Orbital dependent nucleonic pairing in the lightest known isotopes of tin

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By studying the 109\text{Xe} \rightarrow 105\text{Te} \rightarrow 101\text{Sn} superallowed $\alpha$-decay chain, we observe low-lying states in 101\text{Sn}, the one-neutron system outside doubly magic 100\text{Sn}. We find that the spins of the ground state ($J = 7/2$) and first excited state ($J = 5/2$) in 101\text{Sn} are reversed with respect to the traditional level ordering postulated for 103\text{Sn} and the heavier tin isotopes. Through simple arguments and state-of-the-art shell model calculations we explain this unexpected switch in terms of a transition from the single-particle regime to the collective mode in which orbital-dependent pairing correlations dominate.

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Atomic nuclei exhibit a variety of motions, ranging from single-particle (s.p.) behaviour as dictated by the properties of very few nucleons, to collective phenomena which are emergent in character. In the nuclear realm, doubly magic nuclei with closed proton and neutron shells are of particular importance since they provide a shell-model framework 1 through which we explain nuclear behaviour. Among magic species, the self-conjugated ($N = Z = 50$) nucleus 100\text{Sn} is of special significance: revealed experimentally as a short-lived, neutron impoverished system predicted to be the endpoint of the most enhanced $\alpha$-decays known 2. From a theoretical standpoint, 100\text{Sn} is the cornerstone of our understanding of nuclei within the entire 50 $\leq N, Z \leq 82$ region and a perfect laboratory for studying a variety of proton and neutron modes at the limits of particle stability.

The low-energy structure of semi-magic nuclei such as the tin isotopes is usually dominated by pairing correlations, or nucleonic superfluidity 3. In tin isotopes with an even number of neutrons, the $J^\pi = 0^+$ ground state (g.s.) can be viewed as a condensate of monopole (spin zero) Cooper pairs of valence neutrons (zero-quasiparticle state), while the lowest-lying states of neighbouring odd-mass isotopes can be viewed as one-quasiparticle states whose spins are determined by the angular momenta of the orbitals occupied by the unpaired neutron. Experimental data for the known semi-magic isotopic and isotonic chains indicate no exception to this rule, at least in the vicinity of shell closures.

The s.p. neutron states outside the 100\text{Sn} core are believed to be the closely-spaced $d_{5/2}$ and $g_{7/2}$ orbitals, consistent with the idea of pseudo-spin 1, 5. Thus, one expects the g.s. spins for 101\text{Sn}, 103\text{Sn}, 105\text{Sn} to be identical and equal to $J^\pi = 5/2^+$ or $J^\pi = 7/2^+$, depending on whether $d_{5/2}$ or $g_{7/2}$ is the lowest-energy s.p. orbital. One of the main determinations of this work is the ordering of these crucial single-neutron levels in the nucleus 103\text{Sn} from $\alpha$-decay spectroscopy, which reveals that the usual shell-model extrapolation does not apply to the neutron-deficient tin isotopes.

The identification of the two lowest states in 101\text{Sn} was carried out at the Holifield Radioactive Ion Beam Facility, Oak Ridge National Laboratory. The states in 101\text{Sn} were observed by studying the $\alpha$-decay chain 109\text{Xe} \rightarrow 105\text{Te} \rightarrow 101\text{Sn} 2 following heavy ion fusion evaporation reactions of 54\text{Fe} and 58\text{Ni}. Mass A = 109 reaction products were resolved by atomic mass-to-ionic charge ratio and separated from un-reacted primary beam using the Recoil Mass Spectrometer (RMS) 6. The recoiling fusion evaporation residues were implanted at the RMS focal plane into a double-sided silicon strip detector (DSSD) where subsequent radioactive decays were observed. The experiments were instrumented with digital electronics 7 capable of selectively capturing preamplifier double-pulse waveforms from very rapid sequential detector signals. Two campaigns were performed: (a) low-rate high DSSD resolution and (b) high-rate $\alpha$ decay coincidence. For (b) in addition to the DSSD, the ancillary detector suite CARDS 8, comprising four large volume HPGe Clover detectors for $\gamma$-ray detection, was placed around the DSSD chamber. Similar $\alpha$-decay statistics were obtained in both campaigns. The indirect method of producing 104\text{Sn} provided a mechanism for populating the 101\text{Sn} states and unambiguous isotope identification from the characteristic $\alpha$-decays of the parent and grand-
The α-decay and γ-ray energy spectra associated with double-pulse events are shown in Fig. 1. We observe four α-decay transitions. Two α-decays, 3910(10) keV and 4063(4) keV, extracted from the first pulse and assigned as the fine structure and g.s. α-decay, respectively, from 109Xe yielding an excitation energy of 153(11) keV for the first excited state in 105Te, and a third decay, 4711(3) keV, extracted from the second pulse is assigned to 105Te. These decays are consistent with previously published values (3918(9), 4063(4) and 4703(5) keV [2] and 4720(50) keV [3]). Additionally, a fourth heretofore unknown α-decay with energy 4880(20) keV has been extracted from the second pulse thus assigned to 105Te. Assuming 100% α-emission, the measured intensities of these decays yield the α-decay branching ratios of 89(4)% (4711 keV) and 11(4)% (4880 keV). Assuming that these decays populate the ground and first excited states in 101Sn, the measured α-decay energy difference yields a first excited state energy of 170(20) keV.

In coincidence with double-pulse events, we observed two γ-rays at 150(3) keV and 172(2) keV. The weaker 150 keV line is compatible with the depopulation of an excited state in 105Te, whose intensity, after efficiency corrections, corresponds to a 30% α-decay branch. The stronger 172 keV line is compatible with the depopulation of an excited state in 101Sn as indicated by the α-decay energies and is consistent with the γ-ray of 172 keV, previously assigned as the de-excitation of the 1st-excited state in 101Sn [10]. The characteristic double α-decay pulse shapes provide a unique and clean coincidence requirement and allow us to assign this γ-ray unambiguously to 101Sn.

In order to account for the relatively large intensity of the observed 172 keV γ-ray transition, the excited state must be fed by the 4711 keV α-decay. In light of this experimental finding, this α-decay, previously assigned as the g.s.-to-g.s. transition [2, 9] is now reinterpreted as the α-decay from the ground state of 105Te to the first excited state in 101Sn. It is, therefore, the higher-energy 4880 keV decay that should be associated with the g.s.-to-g.s. transition.

The proposed decay scheme for the 109Xe α-decay chain is shown in Fig. 2 (top). Interpreting this scheme within the standard model of α-decay [14] we are bound to conclude that the g.s. spins of 105Te (N = 53) and 101Sn (N=51) must differ from each other, while the spin of the first excited state of 101Sn is equal to the spin of the 105Te g.s. The neighbouring α-decay chain 111Xe→107Te→103Sn [11, 12] is shown in Fig. 2 (bottom). In this case, only

![Fig. 1](image1.png)

![Fig. 2](image2.png)
a very weak, \( I_{\text{a}}=0.5\% \), \( \Delta I = 2 \) fine structure \( \alpha \)-decay branch from the \( ^{107}\text{Te} \) g.s. to the \( ^{103}\text{Sn} \) excited state is observed, with the majority of the emission strength going via the higher-energy \( \Delta I = 0 \), g.s.-to-g.s., transition. We note that while the first excited state in \( ^{103}\text{Sn} \) at 168 keV \([12,13]\) has an energy similar to that in \( ^{101}\text{Sn} \), the \( \alpha \)-decay pattern reveals that the g.s. spins of \( ^{107}\text{Te} \) (\( N = 55 \)) and \( ^{103}\text{Sn} \) (\( N = 53 \)) are the same, contrary to the situation in Fig. 2 (top).

The suggested level inversion observed in the \( ^{107}\text{Te} \rightarrow ^{103}\text{Sn} \) and the \( ^{103}\text{Sn} \rightarrow ^{101}\text{Sn} \) decay chains is unexpected. In \( ^{103}\text{Sn} \), the g.s. spin has been measured as \( 5/2^+ \) with a first excited \( 7/2^+ \) state at 14 keV \([13]\). The energies of the first excited states in \( ^{103,105}\text{Sn} \) have been measured and the g.s. spins tentatively assigned as \( 5/2^+ \), by considering systematic trends and/or theoretical predictions. Some information regarding the ground state in \( ^{101}\text{Sn} \) has been previously obtained. Beta-delayed proton emission measurements \([14]\) resulted in a tentative assignment of a \( J^\pi = 5/2^+ \) g.s.. However, as noted Ref. \([14]\), the low statistics collected and the inherent ambiguities of the model used made this assignment inconclusive and the measurement is not incompatible with a \( J^\pi = 7/2^+ \) g.s.. A \( J^\pi = 5/2^+ \) assignment has also been proposed \([16]\) following the observation of a solitary 172 keV \( \gamma \)-ray. They interpreted the non-observation of higher-energy \( \gamma \)-rays along with extrapolations from systematic trends in favour of \( J^\pi = 5/2^+ \).

In order to develop an understanding of the nature of the observed structural change between \( ^{103}\text{Sn} \) and \( ^{101}\text{Sn} \), we present first a simple two-level model, based on the seniority scheme \([17]\), which conveys much of the physics behind the experimental results. We consider a truncated configuration space, which consists only of the \( g_7/2 \) and \( d_{5/2} \) orbitals. This approximation is justified for the light tin isotopes by the near degeneracy of these states and their large energy separation from the higher-lying orbitals. As the structure of semi magic nuclei is dominated by pairing, we further limit our discussion of \( ^{103}\text{Sn} \) to the two main seniority-one neutron configurations, which are \( (g_7/2)^2 \otimes d_{5/2} \) and \( (d_{5/2})^3 \) for states with \( J^\pi = 5/2^+ \), and \( (d_{5/2})^2 \otimes g_7/2 \) and \( (g_7/2)^3 \otimes d_{5/2} \) for \( J^\pi = 7/2^+ \) states, respectively. The structure of the collective \( J^\pi = 0^+ \) Cooper pair in the ground state of \( ^{102}\text{Sn} \) is determined by the competition between the spacing \( \Delta \varepsilon_{\text{sp}} \approx 172 \) keV of the \( g_7/2 \) and \( d_{5/2} \) levels and the pairing energies of the neutron pairs. The latter are primarily determined by the two diagonal two-body matrix elements (TBME) \( V^{\text{pair}}(l_j) = \langle l_j | V | l_j \rangle \) and one off-diagonal TBME \( V^{\text{pair}}(g_7/2, d_{5/2}) \) representing the scattering of \( J^\pi = 0^+ \) pairs between \( g_7/2 \) and \( d_{5/2} \) shells. These matrix elements are computed \([18]\) from the nucleon-nucleon potentials \( N3LO \) \([19]\) and AV18 \([20]\). For \( N3LO \), we find that \( V^{\text{pair}}(g_7/2) \approx 1.40 \) MeV is significantly larger than \( V^{\text{pair}}(d_{5/2}) \approx 0.84 \) MeV, and their difference \( \Delta V^{\text{pair}} \approx 0.56 \) MeV is considerably larger than \( \Delta \varepsilon_{\text{sp}} \). (Similar results are obtained for AV18.) In addition, since \( \Delta V^{\text{pair}} \approx |V^{\text{pair}}(g_7/2, d_{5/2})| \), substantial mixing between the \( (g_7/2)^3 \otimes d_{5/2} \) occurs.

The resulting structure of the Cooper pair in \( ^{102}\text{Sn} \) is dominated by the \( (g_7/2)^3 \otimes d_{5/2} \) component (about 70%). When coupling the odd neutron to this collective pair to form the low-lying states in \( ^{103}\text{Sn} \), the Pauli blocking kicks in. In the \( 5/2^+ \) state, the weight of the \( (g_7/2)^3 \otimes d_{5/2} \) configuration (33%) becomes enhanced with respect to the \( (g_7/2)^2 \otimes d_{5/2} \) component in \( ^{102}\text{Sn} \), and due to the strong \( g_7/2 \) pairing, produces significant additional binding. Likewise, for the \( 7/2^+ \) state, the weight of the \( (g_7/2)^3 \otimes d_{5/2} \) configuration (33%) becomes significantly reduced, thus lowering the binding. Consequently, it is the strong pairing energy in \( (g_7/2)^3 \otimes d_{5/2} \) and Pauli blocking that sets the \( J^\pi = 5/2^+ \) g.s. spin in \( ^{103}\text{Sn} \). It appears that the region of \( ^{100}\text{Sn} \) is quite unique in exhibiting such a behaviour. A large difference in pairing matrix elements of the pseudo-spin partners, going well beyond the usual \((2j + 1)\) scaling \([17]\) is unexpected from commonly-used phenomenological shell-model interactions.

We substantiate the insights from the two-level model by state-of-the-art configuration interaction (CI) calculations following Ref. \([18]\). These shell-model results are intended to lend further support to the experimental interpretation. In the first calculation variant (V1), we assumed a \( ^{100}\text{Sn} \) core with valence nucleons in \( d_5/2, g_7/2, d_3/2, s_1/2 \) and \( h_{11/2} \) shells. In the second variant (V2), we took an \( ^{88}\text{Sr} \) core (\( N = 50 \)) with valence protons in \( p_1/2, g_9/2 \) shells and valence neutrons in \( d_5/2, g_7/2, d_3/2, s_1/2 \) and \( h_{11/2} \) shells. In V1, the residual interaction was based on AV18 or N3LO nucleon-nucleon potentials and for V2 it was derived from the CD-Bonn potential \([21]\). All these interaction models contain no free parameters and are expected to describe accurately the structure of light systems \([22]\) and nuclei with sufficiently small numbers of valence nucleons. Results of calculations around \( ^{100}\text{Sn} \) using V2, relevant in the context of this study, have been previously discussed \([23,24]\). Overall, V2 gives a very good agreement for \( A \sim 100 \) nuclei.

Figure 3 compares results of V1 and V2 with experiment. In V1, the splitting \( |\Delta \varepsilon_{\text{sp}}| \) has been set to the experimental value of 172 keV. Calculations are performed for both possible orderings of the levels, thus allowing for either \( 7/2^+ \) or \( 5/2^+ \) ground state in \( ^{101}\text{Sn} \). However, regardless of this ordering, the ground state of the heavier Sn isotopes, in particular \( ^{103}\text{Sn} \), always turns out to be \( 5/2^+ \). Assuming a \( 5/2^+ \) ground state in \( ^{103}\text{Sn} \), our calculations overestimate the location of the \( 1^+ \)-excited state in the heavier Sn isotopes by about 200 keV. Assuming a \( 7/2^+ \) ground state, on the other hand, we find excellent agreement between theory and experiment. In V2, based on experimental s.p. energies of \( ^{89}\text{Sr} \) (neutrons) and \( ^{89}\text{Y} \) (protons), the ground state of \( ^{101}\text{Sn} \) is predicted to be \( 7/2^+ \), and this is consistent with the
previous study [25]. Both V1 and V2 reproduce well the experimental parabolic trend of the excited states, including the crossing between $5/2^+$ and $7/2^+$ ground states between $^{109}$Sn and $^{111}$Sn, as well as the energy of the lowest $2^+$ seniority-two state in the even-mass tin isotopes (not shown in Fig. 3).

It has been suggested in Ref. [10] that good agreement can also be achieved assuming a $5/2^+$ g.s. by reducing $V^\text{pair}(g_{7/2})$ by about 30%. Such an adjustment is not consistent with the microscopically derived residual interactions. It cannot be excluded that the $5/2^+$ and $7/2^+$ level inversion occurs between $^{109}$Xe and $^{107}$Te. However, we do not find evidence to support such a scenario. In Fig. 2 we see that the lower-energy $\alpha$-decays from $^{109}$Xe and $^{111}$Xe populate the excited states of $^{105}$Te and $^{107}$Te, respectively, presenting no indication of a change in structure in tellurium. This is supported by the CI calculations for the tellurium isotopes: in V1+AV18 we predict $5/2^+$ for the ground states of $^{105}$Te and $^{107}$Te, regardless of the sign of $\Delta \varepsilon_{\text{sp}}$ or even with 30% reduced $V^\text{pair}(g_{7/2})$ [10].

In conclusion, our $\alpha$-decay studies give strong experimental evidence for a $7/2^+$ g.s and $5/2^+$ first excited state in $^{104}$Sn. This is contrary to what has been previously postulated, based on extrapolation from the heavier tin isotopes. Shell model calculations with realistic interactions, both in a two-level space and in a large configuration space, strongly support our new interpretation. The inversion of the g.s. spins between $^{108}$Sn and $^{110}$Sn is due to the unusually strong pairing interaction between the $g_{7/2}$ neutrons and unusually small energy splitting between the $7/2^+$ and $5/2^+$ states in $^{109}$Sn. The strong pairing in $g_{7/2}$ above $^{100}$Sn can be related to significant contributions from the two-body tensor force, which is expected [26] to produce $7/2^+\rightarrow 5/2^+$ level inversion in $^{109}$Sn, in good agreement with our analysis. The region of proton-rich nuclei above $^{100}$Sn seems to be quite unique in exhibiting such unusual behaviour. Interestingly, another doubly magic isotope of tin, the neutron rich $^{132}$Sn, is a well-behaved shell-model system because of the large energy splitting between single particle orbitals [27].

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[1] M. G. Mayer and J. H. D. Jensen, Nobel Lectures, Physics 1963-1970, Elsevier Amsterdam (1972).
[2] S. N. Liddick et al., Phys. Rev. Lett., 97, 082501 (2006).
[3] D. M. Brink and R. A. Broglia, Nuclear Superfluidity: pairing in finite systems (Cambridge Univ. Press, 2005).
[4] A. Arima, M. Harvey, and K. Shimizu, Phys. Lett. B, 30, 517 (1969).
[5] K. T. Hecht and A. Adler, Nucl. Phys. A, 137, 129 (1969).
[6] C. J. Gross et al., Nucl. Instr. Meth. A, 65, 031303 (2002).
[7] W. Krolas et al., Phys. Rev. C, 65, 051307(R) (2002).
[8] D. Seweryniak et al., Phys. Rev. C, 73, 061301(R) (2006).
[9] D. Seweryniak et al., Phys. Rev. Lett., 99, 022504 (2007).
[10] D. Schardt et al., Nucl. Phys. A, 326, 65 (1979).
[11] D. Seweryniak et al., Phys. Rev. C, 66, 051307(R) (2002).
[12] C. Fahlender et al., Phys. Rev. C, 63, 021307(R) (2001).
[13] G. Gamow, Z. Phys., 51, 204 (1928).
[14] J. Eberz et al., Zeits. Phys. A, 326, 121 (1987).
[15] O. Kavatsyk et al., Eur. Phys. J. A, 31, 319 (2007).
[16] I. Talmi, Simple models of complex nuclei. (Taylor & Francis Ltd, London, 1993).
[17] M. Hjorth-Jensen, T. T. S. Kuo, and E. Osnes, Phys. Rep., 261, 125 (1995).
[18] D. R. Entem and R. Machleidt, Phys. Rev. C, 68, 041001(R) (2003).
[19] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C, 51, 38 (1995).
[20] R. Machleidt, Phys. Rev. C, 63, 024001 (2001).
[21] P. Navratil et al., J. Phys. G, 36, 083101 (2009).
[22] M. Lipoglavsek et al., Phys. Rev. C, 66, 011302(R)

FIG. 3. (colour online) Results of our shell model calculations in variants V1 and V2 for the splitting of the $7/2^+\rightarrow 5/2^+$ states in the neutron-deficient, odd-mass tin isotopes. In V1, the upper (lower) panel assumes the $d_{5/2}$ single-neutron level above (below) the $g_{7/2}$ level. Experimental values (circles) are taken from this work and from Ref. [13] and references quoted therein.

\[ \Delta \varepsilon_{\text{sp}} = +172\text{keV}: \]
\[ ^{101}\text{Sn has } J^* = 5/2^+ \text{ g.s.} \]

\[ \Delta \varepsilon_{\text{sp}} = -172\text{keV}: \]
\[ ^{101}\text{Sn has } J^* = 7/2^+ \text{ g.s.} \]
(2002).
[24] A. Ekstrom et al., Phys. Rev. C, 80, 054302 (2009).
[25] H. Grawe et al., Eur. Phys. J. A, 27 s01, 257 (2006).
[26] T. Otsuka et al., Phys. Rev. Lett., 104, 012501 (2010).
[27] K. L. Jones et al., Nature, 465, 454 (2010).