Effect on Higgs Boson Decays from Large Light-Heavy Neutrino Mixing in Seesaw Models

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Abstract

In seesaw models with more than one generation of light and heavy neutrinos, ν and N, respectively, it is possible to have sizable mixing between them for heavy-neutrino masses of order 100 GeV or less. We explore this possibility further, taking into account current experimental constraints, and study its effect on Higgs-boson decays in the contexts of seesaw models of types I and III. We find that in the type-I case the Higgs decay into a pair of light and heavy neutrinos, $h \rightarrow \nu N$, could increase the total Higgs width in the standard model by up to almost 30% for a relatively light Higgs-boson, which would significantly affect Higgs searches at the LHC. The subsequent prompt decay of $N$ into three light fermions makes this Higgs decay effectively a four-body decay. We further find that, in the presence of the large light-heavy mixing, these four-body Higgs decays can have rates a few times larger than their standard-model counterparts and therefore could provide a potentially important window to reveal the underlying seesaw mechanism.
I. INTRODUCTION

Various experiments have now established that neutrinos have mass and mix with each other [1]. The masslessness of the neutrinos in the minimal standard model (SM) implies that one has to go beyond it to account for this observation. Among a number of possibilities that have been proposed [2–7], the most popular are the seesaw scenarios in which new particles are introduced with masses sufficiently large to make the neutrino masses small.

In the so-called type-I and type-III seesaw models [3, 4], the heavy particles responsible for giving mass to the light neutrinos are neutral fermions, often referred to as heavy neutrinos. Whether the seesaw mechanism can be probed at colliders crucially depends not only on the masses of the heavy neutrinos (as well as their charged partners in the case of type III), but also on the strength of their interactions with SM particles, specifically the mixing between the heavy neutrinos, $N$, and the light ones, $\nu$.

With only one generation of the neutrinos, the size of this light-heavy mixing is of order the square root of their mass ratio, $(m_\nu/m_N)^{1/2}$. Since the light-neutrino mass must be less than an eV or so, the mixing would be very small, less than $10^{-5}$ even for $m_N$ of order 100 GeV. This would make it challenging to test the seesaw mechanism, especially in the type-I model, at colliders. However, in the presence of more than one generation of light and heavy neutrinos, there are circumstances in which the mixing can be much larger [5–8], offering greater hope of observing its effects on various processes. The combination of such large mixing, with $m_N \sim 100$ GeV, and the tiny light-neutrino masses can occur naturally if the underlying theory has some symmetry that is slightly broken [8].

A recent study [9] has explored this possibility of large light-heavy mixing further and considered specific examples in both seesaw scenarios of types I and III. That study also examined some of the implications of the large light-heavy mixing for the single production of the heavy leptons at the LHC via channels such as $q\bar{q}' \to W^* \to lN$ and found that there are interesting prospects for detecting these heavy particles at the LHC.

In the present paper, we consider additional processes where it may be possible to probe the large-mixing effects. In particular, we apply some of the results obtained in Ref. [9] for types-I and -III seesaw to the decays of the Higgs boson into a light ordinary fermion plus one of the new heavy leptons. We show that, with the light-heavy mixing as large as allowed by currently available experimental data, some of these new decay modes of the Higgs boson could give rise to sizable modifications of its decay branching ratios in the SM and therefore could significantly alter Higgs searches at the LHC or other colliders. On the other hand, the new decay modes could serve to open a window to the underlying seesaw mechanism if the Higgs boson is discovered and its decay modes are well measured.

II. LARGE LIGHT-HEAVY MIXING IN TYPE-I SEESAW

In the type-I scenario, the seesaw mechanism is realized by introducing right-handed neutrinos that are singlets under the SM gauge groups and can therefore have large Majorana masses [3]. Following Ref. [9], we assume for definiteness that there are three of these heavy
neutrinos, \( N_{iR} \), responsible for giving mass to the three left-handed light neutrinos, \( \nu_{iL} \). The relevant Lagrangian describing the masses of the neutrinos can be expressed as

\[
\mathcal{L} = -\bar{N}_{iR}(Y_D)_{ij}\tilde{H}^+L_{jL} - \frac{1}{2}\bar{N}_{iR}(M_N)_{ij}(N_{jR})^c + \text{H.c.} ,
\]

where summation over \( i, j = 1, 2, 3 \) is implied, \( Y_D \) is the \((3\times3)\) Yukawa coupling matrix, \( \tilde{H} = i\tau_2H^* \) with \( \tau_2 \) being the usual second Pauli matrix and \( H = (\phi^+(v + h + i\eta)/\sqrt{2})^\dagger \) the Higgs doublet, \( v \) its vacuum expectation value, \( L_{iL} = (\nu_{iL} \, \nu^\dagger_{iL})^\dagger \) is the left-handed lepton doublet, \( M_N \) is the Majorana mass matrix, and \((N_{iR})^c\) denotes the charge conjugate of \( N_{iR} \). The resulting seesaw mass terms are given by

\[
\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\nu_L) \bar{N}_{R} M_{\text{seesaw}}^{\dagger}(N_R)^c + \text{H.c.} ,
\]

where \( \nu_L \) and \( N_R \) are column matrices containing \( \nu_{iL} \) and \( N_{iR} \), respectively, and

\[
M_{\text{seesaw}} = \begin{pmatrix} 0 & m_D^T \\ m_D & M_N \end{pmatrix},
\]

with the Dirac mass matrix \( m_D = vY_D/\sqrt{2} \).

One can relate the weak eigenstates \( \nu_{iL} \) and \((N_{iR})^c\) to the corresponding mass eigenstates by writing

\[
\begin{pmatrix} \nu_L \\ (N_R)^c \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix} , \quad U \equiv \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix} .
\]

where \( \nu_{mL} \) and \( N_{mL} \) are column matrices containing the mass eigenstates. Thus \( U \) is unitary and diagonalizes \( M_{\text{seesaw}} \),

\[
\begin{pmatrix} \hat{m}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix} = U^T M_{\text{seesaw}} U ,
\]

where \( \hat{m}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \) and \( \hat{M}_N = \text{diag}(M_1, M_2, M_3) \). On the other hand, the submatrices \( U_{\nu\nu}, U_{\nu N}, U_{N\nu}, \) and \( U_{NN} \) are not unitary. Assuming that the nonzero elements of \( M_N \) are all much greater than those of \( m_D \), and expanding in terms of \( m_D M_N^{-1} \), one then finds to leading order that \( U_{\nu\nu} \) has small deviations from unitarity, \( U_{\nu N} = m_D^T M_N^{-1}, \quad U_{N\nu} = -M_N^{-1} m_D U_{\nu\nu} \), \( U_{NN} = 1 \), and the reduced light-neutrino mass matrix \( m_\nu \equiv -m_D^T M_N^{-1} m_D \), which can be diagonalized using the unitary Pontecorvo-Maki-Nakagawa-Sakata matrix \( \hat{U}_{\text{PMNS}} \), \( \hat{m}_\nu = \hat{U}_{\text{PMNS}}^T m_\nu \hat{U}_{\text{PMNS}} \). This leads to the leading-order relation

\[
U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^T = -U_{\nu N} \hat{M}_N U_{\nu N}^T .
\]

In terms of the weak eigenstates, the neutrinos couple to the gauge and Higgs bosons in the SM according to

\[
\mathcal{L}' = \left( \frac{g}{\sqrt{2}} \bar{l}_L \gamma^\mu \nu_L W^-_\mu \hat{N}_R m_D \nu_L \frac{h}{v} + \text{H.c.} \right) + \frac{g}{2c_w} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu ,
\]
where \( g = 2m_W/v \) is the usual weak coupling constant and \( c_w = \cos \theta_W \). Using the relations \( U_{Ni}^T m_D = \hat{m}_\nu U_{\nu N}^\dagger \) and \( U_{NN}^T m_D = \hat{M}_N U_{\nu N}^\dagger \) derived from Eq. (5), one can rewrite \( \mathcal{L}' \) in the mass-eigenstate basis as

\[
\mathcal{L}' = \frac{g}{\sqrt{2}} \left( \bar{\ell}_L \gamma^\mu U_{\nu \nu} \nu_{mL} W^-_\mu + \bar{\ell}_L \gamma^\mu U_{\nu N} N_{mL} W^-_\mu + \text{H.c.} \right) \\
+ \frac{g}{2c_w} \left( \bar{\nu}_{mL} \gamma^\mu U_{\nu \nu}^\dagger U_{\nu \nu} \nu_{mL} + \bar{N}_{mL} \gamma^\mu U_{\nu N}^\dagger U_{\nu N} \nu_{mL} \\
+ \bar{\nu}_{mL} \gamma^\mu U_{\nu N}^\dagger N_{mL} + \bar{N}_{mL} \gamma^\mu U_{\nu N}^\dagger U_{\nu N} N_{mL} \right) Z_\mu \\
- \left[ (\nu_{mL})^T \hat{m}_\nu U_{\nu \nu}^\dagger U_{\nu \nu} \nu_{mL} + (N_{mL})^T \hat{M}_N U_{\nu N}^\dagger U_{\nu N} \nu_{mL} \right] + (\nu_{mL})^T \hat{m}_\nu U_{\nu \nu}^\dagger U_{\nu \nu} N_{mL} + (N_{mL})^T \hat{M}_N U_{\nu N}^\dagger U_{\nu N} N_{mL} + \text{H.c.} \right] \frac{h}{v}.
\]

Thus via mixing the heavy neutrinos \( N \) can interact with the SM gauge bosons at tree level.

This Lagrangian indicates that the Higgs-boson coupling to a pair of light and heavy neutrinos, \( h\nu N \), is leading compared to the other Higgs-neutrino couplings, \( h\nu \) and \( hNN \), which are proportional to the tiny light-neutrino masses and of second order in \( U_{\nu N} \), respectively. This dominant coupling generates the decay mode \( h \to \nu N \) if the mass of the heavy neutrino is less than the Higgs mass \( m_h \). Clearly, how important this decay might be would depend on the elements of the matrix \( U_{\nu N} \), which parametrizes the light-heavy mixing. As we will show later, the elements of \( U_{\nu N} \) subject to current experimental constraints can be sufficiently sizable to give rise to significant modifications of the Higgs decay branching ratios in the SM for a relatively light Higgs-boson.

### A. \( h \to \nu N \) decay

From Eq. (8), we obtain the amplitude for \( h \to \nu_i N_j \)

\[
\mathcal{M}(h \to \nu_i N_j) = \frac{g M_j}{2m_W} \bar{\nu}_i \left[ (U_{\nu N}^T U_{\nu N}^*)_{ij} P_L + (U_{\nu \nu} U_{\nu N})_{ij} P_R \right] v_N,
\]

where \( \nu_i \) and \( N_i \) denote the \( i \)th mass-eigenstates of the light and heavy neutrinos, respectively, \( M_i \) is the mass of \( N_i \), and \( P_{L,R} = \frac{1}{2}(1 \mp \gamma_5) \). In deriving this expression, we have made use of the Majorana nature of both neutrinos and neglected contributions from terms in \( \mathcal{L}' \) proportional to the light-neutrino masses. The resulting decay rate for all possible combinations of \( \nu_i N_j \) is

\[
\Gamma(h \to \nu N) = \sum_{i,j=1}^3 \Gamma(h \to \nu_i N_j) = \sum_{i=1}^3 \frac{g^2 m_h M_i^2 (U_{\nu N} U_{\nu N})_{ii}}{32\pi m_W^2} \left( 1 - \frac{M_i^2}{m_h^2} \right)^2,
\]

with \( U_{\nu \nu} U_{\nu \nu}^\dagger \sim 1 \). For our numerical analysis, we will employ some of the results of Ref. [9] which provided specific solutions for \( U_{\nu N} \) having sizable elements and simultaneously satisfying the light-neutrino mass requirement given in Eq. (6).

There are additional sets of constraints that the elements of \( U_{\nu N} \) must satisfy. The first arises from electroweak precision data on processes involving neutral currents conserving lepton
flavor \[1\]. Expressed in terms of \(\epsilon \equiv U_{\nu N}U_{\nu N}^\dagger\), in type-I seesaw the bounds extracted from the data are \[11, 12\]

\[
\begin{align*}
\epsilon_{11} &\leq 3.0 \times 10^{-3}, \\
\epsilon_{22} &\leq 3.2 \times 10^{-3}, \\
\epsilon_{33} &\leq 6.2 \times 10^{-3}.
\end{align*}
\]

(11)

The second set of constraints come from lepton-flavor violating transitions in the charged-lepton sector. Although in type-I seesaw there are no flavor-changing processes involving ordinary charged leptons at tree level, loop-induced ones can occur, such as the radiative decays \(\mu \to e\gamma\), \(\tau \to e\gamma\), and \(\tau \to \mu\gamma\). The bounds determined from the measurements of such transitions are \[11, 13\]

\[
|\epsilon_{12}| \leq 1 \times 10^{-4}, \quad |\epsilon_{13}| \leq 0.01, \quad |\epsilon_{23}| \leq 0.01.
\]

(12)

For heavy neutrinos coupling to the electron, neutrinoless double-beta decay imposes \[14\]

\[
\left| \sum_{i=1}^{3} (U_{\nu N})_{2i}^2 |M_i| \right| \leq 5 \times 10^{-8} \text{ GeV}^{-1}.
\]

(13)

Finally, for \(N\)-mass values between a few GeV and the \(Z\) mass, \(m_Z\), there are also restrictions on the individual elements \((U_{\nu N})_{2i}\) and \((U_{\nu N})_{3i}\) from searches for SM-singlet neutrinos via \(Z \to \nu N\) performed by the L3 and DELPHI experiments at LEP \[15\]. These constraints on \((U_{\nu N})_{2i,3i}\) may be stronger than those inferred from Eqs. (11) and (12), depending on \(M_i\).

To explore the effect of large light-heavy mixing on the decay \(h \to \nu N\), we take some of the examples of \(U_{\nu N}\) from Ref. \[9\]. As discussed therein, the general form of \(U_{\nu N}\) which accommodates the large mixing can be written as \(U_{\nu N} = U_0 + U_\delta\), where \(U_0\) is a rank-one matrix which makes the right-hand side of Eq. (9) vanish exactly and \(U_\delta\) denotes a perturbation matrix with tiny elements fixed to reproduce the light-neutrino masses according to Eq. (6). It follows that the elements of \(U_0\) are not constrained by the light-neutrino masses and can be as large as allowed by the experimental bounds described in the preceding paragraph. We note that, as mentioned earlier, this situation can happen naturally in the presence of some underlying symmetry that is slightly violated \[8\]. Accordingly, in our discussion below we include in \(U_{\nu N}\) only the dominant part, \(U_{\nu N} = U_0\).

Thus, for the first example, we take \[9\]

\[
U_{\nu N} = U_0^a = \begin{pmatrix} a & a & i\sqrt{2}a \\ b & b & i\sqrt{2}b \\ c & c & i\sqrt{2}c \end{pmatrix} \mathcal{R}, \quad \mathcal{R} = \text{diag}(\sqrt{r_1}, \sqrt{r_2}, \sqrt{r_3}),
\]

(14)

where \(a = (0.58 - 0.81 i)\tilde{b}\), \(b = (0.58 + 0.41 i)\tilde{b}\), \(c = (0.58 + 0.41 i)\tilde{b}\), and \(r_i = m_N/M_i\), with \(\tilde{b}\) being a free parameter that has to satisfy the bounds listed above and \(m_N\) taken to be the lightest of \(M_{1,2,3}\). From now on, for simplification we assume that the heavy neutrinos are degenerate, \(M_1 = M_2 = M_3 = m_N\), the situation in the nondegenerate case being qualitatively similar, and so \(\mathcal{R}\) is a unit matrix in the following examples, but not explicitly displayed. We then obtain \(\tilde{b} = 0.006\) to be the largest value allowed by the experimental constraints. Adopting this number, we plot in Fig. 1(a) the ratio of \(\Gamma(h \to \nu N)\) to the total Higgs width \(\Gamma_{h}^\text{SM}\) in
FIG. 1: Ratios of the width of $h \to \nu N$ in type-I seesaw to the total Higgs width in the SM as functions of the Higgs mass $m_h$ for heavy-neutrino mass values $m_N = 70, 80, 90, 100$ GeV and different choices of $U_{\nu N}$ as described in the text.

the SM corresponding to Higgs mass values within the range $100$ GeV $\leq m_h \leq 180$ GeV for $m_N = 70, 80, 90, 100$ GeV. From the curves displayed, the peak of the ratio is seen to be only about 1.3%, corresponding to the $m_N = 70$ GeV curve at $m_h = 120$ GeV. We remark that for lower values of $m_N$, from several to 60 GeV, the LEP searches mentioned earlier impose the strong limits $|(U_{\nu N})_{2i,3i}| \lesssim 0.007$ [15].

As a second example, we can choose [9] $U_{\nu N} = U_{0}^{d} = \begin{pmatrix} 0 & a & ia \\ b & 0 & ib \\ c & ic & 0 \end{pmatrix}$ with $a = -0.82 \bar{a}$, $b = (0.41 + 0.66 \mathbf{i})\bar{a}$, and $c = (0.41 - 0.66 \mathbf{i})\bar{a}$, where $\bar{a}$ is a free parameter subject to the experimental constraints. We find that the maximum allowed value $\bar{a} = 0.0089$ leads to a graph very similar to that in Fig. 1(a). It is worth noting that $U_{\nu N}$ in either Eq. (14) or (15) automatically satisfies the constraint in Eq. (13) for degenerate heavy neutrinos.

It is evident that the effect of light-heavy mixing on $h \to \nu N$ in the preceding examples is not remarkable. This is mainly because of the strict bound on $\epsilon_{12}$ in Eq. (12) which limits the elements of $U_{\nu N}$ to be at most $\sim 0.01$ in size. It turns out that there are other choices of $U_{\nu N}$ which can evade this restriction, one of them being [9] $U_{\nu N} = U_{0}^{e} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & a & ia \\ 0 & b & ib \end{pmatrix}$ with $b = a$. We obtain the largest allowed value of $a$ to be $a = 0.04$ for $m_N \gtrsim 80$ GeV, but $a \sim 0.02$ for $m_N = 70$ GeV from the LEP searches [15]. These numbers lead to $h \to \nu N$ rates which are much larger than those in the earlier examples, and the ratios of these enlarged rates to the SM Higgs total width are shown in Fig. 1(b). More precisely, one observes from the four curves displayed that the ratio can reach as high as 28%, which corresponds to the peak of the
$m_N = 80$ GeV curve at $m_h = 124$ GeV. Another choice that can evade the $\epsilon_{12}$ constraint is

$$U_{\nu N} = U^f_0 = \begin{pmatrix} 0 & a & i a \\ 0 & 0 & 0 \\ 0 & b & i b \end{pmatrix}$$

(17)

with $b = (0.0013 + 1.03i)a$ and $a = 0.02 (0.039)$ for $m_N = 70$ GeV ($\geq 80$ GeV). This results in a plot very similar to that in Fig. [1(b)].

Since the light-heavy mixing also causes the $Z$ coupling to $\nu N$, according to Eq. [5], the decay $Z \rightarrow \nu N$ can happen for $m_N < m_Z$. Therefore, it is important to check if the impact on $Z \rightarrow \nu l$ from the large mixing we are considering is consistent with the precisely measured value of the $Z$ total-width, $\Gamma_{Z}^{\exp} = 2.4952 \pm 0.0023$ GeV [1], which agrees with the standard-model prediction $\Gamma_{Z}^{\text{SM}} = 2.4954 \pm 0.0010$ GeV [16]. From Eq. [8], one derives the amplitude

$$\mathcal{M}(Z \rightarrow \nu_i N_j) = \frac{g \vec{z}_{\nu} \vec{u}_{\nu}}{2c_w} \sum_{i,j=1}^3 \Gamma(Z \rightarrow \nu_i N_j) \sum_{i=1}^3 \frac{g^2(U_{\nu N})_{ii}m_Z^2}{4\pi m_{\nu N}^2} \left(1 - \frac{3m_N^2}{2m_Z^2} + \frac{m_N^4}{2m_Z^4} \right),$$

(18)

upon using the Majorana nature of the neutrinos. One then gets the decay rate

$$\Gamma(Z \rightarrow \nu N) = \sum_{i,j=1}^3 \Gamma(Z \rightarrow \nu_i N_j) = \sum_{i=1}^3 \frac{g^2(U_{\nu N})_{ii}m_Z^2}{4\pi m_{\nu N}^2} \left(1 - \frac{3m_N^2}{2m_Z^2} + \frac{m_N^4}{2m_Z^4} \right),$$

(19)

with $U_{\nu N}U_{\nu N}^\dagger \simeq 1$ as before. Numerically, the choices $U_{\nu N} = U^5_0$ and $U^f_0$ producing the largest effects obtained above yield nearly identical rates: $\Gamma(Z \rightarrow \nu N) = 0.12$, 0.16, and 0.002 MeV for $m_N = 70$, 80, and 90 GeV, respectively. Obviously, each of these $\Gamma(Z \rightarrow \nu N)$ numbers is well within the errors in $\Gamma_Z^{\exp}$ and $\Gamma_Z^{\text{SM}}$. This helps to confirm that our parameter choices for $U_{\nu N}$ already satisfied the LEP and other constraints described earlier in this subsection.

Another process which should be examined if the large light-heavy mixing occurs is the scattering $e^+ e^- \rightarrow \nu N$ followed by the decay $N \rightarrow \nu l^+ l^-$, where the charged leptons $l_{1,2}$ can be equal or different in flavor. For this has the same leptonic final-state as the $W$-pair production process $e^+ e^- \rightarrow WW$, each of the $W$'s subsequently decaying into $\nu l$, which has been well measured at LEP2 [17, 18], its cross-section found to be in accord with the SM expectation. The amplitude for $e^+ e^- \rightarrow \nu N$ is related by crossing symmetry to that for $N \rightarrow \nu l^+ l^-$, to be evaluated later, in Eq. [A3] and proceeds from an $s$-channel $Z$-mediated diagram plus $W$-mediated diagrams in the $t$ and $u$ channels. For completeness, here we write its squared amplitude as

$$|\mathcal{M}(e^+ e^- \rightarrow \nu_i N_j)|^2 = \frac{g^4(l_e^2 + r_e^2)}{4c_w^2} \left|\sum_{i,j=1}^3 (U_{\nu N})_{ij}^2 \left[(t - m_N^2)t + (u - m_N^2)u\right] \left[(t - m_Z^2)t + (u - m_Z^2)u\right] \right|^2
+ \frac{g^4}{4} \left[\sum_{i,j=1}^3 (U_{\nu N})_{ij}^2 \left[(u - m_N^2)t + (t - m_N^2)u\right] \left[(u - m_Z^2)t + (t - m_Z^2)u\right] \right]^2
+ \frac{g^4}{2c_w^2} \sum_{i,j=1}^3 \sum_{i,j=1}^3 (U_{\nu N})_{ij}^2 (U_{\nu N})_{ij}^2 \left[\sum_{i,j=1}^3 (U_{\nu N})_{ij}^2 \left[(u - m_N^2)t + (t - m_N^2)u\right] \left[(u - m_Z^2)t + (t - m_Z^2)u\right] \right]^2,$$

(20)

where we have neglected light-lepton masses and $\Gamma_{W,Z}$ terms, $s = (p_e^+ + p_e^-)^2$, $t = (p_e^+ - p_N)^2$, $u = m_N^2 - s - t$, $l_e = s_w^2 - \frac{1}{2}$, and $r_e = s_w^2$, with $s_w = \sin \theta_W$. We sum this over all
possible $\nu_i N_j$ combinations and then, in order to make comparison with the LEP2 data, apply the resulting cross-section in $\sigma(e^+e^- \to \nu N \to \nu l^+_1 \nu l^-_2) = \sigma(e^+e^- \to \nu N) B(N \to \nu l^+_1 l^-_2)$, employing $B(N \to \nu l^+_1 l^-_2) \sim 0.3$, as we will calculate in the next subsection. We have collected the numbers in Table I for $m_N = 70, 80, 90, 100$ GeV, with $U_{\nu N} = U^e_0$ and $U^f_0$, as before, at the center-of-mass energies $\sqrt{s} = 161, 183, 207$ GeV, which are representative of the measured range. As one can notice from the table, in this case the impact of $U^e_0$ is much smaller than that of $U^f_0$, which is due to the fact that $(U^e_0)_{1j} = 0$. The experimental cross-sections of $e^+e^- \to WW \to \nu l^+_1 \nu l^-_2$ reported by the LEP2 collaborations are consistent with each other [17, 18], and so it suffices to compare with the most recent ones, from OPAL [18], which we have reproduced in Table I after combining the statistical and systematic errors in quadrature. It is evident from this table that all the $\nu N$ contributions, especially the ones arising from $U^e_0$, are well within the uncertainties in the data. Hence our large-mixing results are compatible with the LEP2 measurements.

We have thus demonstrated that, with the large light-heavy mixing subject to current experimental constraints, the new decay mode $h \to \nu N$ can change the total Higgs width in the SM by up to nearly 30% for a relatively light Higgs-boson, especially with $m_h \lesssim 140$ GeV. This would significantly affect the SM expectations in Higgs searches at the LHC. For bigger Higgs masses, $m_h > 2m_W$, as the decay channels into a pair of weak gauge bosons, $h \to WW, ZZ$, become open and start to be dominant, the effect of $h \to \nu N$ due to the large mixing on the Higgs total width would get much reduced, as can be seen also from Fig. [1]. We can mention here that the possibility of $h \to \nu N$ causing important changes to Higgs searches in the presence of the large mixing has also been raised previously in Ref. [6] which proposed radiatively induced neutrino masses and more recently in Ref. [7] in the contexts of other models of light-neutrino mass generation.

Since $h \to \nu N$ is a potentially influential decay mode, it is of interest as well to study the subsequent decays of $N$ in the case of large light-heavy mixing. They may have signatures which are observable and distinguishable from those of the SM. We explore this possibility in the rest of this section.

### Table I: Cross-section of $e^+e^- \to \nu N \to \nu l^+_1 \nu l^-_2$ for $m_N = 70, 80, 90, 100$ GeV with $U_{\nu N} = U^e_0$ in Eq. (16) (columns 2-5) and $U_{\nu N} = U^f_0$ in Eq. (17) (columns 6-9), compared to measured cross-section of $e^+e^- \to WW \to \nu l^+_1 \nu l^-_2$ (last column) at center-of-mass energies $\sqrt{s} = 161, 183, 207$ GeV. All cross-section numbers are in pb.

| $\sqrt{s}$ (GeV) | $m_N$ | $m_N$ | Data [18] |
|------------------|-------|-------|-----------|
| 161 | 70 GeV | 0.001 | 0.003 | 0.005 | 0.004 | 0.013 | 0.047 | 0.050 | 0.040 | 0.28 ± 0.22 |
| | 80 GeV | 0.005 | 0.005 | 0.005 | 0.004 | 0.013 | 0.047 | 0.050 | 0.040 | 0.28 ± 0.22 |
| | 90 GeV | 0.003 | 0.003 | 0.003 | 0.003 | 0.014 | 0.052 | 0.057 | 0.048 | 1.63 ± 0.21 |
| | 100 GeV | 0.002 | 0.003 | 0.003 | 0.002 | 0.014 | 0.056 | 0.063 | 0.055 | 1.83 ± 0.13 |
B. Leading decays of $N$

For $m_N < m_W$, the dominant decay-modes of $N$ are into three light fermions. If $m_h > m_N > m_W$ or $m_Z$, the main $N$ decays are effectively still three-body, as the daughter $W$ or $Z$ promptly decays into a pair of light fermions. We have derived the amplitudes for these decays and collected their expressions in the appendix.

We have computed the corresponding decay rates for $m_N = 70, 80, 90, 100$ GeV and $U_{\nu N} = U_0^e$ in Eq. (16), with the same $b = a$ values as those chosen for Fig. 1(b). Moreover, we have summed the rates over all possible final-states, taking into account the number of colors in final states involving quarks and excluding top-quark contributions. The results are listed in Table II, where $l' \neq l$ in the second decay mode. With $U_{\nu N} = U_0^f$ in Eq. (17) instead, we get similar numbers. One can observe in this table that there are large increases in the rates of some of the modes between $m_N = 80$ and 90 GeV or between $m_N = 90$ and 100 GeV, which are to be expected due to the opening of decay channels with an on-shell $W$ or $Z$.

From the entries in the last row of Table II we can estimate the total widths of $N$ for the different $m_N$ values, namely $\Gamma_N \simeq \Gamma(N \to 3 \text{ fermions})$. For any one of these $m_N$ values, we can then determine how far $N$ is likely to travel after being produced in the Higgs decay $h \to \nu N$ and before decaying in the rest frame of $h$, once $m_h$ is specified. We have found that in this case the largest distance traveled by $N$ is less than $10^{-10}$ m for the $m_h$ values considered here, namely $100 \text{ GeV} \leq m_h \leq 180 \text{ GeV}$. More specifically, in the rest frame of the decaying Higgs boson, the most energetic and longest-lived $N$ corresponds to $m_h = 180\text{ GeV}$, $m_N = 70\text{ GeV}$, and $\Gamma_N \simeq 3 \text{ keV}$, as Table II indicates, and its maximum distance is calculated to be $d_N \simeq 7 \times 10^{-11}$ m.

These considerations imply that the decay $h \to \nu N$ is very quickly followed by $N$ decaying into three light fermions, and hence this decay sequence is effectively a four-body Higgs decay, $h \to \nu f f' f''$, each $f$ being a light fermion. It is interesting to compare these $N$-mediated Higgs decays with their counterparts in the SM, which arise mostly from diagrams mediated by a pair of $W$ or $Z$ bosons, as well as with the other Higgs decay modes in the SM.

| Decay mode | $m_N$ | 70 GeV | 80 GeV | 90 GeV | 100 GeV |
|------------|-------|--------|--------|--------|---------|
| $N \to \nu \nu \bar{\nu}$ | 0.06 | 0.6 | 2 | 16 |
| $N \to \nu l^+ l'^-$ | 0.24 | 3.4 | 38 | 128 |
| $N \to \nu l^+ l^-$ | 0.11 | 1.5 | 19 | 71 |
| $N \to \nu q \bar{q}$ | 0.25 | 2.4 | 7 | 63 |
| $N \to l^- u \bar{d}$ | 0.37 | 5.1 | 57 | 192 |
| $N \to l^+ \bar{u} d$ | 0.37 | 5.1 | 57 | 192 |
| $N \to 3 \text{ fermions}$ | 1.40 | 18.1 | 180 | 662 |
In Fig. 2(a,b,c), we display the ratios of the widths of $h \rightarrow \nu N \rightarrow \nu ff' f''$ to the Higgs total width $\Gamma_{h}^{SM}$ in the SM as functions of the Higgs mass $m_{h}$ for $m_{N} = 80, 90, 100$ GeV and $U_{\nu N} = U_{0}^{c}$ in Eq. (16), with the same $b = a$ values as those chosen for Fig. 1(b). For comparison, Fig. 2(d) shows the branching ratios of $h \rightarrow \nu ff' f''$ in the SM, which are induced by $h \rightarrow WW^{(*)}, ZZ^{*}$ diagrams. In each of these four graphs, the curve labeled $\nu\nu\nu\nu$ corresponds to the rates of the $\nu\nu\nu\nu$ modes, $\nu\nu ll$ to the combined rates of the $\nu\nu l^+ l^-$ and $\nu\nu l^+ l^-$ modes, $\nu\nu qq$ to the rates of the $\nu\nu q\bar{q}$ modes, and $\nu l\bar{u}d$ to the combined rates of the $\nu l^+ \bar{u}d$ and $\nu l^- \bar{u}d$ modes.

Evidently, for $m_{h}$ less than 140 GeV or so, the $N$-mediated contributions to each of the four-body modes graphed in Fig. 2 are comparable to, and can be a few times bigger than, the corresponding SM contributions. This is clearly the case when it comes to the $\nu l\bar{u}d$ and $\nu\nu ll$ curves for the three values of $m_{N}$ considered. We remark that in the plots (a,b,c) we have not included possible interference between the $N$-mediated and SM contributions, but it should be taken into account in a more refined analysis. Nevertheless, this exercise serves to demonstrate the potential importance of the effect of large light-heavy mixing on Higgs decays. Accordingly, if the Higgs boson is detected, with $m_{h} \lesssim 140$ GeV, and its decay modes can be studied with sufficient precision, these four-body Higgs decays may offer useful information on the seesaw mechanism. This information would be complementary to that possibly available from direct

![Graphs showing the ratios of the widths of $h \rightarrow \nu N \rightarrow \nu ff' f''$ to the total Higgs width in the SM as functions of the Higgs mass $m_{h}$ for $m_{N} = 80, 90, 100$ GeV and $U_{\nu N} = U_{0}^{c}$ in Eq. (16), with the same $b = a$ values as for Fig. 1(b).]
searches for $N$ at colliders, such as via $pp \to W^*X \to lNX$ at the LHC \cite{9, 12}. Lastly, it is worth pointing out that, as the $\nu l u d$ and $\nu ll$ curves in Fig. 2(a,b,c) indicate, the rates of these four-body decays are not much smaller than the SM rate of $h \to b\bar{b}$ for the $m_h$ range shown and can be much larger than the rates of other SM modes, such as $h \to c\bar{c}, gg, l^+l^-$.

III. LARGE LIGHT-HEAVY MIXING IN TYPE-III SEESAW

In type-III seesaw the SM-singlet neutrinos in type-I seesaw are replaced by weak-SU(2)$_L$ triplets of right-handed heavy leptons having zero hypercharge \cite{4}. The component fields of each triplet $\Sigma$ and its charge conjugate $\Sigma^c = C\Sigma^T$ are

$$\Sigma = \begin{pmatrix} N^0/\sqrt{2} & E^+ \\ E^- & -N^0/\sqrt{2} \end{pmatrix}, \quad \Sigma^c = \begin{pmatrix} N^{0c}/\sqrt{2} & E^{-c} \\ E^{+c} & -N^{0c}/\sqrt{2} \end{pmatrix}, \tag{21}$$

and the renormalizable Lagrangian for each $\Sigma$ is given by

$$\mathcal{L}_{\text{III}} = \text{Tr}(\Sigma i\partial\Sigma) - \frac{1}{2} \text{Tr}\left(\Sigma M_{\Sigma} \Sigma^c + \Sigma^c M_{\Sigma}^T \Sigma^c\right) - \sqrt{2} \bar{H}^T \Sigma Y_{X^\ell} \Sigma L_{\Sigma} - \sqrt{2} \bar{L}_{\Sigma} Y_{X^\ell}^T \Sigma \bar{H}, \tag{22}$$

where $D_\mu$ is a covariant derivative involving the weak gauge bosons, $M_{\Sigma}$ the mass of the triplet, and $Y_{X^\ell}$ its Yukawa coupling. Defining $E = (E_R^+)^c + E_R^c$ and removing the would-be Goldstone bosons $\eta$ and $\phi^\pm$, one can rewrite $\mathcal{L}_{\text{III}}$ as

$$\mathcal{L}_{\text{III}} = \bar{E}i\partial E + \frac{1}{2} \text{Tr}\left(\Sigma M_{\Sigma} \Sigma^c + \Sigma^c M_{\Sigma}^T \Sigma^c\right) - \frac{1}{2} \left[N_{R}^0 M_{\Sigma} (N_{R}^0)^c + \text{H.c.}\right]$$

$$+ g \left[N_{R}^0 W^+ E_R + (N_{R}^0)^T W^+ E_L + \text{H.c.}\right] - g \bar{E} W_3 E$$

$$- \left[\frac{1}{\sqrt{2}} (v + h) \bar{N}_{R}^0 Y_{X^\ell} \nu_L + (v + h) \bar{E} Y_{X^\ell}^T l_L + \text{H.c.}\right], \tag{23}$$

where $W_3^\mu = -s_w A^\mu + c_w Z^\mu$ is the usual linear combination of the photon and $Z$-boson fields, $N_R = N$, and $E_{L,R} = P_{L,R} E$, with $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$.

From $\mathcal{L}_{\text{III}}$ in the mass-eigenstate basis, one can then write down the relevant terms describing the interactions of the heavy leptons $N$ and $E$ with the Higgs boson. Here we follow the notation of Ref. \cite{9}, where more details on the other terms in the Lagrangian can be found, and also assume that there are three triplets. The Higgs couplings of $N$ are the same as those in the type-I seesaw discussed earlier. The interactions of $E$ are described by

$$\mathcal{L}_E = \frac{-g}{\sqrt{2} m_W^2} \left(\bar{l}_{mL} U_{\nu N} M_{\Sigma} E_{mR} + \bar{E}_{mR} M_{\Sigma} U_{\nu N}^T l_{mL}\right) h + \cdots, \tag{24}$$

where only the relevant part is displayed, $l_{mL}$ and $E_{mR}$ are $(3 \times 1)$ column matrices containing the mass eigenstates of the light and heavy charged-leptons, respectively, and $M_{\Sigma}$ is now a diagonal matrix, $M_{\Sigma} = \text{diag}(M_1, M_2, M_3)$.

The amplitude for $h \to \nu N$ and its decay rate are then those given in Eqs. \cite{9} and \cite{10}. For $h \to l^- E^+$, we have from Eq. \cite{24}

$$\mathcal{M}(h \to l_i^- E_{j}^+) = \frac{g M_j}{\sqrt{2} m_W} (U_{\nu N})_{ij} \bar{u}_l P_R v_E. \tag{25}$$
and so we arrive at

\[
\Gamma(h \rightarrow l^- E^+) = \sum_{i,j=1}^{3} \Gamma(h \rightarrow l_i^- E_j^+) = \sum_i g^2 m_h M_i^2 (U_{\nu N}^\dagger U_{\nu N})_{ii} \left(1 - \frac{M_i^2}{m_h^2}\right)^2,
\]

having used the fact that \(N\) and \(E\) in each triplet have the same mass and neglected the mass of \(l\). Similarly, \(\Gamma(h \rightarrow l^+ E^-) = \Gamma(h \rightarrow l^- E^+).\) Comparing Eqs. (10) and (26), we see that \(\Gamma(h \rightarrow \nu N) = \Gamma(h \rightarrow l^- E^+).\)

As in the type-I case, there are experimental constraints that the elements of \(U_{\nu N}\) must satisfy, besides the requirement in Eq. (6). Expressed in terms of \(\epsilon = U_{\nu N} U_{\nu N}^\dagger\) as before, in type-III seesaw the bounds extracted from electroweak precision data are \([11, 12]\)

\[
\epsilon_{11} \leq 3.6 \times 10^{-4}, \quad \epsilon_{22} \leq 2.9 \times 10^{-4}, \quad \epsilon_{33} \leq 7.3 \times 10^{-4},
\]

whereas from the measurements of lepton-flavor violating transitions \([13]\)
\[
|\epsilon_{12}| \leq 1.7 \times 10^{-7}, \quad |\epsilon_{13}| \leq 4.2 \times 10^{-4}, \quad |\epsilon_{23}| \leq 4.9 \times 10^{-4}.
\]

In addition, direct searches for heavy charged leptons at colliders impose constraints on the mass of \(E\), and hence the mass of \(N\) as well, namely \(M_i \gtrsim 100\) GeV \([1]\).

To explore the effect of large light-heavy mixing on the decays \(h \rightarrow \nu N, l E\), we again adopt some of the examples of \(U_{\nu N}\) from Ref. \([9]\) for illustrations. In addition, we assume that the three triplets are all degenerate, \(M_1 = M_2 = M_3 = m_N = m_E\). As discussed in Ref. \([9]\), the choices of \(U_{\nu N}\) in Eqs. (16) and (17) are also appropriate for type-III seesaw, as they yield \(\epsilon_{12} = 0\), automatically fulfilling the very stringent requirement on \(\epsilon_{12}\) in Eq. (28). For the first one with \(b = a\), we obtain its largest allowed value to be \(a = 0.012\). This leads to the plot in Fig. 3(a) which shows the ratio of the rate sum \(\Gamma(h \rightarrow \nu N) + \Gamma(h \rightarrow l^+ E^-) + \Gamma(h \rightarrow l^- E^+)\) to the Higgs total width \(\Gamma_{\text{SM}}^h\) in the SM as functions of the Higgs mass \(m_h\) for \(m_N = 100, 110\) GeV. For the choice of \(U_{\nu N}\) as in Eq. (17) with \(b = (0.0013 + 1.03) a\), we find that the maximum allowed value \(a = 0.013\) results in somewhat greater rates, as can be seen in Fig. 3(b).

Thus in type-III seesaw the large light-heavy mixing gives rise to modifications of the SM Higgs total width that are modest, roughly only 5%, much smaller than those in the type-I case. This is due to the stronger experimental constraints on the elements of the mixing matrix \(U_{\nu N}\) and also to the lower-limit on the heavy-lepton masses. As a consequence, the Higgs decays into four light fermions in this case would be less sensitive for probing the underlying seesaw mechanism than their type-I counterparts.

**IV. CONCLUSIONS**

In seesaw scenarios with more than one generation of light and heavy neutrinos, it is possible in some circumstances to have sizable mixing between them for heavy-neutrino masses of order 100 GeV or less. We have explored this possibility further, taking into account constraints from currently available experimental data, and considered its effect on Higgs-boson decays in the contexts of the seesaw models of types I and III.
FIG. 3: Ratios of the sum of $h \to \nu N$ and $h \to l^\pm E^\mp$ rates in type-III seesaw to the total Higgs width in the SM as functions of the Higgs mass $m_h$ for heavy-lepton mass values $m_N = m_E = 100, 110$ GeV and two different choices of $U_{\nu N}$ as described in the text.

We have found that the Higgs decay into a pair of light and heavy neutrinos, $h \to \nu N$, in type-I seesaw with large light-heavy mixing could enhance the total Higgs width in the standard model by up to nearly 30% for a relatively light Higgs-boson, with $m_h \lesssim 140$ GeV. This would imply sizable reduction in branching ratio for some of the important Higgs decay modes in the SM and thus could significantly affect Higgs searches at the LHC or other colliders. We have shown that the $N$ produced in $h \to \nu N$ very quickly decays into three light fermions, which makes this Higgs decay effectively a four-body decay. We have further found that, in type-I seesaw with the large mixing, these four-body Higgs decays can have rates comparable to, or a few times larger than, their SM counterparts and therefore may provide a potentially important avenue to uncover the underlying seesaw mechanism. In type-III seesaw, because of stricter experimental constraints the corresponding $h \to \nu N$ decays and the decays involving the charged leptons, $h \to l^\pm E^\mp$, produce only modest, of order 5%, enlargement of the total Higgs width in the SM. All these considerations suggest that, if the Higgs boson is discovered and its decays can be well measured, its decays into a light neutrino plus three other light fermions could serve to probe the seesaw models. Hence such Higgs decays could yield information complementary to that possibly available from the direct searches for these heavy leptons at colliders.

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Appendix A: Amplitudes for decays of $N$ into three light fermions

The decays of $N$ for $m_N < m_W$ are mainly into three light fermions. If $m_h > m_N > m_W$ or $m_Z$, the main $N$ decays are effectively still three-body, as the daughter $W$ or $Z$ will quickly
decay into a pair of light fermions. Thus the relevant diagrams are each mediated by the \( W, Z, \) or Higgs boson. In calculating their decay rates we neglect the light-fermion masses, and so in the amplitudes below we drop terms proportional to light-fermion masses. Consequently, since the Higgs coupling to a light fermion is proportional to its mass, we also drop the contributions of the Higgs-mediated diagrams.

From \( \mathcal{L}' \) in Eq. (3), we obtain the amplitude for \( N_i \rightarrow \nu_j(p_\nu') \nu_k(p_\nu) \bar{\nu}_k(p_\bar{\nu}) \)

\[
\mathcal{M}(N_i \rightarrow \nu_j \nu_k \bar{\nu}_k) = \frac{g^2}{4c_w^2} \bar{u}_\nu i\gamma^\alpha \left[ (U^\dagger_{\nu \nu} U_{\nu \nu})_{ji} P_L - (U^T_{\nu \nu} U^*_{\nu \nu})_{ji} P_R \right] u_N \bar{u}_\nu \gamma_\alpha P_L v_\nu \]  

where no summation over \( k \) is implied and we have employed the Majorana nature of the neutrinos, \( \nu = \nu^c \) and \( N = N^c \). Thus, for \( N_i \rightarrow \nu_j(p_\nu) l_m^-(p_-) l_n^+(p_+) \) with \( m \neq n \), we find

\[
\mathcal{M}(N_i \rightarrow \nu_j l_m^- l_n^+) = \frac{g^2}{2} \frac{(U_{\nu N})_{mj} (U^\dagger_{\nu \nu})_{jm} \bar{u}_\nu \gamma^\alpha P_L u_N \bar{u}_\nu \gamma_\alpha P_L v_l}{m_W^2 - (p_\nu + p_\gamma)^2 - i\Gamma_W m_W} + \frac{g^2}{2} \frac{(U^*_{\nu N})_{mj} (U^T_{\nu \nu})_{jm} \bar{u}_\nu \gamma^\alpha P_R u_N \bar{u}_\nu \gamma_\alpha P_L v_l}{m_W^2 - (p_\nu + p_\gamma)^2 - i\Gamma_W m_W},
\]

where in the second term we have performed a Fierz transformation and a matrix transposition of the charged-lepton part. For \( N_i \rightarrow \nu_j(p_\nu) l_k^- (p_-) l_k^+(p_+) \), we have

\[
\mathcal{M}(N_i \rightarrow \nu_j l_k^- l_k^+) = \frac{g^2}{2} \frac{(U_{\nu N})_{kl} (U^\dagger_{\nu \nu})_{jk} \bar{u}_\nu \gamma^\alpha P_L u_N \bar{u}_\nu \gamma_\alpha P_L v_l}{m_W^2 - (p_\nu + p_\gamma)^2 - i\Gamma_W m_W} + \frac{g^2}{2} \frac{(U^*_{\nu N})_{kl} (U^T_{\nu \nu})_{jk} \bar{u}_\nu \gamma^\alpha P_R u_N \bar{u}_\nu \gamma_\alpha P_L v_l}{m_W^2 - (p_\nu + p_\gamma)^2 - i\Gamma_W m_W} - \frac{g^2}{2c_w^2} \bar{u}_\nu \gamma^\alpha \left[ (U^\dagger_{\nu \nu} U_{\nu \nu})_{ji} P_L - (U^T_{\nu \nu} U^*_{\nu \nu})_{ji} P_R \right] u_N \bar{u}_\nu \gamma_\alpha (l_i P_L + r_i P_R) v_l,
\]

where no summation over \( k \) is implied, \( l_i = \frac{1}{2} + s_w^2 \), and \( r_i = s_w^2 \), with \( s_w = \sin \theta_W \).

There are also decays into final states containing a lepton and a pair of quarks. We derive for \( N_i \rightarrow \nu_j(p_\nu) q(p_q) \bar{q}(p_\bar{q}) \)

\[
\mathcal{M}(N_i \rightarrow \nu_j q \bar{q}) = \frac{g^2}{2c_w^2} \bar{u}_\nu \gamma^\alpha [(U^\dagger_{\nu \nu} U_{\nu \nu})_{ji} P_L - (U^T_{\nu \nu} U^*_{\nu \nu})_{ji} P_R] u_N \bar{u}_\nu \gamma_\alpha (l_q P_L + r_q P_R) v_q,
\]

where \( q \) can be an up-type quark \( u \) or down-type quark \( d \), with

\[
l_u = \frac{1}{2} - \frac{2}{3} s_w^2, \quad r_u = -\frac{2}{3} s_w^2, \quad l_d = -\frac{1}{2} + \frac{1}{3} s_w^2, \quad r_d = \frac{1}{3} s_w^2.
\]

For \( N_i \rightarrow l_j^- (p_-) u(p_u) \bar{d}(p_d) \) we find

\[
\mathcal{M}(N_i \rightarrow l_j^- u \bar{d}) = \frac{g^2}{2} \frac{(U_{\nu N})_{ji} V_{ud} \bar{u}_\nu \gamma^\alpha P_L u_N \bar{u}_\nu \gamma_\alpha P_L v_d}{m_W^2 - (p_u + p_d)^2 - i\Gamma_W m_W},
\]

\[\text{(A6)}\]
where $V_{ud}$ is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The amplitude for $N_i \rightarrow l_j^+ \bar{u}d$ is similar in form.

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