ON THE RIGIDITY THEOREMS FOR LAGRANGIAN TRANSLATING SOLITONS IN PSEUDO-EUCLIDEAN SPACE II

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Abstract. Let \( u \) be a smooth convex function in \( \mathbb{R}^n \) and the graph \( M_{\nabla u} \) of \( \nabla u \) be a space-like translating soliton in pseudo-Euclidean space \( \mathbb{R}^{2n}_n \) with a translating vector \( \frac{1}{n}(a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n) \), then the function \( u \) satisfies
\[
\det D^2 u = \exp \left\{ \sum_{i=1}^{n} -a_i \frac{\partial u}{\partial x_i} + \sum_{i=1}^{n} b_i x_i + c \right\} \quad \text{on} \ \mathbb{R}^n
\]
where \( a_i, b_i \) and \( c \) are constants. The Bernstein type results are obtained in the course of the arguments.

1. Introduction

Consider the logarithmic Monge-Ampère flow, (cf.\cite{17})

\[
\begin{cases}
\frac{\partial u}{\partial t} - \frac{1}{n} \ln \det D^2 u = 0, \quad t > 0, \quad x \in \mathbb{R}^n, \\
u = u_0(x), \quad t = 0, \quad x \in \mathbb{R}^n.
\end{cases}
\]

(1.1)

By Proposition 2.1 in \cite{19}, there exists a family of diffeomorphisms
\[ r_t : \mathbb{R}^n \to \mathbb{R}^n, \]
such that the map
\[ F(x, t) = (r_t(x), Du(r_t(x), t)) \subset \mathbb{R}^{2n}_n, \]
\[ F_0(x) = (x, Du_0(x)) \]
satisfies the mean curvature flow in pseudo-Euclidean space:
\[
\begin{cases}
\frac{dF}{dt} = \vec{H}, \\
F(x, 0) = F_0(x),
\end{cases}
\]
where \( \vec{H} \) is the mean curvature vector of the sub-manifold defined by \( F \).

Definition 1.1. Assume that \( u_0(x) \in C^2(\mathbb{R}^n) \). We call \( u_0(x) \) satisfying condition \( \mathcal{S} \), if
\[
\Lambda I \geq D^2u_0(x) \geq \lambda I, \quad x \in \mathbb{R}^n.
\]
Here \( \Lambda, \lambda \) are two positive constants and \( I \) is the identity matrix.

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The first author established the long time existence result of the logarithmic Monge-Ampère flow [19].

**Proposition 1.1.** Let \( u_0 : \mathbb{R}^n \to \mathbb{R} \) be a \( C^2 \) function which satisfies condition \( S \). Then there exists a unique strictly convex solution of (1.1) such that \( u(x, t) \in C^\infty(\mathbb{R}^n \times (0, +\infty)) \cap C(\mathbb{R}^n \times [0, +\infty)) \) where \( u(\cdot, t) \) satisfies condition \( S \). More generally, for \( l \in \{3, 4, 5 \cdots \} \) and \( \varepsilon_0 > 0 \), there holds

\[
\sup_{x \in \mathbb{R}^n} |D^l u(x, t)|^2 \leq C, \quad \forall t \in (\varepsilon_0, +\infty),
\]

where \( C \) depends only on \( n, \lambda, \Lambda, \frac{1}{\varepsilon_0} \).

More generally, the following decay estimates were derived in [20].

**Proposition 1.2.** Assume that \( u(x, t) \) is a strictly convex solution of (1.1), and \( u(\cdot, t) \) satisfies condition \( S \). Then there exists a positive constant \( C \) depending only on \( n, \lambda, \Lambda, \frac{1}{\varepsilon_0} \), such that for all \( l \in \{3, 4, 5 \cdots \} \) there holds

\[
(1.2) \sup_{x \in \mathbb{R}^n} |D^l u(x, t)|^2 \leq \frac{C}{t^{l-2}}, \quad \forall t \geq \varepsilon_0.
\]

Self-shrinking solutions of the mean curvature flow are determined by the following quasi-linear elliptic systems

\[
(1.3) \vec{H} = -\frac{X_N}{2}.
\]

In the ambient Euclidean space, the self-shrinkers has been considered in [1], [16], [6], [7], [14], [13]. In the ambient pseudo-Euclidean space, the self-shrinking graphs with high codimensions can be seen in [1], [20], [10], [9]. The solutions are hoped to give a better understanding of the flow at type I singularities by Huisken monotonicity formula. Let \( M = \{(x, Du(x))| x \in \mathbb{R}^n \} \) be a space-like submanifold satisfying (1.3) in \( \mathbb{R}^{2n}_+ \) with the induced metric \( u_{ij} dx_i dx_j \). Then up to an additive constant the function \( u \) is a solution to the Monge-Ampère type equation

\[
(1.4) \det D^2 u = \exp \left\{ n(-u + \frac{1}{2} \sum_{i=1}^{n} x_i \frac{\partial u}{\partial x_i}) \right\}.
\]

Q. Ding and Y.L. Xin [9] proved that every classical strictly convex entire solutions of the equation (1.4) must be a quadratic polynomial.

Another important examples of Type II singularities is a class of eternal solutions known as translating solitons. From [3], we see that A. Neves and G. Tian gave examples that exclude the existence of nontrivial translating solutions to Lagrangian mean curvature flow. Some interesting translating solutions were found by D. Joyce, Y.I. Lee and M.P. Tsui [11] with oscillation of the Lagrangian angle arbitrarily small. Recently, Martín, Savas-Halilaj and Smoczyk [8] obtained classification results and topological obstructions for the existence of translating solitons of the mean curvature flow in Euclidean space. In this paper we will classify the translating solutions...
of Lagrangian mean curvature flow under certain convexity assumptions on the generating potential as flat Lagrangian planes in pseudo-Euclidean space $\mathbb{R}^{2n}$.

A.M. Li and the second author [2] showed that every smooth strictly convex solutions of the Monge-Ampère type equation
\[
\det D^2 u = \exp \left\{ -\sum_{i=1}^{n} d_i \frac{\partial u}{\partial x_i} - d_0 \right\}, \quad x \in \mathbb{R}^n
\]
must be a quadratic polynomial where $d_0, d_1, \cdots, d_n$ are constants. Here we consider the following more general Monge-Ampère type equation
\[
(1.5) \quad \det D^2 u = \exp \left\{ \sum_{i=1}^{n} -a_i \frac{\partial u}{\partial x_i} + \sum_{i=1}^{n} b_i x_i + c \right\}
\]
on $\mathbb{R}^n$, where $a_i, b_i$ and $c$ are constants. According to the arguments in [21], the entire solution to (1.5) is a space-like translating soliton to Lagrangian mean curvature flow in pseudo-Euclidean space. As for the authors, it seems that the approach in [18] can’t be applied to here for obtaining the rigidity result of the solutions. So we want to search for new ideas to prove the theorems similar to Jörgens [15], Calabi [12], and Pogorelov [4].

Let $A$ be an $n \times n$ real matrix and define
\[
\Lambda = \left\{ l \mid A^l = I, A^{l-1} \neq I, l \in \mathbb{Z}^+ \right\}, \quad l_A = \min_{l \in \Lambda} l.
\]
Denote $a = (a_1, a_2, \cdots, a_n)$, $b = (b_1, b_2, \cdots, b_n)$ and $(a, b)$ is the inner product of two vectors in $\mathbb{R}^n$. As an application of Proposition 1.2 we can prove that

**Theorem 1.1.** Let $u$ be a smooth strictly convex solution of (1.5) $(n \geq 2)$ where $|a| \neq 0$, $|b| \neq 0$. Suppose that there exists an orthogonal matrix $A$ such that $l_A \geq 3$ and $u(Ax) = u(x)$ for each $x \in \mathbb{R}^n$. If the smallest eigenvalue $\mu(x)$ of $D^2 u$ satisfies
\[
(1.6) \quad \liminf_{x \to \infty} \frac{|x|\mu(x)}{|x|} > \frac{n-1}{|a| \cos \frac{\pi}{l_A}},
\]
then $u(x)$ must be a quadratic polynomial.

By the methods in the proof of Theorem 1.1 we formulate the above result in a more general form when the dimension $n = 1$:

**Corollary 1.1.** Suppose that $u = u(t)$ satisfies
\[
(1.7) \quad u'' = \exp(-a_0 u' + b_0 t + c) \quad \text{on} \quad \mathbb{R},
\]
where $a_0 b_0 > 0$ and there exists $t_0$ such that $u(t - t_0) = u(t_0 - t)$ for each $t \in \mathbb{R}$. Then
\[
u = \frac{b_0}{2a_0} (t - t_0)^2 + \min_{\mathbb{R}} u.
\]

If the potential function $u$ has more symmetry, it is easy to get

**Theorem 1.2.** Let $u$ be a smooth strictly convex radially symmetric solution of (1.5), then $u(x)$ must be a quadratic polynomial.
We outline our proof as follows. In section 2, we provide preliminary results which will be used in the proof of Theorem 1.1. The techniques used in this section are reflective of those in [20], but the corresponding prior estimates to the solutions in the current scenario need modification because the structure of (1.5) is unlike the self-shrinking equation (1.4). In section 3, we give the proofs of the main results.

2. Preliminaries

Straightforward computation gives the relations of (1.1) and (1.5).

**Lemma 2.1.** If $u$ is a smooth strictly convex solution of the PDE (1.5) and define $\tilde{u}(x,t) = u(x-at) + (\langle b, x \rangle - \frac{1}{2}(b,a)t + c)t$. Then $\tilde{u}(x,t)$ satisfies the logarithmic Monge-Ampère flow

\[
\begin{cases}
\frac{\partial \tilde{u}}{\partial t} - \ln \det D^2\tilde{u} = 0, \\
\tilde{u} = u(x),
\end{cases}
\]

$t > 0$, $x \in \mathbb{R}^n$,

$\tilde{u} = u(x)$, $t = 0$, $x \in \mathbb{R}^n$.

An important consequence of the decay estimates (1.2) is the following result.

**Lemma 2.2.** If $u$ is a smooth strictly convex solution of the PDE (1.5) and satisfies condition $\mathcal{S}$. Then $u(x)$ must be a quadratic polynomial.

**Proof.** By Lemma 2.1 and Proposition 1.2, we have

\[
\sup_{x \in \mathbb{R}^n} |D^3\tilde{u}(x,t)|^2 \leq \frac{C}{t}, \quad \forall t \geq \varepsilon_0.
\]

That is

\[
\sup_{x \in \mathbb{R}^n} |D^3u(x-at)|^2 \leq \frac{C}{t}, \quad \forall t \geq \varepsilon_0.
\]

Therefore

\[
\sup_{x \in \mathbb{R}^n} |D^3u(x)|^2 \leq \frac{C}{t}, \quad \forall t \geq \varepsilon_0.
\]

Let $t \to +\infty$ then we obtain $D^3u \equiv 0$ and the claim follows. \qed

To obtain the first rigidity theorem we search that which condition can imply condition $\mathcal{S}$. Denote $B_R$ be a ball centered at 0 with radius $R$ in $\mathbb{R}^n$.

**Lemma 2.3.** Let $u : \mathbb{R}^n \to \mathbb{R}$ be a smooth strictly convex solution to (1.5) and $|a| \neq 0$. Suppose that there exists an orthogonal matrix $A$ such that $l_A \geq 3$ and $u(Ax) = u(x)$ for each $x \in \mathbb{R}^n$. If the smallest eigenvalue $\mu(x)$ of $D^2u$ satisfies (1.6). Then there exists a positive constant $R_0$ such that

\[
D^2u(x) \leq CI, \quad x \in \mathbb{R}^n,
\]

where $C$ is a positive constant depending only on $|a|$, $l_A$, $\mu(x)$ and $\|u\|_{C^2(B_{R_0+1})}$.

**Proof.** Denote

\[
u_i = \frac{\partial u}{\partial x_i}, \quad u_{ij} = \frac{\partial^2 u}{\partial x_i \partial x_j}, \quad u_{ijk} = \frac{\partial^3 u}{\partial x_i \partial x_j \partial x_k}, \ldots
\]
and 

\[ [u^{ij}] = [u_{ij}]^{-1}, \quad L = u^{ij} \frac{\partial^2}{\partial x_i \partial x_j}. \]

Let \( \gamma \) denote a unite vector field. Set

\[ u_{\gamma} = D_{\gamma}u, \quad u_{\gamma\gamma} = D_{\gamma\gamma}^2u. \]

We will prove that

\[ \sup_{x \in \mathbb{R}^n, \gamma \in S^{n-1}} u_{\gamma\gamma} \leq C. \]

By (1.6), there are some constant \( \lambda > \frac{n-1}{|a| \cos \frac{\pi}{l_A}} \) and \( R_0 \), such that

\[ |x| \mu(x) \geq \lambda, \]

for \( |x| > R_0 + 1 \). One can define a family of smooth functions by

\[
f_k(t) = \begin{cases} 
1, & 0 \leq t \leq R_0, \\
\varphi & R_0 \leq t \leq R_0 + 1, \\
- k[t^2 - (R_0 + 1)^2] + \frac{3}{4}, & t \geq R_0 + 1,
\end{cases}
\]

where \( 0 < k \leq 1 \), and \((t, \varphi(t))\) is a smooth curve connecting two points \((R_0, 1), (R_0 + 1, \frac{3}{4})\) satisfying \( \frac{3}{4} \leq \varphi \leq 1 \).

We view \( u_{\gamma\gamma} \) as a function on \( \mathbb{R}^n \times S^{n-1} \). It is easy to see that \( f_k(|x|)u_{\gamma\gamma} \) always attains its maximum at

\[ (p, \xi) \in \{(x, \gamma) \in \mathbb{R}^n \times S^{n-1} | f_k(|x|) > 0\}. \]

Using the definition of \( l_A \) we can choose the maximum point \( x \), denoted by \( p \), such that

\[
\langle x, -a \rangle \geq |a||x| \cos \frac{\pi}{l_A}.
\]

By (1.6), we have \( u_{\gamma\gamma} > 0 \). Let

\[ \eta_k(x) = f_k(|x|), \quad w = \eta_k(x)u_{\xi\xi}. \]

Then at \( p \),

\[
0 \geq Lw = u^{ij}(\eta_k u_{\xi\xi})_{ij} = u^{ij}(\eta_k)_{ij} u_{\xi\xi} + 2u^{ij}(\eta_k)_{i}(u_{\xi\xi})_{j} + \eta_k u^{ij}(u_{\xi\xi})_{ij}.
\]

We assume that

\[ p \in \{x \in \mathbb{R}^n||x| > R_0 + 1\}. \]

By a rotation, we can assume that \( D^2u \) is diagonal at \( p \) with \( \xi \) as the \( x_1 \) direction. In this case, \( u_{\xi\xi} = u_{11} \). Then at \( p \), there holds

\[
(\eta_k u_{11})_{j} = 0, \quad j = 1, 2, \ldots, n.
\]

Hence

\[
(u_{11})_{j} = -u_{11} \frac{(\eta_k)_{j}}{\eta_k}, \quad (\eta_k)_{j} = -\eta_k \frac{(u_{11})_{j}}{u_{11}}, \quad j = 1, 2, \ldots, n.
\]
Clearly, by (2.4),

\[ 2u^{ij}(\eta_k)_{ij}(u_{11})_j = u^{11}(\eta_k)_1 u_{111} + u^{11}(\eta_k)_1 u_{111} + 2 \sum_{i \neq 1} \frac{(\eta_k)_i u_{11i}}{u_{ii}} \]

(2.5)

\[ = - u^{11}(\eta_k)_1 u_{11i} - u^{11}(\eta_k)_{11} - 2 \sum_{i \neq 1} \eta_k u_{11i} - 2 \sum_{i \neq 1} \eta_k u_{11i}. \]

Differentiating the equation (1.5), we have

\[ u^{ij} u_{ij1} = -a_i u_{i1} + b_1, \]

(2.6)

\[ u^{ij} u_{11ij} = \sum_{i,j=1}^n \frac{u_{ij1}}{u_{ii} u_{jj}} - a_i u_{i11}. \]

Substituting (2.5), (2.6) into (2.3) and using

\[ (\eta_k)_i = -2k x_i, \quad (\eta_k)_{ij} = -2k \delta_{ij}, \]

we obtain, at \( p \),

\[ 0 \geq -2k \sum_{i=1}^n u^{ii} u_{11} - \frac{(\eta_k)_i^2}{\eta_k} - \eta_k u_{11}^2 - 2 \eta_k \sum_{i \neq 1} u_{11i}^2 \]

\[ + \eta_k \sum_{i,j=1}^n \frac{u_{ij1}}{u_{ii} u_{jj}} - \eta_k a_i u_{i11}. \]

Note that

\[ \eta_k \sum_{i,j=1}^n \frac{u_{ij1}}{u_{ii} u_{jj}} \geq \eta_k \sum_{i \neq 1} u_{11i}^2 + 2 \eta_k \sum_{i \neq 1} u_{11i}^2. \]

Combining the above two inequalities, we get

\[ 0 \geq -2k \sum_{i=1}^n u^{ii} u_{11} - \frac{(\eta_k)_i^2}{\eta_k} - \eta_k a_i u_{i11}. \]

In view of (2.4),

\[ \eta_k u_{i11} = -u_{11}(\eta_k)_i = 2k x_i u_{11}. \]

Then at \( p \),

\[ \frac{(\eta_k)_i^2}{\eta_k} \geq -2k \sum_{i=1}^n u^{ii} u_{11} - 2k a_i x_i u_{11}. \]

Using (2.2) and \( u_{ii} \geq \frac{\lambda}{|x|} \) for \( i \geq 2 \), it follows from the above arguments that

\[ \frac{4k^2 x_i^2}{\eta_k} \geq -2k - \frac{2k(n-1)}{\lambda} |x| u_{11} + 2k |a| |x| u_{11} \cos \frac{\pi}{l_A}. \]
Noting \( \lambda > \frac{n - 1}{|a| \cos \frac{\pi}{l_A}} \), at \( p \), we have
\[
\frac{1}{(|a| \cos \frac{\pi}{l_A} - \frac{n - 1}{\lambda}) |x|} (\eta_k + 2kx_1^2) \geq \eta_k u_{11}.
\]
Thus if \( p \in \{ x \in \mathbb{R}^n | |x| > R_0 + 1 \} \), then there holds
\[
\max_{x \in \mathbb{R}^n, \gamma \in S^{n-1}} \eta_k u_{\gamma \gamma} \leq \frac{3}{4} + 2k(R_0 + 1)^2 \left( \frac{11}{4} \frac{(R_0 + 1)}{(|a| \cos \frac{\pi}{l_A} - \frac{n - 1}{\lambda}) (R_0 + 1)} \right).
\]
And if \( p \in \{ x \in \mathbb{R}^n | |x| \leq R_0 + 1 \} \), then
\[
(2.8) \quad \max_{x \in \mathbb{R}^n, \gamma \in S^{n-1}} \eta_k u_{\gamma \gamma} \leq \|u\|_{C^2(\bar{B}_{R_0 + 1})}.
\]
From (2.7) and (2.8), by \( k \leq 1 \) we obtain
\[
\max_{x \in \mathbb{R}^n, \gamma \in S^{n-1}} \eta_k u_{\gamma \gamma} \leq \frac{11}{4} \frac{(R_0 + 1)}{(|a| \cos \frac{\pi}{l_A} - \frac{n - 1}{\lambda})} + \|u\|_{C^2(\bar{B}_{R_0 + 1})}.
\]
For any fixed \( x \in \mathbb{R}^n \) and \( \gamma \in S^{n-1} \), let \( k \) converges to 0, then
\[
\frac{3}{4} u_{\gamma \gamma} \leq \frac{11}{4} \frac{(R_0 + 1)}{(|a| \cos \frac{\pi}{l_A} - \frac{n - 1}{\lambda})} + \frac{4}{3} \|u\|_{C^2(\bar{B}_{R_0 + 1})}.
\]
So we obtain
\[
u_{\gamma \gamma} \leq \frac{11}{3} \frac{(R_0 + 1)}{(|a| \cos \frac{\pi}{l_A} - \frac{n - 1}{\lambda})} + \frac{4}{3} \|u\|_{C^2(\bar{B}_{R_0 + 1})}
\]
and inequality (2.1) is proved. \( \square \)

3. PROOF OF THE MAIN RESULTS

Proof of Theorem 1.1:

Introduce Legendre transformation of \( u \):
\[
\hat{x}_i = \frac{\partial u}{\partial x_i}, \quad i = 1, 2, \ldots, n, \quad u^*(\hat{x}_1, \ldots, \hat{x}_n) := \sum_{i=1}^{n} x_i \frac{\partial u}{\partial x_i} - u(x), \quad x \in \mathbb{R}^n.
\]
In terms of \( \hat{x}_1, \ldots, \hat{x}_n, u^*(\hat{x}_1, \ldots, \hat{x}_n) \), one can easily check that
\[
\left( \frac{\partial^2 u^*}{\partial \hat{x}_i \partial \hat{x}_j} \right) = \left( \frac{\partial^2 u}{\partial x_i \partial x_j} \right)^{-1}.
\]
Thus, in view of (2.1),
\[
D^2 u^* \geq \frac{1}{C} I.
\]
By the PDE (1.5) we obtain
\[ \det D^2 u^* = \exp\{a_i \tilde{x}_i - b_i u^*_i - c\}. \]

Since \( u(Ax) = u(x) \) one can verify that \( u^*(A\tilde{x}) = u^*(\tilde{x}) \) for each \( \tilde{x} \in \mathbb{R}^n \). Using Lemma 2.3 we have
\[ D^2 u^* \leq CI. \]

So
\[ \frac{1}{C}I \leq D^2 u \leq CI. \]

An application of Lemma 2.2 yields the desired result. □

**Proof of Corollary 1.1**

Without loss of generality, we assume that \( t_0 = 0 \). The equation (1.7) shows that \( u \) must be strictly convex. For \( n = 1 \), similar to the proof of Lemma 2.3 the inequality (2.2) can be replaced by
\[ \langle t, -a_0 \rangle \geq |a_0||t|. \]

Here we use the symmetry condition. Following the procedure in the proof of Theorem 1.1, we obtain estimates (2.1) for \( n = 1 \) and then we arrive at the conclusion of Corollary 1.1. □

**Proof of Theorem 1.2**

Now we assume that \( u \) is radially symmetric function, then
\[ \Psi = \ln \det(u_{ij}) = \ln u_{rr} + (n - 1)(\ln u_r - \ln r) \]

is also radially symmetric and depends only on \( |x| \). Similar to the arguments in [1], it follows that \( \ln \det(u_{ij}) \) must then attain either a local maximum or a local minimum over any open ball \( B \) in \( \mathbb{R}^n \). From (1.5), we have
\[ u^{ij}\Psi_{ij} - \langle a, D\Psi \rangle = 0. \]

Applying the strong maximum principle to (3.1), we see that \( \Psi \) is constant in \( B \), and hence in \( \mathbb{R}^n \). From the classical Jorgens-Calabi-Pogorelov theorem, we complete the proof of theorem. □

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