Application of fiber Bragg grating sensors for the micro-strain measurement of optical components surface

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Abstract. Fiber Bragg Grating Sensors (FBGS) are gaining increasing attention in the fields of micro-stress analysis, showing high accuracy and sensitivity. In this paper, FBGS which wavelength variations can be converted to strains, are used to measure the micro-strain variation of a plane mirror where the forces acting upon. Both the self-gravity and extrusion forces are applied to the surface of the optical elements, generating inevitable deformation and decreasing the test accuracy of the whole optical system. First, a micro-strain model is built using finite element analysis to obtain the qualitative value of the micro-strain variation. Next, a series experiments are performed to validate the model is effective. A detailed analysis of the micro-strain variation is acquired from the phase generated carrier homodyne demodulation and then the three-step phase-shifting algorithm based on an unbalanced Mach-Zehnder interferometer system. The experimental results match well with the theory. Both the modeling and measurement results indicate that the sensitivity of this method in measuring the micro-strain can be reached as small as $10 \mu \varepsilon$.

1. Introduction
Fiber Bragg gratings (FBGs) have recently been applied in a growing range of sensing applications, such as aerospace, military, petrochemical, structural monitoring, chemical and biomedical sectors [1]. Their many advantages include their compact size which allows for embedding in advanced structures, easily distributed sensing, high precision and sensitivity, wavelength-encoded operation immune to light power fluctuations and system consumption and, in particular, their quasi-linear spectral response to external influences [2,3]. Of these, strain measurement is a major area of interest, and a number of configurations of FBG strain sensors have been demonstrated. The key point of the FBGS sensors systems is to attain the Bragg wavelength shifts caused by the strain. Various demodulation methods for the detection of the Bragg wavelength shift have been conceived. These approaches include techniques based on direct wavelength analysis, phase delay response, and optical intensity measurement [4], most of which involved some form of optical filtering [5]. Filters consisting of tunable filters incorporating matched FBGS, have been widely used [6,7]. Using interference technology can implement real-time measurement. Unbalanced Mach-Zehnder interferometer
changes the wavelength shifts into phase change, has extremely high sensitivity and accuracy. The strain on the surface of the optical component is slight and can not be easily discerned, but it can change the surface shape and is harm to the test accuracy of the whole optical system. So it is essential to measure the strain change of an optical component, better combined with the interferometer.

In this paper, we measure the micro strain on a plane mirror surface due to the extern pressure based on the unbalanced Mach-Zehnder interferometer system based on a $3 \times 3$ fiber coupler. Experimental results are well matched with the results from finite element analysis.

### 2. Principle

From the coupled wave theory, when the phase matching conditions are satisfied, FBG’s Bragg wavelength is (1)

$$\lambda_B = 2n_{\text{eff}} \Lambda$$

(1)

Where $\lambda_B$ is Bragg wavelength; $n_{\text{eff}}$ is effective refractive index of optical fiber spread abroad mode; $\Lambda$ is optical grating period. Here we assume the temperature is constant.

The relation between the shift of the Bragg wavelength of FBG and the axial strain applied to a fiber grating is (2)

$$\frac{\Delta \lambda_B}{\lambda_B} = (1 - \nu) \varepsilon = k \varepsilon$$

(2)

where $P_e = n_{\text{eff}}^2 [P_{12} - \nu(P_{11} + P_{12})]/2$ is the effective photoelastic coefficient of the glass fiber with Poisson ratio $\nu$, $P_{11}$, and $P_{12}$ denote the photoelastic coefficients, and $n_{\text{eff}}$ represents the effective refractive index of the guide mode. For a typical fused-silica fiber, $\nu = 0.16$, $n_{\text{eff}} = 1.46$, $P_{11} = 0.12$, $P_{12} = 0.27$, and thus, $P_e = 0.22$, $k = 0.78$.

![Figure 1 Scheme of measuring FBG system](image-url)

The measuring scheme is shown in figure 1. Take one single fiber-optic interferometer for
Using unbalanced Mach-Zehnder interferometer system, the changes of the reflection wavelength cause the diversification of the light intensity periodically, as equation (3) shows,

$$\Delta \varphi = \frac{2\pi nl}{\lambda} \Delta \lambda = \frac{2\pi nl}{\lambda} k \varepsilon$$  \hspace{1cm} (3)$$

So can measure the strain via the phase information as (4)

$$\varepsilon = \frac{\lambda}{2\pi nlk} \Delta \varphi = G \Delta \varphi$$  \hspace{1cm} (4)$$

Where $G = \lambda / 2\pi nk$ is the slope coefficient.

Thus $\varepsilon$ and $\Delta \varphi$ have a linear relationship. Gauge the three output light intensity we can get the change of the phase $\Delta \varphi$ via (5)

$$\Delta \varphi = \arctan \left( \frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3} \right)$$  \hspace{1cm} (5)$$

The three outer has a 120° phase error between each other.

In practical, is represented the phase difference between strained and zero strain. When the strain is affected, the phase value corresponded with the interference signal is

$$\varphi_\varepsilon = \frac{2\pi n \Delta l}{\lambda + \Delta \lambda} = \frac{2\pi n \Delta l}{\lambda (1 + k \varepsilon)}$$  \hspace{1cm} (6)$$

Thus the phase difference between it and zero strain is

$$\Delta \varphi = \varphi_\varepsilon - \varphi_0 = \frac{2\pi n \Delta l}{\lambda (1 + k \varepsilon)} - \frac{2\pi n \Delta l}{\lambda}$$

$$= \frac{2\pi n \Delta l - k \varepsilon}{\lambda (1 + k \varepsilon)}$$  \hspace{1cm} (7)$$

In the test range, the value of strain is between $0 \sim 1.8 \times 10^{-3}$, obviously $k \varepsilon \ll 1$, so $1 + k \varepsilon \approx 1$. Hence

$$\Delta \varphi = - \frac{2\pi n \Delta l}{\lambda} k \varepsilon$$  \hspace{1cm} (8)$$

The form of equation (8) is the same as the result from formula (3).

3. Finite element model
A round plane mirror is choosed as the experiment object. And a circular groove is used to fix the mirror to avoid shifting.

Firstly, a finite element model is created. The mirror is made of glass, with diameter $d = 66 mm$ and height $h = 10 mm$. Its density is $\rho_1 = 2.6 \times 10^3 kg / m^3$, the Young’s modulus
is $E_1 = 7.2 GPa$, the Poisson’s ratio is $\nu_1 = 0.2$. The material of the groove is steel. Its outer diameter is 120 mm, inner diameter is 66 mm, height is 66 mm, the depth of the groove is 5 mm. The density of the groove is $\rho_2 = 7.8 \times 10^3 \text{kg/m}^3$, the Young’s modulus is $E = 200 GPa$, the Poisson’s ratio is $\nu = 0.3$. The created model is shown as figure 2.

Pressure of 1 N is exerted on the middle point of the mirror surface, and deformation is generated to the mirror, as shown in figure 3.

From the theoretical results, the deformation of the mirror is 1.49 $\mu m$. So the strain of the mirror is $75 \mu \varepsilon$, due to $\varepsilon = \frac{\Delta l}{l}$, where $\Delta l$ is the changed length, and $l$ is the former length, $l = 20 \text{mm}$.

4. Experimental results

To measure strain the FBGS must be fixed to the plane mirror, typically by gluing. Traditionally there are three kinds of adhesives, which were modified acrylate, glass glue and epoxy resin. The modified acrylate is choosed because the FBG sensor fixed by the modified acrylate has a higher linearity and sensitivity than others [8].

The experiment is carried, using the first interferometer, with an instantaneous pressure of 1 N applied in the surface of the mirror. The three outer signal is shown in figure 4, we calculate the changed phase is $\Delta \varphi = 0.6541$, and then $\varepsilon = 64 \mu \varepsilon$. In practical, the relationship between the strain and the changed phase is not strictly linear.

So under the effect of 1 N pressure, the strain changed of the mirror is 64 $\mu \varepsilon$. 4
The same experiment is done using the second interferometer, the output is shown in figure 5. It is calculated $\Delta \phi = 0.4164$, then $\varepsilon = 69 \mu \varepsilon$.

Another experiment using the first interferometer is done again, and the result is $\Delta \phi = 0.6654$ and $\varepsilon = 65 \mu \varepsilon$. These experimental results are well matched with the result from finite element model.

5. Conclusion
We have proposed and demonstrated a FBG strain sensor based on an unbalanced Mach-Zehnder interferometer system. The demodulation that translates the Bragg wavelength of FBGS caused by the strain into the phase information. A linear relationship between the strain and the phase variation is built up. The phase variation is obtained through the three-step phase-shifting algorithm. This method is applied in the micro-strain measurement of a plane mirror. A finite element model is created and a series of experiments are carried up. The results show that this method can be applied in measuring plane mirrors’ surface micro-strain and has a high sensitivity and accuracy. The error between the experiment and the theory is less than $10 \mu \varepsilon$. It can be extended to measure optical components’ surface micro-strain and can be applied in real-time monitoring of interferometers. However, this method has limitation that it is applied to transient measurement, not suitable to gauge strain functioned under long time. This will be our further research.

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