Direct $CP$ violation in semi-leptonic and leptonic decays

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We show that direct $CP$ violation in semi-leptonic and leptonic decays can occur in multi-Higgs doublet extensions of the electroweak standard model with flavor changing neutral currents. For pion and lepton decays this $CP$ violating effects cannot be constrained by experimental data since up to now the branching ratio of the decays $\pi^-$ and $\mu^-$ have not been measured in laboratory.

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I. INTRODUCTION

Recently it was pointed out by Kaplan [1] that the comparison between the polarizations of $\mu^+$ from the decay of $\pi^+$ and of $\mu^-$ from the decay of $\pi^-$ could be used in order to verify if $CP$ is violated in the $\pi \to \mu \to e$ chain decay. Denoting by $A_{\pi^\pm}$ and $A_{\pi^-}$ the oscillation amplitudes for muons from $\pi^+$ and $\pi^-$ respectively it was found from the $g_\mu - 2$ data [3] that

$$-0.01 < A_{CP} \equiv \frac{A_{\pi^+} - A_{\pi^-}}{A_{\pi^+} + A_{\pi^-}} < 0.02. \quad (1)$$

If this asymmetry is confirmed in the future, i.e., $A_{CP} \neq 0$, it means the existence of $CP$ violation in pion and/or muon decays. Hence, we can ask ourselves what sort of models can produce them.

The goal of this work is to point out that multi-Higgs doublet extensions of the $SU(2)_L \otimes U(1)_Y$ model with flavor changing neutral currents (FCNC) in the Yukawa sector and $CP$ violation, through the flavor mixing matrix in the interactions with the vector bosons $W^\pm$ and through the scalar sector (spontaneous or explicit), imply direct $CP$ violation in semi-leptonic and leptonic decays. We also introduce a different way for counting the physical phases in the fermion mixing matrices. Although this way coincides with the usual one, it is more appropriate when there are flavor changing neutral currents in a given model. If there is $CP$ violation but not FCNC the effects are proportional to the fermion mass and therefore negligible. This of course implies constraints coming from de neutral meson parameters, notwithstanding, since there are new mixing angles those constraints do not necessarily imply large mass for both neutral and charged scalars. For instance masses of the order of 150 GeV are still possible in models with similar effects to the present one [4].

In the SM [5] the only source of $CP$ violation is the phase in the mixing matrix $V_{CKM}$ of the vector charged currents [6] or, if we enlarge the Higgs sector it is possible to implement spontaneous or explicit $CP$ violation through the scalar exchange [7]. As we said before, here we will point out an effect which arises when a model has any kind of $CP$ violation and also flavor changing neutral currents (FCNC).

II. MULTI-HIGGS EXTENSIONS OF THE SM

In the electroweak standard model (ESM by short) based on the gauge symmetry $SU(2)_L \otimes U(1)_Y$ and with only one Higgs doublet, the Yukawa interactions in the quark sector are

$$- \mathcal{L}^q_\Phi = \bar{\psi}_L (\Gamma^a D_R' + \Gamma^a \hat{\Phi} U_R') + H.c., \quad (2)$$

with $\Phi = (\phi^+ , \phi^0)^T$, $\hat{\Phi} = i \tau^2 \Phi^*$, and $\Gamma^{d,u}$ being arbitrary complex matrices (Yukawa couplings) in the flavor space, $\psi_L = (U', \psi'_L)$ denotes the doublet of left-handed fields; $D_R'$ and $U_R'$ are gauge singlets; $\tau^2$ is the Pauli matrix and primed fields denote symmetry eigenstates. After the spontaneous symmetry breaking the neutral component of the scalar doublet $\phi^0$ is shifted: $\phi^0 = (v + H^0) / \sqrt{2}$; being $v$ the vacuum expectation value (VEV) and $H^0$ a physical scalar field. Then, the Yukawa neutral interaction reads in the symmetry basis

$$- \mathcal{L}^q_Y = \left( \bar{U}'_L M^u U'_R + D'_L M^d D'_R + H.c. \right) \left( 1 + \frac{H^0}{v} \right), \quad (3)$$

with the quark mass matrices $M^q = v \Gamma^q / \sqrt{2}$. Next, we must diagonalize the quark mass matrices $M^u, M^d$ by using biunitary transformations

$$V^u_L M^u V^u_R = \hat{M}^u, \quad V^d_L M^d V^d_R = \hat{M}^d, \quad (4)$$

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with \( \hat{M}^u \equiv \text{Diag}(m_u, m_c, m_t) \) and \( \hat{M}^d \equiv \text{Diag}(m_d, m_s, m_b) \).

The physical (unprimed) states are related to the symmetry eigenstates as follows:

\[
U'_L = V^u_L U_L, \quad U'_R = V^u_R U_R, \quad D'_L = V^d_L D_L, \quad D'_R = V^d_R D_R,
\]

and with Eqs. (3) and (4) the Yukawa interactions in Eq. (3) become diagonal in the flavor space

\[
-L_N^Y = \left( \hat{U}_L \hat{M}^u U_R + \hat{D}_L \hat{M}^d D_R + \text{H.c.} \right) \left( 1 + \frac{H^0}{v} \right),
\]

It means that there are no flavor changing neutral currents since \( \hat{M}^{u,d} \) are diagonal matrices. This also happens in the neutral currents coupled to the Z\(^0 \) gauge boson. In terms of the physical fields the lagrangian does not depend at all on the \( \hat{V}^u_{\alpha\beta} \) and \( \hat{V}^d_{\alpha\beta} \) matrices which will show here that this is not the case when we have FCNC and the matrices \( \hat{V}^u_\alpha \) and \( \hat{V}^d_\alpha \) appear only as the combination \( V_{\text{CKM}} \equiv \hat{V}^u_{\alpha\beta} \hat{V}^d_{\beta\gamma} V^u_\gamma \) in the charged currents coupled to \( W^+ \):

\[
\hat{U}_L \gamma^\mu F \hat{V}_{\text{CKM}} \hat{D}_L \quad \text{with} \quad U_L = (u, c, b) U^t_L \quad \text{and} \quad D_L = (d, s, b) \tilde{U}_L \quad \text{being mass eigenstates and} \quad V_{\text{CKM}} \quad \text{being an arbitrary unitary matrix}.
\]

Next, it is necessary to determine how many phases in \( V_{\text{CKM}} \) (for simplicity this matrix will be denoted hereafter as \( V \)) are rotation angles. So, the number of phases in \( V \) is \[ 2N (N-1)/2 \]

Although the argument above is correct we will consider a little modified one which seems to be more appropriate when the right-handed mixing matrices \( V^u_{R,\alpha\beta} \) survive in the lagrangian density, like the case in which there are flavor changing neutral currents in the theory.

In the ESM the fermion-neutral gauge boson interactions are flavor as well as helicity conserving. Thus, there is no effect of the rephasing of the left-handed fields. The Yukawa interactions, although they are flavor conserving, are not helicity conserving. However, it is possible to redefine the right-handed quarks exactly with the same phase as the corresponding left-handed ones and the Yukawa term remains unchanged too. That is,

\[
\tilde{u}_{\alpha R} = e^{i\varphi(\alpha)} u_{\alpha R}, \quad \tilde{d}_{3R} = e^{i\varphi(\beta)} d_{3R}.
\]

In terms of the tilded fields, the lagrangian in Eq. (3) is still diagonal, no trace of the phases introduced in Eqs. (3) and (4) survives.

As we said before, the Yukawa couplings \( \Gamma^{u,d} \), or the mass matrices \( M^{u,d} \), are arbitrary complex matrices. It means that they have \( 2N^2 \) real parameters, or \( 2N^2 \) angles and \( N^2 \) phases. On the other hand, the matrices \( V_{L,R} \) are unitary matrices that is, each one of them can have up to \( N(N+1)/2 \) phases. The matrices \( M^{u,d} \) are real and diagonal (with positive eigenvalues). It means that the \( N^2 \) phases of \( \Gamma^u \) (or \( \Gamma^d \)) must be absorbed in the \( N(N+1) > N^2 \) phases of \( V^u_{L} + 1 \) phases of \( V^u_{R} \) plus the phases of \( V^d_{L,R} \).

We see that \( V^u_L \) and \( V^d_R \) do not need to be each one of them general unitary matrix, since in this case they have together more phases than the number needed to diagonalize \( \Gamma^u \). For instance, if we choose \( V^u_L \) to be a general unitary matrix, i.e., with \( N(N+1)/2 \) phases, it is sufficient for \( V^d_R \) to have only \( N(N-1)/2 \) phases; or vice versa, if \( V^d_R \) is the general unitary matrix with \( N(N+1)/2 \) phases, \( V^u_L \) has only \( N(N-1)/2 \) of them (similarly with the \( d \)-like sector).

In the context of the ESM or its extensions without FCNC both selections are indistinguishable. This can easily be seen as follows. In the mixing matrix of the charged currents coupled to the vector bosons \( W^\pm \) only the product \( V \equiv \hat{V}^u_{L} \hat{V}^d_{L} \) appears in the lagrangian. The matrices \( V^{u,d} \) do not appear at all in the lagrangian. Thus, if we had chosen \( V^u_L \) (\( V^d_L \)) as the general unitary matrix, independently of the choice of \( V^u_R \) (\( V^d_R \)) the matrix \( V \) is itself a general unitary matrix with \( N(N+1)/2 \) phases. On the other hand, if we had chosen both \( V^u_L \) and \( V^d_R \) as being unitary matrices both with only \( N(N-1)/2 \) phases, the rest of the phases needed to get real and positive mass eigenvalues must be in the matrices \( V^{u,d} \) and \( V \) only has \( N(N-1) \) phases. The last number has to be equal or less than \( N(N+1)/2 \) for the maximum number of phases allowed for an unitary matrix. Hence, \( N(N-1) < N(N+1)/2 \) for \( N = 2 \); but the number of
phases in $V$ is again $N(N+1)/2$ for $N \geq 3$. If we use now the phase redefinition of the physical fields in Eqs. (6) and (7) the observable phases are as usual for $N \geq 3$ but for the case of $N = 2$ we can have only one phase. It means that we can redefine not $2 \times (N = 2) = 1$ = 3 phase fields but only $2 \times (N = 2) = 2$. The matrix $V_R^u$ has $N(N + 1)/2$ or $N(N - 1)/2$ phases, if the phases of $V_R^d$ are $N(N - 1)/2$ or $N(N + 1)/2$, respectively, (the same for $V_R^{d'}$). The phases will be observable if the matrices $V_{R,i}^{u,d}$ do not disappear from the lagrangian as it is the case when the model has FCNC. Summarizing, $V$ matrices are the same for $V$ to have phases in $W$ and in the charged currents coupled to the vector bosons. The same violating phases appear in both, the Yukawa interactions $\text{CP}$ these charged scalar-quark interactions. The same $V$ phases in $W$ eigenstates, thus it will be possible to have $V_{R,i}^{u,d}$ mixing matrices are diagonalized as follows

$$V_L^{u,i} \sum_i v_i \Gamma_i^u V_R^u = M_u, \quad V_L^{d,i} \sum_i v_i \Gamma_i^d V_R^d = M_d.$$  

(12)

The interaction terms with the neutral scalars are of the form

$$\sum_i \left( \tilde{D}_L V_L^{d,i} \Gamma_i^d v_{R,i}^d D_R \right) h_i^0 + H.c.$$  

(13)

$$\sum_{i,j} \tilde{D}_L O_{ij} D_R h_{ij}^0 + H.c.,$$  

(14)

where

$$(O_{ij})_{\alpha \beta} = \left( V_L^{d,i} \Gamma_i^d v_{R,i}^d O_{ij} \right)_{\alpha \beta}, \quad (i \text{ fixed}).$$  

(15)

The matrices $V_{L,R}$ diagonalize $\sum_i v_i \Gamma_i^d$ but not $v_i \Gamma_i^d$ separately for each $i$; hence we have flavor changing neutral currents coupled to the neutral scalars. Notice that since $\Gamma_i^d$ are arbitrary matrices $V_{L,R}^{d,i} \Gamma_i^d v_{R,i}^d$ have $N^2$ phases in $N \leq n - 1$ of the $\Gamma_i^d$ matrices. We have no more freedom to redefine phases since we have already used it in absorbing the phases of the Cabibbo-Kobayashi-Maskawa matrix $V$, as discussed above. It means that even in the case of $N = 2$ generations we will have four physical phases in $N$ of the matrices $\Gamma_i^d$ appearing in the neutral currents via scalar exchange even if the matrix $V_R^d$ has only one phase.

The charged Yukawa interactions are of the form

$$\sum_i \bar{U}_L V_L^{u,i} \Gamma_i^d v_{R,i}^d D_R \phi_i^\pm + H.c.$$  

(16)

and the same number of phases of Eq. (13) survives here too. Since $\phi_i^\pm$ are symmetry eigenstate fields we can rewrite Eq. (14) in terms of the mass eigenstates $H_i^\pm$ ($\phi_i^\pm = \sum_j K_{ij} H_j^\pm$)

$$\sum_{i,j} \bar{U}_L V_{ij} D_R H_j^\pm + H.c.,$$  

(17)

where we have defined

$$(V_{ij})_{\alpha \beta} = \left( V_L^{u,i} \Gamma_i^d v_{R,i}^d K_{ij} \right)_{\alpha \beta}, \quad (i \text{ fixed})$$  

(18)

with $\alpha = u, c, t, \beta = d, s, b$. Notice that the interactions in Eqs. (13) and (16) (or (14)) are not proportional to the quark masses; even if $O_{ij}$ and $K_{ij}$ were complex matrices, there are $N^2$ phases in the matrices $O$ and $V$ in Eqs. (15) and (18), respectively.

Concerning the charged leptons, they can be rotated like the $d$–like quarks in Eq. (3) but now with $V_{L,R}^d$ instead of $V_{L,R}^u$. In the lepton sector the Yukawa interactions are (with massless neutrinos)
where we have redefined the neutrino fields so that there is no mixing in the charged current coupled to the vector bosons $W^\pm$. The mass matrix for the charged leptons $M^i = \sum_i (v_i/\sqrt{2}) \Gamma^i_\alpha$ is diagonalized as in the case of the quarks $V_{LR}^i M^i V_{LR}^i = \tilde{M}^i$, with $\tilde{M}^i = \text{Diag}(m_e, m_\mu, m_\tau, \ldots)$. Hence, the unitary matrices $V_{LR}^i$ diagonalize $M^i$ but not $v_i \Gamma^i_\alpha$ separately. Although we have redefine the neutrino fields in the charged currents coupled to the vector bosons $W^\pm$, the same is not possible in the interactions with $\phi^\pm$. Hence, we can see from Eq. (19) that even with massless neutrinos we cannot avoid, in general, to have mixing in the charged currents coupled to the charged scalars and FCNC mediated by the neutral scalars in the charged lepton lepton sector as well. If we allow $\Gamma^i_\alpha$ to be general $N \times N$ complex matrices we have $N^2$ phases in the Yukawa interactions of the charged Higgs in the lepton sector.

The currents in Eq. (19) can be written in terms of the physical charged scalar:

$$\sum_{i,j} \left( \bar{\nu}_L V_{ij} V_{LR}^i l_R H_j^+ + \bar{l}_L \mathcal{O}^i_{\alpha j} l_R H_j^0 \right) + H.c.,$$

with

$$\mathcal{O}^i_{\alpha j} = \left( V_{LR}^i \Gamma^i_\alpha V_{LR}^j \right)_{\alpha j}, \quad (i \text{ fixed}),$$

and

$$\mathcal{O}^j_{\alpha j} = \left( V_{LR}^i \Gamma^i_\alpha V_{LR}^j \right)_{\alpha j}, \quad (i \text{ fixed}),$$

where $\alpha, \beta = e, \mu, \tau$.

III. PHENOMENOLOGICAL CONSEQUENCES

An important consequence of this kind of models is that they imply direct CP nonconserving processes. For instance, $\Delta S = 1$ processes like the $K^0_L \rightarrow 2\pi$ decay. In the ESM only penguin diagrams contribute to this sort of processes [8]. In the present context CP violation arises because of the interference of the amplitudes of the diagrams shown in Fig. 1 and that of the standard electroweak model involving W bosons. Similar effect exists in hyperon models [10].

More interesting is the case of CP violation in semi-leptonic and leptonic decays. For instance, $\pi \rightarrow l \nu_l$ (particularly when $l = \mu$), $\tau \rightarrow \mu \nu_\mu \bar{\nu}_\tau$ and $\mu \rightarrow e \nu_e \bar{\nu}_\mu$ decays. Usually it is assumed that the $\pi^+$ decay conserves CP. For massless neutrinos the $CP$ mirror image of the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ is $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$. In the first one the helicity of the muon is negative while in the second one it is positive. Positive pions come to rest then they decay as $\pi^+ \rightarrow \mu^+ \nu_\mu$. Next, the muon after traveling some distance comes to rest and it decays as $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. Events of the chain $\pi^- \rightarrow \mu^- \rightarrow e^-$ are not seen in this form since negative pions coming to rest in any material are attracted by a nucleus and captured at a rate too great for the decay be competitive. Hence, it follows that pions decaying in flight in vacuum are required for a CP test [11]. Similarly for the $\mu^-$ decay.

In models with multi Higgs doublets and FCNC the interference of the amplitudes in Fig. 2 and that of the similar diagram involving the W boson implies CP violation in $\pi^\pm$ decays.

Here we will use the normalization of Ref. [12] which implies $\langle 0|A_\mu(0)|\pi(\vec{q})\rangle = i f_{\pi e} q_\mu$ and $\langle 0|P(0)|\pi(\vec{q})\rangle = i f_{\pi} m_\pi^2/(m_u + m_d)$ [13]. We can define the rate asymmetry

$$\Delta_\pi \equiv \frac{\Gamma_{\pi^+} - \Gamma_{\pi^-}}{\Gamma_{\pi^+} + \Gamma_{\pi^-}} = \frac{\kappa_\pi}{2\Gamma_{\pi^+} - \kappa_\pi},$$

where the difference of the partial width of the respective decays $\pi^+ \rightarrow \mu^+ \nu_\mu$ and $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ ($\Gamma_{\pi^\pm}$) is given by the interference term $\kappa = 4 \text{Im}(M_W M_H)$, times the phase-space factor (which cancel out in the ratio $\Delta_\pi$); $M_W (M_H)$ denotes the invariant amplitude due to the W vector (H scalar) boson. Thus we have:

$$\kappa_\pi = \Gamma_{\pi^+} - \Gamma_{\pi^-} = 4 \sum_j \frac{G_F f_{\pi}^2 m_\mu^2 m_\mu}{8\pi(m_u + m_d) m_{\pi j}^2} \text{Im}(A_j) \sin \delta_j,$$

where we have defined $\Delta \delta = \delta_+ - \delta_-$ with $\delta_+ (\delta_-)$ being CP conserving re-scattering phases for the $\pi^+$ ($\pi^-$),

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.pdf}
\caption{Charged scalar $H^-$ contribution to $K^0_L \rightarrow \pi^+ \pi^-$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.pdf}
\caption{Charged scalar $H^+$ contribution to $\pi^+ \rightarrow \mu^+ \nu_\mu$.}
\end{figure}
respectively, coming from higher order corrections to the diagram in Fig. 2. For instance loop induced correction in the vertex on the right-vertex in Fig. 2 can arise in the model. However they must be of the order of $G_F m_Z^2 \sim 10^{-7}$ [14]. We have also defined $A_j = V_{ud} V^*_{ud} (V^*_{td} / V_{td}) \nu_e$. The contribution of a given scalar $j$ can be suppressed if $\text{Im}(A_j) \sin \Delta \delta$ is small or; if the mass $m_H$ is large. Since the phases $\delta_\pi$ and $\delta_\mu$ vanish at leading order we will assume that $\sin \Delta \delta$ is the main suppression factor. For instance, using $f_\pi \approx 0.131$ GeV, $m_\mu + m_\pi = 10$ MeV, $m_\mu = 106$ MeV [12] and for a $j$ fixed $m_H = 100$ GeV we have

$$\kappa_\pi \approx 1.4 \times 10^{10} \text{Im}(A_j) \sin \Delta \delta \, s^{-1}. \quad (23)$$

We do not know what must be the experimental value of $\kappa_\pi$, since there is no a direct measure of the difference of the partial width of $\pi^+$ with respect to $\pi^-$. (It is always measured the $\Gamma_{\pi^+}$ and it is assumed that the value for $\Gamma_{\pi^-}$ is the same.) However if $\kappa_\pi \ll \Gamma_{\pi}$ we have

$$\Delta_\pi \approx \frac{\kappa_\pi}{\Gamma_{\pi}} \frac{\pi}{2} \approx 1.9 \times 10^{-7}[1.4 \times 10^{10} \text{Im}(A_j) \sin \Delta \delta]$$

$$= 2.66 \times 10^8 \text{Im}(A_j) \sin \Delta \delta \quad (24)$$

with $\text{Im}(A_j) \sin \Delta \delta \approx 10^{-10}$ ($j$ fixed), which is not an unreasonable value (even if $\text{Im}(A_j) \lesssim 1$) for a quantity which arise at higher order, we have an asymmetry $\Delta_\pi$ of the order of $10^{-7}$ as in Ref. [14]. In fact if we assume $\sin \Delta \delta \approx O(10^{-10})$ there is no constraint at all on $\text{Im}(A_j)$ even for a light Higgs scalar ($M_H \sim 100$ GeV). We stress that $\Gamma$’s matrix in Eqs. (21), in principle, are neither unitary nor hermitian, so the most general constraints come from perturbation theory: $|\Gamma|^2/4\pi < 1$. Similar analysis can be done with the $\mu^+$ and $\mu^-$ decays. In this case we can define in analogy with the $\Delta_\pi$ an asymmetry $\Delta_\mu$. However, it is not clear for us what is the relation between $\Delta_\pi$ and $\Delta_\mu$ and the $A_{CP}$ asymmetry in Eq. [3]. Notice that the malmess of the CP violation in the $\pi$ and $\mu$ decays does not implies a small CP violation in the chain $\pi \to \mu e$ since it may exist an CP observable, say $A$, such that

$$A(\pi^+ \to \mu^+ \to e^+) = A(\pi^- \to \mu^- \to e^-), \quad (25)$$

depends only on the weak phases. In calculating the asymmetry in Eq. (25) muons have to be considered as virtual particles [13].

Contributions to $\epsilon_K^{(e)}$ at the tree level constrain $V_{us}$, so, compatibility with data $Re(\epsilon/K) = \left(28 \pm 4.1 \right) \times 10^{-3}$ from KTeV [16] and $(18.5 \pm 7.3) \times 10^{-4}$ from NA48 [17] [18] can be obtained by choosing appropriately this matrix element (for a $M_H = 100$ GeV Higgs scalar) since the $\Delta_\tau$ asymmetry does not constrain it much too.

In the present model there are also contributions to $\epsilon_K^{(e)}$ coming from processes mediated by neutral Higgs bosons and in the $K_L \to \nu \nu \pi$ decay because of the interference of $s \to \bar{u} W^+ \to \bar{u} l \nu$ and $s \to \bar{u} H^+ \to \bar{u} l \nu$. They may be suppressed mainly by the CP conserving phases.

![FIG. 3. Scalar mediated contribution to the $K_L \to \pi \nu \bar{\nu}$ decay.](image)

IV. RARE DECAYS

It is worth to make a remark with respect to the rare neutral kaon decays like $K_L \to \pi^0 e^+ e^-$ [15] and $K_L \to \pi^0 \nu \bar{\nu}$ [20]. Both decays in the standard model violate $CP$ in leading order. In particular the decay $K_L \to \pi^0 \nu \bar{\nu}$ is not only $CP$ violating, but also it does not have the potentially large $2 \gamma$ mediated CP-conserving contributions which occur in the $K_L \to \pi^0 e^+ e^-$ decay [21].

Denoting the $CP$-violating parameter $\bar{\eta}_{\pi \nu \bar{\nu}}$, it has been shown that $0.1 \lesssim \bar{\eta}_{\pi \nu \bar{\nu}} \lesssim 1$ [22], which is much larger than the corresponding $K \to \pi \tau$ parameters. Although in the standard model this decay has a branching ratio $B(K_L \to \pi^0 \nu \bar{\nu}) = 2.78 \times 10^{-11}$ [23] the experimental data give [12]

$$\frac{\Gamma(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma_{\text{total}}} < 4.3 \times 10^{-5}. \quad (26)$$

It means that this decay can be sensitive to new physics. In the standard model the main contributions to the decay $K_L \to \pi^0 \nu \bar{\nu}$ come from penguin and box diagrams. On the other hand, in the present model this decay proceeds via diagrams like that in Fig. 3. The interference of the diagram in Fig. 3 with a similar one with $H^+ \to W^+$ induces $CP$ violating effects.

Independently of the $CP$ issue, using the model independent ratio [24]

$$B(K_L \to \pi^0 \nu \bar{\nu}) < 4.4 \times B(K^+ \to \pi^+ \nu \bar{\nu}) \quad (27)$$

which is valid even if lepton flavor is not conserved and [12]

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = 1.5_{-1.2}^{+3.4} \times 10^{-10},$$

we obtain

$$B(K_L \to \pi^0 \nu \bar{\nu}) < 6.6 \times 10^{-10}. \quad (28)$$

In the present model the decay $K_L \to \pi^0 \nu \bar{\nu}$ arises at the tree-level while the decay $K^+ \to \pi^+ \nu \bar{\nu}$ arises at the 1-loop level. Hence, at first sight it appears that in this
model the inequality in Eq. (27), which assumes only isospin relations, can be evaded and $B(K_L \to \pi^0\nu\bar{\nu}) > B(K^+ \to \pi^+\nu\bar{\nu})$. Notwithstanding, notice that the effective interaction Lagrangian arisen from diagrams like that in Fig. 3 are not of the four-fermion form $(\bar{s}d)(\bar{u}d)$ but of a legitimate six-fermion form. For instance, the strength of the diagram in Fig. 3 is proportional to

$$\sum_j (V_{ij}^2)_{ud} M_K/M_{H_j}^2 M_Z^2.$$  \tag{29}$$

The dimensionless ratio of the strength of the amplitude in Fig. 3 with respect to the four-fermion effective interaction Lagrangian in the standard model, denoted here by $A_{SM}$, is (assuming a fixed $j$)

$$R = \frac{M_{H_j}^4 (V_{ij}^2)_{ud}}{A_{SM}}$$  \tag{30}$$

where

$$A_{SM} = G_F \alpha(M_Z^2) 2 \text{Im} \frac{C}{\sqrt{2}} \pi \sin^2 \theta_W \text{GeV}^{-2},$$  \tag{31}$$

with the values for the parameters in Eq. (21) given in Ref. [23] we obtain $A_{SM} \approx 3.6 \times 10^{-11} \text{GeV}^{-2}$. Hence we have in Eq. (30)

$$R \approx 7.7 \times 10^4 \left(\frac{1 \text{GeV}}{M_{H_j}}\right)^4 (V_{ij}^2)_{ud}. \tag{32}$$

We see that even a relatively light scalar $M_{H_j} > 80$ GeV gives a contribution which is $10^{-3}$ smaller than the standard model 1-loop contributions. The decay $K_L \to \pi^0\nu\bar{\nu}$ was considered in two- and three-Higgs doublet models with and without FCNC in Ref. [23]. There it was shown that the contributions of the charged Higgs bosons for that decay is also smaller than the standard model result and thus unmeasurable.

In the lepton sector the flavor violation effects via the neutral scalar exchange induce not only the usual muon transition $[26]$ but also CP violation, this leaves this system closer to the neutral kaons $[27]$. Notice that in this model there are scalar and pseudoscalar contributions to the $M \to M$ transition $[28]$ If the $(|V_{ij}^2|)_{\mu\nu}$ matrix element is left arbitrary in the pion decay, the CP-violation neutral interactions given in Eqs. (20) can be large enough to be detected by comparing $M \to M$ to $M \to M$ conversions.

There are other exotic decays that are induced by this sort of models. For instance $\mu \to e\gamma$, $\mu \to eee$ and other rare $\tau$ decays. The branching ratio of the first decay above is $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$ $[12]$. In the present model there are contributions coming from both neutral and charged scalars through the interactions in Eqs. (20). For the charged scalars we have

$$B(\mu \to e\gamma) = \frac{\alpha}{48\pi} \left[ \sum_{i,j} \frac{(V_{ij}^j)_{\mu\nu}(V_{ij}^l)_{\nu\nu}}{M_{H_j}^2 G_F} \right]^2, \tag{33}$$

and for accounting the experimental branching ratio we have that (for $i,j$ fixed) in Eq. (13) we have

$$||\langle V_{ij}^j \rangle_{\mu\nu} \langle V_{ij}^l \rangle_{\nu\nu}||^2 < 1.96 \times 10^{-16}. \tag{34}$$

for a scalar mass of 100 GeV we have that $||\langle V_{ij}^j \rangle_{\mu\nu} \langle V_{ij}^l \rangle_{\nu\nu}||^2 < 10^{-8}$. When $i = \mu$ this value is compatible with that needed for saturate the value of ... The decay $\mu \to eee$ has a branching ratio $B(\mu \to eee) < 1.0 \times 10^{-12}$ $[12]$ and it is induced in the present model only by neutral (pseudo) scalars. It means a constraint only on the matrix elements of $O^j_l$ and also on the mass of the neutral Higgs, but both sort of parameters do not appear in the $\pi \to \mu\nu$ decay. This is also valid for the contributions of the neutral scalar to the $\mu \to e\gamma$ decay and also to the constraint coming from the $K_L \to \mu\nu$ decay which has $B(K_L \to e\mu\bar{\nu}) < 3.3 \times 10^{-11}$ $[12]$. For the two doublet case, the process $K_L \to \mu\bar{\nu}$ implies scalar masses of the 30-200 GeV, depending of the ratio of the VEVs $[30]$. There will be also CP violation in another semileptonic decays as $B^0 \to X\nu\ell$, and also in $p\bar{p} \to l^\pm \nu X$ because of the interference of $p\bar{p} \to W^\pm X \to l^\pm \nu X$ with $p\bar{p} \to H^\pm X \to l^\pm \nu X$ but we will not consider them here since the exotic leptonic decay seems to be more restrictive.

There is another source of suppression that we would like to pointed out $[31]$. Suppose the case of two doublets. In this case we have two massive neutral scalars. The charged lepton Yukawa interaction in Eq. (19) can be written as

$$V^1_{L} V^1_{R} \Gamma^1_{L} V^1_{R} = \frac{\sqrt{\gamma}}{v_1} \hat{M}^1 - \frac{v_2}{v_1} V^1_{L} V^1_{R}.$$

It means that the vertex in Eq. (20a) are proportional to

$$V^1_{L} \hat{\Gamma}^1_{L} V^1_{R} \left(-\frac{v_2}{v_1} O_{1j} + O_{2j}\right) + \frac{\sqrt{\gamma}}{v_1} \hat{M}^1 O_{1j}. \tag{36}$$

This implies that there are invariant amplitudes that are proportional to

$$\sum_j \left(-\frac{v_2}{v_1} O_{1j} + O_{2j}\right)^2 \frac{1}{M_{H_j}}. \tag{37}$$

For non-diagonal transitions $\mu \to e, \tau \to \mu$ (the later one appears in the process like $\tau \to \mu e\bar{\nu}$, which also occurs in these sort of models), the mass terms in Eq. (30) do not contribute. Even for diagonal processes, if $v_1$ is of the order of the Fermi scale the mass terms are negligible. It means that one of the matrix elements can be chosen such that the term between parentheses is small, for instance, for the lightest scalar $-(v_2/v_1)O_{1j} + O_{2j} \ll 1$
(j fixed). The other matrix elements are determined by the orthogonality condition but these contributions are suppressed by the mass $M_{H_j}$. For more than two doublets there will be always some vertices that can be suppressed in this way; the other ones can be suppressed by the masses of the scalars. A similar analysis is valid for the pseudoscalar sector. Notice that a light neutral scalar contribution to the $K^0 - \bar{K}^0$ mass difference must be suppressed by the mixing angles of the right-handed matrix $V_R^q$ as it appears in Eq. (13); or/and because of the fact that the phenomenological scalar that couples to quarks and vector bosons $K_{i0}$ implies similar bound on non-diagonal matrix elements.

Finally, let us consider the branching ratio

$$R = \frac{\pi^+ \rightarrow e^+ \nu_e}{\pi^+ \rightarrow \mu^+ \nu_\mu}$$

(38)

which is rather experimentally suppressed and for this reason is an important process to test the $\mu e$ universality

$$R^{exp} = (1.230 \pm 0.004) \times 10^{-4},$$

(39)

and in the present model we have

$$R = R_0 \left[ \frac{1 + (V^*_{j\nu})_{ud}(V^*_{j\nu})_{e\nu} m_e^2/2 - 2 G_F^* m_e M_{H_j}^2}{1 + (V^*_{j\nu})_{ud}(V^*_{j\nu})_{e\nu} m_e^2/2 + 2 G_F^* m_e M_{H_j}^2} \right]^2,$$

(40)

where $R_0$ is the standard model contribution. This implies that even if $|\langle V^*_{j\nu} \rangle_{ud}(V^*_{j\nu})_{e\nu}| \approx 1$ a Higgs with $M_H = 1000$ GeV, as considered in Eq. (22), produces a 0.4% shift in $R$ for any value of $(V^*_{j\nu})_{ud}(V^*_{j\nu})_{e\nu}$. However, since neutrinos are not detected in experiments it implies similar bound on non-diagonal matrix elements. In models where the Higgs scalars couple in proportion to the fermions mass the pion decay implies $M_H = 80$ GeV if the couplings are proportional to the mass of a heavy fermion we have $M_H > 0.5$ TeV.

V. CONCLUSIONS

We would like to stress that the features we have shown in this work can be implemented in other models with complicated Higgs sector and intermediate mass scales. An interesting possibility arises when, by imposing an appropriate discrete symmetry, the scalars coupled to the leptons are different from the scalars coupled to the quarks. In this case we have the so called “leptophilic” Higgs scalars since the VEV of the neutral scalars coupled to the leptons may not be necessarily of the same order of magnitude than the VEVs which give mass to the quarks and vector bosons.

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