Singly heavy baryons in nuclear matter from an SU(3) chiral soliton model

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We investigate how the masses of the singly heavy baryons undergo changes in nuclear matter, based on a medium-modified SU(3) chiral soliton model. Having explained the bulk properties of nuclear matter, we discuss the masses of the singly heavy baryons in nuclear matter. We generalize the vector-meson Lagrangian including the heavy-meson soliton interaction. The mass spectrum of the singly heavy baryon is obtained with the effects of explicit SU(3) symmetry breaking considered as a perturbation. The results show that the mass of the singly heavy baryon mass in nuclear medium is rather sensitive to the medium modifications of the heavy meson mass.

Keywords: chiral soliton model, nuclear matter, medium modification of the heavy hadrons in nuclear medium.

I. INTRODUCTION

Understanding the medium modifications of light hadrons has been one of the most important issues in hadronic and nuclear physics over decades, since it sheds light on the fundamental features of quantum chromodynamics such as the spontaneous breakdown of chiral symmetry and quark confinement (see following reviews [1–5]). It also provides an important clue about how neutron stars are formed [6], in particular, how the two solar-mass problem of the neutron star can be explained. Thus, it is essential to answer the questions as to how properties of the baryons undergo changes in nuclear medium to describe the neutron star. While medium modifications of heavy hadrons in nuclear matter have been much less studied than those of light ones, there have been several works [7–10] on charmed nuclei soon after the charmonium \( J/\psi \) and \( \Sigma_c \) were found [11,12]. Then the heavy baryons in nuclear matter have been sparsely studied [13–17]. Recently, interest in heavy hadrons was renewed as the experiments on them including exotic heavy hadrons have yielded unprecedented findings [15–22] (see a recent review for the status of experiments on nonstandard heavy hadrons [23]). This has also triggered the investigation on heavy baryons in nuclear matter [24–29] (see also a review [30]). Since there are no experimental data on how the heavy baryons undergo changes in nuclear medium and in nuclei, it is of great importance to study the medium modification of them theoretically to guide future experiments.

In the present work, we investigate the in-medium modification of the singly heavy baryons based on an SU(3) chiral soliton model with a bound-state approach [31]. The model is distinguished from a usual bound-state approach in which a heavy baryon emerges as a bound state of a heavy meson and a baryon as a chiral soliton [32–34], regarding both the hyperons and kaons as heavy particles. In Ref. [31], on the other hand, the singly heavy baryons can be constructed as a bound state of a heavy meson and an SU(3) baryon as a soliton. This means that the Wess-Zumino-Witten (WZW) term [37] exists in the model and constrains the allowed representations for the collective Hamiltonian together with the kinetic term of the heavy mesons. What is interesting is that the collective Hamiltonian describes the bosonic soliton as if it had come from the \( N_c - 1 \) valence quarks [35,36]. Then the singly heavy baryon emerges as a coupled state with the single heavy quark. In addition, the model contains the vector-meson degrees of freedom, which improves the semiclassical binding energy. This SU(3) chiral soliton model with a bound-state approach is simpler and clearer than those mentioned previously, so that it can be easily modified in medium. Since we have already constructed the in-medium modified soliton model with vector-meson degrees of freedom [40], it is rather straightforward to modify the effective Lagrangian for this model. The only new ingredient is the heavy-meson degrees of freedom, which was naturally introduced in such a way that it complies with heavy-quark spin symmetry. Thus, we will scrutinize how the in-medium modification of the heavy meson influences the mass spectrum of the singly heavy baryons in nuclear matter. This is a very interesting and remarkable feature of the present approach, since it relates directly the medium modification of the heavy mesons to the singly heavy baryons in the same manner as the change of the vector mesons affects the nucleon and \( \Delta \) isobar in nuclear matter within the Skyrme models.

We sketch the present work as follows: In Section II, we introduce the medium functionals to modify the effective chiral Lagrangian. Since the medium modification of the pion and the vector mesons was done in Ref. [40],

¹ When \( N_c \) is taken to be three, it corresponds to the light diquark. The present model bears a certain similarity to the pion mean-field approach for the singly heavy baryons [38,39].
we follow the same method. We introduce an additional parameter to explain how the heavy degrees of freedom undergo changes in nuclear matter. This parameter governs the dependence of the mass splitting of singly heavy baryons on the nonzero density. In Section III, we show how the in-medium parameters can be fixed by reproducing the bulk properties of nuclear matter. In Section IV, we discuss the numerical results for the mass spectra of the baryon antitriplet and sextet. Section V is devoted to summary and conclusions of the present work.

II. FORMALISM

In the present section we first formulate the medium modifications of the effective chiral Lagrangian and SU(3) light baryons. Then we proceed to implement the changes of the heavy mesons in nuclear matter.

A. Light mesons in nuclear matter

A medium modified Lagrangian of the system consisting of the light pseudoscalar and vector mesons in nuclear matter within the SU(2) framework is given in Ref. [40]. We generalize the idea proposed in Ref. [40], which provides a practical method to describe how the nucleon is modified in nuclear medium, so that we can study the changes of properties of heavy baryons. Thus, we reformulate the medium-modified Lagrangian starting from the free-space Lagrangian presented in Refs. [31, 41]. Note that Ref. [40] without medium modifications is in line with Refs. [31, 41] in the flavor SU(2) case. This means that all the methods developed in Ref. [40] can be easily generalized to the flavor SU(3) case, so that the effective Lagrangian can be written as

$$\mathcal{L}^{\text{light}} = \mathcal{L}_\pi^* + \mathcal{L}_V + \mathcal{L}_{\pi V} + \mathcal{L}_{\pi\omega},$$

(1)

$$\mathcal{L}_\pi^* = \frac{F_{\pi}^*}{8} \text{Tr}[\partial_\mu U \partial_\mu U^\dagger],$$

(2)

$$\mathcal{L}_V = \frac{1}{4} \text{Tr}[F_{\mu\nu}^*(\rho)F_{\mu\nu}^*],$$

(3)

$$\mathcal{L}_{\pi V} = -\frac{m_{\pi}^2}{2g^2} \text{Tr}[(g^* \rho^*_\mu - \bar{v}_\mu^*)^2],$$

(4)

where $F_{\pi}^*$ is the medium-modified pion decay constant with its free space value is $F_\pi = 132 \text{ MeV}$. $m_{\pi}^*$ and $g^*$ stand for the medium-modified mass of the vector mesons and vector coupling constants, respectively. Here the first three terms in Eq. (1) respectively describe pseudoscalar, vector and pseudoscalar-vector interaction terms in nuclear matter in Euclidean space. The first term $\mathcal{L}_\pi^*$, Eq. (2) denotes the well-known Weinberg kinetic term for the pseudoscalar meson fields in a unitary form

$$U(r) = \exp[i\lambda^a \pi^a] = \xi^2,$$

(5)

where $\pi^a$ designate the pseudoscalar meson fields with $a = 1, \ldots, N_f - 1$. In the flavor SU(2), the pion fields are coupled to the spatial coordinates, which is known as the hedgehog Ansatz or more generally the hedgehog symmetry. It provides a minimal generalization of spherical symmetry [42]. So, the pion fields can be expressed as

$$\pi^a(r) = n^a F(r)$$

(6)

with $n^a = x^a/r$ and $r = |r|$. $F(r)$ is called the profile function of the chiral soliton, which will be found by solving the classical equations of motion for $\pi^a$ obtained from the effective Lagrangian. In the case of $N_f = 3$, the simplest generalization can be acquired by the trivial embedding [37]

$$\xi = \left( \begin{array}{c} \exp[i(\hat{n} \cdot \tau F(r))] \\ 0 \\ 1 \end{array} \right),$$

(7)

which preserves the hedgehog symmetry. $\tau$ is the usual Pauli matrices.

The $3 \times 3$ field strength tensor in Eq. (3) is expressed as

$$F_{\mu\nu} = \partial_\mu \rho^*_\nu - \partial_\nu \rho^*_\mu - ig^* [\rho^*_\mu, \rho^*_\nu],$$

(8)

where

$$\rho_\mu^* = \left( \frac{1}{\sqrt{2}} (\omega_\mu + \tau^a \rho_\mu^a) \begin{array}{c} 0 \\ 1 \end{array} \right),$$

$$\rho_4^a = \frac{i a k_a n^b G(r)}{\sqrt{2g^*}} r, \quad \rho_4^0 = 0,$$

$$\omega_1 = 0, \quad \omega_4 = i \omega(r).$$

(9)

Here $G(r)$ and $\omega(r)$ denote respectively the profile functions for the $\rho$ and $\omega$ mesons. The medium-modified vector coupling $g^*$ takes the value $g = 3.93$ in free space. The third term $\mathcal{L}_{V\pi}$, Eq. (4) describes the pseudoscalar-vector interactions, where the currents $v_\mu^\pm$ are written as

$$v_\mu^\pm = \frac{i}{2} (\xi \partial_\mu \xi^\dagger \pm \xi^\dagger \partial_\mu \xi)$$

(10)

and $m_{\pi}^*$ is taken to be the $\rho$ meson mass, i.e. $m_{\pi} = 770 \text{ MeV}$ in free space. Finally, the last term $\mathcal{L}_{\pi\omega}$ represents the WZW term

$$\mathcal{L}_{\pi\omega} = \frac{3}{\sqrt{2}} g^* \omega_\mu B_\mu,$$

(11)

which couples the $\omega_\mu$ meson to the baryon current

$$B_\mu = \frac{e^\mu_\rho}{{24\pi}^2} \text{Tr} \{ (U^\dagger \partial_\rho U)(U^\dagger \partial_\lambda U)(U^\dagger \partial_\beta U) \}.$$

(12)
The profile functions satisfy the following boundary conditions
\[
F(0) = \pi, \quad G(0) = 2, \quad \omega(0) = 0,
F(\infty) = G(\infty) = \omega(\infty) = 0,
\]
which yield the finite energy solutions. The details of the numerical minimization process can be found in Ref. [40].

B. Heavy mesons in nuclear matter

We take the heavy meson part of the Lagrangian from Ref. [31]. Note that the Lagrangian was constructed in a way that heavy-quark flavor-spin symmetry is satisfied. We modify the heavy mesons in the same manner as the light degrees of freedom, whereas the heavy field. The wave functions for the heavy mesons are expressed as
\[
\frac{L_{\text{heavy}}}{M} = iV_\mu \text{Tr}[H(\partial_\mu - i\alpha g^s \rho_\mu - i(1 - \alpha)v_\mu^+)\bar{H}]
+ i\mu \text{Tr}[H\gamma_\mu\gamma_5\nu_\mu\bar{H}]
+ \frac{ic}{m_v} \text{Tr}[H\gamma_\mu\gamma_5F_{\mu\nu}^s(\rho)\bar{H}],
\]
where \( H \) denotes the heavy superfield expressed by
\[
H = \frac{1 - i\gamma_5 V_i}{2}(i\gamma_5 P + i\gamma_\rho Q_\rho), \quad \bar{H} \equiv \gamma_4 H^{\dagger} \gamma_4.
\]
P and \( Q_\rho \) designate respectively the pseudoscalar and vector heavy mesons. \( V_\mu \) stands for the four-vector velocity of the heavy quark, which imposes the superselection rule [33] [34] on the heavy field. The values of the remaining constants are given by \( \alpha \approx -2.9, \ d = 0.53 \) and \( c = 1.6 \), which are determined by using the experimental data [11] [35].

In the rest frame of the heavy meson \( (V_i = 0) \), the \( 4 \times 4 \) heavy superfield \( \bar{H} \) in Eq. (15) has only the nonvanishing elements in the lower-left \( 2 \times 2 \) sub-block:
\[
\bar{H} = \begin{pmatrix}
0 & 0 \\
\bar{H}_{lh} & 0
\end{pmatrix},
\]
where the index \( l \) of the submatrix represents the spin of the light degrees of freedom, whereas \( h \) denotes that of the heavy quark. The wave functions for the heavy field are then written as [11]
\[
\bar{H}_{lh}^b = \left\{ \begin{array}{ll}
\frac{1}{\sqrt{4\pi M}}(\hat{n} \cdot \vec{\tau})_l m_l \epsilon_{lm} u(r) \chi_h & \text{if } b = 1, 2, \\
0 & \text{if } b = 3,
\end{array} \right.
\]
where \( \chi_h \) stands for the heavy-quark spinor and \( u(r) \) is the radial wave function derived in the classical approximation, so that the heavy quark is localized at the origin. Thus, it is given as \( r^2 |u(r)|^2 \approx \delta(r) \), which is an exact expression in the infinite mass limit of the heavy quark \( (m_Q \to \infty) \).

Substituting the Ansatz (17) into the heavy-meson part of the Lagrangian, we obtain the classical binding potential in nuclear matter
\[
W_0^* = -\frac{3d}{2}F'(0) + \frac{3c}{m_v g^s}G''(0) - \frac{\alpha g^s}{\sqrt{2}} \omega(0).
\]
For more details we refer to Ref. [41].

C. Symmetry breaking terms

In Ref. [46], it was shown in detail how the terms for the explicit breaking of flavor SU(3) symmetry can be derived based on the fundamental QCD Lagrangian. The explicit breaking of chiral symmetry is also considered in medium, since it plays an important role in justifying the medium modifications of the nonstrange baryons with regards to the phenomenology of pion-nucleon scattering and the properties of pionic atoms [47] [48] (for more discussions, see also Refs. [49] [50]). We assume isospin symmetry. In addition, we also need to take into account the explicit breaking of flavor SU(3) symmetry breaking. The corresponding medium-modified term in the effective Lagrangian is written as [4]

\[
\mathcal{L}_{SB}^* = \frac{1}{8} m_s^2 F^a_\pi^2 \text{Tr}[\mathcal{M}(U + U^\dagger - 2)]
+ \frac{M_s - M}{2(x - 1)} \text{Tr}(H\xi M\xi H^\dagger) + \text{H.c.},
\]
where \( 3 \times 3 \) matrix \( \mathcal{M} \) with isospin symmetry is given as
\[
\mathcal{M} = T + x S
\]
with the diagonal matrices \( T = \text{diag}(1, 1, 0) \) and \( S = \text{diag}(0, 0, 1) \). The constant \( x \) is defined by the ratio of the currents quark masses
\[
x = \frac{2m_s}{m_u + m_d} = 31.5
\]
and \( M_s - M = m(D^+) - M(D^+) = 100 \text{MeV} \), which are accurate enough up to 2% [11]. In the SU(2) sector the Lagrangian [11] reproduces the standard medium-modified chiral symmetry breaking term.

D. Zero-mode quantization

The quantized solitons appear after the zero-mode quantization, which is performed by rotating the class-

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3 See Eq.(2.10) in Ref. [36] and related discussions.

4 A free-space form of the Lagrangian is given in Ref. [11].
sical static configuration in the following way

\[
\xi(x, t) = A(t)\xi(x)A^\dagger(t), \\
\rho_\mu(x, t) = A(t)\rho_\mu(x)A^\dagger(t), \\
\bar{H}(x, t) = A(t)\bar{H}(x),
\]

where the classical Ansätze are defined in Eqs. (7), (9) and (16), and \(A(t)\) denotes an SU(3) rotational matrix in flavor space. The corresponding angular velocities \(\Omega_K\) of the rotation are defined by

\[
A^\dagger \dot{A} = \frac{i}{2} \sum_{k=1}^{8} \lambda_k \Omega_k,
\]

where the \(\lambda_k\) are the usual Gell-Mann matrices. Substituting Eq. (22) respectively into the light (1) and heavy (14) parts of the Lagrangian, we get the collective Lagrangian

\[
L^* = -M^*_c - (M^* + W_0)P + \frac{\alpha^2}{2} \sum_{i=1}^{3} \Omega_i^2 + \frac{\beta^2}{2} \sum_{k=4}^{7} \Omega_k^2 - \frac{\sqrt{3}}{2} \Omega_8 + \frac{\sqrt{3}}{6} \Omega_8 \chi^\dagger \chi P,
\]

where the heavy meson mass in nuclear medium \(M^*\) by added as an overall energy shift. The medium-modified classical soliton mass is given by the expression \(^5\)

\[
M^*_c = 4\pi \int_0^\infty dr \left\{ \frac{F^*_{\pi,s}^2}{4} (2\sin^2 F + r^2 F'^2) + m^*_{\pi,s} F^*_{\pi,s}^2 r^2 \sin^2 F + \frac{k}{2} F^2 (G - 1 + \cos F)^2 + \frac{1}{2g^*_{t}^2} \left( G'^2 + \frac{G^2 (G - 2)^2}{2r^2} \right) - \frac{r^2}{2} (m^*_{\nu,s}^2 \omega^2 + \omega'^2) + \frac{3g^*_{s}}{2\sqrt{2}\pi^2} \omega F' \sin^2 F \right\}.
\]

Note that in formulating the classical soliton mass we have followed the conventional way of including the symmetry breaking term (the pion mass term in \(M^*_c\)) in Eq. (19). The classical equation of motion, its asymptotic solutions corresponding to the classical soliton configuration, the numerical method, and the solutions are presented in Appendix A. The in-medium modified moments of inertia of the rotating soliton are given by\(^6\)

\[
\alpha^2 = \frac{4\pi}{3} \int dr \left\{ F^*_{\pi,t}^2 r^2 \sin^2 F + \frac{2G^2}{g^*_t^2} + 2kF^*_{\pi,t}^2 \sin^2 \frac{F}{2} \right\},
\]

\[
\beta^2 = 2\pi \int dr \left\{ F^*_{\pi,t}^2 r^2 \sin^2 F + \frac{G^2}{g^*_t^2} + 4kF^2_{\pi,t}^2 \sin^4 F \right\},
\]

The factor \(P\) in Eq. (24) is a projection operator onto the heavy baryon subspace. When \(P\) picks only up the heavy-quark sector, \(W_0\) is the classical binding energy given in Eq. (18). The last term in Eq. (24) originates from the kinetic term for the heavy mesons, i.e. \(iV_{\mu} \bar{T}r (H_{\mu} \bar{H})\). The second lower index \(h\) in the heavy super-field \(17\) stands for the spin of the heavy-quark degree of freedom, so that the heavy-quark spinor satisfies the normalization \(\chi^\dagger \chi = \delta_{hh}\) that appears as the last term in Eq. (24).

The collective Lagrangian does not have the quadratic terms for \(\Omega_8\) since both the chiral field \(U(r)\) and the heavy super-field commute with \(\lambda_8\). The linear-order term with \(\Omega_8\) is associated with the baryon number, which we now discuss. Introducing the canonical conjugate momenta \(R_k\)

\[
R_k = \frac{\partial L}{\partial \dot{\Omega}_k}, \quad k = 1, \ldots, 7,
\]

we find

\[
R_k = \begin{cases} 
-\alpha^2 \Omega_k, & k = 1, 2, 3 \\
-\beta^2 \Omega_k, & k = 4, 5, 6, 7 \\
\frac{1}{\sqrt{3}}, & k = 8.
\end{cases}
\]

The 8\(th\) component of \(R_k\) is constrained by the WZW term and the heavy meson kinetic term. In the usual SU(3) Skyrme model for light baryons, the baryon number arises from the WZW term. So, \(R_8\) is constrained to be \(R_8 = \frac{\sqrt{2}}{2}\), which is identified as the right hypercharge \(Y_R\) which is constrained to be \(R_8 = \frac{2}{\sqrt{3}} R_8 = N_c/3\). This constraint allows one

\(^5\) Note that in the expression of the classical soliton mass only the spatial parts of the constants \(F^*_{\pi,s}\) and \(g^*_s\) appear.

\(^6\) Note that in the expressions of the moments of inertia only the temporal parts of the constants \(F^*_{\pi,t}\) and \(g^*_t\) appear.
to take only the SU(3) representation with zero triality. When we consider the singly heavy baryons as in this work, however, the effect of the kinetic term in the heavy Lagrangian [4] constrains further $R_8 = \frac{1}{\sqrt{3}}$. This has a profound physical meaning. The right hypercharge for the singly heavy baryons is then $Y_R = (N_c - 1)/3$ and this allows us to take the presentations for the singly heavy baryons $3$ with $J = 1/2$ and $6$ with $J = 1/2$ and $J = 3/2$. Moreover, the soliton for the singly heavy baryons appears as that consisting of $N_c - 1$. Thus, even though the present work does not contain the explicit quark degrees of freedom, it has a certain similarity to the pion mean-field approach [38] developed recently.

We then arrive at the collective Hamiltonian of the model in the heavy sector

$$H = M^*_{cl} + (M^* + W^*_0)P$$

$$+ \frac{1}{2} \left( \frac{1}{\alpha^2} - \frac{1}{\beta^2} \right) J_s (J_s + 1)$$

$$+ \frac{1}{6\beta^2} \left[ (p^2 + pq + q^2) + 3(p + q) \right]$$

$$- \frac{1}{2\beta^2} R_8^2$$

(30)

in the $(p, q)$ representation of flavor SU(3) symmetry. Diagonalizing the collective Hamiltonian, we find the collective wave functions for the singly heavy baryons as [31, 51]

$$\Psi_{\text{heavy}}(n, YII_3; J J_3, J_s; A) = \sum_{h=1}^2 C_{M_s}^{J_s} \frac{1}{2} J \langle J_s - M_s \mid \sqrt{\text{dim} N_{D_Y}^{(n)*}} \rangle$$

$$\times (-1)^{J_s - M_s} \sqrt{n} D_Y^{(n)*}_{II_3, Y_R J_s - M_s} (A) \chi_h,$$

(31)

where $J_s$ and $M_s$ are the soliton spin and its $3^{rd}$ component. $C_{M_s}^{J_s} \frac{1}{2} J$ denote the SU(2) clebsch-Gordan coefficients that couple the soliton and the heavy quark.

The effects of explicit flavor SU(3) symmetry breaking are taken as a perturbation [41]. Then the collective Hamiltonian can be written as

$$H = H_{\text{sym}} + H_{\text{br}},$$

(32)

where the part from explicit SU(3) symmetry breaking is expressed as

$$H_{\text{br}} = \tau D_{88}(A), \quad \tau = \tau^*_{\text{light}} + \tau^*_{\text{heavy}},$$

(33)

$$\tau^*_{\text{light}} = \frac{2\pi}{3} (1 - x) m^*_p F^*_\pi \int_0^\infty dr r^2 \sin \frac{F}{2},$$

(34)

$$\tau^*_{\text{heavy}} = \frac{M^*_s - M^*}{3}.$$  

(35)

Here $M_s$ means the heavy meson mass with the $s$ quark. The value of $\tau_{\text{heavy}}$ in free space is fixed by the relation $M_s - M = m(D^*_s) - m(D^+) = m(D^*_s) - m(D^) = 100$ MeV [41].

### III. NUCLEAR MATTER

For simplicity we will consider isospin symmetric infinite nuclear matter in order to discuss the medium effects on the baryon properties. We follow the method presented already in Refs. [40, 50] and introduce the medium functions as

$$F_{\pi, t}^* = \sqrt{\alpha_{p,t} F_{\pi}} \quad g_t^* = \sqrt{\zeta_t g},$$

$$F_{\pi, s}^* = \sqrt{\alpha_{p,s} F_{\pi}} \quad g_s^* = \sqrt{\zeta_s g}.$$  

(36)

The masses of the pseudoscalar and vector mesons undergo the changes in nuclear matter, so that we consider the medium effects for them as [40, 50]

$$m_{\pi}^* = \sqrt{\frac{m_{\pi}}{\alpha_{p,s}}} m_{\pi}, \quad m_{\nu}^* = \sqrt{\frac{m_{\nu}}{\zeta_s m_{\nu}}}.$$  

(37)

In addition, we define four different medium functions by three constants in the following way [50]

$$\alpha_{p,s} = 1 + \frac{C_1 \lambda - C_2 \lambda}{1 + \frac{C_3 \lambda - \sqrt{\zeta_s}}{\sqrt{\zeta_s} - \sqrt{\zeta_t}}},$$

$$\alpha_{p,t} = 1 + \frac{C_1 \lambda - C_2 \lambda}{1 + \frac{C_3 \lambda - \sqrt{\zeta_s}}{\sqrt{\zeta_s} - \sqrt{\zeta_t}}}. $$

(38)

The remaining medium functions will not affect much nuclear matter properties, so that they are chosen to be [40]

$$\alpha_{m} = 1 - \frac{4b_0 \eta \rho}{m^2}, \quad \eta = 1 + \frac{m_{\pi}}{m_N},$$

(39)

where $b_0 = -0.024 m_{\pi}^{-1}$ is taken from the data on pionic atoms [47].

We fit the values of the constants $C_{1, 2, 3}$ by reproducing the bulk properties of isospin symmetric nuclear matter near the normal nuclear-matter density $\rho_0 = 0.16$ fm$^{-3}$. We introduce the normalized nuclear matter density $\lambda = (\rho_p + \rho_n)/\rho_0$ in terms of the proton and neutron distribution densities. Then the volume term in the binding energy formula is defined by

$$\varepsilon_V(\lambda) = \frac{Z M_{p}^*(\lambda) + N M_{n}^*(\lambda)}{A} - \frac{Z M_{p} + N M_{n}}{A}$$

$$= M_N^*(\lambda) - M_N,$$

(40)

where

$$M_N^* = \frac{M_N^*(\lambda) + M_N^*(\lambda)}{2}$$

$$= M_N^*(\lambda) + \frac{3 \sqrt{\alpha^2(\lambda)} - \frac{3}{4} \beta^2(\lambda)}{\sqrt{\zeta_{p,t}(\lambda) \sqrt{1 - \frac{3}{10}}}}.$$  

(41)

Utilizing the stability condition $(\partial \varepsilon_V(\lambda)/\partial \lambda)_{\lambda=1} = 0$, we find the volume energy value $\varepsilon_V(1) = 16$ MeV and the compressibility of the symmetric matter $K_0 =$
\( \{9 \lambda^2 (\partial^2 \varepsilon \lambda^2)\}_{\lambda=1} = 220 \text{MeV} \), which are used for fixing the values of constants in the density functions in Eq. (38)

\[
C_1 = -0.130275, \\
C_2 = 0.488595, \\
C_3 = -0.203271.
\]

The masses of baryons in this model are quite overestimated, e.g. the nucleon mass in free space is \( M_N = 1909 \text{MeV} \). This is well known as a fundamental problem in any chiral soliton models. Thus, instead of using this value, we fix the model parameters in free space such that the masses of the singly heavy baryons are reproduced properly in free space. See Table I for the values of singly heavy-baryon masses in free space in comparison with the experimental data. Following Ref. [40], we introduce the scaling factor \( f_s = M_N/M_N^{\text{exp}} = 1909/1940 = 2.282 \) and use it to reproduce nuclear matter properties at the normal nuclear matter density \( \rho_0 \). The density dependencies of the volume energy for isospin symmetric matter is shown in Fig. 1. The solid curve draws the present results including the inverse scale factor \( f_s^{-1} \), where the binding energy per nucleon at the normal nuclear matter density is given by \( a_0 = \varepsilon_N(1) = -16 \text{MeV} \). It is compared with the Akmal-Pandharipande-Ravenhall (APR) predictions [52] denoted by the blue squares. One can see that the present results are in good agreement with the APR predictions in the density region \( \rho \in [0, 2 \rho_0] \). When the value of the compressibility is taken to be \( K_0 = 220 \text{MeV} \), we get the third derivative parameter of equations of state \( Q_0 = -309.927 \text{MeV} \), which is in qualitative agreement with many other approaches.


![FIG. 1. The dependence of the volume energy on the normalized nuclear matter density \( \lambda = \rho/\rho_0 \). The solid curve depicts the result from this work. The squares represent the Akmal-Pandharipande-Ravenhall (APR) predictions [52].](image)

The density dependence of the pressure in isospin symmetric matter is shown in Fig. 2. While the pressure in isospin symmetric matter is proportional to the first derivative of the volume energy \( p(\lambda) = \rho_0 \lambda^2 (\partial \varepsilon / \partial \lambda) \), the results including the inverse scale factor \( f_s^{-1} \) also holds for it. The present result for the pressure is found to be within the allowed region, given the range of the density \( \rho \in [0, 3 \rho_0] \).

In Fig. 2 we draw also the data extracted from various experiments. The present result is slightly lower than the data from the giant monopole resonance (GMR) experiments [55, 56] for heavy nuclei, which are depicted by the long-dashed curve. Reference [55] analyzed the flow experimental data on \( ^{197} \text{Au} \) nuclei collision, which is illustrated by the red-colored region corresponding to the equation of state for symmetric nuclear matter at zero temperature. The valid range of the pressure was extended by taking into account the mass-radius relation of neutron stars from observational data [56, 57]. In Fig. 1 the corresponding region is denoted by “Flow + 20%”. One can see that the present equation of state lies within the predicted range. The data taken from the kaon production at high-energy nucleus-nucleus collision [53, 54, 58] are drawn as the green-colored region in the range of \( 1.2 \leq \lambda \leq 2.2 \). The present result is also located within this region. We have also considered the two different values of \( K_0 \), i.e., \( K_0 = 240 \text{MeV} \) and \( K_0 = 200 \text{MeV} \) but the results for the pressure are also in agreement with these data. Thus, the present theoretical framework describes the bulk properties of symmetric nuclear matter rather well.

IV. SINGLY HEAVY BARYONS IN NUCLEAR MEDIUM

We now proceed to investigate the singly heavy baryons in nuclear medium. Since the mass of the heavy meson \( M^* \) is involved in the Hamiltonian Eq. (30) as an overall energy shifting factor, we have to examine how the masses of the heavy mesons undergo the change, i.e., to see how \( \Delta M^* = M^* - M \) depends on the nuclear density.
In the present work $M$ is taken to be the $D$-meson mass $M_D$. The properties of heavy mesons and in particular those of the $D$ meson in nuclear environment are discussed within the several theoretical approaches \[53\] \[67\]. A general consensus has not been reached yet, whether the $D$-meson mass would be dropped or raised in nuclear matter. Thus, we will consider in this work three possible cases of the changes of the $D$ meson in the following. We introduce one more density-dependent constant $C_4$ to parametrize the density dependence of the $D$-meson mass in the following linear form

$$M^* = M_D^0 = (1 + C_4 \lambda) M_D.$$ (43)

Moreover, since we focus on the properties of isospin symmetric matter, we ignore the isospin effects on the masses of the heavy mesons. This means that the density dependence of the symmetry breaking coefficient from the heavy-quark sector $\tau_{\text{heavy}}^*$ given in Eq. (35) should also be modified:

$$\tau_{\text{heavy}}^* = (1 + C_4 \lambda) \tau_{\text{heavy}}.$$ (44)

In the present work we consider $C_4$ as a free parameter and examine how the masses of the singly heavy baryons vary with $C_4$ changed. As mentioned previously, we will take the three different cases: the increment of the $D$-meson mass ($C_4 > 0$), no change ($C_4 = 0$), and the dropping mass of the $D$ meson ($C_4 < 0$) in nuclear matter.

We are now in a position to discuss the mass shifts of the singly heavy baryons $\Delta M_B = M_B^* - M_B$ in nuclear matter. We focus our attention on the properties of the singly charmed baryons. Given the different values of free parameter $C_4$, the changes of the heavy baryon masses in nuclear matter at the normal nuclear matter density are shown in Table I. We also provide the values of the masses in free space are also given. All the masses and their shifts are given in units of MeV.

| Baryon | $M_B$, $\rho = 0$ | $\Delta M_B$, $\rho = \rho_0$ |
|--------|------------------|------------------|
| Exp. \[68\] | This work | $C_4 = -0.1$ | $C_4 = 0$ | $C_4 = 0.1$ |
| $\Lambda_c$ | 2286.5 | 2286.0 | $-166.91$ | $21.22$ | $209.34$ |
| $\Xi_c$ | 2469.4 | 2437.8 | $-132.30$ | $54.48$ | $241.25$ |
| $\Sigma_c$ | 2453.5 | 2564.5 | $-86.50$ | $101.54$ | $289.57$ |
| $\Xi'_c$ | 2576.8 | 2646.8 | $-69.89$ | $117.27$ | $304.43$ |
| $\Omega_c$ | 2695.2 | 2721.6 | $-54.23$ | $132.17$ | $318.56$ |

The density dependence of the mass shifts for the baryon antitriplet $\Delta M_B$ are shown in Fig. 3. One can see that the masses of the baryon antitriplet are linearly proportional to the values of $C_4$. We find that those of the baryon sextet exhibit a similar tendency, which are drawn in Fig. 3 with the two different values of $C_4$ given. The masses of the baryon sextet drop less than those of the baryon antitriplet with the negative values of $C_4 = -0.1$ taken. In the case of the null or positive values of $C_4$, those of the baryon sextet grow slower in nuclear matter than those of the baryon antitriplet.

In order to analyze the mass splittings between the baryon sextet with spin 1/2 and 3/2, we need to introduce the hyperfine interaction with the anomalous chromomorphic moment \[68\]

$$H_{L,H}^* = \frac{2}{3} \kappa \frac{m_c}{M^*_{cl}} \mathbf{S}_L \cdot \mathbf{S}_H \equiv \frac{2}{3} \xi \frac{M_{cl}}{M^*_c} \mathbf{S}_L \cdot \mathbf{S}_H,$$ (45)

where $\kappa$ denotes the anomalous chromomorphic moment and $m_c$ stands for the charm quark mass. We assume that $\kappa$ is not influenced in nuclear matter. $\mathbf{S}_{L,H}$ designate the spins of the soliton and heavy quark coming from the heavy meson, respectively. We introduce the coefficient $\xi$ with $M_{cl}$ in free space. We fit the value of $\xi/m_c = \kappa/(m_c M_{cl})$ $\simeq 68.1$ MeV in free space to the experimental values of the masses for the baryon sextet.
The mass shift of the baryon sextet including the hyperfine splitting in nuclear matter are listed in Table I.

One can see that if the anomalous chromomagnetic moment $\kappa$ and the current quark mass $m_c$ are density independent, hyperfine mass splittings are enhanced in nuclear matter because of the soliton mass shown in Eq. (45).

V. SUMMARY AND CONCLUSIONS

In the present work we investigated the medium modification of heavy baryon masses in nuclear matter within the framework of the SU(3) soliton model including the light pseudoscalar and vector mesons, and the heavy meson. We introduced the medium effects in the effective chiral Lagrangian and examined the bulk properties of nuclear matter. The results are consistent with the empirical and experimental data. It was shown that the results for the mass shifts of the singly heavy baryons depend strongly on the medium modification of the heavy meson mass in nuclear matter. In particular, the heavy baryon masses gradually increase if the heavy meson mass remains constant in nuclear matter. On the other hand, if the heavy meson mass becomes larger in nuclear matter, then the baryon masses grow faster as the nuclear matter density increases. With the mass of the heavy meson decreased in nuclear matter, we found that the masses of the singly heavy baryons also lessen as the nuclear matter density increases. We conclude that the mass dropping of the singly heavy baryons crucially depends on information about how the mass of the heavy meson undergoes changes in nuclear matter within the present framework.

ACKNOWLEDGMENTS

The present work was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), Grant-No. 2021R1A2C2093368, 2018R1A5A1025563 (H.-Ch.K.), and 2020R1F1A1067876 (U. Y.).

Appendix A: Equations of Motion

Equations for the profile functions $F$, $G$ and $\omega$ are obtained by the minimization of the classical soliton mass Eq. (25) and have the following forms

$$ F'' = -\frac{2}{r} F' - \frac{1}{\alpha_{p,s} r^2} [2k(G - 1) \sin F + (k - \alpha_{p,s}) \sin 2F] + \frac{\alpha_m}{\alpha_{p,s}} m_s^2 \sin F $$

$$ G'' = \frac{G(G - 1)(G - 2)}{r^2} + \frac{\zeta_s m_c^2 (G - 1 + \cos F)}{r^2} $$

$$ \ddot{\omega} = -\frac{2}{r^2} \dot{\omega} + \frac{3g}{2\sqrt{2\pi^2 F_s}} \frac{F'}{r^2} \sin^2 F $$

where $\ddot{\omega}$ is defined as $\ddot{\omega} = \omega/F_s$. The solution near the origin $r \to 0$ is found to be

$$ F = \pi + \alpha_F r, \quad G = 2 + \alpha_G r^2, \quad \ddot{\omega} = \omega_0 + \alpha_\omega r^2. $$
At large distances $r \to \infty$ one gets
\begin{equation}
F = \frac{\beta_F}{r^2} \left( 1 + \sqrt{\frac{\alpha_p m_r}{\alpha_p m_e}} m_x r \right) e^{-\sqrt{\frac{\alpha_p}{\alpha_p m_r}} m_x r},
\end{equation}
\begin{equation}
G = \frac{\beta_G}{r^2} e^{-2\sqrt{\frac{\alpha_p}{\alpha_p m_r}} m_x r}, \quad \bar{\omega} = \frac{\beta_{G,\omega}}{r^3} e^{-3\sqrt{\frac{\alpha_p}{\alpha_p m_r}} m_x r}.
\end{equation}
The constants $\alpha_{F,G,\omega}$ and $\beta_{F,G,\omega}$ are found by the numerical calculations.

**Appendix B: Solutions**

The numerical calculations for finding the values of $C_{1,2,3}$ are performed by an iteration method. As an initial step the solutions in free space are used (see the first row in Table III). The corresponding solutions in free space are
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Iteration number & $C_1$ & $C_2$ & $C_3$ \\
\hline
0 & -0.110060 & 0.460658 & -0.171721 \\
3 & -0.130249 & 0.488556 & -0.202932 \\
8 & -0.130275 & 0.488595 & -0.203271 \\
9 & -0.130275 & 0.488595 & -0.203271 \\
\hline
\end{tabular}
\caption{The values of medium parameters from the iterations.}
\end{table}

and in nuclear matter at the saturation density are shown in Fig. 5. From Table III one can see that the solutions are almost saturated after the third iteration and not so much different from the free space ones. This is also seen from Fig. 5. Therefore, the factorization of the medium functions from the model functionals can be considered as a possible choice for the qualitative discussions.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{The solutions in free space (solid curves) and in nuclear matter at the saturation density $\rho_0$ (dashed curves).}
\end{figure}

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