PATTERN OF LIFETIMES OF BEAUTY HADRONS AND QUARK-HADRON DUALITY IN HEAVY QUARK EXPANSION

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We discuss (i) the evaluation of the expectation values of four-quark operators assuming that the heavy quark expansion for \(b\) sector converges at the third order in \(1/m_Q\), and (ii) the estimation of the duality breaking short distance nonperturbative corrections to the parton decay rate. We finally point out the implications of the result obtained for the assumption of quark-hadron duality in heavy quark expansion.

1 Introduction

In the heavy quark limit, the inclusive decay rate of weakly decaying heavy hadron \((H_Q)\) can be expressed in an expansion of the weak matrix elements in powers of \(1/m_Q\), where \(m_Q\) is the heavy quark mass (In this article, though we discuss generally for a heavy system, \(b\) or \(c\) but quantitatively refer only to beauty case). At the leading order, the hadronic decay rate is given by the free heavy quark decay rate, universal for all hadrons of given flavour quantum number, \(Q\):

\[ \Gamma_0 = \frac{G_F^2 |V_{KM}|^2 m_Q^5}{192\pi^3} \quad (1) \]

where \(|V_{KM}|\) is the relevant CKM matrix element. The leading order receives corrections due to the motion of the heavy quark inside the hadron \((\lambda_1)\) and the heavy quark spin projection \((\lambda_2)\), which appear at \(1/m_Q^2\) in the expansion:

\[ \lambda_1 = \mu_1^2(H) = \frac{1}{2M_H} \langle H | \bar{Q} (iD)^2 Q | H \rangle \quad (2) \]

\[ \lambda_2 = \mu_2^2(H) = \frac{1}{2M_H} \langle H | \bar{Q} g\sigma G Q | H \rangle \quad (3) \]

Numerically \(\lambda_1(B) = -0.4\ GeV^2\), \(\lambda_1(\Lambda_b) = -0.3\ GeV^2\) and \(\lambda_2(B) = 0.12\ GeV^2\). The term \(\lambda_2\) vanishes for all baryon except \(\Omega_Q\). Thus at \(O(1/m_Q^4)\), the

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total decay rate is the sum of the above terms:

$$\Gamma(H_b) = \Gamma_0 \left\{ 1 - \frac{\lambda_1 - 3\lambda_2}{2m_b^2} + \frac{2\lambda_2}{m_b^2} + \ldots \right\} \quad (4)$$

Thus, at $O(1/m_b^2)$, the decay rate splits up into mesonic and baryonic ones. The decay rate is the same for all $b$ mesons and for all $b$ baryons (except $\Omega_Q$). At this order, the theoretical predictions for the ratio of lifetimes of $B$ mesons agree with the corresponding experimental values, but not the ratio of lifetimes of $\Lambda_b$ baryon and $B$ meson.

| Theory, at $O(1/m_b^2)$ | Experiment |
|-------------------------|------------|
| $\tau(B^-)/\tau(B^0)$ | 1          | 1.04 ± 0.04 |
| $\tau(B_s^+)/\tau(B^0)$ | 1 ± 0.01   | 0.99 ± 0.05 |
| $\tau(\Lambda_b)/\tau(B^0)$ | 0.9        | 0.79 ± 0.06 |

The discrepancy in the case of $\tau(\Lambda_b)/\tau(B^0)$ signifies something interesting beyond the second order. The terms at $O(1/m_Q^3)$ are really interesting because evaluation of the matrix elements elude a concrete understanding of evaluation. Also the discrepancy gives rise to doubt the validity of the underlying assumption of the heavy quark expansion, *quark-hadron duality*. The third order in $1/m_Q$ terms are involving both heavy and light quark fields.

$$C(\mu)\frac{1}{2M_H} \langle H|\langle \bar{Q}\Gamma\mu q)(q\Gamma^\mu Q)H \rangle \quad (5)$$

where the Wilson coefficients, $C(\mu)$, describe the light quark processes such as Pauli interference, Weak annihilation and $W$ scattering and the term $\langle \ldots \rangle$ is, in traditional terms, understood as probability of finding both the heavy and light quarks at zero separation, in other words the wavefunction density at origin, $|\Psi(0)|^2$. The evaluation of the operators depends upon their parameterisation.

In this talk, I discuss the evaluation of the expectation values of four-quark operators from the difference in total decay rates (Sec. 2). It is based on the assumption that at $O(1/m_Q^3)$, the heavy quark expansion converges. The short distance nonperturbative corrections are given in Sec. 3. This signifies that the quark-hadron duality holds good within a few percent of uncertainty due to *actual violation* (Sec. 4).

### 2 Splitting of Total Decay Rates

The estimation of expectation values of four-quark operators (EVFQO) is as follows. Unlike in the charmed sector, the HQE works well for beauty case
because of \(m_b\) being asymptotically heavy. The spectator effects coming from the term at \(O(1/m^3)\) (hereinafter \(m\) refers to the mass of \(b\) quark), is smaller but yet significant. If at all there exists any contribution due to terms beyond \(O(1/m^3)\), then it should be too small to be ignored. Hence, being an asymptotic expansion, the HQE can be assumed to be convergent at \(O(1/m^3)\). This assumption does not hold for charm, since

\[
\frac{16\pi^2}{m^3_c} C(\mu) \langle O_6 \rangle_{H_c} \gg \frac{16\pi^2}{m^3_b} C(\mu) \langle O_6 \rangle_{H_b}
\]

(6)

where \(C(\mu)\) stands for some structure involving \(c_\pm\) and \(\langle O_6 \rangle_H\) the dimension six FQO of hadron. In this background, we make use of the decay rates to obtain the EVFQO.

The \(B\) mesons, \(B^-, B^0\) and \(B^0_s\), are triplet under \(SU(3)_f\) flavour symmetry. Their total decay rate splits up due to its light quark flavour dependence at the third order in the HQE. The differences in the decay rates of the triplet, \(\Gamma(B^0) - \Gamma(B^-), \Gamma(B^0_s) - \Gamma(B^-)\) and \(\Gamma(B^0) - \Gamma(B^0_s)\), are related to the third order terms in \(1/m^3\) by

\[
d\Gamma_{B^0 - B^-} = -\Gamma'_0(1 - x)^2 \\
\times \left\{ Z_1 \frac{1}{3}(c_0 + 6) + (c_0 + 2) \right\} \langle O_6 \rangle_{B^0 - B^-}
\]

(7)

\[
d\Gamma_{B^0_s - B^-} = -\Gamma'_0(1 - x)^2 \\
\times \left\{ Z_2 \frac{1}{3}(c_0 + 6) + (c_0 + 2) \right\} \langle O_6 \rangle_{B^0_s - B^-}
\]

(8)

\[
d\Gamma_{B^0_s - B^0} = -\Gamma'_0(1 - x)^2 \\
\times \left\{ (Z_1 - Z_2) \frac{1}{3}(c_0 + 6) \right\} \langle O_6 \rangle_{B^0_s - B^0}
\]

(9)

where \(d\Gamma_{B^0 - B^-} = d\Gamma(B^0) - \Gamma(B^-), \Gamma'_0 = 2G^2_f|V_{cb}|^2m^2_b/3\pi, c_0 = 2c_+ - c_-\), \(x = m^2_c/m^2_b\) and

\[
Z_1 = \left( \cos^2\theta_c(1 + \frac{x}{2}) + \sin^2\theta_c \sqrt{1 - 4x(1 - x)} \right)
\]

(10)

\[
Z_2 = \left( \sin^2\theta_c(1 + \frac{x}{2}) + \cos^2\theta_c \sqrt{1 - 4x(1 - x)} \right)
\]

(11)

\[
\langle O_6 \rangle \equiv \left\langle \frac{1}{2} \langle \bar{b}\Gamma_\mu b \rangle \left[ \langle \bar{d}\Gamma_\mu d \rangle - \langle \bar{u}\Gamma_\mu u \rangle \right] \right\rangle = \left\langle \frac{1}{2} \langle \bar{b}\Gamma_\mu b \rangle \left[ \langle \bar{s}\Gamma_\mu s \rangle - \langle \bar{q}\Gamma_\mu q \rangle \right] \right\rangle
\]

(12)

with \(q = u, d\). In eqns. (10-12), the \(rhs\) contains the terms corresponding to the unsuppressed and suppressed nonleptonic decay rates and twice the semileptonic decay rates at the third order.
On the other hand, for the triplet baryons, $\Lambda_b$, $\Xi^-$ and $\Xi^0$, with $\tau(\Lambda_b) < \tau(\Xi^0) \approx \tau(\Xi^-)$, we have the relation between the difference in the total decay rates and the terms of $O(1/m^3)$ in the HQE, as

$$d\Gamma_{\Lambda_b-\Xi^0} = \frac{3}{8} \Gamma'(c_{00} - 2) \langle O_6 \rangle_{\Lambda_b-\Xi^0}$$

(13)

where $c_{00} = -c_+ (2c_- + c_+)$. 

2.1 $\tau(\Lambda_b)/\tau(B)$

For the decay rates $\Gamma(B^-) = 0.617 ps^{-1}$, $\Gamma(B^0) = 0.637 ps^{-1}$ and $\Gamma(B_s^0) = 0.645 ps^{-1}$, the EV$_{FQO}$ are obtained for $B$ meson, as an average from eqs. (7-9):

$$\langle O_6 \rangle_B = 8.08 \times 10^{-3} GeV^3.$$

(14)

This is smaller than the one obtained in terms of the leptonic decay constant, $f_B$.

We obtain the EV$_{FQO}$ for the baryon

$$\langle O_6 \rangle_{\Lambda_b-\Xi^0} = 3.072 \times 10^{-2} GeV^3$$

(15)

where we have used the decay rates corresponding to the lifetimes 1.24 ps and 1.39 ps of $\Lambda_b$ and $\Xi^0$ respectively. The EV$_{FQO}$ for baryon is about 3.8 times larger than that of B. For these values

$$\tau(\Lambda_b)/\tau(B) = 0.78$$

(16)

Using the experimental value of $\tau(B^-) = 1.55$ ps alongwith the above theoretical value, the lifetime of $\Lambda_b$ turns out to be

$$\tau(\Lambda_b) = \frac{\Gamma(\Lambda_b)}{\Gamma(B^-) \tau(B^-)} = 1.20 \text{ ps}.$$  

(17)

The predictions here are significant qualitatively. This value may change by few percent due to uncertainties. It is due to structure of currents involved.

3 Renormalon Corrections

3.1 Power Corrections

In this section, we present a study on the renormalons corrections considering the heavy-light correlator in the QCD sum rules approach, assuming that the nonperturbative short distance corrections given by the gluon mass that
is much larger than the QCD scale. We carry out the analysis for both heavy meson and heavy baryon. Our study shows that the short distance nonperturbative corrections to the baryon and the meson differ by a small amount which is significant for the smaller lifetime of the $\Lambda_b$.

Let us consider the correlator of hadronic currents $J$:

$$\Pi(Q^2) = i \int d^4 xe^{iqx} \langle 0|T\{J(x)J(0)\}|0\rangle$$

where $Q^2 = -q^2$. The standard OPE is expressed as

$$\Pi(Q^2) \approx [\text{parton model}](1 + a_1 \alpha_s + a_2 \alpha_s^2 + \ldots) + O(1/Q^4)$$

where the power suppressed terms are quark and gluon operators. The perturbative series in the above equation can be rewritten as

$$D(\alpha_s) = 1 + a_0 \alpha_s + \sum_{n=1}^{\infty} a_n \alpha_s^n$$

where the term in the sum is considered to be the nonperturbative short distance quantity. It is studied by Chetyrkin et al\(^7\) assuming that the short distance tachyonic gluon mass, $\lambda^2$, imitates the nonperturbative physics of the QCD. This, for the gluon propagator, means:

$$D_{\mu\nu}(k^2) = \frac{\delta_{\mu\nu}}{k^2} \rightarrow \delta_{\mu\mu} \left( \frac{1}{k^2 + \Lambda_{QCD}^2} \right)$$

The nonperturbative short distance corrections are argued to be the $1/Q^2$ correction in the OPE.

Let us consider the assumption of the gluon mass $\lambda^2 \gg \Lambda_{QCD}^2$ which is not necessarily to be tachyonic one. The feature of the assumption can be seen with the heavy quark potential

$$V(r) = -\frac{4\alpha(r)}{3r} + kr$$

where $k \approx 0.2 \text{ GeV}^2$, representing the string tension. It has been argued in\(^8\) that the linear term can be replaced by a term of order $r^2$. It is equivalent to replace $k$ by a term describing the ultraviolet region. For the potential in (22),

$$k \rightarrow constant \times \alpha_s \lambda^2$$

5
In replacing the coefficient of the term of $O(r)$ by $\lambda^2$, we make it consistent by
the renormalisation factor. Thus the coefficient $\sigma(\lambda^2)$ is given by:

$$\sigma(\lambda^2) = \sigma(k^2) \left( \frac{\alpha(\lambda^2)}{\alpha(k^2)} \right)^{18/11}$$  \hspace{1cm} (24)

Introduction of $\lambda^2$ brings in a small correction to the Coulombic term. By use of (24),
we specify the effect at both the ultraviolet region and the region characterised by the QCD scale.
Then, we rewrite (20) as

$$D(\alpha) = 1 + a_0 \alpha_s \left( 1 + \frac{k^2}{\tau^2} \right)$$  \hspace{1cm} (25)

where $\tau$ is some scale relevant to the problem and $k^2$ should be read from (24). We would apply this to the heavy light correlator in heavy quark effective
theory.

We should note that in the QCD sum rules approach, the scale involved in is given by the Borel variable which is about 0.5 GeV. But in the heavy quark expansion the relevant scale is the heavy quark mass, greater than the hadronic scale. Thus, there it turns out to be infrared renormalons effects. But, still it represents the short distance nonperturbative property, by virtue of the gluon mass being as high as the hadronic scale.

3.2 Meson

For the heavy light current, $J(x) = \bar{Q}(x)i\gamma_5 q(x)$, the QCD sum rules is already known:

$$\tilde{f}_B^2 e^{-(\Lambda)} = \frac{3}{\pi^2} \int_0^{\omega_c} d\omega \omega^2 e^{-\omega/\tau} D(\alpha_s) - \langle \bar{q}q \rangle + \frac{1}{16\pi^2} \langle g\bar{q}\sigma Gq \rangle + ...$$ \hspace{1cm} (26)

where $\omega_c$ is the duality interval, $\tau$ the Borel variable and $D(\alpha_s)$ as defined in
(21), but of the form defined in (24). It is, corresponding to the particular problem of heavy quarks, given as:

$$D(\alpha_s)_B = 1 + a_B \alpha_s \left[ 1 + \frac{\lambda^2}{\tau^2} \left( \frac{\alpha(\lambda^2)}{\alpha(\tau^2)} \right)^{-18/11} \right]$$ \hspace{1cm} (27)

where $a_B = 17/3 + 4\pi^2/9 - 4\log(\omega/\mu)$, with $\mu$ is chosen to be 1.3 GeV.

With the duality interval of about 1.2-1.4 GeV which is little smaller than the onset of QCD which corresponds to 2 GeV and $\Lambda \geq 0.6$ GeV, we get

$$\lambda^2 \approx 0.35 GeV^2.$$

(28)
3.3 Baryon

For the heavy baryon current

\[ j(x) = \epsilon^{abc}(\bar{q}_1(x)C\gamma_5tq_2(x))Q(x) \]  

(29)

where \( C \) is charge conjugate matrix, \( t \) the antisymmetric flavour matrix and \( a, b, c \) the colour indices, the QCD sum rules is given by

\[ \frac{1}{2}f_{\Lambda_b}^2 e^{\lambda^2/\tau} = \frac{1}{20\pi^4} \int_0^{\infty} d\omega \omega^5 e^{-\omega/\tau} D(\alpha_s)_{\Lambda_b} \]  

(30)

\[ + \frac{6}{\pi^4} E_4^4 \int_0^{\infty} d\omega e^{-\omega/\tau} + \frac{6}{\pi^4} E_6^6 e^{-m_0^2/8\tau^2} \]  

(31)

where

\[ D(\alpha_s)_{\Lambda_b} = 1 - \frac{\alpha_s}{4\pi} a_{\Lambda_b} \left( 1 + \frac{\lambda^2}{\tau^2} \right) \]  

(32)

with \( \eta' = r_1 \log(2\omega/\mu) - r_2 \). With \( f_{\Lambda_b}^2 = 0.2 \times 10^{-3} \text{ GeV}^6 \), \( \langle \bar{q}q \rangle = -0.24^3 \text{ GeV}^3 \), \( \langle g\bar{q}\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle \), \( m_0^2 = 0.8 \text{ GeV}^2 \), \( \langle \alpha_s GG \rangle = 0.04 \text{ GeV}^4 \) and \( D(\alpha_s)_{\Lambda_b} \) is expressed in accordance with power correction factor found in 11. As in the meson case, we obtain

\[ \lambda^2 \approx 0.4 \text{GeV}^2. \]  

(33)

Now we turn to the heavy quark expansion. The total decay rate of a weakly decaying heavy hadron is, at the leading order, given by

\[ \Gamma(H_b) = \Gamma_0 \left[ 1 - \frac{\alpha_s}{\pi} \left( \frac{2}{3} g(x) - \xi \right) \right] \]  

(34)

where

\[ \Gamma_0 = \frac{G_f^2|V_{KM}|^2\alpha_s^2}{192\pi^3} f(x) \]  

(35)

As already mentioned, the power corrections are given by the IR renormalons:

\[ D_{IR} = \hat{a}\alpha_s(1 + \xi) \]  

(36)

\[ \xi = \frac{\lambda^2}{m_0^2} \left( \frac{\alpha(\lambda^2)}{\alpha(m_0^2)} \right)^{11/18} \]  

(37)

In (34), the factor \( \xi \) corresponds to the IR renormalons which corresponds to the square root of the \( \lambda^2 \) term in the above equation. These corrections are estimated to be 0.2\( \Gamma_0 \) and 0.21\( \Gamma_0 \) for \( B \) and \( \Lambda_b \) respectively. This is significant in view of the discrepancy between the lifetimes of \( B \) and \( \Lambda_b \) being 0.2 ps\(^{-1} \) with \( \Gamma(B) = 0.68 \text{ ps}^{-1} \) and \( \Gamma(\Lambda_b) = 0.85 \text{ ps}^{-1} \).
4 Quark-Hadron Duality

The notion of quark-hadron duality (or duality) is variably defined in the literature depending upon the problem at hand. In the case of HQE description of inclusive decays too, there are many definitions. However, the popular and simple formulation is that the hadronic quantities can be obtained in terms of quarks and gluons. As regards the heavy hadron decays, the idea of duality is that the sum of the exclusive decay rates are given by sufficiently inclusive decay rate. It is difficult to provide an analytical or a semi-analytical prescription to establish duality.

We are able to evaluate the EVFQO in a model independent way. Our results for the ratio $\tau(\Lambda_b)/\tau(B)$ is closer to that of the experimental values. The basic idea is the assumption on the convergence of the HQE for beauty case at the third order in $1/m$. If we consider that the lifetime of hadron predicted by HQE can be used as a validation test of duality, then we are led to believe that duality holds good. If there is any discrepancy, it should be few percent and the same can be neglected.

On the other hand, the renormalon contribution to the parton decay rate is found to be less than 10%. This corrections can be construed as duality breaking one. This again validates the assumption of duality. Note that the renormalon corrections here are nonperturbative nature.

5 Final Remarks

Studies show that the theoretical and experimental discrepancy on the ratio of lifetimes of $\Lambda_b$ and $B$ is due to the spectator effects. The basic issue is the estimation of EVFQO. Previous studies leads to partial explanation of the discrepancy. However, in a model independent evaluation has been made which predicts the ratio, $\tau(\Lambda_b)/\tau(B)$, close to the experimental value. It is found in agreement with the one obtained in a crude potential model. Also a QCD sum rules approach predicted the ratio closer to the experimental value.

Further, the notion of duality holds good in the HQE for the beauty case within an amenable few percent. The same cannot be true for the charmed sector.

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