Production of GeV-scale HNL in 3-body decays. 
Comparison with PYTHIA approach

Volodymyr M. Gorkavenko, Yuliia R. Borysenkova 
and Mariia S. Tsarenkova

Faculty of Physics, Taras Shevchenko National University of Kyiv, 
64, Volodymyrs’ka str., Kyiv 01601, Ukraine

Abstract

Despite the undeniable success of the Standard Model of particle physics (SM), there are some phenomena that SM cannot explain. These phenomena indicate that the SM has to be modified. One of the possible ways to extend SM is to introduce heavy neutral leptons (HNL). To search for HNL in intensity frontier experiments, one has to consider HNL production both in 2-body and 3-body decays of some mesons. We verified the possibility to use the PYTHIA approach for the calculation of HNL production in 3-body decays. We conclude that the PYTHIA approach is quite suitable for the estimation of the sensitivity region for the intensity frontier experiments.

Keywords: physics beyond the Standard Model, intensity frontier experiment, HNL.

1 Introduction

The Standard Model of particle physics (SM) \cite{1,2,3} is a theory that describes with high precision the processes of electroweak and strong interactions with the participation of elementary particles. It is consistent up to a very high energy scale (perhaps up to the Planck scale) and it is verified in numerous accelerator experiments up to energy $\sim 15$ TeV. However, SM fails to explain some phenomena such as massiveness of neutrinos (see e.g. \cite{4,5}), dark matter (for reviews see e.g. \cite{6,7,8}), dark energy \cite{9}, baryon asymmetry of the Universe \cite{10}, etc. Therefore, SM is an incomplete theory and it requires an extension. One has to suggest the existence of "hidden" sectors with particles of new physics.

It turns out the mentioned SM problems can be theoretically solved by extending of SM by new particles that can be either heavy or light. Really, neutrino oscillations and the smallness of the active neutrino masses can be explained with help of new particles with sub-eV mass as well as with help of heavy particles of the GUT scale, see e.g. \cite{11}. The same can be said about the baryon asymmetry of the Universe and dark matter problems: physics on the very different scales can be responsible for it, see e.g. \cite{12}.

So, there are two possible answers to the question "why do we not observe particles of new physics in experiments?" The first answer is the following. The new particles are very heavy (e.g., with mass $m_X \gtrsim 100$ TeV) and can not be produced in modern
accelerators like LHC. To detect them one has to build more powerful and more expensive accelerators (we need energy frontier experiments). However, there is another possibility. The particles of new physics can be light particles (with a mass below or of the order of the electroweak scale) that feebly interact with SM particles. The last case is very interesting for the experimental search of the new physics just now, see e.g. \[13\]. To search for the rare interactions of feebly interacting hypothetical particles, we need intensity frontier experiments. These experiments aim to create high-intensity particle beams and use large detectors \[14\]. Several such intensity frontiers experiments have been proposed in recent years: DUNE \[15\], NA62 \[16\] \[17\] \[18\], SHiP \[19\] \[20\], etc.

We do not know what are the properties of the new particles. They can be new scalars, pseudoscalars, vectors or fermions. Each of these options has to be tested in experiments. From a theoretical point of view, there are three possible choices of the new renormalized Lagrangian of the interaction of new particles with SM particles. These interactions are called portals. There are scalar, vector and heavy neutral leptons renormalized portals. It means the new interaction can be observed at any energy scale, including energy scale below EW scale. There are also other portals of high-dimensional operators such as the portal of pseudoscalar particles (axion-like particles), or Chern-Simons like (parity odd) interaction of electroweak gauge bosons with a new vector field \[19\]. These interactions will be less important the lower the energy scale.

In this paper, we will consider extending the SM by neutrino singlets with right chirality, which extremely faintly interact with SM particles. Such right-handed neutrinos are called sterile neutrinos or heavy neutral leptons (HNL).

Renormalized interaction of the right-handed neutrinos with SM particles (HNL portal) is similar to the Yukawa interaction of left-handed quark doublets with singlets of the right-handed quarks in the SM, namely:

\[
\mathcal{L}_{\text{int}} = - \left( F_{\alpha I} \bar{L}_\alpha \tilde{H} N_I + \text{h.c.} \right),
\]

where \( \alpha = e, \mu, \tau \), index \( I \) is from 1 to full number of the sterile neutrinos \( n \), \( L_\alpha \) – doublet of leptons of \( \alpha \)-generation, \( N_I \) – right-handed sterile neutrino, \( F_{\alpha I} \) – new matrix of dimensionless Yukawa couplings, \( \tilde{H} = i\sigma_2 H^* \).

Conditions of invariance of the Lagrangian \[11\] to the transformations of the gauge groups of the SM demand the corresponding charges of the sterile neutrinos to be zero. Therefore, sterile neutrinos are not charged relative to the gauge groups of the SM, which justifies their name.

After electroweak symmetry breaking, Lagrangian \[11\] in the unitary gauge looks like

\[
\mathcal{L}_{\text{int}} = - \left( M_{\alpha I}^D \bar{\nu}_\alpha N_I + \text{h.c.} \right), \quad M_{\alpha I}^D = \frac{v}{\sqrt{2}} F_{\alpha I},
\]

where \( v \approx 246 \text{ GeV} \) is vacuum expectation value of the Higgs field, \( M_{\alpha I}^D \) – Dirac mass terms.

Considering sterile neutrinos as neutral Majorana particles, we can write full Lagrangian of the modified neutrino sector of the SM in the form

\[
\mathcal{L}_{\nu,N} = i \tilde{\nu}_k \not\!p \nu_k + i \tilde{N}_I \not\!p N_I - \left( M_{\alpha I}^D \bar{\nu}_\alpha N_I + \frac{M_I}{2} \tilde{N}_I \gamma^c N_I + \text{h.c.} \right),
\]
where $M_I$ is Majorana mass of $I$ sterile neutrino. Imposing the condition $M_{Ia}^P/M_I \ll 1$, one can perform the diagonalization of the neutrino mass matrix, see e.g. [21, 11], and get mass matrix of the active neutrinos

$$(M_{\nu}^{\text{active}})_{\alpha\beta} = - \sum_{I=1}^{n} \frac{M_{Ia}^P M_{I\beta}^P}{M_I}. \tag{4}$$

The mass matrix for the sterile neutrinos will remain almost unchanged. This mechanism is known as the seesaw mechanism \footnote{This mechanism is called Type-I seesaw mechanism because there are other ways to explain small neutrino masses, see e.g. [11, 23]. In the Type-II seesaw mechanism an extra SU(2) triplet scalar is introduced [24, 25, 26, 27], in the type-III seesaw mechanism an extra fermion in the adjoint of SU(2) is added to the model [28].}, see e.g. [29, 30].

As a result of the neutrino states mixture, the active neutrino states become superposition of the mass states of the active and sterile neutrinos

$$\nu_L = \left(1 - \frac{1}{2}U^U\right) V_1 \nu_{mL} + U^V_2 N^c, \tag{5}$$

where $U_{Ia} = M_{Ia}^P/M_I \ (U = M^{-1}M^P)$ is so called mixing angle ($U_{Ia} \ll 1$), $V_1$ – unitary matrix for diagonalization of the active neutrino mass matrix, $V_2$ – unitary matrix for diagonalization of the sterile neutrino mass matrix (it can be taken as unit matrix in our case). It means that sterile neutrinos interact with SM particles similarly to the active neutrinos:

$$L_{\text{int}} = -\frac{g}{2\sqrt{2}} W^\mu_\nu \sum_{I,\alpha} \bar{N}^c_I U_{I\alpha} \gamma^\mu (1 - \gamma_5) e_\alpha - \frac{g}{4\cos\theta_W} Z^\mu \sum_{I,\alpha} \bar{N}^c_I U_{I\alpha} \gamma^\mu (1 - \gamma_5) \nu_\alpha + \text{h.c.} \tag{6}$$

It should be noted that extension of the SM by HNLs gives an additional source of CP-symmetry violation in the theory, see e.g. [22].

Interest in HNL modification of the SM is conditioned by the model’s ability to explain the smallness of active neutrino masses (due to large values of sterile neutrino masses $M_I$ or small values of Yukawa elements $F_{\alpha I}$) and to describe the generation of the matter-antimatter asymmetry of the Universe due to CP violation in the model [31]. It was shown that Majorana masses of HNLs can be in GeV scale [32, 33, 34] to explain the baryon asymmetry of the Universe.

In 2005 Neutrino Minimal Standard Model (νMSM) model was proposed [33, 35]. In this model, SM is extended by three right-handed neutrinos (heavy neutral leptons) with masses smaller than the electroweak scale. It was shown that 18 new parameters of the model can be chosen in such a way to simultaneously solve problems of neutrino oscillations, baryon asymmetry in the Universe, and dark matter. In this case, the νMSM model requires the existence of two right-handed neutrinos with practically the same masses ($\gtrsim 100$ MeV) and one right-handed neutrino with a relatively small mass in the keV region, see [36] for a review. The lightest right-handed neutrino is the long-lived particle, dark matter candidate. In 2014 the possible manifestation of the lightest right-handed neutrino with mass 7 keV was found in X-ray spectra of the Andromeda galaxy and Perseus galaxy cluster [37, 38].

That is why the extension of the SM by HNLs attracts a lot of attention and interest. This modification of the SM is especially interesting for the GeV-scale HNLs that can be in principle detected in the intensity frontier experiments [14, 19].
The experimental searches for HNLs use a model-independent phenomenological approach, assuming existing of only one HNL and considering that other HNLs do not affect the analysis. For simplicity of analysis, restrictions are imposed on only two free parameters of the model: Majorana mass of the appropriate HNL ($m_N$) and the mixing angle of the interaction of this HNL with only one active neutrino of flavour $\alpha$ ($U_{\alpha}$). While it is usually assumed that the mixing angles of interaction with other active neutrinos are zero.

It should be noted that results of previous numerous experiments almost completely closed the region for HNL with masses below the mass of kaon, see [19, 39] for details. Therefore, HNL with mass $m_N > 0.5$ GeV is of interest to us. The phenomenology of GeV-scale HNL was considered e.g. in [40]. Recent computation of the sensitivity region for HNL search in SHiP experiment was performed in [41].

If we consider the most important production channels of HNLs in the fixed target intensity frontier experiments (such as NA62, SHiP or DUNE), we see that they are the semileptonic decays of $D$, $D_s$, mesons, $B$, $B_s$, $B_c$ mesons and decay of $\tau$ leptons, see [40]. It should be noted that essential contribution to HNL production is given by 3-body decays of mentioned particles, see Fig.1, where we present branchings for decays of $D$ and $B$ mesons as an example. Branchings for decays of other mesons can be found in [40].

Taking into account the number of produced $D$ mesons is sufficiently more than number of the produced $B$ mesons in proton-target collisions (e.g. $\sim 10^{17}$ and $\sim 10^{13}$ correspondingly for 5 years of SHiP experiment operation [40]), it is obvious that HNLs production from $B$ mesons decay can be neglected for HNLs with masses $m_N \lesssim 2$ GeV.

So, 3-body decays of mesons with $\tau$ leptons in the final state are either forbidden (for decays of $D$ mesons) or ineffective (for decays of $B$ mesons). We will denote final charged leptons states as $\ell = e, \mu$.

In SHiP collaboration paper [41] computation of 3-body decay contributions for the formation of sensitivity region for HNL was based on PYTHIA 8. Taking into account that PYTHIA is a program for the generation of high-energy collisions [42], in this paper, we analyze the relevancy of using PYTHIA for the describing of HNLs production in 3-body decays of $B$ and $D$ mesons using the SHiP experiment as an example. The fact is that for describing the semileptonic decays of $D$ mesons PYTHIA uses matrix element

$$|M_{\ell}|^2 = (p_D p_\ell)(p_\nu p_h)$$

(7)
and for describing the semileptonic decays of $B$ mesons PYTHIA uses matrix element

$$|M_{fi}|^2 = (p_B p_\nu)(p_\ell p_h).$$

(8)

However, for accurate consideration we need to use form-factors of meson transitions at least.

We compare the analytical results for HNLs production in 3-body decays with PYTHIA approximation and estimate the error of using PYTHIA in this case.

As it was pointed in [19] HNLs can be produced in $\tau$ lepton decays and these decays are important in the case of dominant mixing with $\tau$ flavour. The main 3-body decay channels of $\tau$ leptons are decays into elementary particles $\tau \rightarrow N_\alpha \bar{\nu}_\alpha$ and $\tau \rightarrow \nu_\tau \ell_\alpha N$, where $\alpha = e, \mu$. In contrast to meson decays, which must be described using the form-factor formalism, these decays can be directly described by the PYTHIA formalism.

The paper is organized in the following way. In Sec.2 we formulate the general strategy to get the domain of parameters that allows hidden particles to be detected in the intensity frontier experiments. In Sec.3 we get the general definitions of a probability density function for the particles production with certain energy in 3-body decays. In Sec.4 we compare PDFs for HNL production in 3-body decays of mesons (in the own reference frame of the mesons) computed using PYTHIA and correct matrix elements. In Sec.5 we derive the energy and polar angle distribution functions of the produced HNLs in the laboratory reference frame. In Sec.6 we find probability of the produced HNL to fall on the detector. In Sec.7 we find probability of the produced HNL to decay inside the vacuum tank before the detector. In Sec.8 we consider the HNLs production in 3-body decays of $\tau$ leptons. Finally, results are summarized in Sec.9. Correct matrix elements for the 3-body decays of mesons are presented in Appendices A and B. Useful kinematic relations for the HNLs in the different reference frames are outlined in Appendix C.

## 2 General strategy

At first, let us remind the general principles of intensity frontier experiments operation on the example of an intended experiment SHiP [43], see Fig.1. A beam line from the CERN SPS accelerator will transmit 400 GeV protons at the SHiP. The proton beam will strike in a Molybdenum and Tungsten fixed target at a center-of-mass energy $E_{CM} \approx 27$ GeV. A great number of light SM particles and hadrons will be produced under such collisions. Hidden particles are expected to be predominantly produced in the decays of the produced hadrons.

The main concept of the SHiP functioning is the following. Almost all the produced SM particles should be either trapped by an absorber or deflected in a magnetic field (muons). Remaining events with SM particles can be rejected using specially developed cuts. If hidden particles will decay into the SM particles inside the decay volume, the latter will be detected. It will mean the existence of hidden particles.

We can estimate the number of hidden particles that can be detected as

$$N_{det} = N_{BSM} \cdot \epsilon_{tot} \cdot P_{decay}.$$ 

(9)

This relation allows us to find a range of parameters $(m_N, \theta_\alpha)$, when HNL can be detected. It is a region where $N_{det} \geq N_{thr_{det}}$. Value $N_{thr_{det}}$ is the threshold value of the detected particles.
when we can affirm the discovery of HNL. It depends on characteristics of the experiment facilities and background level.

Let us explain the meaning of the constituent factors in the (9). Factor \( N_{BSM} \) is the number of hidden particles produced within all time of the SHiP experiment operation. Factor \( \epsilon_{tot} \) is product of the following factors \( \epsilon_{geom} \cdot Br_{vis} \cdot \epsilon_{det} \), where \( \epsilon_{geom} \) is the probability of the produced HNL to move towards the detector, \( \epsilon_{det} \) is the probability of detector to register visible particles, \( Br_{vis} \) is branching of HNL decay into channels, visible for the detectors. Factor \( P_{\text{decay}} \) is the probability of the produced HNL to decay in the volume of the vacuum tank before the detectors.

Therefore, for estimation of the correctness of the contribution of the produced in the 3-body decays HNLs (with PYTHIA approach) to the sensitivity region we have to consider only two factors in (9), namely \( \epsilon_{geom} \) and \( P_{\text{decay}} \).

The probability for the produced HNLs to move towards the detector (\( \epsilon_{geom} \)) can be easily found if we know the distribution function of these particles. The probability of the HNL to decay inside the vacuum tank before the detector is

\[
P_{\text{decay}} = e^{-\frac{L\gamma}{\beta}} - e^{-\frac{(L+\Delta L)\gamma}{\beta}},
\]

where \( L \) is the distance from the target to the vacuum tank, \( \Delta L \) is the length of the vacuum tank, \( \gamma \) is the Lorentz factor, \( \beta \) is velocity of the particle and \( \Gamma \) is the decay width of HNL. Thus, \( P_{\text{decay}} \) depends on the energy distribution function of HNLs, its lifetime, geometry of the experiment, and the coupling constant.

Therefore, to compute both \( \epsilon_{geom} \) and \( P_{\text{decay}} \) we need the energy-angle distribution functions of the produced HNLs. One can easily get these functions in the rest frame of the initial meson. Using energy-angle distribution of the initial mesons, we can get the distribution functions of HNLs in the laboratory reference frame. We assume that in the initial meson’s rest frame the production of HNLs is isotropic.
3 Probability density function for particles produced in 3-body decay

Let us consider 3-body decay $A \rightarrow B + C + N$, where $A$, $B$, $C$ some particles (with masses $m_A$, $m_B$, $m_C$) and $N$ is a sterile neutrino with mass $m_N$.

If the decaying particle ($A$) is a scalar (or we average over its spin states), the differential decay width of the 3-body decay in the rest frame of $A$ particle is defined as, see e.g. [44],

$$d\Gamma = \frac{|M_{fi}|^2}{8m_A(2\pi)^3} dE_N dE_B. \quad (11)$$

Full partial decay width for this channel is given as

$$\Gamma(A \rightarrow BCN) = \int_{E_N^{min}}^{E_N^{max}} \int_{E_B^{min}(E_N)}^{E_B^{max}(E_N)} \frac{|M_{fi}|^2}{8m_A(2\pi)^3} dE_N dE_B, \quad (12)$$

where the boundaries of integration can be found from the condition

$$E_N^{2\text{max/min}} = m_N^2 + \vec{p}_N^2 \pm 2|\vec{p}_B| |\vec{p}_C|,$$

that can be rewritten as solution of equation

$$E_N^{2\text{max/min}} = m_N^2 + E_B^2 - m_B^2 + (m_A - E_N - E_B)^2 - m_C^2 \pm 2\sqrt{(E_B^2 - m_B^2)((m_A - E_N - E_B)^2 - m_C^2)}, \quad (13)$$

namely

$$E_B^{\text{max/min}}(E_N) = \frac{(m_A - E_N)(w^2 + m_B^2 - m_C^2)}{2w^2} \pm \sqrt{E_N^2 - m_N^2 \lambda(w^2, m_B^2, m_C^2)/2w^2}, \quad (15)$$

where $w^2 = m_A^2 + m_N^2 - 2E_N m_A$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ is the Källén function [45].

As one can see, two functions $E_B^{\text{max/min}}(E_N)$ in [15] define the upper and lower boundaries of the region with allowed values of the energy. These functions coincide at the minimal and maximum values of the energy $E_N$ that can be found from the condition for the changing sign term being zero. We get

$$E_N^{min} = m_N, \quad E_N^{max} = \frac{m_A^2 + m_N^2 - (m_B + m_C)^2}{2m_A}. \quad (16)$$

Taking into account (12), we get probability density function (PDF) for production of HNL with certain energy ($E_N$) in the form

$$\text{pdf}(E_N) = \frac{1}{\Gamma(A \rightarrow BCN)} \int_{E_B^{min}(E_N)}^{E_B^{max}(E_N)} \frac{|M_{fi}|^2}{8m_A(2\pi)^3} dE_B. \quad (17)$$
Figure 3: Probability density functions for the energy of HNL (in the rest frame of the initial meson) produced in the reaction $B^\pm \rightarrow D^0 + \ell^\pm + N$ computed correctly (solid line) and in the PYTHIA approximation for decay of $B$ meson [8] (dashed line).

4 PDF for HNL produced in 3-body decay

In this paper, we consider HNL production in 3-body decay of pseudoscalar $B$ and $D$ mesons into other pseudoscalar mesons, namely $B^\pm \rightarrow D^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^0 + \ell^\pm + N$. Also we consider HNL production in 3-body decay of pseudoscalar $B$ and $D$ mesons into vector mesons, namely $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^*(892) + \ell^\pm + N$. Two cases of leptons ($\ell = e, \mu$) in the final states are practically indistinguishable because of the small masses of electron and muon as compared with masses of mesons in the reactions.

Let us compare probability density function in the own frame of the initial meson for production of HNL with a certain energy ($E_N$) computed with help of relation (17) for correct matrix elements (see Appendices A and B) and for matrix elements in PYTHIA approximation, see (7), (8).

We see that PDF computed correctly for reaction of the HNL production in the decay of pseudoscalar meson $B$ into pseudoscalar meson $D^0$ is in good agreement with PDF computed with PYTHIA matrix element for the decay of $B$ mesons [8], see Fig.3. But the PDF computed correctly for reaction of the $B$ meson decay into vector meson $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ is in good agreement not with PDF computed with PYTHIA matrix element for decay of $B$ mesons [8], but with PDF computed with PYTHIA matrix element for decay of $D$ mesons [7], see Fig.4.

We see that PDF computed correctly for reaction of the HNL production in the decay of pseudoscalar meson $D$ into pseudoscalar meson $K^0$ is in good agreement not with PDF computed with PYTHIA matrix element of the $D$ meson decay [7], but with PDF computed with PYTHIA matrix element for decay of $B$ mesons [8], see Fig.5. But the PDF computed correctly for reaction of $D$ meson decay into vector meson $D^\pm \rightarrow K^*(892) + \ell^\pm + N$ is in good agreement with PDF computed with PYTHIA matrix element for decay of $D$ mesons [7], see Fig.6.

5 Distribution functions

Using the program packages PYTHIA and GEANT for simulations of particle collisions, we can get data $Data_h$ for the produced $h$ mesons ($h$ stands for $B$ or $D$ mesons) in proton-
target collisions (along axis $z$) in the SHiP experiment with the appropriate energy ($E_h$) and polar angle ($\theta_h$) values. In the following calculations, data array with $a_B = 6.27 \cdot 10^5$ elements of type $(E_B, \theta_B)$ and data array with $a_D = 10^6$ elements of type $(E_D, \theta_D)$ are used.

We use this data to calculate the distribution functions for the energy ($E_{\text{lab}}^{N}$) and the polar angle ($\theta_{\text{cm}}^{N}$) of the produced HNLs in the laboratory frame with help of Monte-Carlo simulation. Kinematic relations for HNLs between the laboratory and $h$ meson’s own reference frames are presented in Appendix C. Corresponding relations contain dependence on two parameters of $h$ meson in the laboratory reference frame (energy $E_h$ and polar angle $\theta_h$) and four parameters of HNL in the own reference frame of $h$ meson (mass of HNL $m_N$, energy of HNL $E_{\text{cm}}^{N}$, polar and azimuth angles of HNL $\theta_{\text{cm}}^{N}$, $\varphi_{\text{cm}}^{N}$).

It should be noted that in the $h$ meson’s own reference frame, the energy of HNL is defined by energy probability distribution function (17), but directions of motion of the produced HNL are equiprobable because of the isotropic property of $h$ meson decay. So we take these parameters ($y = \cos \theta_{\text{cm}}^{N}$ and $\varphi_{\text{cm}}^{N}$) with randomly chosen values. Set of values ($E_{\text{h}}, \theta_{\text{h}}$) is taken from the elements of $Datah$ with serial number that is taken in a

Figure 4: Probability density functions for the energy of HNL (in the rest frame of the initial meson) produced in the reaction $B^\pm \rightarrow D^\ast(2007)^0 + \ell^\pm + N$ computed correctly (solid line), in the PYTHIA approximation for decay of $B$ meson (dashed line) and in the PYTHIA approximation for decay of $D$ meson (dot-dashed line).

Figure 5: Probability density functions for the energy of HNL (in the rest frame of the initial meson) produced in the reaction $D^\pm \rightarrow K^0 + \ell^\pm + N$ computed correctly (solid line) and in the PYTHIA approximation for decay of $D$ meson (dashed line) and in the PYTHIA approximation for decay of $B$ meson (dot-dashed line).
random way. This way we receive data \((DataN)\) in a form \((E_{lab}^N, \theta_{lab}^N)\).

Using the obtained data, we can now derive probability density functions (PDF) and cumulative distribution functions (CDF) for energy and polar angle for HNL and for initial \(h\) meson also. As example, we present PDF for energy and polar angle (computed via the matrix element from Appendix B) in the laboratory reference frame for HNL with mass 2.6 GeV produced in reaction \(B^{\pm} \rightarrow D^*(2007)^0 + \ell^{\pm} + N\), see Fig.7.

Using the definition of the median value as a value corresponding to the cumulative distribution function equal to 1/2, we get median values of the energy and polar angle for the initial \(B\) and \(D\) mesons produced in the SHiP experiment, namely \(\overline{E_B} \simeq 80\) GeV, \(\overline{\theta_B} \simeq 0.022\) and \(\overline{E_D} \simeq 16.5\) GeV, \(\overline{\theta_D} \simeq 0.022\).

For the produced HNL the corresponding median values depend on its mass and reaction of its production. As example, for HNL with mass 2.6 GeV produced in the reaction \(B^- \rightarrow D^*(2007)^0 + e^- + N\), see Fig.7, median values of energy and polar angle in the laboratory reference frame are \(\overline{E_N} \simeq 40\) GeV and \(\overline{\theta_N} \simeq 0.026\).

6 Factor \(\epsilon_{geom}\)

In this section we consider probability of the produced HNL to move toward the detector \(\left(\epsilon_{geom}\right)\). To do this, we have to find the probability of the polar angle \(\theta_{lab}^N\) of the produced

![Figure 6: Probability density functions for the energy of HNL (in the rest frame of the initial meson) produced in the reaction \(D^{\pm} \rightarrow K^* (892)^0 + \ell^{\pm} + N\) computed correctly (solid line) and in the PYTHIA approximation for decay of \(D\) meson \((\square)\) (dashed line).](image)

![Figure 7: Probability density functions for energy (fig.\(a\)) and polar angle (fig.\(b\)) in the laboratory frame for HNL with mass 2.6 GeV produced in the reaction \(B^{\pm} \rightarrow D^*(2007)^0 + \ell^{\pm} + N\).](image)
Figure 8: We present ratio $\frac{\varepsilon_{\text{PYTH}}}{\varepsilon_{\text{geom}}}$ for HNL produced in reactions a) $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ and b) $D^\pm \rightarrow K^0 + \ell^\pm + N$. Factor $\varepsilon_{\text{geom}}$ was correctly computed, factor $\varepsilon_{\text{PYTH}}$ was computed in PYTHIA approximation. The label PythiaD means that factor $\varepsilon_{\text{PYTH}}$ was computed via matrix element (7). The label PythiaB means computations via matrix element (8).

HNL to be less than the angular size of the detector e.g. for the SHiP experiment $\theta_{\text{detector}} = 0.028$. It is just the value of the cumulative distribution function ($F_{N,\theta}$) for the polar angle of the HNL in the laboratory frame

$$\varepsilon_{\text{geom}}(m_N) = F_{N,\theta}(m_N) = \int_0^{\theta_{\text{detector}}} \text{pdf}(m_N, \theta) d\theta,$$

where $\text{pdf}(m_N, \theta)$ is the PDF on polar angle $\theta$ of the HNL. Value of $\varepsilon_{\text{geom}}$ depends on the mass of HNL and the matrix element for its production.

Result of our computations can be presented in the following way. Factor $\varepsilon_{\text{geom}}$ computed correctly (with help of matrix elements presented in Appendix A and Appendix B) for reactions $B^\pm \rightarrow D^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^*(892) + \ell^\pm + N$ are in good agreement (the relative difference is of order 1% − 2%) with factor $\varepsilon_{\text{geom}}$ computed with help of PYTHIA matrix elements for decay of $B$ and $D$ mesons correspondingly, see (7) and (8). But for reactions $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^0 + \ell^\pm + N$ it is more preferable to use PYTHIA matrix elements for decay of $D$ and $B$ mesons correspondingly, see Fig.8. It should be noted that a similar situation was for the computations PDFs in Sec.4. We pay attention that the production of the HNLs in the $B$ mesons’ decay is effective only for HNL with mass $m_N \gtrsim 2$ GeV, see Introduction.

As one can see from Fig.8 the probabilities of the produced HNL to move towards the detector coincide ($\varepsilon_{\text{PYTH}} = \varepsilon_{\text{geom}}$) for HNL with the maximum kinetically allowed value of the mass. This is how it should be. In the case when HNL has maximum value of mass the initial hadron in its own reference frame decays into three motionless particles. All these particles in the laboratory frame will move in the direction of the initial meson regardless of the matrix element types.

### 7 Factor $P_{\text{decay}}$

The number of produced HNL that can be detected during the period of the SHiP operation is defined by (9). This relation allows us to find a region of parameters $(m_N, U_\alpha)$, with which HNL can be detected.
Behavior of the lower bound of this region for coupling constant ($\theta$) can be estimated analytically. In this case arguments of the exponents in $P_{\text{decay}}$ (10) are small and approximation $e^x \approx 1 + x$ can be used and we get

$$P_{\text{decay}} \approx \Delta L \cdot \Gamma \cdot X(E_N/m_N),$$

(19)

where $X(u) = (u^2 - 1)^{-1/2}$.

With help of (19) we can easily obtain the ratio of $P_{\text{decay}}$ computed for correct matrix elements (see Appendices A and B) and for matrix elements in PYTHIA approximation, see (7) and (8):

$$\frac{P_{\text{decay}}^\text{PYTH}}{P_{\text{decay}}} = \frac{\tilde{X}^\text{PYTH}(E_N/m_N)}{X(E_N/m_N)},$$

(20)

where $\tilde{X}$ is median value of the corresponding function $X$. The median value of the function $X$ was found by computing median of the set of $X$-function values produced by Monte Carlo simulations, see Sec.5.

Results of our computations are similar to the results of the analysis of the $\epsilon_{\text{geom}}$ factor. Factor $P_{\text{decay}}$ computed correctly (with help of matrix elements presented in Appendix A) for reactions $B^\pm \rightarrow D^0 + \ell^\pm + N$ is in good agreement (the relative difference less than 1% for $m_N \geq 2$ GeV) with factor $P_{\text{decay}}$ computed with help of the PYTHIA matrix elements for decay of $B$ mesons, see [8]. Factor $P_{\text{decay}}$ computed correctly (with help of matrix elements presented in Appendix B) for reactions $D^\pm \rightarrow K^*(892) + \ell^\pm + N$ is in agreement (the relative difference less than 5.5%) with factor $P_{\text{decay}}$ computed with help of the PYTHIA matrix elements for decay of $D$ mesons, see [7]. It should be noted that for the reactions $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^0 + \ell^\pm + N$ it is more preferable to use PYTHIA matrix elements for decay of $D$ and $B$ mesons correspondingly, see Fig.9.

8 Conclusions

There are some indisputable phenomena that point to the fact that SM has to be modified and complemented by a new particle (particles). We are sure that there is new physics, but we do not know where to search for it. There are many theoretical possibilities to modify the SM, namely by scalar, pseudoscalar, vector, pseudovector, or fermion particles of new physics. These particles may be substantially heavier than the energy scale of the
present colliders. However, they may also be light (with mass less than the electroweak scale) and feebly interact with the SM particles.

In this paper, we consider HNL extension of the SM. We analyzed the relevance of using the PYTHIA approximation for the describing of GeV-scale HNL production in the most important 3-body decays of $B$, $D$ mesons and $\tau$ leptons that is a topical question for construction of the sensitivity region for experimental search for HNL. We consider this question concerning the SHiP experiment, but our results are applicable and to other intensity frontier experiments.

We conclude that computations of the 3-body decay of $\tau$ leptons with HNL production in PYTHIA approximation coincide with the correct computations, but the matrix elements in PYTHIA approach for describing of the 3-body decay of mesons are just similar to the matrix elements for free quark electroweak decays. Despite this, we have shown PYTHIA approximation has the right to be used.

We consider case of 3-body decays of mesons into light meson, HNL and either electron or muon in final state. These two cases of leptons are practically indistinguishable because of the small masses of electron and muon as compared with masses of mesons in the reactions. Reactions with $\tau$ leptons in the final state are either forbidden (for decays of $D$ mesons) or ineffective (as it was pointed in Introduction HNLs production from $B$ meson decays can be neglected for HNLs with masses $m_N \lesssim 2 \text{ GeV}$).

After analyzing the results of Sec.6 and Sec.7, one can conclude that for the description of the HNL production in the 3-body decay of pseudoscalar meson into another pseudoscalar meson ($B^\pm \rightarrow D^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^0 + \ell^\pm + N$) PYTHIA matrix element (8) is better to use. For description of the HNL production in the 3-body decay of pseudoscalar meson into vector meson ($B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$ and $D^\pm \rightarrow K^*(892)^0 + \ell^\pm + N$) PYTHIA matrix element (7) is better to use.

As it was shown in Sec.2, explicit form of matrix elements for describing 3-body decay affects factors $\epsilon_{\text{geom}}$ and $P_{\text{decay}}$ in the relation (9) that defines the sensitivity region of the intensity frontier experiments. As one can see from Fig.11 due to a suitable choice of PYTHIA matrix elements (7), (8), one can get the smallest difference with the exact matrix element for reaction $B^\pm \rightarrow D^0 + \ell^\pm + N$ (the relative difference less than 0.7%), while the largest irremovable difference is for reaction $D^\pm \rightarrow K^*(892)^0 + \ell^\pm + N$ (the relative difference less than 6%).

Figure 10: We present ratio $\frac{\epsilon_{\text{PYTHIA}} P_{\text{PYTHIA}}}{\epsilon_{\text{geom}} P_{\text{decay}}}$ for HNL produced in 3-body decay of mesons for the most suitable choice of PYTHIA matrix elements. Fig. a) corresponds to $B$ meson decay (line 1: $B^\pm \rightarrow D^0 + \ell^\pm + N$, line 2: $B^\pm \rightarrow D^*(2007)^0 + \ell^\pm + N$). Fig. b) corresponds to $D$ meson decay (line 1: $D^\pm \rightarrow K^0 + \ell^\pm + N$, line 2: $D^\pm \rightarrow K^*(892)^0 + \ell^\pm + N$). Dashed line corresponds to computations via PYTHIA matrix element (7). Solid line corresponds to computations via PYTHIA matrix element (8).
Figure 11: Diagram of the semileptonic decay: a) – decay of pseudoscalar meson \( D^- \) into pseudoscalar meson \( K^0 \) or vector meson \( K^*(892) \), b) – decay of pseudoscalar meson \( B^- \) into pseudoscalar meson \( D^0 \) or vector meson \( D^*(2007) \).

Appendix

A HNL production in semileptonic decays of \( B \) and \( D \) mesons into pseudoscalar mesons

Decay of the electrically charged pseudoscalar meson \( h \) into electrically neutral pseudoscalar meson \( h', \) lepton and HNL \( (h \to h' + \ell + N) \) is derived by weak interaction, see Fig.11. Amplitude of the reaction is

\[
M_{fi} = \theta_\alpha \frac{G_F}{\sqrt{2}} V_{ij}^* \bar{Q}_i \gamma^\nu (1 - \gamma^5) Q_j |h(p)|,
\]

(A.1)

where averaging over the axial quark current gives zero, but averaging over vector quark current can be presented as

\[
\langle h'(p') |\bar{Q}_i \gamma^\nu Q_j |h(p)\rangle = \mathcal{W}_\nu = \left[(p + p')_\nu - \frac{m_h^2 - m_{h'}^2}{q^2} q_\nu \right] f^{hh'}_+(q^2) + \frac{m_h^2 - m_{h'}^2}{q^2} q_\nu f^{hh'}_0(q^2),
\]

(A.2)

where \( f^{hh'}_+(q^2) \) and \( f^{hh'}_0(q^2) \) are form-factors for \( hh' \) transitions.

For \( |M_{fi}|^2 \) summarized over the helicities of the final particles, we have

\[
|M_{fi}|^2 = 4\theta_\alpha^2 G_F^2 |V_{ij}|^2 [2(m_h^2 - m_{h'}^2) f^{hh'}_0(q^2) (k\mathcal{W}) - 2(k\mathcal{W})^2 - (qk)\mathcal{W}^2 + m_N^2 \mathcal{W}^2],
\]

(A.3)

where \( m_h, m_{h'} \) are masses of the corresponding mesons, \( m_N \) – mass of the sterile neutrino.

For consideration of reaction \( B^\pm \to D^0 + \ell^\pm + N \) we use popular parametrization for form-factors of mesons, namely Bourrely-Caprini-Lellouch (BCL) parametrization [46] that takes into account the analytic properties of form-factors (see e.g. [47, 48]),

\[
f(q^2) = \frac{1}{1 - q^2/M_{\text{pole}}^2} \sum_{n=0}^{N-1} a_n \left[ (z(q^2))^n - (-1)^n \frac{n}{N} (z(q^2))^N \right],
\]

(A.4)

where the function \( z(q^2) \) is defined via

\[
z(q^2) = \frac{\sqrt{l_+ - q^2} - \sqrt{l_+ - l_0}}{\sqrt{l_+ - q^2} + \sqrt{l_+ - l_0}}
\]

(A.5)
with
\[ t_+ = (m_h + m_{h'})^2. \]  
(A.6)

The choice of \( t_0 \) and the pole mass \( M_{\text{pole}} \) varies from group to group that performs the analysis. In this work we follow FLAG collaboration [47] and take
\[ t_0 = (m_h + m_{h'}) (\sqrt{m_h} - \sqrt{m_{h'}})^2, \]  
(A.7)
The coefficients \( a_n^+ \) and \( a_n^0 \) are then fitted to the experimental data or lattice results. Their best fit parameter values are given in Table 1.

Table 1: Best fit parameters for the form-factors (A.4) of \( B \to D \) transitions [47].

| \( f \) | \( M_{\text{pole}} \) (GeV) | \( a_0 \) | \( a_1 \) | \( a_2 \) |
|---|---|---|---|---|
| \( f^{DD}_+ \) | \( \infty \) | 0.909 | -7.11 | 66 |
| \( f^{DD}_0 \) | \( \infty \) | 0.794 | -2.45 | 33 |

For consideration of reaction \( D^\pm \to K^0 + \ell^\pm + N \) we use the parametrization for form-factors of mesons given in [49]:
\[ f(q^2) = f(0) - c (z(q^2) - z_0) \left( 1 + \frac{z(q^2) + z_0}{2} \right) \left( 1 - P q^2 \right), \]  
(A.8)
where \( z(q^2) \) is defined by (A.5) and \( z_0 = z(0) \). The best fit parameter values are given in Table 2.

Table 2: Best fit parameters for the form-factors (A.8) of \( D \to K \) transitions [49].

| \( f \) | \( f(0) \) | \( c \) | \( P \) (GeV\(^{-2}\)) |
|---|---|---|---|
| \( f^{DK}_+ \) | 0.7647 | 0.066 | 0.224 |
| \( f^{DK}_0 \) | 0.7647 | 2.084 | 0 |

**B HNL production in semileptonic decays of \( B \) and \( D \) mesons into vector mesons**

Let us consider production of HNL in the semileptonic decay of \( B \) and \( D \) mesons into vector mesons, namely \( B^\pm \to D^*(2007)^0 + \ell^\pm + N \) and \( D^\pm \to K^*(892) + \ell^\pm + N \).

Decay of the electrically charged pseudoscalar meson \( h \) into electrically neutral vector meson \( h' \), lepton and HNL \( (h \to h' + \ell + N) \) is derived by weak interaction, see Fig. 4. Amplitude of the reaction is similar to (A.1), where there is contribution from both vector and axial part of the quark current \( Q_i \gamma_\mu (1 - \gamma^5) Q_j = V_{\mu} - A_{\mu} \), see [40]:

\[ \langle h'_\nu(\epsilon, p') | V_{\mu} | h(p) \rangle = i g(q^2) \epsilon_{\mu\alpha\beta}\epsilon^{*\alpha\beta}(p + p')\sigma(p - p')^\mu = i 2g(q^2) \epsilon_{\mu\alpha\beta}\epsilon^{*\alpha\beta} p^\mu p^\nu \equiv i V_{\mu}, \]  
(B.1)

\[ \langle h'_\nu(\epsilon, p') | A_{\mu} | h(p) \rangle = f(q^2) \epsilon^*_\mu + a_+(q^2)(\epsilon^* \cdot p)(p + p')_\mu + a_-(q^2)(\epsilon^* \cdot p)(p - p')_\mu \equiv A_{\mu}, \]  
(B.2)
We get
\[ \text{Table 3: First part of the table with parameters of form-factors (B.14-B.16) of B and D mesons decays into vector mesons \[50, 51, 52\].} \]

| $h, h'$ | $f_{V}^{hh'}$ | $f_{A_0}^{hh'}$ | $f_{A_1}^{hh'}$ | $f_{A_2}^{hh'}$ | $\sigma_{V}^{hh'}$ | $\sigma_{A_0}^{hh'}$ | $\sigma_{A_1}^{hh'}$ | $\sigma_{A_2}^{hh'}$ |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $D, K^*$ | 1.03           | 0.76           | 0.66           | 0.49           | 0.27           | 0.17           | 0.30           | 0.67           |
| $B, D^*$ | 0.76           | 0.69           | 0.66           | 0.62           | 0.57           | 0.59           | 0.78           | 1.40           |

and $\epsilon_\mu$ is the polarization vector of the vector meson.

For $|M_{fi}|^2$ summarized over the helicities and polarization states of the final particles, we have
\[ |M_{fi}|^2 = 4G_F^2|V_{ij}|^2 \sum_\lambda R^{\mu\nu}[\nabla_\mu \nabla_\nu + A_\mu A_\nu^* + i(A_\mu \nabla_\nu - \nabla_\nu A_\mu^*)], \] (B.3)

where $\lambda$ is the polarization state of the vector meson and
\[ R^{\mu\nu} = (q^\nu - k^\nu)k^\mu - (qk)g^{\mu\nu} + m_N^2 g^{\mu\nu} + (q^\mu - k^\mu)k^\nu - iq_k \epsilon^{ij\nu\mu}. \] (B.4)

Summation over polarization states of the vector meson can be performed directly using relation
\[ \sum_\lambda \epsilon_\alpha (p') \epsilon_\beta (p') = - \left( g_{\alpha\beta} - \frac{p'_\alpha p'_\beta}{m_{h'}^2} \right). \] (B.5)

We get
\[ \sum_\lambda R^{\mu\nu} \nabla_\mu A_\nu^* = 8g^2(q^2) \{ m_B^2 [(p'k)^2 - m_D^2 (qk)] + m_D^2 (pk)^2 + (pp')[-2(p'k)(pk) + (pp')(qk)] \}, \] (B.6)

\[ \sum_\lambda R^{\mu\nu} i(A_\mu \nabla_\nu - \nabla_\mu A_\nu^*) = 8g(q^2)f(q^2)[m_B^2 (p'k) + m_D^2 (pk) - (p'k + pk)(pp')], \] (B.7)

\[ \sum_\lambda R^{\mu\nu} A_\mu A_\nu^* = -R^2 (m_N^2 - qk) \left( m_B^2 - \frac{(pp')^2}{m_D^2} \right) + \frac{2(kR)(kR - qR)}{m_D^2} + 2f(q^2)[(kQ)(qR) + (kR)(qQ - 2kQ) + (RQ)(m_N^2 - qk)] + f^2(q^2) \left( \frac{2(p'k)(p'p - p'k)}{m_D^2} - m_N^2 + qk - 2p'k \right), \] (B.8)

where
\[ R_\mu = a_+(q^2)(p + p')_\mu + a_-(q^2)(p - p')_\mu, \quad Q_\nu = \frac{(pp')}{m_D^2} p'_\nu - p_\nu. \] (B.9)

Form-factors $f(q^2), g(q^2), a_\pm(q^2)$ can be found from the dimensionless linear combinations \[50, 51, 52\]:
\[ V^{hh'}(q^2) = (m_h + m_{h'}) g^{hh'}(q^2), \] (B.10)
\[ A_0^{hh'}(q^2) = \frac{1}{2m_{h'}} \left( f^{hh'}(q^2) + q^2 a_0^{hh'}(q^2) + (m_h^2 - m_{h'}^2) a_+^{hh'}(q^2) \right), \] (B.11)
\[ A_1^{hh'}(q^2) = \frac{f^{hh'}(q^2)}{m_h + m_{h'}}. \] (B.12)
\[ A_2^{hh'}(q^2) = - (m_h + m_{h'}) a_+^{hh'}(q^2), \] (B.13)
Table 4: Second part of the table with parameters of form-factors \((B.14-B.16)\) of \(B\) and \(D\) mesons decays into vector mesons \([50, 51, 52]\).

| \(h, h'\) | \(\xi_{\Delta V}\) | \(\xi_{\Delta A_0}\) | \(\xi_{\Delta A_2}\) | \(M_P^V\) (GeV) | \(M_P^P\) (GeV) |
|----------|----------------|-----------------|----------------|----------------|----------------|
| \(D, K^*\) | 0 | 0 | 0.20 | 0.16 | \(m_{D_s}\) | \(m_{D_s^*}\) |
| \(B, D^*\) | 0 | 0 | 0 | 0.41 | \(m_{B_c}\) | \(m_{B_c^*}\) |

that can be parameterized as

\[
V_{hh'}^{hh'}(q^2) = \frac{f_{V_{hh'}}^{hh'}}{1 - q^2/(M_V^h)^2[1 - \sigma_{A_0}^{hh'}q^2/(M_V^h)^2 - \xi_{A_0}^{hh'}q^4/(M_V^h)^4]}, \tag{B.14}
\]

\[
A_0^{hh'}(q^2) = \frac{f_{A_0}^{hh'}}{1 - q^2/(M_P^h)^2[1 - \sigma_{A_0}^{hh'}q^2/(M_V^h)^2 - \xi_{A_0}^{hh'}q^4/(M_V^h)^4]}, \tag{B.15}
\]

\[
A_{1/2}^{hh'}(q^2) = \frac{f_{A_{1/2}^{hh'}}}{1 - \sigma_{A_{1/2}^{hh'}} q^2/(M_P^{hh'})^2 - \xi_{A_{1/2}^{hh'}} q^4/(M_P^{hh'})^4}. \tag{B.16}
\]

Best fit values of parameters are given in papers \([50, 51, 52]\). The parameters \(f\) and \(\sigma\) are given in Table 3. The parameters \(\xi\) and the pole masses \(M_V, M_P\) are given in Table 4, where \(m_{D_s} = 1.969, m_{D_s^*} = 2.112, m_{B_c} = 6.275, m_{B_c^*} = 6.331\). Mass for \(B_c^*\) was taken from theoretical prediction \([53]\).

C Useful kinematic relations for the HNLs in the different reference frames

Let us consider \(h\) meson with mass \(m_h\) and given 4-momentum \((E_h, \vec{p}_h)\) in the laboratory reference frame. Its velocity in this reference frame (the origin of coordinates is at the point of proton-target collisions, axis \(z\) is directed to the center of detector) is

\[
\vec{V}_{h}^{\text{lab}} = |\vec{V}_{h}^{\text{lab}}| \vec{e}_{h}^{\text{lab}}, \tag{C.1}
\]

where

\[
|\vec{V}_{h}^{\text{lab}}| = \frac{|\vec{p}_h|}{\sqrt{m_h^2 + p_h^2}}, \tag{C.2}
\]

\[
\vec{e}_{h}^{\text{lab}} = (\sin \theta_h \cos \varphi_h, \sin \theta_h \sin \varphi_h, \cos \theta_h).
\]

\(\theta_h\) and \(\varphi_h\) are the polar and azimuth angles of the \(h\) meson’s velocity vector in spherical coordinate system.

Absolute value of HNL’s velocity in the center-of-mass system of the produced particles (own reference frame of the initial \(h\) meson) is

\[
|\vec{V}_{N}^{\text{cm}}| = |\vec{V}_{N}^{\text{cm}}| \vec{e}_{N}^{\text{cm}}, \tag{C.3}
\]

where

\[
|\vec{V}_{N}^{\text{cm}}| = \frac{|\vec{p}_{N,\text{cm}}|}{\sqrt{m_N^2 + p_{N,\text{cm}}^2}}, \tag{C.4}
\]

\[
\vec{e}_{N}^{\text{cm}} = (\sin \theta_N^{\text{cm}} \cos \varphi_N^{\text{cm}}, \sin \theta_N^{\text{cm}} \sin \varphi_N^{\text{cm}}, \cos \theta_N^{\text{cm}}).
\]
\(\theta^m_N\) and \(\varphi^m_N\) are the polar and azimuth angles of the HNL’s velocity vector in the center-of-mass system. It should be noted that in the center-of-mass system of the produced particles the decay of \(h\) meson is isotropic and angles \(\theta^m_N\) and \(\varphi^m_N\) can be arbitrary.

To find value and direction of HNL’s velocity in the laboratory reference frame we have to find components of \(\vec{V}^m_N\) that are parallel and perpendicular to the direction of \(\vec{V}^m_h\), namely \(\vec{V}^m_{N||}\) and \(\vec{V}^m_{N\perp}\).

It’s obvious that \(\vec{V}^m_{N||} = V^m_{N||} \vec{e}^m_{h}\), where

\[
V^m_{N||} = |\vec{V}^m_N| |\vec{e}^m_h| = |\vec{V}^m_N| |\sin\theta^m_N \sin\theta_h \cos(\varphi_h - \varphi^m_N) + \cos\theta^m_N \cos\theta_h| \quad (C.5)
\]

is projection of vector \(\vec{V}^m_N\) on the direction \(\vec{e}^m_h\). Its value can be either positive or negative.

Consider now vector \(\vec{V}^m_{N\perp} = |\vec{V}^m_{N\perp}| \vec{e}^m_{N\perp}\), where

\[
|\vec{V}^m_{N\perp}| = |\vec{V}^m_N| |\vec{e}^m_{N\perp} \times \vec{e}^m_{h}|. \quad (C.6)
\]

Vector \(\vec{V}^m_{N\perp}\) has to lie in the plane of vectors \(\vec{V}^m_N\) and \(\vec{V}^m_h\). The equation of this plane is \(\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0\), where \(\vec{n}\) is the normal vector of the plane \(\vec{n} = \vec{e}^m_S \times \vec{e}^m_B\) and \(\vec{r} - \vec{r}_0\) is vector lying in the plane. Radii \(\vec{r}\) and \(\vec{r}_0\) are meant to be taken with a base in the center of target. We put \(\vec{r}_0\) (point of the \(B\) meson decay) to be a zero vector, because the distance between the point of production of meson and the point of its decay is very small compared with the distance from target to detector. As the vector \(\vec{r}\) we can use the unit vector of direction \(\vec{e}^m_{N\perp}\).

Components of the unit vector \(\vec{e}^m_{N\perp} = (\alpha, \beta, \gamma)\) have to satisfy the following equations

\[
\begin{align*}
\vec{e}^m_{h} \cdot \vec{e}^m_{N\perp} &= \alpha \sin\theta_h \cos\varphi_h + \beta \sin\theta_h \sin\varphi_h + \gamma \cos\theta_h = 0, \\
n_x \alpha + n_y \beta + n_z \gamma &= 0, \\
\alpha^2 + \beta^2 + \gamma^2 &= 1.
\end{align*}
\]  

(C.7)

One can get solution in the form \(\vec{e}^m_{N\perp} = \vec{N} / |\vec{N}|\), where \(\vec{N} = (\alpha, \beta, \gamma)\),

\[
\alpha = \cos\theta^m_N \cos\theta_h \sin\theta_h \cos\varphi_h - \\
- \sin\theta^m_N \left( \cos^2\theta_h \cos\varphi^m_N + \sin^2\theta_h \sin(\varphi_h - \varphi^m_N) \right) \sin\varphi_h, \\
\beta = \cos\theta^m_N \cos\theta_h \sin\theta_h \sin\varphi_h - \\
- \sin\theta^m_N \left( \cos^2\theta_h \sin\varphi^m_N - \sin^2\theta_h \sin(\varphi_h - \varphi^m_N) \right) \cos\varphi_h, \\
\gamma = \sin\theta_h \left( \cos\theta_h \sin\theta^m_N \cos(\varphi_h - \varphi^m_N) - \sin\theta_B \cos\theta^m_N \right)
\]  

(C.8)

and normalized coefficient \(|\vec{N}|\) is equal to \(|\vec{e}^m_N \times \vec{e}^m_{m}\)|. Obtained solution has ambiguity, because system of equations \((C.7)\) is invariant under simultaneous change of sign of all vector's components. This ambiguity can be removed by following condition. If scalar product \(\vec{N} \cdot \vec{V}^m_N\) is positive the sign of vector \(\vec{N}\) is correct, otherwise sign of vector \(\vec{N}\) has to be changed.

So, vector \(\vec{V}^m_{N\perp}\) is simply defined as

\[
\vec{V}^m_{N\perp} = \alpha |\vec{V}^m_{N\perp}| \vec{e}^m_{N\perp} = \alpha |\vec{V}^m_{N\perp}| \vec{N} / |\vec{N}|, \quad \alpha = \text{sgn}[\vec{N} \cdot \vec{e}^m_N]. \quad (C.9)
\]
Now we can find value and direction of HNL’s velocity in the laboratory reference frame
\[ V_{\text{lab}}^{\text{lab}} = \frac{|\vec{V}_h^{\text{lab}}| + V_{N,||}^{\text{cm}}}{1 + V_{N,||}^{\text{cm}} |\vec{V}_h^{\text{lab}}|}, \quad \vec{V}_{N,||}^{\text{lab}} = V_{N,||}^{\text{lab}} \hat{e}_h^{\text{lab}} \]  
(C.10)

\[ |\vec{V}_N^{\text{lab}}| = |\vec{V}_{N,\perp}^{\text{lab}}| \sqrt{1 - \frac{|\vec{V}_h^{\text{lab}}|^2}{1 + V_{N,||}^{\text{cm}} |\vec{V}_h^{\text{lab}}|}}, \quad \vec{V}_{N,\perp}^{\text{lab}} = \alpha |\vec{V}_{N}^{\text{cm}}| \vec{N}, \]  
(C.11)

\[ |\vec{V}_N^{\text{lab}}| = \sqrt{|\vec{V}_{N,||}^{\text{lab}}|^2 + |\vec{V}_{N,\perp}^{\text{lab}}|^2}. \]  
(C.12)

Components of HNL’s velocity vector \( \vec{V}_N^{\text{lab}} \) are
\[ (\vec{V}_N^{\text{lab}})_x = V_{N,||}^{\text{lab}} (\hat{e}_h^{\text{lab}})_x + \alpha |\vec{V}_{N}^{\text{cm}}| (\vec{N})_x = |\vec{V}_N^{\text{lab}}| \sin \theta_N^{\text{lab}} \cos \varphi_N^{\text{lab}}, \]
\[ (\vec{V}_N^{\text{lab}})_y = V_{N,||}^{\text{lab}} (\hat{e}_h^{\text{lab}})_y + \alpha |\vec{V}_{N}^{\text{cm}}| (\vec{N})_y = |\vec{V}_N^{\text{lab}}| \sin \theta_N^{\text{lab}} \sin \varphi_N^{\text{lab}}, \]
\[ (\vec{V}_N^{\text{lab}})_z = V_{N,||}^{\text{lab}} (\hat{e}_h^{\text{lab}})_z + \alpha |\vec{V}_{N}^{\text{cm}}| (\vec{N})_z = |\vec{V}_N^{\text{lab}}| \cos \theta_N^{\text{lab}}, \]  
(C.13)

where \( (\hat{e}_h^{\text{lab}})_i \) and \( (\vec{N})_i \) are the components of vectors \( \text{C.2}, \text{C.8} \) and \( \theta_N^{\text{lab}}, \varphi_N^{\text{lab}} \) are angles of the meson’s velocity vector in laboratory reference frame. Energy of HNL in the laboratory reference frame is
\[ E_N^{\text{lab}} = \frac{m_N}{\sqrt{1 - |\vec{V}_N^{\text{lab}}|^2}}. \]  
(C.14)

In this paper we assume that the experiment facility has a cylindrical symmetry. Therefore, the azimuth angle of \( h \) meson in the laboratory reference frame can be set to zero \( (\varphi_h = 0) \). The energy \( E_N^{\text{lab}} \) and direction of the HNL’s velocity \( (\theta_N^{\text{lab}}, \varphi_N^{\text{lab}}) \) in the laboratory reference frame is defined only by six parameters: energy \( E_h \) and polar angle \( \theta_h \) of \( h \) meson in the laboratory reference frame, two angles \( (\theta_N^{\text{cm}} \text{ and } \varphi_N^{\text{cm}}) \), mass \( m_N \) and energy \( E_N^{\text{cm}} \) of the produced HNL in the center-of-mass system of the produced particles (own reference frame of \( h \) meson).

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