Can one measure timelike Compton scattering at LHC?

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Exclusive photoproduction of dileptons, $\gamma N \rightarrow \ell^+\ell^-$, is and will be measured in ultraperipheral collisions at hadron colliders, such as the Tevatron, RHIC and the LHC. We demonstrate that the timelike deeply virtual Compton scattering (TCS) mechanism $\gamma q \rightarrow \ell^+\ell^- q$ where the lepton pair comes from the subprocess $\gamma q \rightarrow \gamma^* q$ dominates in some accessible kinematical regions, thus opening a new way to study generalized parton distributions (GPDs) in the nucleon. High energy kinematics enables to probe parton distributions at small skewedness. This subprocess interferes at the amplitude level with the pure QED subprocess $\gamma\gamma^* \rightarrow \ell^+\ell^-$ where the virtual photon is radiated from the nucleon.

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I. INTRODUCTION.

Much theoretical and experimental progress has recently been witnessed in the study of deeply virtual Compton scattering (DVCS), i.e., $\gamma^* p \rightarrow \gamma p$, an exclusive reaction where generalized parton distributions (GPDs) factorize from perturbatively calculable coefficient functions, when the virtuality of the incoming photon is high enough \cite{1}. It is now recognized that the measurement of GPDs should contribute in a decisive way to our understanding of how quarks and gluons build hadrons \cite{2}. In particular the transverse location of quarks and gluons become experimentally measurable via the transverse momentum dependence of the GPDs \cite{3}.

The “inverse” process, $\gamma(q) N(p) \rightarrow \gamma^*(q') N(p') \rightarrow l^- (k) l^+ (k') N(p')$ at small $t = (p' - p)^2$ and large timelike virtuality $(k + k')^2 = q^2 = Q^2$ of the final state dilepton, timelike Compton scattering (TCS) \cite{4}, shares many features with DVCS. The Bjorken variable in that case is $\tau = Q^2/s$ with $s = (p + q)^2$. One also defines $\Delta = p' - p$ ($t = \Delta^2$) and the skewness variables $\xi$, $\eta$ as

$$\xi = -\frac{(q + q')^2}{2(p + p') \cdot (q + q')}, \eta = -\frac{(q - q') \cdot (q + q')}{(p + p') \cdot (q + q')},$$

where the approximations hold in the kinematical limit we are working, i.e. in the extended Bjorken regime where masses and $-t$ are small with respect of $Q^2$ ( $s$ is always larger than $Q^2$ ). $x$, $\xi$, and $\eta$ represent plus-momentum fractions (Light-cone coordinates are defined as $v^\pm = \frac{v^0 \pm v^3}{\sqrt{v^0}}$, both proton momenta $p$ and $p'$ moving fast to the right, i.e., having large plus-components).

$$x = \frac{(k + k')^+}{(p + p')^+}, \xi = -\frac{(q + q')^+}{(p + p')^+}, \eta = \frac{(p - p')^+}{(p + p')^+}.$$  \hfill (2)

To leading-twist accuracy one has $\xi = -\eta = -\tau / (2 - \tau)$.

The possibility to use high energy hadron colliders as powerful sources of quasi real photons in ultraperipheral collisions has recently been emphasized \cite{5}. This should allow the study of many aspects of photon proton and photon photon collisions at high energies, already at the Tevatron and at RHIC but in particular at the LHC \cite{6} even if the nominal luminosity is not achieved during its first years of operation. The high luminosity and energies of these photon beams opens a new kinematical domain for the study of TCS, and thus to the hope of determining GPDs in the small skewedness ($\xi$) region, which is complementary to the determination of the large $\xi$ quark GPDs \cite{7} at lower energy electron accelerators such as JLab. Moreover, the crossing from a spacelike to a timelike probe is an important test of the understanding of QCD corrections, as shown by the history of the understanding of the Drell-Yan reaction in terms of QCD.
The physical process where to observe TCS is photoproduction of a heavy lepton pair, $\gamma N \rightarrow \mu^+\mu^- N$ or $\gamma N \rightarrow e^+e^- N$, shown in Fig. 1. As in the case of DVCS, a Bethe-Heitler (BH) mechanism - sometimes called $\gamma\gamma$ process since the lepton pair is produced through the $\gamma(q)\gamma(\Delta) \rightarrow \ell^+\ell^-$ subprocess - contributes at the amplitude level. This amplitude is completely calculable in QED provided one knows the Nucleon form factors at small $t$. This process has a very peculiar angular dependence and overdominates the TCS process if one blindly integrates over the final phase space. One may however choose kinematics where the amplitudes of the two processes are of the same order of magnitude, and either subtract the well-known Bethe-Heitler process or use specific observables sensitive to the interference of the two amplitudes.

The kinematics of the $\gamma(q)N(p) \rightarrow \ell^-(k)\ell^+(k')N(p')$ process is shown in Fig. 2. In the $\ell^+\ell^-$ center of mass system, one introduces the polar and azimuthal angles $\theta$ and $\varphi$ of $\vec{k}$, with reference to a coordinate system with 3-axis along $-\vec{p}'$ and 1- and 2-axes such that $\vec{p}$ lies in the 1-3 plane and has a positive 1-component.

In this paper, we shall examine in detail the feasibility of TCS experiments in ultraperipheral collisions at the LHC. Most conclusions should also apply to the Tevatron and RHIC experimental conditions, once the effective photon fluxes and energies are scaled down to their specific values. We shall work in the leading twist approximation. As in the DVCS case, a gauge invariant treatment necessitates the inclusion of twist 3 effects [7], but we will not address this problem in this paper. We will also stay at the leading order in $\alpha_S$, and thus neglect contributions proportional to gluon GPDs. The motivation of this study has been presented at a recent LHC workshop [8].
II. THE VARIOUS CONTRIBUTIONS

A. The Bethe-Heitler contribution

The Bethe-Heitler amplitude is calculated from the two Feynman diagrams in Fig. 3 where the photon-nucleon vertex is parameterized by the usual Dirac and Pauli form factors $F_1(t)$ and $F_2(t)$, normalizing $F_2(0)$ to be the anomalous magnetic moment of the target. Neglecting masses and $t$ compared to terms going with $s$ or $Q'^2$, the Bethe Heitler contribution to the unpolarized $\gamma p$ cross section is (with $M$ being the proton mass)

$$\frac{d\sigma_{BH}}{dQ'^2 dt d(cos \theta)} \approx \frac{\alpha^2_{em}}{2\pi s^2} \frac{1}{-t} \frac{1 + \cos^2 \theta}{\sin^2 \theta} \left[ \left( F_2 - \frac{t}{4M^2} F_2^2 \right) \frac{2 \Delta t}{t^2} + (F_1 + F_2)^2 \right], \quad (3)$$

provided we stay away from the kinematical region where the product of lepton propagators goes to zero at very small $\theta$. The interesting physics program thus imposes a cut on $\theta$ to stay away from the region where the Bethe Heitler cross section becomes extremely large.

B. The Compton amplitude

In the region where the final photon virtuality is large, the amplitude is given by the convolution of hard scattering coefficients, calculable in perturbation theory, and generalized parton distributions, which describe the nonperturbative physics of the process. To leading order in $\alpha_s$ one then has the dominance of the quark handbag diagrams of Fig. 4. The analysis of these handbag diagrams show the simple relations

$$M^{\lambda+,-\lambda+}|_{TCS} = \left[ M^{\lambda-,\lambda-} \right]_{DVCS},$$

$$M^{\lambda-,\lambda-}|_{TCS} = \left[ M^{\lambda+,-\lambda+} \right]_{DVCS}, \quad (4)$$

between the helicity amplitudes for TCS and DVCS at equal values of $\eta$ and $t$. For instance,

$$M^{+-,-+}|_{TCS} = \sqrt{1 - \eta^2}(\mathcal{H}_1(-\eta, \eta, t) + \tilde{\mathcal{H}}_1(-\eta, \eta, t) - \frac{\eta^2}{1 - \eta^2}(\mathcal{E}_1(-\eta, \eta, t) + \tilde{\mathcal{E}}_1(-\eta, \eta, t))), \quad (5)$$

where the Compton form factors $\mathcal{H}_1, \mathcal{E}_1, \tilde{\mathcal{H}}_1, \tilde{\mathcal{E}}_1$ are defined as

$$\mathcal{H}_1(\xi, \eta, t) = \sum_q e_q^2 \int_{-1}^1 dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H^q(x, \eta, t),$$

$$\mathcal{E}_1(\xi, \eta, t) = \sum_q e_q^2 \int_{-1}^1 dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) E^q(x, \eta, t),$$

$$\tilde{\mathcal{H}}_1(\xi, \eta, t) = \sum_q e_q^2 \int_{-1}^1 dx \left( \frac{1}{\xi - x - i\epsilon} + \frac{1}{\xi + x - i\epsilon} \right) \tilde{H}^q(x, \eta, t),$$

$$\tilde{\mathcal{E}}_1(\xi, \eta, t) = \sum_q e_q^2 \int_{-1}^1 dx \left( \frac{1}{\xi - x - i\epsilon} + \frac{1}{\xi + x - i\epsilon} \right) \tilde{E}^q(x, \eta, t), \quad (6)$$
and \( H^q(x, \eta, t), E^q(x, \eta, t), \tilde{H}^q(x, \eta, t), \tilde{E}^q(x, \eta, t) \) are usual GPDs for a quark of flavour \( q \) and electric charge \( e_q \).

At this order the two processes carry the same information on the generalized quark distributions. This will not be true when higher order contributions are included but we will not consider these corrections here.

The crucial ingredient to estimate the TCS amplitude at large energies is a realistic model of GPDs at small skewedness.

### C. Modelizing GPDs

We anticipate that singlet quark GPDs give the dominant contributions to the TCS amplitude in that domain. Since gluon GPDs only enter the TCS amplitude at the \( O(\alpha_s) \) level, we feel justified in a first step to neglect their contributions and leave its study for a future work. The choice of factorization scale is a major issue since GPD evolution is particularly active in the small \( x \) domain. In this first study of the feasibility of the extraction of the TCS signal, we feel justified to simplify our calculations by using a factorization ansatz for the \( t \) dependence of GPD’s:

\[
\begin{align*}
H^u(x, \eta, t) &= h^u(x, \eta) \frac{1}{2} F_1^u(t) \\
H^d(x, \eta, t) &= h^d(x, \eta) F_1^d(t) \\
H^s(x, \eta, t) &= h^s(x, \eta) F_D(t)
\end{align*}
\]
and a double distribution ansatz for $h^g$ without any D-term:

$$h^g(x, \eta) = \int_0^1 dx' \int_{-1-x'}^{1-x'} dy' \left[ \delta(x - x' - \eta y') q(x') - \delta(x + x' - \eta y') \bar{q}(x') \right] \pi(x', y')$$

$$\pi(x', y') = \frac{3 (1 - x')^2 - y'^2}{4 (1 - x')^3}$$

For the unpolarized distributions $q(x)$ and $\bar{q}(x)$ we take NLO(\overline{\text{MS}}) GRVGJR 2008 parametrization \cite{9}. Their strong dependence of the factorization scale choice for small $x$ is shown on Fig.5. This results in the strong dependence of $h^g$ for small values of $\eta$ as shown on Fig.6.

### III. CROSS SECTION ESTIMATES

Let us now estimate the different contributions to the lepton pair cross section for ultraperipheral collisions at the LHC. Since the cross sections decrease rapidly with $Q'^2$, we are interested in the kinematics of moderate $Q'^2$, say a few GeV$^2$, and large energy, thus very small values of $\eta$. Note however that for a given proton energy the photon flux is higher at smaller photon energy.
A. The Bethe Heitler cross section

The full Bethe Heitler cross section integrated over $\theta \in [\pi/4, 3\pi/4], \varphi \in [0, 2\pi], Q^2 \in [4.5, 5.5] \text{GeV}^2, |t| \in [0.05, 0.25] \text{GeV}^2$, as a function of $\gamma p$ energy squared $s$ is shown on Fig. 7a. We see that in the limit of large $s$ it is constant and equals 28.4 pb. On Fig. 7b, the Bethe Heitler contribution is shown as a function of $Q^2$ when it is integrated over $\varphi$ in the range $[0, 2\pi], -t$ in the range $[0.05, 0.25] \text{GeV}^2$ and for $\theta$ integrated in various ranges $[\pi/3, 2\pi/3], [\pi/4, 3\pi/4]$ and $[\pi/6, 5\pi/6]$. As anticipated, the cross section grows much when small $\theta$ angles are allowed. In the following we will use the limits $[\pi/4, 3\pi/4]$ where the cross section is sufficiently big and does not dominate too much over the Compton process.

B. The TCS cross section

Since the $u$-quark contribution of $H_1$ turns out to dominate the TCS amplitude, we show in Fig. 8 the real and imaginary part of $\mathcal{H}^u$ divided by $\frac{k}{Q^2}(t)$ for various factorization scales and various ranges of $\xi$. We observe that for small values of $\xi$, factorization scale dependence is quite strong. The same is seen on Fig. 9 where the full Compton cross section $\sigma_{\text{TCS}}$ integrated over $\phi$ divided by $\frac{k}{Q^2}F(t)$ is plotted as a function of the photon-proton energy squared $s$. For very high energies $\sigma_{\text{TCS}}$ calculated with $\mu_F^2 = 6 \text{GeV}^2$ is much bigger then with $\mu_F^2 = 4 \text{GeV}^2$. Also predictions obtained using LO and NLO GRVJR2008 PDFs differ significantly.

C. The interference cross section

Since the amplitudes for the Compton and Bethe-Heitler processes transform with opposite signs under reversal of the lepton charge, the interference term between TCS and BH is odd under exchange of the $\ell^+$ and $\ell^-$ momenta. It is thus possible to project out the interference term through a clever use of the angular distribution of the lepton pair. The interference part of the cross-section for $\gamma p \rightarrow \ell^+\ell^- p$ with unpolarized protons and photons is given at leading order by

$$
\frac{d\sigma_{\text{INT}}}{dQ^2 \, dt \, d\cos \theta \, d\varphi} = -\alpha^2_{\text{em}} \frac{1}{4\pi s^2} - t \frac{M}{Q^2} \frac{1}{\tau \sqrt{1 - \tau}} \frac{\cos \varphi}{\sin \theta} \frac{1 + \cos^2 \theta}{\sin \theta} \text{Re} \tilde{M}^{--},
$$

with

$$
\tilde{M}^{--} = 2\sqrt{t_0 - t} \frac{1 - \eta}{M} \left[ F_1 H_1 - \eta (F_1 + F_2) H_1 - \frac{t}{4M^2} F_2 E_1 \right],
$$

where $t_0 = 4t_0^2 M^2/(1 - \eta^2)$. With the integration limits symmetric about $\theta = \pi/2$ the interference term is odd under $\varphi \rightarrow \pi + \varphi$ due to charge conjugation, whereas the TCS and BH cross sections are even. One may thus extract the Compton amplitude through a study of $\int_0^{\pi} d\phi \cos \phi \frac{d\sigma}{d\cos \theta}$.

In Fig. 10 we show the interference contribution to the cross section in comparison to the Bethe Heitler and Compton processes, for various values of photon proton energy squared $s = 10^7 \text{GeV}^2, 10^5 \text{GeV}^2, 10^3 \text{GeV}^2$. We observe that for larger energies the Compton process dominates, whereas for $s = 10^5 \text{GeV}^2$ all contributions are comparable.

D. Rate estimates

As described in [10] the cross section for photoproduction in hadron collisions is given by:

$$
\sigma_{\gamma p} = 2 \int \frac{d\sigma_{\gamma p}(k)}{dk} \frac{dn(k)}{dk} dk
$$

where $\sigma_{\gamma p}(k)$ is the cross section for the $\gamma p \rightarrow \ell^+\ell^- p$ process and $k$ is the photon energy. $\frac{dn(k)}{dk}$ is an equivalent photon flux (the number of photons with energy $k$), and is given by [11]:

$$
\frac{dn(k)}{dk} = \frac{\alpha}{2\pi k} \left[ 1 + (1 - \frac{2k}{\sqrt{s_{pp}}})^2 \right] \left( \ln A - \frac{11}{6} + \frac{3}{A} - \frac{3}{2A^2} + \frac{1}{3A^3} \right)
$$
where: $A = 1 + \frac{0.71 \text{ GeV}^2}{Q_{\text{min}}^2}$, $Q_{\text{min}}^2 \approx \frac{4M^2k^2}{s_{pp}}$ is the minimal squared fourmomentum transfer for the reaction, and $s_{pp}$ is the proton-proton energy squared ($\sqrt{s_{pp}} = 14$ TeV). The relationship between $\gamma p$ energy squared $s$ and $k$ is given by:

$$s \approx 2\sqrt{s_{pp}k}$$  \hspace{1cm} (11)

The pure Bethe - Heitler contribution to $\sigma_{pp}$, integrated over $\theta = [\pi/4, 3\pi/4]$, $\phi = [0, 2\pi]$, $t = [-0.05 \text{ GeV}^2, -0.25 \text{ GeV}^2]$, $Q'^2 = [4.5 \text{ GeV}^2, 5.5 \text{ GeV}^2]$, and photon energies $k = [20, 900]$ GeV gives:

$$\sigma_{pp}^{BH} = 2.9 \text{ pb}.$$  \hspace{1cm} (12)

The Compton contribution (calculated with NLO GRVJR2008 PDFs, and $\mu_F^2 = 5 \text{ GeV}^2$) gives:

$$\sigma_{pp}^{TCS} = 1.9 \text{ pb}.$$  \hspace{1cm} (13)
Figure 9: $\sigma_{TCS}$ as a function of $\gamma p$ c.m. energy squared $s$, for GRVGJR2008 LO (a) and NLO (b) parametrizations, for different factorization scales $\mu_F^2 = 4$ (dotted), 5 (dashed), 6 (solid) GeV$^2$.

We have chosen the range of photon energies in accordance with expected capabilities to tag photon energies at the LHC. This amounts to a large rate of order of $10^5$ events/year at the LHC with its nominal luminosity ($10^{34}$ cm$^{-2}$s$^{-1}$). The rate remains sizeable for the lower luminosity which can be achieved in the first months of run.

IV. CONCLUSION

The operation of LHC as a heavy ion collider will enable us to study TCS on nuclei. Such scattering may occur in a coherent way and its amplitude involves nuclear GPDs [12]. This very interesting subject definitely needs more work. Incoherent TCS which occurs on quasi free neutrons and protons allows to study the GPDs of bound nucleons. Cross sections are then roughly multiplied by $A^2$, where $A$ is the atomic number. One may thus expect sizeable rates in ion-ion collisions at the LHC.

In conclusion, let us stress that timelike Compton scattering in ultraperipheral collisions at hadron colliders opens a new way to measure generalized parton distributions. Our work has to be supplemented by studies of higher order contributions which will involve the gluon GPDs; they will hopefully lead to a weaker factorization scale dependence of the amplitudes.

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[1] D. Müller et al., Fortsch. Phys. 42, 101 (1994); X. Ji, Phys. Rev. Lett. 78, 610 (1997); A. V. Radyushkin, Phys. Rev. D56, 5524 (1997); J. C. Collins and A. Freund, Phys. Rev. D59, 074009 (1999).
[2] M. Diehl, Phys. Rept. 388 (2003) 41; A. V. Belitsky and A. V. Radyushkin, Phys. Rept. 418, 1 (2005); S. Boffi and B. Pasquini, Riv. Nuovo Cim. 30, 387 (2007).
[3] M. Burkardt, Phys. Rev. D 62, 071503 (2000) and Int. J. Mod. Phys. A 18, 173 (2003); J. P. Ralston and B. Pire, Phys. Rev. D 66, 111501 (2002); M. Diehl, Eur. Phys. J. C 25, 223 (2002).
[4] E. R. Berger, M. Diehl and B. Pire, Eur. Phys. J. C 23, 675 (2002).
[5] K. Hencken et al., Phys. Rept. 458, 1 (2008). G. Baur, K. Hencken, D. Trautmann, S. Sadovsky and Y. Kharlov, Phys. Rept. 364, 359 (2002).
[6] D. d’Enterria, M. Klasen and K. Piotrzkowski, Nucl. Phys. Proc. Suppl. B 179, 1 (2008); B. Pire, L. Szymanowski, F. Schwennsen and S. Wallon, [arXiv:0810.3817] [hep-ph].
[7] I. V. Anikin, B. Pire and O. V. Teryaev, Phys. Rev. D 62, 071501 (2000); A. V. Belitsky and D. Müller, Nucl. Phys. B 589 (2000) 611; N. Kivel, M. V. Polyakov, A. Schäfer and O. V. Teryaev, Phys. Lett. B 497 (2001) 73; A. V. Radyushkin
Figure 10: The differential cross sections (solid lines) for $t = -0.2 \text{GeV}^2$, $Q'^2 = 5 \text{GeV}^2$ and integrated over $\theta = [\pi/4, 3\pi/4]$, as a function of $\phi$, for $s = 10^7 \text{GeV}^2$ (a), $s = 10^5 \text{GeV}^2$ (b), $s = 10^3 \text{GeV}^2$ (c) with $\mu_F^2 = 5 \text{GeV}^2$. We also display the Compton (dotted), Bethe-Heitler (dash-dotted) and Interference (dashed) contributions.

[8] B. Pire, L. Szymanowski and J. Wagner, Nucl. Phys. Proc. Suppl. 179-180, 232 (2008).
[9] M. Gluck, P. Jimenez-Delgado and E. Reya, Eur. Phys. J. C 53 (2008) 355.
[10] C. A. Bertulani, S. R. Klein and J. Nystrand, Ann. Rev. Nucl. Part. Sci. 55 (2005) 271
[11] M. Drees and D. Zeppenfeld, Phys. Rev. D 39 (1989) 2536.
[12] E. Berger et al., Phys. Rev. Lett. 87, 142302 (2001); F. Cano and B. Pire, Eur. Phys. J. A 19, 423 (2004); S. Scopetta, Phys. Rev. C 70, 015205 (2004) A. Kirchner and D. Mueller, Eur. Phys. J. C 32, 347 (2003); V. Guzey and M. Strikman, Phys. Rev. C 68, 015204 (2003); V. Guzey, arXiv:0801.3235 [nucl-th] and J. Phys. G 32, 251 (2006); V. Guzey, A. W. Thomas and K. Tsushima, arXiv:0806.3288 [hep-ph]; A. Freund and M. Strikman, Eur. Phys. J. C 33, 53 (2004); S. Liuti and S. K. Taneja, Phys. Rev. C 72, 034902 (2005), *ibid* Phys. Rev. C 72 032201 (2005).