Probing Extra Dimensions with Neutrino Oscillations

P. A. N. Machado\textsuperscript{a}, H. Nunokawa\textsuperscript{b} and R. Zukanovich Funchal\textsuperscript{a} \textsuperscript{*}

\textsuperscript{a}Instituto de Física, Universidade de São Paulo, C. P. 66.318, 05315-970 São Paulo, Brazil

\textsuperscript{b}Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, C. P. 38071, 22452-970 Rio de Janeiro, Brazil

We consider a model where sterile neutrinos can propagate in a large compactified extra dimension (a) giving rise to Kaluza-Klein (KK) modes and the Standard Model left-handed neutrinos are confined to a 4-dimensional spacetime brane. The KK modes mix with the standard neutrinos modifying their oscillation pattern. We examine current experiments in this framework obtaining stringent limits on \(a\).

1. Introduction

The introduction of singlet neutrino fields which can propagate in extra spatial dimensions as well as in the usual three dimensional space may lead to naturally small Dirac neutrino masses, due to a volume suppression. If those singlets mix with standard neutrinos they may have an impact on neutrino oscillations, even if the size of the largest extra dimension is smaller than \(2 \times 10^{-4} \text{m}\) (the current limit from Cavendish-type experiments which test the Newton Law).

2. Theoretical Framework

Here we consider the model discussed in Ref. [1] where the 3 standard model (SM) left-handed neutrinos \(\nu^\alpha_L\) and the other SM fields, including the Higgs \(H\), are confined to propagate in a 4-dimensional spacetime, while 3 families of SM singlet fermions \(\Psi^\alpha\) can propagate in a higher dimensional spacetime with at least two compactified extra dimensions, one of these \((y)\) compactified on a circle of radius \(a\), much larger than the size of the others so that we can in practice use a 5-dimensional treatment.

The singlet fermions have Yukawa couplings \(\lambda_{\alpha\beta}\) with the Higgs and the SM neutrinos leading to Dirac masses and mixings among active species and sterile KK modes. This can be derived from the action
\[
S = \int d^4x \, dy \, i \Psi^\alpha \Gamma^J \partial^J \Psi^\alpha + \int d^4x \, i \tilde{\nu}^\alpha_L \gamma^\mu \partial^\mu \nu^\alpha_L + \int d^4x \, \lambda_{\alpha\beta} H \tilde{\nu}^\alpha_L \Psi^\beta_R(x, 0) + \text{h.c.},
\]

where \(\Gamma^J, J = 0, \ldots, 4\) are the 5-dimensional Dirac matrices, that after dimensional reduction and electroweak symmetry breaking gives rise to the effective neutrino mass Lagrangian
\[
L_{\text{eff}} = \sum_{\alpha, \beta} m^D_{\alpha\beta} \left[ \nu^{(0)}_{\alpha L} \nu^{(0)}_{\beta R} + \sqrt{2} \sum_{N=1}^{\infty} \nu^{(N)}_{\alpha L} \nu^{(N)}_{\beta R} \right] + \sum_{\alpha} \sum_{N=1}^{\infty} \frac{N}{a} \nu^{(N)}_{\alpha L} \nu^{(N)}_{\alpha R} + \text{h.c.},
\]

where the Greek indices \(\alpha, \beta = e, \mu, \tau\), the capital Roman index \(N = 1, \ldots, \infty\), \(m^D_{\alpha\beta}\) is a Dirac mass matrix, \(\nu^{(0)}_{\alpha R}, \nu^{(N)}_{\alpha R}\) and \(\nu^{(N)}_{\alpha L}\) are the linear combinations of the singlet fermions that couple to the SM neutrinos \(\nu^{(0)}_{L}\).

In this context one can compute the active neutrino transition probabilities
\[
P(\nu^{(0)}_{\alpha} \rightarrow \nu^{(0)}_{\beta}; L) = \left| \sum_{i,j,k} \sum_{N=0}^{\infty} U_{\alpha i} U^*_{\beta j} W_{ij}^{(0)N} \right| \left| W_{kl}^{(0)N} \exp \left( \frac{i}{2En^2} \right) \right|^2.
\]
where $U$ and $W$ are the mixing matrices for active and KK modes, respectively. Here $\lambda_j^{(N)}$ is a dimensionless eigenvalue of the evolution equation [1] which depends on the $j$-th neutrino mass ($m_j$), hence on the mass hierarchy, $L$ is the baseline and $E$ is the neutrino energy.

To illustrate what is expected we plot in Fig. [1] the survival probabilities for $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ as a function of $E$. We show the behavior for the normal and inverted mass hierarchy assuming the lightest neutrino to be massless ($m_0 = 0$). The effect of this large extra dimension (LED) depends on the product $m_3a$. We observe that in the $\nu_\mu \to \nu_\mu$ channel the effect of LED is basically the same for normal (NH) and inverted (IH) hierarchies, since in this case all the amplitudes involved are rather large. On the other hand, for $\bar{\nu}_e \to \bar{\nu}_e$ the effect is smaller for NH as in this case the dominant $m_ja$ term is suppressed by $\theta_{13}$.

3. Results

As we can observe in Fig. [1] the main effect of LED is a shift in the oscillation maximum with a decrease in the survival probability due to oscillations to KK modes. This makes experiments such as KamLAND and MINOS, which are currently the best experiments to measure $\Delta m^2_{\text{atm}}$, relevant to our previous analysis of their combined data at 90 and 99% CL (2 dof). When finding these regions all standard oscillation parameters where considered free. To account for our previous knowledge of their values [7], we have added Gaussian priors to the $\chi^2$ function. As expected the limits provided by MINOS ($\nu_\mu \to \nu_\mu$) are basically the same for NH and IH. From their data we obtain $a < 7.3(9.7) \times 10^{-7} \text{ m}$ at the hierarchical case for $m_0 \to 0$ and $a < 1.2(1.6) \times 10^{-7} \text{ m}$ at 90 (99)% CL for degenerate neutrinos with $m_0 = 0.2 \text{ eV}$. We have verified that the inclusion of LED in the fit of the standard atmospheric oscillation parameters does not modify very much the region in the plane $\sin^2 2\theta_{23} \times |\Delta m^2_{\text{atm}}|$ allowed by MINOS data. In fact the best fit point as well as the $\chi^2_{\text{min}}$ remain the same as in the case without LED.

KamLAND data provide, for hierarchical neutrinos with $m_0 \to 0$, a competitive limit only for IH, in this case one gets $a < 8.5(9.8) \times 10^{-7} \text{ m}$ at 90 (99)% CL. For degenerate neutrinos with $m_0 = 0.2 \text{ eV}$ one also gets from KamLAND $a < 2.0(2.3) \times 10^{-7} \text{ m}$ at 90 (99)% CL. The inclusion of LED in the fit of the standard solar oscillation parameters here enlarges the region in the plane $\tan^2 \theta_{12} \times \Delta m^2_{\odot}$ allowed by KamLAND data. The best fit point changes from $\Delta m^2_{\odot} = 7.6 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_{12} = 0.62$ to $\Delta m^2_{\odot} = 8.6 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_{12} = 0.42$, how-

![LED in Vacuum](image1.png)

Figure 1. In the top (bottom) panel we show the survival probability for $\nu_\mu \to \nu_\mu$ as a function of the neutrino energy for $L = 735 \text{ km} (180 \text{ km})$ for $a = 0$ (no LED, black curve) and $a = 5 \times 10^{-7} \text{ m}$ for normal hierarchy (dashed blue curve) and inverted hierarchy (dotted red curve).
ever, the $\chi^2_{\text{min}}$/dof remains the same.

When one combines both experiments the hierarchical limits improve but the degenerate limit remains practically the one given by MINOS.

4. Conclusions

We have investigated the effect of LED in neutrino oscillation data deriving limits on the largest extra dimension $a$ provided by the most recent data from MINOS and KamLAND experiments. For hierarchical neutrinos with $m_0 \rightarrow 0$, MINOS and KamLAND constrain $a < 6.8(9.5) \times 10^{-7}$ m for NH and $a < 8.5(9.8) \times 10^{-7}$ m at 90 (99)% CL for IH. For degenerate neutrinos with $m_0 = 0.2$ eV their combined data constrain $a < 2.1(2.3) \times 10^{-7}$ m at 90 (99)% CL.

We can estimate that the future Double CHOOZ experiment will be able to improve these limits by a factor 2 for the IH and by a factor 1.5 for the degenerate case. Unfortunately NO$\nu$A and T2K cannot improve MINOS limits. See [8] for any detail.

RZF and HN thank Profs. Fogli and Lisi for the cordial invitation to participate of NOW2010. This work has been supported by FAPESP, FAPERJ and CNPq Brazilian funding agencies.

REFERENCES

1. H. Davoudiasl, P. Langacker and M. Perelstein, Phys. Rev. D 65, (2002) 105015 [arXiv:hep-ph/0201128].
2. P. Vahle [MINOS Collaboration], these proceedings.
3. S. Abe et al. [KamLAND Collaboration], Phys. Rev. Lett. 100, (2008) 221803 [arXiv:0801.4589 [hep-ex]].
4. P. Huber et al., Comput. Phys. Commun. 177, (2007) 432 [arXiv:hep-ph/0701187], URL http://www.mpi-hd.mpg.de/~globes.
5. H. Minakata, H. Nunokawa, W. J. C. Teves and R. Zukanovich Funchal, Nucl. Phys. Proc. Suppl. 145, (2005) 45 [arXiv:hep-ph/0501250].
6. J. Kopp, P. A. N. Machado and S. J. Parke, Phys. Rev. D 82 (2010) 113002 [arXiv:1009.0014 [hep-ph]].
7. M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1004 (2010) 056 [arXiv:1001.4524 [hep-ph]].
8. P. A. N. Machado, H. Nunokawa and R. Zukanovich Funchal [arXiv:1101.0003 [hep-ph]].