A Study on Three Rookie Workers’ Assignment Optimization under the Limited-Cycled Model with Multiple Periods

——Law of Rookie First——

Peiya Song, Xianda Kong, Hisashi Yamamoto, Jing Sun, and Masayuki Matsui

Abstract: This paper models and analyzes serial production lines to discuss about the optimal assignment of workers whose ability varies from each other. A limited-cycled model with multiple periods (LCMwMP) is built for evaluating the expected processing cost depending on the risk of delay. In order to minimize the risk of delay, the assignment of the workers, especially rookie worker is focused on because they are the weak point who has the most probability leading to the delay. In previous researches, the workers are separated into two groups by the capacity of processing. And in the case when rookie workers group has only one or two workers, the rules of optimal worker assignment is already proposed by assuming an assembly line as LCMwMP model. In this paper, we continue to deal with LCMwMP model assuming an optimization problem with three rookie workers, for finding an assignment of workers to the line that has a minimum expected processing cost. Then, by the mathematics demonstration, we will prove the validity of these rules found by the numeral experiment. And by some numeral experiments, we will conclude several more rules of optimal assignment. At last, by comparing the rules assumed in this paper with previous research, it can be inspired if we can find out a common rule regardless of the number of rookie workers as a goal of future research.

Key Words: Optimization technique, Production planning, Work management

1. Introduction

After Industrial Revolution in 18th century, the development of the assembly line led to a proliferation of manufacturing inventions and contributed to the substantial fortunes. And till now, in the developing countries such as China, labor-intensive enterprise is still the mainstay industry. We can still benefit from assembly line, which is widely used among labor-intensive enterprise, such as garment factory, for the sake of reducing processing costs and production time. However, even though the reliability of the machine on the assembly line in nowadays surpasses that of centuries ago, the each component of assembly line is still facing the problem of failure during operation. Moreover, laborer is still the main factor of production speed, and not every worker on the assembly line can accomplish the task assigned in time. All these factors will lead to a production delay which can result in financial losses, if we consider the cost of production which fluctuates with the delay, to the manufacturing enterprise, solving the problem mentioned above is important for achieving the goal that processing task can be accomplished efficiently and competently. The efficiency can be lamed not only by the factor of the machines of the each component on the assembly line, but also by the factor of the workers, such as the difference of production speed between veteran and rookie. The former problem is well known as the reliability of a system and the latter is considered as an assembly line balancing problem. And the latter is referred to as a problem of assignment under a limited-cycle model with multiple periods.

2. Literature Review

Salveson established the mathematical formalization of assembly line balancing problem 60 years ago [1], assembly line balancing problem is popularly researched and developed [2]. It assumes a homogeneous skill set to solve the problem of worker assignment in the above literature, that is to say, difference of skill is not considered. Further, the properties of the optimal assignment are not developed. And Yamamoto et al. considered the solutions of this kind of problem [3].

In this problem, the characteristic of the researches of Yamamoto et al.[3] is the setting of constraint condition, (e.g., processing time with a target) which repeats in every multiple periods. If the constraint condition is broken, an expected risk (e.g., penalty cost) will be occurred. Therefore the problem of minimizing the risk in such a situation is called limit-cycle model with multiple periods (LCMwMP).

During the manufacturing process of assembly line, the delay of one task will raise the risk of delay of the posterior task. In other word, the result and efficiency of a certain period (of a production cycle) are influenced not only by the factor which exists in the current period but also by the factor which exists in the previous periods. Therefore, we discuss the minimum expected risk of the case mentioned above, in which the risk depends on the previous situation and occurs repeatedly for
multiple periods. In previous researches, the LCMwMP is classified into various classes, and it has been proposed as a type of problem with ‘a limited-cycle model with dependent multiple periods’, in which the occurrence of an idle or delay in a period depends on the occurrences of them in the other periods by Yamamoto et al.[3],[4].

And according whether the constraint condition (target processing time) reset or not, The LCMwMP is divided into reset model and non-reset model. In a reset model, the target processing time of each periods is the same regardless of the number of idle or delay of previous periods. And in contrary, in a non-reset model, the target processing time of each periods varies with consideration of the idle or delay of previous periods. In the field of reset model, a recursive formula for the total expected risk and an algorithm for optimal assignments based on the branch and bound method are proposed by Yamamoto et al.[5]. Recently, Yamamoto et al.[6] and Kong et al.[7] proposed properties of optimal worker assignment with two kinds of workers in which one special worker exists. Then, Kong et al.[8],[9] proposed properties of optimal worker assignment with two special workers. And in the field of non-reset model, Sun et al.[10] discussed the properties of optimal efficiency switching timing under the model of non-reset limited-cycle model with multiple periods.

After that Song et al. continued to research the properties of optimal assignment with two kinds of workers in which three special workers exist by the numerical experiment with exponential distribution [11] and Erlang distribution [12].

In this paper, the reset model of LCMwMP is used. We figure out the optimal assignment when there are three rookie workers in the processing line and try to summarize the rules of optimal assignment by comparing the similarity of all types of optimal assignment. At first, reset model as a simple model of LCMwMP is introduced. Then, by the mathematic demonstration, it is proved that the rules assumed in previous researches is validity. And by means of the numerical experiment assuming the processing time follows Erlang distribution, it is intuitionally verified that the results are consistent with the rules. Finally, after comparing the rules assumed in this paper with previous research, it can be inspired if we can find out a common rule regardless of the number of rookie workers as a goal of future research.

3. Model Explanation
In this section, reset model, a simple model of the LCMwMP, is described. Then, optimal assignment problem in reset model is defined.

3.1 Reset Model of Limited-Cycle Model with Multiple Periods
The model is considered based on the following assumptions introduced by Kong et al.[9]:

1. Considering an assembly line system, $n$ is the number of process (it may be considered that $n$ is the number of processing station or period).

2. The production is processed in a rotation of period 1, period 2, … and period $n$. One production will be processed by $n$ periods.

3. All of the partially finished productions will be moved to next period and established by time $Z$. Specifically $Z$ is the cycle time of all periods.

4. There are two types of workers, $A$ and $B$. $A$ represents rookie worker whose processing time is longer than others, $B$ represents regular worker. Note that assigning only one worker to each period. The processing time of worker $l$, $T_l$, where $l \in \{A, B\}$, is self-dependent.

5. To sum up, $Z$ is the cycle time of all of the stations. $Z$ is also a kind of limited processing time (or target processing time) of each station. $Z$ is called the target processing time in this research. For $l \in A, B$, when the processing time of worker $l$ is denoted by $T_l$:

$P_l$: The probability of worker $l$ becoming idle, which is $\Pr(T_l \leq Z)$,

$Q_l$: The probability of the worker $l$ becoming delayed, which is $\Pr(T_l > Z)$,

$TS_l$: The expected idle time of the worker $l$, which is $E[(Z - T_l) \cdot I(T_l \leq Z)]$,

$TD_l$: The expected delay time of the worker $l$, which is $E[(T_l - Z) \cdot I(T_l > Z)]$.

where $I(O)$ is an index function and given as follows:

$$I(O) = \begin{cases} 1 & (O \text{ is true}) \\ 0 & (O \text{ is not true}) \end{cases}$$

We suppose the following costs as shown in Figure 1.

In this paper, we consider a fixed target processing time $Z$. Workers’, employment costs and resource will occur whether the processing is done. A processing cost $C_l(\geq 0)$ per unit time will proportional occur to target processing time $Z$. Otherwise, if the processing time is longer than $Z$, overtime work or additional resources will be requested in order to meet the target time $Z$. So the delay cost per unit time $C_d^{(l)}(\geq 0)$ will occur (that is why we call the model a ‘Reset Model’). Meanwhile, if the processing time is shorter than $Z$, work-in-process inventory can be considered before moving to the next process. So the idle cost per unit time $C_i(\geq 0)$ will occur.

As a summary of above, we get:

6. The processing cost per unit time, $C_i(\geq 0)$ for the target processing time limit occurs in each period.

7. When $T_l \leq Z$, the idle cost per unit time, $C_i(\geq 0)$ occurs in each period.

8. When $T_l > Z$, the delay cost per unit time, $C_d^{(l)}(\geq 0)$, occurs in the period if delay occurs in consecutive $i$ processes before its period, for $1 \leq i \leq n$. In this paper, we suppose that the delayed process time of a period can be recovered by the overtime work or spare workers in this period, and $C_d^{(l)}$ is the cost for all of these. Because the cost rises due to the increase of the delay, in this paper, we suppose the $C_d^{(l)}$ is increasing in $i$, that is $0 < C_d^{(i)} < C_d^{(i+1)}$. 

"
3.2 Optimal assignment Problem under Reset Model

We consider that three rookie workers are assigned in this reset model. One of the most important problems is how to assign workers to processes for minimizing the expected cost in \( n \) processes. We call such a problem the optimal assignment problem. For describing the optimal assignment problem, we define the following notations [9]:

For \( 1 \leq i < j < k \leq n \), \( \pi(i, j, k) \): Three untrained workers are assigned in process \( i \), \( j \) and \( k \), \( n - 3 \) regular workers are assigned to other processes.

\( TC(n; \pi(i, j, k)) \): The total costs of processes to \( n \) when workers are assigned by assignment \( \pi(i, j, k) \).

The total cost \( TC(n; \pi(i, j, k)) \) of \( n \) processes when workers assigned according to assignment \( \pi(i, j, k) \) is expressed as

\[
TC(n; \pi(i, j, k)) = nC_rZ + f(n; \pi(i, j, k))
\]

where,

\( f(n; \pi(i, j, k)) \): The expected cost (the sum of the expected idle cost and the expected delay cost) caused in process \( n \).

By using these notations, the optimal assignment problem with multiple periods becomes the problem of obtaining assignment in the following equation:

\[
TC(n; \pi^*) = \min_{1 \leq i < j < k \leq n} TC(n; \pi(i, j, k))
\]

In this paper, we call \( \pi^* \) the optimal assignment.

However, it is easily known from (1) that if the target processing time \( Z \) is constant, the target production cost, \( nC_rZ \), is also constant, so we can simplify (2) to

\[
\min_{1 \leq i < j \leq n} f(n; \pi(i, j, k))
\]

4. Rule of Optimal Assignment

In this section, we consider the rule of optimal assignment under the assumption above and try to prove it by mathematical demonstration.

We demonstrate that it is optimum only if the first process is assigned with rookie worker. And it can be summarized as the following Theorem.

Theorem: Law of Rookie First

When \( C_p^{(i)} \) is increasing in \( i \), if \( Q_A > Q_B \), \( TL_A > TL_B \), then the assignment that a rookie worker is assigned in process \( i \) is optimal.

It means, the expected cost of \( \pi(1, j, k) \) is always smaller than the expected cost of \( \pi(i, j, k) \) (where \( 2 \leq i < j \)), where \( j, k \) is not involved, as a conclusion, \( \pi(1, j, k) \) is the optimal assignment.

It can be explained as Eq. (4).

\[
TC(n; \pi(1, j, k)) < TC(n; \pi(i, j, k)), \text{where} 2 \leq i < j
\]

Here we set a function \( D(i) \) to evaluate the value fluctuation of expected total cost when shift the assignment of the first rookie worker to one process next, which means

\[
D(i) = TC(n; \pi(i + 1, j, k)) - TC(n; \pi(i, j, k))
\]

By the definition of \( TC(n; \pi(i, j, k)) \), we can gain that

\[
D(i) = D_1(i) + D_2(i) + \ldots + D_7(i)
\]

where for \( 2 \leq i < j \),

\[
D_1(i) = \sum_{a=1}^{i-1} ((C_p^{(a+1)} - C_p^{(a+1)}) TL_B Q_B^{(a+1)} (Q_A - Q_B))
\]

\[
D_2(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_3(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_5(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_6(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_7(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

The above proof is inspired by the previous researches [7]-[9], and elucidated in detail in the appendix.

And then because of rookie worker \( A \) has a bad performance of process speed than \( B \), we have inequalities that \( TL_A > TL_B \) and \( Q_A > Q_B \). We can replace \( TL_B \) and \( Q_B \) instead of \( TL_A \) and \( Q_A \). And after such replacement in equation \( D_3(i) \) for example, we can gain new equation with the symbol \( D_3^2(i) \) which is smaller than \( D_3(i) \). As a result, for \( 2 \leq i < j \),

\[
D_3^2(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+2)} (Q_A - Q_B)
\]

\[
D_4^2(i) = (C_p^{(i-1)} - C_p^{(i-1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_5^2(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_6^2(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]

\[
D_7^2(i) = (C_p^{(i+1)} - C_p^{(i+1)}) TL_A Q_B^{(i+1)} (Q_A - Q_B)
\]
From Eq. (7) and Eq. (14), for \(2 \leq i < j\),

\[
D_1(i) + D_2(i) > D_1(i) + D_2(i) = \sum_{a=1}^{n-i} ((C_p(a) - C_p(a+1))T_LBQ_A^a(Q_A - Q_B)) + (C_p(i) - C_p(i+1))T_LBQ_B^{i-1}(Q_A - Q_B)
\]

holds.

From Eq. (9) and Eqs. (15) - (18), for \(2 \leq i < j\),

\[
D_1(i) + D_3(i) + D_4(i) + D_5(i) + D_6(i) > D_1(i) + D_3(i) + D_4(i) + D_5(i) + D_6(i) = \sum_{a=1}^{j-i-2} ((C_p(a) - C_p(a+1))T_LBQ_A^a(Q_A - Q_B)) + ((C_p(i) - C_p(i+1))T_LBQ_B^{i-1}(Q_A - Q_B)) + \sum_{j-i-2}^{j-i-1} ((C_p(a) - C_p(a+1))T_LBQ_B^{a-1}(Q_A - Q_B)) + (C_p(i) - C_p(i+1))T_LBQ_B^{i-2}(Q_A - Q_B)
\]

holds.

From Eq. (19) and Eq. (20), for \(2 \leq i < j\),

\[
D(i) = D_1(i) + D_2(i) + D_3(i) + D_4(i) + D_5(i) + D_6(i) + D_7(i) = \sum_{a=1}^{n-i} ((C_p(a) - C_p(a+1))T_LBQ_A^a(Q_A - Q_B)) + (C_p(i) - C_p(i+1))T_LBQ_B^{i-1}(Q_A - Q_B)
\]

holds.

And, for \(2 \leq i < j\),

\[
D(i) > D(i)
\]

holds.

And from the Eq. (21), it can be known that, when \(n - i - 1 > i\), that is \(i < \frac{n-i-1}{2}\), \(D(i) > 0\), which means the expected cost increases with assigning first rookie worker from period \(i\) to \(i + 1\).

In contrast, when \(n - i - 1 < i\), that is \(i > \frac{n-i-1}{2}\), \(D(i) < 0\), which means the expected cost decreases with shifting first rookie worker from assignment \(i\) to \(i + 1\).

So when changing the first rookie worker’s assignment from period 1 to \(n - 2\), because there are other two rookie workers after him, the expected cost will increase at first, and after \(i > \frac{n-i-1}{2}\), the expected cost will decrease.

But, the difference of expected cost between \(TC(n; \pi(1, j, k))\) and \(TC(n; \pi(n-2, j, k))\) is \(\sum_{i=1}^{n-i} D(i)\), that is

\[
\sum_{i=1}^{n-i} D(i) = TC(n; \pi(n-2, j, k)) - TC(n; \pi(1, j, k))
\]

holds.

When changing the assignment of first rookie worker from period 1 to \(n - 2\), the expected cost transits as shown in Fig. 2. Sum up, For \(2 \leq i \leq n - 2\), \(i < j < k\),

\[
TC(n; \pi(1, j, k)) < TC(n; \pi(i, j, k))
\]

holds.

Therefore, the Law of Rookie First is proved.

5. Numerical Experiment

Although the law of rookie first is demonstrated in previous section, the rules of other two rookie workers are still unclear. If we want to figure out the rule the optimal assignment, it is necessary to find out assignment rules of all the rookie workers. So numerical experiment is conducted for investigate the other rules.

In the following numerical experiment, the processing time of these two kinds of workers follows Erlang distribution though it is expected to summarize rules of optimal assignment regardless of the distribution just as Law of Rookie First.
Erlang distribution. In other word, for general distribution from the results of experiment under a

Table 1 The changing of expected cost when assigning the first rookie worker

| m = 7 | m = 8 | m = 9 | m = 10 | m = 11 | m = 12 |
|-------|-------|-------|--------|--------|--------|
| ABBBAA | 9432.02 | 1686.23 |
| BABBAA | 9933.29 | 1742.80 |
| BBBABA | 10037.52 | 1797.84 |
| BBBBBB | 10679.98 | 1959.53 |
| BBBBBA | 10990.31 | 2545.63 |

Table 2 Optimal assignment when m = 2, μA = 0.1, μB varies from 0.2 to 2.0 under 7 to 12 processes

| μA | μB | n = 7 | n = 8 | n = 9 | n = 10 | n = 11 | n = 12 |
|----|----|------|------|------|------|------|------|
| 0.1 | 0.2 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.3-0.8 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.9 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.0 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.1 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.2 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.3 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.4 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.5 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.6 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.7 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.8-2.0 | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA | ABBBBA |

Table 3 Optimal assignment when n = 9, μA = 0.1, μB varies from 0.2 to 2.0 under shaping parameter varying from 1 to 4

| μA | μB | m = 1 | m = 2 | m = 3 | m = 4 |
|----|----|------|------|------|------|
| 0.1 | 0.2 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.3 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.4 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.5 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.6 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.7 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.8 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 0.9 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |
| 0.1 | 1.0 | ABBBBA | ABBBBA | ABBBBA | ABBBBA |

Table 4 Optimal assignment when m = 2, μA = 0.1, μB varies from 0.2 to 2.0 under shaping parameter varying from 1 to 4

Fig. 2 The change of expected cost when assigning the first rookie worker from period 1 to n 2

However, as a method, it is sensible to seek laws out applicable for general distribution from the results of experiment under a certain distribution.

The processing time of these two kinds of workers follows Erlang distribution. In other word, Ql, the probability of the worker l becoming delayed is

\[ Q_l = \sum_{k=0}^{m-1} \frac{\mu_l Z^k}{k!} e^{-\mu_l Z} \]  

(27)

where \( l \in \{A, B\} \),

1. Rookie worker is A and regular worker is B.
2. \( \mu_l \) is the processing rate. And when \( \mu_l \) is bigger, it means processing speed is higher and lower possibility of delay of processing.
3. \( m \) is the shape parameter. As mentioned in the introduction above, \( m \) can be considered as the quantity of tasks in one process. If the shape parameter \( m = 1 \), Erlang distribution simplifies to the exponential distribution.
4. Z is target processing time, which is mentioned in the model explanation.

And other parameters are assumed as follows. Consecutive Delay Cost: \( C^{(1)}_p = 40, C^{(2)}_p = 80, C^{(3)}_p = 160, C^{(4)}_p = 320, C^{(5)}_p = 640, C^{(6)}_p = 1280, C^{(7)}_p = 2560, C^{(8)}_p = 5120, \) Period \( n = 7, 8, 9 \); Target Processing Time \( Z = 2 \); Idle Cost \( C_s = 20 \).

5.1 Variation trend of expected cost by changing the assignment of the first rookie workers

Under the assumption above, by assigning the second and third worker at the last two processes, we calculate the expected cost when shifting the first rookie worker. And the result is concluded as the Table 1, and showed rules which we feel curious.
In Table 1, it can be known that, in spite of the disparity of speed of the workers, the expected cost reaches lowest value when 1 rookie worker is assigned to the first process. And it verifies the rules mentioned in the chapter 4.

5.2 The relationship between optimal assignment and the disparity of workers’ processing speed

We can get the optimal assignments by calculating the expected cost of all assignments under different condition. Table 1 showed the result when shape parameter $m = 2$, processing rate $\mu_A = 0.1$ and $\mu_B$ varies from 0.2 to 2.0.

Table 2 shows that despite of the number of processes $n$, the varying patterns are similar. If the disparity of processing rate is small, $\pi(1, 2, 3)$ is the optimal assignment which means that these three rookie workers are assigned at the first three processes. We call this kind of assignment as centralized optimal assignment.

And when the disparity becomes great, it is optimal that three rookie workers are as far away from each other as possible which can be represented as $\pi(1, \frac{1}{2}n, n) (n \text{is odd number})$ or $\pi(1, \frac{n}{3}, \frac{2n}{3}) (n \text{is even number})$. We call this kind of assignment as decentralized optimal assignment.

However when the disparity is between the above two conditions, the expected cost of assignment $\pi(1, x, n)$ is the minimal where $1 < x < \frac{1 + \sqrt{5}}{2}$. And this kind assignment is called as transition optimal assignment.

5.3 The relationship between optimal assignment and the shaping parameter

Table 3 showed the influence of shaping parameter to the optimal assignment. By numerical experiment under the same processing periods $n = 9$ and setting the processing rate of rookie worker $\mu_A = 0.1$, four groups of data with variable shaping parameter $m$ are listed by ascending order of $\mu_B$. We can realize that no matter how the shaping parameter changes, the three kinds of optimal assignment is invariant. In other words, we can assume that the new kind of optimal assignment will not appear and these three kinds of optimal assignment will also not disappear in any value of shaping parameter $m$. And this inspires us that these three kinds of optimal assignment are constant even when workers’, processing speed follows other distribution, such as Poisson distribution and Gamma distribution. And in the future researches, we will try to demonstrate this as a law by numerical proof.

However, when $m = 1$, the centralized optimal distribution appears until $\mu_B$ is 0.3, and when $m$ increase to 4, the centralized optimal distribution was expanded to until $\mu_B$ is 1.9. And also the occurrence of transition and decentralized optimal assignment is later.

6. Conclusion

In this paper, we considered the properties of optimal assignment with three rookie workers in LCMwMP. First, we systematically classified and modeled the multi-period problem and defined the optimal workers assignment problem under the reset model. Secondly, by the conclusion of optimal worker assignment with two rookie workers [3][4], we assume the assignment of the case of three rookie workers. Finally, we get conclusions by analyzing the results of numerical experiments.

1. By demonstration, we found the rule that, one rookie worker must be assigned at the beginning of all processes for achieving the optimal assignment.

It means, the expected cost of $\pi(1, j, k)$ is always smaller than the expected cost of $\pi(i, j, k)$.

$$TC(n; \pi(1, j, k)) < TC(n; \pi(i, j, k))$$

2. In spite of the value of parameter and number of processes changes, the three types of optimal assignment is constant. We call $\pi(1, 2, 3)$ centralized optimal assignment which means the three rookie workers are assigned at the first three processes. And $\pi(1, j, n)$, where $1 < j < \frac{1 + \sqrt{5}}{2}$, is called as transition optimal assignment, which implies that two of the rookie workers are assigned at the head and tail of the processing line and another one is assigned at the first half of the line. Finally, $\pi(1, \frac{1}{2}n, n)(\text{when} n \text{is odd number})$ or $\pi(1, \frac{n}{3}, \frac{2n}{3})(\text{when} n \text{is even number})$ is the type of decentralized optimal assignment, which suggests that it is optimal when three rookie workers are assigned as far as possible from each other.

As a goal of future researches, we will try to verify the affecting factors of the optimal assignment by mathematical demonstration. It is also clear that the optimal assignment is strongly influenced by the relation between increasing rate of consecutive delay cost and the disparity of two kinds of worker and we will search for the property of it by theoretic analysis.

References

[1] M. E. Salveson, 1955. The assembly line balancing problem. The Journal of Industrial Engineering, 6(3), pp. 18, 5.
[2] B. Nils, F. Malte, and S. Armin, 2007. A classiﬁcation of assembly line balancing problems, European Journal of Operational Research, 183, pp. 674-693.
[3] H. Yamamoto, M. Matsui, and J. Liu, 2006. A basic study on limited-cycle assembly line balancing problems, European Journal of Operational Research, 183, pp. 674-693.
[4] H. Yamamoto, J. Sun, and M. Matsui, 2010. A study on limited-cycle scheduling problem with multiple periods, Journal of Japan Industrial Management Association, 57(1), pp. 23-31.
[5] H. Yamamoto, J. Sun, and M. Matsui, 2007. A branch and bound method for the optimal assignment during a Limit-cycle problem with multiple periods, Journal of Japan Industrial Management Association, 58(1), pp. 37-43.
[6] H. Yamamoto, J. Sun, M. Matsui, and X. Kong, 2011. A study of the optimal arrangement in the reset limited-cycle problem with multiple periods: with fewer special workers, Journal of Japan Industrial Management Association, 62(5), pp. 239-246.
[7] X. Kong, J. Sun, H. Yamamoto, and M. Matsui, 2010. A Study of an optimal arrangement of a processing system with two kinds of workers in a limited-cycle problem with multiple periods, Proceedings of the 11th Asia Pacific Industrial Engineering and Management Systems Conference, Melaka, Malaysia. (on CD-ROM).
[8] X. Kong, J. Sun, H. Yamamoto, and M. Matsui, 2011. Two special Workers’, optimal assignment with two kinds of workers under a limited-cycle problem with multiple periods. Proceedings of the 21st International Conference of Production Research, Stuttgart, Germany. (on CD-ROM).
[9] X. Kong, J. Sun, H. Yamamoto, and M. Matsui, 2011. Optimal worker assignment with two special workers in limited-cycle
multiple periods. Proceedings of the 2011 Asian Conference of Management Science and Applications, Sanya, China. (on CD-ROM).

[10] J. Sun, H. Yamamoto, and M. Matsui, X. Kong, 2012. Optimal switching frequency in Limited-Cycle with Multiple Periods, Industrial Engineering and Management System, 11(1), pp. 48-53.

[11] P. Song, X. Kong, H. Yamamoto, J. Sun, and M. Matsui, 2014. A Study of Optimal Worker Assignment with Three Rookies under a Limited-Cycle Model with Multiple Periods. Proceedings of the 2014 Asia Pacific Conference on Management Science and Applications, Sanya, China. (on CD-ROM).

[12] P. Song, X. Kong, H. Yamamoto, J. Sun, and M. Matsui, 2014. Numerical Analysis of Three Rookies Assignment Optimization in Limited-Cycled Model with Multiple Periods - the case of Erlang Distribution-, Proceedings of 15th Asia Pacific Industrial Engineering and Management Systems Conference, Ramada Plaza, Jeju Island, Korea. (on CD-ROM).

Appendix

A. Demonstration

When the assignment is \( \pi(i, j, k) \), the expected cost (the sum of the expected idle cost and the expected delay cost) \( f(n; \pi(i, j, k)) \), which caused in process \( n \), is

\[
f((n; \pi(i, j, k)) = C_i(n-3)T_L B + 3TS_A + \sum_{u=1}^{n} \sum_{i=1}^{n} CF(w, a; \pi(i, j, k))
\]

(A. 1)

\[
CF(w, a; \pi(i, j, k)) = \begin{cases} 
C_p \cdot T_L B \cdot P_B \cdot Q_B^{w-1} & w < i \\
C_p \cdot T_L A \cdot P_B \cdot Q_B^{w-1} & w = i \\
C_p \cdot T_L B \cdot P_B \cdot Q_B^{i-1} & i < w < j, a < w - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_B^{i-1} & i < w < j, a = w - i \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & i < w < j, a > w - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & w = j, a < j - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-1} & w = j, a = j - i \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & w = j, a > j - i \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-1} & j < w < k, a < w - j \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-1} & j < w < k, a = w - j \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & j < w < k, w - j < a < w - i \\
C_p \cdot T_L B \cdot P_A \cdot Q_A \cdot Q_B^{w-2} & j < w < k, a = w - i \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-3} & j < w < k, a > w - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-3} & w = k, a < k - j \\
C_p \cdot T_L A \cdot P_A \cdot Q_A^{w-1} & w = k, a = k - j \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & w = k, k - j < a < k - i \\
C_p \cdot T_L A \cdot P_A \cdot Q_A \cdot Q_B^{w-2} & w = k, a = k - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-3} & w = k, a > k - i \\
C_p \cdot T_L A \cdot P_B \cdot Q_A \cdot Q_B^{w-1} & w = k, a < w - k \\
C_p \cdot T_L A \cdot P_A \cdot Q_A^{w-1} & w = k, a = w - k \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-2} & w = k, w - k < a < w - j \\
C_p \cdot T_L B \cdot P_A \cdot Q_A \cdot Q_B^{w-2} & w = k, a = w - j \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-3} & w = k, w - j < a < w - i \\
C_p \cdot T_L B \cdot P_A \cdot Q_A \cdot Q_B^{w-3} & w = k, a = w - i \\
C_p \cdot T_L B \cdot P_B \cdot Q_A \cdot Q_B^{w-4} & w = k, a > w - i \\
\end{cases}
\]

(A. 2)

In case of the difficulty of the definition equation (2), the expected cost cannot be calculated easily. So when we try to investigate which arrangement of first rookie worker is optimal, it is sensible by focusing on the disparity between \( CF(w, a; \pi(i, j, k)) \) and \( CF(w, a; \pi(i + 1, j, k)) \).

\[
CF(w, a; \pi(i + 1, j, k)) =
\]
As mentioned above, we try to find the disparity between 

\[ CF(w, a; \pi(i + 1, j, k)) \]

and 

\[ CD(w, a; \pi(i, j, k)) \]

By knowing this, we can know the changing cost when the first rookie worker is moved to the next period. As a result, we can gain the expected cost figure just like the Fig. 2 so we can know which assignment has the lowest expected cost therefore is the optimal.

\[ CF(w, a; \pi(i + 1, j, k)) - CF(w, a; \pi(i, j, k)) = \]

\[
\begin{align*}
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_B^{-1} & \quad w < i + 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_B^{-1} & \quad w = i + 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_B^{-1} & \quad i + 1 < w < j, a < w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_B^{-1} & \quad i + 1 < w < j, a = w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-2} & \quad i + 1 < w < j, a > w - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-1} & \quad w = j, a < j - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_A^{-1} & \quad w = j, a = j - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-2} & \quad w = j, a > j - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_B^{-1} & \quad j < w < k, a < w - j \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_B^{-1} & \quad j < w < k, a = w - j \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-2} & \quad j < w < k, w - j < a < w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_A^{-2} & \quad j < w < k, a = w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-3} & \quad j < w < k, a > w - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-1} & \quad w = k, a < k - j \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_A^{-1} & \quad w = k, a = k - j \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-2} & \quad w = k, k - j < a < k - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_A^{-2} & \quad w = k, a = k - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-3} & \quad w = k, a > k - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-1} & \quad w > k, a < w - k \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_A^{-1} & \quad w > k, a = w - k \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-2} & \quad w > k, w - k < a < w - j \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_A^{-2} & \quad w > k, a = w - j \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-3} & \quad w > k, w - a < a < w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_A^{-3} & \quad w > k, a = w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-4} & \quad w > k, a > w - i - 1 \\
\end{align*}
\]

(A.3)

\[
\begin{align*}
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_B^{-1} & \quad w = i, w > a \\
C_p^{(a)} \cdot TL_B \cdot Q_B^{-1} & \quad w = i, w = a \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_B^{-1} & \quad w = i + 1, 1 < a < w \\
C_p^{(a)} \cdot TL_A \cdot P_B & \quad w = i + 1, a = 1 \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_B^{-1} & \quad w = i + 1, a = w \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_B^{-1} & \quad i + 1 < w < j, a = w - i \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_B^{-1} & \quad i + 1 < w < j, a = w - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_B^{-1} & \quad w = j, a = w - i \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_B^{-1} & \quad w = j, a = w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-3} & \quad j < w < k, a = w - i \\
C_p^{(a)} \cdot TL_B \cdot P_A \cdot Q_A^{-3} & \quad j < w < k, a = w - i - 1 \\
C_p^{(a)} \cdot TL_A \cdot P_B \cdot Q_A^{-3} & \quad w = k, a = w - i \\
C_p^{(a)} \cdot TL_A \cdot P_A \cdot Q_A^{-3} & \quad w = k, a = w - i - 1 \\
C_p^{(a)} \cdot TL_B \cdot P_B \cdot Q_A^{-4} & \quad w = k, a = w - i - 1 \\
\end{align*}
\]

(A.4)

By calculating summation of all above, we can get the Eq. (6).
Peiya Song is now a master of Graduate School of System Design, Tokyo Metropolitan University, Japan. He received bachelor degree from University of Science and Technology of China, in 2011. He is studying in the field of management systems design, optimization and industrial engineering.

Xianda Kong is an assistant professor of Tokyo Metropolitan University, Japan. And he is also a lecturer of Dokkyo University, Japan. He received his Dr. Eng. degree from Tokyo Metropolitan University, Japan. He is a member of JIMA, IEEE, INFORMS, and IAENG. His research interests include optimal worker assignment in a production system, management engineering.

Hisashi Yamamoto (Member) is a Professor of Tokyo Metropolitan University, Japan. He received the B.S., M.S. and Doctoral degree (Dr. Eng.) in industrial engineering from Tokyo Institute of Technology, Japan. His main research interests are optimizations based on the reliability engineering. He worked as the Chair of IEEE Reliability Society Japan Chapter, the Editor-in-chief of JIMA and REAJ. He was the Chair of APARM2012 program committee, General Co-chair of APARM2014 and a member of IEICE EA committee.

Jing Sun is an assistant professor in Nagoya Institute of Technology, also a guest investigator of Aoyama Gakuin University, Japan. She received her Dr. Eng. degree from The University of Electro-Communications, Japan. Her current research interests are operations research, statistical quality management, and ERP/SCM. She is now the chairman of the international affair committee of Japan Industrial Management Association, a member of board of directors of Asian Association of Management and Applications.

Masayuki Matsui is a Professor in the Kanagawa University, and also the former President of Japan Industrial Management Association (JIMA). He received a Deng in research on conveyor-serviced production systems from Tokyo Institute of Technology, Japan. He was a Visiting Scholar of UC Berkeley and Purdue University in 1996 and 1997. His recent research interests are industrial engineering production and operations management, management theory and ERP/SCM; operations research, quality management and artificial intelligence. He was the Editor (2000', 003) and Director (2003', 005) of the JIMA journal and is a senior member of IIE.