Effect of intermediate Minkowskian evolution on CMB bispectrum

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Abstract. We consider a non-inflationary early Universe scenario in which relevant scalar perturbations get frozen out at some point, but then are defrosted and follow a long nearly Minkowskian evolution before the hot era. This intermediate stage leaves specific imprint on the CMB 3-point function, largely independent of details of microscopic physics. In particular, the CMB bispectrum undergoes oscillations in the multipole $l$ space with roughly constant amplitude. The latter is in contrast to the oscillatory bispectrum enhanced in the flattened triangle limit, as predicted by inflation with non-Bunch-Davies vacuum. Given this and other peculiar features of the bispectrum, stringent constraints imposed by the Planck data may not apply. The CMB 3-point function is suppressed by the inverse duration squared of the Minkowskian evolution, but can be of observable size for relatively short intermediate Minkowskian stage.

Keywords: non-gaussianity, alternatives to inflation, cosmological parameters from CMBR

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1 Introduction

Recently released Planck data favor highly Gaussian primordial perturbations [1]. Although this is in agreement with predictions of the slow roll inflation [2], the picture of the early Universe is still uncertain. In particular, there remains a window for scenarios with relatively strong non-linearities. One reason is that the existing constraints apply to specific types of non-Gaussianity only; other shapes of correlation functions require separate data analyses. Second, one can imagine the situation where fairly large non-linear effects generated during some cosmological epoch get diluted at later times. We entertain both of these possibilities in this paper.

It is well known that the shapes of inflationary bispectra are strongly sensitive to the regime of evolution of scalar perturbations at the time when the non-linearities are at work [3]. Conventionally, it is assumed that perturbations start from vacuum initial conditions, get frozen out at some point and then remain unchanged until the hot era. One then classifies possible non-Gaussianities as follows. If perturbations are superhorizon at the time when non-linearities are important, like in the curvaton [4–8] or modulated reheating scenario [9–12], then the bispectrum is enhanced in the squeezed limit. See ref. [13] for a review. The equilateral bispectrum is characteristic of non-Gaussianities generated in the on-horizon regime [14–22]. There are also scenarios with inflaton fluctuations that start from unconventional (non-Bunch-Davies) initial conditions [23–32]. In this situation, relevant is the evolution in the subhorizon regime at inflation, and the typical outcome is the oscillatory bispectrum enhanced in the flattened triangle limit. Similar oscillatory bispectra originate
Figure 1. Schematic behavior of scalar perturbations. I. Generation epoch: \( \eta < \eta_* \). Towards the end of this epoch, perturbations get frozen (times \( \eta \sim \eta_1 \)) and have nearly flat power spectrum. II. Intermediate stage: \( \eta_* < \eta < \eta_{ex} \). Perturbations oscillate in nearly Minkowski space-time. III. Late stage: \( \eta > \eta_{ex} \). Perturbations are again frozen at time \( \eta = \eta_{ex} \). After \( \eta_{ex} \) perturbations evolve in the standard way (late evolution is not shown). The hot cosmological epoch starts some time after \( \eta_{ex} \).

It may happen that cosmological perturbations have more complicated history than in inflationary models. In this paper we consider the following scenario, see figure 1.

We assume that perturbations are generated at some early cosmological epoch, get frozen before the end of this epoch and have nearly flat power spectrum already at that time. Then there is an intermediate stage when space-time is nearly Minkowskian and perturbations are defrosted and oscillate. They get frozen again at the end of the intermediate stage and stay constant until the beginning of the hot epoch and later. Their further evolution is standard. It is the assumed existence of the intermediate Minkowskian stage that makes our scenario qualitatively different from others.

We note that by itself, the possibility of an early epoch with nearly Minkowski metric is nothing new. Such an epoch is characteristic of a number of alternatives to inflation, e.g., ekpyrotic models [39–42] and Galilean Genesis [43]. In these cases, however, perturbations get frozen once and for all at some time before the hot era, and in this sense their evolution is similar to that in the inflationary theory. Not surprisingly, shapes of resulting bispectra are also similar to ones obtained in versions of inflationary scenario [44–48]. In our case, however, we expect qualitatively distinct shapes of non-Gaussianities.

An example of cosmology with the intermediate Minowskian evolution of scalar perturbations is a version of conformal rolling scenario. In that scenario [42, 43, 49], one assumes that at early times, the space-time geometry is (effectively) Minkowskian, and the theory possesses conformal symmetry. This symmetry is spontaneously broken by a field \( \rho \) of non-zero conformal weight \( \Delta \), which homogeneously rolls away from the conformal point, \( \rho_c \propto |\eta|^{-\Delta} \), where \( \eta \) is the conformal time, \( \eta < 0 \). One also assumes that there is another field \( \theta \) of zero conformal weight. By conformal symmetry, the linear evolution of perturbations \( \delta \theta \) in the background \( \rho_c(\eta) \) is the same as that of massless scalar field in the de Sitter space-time. And the result is the same: perturbations \( \delta \theta \) get frozen at late rolling stage and have flat power spectrum then.

At some point, conformal symmetry becomes irrelevant, and \( \rho_c \) settles down to some condensate value. After that, the field \( \theta \) evolves as massless scalar field minimally coupled
to gravity. Further behavior of its perturbations depends on the cosmological evolution at that time. Generically, there are two sub-scenarios. One is that the modes of interest are already superhorizon in the conventional sense, and perturbations $\delta \theta$ remain frozen until the hot era. Non-linear effects in this sub-scenario have been considered in refs. [48, 50–53]. The situation of our primary interest occurs in the second sub-scenario. It assumes that perturbations $\delta \theta$ evolve non-trivially during long enough epoch between the end of conformal rolling and horizon exit [54]. The metric must be nearly Minkowskian at this stage, otherwise the flat power spectrum generated at conformal rolling would be grossly modified. This second sub-scenario is realized, e.g., if conformal rolling occurs and ends up early at the ekpyrotic contraction stage; the nearly Minkowskian evolution of perturbations $\delta \theta$ takes place in that case from the end of conformal rolling almost to the bounce. As deviations from the Minkowskian metric become strong enough, perturbations $\delta \theta$ leave the horizon, pass unaffected through the bounce and remain unchanged until the beginning of the hot era. At some time at the radiation dominated stage, they convert into adiabatic fluctuations. We assume that the latter literally inherit properties of perturbations $\delta \theta$.

Hereafter, we prefer not stick to any particular early Universe model and consider the general evolutionary picture of figure 1. In addition, we assume that perturbations $\delta \theta$ are massless and non-interacting during the intermediate Minkowskian stage. In this situation, they are related in a simple way to the scale-invariant amplitude $\delta \theta(k, \eta_*)$ existing at the beginning of the intermediate stage $\eta_*$, namely,\footnote{One may wonder if this simple form of the solution survives the horizon crossing at $\eta \sim \eta_{ex}$, when deviation from Minkowski metric becomes strong. The fact that this is indeed so, at least in some cosmological scenarios, has been demonstrated in appendix A of ref. [54]. There, the free propagation of perturbations $\delta \theta$ was considered in the Universe filled with matter with the super-stiff equation of state, $p = w \rho$, $w \gg 1$, like in ekpyrotic models. It was shown that in the limit $w \to \infty$, the evolution of perturbations $\delta \theta$ is effectively Minkowskian all the way up to times $\eta \to 0$, when perturbations $\delta \theta$ are in the superhorizon regime.}

$$\delta \theta(k, \eta) = \delta \theta(k, \eta_*) \cos k(\eta_* - \eta) .$$

Here $k$ is the conformal momentum of a scalar mode, $\eta$ is the conformal time running from $\eta_*$ to the horizon exit $\eta_{ex}$. Without loss of generality we set $\eta_{ex} = 0$, then after the horizon exit one has

$$\delta \theta(k) = \delta \theta(k, \eta_*) \cos k\eta_* .$$

Due to this relation, the form of the primordial bispectrum is related in a simple way to the initial shape function $A(k_1, k_2, k_3)$ generated before the Minkowskian evolution. It reads in terms of the primordial Newtonian potential $\Phi(k)$

$$\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle = \left( \frac{\pi P_\Phi}{2} \right)^{3/2} A(k_1, k_2, k_3) \delta \left( \sum_i k_i \right) \cos(k_1 \eta_*) \cos(k_2 \eta_*) \cos(k_3 \eta_*) .$$

(1.1)

Here $P_\Phi$ is the power spectrum of the scalar perturbation $\Phi(k)$, and we neglect the spectral tilt in what follows; delta function stands for momentum conservation; the normalization factor is chosen for future convenience. Until section 3 we do not concretize the form of the shape function $A(k_1, k_2, k_3)$, considering it as an arbitrary slowly varying function of momenta. This is to show that effects due to the Minkowskian evolution are fairly generic.

The modification of the bispectrum due to the intermediate Minkowskian stage shows up in the Cosmic Microwave Background (CMB) temperature fluctuations. One effect is the
suppression of the CMB bispectrum by the inverse duration squared of this stage. Accordingly, the bispectrum vanishes as the duration of this stage tends to infinity. On the other hand, the bispectrum can be of the observable size for sufficiently short Minkowskian evolution. There is another interesting manifestation: the shape of the 3-point function undergoes oscillations in the multipole $l$ space. These are characterized by nearly constant amplitude in the range $l_2 \leq l_3 \leq l_1 + l_2$ (with the convention $l_1 \leq l_2 \leq l_3$; the bispectrum vanishes for $l_3 > l_1 + l_2$), unlike in inflationary scenarios with non-Bunch-Davies initial conditions [23–30]. This makes us argue that known constraints derived from the Planck data are not directly relevant to the case we study in the present paper.

This paper is organized as follows. In section 2 we study the imprint of the bispectrum (1.1) on CMB. We give analytic calculations of the CMB bispectrum for the cases of sufficiently long and relatively short intermediate Minkowskian stage. Details of calculations can be found in appendices A, B, C and D. In section 3, we discuss the observability issues of the bispectrum with the Planck data. We summarize our findings in section 4.

2 Imprint on CMB

2.1 Generalities

In the multipole representation, CMB temperature coefficients are defined by

$$a_{lm} = \int d\mathbf{n} \delta T(\mathbf{n}) Y_{lm}^*(\mathbf{n}),$$

where $\delta T(\mathbf{n})$ is the temperature fluctuation in the direction $\mathbf{n}$ in the sky, and $Y_{lm}(\mathbf{n})$ are spherical harmonics. Coefficients $a_{lm}$ are related to the primordial Newtonian potential $\Phi(k)$ by

$$a_{lm} = 4\pi i^l \int \frac{dk}{(2\pi)^3/2} \Delta_l(k\eta_0) \Phi(k) Y_{lm}^*(\hat{k}),$$

where $\Delta_l(k\eta_0)$ are standard CMB transfer functions, and $\eta_0$ is the present horizon radius; $\hat{k}$ is the direction of the momentum $k$. Recall that $\Delta_l(y) \propto j_l(y)$, where $j_l$ is the spherical Bessel function. Now, using eq. (1.1) and performing the integration over the third momentum $k_3$, we obtain for the CMB bispectrum

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3} \rangle = i^{l_1+l_2+l_3} \frac{P_{\Phi}^{3/2}}{\eta_0^6} \int dy_1 dy_2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(|y_1 + y_2|) \times Y_{l_1m_1}^*(\theta_1, \phi_1) Y_{l_2m_2}^*(\theta_2, \phi_2) Y_{l_3m_3}^*(\theta_3, \phi_3) \times \cos(y_1z) \cos(y_2z) \cos((y_1 + y_2)z) A(y_1, y_2, y_3),$$

where the angles $(\theta_1, \phi_1)$, $(\theta_2, \phi_2)$ and $(\theta_3, \phi_3)$ correspond to directions of vectors $y_1$, $y_2$ and $y_3 = -(y_1 + y_2)$, respectively. Here we introduced notations

$$y_i \equiv k_i \eta_0, \quad z \equiv -\frac{\eta_*}{\eta_0},$$

and

$$A(y_1, y_2, y_3) \equiv \frac{A(k_1, k_2, k_3)}{\eta_0^6}. $$

For the sake of concreteness, we assume the ordering $l_1 \leq l_2 \leq l_3$ in what follows. And one more comment is in order, before we dig into details of calculations. The parameter $z$
is naturally large, since we are interested in the long Minkowskian evolution, meaning that $z \gg 1$. Leaving aside fine-tuning issues, however, the parameter $z$ is allowed to be not exceedingly large, so we will also consider the case $z \gtrsim 1$. Going into even smaller values of $z$ is most likely not legitimate, as one may ruin the prediction for the scale-invariant power spectrum.\footnote{In more detail, due to the presence of the intermediate stage, the power spectrum gets modified as $P_\Phi \propto \cos^2 y z$. On the observational side this implies order $1/(lz)$ correction to the angular power spectrum $C_l$. For intermediate stage with the duration $z$ larger than unity, the correction is within the error bars due to the cosmic variance $\sim 1/\sqrt{2 l + 1}$. For smaller $z$, however, this is no longer the case.} Note that $y z l \gg 1$ for interesting multipoles in any case.

Despite the complicated structure of the bispectrum, one can perform analytical integration over angles $\theta_i$ and $\phi_i$. To this end, we write the product of three rapidly oscillating cosines as follows
\begin{align}
\cos(y_1 z) \cos(y_2 z) \cos(y_3 z) = \frac{1}{4} \left[ \cos\{(y_1 + y_2 + y_3) z\} + \cos\{(y_1 + y_2 - y_3) z\} + \cos\{(y_1 - y_2 - y_3) z\} + \cos\{(y_2 - y_1 - y_3) z\}\right] , \tag{2.2}
\end{align}

where $y_3 = |y_1 + y_2|$. The contribution due to the first cosine in square brackets is always negligibly small. The reason is that this term rapidly oscillates for all relevant values of the parameter $z$ and CMB multipole numbers. Similar story happens with the contributions due to the other three terms, except for small regions where oscillations are damped. For concreteness, we pick the second cosine in the r.h.s. of eq. (2.2). Its argument has an extremum to the other three terms, except for small regions where oscillations are damped. For shorter duration of the intermediate stage, we should account for variations of spherical harmonics as well.

Depending on the duration of the intermediate stage, we distinguish two cases. For $z \ll l$, one keeps only the product of three rapidly oscillating cosines in eq. (2.1) when finding saddle points. For shorter duration of the intermediate stage, we should account for variations of spherical harmonics as well.

Conservatively, the two cases are separated at $z \sim l_3$, where $l_3$ is the largest multipole number, according to our conventions. In practice, the calculation neglecting the variation of the spherical harmonics can be extended down to $z \sim l_1$ (recall that $l_1 = \min(l_1, l_2, l_3)$). Let us argue for this. We first focus on the contribution due to the second term in eq. (2.2). We are free to integrate first over angles $\theta_1$ and $\phi_1$ associated with the smallest multipole number $l_1$. In that case, the spherical harmonic $Y_{l_1 m_1}$ is not in the game. Variation of the spherical harmonic $Y_{l_1 m_1}$ for sufficiently small values of $\delta \theta_1$ is estimated as $|\delta Y_{l_1 m_1}| \sim |l_1 Y_{l_1 m_1} \delta \theta_1|$. Hence, the inequality $|\delta Y_{l_1 m_1}| \ll |Y_{l_1 m_1}|$ holds in the region (2.4), provided that $z$ is greater

\begin{align}
|\delta \theta_1| \lesssim \frac{1}{\sqrt{N}} \quad \text{and} \quad |\delta \phi_1| \lesssim \frac{1}{\sin \theta_2 \sqrt{N}} , \tag{2.4}
\end{align}

where $N$ is the large dimensionless parameter
\begin{align}
N = \frac{y_1 y_2 z}{2(y_1 + y_2)} . \tag{2.5}
\end{align}
than $l_1$. The estimate for the variation of the spherical harmonic with the largest multipole number reads $|\delta Y_{l_3m_3}| \sim |l_3 Y_{l_3m_3} \delta \theta_3| \sim |l_1 Y_{l_3m_3} \delta \theta_1|$. The second estimate here follows from the triangle relation $k_1 + k_2 + k_3 = 0$. See also eqs. (A.1) and (A.2) of appendix A. We conclude that the variation of the spherical harmonic $Y_{l_3m_3}$ is also sufficiently small in the region (2.4) for $z$ larger than $l_1$.

This discussion is extrapolated to the contribution of the fourth cosine in eq. (2.2) in a straightforward manner. On the other hand, above argument does not work for the contribution of the third term there. However, it gives contribution negligible as compared to the other two in the case of the hierarchy between multipole numbers, $l_1 \ll l_3$. The reason is that the corresponding integral is saturated in the region $y_1 \approx y_2 + |y_1 + y_2| \equiv y_2 + y_3$ (i.e., $y_2 < y_1$ and vector $y_2$ anti-parallel to $y_1$). The transfer functions $\Delta_l(y_2) \propto j_l(y_2)$ and $\Delta_{l_3}(y_3) \propto j_{l_3}(y_3)$ do not vanish only at $y_2 \geq l_2 + 1/2$ and $y_3 \geq l_3 + 1/2$, respectively. Both the shape function $A(y_1, y_2, y_3)$ and transfer function $\Delta_l(y_1)$ are suppressed for $y_1 \approx y_2 + y_3 > l_3 \gg l_1$. Also, the product of the three transfer functions is a rapidly oscillating function of variables $y_i$. These effects lead to quite strong suppression of the term we discuss, see appendix B for details.

We consider the case of long intermediate stage, i.e. $z \gg l_1$, in the following subsection. We generalize the results to the case of arbitrary $z$ (but still $z \gtrsim 1$) in subsection 2.3.

### 2.2 Saddle-point calculation: regime $z \gg l_1$

We first consider the case of sufficiently long intermediate stage, $z \gg l_1$. In this situation, we neglect variations of spherical harmonics and find the saddle points directly from eq. (2.3). We focus on the contribution due to the second cosine in the r.h.s. of eq. (2.2); contributions due to the third and fourth cosines are then obtained by permuting multipole numbers $(l_1, m_1)$ and $(l_2, m_2)$ with $(l_3, m_3)$. The saddle point is at $\delta \theta_1 = 0$ and $\delta \phi_1 = 0$ in terms of variables $\delta \theta_1 = \theta_1 - \theta_2$ and $\delta \phi_1 = \phi_1 - \phi_2$. When integrating in the vicinity of the saddle point, we expand the bispectrum (2.1) in a series in $1/z$.

Naively, the first term in this expansion is of the order $1/z = -\partial_1 / \eta$. However, the suppression is actually stronger. To show this, we consider the inner integral over angles $\theta_1$ and $\phi_1$ in eq. (2.1), write the product of three cosines as in eq. (2.2) and pick the second cosine in the r.h.s. there. To the leading order in $1/z$ we neglect the dependence on $\delta \theta_1$, $\delta \phi_1$ of all other factors in the integrand in eq. (2.1). Using eq. (2.3), we obtain

$$\int d\theta_1 d\phi_1 \cos \left[ \frac{y_1 y_2 z}{2(y_1 + y_2)} \left( \delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2 \right) \right] = \frac{2(y_1 + y_2)}{y_1 y_2 z \sin \theta_2} \left[ \left( \int_{-\infty}^{+\infty} du \cos u^2 \right)^2 - \left( \int_{-\infty}^{+\infty} du \sin u^2 \right)^2 \right] = 0 .$$

The last equality follows from the relation

$$\int_{-\infty}^{+\infty} \cos u^2 du = \int_{-\infty}^{+\infty} \sin u^2 du .$$

We see that the leading contribution to the bispectrum is of the order $1/z^2$. To calculate the 3-point function to this order, one expands all slowly varying functions of $\theta_1$, $\phi_1$ in eq. (2.1) up to terms quadratic in $\delta \theta_1$ and $\delta \phi_1$. At the same time, the argument of the rapidly oscillating cosine must be expanded up to orders $N(\delta \theta_1)^4$, $N(\delta \phi_1)^4$ and $N(\delta \theta_1)^2(\delta \phi_1)^2$. See
appendix A for explicit formulae. It is convenient to split the integral in eq. (2.1) into the sum of two contributions,

\[
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_0 + \delta \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle.
\]

(2.6)

The first term in the r.h.s. involves variations of all functions, except for \(A(y_1, y_2, y_3)\) and \(\Delta_{l_3}(y_3)\). By writing it with the subscript 0, we anticipate that it gives the dominant contribution to the 3-point function. The second term in the r.h.s. accounts for variations of functions \(A(y_1, y_2, y_3)\) and \(\Delta_{l_3}(y_3)\). The expression for the main contribution to the order \(1/z^2\) is given by

\[
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_0 = i^{l_1 + l_2 + l_3} \cdot \frac{\pi P_\Phi^{3/2}}{4z^2} \cdot \int dy_1 dy_2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \times
\]

\[
\times A(y_1, y_2, y_1 + y_2) \cdot B_{l_2 m_2 l_3 m_3}^{l_1 m_1} \cdot \left\{ y_2^2 l_1(l_1 + 1) + y_1^2 l_2(l_2 + 1) - 2y_1 y_2 \sqrt{l_1(l_1 + 1)} \sqrt{l_2(l_2 + 1)} B_{l_2 l_3}^{l_1 l_1} \left( B_{l_2 l_3}^{l_1 l_1} \right)^{-1} + 2y_1 y_2 \right\} +
\]

\[
+ (l_1, m_1 \leftrightarrow l_3, m_3) + (l_2, m_2 \leftrightarrow l_3, m_3);
\]

(2.7)

we refer the reader to appendix A for details of calculations. Constants \(B_{l_2 m_2 l_3 m_3}^{l_1 m_1}\) entering the integrand in eq. (2.7) are given by

\[
B_{l_2 m_2 l_3 m_3}^{l_1 m_1} = \int d\phi d\theta \sin \theta \ Y_{l_1 m_1}^* (\theta, \phi) Y_{l_2 m_2} (\theta, \phi) Y_{l_3 m_3}^* (\pi - \theta, \pi + \phi).
\]

(2.8)

They can be expressed via the Wigner 3j-symbols,

\[
B_{l_2 m_2 l_3 m_3}^{l_1 m_1} = (-1)^{l_1} \sqrt{\frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \\ m_1 & m_2 & m_3 \end{pmatrix}.
\]

(2.9)

Note that the term in the bispectrum written explicitly in eq. (2.7) is symmetric under the interchange of multipole numbers \((l_1, m_1)\) and \((l_2, m_2)\), as it should. Furthermore, its form exhibits statistical isotropy, since this is the property of coefficients \(B_{l_2 m_2 l_3 m_3}^{l_1 m_1}\); see appendix A for discussion of this point. These two observations serve as cross-checks of our computations.

As we pointed out in the end of subsection 2.1, eq. (2.7) can be used down to \(z \sim l_1\), even though the saddle point calculation of the term with permutation \((l_1, m_1) \leftrightarrow (l_3, m_3)\) is not justified for \(z \sim l_1\) and \(l_1 \ll l_3\). In fact, the argument in the end of subsection 2.1 suggests that this term (the first term in the last line of eq. (2.7)) is suppressed at \(l_1 \ll l_3\). We check this explicitly in appendix B.

The correction to the bispectrum due to variations of shape and transfer functions \(A(y_1, y_2, y_3)\) and \(\Delta_{l_3}(y_3)\) is given by

\[
\delta \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \sqrt{2}^{l_1 + l_2 + l_3} \cdot \frac{\pi P_\Phi^{3/2}}{4z^2} \cdot \int dy_1 dy_2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \times
\]

\[
\times A(y_1, y_2, y_1 + y_2) \cdot B_{l_2 m_2 l_3 m_3}^{l_1 m_1} \times y_1 y_2 \cdot \left( \frac{\partial \ln A(y_1, y_2, y_3)}{\partial \ln y_3} + \frac{d \ln \Delta_{l_3}(y_3)}{d \ln y_3} \right)_{y_3 = y_1 + y_2} + (l_1, m_1 \leftrightarrow l_3, m_3) + (l_2, m_2 \leftrightarrow l_3, m_3).
\]

(2.10)
Since $A(y_1, y_2, y_3)$ is a slowly varying function of its arguments, we estimate the corresponding term as $\partial \ln A / \partial \ln y_3 \sim 1$. The estimate for the second term in parenthesis of eq. (2.10) reads

$$\frac{d \ln \Delta_{l_3}(y_3)}{d \ln y_3} \sim y_3 \cdot \frac{|\Delta_{l_3-1}(y_3) - \Delta_{l_3+1}(y_3)|}{|\Delta_{l_3}(y_3)|} \lesssim y_3,$$

which follows from the fact that rapidly varying part of $\Delta_l(y)$ is proportional to the spherical Bessel function $j_l(y)$ and from the relation for the derivative of the spherical Bessel function. Using crude estimate $y_i \sim l_i$, we conclude that the contribution (2.10) is suppressed as compared to one in eq. (2.7) by at least a factor of $l_i^{-1}$.

Equations (2.7) and (2.10) give the explicit expression for the CMB 3-point function to the order $1/z^2$.

### 2.3 Saddle-point calculation: generic case

If the duration of the intermediate stage is not particularly large, spherical harmonics can no longer be treated as smooth functions. Consequently, eq. (2.7) is not valid in this regime. Still, we employ the saddle point technique with the modification that we now search for saddle points of the product of the rapidly oscillating cosines and spherical harmonics. To simplify the problem, we use the approximate form of spherical harmonics valid for large multipole numbers $l \gg 1$,

$$Y_{lm}(\theta, \phi) = \frac{1}{\pi \sin \theta} \cos \left[ \left( l + \frac{1}{2} \right) \theta - \frac{\pi m}{2} + \frac{\pi m}{2} \right] e^{im\phi} + O \left( \frac{1}{l} \right). \quad (2.11)$$

Clearly, this approximation enables one to perform calculations to the leading order in multipole numbers $l_i$ only. Sketch of calculations, tedious but fairly straightforward, is given in appendix C. Here we point out the main technical aspect: due to variations of spherical harmonics, saddle points in terms of the variable $\delta \theta_1 \equiv \theta_1 - \theta_2$ get shifted to $\delta \theta_1 \sim \frac{1}{z}$. This is consistent with the previous subsection: we have for the saddle point value that $\delta \theta_1 \to 0$ as $z \to \infty$. The final expression for the CMB bispectrum reads

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = l_1^{l_1} l_2^{l_2} l_3^{l_3} \cdot \frac{\pi^3/4}{4} \cdot B_{l_2 m_2 l_3 m_3}^{l_1 m_1} \times$$

$$\times \int d y_1 d y_2 \cdot y_1^2 \cdot y_2^2 \cdot \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \cdot A(y_1, y_2, y_1 + y_2) \times$$

$$\times \frac{1}{N} \cdot \sin \left\{ \frac{1}{2} \cdot \frac{y_1^2 l_1^2 + y_2^2 l_2^2}{y_1 y_2 (y_1 + y_2) z} \right\} \cdot \cos \left\{ \frac{l_1 l_2}{(y_1 + y_2) z} \right\} \times$$

$$\times \left[ 1 - \cot \left\{ \frac{1}{2} \cdot \frac{y_1^2 l_1^2 + y_1^2 l_2^2}{y_1 y_2 (y_1 + y_2) z} \right\} \cdot \tan \left\{ \frac{l_1 l_2}{(y_1 + y_2) z} \right\} \cdot \frac{B_{l_3, 1 l_3, 0}^{l_1, 1}}{B_{l_3, 1 l_3, 0}^{l_1, 1}} \right\}$$

$$+ (l_1, m_1 \leftrightarrow l_3, m_3) + (l_2, m_2 \leftrightarrow l_3, m_3),$$

where the large parameter $N$ is given by eq. (2.5). Let us comment on this formula. First, in the limit $z \gg l_1$ we get back to the expression (2.7), as we should. This is a simple cross-check of our computation. Second, in the case $1 \ll z \ll l_1$ the suppression factor is $1/N \sim 1/(l_1 z)$ as compared to $1/z^2$ in eq. (2.7). This implies stronger squeezing of the bispectrum for relatively small $z$. 

− 8 −
2.4 Flattened triangle limit and away from it

The 3-point function given by eq. (2.7) or eq. (2.12) has several peculiar features. Our first observation concerns the behaviour of the bispectrum in the flattened triangle regime where \( l_1 + l_2 - l_3 \ll l_1 \) (recall our convention \( l_1 \leq l_2 \leq l_3 \)). For the sake of concreteness, we specify to the case of long intermediate stage studied in subsection 2.2. In the large \( l \) approximation, the expression in curly brackets in eq. (2.7) takes the form

\[
I^2 \cdot I_2^2 \cdot \left( \frac{y_1}{l_1} - \frac{y_2}{l_2} \right)^2 + 2y_1y_2 \cdot I_1I_2 \cdot \left[ 1 - B_{12;1;0}^{1;1}(B_{120;10}^{1;0})^{-1} \right]. \tag{2.13}
\]

Borrowing results from the next section, we note that the first term here is suppressed due to the effective relation \( y_1/l_1 = y_2/l_2 \) which holds in the flattened triangle regime modulo corrections of the order \( 1/\sqrt{l_1} \). Partial cancellation occurs also in the second term. It is due to the relation

\[
B_{12;1;0}^{1;1} = \frac{\sqrt{l_1(l_1 + 1)}\sqrt{l_2(l_2 + 1)}}{(l_1 + 1)(l_2 + 1)} B_{120;10}^{1;0},
\]

which follows from the analogous relation between the \( 3j \)-symbols. Thus, our bispectrum is suppressed by the factor \( \sim 1/l_1 \), as compared to naive estimate, for configurations with \( l_1 + l_2 - l_3 \ll l_1 \).

The bispectrum is suppressed also away from the flattened triangle limit, but for different reason. The source of this suppression is the integral over three transfer functions, i.e.

\[
\int dy_1dy_2\Delta_{l_1}(y_1)\Delta_{l_2}(y_2)\Delta_{l_3}(y_1 + y_2)\ldots.
\]

We will perform the detailed analysis of this integral in section 3. Here let us make a qualitative observation. The integral over variables \( y_i \) is saturated roughly at \( y_i \sim l_i \). Somewhat loosely, we set arguments of transfer functions at \( l_1, l_2 \) and \( l_1 + l_2 \). Then the bispectrum is roughly proportional to the product \( \Delta_{l_1}(l_1)\Delta_{l_2}(l_2)\Delta_{l_3}(l_1 + l_2) \). The last factor here, \( \Delta_{l_3}(l_1 + l_2) \propto j_{l_3}(l_1 + l_2) \), vanishes for \( l_1 + l_2 < l_3 \) and undergoes rapid oscillations as function of \( l_1 + l_2 - l_3 \) with the amplitude decaying away from the flattened triangle limit. Numerical estimates of section 3 roughly agree with these expectations. Indeed, we will observe rapid oscillations naturally traced back to ones in the transfer functions. The qualification is that their amplitude is nearly constant in the interval \( l_2 \leq l_3 \leq l_1 + l_2 \), rather than decreasing. This is due to the cancellation in the flattened triangle limit we discussed above.

3 Estimates and observational consequences

So far we have found indications for several potentially interesting features: oscillatory behaviour of the bispectrum, suppression both in the flattened triangle limit and away from it. The purpose of the present section is to demonstrate the interplay between these effects. Another purpose is to find the range of the parameter \( z \) where the bispectrum is of observable size. To this end, we specify to the standard local form of the bispectrum generated before the long Minkowskian evolution,

\[
A(k_1, k_2, k_3) = C\frac{\sum_i k_i^3}{\Pi_i k_i^3}, \quad A(y_1, y_2, y_3) = C\frac{\sum_i y_i^3}{\Pi_i y_i^3}. \tag{3.1}
\]
Here the constant $C$ is some combination of parameters inherent in the theory operating before the intermediate stage. Note that the bispectrum of the form (3.1) is a generic prediction of a number of early Universe scenarios including curvaton [12] and conformal Universe [48] models.\footnote{More precisely, in both cases the form of the shape function is $A(y_1, y_2, y_3) \propto \frac{y_3^2}{m_3^2} \left( \frac{y_1}{y_3} - 2 \sum_{i=1}^{2} \frac{y_i}{m_i} - \frac{1}{4} (\gamma + N) \frac{y_3^2}{m_3^2} \right)$, where $y_i = y_1 + y_2 + y_3, \quad \gamma = 0.577$ is the Euler constant, and $N$ is a logarithmic factor. The latter is naturally large. Therefore, the last term in parenthesis dominates, implying the approximate form of the shape function as in eq. (3.1).} In both cases, the constant $C$ is not necessarily small, and can be as large as 10 or 100. Even larger values, however, are at risk of strong coupling between perturbations before the intermediate Minkowski stage.

Because of the statistically isotropic form of eqs. (2.7) and (2.12), it is convenient to introduce reduced bispectrum $b_{l_1 l_2 l_3}$ defined by

$$
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = (-1)^{l_3} P_{l_2 m_2 l_3 m_3}^{l_1 m_1} b_{l_1 l_2 l_3} .
$$

We also make the following simplifications. First, we ignore the correction in eq. (2.6) and focus on the main contribution (2.7), where we neglect terms with permutations. The latter are indeed small, as shown in appendix B. Somewhat loosely, we also set functions $g_l^T (y) = \Delta_l (y) / j_l (y)$ equal to a constant in the integrands in eqs. (2.7) and (2.12), namely,

$$
g_l^T (y) \rightarrow g_{SW}^T \approx 1 / 3 . \quad (3.2)
$$

Although this approximation works only for relatively small multipole numbers from the Sachs-Wolfe (SW) plateau of the CMB angular power spectrum, it is sufficient for our illustrational purposes. The rest of computations can be found in appendix D. Final results are presented for the cases of long and short intermediate stage in subsections 3.1 and 3.2, respectively.

### 3.1 Regime $z \gg l_{\text{min}}$

First, we consider the case of long intermediate stage, $z \gg l_1$. Using eq. (2.7) and following steps outlined in appendix D, we obtain the estimate for the reduced bispectrum

$$
|b_{l_1 l_2 l_3}| \sim \frac{C}{30} \cdot \frac{P_{l_1}^{3 / 2}}{z^2} \frac{|c_{l_1 l_2 l_3}|}{l_1^{5 / 2} \cdot l_2^{1 / 2} \cdot l_3^{3 / 2} \sqrt{l_3}} , \quad (3.3)
$$

where the quantities $c_{l_1 l_2 l_3}$ are negligible for $l_3 > l_1 + l_2$ and otherwise are given by

$$
c_{l_1 l_2 l_3} = l_1 \left[ 1 + \frac{1}{2} \frac{\Delta l^2 / l_1^2}{1 - \Delta l / l_1} - \frac{P_{l_1 l_2 l_3}^{l_1, l_2, l_3}}{P_{l_1 l_2 l_3}^{l_1, l_2, l_3}} \right] F(l_1, \Delta l) . \quad (3.4)
$$

Here $\Delta l \equiv l_1 + l_2 - l_3$ and the function $F(l_1, \Delta l)$ involves the integral over the variable $u_1 \equiv \frac{y_1}{l_1 + l_1}$,

$$
F(l_1, \Delta l) = \left( 1 - \frac{\Delta l}{l_1} \right)^2 \int_1^\infty \frac{d u_1}{u_1^{9 / 2} / \left[ u_1^2 - 1 \right]^{1 / 4} \left( u_1^2 - \left( 1 - \frac{l_1}{\Delta l} \right)^2 \right]^{1 / 4}} \times (3.5)
\times \left[ \cos \left( l_1 f(u_1) - (l_1 - \Delta l) f(\bar{u}_2) \right) - \sin \left( l_1 f(u_1) - (l_1 - \Delta l) f(\bar{u}_2) \right) \right] ,
$$
Figure 2. Structure coefficients $c_{l_1 l_2 l_3}$ defined by eq. (3.4) as functions of $\Delta l \equiv l_1 + l_2 - l_3$ for $l_2 = 600$, $l_1 = 50$ (left) and $l_1 = l_2 = 600$ (right).

Figure 3. Quantity $F(l_1, \Delta l)$ defined by eq. (3.5) as functions of the difference $\Delta l \equiv l_1 + l_2 - l_3$ for $l_1 = 50$ (left) and $l_1 = 600$ (right).

where $\bar{u}_2 = \frac{l_1}{l_1 - l_2} u_1$. The non-trivial part of the bispectrum is encoded in the structure coefficients $c_{l_1 l_2 l_3}$. We plot them in figure 2 as functions of the difference $\Delta l$. For each plot, we keep smaller multipole numbers $l_1$ and $l_2$ fixed and vary the larger one $l_3$, and, consequently, $\Delta l = l_1 + l_2 - l_3$. The main property we observe are oscillations with roughly constant amplitude in the interval $l_2 \leq l_3 \leq l_1 + l_2$. To make things clearer, we also plot the function $F(l_1, \Delta l)$ and the ratio $c_{l_1 l_2 l_3}/F(l_1, \Delta l)$ in figures 3 and 4, respectively. Figure 3 demonstrates the tendency of the amplitude to decrease away from the flattened triangle limit, i.e. with growing $\Delta l$, while figure 4 exhibits the opposite tendency. The two compensate each other and we get the nearly constant amplitude in the end. Another effect seen from figure 2 is the suppression of the bispectrum by a factor $l_1^{-5/2}$ as compared to the situation where the intermediate stage is absent. Indeed, the estimate for coefficients $c_{l_1 l_2 l_3}$ is roughly $c_{l_1 l_2 l_3} \sim 1$. Hence, the bispectrum (3.3) scales as $b_{l_1 l_2 l_3} \propto l_1^{-5/2}$, while the estimate for the local bispectrum is $b_{l_1 l_2 l_3}^{loc} \sim l_1^{-2}$. So, our bispectrum is squeezed stronger than the seed local one. This is a generic feature of models with intermediate Minkowskian stage.

Finally, we would like to address the issue of the observability of our bispectrum with the Planck data. To this end, we consider the Fisher matrix [13]

$$F_{ij} \equiv \sum_{l_1 \leq l_2 \leq l_3} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_i}{\sigma_{l_1 l_2 l_3}^2} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_j}{\sigma_{l_1 l_2 l_3}^2}.$$ (3.6)

Here $\sigma_{l_1 l_2 l_3}^2$ is given by $\sigma_{l_1 l_2 l_3}^2 = C_{l_1} C_{l_2} C_{l_3} D_{l_1 l_2 l_3}$ where $D_{l_1 l_2 l_3}$ takes values 1, 2 and 6, when all $l$’s are different, two are the same and three are the same, respectively. Superscripts $i, j$ refer to particular sources of the non-Gaussianity, i.e. primordial physics, point sources, weak
lensing, Sunyaev-Zel’dovich, emission etc. In what follows, we neglect all sources except for primordial physics, so that the Fisher matrix reduces to a single element. Still, we stick to the standard notion of a ‘matrix’.

To evaluate the sum over numbers \( m_i \) in eq. (3.6), we use the relation

\[
\sum_{m_1 m_2 m_3} \left( \frac{l_1}{m_1} \frac{l_2}{m_2} \frac{l_3}{m_3} \right)^2 = 1.
\]

We estimate the angular power spectrum as \( C_l \sim 6 \cdot 10^{-10}/l^2 \), and 3j-symbols as

\[
\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \sim \frac{1}{\sqrt{l_1 l_2}}.
\]

Now, using eq. (3.3) with \( c_{l_1 l_2 l_3} \sim 1 \), and summing over multipole numbers \( l_1, l_2 \) and \( l_3 \), we obtain an estimate for the Fisher matrix

\[
F \sim \frac{C^2}{1000 \cdot z^4} \frac{l_2^2}{l_0}.
\]  

(3.7)

Here \( \bar{l} \) and \( l_0 \) are the maximum and the minimum multipole numbers used in the analysis, respectively. Note that the dominant contributions to \( F \) come from squeezed configurations with \( l_3 \approx l_2 \sim \bar{l} \) and \( l_1 \sim l_0 \). We set \( \bar{l} = 2000 \) and \( l_0 = 10 \) for estimates. The observation of the bispectrum in the CMB data is possible provided that the Fisher matrix is larger than unity, i.e. \( F \gtrsim 1 \). From eq. (3.7), we obtain phenomenologically interesting window for the parameter \( z \)

\[
l_0 \lesssim z \lesssim 5\sqrt{|C|}.
\]

When writing the left inequality we recalled that the analysis of this subsection is valid only for large enough values of the parameter \( z \), i.e. \( z \gg l_{\text{min}} \sim l_0 \sim 10 \). Therefore, non-vanishing window for the parameter \( z \) in the regime considered requires the constant \( C \) larger than about 100, which we find rather contrived. Things are more optimistic in the situation with relatively short intermediate stage.

### 3.2 Regime \( 1 \lesssim z \lesssim l_{\text{min}} \)

For \( 1 \lesssim z \lesssim l_{\text{min}} \), the estimate (3.3) for the reduced bispectrum is replaced by the following one

\[
|b_{l_1 l_2 l_3}| \sim \frac{C}{30} \frac{P_{\Phi}^{3/2}}{z} \frac{\sqrt{3}}{l_1^{7/2} l_2^{5/2}} |c_{l_1 l_2 l_3}|,
\]

(3.8)
with coefficients $c_{l_1 l_2 l_3}$ given by

$$
c_{l_1 l_2 l_3} = l_1 \left( \frac{1}{l_1} \right) \int_1^\infty \frac{du_1}{u_1^{7/2}} \cdot \frac{1}{u_1^2 - 1}^{1/4} \cdot \frac{1}{u_1^2 - \left( 1 - \frac{\Delta l}{l_1} \right)^2}^{1/2} \times$$

$$
\times \left( \cos[l_1 f(u_1) - (l_1 - \Delta l) f(\bar{u}_2)] - \sin[l_1 f(u_1) - (l_1 - \Delta l) f(\bar{u}_2)] \right) \times$$

$$
\times \sin \left\{ \frac{1}{2} \frac{l_1 l_2}{u_1 l_3^2} \left[ 1 + \left( 1 - \frac{\Delta l}{l_1} \right)^2 \right] \right\} \cdot \cos \left\{ \frac{l_1 l_2}{u_1 l_3^2} \left( 1 - \frac{\Delta l}{l_1} \right) \right\} \times$$

$$
\times \left[ 1 - \cot \left\{ \frac{1}{2} \frac{l_1 l_2}{u_1 l_3^2} \left[ 1 + \left( 1 - \frac{\Delta l}{l_1} \right)^2 \right] \right\} \cdot \tan \left\{ \frac{l_1 l_2}{u_1 l_3^2} \left( 1 - \frac{\Delta l}{l_1} \right) \right\} \cdot \frac{B_{l_2 l_3}^{1,1}}{B_{l_2 l_3}^{10,10}} \right\}
$$

(3.9)

where $\Delta l > 0$ ($c_{l_1 l_2 l_3}$ are again negligible for $\Delta l < 0$). Though this expression has more complicated structure, the qualitative behaviour of coefficients $c_{l_1 l_2 l_3}$ is fairly similar to that in subsection 3.1. This can be readily seen from figure 5. Again we observe oscillations with the roughly constant order one amplitude. Equation (3.8) implies stronger squeezing as compared to the case with the long intermediate stage.

The estimate for the Fisher matrix changes accordingly,

$$
F \sim \frac{C^2}{1000 \cdot \bar{l}^2 \cdot \bar{l}_0^2} \cdot \frac{l_2^2}{l_0^2}
$$

(3.10)

Substituting $\bar{l} = 2000$ and $l_0 = 10$, we obtain a phenomenologically interesting window for the parameter $z$

$$
1 \lesssim z \lesssim |C|
$$

(3.11)

existing now for $C \gtrsim 1$, which is, of course, welcome from the viewpoint of microscopic physics. For even smaller values $|C| \lesssim 1$, however, the bispectrum tends to be of the non-observable size. It is tempting to relax this constraint by extrapolating the analysis down to $l_0 = 2$. In that case the phenomenologically interesting window could be widened by about an order of magnitude. In turn, this would imply an order of magnitude increase in the sensitivity to the parameter $C$. Recall, however, that we are working in the large-$l$ approximation. Therefore, our calculations and, in particular, the estimate (3.10) for the Fisher matrix, are not justified for values of $l_0$ as small as 2.
4 Conclusion

Let us summarize the effects due to the presence of the intermediate Minkowskian evolution:

- The bispectrum is suppressed by the duration of the intermediate stage. We observed that the bispectrum behaves as $\eta_0^2/|\eta_*|^2$ for long enough duration $|\eta_*|$ and as $\eta_0/|\eta_*|$ provided that the duration is relatively short;

- The bispectrum vanishes for $l_1 + l_2 < l_3$ and undergoes oscillations as function of $\Delta l = l_1 + l_2 - l_3 > 0$ (our convention is $l_1 \leq l_2 \leq l_3$). The origin of oscillations can be traced back to oscillations in the CMB transfer functions relating the temperature fluctuation to the Newtonian potential. Normally, these oscillations are averaged out upon integrating over directions of momenta $k$. In our case this does not happen because the bispectrum of scalar perturbations is saturated in the region where $k_2$ is almost parallel to $k_1$.

- Oscillations are characterized by roughly constant amplitude. This fact is quite non-trivial. Indeed, direct translation of the relation $k_3 \approx k_1 + k_2$ into the harmonic space naively implies the preference of flattened triangle configurations obeying the relation $l_3 \approx l_1 + l_2$. This tendency is indeed there, as is clearly seen from figure 3. Unexpectedly, the opposite tendency is also present: there are cancellations occurring for configurations obeying flattened triangle relation. The interplay between the two tendencies is such that the amplitude gets smoothened, and we do not observe any preference of flattened over non-flattened configurations or vice versa. This feature is important for discriminating the models we discussed in this paper from inflationary scenarios, some of which predict the oscillatory behaviour of the bispectrum analogous to ours. For example, shapes somewhat similar to one in eq. (1.1) are considered in refs. [23–30]. These are, however, characterized by the amplitude strongly peaked in the flattened triangle limit.

- In the situation with the long intermediate stage, there is squeezing of the CMB bispectrum as compared to the case where the intermediate stage is absent. Squeezing is particularly strong if the duration of the stage is smaller than the minimum multipole number $l_1$.

We also discussed the sensitivity of the Planck experiment to the bispectrum predicted. We concluded that there is a window for the duration of the intermediate stage, though not particularly wide, where the non-Gaussianity can be of the observable size. Note that the signal of interest, if detected in the CMB data, would not imply the specific early Universe model, but rather indicate the presence of the long Minkowskian evolution before the hot era. This could be of particular importance from the viewpoint of alternatives to inflation, e.g. ekpyrotic scenarios and Galilean Genesis.

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We also expand the cosine,

\[ \cos((y_1+y_2) z) = \cos(N[(\delta \theta_1)^2 + \sin^2 \theta_2 (\delta \phi_1)^2]) \left( 1 - \frac{1}{8} N^2 \sin^2 2\theta_2 (\delta \theta_1)^2 (\delta \phi_1)^4 \right) - \]

\[ - \sin(N[(\delta \theta_1)^2 + \sin^2 \theta_2 (\delta \phi_1)^2]) N \left[ -\frac{\delta \theta_1^4}{12} - \sin^2 \theta_2 \frac{\delta \phi_1^4}{12} \right. \]

\[ - \frac{1}{2} \sin^2 \theta_2 \delta \theta_1^2 \delta \phi_1^2 + \frac{1}{2} \sin 2\theta_2 \delta \theta_1 \delta \phi_1^2 + \frac{y_1 y_2}{4(y_1+y_2)^2} \left( \delta \theta_1^4 + 2 \sin^2 \theta_2 \delta \theta_1^2 \delta \phi_1^2 + \sin^4 \theta_2 \delta \phi_1^4 \right) \]  

(A.3)
Combining these expressions and using integrals
\[
\int_{-\infty}^{+\infty} \cos N x^2 dx = \int_{-\infty}^{+\infty} \sin N x^2 dx = \frac{\sqrt{\pi}}{\sqrt{2}}, \\
\int_{-\infty}^{+\infty} x^2 \cos N x^2 dx = -\int_{-\infty}^{+\infty} x^2 \sin N x^2 dx = -\frac{\sqrt{2\pi}}{4N^{3/2}}, \\
\int_{-\infty}^{+\infty} x^4 \cos N x^2 dx = \int_{-\infty}^{+\infty} x^4 \sin N x^2 dx = -\frac{3\sqrt{2\pi}}{8N^{5/2}},
\]
we obtain for the leading contribution,
\[
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_0 = \frac{\pi P_5^{3/2}}{4\pi^2} \int dy_1 dy_2 (y_1 + y_2)^2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \times
\]
\[
A(y_1, y_2, y_1 + y_2, z) \times
\]
\[
B_{l_1, m_1}^{l_2, m_2, l_3, m_3} \left[ l_1 (l_1 + 1) + \frac{y_1^2}{(y_1 + y_2)^2} l_3 (l_3 + 1) + \frac{2y_1 y_2}{(y_1 + y_2)^2} \right] +
\]
\[
\frac{2y_1}{y_1 + y_2} F_{l_2, m_2, l_3, m_3}^{l_1, m_1}.
\]
Here coefficients $B_{l_1, m_1}^{l_2, m_2, l_3, m_3}$ are given by eq. (2.8), while quantities $F_{l_2, m_2, l_3, m_3}^{l_1, m_1}$ are defined by
\[
F_{l_2, m_2, l_3, m_3}^{l_1, m_1} = -\int d\theta d\phi \sin \theta Y_{l_2}^{* m_2} \left( \frac{\partial Y_{l_1}^{* m_1}}{\partial \theta} \frac{\partial Y_{l_3}^{* m_3}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_{l_1}^{* m_1}}{\partial \phi} \frac{\partial Y_{l_3}^{* m_3}}{\partial \phi} \right).
\]
When deriving eq. (A.4), we used the master equation for spherical harmonics,
\[
\frac{\partial^2 Y_{l m}}{\partial \theta^2} + \cot \theta \cdot \frac{\partial Y_{l m}}{\partial \theta} - \frac{m^2}{\sin^2 \theta} Y_{l m} + l(l + 1) Y_{l m} = 0. \tag{A.5}
\]
Note that eq. (A.4) does not look symmetric under interchange of multipole numbers $(l_1, m_1) \leftrightarrow (l_2, m_2)$. To obtain the bispectrum in the symmetric form we get rid of derivatives of the spherical harmonic $Y_{l_3}^{* m_3}(\theta, \phi)$ by performing integration by parts in eq. (A.4). We also make use of the master equation (A.5) to get rid of the term $l_3 (l_3 + 1) Y_{l_3 m_3}^{*}$. Again performing the integration by parts, we obtain
\[
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle_0 = i^{l_1 + l_2 + l_3} \frac{\pi P_5^{3/2}}{4\pi^2} \int dy_1 dy_2 \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \times
\]
\[
A(y_1, y_2, y_1 + y_2) \left\{ 2y_1 y_2 D_{l_2, m_2, l_3, m_3}^{l_1, m_1} +
\right.
\]
\[
B_{l_1, m_1}^{l_2, m_2, l_3, m_3} \left[ y_1^2 l_1 (l_1 + 1) + y_2^2 l_2 (l_2 + 1) + 2y_1 y_2 \right] \right\} + (l_1, l_2 \leftrightarrow l_3),
\]
Coefficients $D_{l_2, m_2, l_3, m_3}^{l_1, m_1}$ are defined by
\[
D_{l_2, m_2, l_3, m_3}^{l_1, m_1} = \int d\phi d\theta \sin \theta \left[ \frac{\partial Y_{l_1}^{* m_1}}{\partial \theta} \frac{\partial Y_{l_2}^{* m_2}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_{l_1}^{* m_1}}{\partial \phi} \frac{\partial Y_{l_2}^{* m_2}}{\partial \phi} \right] Y_{l_3 m_3}^{*}(\pi - \theta, \pi + \phi). \tag{A.7}
\]
Note that eq. (A.6) is symmetric under the interchange of multipole numbers $(l_1, m_1) \leftrightarrow (l_2, m_2)$.
It is worth noting that eq. (A.6) represents statistically isotropic bispectrum. Namely, the dependence of the latter on numbers \( m_i \) obeys the condition [13]

\[
\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3} \rangle = \sum_{m'_1m'_2m'_3} (a_{l_1m'_1}a_{l_2m'_2}a_{l_3m'_3}) D^{(l_1)}_{m'_1m_1} D^{(l_2)}_{m'_2m_2} D^{(l_3)}_{m'_3m_3} . \tag{A.8}
\]

Here \( (a_{l_1m'_1}a_{l_2m'_2}a_{l_3m'_3}) \) denotes the bispectrum obtained by the rotation of the celestial sphere by Euler angles \( \alpha, \beta \) and \( \gamma \); coefficients \( D^{(l)}_{m'm} = D^{(l)}_{m'm} (\alpha, \beta, \gamma) \) are matrix elements of the rotation matrix \( D(\alpha, \beta, \gamma) \), i.e.

\[
Y_{lm'}(\theta', \phi') = \sum_{m=-l}^{l} D^{(l)}_{m'm} Y_{lm}(\theta, \phi) .
\]

Equation (A.8) is satisfied, since this is the property of coefficients \( B^{l_1m_1}_{l_2m_2;l_3m_3} \) and \( D^{l_1m_1}_{l_2m_2;l_3m_3} \). Indeed, we write the former as in eq. (2.9) and then recall the well known property of the 3j-symbols [13]

\[
\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \sum_{m'_1m'_2m'_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m'_1 & m'_2 & m'_3 \end{pmatrix} D_{m'_1m_1}^{(l_1)} D_{m'_2m_2}^{(l_2)} D_{m'_3m_3}^{(l_3)} .
\]

Clearly, the derivative structure in the integrand of eq. (A.7) also preserves statistical isotropy. This determines the dependence of coefficients \( D^{l_1m_1}_{l_2m_2;l_3m_3} \) on \( m_i \):

\[
D^{l_1m_1}_{l_2m_2;l_3m_3} = \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} D_{l_1l_2l_3} \, ; \tag{A.9}
\]

coefficients \( D_{l_1l_2l_3} \) introduced here are independent of numbers \( m_i \). We use eq. (A.9) to derive coefficients \( D^{l_1m_1}_{l_2m_2;l_3m_3} \) in terms of 3j-symbols. Namely, we evaluate the integral (A.7) for \( m_1 = m_2 = m_3 = 0 \). To this end, we use the following relations for derivatives of spherical harmonics,

\[
\frac{\partial Y_{l0}(\theta)}{\partial \theta} = \sqrt{l(l+1)} Y_{l,1}(\theta, \phi)e^{-i\phi} = -\sqrt{l(l+1)} Y_{l,-1}(\theta, \phi)e^{i\phi} .
\]

Substituting them into eq. (A.7), using eq. (2.9) with \( m_1 = -m_2 = 1 \) and \( m_3 = 0 \) and eq. (A.9), we obtain coefficients \( D_{l_1l_2l_3} \). Substituting the latter back into eq. (A.9), but with arbitrary numbers \( m_i \), we derive the expression of interest,

\[
D^{l_1m_1}_{l_2m_2;l_3m_3} = -\sqrt{l_1(l_1+1)} \sqrt{l_2(l_2+1)} B^{l_1l_1-1}_{l_2l_2l_2} B^{l_2l_2}_{l_2l_2l_2} B^{l_1l_1}_{l_2m_2;l_3m_3} \, . \tag{A.10}
\]

Finally, substituting these into eq. (A.6), we obtain the term written explicitly in eq. (2.7).

B Comments on terms with permutations in eq. (2.7)

In this appendix, we comment on terms with permutations of multipole numbers in the last line of eq. (2.7). We consider the case \( l_3 \sim l_2 \gg l_1 \gg 1 \) and start with the term with

\[\text{We thank D. Karateev for the numerical check of this somewhat heuristic statement.}\]
interchange \((l_1, m_1) \leftrightarrow (l_3, m_3)\),

\[
\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3} \rangle |_{l_1+l_2+l_3} = i^{l_1+l_2+l_3} \frac{\pi P_{l_1}^{3/2}}{4z^2} \int dy_2 dy_3 \Delta_l(y_2 + y_3) \Delta_{l_2}(y_2) \Delta_{l_3}(y_3) \times \\
A(y_2 + y_3, y_2, y_3) B_{l_2}^{l_1m_1} B_{l_3}^{l_2m_2} B_{l_1}^{l_3m_3} \cdot \left\{ y_2^2 l_3(l_3 + 1) + y_2^2 l_2(l_2 + 1) - 2y_2y_3 \sqrt{l_2(l_2 + 1)\sqrt{l_3(l_3 + 1)}} B_{l_2l_3}^{l_10} B_{l_20l_3}^{l_10} \right\}^{-1} + 2y_2y_3 \}
\]

Due to the properties of the transfer functions, the integration range here is effectively \(y_2 \geq l_2 + 1/2, y_3 \geq l_3 + 1/2\). Naively, eq. (B.1) gives the contribution enhanced by a factor \(\sim l_3^2/l_3^3\) as compared to one written explicitly in eq. (2.7). Note, however, that generically shape functions \(A(y_1, y_2, y_3)\) are suppressed for configurations with \(y_1 \approx y_2 + y_3 \gg l_1\) and \(y_2, y_3 \sim l_3\) as compared to configurations with \(y_1 \sim l_1, y_2 \sim l_2 \sim l_3\) and \(y_3 \approx y_1 + y_2 \sim l_3\). In particular, for the standard local shape bispectrum as in eq. (3.1) one has

\[
A^{\text{loc}}(y_2 + y_3, y_2, y_3) |_{y_2, y_3 \sim l_3} \sim \frac{l_1^3}{l_3} \cdot A^{\text{loc}}(y_1, y_2, y_1 + y_2) |_{y_2 \sim y_3 \sim l_3}.
\]

Furthermore, there is the suppression by a factor \(\sim l_1/l_3\) in the transfer function \(\Delta_l(y_2 + y_3)\), i.e.

\[
\Delta_l(y_2 + y_3) |_{y_2 \sim y_3 \sim l_3} \sim \frac{l_1}{l_3} \cdot \Delta_l(y_1) |_{y_1 \sim l_1}.
\]

This follows from the approximate form of the spherical Bessel function (D.1). Finally, for configurations \(y_1 \approx y_2 + y_3\), there is no saddle point in the product of three transfer functions. This gives extra suppression by a factor \(l_1^{3/2}/l_3^2\) as compared to the explicit term in eq. (2.7).

In total, we have the estimate for the contribution to the reduced bispectrum due to the permutation \((l_1, m_1) \leftrightarrow (l_3, m_3)\),

\[
b_{l_1}^{l_1+l_2+l_3} \sim b_{l_1}^{l_1l_2l_3} \cdot \frac{l_1^{5/2}}{l_3^2}.
\]

Two remarks are in order. First, we used the local type of the shape function \(A(y_1, y_1, y_3)\), eq. (3.1), in our estimate. For other types of shape functions, the suppression is generally milder, but it is still there. In particular, the suppression is of the order \(\sqrt{l_1}/l_3\) for the equilateral type of the bispectrum generated before the intermediate stage. Second, we assumed long Minkowskian stage, \(z \gg l_3\). For smaller values of the parameter \(z\), i.e. \(l_1 \ll z \leq l_3\), the expression in the r.h.s. of eq. (B.1) should be replaced by eq. (2.12) (with multipole numbers \((l_1, m_1)\) and \((l_3, m_3)\) permuted). Clearly, additional suppression by a factor \(z/l_3\) takes place in that case. The latter becomes of the order \(l_1/l_3\) for even smaller duration of the intermediate stage, i.e. for \(z \lesssim l_1\).

Finally, let us comment on the term with permuted multipole numbers \((l_2, m_2)\) and \((l_3, m_3)\) in eq. (2.7). It is also suppressed, albeit mildly, as compared to one written explicitly in eq. (2.7). The reason is that the product of the three transfer functions is a rapidly oscillating function for all relevant values of variables \(y_3\). That is, it does not have the saddle point akin to the case discussed in the main text. As a result, we have an estimate for the term of interest,

\[
b_{l_1}^{l_1+l_2+l_3} \sim \frac{1}{\sqrt{l_1}} b_{l_1}^{l_1l_2l_3}.
\]

This estimate works for all relevant values of the parameter \(z\).

Estimates (B.2) and (B.3) justify neglecting terms with permutations in section 3.
C Details of saddle-point calculation in the generic case

Here we comment on the derivation of expression (2.12). To simplify calculations, we use the same trick as in appendix A and calculate the bispectrum for $m_1 = m_2 = m_3 = 0$. Using the property of statistical isotropy, we multiply the result by $B_{l_2m_2,l_3m_3}/B_{l_20,l_30}$ and obtain the bispectrum valid for arbitrary $m_i$.

Using approximate formula (2.11) with $m_1 = m_2 = m_3 = 0$, we write for the product of three spherical harmonics and the cosine

$$Y_{l_10}(\theta_1)Y_{l_20}(\theta_2)Y_{l_30}(\theta_3) \cos \left( N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) =$$

$$= \frac{1}{4} Y_{l_10}(\theta_2)Y_{l_20}(\theta_2)Y_{l_30}(\theta_3) \times$$

$$\times \left[ \cos \left( l_1 \delta \theta_1 + l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) + \cos \left( l_1 \delta \theta_1 - l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) +$$

$$+ \cos \left( l_1 \delta \theta_1 - l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) + \cos \left( l_1 \delta \theta_1 - l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) -$$

$$- \cos \left( l_1 \delta \theta_1 - l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) - \cos \left( + l_1 \delta \theta_1 - l_3 \delta \theta_3 + N[\delta \theta_1^2 + \sin^2 \theta_2 \delta \phi_1^2] \right) +$$

$$+ \ldots .$$

Here dots stand for the combination of terms which is antisymmetric under $\delta \theta_1 \rightarrow -\delta \theta_1$ and vanishes when integrated over $\theta_1$. Saddle point of the first cosine in the third line is at

$$\delta \theta_1 = \frac{y_1(l_3 - l_1) - l_1 y_2}{y_1 y_2 z} .$$

Similar expressions hold for saddle points of other cosines. Now, performing the integration in the vicinity of these saddle points, we obtain

$$\langle a_{l_10}a_{l_20}a_{l_30} \rangle = i^{l_1 + l_2 + l_3} \cdot \frac{\pi P^3_\Phi}{4} \cdot B_{l_20,l_30} \times$$

$$\times \int dy_1 dy_2 \cdot y_1^2 \cdot y_2^2 \cdot \Delta_{l_1}(y_1) \Delta_{l_2}(y_2) \Delta_{l_3}(y_1 + y_2) \cdot A(y_1, y_2, y_1 + y_2) \times$$

$$\times \frac{1}{N} \sin \left\{ \frac{1}{4N} \left( l_1^2 + \frac{y_1^2 y_3^2}{(y_1 + y_2)^2} \right) \right\} \cdot \cos \left\{ \frac{1}{2N} \cdot \frac{y_1 l_1 l_3}{y_1 + y_2} \right\} \times$$

$$\left[ 1 - \cot \left\{ \frac{1}{4N} \left( l_1^2 + \frac{y_1^2 y_3^2}{(y_1 + y_2)^2} \right) \right\} \right] \cdot \tan \left\{ \frac{1}{2N} \cdot \frac{y_1 l_1 l_3}{y_1 + y_2} \right\} \times$$

$$\left( \frac{l_1 + l_2}{l_3} \cdot B_{l_2, -1; l_30}^{l_1,1} (B_{l_30}^{l_10})^{-1} \right) \times$$

$$+ (l_1, m_1 \leftrightarrow l_3, m_3) + (l_2, m_2 \leftrightarrow l_3, m_3) .$$

Finally, integrating by parts and multiplying the result by $B_{l_2m_2,l_3m_3}^{l_1m_1}/B_{l_20,l_30}^{l_10}$, we obtain the expression (2.12) from the main body of the text.

D Comments on the derivation of the estimates (3.3) and (3.8)

In this appendix, we comment on the derivation of the estimates (3.3) and (3.8). For this purpose, we take the integrals over the variables $y_1$ and $y_2$ present in eqs. (2.7) and (2.12).
With the simplification (3.2), transfer functions $\Delta_l(y)$ become proportional to spherical Bessel functions $j_l(y)$. An excellent approximation to spherical Bessel functions in the regime $y > l + 1/2$ is given by

$$j_l(y) = \frac{1}{(l + \frac{1}{2})} \frac{1}{\sqrt{u}} \frac{1}{\sqrt{u^2 - 1}} [ (l + \frac{1}{2}) f(u) - \frac{\pi}{4} ] ,$$  

(D.1)

where

$$u = \frac{y}{l + 1/2}$$

and

$$f(u) = \sqrt{u^2 - 1} - \arccos \left( \frac{1}{u} \right) .$$

In the region $y < l + 1/2$, i.e. for $u < 1$, the spherical Bessel function is exponentially suppressed and we can safely neglect the contribution coming from this region. We evaluate the integral in eq. (2.7) or eq. (2.12) over the variable $u_2$ first. Once again, we employ the saddle point technique. This is legitimate because of the large-$l$ factor in the argument of cosine in eq. (D.1). In the integrand, the product of spherical Bessel functions can be written as follows

$$j_{l_2}(y_2)j_{l_3}(y_3) \approx \frac{1}{2l_2l_3[u_2^3 - u_3^3] \cdot (u_2^3 - 1) \cdot (u_3^3 - 1)]^{1/4} \cos [ l_2 f(u_2) - l_3 f(u_3) ] ,$$  

(D.2)

where

$$u_3 = \frac{l_1}{l_3} u_1 + \frac{l_2}{l_3} u_2 .$$

and we dropped the term with the second cosine which is a rapidly oscillating function for all values of $u_2$. The saddle point of the expression (D.2) is at

$$\bar{u}_2 = \bar{u}_3 = \frac{l_1}{l_3 - l_2} u_1 .$$

We expand the argument of the cosine in eq. (D.2) up to the quadratic term in $\Delta u_2 \equiv u_2 - \bar{u}_2$

$$[ l_2 f(u_2) - l_3 f(u_3) ] = [ l_2 f(\bar{u}_2) - l_3 f(\bar{u}_3) ] + \frac{l_2 (l_3 - l_2)^4}{l_1^3 l_3 u_1^2 \sqrt{u_1^2 - (l_3 - l_2)^2}} \frac{\Delta u_2^2}{l_1^2} .$$

The integral over $u_2$ is now evaluated in a straightforward manner. Final estimates for the bispectrum are presented in the main body of the article in terms of the remaining integral over the variable $u_1$. They are given by eqs. (3.3), (3.4) and (3.5) for the case of long intermediate stage and by eqs. (3.8) and (3.9) for sufficiently short intermediate stage.

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