Quantum Detection of Inertial Frame Dragging

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We study the response function of Unruh De-Witt detectors placed in a slowly rotating shell. We show that the response function picks up the presence of rotation even though the spacetime inside the shell is flat and the detector is locally inertial. Moreover, it can do so when the detector is switched on for a finite time interval within which a light signal cannot travel to the shell and back to convey the presence of rotation.

I. INTRODUCTION

Within any static spherical shell spacetime is flat. By emitting a signal, a pointlike switchable classical antenna at the center of the shell could determine whether or not the shell is present, but only for times longer than the light-crossing time $t_s$ of the shell. Otherwise it is not possible to distinguish its environment from flat Minkowski spacetime.

However, a quantum detector can distinguish between these possibilities for times much smaller than $t_s$. The vacuum state of the quantum field in the shell induces characteristic responses in an Unruh De-Witt (UDW) detector [1], a 2-level quantum system, that differ from the responses induced when the shell is absent [2, 3]. Not only does the vacuum state of a quantum field carry non-local information about the gravitational field of the shell, but the UDW detector can read out that information locally. The ability of such detectors to obtain non-local information about spacetime structure has also been shown in other examples [2–6].

In this paper, we extend the work of [2, 3] by adding rotation to the shell. As before, a classical antenna inside the shell will not be able to detect the rotation in times smaller than $t_s$ in the absence of any other source of energy-momentum. We will show that the quantum UDW detector will be able to do so even in the vacuum state of the quantum field. We consider this to be a quantum detection of inertial frame dragging.

Frame-dragging, also known as the Lense–Thirring effect [7, 8], is a general-relativistic effect that arises due to moving, in particular rotating, matter [9] and 'rotating' gravitational waves [10, 11]. If a gyroscope is located in the vicinity of a rotating body, it will keep its direction with respect to the axes of a local inertial frame at the same place but both the inertial axes and the gyroscope will be rotating with respect to static distant observers (“fixed stars” at asymptotically flat infinity). Its pro-

found explicit manifestation can be seen for a rotating black hole, which drags particles into co-rotation, the dragging becoming so strong inside the ergosphere that no particle there can remain at rest with respect to fixed stars [12].

Inertial frames are also dragged into co-rotation inside rotating bodies. Consider a slowly rotating material shell (see, e.g., [3, 13]). Observers inside the shell who are at rest with respect to distant fixed stars will find that a particle moving inside the shell experiences a Coriolis acceleration (the centrifugal acceleration is of the second order in the shell’s angular velocity). These observers are not inertial, therefore fictitious forces arise. For inertial observers, without looking at or outside the rotating shell, there is no way of determining, by employing classical physics, whether they are surrounded by a rotating shell. They can in principle determine its rotation by, for example, sending out a spherical pulse which, upon reflection, will experience a differential Doppler effect, with different shell latitudes Doppler shifting differently. However, we shall demonstrate that quantum detection of frame dragging is indeed possible for a UDW detector whose location is offset from the center of the shell.

II. SLOWLY ROTATING SHELLS

The metric outside a slowly rotating shell can be written as

$$ds^2 = -f(r)dt^2 + r^2 sin^2 \theta (d\phi - \frac{2Ma}{r^3}dt)^2 + f(r)^{-1} dr^2 + r^2 d\theta^2,$$

where $f(r) = 1 - 2M/r$, $M$ is the mass of the shell and $a = J/M$ is the angular momentum per unit mass. The r-coordinate ranges from $[R, \infty)$, $R$ being the radius of the shell. To first order in $a$, the above metric agrees with the Kerr metric and satisfies the vacuum Einstein’s equations. Inertial frame-dragging is characterized by the function $\omega(r) = g_{\phi\phi}/g_{\phi\phi} = 2J/r^3$, where $J = Ma$ is the fixed total angular momentum as measured at infinity. The gradients of $\omega(r)$ determine the precession of gyroscopes relative to the orthonormal frame of locally

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non-rotating observers \[12\]. On the shell itself, \( r = R \), and \( \varpi_s = 2J/R^3 \).

For an inertial observer inside the shell (who rotates as seen from infinity) spacelike geodesics (for example, \( \phi = 0, \theta = \pi/2, t = \text{constant} \)) connected to fixed points at infinity rotate backwards; the shell is rotating forward (the dragging of the inertial frame becomes complete only if the shell is at its Schwarzschild radius); the fixed stars are rotating backwards. In \[14\] these effects are expressed quantitatively.

The metric (1) must be joined at \( r = R \) to the flat metric inside the shell,

\[
ds^2 = -f(R)dt^2 + r^2 \sin^2 \theta \left( d\phi - \frac{2Ma}{R^3} dt \right)^2 + dr^2 + r^2 d\theta^2.
\]

This metric can be seen to be flat using the coordinate transformation,

\[
\varphi = \phi - \frac{2Ma}{R^3} t,
\]

which transforms the metric (2) to the flat metric in standard coordinates. The stress energy tensor of the shell giving rise to the above spacetime can be found using the Israel junction condition \[15\] and has been well-studied in the literature \[16\].

The coordinates used in \[1\] are (spherical) Lorentzian at infinity and are naturally associated with stationary observers at infinity. All observers at fixed \((r, \theta, \varphi)\) inside the shell rotate rigidly at the rate \( \text{d} \phi/\text{d}t = 2Ma/R^3 \) with respect to observers at rest at infinity (\( \phi = \text{constant} \)). This effect is known as the dragging of inertial frames, first discovered in 1918 by Thirring and Lense \[7\,\[8\].

### III. NORMALIZED MODE SOLUTIONS

We next obtain the normalized mode solutions to the scalar wave equation in the above spacetime to first order in \( a \). For the scalar field \( \Psi \) the wave equation is written as:

\[
\partial_{\mu} \sqrt{-g} g^{\mu \nu} \partial_{\nu} \Psi = 0,
\]

where \( g \) is the determinant of the metric. Upon substituting in the metric \[1\] and \[2\], this equation can be solved by separation of variables. At linear order in \( a \), one can still use the usual mode expansion

\[
\Psi_{\omega \ell m}(t, r, \theta, \phi) = \frac{1}{\sqrt{4\pi \omega}} e^{-i \omega t} Y_{m \ell}(\theta, \phi) \psi(r)
\]

in spherical harmonics \( Y_{m \ell} \). This yields a separated radial equation:

\[
\frac{\alpha}{\beta r^2} \frac{d}{dr} \left( \frac{\alpha}{\beta} r^2 \frac{d\psi}{dr} \right) - \left( \frac{\alpha^2 (\ell + 1)}{r^2} + \gamma + \omega^2 \right) \psi = 0.
\]

The functions \( \alpha, \beta \) and \( \gamma \) are

\[
\begin{align*}
\alpha(r) &= \begin{cases} \sqrt{f(R)} & \text{for } r \leq R \\ \frac{1}{\sqrt{f(R)}} & \text{for } r > R \end{cases} \\
\beta(r) &= \begin{cases} 1, & \text{for } r \leq R \\ 1/\sqrt{f(R)} & \text{for } r > R \end{cases} \\
\gamma(r) &= \begin{cases} \frac{4Mam\omega}{R^2} - \frac{1}{2} \left( \frac{2Mam}{R} \right)^2, & \text{for } r \leq R \\ \frac{2Mam\omega}{R^2} - \frac{1}{2} \left( \frac{2Mam}{R} \right)^2, & \text{for } r > R \end{cases}
\end{align*}
\]

However, the solution outside the shell has to be determined numerically and matched to the solution on the shell. Specifically, we impose continuity of the solution at the shell, \( \psi(R) = j_\ell(\sqrt{\omega(\omega\gamma)}) \). To find the value of \( d\psi/dr |_{R^+} \), we integrate Eq. (5) across the shell, obtaining the condition

\[
\left[ \frac{\alpha(r)}{\beta(r)} \frac{d}{dr} \psi \right]_{r=R} = 0,
\]

where \( \epsilon_r^\mu \) is the radial element of the tetrad and the square brackets represent the difference in the value of the term across the shell. Noting the discontinuity in the coefficient \( \beta(r) \) across the shell as given by expression (6), this yields the required initial conditions \( \psi(R^+) \) and \( \psi'(R^+) \) for numerically solving the radial equation outside the shell.

Finally, to normalize the solution, we will follow the scheme presented in \[2\]. First, the radial equation for \( r > R \) can be rewritten in terms of a new coordinate \( r^* \) such that \( d/dr = \frac{2}{r} d/dr^* \). Further defining \( \rho = r \psi \), the radial equation reads

\[
\frac{d^2}{dr^2} \rho + (\omega^2 - V(r)) \rho = 0,
\]

where

\[
V(r) = \frac{\alpha^2 (\ell + 1)}{r^2} + \gamma + \frac{1}{r} \frac{d}{dr} \left( \frac{\alpha}{\beta} \right).
\]

Asymptotically, \( V(r) \to 0 \) as \( r \to \infty \) and hence \( \psi \sim \sin(\omega r^*)/r^* \). Let the normalized radial solution be denoted as \( \tilde{\psi}_{\omega \ell m}(r^*) = A_{\omega \ell m} \psi(r^*) \). Given any two wave-functions \( \Psi_1, \Psi_2 \), their Klein-Gordon inner product is

\[
(\Psi_1, \Psi_2) = i \int_\Sigma d\sigma n^\mu (\Psi_1^* \nabla_\mu \Psi_2 - \Psi_2^* \nabla_\mu \Psi_1^*),
\]

where \( \Sigma \) is a Cauchy surface with normal \( n^\mu \). A solution will be normalized with respect to the Klein-Gordon inner product if we choose the normalisation constant \( A_{\omega \ell m} \) such that \( A_{\omega \ell m} \psi(r^*) \to 2 \sin(\omega r^*)/r^* \) as \( r^* \to \infty \) \[2\].
IV. UDW DETECTOR RESPONSE

A UDW detector \[1\] is a 2-level quantum mechanical system that interacts locally with a scalar quantum field as it moves along some trajectory \(x(\tau)\) in spacetime. Letting \(\Omega\) denote the energy gap of the detector and \(\hat{\mu}(\tau) = e^{-i\Omega_1 \hat{\sigma}^+} + e^{i\Omega_1 \hat{\sigma}^-}\) its monopole moment (in the interaction picture), the Hamiltonian governing the detector/field interaction is

\[\hat{H}(\tau) = \lambda \chi(\tau) \hat{\mu}(\tau) \otimes \hat{\Psi}(x(\tau)),\]

where \(\hat{\sigma}^\pm\) are the ladder operators, \(\tau\) is the proper time of the detector, and \(\lambda\) is the dimensionless coupling constant. The duration of interaction is controlled by the switching function \(\chi(\tau)\), which we will choose to be

\[\chi(\tau) = \begin{cases} \cos^4(k\tau), & -\frac{\pi}{2k} \leq \tau \leq \frac{\pi}{2k} \\ 0, & \text{otherwise} \end{cases},\]

which has a shape similar to the Gaussian switching function \(\chi_G\) \[3\] used in the static case \[2\], but ensures for some \(k > 0\) that the interaction takes place only between \(\tau \in (-\frac{\pi}{2k}, \frac{\pi}{2k})\). We will denote the total duration of the interaction by \(\Delta \tau = \pi/k\).

If the detector starts off in the ground state and interacts with the quantum vacuum via the above Hamiltonian, there may be a non-zero probability of finding the detector in its excited state after the interaction. The probability of excitation of the detector can be calculated using perturbation theory and is well-known in the literature. It is given by \[17, 18\]

\[P = \lambda^2 \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \chi(\tau_1) \chi(\tau_2) e^{-i\Omega(\tau_2 - \tau_1)} W(x(\tau_1), x(\tau_2)) \times W(x(\tau_1), x(\tau_2))\]

(14)

to second order in \(\lambda\), where \(W(x(\tau_1), x(\tau_2))\) is the Wightman function of the field evaluated along the detector trajectory.

The field operator can be expanded in terms of the normalized field modes \(\hat{\Psi}_{\omega \ell m}\) of the previous section as

\[\hat{\psi}(x(\tau)) = \sum_{\ell, m} \int_0^{\infty} d\omega \hat{a}_{\omega \ell m} \hat{\Psi}_{\omega \ell m}(x(\tau)) + \hat{a}_{\omega \ell m}^\dagger \hat{\Psi}_{\omega \ell m}^\dagger(x(\tau)),\]

(15)

with \(\hat{a}_{\omega \ell m}\) denoting the mode annihilation operators. Let \(|0\rangle\) denote the field vacuum such that \(\hat{a}_{\omega \ell m} |0\rangle = 0\). This corresponds to the vacuum with respect to an observer located at infinity, who is in a non-rotating frame. The Wightman function with respect to this vacuum \(W(x(\tau_1), x(\tau_2)) = \langle 0 | \hat{\psi}(x(\tau_2)) \hat{\psi}(x(\tau_1)) | 0 \rangle\) is given by

\[W(x(\tau_1), x(\tau_2)) = \sum_{\ell, m} \int_0^{\infty} d\omega \hat{\Psi}_{\omega \ell m}^\dagger(x(\tau_1)) \hat{\Psi}_{\omega \ell m}(x(\tau_2)).\]

(16)

From the previous section, we have seen that the normalized mode solutions are given by \(\hat{\Psi}_{\omega \ell m} = \frac{1}{\sqrt{4\pi}} e^{-i\omega t} Y_{\ell m}(\theta, \phi) \hat{\psi}_{\omega \ell m}(r)\). Recall that we are interested in studying how the response of the detector differs when placed respectively in a rotating shell and a stationary shell. A simple choice for the trajectory \(x(\tau)\) of the detector is \(r = r_d, \theta = \pi/2, \varphi = 0\) i.e., \(\phi = 2Mm \pi t\). In this case, noting that \(t = \tau/h\), where \(h = \sqrt{f(R)}\), we find the response function \(F = P/\lambda^2\) of the field in the form

\[F = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \chi(\tau_1) \chi(\tau_2) e^{-i\Omega(\tau_2 - \tau_1)} \sum_{\ell, m} \int_0^{\infty} d\omega \hat{a}_{\omega \ell m}^\dagger(x(\tau_1)) \hat{\Psi}_{\omega \ell m}(x(\tau_2)) \]

\[= \sum_{\ell, m} \int_0^{\infty} \frac{d\omega}{4\pi \omega} \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \chi(\tau_1) \chi(\tau_2) e^{-i(\Omega + \frac{\omega}{h} - \frac{2Mm}{R^3 h})(\tau_2 - \tau_1)} |Y_{\ell m}(\frac{\pi}{2}, 0)|^2 |A_{\omega \ell m}|^2 |j_t(\sqrt{b(\omega)} r_d)|^2,\]

\[= \sum_{\ell, m} \int_0^{\infty} \frac{d\omega}{2\omega} \left| \frac{\omega}{h} - \frac{2Mm}{R^3 h} \right|^2 |A_{\omega \ell m}|^2 |Y_{\ell m}(\frac{\pi}{2}, 0)|^2 |j_t(\sqrt{b(\omega)} r_d)|^2,\]

(17)

where we switched the order of integration since the integrand is smooth and integrated over the \(\tau_1\) and \(\tau_2\) variables, which amounts to performing Fourier transforms

\[\hat{\chi}(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\tau \chi(\tau) e^{-i\gamma \tau}\]

(18)

on the switching functions, noting that \(\hat{\chi}(-y) = \hat{\chi}(y)\) for a real switching function.

We pause to comment that we have computed \[17\] from the modes \(\hat{\Psi}_{\omega \ell m}\) assuming \[5\] is exact. However the metric \[1\] is a valid solution of the Einstein equations only to order \(a\) while the leading corrections to
the Wightman function \( \langle 16 \rangle \) (and thus detector response \( \langle 17 \rangle \)) are of order \( a^2 \). For sufficiently small \( Ma/R^2 \), terms of higher order in \( a \) will not significantly affect our quantitative results, and so we shall plot \( \langle 17 \rangle \) in what follows.

V. RESULTS

We are now ready to look at how rotation of the shell affects the response of UDW detectors. We do this by computing the expression \( \langle 17 \rangle \) numerically, terminating the sum over \( \ell \) at sufficiently large \( \ell \), chosen to give resultant errors not larger than 1%.

![FIG. 1. Detector response against \( \Omega/k \). Shown here is the plot of the difference \( \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}} \) against \( \Omega \) for different (dimensionless) rotation parameters \( a_k \) with \( Mk = 1, Rk = 3, \tau d/k = 0.5 \). The inset shows a zoom-in of the plot around \( \Omega/k = 0 \). The difference \( \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}} \) is small but non-zero, and is more sensitive to the rotation for negative \( \Omega \).
](image)

Fig. 1 shows a plot of \( \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}} \equiv \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{rot}}(a = 0) \) against \( \Omega \) for various (dimensionless) rotation parameters \( a_k \). The difference between the response of a detector placed in a slowly rotating shell \( \mathcal{F}_{\text{rot}} \) and that placed in a static shell \( \mathcal{F}_{\text{stat}} \), though small, is clearly non-zero. The difference is more pronounced when the energy gap \( \Omega/k < 0 \), which physically means that the detector starts off in the excited state. The rotation parameter \( a \) enters the response function \( \mathcal{F} \) in three positions in eq. \( \langle 17 \rangle \): in the Fourier transform of the switching function, in the normalisation constant \( A_{\omega m} \), and in the \( b(\omega) \) of the spherical Bessel function. The net effect of these is an expected increase in \( \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}} \) with \( a \).

We emphasize that the interaction duration \( \Delta \tau_k = \pi \) between the field and detector is less than \( t \), \( \omega = 2(R - r_d)/k = 5 \), the time needed for a light signal to travel from the detector to the shell and back. This is in striking contrast to the classical case, where the fastest way a detector inside the shell (with all possible classical fields in their vacuum states) can detect the presence of rotation is by sending and waiting for a light signal to come back from the shell.

In the top figure of Fig. 2 we plot both \( \mathcal{F}_{\text{rot}}(ak = 0.9) \) and \( \mathcal{F}_{\text{stat}} \) against the detector location \( r_d/R \). The responses peak at some intermediate \( r_d \), in agreement with the results of ref. [3]. From the bottom figure, we see that the detector response increases by more than an order of magnitude as compared to Fig. 1 as \( r_d/R \rightarrow 1 \). We find that the shape of the curves in Fig 1 remains qualitatively the same as \( r_d/R \) increases, though the interaction duration is eventually no longer less than the light crossing time. A detector placed at the origin \( r_d = 0 \) cannot distinguish between a rotating and a static shell. We can understand this explicitly by noting that the rotation parameter \( a \) appears in the radial equation \( \langle 5 \rangle \) through the term \( \gamma \), the azimuthal number \( m \). Hence, it has only nontrivial effects when \( m \neq 0 \). However since \( \theta = 0 \) along the axis of rotation and \( Y_{\ell m}(0, 0) \) is non-zero only when \( m = 0 \), the mode solutions and hence the response function are insensitive to effects of rotation along this axis. As another illustration of this, we plot in Fig. 3 \( \mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}} \) against \( \theta \), the angle measured from the rotation axis. From this, we see that the sensitivity to rotation of detectors placed at the same \( r_d \) increases monotonically as \( \theta \) increases from 0 to \( \pi/2 \).

VI. CONCLUSIONS

Classically, the physical effect of a slowly rotating shell is the dragging of inertial frames. We have shown that this effect can be discerned from local measurements of a quantum particle detector inside the shell, on timescales
much shorter than the light travel time from the detector to the edge of the shell and back.

We note that the gravitational effects inside a rotating material shell are analogous to the electromagnetic effects inside a rotating charged shell; but there are also fundamental differences. For a rotating charged shell, a dipolar magnetic field will be formed inside. Such a field can be observed without the need of quantum detectors, for example as the Larmor precession of charged particles. Within Einstein–Maxwell theory, because of interacting electromagnetic and gravitational perturbations this field would, however, imply a curved spacetime inside the shell (cf. [14], Sec. 2.3), whereas spacetime remains flat for a slowly rotating massive uncharged shell. In the latter case, which is analogous with the quantum vacuum we are considering, the quantum detector outperforms a classical one in detecting rotation of the shell.

By solving the scalar field equation numerically, we have obtained the response function of the detector and seen how it depends on the rotation parameter $a$. Corrections to the metric (1) to higher orders in $a$ will quantitatively modify (17) but will not qualitatively affect our results. Alternatively, we can regard (1) as a ‘kinematic spacetime’ that could be employed in analogue gravity laboratory simulations, in which case our results would hold exactly. Whether or not such effects can be directly detected remains a challenge for future experiments.

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FIG. 3. Plot of $\mathcal{F}_{\text{rot}} - \mathcal{F}_{\text{stat}}$ against $\theta$ for $Mk = 1$, $Rk = 3$, $ak = 0.8$ and $r_\text{dk} = 0.5$.

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