MAGNETIC FIELDS IN COSMOLOGY

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Abstract

Different mechanisms which may possibly explain existence of magnetic fields on astronomically large scales are described. A recently suggested model of magnetic fields generation slightly before hydrogen recombination is discussed in more detail.
1 Introduction

It is known from observations, mostly from Faraday effect, that there are magnetic fields in galaxies with magnitude of a few micro-Gauss and coherence length of the order of galactic size, \( l_{\text{gal}} \sim (\text{a few}) \text{kpc} \). The energy density of such fields is close to the energy density of the cosmic microwave background radiation (CMBR):

\[
\rho_B = \frac{B^2}{8\pi} \sim \rho_{\gamma} \sim 10^{-4} \rho_m \approx 3 \cdot 10^{-34} \text{g/cm}^3
\]

where \( \rho_{\gamma} \) is the energy density of CMBR and \( \rho_m \approx 1.5 \text{ keV/cm}^3 \) is the total cosmological mass/energy density. Though the magnitude of such fields is small in comparison with the fields of stars or even planets (see below) it is very difficult to understand their huge coherence length. Even more puzzling is a possible existence of intergalactic magnetic fields which are 3 orders of magnitude weaker but coherent at a Megaparsec scale, for a review see ref. 1).

For comparison, magnetic field of the earth is \( B_{\oplus} = 0.5 \text{ G} \), magnetic fields of solar type stars can be as large as \( 10^3 \text{ G} \), magnetic fields of white dwarfs may reach \( 10^9 \text{ G} \), and absolute champions are neutron stars with magnetic fields at the level of \( 10^{13} \text{ G} \).

Possible creation mechanisms of large scale galactic or intergalactic fields can be roughly separated into four classes (for reviews see ref. 2):

1. Conventional (astrophysical) mechanism based on stellar ejecta of magnetic fields with subsequent magnetic lines reconnection (from different stars) to create a homogenous on large scale field component. This mechanism was reviewed recently in ref. 3).

2. Processes in the early universe which invoke inflation to stretch the characteristic scale of the field up to galactic or even larger scales. A comprehensive list of references on different versions of these mechanisms can be found in the recent paper 4).

3. Phase transitions in the early universe during which strong magnetic fields could be created but at a very small scale.

4. Relaxation of previously created inhomogeneities. In this process turbulent or laminar flow of primeval plasma with non-zero vorticity could be generated and due to different mobilities of charge carriers vortical electric currents producing magnetic fields would be created. Such mechanism may operate either in the early or relatively late universe.

All these mechanisms either generate magnetic fields at very small scales or of insufficiently large amplitude. In the first case “Brownian” type reconnection of
magnetic field lines is necessary for creation of coherent magnetic fields at large distances. The amplitude of the field in the course of reconnection decreases as $(l_{in}/l_{fin})^{3/2}$ where $l_{in}$ and $l_{fin}$ are the initial and final coherence lengths. Inflationary stretched fields are usually rather weak, though may have a very large coherence length. In both cases galactic dynamo should amplify the seed magnetic fields up to the necessary magnitude.

2 A comment on astrophysical mechanism

The total energy of galactic magnetic field in a large galaxy, e.g. in the Milky Way, is equal to:

$$E_{gal}^{B} = \frac{4}{3} \pi R_{gal}^{3} \rho_{B} \approx 10^{11} M_{\odot}$$  \hspace{1cm} (2)

where $M_{\odot}$ is the solar mass. Magnetic energy of a neutron star with radius $R_{ns} = 10^{6}$cm is equal to $E_{ns}^{B} \approx 10^{-11} M_{\odot} (B/10^{13} G)^{2}$. Magnetic energy of white dwarfs with $R_{wd} = 10^{9}$cm is $E_{wd}^{B} \approx 10^{-10} M_{\odot} (B/10^{9} G)^{2}$. Thus about $10^{11}$ white dwarfs with magnetic field $B = 10^{10}$G or $10^{12}$ neutron stars with $B = 10^{13}$G should be in the Galaxy to feed galactic magnetic field if the energy of the field were not lost in the process of line reconnection. Since the total number of stars in the Galaxy is about $2 \cdot 10^{11}$ it is hardly possible to meet this requirement. Of course this estimate is quite rough and more accurate considerations may be not so negative but still it shows that efficiency of stellar creation of galactic magnetic field should be very high to satisfy energy constraints.

3 Generation of seed magnetic fields in the early universe

It is well known that during inflation very long gravitational waves and large scale scalar field perturbations are generated. A natural question is: "why not electromagnetic fields?" The answer is that scalars and tensor fields are not conformally invariant even in the zero mass case, while photons are. Conformal invariance means that rescaling metric with an arbitrary factor $b(t, r)$ and the fields by the same factor to a power depending on the spin of the field we arrive to formally the same action written in terms of new variables. According to the Parker theorem conformally invariant fields are not generated in conformally flat space-time. Indeed it is known that cosmological Friedman-Robertson-Walker (FRW) background is conformally flat, i.e. after the proper coordinate choice the metric can be written in the form

$$ds^2 = a(\tau, x)^2 \left(d\tau^2 - dx^2\right),$$  \hspace{1cm} (3)
Thus after conformal transformation with the factor $b = a(\tau, \mathbf{x})$ to a proper power we can exclude FRW-gravity for conformally invariant fields. Fortunately, as it has been already mentioned, this cannot be done for scalar (in particular, inflaton) fields. Otherwise cosmological density perturbations would not be generated and we would not be here.

Several possible ways to break conformal invariance of electrodynamics and to create seed magnetic fields have been discussed in the literature:

1. New non-minimal interaction of electromagnetic field with gravity, possibly not gauge invariant\(^7\):

   \[
   \mathcal{L} = C_1 R A_\mu A^\mu + C_2 R_{\mu\nu} A^\mu A^\nu + C_3 R_{\mu\nu\alpha\beta} F^\mu\nu F^\alpha\beta + \ldots \tag{4}
   \]

   where $A_\mu$ is the electromagnetic vector-potential, $F^\mu\nu$ is the Maxwell tensor, $R_{\mu\nu\alpha\beta}$ is the Riemann tensor, $R_{\mu\nu}$ is the Ricci tensor, and $R$ is the curvature scalar.

2. Interaction with a new hypothetical field, dilaton, $\theta$\(^8\):

   \[
   \mathcal{L} = -(1/4) e^\theta F_{\mu\nu} F^{\mu\nu} \tag{5}
   \]

3. Quantum conformal anomaly due to famous triangle diagram which leads to non-zero trace of the energy-momentum tensor of electromagnetic field and breaks in this way conformal invariance of electrodynamics\(^9\):

   \[
   T^\mu_\mu = \kappa F_{\mu\nu} F^{\mu\nu} \tag{6}
   \]

   where the constant coefficient $\kappa$ depends upon the rank of the gauge group and the number of charged fermions. For $SU(N)$ with $N_f$ number of charged fermions it is: $\kappa = (\alpha/\pi)(11N/3 - 2N_f/3)$.

In all these models electromagnetic waves could be generated at inflationary stage and sufficiently large magnetic fields at very large scales would be created to serve as seeds for galactic fields, if one takes appropriate values of coupling constants in the Lagrangians or sufficiently large number of charge particles in the third case. Unsatisfactory features of this approach are introduction of new fields or interactions, though the second case looks quite natural in string inspired theories, while the third one looks good in grand unification models especially keeping in mind that the fine structure coupling constant $\alpha$ becomes quite large, about $1/40$, at the unification scale. The latter makes the mechanism much more efficient.
4 Generation of vorticity perturbations by cosmological inhomogeneities

There are several mechanisms which might create inhomogeneities in the primeval plasma whose relaxation could lead to generation of primordial magnetic fields:

1. First order phase transitions creating bubbles of one phase inside another [10]. Though the magnitude of magnetic fields produced on the boundaries between the phases could be very large, the characteristic scale is extremely small and it is difficult to stretch it up to galactic size.

2. Creation of stochastic inhomogeneities in cosmological charge asymmetry, either electric [11], or e.g. leptonic [12] at large scales which produce turbulent electric currents and, in turn, magnetic fields. The first of these models could be quite efficient but it demands rather unusual physics. For realization of the second model only one rather innocent assumption is necessary, namely, an existence of sterile neutrino, $\nu_s$, very weakly mixed with active ones, $\nu_{e,\mu,\tau}$. If $\nu_s$ is lighter than $\nu_a$ and the mass difference is sufficiently large then the MSW-resonance transition between $\nu_s$ and active neutrinos would take place giving rise to a large, about 0.1, and strongly fluctuating lepton asymmetry in the active neutrino sector [13]. When the wave length of the domain with a large lepton (or anti-lepton) number crossed horizon, the neutrino or antineutrino flux from this domain would induce electric current by scattering on electrons or positrons because of different $\nu_e^-$- and $\nu_e^+$-cross-sections. In this process hydrodynamical flows with large Reynolds numbers would be generated and turbulent vortical currents could be produced. The characteristic wave length of the generated magnetic field is about 100 pc and chaotic line reconnection and large but not unreasonable dynamo amplification are necessary for explanation of the observed galactic fields.

3. Generation of seed magnetic fields slightly before hydrogen recombination epoch [14] through relaxation of the usual density perturbations which are known from observations to exist. This model does not demand any new physics and predicts quite promising amplitude of seed fields at galactic scales. Since the work [14] is new, not yet published, we will discuss it in the next section in some detail.

5 Generation of large scale magnetic fields at recombination epoch

We will consider the period when the universe was already quite old and cool with temperature about 1-100 eV, i.e. somewhat before hydrogen recombination. The usual cosmological density perturbations are known to exist at that
time, with rather small amplitude, $\delta \rho / \rho \sim 10^{-4}$. The motion of the cosmological plasma, or better to say fluid, under pressure forces is governed by the hydrodynamical equation (see e.g. the book [15]):

$$\rho (\partial_t v_i + v_k \partial_k v_i) = -\partial_i p + \partial_k \left[ \eta \left( \partial_k v_i + \partial_i v_k - \frac{2}{3} \delta_{ik} \partial_j v_j \right) + \partial_i (\zeta \partial_j v_j) \right]$$

(7)

where $v$ is the velocity of the fluid element, $\rho$ and $p$ are respectively the energy and pressure densities of the fluid, and $\eta$ and $\zeta$ are the first and second viscosity coefficients. In the case of constant viscosity coefficients this equation is reduced to the well known Navier-Stokes equation. The coefficient $\eta$ is related to the mean free path of particles in fluid as

$$\eta / \rho \equiv \nu = l_f$$

(8)

In what follows we disregard the second viscosity $\zeta$.

The behavior of the solution to eq. (7) crucially depends upon the value of the Reynolds number

$$R_{\lambda} = \frac{v \lambda}{\nu}$$

(9)

where $\lambda$ is the wavelength of the velocity perturbations. If $R \gg 1$, then the fluid motion would become turbulent and non-zero vorticity would be created by spontaneously generated turbulent eddies. In the opposite case of low $R$ the motion is smooth and and no vorticity could be generated in the first order in $\delta \rho$. One would expect that in the case of scalar perturbations vorticity remains zero in any order in $\delta \rho$. However, this is not the case, as we will argue in what follows.

Let us first estimate the Reynolds number of the fluid motion created by the pressure gradient in eq. (7). To this end we assume that the liquid is quasi incompressible and homogeneous, so that the second term in the r.h.s. of this equation can be neglected. This is approximately correct and the obtained magnitude of the fluid velocity is sufficiently accurate. In this approximation eq. (7) reduces to a much simpler one:

$$\partial_t v + (v \nabla) v - \nu \Delta v = -\frac{\nabla p}{\rho}$$

(10)

A comment worth making at this stage. The complete system of equations includes also continuity equation which connects the time variation of energy density with the hydrodynamical flux (see below eq. [19]) and the Poisson equation for gravitational potential induced by density inhomogeneities. We will however neglect the gravitational force and the back reaction of the
fluid motion on the density perturbation. This approximation would give a reasonable estimate of the fluid velocity for the time intervals when acoustic oscillations are not yet developed, i.e. for $t < \lambda / v_s$, where $\lambda$ is the wave length of the perturbation and $v_s$ is the speed of sound (in the case under consideration $v_s^2 = 1/3$). In fact the wave length should be larger than the photon mean free path, to avoid diffusion damping, and the characteristic time interval, as we see below, should be somewhat larger than $\lambda$. So we may hope that our estimates of the velocity are reasonable enough. Neglecting gravitational forces, especially those induced by dark matter would result in a smaller magnitude of the fluid velocity, so the real effect should be somewhat larger.

For small velocities (or sufficiently small wavelengths) we may neglect the second term in the l.h.s. with respect to the third one. In this approximation the equation becomes linear and can be easily solved for the Fourier transformed quantities. Assuming that the parameters are time-independent (though it is not necessary) we obtain:

$$v_k = -\frac{ik}{3k^2 \nu} \delta_k \left[ 1 - \exp(-\nu k^2 t) \right] \tag{11}$$

where $\delta_k = (\delta \rho / \rho)_k$ is the Fourier transform of relative density perturbations, $\delta \rho / \rho$; its natural value is $\sim 10^{-4}$, though it might be much larger at small scales. The coefficient $1/3$ comes from equation of state of relativistic gas, $p = \rho \nu^2/3$.

Therefore, for the Reynolds number we obtain:

$$R_k = \frac{\delta_k}{3 \nu} \left[ 1 - \exp(-\nu k^2 t) \right] \tag{12}$$

If $\delta_k$ is weakly dependent on $k$, then $R_k$ is a monotonically rising function of the wavelength $\lambda = 2\pi/k$. For $t \ll \lambda^2 / \nu$ it takes the value

$$R_k = \frac{t}{3 \nu} \delta_k \ll 1, \tag{13}$$

so the hydrodynamical flow remains laminar and vorticity is not spontaneously generated. Dynamical generation of vorticity is governed by the equation:

$$\partial_t \Omega - \nu \Delta \Omega = -\nabla \times \left( \frac{\nabla p}{\rho} \right) \tag{14}$$

where $\Omega = \nabla \times \mathbf{v}$ and we assume that velocity is small so that the term quadratic in $v$ was neglected. If the r.h.s. is non-vanishing, then $\Omega$ would be non-zero too. However usually pressure density is proportional to the energy density, $p = w \rho$, with a constant coefficient $w$ and hence $\nabla \times (\nabla p/\rho) = 0$. We
can see that this is not so because cosmic plasma consist of different components whose motion is somewhat different. Let us assume that plasma is in local thermal equilibrium with common temperature $T(x)$. This assumption is justified by a large interaction rate between radiation and charged particles. If $T$ would be the only parameter which determines the state of the medium, then vorticity would not be generated because we would have in our disposal only $\nabla T$ and it is impossible to construct non-vanishing $\nabla \times \mathbf{v}$ from the gradient of only one scalar function. However, distributions of charged particles depend upon one more function, their chemical potential:

$$f = \exp \left[ -\frac{E}{T(x)} + \xi(x) \right]$$

where the dimensionless chemical potential $\xi$ can be readily expressed through particle number density $n_e \approx n_B = \beta(x)n_\gamma$ with $\beta(x) = 6 \cdot 10^{-10} + \delta \beta(x)$:

$$\xi(x) = \ln \beta(x) + \text{const}$$

Hence we will find that the source term in the vorticity equation (14) is equal to:

$$S_k \equiv -\epsilon_{ijk} \partial_j \left( \frac{\partial_k \rho}{\rho} \right) = \epsilon_{ijk} \frac{\partial_k \rho_\gamma}{3 \rho_{\text{tot}}} \frac{\partial_j \beta}{\beta} \frac{\rho_b}{\rho_{\text{tot}}}$$

An essential feature here is that the spatial distribution of charged particles does not repeat the distribution of photons and hence the vectors $\nabla \rho_\gamma$ and $\nabla \beta$ are not collinear. This could occur if, for the wavelength corresponding to subgalactic scales, there exist baryon isocurvature fluctuations and thus $\rho(x)$ and $\beta(x)$ have different profiles. As we have mentioned above, different mean free paths of photons and charged particles would maintain such non-collinearity of the order of unity at the scales $\lambda \sim l_\gamma$. Moreover, even in the case of adiabatic perturbations a shift in the distribution of photons and charged particles could also be created because of acoustic oscillations that proceeded with different phases of radiation and matter densities. At the scales $\lambda \leq l_\gamma$ perturbations in the plasma temperature would be erased by the diffusion damping [15], while for $\lambda \gg l_\gamma$ the diffusion processes are not efficient and one would expect self-similar perturbation leading to collinearity of $\nabla \rho_\gamma$ and $\nabla \beta$. On the other hand, when $\lambda$ entered under horizon acoustic oscillations begun which destroyed the self-similarity. Thus the expected wavelengths of vorticity perturbations should be between $l_\gamma < \lambda < H^{-1}$.

Surprisingly vorticity can be also generated (and in the case under consideration even a larger one) if perturbations in plasma are determined by a single scalar function, for example, by $T(t, x)$ because it might be proportional to the product $\partial_t T(t, x) \partial_j T(t', x)$. These two gradients generally are not collinear if
taken at different time moments $t$ and $t'$. To see that, let us start from the Boltzmann equation for the distribution function $f(t, x, E, p)$ of photons:

$$
\left( \frac{\partial}{\partial t} + V \cdot \nabla - H \frac{\partial}{\partial p} + F \frac{\partial}{\partial p} \right) f(t, x, E, p) = I_{\text{coll}}[f_a, f_b, \ldots],
$$

(18)

where $V = p/E$ is the particle velocity (not to be confused with the velocity $v$ of macroscopic motion of the medium, for photons $V = 1$, while $v \ll 1$), $E$ and $p$ are respectively the particle energy and spatial momentum, $H$ is the universe expansion rate, $F$ is an external force acting on particles in question (the latter is assumed to be absent), and $I [f_a, f_b, \ldots]$ is the collision integral depending on the distributions $f_a$ of all participating particles.

At temperatures in eV-range only the Thomson scattering of photons on electrons is essential, so the collision integral is dominated by the elastic term. Integrating both parts of eq. (18) over $d^3p/(2\pi)^3$ we arrive to the continuity equation:

$$
\dot{n}(x) + \nabla J = 0
$$

(19)

where $J$ is the photon flux given by

$$
J \equiv \langle vn \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{p}{E} f
$$

(20)

and $v$ is the average macroscopic velocity of the photon plasma. Using the standard arguments one can derive from eq. (19) the diffusion equation:

$$
\dot{n} = D \Delta n
$$

(21)

where $D \approx l_t/3$ is the diffusion coefficient. We will use this equation below to determine time evolution of the photon temperature $T$.

If the elastic reaction rate $\Gamma_{el} = \sigma_{Th} n_e X_e = 1/l_t$ is sufficiently large, local thermal equilibrium would be established and the photon distribution would be approximately given by

$$
f \approx f_0 = \exp \left( -\frac{E}{T} + \xi \right)
$$

(22)

where the temperature and effective chemical potential could be functions of time and space coordinates: $T = T(t, x)$ and $\xi = \xi(t, x)$, and the photon mean free path is given by $l_t = 30 \text{pc} / X_e(T) T_{eV}^3$, where $T_{eV}$ is the plasma temperature in eV and $X_e$ is a fraction of the free electrons: $X_e(z)$ is practically 1 for $z > 1500$, and sharply decreases for smaller $z$’s, reaching values $\sim 10^{-5}$ at $z < 1000$. 

Evidently $f_0$ annihilates the collision integral. We can find correction to this distribution, $f = f_0 + f_1$, substituting this expression into kinetic equation (18) and approximating the collision integral in the usual way as $-\Gamma_{el} f_1$:

$$(K + \Gamma_{el}) f_1 = -K f_0$$  

(23)

where $K$ is the differential operator, $K = \partial_t + (\mathbf{V} \nabla)$. The solution of this equation is straightforward:

$$f_1(t, x, E, \mathbf{V}) = -\int_0^t dt_1 \exp \left[ -\int_{t_1}^{t} d\tau_2 \Gamma_{el}(t - \tau_2, x - \mathbf{V}\tau_2) \right] K f_0(t - \tau_1, x - \mathbf{V}\tau_1)$$  

(24)

Using this result we can calculate the average macroscopic velocity of the plasma. The calculations are especially simple if elastic scattering rate is high and the integrals are dominated by small values of $\tau_1$. In this case we obtain:

$$v_j(t, x) = \frac{\int d^3p V_j f_1(t, x, E, \mathbf{V})}{\int d^3p f_0(t, x, E)}$$  

(25)

and the vorticity, $\Omega_i = \epsilon_{ijl} \partial_j v_k$ is:

$$\Omega_i \approx 6\epsilon_{ijl} l_\gamma^2 \left( \frac{\partial_j T}{T} \right) \left( \frac{\partial_k T}{T} \right)$$  

(26)

To estimate time derivatives of the temperature we will use the diffusion equation (21), from which we find $\partial_t T = D\Delta T$ and finally obtain for vorticity with the wave vector $k = 2\pi/\lambda$:

$$|\Omega| \approx 2 \left( \frac{\delta T}{T} \right)^2 l_\gamma^3 k^4 \approx 2 \cdot 10^3 \left( \frac{\delta T}{T} \right)^2 \frac{l_\gamma^3}{\lambda^4}$$  

(27)

Since the photon diffusion erases temperature fluctuations at the scales $\lambda < l_\gamma$, the vorticity reaches maximum value near $\lambda \sim l_\gamma$. This magnitude of vorticity is considerably larger than found previously with the source term (17) and we will rely on it in the estimates of magnetic field presented below.

Since the conductivity of cosmic plasma is very high,

$$\kappa = (3/2\alpha) (n_e/n_\gamma) (m_e^2 T),$$  

(28)

the generation of magnetic field by the source currents, created by the cosmological inhomogeneities, is governed by the well know equation of magnetic hydrodynamics:

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\kappa} \nabla \times \mathbf{J}$$  

(29)
The electric current $J$ induced by the relaxation of the density inhomogeneities would contain two components: electronic and protonic. However the first one is surely dominant because it is much easier to drift electrons than heavier protons. This is why a non-zero current can be induced in electrically neutral medium. Of course the motion of electrons would not produce any excess of electric charge because the current could be realized by the flow of the dominant homogeneous part of electron distribution.

The solution of eq. (29) can be roughly written as

$$B \sim \int_{0}^{t} dt \frac{2\pi J}{\lambda \kappa} e^{2\pi v t_{i}/\lambda}$$

(30)

An estimate of magnetic field without pregalactic dynamo enhancement can be easily done if the helical source current is known, $\nabla \times J = e n_{e} \Omega$. With $\Omega$ given by eq. (27) and $B$ by eq. (30) we obtain:

$$\frac{B_{0}}{T^{2}} = 0.24 \cdot 10^{3} (4\pi \alpha)^{3/2} \left( \frac{l}{\lambda} \right) \left( \frac{\gamma_{\lambda}}{\alpha} \right)^{3} \left( \frac{T}{m_{e}} \right)^{2} \approx 10^{-8} T eV$$

(31)

where we took the wavelength equal to the photon mean free path, $\lambda = l_{\gamma}$.

If we take into account that linear compression of pregalactic medium in the process of galaxy formation is approximately $r \sim 10^{2}$, the seed field in a galaxy after its formation would be $r^{2}B_{0}$, i.e. 4 orders of magnitude larger than that given by eq. (31) and, for $T = 1$ eV, a relatively mild galactic dynamo, about $10^{4}$, is necessary to obtain the observed galactic magnetic field of a few micro-Gauss at the scale $l_{B} \sim (100/r)$ kpc = 1 kpc. The seed magnetic fields formed earlier (at higher $T$) would have larger magnitude ($\sim T^{3}$) but their characteristic scale would be smaller by factor $1/T^{2}$. Chaotic line reconnection could create magnetic field at larger, galactic scale $l_{gal}$, but the magnitude of this field would be suppressed by Brownian motion law - it would drop by the factor $(l_{B}/l_{gal})^{3/2}$. It is interesting that according to these results all scales give comparable contributions at $l_{gal}$. This effect may lead to an enhancement of the field but it is difficult to evaluate the latter. Let us also note that magnetic fields generated by the discussed mechanism at the cluster scale, 10 Mpc, should be not larger than $10^{-8}$ $\mu$G if no additional amplification took place.

Larger density perturbations could be helpful for generation of larger magnetic field for which dynamo might be unnecessary. Though much bigger $\delta T$ is not formally excluded at the scale about 100 kpc, but to have them at the level $(\delta T/T)^{2} \sim 10^{-4}$ seems to be too much. A natural idea is to turn to a later stage, to onset of structure formation when $\delta \rho/\rho$ becomes larger than $10^{-2}$. With such density perturbations strong enough magnetic fields might be generated without dynamo amplification. However after recombination the
number density of charge carriers drops roughly by 5 orders of magnitude. Correspondingly $\gamma$ rises by the same amount and the strength of the seed field would be 5 orders of magnitude smaller if density perturbations and the temperature of formation remained the same. However both became very much different. Density perturbations rose as scale-factor, $(\delta \rho/\rho)^2 \sim (T_{eq}/T)^2$, where $T_{eq} \sim 1$ eV is the temperature when radiation domination changed into matter domination and density perturbations started to rise. Since, $B/T^2 \sim T^3$, according to eq. (31), the net effect of going to smaller $T$ is a decrease of $B/T^2$ which would be difficult to cure even by later reionization. Still, as argued in ref. [17], magnetic field generation, driven by anisotropic and inhomogeneous radiation pressure (and in this sense similar to our mechanism) at the epoch of reionization, could end up with the field of about $8 \cdot 10^{-6} \mu$G. This result is 8 orders of magnitude larger than that found in the earlier papers [18] and quite close to ours [31], though these two mechanisms operated during very different cosmological epochs and were based on different physical phenomena.

Generation of magnetic field at recombination was also considered in ref. [19] where a much weaker result was found. This difference can be possibly attributed to the following effects. We considered above an earlier period when the photon mean free path was much smaller than the horizon. It gives a factor about $10^3$ in fluid velocity, eq.(11). Moreover, since in our case the electrons are tightly bound to photons the electron-photon fluid moves as a whole (while protons and ions are at rest) and the electric current induced by macroscopic motion/oscillations of plasma is noticeably larger.

6 Conclusion

Despite many suggested models, the origin of galactic magnetic fields and, especially, intergalactic, if existence of the latter is confirmed, remains mysterious. One class of models is based on known physics and do not invoke any ad hoc assumptions for the explanation of the phenomenon. The explanation based on stellar ejecta possibly encounters serious difficulties because of energy constraints. The mechanism of field generation just before hydrogen recombination looks reasonably good but probably it cannot explain both galactic and intergalactic fields within the frameworks of the standard cosmology with flat spectrum of density perturbations.

Another class of models is based on physical phenomena in the early universe and its different members are spread between inflationary stage to relatively late MeV-epoch. These models manipulate with unknown physics (except possibly the MeV one) and because of that are much less restricted in their possibilities. It is a difficult task to understand what mechanism is indeed responsible for creation of the observed magnetic fields. A critical test would be a possibility of simultaneous explanation of galactic and intergalactic fields.
To this end a confirmation of a possible existence of the latter is of primary importance.

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