Abstract: To solve the problem of wheelset slip and effectively exert locomotive traction, a protection method of anti-slip control with integral sliding mode based on optimal slip ratio is proposed. First, a dynamic model of heavy-haul locomotives is established and a sliding mode observer is designed to observe the real-time adhesion coefficients of heavy-haul locomotives. An extremum seeking algorithm with sliding mode is used to monitor and search the optimal region of slip ratio. Then, a PID compound control is combined to weaken oscillations in the sliding mode extremum seeking. Finally, an integral sliding mode controller with new reaching law is designed to ensure the slip ratio of locomotives running on different rail surfaces are at optimal values, which maximise the utilisation of adhesion between wheels and rails. Simulation results also verify the feasibility and effectiveness of the integral sliding mode variable structure control method based on optimal slip ratio in the anti-slip traction system of vehicles.

1 Introduction

With the development of China's railway and transportation industry, transport volumes are growing rapidly and the demand for traction power and stability safety of heavy-haul locomotives is increasing [1]. The most effective method to prevent slip is to attach an adhesion control system in the traction control of locomotives. At present, locomotives use an adhesion control method [2] that mainly consists of a combined correction method, an acceleration differential method, an adhesion slope method, a model control method, and an intelligent control method. Slip recognition is performed and the peak combined correction method obtained by the driving torque control undergoes relatively large traction loss in adhesion control. The correction system requires a strict setting of boundary conditions. Without substantial experimental research, the set parameters may affect the traction performance of locomotives or even their normal operation. In practical rail surfaces, traditional model control methods cause operational errors when adhesion coefficients change in a small range. By comparison, the acceleration differential method has better rail surface adaptability, with small fluctuations in motor torque and good stability. However, when requirements for filtering methods are high, the interference of noise signals must be reduced as much as possible. These methods are based on optimal adhesion points and are designed to achieve the optimal control of adhesion. These control methods cannot obtain optimal slip ratio values because practical slip ratio is not constant but change with the external environment of locomotives in real time, which will reduce control accuracy. Therefore, to improve the control performance of locomotives, the real-time optimisation of slip ratio is achieved by obtaining adhesion coefficients in real time, and the adhesion between wheels and rails is controlled to reach the optimal value.

The optimal slip ratio between wheels and rails depends on the condition of rail surfaces, the design structure of the locomotive itself, the speed, and other factors. Thus, when different locomotives are running under different conditions, the optimal slip ratio corresponding to the locomotive adhesion coefficients is also different. Therefore, searching the optimal slip ratio online in real time is of great significance [3]. When the adhesion coefficients of different rail surfaces are estimated, the practical slip ratio can be controlled to reach the optimal slip ratio corresponding to the peak value of adhesion coefficients. In general, two methods are used to obtain real-time optimal slip ratio: a model-based method [4] and a method based on the online estimation of extreme values [5]. The model-based method identifies the parameters in the model online according to classic models of adhesion characteristic curves, such as least square estimation [6] and maximum likelihood estimator [7], to find the extremum in mathematical models and obtain the optimal slip ratio. The accuracy of the reference models and the selection of estimation methods have important influences on the control effects. This paper discusses the second method mentioned. The study uses extremum seeking [8] with sliding mode to search for the optimal slip ratio based on observing adhesion coefficients with a sliding mode observer. The extremum seeking algorithm with sliding mode inherits the advantages of sliding mode control with strong robustness. Its convergence rate can be preset and it is not subject to parameter uncertainty and outside interference. This method achieves the optimal adhesion control by searching slip ratio of locomotives online based on meeting certain functional relations between practical adhesion coefficients and creep speed of locomotives. However, the extremum seeking algorithm with sliding mode will oscillate around optimal adhesion points, so it is combined with the PID compound control to attenuate oscillations near extreme points.

Consequently, an integral sliding mode variable structure control is used in this work to optimise the adhesion, which causes the traction control system to perform sliding motion. The well-designed sliding mode has no association with system parameters and disturbances, thus giving the system good robustness. An anti-slip integral sliding mode control system with novel reaching law is designed, and the adhesive force control is achieved by operating locomotives at optimal adhesion points.

2 Basic adhesion theory and dynamic model of heavy-haul locomotives

2.1 Basic adhesion theory

When a train is running, the wheelset rolls forward under the driving torque, and the contact surface between the wheelset and the rail deforms elastically. Under the effect of vertical static load \( P \), the contact surface between the wheel and rail is relatively static with no relative sliding. This condition, called adhesion, forms an
adhesive force on the contact surface and is the only power source for the wheels. A simplified model of a single wheel is shown in Fig. 1.

Owing to the elastic deformation between wheels and rails, when a locomotive is in operation, the wheelset speed $v_d$ is always higher than the locomotive speed $v_t$. In particular, the creep speed \[ v_s = v_d - v_t \] (1)

Adhesion coefficients [9] are defined as follows:

\[ \mu(\lambda) = \frac{F_s}{Wg} \] (2)

Rotational speed of the wheelset is

\[ \omega_d = \omega_m r \] (3)

Slip ratio [10] is defined as follows:

\[ \lambda = \frac{v_s}{v_t} \] (4)

where $W$ is the axle load of locomotives, $F_s$ is the locomotive's adhesion force, $\omega_m$ is the locomotive's rotational speed, $r$ is the wheel radius and $g$ is the gravitational acceleration (9.8 m/s$^2$).

### Table 1 Parameter values of adhesion coefficient calculation

| Road surface      | a    | b    |
|-------------------|------|------|
| dry rail surface  | 0.375| 15   |
| ordinary rail surface | 0.32 | 25   |
| wet rail surface  | 0.275| 40   |

Adhesion coefficient increases with the increase of slip ratio, which reaches the optimal adhesion point $(\mu_{max}, \lambda^*)$ of the adhesion coefficient and is unique. At this point, the rail surface provides maximum adhesion so as to provide maximum traction for the locomotive. When a locomotive runs in the slip region, as the slip ratio continues to increase, the adhesion coefficient does not increase but decreases, causing the locomotive to slip or even idle. Given the complicated relation between adhesion coefficients and slip ratio, when a locomotive runs on different rail surfaces, the optimal adhesion points are also different.

2.3 Establishing the wheelset longitudinal dynamic model

The longitudinal dynamic model of the wheelset is as follows:

\[ M \frac{dv_t}{dt} = F_s - F_d(v_t) \] (6)

\[ F_d(v_t) = (l + mv_t + nv_t^2)Mg \] (7)

where $M$ is the gross mass of the locomotive and the basic resistance during the locomotive's operation, which is proportional to the locomotive's speed $v_t$; and $l, m$ and $n$ are the drag coefficients related to mechanical resistance.

2.4 Establishing the wheelset rotating dynamic model

The wheelset rotating dynamic equation is

\[ J \frac{d\omega_d}{dt} = T - \mu(\lambda)Wgr \] (8)

where $T$ is the driving torque, $R_g$ is the gearbox transmission ratio and $J$ is the wheelset rotational inertia.

3 Designing an adhesion coefficient sliding mode observer

In actual locomotive operation, observing the adhesion coefficients is difficult, though the wheelset speed $\omega_d$ can be measured. Therefore, a state-sliding mode observer can be used to observe the load torque $T_L$. (2) can be used to observe the adhesion coefficients.

The selected wheel speed $\omega_d$ is a state variable with the following state equations:
\[
\begin{align*}
\dot{x}_1 &= \alpha_1 x_1 \\
\dot{x}_2 &= \frac{1}{T} - \frac{1}{T} \dot{R}_g
\end{align*}
\]  
(9)

The sliding mode observer is designed as follows:

\[
\begin{align*}
\dot{x}_1 &= \alpha_1 x_1 \\
\dot{x}_2 &= \frac{1}{T} + \eta \text{sgn}(x_1 - x_2)
\end{align*}
\]  
(10)

where \(\dot{x}_1\) is the observed state value of \(x_1\) and \(\eta\) is a sliding mode gain. The observation error is

\[
e = x_1 - \dot{x}_1
\]  
(11)

The selected sliding mode surface is

\[
s_1 = e
\]  
(12)

Based on state (6) and the designed sliding mode observer (10), for any initial value \(x_0 \in \mathbb{R}^q\), if the sliding mode surface is selected as (12), the adhesion coefficients can be measured by load torque when \(\eta\) is large enough.

According to the Lyapunov stability criterion and sliding mode reaching conditions, the load torque expression is obtained as follows [12]:

\[
\dot{T}_L = -\frac{\eta}{R_g} \text{sgn} e
\]  
(13)

The load torque and adhesion coefficients are related as follows:

\[
\dot{\mu} = \frac{R_g}{W_g \omega} \dot{T}_L
\]  
(14)

The adhesion coefficients can determined indirectly by observing load torque based on (14).

4 Searching the optimal slip ratio of rail surfaces by using the sliding mode extremum seeking-PID compound control algorithm

4.1 Searching optimal slip ratio by using the extremum seeking with sliding mode

According to the relationship between adhesion coefficients and slip ratio, the extremum seeking method [13] with sliding mode is used to search for the optimal estimated value of slip ratio based on adhesion coefficients.

The algorithm principle for searching optimal slip ratio by using the extremum seeking method with sliding mode is shown in Fig. 3.

First, ensure that \(y\) is monotonic with \(g(t)\) with a continuous sliding mode surface and the sinusoidal disturbances signal of the slip ratio is \(\lambda\). The extremum seeking method approaches the optimal slip ratio \(\lambda^*\), which causes the entire system to reach the optimal domain and converge to the optimal value via intra-domain signal interference and oscillation.

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= \mu(\lambda)
\end{align*}
\]  
(15)

where \(x\) is the state variable of the system, \(u(x, \lambda)\) is the control law of the system and \(y\) is the system output.

A reasonable sliding mode surface of slip ratio can be constructed by using:

\[
\begin{align*}
s_2 &= y - g(t) \\
g(t) &= \rho t
\end{align*}
\]  
(16)  
(17)

where \(g(t)\) is the monotone function and \(\rho\) is the constant.

Switching function:

\[
s_2 = \mu(\lambda) - \rho t
\]  
(18)

After the derivation of Formula (18)

\[
s_2 = \frac{d\mu(\lambda)}{dt} \lambda - \rho
\]  
(19)

Let the first derivative of slip ratio be the control law:

\[
\lambda = k \text{sgn} \left( \sin \frac{\pi s_2}{\beta} \right)
\]  
(20)

where \(k\) represent the positive constants, and \(\beta\) is the sine function period. Then

\[
\dot{s}_2 = \frac{d\mu(\lambda)}{dt} \text{sgn} \left( \sin \frac{\pi s_2}{\beta} \right) - \rho
\]  
(21)

where \(\sin \pi s_2/\beta\) takes \(s_2\) as a variable and \(2\beta\) as a period.

When a constant \(k\) exists, the sliding mode reachable condition is

\[
\left| \frac{d\mu(\lambda)}{dt} \right| > \frac{\rho}{k}
\]  
(22)

When a locomotive is running, \(\mu\) increases continuously with the slope \(\rho\) until the extreme value of adhesion coefficients is tracked. \(d\mu(\lambda)/d\lambda\) represents the slope of the adhesion coefficient–slip ratio curve. Moreover, when the value is \(> \rho/k\), the adhesion coefficient continues increasing to approach the curve's maximum point until the above formula cannot be satisfied.

4.2 Sliding mode extremum seeking algorithm with PID compound control

The extremum seeking algorithm with sliding mode several advantages, including a simple control structure, strong robustness and fast system convergence rate. However, it causes the control variables to oscillate around the extreme point. To weaken the oscillation and reduce the loss of locomotive traction power, a sliding mode extremum seeking algorithm combined with PID compound control is designed.

When \(|d\mu/d\lambda| > K\), the extremum seeking control with sliding mode is used to make the system approach the optimal adhesion point quickly. When \(|d\mu/d\lambda| < K\), PID control is used to weaken the oscillation of the sliding mode extremum seeking control near the optimal adhesion point.

The switching conditions for the sliding mode extremum seeking-PID compound control are as follows. The control is automatically switched according to the absolute value \(K\) of the adhesion characteristic curve's slope. Therefore, the system performance is closely related to the selection of the switching point. If it is switched too early, then the advantage of the extremum seeking control with sliding mode cannot be embodied, which may cause a great deviation of the control system. If it is switched too late, then PID control may not be used, but only the
extremum seeking algorithm with single sliding mode will work. Therefore, \(|\mu|/d|\) = K. When a locomotive is on different rail surfaces, it is necessary to reasonably select the absolute value K of the slope of the adhesive characteristic curve so that the locomotive can operate at the optimal adhesion point and the oscillation near this point can be weakened.

5 Designing an integral sliding mode controller with novel reaching law

As the derivation of the integral sliding mode has no second-order terms, the mode is suitable for use as a controller with simple calculations. It also has excellent dynamic performance, anti-interference ability and robustness.

An integral sliding mode controller is designed to fix the slip ratio in optimal position when the locomotive runs on different rail surfaces, such that the adhesion coefficient can reach the optimal value \(\mu_{\text{max}}\). The systematic error is defined as

\[ e = \lambda - \lambda^* \]  

(23)

The integral sliding mode controller uses the switching function to change the control law of the driving torque. The switching surface is selected as

\[ s_1 = c_1 \int edt + e \]  

(24)

where \(c_1\) is a constant \(>0\).

Performing time derivation on (24) derives the following:

\[ \dot{s}_1 = c_1 \dot{e} + e \]  

(25)

To ensure accessible switching surfaces, \(s_1\) must meet the following accessibility conditions:

\[ s_1 \leq 0 \]  

(26)

Integral sliding mode can improve the dynamic quality of the system's normal motion segments by designing various reaching laws. This study adopts the integral sliding mode control with novel reaching law to reduce chattering and reach stability quickly [14].

\[
\begin{aligned}
    s_1 &= -k_3 \arcsin(h(t)) \text{sgn}(s_1) - k_2 s_1 \\
    t &= \infty
\end{aligned}
\]  

(27)

In Formula (27), \(x_i\) is the system state variables, \(s_i\) is the sliding mode surface; \(k_3, k_2, k_1\) and \(b\) are all positive constants; and \(t\) is the time. \(\arcsin(h(\cdot))\) is the inverse hyperbolic sine function whose function value decreases with the decrease of independent variables, in which the closer to zero the independent variable, the greater the slope.

\[ x_i = e \]  

(28)

Substituting (6) and (8) into (27) obtains the driving torque:

\[
T = \frac{J_{\text{el}}^2}{V_1} \left[ - k_3 \arcsin(h(t)) \text{sgn}(s_1) - k_2 s_1 - c_1 e \right] + \dot{\mu} W g r + \frac{J_{\text{el}}^2}{M V_1} (F_i - F_d(t))
\]  

(29)

6 Simulation analysis and experimental verification

A simulation model of the anti-slip adhesion control for heavy-haul locomotives is built in the MATLAB/SIMULINK simulation environment. The control block diagram is shown in Fig. 4.

![Fig. 4 Block diagram of optimal slip ratio control](http://creativecommons.org/licenses/by/3.0/)

Table 2 Model simulation parameters

| Parameters            | Value |
|-----------------------|-------|
| motor rotational inertia \(J, k_1, m^2\) | 434   |
| axle load \(W, t\)     | 30    |
| complete weight \(M, t\) | 6000  |
| wheel radius \(r, m\)  | 0.5   |
| gear ratio \(R_g\)     | 5.2   |
| gravitational acceleration \(g, m/s^2\) | 9.8   |

A contrast simulation is performed to verify the stability of the proposed complex algorithm and attenuate oscillations in extremum seeking it is the anti-slip control based on optimal adhesion points of the extremum seeking with single sliding mode; the other is the anti-slip control based on optimal adhesion points of the sliding mode extremum seeking-PID compound control. Rail surface switching is conducted in the simulation. In the first 10 s, the locomotive runs on the dry rail surface; from 10 to 20 s, it switches to the ordinary rail surface; and from 20 to 30 s, it runs on the wet rail surface. The model simulation parameters are shown in Table 2.

6.1 Anti-slip extremum seeking algorithm with single sliding mode

The sliding mode controller is used to observe adhesion coefficients, and the sliding mode extreme value is used to search for optimal slip ratio in real time. Finally, the integral sliding mode controller is used to make the locomotive run at the optimal adhesion point. Simulation results are shown in Fig. 5. After repeated experiments, the simulation parameters are as follows:

- \(c_1 = 2, k_3 = 5, k_2 = 0.01, b = 1000, k = 8, \beta = 0.35, \rho = 4\)

The simulation results of anti-slip integral sliding mode control based on the extremum seeking with single sliding mode are shown in Fig. 5a. In the figure, the optimal slip ratio of the locomotive on each rail surface is quite high. When the locomotive is on the dry rail surface, the slip ratio quickly approaches the optimal slip ratio. However, after 10 s, when the rail surface switches from dry to ordinary, the slip ratio begins to decrease from 0.28 to 0.17 and then remains unchanged. After 20 s, the slip ratio of the wet rail surface is also slowly stabilised at 0.1. In Figs. 5a and b, the driving torque and adhesion coefficient are generally consistent with the changing trend of the rail surface. However, the chattering of the driving torque is relatively large and the adhesion coefficients are basically stabilised quickly at the optimal adhesion value with little difference compared with practical optimal adhesion values. As shown in the local enlarged drawing of the adhesion coefficient, the chattering is relatively large when the locomotive is entering another rail surface. When the locomotive enters different rail surfaces, the motor control torque decreases rapidly as the adhesion coefficient decreases. The extremum seeking algorithm with sliding mode can search the optimal slip ratio value in real time under the current rail surface condition and adjust the motor control torque in time. Such capabilities ensure that the locomotive always runs in an optimal adhesion state to increase the utilisation of the locomotive's adhesion and make full use of the locomotive's traction power. The locomotive is also prevented from slip under poor rail conditions. The figures illustrate that even if the locomotive is on a wet and low-adhesion rail surface, it can also operate in an adhesion peak area.
To sum up, the designed adhesion coefficient observer can quickly and accurately observe the peak value of adhesion coefficients with good observation performance. Moreover, the extremum seeking algorithm with sliding mode is self-adaptive to the changes of rail surface conditions, enabling it to make full use of the optimal adhesion of locomotives. However, it likewise generates significant chattering near the highest adhesion point, which may not be able to effectively use the locomotive's traction power.

6.2 Sliding mode extremum seeking algorithm–PID compound control for anti-slip

To reduce the adverse effects of high-frequency chattering caused by the extremum seeking with single sliding mode, the said mode is combined with the PID compound control algorithm. The simulation results are shown in Fig. 6, where the simulation parameters are PID controller proportional coefficient $K_P = 100$, integral coefficient $K_I = 5$ and differential coefficient $K_D = 1$.

In Fig. 6a, the extremum seeking with relatively single sliding mode is used, where the oscillation amplitude of the slip ratio on the dry rail surface in the beginning is very small, the convergence speed is fast and the slip ratio reaches the optimal adhesion value within 5 s. On the ordinary and wet rail surfaces, the system reaches the optimal value very quickly. A comparison of Figs. 5b, c and 6b, c shows that the control speed and accuracy of the control torque and adhesion coefficients are greatly improved, whereas the chattering in the system is greatly weakened with fast and smooth changes.

In summary, the designed sliding mode extremum seeking–PID compound control can greatly reduce the chattering in adhesion coefficients, driving torque and slip ratio, as well as has strong robustness and self-adaptive ability against environmental changes.

In addition, it prevents the slip of locomotives and improves the stable operation of locomotive systems.

7 Conclusion

In this study, a sliding mode observer is used to observe the motor load torque and the adhesion coefficients. Then, the extremum seeking algorithm combined with PID control is used to search the optimal slip ratio of the changing rail surfaces. Finally, an integral sliding mode control algorithm with novel reaching law is designed so the system can operate at the optimal adhesion point.

(i) The extremum seeking with sliding mode for optimal slip ratio is an online real-time search. It ensures that locomotives can automatically search for and reach the optimal value under different initial driving conditions so as to adapt to sudden changes in rail surfaces and improve system stability.

(ii) Combining the PID compound control algorithm can effectively reduce oscillations near the optimal slip ratio of the extremum seeking with sliding mode and improve the response speed of the system, thereby improving the control performance of locomotives.

(iii) The integral sliding mode control with novel reaching law makes the locomotive stay in the optimal adhesion point when running on different rail surfaces. This result effectively prevents the locomotives from slip, exerts their traction power and improves the utilisation of their adhesion.

8 Acknowledgments

This work was supported by Natural Science Foundation of China (61773159, 61473117), Hunan Provincial Natural Science Foundation of China (2016JJ5012, 2017JJ4031), Key Laboratory...
for Electric Drive Control and Intelligent Equipment of Hunan Province (2016TP1018). Science and Technology Innovative Research Team in Higher Educational Institutions of Hunan Province.

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