Common Concepts in Nuclear Physics and Ultracold Atomic Gasses

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Abstract. Advances in the exploration of the physics governing extremely cold atomic gasses have induces a flurry of interest in cross-disciplinary application of the concepts that arise in the atomic systems. This is also the case in nuclear physics where analogue to Bose condensates and BCS/BEC crossover are frequently seen in the nuclear literature. We attempt to extract some of the essential features of the fields and comment on which analogues are useful and which are more likely the cause of misconceptions between communities.

1. Introduction

Recently we have witnessed revolutionary gains in experimental capabilities on ultracold atomic gasses. The amount of control that experimentalists exercise over these systems is unprecedented and allows one to simulate interacting quantum systems over a vast range of interesting parameters. The highlights are undoubtedly the achievement of Bose condensates and later the realization of a degenerate Fermi gas in an atomic trap. Through tuning of interaction strength via Feshbach resonances we are now able to stringently test theoretical models of these systems and determine which of these capture the physics of different regimes. A prominent example of this is the so-called BCS/BEC crossover theory. This addresses the situation where one starts from a fermionic system with BCS pairing and then varies the interaction strength to approach a regime where bosonic pairs are the relevant degrees of freedom. The extent to which this system can be described by simple extension of the BCS gap equations is one that can be settled by further research on cold atomic gasses.

These advances have also brought a number of studies that attempt to connect the results from ultracold gasses to other areas of physics. In particular, there has been some work on possible analogies of Bose condensates and BCS/BEC crossover in the field of nuclear physics. We have analyzed the various claims to determine what common concepts are useful in the interest of cross-disciplinary applications. In this paper we will discuss how a particular case of Bose condensate is applied in nuclear and atomic physics, summarizing similarities and differences. This will clarify how and when analogies across the fields can be justified and perhaps enlightening, and also when they are more likely to cause confusion.

2. Ultracold Atomic Gasses

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The potential of modern atomic gas experiments to probe the many-body problem is truly astonishing and experimenters are coming closer to having almost absolute control over these systems. One can vary the particle number and the statistics of the particles, including the interesting regime with mixtures of Bose and Fermi species. The external trapping potential can be accurately tuned and its dimensionality and deformation is also among the parameters. Furthermore, the two-body interactions between particle species has become tunable through the likes of so-called Feshbach resonances. This allows one to pick the interaction strength across a vast range and also its sign, producing both attractive and repulsive two-body terms in the many-body systems. The experimental results can therefore very likely help clarify issues in the many-body problem that are of great relevance, not only in atomic and condensed-matter physics, but also in the nuclear realm. The benefit of an exchange of ideas is therefore obvious but one needs to be extra careful and take measures to make sure that the concepts are meaningful and transferable from one domain to the other.

2.1. Atomic BEC
Bose-Einstein condensates were achieved in atomic alkali gasses in the mid-nineties (see [1] for details and references to the literature). The most famous signal of the transition to the condensed state was the velocity distributions that show distinct peaks at zero momentum below the critical temperature $T_c$. The typical system size was some 100,000 atoms and these were confined by an external trap with characteristic length scale $\hbar \omega = 1 \mu m$. The critical temperature was estimated to be around $T_c \approx 100$ nK. This is extremely low and a $T=0$ description is therefore a very good approximation for many observables. The Bose condensed state turns out to be extremely dilute and the spatial extension is several times the trap length and the kinetic energy is correspondingly small. The two-body interaction in these systems is therefore accurately described by the low-energy scattering length which is of order 10 nm. These facts about the trapped alkali systems explain why the $T=0$ Gross-Pitaevskii equation has been so successful in the theoretical modeling [1].

2.2. Atomic Degenerate Fermi Gasses
A few years later in 1999 the first achievements of degenerate quantum Fermi gasses were seen [1]. The work towards degeneracy of Fermi gasses turns out to be more complicated since they are generally harder to cool. Interactions between the fermions are suppressed by the Pauli principle and therefore the exchange of energy in the system is inhibited, making the usual cooling methods employed for Bose gasses less efficient. However, by use two different spin states that are allowed to interact the degeneracy was finally approached [2]. The experiments with Fermi gasses have largely the same dimensions and particle number as those cited for Bose gasses above. Another very interesting way to cool fermions is through so-called sympathetic cooling where Bosons and Fermions are trapped simultaneously and then cooled by they mutual interaction which is not suppressed by statistical effects. This is a very active area in recent years as this naturally produces Bose-Fermi mixtures that are of great interest to modelers. This could also have some interest for the nuclear physics many-body problem as effective models with nucleons and mesons or (less fundamentally) nucleons and alphas are often used.

2.3. Comparison to Nuclei
When comparing to nuclei one is immediately faced with the finite systems problem. Nuclei have an (often) small number of particle whereas the condensates are approaching the thermodynamic limit. Furthermore, the atomic gasses are neutral systems that do not suffer from the Coulomb effects that often have decisive influence on certain nuclear properties, particularly for light nuclei close to thresholds. The next problem facing any analogues is the fact that nuclei are of course self-bound, ideally there is no external potential. Although this principle is often broken in terms of mean-field models such as Hartree-Fock, one should always be able to restore any broken symmetries (in this case translational). This can cause a number of problems when one tries to define the notions of condensate in finite systems [3]. Another issue one should keep in mind is that most nuclear models
are T=0, although there are some good ones on the market that use finite T. The problem with the latter is that in small systems the effects of transitions of phase can be washed out and be extremely hard to disentangle. This is of course no worry when modeling infinite nuclear systems, such as is done for neutron matter in connection with Neutron Stars.

The above problems should be taken very serious when trying to take advantage of cross-disciplinary concepts and a consensus on what is meaningful and what is not should be reached. The rest of this short paper will thus be used on some recent examples where atomic gas physics concepts have been applied to nuclear systems. We feel that it is in the best interest of the community to point of the potential problem that could arise from a stretching of analogues.

3. Clusters, Mean-Field, and Condensates

The idea of clusters in nuclei has been around from about the time of nuclear structure itself. The point being that one would like to pick the relevant degrees of freedom, whether it be nucleons, deuterons, or alphas to describe the nuclear system. One of the quite successful models is one that attempts to describe N=Z even-even nuclei as a system of alpha particles. These attempts can be viewed from a mean-field perspective: Mean-field descriptions of nuclei imply a product wave function with no correlations. One can then include preferred structures (such as alphas) by putting correlations into such wave functions. In the extreme limit one then gets the purely cluster description. The question becomes whether the system prefers the nucleons constituents to be localized in alphas clusters or delocalized as in a mean-field.

There is a very simple, yet powerful way to think of the competition between localization and delocalization which is due to Mottelson [4]. He defines a quantality parameter, Λ, for any given system which has mutual two-body interactions between its constituents. It is given by

$$\Lambda = \frac{T}{V_0} = \frac{\hbar^2}{4\pi^2 Ma^2 V_0},$$

where $V_0$ and $a$ are the (absolute) value and position of the minimum of two-body interaction, and $M$ is the mass. This ratio of kinetic to potential energy turns out to be the quantity that decided whether a given system becomes a liquid or a solid at T=0. According to the number in [4], the transition happens at $\Lambda \approx 0.1$ with systems below being solid and above being liquid, independently of the statistics of the system constituents. Nucleons have $\Lambda \approx 0.5$ and should therefore be consider in the mean-field category. However, if one attempts to estimate $\Lambda$ for standard alpha-alpha potentials, one find $\Lambda \approx 0.1-0.16$ so it would appear that this is a borderline case just on the verge of solid-like structure. The alpha model is therefore interesting for a three-cluster model like 12C. However, since it is just around the transition between localized solid and delocalized mean-field description, further investigation is needed.

In a recent paper by Tohsaki et al. [5], the authors have proposed that the 0+ Hoyle state close to the triple-alpha continuum in 12C can be viewed as a condensate state of three alphas in mutual s-wave orbits. We have tried to understand this work in connection with the above discussion regarding localization. To define a condensate one should ideally diagonalize the density matrix of the cluster system and then look for a large eigenvalue in the limit of large systems. Since N=3 this would means some number large compared to 3. However, the problem seems to be what one should do about the center-of-mass issue, since there is ambiguity with respect to the choice of coordinates [3]. In this paper we will be as ignorant as possible and simple ignore the center-of-mass and keep a density matrix that is a function of the 3 alpha particle coordinates (ignoring the antisymmetry between nucleons in different alphas which is justified for very dilute states around threshold, such as the Hoyle state).
In figure 1 we show a localized and a delocalized three Gaussian wave function as a very simple $^{12}$C alpha-cluster model. When one diagonalizes the density matrix with respect to the three coordinates indicated in the figure, one finds that the localized left-hand wave function gives three unity eigenvalues (and an infinite number of zeroes) whereas the delocalized one will yield a single non-zero eigenvalue of 3. This assumes that the three Gaussians in the localized wave have no overlap. Of course there is a continuous limit from the no-overlap localized case to the full overlap delocalized case. This indicates that localized alpha particles are in fact the worst-case scenario with respect to finding a condensate state in the system.

The above consideration ignored the fact that one has to produce a rotationally invariant state out of the given cluster wave function in order to preserve the symmetries of the underlying nuclear Hamiltonian. This is usually done by projecting out the 0$^+$ component from the solution. This entails an averaging procedure over all angles in space for the wave function on the left-hand side of figure 1 (the right-hand wave function is of course already a spherically symmetric state). If one assume a small width of the alpha clusters $\Psi_1$ of figure 1, then such an averaging procedure would lead to a wave function that would resemble a hollow sphere. This would give many eigenvalues of the density matrix with small values and would therefore not be a particularly good condensate candidate [3].

It has also been argued [6] that since the wave function of [5] has a large overlap with some traditional cluster states that have a large s-wave component in them, the condensed alpha state of [5] is a good description of the system. Large overlap tells us that the state of [5] apparently gives a decent approximation to the results of more advanced $^{12}$C cluster models that have been around for many years. However, there is still a difference between overlap (comparison of density) and single-particle wave content (through the density matrix) and we feel less convinced about statements based on overlap alone. Even if one did a full density matrix study of the alpha condensed states, then one would still have to address the ambiguities of choice of coordinates that we mentioned above and which will be described in more detail in [3]. Until these issues are resolved, the alpha condensate states seem at best an approximation.

4. Summary
The experimental revolutions seen in atomic gas physics with regards to a vastly improved understanding of the quantum many-body problem are much discussed within several disciplines. However, before attempted exchange of ideas between different fields can become fruitful, it is important that concepts from the different realms are defined and compared in a proper way.

In this paper we have tried to compare some systems from atomic and nuclear physics for similarities and differences and found that whereas some concepts might be superficially related, there
can be intrinsic problem in the definitions that make comparisons less useful or simply misleading. In particular we have considered the recent claims of alpha condensate states in nuclei around threshold, which can be viewed as a simplification/approximation of the typical alpha cluster states that have been discussed in the nuclear physics community for decades. We find that, aside from requiring that nucleons actually combine into alphas, these alphas must also be highly delocalized for the mean-field condensate states to appear, and the question is then if we can realize all of it at once.

Intuitively, cluster-like structure tends towards solids and would contradict the delocalization needed. Using Mottelson’s analysis [4], we concluded from alpha-alpha potentials that these systems are only marginally in the mean-field region. For the interesting Hoyle state in $^{12}$C, recent calculations [7] have also suggested that one will have localization in a $^{8}$Be-alpha configuration so an actual BEC is hard to imagine (localization in $^{8}$Be also means that the Pauli principle cannot be neglected).

Another very important point would be if there are any new observables or different results of known ones that can be predicted from the assumption of an alpha condensate. This seems yet to be proven. This of course means that we have to be very careful in comparing the alpha condensate studies with regular cluster studies.

The authors still feel that cross-disciplinary work is useful and can be crucial for the advancement of physics. The many-body properties that one might hope to gain insight into through controlled experiments on neutral atomic gasses are very likely to help in the modelling of the nuclear system. The realm of neutron matter for application in Neutron stars is one such venue, another is potentially an analogue description of exotic quark matter states that could be simulated in atomic gasses with different spin components and masses. Also with the implementation of optical lattices there is hope that one can get to particle numbers and kinetic-to-interaction energy ratios that are much closer to the nuclear case. From this perspective we believe there could be a bright future of mutual benefit between the disciplines. The paper as presented here is therefore more a cautionary statement that one needs to still be careful about the exchanges.

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