Soft Set Based Intelligent Assistive Model for Multiobjective and Multimodal Transportation Problem

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ABSTRACT The real world conditions of transportation pose new challenges for development of intelligent assistive systems for effective decision making. The ignorance of relationships among various attributes and parameters is one of the major causes of uncertainty in existing models of transportation problems. Fuzzy sets have been explored by various investigators to address the uncertainty issues due to relationships among various attributes. However fuzzy sets are not fully capable of dealing all kinds of uncertainty to provide intelligent solutions of transportation problems. A soft set is explored in this paper to model the uncertainty arising from the relationships of attributes with the parameters in a multimodal and multiobjective transportation problem. The three modes of transportation incorporated in this model are road, rail and air. The real data set is prepared using the tariff, distance and time duration of transport available on websites of transport service agencies. The objectives are to minimize the cost and time duration of transportation. The proposed model evaluates multiple criteria using various combinations of different modes of transport to optimize the objectives of the problem. The model has been illustrated using the real data set using existing methods of solution. The soft set approach is found to be quite effective in dealing with the relationships of attributes with the parameters arising due to multi criteria decision making and leads to intelligent optimal solutions. The proposed model can be used as intelligent assistive system for decision making of multi objective and multimodal transportation problems.

INDEX TERMS Soft set, multi objective and multimodal transportation problem, intelligent assistive systems, optimal solution.

I. INTRODUCTION

A number of decisions are made by professionals in their daily work in an uncertain environment. In fact, the professionals designed the system of human world which contains linguistic variables. The quality of decisions made by the professionals are of critical importance to the health of our living world and should be of great concern to every human being in such environment. In the classical age the decisions are made without the help of quantitative analysis based on an appropriate optimization model under uncertainty. But the manual methods very often lead to decisions quite far from being optimal. In fact many bad decisions are being made daily. Due to this there is a need to find the optimum solution of such decision making problems based on quantitative analysis in an uncertain context.

The traditional tools of various systems are not completely capable to deal with humanistic systems because they are not able to adapt with the reality of fuzziness of human thinking and behavior.

Furthermore, Fuzzy Logic based optimization models facilitate the decision making under the conditions of incomplete information and are able to deal with interdependence between variables and conflicts of interest. The fuzzy optimization models are being used quite oftenly by decision makers for making rational decisions.
TABLE 1. Soft multi objective and multi modal trans-portionation model with soft cost and soft time.

| Source / Destination | 1       | 2       | .. | J       | .. | n       | Supply |
|----------------------|---------|---------|----|---------|----|---------|--------|
|                      | $\tilde{c}_{11}(e) ; \tilde{t}_{11}(e)$ | $\tilde{c}_{12}(e) ; \tilde{t}_{12}(e)$ | .. | $\tilde{c}_{1j}(e) ; \tilde{t}_{1j}(e)$ | .. | $\tilde{c}_{1n}(e) ; \tilde{t}_{1n}(e)$ | $\tilde{a}_1$ |
|                      | $\tilde{c}_{21}(e) ; \tilde{t}_{21}(e)$ | $\tilde{c}_{22}(e) ; \tilde{t}_{22}(e)$ | .. | $\tilde{c}_{2j}(e) ; \tilde{t}_{2j}(e)$ | .. | $\tilde{c}_{2n}(e) ; \tilde{t}_{2n}(e)$ | $\tilde{a}_2$ |
|                      | $\tilde{c}_{i1}(e) ; \tilde{t}_{i1}(e)$ | $\tilde{c}_{i2}(e) ; \tilde{t}_{i2}(e)$ | .. | $\tilde{c}_{ij}(e) ; \tilde{t}_{ij}(e)$ | .. | $\tilde{c}_{in}(e) ; \tilde{t}_{in}(e)$ | $\tilde{a}_i$ |
|                      | $\tilde{c}_{m1}(e) ; \tilde{t}_{m1}(e)$ | $\tilde{c}_{m2}(e) ; \tilde{t}_{m2}(e)$ | .. | $\tilde{c}_{mj}(e) ; \tilde{t}_{mj}(e)$ | .. | $\tilde{c}_{mn}(e) ; \tilde{t}_{mn}(e)$ | $\tilde{a}_m$ |
| Demand               | $b_1$   | $b_2$   | .. | $b_j$   | .. | $b_n$   |        |

The transportation model is a subclass of linear programming model with an objective to transport specific quantities of items from origins to destinations in a specific manner resulting in minimum total transportation cost or time which is subject to the availability and requirement conditions. The cost or time of transporting one unit of item from various origins to various destinations must be known.

The fuzzy sets [1], have become popular for modeling uncertain phenomena in real world problems. Various research workers [2]–[7] have reported developments on various aspects of fuzzy sets and their applications in various fields.

Qiu et al. [8] proposed a technique to solve fuzzy transportation problems using ranking function for non-normal fuzzy numbers and subsequently Kaur and Kumar [9] introduced a systematic procedure to obtain optimal solution to a fuzzy transportation problem based on centroid based defuzzification technique in the same year. A new algorithm based on a ranking function to obtain an optimal solution for the fuzzy transportation problem was introduced by Hussein and Dheyab [10]. Malini and Anathanarayanan [11] introduced a new ranking method to solve the fuzzy transportation problem by converting it to a crisp valued problem.

Chen et al. [12] proposed expected value and chance-constrained goal programming model for the bicriteria solid transportation problem. Cosma et al. [13] designed a heuristic algorithm for two stage transportation problem using linear programming and genetic algorithm. Panicker and Sarin [14] proposed Ant Colony Optimization Approach for multi product and multi fixed transportation problem.

Elumalai et al. [15] and others proposed a new algorithm by applying zero simplex method to obtain the optimal solution for a fuzzy transportation problem based on hexagonal fuzzy numbers using robust ranking method. Hunwisai and Kumam [16] used Robust’s ranking method to transform uncertain data into precise data. Ali and Faraz [17] proposed a fuzzy least cost method to solve a fuzzy triangular transportation problem based on signed distance ranking method. El-Wahed [18] and Thorani and Ravi Shankar [19] proposed fuzzy approaches to solve a multi-objective transportation problem. The fuzzy sets and its variants are not fully adequate to deal with all the kinds of uncertainty present in the attributes of the data sets. Soft set is another tool to address the issues of uncertainty due to the relationships among the parameters and attributes of the data sets.

Molodtsov [20] defined the soft set as a complete parameterization tool. The soft set approach is capable to handle the uncertainties caused due to ignorance of parameters. The soft set theory has been further developed by various authors [21]–[25] This soft set theory is being explored by various research workers for applications in optimization, computer science, engineering, economics etc. In the past soft set approach has not been reported for multi objective and multi modal transportation problems. An attempt has been made in this direction in the present paper. Due to the availability of various choices of the transport services by large number of transport agencies, it is necessary to evaluate all of them at granular levels to have the best options. But the process becomes complex due to various permutations and combinations of choices to be explored for multi criteria decision making. The soft sets are potential tools for such conditions and can be useful to develop models which can form the basis for intelligent assistive systems for decision making.

II. MATHEMATICAL FORMULATION

Let $m$ be the number of sources, $i^{th}$ source can supply certain units of an item and $n$ destinations $(n \neq m)$ with each destination $j$ with requirement of certain units. The cost of transport of an item by various modes from each source to each destination is known in terms of measures of time and distance etc. When the goal is to optimize the cost and time, for multiple modes the problem takes the form of a soft multi-objective multi modal transportation problem.

Mathematically, the soft multi-objective and multi modal transportation problem may be expressed as:

\[
\text{Minimize} \quad \tilde{Z}_k(e) = \sum_{i=1}^{m} \sum_{j=1}^{n} (\tilde{p}_{ij}(e)) x_{ij}(e) \quad (1)
\]

\[
\text{Subject to} \quad \sum_{j=1}^{n} x_{ij}(e) \equiv \tilde{a}_i = 1, 2, \ldots, m \quad (2)
\]

\[
\sum_{j=1}^{m} x_{ij}(e) \equiv \tilde{b}_j = 1, 2, \ldots, n \quad (3)
\]

where $\tilde{Z}_k(e) = \{ \tilde{Z}_1(e), \tilde{Z}_2(e), \ldots, \tilde{Z}_k(e) \}$. 

VOLUME 8, 2020

102647
Here \( \tilde{p}_{ij}(e) \) denotes the attribute/measure value associated with transportation from \( i^{th} \) origin to \( j^{th} \) destination depending on parameter \( e \in E \). The values of the attributes \( \tilde{p}_{ij} \) depend on the set of parameters represented by \( E \). Further \( p_{ij} \) represents attributes like cost, time, distance etc.

The objective function \( \tilde{Z}_1(e) \) denotes the soft cost function,

\[
\tilde{Z}_1(e) = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}(e) x_{ij}(e) \quad (4)
\]

The objective function \( \tilde{Z}_2(e) \) denotes the soft time function,

\[
\tilde{Z}_2(e) = \text{Max} \{ \tilde{t}_{ij}(e), x_{ij}(e) \} \quad (5)
\]

It becomes a bi-objective soft transportation problem, which is represented by using weights of objectives based on the priorities of the objectives as given below:

\[
\tilde{Z}_3(e) = w_1 \text{Max} \left\{ \tilde{t}_{ij}(e), x_{ij}(e) \right\} + w_2 \text{Min} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}(e) x_{ij}(e) \right\} \quad (6)
\]

Subject to \[ \sum_{j=1}^{n} x_{ij}(e) \triangleq \tilde{a}_i, \quad i = 1, 2, \ldots, m \]

\[ \sum_{i=1}^{m} x_{ij}(e) \triangleq \tilde{b}_j, \quad j = 1, 2, \ldots, n \quad (7) \]
TABLE 6. Cost matrix for road transport.

| Origin  | Destination | Distance (In KM) | Basic Fair in Rs Per Metric Tonne Per KM | Cost = Fair * Distance (In RS) | Overhead Charges/handling Charges in Rs per Metric Tonne | Final Cost = Cost + Charges |
|---------|-------------|------------------|----------------------------------------|-------------------------------|-----------------------------------------------------|-----------------------------|
| Delhi   | Bhopal      | 809              | 5                                      | 4045                          | 125                                                 | 4170                        |
|         | Lucknow     | 579              | 5                                      | 2895                          | 151                                                 | 3046                        |
|         | Hyderabad   | 1564             | 4.5                                    | 7038                          | 225                                                 | 7263                        |
|         | Patna       | 1082             | 4.75                                    | 5139.5                        | 135                                                 | 5274.5                      |
| Mumbai  | Bhopal      | 776              | 5                                      | 3880                          | 154                                                 | 4034                        |
|         | Lucknow     | 1442             | 4.75                                    | 6849.5                        | 189                                                 | 7038.5                      |
|         | Hyderabad   | 738              | 5                                      | 3690                          | 250                                                 | 3940                        |
|         | Patna       | 1802             | 4.5                                    | 8109                          | 180                                                 | 8289                        |
| Chennai | Bhopal      | 1443             | 4.75                                    | 6854.25                       | 140                                                 | 6994.25                     |
|         | Lucknow     | 2048             | 4.25                                    | 8704                          | 161                                                 | 8865                        |
|         | Hyderabad   | 627              | 5                                      | 3135                          | 210                                                 | 3345                        |
|         | Patna       | 2043             | 4.25                                    | 8682.75                       | 167                                                 | 8849.75                     |

TABLE 7. Cost matrix for rail transport.

| Origin  | Destination | Distance (KM) | Cost in Rs per Metric Tonne (As per rail tariff) | Overhead Charges/handling Charges in Rs per Metric Tonne | Final Cost= Cost+ Charges (In Rs) |
|---------|-------------|---------------|-----------------------------------------------|-----------------------------------------------------|---------------------------------|
| Delhi   | Bhopal      | 707           | 1034                                         | 1550                                                 | 2584                            |
|         | Lucknow     | 513           | 796                                           | 1950                                                 | 2746                            |
|         | Hyderabad   | 1677          | 2150                                          | 1350                                                 | 3700                            |
|         | Patna       | 1009          | 1514                                          | 1875                                                 | 3389                            |
| Mumbai  | Bhopal      | 843           | 1213                                          | 1850                                                 | 3063                            |
|         | Lucknow     | 1428          | 1932                                          | 1750                                                 | 3682                            |
|         | Hyderabad   | 790           | 1134                                          | 1875                                                 | 3925                            |
|         | Patna       | 1695          | 2150                                          | 1775                                                 | 3925                            |
| Chennai | Bhopal      | 1505          | 2008                                          | 1725                                                 | 3733                            |
|         | Lucknow     | 2064          | 2494                                          | 1650                                                 | 4100                            |
|         | Hyderabad   | 789           | 1134                                          | 1825                                                 | 2959                            |
|         | Patna       | 2344          | 2703                                          | 1950                                                 | 4653                            |

\[ x_{ij} = 0 \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n \]  
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}(e) x_{ij}(e): \text{Total soft cost for shipping from } \text{ith} \text{ source to } \text{jth} \text{ destination.} \]
\[ \max \{ \tilde{t}_{ij}(e), x_{ij}(e) \}: \text{Maximum soft time for shipping from } \text{ith} \text{ source to } \text{jth} \text{ destination among all allocations.} \]

The same soft bi objective transportation problem may be represented in the form of m x n soft matrix (See Table-1) where each cell has soft cost and soft time attributes. (e)

III. THE OPTIMUM SOLUTION OF SOFT MULTI-OBJECTIVE TRANSPORTATION PROBLEM

In this section, the method to solve soft multi-objective transportation problem is presented as follows:
TABLE 8. Cost matrix for air transport.

| Origin  | Destination | By Air | Basic Fair in Rs Per Metric Tonne Per KM | Cost = Fair * Distance (In Rs) | Overhead Charges/handling Charges in Rs per Metric Tonne | Final Cost = Cost + Charges (In Rs) |
|---------|-------------|--------|-----------------------------------------|-------------------------------|----------------------------------------------------------|-------------------------------------|
| Delhi   | Bhopal      | 556.35 | 4.5                                      | 0.5                           |                                                          | 175                                  | 2678.57                             |
| Lucknow |             | 481.87 | 4.5                                      | 224                           |                                                          | 180                                  | 2348.41                             |
| Hyderabad|             | 1230.13| 4.5                                      | 534                           |                                                          | 235                                  | 5770.59                             |
| Patna   |             | 856.35 | 4.5                                      | 385                           |                                                          | 190                                  | 4043.57                             |

TABLE 9. A General Cost and Time matrix of soft transportation problem.

| Source  | Bhopal | Lucknow | Hyderabad | Patna | Supply |
|---------|--------|---------|-----------|-------|--------|
| Delhi   | \{c_{11}(e_1),c_{31}(e_2),c_{11}(e_3)\} | \{c_{12}(e_1),c_{12}(e_2),c_{12}(e_3)\} | \{c_{13}(e_1),c_{13}(e_2),c_{13}(e_3)\} | \{c_{14}(e_1),c_{14}(e_2),c_{14}(e_3)\} | 14 |
| Mumbai  | \{c_{21}(e_1),c_{21}(e_2),c_{21}(e_3)\} | \{c_{22}(e_1),c_{22}(e_2),c_{22}(e_3)\} | \{c_{23}(e_1),c_{23}(e_2),c_{23}(e_3)\} | \{c_{24}(e_1),c_{24}(e_2),c_{24}(e_3)\} | 13 |
| Chennai | \{c_{31}(e_1),c_{31}(e_2),c_{31}(e_3)\} | \{c_{32}(e_1),c_{32}(e_2),c_{32}(e_3)\} | \{c_{33}(e_1),c_{33}(e_2),c_{33}(e_3)\} | \{c_{34}(e_1),c_{34}(e_2),c_{34}(e_3)\} | 26 |
| Demand  | 12      | 10      | 12        |       |        |

TABLE 10. Transportation Matrix for cost and time for transport by combination of modes e_1 = road and e_2 = rail when priority is to minimize Cost.

| Source  | Destination | Bhopal | Lucknow | Hyderabad | Patna | Supply |
|---------|-------------|--------|---------|-----------|-------|--------|
| Delhi   | Cost        | 4170   | 3046    | 3700      | 3389  | 14     |
| Time    |             | 24.30  | 20.10   | 30.50     | 21.30 |        |
| Mumbai  | Cost        | 4034   | 3682    | 3940      | 3925  | 13     |
| Time    |             | 22.15  | 28.50   | 20.30     | 30.40 |        |
| Chennai | Cost        | 3733   | 4144    | 3345      | 4653  | 16     |
| Time    |             | 37.15  | 33.45   | 24.30     | 33.25 |        |
| Demand  |             | 9      | 12      | 10        | 12    | 43     |
there are $k$ parameters, then there will be $2^k - 1$ combinations of these parameters for which there will be $2^k - 1$ single objective transportation formulations. Thus for soft $n$-objectives transportation problem there will be $(n \times (2^k - 1))$ soft single objective transportation formulations. Here the soft bi-objective transportation problem is to be converted into $2 \times (2^k - 1)$ soft single objective transportation formulations.
TABLE 15. Transportation Matrix for cost and time for transport by combination of modes e2 = rail and e3 = air when priority is to minimize Time.

| Source       | Destination | Bhopal | Lucknow | Hyderabad | Patna | Supply |
|--------------|-------------|--------|---------|-----------|-------|--------|
| Delhi        | Time        | 17.3   | 15.15   | 3.4       | 2.15  | 14     |
|              | Cost        | 2584   | 2746    | 5770.59   | 4043.57 | 13    |
| Mumbai       | Time        | 20.15  | 5.15    | 22.3      | 3.3   | 13     |
|              | Cost        | 3063   | 5288.85 | 3009      | 6636.81 | 16    |
| Chennai      | Time        | 5      | 4.1     | 21.2      | 33.25 | 16     |
|              | Cost        | 5354.72| 6692.67 | 2959      | 4653  |        |
| Demand       |             | 9      | 12      | 10        | 12    | 43     |

TABLE 16. Transportation Matrix for cost and time for transport by combination of modes e1 = road; e2 = rail and e3 = air when priority is to minimize Cost.

| Source       | Destination | Bhopal | Lucknow | Hyderabad | Patna | Supply |
|--------------|-------------|--------|---------|-----------|-------|--------|
| Delhi        | Cost        | 2584   | 3046    | 3700      | 4043.57 | 14     |
|              | Time        | 17.3   | 20.1    | 30.5      | 21.5  |        |
| Mumbai       | Cost        | 3113.93| 3682    | 3940      | 3925  | 13     |
|              | Time        | 3.1    | 28.5    | 20.3      | 30.4  |        |
| Chennai      | Cost        | 5354.72| 4144    | 3345      | 4653  | 16     |
|              | Time        | 5      | 33.45   | 2403      | 33.25 |        |
| Demand       |             | 9      | 12      | 10        | 12    | 43     |

TABLE 17. Transportation Matrix for cost and time for transport by combination of modes e1 = road; e2 = rail and e3 = air when priority is to minimize Time.

| Source       | Destination | Bhopal | Lucknow | Hyderabad | Patna | Supply |
|--------------|-------------|--------|---------|-----------|-------|--------|
| Delhi        | Time        | 17.3   | 20.1    | 3.4       | 2.15  | 14     |
|              | Cost        | 2584   | 3046    | 5770.59   | 5274  |        |
| Mumbai       | Time        | 20.15  | 5.15    | 20.3      | 3.3   | 13     |
|              | Cost        | 3063   | 5288.85 | 3940      | 6636.81| 16    |
| Chennai      | Time        | 5      | 4.1     | 24.3      | 33.25 | 16     |
|              | Cost        | 5354.72| 6692.67 | 3345      | 4653  |        |
| Demand       |             | 9      | 12      | 10        | 12    | 43     |

Step 5: Optimum solution \( x_{ij}(e) \) is to be found for each of the soft linear programming problems obtained in step 4 by using any one of the suitable conventional method like simplex, Big-M, Two Phase, etc. or by methods of solution of transportation problems.

Step 6: The optimal solutions obtained for each of the soft single objective transportation formulations forms a set of available options for multi criteria decision making. This set of options is evaluated for multi criteria using dominance principle. As per the dominance principle if the \( i^{th} \) option is superior over the \( j^{th} \) option then the \( j^{th} \) option is discarded.

Step 7: The reduced set of options obtained in step 6 by dominance principle forms the set of best available options or solutions for various conditions or situations. The decision maker can use these options based on his requirements or constraints to arrive at the optimal solution.

A. NUMERICAL EXAMPLE

Problem: As an illustration the following transportation problem is considered in which the goods are to be transported from the cities Delhi, Mumbai and Chennai to the cities Bhopal, Lucknow, Hyderabad and Patna. The three modes of transport (Road, Rail and Air) are available as choice for the industries. The requirements of each destination and availability of each origin are shown in Table 2:

The following Tables represent the time and cost matrix for transport of goods from various origins to various destinations. These tables have been constructed by using data sets of tariff given on websites of some transport agencies.

The problem is to find the optimal solution to the above multi objective and multi modal transportation problem.

B. SOLUTION:

The proposed soft set approach is employed to obtain the solution:

Step 1: It is a balanced Transportation Problem as sum total supply is equal to sum total demand.

Step 2: Since it is a balanced problem, so there is no need to introduce fictitious columns or rows.
TABLE 18. Results of soft transportation problem.

| S. No | Mode | Priority | Allocation | Cost | Time | Optimum results |
|-------|------|----------|------------|------|------|----------------|
| 1     | e1   | Cost     | X12=12     | 36552| 20.10| 203111.5       |
|       |      |          | X14=2      | 10549| 32.50| 55.30          |
|       |      |          | X21=9      | 36306| 22.15| 53098.5        |
|       |      |          | X24=4      | 33156| 38.45| 34350          |
|       |      |          | X33=10     | 33450| 24.30| 37.15          |
| 2     | e1   | Time     | X12=12     | 36306| 22.15| 203111.5       |
|       |      |          | X14=2      | 10549| 38.45| 55.30          |
|       |      |          | X21=9      | 36306| 24.30| 53098.5        |
|       |      |          | X24=4      | 33156| 24.30| 34350          |
| 3     | e2   | Cost     | X12=12     | 21202| 15.15| 140271         |
|       |      |          | X14=2      | 82890| 15.15| 217508.50      |
|       |      |          | X21=9      | 41966| 30.40| 33450          |
|       |      |          | X24=4      | 33450| 30.40| 33450          |
| 4     | e2   | Time     | X12=12     | 21202| 15.15| 140271         |
|       |      |          | X14=2      | 82890| 15.15| 217508.50      |
|       |      |          | X21=9      | 41966| 30.40| 33450          |
|       |      |          | X24=4      | 33450| 30.40| 33450          |
| 5     | e3   | Cost     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 6     | e3   | Time     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 7     | e1,e2| Cost     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 8     | e1,e2| Time     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 9     | e1,e3| Cost     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 10    | e1,e3| Time     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 11    | e2,e3| Cost     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 12    | e2,e3| Time     | X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 13    | e1,e2,e3| Cost| X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |
| 14    | e1,e2,e3| Time| X12=12     | 28181| 1.45 | 154726.01      |
|       |      |          | X14=2      | 8087.1| 2.15 | 4.55           |
|       |      |          | X21=9      | 28025| 3.10 | 33.25          |
|       |      |          | X24=4      | 26547| 3.30 | 33.25          |

Step 3: The given multi objective and multi modal transportation problem is converted into soft transportation problem by considering the parameter e1 = Road; e2 = Train and e3 = Air as given below in Table 9:

Thus various combinations of these parameters e = e1, e2, e3 will be {(e1), (e2), (e3),(e1, e2),( e1, e3),(e2, e3), (e1, e2, e3)}. For these seven combinations seven transportation matrices each for cost and time optimization. Thus it will lead to fourteen transportation matrices in all. The Tables 3 to 8 respectively represent cost and time transportation matrices for each mode e1, e2 and e3.

The transportation matrices for other combinations of e1, e2, e3 are created as follows:

\[ \tilde{c}_{ij}(e_1, e_2) = \min(\tilde{c}_{ij}(e_1), \tilde{c}_{ij}(e_2)) \]

\[ \tilde{t}_{ij}(e_1, e_2) = \min(\tilde{t}_{ij}(e_1), \tilde{t}_{ij}(e_2)) \]
The objective is to determine the options which give minimum cost and minimum time for transportation. Here dominance principle is employed on the optimal solutions obtained and shown in Table 18. The evaluation of these optimal solutions by dominance principle is shown in Tables 19, 20 and 21.

Here in Table 19 if each element of an \(i\)th row is less than or equal to the corresponding elements of the \(j\)th row then the \(i\)th option is superior and \(j\)th option is inferior. Thus \(j\)th option will be discarded.

Step 7: The Table 21 obtained by discarding all the inferior options based on multi criteria from Table 19 now contains the best options. Finally the options available in Table 21 can further be evaluated based on his constraints or requirements to arrive at the best decision.

### IV. DISCUSSION

From the above Table 18 we observe that the optimal allocation, cost and time changes with the mode of transport.
and various combinations of different modes of transportation. This information is of great use to decision makers for evaluating their criteria involving multiple parameters and attributes of transportation problem. In Table 21 it can be seen that if the priority is to optimize the cost then the option at serial number 1 will be selected. Here Serial number 1 in Table 21 corresponds to entry at serial number 11 in Table 18 which corresponds to transportation matrix given in Table 14. This implies that among optimization of all the fourteen transportation models, the optimization of transportation model given in Table 14 gives the minimum cost and will be the best option if cost is the main criteria. The optimal allocations, modes of the transport, the cost and time can be seen at serial number 11 in Table 18 and origins and destinations with per unit cost and time are visible in Table 14. Further if the priority is to optimize the time then the option at serial number 3 in Table 21 will be preferred.

Here Serial number 3 in Table 21 corresponds to entry at serial number 6 in Table 18 which corresponds to transportation matrix given in Table 5. This implies that among optimization of all the fourteen transportation models, the optimization of transportation model given in Table 5 gives the minimum time and will be the best option if time is the main criteria. The optimal allocations, modes of the transport, the cost and time can be seen at serial number 6 in Table 18 and origins and destinations with per unit cost and time are visible in Table 5.

Also if the priority is to optimize both the cost and time almost equally then the option at serial number 2 in Table 21 will be the best option.

Here Serial number 2 in Table 21 corresponds to entry at serial number 5 in Table 18 which corresponds to transportation matrix given in Table 8. This implies that among optimization of all the fourteen transportation models, the optimization of transportation model given in Table 8 gives the better combination of minimum time and cost will be the best option if one can compromise a little on cost and time both. The optimal allocations, modes of the transport, the cost and time can be seen at serial number 5 in Table 18 and origins and destinations with per unit cost and time are visible in Table 8.

The optimal allocations and modes of transport can be seen from Table 18.

The above example gives us the result of multiple criteria for multi objective and multi model transportation system. The decision maker can use weights based on their priorities for each of their parameters and attributes as mentioned in previous sections. A program has been developed for the entire procedure using python libraries pulp, tkinter, math etc and implemented on the numerical example mentioned in this paper.

V. CONCLUSION
A soft set based approach is developed and successfully illustrated for a numerical example based on real data set for multi-objective and multi-model transportation problem. The model can be used to perform multi-criteria decision making for choice of various modes of transport with different objectives. The soft set is able to model the relationship of parameters with the attributes of the problem which makes possible to address the issues of uncertainty arising from degree of relationships involved and generate information for intelligent decision making. The python code developed for the entire problem can serve us an intelligent assistive system for multi-criteria decision making in transportation problem. The proposed model can further extended to various types of transportation problems which the authors as intend to develop in future.

APPENDIX 1
See Tables 10–21.

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