Z\) in at least one distribution \(P\) that factorizes over \(G\) (Theorem 1.2.4 in (Pearl, 2009)).

**Corollary 1** If \((X \not\perp Y|Z)_{P}\) in at least one distribution \(P\) that factorizes over \(G\), then \(d-con_G(X; Y|Z)\). This is the contrapositive of the statement stated in Theorem 1.

### 3.5. Path-Specific Effect

#### 3.5.1. Direct and Indirect Effect

Consider a variable \(X\) and a decision variable \(Y\). Causal effect along the direct path \((X \rightarrow Y)\) is known as Direct effect and along indirect paths \((X \rightarrow \ldots \rightarrow Y)\) is known as Indirect effect (Pearl, 2013). Consider one direct path \(X \rightarrow Y\) and one indirect path \(X \rightarrow M \rightarrow Y\). Now, natural direct effect (NDE) is the effect of path intervening \(X = x\) along the direct path \(X \rightarrow Y\) and \(X = x'\) along the indirect causal paths \(X \rightarrow M \rightarrow Y\) as compared to the effect of path intervening \(X = x'\) along all causal paths. Natural indirect effect (NIE) is the effect of path intervening \(X = x\) along all causal paths as compared to the effect of path intervening \(X = x'\) along all causal paths.

\[
NDE = \mathbb{E}[Y(x, M(x'))] - \mathbb{E}[Y(x')]
\]

(3)

\[
NIE = \mathbb{E}[Y(x)] - \mathbb{E}[Y(x, M(x'))]
\]

(4)

#### 3.5.2. Path Specific Effects: Generalization of Direct and Indirect Effects

Direct and Indirect effects can be generalized into effects along paths of interest \(\pi\) and effect along paths not of interest \(\tilde{\pi}\) respectively (Avin et al., 2005). Using the same intuition as in Section 3.5.1, effect along \(\pi\) a.k.a path-specific effect \(PSE(\pi; X, x')\) is quantified by comparing the effect of intervening \(X = x\) along paths in \(\pi\) and \(X = x'\) along the paths \(\tilde{\pi}\) with the effect of intervening \(X = x'\) along all the causal paths. To mathematically express this statement, we use path-specific nested counterfactual \(Y(\pi; x, x')\) that is a result of intervening \(X = x\) along \(\pi\) and \(X = x'\) along \(\tilde{\pi}\).

**Definition 5** Path-specific nested counterfactual \(Y(\pi; x, x')\) is recursively defined as (Maathuis et al., 2018),

\[
Y(\pi; x, x') = \begin{cases} x, \text{when } Y = X \\ Y(\{W(\pi; x, x')\}, \{W(x')\}, Pa^\pi(Y)), \text{ o.w.} \end{cases}
\]

(7)

where \(Pa^\pi(Y)\) is the set of parents of \(Y\) along an edge which is part of a path in \(\pi\) and \(Pa^\tilde{\pi}(Y)\) is the set of all other parents of \(Y\).
If $Y$ is a variable in $X$, say $X$, then it takes the value $x$ to which $X$ is forcibly set to. If $Y$ is not a variable in $X$, then it is written in terms of its parents each of which is a path-specific nested counterfactual variable. If the parent of $Y$, say $W$, belongs to $Pa^{\pi}(Y)$, then the edge $W \to Y$ does not belong to any path in $\pi$ (see Definition 5) due to which any path starting from $X$ and ending in $W \to Y$ does not belong to $\pi$. Hence, $W$ is measured by intervening $X$ to $x'$ as $x'$ is the value forcibly set along paths $\bar{\pi}$ that are not of interest.

Path specific effect $PSE(\pi, x, x')$ can be expressed on additive and multiplicative scales (Maathuis et al., 2018) as shown below,

**Additive Scale**

$$PSE(\pi, x, x') = E[Y(\pi, x, x')] - E[Y(x')]$$

(8)

**Multiplicative Scale**

$$PSE(\pi, x, x') = \frac{E[Y(\pi, x, x')]}{E[Y(x')]}$$

(9)

**Definition 6** Path Specific Effect can be sensitive to the value taken by the decision variable $Y = y$. $PSE_{Y=y}(\pi, x, x')$ is the effect of observing $Y = y$ after intervening $X=x$ along the paths in $\pi$ and $X=x'$ along the other paths as compared to intervening $X=x'$ along all the causal paths from $X$.

**Additive Scale**

$$PSE_{Y=y}(\pi, x, x') = P[Y(\pi, x, x') = y] - P[Y(x') = y]$$

(10)

**Multiplicative Scale**

$$PSE_{Y=y}(\pi, x, x') = \frac{P[Y(\pi, x, x') = y]}{P[Y(x') = y]}$$

(11)

### 3.5.3. Identification of Path-Specific Effect

**Definition 7** A child $L$ of treatment $A$ is called a recanting witness for $\pi$ (and also by symmetry for $\bar{\pi}$) if there exists a directed path in $\pi$ of the form $A \to L \to \ldots \to Y$ and another directed path in $\bar{\pi}$ of the form $A \to L \to \ldots \to Y$.

**Definition 8 Recanting Witness Criterion:** Consider a Causal Bayesian Network $\mathcal{G}$ with observed variables compatible with $P_*$. If and only if there is no recanting witness for $\pi$ in the Causal Bayesian Network $\mathcal{G}$, the distribution of the corresponding path-specific nested counterfactual $Y(\pi, x, x')$ is identifiable from the observational data where $P(Y(\pi, x, x'))$ is,

$$\sum_{v \cap (x \cup y) \backslash y \in \chi} \prod_{v} P[V_{P_a^{\pi}(V)} \cap X, x'_{P_a^{\bar{\pi}}(V) \cap X}, P_a(V) \backslash X]$$

(12)

Note that when the recanting witness criterion is satisfied, the intersection of $P a^{\pi}(V) \cap X$ and $P a^{\bar{\pi}}(V) \cap X$ would be empty. This helps in unambiguous assignment of the variables in $P a^{\pi}(V) \cap X$ to $x$ and $P a^{\bar{\pi}}(V) \cap X$ to $x'$. Eq. (12) is known as the edge g-formula (Avin et al., 2005), (Maathuis et al., 2018).

### 3.6. Fairness in Causal Bayesian Networks

**Definition 9** Sensitive node $(S)$ is a node in the causal graph $\mathcal{G}$ that can cause discrimination when used to generate data of any of its children. Set of sensitive nodes in $\mathcal{G}$ are denoted by $S_\mathcal{G}$ (marked by red nodes in $\mathcal{G}$)

Sensitive nodes can be attributes of social relevance like race, gender, etc., or other attributes like hair length, height, etc., using which if one generates data of any of its children in $\mathcal{G}$ it can result in discrimination. We make the same assumption as (Zhang et al., 2017) that the sensitive node $S$ has no parents as it is an inherent nature of the individual.

**Definition 10** Unfair edge is a directed edge $S \to X$ in the causal graph $\mathcal{G}$ where $S$ is a sensitive node that causes discrimination when it is used to generate $X$. Set of unfair edges in $\mathcal{G}$ is denoted by $E_\mathcal{G}^{unfair}$.

For instance, in Figure 1, edge $G \to E$ qualifies to be unfair because gender $G$ is a sensitive variable. This edge is unfair if the accused is denied admission based on gender. It is fair if the accused is denied admission because only gender-specific institutions exist in the locality from which the dataset is drawn. Hence, subjective analysis is required to assess the real-world issues and mark an edge whether it is unfair or fair. (Chiappa and Isaac, 2018).

**Definition 11** Unfair paths $(\pi_{S,Y,G}^{unfair})$ are the set of directed paths from a sensitive node $S \in S \subseteq S_\mathcal{G}$ to a decision variable $Y$ in graph $\mathcal{G}$ s.t. at least one unfair edge is present in the directed path.

Since we make the same assumption as (Zhang et al., 2017) that the sensitive node $S$ has no parents as it is an inherent nature of the individual, unfair paths from a sensitive node $S \in S \subseteq S_\mathcal{G}$ to $Y$ are unfair only if the edge from $S$ is unfair. This is because there are no other sensitive nodes in the directed path from $S$ to $Y$ due to the aforementioned assumption.

**Remark 2** $\pi_{S,Y,G}^{unfair} = \{S \to Y | S \in S, S \to Y \in E_\mathcal{G}^{unfair} \} \cup \{S \to A \to \ldots \to Y | S \in S, S \to A \in E_\mathcal{G}^{unfair} \}$ in $\mathcal{G}$.

Unfair paths capture how unfairness propagates from the sensitive nodes onto a destination node. For instance, in Figure 1, $\pi_{G,E,J,G}^{unfair}$ consists of $G \to E \to J$ because gender $G$ is a sensitive attribute and $G \to E$ is unfair. It captures how unfairness in the edge $G \to E$ propagates to $J$. 
4. Modeling the Conditional Probability
\[ \mathbb{P}(X | Pa(X)) \]

In this section, we model the conditional probability distribution \( \mathbb{P}(X | Pa(X)) \) using a function that takes the belief of \( X \) shaped by the dependencies from each of its parents as inputs.

The following lemma helps in visualizing the dependencies in \( \mathbb{P}(X | do(M)) \) along the directed paths that start from any node in \( M \) and end in \( X \) in the causal graph \( \mathcal{G} \).

**Lemma 1** Let \( \mathbb{P}_M \) be a distribution that factorizes over graph \( \mathcal{G}_M \) and let \( D^P_{X|M} = \{(X \not\perp | M; \mathcal{M})_P \mid M \in \mathcal{M}\} \) be the set of possible dependencies in \( \mathbb{P}(X | do(M)) \). If \( (X \not\perp M; \mathcal{M})_P \in D^P_{X|M} \), then \( d\text{-conn}_{\mathcal{G}_M}(X; M; \mathcal{M}) \).

In other words, the active trail between \( X \) and \( M \) given \( M \) in \( \mathcal{G}_M \) is a consequence of the dependency of \( X \) and \( M \) given \( M \) in \( \mathbb{P}_M \).

**Proof:**
When \( \mathbb{P} \) factorizes over \( \mathcal{G} \) and \( \mathbb{P}_M \) factorizes over \( \mathcal{G}_M \),
\begin{align*}
(X \not\perp M; \mathcal{M})_P \in D_X \Rightarrow d\text{-conn}_{\mathcal{G}}(X; M; \mathcal{M}) \Rightarrow (X \not\perp M; \mathcal{M})_P \in D_M \Rightarrow d\text{-conn}_{\mathcal{G}_M}(X; M; \mathcal{M})
\end{align*}

[Corollary 1]
To illustrate the intuition of Lemma 1, consider Figure 3. Let the set of possible dependencies in \( \mathbb{P}(J,R,E,C,T,G) \) be \( D^P_{J,R,E,C,T,G} = \{(J \not\perp R; E,C,T,G)_P, (J \not\perp E; R,C,T,G)_P, (J \not\perp G; R,E,C,T)_P\} \). Each of the dependency in \( D^P_{J,R,E,C,T,G} \) is \( d\text{-connected} \) in \( \mathcal{G} \) when \( \mathbb{P} \) factorizes over \( \mathcal{G} \). \( d\text{-connected} \) paths are the active trails \( R \rightarrow J, E \rightarrow J, C \rightarrow J, T \rightarrow J \) and \( G \rightarrow J \) (Figure 3(a)).

\[ |D^P_{X|R}| = \{|X \not\perp R; \mathcal{M}\}_P \} \]

The dependency in \( D^P_{X|R} \) is \( d\text{-connected} \) in \( \mathcal{G}_R \) when \( \mathbb{P}_R \) factorizes over \( \mathcal{G}_R \). \( d\text{-connected} \) paths are the active trails \( R \rightarrow J, R \rightarrow E \rightarrow J, R \rightarrow T \rightarrow J \) and \( R \rightarrow E \rightarrow T \rightarrow J \) (Figure 3(b)).

**Figure 2.** \( d\text{-connected} \) paths (dotted lines)

**Definition 12** Scaling factor \( S^P_{M \rightarrow X} \) is defined as,
\[ S^P_{M \rightarrow X} = \sum_{m \in \mathcal{M}} \sum_{m' \in \mathcal{M}} \mathbb{P}(X | do(M)) \]

\[ S^P_{M \rightarrow X} = \sum_{x \in X} \sum_{m \in \mathcal{M}} PSE_{X=x'}(\pi_{\text{direct}}, \mathcal{M}, m^\prime)
\]

where \( \pi_{\text{direct}} = \{ M \rightarrow X | M \in \mathcal{M} \} \)

Scaling factor \( S^P_{M \rightarrow X} \) measures the impact of forcibly setting \( M \) to \( m \) along the direct edges from any node in \( M \) to \( X \) regardless of what value \( M \) is forcibly set along the paths that are not direct edges. It is quantified by how probable it is to observe the outcome \( X = x \) when \( M \) is forcibly set to \( m \) along the direct edges \( \{ M \rightarrow X | M \in \mathcal{M} \} \) and to a different value \( m' \) along other paths (measured by the path specific nested counterfactual \( \mathbb{P}(X | \pi_{\text{direct}}, m', m^\prime) \)) as compared to forcibly setting \( M \) to \( m' \) along all the causal paths (measured by the path specific nested counterfactual \( \mathbb{P}(X | \pi_{\text{direct}}, m', m^\prime) \)). The above quantity is averaged across all the different values of \( m' \) to ensure that the impact of forcibly setting \( M \) to \( m \) along the direct paths is indifferent to the value that is forcibly set along the other paths that are not direct edges. Suppose, \( X \) denotes the judicial bail decision; \( J, x \) denotes that the bail was granted \( g \); \( M \) denotes the race \( R \); and \( m \) denotes that the race is African American \( a \). Then, \( S^P_{R \rightarrow a \rightarrow J = g} \) measures the impact of forcibly setting race \( R \) to African American \( a \) along the direct edge \( R \rightarrow J \) resulting in granting of the bail \( g \) irrespective of what value race \( R \) is forcibly set along the other causal paths.

**Properties of the Scaling Factor**

1. \( S^P_{M \rightarrow X} \geq 0 \): Multiplicative measure used in the calculation of \( \mathbb{P}(X | do(M)) \) in \( S^P_{M \rightarrow X} \) ensures that the scaling factor is non-negative which need not be the case when additive measure is used.

2. \( S^P_{M \rightarrow X} \leq 1 \): The normalization factor in the denominator of \( S^P_{M \rightarrow X} \) ensures that the scaling factor is bounded between 0 and 1.

The following theorem helps in visualizing the dependencies in \( S^P_{M \rightarrow X} \mathbb{P}(X | do(M)) \) along the direct edges \( \{ M \rightarrow X | M \in \mathcal{M} \} \) in the causal graph \( \mathcal{G} \).

**Theorem 2** Let \( \mathbb{P}_M \) be a distribution that factorizes over graph \( \mathcal{G}_M \). Heuristically, the set of direct paths \( \pi_{\text{direct}} = \{ M \rightarrow X | M \in \mathcal{M} \} \) in \( \mathcal{G}_M \) are the active trails that are consequences of the set of possible dependencies in \( S^P_{M \rightarrow X} \mathbb{P}(X | do(M)) \).

**Proof:**
We give an informal proof and experimentally validate it later (see Section 7). The set of causal paths \( \{ d\text{-conn}_{\mathcal{G}_M}(X; M; \mathcal{M}) \} \) are the active trails that are consequences of the possible dependencies in \( \mathbb{P}(X | do(M)) \) i.e., \( D^P_{X|M} = \{(X \not\perp M; \mathcal{M})_P \mid M \in \mathcal{M} \} \) [Lemma 1]. Since the scaling factor \( S^P_{M \rightarrow X} \) measures the impact on the outcome \( X \) when \( M \) is forcibly set to
a certain value along the direct paths \( \{M \rightarrow X | M \in M\} \) [Definition 12], \( S^p_{M \rightarrow X} \mathbb{P}(X|do(M)) \) scales down the probability of observing \( X \) due to the dependencies in \( \mathbb{P}(X|do(M)) \) so as to be influenced only along the direct edges \( \{M \rightarrow X | M \in M\} \). Hence, heuristically, the direct paths \( \{M \rightarrow X | M \in M\} \) are the active trails that are consequences of the dependencies in \( S^p_{M \rightarrow X} \mathbb{P}(X|do(M)) \).

\[ \text{Figure 3. Active trail } R \rightarrow J \text{ corresponding to the dependencies in } S^p_{M \rightarrow X} \mathbb{P}(X|do(M)) \text{ with the active trails } \{R \rightarrow E \rightarrow J, R \rightarrow E \rightarrow T \rightarrow J, R \rightarrow T \rightarrow J\} \text{ obscured by the scaling factor } S^p_{M \rightarrow X}. \]

The following theorem models the conditional probability distribution \( \mathbb{P}(X|Pa(X)) \) using a function that inputs the beliefs of \( X \) that can be visualized along the fair paths and each of the unfair paths. Note that the beliefs along the unfair paths are separately fed as input which will later aid in quantifying edge unfairness.

**Theorem 3** Let \( \mathcal{G} \) be a Causal Bayesian Network compatible with \( \mathbb{P}s \). Then,
\[
\mathbb{P}(X|Pa(X)) = \sum_{A \subseteq U_X} \left( \mathbb{P}(X|Pa(X), \mathbf{T}) \right) = \frac{1}{\mathbb{P}(X|do(F_X))} \sum_{A \subseteq U_X} \left( \frac{\mathbb{P}(X|Pa(X), \mathbf{T})}{\mathbb{P}(X|Pa(X))} \right) = \frac{1}{\mathbb{P}(X|do(F_X))} \sum_{A \subseteq U_X} \left( \frac{\mathbb{P}(X|Pa(X), \mathbf{T})}{\mathbb{P}(X|Pa(X))} \right)
\]
where \( U_X \) is the subset of \( Pa(X) \) that are along an unfair edge and \( F_X \) is the subset of \( Pa(X) \) that are along a fair edge given by:
\[
U_X = \{A | A \rightarrow X \in \mathcal{G}_{\text{unfair}} \}
\]
\[
F_X = \{A | A \rightarrow X \notin \mathcal{G}_{\text{unfair}} \}
\]

**Proof:** The set of active trails \( \mathbf{T} \) is \( \text{d-conn}_\mathcal{G}(X; M|Pa(X) \setminus M| Pa(X)) \) i.e. \( \{X \perp M|Pa(X) \setminus M| Pa(X)\} \) [Lemma 1]. The set of active trails \( \mathbf{T}_F \) is \( \text{d-conn}_\mathcal{G}(X; M|Pa(X) \setminus M| Pa(X)) \) i.e. \( \{X \perp M|Pa(X) \setminus M| Pa(X)\} \) [Theorem 1]. The active trail \( A \rightarrow X \) is the consequence of the possible dependency in \( S^p_{A \rightarrow X} \mathbb{P}(X|do(A)) \) and set of active trails \( \mathbf{T}_U = \{A \rightarrow X | A \in U_X\} \) are the consequences of the possible dependencies in \( \bigcup_{A \subseteq U_X} S^p_{A \rightarrow X} \mathbb{P}(X|do(A)) \) [Theorem 1].

4.1. Inputs to \( f \)

Inputs to \( f \) are \( S^p_{F_X \rightarrow X} \mathbb{P}(X|do(F_X)) \) and \( \bigcup_{A \subseteq U_X} S^p_{A \rightarrow X} \mathbb{P}(X|do(A)) \). The possible dependencies in \( S^p_{F_X \rightarrow X} \mathbb{P}(X|do(F_X)) \) are \( D_{F_X} = \{(M \perp X|Pa(X) \setminus M)| M \in F_X \} \) and the possible dependencies in \( \bigcup_{A \subseteq U_X} S^p_{A \rightarrow X} \mathbb{P}(X|do(A)) \) are \( D_{U_X} = \{(M \perp X|Pa(X) \setminus M)| M \in U_X \} \).

Since \( D_{F_X} \bigcup D_{U_X} \) are the possible dependencies in \( \mathbb{P}(X|Pa(X)) \), we can view \( S^p_{F_X \rightarrow X} \mathbb{P}(X|do(F_X)) \) and \( \bigcup_{A \subseteq U_X} S^p_{A \rightarrow X} \mathbb{P}(X|do(A)) \) contributing a fraction of their
we present two instances for \( f \) capturing how these fraction of beliefs interact to produce \( P(X|Pa(X)) \).

### 4.2. Choices for \( f \) and Constraints on the Inputs to \( f \)

We present two instances for \( f \). The list is not limited to these and can be extended as long as \( f \) satisfies the constraints of the conditional probability (Eq. 17, 18).

1. \( f \) is a linear combination in the inputs i.e. \( P(X|Pa(X)) \approx \)
   
   \[
   f(S^F_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^{p}_{A \rightarrow X}P(X|do(A))) = \]
   
   \[
w_{X \rightarrow X} S^F_{X \rightarrow X}P(X|do(F_X)) + \sum_{A \in U_X} w_{A \rightarrow X} S^F_{A \rightarrow X}P(X|do(A)) \]
   
   s.t., \( 0 \leq w_{X \rightarrow X}, w_{A \rightarrow X} \leq 1, \forall A \in U_X \)
   
   \[
w_{X \rightarrow X} + \sum_{A \in U_X} w_{A \rightarrow X} = 1 \]

The weight of the belief inputs \( w_{X \rightarrow X} \) and \( w_{A \rightarrow X} \) are constrained between 0 and 1 since the objective of the mapper \( f \) is to capture the interaction between the fraction of the beliefs given by \( w_{X \rightarrow X} S^F_{X \rightarrow X}P(X|do(F_X)) \) and \( \bigcup_{A \in U_X} S^{p}_{A \rightarrow X}P(X|do(A)) \) and approximate \( P(X|Pa(X)) \).

2. \( f^W = f^W_N \circ \ldots \circ f^W_1 \) is composite function representing a N-layer neural network with \( i^{th} \) layer having \( M_i \) neurons and weights \( W_i \), capturing the non-linear combination of the inputs i.e. \( P(X|Pa(X)) \approx \)

   \[
f_N(\ldots f_1(S^F_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^{p}_{A \rightarrow X}P(X|do(A)))) = \]

   \[
s.t., 0 \leq W_1 \leq 1, \]
   
   \[
f_N : \mathbb{R}^{M_N} \rightarrow [0, 1]|X| \]
   
   \[
   \sum_X f(S^F_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^{p}_{A \rightarrow X}P(X|do(A))) = 1 \]

The weights of the belief inputs \( W_1 \) are constrained between 0 and 1 since the objective of the mapper \( f \) is to capture the interaction between the fraction of the beliefs given by \( W_1[S^F_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^{p}_{A \rightarrow X}P(X|do(A))] \) and approximate \( P(X|Pa(X)) \). Eq. (26) and Eq. (27) ensure that the conditional probability axioms of \( f \) are satisfied. One possibility is to use a softmax function for \( f_N \) to ensure that the outputs of \( f \) satisfy probability axioms.

### 5. Edge and Cumulative Unfairness, Discrimination Discovery in a Linear \( f \)

In this section, we use the model formulated in the previous section to quantify edge unfairness, quantify cumulative unfairness towards a subset of sensitive attributes with respect to a particular decision, and prove that eliminating edge unfairness along all unfair edges results in eliminating cumulative unfairness in any decision towards any subset of sensitive attributes.

If the belief of \( X \) given its parents \( Pa(X) \) is largely shaped by the belief of \( X \) given a sensitive node, say \( A \), then the belief of \( X \) given parents \( Pa(X) \) is indifferent to the other parents \( Pa(X) \setminus A \) resulting in unfairness. Edge unfairness is the fraction of belief of \( X \) given the sensitive node \( A \) along the unfair edge \( A \rightarrow X \) that contributes to the belief of \( X \) given its parents \( Pa(X) \). We use \( S^p_{A \rightarrow X}P(X|do(A)) \) as a proxy that measures the belief of \( X \) given the sensitive node \( A \) along the unfair edge \( A \rightarrow X \) as the active trail that is the consequence of the dependencies in \( S^A_{X \rightarrow X}P(X|do(A)) \) [see Theorem 2].

**Definition 13** Let \( G \) be a Causal Bayesian Network compatible with \( \mathbb{P}^* \). **Edge unfairness** \((U_A \rightarrow X, G)\) in an unfair edge \( (A \rightarrow X)_G \) is quantified as the fraction of \( S^p_{A \rightarrow X}P(X|do(A)) \) contributing to \( P(X|Pa(X)) \) in the mapping \( P(X|Pa(X)) \approx f(S^A_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^p_{A \rightarrow X}P(X|do(A))) \).

The fraction of \( S^p_{A \rightarrow X}P(X|do(A)) \) contributing to \( P(X|Pa(X)) \) in the mapping \( P(X|Pa(X)) \approx f(S^A_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^p_{A \rightarrow X}P(X|do(A))) \) decomposes nicely into its inputs when the mapping is a linear combination in the inputs as discussed below.

**Definition 14** Let \( G \) be a Causal Bayesian Network compatible with \( \mathbb{P}^* \) and \( f \) be a linear combination in the inputs with parameters \( 0 \leq w \leq 1 \). Then, \( w_{A \rightarrow X} \) is the **Edge unfairness** of the unfair edge \( (A \rightarrow X)_G \) as it is the fraction of \( S^A_{A \rightarrow X}P(X|do(A)) \) contributing to \( P(X|Pa(X)) \) in the mapping \( P(X|Pa(X)) \approx f(S^A_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^p_{A \rightarrow X}P(X|do(A))) \) where

\[
f(S^A_{X \rightarrow X}P(X|do(F_X)), \bigcup_{A \in U_X} S^p_{A \rightarrow X}P(X|do(A))) = \]

\[
w_{X \rightarrow X} S^F_{X \rightarrow X}P(X|do(F_X)) + \sum_{A \in U_X} w_{A \rightarrow X} S^p_{A \rightarrow X}P(X|do(A)) \]

s.t., \( 0 \leq w_{F \rightarrow X}, w_{A \rightarrow X} \leq 1, \forall A \in U_X \),

\[
w_{F \rightarrow X} + \sum_{A \in U_X} w_{A \rightarrow X} = 1 \]

See Section 4.1 and 4.2 for the rationale behind enforcing
the constraints for \( f \). \( w_{A \rightarrow X} \) is a property of the edge and does not vary across different settings of \( A \) and \( X \).

**Definition 15** Let \( G \) be a Causal Bayesian Network compatible with \( \mathbb{P}^* \). Then, **Cumulative unfairness** \( C_{y \rightarrow y, G}^{\text{P}} \) in the decision \( Y = y \) made towards the sensitive variables \( S \) is quantified as,

\[
C_{y \rightarrow y, G}^{\text{P}} = \frac{1}{|S|} \sum_{s \in S} PSE_{y \rightarrow y}^{\text{P}}(\pi_{S, Y, G}^{\text{unfair}}, s, s')
\]

where, \(|S|\) is the cardinality of \( S \) without \( s \), \( \pi_{S, Y, G}^{\text{unfair}} \) are the unfair paths in \( G \) (see Definition 11) and \( PSE_{y \rightarrow y}^{\text{P}}(\pi_{S, Y, G}^{\text{unfair}}, s, s') \) is the path-specific effect (see Definition 11).

The definition of Cumulative unfairness is similar to the Path-specific effect to identify discrimination in (Zhang et al., 2017), differing in the usage of multiplicative scale instead of additive scale, to ensure that the cumulative unfairness is a non-negative quantity. The non-negativity of cumulative unfairness helps in differentiating cumulative unfairness w.r.t. to edge unfairness without the usage of modulus operation. Modulus operation can complicate differentiation and interpretation of the result of differentiation.

\( C_{y \rightarrow y, G}^{\text{P}} \) measures the impact of forcibly setting \( S \) to the unfair paths from \( S \) to \( Y \) as compared to the setting when a different value was set along all the paths from \( S \) to \( Y \). It is measured by the probability of observing the outcome \( Y = y \) when \( S \) is forcibly set to \( s \) along the unfair paths from \( S \) to \( Y \) and to a different value along other paths as compared to forcibly setting \( S \) to a different value along all the paths from \( S \). For instance, in Figure 1, \( C_{R=a, G}^{\text{P}} \) measures how probable it is to observe that the bail is granted \( g \) when race \( R \) is forcibly set to African American \( a \) along the unfair paths \( \{ R \rightarrow J, R \rightarrow E \rightarrow J, R \rightarrow T \rightarrow J, R \rightarrow E \rightarrow T \rightarrow J \} \) and to a different value along the other paths as compared to forcibly setting race \( R \) to a different value along all paths from race \( R \).

**Proposition 1** Discrimination exists towards the sensitive attributes \( S \) while making decision \( Y = y \) in the Causal Bayesian Network \( G \) compatible with \( \mathbb{P}^* \) when \( |1 - C_{y \rightarrow y, G}^{\text{P}}| > \epsilon \) where \( C_{y \rightarrow y, G}^{\text{P}} \) is the cumulative unfairness and \( \epsilon \) is the threshold for discrimination.

**Proof:** If \( C_{y \rightarrow y, G}^{\text{P}} = 1 \), forcibly setting \( S \) to \( s \) along unfair paths from \( S \) to \( Y \) and to another value along other paths does not alter the probability of observing \( Y \) as compared to forcibly setting \( S \) to another value along all the paths, on average. In other words, \( C_{y \rightarrow y, G}^{\text{P}} = 1 \) shows that the decision \( Y = y \) is fair towards \( S \) as \( S \)'s is treated similarly to other sensitive groups \( S \)'s, on average, as far as decision \( Y = y \) is concerned. \( C_{y \rightarrow y, G}^{\text{P}} \neq 1 \) shows that the decision \( Y = y \) is either favourable to \( S \)'s \( (C_{y \rightarrow y, G}^{\text{P}} > 1) \) or not favourable to \( S \)'s \( (C_{y \rightarrow y, G}^{\text{P}} < 1) \), on average, as far as decision \( Y = y \) is concerned. An \( \epsilon \)-soft check for discrimination would be to replace \( C_{y \rightarrow y, G}^{\text{P}} = 1 \) with \( |1 - C_{y \rightarrow y, G}^{\text{P}}| \leq \epsilon \) and \( C_{y \rightarrow y, G}^{\text{P}} \neq 1 \) with \( |1 - C_{y \rightarrow y, G}^{\text{P}}| > \epsilon \) where \( \epsilon \) is the threshold for discrimination.

**Theorem 4** Cumulative unfairness \( C_{y \rightarrow y, G}^{\text{P}} \) towards a subset of sensitive variables \( S \) in a decision \( Y = y \) approaches to 1 (no discrimination) when edge unfairness \( U_{e \rightarrow G} = 0 \) in all unfair edges \( e \) from \( S \) and \( \mathbb{P}(X|Pa(X)) \approx f(S_{F \rightarrow X}^{P} \mathbb{P}(X|do(F_X))) \bigcup_{A \in U_X} S_{A \rightarrow X}^{P} \mathbb{P}(X|do(A)) \)

where \( f \) is a linear combination in the inputs with parameters \( 0 \leq w \leq 1 \).

**Proof:**

\[
\mathbb{P}(Y(\pi_{S, Y, G}^{\text{unfair}}, s, s')) = y
\]

\[
= \sum_{V \in (S \cup Y)} \prod_{V \in S} \mathbb{P}(V|S_{Pa \rightarrow (V) \cap S}, S_{Pa \rightarrow (V) \cap S}, Pa(V)\setminus S)
\]

[Eq 12 a.k.a Edge g-formula]

\[
\approx \sum_{V \in (S \cup Y)} \prod_{V \in S} [w_{F \rightarrow X} S_{F \rightarrow X}^{P}(s_{F \rightarrow X} \cap S_{F \rightarrow X}) \setminus V]
\]

\[
\mathbb{P}(V|do(s_{F \rightarrow X} \setminus S_{F \rightarrow X})) + \sum_{A \in U_{F \cap S}} w_{A \rightarrow V} S_{A \rightarrow V}^{P} \mathbb{P}(V|do(A))
\]

+ \sum_{A \in U_{F \cap S}} w_{A \rightarrow V} S_{A \rightarrow V}^{P} \mathbb{P}(V|do(A))

[\text{\textbf{f is a linear combination in inputs}]}

\[
\approx \sum_{V \in (S \cup Y)} \prod_{V \in S} [w_{F \rightarrow X} S_{F \rightarrow X}^{P}(s_{F \rightarrow X} \cap S_{F \rightarrow X}) \setminus V]
\]

\[
\mathbb{P}(V|do(s_{F \rightarrow X} \setminus S_{F \rightarrow X})) + \sum_{A \in U_{F \cap S}} w_{A \rightarrow V} S_{A \rightarrow V}^{P} \mathbb{P}(V|do(A))
\]

[Since \( U_{e \rightarrow G} = 0 \) in all unfair edges \( e \) from \( S \), \( U_{A \rightarrow V, G} = w_{A \rightarrow V} = 0 \), \( \forall A \in U_{V} \cap S \)]

\[
\mathbb{P}(Y(\pi_{S, Y, G}^{\text{unfair}}, s', s') = y) = \mathbb{P}(Y(\pi_{S, Y, G}^{\text{unfair}}, s, s') = y)
\]

[\text{\textbf{f is independent of s from Eq. 35]}]

\[
PSE_{y \rightarrow y}^{\text{P}}(\pi_{S, Y, G}^{\text{unfair}}, s, s') = 1
\]

\[
\mathbb{P}(Y(\pi_{S, Y, G}^{\text{unfair}}, s, s') = y) = \mathbb{P}(Y(\pi_{S, Y, G}^{\text{unfair}}, s', s') = y)
\]

\[
C_{y \rightarrow y, G}^{\text{P}} = \frac{1}{|S|} \sum_{s \in S} PSE_{y \rightarrow y}^{\text{P}}(\pi_{S, Y, G}^{\text{unfair}}, s, s') = 1
\]
A Causal Linear Model to Quantify Edge Unfairness for Unfair Edge Prioritization and Discrimination Removal

Remark 4 A path \( A \) is identifiable from \( x \) or is fair by varying edge unfairness \( S = s \) if it has the potential to amplify cumulative unfairness towards a subset of sensitive \( Y \) with parameters \( 0 \leq w \leq 1 \) and the decision variable, using this algorithm we can find the optimal weights \( w^* \) that represents edge unfairness assuming the model \( f_w \).

Corollary 2 Cumulative unfairness \( C_{S=y, Y}^{\text{unfair}} \) of any subset of sensitive \( S = s \) is identifiable from the observational distribution because there is no recanting witness \( L \) that belongs to both the paths \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \) and \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \). If \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \), then \( A \rightarrow L \) is unfair by the definition of unfair path and hence any other \( A \rightarrow L \rightarrow \ldots \rightarrow Y \) is also an unfair path and cannot be in \( \pi_{S,Y,G}^{\text{unfair}} \).

Corollary 2 points to the fact that eliminating edge unfairness in all unfair edges eliminates cumulative unfairness in any decision made towards any subset of sensitive attributes. But, it does not suggest how to remove discrimination.

Remark 3 \( P(Y = \pi_{S,Y,G}^{\text{unfair}}, X, x') \) is identifiable from the observational distribution because there is no recanting witness \( L \) that belongs to both the paths \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \) and \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \). If \( A \rightarrow L \rightarrow \ldots \rightarrow Y \in \pi_{S,Y,G}^{\text{unfair}} \), then \( A \rightarrow L \) is unfair by the definition of unfair path and hence any other \( A \rightarrow L \rightarrow \ldots \rightarrow Y \) is also an unfair path and cannot be in \( \pi_{S,Y,G}^{\text{unfair}} \).

Notation 1 Let, 
\[
P_{\text{new}}(V|Pa(V)) \approx \sum_{A \in U_V} w^p_{A \rightarrow V} S^p_{A \rightarrow V} \mathbb{P}(V|do(F_X))
\]
(39)
\[
\sum_{A \in U_V} w^p_{A \rightarrow V} S^p_{A \rightarrow V} \mathbb{P}(V|do(A))
\]
(40)

6. Discrimination Removal and Unfair Edges Prioritization in a Linear \( f \)

In this section, we first present the Algorithm 1 that is employed to compute the amount of Edge fairness present in the causal model \( G \). Given the CPTs \( P \), unfair edges \( E^\text{unfair}_G \) and the decision variable, using this algorithm we can find the optimal weights \( w^* \) that represents edge unfairness assuming the model \( f_w \).

Algorithm 1 getEdgeUnfairness(\( G, P, E^\text{unfair}_G, X \))

1. Initialize \( w \) randomly
   \[
   Y \leftarrow \mathbb{P}(X = x|Pa(X) = pa(x))
   \]
2. Compute \( S^p_{\in A \rightarrow \chi} \mathbb{P}(X|do(F_X)) \approx f(S^p_{\in A \rightarrow \chi} \mathbb{P}(X|do(A))) \)
   \[
   Y^w \leftarrow f^w(S^p_{\in A \rightarrow \chi} \mathbb{P}(X|do(F_X)), \bigcup_{A \in U_x} S^p_{\in A \rightarrow \chi} \mathbb{P}(X|do(A)))
   \]
   (Eq. 28)
3. \( w^* \leftarrow \arg \min_w \|Y - Y^w\|^2 \) s.t. Eq. 29 and Eq. 30
   Result: \( w^* \)

We also use the Algorithm 1 to define the following:

1. unfairEdgePriority() (Algorithm 2): It prioritizes the unfair edges based on their edge unfairness and potential to amplify the cumulative unfairness, thereby aiding the policymakers to select the set of unfair edges whose underlying issues need to be addressed.

Algorithm 2 unfairEdgePriority(\( G, P, E^\text{unfair}_G, s, y, w_s, w_u \))

\[
\begin{align*}
\text{for } V \text{ in } V \text{ do} & \quad \text{for } e \text{ in } E^\text{unfair}_G \text{ do} \\
& \quad U_{e,G} = w^*_e \\
& \quad S^p_{e=y} = \frac{\partial C^p_{S=y,Y} \mathbb{P}}{\partial w_e} \bigg|_{w_p} \\
& \quad \text{priority} = w_u U_{e,G} + w_s S^p_{e=y} \\
& \quad \text{priorityList} = \text{priorityList} \cup \{(e, \text{priority})\}
\end{align*}
\]
Result: \text{priorityList}

2. removeDiscrimination() (Algorithm 3): It removes discrimination by regenerating \( P_{\text{new}}(X|Pa(X)) \) from the linear model \( f \) after setting edge unfairness in the unfair edges to 0.
We now compare our approach of discrimination removal with the one proposed in (Zhang et al., 2017). It solves a quadratic programming problem to find a joint distribution that is close to the original distribution such that both direct and indirect discrimination are removed. The inequality constraints in the optimization problem are defined in terms of the path-specific effect that capture the discrimination towards the sensitive attribute $C = c^+$ in the decision $E = e$ as,

$$PSE_{E=e}^{direct}(\pi^{direct}, c^+, c^-) \leq \tau, \quad (42)$$

$$PSE_{E=e}^{indirect}(\pi^{indirect}, c^+, c^-) \leq \tau \quad (43)$$

where $\pi^{direct}$ are the direct paths from $C$ to $E$ and $\pi^{indirect}$ are the indirect path from $C$ to $E$ with $\tau$ being the threshold for discrimination. Eq. 42 and Eq. 43 capture the criterion for direct and indirect discrimination respectively.

Limitations to the Discrimination Removal Procedure (Zhang et al., 2017) assumes the number of sensitive attributes to be one which is a binary attribute and mentions that the methodology proposed can be extended to multiple sensitive attributes and multiple values taken by the sensitive attributes. But it fails to analyze the impact of adding more sensitive attributes on the performance of the optimization problem underlying the discrimination removal procedure.

Adding more sensitive attribute to the causal model increases the number of constraints exponentially in the number of sensitive attributes. Also, each of the constraint increases exponentially in the number of values taken by the sensitive attributes. For example, if we add Race ($R$) attribute, new set of constraints in the discrimination removal algorithm assuming binary attributes are,

$$PSE_{E=e}^{direct}(\pi^{direct}, r^+, r^-) \leq \tau, \quad (44)$$

$$PSE_{E=e}^{indirect}(\pi^{indirect}, c^+, c^-) \leq \tau \quad (45)$$

$$PSE_{E=e}(\pi^{direct}, \{r^+, c^+\}, \{r^-, c^-\}) \leq \tau$$

$$PSE_{E=e}(\pi^{indirect}, \{r^+, c^+\}, \{r^-, c^-\}) \leq \tau \quad (49)$$

Although a quadratic programming problem can be solved in polynomial time, due to the exponentially increasing constraints the time taken to solve the problem also increases. Also, the constraints are non-linear which makes it harder to obtain a feasible point and even maintaining feasibility.

Even approximating individual CPTs ($\mathbb{P}(E|Pa(E))$) instead of entire joint distribution in the objective function will not reduce the number of constraints because the $PSE$ depends on all the CPTs and this approach would not ensure that the joint distribution is free from direct and indirect discriminations.

7. EXPERIMENTS

In this section, we describe the experiments that were performed to validate the model assumption. Consider the causal graph for the criminal recidivism problem in Figure 1. In the section below, we describe this causal model in detail.

7.1. Causal model

We construct the causal graph along the lines of the structural relationships given by the authors in (VanderWeele and Staude, 2011). The difference is that the graph described in their paper contains judge attributes while our work contains accused attributes. That is, we consider judicial decisions made based on case characteristics and attributes of the accused such as criminal’s race, gender, age, etc. In such a scenario, unfairness arises when the bail decision is taken based on the sensitive attributes such as the race and gender of the accused. The values taken by the variables are,

1. Race ($R$): African American(0), Hispanic(1) and White(2).
2. Gender ($G$): Male(0), Female(1) and Others(2).
3. Age ($A$): Old (0)(>35y) and Young (1)(≤ 35y).
4. Education ($E$): Bachelor (0) and Doctorate (1).
5. Training ($T$): Not Employed (0) and Employed (1).
6. Judicial Bail ($J$): bail given (0) and no bail (1).
7. Case Characteristics ($C$): strong (0) criminal history and weak (1) criminal history.

Remark 6 Conditional probability distribution or table (CPT) of attribute $V$ is $\mathbb{P}(V|Pa(V))$.

We now describe how we generated the conditional probability distribution. The joint distribution obtained from the CPTs factorizes over the Causal model given in Figure 1. For this, we first define the following quantities,

1. Parameters: $\theta_{A\rightarrow V} \in [0, 1]$ $\forall V \in V, \forall A \in Pa(V)$ where $\theta_{A\rightarrow V}$ quantifies the direct influence of attribute $A$ on $V$ that is not dependent on the specific values taken by $A$ and $V$. $\theta_{A\rightarrow V}$ is a property of the edge $A \rightarrow V$.
2. Scores: $\lambda_{A=a\rightarrow V=v} \in [0, 1]$ $\forall V \in V, \forall A \in Pa(V)$ where $\lambda_{A=a\rightarrow V=v}$ quantifies the direct influence of attribute $A$ on $V$. It is not only specific to the edge $A \rightarrow V$ but also to the specific values taken by $A$ and $V$. 

Algorithm 3 removeDiscrimination($G, \mathbb{P}, E_{unfair}^G, X$)
\begin{align*}
    \text{w}^* & \leftarrow \text{getEdgeUnfairness}(G, \mathbb{P}, E_{unfair}^G, X) \\
    \text{w} & \leftarrow \{w_{A \rightarrow X} = 0 | (A \rightarrow X) \in E_{unfair}^G, \ w_{A \rightarrow X} \in \text{w}^*\} \cup \\
    \{w_{A \rightarrow X} | (A \rightarrow X) \notin E_{unfair}^G, \ w_{A \rightarrow X} \in \text{w}^*\} \\
    \mathbb{P}_{\text{new}}(X|Pa(X)) & \leftarrow \hat{Y}^w \\
    \text{Result:} \ \mathbb{P}_{\text{new}}(X|Pa(X))
\end{align*}
We generate the CPT of a variable \( V \) by computing the weighted sum of all the scores \( \lambda_{A=a-V=v} \) where the weights are the parameters \( \theta_{A-V} \). That is,

\[
\mathbb{P}(v|pa(V)) = \sum_{A \in pa(V)} \theta_{A-V} \lambda_{A=a-V=v} \tag{50}
\]

For instance, taking an example of Training (T),

\[
\mathbb{P}(T=t|R=r, A=a, G=g) = \theta_{R-T}(R=r \rightarrow T=t) + \theta_{A-T}(A=a \rightarrow T=t) + \theta_{G-T}(G=g \rightarrow T=t) \tag{51}
\]

To ensure that the generated CPTs satisfy the marginality condition, we define the following constraints over the parameters and scores,

\[
\sum_{A \in pa(T)} \theta_{A-T} = 1 \tag{53}
\]

\[
\sum_{t} \lambda_{A=a-T=t} = 1, \ \forall A \in Pa(T) \tag{54}
\]

7.2. Measuring Edge Unfairness

Once the CPTs are constructed for all \( V \in \mathbf{V} \), we fit Eq. 28 for every CPT by solving the constrained least-squares problem (CLSP) to find the optimal solution. We solve CLSP to obtain \( w^* \) by assuming a Linear model for \( f \) which is constrained by 29 and 30. We follow the steps given in Algorithm 1 to obtain \( w^* \). The CLSP is a well-known optimization problem whose implementation is available in SCIKIT-LEARN library in Python.

**Inference:** Now that the parameterized model is defined, we repeat the same algorithm for 625 distinct combinations of \( \{\theta_{A-J}, \theta_{B-T}\mid A \in Pa(J), B \in Pa(T)\} \). In all cases, we found that the optimal weights \( w^* \) a.k.a the edge unfairness obtained are approximately equal to the \( \{\theta_{p_{J}} \mid p_{J} \in Pa(J), P_{J} \in Pa(T)\} \). For example, when we set \( \{\theta_{p_{J}} \mid p_{J} \in Pa(J)\} = \{\frac{1}{|Pa(J)|}\} \) i.e., equal strength to all parents and solve the CLSP, we obtained \( w^* \approx \{0.3, 0.4, 0.3\} \) with a mean squared error \( (e,J) \) of order \( 10^{-3} \). A low MSE indicates that the edge unfairness is quantifiable using a linear model. Non-linear model would be required to capture the edge unfairness if the CPTs we generated by a non-linear combination inputs.

7.3. Minimal Variation in Edge Unfairness when Attribute Values are varied

Edge unfairness is a property of the edge and its value should not depend on the values taken by the attributes corresponding to it. To validate it, we compare the edge unfairness \( w^*_{R-J} \) and \( w^*_{G-J} \) obtained by solving CLSPs with the \( \theta_{p_{J}} \) and \( \theta_{G-J} \) respectively.

**Inference:** From the Figure 5 below, one can infer that the edge unfairness \( w^* \) is insensitive to the specific values taken by the parent attributes as expected. For instance, out of 625 combinations of \( \{\theta_{A-J}, \theta_{B-T}\mid A \in Pa(J), B \in Pa(T)\} \), the optimal weights \( w^*_{R-J} \) obtained in all the models with \( \theta_{R-J} = 0.33 \) are in the range \([0.33, 0.40]\). A small deviation in \( w^*_{R-J} \) shows that \( w^*_{R-J} \) depends only on \( \theta_{R-J} \) and not on the specific values taken by the attributes.

7.4. Impact of Scaling Factor

In the previous section, it was established that the values of edge unfairness \( w^* \) are close to the \( \{\theta_{e \in E_{\text{linear}}}\} \). This section discusses the effect of introducing Scaling factors into the model input. We first show that the inputs to the model \( f \) which determined the Score \( (\lambda_{A=a-V=v}) \) and the Scaling factor increases the correlation even more. For this experiment, \( \mathbb{P}(J = 1|pa(J)) \) is decomposed using CLSP. Here, \( \lambda_{R=0-J=1} \) is varied and the other quantities used to generate \( \mathbb{P}(J = 1|pa(J)) \) are fixed to certain value. For a given \( \lambda_{R=0-J=1} \), the following two quantities are computed,

1. \( \mathbb{P}(J = 1|do(R = 0)) \)
2. \( S^*_{R=0-J=1} \mathbb{P}(J = 1|do(R = 0)) \)

Figure 6 plots both of the above quantities alongside \( \lambda_{R=0-J=1} \).

**Inference:** We observe that both the quantities are linearly dependent on \( \lambda_{R=0-J=1} \). But the latter \( S^*_{R=0-J=1} \mathbb{P}(J = 1|do(R = 0)) \) has a slope=0.55 \( \approx 0.5 \) as compared to the former \( \mathbb{P}(J = 1|do(R = 0)) \) that has a slope=0.23 \( < 0.5 \) and is therefore a better representation of \( \lambda_{R=0-J=1} \). This validates that using Scaling factors in the inputs leads to better approximation of the inputs to \( \lambda_{A=a-V=v} \).

Next, to validate that the scaling factor improves the model performance, CLSP was solved for 625 combinations of \( \{\theta_{A-J}, \theta_{B-T}\mid A \in Pa(J), B \in Pa(T)\} \) and the Mean Squared Errors (MSEs) between the CPT for Judicial bail \( \mathbb{P}(J|R, G, C, E, T) \) and its linear functional approximation.
Another observation from this experiment is that $\delta$ which is a significant decrease in terms of the MSEs 70% purely based on a sensitive attribute, say Race of the criminal, ample of such a case could be the judicial decision made en-extreme cases as illustrated in the above example. One ex-
of $\theta$ model input decreases the MSEs in almost all combinations 625 combinations of $\{\ldots\}$. Example of such a case is $A \in Pa(V)$ and compute $E(P, P^m)$, $\theta$ and $\theta'$ is compared with $\theta$ where $A' \in Pa(V) \setminus A$ are negligible. Example of such a case is $\{\ldots\} = \{\theta_{A \rightarrow J}, \theta_{B \rightarrow J}, \theta_{C \rightarrow J}, \theta_{E \rightarrow J}, \theta_{T \rightarrow J}\}$ = $\{0.81, 0.09, 0.03, 0.03, 0.03\}$. $\delta_j$ is negative only 4% of the 625 combinations of $\theta$. Moreover, all those 4% negative $\delta_j$ are observed in extreme cases as illustrated in the above example. One ex-
ample of such a case could be the judicial decision made entirely based on a sensitive attribute, say Race of the criminal, without considering any other factors. But such situations are highly unlikely to occur in practice.

1. MSEs calculated by feeding the inputs to the model without the use of scaling factor denoted by $e'_j$
2. MSEs calculated by feeding the inputs to the model by using the scaling factor denoted by $e_j$

Inference. Distributions of $e'_j$ and $e_j$ are plotted in Figure 7. Here, the maximum value of $e'_j$ (red bar) is obtained around 0.013 and it is evenly distributed in the range $(0.0, 0.014)$. On the other hand, $e_j$ (blue bars) is skewed in the lower error range i.e., $(0.0, 0.004)$ with the maximum value of $e_j$ (blue bar) obtained around 0.01. This means that the usage of scaling factors in Def. 14 is a better choice because the MSEs distribution is skewed in the lower error range of $(0.0, 0.004)$ with the scaling factor as compared to the other case. To analyze the extent by which MSE is reduced by introducing the Scaling factor, we calculate the percentage decrease in the MSEs $\delta_j$ equal to,

$$\delta_j = \frac{e'_j - e_j}{e'_j}$$

and plot its distribution in the Figure 8. As seen from the Figure 8, majority of the values of $\delta_j$’s are around 60% – 70% which is a significant decrease in terms of the MSEs.

Another observation from this experiment is that $\delta_j$ is negative in few settings. Those specific settings are the instances where $\theta_{A \rightarrow V}$ ($V \in \{J, T\}$) is large for a particular $A \in Pa(V)$ and other $\theta_{A' \rightarrow V}$ where $A' \in Pa(V) \setminus A$ are negligible. Example of such a case is $\{\theta_{A \rightarrow J}, A \in Pa(J)\} = \{\theta_{R \rightarrow J}, \theta_{G \rightarrow J}, \theta_{C \rightarrow J}, \theta_{E \rightarrow J}, \theta_{T \rightarrow J}\}$ = $\{0.81, 0.09, 0.03, 0.03, 0.03\}$. $\delta_j$ is negative only 4% of the 625 combinations of $\theta$. Moreover, all those 4% negative $\delta_j$ are observed in extreme cases as illustrated in the above example. One ex-
ample of such a case could be the judicial decision made entirely based on a sensitive attribute, say Race of the criminal, without considering any other factors. But such situations are highly unlikely to occur in practice.

7.5. Finite data

In this section, we examine the applicability of the proposed approach to realistic scenarios. In most of the real-world settings knowledge of the causal model and the $CPTs$ are not available. Since our work assumes that the causal graph is given, we do dwell on discovering causal structures using a finite amount of data. However, we evaluate the impact of a finite amount of data on $CPTs$ and their impact on calculating edge unfairness. Edge unfairness $w^*(P)$ calculated using Algorithm 1 using true $CPTs$ $P$ is compared with the edge unfairness $w^*(P^m)$ calculated using Algorithm 1 using estimated $CPTs$ $P^m$ where $m$ is the number of samples used for estimation by maximizing the likelihood. $m$ samples are drawn from $P$ randomly.

For a given $m$, the distance between $w^*(P)$ and $w^*(P^m)$ is calculated using Euclidean distance $E(P, P^m) = ||w^*(P) - w^*(P^m)||_2$. We repeat this experiment for different $m$ and compute $E(P, P^m)$. Intuitively, the distance should decrease as $m$ increases, because a large number of i.i.d samples produces a better approximation of the original distribution $P$, thereby reducing the distance between $w^*(P^m)$ and $w^*(P)$. $E(P, P^m)$ is plotted against $m$ in Figure 9 for...
different true distributions $\mathbb{P}$ that are randomly generated (different colours).

![Figure 9. Euclidean Distance $||w^*(\mathbb{P}) - w^*(\mathbb{P}^m)||_2$ (y-axis) vs number of samples $m$ (x-axis)](image)

**Inference:** We observe that $w^*(\mathbb{P}^m)$ move closer to $w^*(\mathbb{P})$ as $m$ increases. Moreover, since $\mathbb{P}$ was randomly generated, we also observe that there exists an empirical bound over $E(\mathbb{P}, \mathbb{P}^m)$ for a given $m$. For instance, in the Figure 9, for $m$ greater than $10^3$, the observed $E(\mathbb{P}, \mathbb{P}^m)$ is always less than 0.04 and this bound shows a decreasing trend as $m$ increases for any true $\mathbb{P}$. Hence, given the number of samples $m$, we can get an empirical upper bound over the error between the true and the estimated edge unfairness which helps in evaluating whether the calculated edge unfairness should be used in real-life scenarios where only a finite amount of data is available. The presence of empirical bound motivates the reader to investigate the possibility of a theoretical bound over $E(\mathbb{P}, \mathbb{P}^m)$. Theoretical bound is not presented in our work.

**8. CONCLUSION**

We have introduced the problem of quantifying overall unfairness in terms of unfairness along an edge. We have quantified unfairness in an edge, say $A \rightarrow X$, by decomposing the conditional probability distribution $\mathbb{P}(X|Pa(X))$ of the destination node $X$ into the beliefs along the fair and unfair edges without modifying the causal graph (Srinivas, 1993; Heckerman, 1993; Heckerman and Breese, 1994). For decomposition, a linear model is assumed that takes beliefs along each of the unfair edge and all fair edges as inputs where the belief along an edge being formulated using interventional distribution and path-specific effects. Since the dependencies in interventional distribution, say $P(X|do(M))$, lead to active trails that emanate from the nodes $M$ and end in $X$, we scale down $P(X|do(M))$ using a scaling factor so as to have active trails along the direct edges from $M$ to $X$. The scaling factor helps in a better approximation of the CPTs. We have formulated overall unfairness in terms of unfairness along edges which helps in evaluating the impact of the latter on the former and also helps in regenerating the joint distribution that is not subjected to unfairness towards any subset of sensitive attributes in any decision. In the future, we aim to evaluate the impact of edge unfairness in the dataset on different stages of machine learning pipeline such as selection, classification, etc. given that the machine learning algorithm is learned using the dataset. We plan to extend our work to settings with unobserved variables and variables that can take continuous values.

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