Laser cooling of a nanomechanical resonator mode to its quantum ground state

I. Wilson-Rae\textsuperscript{1,2}, P. Zoller\textsuperscript{3}, and A. Imamoglu\textsuperscript{2}

\textsuperscript{1} Department of Physics, University of California, Santa Barbara, CA 93106
\textsuperscript{2} Institute of Quantum Electronics, ETH H"onggerberg HPT G12, CH-8093 Z"urich, Switzerland and
\textsuperscript{3} Institut f"ur Theoretische Physik, Universit"at Innsbruck, A-6020 Innsbruck, Austria

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We show that it is possible to cool a nanomechanical resonator mode to its ground state. The proposed technique is based on resonant laser excitation of a phonon sideband of an embedded quantum dot. The strength of the sideband coupling is determined directly by the difference between the electron-phonon couplings of the initial and final states of the quantum dot optical transition. Possible applications of the technique we describe include generation of non-classical states of mechanical motion.

In contrast to bulk semiconductors where lattice vibrations form a reservoir with a continuous spectrum, nano-scale semiconductor structures support sharp mechanical resonances with very high quality factors and frequencies approaching 1 gigahertz\textsuperscript{1}. Unlike optical resonators, the thermal occupancy of these mechanical resonators is well above unity, even when they are cooled to sub-Kelvin temperatures. Presence of thermal noise hinders the study of coherent quantum dynamics in these mechanical systems\textsuperscript{2} and limits their applications to precision measurements\textsuperscript{3}. Here, we propose a technique that would realize laser cooling of a nano-mechanical resonator mode to its motional ground-state where quantum effects will be manifest. The basic idea is to use light scattering and electron-phonon interactions\textsuperscript{4}, such as deformation potential coupling\textsuperscript{5}, in an embedded quantum dot to manipulate a discrete mode of lattice vibrations.

The semiconductor beam structure with an embedded quantum dot (QD) that we analyze is shown in Fig. 1. An infinite length beam supports four phonon branches without an infrared cutoff: two bending branches (in-plane bending and flexural) with quadratic dispersion relations, and a torsional and a compression branch with linear dispersion relations\textsuperscript{6,7,8}. For a finite beam of length \(L\) attached to supports via abrupt junctions, these branches at long wavelengths will develop a series of sharp resonances corresponding to the different harmonics\textsuperscript{8}. In a structure where the thickness of the beam (\(d\)) is smaller than its width (\(b\)) (Fig. 1), the lowest-energy resonance corresponds to the fundamental flexural (vertical bending) mode (\(\hbar \omega_0\)). This will constitute the resonator mode we intend to address and cool to the ground state. For flexural modes with \(\omega_0 \sim 10^9 \text{s}^{-1}\), quality factors (\(Q\)) exceeding 20,000 have been measured\textsuperscript{9}. Though we focus on a bridge geometry (Fig. 1), the analysis of a cantilever structure is completely analogous.

The presence of a zero-dimensional (anharmonic) emitter that can be resonantly excited by a laser, has a spontaneous emission broadened optical transition, and interacts with the acoustic modes is necessary for achieving ground-state cooling. Our discussion will primarily focus on self-assembled InAs QDs\textsuperscript{10} which can satisfy these requirements and can be modeled as two level systems consisting of the empty QD state \(|g\rangle\) and the fundamental exciton state \(|e\rangle\)\textsuperscript{11}. We envision an ideal scenario where processing leaves only one QD inside the beam at a specific location. In assuming a natural line-width limited QD we are ignoring the effect of the surface on the electronic properties of the dot. This is warranted provided the dephasing and/or non-radiative recombination rates arising from the proximity to semiconductor/air interfaces are negligible compared with the spontaneous emission rate.

We observe in Fig. 1 that flexion induces extensions and compressions in the structure\textsuperscript{12}. This longitudinal strain will modify the energy of the electronic transitions corresponding to the different excitations and lead to a sideband excitation. The effect of cooling the modes is that at finite temperature the electronic transitions will be broadened, increasing the coupling of the system to the phonon modes.

![FIG. 1: Top: Schematic diagram of a GaAs bridge of length \(L\), width \(b\) and thickness \(d\) (see II below), with an embedded lens shaped InAs QD (WL denotes the wetting layer). Bottom: Vertical cross-sections through the axis of the bridge illustrating the deformations suffered by a small neighborhood of the QD, when the beam is bended slightly in the vertical direction (\(x\)). I and III correspond respectively to positive and negative deflection and II shows the equilibrium configuration. Each volume element parallel to the axis undergoes a simple extension or compression.]{figure-url}
states confined in the QD through deformation potential coupling. The resulting interaction between a localized carrier and a discrete flexural phonon mode will lead to the appearance of sidebands in optical spectra, originating from phonon assisted photon absorption or emission processes. By tuning a laser field into resonance with the first lower energy (red) sideband of a QD optical transition, we can ensure that absorption of a laser photon will be accompanied by the removal of a phonon from the system.

Before proceeding we note that sideband cooling has been used to cool trapped ions to their motional ground state. Despite the analogy, the physics of the two systems are different. Unlike the case of trapped ions and in marked contrast to optomechanical cooling schemes, the photon recoil plays no role in the laser cooling of a nanomechanical mode using an embedded QD. In the latter system, the (oscillator) strength of the sidebands is determined directly by the difference between the electron-phonon couplings of the initial and final states of the optical transition.

The fact that the modes of interest correspond to acoustic phonons with typical wavelengths $\lambda_p \sim L \gg b, d$ (Fig. 1) orders of magnitude larger than the lattice constant makes thin rod elasticity theory an excellent approximation to describe the phonons inside the beam. The dominant electron-phonon interaction mechanism in our structure will be deformation potential coupling.

$$H_{DP} = \int \! d\bar{r} \left[ D_c \hat{\rho}_a (\bar{r}) - D_v \hat{\rho}_h (\bar{r}) \right] \nabla \cdot \hat{u} (\bar{r}).$$

Here, we have introduced the electron (hole) density operator $\hat{\rho}_a (\hat{\rho}_h)$ and the corresponding deformation potential constants $D_c (D_v)$. The vector field operator $\hat{u} (\bar{r})$ corresponds to the displacement in Lagrangian elasticity. We neglect the difference between the elastic properties and deformation potentials of the strained InAs QDs and the surrounding GaAs matrix.

To proceed we decompose $\hat{u} (\bar{r})$ in terms of a set of orthonormal modes. A possible set consists of discrete modes localized in the beam ($\{\bar{u}_m (\bar{r})\}$) satisfying clamped boundary conditions at the junctions, and a continuum of modes localized in the supports ($\{\bar{u}_q (\bar{r})\}$) satisfying free boundary conditions. In this representation the beam modes only interact with support modes and for long wavelengths ($\lambda_p$) these interactions are weak, making the $\{\bar{u}_m (\bar{r})\}$ correspond to the resonances. As we are just interested in the dynamics of the fundamental mode (described by $\bar{u}_0 (\bar{r}), b_0$), we single it out and diagonalize all the terms in the phonon Hamiltonian which do not involve $b_0$, by transforming the rest of the modes to a new continuum $\{\bar{u}_q (\bar{r})\}$. Thus we obtain a representation consisting of the discrete resonator mode weakly coupled to a reservoir of “background” modes $\{b_q\}$.

We subsequently restrict to the two level model for the QD and integrate over $\bar{r}$ to obtain a coupling term between the QD and the resonator mode of the form: $\hbar \tilde{\omega}_0 |e\rangle \langle e| (b_0 + b_0^\dagger)$, where $\tilde{\omega}_0$ is the bare frequency associated to the discrete resonator mode. Within thin rod elasticity, $\nabla \cdot \hat{u} (\bar{r})$ for a bending mode is well approximated by $(2\pi - 1)\frac{x_0^2}{\omega_0^2}$. We subsequently restrict (1) to the two level model for the QD and integrate over $\bar{r}$ to obtain a coupling term between the QD and the resonator mode of the form: $\hbar \tilde{\omega}_0 |e\rangle \langle e| (b_0 + b_0^\dagger)$, where $\tilde{\omega}_0$ is the bare frequency associated to the discrete resonator mode. Within thin rod elasticity, $\nabla \cdot \hat{u} (\bar{r})$ for a bending mode is well approximated by $(2\pi - 1)\frac{x_0^2}{\omega_0^2}$. We subsequently restrict (1) to the two level model for the QD and integrate over $\bar{r}$ to obtain a coupling term between the QD and the resonator mode of the form: $\hbar \tilde{\omega}_0 |e\rangle \langle e| (b_0 + b_0^\dagger)$, where $\tilde{\omega}_0$ is the bare frequency associated to the discrete resonator mode. Within thin rod elasticity, $\nabla \cdot \hat{u} (\bar{r})$ for a bending mode is well approximated by $(2\pi - 1)\frac{x_0^2}{\omega_0^2}$.

To the extent that one neglects the difference between the bare frequency $\tilde{\omega}_0$ and the actual resonance $\omega_0, \eta^2$ corresponds to the Huang-Rhys parameter of the resonator parameter. Here $p$ and $c_p$ are the density and transverse speed of sound for the material of the beam, and $x_D$ and $x_Z$ are the QD coordinates (Fig. 1). The mode function normalized to the length and the wave-vector of the resonator mode are denoted by $X_0 (k_0 z)$ and $k_0$, respectively. The first factor in Eq. (2) is the square of a characteristic length of the material (2 nm for GaAs). The second factor gives the dependence with the dimensions of the structure and the placing of the QD in the vertical direction. The last factor will depend on whether we have a cantilever or a bridge, and for the latter is of order unity in a neighborhood of the midpoint. Typical parameters (see below) yield $\eta \sim 0.06$.

The Hamiltonian describing our system includes terms corresponding to the interactions between the QD, the resonator mode, the background phonon modes, and the electromagnetic field. To study laser cooling we perform a canonical transformation that eliminates the QD-resonator mode coupling term. After additional coordinate and momentum shifts applied to the phonon modes, we obtain the transformed Hamiltonian:

$$H = H_0 - \hbar \delta \frac{\sigma_\pm}{2} + \hbar \left[ \frac{\Omega}{\sqrt{2} \sigma_\pm} B^\dagger + \sum_k g_k \sigma_+ B^\dagger a_k + \frac{\sigma_\pm}{2} \sum_q \lambda_q b_q + \left( b_0 + b_0^\dagger \right) \sum_q \zeta_q b_q + \text{h.c.} \right],$$

where $H_0 = \hbar [\omega_0 |b_0^\dagger b_0 + \sum_q \omega_q b_q^\dagger b_q + \sum_k (\omega_k - \omega_L) a_k^\dagger a_k]$ and the QD operators are expressed in terms of Pauli matrices (i.e. $|e\rangle \langle e| = 1/2 (1 + \sigma_x)$). We have also defined the polaron operator $B = e^{\eta (b_0 - b_0^\dagger)}$, the detuning of the laser from the fundamental exciton line $\delta$ and its frequency $\omega_L$ and Rabi frequency $\Omega$. The background phonon modes are characterized by annihilation operators $b_q$ and their
couplings to the QD (resonator mode) $\lambda_g (\zeta_g)$. The radiation field is characterized by annihilation operators $a_k$ and their couplings to the QD $g_k$.

The role of $\eta$ in our system is equivalent to the one played by the Lamb-Dicke (LD) parameter in the trapped-ion system $^{12, 17}$. Even though the underlying physics is different, we will refer to the LD regime as the one in which $\eta^2 \equiv \eta^2 (|b_0| b_0 + 1) \ll 1$. We focus on this limit since it is the one of interest for ground state cooling and is well satisfied for realistic structures provided the ambient temperature is low enough. In this regime, both cooling and heating can take place via two paths, where the modification of the motional state of the phonon mode takes place in the laser absorption and spontaneous emission process, respectively (Fig. 2). In contrast to the trapped ion system, these two paths are indistinguishable for the QD-beam structure and exhibit quantum interference.

In this LD regime it is possible to derive a rate equation describing the cooling dynamics of the resonator mode. To this effect we first eliminate the radiation field and subsequently the “background” phonons to obtain a master equation for the QD-resonator system. The first step, that involves the standard Born-Markov approximation, leads to a Liouvillian for the “QD-resonator-background” system that – aside from terms involving the “background” – closely resembles the analogous result for a trapped-ion $^{17}$. The only difference -aside from a trivial interchange of the normal coordinate and momentum of the resonator mode- lies in the dissipative rate equation, and a “rotating wave” part describing the damping of the resonator mode

$$\Gamma / Q$$

that to lowest order in $1 / Q$ is treated on the same footing as $\eta^2$. The resulting rate equation describing the populations $P_n$ of the energy levels of the resonator mode is

$$\dot{P}_n = \eta^2 A_n + \omega_0 n(\omega_0) [nP_{n-1} - (n + 1) P_n] + \eta^2 A_n - \omega_0 n(\omega_0 + 1) [(n + 1) P_{n+1} - n P_n], \quad (4)$$

with $n(\omega_0) = 1 / (e^{\omega_0 / kT} - 1)$. The first term corresponds to heating and the second one to cooling. There are contributions proportional to $1 / Q$ describing the damping arising from coupling to the supports and contributions which involve the scattering of laser light given by $\eta^2 A_\pm = 2 \eta^2 \text{Re} (\sum \nu A^\pm (\sigma_\nu)_{SS})$, with

$$A^\pm = \frac{C}{2} + (\frac{1}{2} R^y - \Gamma C) \cdot (\mp i \omega_0 I - L)^{-1} \cdot (\frac{1}{2} R^y - \Gamma C). \quad (5)$$

We have used for the state space of the QD a basis consisting of the Pauli matrices $\sigma_\nu$ with $\nu > 0$ and the identity $\sigma_0$, and defined: $R^\mu_{\nu \mu} = \frac{1}{2} \text{Tr}_{QD} (\sigma_\nu \sigma_\mu)$, $C_{\nu \mu} = \frac{1}{2} \text{Tr}_{QD} (\sigma_\mu \sigma_\nu + \sigma_\nu \sigma_+)$ and $L_{\nu \mu} = \frac{1}{2} \text{Tr}_{QD} (\sigma_\mu \sigma_\nu)$. Here $L_{QD}$ and $\sigma_\nu_{SS}$ are the Liouvillian and steady state expectation values corresponding to the optical Bloch equations for the QD $^{10}$. We focus on $\delta < 0$ for which there is a net cooling rate $\eta^2 W = \eta^2 (A_+ - A_+) > 0$.

As we start from thermal equilibrium, initially the number of phonons is given by $n_i = n(\omega_0)$. When steady state is reached, the final phonon number $n_f$ will be given by $n_f = (n_i + \eta^2 Q A_+ / \omega_0) / (1 + \eta^2 Q W / \omega_0)$. Therefore there is a threshold value of $n_i = \min \{ A_+ / W \}_{\Omega, \delta}$ below

\[ \text{FIG. 2: Energy level diagram of the QD coupled to the resonator mode for perturbative Rabi frequency.} |g, n\rangle (|e, n\rangle) \text{ denote the state where the QD is in the ground (exciton) state and the resonator mode has n phonons. The cooling (heating) cycle consists of the excitation of the QD and subsequent spontaneous photon emission, and is denoted by solid (dashed) lines. The blue (red) path corresponds to the case where phonon number is changed during the laser absorption (spontaneous emission). If the laser is tuned between the red sideband and the main line these two paths interfere constructively for cooling and destructively for heating (note the relative phase of } \pi \text{ between } \nu_S \text{ and } \nu_L). \]
which laser cooling does not work (Fig. 3), and \( \eta^2 Q >> 1 \) is required to obtain appreciable cooling.

There are two contributions to \( n_f \): the first one is proportional to \( n_i \) and the second one proportional to \( A_+ \).

This leads to two regimes depending on which contribution dominates the behavior of \( n_f \) (Fig. 3). In the first “large \( n_i \)-small \( \eta^2 Q \)” regime the dominant heating mechanism is the coupling to the supports.

In the second “small \( n_i \)-large \( \eta^2 Q \)” regime heating is dominated by the scattering of laser light and the optimal value of \( n_f (\tilde{n}_f) \) is given by \( \tilde{n}_f \approx (\Gamma/4\omega_0)^2 \) (Fig. 3). We find that in both regimes \( \Gamma/\omega_0 < 1 \) is a necessary condition for ground state cooling and \( \tilde{n}_f \lesssim n_i/\eta^2 Q + (\Gamma/4\omega_0)^2 \). Typical parameters we envisage are: \( L = 950\text{nm} \), \( d = 30\text{nm} \) (which correspond to \( \omega_0 \approx 1.2 \times 10^3\text{s}^{-1} \), \( b = 80\text{nm} \), \( x_D = d/4 \), \( z_D = 0 \), \( Q = 3 \times 10^4 \) and \( \Gamma = 3 \times 10^3\text{s}^{-1} \)[1]. These lead to \( \tilde{n}_f = 0.1 \) for a sample held at \( T = 0.1K \) (Fig. 3).

We note that it is possible to realize an all optical measurement of the final temperature. The basic idea is to take two lasers, a first one for cooling with the desired parameters, and a second one for measuring with zero detuning (frequency \( \omega_M \)) and perturbative Rabi frequency (e.g. \( \Omega_M = \Gamma/10 \)). While this second “probe” laser is always on, the first “cooling” laser interacts with the QD only during time windows of duration \( t_c \) (cooling) that alternate with measurement windows of duration \( t_m \ll t_c \). As for realistic parameters optimal cooling occurs at \( \Omega = \omega_0 \), when analyzing the cooling windows the “probe” laser can be neglected. The experimentally measured quantities are the photo-counts \( N_- \) (blue photons) and \( N_+ \) (red photons) around frequencies \( \omega_M + \omega_0 \) and \( \omega_M - \omega_0 \) respectively, during the time-intervals when the “cooling” laser is off. The cooling and heating rates satisfy \( A_+ (\delta) = A_- (-\delta) \). Hence if we assume that there are no background photons, it follows that in the above scenario the occupancy will be given by \( \langle b_0^\dagger b_0 \rangle = N_-/ (N_+ - N_-) \).

We have focused on laser cooling the fundamental resonator mode to its ground state. However we want to stress that one of the main contributions of this letter is to establish a close analogy between this mesoscopic condensed-matter system and a single trapped ion in the LD regime[12,17]. In fact, the analogy extends to cavity-QED; i.e. a single atom in a high-Q optical or microwave cavity[13]. This opens up possibilities that go far beyond cooling[20] and may not be within reach of other techniques for manipulating a nano-resonator. In particular the technique presented here would enable quantum state engineering of non-classical states of motion[21,24] along the lines proposed for the trapped-ion system[21]. Examples are the generation and detection of a Fock state and squeezed states of motion.

One could also envision extending this method to simultaneous cooling of several modes. If successful, this could be used to generate entangled states of mechanical motion. Our results could apply to other systems with sharp vibrational resonances and to different types of zero-dimensional emitters such as localized defects[22].

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FIG. 3: Optimal final number of phonons \( \tilde{n}_f \) v.s. initial number \( n_i \) (solid line) for \( \Gamma/\omega_0 = 1/4, \eta^2 Q = 100 \). The optimum results from searching for the laser parameters (\( \delta_{\text{opt}}, \Omega_{\text{opt}} \)) that minimize \( n_f \). There are three regimes (dotted lines): I) for \( n_i \leq (4\omega_0/\omega) \) laser addressing of the structure is not useful (i.e. \( n_f \geq n_i \)), II) for small \( n_i \) (\( n_i \ll \eta^2 Q(4\omega_0/\omega)^2 \)) the optimum is constant and \( \delta_{\text{opt}} \approx \omega_0, \Omega_{\text{opt}} \ll \omega_0 \) III) for large \( n_i \) (\( n_i \gg \eta^2 Q(4\omega_0/\omega)^2 \)) the optimum is proportional to \( n_i \) with the ratio \( \tilde{n}_f/n_i \) bounded by \( 1/\eta^2 Q \) and \( \delta_{\text{opt}} \approx \omega_0, \Omega_{\text{opt}} \approx \omega_0 \). The rough approximation \( n_i/\eta^2 Q + (\Gamma/4\omega_0)^2 \) is plotted for comparison (dashed line).

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[23] We have estimated Doppler effects and radiation pressure in our system and find them to be negligible.

[24] The structure can be processed so that \( \{x, y, z\} \) (Fig. 1) are oriented along the principal crystal axes. This choice implies that piezoelectricity will only couple the QD to torsion.\(^{\text{[15, 16]}}\) We have bounded the corresponding Huang-Rhys parameters and find them to be negligible.

[25] The smallness of the QD dimensions implies that for phonons characterized by \( \lambda_p \) a QD-phonon interaction \( \propto \nabla \cdot \hat{u}(\vec{r})|_{\vec{r} = \vec{r}_D} \) can be justified without resorting to this approximation\(^{\text{[15]}}\) – \( \vec{r}_D \) is the QD position.

[26] This can be satisfied by processing the structure so that \( b/d > 25/9 \) (Ref. \(^{\text{[6, 7, 8]}}\)).