Comments on Initial Value Formulation

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Abstract. This is the reply given at the conference “Mach’s Principle” at Tübingen in July 1993 to the paper by Isenberg (1993a).

1. On Principles

Isenberg’s (1993) proposal is remarkable not least because it is intended to cover not one or the other aspect of Machian ideas, but a complete formulation of Mach’s Principle. Isenberg gives cogent reasons why the Wheeler-Einstein-Mach (W-E-M) program expresses the important Machian demands, and it is hard to see how it could be improved as a general program, particularly since Isenberg added the nonextendibility requirement, giving a link between Mach’s principle and cosmic censorship.

Isenberg also considers Mach’s principle in the larger context of principles in physics. In this general context, Mach’s principle is somewhat unusual: It cannot easily be disproven, because we know few if any effects that are unequivocally anti-Machian (for example, Ozvath and Schücking 1962). By contrast, the most useful principles in physics naturally have a negative or interdictory aspect. For example, the uncertainty principle forbids certain variables from being simultaneously well-defined, the energy principle forbids perpetual motion, the equivalence principle denies distinction between gravity and inertia, the atomic principle excludes infinite divisibility, and so on. Such a formulation is not only heuristically useful (for example, it saves us from useless speculation about impossible situations) but it can also point the way toward progress in the theory: a negative principle implies a challenge, to find the mechanism or rationale behind the prohibition, and can lead to a new theory in which the principle is automatic, and no longer needs to be stated explicitly.

At first sight the W-E-M principle looks like business as usual (we still do classical general relativity in a way that current lingo might associate with STINO*), and gives no direct motivation to change the theory. The implication is different if we state it as a negation: No spacetime can fail to satisfy the four W-E-M requirements. But of course there are solutions of the classical Einstein equations that are not W-E-Machian. Hence the challenge is to find the mechanism that excludes the offending spacetimes. Thus the W-E-M principle also points the way beyond classical general relativity to new and certainly as yet unfinished business.
2. On Inertia

Many, like Einstein, find something fascinating about the idea that in inertia we feel the rest of the universe at work, and look to Mach’s principle for the real origin of inertia. Does the W-E-M-Isenberg approach finish that business, of formulating the principle? Isenberg tells us that if we know the full spacetime metric near a point, we know all there is to know about inertial frames at that point. In Shimony’s (1992) comparison, you enter Mach’s Store looking on the shelves for various useful and fascinating gadgets, many of them somehow connected with inertia. But in the W-E-M store you find only a general do-it-yourself kit from which you might be able to build your own gadgets. How much more effort is required to build the gadgets we care about out of the W-E-M kit? Let us consider some of the “gadgets” that other authors in this volume might hope to find in Mach’s store.

Prof. Pfister might care about the inertial frame dragging. Suppose we consider a point inside Pfister’s shell. We know the metric there — it is flat. But this knowledge does not tell us all there is to know about the dragging as usually understood (Brill and Cohen 1966, Lindblom and Brill 1974). A true answer about inertia and inertial frames must involve specific frames or coordinates. The W-E-M principle, being a child of general relativity, tends to be hostile to picking out a particular frame — the really significant information is considered to be frame-independent. “Frame not included” is written on the packages in the W-E-M store; but is this not one of the things we expect to get from Mach, not to put into it?

Prof. Raine, whose own formulation of Mach’s principle has been questioned concerning the distinction between matter and gravitational waves, might ask of the W-E-M principle whether there is really a crucial difference between the following two situations: an otherwise closed W-E-M universe containing either a black hole formed by collapse of matter, or an eternal Kruskal black hole, with an asymptotically flat region on the “other side” of the horizon. The former would be called W-E-Machian, and the latter would not, because its $\Sigma^3$ is not compact. But this distinction is not reasonable: since the difference can be extremely small between the physical regions on “this side” of the horizon, and since one cannot look behind a horizon, the Machian nature of a spacetime would be something that could never be ascertained by experiment. Perhaps the attribute Machian should apply to regions in spacetimes, for which it does not matter what happens behind horizons.

If we allow this extension of the W-E-M principle we can treat the following situation, which is more amusing than profound. Suppose Prof. Narlikar asked the question that has a definite answer in his formulation: what is the smallest number of masses in a W-E-Machian $S^3$ universe that is free from other content such as gravitational waves? Suppose we take the absence of wave content to mean that the free data can be chosen to be trivial, and the presence of mass to mean that $n$ asymptotically flat regions behind (apparent) horizons are allowed. Since asymptotically flat regions are conformally equivalent to taking points out of the $S^3$, an appropriate choice for Isenberg’s first set $(\Sigma^3, \lambda, \sigma)$ is $(\mathbb{R}^3$ less $(n - 1)$ points, flat, 0). For $n = 1$ the only regular solution for the Lichnerowicz conformal factor $\phi$ is $\phi = \text{constant}$, which is flat spacetime without horizon, hence without Machian region. For $n = 2$ the solution is $\phi = 1 + \frac{M}{2r}$, with $r = \text{Euclidean distance in } \mathbb{R}^3$ from
the removed point. This is just the single-mass Schwarzschild solution with one horizon, which does not bound a compact Machian region. So one mass is not enough. For \( n = 3 \) we have \( \phi = 1 + \frac{M_1}{r_1^2} + \frac{M_2}{r_2^2} \), which is asymptotically flat in three regions, at \( r_1 \to \infty \), at \( r_2 \to \infty \), and at \( (r_1 \text{ and } r_2) \to \infty \). For small \( M_1, M_2 \) there are only two horizons, not bounding a Machian region. But if \( M_1 \) and \( M_2 \) are chosen large enough (compared to their Euclidean distance), there can be another apparent horizon surrounding the two (Brill and Lindquist 1963). A Machian region then exists between these three horizons. Thus three masses is the answer by this extended W-E-M principle, not unreasonable because three masses usually do define a frame. (Unfortunately in this particular construction they do not, because the solution is rotationally symmetric about an axis through the original \( M_1, M_2 \). In this sense the answer is not better than Narlikar’s two-mass minimum.)

3. On Details

Examples such as the above suggest that the W-E-M principle leaves some room for further refinement. This appears particularly urgent in connection with the distinction between the “first” and “second” set of Cauchy data. The first set should contain variables that can be freely chosen; but in Isenberg’s examples it consists of a \( TT \) tensor \( \sigma \) and transverse fields \( \beta \) and \( \eta \). Because of such transversality requirements these fields are really not free, but themselves subject to constraints. Would it then not be simpler to choose as the first set any constraint-satisfying initial data, so that the second set is empty? If it is allowed to demand transversality of the first set, then why not constraint-satisfaction? Isenberg (1993b) suggests that the former condition is linear and does not essentially restrict free choice, whereas the latter condition is non-linear and implements the Machian determination of the inertial frames. This interpretation itself would of course constitute a (small) refinement of the W-E-M principle, a refinement motivated by a possible physical meaning of the splitting into first and second sets.

Refining the physical meaning of the decomposition of data into the first and second set seems a promising task. It could have interesting physical significance if a particular decomposition were demanded, not just the existence of some decomposition (one of possibly many). For example, in the Lichnerowicz-York decomposition, the vector \( W \) itself does not appear in the “Machian” constraints, only \( LW \) occurs. Perhaps this (or some other, even more Machian) decomposition can give an appropriate, general definition of the frame dragging by means of a vector like \( W \) (which may be related to the shift vector, a coordinate quantity of the type needed really to describe inertia).

It would seem unusual to find that a formulation, one of whose authors is Wheeler, could benefit from greater emphasis on physical meaning, but such are the conclusions to which we are led.

Footnote

*STINO, from stinknormal, name of new popular music phenomenon in Germany, celebrating traditional melodies and folk songs. Perhaps such labels can help us gain recognition in the lay public. (What attention the no-hair theorems might have received if they were identified with skinheads!)
References

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