On the time-scale of quasifission and Coulomb fission

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Abstract. The Coulomb fission may take place in a reaction if the maximum Coulomb excitation energy transfer exceeds the fission barrier of either the projectile or the target nucleus. This condition is satisfied in all the reactions used for the earlier blocking measurements of fission time-scale except for the reaction $^{208}$Pb + natural Ge crystal, where the time-scale is below the measurement limit of the blocking technique < 1 as. Inclusion of Coulomb fission in the data analysis of the blocking experiments leads us to interpret the measured time-scales longer than a few attoseconds (as) (about 1–2.2 as) due to slow Coulomb fission and those shorter than 1 as, as due to quasifission and fast Coulomb fission. Consequently, this finding resolves the critical discrepancies between the fission time-scales measured using the nuclear and blocking techniques. This, in turn, validates the fact that the quasifission and fast Coulomb fission time-scales are indeed of the order of zeptosecond (zs) in accordance with the nuclear experiments and theories. The present results thus provide an essential input to the understanding of the fusion evaporation reaction during the formation of heavy elements.

Keywords. Quasifission; Coulomb fission; fission.

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1. Introduction

The superheavy elements (SHEs) up to the element oganesson (294$^{118}$Og) which has been discovered so far completed the periodic table up to its seventh row [1]. Currently, the synthesis of SHEs for $Z \geq 118$ has become a more significant and challenging field of nuclear research as initial experimental efforts to discover the elements $Z = 119$ and 120 of the eighth row have not yet been successful at FLNR, Dubna and GSI, Darmstadt [2–4]. The failure contradicts the expectation which arose from the observation of higher yields of the synthesised heavy nuclei suggesting increasing stability for SHEs with $Z \geq 110$ [5,6]. It may be quite possible that the evaporation residue cross-sections for $Z > 118$ are presently too low to measure. Special efforts are now being invested to revamp the detection sensitivity of evaporation residue cross-section close to a femtobarn (fb). One may further attribute a strong hindrance to SHE synthesis to other competing faster processes such as quasifission (QF) which causes a fast splitting of the composite system [7]. To understand the underlying mechanism of the QF process, a lot of effort has been put in the measurement of its time-scale [8–20] and theoretical studies [21–31]. Though the QF time-scales measured by the nuclear techniques using the mass–angle distribution [8,9,12,19], the neutron-clock method [10,18] and the giant dipole resonances [11] are in good agreement with the theoretical predictions [21–31]. They are at least two orders of magnitude lower than the values of the order of $10^{-18}$ s as measured by the atomic techniques using the crystal blocking [14–17] and X-ray fluorescence [13,20]. Such long fission time-scales can only be possible if the fission barriers of the isotopes involved in the decay chain are very high which however are in contradiction with the theoretical estimates [32,33]. Further, a recent study [34] shows that a short time-scale from nuclear techniques and a long time-scale from atomic techniques can only be reconciled by an extreme bimodal fission time distribution comprising a 53% long-lived component (up to $10^{-16}$ s) and a 47% short-lived component (up to $10^{-21}$ s), provided the neutron emissions from the accelerating fragments are ignored. However, such
an extreme bimodal distribution appears to be implausible.

In the present work, we investigate the role of Coulomb fission (CF) in the time-scale measurement by crystal blocking experiments [14–17]. The CF is the fission of a nucleus (see the review [35]) which is Coulomb-excited by the time-varying Coulomb field of another nucleus passing in close proximity of the former but staying beyond the range of strong nuclear interaction [36]. Besides the QF process, the CF may also hinder SHE synthesis as shown schematically in figure 1. The CF was first proposed in the 1960s [37] and experimentally verified in the late 1970s [38]. Significantly, the CF phenomenon has been put into application for preparing secondary beams at GSI, Germany for nuclear fission studies in inverse kinematics [39–42].

Coulomb fission (CF) in the time-scale measurement can be observed for two fragments at a distance \( d \) from the crystal row. A sample of \( \Delta E - E \) spectra of the products is also displayed in four zones corresponding to different dominant reaction mechanisms delimited by their contours in figure 2B, which is taken from [46]. These four contours (a–d) denote the events due to projectile like, heavier fission, lighter fission, and target like fragments, respectively. Axial dips can thus be observed for a particular fragment in the corresponding direction and schematically drawn along with the elastic spectrum. The deflection angle does depend on the distance \( d \); the smaller the distance \( d \) the larger the angle \( \psi \). Thus, the shape of the axial dips is related to the lifetime \( \tau \). The change in the minimum yield \([47]\) \( \delta \chi_{\text{min}} \) (see figure 2C) is written as

\[
\delta \chi_{\text{min}} = 2\pi C N a^2 \tau^2,
\]

where \( C \) is a constant \((C \approx 2–3)\), \( N \) is the atomic density of the target and \( a \) is the interatomic distance along the crystal axis. The quantity \( v_\perp \) is the recoil velocity of the compound nucleus perpendicular to the atomic row and is calculated from the kinematics of the reaction assuming complete fusion. The lifetime \( \tau \) can thus be obtained from eq. (1).

2.2 Main experimental findings

The crystal blocking technique has long been used to measure the compound nuclear and fission lifetimes in the range \( 10^{-18}–10^{-16} \) s. In recent years, this method has been extended to measure the lifetimes of heavy-ion induced fusion reactions which give rise to evaporation residues as well as nuclear fission. In what follows, we focus on the heavy-ion reactions leading to the actinides and superactinides only [14–17] and the major results obtained are listed in table 1.

2.3 Comparison with other techniques

The crystal blocking technique is sensitive in the range \( 10^{-18}–10^{-16} \) s. Several other methods are also used to infer the fission time-scales, each with a different range of sensitivity. They include (i) the neutron-clock method or neutron multiplicity measurement [10] which counts the neutrons emitted before the compound nuclei breakup into two fragments. It is sensitive to time-scales from \( 10^{-22} \) to \( 10^{-16} \) s. (ii) The mass–angle distribution of the fragments measures the sticking time of
Figure 1. Synthesis of SHEs. Various notations are as follows: P denotes the projectile, T is the target, CN is the compound nucleus, EVR is the evaporation residue and SF is the spontaneous fission. Different possible processes such as Coulomb excitation followed by Coulomb fission in the pre-capture stage, quasifission and formation of CN at the post-capture stage and fusion–fission and evaporation at post-CN stage are displayed. Finally, $\beta^-$, $\beta^+$, $\alpha$ emissions and SF decay at the post evaporation stage bring the ER to the ground state. Thus, there are three possible outcomes after capture: CF, QF and fusion. Various processes are characterised by the corresponding cross-sections and half-lives. For further details, see the text.

3. Theoretical formalism

3.1 Basics of heavy-ion reactions

Prior to the formation of either the CN or DNS, the projectile slows down once it enters within the interaction barrier zone and starts emitting electromagnetic radiation which can excite the projectile or the target nucleus, the so-called Coulomb excitation (CE) or electromagnetic excitation. The above processes (CN, DNS, CE) are governed by the nucleus–nucleus interaction potential, which includes the Coulomb, nuclear and rotational potentials. This potential along with the surface diffuseness parameter associated with the nuclear potential plays an important role in calculating various nuclear properties.

Normally, the compound nuclei are formed in excited states, which undergo evaporation of neutrons (n), protons (p) and $\gamma$-photons. The evaporation residue cross-section ($\sigma_{EVR}$) for heavy element formation via the fusion evaporation can be evaluated by the DNS model [49] as

$$\sigma_{EVR} = \sum_{\ell=0}^{\infty} (2\ell + 1)\sigma_{cap}(E^*)P_{CN}(E^*, \ell)P_{sur}^{\chi}(E^*, \ell).$$

Here $\sigma_{cap}$ is the partial capture cross-section representing the transition of the colliding nuclei over the Coulomb barrier, while $E^*$ is the excitation energy, $\ell$ is the angular momentum and $P_{CN}$ is the compound nucleus formation probability evaluated as explained in [50].
Table 1. Main findings from the crystal blocking technique using heavy-ion reactions leading to actinides or SHEs.

| Reaction     | $E_p$ (MeV) | Time-scale (as) | Ref. |
|--------------|-------------|-----------------|------|
| $^{32}\text{S} + ^{186}\text{W}$ | 170         | 2               | [15] |
| $^{32}\text{S} + ^{184}\text{W}$ | 170         | 2               | [15] |
| $^{48}\text{Ti} + ^{186}\text{W}$ | 250         | 2               | [15] |
| $^{48}\text{Ti} + ^{184}\text{W}$ | 250         | 2               | [15] |
| $^{58}\text{Ni} + ^{186}\text{W}$ | 315         | 1               | [16] |
| $^{58}\text{Ni} + ^{184}\text{W}$ | 315         | 1               | [16] |
| $^{74}\text{Ge} + ^{184}\text{W}$ | 390         | 2.2             | [16] |
| $^{238}\text{U} + ^{28}\text{Si}$ | 5712        | 1               | [14] |
| $^{238}\text{U} + ^{29}\text{Si}$ | 5712        | 1               | [14] |
| $^{238}\text{U} + ^{30}\text{Si}$ | 5712        | 1               | [14] |
| $^{238}\text{U} + ^{58}\text{Ni}$ | 1575        | 2.2             | [17] |
| $^{238}\text{U} + ^{60}\text{Ni}$ | 1575        | 2.2             | [17] |
| $^{238}\text{U} + ^{61}\text{Ni}$ | 1575        | 2.2             | [17] |
| $^{238}\text{U} + ^{62}\text{Ni}$ | 1575        | 2.2             | [17] |
| $^{238}\text{U} + ^{64}\text{Ni}$ | 1575        | 2.2             | [17] |
| $^{238}\text{U} + ^{70}\text{Ge}$ | 1450        | 2.2             | [17] |
| $^{238}\text{U} + ^{72}\text{Ge}$ | 1450        | 2.2             | [17] |
| $^{238}\text{U} + ^{73}\text{Ge}$ | 1450        | 2.2             | [17] |
| $^{238}\text{U} + ^{75}\text{Ge}$ | 1450        | 2.2             | [17] |
| $^{238}\text{U} + ^{76}\text{Ge}$ | 1450        | 2.2             | [17] |
| $^{208}\text{Pb} + ^{70}\text{Ge}$ | 1281        | < 1             | [17] |
| $^{208}\text{Pb} + ^{72}\text{Ge}$ | 1281        | < 1             | [17] |
| $^{208}\text{Pb} + ^{73}\text{Ge}$ | 1281        | < 1             | [17] |
| $^{208}\text{Pb} + ^{75}\text{Ge}$ | 1281        | < 1             | [17] |
| $^{208}\text{Pb} + ^{76}\text{Ge}$ | 1281        | < 1             | [17] |

Targets used are single crystals. The projectile energy ($E_p$) used for each experiment is also mentioned.

Table 2. Experimental methods employed to measure the time-scale of heavy-ion-induced reactions.

| Expt. method          | Sensitivity (s) | Underlying process | Ref. |
|-----------------------|-----------------|--------------------|------|
| Neutron-clock         | $10^{-22}$-$10^{-16}$ | QF, CF and FF      | [10] |
| GDR-like $\gamma$-ray | $10^{-22}$-$10^{-16}$ | QF, CF and FF      | [11] |
| MA distribution       | $10^{-22}$-$10^{-20}$ | QF only            | [8]  |
| TKE distribution      | $10^{-22}$-$10^{-20}$ | QF only            | [12] |
| Crystal blocking      | $10^{-18}$-$10^{-16}$ | CF and FF          | [14] |
| X-ray fluorescence    | $10^{-18}$-$10^{-16}$ | CF and FF          | [13] |

The first four methods are based on nuclear techniques and the last two belong to the atomic techniques. Sensitivity of a particular technique decides whether it can probe the time-scale of qF, CF or FF. MA stands for mass-angle, GDR for giant dipole radiation and TKE for total kinetic energy.

The survival probability after evaporation of $x$ neutrons is expressed as

$$P_{\text{sur}}(E^*, \ell) = P_{xn}(E^*) \prod_{i=1}^{x} \left( \frac{\Gamma_n}{\Gamma_n + \Gamma_f} \right)_{i,E^*}, \quad (3)$$

where $i$ is the running index for the emitted neutrons, $\Gamma$ is the emission width [51] and $P_{xn}$ is the neutron evaporation probability [50]. It can be evaluated in an approximate manner by assuming that the neutron energy spectrum is $\epsilon \exp(-\epsilon/T)$ and that neutron emission occurs whenever it is energetically possible. If the $ith$ neutron has a binding energy $B_i$, and the nuclear temperature is $T$, the probability that a nucleus with initial excitation $E^*$ will emit exactly $x$ number of neutrons is found to be

$$P_{xn}(E^*) = I(\Delta x, 2x - 3) - I(\Delta x + 1, 2x - 1), \quad (4)$$
Figure 2. (A) Schematic diagram of the crystal blocking technique as reported in [46]. Here \(d\) indicates the distance between the crystal row and the point where the recoiled compound nucleus breaks into two fragments, \(\alpha\) is the angle between the crystal row and the beam axis and \(\psi\) is the angle between the fission fragment direction and the initial direction of the same fragment. (B) the energy loss of the fragment ion in the transmission detector placed in the telescope vs. the residual energy of the fragment in stop detector of the telescope. (C) normalised blocking yield \(\chi_{min}\) as a function of the angle \(\psi\) for an evaporation residue (red curve) corresponding to the dominant reaction (a) of figure 2B and inelastically scattered projectiles (blue curve).

where \(I(z,n)\) is Pearson’s incomplete gamma function, \(I(z,n) = (1/n!) \int_0^z x^n e^{-x} dx\) and \(\Delta x = (E^* - \sum B_j)/T\) is the energy (in units of T) above the threshold for the emission of \(x\) neutrons.

The fusion barrier is evaluated using the following empirical relation based on the experimental data [52]:

\[
B_{fu} = -34488.7618 + 1100.6666z - 14.4066z^2 \\
+ 9.9275 \times 10^{-2}z^3 - 3.7959 \times 10^{-4}z^4 \\
+ 7.6357 \times 10^{-7}z^5 - 6.3136 \\
\times 10^{-10}z^6 \text{ for } 128 \leq z \leq 286. \tag{5}
\]

Here,

\[
z = \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}}
\]

is the Coulomb interaction parameter, where \(Z_1\) and \(Z_2\) are the atomic numbers and \(A_1\) and \(A_2\) are the mass numbers of the projectile and the target, respectively.

3.2 Fundamentals of Coulomb excitation

The Coulomb excitation processes occurs in every nuclear reaction irrespective of the beam energy as portrayed in figure 1. We can calculate the CET in terms of the energy, charge and mass number of the incident projectile, its initial and final velocities, and the excitation energy [36,53]. To start with, the final projectile velocity is given by:

\[
\frac{1}{2} m_1 v_f^2 = E_{lab} - \Delta E' \quad \text{and} \quad \Delta E' = \left(1 + \frac{A_1}{A_2}\right) \Delta E, \tag{6}
\]

where \(A_1\) and \(A_2\) are the nuclear mass numbers of the projectile and the target respectively, \(\Delta E\) is the excitation energy corresponding to a certain state of the rotational band of either the projectile or the target nucleus and \(\Delta E'\) is the excitation energy in the laboratory frame. Introducing a dimensionless parameter

\[
\chi = \frac{\Delta E}{E_{cm}} = \frac{\Delta E'}{E_{lab}}, \tag{7}
\]

we can write the final velocity as

\[
v_f = v_i (1 - \chi)^{1/2} \quad \text{where} \quad v_i = \left(\frac{2E_{cm}}{\mu_0}\right)^{1/2}. \tag{8}
\]
Here \( \mu_0 \) is the reduced mass and \( E_{\text{cm}} \) is the centre of mass energy of the projectile. Note that \( \Delta E \) values are taken from [54]. Using eqs (4) and (5), the classical symmetrised parameter [36], \( a = Z_1 Z_2 e^2 / (\mu_0 v_i v_f) \), can be written as

\[
a = \frac{Z_1 Z_2 e^2}{2 E_{\text{cm}} (1 - \chi)^{1/2}} = \frac{0.7199 Z_1 Z_2}{E_{\text{cm}} (1 - \chi)^{1/2}},
\]

(9)

where \( Z_1 \) and \( Z_2 \) are the charges of the projectile and target nuclei, respectively, \( E_{\text{cm}} \) is given in MeV and \( a \) is obtained in fm. A basic parameter \( \xi \), the so-called dimensionless adiabaticity parameter [36] is defined as the ratio of the collision time and the nuclear excitation time and is given as,

\[
\xi = \frac{0.07905 Z_1 Z_2 \mu_0^{1/2} \Delta E}{(E_{\text{cm}} - \frac{1}{2} \Delta E)^{3/2}} \left( 1 + \frac{5}{32} \left( \frac{\Delta E}{E_{\text{cm}}} \right)^2 + \ldots \right),
\]

(10)

where both \( \Delta E \) and \( E_{\text{cm}} \) are in MeV.

\[\Delta E_{\text{CET}} = \hbar c \beta \xi \gamma \frac{\nu_i v_f}{a - R} \]

(11)

where

\[
\beta = \frac{\nu_i v_f}{c}, \quad \gamma = \frac{1}{(1 - \beta^2)^{1/2}}.
\]

\( \hbar \) is the Planck constant and \( c \) is the speed of light. Further,

\[
R = \frac{1}{\sqrt{2}} \sqrt{R_1^2 + R_2^2}
\]

and

\[
R_i = 1.233 A_i^{1/3} - 0.98 A_i^{-1/3} \quad(i = 1, 2)
\]

[55] correspond to the radii of the projectile and the target nuclei respectively in fm unit. The \( \Delta E_{\text{CET}} \) in MeV unit for high energy collisions (>5 MeV/A) is

\[\Delta E_{\text{CET}} = \hbar c \beta \xi \gamma \frac{\nu_i v_f}{D - a}. \]

(12)

Here the distance of closest approach \( D \) in fm unit is written as

\[
D = \frac{a}{\gamma} \times [1 + \arcsin(\theta_{\text{cm}}^{gr})]
\]

(13)

and \( \theta_{\text{cm}}^{gr} \) is the grazing angle in radian which is given as \[56\]:

\[
\theta_{\text{cm}}^{gr} = 2 \arcsin \left( \frac{\eta}{k_{\text{c}} \times R_{\text{int}} - \eta} \right)
\]

(14)

and \( k_{\text{c}} \) is the reduced wave vector at infinite ion-target separation and it is expressed in fm units as

\[
k_{\text{c}} = 0.2187 \mu_0 \sqrt{\frac{E_{\text{lab}}}{A_1}}.
\]

Here, the Sommerfeld parameter, \( \eta = ak_{\text{c}} \) and the interaction radius in fm units,

\[
R_{\text{int}} = A_1^{1/3} + A_2^{1/3} + 4.59 - \frac{A_1^{1/3} + A_2^{1/3}}{6.35}
\]

[56]. Note that the maximum Coulomb-excitation energy transfer (MCET) is obtained by substituting \( \xi = 1 \) in eqs (11) and (12). In order to evaluate \( a \) from eq. (9), \( \chi \) is estimated by using \( \Delta E \) as obtained approximately from two neighbouring \( \Delta E \)s around \( \xi = 1 \).

### 3.3 Coulomb fission cross-section

In order to compare the experimental CF cross-sections with theoretical predictions, Bonaccorso et al [44] have performed a calculation within the Bertulani and Baur relativistic model [57] of electromagnetic dissociation with fission chosen as the exit channel. The final expression for Coulomb fission cross-section obtained in the above work is as follows:

\[
\sigma_{\text{CF}} = \int \left[ n_{E_1}(\omega) \sigma_{E_1}(\omega) + n_{E_2}(\omega) \sigma_{E_2}(\omega) \right] P_f(\omega) \frac{d\omega}{\omega}.
\]

(15)

Here \( n_{E_1}(\omega) \) is the virtual photon spectra for the electric multipole \( (E_1) \). The excitation multipolarities contributing to the above equation are only \( E_1 \) and \( E_2 \). \( \sigma_{E_1} \) and \( \sigma_{E_2} \) are the photoabsorption cross-sections for dipole and quadrupole multipolarities, respectively. \( P_f(\omega) \) is the fission probability and it is calculated using the formalism of [58]. Note that and above cross-section formula has been used by the LISE++ code [59]. The calculated CF cross-section [44] has shown good agreement with the experimental one [43].

### 3.4 Quasifission and fusion–fission lifetimes

The QF lifetimes are evaluated as follows:

\[
\tau_{\text{QF}} = \frac{1}{\lambda_{\text{QF}}},
\]

(16)

where \( \lambda_{\text{QF}} \) is the QF decay constant and is expressed as

\[
\lambda_{\text{QF}} = \frac{\omega_m}{2\pi \omega_{\text{QF}}} \left( \sqrt{\frac{\Gamma}{2\hbar}} \right) + \frac{\omega_{\text{QF}}^2}{2
\hbar} - \frac{\Gamma}{2\hbar} \right) \times \exp \left( -\frac{B_{\text{QF}}(Z, A, \ell)}{\Theta(Z, A)} \right).
\]

(17)

Here, the quantity \( \Gamma \) denotes an average width of the contributing single-particle states near the Fermi surface and normally, its value is taken as 2 MeV. The quantities \( \omega_m \) and \( \omega_{\text{QF}} \) are the frequencies of the harmonic oscillator and the inverted harmonic oscillator, respectively.
assuming that the dinuclear system undergoes oscillations before QF can occur. These quantities have been evaluated using a set of equations given in [60]. The nuclear temperature $\Theta(Z, A)$ depending on the level density parameter $\alpha' = 0.134A - 1.21 \times 10^{-4}A^2$ is given by

$$\Theta(Z, A) = \sqrt{\frac{E^*(\ell)}{\alpha'}},$$

where $E^*$ is the excitation energy imparted to the compound nucleus. The QF barrier $B_{QF}(Z, A, \ell)$ in the dinuclear system model is expressed as

$$B_{QF}(Z, A, \ell) = V(R_b, Z, A, \beta_{21}, \beta_{22}, \ell) - V(R_m, Z, A, \beta_{21}, \beta_{22}, \ell).$$

Here $\ell$ and $R = R_m$ [60] are the angular momentum and the distance at which the nucleus–nucleus potential is minimum, respectively. $\beta_{21}$ and $\beta_{22}$ are the quadruple deformation parameters. The nucleus–nucleus interaction potential is given by

$$V(R, Z_1, Z_2, \beta_{2i}, \ell) = V_C(R, Z_1, Z_2, \beta_{2i}) + V_{rot}(\beta_{2i})V_N(R, Z_1, Z_2, \beta_{2i}),$$

where $V_C$, $V_N$ and $V_{rot}$ are the Coulomb, nuclear and rotational potentials, respectively and are evaluated using a set of equations given by Soheyli and Khanlari [61]. FF time-scales are valued as follows:

$$\tau_{FF} = \frac{1}{\lambda_{FF}} = \frac{1}{2\pi} \left( \sqrt{\frac{\Gamma_0}{2\hbar}} + \omega^2 - \frac{\Gamma_0}{2\hbar} \right) \times \exp\left( -\frac{B_f(Z, A, \ell)}{\Theta(Z, A)} \right).$$

Here, $\Gamma_0 = 2$ MeV and the fission barrier $B_f$ can be taken from ref. [62].

3.5 QF cross-section

The following equation is used to evaluate the QF cross-sections [64] in the framework of DNS model:

$$\sigma_{QF}(E_{cm}, \beta_p, \alpha_2) = \sum_{\ell=\ell_f}^{\ell_d} (2\ell + 1)\sigma_{cap}(E_{cm}, \ell, \beta_p, \alpha_2) \times (1 - P_{CN}(E_{cm}, \ell, \beta_p, \alpha_2)).$$

where $\ell$ is the angular momentum, $\alpha_2$ is the different orientation angles of the symmetry axis of the deformed target-nucleus, $\beta_p$ is the quadrupole deformation of the projectile nuclei, $P_{CN}$ is the compound nucleus formation probability and $\sigma_{cap}$ is the capture cross-section which is the sum of quasifission, fusion–fission and fast fission cross-sections.

The equations given in this section have been used to calculate various quantities as listed in tables 3 and 4. Since the target is a single crystal consisting of various isotopes with natural abundances, we have considered each and every isotope as a target nucleus being bombarded by the isotopically pure projectile in the calculations given in table 3.

4. Results and discussions

Good agreement between experimental CF cross-sections [43] and the corresponding theoretical predictions [44] has been obtained. Further, the CF phenomenon has found practical implementation in the preparation of secondary beams for nuclear fission studies in inverse kinematics [39–42]. This application also reveals various features of Coulomb fission very well. It may be noted here that for a better probing of the Coulomb excitation process, a large amount of energy has to be transferred to the nuclear system. This requires a relatively short interaction time and a sufficiently large projectile velocity. Consequently, a relativistic theory [72] becomes necessary for a better treatment of the problem. Recently, a variant of such relativistic theories [73] has succeeded in describing the observed fission fragment mass distribution [74]. However, the state-of-the-art relativistic theories [72,73] are yet to be employed for the evaluation of CF cross-sections or lifetimes. Therefore, we discuss here the results obtained using the earlier theories for calculating MCET [36] and the CF cross-sections [44].

We have examined the reactions used for the blocking experiments [14–17] in table 3, where the fission barriers of the CN have been obtained using the prescriptions given in [63,75,76]. Further, the fission barrier of the heavier of either the projectile or the target nucleus ($B_{f+}$) has been calculated using the LISE++ code [59] based on the macroscopic formalism given in [63]. Notice here that the FF lifetimes ($\tau_{FF}$) [67] are $>13$ as, whereas the QF lifetimes ($\tau_{QF}$) [62] are estimated to be $<50$ zs. Thus, both the lifetimes are far from the precisely measured values (1.0–2.2 as) by the blocking technique (see table 1). Consequently the question arises: what time-scale is then measured in the blocking experiments? Comparison of the MCET with the fission barrier ($B_f$) of the heavier partner shows that the former
Table 3. Comparison of the reactions used for blocking experiments [14–17]. The reactions are specified by projectile (Proj.), target (Targ.) and projectile lab energy used ($E_p$), fusion barrier ($V_b$) [65], evaporation channel (chnl) and fission barrier of the compound nucleus ($B_{fc}$). Evaporation residue cross-section ($\sigma_{evr}$) [66], fusion–fission lifetime ($\tau_{FF}$) [67], quasifission barrier [61] and cross-section ($\sigma_{QF}$) [59], quasifission lifetime ($\tau_{QF}$) [62], maximum Coulomb excitation energy transfer (MCET), fission barrier of the heavier, either projectile or target nuclei ($B_{fh}$) [63], Coulomb fission cross-section ($\sigma_{cf}$) [59], highest spin $I$ state that can be populated and the corresponding $\Delta E_{CET}$.

| Proj. | Targ.  | $E_p$ (MeV) | $E^*$ (MeV) | $V_b$ (MeV) | Chnl | $B_{fc}$ (MeV) | $\sigma_{DNS}^{EVR}$ (pb) | $\tau_{FF}$ (as) | $\sigma_{QF}$ (MeV) | $\tau_{QF}$ (zs) | MCET (MeV) | $B_{fh}$ (MeV) | $\sigma_{cf}$ (mb) | $I$ | $\Delta E_{CET}$ (MeV) | Ref. |
|-------|--------|-------------|-------------|-------------|------|--------------|-----------------|----------------|----------------|----------------|------------|---------------|----------------|-----|----------------|------|
| $^{32}$S | $^{186}$W | 170 | 68.5 | 136.8 | 2n | 8.2 | 2.5E6 | 29.3 | 14.3 | 5.34 | 4.5 | 50.5 | 46.1 | 21.4 | 0.1 | 8 | 19.4 | [15] |
| $^{32}$S | $^{184}$W | 170 | 64.6 | 136.5 | 2n | 9.5 | 2.4E6 | 23.2 | 14.3 | 5.24 | 4.3 | 34.5 | 43.6 | 21.1 | 0.1 | 8 | 17.0 | [15] |
| $^{48}$Ti | $^{186}$W | 250 | 58.6 | 183.9 | 3n | 2.2 | 12.7 | 0.62 | 13.9 | 2.81 | 2.9 | 8.9 | 83.2 | 21.6 | 9E–2 | 4 | 11.5 | [15] |
| $^{58}$Ni | $^{186}$W | 315 | 53.5 | 233.1 | 3n | 0.6 | 90.4 | 4.4E–3 | 13.8 | 0.27 | 5.3 | 48.7 | 21.4 | 6.9E–2 | 8 | 19.4 | [15] |
| $^{238}$U | $^{28}$Si | 5712 | 522.2 | 144.2 | 6n | 1.1 | 6.4E–5 | 6.14E–6 | 32.3 | 6.25 | 1.1E–4 | 2.1 | 224.5 | 4.9 | 12.8 | 8 | 4.1 | [14] |
| $^{238}$U | $^{29}$Si | 5712 | 540.4 | 143.7 | 6n | 1.1 | 6E–5 | 7.25E–6 | 32.3 | 6.15 | 1.1E–4 | 1.9 | 242.2 | 4.9 | 12.5 | 8 | 4.2 | [14] |
| $^{238}$U | $^{30}$Si | 5712 | 556.9 | 143.2 | 6n | 0.7 | 5.6E–5 | 7.74E–6 | 31.8 | 5.89 | 1.1E–4 | 1.2 | 257.8 | 4.9 | 12.2 | 8 | 4.4 | [14] |
| $^{238}$U | $^{58}$Ni | 1575 | 87.8 | 280.3 | 4n | 0.3 | 2.9 | 9.78E–3 | 43.7 | 0.67 | 4.5 | 2.9 | 4.2 | 4.0 | 1.6 | 24 | 3.8 | [17] |
| $^{238}$U | $^{60}$Ni | 1575 | 92 | 279.1 | 4n | 0.3 | 2.1 | 2.47E–2 | 43.2 | 0.66 | 4.3 | 3.4 | 4.7 | 4.1 | 0.68 | 22 | 3.6 | [17] |
| $^{208}$Pb | $^{70}$Ge | 1281 | 69.2 | 286.3 | 3n | 0.3 | 1.7E–2 | 4.78E–6 | 41.2 | 1.4E–4 | 1.86 | 3.9 | 4.2 | 3.7 | 1.0 | 24 | 3.5 | [17] |
Table 4. Testing correctness of MCET calculation.

| Proj. | Targ. | $E_p$ (MeV) | MCET (MeV) | $B_f$ (MeV) | $I$ | $\Delta E_CET$ (MeV) | $I'$ | Ref. |
|-------|-------|-------------|------------|-------------|-----|----------------------|-----|------|
| 40Ar  | $^{232}$Th* | 158         | 6.7        | 6.6         | 16  | 5.9                  | 10  | [68] |
| 40Ar  | $^{232}$Th* | 190         | 15.3       | 6.8         | 12  | 6.5                  | 10  | [68] |
| 40Ar  | $^{238}$U*  | 190         | 14.3       | 4.7         | 10  | 4.2                  | 12  | [68] |
| 58Ni  | $^{248}$Cm* | 260         | 6.6        | 2.8         | 10  | 2.4                  | 16  | [69] |
| 136Xe | $^{248}$Cm* | 641         | 9.5        | 2.8         | 8   | 2.2                  | 22  | [69] |
| 116Sn | $^{162}$Dy* | 637         | 90.7       | 29.2        | 8   | 27.5                 | 10  | [70] |

Reactions are specified by projectile (Proj.), target (Targ.) and projectile lab energy ($E_p$), MCET, $B_f$ of either the projectile or the target nucleus, the estimated highest spin state $I$ and experimentally observed state $I'$. The Coulomb excited nucleus is denoted by *.

Figure 3. Relative Coulomb excitation cross-section ($\sigma_{CEC}$) vs. spin $I$. Theoretical calculations are done using the COULEX code [71]. (a) Data for $^{40}$Ar+$^{232}$Th reaction at 158 MeV: the maximum experimental $\sigma_{CEC}$ is found at $I = 4$, but the theoretical counterpart is at $I = 6$ and major trend is similar to each other. However, the theory predicts up to $I = 16$ ought to be observed, but not observed so because of too low cross-section to be measured for $I > 10$, (b) shows the data for $^{40}$Ar+$^{232}$Th reaction at 190 MeV: the maximum experimental $\sigma_{CEC}$ is found at $I = 4$, but the theoretical counterpart is at $I = 6$, but major trend is similar here also. (c) Data for $^{116}$Sn+$^{162}$Dy: the experiment and theory agree very well till $I = 10$, but the transitions for $I > 10$ are also observed due to the large cross-section and large mean lifetime for Coulomb fission. See further details in text.

is higher than the latter for all the cases except for the reactions $^{208}$Pb + Ge single crystal. When the condition $MCET > B_f$ is satisfied, Coulomb fission becomes energetically possible and the fission fragments being detected in the blocking measurements having 1–2.2 as the lifetime may originate from Coulomb fission. On the other hand, the MCET value for the reactions $^{208}$Pb + Ge (see table 3) are far below the fission barrier of $^{208}$Pb and consequently the CF cross-sections are found to be eight orders of magnitude lower than that of QF. Hence, no CF fragments are available in the reactions $^{208}$Pb + Ge single crystal. However, the blocking experiment [17] detects the events characterised by a time-scale $< 1$ as.

The role of thermal vibration of the atoms of the single crystals used as targets has been discussed in the past (for example, see [77] and references therein). According to a calculation [78] based on the above theory, all reactions lasting less than $1 \times 10^{-18}$ s lead to the same value of relative yield of fragments ($\Delta \chi_{\min}$) detected in the precise direction of the crystal axes. This happens because of the distance $d$ (=recoil velocity of the compound nucleus $\times$ fission time-scale) in figure 2 being less than or equal to the amplitude of thermal vibrations. In other words, the lower limit of the atomic time-scale characteristic to Coulombic interaction [79] is about 1 as where the interaction is between the fragment ions and a row of the crystal atoms. This means that any event taking place in less than 1 as, cannot be sensed by the atomic interactions. Hence, monitoring of QF time-scale of the order of zeptoseconds by any atomic methods is not possible.
This aspect will be discussed in detail in a forthcoming work. In contrast, an increasing value of $\Delta x_{\min}$ gives a measure of the time-scale specific to a certain reaction. Such characteristics are found to work satisfactorily in three different reactions studied in inverse kinematics with the blocking technique using identical detector systems [17]. These reactions are $^{208}$Pb + Ge at 6.2 MeV/u, $^{238}$U + Ni at 6.6 MeV/u and $^{238}$U + Ge at 6.1 MeV/u and possibly they lead to the formation of compound nuclei with $Z_{CN} = 114, 120$ and 124, respectively. While $\Delta x_{\min} = 0$ for the first system, $\Delta x_{\min} \neq 0$ for the other two systems. It implies that the time-scale of the first reaction is $< 1$ as, whereas the other two reactions exhibit time-scales $> 1$ as and they have been measured to be $2.2$ as.

The above mentioned observations reveal that the heavy-ion reactions leading to the formation of super-heavy nuclei have the characteristics of a variety of processes. Large time-scales measured for the reactions $^{238}$U + Ni and $^{238}$U + Ge are attributed to the long lifetime components in the scission time distributions. Such a trend is however absent in the reaction $^{208}$Pb + Ge which is ascribed to the long components possibly because of the lower number of neutrons in the CN compared to those predicted for the nearest shell closure ($N = 184$) [17]. A noteworthy point here is that although the Coulomb fission fragments are analogous to the FF as well as the QF fragments, it has not been considered in the analysis so far. If we take the CF process into account, we immediately notice the following fact when the maximum Coulomb-excitation energy transfer (MCET) is compared with the fission cross-sections. The trend however reverses at higher excitation energies [44]. Further, table 3 shows that the CF cross-section is small and the QF cross-section is large if the fission barrier of the heavier partner ($B_{f/h}$) is large irrespective of the excitation energy, e.g., $^{32}$S/$^{48}$Ti+$^{58}$Ni + $^{184,186}$W. If both the CF and QF cross-sections are large, viz., $^{238}$U+$^{58,64}$Ni+$^{70-73,75-76}$Ge, it becomes difficult to distinguish between them in a measurement. But large differences in time-scales can help in differentiating the two processes, as will be discussed later. In contrast, the CF cross-section is four orders of magnitude higher than that of the QF cross-section for the reaction $^{238}$U+$^{28-30}$Si at a very high excitation energy. Hence, the experiment mostly measures the CF fragments and thus the measured time-scales around 2.0 as can only be attributed to the CF process.

Recently, Kannan et al [80] have addressed the fission lifetime for the $^{238}$U+$^{64}$Ni system using a dynamical model which includes the contributions from the nuclear shell effects. The stability of SHEs is very much sensitive to nuclear shell effects and thus to the excitation energy. They have shown that a fission time-scale of $\geq 10^{-18}$ s is possible for this system only if the excitation energy $E^*$ is $\leq 10$ MeV. Moreover, for the excitation energy $E^*$ in the range (88–103) MeV for different isotopes in the natural Ni crystal (see table 3), the estimated time-scale is $10^{-21}$ s. Even with a modified shell damping factor (50% of its unaffected value), the fission time-scale turns out to be $< 10^{-19}$ s (see figure 6 of [80]) in the above calculation. Thus, the fission time-scale is smaller than the time-scale measurement limit of the blocking technique (1 as). Let us now discuss the possible phenomena having such small time-scale.

Oberacker et al [35] discussed the direct coupling of the electric field to the collective motions, can be called $I$ is $10^+$ but the theoretical value is $16^+$. This difference can be attributed to a too low Coulomb-excitation cross-section (CEC) for $I > 10$ to be measured in the experiment as shown in figure 3. The CEC is calculated using the Coulex code [71] and $B(E2↑)$ value of every participating nucleus is taken from NNDC site [54]. In contrast, the experimental $I$ is higher than the theoretical limit in some cases, viz., $^{58}$Ni/$^{136}$Xe + $^{248}$Cm reactions. This can happen because of the high CEC beyond fission barrier and long time-scale of the CF process (2.0 as). Note that the experimental CEC values peak at $I = 4^+$, but the Coulex code predictions occur at $6^+$ for the reactions $^{40}$Ar + $^{238}$U/232Th. However, the relative experimental CEC from $4^+$ to $10^+$ is almost a replica of the theoretical curve from $6^+$ to $12^+$. Both the QF and CF cross-sections are found to be of the order of mb. At low excitation energies, the QF cross-sections are larger than the CF cross-sections. The trend however reverses at higher excitation energies [44]. Further, the electric field to the collective motions, can be called...
as internal excitation, that results in CF and its time-scale is $10^4$ times faster than the nuclear fission. Such fast Coulomb fission (FCF) is inseparable from QF and cannot be probed by the blocking experiments as the lowest limit of this technique is 1 as. In this article, our focus is to know the origin of the measured lifetimes (1–2.2 as) by the blocking techniques. Here the criterion for separating the CF from FF and QF is considered with the timing characteristics. Since the time-scale of QF and FCF is at least less than $10^{-19}$ s, we can infer that the time-scale in the range of 1–2.2 as cannot be either QF or FCF. We have analysed the blocking experiments in terms of MCET and $B_{fh}$. Fortunately, comparison between MCET and $B_{fh}$ saves us and we have successfully analysed all the results obtained from the blocking experiments. This larger time-scale of CF is due to the fission taking place because MCET $> B_{fh}$, and it can be called external excitation. Such type of CF can be named as slow CF (SCF). Hence, CF is of two kinds, FCF and SCF.

One may argue that like CF the QF process may also have two varieties, one is fast and other is slow. However, to have the slow QF, the fission barrier has to be very high and it contradicts the known values of the fission barrier [32,33] as discussed in the introduction itself. Therefore, the measured lifetimes (1–2.2 as) by the blocking techniques may represent mostly the SCF.

The condition MCET $> B_{fh}$ is not only necessary but quite sufficient too at high beam energies to conclude that CF mostly occurs in the blocking experiments. In table 3, we have introduced the FF cross-section to compare with that of CF. However, there is no general trend among the QF, FF and CF cross-sections. The cross-sections vary from reaction to reaction. In contrast, the timing property of these three events is distinctive. We can see in the table that the QF time-scales are of the order of zeptoseconds, whereas the FF time-scale is in the range of a few tens to a few thousands attoseconds. However, the time-scale measured by the blocking experiments is only a few as. This was attributed to QF to comply with the well known fact that the FF time-scale is much longer than this. Nevertheless, such large time-scale is only possible for the QF if the fission barrier is very high and it contradicts the fact as discussed in the introduction itself. Furthermore, the blocking technique cannot go beyond 1 as, as discussed above. Hence, the QF and FCF time-scale cannot be probed by the blocking or any other atomic techniques. Therefore, the measured lifetimes (1–2.2 as) by the blocking techniques may represent mostly the SCF.

One important aspect to discuss is that there are two ways to enhance the CF signal: 1. Collision energies below the Coulomb barrier. 2. High bombarding energies with selection of most peripheral collisions. To avoid the nuclear backgrounds, either small collision energy below the Coulomb barrier or very high beam energy is used to reduce the nuclear events and thus to enhance the QF signals. Otherwise, QF and FF will prevail. On this ground, high energy was used and it caused the high value of MCET, which surpassed $B_{fh}$ and thus in addition to the FCF, the SCF was also evolved.

The QF and CF cross-sections along with the evaporation and FF cross-sections are relevant for the synthesis of SHEs. If the former cross-sections are large and consequently the latter cross-sections are small, SHE production is inhibited. The challenge is to minimise the QF and CF cross-sections and maximise the evaporation residue cross-sections. Moreover, the FF cross-section has to be minimised too and it can be avoided by choosing a CN with a large fission barrier. Hence, occurrence of all these processes QF, CF and FF can be reduced to a good target–projectile combination and appropriate beam energy for performing the reaction.

If the time-scales measured in the blocking experiments are characteristic of the SCF, the time intervals measured using the X-ray fluorescence [13,20] may also represent the same SCF process. This is a crucial point to be further investigated to decipher the fission time-scale fully. This study is now under progress.

We will discuss here an important consequence of the present work. Figure 1 shows that the three potential outcomes from pre-capture and capture are CF, QF and fusion. Further, the CF (FCF + SCF) also plays a hindering role as the QF in the synthesis of SHE since both remove flux from the entrance channel resulting in a fewer number of CN and hence a fewer number of SHEs. Thus, instead of writing the CN formation probability $P_{CN}(E^*, J)$ [81] as

$$P_{CN}(E^*, J) = (1 - P_{QF}). \quad (24)$$

it can be rather written as

$$P_{CN}(E^*, J) = (1 - P_{QF})(1 - P_{CF}). \quad (25)$$

In eq. (24) only the hindrance of QF prevails, whereas both the QF and CF are the cause of hindrance on the formation of superheavy nuclei in eq. (25).

Inclusion of eq. (25) in the dynamical models may drastically reduce $P_{CN}$ at high excitation energies when MCET supersedes $B_{fh}$, and when $\sigma_{CF}$ is either larger than $\sigma_{QF}$ or both are comparable as can be seen in table 3 for many heavy-ion reactions. Consequently, this equation will assist in planning the synthesis of a wider range of superheavy isotopes and in particular to select the beam energy such that the CF can be avoided.
5. Conclusion

CF of the projectile/target nucleus at the pre-capture stage is very significant for any nuclear reaction. Its cross-section can be comparable to that of QF occurring at the post-capture stage. Inclusion of CF has led us to satisfactorily explain the measured time-scales (∼2.0 as) by the crystal blocking technique. The present work reveals that the measured attoseconds time-scale arises due to the slow CF. On the other hand, it validates that the zeptoseconds time-scale can only belong to QF and FCF. We notice that the CF has a significant role in the CN formation probability, and consequently an experimental challenge would be to consider both the CF and QF processes on equal footing for the production of the SHEs. Any appropriate nuclear technique may be attempted to assess perhaps the time-scales of both the slow CF and QF along with the fast CF simultaneously in a single experiment.

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References

[1] W Nazarewicz, Nat. Phys. 14(6), 537 (2018)
[2] C E Dullmann, EPJ Web of Conferences 131, 08004 (2016)
[3] Y T Oganessian et al, Phys. Rev. C 79(2), 024603 (2009)
[4] S Hofmann, J. Phys. G 42(11), 114001 (2015)
[5] J H Hamilton, S Hofmann and Y T Oganessian, Annu. Rev. Nucl. Part. 63, 383 (2013)
[6] Y T Oganessian and V K Utyonkov, Rep. Prog. Phys. 78(3), 036301 (2015)
[7] W U Schoder and J R Huizenga, Treatise on heavy-ion science (Springer, 1984) pp. 113–726
[8] J Toke et al, Nucl. Phys. A 440(2), 327 (1985)
[9] W Q Shen et al, Phys. Rev. C 36(1), 115 (1987)
[10] D J Hinde, D Hilscher and H Rossner, Phys. Rev. C 45(3), 1229 (1992)
[11] J Nestler et al, Phys. Rev. C 51(4), 2218 (1995)
[12] J Velkovska et al, Phys. Rev. C 59(3), 1506 (1999)
[13] H W Wilschut and V L Kravchuk, Nucl. Phys A 734, 156 (2004)
[14] P Goldenbaum et al, Phys. Rev. Lett. 82(25), 5012 (1999)
[15] J U Andersen et al, Phys. Rev. Lett. 99(16), 162502 (2007)
[16] J U Andersen et al, Phys. Rev. C 78(6), 064609 (2008)
[17] M Morjean et al, Phys. Rev. Lett. 101(7), 072701 (2008)
[18] K Ramachandran et al, Phys. Rev. C 73(6), 064609 (2006)
[19] R du Riezr et al, Phys. Rev. Lett. 106(5), 052701 (2011)
[20] M O Fregeau et al, Phys. Rev. Lett. 108(12), 122701 (2012)
[21] K Siwek-Wilczynska, J Wiczynski, R H Siemssen and H W Wilschut, Phys. Rev. C 51(4), 2054 (1995)
[22] J Wiczynski, K Siwek-Wilczynska and H W Wilschut, Phys. Rev. C 54(1), 325 (1996)
[23] A Diaz-Torres, Phys. Rev. C 69, 021603 (2004)
[24] V Zagrebaev and W Greiner, J Phys. Nucl. Part. Phys. 31(7), 825 (2005)
[25] K P Santhosh and V Safoora, Phys. Rev. C 96(3), 034610 (2017)
[26] H C Manjunatha and K N Sridhar, Nucl. Phys. A 962, 7 (2017)
[27] H C Manjunatha and K N Sridhar, Eur. Phys. J. A 53(5), 1 (2017)
[28] K N Sridhar, H C Manjunatha and H B Ramalingam, Phys. Rev. C 98(6), 064605 (2018)
[29] H C Manjunatha, K N Sridhar and N Sowmya, Phys. Rev. C 98(2), 024308 (2018)
[30] K N Sridhar, H C Manjunatha and H B Ramalingam, Nucl. Phys. A 983, 195 (2019)
[31] H C Manjunatha, K N Sridhar and N Sowmya, Nucl. Phys. A 987, 382 (2019)
[32] A Dobrowolski, B Nerlo-Pomorska, K Pomorski and J Bartel, Acta Phys. Pol. B 40(3), 705 (2009)
[33] H C Manjunatha, Indian J. Phys. 92(4), 507 (2018)
[34] A K Sikdar, A Ray and A Chatterjee, Phys. Rev. C 93(4), 041604 (2016)
[35] V E Oberacker, W T Pinkston and H G Kruse, Rep. Prog. Phys. 48(3), 327 (1985)
[36] K Alder, A Bohr, T Haas, B Mottelson and A Winther, Rev. Mod. Phys. 28(4), 432 (1956)
[37] L Wilets, E Guth and J S Tenn, Phys. Rev. 154(4), 1349 (1967)
[38] H Backe et al, Phys. Rev. Lett. 43(15), 1077 (1979)
[39] K H Schmidt et al, Phys. Lett. B 325(3), 313 (1994)
[40] T Aumann et al, Z. Phys. A 352, 163 (1995)
[41] C Bockstiegel et al, Phys. Lett. B 398(3), 259 (1997)
[42] K H Schmidt et al, Nucl. Phys. A 665(3), 221 (2000)
[43] E Piasecki et al, Phys. Lett. B 377(4), 235 (1996)
[44] A Bonaccorso, Z Zelazny and E Piasecki, Z. Phys. Hadr. Nucl. 358(3), 329 (1997)
[45] D S Gemmell and R E Holland, Phys. Rev. Lett. 14(23), 945 (1965)
[46] M Morjean et al, Eur. Phys. J. D 45(1), 27 (2007)
[47] V Nanal, M B Kurup and K G Prasad, Phys. Rev. C 49(2), 758 (1994)
[48] A Bertulani, A Carlos and V Y Ponomarev, Phys. Rep. 321(4), 139 (1999)
[49] G Giardina, S Hofmann, A I Muminov and A K Nasirov, Eurp. Phys. J. A 8(2), 205 (2000)
[50] J D Jackson, *Can. J. Phys.* **34(8)**, 767 (1956)
[51] R Vandenbosch, *Nuclear fission* (Elsevier, 2012)
[52] T Nandi *et al*, *Pramana – J. Phys.* **96**, 1 (2022)
[53] H Wollersheim, *Acta Phys. Polon. B* **42**, (2011)
[54] https://www.nndc.bnl.gov/ensdf/.
[55] P R Christensen and A Winther, *Phys. Lett. B* **65(1)**, 19 (1976)
[56] W W Wilcke *et al*, *At. Data Nucl. Data* **25(5)**, 389 (1980)
[57] C A Bertulani and G Baur, *Phys. Rep.* **164(5)**, 299 (1988)
[58] P Grange and H A Weidenmüller, *Phys. Lett. B* **96(1)**, 26 (1980)
[59] O B Tarasov and D Bazin, *Nucl. Instrum. Methods Phys. Res. B* **376**, 185 (2016)
[60] H C Manjunatha, L Seenappa, P S Damodara Gupta, N Manjunatha, K N Sridhara, N Sowmya and T Nandi, *Phys. Rev. C* **103(2)**, 024311 (2021)
[61] S Soheyli, M Khanlari and Varasteh, *Phys. Rev. C* **94(3)**, 034615 (2016)
[62] M V Khanlari and S Soheyli, *Phys. Rev. C* **95(2)**, 024617 (2017)
[63] A J Sierk, *Phys. Rev. C* **33(6)**, 2039 (1986)
[64] A Nasirov *et al*, *Eur. Phys. J. A* **49(11)**, 1 (2013)
[65] H C Manjunatha and K N Sridhar, *Nucl. Phys. A* **971**, 83 (2018)
[66] W Reisdorf and M Schadel, *Z. Phys. A: Hadrons Nucl.* **343(1)**, 47 (1992)
[67] A K Nasirov *et al*, *Eur. Phys. J. A* **34(3)**, 325 (2007)
[68] J F Stephens, R M Diamond and I Perlman, *Phys. Rev. Lett.* **3(9)**, 435 (1959)
[69] T Czosnyka *et al*, *Nucl. Phys. A* **458(1)**, 123 (1986)
[70] C Y Wu, W V Oertzen, D Cline and M W Guidry, *Annu. Rev. Nucl. Part. Sci.* **40(1)**, 285 (1990)
[71] A Winther and J Boer, *Technical Report* (1965)
[72] I Stetcu, C A Bertulani, A Bulgac, P Magierski and K J Roche, *Phys. Rev. Lett.* **114(1)**, 012701 (2015)
[73] J Diaz-Cortes *et al*, *Phys. Lett. B* **811**, 135962 (2020)
[74] T Enqvist *et al*, *Nucl. Phys. A* **658(1)**, 47 (1999)
[75] S Cohen and W Swiatecki, *Ann. Phys.* **22(3)**, 406 (1963)
[76] C Wong, *Phys. Rev. Lett.* **31(12)**, 766 (1973)
[77] D S Gemmell, *Rev. Mod. Phys.* **46(1)**, 129 (1974)
[78] M Morjean *et al*, *EPJ Web of Conferences* **63**, 02011 (2013)
[79] I P Christov, R Bartels, H C Kapteyn and M M Murnane, *Phys. Rev. Lett.* **86(24)**, 5458 (2001)
[80] M T Kannan *et al*, *Phys. Rev. C* **98(2)**, 021601 (2018)
[81] D J Hinde, M Dasgupta and A Mukherjee, *Phys. Rev. Lett.* **89(28)**, 282701 (2002)