A model for knowledge transfer in a multi-agent organization based on lattice kinetic model

WU Weiwei¹, MA Qian¹,³, LIU Yexin²*, and KIM Yongjun¹

1. School of Management, Harbin Institute of Technology, Harbin 150001, China;  
2. School of Economics and Management, Harbin Institute of Technology at Weihai, Weihai 264209, China;  
3. Beijing Institute of Aerospace Information, Beijing 100854, China

Abstract: A study on knowledge transfer in a multi-agent organization is performed by applying the basic principle in physics such as the kinetic theory. Based on the theoretical analysis of the knowledge accumulation process and knowledge transfer attributes, a special type of knowledge field (KF) is introduced and the knowledge diffusion equation (KDE) is developed. The evolution of knowledge potential is modeled by lattice kinetic equation and verified by numerical experiments. The new equation-based modeling developed in this paper is meaningful to simulate and predict the knowledge transfer process in firms. The development of the lattice kinetic model (LKM) for knowledge transfer can contribute to the knowledge management theory, and the managers can also simulate the knowledge accumulation process by using the LKM.

Keywords: knowledge transfer, multi-agent system, knowledge field (KF), lattice kinetic model (LKM), knowledge diffusion equation (KDE).

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1. Introduction

In the knowledge economy era, knowledge management has become one of the basic conditions for the existence and development of a firm. In order for firms to secure a competitive advantage in the global market competition system, it is essential to continuously improve their knowledge management. In other words, knowledge can be said to be a competitive advantage in which firms can survive and grow in the rapidly changing economic environment. Therefore, many theoretical and practical experts are increasingly interested in research on knowledge management, such as the creation, transformation and sharing of knowledge. The effective development and dissemination of knowledge, has already become a major topic of discussion in the academic and business fields.

Nonaka claimed that knowledge creation is a factor that determines a firm’s competitive advantage by defining knowledge management as a process of creating new knowledge, spreading knowledge to the whole organization and shaping knowledge into products and services [1]. Tagliaventi and Mattarelli suggested that the linkage among people, the linkage between people and information, the ability to transform information into knowledge, and the encouragement of the innovativeness and creativity through knowledge environment are essential elements of knowledge management [2]. McNichols stated that new knowledge comes from an individual and that the core activities of an innovation firm are to transfer the individual knowledge to others, and in particular to make the individual knowledge into organizational knowledge [3].

Several studies have already been conducted to investigate the impact factors of knowledge transfer within an organization. Many scholars characterized the knowledge transfer process by using knowledge stickiness and further analyzed the impact factors [4–6]. Szulanski defined the stickiness of organizational knowledge as the difficulty of knowledge transfer within an organization, and analyzed four major factors affecting it: knowledge characteristics, knowledge source characteristics, knowledge recipient characteristics and context characteristics. Through empirical research, he found out three most important factors, leading to knowledge transfer stickiness, which are the lack of absorptive capacity of the recipients, the causal ambiguity, and the difficulty of communication among knowledge agents [7].

With the development of organizational theory, the field theory in physics has gradually penetrated into social sci-

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*Corresponding author.

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ence research. The field theory refers to the entirety of coexistence considered to be mutually sustainable, and it consists of space, phase, vector and behavior equation [8]. The field theory in organization is a status combination based on a common view of four basic elements, including subject, regulation, carrier and prediction, and it allows group members to interact at a high density.

Most economic management research using the kinetic model is to analyze goods trading in the market. Dragulescu and Yakovenko studied wealth distribution in a closed economy by employing a kinetic exchange model from the point of view of the gaseous kinetic theory for the first time [9]. They regarded mean amount per agent as the effective temperature and assumed that the equilibrium probability distribution of money satisfies the Boltzmann–Gibbs’s condition. Chakraborti and Chakrabarti investigated the saving propensity affecting wealth distribution in market economy based on Boltzmann transport equation [10], and then Matthes and Toscani conducted detailed analysis for their model [11]. Bisi reviewed some major results of the basic kinetic model for wealth distribution in market economy [12].

Recently, there are some attempts to apply physical theories, mainly in terms of the field theory and kinetic theory, into knowledge management. Zhang et al. introduced the concept of knowledge field (KF) and handled the knowledge flow in a similar way as the theoretical model in the electric field [13]. Jun and Xi discussed the mechanism of the knowledge flow by adopting the concepts of knowledge potential, resistance and field strength, and concluded that the knowledge flow process is essentially an interaction of strength and resistance [14]. Lucas and Moll built a Boltzmann-type equation model to explain the knowledge growth and time allocation [15]. Burger et al. applied the Boltzmann mean field game model into knowledge growth based on the model of Lucas and Moll [16,17].

In both natural and social phenomena, the mathematical modeling is very useful to explaining their essence. The equation-based models have been widely used in natural research due to the intuitiveness of expression and the flexibility of solution. However, the application of equation-based models into sociology has not been widely studied. Although there are some equation-based models used to analyze individual agents, such models may cause many problems in computational cost and accuracy when applied to multi-agent systems. The kinetic theory, which is a typical theory in physics, has advantages in application to multi-agent systems because the whole system is considered by statistic-stochastic methods. This is similar to the advantage of the kinetic theory (based on multi-particles) compared to Newtonian dynamics (based on single particle). Therefore, in this paper, we develop a new equation-based model by employing the kinetic theory, which analyzes the knowledge transfer behavior within an organization.

The rest of this paper is arranged as follows. After this introduction, Section 2 explains the knowledge accumulation process in an organization from the perspective of management science, and further analyzes the attributes that affect knowledge transfer. In Section 3, we define a KF similar to the field in physics and establish a knowledge diffusion equation (KDE) that denotes the conceptual model of knowledge transfer mathematically using the lattice kinetic model. In Section 4, numerical experiments to validate the lattice kinetic model (LKM) for knowledge transfer are carried out for different parameters. Section 5 provides the simulation results. Section 6 discusses the results, and Section 7 summarizes the research.

2. Theoretical analysis

2.1 Knowledge accumulation in agent-based organization

In this paper, we consider an individual as a knowledge agent. There are two ways in which an individual accumulates knowledge: one is the accumulation of knowledge based on his/her own level of knowledge; the other is knowledge absorption of partners through interaction with the surrounding agents associated with him/her [18,19]. The knowledge accumulation depends on an individual’s current knowledge level, i.e., the higher knowledge level of an agent is, the higher knowledge increase rate there will be [20]. It is clear that if an agent’s current knowledge level is relatively low, he/she may not have a significant increase of knowledge accumulation. Therefore, agents with a low knowledge level would try to enhance their knowledge level by absorbing knowledge from surroundings. The knowledge transfer in this case is specifically analyzed in Section 2.2 with the attributes affecting knowledge transfer. Besides, the agent’s knowledge accumulation is also negatively affected by effacement of knowledge. For example, according to Ebbinghaus’s forgetting theory, knowledge is lost over time when there is no attempt to maintain memory, so some of the agent’s cumulative knowledge disappears.

2.2 Analyzing knowledge transfer attributes

Knowledge transfer takes place between knowledge providers (or knowledge sources) and knowledge receivers. The knowledge agents have their own attributes and characteristics and do not act blindly but carry on knowledge transfer through interaction with a certain rule. We classify the knowledge transfer attributes into agent attributes, environmental attributes and relationship attributes.
2.2.1 Agent attributes

The agent attribute is unique to the agent who appears when an agent provides or receives knowledge. The appetite for providing and the explaining ability of a provider play important roles in knowledge transfer [21]. In the knowledge transfer process, the provider does not blindly provide knowledge, but rather establishes a specific knowledge provision policy and uses his/her own “virtuosity” to provide knowledge to receivers. Similarly, it is also clear that the absorptive ability of the receiver will influence knowledge transfer [22]. If the receiver is less capable of absorbing or retaining knowledge, then the knowledge transfer time will be longer (i.e., the knowledge transfer rate will vanish) and the knowledge transfer efficiency will be lowered. We include knowledge attributes in agent attributes. Because, although the tacitness, complexity and ambiguity of knowledge are usually included in knowledge attributes, the degree may vary depending on the ability of the agent himself/herself.

2.2.2 Environment attributes

When knowledge is transferred within an organization, the organizational environment, such as size, composition, and culture, will affect the speed and effectiveness of knowledge transfer [23]. Generally, the larger the scale and the more complex the composition, the greater the knowledge stickiness in an organization. If the organization culture is advantageous to knowledge transfer, the knowledge stickiness becomes small. The environment attributes cannot be directly determined by individual agents, and they depend on the common rules or the manager of the organization.

2.2.3 Relation attributes

Factors influencing knowledge transfer also include relationships among agents in addition to the agent attributes and environment attributes [24]. The closer the physical distance among agents in an organization is, the better the transfer of knowledge (especially tacit knowledge) is. Because tacit knowledge is transferred only through face-to-face contact among agents, the knowledge provider has a greater probability of providing more knowledge to the knowledge receiver with a physically closer distance. The relation attributes also contain normative relationship, such as culture, trust and values. If the level of culture and value are similar between interacting agents, they affect the knowledge transfer positively. In addition, the trust relationship between providers and receivers is an essential factor for knowledge transfer. This paper thus introduces a normative distance that implies all of these normative relationships. That is, there is a greater probability that knowledge is transferred between agents whose normative distance is closer. Finally, knowledge distance also affects knowledge transfer. Knowledge distance is a parameter that shows how much the provider and the receiver have common knowledge, and knowledge transfer becomes easier as the knowledge distance becomes closer. Knowledge distance differs from knowledge gap. The knowledge gap represents the differences on certain knowledge, but the knowledge distance includes all the relevant knowledge necessary for obtaining knowledge.

2.3 Parameters used in mathematical formulation

This paper uses some parameters in mathematical formulation, and this section first explains the practical meaning of these parameters.

There are two ways for a department to accumulate knowledge. One is to accumulate the employees’ knowledge by making them learn from each other. For example, an experienced R&D employee teaches inexperienced employees how to accomplish their tasks. The other is to absorb knowledge from outside. For example, the department learns new technologies from universities, other firms, or publications. The knowledge accumulation is affected by some important factors, including knowledge diffusivity, knowledge potential, knowledge depreciation rate, innovation capability, and learning curve coefficient.

Knowledge diffusivity is the speed of the knowledge diffusion of an employee. Knowledge diffusion is the process that shows how an employee interacts with others. When an employee is willing to share his/her knowledge with others, for example teaching the inexperienced or helping his/her colleagues solve problems, his/her knowledge diffusivity is high.

Knowledge potential is a totality of accumulated intangible (intellectual) assets, human resources with abilities and competencies to generate value and add knowledge. Knowledge potential determines the objectivity that flows from high to low diffusion to create knowledge flow and transfer. At the individual level, knowledge potential refers to the ability that an employee can create value by his/her knowledge. In an organization, an employee’s knowledge potential is high if he/she has more innovative work behavior and higher job performance. The knowledge level before accumulation is the initial potential.

Innovation capability is a continuous improvement of capability that an employee possesses to explore and exploit his/her cumulative knowledge for accumulating new knowledge. When an employee can generate new ideas or find new ways of doing work, he/she is regarded as having a high innovation capability.

The learning curve describes how new skills or knowledge can be quickly acquired initially, but subsequent
learning becomes much slower. The learning curve coefficient is the rate between the duration and the degree of effort invested in learning and experience and accumulated knowledge. The learning curve coefficient of an employee is high when he/she has a stronger learning capability.

Knowledge depreciation is the loss of knowledge due to a number of factors, including organizational forgetting, employee turnover, production methods and equipment change, and product obsolescence. The organization’s knowledge is a function of the knowledge embedded in its employees. When an employee leaves the department, the department loses some knowledge. Production methods and equipment change continually over time and so, knowledge of older production methods is less relevant for current production efficiency and labor productivity. Also, product features may change over time resulting in lower effective knowledge.

3. Mathematical formulation

In this section, by employing typical physical theories, such as the field theory and kinetic theory, we model the knowledge transfer process mathematically and build a numerical evolution equation. This formulation is a mathematical and physical expression for the conceptual model described in Section 2, which is included in the category of behavior dynamic research in econophysics.

3.1 KF

The idea of applying the field theory of physics to knowledge management has already been raised by several scholars [13–15]. The physical field begins to be introduced from considering force and matter as a system of physical quantities with different values on every point in a space. In the classical field theory, the space between an object and another object is filled with special material (e.g., fields such as gravitational field or temperature field) which transmits the force. Starting from this concept of the physical field, the KF can be defined as a system consisting of knowledge with different levels for each point (agent) in the space. Like all other fields, KF can also be viewed as a function of time and space. However, the space described in the KF can be handled as a specific space different from that given only by the physical distance. This is because the interactions for knowledge transfer among agents are affected not only by physical distance but also by other factors. Lewin and his followers have developed a sociological field theory that explains human behavior in terms of the relationship with the individual’s present situation (field) by applying the physical field theory to human behavior [25]. In this theory, individuals are the subject of a conscious behavior, and human behavior is determined not by the physical environment but by the psychological environment. In other words, human behavior is determined by the synthesis of internal forces such as attitude, expectation, emotion, and desire, which are influenced by the psychological environment. Therefore, we use the relationship attributes described in Section 2 to construct the coordinates of the special space for the KF, which reflects the influence of the psychological environment. Based on this, we introduce the parameters reflecting the characteristics of the KF.

Physical position coordinate (x-axis) is set by the physical distance among agents in the KF space. It can be regarded as essentially the same coordinates with those in the field theory of physics such as the temperature field, the flow field, the electric field, and the gravitational field. However, different from the position coordinate in physics, it does not mean instantaneous position, but the average position in a certain period. This is because, knowledge is not transferred instantaneously but requires a certain period, and it is impossible to consider agents in the same position, as case of material points in physics.

Normative position coordinate (y-axis) is set by the normative distance among agents in the KF space. As described in Section 2.2, relationship attributes such as the commonality of cultures and values, or trust relationships, affect knowledge transfer among agents. This is similar to the difference in physical location and thus can map the normative position coordinates in the KF space.

Knowledge position coordinate (z-axis) is set by the knowledge distance among the agents in the KF space, which is decided by the environment attributes. Like the physical position and the normative position, the knowledge position coordinate can also be responsible for one axis of the KF coordinates system.

Origin of coordinate (O) can be determined by analyzing the physical location, normative characteristics, and the degree of knowledge ownership of all the agents in the organization and by averaging the values.

Based on the above KF space coordinate system, we can obtain the position vector of any point in the organization, which can be shown as

\[ \mathbf{r} = r_x \mathbf{i} + r_y \mathbf{j} + r_z \mathbf{k} \]  

where, \( r_x \), \( r_y \) and \( r_z \) are components of the physical, normative and knowledge positions, respectively; \( \mathbf{i} \), \( \mathbf{j} \) and \( \mathbf{k} \) indicate unit vectors on \( x \)-axis, \( y \)-axis and \( z \)-axis, respectively.

In general, a vector is defined as the physical quantity that has both magnitude and direction. The position vector is a vector representing the position away from the origin of coordinates in the phase space of the KF. Its start-
ing point is the origin and the ending point is the position (points $P_1$ and $P_2$ in Fig. 1). The phase space is a concept used to represent the instantaneous position of an object. The phase space in the KF represents the position where the agent is located physically, normatively, and intellectually in the knowledge transfer process. In the figure, the position vectors of $P_1$ and $P_2$ are $r_1$ and $r_2$, respectively. As shown in Fig. 1, the three kinds of position coordinate components of $r_1$ and $r_2$ are $(r_{x1}, r_{y1}, r_{z1})$ and $(r_{x2}, r_{y2}, r_{z2})$, and the distances between the two agents are $r_{x2} - r_{x1}$, $r_{y2} - r_{y1}$ and $r_{z2} - r_{z1}$, respectively.

![Fig. 1 Position vector in three dimensions](image)

When considering the KF as a kind of material field, parameters characterizing can be introduced similar to those in physics.

(i) Knowledge potential ($\varphi$): from the viewpoint of social sciences, it refers to the capacity that an agent can create value by an individual’s knowledge. Meanwhile, from the physical viewpoint, it is a scalar quantity that appears because an agent has a specific energy at a specific location.

(ii) Knowledge diffusivity ($D$): it is a scalar quantity influenced by the environment and agent attributes of the KF, which shows the degree of knowledge diffusion.

### 3.2 KDE

In general, many phenomena in various scientific areas are expressed mathematically using well-known partial differential equations. The KDE is developed to describe the knowledge diffusion according to the random motion of micro-particles in physics.

Employing the coordinates system and parameters defined above, similar to thermal diffusion equation (TDE), the KDE can be written by

$$\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial r} \left( D \frac{\partial \varphi}{\partial r} \right).$$

In (2), the left side represents the change of knowledge potential over time, and the right side shows the change according to an agent position. In other words, (2) mathematically explains that knowledge transfers from high-potential agents to low-potential agents. Equation (2) considers only the exchange of knowledge among the agents without the creation or disappearance of knowledge. However, as mentioned in the conceptual model, all the agents constantly accumulate their knowledge based on their current knowledge, while some knowledge disappears by forgetting. Therefore, we would reflect the generation and disappearance of knowledge by adding a source term into (2).

Referring to the organizational learning theory of Epple et al. [26], the rate of the change of knowledge potential by knowledge creation can be written as

$$s_c = \alpha \varphi^\lambda$$

where $\alpha$ is the innovation capability of an agent, which refers to the ability to create knowledge based on one’s cumulative knowledge. $\lambda$ is the learning curve coefficient of an agent. As this value increases, the knowledge amount created by current knowledge exponentially increases. Both $\alpha$ and $\lambda$ are attributes that are unique to an agent.

The damping rate of an agent of knowledge oblivion can be written as

$$s_o = -\delta \varphi$$

where the minus sign represents decrease of the knowledge potential, and $\delta$ is a depreciation coefficient.

Adding (3) and (4) to the right-hand side of (2) yields a complete KDE, which considers creation, transfer, and extinction of knowledge.

$$\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial r} \left( D \frac{\partial \varphi}{\partial r} \right) + \alpha \varphi^\lambda - \delta \varphi$$

### 3.3 LKM

#### 3.3.1 Lattice kinetic equation

The LKM (commonly known as the lattice Boltzmann model in natural sciences) is a numerical model widely used in fluid dynamics [27]. Due to a few advantages, this model is increasingly being applied not only to thermal fluid dynamics but also to many other areas of natural science [28,29], such as plasma physics, material science, electromagnetics, and radiation physics. Based on the general principles of LKM, we first introduce the knowledge potential distribution function $f_k(r, t)$, which can be expressed as

$$\varphi(r, t) = \sum_k f_k(r, t)$$

where $k$ is a subscript indicating an agent that has a direct effect on the agent of interest; $f_k$ is the potential distribution function of the $k$th agent surrounding it (including
knowledge itself). As seen from (6), the knowledge potential of a particular agent is influenced by the potential distribution of the surrounding agents. This reflects the fact that one agent’s level of knowledge is influenced by knowledge levels of oneself and surrounding agents (with near physical, normative and/or knowledge distances).

By using the knowledge potential distribution function, the lattice kinetic equation for (6) can be given \[ \text{(7)} \] by

\[
f_k(r + \Delta r, t + \Delta t) = f_k(r, t) + \omega [f^\text{eq}_k(r, t) - f_k(r, t)] + \Delta t w_k(\alpha \varphi^\lambda - \delta \varphi)
\]

where \( \omega \) is a relaxation factor related to the knowledge diffusivity; \( w_k \) is a weight factor which represents the probability that the surrounding (self-contained) agent affects the node. \( f^\text{eq}_k \) is an equilibrium distribution function, which represents the diffusion equation \[ \text{(20)} \], expressed as

\[
f^\text{eq}_k(r, t) = w_k \varphi(r, t).
\]

The LKM given by (7) is performed by two procedures, collision and streaming \[ \text{(20)} \]. Matlab code for collision and streaming is shown as follows.

\[
f_{\text{col}}(:, :, k) = f(:, :, k) + (f_{\text{eq}}(:, :, k) - f(:, :, k)). \quad \omega \quad \text{omega} + w(k) * (\text{alpha} \ast T \ast \text{gamma} - \text{delta} \ast T);
\]

\[
f_{\text{col}}(:, :, k) = \text{circshift}(f_{\text{col}}(:, :, k), [e(k, 1), e(k, 2)]);
\]

\[
\%
\text{Streaming}:
\]

3.3.2 Lattice arrangement

Lattice selection is very important in the LKM for knowledge transfer. In the case of agent-based modeling, it results in the problem of choosing agents to interact with the attracting agent. For the KF space set up in the previous section, it can be seen that the agent with low knowledge potential receives knowledge from agents whose physical distance, normative distance, and/or knowledge distance are the shortest. Therefore, we can utilize the lattice arrangements used in the lattice Boltzmann method for natural scientific problems. In the following, we introduce the \( \text{DnQm} \) lattices, which is often used in the lattice Boltzmann method, and analyze it from the perspective of knowledge transfer. \( n \) is the number of dimensions, and \( m \) is the number of linkages.

(i) \( \text{D1Q3} \): in one-dimensional (1D) knowledge transfer, a lattice mode in which one agent \( (k = 0) \) interacts with two agents around him/her (Fig. 2). The term of one dimension means that the knowledge agent has the possibility of selecting his/her partner only by one of the three KF coordinates. For instance, when considering a one-dimensional problem with the \( y \)-coordinate, it denotes that the knowledge provider and the knowledge receiver ignore both the physical distance and the knowledge distance, and select the partner only based on the normative position. In this case, \( k \) can take one of the numbers 0, 1 and 2, and the weight factors are set to \( w_0 = 4/6, w_1 = 1/6 \) and \( w_2 = 1/6 \).

![Fig. 2 D1Q3 lattice mode](image)

(ii) \( \text{D2Q9} \): in the two-dimensional (2D) knowledge transfer, a lattice mode that the agent \( (k = 0) \) interacts with the surrounding eight agents (Fig. 3). In the \( \text{D2Q9} \) mode, the agent selects his/her partner by two coordinates. For example, in the \( (x, y) \) coordinate system, the knowledge provider ignores the knowledge distance and selects the physically and/or normatively closest agent as his/her partner. \( k \) is one of the numbers from 0 to 8, and the weight factor is

\[
w_k = \begin{cases} 
4/9, & k = 0 \\
1/9, & k = 1 \text{ to } 4 \\
1/36, & k = 5 \text{ to } 8 
\end{cases}
\]

![Fig. 3 D2Q9 lattice mode](image)

As can be seen from the values of the weight factor, the probability that the agent interacts with the neighboring agents is the same for different KF coordinates. That is, for example of the \( (x, y) \)-system, the agent chooses the partner by the same probability for both the object that is physically away by 1 and the object that is normatively away by 1. Therefore, when scaling the coordinates, the above matters must be considered.

(iii) \( \text{D3Q19} \): in the three-dimensional (3D) knowledge transfer, a lattice mode that the agent \( (k = 0) \) interacts with 18 agents (Fig. 4). The three types of distances between agents all have the same qualifications to choose partners. As in the \( \text{D2Q9} \) lattice mode, the same selection probability should be set for each type of the unit distance. \( k \) can take numbers from 0 to 18, and the weight factor is

\[
w_k = \begin{cases} 
1/3, & k = 0 \\
1/18, & k = 1 \text{ to } 6 \\
1/36, & k = 7 \text{ to } 18 
\end{cases}
\]

![Fig. 4 D3Q19 lattice mode](image)
4. Numerical experiments

In this section, based on the mathematical model obtained above, numerical experiments are conducted on knowledge transfer. The numerical experiments are carried out only on the 2D KF space with the purpose of validating the LKM for knowledge transfer. The simulation is performed by using the Matlab 2016a, where the nodes represent agents within the organization and 2 500 agents are regularly distributed on the 50 × 50 grid.

Parameters used in the experiment are specified as shown in Table 1, so as to reflect the heterogeneity of the agents, by referring to the study of Kim and Park [30,31], and the common principle of the lattice Boltzmann method [27].

| Parameter                        | Variable | Value region       |
|----------------------------------|----------|---------------------|
| Innovation capability           | α        | [0.005, 0.03]       |
| Initial potential               | ϕ₀       | [0.0, 0.1]          |
| Learning curve coefficient      | λ        | [0.02, 0.1]         |
| Depreciation rate               | δ        | [0.001, 0.005]      |
| Knowledge diffusivity           | D        | [0.1]               |

The initial knowledge potential ϕ₀ is a random variable following uniform distribution with the interval [0, 0.1]. The innovation capability, learning curve coefficient and depreciation rate are assumed to be the same for all agents in an organization. The knowledge diffusivity is affected by both environment attributes and agent attributes. Therefore, both the cases where each agent has the same value and that has a random value are considered.

If the agents on the outskirts of the computational domain do not carry out meaningful knowledge transfer to/from outside, then the boundary conditions for the potential can be written as

\[
\varphi|_{\text{outside}} = \varphi|_{\text{outside}}. \quad (9)
\]

Boundary condition for the potential distribution function is set by the non-equilibrium extrapolation scheme [20], which is shown as

\[
f_k|_{\text{outside}} - f_k^{eq}|_{\text{outside}} = f_k|_{\text{outside}} - f_k^{eq}|_{\text{outside}}. \quad (10)
\]

The Matlab code of the boundary condition for the knowledge potential and the potential distribution function is shown below.

```matlab
%%Potential boundary condition:
T(1,:) = T(2,:); T(NX + 1,:) = T(NX,:);
T(:,1) = T(:,2); T(:,NY + 1) = T(:,NY);

%%Distribution function boundary condition:
f(1,:) = f(2,:) + feq(1,:) - feq(2,:);
f(NX + 1,:) = f(NX,:) + feq(NX + 1,:) - feq(NX,:);
f(:,NY + 1,:) = f(:,NY,:) + feq(:,NY + 1,:) - feq(:,NY,:);
f(:,1,:) = f(:,2,:) + feq(:,1,:) - feq(:,2,:);
```

5. Simulation

Multi-agent organization consists of members who want to contribute to the performance of the entire organization by creating and sharing new knowledge. Therefore, if the KF space is defined as a multi-agent organization composed of many agents, environment and mutual relations, it is possible to simulate and analyze the knowledge propagation using the LKM. In this section, we examine the knowledge propagation in KF by using the lattice kinetic simulation for knowledge transfer.

Fig. 5 shows the potential distribution contour when all agents in the knowledge space have the knowledge potential of the interval [0, 0.1]. The knowledge potential of each agent evolves according to time by (6) and (7).

![Fig. 5 Initial potential contour](image_url)

Fig. 6 and Fig. 7 show the knowledge potential contours at \( t = 100 \) and \( t = 6000 \) on different knowledge diffusivities respectively. From the figures, it can be seen that the difference between the maximum and minimum values of knowledge potential is larger for smaller knowledge diffusivity. This is explained by the fact that the smaller the knowledge diffusivity is, the less knowledge sharing among agents there will be.
Fig. 6 Knowledge potential contour over different knowledge diffusivities at $t = 100$ ($\alpha = 0.02, \lambda = 0.05$ and $\delta = 0.001$)

Fig. 7 Knowledge potential contour over different knowledge diffusivities at $t = 6000$ ($\alpha = 0.02, \lambda = 0.05$ and $\delta = 0.001$)

Fig. 8 shows the change in organizational-mean potential with different innovation capabilities. An agent constantly raises his/her knowledge level based on his/her current knowledge level. The higher the innovation capability
of an agent, the greater the increase in his/her knowledge potential, and thus the greater the organizational-mean potential. However, after a certain period of time, due to the balance between knowledge creation and knowledge depreciation, the organizational-mean knowledge potential becomes saturated. It can be seen from Fig. 8 that this saturation phenomenon is reached earlier as the innovation capability is lower.

![Fig. 8 Organization-mean knowledge potential on different innovation capabilities (λ = 0.05, δ = 0.001, and D follows uniform distribution in [0, 1])](image)

Fig. 8 shows the effect of the learning curve coefficient on the knowledge growth by changing values of the learning curve coefficient from 0.02 to 0.1 with 0.02 as an interval. The results show that the time that achieves the saturation of organizational-mean knowledge potential does not make a big difference for different learning curve coefficients. However, the upper limit of the organizational-mean knowledge potential increases with a high learning curve coefficient.

![Fig. 9 Knowledge potential vs. time over different values of the learning curve coefficient (α = 0.02, δ = 0.001, and D follows uniform distribution in [0, 1])](image)

It can be seen from Fig. 10 that the organizational-mean knowledge potential exhibits a very large change with different knowledge depreciation rates. The final organizational-mean knowledge potential in case of the knowledge depreciation rate of 0.001 is almost six times larger than that for the knowledge depreciation rate 0.005. Also, the saturation time occurs at $t = 500$ for $d = 0.005$, whereas at $t = 5000$ for $d = 0.001$. This demonstrates that the smaller the knowledge depreciation rate is, the more advantageous the knowledge growth is.

![Fig. 10 Organization-mean knowledge potential vs time over different values of the depreciation rate (α = 0.02, λ = 0.05, and D follows uniform distribution in [0, 1])](image)

6. Discussions

Since this paper focuses on the knowledge transfer in a multi-agent organization, we take a department in a firm as an example to explain the meanings of the parameters in the model and the results of the model in the real world.

Employees in a new product development department are more willing to share their knowledge with others. For example, a new method of developing products will smoothly spread across all employees in a department when employees mastering this method share the knowledge of the method and their understandings of it with their colleagues.

When employees have good capacities of creating new knowledge based on their cumulative knowledge, the department can easily increase its knowledge stock, and thus raise its knowledge level. The higher the innovation capability is, the greater the increase in knowledge level is, and thus the greater the average knowledge level is. The department can also easily increase its knowledge level when its employees have a strong learning capability. That is, as the value of learning curve coefficient increases, the knowledge amount created by the current knowledge exponentially increases.

However, after a certain period, due to the balance between knowledge creation and knowledge depreciation, the average knowledge level will reach a certain amount
and will not increase. This period will be longer as the innovation capability or the learning capability is higher, and will be shorter as the innovation capability or the learning capability is lower. Compared with the learning capability, the innovation capability exerts greater influences on knowledge accumulation.

7. Conclusions

This paper presents an LKM that can explain knowledge transfer in a multi-agent organization. Based on previous research, the conceptual model for the knowledge transfer in an organization with multi-agents is established, and the knowledge accumulation process and knowledge transfer attributes are analyzed from a perspective of social sciences. The three-dimensional space coordinates system of the KF is constructed on the assumption that knowledge agents in an organization vary in degree of interaction according to the physical location, normative relationship, and nature of possessed knowledge. By introducing the concepts of knowledge potential, we establish the KDE similar to thermal diffusion equation, and add source terms based on the organizational learning theory. On the basis of the lattice Boltzman methodology for fluid dynamics, lattice kinetic modeling for knowledge transfer is constructed and numerical experiments are conducted. The effect of knowledge diffusivity, innovation capability, learning curve coefficient, and depreciation rate on the change of organizational knowledge potential is simulated and analyzed through application of the LKM to the multi-agent organization.

The results of lattice kinetic simulation for knowledge transfer in a multi-agent organization are summarized as follows. The knowledge potential contours reveal visually that the higher the knowledge diffusivity is, the smoother the knowledge transfer in the organization will be. Also, the effects of innovation capability and learning curve coefficient on knowledge transfer are analyzed. The higher the innovation capability is, the higher the knowledge level is, but the longer it takes to reach saturation. This tendency is also similar to the consideration for the influence of the learning curve coefficient. However, the difference can be found in the distribution range. When other parameters are fixed, the knowledge potential over change in the innovation capability is distributed over a wider range. This implies that the influence of the innovation capability is greater, and that firms need to pay more attention to the innovation capability of each agent in order to achieve success in knowledge accumulation.

The significance of this paper can be summarized as follows. First, it provides a new mathematical model to predict a firm’s knowledge behavior and a firm’s knowledge accumulation process. Although mathematical models such as the system dynamic model in various knowledge management studies have been developed to predict and explain the knowledge accumulation process, the models presented in this paper develop knowledge dynamics to a new level by replacing an agent’s behavior with a particle’s motion. Second, the physical theories are employed to mathematically model the knowledge diffusion. Although other examples of introducing mathematical and physical theories into the study of knowledge management can be found, the natural scientific theories employed in these studies are analytical tools, not technical models. Our models physically describe agents’ behavior in a firm based on the similarity of social and physical phenomena. Therefore, this paper establishes the link between knowledge management and physics.

There are also important implications for firms to promote their knowledge accumulation. First, managers should pay enough attention to enhancing employees’ abilities. In order to increase the knowledge accumulation efficiency, it is very important to enhance the learning capability and innovation capability. Thus, managers must present their opinions based on their various competencies. By creating conditions for quality education of their employees through system and culture, etc., managers can enhance the efficiency of knowledge dissemination. Second, because knowledge itself is lost, managers need to pay attention to the creation of new knowledge as well as updating and maintaining existing knowledge and expanding the space that can utilize a wide range of knowledge. Finally, managers may establish management strategies that can take competitive advantage by adopting good models, and forecast and evaluate the knowledge accumulation process. Applying superior mathematical and physical models to knowledge management, driven by rapidly evolving computational and computer technology, many effective measures for business performance will be found. Such applications must be accompanied by a preliminary analysis for the employees’ abilities, relationship among them, knowledge nature and organizational environment.

This paper also has several limitations and future research direction to overcome them as follows. In this paper, we have focused on the development of a new equation model that can explain the diffusion of knowledge in an organization, and have not discussed how to quantify physical distance, normative distance, knowledge distance and knowledge diffusivity. It is expected that our model can be applied to the real situation if it is combined with the empirical study while paying attention to solving these problems in future study. Furthermore, research on the change of the mutual relationship among agents in knowledge transfer process has not been reflected. This problem
can be solved by applying the gaseous kinetic theory similarly to the kinematic exchange model for goods trading in the market [32].

Despite these limitations, it is meaningful that we propose a possibility to quantitatively analyze and predict the knowledge growth characteristics in a multi-agent organization based on a new econophysical model.

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Biographies

WU Weiwei was born in 1978. He is a professor at Harbin Institute of Technology. He received his Ph.D. degree in management from Harbin Institute of Technology. His research interests are technology management and knowledge management.

E-mail: wuweiwei@hit.edu.cn
MA Qian was born in 1983. She is a Ph.D. candidate at School of Management, Harbin Institute of Technology. She is working in Beijing Institute of Aerospace Information as a senior engineering. Her research interests are technology management, knowledge management, and systems engineering. E-mail: 164164851@qq.com

LIU Yexin was born in 1990. He is an assistant professor at Harbin Institute of Technology at Weihai. He received his Ph.D. degree in management from Harbin Institute of Technology. His research interest is technology management. E-mail: liuyexin1990@163.com

KIM Yongjun was born in 1980. He is a Ph.D. candidate at Harbin Institute of Technology. His research interest is knowledge management. E-mail: kimyongjun@hit.edu.cn