Calculation of craters resulting from impact rupture of rock mass using pulse hydrodynamic problem formulation

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Abstract. A liquid–solid hydrodynamic model is used to determine shapes and sizes of craters generated by impact rupture of rocks. Near the impact location, rock is modeled by an ideal incompressible liquid, in the distance—by an absolute solid. The calculated data are compared with the experimental results obtained under impact treatment of marble by a wedge-shaped tool.

Impact rupture of rocks is carried out by jumper bits or rotary–percussion drills. Despite numerous theoretical and experimental studies, the problem connected with the quantitative description and prediction of impact fracture remains yet to be solved as the present paper authors think. One of approaches to solving the problem may be maximum simplification of analytical models which describe the processes using minimum set of input parameters but at practically reasonable accuracy.

In this study, the theoretical determination of sizes of craters made by a wedge tool in soil used a liquid–solid hydrodynamic model earlier applied to blasting craters [1–3]. This model assumes that under impact rupture of soil, strength of soil near the impact tool as opposed to inertia forces can be neglected. In the distance from the tool, soil is assumed hard. The border between the zones of flowing and immovable soil is the flow line at which the critical velocity $V = \text{const}$.

The problem is formulated as finding an unknown section $\Gamma_1$ of the boundary $\Gamma$ (Figure 1) so that the function $\varphi$ satisfies Laplace’s equation in the domain $\Omega$:

$$\Delta \varphi = 0, \quad (1)$$

and the boundary conditions at $\Gamma$:

$$\frac{\partial \varphi}{\partial s} = V \quad \text{at} \quad \Gamma_1, \quad (2)$$

$$\frac{\partial \varphi}{\partial n} = 0 \quad \text{at} \quad \Gamma_1, \quad (3)$$

$$\varphi = 0 \quad \text{at} \quad \Gamma_2, \quad (4)$$

$$\varphi = -\varphi_0 \quad \text{at} \quad \Gamma_3. \quad (5)$$

By symmetry of the crater relative to its axis, Figure 1 only shows the right-hand part. Here, $X, Y$—coordinate axes; $\Gamma_1$—boundary of the crater in soil; $\Gamma_2$—free surface; $\Gamma_3$—boundary of pulse source (edge of the tool); $r_0$—half-width of the tool; $\varphi_0$—potential at the tool boundary; $s$ and $n$—angular position and outward normal drawn to the boundary.
Figure 1. Analytical pulse hydrodynamic model to determine profile of crater.

Transition to dimensionless parameters in (1)–(5) enables understanding the size of the crater depends on one dimensionless parameter:

$$\bar{\phi}_0 = \frac{\phi_0}{r_0 V}.$$  \hspace{1cm} (6)

Inasmuch as a part of the boundary of unknown and is to be determined, we used the iteration algorithm [3–5] and boundary element method. The initial position of the boundary $\Gamma_1$ was set arbitrarily.

The solution of (1)–(5) of Laplace’s equation used the equation [6]:

$$c(\xi)\phi(\xi) + \int_\Gamma \phi(\eta)q'(\xi, \eta) d\Gamma(\eta) = \int q(\eta)\phi'(\xi, \eta) d\Gamma(\eta),$$ \hspace{1cm} (7)

where $c(\xi)$ is constant and equals $\gamma\pi$ ($\gamma$—solid angle at which the boundary $\Gamma$ from the point $\xi$); $\phi(\eta)$ and $q'(\eta)$—potential and derivate of the potential with respect to the outward normal to $\Omega$ at the point $\eta \in \Gamma$, respectively; $\phi'(\xi, \eta)$—solution of Laplace’s equation; $q' = \partial \phi'/\partial n$.

For the plane symmetry used in the calculations, the boundary $\Gamma$ is a boundary of a cylinder. Numerical solution of Eq. (7) used the method by Krylov-Bogolyubov [7] which consists in the substitution of integral equations for algebraic equations. The boundary of the domain $\Omega$ was split into approximately equal linear sections—boundary elements, $\phi$ and $\partial \phi/\partial n$ at each element were assumed constant and related to the middle points of the sections—nodal points. Equation (7) was written in digital form for each boundary element and a system of linear algebraic equations was obtained as a result.

By solving the system, the values of the potential at the nodal points were calculated together with the velocity along $\Gamma_1$ at the intersection of $j$-th and $(j+1)$-th elements from the formula:

$$v_j = 0.5(\phi_{j+1} - \phi_j)/(l_{j+1} + l_j) = 0,$$ \hspace{1cm} (8)

where $\phi_j$, $\phi_{j+1}$ and $l_{j+1}$, $l_j$—potentials at $j$-th and $(j+1)$-th elements and the lengths, respectively. Then, the intersection points of the boundary elements were shifted along the normal drawn to them, and a new curve $\Gamma_1$ was plotted. In case that the solution failed to satisfy (2) at a preset accuracy (deviation of the velocity along the boundary from the critical value was higher than the preset limit), the next iteration was performed. The shift of $\Gamma_1$ on each iteration was fulfilled in two stages. First, the boundary element next the symmetry axis was fixed, and the other elements were shifted, and the velocity $v_j$ on the elements was compared with $v_1$. Then, comparing the velocity on the first section, $v_1$, and the critical velocity $V$, the whole boundary was either expanded or contracted proportionally. For better convergence, the shape of the boundary was regularized. In more details, the procedure is described in [8].
Figure 2a shows the calculated shape of the crater and the approximate profile of the craters in the experiments [9], in Figure 2 there is a picture of the experimental crater.

![Figure 2](image)

**Figure 2.** (a) Calculated and experimental craters and (b) picture of a crater after impact by a wedge tool: ■—calculation, □—experiment.

One of the major issues of calculating craters in the pulse hydrodynamic model is the selection of values of potentials on the surface of pulse sources (in the case under discussion, this is governed by the energy of the wedge tool) and critical velocities governing properties of the medium. Precise theoretical determination of these values is impossible due to abstract nature of the model. A certain relative estimate of the potential $\varphi_0$ in case of a wedge tool can be the value proportional to the tool energy density per unit surface area interacting with the tool edge:

$$\varphi_0 = \kappa \cdot \frac{E}{r_0 l},$$

where $E$—energy of the wedge; $l$—longitudinal size of the wedge (perpendicular to the plane $XY$); $\kappa$—proportionality coefficient.

In the experiments in [9], kinetic energy transmitted by a hammer to a wedge varied from 15 to 125 J. The front edge surface area of the wedge was $2.30 \times 60 = 60$ mm$^2$. The critical velocity was assumed as 1 m/s. The proportionality coefficient for the good correlation of the calculation and test data was chosen as $\kappa = 0.8 \times 10^{-8}$ m$^2$ s/kg.

Figure 2 shows the calculated curves of the crater depths $H0$ in the axis of symmetry, crater radii and wedge energy, the corresponding experimental data and the trend of the experimental results (second-order polynomial curve). The analysis of the plots (refer to Figure 2a) shows that theoretical and experimental depths agree well while the data on crater radii scatter considerably. This is a reflection of a feature of the pulse hydrodynamic model which cannot account for the brittle failure—split-off in walls of the crater, which results in the considerably higher $R$. In principle, the result can be corrected by introducing dependence of the critical velocity $V$ on the potential $\varphi$.

![Figure 3](image)

**Figure 3.** (a) The calculated depth and (b) radius of crater versus the energy of the wedge tool: ■—calculation, □—experiment, ——experimental data trend.
In this manner, within the framework of the pulse hydrodynamic model, the authors have calculated the profile of the craters generated on the half-space surface by an impact wedge. The comparison with the experimental results obtained on a marble block shows that the calculated radii of craters are much smaller. The analytical model can be improved by the introduction of the critical velocity–potential dependence. By selection of the proportionality coefficient between the wedge energy and the potential at the boundary of the pulse source, a good agreement has been reached between the calculation and experiment results on the depth of the craters (scatter within 10–15 %).

The described analytical model and the results may be recommended as an express-method to determine the size of craters formed in solid material under impact loading. Simplicity and low computational efforts make the model adaptable to the models for the assessment of efficiency of impact systems.

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