Prediction of multiple sources exciting a cylinder filled by a heavy fluid using an inverse method

M C Djamaa\textsuperscript{1}, N Ouelaa\textsuperscript{1}, S Guenfoud\textsuperscript{1}, A Rezaiguia\textsuperscript{1}, C Pezerat\textsuperscript{2}

\textsuperscript{1} Laboratoire de Mécanique & Structures (LMS), Université du 8 mai 45, BP. 401, 24000 Guelma, Algérie.
\textsuperscript{2} Laboratoire de Vibrations & Acoustique (LVA), INSA de Lyon, 25 bis, avenue Jean Capelle, 69621 Villeurbanne, France.

Abstract. In this paper, a finite circular cylindrical shell filled with fluid and subjected to multiple excitations (mechanical and acoustical) is studied. The exact solution for vibration is based on the Donnell thin shell theory and the frequency equation of the coupled vibration problem is obtained by taking account of the effect of the inner fluid. In the direct problem, displacements are obtained by the modal method when the cylinder is assumed to be simply supported. The proposed predicting method is based on finite difference principle which makes it possible to express the derivatives of the equation of motion in term of displacements in order to calculate the distribution resulting from excitations. Simulations show that there exists a condition under which deformations can be neglected but it is found that this assumption would cause error in the results when exciting the cylinder below the ring frequency. In order to evaluate the effect of the fluid, some results of the cylindrical shell filled with fluid are compared with those of the shell \textit{in vacuo}. Other numerical results are finally presented using exact and inexact displacements and a regularisation procedure based on the filtering of the wavenumber domain is used to minimise the effect of errors. This technique gives good results and makes the problem stable when the displacements are contaminated by noise resulting from measurement.

1. Introduction

Thin cylindrical shells have wide practical applications as engineering structures like submarine hulls, flow pipes, immersed tubes and aircraft fuselages. Vibration behavior of shells is more complicate that those of beams and plates because the curvature of the shell couples its flexural and extensional stiffness. Also, the complexity enters into the problem by means of complex equations of motion and boundary conditions especially when the shell is coupled with acoustical fluids and subjected to multiple excitations. In this case, the analysis becomes considerably complicate. Cylindrical shell vibrations were analyzed by Flugge [1], followed by Leissa [2] and has been studied later by several researchers [3-6].

In the majority of industrial situations, direct measurements of the interior sources cannot be easily performed. However, it is possible to predict the position and magnitude of these sources by using indirect methods that are based on the measured dynamic response at discrete points on the structure.

\textsuperscript{1} Dr. Djamaa, e-mail: mc_djamaa@yahoo.fr
Unfortunately, measurement errors make the problem unstable and great deviations of the exact solution can be observed. Zhang and Mann III [7] used the FFT to calculate the structural intensity and the force distribution for plates and presented their results before and after windowing and filtering. Karlsson [8] focused his study on the sensitivity of the inverse process and proposed a semi-experimental approach to identify unknown harmonic force amplitudes arising from the response at discrete measurement points. Pezerat and Guyader [9] developed the so called the RIFF method to localize the excitation sources acting on plates using a finite difference method and a regularization step based on filtering and windowing of the force distribution is used to reduce the noise effect. However, few papers dealt with the inverse problem in the case of shell structures rather than plates. Kim and Nelson [10] proposed an optimal regularisation for acoustic source reconstruction deduced from the measured pressure field and the inversion of corresponding matrix of frequency response functions. Liu and Shepard [11] developed an improved method for reconstructing distributed forces acting on a vibrating structure from measured structural response. This new method tends to obtain better reconstruction results while requiring fewer basis functions. In our previous work [12], an extension of the RIFF method to the cylindrical shell excited either by a force or by a monopole is presented. Some assumptions concerning the use of only the radial displacements to reconstruct the force distribution depending to the frequency range.

In this paper, analytical expressions for the equations of motion are derived using Donnell shell theory for a simply supported thin circular cylindrical shell coupled with internal and external fluids and subjected to multiple excitation sources. The exact displacements of the corresponding cylinder are calculated using the analytical expressions and the effect of the uncertainties resulting from measured displacements (here simulated) on the reconstructed distribution is presented. Positive results were obtained either in the case when using exact displacements or in the case when measured displacements are used but after windowing and filtering of the distributions if the filter and the window parameters were suitably defined.

2. Theoretical formulation

Consider a cylindrical shell of finite length $L$, radius $R$ and thickness $h$, with shell cylindrical coordinates $z$, $r$, and $\theta$ as shown in (figure 1). The orthogonal components of displacements of the mid-surface of the shell are represented by $u$, $v$, and $w$ corresponding to $z, \theta$, and radial direction respectively and the cylinder is supposed to be simply supported at its ends.

![Figure 1](image)

**Figure 1.** Schematic of the geometry of the baffled, finite cylindrical shell filled with fluid and immersed in an acoustic medium

The Donnell’s shell theory describes the equations of motion of the shell in terms of a fourth-order differential equation. The equation of motion when the cylindrical shell is subject to a radial force simultaneously with a monopole, takes the following form:
Here $E$ is the Young’s modulus, $h$ is the thickness, $\nu$ is the Poisson’s ratio, $(u,v,w)$ are the displacements, $\rho$ is the shell density, $\omega$ is the excitation frequency, $a$ is the shell radius and $[L]$ is Donnell’s operator. The displacements $(u, v, w)$ are computed by the modal method.

$$ \begin{align*}
  u &= \sum_{\alpha=0}^{1} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \sum_{j=1}^{3} A_{\alpha mj}^u D_{\alpha mj} \sin( n\theta + \alpha \pi / 2) \cos(m \pi / L) \\
  v &= \sum_{\alpha=0}^{1} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \sum_{j=1}^{3} A_{\alpha mj}^v E_{\alpha mj} \cos( n\theta + \alpha \pi / 2) \sin(m \pi / L) \\
  w &= \sum_{\alpha=0}^{1} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \sum_{j=1}^{3} A_{\alpha mj}^w \sin( n\theta + \alpha \pi / 2) \sin(m \pi / L)
\end{align*} $$

(2)

Here $\alpha$ is an integer, its even values indicate the symmetric modes and the odd values the anti-symmetric modes. Mode shapes are identified by the number of circumferential waves ($n = 1, 2, 3, \ldots$), by the number of longitudinal halfwaves ($m = 1, 2, 3, \ldots$) and by $j$ we indicate the type of the mode shape. $D_{\alpha mj}$ and $E_{\alpha mj}$ are the eigenvector components and $A_{\alpha mj}$ is the modal amplitudes.

The forces $F_u, F_v$ and $F_w$ are the distributed force per unit area acting in axial, circumferential and radial directions. Only a single harmonic radial force is considered; therefore $F_u = F_v = 0$.

The external radial distributed load $F_w$ applied on the cylinder is given by:

$$ F_w = F\delta (\theta - \theta_0) (z - z_0) \cos(\omega t) $$

(3)

Where $t$ is the time, $\delta$ is the Dirac delta function, $F$ gives the radial force amplitude, $z$ and $\theta$ give the axial and angular positions of the point of application of the force, respectively.

The acoustical pressure respectively in the external and internal fluids $P_e(Q)\alpha$ and $P_i(Q)\alpha$ are calculated by the following expressions:

$$ P_e(Q)\alpha = \rho_e \int_0^z w(Q) G_e(Q,Q_0) a d\theta dz $$

(4)

$$ P_i(Q)\alpha = -\rho_i \int_0^z G(Q,Q_0)(-\omega^2 + V^2 z \frac{\partial^2}{\partial z^2} - 2 j \omega V z \frac{\partial}{\partial z}) w(Q) a d\theta dz $$

(5)

$\rho_e$ and $\rho_i$ are respectively the densities of the external and internal fluid. $G_e(Q,Q_0)$ and $G_i(Q,Q_0)$ are the Green’s functions of the external and internal fluid.

By taking one of the equations of motion (1) which correspond to the case when the excitation is radial to the cylinder, this equation can be developed as follows:

$$ \frac{Eh}{1-\nu^2} \left[ \begin{array}{c}
\frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial \theta} \\
\frac{\partial w}{\partial z} + h^2 \left( \frac{\partial^4 w}{\partial \theta^4} + \frac{8}{12} \frac{\partial^4 w}{\partial \theta^2 \partial z^2} \right) \right] + \rho h \omega^2 w = -F_w - \left( P_e - P_i \right) $$

(6)

The proposed method permits to approximate the 4th order derivatives in equation (6) using finite difference schemes as follows:

$$ \frac{\partial^4 w}{\partial z^4} \Leftrightarrow \delta^4_j (z) = \frac{1}{\Delta z^4} \left( w_{i+2,j} - 4 w_{i+1,j} + 6 w_{i,j} - 4 w_{i-1,j} + w_{i-2,j} \right) $$

\[3\]
Here $\Delta_z$ and $\Delta_\theta$ denote the distance between two consecutive points respectively in the longitudinal and circumferential directions.

The complete inverse resolution could take into account the deformations $\partial u/\partial z$ and $\partial v/\partial \theta$. Since it is very complicated to achieve simultaneously the measurement of these quantities and the radial displacements at the same points, they are deduced from the first order finite difference schemes.

\[
\frac{\partial u}{\partial z} \Leftrightarrow \frac{1}{2\Delta_z}(u_{i+1,j} - u_{i-1,j}) \quad \text{and} \quad \frac{\partial v}{\partial \theta} \Leftrightarrow \frac{1}{2\Delta_\theta}(v_{i,j+1} - v_{i,j-1})
\]  

(8)

By substituting (8) and (7) into (6), the distribution can be computed for each frequency as follows:

\[
F_q = \frac{E(1+\nu)h}{1-\nu^2} \left( \frac{\nu}{2\Delta_z}(u_{d,j} - u_{i,j}) + \frac{1}{2\Delta_\theta^2}(v_{i+1,j} - v_{i-1,j}) + \frac{w_{j}}{a^2} \left( \frac{1}{12} \delta^i_j(z) + \frac{2}{a^4} \delta^i_j(\theta) + \frac{1}{a^4} \delta^i_j(\theta) \right) - \rho \Phi \omega^2 w_{j} \right)
\]  

(9)

Now, having all displacements calculated or measured on the circular cylindrical shell surface, the distribution resulting from unknown excitations is calculated by developing a FORTRAN program which will also calculate the natural frequencies based on the analytical expressions derived using Donnell shell theory.

3. Numerical results

The numerical simulations were made on a steel cylindrical shell with the characteristics shown in table 1.

| Table 1. Characteristics of the cylindrical shell |
|------------------------------------------------|
| Length $L$ | 1.2m |
| Radius $a$ | 0.4m |
| Thickness $h$ | 0.003m |
| Young’s modulus $E$ | $2.068 \times 10^{11}$Nm$^{-2}$ |
| density $\rho$ | 7850kgm$^{-3}$ |
| Poisson ratio $\nu$ | 0.29 |
| Structural damping $\eta$ | 1% |

3.1. Using exact displacements

Using the exact displacements, the force distribution is calculated using equation (9). The shell is filled with a heavy fluid (water) and excited below the ring frequency (160Hz) and above this frequency (4000Hz).

Case 1:

The shell is excited by two single point forces having the same amplitude (1N) and the same phase. The position of the first applied force is (0.3m, 30°) and the second force position is (0.8m, 60°).

The results are presented in figure 2 where two peaks corresponding to the true position of the applied forces appear clearly. However, the Dirac function lets the amplitude of the forces becoming several times greater than those introduced as excitations.
Figure 2. Force distribution resulting from exact displacements when the structure is excited below the ring frequency (160Hz).

In the case when the frequency is above the ring frequency, the deformations are assumed to be very small and can be neglected from equation (6). This assumption has a considerable impact because the radial displacements can be easily measured by the Scanning Laser Vibrometer.

Figure 3. Force distribution resulting from only exact radial displacements, reconstructed at 4000Hz.

Figure 3 shows the force distribution calculated from only the radial displacements at 4000Hz. It is clear that above the ring frequency, the forces can be easily located because the cylindrical shell will behave as a flat plate as the radius of the curvature is large in comparison to the flexural wavelength.

In order to investigate the influences of the contained fluid on the reconstructed forces, the force distribution is calculated in the case when the shell is in vacuo. The figure 4 shows that the amplitudes of the forces fall three times in comparison with figure 2. In reference [5], some results presented for the same shell vibrating in vacuo show that the existence of the fluid increases the input power flow into the coupled system at low frequencies. Consequently, the vibratory level of the cylinder decreases under the effect of damping of the fluid and the resonance frequencies are shifted towards the low frequencies following to the added fluid mass. From where, the first frequency of the cylindrical shell falls from 144.6 to 69Hz.
As we can observe in figure 4 there is two other reconstructed forces distributed symmetrically from those injected to excite the structure. This configuration can be explained by the presence of symmetric and anti-symmetric modes for the case of cylindrical shell. The presence of the fluid added a damping to the structure damping and then the erroneous forces are strongly smoothed (figure 2).

**Figure 4.** Force distribution resulting from exact displacements when the structure is excited at the frequency of 160Hz but the shell is *in vacuo*.

Case 2:

The shell is excited by a distributed force of 1N m\(^{-2}\) at the axial position of \((0.27-0.33\text{m})\) and at the angular position of \((30^\circ-40^\circ)\). However, the monopole is placed at the inner surface of the shell \((r_s=0.385\text{m})\). His axial position is at \((z_s=0.8\text{m})\) and the angular position is \((\theta_s=60^\circ)\).

The figure 5 shows that both the distributed force and the monopole are located exactly on their exact position. The amplitude of the distributed force is quite equal to the introduced force but the acoustical source magnitude is five times greater than the distributed force magnitude because his radial position is in contact of the cylinder permits him to excite the structure like a single point force. However, if the monopole is placed far of the surface, this one cannot excite the structure and then the location of this source will be very difficult.

**Figure 5.** Force distribution resulting from exact displacements when the structure is excited either by a distributed force and a monopole at the frequency of 160Hz.
3.2. Using measured displacements

To simulate the displacements with uncertainties (measured data), errors are voluntarily added to the exact displacements. The noisy radial displacement is calculated by the following expression:

\[ w_{\text{noisy}} = w_{\text{exact}} + e^\Delta w \]

Here \( \Delta w \) and \( \Delta q \) are two different Gaussian random real numbers. The first has a mean equal to 1 and \( \sigma = 1\% \) and the other has a mean equal to zero and \( \sigma = 1^\circ \).

The noisy force distribution reconstructed at 160Hz (figure 6) shows that the noise dominates the real force because it is amplified by the derivatives of fourth order and the noise must be removed.

![Figure 6](image)

**Figure 6.** Force distribution resulting from measured displacements (1% of noise) when the structure is excited at the frequency of 160Hz. The same as the figure 2.

3.3. Regularization procedure

The present section is concerned with the regularisation of the inverse problem in order to reduce the effect of uncertainties associated with the input displacements. The approach is explicitly detailed in our paper [22]. By a simple filtering, residual forces can appear at the structure boundaries following the limitation of the spatial domain. In this case, windowing must be applied before filtering in order to smooth out the effects these forces.

The spatial response of the filter along both axial and circumferential directions is defined as the product of two sinc functions.

\[
h(z, \theta) = \frac{1}{4\pi^2} \int_{-k_z}^{k_z} \int_{-\pi}^{\pi} h(k_z) h(k_\theta) e^{i k_z z} e^{i k_\theta \theta} dk_z dk_\theta = \frac{\sin(k_z z) \sin(k_\theta a \theta)}{\pi^2 z a \theta}
\]

The filtering procedure consists of the discrete convolution product between the windowed force distribution and the spatial response of the filter calculated.

\[
F_{ij}^{\text{filtered}} = \Delta_z a \Delta_\theta \sum_{k=0}^{N_z-2} \sum_{l=0}^{N_\theta-2} F_{kl} h((i-k)\Delta_z, (j-l)a \Delta_\theta)
\]

Here \( k_z \) and \( k_\theta \) are the cut-off wavenumbers along the axial and circumferential directions respectively. The main difficulty lies in choosing the cut-off wavenumber in both directions when the filtered force distribution should be computed from the noisy force distribution \( F_{ij} \).

Figure 7 shows the force distribution after windowing and filtering of the force distribution. Two peaks corresponding to the exact positions of the forces appear and the noise is completely eliminated.
4. Conclusion

In this work, a predicting method based on the finite difference principle is proposed to identify the position of multiple excitation sources applied on a cylindrical shell from its internal side. At low frequencies, the knowledge of longitudinal and tangential deformations is necessary to achieve the reconstruction process of the input sources but the measurement of these quantities is very difficult to achieve. At frequencies that are equal or higher than the ring frequency, it is shown that it is possible to locate the sources from only the radial displacements. This is due to the vanishing of the coupling between radial and in-plane motion and hence making it easy to take measurements by a Scanning Laser Vibrometer for example. The proposed regularization technique gives positive results when using computer simulations with well defined parameters of the filter.

5. References

[1] Flugge W 1934 Statik and dynamik der schalen. Springer-Verlag Berlin
[2] Leissa A W 1973 Vibration of shells. NASA SP-288, reprinted by the Acoustical society of America through the American Institute of Physics 1993 1-83
[3] Fuller C R 1986 Radiation of sound from an infinite cylindrical elastic shell excited by an internal monopole source Journal of Sound and Vibration 109 2
[4] Lauragnet B and Guyader J L 1985 Sound radiation from finite cylindrical shells partially covered with longitudinal strips of compliant layer Journal of Sound and Vibration. 186 723-42
[5] Ouelaa N, Lauragnet B and Guyader J L 1994 Etude vibro-acoustique d’une coque cylindrique finie remplie de fluide en mouvement uniforme Acta Acustica. 2 275-89
[6] Iakovlev S 2007 Submerged fluid-filled cylindrical shell subjected to a shock wave: Fluid–structure interaction effects Journal of Fluids and Structures. 23 117-42
[7] Zhang Y and Mann III J A 1996 Measuring the structural intensity and force distribution in plates Journal of Acoustical Society of America. 99 345-53
[8] Karlsson S E S 1996 Identification of external structural loads from measured harmonic responses Journal of sound and vibration. 196 1 59-74
[9] Pezerat C and Guyader J L 2000 Force analysis technique: reconstruction of force distribution on plates Acustica united with Acta Acustica. 86 322-32
[10] Kim Y and Nelson P A 2004 Optimal regularisation for acoustic source reconstruction by inverse methods Journal of Sound and Vibration. 275 463-87
[11] Liu Y and Shepard S 2006 An improved method for the reconstruction of distributed force acting on a vibrating structure Journal of Sound and Vibration. 291 369-87
[12] Djamaa M C, Ouelaa N, Pezerat C et Guyader J L 2006 Identification of external forces exciting finite thin cylindrical shell Acta Acustica united with Acustica. 92 3 398-405