Magnetodielectric AC measuring transducer for automation systems in oil refineries

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Abstract: The purpose of the study covers theoretical analysis and mathematical modeling of alternating current measuring transducers based on magnetodielectric cores for automation systems in oil refineries. The paper describes the analytical study of the magnetization curve, creation of the equivalent circuit and thus obtained mathematical model of a transducer, numerical study of the proposed transducer, as well as the contrastive analysis of transducers currently used to measure the current.

1. Introduction

Modern automation systems of oil refineries shall be safe, economic, convenient in service, able to integrate expansions throughout the development of enterprises.

Oil refineries have different technological electrical consumers, among which the most powerful are the pumping units with the capacity of thousands kilowatts. This defines the specifics of oil refinery, namely the set of synchronous and asynchronous motors used in multipurpose compressors and drives of powerful pumping units.

Today the current problem is the control of parameters of power equipment within the automation systems of oil refineries, such as the current. This problem is quite relevant since the operation of power equipment at oil refineries implies a form of network currents significantly different from sinusoidal thus impacting the quality of the network parameters specified by GOST 32144-2013, and this, in turn, has significant effect on the results of current measurements in power circuits of process equipment within the automation systems [1]. In this case the problem of modern measuring current transducers with the least measurement error and ensuring linearity of measurement in the maximum range is becoming ever more urgent [2]. Besides, other important factors include satisfactory physical and operational characteristics of measuring transducers [3]. This work is devoted to modeling and study of the proposed magnetodielectric measuring transducer of alternating current (MDT) based on modern materials.

The distinctive feature of such transducers is that they have the sufficient range and linearity throughout the entire measurement scale [4]. This ensures the creation of intellectual systems of any complexity on their basis [5]. Another absolute advantage is their minimum weight-size parameters. The main problem here is high technological efficiency of MDT production. This is caused by the fact that the MDT production shall ensure full stability of all magnetic characteristics of cores in time at major temperature change.

MDT is based on magnetodielectric cores manufactured from powder carbonyl iron via compaction. This process takes place under high pressure. Cores are made from the mixture of a dielectric filler and iron particles.
Iron-based powder materials appeared at the beginning of the 20th century as a consequence of the study on the need to transfer a telephone signal to the maximum distances at the smallest depressing for pupinization of telephone cables. This material has some disadvantages, including its short lifetime (2 years).

The alternative could be the use of modern materials – Iron Powder of Magnetics (USA). The use of these materials will allow reducing the overall dimensions of measuring transducers.

Figure 1 shows the normal magnetization curve (NMC) for Magnetics material.

![Figure 1. Normal magnetization curve of magnetics](image)

Further task of the study covered analytical expressions approximating the normal magnetization curve with the minimum error.

2. **Modeling of the normal magnetization curve of AC magnetodielectric measuring transducer.**

The following analytical expression was applied to approximate the normal magnetization curve [6]:

\[ B = A \ln(\alpha H + 1) \]  

(1)

where \( B \) and \( H \) – magnetization curve coordinates; 
\( A \) and \( \alpha \) – approximation coefficients.

The more precisely the approximating function and the experimental curve coincide, the more precisely be further calculations of the MDT output voltage. To ensure the minimum approximation error it is necessary to impose additional restrictions on approximated expressions:

\[ \lim_{H \to 0} B(H) = 0 \]  

(2)

\[ \lim_{H \to 0} \frac{dB}{dH} = \mu_\alpha \]  

(3)

where \( \mu_\alpha \) – initial magnetic permeability of NMC.

From the analytical expression (1) it is possible to receive the following expression:

\[ \frac{dB}{dH} = A\alpha \frac{1}{\alpha H + 1} \]  

(4)

At \( H \to 0 \) we get:

\[ \lim_{H \to 0} \frac{dB}{dH} = A\alpha \]  

(5)
From the expressions (3), (5) it is possible to receive the following expression:

$$A = \frac{\mu_H}{\alpha}. \quad (6)$$

If to substitute the expression (6) into the expression (1) we get:

$$B = \frac{\mu_H}{\alpha} \ln(\alpha H + 1). \quad (7)$$

In case of high intensity ($H > 300000$ A/m) the NMC will take a straight form, therefore we can write as follows:

$$B = B_s + \mu_0 H \cdot \quad (8)$$

where $B_s$ – ordinate received by continuation of the saturated site of the magnetization curve; $\mu_0$ – magnetic constant.

$$\lim_{H \to \infty} \frac{dB}{dH} = 0. \quad (9)$$

Let us correct the expression (9) and to satisfy the condition:

$$\lim_{H \to \infty} \frac{dB}{dH} = \mu_0. \quad (10)$$

Let us write the expression for the normal magnetization curve as follows:

$$B = \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H + 1) + \mu_0 H \cdot \quad (11)$$

The formula (11) allows satisfying the main NMC requirements, namely:

$$\lim_{H \to 0} (\alpha H + 1) = 1; \quad \lim_{H \to 0} \frac{dB}{dH} = \mu_0; \quad \lim_{H \to \infty} \frac{dB}{dH} = \mu_0.$$

In the received formula (11) the approximation coefficient $\alpha$, which shall be defined, remains unknown. The least-squares method and NMC coordinates received experimentally can be used for this purpose. Let us calculate the absolute error in this point using the following expression:

$$e_i(\alpha) = \left| \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H_i + 1) + \mu_0 H_i - B_i \right|. \quad (12)$$

where $H_i, B_i$ – NMC coordinates.

The squared error will be equal to:

$$e_i^2(\alpha) = \left( \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H_i + 1) + \mu_0 H_i - B_i \right)^2. \quad (13)$$

To consider all NMC points received experimentally, we can make the aggregate squared error:

$$\hat{O}(\alpha) = \sum_{i=1}^{n} e_i^2 \quad \text{or} \quad \hat{O}(\alpha) = \sum_{i=1}^{n} \left( \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H_i + 1) + \mu_0 H_i - B_i \right)^2. \quad (14)$$

where $n$ – number of experimentally received points.

Minimizing the expression (14) we find parameter $\alpha$, and differentiate this expression according to $\alpha$:

$$\frac{d\hat{O}}{d\alpha}(\alpha) = 2 \sum_{i=1}^{n} \left[ \left( \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H_i + 1) + \mu_0 H_i - B_i \right) \times \left( \frac{\mu_H - \mu_0}{\alpha^2} \ln(\alpha H_i + 1) + \frac{H_i}{\alpha H_i + 1} \right) \right]. \quad (15)$$

Minimization $\Phi(\alpha)$ requires the satisfaction of the necessary condition:

$$\frac{d\hat{O}}{d\alpha}(\alpha) = 0. \quad (16)$$

The substitution of (15) into (16) results in the need to solve the following transcendental equation:
\[
\sum_{i=1}^{n} \left[ \left( \frac{\mu_H - \mu_0}{\alpha} \ln (\alpha H_i + 1) + \mu_0 H_i - B_i \right) \times \left( \frac{\mu_H - \mu_0}{-\alpha^2} \ln (\alpha H_i + 1) + \frac{\mu_H - \mu_0}{\alpha} \frac{H_i}{\alpha H_i + 1} \right) \right] = 0
\]

The solution of the equation (17) by the bisection method at \( \mu_i = 1.22 \cdot 10^{-5} \frac{\tilde{A}i}{I} \) leads to the following result: \( \alpha = 1.13 \cdot 10^{-5} \frac{i}{\tilde{A}} \).

3. Modeling of eclectic equivalent circuit of AC magnetodielectric measuring transducer.

In fact, the magnetodielectric measuring transducer of alternating current is a transformer with a distributed gap operating as a differentiating transformer (Fig. 2). The output parameter of this transducer is the secondary voltage \( u_2 \). This voltage will be equal to the derivative of the measured current taking into account the transformation ratio.

![Figure 2. AC magnetodielectric measuring transducer](image)

Let us consider the equivalent circuit for magnetodielectric transformer presented in Fig. 3 [7].

![Figure 3. Electric equivalent circuit of AC magnetodielectric measuring transducer](image)

The mathematical model represents the system of equations describing the electromagnetic processes of the transducer:

\[ i_1 = i_0 + i_2 \quad \text{(18)} \]

\[ \frac{d\psi}{dt} = (L_{2s} + L_n) \frac{di_2}{dt} + (r_2 + r_n)i_2 \quad \text{(19)} \]

\[ \psi = f(i_0) \]
where $i_0$ – magnetization current; $i_1$ – primary current; $\psi$ – flux linkage; $L_1$ – load inductance; $L_{2s}$ – leakage inductance; $i_2$ – secondary current; $r_2$ – secondary resistance; $r_n$ – load.

The output value of a transducer will be its information signal – $u_2$, while the input signal will represent the measured current $i_1$.

The system of equations describing electromagnetic processes of the transducer (18-20) can be written as follows:

\[ \psi = f(i_0) . \]  
\[ i_1 = i_0 + \frac{u_2}{r_H} . \]  
\[ \psi = f(i_0) . \]  
\[ \frac{d\psi}{dt} = \frac{L_{2s}}{r_H} \frac{dU_2}{dt} + \left(1 + \frac{r_2}{r_H}\right)u_2 . \]

The magnetodielectric core of the transducer shall be made toroidal, then taking into account the uniformity, $L_{2s}$ may be neglected. Considering that $r_H \to \infty$, the external fields may be neglected, therefore $i_1 = i_0$. Hence, the system of equations (21-23) will be as follows:

\[ \frac{d\psi}{dt} = u_2 . \]  
\[ \psi = f(i_0) . \]

First, let us define the flux linkage:

\[ \psi(t) = \Phi(t)w_2, \]

where $\Phi(t)$ – magnetic flux.

\[ \Phi(t) = B(t)S. \]

Earlier [6] expressions and parameters for the core magnetization curve were defined. Proceeding from these expressions let us define the induction.

\[ B = \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H + 1) + \mu_i H \]  

where $H$ – voltage; $\alpha$ – approximation coefficient; $\mu_0, \mu_H$ – magnetic constant and initial magnetic permeability of the core.

The expression for flux linkage, taking into account the expressions (27) and (28), will be as follows:

\[ \psi(t) = w_2 S \left( \frac{\mu_H - \mu_0}{\alpha} \ln(\alpha H + 1) + \mu_i H \right) \]

For the simplest core geometry using the Ampere’s law we get the following:

\[ \oint_{i_0} Hdl = i_1, \quad HL_0 = i_1, \]

where $l_0$ – average length of the magnetic line, hence:

\[ H = \frac{i_1}{l_0} \]

Taking this into account the expression for flux linkage can be as follows:

\[ \psi(t) = w_2 S \left( \frac{\mu_H - \mu_0}{\alpha} \ln \left( \frac{i_1}{l_0} + 1 \right) + \frac{\mu_i}{l_0} \right) \]

If to substitute this expression in the equation (24), we will get:
\[ u_2 = w_2 S \left( \frac{\mu_H - \mu_0}{\alpha} \cdot \frac{\alpha}{l_0} \cdot \frac{di}{dt} + \frac{\mu_0}{l_0} \cdot \frac{di}{dt} \right) \]  

(33)

Let us assume that the primary current changes under the law:

\[ i_1 = I_{im} \left( \cos \varphi \cdot e^{\frac{t}{T_1}} - \cos(wt + \varphi) \right) \]

(34)

where \( \omega \) – angular frequency; \( T_1 \) – time constant; \( I_{im} \) – amplitude of primary current; \( \varphi \) – initial phase.

If to differentiate this law of changing the primary current over time we get the following:

\[ \frac{di_1}{dt} = I_{im} \left( \omega \sin(\omega t + \varphi) - \cos \varphi \cdot \frac{1}{T_1} \cdot e^{\frac{t}{T_1}} \right) \]

(35)

Having substituted these expressions in the equation (33) we get:

\[ u_2 = w_2 S l_{im} \left( \omega \sin(\omega t + \varphi) - \cos \varphi \cdot \frac{1}{T_1} \cdot e^{\frac{t}{T_1}} \right) \cdot \frac{\mu_H - \mu_0}{\alpha l_{im} \left( \cos \varphi \cdot e^{\frac{t}{T_1}} - \cos(wt + \varphi) \right) + \frac{\mu_0}{l_0}} \]

(36)

4. Quantitative assessment of the proposed current transducer

Modeling via standard application program packages was conducted for quantitative assessment of the measurement error of the AC magnetodielectric transducer [8].

The following parameters of the magnetodielectric current transducer were used as a model:

- measured current: 12 kA;
- measured current frequency: \( f_p = 50 \) Hz.
- form and size of the measuring transducer: preliminary calculation showed that the transducer shall have a toroid form and the following dimensions: \( D = 45 \times 10^{-3} \) m; \( d = 38 \times 10^{-3} \) m; \( h = 45 \times 10^{-3} \) m.
- magneto-field vector \( H \) created by primary MDT winding:

\[ H = \frac{\sqrt{2} \cdot I_{1,av} \cdot w_1}{l_{1,av}} \]

(37)

where \( I_1 \) – test current (A); \( l_{cp} \) – length of the average line of a magnetic circuit (m), \( w_1 \) – primary winding turns;

\[ H = 4.041 \times 10^5 \text{ A/m.} \]

- quantity of secondary winding turns:

\[ w_2 = \frac{U_2 \cdot l_{av}}{2 \cdot \pi \cdot S \cdot f_p \cdot I_1 \cdot \mu_\mu} \]

(38)

where \( U_2 \) – secondary voltage (B); \( S \) – cross-section of a magnetic circuit (m²); \( f_p \) – operating frequency (Hz); \( I_1 \) – test current (A); \( I_1 \) – relative magnetic permeability of a magnetic circuit; \( \mu_0 \) – magnetic constant.

Thus, the quantity of winding turns calculated for current \( I = 12 \) kA:

\[ w_2 = 147.8 \]

Hence, we can round to \( w_2 = 148 \) turns.

Fig. 4 shows flow diagrams illustrating different curve amplitudes of current and the corresponding output voltage of the AC magnetodielectric measuring transducer obtained through modeling. In this
case the current values are given in thousands of amperes (kA), while the voltage – in volts (V), time is set in milliseconds.

Modeling of the AC magnetodielectric measuring transducer conducted for the case when the alternating current equals 12 kA.

![MDT flow diagrams at alternating current of 12 kA](image)

The key technical indicator for any current transducer is its measurement error. The quantitative assessment of the measurement error of the AC magnetodielectric transducer can be carried out only when the dependence of secondary voltage on the measured current $u_2(i_1)$ is built. For this purpose, it is sufficient to take one half-cycle of the measured current, integrate the secondary voltage of the AC magnetodielectric measuring transducer, and shift the initial phase of the measured current by $\pi/2$. Fig. 5 shows the analyzed current curve, and Fig. 6 – the dependence $u_2(i_1)$.

![Analyzed current curve](image)

![Dependence of secondary voltage on the measured current](image)

Analyzing Fig. 6 it is possible to conclude that the obtained dependence $u_2(i_1)$ is almost linear. However, to prove this statement it is critical to carry out numerical evaluation of the conversion ratio
error of the AC magnetodielectric measuring transducer. For this purpose, we need to approximate the dependence $u_2(i_1)$ in Fig. 6 with the function $y=kn+b$, obviously having a linear nature.

The standard functions slope and intercept of MathCAD were used for numerical expression of coefficients $k$ and $b$ [9]. Then, the dependence $u_2(i_1)$ and ideal dependence $y(x)$ were compared with each other [10]. Fig. 7 shows the dependence reflecting the quantitative assessment of the fractional error of a conversion ratio.

![Figure 7](image-url)

**Figure 7.** Dependence of the fractional error of a conversion ratio on the measured current

Analyzing the received dependence it is possible to conclude that the measurement error of the AC magnetodielectric transducer makes not more than 1%, provided the measured current does not exceed 12 kA.

5. **Comparative analysis of the proposed measuring transducer and the existing analogs.**

The group of authors compared the proposed AC magnetodielectric measuring transducer with the following types of modern measuring current transducers: current shunt, transformer current transducers, current transducers based on the Hall effect.

The main advantages of the current shunt include the following: relatively low cost and ability to measure when direct-current components are present in the measured circuits. The main disadvantages of current shunts include the following: a compromise when choosing the shunt rating, shunt overheating when measuring high-ampere current, as well as low immunity. Due to the fact that the shunt “is embedded” directly into a power circuit and has significant overheating at high values of the measured currents, its application at oil refineries is strongly limited. The proposed AC magnetodielectric measuring transducer has no such disadvantages.

The advantages of transformer current transducers include the following: galvanic isolation of measurement circuits from information circuits; minimum energy consumption. The disadvantage is the lack of linearity of measurement in a wide range. The proposed AC magnetodielectric measuring transducer allows avoiding excessive measurement errors due to a wide range of linearity (Fig. 7).

Today, current transducers based on the Hall effect are the most modern and widespread measuring transducers in the world since they allow measuring direct and alternating currents with high accuracy and linearity. Besides, they ensure galvanic isolation. The LEM, which is the leader in the market of advanced and high-quality solutions in the field of measurement of electric parameters, are the most widespread transducers [11]. The main disadvantages of all current transducers based on the Hall effect include their high cost and large dimensions [12]. Thus, the HAZ12000-SB current transducer by LEM can measure the alternating current of 12 kA, its overall dimensions are 250 x 160 mm, which is unacceptable in some cases. As shown above, the proposed AC magnetodielectric measuring transducer is more preferable in terms of its mass-dimensional parameters at the minimum measurement errors.

6. **Conclusions**

The study proposes the magnetodielectric measuring transducer of the alternating current. It also describes the mathematical model of the normal magnetization curve of the AC magnetodielectric measuring transducer, which can be used for engineering and scientific analysis of transition processes.
within automated measuring electrical circuits of oil refineries [7]. Besides, it refers to the model allowing calculating the secondary voltage of the magnetodielectric current transducer. Such expressions can be used in the design of measuring parts of the automation systems in oil refineries [8]. Moreover, the study shows that the conversion ratio error within the entire range of the measured currents does not exceed 1%.

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