New Choice for Small Universal Devices: Symport/Antiport P Systems

Sergey Verlan
LACL, Département Informatique, Université Paris Est, 61 av. Général de Gaulle, 94010 Créteil, France
Institute of Mathematics and Computer Science
Academy of Sciences of Moldova, Academiei, 5, MD-2028, Moldova
verlan@univ-paris12.fr

Yurii Rogozhin
Institute of Mathematics and Computer Science
Academy of Sciences of Moldova, Academiei, 5, MD-2028, Moldova
Rovira i Virgili University,
Research Group on Mathematical Linguistics,
Pl. Imperial Tarraco 1, 43005 Tarragona, Spain
rogozhin@math.md

Symport/antiport P systems provide a very simple machinery inspired by corresponding operations in the living cell. It turns out that systems of small descriptional complexity are needed to achieve the universality by these systems. This makes them a good candidate for small universal devices replacing register machines for different simulations, especially when a simulating parallel machinery is involved. This article contains survey of these systems and presents different trade-offs between parameters.

1 Introduction

The idea of symport/antiport P systems comes from simple observations in cell biology. In a living cell, there is a permanent chemical exchange with the environment. Water, ions and other chemicals enter or exit the cell depending on its necessity. Some of these exchanges use a passive transport where no energy is consumed and the chemicals are moved along the chemical gradient, while others use an active transport, which consumes energy in order to move chemicals against the gradient. Very often the active transport uses co-transporters, i.e. molecules that facilitate the penetration of the transported substance through the cell membrane. The most common co-transporters either travel together with the transported substance, in this case we speak about symport, or they are exchanged with the transported substance, in this case we speak about antiport.

This transport mechanism is formalized by symport/antiport P systems [11], [12] which abstract the cell by a set of nested compartments enclosed by membranes and chemicals by a multiset of objects. The symport transport is then represented by a rule \((y, out)\) or \((y, in)\) that specifies that objects present in the multiset \(y\) travel together outside or inside the current compartment. The antiport is formalized by the rule \((x, out; y, in)\) which indicates that objects given by \(x\) and present in the compartment will exchange with objects given by \(y\) situated outside the compartment.

The evolution of a symport/antiport P system is done in a maximally parallel way (other evolution strategies are discussed in [3]), starting from an initial distribution of objects in membranes and the result is obtained by counting objects in some membrane when the system cannot evolve anymore.
Further generalization of the model leads to symport/antiport tissue P systems where the underlying membrane structure is no more represented by a tree as in the case of P systems but by an arbitrary graph corresponding to a tissue of cells. More generalizations and a presentation of P systems (not necessarily using symport and antiport operations) can be found in [12] and [16].

The computational model given by symport/antiport (tissue) P systems is very simple, however it was shown that if a cooperation of three objects is permitted, then one membrane is sufficient to generate all recursively enumerable sets of numbers [7] and [9]. After that other descriptional complexity parameters stared to be investigated, in particular, systems with minimal symport or antiport, where only two objects can cooperate. Such systems are of great interest because the biological variants of symport and antiport involve only two objects in most of the cases. These systems first were investigated in [5], where nine membranes were used to achieve computational completeness. This number was progressively decreased and finally established to two membranes in [3].

Other complexity parameters like the number of used objects or the number of rules were investigated and trade-offs between different parameters were established. In this article we present a survey of different complexity measures and best known results.

2 Definitions

We recall here some basic notions of formal language theory we need in the rest of the paper. We refer to [14] for further details.

We denote by \(\mathbb{N}\) the set of all non-negative integers. Let \(O = \{a_1, \ldots, a_k\}\) be an alphabet. A finite multiset \(M\) over \(O\) is a mapping \(M : O \rightarrow \mathbb{N}\), i.e., for each \(a \in O\), \(M(a)\) specifies the number of occurrences of \(a\) in \(M\). The size of the multiset \(M\) is \(|M| = \sum_{a \in O} M(a)\). A multiset \(M\) over \(O\) can also be represented by any string \(x\) that contains exactly \(M(a)\) symbols \(a_i\) for all \(1 \leq i \leq k\), e.g., by \(a_1^{M(a_1)} \cdots a_k^{M(a_k)}\), or else by the set \(\{a_i^{M(a_i)} \mid 1 \leq i \leq k\}\). For example, the multiset over \(\{a, b, c\}\) defined by the mapping \(a \rightarrow 3, b \rightarrow 1, c \rightarrow 0\) can be specified by \(a^3 b\) or \(\{a, b\}\). An empty multiset is represented by \(\lambda\).

We may also consider mappings \(M\) of form \(M : O \rightarrow \mathbb{N} \cup \{\infty\}\), i.e., elements of \(M\) may have an infinite multiplicity; we shall call them infinite multisets.

In the following we briefly recall the basic notions concerning P systems with symport/antiport rules. For more details on these systems and on P systems in general, we refer to [12].

A P system with symport/antiport of degree \(n\) is a construct

\[
\Pi = (O, \mu, w_1, \ldots, w_n, E, R_1, \ldots, R_n, i_0),
\]

where:

1. \(O\) is a finite alphabet of symbols called objects,
2. \(\mu\) is a membrane structure consisting of \(n\) membranes that are labeled in a one-to-one manner by \(1, 2, \ldots, n\).
3. \(w_i \in O^*\), for each \(1 \leq i \leq n\) is a multiset of objects associated with the region \(i\) (delimited by membrane \(i\))
4. \(E \subseteq O\) is the set of objects that appear in the environment in infinite numbers of copies,
5. \(R_i\), for each \(1 \leq i \leq n\), is a finite set of symport/antiport rules associated with the region \(i\) and which have the following form \((x, in), (y, out), (y, out; x, in)\), where \(x, y \in O^*\).
6. \( i_0 \) is the label of an elementary membrane of \( \mu \) that identifies the corresponding output region.

A symport/antiport P system is defined as a computational device consisting of a set of \( k \) hierarchically nested membranes that identify \( k \) distinct regions (the membrane structure \( \mu \)), where to each region \( i \) there are assigned a multiset of objects \( w_i \) and a finite set of symport/antiport rules \( R_i, 1 \leq i \leq n \). A symport rule \((x, in) \in R_i\) permits to move \( x \) into region \( i \) from the immediately outer region. Notice that rules of the form \((x, in)\), where \( x \in E^+ \) are forbidden in the skin (the outermost) membrane. A symport rule \((x, out) \in R_i\) permits to move the multiset \( x \) from region \( i \) to the outer region. An antiport rule \((y, out; x, in)\) exchanges two multisets \( y \) and \( x \), which are situated in region \( i \) and the outer region of \( i \) respectively.

A computation in a symport/antiport P system is obtained by applying the rules in a non-deterministic maximally parallel manner, i.e. all rules that can be applied together should be applied. Other possibilities not using the maximal parallelism are discussed in [8]. The computation is restricted to moving objects through membranes, since symport/antiport rules do not allow the system to modify the objects placed inside the regions. Initially, each region \( i \) contains the corresponding finite multiset \( w_i \); whereas the environment contains only objects from \( E \) that appear in infinitely many copies.

A computation is successful if starting from the initial configuration it reaches a configuration where no rule can be applied. The result of a successful computation is the natural number that is obtained by counting the objects that are presented in region \( i_0 \). Given a P system \( \Pi \), the set of natural numbers computed in this way by \( \Pi \) is denoted by \( N(\Pi) \).

We denote by \( NOP_n(sym_t, anti_t) \) the family of sets of natural numbers that are generated by a P system with symport/antiport of degree at most \( n > 0 \), symport rules of size at most \( r \geq 0 \), and antiport rules of size at most \( t \geq 0 \). The size of a symport rule \((x, in)\) or \((x, out)\) is given by \(|x|\), while the size of an antiport rule \((y, out; x, in)\) is given by \( \max\{|x|, |y|\} \). We denote by \( NRE \) the family of recursively enumerable sets of natural numbers.

P systems as defined above have an underlying tree-like membrane structure. It is possible to apply a similar reasoning to an arbitrary graph. This leads us to the idea of tissue P systems.

A tissue P system with symport/antiport of degree \( n \geq 1 \) is a construct

\[
\Pi = (O, G, w_1, \ldots, w_n, E, R, i_0),
\]

where \( O \) is the alphabet of objects and \( G \) is the underlying directed labeled graph of the system. The graph \( G \) has \( n + 1 \) nodes and the nodes are numbered from 0 to \( n \). We shall also call nodes from 1 to \( n \) cells and node 0 the environment. There is an edge between each cell \( i, 1 \leq i \leq n \), and the environment. Each cell contains a multiset of objects, initially cell \( i, 1 \leq i \leq n \), contains multiset \( w_i \). The environment is a special node which contains symbols from \( E \) in infinite multiplicity as well as a finite multiset over \( O \setminus E \), but initially this multiset is empty. The symbol \( i_0 \in (1 \ldots n) \) indicates the output cell, and \( R \) is a finite set of rules (associated to edges) of the following forms:

1. \((i, x, j), 0 \leq i \leq n, 0 \leq j \leq n, i \neq j, x \in O^+ \) and not \( i = 0 \) \& \( x \in E^+ \) (symport rules for the communication).
2. \((i, x/y, j), 0 \leq i, j \leq n, i \neq j, x, y \in O^+ \) (antiport rules for the communication).

We remark that \( G \) may be deduced from relations of \( R \). More exactly, \( G \) contains \( n + 1 \) vertices and there is an oriented edge between vertex \( i \) and \( j \) if and only if there is a rule \((i, x, j)\) in \( R \) and edges between \( i \) and \( j \) and \( j \) and \( i \) if and only if there is a rule \((i, x/y, j)\) in \( R \). However, we prefer to indicate both \( G \) and \( R \) because it simplifies the presentation.
The rule \((i,x,j)\) sends a multiset of objects \(x\) from node \(i\) to node \(j\). The rule \((i,x/y,j)\) exchanges multisets \(x\) and \(y\) situated in nodes \(i\) and \(j\) respectively. The size of symport rule \((i,x,j)\) is equal to \(|x|\), while the size of an antiport rule is equal to \(|x| + |y|\).

As in the case of P systems a computational step is made by applying all applicable rules from \(R\) in a non-deterministic maximal parallel way. A configuration of the system is an \((n+1)\)-tuple \((z_0,z_1,\ldots,z_n)\) where each \(z_i, 1 \leq i \leq n\), represents the contents of cell \(i\) and \(z_0\) represents the multiset of objects that appear with a finite multiplicity in the environment (initially \(z_0\) is the empty multiset). The computation stops when no rule may be applied. The result of a computation is given by the number of objects situated in cell \(i_0\), i.e., by the size of the multiset from cell \(i_0\).

We denote by \(NOtP_n\) the family of all sets of numbers computed by tissue P systems with symport/antiport of degree at most \(n\) and which have symport rules of size at most \(p\) and antiport rules of size at most \(q\).

The following theorem shows the basic results for symport/antiport [tissue] P systems:

**Theorem 1** \(NO[t]P_1\)\((\text{sym}_3) = NO[t]P_1\)\((\text{anti}_3) = RE\).

We can also consider accepting (tissue) P systems where an input multiset is placed in some fixed cell/membrane and it is accepted if and only if the corresponding system halts. Theorem \(\square\) holds as well in the accepting case, however it is possible to use a deterministic construction for the proof.

### 3 Size of rules

Theorems from the previous section show that using symport or antiport rules of size three the computational completeness is achieved with only one membrane. The situation changes completely if rules of size two, called minimal antiport or minimal symport rules, are considered – in one membrane or cell, we only get finite sets:

**Theorem 2** \(NO[t]P_1\)\((\text{sym}_1,\text{anti}_2) \cup NO[t]P_1\)\((\text{sym}_2) \subseteq NFIN\).

The theorem follows from the fact that the number of symbols inside the membrane cannot be increased using minimal symport or antiport rules. Hence at least two membranes are needed for the computational completeness. This number is sufficient, as the following result holds.

**Theorem 3** \(NO[t]P_2\)\((\text{sym}_1,\text{anti}_2) = NO[t]P_2\)\((\text{sym}_2) = NRE\).

The proof significantly differs if tissue or tree-like P systems are considered. In the tissue case, the proof is based on the possibility to reach a membrane from another one by two roads, directly or via the environment, which have a different length. In this way, a temporal de-synchronization of pairs of objects is obtained and it can be used to simulate the instructions of a register machine.

Moreover, in the tissue case, we have a deterministic construction for the acceptance of recursively enumerable sets. In the tree-like case it is not possible to use a similar technique, because only the root is connected to the environment, which considerably restricts the accepting power of deterministic P systems:

**Theorem 4** For any deterministic P system with minimal symport and minimal antiport rules (of type sym_2 and anti_2), the number of objects present in the initial configuration of the system cannot be increased during halting computations.
Hence, deterministic P systems with minimal symport and antiport rules with any number of membranes can generate only finite languages.

However, if non-deterministic systems are considered, then it is possible to reach computational completeness for the accepting case with two membranes: an initial pumping phase is performed to introduce a sufficient number of working objects needed to carry out the computation (a non-deterministic guess for the number of working objects is done). After that, the system simulates a register machine thereby consuming the number of working objects.

### 3.1 Generalized Minimal Communication

We can generalize the idea of minimal antiport and symport and introduce the concept of *minimal interaction tissue P systems*. These are tissue P systems where at most two objects may interact, i.e., one object is moved with respect to another one. Such interactions can be described by rules of the form \((a, i)(b, j) \rightarrow (a, k)(b, l)\), which indicate that if symbol \(a\) is present in membrane \(i\) and symbol \(b\) is present in membrane \(j\), then \(a\) will move to membrane \(k\) and \(b\) will move to membrane \(l\). We may impose several restrictions on these interaction rules, namely by superposing several cells. Some of these restrictions directly correspond to antiport or symport rules of size 2.

Below we define all possible restrictions (modulo symmetry): let \(O\) be an alphabet and let \((a, i)(b, j) \rightarrow (a, k)(b, l)\) be an interaction rule with \(a, b \in O, i, j, k, l \geq 0\). Then we distinguish the following cases:

1. \(i = j = k \neq l\): the *conditional-uniport-out rule* sends \(b\) to membrane \(l\) provided that \(a\) and \(b\) are in membrane \(i\).
2. \(i = k = l \neq j\): the *conditional-uniport-in rule* brings \(b\) to membrane \(i\) provided that \(a\) is in that membrane.
3. \(i = j \neq k = l\): the *symport2 rule* corresponds to the minimal symport rule, i.e., \(a\) and \(b\) move together from membrane \(i\) to \(k\).
4. \(i = l \neq j = k\): the *antiport1 rule* corresponds to the minimal antiport rule, i.e., \(a\) and \(b\) are exchanged in membranes \(i\) and \(k\).
5. \(i = k \neq j \neq l\): the *presence-move rule* moves the symbol \(b\) from membrane \(j\) to \(l\), provided that there is a symbol \(a\) in membrane \(i\).
6. \(i = j \neq k \neq l\): the *separation rule* sends \(a\) and \(b\) from membrane \(i\) to membranes \(k\) and \(l\), respectively.
7. \(k = l \neq i \neq j\): the *joining rule* brings \(a\) and \(b\) together to membrane \(i\).
8. \(i = l \neq j \neq k\) or \(i \neq j = k \neq l\): the *chain rule* moves \(a\) from membrane \(i\) to membrane \(k\) while \(b\) is moved from membrane \(j\) to membrane \(i\), i.e., where \(a\) previously has been.
9. \(i \neq j \neq k \neq l\): the *parallel-shift rule* moves \(a\) and \(b\) in independent membranes.

A minimal interaction tissue P system may have rules of several types as defined above. With respect to the computational power of such systems we immediately see that when only antiport1 rules or only symport2 rules are used, the number of objects in the system cannot be increased, hence, such systems can generate only finite sets of natural numbers. However, if we allow uniport rules (i.e., rules of the form \((a, i) \rightarrow (a, k)\) specifying that, whenever an object \(a\) is present in cell \(i\), this may be moved to cell \(k\)), then minimal interaction tissue P systems with symport2 and uniport rules or with antiport1 and uniport rules become tissue P systems with minimal symport or minimal symport and antiport, respectively.

By combining conditional-uniport-in rules and conditional-uniport-out rules, computational completeness can be achieved by simulating a register machine. The best known construction from [2] is
using 14 cells, but it is very probably that this number can be decreased. A register machine may be also simulated by using only the parallel-shift rule with 19 cells [15]. In all other cases, when only one of the types of rules defined above is considered, it is not even clear whether infinite sets of natural numbers can be generated.

Another interesting problem is to investigate how an interaction rule may be simulated by some restricted variants. Such a study may lead to a formulation of sufficient conditions on how combinations of variants of rules \((a,i)(b,j) \rightarrow (a,k)(b,l)\) may guarantee that the system can be realized by using only specific restricted variants of rules in an equivalent minimal interaction tissue P system. After that, a system satisfying sufficient conditions of several restrictions may be automatically rewritten in terms of any corresponding restricted variants. A list of such results can be found in [15].

### 4 Number of Symbols

Another complexity parameter that can be investigated is the number of objects that can be used. The main results for P systems with antiport (and symport) rules can be summarized in the following table:

| objects | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE | NRE |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 5       |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 4       |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 3       | 1   | 2   |     |     |     |     |     |     |     |     |     |     |     |     |
| 2       | C   | 1   | 2   | NRE |     |     |     |     |     |     |     |     |     |     |
| 1       | A   | B   | B   | B   | B   |     |     |     |     |     |     |     |     |     |

In the above table, the class of P systems indicated by A generates exactly \(NFIN\), the class indicated by B generates at least \(NREG\), in the case of C at least \(NREG\) can be generated and at least \(NFIN\) can be accepted, while a class indicated by a number \(d\) can simulate any \(d\)-register machine. The most interesting questions still remaining open are to characterize the families generated or accepted by P systems with only one symbol.

In the tissue case the situation changes as the additional links between every cell and the environment permit to easily simulate a register machine [1]. However, the definition used by the authors is slightly different and it imposes a sequentiality for the communication between two cells, i.e. if two rules that involve same two cells may be applied at the same time, then only one of them will be chosen. The table below shows the obtained results. In the table A indicates that the corresponding family includes at least \(NREG\), and B indicates that the corresponding family can generate more than \(NFIN\).

| objects | NREG | NREG | NREG | NREG | NREG | NREG | NREG | NREG |
|---------|------|------|------|------|------|------|------|------|
| 4       | NREG | NREG | NREG | NREG | NREG | NREG | NREG | NREG |
| 3       | NREG | A    |      |      |      |      |      |      |
| 2       | NREG | A    | NREG | NREG | NREG | NREG | NREG | NREG |
| 1       | NFIN | B    | A    | A    | A    | A    | A    | NRE  |

### 5 Number of Rules

In this section we consider universal symport/antiport P systems having a small number of rules. Such a bound can be obtained if we simulate a universal device for which a bound on the number of rules is already known. Since P systems with antiport and symport rules can easily simulate register machines, it is natural to consider simulations of register machines having a small number of instructions. An
example of such a machine is the register machine $U_{32}$ described in [10], which has 22 instructions (9 increment and 13 decrement instructions). The table below summarizes the best results known on this topic, showing the trade-off between the number of antiport rules and their size:

| number of rules | 73 | 56 | 47 | 43 | 30 | 23 |
|-----------------|----|----|----|----|----|----|
| size of rules   | 3  | 5  | 6  | 7  | 11 | 19 |

The results for columns 1, 4 and 5 were established in [6], while other results are taken from [4]. The last column in this table is particularly interesting, because the register machine $U_{32}$ which was the starting point of the construction uses 25 computational branches.

### 6 Conclusions

Symport/antiport P systems were heavily investigated (there are more than 60 articles on this topic) and a lot of results about them are known, in particular, about systems having low complexity parameters. This information combined with their simple construction makes them an ideal object to be used in universality proofs where they can replace register machines, in particular for parallel computing devices. They are particularly well suited as a simulated device for different classes of P systems which permits to obtain different descriptional complexity improvements.

Even if there are a lot of results on P systems with symport/antiport, there remain a lot of open questions; we would like to highlight the importance of the investigation of generalized minimal communication models as this can show new communication strategies that can be further used in other variants of P systems. Another important topic is the number of rules of universal antiport P systems with one membrane. This is especially interesting because such systems directly correspond to maximally parallel multiset rewriting systems (MPMRS), see [4] for a formal definition of MPMRS. Since almost all types of object-based P systems can be represented in terms of MPMRS, this will give a lower bound on the number of rules needed for an universal P system.

### Acknowledgments

The authors gratefully acknowledge support by the Science and Technology Center in Ukraine, project 4032.

### References

[1] A. Alhazov, R. Freund, M. Oswald: Tissue P systems with antiport rules and a small number of symbols and cells. *Proc. of DLT 2005* (C. de Felice et al., eds.) Palermo, LNCS 3572, Springer, 2005, 100–111.

[2] A. Alhazov, R. Freund, M. Oswald, S. Verlan: Partial versus total halting in P systems. In M.A. Gutiérrez-Naranjo, Gh. Păun, A. Romero-Jimenez, and A. Riscos-Nunez (eds.), *Proceedings of the Fifth Brainstorming Week on Membrane Computing*, Sevilla, 2007, 1–20.

[3] A. Alhazov, Yu. Rogozhin: Towards a characterization of P systems with minimal symport/antiport and two membranes. *Proc. of 7th Int. Workshop on Membrane Computing* (H. J. Hoogeboom, Gh. Păun, G. Rozenberg, A. Salomaa eds.), Leiden, The Netherlands, LNCS 4361, Springer, 2007, 135–153.

[4] A. Alhazov, S. Verlan: Minimization strategies for maximally parallel multiset rewriting systems. *TUCS Technical Report* 862, Turku, 2008.

[5] F. Bernardini, M. Gheorghe: On the power of minimal symport/antiport. *Preproc. of the 3rd Workshop on Membrane Computing* (A. Alhazov, C. Martín-Vide, Gh. Păun, eds.), Tarragona, 2003, 72–83.

[6] E. Csehaj-Vargj, M. Margenstern, Gy. Vaszil, S. Verlan: Small computationally complete symport/antiport P systems. *TCS*, 372 (2007), 152–164.
New Choice for Small Universal Devices: Symport/Antiport P Systems

[7] R. Freund, A. Păun: Membrane systems with symport/antiport: universality results. In [13], 270–287.
[8] R. Freund, S. Verlan: A formal framework for static (tissue) P systems. Proc. of WMC 2008 (G. Eleftherakis et al., eds.), Thessaloniki, Greece, Springer, 2007, LNCS 4860, 271–284.
[9] P. Frisco, H.J. Hoogeboom: Simulating counter automata by P systems with symport/antiport. In [13], 288–301.
[10] I. Korec: Small universal register machines. TCS, 168 (1996), 267–301.
[11] A. Păun, Gh. Păun: The power of communication: P systems with symport/antiport. New Generation Computing, 20 (2002), 295–305.
[12] Gh. Păun: Membrane Computing. An Introduction. Springer, 2002.
[13] Gh. Păun, G. Rozenberg, A. Salomaa, C. Zandron, eds.: Membrane Computing. Intern. Workshop WMC 2002, Curtea de Argeș, Romania, Revised Papers, LNCS 2597, Springer, Berlin, 2003.
[14] G. Rozenberg, A. Salomaa (Eds.): Handbook of Formal Languages. Vol. I-III., Springer, 1997.
[15] S. Verlan, F. Bernardini, M. Gheorghe, M. Margenstern: Generalized communicating P systems. TCS 404 (1-2) (2008), 170–184.
[16] The Membrane Computing Web Page: http://ppage.psystems.eu.