Search for Lensing Signatures from the Latest Fast Radio Burst Observations and Constraints on the Abundance of Primordial Black Holes

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Abstract

The possibility that primordial black holes (PBHs) form some part of dark matter has been considered for a long time but poorly constrained over a wide mass range. Fast radio bursts (FRBs) are bright radio transients with millisecond duration. Their lensing effect has been proposed to be one of the cleanest probes for constraining the presence of PBHs in the stellar-mass window. In this paper, we first apply the normalized cross-correlation algorithm to search and identify candidates for lensed FRBs in the latest public FRB observations, i.e., 593 FRBs, which mainly consist of the first Canadian Hydrogen Intensity Mapping Experiment FRB catalog, and then derive constraints on the abundance of PBHs from the null search result of the lensing signature. For a monochromatic mass distribution, the fraction of dark matter made up of PBHs could be constrained to \(\lesssim 87\%\) for \(\geq 500\ M_\odot\), at the 95\% confidence level by assuming signal-to-noise ratios dependent on the flux ratio threshold for each FRB and that apparently one-off events are intrinsic single bursts. This result would be improved by a factor of 3 when a conventional constant flux ratio threshold is considered. Moreover, we derive constraints on PBHs with a log-normal mass function naturally predicted by some popular inflation models and often investigated with gravitational-wave detections. We find that, in this mass distribution scenario, the constraint from the current public FRB observations is relatively weaker than the one from gravitational-wave detections. It is foreseen that upcoming complementary multimessenger observations will yield considerable constraints on the possibilities of PBHs in this intriguing mass window.

Unified Astronomy Thesaurus concepts: Radio bursts (1339); Strong gravitational lensing (1643); Primordial black holes (1292)

1. Introduction

The cosmological constant plus cold dark matter (\(\Lambda\)CDM) model has explained the evolution of the universe successfully. The scenario where cold dark matter accounts for about a quarter of the total energy density is well consistent with large-scale-structure observations. However, we still know little about the constituents of dark matter. Primordial black holes (PBHs) (Hawking 1971; Carr & Hawking 1974; Carr 1975), which could form in the early universe via different mechanisms, such as enhanced curvature perturbations during inflation (Clesse & García-Bellido 2015; Pt et al. 2018; Chen & Cai 2019; Fu et al. 2019; Motohashi et al. 2020; Ashoorioon et al. 2021), bubble collisions (Hawking et al. 1982), cosmic string (Hogan 1984; Hawking 1989), and domain wall (Caldwell et al. 1996), have been a source of interest for nearly half a century. One reason for this interest is that the mass of PBHs can range from magnitudes small enough for Hawking radiation to be important to the level of a black hole in the center of a galaxy. In contrast, astrophysical processes can only form black holes heavier than a particular mass (around three solar masses).

Moreover, PBHs have also been a source of great astrophysical interest because they are often considered to constitute some part of dark matter. Observational searches for PBHs have been conducted intensively and continuously over several decades. Numerous methods have been proposed to constrain the abundance of PBHs (usually quoted as the fraction of PBHs in dark matter \(f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{Dark Matter}}\)) in various possible mass windows (see Sasaki et al. 2018; Green & Kavanagh 2021 for a review). These constraints include (in)direct observational effects, such as gravitational lensing (Alcock et al. 2001; Tisserand et al. 2007; Griest et al. 2013; Mediavilla et al. 2017; Zumalacarregui & Seljak 2018; Niikura et al. 2019, 2019; Zhou et al. 2021), dynamical effects on ultralight dwarf galaxies (Brandt 2016; Koushiappas & Loeb 2017), nondetections of stochastic gravitational waves (GWs) (Clesse & García-Bellido 2017; Wang et al. 2018; Chen & Huang 2020; De Luca et al. 2020; Hütsi et al. 2021), disruption of white dwarfs (Graham et al. 2015), null detection of scalar-induced GWs (Chen et al. 2020), and the effect of accretion via cosmic microwave background observations (Chen et al. 2016; Ali-Haimoud & Kamionkowski 2017; Aloni et al. 2017; Bernal et al. 2017; Poulin et al. 2017). Constraints on PBHs lighter than \(\sim 10^{-5} M_\odot\) that have already evaporated via Hawking radiation can be indirectly derived from certain features in the extragalactic and Galactic \(\gamma\)-ray backgrounds (Carr et al. 2016; DeRocco & Graham 2019; Laha 2019; Dasgupta et al. 2020; Laha et al. 2020). In addition to these available probes, some other constraints from near-future observations have been proposed, such as gravitational lensing of GWs (Jung & Shin 2019; Diego 2020; Liao et al. 2020; Urrutia & Vaskonen 2021),...
2. Methods

In this section, we briefly introduce the current status of FRB observations, review the FRB lensing theory, and present the method for searching and identifying lensing signatures.

2.1. Fast Radio Burst Observations

The number of verified FRBs is increasing rapidly at the moment owing to the services of several wide-field radio telescopes, such as the CHIME, the Australian Square Kilometre Array Pathfinder, and the Deep Synoptic Array. In particular, the CHIME/FRB Collaboration has recently released a catalog of 535 FRBs detected in less than one year (2018 July 25–2019 July 1) (CHIME/FRB Collaboration 2021). In this catalog, there are 61 bursts from 18 previously reported repeaters. This first large sample observed in a single survey with uniform selection effects is of great value for facilitating comparative and absolute studies of the FRB population. All of these FRBs together with bursts detected by other facilities have been collected and compiled by the Transient Name Server. At this moment, there are about 593 independent events publicly available.

For a detected FRB, one of the most important observational features is the DM, which is theoretically defined as the integration of the electron number density along the traveled path of the radio pulse and, in observations, obtained by measuring the delayed arrival time of two photons with different frequencies. From observed DMs of the first several bursts, which were poorly localized then (Lorimer et al. 2007; Thornton et al. 2013), the cosmological origins of this kind of mysterious flashes were inferred. This inference has been subsequently confirmed by the localization of the first repeater FRB 20121102A to a nearby dwarf galaxy (Chatterjee et al. 2017; Marcote et al. 2017; Tendulkar et al. 2017). Therefore, the distance and the redshift can be roughly derived from the observed DM of a detected FRB, which is usually decomposed into

$$DM = DM_{MW} + DM_{IGM} + \frac{DM_{host} + DM_{src}}{1 + z},$$

where $DM_{MW}$ is the contribution from the Milky Way, and $DM_{host}$ and $DM_{src}$ are contributions from the FRB host galaxy and source environment, respectively. Here, we conservatively adopt the maximum value of $DM_{host} + DM_{src}$ to be 200 pc cm$^{-3}$, which corresponds to the minimum inferred redshift for all host galaxies. In addition, the $DM_{IGM} – z$ relation is given by Deng & Zhang (2014), where $DM_{IGM} \approx 855 + 5z$ (Zhang 2018), considering He ionization history and the fraction of baryon in the intergalactic medium (IGM) $f_{IGM}$ being 0.83. This relation is statistically favored by the five localized FRBs available at that time (Li et al. 2020). The inferred redshifts for the current public FRBs from different facilities are shown in Figure 1. Another important observational feature of FRBs on lensing scales in our

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7 The recent detection of a Galactic FRB in association with a soft gamma-ray repeater suggests that magnetar engines can produce at least some (or probably all) FRBs (Bochenek et al. 2020; CHIME/FRB Collaboration 2020; Lin et al. 2020; Zhang 2020).

8 https://www.chime-frb.ca/catalog

9 https://www.wis-tns.org
The maximum value of the normalized impact parameter can be found by requiring that the two lensed images are greater than some reference value of the flux ratio \( R_l \): \[
\gamma_{\text{max}}(R_{l,\text{max}}) = \frac{1 + R_{l,\text{max}}}{R_{l,\text{max}}} - 2. \tag{5}
\]

The reference value is usually set as a constant, \( R_{l,\text{max}} = 5 \) (Muñoz et al. 2016; Laha 2020; Liao et al. 2020). Recently, Krochek & Kovetz (2021) suggested that this quantity should be dependent on the signal-to-noise ratio (S/N) of each observed FRB. This is reasonable because both the S/N and flux ratio are crucial for identifying a lensed FRB event. Therefore, these two options are considered in our following analysis. For a given source, the lensing optical depth due to a single PBH is

\[
\tau(M_{\text{PBH}}, f_{\text{PBH}}, z_s, w) = \int_0^{z_s} d\zeta (z_l) n_L(f_{\text{PBH}})(1 + z_l)^2 \times \sigma(M_{\text{PBH}}, z_l, z_s, w) \times \frac{D_L D_{\text{LS}}}{D_S} (1 + z_l)^2 \left[ \gamma_{\text{max}}^2(R_{l,\text{max}}) - \gamma_{\text{min}}^2(M_{\text{PBH}}, z_l, w) \right],
\]

where \( n_L \) is the comoving number density of the lens, \( H_0 \) is the Hubble constant in the present universe, \( H(z_l) \) is the Hubble function at \( z_l, f_{\text{PBH}} \) represents the fraction of PBHs in dark matter, and \( \Omega_{\text{Dark Matter}} \) is the current density parameter of dark matter. Now, for a given distribution function \( N(z_s) \) of FRBs, we can calculate their integrated lensing optical depth \( \tau(M_{\text{PBH}}, f_{\text{PBH}}, w) \) as

\[
\tilde{\tau}(M_{\text{PBH}}, f_{\text{PBH}}, w) = \int d\zeta \tau(M_{\text{PBH}}, f_{\text{PBH}}, z_s, w) N(z_s).
\]

If we observe a large number of FRBs, \( N_{\text{FRB}} \), then the number of FRBs that will be lensed is

\[
N_{\text{lensed FRB}} = (1 - e^{-\tilde{\tau}(M_{\text{PBH}}, f_{\text{PBH}}, w)}) N_{\text{FRB}}.
\]

If none of the FRBs is found to be lensed, then the fraction of dark matter in the form of PBHs can be estimated in Equation (8). Now, the newest event number is 593, which holds a statistical meaning. According to the definition, the
expected number of lensed FRBs can be approximated as the sum of the lensing optical depths of all FRBs ($\tau_i \ll 1$):

$$N_{\text{lensed FRB}} = \sum_{i=1}^{N_{\text{frb}}} \tau_i (M_{\text{PBH}}, f_{\text{PBH}}, z_{S,i}, w_i).$$

(9)

The above formalism is valid if the mass distribution of the PBHs follows a monochromatic mass function. It has been theoretically shown that PBHs can also follow an extended mass distribution function (Carr et al. 2017; Bellomo et al. 2018; Laha 2020). We will consider the log-normal mass function of PBHs, and the log-normal mass function is parameterized as

$$P(m, \sigma, m_c) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left(-\frac{\ln^2(m/m_c)}{2\sigma^2}\right),$$

(10)

where $m_c$ and $\sigma$ give the peak mass of $m(p)$ and the width of the mass spectrum, respectively. This mass function is often a good approximation if PBHs result from a smooth symmetric peak in the inflationary power spectrum. Therefore, it can be representative of a large class of extended mass functions. For a log-normal mass distribution, the lensing optical depth can be expressed as

$$\tau(f_{\text{PBH}}, z, w, \sigma, m_c) = \int dm \int_0^{z_L} d\chi (z) n_L(f_{\text{PBH}}) \times (1 + z_L)^2 \sigma(m, z_L, w) P(m, \sigma, m_c)$$

$$\times \frac{3}{2} f_{\text{PBH}} f_{\text{Dark Matter}} \int dm \int_0^{z_L} d\chi \frac{1}{\sqrt{2\pi} \sigma m} \times \exp\left(-\frac{\ln^2(m/m_c)}{2\sigma^2}\right) \frac{H_0^2}{cH(z_L)} \frac{D_L D_{\Delta S}}{D_S} (1 + z_L)^2$$

$$\times [\gamma_{\text{max}}^2(R_{\text{max}}) - \gamma_{\text{min}}^2(m, z_L, w)].$$

(11)

This calculated value of the optical depth can be used to obtain the integrated optical depth in Equation (7).

### 2.3. Searching and Identifying Lensing Signatures

An FRB strongly lensed by a lens mass greater than $\sim 20 M_\odot$ can be separated into two images with an observable time delay of a few milliseconds, and its light curve will appear as two distinct peaks. In this case, the lensing time delay would be comparable with or greater than the duration of the burst and can be read clearly. To detect this, we define the normalized cross-correlation (NCC) of two peaks (Bracewell 1986; Li & Yang 1993; Li et al. 1996; Gonzalez & Woods 2002; Hirose et al. 2006):

$$C(\delta t) = \frac{\int dt I_1(t) I_2(t - \delta t)}{\sqrt{\int dt I_1^2(t) \int dt I_2^2(t - \delta t)}},$$

(12)

where $I_1(t)$ and $I_2(t)$ are the intensities of the two light-curve peaks. They consist of contributions from both signal $S_0$ and noise $N_0$, i.e., $I_1 = S_1 + N_1$. In a specific lensing configuration (high $S/N$, e.g., $S/N \gtrsim 10$, which is necessary to confirm an FRB signal), the light curve of the first peak $I_1(t)$ should be proportional to the second peak $I_2(t)$ with time delay $\Delta t$ and flux ratio $R_\delta$:

$$I_1(t) \propto R_\delta I_2(t - \Delta t).$$

(13)

It is obvious that, for a lensed FRB, NCC will exhibit spikes at different frequency bins with $C(\delta t = \Delta t) \approx 1$. Although for peaks with high $S/N$ the noise $N_i$ always makes $C(\delta t)$ smaller than unity, we would search the lensed candidates of $C(\delta t = \Delta t) \approx 1$ for confirmed FRB signals with considerable $S/N$.

To test the validity of this method, we first simulate a positive case (i.e., a typical lensed signal) for the above-mentioned NCC analysis. In the simulation, we set the frequency range of 400–800 MHz with 16,384 frequency channels and the time resolution of 0.98 ms, which are consistent with characteristics of recently released CHIME FRBs. The simulated dedispersed bursts that follow the Gaussian profile and its background is injected with a Gaussian white noise. We generate two lensed bursts that have the same frequency emission band and pulse structure but different arrival times and intensities. In addition, the $S/N$ of mocked signals is roughly consistent with the averaged level of the CHIME FRB observations. The frequency–time (“waterfall”) plot of the mocked signal is shown in the left panel of Figure 3. Next, we carry out the NCC analysis for the mocked pulses, and the result is presented in the right panel. As expected, for a typical lensed FRB signal, the values of the NCC peak at the same time delay in different frequency bins, which is consistent with the prediction of gravitational lensing theory. Moreover, the lensing time delay of two pulses can be obtained as $\Delta t = 10$ ms with flux ratio $R_\delta = 2$, and we can infer the redshifted lens mass $M_{\text{PBH}}(1 + z_L) = 725.2 M_\odot$ based on Equation (3). Finally, we quote the Pearson correlation coefficient (PCC: $\rho_{12}$) (Pearson 1896; Dunn & Clark 1974; Rodgers & Nicewander 1988) to evaluate the degree of correlation between two peaks (peak1, peak2). In relation to the range of values of $\rho_{12}$, we can distinguish the following cases:

1. $\rho_{12} \sim (0.7, 1)$: It testifies to a strong positive correlation of the dependent peak1 with the independent peak2.
2. $\rho_{12} \sim (0.3, 0.7)$: It testifies to a moderate positive correlation of the dependent peak1 with the independent peak2.
3. $\rho_{12} \sim (0, 0.3)$: It testifies to a weak positive correlation of the dependent peak1 with the independent peak2.

The value of $\rho_{12}$ also could be negative, and its magnitude testifies to the corresponding degree of negative correlation. The PCCs for each frequency bin of the mocked signal are plotted in the right panel of Figure 3. Because the PCC value is sensitive to the $S/N$ level, the mean of PCCs ($\bar{\rho}_{12}$) for frequency bins with $S/N$ larger than a reasonable threshold is derived and also shown in the right panel of Figure 3. For the simulated lensed signal, we obtain $\bar{\rho}_{12} = 0.863$, and it testifies to a strong positive correlation between these two peaks. That is, lensing signatures could be successfully identified by this algorithm.

Furthermore, we simulate a negative case (i.e., a typical unlensed FRB signal) for comparison. In this case, the background is set to the same as the lensed signal. However, the profiles of the two pulses are significantly different. The first pulse is mocked up as a Gaussian profile but the second one is configured as a complicated hybrid of several Gaussian profiles. Moreover, the flux ratios at different frequency bins are also set to vary randomly. The frequency–time (“waterfall”) plot of the mocked unlensed signal is shown in the left panel of Figure 4. Again, we first carry out an NCC analysis for the mocked pulses, and the result is presented in the right panel. It
is found that the time delay maximizing the NCC value is significantly different for different frequency bins. It implies that a specific lensing time delay could not be derived from these two peaks. Then, the PCCs for each frequency bin of this mocked signal are computed and plotted in the right panel of Figure 4. We obtain $\rho_{12} = 0.156$, which verifies a very weak positive correlation between these two peaks. That is, these results show strong evidence of an unlensed signal, which is in excellent agreement with the input of the simulation. Therefore, we conclude that these algorithms are robust for identifying lensing signatures with observable time delay of $\gtrsim$ few milliseconds from FRB observations.

3. Results

In this section, we first present the search result of lensing signatures from the latest FRB observations. Then, we show constraints on the abundance of PBHs derived from this search result.

3.1. Search Result of Lensing Signatures

As suggested in Muñoz et al. (2016) and Liao et al. (2020), a lensed FRB should appear in the dynamic spectrum as two pulses with almost the same profile and only different from each other in flux magnification and time delay. We have carefully checked the dynamic spectra of the latest 593 independent FRBs. Because we target lensing signatures with observable time delays greater than the duration of the burst, we only focus on FRBs with multiple peaks. In addition to the FRBs reported in Liao et al. (2020), FRB 20121002A, FRB 20121102A (repeating), FRB 20170827A, FRB 20180814A (repeating), FRB 20181123B, and FRB 20181123B, we find dozens of other FRBs with double/multiple-peak structures.
For FRBs listed in Table 1, they have been excluded as lensed candidates because of obvious characteristics that significantly violate predictions of gravitational lensing theories in their dynamic spectra, such as the “frequency drift” phenomenon, impossible flux ratio (i.e., the second pulse is brighter than the first one), and different structures of peaks. For example, the dynamic spectra of FRB 20181019A with three peaks clearly show the “frequency drift” phenomenon as mentioned in Liao et al. (2020). FRB 20190124C is not lensed because the pulses show significant different structures. Therefore, S/N-dependent flux ratio thresholds should be used when we calculate the cross section of each FRB (Krochek & Kovetz 2021).

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**Table 1**

| TNS Name        | Repeater Source Name | Frequency Drift | Impossible Flux Ratio | Different Structures of Peaks |
|-----------------|----------------------|-----------------|------------------------|-------------------------------|
| FRB 20180814B   | ✓                    |                 | ✓                      |                               |
| FRB 20180917A   | ✓                    |                 | ✓                      |                               |
| FRB 20181019A   | ✓                    |                 | ✓                      |                               |
| FRB 20181028A   | ✓                    |                 | ✓                      |                               |
| FRB 20181104C   | ✓                    |                 | ✓                      |                               |
| FRB 20181119D   | ✓                    |                 | ✓                      |                               |
| FRB 20181125A   | ✓                    |                 | ✓                      |                               |
| FRB 20181128A   | ✓                    |                 | ✓                      |                               |
| FRB 20181128C   | ✓                    |                 | ✓                      |                               |
| FRB 20181222A   | ✓                    |                 | ✓                      |                               |
| FRB 20181223A   | ✓                    |                 | ✓                      |                               |
| FRB 20181226A   | ✓                    |                 | ✓                      |                               |
| FRB 20181226B   | ✓                    |                 | ✓                      |                               |
| FRB 20181228D   | ✓                    |                 | ✓                      |                               |
| FRB 20190104A   | ✓                    |                 | ✓                      |                               |
| FRB 20190109A   | ✓                    |                 | ✓                      |                               |
| FRB 20190111A   | ✓                    |                 | ✓                      |                               |
| FRB 20190122C   | ✓                    |                 | ✓                      |                               |
| FRB 20190124C   | ✓                    |                 | ✓                      |                               |
| FRB 20190208A   | ✓                    |                 | ✓                      |                               |
| FRB 20190213B   | ✓                    |                 | ✓                      |                               |
| FRB 20190301A   | ✓                    |                 | ✓                      |                               |
| FRB 20190308C   | ✓                    |                 | ✓                      |                               |
| FRB 20190422A   | ✓                    |                 | ✓                      |                               |
| FRB 20190423A   | ✓                    |                 | ✓                      |                               |
| FRB 20190423B   | ✓                    |                 | ✓                      |                               |
| FRB 20190519A   | ✓                    |                 | ✓                      |                               |
| FRB 20190519B   | ✓                    |                 | ✓                      |                               |
| FRB 20190527A   | ✓                    |                 | ✓                      |                               |
| FRB 20190604F   | ✓                    |                 | ✓                      |                               |
| FRB 20190609A   | ✓                    |                 | ✓                      |                               |
| FRB 20190611A   | ✓                    |                 | ✓                      |                               |
| FRB 20190625E   | ✓                    |                 | ✓                      |                               |

**Table 2**

| TNS Name        | Repeater of FRB | Strong NCC | R_{12} |
|-----------------|-----------------|------------|--------|
| FRB 20181117B   | no              | 0.076      |        |
| FRB 20181125A   | no              | −0.013     |        |
| FRB 2018122E    | no              | 0.007      |        |
| FRB 20181224E   | no              | 0.056      |        |
| FRB 20190131D   | no              | 0.030      |        |
| FRB 20190308B   | no              | 0.044      |        |
| FRB 20190308C   | no              | 0.072      |        |
| FRB 20190411C   | no              | 0.210      |        |
| FRB 20190421A   | FRB 20190303A   | no         | 0.008  |
| FRB 20190501B   | no              | −0.065     |        |
| FRB 20190601C   | no              | −0.075     |        |
| FRB 20190605B   | FRB 20180916B   | no         | 0.102  |

Note. “Strong NCC” means that there is a strong cross-correlation at the same time delay for different frequency bins. R_{12} represents the average value of PCC in the frequency bins with high S/N.

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10 Lensing theories predict that lens potential more complicated than the perfect spherically symmetric case may generate >2 images. In this case, the flux ratio criterion alone is not enough to judge whether or not an FRB is lensed. Therefore, for FRB 20190308C and the first two peaks of FRB 20181125A, we further carefully check them with the NCC and PCC methods, and the results are also shown in Table 2. It also suggests that there is no obvious cross-correlation caused by the lensing effect.
Therefore, after carefully checking characteristic features in their dynamic spectra, we find that FRBs with multiple peaks presented in Table 1 are not lensed candidates.

For FRBs listed in Table 2, their characteristic features in dynamic spectra are not enough to identify them as lensed or unlensed candidates because pulses in each FRB show a similar profile and frequency range. In addition, the second pulse of these bursts is fainter than the first corresponding pulse. This is also consistent with the gravitational lensing prediction. Therefore, we apply the NCC and PCC algorithms to identify whether these sources are lensed or unlensed. For example, the dynamic spectrum, and NCC and PCC analysis results of FRB 20190411C are shown in Figure 5. As suggested in the right panel, there is no strong cross-correlation between two peaks in different frequency bins, and the PCC value $\rho_{12} = 0.210$ also demonstrates a weak correlation between two pulses. In Figure 6, we also present the dynamic spectrum, and NCC and PCC analysis results of another example, FRB 20190605B. There is an obvious correlation between two pulses in the frequency range of 400–500 MHz. However, the maximum of NCC happens at different time delays for different frequency bins. Moreover, the PCC value $\rho_{12} = 0.102$ again verifies a weak correlation between two pulses of this burst. Therefore, according to the NCC and PCC analysis results shown in Table 2, no strong evidence of a lensed candidate has been identified in these FRBs.

3.2. Constraints on $\ell_{PBH}$

As shown in Figure 2, there is a two-dimensional distribution of widths and inferred redshifts. We can find that the inferred redshifts of FRBs tend to be concentrated at low redshifts, $z < 1.5$ in Figure 1, which will be one of the factors that have an important influence on the results. We follow standard operating procedure in this paper for constraints on the abundance of PBHs. Here, we take $R_{f,\text{max}} = (S/N)/10$ and
values of the S/N for all currently available FRBs are presented in Figure 7. For the monochromatic mass function, each \((M_{\text{PBH}}, f_{\text{PBH}})\) corresponds to an expected number of lensed FRB signals according to Equations \((6)\) and \((9)\). Because no lensed signal has been found in the current data, the curve in the \((M_{\text{PBH}}, f_{\text{PBH}})\) parameter space that predicts at least three detectable lensed signals should be ruled out at 95% confidence level. As shown in Figure 8, the mass can be tested down to \(\sim 30 M_\odot\), and \(f_{\text{PBH}}\) is gradually constrained to 87% for the mass range \(\geq 500 M_\odot\) at the 95% confidence level by using S/N-dependent flux ratio thresholds. It should be presented that these constraints could become three times stronger when a commonly used constant flux ratio threshold \(R_{f,\text{max}} = 5\) is considered. In Figure 8, we also show the result of constraints on \(f_{\text{PBH}}\) from the FRB observations without CHIME. It suggests that the first CHIME FRB catalog has greatly improved the constraints on the abundance of PBHs. Although current constraints are relatively weak, especially for small masses, there will be much better constraints from a large number of FRBs with high S/N in the near future (Muñoz et al. 2016; Laha 2020; Liao et al. 2020).

Recently, constraints on PBHs in the stellar-mass range from the Bayesian inference method and the log-normal distribution from the GWTC-1/GWTC-2 GW catalog have been extensively studied (Chen et al. 2019; De Luca et al. 2020; Wu 2020; Hütsi et al. 2021; Wong et al. 2021). Therefore, in order to compare constraints on PBHs from FRB observations with those from GW detections in this intriguing mass window, we next derive projected constraints on \(f_{\text{PBH}}\) from the latest FRB observations assuming that the mass of PBHs follows a log-normal distribution. Results are shown in Figure 9. We assume the parameters \(\sigma = 0.2, 0.6, 1.1\) in the log-normal mass function. Analogously, each \((m_c, f_{\text{PBH}})\) corresponds to an expected number of lensed FRB signals according to Equations \((9)\) and \((11)\). Because no lensed signal has been found in the current data, the region above the curve that predicts at least one detectable lensed signal in the \((m_c, f_{\text{PBH}})\) parameter space should be ruled out. We find our results at \(m_c \leq 100 M_\odot\) are still weaker compared with the results from GW detections (De Luca et al. 2020; Hütsi et al. 2021).

4. Conclusions and Discussions

FRBs are one of the most mysterious phenomena in astrophysics. Although we do not yet understand the formation mechanisms of these bursts, some of their unique observational features make them promising probes for astrophysical and cosmological purposes. Meanwhile, the recent detection of GWs from mergers of binary stellar-mass black holes has stimulated great interest in PBHs of this mass range. The gravitational lensing effect of transients with millisecond duration and high event rate, e.g., FRBs, has been put forward as one of the cleanest probes for exploring the properties of PBHs in the mass range \(1–100 M_\odot\). In this paper, we first propose using the normalized cross-correlation and Pearson correlation coefficient algorithms for identifying lensed signatures in FRB observations. Next, we have carefully checked all events with multiple peaks by comparing their observational properties, such as dynamic spectra, morphologies of the pulses, and the flux ratio, with lensing theory predictions. Most of them have been ruled out as lensed FRB candidates. For the
remaining ∼10 FRBs whose waterfalls apparently look like lensed events, we further apply the NCC and PCC algorithms find evidence of them as lensed signals. The tests suggest that there is no strong evidence of a correlation between the peaks of these FRBs. As a result, we conclude that there is no lensing candidate with a time delay greater than the duration in currently available FRB observations.

On the basis of the null search result, we derive direct constraints on the abundance of PBHs from the latest FRB observations and obtain that, for a monochromatic mass distribution, the abundance of PBHs is constrained to \( \leq 87\% \) and \( \leq 26\% \) when an S/N-dependent and a constant flux ratio threshold are considered, respectively. These constraints have been significantly improved owing to the inclusion of the first CHIME FRB catalog. Moreover, we also investigate the constraints by considering a log-normal mass distribution, which is naturally predicted by some popular inflation models, to compare the results constrained from FRB observations with those from GW detections. Although this constraint is weaker compared with the results from GW detections, it will be significantly improved with the rapid increase in the number of FRBs detected by wide-field surveys (like CHIME and DAS-2000) in the near future. As a result, there would be significant overlap between the areas constrained from FRB observations and those from GW detections in the parameter space of the mass distribution. Then, it will be possible to jointly constrain the abundance and mass distribution of PBHs by combining these two kinds of promising multimessenger observations. It is foreseen that these joint constraints will be of great importance for exploring the nature of PBHs in the star-mass range or even their formation mechanisms relating to the physics of the early universe.
