Calculation of benefit reserves based on true m-thly benefit premiums

Riaman, Dwi Susanti, Agus Supriatna, and Budi Nurani Ruchjana
Department of Mathematics, Faculty of Mathematics and Natural Sciences
Universitas Padjadjaran, Indonesia
E-mail: riaman02@yahoo.co.id

Abstract. Life insurance is a form of insurance that provides risk mitigation in life or death of a human. One of its advantages is measured life insurance. Insurance companies ought to give a sum of money as reserves to the customers. The benefit reserves are an alternative calculation which involves net and cost premiums. An insured may pay a series of benefit premiums to an insurer equivalent, at the date of policy issue, to the sum of to be paid on the death of the insured, or on survival of the insured to the maturity date. A balancing item is required and this item is a liability for one of the parties and the other is an asset. The balancing item, in loan, is the outstanding principle, an asset for the lender and the liability for the borrower. In this paper we examined the benefit reserves formulas corresponding to the formulas for true m-thly benefit premiums by the prospective method. This method specifies that, the reserves at the end of the first year are zero. Several principles can be used for the determined of benefit premiums, an equivalence relation is established in our discussion.

1. Introduction
Risk was very close to the soul of man is death. Because death is the end of human life in this world, then people need to plan things that will be accomplished by themselves. One of plan is economic planning. Especially for someone who is already married, matters relating to the economy highly preferred. Such as paying bills, pay for school children, provide for the family, and so forth. Therefore, to reduce the economic burden of the family when someone dies is to buy a life insurance policy. With a life insurance policy, the insurance company (insurer) will compensate financial losses (compensation) experienced by the insured (the policyholder). The insurance product which provides death protection within a specified period is term life insurance. Unfortunately, if a person lives longer until the insurance expires, the premium money will be forfeited.

In life insurance, the insurance company will receive the premiums of the insured person. Premiums received by insurance companies, not just used to provide compensation to policyholders, but there are costs that are required in carrying out their duties. The costs would be borne by the policyholder paid together with net premium and so-called gross premiums. In fact, premiums in the early years were not enough to cover the cost, but the shortfall will be covered by premiums recent years. This situation force insurance companies looking for sources of additional funding to cover the costs of the early years which will then be repaid from premiums a year later. The insurance company has a duty to provide insurance money...
that would be taken as a reserve. Reserves calculation involving net premiums and fees, called reserves are adjusted. There are several methods used to calculate the adjusted reserves, one of which is the method the New Jersey. This paper will discuss the calculation of the adjusted reserves in term life insurance by using New Jersey [1].

2. Main Result
2.1. Mortality Table
Insurance companies use Mortality Table to calculate insurance premiums. This table contains the person’s chances of dying by age of a group of people who are insured (policyholders) and are expected to describe the actual probability of dying from a group of people who are insured.

The number of people who were born at the same time is denoted by \( l_0 \), of a number of these people will be \( l_0 \) of this person, there are \( l_x \) the people who will reach the age of \( x \) years old at the same time. The number of people who died from \( l_x \) people before reaching age \( x + 1 \) is denoted by \( d_x \) thus:

\[
d_x = l_x - l_{x+1}
\]  

(1)

and \( l_x = d_x + d_{x+1} + d_{x+2} + d_{x+3} + \ldots + d_{x+n} \) where \( n \geq 1 \). Probability of person aged \( x \) will die before reaching age \( x + 1 \) year, is denoted by \( q_x \), then:

\[
q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x}
\]

(2)

There are some mortality tables are used in the calculation of insurance. In this paper, the Indonesian Mortality Table 1993 is used.

2.2. Annuity
Annuity is a payment of a certain amount, which is conducted every certain time interval and, on an ongoing basis. Life annuity in which the payment is made at a period of time called the term life annuity. Life annuity futures end with a period of \( n \) years denoted by \( a_{x:|\bar{n}} \) whereas the initial term life annuity is denoted by \( \ddot{a}_{x:|\bar{n}} \), the calculation is as follows:

\[
a_{x:|\bar{n}} = \frac{N_{x+1} - N_{x+n+1}}{D_x}
\]

(3)

\[
\ddot{a}_{x:|\bar{n}} = \frac{N_x - N_{x+n}}{D_x}
\]

(4)

2.3. Discrete Term Life Insurance
Discrete term life insurance is term life insurance, which is repaid at the end of the policy year, the intent of this policy is the final year of the policy at the time of death. Single premium life insurance discrete futures to age \( x \), \( n \)-year term insurance, a sum amount of 1 paid at the end of policy year, denoted by \( A_{x:|\bar{n}} \) [1]

\[
A^1_{x:|\bar{n}} = \frac{M_x - M_{x+n}}{D_x}
\]

(5)

2.4. The Annual Premium
The annual premium is the amount of premiums payment is the same every year. The annual premium \( n \)-year term life insurance, the sum insured 1, payable at the end of the policy year is \( P_{x:|\bar{n}} \) [5]

\[
P^1_{x:|\bar{n}} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}
\]  

(6)
The annual premium whole life insurance with insurance money paid out by the end of the policy and the payment period $m$ years $mP_x$ is

$$mP_x = \frac{M_x}{N_x - N_{x+n}} \quad (7)$$

2.5. Benefit Reserves
Reserves are the amount of money available on the company within the coverage. The calculation of the reserve fund is based on the assumption of annual premium. Reserve fund formed due to excess funds from the annual premium on the value of insurance between the preceding year and the year. Insurance companies sometimes have to pay compensation because the insured dies before the payment period expires. To overcome this problem, the insurer must pay compensation for the use of reserves [3].

Prospective reserves calculation at year $t$ is the value of benefits that will come with the cash value is reduced premiums to come. Mathematically, prospective reserves for term life insurance with a sum insured $1$ is

$$tV_{x:n}^1 = A_{x+t:n-t}^1 - \alpha P_{x:n}^1 \cdot \bar{a}_{x+t:n-t} \quad (8)$$

2.6. Adjusted Reserves
Reserves are adjusted premium reserve calculation that uses the assumptions adjusted premiums. Sources of additional funds to cover the initial costs can be obtained by adjusting the premium reserve. Let $P$ stated net premium for a particular type of insurance. The premium will be replaced with $\alpha$ in the first year followed by a $\beta$ in subsequent years. $\alpha$ and $\beta$ are adjusted premiums. Policyholders pay only net premiums equal each year, the $P +$ fees. Value $\alpha$ and $\beta$ exist only in the actuarial calculations and had nothing to do with the policyholder. $P$ on the one hand, and $\alpha$ and $\beta$ on the other hand are connected by [5]

$$P = \alpha + \beta \quad (9)$$

2.7. Benefit Reserves based on True $m$-thly Benefit Premiums
The benefit reserve formulas corresponding to the formulas for true $m$-thly benefit premiums. The method is based on prospective. [1]

$$hV_{x:n}^{(m)} = A_{x+k:n-k} - hP_{x:n}^{(m)} \cdot \bar{a}_{x+k:n-k}, \quad h < k \quad (10)$$

The difference between $hV_{x:n}^{(m)}$ and $hV_{x:n}$ of limited payment endowment insurance, for $h < k$,

$$hV_{x:n}^{(m)} - hV_{x:n} = hP_{x:n}^{(m)} \cdot \bar{a}_{x+k:n-k} - hP_{x:n}^{(m)} \cdot \bar{a}_{x+k:n-k}$$

$$= hP_{x:n}^{(m)} \cdot \bar{a}_{x+k:n-k}$$

(11)

Based on the assumption of a uniform distribution of deaths in each years of age, equation (15), becomes

$$hV_{x:n}^{(m)} = \beta(m)P_{x:n}^{(m)} V_{x:n}^1$$

In other words, the benefit reserve for an insurance with true $m$-thly benefit premiums equals to the corresponding fully discrete benefit reserve plus a fully discrete benefit reserve over the premium paying period for a fraction, $\beta(m)$, of the true $m$-thly benefit premium for the plan of insurance. [2]
Table 1. Reserves are adjusted by the method of New Jersey to the age of 30 years on a 30 years term life insurance

| IDR 86,991,41 | IDR - |
| IDR 178,165,18 | IDR 257,578,13 |
| IDR 273,196,01 | IDR 350,046,92 |
| IDR 372,649,75 | IDR 446,783,58 |
| IDR 477,094,52 | IDR 548,345,66 |
| IDR 586,518,27 | IDR 654,711,36 |
| IDR 699,811,36 | IDR 764,763,01 |
| IDR 814,897,52 | IDR 876,417,18 |
| IDR 928,997,30 | IDR 986,887,12 |
| IDR 1,040,669,67 | IDR 1,094,719,97 |
| IDR 1,148,952,05 | IDR 1,198,938,87 |
| IDR 1,253,449,30 | IDR 1,299,131,82 |
| IDR 1,354,272,08 | IDR 1,395,390,47 |
| IDR 1,451,234,91 | IDR 1,487,509,27 |
| IDR 1,544,190,16 | IDR 1,575,318,85 |
| IDR 1,631,770,71 | IDR 1,657,429,72 |
| IDR 1,712,003,66 | IDR 1,731,844,44 |
| IDR 1,782,478,85 | IDR 1,796,125,29 |
| IDR 1,840,068,41 | IDR 1,847,113,21 |
| IDR 1,879,459,35 | IDR 1,879,459,35 |
| IDR 1,894,318,08 | IDR 1,894,318,08 |
| IDR 1,878,182,09 | IDR 1,878,182,09 |
| IDR 1,825,924,22 | IDR 1,825,924,22 |
| IDR 1,735,628,19 | IDR 1,735,628,19 |
| IDR 1,601,393,37 | IDR 1,601,393,37 |
| IDR 1,416,615,10 | IDR 1,416,615,10 |
| IDR 1,173,685,80 | IDR 1,173,685,80 |
| IDR 863,666,84 | IDR 863,666,84 |
| IDR 476,462,83 | IDR 476,462,83 |
| IDR - | IDR - |

2.8. Problems

On the basis of the illustrative Life Table under the UDD of each year of age and i = 6%, calculate the following for a 20-year endowment insurance issued to (50) with a unit benefit and true semiannual benefit premiums:

- The benefit reserve at the end of the tenth year if the benefit is payable at the end of the year of death.
- The benefit reserve at the end of the tenth year if the benefit is payable at the moment of the death.

And the solution of the problems, require

\[ A_{60}^{1} = 0.136789, \quad A_{60}^{2} = 0.587984, \quad a_{60}^{1} \]

\[ 10V_{50}^{1} = A_{60}^{1} - P_{50}^{1} \times a_{60}^{1} = 0.0525752 \]

\[ 10V_{50}^{1} = A_{60}^{2} - P_{50}^{2} \times a_{60}^{2} = 0.355380 \]

Under the assumption of a uniform distribution of deaths over each year age, we have
$\alpha_{60:10}^{(2)} = \alpha(2)\overline{a}_{60:10} - \beta(2)(1 - 10 E_{60}) = 7.139240$

Then the benefit reserve, $10V_{50:20}$,

$A_{60:10} - P_{50:20}^{(2)} \times \overline{a}_{60:10} = 0.355822$ or

$10V_{50:20} + \beta(2) P_{50:20}^{(2)} 10 V_{50:20}^{1} = 0.355822$

And the second one need additional calculated value:

$\frac{1}{5} A_{50:20}^{1} = 0.13423835$

$\frac{1}{5} A_{60:10}^{1} = 0.14085233$

$A_{50:20} = 0.36471188$

$P (\overline{A}_{50:20}) = \overline{A}_{50:20} - \lambda_{50:20}^{(1)} = 0.0329873$

$\overline{A}_{60:10} = 0.59204806$

$10V(\overline{A}_{50:20}) = \overline{A}_{60:10} - P (\overline{A}_{50:20}) \overline{a}_{60:10} = 0.3569475$

$10V^{(2)}(\overline{A}_{50:20}) = \overline{A}_{60:10} - P (\overline{A}_{50:20}) \overline{a}_{60:10}^{(2)} = 0.3573937$

$\beta(2) P^{(2)} (\overline{A}_{50:20}^{(2)}) 10 V_{50:20} = 0.000446$

3. Conclusion

The amount of the reserve fund with an annual premium paid will be greater than the reserves for premiums paid m times a year. Or in other words the amount of the reserve fund will be more flexible if the premiums payable m times a year.

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