Dual Lattice Blockspin Transformation and Monopole condensation in QCD $^a$

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Recent studies of confinement based on the idea of abelian monopole condensation are reviewed briefly. Emphasis is placed on the approach to get the effective monopole action using the blockspin transformation on the dual lattice. The trajectory obtained looks to be the renormalized one in $SU(2)$ QCD. A disorder parameter of confinement is constructed. Monopole condensation occurs also in $SU(3)$ QCD.

1 Introduction

It is crucial to understand the mechanism of quark confinement in order to explain hadron physics out of QCD. The 'tHooft idea of abelian projection of QCD is very attractive. The abelian projection is to fix the gauge in such a way that the maximal torus group remains unbroken. After the abelian projection, monopoles appear as a topological quantity in the residual abelian channel. QCD is reduced to an abelian theory with electric charges and monopoles. If the monopoles make Bose condensation, charged quarks and gluons are confined due to the dual Meissner effect.

Based on this standpoint, we have studied quark confinement mechanism and hadron physics performing Monte Carlo simulations of abelian projection in lattice QCD $^1$, $^2$, $^3$, $^4$, $^5$, $^6$, $^7$, $^8$, $^9$, $^{10}$, $^{11}$, $^{12}$, $^{13}$, $^{14}$, $^{15}$, $^{16}$, $^{17}$, $^{18}$, $^{19}$. The aim of the study is to ascertain correctness of the picture, that is, to check if monopole condensation really occurs in QCD. Here I review the results compactly.$^c$

2 Abelian dominance and monopole dominance

Our procedure is as follows:

1. Vacuum configurations of link variables $\{U(s, \mu)\}$ are generated with the Wilson action.

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$^c$We have also studied hadron physics based on an infrared effective Lagrangian constructed directly from QCD on the assumption of the above picture.
Fig. 1 Monopole and photon contributions to the string tension in MA gauge in $SU(2)$ QCD.

Fig. 2 Polyakov loops in MA gauge.

Fig. 3 Critical exponents in $SU(2)$. 

Critical exponents
2. We perform the abelian projection. One of interesting gauges is the maximally abelian (MA) gauge. Define a matrix in $SU(2)$ QCD

$$X(s) = \sum_{\mu} [U(s, \mu)\sigma_3 U^\dagger(s, \mu) + U^\dagger(s - \hat{\mu}, \mu)\sigma_3 U(s - \hat{\mu}, \mu)]$$  \hspace{1cm} (1)

$$= X_1(s)\sigma_1 + X_2(s)\sigma_2 + X_3(s)\sigma_3.$$  \hspace{1cm} (2)

Then a gauge satisfying $X_1(s) = X_2(s) = 0$ is the MA gauge.

3. Separate abelian link variables $u(s, \mu)$ as $V(s)U(s, \mu)V^\dagger(s+\hat{\mu}) = C(s, \mu)u(s, \mu)$ to obtain an ensemble of $\{u(s, \mu)\}$.

4. We construct monopole currents $k_\mu(s)$ following the DeGrand-Toussaint method.

5. We measure expectation values of $U(1) \times U(1)$ invariant operators $O(u(s, \mu))$ and operators composed of monopole currents $O(k_\mu(s))$.

We have found that important features of confinement, i.e., the string tension and the characteristic behaviors of the Polyakov loops are well reproduced in terms of the abelian operators $O(u(s, \mu))$ and also $O(k_\mu(s))$ in MA gauge. Abelian and monopole Polyakov loops are different operators, but they give almost equal critical exponents. See Figs.1~3.

3 Monopole action and condensation

The above abelian dominance suggests that a set of $U(1)^2$ invariant operators $\{O(u(s, \mu))\}$ are enough to describe confinement. Then there must exists an effective abelian action $S_{eff}(u)$ describing confinement. We tried to derive $S_{eff}(u)$ using Schwinger-Dyson equations, but failed to get it in a compact and local form. $S_{eff}(u)$ contains larger and larger loops as $\beta$.

Shiba and Suzuki tried to perform a dual transformation of $S_{eff}(u)$ in $SU(2)$ QCD and to obtain the effective $U(1)$ action in terms of monopole currents, extending the Swendsen method. To study the long range behavior is important in QCD, they have considered also extended monopoles. The extended monopole currents are defined by the number of the Dirac strings surrounding an extended cube:

$$k_\mu^{(n)}(s) = \sum_{i,j,l=0}^{n-1} k_\mu(ns + (n - 1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}),$$  \hspace{1cm} (3)
where \( k_\mu(s) \) is the ordinary monopole current. Considering extended monopoles corresponds to performing a block spin transformation on a dual lattice and so it is suitable for exploring the long range property of QCD.

How about the case of SU(3) QCD? There are two independent (three with one constraint \( \sum_i^3 k_\mu(s) = 0 \) ) currents. When considering the two independent currents, their entropies are difficult to evaluate. Hence we try to evaluate the effective monopole action, paying attention to only one monopole current. Then we can get the monopole action similarly as in SU(2).

The partition function of interacting monopole currents is expressed as

\[
Z = \left( \prod_{s,\mu} \sum_{k_\mu(s) = -\infty}^{\infty} \right) \left( \prod_s \delta_{\delta_\mu^s(s,0) \mu} \right) \exp(-S[k]). \tag{4}
\]

It is natural to assume \( S[k] = \sum_i f_i S_i[k] \). Here \( f_i \) is a coupling constant of an interaction \( S_i[k] \). For example, \( f_1 \) is the coupling of the self energy term \( \sum_{n,\mu} (k_\mu(s))^2 \), \( f_2 \) is the coupling of a nearest-neighbor interaction term \( \sum_{n,\mu} k_\mu(s) k_\mu(s + \hat{\mu}) \) and \( f_3 \) is the coupling of another nearest-neighbor term \( \sum_{n,\mu,\nu} k_\mu(s) k_\mu(s + \hat{\nu}) \).

The monopole actions are obtained locally enough for all extended monopoles considered even in the scaling region. They are lattice volume independent. The coupling constant \( f_1 \) of the self-energy term is dominant and the coupling constants decrease rapidly as the distance between the two monopole currents increases as seen in Figs. 4 and 5.

To study monopole dynamics, we have also studied the length of monopole loops. We have found that the value of the action is proportional to the length \( L \) of the loop and is well approximated by \( f_1 \times L \).

As done in compact QED, the entropy of a monopole loop can be estimated as \( \ln 7 \) per unit loop length. Since the action is approximated by the self energy part \( f_1 L \), the free energy per unit monopole loop length is approximated by \( (f_1 - \ln 7) \). If \( f_1 < \ln 7 \), the entropy dominates over the energy, which means condensation of monopoles. In Figs.6 and 7, \( f_1 \) versus \( \beta \) for various extended monopoles on 24\(^4\) lattice is shown in comparison with the entropy value \( \ln 7 \). Monopole condensation occurs both in SU(2) and SU(3). Each extended monopole has its own \( \beta \) region where the condition \( f_1 < \ln 7 \) is satisfied. When the extendedness is bigger, larger \( \beta \) is included in such a region. Larger extended monopoles are more important in determining the phase transition point.

The behaviors of \( f_i \) are different for different extended monopoles. However, if we plot them versus \( b = n \times a(\beta) \), we get a unique curve as in Fig. 8. The coupling constants seem to depend only on \( b \), not on the extendedness.
nor $\beta$. There is a critical $b_c$ corresponding to critical $\beta^n_c$, i.e., $b_c = na(\beta^n_c)$. On the other hand, the scaling is not yet good in $SU(3)$.

Fig. 4 Monopole action in $SU(2)$.

Fig. 5 Monopole action in $SU(3)$.

Fig. 6 $f_1$ versus $\beta$ in $SU(2)$.

Fig. 7 $f_1$ versus $\beta$ in $SU(3)$.
Now we can derive important conclusions at least in SU(2). Suppose the effective monopole action remains the same for any extended monopoles in the infinite volume limit. Then the finiteness of $b_c = n a (\beta_n^c)$ suggests $\beta_n^c$ becomes infinite when the extendedness $n$ goes to infinity. $SU(2)$ lattice QCD is always (for all $\beta$) in the monopole condensed and then in the quark confinement phase. This is one of what one wants to prove in the framework of lattice QCD.

Notice again that considering extended monopoles corresponds to performing a block spin transformation on the dual lattice. The above fact that the effective actions for all extended monopoles considered are the same for fixed $b$ means that the action may be the renormalized trajectory on which one can take the continuum limit. See Fig. 9. The explicit form of the obtained action will be discussed in elsewhere.

4 Disorder parameter of confinement

Since we have obtained the effective abelian U(1) action from SU(2) QCD, it is possible to define a disorder parameter of confinement following the discussions done in compact QED or in abelian Higgs model.

Following Frölich and Marchetti, we can define the correlation of two monopole operators using the differential form as

$$G_\Psi(x, y) = \frac{1}{Z} \sum_{k \in \mathbb{Z}, \delta^* k = 0} \exp\left[-(v^* k + \Psi^* - \omega, D(v^* k + \Psi^* - \omega))\right],$$
where \( Z = \sum_{k \in Z, \delta^* k = 0} \exp[-(\ast k, D^* k)] \) is the partition function of the monopole current ensemble and \( \ast \Psi = \Psi_3 = d_3 \Delta_3^{-1}(\delta_x - \delta_y) \) is the smeared string and the open current \( \ast \omega \) corresponding to the external monopole sources satisfies \( \delta^* \omega = \delta_x - \delta_y \). Then the disorder parameter can be defined as

\[
G_3 = \lim_{|x-y| \to \infty} G_\Psi(x, y)
\]

Numerically, this shows a typical behavior of the disorder parameter of confinement as shown in Fig. 10. Also one can define another but similar form of the disorder parameter following the discussions by Kennedy and King in noncompact Abelian Higgs model. For details, see the reference.

![Disorder parameter of confinement in SU(2).](image)

Fig. 10 Disorder parameter of confinement in SU(2).
5 Discussions and outlook

A few comments are in order.

1. Gauge independence should be proved if the monopole condensation is the real confinement mechanism. My opinion is the following: gauge independent results will be obtained when we discuss long-range low-energy behaviors, if we go to the scaling region, i.e., for large $\beta$ on larger lattice. Actually new interesting gauges have been found recently which show abelian and monopole dominances as in MA. See the reference 33.

2. How to test the correctness of this idea? The theory predicts the existence of an axial vector glueball-like state $C(J^P = 1^+)$ and a scalar glueball-like state $\chi(J^P = 0^+)$. The masses seem to satisfy $m_c \sim m_\chi$. The masses could not be too heavy. They have to exist under 2 GeV. To evaluate the correlation between the state and the light hadrons in MC simulations of full QCD is very important to derive the total width and the branching ratios.

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