Radiation Efficiency Limits in Direct Antenna Modulation Transmitters

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Abstract—Relative bounds on radiation efficiency are established for time-modulated antenna systems, where radiation is generated by broadband conduction currents impressed using idealized nonradiating time-varying subsystems, such as those found in direct antenna modulation (DAM) transmitters. Analytical and numerical examples demonstrate that the condition of quasi-resonance, common in nearly all practical DAM transmitters, imposes severe restrictions on the otherwise unbounded gains in effective efficiency theoretically achievable by this class of time-modulated transmitters.

Index Terms—Antenna theory, antenna efficiency, electrically small antennas, time-varying circuits.

I. INTRODUCTION

The use of time modulation in an electrically small antenna or its matching network affords the possibility to exceed the strict physical bounds on its linear time-invariant (LTI) performance. Notably, direct antenna modulation (DAM) techniques based on the energy-synchronous modulation of time-varying components have been proposed, simulated, and measured as viable strategies for exceeding the conventional bandwidth-efficiency product limitations of small antennas, e.g., the Chu Q-factor, as it is in purely LTI systems. We do not consider nonconduction-based radiation, e.g., acoustically driven antennas [25]. Whether driven by LTI or non-LTI systems, the physical bounds governing the maximum achievable radiation efficiency by these currents remain the same. However, in non-LTI systems, realized bandwidth may not be directly controlled by limits on radiation Q-factor, as it is in purely LTI systems.

The tradeoff between maximal efficiency and bandwidth for current distributions confined to the design region \( \Omega^2 \) may

With the efficacy of DAM demonstrated by a variety of means, it now becomes prudent to determine whether or not there exist quantitative limits in performance gains achievable by adopting such techniques over traditional transmitters. Here, we formulate the question: for a fixed electrical size, how much more efficient could a DAM system be compared to an optimal resistively broadbanded conventional transmitter achieving the same effective bandwidth? Defined in this way, it may appear that relative DAM efficiency increases indefinitely with decreasing transmitter size due to trends in radiation Q-factor. However, closer inspection suggests that competing trends in loss may limit DAM efficiency gains in the extreme electrically small limit.

The goal of this letter is to calculate bounds on DAM efficiency gains by adapting tools developed for the analysis of conventional LTI systems. Namely, we examine low-frequency trends in the efficiency and Q-factor of small antennas to assess the relative benefit of idealized DAM systems over a broad range of scenarios (e.g., electrical size, tuning assumptions, conductivity models). First, in Section II, a DAM transmitter is decomposed into non-LTI and LTI components and models are proposed for ideal conventional, ideal nonresonant DAM, and ideal QR DAM systems. Then, in Section III, we apply these models to three examples of varying levels of abstraction and generality. In all cases, we observe that the condition of quasi-resonance, much like that of self-resonance in classical antennas [22]–[24], imposes severe efficiency limitations and greatly reduces the potential gains afforded by DAM.

II. MODELING IDEAL LTI AND DAM TRANSMITTERS

Throughout this letter, we assume that conventional and DAM systems are represented by the block diagrams in Fig. 1. In both systems, radiation is produced only by conduction currents induced on the object \( \Omega \). We do not consider nonconduction-based radiation, e.g., acoustically driven antennas [25]. Whether driven by LTI or non-LTI systems, the physical bounds governing the maximum achievable radiation efficiency by these currents remain the same. However, in non-LTI systems, realized bandwidth may not be directly controlled by limits on radiation Q-factor, as it is in purely LTI systems.

The tradeoff between maximal efficiency and bandwidth for current distributions confined to the design region \( \Omega^2 \) may

\( \text{1Due to their dynamic nature, DAM transmitters do not have a classically defined impedance bandwidth. An effective bandwidth may, however, be defined via a proxy measure such as distortion}[20],[21] \)

\( \text{2By proxy, this includes all possible antennas confined to the region } \Omega \)[26].
be rigorously calculated via multibjective optimization methods [27]. These techniques are based on solving deterministic problems formed from operators closely related to the method of moments impedance matrix [26], [28]. Examples of the resulting tradeoffs, in the form of Pareto fronts [29], are shown in Fig. 2. Two Pareto fronts are plotted with and without the requirement of self-resonance, with the latter being less restrictive. Points in bandwidth-efficiency space above each Pareto front are infeasible by LTI transmitters. The self-resonant Pareto front, i.e.,

\[ \eta_{\text{Conv}} = \min\{\eta_0, \eta_{\text{sr}}^{\text{max}}\} \]  

where \( \eta_0 \) is the efficiency at the intersection and minimization explicitly enforces the vertical right boundary of the self-resonant Pareto front in Fig. 2.

B. Ideal Nonresonant DAM Transmitter

Using non-LTI matching and loading networks, it may be possible to impress a broadband signal onto the maximally efficient current distribution feasible within the LTI design region \( \Omega \). If this to be (at least hypothetically) true, then an ideal DAM transmitter with no constraint on resonance or quasi-resonance would achieve the bandwidth \( B_0 \) with the optimal efficiency \( \eta_{\text{DAM}}^{\text{max}} = \eta_{\text{sr}}^{\text{max}} \). The actual topology of the antenna and non-LTI system that would achieve this performance, if it exists, is unknown, but this proposed model serves as a first upper bound on the DAM performance.

C. Ideal QR DAM Transmitter

The aforementioned nonresonant upper bound on radiation efficiency is known to be loose. Additionally, many DAM transmitter architectures rely on resonant tuning to achieve quasi-resonance, e.g., [2], as part of their broadbanding strategy. Here, we incorporate these features by assuming that an ideal QR DAM transmitter may realize the bandwidth \( B_0 \) by impressing a broadband version of the maximum self-resonant efficiency current distribution on the design region \( \Omega \), i.e., \( \eta_{\text{DAM}}^{\text{qr}} = \eta_{\text{sr}}^{\text{max}} \). Example data in Fig. 2 representatively demonstrate that the self-resonant efficiency bound is lower than the nonresonant bound [22]–[24]. Much like in the nonresonant case, the actual shape and existence of a system achieving this efficiency bound is unknown, though its performance establishes a bound on all realizable systems achieving quasi-resonance.

III. EFFECTIVE EFFICIENCY GAINS

With models for the ideal efficiency of conventional, nonresonant DAM, and QR DAM transmitters established, we proceed by studying the relative efficiency of each DAM transmitter with respect to the conventional case, i.e., \( \eta_{\text{DAM}}^{\text{qr}} / \eta_{\text{Conv}} \), and \( \eta_{\text{DAM}}^{\text{nr}} / \eta_{\text{Conv}} \). These quotients represent the highest possible relative gain in efficiency feasible by each class of DAM transmitter over a conventional LTI system.

A. Electrically Small Spherical Shell

As a first example, we analytically study the optimal performance of a spherical shell carrying a TM_{10} (dipole) modal current distribution. Asymptotic forms for the radiation Q-factor
and dissipation factor \( \delta_{nr} \) of this structure in the electrically small \( (ka \to 0) \) limit are [30]

\[
Q_{nr}^r = \frac{3}{2(ka)^3}, \quad Q_{nr}^s = \frac{1}{(ka)^3} \tag{2a}
\]

\[
\delta_{nr}^r = \frac{R_s}{Z_0^r} \frac{9}{4(ka)^2}, \quad \delta_{nr}^s = \frac{R_s}{Z_0^s} \frac{3}{(ka)^3} \tag{2b}
\]

where \( R_s \) and \( Z_0 \) are the shell’s surface resistivity and the impedance of free space, respectively. The nonresonant case is optimal in nonresonant dissipation factor, while the self-resonant case (where the TE_{10} mode is used for tuning) represents both minimum radiation Q-factor and minimum self-resonant dissipation factor. It can be shown that the Pareto front between these parameters is linear in \( \delta Q \)-space between these points [27].

Pareto fronts for shells of three electrical sizes are shown in Fig. 3. For this example calculation, a nondispersive surface resistivity \( R_s = 0.0014 \, \Omega/\square \) is used, corresponding to thick copper at 30 MHz [31], and an objective bandwidth is set at \( B_0 = 0.005 \). The maximal efficiencies \( \eta_{nr}^{\text{max}} \) and \( \eta_{sr}^{\text{max}} \) are denoted by square and triangular markers on each Pareto front. These data show that for all three electrical sizes, the nonresonant DAM efficiency is much higher than the maximally efficient conventional transmitter. This is not the case for DAM transmitters with the constraint of quasi-resonance, which have efficiency that reduces rapidly with decreasing electrical size, eventually passing to the left of the intercept point \( (\eta_0, B_0) \).

The trends observed in Fig. 3 indicate that the “effective efficiency gain” \( \eta/\eta_{\text{Conv}} \) in a DAM system depends highly on whether or not the condition of quasi-resonance is enforced. At moderately small electrical sizes, gains in both forms of DAM are substantial. For extremely small electrical sizes, the rapid increase in losses imposed by resonance overtakes the necessary efficiency sacrifice required to resistively broadband a conventional transmitter to the objective bandwidth \( B_0 \). This is further demonstrated in Fig. 4, where the effective efficiency gain \( \eta/\eta_{\text{Conv}} \) is plotted as a function of electrical size \( ka \) for two objective bandwidths. There clearly observe that gains for nonresonant DAM transmitters are unbounded in the electrically small limit, whereas QR DAM transmitters provide an efficiency benefit only over a limited size range. Naturally, the span of this range depends on the target bandwidth, with more extreme broadbanding leading to gains over broader ranges.

### B. Optimal Substructure Embedded Antenna

As a more complex example utilizing the generality of the Pareto front calculation methods in [27], an embedded antenna is considered where only a portion of the current support is considered controllable. The controllable region \( \Omega_c \) over which currents may be optimized is defined as a rectangular region of dimensions \( h \times \alpha \ell \). The uncontrollable scatterer \( \Omega_u \) is located a distance \( g \) below the controllable region and has dimensions \( h \times \ell \), as shown in Fig. 5. As in the previous example, a nondispersive surface resistance \( R_s = 0.0014 \, \Omega/\square \) is used.

Efficiency gains for several different values of the parameter \( \alpha \) are shown in Fig. 5, where we observe the same trends as in the spherical transmitter. As expected, larger controllable regions lead to smaller DAM performance gains, since the LTI structure is capable of lower radiation Q-factor. Interestingly, the low-frequency trend in QR DAM efficiency gain appears to be only weakly dependent on the geometry parameter \( \alpha \). This can be understood by the rather weak dependence of
the maximally efficiency tuning current Q-factor $Q_L$ on this parameter, consistent with the competing trends in increasing electrical size and decreasing aspect ratio [27].

C. Driven Wire Dipole

As a final example, we step away from Pareto-optimal current analysis and calculate the relative efficiencies of a driven short dipole antenna. Here, we explicitly consider conventional and DAM systems to radiate via the dipole’s driven current distribution, though DAM systems may be designed to impress this current distribution over a broad bandwidth, bypassing the dipole’s narrow impedance bandwidth, e.g., [2]. The impedance $Z_a = R_a + jX_a$, radiation efficiency $\eta_a$, and radiation Q-factor $Q$ (via impedance and efficiency [33]) of a center-fed copper wire dipole with length 2 m and wire radius 1.6 mm are calculated using NEC [34].

At any frequency below its self-resonance, this dipole antenna may be tuned to resonance via an inductance $L$ with Q-factor $Q_L$, giving rise to the total efficiency

$$\eta = \frac{\eta_a R_a}{R_a - X_a/Q_L}. \quad (3)$$

As in the previous example, the conventional efficiency is assumed to be tuned and resistively loaded as needed to obtain a target bandwidth $B_0$, i.e.,

$$\eta_{\text{Conv.}} = \min \{ (B_0Q)^{-1}, \eta \}. \quad (4)$$

An ideal DAM transmitter, which requires the same resonant tuning as in the conventional system (e.g., the dipole OOK systems in [2]–[5]) will, at best, achieve the resonant efficiency $\eta_{\text{DAM}} = \eta$, whereas one without the resonance constraint would achieve the standalone antenna efficiency $\eta_{\text{nr}} = \eta_a$. Note that the latter is equivalent to assuming lossless inductors are available, though rigorous comparison to the conventional case in that particular scenario requires an alteration to (3), so we do not consider this interpretation further in this letter. Taking the ratio of conventional and DAM transmitter efficiencies, with some rearranging, yields

$$\frac{\eta_{\text{nr}}}{\eta_{\text{DAM}}} = \frac{\eta_a R_a}{R_a - X_a/Q_L}, \quad (5a)$$

$$\frac{\eta_{\text{nr}}}{\eta_{\text{Conv.}}} = \max \left\{ \frac{\eta_a Q R_a B_0}{R_a - X_a/Q_L}, 1 \right\}. \quad (5b)$$

Curves are shown in Fig. 6 illustrating the efficiency ratios in (5) for two bandwidths $B_0$ using tuning elements defined by a realistic Q-factor for air-coil inductors [24], [35]

$$Q_L = 150 \sqrt{\omega/(2\pi \cdot 30 \cdot 10^6)}. \quad (6)$$

We observe the same characteristic features as in the previous examples, where efficiency gains in the nonresonant system are potentially unbounded in the low frequency limit while gains in the resonant system are limited and exist only over a finite bandwidth. This example indicates that for realistic loss models, physical size, and frequency ranges, the potential benefits of a DAM transmitter are severely limited when some form of resonance (e.g., quasi-resonance) is required.

[Image 458x696 to 526x712]

Fig. 6. Effective efficiency gains of a 2 m wire dipole antenna.

In the electrically small limit, the dipole antenna’s impedance behavior is accurately4 approximated as

$$R_a \approx R_{\text{rad},0} \frac{\omega^2}{\omega_0^2} + R_{\text{Q},0} \sqrt{\frac{\omega}{\omega_0}} \quad X_a \approx \frac{1}{j\omega C_0} \quad (7)$$

where $R_{\text{rad},0}$, $R_{\text{Q},0}$, and $C_0$ are the radiation resistance at frequency $\omega_0$, the ohmic loss resistance at frequency $\omega_0$, and static capacitance, respectively. Additionally, a general form of (6) where $Q_L = Q_{L,0} \sqrt{\omega/\omega_0}$ may be assumed for the inductor Q-factor. Determining the frequency range over which the self-resonant DAM efficiency in (5a) is greater than unity is best carried out numerically, however the location of the peak in self-resonant efficiency gain may be estimated as

$$\omega_{\text{peak}} = \frac{7}{(6R_{\text{rad},0}C_0 Q_{L,0})^2} \left( \frac{\omega_0^5}{\omega} \right) \quad (8)$$

by assuming that at this frequency radiation and inductor losses far outweigh ohmic losses on the antenna. In Fig. 6 we observe that this approximation, which results in a peak location dependent only on low-frequency asymptotic coefficients, is extremely accurate (<1% error). A similar analysis may be carried out by any small dipole radiator characterized by (7).

IV. CONCLUSION

In this letter, an effective efficiency ratio is used to quantify the advantage of DAM transmitters over their conventional counterparts. By splitting the system into LTI and non-LTI components, many results from optimal LTI antenna theory are applied. From all reported examples, we observe that the requirement of resonance (i.e., quasi-resonance) severely limits the potential benefits of a DAM transmitter. This implies that QR DAM systems, prevalent in the literature, may have limited applicability at extremely low frequencies. The same conclusion holds for any loop based methods, regardless of resonance condition, as their radiation efficiency scales inherently as $(ka)^4$.

Similar concepts may be applied to the analysis of scattering from nonlinear loads [36]–[38], as the underlying bifurcation of a system into linear and nonlinear components is very similar to that used in the antenna problems studied here.

4Note that the skin depth model is used throughout, which is nonphysical in the extreme electrically small limit.
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