Super-Resolution Geomagnetic Reference Map Reconstruction Based on Dictionary Learning and Sparse Representation

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ABSTRACT Geomagnetic matching navigation plays a vital role in the navigation and guidance field, and the construction of geomagnetic reference maps plays a significant role in geomagnetic matching navigation. This paper addresses the problem of generating high-resolution geomagnetic reference maps with limited measured data. Most existing methods can hardly satisfy the accuracy requirements. We base our study on the theory of image super-resolution reconstruction and approach this problem from the perspective of dictionary learning and sparse representation. We propose a method of sparse dictionary initialization based on prior information from rectangular harmonic analysis, and then the K-SVD algorithm is applied to the dictionary training process to improve the performance of the sparse dictionary. Three components of the geomagnetic field are considered for multi-channel sparse representation to enhance the quality of the constructed maps. Experimental results show that our proposed method outperforms other geomagnetic reference map construction methods in the reconstructed precision as well as the robustness to noise.

INDEX TERMS Dictionary learning, geomagnetic reference map, image super-resolution, sparse representation.

I. INTRODUCTION

The mainstream development direction for aircraft navigation technology in recent years has been the satellite navigation and inertial navigation technology [1]–[3]. However, the anti-interference ability of satellite navigation is poor, and the error of inertial navigation accumulates over time. The geomagnetic field changes slowly with time and is not affected by climate and region. Hence, geomagnetic navigation technology has gradually attracted attention as an auxiliary navigation method owing to its strong anti-interference ability [4], [5].

Geomagnetic matching navigation technology uses real-time acquired geomagnetic field data to match the pre-prepared geomagnetic reference map of the target region to determine the location of vehicles [6]. Therefore, the accuracy of the geomagnetic reference map construction plays a decisive role in the performance of geomagnetic navigation [7]. The steps of geomagnetic navigation are shown in Fig. 1. Firstly, the geomagnetic information of target areas is measured to build geomagnetic reference maps in advance. Secondly, flight vehicles measure the real-time geomagnetic strength when they fly over these areas. Finally, they can be positioned by matching their real-time geomagnetic strength with the geomagnetic maps. There are two main methods for constructing geomagnetic reference maps. One is based on the existing physical model of the geomagnetic field, and the other is constructing a gridded geomagnetic reference map based on measured geomagnetic data [8]. The existing International Geomagnetic Reference Magnetic Field (IGRF) and World Magnetic Model (WMM) describe the earth’s primary magnetic field [9]. Since the geomagnetic field model relies on satellite monitoring data and the anomalous field inside the earth is attenuated as the altitude increases, the geomagnetic field model contains a significant deviation from real-time measured geomagnetic data. Geomagnetism based on measured data is more reliable than the geomagnetic field model. Previous work has only focused on interpolation methods for geomagnetic reference map construction. Commonly used methods include cubic spline interpolation, Kriging interpolation, and particle swarm optimization Kriging (PSO-Kriging) interpolation [10]. Although the existing algorithm...
can improve the peak signal-to-noise ratio of the reference map, it is easy to produce an excessively smooth result, and it is difficult to recover the details after smoothing. Therefore, it is essential to improve the resolution of the geomagnetic reference map.

Sparse representation has the potential to extract the textures of signals and is widely used in image processing. Inspired by prior image super-resolution via sparse, we have proposed a high-precision geomagnetic reference map-building methodology based on sparse representation and dictionary learning (SR-GRM). The main contributions of this paper are as follows:

1. The design and implementation of a high-precision geomagnetic reference map-building methodology based on sparse representation and dictionary learning.
2. The method of sparse dictionary initialization based on prior information from rectangular harmonic analysis and experimental results indicate that the proposed method can generate a more qualitative sparse dictionary with a small training set size.
3. The precision of geomagnetic reference map reconstruction for SR-GRM with bicubic interpolation, neighbor embedding, and PSO-Kriging interpolation are compared.
4. The noise robustness of the proposed method is compared with different kinds of other super-resolution methods. The experimental results illustrate that our method can outperform other methods in reducing the impact of noise.

This paper is organized as follows. In section II, we review the relevant research in the field of single image super-resolution, especially sparse representation, and dictionary learning. A detailed description of the proposed SR-GRM algorithm is discussed in section III. In section IV, experimental results demonstrate the efficacy of SR-GRM as a priori for geomagnetic reference map building. Finally, some concluding remarks are given in section V.
In this paper, the three independent components of the geomagnetic field are chosen as geomagnetic north, geomagnetic east, and vertically downward, and are denoted as X, Y, and Z, respectively. Fig. 2 shows the features of the geomagnetic field. Since the total field strength is the main component of interest in geomagnetic navigation, this paper uses the three independent components to reconstruct high-resolution geomagnetic reference maps and then combines them into the total field strength for performance evaluation of our method.

Fig. 3 shows the framework of our solution which includes the three procedures below. (1) Geomagnetic information from the United States Geological Survey (USGS) and International Geomagnetic Reference Field (IGRF) is prepared as data set. Dictionary learning method is used to extract features of the data set. (2) The LR reference maps are constructed by meshing the real-time measured data through Kriging interpolation method. (3) The HR reference maps are reconstructed based on the theory that they have the same sparse codes of given features with the corresponding LR reference maps.

Based on the discussion above, geomagnetic reference maps can be constructed using the concept of super-resolution image reconstruction. The algorithm proposed in this paper includes the following two parts: sparse representation and dictionary learning.

A. SINGLE CHANNEL SPARSE REPRESENTATION

In this paper, the low-resolution reference map, Y, is obtained by blurring and downsampling the high-resolution map X (Eq. (1)).

\[ Y = HLX. \] (1)

where \( H \) is the downsampling matrix, and \( L \) is the blurring filter. It can be seen from Eq. (1) that super-resolution reconstruction is an ill-conditioned problem [21]. Therefore, Eq. (1) is an underdetermined system, and each map corresponds to an infinite number of solutions. Nevertheless, the problem can be solved well under the following constraints.

The first constraint is that the small patches \( x \) of the high-resolution map \( X \) can be sparsely represented by a redundant dictionary \( D_h \), which can be formulated according to Eq. (2).

\[ x \approx D_h \alpha \text{ for some } \alpha \in \mathbb{R}^K \text{ with } |\alpha|_0 \ll K, \] (2)

where \( \alpha \) is the sparse representation coefficient of the HR patch \( x \). Since the LR patch and the corresponding HR patch have the same textures, two dictionaries \( D_l \) and \( D_h \) need to be trained, and the LR and the corresponding HR patches can be sparsely represented with the same coefficients. In other words, a sparse representation of input LR patch \( y \) is obtained with respect to \( D_l \), and the corresponding output HR patch \( x \) will be generated by multiplying \( D_h \) by \( \alpha \). The sparsest representation of the HR patch \( y \) can be acquired from Eq. (3).

\[ \min ||\alpha||_0 \text{ s.t. } ||FD_l \alpha - Fy||_2^2 \ll \varepsilon, \] (3)

where \( F \) is a feature transformation matrix. It is used to extract the high-frequency components of the LR map. Since the high-frequency content can better distinguish the target area from others, \( F \) works as a high-pass filter to rebuild the lost high-frequency components in the HR map.

Although the \( l^0 \)-norm minimization problem in (3) is NP-hard [21], it can be effectively recovered by minimizing the \( l^1 \)-norm while \( \alpha \) are sufficiently sparse. Equation (3) can be reformulated as Eq. (4).

\[ \min ||\alpha||_1 \text{ s.t. } ||FD_l \alpha - Fy||_2^2 \ll \varepsilon. \] (4)

Equation (4) can be equivalently formulated with Lagrange multipliers as Eq. (5).

\[ \min \frac{1}{2} ||FD_l \alpha - Fy||_2^2 + \lambda ||\alpha||_1, \] (5)

where \( \lambda \) balances the residue of the sparse representation to \( y \) and sparsity of coefficients \( \alpha \). Equation (5) is essentially a convex optimization problem and has been excellently solved.

Solving the problem in (5) alone does not guarantee consistency between adjacent map patches. Therefore, when the formula (5) is satisfied, it is also necessary to add a restriction condition that the overlapping portions between the reconstructed adjacent high-resolution image blocks are...
to be consistent. The optimized problem can be expressed by
Eq. (6).
\[ \min \| \alpha \|_1 \quad \text{s.t.} \quad \| F D_l \alpha - F y \|_2 \leq \varepsilon_1 \]
\[ \| P D_h \alpha - \omega \|_2 \leq \varepsilon_2, \quad (6) \]
where \( P \) is the matrix for extracting the overlapping region between
the previously reconstructed HR map and the current patch, and \( \omega \) is composed
of the overlapping part of the previously reconstructed HR map. The optimization in
Eq. (6) can be reformulated as Eq. (7).
\[ \min \lambda \| \alpha \|_1 + \frac{1}{2} \| \tilde{D} \alpha - \tilde{y} \|_2^2, \quad (7) \]
where \( \tilde{D} = \begin{bmatrix} F D_l \\ \beta P D_h \end{bmatrix} \) and \( \tilde{y} = \begin{bmatrix} F y \\ \beta \omega \end{bmatrix} \). \( \beta \) is set to balance the
proximity of the low-resolution input and compatibility with its neighbors. The parameter \( \beta \) is set to 1 in our experiments.
An optimal high-resolution map can be reconstructed by calculating
the solution \( \alpha^* \) from Eq. (7). The final optimization problem can be formulated as Eq. (8).
\[ X^* = \arg \min \| X - X_0 \|_2^2 \quad \text{s.t.} \quad DBX = Y, \quad (8) \]
where \( X^* \) is the optimal reconstructed high-resolution map.

The proposed algorithm of sparse representation is shown in
Fig. 4.

B. MULTI-CHANNEL SPARSE REPRESENTATION
Considering that the high-frequency parts of the HR patches of
the three geomagnetic signal channels are similar, the correlation
between the three channels may be enforced according to
Eq. (9).
\[ \| S_n x_{\mu} - S_n y_\gamma \|_2^2 < \varepsilon_{\mu \gamma}, \quad \mu, \gamma \in \{ n, e, v \}, \mu \neq \gamma, \quad (9) \]
where \( n, e, \) and \( v \) indicate the three channels of the geomagnetic
signal, and the \( S \) matrix is a high-pass filter. For example,
\( S_n x_{e} \) illustrates the high-frequency part in the geomagnetic
north component of the high-resolution map. Rather than considering the sparse codes for different channels independently, they can be jointly determined by reformulating
Eq. (7) as Eq. (10).
\[ [\alpha_n, \alpha_e, \alpha_v] = \arg \min \sum_{\mu \in \{ n, e, v \}} \frac{1}{2} \| D_n \alpha_n - y_\mu \|_2^2 + \lambda \| \alpha \|_1 \]
\[ + \tau [\| D_n \alpha_n - D_e \alpha_e \|_2 \]
\[ + \| D_n \alpha_n - D_v \alpha_v \|_2 \]
\[ + \| S_n D_h \alpha_n - S_e D_h \alpha_e \|_2 \]
\[ + \| S_n D_h \alpha_n - S_v D_h \alpha_v \|_2 ] \quad (10) \]
where the cost function can be formulated as Eq. (11).
\[ L_1 = \sum_{\mu \in \{ n, e, v \}} \left[ \frac{1}{2} \| D_n \alpha_n - y_\mu \|_2^2 + \lambda \| \alpha \|_1 \right] 
+ 2 \tau [\alpha_n^T D_n^T S_n^T S_n D_h \alpha_n] \]
\[ + 2 \tau [\alpha_e^T D_e^T S_e^T S_e D_h \alpha_e] \]
\[ + 2 \tau [\alpha_v^T D_v^T S_v^T S_v D_h \alpha_v]. \quad (11) \]
For simplicity, the regularization parameters \( \lambda \) and \( \gamma \) for all
channels are set to be the same. The high-pass filters \( S_n, S_e \) and \( S_v \) are also assumed to be the same for each channel. It should
be noted that Eq. (11) is reduced to three independent sparse representation
problems when \( \tau = 0 \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Flowchart of the proposed algorithm.}
\end{figure}

In contrast to the single-channel sparse representation problem, the sparse codes in Eq. (10) are no longer independent. Therefore, Eq. (11) presents a more challenging
optimization problem. Next, with the inspiration of [12], [25], a compliant solution for the new problem is proposed.

The following matrices and vectors are defined:
\[ \alpha = \begin{bmatrix} \alpha_n \\ \alpha_e \\ \alpha_v \end{bmatrix}_{3m \times 1} \]
\[ y = \begin{bmatrix} y_n \\ y_e \\ y_v \end{bmatrix}_{3p \times 1}, \quad P = \begin{bmatrix} 0 & 0 & I \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}_{3m \times 3m} \]
\[ D_n = \begin{bmatrix} D_{n_1} & 0 & 0 \\ 0 & D_{n_2} & 0 \\ 0 & 0 & D_{n_3} \end{bmatrix}_{3p \times 3p}, \quad D_h = \begin{bmatrix} D_{h_1} & 0 & 0 \\ 0 & D_{h_2} & 0 \\ 0 & 0 & D_{h_3} \end{bmatrix}_{3p \times 3p} \]
\[ S_n = \begin{bmatrix} S_n & 0 & 0 \\ 0 & S_n & 0 \\ 0 & 0 & S_n \end{bmatrix}_{3p \times 3p}, \quad P_s = \begin{bmatrix} 0 & 0 & I \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}_{3p \times 3p} \]
where \( \alpha_c \) and \( y_c \) constitute a concatenation of sparse codes and LR patches, respectively, in the three channels. \( P \) and \( P_s \) are shifting matrices. The sparse code length of each channel is \( m \), and the size of the HR patches is \( p \). \( D_h \) and \( D_{hs} \) are dictionaries that include the dictionaries of the three channels in their block diagonals. To simplify the formulation, we define \( D_{hs} \) as Eq. (12).

\[
D_{hs} = \begin{bmatrix}
D_{h}^{T} S_{c}^{T} S_{c} D_{hs} & 0 & 0 \\
0 & D_{h}^{T} S_{c}^{T} S_{c} D_{hs} & 0 \\
0 & 0 & D_{h}^{T} S_{c}^{T} S_{c} D_{hs}
\end{bmatrix},
\]

and the cost function can be rewritten as Eq. (13).

\[
L_1 = \frac{1}{2} \|D_h \alpha_c - y_c\|_2^2 + \lambda \|\alpha_c\|_1 + 2 \tau \alpha_c^{T} D_{hs}^{T} S_{c} D_{hs} \alpha_c - 2 \tau \alpha_c^{T} P^{T} D_{hs} \alpha_c
\]

\[
= \alpha_c^{T} \left[ \frac{1}{2} D_{h}^{T} D_{h} + 2 \tau D_{hs}^{T} S_{c} D_{hs} - 2 \tau P^{T} D_{hs} \right] \alpha_c
\]

\[
- y_c^{T} D_{h} \alpha_c + \frac{1}{2} y_c^{T} y_c + \lambda \|\alpha_c\|_1.
\]

For simplification, we define \( D \) as Eq. (14).

\[
D = \frac{1}{2} D_{h}^{T} D_{h} + 2 \tau D_{hs}^{T} S_{c} D_{hs} - 2 \tau P^{T} D_{hs},
\]

and the optimization of \( \alpha_c \) is written as Eq. (15).

\[
\alpha_c^{*} = \arg \min_{\alpha_c} \alpha_c^{T} D \alpha_c - y_c^{T} D \alpha_c + \lambda \|\alpha_c\|_1.
\]

The problem in Eq. (15) is a convex sparsity constrained optimization, which can be solved with the algorithm in [26].

### C. DICTIONARY INITIALIZATION

Before dictionary training, prior geomagnetic information is used for dictionary initialization. Since the collected geomagnetic field information is not equally spaced [7], this paper first models the geomagnetic field of the target area and then grids according to the modeling results represented by Eq. (16).

\[
B_R = B_0 - B_C,
\]

where \( B_R \) is the residual geomagnetic field value, \( B_0 \) is the observed value for the geomagnetic field, and \( B_C \) is the theoretical value calculated from the international geomagnetic reference field (IGRF).

The residual magnetic field values of the target areas can be determined from Eq. (16). In a prior study, a model of the geomagnetic field is obtained by using spherical harmonic analysis (SHA) and rectangular harmonic analysis (RHA). In this paper, geomagnetic reference maps are determined from small areas where the effect of the earth’s curvature is negligible. Therefore, RHA is employed to build a partial geomagnetic model based on \( B_R \). The actual procedure is shown below.

The research object of RHA is a rectangular region. In a space without a magnetic field source, the magnetic position satisfies the Laplace equation (Eq. (17)).

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.
\]

The solution of Eq. (17) can be written as Eq. (18).

\[
V(x, y, z) = Ax + By + Cz + \sum_{q=0}^{N_{max}} \sum_{m=0}^{q} P_{mn}(x, y)e^{i\mu z},
\]

where

\[
\begin{align*}
L_x &= 2\pi / L_x, \\
L_y &= 2\pi / L_y, \\
u &= \sqrt{(mv)^2(w)^2}.
\end{align*}
\]

\[
P_{mn}(x, y) = D_{mn} \cos(mx) \cos(ny) + E_{mn} \sin(mx) \sin(ny) + F_{mn} \sin(mx) \cos(ny) + G_{mn} \sin(mx) \sin(ny).
\]

In Eq. (18), \( x, y, \) and \( z \) are the three-dimensional coordinates of the target points in the rectangular coordinate system, \( V(x, y, z) \) is the maximum truncation order, and \( L_x, L_y \) are the north-south and east-west length of the rectangular area. The center of the target area is selected as the origin of the rectangular harmonic coordinate system. The coordinates are restricted by:

\[
\begin{align*}
-L_x / 2 &\leq x \leq L_x / 2, \\
-L_y / 2 &\leq y \leq L_y / 2.
\end{align*}
\]

and the three components of the magnetic field can be expressed by Eq. (19).

\[
\begin{align*}
B_x &= -\frac{\partial V}{\partial x} = -A + \sum_{q=0}^{N_{max}} \sum_{m=0}^{q} Q_{mn}(x, y)e^{i\mu z}, \\
B_y &= -\frac{\partial V}{\partial y} = -B + \sum_{q=0}^{N_{max}} \sum_{m=0}^{q} R_{mn}(x, y)e^{i\mu z}, \\
B_z &= -\frac{\partial V}{\partial z} = -C + \sum_{q=0}^{N_{max}} \sum_{m=0}^{q} S_{mn}(x, y)e^{i\mu z},
\end{align*}
\]

where

\[
\begin{align*}
Q_{mn}(x, y) &= mv(D_{mn} \sin(mx) \cos(ny)) + E_{mn} \sin(mx) \sin(ny)), \\
R_{mn}(x, y) &= -F_{mn} \cos(mx) \cos(ny)) - G_{mn} \sin(mx) \sin(ny))
\end{align*}
\]

\[
R_{mn}(x, y) = nw(D_{mn} \cos(mx) \sin(ny))
\]
The minimization problem (Eq. (22)) is equal to the approximation of $E_{j_0}$, which can be solved via the singular value decomposition (SVD). However, this may generate a dense vector $x_j^T$, indicating that the non-zeros in $X$ increase. The columns in $E_{j_0}$ should be extracted – those columns where the corresponding entries in $x_j^T$ are non-zero – to keep the sparsity of $X$ sufficient.

Therefore, a restriction matrix $P_{j_0}$ is defined, which multiplies $E_{j_0}$ from the right to preserve the relevant columns. Similarly, we define $(x_{j_0}^R)^T = x_{j_0}^T P_{j_0}$ to preserve the non-zeros of $x_{j_0}^T$ only, and then Eq. (22) can be rewritten as Eq. (24).

$$\min \| E_{j_0} P_{j_0} - d_{j_0}(x_{j_0}^R)^T \|^2_F. \tag{24}$$

An approximation via SVD can be applied to solve Eq. (24), by alternatively updating $d_{j_0}$ and the corresponding coefficients in $x_{j_0}^T$. The concrete solution is shown as follows:

1) Keeping $d_{j_0}$ fixed and updating $x_{j_0}^R$ by solving

$$\hat{x}_{j_0}^R = \arg \min_{x_{j_0}^R} \| E_{j_0} P_{j_0} - d_{j_0}(x_{j_0}^R)^T \|^2_F$$

$$= \frac{P_{j_0} E_{j_0} d_{j_0}}{\|d_{j_0}\|^2_2}. \tag{25}$$

2) Keeping $x_{j_0}^R$ fixed and updating $d_{j_0}$ once Eq. (24) is updated by solving

$$\hat{d}_{j_0} = \arg \min_{d_{j_0}} \| E_{j_0} P_{j_0} - d_{j_0}(x_{j_0}^R)^T \|^2_F$$

$$= \frac{E_{j_0} P_{j_0} x_{j_0}^R}{\|x_{j_0}^R\|^2_2}. \tag{26}$$

The desired dictionary can be achieved after several iterations. Fig. 5 shows the K-SVD method in detail.

IV. EXPERIMENTS

A. EXPERIMENT SETTINGs

Experiments of the proposed method are performed on a personal notebook ThinkPad T450, which has a configuration of an Intel Core i5-6200U CPU @ 2.30 GHz, 4 GB memory. MATLAB 2014a is used for simulation and experimental analysis. The database used in our experiments is a combination of two parts. One part is based on the geomagnetic anomaly data in North America issued by the United States Geological Survey (USGS) in 2002 [24]. The second part is the main magnetic field strength calculated from IGRF. We compare the proposed SR-GRM method with several well-known geomagnetic reference maps and single image super-resolution methods. These include bicubic interpolation, Kriging interpolation, and PSO-Kriging interpolation because they are all widely used in geomagnetic reference map construction. Another method we report results is Neighbor Embedding (NE).

First, the values of the geomagnetic reference maps are normalized to the [0,255] range linearly, according
to Eq. (27), where \( t_i \) denotes the pixel in the geomagnetic reference maps, Max denotes the maximum value, and Min denotes the minimum value in all maps.

\[
t_i = 255 \times \frac{t_i - \text{Min}}{\text{Max} - \text{Min}}
\]  

In our experiments, the input geomagnetic reference maps are magnified by a factor of 2, 3, or 4. For the LR maps, we use 5 \times 5 LR patches with an overlap of 4 pixels between adjacent patches and redundant dictionaries based on the algorithm in [29].

**TABLE 1.** PSNR, SSIM, AND RMSE of signal-channel and multi-channel sparse representation methods.

| Method    | PSNR | SSIM | RMSE |
|-----------|------|------|------|
| Signal-channel | 31.43 | 0.774 | 3.11 |
| Multi-channel   | 31.71 | 0.803 | 2.98 |

**TABLE 2.** PSNR, SSIM, AND RMSE of different methods for super-resolution with magnification factor 2, respect to the original images.

| Method    | PSNR | SSIM | RMSE | MAE | SD  |
|-----------|------|------|------|-----|-----|
| Bicubic   | 31.33 | 0.771 | 3.21 | 2.76 | 3.44 |
| NE        | 31.27 | 0.764 | 3.24 | 2.73 | 3.38 |
| PSO-Kriging | 31.47 | 0.781 | 3.11 | 2.71 | 3.33 |
| CNN       | 31.39 | 0.792 | 3.12 | 2.61 | 3.32 |
| GAN       | 31.36 | 0.783 | 3.06 | 2.57 | 3.28 |
| Our method | 31.71 | 0.803 | 2.98 | 2.51 | 3.23 |

B. EXPERIMENTAL RESULTS

In order to evaluate the obtained super-resolution reference maps quantitatively, five frequently-used image quality matrices are applied, which are the Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Standard Deviation (SD). The total geomagnetic field strength is calculated for quality evaluation using Eq. (20).

In Fig. 6, we compare the reconstruction result of our algorithm with several other methods, including neighbor embedding, bicubic, and PSO-Kriging. The magnification factor is 4X, and the size of the geomagnetic reference map is 128 \times 128. PSO-Kriging interpolation and our method give the best results. Our proposed method better highlights the details of the reference map.

To verify the superiority of multi-channel sparse representation method, single-channel method is used for comparison. The scaling factor is 3. The experimental results are shown in Table 1. Multi-channel method outperforms single-channel method in both PSNR, SSIM and RMSE indicators.

Tables 2–4 show the super-resolution reconstruction results of maps in the validation set. PSNR, SSIM, and RMSE are compared, and our method outperforms the other competing methods. The advantage of our method increases with the magnification factor. Bicubic interpolation is the most commonly used method for its low computational complexity.
FIGURE 6. A geomagnetic reference map reconstructed by a magnification factor of 4: (a) input; (b) the original map; (c) bicubic interpolation; (d) neighbor embedding; (e) PSO-Kriging interpolation; (f) our method.

TABLE 3. PSNR, SSIM, AND RMSE of different methods for super-resolution with magnification factor 3, respect to the original images.

| Method      | PSNR | SSIM | RMSE | MAE | SD  |
|-------------|------|------|------|-----|-----|
| Bicubic     | 28.75| 0.597| 9.22 | 7.69| 9.54|
| NE          | 28.44| 0.586| 9.34 | 7.83| 9.61|
| PSO-Kriging | 29.18| 0.633| 8.65 | 7.46| 8.87|
| CNN         | 29.06| 0.629| 8.75 | 7.49| 8.88|
| GAN         | 29.27| 0.641| 8.71 | 7.51| 8.82|
| Our method  | **29.74**| **0.693**| **8.33**| **7.21**| **8.19**|

However, its performance in super-resolution reconstruction decreases rapidly with large magnification factors. Neighbor embedding generates sharp edges in places, but the textures are blurred among the pixels of LR maps. PSO-Kriging somewhat adapts to the geomagnetic reference map reconstruction but generates undesired smoothing that is not present in the original HR maps. Learning-based methods such as Generative Adversarial Network (GAN) and Convolutional Neural Network (CNN) are also added for comparison. The training set size is the same as that used in the proposed method. Results in Tables 2–4 indicate that the performance of learning-based methods are unsatisfying while the training set is not sufficiently big.

TABLE 4. PSNR, SSIM, AND RMSE of different methods for super-resolution with magnification factor 4, respect to the original images.

| Method      | PSNR | SSIM | RMSE | MAE | SD  |
|-------------|------|------|------|-----|-----|
| Bicubic     | 25.47| 0.466| 21.81| 18.41| 20.41|
| NE          | 25.22| 0.453| 23.01| 19.11| 21.17|
| PSO-Kriging | 26.63| 0.511| 18.65| 16.99| 17.69|
| CNN         | 26.45| 0.502| 18.61| 16.91| 17.57|
| GAN         | 26.66| 0.518| 18.23| 16.78| 17.44|
| Our method  | **27.12**| **0.536**| **15.12**| **13.39**| **15.58**|

So far, we have used a fixed training set of size 50000 for all the experiments. In the following experiment, we verify the importance of prior information for dictionary initialization, which has been discussed in part B of section III. We again trained dictionaries of size 512 atoms and samples of 1600, 3200, 6400, 12800, 25600, 51200 map patches as training sets. The results are also evaluated in terms of PSNR, SSIM, and RMSE. Figures 7–9 show the variation of dictionaries initialized with prior information and random entries versus training set size. When prior information is attached, the dictionary can reach the same reconstruction precision with a relatively small training set size.
FIGURE 7. Effect of training set size on PSNR of redundant dictionaries with prior information and without prior information with a scaling factor of 3.

FIGURE 8. Effect of training set size on SSIM of redundant dictionaries with prior information and without prior information with a scaling factor of 3.

FIGURE 9. Effect of training set size on RMSE of redundant dictionaries with prior information and without prior information with a scaling factor of 3.

An often-made assumption is that the input reference maps are free of noise, which is violated in real geomagnetic data collections. The artifacts introduced in the denoising process may remain and are even magnified after the super-resolution procedure. Different levels of Gaussian noise are attached to the LR reference maps to test the robustness of our method to noise. Similar to [29], the range of the standard deviation of the noise is chosen from 3 to 12 with a magnification factor of 3. Table 5–7 shows the PSNR, SSIM, and RMSE results of reconstruction maps with different levels of noise. The proposed method outperforms the competition in all cases.

V. CONCLUSION

In this paper, we proposed a novel method for geomagnetic reference map reconstruction that makes a considerable improvement over existing reconstruction methods. Besides producing high precision reconstruction results, our method can significantly reduce the impact of noise, which contributes to actual geomagnetic reference map construction. Furthermore, a dictionary initialization method based on prior information of RHA is proposed to train a more qualitative sparse dictionary with a relatively small training set size. In future work, we will investigate the coherence of the three independent components of the geomagnetic reference field and further improve the precision of the geomagnetic reference map.

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