Relaxation of non-order parameter field in directed Ising systems

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We investigate the effect of initial conditions on the dynamic exponents of the interacting monomer-monomer model with infinitely many absorbing states in one dimension. This model exhibits a directed Ising (DI) type transition from an active phase into an absorbing phase. In case of the directed percolation universality class, it has been reported that the non-order parameter as well as the order parameter exhibits critical fluctuations, relaxing algebraically to its natural value with the same scaling exponents. We numerically confirm that this is also valid for the DI universality class. We also observe continuously varying dynamic exponents with a linear dependence on the non-order parameter initial density.

Various kinds of nonequilibrium lattice models exhibiting absorbing phase transitions have been studied extensively during last few decades [1]. Two distinct types of absorbing phase transitions have been identified in one dimension: the directed percolation (DP) and directed Ising (DI) universality class. Most models have been found to belong to the DP class, which involves typically a single absorbing state or multiple absorbing states without any symmetry. DI-type critical behavior appears in models with two equivalent absorbing states or two equivalent group of absorbing states [2,3].

Recently, the question has been addressed whether one can construct initial states that affect the entire temporal evolution of nonequilibrium systems including asymptotic dynamics. Typical examples are the systems that display an absorbing phase transition from an active phase into an absorbing phase with infinitely many absorbing (IMA) states. As in the ordinary order-disorder phase transition models, the order parameter takes a nonzero value in the active phase and vanishes in the absorbing phase. The IMA states can be characterized by a non-order parameter. In the steady state, the non-order parameter approaches algebraically to a natural value at criticality.

Nonuniversal dynamic properties have been reported in various kinds of DP-type nonequilibrium lattice models with IMA states [4,5]. The order-parameter dynamic exponents vary continuously with the initial conditions characterized by the non-order parameter density. Moreover, at criticality, the non-order parameter density also exhibits critical fluctuations, relaxing algebraically to its natural value with the same dynamic exponents [6]. The critical relaxation of the non-order parameter field in DP-type systems with IMA states was recently examined from a field theoretical (Langevin equation) approach [7]. The evolution equation for the order parameter plays the same role in DI systems as well as DP systems.

In this paper, we introduce an interacting monomer-monomer model with IMA states (IMA-IMA model) that belongs to the DI universality class and investigate the effects of the non-order parameter field in one dimension. Using numerical simulations, we show that the non-order parameter plays the same role in DI systems as well as DP systems.

The IMA-IMM model is an interacting monomer-monomer model with two different species of monomers, namely A and B. Monomers A and B are selected with probability p and 1 − p respectively. Monomers can adsorb at a randomly selected vacant site with unit probability for A and 1 − rB for B respectively. An adsorption attempt of a monomer is rejected when both sites adjacent to a selected vacant site are occupied or at least one adjacent site is occupied with a monomer of the same species. A nearest neighbor AB pair reacts and desorbs immediately from the lattice.

\[ A + V \xrightarrow{p} A_s, \]
\[ B + V \xrightarrow{(1-p)(1-r_B)} B_s, \]
\[ A_s + B_s \rightarrow \emptyset. \] (1)

Here the subscript s denotes adsorbed particles, V denotes a vacant site where the adsorption attempt is allowed.

This model has infinitely many absorbing states. Any configuration without a nearest neighbor pair of vacant sites is one of the absorbing states. The number of absorbing states diverges exponentially with system size L as 2L/2. The absorbing states can be divided into two equivalent groups: odd-site occupied and even-site occupied groups. Clearly, these two groups of absorbing states have one-to-one correspondence and the Ising (Z2) symmetry in between. So we expect that this model exhibits the DI-type absorbing phase transition.

We locate the critical line in the \( r_B - p \) phase diagram by dynamic Monte Carlo simulations [8]. For small values of \( r_B \), the system is always absorbing. As \( r_B \) increases,
a window of the active phase appears and divides the absorbing phase by $A$-dominated and $B$-dominated absorbing phases. For example, as $p$ increases along the $\nu_0 = 0.9$ line, the system undergoes two continuous phase transitions from the $B$-dominated absorbing phase into the active phase at $p_{c1} = 0.100(3)$ and finally into the $A$-dominated absorbing phase at $p_{c2} = 0.5058(7)$. We numerically confirm that both transitions are of the DI type as expected [1].

The order parameter $\rho$ is the number density of nearest neighbor pairs of vacant sites, while we define the $A$ particle density as the non-order parameter $\rho_A$. First, we perform static Monte Carlo simulations to measure the natural density of the non-order parameter at both criticalities. We measure the $A$ particle density $\rho_A(t,L)$, averaged over $2 \times 10^3 \sim 5 \times 10^4$ survived samples for system size $L = 2^9 \sim 2^{10}$, $\rho_A(t,L)$ relaxes to the natural density $\rho_A^{nat} \equiv \rho_A(\infty,\infty)$ in the thermodynamic limit. At $p = p_{c1}$ and $p = p_{c2}$, we estimate $\rho_A^{nat} = 0.101(1)$ and 0.458(2), respectively. The distribution function for the $A$ particle density appears to be Gaussian, so the mean density is identical to the most probable density. We measure the most probable density in the long time limit, which turns out to be consistent with the above mean natural density within statistical errors.

Dynamic properties for the non-order parameter $\rho_A(t,L)$ can be extracted by studying its temporal deviation from its steady-state value (natural density) as

$$\Delta \rho_A(t,L) \equiv |\rho_A(t,L) - \rho_A^{nat}|. \quad (2)$$

As in the DP case, we assume that $\Delta \rho_A(t,L)$ follows the same scaling behavior as the order parameter. Using the finite-scaling theory [12] in the steady state, $\Delta \rho_A(t,L)$ scales at criticality as

$$\Delta \rho_A(\infty,L) \sim L^{-\beta/\nu_\perp}. \quad (3)$$

One can also expect the critical short time behavior as

$$\Delta \rho_A(t,\infty) \sim t^{-\beta/\nu_\parallel}, \quad (4)$$

and the characteristic time $\tau_A(L)$ scales as

$$\tau_A(L) \sim L^{\nu_\parallel/\nu_\perp}. \quad (5)$$

From static simulations, we estimate the scaling exponents at both criticalities. At $p = p_{c1}, p_{c2}$, we estimate $\beta/\nu_\perp = 0.46(2), 0.46(3)$ $\beta/\nu_\parallel = 0.26(2), 0.25(2)$, and $\nu_\parallel/\nu_\perp = 1.75(5), 1.90(10)$ (see Fig. 1). As expected, these estimations involve rather large statistical and systematic errors, especially due to inaccuracy of the natural density values. However, these values agree reasonably well with the DI values [13], which confirm our assumption that the non-order parameter exhibits the same type of critical fluctuations as the order parameter.

Dynamic exponents for the order parameter in DP systems with IMA states are known to depend on initial conditions characterized by the non-order parameter. To investigate this nonuniversal dynamic properties in DI systems, we perform dynamic Monte Carlo simulations with various initial conditions. We start with a pair of nearest neighbor vacant sites in the absorbing background which is controlled by the $A$ particle density $\rho_A$.

We measure the survival probability $P(t)$ (the probability that the system is still active at time $t$) and the mean number of pair of vacant sites (order parameter) $N(t)$ averaged over all samples. At criticality, these quantities scale algebraically in long time limit as [14]

$$P(t) \sim t^{-\delta}, \quad N(t) \sim t^{\eta}. \quad (6)$$

The dynamic exponents are in general functions of initial non-order parameter density: $\delta = \delta(\rho_A^0)$ and $\eta = \eta(\rho_A^0)$. With initial configurations of the natural non-order parameter density, the exponents take the ordinary DI values: $\delta(\rho_A^{nat}) = \delta_{DI} \approx 0.285$ and $\eta(\rho_A^{nat}) = \eta_{DI} \approx 0.00$.

At $p = p_{c1}$ with $\nu_0 = 0.9$, we estimate the values of $\delta$ and $\eta$ as $\rho_A^0$ varies from 0 to 0.35 (see Table I). In Fig. 2, we plot the exponent shifts from the DI values ($\delta - \delta_{DI}$ and $\eta - \eta_{DI}$) versus $\rho_A^0 - \rho_A^{nat}$. It shows a linear dependence of the exponent shifts on the deviation of the non-order parameter from the natural density in initial configurations. This linear dependence has been also seen in DP systems.

In summary, we investigated the IMA-IMM model in one dimension, which show the DI-type continuous phase transition from an active phase into an absorbing phase consisting of infinitely many absorbing states. We show that the non-order parameter exhibits critical fluctuations identical to the order parameter. Dynamic exponents $\delta$ and $\eta$ depends linearly on the initial non-order parameter density and coincide with the ordinary DI values only at the natural density.

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![Graph 1](image1)

**FIG. 1.** Temporal dependence of $\Delta \rho_A$ and size dependence of $\Delta \rho_A$ and $\tau_A$ at $p = p_{c1}$. The straight lines are of slope $0.26(=\beta/\nu_\parallel)$, $0.46(=\beta/\nu_\perp)$, and $1.80(=\nu_\parallel/\nu_\perp)$.

![Graph 2](image2)

**FIG. 2.** Initial configuration dependence of the exponents $\delta, \eta$ at $p = p_{c1}$. It shows a linear dependence on the initial non-order parameter density.

| $\rho_0^A$ | $\delta$ | $\eta$ |
|------------|----------|--------|
| 0.00       | 0.352(5) | -0.065(10) |
| 0.05       | 0.322(5) | -0.033(7) |
| 0.10       | 0.293(4) | -0.006(8) |
| 0.15       | 0.258(8) | 0.031(9) |
| 0.20       | 0.223(4) | 0.060(7) |
| 0.25       | 0.193(5) | 0.090(7) |
| 0.30       | 0.155(3) | 0.124(6) |
| 0.35       | 0.122(4) | 0.155(8) |

**TABLE I.** Initial configuration dependence of the dynamic exponents at $p_c = 0.1$. 

![Graph 3](image3)