Heavy colored SUSY partners from deflected anomaly mediation

Fei Wang,\textsuperscript{a,b} Wenyu Wang,\textsuperscript{c} Jin Min Yang,\textsuperscript{b} Yang Zhang\textsuperscript{b}

\textsuperscript{a}Department of Physics and Engineering, Zhengzhou University, Zhengzhou 450000, P. R. China
\textsuperscript{b}State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Academia Sinica, Beijing 100190, P. R. China
\textsuperscript{c}Institute of Theoretical Physics, College of Applied Science, Beijing University of Technology, Beijing 100124, P. R. China

E-mail: feiwang@zzu.edu.cn, wywang@mail.itp.ac.cn, jmyang@itp.ac.cn, zhangyang@itp.ac.cn

Abstract: We propose a deflected anomaly mediation scenario from SUSY QCD which can lead to both positive and negative deflection parameters (there is a smooth transition between these two deflection parameter regions by adjusting certain couplings). Such a scenario can naturally give a SUSY spectrum in which all the colored sparticles are heavy while the sleptons are light. As a result, the discrepancy between the Brookheaven $g_\mu - 2$ experiment and LHC data can be reconciled in this scenario. We also find that the parameter space for explaining the $g_\mu - 2$ anomaly at 1$\sigma$ level can be fully covered by the future LUX-ZEPLIN 7.2 Ton experiment.
Contents

1 Introduction 2
2 A brief review of AMSB 3
3 Deflected AMSB from SUSY QCD 4
4 Reconcile $g_\mu - 2$ and LHC data in deflected AMSB 7
5 Conclusion 12
1 Introduction

As an appealing candidate for the TeV-scale new physics, low energy supersymmetry (SUSY) can give an explanation for the gauge hierarchy problem, realize the gauge coupling unification and provide a viable dark matter candidate. It is remarkable that the 125 GeV Higgs boson recently discovered by the ATLAS [1] and CMS collaborations [2] agrees perfectly with the mass prediction of 115-135 GeV by the Minimal Supersymmetric Standard Model (MSSM). Actually, SUSY can satisfy all current experimental constraints [3] and especially can yield sizable contributions to the muon anomalous magnetic moment which can solve the discrepancy between the E821 experiment at the Brookhaven AGS [4, 5] and the Standard Model (SM) prediction [6].

On the other hand, so far no SUSY partners have been detected at the LHC and the mass limits on squarks and gluinos $m_{\tilde{g}} > 1.5$ TeV for $m_{\tilde{q}} \sim m_{\tilde{g}}$ and $m_{\tilde{q}} \gtrsim 1$ TeV for $m_{\tilde{q}} \gg m_{\tilde{g}}$ have been obtained for the constrained MSSM (CMSSM) [7, 8]. Together with the heavy top squarks required by the 125 GeV Higgs boson mass (the mass bounds from the direct LHC search are not so stringent for top squarks [9]), this indicates rather heavy colored sparticles. Considering the light uncolored sparticles (neutralinos, charginos and smuons) around $O(100)$ GeV required by the explanation of the muon $g-2$ anomaly [12], this poses a tension for the popular CMSSM [13, 14]. So the SUSY spectrum from SUSY breaking seems to have a intricate structure [15]. The origin of SUSY breaking and its mediation mechanism are crucial for the phenomenology.

The anomaly mediated SUSY breaking (AMSB) [16] is one of the most attractive scenarios in supergravity. Not only the sparticle mass spectrum are predicted to be flavor blind and thus automatically solves the SUSY flavor problem, but also the sparticle masses at low energies are insensitive to any high energy theories [18] since the SUSY breaking is mediated through the superconformal anomaly. Unfortunately, the AMSB scenario leads to tachyonic sleptons so that the minimal theory must be extended. The deflected AMSB [17], which introduces a messenger sector in the AMSB, can deflect the Renormalization Group Equation (RGE) trajectory and give new contributions to the soft SUSY breaking terms. The tachyonic slepton problems can be naturally solved by such a deflection.

The SUSY spectrum with heavy colored sparticles and light sleptons can be naturally realized in such a deflected AMSB scenario, especially when the deflection parameter is positive [20, 21]. However, the positive-deflected AMSB model cannot be easily realized and special efforts are need for model building. We propose in this paper a scenario from SUSY QCD in which both positive and negative deflection parameters can be realized and smoothly connected. Messenger sectors can be generated automatically without additional assumptions. With positive-deflected parameters, the tension between $g-2$ anomaly and LHC data can be ameliorated in such an AMSB scenario.

This paper is organized as follows. In Sec. 2, we briefly review the AMSB mechanism. In Sec.3, we propose a scenario which can realize both positive and negative deflected AMSB from SUSY QCD-type theory. A smooth transition can occur for both possibilities and all the contents can origin from a SUSY strong dynamics. In Sec. 4, we examine

\[1\] However, the recent ATLAS Z-peaked excess [10] may indicate a gluino as light as 800 GeV [11]
the parameter space of our deflected AMSB to explain both the LHC results and the Brookhaven $g_\mu - 2$ experiments. Sec. 5 contains our conclusions.

2 A brief review of AMSB

In the MSSM, the SUSY breaking effects can be communicated from some hidden sector to visible sector through gauge [22] or gravitational [23] interactions. Gravitational effects typically lead to sparticle masses from contact terms suppressed by powers of the Planck scale. However, if the two sectors are completely sequestered and these contact terms are absent, the sparticle masses of order $m_{3/2}/(16\pi^2)$ will still be generated due to the superconformal anomaly. Anomaly mediation can be regarded as the pure supergravity contributions to the soft SUSY breaking terms. They are determined by the VEV of the auxiliary compensator field $F_\phi$ within the graviton supermultiplet. The couplings of compensator F-term VEVs to the MSSM are purely quantum effects from the super-Weyl anomaly. The supergravity effects can be studied in the superconformal tensor calculus formalism by the introduction of compensator field [26]. The theory with the compensator can be seen to be equivalent to ordinary non-conformal SUGRA after gauge fixing.

Assume that the only source of SUSY breaking comes from a non-vanishing value of $F_\phi$ of compensator field with

$$\langle \phi \rangle = 1 + \theta^2 F_\phi, \quad (2.1)$$

and $F_\phi \sim m_{3/2}$. The couplings of $\phi$ are restricted by a spurion scale symmetry under which $\phi$ has a mass dimension of $+1$. Therefore, $\phi$ only appears in terms with dimensionful couplings. Although there is no SUSY breaking at tree level, soft masses at loop level emerges because the cut-off in a supersymmetric regulator is a dimensionful coupling which must be made covariant. The Lagrangian of the visible sector can be written as

$$\mathcal{L} = \int d^4\theta Z \left( \frac{\mu}{\Lambda} \right) Q^i e^V Q + \int d^2\theta \left[ S \left( \frac{\mu}{\Lambda} \right) W^a W_a + \lambda Q^3 \right] + h.c. \quad (2.2)$$

After the replacement

$$Z \left( \frac{\mu}{\Lambda} \right) \rightarrow Z \left( \frac{\mu}{\Lambda \sqrt{\phi^4 \phi}} \right), \quad S \left( \frac{\mu}{\Lambda} \right) \rightarrow S \left( \frac{\mu}{\Lambda \sqrt{\phi^4 \phi}} \right), \quad (2.3)$$

and the expansion in $\theta$, the gauginos acquire masses

$$M_\lambda = \frac{g^2}{2} \frac{d g^{-2}}{d \ln \mu} F_\phi = \frac{bg^2}{16\pi^2} F_\phi, \quad (2.4)$$

which are typically at the scale $m_{3/2}$. The sfermions, on the other hand, acquire masses of the form

$$M_f^2 = -\frac{1}{4} \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right) |F_\phi|^2, \quad (2.5)$$

$^2$The analysis in [24] clarifies several physical aspects of AMSB and demonstrates that anomaly mediation of SUSY breaking is in fact not a consequence of any anomaly of the theory [25].
where at leading order

\[ \beta_g = -\frac{bg^3}{16\pi^2}, \quad \gamma = \frac{1}{16\pi^2} \left( 4C_2(r)g^2 - a_1y^2 \right), \quad \beta_y = \frac{y}{16\pi^2}(a_2y^2 - a_3g^2). \] (2.6)

So they give

\[ M_f^2 = \frac{1}{512\pi^4} \left[ 4C_2(r)bg^4 + a_1y^2(a_2y^2 - a_3g^2) \right] |F_\Phi|^2. \] (2.7)

Sfermion masses are in practice family independent but the squared slepton masses are predicted to be tachyonic in this minimal scenario.

## 3 Deflected AMSB from SUSY QCD

Various attempts have been proposed to solve the tachyonic slepton problem for the AMSB. For example, additional gravitational contributions or additional D-term contributions [19, 27] can be added to overcome this problem. It is also possible to generate large Yukawa couplings for sleptons with additional Higgs doublets [28]. An elegant solution is the ‘deflected anomaly mediation’ scenario [17] in which the soft spectrum is modified by the presence of a light modulus (massless in the supersymmetric limit). Such a negatively deflected anomaly mediation scenario tends to release the gaugino hierarchy at the electroweak scale and drag down some of the squark masses, which is not favored by the null search results of sparticles at the LHC. In order to have relatively heavy colored sparticles and light sleptons, we need a positively deflected scenario [20] which gives a possible realization with some particular choice of the power for the singlet \( S \). In our following analysis, we will show that such a positively deflected scenario can be fairly generic in SUSY QCD type theory. The messenger fields, including their couplings, are also naturally obtained from the SUSY QCD dynamics.

We start from a microscopic model of \( SU(N) \) SUSY QCD with \( N_F \) flavor, where we require \( N + 1 < N_F < 3N \) so that the theory is asymptotic free in the UV limit and confines at the scale \( \Lambda \). The global symmetry of the theory is \( SU(N_F)_L \times SU(N_F)_R \times U(1)_V \times U(1)_R \). We can weakly gauge the subgroup of the global symmetry to accommodate the standard model gauge group. In terms of \( SU(N) \times SU(N_F)_L \times SU(N_F)_R \times U(1)_V \times U(1)_R \) group, the quantum number of matter contents \( Q_i, \tilde{Q}_j \) are given as

\[ Q_i \sim (N, N_F, \ 1, \ 1, \ (N_F - N)/N_F), \quad \tilde{Q}_j \sim (\tilde{N}, \ 1, \ \tilde{N}_F, \ -1, \ (N_F - N)/N_F). \] (3.1)

The superpotential in the microscopic ‘electric’ description is introduced as the ISS-type [29]

\[ W = \text{Tr}(m_0 \tilde{Q}_i Q_i), \] (3.2)

which below the confining scale \( \Lambda \) will have an alternative ‘magnetic’ description in terms of \( SU(N_F - N) \) gauge theory via Seiberg duality with the following superpotential

\[ W = -h\bar{\mu}^2 \text{Tr}\Phi + h\text{Tr}q\Phi\bar{q}, \] (3.3)
with \( q, \tilde{q} \) and \( \Phi \) related to the dual baryon \( B \) and meson \( M \), respectively. The parameters are defined as

\[
\Phi = \frac{M}{\sqrt{\alpha \Lambda}}, \quad h = \frac{\sqrt{\alpha \Lambda}}{\Lambda}, \quad \tilde{\mu}^2 = -m_0 \Lambda, \quad \Lambda_m = \tilde{\Lambda}, \quad N_c = N_F - N,
\]

\[
\Lambda_m^{3N - N_F} \Lambda_m^{3N_c - N_F} = (-1)^N c \Lambda_{N_F},
\]

(3.4)

where \( \alpha \) which determines the coupling \( h \) is a dimensionless parameter in the Kahler potential for \( M \).

It is well known that SUSY QCD of vector type does not break SUSY [30]. So this theory leads to a metastable SUSY breaking vacua [29] and at the same time has SUSY preserving vacua at large field value. In our scenario, SUSY breaking arises from the anomaly mediation effects instead of the ISS-type rank conditions. So we will concentrate on the originally SUSY preserving vacua and study the deviations from such a SUSY limit after we taking into account the supergravity effects. Consequently, constraints on the parameters \( \epsilon \equiv \tilde{\mu}/\Lambda_m \) from the lifetime of the meta-stable vacuum in ordinary ISS model will no longer be needed in our scenario.

After integrating out the mass terms \( h\Phi \) of \( \tilde{q}, q \), the low energy superpotential is

\[
W_l = N_c(h N_F \Lambda_m^{3N_c - N_F} \det \Phi)^{1/N_c} - h \tilde{\mu}^2 \text{Tr}(\Phi).
\]

(3.5)

The SUSY breaking effects from \( F_\phi \) also prompts \( F_\Phi \) to be nonzero at large values of \( \Phi \). Adding the compensator field into the previous superpotential, we have

\[
W_l = N_c(h N_F \Lambda_m^{3N_c - N_F} \det \Phi)^{1/N_c} - h \tilde{\mu}^2 \phi^2 \text{Tr}(\Phi),
\]

(3.6)

where all fields within \( \Phi \) have Weyl weight of 1. When \( F_\phi \) is turned on, the tree-level potential for the scalar \( \Phi \) is

\[
V = |F_\Phi|^2 - N_c(h N_F \Lambda_m^{3N_c - N_F} \det \Phi)^{1/N_c} \left(3 - \frac{N_F}{N_c}\right) F_\phi + 2h \tilde{\mu}^2 F_\phi \text{Tr}(\Phi),
\]

(3.7)

which gives the minimum condition of \( \langle \Phi \rangle \propto \tilde{m}_j^3 \)

\[
2 \left(N_F \Lambda_m^{3N_c - N_F} m^{N_F/N_c - 1} - \tilde{\mu}^2 N_F \right) (N_F/N_c - 1) N_F \Lambda_m^{3N_c - N_F/N_c} m^{N_F/N_c - 2} - N_F \Lambda_m^{3N_c - N_F/N_c} m^{N_F/N_c - 1} \left(3 - \frac{N_F}{N_c}\right) F_\phi + 2 \tilde{\mu}^2 N_F F_\phi = 0.
\]

(3.8)

Here we use \( m = h \tilde{m} \). This equation is a transcendental equation which can not be solved exactly. For a large \( N_c \) with \( N_c/N_F \to 1 \) (if the dual description is introduced as the input), we have

\[
m = \frac{(N_F - N_c) \Lambda_m^2}{F_\phi},
\]

(3.9)

which gives

\[
\frac{F_\Phi}{\Phi} = \frac{h^2 N_F (\Lambda_m^2 - \tilde{\mu}^2) F_\phi}{(N_F - N_c) \Lambda_m^2} \approx -\frac{h^2 N_F}{(N_F - N_c) F_\phi},
\]

(3.10)
when $\Lambda_m^2 \gg \tilde{\mu}^2$. The low energy wave function only depends on $\tilde{\Phi} = \Phi/\phi$ with
\begin{equation}
\frac{F_{\tilde{\Phi}}}{\tilde{\Phi}} = \frac{F_{\Phi}}{\Phi} - F_\phi \approx -\frac{h^2 N_F}{(N_F - N_c)}.
\tag{3.11}
\end{equation}
So we can see that we obtain a negatively deflected contribution.

In the limit of $N_F/N_c \to 3/2$ which is $N_F \to 3N$ in the original theory, we have
\begin{equation}
\frac{3}{2} \Lambda_m^{3/2} F_\phi (\sqrt{m})^2 - (N_F \Lambda_m^3 + 2\tilde{\mu}^2 F_\phi) \sqrt{m} + N_F \tilde{\mu}^2 \Lambda_m^{3/2} = 0.
\tag{3.12}
\end{equation}
A positive solution for $\sqrt{m}$ requires
\begin{equation}
\Delta \equiv (N_F \Lambda_m^3 + 2\tilde{\mu}^2 F_\phi)^2 - 6 \Lambda_m^3 F_\phi N_F \tilde{\mu}^2 \geq 0.
\tag{3.13}
\end{equation}
So for $\Lambda_m \gg \tilde{\mu}, F_\phi$, we can obtain
\begin{equation}
m \approx \frac{4}{9} \frac{N_F^2 \Lambda_m^3}{F_\phi^2}, \quad m \approx \frac{\tilde{\mu}^4}{9 \Lambda_m^3}.
\tag{3.14}
\end{equation}
Here only the first solution depends on $F_\phi$ and therefore we keep such an anomaly mediation contribution solution. Then we can obtain the deflection parameter
\begin{equation}
\frac{F_{\tilde{\Phi}}}{\tilde{\Phi}} = \frac{F_{\Phi}}{\Phi} - F_\phi \approx -\left(\frac{3}{2} h^2 + 1\right) F_\phi.
\tag{3.15}
\end{equation}
In this limit, the deflection parameter is still negative.

In the limit $N_F/N_c \to 3$ with $N_F \lesssim 3N_c$ (which amounts to $N_F > 3/2N$ and the theory lies in the conformal window), the ‘magnetic’ theory will no longer be IR free. However, the dynamically superpotential will still have the form (3.5). Following our previous discussions, the resultant cubic equation takes the form
\begin{equation}
2m^3 - 2\tilde{\mu}^2 m + 2 F_\phi \frac{N_F}{N_F} = 0,
\tag{3.16}
\end{equation}
in this limit which can always give negative solutions for $m$ due to the continuous nature of the cubic function. Numerical calculations indicate that the expression
\begin{equation}
\frac{F_\phi}{\Phi} = -\frac{h^2 N_F (m^2 - \tilde{\mu}^2)}{m} \equiv c \frac{h^2}{c} F_\phi,
\end{equation}
always gives a coefficient $c$ less than 1. Therefore we can obtain the deflection parameter for a large $N_c$
\begin{equation}
d \equiv \frac{F_{\tilde{\Phi}}}{\Phi F_\phi} = \frac{F_{\Phi}}{\Phi F_\phi} - 1 \approx c h^2 - 1
\tag{3.18}
\end{equation}
with $0 < c < 1$. Depending on the size of the coupling $h$, the deflection parameter can be positive or negative. We can see that there is a smooth transition between the positive and
negative deflected regions by adjusting the value of \( h \) for general choices of \( N_F \) and \( N_c \). A positive deflection corresponds to a relatively large coupling \( h \).

The general results of the deflected anomaly mediation scenario are given by \([17, 20]\)

\[
\begin{align*}
\frac{m_{\lambda}}{\alpha(\mu)} &= \frac{F_\phi}{2} \left( \frac{\partial}{\partial \ln \mu} - d \frac{\partial}{\partial \ln |\Phi|} \right) \alpha^{-1}(\mu, \Phi), \\
m^2_\lambda(\mu) &= -\frac{|F_\phi|^2}{4} \left( \frac{\partial}{\partial \ln \mu} - d \frac{\partial}{\partial \ln |\Phi|} \right)^2 \ln Z_i(\mu, \Phi), \\
A_i(\mu) &= -\frac{F_\phi}{2} \left( \frac{\partial}{\partial \ln \mu} - d \frac{\partial}{\partial \ln |\Phi|} \right) \ln Z_i(\mu, \Phi). 
\end{align*}
\tag{3.19}
\]

The gaugino masses which acquire an additional contributions from gauge mediation are given by

\[
m_{\lambda}(\mu) = \frac{\alpha_i(\mu)}{4\pi} F_\phi (b_i + d N_F),
\tag{3.20}
\]

with \( b_i \) being the beta functions for the gauge couplings. Similarly, the contribution for sfermions are

\[
m^2_\lambda(\mu) = 2 \sum_{G_i} C_2(r) \left( \frac{\alpha(\mu)}{4\pi} \right)^2 |F_\phi|^2 b_i G(\mu, \Phi),
\tag{3.21}
\]

with

\[
G(\mu, \Phi) = \left( \frac{N_F}{b} - \frac{N_F^2}{b^2} \right) \xi^2 d^2 + \left( \frac{N_F}{b} d + 1 \right)^2,
\tag{3.22}
\]

\[
\xi = \frac{\alpha(\Phi)}{\alpha(\mu)} = \left[ 1 + \frac{b}{4\pi} \alpha(\Phi) \ln \left( \frac{\Phi^\dagger \Phi}{\mu^2} \right) \right].
\tag{3.23}
\]

Here the MSSM beta function are \( b_i = (-33/5, -1, 3) \) and the quadratic Casimir for \( SU(N) \) fundamental representation is \( C_2(N) = (N^2 - 1)/2N \). The trilinear couplings \( A_\lambda \) related to superpotential terms \( \lambda_{ijk} Q_i Q_j Q_k \) are given by \( A_\lambda = (A_{Q_i} + A_{Q_j} + A_{Q_k}) \lambda_{ijk} \) with

\[
A_i(\mu) = 2c_i \left[ -\alpha(\mu) + \frac{d N_F}{b} (\alpha(\Phi) - \alpha(\mu)) \right] \frac{F_\phi}{4\pi} + |y(\mu)|^2 \frac{F_\phi}{32\pi^2}. \tag{3.24}
\]

The generations of higgsino mass \( \mu \) and the soft SUSY breaking \( B_\mu \) are not straightforward and should be seen as an independent problem of anomaly mediation. For example, they can be generated by the mechanism proposed in \([31]\) with additional singlet \( S \). Therefore, we will consider them as free parameters and do not give their explicit expressions in terms of the model inputs.

4 Reconcile \( g_\mu - 2 \) and LHC data in deflected AMSB

The SM prediction of the muon anomalous magnetic moment is

\[
a^{SM}_\mu = 116591834(49) \times 10^{-11}, \tag{4.1}
\]
which is smaller than the experimental result of E821 at the Brookhaven AGS [32]

\[ a_{\mu}^{\text{expt}} = 116592089(63) \times 10^{-11}. \] (4.2)

The deviation is then about 3σ

\[ \Delta a_{\mu}(\text{expt} - \text{SM}) = (255 \pm 80) \times 10^{-11}. \] (4.3)

SUSY can yield sizable contributions to the muon \( g_\mu - 2 \) which dominantly come from the chargino-sneutrino and the neutralino-smuon loop diagrams. At the leading order the analytic expressions for the SUSY contributions are presented in [33]. The \( g_\mu - 2 \) anomaly, which is order \( 10^{-9} \), can be explained for \( m_{\text{SUSY}} = \mathcal{O}(100) \text{ GeV} \) and \( \tan \beta = \mathcal{O}(10) \).

Now we turn to the calculations in the deflected AMSB. From the expression of deflected AMSB spectrum, the gaugino mass ratio can be enlarged in the positive deflected scenario while diminished in the negative deflected case. The gaugino mass ratio at the electroweak scale is given by

\[ M_3 : M_2 : M_1 \approx 12 : 12 : 11.6 \approx 1 : 1 : 1, \] (4.4)

with \( d = -1 \) and \( N_F = 5 \). On the other hand, for the positively deflected AMSB scenario, we have at the electroweak scale

\[ M_3 : M_2 : M_1 \approx -48 : -8 : 1.6 \approx -30 : -5 : 1, \] (4.5)

with \( d = 1 \) and \( N_F = 5 \). So we can see that the \( g_\mu - 2 \) anomaly may be explained in the positively deflected AMSB scenario [34].

We scan the parameter space of our deflected scenarios with \( 3 \geq d \geq -3, \; N_F \geq 5 \) and the messenger scale \( M \). The messenger scale \( M = \Phi \) plays a role of the intermediate threshold between the UV cutoff and the electroweak scale. At the messenger scale \( M \), the gaugino soft masses are given by

\[ m_{\lambda_1}(M) = \frac{\alpha_1(M)}{4\pi} F_\phi (b_1 + d N_F). \] (4.6)

The sfermion masses at the messenger scale \( M \) are

\[
\begin{align*}
\frac{m_{\tilde{Q}_L}^2}{|F_\phi|^2} &= \frac{\alpha_1^2(M)}{(4\pi)^2} 8 G_3 - \frac{\alpha_1^2(M)}{(4\pi)^2} \frac{3}{2} G_2 - \frac{\alpha_1^2(M)}{(4\pi)^2} \frac{11}{50} G_1, \\
\frac{m_{\tilde{U}_L}^2}{|F_\phi|^2} &= \frac{\alpha_1^2(M)}{(4\pi)^2} 8 G_3 - \frac{\alpha_1^2(M)}{(4\pi)^2} \frac{88}{25} G_1, \\
\frac{m_{\tilde{D}_L}^2}{|F_\phi|^2} &= \frac{\alpha_1^2(M)}{(4\pi)^2} 8 G_3 - \frac{\alpha_1^2(M)}{(4\pi)^2} \frac{22}{25} G_1, \\
\frac{m_{\tilde{E}_L}^2}{|F_\phi|^2} &= -\frac{\alpha_1^2(M)}{(4\pi)^2} \frac{3}{2} G_2 - \frac{\alpha_1^2(M)}{(4\pi)^2} \frac{99}{50} G_1, \\
\frac{m_{\tilde{L}_L}^2}{|F_\phi|^2} &= -\frac{\alpha_1^2(M)}{(4\pi)^2} \frac{198}{25} G_1, \\
\frac{m_{\tilde{N}}^2}{|F_\phi|^2} &= \frac{m_{\tilde{L}_L}^2}{|F_\phi|^2} = \frac{m_{\tilde{E}_L}^2}{|F_\phi|^2}, \\
\frac{m_{\tilde{N}}^2}{|F_\phi|^2} &= \frac{m_{\tilde{L}_L}^2}{|F_\phi|^2} - 3 \frac{y_t^2}{(16\pi^2)^2} (\frac{16}{3} g_3^2 + 3 g_2^2 + \frac{13}{15} g_1^2 - 6 y_t^2),
\end{align*}
\] (4.7-4.12)
where we define

\[ G_i = \left( \frac{N_F}{b_i} - \frac{N_F^2}{b_i^2} \right) d^2 + \left( \frac{N_F d + 1}{b_i^2} \right)^2. \tag{4.13} \]

The stop soft masses should also include the yukawa contributions

\[ m_{\tilde{Q}L}^2 = m_{\tilde{Q}L}^2 - \frac{y_t^2}{(16\pi)^2} \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6y_t^2 \right), \tag{4.14} \]

\[ m_{\tilde{t}_L}^2 = m_{\tilde{t}_L}^2 - \frac{2y_t^2}{(16\pi)^2} \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 - 6y_t^2 \right). \tag{4.15} \]

The trilinear soft terms are given by

\[ \frac{A_t}{(F_\phi/2\pi)} = -\frac{8}{3} \alpha_3(M) - \frac{3}{2} \alpha_2(M) - \frac{13}{30} \alpha_1(M) + \frac{1}{8\pi} (6|y_t(M)|^2 + |y_b(M)|^2), \tag{4.16} \]

\[ \frac{A_b}{(F_\phi/2\pi)} = -\frac{8}{3} \alpha_3(M) - \frac{3}{2} \alpha_2(M) - \frac{7}{30} \alpha_1(M) \]

\[ + \frac{1}{8\pi} (|y_t(M)|^2 + 6|y_b(M)|^2 + |y_\tau(M)|^2), \tag{4.17} \]

\[ \frac{A_\tau}{(F_\phi/2\pi)} = -\frac{3}{2} \alpha_2(M) - \frac{9}{10} \alpha_1(M) + \frac{1}{8\pi} (3|y_b(M)|^2 + 4|y_\tau(M)|^2). \tag{4.18} \]

Note that here we use the notation \( g_Y^2 = 3g_1^2/5 \). The tachyonic slepton problem can be solved with the choice of \( d \) and \( N_F \).

The inputs should be seen as the boundary conditions at the messenger scale which, after RGE running to the electroweak scale, should give the low energy spectrum. The free parameters are chosen as \( d, N_F, F_\phi, M, \tan \beta \). We scan over the following ranges of these parameters:

- In our scenario, the value of \( F_\phi \) determines the whole spectrum. Constraints from the gaugino masses indicate that \( F_\phi \) cannot be too low. Thus, we choose \( F_\phi \gtrsim \mathcal{O}(10 \text{TeV}) \). On the other hand, a too heavy \( F_\phi \) will spoil the EWSB and lead to too heavy Higgs mass. In our scan we take the value of \( F_\phi \) in the range \( 10 \text{TeV} < F_\phi < 500 \text{TeV} \).

- The messenger scale \( M \) can be chosen to be below the typical GUT scale \( 10^{16} \text{GeV} \). It should be heavier than the sparticle spectrum. The lower bound is chosen to be \( \mathcal{O}(10 \text{TeV}) \). We note that possible Landau pole problem can possibly be avoided by setting the dynamical scale of the ISS sector to be high enough in a way that is compatible with phenomenological requirements.

- We choose \( N_F \geq 5 \) and \( 3 \geq d \geq -3 \). The value of \( \tan \beta \) is chosen to be \( 40 \geq \tan \beta \geq 2 \).

- The parameter \( \mu \) is chosen to have the same sign as \( M_2 \) because this case gives positive SUSY contributions to \( g_\mu - 2 \).

In our scan we take into account the following collider and dark matter constraints:
(1) The lower bounds of LEP on neutralino and charginos masses, including the invisible decay of $Z$-boson. We require $m_{\tilde{\chi}^{\pm}} > 103$ GeV and the invisible decay width $\Gamma(Z \rightarrow \tilde{\chi}_{0}\tilde{\chi}_{0}) < 1.71$ MeV, which is consistent with the 2$\sigma$ precision electroweak measurement $\Gamma_{inv}^{non-SM} < 2.0$ MeV.

(2) For the precision electroweak measurements, we require the oblique $'S,T,U'$ parameters [35] to be compatible with the LEP/SLD data at 2$\sigma$ level [36].

(3) The combined mass range for the Higgs boson: $123$ GeV $< M_h < 127$ GeV from ATLAS and CMS [1, 2].

(4) The relic density of the neutralino dark matter satisfies the Planck result $\Omega_{DM} = 0.1199 \pm 0.0027$ [37] (in combination with the WMAP data [38]).

In Figs.1-3, we show the samples that survive the above constraints. From these figures we obtain the following observations:

(i) Our scenario can account for both the $g_{\mu} - 2$ anomaly and current Higgs mass measurement at the LHC. It is clear from Fig.1 that in order to solve the $g_{\mu} - 2$ anomaly at 2$\sigma$ level, the Higgs mass can reach 125.5 GeV (see the blue points in the left panel). However, to solve the $g_{\mu} - 2$ anomaly at 1$\sigma$ level, the Higgs boson mass cannot be in the best range $125.09 \pm 0.24$ GeV [39] (the red points in the left panel are upper bounded by 124.5 GeV). Such results are not surprising because the stop sector can give sizeable contributions to Higgs mass only if $A_t$ is large enough which however is controlled by $F_{\phi}$ in our scenario. As shown in the right panel of Fig.1, in the CMSSM the Higgs boson mass is upper bounded by 120 GeV in order to solve the $g_{\mu} - 2$ anomaly at 2$\sigma$ level. So, our scenario is much better in solving the $g_{\mu} - 2$ anomaly and satisfying the Higgs mass measurement.

![Figure 1](image_url)

**Figure 1**: The left and right panels show the scatter plots of the parameter space for our deflected AMSB scenario and the CMSSM, respectively. All the points survive the collider and dark matter constraints (1-4).

(ii) From Fig.2 we see the relations between the deflection parameter $d$, the messenger mass scale $M$ and $N_F$. We find that $M$ is constrained to below $10^{11}$ GeV if the $g_{\mu} - 2$
anomaly is solved at 2σ level. This upper bound on $M$ is lowered to $10^7$ GeV if the $g_\mu - 2$ anomaly is solved at 1σ level. A low $N_F$ value corresponds to a relatively high deflection parameter $d$. Besides, $d$ is constrained in the range $0.7 < d < 3$. It is clear from the right panel of Fig.3 that the value of $\tan \beta$ lies in the range $10 < \tan \beta < 20$ in order to explain the muon $g_\mu - 2$ discrepancy at 1σ level. The value of $F_\phi$ which determines the whole sparticle spectrum is upper bounded by 17 TeV (25 TeV) at 1σ (2σ) level.

![Figure 2](image)

**Figure 2**: Same as Fig.1, but showing the deflection parameter $d$ versus the messenger scale $M$ for our deflected AMSB scenario.

![Figure 3](image)

**Figure 3**: Same as Fig.1, but showing the spin-independent cross section of dark matter (the lightest neutralino) scattering off the nucleon versus the dark matter mass in the left panel and $F_\phi$ versus $\tan \beta$ in the right panel. The LUX [40] limits and the XENON1T [41] and LUX-ZEPLIN 7.2 Ton [42] sensitivities are plotted.

(iii) In the ordinary AMSB scenario, dark matter is mostly wino like which should be 2.7-3 TeV to provide enough cosmic dark matter content. Since the direct detection cross section for the pure wino is extremely small, below $\mathcal{O}(10^{-47}$ cm$^2$), it is very difficult to discover such a wino dark matter via direct detections. In our positively deflected AMSB scenario with $N_F \geq 4$ and $d \geq 1$, the lightest gaugino will in general no longer
be wino. We can see from the gaugino input that wino is always heavier than bino with large $N_F$ and positive $d \sim O(1)$. So the dark matter in our scenario can be either bino-like or higgsino-like. In this case, the dark matter may be accessible at the direct detections. As shown in the left panel of Fig.3, the parameter space for explaining the muon $g_\mu - 2$ discrepancy at $1\sigma$ level can be fully covered by the future LUX-ZEPLIN 7.2 Ton experiment [42].

Finally, we show the details for two benchmark points in Table 1 and Table 2. The benchmark points shown in these tables have $d > 0$ and $d < 0$, respectively.

**Table 1:** A benchmark point with $d > 0$. All the quantities with mass dimension are in GeV.

| $N_F$ | $d$ | $M$ | $F_\phi$ | $\tan\beta$ |
|------|-----|-----|----------|-------------|
| 10   | 1.59| $1.09 \times 10^4$ | $1.33 \times 10^4$ | 15.0 |
| $m_{\tilde{h}_u}^2$ | $m_{\tilde{H}_u}^2$ | $M_1$ | $M_2$ | $M_3$ |
| $6.98 \times 10^4$ | $1.20 \times 10^5$ | $1.82 \times 10^2$ | $5.48 \times 10^2$ | $1.88 \times 10^3$ |
| $m_{\tilde{Q}_1}$ | $m_{\tilde{U}_1}$ | $m_{\tilde{D}_1}$ | $m_{\tilde{L}_1}$ | $m_{\tilde{E}_1}$ |
| $1.30 \times 10^2$ | $1.26 \times 10^3$ | $1.26 \times 10^5$ | $3.46 \times 10^2$ | $1.53 \times 10^2$ |
| $m_{\tilde{Q}_{L,3}}$ | $m_{\tilde{U}_{L,3}}$ | $m_{\tilde{D}_{L,3}}$ | $A_U$ | $A_D$ |
| $1.30 \times 10^2$ | $1.26 \times 10^3$ | $1.26 \times 10^5$ | $-6.58 \times 10^2$ | $-6.50 \times 10^2$ |
| $A_L$ | $A_r$ | $A_t$ | $A_b$ |
| $-1.46 \times 10^2$ | $-1.17 \times 10^2$ | $-2.28 \times 10^2$ | $-5.34 \times 10^2$ |
| $\text{Br}(B \rightarrow X_S\gamma)$ | $\text{Br}(B_S^0 \rightarrow \mu^+\mu^-)$ | $g_\mu - 2$ | $\Omega_\chi h^2$ | $\sigma_F^{SI}$ |
| $3.25 \times 10^{-4}$ | $3.40 \times 10^{-9}$ | $1.82 \times 10^{-9}$ | $0.117$ | $1.09 \times 10^{-12}$ pb |
| $m_{h_1}$ | $m_{\chi_1^0}$ | $m_{\tilde{\tau}_1}$ | $m_{\chi_1^\pm}$ | $m_{\tilde{\chi}_1^0}$ |
| 124.4 | 84.1 | 100.2 | 464.5 | 3949.4 |

5 Conclusion

We proposed a deflected anomaly mediation scenario from SUSY QCD which can lead to both positive and negative deflection parameters. There is a smoothly transition between these two deflection parameter regions by adjusting certain couplings. This scenario can naturally have a SUSY spectrum with heavy colored sparticles and light sleptons. The discrepancy between the Brookheaven $g_\mu - 2$ experiment and the LHC data can be reconciled. We also found that the parameter space for explaining the muon $g_\mu - 2$ discrepancy at $1\sigma$ level can be fully covered by the future LUX-ZEPLIN 7.2 Ton experiment.

Acknowledgement

This work was supported by the Natural Science Foundation of China under grant numbers 11105124, 11105125, 11375001, 11172008, 11275245, 11135003, the Innovation Talent...
Table 2: A benchmark point with $d < 0$. All the quantities with mass dimension are in GeV. In this case the LSP is $\tilde{\tau}$ and thus $\Omega_\chi h^2$ and $\sigma^S_{PP}$ can not be calculated.

| $N_F$ | $d$ | $M$ | $F_\phi$ | $\tan\beta$ |
|-------|-----|-----|----------|------------|
| 10    | -2.66 | $4.57 \times 10^6$ | $1.83 \times 10^4$ | 12.2 |
| $m^2_{\tilde{H}_u}$ | $m^2_{\tilde{H}_d}$ | $M_1$ | $M_2$ | $M_3$ |
| $4.78 \times 10^3$ | $7.12 \times 10^4$ | $-1.02 \times 10^4$ | $-1.44 \times 10^4$ | $-2.54 \times 10^5$ |
| $m_{\tilde{Q}_L}$ | $m_{\tilde{U}_L}$ | $m_{\tilde{D}_L}$ | $m_{\tilde{L}_L}$ | $m_{\tilde{E}_L}$ |
| $8.40 \times 10^2$ | $8.01 \times 10^2$ | $7.99 \times 10^2$ | $2.67 \times 10^2$ | $1.11 \times 10^2$ |
| $m_{\tilde{Q}_L,3}$ | $m_{\tilde{U}_L,3}$ | $m_{\tilde{D}_L,3}$ | $A_U$ | $A_D$ |
| $8.27 \times 10^2$ | $7.73 \times 10^2$ | $7.99 \times 10^2$ | $-7.57 \times 10^2$ | $-7.45 \times 10^2$ |
| $A_L$ | $A_L$ | $A_{\tau}$ | $A_{\ell}$ | $A_b$ |
| $-2.12 \times 10^2$ | $-1.93 \times 10^2$ | $-2.73 \times 10^2$ | $-6.38 \times 10^2$ |
| $Br(B \to X_S \gamma)$ | $Br(B^0_S \to \mu^+ \mu^-)$ | $g_\mu - 2$ | $\Omega_\chi h^2$ | $\sigma^S_{PP}$ |
| $3.27 \times 10^{-4}$ | $3.38 \times 10^{-9}$ | $-2.0 \times 10^{-10}$ | - | - |
| $m_{h_1}$ | $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\tau}_1}$ | $m_{\tilde{g}_1}$ | $m_{\tilde{g}}$ |
| 125.6 | 476.8 | 383.5 | 1231.4 | 5229.1 |

project of Henan Province under grant number 15HASTIT017, the Young-Talent Foundation of Zhengzhou University, the Ri-Xin Foundation of BJUT, and Youth-Talents Foundation of education department of Beijing, and by the CAS Center for Excellence in Particle Physics (CCEPP).

References

[1] G. Aad et al.(ATLAS Collaboration), Phys. Lett. B710, 49 (2012).
[2] S. Chatrchyan et al.(CMS Collaboration), Phys. Lett.B710, 26 (2012).
[3] J. Cao et al., JHEP 1210, 079 (2012) [arXiv:1207.3698 [hep-ph]]; JHEP 1203, 086 (2012) [arXiv:1202.5821 [hep-ph]].
[4] Muon G-2 collaboration, G. Bennett et al., Phys. Rev. D 73 (2006) 072003.
[5] B.L. Roberts, Chin. Phys. C 34 (2010) 741.
[6] K. Hagiwara, A. Martin, D. Nomura and T. Teubner, Phys. Lett. B 649 (2007) 173; T. Teubner et al., Chin. Phys. C 34 (2010) 728; M. Davier et al., Eur. Phys. J. C 66 (2010) 127; Eur. Phys. J. C 66 (2010) 1.
[7] G. Aad et al. (ATLAS collaboration), Phys. Lett. B710 (2012) 67 (2011); Phys. Rev. D 87 (2013) 012008.
[8] S. Chatrchyan et al. (CMS collaboration), Phys. Rev. Lett. 107 (2011) 221804; JHEP 1210 (2012) 018.
[9] C. Han et al., JHEP 1310, 216 (2013) [arXiv:1308.5307 [hep-ph]].
[10] G. Aad et al. [ATLAS Collaboration], arXiv:1503.03290 [hep-ex].
[11] G. Barenboim et al., arXiv:1503.04184 [hep-ph]; U. Ellwanger, arXiv:1504.02244 [hep-ph]; B. Allanach, A. Raklev and A. Kvellestad, arXiv:1504.02752 [hep-ph]; A. Kobakhidze, A. Saavedra, L. Wu and J. M. Yang, arXiv:1504.04390 [hep-ph]; J. Cao, L. Shang, J. M. Yang and Y. Zhang, arXiv:1504.07869 [hep-ph].

[12] J. L. Lopez, D.V. Nanopoulos, X. Wang, Phys. Rev. D 49 (1994)366; U. Chattopadhyay and P. Nath, Phys. Rev. D 53 (1996)1648;

[13] J. Cao et al., Phys. Lett. B 710, 665 (2012) [arXiv:1112.4391 [hep-ph]].

[14] S. Akula, P. Nath, Phys. Rev. D 87, 115022 (2013).

[15] F. Wang, W. Wang and J. M. Yang, arXiv:1504.00505 [hep-ph]; J. Chakrabortty, A. Choudhury and S. Mondal, arXiv:1503.08703 [hep-ph]; K. Kowalska, L. Roszkowski, E. M. Sessolo and A. J. Williams, arXiv:1503.08219 [hep-ph]; K. Harigaya, T. T. Yanagida and N. Yokozaki, arXiv:1501.07447 [hep-ph]; M. A. Ajaib, I. Gogoladze and Q. Shafi, arXiv:1501.04125 [hep-ph]; F. F. Deppisch, N. Desai and T. E. Gonzalo, Front. Phys. 2, 00027 (2014) [arXiv:1403.2312 [hep-ph]]; T. Li, S. Raza, Phys. Rev. D 91, 055016 (2015).

[16] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999); G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998).

[17] A. Pomarol and R. Rattazzi, JHEP 9905, 013 (1999).

[18] I. Jack, D.R.T. Jones, Phys.Lett. B465 (1999) 148-154.

[19] E. Katz, Y. Shadmi, Y. Shirman, JHEP 9908 (1999) 015.

[20] Nobuchika Okada, Phys.Rev. D65 (2002) 115009

[21] Nobutaka Abe, Motoi Endo, Phys.Lett.B564:73-82,2003; Linda M. Carpenter,[arXiv: hep-ph/0510406]

[22] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B 189, 575 (1981); S. Dimopoulos and S. Raby, Nucl. Phys. B 192, 353 (1981); M. Dine and W. Fischler, Phys. Lett. B 110, 227 (1982); M. Dine and A. E. Nelson, Phys. Rev. D48, 1277 (1993); M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D51, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53, 2658 (1996); G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999).

[23] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982); H. P. Nilles, Phys. Lett. B 115, 193 (1982); L. E. Ibanez, Phys. Lett. B 118, 73 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119, 343 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B 120, 346 (1983); J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 121, 123 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 125, 275 (1983); N. Ohta, Prog. Theor. Phys. 70 (1983) 542; L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D 27, 2359 (1983).

[24] M. Dine and N. Seiberg, JHEP 03 (2007) 040.

[25] D. Sanford and Y. Shirman, Phys.Rev.D83:125020(2011).

[26] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B 212, 413 (1983).

[27] I. Jack, D.R.T. Jones, Phys.Lett.B482, 167 (2000).

[28] Z. Chacko, M. A. Luty, I. Maksymyk and E. Ponton, JHEP0004, 001 (2000).

[29] K. Intriligator, N. Seiberg, D. Shih, JHEP0604, 021 (2006).

[30] E. Witten, Nucl. Phys. B 202, 253(1982).
[31] R. Rattazzi, A. Strumia, J. D. Wells, Nucl. Phys. B576, 3 (2000).

[32] G. Bennett, et al., (Muon g-2 Collaboration), Phys. Rev. Lett. 92, 161802 (2004); Phys. Rev. D73, 072003 (2006).

[33] T. Moroi, Phys. Rev. D 53 (1996) 6565 [Erratum ibid. D 56 (1997) 4424].

[34] N. Okada, H. M. Tran, Phys. Rev. D87(2013)035024.

[35] G. Altarelli and R. Barbieri, Phys. Lett. B 253, 161 (1991); M. E. Peskin, T. Takeuchi, Phys. Rev. D 46, 381 (1992).

[36] LEP and SLD Collaborations, Phys. Rept. 427 (2006) 257.

[37] http://www.sciops.esa.int/SA/PLANCK/docs/Planck 2013 results 16.pdf.

[38] J. Dunkley et al.[WMAP Collaboration], Astrophys. J. Suppl. 180, 306 (2009).

[39] ATLAS, CMS Collaborations,[arXiv:1503.07589]

[40] D.S. Akerib et al. [LUX Collaboration], arXiv:1310.8214 [astro-ph.CO].

[41] E. Aprile et al. [XENON100 Collaboration], Phys. Rev. Lett. 109, 181301 (2012).

[42] LUX-Zeplin collaboration, http://lz.lbl.gov/detector/.