Mathematical modeling of dynamic loads on the ground robotic complex of special purpose

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Abstract. A special technique to study the impact of the discharge on the ground robotic complex equipped with small arms in this article is proposed. The development of the technique is based on mathematical modeling of the dynamic system of the robotic complex. The magnitude and nature of dynamic loads on the ground robotic complex are determined. The experimental sample of the ground robotic complex equipped with small arms has been developed by the authors. Measures to ensure a stable position of the complex and methods of compensation for uncontrolled changes in the position of the complex under dynamic loads are proposed. To study the processes of firing, their individual components are considered: the problem of internal ballistics, the process of movement of the retractable parts of the barrel, dynamic loads on the complex. The conducted study gives the chance to create highly effective samples of special equipment. These are, in particular, military ground-based robotic complexes, which will be competitive by increasing the efficiency of their operation.

1. Introduction

Ground robotic complexes equipped with small arms are effective military means. They are widely used in various military operations. Therefore, the study of robotic complexes of this type is relevant.

The issue of developing ground-based robotic systems equipped with small arms is important to ensure the national security of Ukraine.

The issue is related to the important tasks of efficiency increase of the Armed Forces of Ukraine which are of key importance for the national security and defense of Ukraine.

2. Analysis of recent research and publications

Modern ground-based robotic systems are considered and their features are identified in recent studies and publications [1]. Small size of the complexes (about 1 m) and low mass are the cause of complex dynamic processes that take place in them [2]. Ground-based robotic systems are characterized by limited rigidity of the bearing system and drives [3]. It causes possible decrease in the accuracy of positioning systems [4]. Damping systems are recommended in some cases to increase accuracy [5]. The main reason for the low accuracy of the complexes is the uncertain conditions of their support on the ground [6]. It is recommended to increase the accuracy of weapon positioning and, accordingly, the efficiency of its use by special control systems [7].

Issues of guidance accuracy and stability of the position are basic in the creation of military ground-based robotic systems [8]. Effective experimental methods are used to study the accuracy of
robots [9]. It is accepted that the rational method of research is mathematical modeling [10]. Effective methods of modeling high-frequency shock processes have been developed [11].

These results do not take into account the specifics of the dynamic loads on the robotic complex during the shot. Therefore, they have limited application.

A significant number of studies have been devoted to the analysis of loads in artillery systems [12]. Experimental research and modeling are carried out [13]. Methods of experimental measurements and equipment are described in separate publications [14]. Methods of modeling shock loads during shot are given [15]. A significant number of works is devoted to solving problems of internal and intermediate ballistics [16]. Criteria for estimating impact loads on a moving object have been proposed [17]. Some works contain a description of special devices to simulate the action of the shot [18]. Methods for estimating impulse (shock) loads on an object are described [19]. They determine the displacement of an object under the action of irregular shock loads [20]. The significant amount of research does not contain practical methods and tools to ensure the accuracy of military ground-based robotic systems.

3. Defining the purpose and objectives of the study

Thus, one of the previously unresolved parts of the overall problem is to ensure the required firing accuracy of military ground robotic systems. Research in this area is carried out mainly by means of experimental methods. It requires complicated tests in a special firing range and it is not appropriate. Therefore, the aim of the research is mathematical modeling of dynamic processes during the shot of a military ground-based robotic complex and determination of dynamic loads on a ground-based robotic complex.

The tasks of research are based on solving the problem of internal ballistics to determine the kinematic and dynamic parameters of the shot on a robotic complex in the shape of mutually agreed kinematic and dynamic propellers.

4. The basic part of the study

Ground robotic complexes equipped with small arms, as a rule, have a compact design (Figure 1).

Complex and multidimensional dynamic processes occur during firing. Both simplified (elementary) and special methods are used to describe them. Dynamic processes determine the nature and magnitude of dynamic loads on a mobile robotic complex. Therefore, their definition is the main and mandatory factor in the reliability of the complex. Dynamic processes during firing are manifested in the action of kinematic and dynamic force factors.

Dynamic action during firing leads to limited oscillating movements in the dynamic system of the complex. The basic ones are transverse and angular displacements corresponding to the spherical
motion of the complex according to the center of mass C and are described by means of angular coordinates $\psi$, $\theta$, $\phi$. Finally, it leads to transverse and angular displacements of the barrel and equivalent displacements of the dual cut in the plane perpendicular to the axis of the barrel.

To study the processes of firing, their individual components are considered: the problem of internal ballistics; the process of movement of the recoiling parts of the barrel; dynamic loads on the complex. The task of internal ballistics is to determine the kinematic parameters of the bullet in the barrel (Figure 2).

To determine the time change of the dynamic action of a moving bullet, a dynamic calculation of the displacement of the bullet was performed. A simplified scheme of bullet movement in the barrel channel is applied. It was assumed that the force of resistance is proportional to the velocity of the bullet, and the pressure ($P = Re = \text{const}$) is constant in the first approximation. The equation of movement of the bullet in the barrel within the accepted assumptions:

$$m \frac{d^2x}{dt^2} = P_e \cdot f \cdot l(t) - b \frac{dx}{dt},$$  \hspace{1cm} (1)

where $m$ - mass of the bullet; $x$ - displacement of the bullet along the barrel; $P_e$ - equivalent constant value of pressure in the barrel channel; $f$ - the effective area of the end of the bullet which is affected by powder gases; $l(t)$ - single stepped function; $b$ - equivalent coefficient of resistance of the bullet when it moves in the barrel; $t$ - time.

Simplified equation (1) is quite approximate, but it allows you to describe the workflows that accompany the passage of the bullet through the barrel.

Equation (1) is reduced to the standard form by entering the designation:

$$\frac{dx}{dt} = V,$$  \hspace{1cm} (2)

$$\frac{m}{b} = T_v,$$  \hspace{1cm} (3)
Thus, the equation will take the form:

\[ T_V \frac{dV}{dt} + V = K_p \cdot l(t), \]  

\[ T_C \frac{d^2x}{dt^2} + \frac{dx}{dt} = \frac{p_e f}{b} - \frac{F_0}{bV_K}. \]  

Solution of the equation:

\[ V = V_k \left( 1 - e^{-\frac{t}{T_C}} \right), \]  

\[ e^{-\frac{t}{T_C}} = 1 - \frac{V}{V_k}. \]  

Time displacement is determined by integration (Figure 3):

\[ x = \int_0^t V(t) dt = V_c t + T_C V_k \left( e^{-\frac{t}{T_C}} - 1 \right) = V_k \left( t + T_C \left( e^{-\frac{t}{T_C}} - 1 \right) \right). \]  

Figure 3. Time dependence of the bullet displacement in the barrel:

The parameters of the movement of the recoiling parts of the barrel are determined while considering the reciprocating and transverse and angular oscillating motion of the moving parts. We assume linear dynamic models of the minimum level of complexity in the first approximation (see Figure 2b, c).

Dynamic models correspond to the reciprocating displacements of moving parts in the direction \( \xi \) and torsional displacements in the shape of rotation at an angle \( \psi \).

The initial conditions through the velocity \( V_0 \) of mass \( M \) and angular velocity of rotation of the moving parts at the initial moment are determined for dynamic models. They are found from the conditions of momentum conservation and momentum of motion speed

\[ m_k V_k = MV_0, \]  

\[ I_k \omega_k = I \omega_0, \]  

where \( m_k \) is the mass of the bullet and the moment of inertia of the bullet; \( t M, I \) is mass and moment of inertia of moving parts.
Accordingly, the initial conditions for dynamic models are defined at t=0, ξ=0, ψ=0 as
\[ \frac{d\xi}{dt} = V_0 = \frac{m_t V_k}{M}, \quad \frac{d\psi}{dt} = \omega = \frac{I_t \omega_k}{I}. \]

Differential equations of displacement of moving parts at a shot are made:
\[ m \frac{d^2\xi}{dt^2} + b \frac{d\xi}{dt} + c \xi = \delta(t), \quad (12) \]
\[ I \frac{d^2\psi}{dt^2} + b_{\psi} \frac{d\psi}{dt} + c_{\psi} \psi = \delta(t), \quad (13) \]
where b, c are coefficient of resistance and equivalent stiffness of the moving parts of the barrel during recoil; b_{\psi}, c_{\psi} are coefficient of resistance and torsional rigidity of the moving parts of the barrel; δ(t) is delta function.

These equations are transformed to the standard form:
\[ T_{\xi} \frac{d^2\xi}{dt^2} + 2 \xi T_{\xi} \frac{d\xi}{dt} + \xi = K_{\xi} \delta(t), \quad (14) \]
\[ T_{\psi} \frac{d^2\psi}{dt^2} + 2 \psi T_{\psi} \frac{d\psi}{dt} + \psi = K_{\psi} \delta(t), \quad (15) \]
where \( T_{\xi} = \frac{M}{C}, \xi = \frac{b}{2\sqrt{MC}}, \quad K_{\xi} = \frac{1}{C}, \quad T_{\psi} = \frac{I}{C_{\psi}}, \psi = \frac{b_{\psi}}{2\sqrt{IC_{\psi}}}, \quad K_{\psi} = \frac{1}{C_{\psi}} \) the dynamic load acting on the part of the moving parts on the complex will depend on the displacement of the moving parts and the dynamic displacements of the complex.

We will assume the dynamic process of firing to be infinitely fast in comparison with the oscillating process of spatial displacements of the complex as a whole in the first approximation.

The reciprocating and transverse and angular displacement of the moving parts during firing will be determined from the solution of differential equations under the introduced initial conditions. The solution of the equations is as following:
\[ \xi = \frac{K_{\xi}}{T_{\xi} \sqrt{1-\xi^2}}, \quad \psi = \frac{K_{\psi}}{T_{\psi} \sqrt{1-\psi^2}}, \quad e = e^{-\xi t} \]
\[ = \frac{1}{T_{\xi}} \cdot \sin \left( \sqrt{1-\xi^2} \right), \quad e^{-\psi t} \]
\[ = \frac{1}{T_{\psi}} \cdot \sin \left( \sqrt{1-\psi^2} \right). \quad (16) \]

Displacement of the recoiling parts of the table cause a dynamic effect on the complex.
Dynamic loads acting on the complex have the character of a power screw "dynamo", which includes force \( \vec{F} \) and torque \( \vec{M} \). Dynamo parameters are determined on the basis of the dependence
\[ F = C_{\xi} + b \frac{d\xi}{dt}, \quad (18) \]
\[ M = C_{\psi} \psi + b_{\psi} \frac{d\psi}{dt}. \quad (19) \]
After substitution of values of reciprocating and transverse and angular displacements and their derivatives in these formulas, we obtain similar expressions of the load vector

\[ F(t) = \xi(t) + 0.1 \cdot V \xi(t) \]

They are oscillating processes (Figure 4).

**Figure 4.** Relative dynamic loads on the complex caused by the shot: a - module-vector of force directed along the axis of the barrel; b - torque module acting relative to the axis of the barrel

The force factor in the shape of a dynamic screw changes during the shot according to the diagram (Figure 5).

**Figure 5.** Hodograph of the interdependence of the components of the dynamic screw of the module-vector of force \( F \) and torque \( M \)

5. Conclusions

The nature of the spatial displacement of the ground robotic complex under the action of impulse loads due to the shot is established. Measures to ensure a stable position of the complex and methods of compensation for uncontrolled changes in the position of the complex under dynamic loads are proposed. The conducted study gives the chance to create highly effective samples of special equipment, military ground robotic complexes, in particular.

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