COSMOLOGICAL STRUCTURE FORMATION CREATES LARGE-SCALE MAGNETIC FIELDS

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ABSTRACT

This paper examines the generation of seed magnetic fields due to the growth of cosmological perturbations. In the radiation era, different rates of scattering from photons induce local differences in the ion and electron density and velocity fields. The currents due to the relative motion of these fluids generate magnetic fields on all cosmological scales, peaking at a magnitude of $O(10^{-24} \text{ G})$ at the epoch of recombination. Magnetic fields generated in this manner provide a promising candidate for the seeds of magnetic fields currently observed on galactic and extragalactic scales.

Subject headings: cosmology: theory — early universe — galaxies: magnetic fields — large-scale structure of universe

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1. INTRODUCTION

The presence of magnetic fields on galactic and extragalactic scales is a major unsolved puzzle in modern astrophysics. Although the observational evidence supporting the existence of magnetic fields in large-scale structure is overwhelming, there is no consensus as to their origins. The standard paradigm for the creation of these fields is the dynamo mechanism, in which a small initial seed field is amplified by turbulence (the $\alpha$ effect) and/or by differential rotation (the $\omega$ effect). In principle, once a seed field is in place, it should be possible to follow its evolution and amplification from the collapse of structure and the operation of any relevant dynamos.

In recent papers, it has been argued that magnetic fields are generated in the early universe due to induced vorticity (Matarrese et al. 2005; Gopal & Sethi 2005; Takahashi et al. 2005). In this paper, we put forward a new mechanism for the generation of seed fields from cosmological perturbations. We show that the evolution of cosmological perturbations in the prerecombination era produces charge separations and currents on all scales, both of which contribute to magnetic fields, in addition to and independent of any fields generated via vorticity. These seed fields persist until the onset of significant gravitational collapse, at which point field amplification and dynamo processes could magnify such seeds to the $O(\mu\text{G})$ fields observed today.

The generation of magnetic fields from charge separations and currents in the early universe is a necessary consequence of structure formation. This paper calculates the magnitude of these seed fields, and it is shown that these seed fields may be sufficiently strong to account for all of the observed magnetic fields in large-scale structures. The layout of the paper is as follows: § 2 gives an overview of the observational evidence for magnetic fields along with a brief theoretical picture of their generation. Section 2 also contains an explanation of the novel idea that the early stages of structure formation in a perturbed universe generate magnetic fields. Section 3 presents a detailed treatment of cosmological perturbations, with a specific view toward the creation and evolution of local charge separations and currents. Section 4 explicitly calculates the seed magnetic fields that arise via this mechanism as a function of scale and epoch. Finally, § 5 compares the results of this mechanism with competing theories. It also discusses avenues for future investigation of this topic, including possible observational signatures of the fields that would arise from this mechanism.

2. MAGNETIC FIELDS: BACKGROUND

Magnetic fields with approximately microgauss strength are seen in all gravitationally bound or collapsing structures in which the appropriate observations have been made (Widrow 2002). The four major methods used to study astrophysical magnetic fields are synchrotron radiation, Faraday rotation, Zeeman splitting, and polarization of starlight. These observational techniques are detailed in depth in Ruzmaikin et al. (1988), with Faraday rotation often proving the most fruitful of the above methods.

Magnetic fields have been found in many different types of galaxies, in rich clusters, and in galaxies at high redshifts. Spiral galaxies, including our own, appear to have relatively large magnetic fields of $O(10 \mu\text{G})$ on the scale of the galaxy (Fitt & Alexander 1993), with some (such as M82) containing fields up to $\approx 50 \mu\text{G}$ (Klein et al. 1988). Elliptical and irregular galaxies possess strong evidence for magnetic fields (of order $\sim \mu\text{G}$) as well (Moss & Shukurov 1996), although they are much more difficult to observe due to the paucity of free electrons in these classes of galaxies. Coherence scales for magnetic fields in these galaxies, as opposed to in spirals, are much smaller than the scale of the galaxy, of the order of 100–1000 pc. Furthermore, galaxies at moderate ($z \approx 0.4$) and high redshifts ($z \gtrsim 2$) have been observed to require significant ($\sim \mu\text{G}$) magnetic fields to explain their observed Faraday rotations (Kronberg et al. 1992; Athreya et al. 1998). Magnetic fields are also observed in structures larger than individual galaxies. The three main types of galaxy clusters are those with cooling flows, those with radio halos, and those devoid of both. Galaxy clusters with cooling flows are observed to have fields of 0.2–3 $\mu\text{G}$ (Taylor et al. 1994), and the Coma Cluster (a prime example of a radio-halo cluster) is observed to have a field strength $\sim 2.5 \mu\text{G}$ (Kim et al. 1990), while clusters selected to have neither cooling flows nor radio halos still exhibit indications of strong (0.1–1 $\mu\text{G}$) fields (Clarke et al. 2001). There even exists evidence for magnetic fields on extracuster scales. An excess of Faraday rotation is observed for galaxies lying along the filament between the Coma Cluster and the cluster Abell 1367, consistent with an intercluster magnetic field of 0.2–0.6 $\mu\text{G}$ (Kim et al. 1989). On the largest cosmological scales, there exist only upper limits on magnetic fields, arising...
from observations of the cosmic microwave background (Barrow et al. 1997; Yamazaki et al. 2005, 2006) and from nucleosynthesis (Cheng et al. 1994), setting limits that on scales \( \gtrapprox 10 \) Mpc, field strengths are \( \lesssim 10^{-9} \) G.

Observational evidence for magnetic fields is found in galaxies of all types and in galaxy clusters, both locally and at high redshifts, wherever the appropriate observations can be made. A review of observational results can be found in Vallée (1997).

The theoretical picture of the creation of these fields, however, is incomplete. Fields of approximately microgauss strength can be explained by the magnification of an initial, small seed field on galactic (or larger) scales by the dynamo mechanism (Parker 1971; Vainshtein & Ruzmaikin 1971, 1972). A protogalaxy (or protocluster) containing a magnetic field can have its field strength increased by many orders of magnitude through gravitational collapse (Lesch & Chiba 1995; Howard & Kulsrud 1997) and can also be further amplified via various dynamos. Dynamos that can amplify a small seed field into the large fields observed today involve helical turbulence (\( \alpha \)) and/or differential rotation (\( \omega \)). Various types of these dynamos include the mean field dynamo (Steenbeck et al. 1966; Moffatt 1978; Ruzmaikin et al. 1988), the fluctuation dynamo (Kazantzsev et al. 1985; Moss & Shukurov 1996), and merger-driven dynamos (Tribble 1993), among others. However, the dynamo mechanism does not explain the origin of such seed fields.

While the initial seeds that grow into magnetic fields are anticipated to be small, they must still come from somewhere (Zel’’dovich & Novikov 1983), as their existence is not explained by the dynamo mechanism alone. There are many mechanisms that could produce small-strength magnetic fields on astrophysically interesting scales, either through astrophysical or exotic processes (see Widrow 2002 for a detailed review). Exotic processes generally rely on new physics in the early universe, such as a first-order QCD phase transition (Hogan 1983; Quashnock et al. 1989), a first-order electroweak phase transition (Baym et al. 1996; Sigl et al. 1997), broken conformal invariance during inflation (Turner & Widrow 1988; Ashoorioon & Mann 2005), specific inflaton potentials (Ratra 1992), or the presence of charged scalars during inflation (Calzetta et al. 1998; Kandus et al. 2000; Davis et al. 2001). Astrophysical mechanisms, in contrast, are generally better grounded in known physics, although they have difficulty generating sufficiently strong fields on sufficiently large scales. The difference in mobility between electrons and ions admits, under appropriate circumstances, the creation of large-scale currents and magnetic fields. Seed magnetic fields arising from this difference in mobility between electrons and ions could be astrophysically generated via a variety of phenomena, including radiation-era vorticity (Harrison 1970, 1973), vorticity due to gas dynamics in ionized plasma (Biermann 1950; Pudritz & Silk 1989; Subramanian et al. 1994; Kulsrud et al. 1997; Gnedin et al. 2000), stars (Syrovatskii 1970), or active galactic nuclei (Hoyle 1969). Although there are many candidates for producing the seed magnetic fields required by the dynamo mechanism, none have emerged as a definitive solution to the puzzle of explaining their origins.

The mechanism proposed in this paper is that seed magnetic fields are generated by the scattering of photons with charged particles during the radiation era. Unlike the mechanism of Harrison (1970, 1973), which is disfavored (Rees 1987) due to its requirement of substantial primordial vorticity (although see Matarrese et al. [2005] and Takahashi et al. [2005] for an argument that some vorticity is necessary), the fields of interest here are generated by the earliest stages of structure formation, requiring no new physics.

Ions (taken to be protons, for simplicity) and electrons are treated as separate fluids, with opposite charges but significantly different masses. The mass-weighted sums of their density and velocity fields will determine the evolution of the net baryonic component of the universe and should agree with previous treatments (e.g., Ma & Bertschinger 1995). The differences of their density and velocity fields, however, provide a description of a local charge separation and a local current density, both of which contribute to magnetic fields. Since cosmological perturbations, which serve as seeds for structure formation, exist on all scales, it is expected that seed magnetic fields will be generated on all scales by this mechanism. As the magnetic fields generated via this mechanism are in place at very early times, they possess the advantage over other astrophysical mechanisms that these fields will have optimally large amounts of time to be affected by amplification effects and various dynamos. The remainder of this paper focuses on calculating the magnitude of the magnetic fields generated by this process and discussing their cosmological ramifications.

3. COSMOLOGICAL PERTURBATIONS

Although the early universe is isotropic and homogeneous to a few parts in \( 10^5 \) (Bunn & White 1997), it is these small density inhomogeneities, predicted by inflation to occur on all scales (Guth & Pi 1982), that lead to all of the structure observed in the universe today. As it is the early epoch of structure formation that is of interest for the creation of magnetic fields, we calculate the evolution of inhomogeneities in the linear regime of structure formation. The most sophisticated treatment of cosmological perturbations in the linear regime to date is that of Ma & Bertschinger (1995), which provides evolution equations for an inhomogeneous universe containing a cosmological constant, dark matter, baryons, photons, and neutrinos. This section extends their treatment to encompass separate proton and electron components. The mass-weighted sum of protons and electrons will recover the usual baryon component, whereas the difference of the density fields represents a charge separation, and the difference of the velocities is a net current.

The dynamics of any cosmological fluid can be obtained, in general, from the linear Einstein equations (see Peebles & Yu [1970], Silk & Wilson [1980], and Wilson & Silk [1981] for earlier treatments). Although the choice of gauge does not impact the results, the conformal Newtonian gauge leads to the most straightforward calculations. When tensor modes are unimportant, the metric is given by

\[
ds^2 = a^2(\tau) \left[ -(1 + 2\psi) \, dt^2 + (1 - 2\phi) \, dx^i \, dx_i \right],
\]

where \( \psi \approx \phi \) when gravitational fields are weak. The linearized Einstein equations are then

\[
k^2 \phi + 3 \left( \frac{\dot{a}}{a} \right) (\phi + \dot{a} \psi) = 4\pi Ga^2 \delta T^0_0,
\]

\[
k^2 \left( \dot{\phi} + \dot{a} \psi \right) = 4\pi Ga^2 (\dot{\rho} + \dot{P}) \theta,
\]

\[
\frac{\dot{\phi}}{a} (\psi + 2\phi) + \frac{2a}{a} \frac{\dot{a}^2}{a^2} \psi + k^2 (\phi - \psi) = \frac{4\pi}{3} Ga^2 \delta T^i_i,
\]

\[
k^2 (\phi - \psi) = 12\pi Ga^2 (\dot{\rho} + \dot{P}) \sigma,
\]

where \( \sigma \) is the shear stress, which is negligible for nonrelativistic matter but important for photons and neutrinos. A cosmological
fluid that is either uncoupled to the other fluids or mass-averaged among uncoupled fluids in the early universe obeys

\[
\dot{\delta} = -(1+w)(\theta - 3\phi) - 3\frac{\dot{a}}{a}(c_s^2 - w)\delta, \\
\dot{\theta} = -\frac{a}{1-3w}\theta - \frac{\dot{a}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\psi, \tag{3}
\]

where \(\delta\) is defined as the local density relative to its spatial average (\(\delta \equiv \delta \rho/\rho\); \(\theta \equiv ik\psi_\nu\), where \(\psi_\nu\) is the local peculiar velocity; and \(c_s\) is the sound speed of the fluid.

For individual components with additional intercomponent interactions, equation (3) must be augmented to include these interactions. Examples of such interactions include momentum transfer between photons and charged particles and Coulomb interactions between protons and electrons. For protons, electrons, and cold dark matter (CDM), an equation of state \(w = 0\) is assumed, and for radiation and neutrinos, \(w = \frac{1}{3}\). The master equations for each component of interest are computed explicitly in §§3.1–3.5. The evolution equations that appear in this section for baryons, photons, light neutrinos, and cold dark matter are derived in much greater detail, including photon polarization and hydrogen and helium recombination, in both synchronous and conformal Newtonian gauges, in Ma & Bertschinger (1995).

3.1. Cold Dark Matter

The cold dark matter component (denoted by the subscript \(c\)) is collisionless and pressureless; its evolution can be simply read off from equation (3) with \(w = c_s^2 = \sigma = 0\) to be

\[
\dot{\delta}_c = -\theta_c + 3\phi_c, \\
\dot{\theta}_c = -\frac{\dot{a}}{a}\theta_c + k^2\psi_c. \tag{4}
\]

Any cold (i.e., nonrelativistic), collisionless component will behave according to the dynamics given by equation (4).

3.2. Light Neutrinos

For massless (or nearly massless) particles, pressure is non-negligible, and the appropriate equation of state has \(w = \frac{1}{3}\). In addition, the shear term \((\sigma)\) may be important as well. The most accurate way to compute the evolution of such a component of the universe is by integration of the Boltzmann equation, which is given for light neutrinos (denoted by subscript \(\nu\)) by

\[
\frac{\partial F_{\nu}}{\partial \tau} + i k (\hat{k} \cdot \hat{n}) F_{\nu} = 4\left[\dot{\phi} - i k (\hat{k} \cdot \hat{n}) \psi\right], \tag{5}
\]

in Fourier space.

The approximation that neutrinos are massless and uncoupled is very good from an age of the universe of approximately \(t \approx 1\) s until the epoch of recombination. The evolution equations for light neutrinos are then

\[
\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu + 4\phi, \\
\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu + \psi\right), \\
\dot{F}_{\nu} = \frac{k}{2l+1} \left[ F_{\nu(l-1)} - (l + 1) F_{\nu(l+1)} \right], \tag{6}
\]

where \(\sigma_\nu\) is related to \(F_{\nu}\) by \(2\sigma_\nu = F_{\nu,2}\), the index \(l\) governs the final equation for \(l \geq 2\), and \(F_{\nu}\) is defined by the expansion of the perturbations in the distribution function, \(F_{\nu}\).

\[
F_{\nu} = \sum_{l=0}^{\infty} (-1)^l (2l + 1) F_{\nu}(k, \tau) P_l(\hat{k} \cdot \hat{n}), \tag{7}
\]

where \(P_l(\hat{k} \cdot \hat{n})\) are the Legendre polynomials. Equations (5)–(7) are valid for any noncollisional species behaving as radiation.

3.3. Photons

Photons (denoted by subscript \(g\)), although similar to light neutrinos, evolve differently due to their large coupling with charged particles. The differential scattering cross section of photons with electrons is given by

\[
\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{16\pi} (1 + \cos^2 \theta), \tag{8}
\]

where \(\sigma_T\) is the Thomson cross section (Jackson 1998). Photons also scatter with protons, but with a cross section suppressed by a factor of the squared ratio of electron to proton mass, \(m_e^2/m_p^2 \approx 3.0 \times 10^{-9}\).

The polarization-summmed phase-space distribution for photons, \(F_{\gamma}\), is the same as the distribution function for neutrinos (see eq. [7]), with a nonzero difference between the two linear polarization components denoted by \(G_{\gamma}\). The linearized collision operators for Thomson scattering (Bond & Efstathiou 1984, 1987; Kosowsky 1996; Ma & Bertschinger 1995) yield the set of master equations for photons,

\[
\dot{\theta}_\gamma = -\frac{4}{3}\theta_\gamma + 4\phi, \\
\dot{\phi}_\gamma = k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + an_\nu \sigma_T (\theta_\nu - \theta_\gamma), \\
F_{\gamma2} = \frac{8}{15}\theta_\gamma - \frac{3}{5}kF_{\gamma3} - \frac{9}{5}an_\nu \sigma_T \sigma_\gamma + \frac{1}{10}an_\nu \sigma_T (G_{\gamma0} + G_{\gamma2}), \\
F_{\gamma3} = \frac{k}{2l+1} \left[ F_{\gamma(l-1)} - (l + 1) F_{\gamma(l+1)} \right] - an_\nu \sigma_T F_{\gamma1}, \\
\dot{G}_{\gamma m} = \frac{k}{2m+1} \left[ mG_{\gamma(m-1)} - (m + 1)G_{\gamma(m+1)} \right] + an_\nu \sigma_T \left(\frac{1}{10}F_{\gamma0} - \frac{2}{5}G_{\gamma m}\right), \tag{9}
\]

where \(F_{\gamma0} = \delta_\gamma\), \(F_{\gamma1} = 4\theta_\gamma/3k\), \(F_{\gamma2} = 2\sigma_\gamma\), the indices \(l\) and \(m\) are valid for \(l \geq 3\) and \(m \geq 0\), and the subscript \(b\) denotes the baryonic component, which is the mass-weighted sum of the electrons and protons. Electron-photon scattering is so dominant over proton–photon scattering as to render the latter negligible, but the electron–proton coupling (via electromagnetism) is sufficiently strong that, to leading order, those two fluids move in kinetic equilibrium.

3.4. Baryons

The net behavior of the baryonic component can be derived from combining the mass-weighted contributions of the proton and electron fluids. Both protons and electrons contain all of the terms present in the CDM equations (cf. §3.1) but additionally contain important terms arising from the Coulomb interaction and from Thomson scattering. The coupling of the Coulomb
interaction with density inhomogeneities can be calculated through a combination of the electromagnetic Poisson equation,

\[ \nabla^2 \Phi = -\nabla \cdot E = -4\pi \rho_c, \]

where \( \rho_c \) is the electric charge density, and the Euler equation,

\[ \frac{1}{a} \frac{\partial (av)}{\partial t} + \frac{1}{a} (v \cdot \nabla)v = -\frac{1}{a} \nabla \phi - \frac{q}{m} \frac{1}{a} \nabla \Phi + C, \]

where \( q/m \) is the charge-to-mass ratio of the particle in question and \( C \) is the collision operator. The Coulomb contribution appears as \( 4\pi e^2/(n_p - n_e)q_i/m \) in the evolution equation for \( \delta_i \), where \( i \) denotes a species of particle with a mass \( m_i \) and charge \( q_i \). The evolution equations are therefore

\[ \dot{\delta_e} = -\theta_e + 3\dot{\phi}, \]
\[ \dot{\theta}_e = -\frac{\dot{a}}{a} \theta_e + c_e^2 k^2 \delta_e + k^2 \psi + \Gamma_e (\theta_e - \dot{\theta}_e) - \frac{4\pi e^2 a^2}{m_e} (n_p - n_e), \]

for electrons, denoted by subscript \( e \), and

\[ \dot{\delta}_p = \theta_p + 3\dot{\phi}, \]
\[ \dot{\theta}_p = -\frac{\dot{a}}{a} \theta_p + c_p^2 k^2 \delta_p + k^2 \psi + \Gamma_p (\theta_p - \dot{\theta}_p) + \frac{4\pi e^2 a^2}{m_p} (n_p - n_e), \]

for protons, denoted by subscript \( p \), where \( \Gamma_e \) characterizes the rate of momentum transfer due to photon scattering with charged particles. The damping coefficient for electrons (\( \Gamma_e \)) is given by

\[ \Gamma_e = \frac{4\delta_e n_e a^2}{3\rho_e}, \]

and the analogous quantity for protons is smaller by a factor \( \Gamma_p/\Gamma_e = (m_e/m_p)^3 \approx 1.6 \times 10^{-10} \). Note the difference in the sign of the final terms in the equations for \( \dot{\theta}_e \) and \( \dot{\theta}_p \), which will prove important below.

From equations (12) and (13) for electrons and protons the dominant gravitational and electromagnetic combinations can be constructed separately. The remainder of this subsection details the evolution of baryons in the linear regime of a perturbed universe. Baryonic matter can be treated as the combination of electrons and protons; thus, the mass-weighted sum of proton and electron overdensities gives rise to the baryonic perturbations,

\[ \delta_b = m_e \delta_e + m_p \delta_p, \quad \theta_b = m_e \theta_e + m_p \theta_p, \]

where \( m_b = m_p + m_e \). The evolution of baryonic matter follows from a mass-weighted combination of the equations for electrons (eq. [12]) and protons (eq. [13]). To the extent that electrons and protons move together (the tight-coupling approximation) and the difference \( m_p - m_e \) is negligibly small (for baryonic evolution this is an excellent approximation; see § 3.5), the evolution equations for baryonic matter are

\[ \dot{\delta}_b = -\theta_b + 3\dot{\phi}, \]
\[ \dot{\theta}_b = -\frac{\dot{a}}{a} \theta_b + c_e^2 k^2 \delta_b + k^2 \psi + \Gamma_b (\theta_e - \theta_b), \]

where \( \Gamma_b = (m_e \Gamma_p + m_p \Gamma_e)/m_b \approx \Gamma_e m_e/m_p \). In the tight-coupling approximation, the baryon-photon coupling term in equation (16) is driven by the electron-photon interaction, as has been shown by Harrison (1970) and subsequent authors. The equations in equation (16) are identical to the equations for the evolution of baryonic inhomogeneities derived in Ma & Bertschinger (1995).

### 3.5. Charge Separation

From equations (12) and (13), a charge difference component as well as a sum component can be obtained. As the limits on a net electric charge asymmetry in the universe are very strict (Orito & Yoshimura 1985; Massó & Rota 2002; Caprini & Ferreira 2005), we expect that any differences in densities and/or velocities of protons and electrons will not be strong enough to significantly alter the evolution of the other components of the universe. It is therefore expected that the analysis of baryonic matter, detailed in § 3.4, will be unaffected by differences in proton and electron densities and velocities. However, even a small separation may still have important cosmological implications.

The charge difference component (denoted by subscript \( q \)) is the difference between the proton and electron components, such that \( \delta_q = \delta_p - \delta_e \) and \( \theta_q = \theta_p - \theta_e \). The gravitational potential does not affect the evolution of these quantities; gravity acts equivalently on electrons and protons. However, if \( \delta_q \) and \( \theta_q \) are nonzero, resulting electromagnetic fields will act differently on oppositely charged particles. The master equations for the charge-asymmetric component, obtained from equations (12) and (13), are

\[ \dot{\theta}_q = -\frac{\dot{a}}{a} \theta_q + c_e^2 k^2 \delta_q + \frac{4\pi m_e e^2 a^2}{m_e} \delta_q - \Gamma_e (\theta_e - \theta_b + \theta_q), \]

where the approximations \( \Gamma_p \ll \Gamma_e \) and \( m_b \approx m_p \) have been used where applicable. The term \( 4\pi m_e e^2 a^2 \delta_q/m_e \) in equation (17) arises from the Coulomb force acting on charged particles, while the final term, \( \Gamma_e (\theta_e - \theta_b + \theta_q) \), arises from the difference in Thomson scattering between protons and electrons. This final term provides a source for the generation of a charge separation independent of and in addition to any initial charge asymmetry and will create a local charge asymmetry even when there is none initially. In the evaluation of equation (17), the electromagnetic terms dominate the cosmological terms, such that an excellent approximation in the prerecombination universe is

\[ \delta_q = -\theta_q, \quad \dot{\theta}_q = \frac{4\pi m_e e^2 a^2}{m_e} \delta_q - \Gamma_e (\theta_e - \theta_b + \theta_q). \]

For some purposes, it is useful to express the set of equations found in equation (17) as a single ordinary differential equation. This can be accomplished by setting \( \dot{\theta}_q = -\delta_q \) and again by neg-}

\[ t = \left[ \frac{45h^3 \alpha s}{32n^2 G(KT)^4} \right]^{1/2}, \]

\[ \approx N \approx 72.2, \]

\[ N \approx 72.2, \]
in the radiation era, where $\tau_0$ is the age of the universe today and $t \equiv \alpha \tau$, to express all derivatives as derivatives with respect to $a$, denoted by primes (instead of dots). The evolution of $\delta_q$ can be tracked by evolving equation (20) below,

$$\delta''_q + 2N\tau e_0 \frac{1}{a^2} \delta'_q + \frac{16N^2 \pi n_e e^2 a^2}{m_e} \delta_q = 4N^2 \Gamma e_0 \frac{1}{a} (\theta_q - \theta_b),$$

(20)

where the subscript 0 denotes the present value of a quantity. This is simply the equation of a damped harmonic oscillator, with coefficients that change slowly with time compared to damping or oscillation times. The behavior can be characterized as overdamped at the earliest times, critically damped when $a \approx 2.3 \times 10^{-9}$, and underdamped at late times.

Of all the terms in equation (20), only $\theta_q$, $\theta_b$, and $\delta_q$ (and derivatives) are functions of $a$; all other quantities are constant coefficients. Although there does not exist a simple analytic form for $(\theta_q - \theta_b)$ in general, at sufficiently early times there exists the simple approximation

$$\theta_q - \theta_b \approx 6.0 \times 10^{19} k^4 a^5,$$

(21)

valid when the following condition is met:

$$a \leq \begin{cases} 
10^{-5} & \text{for } k \leq 0.1 \text{ Mpc}^{-1}, \\
10^{-6} \left( \frac{1 \text{ Mpc}^{-1}}{k} \right) & \text{for } k \geq 0.1 \text{ Mpc}^{-1}.
\end{cases}$$

Equation (21) is an approximation for a flat LCDM cosmology with cosmological parameters $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.27$, $\Omega_b = 0.044$, and a $^4\text{He}$ mass fraction of $Y = 0.248$. These parameters are used in all subsequent analyses for the calculation of cosmological quantities.

The approximation in equation (21) breaks down at later times. When this occurs, numerical methods must be used to obtain the quantity $(\theta_q - \theta_b)$. The software package COSMICS (Bertschinger 1995) is ideal for performing this computation, as it performs the numerical evolution of equations (4), (6), (9), and (16) concurrently. Computational results for the quantities $\theta_q$ and $\theta_b$ are given by COSMICS, which are valid at all times in the linear regime of structure formation. It is found that when the approximation in equation (21) breaks down, the quantity $(\theta_q - \theta_b)$ grows more slowly initially and proceeds to oscillate at a roughly constant amplitude at later times. These oscillations in the quantity $(\theta_q - \theta_b)$ are closely related to the acoustic oscillations between baryons and photons observed in the cosmic microwave background (Bennett et al. 2003).

Numerical integration of equation (20) can be accomplished in various ways (see Press et al. 1992 for examples). At sufficiently late times (when $a \gg 2.3 \times 10^{-9}$), numerical results indicate that the quasi-equilibrium solution

$$\delta_q = \frac{\sigma \tau m_b}{3 \pi e^2} \left( \frac{\dot{\rho}_{\gamma,0}}{\dot{\rho}_{b,0}} \right) \left( \frac{1}{a^2} \right) (\theta_q - \theta_b),$$

(22)

obtained by neglecting the first two terms in equation (20), is an excellent approximation. With $\theta_q$ and $\theta_b$ given by COSMICS in units of Mpc$^{-1}$, the prefactor in equation (22) can be written as

$$\frac{\sigma \tau m_b}{3 \pi e^2} \left( \frac{\dot{\rho}_{\gamma,0}}{\dot{\rho}_{b,0}} \right) = \frac{8 m_b}{9 m_e} \left( \frac{\Omega_{\gamma,0}}{\Omega_{b,0}} \right) \approx 1.67 \times 10^{-37} \text{ Mpc},$$

(23)

where $r_e = e^2/m_e c^2$ is the classical electron radius. The quantity $\theta_q$ then follows directly from equation (17) to be

$$\theta_q = -\frac{\sigma \tau m_b}{3 \pi e^2} \left( \frac{\dot{\rho}_{\gamma,0}}{\dot{\rho}_{b,0}} \right) \left( \frac{1}{a^2} \right) (\theta_q - \theta_b).$$

(24)

The solutions in equations (22) and (24) remain valid up to the epoch of recombination ($z \approx 1089$).

The results of numerically integrating the equations for $\delta_q$ and $\theta_q$ on various length scales through recombination using the code COSMICS (Bertschinger 1995) are presented as a function of redshift $z$ for comoving scales of $k = 10, 1, 0.1$, and $0.01 \text{ Mpc}^{-1}$ (from top to bottom). The amplitude $\delta_q$ rises as $\sim a^2$ initially, then ceases to grow when the scale of interest enters the horizon and oscillates at an amplitude that first continues to rise slowly, then falls, eventually matching on to the equilibrium solution $\delta_q \propto \theta_q$. Solid lines indicate positive values of $\delta_q$, and dashed lines indicate negative $\delta_q$ [See the electronic edition of the Journal for a color version of this figure.]

The magnitude of the charge separation is small (of the order of $10^{-3}$ in Fig. 1) because the constant in equation (23) is small compared to the Hubble length at decoupling. In the conformal Newtonian gauge the mode amplitudes are defined to be of the order of 1 outside the horizon and need to be multiplied by the COBE (Cosmic Background Explorer) normalization $\delta_H = 1.95 \times 10^{-5}$ of Bunn & White (1997) to obtain the physical quantities of magnetic field strength and magnetic spectral density, as in §4.

Figure 2 shows the spectral density of the charge asymmetry, $4\pi k^3 P_q(k)/(2\pi)^3$, plotted as a function of $k$. For comparison, the spectral density of baryonic matter, $4\pi k^3 P_b(k)/(2\pi)^3$, is also shown. Note the identical scales, $k$, at which $P_q(k)$ and $P_b(k)$ change signs.

The results thus far are accurate up through the epoch of recombination. At this point, however, the universe transitions from a state with ionization fraction $\chi_e \approx 1$ to a state in which the ionization fraction is small, $\chi_e \approx 10^{-4}$ (Peebles 1968), and the universe is transparent to radiation. The calculations of Matarrese et al. (2005) suggest that Silk damping affects only scales $k > 1 \text{ Mpc}^{-1}$. In the absence of interactions
with photons, a charge separation would continue to evolve as
\[
\frac{\dot{\delta}_q}{C_1 \delta_q} = \frac{3}{2} \Omega_b H^2 K^2 a^2 \delta_q,
\]
\[
\text{where } K^2 \text{ is the ratio of the electric to gravitational forces, }
K^2 = \frac{e^2}{G m_p m_e} = 2.26 \times 10^{39}.
\]
A residual charge separation without additional interactions would oscillate (plasma oscillations) with proper frequency \(\omega \approx KH\), with a further slow adiabatic decay of the oscillation amplitude. However, there are many other effects that begin to become important after recombination, including gravitational collapse, dynamo effects, and continued electron-photon scattering, and we do not attempt to compute the subsequent evolution in detail in this paper.

4. MAGNETIC FIELDS

With the results derived in \(\S\) 3.5 for \(\theta_q\) and \(\delta_q\), values for the local current densities and local charge separations can be obtained at any time in the prerecombination universe on all scales. Both \(\delta_q\) and \(\theta_q\) will contribute to magnetic fields, as currents create magnetic fields directly, and the bulk motion of a region of net charge will also produce a magnetic field. For each comoving distance scale (given by the value of \(k\)) and each epoch (determined by the scale factor \(a\)) of the universe, there will be a magnetic field amplitude associated with that scale. This field may serve as the seed for the large-scale magnetic fields observed today.

An expression for magnetic fields can be derived from the currents arising from the relative motion of the protons and electrons in the universe. Magnetic fields can be derived from Maxwell’s equations,
\[
\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = 4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t},
\]
with the current density \(\mathbf{J}\) given by
\[
\mathbf{J} = n_p e \mathbf{v}_p - n_e e \mathbf{v}_e \simeq n_e e \left[ \delta_q \mathbf{v}_b + (1 + \delta_b) \mathbf{v}_q \right],
\]
where \(\mathbf{v}_q \equiv \mathbf{v}_p - \mathbf{v}_e\), and the displacement current is neglected. In the Fourier domain, the curl of Maxwell’s equation for \(\nabla \times \mathbf{B}\) gives a direct expression for the magnetic field as a convolution,
\[
\mathbf{B}(k) = \frac{4\pi n_e e}{m_e} \int \frac{d^3 k'}{(2\pi)^3} \frac{\mathbf{k} \times \mathbf{k}'}{|k'|^2} \times \left[ \theta_b(k') \delta_q(k - k') + \theta_q(k') \delta_b(k - k') \right],
\]
from which the second moment of the magnetic field \(\mathbf{B}(k)\) is
\[
\langle B_i(k_1) B_j(k_2) \rangle = (2\pi)^3 \theta_B(k_1 + k_2) P_{ij} P_B(k),
\]
\[
P_{ij}^\perp = \frac{1}{2} (\delta_{ij} - \hat{k}_i \hat{k}_j),
\]
where \(\delta_{ij}\) is the Dirac delta function and \(P_B(k)\) is the magnetic field power spectrum. Note that the direction parallel to \(\mathbf{k}\) does not contribute to magnetic fields. Therefore, the direction perpendicular
contributions in eq. (30) (the epoch of recombination for a color strength $k$ where $v$ is projected out in equation (29). The power spectrum, $P_B(k)$, is then given by

$$P_B(k) = \left( \frac{4\pi n_e g_e}{\alpha} \right)^2 \int \frac{d^3k'}{(2\pi)^3} |\mathbf{k}|^2 \sin^2 \lambda' \left[ \frac{1}{|\mathbf{k}'|^2} P_{\theta_\phi \theta_\phi}(|\mathbf{k}'|) P_{\theta_\phi \theta_\phi}(|\mathbf{k}''|) - \frac{1}{|\mathbf{k}''|^2} P_{\theta_\phi \theta_\phi}(|\mathbf{k}'|) P_{\theta_\phi \theta_\phi}(|\mathbf{k}''|) + \frac{2}{|\mathbf{k}'|^2} P_{\theta_\phi \theta_\phi}(|\mathbf{k}'|) P_{\theta_\phi \theta_\phi}(|\mathbf{k}'|) - \frac{2}{|\mathbf{k}''|^2} P_{\theta_\phi \theta_\phi}(|\mathbf{k}'|) P_{\theta_\phi \theta_\phi}(|\mathbf{k}''|) \right],$$

(30)

where $\lambda'$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$ and $\mathbf{k}'' \equiv \mathbf{k} - \mathbf{k}'$. The second moment of two general quantities, $\phi$ and $\psi$, is expressed as the power $P_{\phi \psi}(k)$, defined by

$$\langle \phi(k_1) \psi(k_2) \rangle = (2\pi)^3 \delta_D(k_1 + k_2) P_{\phi \psi}(k),$$

(31)

where $k = |k_1| = |k_2|$. All three terms in equation (30) make comparable contributions to the total. Note that although the velocities $v_\theta$ and $v_\phi$ (or $v_\theta$ and $v_\phi$) are derived from potentials, they appear in convolution with density inhomogeneities. For this reason, there is no need for explicitly second-order quantities (such as vorticity or anisotropic stress) to obtain a nonzero magnetic field, an effect similar to the calculation of the kinetic Sunyaev-Zel’dovich effect by Ostriker & Vishniac (1986). A detailed discussion on the application of this method to the computation of a magnetic field in the early universe can be found in Gopal & Sethi (2005).

The spectral density, $4\pi k^3 P_B(k)/(2\pi)^3$, obtained by numerically integrating $\theta_\phi$ and $\delta_\phi$ in equations (22) and (24), provides both a measure of the magnetic field strength on a given scale $k$ and a measure of the energy stored in magnetic fields, $\rho_B = |\mathbf{B}|^2 / 8\pi$. The results for the spectral density of the magnetic field on comoving scales from $10^{-3}$ to $10^2$ Mpc$^{-1}$ at the epoch of recombination are shown in Figure 3. The peak of the density corresponds to a typical magnetic field strength of $10^{-25}$ to $10^{-24}$ G on a comoving scale of $0.1$ Mpc$^{-1}$.

5. DISCUSSION

The major result of this paper has been to demonstrate that seed magnetic fields of cosmologically interesting strengths and scales are necessarily generated by the same processes that cause structure formation. As overdensities in the early universe slowly grow during the radiation era, the differing photon interactions with protons and electrons create charge separations and currents of small magnitudes on all scales. These charge separations and currents grow in magnitude as the universe evolves, causing magnetic fields to grow as well. The magnetic power in a given mode peaks at approximately the time of horizon crossing, oscillating and falling slowly in amplitude after that. The net result is that at the epoch of recombination (and hence prior to any field amplification due to gravitational collapse or dynamo effects), seed magnetic fields of magnitude $O(10^{-24}$ G) are created by the simple dynamics of charged particles.

Other recent discussions of the origins of large-scale magnetic fields have mentioned the possibility that the evolution of cosmological perturbations could be responsible for their generation (Matarrese et al. 2005; Takahashi et al. 2005; Gopal & Sethi 2005; Ichiki et al. 2006). These groups have focused on the effects of vorticity, which arises at second order in cosmological perturbation theory. In addition to the charge separations and currents induced by photon scattering and the Coulomb interaction, as discussed in this work, Takahashi et al. (2005) also considered the contribution of photon anisotropic stress to magnetic fields but found that it is uninteresting on large scales. The overall strength and scale of the magnetic fields reported by these authors seem to differ by large factors from one another. The fields derived in Takahashi et al. (2005) and Ichiki et al. (2006) appear to be unreasonably large and peak at scales where Silk damping ought to be a dominant effect (Matarrese et al. 2005). As shown in the subsequent paragraph, their value for a magnetic field strength of $B \sim 10^{-19}$ G on $O$(Mpc) scales is difficult to reconcile with other estimates. For reasons discussed in the next paragraph, it appears that the estimates for magnetic fields generated by vorticity are most accurately given by Matarrese et al. (2005). When their results are considered on $\sim 10$ Mpc scales, the fields generated are found to be of physically reasonable strengths similar to our own, of the order of $\sim 10^{-25}$ G. Their spectrum is similar to the spectrum of Figure 3 as well, although their spectrum peaks at a slightly higher amplitude, is narrower in shape, and reaches its maximum at a smaller scale.

The approximate magnitude of the field strengths arising from the full calculations of $\S$ 4 can be understood intuitively. From the quasi-equilibrium solution in equation (22), the fractional charge asymmetry ($\delta_\phi$) is of the order of $10^{-33}$ at the scale of the peak of the power spectrum. When this information is combined with the fact that the streaming velocity at decoupling is $v/c \sim 10^{-3}$, a current that yields magnetic field strengths of $O(10^{-25}$ G) is produced. The field strengths expected from vorticity are somewhat less intuitive. Although for an ideal fluid a rotational velocity is not generated from an initially longitudinal velocity if there is no vorticity at linear order (Hwang 1993), this does not remain the case if the imperfect nature of the fluid is properly taken into account (Vishniac 1982). A reasonable “upper bound” estimate of the magnetic field that arises from vorticity can be calculated from the angular momentum of a protogalaxy,
yielding an angular velocity ($\omega$) and therefore a magnetic field. Harrison (1970) showed that magnetic fields of magnitude $B \sim 10^{-4} \omega G$ would arise in the radiation era due to vorticity. From Peebles (1969), an estimate of the angular momentum in the baryonic component of galaxies is $\sim 10^{70}$ g cm$^2$ s$^{-1}$, which yields a magnetic field of $B \sim 10^{-22}$ G for a protogalaxy at recombination. This estimate of the field strength is reasonable as an upper bound, as the angular momentum from Peebles (1969) is for the entire protogalactic system, but the associated magnetic field energy may appear distributed across many smaller scale modes. Furthermore, this estimate is comparable to and consistent with the field strengths found in Mat arrese et al. (2005). The field strengths are also estimated roughly in Gopal & Sethi (2005) and appear to agree with the results in this paper and with Mat arrese et al. (2005). It is therefore our assessment that their paper is the most correct of the works detailing magnetic fields arising from the vorticity generated by cosmological perturbations.

It should not be surprising that the calculational frameworks of this paper and those involving magnetic fields arising from vorticity can lead to comparable results, as both calculations rely on second-order quantities. The calculation of a vorticity in the context of cosmological perturbation theory requires a velocity term that is explicitly second order, whereas § 4 of this paper requires the convolution of two first-order quantities. On dimensional grounds, the two methods should yield comparable results. The results in this paper rely only on fields known to exist, and whether the contribution from charge separations and currents or from vorticity dominates can vary both with scale and from system to system. The usual evolution of cosmological perturbations leads in a straightforward manner to charge separations and currents, which necessarily arise from the differing interactions between protons and electrons with photons. As has been shown in § 4, magnetic fields follow directly from these sources.

A seed field of $O(10^{-24}$ G), as generated via the charge separations and currents induced by structure formation, would provide an excellent candidate for the origins of cosmological magnetic fields. While field amplification due to gravitational collapse is negligible at the epoch of recombination, this will not be the case at all times. At recombination, the universe has only been matter-dominated for a brief time, and thus density perturbations have only grown by a small amount in that time, leading to an insignificant amplification of the field strength. As magnetic flux is frozen in, nonlinear collapse causes $|B|$ to increase by many orders of magnitude (Lesch & Chiba 1995; Howard & Kulshrud 1997). However, the major source of amplification of an initial seed field comes from dynamo effects, as discussed in § 2. The key to solving the puzzle of the origin of cosmic magnetic fields lies in determining whether the seed fields produced by a given mechanism can be successfully amplified into the $O(\mu G)$ fields observed today. A major problem with many of the mechanisms that produce seed fields is that they produce weak fields at times insufficiently early for dynamo amplification to produce fields about as large as microgauss. The Biermann mechanism, for instance, can produce seed fields of the order of $\sim 10^{-19}$ G, but only at a redshift of $z \sim 20$. Although those initial fields are larger than the $\sim 10^{-24}$ G fields produced by the growth of cosmic structure, the fact that magnetic fields from structure formation are in place at $z \sim 10^3$ makes them an extremely attractive candidate for the seeds of cosmic magnetic fields. As argued by Davis et al. (1999), a seed field as small as $10^{-30}$ G at recombination could possibly be amplified into a microgauss field today. Clearly, more work on understanding dynamo amplification is necessary before a definitive solution to the puzzle of cosmic magnetic fields can emerge.

One interesting mechanism worth investigating further is for the cosmic seed fields generated by density perturbations to seed supermassive black holes. It is known that the magnetic field energy in active galactic nuclei and quasars is comparable to the magnetic field energy in an entire galaxy. However, these structures cannot generate their own magnetic fields from nothing; they require a preexisting seed field. It therefore appears to be a reasonable possibility that the seed fields generated by cosmic structure formation could provide the necessary fields to seed supermassive black holes. The resulting amplification via collapse and dynamo effects could explain the origin of large-scale magnetic structures in the universe.

If large-scale magnetic fields exist at the epoch of recombination, they may be detectable by upcoming experiments. The results shown in Figure 3 provide a prediction of large-scale magnetic fields at the epoch of the cosmic microwave background (CMB). Sufficiently large magnetic fields on large scales at recombination may be detectable by Planck (Lewis 2004; Delabrouille 2004), although current estimates of their sensitivity indicate that the field strengths predicted in this paper ($\sim 10^{-24}$ G) would be significantly out of range of Planck’s capabilities ($\sim 10^{-10}$ G). Nonetheless, a knowledge of the field strengths at recombination allow for predictions of CMB photon polarizations and Faraday rotation, both of which could be, at least in principle, observable.

It is also of interest to note that any primordial charge asymmetry or large-scale currents (and therefore magnetic fields) created in the very early universe will be driven away by these dynamics. Equation (20) has an approximate solution for $\delta_0$ that is critically (exponentially) damped at redshift $z \sim 10^9$, capable of reducing any preexisting charge or current by as much as a factor of $\sim 10^{-10}$. As this factor is extraordinarily large, it is easy to conclude that any initial $\delta_q$ or $\theta_q$ will be driven quickly to the value given by equations (22) and (24) at the epoch of critical damping. This result is independent of $k$ and ought to be applicable even to a charge asymmetry on large scales, perhaps approaching that of the present horizon. As a result of this reduction of preexisting charges or currents, the late-time ($z \sim 10^6$) solutions obtained in this paper for charge separations, currents, and magnetic fields ought to be independent of the initial conditions for $\delta_q$, $\theta_q$, and $|B|$ in the early universe.

Overall, the dynamics of ions, electrons, and photons during the radiation era necessarily leads to charge separations and currents on all scales, which in turn generate magnetic fields. These fields supercede any preexisting fields and are in place prior to substantial gravitational collapse. Thus, the creation of charge separations and currents from the evolution of cosmological perturbations emerges as a promising and well-motivated new candidate to explain the origins of cosmic magnetic fields.

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