Landau level broadening in graphene with long-range disorder  
— Robustness of the $n = 0$ level

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Abstract

Broadening of the Landau levels in graphene and the associated quantum Hall plateau-to-plateau transition are investigated numerically. For correlated bond disorder, the graphene-specific $n = 0$ Landau level of the Dirac fermions becomes anomalously sharp accompanied by the Hall transition exhibiting a fixed-point-like criticality. Similarly anomalous behavior for the $n = 0$ Landau level is also shown to occur in correlated random magnetic fields, which suggests that the anomaly is generic to disorders that preserve the chiral symmetry.

Key words: quantum Hall effect, graphene, long-range disorder

1. Introduction

After the seminal discovery of the characteristic quantum Hall effect (QHE) in graphene $^1$, $^2$, attention has been focused on the effect of disorder $^3$, $^4$, $^5$, $^6$, $^7$, $^8$, $^9$, $^{10}$. Randomness, which is important in the QHE in ordinary 2DEG, is particularly crucial since it affects such key factors in the lattice (as opposed to Dirac field) models of graphene such as the chiral (A-B sub-lattice) symmetry $^8$, and the scattering between the valleys (K and K'). Note that the valley degeneracy has a topological origin as the doubling of the massless Dirac fermions on the honeycomb lattice. It is then natural that the nature of disorder crucially affects the electronic structure. A potential disorder (including random site energies) destroys the chiral symmetry, whereas a disorder in bonds respects the symmetry, while the spatial correlation in the disorder controls the inter-valley scattering. In order to explore these, here we adopt the honeycomb lattice rather than the effective Dirac model to investigate how the spatial correlation of disorder affects the Landau level structure, especially the stability of the $n = 0$ Landau level which is essential to the characteristic QHE in graphene.

In the case of a potential disorder with degraded chiral symmetry, the quantum Hall transition at the $n = 0$ Landau level has been shown to be robust in an effective Dirac model $^8$. In such a model, however, the criticality of the transition at $n = 0$ is described by the ordinary quantum Hall transition with nothing special about the criticality at the $n = 0$ transition. In the previous paper $^9$, we have shown that for the bond disorder, which preserves the chiral symmetry, the criticality at $n = 0$ transition is anomalously sensitive to the spatial correlation of disorder. As soon as the correlation length of the bond disorder exceeds few lattice constants of the honeycomb lattice, the $n = 0$ Landau level becomes anomalously sharp and the associated Hall transition exhibits the anomalous criticality, which corresponds to a fixed point of the Dirac fermion with chiral symmetry $^{10}$. In the present paper, we present another example to reinforce our arguments that the anomalous Hall transition is a general property of the chiral-symmetric random systems. Namely, we consider systems with random phases in the transfer integrals, which corresponding to the case of random magnetic fields piercing the hexagons in honeycomb lattice.

2. Models

The tight-binding model for the honeycomb lattice is described by the Hamiltonian $H = \sum_{\langle i,j \rangle} t_{ij} e^{-2\pi i n \theta_{ij}} c_i c_j + \text{h.c.}$, where the transfer integral $t_{ij}$ is real while the Peierls phases $\theta_{ij}$ satisfy the requirement that the sum of the phases around a hexagon is equal to the magnetic flux piercing a hexagon in units of the flux quantum $\phi_0 = h/e$.

Random bonds are introduced as $t_{ij} = t + \delta t_{ij}$, where $t$ and $\delta t_{ij}$ are the uniform and the random components, respectively. The random components $\delta t_{ij}$ are assumed to have a Gaussian distribution, $P(\delta t) = e^{-\delta t^2/2\sigma_t^2}/\sqrt{2\pi}\sigma_t^2$, with a variance $\sigma_t$ and a spatial correlation,

$$ \langle \delta t_{ij} \delta t_{kl} \rangle = \langle \delta t^2 \rangle \exp(-|r_i - r_j|^2/4\eta_t^2), $$

with a correlation length $\eta_t$, where $r_{ij}$ denotes the position of the bond $t_{ij}$ and $\langle \rangle$ the ensemble average.

On the other hand, random magnetic fields are introduced as $\phi(r) = \phi + \delta \phi(r)$. This type of disorder, being another disorder in bonds, also preserves the chiral symmetry. Here $\phi$ represents the uniform part of the magnetic field, while $\delta \phi(r)$ the random magnetic fluxes for hexagons each located at position $r$. We assume that the random component $\delta \phi(r)$ obeys a Gaussian distribution with a variance $\sigma_\phi$ and a spatial correlation

$$ \langle \delta \phi(r_i) \delta \phi(r_j) \rangle = \langle \delta \phi^2 \rangle \exp(-|r_i - r_j|^2/4\eta_\phi^2). $$

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with a correlation length $\eta_0 [11]$. In the following, all lengths are measured in units of the bond length $a$ of the honeycomb lattice.

Let us recapitulate how these types of disorder preserve the chiral symmetry (even for each realization of disorder). There exists a local unitary operator $\gamma$ that anti-commutes with the Hamiltonian, $[\gamma, H] = 0$, which defines the chiral symmetry, and $\gamma^2 = 1$. In the case of the honeycomb lattice, the lattice sites can be decomposed into $A$ and $B$ sublattices. The operator $\gamma$ can then be given as $\gamma = \exp(\pi \sum \delta_{\alpha A} c_i^\dagger c_i)$, where $\sum \delta_{\alpha A}$ denotes the summation over the $A$ sublattice sites. With this choice of $\gamma$ we can readily verify that the fermion operator $c_i$ is transformed as $\gamma c_i \gamma^{-1} = -c_i$ for $i \in A$ and $\gamma c_i \gamma^{-1} = c_i$ for $i \in B$. Due to this chiral symmetry, the energy levels always appear in pairs $\{E, -E\}$ even in the presence of disorder. For instance, if we have an energy eigenstate $\psi_E$ satisfying $H \psi_E = E \psi_E$, the state $\gamma \psi_E$ is an eigenstate with energy $-E$ since $H \gamma \psi_E = -\gamma H \psi_E = -E \gamma \psi_E$. A special situation arises for zero-energy states, for which the eigenstates $\psi_{E=0}$ and $\gamma \psi_{E=0}$ are degenerated. The eigenstates can then be made simultaneous eigenstates $\psi_e = (1 \pm \gamma) \psi_{E=0}$ of the operator $\gamma$ with $\gamma \psi_e \gamma^{-1} = \pm \psi_e$. The eigenstate $\psi_+ \gamma \psi_-$ has non-zero amplitudes only on the $A(B)$ sublattice. The fact that the zero-energy states can be an eigenstate of $\gamma$ implies that the zero-energy states has an extra symmetry, so that the criticality at zero energy can be especially sensitive to the presence or otherwise of the chiral symmetry [12, 13]. Obviously, the potential disorder breaks this symmetry.

The density of states $\langle \rho_0 \rangle = -\sum \text{Im} G_{0i}(E+ie)/Nt$ is evaluated by the Green function $G_{0i}(E+ie) = \langle \hat{i}|(E-H+ie)^{-1}|\hat{i}\rangle$, where $N$ stands for the total number of sites and $e$ an infinitesimal imaginary part of energy for evaluating the Green function numerically. We have carried out calculations for values of $\epsilon/t$ reduced from 0.01 down to $6.25 \times 10^{-4}$ to confirm that the anomaly in the density of states at the $n = 0$ Landau level discussed below is not affected by the value of $\epsilon$. The system size considered is a $L_x \times L_y$ rectangular system with periodic boundaries in $y$ direction, where $x$ axis is assumed to be parallel to the zigzag direction of the honeycomb lattice. In actual calculations, the Landau gauge for the corresponding bricklayer lattice [15] is adopted.

3. Numerical Results — Random Bonds

First, we consider the random bonds in a uniform magnetic field ($\sigma_\phi = 0$). In the previous paper [3], we have clearly demonstrated for such a model that when the correlation length $\eta_1$ exceeds few bond lengths, the $n = 0$ Landau level becomes anomalously sharp and the associated quantum Hall transition shows an almost exact fixed-point-like criticality even in a finite system (Fig. 1). This sharply contrasts with the result for the uncorrelated case $\eta_1 = 0$, for which the $n = 0$ Landau level is broadened in the same way as $n \neq 0$ Landau levels. The anomalous sensitivity to the disorder of correlation occurs only for the $n = 0$ Landau level. We have examined the density of states for other values of $\sigma_\gamma$ and $\phi$, and confirmed that the anomaly at the $n = 0$ Landau level is commonly observed for $\eta_1/a \geq 1$. Here we show the density of states as a function of the Fermi energy and the correlation length $\eta_1$, for $\sigma_\gamma/t = 0.058$ and $\phi/\phi_0 = 1/50$ in Fig. 2. It is again clearly seen, as has been seen in the case of $\sigma_\gamma/t = 0.12$ [9], that the $n = 0$ Landau level becomes anomalously sharp compared to other Landau levels as soon as the correlation length $\eta_1$ is greater than the lattice constant.

In actual graphene samples, the scale of ripples is estimated to be of order of 10 nanometers [19, 20], which is much greater than the lattice constant $a \sim 1.42\text{Å}$ [26], and the magnitude of disorder is likely to be much smaller. Our result therefore clearly indicates that the bond disorder induced by ripples should not broaden the $n = 0$ Landau level.

4. Numerical Results — Random Magnetic Fields

Let us next show the results for another model, namely the random magnetic field model with $\sigma_\phi \neq 0$, where no randomness is assumed for the amplitude of the transfer energies ($\sigma_\gamma = 0$). For this model, we consider both the small random field case, $\sigma_\phi < \phi$ (case 1), and the large random field case, $\sigma_\phi > \phi$ (case 2). For $\sigma_\phi < \phi$ the distinct Landau level structure is present (Fig. 3). The $n = 0$ Landau level is again anomalously sensitive to the spatial correlation of random magnetic fields: When the correlation length $\eta_\phi$ exceeds few lattice constants, the width of the $n = 0$ Landau level becomes a sharp, delta-function-like peak. Indeed, the shape of the $n = 0$ Landau level for $\eta_\phi/a > 0.2$ is almost exactly given by the Lorentzian distribution $(\phi/\phi_0)(\epsilon/\pi)/\left(E^2 + \epsilon^2\right)$, indicative of a zero intrinsic width. This again contrasts with the $n \neq 0$ Landau level.

For $\sigma_\phi > \phi$ (case 2), on the other hand, the Landau level structure is mostly washed out for $n \neq 0$ Landau levels, which is not surprising since the fluctuation of the magnetic field is larger than its mean value. Surprisingly, $n = 0$ Landau level exhibits an anomalous behavior, where the delta-function-like
behavior arises when the correlation length of the random magnetic field exceed few lattice constants, even though the field fluctuation width is much greater than the average field (Fig. 3).

5. Discussions and Conclusions

In order to check that the above anomalies are a consequence of the preserved chiral symmetry, we have also evaluated the density of states for the case of the spatially correlated potential disorder, where the Hamiltonian is given by $H = \sum_{(ij)}(t e^{-2i\eta_j} c_i^+ c_j + \text{h.c.)} + \sum_i \varepsilon_i c_i^+ c_i$, where random site energies $\varepsilon_i$ are assumed to obey a Gaussian distribution with a variance $\sigma^2$ and a spatial correlation $\sigma^2(\eta, \eta') = \sigma^2 \exp(-r^2 / 4\eta^2)$. The result for $\sigma^2 = 0.029$ and $\phi/\phi_0 = 1/41$ in Fig. 5 confirms that no anomaly exists at the $n = 0$ Landau level. As we increase the correlation length $\eta$, keeping the magnitude of disorder $\sigma$ fixed, all the Landau levels are broadened [21].

To summarize, we have investigated the anomalous behavior of the $n = 0$ Landau level of the QHE system on a honeycomb lattice. Two types of bond disorder that preserve the chiral symmetry have been considered, namely the randomness in the magnitude of the transfer integrals (random bonds) and that in their phases (random magnetic fields). It is clearly demonstrated that in both cases the $n = 0$ Landau level is anomalously sensitive to the spatial correlation of disorder, where an anomaly at the $n = 0$ Landau level appears as soon as the correlation length exceeds few bond lengths. This indicates that the absence of the mixing between two valleys is essential to the anomaly at the $n = 0$ Landau level. We have also confirmed that the anomaly does not exist for the correlated potential disorder, where the chiral symmetry is broken. The results suggest that the anomaly at the $n = 0$ Landau level is generic to the graphene system with long-range disorders that preserve the chiral symmetry. This implies that the bond disorder induced by ripples

Figure 2: The density of states for various values of the correlation length $\eta_c/a$ of the random bonds. The system-size is $L_x/(\sqrt{3}a/2) = 5000$, $L_y/(3a/2) = 100$. The imaginary part of energy $\epsilon/t = 6.25 \times 10^{-4}$.

Figure 3: The density of states is plotted for various values of the correlation length $\eta_b/a$ when the fluctuation of the magnetic field is smaller than its mean ($\sigma_b < \phi$). The parameters are $\sigma_b/\phi_0 = 0.0058 < \phi/\phi_0 = 1/41 = 0.024$ for a system size $L_x/(\sqrt{3}a/2) = 5000$, $L_y/(3a/2) = 82$ and the imaginary part of energy $\epsilon/t = 6.25 \times 10^{-4}$.

Figure 4: As above when the fluctuation of the magnetic field is greater than its mean ($\sigma_b > \phi$). The parameters are $\sigma_b/\phi_0 = 0.058 > \phi/\phi_0 = 1/41 = 0.024$ ($\phi < \sigma_b$) for a system size $L_x/(\sqrt{3}a/2) = 5000$, $L_y/(3a/2) = 82$ and the imaginary part of energy $\epsilon/t = 6.25 \times 10^{-4}$. 
Figure 5: The density of states in a random potential for various values of the correlation length $\eta_s/a$. The parameters are $\sigma_s/t = 0.29$, $\phi/\phi_0 = 1/41$ and $\sigma_t = \sigma_\phi = 0$ for a system size $L_x/(\sqrt{3}a/2) = 5000$, $L_y/(3a/2) = 82$ and the imaginary part of energy $\epsilon/t = 6.25 \times 10^{-4}$.

in graphene should not broaden the $n = 0$ Landau level; conversely, if a broadening is observed in experiments, that must be caused by other origins, such as the potential disorder from charged impurities in substrates. Experimentally, a narrower $n = 0$ Landau level of graphene has been reported by measuring the activation energy gaps [22], which may be related to the present analysis. More elaborate analysis on the nature of the disorder is an interesting future problem.

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