Quantum fluctuations of the color flux tube

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The quantum fluctuations of the flux tube joining two static sources in the confining phase of a lattice gauge theory are described by an effective string theory. The predictions of the latter for ratios of Wilson loops of equal perimeter do not contain any free parameters, and can be computed exactly for large Wilson loops. We compare these predictions with numerical results in 3D $Z_2$ gauge theory, finding complete agreement.

1. INTRODUCTION

The flux–tube picture of confinement is now a 20 year old conjecture, which we can summarize in two statements:

• Two static sources in the confined phase of a lattice gauge theory are joined by a color flux tube which is responsible for confinement.

• The flux tube can fluctuate around its equilibrium position; in the rough phase, these fluctuations are massless.

These two hypotheses not only provide an intuitive, physical picture of confinement, but have precise quantitative consequences. In principle, these predictions can be compared with the experimental data for the spectrum of bound states of heavy quarks; in practice, precise tests are currently possible using numerical simulations.

The aim of this work is to show that the precision now available in Monte Carlo data for the simplest lattice gauge theory, namely the 3D $Z_2$ gauge model, allows one to say a final word about the correctness of the fluctuating flux tube picture.

2. THE PREDICTIONS OF THE FLUCTUATING FLUX TUBE MODEL

The natural quantity to be studied in this context is the Wilson loop $W(R, T)$. Suppose two static sources are pulled apart at a distance $R$, kept apart for a (euclidean) time $T$, then put back together. If we assume that the flux tube keeps its equilibrium position, a straight line joining the two sources, the expectation value of the Wilson loop will follow the area law:

$$\langle W(R, T) \rangle \propto e^{-\sigma RT}$$

where $\sigma$ is the string tension. For the interquark potential this corresponds to a linear confining potential:

$$V(R) = \sigma R$$

If, instead, we allow the flux tube to fluctuate around its equilibrium position, we obtain corrections to the area law that can be compared to the results of Monte Carlo simulations. Many different models can be written down to describe the dynamics of the flux tube fluctuations; however, a vast class of these reduce to an effective free string model in the infrared region, i.e. for large Wilson loop. In this region the expectation value of the Wilson loop is given by

$$\langle W(R, T) \rangle \propto e^{-\sigma RT} \int [dX^i] \exp \left\{ -\frac{\sigma}{2} \int d^2 \xi X^i(-\partial^2)X^i \right\}$$

The fields $X^i$, ($i = 2, \ldots, d - 1$) are defined on the rectangle $(0, R) \times (0, T)$ and satisfy Dirichlet boundary conditions.

Work done in collaboration with M. Caselle, R. Fiore, F. Gliozzi and M. Hasenbusch
boundary conditions. They describe the fluctuations of the flux tube in the \( d - 2 \) directions transverse to the Wilson loop.

The functional integral is gaussian and can be performed exactly, using for example \( \zeta \)-function regularization. The result is

\[
\langle W(R, T) \rangle \propto e^{-\sigma R T} \left[ \frac{\eta(i)}{\sqrt{R}} \right]^{d-2}
\]

where \( \tau = i \frac{R}{T} \) and \( \eta(\tau) \) is the Dedekind function, which can be expressed as an infinite product

\[
\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)
\]

in terms of the variable \( q = e^{2\pi i \tau} \). Eq. (4) is the prediction of the effective string model for the expectation value of large Wilson loops. It should be noted that that Eq. (4) does not contain any new adjustable parameters with respect to the simple area law (1).

3. RATIOS OF WILSON LOOPS

Our goal is to compare Eq.(4) with the results of Monte Carlo simulations of the 3D \( Z_2 \) gauge model. The choice of the model is dictated by the need to have very high precision data to obtain an unambiguous result about the validity of the string model. This test could be performed by fitting a set of Monte Carlo data for the Wilson loop with Eq.(4). Taking into account the presence of a perimeter term, this would be a fit with three adjustable parameters. However, to obtain a completely unambiguous test of our ansatz, we decided to follow a different strategy, namely to define an observable quantity for which the predictions of the string model do not contain any adjustable parameters. We considered the following ratios of Wilson loops:

\[
R(L, n) = \frac{\langle W(L + n, L - n) \rangle}{\langle W(L, L) \rangle} e^{-\sigma n^2}
\]

Using Eq.(4) we obtain the prediction of the effective free string model for the ratios \( R(L, n) \), which turns out to depend only on the asymmetry ratio \( t = n/L \):

\[
R(L, n) = F(t) = \left[ \frac{\eta(i)\sqrt{1-t^2}}{\eta(i+1+i^t)} \right]^{1/2}
\]

4. COMPARISON TO THE MONTE CARLO DATA

We performed a Monte Carlo simulation of the 3D \( Z_2 \) gauge model at several values of the coupling \( \beta \), all located in the rough phase and close enough to the deconfinement point to be well inside the scaling region. From these simulations we extracted the expectation values of the Wilson loops. To evaluate \( R(L, n) \), one needs also to know the value of the string tension \( \sigma \). However, precise evaluations of the interface tension in the 3D spin Ising model in the scaling region are available. Using the duality between the \( Z_2 \) gauge model and the Ising spin model in 3D, we can simply plug the Ising model values in the expression (6). In this way we use values of \( \sigma \) that
Table 1
Monte Carlo results for the ratios $R(L, n)$

| $n/L$ | $L$ | $\beta$ | $\sigma$ | $R(L, n)$ | $F(n/L)$ |
|-------|-----|---------|---------|----------|---------|
| 0.2   | 15  | 0.75202 | 0.01023 | 1.0104(17)| 1.00881 |
| 0.25  | 20  | 0.75632 | 0.004779| 1.0166(10)| 1.01453 |
| 0.33  | 12  | 0.74838 | 0.014728| 1.02881(30)| 1.02901 |
| 0.375 | 8   | 0.75245 | 0.009418| 1.0403(31)| 1.03940 |
| 0.45  | 20  | 0.75632 | 0.004779| 1.0684(23)| 1.06588 |
| 0.5   | 20  | 0.75632 | 0.004779| 1.0911(27)| 1.09153 |
| 0.6   | 25  | 0.75632 | 0.004779| 1.165(11)| 1.17667 |

Figure 2. Finite–size effects for small Wilson loop. The prediction of the free string model is $R(L, L/2) = 1.09153 \ldots$ (straight line).

are independent from our Monte Carlo samples.

The results of the comparison are shown in Fig. 1) and Tab. 1). For each value of the asymmetry ratio $t = n/L$ we have displayed the value of the ratio $R(L, n)$ with the largest available physical size $\sigma L^2$. The solid line is the prediction, Eq. (4), of the effective free string model.

Two conclusions can be drawn from these data:

- The fluctuations of the flux tube are quantitatively relevant: it is easy to see that neglecting them the ratios $R(L, n)$ would be predicted to be all equal to one

- The free string model, Eq. (4), describes the fluctuations with great accuracy for large Wilson loops

For Wilson loops smaller than a threshold size of order $\sigma L^2 \sim 1$, rather significant finite–size effects appear. In Fig. (2) we display various data for ratios $R(L, L/2)$, i.e. with the same asymmetry ratio $t = 1/2$, and different sizes $\sigma L^2$.

In conclusion, we have shown that, at large distances, the physics of the flux tube fluctuations is accurately described by a free string model. For shorter distances, finite–size effects are rather significant. Precise data for large Wilson loops are needed to distinguish between the free string model described here and other possible models. For example, in Ref.[3] a fermionic string model was shown to describe the flux–tube fluctuations with an accuracy comparable to or even greater than the free, bosonic string model. The high precision data now available for large Wilson loops allow us to select the bosonic model as the most accurate.

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