A basic orthotropic viscoelastic model for composite and wood materials considering available experimental data and time-dependent Poisson’s ratios

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Abstract. Long-term deformation in creep is of significant engineering importance. For anisotropic materials, such as wood, composites and reinforced concrete, creep testing in several axial directions including shear is necessary to obtain a creep model which is able to predict deformation in the basic orthotropic case. Such a full set of experimental data is generally not available, and simplifying assumptions are typically made to conceive a useful 3D model. These assumptions should preferably be made based on the material behaviour and sound engineering arguments. This problem appears to be addressed in many different ways and sometimes the assumptions are not well justified. In the present study, we examine 3D creep of wood and composite materials. Particular emphasis is made on explaining the choices made in developing the model, considering practicality, incomplete material data and the specific behaviour of wood and composites. An orthotropic linear viscoelastic model is implemented as a material model in a commercial FE software. The constitutive equations are derived in the 1D case using a hereditary approach, then later generalized to the 3D formulation. Guidelines are shown how to implement it into the FE software to predict creep of components and structures. Although the model itself is conventional, the effect of considering time-dependent Poisson’s ratios is investigated here, as well an optimization approach when inserting inevitably asymmetric experimental creep data into the model. As far as the authors know, creep of wooden materials have not been defined using this approach before. The model of interest is calibrated against experimental data. Examples using experimental results from solid wood data and a unidirectional fiber composite are demonstrated. The results show that the model is able to capture the orthotropic behaviour adequately. Orthotropy requires symmetry of the creep compliance matrix, which typically is not the case experimentally. It is shown that in rendering the matrix symmetric, one needs to decide which direction is more important. It is also shown that the frequently employed assumption of constant Poisson’s ratios should be made with caution.

1. Introduction

If a material is subjected to a persistent load, a gradually increasing deformation of the material is introduced, e.g. a wooden shelf loaded by books may increase its deflection as years goes by. This phenomenon refers to a time-dependent material behaviour also called as creep behaviour. If the time-dependent strains are linearly related to the stresses, it is commonly known as
linear viscoelasticity. Over the past years the amount research into the viscoelastic behaviour of composites and timber structures has been significant, given the long-term use of these materials in load carrying structures. A review of previous models is first presented, before showing the advantages of the present approach.

In Refs. [1] and [2] a coincident element method for orthotropic composites is proposed. It utilises two coincident shell elements to model the orthotropic viscoelastic behaviour of a composite laminate. The first element exhibits isotropic and viscoelastic behaviour, while the second element exhibits orthotropic and elastic behaviour. The elements are superimposed in such a way that they are coincident, i.e., the two shell elements share the same nodes and hence deform together. The procedure was used to model a number of published examples of viscoelasticity of laminated composites. It is shown that the coincident element method is a relatively simple and useful tool for modelling orthotropic and viscoelastic response of laminated composites by using a finite element package that only supports isotropic viscoelastic material models. The drawback was that the number of equations to be solved essentially doubled since there are two shell elements describing the same domain. This means, that the elements of each material group will experience the individual stress state under the same displacement field. The resulting stress matrix field in the model is a sum of the stress matrix fields for each group of elements. The model was also restricted to shell problems. Thus the proposed technique is well–suited for the problems of anisotropic viscoelasticity of thin and thick homogeneous and multilayered shells including nonlinear contact problems.

Papers [3], [4] and [5] used the differential approach by combining dashpots and springs in their viscoelastic models for timber structures. These models took into account different phenomena common in wood such as effects of moisture content and mechanosorption. A challenge when using these models is often to find the parameters for the model. In [3], a full 3D constitutive law is proposed for orthotropic nonlinear viscoelastic behaviour. The nonlinearities are taken into account using a generalized Maxwell model with two nonlinear elements. The viscosities in these two Maxwell elements depend on stress level. That parameter is introduced using a danger factor based on the Tsai-Wu criterion and the calculation of an “elastic multiplier”. Viscosity of one of those Maxwell elements also depends on moisture changes. In Ref. [4], in contrary to Ref. [3], a generalized Kelvin model with nonlinear elements was used. In the Kelvin elements, a mechanosorptive dashpot is used for the irrecoverable mechanosorption. The above creep mechanisms are added to the elastic strain and to the hygroexpansion strain. The whole model includes that the mechanical quantities depend on the moisture content. The temperature is considered to be constant during the studied cases.

Papers [6], [7] and [8] used the hereditary approach to exhibit thermoviscoelastic behaviour of composite material to estimate the residual stresses during cool-down. The analysis has been shown to be useful in prediction of residual stresses during production of composite materials and also other thermal environments. It is only needed to characterise the thermo-viscoelastic behaviour of the polymer part of the composite material. The properties of the orthotropic material are calculated in the Laplace domain and used as the input to the subroutine provided to the material model in ANSYS.

3D effects in viscoelasticity are not always strictly accounted for in these models. For instance, none of the above-mentioned models specifically investigated the effect of time-dependent Poisson’s ratios on creep behaviour. Papers such as [7] and [8], assume constant Poisson’s ratios for their models, although that is not often experimentally the case [9]. Another effect that is largely unaddressed is that experiments typically show asymmetric relaxation and compliance matrices [10][11]. In this paper, the hereditary approach is used to represent the viscoelastic behaviour of orthotropic materials. An approach will been shown on how to take into account the actual time-dependent Poisson’s ratios as well as showing how to deal with asymmetric behaviour in a simple manner.
2. Theoretical background

Starting from the fundamentals, the simplified model is derived and explained in this section. As opposed to linear elastic materials, the response of a viscoelastic material depends on the loading history. Performing a relaxation test is a common experimental method to characterize the viscoelastic properties of a material. Characterization may be done by a uniaxial tensile test with a constant strain $\varepsilon_0$. The stress $\sigma(t)$ is then measured as a function of time.

$$\sigma(t) = \int_{0}^{t} E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau. \quad (1)$$

Deriving the above equation is called the hereditary approach to linear viscoelasticity, which allows greater freedom when constructing models than the differential approach that relies on the concepts of certain combinations of springs and dashpots. The essential issue in the hereditary approach is that of superposition. Equation (1) can be generalized to 3D by exchanging the 1D relaxation modulus to a relaxation stiffness matrix using Voigt Notation. Starting from an unloaded and completely relaxed state at $t = 0^-$, the total stress $\sigma$ at time $t$ can be expressed using the convolution integral,

$$\sigma_i(t) = C_{ij}(t)\varepsilon_j(0) + \int_{0}^{t} C_{ij}(t - s)\dot{\varepsilon}_j(s)ds. \quad (2)$$

where $\sigma_i$ and $\varepsilon_i$ are stress and strains in index form using the Voigt notation, and $C_{ij}$ are the components of the time-dependent (relaxation) stiffness matrix. Furthermore, $C_{ij}$ is assumed to be described by a Prony series such as

$$C_{ij}(t) = C_{ij}^\infty + \sum_{k=1}^{M} C_{ij}^{(k)} e^{-\frac{t}{\tau_{ij}^{(k)}}}, \quad (3)$$

where $C_{ij}^\infty = C_{ij}(t \to \infty)$, $\tau_{ij}$ and $C_{ij}^{(k)}$ are material parameters to be determined, and $M$ is the number of Maxwell elements used that is represented by the Prony series.

The stress at an arbitrary time $t$ can be obtained by inserting (3) in (2), which in return can be rearranged such as

$$\sigma_i(t) = C_{ij}^\infty \varepsilon_j(t) + \sum_{k=1}^{M} \sigma_{i,\text{viscous}}^{(k)}(t) \quad (4)$$

where,

$$\sigma_{i,\text{viscous}}^{(k)} = C_{ij}^{(k)} \left( e^{-\frac{t}{\tau_{ij}^{(k)}}} \varepsilon_j(0) + \int_{0}^{t} e^{-\frac{t-s}{\tau_{ij}^{(k)}}} \dot{\varepsilon}_j(s)ds \right) \quad (5)$$

2.1. Finite element implementation

To implement a material model in a commercial finite element software, it is often the case to calculate the stress based on a strain increment and on the information of the previous time steps. That is why the authors have used the relaxation form to derive the viscoelastic constitutive equations. For small time steps $\Delta t$, the strain rate $\dot{\varepsilon}_j$ may be considered constant, and thus eq. (4) transforms into

$$\sigma_{i,\text{viscous}}^{(k)} = e^{-\frac{\Delta t}{\tau_{ij}^{(k)}}} \sigma_{i,\text{viscous}}(t - \Delta t) + \frac{1 - e^{-\frac{\Delta t}{\tau_{ij}^{(k)}}}}{\Delta t/\tau_{ij}^{(k)}} C_{ij}^{(k)} \Delta \varepsilon_j \quad (6)$$
From eq. (6), it is now possible to obtain the viscous stresses from each time step $\Delta t$. It is noted that the viscous stresses at time $t$ is equal to a fraction of the viscous stress from the previous time step plus a stress increment due to the strain increment $\Delta \varepsilon_j$.

Finally, the total stress is the sum of the elastic and viscous contributions,

$$
\sigma_i(t) = C^\infty_{ij} \varepsilon_j(t) + \sum_{k=1}^{M} e^{-\Delta t \tau_{ij}} \sigma_{i,\text{viscous}}(t - \Delta t) + \sum_{k=1}^{M} \frac{1 - e^{-\Delta t \tau_{ij}}}{\Delta t / \tau_{ij}} C^{(k)}_{ij} \Delta \varepsilon_j
$$

(7)

In addition to the stress update, it is necessary to provide the consistent tangent stiffness, also known as the material Jacobian, to the FEM software. The consistent tangent stiffness is necessary to derive the constitutive component of the internal virtual work, where the latter is discretized to formulate the FE equations which is later solved via Newton-Raphson iterations to find the corresponding displacements that yield equilibrium. Deriving eq. (7) w.r.t. strain $\varepsilon_j$, the Jacobian is obtained,

$$
C_{ij}^{\text{tan}} = \frac{\delta \sigma_i(t)}{\delta \varepsilon_j(t)} = C^\infty_{ij} + \sum_{k=1}^{M} \frac{1 - e^{-\Delta t \tau_{ij}}}{\Delta t / \tau_{ij}} C^{(k)}_{ij}
$$

(8)

From eq. (3), it should be noted that this approach will have $12 + 24M$ parameters that need to be defined in order to fully describe the viscoelasticity of a general 3D orthotropic material. Choosing an adequate number of parameters is always a balance. Adding more and more empirical parameters does not add any significant predictive precision given the generally large scatter in measured data. Too few parameters will not capture the main traits of the time-dependent load-deformation behaviour.

2.2. Determination of the model parameters

Given that creep experiments in all direction have been performed, one may easily determine the parameters of the model. The stiffness matrix $\mathbf{C}$ is the inverse of the compliance matrix $\mathbf{D}$ such as

$$
\mathbf{C}^{-1} = \mathbf{D}
$$

(9)

whereas, assuming orthotropic symmetry,

$$
\mathbf{D} = \begin{bmatrix}
\frac{1}{E_1} & -\frac{\nu_{12}}{E_2} & -\frac{\nu_{13}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_3} & 0 & 0 & 0 \\
-\frac{\nu_{13}}{E_3} & -\frac{\nu_{23}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\tau_{112}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\tau_{113}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_{23}}
\end{bmatrix}
$$

(10)

It should be emphasized that each element of the compliance matrix, as well as its components, are time dependent. The time dependent Poisson’s ratios
\[ \nu_{ij} = -\frac{\varepsilon_j(t)}{\varepsilon_i(t)} \]  

are determined by the strain in the lateral direction of the creep test, \( \varepsilon_j \), and the strain in the loading direction of the same creep test, \( \varepsilon_i \). The time dependent Young’s moduli

\[ E_i(t) = \frac{\sigma_i}{\varepsilon_i(t)} \]  

are determined by the constant creep stress \( \sigma_i \) and the aforementioned \( \varepsilon_i \). Determining \( G_{ij} \) is analogous to eq. (12).

It important to keep \( D(t) \) as a symmetric matrix in order to invert it. Thus one should optimize the off-diagonal elements depending on which of the lateral strains that are more important to describe.

Consider two uniaxial tensile creep tests of where constant stresses, \( \sigma_1 \) and \( \sigma_2 \), are applied in each test respectively. The constant stress \( \sigma_1 \) is applied in direction 1, thus resulting into a creep strain \( \varepsilon_1^L(t) \) in the loading direction, and creep strain \( \varepsilon_2^T(t) \) in direction 2 due to the Poisson effect. Similarly, the constant stress \( \sigma_2 \) is applied in direction 2, and thus resulting into a creep strain \( \varepsilon_2^L(t) \) in the loading direction, and creep strain \( \varepsilon_1^T(t) \) in direction 1 due to the Poisson effect. Using eq. (11), \( \nu_{12}(t) \) and \( \nu_{21}(t) \) and using eq. (12), \( E_1(t) \) and \( E_2(t) \) can be calculated. Knowing the Poisson’s ratios as well as the Young’s moduli, will allow us to calculate the off-diagonal elements \( d_{21} \) and \( d_{12} \) in accordance to eq. (10). It is usually expected that \( d_{12} \) and \( d_{21} \) are not equal to each other, and thus an optimization has to be made to make the compliance matrix \( D \) symmetric. E.g., one can define a new diagonal element \( d_{12}^{sym} \) such as

\[ d_{12}^{sym} = xd_{12} + (1 - x)d_{21}, \]  

where \( x \) is decided by minimizing the absolute errors,

\[ \epsilon_1 = || \varepsilon_2^L - d_{12}^{sym}(x)\sigma_1 ||, \quad \epsilon_2 = || \varepsilon_1^T - d_{12}^{sym}(x)\sigma_2 || \]  

In applications, it might be necessary to use some form of weighted absolute errors in cases where there is a specific normal stress component that is dominant throughout the structure. Finally, when \( D(t) \) is fully defined, the elements of the stiffness matrix \( C(t) \) may be fitted by a Prony series described in eq. (3) with an appropriate curve-fitting scheme.

### 3. Examples of implementation

In this section, the proposed model is implemented in the ANSYS subroutine USERMAT and is used to describe the viscoelasticity of common orthotropic materials. The model parameters are determined by calibrating against experimental data from various experimental studies. Two examples of solid wood and one of a unidirectional fiber composite will be demonstrated.

#### 3.1. Fiber reinforced thermoplastic material

Endo et al [11] performed an approximately 3.5 weeks long tensile creep test of a fiber reinforced thermoplastic at a temperature of 23.5 C. Here, two loading directions were performed, parallel and perpendicular relative to the fiber directions. A tensile stress of 12 MPa was applied in both cases. The resulting creep curves are shown in for each loading direction (Fig. 1a and Fig. 1b), showing the creep strains parallel (L) and perpendicular (T) to the fiber direction.
3.2. European beech wood

Ozyhar et al. [10] performed a tensile creep test of European beech wood. In addition to loading in the longitudinal direction, the same test was performed in the tangential and radial directions. The creep tests were carried out in a constant climate test room under controlled environmental conditions, (20°C, 65% RH). The resulting creep curves are shown for each loading direction, showing the creep strains in three perpendicular material axes L, T and R in Fig. 2.

It is readily possible to take into account either fully time dependent or constant (time invariant) Poisson’s ratios in the model. Additionally, one can choose which specific Poisson’s ratios are to be constant or time dependent. This selection can be done by identifying which Poisson’s ratio changes the least in comparison to others. In the experiment of Ozyhar et al., it was identified that $\nu_{RL}$, $\nu_{TL}$ and $\nu_{RT}$ did not change as fast relative to the remaining three Poisson’s ratios, perhaps due to the stiffening radial rays present in the hardwood. Thus, the assumption of having these specific ratios to be time-independent was implemented into the model as well.

![Creep plots from tensile load in L (Endo et al)](image1)

![Creep plots from tensile load in T (Endo et al)](image2)

**Figure 1:** Creep loading according to Endo et al. For (a), the specimen was subjected to a given creep stress of 12 MPa in the longitudinal direction. For (b), the specimen was subjected to a given creep stress of 12 MPa in the perpendicular to the fiber. The resulting creep strains are shown in L and T (perpendicular to the fiber) directions.

4. Discussion

It should be noted that a single 1x1x1 [mm] cube was modelled in FE-software to simulate creep. At this scale, the cellular wood microstructure affects the mechanical behaviour, and the material cannot be regarded as a homogeneous continuum. There is also a risk when using the fitted creep model to simulate larger and complex structures, as the size-effects are not taken into account. For example the polar orthotropy of wood is not considered here, where Cartesian orthotropic behaviour is assumed. The quantification of the error due to size effects is of interest in future studies.

From Fig. 1 and Fig. 2, there is a distinct difference between the assumption of constant and time-dependent Poisson’s ratios. However, the trends are different in the two types of material. In the fiber reinforced thermoplastic material, the assumed constant Poisson’s ratios resulted into larger transverse creep under longitudinal loading direction, see Fig. 1 (a). In contrast, the solid wood example showed smaller transverse creep instead, as can be seen in Fig.
Figure 2: Creep data set from Ozyhar et al. For (a), the specimen was subjected to a given creep stress of 26.2 MPa in the longitudinal direction. For (b), the specimen was subjected with a given creep stress of 5.7 MPa in the tangential direction. For (c), the specimen was subjected to a given creep stress of 11.9 MPa in the radial direction. The resulting creep strains are shown in L, T and R direction for all cases.

2 (a). This difference is of interest and may need some further investigation in the future from a micromechanics viewpoint. Nevertheless, it is clear that applying time-dependent Poisson’s ratios to the model gave similar results compared to experimental data. It was also discovered using the solid wood example, that applying constant Poisson’s ratio of where the longitudinal direction is perpendicular to the loading direction, results to similar results to the assumption of time-dependent ratios. This is simply due to the fact that the Poisson’s ratio is in this case relatively constant comparison to the remaining ones. Thus, it is up to the user of the model to decide which ratios should be taken as constant by comparing the Poisson’s effect in the experimental data. The invariance of certain Poisson’s ratios should be attributed by
the microstructure of the composite or wood material, and could possibly be addressed by micromechanics.

For the sake of convenience, we used only two exponential terms for each component in the C matrix. However, the model allows to have different number of terms for $C_{ij}$, thus it is of future interest to study the the optimal combination. Optimal not only in fitting experimental data, but also out of ease of use and how many parameters are motivated to include given the experimental scatter. At this point, the authors have shown the capabilities of the relatively straightforward hereditary approach on the viscoelastic model of orthotropic materials, as well as how to manage asymmetric experimental data.

5. Conclusions
The following conclusions and recommendations can be made:

- It can be recommended to use two exponential terms ($m=2$) for each component in both transversely isotropic and orthotropic materials, judged by the available experimental data.
- Experimental creep data do generally not comply with a symmetric creep compliance matrix, as required for orthotropic materials. A scheme to render the compliance matrix symmetric is presented in this work.
- If necessary, one may use constant Poisson’s ratio if it has been shown that the Poisson’s ratios are time-independent in the experimental testing. This facilitates the use of the model.

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