Novel time-domain asymptotic-numerical solutions for transient scattered electric field from a coated cylinder covered with a thick dielectric medium

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Abstract: By extending frequency-domain (FD) asymptotic solutions, we develop novel time-domain (TD) asymptotic-numerical solutions (TD-ANSs), which are new reference solutions on engineering applications, for a transient scattered electric field from a two-dimensional coated cylinder covered with a thick dielectric medium. The TD-ANSs newly include the multiple reflection effect passing through the coating medium as compared with a conventional TD-ANS. The TD-ANSs are highly accurate and are useful because they can extract and interpret each pulse wave element from a response waveform including the multiple reflection effect even when pulse wave elements overlap mutually. The validity and usefulness of the TD-ANSs are confirmed by comparing with a reference solution.

Keywords: time-domain asymptotic-numerical solutions (TD-ANSs), transient scattered electric field, coated cylinder, thick dielectric medium

Classification: Electromagnetic theory

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1 Introduction

Analytical and/or numerical studies for frequency-domain (FD) and time-domain (TD) scattered fields from two-dimensional (2-D) impedance [1], dielectric [1], and coated cylinders [2, 3, 4, 5, 6] with a size sufficiently larger than the wavelength have recently been important research subjects in the application area such as the radar cross section (RCS) and the radar target discrimination.

We have proposed FD asymptotic solutions (FD-ASs) for a scattered electric field from a 2-D coated cylinder covered with a dielectric medium whose thickness is large as compared with the wavelength [7, 8, 9]. The FD-ASs are composed of a geometric optical ray (GO) series solution and an extended uniform geometrical theory of diffraction (extended UTD) series solution, each of which includes the multiple reflection effect, arriving at an observation point from a counterclockwise direction and a clockwise direction.
In this paper, by extending the FD-ASs in [8, 9], we develop novel TD asymptotic-numerical solutions (TD-ANSs) [10] for a transient scattered electric field when a high-frequency (HF) pulse wave is incident on a 2-D coated cylinder. We assume that a coating medium is thick as compared with the wavelength for the central angular frequency of an incident pulse wave. The TD-ANSs newly include the multiple reflection effect passing through the coating medium as compared with a conventional TD-ANS [6].

The novel TD-ANSs are highly accurate and are useful new reference solutions on engineering applications because they can extract and interpret each pulse wave element from a response waveform including the multiple reflection effect even when pulse wave elements overlap mutually. The validity and usefulness of the TD-ANSs are confirmed by comparing with a reference solution. The time convention \( \exp(-i\omega t) \) is adopted and suppressed in this paper.

2 Formulation and frequency-domain (FD) asymptotic solutions (FD-ASs) for scattered electric field

Fig. 1 shows 2-D coated cylinder with radius \( \rho = a \) covered with a coating medium 2 \((\varepsilon_2, \mu_0)\) of thickness \( t (= a - b) \), coordinate systems \((x, y, z)\) and \((\rho, \phi)\), and electric line source \(Q(\rho_0, \phi_0)\) and observation point \(P(\rho, \phi)\) located in a surrounding medium 1 \((\varepsilon_1, \mu_0)\). Notation \(\varepsilon_2^n\) denotes a complex permittivity of the medium 2 and is given by \(\varepsilon_2^n = \varepsilon_2 + i\sigma/\omega\) where \(\sigma\) is a conductivity and \(\mu_0\) a permeability of free space. We assume that the coating medium 2 is thick as compared with the wavelength for the central angular frequency of a pulse source function (see (4)).

Figs. 2(a) and 2(b) show geometrical boundaries (GBs) \(GB_p^n\), \(p = 0, 1, 2\), transition regions (TRs) \(TR_p^n\), \(p = 0, 1, 2\), and scattering phenomena when a HF cylindrical wave radiated from source \(Q\) is incident on a coated cylinder from counterclockwise \((n = 0)\) and clockwise directions \((n = -1)\), respectively. Notation \(p\) denotes the number of reflection on the coating medium-conducting interface defined by \(\rho = b\).

HF asymptotic analysis methods for a FD scattered electric field have been discussed in [8]. In Sections 2.1 and 2.2, only the final results needed in the TD asymptotic-numerical analysis method in Section 3.2 are summarized.

![Diagram of 2-D coated cylinder with coordinate systems and electric line source](image_url)

**Fig. 1.** 2-D coated cylinder, and coordinate systems \((x, y, z)\) and \((\rho, \phi)\). \(Q(\rho_0, \phi_0):\) electric line source, \(P(\rho, \phi):\) observation point.
Scattering phenomena for $n = 0$.

Fig. 2. Scattering phenomena from counterclockwise (Fig. 2(a)) and clockwise directions (Fig. 2(b)) when a cylindrical wave radiated from source Q is incident on a coated cylinder. GBs: GB$_n^p$, $p = 0, 1, 2$, and TRs: TR$^p_n$, $p = 0, 1, 2$. Notation $p$ denotes the number of reflection on the coating medium-conducting interface ($\rho = b$). Propagation paths of scattered field elements in Fig. 2(a): DGO$^0_0$ (Q → P$_0$), RGO$^0_0$ (Q → Q$_1$ → P$_0$), RSD$^0_0$ (Q → Q$_2$ → Q$_3$ → P$_1$), RGO$^0_1$ (Q → Q$_1$ → Q$_2$ → Q$_4$ → P$_1$), RSD$^0_1$ (Q → Q$_2$ → Q$_3$ → Q$_6$ → P$_2$), and RGO$^0_2$ (Q → Q$_1$ → Q$_2$ → R$_3$ → Q$_5$ → Q$_6$ → P$_2$), and RSD$^0_2$ (Q → Q$_1$ → R$_1$ → Q$_4$ → R$_3$ → Q$_7$ → P$_2$). Propagation paths of scattered field elements in Fig. 2(b): DGO$^{-1}_0$ (Q → P$_0$), RGO$^{-1}_0$ (Q → Q$_1$ → P$_0$), RSD$^{-1}_0$ (Q → Q$_2$ → Q$_3$ → P$_1$), RGO$^{-1}_1$ (Q → Q$_1$ → Q$_2$ → Q$_4$ → P$_1$), RSD$^{-1}_1$ (Q → Q$_2$ → R$_2$ → Q$_5$ → Q$_6$ → P$_2$), and RGO$^{-1}_2$ (Q → Q$_1$ → R$_1$ → Q$_4$ → R$_3$ → Q$_7$ → P$_2$).
2.1 GO series solution
The GO series solution for the z-component of a FD scattered electric field \( E_z^s(\rho_0, \phi_0, \rho, \phi; \omega) \equiv E_z^s(\omega) \) including the multiple reflected GO (multiple RGO) effect is given by

\[
E_z^s(\omega) \sim E_{z,GO\,series}(\omega) = \sum_{n=-1}^{0} U_{DGO}^n(P)DGO^p(\omega) + \sum_{p=0}^{M_n} U_{RGO}^p(P)RGO^p(\omega) \]  

(1)

where DGO^p(\omega) and RGO^p(\omega) denote a direct GO (DGO) solution and a \( p \) times RGO solution, respectively. The unit step function \( U_j(P), j \equiv DGO^n, RGO^p \) takes the value of 1 (0) when the scattered field element \( j \) can (cannot) reach the point \( P \). Integer \( M_n \) is the truncation number of the multiple RGO series. The readers can find the explicit expressions for the DGO^p(\omega) and RGO^p(\omega) in [8].

The GO series solution in (1) is applicable in the deep lit regions outside the TRs.

2.2 Extended UTD series solution
The extended UTD series solution for the z-component of a FD scattered electric field \( E_z^s(\omega) \) including the multiple reflected and surface diffracted ray (multiple RSD) effect can be expressed as

\[
E_z^s(\omega) \sim E_{z,UTD\,series}(\omega) = \sum_{n=-1}^{0} U_{DGO}^n(P)DGO^p(\omega) + \sum_{p=0}^{N_n} U_{RSD}^p(P)RSD^p(\omega) \]  

(2)

In (2), DGO^p(\omega) and RSD^p(\omega) represent the DGO solution and a \( p \) times RSD solution, respectively. The unit step function \( U_{RSD}^p(P) \) takes the value of 1 (0) when the \( p \) times RSD can (cannot) reach the point \( P \). Integer \( N_n \) is the truncation number of the multiple RSD series. The readers can find the explicit representations for the DGO^p(\omega) and RSD^p(\omega) in [8].

The extended UTD series solution in (2) is applicable in the TRs and in the deep shadow regions far away from the GBs.

3 Novel time-domain (TD) asymptotic-numerical solutions

3.1 Transient scattered electric field integral
The integral \( y(\rho_0, \phi_0, \rho, \phi; t) \equiv y(t) \) for a transient scattered electric field from a coated cylinder can be expressed by the inverse Fourier transform of the product of the FD scattered electric field \( E_z^s(\omega) \) and the frequency spectrum \( S(\omega) \) of a pulse source function \( s(t) \) [6, 11, 12].

\[
y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_z^s(\omega)S(\omega)\exp(-i\omega t)\,d\omega. \]  

(3)

We assume a truncated Gaussian-type modulated pulse source \( s(t) \) [12]:

\[
s(t) = \begin{cases} 
\exp[-i\omega_0(t-t_0) - (t-t_0)^2/(2d^2)] & \text{for } 0 \leq t \leq 2t_0 \\
0 & \text{for } t < 0, t > 2t_0
\end{cases} \]  

(4)
where \( \omega_0 \) denotes a central angular frequency, and \( t_0 \) and \( d \) are constant parameters. The frequency spectrum \( S(\omega) \) of the \( s(t) \) in (4) is given by

\[
S(\omega) = 2d\sqrt{\pi} \text{Re}[\text{erf} \beta(\omega)] \exp[i\omega t_0 - d^2(\omega - \omega_0)^2]
\]

(5)

\[
\beta(\omega) = \frac{t_0}{2d} - id(\omega - \omega_0)
\]

(6)

where the error function \( \text{erf} z \) is defined as [12, 13]

\[
\text{erf} z = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)dt.
\]

(7)

Figs. 3(a) and 3(b) illustrate the real part of the \( s(t) \) in (4) and the absolute value of the \( S(\omega) \) in (5), respectively. Numerical parameters used in the calculations are given in the caption of Fig. 3.

After substituting the FD exact solution \( E_z^e(\omega) \) obtained from the eigen-function expansion [2] and the \( S(\omega) \) in (5) into the integral \( y(t) \) in (3), by applying the fast Fourier transform (FFT) numerical code [14] to the \( y(t) \), we can obtain a reference solution \( y_{\text{reference}}(t) \). Also, the response waveform of \( y(t) \) is obtained from the real part of \( y(t) \), namely, \( \text{Re}[y(t)] \).

In Section 4, the reference solution \( \text{Re}[y_{\text{reference}}(t)] \) is used to confirm the validity and usefulness of the novel TD-ANSs derived in Section 3.2.

### 3.2 TD asymptotic-numerical analysis method

#### 3.2.1 TD-ANS for GO series

Substituting \( E_{z,\text{GO}}^e(\omega) \) in (1) into \( E_z^e(\omega) \) in (3), one obtains the following concise expression [10]:

\[
y(t) \sim y_{\text{TD-ANS}}(t) = y_{\text{GO series}}(t)
\]

\[
= \sum_{n=1}^{M_0} U_{\text{DGO}^n}(P) y_{\text{DGO}^n}(t) + \sum_{p=0}^{M_p} U_{\text{RGO}^p}(P) y_{\text{RGO}^p}(t)
\]

(8)

where \( y_{\text{DGO}^n}(t) \) and \( y_{\text{RGO}^p}(t) \) denote the DGO and the \( p \) times RGO, respectively, and are represented by

\[\text{(a) Real part of } s(t). \quad \text{(b) Absolute value of } S(\omega).\]
\[ y_{DGO}^i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} DGO^\omega(\omega)S(\omega) \exp(-i\omega t) d\omega \]  

(9)

and

\[ y_{RSD}^p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} RSD^\omega_p(\omega)S(\omega) \exp(-i\omega t) d\omega. \]  

(10)

The \( y_i(t) \), \( i \equiv \text{DGO}^n, \text{RGO}^n_p \), expressed by the integrals in (9) and (10) is calculable numerically by applying the FFT numerical code. The TD-ANS for the GO series can be developed by substituting \( y_{DGO}^i(t) \) and \( y_{RGO}^i(t) \) obtained from numerical integrals into (8). Hence the response waveform \( \text{Re}[y_{GO \text{ series}}(t)] \) in (8) associated with (9) and (10) can extract and interpret each pulse wave element from the \( \text{Re}[y_{\text{reference}}(t)] \).

### 3.2.2 TD-ANS for extended UTD series

Replacing \( E^z(\omega) \) in (3) with \( E^z_{\text{extended UTD series}}(\omega) \) in (2) gives the following concise representation [10]:

\[ y(t) \sim y_{\text{TD-ANS}}(t) = y_{\text{extended UTD series}}(t) \]

\[ = \sum_{m=-1}^{\infty} \left[ U_{DGO}(P)y_{DGO}^i(t) + \sum_{p=0}^{N_n} U_{RSD}_D(P)y_{RSD}^p(t) \right] \]  

(11)

where \( y_{DGO}^i(t) \) and \( y_{RSD}^p(t) \) denote the DGO in (9) and the \( p \) times RSD, respectively. The \( y_{RSD}^p(t) \) is expressed by

\[ y_{RSD}^p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} RSD^\omega_p(\omega)S(\omega) \exp(-i\omega t) d\omega. \]  

(12)

The \( y_i(t) \), \( i \equiv \text{DGO}^n, \text{RSD}^p \), represented by the integrals in (9) and (12) is computable numerically by applying the FFT numerical code. The TD-ANS for the extended UTD series can be developed by substituting the numerical results of \( y_{DGO}^i(t) \) and \( y_{RSD}^p(t) \) into (11). Therefore the \( \text{Re}[y_{\text{extended UTD series}}(t)] \) in (11) associated with (9) and (12) can extract and interpret each pulse wave element from the \( \text{Re}[y_{\text{reference}}(t)] \).

### 4 Numerical results and discussions

In this section, we perform numerical calculations required to assess the validity of the TD-ANSs developed in Section 3.2 and to interpret the transient scattering phenomena including the multiple reflection effect when a HF modulated pulse wave is incident on a coated cylinder covered with a thick dielectric material.

Figs. 4(a) and 4(b) show typical response waveform vs. time curves observed in the deep lit region outside the \( \text{TR}_1 \) and in the \( \text{TR}_2 \) adjacent to the \( \text{GB}_n \) (see Fig. 2(a)), respectively. Numerical parameters used in computations are given in the caption of Fig. 4 and the time \( t \) is set \( t = 0 \) when the HF pulse wave (see Fig. 3) is radiated from the source point Q. In this case, the coating medium is six times as thick as the wavelength \( \lambda_0 \) and the GBs at \( \text{GB}^p_n \), \( p = 0, 1, 2 \), are located at \( |\phi - \phi_0| = 109.8^\circ, 137.3^\circ, 164.9^\circ \), respectively.
As shown in Figs. 4(a) and 4(b), response waveforms \( \text{Re}[y_{\text{TD-ANS}}(t)] \) (---: black dashed line) given by

\[
\text{Re}[y_{\text{TD-ANS}}(t)] = \text{Re}[y_{\text{GO series}}(t)] = \text{Re}[y_{\text{DGO0}}(t)] + \sum_{p=0}^{1} \text{Re}[y_{\text{RGO}^p}(t)]
\]

agree very well with the reference solutions \( \text{Re}[y_{\text{reference}}(t)] \) (---: red solid line) in the whole region. Hence the validity of the TD-ANSs is confirmed.

Next, we discuss the interpretation method of the response waveforms in Figs. 4(a) and 4(b) and show the computation times of the TD-ANSs and the reference solution.

In Fig. 4(a), three wave packets are observed in the \( \text{Re}[y_{\text{reference}}(t)] \). Referring to the \( \text{Re}[y_{\text{GO series}}(t)] \) in (13), we can recognize that three wave packets are \( \text{DGO0} \), \( \text{RGO0} \), and \( \text{RGO1} \) in arrival order to the observation point \( P \). As to a computation time, it takes 5 seconds for the \( \text{Re}[y_{\text{TD-ANS}}(t)] \) and 27 seconds for the \( \text{Re}[y_{\text{reference}}(t)] \).

Fig. 4(b) is depicted five wave packets of (i) to (v) in the \( \text{Re}[y_{\text{reference}}(t)] \). As shown in (14), the \( \text{Re}[y_{\text{extended UTD series}}(t)] \) is composed of eight pulse wave elements. However, it is difficult to understand that five wave packets are made up of eight elements. In order to interpret the \( \text{Re}[y_{\text{reference}}(t)] \), we show in Fig. 5 four elements (\( \text{RSD0} \), \( \text{RSD1} \), \( \text{RSD2} \), and \( \text{RSD3} \)) (---: blue solid line) for \( n = 0 \) (Fig. 5(a)), four elements (\( \text{RSD0}^{-1} \), \( \text{RSD1}^{-1} \), \( \text{RSD2}^{-1} \), and \( \text{RSD3}^{-1} \)) (---: blue solid line) for \( n = -1 \) (Fig. 5(b)), and the \( \text{Re}[y_{\text{reference}}(t)] \) (---: red solid line) (Fig. 5(c)). Please note that the scale on the vertical axis in Fig. 5(b) is different from those in Figs. 5(a) and 5(c). By comparing Figs. 5(a) and 5(b) with Fig. 5(c), we can easily identify that the wave packets (i) and (v) are \( \text{RSD0} \) and \( \text{RSD3}^{-1} \), respectively. While, we observe that a bundle of pulse wave (ii) is attributed to the interference of two pulse wave elements \( \text{RSD2} \) and \( \text{RSD0}^{-1} \). In the same manner, we see that two bundles of pulse waves (iii) and (iv) are made up of the interference of \( \text{RSD0} \) and \( \text{RSD2}^{-1} \), and that of \( \text{RSD0} \) and \( \text{RSD3}^{-1} \), respectively. As to a computation time, it takes 43 seconds for the \( \text{Re}[y_{\text{TD-ANS}}(t)] \) and 30 seconds for the \( \text{Re}[y_{\text{reference}}(t)] \).

From the above-mentioned discussions, we can conclude that the novel TD-ANSs in (8) and (11) are useful new reference solutions on engineering applications because they can extract and interpret each pulse wave element from a response waveform including the multiple reflection effect even when pulse wave elements overlap mutually.

However, the computation time of the \( \text{Re}[y_{\text{extended UTD series}}(t)] \) in (14) is later than that of the reference solution \( \text{Re}[y_{\text{reference}}(t)] \). We will plan to solve the problem by applying HF asymptotic analysis methods, such as the saddle point technique [15], to the integrals in (9), (10), and (12) in the future.
Fig. 4. Response waveform vs. time curves. Numerical parameters: $a = 5.0 \text{m}$, $t = 6.0 \lambda_0$ ($= 0.236a$) ($\lambda_0 = 0.196 \text{m}$), $\epsilon_1 = \epsilon_0$, $\epsilon_2 = \epsilon_2 + i\sigma/\omega$, $\epsilon_2 = 3\epsilon_0$, $\sigma = 8.5 \times 10^{-3} \text{S/m}$, source point: $Q(\rho_0, \phi_0) = (1.4a, 0, 0)$, and observation point: $P(\rho, \phi) = (2.4a, \phi)$; $\phi = 25.0^\circ$ (Fig. 4(a)) and $\phi = 160.0^\circ$ (Fig. 4(b)). 

$\cdots$ : $\text{Re}(y_{\text{TD-ANS}}(t))$, $\cdots$ : $\text{Re}(y_{\text{reference}}(t))$. 

Fig. 5. Pulse wave elements ($\text{RSD}_n^0$, $\text{RSD}_n^1$, $\text{RSD}_n^{-1}$, and $\text{RSD}_n^2$) for $n = 0$ (Fig. 5(a)) and ($\text{RSD}_n^1$, $\text{RSD}_n^{-1}$, $\text{RSD}_n^2$, and $\text{RSD}_n^{-2}$) for $n = -1$ (Fig. 5(b)), and $\text{Re}(y_{\text{reference}}(t))$ (Fig. 5(c)). Numerical parameters used in the calculations in Fig. 5 are the same as those in Fig. 4(b). $\cdots$ : pulse wave elements in Figs. 5(a) and (b), $\cdots$ : $\text{Re}(y_{\text{reference}}(t))$ in Fig. 5(c).
5 Conclusion

We have developed the time-domain (TD) asymptotic-numerical solutions (TD-ANSs) for the transient scattered electric field from a coated cylinder covered with a thick dielectric material. By comparing with the reference solution, we confirmed the validity and usefulness of the TD-ANSs.

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