A survey of the consensus for multi-agent systems

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ABSTRACT
Multi-agent systems (MASs) has developed into an emerging complex system science and gradually infiltrated into various fields of social life. The problem of consensus (i.e. all agents eventually to reach an agreement upon a common quantity of interest) is the basis of distributed coordinated control of the MASs, which has attracted tremendous attention from both theoretical and practical perspectives. This paper comprehensively reviews the state-of-the-art development in the consensus of MASs. Firstly, the basic framework and overview of MASs and consensus are discussed. Secondly, the motivations, results and methods of several kinds of consensus problems are introduced, including consensus subjected to communication constraints, leader-following consensus, group consensus, consensus based on trigger mechanism, finite-time consensus, multi-consensus and multi-tracking. Finally, some challenging issues and development trends of the consensus of MASs are considered.

1. Introduction
In recent years, with the deepening of scientific research on the biological behaviour, researchers have had a more profound and intuitive scientific analysis of the group coordination behaviours that are prevalent in biological populations in nature, such as the collaborative division of labour between ant colonies, parade of fish schools, formation of bird groups and cooperative hunting of the herd. Through a large amount of data observation and research, it is shown that the overall intelligent behaviour and actions can be achieved through the local or regional communication and cooperation between the individuals, although the individual’s ability in the group is quite limited. Without the centralized control from the outside world and the internal global information exchange, these groups can present the overall complex behaviour, such as maintaining formation, escaping natural enemies, collaborative attacks and finding food, only through the information exchange with the surrounding individuals. The multi-agent systems (MASs) are derived from the exploration and research of biological behaviours in nature, and it is the refinement and development of the behaviour patterns of biological groups.

Durfee, Lesser, and Corkill (1989) define a MASs as a loosely coupled structure composed of multiple agents, and agents interact with each other to solve problems that cannot be solved by a single agent, due to lack of ability, knowledge or resources, or even low-efficiency problems. The advantages of the MASs over a single agent are (1) ability to perform more complex and dangerous tasks; (2) high efficiency; (3) highly fault tolerant and robust; (4) low cost and easy to develop and so on. MASs can improve the quality and efficiency of complex problems with asynchronous parallel activities between agents. Its loosely coupled structure ensures the reusability and scalability of its components. Its data and resources are dispersed in various agents in the system environment, expressing the distribution of system description problems. Through the coordinated control and collaborative operation of the intelligent group, the effect of MASs far exceeds the cumulative sum of its individual performance. Therefore, the MASs has developed into an emerging complex system science and gradually infiltrated into various fields of social life (Jiang, Liu, & Zhang, 2018). The control methods of studying MASs usually include centralized and distributed. Although the centralized control method is easier to install and implement, when the number of subsystems is large, it requires the central station with sufficient resources to withstand a large amount of communication and computational load. Therefore, the reliability of the central station is relatively high. This type of control is essentially a simple extension of control methods and strategies for the traditional single system. In contrast, the distributed control
method does not rely on the central station function but the control of systems by adopting a complex system structure. Compared with centralized control technology, distributed processing technology has the advantages of high reliability, fast running speed and convenient operation. Research on distributed control of MASs originated from distributed computing, management science, statistical physics and other disciplines, and research in the field of control dates back to the literature (Tsitsiklis & Athans, 1984).

The consensus problem is the basis of distributed coordinated control of the MASs. It has been widely used in cooperative control, formation control, sensor network design and clustering of social insects (Xue, Liu, Gu, Li, & Guan, 2017). Therefore, the consensus problem has become a hot issue in the research of the MASs. The theoretical study of the consensus problem can be roughly divided into three stages. The first stage is mainly based on the simulation of a biological group mechanism, by using computers to simulate some consensus phenomena of natural groups. In 1987, Reynolds built a computer model according to the characteristics of birds, fish and other groups in nature and proposed the famous Boid model. In 1995, Vicsek, Czirók, Ben-Jacob, Cohen, and Shochet proposed a classical model describing the phase shift of self-driven particles from the perspective of statistical mechanics based on the Boid model. The second stage is the initial stage of theoretical research. In 2003, Jadabaie, Lin, and Morse gave a theoretical explanation for the consensus behaviour of Vicsek model by applying graph theory and matrix theory and analysed the effect of graph connectivity on consensus. In 2004, Olfati-Saber and Murray used the properties of the Laplacian matrix to study the consensus problem of the first-order integrator MASs, and formalized the solvability concept and protocol concept of the consensus problem. The theoretical framework of the consensus problem is proposed, which reveals the relationship between the algebraic connectivity of the graph, the consensus convergence rate and the upper bound of the time-delay tolerance in Olfati-Saber and Murray (2004). In 2005, Ren and Beard analysed the consensus problem of the second-order integrator MASs and pointed out the importance of the communication topology including the directed spanning tree for achieving asymptotic agreement. The introduction of the Laplacian matrix has made a qualitative leap in the study of consensus problems from the simulation phase to the theoretical analysis phase. Since then, graph theory has become an important tool for the theoretical analysis of consensus problems, and the study of consensus issues has entered the third stage. In the third stage, the research focuses on the analysis of consensus models, the design of consensus protocols, convergence, equilibrium, and application prospects. Many scholars have applied different model methods and carried out in-depth research and expansion of consensus theory from different directions. The consensus has developed rapidly and yielded fruitful results, and has been widely applied to a variety of scientific and engineering problems, including synchronization of coupled oscillators, formation control, swarm control, optimal cooperative control, clustering, sensor networks, etc. (Lin, Zhang, & Liu, 2018; Zhang, Hu, Liu, Yu, & Liu, 2019).

This paper will introduce mainly the development and research status from the following aspects: consensus subjected to communication constraints, leader-following consensus, group consensus, consensus based on trigger mechanism, finite-time consensus, multiple consensus and multiple tracking. Detailed analysis is made and insightful understanding is given with respect to recent results on the consensus issue of MASs reported in the literature. The remainder of this paper is organized as follows. The consensus subjected to communication constraints is described in Section 2, such as time delays, uncertain communication, saturation, quantization and perturbation. Section 3–7 focus on reviewing the latest theoretical results and their respective advantages and disadvantages about leader-following consensus, group consensus, consensus based on trigger mechanism, finite-time consensus, and multi-consensus and multi-tracking. Section 8 presents some challenging issues.

2. Consensus subjected to communication constraints

In order to achieve consensus and coordinated control of MASs, an important factor is the ability of agents to exchange information by the networks. The intervention of the network not only fundamentally breaks through the limitations of the traditional ‘point-to-point’ signal control, avoids the laying of dedicated lines between control nodes, reduces system wiring, and has many other advantages, such as low cost, easy expansion, flexible structure, easy to diagnose and maintain system, etc. (Hu, Wang, Chen, & Alsaaadi, 2016; Xia, Gao, Yan, & Fu, 2015). However, the network also causes some problems different from the traditional control system. Since there are a large number of information sources in the network, when each node transmits information through the network, the network communication channel is shared in a sharing manner. However, the network bandwidth is limited, and the data traffic in the network changes irregularly. When multiple nodes exchange data through the network, data collision, multi-path transmission, connection interruption, and network congestion often occur
Therefore, time delays and package dropouts inevitably occur, which will affect the performance of the MASs and even lead to its instability (Savino, Souza, & Pimenta, 2018).

In recent years, the research on the consensus of MASs with time delays has been continuously developed. In Sun and Wang (2009), based on the tree transformation method, the necessary and sufficient conditions for the average consensus are established. For the discrete-time MASs with the agent velocity in the non-convex set, Lin, Ren, and Gao design the distributed constraint protocol to study the consensus problem of bounded delay, by using the model transformation and boundedness analysis method (2017). Inspired by the predictive power of nature creature, a small world prediction protocol for the A/R and Vicsek models is designed in Zhang, Chen, and Stan (2011). And for linear dynamic networks without leaders, a distributed predictive control protocol is proposed, which shows that the prediction protocol can improve the convergence speed of consensus and reduce the sampling frequency. In Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009), the consensus problem of MASs with saturated inputs is considered, and distributed predictive control mechanisms and pinning control are used to achieve consensus and improve performance.

Although the literatures (Ferrari-Trecate et al., 2009; Zhang et al., 2011) introduce predictive control methods into MASs, the effects of time-delay on the consensus of MASs are not considered. The first-order and second-order continuous-time MASs with the same constant time-delay are considered in Fang, Wu, and Wei (2012) and Wu, Fang, and She (2012), the weighted average predictive control is introduced to simultaneously increase the upper bound of the maximum tolerance delays and convergence speed. In Wang, Zuo, Lin, and Ding (2017), a zero-input solution is used as the predicted value of the agent’s state in the time-delay period, and the sufficient conditions of the global consensus are given for Lipschitz nonlinear MASs with input delays, based on the Jordan type of the Laplacian matrix. At present, most of the results accept time delays passively, that is, outdated information is used directly to design a protocol (or algorithm). Obviously, outdated information cannot completely and truly reflect the current dynamics of the system (Hu, Chen, & Du, 2014). It is difficult to implement accurate and effective control of the system by using the protocol based on outdated data. Therefore, considering the transmission capability of information in the network environment, it will be a promising research topic on how to overcome actively the impact of time delays on consensus and performance indicators. Tan et al. have introduced the networked predictive control scheme to compensate for the communication delays actively for the discrete-time MASs in Tan and Liu (2013), Tan, Liu, and Shi (2015), Tan, Liu, and Duan (2012), Tan and Liu (2012), Tan, Yin, Liu, Huang, and Zhao (2018), and Li, Tan, and Liu (2016). The state consensus problem of discrete-time homogeneous MASs with time delays is studied under the condition that the states of the agents are unmeasurable, the outputs of the agents are measurable and not fully measurable in Tan and Liu (2013), Tan et al. (2015), Tan et al. (2012), and Tan and Liu (2012). And the output consensus problem of discrete-time heterogeneous MASs with delays is studied in Tan et al. (2018) and Li et al. (2016).

The theoretical design of protocols or algorithms cannot accurately act on the actual object, which greatly limits the further development of the protocols and its engineering application, because almost all physical systems are limited by the operating range of the actuator or device loss (Hu, Wang, & Gao, 2018). That is, actuator saturation constraints or input saturation constraints (Zhang, Li, & Zhao, 2017). So it is necessary to consider the working range of the actual system in the process of designing the protocol. The MASs subjected to saturation constraints is essentially a nonlinear MASs. A distributed adaptive consensus control scheme is proposed for a class of nonlinear MASs with input saturation in Kahkeshi Maryam and Maedeh (2019), based on the minimum learning parameter algorithm and the dynamic surface control method. The global consensus problem for discrete-time MASs with input saturation constraints, and a fixed undirected topology is considered in Yang, Meng, and Johansson (2014). A dual integrator dynamics model with input saturation constraints is established, and a consensus control algorithm is designed in Zhou and Yan (2014). A model based on a dead zone operator is proposed to provide a smooth model of saturated nonlinearity in Shahriari-kahkeshi and Taj (2019), and a consensus strategy is proposed, based on the minimum learning parameter algorithm and the dynamic surface control method. In fact, MASs are often affected by various complex environments, and local information exchange between multiple agents may be interfered by some uncertainties (Hu, Zhang, Yu, Liu, & Chen, 2019; Jenabzadeh & Safarinejadian, 2019). Overcoming the impact of uncertain communication on consensus is of great significance (Hashemi, Askari, Ghasiari, & Kamali, 2017; Kais, Karim, & Tarak, 2017; Xiao & Mu, 2017). A robust feedback controller is designed to ensure the consensus of uncertain MASs with external disturbances in Ramya, Sakthivel, Ren, Lim, and Leelamani (2019), based on interference suppression and Smith predictor scheme. The distributed consensus problem of MASs with parameter uncertainty
is studied in Yang and Li (2019), and an adaptive updating law with time-varying parameter is designed. Wang et al. propose a metamorphic adaptive low-gain feedback approach to investigate the semi-global robust tracking consensus problem of uncertain MASs with input saturation in Wang, Chen, and Zhang (2019). Xu, Peng, and Guo (2018) investigated the consensus problem for a class of nonlinear MASs with stochastic uncertainties and disturbances; a novel impulsive control protocol is presented to reduce the control cost effectively. The $H_{\infty}$ PID feedback for an arbitrary-order delayed multi-agent system is investigated to improve the system performance, based on the extended Hermite-Biehler theorem in Ou, Chen, Zhang, and Zhang (2014). The consensus problem of a class of MASs with uncertain topology and partially unknown control directions is studied in Chen, Li, Zhang, and Wei (2019). Under the assumptions that the uncertain topology is a fuzzy joint connection and only a small number of followers can access the leader information, some new control protocols are proposed to solve the consensus problem of the first-order and second-order nonlinear MASs.

The exchange of information between agents is usually limited by the capacity of the communication channel. When the information to be transmitted exceeds the communication carrying capacity, the performance of the system may be degraded or even unstable. In order to solve the constraints caused by limited communication bandwidth, the quantization information is often encoded at the transmitting end and correspondingly decoded at the receiving end, which will introduce quantization error and strong nonlinear factors (Hu, Wang, Liu, & Zhang, 2019; Hu, Wang, Shen, & Gao, 2013; Meng, Zhao, & Lin, 2013; Wang, Dong, & Wang, 2017). Under the quantization effect, all first-order nonlinear agents can reach a consensus by using an edge-based adaptive protocol in Li, Ho, and Li (2018). By constructing a novel dynamic quantizer, a distributed protocol via sampled and quantized data is designed to solve the consensus problem of the continuous-time linear MASs in Ma, Ji, and Sun (2018). The distributed preamble fixed-time quantization consensus problem of nonlinear MASs is considered in Zhang, Hu, and Huang (2019), based on impulse control. A neuro-based robust adaptive consensus control scheme for a class of uncertain nonstrict-feedback MASs is proposed in the presence of input quantization and unmodelled dynamics in Qin, He, and Li (2019). A distributed dynamic output feedback protocol is proposed for the MASs with structured uncertainty and external disturbance in Xue, Wu, and Yuan (2019), which utilizes not only relative output information of neighbouring agents but also relative state information of neighbours. The non-fragile consensus control problem is studied for a class of nonlinear MASs with uniform quantization and randomly occurring deception attacks in Wu, Hu, and Chen (2019).

3. Leader-following consensus

In recent years, the leader-following problem of the MASs has also received extensive attention (Tan, Liu, & Duan, 2010). According to the different properties of the leader, a leader-following consensus problem can be categorized as a real leader case and a virtual leader case. A leader-following consensus protocol is adopted to solve the consensus problem of heterogeneous multi-agent systems with time-varying communication and input delays in Dai, Lin, and Liu (2014). The distributed tracking control problem for first-order agents with multiple dynamic leaders and directed Markovian switching topologies has been investigated in Li, Xie, and Zhang (2015). The leader-following consensus problem for second-order MASs is studied in Zhang and Duan (2018) and Zhu and Cheng (2010). And Su presents a novel distributed internal model approach to further study the leader-following rendezvous problem for double-integrator MASs subject to both external disturbances and uncertainties in Su (2015). And both the distributed full and partial state feedback control without velocity measurement have been investigated. Ding, Han, and Guo (2013) investigate network-based leader-following consensus for a distributed MASs. Liu and Huang (2018) further study the leader-following attitude consensus problem of multiple rigid body systems subject to a jointly connected switching communication network. Combining a feed-forward control method with an adaptive control approach, a new adaptive distributed controller is proposed for multiple uncertain Euler–Lagrange systems, which can adapt to arbitrary bounded non-uniform time-varying communication delay and directed switching communication network in Lu and Liu (2018). A control scheme based on distributed robust adaptive neural network is designed to ensure that the uniform output tracking errors between followers and leaders are semi-globally uniformly and ultimately bounded in Shen and Shi (2015), avoiding the classical ‘explosion of complexity’ problem in a standard back-stepping design. For multiple rigid spacecraft systems, whose attitude is represented by the unit quaternion, a nonlinear distributed observer is established to achieve the leader-following consensus in Cai and Huang (2014). Lu, Chen, and Chen (2016) present two non-smooth leader-following formation protocols for non-identical Lipschitz nonlinear MASs. By introducing local estimators for the bounds of reference trajectory and a filter, a new backstepping based smooth distributed adaptive
control protocol is proposed to achieve leader-following consensus control for high-order nonlinear MASs in Huang, Song, Wang, Wen, and Li (2017). Contrary to the previous studies on leader-following consensus, the Caputo fractional MASs cover bounded and unbounded time-dependent Lipschitz coefficients in Almeida, Girejko, Hristova, and Malinowska (2019). A constrained control protocol is designed for the nonlinear MASs with input constraint in Deng, Sun, and Liu (2019). The exponential leader-following consensus problem is investigated for a class of nonlinear stochastic MASs with partial mixed impulses in Tang, Gao, Zhang, and Kurths (2015). The global leader-following consensus problem for the MASs with bounded controls has been studied in Zhao and Lin (2016). Under a fixed directed graph, the leader-following output consensus problem is investigated for a class of nonlinear MASs in Hua, Li, and Guan (2019). For high-order stochastic nonlinear MASs, the dynamic gain in the controller is used to compensate the time-varying coefficients of the nonlinear function in You, Hua, Yu, and Guan (2019). A distributed adaptive state feedback control law is introduced to make leader-following consensus for a class of uncertain nonlinear MASs under jointly connected directed switching networks in Liu and Huang (2017).

4. Group consensus

In many practical situations, a group of agents must be able to sense and respond to unexpected situations or any changes when a cooperative task is implemented. Besides, different agreements of agents may be caused by different task distributions in cooperative control. Therefore, it is an important issue that appropriate protocols are designed to make agents reach different consensus values. This problem is called group consensus problem, which is more suitable for dealing with collaborative control problems (Xia, Huang, & Shao, 2010; Yu & Wang, 2009a, 2009b). As one of the hot topics in the distributed control of MASs, the group consensus problem of MASs has broad applications in multi-robot manipulators, satellite clusters, vehicle formations and so on (Li, Duan, & Tan, 2011; Tan, Liu, & Duan, 2011).

Recently, great deals of excellent research results on group consensus have emerged constantly. Miao and Ma (2015) investigate group consensus for the first-order discrete-time or continuous-time MASs with nonlinear input constraints. Kim, Park, and Choi (2014) investigate the group average-consensus and group formation-consensus problems for first-order MASs by using average matrices. Liu and Zhou (2014) investigate the impulsive group consensus problems of second-order MASs under directed network topology with acyclic partition, and then some criteria on convergence for such algorithms are established. Gao, Hu, Shen, and Jiang (2019) investigate the group consensus for leaderless MASs. When cyber-attacks are recoverable, the sufficient conditions of the group consensus for the MASs subjected to cyber-attacks are given. The leader-following group consensus problem of second-order MASs is discussed in Ma, Wang, and Miao (2014) and Shi, Cui, and Xie (2017). Ning and Lin introduce an approach of clustering, based on the group consensus of dynamic linear high-order MASs in (2015). Zhao and Park (2014) investigate the group consensus problem by model transformation for discrete-time MASs with a fixed topology and stochastic switching topologies. The cluster consensus of heterogeneous MASs is studied in Chen, Wang, Zhang, and Lewis (2018), by using the linear small gain theory, the output regulation theory and small gain theory. Zheng and Wang (2015) consider the group consensus problem of heterogeneous MASs, in which a novel protocol is proposed, the state transformation method is used and an equivalent system is obtained. Some corresponding sufficient conditions are obtained to achieve group consensus of heterogeneous MASs with fixed and switching topologies in Wen, Huang, Wang, Chen, and Peng (2015). Hou, Xiang, and Ding (2019) consider the group consensus problem for nonlinear MASs, which shows that the consensus can be achieved in both discrete time and continuous time. A reverse group consensus problem for the dynamic agents in the cooperation-competition network is investigated in Hu, Yu, Wen, Xuan, and Cao (2016), which can be divided into two sub-networks. It is found that the reverse group consensus problem can be achieved if the mirror graph is strongly connected. A distributed cooperative control of MASs is proposed for distributed generators clusters in multi-microgrids in Shen, Xu, and Yao (2018). The proposed control method presets the pinned consensus values for multiple MGs considering global cooperation and realizes a pinning based group consensus for distributed generators.

In addition, some interesting and excellent achievements also have been achieved to deal with the group consensus problem for MASs with time delays in recent years. Ma et al. mainly investigate the second-order group consensus for MASs with time-varying delays based on using the second-order neighbours’ information in (2014). Li (2019) studies the reverse group consensus problem for second-order MASs with delayed nonlinear dynamics and intermittent communication in the cooperation-competition networks. The group consensus problem of MASs with time-delay is studied in Du, Wang, and Zhao (2015). Weighted group consensus problem of MASs is investigated in Du et al. (2015). A state-based predictive approach for group consensus
controllers for MAS with time-varying delays is proposed in An, Liu, and Tan (2018), and the criteria for group consensus are presented. He and Wang (2016) studied the weighted group consensus of MASs with bipartite topologies through adjusting the proportion of the current states and the delay states in the control algorithms, which is able to enlarge the upper bound on the maximum time-delay of weighted group consensus. The group consensus problem of nonlinear MASs with delayed Lurie-type dynamics is investigated in Guo et al. (2015), and a pinning control scheme is designed under an undirected communication graph. Wen, Yu, Peng, and Wang (2016) investigate the dynamics group consensus problem of heterogeneous MASs with time delays, in which agents’ dynamics are modelled by single integrators and double integrators.

5. Consensus based on trigger mechanism

In the theoretical study of MASs, it is usually assumed that there is abundant energy, excellent computing power, and real-time communication. However, in practical applications, the computing power and communication capabilities of a single agent depend on its embedded digital microprocessor, and the energy comes from the embedded battery. The resources of the MASs include the computing power, communication capability, and energy reserve of the agent. Excessive calculations and communication will cause the agent to be busy, unable to respond to other work, or even not working properly, which will affect the normal operation of the entire system. Moreover, the energy of the agent is limited, and excessive calculation and communication consume a lot of energy. Studies have shown that wireless communication will consume up most of the energy of the sensor (Nada, Bousbia-Salah, & Bettayeb, 2018). Exhaustion of energy will cause agents to fail to work, affect the performance of the MASs, and even cause the system to crash. In order to take advantage of the distribution and robustness of MASs, it is especially important to reduce communication and computing as much as possible. Therefore, when designing the control strategy, it is necessary to fully consider the utilization of the system’s own energy and network resources, which makes the MASs cooperative control design more challenging. How to reduce the utilization of MASs resources? The most direct method is to reduce the amount of information exchange between agents by designing a transmission strategy. It is well known that the use of digital signal control methods can save more information exchange and computing resources than continuous signal control (Zou, Wang, & Zhou, 2017). Digital signal triggering methods usually include time triggering and event triggering (Hu, Wang, Alsadi, & Hayat, 2017). The former mainly refers to the traditional sampling control, that is, the measurement and control updates of the system are periodic, and the control information remains unchanged during the period by the zero-order holder. The latter determines whether data is sent by judging a given event condition. The advantage of periodic sampling is easier to implement in the analysis and design. In fact, ‘after a certain period of time’ can also be regarded as an event, then time triggering can be a special case of event triggering. Through the design of the event and trigger response, the event-triggered mechanism can be better than the time-triggered mechanism to save the resource (Hu, Liu, Zhang, & Liu, 2020).

The time-triggered strategy usually refers to the method of sample control, that is, information measurement and control task execution are performed periodically. The consensus problem under the sampling control framework is called sampling consensus. Due to the infinite sampling period, information transfer between agents is not possible. Therefore, how to select the sampling period to ensure the consensus is the main research content of sampling consensus. At present, there is a large amount of literature on the sampling consensus of MASs. Initially, the sampling consensus study was mainly for the first-order integrator MASs (Xie, Liu, Wang, & Jia, 2009a, 2009b). Two sampling consensus algorithms are proposed for the second-order integrator MASs with directed topology in Cao and Ren (2010). A distributed consensus protocol is designed based on the current and past location sampling information, and the necessary and sufficient conditions are given to ensure the consensus of the second-order MASs in Yu, Zheng, Chen, Ren, and Cao (2011). When the current location information is not available, only the sampling information of the position and velocity is used to design the protocol and necessary and sufficient conditions for the sampling consensus are obtained in Yu, Zhou, Yu, Lü, and Lu (2013). A novel consensus protocol is proposed to achieve the state consensus for any large sampling interval in Xiao and Chen (2012). And the sampling interval is required to have a lower bound when the sampling interval is aperiodic. For the second-order MASs with nonlinear dynamic and directed topology, the algorithm for determining the maximum allowable sampling interval is given in Wen, Duan, Yu, and Chen (2013). The above literatures assume that all agents synchronize data updates simultaneously, and clock synchronization techniques are required. However, it is sometimes difficult to guarantee sampling synchronism due to communication technology, external disturbances and so on. Therefore, it is necessary to design an asynchronous sampling consensus algorithm, that is, each agent can update data
6. Finite-time consensus

Convergence speed is an important performance index for the consensus. The consensus algorithm depends on the convergence rate. Many researchers choose suitable communication topology to obtain a higher speed by the optimal vertex configuration. At present, most of the consensus algorithms are asymptotical consensus algorithms, that is to say, the optimal index value of convergence rate is in an infinite time, and the states of all agents cannot be consistent within a limited time. However, many practical control systems require more stringent convergence time and fast dynamic response, and dynamically move to the equilibrium point of the system or achieve zero tracking error within a finite time (Niamsup & Phat, 2018). For example, a brake control system requires that the vehicle’s speed reaches zero or the vehicle reaches a specified position within a limited time. Because the finite-time consensus algorithm has the advantages of fast convergence, strong anti-interference and excellent robustness to uncertain factors, finite-time consensus has a stronger engineering application background. However, the finite-time control problem is difficult for theoretical analysis, which is non-smooth in the sense of time invariance. Due to the lack of effective analysis tools, the design and analysis of finite-time consensus algorithms are much more difficult than asymptotical consensus. Therefore, it is an important engineering research topic to propose effective design and analysis methods for the finite-time consensus problem.

At present, the main methods for studying finite-time consensus can be divided into two types. One is the homogeneous theory. The homogeneous theory includes three steps: the first step is to prove that the system is globally asymptotically stable by constructing the Lyapunov functional and combining with the Barbalat lemma or Lasalle invariant principle under the given consensus protocol; the second step is to prove that the system is locally finite-time stable by using the homogeneous theory; the third step is to infer that the system can achieve global stability in finite time by combining with the first two steps. The other is constructing Lyapunov functional, by which the finite-time consensus can be proved, and the upper bound of the time is also obtained (Hu, Zhang, Kao, Liu, & Chen, 2019; Li, Liu, Sun, & Tan, 2019).

The finite-time consensus protocol can be divided into two categories: discontinuous protocol and continuous protocol (Zheng & Tie, 2014). The discontinuous protocol mainly includes a switching protocol and an ultimate sliding mode protocol. For the MASs with a dynamical leader, Meng et al. have designed a distributed observer based
on a super-twisting algorithm to solve the finite-time consensus tracking problem of the system in Meng and Lin (2014). Considering the dynamic leader with unknown acceleration and the follower with bounded disturbance, a nonlinear consensus protocol based on non-singular terminal sliding mode algorithm is designed to drive that states of the followers converge to the corresponding state of the leader for a limited time in He, Wang, and Yu (2015). For the MASs consisting of Euler–Lagrangian dynamics, Ghasemi and Nersesov (2014) design a non-smooth sliding surface-based protocol. When the system trajectory slides on the sliding surface, the states of the system reach the specified position for collaboration control.

Due to the existence of discontinuous control items, there is chattering in the system, which is undesirable in practical systems. Therefore, researchers begin to design continuous controllers to avoid chattering. The continuous protocol mainly includes single-fraction power protocol, homogeneous finite-time protocol, high-order sliding mode protocol, and augmented integral protocol. Under the condition of switching topology with tree structure, Xiao, Wang, and Chen (2014) propose a novel nonlinear continuous protocol for MASs with unknown internal dynamics. In the case of unmeasurable speed and input saturation, Zhang, Jia, and Matsuno (2014) design first-order and high-order finite-time observers by using the homogeneous theory to analyse the stability of the closed-loop system, so that the states of all followers can converge to the leaders’ states in finite time. Wang, Li, and Shi (2014) design a distributed finite-time protocol by using the power-integration technique to ensure that all states of the followers can converge into a convex set composed of leader states, and the algorithm is also applicable to the case of multiple static leaders. Zuo and Tie (2014) propose a class of global continuous time-invariant protocol for first-order integrator MASs such that the convergence time can be designed and estimated off-line. Some consensus algorithms depend on output information or the complete state of neighbours, Chen, Lewis, and Xie (2011) propose an algorithm only requiring the relative state measurements, by using the binary consensus protocol and the pinning control scheme. A continuous nonlinear distributed protocol is proposed to achieve a finite-time consensus of heterogeneous systems in Li, Ren, and Liu (2016). A virtual velocity is introduced to the protocol of second-order MASs, which can be tracked by the real velocity in finite time in Feng and Zheng (2018). The finite-time consensus of heterogeneous second-order MASs with measurable and unmeasurable velocity is studied in Wang and Xiao (2012). Zhang and Yang (2013) consider the finite-time consensus problems of second-order MASs with one and multiple leaders under a digraph. A new nonsingular finite-time terminal sliding-mode control method is proposed for second-order MASs with disturbances in Zhou, Zhou, Chen, and Chen (2018). Based on the pinning error function vector, robust distributed finite-time consensus is given. Wang and Song (2018) present a distributed and smooth finite-time control scheme to achieve leader-following consensus under the topology containing a directed spanning tree. By using the recursive method, the finite-time consensus of high-order uncertain nonlinear MASs is guaranteed by non-Lipschitz continuous control laws in Hua, You, and Guan (2016). The global finite-time consensus tracking problem for uncertain second-order MASs with input saturation is studied in Yang, Eng, Dimarogonas, and Johansson (2013). A distributed controller based on a sliding mode observer is proposed to realize global finite-time consensus tracking with a limited control input. Zheng, Chen, and Wang (2011) give the nonlinear consensus protocol for MASs with Gaussian white noises and define the concept of probability finite-time consensus. The problem of leader-follower finite-time consensus for a class of time-varying nonlinear MASs is studied in Liu and Liang (2016).

7. Multi-consensus and multi-tracking

Due to the interaction between agents and environments, MASs generate complex clustering behaviour. Researchers have a strong interest in the clustering behaviour of MASs, and have obtained some meaningful research results, and applied them to the fields of traffic control (Abdoos & Mozayani, 2013; Balaji & Srinivasan, 2010), flexible manufacturing (Nejad & Sugimura, 2010), intelligent robots (Lopez-Ortega & Villamarin, 2009) and collaborative expert systems (Chiddarwar & Babu, 2011). In a multi-agent network, its nodes are autonomous individuals with certain intelligence. When multiple agents collaborate to complete a complex task, the evolution of the MASs presents multiple coordinated states in some stages due to different task assignments or changes in the environment, in which the agent is located. Multi-consensus means that MASs present multi-consensus states under appropriate distributed control protocols. These states may be related to the grouping of MASs or may not require grouping of MASs. Multi-tracking system collaboratively tracks multiple desired orbits (or virtual leaders) under appropriate distributed control protocols. Cluster behaviours of MASs, such as multi-consensus, multi-tracking, multi-swarming and other clustering behaviours, have important theoretical significance for exploring the theory of complexity (Xu, Zhao, Yang, Gui, & He, 2017).
In recent years, many outstanding research results in multi-consensus and multi-tracking have emerged. Compared with the group consensus problem, the multi-consensus problem includes more rich content and is closer to the actual engineering problem; in other words, the group consensus problem is a special case of multi-consensus problems. The multi-consensus problem of the MASs with nonlinear protocols is investigated in Li and Guan (2013). Cui and Xie (2016) introduce a protocol under the assumption that all subgroups satisfy the intra-balance condition, and focus on the group consensus tracking problem of continuous-time second-order MASs. Xie and Shi (2017) introduce a control protocol that divides the whole system into subgroups with multiple leaders and study discrete-time second-order multi-agent groups tracking with Markov switching topology. Based on the internal model principle, Zhang et al. transform the global robust group output adjustment problem into the global robust dynamic stability problem of the MASs. The global robust group output adjustment problem for MASs with uncertain second-order nonlinear dynamics is studied in Zhang and Liu (2018).

In the wireless networks, a multi-tracking protocol is proposed, in which the source node tracks the progress and ‘cooperation’ of the neighbours to improve their end-to-end delay and overall network performance. For small self-organizing wireless networks with node failures, Yanmaz and Karrels study the performance of multi-tracking routing protocols in (2008). Han et al. study the multi-tracking problem of the first-order multi-agent network through self-trigger control in Han, Guan, and Chen (2015). The states of multiple agents in each second-order sub-network asymptotically converge to the same desired trajectory in Han and Zhang (2014). Han and He (2016) introduce the concept of intelligence to characterize the level of proxy intelligence, and propose a distributed switching pulse protocol by using sampling position data and sampling speed data alternately. A control protocol is designed to achieve multi-tracking of bounded variables, where the final tracking error is proportional to the sampling period in Han, Guan, and Li (2016). Chen, Guan, and He (2015) study the multi-tracking problem of second-order MASs based on sampling position information. Based on the fast terminal sliding mode control method, Han and Guan (2017) propose a distributed finite-time formation tracking protocol and study the finite time formation tracking control problem of MASs. Wei and Yi (2016) transform the multi-consensus problem into the stability problem of the extended error system. A new nail-like consensus protocol with non-periodic intermittent effects is designed in Huang and Jiang (2018). Franchi, Giordano, and Michieletto study the choice of online leaders in (2019).

For the multi-consensus and multi-tracking problem of high-order MASs, researchers have conducted in-depth research (Monaco & Celsi, 2019; Qin, Ma, & Yu, 2018). Peng, Wang, and Zhang (2014) propose a new iterative learning method for collaborative tracking and estimation of linear MASs with dynamic leaders. Amini and Azarbahram study a new method for achieving consensus for a nonlinear MASs, by using a fixed-order non-fragile dynamic output feedback controller in (2016). Hu and Guan (2016) adopt a node clustering scheme to ensure a relatively high degree of connectivity within each potential subgroup and use some exclusion effects to deal with subgroup outbound links. Zhang, Liu, and Wang (2016) convert the multi-tracking control problem of MASs into the zero-stationary error control problem of some independent subsystems and study the multi-tracking control of high-order heterogeneous MASs. Yan and Yu (2017) study event-triggered tracking control of a coupled-group MASs. Under the Lipschitz condition, Pei and Chen (2018) studied the consensus tracking problem of heterogeneous MASs with fixed topology. Zhang and Han (2018) propose a distributed pulse protocol to study the robust multi-tracking problem for heterogeneous MASs with uncertain nonlinear and disturbance.

8. Conclusions and prospects

Over the past few years, the consensus of multi-agent systems has attracted much attention from various scientific communities. Up until now, many algorithms have been well designed to guarantee that agents converge to the common value. However, the existing results and methodologies have some limitations from strict assumptions and special requirements. And taking into account the impact of time-varying, perturbation, various uncertainties, nonlinearities, diversity of agent’s structure and other more complex factors, there still exist a number of challenging research topics in further investigations.

Inter-group communication between different groups has an important impact on group consensus. The intra-balance communication constraints critically limit the application scope for group consensus. So it is worthwhile to study how to properly design and optimize communication between different groups of agents. In addition, new control methods should be explored to achieve group consensus, such as impulsive control, pinning control, adaptive control, etc.

There are many studies on the problem of finite-time consensus for continuous-time MASs in existing literatures, but few researches are discussed in the case of discrete-time MASs. To the best of our knowledge, the
problem of finite-time consensus for discrete-time heterogeneous nonlinear MASs has not been adequately investigated.

Although some event-triggered consensus issues of MASs have been well addressed in the literature, it is a difficult point for how to design the dynamical event-triggered mechanism due to coupled information among the agents. The event-triggered mechanism can reduce the sampling actions and/or control updates, yet decrease the convergence rate at the same time. Hence, it would be a promising topic to design a suitable event-triggered mechanism to achieve consensus in a finite time. That is, the convergence rate is fast, and the utilization of communication and computation resources are low.

The problem of consensus has been investigated for linear MASs with time delays, uncertain communication, saturation, quantization and perturbation. However, it will need in-depth research for nonlinear MASs with communication constraints, uncertainties and perturbations.

The various consensus issues of multi-agent systems have yielded fruitful results in theory, which contain consensus subjected to communication constraints, leader-following consensus, group consensus, consensus based on trigger mechanism, finite-time consensus, multi-consensus and multi-tracking and so on. However, there are still gaps in how to apply mature theoretical results to actual engineering systems. Therefore, the engineering application of the consensus algorithms has a long way to go.

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