Similar Structure of Solution for the Model of Nonlinear Seepage in Multilayer Composite Reservoir

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ABSTRACT

Aimed at the multilayer composite reservoir, the model of nonlinear seepage is established in different outer boundary (infinity, closed, constant pressure) conditions. Through linearization and Laplace transform of the model, boundary value problem of the nonlinear seepage model is transformed into the boundary value problem of ordinary differential equation, and then constructing similar kernel function, similar structure of solution of reservoir pressure in the Laplace space is obtained in different outer boundary conditions for linearized model, this research provides a theoretical basis for well test analysis.

INTRODUCTION

Basic equation of seepage often neglects the quadratic gradient term in traditional seepage model for reservoir. However, with the extension of well test time, ignoring the quadratic gradient term will produce certain error, therefore traditional seepage model can not really reflect seepage law of reservoir. In recent years, with the deepening and perfect of the seepage theory, the study of nonlinear seepage theory has been widely attention, and it has become a new development direction for modern seepage mechanics [1]. But the solution of nonlinear seepage model has always been a big problem in the development of seepage theory.

In the past several decades, many researchers have studied the solution of nonlinear seepage model, and they also achieved certain results. Considering the influence of quadratic pressure gradient, Yonglu Jia[2] solved the seepage model by using different methods. The seepage model which was considering the influence of well bore storage and quadratic pressure gradient for heterogeneous medium reservoir was established by W.W-G YEH[3], the approximate solution of model was obtained by using implicit differential method.

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On the basis of above research, considering the influence of well bore storage, the model of nonlinear seepage for multilayer composite reservoir was built in three kinds of outer boundary conditions, by using variable substitution; Laplace transform and constructing similar kernel function, similar structure of solution was obtained for the linearized seepage model, and then similar structure theory\[4-5\] is generalized for solving the model of nonlinear seepage. This study provides the convenience for well test analysis in petroleum engineering at the same time.

BUILDING THE MODEL OF NONLINEAR SEEPAGE IN MULTILAYER COMPOSITE RESERVOIR

In order to build the model of nonlinear seepage in multilayer composite reservoir, the basic assumptions are included as follows:

1. \(N\) layers are level and equal thickness in multilayer composite reservoir, and interlayer has not crossflow.

2. The fluid of every layer is homogeneous and weakly compressible.

3. Initial pressure of every layer is \(p_{j0}\) (\(j = 1, 2, \ldots, N\)).

4. A well of radius \(r_w\) (the well radius of \(j\) layer is \(r_{wj}\), and \(r_w = \frac{1}{N} \sum_{j=1}^{N} r_{wj}\)) is opened in the center of reservoir, fixed yield is \(q\) (the yield of \(j\) layer is \(q_j\), \(q = \frac{1}{N} \sum_{j=1}^{N} q_j\)), the boundary radius is \(\alpha r_{wj}\) (\(\alpha > 1\)) in the inner region, the boundary radius is \(R_j\) \((R_j > \alpha r_{wj})\) in the outer region.

5. Formation fluid is single-phase, and fluid obeys Darcy’s law.

6. Considering the influence of well bore storage and neglecting the influence of capillary force and gravity.

7. Compressibility coefficient for fluid of interface is equal \(C_{P_{j1}} = C_{P_{j2}}\), and fluid is dynamic equilibrium. The model of nonlinear seepage in multilayer composite reservoir is built as follows:

Basic Equations of Seepage

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P_{j1}}{\partial r} \right) - C_{P_{j1}} \left( \frac{\partial P_{j1}}{\partial r} \right)^2 = \frac{1}{\eta_{j1}} \frac{\partial P_{j1}}{\partial t} \quad \left( r_{wj} < r < \alpha r_{wj}, t > 0, j = 1, 2, \ldots, N \right) \tag{1}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P_{j2}}{\partial r} \right) - C_{P_{j2}} \left( \frac{\partial P_{j2}}{\partial r} \right)^2 = \frac{1}{\eta_{j2}} \frac{\partial P_{j2}}{\partial t} \quad \left( r > \alpha r_{wj}, t > 0, j = 1, 2, \ldots, N \right) \tag{2}
\]

Initial condition

\[P_{j1}(r, 0) = P_{j2}(r, 0) = 0 \quad \left( j = 1, 2, \ldots, N \right) \tag{3}\]

Inner boundary condition

\[P_w(t) = P_{j1}(r_{wj}, t), \quad 2\pi \sum_{j=1}^{N} e_j \left( r \frac{\partial P_{j1}}{\partial r} \right) \bigg|_{r=r_{wj}} = -Bq + C \frac{dP_w}{dt} \quad \left( j = 1, 2, \ldots, N \right) \tag{4}\]

Interface boundary condition

\[P_{j1}(\alpha r_{wj}, t) = P_{j2}(\alpha r_{wj}, t), \quad \left. \frac{\partial P_{j1}}{\partial r} \right|_{r=\alpha r_{wj}} = \frac{1}{\lambda_{j2}} \left. \frac{\partial P_{j2}}{\partial r} \right|_{r=\alpha r_{wj}} \quad \left( j = 1, 2, \ldots, N \right) \tag{5}\]
Outer boundary conditions when outer boundary is infinity
\[ P_{j2}(\infty, t) = 0 \quad (j = 1, 2, \cdots, N) \]  
(6)

When outer boundary is closed
\[ \left. \frac{\partial P_{j2}}{\partial r} \right|_{r=R_j} = 0 \quad (j = 1, 2, \cdots, N) \]  
(7)

When outer boundary is constant pressure
\[ P_{j2}(R_j, t) = 0 \quad (j = 1, 2, \cdots, N) \]  
(8)

Where \( P_{jm} = p_{oj} - p_{jm}; \eta_{jm} = \frac{k_{jm}}{\varphi_{jm} \mu_{jm} C_{jm}} \); \( \lambda_{j12} = \left( \frac{k_{j1}}{\mu_{j1}} \right)_{j2}; \varepsilon_{j1} = \frac{k_{j1 h_{j1}}}{\mu_{j1}} \); \( j = 1, 2, \cdots, N \)

\( (m=1,2) \). The subscript "1" indicates the inner zone and subscript "2" indicates the outer zone.

THE LINEARIZATION OF THE MODEL

In order to solve equation (1) and (2). Firstly, equations are linearized, we ensure that outer boundary condition and initial condition of nonlinear seepage model in multilayer composite reservoir are homogeneity at the same time.

Let \( P_{im} = -\frac{1}{C_{pim}} \ln (u_{im} + 1) \)(m = 1, 2), \( P_{in} = -\frac{1}{C_{pin}} \ln (u_{in} + 1) \). Therefore the seepage equations (1) - (10) are converted into linear form as follows:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_{j1}}{\partial r} \right) = \frac{1}{\eta_{j1}} \frac{\partial u_{j1}}{\partial t} \quad \left( r_{wj} < r < \alpha r_{wj}, t > 0, j = 1, 2, \cdots, N \right) \]  
(9)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_{j2}}{\partial r} \right) = \frac{1}{\eta_{j2}} \frac{\partial u_{j2}}{\partial t} \quad \left( r > \alpha r_{wj}, t > 0, j = 1, 2, \cdots, N \right) \]  
(10)

\[ u_{j1}(r, 0) = u_{j2}(r, 0) = 0 \quad , u_w(t) = u_{j1}(r_{wj}, t), \quad u_w(t) = u_{j1}(r_{wj}, t) \]  
(11)

\[ 2\pi \sum_{j=1}^N q_{ji} \frac{1}{C_{pj1}} \frac{\partial u_{ji}}{\partial r} \rvert_{r=r_{wj}} = Bq(u_w + 1) + \frac{C_{pj1}}{C_{pj1}} \frac{du_w}{dt} \]  
(12)

\[ u_{j1}(r_{wj}, t) = u_{j2}(r_{wj}, t), \quad \frac{\partial u_{j1}}{\partial r} \rvert_{r=r_{wj}} = \frac{1}{\lambda_{j12}} \frac{\partial u_{j2}}{\partial r} \rvert_{r=r_{wj}} \]  
(13)

\[ u_{j2}(\infty, t) = 0 \quad , \quad \frac{\partial u_{j2}}{\partial r} \rvert_{r=R_j} = 0 \quad , \quad u_{j2}(R_j, t) = 0 \]  
(14)

LAPLACE TRANSFORM OF THE MODEL

Applied to linearization model (9)-(14) by employing the Laplace transform method on dimensionless time \( \tau \).

Let \( \overline{u}_{jk}(r, z) = \int_0^\infty e^{-z\tau} u_{jk}(r, \tau) d\tau, (k = 1, 2) \), then boundary value problem of ordinary differential equations in Laplace space is obtained as follows:

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\overline{u}_{j1}}{dr} \right) = \frac{z}{\eta_{j1}} \overline{u}_{j1} \quad \left( r_{wj} < r < \alpha r_{wj}, t > 0, j = 1, 2, \cdots, N \right) \]  
(15)
The solution of the model

Equations (15) and (16) are modified Bessel equation of 0 order, therefore their general solutions [3-4] are given as follows:

\[ \bar{u}_j = A_{j1}K_0 \left( \frac{z}{\eta_{j1}} \right) + B_{j1}I_0 \left( \frac{z}{\eta_{j1}} \right) \]

\[ \bar{u}_j = A_{j2}K_0 \left( \frac{z}{\eta_{j2}} \right) + B_{j2}I_0 \left( \frac{z}{\eta_{j2}} \right) \]

Where \( I_0() \), \( K_0() \) are first and second kind of modified Bessel functions of \( h \) order respectively, \( A_{j1} \), \( B_{j1} \), \( A_{j2} \), \( B_{j2} \) are constant, which are determined by conditions (17)-(21).

The following formulas are obtained by putting the formula (22) and (23) plug into formula (17)-(21).

\[ \bar{u}_v = A_{j1}K_0 \left( \frac{z}{\eta_{j1}} \right) + B_{j1}I_0 \left( \frac{z}{\eta_{j1}} \right) \]

\[ 2\pi \sum_{j=1}^{N} \bar{u}_j \left( \frac{z}{\eta_{j1}} \right) = B_1 \left( \frac{z}{\eta_{j1}} \right) + \frac{Cz}{C_{j1}} \bar{u}_v \]

\[ \bar{u}_j = A_{j2}K_0 \left( \frac{z}{\eta_{j2}} \right) + B_{j2}I_0 \left( \frac{z}{\eta_{j2}} \right) \]

\[ \bar{u}_v = A_{j1}K_0 \left( \frac{z}{\eta_{j1}} \right) + B_{j1}I_0 \left( \frac{z}{\eta_{j1}} \right) \]

\[ 2\pi \sum_{j=1}^{N} \bar{u}_j \left( \frac{z}{\eta_{j1}} \right) = B_1 \left( \frac{z}{\eta_{j1}} \right) + \frac{Cz}{C_{j1}} \bar{u}_v \]

\[ \bar{u}_j = A_{j2}K_0 \left( \frac{z}{\eta_{j2}} \right) + B_{j2}I_0 \left( \frac{z}{\eta_{j2}} \right) \]

Similar kernel function \( \psi(r,z) \) is defined as follows:
\[ \psi (r, z) = \frac{1}{\lambda_{j_2}} \psi_{0,0} \left( r, \alpha r_{w_j}, \frac{z}{\eta_{j_2}} \right) + \psi^* \left( \alpha r_{w_j}, z \right) \psi_{0,1} \left( r, \alpha r_{w_j}, \frac{z}{\eta_{j_2}} \right) + \frac{1}{\lambda_{j_1}} \psi_{0,0} \left( r, \alpha r_{w_j}, \frac{z}{\eta_{j_1}} \right) + \psi^* \left( \alpha r_{w_j}, z \right) \psi_{1,1} \left( r, \alpha r_{w_j}, \frac{z}{\eta_{j_2}} \right) \]

When the outer boundary is infinity:

\[ \psi^* (r, z) = \frac{K_0 \left( r \sqrt{z} \right)}{\sqrt{\eta_{j_2} K_1 \left( \alpha r_{w_j} \sqrt{z} \right)}} \]

When the outer boundary is constant pressure:

\[ \psi^* (r, z) = \frac{\psi_{0,0} \left( r, R_j, \frac{z}{\eta_{j_2}} \right)}{\sqrt{\eta_{j_2} \psi_{1,0} \left( \alpha r_{w_j}, R_j, \frac{z}{\eta_{j_2}} \right)}} \]

Where \( A_{j_1}, B_{j_1}, A_{j_2}, B_{j_2} \) are obtained by solving equations (20)-(28), Laplace space solution of reservoir pressure is obtained by putting the \( A_{j_1}, B_{j_1}, A_{j_2}, B_{j_2} \) plug into formula (22) and (23) for linearized model. The following is the form of continued fraction multiplication is obtained by constructing similar kernel function, namely similar structure.

When \( r_{w_j} < r < \alpha r_{w_j} \), solution of reservoir pressure distribution in the Laplace space is obtained for linearized model as follows:

\[ \bar{u}_{j_1} = \frac{-Bq \times \psi (r, z) \psi (r_{w_j}, z)}{Bq + \frac{Cz}{C_{p_{j_1}}} + 2\pi \sum_{j=1}^{\infty} \frac{e_{j_1} r_{w_j}}{C_{p_{j_1}}} \sqrt{\frac{z}{\eta_{j_1}}} \frac{1}{\psi (r_{w_j}, z)}} \]  (29)

When \( r > \alpha r_{w_j} \), solution of reservoir pressure distribution in the Laplace space is obtained for linearized model as follows:

\[ \bar{u}_{j_2} = \frac{-Bq \times 1}{Bq + \frac{Cz}{C_{p_{j_1}}} + 2\pi \sum_{j=1}^{\infty} \frac{e_{j_1} r_{w_j}}{C_{p_{j_1}}} \sqrt{\frac{z}{\eta_{j_1}}} \frac{1}{\psi (r_{w_j}, z)}} \times \psi_{0,1} \left( \alpha r_{w_j}, \alpha r_{w_j}, \frac{z}{\eta_{j_1}} \right) \times \psi^* (r, z) \times \psi_{1,0} \left( r_{w_j}, \alpha r_{w_j}, \frac{z}{\eta_{j_1}} \right) \]  (30)

**CONCLUSION AND UNDERSTANDING**

(1) With regard to the model of nonlinear seepage in multilayer composite reservoir, through linearization and Laplace transform, similar structure of solution
of reservoir pressure distribution is still obtained in the Laplace space for linearized model, it can better carry out well test analysis in Laplace space.

(2) For similar structure (29) and (30), it is easy to analyze the influence of well bore storage and outer boundary conditions for the linearized solution in the Laplace space, therefore it provides theoretical guidance for well test analysis.

(3) Nonlinear seepage is a widespread physics phenomenon in nature, the study of nonlinear seepage model for reservoir can better reflect the law of seepage.

(4) Generally speaking, linearized solution of reservoir pressure distribution in the Laplace space can convert into numerical solution in corresponding real space by using formula of Stehfest numerical inversion[5], this conversion fully satisfy the needs of well test analysis in petroleum engineering.

EXPLANATION OF SYMBOLS

$p_i$ is initial pressure of each layer in the reservoir (MPa); $r_w$ is well radius (m);
$r_{w_j}$ is well radius of the $j$ layer (m); $p_j$ is pressure of the $j$ layer at time $t$ (MPa);
$P_j$ is pressure drop of the $j$ layer (MPa); $C_{nj}$ is coefficient of compressibility for fluid of the $j$ layer of the reservoir; $C$ is coefficient of wellbore storage ($m^3/MPa$);
$B$ is coefficient of crude oil volume ($m^3/m^3$); $k_j$ is permeability of the $j$ layer ($um^2$);
$C_b$ is coefficient of integrated compression for the $j$ layer ($1/MPa$);
$\phi_j$ is porosity of the $j$ layer ($\%$); $\mu_j$ is fluid viscosity of the $j$ layer (MPa·s);
$h_j$ is thickness of the $j$ layer (m); $q_j$ is production of the $j$ layer ($m^3/d$);
$q$ is production ($m^3/d$); $R_j$ is outer boundary radius of the $j$ layer (m);
$R$ is outer boundary radius (m); $t$ is time (h); $Z$ is variable of Laplace space.

ACKNOWLEDGMENTS

Scientific Research Fund of Sichuan Provincial Education Department of China under Grant No.12ZA164, Scientific Research Fund of Sichuan Provincial Science & Technology Department of China under Grant No.2015JY0245.

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