A damage model for fracking

J Quinn Norris¹, Donald L Turcotte² and John B Rundle¹,²,³

Abstract
Injections of large volumes of water into tight shale reservoirs allow the extraction of oil and gas not previously accessible. This large volume “super” fracking induces damage that allows the oil and/or gas to flow to an extraction well. The purpose of this paper is to provide a model for understanding super fracking. We assume that water is injected from a small spherical cavity into a homogeneous elastic medium. The high pressure of the injected water generates hoop stresses that reactivate natural fractures in the tight shales. These fractures migrate outward as water is added creating a spherical shell of damaged rock. The porosity associated with these fractures is equal to the water volume injected. We obtain an analytic expression for this volume. We apply our model to a typical tight shale reservoir and show that the predicted water volumes are in good agreement with the volumes used in super fracking.

Keywords
Damage, hydraulic fracturing, fracture, fracking, induced permeability

Introduction
Injections of large volumes of water into tight shale reservoirs allow the extraction of oil and gas not previously accessible. The large volume injections were made possible by the use of “slickwater” beginning in the 1990s. Slickwater includes additives that reduce the water’s viscosity by an order of magnitude. This reduces the resistance to flow. This large volume “super” fracking induces distributed damage that allows oil and/or gas to flow to an extraction well.

In order to understand super fracking, it is necessary to understand the history and structure of tight shale reservoirs. During deposition, the depth and temperature of the shale increase. The increased temperature first converts the carbon to oil and subsequently to gas. This thermally activated conversion generates high fluid pressures that generate natural hydrofractures which

¹Department of Physics, University of California-Davis, Davis, USA
²Department of Earth and Planetary Sciences, University of California-Davis, Davis, USA
³Santa Fe Institute, Santa Fe, USA

Corresponding author:
J Quinn Norris, Department of Physics, University of California-Davis, Davis, USA.
Email: jqnorris@ucdavis.edu
allow some of the oil and gas to escape the reservoir and reduce the pressure. However, a large fraction of the oil and/or gas remain in the reservoir.

Subsequently, the natural hydrofractures are sealed by the deposition of silica or carbonates. This sealing leads to a “tight” (low permeability) reservoir. The injection of slickwater during a super frack opens the preexisting sealed fractures allowing the oil and/or gas to migrate to the production well.

The purpose of this paper is to present an idealized model for the damage generated in a super frack. We assume water is injected from a small spherical cavity in a homogeneous elastic medium. The high pressure of the injected slickwater generates hoop stresses that reactivate the sealed natural fractures in the tight shales. These fractures migrate outward as slickwater is added creating a spherical shell of damaged rock. The porosity generated is equal to the water volume injected. We obtain an analytic expression for the volume. We apply our model to a typical tight shale reservoir and show that the predicted volumes are in good agreement with the volumes of slickwater used in a super frack.

Shales that are source rocks for hydrocarbons are known as black shales due to their color. Black shales contain some 2–20% porosity filled with organic material. Typical grain sizes are less than 4 \( \mu \text{m} \), and capillary forces strongly restrict granular flows of fluids. With increasing burial depth, the increasing temperature first produces oil from the organic material (the oil window) and at higher temperatures the oil breaks down to produce gas (the gas window).

The generation of oil and gas in black shales increases the fluid pressure resulting in extensional hydraulic fractures (natural fracking). Secor (1965) described these fractures as extensional fractures perpendicular to the least principle stress direction. A consequence of this natural fracking is the joint (fracture) sets that are found in all black shales in which oil and gas have been generated (Olson et al., 2009). Engelder et al. (2009) have carried out extensive studies of the joint sets and found that they tend to be planar, parallel, and quasi-periodic with spacings in the range 0.1–3.0 m.

Natural fracking provides fracture permeability in shales. Granular permeability, although very low, allows oil and gas to flow to the closely spaced joints. The joint sets provide pathways for the vertical migration of the oil and gas. This oil migration has two consequences: (1) The oil flows upward into reservoirs of high permeability strata (often porous sandstones) overlain by very low permeability strata that trap the oil and gas. A large fraction of traditional oil and gas production has been from these reservoirs that are relatively easy to access. (2) A second consequence of the vertical migration is surface hydrocarbon seeps.

A tight shale formation is defined to be a shale in which the natural fractures do not yet exist, or have been sealed, often by pressure solution and deposition of silica or carbonates. The fracture permeability is very low. In order to extract oil and gas from tight shale formations, super fracking was developed. Fracking or hydraulic fracturing is the high pressure injection of water to create one or more open fractures in the target reservoir. A perforation is made in the well casing and high pressure water is pumped at high pressure through the perforation. The objective is to create hydraulic fractures through which the oil and gas can migrate to production wells (Fjaer et al., 2008; Yew, 1997).

It is important to distinguish between two types of fracking. Traditional (low volume) fracking typically uses 75–300 \( \text{m}^3 \) of water. Guar gum or hydroxyethyl cellulose is added to increase the viscosity of the water. The objective is to create a single or, at most, a few large fractures through which oil and gas will flow to the production well. A large volume of “proppant” (generally sand) is also injected in order to keep the fracture open. Traditional fracking is not effective in tight reservoirs. It is estimated that some 80% of the producing wells in the United States have been subjected to traditional fracking (Montgomery and Smith, 2010).
The second process is super (high volume) fracking, the primary advance that has made tight shale production possible. A typical super fracking injection uses $7.5 \times 10^3 \text{ m}^3$ to $10^4 \text{ m}^3$ of water, approximately 100 times as much water as in a traditional fracking injection. The development that has made high volume fracking possible is the use of “slickwater” as the injection fluid. “Slickwater” is a fluid in which the viscosity of the water is reduced by the addition of chemical additives, usually polyacrylamide (Curtis, 2002). This practice allows the injection of much larger volumes of water at the same injection pressure because of the reduction of viscosity and resultant resistance to flow of the water. Super fracking is illustrated schematically in Figure 1.

The objective of super fracking is to create a large volume of damage in the reservoir, i.e. to create a widely distributed network of fractures through which oil and gas can migrate to the production well (Busetti et al., 2012a, 2012b). The production well is drilled vertically until it reaches the target strata including the production reservoir. Using directional drilling, the well is then extended horizontally into the target strata. The horizontal extension is typically several kilometers in length. It is desirable to target relatively deep, 3–5 km, reservoirs so that there is high lithostatic pressure to drive the fluid out. Plugs or “packers” are used to block off a section of the well, and explosives are used to perforate the well casing. Super fracking injects “slickwater” through the blocked off perforation to create distributed hydrofractures as illustrated in Figure 1(b). A sequence of super fracking injections are carried out as shown.

Super fracking creates a distribution of microseismicity that documents the complex fracture network that is being generated. In order to document the area that is being fractured, it is now standard practice to drill one or more vertical monitoring wells, with seismometers distributed along their lengths. These seismometers can locate the microseismicity in real time and the results are used to control the rates of injection. Figure 2 shows a typical example from the Barnett Shale in Texas.
This map shows the epicenters from a four-stage super fracking treatment as well as the locations of the vertical and horizontal components of the injection (production) well and the vertical monitoring well. The first and second injections produced relatively narrow clusters of seismicity while the third and fourth injections produced much broader clusters indicating less localized fractures. The narrow clusters probably resulted from the orientation of the least principal compressional stresses parallel to the horizontal injection well. The waveforms of the larger events discriminate between deviatoric tensional failures or shear failures. These observations generally have large s-wave amplitudes relative to p-wave amplitudes indicating shear failures (Maxwell, 2011; Rutledge et al., 2004). The conclusion is that most of the events occur on preexisting healed natural fractures with a regional shear stress component generating shear displacement.

It is of interest to compare the role of super fracking in the extraction of gas from two relatively old tight black shales. We first consider the Barnett Shale in Texas, the site of the original development of super fracking injections of slickwater. Production rapidly accelerated in the early 2000s with the refinement of super fracking technology. So far some 8000 wells have been drilled, about 90% since the year 2000. Most of these wells are horizontal and have been subject to super fracking.

The Barnett Shale is a black shale of Late Mississippian age (323–340 Myr) located in the Fort Worth Basin. The organic carbon concentration in productive Barnett Shale ranges from less than

![Figure 2](image-url)
0.5% to more than 6.0%, with an average of 4.5% by weight. Depths of production range from about 1.5 to 2.5 km. The production formation has a maximum thickness of about 300 m, is relatively flat lying, and has only slight tectonic deformations.

Most natural hydraulic fractures in the Barnett Shale have been completely sealed by carbonate deposition (Gale et al., 2007). The bonding between the carbonate and shale is weak so that it is relatively easy for the super fracking injection to open the sealed fractures. There is strong evidence that open natural fractures prevent super fracking injections from creating distributed fractures. The injected slickwater leaks through the natural fractures without producing further damage.

We next consider the Antrim Shale in Michigan. The Antrim Shale is a tight black shale of Upper Devonian age (354–370 Myr) in the horizontally stratified Michigan Basin, Michigan. In terms of age, black shale deposition, and tectonic setting, the Antrim Shale is very similar to the Barnett Shale. Gas in the Antrim Shale is produced from some 9000 wells, almost entirely traditional wells using traditional production techniques. This production utilizes open natural fractures. Most fractures in the Antrim Shale are uncemented (Curtis, 2002). Because of this existing fracture permeability, super fracking does not appear to be effective. The low viscosity water used in a super fracking injection migrates through the preexisting, open natural fractures without effectively increasing the permeability. The inability of super fracking to increase production can explain the decline in production of gas from the Antrim Shale at the same time that gas production from the Barnett Shale was rapidly increasing.

Our model

In order to better understand the fundamental processes associated with super fracking, we will consider a relatively simple spherically symmetric problem. This problem is illustrated in Figure 3. Fluid is injected from a spherical fluid-filled cavity with radius \( r_c \). Initially the cavity has a radius \( r_{c0} \).
and it is embedded in a uniform infinite elastic medium. We assume the medium and fluid are initially at a uniform lithostatic pressure $p_L = \rho gh$ where $h$ is the depth of the cavity. This assumption assumes that the radius $r_e$ of the region influenced by a high fluid pressure is small compared with the depth $h$ (generally a good approximation). In the solution given below, all pressures, stresses, and strains are given as variations from the uniform background conditions, pressure $p_L$. In order to initiate fluid fracturing, the fluid pressure $p_0 + p_L$ is increased. At relatively low excess fluid pressures $p_0$, the surrounding rock deforms elastically. The elastic solution for the spherically symmetric stress and strain fields resulting from a pressurized (excess pressure $p_0$) fluid-filled cavity is given by Galanov et al. (2008)

$$\sigma_r = p_0 \left( \frac{r_e}{r} \right)^3, \quad \sigma_h = -\frac{p_0}{2} \left( \frac{r_e}{r} \right)^3$$  \hspace{1cm} (1) 

$$\epsilon_r = \frac{(1 + \nu) p_0}{E} \left( \frac{r_e}{r} \right)^3, \quad \epsilon_h = -\frac{(1 + \nu)}{2E} p_0 \left( \frac{r_e}{r} \right)^3$$  \hspace{1cm} (2)

where $r$ is the radial distance from the center of the fluid-filled cavity, $r_e$ is the inner radius of the elastic region, $\sigma_r$ and $\epsilon_r$ are the compressional radial components of stress and strain, and $\sigma_h = \sigma_\theta = \sigma_\phi$ and $\epsilon_h = \epsilon_\theta = \epsilon_\phi$ are the compressional hoop (azimuthal) components of the spherically symmetric stress and strain fields. The elastic deformation is controlled by Young’s modulus $E$ and Poisson’s ratio $\nu$.

The excess fluid pressure $p_0$ is increased to the value $p_d$ at which damage (micro-cracking) occurs. The hoop stress becomes tensional when $\sigma_h = -p_L$. We assume that damage occurs when the maximum elastic hoop stress $\sigma_{hd}$ is a specified fraction $f$ of the lithostatic pressure $p_L$

$$\sigma_{hd} = -fp_L$$  \hspace{1cm} (3)

with $0 < f < 1$. This condition allows shear fractures to develop in the damage zone even though both $\sigma_r$ and $\sigma_h$ are compressional. If $p_0 < p_d$, damage does not occur and no fluid penetrates the elastic region. The stress and strain distribution in the elastic region is given by equations (1) and (2) with $r_e = r_c$.

As more fluid is injected, a spherical shell of damaged rock is created with an outer radius $r_e$ (the inner radius of the elastic region). We assume that the damaged shell of rock has connected radial fractures that allow fluid to penetrate and eliminate differential hoop stresses in the damaged shell. We further assume that the injection rate of fluid, controlled by the injection pumps, is sufficiently small that the pressure drop associated with the fluid flow through the damaged shell can be neglected. Without hoop stresses, the stress field in the damaged shell is isotropic and equal to the excess fluid pressure $p_0 = p_d$. From equations (1) and (3) the components of stress and strain at the inner boundary of the elastic region ($r = r_e$) are given by

$$\sigma_r(r_e) = p_d, \quad \sigma_h(r_e) = -\frac{p_d}{2}$$  \hspace{1cm} (4)

$$\epsilon_r(r_e) = \frac{(1 + \nu)}{E} p_d, \quad \epsilon_h(r_e) = -\frac{(1 + \nu)}{2E} p_d$$  \hspace{1cm} (5)
From equations (3) and (4) the fluid pressure $p_d$ in excess of the lithostatic pressure required to generate damage is given by

$$p_d = 2fp_L \tag{6}$$

As stated earlier, this value is independent of the thickness of the damaged shell. In our solution, we specify the inner radius of the elastic region $r_e$ and will determine the required volume of injected fluid necessary to generate the required porosity.

Outside a sphere with radius $r_e$, the rock behaves elastically and equations (1) and (2) are applicable. The sphere of radius $r_e$ contains the initial spherical cavity, damaged rock, and injected fluid. As stated earlier, we assume that the excess pressure in this region is uniform with a value $p_d$ and it is independent of $r_e$. We further assume that the radius of the damaged region is large compared to the initial radius of the spherical cavity, that is $r_e \gg r_{i0}$. This allows us to neglect the volume of the cavity compared with the volume of the damaged region.

We now determine the volume of fluid $\Delta V_f$ required to produce a damaged sphere with radius $r_e$. This volume has two contributions $\Delta V_f = \Delta V_1 + \Delta V_2$. The first contribution $\Delta V_1$ is due to the increase in volume of the damaged region due to the compression of the rock in the elastic region. The second contribution $\Delta V_2$ is due to the compression of the damaged rock due to the excess pressure $p_d$.

The increase in the volume of the damaged shell $\Delta V_1$ due to the increase in the radius of the elastic region from $r_{e0}$ to $r_e$ is given by

$$\Delta V_1 = 4\pi r_{e0}^2(r_e - r_{e0}) \tag{7}$$

assuming the condition for linear elasticity that $\frac{(r_e - r_{e0})}{r_{e0}} \ll 1$. The change in radius is related to the elastic hoop (azimuthal) strain at $r = r_{e0}$ by

$$r_e - r_{e0} = -r_{e0}\epsilon_h(r_{e0}) \tag{8}$$

Substitution of equations (5) and (6) into equation (8) gives

$$r_e - r_{e0} = \frac{r_{e0}(1 + v)fp_L}{E} \tag{9}$$

And substitution of equation (9) into equation (7) gives the increase in the volume of the damaged shell due to the decrease in the volume of the elastic region

$$\Delta V_1 = \frac{4\pi r_{e0}^3(1 + v)fp_L}{E} \tag{10}$$

This relation gives the volume change due to the compression of the elastic region $\Delta V_1$ as a function of the inner radius of the elastic region $r_{e0}$.

We now turn to the second contribution to the volume of fluid required $\Delta V_2$. This contribution is due to the compression of the damaged rock due to the excess fluid pressure $p_d$. As discussed earlier we assume that the damaged rock has a uniform increase in pressure $p_d$. The compressibility of the damaged rock is given by

$$\beta = \frac{3(1 - 2v)}{E} \tag{11}$$
The increase in the volume of the damaged shell $\Delta V_2$ due to the decrease in the volume of the damaged rock is given by

$$\Delta V_2 = \frac{4}{3} \pi r_c^3 \beta p_d$$  \hspace{1cm} (12)$$

In writing this relation, we have neglected the volume of the spherical fluid-filled cavity, i.e. $r_c^3 \ll r_e^3$. Substitution of equations (6) and (11) gives

$$\Delta V_2 = \frac{8 \pi r_c^3 (1 - 2\nu) f_p L}{E}$$  \hspace{1cm} (13)$$

The required volume of fracking fluid $\Delta V_f$ needed to generate a damage region of radius $r_e$ is given by

$$\Delta V_f = \Delta V_1 + \Delta V_2 = \frac{12 \pi r_c^3 (1 - \nu) f_p L}{E}$$  \hspace{1cm} (14)$$

The volume $\Delta V_f$ is the volume generated by damage (fractures) in the spherical shell.

We relate the damage volume $\Delta V_f$ generated by the fluid pressure $p_d$ to the elastic volume change that is generated by the fluid pressure by introducing the damage variable $\alpha$ defined by

$$\frac{\Delta V_f}{\frac{4}{3} \pi r_c^3} = \left(\frac{\alpha}{1 - \alpha}\right) \beta p_d$$  \hspace{1cm} (15)$$

When $\alpha = 0$ we have $\Delta V_f = 0$ and there is no damage. In the limit $\alpha \to 1$, we have $\Delta V_f \to \infty$ and the damaged rock disintegrates. Substitution of equations (6) and (11) into equation (14) gives

$$\frac{\Delta V_f}{\frac{4}{3} \pi r_c^3} = \frac{3}{2} \left(\frac{1 - \nu}{1 - 2\nu}\right) \beta p_d$$  \hspace{1cm} (16)$$

Comparison of equations (15) and (16) gives

$$\alpha = \frac{3(1 - \nu)}{(5 - 7\nu)}$$  \hspace{1cm} (17)$$

For shale we take $\nu = 0.17$ (Engelder and Lacazette, 1990) and find that $\alpha = 0.654$. The damage variable is only a function of the Poisson’s ratio $\nu$ and does not depend on the thickness of the damaged shell as long as $r_c \ll r_e$.

We next obtain an estimate for the permeability in the damaged region. To do this, we utilize an idealized cubic matrix model for the region (Turcotte and Schubert, 2014). We consider a block model in which the damaged region is made up of cubic blocks of rock with dimension $b$. The walls of each cubic block are channels of uniform width $\delta$. These channels approximate the joint sets generated by natural fracking. The porosity $\phi$ of this geometry is given by

$$\phi = \frac{\Delta V_f}{\frac{4}{3} \pi r_c^3} = 3 \frac{\delta}{b}$$  \hspace{1cm} (18)$$
We assume that a pressure gradient $\frac{dP}{dx}$ is applied perpendicular to one face of the cubes. Assuming that the flow through the channel is laminar, the mean flow velocity in the channel is given by Turcotte and Schubert (2014)

$$\bar{u} = -\frac{\delta^2 dP}{12\mu \, dx}$$  \hspace{1cm} (19)

where $\mu$ is the fluid viscosity. The Darcy velocity $u_d$ for flow through the damaged rock is given by

$$u_d = \frac{2\delta \bar{u}}{b^2}$$  \hspace{1cm} (20)

Combining equations (19) and (20) gives

$$u_d = \frac{-\delta^3 dP}{6b\mu \, dx}$$  \hspace{1cm} (21)

Using the definition of permeability we obtain (Turcotte and Schubert, 2014)

$$k = \mu u_d \frac{dP}{dx} = \frac{\delta^3}{6b}$$  \hspace{1cm} (22)

More detailed geometrical models differ by numerical factors that are of order 1. We next consider a specific example related to super fracking.

An example

In order to test our model, we will consider a specific example. Our model is certainly only approximately valid. We assume the damage front is spherical. From the microseismicity illustrated in Figure 2, the rupture region is directional and individual fractures propagate into the initially undamaged rock. Nevertheless, we believe it is appropriate to compare the volume of water injected in a super frack with the volume of water predicted by our analysis. Doing this is clearly very approximate.

A typical well diameter is 0.30 m and we will take the radius of our initial fluid-filled cavity $r_{c0} = 0.15$ m. For the properties of the oil (gas) shale, we take the density $\rho = 2,620$ kg m$^{-3}$, Young’s modulus $E = 65$ GPa, and Poisson’s ratio $\nu = 0.17$ (Engelder and Lacazette, 1990). As a typical tight shale reservoir, we will take the Barnett shale in Texas that was previously discussed. Production depths are in the range 2–3 km and shale layer thicknesses are in the range 10–300 m. For our example, we will take $h = 2.5$ km. The lithostatic pressure in the reservoir is $p_L = \rho gh = 64$ MPa.

We first give the volume of water required to frack a spherical shell of radius $r_e$ using equation (14). The dependence of the fluid volume $\Delta V_f$ on the damage radius $r_{c0}$ is given in Figure 4 taking $f = 1.0$ and 0.5. As discussed in the “Introduction” section, the typical range of water volumes used in super fracks is $7.5 \times 10^3$ m$^3$ to $11.0 \times 10^3$ m$^3$. This range is also shown in Figure 4. Our results indicate that this volume of water would produce a sphere of damaged shale with a radius in the range 60–90 m.

We now estimate the damage volume for the Stage 1 frack illustrated in Figure 2. Clearly the horizontal distribution of damage associated with microseismicity is not circular, but we approximate the area to be $300 \times 75 = 2.25 \times 10^4$ m$^2$. We also approximate the vertical height of damage to
be 75 m. A frack is carried out to remain within the target strata, a reasonable estimate for this thickness should be about 75 m. Taking these values the total damage volume is estimated to be

$$1.7\times10^6\text{ m}^3$$

the radius of sphere of this volume is $r = 74\text{ m}$. This is within the range of values given in Figure 4. Our comparison is certainly approximate both in terms of the shape of the damage region and the uniformity of damage in the region. But we suggest that the agreement indicates our basic approach to calculating the porosity (water) volume is approximately correct.

From equations (14) and (18), the porosity of the damaged region is

$$\phi = \frac{9(1-v)}{E}f_p L$$

Taking values given earlier, we find that the porosity with $f = 1$ is $\phi = 7.4 \times 10^{-3}$ and with $f = 0.5$ $\phi = 3.7 \times 10^{-3}$. With our assumptions, these values are independent of the volumes of the damaged rock. For the estimated damage volume given earlier $1.7\times10^6\text{ m}^3$, the pore (fluid) volume ranges from $6\times10^3$ to $12\times10^3\text{ m}^3$. These are in good agreement with the typical range of water volumes used in super fracks given earlier $7.5 \times 10^3$ to $11 \times 10^3\text{ m}^3$. For our assumed grid of cubic fractures, the open fracture width from equation (18) is

$$\delta = \frac{b\phi}{3}$$

From this result we can determine the range of fracture thicknesses if the fracture spacings $b$ are specified.

Based on the measured fractures spacings given by Engelder et al. (2009) we take $b$ to be in the range 0.1–3 m. Substitution of these values into equation (24) we find fracture thicknesses of $\delta = 0.25$ and 7.4 mm for $f = 1$ and $\delta = 0.12$ and 3.68 mm for $f = 0.5$. For our model, the permeability is given by equation (22). For fracture spacings $b = 0.1 \text{ m}$ and $b = 3.0 \text{ m}$, we find permeabilities $k = 2.5 \times 10^{-11}\text{ m}^2$ and $k = 2.2 \times 10^{-8}\text{ m}^2$ for $f = 1$ and $k = 3.1 \times 10^{-12}\text{ m}^2$ and $k = 2.8 \times 10^{-9}\text{ m}^2$ for $f = 0.5$. Our range of permeabilities are about $k = 2 \times 10^{-8}\text{ m}^2$ to $k = 3 \times 10^{-12}\text{ m}^2$. Typical sandstone reservoirs have permeabilities $k$ in the range $10^{-12}$ to $10^{-14}\text{ m}^2$. Our results are very
approximate, but we suggest that the reasonable values obtained indicate our basic method for obtaining the fluid volume and porosity is appropriate.

**Discussion**

Large volume “super” fracking is effective in extracting oil and/or gas from tight shale reservoirs. In a super frack some $10^4$ m$^3$ of water is injected from a well. It is important that the water includes additives that reduce its viscosity creating “slick” water. Studies of the resulting microseismicity indicate that the injected water migrates along preexisting fractures resulting in small shear seismic displacements. The objective of a super frack is to create a network of fractures with significant permeability to allow the oil and/or gas to migrate to the well when the injected water is removed.

The purpose of this paper is to present a relatively simple model for super fracking. We consider a spherically symmetric problem with fluid injection from a small spherical cavity (modeling the injection from a perforated well bore). The high pressure of the injected water generates hoop stresses that reactivate preexisting natural fractures in the tight shale. These fractures migrate outward as water is added creating a spherical shell of damaged rock. We neglect the pressure drop associated with this flow so that the radius of the damaged shell is controlled by the volume of water injected.

In order to test our model, we have considered a specific example typical of a tight shale reservoir. The fluid volumes predicted by our model are compared with actual super fracking volumes in Figure 4. It is seen that there is quite good agreement.

Our model requires a number of assumptions that are certainly subject to uncertainties. Examples include:

1. We assume that the damaged region is spherical. This is only approximately valid since the individual reactivated fractures will tend to propagate into the elastic region making a gradational boundary for the damaged region.
2. We assume that the fluid pressure in the damaged region is constant. The pressure drop associated with the fluid flow in this region is neglected. An extension of our model could include a radially dependent fluid pressure in the damaged region.
3. We assume that the fracking-induced permeability is associated with the reactivation of the preexisting natural fractures. Another possible mechanism is that the super fracking breaks up (comminution) the shale. This would induce permeability on a much smaller scale but would also require large amounts of energy. We believe that the observed microseismicity favors our model. Also, the role of natural fractures in the extraction of shale oil and gas has been discussed in considerable detail by Engelder et al. (2009).
4. We assume that the initial stress state is isotropic. This is clearly not the case and the stress variability will focus on the individual fractures. In general, fractures will open perpendicular to the minimum principal stress. In shale reservoirs, the maximum principal stress is generally vertical. Horizontal wells are generally drilled in the direction of the minimum principal stress so fractures will tend to be oriented perpendicular to the horizontal well. Observations of super fracking-induced microseismicity (Maxwell, 2011) indicate considerable deviations in fracture orientation. A future extension of our present study could include a nonisotropic stress field.

We recognize that our model is highly idealized. However, we believe it provides a basis for understanding the role of super fracking in generating permeability and provides a simple analytical relationship between the volume of fluid injected and the volume of the reservoir that is damaged.
Our basic assumption is that the high pressure of the injected fluid generates porosity due to the compression of the elastic media. This porosity is primarily associated with the rupture and opening of preexisting, sealed natural fractures. Only if the natural fractures are sealed can the fluid have a sufficient overpressure to create the required damage and associated permeability for the extraction of oil and gas for tight black shales.

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