Electromagnetic Duality on The Light-Front in The Presence of External Sources

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(April 13, 1999)

Abstract

We investigate the issue of electromagnetic duality on the light front. We work with Zwanziger’s theory of electric and magnetic sources which is appropriate for treating duality. When quantized on the light-front in the light front gauge, this theory yields two independent phase space degrees of freedom, namely the two transverse field components, the right number to describe the gauge field sector of normal light-front QED and also the appropriate commutator between them. The electromagnetic duality transformation formulated in terms of them is similar in form to the Susskind transformation proposed for the free theory, provided one identifies them as the dynamical field components of the photon on the light-front in the presence of magnetic sources. The Hamiltonian density written in terms of these components is invariant under the duality transformation.

PACS: 12.20.-m, 11.15.-q, 11.10.Ef

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There has been a resurgence of interest in a new aspect of light-front theory in recent years with the advent of Matrix Theory [1] which can be formulated in light front coordinates [2]. As duality is an important concept and tool in Matrix Theory, the question of whether one can formulate it in light-front coordinates is important. In this paper we attempt to formulate electromagnetic duality for a U(1) gauge field theory in the presence of interactions in 3 + 1 dimensions. In a recent paper [3], it has been shown that when the free theory is formulated in light-front coordinates and in the light-front gauge, there exists a transformation known as the Susskind transformation [4] on the transverse components of the potential, which gives the electromagnetic duality transformation. However, it has been shown [3] that the Susskind transformation doesn’t hold for the interacting theory if one tries to incorporate it within the existing framework of ordinary light-front QED.

For a duality symmetric interacting theory there should be both electric and magnetic sources present. In this case, one has to consider a theory which deals with them. There exist several such theories [9]. In this paper, we have worked with a well known theory of magnetic monopoles namely that of Zwanziger [5]. It employs two potentials. We have quantized this theory on the light front in the light front gauge and shown that, out of sixteen phase space degrees of freedom that one starts with, two transverse field components are the only dynamical degrees of freedom. This is the correct number to describe the electromagnetic theory on the light front. We have shown that they obey the same commutation relation as that between the transverse components of the vector potential in light front QED. Thus Zwanziger’s theory is the theory of the photon on the light front in the presence of a magnetic current. We have formulated the duality transformation in terms of these two dynamical field components. This is a transformation between the transverse components of two different potentials. If one identifies them as the two independent field components of the photon on the light front in the presence of a magnetic current, this duality transformation has a form

I. INTRODUCTION
similar to the Susskind transformation. Thus under this identification, one can interprete it as the Susskind transformation for the interacting theory.

II. ELECTROMAGNETIC DUALITY FOR THE FREE THEORY IN LIGHT-FRONT COORDINATES

The light-front coordinates are given by, \( x^+ = x^0 + x^3, x^- = x^0 - x^3 \), where \( x^+ \) is the light-front time and \( x^- \) is the longitudinal coordinate. The inner product of two four-vectors, \( a, b \) is defined as,

\[
a^\mu b_\mu = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - a^i b^i \tag{2.1}
\]

where \( i = 1, 2 \).

The Lagrangian density for the free electromagnetic theory is,

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2.2}
\]

where \( F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \).

In light-front coordinates and in the light-front gauge \( A^+ = 0 \), the Lagrangian density can be written as,

\[
\mathcal{L} = \frac{1}{8} (\partial^+ A^-)^2 + \frac{1}{2} \partial^+ A^i \partial^- A^i - \frac{1}{2} \partial^+ A^i \partial^A^i A^- - \frac{1}{4} (\partial^i A^j - \partial^j A^i)^2 \tag{2.3}
\]

where \( \partial^+ = 2 \frac{\partial}{\partial x^+}, \partial^- = 2 \frac{\partial}{\partial x^-} \) and \( \partial^i = - \frac{\partial}{\partial x^i} \). The momenta conjugate to the fields are constrained,

\[
\pi_i = \frac{\partial \mathcal{L}}{\partial \partial^- A^i} = \frac{1}{2} \partial^+ A^i \tag{2.4}
\]

\[
\pi_- = \frac{\partial \mathcal{L}}{\partial \partial^- A^-} = 0 \tag{2.5}
\]

It can be shown that, in this gauge \( A^- \) is a constrained field and can be eliminated using the equation of constraint,

\[
(\partial^+)^2 A^- = 2 \partial^+ \partial^i A^i \tag{2.6}
\]
So there are only two independent degrees of freedom, $A^i$ (i=1,2) in the theory.

The free electromagnetic theory on the light-front is a constrained theory and has been quantized using the Dirac procedure \cite{7} or the reduced phase space method \cite{8}. It can be shown that the two dynamical field components, $A^i$ obey the canonical equal (light-front) time commutator given by,

$$[A^i(x), A^j(y)]_{x^+ = y^+} = -i \frac{1}{4} \delta^{ij} \epsilon(x^+ - y^+ \delta^2(x^\perp - y^\perp) (2.7)$$

The dual tensor is defined as,

$$F^{d\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho} (2.8)$$

where $\epsilon^{\mu\nu\lambda\rho}$ is totally antisymmetric, $\epsilon^{+\perp\perp+} = 2$. It can be shown \cite{3} that under the following transformation of the transverse components,

$$A^j \rightarrow \tilde{A}^j = -\epsilon^{ji} A^i (2.9)$$

the electromagnetic field components undergo the duality transformation,

$$- F^{d\mu\nu}(A^\perp) = F^{\mu\nu}(\tilde{A}^\perp), \quad F^{\mu\nu}(A^\perp) = F^{d\mu\nu}(\tilde{A}^\perp) (2.10)$$

So the transformation given by Eq.(2.9) known as the Susskind transformation is the duality transformation for the free theory.

Since duality symmetry necessitates the introduction of a magnetic current for the interacting theory, one has to work with a theory which deals with it. There are many such theories \cite{9}. In this paper, we work with the theory of Zwanziger, in which the duality transformation can be formulated in terms of the two four potentials involved. So we formulate this theory on the light-front and show that on quantizing it using the Dirac procedure one gets the right number of dynamical degrees of freedom (two) and also the normal light-front commutator between them. The duality transformation can then be formulated solely in terms of these dynamical degrees of freedom and it is similar in form to the Susskind transformation.
III. ZWANZIGER’S FORMULATION OF ELECTROMAGNETIC THEORY
WITH ELECTRIC AND MAGNETIC CURRENTS

The Maxwell equations in the presence of magnetic charges are,

\[ \partial_\mu F^{\mu\nu} = j_\nu^e, \quad \partial_\mu F^{d\mu\nu} = j_\nu^g \] (3.1)

where \( \partial_\mu j_\mu^e = \partial_\mu j_\mu^g = 0. \)

Zwanziger showed that the Maxwell equations can be derived from the following Lagrangian,

\[ L = \frac{1}{2} n_\alpha (\partial \wedge q_1)^\alpha - \frac{1}{2} \epsilon_{ab} n_\alpha (\partial \wedge q_a)^a \epsilon_{\alpha\gamma} n_\gamma \] (3.2)

with the identification

\[ F^{\mu\nu} = n^\alpha (\partial \wedge q_1)^\alpha - n^\nu \epsilon_{\alpha\beta} n_\gamma \partial \wedge q_2^{\gamma\beta} \] (3.3)

where \( a \) runs over 1 and 2 which correspond to two vector potentials, and \( n^\alpha \) is a fixed vector which is usually chosen to be spacelike. \( (\partial \wedge q_b)^d \) denotes the dual of \( (\partial \wedge q_b) \) as in Eq. (2.8) and the wedge product is defined as, \( (A \wedge B)^\mu = A^\mu B^\nu - A^\nu B^\mu \). This Lagrangian is not manifestly covariant because of the presence of the fixed vector \( n^\mu \). Covariance is regained only for quantized values of the coupling constants \( (e_n g_m - e_m g_n) \) where \( e_n \) and \( g_n \) are electric and magnetic charges. Moreover, this theory has two vector potentials and hence has sixteen phase space degrees of freedom. But the number of independent phase space degrees of freedom for the gauge field sector of QED is four. So the theory is highly constrained. Zwanziger had quantized it by introducing a gauge fixing term. Later Balachandran et.al. did the quantization using the Dirac procedure by making a specific choice of the fixed vector \( n^\mu \). The duality transformation is given in this theory in terms of the two potentials and the currents as follows:

\[ q_\alpha^\mu \rightarrow \epsilon_{ab} q_b^\mu, \quad j_\alpha^\mu \rightarrow \epsilon_{ab} j_b^\mu \] (3.4)

We now formulate Zwanziger’s theory in light-front coordinates.
IV. ZWANZIGER’S THEORY IN LIGHT-FRONT COORDINATES

We take $n^\mu$ to be spacelike and choose,

$$n^\mu = (n^+, n^-, n^1, n^2) = (0, 0, 1, 0)$$ (4.1)

The Lagrangian in light-front coordinates for this choice of $n^\mu$ becomes,

$$L = \frac{1}{2}(\partial \wedge q_a)^{1+}(\partial \wedge q_a)^{1-} + \frac{1}{2}(\partial \wedge q_a)^{12}(\partial \wedge q_a)^{12} + \frac{1}{4}\epsilon_{ab}(\partial \wedge q_a)^{1+}(\partial \wedge q_b)^{2-}$$

$$- \frac{1}{4}\epsilon_{ab}(\partial \wedge q_a)^{1-}(\partial \wedge q_b)^{2+} - \frac{1}{4}\epsilon_{ab}(\partial \wedge q_a)^{12}(\partial \wedge q_b)^{+-}$$

$$- j_a^\mu q_{a\mu}$$ (4.2)

We take $j_a^\mu$ to be currents due to external sources. The Lagrangian equations of motion are,

$$-\epsilon_{ab}\partial^+ (\partial \wedge q_b)^{12} + \partial^1 (\partial \wedge q_a)^{1+} + \epsilon_{ab}\partial^2 (\partial \wedge q_b)^{1+} = j_a^+$$ (4.3)

$$\frac{1}{2}\partial^+ (\partial \wedge q_a)^{1-} + \frac{1}{2}\partial^- (\partial \wedge q_a)^{1+} - \partial^2 (\partial \wedge q_a)^{12} = j_a^1$$ (4.4)

$$-\frac{1}{2}\epsilon_{ab}\partial^1 (\partial \wedge q_b)^{+-} + \partial^1 (\partial \wedge q_a)^{12} = j_a^2$$ (4.5)

$$\epsilon_{ab}\partial^- (\partial \wedge q_b)^{12} + \partial^1 (\partial \wedge q_a)^{1-} - \epsilon_{ab}\partial^2 (\partial \wedge q_b)^{1-} = j_a^-$$ (4.6)

It can be verified easily from Eq.(3.3) that the $F^{\mu\nu}$ for this choice of $n^\mu$ becomes,

$$F^{\mu\nu} = -\delta^{\mu1}(\partial \wedge q_1)^{1p} + \delta^{p1}(\partial \wedge q_1)^{1\mu} - \epsilon^{\mu\nu13}(\partial \wedge q_2)^{1\beta}$$ (4.7)

Using this expression one can verify that Eqs.(4.3)-(4.6) are the two sets of Maxwell equations in light-front coordinates. Since Eq.(4.3) does not involve a light-front time derivative $\partial^- = \frac{\partial}{\partial x}$, it gives two constraint equations.

In order to describe electromagnetic theory on the light-front, Zwanziger’s theory has to have more constraints than the equal-time case, since the number of phase space variables is two for the gauge field sector of QED in light-front coordinates [12], instead of four as in the equal time formulation. To quantize such a highly constrained theory, we use the Dirac procedure in which the Poisson bracket in the classical theory is replaced by a new object called the Dirac bracket which gives the commutator in the quantum theory.
We first calculate the momenta canonically conjugate to the fields $q^\mu_a$, defined as,

$$\pi_{\mu a} = \frac{\partial L}{\partial (\partial^- q^\mu_a)}$$  \hspace{1cm} (4.8)

Neither of these expressions turn out to have any light-front time derivatives and hence give rise to what are known as primary constraints. These are,

$$C_{a1} = \pi_a$$  \hspace{1cm} (4.9)

$$C_{a2} = \pi_a - \frac{1}{4} \epsilon_{ba} (\partial \land q_b)^{12}$$  \hspace{1cm} (4.10)

$$C_{a3} = \pi_a - \frac{1}{2} (\partial \land q_a)^{1+} - \frac{1}{4} \epsilon_{ba} (\partial \land q_b)^{2+}$$  \hspace{1cm} (4.11)

$$C_{a4} = \pi_a + \frac{1}{4} \epsilon_{ba} (\partial \land q_b)^{1+}$$  \hspace{1cm} (4.12)

For $a = 1, 2$ there are thus eight primary constraints. In Dirac’s method the constraints are set to zero only after evaluating all the Poisson brackets.

The canonical Hamiltonian density is defined as,

$$H_C = \pi_{\mu a} q^\mu_a - L$$  \hspace{1cm} (4.13)

Where $\pi_{\mu a}$ are obtained from Eqs. (4.9) - (4.12) with the primary constraints set to zero. The primary Hamiltonian is defined as,

$$H_P = \frac{1}{2} \int dx^- d^2 x^\perp [H_C + v_{aa} C_{aa}]$$  \hspace{1cm} (4.14)

where $v_{aa}$ are eight Lagrange multipliers subject to certain consistency conditions.

In this case the primary Hamiltonian is given by,

$$H_P = \frac{1}{2} \int dx^- d^2 x^\perp \{ [-8(\pi^+)^2 - (\partial^1 \pi_{1a} + \partial^2 \pi_{2a} + \partial^+ \pi_{+a})q_a^- + j^\mu_a q_{\mu a}] + v_{aa} C_{aa} \}$$  \hspace{1cm} (4.15)

The Poisson bracket between two quantities $A$ and $B$ is defined as,

$$[A(x), B(y)]_P = \frac{1}{2} \int dz^- d^2 z^\perp \left[ \frac{\partial A(x)}{\partial q^\mu (z)} \frac{\partial B(y)}{\partial \pi^\mu (z)} - \frac{\partial B(y)}{\partial q^\mu (z)} \frac{\partial A(x)}{\partial \pi^\mu (z)} \right]$$  \hspace{1cm} (4.16)

Evaluating the Poisson brackets of the constraints $C_{aa}$ with the primary Hamiltonian we find that $[C_{a1}, H_P]_P$ when required to be zero give two secondary constraints.
\[ C_{a5} = \partial^1 \pi_{1a} + \partial^2 \pi_{2a} + \partial^+ \pi_{+a} - \frac{1}{2} j_a^+ \]  

(4.17)

We also find that the constraints \( C_{a1} \) and \( C_{a5} \) are first class constraints since they give vanishing Poisson brackets with all other constraints including themselves. The remaining ones are second class. Requiring the Poisson brackets of the second class constraints with \( H_P \) to be zero we get some relations among the Lagrange multipliers \( v_{aa} \) for the second class constraints from which they can be determined consistently. Poisson bracket of the secondary constraints \( C_{a5} \) with \( H_P \) do not give any more constraints. Thus we find that there are ten constraints in the theory of which four are first class.

The presence of first class constraints indicates that there is gauge freedom in the theory. Since there are four first class constraints one has to introduce four gauge conditions. They are to be chosen such that they give non-zero Poisson brackets with the first class constraints and at the same time are consistent with the equations of motion. We choose the light-front gauge, \( q_a^+ = 0 \). It can be easily checked that these gauge constraints give non-zero Poisson brackets with the first class constraints, \( C_{a5} \). We find that in this gauge, two of the equations of motion given by Eq.(4.5) become constraint equations. We take these to be the other two gauge conditions. So we get the gauge constraints,

\[ C_{a6} = q_a^+ \]  

(4.18)

\[ C_{a7} = \frac{1}{2} \epsilon_{ba} \partial^1 \partial^+ q_b^- + \partial^1 (\partial \wedge q_a)^{12} - j_a^2 \]  

(4.19)

\( C_{a7} \) give non-zero Poisson bracket with the other first class constraints, \( C_{a1} \). Thus, after the gauge constraints are introduced, the first class constraints become second class. There are now fourteen second class constraints and the number of independent degrees of freedom is two, as in ordinary light-front QED.

This fact can also be observed from the Lagrangian equations of motion. In the light-front gauge, \( q_a^+ \) can be eliminated using Eq.(4.5). As mentioned earlier, Eq.(4.3) also represents two constraint equations since they do not contain any light-front time derivative. Hence out of four field components \( q_i^a \) two can be eliminated using them. Thus there are only two dynamical degrees of freedom since all the momenta are constrained.
We now proceed with the Dirac formalism and construct the 14 by 14 constraint matrix, the \(ij\)th element of which is given by,

\[
(C)_{ij,ab} = [C_{ai}, C_{bj}]_P
\]  

(4.20)

The constraint matrix is given in the appendix A. It is non-singular and hence can be inverted. The Dirac bracket between two quantities \(A\) and \(B\) is given by,

\[
[A, B]_D = [A, B]_P - \frac{1}{2} \int dx^- d^2 x^\perp \frac{1}{2} \int dy^- d^2 y^\perp [A, C_{ai}(x)]_P C^{-1}_{ij,ab}(x, y) [C_{bj}(y), B]_P
\]  

(4.21)

where \(C^{-1}\) is the inverse of the constraint matrix.

We now calculate the Dirac brackets between the field components, \(q^i_a\). We see that only the constraints \(C_{a3}, C_{a4}\) and \(C_{a5}\) give non-zero Poisson brackets with these field components. So the block of the inverse matrix that contributes to these Dirac brackets is given below,

\[
\begin{pmatrix}
C^{-1}_{33} I & i\sigma_2 C^{-1}_{33} & 0 \\
- i\sigma_2 C^{-1}_{33} & C^{-1}_{33} I & i\sigma_2 C^{-1}_{24} C^{-1}_{56} C^{-1}_{26} \\
0 & i\sigma_2 C^{-1}_{24} C^{-1}_{56} C^{-1}_{26} & 0
\end{pmatrix}
\]

The Dirac bracket between the field components are given by,

\[
[q^i_a(x), q^j_b(y)]_D = \frac{1}{4} \delta^{ij} \delta^2 \delta_{ab} C^{-1}_{33}(x, y) + \frac{1}{4} \delta^{ij} \delta^2 \epsilon_{ab}(C^{-1}_{33})(x, y)
\]

\[
+ \frac{1}{2} \epsilon_{ab}(C^{-1}_{33})(x, y) + \delta^{ij} \delta^2 \delta_{ab}(C^{-1}_{33})(x, y)
\]

\[
- \frac{1}{2} \epsilon_{ab}(C^{-1}_{24} C^{-1}_{56} C^{-1}_{26})(x, y) - \delta^{ij} \epsilon_{ab}(C^{-1}_{24} C^{-1}_{56} C^{-1}_{26})(x, y)
\]  

(4.22)

From this expression we find,

\[
[q^1_a, q^1_b]_D = \frac{1}{4} \delta_{ab} \epsilon(x^- - y^-) \delta^2(x^\perp - y^\perp) = [q^2_a, q^2_b]_D
\]  

(4.23)

\[
[q^1_a, q^2_b]_D = -\frac{1}{4} \epsilon_{ab} \epsilon(x^- - y^-) \delta^2(x^\perp - y^\perp) = -[q^2_a, q^1_b]_D
\]  

(4.24)

Here we have used,

\[
\frac{1}{2} \int dz^- d^2 z^\perp f^{-1}(x, z) f(z, y) = \delta(x^- - y^-) \delta(x^\perp - y^\perp)
\]  

(4.25)
where $f(x, y)$ is a function of $x, y$. This gives,

$$C_{33}^{-1}(x, y) = \frac{1}{4}\epsilon(x^- - y^-)\delta^2(x^+ - y^+)$$  \hspace{1cm} (4.26)$$

$$\left(C_{24}C_{56}\right)^{-1}(x, y)) = \frac{1}{4}\epsilon(x^- - y^-)\epsilon(x^1 - y^1)\delta(x^2 - y^2)$$  \hspace{1cm} (4.27)$$

where we have used the antisymmetry property of the Dirac brackets, $[q_a^1, q_b^1]_D$ and $[q_a^2, q_b^2]_D$.

In the quantum theory the Dirac brackets are replaced by $(-i)$ times the commutator. Then from Eq.(4.23) we find that both the Dirac brackets $[q_a^1, q_b^1]_D$ and $[q_a^2, q_b^2]_D$ give the canonical commutator between the transverse field components $A^i$ in normal light front QED. We can thus take either $q_a^1$ or $q_a^2$ as the two dynamical degrees of freedom and all the others as constrained variables and Zwanziger’s theory expressed in terms of them is the theory of the photon on the light front in the presence of magnetic sources. After all the Dirac brackets are evaluated, the constraints can be set to zero. Then the only relevant bracket is that between the dynamical field components. We take $q_a^1, a = 1, 2$ as the dynamical field components. It is interesting to note that they are the same components of two different potentials. In terms of these the Hamiltonian density can be written as,

$$\mathcal{H} = -\frac{1}{2}\left[\epsilon_{ab}\partial^1 q_b^1 - \partial^2 q_a^1 + \frac{1}{\partial^+}\epsilon_{ab}\partial^1 j_b^1\right]^2 - j_a^1 q_a^1 - \epsilon_{ab} j_a^2 q_b^1 - \epsilon_{ab} j_a^1 q_b^2 - \frac{1}{\partial^+} \frac{1}{\partial^1} j_b^1$$  \hspace{1cm} (4.28)$$

The Hamiltonian is invariant under the duality transformation given by Eq.(3.4), which can be written solely in terms of the dynamical field components as,

$$q_a^1 \rightarrow \epsilon_{ab} q_b^1, \quad j_a^\mu \rightarrow \epsilon_{ab} j_b^\mu$$  \hspace{1cm} (4.29)$$

It is clear that the above duality transformation is similar in form to the Susskind transformation, however, it is a transformation between the same component of two different potentials, whereas the Susskind transformation for the free theory is between the two transverse components of a single potential. Since $q_a^1$ are the only two dynamical field components and they obey the same commutation relation as the $A^i$s, one can identify them as the two dynamical field components that describe the photon on the light-front in the presence of a magnetic current. In that sense, the duality transformation can be interpreted as the Susskind transformation for the interacting theory.
To summarize, in this paper we have formulated electromagnetic duality on the light-front in the presence of interactions. We have worked with Zwanziger’s theory with electric and magnetic currents in light-front coordinates for a specific choice of $n^\mu$. We have quantized the theory following the Dirac procedure and shown that the number of independent degrees of freedom is two as it should be for the gauge field in light-front QED and they obey the canonical light-front QED commutator. The duality transformation given in terms of the two dynamical field components can be interpreted as the Susskind transformation, provided we identify these dynamical components as those which describe a photon on the light-front in the presence of a magnetic current.

We would like to thank Prof. A. Harindranath for bringing the problem of electromagnetic duality on the light-front to our attention. We are grateful to Prof. P. Mitra, Prof. A. Harindranath and also to Ananda Dasgupta for helpful discussions.

**APPENDIX A: THE CONSTRAINT MATRIX**

The constraint matrix is given by,

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & i\sigma_2 C_{17} \\
0 & 0 & C_{23}I & i\sigma_2 C_{24} & 0 & C_{26}I & 0 \\
0 & C_{23}I & C_{33}I & 0 & 0 & 0 & C_{37}I \\
0 & -i\sigma_2 C_{24} & 0 & 0 & 0 & 0 & C_{47}I \\
0 & 0 & 0 & 0 & 0 & C_{56}I & 0 \\
0 & -C_{26}I & 0 & 0 & C_{56}I & 0 & 0 \\
i\sigma_2 C_{17} & 0 & -C_{37}I & -C_{47}I & 0 & 0 & 0
\end{pmatrix}
$$

where we have used $(i\sigma_2)_{ab} = \epsilon_{ab}$ and $I$ is the two by two identity matrix. The various nonzero elements $C_{ij}$ are,

$$
C_{17} = -\frac{1}{2} \partial_y^1 \partial_y^+ \delta(x^- - y^-) \delta^2(x^+ - y^+) \\
C_{23} = \frac{1}{2} \partial_y^1 \delta(x^- - y^-) \delta^2(x^+ - y^+)
$$
\[C_{24} = -\frac{1}{2} \partial^1 y \delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A3)}\]
\[C_{26} = -\delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A4)}\]
\[C_{33} = -\partial^1 y \delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A5)}\]
\[C_{37} = \partial^1 y \partial^2 y \delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A6)}\]
\[C_{47} = -\partial^1 y \partial^1 y \delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A7)}\]
\[C_{56} = \partial^1 y \delta(x^- - y^-) \delta^2(x^\perp - y^\perp) \quad \text{(A8)}\]
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