Based on Prospect Theory of Project Priority Selection

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Abstract. Under the background of new power reform, the project investment of power grid enterprises is faced with great challenges, so it is urgent to make scientific allocation of power grid investment to improve the return rate of investment. This paper presents a project priority comparison model based on prospect theory, which can well balance the influence of qualitative and quantitative factors on the portfolio scheme and ensure that the selected portfolio scheme meets the constraints of investment capacity, power grid reliability, capacity ratio and regional development. This model can provide some reference for power grid investment.

Keywords: Prospect theory, Power grid investment, Priority selection.

1. Introduction

In the new reform situation of the power grid, the electricity selling side has been opened up, the pricing mode of transmission and distribution has been changed, the prices setting in a larger range has been guided by market gradually, and the permitted income of the company has been verified based on the effective assets. The implementation of this series of policies undoubtedly brings great challenges to the investment of power grid enterprises. Therefore, under the background of the new power reform, power grid enterprises urgently need to make scientific allocation of project investment, so as to reduce investment risks and improve the returns of investment [1].

At present, there have been some studies on portfolio optimization of power grid investment projects in the field of power grid investment decision-making, such as literature [1-4]. But the existed research only considered the quantitative factors that affect the investment decision of power grid. In fact, there are many factors affecting the grid portfolio, both qualitative and quantitative, and the mere consideration of quantitative factors will greatly reduce the effectiveness and reliability of the model. For this reason, this paper establishes a project priority comparison model based on prospect theory, which can not only balance the impact of quantitative and qualitative factors on the project portfolio, but also could ensure that the selected portfolio solutions meet the constraints of investment capacity, power grid reliability, capacity ratio, regional development and so on.
2. Power grid investment project priority setting

2.1. Construction of priority index system.
The construction of index system aims to quantify the profit of power grid investment projects, and then determine the priority of investment. The higher the profit value is, the higher the priority is. The comprehensive index system of investment assessment for this research was established by referring to the principles of SMART index system construction. As Table 1, which including four first-level indicators of project importance, technology, economic benefit, social environment and project maturity. Each first-level indicator contains several second-level indicators.

| first level indicators | NO. | second level indicators | NO. | type | classification |
|------------------------|-----|-------------------------|-----|------|----------------|
| project importance     | A   | reduce the number of heavy-duty main transformers | A1  | quantitative | Benefit type |
|                        |     | reduce the number of lines whose length exceeds the limit | A2  | quantitative | Benefit type |
| technology             | B   | technical innovation level of the project | B1  | qualitative  | Benefit type |
|                        |     | risk level              | B2  | qualitative  | Cost type    |
| economic               | C   | expected net present value | C1  | quantitative | Benefit type |
|                        |     | the new electricity sales of Unit investment | C2  | quantitative | Benefit type |
|                        |     | investment amounts      | C4  | quantitative | Cost type    |
| the social environment | D   | noise                   | D1  | quantitative | Cost type    |
|                        |     | the new quantity of employment | D2  | quantitative | Benefit type |
|                        |     | the area of Vegetation destruction | D3  | quantitative | Cost type    |
| project maturity       | E   | Pre-project planning ability | E1  | qualitative  | Benefit type |
|                        |     | ability to participate team management and control | E2  | qualitative  | Benefit type |
|                        |     | ability to control bidding and tendering | E3  | qualitative  | Benefit type |

2.2. Description of quantitative and qualitative indicators.
Setting \( S = \{s_1, s_2, ..., s_m\} \) is a set of \( m \) alternative items, \( U = \{u_1, u_2, ..., u_n\} \) is the index set in \( n \) comprehensive index system; \( x_{ij} \) represents the attribute value of the \( i \)th item under the \( j \)th evaluation index.

2.2.1. The profit /loss of quantitative index and its probability description. In the market environment, there are uncertainties in electricity sold, power supply cost, line loss rate and power supply reliability, etc., so there are uncertainties in each index. In this paper, the interval number method is adopted to describe the uncertainty of each quantitative index. Setting the interval of \( x_{ij} \) is \([x_{L,i,j}, x_{U,i,j}]\), of which \( x_{L,i,j} \) and \( x_{U,i,j} \) represent \( x_{ij} \) upper and lower limits respectively.

Suppose \( x_{ij} \) is uniformly distributed in the interval \([x_{L,i,j}, x_{U,i,j}]\), then the probability density function is

\[
   f_{ij} = 1/(x_{U,i,j} - x_{L,i,j}) \quad (1)
\]
Let $m_{U,j} = \max_{1 \leq i \leq m} x_{U,ij}$, $m_{L,j} = \min_{1 \leq i \leq m} x_{L,ij}$

$$y = \begin{cases} \frac{x - m_{L,j}}{m_{U,j} - m_{L,j}} & \text{Indicator } j \text{ is of benefit type} \\ \frac{m_{U,j} - x}{m_{U,j} - m_{L,j}} & \text{Indicator } j \text{ is of cost type} \end{cases}$$ \hspace{1cm} (2)

The normalized value of the attribute $y_{ij} = [y_{L,ij}, y_{U,ij}]$. Probability density function $f'_{ij} = \frac{1}{(y_{U,ij} - y_{L,ij})}$. For normalized indices $j$, choose its reference value as $y_{ref,j} = \frac{1}{2m} \sum_{l=1}^{m} (y_{U,ij} + y_{L,ij})$.

If $y_{L,ij} \geq y_{ref,j}$, then

$$\begin{cases} r_{ij} = \int_{y_{L,ij}}^{y_{U,ij}} (y_{ij} - y_{ref,j}) f'_{ij} dy \\ d_{ij} = 0 \\ l_{ij} = 0 \end{cases} \quad p_{ij} = 1$$ \hspace{1cm} (3)

If $y_{U,ij} \leq y_{ref,j}$, then

$$\begin{cases} r_{ij} = 0 \\ d_{ij} = \int_{y_{L,ij}}^{y_{U,ij}} (y_{ref,j} - y_{ij}) f'_{ij} dy \\ l_{ij} = 1 \end{cases} \quad p_{ij} = 0$$ \hspace{1cm} (4)

If $y_{L,ij} < y_{ref,j} < y_{U,ij}$, then

$$\begin{cases} r_{ij} = \int_{y_{ref,j}}^{y_{U,ij}} (y_{ij} - y_{ref,j}) f'_{ij} dy \\ d_{ij} = \int_{y_{L,ij}}^{y_{ref,j}} (y_{ref,j} - y_{ij}) f'_{ij} dy \\ l_{ij} = \int_{y_{ref,j}}^{y_{U,ij}} (y_{ref,j} - y_{ij}) f'_{ij} dy \end{cases}$$ \hspace{1cm} (5)

2.2.2. The profit/loss of qualitative index and its probability description. As for the qualitative index, this paper uses the triangle fuzzy number method to quantify it. The fuzzy language and corresponding triangle fuzzy Numbers are shown in Table 2.

| Fuzzy language   | Triangular fuzzy Numbers | Fuzzy language   | Triangular fuzzy Numbers |
|------------------|--------------------------|------------------|--------------------------|
| Extreme low      | (0,0,0.1)                | Very low         | (0.0.1,0.2)               |
| Very low         | (0.1,0.2,0.3)            | Lower            | (0.2,0.3,0.4)             |
| Lower            | (0.1,0.2,0.3)            | General          | (0.4,0.5,0.6)             |
| The lower        | (0.2,0.3,0.4)            | Special high     | (0.8,0.9,1.0)             |
| Slightly lower   | (0.3,0.4,0.5)            | Extreme high     | (0.9,1.0,1.0)             |
| general          | (0.4,0.5,0.6)            |                  |                          |

For fuzzy attribute values $x_{ij}$, the linguistic values about the form of triangular fuzzy Numbers $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$, $a_{ij} \leq b_{ij} \leq c_{ij}$, its membership function is as follows:
\[
\varphi_{ij}(x) = \begin{cases} 
0 & x < a_{ij} \text{ or } x > c_{ij} \\
\frac{x - a_{ij}}{b_{ij} - a_{ij}} & a_{ij} \leq x < b_{ij} \\
\frac{c_{ij} - x}{c_{ij} - b_{ij}} & b_{ij} \leq x \leq c_{ij} 
\end{cases} \tag{6}
\]

Let \( m_{u,j} = \max_{1 \leq s \leq m} c_{ij} \), \( m_{l,j} = \min_{1 \leq s \leq m} a_{ij} \)

\[
y = \begin{cases} 
\frac{x - m_{l,j}}{m_{u,j} - m_{l,j}} & \text{Indicator } j \text{ is of benefit type} \\
\frac{m_{u,j} - x}{m_{u,j} - m_{l,j}} & \text{Indicator } j \text{ is of cost type} 
\end{cases}
\tag{7}
\]

The normalized value of the attribute \( y_{ij} = (a'_{ij}, b'_{ij}, c'_{ij}) \), grade of membership is \( \varphi_{ij}'(y) \), which form is same as \( \varphi_{ij}(x) \).

\[
\varphi_{ij}'(y) = \begin{cases} 
0 & y < a'_{ij} \text{ or } y > c'_{ij} \\
\frac{y - a'_{ij}}{b'_{ij} - a'_{ij}} & a'_{ij} \leq y < b'_{ij} \\
\frac{c'_{ij} - y}{c'_{ij} - b'_{ij}} & b'_{ij} \leq y \leq c'_{ij} 
\end{cases} \tag{8}
\]

By linear transformation of \( \varphi_{ij}'(y) \), the degree of membership is transformed into probability density function, which needs to make \( \int_{-\infty}^{+\infty} q \varphi_{ij}'(y) \, dy = 1 \). Therefore, the probability density function of normalized form of triangular fuzzy number is \( f'_{ij} = q \varphi_{ij}'(y) \), of which \( q = 2/(c'_{ij} - a'_{ij}) \), distribution function \( F'_{ij} = \int_{-\infty}^{y} \varphi_{ij}'(y) \, dy \). The reference value of index \( j \) is \( y_{ref,j} = \sum_{i=1}^{m} b'_{ij}/m \).

If \( a'_{ij} \geq y_{ref,j} \), then

\[
\begin{cases} 
\begin{array}{c}
\frac{c'_{ij}}{a'_{ij}} \left( y_{ij} - y_{ref,j} \right) f'_{ij} \, dy & l_{ij} = 1 \\
d_{ij} = 0 & l_{ij} = 0
\end{array}
\end{cases} \tag{9}
\]

If \( c'_{ij} \leq y_{ref,j} \), then

\[
\begin{cases} 
\begin{array}{c}
r_{ij} = 0 & p_{ij} = 0 \\
d_{ij} = \int_{a'_{ij}}^{c'_{ij}} \left( y_{ref,j} - y_{ij} \right) f'_{ij} \, dy & l_{ij} = 1
\end{array}
\end{cases} \tag{10}
\]

If \( a'_{ij} < y_{ref,j} < c'_{ij} \), then
2.3. Prospect theory is used to obtain the prospect value of the index.

In prospect theory, the value function can be used to convert relative gain/loss value into decision value. In this research, the given value functions of gains and loss are:

\[
\begin{align*}
    v_{r,ij} &= (r_{ij})^\alpha \\
    v_{d,ij} &= -\lambda (-d_{ij})^\beta
\end{align*}
\]

Where, \(\alpha\) and \(\beta\) reflect decision makers' attitudes towards gains and losses respectively. \(0 \leq \alpha < 1, 0 \leq \beta \leq 1\); \(\lambda\) reflects the decision maker's loss aversion; \(\lambda > 1\); \(r_{ij}\) and \(d_{ij}\) represent the gains and losses of relative reference values respectively.

Convert the probabilities of gains and losses into decision weights:

\[
\begin{align*}
    \pi_{r,ij}(p_{ij}) &= \frac{(p_{ij})^r}{((p_{ij})^r + (1 - p_{ij})^r)^{\frac{1}{r}}} \\
    \pi_{l,ij}(l_{ij}) &= \frac{(l_{ij})^\mu}{((l_{ij})^\mu + (1 - l_{ij})^\mu)^{\frac{1}{\mu}}}
\end{align*}
\]

Where, \(\pi_{r,ij}\) is the decision weight of risk return; \(\pi_{l,ij}\) is the decision weight of risk loss; \(r\) and \(\mu\) the attitude of decision makers in the face of profit risk and loss risk respectively; \(p_{ij}\) and \(l_{ij}\) represents the probability of relative gain and relative loss respectively. The foreground value \(v_{ij}\) of the \(j\)th index of alternative project \(i\) is defined as:

\[
v_{ij} = \pi_{r,ij}v_{r,ij} + \pi_{l,ij}v_{d,ij}
\]

2.4. Index weight calculation.

The weight of each index is calculated by the coefficient of variation method.

Suppose \(\mu_j\) and \(\sigma_j\) present the mean value and standard deviation of the \(j\)th index of all candidate items respectively.

\[
\mu_j = \frac{\sum_{i=1}^{m} v_{ij}}{m}
\]

\[
\sigma_j = \sqrt{\frac{\sum_{i=1}^{m} (v_{ij} - \mu_j)^2}{m}}
\]

Then, the coefficient of variation of index J is defined as the ratio of its standard deviation to its mean value.
\[ CV_j = \frac{\sigma_j}{\mu_j} \]  

(17)

Based on the coefficient of variation, the weight of index \( j \) is determined as

\[ \lambda_j = \frac{CV_j}{\sum_{j=1}^{m} CV_j} \]  

(18)

2.5. **VIKOR-based priority setting method for power grid investment projects.**

This project adopts VIKOR method to set the priority of investment projects. VIKOR is a multi-attribute decision making method approaching the ideal point. By maximizing the group utility and minimizing the individual regret, the compromise ranking of the limited project portfolio is carried out, and then the compromise solution of each index of the project portfolio is obtained. The specific setting steps are as follows:

1) Calculating Project \( i \) group utility \( S_i \) and individual regret \( R_i \)

\[ S_i = \sum_{j=1}^{m} \lambda_j \frac{v_{\text{max},j} - v_{ij}}{v_{\text{max},j} - v_{\text{min},j}} \]  

(19)

\[ R_i = \max \left\{ \lambda_j \frac{v_{\text{max},j} - v_{ij}}{v_{\text{max},j} - v_{\text{min},j}} \right\} \]  

(20)

where, \( v_{\text{max},j} = \max_{1 \leq s \leq m} v_{ij}, v_{\text{min},j} = \min_{1 \leq s \leq m} v_{ij} \).

2) Calculate the combined utility \( Q_i \) for item \( i \) based on the values of \( S_i \) and \( R_i \)

\[ Q_i = \eta \frac{S_i - S_{\text{min}}}{S_{\text{max}} - S_{\text{min}}} + (1 - \eta) \frac{R_i - R_{\text{min}}}{R_{\text{max}} - R_{\text{min}}} \]  

(21)

Where, \( S_{\text{min}} = \min_i S_i, S_{\text{max}} = \max_i S_i, R_{\text{min}} = \min_i R_i, R_{\text{max}} = \max_i R_i \); \( \eta \) is the compromise coefficient between decision makers’ maximization of group utility and minimization of individual regret, which is between \([0, 1]\).

3) According to the comprehensive utility \( Q_i \), the priority order of all alternative items is obtained from small to large, that is, the smaller \( Q_i \) is, the higher the priority order is.

3. **The priority comparison model of power grid investment project based on prospect theory.**

This model considers power grid investment decisions in \( h \) regions, where \( k = 1, \ldots, h \) and contains \( p \) alternative items, and the objective is to determine the optimal project portfolio for each region under the constraints of investment capacity, grid reliability, load demand, regional development, and project correlation.

3.1. **Objective function**

\[ \min Q = \sum_{k=1}^{h} \sum_{l=1}^{p} p_{kl} Q_{kl} \]  

(22)

Where, \( Q_{kl} \) represents the comprehensive utility value of the \( l \) item in region \( k \), which can be calculated by formula (2-10). \( x_{kl} \) is a 0-1 type decision variable, and \( x_{kl} = 1 \) represents the \( l \)th project of investment area \( k \), otherwise \( x_{kl} = 0 \).
3.2. Constraints condition

3.2.1. Constraint on investment capacity. The investment capacity constraint requires that the total investment amount of all proposed investment projects should not exceed the company's investment capacity.

\[ \sum_{k=1}^{h} \sum_{i=1}^{p_i} x_{kl}v_{kl} \leq V_{\text{max}} \]  

Where, \( v_{kl} \) represents the investment amount (ten thousand yuan) of the \( i \) alternative project in District \( k \); \( V_{\text{max}} \) represents the maximum investment capacity (ten thousand yuan) of the company during the planning period.

3.2.2. Grid reliability constraint. Capacity-load ratio refers to the ratio of the total capacity of substation equipment in a power supply area to the load in the power supply area. Too high capacity ratio will reduce the investment efficiency of power grid, too low will reduce the adaptability of power grid. Therefore, the load capacity ratio should be controlled within a certain range according to the policy.

\[ r_{\text{min}} \leq \frac{\sum_{i=1}^{p_i} x_{kl}c_{kl} + C_{k}}{L_{k}} \leq r_{\text{max}} \]  

Where, \( r_{\text{min}} \) represents the lower limit of the capacity-load ratio; \( r_{\text{max}} \) represents the upper limit of the capacity ratio; \( L_{k} \) and \( C_{k} \) represent the average mainline load (MW) and old l capacity (MW) of the \( k \) power grid. \( c_{kl} \) represents the new capacity (MW) of the \( l \) th alternative project in the district \( k \).

3.2.3. Load demand constraint. Grid investment activities should ensure that the new grid capacity meets the load demand of the region during the planning period.

\[ \sum_{j=1}^{p_i} x_{kl}c_{kl} \geq D_{k} \]  

Where, \( D_{k} \) represents the newly added load demand (MW) of region \( k \).

3.2.4. Regional development constraint. Let \( \delta_{k} \) represent the minimum investment ratio of region \( k \), then regional development constraints require that the power grid investment in each region should not be lower than \( \delta_{k}V_{\text{max}} \).

\[ \sum_{j=1}^{p_i} x_{kl}v_{kl} \geq \delta_{k}V_{\text{max}} \]  

Where, \( \delta_{k} \) satisfies the condition:

\[ \delta_{1} + \delta_{2} + \cdots + \delta_{n} \leq 1 \]  

4. Conclusion
Aiming at the investment problem of power grid project, this paper establishes the priority comparison model of power grid investment project based on prospect theory to optimize the investment project portfolio scheme. The innovation of this model is that it can integrate the influence of quantitative and
qualitative factors on the project portfolio, and simultaneously consider the maximization of group utility and the minimization of individual regret to make the compromise ranking for the limited project portfolio, so as to obtain the compromise solution of each index in the project portfolio. In addition, the model can also ensure that the selected portfolio schemes can meet mandatory constraints such as investment capacity, power grid reliability and load ratio, and consider the influence of regional development on power grid investment, allowing the investment to be inclined to some regions.

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