Two-Particle Scattering and Resistivity of Rashba Electron Gas

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We calculate the electrical resistivity of a two-dimensional electron gas that results from two-particle collisions and strong Rashba spin-orbit coupling. When combined with impurity scattering, the two-particle correction to the resistivity is proportional to the square of temperature $T$ if only the lower helicity band is filled, but the $T^2$ term vanishes if the Fermi level is above the Dirac point. In the absence of impurities, two-particle collisions do not contribute to resistivity.

It is well known that electron-electron scattering does not affect the resistivity of Galilean-invariant Fermi liquids because the current is proportional to the total momentum of electrons, which is conserved by interparticle collisions. However in realistic materials the electron-electron scattering may contribute to the resistivity through several mechanisms, which result in its $T^2$ temperature dependence. First of all, it is Umklapp scattering, which conserves the quasimomentum up to a reciprocal lattice vector $[1, 2]$. Baber $[3]$ suggested that even normal collisions may result in the $T^2$ contribution to the resistivity of multi-band metals if the effective masses of electrons in the bands are essentially different. The combined action of electron–impurity and interband electron–electron scattering was considered in a large number of papers both for two- (2D) and three-dimensional systems $[4–6]$. Recently, this interplay was analyzed for anisotropic Fermi surfaces $[7–9]$. It was found that the $T^2$ term is absent for simply connected convex Fermi surfaces, but is present if they are concave or multiply connected. Based on these findings, one may conclude that the existence of $T^2$ contribution to the resistivity from two-particle scattering for multiply connected Fermi surfaces depends only on their shape, but this is not the case. We demonstrate this using spin-orbit-coupled 2D electron gas as an example.

In last decades, electric transport in 2D systems with Rashba spin-orbit coupling $[10]$ became a subject of intensive investigations. This coupling spin-splits the dispersion curve into the upper and lower helicity bands. It was found that in the low-density regime when only the lower band is filled, the impurity-related resistivity exhibits an unconventional electron-density dependence $[11–13]$. Apparently, this system is not Galilean-invariant and is described at the same time by a minimum number of independent parameters. Therefore it is of interest to calculate its resistivity caused by two-particle collisions.

In this paper, we consider the effect of electron–electron scattering in a generic Rashba spin-orbit coupled 2D electron gas at temperatures smaller than the coupling energy. Using the Boltzmann equation, we calculate the resistivity of this system both in the presence of impurities and in the pure case at high and low electron densities. Despite the doubly-connected Fermi surface, the contribution to the resistivity from the electron–electron collisions in the absence of impurities is zero.

Consider a two-dimensional electron gas lying in the $xy$ plane, so its unperturbed Hamiltonian is of the form

$$\hat{H} = \frac{p_x^2 + p_y^2}{2m} + \alpha (\hat{\sigma}_x \hat{p}_y - \hat{\sigma}_y \hat{p}_x),$$

where $\alpha$ is the Rashba coupling constant and $\hat{\sigma}_{x,y}$ are the Pauli matrices. The diagonalization of this Hamiltonian results in two branches of the spectrum

$$\varepsilon_{\pm}(p) = \frac{p_x^2 + p_y^2}{2m} \pm \alpha \sqrt{p_x^2 + p_y^2},$$

which correspond to the two rotationally symmetric nonparabolic energy bands with opposite helicities that intersect only in one point at $p = 0$ (the Dirac point, see Fig. 1). The corresponding wave functions are spinors

$$\Psi_{\pm}(r) = \frac{1}{\sqrt{2}} e^{i pr/h} \left( \begin{array}{c} e^{i \chi p/2} \\ \pm e^{-i \chi p/2} \end{array} \right),$$

where $\chi_p = \arctan(p_x/p_y)$ so that the spin component perpendicular to $p$ is $\pm 1/2$. The position of the Fermi level $E_F$ may be tuned by external electrostatic gates, so it can cross either only the lower band, or both bands.

The electron–impurity and electron–electron scattering is assumed to be weak, so the charge transport may
be described by the Boltzmann equation in the basis of eigenstates of Hamiltonian $H$. In the linear approximation in the electric field $E$, it is of the standard form

$$eE \frac{\partial \epsilon_\nu}{\partial p} \frac{d\bar{f}}{d\epsilon_\nu} = I^{\text{imp}}_\nu(p) + I^{\text{ee}}_\nu(p),$$

where $\nu = \pm$ is the branch index and $\bar{f}$ is the equilibrium Fermi distribution. The electron–impurity collision integral may be written in the Born approximation as

$$I^{\text{imp}}_\nu(p) = n_i \sum_{\nu'} \int \frac{d^2p'}{(2\pi\hbar)^2} \left[ U^{\nu'\nu}_{pp'} \right]^2 \delta(\epsilon_\nu - \epsilon_{\nu'}) \times [f_\nu'(p') - f_\nu(p)],$$

where $n_i$ is the concentration of impurities and $U^{\nu'\nu}_{pp'}$ is the matrix element of the impurity potential between the electron states $(p, \nu)$ and $(p', \nu')$. In the case of point-impurity potentials with a potential $U(r) = U_0 \delta(r)$, one easily obtains that

$$\left[ U^{\nu'\nu}_{pp'} \right]^2 = \frac{1}{2} U_0^2 \left[ 1 + \nu \nu' \cos(\vec{p}, \vec{p}') \right].$$

The electron–electron collision integral is of the form

$$I^{\text{ee}}_\nu(p) = \sum_{\nu_1} \sum_{\nu_2} \sum_{\nu_3} \int \frac{d^2p_1}{(2\pi\hbar)^2} \int \frac{d^2p_2}{(2\pi\hbar)^2} \int d^2p_3 \times \delta(p + p_1 - p_2 - p_3) \delta(\epsilon_{\nu_1} + \epsilon_{\nu_2} - \epsilon_{\nu_3}) \times W^{\nu_1\nu_2\nu_3}_{pp_1, pp_2, pp_3} \times [1 - f](1 - f_1) f_2 f_3 - f f_1 (1 - f_2)(1 - f_3).$$

We assume that due to the screening by a nearby gate, the interaction potential is short-ranged and may be written as $V(r - r') = V_0 \delta(r - r')$. Calculating the difference between the matrix elements of direct and exchange interactions between the states $\nu_1$ and squaring it results in the expression for the scattering probability

$$W^{\nu_1\nu_2\nu_3}_{pp_1, pp_2, pp_3} = \pi \frac{V_0^2}{\hbar} \left[ 1 - \nu \nu_1 \cos(\vec{p}, \vec{p}_1) \right] \times \left[ 1 - \nu_2 \nu_3 \cos(\vec{p}_2, \vec{p}_3) \right].$$

It is convenient to replace the momentum variable $p$ by the energy $\epsilon$ measured from $E_F$ and the angle $\varphi$ measured from the direction of electric field. The solution of Eq. (3) in band $\nu$ is sought in the standard form

$$f_\nu(\epsilon, \varphi) = \bar{f}(\epsilon) + C_\nu(\epsilon) \bar{f}(\epsilon) \left[ 1 - \bar{f}(\epsilon) \right] \cos \varphi,$$

where $C_\nu$ describes the correction to $\bar{f}$ in the electric field. A substitution of this ansatz into Eq. (4) results in the linearized collision integral

$$I^{\text{ee}}_{\nu}(\epsilon, \varphi) = \sum_{\nu_1} \sum_{\nu_2} \sum_{\nu_3} \int \frac{d\epsilon_1}{|v_{\nu_1}|} \int \frac{d\epsilon_2}{|v_{\nu_2}|} \int \frac{d\epsilon_3}{|v_{\nu_3}|} \times \delta(\epsilon + \epsilon_1 - \epsilon_2 - \epsilon_3) (1 - \bar{f})(1 - \bar{f}_1) \bar{f}_2 \bar{f}_3 \times (\Omega_2 C_{\nu_2} + \Omega_3 C_{\nu_3} - \Omega_1 C_{\nu_1} - \Omega_{\nu} C_{\nu}) \cos \varphi,$$

where the quantities

$$\Omega_{\nu}(\epsilon, \nu_2, \nu_3) = \int \frac{d^2p_1}{(2\pi\hbar)^2} \int \frac{d^2p_2}{(2\pi\hbar)^2} \int d^2p_3 \times \pi^{\nu_1\nu_2\nu_3}_{pp_1, pp_2, pp_3} \cos(\vec{p}, \vec{p}_1) \delta(p + p_1 - p_2 - p_3) \times \delta(\bar{f}_1 - p_1 - p_2) \delta(\bar{f}_2 - p_2 - p_3) \delta(\bar{f}_3 - p_3 - p_0),$$

include both the scattering parameters and the effective phase volume available for the scattering, $\nu_0(\epsilon_i)$ is the solution of equation $\epsilon_\nu(p) = E_F + \epsilon_i$, and $v_\nu = (dp_\nu/d\epsilon_i)^{-1}$ is the corresponding velocity.

The interband scattering is relevant only if the Fermi level crosses both helicity bands. However if it is located below the Dirac point, the Fermi surface is still doubly connected because of nonmonotonic $\epsilon_\nu(p)$ dependence. In this case, one can use the above equations by replacing labels $\nu_\pm$ with $\lambda = \pm$ that correspond to the ascending and descending portions of this curve. The products $\nu_\nu$ and $\nu_\nu \nu_\nu$ must be set equal to 1 and the remaining $\nu_i$ must be replaced by $\lambda_i$. However it should be kept in mind that $\nu_\nu (\epsilon) = -\nu_\nu (\epsilon)$, while $v_\nu (\epsilon) = v_\nu (\epsilon)$. Depending on the sign of $E_F$, we denote either $\nu$ or $\lambda$ by $\mu$ where it does not lead to a confusion and imply that $\bar{\nu}$ reverses the sign of $\nu$ or sense of $\lambda$. For example, the equation for the current density may be written as

$$j = \frac{e}{4\pi^2 \hbar^2} \sum_{\mu} \text{sgn} v_\mu \int d\epsilon \bar{f}(1 - \bar{f}) p_\mu C_{\mu}.$$
where $\Gamma_2(T) = V_0^2T^2\langle p_\mu + p_{-\mu}\rangle/32\pi^2\hbar^5|v_\mu|^3$ and $E_{SO} = \alpha^2/2\mu$ is the characteristic energy of spin-orbit coupling. The dimensionless functions $\Phi_\mu$ represent the phase volume available for the scattering and can be calculated only numerically (see Fig. 2 and Appendix for details). Except for the explicit form of $\Gamma_2$ and $\Phi_\mu$, Eq. (15) does not depend on the presence of spin-orbit scattering, the system of kinetic equations (4) with $I^{ee}$ given by Eq. (15) becomes degenerate and has no stationary solution.

To overcome this difficulty, we consider the case where the impurity scattering is strong and $I^{ee}$ may be treated as a perturbation. The solution of Eq. (1) is sought as a sum $f = \tilde{f} + \delta f^{imp} + \delta f^{ee}$, where $\delta f^{imp}$ is given by Eq. (9) with $C^{imp}_\mu$ from Eq. (14), and $\delta f^{ee}$ is the solution of equation

$$I^{imp}_{\mu}\{\delta f^{ee}\} + I^{ee}_{\mu}\{\delta f^{imp}\} = 0.$$

This equation is easily solved for $C^{ee}_\mu$ and the correction to the current is calculated using Eq. (14). Below the Dirac point, it equals

$$\delta J^{ee} = -\frac{2\pi e^2E}{3\hbar^2}\Gamma_2V_0^{\mu}v^p_{p>\mu}\frac{p^2_\mu\Phi_< + p^2_\mu\Phi_>}{(p_\mu + p_{-\mu})^3}.$$

The inelastic correction to the current tends to zero as $E_F \to 0$ and vanishes above $E_F$ because $C_\mu$ from Eq. (14) turn $I^{ee}_{\mu}\{\delta f^{imp}\}$ into zero. Actually this is a consequence of equal slopes of the two dispersion curves $\varepsilon_+(p)$ and $\varepsilon_-(p)$ at the same energy. The correction logarithmically diverges at the bottom of the lower helicity band because of singularity in $\Phi_\geq$ due to head-on collisions of electrons on the different Fermi contours as $p_{\geq}$ and $p_{<}$ approach each other. This implies that the perturbative result Eq. (17) breaks down in this limit.

The contribution to the resistivity from electron–electron scattering is larger than that from electron–phonon scattering, which is proportional to $T^{4.5}$ at low temperatures. The possibility of observing it depends on the quality of the samples. A good candidate for such experiments is 2D electron gas in InAs, which exhibits a strong spin-orbit coupling with Rashba parameter $\alpha = 1.2\text{ eV}$. The electron-electron scattering effects are more prominent at low concentrations when only the lower helicity band is filled. For the electron concentration $3 \times 10^{10} \text{ cm}^{-2}$, the gas–gate distance of 20 nm, and for the elastic mean free path of 800 nm reported very recently in InAs 2D electron gas in Ref. [17], the temperature-dependent correction to the resistivity may be of the same order as the impurity-induced resistivity already at $T = 2 \text{ K}$. Therefore it may be observable for realistic parameters of the system.

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Expressions for $\Phi_\equiv$

The quantities $\Phi_\equiv$ that appear in the electron–electron collision integral Eq. (15) below the Dirac point are obtained as the sums of integrals

\[ \Phi_\equiv = \sum_{i=1}^{8} \zeta_i(x), \quad \Psi_\equiv = \sum_{i=1}^{8} \zeta_i(1/x), \quad \tag{18} \]

where

\[ \zeta_1(x) = 0, \quad \tag{19a} \]

\[ \zeta_2(x) = \zeta_3(x) = \int_{-\pi}^{\pi} d\varphi \frac{1 + x^2 + 2 \cos \varphi}{4x} \left| \text{Re} \left( \frac{(1 + x)^2/2 - 1 - \cos \varphi}{1 + \cos \varphi - (x - 1)^2/2} \right) \right|, \quad \tag{19b} \]

\[ \zeta_4(x) = \frac{1}{x} \int_{-\pi}^{\pi} d\varphi \left( 1 - \cos \varphi \right) \left| \text{Re} \sqrt{2x^2 - 1 - \cos \varphi}(1 + \cos \varphi) \right|, \quad \tag{19c} \]

\[ \zeta_5(x) = -x \int_{-\pi}^{\pi} d\varphi \frac{(1 - \cos \varphi) \cos \varphi}{x} \left| \text{Re} \sqrt{3/x - x - 2 \cos \varphi} \right|, \quad \tag{19d} \]

\[ \zeta_6(x) = \zeta_7(x) = x \int_{-\pi}^{\pi} d\varphi \frac{(1 - \cos \varphi)^2}{x} \frac{\sqrt{1 - \cos^2 \varphi}}{x + 1/x + 2 \cos \varphi}, \quad \tag{19e} \]

\[ \zeta_8(x) = \int_{-\pi}^{\pi} d\varphi \frac{(1 - \cos \varphi)}{x} \left| \text{Re} \sqrt{3x - 1/x - 2 \cos \varphi} \right|, \quad \tag{19f} \]

and $x = p_\equiv/p_\cong$. The transition to the dependence on $E_F$ below the Dirac point is performed by means of equation

\[ x = \frac{1 + \sqrt{1 + E_F/E_{SO}}}{1 - \sqrt{1 + E_F/E_{SO}}}, \quad \tag{20} \]

Quantities $\Phi_\pm$ that appear in the electron–electron collision integral Eq. (15) above the Dirac point are represented by the sums of expressions

\[ \Phi_- = \sum_{i=1}^{8} \xi_i(y), \quad \Phi_+ = \sum_{i=1}^{8} \xi_i(1/y), \quad \tag{21} \]
where

\[ \xi_2(y) = \xi_3(y) = \int_{-\pi}^{\pi} d\varphi \ (1 - \cos \varphi) \frac{2 y^2 \cos \varphi + y^2 + 1}{4 y} \left| \text{Re} \sqrt{\frac{(y^{-1} - 1)^2/2 - 1 - \cos \varphi}{1 + \cos \varphi - (y^{-1} + 1)^2/2}} \right|, \]

\[ (22a) \]

\[ \xi_4(y) = \int_{-\pi}^{\pi} d\varphi \ (1 - \cos \varphi) \sqrt{\cos \varphi + 1} \left| \text{Re} \sqrt{2 - y^2 (1 + \cos \varphi)} \right|, \]

\[ (22b) \]

\[ \xi_5(y) = -\int_{-\pi}^{\pi} d\varphi \frac{\cos \varphi (\cos \varphi + 1)}{y} \left| \text{Re} \sqrt{\frac{3y - 1/y - 2 \cos \varphi}{y + 1/y + 2 \cos \varphi}} \right|, \]

\[ (22c) \]

\[ \xi_6(y) = \xi_7(y) = \int_{-\pi}^{\pi} d\varphi \frac{\cos \varphi (\cos \varphi + 1)^2}{y^2 + 2y \cos \varphi + 1}, \]

\[ (22d) \]

\[ \xi_8(y) = \int_{-\pi}^{\pi} d\varphi \ (\cos \varphi + 1) \left| \text{Re} \sqrt{\frac{3/y - y - 2 \cos \varphi}{y + 1/y + 2 \cos \varphi}} \right|, \]

\[ (22e) \]

\[ \xi_9(y) = \xi_{10}(y) = \int_{-\pi}^{\pi} d\varphi \ (\cos \varphi + 1) \left| \text{Re} \sqrt{\frac{3/y - y - 2 \cos \varphi}{y + 1/y + 2 \cos \varphi}} \right|, \]

\[ (22f) \]

and \( y = p_-/p_+ \). The transition to the dependence on \( E_F \) above the Dirac point is performed by means of equation

\[ y = \sqrt{\frac{E_F/E_{SO} + 1 + 1}{E_F/E_{SO} + 1 - 1}}, \]

\[ (23) \]

Overall dependences of quantities \( \Phi_\geq \) and \( \Phi_\pm \) on \( E_F \) are shown in Fig. 2 of the paper. At \( E_F = -E_{SO} \), both \( \Phi_\leq \) and \( \Phi_\geq \) exhibit a logarithmic singularity

\[ \Phi_\leq(E_F/E_{SO}) = 16 \ln \left( \frac{E_{SO}}{E_F + E_{SO}} \right). \]

\[ (24) \]

At the Dirac point \( E_F = 0 \), \( \Phi_\geq \) smoothly join \( \Phi_\pm \), so that

\[ \Phi_\leq(0) = \Phi_+(0) = 16 + 2\sqrt{3}\pi, \quad \Phi_\geq(0) = \Phi_-(0) = 0. \]

\[ (25) \]

In the limit \( E_F/E_{SO} \to \infty \), both \( \Phi_+ \) and \( \Phi_- \) tend to the same limiting value

\[ \Phi_+(\infty) = \Phi_-(\infty) = 16. \]

\[ (26) \]