New approach to measuring the IVC of non-linear inertia-free elements

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Abstract. A technique for measuring the current-voltage characteristics (IVC) of a nonlinear inertia-free element from the amplitude spectrum of a current curve obtained by decomposing it into a Fourier series is given. The initial data for the method are the curves of current and voltage on a nonlinear element, measured for different points in time. The IVC is represented as a ratio of power polynomials whose coefficients are determined by the Cholesky method. Examples of measurement of the current-voltage characteristics are given, estimates of the relative and absolute error of the approach are given. The advantage of the proposed approach in comparison with the existing ones is the better accuracy of the reproduction of the IVC.

1. Introduction
Evaluation of the technical condition of electrical equipment is an urgent task of electrical engineering. Electrical networks directly depend on the accuracy of determining the parameters of power elements. Changes in the current-voltage characteristics of electrical equipment, due to its performance, lead to a decrease in the efficiency of its operation, as well as to errors in the operation of relay protection and automation devices, which in turn require the safety of operation of the power grid complex [1-5].

Modern methods of measuring the IVC in most applications use a polynomial approximation of the spectrum of the current curve [6-9]. However, in areas of sharp nonlinearity, this approximation does not ensure the adequacy of the design model. In the proposed method, for approximation, the ratio of polynomials is used, which allows, by varying the degree of the numerator or denominator polynomials, to provide the best accuracy of the IVC reproduction. This is most clearly manifested when it is necessary to faithfully reproduce the asymptotic behavior of IVC. Thus, with an unlimited increase in the IVC, the predominance of the degree of the numerator over the degree of the denominator provides the desired growth rate, the property of the IVC tendency toward zero or a constant value is also provided by varying the degrees of the polynomials of the numerator and the denominator.

2. Method for the determining the IVC
Consider a nonlinear inertia-free element with IVC. The approximating function for the current is the equation:
\[ i = a_0 + \sum_{n=0}^{N-1} a_n u^n + \sum_{m=0}^{M-1} b_m u^m \] (1)

where \( i \) and \( u \) are current and voltage of the element under study; \( a_n, b_m \) \( n=1,N, m=1,M \) are the coefficients of the polynomials of the numerator and denominator, which are the desired quantities in our approach.

Let the curves \( i(t), u(t) \) measured and laid out in a Fourier series:

\[ i(t) = I_0 + \sum_{k=1}^{N/2} \left[ I_{1.k.m} \sin(\omega t) + I_{2.k.m} \cos(\omega t) \right], \quad u(t) = U_0 + \sum_{k=1}^{N/2} \left[ U_{1.k.m} \sin(\omega t) + U_{2.k.m} \cos(\omega t) \right] \]

It is important here that the amplitudes \( I_{1,k,m}, I_{2,k,m}, U_{1,k,m}, U_{2,k,m} \) are known numbers. Substituting expressions for \( i(t), u(t) \) in (1), performing a transition from the powers of trigonometric functions to multiple angles and equating the terms with sines and cosines with the same \( \omega k \), get the system of equations to determine the coefficients \( a_n, b_m \) \( n=1,N, m=1,M \). The form of this system of equations for \( N=2, M=1, N_I=3, N_U=2, i_0=0 \) is below.

\[
\begin{bmatrix}
-2, & -2U_{10}, & -2U_{11}^2 - U_{12}^2 - U_{13}^2 - U_{14}^2 - U_{15}^2, & U_{16} I_k + U_{17} I_{k1} + U_{18} I_{k2} + U_{19} I_{k3}, \\
0, & -2U_{20}, & -2U_{21}^2 - U_{22}^2 - U_{23}^2 - U_{24}^2 - U_{25}^2, & U_{26} J_k + U_{27} J_{k1} + U_{28} J_{k2} + U_{29} J_{k3}, \\
0, & -2U_{30}, & -2U_{31}^2 - U_{32}^2 - U_{33}^2 - U_{34}^2 - U_{35}^2, & 2U_{36} J_k + U_{37} J_{k1} + U_{38} J_{k2} + U_{39} J_{k3}, \\
0, & 0, & -2U_{40}, & U_{41} I_k + U_{42} I_{k1} + U_{43} I_{k2} + U_{44} I_{k3}, \\
0, & 0, & 0, & U_{45} I_k + U_{46} I_{k1} + U_{47} I_{k2} + U_{48} I_{k3}, \\
0, & -2U_{50}, & -2U_{51}^2 - U_{52}^2 - U_{53}^2 - U_{54}^2 - U_{55}^2, & 2U_{56} J_k + U_{57} J_{k1} + U_{58} J_{k2} + U_{59} J_{k3}, \\
0, & 0, & 0, & U_{51} I_k + U_{52} I_{k1} + U_{53} I_{k2} + U_{54} I_{k3}, \\
0, & -2U_{60}, & -2U_{61}^2 - U_{62}^2 - U_{63}^2 - U_{64}^2 - U_{65}^2, & 2U_{66} J_k + U_{67} J_{k1} + U_{68} J_{k2} + U_{69} J_{k3}, \\
0, & 0, & 0, & U_{61} I_k + U_{62} I_{k1} + U_{63} I_{k2} + U_{64} I_{k3}, \\
0, & -2U_{70}, & -2U_{71}^2 - U_{72}^2 - U_{73}^2 - U_{74}^2 - U_{75}^2, & 2U_{76} J_k + U_{77} J_{k1} + U_{78} J_{k2} + U_{79} J_{k3}, \\
0, & 0, & 0, & U_{71} I_k + U_{72} I_{k1} + U_{73} I_{k2} + U_{74} I_{k3}, \\
0, & 0, & 0, & 0
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\\na_2 \\\na_3 \\\nb_0 \\\nb_1 \\\nb_2 \\\nb_3
\end{bmatrix}
\]

In this system 11 equations and 4 unknowns. Incompleteness of the system of equations for the coefficients \( a_n, b_m \) \( n=1,N, m=1,M \) typical of the tasks we are considering. To determine the desired coefficients we use the method of least squares.

\[
(A^T A) \cdot X = A^T B
\] (3)

where \( A, X \) and \( B \) - equation coefficients shown in (2).

System of equations (3) defined and contains 11 equations. Its solution is produced by the Cholesky method [10-13]. The calculations are implemented in the software environment Maple, that is, the expressions for the coefficients are \( a_n, b_m \) \( n=1,N, m=1,M \) obtained analytically and exclude the error of the solution.

3. The results of the study of the accuracy of the method and their discussion

For the study of our approach, we consider a nonlinear element, whose characteristic is analytically given \( i(u) = \arctg(u/10) \). Connecting to its terminals a source of sinusoidal EMF, frequency 50 Hz and
amplitude $10 \, \text{V}$ ($u(t) = 10 \sin \omega t$), remove the current dependence on time and decompose it in a Fourier series, keeping 5 harmonics in the decomposition:

$$i(t) = 0.828427 \sin \omega t + 0.0473787 \sin 3\omega t + 0.0048774 \sin 5\omega t$$

Here we do not discuss the issues of accuracy of determining the expansion coefficients, and methods for increasing it, which we discussed in detail in [14-17]. Let be $N = 3$ and $M = 2$. Then, substituting $i(t), u(t)$ in (1), get the system of equations to determine the coefficients $a_n, \ b_n,$ $n=1, N, \ m=1, M$, from 8 equations with 6 unknowns - $a_0, a_1, a_2, a_3, b_1, b_2$. The terms of these equations, which do not contain unknowns, are transferred to the right-hand side. As a result, a system of equations similar to (2) is obtained:

$$
\begin{pmatrix}
-1, & 0, & -50, & 0, & 4.1421, & 0, \\
0, & -10, & 0, & -750, & 0, & 60.9476, \\
0, & 0, & 0, & 250, & 0, & -18.7637, \\
0, & 0, & 0, & 0, & 0, & -0.9406, \\
0, & 0, & 0, & 0, & 0, & -0.1219, \\
0, & 0, & 50, & 0, & -3.9052, & 0, \\
0, & 0, & 0, & 0, & -0.2125, & 0, \\
0, & 0, & 0, & 0, & -0.0244, & 0,
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
a_2 \\
a_3 \\
b_1 \\
b_2
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
-0.8284 \\
-0.0474 \\
-0.0049 \\
0 \\
0
\end{pmatrix}
$$

Solving by the Cholesky method a system similar to (3), we get:

| $a_0$ | $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ |
|------|------|------|------|------|------|
| -2.4E-08 | 0.09989 | -3.5E-09 | 0.000187 | -5.3E-08 | 0.0051 |

As can be seen from Table 1, the coefficients $a_0, a_2$ and $b_1$ are small. Neglecting them, we write the expression approximating the IVC:

$$i_{\text{approx}} = 10^{-2} \frac{99.89 + 0.187u^2}{1 + 0.0051u^2} u$$

Determine the relative error $E_i(u)$ approximations of the IVC as follows:

$$E_i(u) = 100\% \left| \frac{\arctg \frac{u}{10} - i_{\text{approx}}}{\arctg \frac{u}{10}} \right| \int_0^{10} \frac{1}{10} \arctg \frac{u}{10} \, du$$

where the denominator is the average value approximated by the IVC over the entire voltage change interval.

Error $E_i(u)$ reproduction IVC is shown in Figure 1. The reproduction error of the IVC for the case when we do not neglect coefficients $a_0, a_2$ and $b_1$ shown in the same figure with a dotted line. As expected, the error curves are almost indistinguishable.

Consider further the IVC of the form $i(t) = \text{tg}(\omega t/10)$. Connecting to its terminals a source of sinusoidal EMF, frequency 50 Hz and amplitude 10 \, \text{V} ($u(t) = 10 \sin \omega t$), remove the current dependence on time and decompose it in a Fourier series, keeping 5 harmonics in the decomposition:

$$i(t) = 1.38039 \sin \omega t + 0.154722 \sin 3\omega t + 0.0198543 \sin 5\omega t$$
In this example we take the polynomial of order as the approximating function \( N = 5 \) and \( M = 0 \). Get the system of equations:

\[
\begin{bmatrix}
-1, & 0, & -50, & 0, & -3750, & 0, \\
0, & -10, & 0, & -750, & 0, & -62500, \\
0, & 0, & 0, & 250, & 0, & 31250, \\
0, & 0, & 0, & 0, & 0, & -6250, \\
0, & 0, & 0, & 50, & 0, & 5000, \\
0, & 0, & 0, & 0, & 0, & -1250
\end{bmatrix}
\begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\alpha_5
\end{bmatrix}
= \begin{bmatrix}
0 \\
-1.3804 \\
-0.1547 \\
0 \\
0 \\
0
\end{bmatrix}
(5)
\]

The approximation coefficients found by the Cholesky method are listed in Table 2.

| \( \alpha_0 \) | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) | \( \alpha_5 \) |
|---|---|---|---|---|---|
| -1.42E-06 | 1.02E-01 | 1.81E-06 | 2.22E-04 | -1.92E-08 | 3.18E-06 |

Neglecting \( \alpha_0, \alpha_2, \alpha_4, \alpha_5 \) and substituting \( \alpha_1, \alpha_3 \) and in (1), will get:

\[
\begin{aligned}
\alpha_{approx} &= 10^{-6}u\left[-1.42 + \left(1.02 \cdot 10^2 + 1.81u + 22.2u^2 + 3.18u^4\right)\right].
\end{aligned}
\]

The relative error of approximation is represented by the function:

\[
E_2(u) = 100% \left| \frac{\tan \frac{u}{10} - \alpha_{approx}}{\frac{1}{10} \int_0^{10} \tan \frac{u}{10} \, du} \right|.
\]

The reproduction error of IVC \( E_1(u) \) shown in Figure 2. The reproduction error of the IVC for the case when we do not neglect coefficients \( \alpha_0, \alpha_2, \alpha_4, \alpha_5 \) shown in the same figure with a dotted line.

![Figure 1. Relative errors in the reproduction of the IVC element.](image-url)
Figure 2. Relative errors in the reproduction of the IVC element.

4. Conclusions
The results of testing showed that for a real IVC of the tangent and arctangent the absolute error of the model and the real curve does not exceed 0.0155% and 0.00014%, respectively.

The proposed approach allows the measurement of the IVC for 1–2 periods of the fundamental frequency. This, in particular, makes it possible to damp the interfering influence of such factors as a change in the electromagnetic environment or a change in the temperature of an element during an experiment.

The approach, when using modern technology of digital signal processing, is highly accurate, superior, in the authors' opinion, to the accuracy achievable in traditional measurement methods and ease of implementation.

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