Quantum Nondemolition Photon Counting With a Hybrid Electromechanical Probe

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(Dated: August 26, 2020)

Quantum nondemolition (QND) measurements of photons is a much pursued endeavor in the field of quantum optics and quantum information processing. Here we propose a novel hybrid optoelectromechanical platform that integrates a cavity optomechanical system with a single electron transistor for QND photon counting. Building upon a mechanical-mode-mediated nonperturbative dispersive coupling between electrons and photons, our protocol performs the QND photon counting measurement by means of the current-voltage characteristics of the single electron transistor. In particular, we show that the peak voltage shift of the differential conductance is linearly dependent on the photon occupation number, thus providing a sensitive measure of the photon number, especially in the strong optomechanical coupling regime. Given that our proposed hybrid system is compatible with state-of-the-art experimental techniques, we discuss immediate implementations and anticipate applications in quantum optics and polariton physics.

Introduction.— Over the past decades, quantum nondemolition (QND) measurements [1–5] have been implemented in many experiments that require ultimate sensitivity, such as gravitational-wave detection [6], with a growing recent interests in quantum information processing and storage applications [7]. Particularly, in the quantum optics community, it is well appreciated that a QND measurement scheme can evade the adverse effect of quantum back-action during the measurement [1–3], such that an experiment observable can be repeatedly measured without perturbing the underlying quantum state. Therefore, exploiting the theory of QND measurements, many researchers have demonstrated novel strategies for probing quantum characteristics, including quantum fluctuation sensing [8–10] and photon counting [11–27], pushing the limit of quantum technology.

Conditions for an ideal QND measurement were formulated by Imoto et al. [28]. As far as a general quantum measurement is concerned, one usually considers a signal observable \( O_s \) of the measured system (with the unperturbed Hamiltonian \( H_s \)) and a readout observable \( O_p \) of the probe system (with the unperturbed Hamiltonian \( H_p \)) coupled through an interaction operator \( H_I \). Following the strict definition of an ideal QND-type measurement [28], \( O_s \) and \( O_p \) are a QND pair if the following mathematical conditions are satisfied: (i) \( H_I = H_I(O_s) \), (ii) \([H_I, O_s] = 0\), (iii) \([H_I, O_p] = 0\), and (iv) \([H_s, O_p] = 0\). In the context of photon number measurements, to achieve QND-type detection of photons—namely to avoid any optical absorption of photons during the measurement—one common choice is to probe the photon of the measured system \( (O_s = a_s^a a_s^a \text{ as the system photon number operator}) \) by another optical mode \( (O_p = a_p - a_p^a) \), where \( a_p \) is the annihilation operator of the probe photon) with a non-linear interaction Hamiltonian (e.g. in a Kerr medium \( H_I \propto a_s^a a_s^a a_p a_p \)) using optical interferometers [16, 17, 28, 29]. Nevertheless, such measurement schemes are almost always restricted to certain medium-dependent frequencies at which strong non-linearity can emerge.

Recent advances in quantum optics have enabled QND photon number measurements that do not rely on the material non-linearity through strong light–matter interactions using cavity or circuit quantum electrodynamical systems [11, 15, 18–26, 30]. In these QND schemes, cavity photons are coupled to a probe atomic system (e.g. a two-level system) with large atom-cavity detuning so that the cavity photon frequency is off-resonant with the electronic transition of the atom. One key point for such QND measurements is that, in the large detuning limit, the Stark shift of the atomic transition as induced by the atom-photon coupling is approximately linear-dependent on the photon number [31], so that the dispersive phase shift of the atom can be measured and served as a QND readout of the photon number. Such a readout is dispersive (i.e. the phase shift depends on the atom-cavity detuning) and perturbative (one neglects the higher-order terms to the interaction Hamiltonian in the large detuning limit). That being said, it is known that the higher-order corrections will inevitably cause the measurement to demolish the measured quantum state, thereby fundamentally restricting the applicability of a perturbative dispersive readout [32–34]. In practice, such measurement-induced demolition leads to a progressive damage to the qubit and cavity states as one aims to continuously monitor the qubit [31, 33].

With this background in mind, it is clear that to achieve an ideal QND photon measurement, one needs a non-perturbative dispersive coupling that does not rely on the linear dispersive limit [35]. In this letter, we propose a novel QND photon counting measurement scheme using a hybrid optoelectromechanical protocol that integrates the measured cavity with an electromechanical probe; see Fig. 1 for an illustration. Cavity photons interact with the mechanical resonator of the probe through the radiation-pressure force [36] which imprints the photon occupation in the mechanical motion. A single electron transistor (SET) [37, 38], being the electronic component of the probe, is exploited to measure the mechanical motion to which it is capacitively coupled. The exquisite
Figure 1. A possible realization of the proposed hybrid quantum system. The electromechanical probe consists of a mechanical resonator with frequency \( \omega_b \) capacitively coupled to a single electron transistor comprising a source (S), a drain (D) and an island (I). A microtoroidal cavity supporting an optical whispering gallery mode with frequency \( \omega_c \) and a finite photon occupation is coupled to the electromechanical probe with its mechanical mode through a radiating-pressure force.

charge sensitivity of the SET guarantees an ultra-sensitive measure of the mechanical motion [39–42]. Most importantly, we demonstrate that this hybrid optoelectromechanical system yields a mechanical-mode-mediated non-perturbative dispersive coupling between the cavity photons and the electrons in the SET, enabling a single-shot QND readout of photon number via charge current measurements of the SET. To make our QND photon counting protocol clear, we discuss its immediate implementations with currently available experimental conditions for optomechanical [36, 43–51] and electromechanical systems [39–41, 52–57]. In the strong optomechanical coupling regime, we show that the voltage shift of the differential conductance peak provides a sensitive measure for the number of photons stored in the cavity.

Hybrid optoelectromechanical system.– We consider an optoelectromechanical system \( H \) which includes a high-quality cavity with a single photon mode (\( a, a^\dagger \)) to be measured, and the electromechanical probe which consists of a mechanical resonator mode (\( b, b^\dagger \)), the SET island (\( d, d^\dagger \)) and the source (S) and drain (D) electrodes (\( c_{kv}, c_{kv}^\dagger \) for \( v = S, D \)). Here we denote each component by their annihilation and creation operators respectively. This Hamiltonian \( H \) can be written as (setting \( h = 1, e = 1, k_B = 1 \) and Fermi energy \( \epsilon_F = 0 \) hereafter)

\[
H = \epsilon_0 d^\dagger d + \omega_c a^\dagger a + \omega_b b^\dagger b \\
- g_0 a^\dagger a (b^\dagger + b) + \lambda d^\dagger d (b^\dagger + b) \\
+ \sum_{k,v=S,D} \left[ \epsilon_{kv} c_{kv}^\dagger c_{kv} + t_{kv}(c_{kv}^\dagger d^\dagger + d c_{kv}) \right].
\]  

where \( \omega_b \) and \( \omega_c \) are the frequency of the mechanical mode and the cavity photon, respectively. \( g_0 \) is the single-photon coupling strength arising from the radiation-pressure coupling between the cavity and mechanical modes [36]. \( \lambda \) denotes the coupling strength between the mechanical mode and the electronic conductor. Here the SET is assumed to be in the sequential tunneling regime such that it can be described by a single-level island at electrostatic energy level \( \epsilon_0 = 1/(2C_S) \) with the total capacitance \( C_S \), coupled to a collection of electrons in the two electrodes with energies \( \epsilon_{kv}, v = S, D \). This coupling is characterized by the spectral density defined as \( \Gamma_v(\epsilon) = \pi \sum_k t_{kv}^2 \delta(\epsilon - \epsilon_{kv}) \). Throughout the study, we consider the wide-band limit, \( \Gamma_v(\epsilon) = \Gamma_v [58] \). For simplicity, we focus on a normal SET, rather than a superconducting one [59].

We further include a dissipation Hamiltonian, \( H_{\text{tot}} = H + H_{\text{diss}} \), where \( H_{\text{diss}} \) represents the damping of the mechanical mode by its thermal environment at an ambient temperature \( T_0 \); this dissipation term will be treated at the level of an input-output theory [60]. Here we do not include the cavity mode damping (as induced by photon losses due to imperfect reflection for instance) with the understanding that typical QND measurements are performed within a time scale faster than that of such decay process [18]. While, in general, the mechanical damping occurs on a time scale that is much slower than that of cavity photon decay process, we keep it since it plays a crucial role for determining the current-voltage characteristics of the SET [60].

To reveal that the mechanical mode mediates an electron-photon dispersive coupling, we introduce a unitary transformation

\[
\mathcal{G} = \exp \left[ -g_0 (b^\dagger - b)a^\dagger a/\omega_b \right] \otimes \exp \left[ \lambda (b^\dagger - b)d^\dagger d/\omega_b \right]. \tag{2}
\]

The transformed system Hamiltonian \( \tilde{H} \equiv \mathcal{G} H \mathcal{G}^\dagger \) reads

\[
\tilde{H} = \left( \epsilon_0 - \frac{\lambda^2}{\omega_b} \right) d^\dagger d + \omega_c a^\dagger a + \omega_b b^\dagger b \\
- \frac{g_0^2}{\omega_b} (a^\dagger a)^2 + \frac{2\lambda g_0}{\omega_b} a^\dagger a d^\dagger d \\
+ \sum_{k,v=S,D} \left[ \epsilon_{kv} c_{kv}^\dagger c_{kv} + t_{kv}(c_{kv}^\dagger d^\dagger + d c_{kv}) \right]. \tag{3}
\]

Here \( \tilde{d} \equiv D^\dagger d \) with a displacement operator defined as \( \mathcal{D}_\lambda = \exp[(b^\dagger - b)\lambda/\omega_b] \). It should be noted that we account for the full radiation-pressure coupling, rather than its linearized form [36]. The effect of this transformation on \( H_{\text{diss}} \) is negligible as the coupling between the mechanical resonator and thermal environment is typically weak [36, 60].

From Eq. (3), it is evident that the transformed Hamiltonian yields a dispersive coupling \( H_I = 2g_0^2/\omega_b a^\dagger a d^\dagger d \). We emphasize that this interaction is non-perturbative, namely no Hamiltonian truncation is involved [31, 35]. Generally, \( H_I \) can be interpreted as either an electron-number-dependent shift of the cavity frequency or, vice versa, a photon-number-dependent shift of the electronic level. The transformation also generates an effective Kerr non-linear term \( H_K = -g_0^2 (a^\dagger a)^2/\omega_b \) between the cavity photons. However, \( H_K \) conserves the cavity photon number and hence has no impact on the QND photon counting.
Next, since we neglected cavity losses during the measurement process, the photon occupation $n_p \equiv \langle a^\dagger a \rangle$ can be considered a time-independent observable. Therefore, for a given photon number $n_p$, the dispersive coupling leads to a renormalized electrostatic energy of the SET island

$$
\tilde{\varepsilon}_n = \varepsilon_0 - \frac{\lambda^2}{\omega_b} + 2\frac{\lambda g_0}{\omega_b} \tilde{n}_p. \tag{4}
$$

In the SET, this renormalized electronic level $\tilde{\varepsilon}_n$ sets the condition for resonant electron transport. Thus, a shift in the electronic energy leads to a linear shift of the bias voltage at which resonant transport occurs.

**Current-voltage characteristics of the SET.** The steady state charge current $I_{\text{SET}}$ serves to probe the photon number. Based on a generalized input-output method [60–62], we arrive at the current expression out of the source

$$
\langle J_S \rangle = \frac{4\Gamma_S^2 \Gamma_D}{\Gamma} \int \frac{d\varepsilon}{2\pi} G(\varepsilon) [n_\text{eff}^S(\varepsilon) - n_\text{eff}^D(\varepsilon)]. \tag{5}
$$

Here, $\Gamma = \Gamma_S + \Gamma_D$ and $n_\text{eff}^S(\varepsilon) = \left[ \exp\left(\left(\varepsilon - \mu_e \right)/T_0 \right) + 1 \right]^{-1}$ is the Fermi-Dirac distribution of the $v$th lead with $\mu_e$ the chemical potential and $T_0$ the ambient temperature. The generalized transmission function reads

$$
G(\varepsilon) \equiv \text{Re} \left[ \int_0^\infty d\tau e^{-i\varepsilon_n\tau} B_\lambda(\tau) \right]. \tag{6}
$$

Here, ‘Re’ takes the real part, $\tilde{\varepsilon}_n$ is given by Eq. (4) and $B_\lambda(\tau)$ denotes a mechanical mode correlation function [60]

$$
B_\lambda(\tau) = \exp \left[ -\frac{\lambda^2}{\omega_b^2} \int \frac{d\omega}{\pi} \frac{\gamma_{\text{eff}}}{\gamma_{\text{eff}}^2 + (\omega - \omega_b)^2} \times \left( \coth (\omega/2T_{\text{eff}}) (1 - \cos \omega \tau) + i \sin \omega \tau \right) \right]. \tag{7}
$$

To arrive at Eqs. (5)–(7), we employ an effective bath description [63–66] to capture the dynamics of the mechanical mode. Based on the time scale separation between the fast electron dynamics and the slow mechanical motion, this treatment allows us to include both the intrinsic thermal dissipation of the mechanical mode (with damping rate $\gamma_0$ and temperature $T_0$) and the back-action from the conducting electrons (as approximated in terms of an extra thermalized bath with damping rate $\gamma_1$ and temperature $T_1$) [60, 63, 65, 66]. Consequently, the overall effect of the thermal dissipation and the back-action on the mechanical mode is described by an effective damping rate $\gamma_{\text{eff}} \equiv \gamma_0 + \gamma_1$ from an effective thermal bath characterized by an effective temperature $T_{\text{eff}} = (\gamma_0 T_0 + \gamma_1 T_1)/\gamma_{\text{eff}}$ [43, 63–65]. Typically, $\gamma_1/\gamma_0 \approx 20 - 50$ [43, 67].

**QND photon counting scheme.** We are now ready to formulate a QND photon counting measurement protocol using the current-voltage characteristics of the SET as a read-out observable. First, we verify that, given the dispersive coupling $H_I = \frac{2\lambda g_0}{\omega_b} n_p d^\dagger d$, the cavity photon number operator $n_p \equiv a^\dagger a$ and the charge current operator $J_S \equiv i \sum_k t_{k,S} (c_{k,S}^\dagger d - d^\dagger c_{k,S})$ satisfy the conditions of QND measurements [28], i.e. (i) $H_I = H_I(n_p)$, (ii) $[H_I, n_p] = 0$, and (iii) $[H_I, J_S] \neq 0$. Therefore, one can infer the photon number without disturbing the cavity field from a charge current measurement.

In practice, we propose the following two-step protocol to determine the photon number confined in the cavity by contrasting the following measurements: (1) By coupling an empty cavity to the electromechanical probe, we determine the peak position of the differential conductance $\partial J_S/\partial V$ of the SET (denoted as the reference voltage $V_0^* \equiv 0$). The peak position corresponds to the island energy $\varepsilon_0 - \frac{\lambda^2}{\omega_b}$. (2) Injecting a photonic field with a finite yet unknown photon occupation $\tilde{n}_p$ to the empty cavity [24], the peak of the differential conductance will shift and appear at voltage bias $V_n^* \equiv \varepsilon_0 - \frac{\lambda^2}{\omega_b}$, corresponding to the renormalized island energy $\tilde{\varepsilon}_n$.

Following this protocol, the photon occupation of the cavity can be simply inferred from the voltage difference $V_n^* - V_0^*$,

$$
\tilde{n}_{p, \text{measure}} = \frac{V_n^* - V_0^*}{2\Delta \varepsilon}. \tag{8}
$$

This constitutes one of main results of our work. Here, for simplicity, we assume a symmetric bias drop for the SET, with the understanding that our scheme is not limited to this scenario [60]. The resolution of the photon number measurement is determined by $\Delta \varepsilon \equiv 2\lambda g_0/\omega_b$. Such a QND photon counting can reach high sensitivity by increasing either the optomechanical coupling strength ($g_0$) or the electromechanical counterpart ($\lambda$), or ideally, both. Although here we rely on the differential conductance for the measurement of the voltage shift $V_n^* - V_0^*$, one can also resort to the second-order derivative $\partial^2 J_S/\partial V^2$ and identify the voltage values $V_n^*$ from its node, which also marks the onset of resonant transport.

**Experimental feasibility.** We now discuss the feasibility of the proposed hybrid platform with state-of-the-art nanoscale fabrication technologies for quantum cavity optomechanical systems [36, 43–51] and electromechanical counterparts [39–41, 52–57, 68, 69].

First, we justify the assumption that $\tilde{n}_p$ remains time-independent during the charge current measurement. For a typical SET, the electrostatic capacitance is $C_S \approx 400 \text{ aF}$ [38, 40] and the total junction resistance is $R \approx 100 \text{ k}\Omega$, so that the electron tunneling time can be estimated by $\tau_e = 2RC_S \approx 0.1 \text{ ns}$ [63]. We consider a high-quality cavity in which the cavity photon decays at a damping rate ($\kappa/2\pi$) of the order of MHz [45, 48–50] and the lifetime is about $1/\kappa \approx 1 \mu s$.

Second, as far as the peak voltage shift is concerned, the photon number resolution measured by Eq. (8) is determined by $\Delta \varepsilon = 2\lambda g_0/\omega_b$. Here we choose the electro-mechanical coupling to be weak (typically $\lambda/\omega_b \sim 0.1$ [70]) so that the effective bath description is valid [65, 66]. So far, the single-photon optomechanical coupling strength can reach $g_0/2\pi = 3.4 \text{ kHz}$ for an optomechanical microresonator system [50]. Thus, in practice, the estimate of the single-photon electrostatic energy shift is $\Delta \varepsilon \approx 10^{-12} \text{ eV}$, which can be sensed by
the ultra-high energy sensitivity of SET which is as low as a few $h \approx 10^{-16}$ eV · s $^{[71]}$.

Finally, notwithstanding, one legitimate concern is that zero-point quantum fluctuation and weak optomechanical coupling $^{[36]}$ may make this QND measurement ineffective at a single-photon level. Indeed, we focus here on experiments carried out with a large photon number (at least on the order of $\bar{n}_p \approx 10^7$) for achieving a strong optomechanical coupling $^{[50]}$, i.e. $g_0/\sqrt{\bar{n}_p} > \kappa$. In this multi-photon scenario, our QND measurement protocol should yield a peak voltage shift of the order $\bar{n}_p \Delta \epsilon \sim 10^{-6}$ eV with peak broadening determined by $\Gamma \sim \pi^{-1} \approx 10^{-6}$ eV and the effective temperature $T_{\text{eff}}$. Hence, under these experimental conditions, the effect of zero-point quantum fluctuation should not prevent us to observe a clear differential conductance peak shift.

**Proof-of-principle simulation.** To access the efficacy of the proposed QND photon counting scheme in the strong optomechanical coupling regime, we provide a proof-of-principle simulation of Eq. (8) for given photon occupation $\bar{n}_p$ using Eqs. (5)–(7). Fig. 2 depicts the differential conductance peak compared with that obtained when the cavity is empty, namely, $\bar{n}_p = 0$, thereby demonstrating the feasibility of the proposed QND photon counting scheme under current experimental conditions.

Although intriguing, there are few remarks that are worth mentioning: (i) We have adopted the relation $T_f = V/4$, see definitions below Eq. (7), where $V$ is the applied voltage bias $^{[63, 65]}$. Strictly speaking, this expression is valid for large voltage bias of the order of $e_0$ $^{[63, 65]}$. Hence, quantitatively, our calculation overestimates the back-action from the conducting electrons to the mechanical mode at low voltages. However, this overestimation should have a negligible effect on demonstrating our QND protocol as we are interested in the resonance peak of the differential conductance rather than its broadening. (ii) The proposed QND photon counting scheme should be equally applicable in superconducting SET with modified electron tunneling rates $^{[65]}$. Hence we expect that the voltage shift due to a finite photon occupation can still serve as an accessible QND measure in the superconducting case.

**Conclusion.** We proposed an experimentally feasible QND measurement for cavity photon counting using an electromagnetic probe. Our scheme builds upon a mechnaical-mode-mediated coupling between cavity photons and conducting electrons, enabling a single-shot QND readout of photon number through the measurement of the SET charge current. We further demonstrated the feasibility of the measurement protocol by simulating the current-voltage characteristics of the SET in a strong optomechanical coupling regime achieved with a large photon occupation.

Looking forward, we expect that a single-photon QND measurement with the proposed hybrid scheme can be realized in the near future as many experimental advances of cold atom $^{[72, 73]}$, photon-crystal $^{[74]}$, and microwave $^{[48, 49, 75]}$ optomechanical systems have already showed great promise to reach the strong single-photon coupling regime. With the ability to measure few photons, we should be able to non-destructively identify different cavity photon statistics, such as Poisson and Bose-Einstein distributions, by repeating the QND measurement and depicting the photon number histogram. Furthermore, if we include molecular systems within the cavity, this QND measurement may provide a direct probe to investigate polariton excitations (hybrid light-matter excitation when the molecular system is strongly coupled to cavity photons), rather than relying on far-field photon emission $^{[76]}$. Lastly, while we have conveniently neglected time-dependence of the photon occupation in the present paper, extension of this QND scheme for observing multi-photon correlation functions should reveal more quantum properties, such as photon bunching and antibunching $^{[77]}$, representing an exciting new direction for quantum optics.

**Acknowledgement.** The authors thank Prof. Abraham Nitzan for insightful discussions and constructive comments. J. Liu and D. Segal acknowledge support from the Natural Sciences and Engineering Research Council (NSERC) of Canada Discovery Grant and the Canada Research Chairs Program.
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[59] We point out that Eq. (1) is applicable in the sequential tunneling regime regardless of whether the SET is superconducting or not [65, 66]. The only difference is that in the case of a superconducting SET, one should interpret $d$ as the occupation operator for quasi-particles. For our case, although we are in the superconducting regime ($T \sim 100$ mK), the normal-state description is still applicable as one can apply an out-of-plane magnetic field to turn a superconducting SET to a normal one [39].

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Supplemental material: Quantum Nondemolition Photon Counting With a Hybrid Electromechanical Probe

In this supplementary material we present the derivation of the steady state charge current expression used in the main text by resorting to a generalized input-output method [61, 62].

I. CURRENT-VOLTAGE CHARACTERISTICS OF SINGLE ELECTRON TRANSISTORS

In this study, we consider a hybrid optoelectromechanical system, which includes a quantum cavity coupled to a mechanical resonator, itself interacting with a single electron transistor (SET) that acts as an electromechanical probe to the photon number. For the sake of completeness, we first write down the total Hamiltonian \( H_{\text{tot}} = H_0 + H_E + H_{\text{diss}} \) (setting \( \hbar = 1, e = 1, k_B = 1 \) and Fermi energy \( \epsilon_F = 0 \) hereafter),

\[
H_0 = \epsilon_0 d^\dagger d + \omega_c a^\dagger a + \omega_b b^\dagger b - g_0 a^\dagger a(b^\dagger + b) + \lambda d^\dagger d(b^\dagger + b), \\
H_E = \sum_{k,v=u,S,D} \left[ \epsilon_{kv} c_{kv}^\dagger c_{kv} + t_{kv}(c_{kv}^\dagger d + d^\dagger c_{kv}) \right], \\
H_{\text{diss}} = \sum_j \omega_j r_j^\dagger r_j + \sum_j \eta_j (r_j^\dagger b + b^\dagger r_j). 
\]

(S1)

Here, \( H_0 \) accounts for the high-quality single mode cavity of frequency \( \omega_c \) with an annihilation operator \( a \), the high-quality mechanical resonator of frequency \( \omega_b \) with an annihilation operator \( b \), a SET island of an electrostatic energy \( \epsilon_0 \) with an annihilation operator \( d \), and a radiation-pressure optomechanical coupling as well as an electromechanical interaction characterized by coupling strengths \( g_0 \) and \( \lambda \), respectively. \( H_E \) contains the electron source (S) and drain (D) of the SET, together with electron tunneling between the island and the electrodes. Finally, \( H_{\text{diss}} \) accounts for the damping of the mechanical mode induced by its intrinsic thermal environment modelled as a harmonic thermal bath with annihilation operators \( r_j \) and frequencies \( \omega_j \). \( \eta_j \) denotes the coupling strength between the mechanical mode and the \( j \)th harmonic oscillator of the thermal bath. We assume that the interaction between the mechanical mode and the thermalized modes is rather weak such that the rotating wave approximation is justified. The influence of thermal bath, acting on the mechanical mode, is characterized by the spectral density function \( \gamma_0(\omega) = \pi \sum_j \eta_j^2 \delta(\omega - \omega_j) \).

The transformed Hamiltonian \( \tilde{H}_{\text{tot}} \equiv G H_{\text{tot}} G^\dagger = \tilde{H}_0 + \tilde{H}_E + \tilde{H}_{\text{diss}} \) under a unitary transformation generated by the operator \( G = \exp \left[ -g_0(b^\dagger - b)a^\dagger a/\omega_b \right] \otimes \exp \left[ i(\lambda(b^\dagger - b)d^\dagger d/\omega_b) \right] \) becomes

\[
\tilde{H}_0 = \left( \epsilon_0 - \frac{\lambda^2}{\omega_b} \right) d^\dagger d + \omega_c a^\dagger a + \omega_b b^\dagger b - \frac{g_0^2}{\omega_b} (a^\dagger a)^2 + \frac{2\lambda g_0}{\omega_b} a^\dagger a d^\dagger d, \\
\tilde{H}_E = \sum_{k,v=u,S,D} \left[ \epsilon_{kv} c_{kv}^\dagger c_{kv} + t_{kv}(c_{kv}^\dagger \tilde{d} + d^\dagger c_{kv}) \right].
\]

(S2)

Here \( \tilde{d} \equiv D_\Lambda d \) denotes a polaron operator with a displacement operator defined as \( D_\Lambda = \exp( b^\dagger - b ) \lambda/\sqrt{\omega_b} \). We neglect the effect of this transformation on \( H_{\text{diss}} \). To be precise, we ignore the term \( \sum_j \eta_j (\lambda d^\dagger d/\omega_b - g_0 a^\dagger a/\omega_b)(r_j^\dagger + r_j) \) in the transformed Hamiltonian. This omission is justified in the present study since the energies \( \eta_j \lambda/\omega_b \) and \( \eta_j g_0/\omega_b \) are assumed small, by noting that the coupling between the high-quality mechanical mode and its thermal environment should be rather weak and \( \lambda, g_0 \ll \omega_b \).

Adopting a recently developed generalized input-output method for electronic systems [61, 62], we treat the hybrid quantum system within a unified input-output picture. As the system \( \tilde{H}_0 \) contains both fermionic and bosonic operators, we should treat them separately. To this end, we use the notations \( \{A,B\} \equiv \{A,B\}_- \) and \( \{A,B\} \equiv \{A,B\}_+ \) for the quantum commutator and anti-commutator, respectively. The corresponding Heisenberg-Langevin equation (HLE) that governs the dynamical evolution of system operators reads [61]

\[
\dot{\mathcal{O}} = i[\tilde{H}_0, \mathcal{O}]_- - i \sum_{v=u,S,D} L_v^\mu_- - i\chi.
\]

(S3)

Here, \( \dot{\mathcal{A}} \) denotes a time derivative of operator \( \mathcal{A} \). Explicit forms for the superoperators \( L_v^\mu \) and \( \chi \) are obtained from an input-output description of the electron tunneling Hamiltonian in \( \tilde{H}_E \) and the thermal damping of the mechanical mode by \( \tilde{H}_{\text{diss}} \), respectively [61],

\[
L_v^\mu \equiv \left( d^\dagger \Gamma_v d^\dagger + \sqrt{2\pi} d^\dagger_{\text{in}} \right) \left[ \mathcal{O}, d \right]_+ + \left[ \mathcal{O}, d^\dagger \right]_+ \left( -i\Gamma_v d^\dagger + \sqrt{2\pi} d_{\text{in}} \right), \\
\chi \equiv \left( i\gamma_0 b + \sqrt{2\pi} b_{0,\text{in}} \right) \left[ \mathcal{O}, b \right]_- + \left[ \mathcal{O}, b^\dagger \right]_- \left( -i\gamma_0 b^\dagger + \sqrt{2\pi} b_{0,\text{in}} \right).
\]

(S4)
Here $\gamma_0 \equiv \gamma_0(\omega_0)$ denotes a damping rate for the mechanical mode induced by its thermal bath, $\Gamma_v(\epsilon) = \pi \sum k t^2_{kv} \delta (\epsilon - \epsilon_{kv})$ is the spectral density function of electrons in the two metals. In the above equation, the top signs apply if $O$ is a fermionic operator; the bottom signs apply if $O$ is bosonic. We remark that the form of $\mathcal{L}_i^\nu$ is exact in the wide-band limit, whereas $X$ is obtained by assuming a Markovian thermal bath with $\gamma_0(\omega)$ assumed a constant at the vicinity of $\omega_0$. We have defined input fields as follows

$$d_{in}^\nu(t) = \frac{1}{\sqrt{2\pi}} \sum_k t_{kv} e^{-i\epsilon_{kv}(t-t_0)} c_{kv}(t_0),$$

$$b_{0,in}(t) = \frac{1}{\sqrt{2\pi}} \sum_j \eta_j e^{-i\omega_j (t-t_0)} r_j(t_0).$$

(S5)

Here $t_0$ is the initial time at which the dynamical evolution begins.

As can be seen, the definitions of input fields in terms of environment operators at the initial time ensure that they can be specified as initial conditions. We prepare the initial state of the hybrid system to be such that, at $t = t_0$, the SET island, the mechanical mode, and metallic leads have the same ambient temperature, and the Bose-Einstein distribution function $n_{b,0}(\omega) = [\exp(\omega/T_0) - 1]^{-1}$, respectively. We assume that the intrinsic thermal environment of the mechanical mode and metallic leads have the same ambient temperature $T_0$. By doing so, the noise correlators associated with the input fields are given by [61]

$$\langle b_{0,in}(t') b_{0,in}(t) \rangle = \gamma_0 \int \frac{d\omega}{2\pi^2} e^{-i\omega(t-t')} n_{b,0}(\omega),$$

$$\langle b_{0,in}(t) b_{0,in}^{\dagger}(t') \rangle = \gamma_0 \int \frac{d\omega}{2\pi^2} e^{-i\omega(t-t')} [1 + n_{b,0}(\omega)],$$

$$\langle d_{in}^{\nu,\dagger}(t') d_{in}^\nu(t) \rangle = \delta_{v,v'} \Gamma_v \int \frac{d\epsilon}{2\pi^2} e^{-i\epsilon(t-t')} n_{\nu}(\epsilon),$$

$$\langle d_{in}^\nu(t) d_{in}^{\nu,\dagger}(t') \rangle = \delta_{v,v'} \Gamma_v \int \frac{d\epsilon}{2\pi^2} e^{-i\epsilon(t-t')} \left[1 - n_{\nu}(\epsilon)\right].$$

(S6)

In obtaining the first two correlation functions, we have approximated $\gamma_0(\omega) \approx \gamma_0$, which is valid in the Markovian limit. The output fields are related to the input fields via the so-called input-output relations

$$b_{0,out}(t) = b_{0,in}(t) - i\sqrt{\frac{2}{\pi}}\gamma_0 b(t),$$

$$d_{out}^\nu(t) = d_{in}^\nu(t) - i\sqrt{\frac{2}{\pi}}\Gamma_v d(t).$$

(S7)

The above relations imply that it is sufficient to work with input fields in the context of input-output theory.

As the photon occupation $\bar{n}_\nu \equiv \langle a^\dagger a \rangle$ is time independent during the charge current measurement, we focus here on the dynamical evolution of electron and mechanical mode. Using the HLE Eq. (S3), we first find

$$\dot{b}(t) = -i(\omega_b + \gamma_0) b(t) - i\sqrt{2\pi} b_{0,in}(t) + i\frac{\lambda}{\omega_b} \sum_v \left( i\Gamma_v d(t) + \sqrt{2\pi} d_{in}^\nu(t) \right) \tilde{d}(t)$$

$$= -i\frac{\lambda}{\omega_b} \sum_v \tilde{d}(t) \left(-i\Gamma_v \tilde{d}(t) + \sqrt{2\pi} d_{in}^\nu(t) \right)$$

$$= -i\frac{\lambda}{\omega_b} \sum_v \left[ 2\sqrt{2\pi} \text{Im} \left( \tilde{d}^\nu(t) d_{in}^\nu(t) \right) - 2\Gamma_v \tilde{d}(t)d(t) \right]$$

$$= -i\frac{\lambda}{\omega_b} \sum_v \left( \bar{n}_\nu b(t) - i\sqrt{2\pi} b_{0,in}(t) + \frac{\lambda}{\omega_b} \sum_v J_v(t) \right),$$

where we have utilized the relations $[b, \tilde{D}^\nu] = -\frac{\lambda}{\omega_b} D^\nu$ and $[b, D_\lambda] = \frac{\lambda}{\omega_b} D_\lambda$. 'Im' takes the imaginary part. $J_v$ is the formal definition of charge current operator out of $v$-lead [61]

$$J_v = 2\sqrt{2\pi} \text{Im} \left( \tilde{d}^\nu (t) d_{in}^\nu(t) \right) - 2\Gamma_v \tilde{d}(t)d(t).$$

(S9)

Clearly, the term $\frac{\lambda}{\omega_b} \sum_v J_v(t)$ in Eq. (S9) represents the backaction from the conducting electrons arising due to the coupling of the mechanical mode to the SET. In the steady state limit, we have $\sum_v J_v = 0$ because of charge conservation. However, at
transient times, $\Sigma_v J_v(t)$ is generally nonzero. To account for this dissipation source which will in turn affect the current-voltage characteristics of the SET through the mechanical backaction, we need a faithful treatment of backaction from the conducting electrons.

Technically speaking, this coupled dynamical problem is challenging to solve even numerically. To simplify the problem while taking into account the backactions, we resort to an effective treatment motivated by a significant time-scale separation between electron tunneling ($\sim 10^{-10}$ s) and mechanical motion ($\sim 10^{-6}$ s) [42, 43, 63–66]: For a slow mechanical motion, an adiabatic approximation is valid and the SET acts as a thermalized environment characterized by a temperature $T_1$ and it induces an extra damping rate $\gamma_1$ on the mechanical mode. Particularly, $T_1$ is set by the source-drain voltage bias $[63, 65, 66]$ and $\gamma_1/\gamma_0 \sim 20–50$ [43, 67]. Altogether, the mechanical mode experiences damping due to its direct thermal bath (temperature $T_0$ and decay rate $\gamma_0$) and from the electronic compartment (temperature $T_1$ and decay rate $\gamma_1$). These two processes sum up to a total effective damping with an effective damping rate $\gamma_{\text{eff}} \equiv \gamma_0 + \gamma_1$ and an effective temperature $T_{\text{eff}} = (\gamma_0 T_0 + \gamma_1 T_1)/\gamma_{\text{eff}}$ [64, 65]. In doing so, the effective equation of motion for $b$ becomes

$$
\dot{b} = -(i\omega_b + \gamma_{\text{eff}}) b(t) - i\sqrt{2\pi} \langle \hat{b}_{\text{in},\text{eff}}(t) \rangle - i\sqrt{2\pi} \langle \hat{b}_{\text{in},\text{eff}}(t) \rangle,
$$

(S10)

where the effective input field is determined by the following correlation functions

$$
\langle \hat{b}_{\text{in},\text{eff}}^\dagger(t) \hat{b}_{\text{in},\text{eff}}(t) \rangle = \gamma_{\text{eff}} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} n_{b,\text{eff}}(\omega),
$$

and

$$
\langle \hat{b}_{\text{in},\text{eff}}(t) \hat{b}_{\text{in},\text{eff}}^\dagger(t') \rangle = \gamma_{\text{eff}} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} [1 + n_{b,\text{eff}}(\omega)],
$$

(S11)

with $n_{b,\text{eff}}(\omega) = [\exp(\omega/T_{\text{eff}}) - 1]^{-1}$. Eq. (S10) will be adopted to calculate the mechanical correlation function involved in the charge current in the below.

As for the electronic operator, we have

$$
\dot{d}(t) = - \left[ i \left( \epsilon_0 - \frac{\lambda^2}{\omega_b} + \frac{2\lambda g}{\omega_b} n_p \right) + \sum_v \Gamma_v \right] d(t) - i\sqrt{2\pi} \sum_v \mathcal{D}_\lambda(t) d_{in}^v(t) + \mathcal{D}_\lambda(t) d_{in}^v(t) \right],
$$

(S12)

here we have defined $\Gamma \equiv \sum_v \Gamma_v$ and

$$
\epsilon_n \equiv \epsilon_0 - \frac{\lambda^2}{\omega_b} + \frac{2\lambda g}{\omega_b} n_p.
$$

(S13)

We note that the dynamical evolution of displacement operator $\mathcal{D}_\lambda(t)$ is now determined by the effective description Eq. (S10).

The average charge current out of the source in the steady state limit reads

$$
\langle J_S \rangle = 2\sqrt{2\pi} \text{Im} \left\langle \hat{d}\hat{d}_{in}^\dagger \right\rangle - 2\Gamma_S \left\langle \hat{d} \hat{d} \right\rangle.
$$

(S14)

Here, the ensemble average are evaluated with respect to an initial factorized state where the metallic leads and mechanical thermal environment are in their thermal equilibrium states. To get the explicit form of $\langle J_S \rangle$, we solve Eq. (S12) in the steady state limit of $t_0 \to -\infty$:

$$
\langle J_S \rangle = 2\sqrt{2\pi} \sum_v \int_{-\infty}^{t} e^{-(\Gamma + i\epsilon_n)(t-\tau)} \mathcal{D}_\lambda(\tau) d_{in}^v(\tau).
$$

(S15)

We first evaluate the ensemble average $\langle \hat{d}\hat{d}_{in}^\dagger \rangle$ on the right-hand-side (RHS) of Eq. (S14) by using the correlation functions for input fields listed in Eqs. (S6) for $d_{in}^v$ and (S11) for $b_{in}$:

$$
\langle \hat{d}\hat{d}_{in}^\dagger \rangle = i\sqrt{2\pi} \int_{-\infty}^{t} d\tau e^{-(\Gamma + i\epsilon_n)(t-\tau)} \langle d_{in}^\dagger(\tau) \mathcal{D}_\lambda(\tau) d_{in}(\tau) \rangle
$$

$$
= \int_{-\infty}^{t} d\tau \frac{n_{in}^S(\tau)}{2\pi^2} \int_{0}^{\infty} e^{-(\Gamma + i\epsilon_n + \tau)} B_\lambda^*(\tau) d\tau,
$$

(S16)

where we have decoupled the electron and mechanical mode correlations by noting that Eq. (S10) does not contain any electronic operators,

$$
\langle d_{in}^{S,\dagger}(\tau) \mathcal{D}_\lambda(\tau) d_{in}^S(\tau) \rangle = \langle d_{in}^{S,\dagger}(\tau) d_{in}^S(\tau) \rangle \langle \mathcal{D}_\lambda(\tau) \mathcal{D}_\lambda(\tau) \rangle
$$

(S17)
and introduced a mechanical mode correlation function \( B_\lambda(t - \tau) = \langle \mathcal{D}_\lambda^\dagger(t) \mathcal{D}_\lambda(\tau) \rangle \) whose detailed form reads [61]:

\[
B_\lambda(\tau) = \exp \left[ -\frac{\lambda^2}{\omega_b^2} \int \frac{d\omega}{\pi} \frac{\gamma_{\text{eff}}}{\omega^2 + (\omega - \omega_b)^2} \left( \coth(\omega/2\gamma_{\text{eff}})(1 - \cos \omega \tau) + i \sin \omega \tau \right) \right].
\] (S18)

Similarly, we find

\[
\langle \alpha^d \alpha^d \rangle = 2 \int \frac{d\epsilon}{2\pi} \frac{\Gamma_S n_F^S(\epsilon) + \Gamma_D n_F^D(\epsilon)}{\Gamma} \int_{-\infty}^{t} d\tau e^{i(\epsilon - \epsilon_\alpha)(\tau - \tau')} e^{-\Gamma(2t - \tau - \tau')} B_\lambda(\tau - \tau')
\]

\[
= 4 \int \frac{d\epsilon}{2\pi} \frac{\Gamma_S n_F^S(\epsilon) + \Gamma_D n_F^D(\epsilon)}{\Gamma} \text{Re} \left[ \int_{-\infty}^{t} d\tau' e^{i(\epsilon - \epsilon_\alpha)(\tau - \tau')} e^{-\Gamma(2t - \tau - \tau')} B_\lambda(\tau - \tau') \right]
\]

\[
= 2 \int \frac{d\epsilon}{2\pi} \frac{\Gamma_S n_F^S(\epsilon) + \Gamma_D n_F^D(\epsilon)}{\Gamma} \text{Re} \left[ \int_{0}^{\infty} d\tau e^{-\Gamma(\epsilon - i\epsilon_\alpha - i\epsilon)\tau} B_\lambda(\tau) \right].
\] (S19)

Here, “Re” takes the real part. Altogether, we find

\[
\langle J_S \rangle = \frac{4\Gamma_S \Gamma_D}{\Gamma} \int \frac{d\epsilon}{2\pi} \text{Re} \left[ \int_{0}^{\infty} d\tau e^{-(\Gamma + i\epsilon_\alpha - i\epsilon)\tau} B_\lambda(\tau) \right] [n_F^S(\epsilon) - n_F^D(\epsilon)],
\] (S20)

which is the charge current expression that we use in the main text. Notably, we can identify an effective transmission function in the integral. It depends on the mechanical mode autocorrelation function, and it includes the backaction of electrons through an effective-bath approximation.

\[\text{II. MEASURING PHOTON NUMBER WITH AN ASYMMETRIC BIAS DROP}\]

In general, we can express the chemical potentials of the electrodes (source and drain) as

\[
\mu_S = \alpha V, \quad \mu_D = -(1 - \alpha)V.
\] (S21)

Here \( V \) is the voltage bias across the SET and \( \alpha \in [0, 1] \) characterizes the asymmetry of bias drop and can be determined by experiments. For the SET, we can have a phenomenological expression

\[
\alpha = \frac{R_S}{R_S + R_D}
\] (S22)

with \( R_v \) \((v = S, D)\) the junction resistance of the corresponding island-lead interface. If \( R_S = R_D \), we recover the symmetric bias drop considered in the main text.

With the above voltage splitting given by Eq. (S21), the resonant electron transport occurs when the following condition

\[
\mu_S = \tilde{\epsilon}_n
\] (S23)

is fulfilled. Here, \( \tilde{\epsilon}_n \) is given by Eq. (S13). From the above equation, we find

\[
V_n^* = \frac{\tilde{\epsilon}_n}{\alpha},
\] (S24)

which yields the following expression for the measured photon number

\[
\bar{n}_{p,\text{measure}} = \frac{\alpha}{\Delta\epsilon} (V_n^* - V_0^*)
\]

\[
= \frac{R_S}{(R_S + R_D)\Delta\epsilon} (V_n^* - V_0^*).
\] (S25)

Here, \( \Delta\epsilon \equiv 2\lambda g_0/\omega_b \). The symmetric case considered in the main text is recovered when \( R_S = R_D \).