BLACK HOLE SPIN EVOLUTION: IMPLICATIONS FOR SHORT-HARD GAMMA-RAY BURSTS AND GRAVITATIONAL WAVE DETECTION

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ABSTRACT

The evolution of the spin and tilt of black holes in compact black hole–neutron star and black hole–black hole binary systems is investigated within the framework of the coalescing compact star binary model for short gamma ray bursts via the population synthesis method. Based on recent results on accretion at super critical rates in slim disk models, estimates of natal kicks, and the results regarding fallback in supernova models, we obtain the black hole spin and misalignment. It is found that the spin parameter, $a_{\text{spin}}$, is less than 0.5 for initially nonrotating black holes and the tilt angle, $i_{\text{tilt}}$, is less than 45° for 50% of the systems in black hole–neutron star binaries. On comparison with the results of black hole–neutron star merger calculations we estimate that only a small fraction (~0.01) of these systems can lead to the formation of a torus surrounding the coalesced binary potentially producing a short-hard gamma ray burst. On the other hand, for high initial black hole spin parameters ($a_{\text{spin}} > 0.6$) this fraction can be significant (~0.4). It is found that the predicted gravitational radiation signal for our simulated population does not significantly differ from that for nonrotating black holes. Due to the (1) insensitivity of signal detection techniques to the black hole spin and the (2) predicted overall low contribution of black hole binaries to the signal we find that the detection of gravitational waves are not greatly inhibited by current searches with nonspinning templates. It is pointed out that the detection of a black hole–black hole binary inspiral system with LIGO or VIRGO may provide a direct measurement of the initial spin of a black hole.

Subject headings: binaries: close — black hole physics — gamma rays: bursts — gravitational waves

Online material: color figure

1. INTRODUCTION

Double compact object binaries have attracted much attention in recent years as primary sources of gravitational wave radiation (GR) and as potential sites for the short-hard gamma ray burst (GRB) phenomenon. As such, intensive searches for the inspiral signal from double neutron stars (NS-NS), double black holes (BH-BH), and mixed black hole–neutron star (BH-NS) systems are currently underway at ground based GR observatories (e.g., LIGO or VIRGO; for a recent review see Kalogera et al. 2007). Of these systems, the mergers of NS-NS and BH-NS systems have been suggested to produce short-hard GRBs (for a recent review see Nakar 2007). However as new NS-NS binaries are discovered (e.g., Lorimer 2005), there has yet to be a detection of a BH-NS or BH-BH system.

Despite the accumulation of observational data on short-hard GRBs from the HETE-2 and Swift satellites, the origin of these GRBs still remains elusive. Theoretical predictions of the compact merger model have been compared to observations in the hope of identifying the possible progenitor (e.g., Nakar et al. 2006; Belczynski et al. 2006), but some specific cases of the model are still lacking. For example, to assess the validity of BH-NS merger as a short-hard GRB progenitor, it is usually assumed that all these mergers produce a GRB. However, recent hydrodynamical simulations (e.g., Setiawan et al. 2004; Faber et al. 2006; Shibata & Uryu 2007; Rantsiou et al. 2008) indicate that only a fraction of these binaries can potentially produce a GRB. Specifically, the existence of a thick torus of material is required such that it can be rapidly accreted onto the central BH to produce a GRB. To satisfy this constraint, the NS must be disrupted prior to its final plunge below the BH event horizon with sufficient angular momentum such that the remnant material can form an accretion torus. In this case, only compact systems with specific parameters (spin, mass ratio, and BH spin tilt with respect to the orbital angular momentum axis) can provide the requisite initial conditions for GRB production. Accordingly, we report on the results of our investigation of the spin magnitude and tilt in merging BH-NS binaries to estimate the fraction of BH-NS systems that can potentially produce short-hard GRBs.

The spin of the BH in BH-BH and BH-NS binaries is of special interest in the search for gravitational wave signatures in existing data streams from ground-based observatories. On the one hand, population synthesis studies (e.g., Fryer et al. 1999; Nelemans et al. 2001; Belczynski et al. 2007) attempt to provide realistic merger rates and the characteristic properties of the merging binaries, while, on the other hand, detailed general relativistic calculations of mergers (e.g., Baker et al. 2007; Buonanno et al. 2007) attempt to predict the exact shape of the GR signal. These latter studies are essential for guiding the search for the inspiral signal that currently involves the use of a limited number of precalculated gravitational wave templates (e.g., Abbott et al. 2005, 2006). It is well known that if the spin of a BH is significant and if it is misaligned with respect to the orbital angular momentum axis, the shape of GR signal will differ drastically from the nonspinning/aligned case (Apostolatos et al. 1994; Kidder 1995; see also § 2.5). Yet, so far the search methods presented in the literature (e.g., Abbott et al. 2005, 2006) employ nonspinning templates only.
The previous studies of BH spins in compact binaries in the context of BH-NS systems were carried out by Kalogera (2000), Granden et al. (2004), and O’Shaughnessy et al. (2005). The spin-up of the BH via binary evolutionary processes was estimated, and it was found that accretion in the common envelope (CE) phase was a major contributing factor in significantly spinning up the BHs. These early results are reassessed and extended here since (1) now only very few BH-BH progenitor systems are predicted to evolve through the CE phase (Belczynski et al. 2007), (2) for systems evolving through the CE phase the Bondi-Hoyle accretion mode may overestimate the accretion (and spin-up) rates (see § 2.2), and (3) during the stable mass transfer phases the degree of spin-up requires reevaluation in view of the possibility that the BH can accept mass at rates significantly exceeding the Eddington accretion rate (e.g., Abramowicz et al. 1988; Ohsuga et al. 2005; Ohsuga 2007). Since the predictions of compact object spin misalignment depend critically on the natal kick distribution, we make use of the most recent work on natal kicks by Hobbs et al. (2005). Finally, we also extend previous studies to include the double black hole binary population.

In § 2, the various elements of our model are described. The results of our BH spin calculations are presented in § 3. Finally, in § 4 we conclude and discuss the implications of our findings.

2. MODEL

2.1. Population Synthesis Model

Binary population synthesis is used to calculate the populations of close BH-NS and BH-BH binaries that merge within 10 Gyr. The formation of double compact objects is modeled via binary evolutionary processes in the absence of stellar dynamical processes (e.g., such as in the cores of globular clusters). The formation of BH-BH systems in dense environments was recently studied by Portegies Zwart & McMillan (2000), O’Leary et al. (2006), and Sadowski et al. (2008). No relevant studies are available for BH-NS binaries, as their formation rate in dense environments is probably negligible or small, as was predicted for NS-NS systems (e.g., Phinney 1991; Windl et al. 2006).

Our population synthesis code, StarTrack, was initially developed to study double compact object mergers in the context of GRB progenitors (Belczynski et al. 2002a) and gravitational wave inspiral sources (Belczynski et al. 2002a). In recent years StarTrack has undergone major updates and revisions in the physical treatment of various binary evolution phases, and especially the mass transfer phases. The new version has already been tested and calibrated against observations and detailed binary mass transfer calculations (Belczynski et al. 2008a) and has been used in various applications (e.g., Belczynski & Taam 2004; Belczynski et al. 2004, 2005, 2006, 2007). The physics updates that are most important for compact object formation and evolution include: a full numerical approach for the orbital evolution due to tidal interactions, calibrated using high-mass X-ray binaries and open cluster observations, a detailed treatment of mass transfer episodes fully calibrated against detailed calculations with a stellar evolution code, updated stellar winds for massive stars, and the latest determination of the natal kick velocity distribution for neutron stars (Hobbs et al. 2005). For helium star evolution, which is of a crucial importance for the formation of double neutron star binaries (e.g., Ivanova et al. 2003; Dewi & Pols 2003), we have applied a treatment matching closely the results of detailed evolutionary calculations. If the helium star fills its Roche lobe, the systems are examined for the potential development of a dynamical instability, in which case they are evolved through a CE phase; otherwise, a highly nonconservative mass transfer ensues. We treat CE events using the energy formalism (Webbink 1984), where the binding energy of the envelope is determined from the set of He star models calculated with the detailed evolutionary code by Ivanova et al. (2003). In case the CE is initiated by a star crossing the Hertzsprung gap (HG) we assume a merger and abort further binary evolution. This is due to the fact that there is no clear core-envelope boundary (and no entropy jump as for more evolved stars) in the interior structure of HG donors to facilitate the formation of a remnant binary system. As a consequence, a large decrease in the formation efficiency of close double compact binaries results (Belczynski et al. 2007). For a detailed description of the revised code we refer the reader to Belczynski et al. (2008a).

Since the study of Belczynski et al. (2007) there was an update of rejuvenation treatment for main sequence stars, and the method presented by Tout et al. (1997) was introduced as described in detail in Belczynski et al. (2008a; see their § 5.6). The resulting changes for close BH-NS binaries are negligible. For close BH-BH binaries we note ~40% drop in coalescence rates, a drop that is insignificant compared to over 2 orders of magnitude uncertainty in the model rates (Belczynski et al. 2007).

2.2. BH Accretion Model: Spin Magnitude

The pioneering studies of accretion onto a BH began with the seminal work of Shakura & Sunyaev (1973) and Thorne (1974). In this study, we employ the formalism presented by Brown et al. (2000) to describe the spin evolution of a BH as taken from the energy and angular momentum derived from the Kerr metric (Boyer & Lindquist 1967). The Boyer-Lindquist coordinates allow for a continuous transformation across the double horizons of the Kerr metric, preserving the essential singularity at $r = 0$, and also allowing us to address the stress-energy tensor inside the BH. As such, the angular momentum is calculated for the entire mass of the BH resulting in the following evolution equations.

The BH spin parameter is defined as

$$ a_{\text{spin}} = \frac{J_{\text{BH}}}{M_{\text{BH}} G} \tag{1} $$

where $M_{\text{BH}}$ denotes the BH mass, $J$ its angular momentum, and $G$ and $c$ are the gravitational constant and the speed of light, respectively. The angular momentum, $I$, and energy, $E$, of the accreted material with rest mass $M_{\text{rest}}$ can be expressed as

$$ I = \left[ R_{\text{iso}}^2 - a_{\text{sp}} \sqrt{2 R_{\text{BH}} R_{\text{iso}} + a_{\text{sp}}^2} \right] \left[ R_{\text{iso}} (R_{\text{iso}}^2 - \frac{1}{2} R_{\text{BH}} R_{\text{iso}} + a_{\text{sp}} \sqrt{2 R_{\text{BH}} R_{\text{iso}}})^{1/2} \right] \times c \sqrt{R_{\text{BH}} R_{\text{iso}}} \frac{R_{\text{iso}}}{2} M_{\text{rest}}, \tag{2} $$

$$ E = \left[ R_{\text{iso}}^2 - R_{\text{BH}} R_{\text{iso}} + a_{\text{sp}} \sqrt{2 R_{\text{BH}} R_{\text{iso}}/2} \right] \left[ R_{\text{iso}} (R_{\text{iso}}^2 - \frac{1}{2} R_{\text{BH}} R_{\text{iso}} + a_{\text{sp}} \sqrt{2 R_{\text{BH}} R_{\text{iso}}})^{1/2} \right] \times c^2 M_{\text{rest}}, \tag{3} $$

where $R_{\text{BH}} = 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius of a BH, $a_{\text{sp}} = J/M_{\text{BH}} c = a_{\text{spin}} (GM_{\text{BH}}/c^2)$, and the last stable orbit radius is calculated from

$$ R_{\text{iso}} = \frac{R_{\text{BH}}}{2} \left\{ 3 + z_2 - [(3 - z_1)(3 + z_1 + 2z_2)]^{1/2} \right\}, \tag{4} $$

where $z_1 = \frac{2}{3} (1 - a_{\text{sp}}) R_{\text{BH}}$, $z_2 = \frac{1}{6} a_{\text{sp}}^2 R_{\text{BH}}$, and $R_{\text{iso}}$, $R_{\text{BH}}$, and $z_1$, $z_2$ are evaluated at the last stable orbit radius.
with
\[
\begin{align*}
z_1 &= 1 + \left( 1 - \frac{4a_{sp}^2}{R_{BH}} \right)^{1/3} \left[ \left( 1 + \frac{2a_{sp}}{R_{BH}} \right)^{1/3} + \left( 1 - \frac{2a_{sp}}{R_{BH}} \right)^{1/3} \right], \\
z_2 &= \left( 3 \frac{4a_{sp}^2}{R_{BH}} + z_1^2 \right)^{1/2}.
\end{align*}
\]

The accretion of \( M_{\text{rest}} \) onto a BH changes its gravitational mass to
\[
M_{BH,f} = M_{BH,i} + \frac{E}{c^2},
\]
where the indices \( i \) and \( f \) correspond to the initial (preaccretion) and final (postaccretion) values. The accretion of \( M_{\text{rest}} \) onto a BH changes its angular momentum to
\[
J_f = J_i + l,
\]
where the initial angular momentum is obtained from equation (1) \( J_i = \alpha_{\text{spin},i} M_{BH,i} G/c \), and the new BH spin is calculated from
\[
\alpha_{\text{spin},f} = \frac{J_f c}{M_{BH,f}^2 G}.
\]

Since the BH spins at formation are unknown, we perform our calculations for a wide range of the initial values of spin parameter, including the nonspinning BH case (\( \alpha_{\text{spin},i} = 0 \)) as well as rapidly rotating BH cases (\( \alpha_{\text{spin},i} > 0.9 \)). We refer to case of \( \alpha_{\text{spin},i} = 0 \) as low spin, \( \alpha_{\text{spin},i} = 0.55 \) as moderate spin, and \( \alpha_{\text{spin},i} = 0.9 \) as high spin.

Given the mass transfer rates obtained from the population synthesis, we make use of calculations of supercritical accretion flows around BHs to estimate the mass accretion rate (e.g., Abramowicz et al. 1988; Ohsuga et al. 2002, 2005; Ohsuga 2007). Recently, Ohsuga (2007) demonstrated that photon trapping in slim accretion disk models is important, allowing accretion rates significantly exceeding the critical value (\( M_{\text{crit}} \), see below). In particular, for a flow rate (e.g., transfer rate from a donor star \( M_{\text{don}} \)) of \( \sim 1000 \) \( M_{\text{crit}} \), matter was accreted at a rate at the level of \( M_{\text{acc}} \sim 100 \) \( M_{\text{crit}} \), with the remaining matter lost in disk outflow due to strong radiation pressure effects. For lower flow rates (\( M_{\text{don}} \leq 100 \) \( M_{\text{crit}} \)), the disk undergoes limit cycle oscillations. We fit\(^6\) the recent results presented by Ohsuga (2007; see his Fig. 4) to obtain a prescription for the mass accretion rate onto a BH as follows

\[
\log \left( \frac{\dot{M}_{\text{acc}}}{M_{\text{crit}}} \right) = \begin{cases} 
\log \left( \frac{|\dot{M}_{\text{don}}|}{M_{\text{crit}}} \right) & |\dot{M}_{\text{don}}| \leq M_{\text{crit}} \\
0.544 \log \left( \frac{|\dot{M}_{\text{don}}|}{M_{\text{crit}}} \right) & M_{\text{crit}} < |\dot{M}_{\text{don}}| \leq 10M_{\text{crit}}, \\
0.934 \log \left( \frac{|\dot{M}_{\text{don}}|}{M_{\text{crit}}} \right) & |\dot{M}_{\text{don}}| > 10M_{\text{crit}}, \\
-0.380 & |\dot{M}_{\text{don}}| > 10M_{\text{crit}},
\end{cases}
\]

\(^6\) The mass accretion rates onto a BH were presented only graphically in Ohsuga (2007).

where we adopt \( M_{\text{crit}} = 2.6 \times 10^{-8} (M_{\text{BH}}/10 \, M_{\odot}) \) \( M_{\odot} \) yr\(^{-1}\) from Ohsuga (2007).\(^7\) At the lowest mass transfer rates \( M_{\text{don}} \ll M_{\text{crit}} \), it is assumed that all the matter transferred is accreted by the black hole. It is noted that the accretion limits adopted from Ohsuga (2007) are based not on full GR calculations, but on calculations performed using a gravitational potential for nonrotating black holes developed by Paczynsky & Wiita (1980). We have also adopted the Ohsuga (2007) results for the viscosity parameter of \( \alpha = 0.5 \). The introduction of black hole spin in the Ohsuga (2007) calculations would tend to decrease the limiting accretion rate. The effect would be especially important for rapidly rotating black holes. Therefore, our results provide upper limits on accretion and mass accumulation on black holes in binary systems.

In the case of a NS accretor, the accretion is limited to the critical Eddington accretion rate. The critical Eddington rate for NS is taken to be \( 1.7 \times 10^{-8} M_{\odot} \) yr\(^{-1}\) in case of hydrogen accretion and \( 2.9 \times 10^{-8} M_{\odot} \) yr\(^{-1}\) in case of heavier element accretion.

The above prescriptions (BH and NS accretion) are adopted for the case of dynamically stable Roche lobe overflow (RLOF). The transferred material that is not accreted onto the compact object is assumed to be ejected from the system, carrying away the specific orbital angular momentum of the compact object. The subsequent change in the binary orbit is readily obtained (e.g., Belczynski et al. 2008a).

In the case of mass transfer proceeding on a dynamical timescale, a CE phase will develop. The amount of mass accreted during this phase, denoted as \( \Delta M_{\text{acc}} \), is taken to be given by

\[
\Delta M_{\text{acc}} = f_{\text{CE}} \Delta M_{\text{Bondi}}
\]

where \( \Delta M_{\text{Bondi}} \) is the amount of mass accreted if accretion proceeds at the Bondi-Hoyle rate, and \( f_{\text{CE}} = 0.1 \) is a scaling factor adopted for both neutron star and black hole accretors (e.g., Ricker & Taam 2008). Since the matter within an accretion cylinder is characterized by density and velocity gradients, accretion is not spherically symmetric. The numerical calculations of Ruffert (1999) confirm that the accretion rate can be significantly lower than the Bondi-Hoyle rate. We use a numerical approach, presented in Belczynski et al. (2002b, see their Appendix), to estimate \( \Delta M_{\text{Bondi}} \). For a NS accretor, the amount of mass accreted is sufficient to mildly reheat the pulsar (e.g., Zdunek et al. 2002; Jacoby et al. 2005). This particular approach results in a surprisingly good agreement between the observed and predicted masses of pulsars in Galactic double neutron star systems (Belczynski et al. 2008b). As only part of the donor envelope is accreted onto the compact object, the remainder is ejected at the expense of the change in orbital energy. We use the standard energy balance (e.g., Webbink 1984) in which fully efficient energy transfer from the orbit to the envelope (\( \alpha_{\text{CE}} = 1 \)) is assumed to calculate the change of orbital separation.

An example of the spin magnitude evolution for a BH with initial mass of \( 10M_{\odot} \) is illustrated in Figure 1. In the upper panel, we consider the BH spin evolution for several different mass transfer rates \( M_{\text{don}} = 1, 10, 100, 1000 \) \( M_{\odot} \) yr\(^{-1}\), where \( M_{\text{crit}} = 2.6 \times 10^{-8} \) \( M_{\odot} \) yr\(^{-1}\) is the critical accretion rate for a 10 \( M_{\odot} \) BH. For an initially nonspinning BH (i.e., \( \alpha_{\text{spin}} = 0 \)), it is clear that a prolonged RLOF phase (\( \sim 200 \) Myr) is required to significantly spin-up the BH (\( \alpha_{\text{spin}} \geq 0.9 \)) if the mass transfer proceeds at the critical rate (\( M_{\text{crit}} \)). If mass transfer is as high as \( 100M_{\text{crit}} \), then a shorter time (\( \sim 1–10 \) Myr) is required to significantly increase BH spin. In the lower panel of Figure 1 we display the BH spin-up as a function of accreted mass (e.g., in CE phase) for the three

\(^7\) Ohsuga (2007) expresses critical mass flow rate in terms of \( L_{\text{Edd}}/c^2 \), where \( L_{\text{Edd}} \) is the critical Eddington luminosity and \( c \) is the speed of light.
different initial BH spin values of \( \alpha_{\text{spin}} = 0, 0.55, \) and 0.9. It is easily seen that if the BH initial spin is small/moderate, a significant amount of mass must be accreted (\( \geq 4 - 7 M_\odot \)) to increase the spin to high values (\( \alpha_{\text{spin}} \geq 0.9 \)).

2.3. Supernova Explosion Model: Spin Tilt

To estimate the degree of misalignment (tilt) of the BH spin axis relative to the orbital angular momentum axis of the binary system, we assume no tilt initial conditions. That is, (1) the spin axes of both stars in the binary system have spins that are parallel to the orbital angular momentum axis, and (2) once a compact object is formed in a core collapse supernova explosion the spin direction is preserved (i.e., the BH spin preserves the same direction as the spin of collapsing star). In this case, the tilt results from the change of the orbital plane due to the natal kick the compact object receives in the core collapse supernova explosion.

We assume that the distribution of natal kicks is isotropic and the magnitude is obtained from a single Maxwellian with \( \sigma = 265 \text{ km s}^{-1} \) (Hobbs et al. 2005) modified in the following way

\[
V_{\text{kick}} = (1 - f_b) V_1
\]

where \( V \) is the kick magnitude drawn from the Hobbs et al. (2005) distribution, and \( f_b \) is a fallback parameter, i.e., the fraction (from 0 to 1) of the stellar envelope that falls back onto the compact object. For a NS compact object, no fallback is assumed (energetic SN explosion) and full kicks are applied (\( f_b = 0 \)). On the other hand, for the most massive BHs, formed silently (no SN explosion) in a direct collapse (\( f_b = 1 \)) of a massive star to a BH, it is assumed that no natal kick is imparted. This would occur for the most massive stars (initial, zero-age main sequence stars with masses \( \geq 40 M_\odot \); Fryer 1999; Fryer & Kalogera 2001) that form massive BHs (\( M_{\text{BH}} \geq 9 M_\odot \); Belczynski et al. 2008a).

Lower mass BHs are formed accompanying a SN explosion; however, only a fraction of the progenitor envelope is ejected and the rest is retained by the newly formed BH. In these cases the kick is decreased by an amount dependent on the expected fall back of mass (for more details see Belczynski et al. 2008a).

The effect of the supernova explosion on the binary orbit is followed in the general case of eccentric orbits. In particular, we chose a random position on the orbit where the explosion takes place and use evolutionary formulæ to estimate the mass of the compact object. The mass ejection is assumed to be spherical, and the expelled material is removed, carrying the specific orbital angular momentum of the exploding star. The newly formed compact object receives the natal kick, changing the direction and magnitude of its velocity around its companion star. If the resulting orbit is unbound, the binary evolution is terminated, and the two stars are followed as single objects. However, for a bound orbit the orbital parameters are recalculated to include the inclination of the orbit. Here, the change of the orbital inclination is equal to the change in direction of the compact object spin with respect to the orbital angular momentum direction. In the case of double compact objects a progenitor system experiences two core collapse supernova events and the respective tilts of the first- and second-born compact object are then given by

\[
\begin{align*}
\theta_{\text{tilt,1}} &= \Delta \theta_{\text{SN1}} + \Delta \theta_{\text{SN2}}, \text{ first born}, \\
\theta_{\text{tilt,2}} &= \Delta \theta_{\text{SN1}} + \Delta \theta_{\text{SN2}}, \text{ second born},
\end{align*}
\]

where \( \Delta \theta_{\text{SN1}} \) and \( \Delta \theta_{\text{SN2}} \) denote the relative change of the orbital inclination in a first and a second core collapse supernova event, respectively. In previous studies (Kalogera 2000; Grandclement et al. 2004; O’Shaughnessy et al. 2005) only BH-NS systems were considered and only the spin evolution of the BH was followed.\(^8\)

It is assumed that the spin of the first-born compact object (BH) is aligned with orbit at the time of second core collapse supernova event (i.e., \( \theta_{\text{tilt,1}} = \Delta \theta_{\text{SN2}} \)). This assumption is based on the fact that a mass transfer episode that occurs between two supernova phases, will tend to align the spin of a BH with orbital angular momentum axis. Alignment for the progenitor of second-born compact object can also occur in the same manner during a mass transfer episode between the two core collapse supernova events when the progenitor filling its Roche lobe is subject to strong tidal interactions. For this case, it is likely that \( \theta_{\text{tilt,2}} = \Delta \theta_{\text{SN2}} \) for BH-NS systems and for some BH-BH binaries. We point out that this assumption may not be justified in evolutionary scenarios of BH-NS formation where only very little mass (\( \sim 0.5 M_\odot \)) is accreted onto a massive BH (\( \sim 10 M_\odot \)), or in the case of BH-BH formation where mass transfer does not occur between the two core collapse supernova events (Belczynski et al. 2007, see their Table 1, model A). We shall discuss the effect of the above assumptions on the spin tilt in the formation of double compact object binaries in §2.4 and 2.5.

2.4. Short-Hard GRB Model

The mergers of BH-NS binaries have been proposed to give rise to short-hard GRBs. A common ingredient in such theoretical

\(^8\) The spin of a NS is small compared to that for a BH in BH-NS binaries with a massive BH (e.g., see Kalogera 2000, and references therein). However, we follow the spin of both components since we also include BH-BH systems in this study.
models is the requirement of a thick torus surrounding the BH. The subsequent accretion of matter in the torus releases the gravitational potential energy required to power the GRB. Setiawan et al. (2004) suggest that $n$-annihilation in a low density funnel above the BH along its spin axis can deposit energy at a rate of $\sim 10^{50}$ erg s$^{-1}$, accounting for a total energy release of some $10^{49}$ erg for the case of a torus characterized by a mass $\approx 0.1 M_\odot$ and a high viscosity orbiting a BH with spin $a_{\text{spin}} \sim 0.6$. Recent calculations of BH-NS mergers carried out using a general relativistic treatment by three independent groups (Shibata & Uryu 2007; Faber et al. 2006; Rantsiou et al. 2008) have explored the outcome of a merger event as a function of mass ratio, equation of state for the NS, and the misalignment of BH spin with respect to the orbital angular momentum axis.

For BH-NS mergers with a BH in the mass range of $\sim 3$–$4 M_\odot$, Shibata & Uryu (2007) conclude that the disruption of a NS by a low-mass BH will lead to the formation of a low-mass disk ($\sim 0.1 M_\odot$) around the BH, which could potentially power a short-hard GRB. The formation of a massive disk was not found in their simulations, leading them to conclude that systems with massive disks ($\sim 1 M_\odot$) cannot be formed in BH-NS mergers with a nonrotating BH. Similar results were found by Faber et al. (2006), who employed a fully relativistic treatment in their simulations of BH-NS mergers for low mass ratio ($q = 0.1$) systems. In particular, most of the infalling NS mass is accreted promptly onto the BH, with a fraction ($\sim 25\%$) of NS remaining bound in the form of a disk.

In contrast to these two groups, who assumed a nonspinning BH, Rantsiou et al. (2008) investigated the effect of the BH spin angular momentum and the BH spin misalignment for BH-NS mergers with mass ratio $q \sim 0.1$ (i.e., for a $15 M_\odot$ BH). They found that both BH spin and their tilts play an important role in the outcome of the merger. Specifically, only for high BH spins ($a_{\text{spin}} > 0.9$) and tilts in the range of $20^\circ$–$40^\circ$, can the merger result in the ejection of significant fraction (up to $40\%$) of NS mass, part of which will remain bound to form a thick torus of mass $\sim 0.1 M_\odot$ around the BH.

A key issue for the outcome of a BH-NS merger is the relative position of the innermost stable circular orbit (ISCO), denoted by $R_{\text{ISCO}}$, of the BH and the disruption radius (or tidal radius $R_{\text{tid}}$) of the inspiraling NS. Disruption must occur well outside the ISCO, for otherwise, the entire NS plunges below the event horizon of a BH, leaving no material to initiate a GRB. Therefore, a necessary condition for the production of a GRB is $R_{\text{tid}} > R_{\text{ISCO}}$. However, this is not a sufficient condition, since (1) the disrupted material must form an accretion torus around BH and (2) there must be sufficient material ($\sim 0.01$–$0.3 M_\odot$; e.g., Ruffert & Janka 1999) in the torus to power a GRB. To provide for sufficient material, $R_{\text{tid}} \gtrsim 2R_{\text{ISCO}}$. The high inclination orbits ($i_{\text{incl}} \gtrsim 40^\circ$–$90^\circ$) are excluded since material not accreted by the BH in these mergers is ejected and does not form an accretion torus around the BH. We note that the position of the ISCO not only depends on the BH spin, but also on its tilt with respect to the orbital angular momentum vector. Furthermore, the relative position of $R_{\text{tid}}$ and $R_{\text{ISCO}}$ depends on the mass ratio of the NS to BH. Figure 2 illustrates the dependence and provides insight on the binary parameters that could allow for the formation of a disk/torus around the BH. Based on Figure 2 and the results of the available simulations (see above) for BH-NS mergers some constraints on the characteristics of BH-NS binaries can be placed on those mergers that could potentially power a short-hard GRB. Table 1 summarizes our criteria for a short-hard GRB production from BH-NS mergers.
The ISCO for a particle in an orbit of inclination \( i \) around a BH of mass \( M \) and arbitrary Kerr parameter \( a_{\text{sp}} \) (defined after eq. [3]) is found by solving the set of equations

\[
R = 0 = R' = R''
\]  

(13)

where \( R \) is defined by

\[
\Sigma^2 \left( \frac{dr}{d\tau} \right)^2 = [E(r^2 + a_{\text{sp}}^2) - a_{\text{sp}} L_z]^2 - \Delta [r^2 + (L_z - a_{\text{sp}} E)^2 + Q] \equiv R,
\]

(14)

with \( \Sigma = r^2 + a_{\text{sp}}^2 \cos^2 \theta \) and \( \Delta = r^2 - 2Mr + a_{\text{sp}}^2 \) (Carter 1968; Hughes 2000). The condition \( R = 0 \) defines a circular orbit, an orbit of constant radius \( r_0 \). The condition on the first derivative \( R' = 0 \) means that the particle’s radial acceleration is also zero at \( r_0 \) as required for an orbit to remain circular. A condition on the second derivative \( R'' = 0 \) in general guarantees that the orbit is stable (actually determining that the effective potential has a minimum at that point) and if specifically \( R'' = 0 \) then the orbit has the smallest allowed radius. In the above equation \( E \) and \( L_z \) are the conserved energy and conserved angular momentum per unit rest mass, respectively, and \( Q \) is the Carter constant. One can numerically find the ISCO-inclination dependence for orbits around a BH of given \( a_{\text{sp}} \) by varying the orbit’s inclination \( i \) [defined as \( \cos (i) = L_z/(L_z^2 + Q)^{1/2} \)] from \( 0^\circ \) to \( 180^\circ \) and solving equation (13) for \( L_z, e, \) and \( r \).

### 2.5. Gravitational Inspiral Signal

The gravitational wave signal resulting from an inspiral of a stellar mass compact object binary (BH-NS or BH-BH) can be detected by ground-based interferometric detectors, such as LIGO or VIRGO. These signals are strongly affected by the presence of nonparallel spin of the compact object with respect to the orbital angular momentum direction in the binary, mostly due to the orbital precession that is induced by the spin-orbit interaction. In Figure 3 we present a comparison of the gravitational wave signals for spinning and nonspinning BHs. The inspiral is presented in terms of amplitude strain \( h \) (relative test mass shift) at the output of a given detector, defined by a linear combination of two independent polarization states of the gravitational wave \( (h_+, h_\times) \) convolved with the interferometric antenna pattern (e.g., Apostolatos et al. 1994).

Figure 3 shows a noiseless waveform shape in the time domain, as it would be detected by one of the LIGO detectors and the VIRGO detector. The signal is detectable once the gravitational wave frequency enters the detector band at 40 Hz for LIGO or 30 Hz for VIRGO until the binary reaches the ISCO. The calculation was performed for a binary consisting of a 10 \( M_\odot \) BH and a 1.4 \( M_\odot \) NS, at a distance of 30 Mpc. For reference, the top panel shows the signal for the nonspinning case, while in the bottom panels, the BH is characterized by \( a_{\text{spin}} = 0.1 \) and 0.3 with a misalignment angle between spin and orbital angular momentum taken to be \( i_{\text{ini}} = 35^\circ \). Here, the NS is assumed to have negligible spin angular momentum. The waveforms were created in the 1.5 post Newtonian approximation for the phase and Newtonian amplitudes based on a simple precession model (Apostolatos et al. 1994) to describe the effect of spin. The conversion of the global signal to the local signal for each detector was performed using the network routines of the Monte Carlo code developed by Röver et al. (2006).
Fig. 3.—Gravitational radiation inspiral signal ($h$–GR wave strain) of BH-NS binary with 10 $M_\odot$ BH and 1.4 $M_\odot$ NS at a distance of 30 Mpc. The signal was calculated for LIGO and VIRGO detectors for nonspinning case (unrealistic) and low-spin cases with $a_{\text{spin}} = 0.1, 0.3$, and moderate tilt of $i_{\text{tilt}} = 35^\circ$. Note that as compared with a nonspinning case there is only a small difference in signal for $a_{\text{spin}} = 0.1$, and that the difference becomes more pronounced at higher spins $a_{\text{spin}} = 0.3$. However, only a very few black holes ($\sim 1\%$) in BH-NS systems accrete enough to be spun-up to attain spins above $a_{\text{spin}} = 0.3$ (provided that black holes were initially nonspinning; see top of Fig. 5).
phase. Had a full Bondi-Hoyle accretion rate been adopted, the accreted mass would have increased to $\sim 1 M_\odot$. Even such an amount is insufficient to significantly increase the spin of a massive BH (see Fig. 1).

The distribution of final BH spins are presented in Figure 5 for the three different initial conditions: nonspin ($a_{\text{spin}} = 0$), moderate spin ($a_{\text{spin}} = 0.55$), and high spin ($a_{\text{spin}} = 0.9$). It can be seen that, in all cases, BHs in BH-NS binaries increase their spins through accretion. For the initially nonspinning case the average final spin is $a_{\text{spin}} = 0.07$, for moderate rotators $a_{\text{spin}} = 0.59$, while for initially rapidly rotating BHs we obtain $a_{\text{spin}} = 0.91$. The amount of spin-up decreases with the initial value of spin, as it is more difficult to increase the spin of rapidly rotating objects. However, the robust conclusion may be reached in case of BH-NS binaries, that independent of initial conditions, it is expected that all BHs are spinning. Although accretion can only enhance the spin to a rather limited degree, some BHs can spin rapidly provided they were born with high spin.

The characteristic properties of close BH-NS binaries are illustrated in Figure 6. Most of the systems host massive BHs with masses $M_{\text{BH}} \sim 10 M_\odot$, leading to the rather extreme mass ratio in these systems ($q = M_{\text{NS}}/M_{\text{BH}} \sim 0.14$), as most of neutron stars have a mass $M_{\text{NS}} \sim 1.3 M_\odot$.

The origin of the high-mass BHs in our population is due to binary evolutionary effects which preferentially select the formation of high-mass BHs in these systems. Specifically during their evolution most of the BH-NS progenitors proceed without any major interaction until the occurrence of the first supernova explosion. The more massive primary does not initiate a mass transfer episode, but for most cases (98% see “acc1” and “acc2” channels in Table 2) the less massive secondary is involved in such a phase. This evolution is related to the high mass ratio of the progenitor systems and is connected to the dependence of the rate of mass loss from stellar winds on the mass of the star. For example, neutron stars are formed from the stars with initial masses of $\sim 10$–20 $M_\odot$, while black holes require higher initial masses. If the primary is in the low initial mass range of BH formation ($\sim 20$–70 $M_\odot$), then the BH receives a significant kick that tends to disrupt the binary. However, if the primary is more massive (≥70 $M_\odot$), BH formation proceeds through a direct collapse and does not disrupt the binary. Such massive stars lose their hydrogen-rich envelopes very early in their evolution and become naked helium stars, leading to the result that they never attain large radii ($R_{1,\text{max}} \leq 200$–300 $R_\odot$). However, the secondary stars (the progenitors of NSs) do not lose their hydrogen-rich envelopes and they increase in size with time. Eventually, during the core helium burning phase, the stars may reach radii ($R_{1,\text{max}} \geq 300$–400 $R_\odot$) sufficient for them to fill their Roche

### Table 2

**Accretion History for BH Binary Progenitors**

| Name       | Efficiency | Formation History | Mass Accreted | $a_{\text{spin,init}}$ |
|------------|------------|-------------------|---------------|-------------------------|
| acc1 (BH-NS) | 73%        | SN1-CE-SN2        | 0.10          | 0.0                     |
| acc2 (BH-NS) | 26%        | SN1-CE-RLOF-SN2   | 0.28          | 0.0                     |
| acc3 (BH-NS) | 0.5%       | SN1-RLOF-SN2      | 0.79          | 0.0                     |
| acc4 (BH-NS) | 0.5%       | SN1-SN2           | No accretion  | 0.0                     |
| acc5 (BH-BH) | 70%        | SN1-SN2           | No accretion  | 0.0                     |
| acc6 (BH-NS) | 28%        | SN1-CE-SN2        | 0.14          | 0.0                     |
| acc7 (BH-BH) | 2%         | SN1-RLOF-SN2      | 0.73          | 0.0                     |

* The evolutionary history after the first supernova (SN) that forms a BH is listed. Mass accretion may occur in the common envelope (CE) and/or during stable Roche lobe overflow (RLOF) phase.

* Average accreted mass listed. For the full distribution see Figs. 4 and 8.
lobes, thereby initiating a CE phase that leads to orbital shrinkage and to the formation of the tight BH-NS system. The CE phase itself can also favor the selection of high-mass BHs since higher mass BHs increase the probability for CE survival due to the greater orbital energy available for ejection of the common envelope. Hence, the high-mass BHs found in the BH-NS binaries are the combined result of the initial high mass ratio and the common envelope evolution operating on the progenitor systems.

The tilt distribution of BHs is also presented, revealing a drop off with the increasing tilt, with about ~50% of the systems characterized by rather low-to-moderate tilts ($i_{\text{tilt}} < 45^\circ$). Here, we have assumed that both supernovae contribute to the tilt of a BH (see eq. [12]). That is, the mass transfer phase between the occurrence of supernovae (found for the vast majority of BH-NS progenitors) does not lead to alignment of the BH spin with respect to the orbital angular momentum. Had we allowed for such the alignment, the results would not change significantly: the distribution of tilts would look very similar, but with tilts shifted slightly to lower values (i.e., ~50% of systems with $i_{\text{tilt}} < 40^\circ$). Most of the BHs (~65%) in BH-NS binaries are formed directly (with no natal kick), while the majority of the remaining BHs are formed with a small kick (see § 2.3). Therefore, the first SN explosion (forming BH) does not induce a large (if any) change of orbital inclination. The tilts mostly originate from the second SN explosion, in which the NS is formed (full natal kick). Only a small percentage (~10%–15%) of systems have tilts that are very small ($i_{\text{tilt}} < 5^\circ$) independent of the potential alignment between the two supernova events.

Given the results of our BH spin calculations presented in Figures 5 and 6, in combination with the GRB formation criteria listed in Table 1, the fraction ($f_{\text{GRB}}$) of BH-NS mergers that can potentially produce a short-hard GRB can be estimated. This fraction is shown as a function of the initial BH spin in Figure 7. For low initial BH spins ($a_{\text{spin}} < 0.5$), only a very small fraction ($f_{\text{GRB}} \sim 1\%$) of BH-NS mergers can potentially produce a GRB, while for high initial spins ($a_{\text{spin}} > 0.6$) the fraction becomes significant ($f_{\text{GRB}} \sim 40\%$). The transition occurs for intermediate BH initial spins ($a_{\text{spin}} \sim 0.55$). In Table 1 we have identified the three separate GRB formation criteria for BHs in the various mass ranges and list the specific fractions of BH-NS systems that satisfy the criteria in Table 3. The fractions are presented for the three representative initial BH spins ($a_{\text{spin}} = 0.0, 0.55, and 0.9$). The vast majority (92%) of BHs fall within the pop2 group ($M_{\text{BH}} = 7–11 M_\odot$), while only a small fraction of BHs (8%) fall within pop1 (see also Fig. 6). Note that no BHs are formed in close BH-NS binaries with mass over 11 $M_\odot$ (at solar metallicity).

### Table 3

| Criterion | $M_{\text{BH}}$ | $q$ | $i_{\text{tilt}}$ | $a_{\text{spin}}$ |
|-----------|----------------|----|------------------|------------------|
| $a_{\text{spin}} = 0$ | | | | |
| pop1 | 0.08 | 0.02 | 0.01 | 0.01 |
| pop2 | 0.92 | 0.86 | 0.41 | 0 |
| pop3 | | | | |
| $a_{\text{spin}} = 0.55$ | | | | |
| pop1 | 0.08 | 0.02 | 0.01 | 0.01 |
| pop2 | 0.92 | 0.86 | 0.41 | 0.06 |
| pop3 | | | | |
| $a_{\text{spin}} = 0.9$ | | | | |
| pop1 | 0.08 | 0.02 | 0.01 | 0.01 |
| pop2 | 0.92 | 0.86 | 0.41 | 0.41 |
| pop3 | | | | |

*Note.—The fraction of BH-NS mergers that satisfy the GRB criteria presented in Table 1 are listed. Imposition of the subsequent criteria (on BH mass, mass ratio, tilt, and spin magnitude) reduces the total fraction.*

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Fig. 7.—Fraction of BH-NS mergers that can produce a short-hard GRB according to the criteria presented in § 2.4. Note the strong dependence on the assumed initial BH spin; only a very small fraction (~1%) of the mergers can produce GRB for low initial spins, while significant fraction (~40%) is found for high initial BH spins. [See the electronic edition of the Journal for a color version of this figure.]

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![Graph showing distribution of BH mass, mass ratio, and tilt of BH spin in close BH-NS binaries for a model with the moderate initial BH spin ($a_{\text{spin}} = 0.55$). Note that the high masses of BHs result in an extreme mass ratio distribution and that moderate tilts dominate ($i_{\text{tilt}} < 40^\circ$ for ~50% of systems).](image1)

![Graph showing distribution of BH mass ratio (q) and tilt ($i_{\text{tilt}}$) for BH-NS binaries with moderate initial BH spin.](image2)

![Graph showing distribution of BH mass ratio (q) and tilt ($i_{\text{tilt}}$) for BH-NS binaries with high initial BH spin.](image3)
so there are no systems in pop3. Consider the dominating population pop2, where the majority (86%) of systems also satisfy the mass ratio criterion \((0.13 < q < 0.2)\). By imposing the requirement on the BH tilt \((<40°)\) the fraction of potential GRB candidates is reduced to \(\sim 0.41\). To determine the estimated rate for short hard GRBs, a criterion on the final BH spin must be imposed. For this last constraint, we have shown that the final spin depends sensitively on the initial BH spin (with a moderate increase from binary accretion). Since the spin must be quite large to produce a GRB \((a_{\text{spin}} > 0.6)\), systems with low initial spins do not satisfy the spin criterion and no GRBs are predicted for pop2 \((f_{\text{GRB}} = 0\%)\). If, on the other hand moderate-to-large initial spins are assumed, the fraction can be as high as \(f_{\text{GRB}} = 6\% - 41\%\). The fraction, \(f_{\text{GRB}} = 1\%\), for low initial BH spins marked in Figure 7 originate from the small number of BH-NS systems fulfilling the criteria in group pop1. Note that these criteria limit only the BH mass, mass ratio and tilt, but are independent of BH spin. The contribution from group pop1 is small because only a very small fraction of BH-NS systems are predicted to host low-mass BHs \((2.5 - 5 M_\odot)\).

### 3.2. BH-BH Binaries

Similar calculations to the BH-NS binaries were performed for the BH-BH accretion histories are listed for BH-BH progenitors in Table 2 as well. It is found that most \((70\%)\) of these progenitors do not evolve through a CE or a RLOF phase after the first BH was formed. These systems are moderately wide \((\sim 700 - 900 R_\odot)\) and eccentric \((0.1 - 0.2)\) after the first supernova explosion. The massive secondary \((\sim 50 M_\odot)\) evolves losing most of its mass in a stellar wind finally forming a helium star \((\sim 20 M_\odot)\) that continues to lose mass until the time of the second supernova explosion \((\sim 10 M_\odot)\). In this case, the secondary never fills its Roche lobe. If the secondary collapses to form a second BH, such a system would be too wide to coalesce within 10 Gyr. However, for a small to a moderate natal kick (see § 2.3) associated with the explosion, the kick may be sufficiently large to induce an extremely high eccentricity for some systems \((\sim 0.99)\), decreasing the size of the orbit to \(\sim 400 - 500 R_\odot\). With the two massive black holes \((\sim 6 - 10 M_\odot)\); see Fig. 10, below) such a system will lose sufficient angular momentum through emission of gravitational radiation to coalesce within 10 Gyr (e.g., Peters 1964).

A significant, but small, percentage \((28\%)\) of the progenitor systems result in the accretion onto the BH during a CE phase. The average mass accreted onto the BH \((0.14 M_\odot)\) is slightly higher than for BH-NS progenitors \((0.10 M_\odot)\) as BH-BH progenitors are more massive, and a larger mass reservoir is available during the CE phase. Only a very small percentage \((2\%)\) of systems evolves through a phase of RLOF where accretion occurs onto the BH. The distribution of accreted masses is presented in Figure 8.\(^{11}\)

In Figure 9 the distribution of BH spins is illustrated for BH-BH binaries for several different assumptions on the initial spin. Since only the first-born BH can evolve through an accretion phase, these results only apply to these BHs. Only a small fraction \((0.3)\) evolve through the accretion phase increasing their spins, while the remaining majority and the second-born BHs remain at their initial spin. For the initially nonspinning case, the average final BHs spin is \(a_{\text{spin}} = 0.020\), whereas for moderate

\(^{11}\) We note that only very few systems are studied \((a_{\text{spin}})\) of close BH-BH binaries out of which only small fractions are encountering CE/RLOF between two supernova events. Therefore, our distributions are burdened with large statistical errors, and are presented to give an approximate range of the accreted mass rather than a detailed shape. The number of BH-BH binaries cannot be easily increased as these systems are very rare (see Belczynski et al. 2007), and the population presented here is the result of the evolution of \(40 \times 10^6\) massive binaries \((\sim 50\%\) of the Galactic disk), a calculation that consumed about 2 months on 50 fast processors.
and high initial spin the final average BH spins are \(a_{\text{spin}} \sim 0.561\) and \(\geq 0.904\), respectively. The average BH spin-up in BH-BH binaries is smaller than for BH-NS systems due to (1) much smaller fraction of accreting BHs in BH-BH binaries, and (2) the fact that BHs in BH-BH binaries accrete mostly through CE, while significant fraction of BHs in BH-NS binaries accrete additionally (to CE) in RLOF, thus enhancing their spin-up. Note also, that the highest attainable spins for BHs in BH-BH binaries are close to their initial spins.

The physical properties of merging BH-BH binaries are shown in Figure 10. In the upper panel, the mass distribution for the first-born and second-born BH are displayed separately. The average mass of the first-born BH (\(\sim 6.7 M_\odot\)) is smaller than the second-born BH (\(\sim 8.3 M_\odot\)). This is a result of the initial mass transfer phase encountered for many BH-BH progenitors. This mass transfer occurs when the secondary is still on main sequence, and the evolved primary transfers mass and rejuvenates the secondary. Rejuvenation leads to an increase of the secondary core mass, to supply sufficient energy for the ejection of the envelope. The evolved BH then becomes a BH-BH progenitor.

Comparison to Earlier Work

The results of our study differ from the recent work of O’Shaughnessy et al. (2005), also based on the StarTrack code, and reflect the introduction of additional input physics and a different implementation of the code. In particular, O’Shaughnessy et al. (2005) find that the masses of BHs in BH-NS binaries range from \(2-15 M_\odot\) and that the accretion of mass onto BHs can amount to as much as \(\sim 5 M_\odot\) (see their Fig. 1). This leads to a significant increase in the BH spin independent of the initial BH spin (see their Figs. 2 and 3). We note that the majority of the BHs in BH-NS binaries in O’Shaughnessy et al. (2005) are of low mass, starting as heavy NSs (\(\leq 2 M_\odot\)) that accreted sufficient mass to exceed the maximum NS mass limit (adopted to be \(2 M_\odot\)). In addition, the full Bondi-Hoyle accretion onto the compact objects (both NS and BH) in the CE phase was adopted. In contrast, a higher NS mass limit (\(2.5 M_\odot\)) is adopted here, guided by the recent NS mass estimates (e.g., \(\sim 2.7 M_\odot\)) that were used in O’Shaughnessy et al. (2005). This large difference stems from the combination of the following. Only 10% of the accretion in the CE phase is assumed, guided by the hydrodynamical simulations of Ruffert (1999) and recent estimates of Ricker & Taam (2008) and noting that only a modest accretion takes place during the CE phase (see below). The combination of these two effects depletes O’Shaughnessy et al. (2005) BH-NS population by factors of \(\sim 4\)–\(5\) leaving most systems with high-mass BH (i.e., extreme mass ratios, as observed in our current study). The highest amount of accretion onto a BH in the CE phase found here is \(\sim 0.2 M_\odot\) (see Fig. 4 or Fig. 8), while it reaches \(\sim 5 M_\odot\) in O’Shaughnessy et al. (2005). This large difference stems from the combination of the following. Only 10% of the accretion in the CE phase is assumed, guided by the hydrodynamical simulations of Ruffert (1999) and recent estimates of Ricker & Taam (2008) and noting that only a modest accretion is permitted in the CE phase if one is to reproduce the observed NS mass spectrum (Belczynski et al. 2008b). In addition, we have only considered a CE model with a high efficiency of envelope ejection \((\alpha_{CE} = 1)\); e.g., see Webbink (1984) while O’Shaughnessy et al. (2005) use an entire spectrum of efficiencies \((\alpha_{CE} = 0\,–\,1)\). It is to be noted that for typical BH-NS progenitors evolving through the CE phase, a change of the efficiency from our adopted value \((\alpha_{CE} = 1)\) to much smaller values of 0.1 and 0.01 leads to a factor of 2 and 4 increase in the amount of accreted mass onto the BH, respectively. This is due to a fact that for smaller CE efficiencies the compact object sinks further into the donors envelope, resulting in greater accretion of mass, to supply sufficient energy for the ejection of the envelope. We note that very small CE efficiency values \((\alpha_{CE} \sim 0.01)\) tend to eliminate (through CE mergers) most of NS-NS progenitors, and lower the NS-NS predicted merger rates below the empirically estimated rates. Finally, O’Shaughnessy et al. (2005) allow for a wide range of input parameters (such as decreased stellar
winds, or fully conservative mass transfer) that eventually lead to more massive donors at the time of the CE phase. This provides a larger mass reservoir for accretion onto the BH; however, this effect is less significant than the other two effects mentioned above.

4. DISCUSSION

The evolution of the progenitors of close (coalescing) BH-NS and BH-BH binaries has been investigated with particular emphasis on the spin and tilt of the BH members in these systems. The results of our population synthesis have been used to estimate the fraction of BH-NS mergers that can potentially produce a short-hard GRB within the context of the coalescing binary model for these GRBs. It is found that close BH-NS systems form with rather massive BHs ($\sim 10 M_\odot$), resulting in a mass ratios typically of the order of 0.14. Accretion onto BHs in the progenitors of close BH-NS binaries leads to a small increase in the BH spin. Thus, the final BH spin is only a weak function of binary accretion and primarily depends on its unknown initial value. The misalignment of the BH spin with respect to the direction of the binary angular momentum is found to be moderate ($\leq 45^\circ$ for 50% of systems) and is mainly induced by the second SN explosion that forms the NS in the system.

By combining our results with the recent hydrodynamical calculations of BH-NS mergers, we estimate the fraction of these mergers which can potentially produce a short-hard GRB. It is found that only a very small percentage ($\sim 1\%$) of BH-NS mergers can produce GRB if the initial BH spins are small ($a_{\text{spin}} < 0.5$). This percentage increases to $\sim 40\%$ if the initial BH spins are high ($a_{\text{spin}} > 0.6$). We point out that an estimate of a GRB rate originating from BH-NS mergers should take into account the reduction factor of the order of $\sim 2.5 - 100$ compared to previous estimates. Given this reduction factor to the already low BH-NS merger rate the observed short-hard GRBs are difficult to reconcile with the BH-NS merger scenario only.

In our calculations we have used a uniform distribution for the direction of the natal kicks. However, some recent studies have pointed out that there may exist NS spin–kick velocity correlation that would favor polar kicks (along spin axis) over uniformly distributed kicks (e.g., Rankin 2007; Willems et al. 2008; Postnov & Kuranov 2008). The effect of polar kicks on BH-NS progenitors is twofold: (1) it reduces the number of BH-NS systems as polar kicks tend to disrupt binaries more easily than uniform kicks, and (2) the tilt angle (on average) is larger for surviving binaries, as the NS is kicked straight out of its orbital plane. Both of the above decrease the number of potential BH-NS GRB progenitors, further supporting our conclusion that these systems are unlikely to be responsible for the majority of short-hard GRBs.

The spin evolution of BHs in close BH-BH binaries was also investigated. In this case, only a fraction (0.3) of BHs in these systems slightly increase their spins due to an accretion phase in the binary progenitor. This small fraction is due to the fact that many BH-BH systems do not experience a mass transfer episode after the first BH is formed. As described in §3.3, almost all BHs in close BH-NS binaries increase their spin (even for initially nonspinning BHs, the average final BH spin is found to be $a_{\text{spin}} \sim 0.1$). However, the change of spin is rather insignificant and does not drastically alter the shape of the GR inspiral signal. As illustrated in Figure 3, a small spin magnitude ($\sim 0.1$) with a moderate tilt to the orbital angular momentum vector does have a large effect on the inspiral waveform. Only for higher spins ($\geq 0.3$) can the waveform be significantly altered. However, only a very small fraction ($\sim 1\%$) of initially nonspinning BHs can attain such spins (see Fig. 5).

Currently (Abbott et al. 2005, 2006), nonspinning templates are used in the search for an inspiral signal in LIGO data streams. Grandclement et al. (2004) estimated that the loss of BH-NS inspiral detection due to use of nonspinning templates cannot be greater than $\sim 30\%$. As this estimate employed the possibility that all BHs in BH-NS binaries may have misaligned spins, and as we have demonstrated that (1) misalignment is rather moderate and (2) accretion does not significantly increase BH spin, this loss estimate should be treated as an upper limit. For BH-BH binaries the potential detection loss is expected to be even smaller, as most of these binaries are predicted to have BH spins aligned with the orbital spin, and the accretion is insignificant in BH spin evolution. Although the detection of BH-NS and BH-BH binaries is not significantly affected by the use of nonspinning templates, the parameter estimation for a detected system should incorporate techniques that allow for spinning and misaligned BHs. Our estimates of the BH spin can be used as a guide for the initial conditions in hydrodynamical and detailed relativistic simulations of the BH-BH and BH-NS mergers and their expected gravitational wave signature. In addition, BH spin is an important parameter required in the estimation of the gravitational radiation recoil produced by the merger of two spinning black holes (see Baker et al. 2007, and references therein).

Finally, we point out that the measurement of BH spin for the inspiraling BH binary, can yield a value of initial BH spin, because the majority of BH-BH binaries are mostly unaffected by accretion spin-up. This may be the most direct way to infer the initial BH spin. We note, however, that BH-BH binaries are predicted to be very rare in field populations and may be difficult to observe as only $\sim 2$ detections per year for advanced LIGO (Belczynski et al. 2007) are expected. If significantly greater numbers of BH-BH binaries are detected, then they most probably originate from dynamical formation in globular clusters (Sadowski et al. 2008). In this case a large misalignment (formation through tidal capture/exchange) for BH-BH binaries is possible.

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