Distribution of epicenters in the Olami-Feder-Christensen model

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Abstract

We found that the well established Olami-Feder-Christensen (OFC) model for the dynamics of earthquakes is able to reproduce a new striking property of real earthquake data. Recently, Abe and Suzuki found that the epicenters of earthquakes could be connected to generate a graph, with properties of a scale-free network of the Barabási-Albert type. However, only the non conservative version of the Olami-Feder-Christensen model is able to reproduce this behavior. The conservative version, instead, behaves like a random graph. Those findings, besides indicating the robustness of the model to describe earthquake dynamics, reinforce that conservative and nonconservative versions of the OFC model are qualitatively different, and propose a completely new dynamical mechanism that, without an explicit rule of preferential attachment, is able to generate a free scale network. The preferential attachment is in this case a “by-product” of the long term correlations built by the self-organized critical state. We believe that the detailed study of the properties of this network can reveal new aspects of the dynamics of the OFC model, contributing to the understanding of self-organized criticality in non conserving models.

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The concept of self-organized criticality was introduced by Bak, Tang and Wiesenfeld [1] in 1987, as a possible explanation of scale invariance in nature. To illustrate their basic ideas, they presented a cellular automaton model, the sandpile model, so called because of a possible analogy between its dynamical rules and the movement of sand or snow in avalanches. Since this seminal work, a great number of cellular automata and coupled map models have been investigated, in an attempt to elucidate the essential mechanisms hidden in such a wide class of different non-linear phenomena whose statistics of events (or avalanches) are governed by power-laws. However, up to now, one still lacks from a general theoretical framework for self-organized criticality. Success in analytical investigations have been achieved in many models. For a revision see, for instance, [2, 3].

In this context, a model that has been widely studied in the literature is the Olami-Feder-Christensen (OFC) model for the dynamics of earthquakes. The original OFC model, introduced in 1992 [4], is a two-dimensional coupled map model defined on a square lattice, whose dynamical rules were inspired in a spring-block model proposed to describe the dynamics of earthquakes. Earthquakes, in the real world, are associated with many power-laws, the most known of them being the Gutenberg-Ritcher law for the distribution of avalanche energies. The OFC model assigns - to each site of a square lattice - a real variable $z_{i,j}$ (energy or tension), initially chosen at random in the interval $[0, z_c)$, where $z_c$ is a threshold value. $z_{i,j}$ increases slowly throughout the lattice and each time that, for a given site, $z_{i,j}$ exceeds $z_c$, the system relaxes. A fraction $\alpha z_{i,j}$ of the tension of site $(i, j)$ is then distributed to each of its nearest neighbors. As a consequence, the tension of some of its neighbors may also exceed $z_c$, generating an ‘avalanche’ that will only stop when $z_{i,j} < z_c$ again for all sites of the lattice. We have assumed open boundaries in our simulations.

Within the OFC model there is a dissipation parameter $\alpha$. If $\alpha = 0.25$ the total tension in the lattice, $\sum z_{i,j}$, is conserved during the avalanching process, in the bulk of the lattice (there is always dissipation in the boundaries). But if $\alpha < 0.25$ there is some dissipation also in the bulk of the system. Because of those facts, this model has been widely studied in literature: it is, at the same time, a prototype of self-organization in systems with non-conservative relaxation rules (the existence of SOC in the non-conservative models is, up to now, not well understood [5, 6, 7, 8, 9]) and also a paradigm of the success of SOC ideas, since it is able to reproduce important aspects of the dynamics of earthquakes.

Recently, Abe and Suzuki [10] observed a new power-law in the statistics of earthquakes.
They analyzed earthquake data from both the district of southern California and Japan, connecting their epicenters in order to generate a graph. Each area analyzed was divided into small cubic cells; they associated to each of these cells a node every time an earthquake started inside it. The epicenters of two successive earthquakes were linked, defining an edge. In this way the data has been mapped into a complex growing graph that behaves like a scale-free network of the Barabási-Albert type \cite{11}. The degree distribution of the graph decays as a power-law. The clustering coefficient and the diameter of a cluster were also calculated, showing small-world network properties \cite{12}. These features have revealed a novel aspect of earthquakes as a complex critical phenomenon.

In this paper we studied the Olami-Feder-Christensen model to see if it could also predict this new striking behavior. We found that the non conserving version of the model reproduces the behavior of experimental data, even for a very small degree of non conservation. The degree distribution of the evolving network formed by its epicenters is scale free. However, the conservative version of the model has a qualitatively different behavior, more similar to a random graph, whose degree distribution is Poisson, indicating that most of the nodes have the same degree and - although random - the corresponding network is much more homogeneous. These results are in agreement with some recent observations, reinforcing that conservative and non conservative versions of the OFC model are quite different. Hergarten and Neugebauer (2002) \cite{13} studying the efficiency of the OFC model to predict foreshocks and aftershocks, de Carvalho and Prado (2003) \cite{14}, studying the transient behavior of the OFC model and Miller and Boulter (2003) \cite{15} studying the distribution of values at which supercritical sites topple have also reported qualitatively different behaviors between the conservative and non conservative OFC model.

In a complex graph, the edges are not distributed in an regular way and not all nodes have the same number of edges. One possible way to characterize complex networks is through its distribution function $P(k)$, which gives the probability that a random selected node has exactly $k$ edges. $k$ is called the degree of the node. In a random graph, since the edges are placed randomly among the nodes, the majority of nodes have approximately the same degree, close to the average connectivity $\langle k \rangle$, and the distribution $P(k)$ is a Poisson distribution with a peak at $P(\langle k \rangle)$. Most complex networks, however, have a distribution function $P(k)$ that deviates significantly from a Poisson distribution. In particular, for a large number of networks, associated with a wide class of systems, ranging from the world
wide web to metabolic networks, $P(k)$ has a power-law tail, $P(k) \sim k^{-\gamma}$. Such networks are called scale free \cite{11}, and have called the attention of many researchers in the last years.

We simulated the OFC model in a square lattice, building graphs with a procedure very similar to what has been employed by Abe and Suzuki. Each site that gives birth to a new avalanche is an epicenter; each epicenter defines a node, and every node is then connected to the node where the next epicenter occurs, establishing a link or edge between them. After many avalanches this procedure generates a complex network (or graph), and we have studied some of its statistical properties.

After eliminating a transient of at least $10^6$ events, we calculate numerically the distribution function $P(k)$ for the graph constructed from the time sequence of epicenters in the OFC model, for different values of $\alpha$ and different lattice sizes. As the first and last sites are the only ones with an odd number of edges, they were eliminated. Our results for the distribution $P(k)$ can be seen in figure 1. It is clear that, if $\alpha < 0.25$ (figure 1a), the distribution is scale-free for some decades, with an exponent $\gamma$ that varies linearly with $\alpha$ (see figure 2), at least for values of $\alpha$ not too far from the conservative regime. The network grows toward the inside of the lattice, with the most connected sites in the borders and the most inner sites being the last ones being added to it (see figure 3a). The complex structure, however, is not a boundary effect. If we take out the border sites and adjusts the scale, we see that the same spatial structure is reproduced (figure 3b). Because one needs a growing network to observe the scale free-behavior \cite{11}, after a certain number of events, as a consequence of the finite size of the lattice, most of the sites of the lattice have already become part of the network. At this point the scale free behavior starts to break.

If the system is conservative, however, the distribution function $P(k)$ has a well defined peak, indicating a higher degree of homogeneity among the nodes (figure 1b). Figures 4a and 4b, that shall be compared with figure 3, shows the spatial distribution of connectivities in the lattice. As expected, it is much more homogeneous. This homogeneous behavior is not destroyed if we vary the statistic of events.

Finally, our findings seems also to be robust with respect to the cell size. If we increase the size of the cell, defining, for instance, four adjacent sites of the lattice as a unique cell, there is no change in the results, not even in the exponent $\gamma$ that characterizes the degree distribution $P(k)$, as shown in figure 5.

In conclusion, we have shown that the non conservative version of the Olami-Feder-
Christensen model is able to reproduce the scale free network associated to the dynamics of the epicenters observed on real earthquake data. The conservative version of the model displays a qualitatively different behavior, being more close to a random graph. The smallest degree of non conservation seems to be enough to change the behavior of the model, since for \( \alpha = 0.249 \) we see that \( P(k) \) has already a well defined power law behavior for some decades. Those findings, besides giving an indication of the robustness of this model to reproduce the dynamics of earthquakes, reproducing the experimental findings of Abe and Suzuki, present a completely new dynamical mechanism to generate a free scale network. There is no explicit rule of preferential attachment, and the preferential attachment observed in the network is a signature of the model dynamics. We hope that a complete study of the properties of the network can help to solve some still controversial aspects of the Olami-Feder-Christensen model and of self-organized critical behavior, and can be interesting and useful even if a more detailed study of earthquake data comes to show in the future that the the results reported by Abe and Suzuki are not universal.

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Figure Captions

**Figure 1:** Normalized degree distribution $P(k/I)$ for different values of $\alpha$. $I$ is the total number of epicenters. (a) non conservative regime: the results show a free scale network behavior in all cases. The curves for $\alpha < 0.249$ have been shifted upwards along the $y$ axis for clarity, otherwise they would all coincide. In all cases $L = 200$ and the number of epicenters registered is $10^5$. (b) Conservative regime: the degree distribution is similar to a random graph; in this case we have $L = 200$ and an statistics of $10^6$ events. Lowering the statistics does not change this behavior.

**Figure 2:** Exponent $\gamma$, that characterizes the power law behavior of $P(k)$, for different values of $\alpha$. $\gamma$ seems to increase linearly with $\alpha$. In all cases $L = 200$ the number of epicenters is $10^5$.

**Figure 3:** Spatial distribution of node degrees in the non conservative case, for $\alpha = 0.249$, $L = 200$ and $10^5$ events. Sites associated with nodes of higher degree are darker and, as one can see, are closer to the boundaries. Figure (b) is a blow up of (a). The 20 sites closer to the boundaries have not been plotted and the scale has been changed in order to show the details. We can see that the structure of the network is reproduced and is not a boundary effect.

**Figure 4:** Spatial distribution of node degrees for the conservative case. Sites associated with nodes of higher degree are darker, $L = 200$ and the number of epicenters is $10^6$. (a) The same scale of figure 3a has been used. (b) The scale has been changed to reveal details of the structure of the network that, in this case, is much more homogeneous and quite different than the one observed in the non conservative regime.

**Figure 5:** The normalized degree distribution $P(k/I)$ for $\alpha = 0.249$, $L = 200$ and $10^5$ events, for different cell sizes. $I$ is the total number of epicenters. (a) $L = 200$ and each site of the lattice defines a cell. (b) $L = 400$ and each four adjacent sites are in the same cell. The curve has been shifted upwards in the axis $y$ for clarity.
Figure 1
Figure 2
Figure 5