INTRODUCTION

The quantum Hall effect (QHE), which was first observed in two-dimensional (2D) electron gas systems, demonstrates the first example of a quantum state that is topologically distinct from other condensed matters. Under a strong external magnetic field, the QHE edge states emerging at the sample boundary exhibit the chiral dissipationless transport nature with a vanishing longitudinal resistance and a quantized Hall conductance of $\frac{e^2}{h}$ (where $v$ is the Landau level index, $e$ is the electron charge, and $h$ is the Planck constant). Such a transport signature, if the requirement of high magnetic field can be removed, would unveil tremendous opportunities in low-power electronic applications (1). This inspires an upsurge in searching for a zero-field version of QHE for more than three decades. One such version is known as the quantum anomalous Hall effect (QAHE) (2–6). A topologically nontrivial electronic structure with time-reversal symmetry broken by magnetic moment instead of an external magnetic field is one of the most promising systems to host the QAHE. Recently, the QAHE was demonstrated experimentally in magnetically doped topological insulators (MTIs), e.g., Cr/V:(Bi,Sb)$_2$Te$_3$, where the utilization of both the intrinsic strong spin-orbit coupling (SOC) and the incorporated long-range out-of-plane ferromagnetic (FM) order give rise to the chiral edge state with a nonzero Chern number (7, 8). However, to date, the observation of the QAHE state is yet limited to extremely low temperatures (below 1 K), although some host MTIs have a Curie temperature above 170 K (9, 10). As a result, the underlying physics of this temperature limit has remained to be an open question so far (9, 11–16).

To address this issue, earlier attempts have been focusing on understanding the FM origins and exploring the possible band structure modifications caused by the magnetic dopants. From the FM ordering aspect, the presence of superparamagnetic domains and the bulk valence band near the Fermi level could also introduce additional dissipative conduction channels and hence deteriorate the transport signature of the QAHE (18–20). These findings all suggest sophisticated coupling mechanisms among the SOC, FM ordering, dopant profile, and film structure in realizing the QAHE (21). Consequently, a QAHE platform in which all these key parameters can be manipulated individually is of great demand before pushing the limit of the QAHE toward room temperature. In this work, a set of MTI thin films are prepared with a wide tuning range of thickness and doping profile, with which the limiting factors to the onset of the QAHE can be systematically investigated. An intricate interplay between the FM order and the band structure is demonstrated by investigating the quantum transport signature, activation behavior, magneto-optic response, and the QAHE phase transition diagram. The activation behavior of the QAHE is found to be closely related to the gap between the Fermi level and its nearest bulk/impurity band edge. The tracing of such a gap in samples with various thicknesses and the external magnetic field points to different origins of temperature limiting factors. This QAHE activation gap can serve as a benchmark of the QAHE and provide a guide for searching high-temperature QAHE material and heterostructures toward robust and functional QAHE states.

RESULTS AND DISCUSSION

Global phase diagram of QAHE in various thicknesses

Using an optimized MTI thin-film growth procedure established from our previous work (11, 22), a set of high-quality (Cr$_{0.12}$Bi$_{0.88}$Sb$_{0.62}$)$_2$Te$_3$ samples with film thickness ranging from 6 to 10 quintuple layers (QLs) were epitaxially grown with the Fermi level positioned inside
the surface magnetic exchange gap without additional gate tuning. Figure 1 summarizes the magneto-transport results of three uniformly doped \((\text{Cr}_{0.12}\text{Bi}_{0.26}\text{Sb}_{0.62})\text{Te}_3\) samples with film thicknesses of 6, 8, and 10 QLs, respectively. At 20 mK, all three samples exhibit a fully quantized Hall resistance \(\rho_{xy} = \pm h/e^2\) at zero magnetic field, highlighting the realization of QAHE. The resistance fluctuation close to zero magnetic field is a result of unstable temperature, with more details explained in the Supplementary Materials.

Notably, the detailed quantum conductance and topological phase transition exhibit distinct features among the three samples. Specifically, at the extreme 2D limit, i.e., the six-QL MTI sample, the top and bottom surfaces will strongly hybridize, resulting in the topological quantum phase transition between the \(\pm h/e^2\) states (i.e., with Chern number \(C = \pm 1\), respectively) \((23)\). As discussed in our previous work \((22, 23)\), the surface hybridization gap due to the quantum confinement of the six-QL MTI sample would compete with the magnetic exchange gap, whose magnitude is determined by the FM order. Accordingly, when the FM moment is minimized during the magnetization reversal, the dominant hybridization gap would give rise to a topologically trivial insulating state with \(C = 0\). Experimentally, this emergent topological quantum transition is shown in Fig. 1 (D and G), where \(\sigma_{xx}\) of the six-QL sample goes through two peaks separating the \(C = \pm 1\) and 0 states (all of which have zero \(\sigma_{xx}\) value), and \(\sigma_{xy}\) develops zero-conductance Hall plateaus at \(C = 0\) between \(\pm h/e^2\) states during each sweep (the orange/blue colored traces represent the different field sweep directions). Here, it is noted that the \(C = 0\) insulating state

![Fig. 1. QAHE realized in different thicknesses of MTIs. Curves in red indicate the magnetic field sweeping from the positive to the negative direction, and blue curves indicate the reverse trace. (A to C) Hall resistivity versus external magnetic field for samples with thicknesses of 6, 8, and 10 QLs, respectively. All samples show Hall resistivity quantized at \(\pm h/e^2\) at zero magnetic field. For (A), the transition part of the data is not shown due to the sample's extremely insulating behavior. (D to F) Longitudinal conductivity for the same set of MTIs. At zero magnetic field, 8 QLs show vanishing longitudinal conductivity and 10 QLs still have a finite longitudinal conductivity, yet the 6-QL MTI shows a split double-peak behavior around the coercive field and nonzero longitudinal conductivity. (G to I) Hall conductivity for the same set of MTIs. At zero magnetic field, 8- and 10-QL MTIs show quantized \(\sigma_{xy}\) at \(e^2/h\) when the external field is approaching zero. (J) Schematic drawing of the MTI samples. (K) Global phase diagram of QAHE for MTIs' thicknesses ranging from 5 to 10 QLs. For five-QL MTI, the hybridization gap dominates the system and drives it into a trivial insulator. For six-QL MTI, the competition between hybridization gap and exchange gap during transition forms a unique double-semicircle transition curve. For 8- and 10-QL samples, the phase transition only happens between the \(C = \pm 1\) states.

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states exhibit a noisy and high resistance behavior (shown in fig. S1); this can be explained by percolation theory, where the sample entered a random domain state during the switching. The resistance as high as 10 megohms is also, by far, the most insulating state reported in the QAHE systems (24, 25). Furthermore, this insulating state (σxx, σxy) = (0, 0) is found to be present across a wide magnetic field range (i.e., ~0.2 T) which not only manifests the unique metal-insulator switching of the QAHE state in the 2D limit (22) but also offers the opportunity for searching exotic critical quantum phases, for instance, the chiral Majorana edge modes in the QAH/superconductor hybrid system between \( C = \pm 1 \) and \( C = 0 \) states (26, 27). On the contrary, in thicker MTIs where the hybridization gap becomes negligible, the system is always topologically nontrivial with either Chern number \( C = \pm 1 \). Such a scenario is demonstrated in both the 8- and 10-QL samples, where a sharp transition between the \(+h/e^2\) and the \(-h/e^2\) edge states takes over in the QAHE hysteresis loop without the presence of any intermediate insulating states, as shown in Fig. 1 (E, F, H, and I).

By achieving the QAH states across a thickness range, we can complete a thickness-dependent QAH phase diagram. As highlighted in Fig. 1K, the \((\sigma_{xy}, \sigma_{xx})\) plot of the six-QL MTI sample follows a double semicircular transition that continuously connects the \((\pm h/e^2, 0)\) and \((0, 0)\) states, while the 8- and 10-QL samples only exhibit two stable states at \((\pm h/e^2, 0)\). Under the fixed magnetic sweeping speed (0.1 T/min), the eight-QL film shows a single-domain behavior where the transition between the two quantized \( C = \pm 1 \) states is fast, with only a few discrete \((\sigma_{xy}, \sigma_{xx})\) points located along the meta-stable transition route, while the \((\sigma_{xy}, \sigma_{xx})\) plot of the 10-QL sample displays a more continuous feature and the transition loop is found to be slightly deviated from the perfect semicircle contour, which may suggest the presence of additional dissipative conduction path (given that the 10-QL film may possibly have the largest bulk contribution). We notice that when the quantum confinement dominates in the ultrathin MTI film (i.e., five QLS), the hybridization gap [~50 meV according to (28, 29)] will drive the host QAHE system into a topologically trivial insulating state regardless of the external magnetic field (30–32). Under such circumstances, the five-QL phase diagram reduces to a single point, where \( \sigma_{xy} \) and \( \sigma_{xx} \) both are equal to zero corresponding to the fact that the Fermi level rests inside the trivial surface and bulk gap. In short, the thickness-dependent QAHE results, along with the corresponding QAH phase diagram shown in Fig. 1K, signify the importance of film thickness (or equivalently, the magnetic exchange gap and hybridization gap), thus implying the possibility to enhance the QAH temperature and manipulate QAH states via structural engineering, for which we will discuss in the “Toward high-temperature and functional QAHE” section.

**QAH activation gap**

In both integer and fractional QHE, the activation energy scale, which is usually extracted from the temperature-dependent \( \sigma_{xx} \) plot, has been widely used to determine the robustness of such quantum state against thermal fluctuation (33–35). Sharing the same topological nature (36), the QAH activation gap \( \Delta \) also carries similar information as its QHE counterpart; hence, it can be investigated by using the same approach. Application wise, this activation gap size, which serves as a benchmark parameter, dictates the viable temperature range to use the dissipationless nature of QAH states. Inspired by the unique thickness-dependent QAH transport behaviors obtained in Fig. 1, we further traced the QAH activation gap in these MTI thin films with different magnetic field responses to better understand the underlying mechanisms, as well as to probe the possible temperature limiting factors. In particular, we attempted to use a single–activation gap Arrhenius equation \( \sigma_{xx} = \sigma_0 \cdot \exp(-\Delta/k_B T) \) to fit the data, yet only succeeded in the high field regime in the six-QL sample, as shown in Fig. 2A. Alternatively, when the external field was smaller than 0.15 T, an additional dissipative conductivity term \( \sigma_1 \) that does not follow the Arrhenius equation had to be included to fit the data such that

\[
\sigma_{xx} = \sum_{i=0,1} \sigma_i \cdot \exp(-\frac{\Delta_i}{k_B T}), \Delta_1 = 0
\]

In contrast, the QHE activation behaviors can usually be described by one or few activation energies, which follow Arrhenius equation in either graphene, oxide material, or 2D electron gas systems (34, 37–39). Meanwhile, both \( \Delta_0 \) and \( \sigma_1 \) have a strong field dependence, where the increase of the applied magnetic field results in a larger activation gap and a smaller dissipative component, as depicted in Fig. 2 (D and E). Therefore, we may conclude that to drive the system into a near-perfect QAH state with minimized \( \sigma_1 \), an additional external field (>0.15 T) is required to suppress the dissipative channel caused by the weak ferromagnetism and existence of superparamagnetism in the six-QL sample, as we will elaborate in the next section.

On the other hand, when the thin-film thickness is thicker than six QLS, the activation feature markedly changes. As compared with the six-QL sample, the activated conduction behaviors of both the 8- and 10-QL samples always include both the dissipative conductivity \( \sigma_1 \) across the entire magnetic field range (0 to 5 T), as shown in Fig. 2D. Unlike the six-QL case, the magnitude of \( \sigma_1 \) in thicker MTI samples is found to gradually increase under larger magnetic field, hence implying a different origin of the dissipative channel rather than the unstable ferromagnetism, as illustrated in Fig. 2D. This phenomenon is commonly observed in metallic systems, where the positive magnetoresistance background is related to the Fermi level crossing the trivial band edges. Besides, it is noted that our results agree well with the recent angle-resolved photoemission spectroscopy (ARPES) study (18) focusing on the band structure near the Dirac point of the MTI, which showed the overlapping of the nontrivial surface band with the trivial bulk valence band.

Meanwhile, as summarized in Fig. 2E, the QAH activation gaps \( \Delta_0 \) in the cases of 8- and 10-QL MTIs are both negatively related to the external magnetic field, which are opposite to the 6-QL scenario. This field-dependent feature highlights the pronounced splitting of the bulk valence band under high magnetic field in the thicker MTIs (40). In general, magnetic dopants can induce an energy splitting between up and down spins, which has been well studied in diluted magnetic semiconductors (40, 41). This splitting comes from the combined effect of the on-site exchange energy and the Zeeman energy given by the applied field. Here, the MTI system, without considering its topological property, also falls in the category similar to the diluted magnetic semiconductor system (42). This band splitting can affect MTI’s transport properties if they are included in the thermal window. Here, we summarize and propose a possible band structure picture as shown in Fig. 2F. In the case of six QLS, the bulk-band splitting is pushed away from the surface bands due to the vertical quantum confinement and thus does not strongly affect the transport. However, when thicker than six QLS, the band edges are closer to the Fermi level, and the activation gap \( \Delta_0 \) may be strongly
modulated. An increase in the applied field can enlarge the spin splitting, and therefore, $\Delta_0$ decreases. Furthermore, we need to point out that the 8-QL sample shows almost five times larger activation gap and one order of magnitude smaller dissipative term than the 10-QL sample. In addition, thicker MTI samples have another trivial longitudinal resistance component, which is the side surface conduction that has been reported before (11). However, this effect has no magnetic field dependence and is not included in our band structure discussion. To sum up, the marked transport result difference again manifests the critical thickness limiting factor in the (Bi,Sb)$_2$Te$_3$-based QAH system, where the inevitable increased bulk contribution finally prevents the detection of QAHE in even thicker MTI samples.

**Magneto-optic Kerr effect**

The above transport experiments of the six-QL MTI sample revealed the importance of MTI’s ferromagnetism in reaching full quantization. Unlike the 8- and 10-QL MTI samples, it is noted that the conductance $\sigma_{xx}$ and $\sigma_{xy}$ of the 6-QL MTI deviate from the quantized value before the external field approaches zero, as shown in Fig. 1 (D and G). Therefore, the need of an additional small external field to realize full quantization may imply the existence of soft magnetic order in the six-QL MTI sample. To directly address this magnetic order, we further performed magneto-optic Kerr effect (MOKE) measurements on one of our six-QL MTI samples. The magneto-optical setup is constructed with a loop-less fiber-optic Sagnac interferometer for better sensitivity, which can accurately measure time-reversal symmetry breaking event down to 10 nano-radians (43). Figure 3 (A and B) summarizes both the temperature-dependent Kerr rotation angle and the anomalous Hall resistance $R_{xy}$ results, which are taken simultaneously (see Materials and Methods). It is observed that the magnetic hysteresis behavior revealed by both results is highly consistent across a wide temperature range (70 mK to 28 K) in terms of the coercive field size. Yet, we need to emphasize the major difference between the two sets of data: When the system temperature decreases, the up-ramping of magnetic field quickly drives the Kerr signal to a saturation value around $\sim 10^{-3}$ rad that almost remains the same for up to 4.3 K, as shown in Fig. 3A (i.e., indicating that the magnetization of the MTI sample is fully saturated); in contrast, the $R_{xy}$ value will not reach the quantized $h/e^2$ value until $T < 700$ mK, as shown in Fig. 3B. This finding may suggest that the robust macroscopic FM order alone does not necessarily guarantee the dissipationless quantum transport in our MTI system.

As mentioned in the previous section, we have identified the presence of a dissipative conductivity $\sigma_1$ when the applied magnetic field is smaller than 0.15 T and related this term to the weak ferromagnetism in the six-QL MTI. Using optical technique, here, we are able
to confirm the existence of superparamagnetism in our sample, as some groups have reported similar findings using different techniques. The first evidence is in the hysteresis loop scan shown in Fig. 3 (A and B), where coercivity nearly vanishes above 12 K for both optical and transport measurements, although there is still a noticeable magnetic moment. This is often observed in a low dimensional magnetic system (44, 45).

Followed by the measurements tracing both temperature-dependent Kerr signal and $R_{xy}$ signals by using different field cooling/warming schemes as shown in Fig. 3 (C to F), the second evidence is that in the linear-scale plot (Fig. 3, C and D), the temperature-dependent Kerr rotation signal during the field cooling ($\mu_0 H = 400$ and 12 mT) process does not follow classic Curie-Weiss type of spontaneous magnetization behavior in a typical ferromagnet (detailed information of a typical ferromagnet SrRuO$_3$ is provided in fig. S2) (46). The evident discrepancy between the two field-cooling curves (red and green in Fig. 3, C and D) acquired under different magnetic fields suggests that the magnetism of six-QL MTI is not strong enough to yield a robust order without the assistance of external field.

The third evidence is that during the succeeding zero-field warming process (step 2 curve in Fig. 3, C and D, respectively), the curves’ deviation from the saturation magnetization starts as early as 400 mK (blue trace in Fig. 3, E and F; $T = 400$ mK is highlighted with dashed line), which is almost two orders of magnitude lower than MTI’s perceived Curie temperature (~20 K). In addition, the zero-field warming curves for Kerr and $R_{xy}$ signal go to near zero when the base temperature reaches above 14 K. It may be attributed to the spin relaxation due to the competition between the magnetic anisotropy energy $KV$ ($K$ is the magnetocrystalline anisotropy constant and $V$ is the nanomagnet volume) versus the thermal fluctuation energy $k_B T$, as the origin of superparamagnetism (47). In addition, we observed a rather abnormal sign change of both the Kerr rotation signal and the Hall voltage during the zero-field warming process within the [6 K, 14 K] regime. The reason of this magnetization flipping at elevated temperature is not clear yet, but it has also been observed in some magnetic systems with complicated spin configurations (48, 49). The effect arising from the antiferromagnetically coupled Cr/Sb and Te moment identified using x-ray magnetic circular dichroism technique.
(50, 51) might participate in this phenomenon, which requires further investigation.

To conclude this part, we confirmed the existence of superparamagnetism in the six-QL uniform-doped MTI. The magnetic anisotropy energy scale of the superparamagnetism when converted to temperature (TKV) should be much lower than the Curie temperature TC, given that the onset of QAHE happens only at T < 700 mK. As a result, superparamagnetic domains may form inside the film (20) over a relatively wide temperature range, and the misalignment of such domains may account for the small QAHE activation gap Δm as well as the parallel dissipative conductivity σ∥ in the hybridized 2D MTI system. Therefore, to solve this problem, major effort should focus on improving MTIs with not only more robust but also more uniform FM order.

**Toward high-temperature and functional QAHE**

So far, we have identified two methods that may help to increase the QAHE gap, namely, the uniform single-domain FM texture and the minimized dissipative bulk conduction channel. For the case of thicker MTI films, the realization of QAHE is limited by the intrinsic band structure (i.e., the small thermal window between the Fermi level and the bulk valence band as shown in Fig. 2F). Therefore, to enlarge the thermal window, additional elemental doping [e.g., Sn or S (52, 53)] is required to modify the bulk valence band. However, the involvement of such high vapor pressure elements would inevitably make it more difficult to precisely control the chemical composition of the MTI sample during sample growth (22). Alternatively, as we demonstrated in the “QAHE activation gap” section, the 2D MTI system turns out to be a better platform for the pursuit of high-temperature QAHE, because the parallel bulk conduction is minimized and the dissipative conductivity σ∥ (due to the multi-domain and superparamagnetic texture) can be suppressed with the assistance of the external magnetic field. Accordingly, to further stabilize the magnetic domain and to increase the QAHE exchange gap Δm at zero magnetic field, we introduce structural engineering on the 2D MTI system by modifying the dopant profile in the six-QL candidates.

To experimentally validate the above proposal, we prepared two additional 2D MTI samples with different structures: one is a uniformly doped six QLs (Cr0.16Bi0.25Sb0.59)2Te3 with higher Cr doping of 16% and the other one is a trilayer structure sample, where both the top and bottom surfaces (i.e., with a thickness of one QL) are heavily doped by Cr (~24%), while the bulk four QLs remain as the (Cr0.12Bi0.26Sb0.62)2Te3 composition, as illustrated in Fig. 4A. It is found that the higher uniform Cr-doping MTI showed similar transport behavior as the one with original recipe, as shown in Fig. 4D. In contrast, for the trilayer modulation-doped MTI, the activated conduction plot shown in Fig. 4B is governed by the single-gap picture without the presence of the dissipative σ∥ contribution even at zero magnetic field. More notably, the measured QAHE activation gap Δm = 80 μeV is more than four times larger than the uniform MTI counterpart at zero field, and is almost insensitive to external magnetic field, as highlighted in Fig. 4C. Concurrently, the longitudinal and Hall conductance in this trilayer six-QL MTI sample perfectly quantized at (σxx, σxy) = (0, ±ℏe2/h) at zero field (shown in Fig. S4), hence manifesting the formation of a robust spontaneous FM order. Here, we need to point out that the onset of QAHE in the trilayer sample is still around T = 1 K due to the relatively small magnetic exchange gap. [Several studies suggested the inhomogeneity of magnetic dopants could effectively smear out the exchange gap (54, 55).] Nevertheless, the use of the trilayer configuration by structural engineering unveils an effective way to a more robust QAHE state and potentially can lead to higher QAHE onset temperature.

In addition, it is also noted that while a more robust remnant quantized state in the modulation-doped six-QL MTI films was achieved, the flat zero-Hall plateaus have narrowed down to 0.04 T, as shown in Fig. 4D. As discussed above, the exchange gap by modulation doping effectively modulates the relative strength between Δm (exchange gap size) and m0 (hybridization gap size) especially during the magnetization reversal process, where the filling factor C is related to parameters as (23)

\[ C = \begin{cases} \frac{\Delta_m}{|\Delta_m|}, & \text{for } |\Delta_m| > |m_0| \\ 0, & \text{for } |\Delta_m| < |m_0| \end{cases} \]

The size of the zero-Hall plateaus is a key factor to successfully observing the chiral Majorana edge mode. By using different doping profile, we are able to control the width effectively. Consequently, this doping strategy can serve as an additional degree of freedom to control the chiral Majorana edge mode based on the MTI/superconductor heterostructure (27).

**CONCLUSION**

In conclusion, we have studied the temperature limiting factors of QAHE states by quantitatively investigating the quantum transport behavior across a range of thickness and doping of MTI as well as their magneto-optic response. We showed that in the hybridized 2D limit (six QLS), the QAHE onset temperature is mostly limited by the formation of superparamagnetic states. By introducing proper modulation doping of Cr element, we successfully enlarged the exchange gap and stabilize the ferromagnetism near zero field. However, the QAHE gap size is still limited to around 1 K. We believe that the small gap size problem cannot be fully resolved by increasing the doping level alone. Because, first, magnetic doping is capped by solubility limit; second, the associated decrease in SOC when Cr substitutes the heavier element (e.g., Bi or Sb) in MTI will restrict the upper bound of magnetic doping. Beyond the doping level limit, the statistical random fluctuation of magnetic dopant is also inevitable in molecular beam epitaxy (MBE) as well as any other low-temperature epitaxy technique. This effect greatly limits the global nontrivial exchange gap size, which explains the experimental discrepancy between the local 30-meV mass gap measured by scanning tunneling microscopy (56, 57) and a much smaller average activation gap size measured using transport method. On the other hand, for thicker MTI samples (8 and 10 QLS) with more robust ferromagnetism, the QAHE state is affected by the bulk effect and the activation gap size is vulnerable to the vicinity of the Fermi level to the bulk valence band. As a result, the onset temperature of QAHE does not exhibit clear advantage over the 2D MTI samples.

On the basis of the results presented in this work, to further increase the QAHE gap as well as its onset temperature, here, we propose two approaches. Internally, we could dope extra element that fulfills any of the following criterion: (i) element that introduces large SOC, (ii) element that promotes long-range coupling of the existing magnetic dopants, and (iii) elements that can suppress the side
valence band near the Dirac point. Externally, we could interface highly ordered FM or anti-FM insulator to counter the inhomogeneity issue and raise MTI’s ordering temperature through effective exchange interactions (58–60). In addition, searching for new materials with intrinsic ferromagnetism and topological surface state has also shown promising progress especially in the MnBi₂Te₄ system (61).

Beyond the QAHE observed in current material system, we demonstrated the importance of structural engineering to functionalize QAH states, especially for the study of the chiral Majorana edge mode and axion insulators (62, 63). By optimizing the thickness and doping profile of MTI, we could control the width of the zero-Hall plateaus that adds an additional dimension to manipulate the chiral Majorana edge states, which can be potentially used in fault-tolerant topological quantum computations.

**MATERIALS AND METHODS**

**Material growth**

MTI thin films were grown by the MBE method in an ultrahigh vacuum Perkin-Elmer system. Semi-insulating GaAs(111)B substrate was first cleaned ultrasonically by acetone and deionized water. After being loaded into the vacuum chamber, the substrate was first heated up to 580°C under Se-rich atmosphere to remove the native oxide. Then, the substrate was cooled down to growth temperature. During the growth, the substrate was maintained at 200°C growth temperature. The epitaxial growth was monitored by reflection high-energy electron diffraction (RHEED) to optimize the growth condition. After the growth, the substrate was cooled down to room temperature and 2-nm Al was evaporated from a Knudsen cell to passivate the MTI before being taken out of the ultra-high vacuum (UHV) chamber.

**Transport measurement**

Magneto-transport at 0.02 K < T < 0.3 K was performed in He3/He4 dilution fridge. AC bias (10 nA) was applied by sourcing 1 V RMS across a 100-megohm reference resistor at 7.351-Hz measurement frequency. The drain current, longitudinal voltage \( V_{xx} \), and Hall voltage \( V_{yx} \) were measured with SR830 lock-in amplifiers.

**Optical measurement**

The magneto-optic Kerr rotation experiment was done using a fiber-optic Sagnac interferometer with \( 10^{-8} \) rad DC Kerr sensitivity. The system is integrated with a dilution fridge to allow concurrent optical and transport measurement at milli-kelvin temperature range.

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/6/25/eaaz3595/DC1

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