Gluon exchange and $p_T$ distribution

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Abstract

It is shown that even if a quark $q$ is on the mass shell, the quark $q'$ created via vacuum pair production in the process $q \rightarrow h + q'$ has wide distribution for its spacelike virtuality, and so at the end of daughter string ($q'\overline{q}$) there is a parton having wide distributed spacelike virtuality. However, the essence of the parton model is to regard a high energy hadron-like cluster as a collection of quasifree partons. Therefore we use a set of some principles of string model, formulated, for example, by Andersson and Nilsson, that gluons move like localised particles carrying energy momentum between quark and antiquark at the endpoints of string, and so we take into account they go on the mass shell before the break of daughter string occurs. It is stressed, the gluons carry also transverse momentum which could be connected with spacelike virtuality of quark at the end of string. Some approximations to describe the gluon exchange are studied and results of Monte Carlo calculations for antiproton-proton interactions are given.
1 Introduction

Well known concepts of a quark confinement and quantum mechanical tunneling of quark-antiquark pair from the vacuum are used actively in many models of multiparticle production \([1]-[3]\). An example of quantitative description of quark confinement was given in the models \([1]-[2]\) where the evolution in space and time for quark and antiquark at the endpoints of string occurs according to classical string theory and the quark-antiquark colourless system is confined by constant magnitude force which determines the breakpoint of string. The other description for evolution of same colourless system of confined quarks is given in the models \([3]-[5]\) where already the momentum energy representation and scaling distributions\(^1\) are used to describe the string breaking. In this fragmentation procedure there is the parton approach, but in that way a quark confinement means also that an interaction has to be between the quark and antiquark at the endpoints of string. So, we are going now to construct a Monte Carlo model which takes into account a quantitative description for interaction between the quark and antiquark at the endpoints of string according to concepts of the models \([3]-[5]\). It has to be stressed, our new point of view on the process is that we suppose the interaction between the both endpoints of string precedes each break of string into daughter string and hadron\(^2\).

Let us remind the Monte Carlo string breaking scheme exploited in well-known models \([3]-[5]\) where a conversion of quark \(q\), which is endpoint of primary string, into hadrons \(h\) (Fig.1a) is

\(^1\)i.e. these distributions do not depend on the energy of the (anti)(di)quark at the endpoint of string.

\(^2\)or into two daughter hadrons
an iteration of an elementary process presented in Fig.1b: quark q is converted into hadron h and quark q’ via the vacuum pair production. Therefore the produced q’ does not lie on the mass shell and so in the next conversion presented in Fig.1c (q’ → h + q’’) the virtuality of quark q” rises above the virtuality of quark q’, i.e. the virtuality of each produced quark grows with the number of conversions presented in Fig.1a. It does not take into account in the models [3]-[5] where the scaling distributions for the fraction z of the longitudinal and transverse momentum projections carried by hadron h do not depend on virtuality of quark q (Fig.1b) converted into daughter hadron h and quark q’. However, as it is shown in Fig.2, the produced quark q’ (diquark qq’) has wide distribution for its mass squared invariant $m^2_{qq'} = E^2_{q'} - P^2_{q'}$ in negative region even if quark q (see Fig.1b) lies on the mass shell in each its conversion into daughter hadron h and quark q’ (diquark qq’). For example, if quark q is on the mass shell in each conversion (Fig.1b), the average values of the quark mass squared invariants $m^2_{u(qq')} = E^2_u - P^2_u$ for daughter u-quark, s-quark and diquark produced in the processes $q → h + q'_u$, $q → h + q'_s$ and $q → h + qq'$ are $(-0.724 \pm 0.009) GeV^2$, $(-1.020 \pm 0.012) GeV^2$ and $(-1.940 \pm 0.022) GeV^2$ respectively.

Furthermore, in the fragmentation schemes of the Monte Carlo models [3]-[5] two partons at the ends of primary string are con-

3 In the Monte Carlo models [3]-[5] the energy and momentum vector of produced quark q’ are determined as the differences between energies and momentum vectors of q and h.

4 In the models [3]-[5] there is not conversion of quark (into hadron and quark), if it has the negative energy. Therefore we can choose the quark mass squared invariant $m^2_q = E^2_{quark} - P^2_{quark}$ as a quantitative characteristic for virtuality of quark.

5 In the model [4] the fraction of longitudinal end transverse energy carried by hadron h are generated instead of fraction of longitudinal end transverse momentum.

6 These numbers will move farther into negative region, if quark q does not lie on the mass shell in the conversion $q → h + q'$. 

3
verted into two jets of hadrons independently (Fig.1d), i.e. any interaction between partons at the ends of each daughter string does not take into account. Therefore we try to describe here a Monte Carlo version for these interactions included in the model without contradiction to a set of concepts of the string model formulated by B. Andersson and A. Nilsson in the paper [6], which are:

\[ \text{quarks, } q, \text{ and antiquarks, } \bar{q}, \text{ are treated as excitations at the endpoints of the force field and gluons, } g\text{'s, are internal excitations. The massless relativistic string is used as a (semi-classical) model for the force field. The above-mentioned excitations move like localised particles carrying energy momentum.} \]

Even for that quasi-classical approximation the string interaction is realised by gluons, \( g \text{'s, which move like localised particles carrying energy momentum between the endpoints of string. It is like a classical parton model approximation} \] and it gives us a way to construct a some model for confinement interaction. Therefore let us suppose the following scenario for confinement interaction and string breaking. The endpoints of string are quark, \( q' \text{' and antiquark, } \bar{q}. \text{ They interact each other via gluon exchange. The gluons, } g\text{'s, carry energy momentum, so the energy momentum of partons at the endpoints of each daughter string is changed. As a result both partons turn out to be on the mass shell} \]

\[ \text{7where quarks and gluons are localised particles carrying some parts of energy-momentum of hadron} \]

\[ \text{8If one of them had been produced from the vacuum (} q' \text{‘ in Fig.1b), it would not have lain on the mass shell (Fig.2) before gluon exchange (circle in Fig.3b) occurs between both partons (} q' \text{‘ and } \bar{q} \text{ in Fig.3b) at the endpoints of string.} \]
to find their new energies and momenta in the rest frame of the string where then a conversion (for example\footnote{The conversions of quark $q' \to h + q''$ and antiquark $\bar{q} \to h + \bar{q'}$ at the endpoints of string are generated with equal probabilities.}) of quark $q' \to h + q''$ is generated. This algorithm for produced daughter string tensed already by antiquark $\bar{q}$ and quark $q''$ produced from vacuum is repeated up to the mass of string becomes low (see Sect.4) and decay of string into two hadrons occurs. In frame of this scenario (anti)(di)quark (parton at the endpoint of string) turns out to be on the mass shell before it is converted into hadron and quark (Fig.1b). So, there is not any problem connected with virtuality and simultaneously we take into account the interaction between quark and antiquark at the endpoints of each daughter string produced during cascade decay (Fig.3b) of primary string.

The mechanism for string breaking is visioned via the tunneling of $q' - \bar{q'}$ pair from vacuum (Fig.1b). Usually the Gauss law is used to describe the distribution for the transverse momentum projections $p_x, p_y$ of vacuum quark. So, if we use the Gaussian distribution which has zero center parameter\footnote{i.e. the summary transverse momentum of the vacuum quark-antiquark pair is equal to zero.}, we have to generate $p_x$ and $p_y$ of vacuum quark $q'$ in the system where the transverse momentum of converted quark $q$ (Fig.1b) is zero\footnote{it is the $K_{decay}$ frame determined in Sect.4}. To generate the $x$- and $y$-projections for $u(d)$-quark, $s$-quark and di-quark from vacuum pair we use, for Gaussian distribution, the widths

$$
\sigma_{u(d)}^{\text{vac}} = 0.15 \text{ GeV}, \quad \sigma_s^{\text{vac}} = 0.25 \text{ GeV}, \quad \sigma_{\text{diquark}}^{\text{vac}} = 0.35 \text{ GeV} \quad (1)
$$

respectively.

The other process is gluon exchange between the quarks at
the endpoints of string. The gluons carry also the transverse momentum \( p_T \). The Gauss law usually is used to describe the distributions of the \( p_{Tx} \) and \( p_{Ty} \) projections in the non-perturbative process, if the transverse momentum exchanges are considered as uncorrelated. However the string interaction is gluon exchange between partons having long tail distributions for (di)quark mass squared invariant \( m_{q^*(qq')}^2 \) (see Fig.2). Therefore if we use the Gauss law to describe \( p_T \) distribution for the gluon exchange process\(^{12}\), the Gauss width parameter might be connected with spacelike virtuality of partons at the endpoints of string. It seems to be reasonable. Namely, gluon exchange between partons, having big spacelike virtuality (Fig.2), can lead to different \( p_T \) because after interaction each other the both partons turn out to be on the mass shell. However, the essence of the parton model is to regard a high energy cluster (hadron, string) as a collection of quasifree partons. So, if both partons at the ends of string are on the mass shell, the transferred \( p_T \) of gluons has to be zero. The variable which has the similar properties is the difference between the quark mass squared invariant \( m_{q^2} = E_{\text{quark}}^2 - \vec{P}_{\text{quark}}^2 \) and the fixed quark mass squared \( m_{q}^2 \) taken from \(^{3}\) at the given type of quark \( q \). This difference equals to zero (i.e. \( m_{q^2} = m_{q}^2 \)), if quark is on the mass shell, and so the variable\(^{13}\)

\[
\sigma_q = \left| m_{q^2}^* - m_{q}^2 \right|^{0.5}
\]

(2)

could be chosen as the width parameter for Gaussian distribution of \( x-, y\)-projections of parton transverse momentum after gluon exchange. Of course, this approximation is discussionable, so in

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\(^{12}\) in string rest frame where quark and antiquark at the endpoints of string have zero transverse momenta and move in opposite directions (it is \( K_{\text{exchange}} \) frame determined in Sect.3).

\(^{13}\) Here \( q \) is parton which does not lie on the mass shell before gluon exchange occurs.
our calculations we chose more simple approximation. Instead of the variable $m_q^2$ we took its average values $< m_q^2 >$ given above. Namely, instead of eq.(2) we used

$$\sigma_q = |< m_q^2 > - m_q^2 |^{0.5} ,$$

(3)

where $< m_q^2 >$ equals to $(-0.724 \pm 0.009) GeV^2$, $(-1.020 \pm 0.012) GeV^2$ and $(-1.940 \pm 0.022) GeV^2$ for $u(d)$-quark, $s$-quark and diquark respectively. Now from eq.(3) one can get

$$\sigma_{u(d)} = 0.92 GeV, \quad \sigma_s = 1.13 GeV, \quad \sigma_{diquark} = 1.48 GeV .$$

(4)

These values, describing the $p_T$ distribution of quark after gluon exchange, are in a few times more than the same values (see (1)) describing the $p_T$ distribution of quark from vacuum pair. Therefore in our calculations we put the relations

$$\sigma_{u(d)} = 3 \cdot \sigma_{u(d)}^{vac} , \quad \sigma_s = 3 \cdot \sigma_s^{vac} , \quad \sigma_{diquark} = 3 \cdot \sigma_{diquark}^{vac} .$$

(5)

2 The formation of primary strings

The initial stage of soft hadron-hadron collision is the formation of colourless strings. In our calculations we use the Dual Parton Model (DPM) \cite{7-10} to form the primary strings. In DPM the primary colourless strings are stretched by partons of target and projectile (see Fig.4). In our model the number $l$ of pair of primary strings is generated firstly according to theoretical

\footnote{A (d)quark at the endpoint of string occurs from the vacuum pair where usual relation between quark flavours is $u : d : s = 3 : 3 : 1$, and so the average mass of diquark is 0.49 GeV. To calculate this average value we put $m_{ud} = 0.45 GeV$, $m_{ds} = 0.62 GeV$, $m_{ss} = 0.79 GeV$.}

\footnote{In the presented model we do not take into account the (semi)hard interactions between the partons of projectile and target.}
distribution \[7\] (see Appendix 1). The flavours of sea quarks in projectile and target (see Fig.4) and the flavours of quarks, produced from vacuum at the decay of string (see Sect.4), are generated according to relation

\[ u : d : s = 3 : 3 : 1. \]  

(6)

The initial multidimensional distribution of partons on the part \(x_i\) of longitudinal momentum of projectile in the centre mass system is given by \[8\], \[10\]

\[
x_i^{-0.5} \left( x_2^2 + \mu_2^2 / P^2 \right)^{-0.5} \ldots \left( x_{2l-1}^2 + \mu_{2l-1}^2 / P^2 \right)^{-0.5} x_{2l}^{1.5} \ dx_1 \ dx_2 \ldots \ dx_{2l}
\]

(7)

at \(0 \leq x_i \leq 1\), where \(P\) is the momentum of proton in c.m.s.; indeces 1 and \(2l\) are related to the valence quark and diquark correspondingly, the remaining indices are related to sea quarks and antiquarks; \(\mu_i\) is the mass of \(i\)-th quark or antiquark (\(\mu_u = \mu_d = 0.34\) GeV, \(\mu_s = 0.51\) GeV). The \(x'_i\)-values of target partons are generated identically. For realization of longitudinal momentum conservation after \(x_i\) and \(x'_i\) generation we substitute

\[
x_i / \sum_{j=1}^{2l} x_j \rightarrow x_i , \quad x'_i / \sum_{j=1}^{2l} x'_j \rightarrow x'_i .
\]

(8)

Taking into consideration the energies of projectile quark (antiquark) \(\varepsilon_i = (\mu_i^2 + x_i^2 P^2)^{0.5}\) and target quark (antiquark) \(\varepsilon'_i = (\mu_i'^2 + x'_i^2 P^2)^{0.5}\), one can find the energy \(E_i\), longitudinal momentum \(P_i\) and mass \(M_i\) of every \(i\)-th string. The transverse momenta of partons are not accounted for. Then we control for every \(i\)-th string the fulfilment of inequalities
\[ E_i > |P_i|, \quad M_i > \mu_{i1} + \mu_{i2}, \quad M_i > M_{i1}^{\text{min}} + M_{i2}^{\text{min}}, \quad (9) \]

where \( \mu_{i1} \) and \( \mu_{i2} \) are masses of the partons, stretching the string, and the \( M_{ij}^{\text{min}} \) is minimum mass of the \( j \)-th hadron with the fixed quark composition, which can be produced by decay of the \( i \)-th string\[16\]. The Monte Carlo event is rejected, if only one inequality in eq.(9) is not fulfilled. The share of rejected events depends on the \( l \) value, so the distribution of unrejected events over \( l \) differs from the initial distribution \[7\]. However the share of rejected MC events grows with the number \( l \) of primary strings. Therefore the difference between the theoretical distribution and the distribution for Monte Carlo events on \( l \) is only for large \( l \) which have a little share (about a few percents) in theoretical distribution. So, for a first approximation the both distributions are same\[17\] each other.

\section{Gluon exchange}

There are many feasible ways to calculate the gluon exchange between both partons at the ends of string. One of them is given in this Section.

Before every break, for example, of secondary string of mass \( M \), occurs, we know following its characteristics (in the \( K_0 \) c.m.s. of antiproton-proton collisions): the scheme of the decay of secondary string (see Sect.4 and Figs.3,5), the flavours of the partons participating in this decay, the mass \( M \), the energy \( E_0 \) and the

\[16\]See Sect.4.
\[17\]For example, the share of rejected Monte Carlo events is 0.10 at \( \sqrt{s}=27 \text{ GeV} \) and 0.03 at \( \sqrt{s}=540 \text{ GeV} \).
momentum vector $\vec{P}_0$ of the string and the energies $\varepsilon_{01}$, $\varepsilon_{02}$ and momentum vectors $\vec{p}_{01}$, $\vec{p}_{02}$ of first and second partons\[18\] stretching the string\[19\].

As it was discussed in Sect.1, it is convenient way to generate the gluon exchange in the string rest frame. So, the kinematical characteristics of both partons should be transformed from $K_0$ to string rest frame. It is described below in details:

**Transformation** from $K_0$ to $K_L$ frame where the longitudinal momentum $P_{z0}$ of string is equal zero:

This $K_L$ frame moves with $\beta_{z0} = P_{z0}/E_0$ velocity along the $z_0$ axis in the $K_0$ frame and with Lorentz-factor $\gamma_0 = E_0/M_\perp = E_0/(E_0^2 - P_{z0}^2)^{0.5}$, where $M_\perp = (M^2 + P_{\perp0}^2)^{0.5}$ is transverse mass of string.

**Transformation** from $K_L$ to $K'$ frame where the longitudinal momentum $P_{z0}$ of string is equal zero and $x'$ axis is along the transverse momentum vector of string:

In the $K_L$ frame, the string has transverse momentum $\vec{P}_{\perp0}$ with projections $P_{x0}$ and $P_{y0}$. We turn the $K_L$ frame (in the transverse $xy$ plane) to obtain the $x_L$ axis along the vector $\vec{P}_{\perp0}$. The parameters for this transformation are $\cos \varphi_\perp = P_{x0}/P_{\perp0}$, $\sin \varphi_\perp = P_{y0}/P_{\perp0}$.

**Transformation** from $K'$ to $K_{\text{rest}}$ frame where the momentum\[18\]One of them can have a big specelike virtuality.\[19\]Of course, the energies and momentum vectors of partons determine the mass $M$, the energy $E_0$ and the momentum vector $\vec{P}_0$ of the string.
of string is equal zero:

Let the $K'$ frame move with $\vec{\beta}_{\perp 0} = \vec{P}_{\perp 0}/M_{\perp}$ velocity and with Lorentz-factor $\gamma_{\perp 0} = E_{\perp 0}/M = M_{\perp}/M$, where $E_{\perp 0}$ is energy of string in $K_L$ frame. Thereby we obtain the $K_{\text{rest}}$ rest frame of the string.

Transformation from $K_{\text{rest}}$ to $K_{\text{exchange}}$ frame where the $z_{\text{exchange}}$ axis is along one of two parton momentum vectors which are before gluon exchange occurs:

The momentum vectors of partons are transformed from $K_0$ to $K_{\text{rest}}$ frame. The longitudinal $z_{\text{rest}}$ axis of $K_{\text{rest}}$ frame is parallel to the longitudinal $z_0$ axis of $K_0$ c.m.s. of both collising particles. We turn the $K_{\text{rest}}$ frame to obtain the $z_{\text{exchange}}$ axis along to the momentum vector of one of two partons and the $y_{\text{exchange}}$ axis along to the vector product $\vec{z}_{\text{exchange}} \times \vec{z}_{\text{rest}}$. Thereby we obtain the $K_{\text{exchange}}$ frame where we can determine the new parton momentum vectors arising after gluon exchange, which leads to transition of two partons on the mass shell (see Sect.1).

At the transition of both partons on the mass shell the modulus of momentum of the parton in the $K_{\text{exchange}}$ frame changes. The energies of the partons in the rest frame of string $K_{\text{exchange}}$ become equal to

$$
\varepsilon_{1\text{exchange}} = \frac{M^2 + \mu_1^2 - \mu_2^2}{2M}, \quad \varepsilon_{2\text{exchange}} = \frac{M^2 + \mu_2^2 - \mu_1^2}{2M}, \quad (10)
$$

\footnote{The parameters for transformation of momentum projections from $K_{\text{exchange}}$ to $K_{\text{rest}}$ frame are given in Appendix 2.}
where $\mu_1$ and $\mu_2$ are the masses of partons. The diquark masses: $m_{du} = 0.45\text{GeV}$, $m_{us} = 0.62\text{GeV}$, $m_{ss} = 0.79\text{GeV}$ \cite{3} are used.

As it has been discussed above in Sect.1, the Gauss parameter should be connected with space like virtuality of parton, for example it might be the law similar to one given by (2). However, in our calculations we have taken an approximation (5). It means that Gauss parameter should be connected with flavour of parton which does not lie on the mass shell before gluon exchange. Therefore the $p_{x1}^{\text{exchange}}$ and $p_{y1}^{\text{exchange}}$ components of the transverse momentum of one of two partons in the rest frame of string $K_{\text{exchange}}$ are generated according to Gauss law

$$r_1(r_2) = \sigma^{-1}(2\pi)^{-0.5} \int_{-\infty}^{\infty} p_{x}^{\text{exchange}}(p_{y}^{\text{exchange}}) \exp(-x^2/2\sigma^2) \; dx, \quad (11)$$

where $r_i$ is uniformly distributed random number in the range $(0,1)$ and $\sigma$ is parameter, which is $\sigma_u(d)$, $\sigma_s$ or $\sigma_{\text{diquark}}$ from (5).

Now one can determine new longitudinal projections of the parton momentum vectors in the rest frame of string $K_{\text{exchange}}$ and transform the momentum vectors of both partons from $K_{\text{exchange}}$ to $K_{\text{rest}}$ (see Appendix 3).

4 The decay of string

We follow the models \cite{3}-\cite{5} and suppose that the cascade breaking of string is described by the iteration of decay of string into hadron and string (Fig.3), i.e. by the iteration of conversion of (di)quark into hadron and (di)quark (see Fig.5) via vacuum pair production.

As it has been discussed in Sect.1, we should generate the
transverse projections $p_x$ and $p_y$ of vacuum quark $q'$ in the system where the transverse momentum of parent quark $q$ (Fig.1b) is zero. So, the kinematical characteristics of both partons should be transformed from $K_{rest}$ to this ”decay” frame. The possible way to do that is :

*Transformation* from $K_{rest}$ to $K''$ frame where $x''$ axis is along the transverse momentum vector of quark $q$ in the $K_{rest}$ system :

In the $K_{rest}$ frame, the quark $q$ has transverse momentum $\vec{p}_{\perp rest}$ with projections $p_x^{rest}$ and $p_y^{rest}$. We turn the $K_{rest}$ frame (in the transverse $xy$ plane) to obtain the $x_{rest}$ axis along the vector $\vec{p}_{\perp rest}$. The parameters for this transformation are $\cos \varphi = \frac{p_x^{rest}}{p_{\perp rest}}$, $\sin \varphi = \frac{p_y^{rest}}{p_{\perp rest}}$.

*Transformation* from $K''$ to $K_{decay}$ frame where the transverse momentum of quark $q$ is equal zero :

Let the $K''$ frame move with $\vec{\beta}'' = \vec{p}_{\perp rest}/\varepsilon''$ velocity where $\varepsilon''$ is the energy of quark $q$ in $K''$ ($K_{rest}$) frame. Thereby we obtain the $K_{decay}$ frame, where the conversion $q \rightarrow h + q'$ is generated and where the transverse momentum of quark $q$ is equal zero, and so we are sure that the functions, describing this conversion, can not depend on the transverse momentum of quark $q$.

Three schemes of conversions of the (di)quark into hadron and (di)quark are taking into consideration in our modeling (see Fig.5). The $a$ and $b$ types of conversions is generated according to the relation $a : b = 0.85 : 0.15$.

The flavours of quarks, produced from vacuum at the decay of

\footnote{It might be either of the two partons taken with equal probability.}
string, are generated according to the relation (6) \textsuperscript{3}. 

In Monte Carlo simulation of decay (conversion) of a parton into another parton and hadron (Fig.1b) we have to take into account that fixed quark content of hadron might correspond to a few types of particles. Therefore there are some decay modes for the given quark content of hadron. We attribute a weight to each decay mode. This weight is equal to the product of two factors. The spin factor is equal to \((2J + 1)\), where \(J\) is spin of the hadron. The \(SU_3\)-factor is taken into consideration, if there are several hadrons with the same quark content, spin and parity. For example, \(SU_3\)-factor of \(\eta\)-meson which is formed from \(\bar{u}u\)-pair is equal to \(1/6\), and the same \(SU_3\)-factor of \(w\)-meson is equal to \(1/2\).

If the hadron \(h\) is resonance, its mass is generated, after the generation of the decay mode, according to the Breit-Wigner distribution.

In the \(K_{\text{decay}}\) frame the \(P_{x \text{ had}}^{\text{decay}}\) and \(P_{y \text{ had}}^{\text{decay}}\) components of the transverse momentum \(p_{\perp \text{ had}}^{\text{decay}}\) of daughter hadron \(h\) are generated according to the Gauss law (see Sect.1)

\[
r_1(r_2) = \sigma^{-1}(2\pi)^{-0.5} \int_{-\infty}^{P_{x \text{ had}}^{\text{decay}}(P_{y \text{ had}}^{\text{decay}})} \exp\left(-x^2/2\sigma^2\right) \, dx , \quad (12)
\]

where \(r_i\) is uniformly distributed random number in the range \((0,1)\) and \(\sigma\) is parameter taken from \((1)\).

At the conversion of parton \(q\) (Figs.1b,3a) of the string \(S\) (Fig.3a) into daughter hadron \(h\) in the \(K_{\text{decay}}\) system we generate the convenient Lorentz-invariant variable

\[
z = \frac{E_{\text{had}}^{\text{decay}} + P_{z \text{ had}}^{\text{decay}}}{\varepsilon_{\text{parton}}^{\text{decay}} + P_{z \text{ parton}}^{\text{decay}}} . \quad (13)
\]
In (13) $E_{\text{had}}^{\text{decay}}$ and $P_{z \text{had}}^{\text{decay}}$ are the energy and the momentum projection (on $z_{\text{decay}}$ axis) of daughter hadron, $\varepsilon_{\text{parton}}^{\text{decay}}$ and $P_{z \text{parton}}^{\text{decay}}$ are the same values of the parton.

The distribution density of $z$ variable is parameterized by the form

$$F(z) \propto z , \quad 0 < z < 1 . \quad (14)$$

After determination of the momentum and energy of $q'$ we know all kinematical characteristics of both partons ($\bar{q}$ and $q'$) at the ends of daughter string $S'$ (Fig.3a), and so we can find its energy $E'$ and momentum vector $\vec{P}'$ and control the inequality

$$E' > P' . \quad (15)$$

If the inequality (15) is fulfilled for daughter string $S'$, we determine its mass $M' = \sqrt{E'^2 - P'^2}$ and control the inequality

$$M' > \mu_1 + \mu_2 , \quad (16)$$

where $\mu_1$ and $\mu_2$ are fixed (di)quark masses (taken from [3]) of the partons ($q'$ and $\bar{q}$ in Fig.3a), tensing the daughter string ($S'$ in Fig.3a).

If the inequality (16) is fulfilled, we control the possibility to be decay of daughter string into two hadrons. So, the decay scheme of daughter string $S'$ (Fig.6) into two hadrons and the flavours of quarks, produced in this decay from vacuum are generated.

Then we control the inequality

$$M' > M_1^{\text{min}} + M_2^{\text{min}} , \quad (17)$$

where $M_1^{\text{min}}$ and $M_2^{\text{min}}$ are minimum masses of the 1-st and 2-nd hadrons with the fixed quark composition, which can be produced by decay of the daughter string $S'$.

\[22\] The decay scheme (Fig.6) is generated according to the relation $a : b = 0.85 : 0.15$. 

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If the inequalities (15)-(17) are fulfilled, the energies and momentum vectors of daughter hadron and partons of daughter string are transformed from the $K_{\text{decay}}$ to $K_0$ frame.

If one of three inequalities (15)-(17) is not fulfilled, instead of the decay of mother string $S$ into daughter string and hadron (Fig.3a), the decay of mother string into two hadrons (Fig.6) is generated in the $K_{\text{rest}}$ frame. In that case there are some decay modes for the given quark content of these hadrons as it was already discussed above.

The $M_1$ and $M_2$ masses of resonances are generated after the generation of the decay mode. For example, if string decays into two resonances and the mass of first resonance must be generated at first, the $M_1$ and $M_2$ masses can be generated according to the Breit-Wigner distributions in intervals

$$M_1^{\text{min}} < M_1 < M - M_2^{\text{min}},$$

$$M_2^{\text{min}} < M_2 < M - M_1,$$

where $M_i^{\text{min}}$ is maximum sum of masses of particles produced by the decay of $i$-th resonance. In the $K_{\text{rest}}$ frame the square of transverse momentum of resonance $(P_{\text{rest}}^\perp)^2 = (P_{x1}^{\text{rest}})^2 + (P_{y1}^{\text{rest}})^2$ is generated in the interval $(0, P^2)$, and the azimuthal angle is generated uniformly in the interval $(0, 2\pi)$. The momentum of resonance is supposed to have the sharp angle with momentum vector of its parent parton. The decays of unstable hadrons into stable and quasistable particles are generated, for example, according to algorithm given in the Appendix 2 from Ref. [11]. The momenta of stable and quasistable particles are transformed from the $K_{\text{rest}}$ to $K_0$ frame.
The decay of the daughter string is generated also as the decay of the mother string. The cascade is generated up to the decay of all strings into hadrons.

5 Conclusion

The string decay is based on the vacuum pair production which could be understood as a result of the confinement interaction. A quantitative description for quark confinement was given in the models [1]-[2] of multiparticle production where the quark-antiquark colourless system is confined by constant magnitude force which determines the breakpoint of string. Here we try to construct a possible scenario for quantitative description of the quark confinement interaction which is reason for vacuum pair production in the widely explored process of conversion of quark $q$ into hadron $h$ and quark $q'$ (Fig.1b). In other words, instead of the process described by the diagram $\text{23}$ in Fig.1d we use the diagram in Fig.3 where we add an interaction (marked by the circle) to the process of conversion $q \rightarrow h + q'$ which is seemed as a result of the confinement interaction. To describe the confinement interaction we try to use an essential idea of the parton model which regards a high energy cluster (hadron,string) as a collection of quasifree partons, and we try to follow some concepts of the string model formulated in [3] where the gluons move like localised particles carrying energy momentum between quark and antiquark at the endpoints of string, and so we take into account they go on the mass shell before the break of daughter string occurs (Fig.3). In the frame of that scenario there is not any problem connected

\footnote{It is an independent conversion of both partons $q$ and $\bar{q}$ at the ends of string into two jets of hadrons $h$.}
with virtuality\textsuperscript{24} and simultaneously we try to take into account the interaction between the partons at the ends of each string (colourless system). At the gluon exchange the gluons carry also transverse momentum which could be connected with spacelike virtuality of quark (Fig.2) at the end of string. To describe the transverse \(x\)- and \(y\)-projections of parton momentum at string rest frame (see Sect.3) after gluon exchange we choose\textsuperscript{25} the Gauss law and an approximation (5).

Some results of Monte Carlo calculations for antiproton-proton interactions are given in Fig.7, where the transverse momentum distribution for the secondary Monte Carlo particles depends on the energy of interaction. It is because of kinematical correlations between the ends of string (confinement interaction). Namely, in frame of presented scenario, there are two Gauss laws which describe the distributions for the transverse momentum projections in two different processes. The first Gauss law is applyed to the processes given in Figs.5,6 (the vacuum pair production). The second one is applyed to the gluon exchange (circle in Fig.3). At the small energy of interaction of primary proton and antiproton the most part of number of all primary strings (Fig.4) have the decays directly into two hadrons (Fig.6). If the c.m. energy \(\sqrt{s}\) grows, the average number of gluon exchanges (confinement interactions) grows with the number of cascade decays (Fig.3) of each primary string (Fig.4). According to the approximation (5), that means, more big \(p_T\) of particles occur.

\textsuperscript{24}The quark \(q'\) is produced from vacuum and so even if a quark \(q\) (Fig.1b) is on the mass shell, the quark \(q'\) has wide distributed spacelike virtuality (see Fig.2) in its conversion \(q' \rightarrow h + q''\) in the models based on the diagram shown in Fig.1d.

\textsuperscript{25}See Sect.1
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Appendix 1

The inelastic nondiffractive cross section corresponding to formation of \( l \) pairs of primary quark-gluon strings at the energy \( \sqrt{s} \) in c.m.s. has been taken from \[7\] :

\[
\sigma_l = \frac{\sigma_P}{l_z} \left( 1 - \exp(-z) \sum_{k=0}^{l-1} \frac{z^k}{k!} \right),
\]

where

\[
\sigma_P = 8\pi\gamma \exp(\varepsilon\Delta), \quad \gamma = 3.64\text{GeV}^{-2}, \quad \varepsilon = \ln(s/s_0), \quad s_0 = 1\text{GeV}^2, \quad \Delta = 0.068, \quad z = 2c\gamma \exp(\varepsilon\Delta)/(R^2 + \alpha'_P\varepsilon), \quad c = 1.5, \quad R^2 = 3.56\text{GeV}^{-2}, \quad \alpha'_P = 0.25\text{GeV}^{-2}.
\]

In calculation of probabilities \( W_l = \sigma_l/\sigma \) the inelastic nondiffractive cross section \( \sigma \) has been determined by formula

\[
\sigma = \sigma_P \sum_{\nu=1}^{\infty} \frac{(-z)^{\nu-1}}{\nu \cdot \nu!},
\]

and at any energy \( \sqrt{s} \) in c.m.s. it is taken into account up to \( l=15 \) pairs of primary strings.

Appendix 2

The parameters to transformation of momentum projections from \( K_{\text{exchange}} \) to \( K_{\text{rest}} \) frame are
\[
\cos \theta = \frac{-p_z^{\text{rest}}}{\sqrt{(p_x^{\text{rest}})^2 + (p_y^{\text{rest}})^2 + (p_z^{\text{rest}})^2}},
\]

\[
\cos \varphi = \frac{-p_x^{\text{rest}}}{\sqrt{(p_x^{\text{rest}})^2 + (p_y^{\text{rest}})^2}},
\]

\[
\sin \varphi = \frac{-p_y^{\text{rest}}}{\sqrt{(p_x^{\text{rest}})^2 + (p_y^{\text{rest}})^2}},
\]

where \(p_x^{\text{rest}}, p_y^{\text{rest}}, p_z^{\text{rest}}\) are the momentum projections (in the \(K^{\text{rest}}\) frame) of one of two partons before gluon exchange.

**Appendix 3**

New parton momentum projections, arising after gluon exchange, are transformed from the \(K_{\text{exchange}}\) to \(K_{\text{rest}}\) frame according to formulas

\[
(p_x^{\text{rest}})^{\text{new}} = -p_x^{\text{exchange}} \cos \theta \cos \varphi - p_y^{\text{exchange}} \sin \varphi - p_z^{\text{exchange}} \sin \theta \cos \varphi,
\]

\[
(p_z^{\text{rest}})^{\text{new}} = p_x^{\text{exchange}} \sin \theta - p_z^{\text{exchange}} \cos \theta,
\]

\[
(p_y^{\text{rest}})^{\text{new}} = -p_x^{\text{exchange}} \cos \theta \sin \varphi + p_y^{\text{exchange}} \cos \varphi - p_z^{\text{exchange}} \sin \theta \sin \varphi,
\]

where \(p_x^{\text{exchange}}, p_y^{\text{exchange}}, p_z^{\text{exchange}}\) are the momentum projections of one of two partons arising after gluon exchange in the \(K_{\text{exchange}}\) frame, and \(\theta, \varphi\) angles are determined in Appendix 2.
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Figure Captions

Fig.1. a The diagram of conversion of the fast quark $q$ into hadrons $h$ in the models [3]-[5]. b. An elementary Monte Carlo process of the models [3]-[5]: $q \rightarrow h + q'$. c. An elementary Monte Carlo process $q' \rightarrow h + q''$. d. The diagram of independent conversion of the quark and antiquark at the ends of string into two jets of hadrons in the models [3]-[5]. The double line is the diquark.

Fig.2 a,b. The distributions for the mass squared invariant $m_{q'}^2 = E_{q'}^2 - \vec{P}_{q'}^2$ of $q'_u$-quark (a), $q'_s$-quark (b) produced in the process $q \rightarrow h + q'$, if quark $q$ (see Figs.1b,5a) lies on the mass shell in each its conversion into daughter hadron $h$ and quark $q'$. c. The distribution for the mass squared invariant $m_{qq'}^2 = E_{qq'}^2 - \vec{P}_{qq'}^2$ of diquark $qq'$ produced in the process presented in Fig.5b, if quark $q$ lies on the mass shell in each its conversion into daughter hadron $h$ and diquark $qq'$.

Fig.3 a The diagram of the gluon exchange (marked by circle) between the quark $q$ and antiquark $\bar{q}$ at the ends of secondary string $S$ which then has decay into daughter hadron $h$ and string $S'$ via conversion $q \rightarrow h + q'$. b The diagram of the process $S' \rightarrow S'' + h$ which is like one given in a.

Fig.4 The formation of four ($l=2$) primary strings $S_1$, $S_2$, $S_3$, $S_4$ by partons of projectile and target in inelastic $\bar{p}p$-interaction. The double line is the diquark.
Fig. 5  Three schemes of conversion of the quark into meson and quark (a), quark into baryon and diquark (b), diquark into baryon and quark (c), via vacuum pair production. The double line is the diquark.

Fig. 6  Three schemes of decay of string into two daughter hadrons $h_1$ and $h_2$ via vacuum pair production. The double line is the diquark.

Fig. 7  Transverse momentum distribution for charged particles produced in $p\bar{p}$-interactions at $\sqrt{s}=546$ GeV. Experimental data are from [12].
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