Top quark threshold production in $\gamma\gamma$ collision in the next-to-leading order.

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Abstract

The total cross section of the top quark-antiquark pair production near threshold in $\gamma\gamma$ collision is computed analytically up to the next-to-leading order in perturbative and nonrelativistic expansion for general photon helicity. The approximation includes the first order corrections in the strong coupling constant and the heavy quark velocity to the nonrelativistic Coulomb approximation.

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1 Introduction.

Theoretical study of the top quark-antiquark pair production near the two-particle threshold is based on the key observation that the relatively large width $\Gamma_t$ of the top quark serves as a natural infrared cutoff for long distance strong interaction effects [1]. Quantitatively, the top quark width is mainly saturated by the decay channel $t \to Wb$ and is equal to $\Gamma_t = 1.43$ GeV. Near the production threshold the nonrelativistic approximation is accurate and the relevant scale $\sqrt{\Gamma_t m_t} = 15.8$ GeV, $m_t$ being the top quark mass $m_t = 175$ GeV, is much larger than $\Lambda_{QCD}$ that makes perturbative QCD applicable for the theoretical description of the threshold top quark physics [1, 2, 3]. On the other hand the Next Linear Collider provides an opportunity of experimental study of high energy $\gamma\gamma$ interactions which can be used for the top quark production. Using the laser backscattering method one can obtain $\gamma\gamma$ colliding beams with the energy and luminosity comparable to those in $e^+e^-$ collisions [4]. Because of high precision which can be achieved in such experiments and the high accuracy of the theoretical description, the process of top quark pair production in $\gamma\gamma$ collisions near the two-particle threshold has been recognized as a promising place for thorough investigation of quark interactions and for determining the numerical values of the strong coupling constant $\alpha_s$, the top quark mass $m_t$, and the top quark width $\Gamma_t$. Moreover, the strong dependence of the cross section on the photon helicities extends possibilities of studying the details of the top quark threshold dynamics [4].

As is well known the ordinary finite order perturbation theory of QCD breaks down in the threshold region of particle production and the resummation of the Coulomb effects is necessary that can be systematically done in the framework of nonrelativistic QCD (NRQCD) [5]. The threshold effects are treated exactly by using the nonrelativistic Schrödinger equation with effective parameters. This equation can be also obtained from the corresponding Bethe-Salpeter equation in the nonrelativistic limit. Relativistic effects for the top quark are rather small and can be treated as corrections. Note that the characteristic scale of the Coulomb
effects for the top production $\alpha_s m_t$ is comparable numerically with the cutoff scale $\sqrt{\Gamma_t m_t}$ so these effects are not suppressed by the top quark width. Quantitatively, Coulomb effects require to sum up power terms in the variable (up to color group factors) $\alpha_s \sqrt{m_t/\Gamma_t} \sim 1.1$ which is not small and the corresponding function is not accurately approximated by the finite Taylor series. The determination of the higher order strong coupling corrections and relativistic corrections in this case requires the perturbative expansion to be performed near the Coulomb approximation rather than near the Green functions of a free theory. Technically, it makes the calculation rather involved. Only recently an essential progress has been achieved in analytical evaluation of the next-to-leading order (NLO) and the next-to-next-to-leading order (NNLO) corrections to the heavy quark vacuum polarization function near the threshold which enters the analysis of the heavy quark threshold production in $e^+e^-$ annihilation [6, 7, 8, 9, 10, 11]. The corrections are found to be relatively large in the case of the top quark production and very large in the case of the bottom quark threshold sum rules. One can expect the similar effect in the $\gamma\gamma$ threshold production of heavy quarks. Therefore, from phenomenological point of view, the calculation of $\alpha_s$ and relativistic corrections to the corresponding cross section is necessary for accurate quantitative study of the process. A part of the NLO corrections (the one to the cross section for the colliding photons of the same helicity) has been studied in ref. [2] using the numerical algorithm [3]. The complete analytical result for the corrections is still absent.

In the present paper we formulate the relevant (NRQCD) framework for the calculation of the higher order corrections and obtain analytically the NLO corrections which include the first order corrections in the strong coupling constant and the heavy quark velocity to the nonrelativistic Coulomb approximation for general photon helicity. The analytical result in the case of the opposite helicity colliding photons is of special importance because the available numerical methods are not applicable in this case [2]. We evaluate the relativistic $\Gamma_t \alpha_s$ corrections to the cross section for the opposite helicity colliding photons that cannot be
found within the pure nonrelativistic approximation and require an additional information on the relativistic structure of the full theory. The procedure of matching to the full theory is used to recover the necessary parameters of effective NRQCD. We present also the analytical NNLO result for the parameters of the resonances in the cross sections for $\gamma\gamma \to t\bar{t}$ and $e^+e^- \to t\bar{t}$ top quark threshold production.

The paper is organized as follows. In the next Section we formulate the nonrelativistic approximation for the total cross section of top quark pair production in $\gamma\gamma$ collisions. In Section 3 corrections to the nonrelativistic Green function and its derivative which enter the expression for the cross section are considered. In Section 4 we discuss the effects of nonvanishing top quark width. In Section 5 we present the NNLO analysis of the resonance structure of the cross section. The last Section contains the results of numerical analysis and our conclusions.

2 The nonrelativistic expansion for the cross section.

We fix our notation of kinematics for the top quark pair production in $\gamma\gamma$ collisions for working out the nonrelativistic expansion for the cross section as follows. The collision process is given by

$$\gamma(q_1, \epsilon(q_1))\gamma(q_2, \epsilon(q_2)) \to t(p_1, \sigma_1)\bar{t}(p_2, \sigma_2)$$

where $q_1, q_2$ are momenta of initial photons, $p_1, p_2$ are momenta of the final state fermions (top quark-antiquark pair), $q_1 + q_2 = p_1 + p_2$, $(q_1 + q_2)^2 = s$ is the square of the total energy of two photon beams in the photons zero-momentum frame. Here $\epsilon^\lambda(q_i)$ are the photon polarization vectors, $\sigma_{1,2}$ are spin variables of the fermions. After reducing the photon states the amplitude of $\gamma\gamma \to t\bar{t}$ transition reads

$$\mathcal{M}(\gamma\gamma \to t\bar{t}) = \langle eQ_1)^2e^{\nu}(q_1)e^{\nu}(q_2)\langle t\bar{t}|T_{\mu\nu}|0 \rangle$$

(1)
where $eQ_t, Q_t = 2/3$, is the top quark electric charge. The quantity $T_{\mu\nu}$ in eq. (1) is a Fourier transform of the $T$-product of two top quark electromagnetic (vector) currents $J_\lambda = \bar{t}\gamma_\lambda t$

$$T_{\mu\nu}(q) = i \int T J_\mu(x/2) J_\nu(-x/2) e^{iqx} dx ,$$

where $q = (q_1 - q_2)/2$ is the difference between the phonon momenta.

The $T$-product of the electromagnetic currents in eq. (2) can be expanded into a series over the local operators using a consistent expansion in $x$ at small $x$ within the Wilson operator product expansion [13]. Being inserted in eq. (2) it converts into a power series in the variable $1/(m_t^2 - q^2)$. Because $q^2 = -s/4$ where $s$ is a total photon energy squared the expansion is not singular near the fermion-antifermion threshold where $s \sim 4m_t^2$. In fact, in the threshold region $q^2 \sim -m_t^2$ and the corresponding expansion becomes an expansion in $1/m_t^2$. The leading term of the operator product expansion (a contribution of the local operator of dimension three) reads [13]

$$T^{(3)}_{\mu\nu} = \frac{2i\epsilon_{\mu\nu\alpha\beta} q^\alpha \bar{t}\gamma^\beta \gamma_5 t}{m_t^2 - q^2} .$$

Next contributions to the transition amplitude are related to the dimension four local operators of the operator product expansion which have the following explicit form

$$T^{(4a)}_{\mu\nu} = \frac{\bar{t}\gamma_\mu D_\nu t}{m_t^2 - q^2} ,$$

$$T^{(4b)}_{\mu\nu} = \frac{2q^\alpha m_t \bar{t}\sigma_{\mu\nu} D_\alpha t}{(m_t^2 - q^2)^2} ,$$

$$T^{(4c)}_{\mu\nu} = \frac{2i q^\alpha \bar{t} q_{(\mu} \gamma_{\nu)} D_\alpha t}{(m_t^2 - q^2)^2} ,$$

where $D_\alpha$ is a covariant derivative containing the gluon field. Note that operator (4) does not contribute to the amplitude because $q^\mu e_{\mu}(q_i) = 0$. The operator product expansion is organized in such a way that near the two-particle threshold the matrix element of a higher dimension operator between the vacuum and $t\bar{t}$ state is suppressed by either the quark pair
energy \( E = \sqrt{s} - 2m_t \) counted from the threshold or explicitly by powers of \( \alpha_s \). Thus, for the transition amplitude the operator product expansion converts into a simultaneous expansion in \( \alpha_s \) and in \( \beta = \sqrt{1 - 4m_t^2/s} \), the parameter of nonrelativistic near-threshold expansion. The operators in eqs. (3–6), however, are relativistic ones. Therefore, to obtain the consistent nonrelativistic expansion of the amplitude one has to expand these operators in a series over the nonrelativistic operators in the framework of NRQCD [14]. The nonrelativistic expansion of the operators entering eqs. (3, 4) which are necessary for the further analysis up to a perturbative normalization factor reads

\[
\bar{t}\gamma^0\gamma_5 t = \varphi^\dagger \chi + \ldots \tag{7}
\]

\[
i\bar{t}\gamma_{(i} \vec{D}_{j)} t = i\varphi^\dagger \sigma_{(i} \vec{D}_{j)} \chi + \ldots \tag{8}
\]

where \( \varphi (\chi) \) is the nonrelativistic two component top (antitop) quark spinor, \( \sigma_i \) is the Pauli matrix and ellipsis stands for the higher dimension operators. Note that only the time component of the current in eq. (3) and space components of the tensor in eq. (4) contribute in the leading order of nonrelativistic expansion.

Let us turn to the calculation of the cross section. We deal with the normalized cross section

\[
R(s) = \frac{\sigma(\gamma\gamma \rightarrow tt)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}
\]

where the standard lepton cross section

\[
\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2_{QED}}{3s}
\]

is just a kinematic normalization factor.

In the case of the same helicity colliding photons only the operator of dimension three contributes to the amplitude up to NLO. Using the leading term of the nonrelativistic expansion for the quark current in eq. (3) one arrives at the following representation for the cross section [2]

\[
R^{++}(E) = 6Q_t^4N_cC_h^{++}(\alpha_s) \left( \frac{m_t^2}{4\pi} \right)^{-1} \text{Im} G(0,0,k) \tag{9}
\]
where $G(x, y, k) (k^2 = -m_tE)$ is the nonrelativistic Green function which accounts for the soft (threshold) effects and

$$C_h^{++}(\alpha_s) = 1 - \left(5 - \frac{\pi^2}{4}\right) \frac{C_F \alpha_s(\mu_h)}{\pi}$$

is the NLO perturbative coefficient matching correlators of relativistic and nonrelativistic currents \cite{15} with $\alpha_s$ being taken at “hard” normalization scale $\mu_h$. The higher dimension operators in the expansion of eq. \eqref{eq:2} and in the nonrelativistic expansion \eqref{eq:7} lead to relative $O(\beta^2)$ corrections to the cross section.

For the opposite helicity colliding photons the amplitude up to NLO is determined by the operator given in eq. \eqref{eq:4}. By using the leading term of eq. \eqref{eq:8} for the corresponding cross section one gets \cite{2}

$$R^{+-}(E) = 8Q_t^4 N_c C_h^{+-}(\alpha_s) \left(\frac{m_t^4}{4\pi}\right)^{-1} \partial_x \partial_y \text{Im} G(x, y, k) |_{y=0, x=0}$$

with the perturbative coefficient being \cite{10}

$$C_h^{+-}(\alpha_s) = 1 - 4 \frac{C_F \alpha_s(\mu_h)}{\pi}.$$ 

Note that the leading contribution to $R^{+-}(E)$ comes from the quark pair with the orbital momentum $l = 1$ because $l = 0$ state cannot be produced by the opposite helicity photons with the total angular momentum $J = 2$ while the dimension three operator \eqref{eq:3} interpolates only $l = 0$ states and, therefore, does not contribute to $R^{+-}$ cross section. The dimension four operator \eqref{eq:5} does not contribute to $R^{+-}$ in the leading order because it has the parity $P = -1$ while the quark pair with the orbital momentum $l = 1$ in the final state has the parity $P = 1$. In full analogy with the $R^{++}$ case the higher dimension operators lead only to relative $O(\beta^2)$ corrections to $R^{+-}$ cross section.

The nonrelativistic Green function appearing in eqs. \eqref{eq:3}-\eqref{eq:10} in the considered approximation satisfies the Schrödinger equation

$$\left(-\frac{\partial^2}{m_t^2} + V_C(x) + \Delta_1 V(x) + \frac{k^2}{m_t^2}\right) G(x, y, k) = \delta(x - y)$$

\cite{14}
where $x = |x|$, $V_C(x) = -C_F \alpha_s(\mu_s)/x$ is the Coulomb potential with $\mu_s$ being the soft normalization scale which determines the strong coupling constant. In the NLO one has to take into account the first order perturbative QCD corrections to the Coulomb potential $\Delta_1 V$ which is of the following form [19]

$$\Delta_1 V(x) = \frac{\alpha_s}{4\pi} V_C(x)(C^1_0 + C^1_1 \ln(x\mu_s)) = \frac{\alpha_s}{4\pi} V_C(x)C^1_1 \ln(x\mu_1)$$

where

$$C^1_0 = a_1 + 2\beta_0 \gamma_E, \quad C^1_1 = 2\beta_0,$$

$$a_1 = \frac{31}{9} C_A - \frac{20}{9} T_F n_f, \quad \mu_1 = \mu_s \exp\left(\frac{C^1_0}{C^1_1}\right).$$

The color symmetry $SU(3)$ group invariants for QCD are $C_A = 3$, $C_F = 4/3$, $T_F = 1/2$, $n_f = 5$ is the number of light fermion flavors, and $\beta_0 = 11 C_A/3 - 4 T_F n_f/3$ is the first coefficient of $\beta$-function. Here $\gamma_E = 0.577216\ldots$ is the Euler constant.

Note that up to an overall normalization factor the expression for $R^{++}$ up to NLO coincides with the normalized cross section of the top quark threshold production in $e^+e^-$ annihilation.

In zero order in $\alpha_s$ eqs. (9,10) are reduced to Born approximation for corresponding cross sections and contain only continuous spectrum starting from the two-particle threshold

$$R^{++}(\beta) = 6 Q_t^4 N_c \beta \theta(s - 4m_t^2) + O(\beta^3),$$

$$R^{+-}(\beta) = 8 Q_t^4 N_c \beta^3 \theta(s - 4m_t^2) + O(\beta^5).$$

The full relativistic result for the cross sections $R^{++}(\beta)$ and $R^{+-}(\beta)$ in tree approximation which sums up all powers of $\beta$ was obtained in ref. [17]. At the one loop level the analytical result has been recently obtained as a power expansion in $\beta$ up to $\beta^{10}$ order [18].
3 The nonrelativistic Green function beyond the leading order.

In this section we describe the technique for computing the nonrelativistic Green function beyond the leading order. The corrections are found within the perturbation theory around the Coulomb solution as a leading order approximation relevant for computation of the cross section near the threshold. The developed technique is applicable for computing corrections to the nonrelativistic Green function with any orbital momentum $l$ and is a direct generalization of the method suggested in [6, 9, 11].

The nonrelativistic Green function has the standard partial wave decomposition

$$G(x, y, k) = \sum_{l=0}^{\infty} (2l + 1)(xy)^l P_l(xy) G_l(x, y, k)$$  \hspace{1cm} (13)

where $P_l(z)$ is a Legendre polynomial. The partial waves of the Green function of the pure Coulomb Schrödinger equation $G^C(x, y, k)$ are

$$G^C_l(x, y, k) = \frac{m_t k}{2\pi} (2k)^{2l} e^{-k(x+y)} \sum_{m=0}^{\infty} \frac{L_m^{2l+1}(2kx) L_m^{2l+1}(2ky) m!}{(m + l + 1 - \nu)(m + 2l + 1)!}$$  \hspace{1cm} (14)

where $\nu = \lambda/k$, $\lambda = \alpha_s C_F m_t/2$, $L_m^\alpha(z)$ is a Laguerre polynomial

$$L_m^\alpha(z) = \frac{e^z z^{-\alpha}}{m!} \left( \frac{d}{dz} \right)^m (e^{-z} z^{m+\alpha})$$

and $\alpha_s$ is taken at the soft scale $\mu_s$. At $y = 0$ the partial waves take the form

$$G^C_l(x, 0, k) = \frac{m_t k}{2\pi} (2k)^{2l} e^{-kx} \frac{(l + 1 - \nu) U(l + 1 - \nu, 2l + 2, 2kx)}{\Gamma(l + 1 + \nu) U(l + 1 - \nu, 2l + 2, 2kx)}$$  \hspace{1cm} (15)

where $U(a, b, z)$ is the confluent hypergeometric function.

Only $l = 0$ component of the Green function contributes to its value at the origin

$$G(0, 0, k) = G_0(0, 0, k).$$

For the Coulomb Green function we find an explicit expression in the form

$$G^C_0(x, 0, k) \big|_{x \to 0} = \frac{m_t}{4\pi} \left( \frac{1}{x} - 2\lambda \ln(2x \mu_f) - 2\lambda \left( \frac{k}{2\lambda} + \ln \left( \frac{k}{\mu_f} \right) \right) \right.$$
\[ +2\gamma_E - 1 + \Psi_1 (1 - \nu) \]  

where \( \Psi_n(z) = d^n \ln \Gamma(z)/dz^n \) and \( \Gamma(z) \) is the Euler \( \Gamma \)-function. In the short distance limit \( x \to 0 \) the Coulomb Green function \( G^C(x, 0, k) \) has \( 1/x \) and \( \ln x \) divergent terms. These terms, however, are energy independent and do not contribute to the cross section. Hence they can be subtracted. In eq. (16) \( \mu_f \) is an auxiliary parameter. Though an accurate definition of this parameter is important in the NNLO analysis [7, 10] it does not enter the cross section of top production for the stable top quark, or in zero width approximation.

The derivative of the Green function at the origin is saturated with its \( l = 1 \) component

\[ \partial_x \partial_y G(x, y, k)|_{y=0, x=0} = 9G_1(0, 0, k) \]

and for the Coulomb Green function we have

\[
\begin{align*}
G_1^C(0, 0, k) &= \frac{m_t}{36\pi} \left( \frac{3}{x^3} + \frac{3\lambda}{x^2} + \frac{6\lambda^2 - 3k^2}{2x} + 2\lambda(k^2 - \lambda^2) \ln(2x\mu_f) 
+ \lambda \left( 2(k^2 - \lambda^2) \left( \frac{k}{2\lambda} + \ln \left( \frac{k}{\mu_f} \right) + 2\gamma_E - \frac{11}{6} + \Psi_1 (1 - \nu) \right) + \frac{k^2}{2} \right) \right).
\end{align*}
\]

One can immediately see that this expression has no lowest pole present in eq. (16). In the short distance limit \( x \to 0 \) the derivative of Coulomb Green function (or partial wave with \( l = 1 \)) has \( 1/x^n \) (\( n = 1, 2, 3 \)) and \( \ln x \) singularities. In contrast to Coulomb Green function itself some of the divergent terms now are \( k \) dependent but also do not contribute to the cross section for \( \Gamma_t = 0 \) because they have no discontinuity across the physical cut in the complex energy plane in the zero-width approximation. The case of non-zero top quark width needs more detailed analysis given in the next section.

The solution to eq. (14) with the logarithmic correction to the Coulomb potential can be found within the standard nonrelativistic perturbation theory around the Coulomb Green function taken as a leading order approximation

\[ G(x, y, k) = G^C(x, y, k) + \Delta_1 G(x, y, k), \]
\[ \Delta_1 G(x, y, k) = - \int G^C(x, z, k) \Delta_1 V(z) G^C(z, y, k) dz . \] (18)

The calculation goes along the lines developed in refs. [6, 9, 11]. Only \( l = 0 \) component of eq. (13) is necessary for the calculation of the correction to the Green function at the origin.

The integral in eq. (18) diverges at \( x, y = 0 \). This divergence is induced by the singular \( 1/x \) term in eq. (16) which is related to the singular behavior of the free (\( \alpha_s = 0 \)) Green function \( G^F(x, y, k) \) in the limit \( y = 0, x \to 0 \). The divergent part can be separated in the following way. Eq. (18) for \( x, y = 0 \) can be written in the form

\[ \Delta_1 G(0, 0, k) = \Delta_1 G_0(0, 0, k) = -\left( \int (G^C_0(0, x, k) - G^F_0(0, x, k))^2 \Delta_1 V(x) dx \right. \]

\[ + 2\int (G^C_0(0, x, k) - G^F_0(0, x, k))G^F_0(x, 0, k) \Delta_1 V(x) dx + \int G^F_0(0, x, k)^2 \Delta_1 V(x) dx \) . (19)

By using representation (13) for \( G^C_0(x, y, k) \) and the same expression with \( \alpha_s = 0 \) for \( G^F_0(x, y, k) \) one gets

\[ \Delta_1 G_0(0, 0, k) = -\left( \frac{m_t k}{2\pi} \right)^2 \left( \sum_{m,n=0}^{\infty} F(m)F(n) \int e^{-2kx} L^1_m(2kx)L^1_n(2kx) \Delta_1 V(x) dx \right. \]

\[ + 2\sum_{m=0}^{\infty} F(m) \int \frac{e^{-2kx} L^1_m(2kx) \Delta_1 V(x)}{2kx} dx + \int \frac{e^{-2kx} \Delta_1 V(x)}{(2kx)^2} dx \) (20)

where

\[ F(m) = \frac{\nu}{(m + 1)(m + 1 - \nu)} \]

and we used the properties of the Laguerre polynomial

\[ L^\alpha_m(0) = \frac{\Gamma(m + \alpha + 1)}{\Gamma(\alpha + 1)\Gamma(m + 1)} , \sum_{m=0}^{\infty} \frac{L^\alpha_m(z)}{m + \alpha} = z^{-\alpha}\Gamma(\alpha). \]

Only the last term is divergent in eq. (20). After regularization the divergent integral in that term becomes

\[ \int_0^\infty \frac{e^{-2kx} \ln(\mu x)}{x} dx = -\gamma E L(k) + \frac{1}{2} L(k)^2 + \ldots \]

\(^1\)The correction to the Green function at the origin has been also studied using the numerical algorithm in refs. [12, 13, 14, 15].
where $L(k) = -\ln(2k/\mu_1)$ and ellipsis stands for the divergent part. The divergent part being $k$ independent does not contribute to the spectral density. Two finite integrals in eq. (19) are

$$\int_0^\infty e^{-z}L_n^1(z)L_m^1(z)\ln(z)dz = \begin{cases} (m+1)\Psi_1(m+2), & m = n \\ -\frac{n+1}{m-n}, & m > n \end{cases}$$

$$\int_0^\infty e^{-z}L_m^1(z)\ln(z)dz = -2\gamma_E - \Psi_1(m+1).$$

(21)

To compute the first integral we rewrite it in the form

$$\int_0^\infty e^{-z}L_n^1(z)L_m^1(z)\ln(z)dz = \frac{d}{d\varepsilon} \left( \int e^{-z}L_n^1(z)L_m^1(z)z^{1+\varepsilon}dz \right)\bigg|_{\varepsilon=0}. \quad (22)$$

By using the relations

$$L_\beta^\alpha(z) = \sum_{n=0}^m \frac{\Gamma(\beta - \alpha + n)}{\Gamma(\beta - \alpha)\Gamma(n+1)} L_{m-n}^\alpha(z),$$

$$\int_0^\infty e^{-z}L_n^\alpha(z)L_m^\alpha(z)z^\alpha dz = \delta_{mn} \frac{\Gamma(m + \alpha + 1)}{\Gamma(m+1)}$$

for $\beta = 1$, $\alpha = 1 + \varepsilon$ the integration in the right hand side of eq. (22) can be performed analytically. Then taking the derivative in $\varepsilon$ at $\varepsilon = 0$ we get the first line of eq. (21). The second integral in eq. (21) can be computed using the same technique. Thus the final result for the correction is

$$\Delta_1 G_0(0,0,k) = \frac{\alpha_s \beta_0 \lambda m_t}{2\pi} \left( \sum_{m=0}^\infty F(m)^2 (m+1) (L(k) + \Psi_1(m+2)) - 2 \sum_{m=1}^\infty \sum_{n=0}^{m-1} F(m) 
\times F(n) \frac{n+1}{m-n} + 2 \sum_{m=0}^\infty F(m) (L(k) - 2\gamma_E - \Psi_1(m+1)) - \gamma_E L(k) + \frac{1}{2} L(k)^2 \right).$$

(23)

Note that the sum in the finite limits in this equation can be reexpressed through some combination of $\psi$ functions of different arguments. This however is not important for numerical analysis at present when the actual calculation is performed with computers.
The calculation of the correction to the derivative of the Green function at the origin (or partial wave with \(l = 1\)) is a bit more cumbersome. The corresponding expression reads
\[
\partial_x \partial_y \Delta_1 G(x, y, k)|_{x,y=0} = 9 \Delta_1 G_1(0, 0, k) = -9 \int G_1^C(0, x, k)^2 \Delta_1 V(x) x^2 \, dx. \tag{24}
\]
Eq. (17) contains \(1/x^n\) \((n = 1, 2)\) singular terms with the coefficients depending on \(\alpha_s\) which lead to the divergence of the integral in eq. (24). Hence this is not enough to subtract just the free Green function from the Coulomb one to separate the divergent part of eq. (24) as has been done in the analysis of the correction to the Green function at the origin. The analog of eq. (19) now reads
\[
\Delta_1 G_1(0, 0, k) = -\left( \int (G_1^C(x, 0, k) - D(x, k))^2 \Delta_1 V(x) x^2 \, dx \right)
+ 2 \int (G_1^C(x, 0, k) - D(x, k)) D(x, k) \Delta_1 V(x) x^2 \, dx + \int D(x, k)^2 \Delta_1 V(x) x^2 \, dx \tag{25}
\]
where
\[
D(x, k) = \frac{m_t k^3}{3\pi} e^{-kx} H(2kx),
\]
\[
H(z) = \frac{2}{z^3} + \frac{1 + \nu}{z^2} + \frac{\nu(1 + \nu)}{2z}. \tag{26}
\]
By using the relation
\[
H(z) = \sum_{m=0}^{\infty} L_m^3(z) \left( \frac{\nu(\nu + 1)}{2(m + 1)} + \frac{1 - \nu^2}{m + 2} + \frac{\nu(\nu - 1)}{2(m + 3)} \right)
\]
eq (25) can be transformed into
\[
\Delta_1 G_1(0, 0, k) = - \left( \frac{m_t k^3}{3\pi} \right)^2 \left( \sum_{m,n=0}^{\infty} \bar{F}(m) \bar{F}(n) \int e^{-2kx} L_m^3(2kx) L_n^3(2kx) \Delta_1 V(x) x^2 \, dx \right)
+ 2 \sum_{m=0}^{\infty} \bar{F}(m) \int e^{-2kx} L_m^3(2kx) H(2kx) \Delta_1 V(x) x^2 \, dx + \int e^{-2kx} H(2kx)^2 \Delta_1 V(x) x^2 \, dx \tag{27}
\]
where
\[
\bar{F}(m) = \frac{\nu(\nu^2 - 1)}{(m + 2 - \nu)(m + 1)(m + 2)(m + 3)}
\]
and the divergence is contained in the last term. After a regularization this term can be
directly computed with the result
\[
I(k) = - \left( \frac{C_F \beta_0 \alpha_s^2}{2\pi} \right)^{-1} \frac{4k^4}{\pi} \int e^{-2kx} H(2kx)^2 \Delta_1 V(x)x^2 dx = -\frac{(\gamma_E - 1)^2}{2} - \frac{\pi^2}{12}
\]
\[- (4 - 3\gamma_E)\nu + \frac{1 - 9\gamma_E + 6\gamma_E^2 + \pi^2}{4} \nu^2 + \frac{1 - 3\gamma_E}{2} \nu^3 + \frac{1 - \gamma_E}{4} \nu^4
\]
\[+ \left( \gamma_E - 1 - 3\nu + \frac{9 - 12\gamma_E}{4} \nu^2 + \frac{3}{2} \nu^3 + \frac{1}{4} \nu^4 \right) L(k) + \left( -\frac{1}{2} + \frac{3}{2} \nu^2 \right) L(k)^2 + \ldots
\]
where dots stand for divergent term which has no imaginary part and does not contribute
to the spectral density in zero width approximation. A set of finite integrals appearing in
eq. (25) can be computed by the technique which has been already applied for the analysis
of the correction to the $l = 0$ partial wave. The results for relevant integrals read
\[
\int_0^\infty e^{-z} L_n^3(z)L_n^3(z) \ln(z)z^3 dz = \begin{cases} (m+1)(m+2)(m+3)\Psi_1(m+4), & m = n \\
-(n+1)(n+2)(n+3) \frac{1}{m-n}, & m > n \end{cases}
\]
\[J_0(m) = \int_0^\infty e^{-z} L_n^3(z) \ln(z) dz = -2\Psi_1(m+1) - 4\gamma_E + 3,
\]
\[J_1(m) = \int_0^\infty e^{-z} L_n^3(z) \ln(z)z dz = (m+1)(-\Psi_1(m+2) - 2\gamma_E + 2),
\]
\[J_2(m) = \int_0^\infty e^{-z} L_n^3(z) \ln(z)z^2 dz = \frac{(m+1)(m+2)}{2} \left( -\Psi_1(m+3) - 2\gamma_E + \frac{3}{2} \right).
\]
Substituting these formulae in eq. (25) we arrive at the final result for the correction to the
$l = 1$ partial wave at the origin
\[
\Delta_1 G_1(0,0,k) = \frac{\alpha_s \beta_0}{2\pi} \frac{\lambda_m k^2}{18\pi} \left( \sum_{m=0}^{\infty} \tilde{F}(m)^2(m+1)(m+2)(m+3)(L(k) + \Psi_1(m+4))
\right.
\]
\[-2 \sum_{m=1}^{\infty} \sum_{n=0}^{m-1} \tilde{F}(m)\tilde{F}(n) \frac{(n+1)(n+2)(n+3)}{m-n} + 2 \sum_{m=0}^{\infty} \tilde{F}(m) \left( 2J_0(m) + (m+1)(m+2)L(k)
\right.

\left. + (1 + \nu)(J_1(m) + (m+1)L(k)) + \nu(\nu+1) \frac{1}{2} (J_2(m) + 2L(k)) \right) + I(k) \bigg) .
\]
Note that it is quite straightforward to extend our method to calculation of the correction
to a component of the Green function with arbitrary $l$. 


The Green function in every order of the nonrelativistic perturbation theory has to be written in the form which includes only single poles in the energy variable and is more natural for the Green function of a nonrelativistic Schrödinger equation. For the Green function at the origin this representation reads

$$G(0, 0, E) = \sum_{m=0}^{\infty} \frac{|\psi_m(0)|^2}{E_m^{(0)} - E} + \frac{1}{\pi} \int_{E}^{\infty} \frac{|\psi_{E'}(0)|^2}{E' - E} dE'$$

where $\psi_{m,E'}(0)$ is the wave function at the origin, $E_m^{(0)}$ is a $l = 0$ bound state energy, the sum goes over bound states and the integration in the second term is performed over the state of continuous part of the spectrum. In this way the corrections to Green function stemming from the discrete part of the spectrum can be reduced to the correction to the Coulomb bound state energy levels

$$E_m^{(0)} = -\frac{\lambda^2}{m_t(m+1)^2} \left( 1 + \frac{\alpha_s}{4\pi} 2C_1^1 [L(m) + \Psi_1(m+2)] \right)$$

and to the values of Coulomb bound state wave functions at the origin

$$|\psi_m(0)|^2 = \frac{\lambda^3}{\pi(m+1)^3} \left( 1 + \frac{\alpha_s}{4\pi} C_1^1 \left( 3L(m) - 1 - 2\gamma_E + \frac{2}{m+1} + \Psi_1(m+2) \right) - 2(m+1)\Psi_2(m+1) \right)$$

where

$$L(m) = \ln \left( \frac{(m+1)\mu_1}{C_F \alpha_s m_b} \right).$$

For the derivative of the Green function at the origin one has

$$\partial_x \partial_y \Delta_1 G(x, y, E)|_{x,y=0} = \sum_{m=0}^{\infty} \frac{|\psi_{m,E'}(0)|^2}{E_m^{(1)} - E - i0} + \frac{1}{\pi} \int_{E}^{\infty} \frac{|\psi_{E'}(0)|^2}{E' - E - i0} dE'$$

where

$$|\psi_{m,E'}(0)|^2 = \partial_x \psi_{m,E'}^*(x) \partial_y \psi_{m,E'}(y)|_{x,y=0}$$

and $E_m^{(1)}$ is a $l = 1$ bound state energy. In NLO approximation these quantities read

$$E_m^{(1)} = -\frac{\lambda^2}{m_t(m+2)^2} \left( 1 + \frac{\alpha_s}{4\pi} 2C_1^1 [L(m+1) + \Psi_1(m+4)] \right).$$
\[ |\psi_m'(0)|^2 = \frac{\lambda^5}{\pi} \frac{(m+1)(m+3)}{(m+2)^5} \left( 1 + \frac{\alpha_s}{4\pi} C_1 \left( -\frac{\pi^2}{3} (m+2) - 1 + 5L(m+1) + 5\Psi_1(m+4) \right) + 2 \sum_{n=0}^{m-1} \frac{(n+1)(n+2)(n+3)}{(m+1)(m+3)(m-n)^2} \right). \]

(36)

The continuum contributions in eqs. (31, 34) can be directly found by subtracting the discrete part of these equations expanded around the Coulomb approximation up to NLO from the result obtained within the nonrelativistic perturbation theory for the Green function and its derivative at the origin.

4 Effects of the finite width of top quark.

As has been already mentioned only sufficiently large \(t\)-quark decay width suppresses the nonperturbative effects of strong interactions at large (\(\sim 1/\Lambda_{QCD}\)) distances and makes the perturbation theory applicable for the description of the \(t\)-quark threshold dynamics. The accurate description is achieved in the framework of NRQCD that provides the adequate leading order approximation for the problem. However main features of the physical situation can be reflected in a simpler approach to which we restrict ourselves in the present paper. According to the prescription of ref. [1] the non-zero width can be taken into account by direct replacing \(E \to E + i\Gamma_t\) in the Green function near the production threshold. This is justified because the threshold dynamics is nonrelativistic one and is rather insensitive to the hard momentum details of \(t\)-quark decays. This prescription is not equivalent to the full NRQCD description in higher orders of perturbation theory in the strong coupling constant. Also, strictly speaking this prescription is valid in its simple form only when the complete factorization of the hard and soft contributions to the cross section takes place. This is realized in the case of \(R^{++}\) cross section. In the case of \(R^{+-}\) cross section, however, the nonrelativistic approximation is not able to describe properly the entire effect of the non-zero top quark width [2]. Indeed, eq. (17) for a nonvanishing width in the limit \(x \to 0\) has the
divergent imaginary part with the leading power singularity \( \sim \Gamma_t/x \) related to the free Green function singularity and logarithmic singularity \( \sim \Gamma_t \alpha_s \ln x \) produced by the one Coulomb photon exchange. This clearly indicates that the coefficient in front of the constant term linear in \( \Gamma_t \) in the cross section gets a contribution from large momentum region and cannot be obtained within the nonrelativistic theory. This constant can be obtained within the full relativistic theory. The correct treatment of the problem consists in matching NRQCD expression for this part of the cross section to the one obtained in full theory by direct inserting the (complex) mass operator into the \( t \)-quark propagator. In the leading order in \( \alpha_s \) this has been done semiphenomenologically in refs. [2, 12]. In the present paper we limit ourselves to formulas obtained in [2, 12] that are sufficient from phenomenological point of view. The contribution linear in the top quark width reads

\[
R_{t}^{\pm} = 64\pi Q_t^4 N_c \int \frac{p^2}{m_t^2} d\rho
\]

\[
= \frac{8Q_t^4N_c\Gamma_t}{m_t} \int_{(m_W+m_b)^2}^{m_t^2} \frac{((9m_t^2-p^2)(m_t^2-p^2))^{3/2}}{(m_t^2-p^2)^2 + 4m_t^2\Gamma_t(p^2)^2} \frac{\Gamma_t(p^2)dp^2}{32\pi m_t m_t} = 0.185 \frac{8Q_t^4N_c\Gamma_t}{m_t}
\]

(37)

where \( p^2 \) is the Born amplitude squared, \( d\rho \) is the relativistic phase space of two unstable particles for \( E = 0 \) (see ref. [12]), \( \Gamma_t(p^2) \) is the imaginary part of the \( t \)-quark mass operator \( (\Gamma_t(m_t^2) = \Gamma_t) \) and the leading \( t \to Wb \) decay channel is taken into account. In eq. (37) \( m_W = 80.3 \) GeV is the \( W \)-boson mass [20] and \( m_b = 4.8 \) GeV is the \( b \)-quark pole mass [11]. In the final expression for the cross section this contribution has to be multiplied by the hard normalization coefficient \( C_f^{\pm}(\alpha_s) \). Numerically this contribution to the cross section is suppressed in comparison with one of the regular pure nonrelativistic terms of eq. (17) which saturate the total result.

To find the coefficient in front of the \( \Gamma_t \alpha_s \) term in the imaginary part of eq. (17) or, equivalently, to fix the parameter \( \mu_f \) one has to know the one-gluon correction to Born approximation. This correction can be obtained from QED result [21] and for \( E = 0 \) is equal
to $\pi \lambda |p|$. A corresponding contribution to the cross section reads

$$R^{\alpha_s \Gamma_t} = 64\pi^2 Q^4_t N_c \int \frac{\lambda|p|}{m_t^2} d\rho$$

$$= \frac{8Q^4_t N_c \lambda \Gamma_t}{m_t^2} \int_{(m_{W}+m_b)^2}^{m_t^2} \frac{(9m_t^2 - p^2)(m_t^2 - p^2)}{(m_t^2 - p^2)^2 + 4m_t^2 \Gamma_t(p^2)^2} \frac{\Gamma_t(p^2)dp^2}{8\Gamma_t m_t^2} = \frac{8Q^4_t N_c \lambda \Gamma_t}{m_t^2} \left( \ln \left( \frac{m_t}{\Gamma_t} \right) - 2.07 \right)$$

(38)

where the term with the logarithmic dependence on $\Gamma_t$ is separated and computed analytically within the approximation $\Gamma_t(p^2) = \Gamma_t$. Following the general line of the effective field theory approach one has to equate the contribution to the cross section determined by the $O(\Gamma_t \alpha_s)$ terms in the imaginary part of eq. (17) at $E = 0$ to the relativistic expression (38). This equation results in the following matching relation

$$-\ln \left( \frac{m_t \Gamma_t}{\mu_f^2} \right) - 2\gamma_E + \frac{19}{6} = \ln \left( \frac{m_t}{\Gamma_t} \right) - 2.07$$

which fixes the auxiliary parameter $\mu_f = 0.13 m_t$. Note that in this relation the logarithmic dependence of eq. (17) on $\Gamma_t$ exactly matches one of eq. (38) so $\mu_f$ is related only to the hard scale of the process as one expects from the general ground.

The above analysis cannot be considered as a completely relativistic one – Born amplitude and the Coulomb one-gluon correction are taken in the leading order of the nonrelativistic expansion though the relativistic phase volume is used. This, however, is justified because the integrals in eqs. (37, 38) are saturated within the region $|p| \sim \sqrt{m_t \Gamma_t} << m_t$ ($p^2 \sim m_t^2$) where the nonrelativistic approximation works well. By the same reason the function $\Gamma_t(p^2)$ in these equations can be well approximated by its constant value $\Gamma_t$.

Note that for non-vanishing width some of the singular terms in eq. (28) which describes the correction to the Green function also have imaginary part and contribute to the cross section. This contribution, however, has very weak energy dependence and is suppressed by an extra power of $\alpha_s$. Thus, this is beyond the accuracy of our approximation.
5 Resonances of the cross section in the next-to-next-to-leading order.

Though the complete NNLO analysis of the cross section is not still available, some important parameters of the cross section can be found in NNLO approximation. Indeed, the position of the poles of the nonrelativistic Green function which correspond to resonances in the cross section can be determined within the nonrelativistic approximation \[9, 10, 11, 22\]. This has been explicitly done for the spin triplet states in refs. \[10, 11, 22\]. This analysis can be directly generalized to the spin singlet states which are produced in the two photon collisions. The only difference is in the correction induced by the part of the Breit potential responsible for the hyperfine splitting. We, however, do not need the complete result because the most of resonances are smoothed out by the relatively large top quark width. The numerical analysis shows that only the ground state resonance in \(l = 0\) partial wave cross section \(R^{++}(E)\) is distinguishable. Its separation from others is not completely covered by the infrared cutoff provided by the top quark width. Indeed, using the pure Coulomb formulas for estimates within the order of magnitude we find

\[|E_0^{(0)} - E_1^{(0)}| = \frac{3\lambda^2}{4m_t} \approx 0.6 \text{ GeV}\]

to be compared with the top quark width \(\Gamma_t = 1.43 \text{ GeV}\). The spacing between next radial excitations for \(l = 0\) partial wave (which is also equal to the first one for \(l = 1\) partial wave) is much smaller

\[|E_1^{(0)} - E_2^{(0)}| = |E_0^{(1)} - E_1^{(1)}| = \frac{5}{36} \frac{\lambda^2}{m_t} \approx 0.11 \text{ GeV}\]

and is completely smeared out with the top quark width. Therefore it is numerically justified to treat separately only the first resonance in \(l = 0\) partial wave cross section and to sum all corrections to the denominator of the Green function to avoid the appearance of the double pole in the correction which is large at the resonance energy. For all other states this procedure makes no numerical difference because the infrared cutoff is sufficiently large to
smear out all picks in the cross section. For $R^+(E)$ cross section the lowest contributing state has $l = 1$ and is rather close to other states and therefore is indistinguishable as a separate contribution.

Using the results of refs. [9, 10, 11, 22] it is straightforward to find the energy of this resonance in NNLO approximation counted from the threshold

$$E_{\text{res}}(\gamma \gamma \rightarrow t\bar{t}) \equiv E_\text{res}^{(0)} = -\frac{\lambda^2}{m_t^2} \left(1 + \frac{\alpha_s}{4\pi} 2C_1^2 (L(\lambda) + 1 - \gamma_E) + \frac{\alpha_s^2}{4\pi} \left(2C_2^2 (L_1(\lambda) + 1 - \gamma_E)ight)

+ (C_1^1)^2 \left((L(\lambda) - \gamma_E)^2 + 1 - \frac{\pi^2}{3} - \Psi_3(1)\right) + 2C_2^2 \left((L_2(\lambda) + 1 - \gamma_E)^2 - 1 + \frac{\pi^2}{6}\right)ight)

+ C_2^2 (\frac{C_A}{C_F} + \frac{21}{16}) .$$  \hspace{1cm} (39)

where

$$L_1(\lambda) = \ln \left(\frac{\mu_e C^2 / C_0^2}{\ln(\mu_s / 2\lambda)}\right), \quad L_2(\lambda) = \ln \left(\frac{\mu_s}{2\lambda}\right),$$

$$C_0^2 = \left(\frac{\pi^2}{3} + 4\gamma_E^2\right) \beta_0^2 + 2(\beta_1 + 2\beta_0 a_1) \gamma_E + a_2,$$

$$C_1^2 = 2(\beta_1 + 2\beta_0 a_1) + 8\beta_0^2 \gamma_E, \quad C_2^2 = 4\beta_0^2,$$

$$a_2 = \left(\frac{4343}{162} + 6\pi^2 - \frac{\pi^4}{4} + \frac{22}{3} \zeta(3)\right) C_A^2 - \left(\frac{1798}{81} + \frac{56}{3} \zeta(3)\right) C_A T_F n_f

- \left(\frac{55}{3} - 16 \zeta(3)\right) C_F T_F n_f + \left(\frac{20}{9} T_F n_f\right)^2 ,$$

$$\beta_1 = \left(\frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F n_f - 4 C_F T_F n_f\right) \zeta(3) .$$

and $\zeta(z)$ is the Riemann $\zeta$-function. This value is related to the energy of the bound state resonance of the top quark production in $e^+e^-$ annihilation by the hyperfine splitting

$$E_{\text{res}}(e^+e^- \rightarrow t\bar{t}) - E_{\text{res}}(\gamma \gamma \rightarrow t\bar{t}) = \frac{4 \lambda^2}{3 m_t} C_2^2 \alpha_s^2 .$$  \hspace{1cm} (40)

This NNLO result provides us with the information on the convergence of the perturbative expansion and, in particular, restricts the allowed region of the normalization point which can be used for reliable estimates. In fact, the normalization points of $\alpha_s$ entering
the coefficients $C_h$ and the nonrelativistic Green function can be different when NNLO corrections are considered. The difference between the normalization points of the hard and soft corrections can be noticed only in higher orders of perturbative expansion. This gives an additional possibility to improve the convergence of the perturbation theory. The typical hard scale of the problem is the heavy quark mass $m_t$. The soft physical scale of the problem is determined by the natural infrared cutoff related to the top quark width $\sqrt{m_t \Gamma_t}$ that measures the distance to the nearest singularity in the complex energy plane and/or by the characteristic scale of the Coulomb problem $\lambda$ i.e. $\mu_s \sim 15$ GeV. For top quark both scales are rather close to each other that makes possible a uniform description both perturbative QCD and Coulomb resonance effects. Indeed, for $\mu_s \sim 15$ GeV the NLO correction to the resonance energy reaches its minimal magnitude (see Fig. 1). However, from Fig. 1 we see that at this scale the NNLO correction is large and the series for the resonance energy diverges. At first glance this seems to contradict our physical intuition. However, since the normalization scale is defined in a rather artificial $\overline{\text{MS}}$ scheme that has little to do with peculiarities of $t\bar{t}$ physics being originally designed for describing a massless quark approximation in $e^+e^-$ annihilation, there is no reason for a literal coincidence of $\mu_s$ parameter with any physical scale of the process. The relative weight of the NNLO corrections is stabilized at $\mu_s \sim 40$ GeV where the series for the resonance energy looks like $E_{\text{res}} = E_{\text{LO}}^{\text{res}}(1 + 0.6 + 0.5)$. Moreover, for the larger values of the normalization point the $\mu$ dependence of the cross section becomes small (see the numerical analysis of the next Section). Thus our analysis implies the optimal choice of normalization point in the range $\mu_s \gtrsim 40$ GeV. The convergence of the series, however, is not fast and the uncertainty related to the truncation of the series exceeds those due to the nonperturbative effects which are suppressed parametrically as $(\Lambda_{QCD}/\lambda)^4$ \[23\]. This implies relatively large NNLO corrections to the $R^{++}$ cross section (similar effect takes place in $e^+e^- \rightarrow t\bar{t}$ top quark threshold production \[4, 8\]). Note that the apparent convergence for the resonance energy can be slightly improved by changing the
definition of the top quark mass and using \( \overline{\text{MS}} \) scheme for defining the mass parameter of the top quark instead of its (perturbative) pole mass the convergence of the series for the resonance energy becomes better.

6 Summary and conclusion.

Thus in the present paper the total cross section of the top quark pair production near the threshold in \( \gamma \gamma \) collision is computed analytically up to the next-to-leading order in perturbative and nonrelativistic expansion for general photon helicity with the top quark width being taken into account.

To demonstrate the significance of the NLO corrections we plot the functions \( R^{++}(E) \) and \( R^{+-}(E) \) in Born \( (\alpha_s = 0) \), leading order\(^2\) and NLO approximations in Fig. 2 and Fig. 3. We use \( m_t = 175 \text{ GeV} \), \( \Gamma_t = 1.43 \text{ GeV} \) and \( \alpha_s(m_Z) = 0.118 \) as typical numerical values of corresponding parameters \(^2\), \( \mu_h = m_t \) for the hard normalization scale and \( \mu_s = 25 \text{ GeV} \), 50 GeV, and 75 GeV for the soft normalization scale (in the leading order approximation the upper curves correspond to \( \mu_s = 25 \text{ GeV} \) and lower curves correspond to \( \mu_s = 75 \text{ GeV} \)). The results are almost independent of \( \mu_h \) and their dependence on the soft normalization scale decreases with increasing \( \mu_s \).

The main results that we have obtained from the study of the NLO corrections to the total cross section of the top quark-antiquark pair production near threshold in \( \gamma \gamma \) collision are:

- The typical size of the NLO corrections is 20% for \( R^{++}(E) \) and 10% for \( R^{+-}(E) \).

- Inclusion of the NLO corrections leads to considerable stabilization of the theoretical results for the cross sections against changing the normalization point.

\(^2\)In the leading order approximation \( C_h^{++} = C_h^{+-} = 1 \) and the Coulomb Green function is used.
• Relatively large top quark width smears out all the resonances in $R^{+-}(E)$ and only the ground state resonance survives in $R^{++}(E)$. The inclusion of the higher order corrections makes this peak more distinguishable. Thorough experimental study of the location and shape of this pick can provide precise data for extraction of the mass and width of top quark and the numerical value of the strong coupling constant.

• The NNLO correction to the resonance energy in $\gamma\gamma \rightarrow t\bar{t}$ top quark threshold production is relatively large that implies large NNLO corrections to the $R^{++}$ cross section.

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Figure captions

**Fig. 1.** The relative weight of the NLO corrections $E_{NLO}^{res}/E_{LO}^{res} - 1$ (solid line) and NNLO corrections $E_{NNLO}^{res}/E_{NLO}^{res} - 1$ (dotted line) to the resonance energy as a function of the normalization point $\mu_s$.

**Fig. 2.** The normalized total cross section $R^{++}(E)$ of top quark pair production for the colliding photons of the same helicity in leading order (dotted lines) and NLO (bold solid lines) for $\mu_s = 25$ GeV, 50 GeV and 75 GeV. The normal solid line corresponds to the Born approximation.

**Fig. 3.** The normalized total cross section $R^{+-}(E)$ of top quark pair production for the colliding photons of the opposite helicities in leading order (dotted lines) for $\mu_s = 25$ GeV, 50 GeV and 75 GeV and NLO (bold solid lines) for $\mu_s = 50$ GeV (the NLO result is given for only one normalization point because it has very weak dependence on the normalization scale). The normal solid line corresponds to the Born approximation.
Fig. 2.

Fig. 3.