SUBSTRUCTURE IN DARK HALOS: ORBITAL ECCENTRICITIES AND DYNAMICAL FRICTION
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ABSTRACT

The virialized regions of galaxies and clusters contain significant amounts of substructure; clusters have hundreds to thousands of galaxies, and satellite systems and globular clusters orbit the halos of individual galaxies. These orbits can decay owing to dynamical friction. Depending on their orbits and their masses, the substructures either merge, are disrupted, or survive to the present day. We examine the distributions of eccentricities of orbits within mass distributions similar to those we see for galaxies and clusters. A comprehensive understanding of these orbital properties is essential to calculate the rates of physical processes relevant to the formation and evolution of galaxies and clusters. We derive the orbital eccentricity distributions for a number of spherical potentials. These distributions depend strongly on the velocity anisotropy, but only slightly on the shape of the potential. The eccentricity distributions in the case of an isotropic distribution function are strongly skewed toward high eccentricities, with a median value of typically ~0.6, corresponding to an apocenter-to-pericenter ratio of 4.0. We also present high-resolution N-body simulations of the orbital decay of satellite systems on eccentric orbits in an isothermal halo. The dynamical friction timescales are found to decrease with increasing orbital eccentricity because of the dominating deceleration at the orbit's pericenter. The orbital eccentricity stays remarkably constant throughout the decay; although the eccentricity decreases near pericenter, it increases again near apocenter, such that there is no net circularization. We briefly discuss several applications for our derived distributions of orbital eccentricities and the resulting decay rates from dynamical friction. We compare the theoretical eccentricity distributions to those of globular clusters and galactic satellites for which all six phase-space coordinates (and therewith their orbits) have been determined. We find that the globular clusters are consistent with a close-to-isotropic velocity distribution, and they show large orbital eccentricities because of this (not in spite of this, as has been previously asserted). In addition, we find that the limited data on the Galactic system of satellites appears to be different and warrants further investigation as a clue to the formation and evolution of our Milky Way and its halo substructure.

Subject headings: celestial mechanics, stellar dynamics — dark matter — galaxies: interactions — galaxies: kinematics and dynamics — galaxies: structure — globular clusters: general — methods: numerical

1. INTRODUCTION

In the standard cosmological model, galaxies and clusters form by hierarchical clustering and merging of small density perturbations that grow by gravitational instability. In this standard picture, the mass of the universe is dominated by dissipationless dark matter that collapses to form dark halos, inside of which the luminous galaxies form. It was once assumed that the previous generation of substructure was erased at each level of the hierarchy (White & Rees 1978). However, high-resolution N-body simulations have recently shown that some substructure is preserved at all levels (Moore, Katz, & Lake 1996a; Klypin et al. 1997; Tormen, Bouchet, & White 1997; Brainerd, Goldberg, & Villumsen 1998; Moore et al. 1998a; Ghigna et al. 1998; Tormen, Diaferio, & Syer 1998). This is consistent with observations that reveal substructure in a variety of systems: globular clusters within galaxies, distant satellites and globulars in the halos of galaxies, and galaxies within clusters. This substructure evolves as it is subjected to the forces that try to dissolve it: dynamical friction, tides from the central objects, and impulsive collisions with other substructures. The timescales for survival and the nature of the dissolusion guide our understanding of the formation processes, most of which depend on the nature of the orbits. Tides strip satellites on elongated orbits. An elongated orbit and a circular one clearly evolve differently owing to dynamical friction, especially if there is a disk involved. The disk heating similarly depends on the orbits of the satellites. The nature of the orbits also affects the nature and persistence of tidal streams at breakup and the mutual impulsive collisions of structures (galaxy harassment).

Since the clustering and merging of halo substructures is one of the cornerstones of the hierarchical structure formation scenario, a comprehensive understanding of their orbital properties is of invaluable importance when seeking to understand the formation and evolution of structure in the universe. We have found that the properties of orbits within spherical, fully relaxed systems has received little attention and is often misrepresented. Hence, the first goal of this paper is to derive a statistical characterization of the orbits in potential/density distributions that describe galaxies within clusters, globular clusters within galaxies, etc. We find that orbits are far more elongated than typically characterized. Recently, Tormen (1997) and Ghigna et al. (1998) used high-resolution N-body simulations to investigate the orbital properties of halos within clusters. They found that orbits of the subhalos are strongly elongated with a median apocenter-to-pericenter ratio of approximately 6. We compare our results on equilibrium spherical
inconsistent change in orbital eccentricity with time. We use fully self-
consistent potentials with the distribution of orbits in the cluster that
was simulated by Ghigna et al. in a cosmological context and show that the orbital eccentricities of the subhalos are consistent with an isothermal halo that is close to isotropic.

In the second part of this paper, we use high-resolution N-body simulations to calculate the dynamical friction on eccentric orbits. Past studies have compared numerical simulations with Chandrasekhar’s dynamical friction formula (see, e.g., White 1978, 1983; Tremaine 1976, 1981; Lin & Tremaine 1983; Tremaine & Weinberg 1984; Bontekoe & van Albada 1987; Zaritsky & White 1988; Hernquist & Weinberg 1989; Cora, Muzzio, & Vergne 1997). Comprehensive overviews of these studies with discussions regarding the local versus global nature of dynamical friction can be found in Zaritsky & White (1988) and Cora et al. (1997). Most of the studies followed the decay of circular or only slightly eccentric orbits. The two exceptions are Bontekoe & van Albada (1987) and Cora et al. (1997). The former examined the orbital decay of a “grazing encounter” of a satellite on an elliptical orbit that grazes a larger galaxy at its pericenter. In this case, dynamical friction occurs only near pericenter. The pericentric radius remains nearly fixed with significant circularization of the orbit in just a few dynamical times (see also Colpi 1998 for an analytical treatment based on linear response theory). Cora et al. followed satellites on eccentric orbits that were completely embedded in a dark halo, but they didn’t discuss the dependence of decay time on orbital eccentricity or the change in orbital eccentricity with time. We use fully self-consistent N-body simulations with 50,000 halo particles to calculate dynamical friction on eccentric orbits. We show that the timescale for dynamical friction is shorter for more eccentric orbits but that the dependence is considerably weaker than claimed previously by Lacey & Cole (1993) based on an analytical integration of Chandrasekhar’s formula. In addition, we show that, contrary to common belief, dynamical friction does not lead to circularization. All in all, dynamical friction leads to only a very moderate change in the distribution of orbital eccentricities over time.

In §2 we derive the distributions of orbital eccentricities for a number of spherical densities/potentials using both analytical and numerical methods. Section 3 describes our N-body simulations of dynamical friction on eccentric orbits in an isothermal halo. In §4 we discuss a number of applications. Our results and conclusions are presented in §5.

2. ORBITAL ECCENTRICITIES

2.1. The Singular Isothermal Sphere

The flat rotation curves observed for spiral galaxies suggest that their dark halos have density profiles that are not too different from isothermal. Hence, we start our investigation with the singular isothermal sphere, whose potential, \( \Phi \), and density, \( \rho \), are given by

\[
\Phi(r) = V_c^2 \ln(r), \quad \rho(r) = \frac{V_c^2}{4\pi Gr^2}.
\]  

Here \( V_c \) is the circular velocity, which is constant with radius.

2.1.1. Analytical Method

For non-Keplerian potentials, in which the orbits are not simple ellipses, it is customary to define a generalized orbital eccentricity, \( e \), as

\[
e = \frac{r_+ - r_-}{r_+ + r_-}.
\]  

Here \( r_- \) and \( r_+ \) are the pericenter and apocenter, respectively. For an orbit with energy \( E \) and angular momentum \( L \) in a spherical potential, \( r_- \) and \( r_+ \) are the roots for \( r \) of

\[
\frac{1}{r^2} + \frac{2[\Phi(r) - E]}{L^2} = 0
\]  

(Binney & Tremaine 1987). For each energy, the maximum angular momentum is, for a singular isothermal sphere, given by \( L_s(E) = r_c(E) V_c \). Here \( r_c(E) \) is the radius of the circular orbit with energy \( E \) and is given by

\[
r_c(E) = \exp (\frac{E - V_c^2/2}{V_c^2}).
\]  

Upon writing \( L = \eta L_s(E) \ (0 \leq \eta \leq 1) \), one can rewrite equation (3) for a singular isothermal sphere such that the apocenter and pericenter are given by the roots for \( x = r/r_c \) of

\[
\frac{1}{x^2} + \frac{2}{\eta^2} \ln(x) - \frac{1}{\eta^2} = 0.
\]  

As might be expected in this scale-free case, the ratio \( r_+/r_- \) depends only on the orbital circularity \( \eta \) and is independent of energy. This dependence is shown in Figure 1.

At a certain radius, the average of any quantity \( S \) is determined by weighting it by the distribution function (hereafter

\[\text{FIG. 1.—Eccentricity as function of the orbital circularity } \eta \text{ for orbits in a singular isothermal sphere.}\]
DF) and integrating over phase space. For the singular isothermal sphere, this yields

\[ \bar{S}(r) = \frac{4\pi}{r^2\rho(r)} \int_0^\infty dE \int_{\Phi(r)}^{\Phi_0} \frac{L dL}{\sqrt{2(E - \Phi)}}. \]  

(6)

In what follows, we consider the family of quasi-separable DFs

\[ f(E, L) = g(E)h_a(\eta). \]  

(7)

This approach makes the solution of equation (6) analytically tractable. The general properties of spherical galaxies with this family of DFs are discussed in detail by Gerhard (1991, hereafter G91). We adopt a simple parameterization for the function \(h_a(\eta)\):

\[ h_a(\eta) = \begin{cases} \frac{\tanh \left( \frac{\eta}{a} \right)}{\tanh \left( \frac{1}{a} \right)} & \text{if } a > 0 \\ 1 & \text{if } a = 0 \\ \frac{\tanh \left( \frac{1 - \eta}{a} \right)}{\tanh \left( \frac{1}{a} \right)} & \text{if } a < 0 \end{cases}. \]  

(8)

For \(a = 0\), the DF is isotropic. Radially anisotropic models have \(a < 0\), whereas positive values of \(a\) correspond to tangential anisotropy. For a quasi-separable DF of the form of equation (7), and the eccentricity \(e\), which depends on \(\eta\) only, equation (6) yields

\[ \bar{\varepsilon}(r) = \frac{4\pi}{r^2\rho(r)} \int_0^\infty dE g(E) L_c(E) \times \int_0^{\eta_{\text{max}}} h_a(\eta) e(\eta) \frac{\eta d\eta}{\sqrt{\eta_{\text{max}}^2 - \eta^2}}, \]  

(9)

where

\[ \eta_{\text{max}} = \frac{r\sqrt{2(E - \Phi)}}{L_c(E)}. \]  

(10)

For a singular isothermal sphere, G91 has shown that the energy dependence of the DF is given by

\[ g(E) = \frac{\exp \left( \frac{1}{16\pi^2 GV_c u_H^2} \right)}{16\pi^2 GV_c u_H^2} \exp \left[ -\frac{2E}{V_c^2} \right], \]  

(11)

where

\[ u_H = \int_0^\infty du \exp \left( -u \right) \int_0^{\eta_{\text{max}}} h_a(\eta) \frac{\eta d\eta}{\sqrt{\eta_{\text{max}}^2 - \eta^2}}. \]  

(12)

Here \(\eta_{\text{max}}\) depends on \(u\) only and is given by

\[ \eta_{\text{max}} = \sqrt{2u} \exp \left( -u + \frac{1}{2} \right). \]  

(13)

Substitution of equations (11) and (12) into equation (9) yields [upon substituting \(u = (E - \Phi)/V_c^2\)]

\[ \bar{\varepsilon}(r) = \bar{\varepsilon} = \frac{1}{u_H} \int_0^\infty du \exp \left( -u \right) \int_0^{\eta_{\text{max}}} h_a(\eta) e(\eta) \frac{\eta d\eta}{\sqrt{\eta_{\text{max}}^2 - \eta^2}}. \]  

(14)

Note that, because of the scale-free nature of the problem, this average is independent of radius.

We have numerically solved this integral as a function of the anisotropy parameter \(a\). The discontinuous behavior at \(a = 0\) is due to our particular choice for the function \(h_a(\eta)\). Negative and positive \(a\) correspond to radial and tangential anisotropy, respectively. In the isotropic case \((a = 0)\), the average eccentricity is \(\bar{\varepsilon} = 0.55\).

2.1.2. Numerical Method

To examine the actual distribution of eccentricities, rather than just calculate moments of the distribution, we Monte Carlo sample the quasi-separable DF of equation (7) for orbits in a singular isothermal potential and calculate their eccentricities. We provide a detailed description of this method in the Appendix.

The normalized distribution functions of orbital eccentricities for three different values of the anisotropy parameter \(a\) are shown in Figure 3 (upper panels). The lower panels show the corresponding distributions of apocenter-to-pericenter ratios. Each distribution is computed from a Monte Carlo simulation with \(10^6\) orbits. The thin vertical lines in Figure 3 show the 20th (dotted lines), 50th (solid lines), and 80th (dashed lines) percentile points of the distributions. The average eccentricity for the isotropic case \((a = 0)\) computed from the Monte Carlo simulations is \(\bar{\varepsilon} 
\approx 0.55\), in excellent agreement with the value determined in §2.1.1. About 15% of the orbits in the isotropic singular isothermal sphere have apocenter-to-pericenter ratios larger than 10, whereas only \(\sim 20\%\) of the orbits have \(r_+/r_- < 2\). Note however that these numbers depend strongly on the velocity anisotropy.

2.2. Tracer Populations

In the previous two sections, we concentrated on the self-consistent case of a singular isothermal halo and the corresponding density distribution that follows from the Poisson equation. Tracer populations, however, do not necessarily follow the self-consistent density distribution. Consider a
tracer population in a singular isothermal sphere potential with a density distribution given by

$$\rho_{\text{trace}}(r) = \rho_0 \left( \frac{r}{r_0} \right)^{-\alpha}$$

(15)

(the self-consistent case corresponds to $\alpha = 2.0$). If we consider the same quasi-separable DF as in § 2.1 (i.e., eq. [7]), the energy dependence of the DF becomes

$$g(E) = \frac{\exp{(1)}\rho_0 r_0^\alpha}{4\pi V_c^2 u_{\text{hl},z}} \exp\left( -\frac{zE}{V_c^2} \right),$$

(16)

with

$$u_{\text{hl},z} = \int_0^{\rho_0} du \exp\left[ ((1-\alpha)u) \int_0^{\eta_{\text{max}}} \frac{\eta \, d\eta}{\sqrt{\eta_{\text{max}} - \eta}} \right].$$

(17)

The average eccentricity can be computed by substituting equations (16) and (17) into equation (9), thereby using the expression $\Phi(r)$ given by equation (1). In Figure 4 we plot the average eccentricity thus computed as a function of the power-law slope $\alpha$. As can be seen, the orbital eccentricities depend only mildly on the slope of the density distribution. Note that for $\alpha > 3$, the mass within any radius $r > 0$ is infinite, whereas in the case $\alpha < 3$, the mass outside any such radius is infinite. These differing properties in the two regimes are apparent in the behavior of the median eccentricity seen in Figure 4, with the intermediate case of $\alpha = 3$ yielding a minimum. Because of the infinities involved, the cases examined may not accurately represent true dark halos. After giving an example of how such infinities cause problems, in the subsequent sections we examine other, more realistic, spherical potentials with finite masses.

2.3. Stellar Hydrodynamics and the Virial Theorem in the Isothermal Potential

The difference between infinite and finite samples is evident when comparing the equation of stellar hydrody-
namics to the virial theorem. For a sphere with an isotropic DF, the former takes the simple form for the one-dimensional velocity dispersion $\sigma$:

$$\frac{d}{dr}(\rho \sigma^2) = -\rho \frac{d\Phi}{dr}. \quad (18)$$

For the tracer population of equation (15) and an isothermal potential, this simplifies to

$$\sigma^2 = -V_T^2 \left(\frac{d \ln \rho}{d \ln r}\right)^{-1} = \frac{V_c^2}{\pi}. \quad (19)$$

In contrast, the virial theorem states that twice the kinetic energy is equal to the virial. Adopting particle masses of unity yields

$$\sum \nu^2 = \sum F \cdot r = \sum \frac{V_c^2}{r} r. \quad (20)$$

Since, the expected value of $\nu^2$ is $3 \sigma^2$, this reduces to

$$\sigma^2 = \frac{V_c^2}{3}. \quad (21)$$

This difference results from the assumption of finite versus infinite tracers. The answers match only for $\alpha = 3$, where the divergence in mass is only logarithmic at both $r \to 0$ and $r \to \infty$. Even the “self-consistent” case of $\alpha = 2$ has a problem that is pointed out in problem [4-9] of Binney & Tremaine (1987). The kinetic energy per particle is $V_c^2$ in a model with only circular orbits and $3V_T^2/2$ if they are isotropic, yet they have the same potential and must satisfy the virial theorem. In §4.2, we return to this issue as we examine a case in which the equations of stellar hydrodynamics have been used on astrophysical objects for which a subsample of a finite number of global tracers has been observed. We now turn to the calculation of the distribution of eccentricities for finite sets of tracers.

2.4. Truncated Isothermal Sphere with Core

Given the infinite mass and central singularity of the singular isothermal sphere, dark halos are often modeled as truncated, nonsingular, isothermal spheres with a core:

$$\rho(r) = \frac{M}{2\pi^{1/2} r_t^{3/2}} \frac{\exp \left(-r^2/r_t^2\right)}{r^2 + r_c^2}. \quad (22)$$

Here $M, r_t,$ and $r_c$ are the mass, truncation radius, and core radius, respectively, and $\gamma$ is a normalization constant given by

$$\gamma = 1 - \sqrt{\pi} \left(\frac{r_c}{r_t}\right) \exp \left(\frac{r_t^2}{2r_c^2}\right)[1 - \text{erf}(r_c/r_t)]. \quad (23)$$

Since for this density distribution the DF is not known analytically [i.e., this requires the knowledge of $\rho(\Phi)$ in order to solve the Eddington equation], and since this density distribution is no longer scale free, we have to use a different approach in order to determine the distribution of orbital eccentricities. We employ the method described by Hernquist (1993). We randomly draw positions according to the density distribution. The radial velocity dispersion is computed from the second-order Jeans equations, assuming isotropy, i.e., assuming $f = f(E)$:

$$\overline{v_r^2}(r) = \frac{1}{\rho(r)} \int_r^\infty \rho(r) \frac{d\Phi}{dr} dr. \quad (24)$$

We compute local velocities $v$ by drawing randomly a unit vector and then a magnitude from a Gaussian distribution whose second moment is equal to $v_T^2$, truncated at the local escape speed. Once the six phase-space coordinates are known, the energy and angular momentum are calculated, providing the apocenter and pericenter of the orbit by solving equation (3). The resulting distributions of orbital eccentricities are not rigorous, since the velocity field has not been obtained from a stationary DF but rather used only the second moments. However, as demonstrated by Hernquist (1993), N-body simulations run from these initial conditions are nearly in equilibrium (see also §3.2), which suggests that the eccentricity distributions derived are sufficiently close to the actual equilibrium distributions.

In Figure 5, we plot the 20th (dotted lines), 50th (solid lines), and 80th (dashed lines) percentile points of the distributions of eccentricity (left panel) and apocenter-to-pericenter ratio (right panel), as functions of $r_i/r_c$. For $r_i = r_c$ the distribution of eccentricities is almost symmetric, with the median equal to 0.50. When $r_i/r_c$ increases, the distribution becomes more and more skewed toward high-eccentricity orbits; the distribution closely approaches that of the singular isothermal sphere in the limit $r_i/r_c \to \infty$.

2.5. Steeper Halo Profiles

To examine the dependence of the orbital eccentricities on the actual density distribution of the halos, we determine the eccentricity distributions of two well-known models.
with steeper outer density profiles ($\rho \propto r^{-4}$):

$$\rho_H(r) = \frac{M}{4\pi} \frac{a}{r^2(r+a)^2}$$

and

$$\rho_J(r) = \frac{M}{2\pi} \frac{a}{r(r+a)^3},$$

where $M$ is the total mass. These profiles differ only in the steepness of the central cusp: the former one, known as the Jaffe (1983) profile, has a $r^{-2}$ cusp, whereas the latter, known as the Hernquist (1990) profile, has a shallower $r^{-1}$ cusp. We use the technique described in § 2.4 to compute the distributions of orbital eccentricities for isotropic DFs $f(E)$.

The results are shown in Figure 6, where we plot the normalized eccentricity distributions for the Jaffe and Hernquist spheres. For comparison the results for the singular isothermal sphere with isotropic DF are reproduced as well. The three distributions are remarkably similar (the differences are best appreciated by comparing the thin lines indicating the percentile points). The distributions are progressively skewed toward higher eccentricities in the sequence isothermal, $\rho_H(r)$, $\rho_J(r)$, but only moderately so.

Navarro, Frenk, & White (1995, 1996, 1997) have used cosmological simulations to argue that the outer density profiles of dark halos decline as $r^{-3}$. Their profiles are likely created by a variety of numerical artifacts (Moore et al. 1998b). However, our results suggest that such debates will not significantly alter the expected distributions of eccentricities; velocity anisotropy is far more important than the details of the density profile.

3. ORBITAL DECAY OF ECCENTRIC ORBITS BY DYNAMICAL FRICTION

The orbits of substructures within halos change owing to dynamical friction. Chandrasekhar (1943) derived the local deceleration of a body with mass $M$ moving through an infinite and homogeneous medium of particles with mass $m$. The deceleration is proportional to the mass $M$, such that more massive subhalos sink more rapidly. As long as $M \gg m$, the frictional drag is proportional to the mass density of the medium but independent of the mass $m$ of the constituents. For an isotropic, singular isothermal sphere, the deceleration is

$$\frac{dv}{dt} = -\frac{GM_s}{r^2} \ln \Lambda \left( \frac{v}{V_c} \right)^2$$

$$\times \left\{ \text{erf} \left( \frac{v}{V_c} \right) - \frac{2}{\sqrt{\pi}} \left( \frac{v}{V_c} \right) \exp \left[ -\left( \frac{v}{V_c} \right)^2 \right] \right\} \hat{e}_r,$$  \hspace{1cm} (27)

with $M_s$ and $v$ the mass and velocity of the object being decelerated, $r$ the distance of that object from the center of the halo, $\ln \Lambda$ the Coulomb logarithm, and $\hat{e}_r$ the unit velocity vector (Tremaine 1976; White 1976a).

As mentioned earlier, remarkably little attention has been paid to the effects of dynamical friction on eccentric orbits—our goal for the rest of this section. Our main objective is to use both analytical and numerical tools to investigate the change of orbital eccentricity with time and the dependence of the dynamical friction timescale on the (intrinsic) eccentricity. Unlike most previous studies, we will not focus on testing Chandrasekhar’s dynamical friction formula or on studying the exact cause of the deceleration (i.e., local or global), as these have been the topic of discussion in many previous papers.

3.1. The Time Dependence of Orbital Eccentricity

We investigate the rate at which orbital eccentricity changes because of dynamical friction. For simplicity, we focus on the evolution of orbital eccentricity in the singular isothermal sphere, for which

$$\frac{de}{dt} = \frac{d\eta}{dt} \frac{de}{d\eta},$$ \hspace{1cm} (28)

with $\eta = L/L_s(E)$ (see § 2.1). Using equation (4), we find

$$\frac{d\eta}{dt} = \eta \left( \frac{1}{L} \frac{dL}{dt} - \frac{1}{V_c^2} \frac{dE}{dt} \right).$$ \hspace{1cm} (29)
Because of dynamical friction, the energy and angular momentum are no longer conserved and

\[ \frac{dE}{dt} = v \frac{dv}{dt} \]  

(30)

and

\[ \frac{dL}{dt} = r \frac{dv}{dt}. \]  

(31)

Since the frictional force acts in the direction opposite of the velocity,

\[ \frac{dv}{dt} = \frac{v}{v} \frac{dv}{dt}. \]  

(32)

Upon combining equations (28)–(32), we find

\[ \frac{de}{dt} = \frac{\eta}{v} \frac{dv}{dn} \left[ 1 - \left( \frac{v}{V_c} \right)^2 \right] \frac{dv}{dt}. \]  

(33)

Here \( dv/dt \) is the frictional deceleration given by equation (27). Since both \( dv/dt < 0 \) and \( de/dn < 0 \) (see Fig. 1), we can immediately derive the sign of \( dv/dt \) at apocenter and pericenter. At apocenter \( v < V_c \), such that \( dv/dt > 0 \), whereas at pericenter \( v > V_c \) and thus \( dv/dt < 0 \). This explains the circularization found for “grazing encounters” (Bontekoe & van Albada 1987), as dynamical friction happens only near pericenter. Equation (27) shows that \( dv/dt \propto r^{-2} \), and the change in eccentricity is thus larger at pericenter than at apocenter. However, the time spent near pericenter is shorter than near apocenter, so that the evolution of the eccentricity can not be determined by inspection. In the next sections, we use numerical simulations.

### 3.2. N-Body Simulations

We perform a set of fully self-consistent \( N \)-body simulations with a large number of particles in order to examine the effects of dynamical friction on eccentric orbits in a massive halo. The halo is modeled by a truncated isothermal sphere (eq. [22]), with total mass of unity, a core radius \( r_c = 1 \), and a truncation radius \( r_t = 50 \). Scaled to the Milky Way, we adopt a unit of mass of \( 10^{12} M_\odot \) and a unit of length of 4 kpc. With the gravitational constant set to unity, the units of velocity and time are \( 10^{37} \) km s\(^{-1}\) and 3.8 Myr, respectively.

The initial velocities of the halo particles are set up as described in § 2.4, following the procedure of Hernquist (1993). Because of this particular method the halo is not necessarily in equilibrium, nor is it expected to be virialized. In order to remove effects of the halo’s virialization on the decay of the orbiting substructure, we first simulate the halo in isolation for 10 Gyr. At the end, the halo has nicely settled in virial equilibrium. Figure 7 shows the initial density profile compared to that after 10 Gyr. As can be seen, and as already pointed out by Hernquist (1993), the density profile has not changed significantly after 10 Gyr.

We are interested in the effects of dynamical friction on galactic objects that range from \( \sim 10^6 M_\odot \) (globular clusters) to \( \sim 10^{12} M_\odot \) (a massive satellite). To simulate dynamical friction on an object of mass \( M_s \) orbiting in a halo of mass \( M_h \), we require \( M_s \gg M \). In our simulations, we insist that \( N \gtrsim 10 M_s / M \). A simulation of the orbital decay of a globular cluster in a galactic halo of \( 10^{12} M_\odot \) thus requires \( N \gtrsim 10^7 \), clearly too large a number for practical purposes. We run our simulations with \( N = 50,000 \) particles, roughly an order of magnitude increase over most previous work. The highest resolution self-consistent simulations aimed at investigating dynamical friction have so far been performed by Hernquist & Weinberg (1989) and Tormen et al. (1998), both using \( \sim 20,000 \) particles. Hernquist & Weinberg focused only on circular orbits, whereas Tormen et al. investigated the evolution of subhalos on realistic, eccentric orbits in a simulated cluster. In their simulations, Tormen et al. followed the simultaneous evolu-
We discuss the influence of the number of halo particles in a single satellite of constant mass embedded in a large halo. We therefore follow the effects of dynamical friction only, and in particular on the dependence of orbital eccentricity. In addition to dynamical friction, these subhalos within the cluster environment. In particular, we discuss its influence on dynamical friction and the scaling is set so that a satellite with mass similar to that of the Large Magellanic Cloud (LMC) has a softening length comparable to the LMC’s effective radius (de Vaucouleurs & Freeman 1972). This choice is somewhat arbitrary, and we discuss its influence on dynamical friction timescales in §3.3.3.

All simulations are run for 15 Gyr on 2 or 3 DEC Alpha processors, each requiring about 48 hr of wall-clock time. Energy conservation was typically of order 1% over the total length of the simulation.

### Table 1

**Parameters of the N-Body Simulations**

| Model (1) | $M_s$ ($\times 10^{12} M_\odot$) (2) | $e_0$ ($\times 4$ kpc) (3) | $r_s$ ($\times 4$ kpc) (4) | $\epsilon_s$ ($\times 4$ kpc) (5) | $N_h$ (6) |
|-----------|----------------------------------|----------------|----------------|----------------|---------|
| 1.......... | $2 \times 10^{-4}$ | 0.8 | 40.0 | 0.18 | $5 \times 10^4$ |
| 2.......... | $2 \times 10^{-3}$ | 0.8 | 40.0 | 0.40 | $5 \times 10^4$ |
| 3.......... | $2 \times 10^{-2}$ | 0.8 | 40.0 | 0.86 | $5 \times 10^4$ |
| 4.......... | $2 \times 10^{-1}$ | 0.6 | 37.3 | 0.86 | $5 \times 10^4$ |
| 5.......... | $2 \times 10^{-3}$ | 0.3 | 31.4 | 0.86 | $5 \times 10^4$ |
| 6.......... | $2 \times 10^{-2}$ | 0.0 | 24.6 | 0.86 | $5 \times 10^4$ |
| 7.......... | $2 \times 10^{-3}$ | 0.8 | 40.0 | 1.72 | $5 \times 10^4$ |
| 8.......... | $2 \times 10^{-2}$ | 0.8 | 40.0 | 0.43 | $5 \times 10^4$ |
| 9.......... | $2 \times 10^{-2}$ | 0.8 | 40.0 | 0.22 | $5 \times 10^4$ |
| 10.......... | $2 \times 10^{-2}$ | 0.6 | 37.3 | 0.86 | $2 \times 10^4$ |
| 11.......... | $2 \times 10^{-2}$ | 0.6 | 37.3 | 0.86 | $1 \times 10^5$ |

**Notes.**—Col. (1): Model ID by which the different simulations are addressed in the text. Cols (2), (3), (4), and (5): Masses, initial eccentricities, and apocentric radii from which the satellites are started and the softening lengths of the satellites in each model, respectively (all in model units). Col. (6): Number of halo particles used in the simulations. All satellites have the same initial specific energy. The satellites in models 1–6 and 10 and 11 all have the same average density.
determine the radial turning points of the orbit and compute the approximate eccentricity that we assign to a time midway between the turning points (open circles in Figs. 9 and 10).

In Figure 8 we plot the trajectories of the satellites for models 1–6. Both the x-y (upper panels) and x-z projections (smaller lower panels) are shown. The three trajectories plotted on the top vary in satellite mass, whereas those at the bottom vary in their initial orbital eccentricity (see Table 1). The time dependences of galactocentric radius, eccentricity, energy, and angular momentum for various models are shown in Figures 9 and 10. Energies are scaled by the central potential \( \Phi_0 \), and angular momenta by their value at \( t = 0 \). Eccentricities are plotted only up to \( t_{0.8} \) (see below), after which the satellite has virtually reached the halo’s center. Models 1 and 2 reveal an almost constant orbital eccentricity. In models 3–9, in which the satellite mass is equal to \( 2 \times 10^{10} M_\odot \), the eccentricity reveals a saw-tooth behavior, such that eccentricities decrease near pericenter and increase near apocenter. This is in perfect agreement with equation (33). It is remarkable that the net effect of \( dE/dt \) is nearly zero: the eccentricity does not change significantly. The alternative definition of eccentricity based on observed turning points (open circles) shows similar results. Small deviations are due to a change of the halo potential induced by the decaying satellite and the heuristic assignment of a time to the value found from monitoring the radial turning points. The absence of circularization in our simulations is not in agreement with the results of Tormen et al. (1998), who found that more massive satellites experience larger amounts of orbital circularization. This inconsistency is most likely due to the fact that Tormen et al. take mass loss due to tidal stripping into account and have their subhalos suffer from frequent encounters with other subhalos. We, on the other hand, have considered a single satellite of constant mass. In addition to these intrinsic differences, Tormen et al. plotted the change in the median orbital circularity of a number of satellites in a certain mass bin. As we discuss in § 3.3.2 below, the dynamical friction time is smaller for more eccentric orbits. Therefore, the mean eccentricity of a number of orbits with different initial eccentricities will always decrease, i.e., become more circular, as the more eccentric ones have shorter survival times. This, however, does not mean that any individual orbit undergoes circularization. It remains to be investigated in more detail if the circularization found by Tormen et al. is mainly due to their averaging procedure, is a result from mass loss owing to tidal stripping, whether it is caused by encounters with other satellites, or a combination of these effects.

The change in energy with time reveals a step-wise behavior, indicating that the pericentric passages dominate the satellite’s energy loss. Note that the energy of the satellites in models 3–9 does not become equal to \( \Phi_0 \) once the satellite reaches the center of the potential well. This owes to the deposition of energy into the halo particles by the satellite. The details of this process will be examined in a future paper.

Because of the elongation of the orbits, the galactocentric distance is not a meaningful parameter to use to characterize the decay times. Instead we use both the energy and the angular momentum. While the energy is well defined, it

![Fig. 8.—Orbits of the satellites in models 1–6. Both (larger panels) the x-y and (smaller panels) the x-z projections are shown. The solid dot in each panel indicates the initial position from which the satellite is started. Parameters for the different models are listed in Table 1.](image-url)
changes in an almost stepwise fashion (see Figs. 9 and 10). The angular momentum depends on the precise position of the halo's center, which may be poorly determined when the satellite induces an \( m = 1 \) mode. We define the following characteristic times: \( t_{0.4} \), \( t_{0.6} \), and \( t_{0.8} \), defined as the time when the satellite's energy reaches 40\%, 60\%, and 80\% of \( \Phi_0 \), respectively, and \( t_{3/4} \), \( t_{1/2} \), and \( t_{1/4} \), when the angular momentum is reduced to one-quarter, one-half, and three-quarters of its initial value. These timescales are listed in Table 2. Because of the instantaneous introduction of the

![Table 2](attachment:table2.png)

**Notes.**—The characteristic timescales in Gyr for dynamical friction as defined in § 3.3. Col. (1): Model ID. Cols. (2), (3), and (4): Characteristic timescales for energy loss. Cols. (5), (6), and (7): Characteristic timescales for angular momentum loss.
satellite in the virialized halo potential, the absolute values of these timescales may be off by a few percent. However, we are mainly interested in the variation of the dynamical friction time as function of the orbital eccentricity. We believe that the instantaneous introduction of the satellite will not have a significant influence on this behavior, as this is only a second-order effect.

3.3.1. Influence of Satellite Mass

Models 1, 2, and 3 start on the same initial orbit but vary in both the mass and size of the satellite. Satellite masses correspond to $2 \times 10^8 M_\odot$, $2 \times 10^9 M_\odot$, and $2 \times 10^{10} M_\odot$ for models 1, 2, and 3 respectively. The sizes of the satellites, e.g., their softening lengths, are set by equation (34), such that all satellites have the same mean density. The mass and size of the satellite in model 3 correspond closely to those of the LMC. It is clear from a comparison of models 1, 2, and 3 that dynamical friction by the Galactic halo is negligible for satellites with masses $\lesssim 10^9 M_\odot$. Thus, globular clusters and the dwarf spheroidals in the Galactic halo are not expected to have undergone significant changes in their orbital properties induced by dynamical friction by the halo; the only two structures in the Galactic halo that have experienced significant amounts of dynamical friction by the halo are the LMC and the SMC, with masses of $\sim 2 \times 10^{10} M_\odot$ and $\sim 2 \times 10^9 M_\odot$, respectively (Schommer et al. 1992). This is in excellent agreement with the results of Tormen et al. (1998). Note that we have neglected the action of the disk and bulge, which apply a strong torque on objects passing nearby. This can result in an enhanced decay of the orbit, not taken into account in the simulations presented here.

3.3.2. Influence of Orbital Eccentricity

Models 3, 4, 5, and 6 differ only in the eccentricity of their initial orbit; satellites on less eccentric orbits are started from smaller apocentric radii, such that the initial energy is the same in each case. The characteristic friction timescales (defined in § 3.3 and listed in Table 2) as a function of initial eccentricity are shown in Figure 11. The decay time decreases with increasing eccentricity, the exact rate of which depends on the characteristic time employed. The timescales for the most eccentric orbit (model 3) are a factor 1.5–2 shorter than for the circular orbit (model 6). This exemplifies the importance of a proper treatment of orbital eccentricities, since decay times based on circular orbits alone overestimate the timescales of dynamical friction for typical orbits in a dark halo by up to a factor of 2.

Using Chandrasekhar’s dynamical friction formula, and integrating over the orbits, Lacey & Cole (1993) found that, for a singular isothermal sphere, the dynamical friction time is proportional to $\eta^{0.78}$ (with $\eta$ the orbital circularity defined in § 2.1.1). The dotted lines in Figure 11 correspond to this dependence, normalized to $t_{0.6}$ and $t_{1.2}$ for the orbit with an intrinsic eccentricity of 0.6. As can be seen, our results seem to suggest a somewhat weaker dependence of the dynamical friction time on orbital eccentricity. Combining all the different characteristic decay times listed in Table 2, we obtain the best fit to our $N$-body results for $t \propto \eta^{0.53 \pm 0.01}$. This difference with respect to the results of
Lacey & Cole is most likely due to the fact that their computations ignore the global response of the halo to the decaying satellite.

### 3.3.3. Influence of Satellite Size

Since the adopted softening length of a satellite, $\epsilon_s$, is somewhat arbitrary (see § 3.2), we examine the effect of this choice by running a set of models (3, 7, 8, and 9) that differ only in this quantity. The characteristic friction timescales as a function of $\epsilon_s$ are shown in Figure 12. In Chandrasekhar's formalism, the dynamical friction timescale depends on the inverse of the Coulomb logarithm $\ln \Lambda$. This is often approximated by the logarithm of the ratio of the maximum and minimum impact parameters for the satellite. Taking $b_{\text{max}} = r_s = 200$ kpc and $b_{\text{min}} = \epsilon_s$, the expected ratios between the timescales of models 3, 7, 8, and 9 can be determined. The dotted lines in Figure 12 shows these predictions scaled to the characteristic times of model 3. It is apparent, however, that these simulations are not accurately represented by this simple approximation. This is not surprising as the choices for the minimum and maximum impact parameters are somewhat arbitrary. Because of the radial density profile of the halo, $b_{\text{max}}$ is likely to depend on radius. Furthermore, approximating $b_{\text{min}}$ by the satellite's softening length is appropriate only for large values of $\epsilon_s$; in the limit at which the satellite becomes a point mass, $b_{\text{min}}$ should be taken as $GM_s/\langle v^2 \rangle$, where $\langle v^2 \rangle^{1/2}$ is the rms velocity of the background, a quantity that again depends on the galactocentric distance of the satellite (White 1976b). The predictions in Figure 12 are thus as good as we might have hoped for. The important point we wish to make here is that deviations from the softening lengths given by equation (34), within reasonable bounds, do not significantly modify the characteristic dynamical friction timescales presented here.

### 3.3.4. Influence of Number of Halo Particles

In order to address the accuracy of our simulations, we have repeated model 4 with both 20,000 and 100,000 halo particles. We henceforth refer to these simulations as models 10 and 11, respectively (see Table 1). Both these models are first evolved for 10 Gyr without a satellite to let the halo virialize and reach equilibrium. As for the halo with 50,000 particles, the density distribution after 10 Gyr has not changed significantly.

A comparison of the results of models 4, 10, and 11 is shown in Figure 13. The solid lines correspond to model 4, the dotted lines to model 10 ($N = 20,000$), and the dashed lines to model 11 ($N = 100,000$). Clearly our results are robust, with only a negligible dependence on the number of halo particles. The main differences occur at later times, when the satellite has virtually reached the center of the halo. In models 4 and 11 the satellite creates a core in the halo, which causes $E/\Phi_0 < 1$ in the center. Model 10 has insufficient particles to resolve this effect, which explains the differences in both the satellite's energy and its eccentricity at later times with respect to models 4 and 11. In Table 2 we list the different characteristic timescales for model 10 and 11. They are similar to those of model 4 to an accuracy of $\sim 4\%$ for model 10 and $\sim 1\%$ for model 11. Finally we note
that our main conclusion, that there is no net amount of circularization, holds for different numbers of halo particles.

4. APPLICATIONS

4.1. Orbits of Globular Clusters

Odenkirchen et al. (1997, hereafter OBGT) used Hipparcos data to determine the proper motions of 15 globular clusters so that all six of their phase-space coordinates are known. Their velocity dispersions show a slight radial anisotropy with $(\sigma_r, \sigma_\theta, \sigma_z) = (127 \pm 24, 116 \pm 23, 104 \pm 20) \text{ km s}^{-1}$. OBGT integrated the orbits using a model for the Galactic potential (Allen & Santillan 1991) that approaches isothermality at large radii. They find a median eccentricity of $e = 0.62$ and conclude that the globulars are preferentially on orbits of high eccentricity.

We can compare their results with the eccentricity distributions expected for a power-law tracer population in a singular isothermal halo. We use the technique described in the Appendix to determine the distribution of orbital eccentricities given a logarithmic slope of the density distribution $\alpha$ and an anisotropy parameter $a$. We compare the cumulative distribution to OBGT's sample using the K-S test (see, e.g., Press et al. 1992). We determine the probabilities that the OBGT sample is drawn randomly from such a distribution using a $100 \times 100$ grid in $(\alpha, a)$ space with $\alpha \in [2, 6]$ and $a \in [-2, 2]$ and show the contour plot of these probabilities in Figure 14. The contours correspond to the 10%, 20%, ..., 80%, and 90% probability levels, whereby the latter is plotted as a thick contour. Clearly, the slope of the density distribution, $\alpha$, is poorly constrained as the dependence is mild (see §2.2). However, the velocity anisotropy is well constrained, and we find that a small amount of radial anisotropy is required in order to explain the observed eccentricities of the globular clusters. This is in excellent agreement with the velocity dispersions obtained directly from the data. The solid dot in Figure 14 corresponds to the best-fitting model with $\alpha = 3.5$, the value preferred by Harris (1976) and Zinn (1985). In Figure 15 we plot the cumulative distribution of the eccentricities from the OBGT sample (thin lines) with the cumulative distribution for the model represented by the dot in Figure 14. The K-S test yields a probability of 94.3% that the OBGT sample of orbits is drawn randomly from the probability distribution represented by the thick line.

We conclude that the distribution of eccentricities is just what one expects from a population with a mild radial velocity anisotropy; there is no “preference” for high-
eccentricity orbits as suggested by OBGT. The potential used by OBGT to integrate their orbits deviates significantly from a spherical isothermal in the center, where the disk and bulge dominate. Since the OBGT sample is limited to nearby globulars within approximately 20 kpc of the Sun, the deviations are likely significant. However, the good agreement between the distribution of eccentricities for the OBGT sample, based on the axisymmetric potential of Allen & Santillan (1991), and a spherical isothermal potential suggests that the differences between these two potentials have only a mild influence on the distribution of orbital eccentricities.

4.2. Kinematics in ω Centauri

Norris et al. (1997) examined the dependence of kinematics on metal abundance in the globular cluster ω Centauri (ω Cen) and found that the characteristic velocity dispersion of the most calcium-rich stars is \( \sim 8 \) km s\(^{-1}\), while that of the calcium-poor stars is \( \sim 13 \) km s\(^{-1}\). The metal-rich stars are located closer to the middle, where the velocity dispersion is largest, and the authors note that there is evidence for rotation in the metal-poor sample (at \( \sim 5 \) km s\(^{-1}\)) but not in the metal-rich sample. They use all these facts to conclude that “The more metal-rich stars in ω Cen are kinematically cooler than the metal-poorer objects.” The metal-rich stars in their sample live in the part of the cluster where the inferred circular velocity is greatest.

Hence, they have an average value of \( \mathbf{F} \cdot \mathbf{r} \) that is greater than that for the metal-poor stars. By the virial theorem, their mean kinetic energy must be higher (eq. [20]), yet Norris et al. find just the opposite, with the average kinetic energy of a metal-poor star being more than twice that of a metal-rich star.

Since we are not about to abandon the virial theorem, we can only conclude that, if the measurements of the dispersions are correct, the kinetic energy per metal-rich star must be at least that of the metal-poor stars, implying a much greater kinetic energy in the plane perpendicular to the line of sight than observed along the line of sight. The straightforward way for this to occur is a rotating disk seen face-on. The rotation must be large enough that \( \nu_\text{rot}^2 + 3\sigma^2 \) is at least as large as is seen for the metal-poor stars. This implies that the metal-rich stars have a rotation velocity of \( \gtrsim 18 \) km s\(^{-1}\). The metal-rich component of ω Cen must be a rotating disk that is more concentrated than the metal-poor stars. This is exactly the signature that Norris et al. would have ascribed to self-enrichment of the cluster (Morgan & Lake 1989). The rotation signature would be visible as a proper motion of 0′064 century\(^{-1}\). Rotation of a smaller magnitude has been detected in M22 by Peterson & Cudworth (1994). Note that a face-on disk in ω Cen, which is the Galactic globular with the largest projected flattening, implies a triaxial potential.

Norris et al. did not realize that their observations presented this dynamical puzzle. Instead, they believed that the difference could result from the relative radial profiles of the two components as might be seen in the equations of stellar hydrodynamics (eq. [19]); i.e., the density distribution of the metal-rich component must fall off more rapidly with radius.
than for the metal poor component. They used the observations to argue that ω Cen was the product of a merger of previous generations of substructure. However, we argue that such a merger would not have the signatures that they see. The metal-poor stars are the overwhelming majority of the stars. Conservation of linear momentum thus implies that the mean radius of the stars that were in the small (metal-rich) object will be greater than those that were in the large (metal-poor) one. Conservation of angular momentum furthermore implies that the rotation velocity of the stars that were in the small (metal-rich) object will be greater than those that were in the large (metal-poor) one. These signatures are exactly opposite of those claimed by Norris et al. to be consistent with the merger model.

4.3. Sinking Satellites and the Heating of Galaxy Disks

The sinking and subsequent merging of satellites in a galactic halo with an embedded thin disk has been studied by numerous groups (e.g., Quinn & Goodman 1986; Toth & Ostriker 1992; Toth, Quinn, Hernquist, & Fullager 1993; Walker, Mihos, & Hernquist 1996; Huang & Carlb erg 1997). The timescale for merging clearly depends on the eccentricity of its orbit. Once the satellite interacts with the disk, the sinking accelerates. Several of the above studies assumed that circularization owing to dynamical friction by the halo is efficient and examined satellites that started on circular orbits at the edge of the disk. However, we have shown that circularization is largely a myth; satellites will have large eccentricities when they reach the disk. Quinn & Goodman (1986) followed a satellite with a "typical" eccentricity in an isotropic singular isothermal sphere. They derived $e \approx 0.47$ or $r_e/r_\ast = 2.78$ for this "characteristic" orbit based on some poorly founded arguments. This ratio, however, is significantly smaller than the true median value of $r_e/r_\ast \sim 3.5$; approximately 63% of the orbits have $e > 0.47$. Huang & Carlberg (1997), in an attempt to be as realistic as possible when choosing the initial orbital parameters, started their satellite well outside the disk on an eccentric orbit. However, the eccentricity is only 0.2, clearly too low to be considered a typical orbit.

The eccentricity of the orbit has two effects; more eccentric orbits decay more rapidly in the halo and they touch the disk at an earlier time. The sinking timescales for the satellite caused by its interaction with the disk are more rapid when the difference in velocities between the satellite and the disk stars is smaller; satellites on prograde, circular orbits in the disk decay fastest (see Quinn & Goodman 1986 for a detailed discussion). Thus whereas more eccentric orbits reach the disk sooner, they are less sensitive to the disk interaction because of their high velocities at pericenter. Less eccentric orbits, on the other hand, require a longer time to reach the disk, but once they do their onward decay is rapid (if the orbit is prograde). The exact dependence of the timescales and disk heating on the orbital eccentricities awaits simulations.

We can compare our results to the few satellite orbits determined from proper motions. Johnston (1998) integrated the orbits of the LMC, Sagittarius, Sculptor, and Ursa Minor in the Galactic potential and found apocenter-to-pericenter ratios of 2.2, 3.1, 2.4, and 2.0, respectively. Using the K-S test, we find that these eccentricities are so small that there is only a 8.7% probability that this subsample is drawn from the isotropic model of an isothermal sphere. As shown in previous sections the distribution is relatively insensitive to the profiles of the underlying potential and the density distribution of the tracer population. This therefore implies that the velocity distribution is very strongly tangential. Since we expect that collapsed halos would produce states with dispersions that are preferentially radial, we suspect that the system of Galactic satellites has been strongly altered with satellites on more eccentric orbits having been destroyed owing to their faster dynamical friction timescales. In § 3, we found that more eccentric orbits have smaller dynamical friction timescales, but the effect is only modest (less than a factor two). Furthermore, all the satellites except the Magellanic clouds have masses $\lesssim 10^9 M_\odot$, and the dynamical friction owing to the halo is almost negligible for these systems. To get the strong effect that is seen, we have to appeal to the Milky Way disk to accelerate the decay and/or tidally disrupt the satellites on the more eccentric orbits. The problem with this solution, however, is that most studies of sinking satellites have shown that they lead to substantial thickening of the disk. Further studies are needed to investigate whether a disk can indeed yield the observed distribution of orbital eccentricities for the (surviving) satellites without being disrupted itself. We are currently using high-resolution N-body simulations to investigate this in detail. Finally, we must note that the sample of satellites with proper motions is small and the proper motions themselves have large errors, which bias results toward large transverse motions. Hence, we close this section with the all too common lament of the need for more data with more precision as well as better simulations that include the disk.

4.4. Tidal Streams

The tidal disruption of satellites orbiting in a galactic halo creates tidal streams (see, e.g., McGlynn 1990; Moore & Davis 1994; Oh, Lin, & Aarseth 1995; Piatek & Pryor 1995; Johnston, Spergel, & Hernquist 1995). These streams are generally long-lived features outlining the satellite’s orbit (Johnston, Hernquist, & Bolte 1996). Clearly, a proper understanding of the orbital properties of satellites is of great importance when studying tidal streams and estimating the effect they might have on the statistics of microlensing (see, e.g., Zhao 1998).

An interesting result regarding tidally disrupted satellites was reached by Kroupa (1997) and Klessen & Kroupa (1998). They simulated the tidal disruption of a satellite without dark matter orbiting a halo. After tidal disruption, a stream sighted along its orbit can have a spread in velocities that mimics a bound object with a mass-to-light ratio that can be orders of magnitude larger than the actual stellar mass-to-light ratio. They show that one can sight down such a stream only if the satellite was on an eccentric orbit. The chance of inferring such a large mass-to-light ratio thus increases with eccentricity. Klessen & Kroupa conclude that the high inferred mass-to-light ratios in observed dwarf spheroidal galaxies could occur from tidal streams (rather than from a dark matter halo surrounding the satellite) if the orbital eccentricities exceed $\sim 0.5$. We find that $\sim 60\%$ of the orbits obey this criterion even without any radial anisotropy (e.g., Fig. 3). We can thus not rule out satellites without dark matter based on the distribution of

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* Even if we adopt the maximum amount of tangential anisotropy allowed by our simple parameterization of $h, (\eta)$ (i.e., $\alpha \rightarrow \infty$), the K-S probability does not exceed 20%.
orbits. However, we wish to point out that a comparison of numerical simulations of the Sagittarius dwarf with the detailed kinematic data of this tidally disturbed galaxy leads Ibata & Lewis (1998) to conclude that this satellite has to be embedded in a massive dark halo.

4.5. Clusters of Galaxies

Using very high resolution cosmological N-body simulations, Ghigna et al. (1998) were able to resolve several hundred subhalos within a rich cluster of galaxies. They examined, among others, the orbital properties of these dark matter halos within the larger halo of the cluster and found that the subhalos followed the same distribution of orbits as the dark matter particles (i.e., those particles in the cluster that are not part of a subhalo). Ghigna et al. report a median apocenter-to-pericenter ratio of 6 (see also Tormen et al. 1998). We have used the orbital parameters of the subhalos of the cluster analyzed by Ghigna et al. to examine the orbital properties in some more detail. When we consider only subhalos whose apocenters are less than the virial radius of the cluster, \( r_{200} \), we obtain a sample of 98 halos with a median apocenter-to-pericenter ratio of 3.98. Using the K-S test as in § 4.1, we obtain a best fit to the distribution of orbital eccentricities for an anisotropy parameter of \( a \approx -0.04 \): the virialized region of the cluster is very close to isotropic. Throughout we consider only a tracer population with \( \alpha = 2 \), since this is in reasonable agreement with the observed number density of subhalos. Furthermore, the results presented here are very insensitive to the exact value of \( \alpha \), similar to what we found in § 4.1. In Figure 16 we plot the cumulative distribution of the orbital eccentricities of the 98 halos with \( r_+ < r_{200} \) (thin line) together with the same distribution for an isothermal sphere with our best-fitting anisotropy parameter \( a = -0.04 \). The K-S test yields a probability of 61.3% that the two data sets are drawn from the same distribution.

When analyzing all subhalos with \( r_+ < 2 r_{200} \), we obtain a sample of 311 halos with a median apocenter-to-pericenter ratio of 4.64 and a best-fitting value for the anisotropy parameter of \( a = -0.27 \). Clearly, the periphery of the cluster, which is not yet virialized, is more radially anisotropic with more orbits on more eccentric orbits.

The N-body simulations in § 3.2 are easily scaled to clusters of galaxies, considering the virialized regions of both galaxies and clusters. If we adopt a cluster mass of \( 10^{15} M_\odot \) and take the timescale to be the same as for the Milky Way simulation, the truncation radius becomes \( r_t = 2 \) Mpc. In § 3.3.1 we found that only objects with masses greater than \( \sim 0.1\% \) of the halo mass, \( M_{\text{halo}} \gtrsim 10^{12} M_\odot \) in the cluster, experience significant dynamical friction in a Hubble time, so, only the most massive galaxies experience any orbital decay.

Recently, Moore et al. (1996b) showed that high-speed encounters combined with global cluster tides—galaxy harassment—causes the morphological transformation of disk systems into spheroids (see also Moore, Lake, & Katz 1998b). Moore et al. limited themselves to mildly eccentric orbits with \( r_+ / r_0 = 2 \), but they noted correctly that this was a low value compared to the typical eccentricity. Their choice was made in order to be conservative and to underestimate the effect of harassment, as the effect increases with orbital eccentricity. They also felt that any larger value would stretch the reader’s credulity, as they could refer to no clear presentation of the likely distribution of orbital eccentricities. Our results imply that the effects of harassment were underestimated in their study.

4.6. Semianalytical Modeling of Galaxy Formation

Over the past couple of years several groups have developed semianalytical models for galaxy formation within the framework of a hierarchical clustering scenario of structure formation (see, e.g., Kauffmann, White, & Guiderdoni 1993; Cole et al. 1994; Heyl et al. 1995; Baugh, Cole & Frenk 1996; Somerville & Primack 1998). The general method of these models is to use the extended Press-Schechter formalism (Bower 1991; Bond et al. 1991; Lacey & Cole 1993) to create merging histories of dark matter halos. Simplified yet physical treatments are subsequently used to describe the evolution of the baryonic gas component in these halos. Using simple recipes for star formation and feedback, coupled to stellar population models, finally allows predictions for galaxies to be made in an observational framework.

A crucial ingredient of these semianalytical models is the treatment of mergers of galaxies. When two dark halos merge, the fate of their baryonic cores, i.e., the galaxies, depends on a number of factors. First of all, dynamical friction causes the galaxies to spiral to the center of the combined dark halo, thus enhancing the probability that the baryonic cores collide. Secondly, whether or not such a collision results in a merger depends on the ratio of the internal velocity dispersion of the galaxies involved to the encounter velocity. Both depend critically on the masses involved and on the orbital parameters. The dependence on the orbital eccentricities is addressed by Lacey & Cole.
order of 70% line friction times by approximately 10%.

where is the friction time for a circular orbit and is a in a singular isothermal sphere with anisotropy parameter . This average time is plotted as function of (see eq. [35]). The solid line corresponds to , consistent with the results from our numerical simulations, whereas the dashed line corresponds to , as predicted by Lacey & Cole (1993).

In the actual semianalytical modeling, the merger timescales are defined by simple scaling laws that depend on the masses only but ignore the orbital parameters. The eccentricity distributions presented here, coupled with the dependence of dynamical friction times on orbital eccentricity. Furthermore, as emphasized in § 4.3, our current understanding of the damaging effect of sinking satellites on thin disks is not well established and may have been overestimated in the past (Huang & Carlberg 1997).

In the actual semianalytical modeling, the merger timescales are defined by simple scaling laws that depend on the masses only but ignore the orbital parameters. The eccentricity distributions presented here, coupled with the dependence of dynamical friction times on orbital eccentricity, may prove helpful in improving the accuracy of the merging timescales in the semianalytical treatments of galaxy formation. As an illustrative example, we calculate the average dynamical friction time

where is the friction time for a circular orbit and is the normalized distribution function of orbital circularities in a singular isothermal sphere with anisotropy parameter . This average time is plotted as function of in Figure 17 (solid line). For comparison we also plotted the average times obtained by assuming a dependence (dashed line). The average dynamical friction time is typically on the order of 70%–80% of that of the circular orbit. The stronger dependence of Lacey & Cole underestimates the typical friction times by approximately 10%.

5. CONCLUSIONS AND DISCUSSION

This paper has presented the distributions of orbital eccentricities in a variety of spherical potentials. In a singular isothermal sphere, the median eccentricity of an orbit is , corresponding to an apocenter-to-pericenter ratio of 3.55. About 15% of the orbits have , whereas only 20% have moderately eccentric orbits with . These values depend strongly on the velocity anisotropy of the halo. Collapse is likely to create radially biased velocity anisotropies that skew the distribution to even higher eccentricities. We also examined the distributions of orbital eccentricities of isotropic tracer populations with a power-law density distribution ( ) and found only modest dependence on . Because of the unnatural physics of the singular isothermal sphere, we examined more realistic models and found that they differed only slightly from the isothermal case.

We stress that these eccentricity distributions apply only to systems in equilibrium. If a tracer population has not yet fully virialized in the halo’s potential, its orbital eccentricities can be significantly different from the virialized case. Cosmological simulations of galaxy clusters and satellite systems around galaxies show prolonged infall, and recent mergers can produce correlated motions. Hence, care must be taken in applying our results to such systems.

Objects with mass fractions greater than 0.1% experience significant orbital decay owing to dynamical friction. We used high-resolution N-body simulations with 50,000 particles to examine the sinking and (lack of) circularization of eccentric orbits in a truncated, nonsingular isothermal halo. We derived, and numerically verified, a formula that describes the change of eccentricity with time; dynamical friction increases the eccentricity of an orbit near pericenter but decreases it again near apocenter, such that no net amount of circularization occurs. Note, however, that we ignored tidal stripping of the satellite, which causes the satellite to loose mass and may result in orbital circularization (see § 3.3). The energy loss owing to dynamical friction is dominated by the deceleration at pericenter resulting in moderately shorter sinking timescales for more eccentric orbits. We find a dependence of the form ; the average orbital decay time for an isotropic, isothermal sphere is ~75% of that of the circular orbit. This dependence is weaker than the predictions of Lacey & Cole (1993), who found based on analytical integrations of Chandrasekhar’s dynamical friction formula. Since this analytical treatment ignores the global response of the halo to the decaying satellite, we believe our results to be more accurate. This relatively weak dependence of decay times on eccentricity, together with the absence of any significant amount of circularization, implies that dynamical friction does not lead to strong changes in the distribution of orbital eccentricities. When scaling the simulations to represent the orbiting of satellites in the Galactic halo, we find that the LMC and the SMC are the only objects in the outer Milky Way halo that have experienced significant amounts of energy and angular momentum loss by dynamical friction.

The distribution of orbital eccentricities is important for several physical processes, including timescales for the sinking and destruction of galactic satellites, structure and evolution of streams of tidal debris, harassment in clusters of galaxies, and mass estimates based on the dynamics of the system of globular clusters. The results presented here may prove particularly useful for improving the treatment of galaxy mergers in semianalytical models of galaxy formation. In § 4 we showed that the distribution of orbital eccentricities of a subsample of the Galactic globular cluster

$\langle t/t_0 \rangle = \int_0^1 d\eta \eta^{0.53} p_a(\eta)$,
system is consistent with that of a slightly radially anisotropic \( r^{-3.3} \) tracer population in an isothermal potential. A similar result was found for the subhalos of galaxies in a high-resolution, cosmological \( N \)-body simulation presented by Ghigna et al. (1998). However, the Milky Way satellites are not consistent with this distribution but show a bias toward circularity that may have been caused by dynamical friction and/or tidal disruption by the Galactic disk. We expect that additional data together with simulations that include the disk will lead to stringent constraints on the formation and evolution of substructure in the Milky Way.

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APPENDIX

A MONTE CARLO METHOD TO COMPUTE DISTRIBUTIONS OF ORBITAL ECCENTRICITIES IN A SINGULAR ISOHERMAL POTENTIAL

To determine the distribution of orbital eccentricities in a spherical potential, one must sample orbits according to the distribution function \( f(E, L) \). In this appendix, we describe a Monte Carlo method that samples a quasi-separable distribution function (eq. [7]). Once the energy and angular momentum of an orbit are known, the eccentricity is easily determined (eq. [10]). For a singular isothermal density distribution with a quasi-separable distribution function of the form given by equation (7), the density can be written as

\[
\rho(r) = \frac{4\pi}{r^2} \int_0^{\eta_{\text{max}}} dE g(E) L_\nu(E) \int_0^{\eta_{\text{max}}} \eta \, \frac{d\eta}{\sqrt{\eta^2 - \eta^2}}.
\]

The joint probability distribution of \((E, \eta)\) at a fixed radius \(r_0\) is therefore

\[
\mathcal{P}(E, \eta) = \frac{4\pi}{r_0 \rho(r_0) g(E) L_\nu(E)} \frac{\eta h(\eta)}{\sqrt{\eta_{\text{max}}^2 - \eta^2}},
\]

with \(E > \Phi(r_0)\) and \(\eta \in [0, \eta_{\text{max}}]\) (cf. van der Marel, Sigurdsson, & Hernquist 1997).

Because of the quasi-separable nature of the DF, the probability function for the energies \(E\) can be separated:

\[
\mathcal{P}(E) = \frac{4\pi}{r_0 \rho(r_0) g(E) L_\nu(E)} \frac{r_0}{u_\eta V_c^2} \exp \left( -\frac{E}{V_c^2} \right).
\]

The normalized, cumulative probability distribution of the energy is

\[
\hat{\mathcal{P}}(< E) = 1 - \exp \left[ \Phi(r_0) - \frac{E}{V_c^2} \right].
\]

Using the inversion of \(\hat{\mathcal{P}}(< E)\), energies are selected by drawing a random number \(R\) and assigning an energy

\[
E = \Phi(r_0) - V_c^2 \ln (1 - R).
\]

Once the energy is known, the maximum value of \(\eta\) corresponding to that energy can be calculated using equation (10). For our random number \(R\) this yields

\[
\eta_{\text{max}} = \sqrt{2} \exp \left( \frac{1}{2} \right) \sqrt{-\ln (1 - R)}.
\]

Note that \(\eta_{\text{max}}\) is independent of the radius \(r_0\).

In order to draw \(\eta\) from the probability distribution

\[
\mathcal{P}(\eta) = \frac{\eta h(\eta)}{\sqrt{\eta_{\text{max}}^2 - \eta^2}},
\]

we choose the comparison probability distribution

\[
\mathcal{P}_{\text{comp}}(\eta) = \frac{\eta_{\text{max}}}{\sqrt{\eta_{\text{max}}^2 - \eta^2}},
\]

which has the property \(\mathcal{P}_{\text{comp}}(\eta) \geq \mathcal{P}(\eta)\) for \(0 \leq \eta \leq \eta_{\text{max}}\), as long as \(h(\eta) \leq 1\). Trial values for \(\eta\) are drawn from the probability distribution \(\mathcal{P}_{\text{comp}}(\eta)\) by inversion of its normalized, cumulative probability distribution,

\[
\hat{\mathcal{P}}_{\text{comp}}(< \eta) = \frac{2}{\pi} \arcsin \left( \frac{\eta}{\eta_{\text{max}}} \right),
\]
and the rejection method (see, e.g., Press et al. 1992) is used to decide whether or not this trial value should be accepted.

The eccentricity of a random orbit in an anisotropic, singular isothermal sphere is thus obtained as follows.

1. Draw random numbers \( R_1 \) and \( R_2 \).
2. Calculate \( \eta_{\text{max}} = \sqrt{2 \exp (1)} \sqrt{1 - (R_1)^2} \) \( \sqrt{-\ln (1 - R_1)} \). If \( R_2 > \eta_{\text{max}} \), return to step 1. This takes care of the fact that \( \mathcal{P}(E) \) and \( \mathcal{P}(\eta) \) are not independent: \( \mathcal{P}(\eta) \) depends on \( \eta_{\text{max}} \), which is a function of energy.
3. Draw random numbers \( R_3 \) and \( R_4 \).
4. Calculate \( \eta_{\text{try}} = \eta_{\text{max}} \sin (2 R_3) \). If \( R_4 > \mathcal{P}_{\text{comp}}(\eta_{\text{try}})/\mathcal{P}(\eta_{\text{try}}) \), return to step 3
5. Accept \( \eta_{\text{try}} \) and compute the corresponding eccentricity by solving for the roots of equation (5).

When determining the distribution of orbital eccentricities for a power-law tracer population in a singular isothermal potential (see § 2.2), one can use the same method as outlined above but with the equation in step 2 replaced by

\[
\eta_{\text{max}} = \sqrt{2 \exp (1)} \sqrt{1 - (R_1)^2} \sqrt{-\ln (1 - R_1)},
\]

where \( \beta = 1/(\alpha - 1) \).

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