Modified Supergravity and Early Universe: the Meeting Point of Cosmology and High-Energy Physics

Sergei V. Ketov \textsuperscript{a,b}

\textsuperscript{a} Department of Physics, Tokyo Metropolitan University, Minami-ohsawa 1-1 Hachioji-shi, Tokyo 192-0397, Japan
\textsuperscript{b} Kavli Institute for the Physics and Mathematics of the Universe (IPMU), The University of Tokyo, Kashiwanoha 5-1-5, Kashiwa-shi, Chiba 277-8568, Japan
ketov@phys.se.tmu.ac.jp

Abstract

We review the new theory of modified supergravity, dubbed the $F(R)$ supergravity, and some of its recent applications to inflation and reheating in the early universe cosmology. The $F(R)$ supergravity is the $N = 1$ locally supersymmetric extension of the $f(R)$ gravity in four space-time dimensions. A manifestly supersymmetric formulation of the $F(R)$ supergravity exist in terms of $N = 1$ superfields, by using the (old) minimal Poincaré supergravity in curved superspace. We find the conditions for stability, the absence of ghosts and tachyons. Three models of the $F(R)$ supergravity are studied. The first example is devoted to a recovery of the standard (pure) $N = 1$ supergravity with a negative cosmological constant from the $F(R)$ supergravity. As the second example, a generic $R^2$ supergravity is investigated, and the existence of the AdS bound on the scalar curvature is found. As the third (and most important) example, a simple viable realization of chaotic inflation in supergravity is found. Our approach is minimalistic since it does not introduce new exotic fields or new interactions, beyond those already present in (super)gravity. The universal reheating mechanism is automatic. We establish the consistency of our approach and also apply it to preheating and reheating after inflation. The Higgs inflation and its correspondence to the Starobinsky inflation are established in the context of supergravity. We briefly review other relevant issues such as non-Gaussianity, $CP$-violation, origin of baryonic asymmetry, lepto- and baryo-genesis. The $F(R)$ supergravity has promise for possible solutions to those outstanding problems too.

Keywords: inflation, reheating, supergravity, superspace, Higgs particle

PACS numbers: 98.80.Cq, 04.65.+e, 04.62.+v, 98.80.Hw
## CONTENTS

1. Introduction and Motivation 3
2. Starobinsky Approach to Inflation 5
3. $f(R)$ Gravity 6
4. Inflationary Theory and Observations 8
5. Supergravity and Superspace 11
6. $F(\mathcal{R})$ Supergravity in Superspace 13
7. No-scale $F(\mathcal{R})$ Supergravity 17
8. Fields from Superfields in $F(\mathcal{R})$ Supergravity 18
9. Generic $\mathcal{R}^2$ Supergravity, and AdS Bound 20
10. Inflationary Model in $F(\mathcal{R})$ Supergravity 25
11. More about Inflationary Dynamics in our Model 29
12. Facing Observational Tests 32
13. Effective Scalar Potential for Preheating 36
14. Preheating after Inflation 39
15. Current Status of our Model 44
16. Cosmological Constant in $F(\mathcal{R})$ Supergravity 45
17. Nonminimal Scalar-Curvature Coupling in Gravity and Supergravity, and Higgs Inflation 49
18. Quantum Particle Production (Reheating) 56
19. Conclusion 59
20. Outlook: $CP$-violation, Baryonic Asymmetry, Lepto- and Baryo-genesis, Non-Gaussianity, and Experimental Tests 61

Acknowledgements 63
Appendix: Scalar Potential in Generic $F(\mathcal{R})$ Supergravity 64
References 65
1 Introduction and Motivation

A brief history of our universe in pictures is nicely summarized in the NASA website of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite mission [1]. Very recently (March 2013) more data about the observational constraints on inflation has become available from the PLANCK satellite mission [2]. In this review paper we focus on a field-theoretical description of the inflationary phase of early universe and its post-inflationary dynamics (pre-heating and re-heating) in the context of modified supergravity proposed and studied in Refs. [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

Cosmological inflation (a phase of ‘rapid’ quasi-exponential accelerated expansion of universe) [17, 18, 19, 20, 21] predicts homogeneity of our universe at large scales, its spatial flatness, large size and entropy, and the almost scale-invariant spectrum of cosmological perturbations, in good agreement with the ongoing WMAP and PLANCK measurements of the CMB radiation spectrum [22, 23, 24]. Inflation is also the only known way to generate structure formation in the universe via amplifying quantum fluctuations in vacuum. See, e.g., Refs. [25, 26, 27, 28, 29] for a comprehensive review of inflationary physics and mathematics.

However, inflation is just the cosmological paradigm, not a theory! The known field-theoretical mechanisms of inflation use a slow-roll scalar field $\phi$ (called inflaton) with proper scalar potential $V(\phi)$.

The scale of inflation is well beyond the electro-weak scale, ie. it is well beyond the Standard Model of Elementary Particles! Thus the inflationary stage in the early universe is the most powerful High-Energy Physics (HEP) accelerator in Nature (up to $10^{10}$ TeV). Therefore, inflation is the great and unique window to very HEP!

The nature of inflaton and the origin of its scalar potential are the big mysteries.

In this paper the units $\hbar = c = 1$ and $M_{Pl} = \kappa^{-1} = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV, and the spacetime signature $(+, -, -, -)$ are used. See ref. [30] for our use of Riemann geometry of a curved spacetime.

The CMB radiation picture from the WMAP and PLANCK are the main source of data about early universe. Deciphering it in terms of density perturbations, gravity wave polarization, power spectrum and its various indices is a formidable task. It also requires the heavy mathematical formalism based on General Relativity [27, 28, 29]. Fortunately, we do not need much of that formalism for our purposes, since the relevant indices can also be introduced in terms of the inflaton scalar potential (Sec. 4). We assume that inflation did happen. There exist many inflationary models — see eg., Ref. [26] for their description and comparison (without supersymmetry). Our aim is a viable theoretical description of inflation in the context of supergravity and its relation to HEP of elementary particles beyond the SM.

The main Cosmological Principle of a spatially homogeneous and isotropic $(1+3)$-dimensional universe (at large scales) gives rise to the FLRW metric

$$ds^2_{\text{FLRW}} = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$ (1.1)
where the function \( a(t) \) is known as the scale factor in ‘cosmic’ (comoving) coordinates \((t, r, \theta, \phi)\), and \( k \) is the FLRW topology index, \( k = (-1, 0, +1) \). The FLRW metric (1.1) admits the six-dimensional isometry group \( G \) that is either \( SO(1, 3) \), \( E(3) \) or \( SO(4) \), acting on the orbits \( G/SO(3) \), with the spatial three-dimensional sections \( H^3 \), \( E^3 \) or \( S^3 \), respectively. The Weyl tensor of any FLRW metric vanishes, \( C_{\mu \nu \lambda \rho}^{\text{FLRW}} = 0 \) (1.2) where \( \mu, \nu, \lambda, \rho = 0, 1, 2, 3 \). The early universe inflation (acceleration) means

\[
\ddot{a}(t) > 0 \quad \text{or equivalently,} \quad \frac{d}{dt} \left( \frac{H^{-1}}{a} \right) < 0
\]

where \( H = \dot{a}/a \) is called Hubble function, and \( \frac{H^{-1}}{a} \) is called Hubble radius. The latter describes the causally connected region whose size is decreasing during inflation. We take \( k = 0 \) for simplicity. The amount of inflation (called the \textit{e-foldings} number) is given by

\[
N_e = \ln \left( \frac{a(t_{\text{end}})}{a(t_{\text{start}})} \right) = \int_{t_{\text{start}}}^{t_{\text{end}}} H \, dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi_{\text{end}}} \frac{V}{V'} \, d\phi
\]

It is well recognized by now that one has to go beyond the Einstein-Hilbert action for gravity, both from the experimental viewpoint (because of dark energy) and from the theoretical viewpoint (because of the UV incompleteness of quantized Einstein gravity and the need of its unification with the Standard Model of Elementary Particles).

In our approach the origin of inflation is geometrical or gravitational, i.e. is closely related to space-time and gravity. It can be technically accomplished by taking into account the higher-order curvature terms on the left-hand-side of Einstein equations (modified gravity), and extending gravity to supergravity. The higher-order curvature terms are supposed to appear in the gravitational effective action of Quantum Gravity. Their derivation from Superstring Theory may be possible too. The true problem is a selection of those high-order curvature terms that are physically relevant and/or derivable from a fundamental theory of Quantum Gravity.

There are many phenomenological models of inflation in the literature, which usually employ some new fields and new interactions. It is, therefore, quite reasonable and meaningful to search for the \textit{minimal} inflationary model building, by getting the most economical and viable inflationary scenarios. We are going to use the approach proposed the long time ago by Starobinsky [17, 18], which is also known as the (chaotic) \( R^2 \)-inflation. We assume that the general coordinate invariance in spacetime is fundamental, and it should not be sacrificed. Moreover, it should be extended to the more fundamental, local supersymmetry that is known to imply the general coordinate invariance. It thus leads us to supergravity which, in addition, automatically has several viable candidates for \textit{Dark Matter} particle (see Sec. 20 for more).

On the theoretical side, the available inflationary models may be also evaluated with respect to their “cost”, i.e. against what one gets from a given model in relation
to what one puts in! Our approach does not introduce new fields, beyond those already present in gravity and supergravity. We also exploit (super)gravity interactions only, 

Before going into details, let us address two common prejudices and objections.

The higher-order curvature terms are usually expected to be relevant near the space-time curvature singularities. It is also quite possible that some higher-derivative gravity, subject to suitable constraints, could be in the effective action of a quantized theory of gravity, ¹ like e.g., in String Theory. However, there are also some common doubts against the higher-derivative terms, in principle.

First, it is often argued that all higher-derivative field theories, including the higher-derivative gravity theories, have ghosts (i.e. are unphysical), because of Ostrogradski theorem (1850) in Classical Mechanics. As a matter of fact, though the presence of ghosts is a generic feature of the higher-derivative theories indeed, it is not always the case, while many explicit examples are known (Lovelock gravity, Euler densities, some \( f(R) \) gravity theories, etc.) — see e.g., ref. [32] for more details. In our approach, the absence of ghosts and tachyons is required, while it is also considered as one of the main physical selection criteria for the “good” higher-derivative field theories.

Another common objection against the higher-derivative gravity theories is due to the fact that all the higher-order curvature terms in the action are to be suppressed by the inverse powers of \( M_{\text{Pl}} \) on dimensional reasons and, therefore, they seem to be ‘very small and negligible’. Though it is generically true, it does not mean that all the higher-order curvature terms are irrelevant at all scales much less than \( M_{\text{Pl}} \). For instance, it appears that the quadratic curvature terms have dimensionless couplings, while they can easily describe the early universe inflation (in the high-curvature regime). A non-trivial function of \( R \) in the effective gravitational action may also ‘explain’ dark energy in the present universe [33, 34, 35].

## 2 Starobinsky approach to inflation

The Starobinsky models were the first inflationary models introduced as early as 1980 [17, 18]. Remarkably, they are still viable, being consistent with all cosmological observations at present. To say more, they are currently preferred by the most recent WMAP9 and PLANCK observational data [23, 24]. In this section we approach the Starobinsky models from the very different (formal) perspective.

It can be argued that it is the scalar curvature-dependent part of the gravitational effective action that is most relevant to the large-scale dynamics \( H(t) \). Here are some simple arguments.

In four spacetime dimensions all the independent quadratic curvature invariants

¹To the best of our knowledge, this proposal was first formulated by A.D. Sakharov in 1967 [31].
are $R^\mu\nu\lambda\rho R_{\mu\nu\lambda\rho}$, $R^\mu\nu R_{\mu\nu}$ and $R^2$. However, the Gauss-Bonnet combination

$$\int d^4 x \sqrt{-g} \left( R^\mu\nu\lambda\rho R_{\mu\nu\lambda\rho} - 4 R^\mu\nu R_{\mu\nu} + R^2 \right)$$

(2.1)

is topological (ie. a total derivative) for any metric, while

$$\int d^4 x \sqrt{-g} \left( 3 R^\mu\nu R_{\mu\nu} - R^2 \right)$$

(2.2)

is also topological for any FLRW metric, because of eq. (1.2). Hence, the FLRW-relevant quadratically-generated gravity action is $(8\pi G_N = 1)$

$$S = -\frac{1}{2} \int d^4 x \sqrt{-g} \left( R - R^2 / M^2 \right)$$

(2.3)

This action is known as the simplest Starobinsky model [17, 18]. Its equations of motion allow a stable inflationary solution, and it is an attractor! When $H \gg M$, one finds

$$H \approx \left( \frac{M}{6} \right)^2 (t_{\text{end}} - t)$$

(2.4)

It is the particular realization of chaotic inflation (ie. with chaotic initial conditions) [36] with a Graceful Exit.

In the case of a generic gravitational action with the higher-order curvature terms, the Weyl dependence can be excluded due to eq. (1.2) again. A dependence upon the Ricci tensor may also be excluded since, otherwise, it would lead to the extra propagating massless spin-2 degree of freedom (in addition to a metric) described by the field $\partial L / \partial R_{\mu\nu}$. The higher derivatives of the scalar curvature in the gravitational Lagrangian $L$ just lead to more propagating scalars [37], so we simply ignore them for simplicity in what follows.

### 3 $f(R)$ Gravity

The Starobinsky model (2.3) is the special case of the $f(R)$ gravity theories [33, 34, 35] having the action

$$S_f = -\frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} \tilde{f}(R)$$

(3.1)

In the absence of extra matter, the gravitational (trace) equation of motion is of the fourth order with respect to the time derivative,

$$\frac{3}{a^3} \frac{d}{dt} \left( a^3 \frac{d\tilde{f}'(R)}{dt} \right) + R \tilde{f}'(R) - 2 \tilde{f}(R) = 0$$

(3.2)
where we have used $H = \frac{a}{a}$ and $R = -6(\dot{H} + 2H^2)$. The primes denote the derivatives with respect to $R$, and the dots denote the derivative with respect to $t$. Static de-Sitter solutions correspond to the roots of the equation [38]

$$R\tilde{f}'(R) = 2\tilde{f}(R)$$

The 00-component of the gravitational equations is of the third order with respect to the time derivative,

$$3H \frac{d\tilde{f}'}{dt} - 3(\dot{H} + H^2)\tilde{f}'(R) - \frac{1}{2}\tilde{f}(R) = 0 \quad (3.3)$$

The (classical and quantum) stability conditions in $f(R)$ gravity are well known [33, 34], and are given by (in our notation)

$$\tilde{f}'(R) > 0 \quad \text{and} \quad \tilde{f}''(R) < 0 \quad (3.4)$$

respectively. The first condition (3.4) is needed to get a physical (non-ghost) graviton, while the second condition (3.4) is needed to get a physical (non-tachyonic) scalaron (see Sec. 9 for more).

Any $f(R)$ gravity is known to be classically equivalent to the certain scalar-tensor gravity having an (extra) propagating scalar field [39, 40, 41]. The formal equivalence can be established via the Legendre-Weyl transformation to be described below.

First, the $f(R)$-gravity action (3.1) can be rewritten to the form

$$S_A = -\frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{AR - Z(A)\} \quad (3.5)$$

where the real scalar (or Lagrange multiplier) $A(x)$ is related to the scalar curvature $R$ by the Legendre-like transformation:

$$R = Z'(A) \quad \text{and} \quad \tilde{f}(R) = RA(R) - Z(A(R)) \quad (3.6)$$

with $\kappa^2 = 8\pi G_N = M_{\text{Pl}}^{-2}$.

Next, a Weyl transformation of the metric,

$$g_{\mu\nu}(x) \rightarrow \exp \left[ \frac{2\kappa\phi(x)}{\sqrt{6}} \right] g_{\mu\nu}(x) \quad (3.7)$$

with arbitrary field parameter $\phi(x)$ yields

$$\sqrt{-g} R \rightarrow \sqrt{-g} \exp \left[ \frac{2\kappa\phi(x)}{\sqrt{6}} \right] \left\{ R - \sqrt{\frac{6}{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu}\phi \right) \kappa - \kappa^2 g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi \right\} \quad (3.8)$$

Therefore, when choosing

$$A(\kappa\phi) = \exp \left[ \frac{-2\kappa\phi(x)}{\sqrt{6}} \right] \quad (3.9)$$
and ignoring a total derivative in the Lagrangian, we can rewrite the action to the form

\[
S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left\{ -\frac{R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2\kappa^2} \exp \left[ \frac{4\kappa \phi(x)}{\sqrt{6}} \right] Z(A(\kappa \phi)) \right\}
\]

in terms of the physical (and canonically normalized) scalar field \( \phi(x) \), without any higher derivatives and ghosts. As a result, one arrives at the standard action of the real dynamical scalar field \( \phi(x) \) \textit{minimally} coupled to Einstein gravity and having the scalar potential

\[
V(\phi) = -\frac{M_{P1}^2}{2} \exp \left\{ \frac{4\phi}{M_{P1} \sqrt{6}} \right\} Z \left( \exp \left[ -\frac{2\phi}{M_{P1} \sqrt{6}} \right] \right)
\]

In the context of the inflationary theory, the \textit{scalaron} (= scalar part of spacetime metric) \( \phi \) can be identified with inflaton. This inflaton has the clear origin as the spin-0 part of spacetime metric, and may also be understood as the conformal mode of the metric in Minkowski or (A)dS vacuum.

In the Starobinsky case, \( \tilde{f}(R) = R - R^2/M^2 \), the inflaton scalar potential reads

\[
V(y) = V_0 \left( e^{-y} - 1 \right)^2
\]

where we have introduced the notation

\[
y = \sqrt{\frac{2}{3}} \frac{\phi}{M_{P1}} \quad \text{and} \quad V_0 = \frac{1}{8} M_{P1}^2 M^2
\]

It is worth noticing here the appearance of the inflaton vacuum energy \( V_0 \) driving inflation. The end of inflation (Graceful Exit) is also clear: the scalar potential (3.12) has a very flat (slow-roll) ‘plateau’, ending with a ‘waterfall’ towards the minimum (Fig. 1).

It is worth emphasizing that the inflaton (scalaron) scalar potential (3.12) is derived here by merely assuming the existence of the \( R^2 \) term in the gravitational action. The Newton (weak gravity) limit is not applicable to early universe (including its inflationary stage), so that the dimensionless coefficient in front of the \( R^2 \) term does not have to be very small at early time. It distinguishes the primordial ‘Dark Energy’ driving inflation in the early Universe from the ‘Dark Energy’ responsible for the present universe acceleration.

### 4 Inflationary Theory and Observations

The \textit{slow-roll} inflation parameters are defined by

\[
\varepsilon(\phi) = \frac{1}{2} M_{P1}^2 \left( \frac{V'}{V} \right)^2 \quad \text{and} \quad \eta(\phi) = M_{P1}^2 \frac{V''}{V}
\]
A necessary condition for the slow-roll approximation is the smallness of the inflation parameters
\[ \varepsilon(\phi) \ll 1 \quad \text{and} \quad |\eta(\phi)| \ll 1 \quad (4.2) \]
The first condition implies \( \ddot{\phi}(t) > 0 \). The second one guarantees that inflation lasts long enough, via domination of the friction term in the inflaton equation of motion, \( 3H \dot{\phi} = -V' \).

The CMB temperature fluctuations [1, 22] have the scale \( \delta T / T \approx 10^{-5} \) at the WMAP normalization of 500 Mpc. Actually, the scalar \( (\rho_s) \) and tensor \( (\rho_t) \) perturbations of metric do decouple. The scalar perturbations couple to the density of matter and radiation, so they are responsible for the inhomogeneities and anisotropies in the universe. The tensor perturbations (or gravity waves) also contribute to the CMB, while their experimental detection would tell us much more about inflation. The CMB radiation is expected to be polarized due to Compton scattering at the time of decoupling [42, 43].

The primordial (Zeldovich-Harrison) spectrum is proportional to \( k^{n-1} \), in terms of the comoving wave number \( k \) and the spectral index \( n \), in the 2-point function (observable)
\[ \left\langle \frac{\delta T(x)}{T} \frac{\delta T(y)}{T} \right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n-1} \quad (4.3) \]

In theory, the slope \( n_s \) of the scalar power spectrum, associated with the density perturbations, \( \left( \frac{\delta \rho}{\rho} \right)^2 \propto k^{n_s-1} \), is given by \( n_s = 1 + 2\eta - 6\varepsilon \), the slope of the tensor primordial spectrum, associated with gravitational waves, is \( n_t = -2\varepsilon \), and the tensor-to-scalar ratio is \( r = \delta \rho_s / \delta \rho_t = 16\varepsilon \) (see eg., ref. [26]).

It is straightforward to calculate those indices in any inflationary model with a given inflaton scalar potential. In the case of the Starobinsky model and its scalar

![Figure 1: The inflaton scalar potential \( v(x) = (e^y - 1)^2 \) in the Starobinsky model, after \( y \to -y \)]
potential (3.12), one finds [44, 45, 8]

\[ n_s = 1 - \frac{2}{N_e} + \frac{3\ln N_e}{2N_e^2} - \frac{2}{N_e^2} + \mathcal{O} \left( \frac{\ln^2 N_e}{N_e^3} \right) \]  

(4.4)

and

\[ r \approx \frac{12}{N_e^2} \approx 0.004 \]  

(4.5)

with \( N_e \approx 55 \). The very small value of \( r \) is the sharp prediction of the Starobinsky inflationary model towards \( r \)-measurements in a future.

Those theoretical values are to be compared to the observed values of the CMB radiation. For instance, the WMAP7 observations [22] yield

\[ n_s = 0.963 \pm 0.012 \quad \text{and} \quad r < 0.24 \]  

(4.6)

with the 95\% level of confidence.

The most recent PLANCK data yields [24]

\[ n_s = 0.960 \pm 0.007 \quad \text{and} \quad r < 0.11 \]  

(4.7)

also with the 95\% level of confidence.

The amplitude of the initial perturbations, \( \Delta_R^2 = M_{Pl}^4 V/(24\pi^2 \varepsilon) \), is also the physical observable whose experimental value is known since 1992 due to the Cosmic Background Explorer (COBE) satellite mission [46]:

\[ \left( \frac{V}{\varepsilon} \right)^{1/4} = 0.027 \ M_{Pl} = 6.6 \times 10^{16} \text{ GeV} \]  

(4.8)

It determines the normalization of the \( R^2 \)-term in the action (2.3)

\[ \frac{M}{M_{Pl}} = 4 \cdot \sqrt{\frac{2}{3}} \cdot (2.7)^2 \cdot \frac{e^{-y}}{(1 - e^{-y})^2} \cdot 10^{-4} \approx (3.5 \pm 1.2) \cdot 10^{-6} \]  

(4.9)

The inflaton mass is given by \( M_{inf} = M/\sqrt{6} \), and there are no free parameters left.

The main theoretical lessons we can draw from that are:

(i) the main discriminants amongst all inflationary models are given by the values of \( n_s \) and \( r \);  
(ii) the Starobinsky model (1980) of chaotic inflation is very simple and economic. It uses gravity interactions only. It predicts the origin of inflaton and its scalar potential. It is still viable and consistent with all known observations. Inflaton is not charged (singlet) under the SM gauge group. The Starobinsky inflation has an end (Graceful Exit), and gives the simple explanation to the WMAP-observed value of \( n_s \). The key difference of Starobinsky inflation from the other standard inflationary models (having \( \frac{1}{2}m^2 \phi^2 \) or \( \lambda \phi^4 \) scalar potentials) is the very low value of \( r \) — see Fig. 2 for comparison and ref. [47] for more details;
(iii) the viable inflationary models, based on \( \tilde{f}(R) = R + \hat{f}(R) \) gravity, turn out to be close to the simplest Starobinsky model (over the range of \( R \) relevant to inflation), with \( \hat{f}(R) \approx R^2 A(R) \) and the slowly varying function \( A(R) \) in the sense

\[
|A'(R)| \ll \frac{A(R)}{R} \quad \text{and} \quad |A''(R)| \ll \frac{A(R)}{R^2}
\]  

(4.10)

5 Supergravity and Superspace

Supersymmetry (SUSY) is the leading proposal to new physics beyond the SM. Therefore, it is quite natural to unify inflation with high-energy particle physics in the context of supersymmetry.

SUSY is the symmetry between bosons and fermions. SUSY is the extension of Poincaré symmetry of spacetime, and is well motivated in HEP beyond the SM. Supersymmetry is also needed for consistency of strings. Supergravity (SUGRA) is the theory of local supersymmetry that automatically implies general coordinate invariance. Hence, considering inflation with supersymmetry necessarily leads to supergravity. As a matter of fact, most of studies of superstring- and brane-cosmology are also based on their effective description in the four-dimensional \( N = 1 \) supergravity.

It is not our purpose to give a detailed account of SUSY and SUGRA, because of the existence of several good textbooks — see eg., refs. [48, 49, 50]. In this Section we recall only the basic facts about \( N = 1 \) supergravity in four spacetime dimensions, which are needed for our purposes.

A concise and manifestly supersymmetric description of SUGRA is provided by Superspace. In this section the natural units \( c = \hbar = \kappa = M_{\text{Pl}} = 1 \) are used for more simplicity.

Supergravity needs a curved superspace. However, they are not the same, because one has to reduce the field content to the minimal one corresponding to an off-shell
supergravity multiplet. It can be done by imposing certain off-shell constraints on
the supertorsion tensor in curved superspace [48, 49, 50]. An off-shell supergravity
multiplet has some extra (auxiliary) fields with noncanonical dimensions, in addition to
physical spin-2 field (metric) and spin-3/2 field (gravitino). It is worth mentioning that
imposing the off-shell constraints is independent upon writing a supergravity action.

One may work either in a full (curved) superspace or in a chiral one. There are
some practical advantages of using the chiral superspace, because it helps us to keep
the auxiliary fields unphysical (i.e. nonpropagating). The chiral superspace is more
closely related to supergravity in components (in a Wess-Zumino gauge).

The chiral superspace density reads
\[ E(x, \theta) = e(x) \left[ 1 + i \theta \sigma^a \bar{\psi}_a(x) - \theta^2 \left( B^*(x) + \bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b \right) \right], \tag{5.1} \]
where \( e = \sqrt{-\det g_{\mu\nu}} \) is a spacetime metric, \( \psi^a_\alpha = e^\mu_a \psi^\alpha_\mu \) is a chiral gravitino,
\( B = S + iP \) is the complex scalar auxiliary field. We use the lower case middle greek
letters \( \mu, \nu, \ldots = 0, 1, 2, 3 \) for curved spacetime vector indices, the lower case early
latin letters \( a, b, \ldots = 0, 1, 2, 3 \) for flat (target) space vector indices, and the lower case
early greek letters \( \alpha, \beta, \ldots = 1, 2 \) for chiral spinor indices.

A solution to the superspace Bianchi identities together with the constraints defin-
ing the \( N = 1 \) Poincaré-type minimal supergravity theory results in only three
covariant tensor superfields \( R, G_{\alpha} \) and \( W_{\alpha \beta \gamma} \), subject to the off-shell relations [48, 49, 50]:
\[ G_{\alpha} = \bar{G}_{\alpha}, \quad W_{\alpha \beta \gamma} = \bar{W}_{(\alpha \beta \gamma)}, \quad \nabla_{\alpha} R = \nabla_{\alpha} W_{\alpha \beta \gamma} = 0, \tag{5.2} \]
and
\[ \nabla_{\alpha} G_{\alpha \alpha} = \nabla_{\alpha} R, \quad \nabla_{\gamma} W_{\alpha \beta \gamma} = \frac{i}{2} \nabla_{\alpha} \bar{G}_{\beta \alpha} + \frac{i}{2} \nabla_{\beta} \bar{G}_{\alpha \alpha}, \tag{5.3} \]
where \( (\nabla_{\alpha}, \bar{\nabla}_{\alpha}, \nabla_{\alpha} \bar{\nabla}_{\alpha}) \) stand for the curved superspace \( N = 1 \) supercovariant deriva-
tives, and the bars denote complex conjugation.

The covariantly chiral complex scalar superfield \( R \) has the scalar curvature \( R \) as the
coefficient at its \( \theta^2 \) term, the real vector superfield \( G_{\alpha \alpha} \) has the traceless Ricci tensor,
\( R_{\mu \nu} + R_{\nu \mu} - \frac{1}{2} g_{\mu \nu} R \), as the coefficient at its \( \theta \sigma^a \bar{\sigma}^b \theta \) term, whereas the covariantly chiral,
complex, totally symmetric, fermionic superfield \( W_{\alpha \beta \gamma} \) has the self-dual part of the
Weyl tensor \( C_{\alpha \beta \gamma \delta} \) as the coefficient at its linear \( \theta^4 \)-dependent term.

A generic Lagrangian representing the supergravitational effective action in (full)
superspace, reads
\[ \mathcal{L} = \mathcal{L}(R, G, W, \ldots) \tag{5.4} \]
where the dots stand for arbitrary supercovariant derivatives of the superfields.

The Lagrangian (5.4) it its most general form is, however, unsuitable for physical
applications, not only because it is too complicated, but just because it generically
leads to the “propagating auxiliary” fields whose physical interpretation is unclear.
The important physical condition of keeping the supergravity auxiliary fields to be
truly auxiliary (i.e. nonphysical or nonpropagating) in field theories with the higher
derivatives was dubbed the ‘auxiliary freedom’ in refs. [51, 52]. To get the supergravity
actions with the ‘auxiliary freedom’, we employ the chiral (curved) superspace.
6 \textit{F(\mathcal{R})} Supergravity in Superspace

Here we focus on the scalar-curvature-sector of a generic higher-derivative supergravity (5.4), which is most relevant to the FRLW cosmology, by ignoring the tensor curvature superfields $\mathcal{W}_{\alpha\beta\gamma}$ and $G_{\alpha\beta}$, as well as the derivatives of the scalar superfield $\mathcal{R}$. Then there is only one candidate for a locally supersymmetric action in the chiral curved superspace,

$$S_F = \int d^4 x d^2 \theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \quad \text{(6.1)}$$

governed by a chiral analytic function $F(\mathcal{R})$.\footnote{The field construction of this theory by using the 4D, $N = 1$ superconformal tensor calculus was given in ref. [53].} Besides having the manifest local $N = 1$ supersymmetry, the action (6.1) has the auxiliary freedom since the auxiliary field $B$ does not propagate. It distinguishes the action (6.1) from other possible truncations of eq. (5.4). The action (6.1) gives rise to the spacetime torsion generated by gravitino, while its bosonic terms have the form

$$S_f = -\frac{1}{2} \int d^4 x \sqrt{-g} \hat{f}(\mathcal{R}) \quad \text{(6.2)}$$

Hence, eq. (6.1) can be considered as the locally $N = 1$ supersymmetric extension of the $f(\mathcal{R})$-type gravity (Sec. 3). However, in the context of supergravity, the ‘super-symmetrizable’ bosonic functions $\hat{f}(\mathcal{R})$ are very restrictive (see Secs. 9 and 10).

The superfield action (6.1) is classically equivalent to

$$S_V = \int d^4 x d^2 \theta \mathcal{E} [Z \mathcal{R} - V(Z)] + \text{H.c.} \quad \text{(6.3)}$$

with the covariantly chiral superfield $Z$ as the Lagrange multiplier superfield. Varying the action (6.3) with respect to $Z$ gives back the original action (6.1) provided that

$$F(\mathcal{R}) = Z \mathcal{R} - V(Z(\mathcal{R})) \quad \text{(6.4)}$$

where the function $Z(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(Z) \quad \text{(6.5)}$$

Equations (6.4) and (6.5) define the superfield Legendre transform, and imply

$$F'(\mathcal{R}) = Z(\mathcal{R}) \quad \text{and} \quad F''(\mathcal{R}) = Z'(\mathcal{R}) = \frac{1}{V''(Z(\mathcal{R}))} \quad \text{(6.6)}$$

where $V'' = d^2 V/dZ^2$. The second formula (6.6) is the duality relation between the supergravitational function $F$ and the chiral superpotential $V$.

A supersymmetric (local) Weyl transform of the acton (6.3) can be done entirely in superspace. In terms of the field components, the super-Weyl transform amounts to
a Weyl transform, a chiral rotation and a (superconformal) $S$-supersymmetry transformation [54]. The chiral density superfield $\mathcal{E}$ appears to be the chiral compensator of the super-Weyl transformations,

$$\mathcal{E} \rightarrow e^{3\Phi} \mathcal{E}$$

(6.7)

whose parameter $\Phi$ is an arbitrary covariantly chiral superfield, $\nabla^{\alpha} \Phi = 0$. Under the transformation (6.7) the covariantly chiral superfield $\mathcal{R}$ transforms as

$$\mathcal{R} \rightarrow e^{-2\Phi} \left( \mathcal{R} - \frac{1}{3} \nabla^2 \right) e^{\Phi}$$

(6.8)

The super-Weyl chiral superfield parameter $\Phi$ can be traded for the chiral Lagrange multiplier $Z$ by using a generic gauge condition

$$Z = Z(\Phi)$$

(6.9)

where $Z(\Phi)$ is a holomorphic function of $\Phi$. It results in the action

$$S_\Phi = \int d^4xd^2\theta \ E^{-1} e^{\Phi+\Phi} \left[ Z(\Phi) + \text{H.c.} \right] - \int d^4xd^2\theta \ e^{3\Phi} V(Z(\Phi)) + \text{H.c.}$$

(6.10)

Equation (6.10) has the standard form of the action of a chiral matter superfield coupled to supergravity,

$$S[\Phi, \bar{\Phi}] = \int d^4xd^2\theta \ E^{-1} \Omega(\Phi, \bar{\Phi}) + \left[ \int d^4xd^2\theta \ E P(\Phi) + \text{H.c.} \right]$$

(6.11)

in terms of the non-chiral potential $\Omega(\Phi, \bar{\Phi})$ and the chiral superpotential $P(\Phi)$. In our case (6.10) we find

$$\Omega(\Phi, \bar{\Phi}) = e^{\Phi+\Phi} \left[ Z(\Phi) + \bar{Z}(\bar{\Phi}) \right], \quad P(\Phi) = -e^{3\Phi} V(Z(\Phi))$$

(6.12)

The Kähler potential $K(\Phi, \bar{\Phi})$ is given by

$$K = -3 \ln(-\frac{\Omega}{3}) \quad \text{or} \quad \Omega = -3e^{-K/3}$$

(6.13)

so that the action (6.11) is invariant under the supersymmetric (local) Kähler-Weyl transformations

$$K(\Phi, \bar{\Phi}) \rightarrow K(\Phi, \bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}), \quad P(\Phi) \rightarrow -e^{-\Lambda(\Phi)} P(\Phi)$$

(6.14)

with the chiral superfield parameter $\Lambda(\Phi)$. It follows that

$$\mathcal{E} \rightarrow e^{\Lambda(\Phi)} \mathcal{E}$$

(6.15)

The scalar potential in terms of the usual fields is given by the standard formula [55]

$$V(\phi, \bar{\phi}) = e^K \left\{ \left| \frac{\partial P}{\partial \Phi} + \frac{\partial K}{\partial \Phi} \right|^2 - 3 |P|^2 \right\}$$

(6.16)
where all the superfields are restricted to their leading field components, \( \Phi = \phi(x) \), and we have introduced the notation

\[
\left| \frac{\partial P}{\partial \Phi} + \frac{\partial K}{\partial \Phi} \right|^2 \equiv |D_\Phi P|^2 = D_\Phi P (K^{-1}) \bar{D}_\Phi \bar{P}
\]  

(6.17)

with \( K_{\Phi \bar{\Phi}} = \partial^2 K/\partial \Phi \partial \bar{\Phi} \). Equation (6.16) can be simplified by making use of the Kähler-Weyl invariance (6.14) that allows one to choose a gauge

\[ P = 1 \]  

(6.18)

It is equivalent to the well known fact that the scalar potential (6.16) is actually governed by the single (Kähler-Weyl-invariant) potential

\[ G(\Phi, \bar{\Phi}) = \Omega + \ln |P|^2 \]  

(6.19)

In our case (6.12) we find

\[ G = e^{\Phi + \bar{\Phi}} \left[ \mathcal{Z}(\Phi) + \mathcal{Z}(\bar{\Phi}) \right] + 3(\Phi + \bar{\Phi}) + \ln(V(\mathcal{Z}(\Phi))) + \ln(V(\mathcal{Z}(\bar{\Phi})) \]  

(6.20)

So let us choose a gauge by the condition

\[ 3\Phi + \ln(V(\mathcal{Z}(\Phi))) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = e^{-3\Phi} \]  

(6.21)

that is equivalent to eq. (6.18). Then the G-potential (6.20) gets simplified to

\[ G = e^{\Phi + \bar{\Phi}} \left[ \mathcal{Z}(\Phi) + \mathcal{Z}(\bar{\Phi}) \right] \]  

(6.22)

There is the correspondence between a holomorphic function \( F(R) \) in the supergravity action (6.1) and a holomorphic function \( \mathcal{Z}(\Phi) \) defining the scalar potential (6.16),

\[ V = e^G \left[ \left( \frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} \right)^{-1} \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}} - 3 \right] \]  

(6.23)

in the classically equivalent scalar-tensor supergravity. More simplifications are possible in a particular gauge and for a particular model — see Sec. 13.

To the end of this section, we comment on the standard way of the inflationary model building by a choice of \( K(\Phi, \bar{\Phi}) \) and \( P(\Phi) \) — see eg., refs. [56, 57] for a review.

The factor \( \exp(K/M_P^2) \) in the \( F \)-type scalar potential (6.16) of the chiral matter-coupled supergravity, in the case of the canonical Kähler potential, \( K \propto \Phi \bar{\Phi} \), results in the scalar potential \( V \propto \exp(|\Phi|^2 / M_P^2) \) that is too steep to support chaotic inflation. Actually, it also implies \( \eta \approx 1 \) or, equivalently, \( M_{\text{inflaton}}^2 \approx V_0/M_P^2 \approx H^2 \). It is known as the \( \eta \)-problem in supergravity [58].

As is clear from our discussion above, the \( \eta \)-problem is not really a supergravity problem, but it is the problem associated with the choice of the canonical Kähler potential for an inflaton superfield. The Kähler potential in supergravity is a (Kähler)
gauge-dependent quantity, and its quantum renormalization is not under control. Unlike the one-field inflationary models, a generic Kähler potential is a function of at least two real scalar fields, so it implies a nonvanishing curvature in the target space of the \textit{non-linear sigma-model} associated with the Kähler kinetic term. \(^3\) Hence, a generic Kähler potential cannot be brought to the canonical form by a field redefinition.

To solve the \(\eta\)-problem associated with the simplest (naive) choice of the Kähler potential, one may assume that the Kähler potential \(K\) possesses some shift symmetries (leading to its flat directions), and then choose inflaton in one such flat direction \([60]\). However, in order to get inflation that way, one also has to add (\textit{ad hoc}) the proper inflaton superpotential breaking the initially introduced shift symmetry, and then stabilize the inflationary trajectory with the help of yet another matter superfield.

The possible alternative is the \(D\)-term mechanism \([61]\), where the inflaton particle belongs to the matter \textit{gauge} sector and, as a result, inflation is highly sensitive to gauge charges \([61]\). This mechanism is not related to spacetime and gravity.

It is worth mentioning that in the (perturbative) superstring cosmology one gets the Kähler potential (see e.g., refs. \([62, 63]\))

\[
K \propto \log(\text{moduli polynomial})_{\text{CY}}
\]

\((6.24)\)

over a Calabi-Yau (CY) space in the type-IIB superstring compactification, thus avoiding the \(\eta\)-problem but leading to a plenty of choices (“embarrassment of riches”) in the String Landscape and the associated high unpredictability.

Finally, one still has to accomplish stability of a given inflationary model in supergravity against quantum corrections. Such corrections can easily spoil the flatness of the inflaton potential. The Kähler kinetic term is not protected against quantum corrections, because it is given by a full superspace integral (unlike the chiral superpotential term). The \(F(R)\) supergravity action \((6.1)\) is given by a \textit{chiral} superspace integral, so that it is protected against the quantum corrections given by full superspace integrals.

To conclude this section, we claim that an \(N = 1\) locally supersymmetric extension of \(f(R)\) gravity is possible. It is non-trivial because the auxiliary freedom has to be preserved. The new supergravity action \((6.1)\) is classically equivalent to the standard \(N = 1\) Poincaré supergravity coupled to a \textit{dynamical} chiral matter superfield, whose Kähler potential and the superpotential are dictated by a single \textit{holomorphic} function. Inflaton can be identified with the real scalar field component of that chiral matter superfield originating from the supervielbein, and thus has the geometrical origin.

The action \((6.1)\) has yet another natural extension in the chiral curved superspace due to the last equation \((5.2)\), namely,

\[
S_{\text{ext}} = \int d^4x d^2\theta \mathcal{E} F(R, W) + \text{H.c.}
\]

\((6.25)\)

where \(W_{\alpha\beta\gamma}\) is the \(N = 1\) covariantly-chiral Weyl superfield of the \(N = 1\) superspace supergravity \([16]\). In Superstring Theory the Weyl-tensor-dependence of the perturbative gravitational effective action is unambiguously determined by the superstring

\(^3\)See eg., ref. \([59]\) for more about the non-linear sigma-models.
scattering amplitudes or by the super-Weyl invariance of the corresponding non-linear sigma-model (see eg., ref. [59]). However, the action of the type (6.25) may only be generated from superstrings nonperturbatively.

A possible connection of $F(\mathcal{R})$ supergravity to the Loop Quantum Gravity was investigated in ref. [5].

7 No-scale $F(\mathcal{R})$ Supergravity

In this section investigate a possibility of spontaneous supersymmetry breaking without fine tuning by imposing the condition of the vanishing scalar potential. Those no-scale supergravities are the starting point of many phenomenological applications of supergravity to HEP and inflationary theory, including superstring theory applications — see eg., refs. [64, 65] and the references therein.

The no-scale supergravity arises by demanding the scalar potential (6.16) to vanish. It results in the vanishing cosmological constant without fine-tuning [66]. The no-scale supergravity potential $G$ has to obey the non-linear 2nd-order partial differential equation, which follows from eq. (6.23),

$$3 \frac{\partial^2 G}{\partial \Phi \partial \bar{\Phi}} = \frac{\partial G}{\partial \Phi} \frac{\partial G}{\partial \bar{\Phi}}$$  \hspace{1cm} (7.1)

A gravitino mass $m_{3/2}$ is given by the vacuum expectation value [49]

$$m_{3/2} = \langle e^{G/2} \rangle$$  \hspace{1cm} (7.2)

The well known exact solution to eq. (7.1) is given by

$$G = -3 \log(\Phi + \bar{\Phi}) + \text{const}.$$  \hspace{1cm} (7.3)

In the recent literature the no-scale solution (7.3) is usually modified by other terms, in order to describe the universe with a positive cosmological constant — see e.g., the KKLT mechanism [67].

To appreciate the difference between the standard no-scale supergravity solution and our ‘modified’ supergravity, it is worth noticing that demanding eq. (7.1) gives rise to the first-order non-linear partial differential equation

$$3 \left( e^\Phi X' + e^{\bar{\Phi}} \bar{X}' \right) = \left| e^\Phi X' + e^{\bar{\Phi}} \bar{X}' \right|^2$$  \hspace{1cm} (7.4)

where we have introduced the notation

$$Z(\Phi) = e^{-\Phi} X(\Phi), \quad X' = \frac{dX}{d\Phi}$$  \hspace{1cm} (7.5)

in order to get the differential equation in its most symmetric and concise form.
Accordingly, the gravitino mass (7.2) is given by

\[ m_{3/2} = \left\langle \exp \frac{1}{2} \left( e^\Phi X + e^\Phi \bar{X} \right) \right\rangle \quad (7.6) \]

We are not aware of any non-trivial holomorphic exact solution to eq. (7.4). However, should it obey a holomorphic differential equation of the form

\[ X' = e^\Phi g(X, \Phi) \quad (7.7) \]

with a holomorphic function \( g(X, \Phi) \), eq. (7.4) gives rise to the functional equation

\[ 3 (g + \bar{g}) = \left| e^\Phi g + \bar{X} \right|^2 \quad (7.8) \]

Being restricted to the real variables \( \Phi = \bar{\Phi} \equiv y \) and \( X = \bar{X} \equiv x \), eq. (7.4) reads

\[ 6x' = e^y(x' + x)^2, \quad \text{where} \quad x' = \frac{dx}{dy} \quad (7.9) \]

This equation can be integrated after a change of variables,

\[ y = \int^{u} \frac{d\xi}{3 \pm \sqrt{3(3 - 2\xi)}} = \mp \sqrt{1 - \frac{2}{3}u} + \ln \left( \sqrt{3(3 - 2u)} + 3 \right) + C. \quad (7.10) \]

It follows

\[ y = \frac{\int^{u} \frac{d\xi}{3 \pm \sqrt{3(3 - 2\xi)}}}{3 \pm \sqrt{3(3 - 2\xi)}} = \mp \sqrt{1 - \frac{2}{3}u} + \ln \left( \sqrt{3(3 - 2u)} + 3 \right) + C. \quad (7.12) \]

### 8 Fields from Superfields in \( F(\mathcal{R}) \) Supergravity

For simplicity, now we set all fermionic fields to zero, and keep only bosonic field components of the superfields. It greatly simplifies all equations but makes supersymmetry to be manifestly broken. Of course, SUSY is restored after adding back all the fermionic terms.

Applying the standard superspace chiral density formula [48, 49, 50]

\[ \int d^4x d^2\theta \mathcal{E} \mathcal{L} = \int d^4x e \left\{ \mathcal{L}_{\text{last}} + B \mathcal{L}_{\text{first}} \right\} \quad (8.1) \]

to the action (6.1) yields its bosonic part in the form

\[ (-g)^{-1/2}L_{\text{bos}} \equiv f(R, \bar{R}; X, \bar{X}) = F'(\bar{X}) \left[ \frac{1}{2} R_* + 4\bar{X}X \right] + 3XF(\bar{X}) + \text{H.c.} \quad (8.2) \]
where the primes denote differentiation with respect to the given argument. We have used the notation

\[
X = \frac{1}{3} B \quad \text{and} \quad R_* = R + \frac{i}{2 \varepsilon} e_{abcd} R_{abcd} \equiv R + i \tilde{R}
\] (8.3)

The \( \tilde{R} \) does not vanish in \( F(\mathcal{R}) \) supergravity, and it represents the pseudo-scalar superpartner of the real scalaron field in our construction.

Varying eq. (8.2) with respect to the auxiliary fields \( X \) and \( \bar{X} \),

\[
\frac{\partial L_{\text{bos}}}{\partial X} = \frac{\partial L_{\text{bos}}}{\partial \bar{X}} = 0
\] (8.4)

gives rise to the algebraic equations on the auxiliary fields,

\[
3F + X(4F' + 7F') + 4\bar{X}XF'' + \frac{1}{3} F''R_* = 0
\] (8.5)

and its conjugate

\[
3\bar{F} + \bar{X}(4\bar{F}' + 7\bar{F}') + 4\bar{X}\bar{F}'' + \frac{1}{3} \bar{F}''\bar{R}_* = 0
\] (8.6)

where \( F = F(X) \) and \( \bar{F} = \bar{F}(\bar{X}) \). The algebraic equations (8.5) and (8.6) cannot be explicitly solved for \( X \) in a generic \( F(\mathcal{R}) \) supergravity.

To recover the standard (pure) supergravity in our approach, let us consider the simple special case when

\[
F'' = 0 \quad \text{or, equivalently,} \quad F(\mathcal{R}) = f_0 - \frac{1}{2} f_1 \mathcal{R}
\] (8.7)

with some complex constants \( f_0 \) and \( f_1 \), where \( \text{Re} f_1 > 0 \). Then eq. (8.5) is easily solved as

\[
\bar{X} = \frac{3 f_0}{5 (\text{Re} f_1)}
\] (8.8)

Substituting this solution back into the Lagrangian (8.2) yields

\[
L = -\frac{1}{3} (\text{Re} f_1) R + \frac{9 |f_0|^2}{5 (\text{Re} f_1)} \equiv -\frac{1}{2} M_{\text{Pl}}^2 R - \Lambda
\] (8.9)

where we have introduced the reduced Planck mass \( M_{\text{Pl}} \), and the cosmological constant \( \Lambda \) as

\[
\text{Re} f_1 = \frac{3}{2} M_{\text{Pl}}^2 \quad \text{and} \quad \Lambda = -\frac{6 |f_0|^2}{5 M_{\text{Pl}}^2}
\] (8.10)

It is the standard pure supergravity with a \textit{negative} cosmological constant [48, 49, 50].
9  **Generic $\mathcal{R}^2$ supergravity, and AdS Bound**

The simplest non-trivial $F(\mathcal{R})$ supergravity is obtained by choosing $F'' = \text{const.} \neq 0$ that leads to the $\mathcal{R}^2$-supergravity defined by a generic quadratic polynomial in terms of the scalar supercurvature [10].

Let us recall that the stability conditions in $f(\mathcal{R})$-gravity are given by eqs. (3.4) in the notation (3.1). In the notation (8.2) used here, i.e. when $f(\mathcal{R}) = -\frac{1}{2} M_{Pl}^2 \tilde{f}(\mathcal{R})$, one gets the opposite signs,

\[ f'(\mathcal{R}) < 0 \]  \hspace{1cm} (9.1)  

and

\[ f''(\mathcal{R}) > 0 \]  \hspace{1cm} (9.2)

The first (classical stability) condition (9.1) is related to the sign factor in front of the Einstein-Hilbert term (linear in $\mathcal{R}$) in the $f(\mathcal{R})$-gravity action, and it ensures that graviton is not a ghost. The second (quantum stability) condition (9.2) guarantees that scalaron is not a tachyon.

Being interested in the inflaton (scalaron) part of the bosonic $f(\mathcal{R})$-gravity action that follows from eq. (8.2), we set gravitino to zero and the scalar $X$ to be real, which also implies the real $\mathcal{R}$ or $\mathcal{R}^*$.

In this Section we investigate a generic quadratically generated Ansatz (with $F'' = \text{const.} \neq 0$) that leads to the simplest non-trivial toy-model of $F(\mathcal{R})$ supergravity with the master function

\[ F(\mathcal{R}) = f_0 - \frac{1}{2} f_1 \mathcal{R} + \frac{1}{2} f_2 \mathcal{R}^2 \]  \hspace{1cm} (9.4)

having three coupling constants $f_0$, $f_1$, and $f_2$. We take all of them to be real, since we ignore this potential source of $CP$-violation here (see, however, the Outlook in Sec. 20). As regards the mass dimensions of the quantities introduced, we have

\[ [F] = [f_0] = 3, \quad [\mathcal{R}] = [f_1] = 2, \quad \text{and} \quad [\mathcal{R}] = [f_2] = 1 \]  \hspace{1cm} (9.5)

The bosonic Lagrangian (8.2) with the function (9.4) reads

\[ (-g)^{-1/2} L_{bos} = 11 f_2 X^3 - 7 f_1 X^2 + \left( \frac{2}{3} f_2 \mathcal{R} + 6 f_0 \right) X - \frac{1}{3} f_1 \mathcal{R} \]  \hspace{1cm} (9.6)

Hence, the auxiliary field equation (8.4) takes the form of a quadratic equation,

\[ \frac{43}{2} f_2 X^2 - 7 f_1 X + \frac{1}{3} R f_2 + 3 f_0 = 0 \]  \hspace{1cm} (9.7)
whose solution is given by

\[ X_\pm = \frac{7}{3 \cdot 11} \sqrt[2]{\frac{f_1}{f_2}} \pm \sqrt{\frac{2 \cdot 11}{7^2}} (R_{\text{max}} - R) \]  

(9.8)

where we have introduced the maximal scalar curvature

\[ R_{\text{max}} = \frac{7^2}{2 \cdot 11} \frac{f_1^2}{f_2^2} - 3^2 \frac{f_0}{f_2} \]  

(9.9)

Equation (9.8) obviously implies the automatic bound on the scalar curvature (from one side only). In our notation, it corresponds to the (AdS) bound on the scalar curvature from above,

\[ R < R_{\text{max}} \]  

(9.10)

The existence of the built-in maximal (upper) scalar curvature (or the AdS bound) is a nice bonus of our construction. It is similar to the factor \( \sqrt{1 - v^2/c^2} \) in Special Relativity. Yet another close analogy comes from the Born-Infeld non-linear extension of Maxwell electrodynamics, whose (dual) Hamiltonian is proportional to \[ (1 - \sqrt{1 - \frac{E^2}{E_{\text{max}}^2} - \frac{H^2}{H_{\text{max}}^2} + (\vec{E} \times \vec{H})^2/\frac{E_{\text{max}}^2}{H_{\text{max}}^2}}) \]  

(9.11)

in terms of the electric and magnetic fields \( \vec{E} \) and \( \vec{H} \), respectively, with their maximal values. For instance, in String Theory one has \( E_{\text{max}} = H_{\text{max}} = (2\pi \alpha')^{-1} [59]. \)

Substituting the solution (9.8) back into eq. (9.6) yields the corresponding \( f(R) \)-gravity Lagrangian

\[ f_\pm(R) = \frac{2 \cdot 7 f_0 f_1}{11} - \frac{2 \cdot 7^3}{3^3 \cdot 11^2} f_2^3 \]

\[ - \frac{19}{3^2 \cdot 11} f_1 R \mp \sqrt{\frac{2}{11}} \left( \frac{2^2}{3^3} f_2 \right) (R_{\text{max}} - R)^{3/2} \]  

(9.12)

Expanding eq. (9.12) into power series of \( R \) yields

\[ f_\pm(R) = -\Lambda_\pm - a_\pm R + b_\pm R^2 + O(R^3) \]  

(9.13)

whose coefficients are given by

\[ \Lambda_\pm = \frac{2 \cdot 7}{3^2 \cdot 11} f_1 \left( R_{\text{max}} - \frac{7^2}{2 \cdot 3 \cdot 11} f_2^2 \right) \pm \sqrt{\frac{2}{11}} \left( \frac{2^2}{3^3} f_2 \right) R_{\text{max}}^{3/2} \]  

(9.14)

\[ a_\pm = \frac{19}{3^2 \cdot 11} f_1 \mp \sqrt{\frac{2}{11}} R_{\text{max}} \left( \frac{2}{3^2} f_2 \right) \]  

(9.15)
and
\[ b_\pm = \mp \sqrt{\frac{2}{11 R_{\text{max}}} \left( \frac{f_2}{2 \cdot 3^2} \right)} \quad (9.16) \]

Those equations greatly simplify when \( f_0 = 0 \). One finds [7, 10]
\[ f_\pm^{(0)}(R) = -\frac{5 \cdot 17}{2 \cdot 3^2 \cdot 11} M_{\text{Pl}}^2 R + \frac{2 \cdot 7}{3^2 \cdot 11} M_{\text{Pl}}^2 (R - R_{\text{max}}) \left[ 1 \pm \sqrt{1 - R/R_{\text{max}}} \right] \quad (9.17) \]
where we have chosen
\[ f_1 = \frac{3}{2} M_{\text{Pl}}^2 \quad (9.18) \]
in order to get the standard normalization of the Einstein-Hilbert term that is linear in \( R \). Then, in the limit \( R_{\text{max}} \to +\infty \), both functions \( f_\pm^{(0)}(R) \) reproduce General Relativity. In another limit \( R \to 0 \), one finds a \textit{vanishing} or \textit{positive} cosmological constant,
\[ \Lambda^{(0)}_\pm = 0 \quad \text{and} \quad \Lambda^{(0)}_+ = \frac{2^2 \cdot 7}{3^2 \cdot 11} M_{\text{Pl}}^2 R_{\text{max}} \quad (9.19) \]

The stability conditions are given by eqs. (9.1), (9.2) and (9.3), while the 3rd condition implies the 2nd one. In our case (9.12) we have
\[ f_\pm'(R) = -\frac{19}{3^2 \cdot 11} f_1 \pm \sqrt{\frac{2}{11} \left( \frac{2}{3^2} f_2 \right)} \sqrt{R_{\text{max}} - R} < 0 \quad (9.20) \]
and
\[ f_\pm''(R) = \mp \left( \frac{f_2}{3^2} \right) \sqrt{\frac{2}{11(2 R_{\text{max}} - R)}} > 0 \quad (9.21) \]
while eqs. (9.3), (9.4) and (9.8) yield
\[ \pm \sqrt{\frac{2 \cdot 11}{7^2} (R_{\text{max}} - R)} < \frac{19}{2 \cdot 7} f_1 \frac{f_2}{f_2} \quad (9.22) \]

It follows from eq. (9.21) that
\[ f_2^{(+)} < 0 \quad \text{and} \quad f_2^{(-)} > 0 \quad (9.23) \]
Then the stability condition (9.2) is obeyed for any value of \( R \).

As regards the \((-\)-case), there are two possibilities depending upon the sign of \( f_1 \). Should \( f_1 \) be \textit{positive}, all the remaining stability conditions are automatically satisfied, i.e. in the case of both \( f_2^{(-)} > 0 \) and \( f_1^{(-)} > 0 \).

Should \( f_1 \) be \textit{negative}, \( f_1^{(-)} < 0 \), we find that the remaining stability conditions (9.20) and (9.22) are \textit{the same}, as they should, while they are both given by
\[ R < R_{\text{max}} - \frac{19^2}{23 \cdot 11} f_2^2 \frac{f_1^2}{f_2^2} = -\frac{3 \cdot 5}{23 \cdot 11} f_2^2 \frac{f_1^2}{f_2^2} = \frac{3^2 f_0}{f_2} \equiv R_{\text{ins}}^{\text{max}} \quad (9.24) \]
As regards the (+)-case, eq. (9.22) implies that \( f_1 \) should be negative, \( f_1 < 0 \), whereas then eqs. (9.20) and (9.22) result in the same condition (9.24) again.

Since \( R_{\text{max}}^{\text{ins}} < R_{\text{max}} \), our results imply that the instability happens before \( R \) reaches \( R_{\text{max}} \) in all cases with negative \( f_1 \).

As regards the particularly simple case (9.17), the stability conditions allow us to choose the lower sign only.

A different example arises with a negative \( f_1 \). When choosing the lower sign (ie. a positive \( f_2 \)) for definiteness, we find

\[
\mathcal{F}_-(R) = - \frac{2 \cdot 7}{11} f_0 \left| \frac{f_1}{f_2} \right| + \frac{2 \cdot 7^3}{3^2 \cdot 11^2} \left| \frac{f_1^3}{f_2^2} \right|
+ \frac{19}{3^2 \cdot 11} |f_1| R + \sqrt{\frac{2}{11}} \left( \frac{2^2}{3^3} f_2 \right) (R_{\text{max}} - R)^{3/2}
\]

(9.25)

Demanding the standard normalization of the Einstein-Hilbert term in this case implies

\[
R_{\text{max}} = \frac{3^4 \cdot 11}{2^3 f_2^2} \left( \frac{M_{\text{Pl}}^2}{2} + \frac{19}{3^2 \cdot 11} |f_1| \right)^2
\]

(9.26)

where we have used eq. (9.15). It is easy to verify by using eq. (9.14) that the cosmological constant is always negative in this case, and the instability bound (9.24) is given by

\[
R_{\text{max}}^{\text{ins}} = \frac{3^4 \cdot 11 M_{\text{Pl}}^2}{2^3 f_2^2} \left( \frac{M_{\text{Pl}}^2}{2^2} + \frac{19}{3^2 \cdot 11} |f_1| \right) < R_{\text{max}}
\]

(9.27)

The \( \mathcal{F}_-(R) \) function of eq. (9.12) can be rewritten to the form

\[
f(R) = \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2} - \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\text{max}} - \frac{19}{3^2 \cdot 11} f_1 R + f_2 \sqrt{\frac{2^5}{3^6 \cdot 11}} (R_{\text{max}} - R)^{3/2}
\]

(9.28)

where we have used eq. (9.9). There are three physically different regimes:

(i) the high-curvature regime, \( R < 0 \) and \( |R| \gg R_{\text{max}} \). Then eq. (9.28) implies

\[
f(R) \approx -\Lambda_h - a_h R + c_h |R|^{3/2}
\]

(9.29)

whose coefficients are given by

\[
\Lambda_h = \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\text{max}} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2}
\]

\[
a_h = \frac{19}{3^2 \cdot 11} f_1
\]

\[
c_h = \sqrt{\frac{2}{11}} \left( \frac{2^2}{3^3} f_2 \right)
\]

(9.30)
(ii) the low-curvature regime, $|R/R_{\text{max}}| \ll 1$. Then eq. (9.28) implies

$$f(R) \approx -\Lambda_l - a_l R,$$

(9.31)

whose coefficients are given by

$$\Lambda_l = \Lambda_h - \left(\frac{2}{11}\right)^{3/2} \frac{2^2 f_2}{f_2^3},$$

$$a_l = a_h + \left(\frac{2}{11}\right)^{3/2} \frac{2^2 f_2}{f_2^3} = a_\Lambda = \frac{M_{\text{Pl}}^2}{2},$$

(9.32)

where we have used eq. (9.15).

(iii) the near-the-bound regime (assuming that no instability happens before it), $R = R_{\text{max}} + \delta R$, $\delta R < 0$, and $|\delta R/R_{\text{max}}| \ll 1$. Then eq. (9.28) implies

$$f(R) \approx -\Lambda_b + a_b |\delta R| + c_b |\delta R|^{3/2}$$

(9.33)

whose coefficients are

$$\Lambda_b = \left(\frac{1}{3}\right) f_l R_{\text{max}} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2},$$

$$a_b = a_h,$$

$$c_b = \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3 f_2}\right),$$

(9.34)

The cosmological dynamics may be either directly derived from the gravitational equations of motion in the $f(R)$-gravity with a given function $f(R)$, or just read off from the form of the corresponding scalar potential of a scalaron (see below). For instance, as was demonstrated in ref. [7] for the special case $f_0 = 0$, a cosmological expansion is possible in the regime (i) towards the regime (ii), and then, perhaps, to the regime (iii) unless an instability occurs.

However, one should be careful since our toy-model (9.4) does not pretend to be viable in the low-curvature regime, eg., for the present Universe. Nevertheless, if one wants to give it some physical meaning there, by identifying it with General Relativity, then one should also fine-tune the cosmological constant $\Lambda_l$ in eq. (9.32) to be “small” and positive. We find that it amounts to

$$R_{\text{max}} \approx \frac{3^4 \cdot 7^2 \cdot 11}{25 \cdot 19^2} \frac{M_{\text{Pl}}^4}{f_2^2} = R_{\Lambda=0}$$

(9.35)

with the actual value of $R_{\text{max}}$ to be “slightly” above of that bound, $R_{\text{max}} > R_{\Lambda=0}$. It is also possible to have the vanishing cosmological constant, $\Lambda_l = 0$, when choosing $R_{\text{max}} = R_{\Lambda=0}$. It is worth mentioning that it relates the values of $R_{\text{max}}$ and $f_2$. 


The particular $\mathcal{R}^2$-supergravity model (with $f_0 = 0$) was introduced in ref. [7] in an attempt to get viable embedding of the Starobinsky model into $F(\mathcal{R})$-supergravity. However, it failed because, as was found in ref. [7], the higher-order curvature terms cannot be ignored in eq. (9.17), i.e., the $R^n$-terms with $n \geq 3$ are not small enough against the $R^2$-term. In fact, the possibility of destabilizing the Starobinsky inflationary scenario by the terms with higher powers of the scalar curvature, in the context of $f(\mathcal{R})$ gravity, was noticed earlier in refs. [68, 69]. The most general Ansatz (9.4), which is mostly quadratic in the supercurvature, does not help for that purpose either.

For example, the full $f(\mathcal{R})$-gravity function $f_-(\mathcal{R})$ in eq. (9.17), which we derived from our $\mathcal{R}^2$-supergravity, gives rise to the inflaton scalar potential

$$V(y) = V_0 (11 e^y + 3)(e^{-y} - 1)^2$$

where $V_0 = (3^3/2^6)M_{Pl}^4/f_2^2$. The corresponding inflationary parameters

$$\varepsilon(y) = \frac{1}{3} \left[ \frac{e^y (11 + 11e^{-y} + 6e^{-2y})}{(11e^y + 3)(e^{-y} - 1)} \right]^2 \geq \frac{1}{3}$$

and

$$\eta(y) = \frac{2}{3} \frac{(11e^y + 5e^{-y} + 12e^{-2y})}{(11e^y + 3)(e^{-y} - 1)^2} \geq \frac{2}{3}$$

are not small enough for matching the WMAP observational data. A solution to this problem is given in the next section.

### 10 Chaotic inflation in $F(\mathcal{R})$ Supergravity

Let us further generalize our Ansatz and consider a new $F(\mathcal{R})$ function having the cubic form

$$F(\mathcal{R}) = -\frac{1}{2}f_1 \mathcal{R} + \frac{1}{2}f_2 \mathcal{R}^2 - \frac{1}{6}f_3 \mathcal{R}^3$$

whose real (positive) coupling constants $f_{1,2,3}$ are of (mass) dimension 2, 1 and 0, respectively. Our conditions on the coefficients are

$$f_3 \gg 1, \quad f_2^2 \gg f_1$$

The first condition is needed to have inflation at the curvatures much less than $M_{Pl}^2$ (and to meet observations), while the second condition is needed to have the scalaron (inflaton) mass be much less than $M_{Pl}$, in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time.

The bosonic action is given by eq. (8.2). For a real scalaron it reduces to

$$L/\sqrt{-g} = 2F' \left[ \frac{1}{3} \mathcal{R} + 4X^2 \right] + 6XF$$

25
so that the real auxiliary field is a solution to the algebraic equation

\[ 3F + 11F'X + F'' \left[ \frac{1}{3}R + 4X^2 \right] = 0 \]  

(10.4)

Stability of the bosonic embedding in supergravity requires \( F'(X) < 0 \) (Sec. 9). In the case (10.1) it gives rise to the condition \( f_2^2 < f_1 f_3 \). For simplicity here, we will assume a stronger condition,

\[ f_2^2 \ll f_1 f_3 \]  

(10.5)

Then the second term on the right-hand-side of eq. (10.1) will not affect inflation, as is shown below. However, it will be quite important for reheating (see Secs. 13 and 14).

Equation (10.3) with the Ansatz (10.1) reads

\[ L = -5f_3X^4 + 11f_2X^3 - (7f_1 + \frac{1}{3}f_3R)X^2 + \frac{2}{3}f_2RX - \frac{1}{3}f_1R \]  

(10.6)

and gives rise to a cubic equation on \( X \),

\[ X^3 - \left( \frac{33f_2}{20f_3} \right) X^2 + \left( \frac{7f_1}{10f_3} + \frac{1}{30}R \right) X - \frac{f_2}{30f_3}R = 0 \]  

(10.7)

We find three consecutive (overlapping) regimes.

• The high curvature regime including inflation is given by

\[ \delta R < 0 \text{ and } \frac{|\delta R|}{R_0} \gg \left( \frac{f_2}{f_1 f_3} \right)^{1/3} \]  

(10.8)

where we have introduced the notation \( R_0 = 21f_1/f_3 > 0 \) and \( \delta R = R + R_0 \). With our sign conventions we have \( \dot{R} < 0 \) during the de Sitter and matter dominated stages. In the regime (10.8) the \( f_2 \)-dependent terms in eqs. (10.6) and (10.7) can be neglected, and we get

\[ X^2 = -\frac{1}{30} \delta R \]  

(10.9)

and

\[ L = -\frac{f_1}{3}R + \frac{f_3}{180} (R + R_0)^2 \]  

(10.10)

It closely reproduces the Starobinsky inflationary model (Sec. 2) since inflation occurs at \( |R| \gg R_0 \). In particular, we can identify

\[ f_3 = \frac{15M_{P1}^2}{M_{\text{inf}}^2} \]  

(10.11)

It is worth mentioning that we cannot simply set \( f_2 = 0 \) in eq. (10.1) because it would imply \( X = 0 \) and \( L = -\frac{f_1}{3}R \) for \( \delta R > 0 \). As a result of that the scalar degree of freedom would disappear that would lead to the breaking of a regular Cauchy evolution. Therefore, the second term in eq. (10.1) is needed to remove that degeneracy.
The intermediate (post-inflationary) regime is given by
\[ \frac{\delta R}{R_0} \ll 1 \] (10.12)

In this case \( X \) is given by a root of the cubic equation
\[ 30X^3 + (\delta R)X + \frac{f_2 R_0}{f_3} = 0 \] (10.13)

It also implies that the 2nd term in eq. (10.7) is always small. Equation (10.13) reduces to eq. (10.9) under the conditions (10.8).

The low-curvature regime (up to \( R = 0 \)) is given by
\[ \delta R > 0 \quad \text{and} \quad \frac{\delta R}{R_0} \gg \left( \frac{f_2}{f_1 f_3} \right)^{1/3} \] (10.14)

It yields
\[ X = \frac{f_2 R}{f_3(R + R_0)} \] (10.15)

and
\[ L = -\frac{f_1}{3} R + \frac{f_2^2 R^2}{3 f_3(R + R_0)} \] (10.16)

It is now clear that \( f_1 \) should be equal to \( 3M_{Pl}^2/2 \) in order to obtain the correctly normalized Einstein gravity at \( |R| \ll R_0 \). In this regime the scalaron mass squared is given by
\[ \frac{1}{3 |f''(R)|} = \frac{f_3 R_0 M_{Pl}^2}{4 f_2^2} = \frac{21 f_1}{4 f_2} M_{Pl}^2 = \frac{63 M_{Pl}^4}{8 f_2^2} \] (10.17)

in agreement with the case of the absence of the \( R^3 \) term, studied in the previous section. The scalaron mass squared (10.17) is much less than \( M_{Pl}^2 \) indeed, due to the second inequality in eq. (10.2), but it is much more than one at the end of inflation (\( \sim M^2 \)).

It is worth noticing that the corrections to the Einstein action in eqs. (10.10) and (10.16) are of the same order (and small) at the borders of the intermediate region (10.12).

The roots of the cubic equation (10.7) are given by the textbook (Cardano) formula [70], though that formula is not very illuminating in a generic case. The Cardano formula greatly simplifies in the most interesting (high curvature) regime where inflation takes place, and the Cardano discriminant is
\[ D \approx \left( \frac{R}{90} \right)^3 < 0 \] (10.18)
It implies that all three roots are real and unequal. The Cardano formula yields the roots

\[ X_{1,2,3} \approx \frac{2}{3} \sqrt{-\frac{R}{10}} \cos \left( \frac{27}{4f_3 \sqrt{-10R/f_2^2}} + C_{1,2,3} \right) + \frac{11f_2}{20f_3} \]  

where the constant \( C_{1,2,3} \) takes the values \((\pi/6, 5\pi/6, 3\pi/2)\).

As regards the leading terms, eqs. (10.6) and (10.19) result in the \((-R)^{3/2}\) correction to the \((R + R^2)\)-terms in the effective Lagrangian in the high-curvature regime \(|R| \gg f_2^2/f_3^2\). In order to verify that this correction does not change our results under the conditions (10.8), let us consider the \(f(R)\)-gravity model with

\[ \tilde{f}(R) = R - b(-R)^{3/2} - aR^2 \]  

whose parameters \(a > 0\) and \(b > 0\) are subject to the conditions \(a \gg 1\) and \(b/a^2 \ll 1\). It is easy to check that \(\tilde{f}'(R) > 0\) for \(R \in (-\infty, 0]\), as is needed for (classical) stability.

Any \(f(R)\) gravity model is classically equivalent to the scalar-tensor gravity with certain scalar potential (Sec. 3). The scalar potential can be calculated from a given function \(f(R)\) along the standard lines (Sec. 3). We find (in the high curvature regime)

\[ V(y) = \frac{1}{8a} (1 - e^{-y})^2 + \frac{b}{8\sqrt{2a}} e^{-2y} (e^y - 1)^{3/2} \]  

in terms of the inflaton field \(y\). The first term of this equation is the scalar potential associated with the pure \((R + R^2)\) model, and the 2nd term is the correction due to the \(R^{3/2}\)-term in eq. (10.20). It is now clear that for large positive \(y\) the vacuum energy in the first term dominates and drives inflation until the vacuum energy is compensated by the \(y\)-dependent terms near \(e^y = 1\).

It can be verified along the lines of ref. [44] that the formula for scalar perturbations remains the same as that for the model (2.3), i.e. \(\Delta_R^2 \approx N^2 M_{\text{inf}}^2 / (24\pi^2 M_{\text{Pl}}^2)\), where \(N\) is the number of e-folds from the end of inflation. So, to fit the observational data, one has to choose

\[ f_3 \approx 5N^2 / (8\pi^2 \Delta_R^2) \approx 6.5 \cdot 10^{10} (N_e/50)^2 \]  

Here the value of \(\Delta_R\) is taken from ref. [22] and the subscript \(\mathcal{R}\) has a different meaning from the rest of this review.

We conclude that the model (10.1) with a sufficiently small \(f_2\) obeying the conditions (10.2) and (10.5) gives a viable realization of the chaotic \((R + R^2)\)-type inflation in supergravity. The only significant difference with respect to the original \((R + R^2)\) inflationary model is the scalaron mass that becomes much larger than \(M\) in supergravity, soon after the end of inflation when \(\delta R\) becomes positive. It makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of \(\mathcal{R}\) may also be considered, instead of the limited Ansatz (10.1). The only necessary condition for embedding inflation is that \(f_3\) should
be anomalously large. When the curvature grows, the $R^3$-term should become important much earlier than the convergence radius of the whole series without that term. Of course, it means that viable inflation does not occur for any function $F(R)$ but only inside a small region of measure zero in the space of all those functions. However, the same is true for all known inflationary models, so the very existence of inflation has to be taken from the observational data, not from a pure thought.

The results of this section can be considered as the viable alternative to the earlier proposals [60, 61] for realization of chaotic inflation in supergravity. But inflation is not the only target of our construction. As is well known [17, 18, 71], the scalaron decays into pairs of particles and anti-particles of quantum matter fields, while its decay into gravitons is strongly suppressed [72]. It thus represents the universal mechanism of viable reheating after inflation and provides a transition to the subsequent hot radiation-dominated stage of the universe evolution. In its turn, it leads to the standard primordial nucleosynthesis (BBN) after. In $F(R)$ supergravity the scalaron has a pseudo-scalar superpartner that may be the source of a strong $CP$-violation and then, subsequently, lepto- and baryo-genesis that may lead to baryon (matter-antimatter) asymmetry [73, 74, 75, 76] — see Sec. 20 for more.

### 11 More about Inflationary Dynamics in our Model

The supersymmetric extension of the simplest $R^2$-type inflationary model in the previous section has some important improvements against the original Starobinsky’s model, because it is characterized by two mass scales of a scalar degree of freedom (scalaron): $M$ (associated with the inflationary era) and $m$ (associated with the pre-heating era). They correspond to two free real parameters $f_2$ and $f_3$ in our Ansatz (10.1). The allowed values of the masses $M$ and $m$ can be derived from the amplitude of the CMB temperature anisotropies. In the previous section the viability of our model was established only in certain limit of its parameter space. Here we show that our model is consistent with the joint observational constraints of the WMAP and the PLANCK in the regime where a sufficient amount of inflation (with the number of e-foldings larger than 50) is realized. We also find observational bounds on the parameter values. In the low-energy regime relevant to preheating, we derive the effective scalar potential in the presence of a pseudo-scalar field $\chi$ coupled to the inflaton (scalaron) field $\phi$ (the field $\chi$ was ignored in the previous section). This potential is employed for numerical analysis of the preheating stage after inflation. If $m$ is much larger than $M$, we find that there exists the preheating stage in which the field perturbations $\delta \chi$ and $\delta \phi$ rapidly grow by a broad parametric resonance by which the both field perturbations $\delta \chi$ and $\delta \phi$ are amplified (Sec. 14). The dynamics of reheating appears to be different from that in the original Starobinsky’s $f(R)$ model and, in fact, more efficient.

---

$^4$Compared to the earlier sections, we rescale $M$ by the factor of $\sqrt{6}$ here, in order to make it equal to the scalaron mass during inflation.
In order to recover the standard behaviour of General Relativity in the low-energy regime we require that $f_1 = 3M_{Pl}^2/2$. The mass squared of the scalar degree of freedom is given by $m^2 = 1/(3f''(R))$, where $f(R)$ is related to the Lagrangian $L(R)$ as $L(R) = -M_{Pl}^2 f(R)/2$. According to Sec. 10 in the limit $|R| \ll R_0$ we have

$$m^2 = \frac{21f_1M_{Pl}^2}{4f_2} = \frac{63M_{Pl}^4}{8f_2} \quad (11.1)$$

In the high-curvature regime the scalaron mass squared is given by

$$M^2 = \frac{15M_{Pl}^2}{f_3} \quad (11.2)$$

Hence, the constants $f_{1,2,3}$ can be expressed by using the three mass scales $M_{Pl}, m$, and $M$, as follows:

$$f_1 = \frac{3}{2}M_{pl}^2, \quad f_2 = \sqrt{\frac{63}{8} \frac{M_{Pl}^2}{m}}, \quad f_3 = \frac{15M_{Pl}^2}{M^2} \quad (11.3)$$

The conditions $f_2^2 < f_1 f_3$, $f_3 \gg 1$ and $f_2^2 \gg f_1$ of Sec. 10 translate into

$$m > \sqrt{\frac{7}{20}} M, \quad M \ll M_{Pl}, \quad m \ll M_{Pl} \quad (11.4)$$

respectively.

The high-energy regime (A) satisfies the condition $|R| \gg R_0$ with the flat FLRW background described by the line element $ds^2 = dt^2 - a^2(t)dx^2$. It is convenient to introduce the following dimensionless functions:

$$\alpha \equiv \frac{M^2}{mH}, \quad \beta \equiv \frac{M^2}{H^2} \quad (11.5)$$

and represent $R_0$ as $R_0 = 21M^2/10$. During inflation the functions (11.5) should satisfy the conditions $\alpha \ll 1$ and $\beta \ll 1$ (see below). In eq. (10.10) the term $f_3 R^2/180$ is the dominant contribution during inflation. Hence, we neglect the higher-order terms beyond that of the first (linear) order in $\alpha$ and $\beta$. Then the Lagrangian following from eq. (10.10) is given by

$$f(R) \simeq \frac{3}{10}R - \frac{R^2}{6M^2} - \frac{3\sqrt{105}}{100} \frac{(-R)^{3/2}}{m}. \quad (11.6)$$

We assume that the Lagrangian (11.6) is valid by the end of inflation.

In the flat FLRW spacetime the field equations of motion are

$$3\mathcal{F}H^2 = (f - R\mathcal{F})/2 - 3H\dot{\mathcal{F}}, \quad (11.7)$$

$$-2\mathcal{F}\dot{H} = \ddot{\mathcal{F}} - H\dot{\mathcal{F}}, \quad (11.8)$$
where $\mathcal{F} \equiv f'(R)$. It is useful to define the new slow-roll parameters as [34]

$$
\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\dot{\mathcal{F}}}{2H\mathcal{F}}, \quad \epsilon_3 \equiv \frac{\ddot{\mathcal{F}}}{H\mathcal{F}} \quad (11.9)
$$

which satisfy $|\epsilon_i| \ll 1 (i = 1, 2, 3)$. It follows from eq. (11.8) that

$$
\epsilon_1 = -\epsilon_2 (1 - \epsilon_3) \quad (11.10)
$$

In what follows we carry out the linear expansion in terms of the variables $\epsilon_i (i = 1, 2, 3)$, $\alpha$, $\beta$, and $s \equiv \dot{\bar{H}}/(\bar{H}\dot{\bar{H}})$.

For the Lagrangian (11.6) we have

$$
\mathcal{F} = \frac{4H^2}{M^2} \left(1 + \frac{27\sqrt{35}}{400} \alpha + \frac{3}{40} \beta - \frac{1}{2} \epsilon_1\right), \quad (11.11)
$$

$$
\dot{\mathcal{F}} = -\frac{8H^3}{M^2} \epsilon_1 \left(1 + \frac{27\sqrt{35}}{800} \alpha + \frac{1}{4} s\right) \quad (11.12)
$$

Then the variable $\epsilon_2$ is given by

$$
\epsilon_2 = -\epsilon_1 \left(1 - \frac{27\sqrt{35}}{800} \alpha - \frac{3}{40} \beta + \frac{1}{2} \epsilon_1 + \frac{1}{4} s\right) \quad (11.13)
$$

Comparing this with eq. (11.10) we obtain

$$
\epsilon_3 = -\frac{27\sqrt{35}}{800} \alpha - \frac{3}{40} \beta + \frac{1}{2} \epsilon_1 + \frac{1}{4} s \quad (11.14)
$$

Similarly, eq. (11.7) gives the following relations:

$$
\epsilon_1 = \frac{3\sqrt{35}}{200} \alpha + \frac{1}{20} \beta \quad (11.15)
$$

and

$$
\epsilon_2 = -\frac{3\sqrt{35}}{200} \alpha - \frac{1}{20} \beta \quad (11.16)
$$

Equation (11.15) is equivalent to

$$
\dot{H} = -\frac{3\sqrt{35}}{200} \frac{M^2}{m} \left(H + \frac{10m}{3\sqrt{35}}\right) \quad (11.17)
$$

This differential equation can be easily integrated. It yields

$$
H(t) = \left(H_i + \frac{10m}{3\sqrt{35}}\right) \exp\left[\frac{3\sqrt{35}}{200} \frac{M^2}{m} (t_i - t)\right] - \frac{10m}{3\sqrt{35}} \quad (11.18)
$$

where $H_i$ is the initial value of $H$ at $t = t_i$. So we find

$$
s = -\frac{3\sqrt{35}}{200} \alpha \quad (11.19)
$$
Substituting eqs. (11.15) and (11.19) into eq. (11.14) we obtain

$$\epsilon_3 = - \frac{3\sqrt{35}}{100} \alpha - \frac{1}{20} \beta$$

(11.20)

The end of inflation \((t = t_f)\) is identified by the condition \(\epsilon_1 = 1\). By using the solution (11.18), we have

$$t_i - t_f = \frac{200m}{3\sqrt{35}M^2} \ln \left( \frac{63M^2}{80m(3\sqrt{35}H_i + 10m)} \right)$$

$$\times \left[ 1 + \frac{800}{63} \left( \frac{m}{M} \right)^2 + \sqrt{1 + \frac{1600}{63} \left( \frac{m}{M} \right)^2} \right]$$

(11.21)

We define the number of e-foldings from the onset of inflation \((t = t_i)\) to the end of inflation \((t = t_f)\) as \(N(t_i) \equiv \int_{t_i}^{t_f} H dt\). From eqs. (11.18) and (11.21) we can express \(N(t_i)\) in terms of \(H_i, M,\) and \(m\). The number of e-foldings \(N\) corresponding to the time \(t\) can be derived by replacing \(H_i\) in the expression of \(N(t_i)\) for \(H\). It follows that

$$N = \frac{1}{126\alpha^2} \left[ 3\alpha(80\sqrt{35} - 21\alpha - \sqrt{7(63\alpha^2 + 1600\beta)}) \right]$$

$$- 400\beta(8\ln 2 + 3\ln 5) + 800\beta$$

$$\times \ln \left( \frac{\sqrt{7(63\alpha^2 + 800\beta)} + 21\alpha\sqrt{63\alpha^2 + 1600\beta}}{21\alpha + 2\sqrt{35}\beta} \right)$$

(11.22)

In the limit \(\alpha \to 0\) one has \(N \to 10/\beta - 1/2\), ie. \(\beta \to 20/(2N + 1)\). In this case the \(R^2/(6M^2)\) term in the Lagrangian (11.6) dominates over the dynamics of inflation, which corresponds to the Starobinsky’s \(f(R)\) model. In another limit \(\beta \to 0\) it follows that \(N \to 40\sqrt{35}/(21\alpha) - 1\), ie. \(\alpha \to 40\sqrt{35}/[21(N + 1)]\). Then we obtain the following bounds on \(\alpha\) and \(\beta\):

$$0 < \alpha < \frac{40\sqrt{35}}{21(N + 1)}, \quad 0 < \beta < \frac{20}{2N + 1}$$

(11.23)

In order to realize inflation with eg., \(N = 60\), the two variables need to be in the range \(0 < \alpha < 0.185\) and \(0 < \beta < 0.165\). For the number of e-foldings relevant to the CMB temperature anisotropies \((50 \lesssim N \lesssim 60)\) the slow-roll parameters given in eqs. (11.15), (11.16), (11.20) are much smaller than unity, so that the slow-roll approximation employed above is justified.

## 12 Facing Observational Tests

In this section we study more closely whether the \(f(R)\) model (11.6) satisfies the observational constraints of the CMB temperature anisotropies. The power spectra of
scalar and tensor perturbations generated during inflation based on \( f(R) \) theories were calculated in ref. [44].

The scalar power spectrum of the curvature perturbation is given by [34]

\[
P_s = \frac{1}{24\pi^2 F} \left( \frac{H}{M_{pl}} \right)^2 \frac{1}{\xi^2}
\]

(12.1)

Using eqs. (11.11), (11.15), and (11.16), it follows that

\[
P_s \simeq \frac{1250}{3\pi^2} \left( \frac{M}{M_{pl}} \right)^2 \left( 3\sqrt{35\alpha} + 10\beta \right)^{-2}
\]

(12.2)

where in the expression of \( F \) we have neglected the terms \( \alpha \) and \( \beta \) relative to 1. Using the WMAP7 normalization \( P_s = 2.4 \times 10^{-9} \) at the pivot wave number \( k_0 = 0.002 \) Mpc\(^{-1}\) [22], the mass \( M \) is constrained to be

\[
M \simeq 7.5 \times 10^{-6} \left( 3\sqrt{35\alpha} + 10\beta \right) M_{Pl}
\]

(12.3)

In the limit \( \alpha \rightarrow 0 \) and \( \beta \rightarrow 20/(2N+1) \) we have \( M/M_{Pl} = 7.5 \times 10^{-4}/(N+1/2) \). In another limit \( \alpha \rightarrow 40\sqrt{35}/[21(N+1)] \) and \( \beta \rightarrow 0 \) it follows that \( M/M_{Pl} = 1.5 \times 10^{-3}/(N+1) \). In the intermediate regime characterized by eq. (11.23) we can numerically find the values of \( \alpha \) and \( \beta \) for given \( N \) satisfying the constraint (11.22), which allows us to evaluate \( M \) from eq. (12.3). From eq. (11.5) the mass scale \( m \) is also known by the relation \( m = (\sqrt{\beta/\alpha}) M \).

In Fig. 3 we plot \( M \) and \( m \) versus \( \alpha \) in the regime \( 10^{-4} \leq \alpha \leq 0.18 \) for \( N = 55 \). In this case \( \alpha \) is bounded to be \( 0 < \alpha < 0.201 \) from Eq. (11.23). The mass \( M \) weakly depends on \( \alpha \) with the order of \( 10^{-5} M_{Pl} \), while \( m \) changes significantly depending on the values of \( \alpha \). For \( \alpha \) much smaller than 1 we have \( m \gg M \), while \( m \) is of the same order as \( M \) for \( \alpha \gtrsim 0.1 \). We recall that there is the condition \( m > \sqrt{7}/20 M \). For \( N = 55 \) this condition gives the upper bound \( \alpha < 0.178 \).

The scalar spectral index \( n_s \) can be defined by \( n_s = 1 + d \ln P_s / d \ln k \), which is evaluated at the Hubble radius crossing \( k = aH \) (where \( k \) is a comoving wave number) [77, 78, 79]. In \( f(R) \) gravity it is given by [34]

\[
n_s = 1 - 4\epsilon_1 + 2\epsilon_2 - 2\epsilon_3
\]

(12.4)

By using eqs. (11.15), (11.16) and (11.20), we obtain

\[
n_s = 1 - \frac{3\sqrt{35}}{100} \alpha - \frac{1}{5} \beta
\]

(12.5)

The tensor power spectrum is given by [34]

\[
P_t = \frac{2}{\pi^2 F} \left( \frac{H}{M_{pl}} \right)^2
\]

(12.6)
Figure 3: The two masses $M$ and $m$ versus the variable $\alpha$ in the regime $10^{-4} \leq \alpha \leq 0.18$ for the number of e-foldings $N = 55$. We also show the upper bound $\alpha_{\text{max}} = 0.201$ determined by Eq. (11.23). $M$ is weakly dependent on $\alpha$ with the order of $10^{-5} M_{\text{Pl}}$, whereas $m$ strongly depends on $\alpha$. The condition $m > \sqrt{7/20} M$ is satisfied for $\alpha < 0.178$.

From Eqs. (12.1) and (12.6) the tensor-to-scalar ratio is

$$r \equiv \frac{T}{P} = 48 \epsilon_2^2 = \frac{3}{2500} \left( 3\sqrt{35} \alpha + 10 \beta \right)^2 .$$  \hspace{1cm} (12.7)

In the limit $\alpha \to 0$ and $\beta \to 20/(2N+1)$ the observables (12.5) and (12.7) reduce to

$$n_s(\alpha \to 0) = 1 - \frac{4}{2N + 1},$$  \hspace{1cm} (12.8)

$$r(\alpha \to 0) = \frac{48}{(2N + 1)^2} ,$$  \hspace{1cm} (12.9)

which agree with those in the Starobinsky's $f(R)$ model [44]. For $N = 55$ one has $n_s(\alpha \to 0) = 0.964$ and $r(\alpha \to 0) = 3.896 \times 10^{-3}$. In another limit $\alpha \to 40\sqrt{35}/[21(N + 1)]$ and $\beta \to 0$ it follows that

$$n_s(\beta \to 0) = 1 - \frac{2}{N + 1} ,$$  \hspace{1cm} (12.10)

$$r(\beta \to 0) = \frac{48}{(N + 1)^2} .$$  \hspace{1cm} (12.11)

For $N = 55$ one has $n_s(\beta \to 0) = 0.964$ and $r(\beta \to 0) = 1.531 \times 10^{-2}$. While the scalar spectral indices (12.8) and (12.10) are practically identical for $N \gg 1$, $r(\beta \to 0)$
The three thick lines show the theoretical values of $n_s$ and $r$ for $N = 50, 60, 70$ with $\alpha$ ranging in the region (11.23). The thin solid curves are the 1$\sigma$ (inside) and 2$\sigma$ (outside) observational contours constrained by the joint data analysis of WMAP7, BAO, and HST. For $\alpha \to 0$, $n_s$ and $r$ are given by Eqs. (12.8) and (12.9). In the limit $\beta \to 0$, $n_s$ and $r$ approach the values given in Eqs. (12.10) and (12.11).

This shows that both $n_s$ and $r$ increase for larger $\alpha$ satisfying the condition $\alpha \ll \beta$. As
we see in Fig. 4, \( n_s \) switches to decrease at some value of \( \alpha \), whereas \( r \) continuously grows toward the asymptotic value given in Eq. (12.11).

From Fig. 4 we find that the \( f(R) \) model (11.6) in which \( \alpha \) is in the range (11.23) is inside the 1\( \sigma \) observational contour. The condition \( m > \sqrt{7/20} M \) provides the constraints \( \alpha < 0.194, \alpha < 0.165, \alpha < 0.143 \) for \( N = 50, 60, 70 \) respectively, while the bound (11.23) in each case corresponds to \( \alpha < 0.221, \alpha < 0.185, \alpha < 0.159 \). When \( N = 60 \) the scalar spectral index and the tensor-to-scalar ratio are \( n_s = 0.969, r = 0.0110 \) for \( \alpha = 0.165 \) and \( n_s = 0.967, r = 0.0129 \) for \( \alpha = 0.185 \), which are not very different from each other. For the background in which inflation is sustained with the number of e-foldings \( N > 50 \) the model is consistent with the current observations.

Note that the nonlinear parameter \( f_{NL} \) of the scalar non-Gaussianities is of the order of the slow-roll parameters in \( f(R) \) gravity [82] — see also Sec. 20. Hence, in current observations, this does not provide additional constraints to those studied above.

### 13 Effective Scalar Potential for Preheating

In this section we derive the effective scalar potential and the kinetic terms of a complex scalar field in the low-energy regime (B) characterized by \( |R| \ll R_0 \). In doing so, let us return to the original \( F(R) \) supergravity action (6.1) and perform the superfield Legendre transformation — see Sec. 6. As is usual, we temporarily set \( M_{Pl} = 1 \) to simplify our calculations. The Legendre transform yields the equivalent action

\[
S = \int d^4x \ d^2\theta \ \mathcal{E} [-\mathcal{Y}R + Z(\mathcal{Y})] + \text{H.c.},
\]

where we have introduced the new covariantly chiral superfield \( \mathcal{Y} \) and the new holomorphic function \( Z(\mathcal{Y}) \) related to the function \( F \) as

\[
F(R) = -R\mathcal{Y}(R) + Z(\mathcal{Y}(R))
\]

The equation of motion of the superfield \( \mathcal{Y} \), which follows from the variation of the action (13.1) with respect to \( \mathcal{Y} \), has the algebraic form

\[
\mathcal{R} = Z'(\mathcal{Y}),
\]

so that the function \( \mathcal{Y}(\mathcal{R}) \) is obtained by inverting the function \( Z' \). Substituting the solution \( \mathcal{Y}(\mathcal{R}) \) back into the action (13.1) yields the original action (6.1) because of eq. (13.2). We also find

\[
\mathcal{Y} = -F'(\mathcal{R})
\]

The inverse function \( \mathcal{R}(\mathcal{Y}) \) always exists under the physical condition \( F'(\mathcal{R}) \neq 0 \). As regards the \( F \)-function (10.1), eq. (13.4) yields a quadratic equation with respect to \( \mathcal{R} \), whose solution is

\[
\mathcal{R}(\mathcal{Y}) = \frac{\sqrt{14} M^2}{20 m} \left[ 1 - \sqrt{1 + \frac{80 m^2}{21 M^2} (Y - 3/4)} \right],
\]

36
where we have used the parametrization (11.3). Equation (13.5) is also valid for the leading complex scalar field components \( R| = \bar{B}/3 = \bar{X} \) and \( Y| \equiv Y \), where \( Y \) is the complex scalaron field.

The kinetic terms of \( Y \) are obtained by using the identity

\[
\int d^4x d^2\theta \mathcal{E} Y R + \text{H.c.} = \int d^4x d^4\theta E^{-1}(Y + \bar{Y}), \quad (13.6)
\]

where \( E^{-1} \) is the full curved superspace density \([48, 49, 50]\). Therefore, the Kähler potential reads

\[
K = -3 \ln (Y + \bar{Y}) \quad (13.7)
\]

up to an additive constant. It gives rise to the kinetic terms

\[
\mathcal{L}_{\text{kin}} = \left. \frac{\partial^2 K}{\partial Y \partial \bar{Y}} \right|_{Y=\bar{Y}} \partial_\mu Y \partial^\mu \bar{Y} = 3 \left( \frac{\partial_\mu y}{(Y + \bar{Y})^2} \right)^2 = 3 \frac{\left( \partial_\mu y \right)^2 + \left( \partial_\mu z \right)^2}{4y^2}, \quad (13.8)
\]

where we have used the notation \( Y = y + iz \) in terms of the two real fields \( y \) and \( z \). The imaginary component \( z \) corresponds to a pseudo-scalar field. The kinetic terms (13.8) represent the non-linear sigma model \([59]\) with the hyperbolic target space of (real) dimension two, whose metric is known as the standard Poincaré metric. The kinetic terms are invariant under arbitrary rescalings \( Y \rightarrow Ay \) with constant parameter \( A \neq 0 \).

The effective scalar potential \( V(Y, \bar{Y}) \) of a complex scalaron \( Y \) in the regime (B), where supergravity decouples (it corresponds to rigid supersymmetry) is easily derived from eq. (13.1) when keeping only scalars (i.e. ignoring their spacetime derivatives together with all fermionic contributions) and eliminating the auxiliary fields, near the minimum of the scalar potential. We find

\[
V = \frac{21}{2} |Z'(Y)|^2 = \frac{21}{2} |R(Y)|^2, \quad (13.9)
\]

which gives rise to the chiral superpotential

\[
W(Y) = \sqrt{\frac{21}{2}} Z(Y) \quad (13.10)
\]

The superfield equations (13.7) and (13.10) are model-independent, i.e. they apply to any function \( F(R) \) in the large \( M_{Pl} \) limit, near the minimum of the scalar potential with the vanishing cosmological constant. The exact scalar potential including the supergravity effects is derived in Appendix A, but it is not very illuminating.

There is no field redefinition that would bring all the kinetic terms (13.8) to the free form. The canonical (free) kinetic term of a real scalaron \( y \) alone can be obtained via the field redefinition

\[
y = A \exp(-\sqrt{2/3} \phi) \quad (13.11)
\]
The scalaron potential vanishes at \( y = 3/4 \). Demanding that this minimum corresponds to \( \phi = 0 \), we have \( A = 3/4 \) and hence \( y = (3/4) \exp(-\sqrt{2/3} \phi) \). Defining a rescaled field \( \chi \) as \( \chi = \sqrt{8/3} z \), the kinetic term (13.8) can be written as

\[
L_{\text{kin}} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} e^{2\sqrt{2/3} \phi/M_{\text{pl}}} (\partial_\mu \chi)^2.
\]

Here and in what follows we restore the reduced Planck mass \( M_{\text{pl}} \).

The total potential (13.9) including both the fields \( \phi \) and \( \chi \) is given by

\[
V(\phi, \chi) = \frac{147 M^4 M_{\text{pl}}^2}{400 m^2} \left| \sqrt{B(\phi) + iC(\chi)} - 1 \right|^2,
\]

where

\[
B(\phi) = 1 + \frac{20 m^2}{7 M^2} \left( e^{-\sqrt{2/3} \phi/M_{\text{pl}}} - 1 \right),
\]

\[
C(\chi) = \frac{80 m^2}{21 M^2} \sqrt{\frac{3}{8} \chi M_{\text{pl}}}.
\]

In order to express (13.13) in a more convenient form we write \( \sqrt{B(\phi) + iC(\chi)} = p + iq \), where \( p \) and \( q \) are real. This gives the relations

\[
p^2 - q^2 = B(\phi) \quad \text{and} \quad 2pq = C(\chi).
\]

Solving these equations for \( p \), we find

\[
p = \frac{1}{\sqrt{2}} \left[ B(\phi) + \sqrt{B^2(\phi) + C^2(\chi)} \right]^{1/2},
\]

where we have chosen the solution \( p > 0 \) to recover \( p = \sqrt{B(\phi)} \) for \( B(\phi) > 0 \) in the limit \( C(\chi) \to 0 \). Then the field potential (13.13) reads

\[
V(\phi, \chi) = \frac{147 M^4 M_{\text{pl}}^2}{400 m^2} \left[ 1 + \sqrt{B^2(\phi) + C^2(\chi)} - \sqrt{2} \left\{ B(\phi) + \sqrt{B^2(\phi) + C^2(\chi)} \right\}^{1/2} \right].
\]

In the absence of the pseudo-scalar \( \chi \) the potential (13.17) reduces to

\[
V(\phi) = \frac{147 M^4 M_{\text{pl}}^2}{400 m^2} \left[ 1 + |B(\phi)| - \sqrt{2} \left\{ |B(\phi)| + |B(\phi)| \right\}^{1/2} \right]
\]

For the field \( \phi \) satisfying the condition \( B(\phi) < 0 \) it follows that

\[
V(\phi) = \frac{21 M^2 M_{\text{pl}}^2}{20} \left( 1 - e^{-\sqrt{2/3} \phi/M_{\text{pl}}} \right),
\]

which approaches the constant \( V(\phi) \to 21 M^2 M_{\text{pl}}^2 / 20 \) in the limit \( \phi \to \infty \). Defining the slow-roll parameter \( \epsilon_V = (M_{\text{pl}}^2/2)(V_{\phi}/V)^2 \), we have

\[
\epsilon_V = \frac{x^2}{3(1 - x)^2}, \quad x = e^{-\sqrt{2/3} \phi/M_{\text{pl}}}
\]
The end of inflation is characterized by the criterion $\epsilon_V = 1$. This gives $x_f = e^{-\sqrt{2/3}} \phi_f / M_{pl} = (3 - \sqrt{3})/2$ and hence $\phi_f = 0.558 M_{pl}$. For $m > M$ the condition $B(\phi_f) < 0$ is satisfied, so that the potential (13.19) is valid at the end of inflation. If $m$ is close to the border value $\sqrt{7}/20 M$, then the potential (13.19) is already invalid at the end of inflation.

For small $\phi$ satisfying the condition $B(\phi) > 0$ the potential (13.18) reads

$$V(\phi) = \frac{147 M^4 M_{pl}^2}{400 m^2} \left[ 1 - \sqrt{1 + \frac{20 m^2}{7 M^2} \left( e^{-\sqrt{2/3}} \phi/M_{pl} - 1 \right)} \right]^2$$  (13.21)

In this case Taylor expansion around $\phi = 0$ gives rise to the leading-order contribution $V(\phi) = m^2 \phi^2 / 2$. Reheating occurs around the potential minimum through the oscillation of the canonical field $\phi$.

The full effective potential involving the interaction between the fields $\phi$ and $\chi$ is given by eq. (13.17). Expanding the potential (13.17) around $\phi = \chi = 0$ and picking up the terms up to fourth-order in the fields, we obtain

$$V(\phi, \chi) \simeq \frac{1}{2} m^2 \phi^2 + \frac{\sqrt{6} m^2 (10 m^2 - 7 M^2)}{42 M^2 M_{pl}} \phi^3 + \frac{(1500 m^4 - 1260 m^2 M^2 + 343 M^4) m^2}{1764 M^4 M_{pl}^2} \phi^4$$

$$+ \frac{1}{2} m^2 \chi^2 - \frac{25 m^6}{49 M^4 M_{pl}^2} \chi^4 + \frac{5 \sqrt{6} m^4}{21 M^2 M_{pl}^2} \phi \chi^2 + \frac{5 m^4 (10 m^2 - 7 M^2)}{147 M^4 M_{pl}^2} \phi^2 \chi^2$$  (13.22)

The scalaron $\phi$ is coupled to the pseudo-scalar $\chi$ through the interaction given in the second line of eq. (13.22).

## 14 Preheating after Inflation

Here we study the dynamics of preheating for the two-field system described by the kinetic term (13.12) and the effective potential (13.17). The background equations of motion on the flat FLRW background are

$$3 M_{pl}^2 H^2 = \dot{\phi}^2 / 2 + e^{2 b} \dot{\chi}^2 / 2 + V ,$$  (14.1)

$$\ddot{\phi} + 3 H \dot{\phi} + V_{,\phi} - b_{,\phi} e^{2 b} \dot{\chi}^2 = 0 ,$$  (14.2)

$$\ddot{\chi} + (3 H + 2 b_{,\phi} \dot{\phi}) \dot{\chi} + e^{-2 b} V_{,\chi} = 0 ,$$  (14.3)

where $b(\phi) = \sqrt{2/3} \phi / M_{pl}$ and "$,\phi$" represents a partial derivative with respect to $\phi$.

In Fourier space the field perturbations $\delta \phi_k$ and $\delta \chi_k$ with the comoving wave num-
ber $k$ obey the following equations:

$$
\ddot{\delta \phi}_k + 3H \dot{\delta \phi}_k + \left[ k^2 / a^2 + V,\phi \phi - (2b^2_\phi + b_\phi \phi) e^{2b_\chi} \right] \delta \phi_k \\
= -V,\phi \delta \chi_k + 2b_\phi e^{2b_\chi} \dot{\delta \chi}_k, \quad (14.4)
$$

$$
\ddot{\delta \chi}_k + (3H + 2b_\phi \phi) \dot{\delta \chi}_k + \left[ k^2 / a^2 + e^{-2b_\chi} V,\chi \chi \right] \delta \chi_k \\
= -e^{-2b_\chi} (V,\phi \chi - 2b_\phi \phi V,\chi + 2b_\phi \phi e^{2b_\chi} \dot{\delta \phi}_k - 2b_\phi \dot{\chi} \dot{\delta \phi}_k) \quad (14.5)
$$

The derivative $V,\chi$ of the potential (13.17) vanishes at $\chi = \pm \chi_c$, where

$$
\chi_c = \sqrt{210M^2 \left[ 1 - e^{-\sqrt{2/3} \phi / M_{pl}} - \frac{21}{80} \left( \frac{M}{m} \right)^2 \right]}^{1/2} M_{pl} \quad (14.6)
$$

The local minima exist in the $\chi$ direction provided that

$$
\phi > \sqrt{\frac{3}{2}} \ln \left[ 1 - \frac{21}{80} \left( \frac{M}{m} \right)^2 \right]^{-1} M_{pl} \equiv \phi_c, \quad (14.7)
$$

whereas they disappear for $\phi < \phi_c$. In Fig. 5 we plot the potential (13.17) with respect to $\phi$ and $\chi$ for $m = 1.14 \times 10^{-4} M_{pl}$ and $M = 1.62 \times 10^{-5} M_{pl}$. Since $\phi_c = 6.5 \times 10^{-3} M_{pl}$ in this case, the potential has the local minima in the $\chi$ direction for $\phi > 6.5 \times 10^{-3} M_{pl}$. From eq. (14.6) the field value $\chi_c$ increases for larger $\phi$. For the model parameters used in Fig. 5, for example, one has $\chi_c = 0.028 M_{pl}$ at $\phi = 0.1 M_{pl}$ and $\chi_c = 0.059 M_{pl}$ at $\phi = 0.5 M_{pl}$.

If the initial conditions of the fields are $0 < \chi < \chi_c$ and $\phi > \phi_c$, the field $\chi$ grows toward the local minimum at $\chi = \chi_c$. After $\phi$ drops below $\phi_c$, the field $\chi$ approaches the global minimum at $\chi = 0$. In Fig. 6 we show one example for the evolution of
the background fields $\phi$ and $\chi$ with the same values of $m$ and $M$ as those in Fig. 5. The energy density of the field $\chi$ catches up to that of the inflaton around the onset of reheating.

As we see in eq. (14.7), the critical field value $\phi_c$ gets smaller for increasing $m/M$. Hence, for larger $m/M$, the potential (13.17) possesses the local minima at $\chi = \pm \chi_c$ for a wider range of $\phi$. The potential in the region $|\chi| < \chi_c$ can be flat enough to lead to inflation by the slow-roll evolution of the field $\chi$, even if $\phi$ is smaller than $\phi_f = 0.558 M_{\text{pl}}$. For larger ratio $m/M$ inflation ends with the field value much smaller than $\phi_f$. If $m/M = 20$ and $m/M = 83$, for example, the amplitudes of the field $\phi$ at the onset of oscillations are $\phi_i = 1.5 \times 10^{-2} M_{\text{pl}}$ and $\phi_i = 5.0 \times 10^{-3} M_{\text{pl}}$, respectively.

Let us consider the regime where the condition

$$
\left( \frac{m}{M} \right)^2 \frac{|\phi|}{M_{\text{pl}}} \ll 1
$$

(14.8)

is satisfied. Then the potential (13.22) is approximately given by $V(\phi, \chi) \simeq m^2 \phi^2/2 + m^2 \chi^2/2$, in which case both $\phi$ and $\chi$ have the same mass $m$. This gives rise to the matter-dominated epoch (where $H = 2/(3t)$) driven by the oscillations of two massive scalar fields. From eq. (14.2) we have that $\ddot{\phi} + (2/t)\dot{\phi} + m^2 \phi \simeq 0$, whose solution is

$$
\phi(t) \simeq \frac{\pi}{2mt} \phi_i \sin(mt).
$$

(14.9)

Here the initial field value $\phi_i$ corresponds to the time $t_i = \pi/(2m)$.

Figure 6: Evolution of the background fields $\phi^2$ and $\chi^2$ (both are normalized by $M^2_{\text{pl}}$) for $m = 1.14 \times 10^{-4} M_{\text{pl}}$ and $M = 1.62 \times 10^{-5} M_{\text{pl}}$ with the initial conditions $\phi = 0.55 M_{\text{pl}}$, $\chi = 10^{-3} M_{\text{pl}}$, $\dot{\phi} = -1.6 \times 10^{-2} m M_{\text{pl}}$, and $\dot{\chi} = 1.5 \times 10^{-3} m M_{\text{pl}}$. 


Figure 7: Evolution of the field perturbations $\delta \phi_k = k^{3/2} \delta \phi_k / M_{pl}$ and $\delta \chi_k = k^{3/2} \delta \chi_k / M_{pl}$ with the wave number $k = m$ for $m = 1.16 \times 10^{-3} M_{pl}$ and $M = 1.39 \times 10^{-5} M_{pl}$. We choose the background initial conditions $\phi = 0.1 M_{pl}, \chi = 1.0 \times 10^{-3} M_{pl}, \dot{\phi} = -8.48 \times 10^{-4} m M_{pl}$, and $\dot{\chi} = 1.18 \times 10^{-5} m M_{pl}$.

In order to discuss the dynamics of the field perturbations in eqs. (14.4) and (14.5) we define the two frequencies $\omega_\phi$ and $\omega_\chi$, as $\omega_\phi^2 = k^2/a^2 + V_{,\phi} - (2b_\phi^2 + b_{,\phi\phi})e^{2b} \dot{\chi}^2$ and $\omega_\chi^2 = k^2/a^2 + e^{-2b} V_{,\chi\chi}$. As long as the condition (14.8) is satisfied, it is sufficient to pick up the terms up to cubic order in fields. It then follows that

$$\omega_\phi^2 \simeq \frac{k^2}{a^2} + m^2 + \frac{\sqrt{6} m^2 (10m^2 - 7M^2)}{7M^2 M_{pl}} \phi,$$

$$\omega_\chi^2 = \frac{k^2}{a^2} + m^2 e^{-2b} + \frac{10\sqrt{6} m^4}{21M^2 M_{pl}} e^{-2b} \phi,$$

where, in eq. (14.10), we have neglected the contribution of the term $-(2b_\phi^2 + b_{,\phi\phi})e^{2b} \dot{\chi}^2$.

We introduce the rescaled fields $\delta \varphi_k = a^{3/2} \delta \phi_k$ and $\delta X_k = a^{3/2} e^b \delta \chi_k$ to estimate the growth of perturbations in the regime (14.8). Neglecting the contributions of the r.h.s. of eqs. (14.4) and (14.5) and also using the approximation $e^{-2b} \simeq 1$ in the regime $H \ll m$, the field perturbations $\delta \varphi_k$ and $\delta X_k$ obey the following equations

$$\frac{d^2}{dz^2} \delta \varphi_k + [A_k - 2q_\phi \cos(2z)] \delta \varphi_k \simeq 0,$$

$$\frac{d^2}{dz^2} \delta X_k + [A_k - 2q_\chi \cos(2z)] \delta X_k \simeq 0,$$
Figure 8: Evolution of the field perturbations with the wave number $k = m$ for $m = 2.89 \times 10^{-4} M_{\text{pl}}$ and $M = 1.46 \times 10^{-5} M_{\text{pl}}$. We choose the background initial conditions $\phi = 0.1 M_{\text{pl}}$, $\chi = 1.0 \times 10^{-3} M_{\text{pl}}$, $\dot{\phi} = -7.35 \times 10^{-3} m M_{\text{pl}}$, and $\dot{\chi} = 6.85 \times 10^{-4} m M_{\text{pl}}$.

where $2z = mt + \pi/2$. The quantities $A_k$, $q_\phi$, and $q_\chi$ are given by

$$A_k = 4 + 4 \frac{k^2}{m^2 a^2},$$

$$q_\phi = \frac{20 \sqrt{6}}{21} \left( 1 - \frac{7 M^2}{10 m^2} \right) \left( \frac{m}{M_{\text{pl}}} \right)^2 \frac{\phi_i}{m} \frac{\pi/2}{mt},$$

$$q_\chi = \frac{20 \sqrt{6}}{21} \left( \frac{m}{M_{\text{pl}}} \right)^2 \frac{\phi_i}{m} \frac{\pi/2}{mt},$$

which are time-dependent.

Equations (14.12) and (14.13) are the so-called Mathieu equations describing the parametric resonance caused by oscillations of the field $\phi$ [83, 84, 85, 86]. In the regime (14.8) both $q_\phi$ and $q_\chi$ are smaller than 1 for $t \geq t_i = \pi/(2m)$. In this case the resonance occurs in narrow bands near $A_k = l^2$, where $l = 1, 2, \cdots$ [85, 86, 87]. As the physical momentum $k/a$ redshifts away, the field perturbations approach the instability band at $A_k = 4$. Although $\delta \phi_k$ and $\delta \chi_k$ can be amplified for $A_k \approx 4$ and $q_\phi \lesssim 1$, $q_\chi \lesssim 1$, this narrow parametric resonance is not efficient enough to lead to the growth of $\delta \phi_k$ and $\delta \chi_k$ against the Hubble friction [85, 86].

If the initial field $\phi_i$ satisfies the condition $(m/M)^2 |\phi_i|/M_{\text{pl}} \gg 1$, the quantities $q_\phi$ and $q_\chi$ are much larger than 1 at the onset of reheating. This corresponds to the so-called broad resonance regime [85, 86] in which the perturbations $\delta \phi_k$ and $\delta \chi_k$ can grow even against the Hubble friction. We caution, however, that eqs. (14.12) and (14.13) are no longer valid because the background solution (14.9) is subject to
change due to the effect of higher-order terms in the potential (13.17). Still, the non-adiabatic particle production occurs around the potential minimum ($\dot{\phi} = 0$) [85, 86]. In this region the dominant contribution to the potential is the quadratic term $m^2 \dot{\phi}^2/2$. Hence, it is expected that preheating can be efficient for the values of $q_\phi$ and $q_\chi$ much larger than 1 at the onset of the field oscillations.

We numerically solve the perturbations equations (14.4) and (14.5) together with the background equations (14.1), (14.2), and (14.3) for the full potential (13.17) without using the approximate expression (13.22). In Figs. 7 and 8 we plot the evolution of the field perturbations $\delta \phi_k$ and $\delta \chi_k$ with the wave number $k = m$ for two different choices of the parameters $m$ and $M$ (which are constrained by the WMAP normalization in Fig. 3). The initial conditions of the perturbations are chosen to recover the choices of the parameters $m$ and $M$ (which are constrained by the WMAP normalization in Fig. 3). The initial conditions of the perturbations are chosen to recover the vacuum state characterized by $\delta \varphi(t_0) = e^{-i\omega_\phi t_0}/\sqrt{2\omega_\phi}$ and $\delta X_k(t_0) = e^{-i\omega_\chi t_0}/\sqrt{2\omega_\chi}$.

Figure 7 corresponds to the mass scales $m = 1.16 \times 10^{-3} M_{\text{pl}}$ and $M = 1.39 \times 10^{-5} M_{\text{pl}}$, i.e., the ratio $m/M = 83$. The field value at the onset of oscillations is found to be $\phi_i = 5.0 \times 10^{-3} M_{\text{pl}}$, in which case $q_\phi(t_i) = 244$ and $q_\chi(t_i) = 81$. Figure 7 shows that both $\delta \phi_k$ and $\delta \chi_k$ rapidly grow by the broad parametric resonance. The growth of the field perturbations ends when $q_\phi$ and $q_\chi$ drop below 1.

Figure 8 corresponds to the ratio $m/M = 20$, in which case $\phi_i = 1.5 \times 10^{-2} M_{\text{pl}}$, $q_\phi(t_i) = 41$, and $q_\chi(t_i) = 4.6$. Compared to the evolution in Fig. 7, preheating is less efficient because of the smaller values of $q_\phi(t_i)$ and $q_\chi(t_i)$. The parameter to control the efficiency of preheating is the mass ratio $m/M$. For larger $m/M$ the creation of particles tends to be more significant. For the mass $m$ smaller than $10^{-4} M_{\text{pl}}$ the field perturbations $\delta \phi_k$ and $\delta \chi_k$ hardly grow against the Hubble friction because they are not in the broad resonance regime.

In our numerical simulations we did not take into account the rescattering effect between different modes of the particles. The lattice simulation [88, 89, 90, 91] is required to deal with this problem. It will be of interest to see how the non-linear effect can affect the evolution of perturbations at the final stage of preheating.

## 15  Current Status of our Model

In the preceding sections we studied the viability of the $f(R)$ inflationary scenario in the context of $F(\mathcal{R})$ supergravity. In the high-energy regime characterized by the condition $|R| \gg R_0$ there is a correction of the form $(-R)^{3/2}/m$ to the function $f(R) = 3R/10 - R^2/(6 M^2)$. Introducing the dimensionless functions $\alpha$ and $\beta$ in eqs. (11.5), we showed that these are constrained to be in the range (11.23) to realize inflation with the number of e-foldings $N$.

The masses of the scalaron field in the regimes $|R| \gg R_0$ and $|R| \ll R_0$ are approximately given by $M$ and $m$, respectively. From the WMAP normalization of the CMB temperature anisotropies we derived $M$ and $m$ as a function of $\alpha$ in Fig. 3. The weak dependence of $M$ with respect to $\alpha$ means that the term $-R^2/(6 M^2)$ needs to dominate over the correction $(-R)^{3/2}/m$ during inflation. We also showed that the
model is within the 1σ observational contour constrained from the joint data analysis of WMAP7, BAO, and HST, by evaluating the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$.

In the presence of the pseudo-scalar field $\chi$ coupled to the scalaron field $\phi$ we derived the effective potential (13.17) and their kinetic energies (13.12) in the low-energy regime ($|R| \ll R_0$). Provided that the condition (14.7) is satisfied, the effective potential has two local minima at $\chi = \pm \chi_c$. Around the global minimum at $\phi = \chi = 0$ the system is described by two massive scalar fields with other interaction terms given in Eq. (13.22). Even if $\chi$ is initially close to 0, $\chi$ typically catches up to $\phi$ around the onset of the field oscillations (see Fig. 6).

In the regime where the field $\phi$ is in the range (14.8) we showed that both the field perturbations $\delta \varphi_k = a^{3/2} \delta \phi_k$ and $\delta \chi_k = a^{3/2} e^b \delta \chi_k$ obey the Mathieu equations (14.12) and (14.13). This corresponds to the narrow resonance regime in which $q_\phi$ and $q_\chi$ are smaller than the order of unity. The broad resonance regime is characterized by the condition $(m/M)^2 |\phi|/M_{\text{pl}} \gg 1$, but in this case the expansion (13.22) of the effective potential around the minimum is no longer valid. In order to confirm the presence of the broad resonance we numerically solved the perturbation equations (14.4) and (14.5) for the full potential (13.17). Indeed we found that preheating of the both perturbations $\delta \phi_k$ and $\delta \chi_k$ is efficient in this regime. As we see in Figs. 7 and 8, the broad parametric resonance is more significant for larger values of $m/M$.

Our results lend compelling support to the phenomenological viability of the bosonic sector of $F(R)$ supergravity, in addition to its formal consistency. It is also worthwhile to recall that supergravity unifies bosons and fermions with General Relativity, highly constrains particle spectrum and interactions, has the ideal candidate for a dark matter particle such as the lightest super-particle (see Sec. 20). It may also be deduced from quantum gravity such as superstring theory. The $F(R)$ supergravity action (6.1) is truly chiral in superspace, so that it is expected to be protected against quantum corrections, which is important for stabilizing the masses $M$ and $m$ in quantum theory.

16 Cosmological Constant in $F(R)$ Supergravity

The Standard ($\Lambda$-CDM) Model in cosmology gives a phenomenological description of the observed Dark Energy (DE) and Dark Matter (DM). It is based on the use of a small positive cosmological constant $\Lambda$ and a Cold Dark Matter (CDM), and is consistent with all observations coming from the existing cosmological, Solar system and ground-based laboratory data. However, the $\Lambda$-CDM Model cannot be the ultimate answer to DE, since it implies its time-independence. For example, the ‘primordial’ DE responsible for inflation in the early Universe was different from $\Lambda$ and unstable. The dynamical (ie. time-dependent) models of DE can be easily constructed by using the $f(R)$ gravity theories, defined via replacing the scalar curvature $R$ by a function $f(R)$ in the gravitational action. The $f(R)$ gravity provides the self-consistent non-trivial alternative to the $\Lambda$-CDM Model. The viable $f(R)$-gravity-based models of
the current DE are known [92, 93, 94], and the combined inflationary-DE models are possible too [34].

The natural question arises, whether $F(R)$ supergravity is also capable to describe the present DE and eg., a positive cosmological constant. It is non-trivial because the standard (pure) supergravity can only have a zero or negative cosmological constant. In this section we further extend the Ansatz used in Sec. 10 for the $F$-function, and apply it to get a positive cosmological constant in the regime of a low spacetime curvature.

Throughout this section we again use the units $c = \hbar = M_{Pl} = 1$. We recall that an AdS-spacetime has a positive scalar curvature, and a dS-spacetime has a negative scalar curvature in our notation.

The embedding of $f(R)$ gravity into $F(R)$ supergravity is given by (Sec. 8)

$$f(R) = f(R, X(R)) \quad (16.1)$$

where the function $f(R, X)$ (or the gravity Lagrangian $\mathcal{L}$) is defined by

$$\mathcal{L} = f(R, X) = 2F'(X) \left[ \frac{1}{3} R + 4X^2 \right] + 6XF(X) \quad (16.2)$$

and the function $X = X(R)$ is determined by solving an algebraic equation,

$$\frac{\partial f(R, X)}{\partial X} = 0 \quad (16.3)$$

The cosmological constant in $F(R)$ supergravity is thus given by

$$\Lambda = -f(0, X_0) \quad (16.4)$$

where $X_0 = X(0)$. It should be mentioned that $X_0$ represents the vacuum expectation value of the auxiliary field $X$ that determines the scale of the supersymmetry breaking. Both inflation and DE imply $X_0 \neq 0$.

To describe DE in the present Universe, ie. in the regime of a low spacetime curvature $R$, the function $f(R)$ should be close to the Einstein-Hilbert (linear) function $f_{EH}(R)$ with a small positive $\Lambda$,

$$|f(R) - f_{EH}(R)| \ll |f_{EH}(R)|, \quad |f'(R) - f_{EH}'| \ll 1, \quad |Rf''(R)| \ll 1 \quad (16.5)$$

ie. $f(R) \approx -\frac{1}{2} R - \Lambda$ for small $R$ with the very small and positive $\Lambda \approx 10^{-118}(M_{Pl}^4)$.

Equations (16.2) and (16.4) imply

$$\Lambda = -8F''(X_0)X_0 - 6X_0F(X_0) \quad (16.6)$$

where $X_0$ is a solution to the algebraic equation

$$4X_0^2 F''(X_0) + 11X_0F'(X_0) + 3F(X_0) = 0 \quad (16.7)$$
As is clear from eq. (16.6), to have \( \Lambda \neq 0 \), one must have \( X_0 \neq 0 \), ie. a (spontaneous) supersymmetry breaking. However, in order to proceed further, we need a reasonable Ansatz for the \( F \)-function.

The simplest opportunity is given by expanding the function \( F(R) \) in Taylor series with respect to \( R \). Since the \( N = 1 \) chiral superfield \( R \) has \( X \) as its leading field component (in \( \theta \)-expansion), one may expect that the Taylor expansion is a good approximation as long as \( |X_0| \ll (M_{\text{Pl}})^{-1} \). As was demonstrated in Sec. 10, a viable (successful) description of inflation is possible in \( F(R) \) supergravity, when keeping the cubic term \( R^3 \) in the Taylor expansion of the \( F(R) \) function. It is, therefore, natural to expand the function \( F \) up to the cubic term with respect to \( R \), and use it as our Ansatz here,

\[
F(R) = f_0 - \frac{1}{2} f_1 R + \frac{1}{2} f_2 R^2 - \frac{1}{6} f_3 R^3 \tag{16.8}
\]

with some real coefficients \( f_0, f_1, f_2, f_3 \). The Ansatz (16.8) differs from the one used in eq. (10.1) by the presence of the new parameter \( f_0 \) only. It is worth emphasizing here that \( f_0 \) is not a cosmological constant because one still has to eliminate the auxiliary field \( X \). The stability conditions (Sec. 9) imply

\[
f_1 > 0 \quad , \quad f_2 > 0 \quad , \quad f_3 > 0 \tag{16.9}
\]

and

\[
f_2^2 < f_1 f_3 \tag{16.10}
\]

Inflation requires \( f_3 \gg 1 \) and \( f_2^2 \gg f_1 \).\(^5\) As was already found in Sec. 10, in order to meet the WMAP observations, the parameter \( f_3 \) should be approximately equal to \( 6.5 \cdot 10^{10} (N_c/50)^2 \). The cosmological constant in the high-curvature regime does not play a significant role in early universe, so it can be ignored.

In the low curvature regime, in order to recover the Einstein-Hilbert term, one has to fix \( f_1 = 3/2 \) (Sec. 10). Then the Ansatz (16.8) leads to the gravitational Lagrangian

\[
f(R, X) = -5 f_3 X^4 + 11 f_2 X^3 - \frac{1}{3} f_3 \left( R + \frac{63}{2 f_3} \right) X^2 + \left( 6 f_0 + \frac{2}{3} f_2 R \right) X - \frac{1}{2} R \tag{16.11}
\]

and the auxiliary field equation

\[
X^3 - \frac{33 f_2}{20 f_3} X^2 + \frac{1}{30} \left( R + \frac{63}{2 f_3} \right) X - \frac{1}{30 f_3} (f_2 R + 9 f_0) = 0 \tag{16.12}
\]

whose formal solution is available via the standard Cardano-Viète formulae [70].

In the low-curvature regime we find a cubic equation for \( X_0 \) in the form

\[
X_0^3 - \left( \frac{33 f_2}{20 f_3} \right) X_0^2 + \left( \frac{21}{20 f_3} \right) X_0 - \left( \frac{3 f_0}{10 f_3} \right) = 0 \tag{16.13}
\]

\(^5\)The stronger condition \( f_2^2 \ll f_1 f_3 \) was used in Sec. 10 for simplicity.
'Linearizing' eq. (16.13) with respect to $X_0$ brings the solution $X_0 = 2f_0/7$ whose substitution into the action (16.11) gives rise to a negative cosmological constant, $\Lambda_0 = -6f_0^2/7$. This way we recover the standard supergravity case.

Equations (16.11) and (16.13) allow us to write down the exact eq. (16.4) for the cosmological constant in the factorized form

$$\Lambda(X_0) = -\frac{11f_2}{4}X_0(X_0 - X_-)(X_0 - X_+)$$

where $X_{\pm}$ are the roots of the quadratic equation $x^2 - \frac{21}{11f_2}x + \frac{18f_0}{11f_2} = 0$, ie.

$$X_{\pm} = \frac{21}{22f_2} \left[ 1 \pm \sqrt{1 - \frac{23 \cdot 11}{72} f_0 f_2} \right]$$

(16.15)

Since $f_0 f_2$ is supposed to be very small, both roots $X_{\pm}$ are real and positive.

Equation (16.14) implies that $\Lambda > 0$ when either (I) $X_0 < 0$, or (II) $X_0$ is inside the interval $(X_-, X_+)$. By using Mathematica we were able to numerically confirm the existence of solutions to eq. (16.13) in the region (I) when $f_0 < 0$, but not in the region (II). So, to this end, we continue with the region (I) only. All real roots of eq. (16.13) are given by

$$(X_0)_1 = 2\sqrt{-Q} \cos \left( \frac{\vartheta}{3} \right) + \frac{11f_2}{20f_3} ,$$

$$(X_0)_2 = 2\sqrt{-Q} \cos \left( \frac{\vartheta + 2\pi}{3} \right) + \frac{11f_2}{20f_3} ,$$

$$(X_0)_3 = 2\sqrt{-Q} \cos \left( \frac{\vartheta + 4\pi}{3} \right) + \frac{11f_2}{20f_3} ,$$

(16.16)

in terms of the Cardano-Viète parameters

$$Q = -\frac{11f_2}{2^2 \cdot 5f_3} - \frac{7^2}{2^4 \cdot 5^2 f_3^2} \approx -\frac{11f_2}{20f_3} ,$$

$$\hat{R} = -\frac{3 \cdot 7 \cdot 11f_2}{2^5 \cdot 5^2 f_3^2} + \frac{3f_0}{2^2 \cdot 5f_3} + \frac{11^3f_2^3}{2^6 \cdot 5^3 f_3^3} \approx -\frac{1}{20f_3} \left( -\frac{21}{2}Q + 3f_0 \right)$$

(16.17)

and the angle $\vartheta$ defined by

$$\cos \vartheta = \frac{\hat{R}}{\sqrt{-Q^3}}$$

(16.18)

The Cardano discriminant reads $D = \hat{R}^2 + Q^3$. All three roots are real provided that $D < 0$. It is known to be the case in the high-curvature regime (Sec. 10), and it is also the case when $f_0$ is extremely small. Under our requirements on the parameters the angle $\vartheta$ is very close to zero, so the relevant solutions $X_0 < 0$ are given by the 2nd and 3rd lines of eq. (16.16), with $X_0 \approx f_0/10$.
We thus demonstrated that it is possible to have a positive cosmological constant (at low spacetime curvature) in the particular $F(R)$ supergravity without its coupling to super-matter, as described by the Ansatz (16.8). The same Ansatz is applicable for describing a viable chaotic inflation in supergravity (at high spacetime curvature). A positive cosmological constant was achieved as the non-linear effect (with respect to the superspace curvature and spacetime curvature) in the narrow part of the parameter space (it is, therefore, highly constrained). It also implies the apparent violation of the Strong Energy Condition in our model.

Of course, describing the DE in the present Universe requires enormous fine-tuning of our parameters in the $F$-function. However, it is the common feature of all known approaches to the DE. Our analysis does not contribute to ‘explaining’ the smallness of the cosmological constant. Yet another attempt for describing DE by an $F(R)$ supergravity model with spontaneous breaking of supersymmetry was proposed in ref. [15].

17 Nonminimal Scalar-Curvature Coupling in Gravity and Supergravity, and Higgs inflation

One can pursue different strategies in a theoretical search for inflaton. For instance, inflaton may be either a new exotic particle or something that we already know ‘just around the corner’. In this review we advocate the second “economical” approach. Besides the Starobinsky inflation another “economical” approach is given by the so-called Higgs inflation [95, 96, 97].

According to the cosmology textbooks, a Higgs particle of the Standard Model cannot serve as inflaton because the SM parameters are $\lambda \approx 1$, $m_H \approx 10^2 \text{GeV}$, and $(\delta T/T) \approx 1$, whereas inflation requires (see Sec. 4) $\lambda \approx 10^{-13}$, $m_{\text{inf}} \approx 10^{13} \text{GeV}$, and $(\delta T/T) \approx 10^{-5}$. Nevertheless, it is possible to reach the required values when assuming that Higgs particle is nonminimally coupled to gravity [95, 96, 97]. For instance, adding the nonminimal coupling of the Higgs field to the scalar spacetime curvature is natural in curved spacetime because it is required by renormalization [98].

In this section we compare the inflationary scalar potential, derived by the use of the nonminimal coupling [95, 96, 97], with the scalar potential that follows from the $(R + R^2)$ inflationary model (Sec. 2), and confirm that they are the same. Then we also upgrade that equivalence to supergravity. In this section we set $M_{Pl} = 1$ too.

The original motivation of Refs. [95, 96, 97] is based on the assumption that there is no new physics beyond the Standard Model up to the Planck scale. Then it is natural to search for the most economical mechanism of inflation by identifying inflaton with Higgs particle. We assume that there is the new physics beyond the Standard Model, and it is given by supersymmetry. Then it is quite natural to search for the most economical mechanism of inflation in the context of supergravity. Moreover, we do not have to identify our inflaton with a Higgs particle of the Minimal Supersymmetric
Standard Model. Let us begin with the 4D Lagrangian

\[ L_J = \sqrt{-g_J} \left[ -\frac{1}{2}(1 + \xi \phi_J^2) R_J + \frac{1}{2} g_{\mu \nu}^J \partial_\mu \phi_J \partial_\nu \phi_J - V(\phi_J) \right] \]  

(17.1)

where we have introduced the real scalar field \( \phi_J(x) \), nonminimally coupled to gravity (with the coupling constant \( \xi \)) in Jordan frame, with the Higgs-like scalar potential

\[ V(\phi_J) = \frac{\lambda}{4}(\phi_J^2 - v^2)^2 \]  

(17.2)

The action (17.1) can be rewritten to Einstein frame by redefining the metric via a Weyl transformation,

\[ g_{\mu \nu}^E = \frac{g_{\mu \nu}^J}{(1 + \xi \phi_J^2)} \]  

(17.3)

It gives rise to the standard Einstein-Hilbert term \( -(1/2)R \) for gravity in the Lagrangian. However, it also leads to a nonminimal (or noncanonical) kinetic term of the scalar field \( \phi_J \). To get the canonical kinetic term, a scalar field redefinition is needed, \( \phi_J \rightarrow \varphi(\phi_J) \), subject to the condition

\[ \frac{d\varphi}{d\phi_J} = \sqrt{\frac{1 + \xi (1 + 6\xi) \phi_J^2}{1 + \xi \phi_J^2}} \]  

(17.4)

As a result, the non-minimal theory (17.1) is classically equivalent to the standard (canonical) theory of the scalar field \( \varphi(x) \) minimally coupled to gravity,

\[ L_E = \sqrt{-g} \left\{ -\frac{1}{2} R + \frac{1}{2} g_{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\} \]  

(17.5)

with the scalar potential

\[ V(\varphi) = \frac{V(\phi_J(\varphi))}{[1 + \xi \phi_J^2(\varphi)]^2} \]  

(17.6)

Given a large positive \( \xi \gg 1 \), in the small field limit one finds from eq. (17.4) that \( \phi_J \approx \varphi \), whereas in the large \( \varphi \) limit one gets

\[ \varphi \approx \sqrt{\frac{3}{2}} \log(1 + \xi \phi_J^2) \]  

(17.7)

Then eq. (17.6) yields the scalar potential:

(i) in the very small field limit, \( \varphi < \sqrt{\frac{3\xi}{8}} \), as

\[ V_{vs}(\varphi) \approx \frac{\lambda}{4} \varphi^4 \]  

(17.8)

(ii) in the small field limit, \( \sqrt{\frac{3\xi}{8}} < \varphi \ll \sqrt{\frac{3}{2}} \), as

\[ V_s(\varphi) \approx \frac{\lambda}{6\xi^2} \varphi^2, \]  

(17.9)
(iii) and in the large field limit, $\varphi \gg \sqrt{\frac{2}{3}} \xi^{-1}$, as

$$V(\varphi) \approx \frac{\lambda}{4\xi^2} \left( 1 - \exp \left[ -\sqrt{\frac{2}{3}} \varphi \right] \right)^2$$

(17.10)

We have assumed here that $\xi \gg 1$ and $v \xi \ll 1$.

Identifying inflaton with Higgs particle requires the parameter $v$ to be of the order of weak scale, and the coupling $\lambda$ to be the Higgs boson self-coupling at the inflationary scale. The scalar potential (17.10) is perfectly suitable to support a slow-roll inflation, while its consistency with the WMAP normalization condition (Sec. 4) for the observed CMB amplitude of density perturbations at the e-foldings number $N_e = 55$ gives rise to the relation $\xi/\sqrt{\lambda} \approx 5 \cdot 10^4$ [95, 96, 97].

The scalar potential (17.9) corresponds to the post-inflationary matter-dominated epoch described by the oscillating inflaton field $\varphi$ with the frequency

$$\omega = \sqrt{\frac{\lambda}{3}} \xi^{-1} = M_{\text{inf}}$$

(17.11)

When gravity is extended to 4D, $N = 1$ supergravity, any physical real scalar field should be complexified, becoming the leading complex scalar field component of a chiral (scalar) matter supermultiplet. In a curved superspace of $N = 1$ supergravity, the chiral matter supermultiplet is described by a covariantly chiral superfield $\Phi$ obeying the constraint $\nabla_\alpha \Phi = 0$. The standard (generic and minimally coupled) matter-supergravity action is given by in superspace by eqs. (6.11) and (6.13), namely,

$$S_{\text{MSG}} = -3 \int d^4x d^2\theta E^{-1} \exp \left[ -\frac{1}{3} K(\Phi, \overline{\Phi}) \right] + \left\{ \int d^4x d^2\theta E W(\Phi) + \text{H.c.} \right\}$$

(17.12)

in terms of the Kähler potential $K = -3 \log(-\frac{1}{3} \Omega)$ and the superpotential $W$ of the chiral supermatter, and the full density $E$ and the chiral density $\mathcal{E}$ of the superspace supergravity (Sec. 5).

The non-minimal matter-supergravity coupling in superspace reads

$$S_{\text{NM}} = \int d^4x d^2\theta \mathcal{E} X(\Phi) \mathcal{R} + \text{H.c.}$$

(17.13)

in terms of the chiral function $X(\Phi)$ and the N=1 chiral scalar supercurvature superfield $\mathcal{R}$ obeying $\nabla_\alpha \mathcal{R} = 0$. In terms of the field components of the superfields the non-minimal action (17.13) is given by

$$\int d^4x d^2\theta \mathcal{E} X(\Phi) \mathcal{R} + \text{H.c.} = -\frac{1}{6} \int d^4x \sqrt{-g} X(\phi_c) R + \text{H.c.} + \ldots$$

(17.14)

stand for the fermionic terms, and $\phi_c = |\Phi| = \phi + i\gamma$ is the leading complex scalar field component of the superfield $\Phi$. Given $X(\Phi) = -\xi \phi^2$ with the real coupling constant
\[ S_{\text{NM, bos.}} = \frac{1}{6}\xi \int d^4x \sqrt{-g} \left( \phi^2 - \gamma^2 \right) R \] (17.15)

It is worth noticing that the supersymmetrizable (bosonic) non-minimal coupling reads
\[ \left[ \phi_c^2 + (\phi_c^\dagger)^2 \right] R, \text{ not } (\phi_c^\dagger \phi_c) R. \]

Let us now introduce the manifestly supersymmetric nonminimal action (in Jordan frame) as
\[ S = S_{\text{MSG}} + S_{\text{NM}} \] (17.16)

In curved superspace of \( N = 1 \) supergravity the (Siegel’s) chiral integration rule
\[ \int d^4x d^2\theta \mathcal{L}_{\text{ch}} = \int d^4x d^4\theta E^{-1} \mathcal{L}_{\text{ch}} \] (17.17)

applies to any chiral superfield Lagrangian \( \mathcal{L}_{\text{ch}} \) with \( \nabla_\alpha \mathcal{L}_{\text{ch}} = 0 \). It is, therefore, possible to rewrite eq. (17.13) to the equivalent form
\[ S_{\text{NM}} = \int d^4x d^4\theta E^{-1} \left[ X(\Phi) + \bar{X}(\bar{\Phi}) \right] \] (17.18)

We conclude that adding \( S_{\text{NM}} \) to \( S_{\text{MSG}} \) is equivalent to the simple change of the \( \Omega \)-potential as (cf. ref. [99])
\[ \Omega \rightarrow \Omega_{\text{NM}} = \Omega + X(\Phi) + \bar{X}(\bar{\Phi}) \] (17.19)

It amounts to the change of the Kähler potential as
\[ K_{\text{NM}} = -3 \ln \left[ e^{-K/3} - \frac{X(\Phi) + \bar{X}(\bar{\Phi})}{3} \right] \] (17.20)

The scalar potential in the matter-coupled supergravity (17.12) is given by eq. (6.23),
\[ V(\phi, \bar{\phi}) = e^G \left[ \left( \frac{\partial^2 G}{\partial\phi\partial\bar{\phi}} \right)^{-1} \frac{\partial G}{\partial\phi} \frac{\partial G}{\partial\bar{\phi}} - 3 \right] \] (17.21)

in terms of the Kähler-gauge-invariant function (6.19), ie.
\[ G = K + \ln |W|^2 \] (17.22)

Hence, in the nonminimal case (17.16) we have
\[ G_{\text{NM}} = K_{\text{NM}} + \ln |W|^2 \] (17.23)

Contrary to the bosonic case, one gets a nontrivial Kähler potential \( K_{\text{NM}} \), ie. a \textit{Non-Linear Sigma-Model} (NLSM) as the kinetic term of \( \phi_c = \phi + i\gamma \) (see ref. [59] for
more about the NLSM). Since the NLSM target space in general has a nonvanishing curvature, no field redefinition generically exist that could bring the kinetic term to the free (canonical) form with its Kähler potential $K_{\text{free}} = \Phi \Phi$.

Let’s now consider the full action (17.16) under the slow-roll condition, i.e. when the contribution of the kinetic term is negligible. Then eq. (17.16) takes the truly chiral form

$$S_{\text{ch.}} = \int d^4 \theta d^2 \theta \mathcal{E} \left[ X(\Phi) \mathcal{R} + W(\Phi) \right] + \text{H.c.}$$

(17.24)

When choosing $X$ as the independent chiral superfield, $S_{\text{ch.}}$ can be rewritten to the form

$$S_{\text{ch.}} = \int d^4 \theta d^2 \theta \mathcal{E} \left[ X \mathcal{R} - Z(X) \right] + \text{H.c.}$$

(17.25)

where we have introduced the notation

$$Z(X) = -W(\Phi(X))$$

(17.26)

In its turn, the action (17.25) is equivalent to the chiral $F(\mathcal{R})$ supergravity action (6.1), whose function $F$ is related to the function $Z$ via Legendre transformation (Sec. 6)

$$Z = X \mathcal{R} - F, \quad F'(\mathcal{R}) = X \quad \text{and} \quad Z'(X) = \mathcal{R}$$

(17.27)

It implies the equivalence between the reduced action (17.24) and the corresponding $F(\mathcal{R})$ supergravity whose $F$-function obeys eq. (17.27).

Next, let us consider the special case of eq. (17.24) when the superpotential is given by

$$W(\Phi) = \frac{1}{2} m \Phi^2 + \frac{1}{6} \tilde{\lambda} \Phi^3$$

(17.28)

with the real coupling constants $m > 0$ and $\tilde{\lambda} > 0$. The model (17.28) is known as the Wess-Zumino (WZ) model in 4D, $N = 1$ rigid supersymmetry. It has the most general renormalizable scalar superpotential in the absence of supergravity. In terms of the field components, it gives rise to the Higgs-like scalar potential.

For simplicity, let us take a cubic superpotential,

$$W_3(\Phi) = \frac{1}{6} \tilde{\lambda} \Phi^3$$

(17.29)

or just assume that this term dominates in the superpotential (17.28), and choose the $X(\Phi)$-function in eq. (17.24) in the form

$$X(\Phi) = -\xi \Phi^2$$

(17.30)

with a large positive coefficient $\xi, \xi > 0$ and $\xi \gg 1$, in accordance with eqs. (17.14) and (17.15).

Let us also simplify the $F$-function of eq. (10.1) by keeping only the most relevant cubic term,

$$F_3(\mathcal{R}) = -\frac{1}{6} f_3 \mathcal{R}^3$$

(17.31)
It is straightforward to calculate the $Z$-function for the $F$-function (17.31) by using eq. (17.27). We find

$$-X = \frac{1}{2} f_3 R^2 \quad \text{and} \quad Z'(X) = \frac{\sqrt{-2X}}{f_3} \quad (17.32)$$

Integrating the last equation with respect to $X$ yields

$$Z(X) = \frac{2}{3} \sqrt{\frac{2}{f_3}} (-X)^{3/2} = -\frac{2\sqrt{2}}{3} \frac{\xi^{3/2}}{f_3^{1/2}} \Phi^3 \quad (17.33)$$

where we have used eq. (17.30). In accordance to eq. (17.26), the $F(R)$-supergravity $Z$-potential (17.33) implies the superpotential

$$W_{KS}(\Phi) = \frac{2\sqrt{2}}{3} \frac{\xi^{3/2}}{f_3^{1/2}} \Phi^3 \quad (17.34)$$

It coincides with the superpotential (17.29) of the WZ-model, provided that we identify the couplings as

$$f_3 = \frac{32 \xi^3}{\tilde{\lambda}^2} \quad (17.35)$$

We conclude that the original nonminimally coupled matter-supergravity theory (17.16) in the slow-roll approximation with the superpotential (17.29) is classically equivalent to the $F(R)$-supergravity theory with the $F$-function given by eq. (17.31) when the couplings are related by eq. (17.35).

The inflaton mass $M$ in the supersymmetric case, according to eqs. (10.11) and (17.35), is given by

$$M_{\text{inf}}^2 = \frac{15 \tilde{\lambda}^2}{32 \xi^3} \quad (17.36)$$

Since the value of $M_{\text{inf}}$ is fixed by the WMAP normalization (Sec. 4), the value of $\xi$ in the supersymmetric case is $\xi_{\text{susy}}^3 = (45/32) \xi_{\text{bos}}^2$, or $\xi_{\text{susy}} \approx 10^3$, i.e. is lower than that in the bosonic case. We have assumed here that $\tilde{\lambda} \approx O(1)$.

The established equivalence begs for a fundamental reason. In the high-curvature (inflationary) regime the $R^2$-term dominates over the $R$-term in the Starobinsky action (2.3), while the coupling constant in front of the $R^2$-action is dimensionless (Sect. 2). The Higgs inflation is based on the Lagrangian (17.1) with the relevant scalar potential $V_4 = \frac{1}{4} \lambda \phi^4$ (the parameter $v$ is irrelevant for inflation), whose coupling constants $\xi$ and $\lambda$ are also dimensionless. Therefore, both relevant actions are scale invariant. Inflation breaks that symmetry spontaneously.

The supersymmetric case is similar: the nonminimal action (17.24) with the $X$-function (17.30) and the superpotential (17.29) also have only dimensionless coupling constants $\xi$ and $\tilde{\lambda}$, while the same is true for the $F(R)$-supergravity action with the
The basic field theory model, describing both inflation and the subsequent reheating, reads (see e.g., eq. (6) in ref. [85, 86])

\[
L/\sqrt{-g} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \frac{1}{2} \partial_\mu \gamma \partial^\mu \gamma - \frac{1}{2} m^2 \gamma^2 + \frac{1}{2} \xi R \gamma^2 + \bar{\psi}(i\gamma^\mu \partial_\mu - m_\psi)\psi
\]

\[
- \frac{1}{2} g^2 \phi^2 \gamma^2 - h(\bar{\psi}\psi)\phi
\]

(17.37)

with the inflaton scalar field \(\phi\) interacting with another scalar field \(\gamma\) and a spinor field \(\psi\). The nonminimal supergravity theory (17.16) with the Wess-Zumino superpotential (17.28) can be considered as the \(N = 1\) locally supersymmetric extension of the basic model (17.37) after rescaling \(\phi_c\) to \((1/\sqrt{2})\phi_c\) and identifying \(\tilde{\xi} = -\frac{1}{3} \xi\) because of eq. (17.15). Therefore, pre-heating (i.e. the nonperturbative enhancement of particle production due to a broad parametric resonance [85, 86]) is a generic feature of our supergravity models.

The axion \(\gamma\) and fermion \(\psi\) are both required by supersymmetry, being in the same chiral supermultiplet with the inflaton \(\phi\). The scalar interactions are

\[
V_{\text{int}}(\phi, \gamma) = m\lambda\phi(\phi^2 + \gamma^2) + \frac{\lambda^2}{4}(\phi^2 + \gamma^2)^2
\]

(17.38)

whereas the Yukawa couplings are given by

\[
L_{\text{Yu}} = \frac{1}{2} \lambda\phi(\bar{\psi}\psi) + \frac{1}{2} \lambda\gamma(\bar{\psi}\gamma_5\psi)
\]

(17.39)

Supersymmetry implies the unification of couplings since \(h = -\frac{1}{2} \lambda\) and \(g^2 = \lambda^2\) in terms of the single coupling constant \(\tilde{\lambda}\). If supersymmetry is unbroken, the masses of \(\phi, \gamma\) and \(\psi\) are all the same. However, inflation already breaks supersymmetry, so the spontaneously broken supersymmetry is appropriate here.

To conclude, inflationary slow-roll dynamics in Einstein gravity theory with a nonminimal scalar-curvature coupling can be equivalent to that in the certain \(f(R)\) gravity theory. We just extended that correspondence to \(N = 1\) supergravity. The nonminimal coupling in supergravity can be rewritten in terms of the standard (‘minimal’) \(N = 1\) matter-coupled supergravity, by using their manifestly supersymmetric formulations in curved superspace. The equivalence relation between the supergravity theory with the nonminimal scalar-curvature coupling and the \(F(R)\) supergravity during slow-roll inflation is, therefore, established.

The equivalence is expected to hold even after inflation, during initial reheating with harmonic oscillations. In the bosonic case the equivalence holds until the inflaton...
field value is higher than $\omega \approx M_{Pl}/\xi_{bos} \approx 10^{-5}M_{Pl}$. In the supersymmetric case we have the same bound $\omega \approx M_{Pl}/\xi_{susy}^{2/3} \approx 10^{-5}M_{Pl}$.

The Higgs inflation and the renormalization group can be used to compute the mass of a Higgs particle in the Standard Model by descending from the inflationary scale to the electro-weak scale. For example, in the two-loop approximation one finds [100]

$$129 \text{ GeV} < m_{H} < 194 \text{ GeV}$$

with the theoretical uncertainty of about $\pm 2 \text{ GeV}$. It is to be compared to the observed Higgs mass at the Linear Hadron Collider (LHC) in 2012 [101]

$$\text{LHC (ATLAS)} : m_{H} = 126 \pm 0.8 \text{ GeV}$$

Therefore, the bosonic Higgs inflation is (almost) ruled out. It is worth noticing that in a supersymmetric extension of the SM (like the MSSM and NMSSM) there are more particles, when compared to the bosonic SM. Hence, the SUSY renormalization group trajectory is going to be steeper, while the theoretical SUSY bounds on the Higgs mass at the electro-weak scale are going to be lower than those in eq. (17.40).

18 Quantum Particle Production (Reheating)

Reheating is a transfer of energy from inflaton to ordinary particles and fields. It took place after inflation but before BBN and hot radiation domination. All particles in the universe are believed to be created via the inflaton decay soon after the inflation. The leading channel of the particle production is preheating (due to the nonperturbative parametric resonance). The resonance eventually disappeared when the inflaton field became sufficiently small, and it was replaced by perturbative decay. The reheating provided initial conditions for the BBN that began after the first 3 minutes (such as the initial temperature of baryogenesis, DM abundance, relic monopoles and gravitinos, etc.). Both preheating and reheating are highly model-dependent. In our approach we advocate the (super)gravitational preheating and reheating due to the universal coupling of (super)inflaton to conformally noninvariant fields (see also Ref. [102]).

The classical solution (neglecting particle production) near the minimum of the inflaton scalar potential reads

$$a(t) \approx a_{0}\left(\frac{t}{t_{0}}\right)^{2/3} \quad \text{and} \quad \varphi(t) \approx \left(\frac{M_{Pl}}{3M_{inf}}\right)\cos\left[M_{inf}(t - t_{0})\right]$$

A time-dependent classical spacetime background leads to quantum production of particles with masses $m < \omega = M_{inf}$ [98]. Actually, the amplitude of $\varphi$-oscillations decreases much faster [85, 86], namely, as

$$\exp\left[-\frac{1}{2}(3H + \Gamma)t\right]$$
via inflaton decay and the universe expansion, as the solution to the inflaton equation

\[ \ddot{\phi} + 3H \dot{\phi} + (m^2 + \Pi)\phi = 0 \]  

(18.3)

Here \( \Pi \) denotes the polarization operator that effectively describes particle production. Unitarity (optical theorem) requires \( \text{Im}(\Pi) = m \Gamma \). The assumption \( m \gg H \) was used here [85, 86].

The Starobinsky model (in Jordan frame) with the action

\[ S = \int d^4x \sqrt{-g_J} f_S(R_J) + S_{\text{SM}}(g_{\mu\nu}^J, \psi) \]  

(18.4)

after the conformal transformation to Einstein frame takes the form

\[ S = S_{\text{scalar–tensor gravity}}(g_{\mu\nu}, \phi) + S_{\text{SM}}(g_{\mu\nu} e^{-\sigma \phi}, \psi) \]  

(18.5)

so that the inflaton \( \phi \) couples to all non-conformal terms and fields \( \psi \), due to the universality of gravitational interaction. Therefore, the Starobinsky inflation automatically leads to the universal mechanism of particle production.

For example, let us consider the scalar and spinor fields in the Jordan frame, with the action

\[ S = -\frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}} \]  

(18.6)

where the matter is represented by the standard Klein-Gordon and Dirac actions, \( S_{\text{matter}} = S_{\text{KG}} + S_{\text{Dirac}} \), with the minimal coupling to gravity,

\[ S_{\text{KG}} = \int d^4x \sqrt{-g} \left( \frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi \phi^2 \right) \]  

(18.7)

and

\[ S_{\text{Dirac}} = \int d^4x \sqrt{-g} \left( i \bar{\psi} \tilde{D} \psi - m_\psi \bar{\psi} \psi \right) \]  

(18.8)

After rewriting the full action to the Einstein frame by a Weyl transformation of the metric with the scalaron field \( \phi \),

\[ g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}, \quad \Omega(\phi) = \exp \left[ \sqrt{2/3} \phi / M_\text{Pl} \right], \]  

(18.9)

and rescaling the matter scalar and spinor fields to get their canonical kinetic terms as

\[ \phi \rightarrow \tilde{\phi} = \Omega^{-1/2} \phi, \quad \psi \rightarrow \tilde{\psi} = \Omega^{-3/4} \psi, \]  

(18.10)

where we have used \( \tilde{D} = \gamma^\mu D_\mu = e_a^\mu \gamma^\alpha D_\mu \) and \( \tilde{D} \rightarrow \Omega^{-1/2} \tilde{D} \), one finds

\[ S = S_{\text{quintessence}}[\phi, \tilde{g}] + S_{\text{KG}}[\tilde{\phi}, \tilde{g}, \phi] + S_{\text{Dirac}}[\tilde{\psi}, \tilde{g}, \phi] \]  

(18.11)
where \[ S_{KG}[\tilde{\varphi}, \tilde{g}, \varphi] = \int d^4x \sqrt{-\tilde{g}} \left( \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\varphi} \partial_\nu \tilde{\varphi} - \frac{1}{2} \Omega^{-1} m_\varphi^2 \tilde{\varphi}^2 + \frac{\tilde{\varphi}^2}{12M^2_{Pl}} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{\tilde{\varphi}^2}{\sqrt{6}M_{Pl}} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) \] (18.12)

and \[ S_{Dirac}[\tilde{\psi}, \tilde{g}, \varphi] = \int d^4x \sqrt{-\tilde{g}} \left( i\tilde{\psi} \tilde{D} \tilde{\psi} - \Omega^{-1/2} m_\psi \tilde{\psi} \tilde{\psi} \right) \] (18.13)

As is clear from those equations, all interactions with inflaton are suppressed by the factors of \( M_{Pl} \). Hence, they are only relevant for the large \( \phi \)-values comparable to \( M_{Pl} \). Those interactions (and decay rates) are sensitive to the mass and spin of the created particles. The conformal couplings to not contribute to the inflaton decay [106]. In particular, the dominant contribution to the inflaton decay rate in the scalar channel comes from the 3rd term in the action \( S_{KG}[\tilde{\varphi}, \tilde{g}, \varphi] \) of eq. (18.12). The only contribution to the inflaton decay rate in the spinor channel comes from the mass term in \( S_{Dirac}[\tilde{\psi}, \tilde{g}, \varphi] \) of eq. (18.13).

The perturbative decay rates of the inflaton into a pair of scalars \((s)\) or into a pair of spin-1/2 fermions \((f)\) are given by [17, 18, 103]

\[
\Gamma_{\varphi \to ss} = \frac{M_3^3}{192\pi M^2_{Pl}} \quad \text{and} \quad \Gamma_{\varphi \to ff} = \frac{M_{inf} M_f^2}{48\pi M^2_{Pl}},
\] (18.14)

respectively. The perturbative decay rate of the inflaton into a pair of gravitino is [104]

\[
\Gamma_{\varphi \to 2\psi_{3/2}} = \frac{|G_{\varphi \psi}|^2}{288\pi} \frac{M^5_{inf}}{m_{3/2}^2 M^2_{Pl}},
\] (18.15)

Being proportional to \( M^5_{inf} \), eq. (18.15) may lead to the cosmologically disastrous gravitino overproduction in early universe [105], if the gravitino mass is relatively small (under 100 GeV). In the case of the large-field inflation, when the inflaton expectation value has the order of the Planck mass (it includes the Starobinsky inflation), one can demonstrate that eq. (18.15) reduces to the scalar decay rate (18.14) proportional to \( M^3_{inf} \) [105].

The energy transfers by the time \( t_{reh} \geq \left( \sum_{s,f} \Gamma_{s,f} \right)^{-1} \). The reheating temperature is given by [45, 106]

\[
T_{reh} \propto \sqrt{\frac{M_{Pl} \Gamma}{(\#d.o.f.)^{1/2}}} \approx 10^9 \text{ GeV}
\] (18.16)

that gives the maximal temperature of the primordial plasma.

In the context of supergravity coupled to the supersymmetric matter (like MSSM) gravitino can be either LSP (= the lightest sparticle) or NLSP (= not LSP). In the
LSP case (that usually happens with gauge mediation of supersymmetry breaking and $m_{3/2} \ll 10^2$ GeV) gravitino is stable due to the R-parity conservation. If gravitino is NLSP, then it is unstable (it usually happens with gravity- or anomaly- mediation of supersymmetry breaking, and $m_{3/2} \gg 10^2$ GeV). Unstable gravitino can decay into LSP. See ref. [107] for a review of mediation of supersymmetry breaking from the hidden sector to the visible sector.

Stable gravitino may be the dominant part of Cold Dark Matter (CDM) [108]. There exist severe Big Bang Nucleosynthesis (BBN)\(^6\) constraints on the overproduction of $^3He$ in that case, which give rise to the upper bound on the reheating temperature of thermally produced gravitinos, $T_{\text{reh}} < 10^{5.5}$ GeV [110, 111]. The reheating temperature (18.16) is unrelated to that bound because it corresponds to the much earlier time in the history of the Universe.

When gravitino is NLSP of mass $m_{3/2} \gg 10^2$ GeV, the BBN constraints are drastically relaxed because the gravitino lifetime becomes much shorter than the BBN time (about 1 sec) [110, 111]. In that case the most likely CDM candidate is MSSM neutralino, while the reheating temperature may be as high as $10^{10}$ GeV [111].

An overproduction of gravitinos from inflaton decay and scattering processes should be avoided, in order to prevent overclosure of the universe. The cosmological constraints on gravitino abundances were formulated in ref. [105]. Those constraints are very model-dependent.

The rate of decay changes with time, along with the decreasing amplitude of inflaton oscillations. It stops when the decay rate becomes smaller than the production rate. The reheating transfers most of energy to radiation, and leads to a radiation-dominated universe with $a \propto t^{1/2}$.

In the matter-coupled $F(\mathcal{R})$ supergravity with the action

$$S = \left[ \int d^4x d^2\theta \mathcal{E} F(\mathcal{R}) + \text{H.c.} \right] + S_{\text{SSM}}(E, \Psi)$$

(18.17)

after the super-Weyl transformation, $\mathcal{E} \rightarrow \mathcal{E} e^{3\phi}$, we get

$$S = S_{\text{scalar--tensor supergravity}}(E, \Phi) + S_{\text{SSM}}(e^{\Phi+i\Phi} E, \Psi)$$

(18.18)

so that the superscalaron $\Phi$ is universally coupled to the SSM matter superfields $\Psi$.

### 19 Conclusion

- A manifestly 4D, $N = 1$ supersymmetric extension of $f(R)$ gravity exist, it is chiral and is parametrized by a holomorphic function. An $F(\mathcal{R})$ supergravity is classically equivalent to the scalar-tensor theory of a chiral scalar superfield (with certain Kähler potential and superpotential) minimally coupled to the $N = 1$ Poincaré

---

\(^6\)See ref. [109] for a review of BBN.
supergravity in four spacetime dimensions (with nontrivial \( G \) and \( K \)), i.e. the \( N = 1 \) supersymmetric quintessence.

The classical equivalence between the \( F(\mathcal{R}) \) supergravity and the quintessence \( N=1 \) supergravity has the same physical contents as the classical equivalence between \( f(\mathcal{R}) \) gravity and scalar-tensor gravity, i.e. the same inflaton scalar potential and, therefore, the same inflationary dynamics. However, the physical nature of inflaton in the \( f(\mathcal{R}) \) gravity and the scalar-tensor gravity is very different. In the \( f(\mathcal{R}) \) gravity the inflaton field is the spin-0 part of metric, whereas in a generic scalar-tensor gravity inflaton is a matter particle. The inflaton interactions with other matter fields are, in general, different in both theories. It gives rise to the different inflaton decay rates and different reheating, i.e. implies different physics in the post-inflationary universe.

Similar remarks apply to the equivalence between Higgs inflation and Starobinsky inflation (Sec. 12). The equivalence does not have to be valid after inflation. For example, the reheating temperature \( T_{\text{reh}} \) after the Higgs inflation is about \( 10^{13} \) GeV \([95, 96, 97]\), whereas after the Starobinsky inflation one has \( T_{\text{reh}} \approx 10^9 \) GeV \([18]\), or the one order more in the supersymmetric case.

- It is expected that the classical equivalence is broken in quantum theory because the classical equivalence is achieved via a non-trivial field redefinition (Secs. 3 and 6). When doing that field redefinition in the quantum path integrals defining those quantum theories (unter their unitarity bounds), it gives rise to a non-trivial Jacobian that already implies the quantum inequivalence, even before taking into account renormalization.\(^7\)

In the supergravity case, there is one more reason for the quantum inequivalence between the \( F(\mathcal{R}) \) supergravity and the classically equivalent quintessence supergravity. The Kähler potential of the scalar superfield is described by a full superspace integral and, therefore, it receives quantum corrections that can easily spoil classical solutions describing an accelerating universe (those corrections are not under control). It was the reason for introduction of flat directions in the Kähler potential and popular realizations of inflation in supergravity by the use of a chiral scalar superpotential along the flat directions \([58, 60, 57]\). The \( F(\mathcal{R}) \) supergravity action is truly chiral, so that the function \( F(\mathcal{R}) \) is already protected against the quantum perturbative corrections given by full superspace integrals. It is the important part of physical motivation for \( F(\mathcal{R}) \) supergravity. It also explains why we consider \( F(\mathcal{R}) \) supergravity as the viable and self-consistent alternative to the Kähler flat directions for realizing slow-roll inflation in supergravity. Of course, one can also consider both ways together \([102]\).

- The Starobinsky model of chaotic inflation can be embedded into \( F(\mathcal{R}) \) supergravity. It is the viable realization of chaotic inflation in supergravity, and gives a simple solution to the \( \eta \)-problem.

- A simple extension of our inflationary model (Sec. 16) has a positive cosmological constant in the regime of low spacetime curvature (Secs. 10 and 11). It is non-trivial because the standard supergravity with usual matter can only have a nega-

---

\(^7\)See ref. \([112]\) for the first steps of quantization with a higher time derivative.
tive or vanishing cosmological constant \[113\]. It happens because the usual (known) matter does not violate the **Strong Energy Condition** (SEC) \[114\]. A violation of the SEC is required for an accelerating universe, and is easily achieved in \( f(R) \) gravity due to the fact that the quintessence field in \( f(R) \) gravity is part of metric (i.e. the unusual matter). Similarly, the quintessence scalar superfield in \( F(R) \) supergravity is part of super-vielbein, and also gives rise to a violation of the SEC.

In the \( F(R) \) supergravity model we considered (Secs. 10 and 11), the effective \( f(R) \) gravity function in the high-curvature regime is essentially given by the Starobinsky function \((-\frac{M^2_{Pl}}{2}R + \frac{M^2_{Pl}}{12M^2_{inf}}R^2)\). In the low-curvature regime it is essentially given by the Einstein-Hilbert function with a cosmological constant, \((-\frac{M^2_{PL}}{2}R - \Lambda)\). Therefore, our model has a cosmological solution describing an inflationary universe of the quasi-dS type with \( H(t) = (M^2_{inf}/6)(t_{end} - t) \) at early times \( t < t_{end} \), and an accelerating universe of the dS-type with \( H = \Lambda \) at late times.

The dynamical chiral superfield in \( F(R) \) supergravity may be identified with the dilaton-axion chiral superfield in quantum 4D Superstring Theory, when demanding the \( SL(2, \mathbb{Z}) \) symmetry of the effective action. As is well known, String Theory supports the higher-derivative gravity. In particular, the required \( R^2A(R) \) terms may appear in the (nonperturbative) gravitational effective action after superstring compactification (with fluxes, after moduli stabilization). The problem is how to get the anomalously large coefficient in front of the \( R^3 \)-term in the effective \( F(R) \) supergravity theory that would be consistent with the superstring dynamics.

Supersymmetry in \( F(R) \) supergravity is broken by inflation but is restored near the minimum of the scalar potential. The anomaly- or gravitationally-mediated supersymmetry breaking (in the hidden sector) may serve as the important element for the new particle phenomenology (beyond the Standard Model) based on the matter-coupled \( F(R) \) supergravity theory.

### 20 Outlook: \( CP \)-violation, Baryonic Asymmetry, Lepto- and Baryo-genesis, Non-Gaussianity, Tests

The observed part of our Universe is highly \( C \)- and \( CP \)-asymmetric (no antimatter). Inflation naturally implies a dynamical origin of the baryonic matter predominance due to a nonconserved baryon number. The main conditions for the dynamical generation of the cosmological baryon asymmetry in early universe were formulated in Ref. [31]:

1. nonconservation of baryons (cf. SUSY, GUT, EW theory),
2. \( C \)- and \( CP \)-symmetry breaking (confirmed experimentally),
3. deviation from thermal equilibrium in initial hot universe.
The first condition is clearly necessary. And (in theory) there is no fundamental reason for the baryon number conservation. The baryon asymmetry should have originated from spontaneous breaking of the $CP$ symmetry that was present at very early times, so is the need for the second condition. Then the third condition is required by the $CPT$ symmetry, when the $CP$-violation is compensated by the $T$-violation, so it has to be no thermal equilibrium.

There exist many scenarios of baryogenesis (see ref. [74] for a review), all designed to explain the observed asymmetry (BBN, CMB):

$$\beta = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.0 \pm 0.5) \cdot 10^{-10} \quad (20.1)$$

Here $n_B$ stands for the concentration of baryons, $n_{\overline{B}}$ for the concentration of antibaryons, and $n_\gamma$ for the concentration of photons.

Perhaps, the most popular scenario is the nonthermal baryo-through-lepto-genesis [75, 73], i.e., a creation of lepton asymmetry by L-nonconserving decays of a heavy ($m \approx 10^{10}$ GeV) Majorana neutrino, and a subsequent transformation of the lepton asymmetry into the baryonic asymmetry by $CP$-symmetric, B-nonconserving and (B-L)-conserving electro-weak processes.

The thermal leptogenesis requires the high reheating temperature, $T_{\text{reh}} \geq 10^9$ GeV [115], which is consistent with eq. (18.16).

The matter-coupled $F(\mathcal{R})$ supergravity theory may contribute towards the origin and the mechanism of $CP$-violation and baryon asymmetry, because

- complex coefficients of $F(\mathcal{R})$-function and the complex nature of the $F(\mathcal{R})$ supergravity are the simple source of explicit $CP$-violation and complex Yukawa couplings;
- the nonthermal leptogenesis is possible via decay of heavy sterile neutrinos (FY-mechanism) universally produced by (super)scalaron decays, or via neutrino oscillations in early universe [116];
- the existence of the natural Cold Dark Matter candidates (gravitino, axion, inflatino or, maybe, inflaton itself!) in $F(\mathcal{R})$ supergravity;
- as is well known, non-Gaussianity is a measure of inflaton interactions described by its 3-point functions and higher – cf. eq. (4.3). The non-Gaussianity parameter $f_{NL}$ is defined in terms of the (gauge-invariant) comoving curvature perturbations as

$$\hat{\mathcal{R}} = \hat{\mathcal{R}}_{\text{gr}} + \frac{3}{5} f_{NL} \hat{\mathcal{R}}_{\text{gr}}^2 \quad (20.2)$$

The non-Gaussianity was not observed yet, though it is expected. As regards the single-field inflationary models, they predict [117]

$$f_{NL} = \frac{5}{12} (1 - n_s) \approx 0.02 \quad (20.3)$$

The Starobinsky inflation is known to yield highly Gaussian fluctuations, which is consistent with the recent Planck (2013) data [24].
Finally, we would like to comment on possible testing of $f(R)$ gravity and $F(R)$ supergravity in Solar system and ground-based experiments.

As regards the large-scale structure of the present universe, the scalaron (i.e. the dynamical spin-0 part of metric) may be responsible for its acceleration or Dark Energy. However, since scalaron is universally coupled to all matter with gravitational strength, it may lead to an unacceptable violation of the equivalence principle. To avoid it, the scalaron should be “screened off” on the Solar system scales, because of the strong observational constraints from experimental tests of the equivalence principle [118, 119]. Moreover, it should not give rise to a large violation of the equivalence principle in ground-based (on Earth) laboratories, because of the tight constraints on the fifth fundamental force in Nature [120].

A natural solution to both problems is provided by Chameleon Cosmology [121, 122], because the effective scalaron mass is dependent upon a local matter density $\rho$ (see also refs. [123, 124]). The effective scalar potential of the scalaron (Chameleon) field takes the form

$$V_{\text{eff}}(\varphi) = V(\varphi) + \rho \exp(\beta \varphi/M_{\text{Pl}})$$

(20.4)

where the parameter $\beta$ is of the order 1. The exponential factor here arises due to the universal coupling of the scalaron to the matter of density $\rho$ — see eq. (18.5). As a result, the effective Chameleon mass is about $\rho$, so that in a sufficiently dense environment one can evade the observational constraints on the equivalence principle and the fifth force.

**Acknowledgements**

I am grateful to my collaborators: S.J. Gates Jr., A.A. Starobinsky, S. Tsujikawa, T. Terada and N. Yunes and my students: S. Kaneda and N. Watanabe, for their efforts. I wish to thank the Theory Division of CERN in Geneva, the Institute of Theoretical Physics in Hannover, the Heisenberg Institute of Physics in Munich, the Einstein Institute of Gravitational Physics in Potsdam, the DESY Theory Group in Hamburg and the Center of Theoretical Physics in Marseille for their kind hospitality extended to me during preparation of this paper. I also thank Hiroyuki Abe, Joseph Buchbinder, Gia Dvali, Antonio De Felice, Richard Grimm, Koichi Hamaguchi, Artur Hebecker, Simeon Hellerman, Norihiro Iizuka, Satoshi Iso, Renata Kallosh, Sergey Kuzenko, Kazunori Kohri, Kei-ichi Maeda, Mihail Shaposhnikov, Liam McAllister, Stefan Theisen, Roland Triay, Alexander Westphal, Bernard de Wit, Masahide Yamaguchi, Tsutomu Yanagida and Norimi Yokozaki for useful discussions.

This work was supported in part by the TMU Graduate School of Science and Engineering in Tokyo, the World Premier International Research Center Initiative of MEXT in Japan, the German Academic Exchange Service (DAAD), the Max-Planck Institute of Physics in Munich, the Max-Planck Institute of Gravitational Physics in Potsdam, and the SFB 676 of the University of Hamburg and DESY in Germany.
Appendix A: Scalar Potential in $F(\mathcal{R})$ Supergravity

The exact Kähler potential and the superpotential in a generic $F(\mathcal{R})$ Supergravity described by the action (6.1) with the fixed chiral compensator are found in Sec. 13 — see eqs. (13.7) and (13.1), respectively. It is, therefore, straightforward to compute the full scalar potential by the use of eqs. (6.16) or (6.23), with all gravitational corrections included.

Equation (6.23) in the units with $M_{\text{Pl}} = 1$ for the chiral superpotential $Z(\mathcal{Y})$ reads

$$V = e^G \left[ \frac{\partial G}{\partial \mathcal{Y}} \left( \frac{\partial^2 G}{\partial \mathcal{Y}^2} \right)^{-1} \frac{\partial G}{\partial \bar{\mathcal{Y}}} - 3 \right]_{\mathcal{Y} = \mathcal{Y}} \tag{A.1}$$

in terms of the Kähler gauge-invariant function $G(\mathcal{Y}, \bar{\mathcal{Y}}) = K(\mathcal{Y}, \bar{\mathcal{Y}}) + \ln |Z(\mathcal{Y})|^2$. Substituting the Kähler potential of eq. (13.7) yields the scalar potential in the form

$$V = \frac{1}{3(\mathcal{Y} + \bar{\mathcal{Y}})} \left\{ \left| \frac{\partial Z}{\partial \mathcal{Y}} \right|^2 - \frac{3}{\mathcal{Y} + \bar{\mathcal{Y}}} \left( Z \frac{\partial Z}{\partial \mathcal{Y}} + \bar{Z} \frac{\partial \bar{Z}}{\partial \bar{\mathcal{Y}}} \right) \right\} \tag{A.2}$$

In the case of the cubic Ansatz (10.1) for the $F(\mathcal{R})$ function, we find

$$Z(\mathcal{Y}) = \sqrt{\frac{14}{60}} \frac{M^2}{m} \left\{ 3(\mathcal{Y} - 3/4) - 2(\mathcal{Y} - 3/4) \sqrt{1 + \frac{80m^2}{21M^2}(\mathcal{Y} - 3/4)} \right. \\
+ \left. \frac{21M^2}{40m^2} \left( 1 - \sqrt{1 + \frac{80m^2}{21M^2}(\mathcal{Y} - 3/4)} \right) \right\} \tag{A.3}$$

When substituting it into eq. (A.2) one arrives at a very lengthy formula for the scalar potential with many square roots, which is not very illuminating. It is therefore, no surprise that such scalar potentials were not investigated earlier.

We would like to emphasize that in our approach there is no need to use the scalar potential because it is much easier to work in the original picture with the $F$-function. See recent refs. [125, 126] for the different (non-minimal) approaches to the Starobinsky inflation in supergravity by the use of two or three chiral superfields.

References

[1] Wilkinson Microwave Anisotropy Probe (WMAP), http://map.gsfc.nasa.gov
[2] Planck Science Team Home, http://www.rssd.esa.int/index.php?project=planck
[3] S.J. Gates, Jr., and S.V. Ketov, Phys. Lett. B674 (2009) 59
[4] S.V. Ketov, Class. and Quantum Grav. 26 (2009) 135006
[5] S.J. Gates, Jr., S.V. Ketov and N. Yunes, Phys. Rev. D80 (2009) 065003
[6] S.V. Ketov, AIP Conf. Proc. 1241 (2010) 613
[7] S.V. Ketov, Phys. Lett. B692 (2010) 272
[8] S. Kaneda, S.V. Ketov and N. Watanabe, Mod. Phys. Lett. A25 (2010) 2753
[9] S. Kaneda, S.V. Ketov and N. Watanabe, Class. and Quantum Grav. 27 (2010) 145016
[10] S.V. Ketov and N. Watanabe, JCAP 1103 (2011) 011
[11] S.V. Ketov and A.A. Starobinsky, Phys. Rev. D83 (2011) 063512
[12] S.V. Ketov and N. Watanabe, Phys. Lett. B705 (2011) 410
[13] S.V. Ketov and A.A. Starobinsky, JCAP 1208 (2012) 022
[14] S.V. Ketov and S. Tsujikawa, Phys. Rev. D86 (2012) 023529
[15] S.V. Ketov and N. Watanabe, Mod. Phys. Lett. A27 (2012) 1250225
[16] S.V. Ketov and T. Terada. New actions for modified gravity and supergravity, arXiv:1304.4319 [hep-th]
[17] A.A. Starobinsky, Phys. Lett. B91 (1980) 99
[18] A.A. Starobinsky, Nonsingular model of the Universe with the quantum-gravitational de Sitter stage and its observational consequences, in the Proceedings of the 2nd Intern. Seminar “Quantum Theory of Gravity” (Moscow, 13–15 October, 1981); INR Press, Moscow 1982, p. 58 (reprinted in “Quantum Gravity”, M.A. Markov and P.C. West Eds., Plemum Publ. Co., New York, 1984, p. 103)
[19] A.H. Guth, Phys.Rev. D23 (1981) 347
[20] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220
[21] A.D. Linde, Phys. Lett. B108 (1982) 389
[22] E. Komatsu et al., (WMAP7), Astrophys. J. Suppl. 192 (2011) 18
[23] G. Hinshaw et al., Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results, arXiv:1212.5226 [astro-ph]
[24] P. Ade at al. (Planck Collaboration, 2013), arXiv:1303.5083, 5082 and 5076 [astro-ph]
[25] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood, Chur, Switzerland, 1990

[26] A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large-scale Structure*, Cambridge University Press, Cambridge, 2000

[27] V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, Cambridge, 2005

[28] A.R. Liddle and D.H. Lyth, *The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure*, Cambridge University Press, 2009

[29] D.S. Gorbunov and V.A. Rubakov, *Introduction to the Theory of the Early Universe*, World Scientific, in two Volumes, 2010 and 2011

[30] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press, Oxford, 2002

[31] A.D. Sakharov, Pis’ma ZhETF 5 (1967) 32

[32] R.P. Woodard, Lect. Notes Phys. 720 (2007) 403

[33] T.P. Sotiriou and V. Varaoni, Rev. Mod. Phys. 82 (2010) 451

[34] A. De Felice and S. Tsujikawa, Living Rev. Rel. 13 (2010) 3

[35] S. Capozziello and M. De Laurentis, Phys. Rept. 509 (2011) 167

[36] A.D. Linde, Phys. Lett. B 129 (1983) 177

[37] D. Wands, Class. and Quantum Grav. 11 (1994) 269

[38] V. Müller, H.-J. Schmidt and A.A. Starobinsky, Phys. Lett. B202 (1988) 198

[39] B. Whitt, Phys. Lett. B145 (1984) 176

[40] J.D. Barrow and S. Cotsakis, Phys. Lett. B214 (1988) 515

[41] K.-I. Maeda, Phys. Rev. D39 (1989) 3159

[42] S. Dodelson, *Modern Cosmology*, Elsevier, 2003

[43] S. Weinberg, *Cosmology*, Oxford Univ. Press, 2008

[44] V.F. Mukhanov and G.V. Chibisov, JETP Lett. 33 (1981) 532

[45] A.A. Starobinsky, Sov. Astron. Lett. 9 (1983) 302

[46] G.F. Smoot, Astrophys. J. 396 (1992) L1; http://en.wikipedia.org/wiki/Cosmic.Background.Explorer

66
[47] A. Linde, M. Noorbala and A. Westphal, JCAP 1103 (2011) 013

[48] S.J. Gates, Jr., M.T. Grisaru, M. Roček and W. Siegel, *Superspace or 1001 Lessons in Supersymmetry*, Benjamin-Cummings Publ. Company, 1983

[49] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton Univ. Press, 1992

[50] I.L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity*, IOP Publ., 1998

[51] S.J. Gates, Jr., Phys. Lett. B365 (1996) 132

[52] S.J. Gates, Jr., Nucl. Phys. B485 (1997) 145

[53] S. Cecotti, Phys. Lett. B190 (1987) 86

[54] P.S. Howe and R.W. Tucker, Phys. Lett. B80 (1978) 138

[55] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B147 (1979) 105

[56] D. Lyth and A. Riotto, Phys. Rept. 314 (1999) 1, hep-ph/9807278

[57] M. Yamaguchi, Class. and Quantum Grav. 28 (2011) 103001

[58] H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. D50 (1994) 2356

[59] S.V. Ketov, *Quantum Non-linear Sigma-models*, Springer-Verlag, 2000

[60] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. 85 (2000) 3572

[61] P.B. Binetruy and G. Dvali, Phys. Lett. B388 (1996) 241

[62] P. Berglund and G. Ren, *Multi-field inflation from string theory*, arXiv:0912.1397 [hep-th]

[63] P. Berglund and G. Ren, *Non-Gaussianity in string cosmology: a case study*, arXiv:1010.3261 [hep-th]

[64] M. R. Douglas and S. Kachru, Rev. Mod. Phys. 79 (2007) 733

[65] R. Kallosh, Lecture Notes Phys. 738 (2008) 119

[66] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B133 (1983) 61

[67] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D68 (2003) 046005
[68] K.-I. Maeda, Phys. Rev. **D37** (1988) 858
[69] A.L. Berkin and K.-I. Maeda, Phys. Lett. **B245** (1990) 348
[70] M. Abramowitz and I.A. Stegun, Eds., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York, Dover, 1972, p. 17
[71] L.A. Kofman, A.D. Linde and A.A. Starobinsky, Phys. Lett. **B157** (1985) 361
[72] A.A. Starobinsky, JETP Lett. **34** (1981) 438
[73] M. Fukugita and T. Yanagida, Phys. Lett. **B174** (1986) 45
[74] M. Dine and A. Kusenko, Rev. Mod. Phys. **76** (2004) 1
[75] W. Buchmüller, R.D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. **55** (2005) 311
[76] W. Buchmüller, K. Schmitz and G. Vertongen, Phys. Lett. **B693** (2010) 421
[77] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev. Mod. Phys. **69** (1997) 373
[78] D. H. Lyth and A. Riotto, Phys. Rept. **314** (1999) 1
[79] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78** (2006) 537
[80] W. Percival et al., Mon. Not. R. Astron. Soc. **401** (2010) 2148
[81] A. G. Riess et al., Astrophys. J. **699** (2009) 539
[82] A. De Felice and S. Tsujikawa, Phys. Rev. D **84** (2011) 083504
[83] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **42** (1990) 2491
[84] Y. Shtanov, J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **51** (1995) 5438
[85] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195
[86] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258
[87] S. Tsujikawa, K.-i. Maeda and T. Torii, Phys. Rev. D **60** (1999) 063515
[88] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **77** (1996) 219
[89] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **79** (1997) 1607
[90] T. Prokopec and T. G. Roos, Phys. Rev. D **55** (1997) 3768
[91] G. N. Felder and I. Tkachev, Comput. Phys. Commun. 178 (2008) 929
[92] A.A. Starobinsky, JETP Lett. 86 (2007) 157
[93] W. Hu and I. Sawicki, Phys. Rev. D76 (2007) 064004
[94] A. Appleby and R. Battye, Phys. Lett. B654 (2007) 7
[95] F.L. Bezrukov and M. Shaposhnikov, Phys. Lett. B659 (2008) 703
[96] F. Bezrukov, AIP Conf. Proc. 1241 (2010) 511
[97] F. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, JHEP 1101 (2011) 01
[98] N.D. Birrell and P.C.W Davies, Quantum Fields in Curved Space, Cambridge Univ. Press, 1982
[99] M.B. Einhorn and D.R.T. Jones, JHEP 1003 (2010) 026
[100] M. Shaposhnikov, Cosmological Inflation and the Standard Model, invited talk at the DESY Theory Workshop “Cosmology Meets Particle Physics”, 27–30 September 2011, Hamburg, Germany; http://th-workshop2011.desy.de/e98837/e98838/
[101] ATLAS Collaboration, Phys. Lett. B716 (2012) 1
[102] Y. Watanabe and J. Yokoyama, Gravitational modulated reheating and non-Gaussianity in supergravity $R^2$ inflation, arXiv:1303.5191
[103] A. Vilenkin, Phys. Rev. D32 (1985) 2511
[104] M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. Lett. 96 (2006) 211301
[105] M. Endo, F. Takahashi and T.T. Yanagida, Phys. Rev. D76 (2007) 083509
[106] D.S. Gorbunov and A.G. Panin, Phys. Lett. B700 (2011) 157
[107] G.F. Giudice and R. Rattazzi, Phys. Rep. 322 (1999) 419
[108] M. Bolz, A. Brandenburg and W. Buchmüller, Nucl. Phys. B606 (2001) 518
[109] F. Iocco, G. Mangano, G. Miele, O. Pisanti and P.D. Serpico, Phys. Rep. 472 (2009) 1
[110] M. Kawasaki, K. Kohri and K. Moroi, Phys. Rev. D71 (2005) 083502
[111] M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D78 (2008) 065011
[112] S.V. Ketov, G. Michiaki and T. Yumibayashi, *Quantizing with a higher time derivative*, in “Aspects of Quantum Field Theory”, InTech Publishers, 2012; arXiv:1110.1155 [hep-th]

[113] G.W. Gibbons, *Aspects of Supergravity*, Lectures at GIFT Seminar on Theoretical Physics, San Feliu de Guixols, Spain, June 4–11, 1984; published in the Proceedings “Supersymmetry, Supergravity and Related Topics”, edited by F. Del Aguila et al., Singapore, World Scientific, 1985

[114] S. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, 1973, par. 4.3

[115] W. Buchmüller, P. Di Bari and M. Plumacher, Annals of Phys. 315 (2005) 305

[116] E.K. Akhmedov, V.A. Rubakov and A.Y. Smirnov, Phys. Rev. Lett. 81 (1998) 1359

[117] J.M. Maldacena, JHEP 0305 (2003) 013

[118] C.M. Will, *Theory and Experiment in Gravitational Physics*, New York: Basic Books/Perseus Group, 1993

[119] C.M. Will, *The confrontation between General Relativity and Experiment*, Living Rev. Rel. 4 (2011) 4

[120] E. Fischbach and C. Talmadge, *The Search for Non-Newtonian Gravity*, New York, Springer-Verlag, 1999

[121] J. Khoury and A. Weltman, Phys. Rev. D69 (2004) 044026

[122] J. Khoury and A. Weltman, Phys. Rev. Lett. 93 (2004) 171104

[123] T. Damour and A.M. Polyakov, Nucl. Phys. B423 (1994) 532

[124] T. Damour and A.M. Polyakov, Gen. Rel. Grav. 26 (1994) 1171

[125] J. Ellis, D.V. Nanopoulos and K.A. Olive, *A no-scale supergravity realization of the Starobinsky model*, arXiv:1305.1247 [hep-th]

[126] R. Kallosh and A. Linde, *Superconformal generalizations of the Starobinsky model*, arXiv:1306.3214 [hep-th].