Parameter Estimation in Electrical Power Systems Using Prony’s Method

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Abstract. This paper presents the basic principle of Prony’s method, which is used for determining electrical power system parameters such as phasors, angular speeds as well as damping factors in a large frequency range. For fixed sampling frequencies, the approach to finding an optimal model order and to extracting transient parameters for further identification of different system effects is shown. Furthermore, it is shown that the method can be applied to the diverse plausibility checks for a confirmation of the result obtained using a conventional Fourier based approach. It will be presented that the significant advantage of the Prony algorithm in comparison to Fourier based methods is the possibility of computing an accurate frequency and in determining a damping factor.

1. Introduction

Power system protection technology has undergone large steps in development from electro-mechanics to digital devices. Thanks to that the power system protection became more sensitive, reliable and faster. This contributes to a high network stability and security. Modern digital protection systems are highly complex and require a large degree of knowledge in measurement and signal processing techniques. In order for the protection devices to work accurately, the precise determination of the network state is necessary. This is done primarily through so called phasor quantities determined by Fourier-based algorithms. These provide good results for many applications, but in dynamic processes, they have a limited performance. Besides the FFT, other methods used for calculating network parameters and quantities can be applied successfully. Prony’s method is one of these, which makes a prediction on the dynamics of a system using a mathematical model and a limited amount of measurement values. As such, it can be used to determine the phasors and frequency components of an electrical network. This publication examines whether Prony’s method is applicable to modern protection systems and whether it can be used to identify different power system effects.

First examinations of Prony’s method in the context of electrical power systems were published in 1990 [1]. The authors describe applications of the method and first results. Furthermore, they compare the methods against other known mathematical models. Moreover, Prony’s method is primarily used for spectral analysis of distorted signals as in [6], as well as in the area of power electronics, to determine the harmonics and inter-harmonics in frequency inverters [7]. In [8] Prony’s method is shortly explained and used for the analysis of fault currents. Statements are made about the model order, length of the measuring window, and the sampling frequency. In this context, [9] and [10]
should also be mentioned. They also deal with the choice of the sampling frequency, in order to improve the efficiency of Prony’s method. Fault currents were also analyzed in [11, 12, 13, 14]. In [11] the authors analyzed phase-to-ground faults using Prony’s method in isolated and compensated power systems. Symmetrical fault-currents are examined in [12]. The authors use the presence of a decaying DC component in the currents as an indication of a symmetrical fault. Inrush currents are examined in [13] with Prony’s method and are compared to the FFT-analysis. In [14] Prony’s method is brought into conjecture with the reconstruction of currents, which are distorted by transformer saturation.

Prony’s method is also examined in the context of fault-location algorithms. In [15, 16] the authors use the calculated frequencies and magnitudes of the currents during the process to calculate the fault location in DC networks. Other methods for determining the fault location in transmission and distribution networks are presented in [17, 18, 19, 20].

In connection with the identification of power swing effects, Prony’s method appears, among others, in [21]. The author also describes the suppression of these effects using Power System Stabilizers [21].

The paper is organized as follows: Section 2 gives an introduction into Prony’s method. Specifically, the classic variant of Prony’s method and the least square Prony method are presented. In Section 3 Prony’s method is used for the identification of different power system effects. Section 4 gives a conclusion and an outlook.

2. Prony’s method

Prony’s method is a parametric identification procedure which was developed in 1795 by Gaspard de Prony. In Figure 1 Prony’s method is represented schematically with inputs and outputs.

![Figure 1. Input-output diagram of Prony’s method](image)

The method is based on the assumption that a discrete series of N measured values \( y[1], \ldots, y[N] \) of any sinusoidal or exponentially damped signal can be approximated by a linear combination of \( p \) exponential terms. The calculated signal components consist of positive, as well as negative frequencies. This can be interpreted as a phasor rotating one or the other way. The usage of Euler’s formula relates the components to scalar and time-discrete values \( \hat{y}[n] \).

\[
\hat{y}[n] = \sum_{k=1}^{p} A_k \cdot e^{(\alpha_k+j\omega_k)(n-1)T_s} \cdot e^{j\varphi_k}
\]  

where:

- \( n \) - \( 1, 2, \ldots, N \); \( N \) is the length of the signal
- \( p \) - Number of exponentials / model order
- \( T_s \) - Sampling period in [s]
- \( A_k \) - Magnitude
- \( \alpha_k \) - Damping factor in [1/s]
- \( \omega_k \) - Angular velocity in [rad/s]
- \( \varphi_k \) - Initial phase angle in [rad]
The discrete-time function in equation (1) can be simplified by separation into time-dependent and time-independent components:

\[ \hat{y}[n] = \sum_{k=1}^{p} h_k \cdot z_k^{n-1} \]  

(2)

The time-independent component is introduced as the complex magnitude \( h_k \), defined as:

\[ h_k = A_k \cdot e^{i\varphi_k} \]  

(3)

The complex exponent \( z_k \) is a time-dependent parameter, defined as:

\[ z_k = e^{(\alpha_k + j\omega_k)T_s} \]  

(4)

In order to represent the signal in a parametric form, the deviation of the model output from the original signal needs to be produced. The model error \( e[n] \) is calculated for each data point.

\[ e[n] = y[n] - \hat{y}[n] = y[n] - \sum_{k=1}^{p} h_k \cdot z_k^{n-1} \]  

(5)

A better indicator is the root mean square error \( V \) over \( N \) measurement points. This will be the criterion for the model’s quality.

\[ V = \frac{1}{N} \sum_{n=1}^{N} \sqrt{e[n]^2} \]  

(6)

The goal function minimization process leads to a nonlinear problem. The idea of Prony’s method is to solve the difficult nonlinear problem by using linear equations. Therefore, Prony proposed a method which considers the nonlinear aspect with the help of a polynomial factorization process.

### 2.1. Classic variant of Prony’s method

The classic variant uses twice as many sampled data \( N \) as exponential components \( p \) to exactly model \( 2p \) complex parameters \( h_1, \ldots, h_p \) and \( z_1, \ldots, z_p \).

\[ y[n] = \sum_{k=1}^{p} h_k \cdot z_k^{n-1} \]  

(7)

Equation (7) shows the aim of Prony’s method. A detailed expression for \( 1 \leq n \leq p \) is given in equation (8):

\[
\begin{bmatrix}
1 & 1 & \cdots & 1 \\
z_1 & z_2 & \cdots & z_p \\
\vdots & \vdots & \ddots & \vdots \\
z_1^{p-1} & z_2^{p-1} & \cdots & z_p^{p-1}
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_p
\end{bmatrix}
= 
\begin{bmatrix}
y[1] \\
y[2] \\
\vdots \\
y[p]
\end{bmatrix}
\]  

(8)

Assuming the knowledge of the elements \( z \), the complex magnitude vector \( h \) can be solved through the linear equations. Furthermore, Prony proposed that equation (14) is the solution of a homogeneous linear differential equation and defines a polynomial \( F(z) \) of degree \( p \) that has the \( z_k \) exponents as its roots [2, 3]:

\[ F_p(z) = \prod_{k=1}^{p} (z - z_k) = (z - z_1) \cdot (z - z_2) \cdot \cdots \cdot (z - z_p) \]  

(9)
The polynomial can be represented as a sum after utilizing the fundamental theorem of algebra:

\[ F_p(z) = \sum_{m=0}^{p} a[m] \cdot z^{p-m} = a[0] \cdot z^p + a[1] \cdot z^{p-1} + \cdots + a[p-1] \cdot z + a[p] \]  

(10)

The coefficient \(a[0] = 1\) is defined. A shift in the index in equation (7) from \(n\) to \(n-m\) and a multiplication by the parameter \(a[m]\) results in equation (11) [2]:

\[ a[m] \cdot y[n-m] = a[m] \cdot \sum_{k=1}^{p} h_k \cdot z_k^{n-m-1} \]  

(11)

The sum of the products results in equation (12) for \(p + 1 \leq n \leq 2p\):

\[ \sum_{m=0}^{p} a[m] \cdot y[n-m] = \sum_{m=0}^{p} a[m] \cdot \left[ \sum_{k=1}^{p} h_k \cdot z_k^{n-m-1} \right] = \sum_{k=1}^{p} h_k \cdot \sum_{m=0}^{p} a[m] \cdot z_k^{n-m-1} \]  

(12)

The substitution of

\[ z_k^{n-m-1} = z_k^{n-p-1} \cdot z_k^{p-m} \]  

(13)

results in:

\[ \sum_{m=0}^{p} a[m] \cdot y[n-m] = \sum_{k=1}^{p} h_k \cdot z_k^{n-p-1} \cdot \sum_{m=0}^{p} a[m] \cdot z_k^{p-m} = 0 \]  

(14)

This equation is the desired linear differential equation. Looking at the summation in equation (14), the defined polynomial in equation (10) can be identified. By determining the roots \(z_k\) we obtain the sought zeros. Equation (14) can be solved exactly for the coefficients \(a[m]\) that are represented by \(p\) equations.

\[
\begin{bmatrix}
y[p] \\
y[p+1] \\
y[p+2] \\
\vdots \\
y[2p]
\end{bmatrix}
\begin{bmatrix}
y[p-1] \\
y[p] \\
y[p+1] \\
\vdots \\
y[2p-1]
\end{bmatrix}
\begin{bmatrix}
y[p-2] \\
y[p-1] \\
y[p] \\
\vdots \\
y[p-2]
\end{bmatrix}
\begin{bmatrix}
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\cdots 
\end{bmatrix}
= \begin{bmatrix}
y[0] \\
a[0] \\
a[1] \\
a[2] \\
a[p]
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(15)

Equation (15) shows an over-determined, linear Toeplitz-type system. To determine a solution vector, the top line in the equation needs to be deleted and the first column needs to be substracted.

\[
\begin{bmatrix}
y[p] \\
y[p+1] \\
y[p+2] \\
\vdots \\
y[2p-1]
\end{bmatrix}
\begin{bmatrix}
y[p-1] \\
y[p] \\
y[p+1] \\
\vdots \\
y[2p-2]
\end{bmatrix}
\begin{bmatrix}
y[p-2] \\
y[p-1] \\
y[p] \\
\vdots \\
y[p-2]
\end{bmatrix}
\begin{bmatrix}
\cdots \\
\cdots \\
\cdots \\
\cdots \\
\cdots 
\end{bmatrix}
= \begin{bmatrix}
y[0] \\
a[1] \\
a[2] \\
\vdots \\
a[p]
\end{bmatrix}
= \begin{bmatrix}
y[p+1] \\
y[p+2] \\
\vdots \\
y[2p]
\end{bmatrix}
\]

(16)

The basic procedure shown in Figure 2 can be separated into three steps:
1. Step: Solve equation (16) and determine the parameters $a[0] \ldots a[p]$, which are the polynomial's coefficients in equation (10).

2. Step: Find the roots of the characteristic polynomial in equation (10). These roots represent the time-dependent parameter vector $z_k$ in equation (7). The damping factors and the frequency components can be determined with the aid of $z_k$.

The $k^{th}$ frequency is defined as:

$$f_k = \frac{\arctan\left(\frac{\text{Im}(z_k)}{\text{Re}(z_k)}\right)}{2 \cdot \pi \cdot T_s}$$

(17)

Further the $k^{th}$ damping factor can be calculated:

$$\alpha_k = \text{Re}\left(\frac{\ln(z_k)}{T_s}\right)$$

(18)

3. Step: Solve the system of linear equations (8) and determine the $k^{th}$ magnitude $A_k$ and the $k^{th}$ initial phase angle $\varphi_k$.

$$A_k = |h_k|$$

(19)

$$\varphi_k = \arctan\left(\frac{\text{Im}(h_k)}{\text{Re}(h_k)}\right)$$

(20)
2.2. Least squares Prony’s method
If the amount of data points is greater than the number of parameters \( 2p \) then the sampled data can only be approximated but not interpolated by exponential functions. This approach is referred to as the minimization of the forward prediction.

\[
e[n] = \sum_{m=1}^{p} a[m] \cdot y[n - m]
\]  
(21)

The identification of the relevant parameters \( a[m] \) corresponds to the classic variant of Prony’s method. The zeros of the characteristic polynomial with the coefficients \( a[m] \) are \( z_1, ..., z_p \). This results in the following system of equations:

\[
\begin{bmatrix}
1 & 1 & \cdots & 1 \\
z_1^1 & z_2^1 & \cdots & z_p^1 \\
\vdots & \vdots & \ddots & \vdots \\
z_1^{N-1} & z_2^{N-1} & \cdots & z_p^{N-1}
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_p
\end{bmatrix}
=
\begin{bmatrix}
y[1] \\
y[2] \\
\vdots \\
y[N]
\end{bmatrix}
\]  
(22)

To obtain the complex magnitudes \( h_1, ..., h_p \), the following calculation is applied:

\[
Z \cdot h = y
\]  
(23)

\[
Z' \cdot Z \cdot h = Z' \cdot y
\]  
(24)

\[
h = (Z' \cdot Z)^{-1} \cdot Z' \cdot y
\]  
(25)

where \( Z \in \mathbb{C}^{N \times p} \), \( h \in \mathbb{C}^{p \times 1} \) and \( y \in \mathbb{R}^{N \times 1} \).

The remaining calculation steps are analog to the classic variant of Prony’s method.

3. Analysis of the power system quantities
Electrical Power Systems, in particular medium and high voltage systems, are naturally time variant due to event-driven switching of power lines and network areas. Thus, based on a local data acquisition in protection devices the structure and electrical parameters of the circuit diagram (of the electrical power system) change arbitrarily. For this reason, the model based approaches like state observer are not straightforward applicable to estimate states and system parameters like damping factors. During normal power system operation, currents and voltages are pure sinusoidal curves containing one fundamental component. However, it cannot be avoided that unforeseeable disturbances appear in power system. These contribute to significant changes in current and in voltage waves which include fundamental and periodic components. It can be noted that, during disturbances, the frequency, phase, magnitude, as well as the damping of undesired oscillations in current and voltage, are unknown. The commonly used Fourier-Transformation based methods do not deliver some signal parameters in such situations (they must be assumed and depend on the measurement window in which the analysis takes place) or their qualities are not acceptable. A very good alternative to the Fourier-Transformation seems to be the Prony-Algorithm. The description of the signal consists of its decomposition into exponential functions, including all periodic parameters (frequency, phase, magnitude and the damping factor) to be estimated. Based on the decomposed signal through Prony’s method and analysis of the characteristic components, it can be concluded which effect is occurring in the network. The result of Prony’s method does not depend on the measurement windows, which is the case for a Fourier-Transformation. The significant advantage of Prony’s method over a Fourier-Transformation, is the possibility of online estimation of the damping factor of the periodic signal. It
can be used in a plausibility check for a confirmation of the result. Especially the damping factor might show heavy fluctuations and still it corresponds to a realistic measurement.

The data analysis with Prony’s method is performed in such way that a measurement window with a fixed length is chosen. The analysis is then subsequently carried out for every new data point. The number of frequency components delivered by Prony’s method correlates with the data window length. Following this, only the more significant components arranged by magnitude will be considered.

\[
magnitude(f_1) \geq magnitude(f_2) \geq magnitude(f_3) \geq magnitude(f_4)
\]

3.1. Short circuit analysis

In order to simplify the consideration, the simulated fault current is analyzing using Prony’s method. The pre-fault condition is a current containing only a fundamental component with a frequency of 50 Hz and a magnitude of 1 A. From fault inception, an increasing current in the 50 Hz component and an additional DC component can be seen. This DC component can be interpreted as unwanted noise in the conventional Fourier-based phasor calculation. The expectation is that Prony’s method will estimate the fundamental component of 50 Hz with higher accuracy.

Figure 3. Prony parameter analysis of short circuit: \( p = 3; N = 6, f_S = 1 \) kHz
In Figure 3, the results of the fault analysis using Prony’s method is presented. The measurement window length is 6 ms = 6 samples. The resulting model order is $p = 3$. This chosen model order allows for the estimation of two periodic components. During the power system fault, only the fundamental and DC component appears. Thus, the order of 3 is sufficient to accurately identify the signal parameters. This is a simulated case, therefore the signal does not include additional distortion or noise. All parameters obtained using Prony’s method are presented with the appropriate time assignments. The presented results are correct and confirm the abilities of Prony’s method. The analysis of the damping factor of the DC component shows that the power system time constant $\tau$ can be estimated. The calculation is only possible if there is a low fluctuation in this parameter. From this example the time constant can be extracted numerically.

\[
e^{\alpha_2} = e^{-\frac{1}{\tau}}
\]

\[
\tau = \frac{-1}{\alpha_2}
\]

\[
\tau = \frac{-1}{\frac{1}{20}} = 50 \text{ ms}
\]

Prony’s method enables an identification of power system fault components and a description in form of phasor quantities. Also, the very short measurement window does not limit the application of Prony’s method in the field of power system protection and monitoring.

3.2. Inrush analysis:
When energizing a power transformer, large inrush currents can occur. These currents vary in shape and magnitude with the voltage angle and with the transformer core’s premagnetization. The voltage switching moment has an impact on the magnetic flux in every phase. In addition, the residual magnetism (remanence) affects the inrush current flow. Peak currents of 8 times the nominal current can be expected. Current differential protection is one of the most selective protection schemes. It is commonly used in medium and high voltage power systems. The principle of differential protection is based on Kirchhoff’s first law. The protected zone is defined by the current transformer (CT) positions. All devices process distributed current measurements at the terminals. The complex current sum of all terminals is close to zero in a normal operating state. If the complex current sum is not zero, then an internal fault is detected and the protection devices trip the circuit breaker. The protected object is then disconnected from the power system. The occurrence of inrush currents within the protected zone might lead to a malfunction in the protection devices. The magnetizing current remains within the protected zone and the transformer is energized. A common method to cope with inrush events is to increase the restraint current or to block the differential protection’s operation. The inrush detection can be triggered by a high proportion of the second harmonic in the current signal.
Selecting a measurement window length and model order:
Selecting the right measurement window length is of great importance in order to parametrically describe a signal sequence. The following figure shows the spectral decomposition of an inrush current signal. Short data windows lead to unstable and jittering frequency components but an increasing window length generates stable results. A window length of 40 ms (= 40 samples) provides the best results for that case. The frequency components contained in the signal can be extracted in a steady-state manner.

![Figure 4. Selection of model order and measurement window](image)

The analysis of the frequency components phase angle, magnitude and damping are depicted in figure 5. It can be shown that the signal contains a high second and third harmonic component. Also, it can be seen that the frequency components vary in time. The damping constant \( \alpha \) at a model order of \( p = 20 \) yields values between 5 [1/s] and -15 [1/s]. This indicates an increasing oscillation or a decreasing oscillation, respectively. A data window at the length of approximately one period of the fundamental frequency yields results which are nearly constant over time.
Figure 5. Analysis of a typical inrush current: $p = 20, N = 40, f_s = 1 \text{ kHz}$

The analysis of a power system fault and an inrush current utilizing Prony’s method delivers substantial differences. In most cases, a power system fault consists of a large ratio of a fundamental frequency. Evaluating the second harmonic content can be an appropriate indicator to detect inrush events and to distinguish between other power system effects. For example, if the current is affected by CT saturation it can be complicated to discriminate these two effects. To achieve more solid results the measuring windows length could be increased. A windows length of 40 ms delivers a robust and reliable spectral decomposition. The inrush detection can be carried out for 30 ms after the energization. To achieve a better real-time performance it is possible to decrease the sampling rate (downsampling). The Nyquist-Shannon theorem must still be fulfilled.
4. Conclusion

The accuracy of the result using Prony’s method strongly depends on the chosen model order, the sampling frequency, and the measurement window. Parameters that were chosen badly, lead to an inaccurate result, and thus to a large deviation from the original signal. Prony’s method is not able to describe the input signal in spectral components in this case. Another effect which has to be considered is the algorithmic performance. Large data windows lead to a huge computational effort.

An important advantage of Prony’s method in comparison to the current method is that the frequencies being calculated are not limited in their bandwidth. The frequency components are the result of a minimization process and can take non-integer values. An application which benefits from this fact can be the power swing detection. Under optimal circumstances the results can be calculated within shorter measurement windows than it is possible with classic methods and is available only a few seconds after the start of an event. More complicated signal sequences will need more time for evaluation. Prony’s method is able to create a new cluster of signal processing algorithms and to accelerate and improve current techniques. In the future, these methods can have a share in power system protection algorithms and the network stability and reliability can be further increased.

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