Analyses and Solutions of Errors on GPS/GLONASS Positioning

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1 Introduction

The potential 48-satellite constellation offered by the combination of observations from both GPS and GLONASS has extracted considerable interest of existing GPS users and communities all over the world. However, the combination of these satellite systems raises problems that must be considered. The system time and reference frame of GLONASS are different from that of GPS. Moreover, to distinguish among individual satellites, GLONASS satellites employ different frequencies to broadcast their navigational information, which make existing GPS data processing software unable to process GLONASS observations.

A lot of work has been done to deal with the forenamed problems[1-5]. This paper mainly analyzes the major errors and discusses their reduction approaches with respect to combined GPS/GLONASS positioning.

2 Resolving the difference of system time

In combined GPS/GLONASS data processing, the difference between the two system times must be considered. Otherwise, systematic errors will affect the combined positioning solution. To determine this difference in the time reference systems, a number of procedures are available, and two of them will be discussed as follows.

2.1 Introducing a second receiver clock offset

Different receiver clock offsets are introduced with respect to GPS and GLONASS system time. These two clock offsets are

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ABSTRACT  This paper focuses mainly on the major errors and their reduction approaches pertaining to combined GPS/GLONASS positioning. To determine the difference in the time reference systems, different receiver clock offsets are introduced with respect to GPS and GLONASS system time. A more desirable method for introducing an independent unknown parameter of fifth receiver, which can be canceled out when forming difference measurements, is discussed. The error of orbit integration and the error of transformation parameters are addressed in detail. Results of numerical integration are given. To deal with the influence of ionospheric delay, a method for forming dual-frequency ionospheric free carrier phase measurements is detailed.
instantaneously determined at each observation epoch together with three unknowns of the receiver position.

The simplified non-linear observation equation for a pseudorange observation to satellite $S$ of an arbitrary system (GPS or GLONASS) by an observer $R$ can be written as:

$$PR_S^R = \rho_R^S + c \cdot \delta t_R^S - c \cdot \delta t_S$$  \hspace{1cm} (1)

Expanded with Taylor series around an approximate position $P_0$, we obtain the linearized equation

$$\begin{align*}
PR_S^R &= \rho_0^S + \frac{x_0 - x_S}{\rho_0^S} \cdot (x_R - x_0) + \\
&\quad \frac{y_0 - y_S}{\rho_0^S} \cdot (y_R - y_0) + \quad \frac{z_0 - z_S}{\rho_0^S} \cdot (z_R - z_0) + \\
&\quad c \cdot \delta t_R^S - c \cdot \delta t_S
\end{align*}$$  \hspace{1cm} (2)

where $PR_S^R$ is observed pseudorange; $c$ is the speed of light; $x_0, y_0, z_0$ are the approximate coordinates of receiver; $x_R, y_R, z_R$ are the receiver’s true coordinates to be determined; $x_S, y_S, z_S$ are the coordinates of satellite; $\delta t_R$ is the receiver clock offset with respect to system time; $\delta t_S$ is the satellite clock offset with respect to system time; $\rho_0^S = \sqrt{(x_0 - x_S)^2 + (y_0 - y_S)^2 + (z_0 - z_S)^2}$ is the geometric distance between the approximate position and the satellite position.

With the receiver clock error $\delta t_R = t_R - t_{SP}$ (where $t_{SP}$ is the GPS or GLONASS system time, respectively) as one of the unknowns, it is clear that in combined GPS/GLONASS processing two receiver clock offsets have to be introduced, one for the receiver offset with respect to GPS time and the other for that with respect to GLONASS time. Thus two different observation equations are obtained for a GPS satellite $i$ and a GLONASS satellite $j$:

$$\begin{align*}
PR_R^i &= \rho_0^i + \frac{x_0 - x_i}{\rho_0^i} \cdot (x_R - x_0) + \\
&\quad \frac{y_0 - y_i}{\rho_0^i} \cdot (y_R - y_0) + \quad \frac{z_0 - z_i}{\rho_0^i} \cdot (z_R - z_0) + \\
&\quad c \cdot \delta t_{R, GPS}^i - c \cdot \delta t_i^i
\end{align*}$$  \hspace{1cm} (3)

$$\begin{align*}
PR_R^j &= \rho_0^j + \frac{x_0 - x_j}{\rho_0^j} \cdot (x_R - x_0) + \\
&\quad \frac{y_0 - y_j}{\rho_0^j} \cdot (y_R - y_0) + \quad \frac{z_0 - z_j}{\rho_0^j} \cdot (z_R - z_0) + \\
&\quad c \cdot \delta t_{R, GLONASS}^j - c \cdot \delta t_j^j
\end{align*}$$  \hspace{1cm} (4)

where $\delta t_{R, GPS} = t_R - t_{GPS}^i$ and $\delta t_{R, GLONASS} = t_R - t_{GLONASS}^j$. $t_{GPS}$ is the system time of GPS and $t_{GLONASS}$ is the system time of GLONASS.

Due to the one more unknown as compared with GPS solely positioning, an additional (fifth) observation is necessary to obtain a positioning solution. Since the combined use of GPS and GLONASS approximately doubles the number of observations, the sacrifice of one observation can easily be accepted. Eqs. (3) and (4) can be used to form the normal equation in order to resolve the five unknowns with conventional methods, such as least square adjustment and Kalman Filtering.

It should be noted that a solution of these equations would be possible only if there are observations of both GPS and GLONASS satellites. If all but one observed satellites are from one system, with only one satellite observation from the second system, this additional observation only contributes to the second receiver clock offset and has no influence on the computed position.

For this method, tests have been performed with 30 hours real data obtained with Legacy GPS/GLONASS dual-frequency receivers in Nov. 1999,
and more details are shown in Reference [6]. Compared with the next algorithm, the result of GLONASS time offsets is subtracted from GPS time offsets, and the RMSs of these differences are calculated as shown in Fig. 1. The average of RMSs of differences is about 40ns (1 sigma), slightly higher than that reported by others [1].

2.2 Introducing the difference of system time

Starting with the pair of Eqs. (3) and (4), the receiver clock offset with respect to GLONASS system time can be rewritten as:

$$\delta t_{R, \text{GLONASS}} = t_R - t_{\text{GLONASS}} = t_R - t_{\text{GPS}} + t_{\text{GPS}} - t_{\text{GLONASS}}$$

Eq. (4) then transforms to

$$P_R = \rho_0 + \frac{x_0 - x'}{\rho_0} \cdot (x_R - x_0) + \frac{y_0 - y'}{\rho_0} \cdot (y_R - y_0) + \frac{z_0 - z'}{\rho_0} \cdot (z_R - z_0) + c \cdot \delta t_{R, \text{GPS}} + c(t_{\text{GPS}} - t_{\text{GLONASS}}) - c \cdot \delta t$$

 Principally, this method is equivalent to the one described in the former section, but it is more desirable. The fifth unknown parameter \( t_{\text{GPS}} - t_{\text{GLONASS}} \) as the difference of system time is independent of receivers. When differences of the same kind of measurements are formed between two receivers, this unknown will be canceled out.

Similar to the case of two separated receiver clock offsets, a solution of these equations would be possible only if there are observations of both GPS and GLONASS satellites. If all but one observed satellites are from one system, with only one satellite observation from the second system, this additional observation only contributes to the difference of system time frames and has no influence on the computed position. It is reasonable in both approaches to neglect the only one observation from the second satellite system in practical data processing.

Test has also been done with the same data as in former section. The RMSs of differences of system time between GLONASS and GPS are shown in Fig. 2. Clearly, the average of RMSs of differences is very close to that described in former section, but the deviations are less than that of the former ones, which shows that the introduction of the difference of system time is more desirable.

3 Orbit integration and coordinate transformation

Since GPS navigation and positioning have become the standard in western countries and WGS84 is more widely utilized than PZ-90, it is considered best to transform GLONASS satellite position from PZ-90 to WGS84 in combined navigation and positioning, thus the user position is also obtained in WGS84.

GLONASS broadcast ephemerides contain the satellite position in PZ-90 at a reference time, together with the satellite velocity and its acceleration due to luni-solar attraction. To obtain GLONASS satellite position at an epoch other than reference time, the satellite’s equation of motion has to be integrated.

The error of GLONASS satellite coordinate is mainly composed of the error of orbit integration and the error of transformation parameters.
3.1 Numerical integration

According to Newton's laws of motion, the motion of a satellite orbiting the earth is determined by the forces acting on it. The primary force acting on satellite is caused by Earth's gravity field potential. Expanding the non-spherical part of the gravitational potential into spherical harmonics, taking the influence of Earth rotation into account, assuming the acceleration of the satellite due to lunar and solar gravitation to be constant over a short time span of integration, and ignoring all other insignificant forces, the GLONASS satellite's equation of motion can be finally written as follows[6]:

\[
\begin{align*}
\frac{dV_x}{dt} &= -\frac{GM}{r^3}x + \frac{3}{2}C_20 \frac{GMa_z}{r^5}x \left(1 - \frac{5a_z^2}{r^2}\right) + x_{LS} + a_t^2 x + 2a_tV_y \\
\frac{dV_y}{dt} &= -\frac{GM}{r^3}y + \frac{3}{2}C_20 \frac{GMa_z}{r^5}y \left(1 - \frac{5a_z^2}{r^2}\right) + y_{LS} + a_t^2 y - 2a_tV_x \\
\frac{dV_z}{dt} &= -\frac{GM}{r^3}z + \frac{3}{2}C_20 \frac{GMa_z}{r^5}z \left(1 - \frac{5a_z^2}{r^2}\right) + z_{LS}
\end{align*}
\]

where \(x, y, z\) are the coordinates of satellite; \(a_t, a_t^{LS}, z_{LS}\) are the luni-solar acceleration; \(r = \sqrt{x^2 + y^2 + z^2}\) is the distance of satellite to the center of Earth; \(a_t = 6378.136\) m is equatorial radius of Earth; \(GM = 3.986 \times 10^{14}\) m\(^3\)s\(^{-2}\) is gravitational constant of Earth; \(C_20 = -1.082\) is the second zonal coefficient; \(\omega = 7.292 \times 10^{-5}\) s\(^{-1}\) is the rotation rate of Earth.

The discrepancy between forward and backward integration for 15 min is shown in Fig. 3. The fourth order Runge-Kutta method is used for the numerical integration. Table 1 shows the discrepancies between a forward integration (initial value is the broadcast position for epoch \(i\)) and a backward integration (initial value is the broadcast position for epoch \(i + 30\) min) for the epoch \(i + 15\) min, using three different integration step widths (0.1 s, 1 s and 30 s). The 0.1 s, 1 s and 30 s integration step widths lead to nearly identical results, with the root-mean-squares (RMS) no more than 1 m for all components.

| Step(s) | Components | Max. | Min. | Average | RMS |
|---------|------------|------|------|---------|-----|
| 0.1     | X          | 1.220| -1.004| 0.004   | 0.801|
|         | Y          | 0.342| -2.158| -0.698  | 0.744|
|         | Z          | 1.449| -1.220| 0.469   | 0.955|
| 1.0     | X          | 1.220| -1.014| 0.004   | 0.801|
|         | Y          | 0.342| -2.158| -0.698  | 0.744|
|         | Z          | 1.449| -1.243| 0.469   | 0.955|
| 30.0    | X          | 1.220| -1.004| 0.126   | 0.855|
|         | Y          | 0.342| -2.158| -0.634  | 0.706|
|         | Z          | 1.457| -1.251| 0.382   | 0.951|

3.2 Coordinate transformation

Considering the 3D transformation between PZ-90 and WGS84, the well-known 7-parameter Helmert transformation can be used:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{\text{WGS84}} = \begin{bmatrix}
\frac{dX_0}{dm} \\
\frac{dY_0}{dm} \\
\frac{dZ_0}{dm}
\end{bmatrix} + \begin{bmatrix}
1 & \beta_x & \beta_y \\
-\beta_x & 1 & \beta_z \\
-\beta_y & -\beta_z & 1
\end{bmatrix} \begin{bmatrix}
U \\
V \\
W
\end{bmatrix}_{\text{PZ-90}}
\]

where \(dX_0, dY_0, dZ_0\) are the coordinates of the origin of PZ-90 in frame WGS84; \(\beta_x, \beta_y, \beta_z\) are the differential rotations around the axes \((U, V, W)\), respectively; \(dm\) is the differential scale change.

There are several possible methods to determine the transformation parameters from PZ-90 to WGS84. Three points known in both systems are mathematically sufficient to calculate the desirable parameters. However, as much points as possible are desired to obtain a good quality of the derived parameters. For the computation of these parame-
ters, ground-based techniques and space-based techniques are used by researchers\(^7\).

One of the most desirable parameters is given by Rossetti et al. (1996). When applied to the station coordinates, this transformation yields a residual of 30-40 cm RMS.

\[ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{WGS84}} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} + \begin{bmatrix} U \\ V \\ W \end{bmatrix}_{\text{PZ-90}} \]

It is given by Evgeny Bykhanov that an internal coordinate system transformation accuracy of 0.3 m can be achieved\(^\text{11}\). The accuracy of PZ-90 relative to WGS84 is confirmed by Earth rotation data collected in the system control center from GLONASS observations. These ERP (GL) differs from the respective ERP (IERS) by no more than 0.3 m (1 sigma).

The error with respect to GLONASS satellite coordinate is mainly composed of the error of orbit integration and the error of transformation parameters. So the synthetic influence of the two errors is about 1 m (1 sigma), which is sufficient for close range differencing navigation and positioning. Moreover, the international GLONASS Experiment (IGEX) combined GPS/GLONASS precise ephemerides are available in post processing positioning.

4 Ionospheric correction

Similar to GPS positioning, ionospheric delay is also one of the major constraints in combined GPS/GLONASS precise positioning. But the ionospheric delay of L1 or L2 signal varies with different GLONASS satellites. Whereas it is identical for GPS satellites, a reduction method applicable for combined GPS/GLONASS positioning has to be developed. Since GLONASS grants full access to the L2 frequency, it enables the properly equipped user to deal with the ionosphere problem by using dual-frequency ionospheric free carrier phase measurements.

The ionospheric path delay of a GPS or GLONASS satellite signal depends on the electron content of the ionosphere, the frequency of the signal and the distance that the signal travels through the ionosphere, which in turn depends on the satellite elevation. It can be written as:

\[ c \cdot \delta r_R^{\text{IONO}} = \frac{1}{\cos z} \frac{40.3 \text{m}^2/\phi^2}{f^2} \cdot \text{TEC} \] (10)

where \( z \) is the zenith distance of signal at ionospheric piercing point; \( f \) is the frequency of carrier signal; TEC is the total electron content of ionosphere.

The GLONASS observation equation for carrier phase measurements from receiver \( R \) to satellite \( S \) scaled in cycles can be written as:

\[ \varphi_R^S = \frac{1}{\varphi^{\text{PR}}_R + N^R_S + f^S \cdot \delta t_R - f^S \cdot \delta t^S + f^S \cdot \delta t^{\text{Trop}}_R - f^S \cdot \delta t^{\text{IONO}}_R + \varepsilon^R_S} \]

where \( \varphi^{\text{PR}}_R = \sqrt{(x_R - x^S)^2 + (y_R - y^S)^2 + (z_R - z^S)^2} \), and \( x_R, y_R, z_R \) are the coordinates of receiver; \( x^S, y^S, z^S \) are the coordinates of satellite; \( f^S \) is the wavelength of carrier signal of satellite \( S \); \( f^S \) is the frequency of satellite signal; \( \varphi^{\text{PR}}_R \) is the carrier phase measurement of receiver \( R \) to satellite \( S \); \( N^R_S \) is the carrier phase ambiguity of receiver \( R \) to satellite \( S \); \( \delta t_R \) is the receiver clock offset with respect to system time; \( \delta t^S \) is the satellite clock offset with respect to system time; \( \delta t^{\text{Trop}}_R \) is the tropospheric delay of signal; \( \delta t^{\text{IONO}}_R \) is the ionospheric advance of signal; \( \varepsilon^R_S \) is the measurement noise.

To ensure the ionospheric free linear combination of carrier phase measurements in the order of magnitude of truly measured values, this combination can then be written as:

\[ k_1 \cdot \varphi_{R,R}^S = \frac{k_1}{k_1 + k_2} \cdot \varphi_{R,L_1}^S \cdot \frac{k_2}{k_1 + k_2} \cdot \varphi_{R,L_2}^S \] (12)

where \( k_1 \) and \( k_2 \) are the arbitrary factors to be determined in such a way that \( \varphi_{R,R}^S \) no longer contains any influence of the ionosphere.

Assuming that the ionospheric influence in Eq. (11) on this linear combination disappear:

\[ k_1 \cdot f_{L_1} \cdot \delta t^{\text{IONO}}_R (f_{L_1}) + k_2 \cdot f_{L_2} \cdot \delta t^{\text{IONO}}_R (f_{L_2}) = 0 \] (13)

For convenience, choose \( k_1 = 1 \), then obtain \( k_2 = \)
By inserting Eq. (10) we get

\[ k_2 = -\frac{f_{L_2}}{f_{L_1}}. \]

So the ionospheric free linear combination of carrier phase measurements (12) can be rewritten as:

\[ \varphi_{L_1}^b = \frac{f_{L_1}}{f_{L_1} - f_{L_2}} \varphi_{L_1} - \frac{f_{L_2}}{f_{L_1} - f_{L_2}} \varphi_{L_2}. \]

(14)

As for GLONASS satellite, \( k_2 = -\frac{f_{L_2}}{f_{L_1}} = -\frac{7}{9} \) is a non-integer value, the ionospheric free linear combination ambiguity \( N_{L_1}^b = \frac{f_{L_1}}{f_{L_1} - f_{L_2}} \cdot N_{L_1} \),

\[ N_{L_1}^b \cdot N_{L_2}^b \cdot N_{L_2} = \frac{1}{1 - f_{L_2}/f_{L_1}}. \]

is no longer an integer value. To retain the integer nature of this value, sometimes the so-called \( L_0 \) combination \( \varphi_{L_0}^b = 9 \cdot \varphi_{L_1} - 7 \cdot \varphi_{L_2} \) can be used as the ionospheric free linear combination. However, under the assumption that the measurement noise of \( L_1 \) carrier phase is identical to that of \( L_2 \) carrier phase, \( \sigma_{\varphi_{L_1}} = \sigma_{\varphi_{L_2}} = \sigma_{\varphi} \), the noise of this combination is

\[ \sigma_{\varphi_{L_0}} = \sqrt{9^2 + 7^2} \cdot \sigma_{\varphi} \approx 11.4 \cdot \sigma_{\varphi}. \]

The wavelength \( \lambda_{L_0} \) of this combination is

\[ \lambda_{L_0} = \frac{c}{9 f_{L_1} - 7 f_{L_2}} = \frac{1}{9 - 7 f_{L_2}/f_{L_1}} = \lambda_{L_1} \cdot \left( \frac{9}{9^2 - 7^2} \right) = 0.28125 \cdot \lambda_{L_1} \]  

(15)

this is about 5.26 cm for GLONASS frequency No. 1.

For GPS, \( k_2 = -\frac{f_{L_2}}{f_{L_1}} = -\frac{60}{77} \), the \( L_0 \) combination noise is

\[ \sigma_{\varphi_{L_0}} = \sqrt{77^2 + 60^2} \cdot \sigma_{\varphi} \approx 97.6 \cdot \sigma_{\varphi} \]

at a wavelength of \( \lambda_{L_0} = \frac{c}{77 f_{L_2} - 60 f_{L_1}} \approx 0.033 \lambda_{L_1} \), which corresponds approximately to 0.6 cm.

High noise and small wavelength have precluded this combination from having any significant importance for GPS carrier phase positioning. But for GLONASS, these values are much more favorable. Today’s GLONASS receivers provide a noise level around 0.5-1 mm (1 sigma) for carrier phase measurements[9]. This would mean a noise level of 0.57-1.14 cm for the \( L_0 \) combination, well below the 5.26 cm wavelength of the \( L_0 \) signal.

30 hours’ real data are used to test the performance of the ionospheric free linear combination. The ionospheric free observations of GPS are created with the standard algorithm used by most software such as GAMIT or BERNESE, observations of GLONASS are created with forenamed algorithm, 30 hours’ real data are averagely subdivided into 30 segments. The baseline length is calculated and compared with the “true length” obtained with the 30 hours’ data. Deviations of each hour’s baseline length are shown in Fig. 4.

It can be seen that the differences between one hour’s results and the “true length” are all below 4 mm, and the RMS of these differences is about 2 mm, higher than the results given in Reference [6].

Fig. 4 Results of baseline length with 30 hours’ real data
5 Conclusion

In combined GPS/GLONASS data processing, the differences between the system times must be accounted for. Two procedures to reduce the error of time reference differences between GPS and GLONASS are discussed. As can be seen from the real data tests, the method of introducing the fifth unknown as the difference of system time is more desirable.

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