Constraints on a Stochastic Background of Primordial Magnetic Fields with WMAP and South Pole Telescope data

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We constrain a stochastic background of primordial magnetic fields (PMF) by its contribution to the cosmic microwave background (CMB) anisotropy angular power spectrum with the combination of WMAP 7 year and South Pole Telescope (SPT) data. The contamination in the SPT data by unresolved point sources and by the Sunyaev Zeldovich (SZ) effect due to galaxy clusters has been taken into account as modelled by the SPT collaboration. With this combination of WMAP 7 yr and SPT data, we constrain the amplitude Gaussian smoothed over 1 Mpc scale of a stochastic background of non-helical PMF to $B_{1\text{Mpc}} < 3.5 \text{ nG}$ at 95% confidence level, improving on previous bounds. Our analysis shows that SPT data up to $\ell = 3000$ bring an improvement of almost a factor two with respect to results with previous CMB high-$\ell$ data. We then discuss the forecasted impact from unresolved points sources and SZ effect for PLANCK capabilities in constraining PMF.

I. INTRODUCTION

Current CMB anisotropy measurements lead to upper limits on the amplitude of a stochastic background of primordial magnetic fields generated before nucleosynthesis [1,5]. Indeed, a stochastic background of PMF generates all types of magnetized linear perturbations [7,8]: scalar [1,7,12,14], vector [15–17] and tensor [16,18,19] and all these contribute to the CMB anisotropy pattern in temperature and polarization. CMB constraints on PMF with the angular power spectrum agree with those from their effect on the reionization epoch [20]. PMF modelled as a fully inhomogeneous component have also a non zero higher statistical moments, which can be used as useful probes, such as the magnetized bispectrum [21,22] and the magnetized trispectrum [23].

In our previous works [3,7,12] we have refined the computation of magnetized CMB anisotropies. In Ref. [3] we have computed the constraints coming from CMB data by WMAP7 in combination with data from ACBAR [24], QUaD [25] and BICEP [26] updating previous investigations [1,2,5].

In this work we use the publicly available CMB anisotropy data at high multipoles as those from the South Pole Telescope (SPT) [27,28] to further constrain a stochastic background of PMF. Constraints on PMF from CMB anisotropies at high multipoles, $\ell \sim 3000$, are not a straightforward extension of those derived at larger angular scales. Small angular scale are in fact polluted by extragalactic contamination [28,30,31] and secondary anisotropies, such as Sunyaev-Zeldovich [37,38]. In order to fully exploit small scale CMB data to constrain PMF it is necessary to model the residual foreground contamination to the angular power spectrum.

II. STOCHASTIC BACKGROUND OF PMF AND MAGNETIZED CMB ANISOTROPIES

We follow the same methodology used in our previous papers to compute the PMF contribution to CMB anisotropies. We model a stochastic background of PMF as a fully inhomogeneous component with a power-law power spectrum $P_B(k) = A k^{n_B}$, where $A$ is the amplitude and $n_B$ is the spectral index with $n_B > -3$. Our convention for the Fourier transform of the two point correlation function for a stochastic background is:

$$\langle B_i(k) B_j^*(k') \rangle = (2\pi)^3 \delta(k - k') \delta_{ij} - k_i \bar{k}_j \frac{P_B(k)}{2}$$

We assume the MHD limit in which $B(x, \tau) = B(x)/a(\tau)^2$ with $a(\tau)$ being the scale factor (normalized to $a_0 = 1$ today) and $\tau$ the conformal time. As convention, we use the amplitude of the magnetic fields smoothed over $\lambda$ as a sampling parameter:

$$B_\lambda^2 = \int_0^{\infty} \frac{dk k^2 e^{-k \lambda^2}}{4\pi^2 \lambda^{n_B+3}} P_B(k)$$

This smoothed amplitude on a scale of 1Mpc can be easily connected with measurements of magnetic fields in clusters of galaxies, but we will also discuss the implications of our results for alternative definitions of the amplitude of the stochastic background of PMF. PMF survive the Silk damping but are damped on smaller scales by radiation viscosity [15,29]. We model this damping with a sharp cut off in the power spectrum at the scale $10^3$ Mpc:

$$k_D = \alpha \left( \frac{B_\lambda}{\text{nG}} \right)^{\frac{1}{n_B+5}} \left( \frac{2\pi}{\lambda/\text{Mpc}} \right)^{-\frac{1}{n_B+3}} \frac{h_{n_B+5}}{\text{Mpc}^{-1}}$$

where $\alpha = (2.9 \times 10^4)^{1/n_B+5}$. 

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A stochastic background of PMF acts as a fully inhomogeneous source to metric scalar, vector and tensor perturbations. The source terms are given by the PMF energy momentum tensor and Lorentz Force in Fourier space which are convolution of the PMF. In [16, 18].

In [3, 7], we presented the analytical exact results for the PMF EMT spectra for specific values of space which are convolution of the PMF [16, 18]. To calculate the PMF contribution to CMB anisotropies in a continuous range of \( n_B \) we will use the approximations of [3]. We use the initial conditions for cosmological fluctuations as given in [3, 7]. For scalar perturbations we consider the compensated mode described in [3, 7, 12]. The scalar magnetized perturbations are the dominant PMF contribution on small angular scales, whereas the vector magnetized perturbations represent the dominant PMF contribution on small angular scales. On these scales the primary CMB is suppressed by the Silk damping, making the vector magnetic mode the dominant contribution. To constrain the PMF amplitude we neglect the tensor contribution, since it is subdominant with respect to scalar and vector ones [3, 7].

III. ASTROPHYSICAL CONTAMINATION OF CMB DATA ON SMALL ANGULAR SCALES

It is well understood and proved that CMB data are a fundamental tool to constrain a stochastic background of PMF. Considering the nature of their impact on CMB angular power spectrum it is obvious that the resolution of the data tighter should be the constraints on PMF. But data on small angular scales are also affected by contamination from astrophysical sources. In particular for SPT data the astrophysical contamination is given by residual extragalactic point sources and cluster of galaxies. Both radio and infrared galaxies contribute with a Poissonian term which is due to their random distribution in the sky. The Poissonian term is simply given by a flat angular power spectrum whose amplitude is determined by the source number counts integrated in flux densities and the flux density detection threshold [31–33]. Infrared galaxies together with the Poissonian term contributes also with a clustering term. The clustering is much more complex than the Poissonian term and can be modelled in different ways with increasing complexity [30, 31, 34, 35]. The galaxy clusters contribute with the Sunyaev-Zeldovich effect (SZ) which can be divided into thermal [37] and kinetic [38] contributions. In the case of SPT data both contributions have been considered in a single SZ term [27].

For our analysis we use the templates given by the SPT collaboration for the 150 GHz data [27, 39]. The templates are characterized by one amplitude parameter each, therefore we account for three new parameters in the analysis. The amplitudes of the templates of SPT data are obtained from the SPT own measurements of the extragalactic and SZ contributions on very small angular scales (3000 < \( \ell < 9000 \)) where the angular power spectrum is completely dominated by astrophysical contamination [28]. In Fig. 1 we show the comparison between the total astrophysical contribution predicted for SPT and the total magnetic contribution.

IV. RESULTS

In the present work we derive the constraints on PMF performing a combined analysis of the WMAP 7 year [40, 41] and SPT data following Ref. [27]. We use the latest WMAP likelihood code (version v4p1) and associated data available at [http://lambda.gsfc.nasa.gov/]. We modify the WMAP likelihood by excluding the temperature bandpowers between \( \ell = 800 \) and 1200. We use the SPT data release relative to the observation of 790 square degrees of the sky at 150 GHz during 2008 and 2009. The data spans the \( \ell \) range from 650 to 3000. In order to decrease the correlations between the two data sets we excluded the SPT bandpowers for \( \ell < 800 \) and we used WMAP 7 years data in temperature up to \( \ell = 800 \).

We develop an extension of CosmoMC [42] in order to compute the Bayesian probability distribution of cosmological parameters, including the magnetic ones. In order to use the small scale SPT data we introduced the contribution of astrophysical contaminations following the scheme given by the SPT collaboration [27]. We modified the code following the procedure given in [39].

We vary the baryon density \( \omega_b = \Omega_b h^2 \), the cold dark matter density \( \omega_c = \Omega_c h^2 \) (with \( h \) being \( H_0/100 \text{km s}^{-1} \text{Mpc}^{-1} \)), the reionisation optical depth \( \tau \) (not to be confused with the conformal time \( \tau \)), the ratio...
of the sound horizon to the angular diameter distance at decoupling $\theta$, $\ln(10^{10} A_s)$, $n_S$ and the magnetic parameters $B_{1\text{Mpc}}$ (in units of 10 nG) and $n_B$. As priors we use $[0,10]$ for $B_{1\text{Mpc}}/(10 \text{nG})$ and $[-2.9,3]$ for $n_B$ (>−3 in order to avoid infrared divergencies in the PMF EMT correlators). Together with cosmological and magnetic parameters we varied also the parameters describing the astrophysical residual contributions which are associated with the three templates for astrophysical contributions: $D_{3000}^\text{PS}$, $D_{500}^\text{PS}$, $D_{500}^\text{Cl}$. We use the prior $[0,100]$ for the three astrophysical parameters.

We assume a flat universe, a CMB temperature $T_{CMB} = 2.725 \text{K}$ and we set the primordial Helium fraction to $y_{\text{He}} = 0.24$. We restrict our analysis to three massless neutrinos (a non-vanishing neutrino mass leads to a large scale enhancement in the power spectrum of CMB anisotropies in the presence of PMF [8] and would not change our results). The pivot scale of the primordial scalar was set to $k_*= 0.05 \text{ Mpc}^{-1}$. In order to match the data we lensed the primary CMB angular power spectrum using the lensing tool included in CosmoMC, we have not considered the lensing of magnetized angular power spectrum. We sample the posterior using the Metropolis-Hastings algorithm [13] generating four parallel chains and imposing a conservative Gelman-Rubin convergence criterion [14] of $R−1<0.01$.

The results of the analysis performed with the combination of WMAP 7 and SPT data show constraints on cosmological parameters in agreement with the ones obtained in [27] since the PMF contribution does not modify the constraints on standard parameters [3]. In Fig. 2 we show the marginalized posterior probabilities for the PMF parameters; we obtain $B_{1\text{Mpc}} < 3.5 \text{ nG}$ and $n_B < −0.3$ at 95% confidence level. The magnetic parameters are not degenerate with the astrophysical ones as shown in the two dimensional plots in Fig. 3.

We note the improvement given by SPT with respect to our previous analysis with WMAP 7 and a combination of small angular scale data [3] which included ACBAR [24], BICEP [20] and QUaD [23]. We considered ACBAR [24] data up to $\ell=2000$ with constraints: $B_{1\text{Mpc}} < 5.0 \text{ nG}$ and $n_B < −0.1$ at 95% confidence level.

Similar CMB constraints - of the order of 6 nG at 95% confidence level - with similar data sets were obtained in [3].

We now discuss the implications of our results for alternative definitions of the amplitude of the stochastic background of PMF. The mean square magnetic field defined as:

$$\langle B^2 \rangle = \int_0^{k_D} dk \frac{k^2}{2\pi^2} P_B(k) = \frac{A}{2\pi^2(n_B+3)} k_D^{n_B+3} \quad (4)$$

has also been used as an effective amplitude to be compared with observations [42, 46]. This alternative definition is a non-local quantity, strongly dependent on the damping scale and unrelated to local astrophysical measurements, but useful in the context of nucleosynthesis [47]. We derive the WMAP 7 + SPT constraint $\sqrt{\langle B^2 \rangle} < 29 \text{ nG}$ for the choice of $k_D$ in Eq. (3) such CMB constraint is 30 times tighter than the one derived from Big Bang Nucleosynthesis, i.e. $\sqrt{\langle B^2 \rangle} < 840 \text{ nG}$ [49]. Another possible definition for the amplitude of the stochastic background of PMF which takes into account
We have derived the constraints on a stochastic background of PMF by using the CMB temperature anisotropy measurements at high multipoles by SPT. This study is motivated by the fact that the PMF contribution to CMB anisotropies is not suppressed by Silk damping as the primary anisotropies.

In order to not introduce biases in the magnetic parameter constraints we need to consider the contamination by astrophysical residuals of the SPT data. The dominant contributions are given by unresolved point sources and the SZ effect for the galaxy clusters. We have considered both Poissonian and clustering terms for point sources and the SZ effect for the galaxy cluster contribution. To model the contributions to the angular power spectrum of the three signals we...
have used the templates provided by the SPT collaboration [27,29]. We performed a MCMC analysis with the eleven cosmological, magnetic and astrophysical parameters and we constrain $B_{1\text{Mpc}} < 3.5 \text{nG}$. The results do not show any strong degeneracy between magnetic and astrophysical parameters which is compatible with the multipole range of SPT data ($\ell_{\text{max}} \sim 3000$) used. Comparing these results with the previous constraints with data by WMAP7, ACBAR, QUAD and BICEP [3,5], which were of the order of $B_{1\text{Mpc}} < 5 \text{nG}$, we note a drastic improvement in the constraint on $B_{1\text{Mpc}}$ with the use of SPT data. We have shown how the current CMB constraints for our choice of $k_D$ are by far tighter than those derived from BBN for all the $n_B$ considered here.

We have also updated the expected constraints from Planck by including the astrophysical contamination at small angular scales following the treatment in [30]. The results we obtained show a (expected) degradation of the constraints on PMF due to the presence of extragalactic contributions: $B_{1\text{Mpc}} < 3.6 \text{nG}$, compared to the previous constrain: $B_{1\text{Mpc}} < 2.4 \text{nG}$ (obtained taking into account only noise and sensitivity). The results presented here confirmed a previously noted trend which prefer negative $n_B$. Since $n_B > 0$ is mainly related to causal generation mechanism, we have shown again how causal fields are allowed with an amplitude much smaller than the nGauss level.

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