$D$-wave charmonium production in $B$ decays

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Abstract

The calculation of $D$-wave charmonium production rates in $B$ meson decays under the NRQCD factorization formalism is presented. We find that inclusion of the color-octet contributions permits us to detect the $D$-wave charmonium states in $B$ decays at present experimental facilities. The same amount signals of $2^{--}$ $D$-wave state as that of $\psi'$ could be observed at LEP and CESR. We also predict the relative production rates for four $D$-wave states are $\delta^c: \delta_1^c: \delta_2^c: \delta_3^c = 2.5: 3: 5: 7$, where $\delta^c$, $\delta_1^c$, $\delta_2^c$, $\delta_3^c$ represent respectively the $2^{++}$, $1^{--}$, $2^{--}$, $3^{--}$ states.

PACS number(s): 12.25.Hw, 12.38.Lg, 14.40.Nd
I. INTRODUCTION

Studies of heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. In recent years, a rigorous framework for treating quarkonium production and decays has been advocated by Bodwin, Braaten and Lepage in the context of nonrelativistic quantum chromodynamics (NRQCD) [1]. In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. This factorization formalism provides a new production mechanism called the color-octet mechanism, in which the heavy-quark and antiquark pair is produced at short distance in a color-octet configuration and subsequently evolves nonperturbatively into physical quarkonium state. This mechanism is first considered to cancel the infrared divergence in the calculation of $P$-wave charmonium production in $B$ decays [2]. But the most important progress is that by including the color-octet production mechanism one might explain the $\psi'$ ($J/\psi$) surplus measured by CDF at the Tevatron [3]. In the past few years, applications of the NRQCD factorization formalism to $J/\psi(\psi')$ production at various experimental facilities have been studied [4].

Recently, a calculation of $D$-wave charmonium production in $Z^0$ decays suggests a crucial test of color-octet production mechanism [5]. The authors show that the production rates due to color-octet gluon fragmentation to $D$-wave charmonium states are $2 \sim 3$ orders larger than that due to the dominant color-singlet quark fragmentation [6]. They also predict that at the Tevatron the same amount of magnitude of direct production rate of $2^{-}$ $D$-wave charmonium state as $\psi'$ could be observed [7]. All these progresses show that the color-octet mechanism is crucial important to $D$-wave charmonium production because the color-octet contributions are over two orders larger than the color-singlet contributions. While in $S$- or $P$-wave states production, the situations are not so critical. This huge divergences between color-octet and color-singlet contributions in $D$-wave states production are most helpful in distinguishing the two production mechanisms in experiments and will provide a crucial test of color-octet production mechanism. On the other hand, even if the color-singlet model predicts the $D$-wave production rates too small to be visible, they could now be detected after and only after including the color-octet production mechanism. So it is the color-octet mechanism that could make it possible to search for the $D$-wave heavy quarkonium states,
and then will complete the studies of charmonium and bottomonium families.

In this paper, we will calculate $D$-wave charmonium production rates in $B$ decays, as a complementation to previous studies [4] [7], and also as an independent determination of the color-octet matrix elements associated with color-octet $D$-wave charmonium production. Color-singlet $S$- and $P$-wave charmonium production in $B$ decays have been studied in the literatures [8]. The color-octet $S$- and $P$-wave production have also been investigated [2] [3] [10]. From the following calculations, we will show that the color-octet production mechanism is also crucial important to $D$-wave charmonium production in $B$ decays. The rest of the paper is organized as follows. In Set.II, we give the factorization formulas under the NRQCD formalism. In Sec.III, we estimate the branching fractions of $D$-wave charmonium production in $B$ decays by assuming that the values of color-octet matrix elements are taken by the NRQCD velocity scaling rules. We also discuss the detection of $D$-wave charmonium states in $B$ decays at present experimental facilities in this section. A short summary will be given in Sec.IV.

II. FACTORIZATION FORMULA

According to the NRQCD factorization formalism, the partial width for the inclusive production of a given charmonium state $H$ in $b$ decays has the following factorization form,

$$\Gamma(b \rightarrow H + X) = \sum_n \hat{\Gamma}(b \rightarrow c\bar{c}(n) + X) \langle \mathcal{O}_n^H \rangle. \quad (1)$$

Here, $\hat{\Gamma}_n$ is the short-distance subprocess rate for producing a $c\bar{c}$ pair in configuration denoted by $n$ (including the angular momentum $2S+1L_J$ and the color index 1 or 8). $\langle \mathcal{O}_{n}^{H} \rangle$ is the long-distance nonperturbative matrix element demonstrating the probability of the $c\bar{c}$ pair in $n$ configuration evolving into a physical charmonium state $H$. While $\hat{\Gamma}_n$ can be calculated perturbatively as an expansion in coupling constant $\alpha_s(M_b)$, $\langle \mathcal{O}_{n}^{H} \rangle$ is a nonperturbative parameter, and practically can only be determined by fitting to the experimental data. Fortunately, under the framework of NRQCD, the relative size of $\langle \mathcal{O}_{n}^{H} \rangle$ can be roughly estimated by using its scaling property in $v^2$ (controlled by the velocity scaling rules), where $v$ is the typical relative velocity of the heavy quark inside the bound state $H$.

The short-distance subprocesses $b \rightarrow c\bar{c}q$ (with $q = s, d$) are described by the effective Hamiltonian [2]
\[ H_{\text{eff}} = -\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^* \left( \frac{2C_+ - C_-}{3} \bar{c}\gamma_\mu(1 - \gamma_5)c\bar{s}\gamma_\mu(1 - \gamma_5)b \right) 
+ (C_+ + C_-)\bar{c}\gamma_\mu(1 - \gamma_5)T^a\bar{c}\gamma_\mu(1 - \gamma_5)b, \]

where \( G_F \) is the Fermi constant and \( V_{ij} \)'s are KM matrix elements. The coefficients \( C_+ \) and \( C_- \) are Wilson coefficients at the scale of \( \mu = M_b \). To leading order of \( \alpha_s(M_b) \) and to all orders of \( \alpha_s(M_b)\ln(M_W/M_b) \), they are

\[ C_+(M_b) \approx \left[ \frac{\alpha_s(M_b)}{\alpha_s(M_W)} \right]^{-6/23}, \]
\[ C_-(M_b) \approx \left[ \frac{\alpha_s(M_b)}{\alpha_s(M_W)} \right]^{12/23}. \]

In this effective Hamiltonian Eq. (2), the first term provides the color-singlet contribution of charmonium production in \( b \) decays, \( i.e. \), it produces \( c\bar{c} \) pair in a color-singlet state, while the second term produces \( c\bar{c} \) pair in a color-octet state and provides the color-octet contribution.

To \( D \)-wave charmonium production, as argued in [5] [7], the color-singlet and color-octet processes are both scaled with the same orders in \( v^2 \). In \( b \) decays, the color-octet contributions include \( ^1S_0^{(8)} \) subprocess for spin-singlet and \( ^3S_1^{(8)} \) and \( ^3P_1^{(8)} \) subprocesses for spin-triplet \( D \)-wave states. In the factorization formula Eq. (1), the associated matrix elements are all in \( v^7 \), and they have following relations according to the NRQCD velocity scaling rules,

\[ \langle O_1^{\delta^c} (^1D_2) \rangle \sim M_c^7 v^7, \quad \langle O_8^{\delta^c} (^1S_0) \rangle \sim M_c^3 v^7; \]
\[ \langle O_1^{\delta^f} (^3D_J) \rangle \sim M_c^7 v^7, \quad \langle O_8^{\delta^f} (^3S_1) \rangle \sim M_c^3 v^7, \quad \langle O_8^{\delta^f} (^3P_1) \rangle \sim M_c^5 v^7. \]

Here, the symbol \( \delta^c \) represents the physical spin-singlet \( D \)-wave charmonium state, and \( \delta^f_J \) \( (J = 1, 2, 3) \) for spin-triplet \( 1^{--}, 2^{--}, 3^{--} \) states. The notations \( ^{2S+1}D_J \) represent the \( c\bar{c} \) pairs configurations with angular momentum \( L = 2 \). Because the color-singlet and color-octet processes are all in the same orders in \( v^2 \), at leading order in \( \alpha_s \) and \( v^2 \), all these processes must be taken into account for a consistent calculation. The color-singlet matrix elements \( \langle O_1^{\delta^c} (^3D_J) \rangle \) can be related to the second derivative of the nonrelativistic radial wave function at the origin \( |R''_D(0)|^2 \) for \( D \)-wave by

\[ \langle O_1^{\delta^c} (^1D_2) \rangle = \frac{75N_c}{4\pi} |R''_D(0)|^2, \]
\[ \langle O_1^{\delta^c} (^3D_J) \rangle = \frac{15(2J + 1)N_c}{4\pi} |R''_D(0)|^2. \]
The wave function may be evaluated by potential model so that the color-singlet matrix elements are fixed. Whereas the color-octet matrix elements \( \langle O_8^{(3S_1)} \rangle \) and \( \langle O_8^{(3P_1)} \rangle \) are free parameters and must be determined by experiment.

For the spin-singlet \( D \)-wave charmonium state, at leading order in \( \alpha_s \), the color-singlet subprocess can not contribute to its production in \( b \) decays. At leading order in \( v^2 \), the production can only come from the color-octet \( 1S_0 \) subprocess. Using the effective Hamiltonian(2), one can calculate the short-distance coefficient [9], and get

\[
\Gamma(b \to c\bar{c}(1S_0) + s, d \to \delta^c + X) = \frac{3 \langle O_8^{(1S_0)} \rangle}{2M_c^2} (C_+ + C_-)^2 \hat{\Gamma}_0, \tag{9}
\]

with

\[
\hat{\Gamma}_0 = |V_{cb}|^2 \frac{G_F^2}{144\pi} M_b^3 M_c (1 - \frac{4M_c^2}{M_b^2})^2. \tag{10}
\]

Here the relation \( |V_{cs}|^2 + |V_{cd}|^2 \approx 1 \) has been used.

For spin-triplet \( D \)-wave states, at leading order in \( \alpha_s \), the color-singlet process can only contribute to the production of state \( 3D_1 \). Other two states \( (3D_2, 3D_3) \) can not be produced in color-singlet process and can only be produced in color-octet processes.

We first calculate the color-singlet contribution to \( 3D_1 \) charmonium production in \( b \) decays. This color-singlet production rate can be calculated by making use of the covariant formalism [3]. From standard BS wave function in nonrelativistic approximation for vector mesons, the bound state wave function can be project out as,

\[
\Phi(P, \vec{q}) = \sum_{sm} \langle JM|1sLm\rangle J(1 + \frac{P}{M})\psi_Lm(\vec{q}). \tag{11}
\]

To \( D \)-wave function, \( \Phi(P, \vec{k}) \) must be expanded to the second order in the relative momentum \( \vec{q} \). The first and second derivative of the wave function are

\[
\Phi_\alpha(\vec{q}) = \frac{-1}{2M_c^2 M} \sum_{sm} \langle JM|1sLm\rangle M_c \gamma_\alpha \frac{1}{M+P} \psi_Lm(\vec{q}); \tag{12}
\]

\[
\Phi_{\alpha\beta}(\vec{q}) = \frac{-1}{2M_c^2 M} \sum_{sm} \langle JM|1sLm\rangle \gamma_\alpha \frac{1}{M+P} \gamma_\beta \psi_Lm(\vec{q}). \tag{13}
\]

After integrating over the relative momentum \( \vec{q} \), the orbit angular momentum part of the wave function will depend on the radial wave function or its derivatives at the origin \( R_S(0), \)
$R'_p(0)$, and $R''_D(0)$, respectively, for $S$-wave ($L = 0$), $P$-wave ($L = 1$), and $D$-wave ($L = 2$) states,

$$
\int \frac{d^3q}{(2\pi)^3} \psi_{00}(q) = \frac{1}{\sqrt{4\pi}} R_S(0),
$$

$$
\int \frac{d^3q}{(2\pi)^3} q^\alpha \psi_{1m}(q) = i\epsilon^\alpha \frac{3}{4\pi} R'_p(0),
$$

$$
\int \frac{d^3q}{(2\pi)^3} q^\alpha q^\beta \psi_{2m}(q) = \epsilon_m^{\alpha\beta} \sqrt{\frac{15}{8\pi}} R''_D(0),
$$

where the polarization tensor’s label $m$ is magnetic quantum number. For the spin-triplet case where $J = 1, 2, 3$, using explicit Clebsch-Gordan coefficients, there are following relations for three cases \[11\].

$$
\sum_{sm} \langle 1J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_{\rho}^{(s)} = -\frac{3}{20} \left[ (g_{\alpha\rho} - \frac{p_{\alpha\rho}}{4M_c^2}) \epsilon_{\beta}^{(J_z)} + (g_{\beta\rho} - \frac{p_{\beta\rho}}{4M_c^2}) \epsilon_{\alpha}^{(J_z)} \right],
$$

$$
\sum_{sm} \langle 2J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_{\rho}^{(s)} = \frac{i}{2\sqrt{6}M_c} \left( \epsilon_{\alpha\sigma} \epsilon_{\tau\rho\sigma} p^\tau g^{\sigma\sigma'} + \epsilon_{\beta\sigma} \epsilon_{\tau\rho\sigma} p^\tau g^{\sigma\sigma'} \right),
$$

$$
\sum_{sm} \langle 3J_z | 1s2m \rangle \epsilon_{\alpha\beta}^{(m)} \epsilon_{\rho}^{(s)} = \epsilon_{\alpha\beta\rho}^{(J_z)}.
$$

Here, $\epsilon_{\alpha}$, $\epsilon_{\alpha\beta}$, $\epsilon_{\alpha\beta\rho}$ are spin-one, spin-two and spin-three polarization tensors which obey the projection relations \[11\]

$$
\sum_{m} \epsilon_{\alpha}^{(m)} \epsilon_{\beta}^{(m)} = (-g_{\alpha\beta} + \frac{p_{\alpha\beta}}{4M_c^2}) \equiv \mathcal{P}_{\alpha\beta},
$$

$$
\sum_{m} \epsilon_{\alpha\beta}^{(m)} \epsilon_{\alpha'\beta'}^{(m)} = \frac{1}{2} \left[ \mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\beta'} + \mathcal{P}_{\alpha\beta'} \mathcal{P}_{\beta\alpha'} \right] - \frac{1}{3} \mathcal{P}_{\alpha\beta} \mathcal{P}_{\alpha'\beta'},
$$

$$
\sum_{m} \epsilon_{\alpha\beta\rho}^{(m)} \epsilon_{\alpha'\beta'\rho'}^{(m)} = \frac{1}{6} \left[ \mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\beta'} \mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\alpha'} \mathcal{P}_{\beta\beta'} \mathcal{P}_{\beta'\rho} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\beta'} \mathcal{P}_{\rho\rho'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\rho} \mathcal{P}_{\rho'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\rho'} \mathcal{P}_{\rho'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\beta'} \mathcal{P}_{\rho'\rho'} \right]
$$

$$
- \frac{1}{15} \left[ \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\rho'} \mathcal{P}_{\rho'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\rho} \mathcal{P}_{\rho'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\rho'} \mathcal{P}_{\beta'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\beta'} \mathcal{P}_{\rho'\rho'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\rho\rho'} \mathcal{P}_{\beta'\beta'} + \mathcal{P}_{\alpha\beta} \mathcal{P}_{\beta'\beta'} \mathcal{P}_{\rho'\rho'} \right].
$$

Using these relations, the matrix elements $<0| (c \bar{c})_{V-A} |3J \rangle$ can be calculated out,
\[ < 0 | (c \bar{c})_{V-A} |^3 D_1 > = \epsilon_{\mu} \frac{20 \sqrt{3}}{\sqrt{2\pi}} \frac{R_D''(0)}{\sqrt{M^3}}. \]  
\[ (23) \]
\[ < 0 | (c \bar{c})_{V-A} |^3 D_2 > = 0, \]  
\[ (24) \]
\[ < 0 | (c \bar{c})_{V-A} |^3 D_3 > = 0. \]  
\[ (25) \]

Adopting this results, we obtain the partial width for color-singlet \( \delta_1^c \) production in \( b \) decays,

\[ \Gamma(b \rightarrow c \bar{c}(^3 D_1^{(1)}) + s, d \rightarrow \delta_1^c + X) = \frac{25}{6\pi^2} \frac{|R_D''(0)|^2}{M_0^6} M_b^3 (2C_+ - C_-)^2 (1 + \frac{2M^2}{M_b^2}) (1 - \frac{M^2}{M_b^2})^2 \]  
\[ (26) \]
\[ = \frac{5}{9} \frac{\langle O_8^{GS}(^3 D_1) \rangle}{M_c^4} (2C_+ - C_-)^2 (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0. \]  
\[ (27) \]

\[ \Gamma(b \rightarrow (c \bar{c})_s + s, d \rightarrow \delta_1^c + X) = \frac{\langle O_8^{GS}(^3 S_1) \rangle}{2M_c^4} + \frac{\langle O_8^{GS}(^3 P_1) \rangle}{M_c^4} (C_+ + C_-)^2 (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0, \]  
\[ (28) \]
\[ \Gamma(b \rightarrow (c \bar{c})_s + s, d \rightarrow \delta_2^c + X) = \frac{\langle O_8^{GS}(^3 S_1) \rangle}{2M_c^4} + \frac{\langle O_8^{GS}(^3 P_1) \rangle}{M_c^4} (C_+ + C_-)^2 (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0, \]  
\[ (29) \]
\[ \Gamma(b \rightarrow (c \bar{c})_s + s, d \rightarrow \delta_3^c + X) = \frac{\langle O_8^{GS}(^3 S_1) \rangle}{2M_c^4} + \frac{\langle O_8^{GS}(^3 P_1) \rangle}{M_c^4} (C_+ + C_-)^2 (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0. \]  
\[ (30) \]

Adding the color-singlet and color-octet contributions together, for \( \delta_1^c \) production, the partial width is

\[ \Gamma(b \rightarrow \delta_1^c + X) = (1 + \frac{8M^2}{M_b^2}) \hat{\Gamma}_0 \left[ \frac{5}{9M_b^6} \langle O_1^{GS}(^3 D_1) \rangle (2C_+ - C_-)^2 + \frac{\langle O_8^{GS}(^3 S_1) \rangle}{2M_c^4} + \frac{\langle O_8^{GS}(^3 P_1) \rangle}{M_c^4} (C_+ + C_-)^2 \right]. \]  
\[ (31) \]

**III. NUMERICAL ESTIMATION AND DISCUSSION OF THE DETECTION**

In the numerical calculations, we take the input parameters as

\[ M_b \approx M_B = 5.3 GeV, \quad M = 2M_c = 3.8 GeV, \quad \alpha_s(M_b) = 0.20, \quad \alpha_s(M_c) = 0.116, \]  
\[ (32) \]

and then,
$$C_+(M_b) = 0.87, \quad C_-(M_b) = 1.34. \quad (33)$$

By using the standard method [2], we obtain the inclusive branching fractions for $\delta_c$ and $\delta_J$

$$BR(B \to \delta^c + X) = 0.76 \frac{\langle \mathcal{O}_8^{\delta^c (1S_0)} \rangle}{M_c^3}, \quad (34)$$

$$BR(B \to \delta^c_1 + X) = 0.019 \frac{\langle \mathcal{O}_8^{\delta^c_1 (3D_1)} \rangle}{M_c^3} + 1.03 \frac{\langle \mathcal{O}_8^{\delta^c_1 (3S_1)} \rangle + \langle \mathcal{O}_8^{\delta^c_1 (3P_1)} \rangle}{2M_c^3}, \quad (35)$$

$$BR(B \to \delta^c_2 + X) = 1.7 \frac{\langle \mathcal{O}_8^{\delta^c_2 (3S_1)} \rangle}{2M_c^3} + \frac{\langle \mathcal{O}_8^{\delta^c_2 (3P_1)} \rangle}{M_c^5}, \quad (36)$$

$$BR(B \to \delta^c_3 + X) = 2.4 \frac{\langle \mathcal{O}_8^{\delta^c_3 (3S_1)} \rangle + \langle \mathcal{O}_8^{\delta^c_3 (3P_1)} \rangle}{2M_c^3}, \quad (37)$$

where the heavy quark spin symmetry relations for the color-octet matrix elements have been used,

$$\langle \mathcal{O}_8^{\delta^c_j (3S_1)} \rangle \approx \frac{2J + 1}{3} \langle \mathcal{O}_8^{\delta^c_j (3S_1)} \rangle, \quad (38)$$

$$\langle \mathcal{O}_8^{\delta^c_j (3P_1)} \rangle \approx \frac{2J + 1}{3} \langle \mathcal{O}_8^{\delta^c_j (3P_1)} \rangle. \quad (39)$$

From the above results, we can see that the color-octet contributions are crucial important to $D$-wave charmonium production in $B$ decays. The color-singlet contribution is not vanishing only in the case of $J = 1$. Furthermore, in that case, the color-singlet contribution is too small compared with the color-octet contributions, because the color-octet matrix elements are at the same order in $v^2$ as the color-singlet matrix element but the short-distance coefficients in the color-octet terms are about 50 times larger than that in the color singlet term. So, the color-octet contributions are much more important than the color singlet contribution.

The values of color-singlet matrix elements are gotten from the potential model calculation $|R_D''(0)|^2 = 0.015 GeV^7$ [12]. Using the relations Eqs.(8-9), we get

$$\langle \mathcal{O}_1^{\delta^c (1D_2)} \rangle = 0.27 GeV^7, \quad \langle \mathcal{O}_1^{\delta^c (3D_1)} \rangle = 0.16 GeV^7. \quad (40)$$

We estimate the size of the color-octet matrix elements by using the naive NRQCD velocity scaling rules,

$$\left\langle \frac{\mathcal{O}_8^{\delta^c (3S_1)}}{M_c^3} \right\rangle \approx \left\langle \frac{\mathcal{O}_8^{\delta^c (3P_1)}}{M_c^5} \right\rangle \approx \left\langle \frac{\mathcal{O}_8^{\delta^c (3D_1)}}{M_c^7} \right\rangle = 0.0018, \quad (41)$$

$$\left\langle \frac{\mathcal{O}_8^{\delta^c (1S_0)}}{M_c^5} \right\rangle \approx \left\langle \frac{\mathcal{O}_1^{\delta^c (1D_2)}}{M_c^7} \right\rangle = 0.0030. \quad (42)$$
Here, these equation only make sense in the estimate of order of magnitudes (we may also use another estimate for these relations as in Ref. [7], in that case, the results of this paper will be enhanced about one order). Adopting these matrix elements values, the predicted branching fractions for $D$-wave charmonium states are,

\[
BR(B \to \delta c + X) = 0.23\%,
\]

(43)

\[
BR(B \to \delta_1^c + X) = 0.28\%,
\]

(44)

\[
BR(B \to \delta_2^c + X) = 0.46\%,
\]

(45)

\[
BR(B \to \delta_3^c + X) = 0.65\%.
\]

(46)

We can easily find that the relative production rates predicted above are $\delta^c : \delta_1^c : \delta_2^c : \delta_3^c = 2.5 : 3 : 5 : 7$. If we do not take into account the color-octet mechanism the relative rates would be $\delta^c : \delta_1^c : \delta_2^c : \delta_3^c = 0 : 1 : 0 : 0$.

Among the three triplet states of $D$-wave charmonium, $\delta_2^c$ is the most prominent candidate to be discovered firstly. It is a narrow resonance, and its branching fraction of the decay mode $J/\psi\pi^+\pi^-$ is estimated to be \[5\]

\[
B(\delta_2^c \to J/\psi\pi^+\pi^-) \approx 0.12,
\]

(47)

which is only smaller than that of $B(\psi' \to J/\psi\pi^+\pi^-)$ by only a factor of 3. Therefore the decay mode $\delta_2^c \to J/\psi\pi^+\pi^-$ could be observable if the production rate of $\delta_2^c$ is of the same order as $\psi'$. Comparing the predicted production rate of $2^- D$-wave charmonium \[13\] with that of $\psi'$ \[3\], we can see that the former has the same amount of that of the latter. At CESR, the CLEO collaboration have observed strong signals of $\psi'$ by $J/\psi\pi\pi$ triggering \[13\].

Recently, at LEP, the OPAL collaboration have also detected $\psi'$ by the same trigger with higher statistics \[14\]. Therefore, we would expect the detection of $2^- D$-wave charmonium state at these two machines will soon be obtained.

The other two states of the spin-triplet, $\delta_1^c$ and $\delta_3^c$ are above the open channel threshold and are not narrow, and therefore are difficult to detect. However, the analysis of $J/\psi\pi^+\pi^-$ spectrum in $Z^0$ decays performed by the OPAL collaboration shows some events above the background around 3.77$GeV$ besides the events peak at $\psi'(3686)$ \[14\]. If these events are finally confirmed to be associated with the $1^{--}$ $D$-wave charmonium state $\psi(3770)$, they may be mostly from $b$ decays. Moreover, $\psi(3770)$ could also be seen in a $D\bar{D}$ (charmed meson pair) final state.
At the Tevatron, about $10^9 b$ quarks are produced with an accumulated luminosity of $100Pb^{-1}$, which implies more than $10^6 D$-wave charmonium particles could be produced. The estimated combined branching ratio $B(\delta_c^2 \rightarrow J/\psi\pi^+\pi^-, J/\psi \rightarrow \mu^+\mu^-) \approx 7 \times 10^{-3}$ shows that more than $10^3$ events of $2^{--} D$-wave charmonium will be detected at this collider. Furthermore, the combined branching ratio $B(\delta_c \rightarrow 1P_1\gamma, 1P_1 \rightarrow J/\psi\pi^0, J/\psi \rightarrow \mu^+\mu^-)$ has been estimated to be about $10^{-4}$ [15]. This shows that more than $10^2$ events of this particle can also be detected.

IV. SUMMARY

We have calculated $D$-wave charmonium production rates in $B$ decays in this paper. We find that the inclusion of color-octet production mechanism can help us to obtain the $D$-wave states production rates at an observable level. In our calculations, the values of the color-octet matrix elements are taken by assuming that the NRQCD velocity scaling rules are valid in these cases. Practically, these matrix elements can be determined by fitting the theoretical prediction rates to the experimental data. In [3, 7], these matrix elements also appear in the calculations. If $D$-wave charmonium states are detected in the future, all these matrix elements can be extracted from experimental data, and the comparison of the results from different experiments will provide another important test of the universality of the color-octet matrix elements, the NRQCD velocity scaling rules, and further the NRQCD factorization formalism.
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