Strong decays $B_{s0} \to B_s \pi$ and $B_{s1} \to B_s^* \pi$ with light-cone QCD sum rules

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Abstract

In this article, we calculate the strong coupling constants $g_{B_{s0}B_s \eta}$ and $g_{B_{s1}B_s^* \eta}$ with the light-cone QCD sum rules. Then we take into account the small $\eta - \pi^0$ transition matrix according to Dashen’s theorem, and obtain the small decay widths for the isospin violation processes $B_{s0} \to B_s \eta \to B_s \pi^0$ and $B_{s1} \to B_s^* \eta \to B_s^* \pi^0$. We can search the strange-bottomed $(0^+, 1^+)$ mesons $B_{s0}$ and $B_{s1}$ in the invariant $B_s \pi^0$ and $B_s^* \pi^0$ mass distributions respectively.

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1 Introduction

Recently, the CDF Collaboration reports the first observation of two narrow resonances consistent with the orbitally excited $P$-wave $B_s$ mesons using $1 \mathrm{fb}^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.96 \mathrm{TeV}$ collected with the CDF II detector at the Fermilab Tevatron [1]. The masses of the two states are $M(B_{s1}^*) = (5829.4 \pm 0.7) \mathrm{MeV}$ and $M(B_{s2}^*) = (5839.7 \pm 0.7) \mathrm{MeV}$, and they can be assigned as the $J^P = (1^+, 2^+)$ states in the heavy quark effective theory [2]. The D0 Collaboration reports the direct observation of the excited $P$-wave state $B_{s2}^*$ in fully reconstructed decays to $B^+ K^–$. The mass of the $B_{s2}^*$ meson is measured to be $(5839.6 \pm 1.1 \pm 0.7) \mathrm{MeV}$ [3]. While the $B_s$ states with spin-parity $J^P = (0^+, 1^+)$ are still lack experimental evidence.

The masses of the $B_s$ mesons with $(0^+, 1^+)$ have been estimated with the potential quark models, heavy quark effective theory and lattice QCD [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], the values are different from each other. In our previous work [17], we study the masses of the strange-bottomed $(0^+, 1^+)$ mesons with the QCD sum rules, and observe that the central values are below the corresponding $BK$ and $B^*K$ thresholds respectively. The decays $B_{s0} \to BK$ and $B_{s1} \to B^*K$ are kinematically forbidden. In previous works, the mesons $f_0(980)$, $a_0(980)$, $D_{s0}$, $D_{s1}$, $B_{s0}$ and $B_{s1}$ are taken as the conventional $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ states respectively, and the values of the strong coupling constants $g_{f_0KK}$, $g_{a_0KK}$, $g_{D_{s0}DK}$, $g_{D_{s1}D^*K}$, $g_{B_{s0}BK}$ and $g_{B_{s1}B^*K}$ are calculated with the light-cone QCD sum rules [18, 19, 20, 21, 22, 23]. The large values of the strong coupling constants support the hadronic dressing mechanism [24, 25, 26]. Those mesons may have small $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ kernels of the typical $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ mesons size respectively, strong couplings to the virtual intermediate

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hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ mesons in the potential quark models, enrich the pure $q\bar{q}$, $c\bar{s}$ and $b\bar{s}$ states with other components [14, 18, 19, 20, 21, 22, 23, 27, 28, 29].

The $P$-wave heavy mesons $B_{s0}$ and $B_{s1}$ can decay through the isospin violation precesses $B_{s0} \rightarrow B_s \eta \rightarrow B_s \pi^0$ and $B_{s1} \rightarrow B_s^* \eta \rightarrow B_s^* \pi^0$, respectively. The $\eta - \pi^0$ transition matrix is very small according to Dashen’s theorem [30], $t_{\eta\pi} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003 \text{GeV}^2$, they may be very narrow. In this article, we calculate the values of the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules, and study the strong isospin violation decays $B_{s0} \rightarrow B_s \pi^0$ and $B_{s1} \rightarrow B_s^* \pi^0$. In previous work [31], the authors calculate the strong coupling constants $g_{D_{s0}D_s\eta}$ and $g_{D_{s1}D_s^*\eta}$ with the light-cone QCD sum rules, then take into account the $\eta - \pi^0$ mixing and calculate their pionic decay widths.

The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes (which classified according to their twists) instead of the vacuum condensates [32, 33, 34, 35, 36, 37]. The non-perturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal [38, 39, 40].

The article is arranged as: in Section 2, we derive the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

## 2 Strong coupling constants $g_{B_{s1}B_s^*\eta}$ and $g_{B_{s0}B_s\eta}$ with light-cone QCD sum rules

In the following, we write down the definitions for the strong coupling constants $g_{B_{s0}B_s\eta}$ and $g_{B_{s1}B_s^*\eta}$ respectively,

$$
\langle B_{s1} | B_s^* \eta \rangle = -i g_{B_{s1}B_s^*\eta} \eta^* \cdot \epsilon,
\langle B_{s0} | B_s \eta \rangle = g_{B_{s0}B_s\eta},
$$

(1)

where the $\epsilon_\mu$ and $\eta_\mu$ are the polarization vectors of the mesons $B_s^*$ and $B_{s1}$ respectively. The interactions among the bottomed $(0^- , 1^-)$, $(0^+ , 1^+)$ mesons and the light
pseudoscalar mesons can be described by the phenomenological lagrangian [41],

\[ \mathcal{L} = ih \text{Tr} \left[ S_\mu \gamma_\mu \mathcal{A}_{\mu a} \bar{H}_a \right] + h.c., \]

\[ S_a = \frac{1+i\gamma_5}{2} \left[ B^a_\mu \gamma_\mu - B_{a0} \right], \]

\[ H_a = \frac{1+i\gamma_5}{2} \left[ B^a_\mu \gamma_\mu - i\gamma_5 B_a \right], \]

\[ \bar{H}_a = \gamma^0 H_a^\dagger \gamma^0, \]

\[ \mathcal{A}_\mu = \frac{1}{2} \left( L^\dagger \partial_\mu L - R^\dagger \partial_\mu R \right), \]

\[ L = R^\dagger = \exp \left[ i \mathcal{M} \right], \]

\[ \mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\ \frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\eta_0}{\sqrt{2}} - \frac{\tilde{K}}{\sqrt{2}} \\ K^- & \tilde{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}, \]  

where the \( a \) and \( b \) are the flavor indexes for the light quarks, \( v^2 = 1 \), and the \( h \) is the strong coupling constant. From the phenomenological lagrangian, we can obtain \( g_{B_1 B_1^* \eta} \propto ih \) and \( g_{B_0 B_0^* \eta} \propto h \). The hadronic matrix elements \( \langle B_{s1} | B_{s1}^* \eta \rangle \) and \( \langle B_{s0} | B_{s0} \eta \rangle \) have a relative phase factor \( i \), furthermore, we take the definition \( \langle B_{s1} | B_{s1}^* \eta \rangle = -ig_{B_1 B_1^* \eta}^* \cdot \epsilon \) as the corresponding one \( \langle B_{s1} | B_{s1}^* K \rangle = -ig_{B_1 B_1^* K}^* \cdot \epsilon \) in Ref. [23], where a negative sign is chosen to guarantee that the strong coupling constant \( g_{B_1 B_1^* K} \) has positive value. The expressions in Eq.(1) are the correct formula, although there are other definitions [31]. In literatures, the super-field \( H_a \) are usually defined as \( H_a = \frac{1+i\gamma_5}{2} \left[ B^a_\mu \gamma_\mu - \gamma_5 B_a \right] \), the \( i \) companied with the pseudoscalar mesons \( B_a \) is missed, therefor the \( i \) in Eq.(1) disappears. Here we take the correct expression given by A. V. Manohar and M. B. Wise in the book ”Heavy Quark Theory” [42].

We study the strong coupling constants \( g_{B_{s1} B_{s1}^* \eta} \) and \( g_{B_{s0} B_{s0}^* \eta} \) with the two-point correlation functions \( \Pi_{\mu\nu}(p, q) \) and \( \Pi_\mu(p, q) \) respectively,

\[ \Pi_{\mu\nu}(p, q) = i \int d^4x \, e^{-iq\cdot x} \left\langle 0 \left| T \left\{ J_\mu^V(0) J_\nu^{A\dagger}(x) \right\} \right| \eta(p) \right\rangle, \]  

\[ \Pi_\mu(p, q) = i \int d^4x \, e^{-iq\cdot x} \left\langle 0 \left| T \left\{ J_\mu^S(0) J_\nu^{S\dagger}(x) \right\} \right| \eta(p) \right\rangle, \]

\[ J_\mu^V(x) = \bar{s}(x) \gamma_\mu b(x), \]

\[ J_\mu^A(x) = \bar{s}(x) \gamma_\mu \gamma_5 b(x), \]

\[ J_\mu^S(x) = \bar{s}(x) \gamma_\mu \gamma_5 b(x), \]

\[ J^S(x) = \bar{s}(x) b(x), \]

where the currents \( J_\mu^V(x) \), \( J_\mu^A(x) \), \( J_\mu^S(x) \) and \( J^S(x) \) interpolate the strange-bottomed mesons \( B_{s1}^*, B_{s1}, B_s \) and \( B_{s0} \) respectively, the external \( \eta \) meson has four momentum
with $p^2 = m_{\eta}^2$. The $J_{\mu}^5(x)$ and $J_{\mu}^A(x)$ are the same current, we take different notations to denote that the contributions from the pseudoscalar meson and axial-vector meson are taken respectively.

The correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_{\mu}(p, q)$ can be decomposed as

$$
\Pi_{\mu\nu}(p, q) = i\Pi_A(p, q)g_{\mu\nu} + i\Pi_{A1}(p, q)p_\mu q_\nu + i\Pi_{A2}(p, q)p_\nu q_\mu + i\Pi_{A3}(p, q)q_\mu q_\nu ,
$$

$$
\Pi_\mu(p, q) = i\Pi_S(p, q)q_\mu + i\Pi_{S1}(p, q)p_\mu
$$

(6)
due to the Lorentz invariance. We choose the tensor structures $g_{\mu\nu}$ and $q_\mu$ for analysis in this article.

According to the basic assumption of current-hadron duality in the QCD sum rules approach $[38, 39, 40]$, we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J^V_\mu(x), J^A_\mu(x), J^5_\mu(x)$ and $J_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}(p, q)$ and $\Pi_\mu(p, q)$ to obtain the hadronic representations. After isolating the ground state contributions from the pole terms of the mesons $B_s^*, B_{s1}, B_s$ and $B_{s0}$, we get the following results,

$$
\Pi_{\mu\nu} = \frac{\langle 0|J^V_\mu(0)|B_s^*(q + p)\rangle\langle B_s^*|B_{s1}\eta\rangle\langle B_{s1}(q)|J^A_\nu(0)|0\rangle}{[M_{B_s^*}^2 - (q + p)^2][M_{B_{s1}}^2 - q^2]} + \frac{\langle 0|J^V_\mu(0)|B_s^*(q + p)\rangle\langle B_s^*|B_s\eta\rangle\langle B_s(q)|J^A_\nu(0)|0\rangle}{[M_{B_s^*}^2 - (q + p)^2][M_{B_s}^2 - q^2]} + \frac{\langle 0|J^V_\mu(0)|B_{s0}(q + p)\rangle\langle B_{s0}|B_s\eta\rangle\langle B_s(q)|J^A_\nu(0)|0\rangle}{[M_{B_{s0}}^2 - (q + p)^2][M_{B_{s0}}^2 - q^2]} + \cdots,
$$

$$
= -\frac{ig_{B_s^*B_{s1}\eta}f_{B_s^*}f_{B_{s1}}M_{B_s^*}M_{B_{s1}}}{[M_{B_s^*}^2 - (q + p)^2][M_{B_{s1}}^2 - q^2]} \left[ -g_{\mu\nu} + \frac{(p + q)_\mu(p + q)_\lambda}{M_{B_s^*}^2} \right] \left[ -g_{\lambda\nu} + \frac{q_\lambda q_\nu}{M_{B_{s1}}^2} \right] \left[ -g_{\mu\lambda} + \frac{(p + q)_\mu(p + q)_\lambda}{M_{B_s}^2} \right] \left[ -g_{\nu\lambda} + \frac{(p + q)_\nu(p + q)_\lambda}{M_{B_{s0}}^2} \right] p^\lambda q_\nu + \cdots
$$

$$
= -\frac{ig_{B_s^*B_{s1}\eta}f_{B_s^*}f_{B_{s1}}M_{B_s^*}M_{B_{s1}}}{[M_{B_s^*}^2 - (q + p)^2][M_{B_{s1}}^2 - q^2]} g_{\mu\nu} + \cdots,
$$

(7)
where the following definitions for the weak decay constants have been used,

\[ \Pi_\mu = \frac{\langle 0 | J^5_{\mu}(0) | B_s(q + p) \rangle \langle B_s | B_{s0} \eta \rangle \langle B_{s0} | J^{S\dagger}(0) | 0 \rangle}{[M_{B_{s0}}^2 - (q + p)^2] [M_{B_{s0}}^2 - q^2]} \]

\[ + \frac{\langle 0 | J^5_{\mu}(0) | B_{s1}(q + p) \rangle \langle B_{s1} | B_{s0} \eta \rangle \langle B_{s0} | J^{S\dagger}(0) | 0 \rangle}{[M_{B_{s1}}^2 - (q + p)^2] [M_{B_{s0}}^2 - q^2]} + \ldots, \]

\[ = \frac{ig_{B_{s0}B_{s1}\eta} f_{B_s} f_{B_{s0}} M_{B_{s0}}}{[M_{B_{s}}^2 - (q + p)^2] [M_{B_{s0}}^2 - q^2]} (p + q)_\mu \]

\[ + iC_3 \left[ -g_{\mu\lambda} + \frac{(p + q)_\mu(p + q)_\lambda}{M_{B_{s1}}^2} \right] p_\lambda + \ldots, \]

\[ = \frac{ig_{B_{s0}B_{s1}\eta} f_{B_s} f_{B_{s0}} M_{B_{s0}}}{[M_{B_{s}}^2 - (q + p)^2] [M_{B_{s0}}^2 - q^2]} q_\mu + iC_3 \frac{M_{B_{s0}}^2 + m_\eta^2 - M_{B_{s1}}^2}{2M_{B_{s1}}^2} q_\mu + \ldots, \tag{8} \]

where the following definitions for the weak decay constants have been used,

\[ \langle 0 | J^5_{\mu}(0) | B_s^*(p) \rangle = f_{B_s^*} M_{B_s^*} \epsilon_\mu, \]

\[ \langle 0 | J^A_{\mu}(0) | B_{s1}(p) \rangle = f_{B_{s1}} M_{B_{s1}} \eta_\mu, \]

\[ \langle 0 | J^5_{\mu}(0) | B_{s0}(p) \rangle = i f_{B_{s0}} M_{B_{s0}} \mu, \]

\[ \langle 0 | J^S_{\mu}(0) | B_{s0}(p) \rangle = f_{B_{s0}} M_{B_{s0}}, \]

\[ \langle 0 | J^0_{\mu}(0) | B_{s0}(p) \rangle = f_{B_{s0}} p_\mu. \tag{9} \]

We introduce the notations \( C_i \) for simplicity, the explicit expressions are neglected as the contributions can be deleted with suitable tensor structures. The term proportional to the \( C_3 \) is greatly suppressed by the small numerical factor \( \frac{M_{B_{s0}}^2 + m_\eta^2 - M_{B_{s1}}^2}{M_{B_{s1}}^2} \), and the contributions from the axial-vector meson can be neglected safely in Eq.(8).

We choose the tensor structure \( g_{\mu\nu} \) to avoid the contaminations from the scalar meson \( B_{s0} \) and the pseudoscalar meson \( B_s \) in the sum rule for the strong coupling constant \( g_{B_{s1}B_{s1}\eta} \). In deriving the sum rule for the strong coupling constant \( g_{B_{s0}B_{s1}\eta} \), we choose the axial-vector current \( J^5_{\mu}(x) \) to interpolate the pseudoscalar meson \( B_s \), although there are contaminations from the axial-vector meson \( B_{s1} \), the contaminations are tiny and can be neglected safely if we choose the tensor structure \( q_\mu \). If we choose the pseudoscalar current \( J^S(x) = \bar{s}(x)i\gamma_5c(x) \) to interpolate the pseudoscalar meson \( B_s \), the axial-vector mesons have no contaminations, I fail to take notice of this fact at beginning of the work.

We perform the operator product expansion for the correlation functions \( \Pi_{\mu\nu}(p, q) \) and \( \Pi_\mu(p, q) \) in perturbative QCD theory, and obtain the analytical expressions at the level of quark-gluon degrees of freedom. In calculation, the two-particle and three-particle \( \eta \) meson light-cone distribution amplitudes have been used [43, 44], the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and are calculated with the QCD sum rules [43, 44]. In this article, the energy scale \( \mu \) is chosen to be \( \mu = 1 \text{GeV} \), one
can choose another typical energy scale $\mu = \sqrt{M^2_B - m_b^2} \approx 2.4\text{GeV}$. The light-cone distribution amplitudes are calculated at the energy scale $\mu = 1\text{GeV}$ with the QCD sum rules, evolution of the coefficients to larger energy scales with the (complex) renormalization group equation which concerns approximations in one or other ways, additional uncertainties are introduced. The physical quantities would not depend on the special energy scale we choose, we expect that scale dependence of the input parameters is canceled out approximately with each other, the values of the strong coupling constants which are calculated at the energy scale $\mu = 1\text{GeV}$ can make robust predictions. Furthermore, in the heavy quark limit, the bound energy of the strange-bottomed $(0^+, 1^+)$ mesons is about $\Lambda = \frac{3M_{B^*} + M_{B^0}}{4} - m_b \approx 1\text{GeV}$, which can serve as a typical energy scale and validate our choice.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions $\Pi_A$ and $\Pi_S$ at the level of quark-gluon degrees of freedom. The masses of the strange-bottomed mesons are $M_{B^1} = 5.72\text{GeV}$, $M_{B^0} = 5.70\text{GeV}$, $M_{B^*} = 5.412\text{GeV}$ and $M_{B_s} = 5.366\text{GeV}$,

$$\frac{M^2_{B^1}}{M^2_{B^1} + M^2_{B^*_s}} \approx \frac{M^2_{B^0}}{M^2_{B^0} + M^2_{B_s}} \approx 0.53,$$

there exists an overlapping working window for the two Borel parameters $M^2_1$ and $M^2_2$, it’s convenient to take the value $M^2_1 = M^2_2$. We introduce the threshold parameter $s_0$ (denotes $s^0_S$ and $s^0_A$) and make the simple replacement,

$$e^{-\frac{m^2 + u_0(1 - u_0)}{M^2}} \rightarrow e^{-\frac{m^2 + u_0(1 - u_0)}{M^2}} - e^{-s_0},$$

to subtract the contributions from the high resonances and continuum states [45], finally we obtain the sum rules for the strong coupling constants $g_{B^0B_s\eta}$ and $g_{B^*B_s\eta}$ respectively\[2].

\[2\] For example, we use the notation $(A_\parallel + A_\perp)(1 - \alpha - \beta, \alpha, \beta)$ to represent $A_\parallel(1 - \alpha - \beta, \alpha, \beta) + A_\perp(1 - \alpha - \beta, \alpha, \beta)$. Other expressions can be understood in the same way.
\[ g_{B_0 B_\eta} = \frac{1}{f_{B_0} f_{B_0} M_{B_0}} \exp \left( \frac{M_{B_0}^2}{M_1^2} + \frac{M_{B_0}^2}{M_2^2} \right) \left\{ \exp \left( -\frac{\Xi}{M^2} \right) - \exp \left( -\frac{s_{S}}{M^2} \right) \right\} \]

\[ \frac{f'_{\eta} m_{\eta}^2 M^2}{m_s} \left[ \varphi_p(u_0) - \frac{d \varphi_\sigma(u_0)}{6du_0} \right] + \exp \left( -\frac{\Xi}{M^2} \right) \left[ -m_b f'_{\eta} m_{\eta}^2 \int_0^{u_0} dt B(t) \right. \]

\[ + f'_{3\eta} m_{\eta}^2 \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_s} d\alpha_g \varphi_{3\eta}(1 - \alpha_s - \alpha_g, \alpha_g, \alpha_s) \frac{2(\alpha_s + \alpha_g - u_0) - 3\alpha_g}{\alpha_g^2} \]

\[ - \frac{2m_b f'_{\eta} m_{\eta}^4}{M^2} \int_{1-u_0}^{1} d\alpha_g \frac{1 - u_0}{\alpha_g^2} \int_0^{1-\beta} d\beta \Phi(1 - \alpha - \beta, \beta, \alpha) \]

\[ + \frac{2m_b f'_{\eta} m_{\eta}^4}{M^2} \left( \int_0^{1-u_0} d\alpha_g \int_{u_0-\alpha_g}^{u_0} d\alpha_s \int_0^{\alpha_s} d\alpha + \int_{1-u_0}^{1} d\alpha_g \int_{u_0-\alpha_g}^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \right) \]

\[ \frac{\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha)}{\alpha_g} \right\} \],

(11)
\[ g_{B_1 B_2 \eta} = \frac{1}{f_{B_1} f_{B_2} f_{B_1 B_2 B_1}} \exp \left( \frac{M_{B_2}^2 + M_{B_2}^2}{M_1^2} \right) \left\{ \left[ \exp \left( -\frac{\Xi}{M^2} \right) - \exp \left( -\frac{s_A^0}{M^2} \right) \right] \right. \\
\left. \quad + f'_\eta \left[ \frac{m^2 \eta M^2}{m_s} \varphi_p(u_0) + \frac{m^2 \eta (M^2 + m^2_b)}{8} \frac{d}{du_0} A(u_0) - \frac{M^4}{2} \frac{d}{du_0} \phi_\eta(u_0) \right] \right\} \] \\
- \exp \left( -\frac{\Xi}{M^2} \right) \left[ f'_\eta m^2 \eta \int_{u_0}^1 du_0 \frac{d}{du_0} \frac{A_\| - V_{\|}}{(1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g)} \right] \\
+ m^2 \eta \int_0^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g \frac{u_0 f'_\eta m^2 \eta \Phi + f'_\eta m_b \varphi_{3\eta}}{\alpha_g} \] \\
+ f'_\eta m^2 \eta M^2 \frac{d}{du_0} \int_0^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g A_\| \left( 1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g \right) \] \\
- f'_\eta m^2 \eta M^2 \frac{d}{du_0} \int_0^{u_0} d\alpha_s \int_{u_0 - \alpha_s}^{1 - \alpha_s} d\alpha_g A_\| \left( 1 - \alpha_s - \alpha_g, \alpha_s, \alpha_g \right) \] \\
+ f'_\eta m^4 \eta \left( \int_{u_0 - \alpha_g}^{1 - \alpha_g} d\alpha_g \right) \] \\
\left[ \frac{1}{\alpha_g} \left( 3 - \frac{2m_b^2}{M^2} \right) \Phi + \frac{4m_b^2 \alpha_s + \alpha_g - u_0}{\alpha_g^2} \right] (A_\| + A_\perp) \right) \left( 1 - \alpha - \alpha_g, \alpha_s, \alpha_g \right) \] \\
- f'_\eta m^4 \eta u_0 \int_0^{1 - u_0} \] \\
\left[ \Phi(1 - \alpha - \alpha_g, \alpha_s, \alpha_g) \right] \] \\
- f'_\eta m^4 \eta \int_{1 - u_0}^1 d\alpha_g \int_{u_0 - \alpha_g}^{\alpha_g} d\beta \int_0^{1 - \beta} d\alpha \left[ \Phi(1 - \alpha - \beta, \alpha, \beta) \frac{1 - u_0}{\alpha_g^2} \right] \left( 4 - \frac{2m_b^2}{M^2} \right) \] \\
+ \frac{4m_b^2 (1 - u_0)^2}{\alpha_g^2} \left( A_\| + A_\perp \right) \left( 1 - \alpha - \alpha_g, \alpha_s, \alpha_g \right) \] \\
+ f'_\eta m^4 \eta \int_{1 - u_0}^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1 - \beta} \] \\
\left[ \Phi(1 - \alpha - \beta, \alpha, \beta) \frac{u_0 (1 - u_0)}{\alpha_g^2} \right] \} , \] \\
(12)

where

\[ \Phi(\alpha_i) = A_\|(\alpha_i) + A_\perp(\alpha_i) - V_\|(\alpha_i) - V_\perp(\alpha_i) , \] \\
\[ \Xi = m_b^2 + u_0 (1 - u_0) m^2_\eta , \] \\
\[ u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \] \\
\[ M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} . \] \\
(13)
values of the strong coupling constants \( g \) are shown in Fig. 1. The Borel parameters are chosen as \( \lambda, g \).

Figure 1: The strong coupling constants \( g_{B_{s1}B_{s}^*\eta}(A) \) and \( g_{B_{s0}B_{s}^*\eta}(B) \) with the parameter \( M^2 \).

3 Numerical result and discussion

The input parameters are taken as \( m_s = (140 \pm 10)\text{MeV}, m_b = (4.7 \pm 0.1)\text{GeV}, \lambda_3 = 0.0, a_1 = 0.0, f_{3\eta} = (0.40 \pm 0.12) \times 10^{-2}\text{GeV}^2, \omega_3 = -3.0 \pm 0.9, \eta_4 = 0.5 \pm 0.2, \omega_4 = 0.2 \pm 0.1, a_2 = 0.20 \pm 0.06 \) \cite{13, 14}, \( f_\eta = 0.145\text{GeV}, m_\eta = 0.548\text{GeV}, f'_\eta = -\frac{2}{\sqrt{6}} f_\eta, f'_{3\eta} = -\frac{2}{\sqrt{6}} f_{3\eta}, M_{B_s} = 5.366\text{GeV}, M_{B_{s0}} = 5.412\text{GeV} \) \cite{16}, \( M_{B_{s1}} = (5.70 \pm 0.11)\text{GeV}, M_{B_{s2}} = (5.72 \pm 0.09)\text{GeV}, f_{B_{s0}} = f_{B_{s1}} = (0.24 \pm 0.02)\text{GeV} \) \cite{17}, \( f_{B_{s2}} = f_{B_{s}} = (0.19 \pm 0.02)\text{GeV} \) \cite{37, 17, 48}, \( s_\omega = (37 \pm 1)\text{GeV}^2 \) and \( s_A = (38 \pm 1)\text{GeV}^2 \) \cite{17}. The Borel parameters are chosen as \( M^2 = (5 - 7)\text{GeV}^2 \), in this region, the values of the strong coupling constants \( g_{B_{s1}B_{s}^*\eta} \) and \( g_{B_{s0}B_{s}^*\eta} \) are rather stable, which are shown in Fig. 1.

In the limit of large Borel parameter \( M^2 \), the strong coupling constants \( g_{B_{s1}B_{s}^*\eta} \) and \( g_{B_{s0}B_{s}^*\eta} \) take up the following behaviors respectively,

\[
g_{B_{s0}B_{s}^*\eta} \propto \frac{M^2 \phi_p(u_0)}{f_{B_{s0}} f_{B_{s}}},
\]

\[
g_{B_{s1}B_{s}^*\eta} \propto \frac{m_b M^2 \phi_p(u_0)}{f_{B_{s2}} f_{B_{s1}}}. \tag{14}
\]

It is not unexpected, the contributions from the two-particle twist-3 light-cone distribution amplitude \( \phi_p(u) \) are greatly enhanced by the large Borel parameter \( M^2 \), (large) uncertainties of the relevant parameters presented in above equations have significant impact on the numerical results. The contribution from the two-particle twist-3 light-cone distribution amplitude \( \phi_p(u_0) \) is zero due to symmetry property.

Taking into account all the uncertainties of the input parameters, finally we obtain the numerical values of the strong coupling constants, which are shown in

\[
\begin{align*}
\eta_3 = & 0.5 \\
\eta_4 & = 0.2 \\
a_2 & = 0.20 \\
f_{3\eta} & = 0.40 \\
M_{B_{s0}} & = 5.412 \\
M_{B_{s1}} & = 5.70 \\
M_{B_{s2}} & = 5.72 \\
f_{B_{s0}} & = 0.24 \\
f_{B_{s1}} & = 0.24 \\
f_{B_{s2}} & = 0.19 \]
\]
Fig.1,

\[ |g_{B_{s1}B_{s1}\eta}| = (17.8 \pm 5.8) \text{GeV}, \]
\[ |g_{B_{s0}B_{s0}\eta}| = (20.1 \pm 7.2) \text{GeV}, \]

the uncertainties are large, about 30%. Taking into account the small \( \eta - \pi^0 \) transition matrix according to Dashen’s theorem [30], \( t_{\eta\pi} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003 \text{GeV}^2 \), we can obtain the narrow decay widths.

\[ \Gamma_{B_{s1}B_{s1}\pi} = \frac{p_1}{24\pi M_{B_{s1}}^2} \sum_{\lambda} \sum_{\lambda'} \left| \frac{g_{B_{s1}B_{s1}\eta} \eta^*(\lambda) \cdot \epsilon(\lambda') t_{\eta\pi}}{m_{\pi}^2 - m_{\eta}^2} \right|^2 = (5.3 - 20.7) \text{KeV}, \]
\[ \Gamma_{B_{s0}B_{s0}\pi} = \frac{p_2}{8\pi M_{B_{s0}}^2} \left| \frac{g_{B_{s0}B_{s0}\eta} t_{\eta\pi}}{m_{\pi}^2 - m_{\eta}^2} \right|^2 = (6.8 - 30.7) \text{KeV}, \]

\[ p_1 = \frac{\sqrt{[M_{B_{s1}}^2 - (M_{B_{s1}^*} + m_{\pi})^2] [M_{B_{s1}}^2 - (M_{B_{s1}^*} - m_{\pi})^2]}}{2M_{B_{s1}}^2}, \]
\[ p_2 = \frac{\sqrt{[M_{B_{s0}}^2 - (M_{B_{s0}} + m_{\pi})^2] [M_{B_{s0}}^2 - (M_{B_{s0}} - m_{\pi})^2]}}{2M_{B_{s0}}^2}, \]

which are consistent with the ones obtained from the analysis of the unitarized two-meson scattering amplitudes with the heavy-light chiral lagrangian, \( \Gamma_{B_{s1}B_{s1}\pi} = 10.36 \text{KeV} \) and \( \Gamma_{B_{s0}B_{s0}\pi} = 7.92 \text{KeV} \) [49, 50]. We can search the strange-bottomed \((0^+, 1^+)\) mesons \( B_{s0} \) and \( B_{s1} \) in the invariant \( B_s\pi^0 \) and \( B_{s1}\pi^0 \) mass distributions respectively, just like the BaBar and CLEO Collaborations observed the strange-charmed \((0^+, 1^+)\) mesons \( D_{s0} \) and \( D_{s1} \) in the invariant \( D_s\pi^0 \) and \( D_{s1}\pi^0 \) mass distributions respectively [51, 52].

4 Conclusion

In this article, we calculate the strong coupling constants \( g_{B_{s0}B_{s0}\eta} \) and \( g_{B_{s1}B_{s1}\eta} \) with the light-cone QCD sum rules. Then we take into account the small \( \eta - \pi^0 \) transition matrix according to Dashen’s theorem, and obtain the small decay widths. We can search the strange-bottomed \((0^+, 1^+)\) mesons \( B_{s0} \) and \( B_{s1} \) in the invariant \( B_s\pi^0 \) and \( B_{s1}\pi^0 \) mass distributions respectively.

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Appendix

The light-cone distribution amplitudes of the \( \eta \) meson are defined by

\[
\langle 0| \bar{s}(0) \gamma_\mu \gamma_5 s(x) | \eta(p) \rangle = i f^i_\eta p_\mu \int_0^1 du e^{-iup} \left\{ \phi_\eta(u) + \frac{m^2_{\eta} x^2}{16} A(u) \right\} + f^i_\eta m^2_{\eta} \frac{ix_\mu}{2p \cdot x} \int_0^1 du e^{-iup} B(u),
\]

\[
\langle 0| \bar{s}(0) i \gamma_5 s(x) | \eta(p) \rangle = \frac{f^i_\eta m^2_{\eta}}{m_s} \int_0^1 du e^{-iup} \varphi_p(u),
\]

\[
\langle 0| \bar{s}(0) \sigma_{\mu\nu} \gamma_5 s(x) | \eta(p) \rangle = i(p_\mu x_\nu - p_\nu x_\mu) \frac{f^i_\eta m^2_{\eta}}{6m_s} \int_0^1 du e^{-iup} \varphi_\varphi(u),
\]

\[
\langle 0| \bar{s}(0) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(vx) s(x) | \eta(p) \rangle = f^i_\eta \{ (p_\mu p_\nu g_{\nu\beta}^\perp - p_\nu p_\alpha g_{\mu\beta}^\perp) - (p_\mu p_\beta g_{\nu\alpha}^\perp - p_\nu p_\alpha g_{\mu\beta}^\perp) \} \int D\alpha_i \varphi_3 n(\alpha_i) e^{-ipx(\alpha_s + v_{\alpha\beta})},
\]

\[
\langle 0| \bar{s}(0) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) s(x) | \eta(p) \rangle = p_\mu \frac{f^i_\eta p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} m^2_{\eta} f^i_\eta \int D\alpha_i A_{\parallel}(\alpha_i) e^{-ipx(\alpha_s + v_{\alpha\beta})} + f^i_\eta m^2_{\eta} (p_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}) \int D\alpha_i A_{\perp}(\alpha_i) e^{-ipx(\alpha_s + v_{\alpha\beta})},
\]

\[
\langle 0| \bar{s}(0) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) s(x) | \eta(p) \rangle = p_\mu \frac{f^i_\eta p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} m^2_{\eta} f^i_\eta \int D\alpha_i V_{\parallel}(\alpha_i) e^{-ipx(\alpha_s + v_{\alpha\beta})} + f^i_\eta m^2_{\eta} (p_\beta g_{\alpha\mu} - p_\alpha g_{\beta\mu}) \int D\alpha_i V_{\perp}(\alpha_i) e^{-ipx(\alpha_s + v_{\alpha\beta})},
\]

where the operator \( \tilde{G}_{\alpha\beta} \) is the dual of the \( G_{\alpha\beta} \), \( \tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} G^{\mu\nu} \) and \( D\alpha_i \) is defined as\( D\alpha_i = d\alpha_i d\alpha_i d\alpha_i d\alpha_i \delta(1 - \alpha_s - \alpha_g - \alpha_s) \). The light-cone distribution amplitudes are
parameterized as

\[
\begin{align*}
\phi_\eta(u) &= 6u(1-u) \left\{ 1 + a_1 C_1^3(2u-1) + a_2 C_2^3(2u-1) \right\}, \\
\varphi_p(u) &= 1 + \left\{ 30\eta_3 - \frac{5}{2} \rho^2 \right\} C_2^3(2u-1) \\
&\quad + \left\{ -3\eta_3 \omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2 \right\} C_4^3(2u-1), \\
\varphi_\sigma(u) &= 6u(1-u) \left\{ 1 + [5\eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2] C_2^3(2u-1) \right\}, \\
\varphi_3\eta(\alpha_i) &= 360\alpha_s\alpha_s\alpha_s^2 \left\{ 1 + \chi_3(\alpha_s - \alpha_s) + \omega_3 \frac{1}{2} (7\alpha - 3) \right\}, \\
V_\parallel(\alpha_i) &= 120\alpha_s\alpha_s\alpha_s\alpha_g \left[ v_{00} + v_{10} (3\alpha_g - 1) \right], \\
A_\parallel(\alpha_i) &= 120\alpha_s\alpha_s\alpha_s\alpha_g a_{10}(\alpha_s - \alpha_s), \\
V_\perp(\alpha_i) &= -30\alpha_s^2 \left\{ h_{00}(1 - \alpha_g) + h_{01} \left[ \alpha_g(1 - \alpha_g) - 6\alpha_s\alpha_s \right] \\
&\quad + h_{10} \left[ \alpha_g(1 - \alpha_g) - \frac{3}{2} (\alpha_s^2 + \alpha_s^2) \right] \right\}, \\
A_\perp(\alpha_i) &= 30\alpha_s^2 (\alpha_s - \alpha_s) \left\{ h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5\alpha_g - 3) \right\}, \\
A(u) &= 6u(1-u) \left\{ \frac{16}{15} + \frac{24}{35} a_2 + 20\eta_3 + \frac{20}{9} \eta_4 \\
&\quad + \left\{ -\frac{1}{15} + \frac{1}{16} \right\} \eta_3 \omega_3 - \frac{10}{27} \eta_4 \right\} C_2^3(2u-1) \\
&\quad + \left\{ -\frac{11}{210} a_2 - \frac{4}{135} \eta_3 \omega_3 \right\} C_4^3(2u-1) \right\} + \left\{ -\frac{18}{5} a_2 + 21\eta_4 \omega_4 \right\} \\
\{2u^3(10 - 15u + 6u^2) \log u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u} \\
&\quad + u\bar{u}(2 + 13u\bar{u}) \right\}, \\
g_\eta(u) &= 1 + g_2 C_2^3(2u-1) + g_4 C_4^3(2u-1), \\
B(u) &= g_\eta(u) - \phi_\eta(u), \\
\end{align*}
\]
where
\begin{align}
    h_{00} &= v_{00} = -\frac{\eta_4}{3}, \\
    a_{10} &= \frac{21}{8} \eta_4 \omega_4 - \frac{9}{20} a_2, \\
    v_{10} &= \frac{21}{8} \eta_4 \omega_4, \\
    h_{01} &= \frac{7}{4} \eta_4 \omega_4 - \frac{3}{20} a_2, \\
    h_{10} &= \frac{7}{2} \eta_4 \omega_4 + \frac{3}{20} a_2, \\
    g_2 &= 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \\
    g_4 &= -\frac{9}{28} a_2 - 6 \eta_3 \omega_3, \\
\end{align}

here \( C^2_2(\xi) \), \( C^4_4(\xi) \) and \( C^3_2(\xi) \) are Gegenbauer polynomials, \( \eta_3 = \frac{f_{s_0}}{f_0} \frac{m_s}{m_0} \) and \( \rho^2 = \frac{m_0^2}{m_s^2} \). [13, 44].

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