Developing high-performance software for studying radon transport processes

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Abstract. A mathematical model of the three-dimensional problem of diffusion-advection of radon in piecewise homogeneous layered mediums with inclusions is proposed. The model takes into account anisotropy of diffusion properties of subregions of geological mediums. A combined method of solving the problem with the methods of integral transformations, integral representations, and boundary integral equations is proposed. Results of computer modeling with known models for the case of homogeneous piecewise layered mediums are presented. Results of computational experiments for studying processes of radon transport in anisotropic layered mediums with inclusions are presented. Additionally, the use of OpenMaple technology for developing high-performance software with language C# for studying radon transport processes is discussed.

1. Introduction
Transfer of matter in diffused layered mediums is a subject of studying in different fields of science: Geology, Geophysics, Medicine, and Ecology. In the field of Medicine, the most relevant is the problem of finding the correct dose of medicine to reach the required concentration of the substance in a human body in the necessary time frame. In the fields of Geology and Geophysics, actual problems relate to mass transport of radon, byproducts of its decay and measuring properties of radon fields. Solving these problems helps in searching of deposits of radioactive and fossil fuel, in geological mapping, prediction of rock bursts and tectonic hazards, ecological evaluation of areas for construction, evaluation of sanitary conditions of living and commercial spaces, etc.

Accurate modeling of radon transport processes in a geological medium is a difficult problem. Geological mediums are uneven and layered with noticeably different physical and geological properties [1].

It should be noted that two views exist about the mechanisms of radon fields formation and properties specific to radon fields which should be measured. First states that the most important information about active objects and geological structures is carried out by radon concentration in soil and/or atmospheric air. Another opposes stating that only radon flow through the original ground is able to provide reliable information about radon sources and deep structures through which radon flows and creates radon anomalies [2].
Mathematical modeling of processes of radon distribution in soil and its overflow to the low atmosphere is a task associated with parabolic boundary problems of mathematical physics. Developing mathematical models, algorithms, and software for computing processes of radon distribution is the actual problem having high practical value.

Literature review on the subject shows a big number of works, which make and study mathematical models of radon transport processes. Most of them are based on one-dimensional, diffusion or diffusion-convective mathematical models in homogeneous geological mediums [1-9]. However, nowadays one can find studies on computation of volume activity of radon for three-dimensional models. This shows the importance of scientific work on radon topic.

2. Problem statement and method of its solution

Without limiting the generality of the discussion let us look into a horizontal-layered model of medium with local inclusions (Figure 1).

Let horizontal-layered medium be divided by smooth parametrically set boundaries \( \gamma_{i,0} = \frac{1}{2} \left( \frac{z_j}{z_i} \right) \) (\( i = 0, N \)) – smooth parametric boundaries of horizontal-layered medium, divided by them into horizontal layers \( \Omega_{i,0}, \Omega_{i,1}, \ldots, \Omega_{i,N,0} \), filled with matter, diffusion properties of which are described by symmetric tensors \( D_{i,0} = \begin{pmatrix} d_{i,xx}^{0} & d_{i,xy}^{0} & d_{i,xz}^{0} \\ d_{i,yy}^{0} & d_{i,yy}^{0} & d_{i,zz}^{0} \\ d_{i,xz}^{0} & d_{i,yy}^{0} & d_{i,zz}^{0} \end{pmatrix} \) and speeds of advection \( v_{0,0}, v_{1,0}, v_{2,0}, \ldots, v_{N,0} \) respectively. Every layer \( \Omega_{i,0} \) contains \( M_i \) local inclusions \( \Omega_{i,j} \) with boundaries \( \gamma_{i,j} \), filled with matter, physical properties of which are described by constant symmetric tensors of diffusion \( D_{i,j} = \begin{pmatrix} d_{i,j,xx} & d_{i,j,xy} & d_{i,j,xz} \\ d_{i,j,xy} & d_{i,j,yy} & d_{i,j,yy} \\ d_{i,j,xz} & d_{i,j,yy} & d_{i,j,zz} \end{pmatrix} \) and speeds of advection \( v_{i,j} \), \( i = 0, N, j = 1, M_i \).

The mathematical model of radon transport in the area of study \( \Omega = \bigcup_{i=0}^{N} \bigcup_{j=0}^{M_i} \Omega_{i,j} \subset \mathbb{R}^3 \) can be presented as a boundary value problem:
\[
\frac{\partial A_{i,j}(P,t)}{\partial t} = \text{div}(D_{i,0} \nabla A_{i,j}(P,t)) + v_{i,0} \frac{\partial A_{i,j}(P,t)}{\partial z} - \lambda(A_{i,j}(P,t) - A_{i,x}), \quad P = P(x, y, z) \in \Omega_{i,j}, i = 0, N, j = 0, M_{i};
\]

\[
\left(\frac{D_{i,0} \nabla A_{i,0}(P,t)}{\gamma_{i,0}} + v_{i,0} A_{i,0}(P,t)\right) = \left(\frac{D_{i+1,0} \nabla A_{i+1,0}(P,t)}{\gamma_{i,0}} + v_{i,0} A_{i+1,0}(P,t)\right), \quad i = 0, N - 1; \quad A_{i,0}(P,t) = A_{i+1,0}(P,t), \quad i = 0, N - 1;
\]

\[
\left(\frac{D_{i,0} \nabla A_{i,j}(P,t)}{\gamma_{i,j}} + v_{i,0} A_{i,j}(P,t)\right) = \left(\frac{D_{i+1,0} \nabla A_{i+1,j}(P,t)}{\gamma_{i,j}} + v_{i,0} A_{i+1,j}(P,t)\right), \quad i = 0, N, j = 1, M_{i};
\]

\[
\lim_{z \to \infty} A_{i,0}(P,t) = A_{N,0}, \quad \lim_{z \to \infty} A_{i,0}(P,t) = 0;
\]

\[
\lim_{P \in \Omega_{i,j}, \sqrt{x^2 + y^2} \to \infty} A_{i,0}(P,t) = A_{i,j}(P,t) = A_{i,j}(P,0) = 0, i = 0, N, j = 0, M_{i}.
\]

Here \(A_{i,j}(P,t)\) is a function of radon volume activity in soil, \(A_{i,j}(P,t) \in C_{p}^{2}(\Omega) \cap C_{p}^{1}(\Omega_t) \cap C_{p}^{1}(\Omega_t)\); \(\lambda\) – decay constant of radon; \(A_{i,0}\) – volume activity of radon in radioactive equilibrium with radium \((^{226}\text{Rn})\) at specified depth in soil at \(i\)th layer, which is equal to \(A_{i,0} = K_{i,\text{em}} A_{i,\text{Ra}} P_{i,s} (1 - \eta_{i})\), where \(K_{i,\text{em}}\) – radon emanation coefficient, \(A_{i,\text{Ra}}\) – specific activity \(^{226}\text{Rn}\), \(P_{i,s}\) – density of solid particles, \(\eta_{i}\) – soil porosity, \(A_{i,j}(P,t)\) – normal radon field, describing diffusion-advection of radon in layered medium without inclusions. Variables \(t \geq 0\) \(-\) time.

If layer \(\Omega_{i,0}\) is a low atmosphere, then in the problem (1) \(A_{i,0} = 0\). At \(M_{i} > 0\) inclusions \(\Omega_{i,1}, \ldots, \Omega_{i,M_{i}}\) could describe living and industrial buildings.

A combined method of solving this problem with the methods of integral transformations, integral representations, and boundary integral equations was proposed. Algorithm for calculating the field of radon volume activity was built [10].

Let’s present the desired function of volume activity of radon in soil \(A_{i,j}(P,t)\) as a sum of two auxiliary functions of normal \(A_{i}(P,t)\) and anomalous \(A_{i,j}(P,t)\) fields:

\[
A_{i,j}(P,t) = A_{i}(P,t) + \bar{A}_{i,j}(P,t), i = 0, N, j = 0, M_{i},
\]

where the normal radon field is defined as a boundary problem and describes radon field in horizontal-layered medium with inclusions:

\[
\frac{\partial A_{i}(P,t)}{\partial t} = \text{div}(D_{i,0} \nabla A_{i}(P,t)) + v_{i,0} \frac{\partial A_{i}(P,t)}{\partial z} - \lambda(A_{i}(P,t) - A_{i,x}), \quad P \in \Omega_{i,0}, i = 0, N;
\]

\[
\left(\frac{D_{i,0} \nabla A_{i,0}(P,t)}{\gamma_{i,0}} + v_{i,0} A_{i,0}(P,t)\right) = \left(\frac{D_{i+1,0} \nabla A_{i+1,0}(P,t)}{\gamma_{i,0}} + v_{i,0} A_{i+1,0}(P,t)\right), \quad i = 0, N - 1;
\]

\[
\lim_{z \to \infty} A_{i,0}(P,t) = A_{N,0}, \quad \lim_{z \to \infty} A_{i,0}(P,t) = 0;
\]

\[
\lim_{P \in \Omega_{i,0}, \sqrt{x^2 + y^2} \to \infty} A_{i,0}(P,t) = A_{i}(P,t) = A_{i}(z,t), i = 0, N; \quad A_{i}(P,0) = 0, i = 0, N,
\]
where $\tilde{A}_i(z,t) - \text{volume activity of radon in piecewise-homogeneous horizontal-layered medium with flat-parallel boundaries } z = z_i, i = 0, N - 1 \text{ and diffusion coefficients } \tilde{d}_i = d_i^{i0}, i = 0, N. \text{ It should be noted that in the case of homogeneous medium with flat-parallel boundaries a function of normal radon field } A_i(P,t) \text{ matches a function } \tilde{A}_i(z,t). \text{ A way of finding } \tilde{A}_i(z,t) \text{ is described in [5].}

With the problem (3) anomalous radon field satisfies the next boundary problem:

$$\frac{\partial \tilde{A}_{i,j}(P,t)}{\partial t} = \text{div}(D_{i,j} \nabla \tilde{A}_{i,j}(P,t)) + v_{i,j} \frac{\partial \tilde{A}_{i,j}(P,t)}{\partial z} - \lambda \tilde{A}_{i,j}(P,t), \ P \in \Omega_{i,j}, i = 0, N, j = 0, M_i;$$

$$\left(\frac{D_{i,j} \nabla \tilde{A}_{i,j}(P,t)}{\partial y} + v_{i,j} \tilde{A}_{i,j}(P,t)\right)_{\Gamma_{i,j}} = \left(\frac{D_{i+1,j} \nabla \tilde{A}_{i+1,j}(P,t)}{\partial y} + v_{i+1,j} \tilde{A}_{i+1,j}(P,t)\right)_{\Gamma_{i,j}}, i = 0, N - 1;$$

$$\frac{D_{i,j} \nabla \tilde{A}_{i,j}(P,t)}{\partial y} + v_{i,j} \tilde{A}_{i,j}(P,t)\right)_{\Gamma_{i,j}} = \left(\frac{D_{i,j} \nabla \tilde{A}_{i,j}(P,t)}{\partial y} + v_{i,j} \tilde{A}_{i,j}(P,t)\right)_{\Gamma_{i,j}},\ i = 0, N, j = 1, M_i;$$

$$\lim_{P \to \infty} \tilde{A}_{i,j}(P,t) = 0, i = 0, N;$$

$$\tilde{A}_{i,j}(P,0) = 0, i = 0, N, j = 0, M_i.$$

The function $\psi_{i,j}(P,t)$ of type (*) is found out by applying the equation (2) to boundary properties of the original problem (1).

Let’s make a substitution in the problem (4):

$$\tilde{A}_{i,j}(P,t) = e^{-\lambda t} u_{i,j}(P',t),$$

where $P' = (x, y, z'), \ z' = z + v_{i,j}t.$

It gives the following problem:

$$\frac{\partial u_{i,j}(P',t)}{\partial t} = \text{div}(D_{i,j} \nabla u_{i,j}(P',t)), \ P' \in \Omega_{i,j}, i = 0, N, j = 0, M_i;$$

$$\left(\frac{D_{i,j} \nabla u_{i,j}(P',t)}{\partial y} + v_{i,j} u_{i,j}(P',t)\right)_{\Gamma_{i,j}} = \left(\frac{D_{i+1,j} \nabla u_{i+1,j}(P',t)}{\partial y} + v_{i+1,j} u_{i+1,j}(P',t)\right)_{\Gamma_{i,j}}, i = 0, N - 1;$$

$$\frac{D_{i,j} \nabla u_{i,j}(P',t)}{\partial y} + v_{i,j} u_{i,j}(P',t)\right)_{\Gamma_{i,j}} = \left(\frac{D_{i,j} \nabla u_{i,j}(P',t)}{\partial y} + v_{i,j} u_{i,j}(P',t)\right)_{\Gamma_{i,j}},\ i = 0, N, j = 1, M_i;$$

$$u_{i,j}(P',t)\right)_{\Gamma_{i,j}} = u_{i,j}(P',t)\right)_{\Gamma_{i,j}},\ i = 0, N, j = 1, M_i;$$

$$\lim_{P \to \infty} u_{i,j}(P',t) = 0, i = 0, N;$$

$$u_{i,j}(P',0) = 0, i = 0, N, j = 0, M_i.$$

Let’s apply to the problem (6) a method of solving, described in the work [11], using the Laplace integral transform:

$$F(P',s) = \int_0^\infty u(P',t)e^{-st}dt$$

with the Riemann-Mellin inversion formula.
\[ u(P', t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} F(P', s)e^{st}ds. \]  

(8)

It the formula (8) the limits of integration \((\sigma - i\varepsilon; \sigma + i\varepsilon)\) mean, that integration could be done at any line, parallel to axis \(\text{Im}(s)\) and crossing the axis \(\text{Re}(s)\) at the point \(\sigma > \sigma_0\). This means that \(s = \sigma + i\omega\), where \(\sigma(\sigma > \sigma_0)\) – is a constant, \(\omega(-\infty < \omega < +\infty)\) – a variable of integration.

Next, we will get a set of boundary problems with respect to the complex parameter \(s\):

\[
div(D_{i,j} \overline{F}_{i,j}(P', s)) - sF_{i,j}(P', s) = 0, \quad P' \in \Omega_{i,j}, \quad i = 0, N, \quad j = 0, M_j;
\]

\[
((D_{i,0} \overline{F}_{i,0}(P', s), \overline{n}) + v_{i,0} F_{i,0}(P', s)) \big|_{\gamma_{i,0}'} = ((D_{i,1} \overline{F}_{i,1}(P', s), \overline{n}) + v_{i,1} F_{i,1}(P', s)) \big|_{\gamma_{i,1}'}, \quad i = 0, N - 1;
\]

\[
F_{0,0}(P', s) \big|_{\gamma_{0,0}'} = F_{1,0}(P', s) \big|_{\gamma_{1,0}'}, \quad i = 0, N - 1;
\]

\[
((D_{i,j} \overline{F}_{i,j}(P', s), \overline{n}) + v_{i,j} F_{i,j}(P', s)) \big|_{\gamma_{i,j}'} = ((D_{i,0} \overline{F}_{i,0}(P', s), \overline{n}) + v_{i,0} F_{i,0}(P', s) + F_{0,j}(P', s)) \big|_{\gamma_{i,j}'}, \quad i = 0, N, \quad j = 1, M_j;
\]

\[
\lim_{P' \to \infty} F_{i,j}(P', s) = 0, \quad i = 0, N,
\]

where functions \(F_{\psi_{i,0}}(P', s)\) and \(F_i(P', s)\) – are images of the functions \(\psi_{i,0}(P', t)\) and \(A_i(P', t)\) at transformation (7) respectively.

The problem (9) is a boundary problem of differential equations in partial elliptical derivative. For this let us look into the problem for the Green’s function \(G(P, Q)\) – function of the point source, located at the random point \(Q(x_q, y_q, z_q)\) and generating diffusion field of single intensity in accommodating space (in layered medium without inclusions):

\[
div(D_{i,0} \overline{G}_{i,0}(P', Q)) - sG_{i,0}(P', Q) = -\delta(P', Q), \quad P' \in \Omega_{i,0}, \quad i = 0, N;
\]

\[
((D_{i,0} \overline{G}_{i,0}(P', Q), n) + v_{i,0} G_{i,0}(P', Q)) \big|_{\gamma_{i,0}'} = ((D_{i,1} \overline{G}_{i,1}(P', Q), n) + v_{i,1} G_{i,1}(P', Q)) \big|_{\gamma_{i,1}'}, \quad i = 0, N - 1;
\]

\[
G_{0,0}(P', Q) \big|_{\gamma_{0,0}'} = G_{1,0}(P', Q) \big|_{\gamma_{1,0}'}, \quad i = 0, N - 1;
\]

\[
\lim_{P' \to \infty} G_{i,0}(P', Q) = 0, \quad i = 0, N.
\]

Then integral representation of the problem (9) in the field \(\Omega\) could be represented as:

\[
F(P', s) = \sum_{i=0}^{N} \sum_{j=1}^{M_j} \int_{\gamma_{i,j}'} F_i(Q, s)[(v_{i,0} - v_{i,j})G_{i,0}(P', Q) + ((D_{i,0} - D_{i,j})\overline{G}_{i,0}(P', Q), \overline{n}_Q)]dy_{i,j} +
\]

\[
+ \sum_{i=0}^{N} \sum_{j=1}^{M_j} \int_{\gamma_{i,j}'} F_{\psi_{i,0}}(Q, s)G_{i,0}(P', Q)dy_{i,j}.
\]

(11)

Here \(\overline{n}_Q\) is a vector of external normal to the boundary of inclusion at the point \(Q\).

From (11) follows that the solution of the problem (9) could be found at any point \(P'\) of piecewise-constant anisotropic layer \(\Omega_{i,j} (i = 0, N, j = 1, M_j)\), if the solution of the problem (10) – the
Green’s function \( G_{i,0}(P',Q) \), \( i=0,N \), and there will be known the boundary values for the function \( F_{i,j}(Q,s) \) at internal boundaries subareas, not included to the problem for the Green’s function.

Putting (11) in the point \( P' \) at each of these boundaries, we will get the system of integral equations of Fredholm of the second kind with respect to unknown boundary values of function \( F_{i,j}(Q,s) \):

\[
F(P',s) - \sum_{i=0}^{N} \sum_{j=1}^{M_i} \int_{\gamma_{i,j}} F_{i,j}(Q,s) \left[ (\nu_{i,0} - \nu_{i,j}) G_{i,0}(P',Q) + ((D_{i,0} - D_{i,j}) \bar{V} G_{i,0}(P',Q), \bar{n}_Q) \right] d\gamma_{i,j} = 0,
\]

\[
= \sum_{i=0}^{N} \sum_{j=1}^{M_i} \int_{\gamma_{i,j}} F_{i,j}(Q,s) G_{i,0}(P',Q) d\gamma_{i,j}.
\]

So, the algorithm for solving the original problem (1) is following:

Step 1. Define a normal radon field \( \bar{A}_i(z,t) \) in horizontal layered piecewise homogeneous medium with flat-parallel boundaries \( z = z_i = \text{const}, i = 0,N \), and diffusion coefficients \( \bar{d}_i = d_{zz}^{i,0}, i = 0,N \), and speeds of advection \( \nu_{i,0}, i = 0,N \) with algorithm described in [5].

Step 2. If the boundaries of the layers \( z = \gamma_{i,0}(x,y) = z_i = \text{const} \), i.e. the medium has flat-parallel boundaries, then the solution for the problem (3) for normal radon field has been found: \( A_i(P,t) = \bar{A}_i(z,t) \). Otherwise, the problem (3) should be solved with, for example, the method of integral equations, forming them at the boundaries \( \gamma_{i,0}(x,y) \neq z_i \).

Step 3. Compute the functions \( \psi_{i,0}(P',t) \) at the boundaries of inclusions \( \gamma_{i,j}, i = 0,N, j = 1,M_i \) with a formula (*)

Step 4. For every value of a parameter \( s \) set of quadrature nodes numerical transformation of Laplace (with the algorithm in [12]) with the formula (7):

Step 4.1. Find images \( F_{\psi_{i,0}}(P',s) \) of the functions \( \psi_{i,0}(P',t) \) at transformations (5) and (7).

Step 4.2. Find solution for a problem (10) with the Green’s function. It can be derived analytically for the case of homogeneous layers with flat parallel boundaries, for example, with the Hankel integral transform.

Step 4.3. Form a system (12) and find its solution – boundary (at the boundaries of inclusions) values of a function \( F_{i,j}(Q,s) \).

Step 4.4. With the formula (11) find solution for the problem (9) – a function \( F_{i,j}(P',s) \).

Step 4.5. Form the term of quadrature formula for integral (8), computing functions \( u_{i,j}(P',t) \).

Step 5. Find anomalous field \( \bar{A}_{i,j}(P,t) \) with the formula (5).

Step 6. The solution of the original problem (1) is a function \( A_{i,j}(P,t) \) derived with the formula (2).

Suggested combined methods and algorithms are the development of the theory of solving of boundary problems for heat mass transfer equations in piecewise-homogeneous anisotropic mediums. They allow solving practical problems for studying radon transport processes in three-dimensional piecewise-homogeneous anisotropic layered mediums with anisotropic inclusions.

3. Experimental results

In accordance with the suggested method of solving the problem was developed a specialized software in the computer system Maple. It uses numerical algorithms of finding a normal radon field,
Green’s function in piecewise-homogeneous horizontal-layered medium with flat-parallel boundaries, the Laplace integral transform and the function of anomalous radon field with inclusions.

With the help of the developed software complex was made a comparative analysis of results of computer modeling with known models for the case of homogeneous piecewise mediums. For example, diffusion-advection model of radon transport for five-layered horizontal-layered medium with flat-parallel boundaries were tested. It has following parameters [1]:

\[
\begin{align*}
 n &= 5 \quad \text{number of layers}; \\
 \lambda &= 2.1 \cdot 10^{-6} \text{ s}^{-1} \quad \text{radon decay constant}; \\
 z_0 &= 0 \text{ m}, \ z_1 = 1 \text{ m}, \ z_2 = 3 \text{ m}, \ z_3 = 6 \text{ m} \quad \text{boundaries}; \\
 d_{0.0} &= 1 \cdot 10^6 \text{ m}^2/\text{s}, \ d_{1.0} = d_{2.0} = d_{3.0} = d_{4.0} = 3 \cdot 10^{-6} \text{ m}^2/\text{s} \quad \text{diffusion coefficients in every layer respectively}; \\
 v_{0.0} &= 0 \text{ m/s}, \ v_{1.0} = v_{2.0} = v_{3.0} = v_{4.0} = 4 \cdot 10^{-6} \text{ m/s} \quad \text{speeds of advection in every layer respectively}; \\
 \rho_1 = \rho_2 = \rho_3 = \rho_4 &= 2700 \text{ kg/m}^3 \quad \text{density of soil particles for every layer}; \\
 \eta_1 = \eta_2 = \eta_3 = \eta_4 &= 0.45 \quad \text{soil porosity for every layer}; \\
 K_{1,em} = K_{2,em} = K_{3,em} = K_{4,em} &= 0.2 \quad \text{soil emanation coefficients for every layer}; \\
 A_{1, Ra} &= 90 \text{ Bq/kg}, A_{2, Ra} = 4 \text{ Bq/kg}, A_{3, Ra} = 30 \text{ Bq/kg}, A_{4, Ra} = 1000 \text{ Bq/kg} \quad \text{specific activity of} \ 226\text{Ra for every layer.}
\end{align*}
\]

Distribution curves of volume activity of radon at different time intervals \( t \): \( 10^5 \text{ s} \); \( 2 \cdot 10^5 \text{ s} \); \( 3 \cdot 10^5 \text{ s} \); \( 10^6 \text{ s} \) are shown on figure 2.

![Figure 2. Distribution curves for volume activity of radon at different time intervals t.](image)

Comparative analysis of the results: for curve, corresponding to the moment of time \( t = 10^6 \text{ s} \), is shown in the table 1, where \( z \) – depth, m; \( A_{gr} \) – values of volume activity of radon, shown in the work [1], kBq/m\(^3\); \( A_{num} \) – values of volume activity of radon, found by the developed algorithms, kBq/m\(^3\); \( \Delta \) – absolute error, \%; \( \delta \) – relative error, \%.
Data in the table 1 show that the maximum relative error for the curve at the moment \( t = 10^6 \) s doesn’t exceed 4%.

Therefore, the results qualitatively and quantitatively are similar with the results shown in the considered work.

Table 1. Comparative analysis of the results at the moment \( t = 10^6 \) s.

| \( z \), m | \( A_{ex} \), kBq/m\(^3\) | \( A_{num} \), kBq/m\(^3\) | \( \Delta \), % | \( \delta \), % |
|----------|-----------------|-----------------|----------|----------|
| 0        | 0.0000          | 0.0001          | 0.01     | –        |
| 1        | 2.3629          | 2.2798          | 8.31     | 3.52     |
| 2        | 0.4401          | 0.4465          | 0.64     | 1.45     |
| 3        | 0.9790          | 0.9736          | 0.54     | 0.55     |
| 4        | 2.8929          | 2.8628          | 3.01     | 1.04     |
| 5        | 27.6425         | 27.3560         | 28.65    | 1.04     |
| 6        | 149.4550        | 149.3438        | 11.12    | 0.07     |
| 7        | 271.0907        | 271.3304        | 23.97    | 0.09     |
| 8        | 295.2742        | 295.7716        | 49.74    | 0.17     |
| 9        | 297.1825        | 296.9868        | 19.57    | 0.07     |
| 10       | 297.2752        | 297.0000        | 27.52    | 0.09     |

In the work [13] are shown results of computational experiments for studying radon transport processes in piecewise-homogeneous anisotropic layered mediums with inclusions and mutual influence of the parameters of the model. It was found that consideration of anisotropy in modeling of radon transport processes in geological mediums leads to the substantial change of field of volume activity and is an important factor, required for describing the mathematical model of the field in real geological mediums.

Numerical computation of function of distribution of volume activity of radon in piecewise-homogeneous flat-parallel horizontal-layered mediums with sphere-like inclusion \( \Omega_{4,1} \) of radius \( R = 0.5 \) m with the center in the point (1,1,7) (figure 3).

![Figure 3. Geometry of the medium.](image)
Values of medium parameters correspond to the case of diffusion-advection model of radon transport for five-layered horizontal-layered medium with flat parallel boundaries discussed above. Physical properties of this inclusion is described with the symmetric tensor of diffusion

$$
D_{4,1} = \begin{pmatrix}
3 \times 10^{-5} & 0 & 0 \\
0 & 3 \times 10^{-5} & 0 \\
0 & 0 & 3 \times 10^{-5}
\end{pmatrix}
$$

and the speed of advection with a value of \( v_{4,1} = 4 \times 10^{-5} \) m/s.

Results of the numerical modeling are shown on Figure 4.

On figure 4 (a) is shown a function graph of normal radon field at \( t = 10^6 \) s. Influence of inclusion is shown on figure 4 (b).

![Figure 4. Function graph of volume activity of radon.](image)

Adequacy of the provided theoretical solutions is better to test on laboratory models which allow for accurate testing of all input parameters. In the work [14] are shown the results of comparative analysis of data for computational and real experiments, which were taken in the lab of Institute of Geophysics, UB RAS, for studying processes of radon transport in piecewise-homogeneous horizontal-layered mediums. They showed adequacy and accuracy of the suggested models and the method of solving the problem.

However, the developed software was not effective (unacceptably slow generates the value of estimated field at the point of the study) and, to use it in the consequent studies for solving the problems of reverse geophysical practical problems, a radical paradigm shift was required: algorithm optimization, the use of new technology of parallelization for multiprocessor systems. For this reason, the authors of the study decided to develop high-performance software with object-oriented language C# and OpenMaple technology.

OpenMaple technology is a set of functions, which gives access to different data types used by Maple itself; additionally, it has a variety of functions to work with data. Using this technology gives two advantages. It allows reusing of the functions written for Maple and it gives the high performance of object-oriented language C#.

The special module for numerical inversion of the Laplace transform (in accordance with the algorithm [12]) was made with OpenMaple and the language C#. It uses built-in mathematical
functions of C#, functions for working with complex numbers of external library ALGLIB and some built-in functions of Maple system, which were called with special commands of OpenMaple.

The comparison graph of the performance of the program (in milliseconds) in Maple and in Visual Studio depending on the number of nodes for general quadrature formula with the highest precision is shown on figure 5.

![Figure 5. Comparison graph of the performance of the program.](image)

4. Conclusions
The obtained results already allow (with the example of one program module) to confirm that the use of the language C# and OpenMaple technology gives a substantial increase in the performance of computations.

Currently computations of density of radon flow are carried out. Additionally, discussions on using the developed algorithm for solving reverse boundary problem in variational methods of searching approximated splines of boundaries of geological mediums as the extremal functions of the Tikhonov regularization are ongoing.

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