Study on effective anchorage length of anchor cable based on Gauss's function

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Abstract. In the current relevant codes and technical standards, the design of anchor cables (bars) is based on the assumption that the shear stress is distributed uniformly along the anchor segment. However, according to this assumption, the super-deep pit need a large uplift bearing capacity, which will lead to the long anchor segment and has great hidden danger in the actual engineering design. Therefore, this paper, based on the previous derived shear stress function between τ and the anchor length, we get a research suitable model-Gauss curve distribution model through the analysis of saturated clay drawing data, and strata anchor shear stress distribution is more suitable for the Gauss function model in saturated clay; at the same time, through the FLAC numerical simulation of Jinan Government Affairs Center project, we obtained the optimal anchor effective anchor length is about 10~12m, the foundation pit anchor is optimized.

1. Introduction

The study about the shear stress distribution of anchor has attracted the attention of many scholars. Based on the assumption of uniform shear stress, the design often leads to the length of bolt is very long in ultra-deep foundation pit. The research confirmed that when the length of the bolt exceeds a certain value, the rear end of the bolt produces a “whipping effect”. The extra long anchor force is very small or not, resulting in actual anchor pullout is much smaller than the design capacity. And it will make the design in a great hidden danger.

The effective anchoring length of the anchor has given many scholars a consensus. The Canadian Phillip proposed that the frictional resistance of the anchor should be distributed in a power function along the length of the anchor. On this basis, the function expression of ultimate bearing capacity of rock bolt is deduced. With the help of the assumption that the shear force between the rock and the anchor is linearly related to the shear displacement, Zhang Jiru and Tang Baofu established the hyperbolic function model of load transfer and verified the distribution rule by drawing test in clay layer.

Zhang Youpa and others obtained the conclusion that the effective anchor length of anchor cable is less than 6m under 180kN through the method of calculating the tangential displacement of the rod and it is based on the fact that the smaller the shear displacement of the anchor section, the better the bearing capacity of the anchor in the case of the same pull force.
Based on the analysis of the stress distribution of the anchorage section of the prestressed anchor cable, the length of the anchorage section is studied to determine the effective anchorage length of the prestressed anchor in the saturated clay layer, and the results are used to guide the similar to the foundation pit support in the design of prestressed anchor, construction, and further understanding of the transmission mechanism of bolt load.

2. Study on length effect of anchorage segment of anchor cable

At present, the design and calculation of domestic and foreign experts and related technical standards are used in the design method mainly is the average shear stress at the second interface, and shear stress is not evenly distributed. Therefore, this paper mainly considers the anchoring effect caused by the shear stress of the second interface.

In this paper, we use the similarity model test data and the Origin data processing software to draw the corresponding anchor length under the axial force distribution map:

![Fig.1 Distribution of axial force when bolt length is 15 m](image1)

![Fig. 2 Distribution of axial force when bolt length is 20m](image2)

From the above data chart analysis, we can see: in the test, the variations are very similar to the two-parameter Gaussian function curves (1) and the two-parameter complex power function (2) curves described in [2,3], so we will use these two functions to curve the axial force fit.

Two parameter Gauss function curve:

\[ N(x) = A \exp\left(\frac{-x^2}{2w^2}\right) \]  

(1)

Two parameter composite power function:

\[ N(x) = \frac{B}{1 + gx^2} \]  

(2)

In the formula: \( N(x) \) - the shaft force at the outer end \( x \) of the anchorage section;

\( A, B, w, g \) - parameters in the function.

The experimental data of anchorage length of 15m and 20m are fitted by data processing software Matlab. The Gaussian function model is used to derive the formula,

\[ N(x) = P \exp\left(\frac{-x^2}{2L_m^2w^2}\right) \]  

(3)

In fact, when the axial force of the anchor segment falls to less than 10% of the drawing force \( P \) in a large number of field tests and works[2], the axial force at the back of the anchor segment is very small and it can be considered invalid:
\[ P \exp \left(-\frac{1}{2w^2} \right) = 0.1P \]  \hfill (4)

According to the above formula, the parameter \( w \) is determined to be 0.466.

According to the static equilibrium:

\[ N(x) + dN - N(x) + \tau(x)2\pi R dx = 0 \]  \hfill (5)

(4) is brought into (5) and the relationship between the shear stress \( \tau(x) \) and the length of the anchoring section \( L_m \) is obtained after the simplification:

\[ \tau(x) = \frac{Px}{2\pi RL_m w^2} \exp \left( -\frac{x^2}{2L_m^2 w^2} \right) \]  \hfill (6)

When \( x = w \):

\[ \tau_p = \frac{P}{2\pi w RL_m} e^{-1/2L_m^2} \]  \hfill (7)

From the distribution formula of shear stress, it can be seen: when \( x = w \), the shear stress of the anchor reaches apex and gets formula (7), on the basis of two parameter Gauss's function, the formula of anchor length is obtained. In order to make the bolt safe, it requires that the peak shear stress between grout and soil is not more than the allowable shear strength value \([\tau]\). When the anchor segment \( L_m \geq 6m \), in the (7) can be approximately equal to 1. The formula for finding the length of anchorage is obtained, after simplification and finishing:

\[ L_m \geq \frac{P}{2\pi wR[\tau]} \]  \hfill (8)

\( w = 0.466 \).

Using the above formula to calculate the effective length of anchor cable about foundation pit engineering of Jinan municipal service center building, and the model soil parameters are shown in Table 1.

| No. | Project               | \( \gamma \) (kN/m) | E (MPa) | C (kPa) | \( \Phi \) (°) |
|-----|-----------------------|---------------------|---------|---------|--------------|
| 1   | miscellaneous fill    | 1.5                 | 24      | 8.0     | 100          |
| 2   | scree                 | 2.3                 | 72      | 5.0     | 350          |
| 3   | clay1                 | 6.1                 | 30      | 190     | 150          |
| 4   | clay2                 | 4.5                 | 36      | 370     | 180          |
| 5   | Silt                  | 1.2                 | 24      | 19.5    | 200          |
| 6   | Intense weathering1   | 3.0                 | 120     | 200     | 300          |
| 7   | Intense weathering2   | 16.4                | 150     | 200     | 320          |

The excavation depth of the foundation pit is 16.0m. The support of foundation pit is pile + prestressed anchor and support pile (Fig.3). The horizontal distance of the anchor is 2.0m, and the front angle of the prestressed anchor is 20°. The length of the support pile is 23.0m, embedded depth is 11m, diameter is 1000mm, and pile distance is 1.5m. The sketch map of support structure as shown in Figure 2:
3. Finite element simulation

3.1. General situation of Engineering

Jinan City Government Service Center is located in Weier Road, the east side of the building is Station Street, the south is Jinger Road, the north is Shengping Street.

There is a East-West air-raid shelter on the south side of the foundation pit, its top elevation is about 6.0m underground, and the bottom elevation is about 9.0m underground. After the construction unit assorted with the air defense department, the shelter can be abandoned.

There are power lines on the east side of the Zhanqian street. The depth is 1.5m.

3.1.1. Comparative analysis. We have access to geological survey report and related information to get the parameters of Table 2. And we obtained the effective anchoring length of 10~12m in the design of the pile anchor foundation pit support of Jinan Municipal Government Service Center through the formula (8).

| Parameter | R (m)  | [τ] (MPa) | P (kN) | w  |
|-----------|--------|-----------|--------|----|
| number    | 0.075  | 75~85     | 200    | 0.466 |

The Contrast actual field monitoring data and the monitoring data of anchor length by numerical simulation and the analytic formula. Following picture (ZD11 is the measured data):

![Fig. 3 The profile of support structure](image_url)

![Fig. 4 Comparison curve of horizontal displacement of pile](image_url)

![Fig. 5 Comparison curve of horizontal displacement of pile body](image_url)
We can see from Fig.4 and Fig.5 that the maximum horizontal displacement of pile top is 6.94mm in actual monitoring data, the maximum horizontal displacement of pile is 16.26mm. Although the simulated monitoring data is different from the actual monitoring data. But the trend is consistent, and it shows that the numerical simulations are valid. When the effective anchor length is 8m, the maximum horizontal displacement of pile top is 16.83mm, and when the effective anchor length is 12m, the maximum horizontal displacement of pile top is 14.00mm, the change rate is smaller when the effective anchor length is 12m. There is a gap compared with the actual site monitoring data when the effective anchorage length is 8m. But the trend is consistent and the data are within the warning value.

| Table 3 Anchor parameters |
|---------------------------|
| Original design | Free segment | Anchor length | Anchor length 8m(12m) | Free segment | Anchor length |
|-------------------|-------------|---------------|------------------------|-------------|---------------|
| First anchor      | 10          | 18            | First anchor           | 10          | 8(12)         |
| Second anchor     | 8           | 21            | Second anchor          | 8           | 8(12)         |
| Third anchor      | 6           | 23            | Third anchor           | 6           | 8(12)         |
| Fourth anchor     | 5           | 16            | Fourth anchor          | 5           | 8(12)         |

Fig. 6 Comparison of axial force

Table 6 is the parameters of the original design and optimization of anchor cables. Fig.6 is pit excavation to the bottom. Comparison of the axial forces between the original design and the effective anchorage length 8m and 12m, we can see from the picture, the axial force of anchor remains unchanged or varies little after a certain range with the increase of anchor length. We believe that exceeding anchor segment length is of little effect.
By the first shear stress and anchorage length formula (6) can be obtained:

$$
\int_0^x \tau(x) \, dx = \int_0^x \frac{P_x}{2\pi R w^2} \exp \left( -\frac{x^2}{2w^2} \right) \, dx
$$

(9)

Simplify the anchoring force of the anchor:

$$
P_m = -\frac{P}{2\pi R} \left( e^{-\frac{x^2}{2w^2}} \right)_{x=0}
$$

(10)

After further analysis and optimization, we carry out the solution analysis through Lizheng and formula (10) about the effective length of 10m and 12m is obtained by analytic method through.

We can get that the pull-out force of the anchorage section length obtained by formula (10) does not satisfy the requirement. Therefore, we should optimize the cable pitch, and the result is as follows:

| NO. | Type  | horizontal distance (m) | Vertical distance (m) | Incidence angle (°) | Total length (m) | Anchor segment (m) |
|-----|-------|-------------------------|-----------------------|---------------------|------------------|-------------------|
| 1   | anchor| 1.5                     | 3.0(3.0)              | 15.0                | 20(22)           | 10(12)            |
| 2   | anchor| 1.5                     | 2.5(3.0)              | 15.0                | 18(20)           | 10(12)            |
| 3   | anchor| 1.5                     | 2.5(3.0)              | 20.0                | 16(18)           | 10(12)            |
| 4   | anchor| 1.5                     | 2.0(3.0)              | 20.0                | 15(17)           | 10(12)            |
| 5   | anchor| 1.5                     | 2.0(3.0)              | 20.0                | 15(17)           | 10(12)            |

Through the analysis, the optimized anchor meets the requirements of bearing capacity. Considering the above optimization, it can be concluded that when the effective length of anchor section is 10~12m, the retaining of foundation pit is more appropriate.

4. Summary

Through the above research, we can draw the following conclusions:

(1) Based on previous studies, the function is derived between shear stress $\tau$ and the anchor length, through the analysis of saturated clay drawing data, we get suitable mode-Gauss curve distribution model. Based on it, the formula of effective length in saturated clay bolt is derived:

$$
L_m \geq \frac{P}{2\pi w R [\tau]} \quad (L_m \geq 6)
$$

(2) Under the same geological conditions, the optimal effective anchorage length of the cable is about 10~12m.

(3) Through the study, the vertical space of the anchor cable is optimized to meet the bearing capacity requirements of the foundation pit support in Jinan city government service center. And finally the optimal effective anchorage length is about 10~12m.

(4) The shear stress distribution of anchor in saturated clay stratum is more suitable for Gauss’s function model. As the anchor section provides most of the bearing capacity, more than the effective length of the anchor part mainly does not bear the load, and long length caused a waste. Therefore, the design of the anchor cable should be fully considered about the effective length of the anchorage section.
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