We discuss single top quark production via $u^i \bar{d}^j \to t \bar{b}$ at the Fermilab Tevatron in the minimal supersymmetric model with $R$-parity violation. We find that within the allowed range of coupling constants, the lepton-number violating couplings can give rise to observable effects when the slepton mass lies in a specific narrow range. For the baryon-number violating couplings, the contribution to the production rate can be quite large in the presently allowed range of the coupling constants. We show that the measurement of single top production at the upgraded Tevatron can be used to constrain a linear combination of products of the $R$-parity violating couplings.

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1. Introduction

Single top quark production through $W$-gluon fusion\cite{1} $g + W \rightarrow t + \bar{b}$, and quark-antiquark annihilation via virtual $s$-channel $W$\cite{2} $u^i + \bar{d}^j \rightarrow t + \bar{b}$, where $i, j$ are the generation indices, are interesting to study at the Tevatron. In contrast to the QCD process of $t\bar{t}$ pair production, the single top production involves the electroweak interaction and can, therefore, be used to probe the electroweak theory. In particular, the quark-antiquark annihilation subprocess can be used to study models of new physics in connection with the top quark.

It has been shown\cite{3} that the signal for $u^i \bar{d}^j \rightarrow t\bar{b}$ is potentially observable at the Tevatron with 2-3 fb$^{-1}$ integrated luminosity, although it will be overwhelmed at the LHC by the large background from $t\bar{t}$ production plus single tops from the $W$-gluon fusion\cite{1}. Compared to $Wg \rightarrow t\bar{b}$, $u^i \bar{d}^j \rightarrow t\bar{b}$ has the advantage that the cross section can be calculated reliably because the quark and antiquark structure functions at the relevant values of $x$ are better known than the gluon structure functions entering in the calculation for the $W$-gluon fusion cross section. With a 30 fb$^{-1}$ data sample from Run 3 at the upgraded Tevatron with $\sqrt{s} = 2$ TeV, it is possible to measure the ratio of single top production ($u^i \bar{d}^j \rightarrow W \rightarrow t\bar{b}$) and Drell-Yan cross section ($u^i \bar{d}^j \rightarrow W \rightarrow l\nu$) to an accuracy of $\pm 8\%$ \cite{4}. Thus new physics effects that produce larger than 16\% effect on the cross section ratio should be detectable\cite{4}.

In the Standard Model, the cross section of single top quark production $u^i \bar{d}^j \rightarrow t\bar{b}$ has been calculated to one loop order\cite{5}. Recently, this process has been used to study the effects of model-independent new physics involving the third-family quarks\cite{6} and it was shown that at the upgraded Tevatron it can be a powerful probe of new physics. Effects in specific new physics models on this production process were evaluated in Refs.\cite{7,8}. In the general two-Higgs-doublet model and the minimal supersymmetric model (MSSM) with $R$-parity conservation\cite{7,9,10}, the contribution to the production rate can reach the observable level only for $\tan \beta < 1$ which is generally not regarded as viable. In most of the new physics models considered in the literature the enhancement effect of the single top quark production is smaller than 15\% and some of the models can even suppress the production rate\cite{8}. An exception to this general result can exist, as analysed in Ref.\cite{8}, in a non-commuting ETC model with an extra weak gauge boson of
mass no less than 500 GeV.

In this paper we focus on the $R$-parity violating MSSM\cite{11} and evaluate the effect of $R$-parity violating couplings on single top quark production at the Tevatron. While this is an interesting problem in its own right, the recent anomalous events at HERA provide an additional motivation for the study of the $R$-parity violating supersymmetric couplings. The HERA data showed excess events in deep-inelastic positron-proton scattering at high-$Q^2$ and high $x$, which are in apparent conflict with the Standard Model expectations\cite{12}. The excess events have been interpreted as evidence of $R$-parity breaking supersymmetry\cite{13}. Hence the examination of effects of $R$-parity breaking supersymmetry in other processes are desirable. In Sec.2 we present the Lagrangian for $R$-parity violating couplings. In Sec.3 we evaluate the effects of $R$-parity violating couplings on the production rate of single top quark. In Sec.4 we present the numerical results and some discussions.

2. Lagrangian of $R$-parity violating couplings

In supersymmetric models, the $R$-parity of a field with spin $S$, baryon-number $B$ and lepton-number $L$ is defined to be

\[ R = (-1)^{2S+3B+L}. \]  

(1)

$R$ is +1 for all the SM particles and -1 for all super particles. $R$-parity invariance is often imposed on the Lagrangian in order to maintain the separate conservation of baryon-number and lepton-number. Imposition of $R$-parity conservation has some important consequences; super particles must be produced in pairs in collider experiments and the lightest super particle (LSP) must be absolutely stable. Thus the LSP provides a good candidate for cold dark matter.

Despite the above mentioned attractive feature of $R$-parity conservation, the conservation is not dictated by any fundamental principle such as gauge invariance and there is no compelling theoretical motivation for it. The most general superpotential of the MSSM, consistent with $SU(3) \times SU(2) \times U(1)$ gauge symmetry and supersymmetry, can be written as

\[ W = W_R + W_H. \]  

(2)
where $W_R$ is the $R$-parity conserving part while $W_{\bar{R}}$ violates the $R$-parity. They are given by

\begin{align}
W_R &= h_{ij} L_i H_2^c + h'_{ij} Q_i H_2^c + h''_{ij} Q_i H_1^c, \\
W_{\bar{R}} &= \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i D_j^c D_k^c + \mu_i L_i H_2. 
\end{align}

Here $L_i(Q_i)$ and $E_i(U_i, D_i)$ are the left-handed lepton (quark) doublet and lepton (quark) singlet chiral superfields. $i, j, k$ are generation indices and $c$ denotes a charge conjugate field. $H_{1,2}$ are the chiral superfields representing the two Higgs doublets. Note that the term $\mu_i L_i H_2$ can be rotated away by a redefinition\(^2\) of the Higgs $H_1$ and that of the leptonic $L_i$ superfields\(^{[15]}\).

In the $R$-parity violating superpotential Eq.(4), the $\lambda$ and $\lambda'$ couplings violate lepton-number conservation, while the $\lambda''$ couplings violate baryon-number conservation. $\lambda_{ijk}$ is antisymmetric in the first two indices and $\lambda''_{ijk}$ is antisymmetric in the last two indices. While it is theoretically possible to have both baryon-number and lepton-number violating terms in the Lagrangian, the non-observation of proton decay imposes very stringent conditions on their simultaneous presence\(^{[16]}\). We, therefore, assume the existence of either $L$-violating couplings or $B$-violating couplings, but not the coexistence of both. We calculate the effects of both types of couplings.

In terms of the four-component Dirac notation, the Lagrangian of the $\lambda'$ and $\lambda''$ couplings that affect single top production at the Tevatron are given by

\begin{align}
\mathcal{L}_{\lambda'} &= -\lambda'_{ijk} \left[ \bar{\nu}_L^i d_R^k u_L^j + \bar{d}_L^j d_R^k \nu_L^i + (d_R^k)^* (\bar{\nu}_L^i)^c d_L^j \\
&\quad - \bar{e}_L^i d_R^k u_L^j - \bar{u}_L^j d_R^k \bar{e}_L^i - (d_R^k)^* (\bar{e}_L^i)^c u_L^j \right] + h.c., \\
\mathcal{L}_{\lambda''} &= -\lambda''_{ijk} \left[ \bar{d}_R^k (\bar{u}_L^i)^c d_L^j + \bar{d}_R^j (\bar{d}_L^k)^c u_L^i + \bar{u}_R^i (\bar{d}_L^k)^c d_L^j \right] + h.c.
\end{align}

The terms proportional to $\lambda$ are not relevant to our present discussion and will not be considered here. As discussed in the proceeding paragraph, we assume that $\mathcal{L}_{\lambda'}$ and $\mathcal{L}_{\lambda''}$ are not present simultaneously. The Lagrangians given above can give rise to interesting effects in low energy processes.

In the presence of $R$-parity violation, the phenomenology of MSSM changes considerably. For instance, the LSP is no longer stable and their decays can lead to interesting phenomenology.

\(^2\)Such redefinition does not leave the full Lagrangian invariant\(^{[14]}\), but it has no relevant consequences to our analysis.
in collider experiments. Several authors have investigated the phenomenological implications of $R$-parity violating couplings at various colliders[17]. Constraints on the $R$-parity violating couplings have been obtained from perturbative unitarity[18,19], $n - \bar{n}$ oscillation[19,20], $\nu_e$-Majorana mass[21], neutrino-less double $\beta$ decay[22], charged current universality[23], $e - \mu - \tau$ universality[23], $\nu_\mu - e$ scattering[23], atomic parity violation[23], $\nu_\mu$ deep-inelastic scattering[23], $K$-decay[24,25], $\tau$-decay[26], $D$-decay[26], $B$-decay[27-29] and $Z$-decay at LEP I[30,31].

As was pointed out in Ref.[24], transforming the Lagrangian from the gauge basis to the quark mass basis can lead to a flavor changing neutral current (FCNC) in the up or the down quark sector even under the assumption of one $R$-parity violating coupling. FCNC processes can therefore provide stringent constraints on the $R$-parity violating couplings.

3. Production rate of single top quark in $R$-parity violating MSSM

The Feynman diagrams for the single top quark production at the tree level parton process, $u^i(p_1) + \bar{d}^j(p_2) \to t(p_3) + \bar{b}(p_4)$, are shown in Fig.1. The diagrams for the $L$-violating couplings and $B$-violating couplings are shown in Fig.1(b) and Fig.1(c), respectively. We assume that figures 1(b) and 1(c) do not exist simultaneously. Note that the $B$-violating couplings can also give rise to a $s$-channel diagram $u^i + \bar{d}^j \to \tilde{d}^j \to t + b$. But this is a different process which does not interfere with the SM diagram $u^i + \bar{d}^j \to W^+ \to t + \bar{b}$ and it is relatively suppressed by the sea structure functions. Therefore, its contribution is expected to be much smaller at the Tevatron than the $t$-channel diagram in Fig.1(c).

The amplitudes for Fig.1(a,b,c) denoted by $M_{ij}^0$, $\delta M_{ij}^N$ and $\delta M_{ij}^{\lambda''}$, respectively, are given by

\[
M_{ij}^0 = i \frac{g^2}{2} \frac{K_{ij}}{s - M_W^2} \bar{u}(p_2) \gamma_\mu P_L u(p_1) \bar{u}(p_3) \gamma^\mu P_L v(p_4),
\]

\[
\delta M_{ij}^N = -i \chi_{kij} \chi_{k\bar{s}3} \frac{1}{s - M_{\tilde{W}_k^L}^2 + i \Delta_{\tilde{W}_k^L} \Gamma_{\tilde{W}_k^L}} \bar{u}(p_2) P_L u(p_1) \bar{u}(p_3) P_R v(p_4),
\]

\[
\delta M_{ij}^{\lambda''} = -i \chi_{kij} \chi_{k\bar{s}3} \frac{1}{t - M_{\tilde{d}_R^j}^2} \left[ \bar{u}^c(p_1) P_L v(p_4) - \bar{v}^c(p_4) P_L u(p_1) \right] \times \left[ \bar{v}(p_2) P_R u^c(p_3) - \bar{u}(p_3) P_R v^c(p_2) \right].
\]
Here \( P_{L,R} \equiv (1 \mp \gamma_5)/2 \), \( i = (p_1 - p_4)^2 \), and the sum over \( k = 1, 2, 3 \) is implied. \( K_{ij} \) are the KM matrix elements, \( \hat{s} \) is the center-of-mass energy squared for the parton-level process, and \( M_{\tilde{e}_R^k} \) \( M_{\tilde{e}_L^k} \) are the masses of squark and slepton, respectively.

The SM parton-level cross section at tree level is

\[
\hat{\sigma}_{ij}^0 = \frac{g^4|K_{ij}|^2}{384\pi} \frac{(\hat{s} - M_{\tilde{e}_R^i}^2)^2}{\hat{s}^2(\hat{s} - M_W^2)^2}(2\hat{s} + M_t^2),
\]

where we have neglected the masses of the bottom quark and the initial partons \(^3\). The contribution of \( L\)-violating couplings to the parton-level cross section is given by

\[
\Delta\hat{\sigma}_{ij}^\prime = \frac{1}{64\pi} \frac{(\hat{s} - M_{\tilde{e}_R^i}^2)^2}{\hat{s}} \left| \sum_k \frac{\lambda_{ij}^k \lambda_{k33}}{\hat{s} - M_{\tilde{e}_L^k}^2 + iM_{\tilde{e}_R^k} \Gamma_{\tilde{e}_R^k}} \right|^2,
\]

where \( \Gamma_{\tilde{e}_R^k} \) is the width of the charged slepton \( \tilde{e}_L^k \). In the R-parity conserving MSSM, the charged sleptons \( \tilde{e}_L^k \) will decay into charginos and neutralinos via the processes \( \tilde{e}_L^k \rightarrow \nu_{\ell^k} + \bar{\chi}_j^\pm \) \( (j = 1, 2) \) and \( \tilde{e}_L^k \rightarrow e^k + \bar{\chi}_j^0 \) \( (j = 1, 2, 3, 4) \), where \( \bar{\chi}_j^\pm \) and \( \bar{\chi}_j^0 \) represent a chargino and neutralino, respectively\(^[32]\). However, in the R-parity violating MSSM, the slepton can also have the decay modes \( \tilde{e}_L^k \rightarrow \bar{u}^j + d^i \) \( (i, j = 1, 2, 3) \) via the \( \lambda' \) couplings. The partial widths are given by

\[
\Gamma(\tilde{e}_L^k \rightarrow \nu_{\ell^k} + \bar{\chi}_j^\pm) = \frac{g^2}{16\pi M_{\tilde{e}_L^k}^3} |U_{j1}|^2 \left( M_{\tilde{e}_L^k}^2 - M_{\bar{\chi}_j^\pm}^2 \right)^2,
\]

\[
\Gamma(\tilde{e}_L^k \rightarrow e^k + \bar{\chi}_j^0) = \frac{g^2}{8\pi M_{\tilde{e}_L^k}^3} \left| s_W N_{j1} + \frac{1}{c_W} \left( \frac{1}{2} - s_W^2 \right) N_{j2} \right|^2 \left( M_{\tilde{e}_L^k}^2 - M_{\bar{\chi}_j^0}^2 \right)^2,
\]

\[
\Gamma(\tilde{e}_L^k \rightarrow \bar{u}^j + d^i) = \frac{(\lambda_{ij}^k)^2}{16\pi M_{\tilde{e}_L^k}^3} \left( M_{\tilde{e}_L^k}^2 - M_{\bar{u}^j}^2 \right)^2,
\]

where \( s_W \equiv \sin \theta_W, c_W \equiv \cos \theta_W \) and the masses of the lepton \( e^k \) and down-type quark \( d^i \) are neglected. The masses of charginos and neutralinos, and the matrix elements \( U_{ij} \) and \( N_{ij}' \) depend on the SUSY parameters \( M_2, M_1, \mu, \) and \( \tan \beta \)[10]. Here, \( M_2 \) and \( M_1 \) are the masses of gauginos corresponding to \( SU(2) \) and \( U(1) \), respectively. \( \mu \) is the coefficient of the \( H_1H_2 \) mixing term in the superpotential, and \( \tan \beta = v_2/v_1 \) is the ratio of the vacuum expectation values of the two Higgs doublets.

\(^3\)We neglected the contribution of third-family sea quark in the initial states.
The contribution of the $B$-violating couplings to the cross section is given by

$$\Delta \hat{\sigma}_{ij}^{\lambda'} = \frac{g^2}{24\pi} \lambda''_{ijk} \lambda''_{kij} K_{ij} \frac{\hat{s} - M^2_t}{\hat{s}^2 (\hat{s} - M^2_W)} \left[ \frac{\hat{s} + M^2_t - 2M^2_{d_R}}{\hat{s} - M^2_t} \right] \left[ \frac{M^2_{d_R} - M^2_t}{\hat{s} - M^2_t} \right] \left[ \frac{\hat{s} + M^2_{d_R} - M^2_t}{M^2_{d_R}} \right] \log \frac{\hat{s} + M^2_t - 2M^2_{d_R}}{M^2_{d_R}},$$

where the sum over $k = 1, 2, 3$ is implied.

For the $B$-violating couplings, we keep only the interference term $\delta M_{\lambda'} M_0^1$. But for the $L$-violating couplings, we kept the higher order term $|\delta M_{\lambda'}|^2$ since the interference term $\delta M_{\lambda'} M_0^1 \sim m_d m_b$ which drops out when the mass of $d^j$ is neglected.

The total hadronic cross section for the production of single top quark is obtained by

$$\sigma(s) = \sum_{i,j} \int_{\tau_0}^{1} \frac{d\tau}{\tau} \left( \frac{1}{s} \frac{dL_{ij}}{d\tau} \right) (\hat{s} \hat{\sigma}_{ij})$$

where $s$ is center-of-mass energy squared, $\tau_0 = (M_t + M_b)^2 / s$ and $\tau$ is defined by $\tau = x_1 x_2$ with $x_{1,2}$ denoting the longitudinal momentum fractions of the initial partons $i$ and $j$, respectively. The quantity $dL_{ij}/d\tau$ is the parton luminosity defined by

$$\frac{dL_{ij}}{d\tau} = \int_{\tau}^{1} \frac{dx_1}{x_1} \left[ f_i^A(x_1, \mu) f_j^B(\tau/x_1, \mu) + (A \leftrightarrow B) \right]$$

where $A$ and $B$ denote the incident hadrons and the functions $f_i^A$ and $f_j^B$ are the usual parton distributions.

5. Numerical results and conclusion

In our numerical calculation, we use the CTEQ3L parton distribution functions with $\mu = \sqrt{s}$, and assuming $M_t = 175$ GeV and $\sqrt{s} = 2$ TeV. The KM matrix elements $K_{12} = -K_{21} = 0.22$ are used in our calculation.

5.1 $L$-violating couplings
For the contribution of $L$-violating couplings, we assume the masses of sleptons $\tilde{e}^k_L$ to be degenerate. The contribution of $L$-violating couplings to the cross section is sensitive to the width of the charged slepton $\tilde{e}^k_L$, which depends on the SUSY parameters $M_2, M_1, \mu, \tan \beta,$ and all $\lambda'$ couplings.

In our calculation we use the GUT relation $M_1 = \frac{5}{3} \frac{g'^2}{g^2} M_2 \approx \frac{1}{2} M_2$. Since the masses of charginos and neutralinos are not sensitive to $\tan \beta$, we fix $\tan \beta = 2$ and retain $M_2$ and $\mu$ as variables.

The upper limits of the $L$-violating couplings for the squark mass of 100 GeV are given by

$$|\lambda'_{kij}| < 0.012, \quad (k, j = 1, 2, 3; i = 1, 2),$$  \hspace{1cm} (18)

$$|\lambda'_{13j}| < 0.16, \quad (j = 1, 2),$$  \hspace{1cm} (19)

$$|\lambda'_{133}| < 0.001,$$  \hspace{1cm} (20)

$$|\lambda'_{23j}| < 0.16, \quad (j = 1, 2, 3),$$  \hspace{1cm} (21)

$$|\lambda'_{33j}| < 0.26, \quad (j = 1, 2, 3),$$  \hspace{1cm} (22)

The first set of constraints come from the decay $K \to \pi\nu\nu$ with FCNC processes in the down quark sector[24]. The second and forth set of constraints are obtained from the semileptonic decays of $B$-meson[29]. The third constraint, i.e., that on the coupling $\lambda'_{133}$, is obtained from the Majorana mass that the coupling can generate for the electron type neutrino[21]. The last set of limits have been derived from the leptonic decay modes of the $Z[30]$.

Also the following constraints are derived for the combinations of $\lambda'$ couplings

$$\lambda'_{13i}\lambda'_{12i}, \quad \lambda'_{23j}\lambda'_{22j} < 1.1 \times 10^{-3}, \quad (i = 1, 2; j = 1, 2, 3),$$  \hspace{1cm} (23)

$$\lambda'_{11n}\lambda'_{12n}, \quad \lambda'_{in2}\lambda'_{jn1} < 10^{-5}, \quad (i, j, n = 1, 2, 3),$$  \hspace{1cm} (24)

$$\lambda'_{111}\lambda'_{212}, \quad \lambda'_{112}\lambda'_{211}, \quad \lambda'_{121}\lambda'_{222} < 10^{-7},$$  \hspace{1cm} (25)

$$\lambda'_{122}\lambda'_{221}, \quad \lambda'_{131}\lambda'_{232}, \quad \lambda'_{132}\lambda'_{231} < 10^{-7},$$  \hspace{1cm} (26)

where the first set of constraints are derived in Ref.[29] and the other derived in Ref.[25].

Inputing the upper limits of the relevant $L$-violating couplings for squark mass of 100 GeV, we get the maximum contribution of $L$-violating couplings which are shown in Fig.2 and Fig.3.
Figure 2 shows the histogram of the differential cross section versus the invariant mass of the $t\bar{b}$ system over a bin size of 10 GeV with the favorable parameters $M_2 = -\mu = 200$ GeV. The solid line is for the standard model. To illustrate the contributions of sleptons of different masses, we superpose the effects of three sleptons on the same curve. The dashed, dotted and dash-dotted lines are the standard model plus the slepton contributions for three different slepton masses respectively: 230 GeV, 300 GeV and 350 GeV. The resonance behavior is already manifested. Because of their narrow widths, for each slepton the contributions of the $\lambda'$-couplings are negligible for a couple of bins away from the resonance. This will help to identify the signal of the slepton production.

Figure 3 shows the ratio of the total slepton contribution integrated over $\hat{s}$ to that of the standard model as a function of the slepton mass. For slepton mass lighter than 180 GeV, the contribution is very small. When the slepton mass becomes heavier than 180 GeV, which is the threshold for the slepton to decay into $t$ and $\bar{b}$, the contribution increases sharply and reaches its maximum sizes at the slepton mass of about 230 GeV. Then the effect drops quickly with further increase in the slepton mass. The large slepton mass suppression can be understood as follows: When the slepton mass is large the parton cross section contributions coming mainly from $\hat{s}\sim$-mass of slepton require large momenta from the initial partons which is suppressed by their structure functions. An additional suppression is caused by the increase of the slepton width when the slepton mass increase.

Figure 3 also shows that the contribution to the production rate increases with the increase of $M_2$ and $\mu$. This is because with the increase of $M_2$ and $\mu$, the chargino and neutralino masses increase and thus the width of the slepton decreases. The overall contribution to the production rate can exceed 20% when slepton mass lies in a narrow range (200 GeV ~ 270 GeV) and $M_2 = -\mu > 200$ GeV. As shown in Fig.2, the contribution is confined within a bin of 10 ~ 20 GeV. So, one can use the upgraded Tevatron to search for enhanced single top quark production so as to further constrain the $L$-violating couplings in this mass range of sleptons.

5.2 $B$-violating couplings

In our calculation, we assume the masses of $\tilde{d}_R^k$ to be degenerate. Then the contribution of
$B$-violating couplings can be parametrized as

$$\frac{\Delta \sigma^{\lambda''}}{\sigma_0} = F''_{11} \lambda''_{132} \lambda''_{312} + F''_{12} \lambda''_{113} \lambda''_{312} + F''_{21} \lambda''_{223} \lambda''_{312} + F''_{22} \lambda''_{213} \lambda''_{312};$$  \hspace{1cm} (27)

where $F''_{ij}$ depend on squark mass and are shown in Fig.4. From Fig.4 we see that $F''_{21}$ and $F''_{22}$ are the same and negligibly small. Since $F''_{12}$ is smaller than $F''_{11}$ and $\lambda''_{113} \leq 10^{-4}$ [19,20], we can also neglect the term $F''_{12} \lambda''_{113} \lambda''_{312}$.

For squark mass of 100 GeV, we take the upper limits for other $B$-violating couplings as given by [18,19,31] and are derived from perturbative unitarity and $Z$ decay.

$$\lambda''_{312} < 0.97,$$  \hspace{1cm} (28)

$$\lambda''_{132}, \lambda''_{223}, \lambda''_{213} < 1.25,$$  \hspace{1cm} (29)

From these upper limits, we find the upper limit of the contribution to be $\Delta \sigma^{\lambda''}/\sigma_0 < 2752\%$. This shows that the contribution can be quite large in the allowed region of $B$-violating couplings. Of course, in this upper limit of couplings, our results are no longer reliable and the higher order terms such as $(\lambda''_{132} \lambda''_{312})^2$ have to be included.

Neglecting the terms proportional to $F''_{12}$, $F''_{21}$ and $F''_{22}$ we present in Fig.5 the plot corresponding to $\Delta \sigma/\sigma_0 = 20\%$ in the $(\lambda''_{132} \lambda''_{312}, M_{\tilde{q}})$ plane. The region above the plot corresponds to $\Delta \sigma/\sigma_0 > 20\%$ while the region below the plot corresponds to $\Delta \sigma/\sigma_0 < 20\%$. From Fig.5 we see that if we assume an observable level of 20$\%$ at the upgraded Tevatron, the coupling $\lambda''_{132} \lambda''_{312}$ can be probed down to 0.01, 0.02, 0.03, 0.04, 0.06, 0.08, 0.1 and 0.13 for $M_{\tilde{q}}$ = 100, 200, 300, 400, 500, 600, 700 and 800 GeV, respectively. As mentioned above, the present upper limit is $\lambda''_{132} \lambda''_{312} < 1.25 \times 0.97 \approx 1.2$ for squark mass of 100 GeV. So, the single top production process at the upgraded Tevatron can be used meaningfully to probe the product of the $B$-violating couplings $\lambda''_{132}$ and $\lambda''_{312}$.

In summary, we studied the single top quark production via $u\bar{d} \rightarrow t\bar{b}$ at the Fermilab Tevatron in $R$-parity violating supersymmetry. We found that within the allowed range of the coupling constants, the effects of the $L$-violating $\lambda'$ couplings can be observed at the upgraded...
Tevatron only for slepton mass in a narrow range. As shown in Fig.2 a distinctive resonance is associated with the enhanced production of single top. This can also serve as a signal for the production of slepton via $L$-violating couplings or further constraint to their coupling strengths. For $B$-violating $\lambda''$ couplings, in the allowed range of the relevant coupling constants Eqs.(28,29), the contribution to the production rate can reach the observable level for a wide range in squark mass. So the upgraded Tevatron can make a powerful probe for the product of the $B$-violating couplings $\lambda''_{132}$ and $\lambda''_{312}$. Failure to observe a signal of enhancement to the single top production will be an indication of small $B$-violating couplings as shown in Fig.5. We note that owing to the linear vs quadratic dependence of the products of $R$-parity violating couplings, the $B$-violating couplings will be a more sensitive probe than the $L$-violating couplings.

It should be noted that the sparticle exchange effect in single top quark production explored in the present article is complementary to the approach of direct sparticles production at Tevatron. This latter approach can also yield a mass limit on the sparticle. In the case of the $L$-violating coupling, a lower limit of 100 GeV on the mass of squark/ gluino has been obtained in [34] and the limit can possibly be pushed up to 250 GeV for the accumulated dilepton data from RUN 1 [35]. But this sparticle production with the $L$-violating coupling does not allow a direct measurement of the slepton mass. If we use the MSSM with grand unification, in which the squark and slepton are related, a similar limit for the slepton mass can also be obtained as for the squark, i.e., a 100/250 GeV mass bound can be set. The slepton pair production in R-parity conserving MSSM is also a limited probe of the slepton mass at the Tevatron [36]. However, our approach allows a direct reach of the slepton mass, albeit a limited range of values. For the $B$-violating SUSY model corresponding to the $\lambda''$, the Tevatron reach of squark mass by direct production of the squark is very limited. The approach presented in this article can probe a wide range of the ratio of the product of the $\lambda''$ and the squark mass.

We conclude by noting that the $s$-channel squark intermediate process, $u^i + d^j \rightarrow \tilde{d}^k_R \rightarrow t + b$, is small at the Tevatron as pointed out early. But this process may not be small in the environment of $pp$ collision of LHC. This process has an interesting distinctive signal of double $b$ production and is currently under investigation.