The formation of strip-like vortex motion by surface gravity waves

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Abstract. We studied experimentally the generation of vortex flow by non-collinear gravity waves with a frequency of 2.34 Hz. The vortices formed on the water surface have the form of stripes, the width $L = \pi/(2k \sin \theta)$ of which is determined by the wave vector $k$ and the angle between them, and the length is determined by the size of the system. We demonstrate that the measured dependence $\Omega(t)$ can be described within the recently developed model that considers the Eulerian contribution to the generated vortex flow and the effect of surface contamination.

Introduction

Waves on the water surface have been the subject of research for quite some time. Detecting particles are used to study experimentally horizontal transport. The Lagrangian velocity for several surface waves propagating in arbitrary directions [1] has nonzero vertical vorticity $\Omega = \partial_x V_y - \partial_y V_x$, which permits creating horizontal drift flows of various spatial structures [2-4].

If traveling and standing waves excited on the liquid surface intersect at a small angle, a vortex flow is formed in the form of stripes. The length of the strips is close to the length of the experimental cell, and the characteristic distance between the stripes is determined by the length of pump waves and the angle between them. Vertical vorticity generated by the flow on the liquid surface can be described by the sum of two terms [5,6]:

$$\Omega = \Omega_E + \Omega_S.$$ (1)

The first term, the Eulerian contribution, represents an average velocity of the liquid, while the second term represents the Stokes drift in an ideal liquid [7] in the case of non-collinear waves. These two contributions have different stationary amplitudes and characteristic time scales. The situation was analyzed in detail theoretically in [8], and it was found for two surface waves propagating on the surface of infinitely deep water at an angle of $2\theta$ to each other that the ratio of steady-state amplitudes is $\Omega_E/\Omega_S = 1/ \sin \theta$. The characteristic Stokes drift time $T_S$ can be estimated as half of the wave decay time since the Stokes drift is quadratic in amplitude. The characteristic time of the Eulerian contribution can be estimated as $T_E = L^2/(\pi^2 \nu)$, where $\nu$ is the kinematic viscosity of the liquid, $L$ is the horizontal dimensions of the slow flow, and $k$ is the wavenumber of excited surface waves.

For typical experimental conditions, $T_E \gg T_S$ due to additional dissipation of wave motion near the system boundaries [6, 9] and surface contamination [5, 10]. The latter factor also significantly affects the amplitude of the Eulerian contribution $\Omega_E$ on the liquid surface [5, 6], but in all cases, the theory predicts its universal dependence on time:

$$\Omega_E(t) \propto \text{erf}\left(\sqrt{t/T_E}\right),$$ (2)

where erf is the error function. Expression (2) is valid for a liquid of finite depth $d \gg 1/k$, but the applicability is limited in time $t \ll d^2/\nu$, during which the Eulerian flow does not have time to penetrate to the bottom due to viscous diffusion.
The experiments aimed to observe the formation of strip-like vortex flows, study the development of vorticity on the water surface at a pump frequency of 2.34 Hz, as well as study the formation of the Eulerian vorticity \( \Omega_E \) on the liquid surface [5].

1. Experimental Methods

The investigations were performed on an experimental setup, the scheme of which is presented in Figure 1. The setup consists of a bath a length of 70 cm, a width of 70 cm, and a height of 25 cm; it was made from 10-mm-thick glass. Distilled water was poured into the bath to the level of 10 cm. The bath was filled with distilled water to the level of 10 cm. The bath was placed on a Standa vibration isolation table with an air suspension.

Surface gravity waves were excited by wavemakers consisting of drives (Pioneer TS-W254R subwoofers with a nominal power of 250 W) and plungers (stainless steel tube 1 cm in diameter and 30 cm in length). One of the plungers was placed at an angle of 19° to the bath wall, the second was located parallel to the bath wall at a distance of 1 cm from it. Sinusoidal signals with a frequency of 2.34 Hz were generated by a dual-channel generator, amplified, and supplied to the subwoofers. The phase difference between the signals was \( \psi = \pi/2 \). The surface was recorded using a Canon EOS 70D camera at 24 fps. The vortex motion was recorded by decorating the surface with white polyamide PA12 particles with an average diameter of about 30 \( \mu m \), that were illuminated by an LED strip located along the bath perimeter. The PIVLab code [11] for MATLAB permitted obtaining the field of horizontal velocity associated with the motion of the detecting particles, after which the vertical vorticity \( \Omega \) was calculated. At the same time, vertical oscillations of the liquid surface were recorded using a recently developed technique [12], that is based on restoring the surface curvature by analyzing optical distortions of a contrast image at the bottom of the bath.

The distribution of vertical vorticity on the liquid surface and the spectrum of waves in the k-space were found by processing the obtained data by the Fourier transform with the Hann window function. The Hann function rolls off at the edges, so its application allows us to weaken the finite size effects and achieve periodic boundary conditions for the signals. Then we calculated the wave elevation and vorticity amplitudes based on the maximum values of corresponding peaks in the k-space. The registration method and data processing algorithms were discussed in detail earlier, see [13].

2. Results

Figure 2 (a) shows the tracks of polyamide particles 100 seconds after the start of pumping. The vortices have an elongated shape, and their inclination to the X axis is \( \theta = 18^\circ \). The width of these three vortices is \( L = 23 \) cm, and the length is limited by the size of the system. Figure 2 (b) illustrates the corresponding distribution of vertical vorticity on the water surface. The middle stripe has vorticity of the opposite sign with respect to the other two stripes. The data presented in Figure 2 correspond to the time
instant $t = 1000$ seconds after the start of pumping, and the amplitude of the wave propagating along the X axis in the stationary mode is 0.43 mm.

Figure 2. (a) – tracks of polyamide particles located on the liquid surface, obtained 1000 seconds after the pumping was switched on. (b) – the corresponding distribution of vertical vorticity on the water surface. The amplitude of the wave propagating along the X axis in the stationary regime is 0.43 mm.

Figure 3a shows the distribution of the energy of vertical oscillations at a frequency of 2.34 Hz in the k-space at the time $t = 1000$ seconds. On the water surface, two standing waves are excited, the energies of which differ by several times, and the angle between the waves is $2\theta \approx 36^\circ$ to the X axis. The wavenumber of both waves is $k \approx 0.22 \text{ cm}^{-1}$. Note that $kd \approx 2.2$, therefore, we can assume that the waves propagate in deep water.

Figure 3b illustrates the distribution of vertical vorticity in the wave vector space. Two bright peaks correspond to an elongated vortex structure inclined at an angle $\theta \approx 18^\circ$ to the X axis and shown earlier in Figure 2a. The width of the most intense vortex structure should be equal to $L = \frac{\pi}{2k \sin \theta} \approx 23 \text{ cm}$, which is in good agreement with the results presented in Figure 2.

Next, the amplitude of the rise of waves $H_1$ and $H_2$ and the amplitude of vorticity were calculated; they corresponded to the most intense vortex structure in the k-space. Let $H_1$ – be the amplitude of the wave propagating in the X direction, and $H_2$ – be the amplitude of the wave propagating at an angle of $2\theta \approx 36^\circ$.

Figure 4a shows the time dependence of $H_1H_2$ for different pump intensities. The nonmonotonicity of this dependence is due to a small discrepancy between the pump frequency and the frequency of the resonant mode. The beating of the oscillation amplitude completely dies out after about 100 seconds, after which the product of the amplitudes becomes constant.

Figure 3. Distribution of (a) wave energy and (b) vertical vorticity in the k-space obtained 1000 seconds after the start of pumping. The amplitude of the wave propagating along the X axis in the stationary mode is 0.43 mm. The ends of the black lines indicate the positions of the theoretical
peaks using the dispersion law $\omega^2 = gk \tanh(kd)$.

![Figure 4](image_url)

**Figure 4.** Time dependence of the product of amplitudes $H_1H_2(t)$ (a) and dependence of the amplitude of the vertical vorticity $\Omega(t)$ on the liquid surface (b). Different curves correspond to different values of the amplitude $H_1$ of the wave propagating along the X axis in the stationary mode: 1 – 0.17 mm, 2 – 0.29 mm, 3 - 0.43 mm, 4 – 0.57 mm, 5 – 0.72 mm, 6 – 0.85 mm.

Figure 4b shows the dependence $\Omega(t)$. At times shorter than 100 seconds, it follows changes in the product $H_1H_2(t)$ of the wave amplitudes. At longer times, the wave amplitude does not change, and the vorticity $\Omega(t)$ continues to increase. Thus, we observe the birth of the Eulerian contribution $\Omega_E(t)$ since the Stokes drift contribution $\Omega_St$ remains at a constant level.

3. Discussion

Let us direct the X’ axis in the direction of the stripes shown in Figure 2 and direct the Y’ axis - perpendicular to it. In this coordinate system, the surface elevation can be written as:

$$h(t, x', y') = H_1 \cos(\omega t) \cos(kx'\cos\theta - ky'\sin\theta) + H_2 \cos(\omega t + \psi) \cos(kx'\cos\theta + ky'\sin\theta), \quad (3)$$

where $\psi$ is the phase shift between the excited waves. In these experiments, the value of $\psi$ was close to $-\pi/2$.

Next, consider the vertical vorticity corresponding to the two brightest peaks in Figure 3b. The contribution associated with the Stokes drift is:

$$\Omega_S = \frac{a}{2} k^2 H_1 H_2 \sin(\theta) \cos(\omega t) \cos(2ky'\sin\theta), \quad (4)$$

see [8, Eq. E2]. The Eulerian contribution to vertical vorticity is determined by the following expression:

$$\Omega_E = \left( 2 + \frac{\varepsilon^2}{2\sqrt{2}\gamma(\varepsilon^2 - \sqrt{2} + 1)} \right) a k^2 H_1 H_2 \cos^3(\theta) \cos(2ky'\sin\theta) \text{erf}\left(\sqrt{t/T_E}\right). \quad (5)$$

Here we introduced $\gamma = \sqrt{\nu k^2/\omega}$ and assumed that the liquid surface might be covered with a thin insoluble liquid film due to contamination [10]. The presence of such a film on the liquid surface can increase significantly the intensity of a vortex flow. The properties of this film can be described by the dimensionless compression modulus $\varepsilon \geq 0$ introduced in [5] and characterizing the properties of the film.

To explain the measured dependence $\Omega(t)$ we use equation (1), substituting relations (4) and (5) into it. The result is shown in Figure 4b with a black curve. The optimal ratio corresponds to the film compression modulus $\varepsilon = 0.37$ and $T_E = 1000$ seconds.

Let us estimate the relative contributions of the Stokes drift and the Eulerian term. At $T = 100$ seconds and with an amplitude of $H_1 = 0.43$ mm, the Eulerian contribution is $\Omega_E = 5 \times 10^{-3}$ s$^{-1}$, while the Stokes drift is $\Omega_S = 5 \times 10^{-4}$ s$^{-1}$. At $T = 1000$ seconds and the same wave amplitude, the Eulerian contribution is $\Omega_E = 10^{-2}$ s$^{-1}$, and the Stokes drift remains the same. It should be noted that the ratio of the Stokes and Euler contributions to vertical vorticity does not depend on the wave amplitude if the Reynolds number is not very high since both terms are proportional to the product $H_1H_2(t)$, see (4) and (5).

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4. Conclusion

It has been shown experimentally that non-collinear waves propagating at an angle of $2\theta$ on the water surface form strip-like vortex flows. The width of the stripes can be estimated as $L = \pi/(2k \sin \theta)$, and their length is limited by the size of the system.

The Eulerian contribution has relatively slow kinetics compared to the Stokes drift. An increase in the Eulerian contribution is well described by dependence (2). This contribution leads to an increase in the total vorticity $\Omega(t)$, which can be seen in Figure 4b. Quantitative agreement with the theory was achieved only when possible contamination of the liquid surface was taken into account [5].

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