Gravitational Waves Propagation through the Stochastic Background of Gravitational Waves

Frajuca, C.\textsuperscript{1}, Bortoli, F.S.\textsuperscript{1}, Nakamoto, F.Y.\textsuperscript{1} and Santos, G.A.\textsuperscript{1}
\textsuperscript{1} Sao Paulo Federal Institute – Sao Paulo, SP, Brazil

E-mail: Frajuca@gmail.com

Abstract: With the recent claim that gravitational waves were finally detected and with other efforts around the world for GWs detection, it is reasonable to imagine that the relic gravitational wave background could be detected in some time in the future and with such information gather some hints about the origin of the universe. But, it’s also be considered that gravity has self-interaction, with such assumption it’s reasonable to expect that these gravitational wave will interact with the relic or non-relic GW background by scattering, for example. Such interaction should decrease the distance which such propagating waves could be detected. The propagation of gravitational waves (GWs) is analyzed in an asymptotically de Sitter space by the perturbation expansion around Minkowski space using a scalar component. Using the case of de Sitter inflationary phase scenario, the perturbation propagates through a FRW background. The GW, using the actual value for the Hubble scale ($H_0$), has a damping factor with a very small value for the size of the observational universe; the stochastic relic GW background is given by a dimensionless function of the frequency. In this work we analyze this same damping including the gravitational wave background due to astrophysical sources such background is 3 orders of magnitude bigger in some frequencies and produces a higher damping factor.

1. Introduction
The detection of gravitational waves came after a long road of experiments planned in 2010 [1], in 2016 finally the detection was made [2,3]. Gravitational waves got a very strong evidence with the PSR B1913+16 (also known as PSR J1915+1606, PSR 1913+16, and the Hulse–Taylor binary after its discoverers) is a pulsar (a radiating neutron star) which together with another neutron star thus forming a binary star system. PSR 1913+16 was the first binary pulsar to be discovered and its orbital period is decreasing with time due the emission of gravitational waves [4]. The first attempts to gravitational wave detection starts in the early sixties [5] with the resonant mass gravitational wave detection [6,7,8,9]. The efforts towards the detection of gravitational waves using this kind of detector culminated with the spherical detector where six sensors are connected to the surface of the sphere, arranged according to a work based on the distribution of Merkowitz and Johnson [10,11], these transducers are located as if they were in the center of 6 pentagons connected in a surface corresponding to half dodecahedron [12,13]. Each transducer mechanically amplifies the motion occurring at the detection rate of the detector in the region of the sphere in which it is connected. The already amplified movement excites the membrane of one resonant cavity.

The Brazilian efforts on the field can be seen in [14,15,16,17,18,19,20,21,22,23,24,25,26,27].
The theory postulates that gravitons are always attractive (gravity never repels), acting beyond any distance (gravity is universal) and come from an unlimited number of objects. Therefore, if graviton exists, it must be a boson with spin equal to two, and must have a zero rest mass, according to Quantum Mechanics.

The gravitons were postulated after the great success of quantum theory (especially the Standard Model) in describing the behavior of all other forces known to nature as transmitted by elementary particles: electromagnetism through the photon, the strong nuclear force by gluons, and the weak nuclear force by the W and Z bosons. The hypothesis is that the gravitational force is similarly transmitted by an elementary particle, which has not yet been discovered yet, called graviton, rather than described in terms of space-time curvature as general relativity. At the classical limit, both approaches provide identical results, and compatible with Newton's law of gravitation.

The granulation of quantum theory is not compatible with the uniformity of Einstein's general relativity. These problems, along with some conceptual puzzles, have led many physicists to believe that a more complete theory than general relativity should regulate behavior close to Planck's length. String theory finally emerged as the most promising solution, it is the only known theory in which the quantum correction of any order for graviton dispersion is finite, and is always stable. But it is totally conjectural because string theory is not validated through experiments since its range of energies is far beyond what our experimental ability allows.

For this reason it is tried in this work to obtain a form to obtain results similar to the one of graviton scattering however using the theory of general relativity with a scalar component to try to prove the validity of this method and in the future to apply the same method using only generative relativity.

Gravitational waves can be seen as the coherent state of many gravitons, with both electromagnetic waves being coherent states of photons.

2. The scope of this work
With the detection of gravitational wave its is reasonable to imagine that the relic gravitational wave background could be detected in some time in the future and with such information gather some hints about the origin of the universe [28,29]. Some calculations in the amplitude of some background has been made, as can be seen on figure 1.

But, it’s need to be considered that gravity has self-interaction, it means that graviton interacts with gravitons [30, 31] as can be seem in figure 2. With such assumption it’s reasonable to expect that these gravitational wave will interact with the relic or non-relic GW background by scattering, making some of the gravitational wave that is traveling in space loses coherence and deviate in another direction. Such interaction should decrease the distance which such propagating waves could be detected or at least making the signal weaker what makes it appear that the source is far away.
Figure 1. The gravitational wave background and the possible detector for each frequency band as can be found in the web page of Caltech [http://www.tapir.caltech.edu/~teyvet/Waves/gwave_spectrum.html]. SMBHB means Supermassive Black Hole Binaries; WDB means White Dwarf Binaries; EMRI means Extreme Mass Ratio Inspirals and NSB means Neutron Star Binaries.

Figure 2. Four-graviton scattering in the first two orders of the perturbation theory (panels a and b, respectively).
The propagation of gravitational waves (GWs) is analyzed in an asymptotically de Sitter space by the perturbation expansion around Minkowski. The stochastic relic GW background is given by a dimensionless function of the frequency [28 and 29]. Another way to characterize such background could be seen in [32, 33]. Using the case of de Sitter inflationary phase scenario [28, 29, 34, 35], the perturbation propagates through a FRW background with a metric

\[ ds^2 = a^2(\eta) \left( -d\eta^2 + dx^2 \right) \] (1)

where the negative term is the conformal time.

The GW is represented by

\[ h_{\mu\nu} = e_{\nu}^\alpha \phi(\eta) e^{ik \cdot x} \] (2)

The perturbation propagation metric could be expressed as

\[ g_{\mu\nu} = a^2(\eta) \left( -d\eta^2 + dx^2 + h_{\mu\nu} dx^\mu dx^\nu \right) \] (3)

The amplitude of the GW must satisfy the following equation:

\[ \left( \frac{d^2}{d\eta^2} + 2\frac{da}{d\eta} \frac{d}{d\eta} + |k|^2 \right) \phi = 0 \] (4)

the relevant result is the one corresponding to the de Sitter inflationary phase, where the scale factor behaves as

\[ a \propto e^{H_{ds} t} \] (5)

in terms of the physical time \( t \). In this scenario the solution becomes

\[ \phi(\eta) = \frac{a(\eta_1)}{a(\eta)} \left( 1 + iH_{ds} \eta \right)^{-1} e^{-ik(\eta - \eta_1)} \] (6)

here \( H_{ds} \) is the Hubble factor during the de Sitter phase.

It shows that, according to [35] the amplitude has a damping effect that goes with

\[ \phi \propto e^{-H_{ds} t} \] (7)

where \( H_{ds} \) is the Hubble parameter in the De Sitter phase. It shows that after a distance equal to \( 1/H_{ds} \) the amplitude of the wave decreases to a factor of \( 1/e \).

Now if the actual value for the Hubble scale \( H_{obs} \), has a damping factor with a very small valor for the size of the observational universe. In this work we analyze this same damping including the gravitational wave background due to astrophysical sources. Such background is 3 orders of magnitude bigger in some frequencies and produces a considerable damping at shorter distances.

These results are presented in [34] and gave the authors the idea of developing this work, nevertheless it was done using the cosmological constant and not the GW background.
3. Results
This shows that the GW, using the actual value for the Hubble scale $H_0$, the amplitude of the GW decreases by a factor of $1/e$ after 100BLY propagating in the relic gravitational wave background.

What will happen if the gravitational background relic has the values that the GW background has today? Figure 1 shows that the gravitational wave background has a amplitude bigger than the relic gravitational background by a factor of $10^{1.5}$ in strain $h$ in all the wavelengths. The stochastic GW background is a energy density distribution $\rho_{GW}$, which is proportional to the GW strain squared $\langle h^2 \rangle$, and the critical density to close the universe is given by:

$$\rho_c = \frac{3 H_0^2}{8 \pi G}$$  \hspace{1cm} (8)

the gravitational wave relic is part of the critial density and depends in $h^2$, as the authors want to increase value of the gravitational wave relic to the value of the gravitational wave background, let assume that the critical density increases by the same factor, then $H_0^2$ should increase by the same order of $h^2$.

This implies that the GW, using the actual value for the Hubble scale $H_0$, correct by a factor of $10^{1.5}$, the amplitude of the GW decreases by a factor of $1/e$ after about 3BLY.

4 Conclusions
Taking in consideration that gravitons should scatter as they interact with other gravitons, gravitational waves should do the same as they interact with the gravitational wave background. This effect is small but as gravitational waves travel very long distances the effect should become important.

This is just a preliminary result and much work is yet to be done.

As the gravitational wave measured in [1,2] was identified to be occurred in a distance of 1BLY without taking this effect into account, it could be located in a distance 30% closer, if the approximations used here are correct.

The next step to this work is to make the same calculations using only General Relativity without the use of scalar components.

Acknowledgements
Carlos Frajuca acknowledges FAPESP for grant #2013/26258-4 and grant #2006/56041-3.

References
[1] The Gravitational Waves International Committee Roadmap (GWIC). A global pan. 2010. Glasgow: University of Glasgow - Department of Physics and Astronomy - Kelvin Building (G12 8QQ), 117p.
[2] LIGO Scientific Collab. and Virgo Collab. (Abbott B P et al.) 2016 Phys.Rev. Lett., 116 (6): 061102
[3] Castelvecchi D, Witze A Nature News doi:10.1038/nature.2016.19361. Retrieved 11 February 2016[2]
[4] Taylor J H, Hulse R A, Fowler L A, Guallahorn G E, Rankin J M 1976 Astrophysical Journal 206 L53
[5] Weber J. 1960 Physical Review 117 306
[6] Richard J P 1984 Physical Review Letters 167 165
[7] Richard J P Proc. Second Marcel Grossmann Meeting on General Relativity, edited R.Ruffini, (North-Holland, Amsterdam).
[8] Frajuca C et al 2004 Class. Quantum Grav. 21 1107
[9] Frossati G Proc. First International Workshop for an Omnidirectional Gravitational Radiation Observatory, W.F. Veloso, Jr., O.D. Aguiar and N.S. Magalhaes, editors (Singapore, World Scientific, 1997).
[10] Merkowitz S M and Johnson W W 1997 Phys. Rev. D 56 7513
[11] S. M. Merkowitz and W. W. Johnson 1993 Phys. Rev. Lett. 70, 2367
[12] Frajuca, C. “Otimização de transdutores de dois modos mecânicos para detectores de ondas gravitacionais.” São Paulo. 97 p. PhD Dissertation - Universidade de Sao Paulo, 1996.
[13] Frajuca C. et al , Proc. 3rd Edoardo Amaldi Conference on Gravitational Waves (Pasadena, USA, July 1999). AIP Conf. Proc. 523 (New York, AIP) p.417
[14] Aguiar O D et all 2011 Journal of Physics: Conference Series 363 012003
[15] O. D. Aguiar et. al 2005 Class. Quantum Grav. 22, 209
[16] Frajauca et al 2002 Class. Quantum Grav. 19 1961
[17] Magalhaes N S et al 1997 Astrophysical Journal 475, 462
[18] Magalhaes N S et al 1995 MNRAS 274, 670
[19] Frajauca C, Bortoli F S, Magalhaes N S 2005 Brazilian Journal of Physics 35 1201
[20] Aguiar O D et al. 2006 Journal Class. Quantum Grav. 23, 239
[21] Frajauca C, Bortoli F S, Magalhaes N S 2006 Journal of Physics: Conference Series 32 319
[22] Frajauca C, Bortoli F S, Magalhaes N S 2005 Brazilian Journal of Physics 35 1201
[23] Aguiar O D et al., Class. Quantum Grav. 21 459, 2004.
[24] Frajauca C, Magalhaes N S, Horiguti A M 2008 Journal of Physics: Conference Series 122 012029
[25] Bortoli F S et al 2010 Journal of Physics: Conference Series 228 012011.
[26] Magalhaes et al 1997 Gen. Relat. Grav. 29 1511
[27] Magalhaes et al 1997 Astroph. J. 475 462
[28] Allén B The stochastic gravity-wave background: Sources and Detection, in Relativistic Gravitation and Gravitational Radiation (Les Houches, 1995), p. 373
[29] Maggiore M 2000 Phys. Rep. 331 283
[30] Tsvi P 2016 International Journal of Modern Physics D 25 1644020
[31] Anirban B 2016 CQG 33 125028
[32] Capozziello S, Corda C, De Laurentis M 2007 Mod. Phys. Lett. A 22 2647
[33] Capozziello S, Corda C, De Laurentis M 2007 Mod. Phys. Lett. A 22 1097
[34] Nowakowski M, Arraut I 2010 Acta Physica Polonica B 41 911
[35] Arraut I 2013 Modern Physics Lettes A 28 1350019