Bag Picture of the Excited QCD Vacuum with Static $Q\bar{Q}$ Source

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The gluon excitations of the QCD vacuum are investigated in the presence of a static quark-antiquark source. It is shown that the ground state potential and the excitation spectrum of dynamical gluon degrees of freedom, as determined in our lattice simulations, agree remarkably well with model predictions based on the dielectric properties of the confining vacuum described as a dual superconductor. The strong chromoelectric field of the static $Q\bar{Q}$ source creates a bubble (bag) in the condensed phase where weakly interacting gluon modes can be excited. Some features and predictions of the bag model are presented and the chromoelectric vortex limit at large quark-antiquark separation (string formation) is briefly discussed.

1. Introduction

The project which we initiated eighteen months ago to study the gluon excitations of the QCD vacuum in the presence of a static quark-antiquark source has two distinct goals. Our first objective was to determine the spectrum of hybrid $c\bar{c}g$ and $b\bar{b}g$ states with results reported in Ref. [1]. Early predictions of these states in the Born-Oppenheimer approximation were based on the bag picture of the QCD vacuum in Ref. [2], and additional phenomenological observations were made in Ref. [3].

The investigation of gluon excitations around a static quark-antiquark pair in the QCD vacuum has important implications on our conceptual understanding of the quark confinement mechanism about which very little is known at the present. In this talk we will address this second goal within the context of a simplified picture of the QCD vacuum which in the first tests appears to agree surprisingly well with simulation results.

2. Dielectric vacuum and bag formation

There is little doubt that some sort of a bag is formed when a static $Q\bar{Q}$ pair is inserted in the physical vacuum at a separation $r \ll 1$ fm where asymptotic freedom holds. The strong chromoelectric dipole field, $E_0 = \frac{2\Omega_{Q\bar{Q}} g(r)}{R^3}$, at a distance $R$ from the dipole source, suppresses the microscopic nonperturbative condensate before the field strength drops to some typical confinement scale $E_{\text{critical}} \sim \Lambda^2_{\text{QCD}}$ at a distance $R_b$ which we will identify qualitatively as the bag radius of confinement ($g(r)$ is the coupling constant, or color charge). At the confinement scale, the perturbative vacuum bubble which is sustained by the strong dipole field has to be replaced by the nonperturbative condensate of the physical vacuum. Within the bubble (bag) we should be able to apply perturbation theory for gluonic corrections to the dipole field to recover the running Coulomb law. The size of the bubble $R_b$ can be estimated from the relation $E_{\text{critical}} \sim \frac{2\Omega_{Q\bar{Q}} g(r)}{R_b^3} \sim \Lambda^2_{\text{QCD}}$. In the bag model, as explained in Refs. [4] and [5], we assume a simple confinement picture for the interface between the two phases of the vacuum. Inside the bag the chromoelectric vacuum permeability $\epsilon$ is set to one (perturbative vacuum). In the confining gluon condensate of the physical vacuum $\epsilon = 0$ is assumed (dielectric vacuum) which is expected to emerge from the microscopic theory of a dual nonabelian superconductor in QCD. A sharp boundary is assumed to separate the bag from the physical vacuum with surface energy $\sigma$ per unit area and volume energy $B$ per unit volume. The value
of $B$ is related to the gluon vacuum condensate by the relation $B = -\frac{3}{8} \langle |\pi^a F^a_{\mu\nu} F^a_{\mu\nu}| \rangle$ [5]. Based on QCD sum rules and heavy QQ spectroscopy, the value of $B$ is determined to be in the range $B^{1/4} \sim 250 - 350$ MeV [5]. The interface energy at the deconfining transition temperature of quenched QCD has been determined in the $\sigma \sim 5 - 10$ MeV/fm$^2$ range [6] which is small in comparison with the volume energy and neglected in the calculations we present.

3. Adiabatic bag picture

The adiabatic method we apply here is a variational principle for the total energy of the bag (for details we refer to Refs. [2] and [4]). In the results we present here an ellipsoidal shape is used in the variational calculations which is adequate within a few percent accuracy. With an effective coupling constant $\alpha_s$ inside the bag, and with the choice of $B$ given in Fig. 1, we first solve the bag equations in Coulomb gauge for the ground state when dynamical gluons are not excited. The variation of the minimal bag shape as a function of the QQ separation is depicted in Fig. 1. Various bag shapes in the presence of gluon excitations are shown in Fig. 2. The notation for the gluon quantum numbers is explained in Ref. [1].

The bag model in the adiabatic approximation predicts the gluon excitations without free parameters. Fig. 3 and Fig. 4 compare the bag model predictions with our simulation results. The agreements are quite remarkable.

4. Chromoelectric vortex (string) limit

In the adiabatic approximation there is an exact vortex solution to the bag equations which describes a homogeneous chromoelectric flux with an intrinsic radius $R_{vortex} = (8\alpha_s/3\pi B)^{1/4}$, where $\alpha_s$ and $B$ are the only two parameters of the model. The vortex energy per unit length (string tension, or slope of the linear part of the QQ potential) is given by $\kappa = \sqrt{32\pi\alpha_s B/3}$. The vortex limit is illustrated in Fig. 3. As it was shown in Ref. [4] this vortex has massive intrinsic gluon excitations along the “waveguide” of the vortex and collective string excitations which correspond to

\[
\begin{align*}
\Sigma^+_s & \quad \text{Bag Shapes} \\
\alpha &= 0.23 \\
B^0 &= 315 \text{ MeV}
\end{align*}
\]

\[
\begin{array}{c}
\text{transverse size} \\
1/\text{GeV units}
\end{array}
\]

\[
\begin{array}{c}
r (1/\text{GeV})
\end{array}
\]

\[
\begin{array}{c}
r=0.4/\text{GeV} \\
r=1/\text{GeV} \\
r=2/\text{GeV}
\end{array}
\]

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet
\end{array}
\]

Figure 1. The shape of the bag in the $\Sigma^+_s$ ground state is depicted as a function of quark-antiquark separation in the ellipsoidal approximation. The black dots designate the locations of the Q and $\bar{Q}$ sources. The oblate shape at small separation is determined by the dominant dipole field. The transverse size of the asymptotic vortex solution is reached at $r \sim 1$ fm separation.

\[
\begin{align*}
\Sigma^+_s & \quad \text{Bag Shapes} \\
\Pi^+_u & \quad \text{Oblate} \\
\Delta_u & \quad \text{spherical}
\end{align*}
\]

\[
\begin{array}{c}
r=0.8 \text{ fm}
\end{array}
\]

\[
\begin{array}{c}
r=2.4 \text{ fm}
\end{array}
\]

Figure 2. Bag shapes with gluon excitations are compared with the shape of the bag in its ground state at two different quark-antiquark separations.
Figure 3. The bag model predictions for the CP even $V_{\Pi_{\Lambda}}(r)$ and CP odd $V_{\Pi_{u}}(r)$ excitations with $\Lambda = 1$ angular momentum projection on the $Q \bar{Q}$ axis are depicted as the solid curves in units of the hadronic scale parameter $r_0$ (defined in [1]) against the quark-antiquark separation $r$.

Figure 4. The CP even $V_{\Delta_{\Lambda}}(r)$ and CP odd $V_{\Delta_{u}}(r)$ $\Lambda = 2$ excitations are depicted together with $V_{\Sigma^{+}_{\Lambda}}(r)$ and $V_{\Pi_{u}}(r)$.

Figure 5. The shape of the bag in the $\Sigma^{+}_{\Lambda}$ ground state is depicted as a function of quark-antiquark separation. The exact vortex solution is shown by the dashed lines.

The bag model provides a first attempt for a unified low energy effective theory to capture the physics of quark confinement at small and large $Q \bar{Q}$ separations. Effective string models focus on the long range part of the picture [8]. It remains a challenge to understand the more detailed connection between the two approaches.

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