Reanalysis of the $e^+e^- \rightarrow \eta\gamma$ reaction cross section

M. N. Achasov,1, 2 K. I. Beloborodov,1, 2 A.V. Berdyugin,1 A. G. Bogdanchikov,1 A. D. Bukin,1, 2 D. A. Bukin,1 T. V. Dimova,1 V. P. Druzhinin,1, 2 V. B. Golubev,1, 2 A. A. Korol,1 S. V. Koshuba,1 E. V. Pakhtusova,1 S. I. Serednyakov,1, 2 Z. K. Silagadze,1, 2 and A. V. Vasiliev1, 2

1Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
2Novosibirsk State University, Novosibirsk, 630090, Russia

In the experiment with the SND detector at the VEPP-2M $e^+e^-$ collider measuring the $e^+e^- \rightarrow \eta\gamma$ cross section in the energy range $0.6 < \sqrt{s} < 1.38$ GeV the reanalysis of data is performed. The goal is to improve the accuracy of the previous results by analysing ambiguities in the approximation of energy dependence of the $e^+e^- \rightarrow \eta\gamma$ cross section, which were not taken into account in our previous study. We report new results on the approximation of the $e^+e^- \rightarrow \eta\gamma$ cross section based on Vector Dominance Model under new model parameter assumptions.

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In our previous work [1] we performed a measurement of the $e^+e^- \rightarrow \eta\gamma$ reaction cross section for two $\eta$-meson decay modes: $\eta \rightarrow 3\pi^0$ and $\eta \rightarrow \pi^+\pi^-\pi^0$. The measured cross section was approximated within Vector Dominance Model (VDM). In this approximation the fact that with the free magnitudes and phases of all four $\rho$, $\omega$, $\phi$, and $\rho'$ resonances, the minimized likelihood function has several minima and thus, a choice of the right minimization solution is ambiguous, was ignored. Additional physical restrictions must be applied to, at least, the parameters of the $\rho'$ resonance, in order to make a right choice of the minimization solution.

In this work the $\rho'$-meson parameters were estimated from the experimental data on the $e^+e^- \rightarrow \eta\eta$ cross section [2, 3]. The cross section value in the $\rho'$-resonance maximum obtained from the approximation of the $e^+e^- \rightarrow \eta\eta$ cross section can be translated to the $e^+e^- \rightarrow \rho' \rightarrow \eta\gamma$ cross section using VDM as in Ref. [3]. The approximation of the data on the $e^+e^- \rightarrow \eta\eta$ cross section was done with three different models describing the energy dependence of the $\rho'$ resonance width: $1 - B(\rho' \rightarrow \omega\pi) = 0.5$, $B(\rho' \rightarrow \pi^+\pi^-) = 0.5$; $2 - B(\rho' \rightarrow \omega\pi) = 1$; and $3 - B(\rho' \rightarrow \pi^+\pi^-) = 1$. Under these assumptions the resulting mass, width, and maximum cross section of the $\rho'$ resonance varied within $1440 - 1520$ MeV, $220 - 400$ MeV, and $0.08 - 0.13$ nb, respectively. All three $\rho'$ parameter sets were used in the $e^+e^- \rightarrow \eta\gamma$ cross section approximation. The difference in the experimental results was included in the systematic uncertainty of the final result. The cross section at the $\rho'$ resonance maximum acquired an additional 20% systematic error from the uncertainty of the cross section calculation in the VDM.

Natural values for the phase of the $\rho'$ amplitude are 0 or 180 degrees. The phase shift from these values can be induced by the $\rho$ and $\rho'$ mixing via common decay channels, for example, $\rho, \rho' \rightarrow \omega\pi^0$. In the model considered in Ref. [3] it was shown that mixing is strongly energy dependent. For the approximation of the data on $e^+e^- \rightarrow \eta\gamma$ cross section the $\varphi_{\rho'}$ value at the $\phi$ resonance maximum is important. Since at this energy the $\rho - \rho'$ mixing is small, no significant phase shifts from 0 or 180 degrees are expected. Two approximations with the $\varphi_{\rho'}$ phase fixed at these values were considered. In order to estimate a corresponding systematic uncertainty of the approximation parameters due to the $\rho'$ phase uncertainty, we varied $\varphi_{\rho'}$ by $\pm 20^\circ$ with respect to its initial values.

The fitting results with the $\rho'$ parameters fixed to described above values are presented in Table I. Excluded from Table I are two solutions with the phase $\varphi_{\omega} \sim 180^\circ$. The values of $B(\omega \rightarrow \eta\gamma) \simeq (28 \pm 1) \times 10^{-4}$ corresponding to these values of $\varphi_{\omega}$ contradict the result of Ref. [3], $B(\omega \rightarrow \eta\gamma) = (3.1 \pm 1.1) \times 10^{-4}$, where the contribution from $\rho - \omega$ interference was suppressed by event selection cuts. The quark model predictions for the phases of $\omega$ and $\phi$ mesons are $0^\circ$ and $180^\circ$, respectively. The values close to those were obtained, for example, for the processes $e^+e^- \rightarrow 3\pi$ [7] and $e^+e^- \rightarrow \pi^0\gamma$ [8]. The difference between the observed phases and quark model predictions of $0^\circ$ and $180^\circ$ for these processes can be described by $\phi - \omega$ and $\rho - \omega$ mixing [8]. Estimations show, that the contribution of the $\phi - \omega$ mixing in the $\eta\gamma$ channel is small. Taking into account that $\phi - \rho$ mixing is small ($B(\phi \rightarrow \pi^+\pi^-) \sim 10^{-4}$), the expected deviation of the $\phi$-meson phase from 180$^\circ$ does not exceed few degrees. The $\rho - \omega$ mixing leads to the $\omega$-meson phase $\varphi_{\omega} \sim 15^\circ$. Since for the solution corresponding to the $\varphi_{\rho'} = 180^\circ$ the value of $\varphi_{\phi}$ differs from its expectation by more than 3.5$^\circ$, for our final result we take the solution corresponding to $\varphi_{\rho'} = 0^\circ$. It should be noted that the same choice of the $\rho'$ phase was used in the work [3].

Using the world-average values [10] for the $\rho$, $\omega$, and $\phi$ masses and $\rho, \omega, \phi \rightarrow e^+e^-$ decay probabilities, we can calculate the following values for the decay probabilities into the $\eta\gamma$ final state:

$$B(\rho \rightarrow \eta\gamma) = (2.82 \pm 0.30 \pm 0.17) \times 10^{-4},$$
$$B(\omega \rightarrow \eta\gamma) = (4.33 \pm 0.44 \pm 0.17) \times 10^{-4},$$

(1)
TABLE I: Results of the approximation. The first error is statistical, the second is systematic.

| $\varphi_{\mu'}$ | $0^\circ$ | $180^\circ$ |
|------------------|----------|-----------|
| $\sigma_{\rho \to \eta \gamma}$ | $(0.322 \pm 0.034 \pm 0.019)$ nb | $(0.319 \pm 0.032 \pm 0.019)$ nb |
| $\sigma_{\omega \to \eta \gamma}$ | $(0.744 \pm 0.075 \pm 0.027)$ nb | $(0.816 \pm 0.081 \pm 0.029)$ nb |
| $\sigma_{\phi \to \eta \gamma}$ | $(57.21 \pm 0.95 \pm 1.66)$ nb | $(59.79 \pm 0.75 \pm 1.73)$ nb |
| $\varphi_{\omega}$ | $(6.9 \pm 7.7 \pm 2.3)^0$ | $(18.3 \pm 7.7 \pm 2.4)^0$ |
| $\varphi_{\phi}$ | $(166 \pm 18 \pm 8)^0$ | $(219 \pm 7 \pm 9)^2$ |

$B(\phi \to \eta \gamma) = (1.364 \pm 0.023 \pm 0.044) \times 10^{-2}$.

The systematic error on the branching fractions and cross sections quoted in Table I and Eq. (1) includes a 1.9% uncertainty of the detection efficiency, 2% uncertainty of the integrated luminosity, and the model uncertainty equal to 5.2% for $\sigma_{\rho \to \eta \gamma}$, 2.3% for $\sigma_{\omega \to \eta \gamma}$, and 0.9% for $\sigma_{\phi \to \eta \gamma}$. The systematic uncertainty of the decay probability also includes the error on the $\rho, \omega, \phi \to e^+e^-$ branching fractions.

Using the results of the cross section approximation we calculated the radiative correction and its model uncertainty. The model uncertainty is included into a systematic error on the total cross section (Tables II and III) together with the errors on detection efficiency and integrated luminosity. The model error does not exceed 1% for all energy points below 1040 MeV. At the energy of about 1100 MeV near the cross section minimum it reaches a 17% level.

In comparison with the results published in Ref. [1], $B(\rho \to \eta \gamma)$ increased by 1.4σ, $B(\omega \to \eta \gamma)$ decreased by 0.7σ, the change in $B(\phi \to \eta \gamma)$ is less than 0.1σ. The alterations of the measured cross section values are statistically insignificant in the energy range below 1030 MeV. For energies in the range 1030–1050 MeV and above 1200 MeV the difference is about one statistical standard deviation, but in the energy range 1050–1200 MeV, where the reaction cross section is small, the difference between the cross sections corresponds to six statistical standard deviations.

I. CONCLUSION

In Table IV the decay probabilities presented in our previous paper [1], the results of this work, and other most precise experiments are listed. The results on decay branching fractions obtained in this work differ from [1] by about one standard deviation, which proves the stability of the results under different model assumptions.

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The first error is statistical, the second — systematic.

### TABLE III: Cross section ($\sigma$) of the process $e^+e^- \rightarrow \eta\gamma$ measured in the decay mode $\eta \rightarrow 3\pi^0$. $E$ is center-of-mass energy. The first error is statistical, the second — systematic.

| $E$, MeV | $\sigma$, nb | $E$, MeV | $\sigma$, nb | $E$, MeV | $\sigma$, nb |
|---------|-------------|---------|-------------|---------|-------------|
| 600.00  | $< 0.36$ 90\% CL | 794.24  | $0.58^{+0.17}_{-0.13} \pm 0.02$ | 1019.62 | $54.14 \pm 0.99 \pm 1.52$ |
| 630.00  | $< 0.19$ 90\% CL | 800.29  | $0.56^{+0.14}_{-0.11} \pm 0.02$ | 1020.58 | $41.37 \pm 1.60 \pm 1.16$ |
| 660.00  | 0.059$^{+0.079}_{-0.038} \pm 0.002$ | 810.26  | $0.29^{+0.06}_{-0.06} \pm 0.01$ | 1021.64 | $23.37 \pm 1.16 \pm 0.66$ |
| 690.00  | $< 0.10$ 90\% CL | 820.00  | $0.43^{+0.12}_{-0.09} \pm 0.01$ | 1022.78 | $12.30 \pm 0.66 \pm 0.35$ |
| 720.00  | 0.22$^{+0.07}_{-0.05} \pm 0.01$ | 840.00  | $0.25^{+0.06}_{-0.05} \pm 0.01$ | 1027.76 | $2.84 \pm 0.19 \pm 0.08$ |
| 750.26  | 0.33$^{+0.14}_{-0.10} \pm 0.01$ | 880.00  | $0.19^{+0.08}_{-0.06} \pm 0.01$ | 1033.70 | $0.86 \pm 0.09 \pm 0.03$ |
| 760.29  | 0.36$^{+0.14}_{-0.10} \pm 0.01$ | 920.00  | $0.21^{+0.06}_{-0.06} \pm 0.01$ | 1039.68 | $0.34 \pm 0.04 \pm 0.01$ |
| 764.31  | 0.32$^{+0.13}_{-0.09} \pm 0.01$ | 940.00  | $0.27^{+0.08}_{-0.06} \pm 0.01$ | 1049.76 | $0.12 \pm 0.02 \pm 0.01$ |
| 770.31  | 0.54$^{+0.15}_{-0.12} \pm 0.02$ | 950.00  | $0.16^{+0.09}_{-0.06} \pm 0.01$ | 1059.76 | $0.072 \pm 0.011 \pm 0.005$ |
| 774.23  | 0.77$^{+0.20}_{-0.16} \pm 0.02$ | 958.00  | $0.30^{+0.14}_{-0.11} \pm 0.01$ | 1078.54 | $0.032^{+0.009}_{-0.007} \pm 0.003$ |
| 778.21  | 1.49$^{+0.25}_{-0.22} \pm 0.04$ | 970.00  | $0.23^{+0.11}_{-0.08} \pm 0.01$ | 1099.92 | $0.016^{+0.007}_{-0.005} \pm 0.001$ |
| 780.14  | 1.56$^{+0.23}_{-0.20} \pm 0.04$ | 980.00  | $0.24^{+0.19}_{-0.12} \pm 0.01$ | 1131.58 | $0.020^{+0.009}_{-0.007} \pm 0.003$ |
| 781.16  | 2.17$^{+0.25}_{-0.23} \pm 0.06$ | 984.10  | $0.42^{+0.12}_{-0.09} \pm 0.01$ | 1182.96 | $0.020^{+0.010}_{-0.009} \pm 0.003$ |
| 782.06  | 2.11$^{+0.17}_{-0.16} \pm 0.06$ | 1003.82 | $1.61 \pm 0.19 \pm 0.05$ | 1227.34 | $0.015^{+0.035}_{-0.012} \pm 0.001$ |
| 783.17  | 1.98$^{+0.20}_{-0.18} \pm 0.06$ | 1009.68 | $3.20 \pm 0.30 \pm 0.09$ | 1271.68 | $0.051^{+0.049}_{-0.028} \pm 0.001$ |
| 784.27  | 1.83$^{+0.22}_{-0.20} \pm 0.05$ | 1015.64 | $15.15 \pm 0.86 \pm 0.42$ | 1315.44 | $0.021^{+0.048}_{-0.017} \pm 0.001$ |
| 785.40  | 1.56$^{+0.26}_{-0.23} \pm 0.04$ | 1016.70 | $25.06 \pm 1.22 \pm 0.70$ | 1360.44 | $0.022^{+0.039}_{-0.014} \pm 0.001$ |
| 786.21  | 1.30$^{+0.22}_{-0.19} \pm 0.04$ | 1017.66 | $35.55 \pm 1.73 \pm 1.00$ |            |            |
| 790.22  | 0.83$^{+0.21}_{-0.17} \pm 0.02$ | 1018.64 | $53.75 \pm 1.54 \pm 1.51$ |            |            |

The first error is statistical, the second — systematic.

### TABLE IV: The $\rho, \omega, \phi \rightarrow \eta\gamma$ decay branching fractions.

| Source | Published [1] | This work | Previous measurements |
|--------|-------------|---------|----------------------|
| $B(\rho \rightarrow \eta\gamma) \cdot 10^4$ | $2.40 \pm 0.25 \pm 0.07$ | $2.82 \pm 0.30 \pm 0.17$ | $3.28 \pm 0.37 \pm 0.23$ [4] |
| $B(\omega \rightarrow \eta\gamma) \cdot 10^4$ | $4.63 \pm 0.46 \pm 0.13$ | $4.33 \pm 0.44 \pm 0.17$ | $5.10 \pm 0.72 \pm 0.34$ [4] |
| $B(\phi \rightarrow \eta\gamma) \cdot 10^2$ | $1.362 \pm 0.019 \pm 0.035$ | $1.364 \pm 0.023 \pm 0.044$ | $1.338 \pm 0.012 \pm 0.052$ [11] |