The Mass of the Dilaton

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Abstract

It is shown that, in a theory where the dilaton is coupled to a Yang-Mills gauge field which enters a confining phase at scale \( \Lambda \), the dilaton may grow a mass

\[
m_{\text{dilaton}} \sim \frac{\Lambda^2}{m_{\text{Pl}}} \sim (m_{\text{SUSY}}^2 m_{\text{Pl}})^{1/3} \sim 10^8 \text{ GeV}.
\]

This allows ample time for decay before the electroweak era if \( m_{\text{SUSY}} \approx 1 \text{ TeV} \), and circumvents cosmological problems normally associated with its existence.
Introduction

The dilaton is a spin-0 field which is associated with the graviton and with an axion as a massless excitation in string theory \[1\]. In the field theoretic limit it persists as a massless spin-0 field in the no-scale supergravity theory. A truly massless dilaton is of course incompatible with the success of Newtonian gravity, since it couples as a full strength Brans-Dicke field. However, even if a dynamical mechanism were to be found to stabilize the dilaton potential \[2\], perhaps generating a dilaton mass of the order of the the SUSY-breaking \(m_{SUSY} \sim 1\) TeV, severe cosmological problems would still persist. Most notably, the late decay time would cause a substantial reheating well after the nucleosynthesis era \[3\], vitiating a major success of the standard cosmological model. All of this could conceivably be modified if the dilaton were coupled to a field whose interactions are strong: assuming that the dilaton masslessness is not protected by a strict gauge principle, then mass growth could occur as a result of mixing with composite spin-0 objects in the strongly-interacting sector. This is the mechanism I will consider in this work.

Effective Potential

I begin by presenting the minimal components of the model necessary for this work. I assume that the dilaton field \(S\) is universally coupled to all (hidden and unhidden) gauge fields \(F^i_{\mu\nu}\), giving a contribution to the Lagrangian

\[
\mathcal{L}_{SF} = S \sum_i F^i_{\mu\nu} F^i_{\mu\nu}. \quad (1)
\]

The dilaton field is assumed to develop a vev \(\langle S\rangle = 1/g^2\), leaving an interaction with the remaining (quantum) field

\[
\mathcal{L}_{\sigma F} = \frac{1}{m_{Pl}} \sigma \sum_i F^i_{\mu\nu} F^i_{\mu\nu}, \quad (2)
\]

where \(\sigma = S - \langle S\rangle\), and the gauge fields in Eq. (2) are now normalized with canonical kinetic energy. I now assume that at some scale \(\Lambda\) the gauge coupling becomes strong. What I have mostly in mind, of course, is the hidden-gaugino condensate mechanism for generating SUSY-breaking \[4\]. In such models, one commonly obtains

\[
m_{gravitino} \approx m_{SUSY} \approx \frac{\Lambda^3}{m_{Pl}^2}, \quad (3)
\]
so that $\Lambda \sim 10^{14}\text{GeV}$. In any case, the spectrum at scales below $\Lambda$ will now contain glueballs, at least one of which will have spin-0, and which will be denoted by $G$. As a result of (2), there will be $\sigma$--$G$ mixing, which can be parametrized by a mixing term

$$V_{\text{mix}} = -\mu \sigma G .$$  \hspace{1cm} (4)

The coupling (2) and dimensional considerations imply

$$\mu = \kappa \frac{\Lambda^3}{m_{\text{Pl}}} ,$$ \hspace{1cm} (5)

with $\kappa \sim O(1)$. To complete the effective potential for the $\sigma$--$G$ system, I introduce glueball and dilaton mass terms, and some quartic terms to stabilize the potential:

$$V(\sigma, G) = \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} m_G^2 G^2 - \mu \sigma G + \frac{1}{4} \lambda_\sigma \sigma^4 + \frac{1}{4} \lambda_G G^4 .$$ \hspace{1cm} (6)

In Eq. (6), $m_{\sigma}^2, \lambda_\sigma$ and $\lambda_G$ are unknown. For the glueball mass, along with the estimates (3) and (5), we expect

$$m_G = \alpha \Lambda ,$$ \hspace{1cm} (7)

with $\alpha \sim O(1)$. Although the parameters $m_{\sigma}^2, \lambda_\sigma, \lambda_G$ have been introduced in complete ignorance of their origin and magnitude, it will turn out, very remarkably, that (a) only $\lambda_\sigma$ will be required to be non-zero and (b) as long as $\lambda_\sigma > O((\Lambda/m_{\text{Pl}})^4)$, the mass eigenvalues will be determined entirely by $m_G$ and $\mu$, and will not depend on $\lambda_\sigma$ to leading order in $\Lambda/m_{\text{Pl}}$.

**Mass Spectrum: Simplified Case**

I first note that, in the absence of the mixing term, the potential has a minimum at $\sigma = G = 0$. The mixing term shifts this minimum, as well as the mass eigenvalues. We can obtain a general idea of the result of this by considering a simplified situation, in which $\lambda_G = m_{\sigma}^2 = 0$. The potential then becomes

$$V^{(0)} (\sigma, G) = \frac{1}{2} m_{G}^2 G^2 - \mu \sigma G + \frac{1}{4} \lambda_\sigma \sigma^4 .$$ \hspace{1cm} (8)

The stationary conditions are

$$\mu G - \lambda_\sigma \sigma^3 = 0$$

$$m_G^2 G - \mu \sigma = 0 .$$ \hspace{1cm} (9)
These are consistent with either \( \sigma = G = 0 \), or with
\[
\bar{\sigma} = \pm \frac{1}{\sqrt{\lambda_\sigma}} \left| \frac{\mu}{m_G^2} \right|
\]
\( \bar{G} = \pm \frac{1}{\sqrt{\lambda_\sigma}} \left| \frac{\mu}{m_G^2} \right| . \)  
(10)
It is simple to check that the zero-field point \((0,0)\) is a saddle point of \( V^{(0)} \), whereas \((\bar{\sigma}, \bar{G})\) lies below that for the zero field values. Thus \((\bar{\sigma}, \bar{G})\) is a true minimum, with energy
\[
V^{(0)} (\bar{\sigma}, \bar{G}) = -\frac{1}{4} \left( \frac{1}{\lambda_\sigma} \right) \left( \frac{\mu}{m_G^2} \right)^4 . \]  
(11)
It is important to note that as long as \( \lambda_\sigma \gg (\Lambda/m_{Pl})^4 \), the shift in \( S \) due to the new vacuum is \( \delta S = \bar{\sigma}/m_{Pl} \ll 1/g^2 \).

One may wish to constrain the parameters so that the shift in vacuum value of energy be \( \lesssim O(|F|^2) \) where the \( F \) is a SUSY-breaking \( F \)-term, of \( O(m_{Pl}m_{SUSY}) \). From (3), this is equivalent to requiring
\[
\left| V^{(0)} (\bar{\sigma}, \bar{G}) - V^{(0)}(0,0) \right| \lesssim \frac{\Lambda^6}{m_{Pl}^2} . \]  
(12)
Eq. (12), combined with (3), (7), and (11), implies \( \lambda_\sigma \gg O((\Lambda/m_{Pl})^2) \).

Using (10) and (8) one may calculate the quadratic fluctuation matrix \( i.e., \) the \( \sigma-G \) (mass)\(^2\) matrix \( \mathcal{M}^2 \) about \((\bar{\sigma}, \bar{G})\):
\[
\left( \mathcal{M}^2 \right)_{\bar{\sigma}\bar{G}} = \begin{pmatrix} 3(\mu^2/m_G^2) & -\mu \\ -\mu & m_G^2 \end{pmatrix} . \]  
(14)
There is a remarkable aspect about this result: although a non-zero value of \( \lambda_\sigma \) was required in order to establish the non-trivial vacuum, the fluctuations about this vacuum do not depend on the value of \( \lambda_\sigma \). This parameter has acted as a regulator, disappearing in the physical masses (although not in the vacuum energy). The eigenvalues may be immediately calculated, and are most transparent in leading order in \( (|\mu|/m_G^2) \sim \Lambda/m_{Pl} \):
\[
m_{\sigma'} \simeq \sqrt{2} \frac{|\mu|}{m_G} = \sqrt{\frac{2\kappa}{\alpha}} \frac{\Lambda^2}{m_{Pl}}
\]
\[
m_{G'} \simeq m_G = \alpha \Lambda . \]  
(15)
Mass Spectrum: General Case

The treatment of the more general case is slightly more complicated, but the result is the same. In what follows, it will be convenient to set $m_{Pl} = 1$ until the end, and to define

$$\epsilon \equiv \frac{\Lambda}{m_{Pl}} ,$$

In accordance with Eqs. (5) and (7),

$$\mu \sim \epsilon^3 , \quad m_G^2 \sim \epsilon^2 .$$

Typically, models of dilaton potentials prior to mixing with glueball give

$$m^2_{\sigma} \sim m^2_{SUSY} \sim \epsilon^6 .$$

Beginning with the potential (8), it is apparent that a tachyonic instability develops in the quadratic sector if

$$m^2_{\sigma} \cdot m^2_G < \mu^2 ,$$

which will certainly be the case with the estimates in Eqs. (17) and (18). In such a case, we search for the displaced vacuum and spectrum for (8). The extremum conditions following from (8) are

$$m^2_{\sigma} \cdot \sigma - \mu G + \lambda_{\sigma} \sigma^3 = 0$$
$$m^2_G \cdot G - \mu \sigma + \lambda_G G^3 = 0 .$$

These allow the solution $\sigma = G = 0$, with $V = 0$; this is a saddle point if (19) is true. Alternately, there is a non-trivial solution which follows from (20). On eliminating $G$ via

$$G = \frac{\sigma}{\mu} \cdot \left( m^2_{\sigma} + \lambda_{\sigma} \sigma^2 \right) \equiv \frac{\sigma}{m^2_G} \cdot \frac{\mu^2}{m^2_G} \cdot x ,$$

in terms of a dimensionless variable $x$, one obtains from Eq. (20) the quartic equation

$$f(x) = \left( \frac{\mu}{m^2_G} \right)^4 \left( \frac{\lambda_G}{\lambda_{\sigma}} \right) \left( x^4 - \left( \frac{m^2_{\sigma} m^2_G}{\mu^2} \right) x^3 \right) + x - 1 = 0 .$$
The analysis of the solution spectrum of (22) goes as follows: We first note that, in terms of the small quantity $\epsilon$ (Eq. (16)),

\[
\left(\frac{\mu}{m_G^2}\right)^4 \sim \epsilon^4
\]

\[
\frac{m_\sigma^2 m_G^2}{\mu^2} \sim \epsilon^2 .
\] (23)

It is then easy to show that (i) $f(0) < 0$, $f(x \geq 1) > 0$; (ii) if $\lambda_\sigma > O(\epsilon^{10})$, then $f(x)$ is a monotonically increasing function for $x \geq 0$. In that case, there is one and only one root for positive $x$, and we can obtain it by expanding around $x = 1$. After some algebra, one finds that the stationary point occurs at

\[
\bar{x} \simeq 1 + O(\epsilon^4) .
\] (24)

After this, the analysis proceeds exactly as in the simplified case. Once more, the stationary point $(\bar{\sigma}, \bar{G})$ in field space is given by Eq. (10) up to small corrections, and the energy density at the true vacuum $(\bar{\sigma}, \bar{G})$ is given (up to corrections of $O(\epsilon^4)$) by (11). Thus, in the general case, the energy density at $(\bar{\sigma}, \bar{G})$ is lower than at $\sigma = G = 0$. The masses (to leading order in $\epsilon$) are as before:

\[
m_{\sigma'} \simeq \sqrt{2} \frac{|\mu|}{m_G} = \frac{\sqrt{2} \kappa}{\alpha} \frac{\Lambda^2}{m_{Pl}}
\]

\[
m_G' \simeq m_G = \alpha \Lambda .
\] (15)

The result of all this is a dilaton mass

\[
m_{\text{dilaton}} = m_{\sigma'} \sim (m_{\text{SUSY}} m_{Pl})^{1/4} \simeq 10^8 \text{GeV} .
\] (25)

The Decay of the Dilaton

From the mass matrix (14) and the eigenvalues (15), it is seen that the dilaton eigenstate $\sigma'$ differs from the uncoupled $\sigma$ by terms of $O(\epsilon)$:

\[
\sigma' \simeq \sigma + (\mu/m_G^2)G .
\] (26)
so that the coupling of the dilaton to standard model gauge fields can be obtained from (2). This immediately yields a lifetime

$$\tau_{\text{dilaton}} \approx \frac{m_{Pl}^2}{m_d^2}, \quad \approx \frac{m_{Pl}^5}{\Lambda^6}, \quad \approx \frac{m_{Pl}^2}{m_{\text{SUSY}}},$$

(27)

so that the decay of the dilaton precedes (or is coincident with) the electroweak era [6]. Thus, even if a large amount of energy resides in the oscillations of the dilaton field about its minimum, the reheating caused by its decay would not endanger nucleosynthesis [3] or even anomalous baryosynthesis [7], [8].

Conclusions

It has been shown that the mixing of a dilaton with a heavy scalar glueball, as determined by the canonical coupling of the dilaton, will result in a dilaton of mass \( \sim \Lambda^2/m_{Pl} \), where \( \Lambda \) is the confinement scale of the gauge theory. The result is due to a specific interplay of the mass of the glueball and the size of the coupling. A quartic stabilization of the dilaton potential is necessary for the dynamics, but the magnitude of the coupling plays no role (to leading order in \( \Lambda/m_{Pl} \)) in determining the mass or the coupling of the physical dilaton. Finally, the large mass of the dilaton is shown to circumvent the usual cosmological problems associated with its existence.

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