Square-rebar corrosion-induced cover cracking and its time prediction for historical reinforced concrete buildings in China

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ABSTRACT
Square-rebars have been used in many historical reinforced concrete buildings worldwide. The durability and structural behavior of such historical buildings should be evaluated regularly and accurately. The prediction of the corrosion-induced cracking time is a critical element for the durability evaluation work. This study demonstrates a mathematical model for the corrosion-induced cover cracking for square-rebars and its time prediction method. The research methodology includes theoretical derivation and experimental validation. First, a simplified model that describes the corrosion-induced cover cracking for square-rebars is proposed, and then, Faraday’s law is combined with the model to develop a time prediction model for corrosion-induced cover cracking. Finally, an accelerated corrosion experiment for square-rebars is performed to validate the applicability and accuracy of the prediction model.

1. Introduction
The durability of reinforced concrete (RC) buildings is directly affected by the corrosion of the steel rebars. The expanded corrosion products can apply pressure to the concrete around the steel rebars, resulting in the development of stress in the concrete, the cover of which eventually cracks. The generation of a crack can gradually reduce the carrying capacity of the component. Therefore, the time for the conservation and protection of RC buildings is usually determined by the corrosion-induced cover cracking (Liu and Weyers 1998; Vu, Stewart, and Mullard 2005).

In order to assess the serviceability limit status of RC buildings, it is necessary to accurately predict the corrosion-induced cover cracking time. Several studies have been published concerning the problems related to the corrosion-induced cover cracking time. These studies include experimental studies (Andrade, Alonso,
and Molina 1993; Vu, Stewart, and Mullard 2005; Lu, Jin, and Liu 2011; Mullard and Stewart 2011; Ye et al. 2018; Chen, Baji, and Li 2018; Wang et al. 2020), theoretical analyses (El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011; Mullard and Stewart 2011; Malumbela, Alexander, and Mayo 2011), and numerical models (Castorena et al. 2008; Xiao, Ying, and Shen 2012; Chernin and Val 2011; Srubar 2015; Stambaugh, Bergman, and Srubar 2018). Nevertheless, all these published studies have focused only on round-rebars, while square-rebars have been ignored. Although currently the steel material of RC structures is dominated by round-rebars, historically, square-rebars have been widely used around the world, especially in Europe (Lee 1915; Du 1934), America (Alexander 2000; Meinheit and Felder 2014) and China (Du 1934). From 1912 to 1949, China imported a large number of square-rebars from Europe and America (Du 1934), and built many RC buildings. Many of these buildings are still in service today, and the majority have been listed as protected buildings. The durability evaluation of these historical RC buildings has become a critical problem for future services, and the prediction of the corrosion-induced cover cracking time for square-rebar cases is an essential element of the durability evaluation work.

Although new square-rebar RC structures will not be erected anymore, there is a lot of historical RC buildings with square-rebars around the world. Due to their historical and cultural values, these buildings will be preserved in the future. All reinforcement measurements on these structures are under rigorous control and supervision due to the protection requirements for important historical RC buildings in China. While the old concrete of the components can be removed and repoured, the vintage steel rebars (not only the square ones but also some other types of steel rebars used historically) cannot be replaced by new ones. More specifically, the vintage steel rebars need to be refreshed in situ and maintained for a long time. Thus, the structural behavior of reinforced or refreshed concrete components will be always related to the square-rebars. However, for the reinforcement and protection of these historical RC buildings in China, currently, durability evaluation methods and structural theories based on round-rebar cases are used, even in the cases where the concrete structures contain square-rebars. This inaccuracy can probably cause economic waste or even structural problems. Therefore, the corrosion-induced cracking of square-rebar components and the related problems need to be investigated, which will have long-term engineering value.

Researchers and engineers have not paid too much attention to square-rebars. The Concrete Reinforcing Steel Institute (Meinheit and Felder 2014) in the USA published a book that summarized the uses of different types of vintage steel rebars in the past. It stated that the characteristics of the different steel rebars aimed at improving the bond properties between steel rebar and concrete. The historical material in the book confirmed that square-rebars was used in Europe and America far and wide. Chun and Pan (2014) proposed a criterion for calculating the residual functional life of historical RC structures in China; however, a critical equation was based on experiments for round-rebars. Chun et al. (2014) tested 66 vintage steel rebars (30 round-rebars and 36 square-rebars), and found that, while the height of the ribs of these vintage steel rebars can meet the regulations of current standards, the space between the ribs and the relative rib area cannot. This finding indicated that the bond-slip property between vintage steel rebar and concrete might not meet the requirements of current standards, which was eventually proved by Zhang, Chun, and Jin et al. (2020). Dong, Chun, and Xu et al. (2017) performed an accelerated corrosion experiment on eight concrete specimens with old square-rebars. The critical corrosion depth of the square-rebars at cover cracking and the corrosion products of the old steel rebars were investigated. Although the theoretical model was not investigated, their study results indicated that the square-rebar RC component was different from the round-rebar RC component regarding the corrosion-induced cracking problem.

Most of the previous studies on steel corrosion and corrosion-induced cracking have been focused on round-rebars; and all the proposed prediction models are for round-rebars; while there are only few studies concerning square-rebars, which have been mainly focused on the material and mechanical properties. The biggest difference between round-rebar and square-rebar is the rebar section shape, which can directly determine the mechanical model on the problem of rebar corrosion-induced cracking. The mechanical model can directly influence the cracking time. Thus, the existed prediction models for round-rebars are not applicable to square-rebars. In this study, a cracking time prediction model for square-rebar is presented. The predicted time concern the period from corrosion initiation of the steel to cover cracking. In addition, a corresponding accelerated corrosion experiment for square-rebar RC components has been performed to demonstrate the accuracy of the proposed prediction method.

2. Materials and methods

The research methodology followed in this study includes theoretical derivation and experimental validation. First, an accelerated corrosion experiment for
3. Corrosion-induced cover cracking analysis

3.1. Prediction model of the corrosion process of steel rebars

In previous studies (Tuutti 1980; Weyers 1998), the corrosion process of steel rebars in concrete has been considered as a three-stage process: free expansion of the corrosion products, pressure-induced, and cover cracking. However, this division is too complicated for engineering applications. For the durability evaluation work, the final cracking time is the critical index for assessing the functional life of RC structures during the entire corrosion process. In order to reduce the complexity for engineering applications, in this study, the predicted corrosion process was divided into two stages (Figure 2). In the first stage of the three-stage theory, some corrosion products fill the voids and pores around the steel rebar, which is a very thin layer (Alkhalaif and Page 1979; Weyers 1998). In this study, this point has been taken as the end of the corrosion initiation, and then, the corrosion products start to apply pressure to the concrete around the rebar. The endpoint of the second stage is when the cover cracks. The duration of the entire process presented in Figure 2 is the same as that in the three-stage theory.

3.2. Basic assumption

The corrosion-induced cover cracking model was developed based on the elasticity theory. Except for the basic assumptions of the elasticity theory, in this study, the following assumptions have been made:

1. The problem is considered as a plane strain problem in the elasticity theory (Vu, Stewart, and Mullard 2005; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).
2. The corrosion process of the steel rebar is spatially uniform; thus, the pressure developed by the corrosion products around the steel-concrete...
interfaces is uniform (Bazant 1979; Liu and Weyers 1998; Bhargava et al. 2005; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).

(3) The stresses and strains in the concrete are induced only by the steel rebar corrosion products (Bazant 1979; Liu and Weyers 1998; Bhargava et al. 2005; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).

(4) The concrete material is isotropic and linear elastic (Liu and Weyers 1998; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).

(5) Around the steel rebar, there is a porous zone which acts as a transition zone (Alkhalaf and Page 1979; Weyers 1998), and its thickness is uniform (Liu and Weyers 1998; Bhargava et al. 2005; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).

(6) No corrosion products enter into the crack before the crack reaches the concrete cover surface (El Maaddawy and Soudki 2007; Michel et al. 2011; Zhao, Wu, and Jin 2013).

3.3. Simplified mechanical model

According to previous studies, the theoretical mechanical models for corrosion-induced cover cracking with round-rebar cases can be mainly divided into two types, i.e., thick-walled cylinder models (Zhao and Jin 2006; El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011; Malumbela, Alexander, and Moyo 2011; Amin et al. 2013; Roshan, Noel, and Martin-Perez 2020) and double-layer thick-walled cylinder models (Pantazopoulos and Papoulias 2001; Bhargava et al. 2006; Chernin, Val, and Volokh 2010), the majority of which have been based on the elasticity theory. In the thick-walled cylinder model, the analysis objective is a single cylinder, while in the double-layer thick-walled cylinder model, the analysis objectives are two coaxial cylinders. The double-layer thick-walled cylinder model is derived by adding the displacement coordination conditions on the expanded boundary based on the thick-walled cylinder model. Although it is slightly more developed from the thick-walled cylinder model, it is based on the same assumptions. Since the thick-walled cylinder model is the essential one, it is better to use it to develop the square-rebar case model for the first theoretical research. Consequently, the ideal model for the square-rebar case is similar to the thick-walled square tube model, and can be considered as a plane square-hole problem. While the thick-walled square tube model can be solved through a conformal mapping method by complex variable functions (Muskhelishvili and Zhao 1958), the derivation and analytical solution are too complicated for engineering applications. In the elasticity theory, the most important issue in the plane square-hole problem

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**Figure 3.** Corrosion and cover cracking process: (a) corrosion initiation stage; (b) pressure induced by the corrosion products; (c) concrete cover cracking.

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**Figure 4.** Deduction process: (a) concrete specimen with a square-rebar; (b) isolated body; (c) simplified mechanical model.
is the stress concentration at the corners. However, according to the experimental observations in this study, the corrosion-induced cover crack starts from the edge of the rebar section, forming an almost vertical crack rather than an inclined crack from the corner of the rebar section. Based on the experimental evidence, in reality, the stress concentration on the corners is not the critical problem for the model; thus, the first corrosion-induced crack can be ideally considered to be perpendicular to the edge of the square-rebar section, and the foot point on the middle point of the rebar section edge. The corrosion-induced cover cracking process of the square-rebar case is illustrated in Figure 3.

To describe the above process mathematically, a simplified mechanical model based on the elasticity theory is proposed. A portion of the area around the steel rebar can be taken as an isolated body for mathematical analysis. Based on the symmetry characteristics, the problem can be simplified as a single-span statically indeterminate beam model with a trapezoidal shape in plane. The two bevel edges of the trapezoid should be fixed. The simplified model and the deduction process are presented in Figure 4.

For the model presented in Figure 4, its thickness is considered as a unit length, and its physical strength is excluded; thus, it can be considered as a plane strain problem in the elasticity theory. A uniform load is applied on the upper boundary (by the swell of the corrosion products), while the lower boundary (the boundary of the concrete on the cover side) is free. Therefore, the lower boundary condition can be given by:

\[ (\sigma_y)_{y=h_2/2} = -q, \quad (\sigma_y)_{y=-h_2/2} = 0 \]  

(1)

From Equation (1), it can be assumed that \( \sigma_y \) is not related to \( x \), which is only related to \( y \); thus, \( \sigma_y = f(y) \).

Since \( \sigma_y = \frac{\partial \varphi}{\partial y} \), the trial Airy stress function should be:

\[ \varphi = \frac{1}{2} x^2 f(y) + x f_0(y) + f_1(y) \]  

(2)

A basic solution for the trial Airy stress function can be found in many standard textbooks on the theory of elasticity (e.g., Timoshenko and Gere 2009)

\[ \sigma_x = \frac{6q}{h^3} x^2 + \frac{4q}{h^3} y^3 + 6Hy + 2K \]  

(3)

\[ \sigma_y = \frac{2q}{h^3} y^3 + \frac{3q}{2h} - \frac{q}{2} \]  

(4)

\[ \tau_{xy} = \frac{6q}{h^3} x y^2 - \frac{3q}{2h} x \]  

(5)

where \( q \) is the pressure on the upper edge; \( h \) is the height of the trapezoid in Figure 4(c); and \( H \) and \( K \) are two unknown constants, which can be determined by introducing physics equations:

\[ \varepsilon_x = \frac{1}{E} (\sigma_x - \mu \sigma_y), \quad \varepsilon_y = \frac{1}{E} (\sigma_y - \mu \sigma_x), \quad \gamma_{xy} = 2 \frac{(1+\mu)}{E} \tau_{xy} \]  

and geometrical equations:

\[ \varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial u}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]  

where \( \mu \) is the Poisson’s ratio of the material and \( E \) is its elastic modulus. By substituting Equations (3–5) into the physics and geometrical equations, the following relationships can be obtained:

\[ \frac{\partial u}{\partial x} = \frac{1}{E} \left[ \frac{6q}{h^3} x^2 y + \frac{4q}{h^3} y^3 + 6Hy + 2K \right] - \frac{\mu}{6h^3} \left[ \frac{3q}{h^3} y^3 + \frac{3q}{2h} y - \frac{q}{2} \right] \]  

(6)

\[ \frac{\partial v}{\partial y} = \frac{1}{E} \left[ -\mu \left( \frac{6q}{h^3} x^2 y + \frac{4q}{h^3} y^3 + 6Hy + 2K \right) - \frac{2q}{h^3} y^3 \right] \]  

\[ + \frac{3q}{2h} y - \frac{q}{2} \]  

(7)

\[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2 \left( 1 + \mu \right) \frac{q}{E} \left( \frac{6q}{h^3} x^2 y^2 - \frac{3q}{2h} x \right) \]  

(8)

According to Equation (6):

\[ u = \frac{1}{E} \left[ \frac{2q}{h^3} x^2 y + \frac{4q}{h^3} y^3 + 6Hy + 2K \right] - \mu x \left( \frac{6q}{h^3} y^3 + \frac{3q}{2h} y - \frac{q}{2} \right) + g_1(y) \]  

(9)

According to Equation (7):

\[ \nu = \frac{1}{E} \left[ -\mu \left( \frac{3q}{h^3} x^2 y^2 + \frac{q}{2h} y^4 + 3Hy^2 + 2Ky \right) - \frac{q}{2h} y^4 \right] + \frac{3q}{2h} y^2 - \frac{q}{2} y + g_2(x) \]  

(10)

Then, by substituting Equations (9, 10) into Equation (8), it can be obtained:

\[ -\frac{2q}{h^3} x^3 + 6Hy + 6 + 3\mu \frac{q}{2h} x + g_2^*(x) = -g_1^*(y) \]  

(11)

Since the left side of the equation is a function with \( x \) and the right side is a function with \( y \), both sides should be equal to the same constant, which is considered as \( \omega \). Consequently, two new expressions can be obtained:

\[ g_1(y) = -\omega y + u_0 \]  

(12)

\[ g_2(x) = \frac{q}{2h} x^3 - 3Hy^2 - \frac{6 + 3\mu}{2h} qx^2 + \omega x + v_0 \]  

(13)

By substituting Equations (12, 13) into Equations (9, 10):

\[ u = \frac{1}{E} \left[ \frac{2q}{h^3} x^2 y + \frac{4q}{h^3} y^3 + 6Hy + 2K \right] - \mu x \left( \frac{6q}{h^3} y^3 + \frac{3q}{2h} y - \frac{q}{2} \right) - \omega y + u_0 \]  

(14)
\[ v = \frac{1}{E}\left[ -\mu \left( -\frac{3q}{h^3} x^2 y^2 + \frac{q}{h^3} y^4 + 3Hy^2 + 2Ky \right) - \frac{q}{2h^3} y^2 + \frac{3q}{4h} y^2 - \frac{q}{2} y + \frac{q}{2h^3} x^4 - 3Hx^2 - \frac{6 + 3\mu}{4h} qx^2 + \omega y + v_0 \right] \]

(15)

In Equations (14, 15), \( u_0 \), \( v_0 \), and \( \omega \) are all constants that represent the rigid displacement. According to the symmetry, \( u \) is an odd function of \( x \) and \( v \) is an even function of \( x \); therefore, \( u_0 = 0 \) and \( \omega = 0 \). The boundary conditions for both bevel edges are fixed ends, and in the true sense, all points on the ends cannot move or rotate. Nevertheless, the fixed end is impossible in many realistic problems to obtain a polynomial solution. As a result, the boundary condition is simplified such that the middle point of the bevel edge is fixed, and cannot move or rotate in the horizontal direction (Fan, Gao, and Li 2006). Considering the displacement boundary condition for the left side and that \( N = \frac{d}{2} + \frac{3}{4} \), then:

\[ (u)_{x=y=\frac{d}{2}+\frac{1}{2}, y=0} = 0, \quad (v)_{x=y=\frac{d}{2}+\frac{1}{2}, y=0} = 0, \quad \left( \frac{\partial v}{\partial x} \right)_{x=y=\frac{d}{2}+\frac{1}{2}, y=0} = 0 \]

(16)

Subsequently, \( v_0, H, \) and \( K \) can be calculated:

\[ v_0 = \frac{qN^2}{2h^3}, \quad H = \frac{qN^2}{3h^3} - \frac{(2 + \mu)q}{4h}, \quad K = -\frac{\mu q}{4} \]

(17)

where \( d \) is the length of the upper edge and \( l \) is the length of the long edge. The solution for the stress can be obtained as follows:

\[ \sigma_x = -\frac{6q}{h^3} x^2 y + \frac{4q}{h^3} y^3 + \left( 2\frac{q}{h^3} N^2 - \frac{6 + 3\mu}{2h} q \right) y - \frac{\mu q}{2} y \]

(18)

\[ v = \frac{1}{E}\left[ -\mu \left( -\frac{3q}{h^3} x^2 y^2 + \frac{q}{h^3} y^4 + \frac{4q}{h^3} y^4 - \frac{qN^2}{2h} y^2 + \frac{q}{2h^3} x^4 - \frac{6 + 3\mu}{4h} qx^2 + \frac{\mu q}{2} x^2 + \frac{q}{2h^3} y^4 + \frac{qN^2}{2h} y^2 \right) \right] \]

(19)

### 3.4. Pressure induced by the corrosion products

According to the basic assumptions made above, a uniform layer of corrosion products can lead to the development of a uniform pressure between steel rebar and concrete, inducing a uniform displacement of the concrete interface. In this part, the displacement (Equation (19)) can be converted into a uniform displacement by dividing the total area of displacement with the length of the edge:

\[ \delta_c = \delta_s = \int_{-\frac{d}{2}}^{\frac{d}{2}} (v)_{y=\frac{d}{2}+\frac{1}{2}, y} dy/d \]

\[ = \frac{q}{E_{ef}} \int_{-\frac{d}{2}}^{\frac{d}{2}} \left[ \frac{d^4}{16h^3} + \frac{w^2}{16} - \frac{N^2 d^2}{12h^3} + \frac{\mu N^2}{4h} - \frac{(5\mu - \mu^2)h}{16} + \frac{N^4}{2h^3} + \frac{13h}{32} \right] \]

(20)

where \( \delta_c \) is the displacement of the concrete interface; \( \delta_s \) is the assumed average displacement of the concrete interface; \( q \) is the pressure induced by the expansion of the corrosion products, which causes \( \delta_c \); and \( E_{ef} \) is the effective elastic modulus of the concrete cover. The relationship between \( \delta_c \) and \( q \) can be expressed as follows:

\[ \delta_c = \psi q_r \]

(21)

where \( \psi \) is a constant factor that relates the displacement of the concrete to the swelling force applied by the corrosion products.

\[ \psi = \frac{1}{E_{ef}} \left( \frac{d^4}{16h^3} + \frac{w^2}{16} - \frac{N^2 d^2}{12h^3} + \frac{\mu N^2}{4h} - \frac{(5\mu - \mu^2)h}{16} + \frac{N^4}{2h^3} + \frac{13h}{32} \right) \]

(22)

Figure 5 shows the changed situation around the square-rebar for the corrosion-induced cover cracking model. It is assumed that \( D' = D + 2\delta_0 \), where \( D \) is the edge width of the square-rebar and \( \delta_0 \) is the thickness of the porous zone as mentioned above in the basic assumptions. Moreover, it is assumed that \( C' = C - \delta_0 \), where \( C \) is the thickness of the concrete cover. Thus, \( D' \)
is considered as the length of the $q_r$, $C'$ as the height of the trapezoid, and $N = \frac{C}{2} + \frac{C'}{2}$. Equation (22) can be rewritten as:

$$
\psi = \frac{1}{E_{ct}} \left\{ \frac{D^4}{16C^2} + \frac{\mu D^2}{16C} - \frac{N^2 D^2}{12C^3} - \frac{\mu N^2 C'}{4C} + \frac{(5 \mu - \mu^2) C'}{16} \right\} + \left\{ \frac{N^2}{2C} + \frac{13C}{32} \right\}
$$

(23)

$$
\delta_r = \delta_l + \delta_0 + \delta_c
$$

(24)

Based on Figure 5, the relationship of the volume can be written as:

$$
\frac{M_r}{\rho_r} - \frac{M_{loss}}{\rho_s} = [D + 2(\delta_r - \delta_l)]^2 - D^2
$$

(25)

where $M_r$ is the mass of the corrosion products; $M_{loss}$ is the mass of the lost steel; $\rho_r$ is the mass density of the corrosion products; and $\rho_s$ is the mass density of the steel. By substituting Eq. (24) into Eq. (25), the relationship becomes:

$$
\frac{M_r}{\rho_r} - \frac{M_{loss}}{\rho_s} = 4D(\delta_0 - \delta_c) + 4(\delta_0 - \delta_c)^2
$$

(26)

For the following calculations, the term $4(\delta_0 - \delta_c)^2$ can be neglected, since $(\delta_0 - \delta_c)^2 < < D$. Assuming that $M_r = \gamma_1 M_{loss}$ and $\rho_r = \gamma_2 \rho_s$, then, Eq. (25) can be rewritten as:

$$
\frac{(\gamma_1 - \gamma_2)M_{loss}}{\gamma_2 \rho_s} = 4D(\delta_0 + \delta_c)
$$

(27)

By combining Equations (21), (23), and (27), $q_r$ can be calculated as:

$$
q_r = \frac{1}{\psi} \left\{ \frac{(\gamma_1 - \gamma_2)M_{loss}}{4\gamma_2 \rho_s D} - \delta_0 \right\}
$$

(28)

### 3.5. Critical condition for the pressure at the cracking time

The critical status for the corrosion-induced cover cracking is illustrated in Figure 6. In this study, the generation of the crack is considered to be induced by the uniform pressure applied by the corrosion products.

The critical pressure is called $q_{ct}$ and can be calculated by:

$$
q_{ct} = 2Cf_t
$$

(29)

where $f_t$ denotes the concrete tensile strength. Since $(\delta_0 + \delta_c) < < D$ and $(\delta_0 + \delta_c) < < C$, $q_{ct}$ can be given by:

$$
q_{ct} = \frac{2Cf_t}{D}
$$

(30)

By combining Equations. (23), (28), and (30), the relationship for the critical condition can be given by:

$$
\frac{2Cf_t}{D} = \frac{1}{\psi} \left\{ \frac{(\gamma_1 - \gamma_2)M_{loss}}{4\gamma_2 \rho_s D} - \delta_0 \right\}
$$

(31)

### 3.6. Predicted time of corrosion-induced cover cracking

In order to introduce the time factor into Eq. (31), Faraday’s law needs to be used, and the relationship is built based on the mass of the corroded steel. This method has been adopted by several studies on prediction models of the corrosion-induced cracking time (El Maaddawy and Soudki 2007; Lu, Jin, and Liu 2011).

In addition, some experimental results (Andrade, Alonzo, and Molina 1993; Cabrera 1996; Martin-Pérez 1999) have also demonstrated that the amount of the corrosion products follows approximately the Faraday’s law, the expression of which is as follows:

$$
M_{loss,t} = \frac{Ml t}{2F}
$$

(32)

where $M_{loss,t}$ is the total mass of steel lost to corrosion; $M$ is the atomic mass of iron (56 g); $l$ is the current (A); $t$ is the time (s); $z$ is the ionic charge; and $F$ is the Faraday’s constant (96, 500 A-s). According to the composition analysis of corrosion products for this type of square-rebars by Dong, Chun, and Xu et al. (2017), the value of the ionic charge $z$ is approximately 2, which is also the value used in this study. Therefore, the time $t$ can be obtained by:

$$
t = \frac{24125(M_{loss,t}/a)}{7i}
$$

(33)

where $i$ is the current density (A/cm²) and $a$, is the surface area of the steel rebar. In a previous study (El Maaddawy and Soudki 2007), the factor $m_1$, which
denotes the percentage loss of steel mass, was introduced to combine the governing equation at the cracking time and the time expressed by Faraday’s law.

\[ m_1 = 100 \frac{M_{\text{loss}}}{M_{\text{st}}} \]  

(34)

where \( M_{\text{loss}} \) is the mass of steel lost to corrosion products per unit length and \( M_{\text{st}} \) is the original mass of steel per unit length prior to corrosion initiation. Thus, it can be derived that:

\[ \frac{M_{\text{loss}}}{\rho_s} = \frac{M_{\text{loss}}}{M_{\text{st}}} \cdot \frac{M_{\text{st}}}{\rho_s} = \frac{m_1 A_b}{100} = \frac{m_1 D^2}{100} \]  

(35)

\[ \frac{M_{\text{loss}}}{a_s} = \frac{m_1 D^2 \rho_s}{100 \times 4D} = \frac{m_1 D \rho_s}{400} \]  

(36)

where \( A_b \) is the original cross-sectional area of the steel rebar. By combing Equations (33) and (36) with \( \rho_s = 7.85 \text{ g/cm}^3 \), it can be obtained:

\[ m_1 = 100 \frac{M_{\text{loss}}}{M_{\text{st}}} \]  

(34)

\[ \frac{M_{\text{loss}}}{\rho_s} = \frac{M_{\text{loss}}}{M_{\text{st}}} \cdot \frac{M_{\text{st}}}{\rho_s} = \frac{m_1 A_b}{100} = \frac{m_1 D^2}{100} \]  

(35)

\[ \frac{M_{\text{loss}}}{a_s} = \frac{m_1 D^2 \rho_s}{100 \times 4D} = \frac{m_1 D \rho_s}{400} \]  

(36)

where \( A_b \) is the original cross-sectional area of the steel rebar. By combing Equations (33) and (36) with \( \rho_s = 7.85 \text{ g/cm}^3 \), it can be obtained:
Validation

4.1. square-rebars

The end of the rebar is illustrated in Figure 7. The mix of the concrete was performed according to historical books (Zhao 1935; Chen 1936; Zhang 1944). The mass ratio of water, cement, sand, and stone was 0.35:0.55:0.95:1.71. All concrete specimens were of same size, i.e., 150 mm×200 mm×150 mm. A sketch of the specimen is illustrated in Figure 8. One end of the steel rebar was linked with the electric source. The cover thickness of the specimens was determined by historical books (Zhao 1935; Zhang 1944). More specifically, between 1912 and 1949, the cover thickness of the beam components was usually 38 mm. Detailed information on the different specimens used in the experiment is listed in Table 1.

The accelerated corrosion of the steel rebars was achieved electrochemically. The value of the impressed direct current was 100 μA/cm² (Alonso et al. 1998; Vu, Stewart, and Mullard 2005; Lu, Jin, and Liu 2011), and the electrolyte was 5% NaCl solution (Vu, Stewart, and Mullard 2005; Lu, Jin, and Liu 2011). The experimental setup is illustrated in Figure 9. An electric wire was welded on the end of the rebar. After welding, the side surfaces of the specimens and the exposed rebars were coated with 2-mm-thick epoxy resin. Subsequently, half the height of the specimens was immersed in the solution for 72 hours. After the upper surfaces of the specimens were totally moist, the current was supplied. The critical width of the crack was set as 0.1 mm (Andrade, Alonso, and Molina 1993; Dong, Chun, and Xu et al. 2017), which also marked the end of the experiment.

4.2. Results and discussion

For the calculations, the value of the effective elastic modulus of the concrete $E_{cr}$ in Eq. (23) is equal to $E/(1 + \phi_c)$, and $\phi_c$ is the concrete creep coefficient. The review article by Liang and Wang (2020) listed eight published papers on the problem of corrosion-induced cracking prediction, and the values of the concrete creep coefficient varies from 0 to 3. In three of them, the value of the concrete creep coefficient, $\phi$, was taken as 2.0, which was the most widely accepted by the researchers; thus, this value was also adopted in this study. For the concrete, a Poisson's ratio of 0.35 was determined. The creep coefficient of the concrete, $\gamma_{cr}$, was taken as 0.25, which was the most widely accepted by the researchers; thus, this value was also adopted in this study. For the concrete, a Poisson's ratio of 0.35 was determined.

$$T = \frac{1879.2m_1D}{i} \quad (37)$$

where $T$ is the time (h) and $i$ is the current density ($\mu A/cm^2$).

By combining Equations. (23), (31), (35), and (37), the time from corrosion initiation to corrosion-induced cover cracking $T_{cr}$ (h) can be predicted by:

$$T_{cr} = \frac{751680\gamma_2}{i(Y_1 - Y_2)} \left( \frac{2C_T\gamma}{D} + \delta_0 \right) \quad (38)$$

Table 2. Comparison between predicted and experimental results

| Item | Tested time (h) | Our study | Error | *Maaddawy’s model (El Maaddawy and Soudki 2007) | *Lu’s model (Lu, Jin, and Liu 2011) |
|------|----------------|-----------|-------|---------------------------------------------|----------------------------------|
| A-1  | 242            | 204–234   | −3.3% | 142–176                                     |                                  |
| A-148| 261            | 221–251   | −3.8% | 147–181                                     |                                  |
| A-152| 287            | 255–270   | −5.9% | 153–187                                     |                                  |
| A-3  | 311            | 259–289   | −7.1% | 158–192                                     |                                  |
| A-153| 336            | 279–309   | −8.0% | 164–198                                     |                                  |
| A-5  | 213            | 170–200   | −6.1% | 146–180                                     |                                  |
| B-1  | 220            | 183–213   | −3.3% | 151–185                                     |                                  |
| B-138| 236            | 196–226   | −4.2% | 156–191                                     |                                  |
| B-144| 248            | 210–240   | −3.3% | 162–196                                     |                                  |
| B-147| 260            | 225–255   | −1.9% | 167–201                                     |                                  |

The two prediction models are for round-rebar cases. They are both based on the rebar diameter to predict the time. The results in this table were calculated by replacing the diameter of the round-rebar with the width of the square-rebar.
ratio, $\mu$, of 0.18 was used in this study, which has been adopted by many researchers (Liang and Wang 2020). Although some parameters concerning the corrosion products of square-rebars have not been studied extensively yet, Chun et al. (2014) tested 36 square-rebars, and found that they were hot-rolled, which is the same technique followed for the current hot-rolled round-rebars. Additionally, the chemistry contents of square-rebars can basically meet the requirements described in current standards (European Committee for Standardization 2004; State Administration of Quality Supervision, Inspection and Epidemic, & China Standardization Administration 2018). Thus, parameters like the mass density of the corrosion products and steel, and the relationship between corrosion products and steel can be referenced from the related research on round-rebars. The relationship between corrosion products and steel $\gamma_1$ was taken as 1.747 (Liu and Weyers 1998), and the mass density of the corrosion products and steel $\gamma_2$ was taken as 0.5 (Liu and Weyers 1998). Moreover, the thickness of the porous zone $\delta_0$ is typically in the range of 10–20 $\mu$m (El Maaddawy and Soudki 2007). The predicted time is given as a range, and the lower and upper values, which result from $\delta_0$, are equal to 10 and 20 $\mu$m, respectively. In Table 2, except for the results of the presented models, the predicted time results from two additional prediction models for round-rebar cases are also presented.

All cracked specimens after the end of the experiment are demonstrated in Figure 10. While the crack was not strictly in the projection range of the square-rebar edge in every specimen, all cracks started from positions within the rebar edges rather than the corners. In all specimens the crack was perpendicular to the rebar edge. Consequently, the path of the crack in the concrete can be ideally considered to be perpendicular to the edge of the rebar section, which was the source of the assumption in Section 3.3.

The time results predicted by the two previous models for round-rebar cases were clearly not accurate for the square-rebar case. By comparing the time predicted by the proposed model and the experimental results, it can be observed the errors were less than 8%, which is an acceptable error for structural engineering applications. However, it is worth noticing that, for all specimens, the experimental times were longer than the predicted ones. The error can be attributed to reasons related to the limitations of the experiments and the complexity of the steel corrosion process, as follows:

1. In the accelerated corrosion process, some parts of the corrosion products may have permeated into the concrete, rather than being all attached on the steel rebar. The loss of corrosion products may have caused a little decrease in the corrosion-induced force.
2. At the crack initiation stage, there might be more than one crack around the steel-concrete interface; however, only one propagated to the surface of the cover, and the others remained at the initiation status (very tiny). These tiny cracks released some of the corrosion-induced pressure.
3. The predicted time does not include the damage process of the passivation film on the steel surface. Although it was relatively short for the accelerated corrosion experiment (usually only several hours), it introduced some errors.
4. The crack of the epoxy resin on the concrete surface may have released some of the corrosion-induced pressure.

Figure 10. Cracked concrete specimens after the electrochemically accelerated corrosion experiment.
Overall, within the acceptable error range, the experimental results proved that the proposed mathematical model can generally express the corrosion-induced cracking process of square-rebars in concrete, and provide a reasonable prediction of the time from corrosion initiation to corrosion-induced cover cracking.

5. Conclusions

A model for predicting the time from corrosion initiation to corrosion cracking was developed for the square-rebar case, and its rationality and applicability were validated by a corresponding laboratory-based electrochemical experiment. Its accuracy was qualified for engineering applications. Additionally, a mechanical model of the corrosion-induced cracking for square-rebar cases was also presented in the derivation process. Based on the results of the present work, the following conclusions can be drawn:

1. In square-rebar RC components, the first corrosion-induced crack often appears directly above the midpoint of the side of the square section, rather than being diagonal and starting from the corner of the rebar section. The direction of the crack propagation is perpendicular to the direction of the rebar length.
2. The previous time-prediction methods for round-rebar cases are not suitable for square-rebar cases. The corrosion-induced cover cracking model for the square-rebar case can be considered as a thick-walled square tube model in the elasticity theory, and the solution can be approximately obtained based on its symmetry.
3. The corrosion-induced cracking time for square-rebar RC components is proportional to the ratio of the concrete cover thickness and the rebar section width, and inversely proportional to the corrosion current density.

The current outcomes are the preliminary study to reveal the square-rebar corrosion-induced cracking and its time prediction. Since the proposed model has been verified effectively, the developed model and the further experiment will be studied in the future.

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