Pricing of premiums for equity-linked life insurance based on joint mortality models

Riaman*, K Parmikanti, I Irianingsih, S Supian
Departemen Matematika, FMIPA Unpad, Bandung 45363, INDONESIA

*E-mail: riaman@unpad.ac.id

Abstract. Life insurance equity-linked is a financial product that not only offers protection, but also investment. The calculation of equity-linked life insurance premiums generally uses mortality tables. Because of advances in medical technology and reduced birth rates, it appears that the use of mortality tables is less relevant in the calculation of premiums. To overcome this problem, we use a combination mortality model which in this study is determined based on Indonesian Mortality table 2011 to determine the chances of death and survival. In this research, we use the Combined Mortality Model of the Weibull, Inverse-Weibull, and Gompertz Mortality Model. After determining the Combined Mortality Model, simulators calculate the value of the claim to be given and the premium price numerically. By calculating equity-linked life insurance premiums well, it is expected that no party will be disadvantaged due to the inaccuracy of the calculation result.

1. Introduction
Equity-linked life insurance is one of the development products of traditional life insurance. Traditional life insurance only benefits when the insured person (the insured) is at risk. Surely it is related to the benefits of protection offered by life insurance. But as financial developments, life insurance allows not only the benefits of protection but also investment. This is driven by the increasing public awareness of the importance of investment. Insurance that provides protection and investment benefits is then called equity-linked life insurance. In equity-linked life insurance, there are two risks to be studied, namely payoff uncertainty and time uncertainty [11].

To overcome payoff uncertainty, commonly used stock option pricing models such as model [4] or [10]. However, to cover uncertainty of time, always use the death table to determine the life insurance premium associated with equity. It appears that the use of mortality tables is less relevant as medical advances in the medical field and decline in birth rates. There are two main areas of weakness from the use of mortality tables. First, the mortality table shows only historical data, so it can`t get Forecasting mortality rates to evaluate insurance. Secondly, mortality tables do not allow for highly accurate mortality information between the two ages because mortality tables are presented discretely, whereas the death process occurs continuously. Based on these reasons, it is assumed that the premium price offered is lower than it should be. This in the long term can lead to even bankruptcy losses for life insurance companies [11].

To overcome this, a combined mortality table was used to replace the role of mortality tables in the calculation of death and survival chances. Based on the journals written by [1] and [6], it is classified that the Weibull type mortality function for children, the Inverse-Weibull type for teenagers, and
Gompertz types. This is evident from the analysis of mortality charts of the United States and Taiwan. In this study, Indonesia mortality chart 2011 was used by AAJI (Association of Indonesian Life Insurance) to determine the Combined Mortality Model suitable for use in Indonesia and its effect on the calculation model of equity-linked life insurance premium. Then a simulation of the value of claims and equity-linked life insurance premium price with combined Mortality Model was obtained.

2. Mortality Models

The combined mortality model is a model formed on the basis of a combination of mortality. This combined mortality model uses the mortality function of the Weibull, Inverse-Weibull, and Gompertz distributions. Based on [11] as well as a combined mortality study by [6] which states that the mortality rate of children follows the Weibull distribution, adolescents follow the Inverse-Weibull distribution, and Adult following the Gompertz distribution. The combined mortality model has the following formula:

\[ s(x) = \sum_{k=1}^{m} \psi_k s_k(x) \]

Where \( s(x) \) is a combined of \( m \) survival function, and \( \psi_k \) indicates the probability of death by cause \( k \), that satisfy \( \psi_1 + \psi_2 + \cdots + \psi_k = 1 \), and \( s_k(x) \) is an opportunity of survival until the age of \( x \), cause of death \( k \). In this case \( k = 1, 2, 3 \). Where \( k = 1 \) indicates mortality due to children factor, \( k = 2 \) indicates death due to adolescent factor, and \( k = 3 \) shows death due to adult factor. \( s_k(x) \) is the probability of survival of factors \( k \) in age \( x \). For children, weibull distribution is used, while adolescents use Inverse-Weibull distribution, and adults use Gompertz distribution, then the following mortality model is obtained [5]:

\[ s_1(x) = \exp \left\{ -\left( \frac{x}{m_1} \right)^{\frac{m_1}{\sigma_1}} \right\} \]

\[ s_2(x) = 1 - \exp \left\{ -\left( \frac{x}{m_2} \right)^{\frac{m_2}{\sigma_2}} \right\} \]

\[ s_3(x) = \exp \left\{ -\left( \frac{m_3}{\sigma_3} \right) - e^{\left( \frac{x - m_3}{\sigma_3} \right)} \right\} \]

Of course to determine the model, overall need to know the value of the parameters contained in it, that is \( \psi_1, \psi_2, \psi_3, m_1, m_2, m_3, \sigma_1, \sigma_2, \sigma_3 \). To determine the value of each of these parameters, a method such as [6] is used, that is, by minimizing the loss function as follows:

\[ \sum_{x=0}^{111} (1 - \frac{\hat{q}_x}{q_x})^2 \]
Where \( q_x \) is the probability of death taken from Indonesian Mortality Table 2011. And \( \tilde{q}_x \) represents the probability of death to be estimated and given the symbol \( \tilde{q}_x = 1 - (\tilde{s}(x + 1)/\tilde{s}(x)) \). For \( \tilde{s}(x) \), a combined mortality model is used, i.e. [3]

\[
\tilde{s}(x) = \psi_1 \exp \left\{ -\left( \frac{x}{m_1} \right)^{\frac{m_1}{\sigma_1}} \right\} + \psi_2 \left\{ 1 - \exp \left\{ -\left( \frac{x}{m_2} \right)^{-\frac{m_2}{\sigma_2}} \right\} \right\} + \psi_3 \exp \left\{ -\left( \frac{x-m_1}{\sigma_3} \right) e^{\frac{x-m_1}{\sigma_3}} - e^{-\frac{x-m_1}{\sigma_3}} \right\}
\]

The result that the minimum value of the loss function is obtained by estimating the parameters are as follows:

| No. | Parameters | Values                      |
|-----|------------|-----------------------------|
| 1   | \( \psi_1 \) | 0.49812898863306            |
| 2   | \( \psi_2 \) | 0.501871002123818           |
| 3   | \( \psi_3 \) | 7.87648632878604 \times 10^{-9} |
| 4   | \( m_1 \)   | 0.00221415081475328         |
| 5   | \( m_2 \)   | 0.0166257778526924          |
| 6   | \( m_3 \)   | 0.00644796904262000         |
| 7   | \( \sigma_1 \) | 0.839543372660222           |
| 8   | \( \sigma_2 \) | 5.39644740308355            |
| 9   | \( \sigma_3 \) | 2.60032623812453 \times 10^{-8} |

Substitute the values of these parameters to \( s(x) \) so that the combined mortality model based on the Indonesian mortality table is as follows:

\[
s(x) = 0.49812898863306 \cdot \exp \left\{ -\left( \frac{x}{0.00221415081475328} \right)^{2.637327489 \times 10^{-3}} \right\} + 0.501871002123818 \left\{ 1 - \exp \left\{ -\left( \frac{x}{0.0166257778526924} \right)^{-3.080874622 \times 10^{-3}} \right\} \right\}
\]
The model is a combined mortality model that will be used to determine the premium value of equity-linked life insurance. The model obtained is certainly capable of measuring the specific chance of death and survival, which is of course useful to determine the possibility of a claim occurring and useful in the calculation of equity-linked life insurance premiums. This can be seen from the use of the Combined Mortality Model in the model of equity-linked life insurance premium pricing on equations, [7], [9].

\[ D_t = R_{pv}(t,T)s(T \mid t) + \int_t^T R_{pv}(t,u)f(s \mid u)ds \]

Here are assumptions used in numerical simulations:

1. The interest used is constant, i.e., the average of the BI rate data obtained, that is equal to 0.067 or 6.7%.
2. Purchase of shares is only 1 lot (100 pieces).
3. Allocated value for risk free investment of IDR500,000.00.
4. Age of insurance entry is 20 years, 25 years, 35 years old with 55 years of retirement age.
5. The correlation between portfolio and interest rate is 0.

Simulation of Equity-Linked Life Insurance Premium Price Determination by using Numerically Combined Mortality Model is as follows:

First, we will determine the value of the expected present value of the claim. Calculations performed directly for three ages of pre-determined insurance. Thus, with [1], we will find the value of \( R_{pv}(0,35), R_{pv}(0,30), R_{pv}(0,25) \). For all three have the same portfolio, namely:

\[ y(\omega) = 100 \cdot S(0) + 500000 \]

Based on the stock data obtained, used the value until the last close of the data, which is \( S(0) = 11675 \). So obtained value of the \( y(\omega) = 1667500 \).

Next will be calculated the value of the variance between the portfolio and the interest rate, namely:

\[ \sigma_p^2 = \sigma_r^2 + \sigma_y^2(s) - 2 \rho \sigma_r \sigma_y(s) \]

Since \( \rho = 0 \) and \( r \) are constant, then we get:

\[ \sigma_p^2 = \sigma_r^2 + \sigma_y^2 \]

Based on the analysis of BI rates data, it is found that \( \sigma_r^2 = 0.0000082892 \). Based on the analysis of stock data, it is found that the variance of stock is 0,022089325. Then calculated the variance of the portfolio, i.e. by[2]:

\[ n^2 \text{var}(S(t)) = 100^2 \cdot 0,022089325 = 220,89325 \]

So obtained:

\[ \sigma_p^2 = \sigma_r^2 + \sigma_y^2 = 0.0000082892 + 220,89325 = 220,8932582892 \]

Calculated the value of the standard deviation, namely:

\[ \sigma_p = \sqrt{220,8932582892} = 14,8624782 \]

Next will be determined zero coupon bond to be used [8]. It was previously assumed that the interest rate was constant so obtained:
\[
B(t, u) = E \left( \frac{1}{\int r(v) dv} \right) = E \left( e^{r(u-t)} \right) = e^{r(u-t)}
\]

In this numerical simulation, \( M = 500000 \) is used, so that 
\( \theta = M / \gamma(t) = 500000 / 1667500 = 0.299850075 \) is obtained

The amount of claim can be specified for each entry age as follows:
\[
R_{pv} (0,30) = 1667500 \cdot 1 - 0,1873081795 \cdot 500000 \cdot 0 + 0,1873081795 \cdot 500000 = 1761154,09
\]
Furthermore, based on the value of expected present value, will be determined the premium value of each age category entered. By using a computer program, the following results are obtained:

Table 2 Numerical Calculation Results

| No. | Entry Age | Value of Claim | Price of Premium |
|-----|-----------|----------------|------------------|
| 1   | 20 years  | IDR 1715423,601| IDR 844883,5799  |
| 2   | 25 years  | IDR 1734494,337| IDR 854557,5769  |
| 3   | 30 years  | IDR 1761154,09 | IDR 868002,1250  |

3. Conclusion
Based on the analysis of the Indonesian mortality rate in 2011, we obtained a composite mortality model in accordance with the Indonesian Mortality Model with model parameters that determine the amount of claims emerging and the calculation of the premium price. With the Combined Mortality Model, the model of claim calculation and premium is performed numerical simulation with various assumptions and obtained results as shown in Table 2. These results are only the initial stage, not yet until the analysis phase.

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