ASCCC Fractal and Its Application in Antenna Miniaturization

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Abstract
In this chapter, ASCCC fractal is defined. The name “ASCCC” is based on the process that the fractal is built. It is made by adding and subtracting circles to the circumference of a circle. Then the necessary formulas to build up the first and higher orders of ASCCC fractal are derived. By calculating the perimeter of each order, it is shown that the ASCCC fractal has a great capability in antenna miniaturization. Based on first-order ASCCC fractal, a systematic approach is designed to miniaturize an antipodal dipole at any arbitrary frequency. Then the proposed method is applied at band LTE13 (746–787 MHz), which is controversy for mobile antenna, because it causes the size of a common antenna to become very large for a handheld mobile. It is illustrated that not only the ASCCC fractal is successful in miniaturization of dipole antenna, but also it is very good at improving the antenna’s efficiency in comparison with its counterparts like Koch dipole/monopole.

Keywords: fractal antenna, antenna miniaturization, antenna’s efficiency, antipodal dipole antenna, mobile antenna

1. Introduction
Nowadays, there is demand for antennas which fit in small space while have good radiation performance. Therefore, miniaturization techniques are inevitable in antenna design. Most of miniaturization techniques are based on slot loading, lumped loading, material loading, meandering, using fractal shapes or meta-materials. Generally, these techniques cause radiation efficiency and bandwidth to reduce. The antenna performance can be improved if the available volume within the Chu’s sphere is used effectively. Fractal, meander and volumetric antennas are based on this method [1]. However, volumetric antennas are not suitable for
planar structures. The meander antennas [2] and some fractal antennas such as Hilbert [1] and Koch dipole/monopole [3, 4] have some sections of cancelling current from adjacent conductors that cause the efficiency not to improve significantly. Furthermore, the resonance frequency cannot be found analytically because the physical length is not equivalent with electrical length [1, 2].

In this chapter, a novel fractal named adding and subtracting circles to the circumference of a circle (ASCCC) is defined and the required formulas are derived to build it. The ASCCC fractal is made by adding and subtracting an even number of circles on circumference of a circle, in brief named as adding and subtracting circles to the circumference of a circle (ASCCC). Then, a procedure is shown to miniaturize an antipodal dipole based on first order of ASCCC fractal at any arbitrary frequency. A formula is extracted to determine the resonance frequency of the ASCCC dipole with excellent precision. The proposed procedure is used to design a mobile antenna at challenging band of LTE13 (746–787 MHz). Because of low frequency nature of LTE13, the in-building penetration and area coverage are very good [6]. On the other hand, the size of antenna becomes so large at LTE13 that it is not suitable for a handheld mobile [7]. Therefore, some miniaturization techniques should be applied to the design. One of the great advantages of ASCCC dipole antenna is using the Chu’s sphere so effectively that the antenna’s efficiency improves considerably in addition to antenna miniaturization. Actually, the currents in adjacent teeth of ASCCC fractal dipole do not weaken the effect of each other, so very good efficiency is obtained. This advantage also makes the physical length to be approximately equal with electrical length.

The design is simulated by full-wave software (Ansoft-HFSS version 15). The results of simulation and measurement are in very good agreement. The efficiency of the proposed dipole antenna is higher than the existing works at LTE13 for handheld mobile antenna [8–25]. Also, the design obtains 40% size reduction compared with a common dipole. Furthermore, the ASCCC design has advantages of being planar and vialess [5].

In Section 2, the ASCCC fractal is explained. Then in Section 3, a procedure is shown to use ASCCC fractal in arms of an antipodal dipole. Theoretically, how to design an ASCCC dipole antenna for a special band is discussed. Next, a mobile antenna is designed, simulated and measured at LTE13. Finally, the conclusion is presented in Section 4.

2. ASCCC fractal

ASCCC fractal is based on adding and subtracting an even number of circles alternately on circumference of an initial circle. In brief, it is named as adding and subtracting circles to the circumference of a circle (ASCCC). It should be noted that radius of secondary circles ($R_2$) must be smaller than the radius of initial circles ($R_1$). To make ASCCC fractal clear, firstly consider a circle with radius $R_1$ as shown in Figure 1(a). Then, arbitrary even numbers of circles ($n_1$) with radius $R_2$ ($R_2 < R_1$) are placed to the circumference of Figure 1(a) such that their centres are on the initial circle and two adjacent circles have two common points that one of them is on the circumference of initial circle and the other one is inside of it. These conditions lead $n_1$ to be at least 4 (for $n_1 = 2$, two adjacent circles have two common points but both of them are on the circumference of initial circle and $R_2 > R_1$) and secondary circles cover whole circumference of
the initial circle. For example, Figure 1(b) shows 20 secondary circles that have been placed on an initial circle with $R_1 = 21$ m. Figure 1(c) illustrates how secondary circles are added and subtracted alternately. The radius of the secondary circles ($R_2$) is calculated as follows. Firstly, it is supposed that each secondary circle occupies $2\theta$ angle on the initial circle as in Figure 2(a). Two radiuses of $R_1$ and one radius of $R_2$ can make an isosceles triangle with a $\theta$ vertex angle as in Figure 2(b). Then, its two leg length and base length are equal to $R_1$ and $R_2$, respectively. $R_2$ is determined by a trigonometric relationship as in Eq. (1) with respect to Figure 2(c). It should be noted that the value of $\theta$ is known because $2\theta$ is related to $n_1$ as in Eq. (2). If Eq. (2) is substituted in Eq. (1) and some simplifications are done, the $R_2$ can be written as in Eq. (3).

\[
\sin \frac{\theta}{2} = \frac{R_2}{2R_1} \Rightarrow R_2 = 2R_1 \sin \frac{\theta}{2} \quad (1)
\]

\[
2\theta = \frac{2\pi}{n_1} \Rightarrow \theta = \frac{\pi}{n_1} \quad (2)
\]

\[
R_2 = 2R_1 \sin \left(\frac{\pi}{2n_1}\right) \quad (3)
\]

Zero, first and second orders of ASCCC fractal for $R_1 = 21$ mm, $n_1 = 12$ and $n_2 = 10$ are shown in Figure 3(a)–(c). Figure 4(a)–(c) illustrates the stages of producing Figure 3(a)–(c).

Perimeter of the first-order ASCCC ($P_1$) is equal to perimeter of $n_1/2$ full circle with radius $R_2$. So, it is determined by Eq. (4). To understand clearly Eq. (4), firstly consider two adjacent circles shown in Figure 1(b). One of them is supposed to be added (united) and another one is subtracted from the initial circle. Therefore, the effect of these two adjacent circles on the perimeter of Figure 1(c) is equal to the circumference of one full circle with radius $R_2$. Since the total number of secondary circles is $n_1$, the total perimeter of Figure 1(c) is equal to $(n_1/2)(2\pi R_2)$.

\[
P_1 = \frac{n_1}{2}(2\pi R_2) \quad (4)
\]

For calculating the perimeter of the second-order ASCCC fractal ($P_2$), as it is made of $(n_1/2)$ full-circle that each has $n_2/2$ full-circles with radius $R_3$; therefore, its perimeter is equal to the perimeter of a total number of $(n_1/2)(n_2/2)$ full-circle with radius $R_3$ as in Eq. (5).
Figure 2. (a) A secondary circle occupies $2\theta$ angle on the initial circle, (b) the radiuses of initial circle and secondary circle make an isosceles triangle with a $\theta$ vertex angle and (c) $R_1$ and $R_2$ in the isosceles triangle.

Figure 3. ASCCC fractal for $R_1 = 21$ mm, $n_1 = 12$ and $n_2 = 10$ (a) initial circle with $R_1 = 21$ mm, (b) first-order ASCCC with $n_1 = 12$ and (c) second-order ASCCC with $n_2 = 10$ [5].

Figure 4. An illustration for building of second-order ASCCC fractal (a) for simplicity, third-order circles are placed only on inner (outer) edge of secondary circles which are supposed to be subtracted (added), (b) Secondary circles are added and subtracted alternatively and (c) third circles are added and subtracted alternatively [5].
Eqs. (6) and (7) show the ratio of $P_1$ and $P_2$ to the perimeter of initial circle ($P_0$), respectively. If $n_1$ and $n_2$ have large values, sine function could be approximated by its argument. Then, Eqs. (6) and (7) can be written as $(\pi/2)$ and $(\pi/2)^2$, respectively. Therefore, the perimeter of ASCCC fractal can be multiplied by $(\pi/2)$ in each order.

\[
\frac{P_1}{P_0} = \frac{2n_1 \pi R_1 \sin (\pi/2n_1)}{2\pi R_1} = n_1 \sin (\pi/2n_1) \tag{6}
\]

\[
\frac{P_2}{P_0} = \frac{2n_1 n_2 \pi R_1 \sin (\pi/2n_1) \sin (\pi/2n_2)}{2\pi R_1} = n_1 n_2 \sin (\pi/2n_1) \sin (\pi/2n_2) \tag{7}
\]

Now, it is time to compare $P_1$ (the perimeter of the first-order ASCCC with initial ($R_1$) and secondary ($R_2$) radiiuses) to the circumference $C_1$ of a common circle with radius $R_1 + R_2$ that occupies the same space on a board. Eq. (8) shows the ratio of $P_1$ to $C_1$. Eq. (9) presents the solutions of Eq. (8) for different $n_1$. When the argument of sine is much smaller than unity, the sine can be approximated to its argument. Therefore, approximation $\sin (\pi/2n_1) = \pi/2n_1$ is used for $n_1 \geq 10$. As it is seen, the ratio [Eq. (9)] is greater than one for $n_1 \geq 6$, so $P_1$ is larger than $C_1$. Therefore, if a way is found to force the current to travel the perimeter $P_1$, antenna miniaturization is obtained for $n_1 \geq 6$ [5].

\[
\frac{P_1}{C_1} = \frac{2n_1 \pi R_1 \sin (\pi/2n_1)}{2\pi (R_1 + R_2)} = \frac{n_1 \sin (\pi/2n_1)}{1 + 2 \sin (\pi/2n_1)} \tag{8}
\]

\[
\frac{P_1}{C_1} = \begin{cases} 
0.867 & n_1 = 4 \\
1.023 & n_1 = 6 \\
1.122 & n_1 = 8 \\
1.191 \leq \frac{\pi/2}{1 + (\pi/n_1)} \leq \frac{\pi}{2} & n_1 \geq 10 
\end{cases} \tag{9}
\]

### 3. An application of ASCCC fractal in antenna miniaturization

In this section, it is shown that an antipodal dipole antenna is miniaturized by applying the first-order ASCCC fractal to arms of the dipole antenna. The procedure could be applied to any arbitrary frequency [5].

#### 3.1. The proposed design

In this section, it is shown that an antipodal dipole antenna is miniaturized by applying the first-order ASCCC fractal to arms of the dipole antenna. Figure 5(a)–(d) presents the utilized method. In the first step, two first-order ASCCC fractals with the same $n_1$ but different $R_1$ are designed (Figure 5(a)). To distinguish $R_1$ of fractals, $R_{1i}$ is chosen for inner fractal and $R_{1o}$ for outer fractal. In the next step, the inner fractal is subtracted from the outer fractal as shown in Figure 5(b). Then, the shape is split into two equal halves as shown in Figure 5(c). Finally, each...
half is employed as an arm in a balanced antipodal dipole as illustrated in Figure 5(d). The first
resonance of the proposed dipole is calculated by Eq. (10). In Eq. (10), \( c \) is the speed of light and \( \lambda \) is the resonance wavelength.

\[
f = \frac{c}{\lambda}
\]  

(10)

In a common dipole antenna, the length in which current travels along the two arms is equal to \( \lambda/2 \) of the first resonance wavelength. In calculation of \( \lambda/2 \), it should be noted that current tends to travel the shortest path. In Figure 5(d), the current is confined to area between inner and outer fractals. The perimeter of inner fractal is shorter than the perimeter of outer one. Therefore, the inner fractal perimeter is more likely to be tracked. To be sure that current does not find any shorter path than the inner perimeter, \( R_{1o} \) should be chosen as close to \( R_{1i} \) as possible. As a result, the current travelling length is approximately equal to the inner fractal perimeter that is determined by using Eq. (4). Then, the resonance frequency could be written as in Eq. (11). In Eq. (11), \( P_{1i} \) is the perimeter of inner fractal in Figure 5(d).

\[
f = \frac{c}{\frac{\lambda}{2} \times \frac{P_{1i}}{2}} = \frac{c}{4\pi R_{1i} \sin (\pi/2n_{1})}
\]  

(11)

To design a balanced feedline, the method described in Refs. [26] and [27] is used. The line parameters are given in Figure 6. The exponential part of line is made by Eqs. (12) and (13). \( W_{cps} \) is equal to \((R_{1o} + R_{2o}) - (R_{1i} + R_{2i})\). \( L_{exp} \), \( L_{mv} \), \( W_{gnd} \) and \( p \) are arbitrary parameters that are chosen with respect to a good \( S_{11} \) result.

\[
y = \pm \left[ A \times \exp(px) + \left(\frac{W_{cps}}{2} - A\right)\right]
\]  

(12)

\[
A = \frac{W_{gnd} - W_{cps}}{2\left(\exp(p \times L_{exp}) - 1\right)}
\]  

(13)

3.2. Simulation and measurement results

The method described in Section 3.1 is used to design a handset mobile antenna at the LTE13 band (746–787 MHz). The antenna is printed on an FR4 substrate with \( \varepsilon_r = 4.4 \) and tan
Figure 6. Geometry and parameters of the balanced feedline.
δ = 0.02. Firstly, an initial resonance frequency within band LTE13 should be picked out to determine \( \lambda/2 \) by Eq. (10). As the length \( \lambda/2 \) is approximately equal to the perimeter of inner ASCCC, as shown in Figure 5(a), so \( R_{1i} \) is determined by Eq. (4). To stay in safe side, a frequency of 750 MHz is picked out for the initial design because a good \( S_{11} \) in lower frequencies needs longer length in arms while preparing longer length is harder to obtain. Please note that the value of \( n_1 \) is arbitrary. The larger \( n_1 \) results in the smaller \( R_{2i} \), so a more compact design is obtained. In the simulations, feedline parameters have been chosen as: \( L_m = 5 \) mm, \( L_{exp} = 25 \) mm, \( W_{gnd} = 20 \) mm, \( W_{ms} = 3.04 \) mm and \( p = 150 \).

\( R_{1i} = 20.28 \) mm is found for \( n_1 = 20 \) at 750 MHz. To determine a value for \( R_{1o} \), some simulations are done for different \( R_{1o} \) radiiuses (\( R_{1o} = 23, 24.5 \) and 26 mm). The simulated \( S_{11} \) results are shown in Figure 7. As seen, a smaller \( R_{1o} \) makes a better confinement of current to the inner fractal perimeter; therefore, the resonance frequency is closer to the initial design. On the other hand, a bigger \( R_{1o} \) results in better \( S_{11} \) and wider bandwidth because of a larger radiating area. However, \( R_{1o} = 23 \) mm cannot be chosen. Although resonance frequency is closer to the initial design, the whole band of LTE13 cannot be covered. The problem could be tackled as follows. If a bigger \( R_{1i} \) is chosen, the resonance frequency is lowered, so more freedom is prepared to pick out larger values for \( R_{1o} \) that could somehow compensate for lowering of frequency while enough bandwidth and good \( S_{11} \) are obtained at the LTE13 band. By a little try and error, it is found the whole LTE13 band could be covered with good \( S_{11} \) for \( R_{1i} = 20.5 \) mm and \( R_{1o} = 25.5 \) mm. As it is seen, the selected \( R_{1i} \) is so close to the initial design (\( R_{1i} = 20.28 \) mm) and the proposed formulas prepare very good primary guess.

A fabricated prototype of the proposed antenna is shown in Figure 8. The overall size of the printed antenna is \( 62 \times 115 \times 1.6 \) mm\(^3\) that is suitable for a handheld mobile. The simulated and measured results of \( S_{11} \) are presented in Figure 9. It is seen that there is very good agreement between them. The small resonance at 1.045 GHz is due to the type of feedline.

![Figure 7. S11 parameter for different R1o (R1i = 20.28 mm, n1 = 20) [5].](image-url)
Figure 8. Fabricated prototype of the proposed design [5].

Figure 9. Simulation and measurement results of $S_{11}$ against frequency [5].
Figure 10. Simulated 3D radiation patterns at 769 MHz [5].

Figure 11. Simulated and measured radiation patterns at 769 MHz (a) E-plane and (b) H-plane [5].

Figure 12. Simulated and measured radiation efficiency for LTE13 [5].
Figures 10 and 11 present the 3-D and polar radiation patterns of the proposed antenna at 769 MHz, respectively. As they show, the antenna has a dipolar radiation pattern. Figure 12 shows the efficiency of antenna. The measured efficiency is obtained by the improved Wheeler-cap method [28]. Antenna efficiency varies from 79.28 to 88.01%. As it is seen, the antenna has very high efficiency at LTE13, on the contrary of the other designs for this band that are listed in Table 1 [8–25].

Finally, the antenna exhibits 40% size reduction in comparison with a common dipole. This is evidence that the proposed procedure is a good technique in antenna miniaturization [5].

4. Conclusion

In this chapter, ASCCC fractal is defined and its driving formulas are extracted. It is shown that ASCCC fractal has a great potential in antenna miniaturization and improving efficiency. A miniaturization method was designed for a dipole antenna at any arbitrary frequency. Then, the method applied to the dipole antenna at band LTE13 which is very challenging for reduction in size of mobile antennas. The total size of antenna is $62 \times 115 \times 1.6 \text{ mm}^3$, which is appropriate for handheld mobiles. The efficiency of antenna is greater than 79% with $S_{11}$ better than $-10$ dB. The amount of efficiency is considerably higher than existing works.

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