A comparison between models of gravity induced decoherence of the wavefunction

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Abstract. It has already been suggested that quantum theory needs to be reformulated or modified in order to explain the measurement process and the successive collapse of the wavefunction. However, there are also models of another type which keep quantum theory intact and instead modify the classical gravity by introducing stochasticity to it. These models suggest that there is a fluctuation in the background gravitational field which eventually results in the decoherence of the wavefunction. These fluctuations limit the precision with which one can measure the properties of a spacetime geometry with a quantum probe. Two similar models along this line have been suggested by Karolyhazy (K-model) and Diósi (D-model). They are based upon apparently different spacetime bounds. The results obtained for the coherence length are also somewhat different. In this article, we show that, given certain conditions apply, the minimal spacetime bounds in these two models are equivalent. We also derive the two-point correlation for the fluctuation potential in K-model which turns out to be non-white, unlike in D-model, where the corresponding correlation is white noise in time. In our opinion, this is the origin of discrepancy in the predictions of the two models. We argue that the noise correlation cannot be determined uniquely from a given spacetime bound.

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1. Introduction
Quantum theory is a linear theory governed by the Schrödinger equation that allows a superposition of two possible quantum states of an object. This feature has been observed in Young’s double slit experiment and various other interference experiments. But one problem that still needs an explanation is the “quantum measurement problem” as we know it. What happens when we attempt to measure the properties of a “quantum” particle using a classical apparatus? Why do we get one of the possible states with a definite probability instead of observing a superposed state of the possible outcomes as predicted by quantum theory? Clearly, there is an inherent randomness in this theory, the origin of which is not quite obvious. The probability of getting a particular outcome is given by the Born probability rule. Why does the collapse occur during a measurement process of a quantum object and what is the reason of emergence of the Born probability: these two questions together are referred to as the “quantum measurement problem” [1, 2].
Many works on this have been going on to explain properly how the randomness came into a theory which seems to be deterministic and the superposition is “lost” when a classical object is involved. One of the first resolutions came in the form of Copenhagen interpretation \[3\] which says that quantum mechanics and classical theory - they have a mutually exclusive domain of validity i.e. laws of quantum mechanics do not hold for macro-objects, without giving proper explanation of what a macro-object is and where the divide lies. Later on, von Neumann \[4\] gave a more refined form of this interpretation. Next explanation was the decoherence of the wave function \[5–7\] which is the role played by the environment to destroy the coherence of the states through interaction with the system. This was supplemented by the many-worlds interpretation \[8, 9\] saying that observers belonging to different branches of the universe see different outcomes. These two together was an attempt to explain decoherence along with Born’s rule. Though this theory is far from perfection, it is still an accepted view because it does not modify quantum theory. Bohmian mechanics is also an alternative explanation \[10, 11\] according to which the quantum evolution is deterministic, the randomness being attached to the choice of initial states itself. Another view is that quantum theory is not complete, it is only approximation to a more general nonlinear dynamical theory whose dynamics is stochastic. The most convincing nonlinear model is known as Continuous Spontaneous Localization (CSL) which has been able to explain Born’s rule successfully by invoking a non-linear and stochastic modification to the Schrödinger equation \[12\]. It seems very promising apart from the fact that it does not explain the origin of such modifications.

A more fundamental reason for collapse is still being searched and various proposed models are being investigated. One possible solution seems to be the connection of gravity with the collapse process. As gravity is a universal force, it has effects on both micro and macro objects. It is proposed that macro-objects, being heavier, are affected by gravity more and that is somehow connected to the spontaneous collapse whereas in micro-objects, the effect is minimum so that they are still governed by quantum mechanics. Various gravity induced collapse models have been proposed, the central idea of which is that the intrinsic quantum fluctuations in the motion of an object induce a haziness in the structure of spacetime. This fluctuation manifests itself in the form of a universal stochastic noise that results in the decoherence of the wave function. While Karolyhazy and others introduce a family of metrics to represent such noise \[13–16\], Diósi brings in the concept of a spacetime averaged classical Newtonian field \[17–19\] and the uncertainty in the measurement of this field by a quantum probe as we shall discuss in the next section. The first one will be called as the K-model and the second one will be referred to as the D-model. Both the models start with a similar spirit but end up getting different results. In our work, we find what could be responsible for such discrepancies. It is worthwhile to note that these models can only predict localization, but they cannot explain the Born probability rule. Beyond the scope of our present work is the ramification \[19\] of the D-model \[18\], similar to CSL, which can reproduce the Born rule.

2. Wavefunction collapse and gravity

Even though the idea may not seem intuitive, there is a possibility that gravity has an important role to play in the reduction of a wave vector and thus resulting collapse. Many theories have been suggested on this as possible mechanism of the collapse none of which has yet been confirmed through experiments. In this section we shall give a brief overview of two such models: K-model (proposed by F. Karolyhazy) and D-model (proposed by L. Diósi).
2.1. K-model (Karolyhazy et. al)

Karolyhazy shows that to measure a certain length \( s = cT \) in flat spacetime using a clock, there will be an inherent uncertainty involved with this measurement arising from the quantum uncertainty of the clock and is given by

\[
\Delta s^3 \approx l_p^2 s
\]

where \( l_p \) is Planck’s length. To formulate a mathematical model for this, he describes the spacetime metric as fluctuating around the flat Minkowski metric. In his model, the fluctuation has been described by a family of matter-free metrics \( \{ g_{\mu\nu}^{\beta} \} \), each containing a perturbation \( \gamma_{\mu\nu}^{\beta} \) around the Minkowskian. The uncertainty in the line segment \( s = cT \) is defined in this scheme as

\[
\Delta s = \left[ \langle (s - s_{\beta})^2 \rangle \right]^{1/2}
\]

where \( s \) is the average length over the family of metrics and \( \langle \rangle \) denotes the stochastic average. The fluctuation term \( \gamma_{\mu\nu}^{\beta} \) is assumed to satisfy the wave equation and hence has a solution of the form,

\[
\gamma_{\beta}(x, t) = \frac{1}{\sqrt{l^3}} \sum_k \left[ c_{\beta}(k)e^{i(k \cdot x - \omega t)} + c.c \right]
\]

where \( l \) the length of an arbitrarily chosen large box and \( \omega = ck \). The model assumes that the coefficients \( c_{\beta}(k) \) vary around an average value zero while moving through the family and take values independently from each other i.e.

\[
\langle c_{\beta}(k) \rangle = 0, \quad \langle c_{\beta}^2(k) \rangle = 0, \quad \langle c_{\beta}(k)c_{\beta}^*(k') \rangle = \delta_{k,k'}(f(k))^2
\]

where \( f(k) \) needs to be determined. It was shown that in order to get the K-model uncertainty correctly we must have

\[
f(k) = l_p^{2/3} k^{-5/6}.\]

Due to this metric fluctuation, when a wavefunction propagates through this spacetime, a phase will be introduced to the wavefunction with respect to the free wavefunction in the absence of any fluctuation. One can calculate the variance of relative phase between two spacetime points over the family of metrics and this gives a measure of decoherence. When this becomes of the order of \( \pi \), we say that coherence is lost between the parts of the wavefunction at the two spacetime points. The corresponding spatial separation between these points is called the coherence length \( a_c \) and the time required for it is called the decoherence time \( \tau_c \). Karolyhazy and Frenkel have calculated the coherence length and decoherence time for point masses and extended objects \([16]\).

For a proton, it turns out

\[
a_c \approx 10^{25} cm, \quad \tau_c \approx 10^{53} s.
\]

On the other hand, for a ball of radius, say, 1 cm and having density 1g/cc, the quantities calculated are

\[
a_c \approx 10^{-16} cm, \quad \tau_c \approx 10^{-4} s.
\]
2.2. D-model (L. Diósi)

Diósi model works with the weak field Newtonian limit of GR where the metric becomes

\[ g_{ij} = \begin{cases} 
1 + \frac{2\phi}{c^2} & i = j = 0 \\
-\delta_{ij} & i, j = 1, 2, 3 
\end{cases} \]  

(8)

\( \phi \) representing the background potential field.

This model questions the accuracy with which a Newtonian gravitational field can be measured using a quantum probe which obeys uncertainty principle. The gravitational field which is to be measured by a quantum probe is given by

\[ g(r, t) = -\nabla \phi(r, t). \]  

(9)

The quantum probe obeys uncertainty principle and this uncertainty in its motion back-reacts onto the gravitational field which introduces an uncertainty in the measured field. The uncertainty in the measured field \( \tilde{g} \) is given by \( \delta \tilde{g} \) and its variance follows the Diósi bound

\[ (\delta \tilde{g})^2 \geq \frac{G\hbar}{VT} \]  

(10)

where the measured field \( \tilde{g} \) is the average of the gravitational field \( g \) over the entire volume \( V \) of the measuring probe and over the measuring time \( T \).

The two point correlation function is then determined as

\[ \langle \phi(r, t)\phi(r', t') \rangle = \frac{\hbar G}{|r - r'|} \delta(t - t'). \]  

(11)

Given this noise correlation, one can write down the Schrödinger equation and derive the master equation for the density matrix between two configurations \( |X\rangle \) and \( |X'\rangle \) as

\[ \langle X| \dot{\rho} |X'\rangle = -i \hbar^{-1} \langle X| [\hat{H}, \rho(t)] |X'\rangle - [\tau_d(X, X')]^{-1} \langle X| \dot{\rho}(t) |X'\rangle \]  

(12)

where \( \hat{H} \) is the Hamiltonian, the decay time \( \tau_d(X, X') \) between the two configurations is given by the following equation

\[ [\tau_d(X, X')]^{-1} = \frac{G}{2\hbar} \int d^3r d^3r' |f(r|X) - f(r|X')||f(r'|X) - f(r'|X')| \]  

(13)

The same decay time was independently proposed by Penrose [20].

Corresponding to this decay time, there is a critical length \( a_{crit} \) beyond which the damping term becomes important. Assuming a spherical mass distribution of radius \( R \), Diósi has calculated both the decoherence time and the critical length [18]. Surprisingly, the results obtained in this model are quantitatively very different from the K-model predictions. For a proton, the critical length and decay time come out to be

\[ a_{crit} \approx 10^6 \text{cm}, \quad \tau_d \approx 10^{15} \text{s}. \]  

(14)

In the rest of this paper, we shall try to understand the origin of this discrepancy and shall investigate whether any of these models gives the estimates correctly or both are lacking in some way.
3. Comparison of the two models
In the previous section we have seen that the predictions of the K-model and D-model differ by orders of magnitude despite relying on the same underlying physics. At this point, it is worthwhile to note that even though the background physics is same for these two models, they are apparently different in the methodology. Also the spacetime bounds look completely different and seem unrelated. In this section, we will systematically investigate both the models to find at what stage the actual difference sets in. We will first start by comparing the two apparently different spacetime bounds to see if they are related. Then we will compare the two different approaches taken in the two models and lastly we will compare the nature of the noise correlations assumed in these models.

3.1. Equivalence of K-model and D-model bounds
Here we show explicitly that the spacetime averaged potential in the D-model also implies the K-model spacetime bound, provided the white noise correlation is assumed [21]. We calculate the stochastic world line length similar to in K-model but now with a spacetime averaged potential:

\[ s = \int_0^T \sqrt{1 + \frac{2\tilde{\phi}}{c^2}} \, c \, dt. \]  

(15)

Hence,

\[ s - s_{\text{avg}} \simeq c \int_0^T \tilde{\phi} \, dt \]  

(16)

where \( s_{\text{avg}} = cT \). Note that here \( \tilde{\phi} \) still represents a c-number stochastic variable even though it has been averaged over space and time. Next we calculate \( \Delta s^2 = \langle (s - s_{\text{avg}})^2 \rangle \) as before:

\[ \Delta s^2 = \frac{1}{c^2} \left( \int_0^T \tilde{\phi} \, dt' \int_0^T \tilde{\phi} \, dt'' \right) \]

. Assuming that \( \tilde{\phi} \) does not change much within the measuring time \( T \), we can directly integrate over time

\[ \Delta s^2 = \frac{T^2}{c^2} \langle \tilde{\phi}^2 \rangle \]

. Using the white noise correlation, \( \langle \tilde{\phi}^2 \rangle \) is calculated which has been described in [21]. Using the result, we get the uncertainty in measuring the length \( s = cT \) as

\[ \Delta s^2 \approx \frac{l_p^2}{R} s. \]  

(17)

At this stage, we make the claim that \( R \) has to be of the order of \( \Delta s \) to get the minimal uncertainty. Increasing \( R \) will reduce the uncertainty but \( R \) cannot go beyond \( \Delta s \) as that would make the imprecision to be \( R \) itself, not \( \Delta s \). So, \( R \sim \Delta s \) would be the optimal choice which reduces the above bound essentially to the K-bound i.e,

\[ \Delta s^3 \sim l_p^2 s. \]  

(18)

We thus see that the minimal spacetime bounds in the D-model and in the K-model, even though they appear quite different, are essentially equivalent.
3.2. Phase variance and master equation

In Karolyhazy model, we have seen that the phase variance method was adopted to get the coherence length and time [16]. Diósi, in his model, on the other hand followed the conventional method of deriving the master equation for the density matrix. The two approaches are completely different. In this section we try to see if they produce the same result. We check whether phase variance method is a valid approach which is able to produce similar results obtained from master equation.

We start with the Diósi’s choice of potential and, instead of writing the master equation, we now calculate the phase accumulated by the wave function. In Diósi’s model, the potential for a given configuration X is given as follows:

\[ U(X,t) = \int_{vol} \phi(x,t)f(x|X)\,d^3x. \]  

(19)

The phases accumulated at time t for two different configurations X and X’ are,

\[ \delta(X,t) = \frac{1}{\hbar} \int_{0}^{t} \int_{vol} \phi(x',t')f(x'|X)\,d^3x'\,dt' \]

(20)

\[ \delta(X',t) = \frac{1}{\hbar} \int_{0}^{t} \int_{vol} \phi(x'',t'')f(x''|X')\,d^3x''\,dt'' \]

(21)

We now follow the phase variance approach followed by Karolyhazy and collaborators and evaluate the variance \( \langle \delta(X,t) - \delta(X',t) \rangle^2 \). When this becomes \( \sim \pi^2 \), we get an estimate for the decoherence time. Calculations show [21] that the decoherence time becomes,

\[ \tau_d = \left[ \frac{G}{\pi^2\hbar} \int d^3x d^3x' \frac{|f(x|X) - f(x|X')|\,|f(x'|X) - f(x'|X')|}{|x - x'|} \right]^{-1} \]

(22)

which is same as the decoherence time obtained by Diósi using the master equation apart from some constant factors [22]. Thus, use of master equation or phase variance method gives similar results.

3.3. Noise two-point correlation in K-model

Having tested all possible sources of discrepancy, we now compare the noise two-point correlations of the two models. K-model confines its calculations to the Fourier space without referring to any noise correlation. In this section, we calculate the noise correlation for this model and figure out differences with the white noise prediction of D-model. Now the family \( \gamma_\beta \) is to be thought of as a stochastic potential with zero mean whose two point correlation is such that when a length \( s = cT \) is measured in the presence of such a potential, it exhibits an uncertainty given by [1].

Using the Fourier expansion [3] we compute \( \langle \gamma_\beta(x,t)\gamma_\beta(x',t') \rangle \) (see [21] for the detailed calculations) and show that in the limit \( l \to \infty \), this becomes,

\[ \langle \gamma_\beta(x,t)\gamma_\beta(x',t') \rangle = \frac{l^{4/3}}{4\pi^2r} \Gamma(1/3) \left[ \frac{1}{r + c|\tau|} + \frac{\text{sign}(r - c|\tau|)}{|r - c|\tau|^{1/3}} \right] \]

(23)

where we have used \( r = |x - x'| \) and \( \tau = t - t' \). This form was also reported earlier in [23]. This is evidently not white noise, and is predicted to be the feature responsible for the difference in
the results obtained for localization length in the K-model and the D-model.

The Schrödinger equation now can be written for a mass distribution without projecting it to the position basis,

\[ i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \left[H + \frac{c^2}{2} \int dx' \hat{f}(x') \gamma(x', t)\right] |\psi(t)\rangle \]  

(24)

where \( \hat{f} \) represents the local mass density operator.

The corresponding non-Markovian master equation can be written using the perturbative results of [24] as follows,

\[ \frac{d\rho(t)}{dt} = -\frac{i}{\hbar} [H, \rho(t)] - \left(\frac{c^2}{2\hbar}\right)^2 \int dx dx' \int_0^t ds \langle \gamma_\beta(x, t) \gamma_\beta(x', s) \rangle \left[\hat{f}(x), \left[\hat{f}(x'; s-t), \rho(t)\right]\right] \]  

(25)

where \( \hat{f}(x; s-t) \) is the local mass density operator in the interaction picture evolved up to time \( s-t \). Calculation of decoherence time from this master equation is not very straightforward.

4. Conclusion

In this paper we tried to compare the K-model and D-model by treating them in a similar approach. We argued that the spacetime uncertainty bounds in the two models are essentially equivalent to each other given that the uncertainty is minimal. We also treated the Diósi potential with the phase variance method approach adopted by Karolyhazy and showed that it generates the same results as the master equation scheme does. Up to this point, both the models do not exhibit any discrepancy. The major discrepancy shows up when we calculate the two point correlation function in the K-model. We showed that this noise correlation is non-white noise unlike in D-model where the correlation is white noise in time. Consequently, the master equation for the density matrix in K-model becomes non-Markovian. We argued that this basic difference in the choice of the noise is possibly the reason which leads to different results predicted by these models.

We argue that the noise correlation cannot be predicted uniquely from the spacetime bound and many choices are possible, each of which is likely to give different results for the decoherence time scale. White noise maybe the simplest choice, but there seems no physical reason why gravitational effects must conform to white noise. Thus it appears that additional criteria, apart from the minimal bound, are essential to precisely define a model of gravity induced decoherence with a unique choice of the noise potential. Nonetheless, it can be said that the role of gravity in decoherence is fundamentally suggested, and further investigation of this problem is highly desirable.

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