Propagation acoustic signal in the multifractional gas suspension

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Abstract. The dispersion and dissipation of the acoustic signal in a multifraction gas suspension is investigated. A closed linearized system of differential equations of the perturbed motion of a multifraction gas suspension and the analytic dependence of the complex wave number on the perturbation frequency are presented. Using the fast Fourier transform method, calculations were performed on the propagation of pulsed perturbations in the considered media. The influence of the parameters of the dispersed phase on the dispersion and dissipation of the acoustic signal is analyzed.

1. Introduction
Interest in the study of multiphase media is associated with their wide distribution in nature and technology and is dictated by industrial and environmental needs. Dispersed mixtures can be separated from a variety of heterogeneous media, which are a mixture of several phases, one of which is various inclusions (droplets, solid particles - aerosols, gas suspensions, etc. In the monograph [1], various problems of mechanics and thermophysics of multiphase media of various structures are considered, the basic equations are given, and methods for describing the interfacial interaction are considered. The work [2] is devoted to the problems of studying two-phase flows with solid particles, drops and bubbles. In [3], the problems of the formation of regions of increased concentration of the dispersed phase in multiphase flows were considered. The monograph [4] is an introduction to the mechanics of dispersed mixtures of bubbles, droplets and solid particles with a gas or liquid. For their description, a unified approach is used, methods for describing the movement of inclusions with regard to the interfacial exchange of momentum and heat are described in detail. Particular attention is paid to the problems of attenuation and dispersion of acoustic waves, several main points of view are discussed. The fundamentals of the developed theory of the propagation of plane acoustic waves in gas mixtures with vapor and liquid drops from a unified standpoint of the mechanics of multiphase media are described in the monograph [5]. The main attention is paid to the study of the influence of phase transformations on the processes of dispersion and dissipation of disturbances. Mathematical models are proposed, the most common dispersion relations are derived, some special cases are analyzed, areas of applicability are considered. For the first time, the dynamics of pulsed waves of small amplitude in monodisperse vapor-gas-droplet mixtures was studied in [6]. Wave-type evolution equations that describe the propagation of linear waves in monodisperse suspensions with phase transitions are obtained and analyzed. In [7], the anomalous effect of the nonmonotonic dependence of
the dissipation of weak harmonic and pulsed perturbations on the mass concentration of droplets \( m \) in monodisperse aerosols with heat and mass transfer was studied. It has been established that in a certain range of variation of \( m \) and perturbation frequencies, a decrease in perturbation attenuation is observed with increasing concentrations of droplets, which are the source and main cause of wave dissipation. The propagation of spherical and cylindrical waves of small amplitude in polydisperse fogs with phase transformations was first considered in [8]. In [9], the propagation of acoustic waves of various geometry in two-fraction gas suspension with particles of different materials and sizes without taking into account phase transformations was studied. The propagation features of plane, cylindrical, and spherical waves of small amplitude in vapor-gas-droplet mixtures with solid particles are analyzed in [10], [11], [12].

This paper discusses the propagation of an acoustic signal in multifraction gas suspension.

2. Basic equations, dispersion equation and impulses.

Similar to [5], the linearized system of equations of disturbed motion of a multifractional polydisperse gas suspension, in the coordinate system relative to which the unperturbed medium is at rest, can be written as:

\[
\begin{align*}
\frac{\partial \rho'_i}{\partial t} + \rho_{i0} \left( \frac{\partial \nu'_i}{\partial x} + \frac{v'_i}{r} \right) &= 0, \quad \frac{\partial \rho'_i}{\partial t} + \rho_{i0} \left( \frac{\partial \nu'_i}{\partial x} + \frac{v'_i}{r} \right) = 0, \quad (j = 2, N, i = 1, M_j) \\

\rho_{i0} \frac{\partial \nu'_i}{\partial t} + \rho_{i0} \sum_{j=2}^{N} n_{j0} f_{j2} - &\sum_{j=2}^{N} n_{j0} f_{ij}, \quad (j = 2, N, i = 1, M_j) \\

\frac{\partial T'_i}{\partial t} &= \frac{1}{\rho_{i0} c_{\gamma1}} \frac{\partial \rho'_i}{\partial t} - \sum_{j=2}^{N} n_{j0} T'_j - T'_i, \quad (j = 2, N, i = 1, M_j) \\

\frac{\partial T'_j}{\partial t} &= -\sum_{j=2}^{N} n_{j0} f_{j2} - T'_j, \quad (j = 2, N, i = 1, M_j) \\

\sum_{j=2}^{N} n_{j0} \left( \frac{T'_j - T'_i}{\tau_{ij}} \right) + \sum_{j=2}^{N} n_{j0} \left( \frac{T'_j - T'_i}{\tau_{ij}} \right) &= 0, \quad \bar{\chi}_j = \frac{c_{\gamma1}}{c_{\gamma1}}, \quad (j = 2, N, i = 1, M_j) \\

p'_j &= \frac{C_i}{\gamma_1} \rho'_j + \frac{p_0}{T_0} T'_j, \\

f_{ji} &= \frac{4}{3} \pi r_j \nu_j, \quad f_{ji} &= 6 \pi r_j \nu_j, \quad f_{ji} = B_{ji} \int_{-\infty}^{t} (v_i - v_j) \frac{d\tau}{\sqrt{t - \tau}}, \quad B_{ji} = 6 r_j \sqrt{\pi \nu_j \rho_{i0}} \\

\rho_{i0} &= \alpha_{i0} \rho_{i0}, \quad \alpha_{i0} = \frac{4}{3} \pi r_j \nu_j, \quad (j = 2, N, i = 1, M_j), \quad \alpha_{i0} + \sum_{j=2}^{N} \alpha_{i0} = 1
\end{align*}
\]

The system of equations (1) for the parameter \( \theta = 0 \) describes plane waves in a Cartesian coordinate system, when \( \theta = 1 \) - cylindrical waves in a cylindrical coordinate system, when \( \theta = 2 \) - spherical waves in a spherical coordinate system. Here and further, Variables with index "1" refer to the carrier phase, with index "ji", \( (j = 2, N, i = 1, M_j) \) – to a particle of the \( j \)-th type of the \( i \)-th radius, \( r_j \) - radius of inclusions, \( n_{j0} \) – the number of particles of the \( j \)-th type of the \( i \)-th radius per unit volume, \( T_i \) – temperature carrier phase, \( T_{ji} \) – temperature in the near-surface \( \Sigma \) layer of a particle of the \( j \)-th type of the \( i \)-th radius, \( T_{ji} \) – the temperature of the \( j \)-th type particle of the \( i \)-th radius, \( \nu_{ji} \) and \( \beta_{ji} \) - dimensionless (Nusselt number) and dimensional coefficients of heat transfer of the carrier phase with the \( j \)-th type particle interface of the \( i \)-th radius, \( \nu_{ji} \) and \( \beta_{ji} \) - dimensionless (Nusselt number) and
dimensional heat exchange coefficients of the \( j \)-th type \( i \)-th radius of the interface, \( \lambda \) – the coefficient of thermal conductivity.

Assuming that the system of equations (1) has a non-trivial solution, the following analytical dependence of the complex wave number on the perturbation frequency is obtained:

\[
\left( \frac{C_iK_m}{\omega} \right)^2 = V(\omega)D(\omega),
\]

where

\[
V(\omega) = 1 + \sum_{j=2}^{N} \sum_{i=1}^{M} m_{ji} \left( 1 - i\omega \tau_{ji} \right), \quad D(\omega) = 1 + \left( \gamma_1 - 1 \right) \sum_{j=2}^{N} \sum_{i=1}^{M} \frac{m_{ji}}{1 - i\omega \tau_{ji}}.
\]

We study the features of the propagation of an acoustic signal in a multifractional gas suspension. In the framework of linear analysis in accordance with the principle of superposition, an acoustic signal with any predetermined degree of accuracy can be represented as the sum of superimposed harmonic waves, the characteristic frequencies of which lie in a certain interval. The effects associated with the inharmonicity of the acoustic signal can be defined as the sum of the effects created separately for each of the harmonic components. Due to the difference in the phase velocities of the harmonic components and the different attenuation coefficients of the disturbances of different frequencies, the shape of the initial signal will change during propagation, while its amplitude will decrease.

The evolution of an random space-time acoustic signal is described by the formula [5]

\[
p(x,t) = \int_{-\infty}^{\infty} P(\omega) \exp\left[ i K_m(\omega)x - \omega t \right] d\omega
\]

where the spectral function \( P(\omega) \) is located at a given initial signal

\[
P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} p(0,t) \exp\left[ i\omega t \right] d\omega
\]

Formula (3) defines the inverse Fourier transform of the function \( P(\omega) \), formula (4) defines the direct Fourier transform of the function \( p(0,t) \).

3. Results.

As an example, we consider the propagation of an acoustic signal in a three-fractional mixture of air with particles of sand, ice and aluminum at a temperature \( T_0 = 273 \) K and pressure \( p_0 = 0.1 \) MPa and the following thermal and physical parameters for the carrier gas - \( \rho_0 = 1.19 \) kg/m\(^3\), \( C_i = 343 \) m/s, \( c_{\mu_1} = 1007 \) m\(^2\)/s\(^2\)-K, \( \lambda_1 = 0.0258 \) kg\(\cdot\)m/s\(^3\)-K, \( \mu_1 = 1.81 \times 10^{-3} \) kg/m\(\cdot\)s, \( \gamma_1 = 1.4 \), for aluminum particles - \( r_a = 10^{-6} \) m, \( \rho_{2a} = 2700 \) kg/m\(^3\), \( c_{2a} = 896 \) m\(^2\)/s\(^2\)-K, \( \lambda_{2a} = 209 \) kg\(\cdot\)m/s\(^3\)-K, for ice particles - \( r_i = 10^{-5} \) m, \( \rho_{2i} = 1000 \) kg/m\(^3\), \( c_{2i} = 4180 \) m\(^2\)/s\(^2\)-K, \( \lambda_{2i} = 0.602 \) kg\(\cdot\)m/s\(^3\)-K, for sand particles - \( r_s = 8 \times 10^{-6} \) m, \( \rho_{2s} = 1630 \) kg/m\(^3\), \( c_{2s} = 795.492 \) m\(^2\)/s\(^2\)-K, \( \lambda_{2s} = 0.00301496 \) kg\(\cdot\)m/s\(^3\)-K.
Figure 1. Influence of mass content on the evolution of a plane impulsive perturbation of rectangle form.

In fig. 1-6 show the effect of a three-fractional composition of the mixture and a different process geometry on the evolution of a Gaussian, rectangular and triangular acoustic signal in a mixture of air with aluminum particles, sand and ice with a mass content of aluminium particles $m_a = 0.15$, sand particles - $m_s = 0.2$ and ice particles - $m_i = 0.25$ (line I), with a mass content of aluminium particles $m_a = 0.2$, sand particles - $m_s = 0.25$ and ice particles - $m_i = 0.35$ (line II), with a mass content of aluminium particles $m_a = 0.25$, sand particles - $m_s = 0.3$ and ice particles - $m_i = 0.35$ (line III). The calculated profiles are constructed at a distance of 4 m and 8 m from the place of initiation of pulse, respectively. The calculations were carried out using the relation (2), using the fast Fourier transform routines [13].

The figures show that the impulse, which initially had a rectangular or triangular shape, when distributed under the influence of dispersion and dissipation takes a bell-shaped form or a Gaussian curve, which corresponds to the analysis given in [5] for a monodisperse suspension.
Figure 2. Influence of mass content on the evolution of a plane impulsive perturbation of triangle form.

Figure 3. Influence of mass content on the evolution of a plane impulsive perturbation of Gaussian form.
Figure 4. Influence of mass content on the evolution of rectangular impulse perturbations in the cylindrical case

Figure 5. Influence of mass content on the evolution of triangular-shaped impulse perturbations in the cylindrical case
Figure 6. Influence of mass content on the evolution of a pulsed perturbation of a Gaussian form in the spherical case.

The figures show that an increase in the mass content of inclusions leads to both stronger attenuation and a stronger change in the shape of the pulsed disturbance, due to the greater dispersion of the sound velocity and wave dissipation.

In fig. 7 shows influence of the three-fractional composition of the dispersed phase on the evolution of the acoustic signal in a mixture of air with particles of aluminium, sand and ice (I) with a mass content of aluminium particles $m_a = 0.1$, sand particles $m_s = 0.1$ and ice particles $m_l = 0.1$, for a two-fraction gas suspension with aluminium particles $m_a = 0.15$ and sand particles $m_s = 0.15$ (II), and for a monodisperse air mixture with ice particles $m_l = 0.3$ (III).
Figure 7. Evolution of a plane impulse perturbation of a Gaussian form in a three-fractional gas suspension with aluminum, sand and ice (I) particles, a two-fraction gas suspension with aluminum and sand (II) particles, monodisperse gas mixtures with ice (III) particles.

Conclusion.

This paper presents a closed system of linear differential equations of motion for a multifractional gas suspension. An analytical dependence of the complex wave number on the perturbation frequency is derived. Using the fast Fourier transform method, calculations were performed for the propagation of impulsive perturbations in the dispersed systems considered. The influence of the multi-fractional nature of the medium and the form of a pulsed disturbance on its attenuation is shown. It is established that the presence of inclusions of different materials and sizes significantly affects the dynamics of the acoustic signal in multi-fractional gas mixtures, which must be taken into account when developing methods of acoustic diagnostics of considered media.

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