The Consistent Result of Cosmological Constant From Quantum Cosmology and Inflation with Born-Infeld Scalar Field

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Abstract

The Quantum cosmology with Born-Infeld(B-I) type scalar field is considered. In the extreme limits of small cosmological scale factor the wave function of the universe can also be obtained by applying the methods developed by Hartle-Hawking(H-H) and Vilenkin. H-H wave function predicts that most Probable cosmological constant \(\Lambda\) equals to \(\frac{1}{\eta}\) (\(\eta\) equals to the maximum of the kinetic energy of scalar field). It is different from the original results(\(\Lambda = 0\)) in cosmological constant obtained by Hartle-Hawking. The Vilenkin wave function predicts a nucleating universe with largest possible cosmological constant and it is larger than \(1/\eta\). The conclusions have been nicely to reconcile with cosmic inflation. We investigate the inflation model with B-I type scalar field, and find that \(\eta\) depends on the amplitude of tensor perturbation \(\delta_h\), with the form \(\eta \simeq \frac{m^2}{12\pi \left(\frac{\Phi}{\delta_h}\right)^2 - 1}\). The vacuum energy in inflation epoch depends on the tensor-to-scalar ratio \(\frac{\delta_h}{\delta}\). The amplitude of the tensor perturbation \(\delta_h\) can, in principle, be large enough to be discovered. However, it is only on the border of detectability in future experiments. If it has been observed in future, this is very interesting to determine the vacuum energy in inflation epoch.

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1 Introduction

Astronomical observations indicate that the cosmological constant is not zero and it has the same order of magnitude as matter energy density (density parameter $\Omega_\Lambda$ of the vacuum energy $\sim 0.73$).

Before 1998 year a crude experimental upper bound on $\Lambda$ or vacuum energy density $\rho_V$ is provided as $\rho_V \lesssim 10^{-29} g/cm^3 \sim 10^{-47} GeV^4$[1]. However when the universe approximates to planck scale the energy difference between the symmetry and broken symmetry phase of vacuum is $10^{18} GeV$. The effective vacuum energy density $(10^{18} GeV)^4$ exceeds observational limit by some 120 orders of magnitude. There are many number of symmetries which seem to be broken in the present universe, including chiral symmetry, electroweak symmetry and possibly supersymmetry. Each of these would give a contribution to $\rho_V$ that would exceed the upper limit by at least forty orders of magnitude. It is very difficult to believe that the cosmological constant is fine tune so that after all the symmetry breakings, the effective vacuum energy density satisfies the upper bound. What one would like to is some mechanism by which the cosmological constant $\Lambda$ could relax to near zero. Weinberg has described five approaches to find such a mechanism, including Anthropic considerations, superstrings and supersymmetry, Adjustment Mechanisms, changing Gravity, Quantum Cosmology. At present, the approaches which based on quantum cosmology is most promising[1]. In 1984 Hawking described how in quantum cosmology there could arise a distribution of values for the cosmological constant. Hawking introduces a 3-form gauge field $A_{\mu\nu\eta}$ or scalar field $\phi$. According to the general ideas of Euclidean quantum cosmology, he obtained that probability density is proportional to $e^{3\pi/G\Lambda}$. The probability density has an infinite peak for $\Lambda \rightarrow 0^+$. The most probable cosmological constant will be those with very small values[2]. Coleman considers the effect of topological fixtures known as wormholes. He argued that if wormholes exist, they have the effect of making the cosmological constant vanish[3]. However these conclusions have been hard to reconcile with cosmic inflation[4].

Recently, the problem of cosmological constant based on quantum cosmology has been investigated by many authors. Kalinin and Melnikov discuss quantization of closed isotropic cosmological model with a cosmological constant which is realized by the Wheeler-Dewitt (WDW) equation. It is shown that such quantization leads to interesting results, in particular, to a finite lifetime of the system, and appearance of the universe as penetration via the barrier. These purely quantum effects appear when cosmological constant is larger than zero[5]. Capozziello and Gavattini have defined the cosmological constant is an eigenvalue of WDW equation with f(R) theories of gravity. The explicit calculation is performed for a schwarzschild metric where one-loop energy is derived by the zeta function regularization method and renormalized running cosmological constant is obtained [6]. There are papers on the quantum cosmology by Lemos and Monerat et.al[7]. The quantization of the Friedmann-Robertson-Walker spacetime in the presence of negative cosmological constant was used in the papers. They have concluded that there are solutions which avoid singularities (big bang, big crunch) at the quantum level[7]. Moguigan have discussed a seesaw mechanism of cosmological constant in the context of quan-
tum cosmology[8]. The FRW quantum cosmology in the non-Abelian Born-Infeld theory has been discussed by Moniz[9]. Reference [10] have discussed Quantum birth of the universe in gravitational theory of varying cosmological constant.

In the 1930’s Born and Infeld[20] attempted to eliminate the divergent self energy of the electron by modifying Maxwell’s theory. Born-Infeld electrodynamics follows from the lagrangian $L_{BI} = b^2 \left( \sqrt{1 - \left(1/2b^2\right) F_{\mu\nu} F^{\mu\nu} - 1} \right)$, Where $F_{\mu\nu}$ is the electromagnetic field tensor. Our B-I type lagrangian has been first proposed by Heisenberg in order to describe the process of meson multiple production connected with strong field regime[21]. The Born-Infeld type action also appears in string theory[22]. In the important paper[23] it was demonstrated that the leading-order term in the expansion in $\partial F$ of the condition of conformal invariance of the open string sigma model follows indeed from the BI action. Static and spherically symmetric solutions of the B-I type scalar field have been recently investigated qualitatively by Oliveiva[24]. Furthermore our B-I type scalar field lagrangian is the special case of tachyon lagrangian $u(\varphi) \sqrt{1 - g_{\mu\nu} \varphi,\mu \varphi,\nu}$ when the potential $u(\varphi) = \text{constant}$. V.Mukhanov and A.Vikman[18] have investigated inflation with an analogous scalar field lagrangian $\alpha^2 \left( \sqrt{1 + \frac{g_{\mu\nu} \varphi,\mu \varphi,\nu}{\alpha^2}} - 1 \right) - V(\varphi)$. The speed of cosmological perturbations $c_s^2 > 1$, which is different with our B-I scalar field lagrangian(see Eq(34)). We have investigated quantum cosmology and dark energy model in our B-I type scalar field[11].

In this paper, we combine inflation with quantum cosmology in the B-I type scalar field and obtain interesting results that the vacuum energy(corresponding cosmological constant) in inflation epoch depends on the tensor-to-scalar ratio $\frac{\delta_h}{\delta_\Phi}$.

The present paper is organized as follows. In section 2 we consider quantum cosmology with B-I type scalar field, obtain the wheeler-Dewitt(W-D) equation of our B-I scalar field model. In section 3 we apply Hartle-Hawking’s method to obtain the wave function of the universe. The Vilenkin’s quantum tunneling approach is also considered. The probability density obtained from Hartle-Hawking method is proportional to $e^{\frac{3\pi}{G} (\Lambda - \frac{1}{\eta})}$. The probability density has an infinite peak for $\Lambda \to 1/\eta$(the maximum kinetic energy $\frac{1}{2} \dot{\delta}_h^2$ is $\frac{1}{2\eta}$). i.e, the vacuum energy equals to two time of maximum kinetic energy of scalar field. The Vilenkin wave function predicts a nucleating universe with largest possible cosmological constant and it is larger than $1/\eta$. In section 4 we have discussed the inflation with B-I type scalar field. We find that $\eta$ depends on the amplitude of tensor perturbation $\delta_h$, with the form $\frac{1}{\eta} \simeq m^2/12\pi[\left(\frac{9\delta_h^2}{N\delta_\Phi}\right)^2 - 1]$. We conclude our results in the last Section.

## 2 WD Equation with B-I type Scalar Field

The action of the gravitational field interacting with a Born-Infeld type scalar field is given by

$$ S = \int \frac{R}{4\pi G} \sqrt{-g} d^4 x + \int L_s \sqrt{-g} d^4 x $$

(1)
where we have chosen units so that $c = 1$, $R$ is the Ricci Scalar curvature and the lagrangian $L_s$ of the B-I scalar field\cite{11} is

$$L_s = \frac{1}{\eta}(1 - \sqrt{1 - \eta g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi}) - V(\varphi)$$

(2)

Where $\eta$ is a constant and $V(\varphi)$ is the potential of vacuum field. We shall consider that the potential of the vacuum field $V(\varphi)$ is a constant. Therefrom a physical point of view, it is equivalent to cosmological constant.

At the planck time, the quantum effects played the main role in the universe. So it is suitable to describe the dynamics and evolution of very early universe by using the cosmological wave function $\Psi(h_{ij}, \varphi)$ defined on the superspace of all three-metrics $h_{ij}$ and material fields $\varphi$. In the superspace it satisfies the Wheeler-DeWitt(WD) equation:

$$\hat{H}\Psi = 0$$

(3)

$\hat{H}$ is a second-order differential operator in the superspace. In principle, $\Psi(h_{ij}, \varphi)$ should contain the answer to all meaningful questions one can ask about the evolution of the very early universe. In order to find out the solution of the WD equation, we shall apply the minisuperspace model—a Robertson-Walker(RW) space-time metric. The B-I type scalar field is given by Eq.(2). In the minisuperspace there are only two degrees of freedom: $a(t)$ and $\varphi(t)$. The RW space-time metric is

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 + kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

(4)

Using Eq.(2) and by integrating with respect to space-components the action (1) becomes (the upper-dot means the derivative with respect to the time t):

$$S = \int \frac{3\pi}{4G}(1 - a^2)dt + \int 2\pi^2 a^3 \left[\frac{1}{\eta}(1 - \sqrt{1 - \eta \dot{\varphi}^2}) - V\right]dt = \int \mathcal{L}_g dt + \int \mathcal{L}_s dt$$

(5)

From the Euler-lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_s}{\partial \dot{\varphi}}\right) - \frac{\partial \mathcal{L}_s}{\partial \varphi} = 0$$

(6)

We can obtain

$$\dot{\varphi} = \frac{c}{\sqrt{a^6 + \eta c^2}}$$

(7)

where $c$ is integral constant. From the above equation we know that cosmological scale factor $a(t)$ is very large or small when $\dot{\varphi}$ is very small or large respectively. The maximum of kinetic energy is $1/2\eta$ from Eq.(7).

To quantize the model, we first find out the canonical momenta $P_a = \partial \mathcal{L}_g / \partial \dot{a} = -(3/2G)aa\dot{a}$, $P_\varphi = \partial \mathcal{L}_\varphi / \partial \dot{\varphi} = 2\pi^2 a^3 \dot{\varphi} / \sqrt{1 - \eta \dot{\varphi}^2}$ and the Hamiltonian $H = P_a \dot{a} + P_\varphi \dot{\varphi} - \mathcal{L}_g - \mathcal{L}_s$. $H$ can be written as the follows

$$H = -\frac{G}{3\pi a}P_a^2 - \frac{3\pi}{4G}a[1 - \frac{8\pi G}{3}a^2 V(\varphi)] - \frac{2\pi^2 a^3}{\eta} \left[1 - \sqrt{1 + \frac{\eta P_\varphi^2}{4\pi^2 a^6}}\right]$$

(8)
For $\dot{\varphi}^2 \ll \frac{1}{\eta}$, the Hamiltonian Eq.(8) can be simplified by using the Taylor expansion, and the terms smaller than $\dot{\varphi}^6$ can be ignored, so the Hamiltonian becomes

$$H = -\frac{G}{3\pi a} P_{\varphi}^2 - \frac{3\pi}{4G} a \{ 1 - \frac{8\pi G}{3} a^2 V(\varphi) \} + \frac{P_{\varphi}^2}{4\pi^2 a^3} - \frac{\eta P_{\varphi}^4}{64\pi^4 a^9}$$  \hspace{1cm} (9)$$

If $\dot{\varphi}$ is very large($\dot{\varphi}^2 \sim 1/\eta$), Eq.(8) becomes

$$H = -\frac{G}{3\pi a} P_{\varphi}^2 - \frac{3\pi}{4G} a \{ 1 - \frac{8\pi G}{3} a^2 [V(\varphi) - \frac{1}{\eta}] \}$$  \hspace{1cm} (10)$$

The WD equation is obtained from $\hat{H}\psi = 0$, Eqs.(9) and (10) by replacing $P_a \rightarrow -i(\partial/\partial a)$ and $P_{\varphi} \rightarrow i(\partial/\partial \varphi)$. Then we obtain

$$\left[ \frac{\partial^2}{\partial a^2} + \frac{p}{a} \frac{\partial}{\partial a} - \frac{1}{a^2} \frac{\partial^2}{\partial \tilde{\Phi}^2} - \frac{\eta}{16\pi^4 a^8} \frac{\partial^4}{\partial \tilde{\Phi}^4} - U(a, \tilde{\Phi}) \right] \psi = 0$$  \hspace{1cm} (11)$$

and

$$\left[ \frac{\partial^2}{\partial a^2} + \frac{p}{a} \frac{\partial}{\partial a} - u(a, \tilde{\Phi}) \right] \psi = 0$$  \hspace{1cm} (12)$$

where $\tilde{\Phi}^2 = 4\pi G \varphi^2/3$ and the parameter $p$ represent the ambiguity in the ordering of factor $a$ and $\partial/\partial a$ in the first term of Eqs.(9) and (10). The variation of $p$ does not affect the solution of Eqs.(9) and (10). In following discussion, we shall set $p = -1[12,16]$. We have also denoted

$$U(a, \tilde{\Phi}) = \left( \frac{3\pi}{2G} \right)^2 a^2 \{ 1 - \frac{8\pi G}{3} a^2 [V(\tilde{\Phi}) - \frac{1}{\eta}] \}$$  \hspace{1cm} (13)$$

$$u(a, \tilde{\Phi}) = \left( \frac{3\pi}{2G} \right)^2 a^2 \left[ 1 - \frac{8\pi G}{3} a^2 [V(\tilde{\Phi}) - \frac{1}{\eta}] \right]$$  \hspace{1cm} (14)$$

Eqs.(11) and (12) are the WD equations corresponding to the action (1) in the case of large and small scale factor $a$ respectively. If scale factor $a$ is very large, the general solution of Eq.(1) is given by

$$\psi(a, \varphi) \sim \frac{a}{a_0} Z_a \left( \frac{2\tilde{\mu} a^3}{3} \right) e^{-k\varphi}$$  \hspace{1cm} (15)$$

Where $a_0$ is the planck length, $k$ is an arbitrary constant. and $\tilde{\mu} = 2\pi^2 V(\varphi)$. The $\psi(a, \varphi)$ is an oscillatory function with respect to scale factor $a[11]$.

3 The Vilenkin and Hartle-Hawking Method

Next we will use Vilenkin’s quantum tunneling[12] approach to consider the cosmology in case of very large $\varphi$(correspondingly very small $a(t)$). Eq.(12) has the form of a one-dimensional Schrödinger equation for a "particle" described by a coordinate $a(t)$, which is zero energy and moves in a potential $u$. The classically allowed region is $u \leq 0$ or $a \geq H^{-1}$, with $H = [8\pi G(V - \frac{1}{\eta})]^{1/2}$. In this region, disregarding the pre-exponential factor, the WKB solutions of Eq.(12) are

$$\psi^{(1)}_\pm(a) = exp\{ [\pm i \int_{H^{-1}}^a P(a') da'] + \frac{i\pi}{4} \}$$  \hspace{1cm} (16)$$
The under-barrier ($a < H^{-1}$, classically forbidden or Euclidean region) solutions are

$$\psi^{(2)}_{\pm}(a) = \exp\left[\pm \int_{a}^{H^{-1}} |P(a')| da'\right]$$  \hspace{1cm} (17)

where $P(a) \equiv \sqrt{|-u(a)|}$.

The classical momentum conjugate to $a$ is $P_a = -a \dot{a}$. For $a > H^{-1}$, we have

$$(-i \frac{d}{da})\psi^{(1)}_{\pm}(a) = \pm P(a)\psi^{(1)}_{\pm}(a)$$  \hspace{1cm} (18)

and thus $\psi^{(1)}_{-}(a)$ and $\psi^{(1)}_{+}(a)$ describe the expanding and contracting universe respectively. The tunneling boundary condition requires that only the expanding component should be present at large $a$,

$$\psi_T(a > H^{-1}) = \psi^{(1)}_{-}(a)$$  \hspace{1cm} (19)

The under-barrier wave function is found from WKB connection formula

$$\psi_T(a < H^{-1}) = \psi^{(2)}_{+}(a) - \frac{i}{2} \psi^{(2)}_{-}(a)$$  \hspace{1cm} (20)

The growing exponential $\psi^{(2)}_{+}(a)$ and the decreasing exponential $\psi^{(2)}_{-}(a)$ have comparable amplitudes at the nucleation point $a = H^{-1}$, but away from that point the decreasing exponential dominates

$$\psi_T(a < H^{-1}) \approx \psi^{(2)}_{+}(a) = \exp\left[\frac{\pi}{2GH^2}(1 - H^2a^2)^{\frac{3}{2}}\right]$$  \hspace{1cm} (21)

The "tunneling amplitude" is proportional to

$$\frac{\psi_T(H^{-1})}{\psi_T(0)} = e^{-\frac{\pi}{2Gh^2}}$$  \hspace{1cm} (22)

The corresponding probability density

$$P_T \propto e^{-\frac{\pi}{2Gh^2}}$$  \hspace{1cm} (23)

From Eq.(23) we obtain the result that the tunneling wave function predicts a nucleating universe with the largest possible vacuum energy (i.e., the largest possible cosmological constant), but cosmological $\Lambda$ must be larger than $1/\eta$. In other word, the vacuum energy is larger than two time of the maximum of kinetic energy of scalar field. It is correct condition for the inflation. Eqs.(21,23) can be obtain by an alternative method, devised by Zeldovich and starobinsky[13], Rubakov[14] and Linde[15]. The Eq.(23) predicts that a typical initial value of the field $\varphi$ is given by $V(\varphi) \sim M_p^4$(if one does not speculate about the possibility that $V(\varphi) \gg M_p^4$), which leads to a very long stage of inflation.

The Hartle-Hawking(H-H) no boundary wave function is given by the path integral[16]

$$\psi_{HH} = \int [dg][d\varphi] e^{-SE(g,\varphi)}$$  \hspace{1cm} (24)
In order to determine $\psi_{HH}$, we assume that the dominant contribution to the path integral is given by the stationary points of the action (the instantons) and evaluates $\psi_{HH}$ simply as $\psi_{HH} \sim e^{-S_E|_{\text{saddle-point}}}$. When $\dot{\phi}$ is very large, $\dot{\phi}^2 \sim 1/\eta$, from action (5) we can obtain

$$S = \int \frac{3\pi}{4G}[(1 - \dot{a}^2)a]dt - \int 2\pi^2 a^3(V - \frac{1}{\eta})dt$$

The corresponding Euclidean action $S_E = -i(S)_{\text{continue}}$ is

$$S_E = \int \frac{3\pi}{4G}[1 + (\frac{da}{d\tau})^2]d\tau + \frac{3\pi}{4G} \int a^3 H^2 d\tau$$

Where $H^2 = \frac{8\pi G}{3}(V - \frac{1}{\eta})$ and $\tau = it$. From action (25), we can also obtain that the $a(t)$ satisfying the following classical equation of motion

$$-\frac{\ddot{a}}{a} - 1 + H^2 a^2 = 0$$

The solution of Eq.(27) is the de-Sitter space with $a(t) = H^{-1}cosh(Ht)$. The corresponding Euclidean version (replacing $t \rightarrow -i\tau$) of Eq.(27) is

$$\left(\frac{da}{d\tau}\right)^2 - 1 + H^2 a^2 = 0$$

The solution of Eq.(28) is

$$a(\tau) = H^{-1}sin(H\tau)$$

We consider a saddle point approximation to the path integral (24), use Eqs.(26,29), and obtain

$$\psi_{HH}(a) \propto \exp[-\frac{\pi}{2GH^2}(1 - H^2 a^2)^{\frac{3}{2}}]$$

The only one difference between the H-H’s wave function (30) and Vilenkin’s wave function (21) is the sign of the exponential factor. The H-H’s wave function (30) gives out the probability density

$$P_{HH} \propto e^{\frac{\pi}{G\eta^2}}$$

The H-H’s probability density (31) is the same as Vilenkin’s one (23), except a sign in the exponential factor. The probability density (31) is peaked at $V - (1/\eta) = 0$ (note, here $H^2 = \frac{8\pi G}{3}(V - 1/\eta)$) and it predicts a very possible universe with a positive cosmological constant $\frac{1}{\eta}$. The corresponding vacuum energy equals to two time of maximum of kinetic energy of scalar field. It predicts a correct condition for inflation cosmology. It is different from previous result predicted by Hawking that cosmological constant equals zero [2].

### 4 The Inflation with B-I type scalar field

The lagrangian of B-I type scalar field

$$L = K(X) - V = \frac{1}{\eta}[1 - \sqrt{1 - 2\eta X}] - \frac{1}{2}m^2 \phi^2$$
We assume that for the homogenous scalar field $X = \frac{1}{2} \phi^2$. Here the $L$ plays the role of pressure $p$. The corresponding energy density 

$$\rho = 2XL_X - L = \frac{1}{\eta}[(1 - 2\eta X)^{-\frac{1}{2}} - 1] + \frac{1}{2}m^2\phi^2$$

(33)

The effective speed of sound (i.e., speed of propagation of the cosmological perturbation) is 

$$c_s^2 = \frac{p, X}{\rho, X} = 1 - 2\eta X$$

(34)

The stability condition with respect to the high frequency cosmological perturbation requires $c_s^2 > 0$. We can find that $c_s^2 < 1$ and when $\eta \to 0$, $c_s^2 = 1$. In the nonlinear scalar field model considered by Mukhanov and Vikman[18], the speed of sound $c_s^2 > 1$. Let us consider a R-W space-time with small perturbations:

$$ds^2 = (1 + 2\Phi)dt^2 - \frac{a(t)^2}{(1 - 2\Phi)\delta_{ik} + h_{ik}}dx^i dx^k$$

(35)

Where $\Phi$ is the gravitational potential characterizing scalar metric perturbations and $h_{ij}$ is a traceless, transverse perturbations describing the gravitational waves. The equation of Einstein gravitation field:

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho$$

(36)

where $H^2 = (\dot{a}/a)^2$, the dot denotes the derivative with respect to time $t$. In inflation epoch, the term $k/a^2$ in (36) becomes negligibly small compared with $H^2$.

The effective energy-moment conservation law is

$$\dot{\rho} + 3H(\rho + p) = 0$$

(37)

We obtain the following equation for scalar field by varying action of B-I type scalar field.

$$\ddot{\phi} + 3c_s^2H\dot{\phi} + \frac{V,\phi}{\rho, X} = 0$$

(38)

From the Eqs.(36,38) of the field motion it is clear that if following slow-roll conditions

$$XK, X \ll V, \quad and \quad K \ll V, \quad |\dot{\phi}| \ll \frac{V,\phi}{\rho, X}$$

(39)

and satisfied for at least 75 e-folds then we have a successful slow-roll inflation due to the potential $V(\phi)$. Considering the canonical scalar field with $K = X$, one can take a flat potential $V(\phi)$ so that $X \ll V$ (for more than 75 e-folds). It is the standard slow-roll inflation[17] and in this case $c_s = 1$. In contrast to ordinary slow-roll inflation one can have any speed of sound and $c_s < 1$. It is important to the amplitude of the final scalar perturbations (during the postinflationary, radiation-dominated epoch) and the ratio of tensor to scalar amplitudes on supercurvature scales are given by[18]

$$\delta_s^2 \simeq \frac{64}{81}\left[\frac{\rho}{c_s(1 + p/\rho)}\right]_{c_s \approx Ha}$$

(40)
\[ \frac{\delta_h^2}{\delta_\Phi^2} \simeq 27[c_s(1 + p/\rho)]_{k \simeq H a} \]  

Here it is worthwhile reminding that all physical quantities on the right hand side of Eqs.(40,41) have to be calculated during inflation at the moment when perturbations with wave number \( k \) cross corresponding horizon: \( c_s k \simeq H a \) for \( \delta_\Phi \) and \( k \simeq H a \) for \( \delta_h \) respectively. The amplitude of the scalar perturbations \( \delta_\Phi \) is a free parameter of the inflationary theory which is taken to fit the observations \( 10^{-5} \).

In the slow-roll regime, Eqs(36,38) reduce to

\[ H \simeq \sqrt{\frac{4\pi}{3} m \varphi} \]  

\[ 3 p_X H \dot{\varphi} + m^2 \varphi \simeq 0 \]  

From Eq.(42,43) we can obtain a slow-roll solution \( \dot{\varphi} \simeq -\frac{mc_I}{\sqrt{12\pi}} \), then we obtain

\[ \varphi = \varphi_0 - \frac{mc_I}{\sqrt{12\pi} t} \]  

Where \( c_I = (1 + \frac{m^2}{12\pi})^{-1/2} \), it is the sound speed during inflation. The sound speed is smaller than the speed of light, approaching it as \( \eta \to 0 \). The effective energy density and pressure are given by

\[ \rho = \frac{1}{\eta} \left[ \frac{1}{c_I} - 1 \right] + \frac{1}{2} m^2 \varphi^2, \quad p = \frac{1}{\eta} [1 - c_I] - \frac{1}{2} m^2 \varphi^2 \]  

To determine \( a(\varphi) \) we use \( \dot{\varphi} \simeq -\frac{mc_I}{\sqrt{12\pi}} \) to rewrite the equation (42) as

\[ -\frac{mc_I}{\sqrt{12\pi}} \frac{dlna}{d\varphi} \simeq \sqrt{\frac{4\pi}{3} m \varphi} \]  

and obtain

\[ a(\varphi) \simeq a_f exp\left[\frac{2\pi}{c_I}(\varphi_f^2 - \varphi^2)\right] \]  

Where \( a_f \) and \( \varphi_f \) are the values of the scale factor and scalar field at the end of inflation. The inflation is over when \( (\rho + p)/\rho \simeq c_I/(6\pi)^{1/2} \) becomes of order unity, that is, at \( \varphi \sim \varphi_f = (c_I/6\pi)^{1/2} \). After that the field \( \varphi \) begins to oscillate and decays. Given a number of e-folds before the end of inflation \( N \), we find that at this time \( 2\pi \varphi_f^2/c_I \sim N \), hence \( (\rho + p)/\rho \simeq 1/3N \) does not depend on \( c_I \). Thus, for a given scale, which cross the Hubble scale \( N \) e-folds before the end of inflation, the tensor-to-scalar ratio is[18]

\[ \frac{\delta_n^2}{\delta_\Phi^2} \simeq 27[c_I(1 + p/\rho)] \simeq \frac{9c_I}{N} \]  

Next one can estimate the mass \( m \) which is necessary to produce the observed \( \delta_\Phi \sim 10^{-5} \). Using \( (2\pi \varphi^2/c_I) \simeq N \), \( (\rho + p)/\rho \simeq 1/3N \) and from Eq.(40) one can obtain \( m \simeq 3\sqrt{3\pi} \delta_\Phi/4N \). Then one can obtain \( m \sim 2.4 \times 10^{-7} \) for \( N \sim 75 \). It is similar to the usual chaotic inflation[19]. The spectral index of scalar perturbations is

\[ \frac{\delta_h^2}{\delta_\Phi^2} \simeq 27[c_s(1 + p/\rho)] \simeq \frac{9c_I}{N} \]
\[ n_s - 1 \simeq -3\left(1 + \frac{p}{\rho}\right) - H^{-1}\frac{d\ln(1 + \frac{p}{\rho})}{dt} = -\frac{2}{N} \] (49)

This is exactly the same tilt as for the usual chaotic inflation. Finally we obtain by Eq.(48) and 
\[ c_I = (1 + \frac{m^2}{12\pi})^{-\frac{1}{2}} \]

\[ \frac{1}{\eta} \simeq \frac{m^2}{12\pi\left[1 + \left(\frac{9\rho}{N\delta}\right)^2 - 1\right]} \] (50)

![Graph](image)

Fig1: The value of \( \frac{\delta h}{\delta \Phi} \) with \( \frac{1}{\eta} \). The horizontal axis represents \( \log_{10}\frac{1}{\eta} \) and the vertical axis represents \( \frac{\delta h}{\delta \Phi} \).

We list the details in TABLE 1:

| \( \frac{1}{\eta} \) | \( 1 \) | \( 10^{-3} \) | \( 10^{-5} \) | \( 10^{-7} \) | \( 10^{-9} \) | \( 10^{-11} \) |
|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \frac{\delta h}{\delta \Phi} \) | 0.3464101614 0.3464101614 0.3464101614 | 0.3464101602 | 0.3464100291 | 0.3463969309 |
| \( \frac{1}{\eta} \) | \( 10^{-13} \) | \( 10^{-15} \) | \( 10^{-17} \) | \( 10^{-19} \) | \( 10^{-21} \) | \( 10^{-23} \) |
| \( \frac{\delta h}{\delta \Phi} \) | 0.3450994646 0.2747269024 0.09836927818 0.03115782485 0.003115787144 |

From our Vilenkin and Hartle-Hawking wavefunction(Eqs.(21,30) we obtain that \( \Lambda \geq \frac{1}{\eta} \). In inflation epoch, the evolution of the field \( \varphi \) is very slow, so that this field acts only as a cosmological constant \( \Lambda(\varphi) = \frac{8\pi V(\varphi)}{M_P^4} \), where \( M_P \) is the plank mass[19]. Noted that \( \Lambda \geq \frac{1}{\eta} = 1, 10^{-3}, \ldots, 10^{-23} \) correspond to the vacuum energy \( \rho_V \geq \frac{M_P^4}{8\pi}, \frac{10^{-3}M_P^4}{8\pi}, \ldots, \frac{10^{-23}M_P^4}{8\pi} \). From the Fig.1 or Table 1 we can conclude that only when the vacuum energy \( \rho_V < \frac{10^{-14}M_P^4}{8\pi} \) the value of \( \frac{\delta h}{\delta \Phi} \) begins decrease rapidly. When \( \rho_V \to 0, \frac{\delta h}{\delta \Phi} \to 0 \). The tensor perturbations can be seen indirectly in the B-mode of the CMB polarization. The amplitude of the tensor perturbations can, in principle, be large enough to be observed. However, it is only on the border of detectability.
in future experiments. If it has been observed in future, this is very interesting to define the cosmological constant.

5 Conclusion

Weinberg has described five directions that have been taken in trying to solve the problem of the cosmological constant. The five approaches respectively are Anthropic considerations, superstrings and supersymmetry, Adjustment Mechanisms, changing Gravity, Quantum Cosmology. At present, all of the five approaches to the cosmological constant problem remain interesting. The approach which based on quantum cosmology is most promising[1]. In quantum cosmology with B-I type scalar field, our Hartle-Hawking wave function predicts that the most probable value of cosmological constant is \( \frac{1}{\eta} \) while our Vilenkin wavefunction predicts \( \Lambda > \frac{1}{\eta} \). These results are correct condition for inflation cosmology. The parameter \( \eta \) can be obtained by the tensor perturbation from our inflation model. The tensor perturbations can be seen indirectly in the B-mode of the CMB polarization. The amplitude of the tensor perturbations can, in principle, be large enough to be observed. However, it is only on the border of detectability in future experiments. If it has been observed in future, this is very interesting to determine the cosmological constant in the inflation epoch.

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