Scalar field perturbations in Fresh inflation

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The model of fresh inflation with increasing cosmological parameter provides sufficient e-folds to solve the flatness/horizon problem and the density fluctuations agree with experimental values. In this model the temperature increases during fresh inflation and reach its maximum value when inflation ends where the N-number of e-folds is $N(t_e) \gg 60$. The most important characteristic of this model is that provides a natural transition between the end of inflation and the epoch when the universe is radiation dominated.

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Inflation is one of the most reliable concepts in modern cosmology. The first model of inflation was proposed by A. Starobinsky [1]. A much simpler inflationary model with a clear motivation was developed by A. Guth [2], in order to solve some of the shortcomings of the big bang theory, and in particular, to explain the extraordinary homogeneity of the observable universe. However, the universe after inflation in this scenario becomes very inhomogeneous. These problems were sorted out by A. Linde in 1983 with the introduction of chaotic inflation [3]. In this scenario inflation can occur in theories with potentials such as $V(\phi) \sim \phi^n$. It may begin in the absence of thermal equilibrium in the early universe, and may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily solved. Although a justification from first principles for dissipative effects has not been firmly achieved in the framework of inflation, such effects should not be ruled out on the basis of readiness alone. Much work can be done on phenomenological grounds as, for instance, by applying nonequilibrium thermodynamic techniques to the problem or even studying particular models with dissipation. An interesting example of the latter case is the warm inflationary picture [4,5]. As in new inflation [6,7], a phase transition driving the universe to an inflationary period dominated by the scalar field potential is assumed. A standard phenomenological frictionlike term $\Gamma \dot{\phi}$ is inserted into the scalar field equation of motion to represent a continuous energy transfered from $\phi$ to the radiation field. This persistent thermal contact during inflation is so finely adjusted that the scalar field evolves all the time in a damped regime generating an isothermal expansion. As a consequence, the subsequent reheating mechanism is not needed and thermal fluctuations produce the primordial spectrum of density perturbations [8]. More recently was demonstrated that isentropic and warm pictures are just extreme cases of an infinite two-parametric family of possible inflationary scenarios [9]. Very recently, a dissipative mechanism which emerges in generic interacting quantum field systems and leads to robust warm inflation was developed [10].

Two years ago, a new scenario called fresh inflation was introduced [11,12]. It can be viewed as a unification of both chaotic and warm inflation scenarios with constant [11] and increasing [12] $F$-cosmological parameter. As in chaotic inflation the universe begins from an unstable primordial matter field perturbation with energy density nearly $M_p^4$ ($M_p = 1.2 \times GeV$ is the Planckian mass) and chaotic initial conditions. Furthermore, initially the universe there is not thermalized so that the radiation energy density when inflation starts is zero [$\rho_r(t = t_0) = 0$]. It must to be point out that quantum field theory first principle calculations of radiation generated from a zero temperature state has been done in [13], but this formalism is realized in an non-expanding Minkowski spacetime.

As initial time we understand the Planckian time $G^{1/2}$, where $G$ is the gravitational constant. Later, the universe will describe a second-order phase transition. In other words, the inflaton rolls down towards the minimum energetic configuration. Particle production and thermalization occur together during the rapid expansion of the universe, so that the radiation energy density grows during fresh inflation ($\dot{\rho}_r > 0$). The interaction between the inflation field and the particles produced during inflation provides slow-rolling of the inflaton field. So, in the fresh inflationary model (also in warm inflation), the slow-roll conditions are physically well justified. The decay width of the $\phi$-field grows with time, so when the inflaton approaches to the minimum of the potential there is no oscillation around the minimum energetic configuration. Hence, the reheating period does not happen in fresh inflation. This model attempts to

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build a bridge between the standard and warm inflationary models, beginning from chaotic initial conditions which provides naturality. In fresh inflation with constant cosmological parameter the universe can be seen as inflating in a 4D Friedmann-Robertson-Walker (FRW) metric embedding in a 5D metric in apparent vacuum when the fifth coordinate of the metric is non-compact and remains constant [14]. Furthermore, can be demonstrated the possibility that violation of baryon number conservation [15] and ultralight particle creation [16] can occur during the period out-of-equilibrium, i.e., before the thermal equilibrium to be restored at the end of fresh inflation. Gauge-invariant metric fluctuations in a fresh inflationary model with increasing cosmological parameter was considered in [12]. The main aim of this paper is the study of the evolution of density perturbations in such that model.

We describe fresh inflation with a Lagrangian for a $\phi$-scalar field minimally coupled to gravity, which also interacts with another $\psi$-scalar field by means of $\mathcal{L}_{\text{int}} = -g^{\mu\nu}\partial_\mu\phi^2\psi^2$, is

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}\left(\nabla \phi\right)^2 - V(\phi) + \mathcal{L}_{\text{int}}\right],$$

where $R = 6(a\ddot{a} + \dot{a}^2)/a^2$ is the scalar curvature, $a$ is the scale factor of the universe and $g$ is the determinant of the metric tensor $g^{\mu\nu}$ with $\mu, \nu = 0, 1, 2, 3$. In this paper I consider a FRW metric for a spatially flat, isotropic and homogenous universe described by the line element $ds^2 = -dt^2 + a^2(\theta)dr^2$. If $\delta = \dot{\rho}_r + 4H\rho_r$ describes the interaction between the inflaton and the bath for a $\gamma = 4/3$-fluid which expands with a Hubble parameter $H = \dot{a}/a$ and radiation energy density $\rho_r$, hence the equations of motion for $\phi$ and radiation energy density are

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \frac{\delta}{\phi} = 0, \quad \dot{\rho}_r + 4H\rho_r - \delta = 0. \quad (2)$$

Here, $\delta = \Gamma(\theta)\phi^2$ describes a Yukawa interaction and the $\phi$-decay width is $\Gamma(\theta) = [g_{\text{eff}}^4/(192\pi)]\theta$ [17]. Furthermore, $\theta \sim \rho_r^{1/4}$ is the temperature of the bath. The cosmological parameter $F = (p_t + \rho_t)/\rho_t$ describes the evolution of the universe during inflation

$$F = \frac{2H}{3H^2} = \frac{\dot{\phi}^2 + \frac{2}{3}\rho_r}{\rho_r + \frac{\dot{\phi}^2}{2} + V}, \quad (3)$$

where the total pressure and energy density are given respectively by

$$p_t = \frac{\dot{\phi}^2}{2} + \frac{\rho_r}{3} - V(\phi), \quad \rho_t = \rho_r + \frac{\dot{\phi}^2}{2} + V(\phi). \quad (4)$$

In the previous works [11] only was considered the case where the cosmological parameter $F$ is a constant. However, as we can see in eq. (3), during inflation the potential energy density decreases, so that the radiation energy density becomes more important in $F$. This means that $F$ must be increasing during fresh inflation, but of course, always remaining below 4/3, which corresponds to a radiation dominated universe. We can write $\rho_r$ and $V(\phi)$ as a function of $\phi$ [11]

$$\rho_r = \left(\frac{3F}{4 - 3F}\right)V - \frac{27}{8}\left(\frac{H^2}{H'}\right)^2\frac{(2F - F'^2)}{(4 - 3F)}V(\phi), \quad V(\phi) = \frac{3}{8\pi G}\left[\left(\frac{4 - 3F}{4}\right)H^2 + \frac{3\pi G}{2}F^2\left(\frac{H^2}{H'}\right)^2\right], \quad (5)$$

where $F$ is a function of $\phi$ and the time evolution of $\phi$ is described by the equation

$$\dot{\phi} = -\frac{3H^2}{2H'}F(\phi). \quad (6)$$

We consider in the second equation of (5) the potential $V(\phi) = [\mathcal{M}^2(0)/2]\phi^2 + [\lambda^2/4]\phi^4$, where $G = M_p^{-2}$ is the gravitational constant and $M_p = 1.2 \times 10^{19}$ GeV is the Planckian mass. The inflaton field is really an effective field described by $\phi = (\phi, \phi_i)^{1/2}$. Furthermore, $\mathcal{M}^2(0)$ is given by $M_p^2$ plus renormalization counterterms in the initial potential $\frac{1}{2}\mathcal{M}_0^2(\phi, \phi_i) + \frac{\lambda^2}{2}(\phi, \phi_i)^2$ [18], the effective potential is $V_{\text{eff}}(\phi, \theta) = [\mathcal{M}^2(\theta)/2]\phi^2 + [\lambda^2/4]\phi^4$. Here, $\theta$ is the temperature and $\mathcal{M}^2(\theta) = \mathcal{M}^2(0) + \frac{(n + 2)}{12}\lambda^2\theta^2$, such that $V_{\text{eff}}(\phi, \theta) = V(\phi) + \rho_r(\phi, \theta)$. The temperature increases with the expansion of the universe because the inflaton transfers radiation energy density to the bath with a rate larger than the expansion of the universe. So, the number of created particles $n$ [for $\rho_r = (\pi^2/30)g_{\text{eff}}\theta^4 = (\mathcal{M}^2(\theta) - \mathcal{M}^2(0))\phi^2$], is given by
\[ (n + 2) = \frac{2\pi^2}{3\lambda^2 g_{\text{eff}}^2} \frac{\theta^2}{\phi^2}, \]  

where \( g_{\text{eff}} \) denotes the effective degrees of freedom of the particles and it is assumed that \( \phi \) has no self-interaction. Note that eq. (7) only depends on the choice of \( V_{\text{eff}} \) and \( \mathcal{M}^2(\theta) \), and not on \( H \) and \( F \). (For a more detailed explanation of eq. (7) the reader can see the papers [11].) The density fluctuations are given by

\[
\frac{\langle \delta \rho_t \rangle}{\rho_t} \simeq \Delta(\phi) \langle \delta \phi^2 \rangle^{1/2},
\]

where \( \delta \phi^2 \) are the squared fluctuations of the inflaton field and \( \Delta(\phi) = \frac{V_{\text{eff}}}{\phi^2 + 4/3 \rho_t} \). The notation \( \langle ... \rangle \) denotes the averaging with respect to a gaussian distribution. Since the temperature of the thermal bath is \( \theta \), we can consider the Langevin equation for the field fluctuations \( \delta \phi \), with a white and gaussian noise \( \kappa(t) \), due to the stochastic interaction of the field \( \phi \) with the thermalized environment

\[
\delta \phi \simeq \frac{H^2}{3H + \Gamma} \delta \phi + \kappa(t).
\]

Here, we have made use of the fluctuation dissipation theorem [19], and the fact that the slow-roll condition holds. This fact implies that \( H^2 \gg V'' \). The diffusion coefficient for the fluctuations with a wavelength given by the Hubble radius \( H^{-1} \), is (see the first paper in [8]) \( D = \frac{3}{2\pi} \frac{H^3}{3H + \Gamma} \), such that the squared mean fluctuations of the field are [20]

\[
\langle \delta \phi^2 \rangle \sim \sqrt{3Ht} \theta.
\]

This relation is only valid when the system is thermalized, i.e., in the last stages of fresh inflation for which the relation \( \Gamma > H \) holds.

As an example we consider the particular case where the Hubble and cosmological parameters are \( H(\phi) = A \phi^2 \) and \( F(\phi) = B \phi^{-2} \), being \( A \) and \( B \) \( G^{1/2} \) and \( G^{-1} \) dimensional constants which can be obtained by replacing \( H \) and \( F \) in eq. (5)

\[
A^2 = \lambda^2 8\pi G \frac{4}{3}, \quad B = \frac{3\lambda + \sqrt{9\lambda^2 + 48\pi G \mathcal{M}^2(0)}}{3\lambda G}.
\]

From eq. (6) we obtain the time evolution for the inflaton field \( \phi(t) = \phi^{(s)} e^{-\frac{\mathcal{M}^2(\theta)}{H}} \), where \( \phi^{(s)} \) is its initial value. It must be sufficiently large to assures at least, the 50–60 e-folds needed during inflation before transcurred \( (10^{10} – 10^{12}) \) Planckian times. Since \( H = \dot{a}/a \), the time evolution of the scale factor will be \( a = a_0 \ e^{-\frac{\mathcal{M}^2(\theta)}{H}} \), which increases with time due to the decreasing of \( \phi(t) \). Furthermore, the temperature, written as a function of the field is obtained from the second equation in (2)

\[
\theta(\phi) = \left\{ 4\pi \phi(8\phi - 3B) [16\mathcal{M}^2(0) + \phi^2 (8\lambda^2 - 18A^2B) + 9A^2B^2] \right\} \left[ BA g_{\text{eff}}^4 \left( 4\phi^2 - 3B \right) \right]^{-1},
\]

which is zero when fresh inflation starts. The \( \phi \)-value at this time is \( \phi^{(s)} = 3B/8 \). The function \( \Delta(\phi) \) in eq. (8) is

\[
\Delta(\phi) = \frac{3}{4\phi} \left( \pi GB - 2 \right).
\]

Hence, from eqs. (13) and (10) one obtains the density fluctuations \( \frac{\langle \delta \rho_t \rangle}{\rho_t} \) in eq. (8)

\[
\frac{\langle \delta \rho_t \rangle}{\rho_t} \sim \frac{3(\pi GB - 2)}{4\phi} \left[ \frac{\pi \phi^3 (3B - 8\phi)}{g_{\text{eff}}^4 B (4\phi^2 - 3B)} \right]^{1/2} \left[ 2\phi^2 (9A^2B - 4\lambda^2) - 16\mathcal{M}^2(0) - 9A^2B^2 \right]^{1/2}.
\]

Figures (1), (2) and (3) show respectively the evolution of the temperature \( \theta(t) \), density fluctuations \( \langle \delta \rho_t \rangle / \rho_t \) (as a function of the number of e-folds) and the number of created particles in eq. (7) (as a function of time). These graphics were made using the values \( \mathcal{M}(0) = 1.5 \times 10^{-9} \) \( G^{-1/2} \), \( \lambda = 4 \times 10^{-15} \) and \( g_{\text{eff}} = 25 \). With these parameter values we obtain respectively \( A = 0.578 \times 10^{-14} \) \( G^{1/2} \) and \( B = 488726.728 \) \( G^{-1} \) in eqs. (11). Notice that fresh inflation ends at \( t_e \simeq 2 \times 10^6 \) \( G^{1/2} \), when \( \theta(t_e) \simeq 6 \times 10^{-8} \) \( G^{-1/2} \) [see Fig. (1)] and \( \langle \delta \rho_t \rangle / \rho_t |_{N(t_e)} \simeq 10^{-5} \) [see Fig. (2)] reach their maximum values. The \( \phi \)-value when fresh inflation ends is given by \( \phi^{(e)} = \phi(t_e) \simeq 8 \times 10^5 \) \( G^{-1/2} \). Note
that $N(t_e) \gg 60$ at this moment so that the model here studied solve the flatness/horizon problems. Furthermore, $\phi$ takes transplanckian values during fresh inflation with increasing cosmological parameter. It could be a problem for this model because there is no clear the validity of conventional quantum field theory in the transplanckian regime.

Finally, Fig. (3) shows the evolution of the number of created particles, which grows to reach its asymptotic value $n \simeq 9 \times 10^6$, at the end of the inflationary period.

To summarize, the conditions on $\rho_r$ lead to two different types of thermodynamic regimes of inflation. At the beginning $\rho_r \simeq 0$ and the expansion is isentropic during inflation, but the temperature increases because $\Gamma$ is sufficiently strong to make $\dot{\rho}_r > 0$. This is the case of fresh inflation, in which this first regime is followed by a non-isentropic period in which $\rho_v > \rho_r > 0$, so that the temperature of the universe may still be sizable. In this regime, conversion of vacuum energy into radiation energy occurs during the inflationary period and the inflationary regime smoothly terminates into a radiation dominated regime without an intermediate reheating period. The toy model here studied shows that fresh inflation can be a feasible alternative model to standard inflation [3]. The most important characteristic of this model is that shows a natural transition between the end of inflation and the epoch when the universe is radiation dominated. To conclude, a more exhaustive calculation in the inflation/radiation-dominated interphase should include cold dark matter (CDM) [21] in this model of fresh inflation with increasing cosmological parameter.

[1] A. A. Starobinsky, JEPT Lett. 30, 682 (1979); Phys. Lett. B91, 99 (1980).
[2] A. Guth, Phys. Rev. D23, 347 (1981).
[3] A. D. Linde, Phys. Lett. B129, 177 (1983).
[4] A. Berera, Phys. Rev. Lett. 75, 3218 (1995); Phys. Rev. D54, 2519 (1996); Phys. Rev. Lett. 83, 264 (1999).
[5] M. Bellini, Phys. Lett. B428, 31 (1998); Class. Quant. Grav. 16, 2393 (1999); Nucl. Phys. B563 245 (1999); Class. Quant. Grav. 17, 145 (2000);
[6] A. D. Linde, Phys. Lett. B108, 389 (1982).
[7] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
[8] A. Berera and L. Z. Fang, Phys. Rev. Lett. 74, 1912 (1995); M. Bellini, Phys. Rev. D58, 103518 (1998); W. Lee and L. Z. Fang, Phys. Rev. D59, 083503 (1999); A. N. Taylor and A. Berera, Phys. Rev. D62, 083517 (2000).
[9] J. M. F. Maia and J. A. S. Lima, Phys. Rev. D60, 101301 (1999).
[10] A. Berera and R. O. Ramos, E-print: hep-ph/0210301.
[11] M. Bellini, Phys. Rev. D63, 123510 (2001); Phys. Rev. D64, 123508 (2001).
[12] M. Bellini, Phys. Rev. D67, 027303 (2003).
[13] A. Berera and Runde O. Ramos, Phys. Rev. D63, 103509 (2001).
[14] M. Bellini, Gen. Rel. Grav. 35, 35 (2003).
[15] M. Bellini, Gen. Rel. Grav. 34, 2127 (2002).
[16] M. Bellini, Nuovo Cim. 117B, 653 (2002).
[17] A. Berera, M. Gleiser and R. O. Ramos, Phys. Rev. D58, 123508 (1998).
[18] M. Weinberg, Phys. Rev. D9, 3357 (1974).
[19] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977).
[20] A. Berera, Nucl. Phys. B585, 666 (2000).
[21] L. Amendola et al., Phys. Rev. Lett. 88, 211302 (2002).

**Fig. 1:** Evolution of the temperature $\theta(t)$ during inflation. The maximum, with parameters $\mathcal{M}(0) = 1.5 \times 10^{-9} \, G^{-1/2}$, $\lambda = 4 \times 10^{-15}$ and $g_{\text{eff}} = 25$, occurs at the end of fresh inflation (i.e., around $t_e \simeq 2 \times 10^8 \, G^{1/2}$).

**Fig. 2:** Evolution of density fluctuations $<\delta \rho_e>/\rho_e$ as a function of the number of $e$-folds, where $N(t_e) \gg 60$. The maximum at the end of inflation is of the order of $10^{-5}$.

**Fig. 3:** Temporal evolution of $n$ which shows that the number of created particles grows with time to reach the value of $n \simeq 9 \times 10^6$ at the end of inflation.
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