Hres O.M./Гресь О.М.
Student / студент

Morozov A.S./Морозов А.С.
c.t.s., as.prof/к.т.н./доц.

ORSID:0000-0001-5769-489X

National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”,
Peremogy ave.,37,Kyiv,Ukrane,03056

Abstract: Due to the rapid development of printing, the improvement of such a widespread product as the paper is urgent. This article is considered a new way of producing paper based on a specific fractal structure that can be hypothetically reproduced using 3D printing. We offer materials and printing methods that are suitable for creating paper, and give their characteristics. The article also analyzes the advantages of such a paper, its features, etc.

Keywords: paper, fractals, 3D printing technologies, composite materials for 3D printer.

Introduction

Surprisingly, fractals open up new possibilities in the field of polygraphy. As it turns out, fractals are capable of creating heavy-duty material. Recently, a group of scientists led by Chung Mao discovered that fractal structures could be used to create new generation building materials. Thus, using a 3D printer and fractal geometry, scientists reproduced the experimental structure of the polymer resin - the shape was 10 times stronger than conventional steel. This discovery gives us many opportunities, especially in printing [1].

Hypothetically, if we put fibers together in the form of fractals, you get paper that’s much more durable.

The results of the research

Using fractal geometry in paper production, it is possible to create a new strong paper. Fractal — any of various extremely irregular curves or shapes for which any suitably chosen part is similar in shape to a given larger or smaller part when
magnified or reduced to the same size [2].

To work with fibers, they must be long and durable. Natural fibers are of limited length: cotton - 6-51 mm; linen - 250-1000 mm; wool - 10-250 mm; natural silk - 400-1000 mm. The longest are synthetic fibers. The length of such fibers is 18 - 70 km [3]. Given that synthetic fibre is a chemical fibre that is formed from synthetic polymers such as polyamides, polyesters, polyacrylonitrile, etc., it seems possible to produce strong synthetic paper with fractal fibre structure using 3Dprint. Modern 3D printers are able to print with different materials: plastic, metal powder, and even chocolate [4]. So, when we print synthetic fibers, we get synthetic paper.

Synthetic paper is printed and written like regular paper. But synthetic paper has the properties of plastic: resistant to tearing, waterproof and does not burn. Synthetic paper is suitable for digital, inkjet and offset printing. Hypothetically, the fractal structure of paper allows the formation of reliable, lightweight, flexible and durable products due to the single layer of paper [5].

Table 1 presents the proposed materials with which we can print fractal structure, printing methods for specific material and their characteristics respectively [9,10,11,12]. Table 2 presents the proposed fractals and their mathematical description [13,14,15,16].

| №  | proposed materials | Material characteristic | Proposed printing methods | Printing characteristic |
|----|--------------------|-------------------------|--------------------------|------------------------|
| 1  | Polyamides (nylons)| 1) Light weight; 2) Elastic; 3) Wear-resistant; 4) moisture-proof; 5) High resistance to many chemicals. | 1) FDM 2) SLA or DLP | 1) **FDM** is a material extrusion method of additive manufacturing where materials are extruded through a nozzle and joined together to create 3D objects. A typical FDM 3D printer, takes a polymer-based filament and forces it through a heated nozzle, which melts the material and deposits it in 2D layers on the build platform. While still warm, these layers fuse with each other to eventually create a three-dimensional part. 2) **SLA** is a form of 3D printing technology used for creating models, prototypes, patterns, and production parts in a layer by layer fashion using photochemical processes by which light causes chemical monomers and oligomers to cross-link together to form polymers. Those polymers then make up the body of a three-dimensional solid. **DLP** stands for digital light processing, and is a type of vat polymerization. Vat polymerization 3D printing technologies make use of a (liquid) photopolymer resin which is able to cure (solidify) under a light source. |
ELASTAN (TPU) | FDM
---|---
1) elastic; 2) High wear resistance; 3) High resistance to impact; 4) excellent solvent resistance; 5) good resistance to weather.

PRIMALLOY | FDM or FFF
---|---
1) Soft rubber material; 2) High temperature resistance; 3) High mechanical strength; 4) High resistance to cleaning; 5) Unique flexibility without loss of strength.

Biopaste from lignin and cellulose | ...
---|---

| Proposed fractal structure | Fractal characteristic |
|---------------------------|-----------------------|
| Heighway dragon (also known as the Harter–Heighway dragon or the Jurassic Park dragon) | The Heighway dragon is also the limit set of the following iterated function system in the complex plane: $f_1(z) = \frac{(1+i)z}{2}$, $f_2(z) = 1 - \frac{(1-i)z}{2}$, with the initial set of points $S_0 = \{0;1\}$. Using pairs of real numbers instead, this is the same as the two functions consisting of

$f_1(x,y) = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$f_2(x,y) = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos 135^\circ & -\sin 135^\circ \\ \sin 135^\circ & \cos 135^\circ \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

| Lévy C curve | If using an iterated function system, then the construction of the C curve is a bit easier. It will need a set of two "rules" which are: two points in a plane (the translators), each associated with a scale factor of $1/\sqrt{2}$. The first rule is a rotation of $45^\circ$ and the second $-45^\circ$. This set will iterate a point $[x, y]$ from randomly choosing any of the two rules and use the parameters...
associated with the rule to scale/rotate and translate the point using a 2D-transform function. Put into formulae:

\[ f_1(z) = \frac{(1 - i)z}{2} \]
\[ f_2(z) = 1 + \frac{(1 + i)(z - 1)}{2} \]
from the initial set of points \( S_0 = \{0;1\} \).

**Pythagoras tree**

The construction of the Pythagoras tree begins with a square. Upon this square are constructed two squares, each scaled down by a linear factor of \( \frac{\sqrt{2}}{2} \), such that the corners of the squares coincide pairwise. The same procedure is then applied recursively to the two smaller squares, ad infinitum.

**Koch snowflake** (also known as the Koch curve, Koch star, or Koch island)

Each iteration multiplies the number of sides in the Koch snowflake by four, so the number of sides after \( n \) iterations is given by:

\[ N_n = 4^n \]

If the original equilateral triangle has sides of length \( s \), the length of each side of the snowflake after \( n \) iterations is:

\[ S_n = \frac{s}{3^n} = s \left( \frac{1}{3} \right)^n \]

an inverse power of three multiple of the original length. The perimeter of the snowflake after \( n \) iterations is:

\[ P_n = N_n \cdot S_n = 3 \cdot s \cdot \left( \frac{4}{3} \right)^n \]

Note that even in the absence of a 3D printer, it is possible to create any model using "Shapeways" or "Ponoko". We can reproduce a fractal using engineering calculation software (PTC Mathcad), 3D CAD software (Creo), and a 3D printer. [https://community.ptc.com/t5/Mathcad/ct-p/PTCMathcad](https://community.ptc.com/t5/Mathcad/ct-p/PTCMathcad).

Typically, to create a specific fractal, you need to describe it using a mathematical formula. To print it, you need to export the matrix and create the STL file. In PTC Mathcad you can export photos as BMP for instance, and then, using any free online tool, convert information to STL for 3D printing. Then you need to use Creo to prepare the fractal for printing. PTC Creo, formerly known as Pro/ENGINEER, is 3D modeling software used in mechanical engineering, design, manufacturing, and in CAD drafting service firms. It was one of the first 3D CAD modeling applications that used a rule-based parametric system. Creo Parametric should not to be confused with Creo Elements/Direct ModelingCreo Elements can be used to create a complete 3D digital model of manufactured goods. The models consist of 2D and 3D solid model data which can also be used downstream in finite element analysis, rapid prototyping, tooling design, and CNC manufacturing [8].
Figure 1 — PTC Creo Parametric

Figure 2 — fractals in Mathcad and the Creo

The printing process can be seen in the Makerbot software [6,7].

Figure 3 — Extruded model in Makerbot software

But this is just one of many ways to print fractals. Today there are a lot of programs for creating various fractals and many 3D printers that could print them. In this case, the method of printing Mandelbrot fractal was introduced. Any other fractals can be printed in the same way.

The following figures depict the Harter–Heighway dragon fractal in the software and printed version.
Figure 4 — by Joshua DJad91 • Mason Dixon Line

Figure 5 — by Rodrigo HerreraFrogmaker17 • Guadalajara, Mexico

**Conclusions**: by creating paper with a fractal structure, it is possible to achieve a super-strong material that would hypothetically be stronger than conventional paper. The proposed manufacturing method is based on a radically new fractal structure and the application of 3D printing.

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