Strange stars at finite temperature

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Abstract.
We calculate strange star properties, using large \(N_c\) approximation with built-in chiral symmetry restoration (CSM). We used a relativistic Hartree Fock mean field approximation method, using a modified Richardson potential with two scale parameters \(\Lambda\) and \(\Lambda'\), to find a new set of equation of state (EOS) for strange quark matter. We take the effect of temperature (T) on gluon mass, in addition to the usual density dependence, and find that the transition T from hadronic matter to strange matter is 80 MeV. Therefore formation of strange stars may be the only signal for formation of QGP with asymptotic freedom (AF) and CSM.

1. Introduction
There have been some exciting developments recently since the four groups BRAHMS, PHENIX, PHOBOS and STAR have analyzed RHIC data. These four groups reporting on RHIC, with gold on gold, show that quark gluon plasma with asymptotic freedom and chiral symmetry restoration may never be realized in RHIC although a non-hadronic phase is reached. Unfortunately, their conclusions are negative in so far as finding of asymptotically free chirally symmetric QCD state is not possible in these reactions - although a new phase is formed which is quite distinct from the hadronic phase. This phase is strongly interacting and is not fully understood theoretically but it is certainly not QGP with asymptotic freedom and chiral symmetry restoration.

Two decades back Witten\(^1\) has proposed the existence of strange matter and strange stars, and even today it is still difficult to prove or disprove the existence of a state of strange quark matter in its purest form. In literature, there are several EOSs for strange quark matter, starting from the Bag model\(^2\)\(^3\)\(^4\) to the recent models like the mean field model with interacting quarks\(^5\), the perturbative QCD approach\(^6\), chiral chromodielectric model\(^7\), Dyson-Schwinger model\(^8\), etc. Subsequently, Rajagopal and Wilczek combined asymptotic freedom and BCS theory to arrive at the color-flavor locked state of quark matter\(^9\), and ever since, a lot of studies on this state and their application to quark matter EOSs are made\(^10\). Here, we developed a set of new EOSs using a two parameter interaction potential.

2. The Model
The original qq potential of Richardson\(^11\) was designed to obtain the mass spectrum of heavy mesons (Charmonium and Upsilon). It takes care of two features of qq force, AF and confinement, however, with the same scale, \(\Lambda\):

\[
V_{ij} = \frac{12\pi}{27} \frac{1}{\ln(1 + (k_i - k_j)^2 / \Lambda^2)} \frac{1}{(k_i - k_j)^2}
\]  \hspace{1cm} (1)
with $\Lambda = 400$ MeV. It was further applied to light meson spectroscopy and baryon properties with the same value of $\Lambda$ [12, 13]. In strange stars, the AF part is important and there $\Lambda$ would be much smaller, around 100 MeV which is the scale for asymptotic freedom as obtained from perturbative QCD. Indeed, for the self bound high density strange quark matter Dey et al. [5] found that $\Lambda$ needed to be $\sim 100$ MeV. Quarks are deconfined at high density, as the bare potential is screened. Confinement is softened and the AF part takes over. This bare potential in a medium will be screened due to gluon propagation. The temperature dependence of the screening in quark matter is taken from Alexanian & Nair [14]. The inverse Debye screening length (the gluon mass) becomes:

$$ (D^{-1})^2 = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s} k_i f \sqrt{(k_i f)^2 + m_i^2} + 7.14 \alpha_0 T $$

(2)

where $k_i f$, the Fermi momentum of the $i$-th quark is obtained from the corresponding number density $k_i f^2 = (n_i \pi^2)^{1/3}$ and $\alpha_0$ is the perturbative quark gluon coupling.

In our model, chiral symmetry restoration at high density is incorporated by assuming that the quark masses are density dependent:

$$ M_i = m_i + M_Q \text{sech} \left( \frac{n_B}{N n_0} \right), \quad i = u, d, s. $$

(3)

where $n_B = (n_u + n_d + n_s)/3$ is the baryon number density; $n_0 = 0.17$ fm$^{-3}$ is the normal nuclear matter density; $n_u, n_d, n_s$ are number densities of u, d and s quarks respectively and $N$ is a parameter. The current quark masses ($m_i$) are taken as: $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV. It is ensured that in strange matter, the chemical potentials of the quarks satisfy $\beta$ equilibrium and charge neutrality conditions. The parameters $M_Q$ and $N$ are adjusted in such a way that the minimum value of $E/A$ for u,d,s quark matter is less than that of Fe$^{56}$, so that u,d,s quark matter can constitute stable stars. The minimum value of $E/A$ is obtained at the star surface where the pressure is zero. The surface is sharp since strong interaction dictates the deconfinement point. However, the minimum value of $E/A$ for u-d quark matter is greater than that of Fe$^{56}$ so that Fe$^{56}$ remains the most stable element in the non-strange world.

Recently the magnetic moments of $\Delta^{++}$ and $\Omega^-$ with three u and three s valence quarks respectively have been found in accurate experiments. These sensitive properties are fitted with a two parameter modified Richardson potential with different scales for confinement ($\sim 350$ MeV) and AF ($\sim 100$ MeV) [15]. The baryonic properties depend more on the confinement part and less on the AF part as baryons are confined quark systems. It is natural to apply this new potential to star calculation since they are constrained by baryonic data.

In our present calculation, we have modified the Richardson potential as:

$$ V_{ij} = \frac{12\pi}{27} \left[ \frac{1}{\ln(1 + \frac{(k_i - k_j)^2}{\Lambda'^2})} - \frac{\Lambda'^2}{(k_i - k_j)^2} \right] \times \frac{1}{(k_i - k_j)^2} $$

(4)

with $\Lambda'$ taking care of the confinement property and $\Lambda$ that of asymptotic freedom.

The term $\left( \frac{1}{\ln(1 + \frac{(k_i - k_j)^2}{\Lambda'^2})} - \frac{\Lambda'^2}{(k_i - k_j)^2} \right)$ is asymptotically zero for large momentum transfer $Q^2 = (k_i - k_j)^2$ and the term $\frac{\Lambda'^2}{Q^2}$ explains the confinement reducing to a linear confinement for small $Q^2$. The appropriate values of $\Lambda$ and $\Lambda'$ as obtained from a fit to baryonic properties calculations [15] are $\Lambda \sim 100$ MeV and $\Lambda' \sim 350$ MeV, which we used in the present calculation.
Finite temperature $T$ is incorporated in the system through the Fermi function \cite{16}:

$$FM(k, T) = \frac{1}{e^{(\epsilon - \epsilon_F)/T} + 1}$$  \hspace{1cm} (5)

with the flavour dependent single particle energy

$$\epsilon_i = \sqrt{k^2 + M_i(\rho)^2} + U_i(k).$$  \hspace{1cm} (6)

$$I = \frac{\gamma}{2\pi^2} \int_0^{\infty} \phi(\epsilon)k^2FM(k, T)dk$$  \hspace{1cm} (7)

$I$ = number density for $\phi(\epsilon) = 1$ and $I$ = energy density for $\phi(\epsilon) = \epsilon$. $\gamma$ is the spin-colour degeneracy, equal to 6.

The system is highly degenerate even at very high $T$ which is around 80 MeV since the chemical potential is very high, of order several hundred MeV. The entropy is calculated as follows:

$$S(T) = \int_0^{\infty} k^2[FM(k, T)\ln(FM(k, T)) + (1 - FM(k, T))\ln(1 - FM(k, T))]dk$$  \hspace{1cm} (8)

The pressure is calculated from the free energy $f = \epsilon - Ts$ as follows:

$$P = \sum_i \rho_i \frac{\partial f_i}{\partial \rho_i} - f_i$$  \hspace{1cm} (9)

With the obtained EOS (pressure vs density), we solve the Tolman Oppenheimer Volkov equation, to get the stellar structure.

\underline{3. Conclusions and summary:}
A set of new EOS for strange matter is presented, using the Richardson potential with the value of $\Lambda' \simeq 300$ to 350 $MeV$ and the value of $\Lambda = 100$ $MeV$, the two scales for confinement and
Figure 3. Mass of the gluon (inverse Debye screening length) increases with density and also with temperature.

Figure 4. The entropy per baryon decrease with density. In the nuclear matter limit (say $1.5 \times$ normal nuclear matter density), it is found to match exactly with the experimental results.

AF respectively. This is then a good inter-quark potential as it also explains both the properties of the deconfined quark matter (our present work) and the properties of confined $3u$ and $3s$ baryons[15]. Although the confinement part $\Lambda'$ is stronger, leading to a sharp surface, - it is softened by medium effect, developing a screening length. Inside the star, the AF part is more important. We have found that for a wide range of parametric variation, the strange matter EOS gives minimum energy of $E/A$, much less than compared to $(E/A)_{Fe^{56}} = 930.6$ MeV which ensures that the system is absolutely stable, and unlike neutron star like structure, it is not gravitational force alone that binds the system. However, for this present article, we have presented with only one such EOS. Considering temperature dependent screening of the potential, we have found that strange stars can sustain stable configurations up to a temperature of 80 MeV; this value of the temperature is very close to Witten’s scenario of cosmic phase separation.

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