Lopsided optical diffraction in loop electromagnetically induced grating

Da Huo,1 Shuo Hua,1,2 Xue-Dong Tian,3,* and Yi-Mou Liu1,†

1Center of Quantum Sciences and School of Physics, Northeast Normal University, Changchun 130024, P. R. China.
2School of Physics, Beihua University, Jilin 132013, P. R. China.
3College of Physics Science and Technology, Guangxi Normal University, Guilin 541004, P. R. China.

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We propose a theoretical scheme in a cold Rubidium-87 (87Rb) atomic ensemble with a non-Hermitian optical structure, in which a lopsided optical diffraction grating can be realized just with the combination of single spatially periodic modulation and loop-phase. Parity-time (PT) symmetric and parity-time antisymmetric (APT) modulation can be switched by adjusting different relative phases of the applied beams. Both PT symmetry and APT antisymmetry in our system are robust to the amplitudes of coupling fields, which allows optical response to be modulated precisely without symmetry breaking. Our scheme shows some nontrivial optical properties, such as lopsided diffraction, single-order diffraction, asymmetric Dammann-like diffraction, etc. Our work will benefit the development of versatile non-Hermitian/asymmetric optical devices.

I. INTRODUCTION

A system with non-Hermitian Hamiltonian being commutative with the parity-time operator (\(\{PT, H\} = 0\)), proposed by Bander et al, has a real eigenenergy spectrum and some novel properties under certain conditions [1]. Due to the similarity between Schrodinger’s equation and the optical Helmholtz equation, the optical system with out-of-phase spatial modulation is a good platform to simulate the system with PT symmetry, which is named the non-Hermitian optical system [2]. Corresponding to the potential condition \(V(x) = V^*(-x)\) satisfied in the PT-symmetry system, the non-Hermitian optical system needs to satisfy complex refractive index condition \(n(x) = n^*(-x)\) [3–6]. Similarly, optical PT-antisymmetry is realized in optical media with instead condition \(n(x) = -n^*(-x)\) [7–11]. In recent years, non-Hermitian optical structures based on discrete systems such as optical waveguide [12, 13], hybrid optical micro-cavity [14, 15], electrical circuit resonators [16] as well as continuous optical media such as cold atomic ensemble with spatially periodic modulation [5–10], have been implemented successively both in experimental and theoretical works. Moreover, series of new applications or properties have been reported as optical Bloch oscillation [17, 18], photon or phonon laser [19–21], unidirectional/bidirectional invisibility/optical cloaks [22–28], and so on.

The spectroscopic device, such as diffraction grating, has been a significant branch of optical devices since Newton’s era. As an important tool for spectral analysis and optical imaging, it has been playing an important role in many fields such as physics, chemistry, astronomy, biology, etc., [29–32]. Electromagnetically induced grating (EIG) [33–35] based on electromagnetically induced transparency (EIT) [36], makes it possible to tune the diffraction patterns dynamically. Hybrid modulation EIG schemes with traditional amplitude/phase modulations and untraditional ones (nonlinear or nonlocal modulation) have improved diffraction efficiency while greatly expanding the versatility of the grating [37–42]. In recent years, combined with the non-Hermitian optical modulation, many schemes of one-/two-dimensional asymmetric optical diffraction gratings have been proposed serially [43–50], and even similar applications of asymmetric scattering have appeared in the field of acoustics [51].

However, due to rigid realization conditions of PT/APT symmetry, there is still a great hindrance to the realization of precise and flexible dynamic operation, especially for some special optical diffractions. In most previous schemes, dual spatial periodic modulation (via amplitude, detuning of coupling field or atomic density) has been used to achieve two goals, including the realization of PT/APT symmetry and construction of a grating structure. It results in the lack of accurate modulation capabilities with the protection of PT/APT symmetry. Therefore, a method for the preparation of non-Hermitian EIG with simple structure (easy to analyze), dynamic control ability, and protection of optical non-Hermitian symmetry is extremely necessary and desired.

In this paper, we propose a theoretical scheme with a four-level loop-\(\overline{N}\) configuration in a cold 87Rb atomic ensemble, in which an EIG with PT-symmetric/-antisymmetric structure can be realized via a single spatially periodic detuning modulation. PT-symmetric and PT-antisymmetric structures can be easily switched by adjusting different relative phases of the applied beams. In addition, the non-Hermitian optical symmetry in our system is robust to variation of coupling amplitude, giving our scheme the ability of dynamic modulation with symmetry protection. Considering the contribution of higher-order scattering, which violates Friedel’s law [52–54], we also discuss the realization condition of asymmetric diffraction. The perfect lopsided diffraction attained here is explained as the cooperative result of higher-order scattering and spatial Kramers-Kronig relation [55–58]. Moreover, we can also achieve single-order and Dammann-like asymmetric diffraction with our scheme.

This work is organized through the following Sec. II, where we describe the background model, and Sec. III, where we discuss the robustness of optical non-Hermitian symmetry, far-field Fraunhofer lopsided diffraction arising from the combination of higher-order scattering, and spatial Kramers-Kronig relation also with discussions on special asymmetric diffrac-
tions based on our scheme. We summarize, at last, our conclusions in Sec. IV.

II. MODEL AND EQUATIONS

We start by considering an ensemble of cold $^{87}$Rb atoms driven into a four-level loop-$N$ configuration, by four coherent fields with frequencies (amplitudes) $\omega_p (E_p)$, $\omega_c (E_c)$, $\omega_d (E_d)$ and $\omega_m (E_m)$ as shown in Fig. 1(a). The weak probe field $\omega_p$ interacts with transition $|g\rangle \leftrightarrow |e\rangle$, while the strong control fields $\omega_c$ and $\omega_d$ act upon transitions $|m\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |b\rangle$, respectively. The two ground states $|g\rangle$ and $|m\rangle$ are interacted by an equivalent microwave field with $\omega_m$. The half Rabi frequencies are defined as $\Omega_p = E_p \cdot \Phi_{ge}/2h$, $\Omega_c = E_c \cdot \Phi_{me}/2h$, and $\Omega_d = E_d \cdot \Phi_{mb}/2h$ with $\Phi_{ij}$ being transition frequencies and $\Phi_{ge}$ being dipole moments. In the rotating-wave and electric-dipole approximations, the Hamiltonian for our loop-$N$-type cold atoms can be written down as $H = H_a + Y_{af}$ containing an unperturbed atomic Hamiltonian $H_a$ and an atom-field interaction Hamiltonian $Y_{af}$:

$$
H_a = -\hbar \sum_j^{N} [\delta_p \sigma_{ee}^j + (\delta_p - \Delta_c) \sigma_{em}^j + (\delta_p - \Delta_e + \delta_d) \sigma_{eb}^j],
$$

$$
Y_{af} = -\hbar \sum_j^{N} [\Omega_p \sigma_{eg}^j + \Omega_c \sigma_{em}^j + \Omega_d \sigma_{bm}^j + \Omega_m \sigma_{mg}^j + H.c.],
$$

(1)

with $\sigma_{\mu\nu}$ being the projection ($\mu = \nu$) or transition ($\mu \neq \nu$) operators, $\delta_p = \omega_p - \omega_{eg}$, $\Delta_c = \omega_c - \omega_{em}$, $\delta_d = \omega_d - \omega_{mb}$, $\Delta_e = \omega_e - \omega_{me}$ being the probe (coupling) detunings. Considering the initial phases $\phi_p$, $\phi_c$ and $\phi_m$ of the probe, coupling and microwave fields, we rewrite half Rabi frequencies as $\Omega_p = \tilde{\Omega}_p e^{i\phi_p}$, $\Omega_c = \tilde{\Omega}_c e^{i\phi_c}$ and $\Omega_m = \tilde{\Omega}_m e^{i\phi_m}$, where $\tilde{\Omega}_p$, $\tilde{\Omega}_c$ and $\tilde{\Omega}_m$ are chosen to be real, and obtain the density matrix equations (DMEs):

$$
\partial_t \rho_{gg} = [\gamma_{mc} + i\Delta_c] \rho_{gg} + i\Omega_p \rho_{mg} - e^{-i\phi_p} \rho_{gm},
$$

$$
\partial_t \rho_{ee} = -[\gamma_{em} + i\Delta_e] \rho_{ee} + i\Omega_p \rho_{ge} + e^{i\phi_p} \rho_{eg},
$$

$$
\partial_t \rho_{bb} = -[\gamma_{mb} + i\Delta_d] \rho_{bb} + i\Omega_p \rho_{mb} - e^{-i\phi_p} \rho_{bm},
$$

$$
\partial_t \rho_{gm} = [\gamma_{mg} + i(\delta_p - \Delta_c)] \rho_{gm} + i\Omega_p \rho_{em} - i\Omega_p \rho_{em} + i\Omega_d \rho_{gb} + i\tilde{\Omega}_m e^{i\phi_p} \rho_{gm} - i\tilde{\Omega}_m e^{i\phi_p} \rho_{gm},
$$

$$
\partial_t \rho_{ge} = [\gamma_{ge} + i\delta_p] \rho_{ge} + i\Omega_c \rho_{gz} - i\tilde{\Omega}_m e^{i\phi_p} \rho_{me} + i\tilde{\Omega}_p \rho_{gg} - e^{i\phi_p} \rho_{eg},
$$

$$
\partial_t \rho_{gb} = [\gamma_{gb} + i(\delta_p - \Delta_e + \delta_d)] \rho_{gb} + i\Omega_d \rho_{gm} - i\tilde{\Omega}_m e^{i\phi_p} \rho_{mb} + i\Omega_p \rho_{eb},
$$

$$
\partial_t \rho_{me} = [\gamma_{me} + i\Delta_e] \rho_{me} + i\Omega_p \rho_{mg} - i\Omega_d \rho_{be} - i\Omega_m \rho_{ge} + i\Omega_c \rho_{zm} - \tilde{\Omega}_m (\rho_{mm} - \rho_{ee}),
$$

$$
\partial_t \rho_{mb} = [\gamma_{mb} + i\delta_d] \rho_{mb} - i\Omega_c \rho_{eb} + i\Omega_m e^{-i\phi_p} \rho_{gb} + i\Omega_d \rho_{zm} - \tilde{\Omega}_m (\rho_{mm} - \rho_{bb}),
$$

$$
\partial_t \rho_{eb} = [\gamma_{eb} + i(\delta_d - \Delta_c)] \rho_{eb} - i\Omega_c \rho_{mb} - i\Omega_p \rho_{gb} + i\tilde{\Omega}_d \rho_{em}.
$$

(2)

with $\Phi_{ij} = \Phi_{ji}$. Here $\Phi = \phi_m + \phi_c - \phi_p$ is the relative loop-phase of the three fields.

Setting the time derivatives as zero and utilizing the perturbation method (for $\varepsilon \Omega_p$), the steady-state solution can be attained by solving Eq. (2) analytically:

$$
\rho_{ge} = \rho_{eg}^{(0)} + \varepsilon \rho_{eg}^{(1)} + \varepsilon^2 \rho_{eg}^{(2)} + \ldots + \varepsilon^n \rho_{eg}^{(n)} + \ldots,
$$

$$
\rho_{eg}^{(0)} = \frac{\Omega_m \Omega_c}{R_{eg} + \Omega_p \Omega_c} \left[ \rho_{mm}^{(0)} - \rho_{eg}^{(0)} \right],
$$

$$
\rho_{eg}^{(1)} = \frac{e^{-i\phi_p} \Omega_p}{R_{eg} + \Omega_p \Omega_c} \left[ \rho_{mm}^{(0)} - i\rho_{eg}^{(0)} \right],
$$

with $R_{\mu\nu} = \gamma_{\mu\nu} + i(\omega_{\mu} - \omega_{\nu})$ being the effective coherent relaxation rate between state $|\mu\rangle$ and state $|\nu\rangle$ ($\{\mu, \nu\} = \{g, e, m, b\}$). With condition $\Omega_p \ll \Omega_{m,d}$ being satisfied, it is reasonable to take the approximation $\rho_{eg}^{(0)} \approx \rho_{eg}^{(0)} + \varepsilon \rho_{eg}^{(1)} \Omega_p e^{(0)}$, where we consider perturbation $\Omega_p \to \Omega_p e^{(0)}$ and just keep to the first-order.

The linear probe susceptibility satisfies $\chi_p(\omega) = \frac{N_{eg}^2}{2\omega_{eg}^2} e^{-i\phi_p} e^{i\phi_c} \mathbf{g}(\omega)/\Omega_p \Re \chi_p$ and $\Im \chi_p$ are used to represent the real and imaginary parts of probe susceptibility, describing the dispersion and absorption/gain ($\Im \chi_p > 0 / \Im \chi_p < 0$) properties, respectively.

Aiming to implement a nontrivial EIG with asymmetric diffraction patterns, $\Delta_c$ needs to be periodically modulated as:

$$
\Delta_c(x) = \Delta_{c0} \cdot \sin \left[ \frac{\pi \lambda_p (x - x_0)}{\alpha} \right].
$$

(4)

For a thickness of thickness $L$ along the $z$ direction and modulated in the $x$ direction, the transmission function of a probe beam takes the form:

$$
T_L(x) = T_a(x) \cdot T_p(x),
$$

(5)

where $T_a(x) = e^{-k_p \Im \chi_p(x)L} \L e$.$(T_p(x) = e^{ik_p \Re \chi_p(x)L})$ denotes the amplitude (phase) component with $k_p = 2\pi/\lambda_p$ being the probe wave vector and $\lambda_p$ being the probe wavelength. The Fourier transformation of $T_L(x)$ then yields the Fraunhofer or far-field intensity diffraction equation

$$
I_p(\theta_n) = \left| \sum_{p} \left[ \chi_p^0(\theta_n) \right]^2 \sin^2 (M \pi R \sin \theta_n) \right|,
$$

(6)
with

$$E_I^{(n)}(\theta_n) = \int_{-\alpha/2}^{\alpha/2} T_{\alpha}(x)e^{i2\pi xR\sin\theta_n}dx,$$

with $R = a/\lambda_p$. In addition, $\theta_n$ denotes the $n$th order diffraction angle of probe photons with respect to the $z$ direction while $M$ represents the ratio between the beam width $\omega_p$ and the grating period $a$ ($M = \omega_p/a$). The $n$th-order diffracted probe field will be found at an angle determined by $n = R\sin\theta_n \in \{\ldots, -1, 0, +1, \ldots\}$.

### III. RESULTS AND DISCUSSION

In this section, to implement analytical/numerical calculations based on the above equations, the four atomic levels in Fig. 1(a) are assumed as $|g\rangle \equiv |5S_{1/2}, F = 2\rangle$ and $|m\rangle \equiv |5S_{1/2}, F = 1\rangle$, as well as $|e\rangle \equiv |5P_{1/2}, F = 1\rangle$ and $|b\rangle \equiv |5P_{3/2}, F = 0\rangle$ for cold $^{87}$Rb atoms. Then the decay rates are $\Gamma_{eg} = \Gamma_{em} = \Gamma_{dm} \approx 3.0 \times 2\pi$ MHz, with the dephasing rates $\gamma_{eg} = \gamma_{em} \approx 3.0 \times 2\pi$ MHz, $\gamma_{md} = \gamma_{gd} = \Gamma_{dm}/2$, and $\gamma_{mg} \approx 1.0$ kHz. In addition, the atomic medium length is $L = 20.0$ $\mu$m and the density is $N = 4.0 \times 10^{10}$ $\text{cm}^{-3}$.

#### A. $PT$-symmetry/-antisymmetry based on single periodic modulation

In the beginning, we apply two control fields ($\Omega_e = \Omega_d = 5.0 \times 2\pi$ MHz) and an equivalent microwave field ($\Omega_m = 0.5 \times 2\pi$ MHz), which are all traveling waves (TW). Figure 2 shows the absorption ($\text{Im}[\chi_p(\omega)]$) and dispersion ($\text{Re}[\chi_p(\omega)]$) spectra of the system, with detunings $\Delta_\epsilon = \delta_d = 0$ and different loop phases ($\Phi = 0, \pi/2, \pi$ and $3\pi/2$). Evidently, $\text{Im}[\chi_p(\omega)]$ and $\text{Re}[\chi_p(\omega)]$ are odd (even) function of frequency $\omega$ ($\delta_p$) with loop phase $\Phi = n\pi$ as shown in Fig. 2(a) and (c). While the parity of $\text{Im}[\chi_p(\omega)]$ and $\text{Re}[\chi_p(\omega)]$ are opposite if the loop phase is chosen as $\Phi = (2n - 1)\pi/2$ in Fig. 2(b) and (d), with $n \in \mathbb{Z}$.

Figure 3 shows absorption and dispersion spectra via $x$ coordinate when we apply $\Omega_e$ as a standing wave (SW) coupling field, instead of a TW coupling field, with different loop phases $\Phi$. Here we choose the detuning of coupling field as in Eq. (4) with $\Delta_{\omega} = 4.0 \times 2\pi$ MHz. Under the two-photon resonance condition ($\delta_p = 0$), it can be attained that $\delta_p(x) = -\Delta_{\omega} \cdot \sin[\frac{\pi a(x-x_0)}{\lambda}]$. Then it is easy to get $\chi_p(\omega) = \chi_p[\delta_p(x)] \rightarrow \chi_p(x)$, indicating the parity characteristic transfer from frequency domain to spatial domain [See Appendix A]. It is obvious that $\text{Im}[\chi_p(x)]$ and $\text{Re}[\chi_p(x)]$ are odd (even) and even (odd) functions of $x$ in Fig. 3(a)-(d) with $\Phi = 0, \pi$ ($\Phi = \pi/2, 3\pi/2$), respectively, so the system satisfies optical $PT$ symmetry ($PT$-antisymmetry).

Switching between $PT$-symmetric modulation and $PT$-antisymmetric modulation by adjusting the loop-phase is convenient in our scheme [See Fig. 3, Eq. (A2), and Table. I]. With the loop phase being chosen as $\Phi = 2m\pi$ or $\Phi = (2m + 1)\pi$ ($m \in \mathbb{Z}$), the system is under $PT$-symmetric modulation. The real (imaginary) parts $\text{Re}[\chi_p(x)]$ ($\text{Im}[\chi_p(x)]$) of
the susceptibilities in these two cases with different phases just have the opposite signs. It is worth noting that except loss case (normal APT) with \( \Phi = \frac{(4m+1)\pi}{2} \), APT antisymmetric cases also include gain case (abnormal APT) with \( \Phi = \frac{4m\pi}{2} \), which is uncommon in the continuous medium schemes. The gain APT structures will provide an alternative scheme with high flexibility for some potential applications which require both considerable gain and out-of-phase modulation.

The parity of the system susceptibility under the loop-phase control makes it possible to achieve both optical \( \mathcal{PT} \)-symmetry and \( \mathcal{PT} \)-antisymmetry based on single spatial periodic modulation solely, which is quite different from the previous schemes [5–10, 43–50]. Especially for \( \mathcal{PT} \)-symmetry in these continuous periodic systems, the dual spatial periodic modulation, generally provided by multiple SW fields or atomic lattice systems (spatial density modulation), is a necessary condition. Notably, single spatial periodic modulation dramatically reduces the system’s complexity and makes it possible to analytically analyze the relationship between non-Hermitian optical symmetry (\( \mathcal{PT} \)-symmetry/-antisymmetry) and asymmetric diffraction in a system with continuously varying complex susceptibility.

Robustness of non-Hermitian optical symmetry is necessary to be considered for a system that requires precise control, as the symmetry needs to be satisfied strictly to implement some special optical functions (such as perfect lopsided diffraction) [See Sec. III B]. In previous works, the non-Hermitian symmetries are very fragile to even a slight perturbation of parameters such as coupling amplitudes. Here we consider the robustness of our system with varying coupling intensity \( \Omega_n \) in Fig. 3. It is obvious that the system has a large dynamic tuning range here (\( \Omega_n \in [2.2, 10.0] \times 2\pi \text{ MHz} \)), with the protection of non-Hermitian optical symmetry. Similarly, non-Hermitian symmetry robustness to other varying coupling amplitudes (\( \Omega_d \) and \( \Omega_m \)) are shown in Fig. 4. The above conclusion can also be analytically supported by Eq. (A1). Compared to the optical depth (OD) modulation (via initial preparation of atomic density \( N \) and medium length \( L \)) in the previous work [44], the amplitude modulation in our scheme is more feasible with a large dynamic modulation range under non-Hermitian optical symmetry protecting, meaning the possibility to achieve some special optical functions.

## B. Lopsided, Single-order, and Dammann-like asymmetric Diffractions

After constructing the tunable optical \( \mathcal{PT}/\mathcal{APT} \) symmetry of the system with the loop phase, we try to analyze the diffraction characteristics of our system in this subsection. The optical diffraction characteristics of this non-Hermitian EIG can be studied by injecting a probe field along the \( z \)-axis which is perpendicular to the direction of spatial periodic modulation (\( x \)-axis). Figs. 5(a1–d1) give dispersion and absorption (or gain) spectra under the out-of-phase (\( \mathcal{PT} \)-symmetric/-antisymmetric) modulations for different loop-phases, with same parameters as in Fig. 2 except \( \Delta_{\omega}/2\pi = 4.0 \text{ MHz} \). Figs. 5(a2–d2) show that the probe beam is only diffracted into the negative angles in the range \( \theta \in (\pi/6, 0) \) with loop phase \( \Phi = 0, \pi/2, \pi, 3\pi/2 \), respectively. Fig. 5(b2) displays normal APT case, that is, the system is pure dissipative. Instead, Fig. 5(d2) exhibits an unconventional optical \( \mathcal{PT} \)-antisymmetric case with optical gain, which is also quite different from the \( \mathcal{PT} \) symmetric case [See Fig. 5(a2) and Fig. 5(c2)] with the balance of gain and loss.

It is known that diffraction peaks occur at discrete angles \( \theta_n \) determined by \( k_n = k_p \sin \theta_n = 2n\pi/a \) or \( n = R \sin \theta_n \), where we choose \( R = a/\lambda, M = \omega_B/a = 10 \). Combined with Eq. (5), we could focus on the \( n \)-th order diffraction by examining \( E_n = E_p(\theta_n) \) for \( n \neq 0 \). For sim-
TABLE I. non-Hermitian Spatial Modulation

| Symmetry Type | Loop-phase \{\Phi\} | Asymmetric Diffraction spectra | Gain/Loss |
|---------------|--------------------|-------------------------------|-----------|
| \( P^T \)    | \( \{m \pi\}, m \in \mathbb{Z} \) | Near Lopsided (\( \Omega_c > 2 \gamma_0 \)) | Both (balance) |
| normal \( APT \) | \( \{4m + 1\pi/2\} \) | Perfect Lopsided (\( \Omega_c > 2 \gamma_0 \)) | Loss (pure) |
| abnormal \( APT \) | \( \{4m - 1\pi/2\} \) | Irregular (\( \Omega_c > 2 \gamma_0 \)) | Gain (pure) |

FIG. 5. Dispersion \( \text{Re}[\chi_p(x)] \) (blue-solid curves) and absorption \( \text{Im}[\chi_p(x)] \) (orange-dotted curves) spectra of gratings with lopsided diffraction patterns versus \( x \) with \( \Delta \chi = \Delta_{\text{p}} \sin[\pi \lambda_p(x - x_0)/a] \) and \( \delta_p = 0 \) for \( (a_1) \Phi = 0 \), \( (b_1) \Phi = \pi \), \( (c_1) \Phi = \pi/2 \), \( (d_1) \Phi = 3\pi/2 \), respectively. Moreover, \( (a_2)-(d_2) \) are the corresponding gratings of our system for \( \gamma_p(x) = 2n x \), we can make a power series expansion of Eq. (7) and obtain

\[
\mathcal{E}_n = \int_{-\alpha/2}^{+\alpha/2} dx \cdot e^{-\gamma_n(x)} [1 + \alpha(x) + \frac{\alpha(x)^2}{\alpha} + \frac{\alpha(x)^3}{6} \ldots + \frac{i \beta(x)^m}{m!} \ldots],
\]

where it is easy to find \( |\alpha(x)| \sim |\beta(x)| < 1 \) (\( N = 4.0 \times 10^{10}\) cm\(^{-1}\) and \( L = 20 \mu m \)).

FIG. 6. Diffraction intensity \( I_p(\theta) \) versus the diffraction angle and the amplitude of coupling field \( \Omega_c \). (a), negative order diffraction intensity \( I_p(-\theta_n) \) \( (b_1) \), positive order diffraction intensity \( I_p(\theta_n) \) \( (b_2) \) and asymmetric diffraction coefficient \( \eta_n \) versus \( \Omega_c \). (c), with the same parameters as in Fig. 5 except \( \Phi = 0 \). The position of the red line corresponds to \( \Omega_c/2\pi = 3.5 \) MHz.

where the scattering factor \( \varepsilon_n \) is small enough to keep only the first- and second-order scattering terms [59]. It is easy to find that \( f_n = f_n'' \), \( f_n'' = f_n'' \), \( g_n = g_n'' \), and \( g_n'' = -g_n'' \), and we can write down the intensities \( I_{\pm n} \approx |f_n \varepsilon_n + g_n \eta_n|/|f_n'' \varepsilon_n + g_n'' \eta_n|/|2|^2 + |f_n'' \varepsilon_n + g_n'' \eta_n|/|2|^2 \) for the \( \pm n \)th diffraction orders. Accordingly, the intensity contrast ratio can be introduced as

\[
\eta_n = \frac{I_n - I_{-n}}{I_n + I_{-n}} \approx \frac{f_n' \cdot g_n'' - f_n'' \cdot g_n'}{I_n'' + I_{-n}''} \cdot \varepsilon_n, \quad (11)
\]
to evaluate the degree of asymmetric diffraction.

Considering \( \gamma_n(-x) = -\gamma_n(x) \), it is not difficult to find \( \eta_0 = 0 \) \( (I_n = I_{-n}) \) with \( f_n' \cdot g_n'' = f_n'' \cdot g_n' \), indicating the symmetric diffraction occurs in the system, in the case of spatial even symmetry with \( \text{Im}[\chi_p(x)] = \text{Im}[\chi_p(-x)] \) and \( \text{Re}[\chi_p(x)] = \text{Re}[\chi_p(-x)] \) (in-phase modulation). On the contrary, if the system is deviated from in-phase modulation, with \( f_n' \cdot g_n'' \neq f_n'' \cdot g_n' \), we can obtain the asymmetric diffraction angle spectra. Moreover, if optical systems satisfy \( \mathcal{PT}-\text{symmetry} \) or \( \mathcal{PT}-\text{antisymmetry} \) (out-of-phase) there will be a large possibility to achieve \( \eta_n \neq 1 \), named the perfect asymmetric diffraction or lopsided diffraction [See Appendix B].

FIG. 6(a) shows the asymmetric diffraction angular spectra \( I_p(\theta) \) with varying coupling amplitude \( \Omega_c \) of our system for the \( \mathcal{PT} \)-symmetric case (\( \Phi = 0 \)). Panels (b1) and (b2) show the \( \pm n \)th order diffraction intensity \( I_p(\pm \theta_n) \) \( (n = 1, 2, 3) \) varying with the coupling amplitude (\( \Omega_c/2\pi \in (0, 10) \) MHz). Moreover, we show the intensity contrast ratio (asymmetry diffraction coefficient) \( \eta_n \) versus \( \Omega_c \) in Fig. 6(c). Comparing the three panels of Fig. 6, it is easy to find that, in the range

\[
\mathcal{E}_n \approx [f_n' \varepsilon_n - g_n'' \varepsilon_n^2/2] + i[f_n'' \varepsilon_n + g_n'' \varepsilon_n^2/2], \quad (10)
\]
of $\Omega_c/2\pi \in (2.0, 5.0)$ MHz, a high degree of asymmetric diffraction is obtained ($\eta_n > 0.95$ and $\rightarrow 1$), with one-sided non-zero diffraction intensity ($I_y(\theta_n) > 0.2$). Particularly, we can get $\eta_n > 0.97$ and $I_y(\theta_n) > 0.6$, which is extremely close to the optimal situation for $\mathcal{PT}$-symmetric case, at the red line position ($\Omega_c/2\pi = 3.5$ MHz).

Next, we further discuss asymmetric diffraction of the system with different loop phases. The curves of asymmetry diffraction coefficients versus $\Omega_c$ for different diffraction orders $\eta_n$ ($n = \{1, 2, 3\}$) are shown in Fig. 7 with different loop phases $\Phi$. Comparing the $\mathcal{PT}$ symmetric modulation cases in Fig. 7(a) and Fig. 7(c), we can obtain the asymmetry diffraction coefficients $\eta_{1,2} \rightarrow 1$ ($0.85 < \eta_3 < 0.95$) in the coupling amplitude range $\Omega_c/2\pi > 2.0$ MHz, and values of $\eta_{1,2}$ reach the maximum at $\Omega_c/2\pi = 3.5$ MHz; so these cases can be named as \textit{near lopsided} diffractions in Table 1.

Figure 7(b) shows asymmetry diffraction coefficients $\eta_n$ in \textit{normal} $\mathcal{PT}$-antisymmetric case with $\Phi = \pi/2$. Obviously, the \textit{perfect lopsided} diffraction effect has been achieved, that is, $\eta_n \equiv 1$ ($n = \{1, 2, 3\}$) if the coupling amplitude is chosen as $\Omega_c/2\pi > 2.0$ MHz. Employing the same parameters as in Fig. 5 (b), for the pure-loss optical system, we can easily obtain

$$\eta_n^{\mathcal{APT}} = \left[ \frac{f_n'g_n'}{f_ng_n'} \right] \cdot \varepsilon_n = \left[ \frac{g_n'}{f_n'} \right] \cdot \varepsilon_n \quad (12a)$$

$$\approx \left[ \frac{\int_0^{a/2} dx \beta(x) \sin[2n\pi x]}{\int_0^{a/2} dx [1 + \alpha(x) \cos[2n\pi x]]} \right] \cdot \varepsilon_n \quad (12b)$$

$$= \left[ \frac{\int_0^{a/2} dx \alpha(x) \cos[2n\pi x]}{\int_0^{a/2} dx \alpha(x) \cos[2n\pi x]} \right] = 1, \quad (12c)$$

with $\varepsilon_n = 1$ from Eq. (B3). Here we also use $\beta(x) \propto \int \alpha(x) dx$ ($\alpha(x) \propto \int \beta(x) dx$) from the spatial Kramers-Kronig relations of susceptibility [55–58]

$$\text{Re}[\chi_p(x)] = \frac{1}{\pi} \mathbf{P} \int \text{Im}[\chi_p(x')] dx' \quad (13),$$

$$\text{Im}[\chi_p(x)] = \frac{1}{\pi} \mathbf{P} \int \text{Re}[\chi_p(x')] dx' \quad (13),$$

where $\mathbf{P}$ indicates the principal value of the integral. \textit{Perfect lopsided} diffraction is the cooperative result of multiple higher-order scattering, which can be explained by Eq. (10) and Eq. (11), and the spatial Kramers-Kronig relations. For the gain $\mathcal{APT}$ case, \textit{irregular} asymmetric diffraction is shown in Fig. 7(d). The perfect lopsided diffraction condition will be broken if the condition of series expansion is not satisfied any longer, owing to the large optical gain ($\alpha(x) \sim \beta(x) \gg 1$).

With the tunable ability of lopsided diffraction in our scheme, we could discuss some special diffractions further. \textit{Single-order diffraction} is displayed in Fig. 8 (a1) and (b1) for the optical $\mathcal{APT}$ case ($\Phi = \pi/2$), with showing the susceptibility and diffraction angle spectrum of the one-dimensional case (1D), respectively. Here the parameters are chosen as $\Omega_c = 1.05\gamma_\Omega$, $\Omega_c = 3.0\gamma_\Omega$, and $\Omega_c = 0.7\gamma_\Omega$ ($\gamma_\Omega = 1.0 \times 2\pi$ MHz). It is obvious that the probe beam is only diffracted into the $-1$st order with a considerable diffraction efficiency (intensity) $I_{-1} \approx 0.597$. In addition, the above conclusion can be extended to the two-dimensional case (2D). We plot the single-order asymmetric diffraction for 2D case in Fig. 8(b1f) with spatial periodic modulation in both directions ($\Delta_x = \Delta_{xx} \sin[\pi \lambda_p(x - x_0)]$ and $\Delta_{yy} \sin[\pi \gamma_\Omega(y - y_0)/a]$). Here, only a single case ($\Delta_{xx}/2\pi = 4.0$ MHz and $\Delta_{yy}/2\pi = 0.1$ MHz) is given to illustrate the functionality of our scheme. In fact, in combination with the two-dimensional Hermitian/non-Hermitian hybrid modulation [49], there will be other diffraction modes of single-order asymmetric diffraction.

Identically, through continuous parameter modulation for several coupling fields ($\Omega_{c,d,m}$, $\Delta_{x,y,d}$), the special asymmetric diffraction with multiple equal intensity diffusion orders can be easily accomplished. As the simplest example, the case with two equal-intensity diffraction orders is shown in Fig. 8 (a2) and (b2). Here we first check the susceptibilities and diffraction angle spectra in the 1D case to ensure the satisfaction of non-Hermitian optical symmetry and lopsided diffraction conditions. In the same way, following the 2D non-Hermitian hybrid modulation methods [49], an array of diffraction beams with equal intensity will be implemented, named asymmetric Dammann-like diffraction grating, the analog of the Dammann gratings [60–64] in the asymmetric case. These special asymmetric diffractions and the implementation methods will greatly facilitate the development and application of scattering-type asymmetric optical devices.

**IV. CONCLUSIONS**

In summary, the ensemble of cold $^{87}$Rb atoms driven into a four-level loop-$\mathcal{N}$ configuration can provide an interesting venue to realize non-Hermitian EIG with various symmetry features. Firstly, the scheme is proposed to prepare
Moreover, based on Eq. (3) we can easily attain the relationship between real (imaginary) part $\text{Re}[\chi_p]$ ($\text{Im}[\chi_p]$) of susceptibility and space coordinates $x$: 

$$A_0[\delta_p(x)^{nA0}] \simeq \gamma(\Omega_m \Omega_e - \Omega_q)^2 \Omega_m \Omega_e, \quad n_{A0} \to 2,$$

$$B_0[\delta_p(x)^{nB0}] \simeq \gamma(\Omega_m \Omega_e - \Omega_q)^2 \Omega_m \Omega_e, \quad n_{B0} \to 1,$$

$$A_1[\delta_p(x)^{nA1}] \simeq \Omega_m \Omega_e \gamma(\gamma + \gamma_m) \Omega_m \Omega_e \gamma(\Omega_m \gamma^2 + \delta_p^2)^2 \gamma_m(\gamma^2 + \delta_p^2), \quad n_{A1} \to 3,$$

$$B_1[\delta_p(x)^{nB1}] \simeq \Omega_m \Omega_e \gamma(\gamma + \gamma_m) \Omega_m \Omega_e \gamma(\Omega_m \gamma^2 + \delta_p^2)^2 \gamma_m(\gamma^2 + \delta_p^2), \quad n_{B1} \to 2.$$  

(A1)
The real and imaginary parts of \( g_{eg}^{(1)} \) introduced by the loop structure finally determine the parity of susceptibility of the system. Hence the non-Hermitian optical (\( PT^- \) or \( APT^- \)) symmetry can be easily modulated by loop-phase \( \Phi \). Significantly, from Eq. (A1), we can find the varying coupling amplitudes \( \Omega_\mu (\mu = \{c, d, m\}) \) have no impact on the parity of susceptibility of our system under the reasonable condition.

\[
\chi_p(x) \propto A_0[g_{eg}^{(0)}(x)] - iB_0[g_{eg}^{(0)}(x)] + e^{-i\Phi}A_1[g_{eg}^{(1)}(x)] - ie^{-i\Phi}B_1[g_{eg}^{(1)}(x)];
\]

\[
\text{Re}[\chi_p(x)] \propto \begin{cases} 
A_0[\delta_p(x)^2] + A_1[\delta_p(x)^3], & \Phi = n\pi, \text{ Odd} \\
A_0[\delta_p(x)^2] + B_1[\delta_p(x)^2], & \Phi = \frac{(2n-1)\pi}{2}, \text{ Even} \\
B_0[\delta_p(x)^1] + B_1[\delta_p(x)^2], & \Phi = n\pi, \text{ Even} \\
B_0[\delta_p(x)^1] + A_1[\delta_p(x)^3], & \Phi = \frac{(2n-1)\pi}{2}, \text{ Odd}. 
\end{cases}
\]

\[
\text{Im}[\chi_p(x)] \propto \begin{cases} 
B_0[\delta_p(x)^1] - B_1[\delta_p(x)^2], & \Phi = n\pi, \text{ Even} \\
B_0[\delta_p(x)^1] - A_1[\delta_p(x)^3], & \Phi = \frac{(2n-1)\pi}{2}, \text{ Odd}. 
\end{cases}
\]

Appendix B: Asymmetric Diffraction

In this part, we will try to calculate \( \eta_n \) according to the spatial symmetry of the system. Firstly, for general Hermitian optical media, the real and imaginary parts of susceptibility are even functions of \( x \), \( \text{Im}[\chi_p(x)] = \text{Im}[\chi_p(-x)] \) and \( \text{Re}[\chi_p(x)] = \text{Re}[\chi_p(-x)] \). We can obtain \( \alpha(x) = \alpha(-x) \) and \( \beta(x) = \beta(-x), \) where \( \alpha(x) = k_p \text{Im}[\chi_p(x)] L \) and \( \beta(x) = k_p \text{Re}[\chi_p(x)] L \). Combined with the above relationship and Eq. (9), the scattering coefficients read:

\[
f'_n = \int_0^{\alpha/2} dx \cdot [1 + \alpha(x) + \frac{\alpha(x)^2}{2}] \beta(x) \sin[\gamma_n(x)] - \int_0^{\alpha/2} dx \cdot [1 + \alpha(x) + \frac{\alpha(x)^2}{2}] \beta(x) \sin[\gamma_n(x)] = 0,
\]

\[
g''_n = \int_0^{\alpha/2} dx \cdot [1 + \alpha(x) + \frac{\alpha(x)^2}{2}] \beta(x)^2 - \frac{2}{2} \sin[\gamma_n(x)] - \int_0^{\alpha/2} dx \cdot [1 + \alpha(x) + \frac{\alpha(x)^2}{2}] \beta(x)^2 - \frac{2}{2} \sin[\gamma_n(x)] = 0.
\]

 Apparently, we can get the result \( \eta_n = 0 \) for each nth diffraction order, meaning a symmetric diffraction behavior of the Hermitian grating.

In a similar fashion, with non-Hermitian modulations, we can get

\[
[f'_n \cdot g'_n]_{PT} = \int_0^{\alpha/2} dx \cdot 2\alpha(x)\beta(x) \sin[\gamma_n(x)] \times \int_0^{\alpha/2} dx \cdot [1 + \alpha(x)^2/2] [\beta(x)^2 - 2] \cos[\gamma_n(x)],
\]

\[
[f''_n \cdot g''_n]_{PT} = \int_0^{\alpha/2} dx \cdot [1 + \alpha(x)^2/2] \beta(x) \sin[\gamma_n(x)] \times \int_0^{\alpha/2} dx \cdot \alpha(x) [\beta(x)^2 - 2] \cos[\gamma_n(x)],
\]

\[
[f'_n \cdot g'_n]_{APT} = \int_0^{\alpha/2} dx \cdot [2 + 2\alpha(x) + \alpha(x)^2/2] \beta(x) \sin[\gamma_n(x)] \times \int_0^{\alpha/2} dx \cdot [1 + \alpha(x) + \alpha(x)^2/2] [\beta(x)^2 - 2] \cos[\gamma_n(x)],
\]

\[
[f''_n \cdot g''_n]_{APT} = 0,
\]

with \( \alpha(x) \rightarrow \text{odd}, \beta(x) \rightarrow \text{even} (PT^-\text{symmetric case}) \) and \( \alpha(x) \rightarrow \text{even}, \beta(x) \rightarrow \text{odd} (PT^-\text{antisymmetric case}), \) respectively. After simplification, we can attain the relationship between the asymmetry coefficients and the scattering coeffi-
Thus, we know that when the system satisfies non-Hermitian symmetry, $\eta_n^{PT}$ ($\eta_n^{APT}$) is unequal to 0, which causes asymmetric diffraction behavior of the atomic grating. Moreover, the contribution of high-order scattering can be influenced by adjusting parameters such as coupling amplitudes. It gives a method to achieve adjusting the degree of asymmetry accurately.

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