Higher Derivative Terms in Three Dimensional
Supersymmetric Theories

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Abstract

In this work, we systematically analyze higher derivative terms in the effective actions for three dimensional scalar field theories with $\mathcal{N} = 1$ supersymmetry. In these effective actions, we show that the auxiliary fields do not acquire kinetic terms and their effective actions can be expressed in terms of physical fields. We use the derivative expansion to generate four, five and six dimensional terms for $\phi^6$ scalar field theory with $\mathcal{N} = 1$ supersymmetry. We show that along with pure fermionic terms, there are various five and six dimensional topological terms that contain bosonic and fermionic fields. Finally, we use these results to obtain higher derivative topological terms in the effective action for two M2-branes. Thus, using an off-shell formalism, we obtain several higher derivative topological terms, which we compare with an earlier study that was done using an on-shell formalism.

1 Introduction

Three dimensional supersymmetric field theories are interesting as they have been analyzed as examples of the $AdS_4/CFT_3$ correspondence in M-theory. The $OSp(8\vert 4)$ symmetry of the eleven dimensional supergravity on $AdS_4 \times S_7$ is realized as $\mathcal{N} = 8$ supersymmetry of the boundary superconformal field
theory. This boundary superconformal field theory describes a system of multiple M2-branes. Furthermore, this boundary theory is constrained not to have any on-shell degrees of freedom coming from the gauge fields. All these properties are satisfied by BLG theory [20, 22, 23, 24, 25]. The BLG theory only describes two M2-branes. However, it has been possible to construct a generalization of the BLG theory called the ABJM theory [26, 27, 28, 29]. The ABJM theory is thought to describe multiple M2-branes, and it reduces to the BLG theory for two M2-branes. Even though the ABJM theory has only $\mathcal{N} = 6$ supersymmetry, it is expected that its supersymmetry might get enhanced to full $\mathcal{N} = 8$ supersymmetry [30]. Just as the higher derivative correction to the D2-brane action can be written in form of Dirac-Born-Infeld action, it is possible to write higher derivative corrections to the ABJM theory. This can be done by writing the matter part of the ABJM theory in form of a gauge covariantized Nambu-Goto action. It may be noted that higher derivative corrections to this non-linear extension of the ABJM model have also been studied [31]. It has been shown that the Mukhi-Papageorgakis higgs mechanism can be used to determine higher derivative corrections to the BLG effective action[15] on-shell.

It may be noted that apart from the application to the physics of M2-branes and D2-branes, the addition of higher derivative corrections is interesting in its own right. Recently a generic three dimensional supersymmetric gauge theory coupled to matter fields has been constructed [32]. Under various limits this generic action reduces to the supersymmetric Maxwell theory, supersymmetric Maxwell-Chern-Simons, and supersymmetric Chern-Simons theories with matter fields. A generic three dimensional higher derivative superfield theory for self interacting scalar superfields has also been constructed [33]. In fact, in this analysis the self interacting higher derivative actions for real and complex scalar superfields have been studied.

Furthermore, supersymmetric theories with higher derivative terms play an important role in various cosmological models [1]. In the Dirac-Born-Infeld inflation, a scalar field describes the position of a brane plays the role of the inflaton field and causes an accelerated expansion of the universe [2, 3]. The higher derivatives in the action cause new dynamics to arise and lead to equilateral-type non-gaussianity in the primordial density fluctuations [4]. Furthermore, higher derivatives play an important role even in ekpyrotic universes [5, 6]. In these universes big bang is produced by the collision of branes in the bulk space. The phase transition of cosmologies from a phase of contraction to a phase of expansion is the main idea behind these models. This phase transition requires the violation of null energy condition. This conditions can be violated if the sum of the pressure and the effective energy density is negative. However, this leads to the existence of ghosts. It is
possible to overcome this problem through adding higher derivative terms [7, 8], as a result of ghost condensates [9, 10].

Higher derivative corrections coming from this approach have been studied for four dimensional field theories with $\mathcal{N} = 1$ supersymmetry [11, 12, 13, 14]. Such terms have also been studied in harmonic superspace in four dimensions [16, 17, 18, 19]. In this work, we analyze higher derivative terms occurring generated in derivative expansion of three dimensional $\mathcal{N} = 1$ supersymmetric field theory.

In this work we consider the derivative expansion of the $\phi^6$, $\mathcal{N} = 1$ supersymmetric effective action. We obtain various four, five and six dimensional higher derivative terms for this theory. We show that pure fermionic and mixed topological terms with five and six mass dimensions exist in the effective action of this theory. Finally, we use these results to obtain higher derivative contributions to the effective action for two M2-branes. We obtain several higher derivative terms which we compare to an earlier study that has been done on-shell.

The remaining paper is organized as follows. In section 2, we use derivative expansion of the effective action of a real superfield in three dimensions to show that supersymmetric higher derivative (HD) terms will not produce a kinetic energy term for the auxiliary field, therefore, the theory can be expressed in terms of the physical fields. We give a simple example of free real superfield and calculate the HD terms in this case. In section 3, we use derivative expansion to generate HD terms with mass dimensions four, five and six in the supersymmetric $\phi^6$ theory with a single superfield. In section 4, we apply derivative expansion of the supersymmetric $\phi^6$ theory with multiple superfields to BLG theory to generate all topological six dimensional HD terms of two M2-branes effective actions and compare our result to the previous studies. In this results. In the last section we present our conclusion.

2 Auxiliary Fields

In this section, we use derivative expansion of the effective action of a real superfield in three dimensions to show that supersymmetric higher derivative (HD) terms will not produce a kinetic energy term for the auxiliary field, therefore, the theory can be expressed in terms of the physical fields. We will give a simple example of free real superfield and calculate the HD terms in this case.

Before presenting our argument for the supersymmetric theories, let us analyse the leading order higher derivative terms for a non-supersymmetric theory. These terms will correct the kinetic energy term in the low energy...
effective action of the theory. A natural frame work to study a set of HD terms in a particular theory is the derivative expansion of the low energy effective action, which reproduces the theory in the infrared limit. We will now perform such an expansion for a free massless scalar field ($\phi$) theory in three dimensions. It is important to list the mass dimension of various fields, since in effective field theories, we consider the action up to a particular dimension which are suppressed by some microscopic length scale $l$. From kinetic terms of various fields one can obtain the mass dimension for each field and derivative as, $[\phi] = 1/2$, $[\partial] = 1$, $[m] = 1$. It is important to state here that even if we are only interested in studying six dimensional terms, for example, we have to include four and five dimensional terms for the consistency of the low energy effective action expansion.

Now consider a derivative expansion of a low-energy effective action of a real superfield $\Phi$ in a generic $\mathcal{N} = 1$ supersymmetric theory

$$S_t = \int d^2 \theta d^3 x \mathcal{L}(\Phi, D\Phi, ...). \tag{1}$$

Since the length scale $l$ controls the derivative expansion, the leading order term has three mass dimensions. After integrating over the fermionic coordinates we get

$$S_t = S_0 + l S_1 + l^2 S_2 + O(l^3) \tag{2}$$

where $S_n = S_n(\phi, \psi, F)$; $n = 0, 1, 2, ...$ are functions of the component fields and have $n + 3$ mass dimensions. Therefore, $S_0$ must be at most quadratic in the auxiliary field $F$, since $\Phi = \phi + \theta \psi - \theta^2 F$, i.e., $[F] = 3/2$. This means the general form of $S_0$ is given by

$$S_0 = \int d^3 x \left[ \frac{\alpha}{2} F^2 + g(\phi, \psi) F + k(\phi, \psi) \right]. \tag{3}$$

Now let us obtain the field equation of $F$, it reads

$$S_t' = S_0' + l S_1' + l^2 S_2' + O(l^3) \tag{4}$$

where $S' = \frac{\delta S}{\delta F}$. Expanding $F$ in terms of $l$, one obtains

$$F = F_0 + l F_1 + l^2 F_2 + O(l^3). \tag{5}$$

Using equation (5) and expand $S_1'$ and $S_2'$ in terms of $l$ we get the following equations

$$\alpha F_0 + g(\phi, \psi) = 0,$$
$$\alpha F_1 + S_1'(\phi, \psi, F_0) = 0,$$
$$\alpha F_2 + S_2'(\phi, \psi, F_0) + S_1''(\phi, \psi, F_0) F_1 = 0. \tag{6}$$

\footnote{Here we use the notation of Ref.[34]}
These equations show that all the fields $F_n$'s, can be all expressed as functions of the physical fields $\phi$ and $\psi$, therefore, $F$ has no kinetic term and can be written entirely in terms of $\phi$ and $\psi$ independent of the form of the higher derivative terms $S_n$. In fact, one can extend this argument to $N = 1$ $D = 4$ chiral and vector superfields in low-energy effective actions which we will report elsewhere [21].

3 Application

Now we will apply the above argument to the supersymmetric action of a free massless real superfield $\Phi$ with higher derivative terms. Consider a real superfield $\Phi(x, \theta) = \phi + \theta^a \psi_\alpha - \theta^2 F$ [34], whose action is given by

$$S_0 = - \int d^3x d^2\theta \left[ \frac{1}{2} (D^\alpha \Phi)^2 \right].$$

(7)

In component form the above action can be written as

$$S_0 = \int d^3x \frac{1}{2} [F^2 + i \psi^\alpha \partial^\alpha \psi_\beta + \phi \Box \phi].$$

(8)

Using field equation of the auxiliary field $F = 0$, which follows from this action, we can write the action of the component fields as,

$$S_0 = \int d^3x \frac{1}{2} [i \psi^\alpha \partial^\alpha \psi_\beta + \phi \Box \phi].$$

(9)

Now considering the above action with four and five dimensions HD terms, we obtain the following total action, The HD terms up to five dimension terms reads

$$S_t = \int d^3x \; d^2\theta \left[ \Phi D^2 \Phi + \alpha l (D^2 \Phi)^2 + \beta l^2 D^2 \Phi \Box \Phi \right] + O(l^3),$$

(10)

where $\alpha$ and $\beta$ are some couplings. Expanding the above action in component one gets

$$S_t = \int d^3x \; \frac{1}{2} [F^2 + \phi \Box \phi + i \psi^\alpha \partial^\alpha \psi_\beta] + \alpha l [\psi^\beta \Box \psi_\beta + 2F \Box \phi]$$

$$+ \beta l^2 \left[ \phi \Box^2 \phi - i \psi^\alpha \partial^\alpha \psi_\beta + F \Box F \right] + O(l^3).$$

(11)

It may be noted that the auxiliary field has a kinetic-like term which is a common term if we consider supersymmetric HD terms in most of theories. These terms might leads to a nonunitary theory if $\beta < 0$, which leaves the
vacuum of the field theory unstable. Since microscopic scale $l$ controls the
derivation expansion of the effective action and suppresses HD terms, it is
natural to expand the fields in terms of $l$. Also since all field fluctuations
larger than $1/l$ has been integrated out, the only field fluctuations we have in
the should be less than $1/l$. The field equation of the auxiliary field is given
by
\[ F + 2 \alpha l \Box \phi + \beta l^2 \Box F = 0. \]
(12)

It is natural to expand $F$ in terms of $l$
\[ F = F_0 + lF_1 + l^2 F_2 + O(l^3). \]
(13)

Now using field equation one gets,
\[ F_0 = 0, \]
\[ F_1 = -2 \alpha \Box \phi, \]
\[ F_2 = -\beta \Box F_0 = 0. \]
(14)

Now the total action reads
\[ S_t = \int d^3x \frac{1}{2} [\phi \Box \phi + i \psi^\alpha \partial_\alpha \beta \psi_\beta] + \alpha l \psi^\beta \Box \psi_\beta \]
\[ + l^2 [(\beta - 2 \alpha^2) \phi \Box^2 \phi - i \beta \psi_\alpha \partial_\alpha \beta \Box \psi_\beta] + O(l^3). \]
(15)

It is not surprising that supersymmetry requires the existence of addi-
tional four dimension terms, and such terms did not exist in the non-
supersymmetric version of the action, since it contains the fermionic field $\psi$.
However, the interesting result here is that four dimension terms include an
interesting fermionic topological term, $i \epsilon^\mu\nu\sigma \partial_\mu \psi_\beta \partial_\nu \psi_\gamma (\gamma_\sigma)^\beta_\gamma$. Even though,
it is a total derivative for four mass-dimension terms, it is clear that this
term shows the possibility of having a five dimensional topological term in
an interacting theory that includes boson and fermion fields. Indeed, we
will see that in the coming section, where we show the possibility of hav-
ing a pure fermionic topological term with six mass dimensions of the form,
$\psi^2 \epsilon^\mu\nu\sigma \partial_\mu \psi_\beta \partial_\nu \psi_\gamma (\gamma_\sigma)^\beta_\gamma$.

### 4 Interacting Theory

Now we will analyze higher derivative terms for the interacting supersym-
metric $\phi^6$ theory, which is renormalizable in three dimensions. The super-
symmetric action for this theory can be written as
\[ S_0 = \int d^3x d^2 \theta \left[ -\frac{1}{2} (D^\alpha \Phi)^2 + \frac{\lambda}{4!} \Phi^4 \right]. \]
(16)
In component form the above action can be written as

$$S_0 = \int d^3 x \frac{1}{2} [F^2 + i \psi^\alpha \partial_\alpha \psi^\beta + \phi \Box \phi] + \frac{\lambda}{2} \phi^2 \psi^2 + \frac{\lambda}{3!} F \phi^3.$$  \hspace{1cm} (17)

Using the field equation for the auxiliary field $F = - \lambda \phi^3/3!$, we obtain

$$S_0 = \int d^3 x \frac{1}{2} [i \psi^\alpha \partial_\alpha \psi^\beta + \phi \Box \phi] + \frac{\lambda}{2} \phi^2 \psi^2 - \frac{\lambda^2}{2(3!)^2} \phi^6.$$ \hspace{1cm} (18)

Now we will write the low energy effective theory with all possible HD terms consistent with the symmetries up to dimension five. The list of independent four and five dimensional terms is given by

$$S_1 = l \int d^2 \theta \left[ c_1 (D^2 \Phi)^2 + c_2 \Phi^2 (D_\alpha \Phi)^2 + c_4 \Phi^6 \right],$$

$$S_2 = l^2 \int d^2 \theta \left[ c_5 D^2 \Phi \Box \Phi + c_6 \Phi^2 (D^2 \Phi)^2 + c_7 \Phi D^2 \Phi (D_\alpha \Phi)^2 \right. + c_8 (D_\alpha \Phi)^4 + c_9 \Phi^4 (D_\alpha \Phi)^2 + c_{10} \Phi^8 \right].$$ \hspace{1cm} (19)

Now we can list all the four dimensional terms in component fields as

$$\int d^2 \theta (D^2 \Phi)^2 = \psi_\alpha \Box \psi^\alpha + 2F \Box \phi,$$

$$\int d^2 \theta \Phi^2 (D_\alpha \Phi)^2 = \phi^2 \left[ F^2 + i \psi^\alpha \partial_\alpha \psi^\beta + \phi \Box \phi \right] + \psi^4 + 2\psi^2 \phi F,$$

$$\int d^2 \theta \Phi^6 = 30 \psi^2 \phi^4 + 6F \phi^5.$$ \hspace{1cm} (20)

We can also list all the five dimensional terms in component fields as

$$\int d^2 \theta (D^2 \Phi) \Box \Phi = \phi \Box^2 \phi - i \psi_\alpha \partial^\alpha \beta \Box \psi^\beta + F \Box F,$$

$$\int d^2 \theta (D^2 \Phi)^2 \Phi^2 = \phi^2 \left[ \psi^\beta \Box \psi^\beta + i \epsilon^{\mu \nu \sigma} \partial_\mu \psi^\beta \partial_\nu \psi^\gamma (\gamma_\sigma)^{\beta} \right.$$

$$+ 2F \Box \phi] + 2 \phi F^3 +$$

$$+ 2 \psi^2 F^2 + i 8 \phi \psi_\alpha \partial^\alpha \psi^\beta \phi F,$$

$$\int d^2 \theta (D^2 \Phi) \Phi (D_\alpha \Phi)^2 = F^2 \phi^2 - i \psi^2 \psi^\alpha \partial^\alpha \psi^\beta + \phi \Box \phi \psi^2 +$$

$$+ i \phi F \psi^\alpha \partial^\alpha \psi^\beta - \phi \psi^\gamma \partial_\mu \psi_\gamma \partial^\mu \phi$$
\[ + i \epsilon^{\mu \nu \sigma} \phi \psi^\beta (\gamma_\alpha)^\beta_\beta \partial_\mu \psi_\alpha \partial_\nu \phi, \]
\[ \int d^2 \theta (D_\alpha \Phi)^4 = \psi^2 [\partial_\mu \phi \partial^\mu \phi + 2 F^2], \]
\[ \int d^2 \theta \Phi^4 (D_\alpha \Phi)^2 = 12 \psi^4 \phi^2 + 4 \psi^2 \phi^3 F + 8 \psi^2 \phi^3 F \]
\[ + \phi^4 [F^2 + i \psi^\alpha \partial^\alpha \psi^\beta + \phi \Box \phi], \]
\[ \int d^2 \theta \Phi^6 = 56 \psi^2 \phi^6 + 8 \phi^7 F. \]  

(21)

It is interesting to notice the existence of topological term that mixes scalars and fermions in five dimensional terms. Usually these terms have quantized coupling and non-renormalized which makes them important very interesting terms and important tools to study the non-perturbative nature of a given theory. The existence of a single scalar field causes some topological terms to vanish for example, terms which involve only scalar fields as well as mixed terms such as \( \psi_\beta (\gamma_\alpha)^\beta_\beta \psi^\gamma \epsilon^{\mu \nu \sigma} \partial_\mu \phi \partial_\nu \phi \). In the next section we will see that upon having more than one component for the superfield these terms becomes nonvanishing.

Now the list of six-dimension terms in supersymmetric \( \phi^6 \) theory is given by

\[ S_3 = l^3 \int d^3 x d^2 \theta [c_11 (D^2 \Phi)^3 \Phi + c_{12} (D^2 \Phi)^2 \Phi^4 + c_{13} (D^2 \Phi) \Phi^7 \]
\[ + c_{14} (D_\alpha \Phi)^2 D^2 \Phi \Phi^3 + c_{15} (D_\alpha \Phi)^2 (D^2 \Phi)^2 \]
\[ + c_{16} D^2 \Phi (\Box \Phi) \Phi^2 + c_{17} (\Box \Phi)^2 + c_{18} (\Box \Phi) \Phi^5 + c_{19} \Phi^{10}] \]  

(22)

These terms will give the following contributions in terms of component fields

\[ \int d^2 \theta (D^2 \Phi)^3 \Phi = 3 F^2 [i \psi^\alpha \partial^\alpha \psi_\alpha + F^2 + \phi \Box \phi] \]
\[ - 6 \phi F \partial_\mu \psi^\alpha \partial^\mu \psi_\alpha \]
\[ + i 6 \phi F \epsilon^{\mu \nu \sigma} \partial_\mu \psi^\alpha (\gamma_\sigma)_{\alpha \beta} \partial_\nu \psi^\beta, \]
\[ \int d^2 \theta (D^2 \Phi)^2 \Phi^4 = 2 F^3 \Phi^3 + 3 \psi^2 \phi^2 F^2 + i 8 \psi^\alpha \partial^\alpha \psi^\beta \phi^3 F \]
\[ + 2 \phi^4 F \Box \phi - \frac{1}{2} \phi^4 \partial_\mu \psi^\beta \partial^\mu \psi^\beta \]
\[ - \frac{i}{2} \phi^4 \epsilon^{\mu \nu \sigma} (\gamma_\sigma)_{\alpha \beta} \partial_\mu \psi^\beta \partial_\nu \psi^\alpha \]
\[ + 4 i F \phi^3 \psi^\alpha \partial^\alpha \psi^\beta, \]
\[ \int d^2 \theta (D^2 \Phi) \Phi^7 = \phi^7 \Box \phi - 7 i \phi^6 \psi^\alpha \partial^\alpha \psi^\beta + 7 F^2 \phi^6 \]
\[ \int d^2 \theta (D_\alpha \Phi)^2 (D^2 \Phi) \Phi^3 = \]
\[ + 42 \psi^2 F \phi^5, \]
\[ \phi^3 \Box \psi^2 - \frac{i}{4} \phi^3 \epsilon^{\mu \nu \sigma} (\gamma_\sigma)_\alpha^\beta \partial_\mu \psi \partial_\nu \psi^\alpha \]
\[ = \frac{1}{4} \phi^3 \psi \partial_\mu \psi \partial^\mu \phi + \frac{i}{4} \phi^3 F \psi^\alpha \partial_\alpha \psi \beta \]
\[ + 3i \phi^2 \psi^2 \partial_\alpha \psi \beta - 2 F^3 \phi^3 \]
\[ + F^3 \partial_\mu \phi \partial_\nu \phi \phi + \frac{3}{2} F^2 \psi^2 \phi^2 + 6 F \psi^4 \phi, \]
\[ \int d^2 \theta (D_\alpha \Phi)^2 (D^2 \Phi)^2 = \]
\[ F \psi^2 \Box \phi - \frac{i}{2} \psi^2 [\partial_\mu \psi \partial_\mu \psi^\alpha] \]
\[ + i \epsilon^{\mu \nu \sigma} (\gamma_\sigma)_\alpha^\beta \partial_\mu \psi \partial_\nu \psi^\alpha \]
\[ - \frac{3}{2} F^2 \psi^\alpha \partial_\alpha \psi \beta + \frac{3}{2} F [\partial_\mu \psi \partial_\mu \phi \psi^\alpha] \]
\[ + i \epsilon^{\mu \nu \sigma} (\gamma_\sigma)_\alpha^\beta \partial_\mu \psi \partial_\nu \phi \psi^\alpha, \]
\[ \int d^2 \theta (D^2 \Phi) \Box \Phi^2 = \]
\[ (\Box \phi)^2 \phi^2 - 2 i \psi_\alpha \partial_\beta \psi \phi \Box \phi \]
\[ + F \Box F \phi^2 + 2 F \psi_\beta \Box \psi \beta \]
\[ + F \Box F \phi^2 + 2 F^2 \phi \Box \phi - 2 F \psi^2 \Box \phi, \]
\[ \int d^2 \theta (\Box \Phi)^2 = \]
\[ 2 \Box F \phi + \Box \psi_\alpha \Box \psi^\alpha, \]
\[ \int d^2 \theta (\Phi)^{10} = \]
\[ \frac{45}{2} \phi^8 \phi^2 + 10 F \phi^9, \]
\[ \int d^2 \theta (\Box \Phi) \Phi^5 = \]
\[ 2 \phi^5 \Box F + 5 \phi^4 \psi_\alpha \Box \psi^\alpha \]
\[ + 5 F \phi^4 \Box \phi + 20 \phi^3 \psi^2 \Box \phi. \]

Notice the existence of nonvanishing purely fermionic topological terms as well as mixed topological terms. With more than one superfield, these terms should play an important role in the effective action of M2-branes.

As we have mentioned before, having a single superfield forces certain topological terms to vanish as a result of antisymetrizing spacetime derivatives in such terms. Let us list few examples of those terms keeping in mind that some of them (dimension 6 terms) will appear in the following section. Thus, we can write the five dimensions term as

\[ \int d^2 \theta (D_\beta \Phi)(D^\beta D_\rho \Phi)(D^\rho \Phi) \Phi \]
\[ \supset \epsilon^{\mu \nu \sigma} \partial_\mu \phi \partial_\nu \phi \psi_\beta (\gamma_\sigma)_\alpha^\beta \psi^\alpha + \epsilon^{\mu \nu \sigma} \phi \partial_\mu \phi \partial_\nu \phi \partial_\sigma \phi, \]  

(24)
and the six dimensional term as

\[ \int d^2 \theta \Phi^3 (D_\beta \Phi)(D^\beta D_\rho \Phi)(D^\rho \Phi) \]
\[ \supset \epsilon^{\mu \nu \sigma} \phi^2 \partial_\mu \phi \partial_\nu \phi \psi^3 (\gamma_\alpha)^\alpha_\beta \psi_\alpha + \epsilon^{\mu \nu \sigma} \phi^3 \partial_\mu \phi \partial_\nu \phi \partial_\rho \phi. \]  

(25)

5 Topological Terms for M2-Branes

In this section we are going to use the above analysis to argue for the existence of new topological HD terms in the effective action of M2-branes. The BLG theory describes the physics of two M2 branes. Here we review the \( \mathcal{N} = 1 \) superspace formalism of such a theory [35]. Fields in the BLG theory take values in a Lie 3-algebra rather than a conventional Lie algebra. A Lie 3-algebra is a vector space endowed with a trilinear product,

\[ [T^a, T^b, T^c] = f^{abc}_d T^d, \]

(26)

The structure constants of this Lie 3-algebra are totally antisymmetric in \( a, b, c \). They also satisfy the Jacobi identity [36],

\[ f^{[abc}_{g} f^{deg}_h = 0. \]

(27)

The metric of this Lie 3-algebra can be defined by taking the trace over the Lie 3-algebra indices,

\[ h^{ab} = Tr(T^a T^b). \]

(28)

It is also possible to define a symmetrised trace of four Lie 3-algebra generators as

\[ Str(T^a T^b T^c T^d) = m h^{[ab} h^{cd]}, \]

(29)

where \( m \) is a constant. For the Lorentz Lie 3-algebra, it is possible to consider a set of generators corresponding to a compact subgroup of the full symmetry group. Hence, we can choose the generators of a \( SU(2) \) Lie algebra, and write [15]

\[ Tr(T^a T^b) = \frac{1}{2} \delta^{ab}, \]

\[ Str(T^a T^b T^c T^d) = \frac{1}{4} \delta^{(ab} \delta^{cd)). \]

(30)

The gauge fields are valued in the Lie 3-algebra, \( \Gamma^a_{ab} T^a T^b = \Gamma^a \). The BLG theory has been written as \( \mathcal{N} = 1 \) supersymmetric theory in terms of a real superfield \( \Phi^I_a = \phi^I_a + \theta^a \psi^I_{\alpha a} - \theta^2 \Phi^I_a \), with \( I = 1, 2, \ldots, 8 \), where \( a \) is
the three-algebra index with a structure constant $f_{abcd}$. So, we can write the action for the BLG theory as [35]

$$S_0 = -\int d^3x d^2\theta \left[ \frac{1}{4} (D^a \Phi^I_d + f^{abc}_d \Gamma^a_{ab} \Phi^I_c)^2 ight] + \frac{1}{8} f^{abcd} (D^a \Gamma^b_{ab}) (D^b \Gamma^c_{cd}) + \frac{1}{6} f^{cda} f^{efg} (D^a \Gamma^b_{ab}) \Gamma^c_{cd} \Gamma^e_{ef} + \frac{1}{24} f^{abcd} C_{IJKL} \Phi^I_a \Phi^J_b \Phi^K_c \Phi^L_d].$$ (31)

The component field definitions are as

$$\Phi^I_a = \phi^I_a, \quad D^a \Phi^I_a = \psi^I_a, \quad D^2 \Phi^I_a = F^I_a, \quad \Gamma_{ab} = \chi_{ab}, \quad \frac{1}{2} D^a \Gamma_{ab} = B_{ab}, \quad \Gamma_{ab} = 2\lambda_{ab} - i \partial^\beta \chi_{ab}, \quad D^a \Gamma_{ab} = i (\gamma_\mu)_{ab} A^\mu_{ab} - \delta_{ab} B_{ab}, \quad D^2 \Gamma_{ab} = 2\lambda_{ab} + i \partial^\beta \chi_{ab},$$

$$D^a \Gamma_{ab} = i (\gamma_\mu)_{ab} A^\mu_{ab} + \delta_{ab} B_{ab}, \quad D^a \Gamma_{ab} = 2\lambda_{ab} + i \partial^\beta \chi_{ab}.$$ (32)

An octonion algebra \(\{1, e_i\}\), with \(i = 1, \ldots, 7\), such that \(e_i e_j = e_{ijk} e_k - \delta_{ij}\), has been used to defined \(C_{IJKL}\). This is done by taking a totally antisymmetric tensor \(e_{ijk}\). The seven dimensional dual of this is \(e_{ijkl} = \frac{1}{6} \epsilon_{ijkmn} e^{mno}\). Now it is possible to construct an \(SO(7)\) invariant tensor \(C_{IJKL I,J,K,L=1,..,8}\) which is self dual in eight dimensions, \(C_{ijkl} = c_{ijk} C_{ijkl} = c_{ijkl}\). This octonionic structure constants can be used to construct \(SO(8)\) gamma matrices [35]. So, we can use write \((\Gamma^I)_{A\dot{A}} = c^I_{A\dot{A}} + \delta_{8A} \delta_{A\dot{A}} - \delta_{8A} \delta_{A\dot{A}}\), where \(i = 1, \ldots, 7\) and \(A, \dot{A} = 1, \ldots, 8\). We also have \((\Gamma^8)_{A\dot{A}} = \delta_{AA}, c_{8A} = c^I_{A\dot{A}} = 0\). Here we have defined \(\hat{\Gamma}^I_{A\dot{A}} = (\Gamma^T)^I_{A\dot{A}}\), and \(\Gamma^I \hat{\Gamma}^J + \Gamma^J \hat{\Gamma}^I = 2\delta^{IJ}\). Now the clifford algebra can be written as

$$\gamma^I \gamma^J + \gamma^J \gamma^I = 2\delta^{IJ},$$ (33)

where

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I_{A\dot{A}} \\ \hat{\Gamma}^I_{A\dot{A}} & 0 \end{pmatrix}.$$ (34)

Now we can also write

$$\Gamma^{IJ}_{AB} = \frac{1}{2} \left( \Gamma^I_{AA} \hat{\Gamma}^J_{AB} - \Gamma^J_{AA} \hat{\Gamma}^I_{AB} \right) = C^{IJ}_{AB} + \delta^I_A \delta^J_B - \delta^I_B \delta^J_A.$$ (35)
The effective M2 brane action can be expanded in terms of Planck length $l_p$ as follows

$$S_{BLG} = S_0 + l_p^3 S_3 + \ldots$$

(36)

Therefore, the first correction to the leading contribution is of dimension six. This is why we have expanded our effective action in the general $\phi^6$ theory up to such order. The action of the theory without the gauge field, i.e., the Higgs branch, is given by

$$S_0 = -\int dx^3 d^2 \theta \text{Tr} \left( (D_\alpha \Phi)^2 + \frac{1}{12} [\Phi^I, \Phi^J, \Phi^K] \Phi^L C_{IJKL} \right).$$

(37)

Since the leading correction is of order $O(l_p^3)$, the field equation for the auxiliary field is given by

$$F_a = -\frac{1}{6} f^{abcd} C_{IJKL}^I \Phi_b^I \Phi_c^J \Phi_d^K \Phi_L^L + O(l_p^3).$$

(38)

The nonvanishing six dimensional topological terms can be classified as bosonic, fermionic and mixed terms. Here we list all such terms.

Now we find the following bosonic terms,

$$L_{b_1} = \int d^2 \theta \text{Tr} \left( C_{IJKL} C_{J'K'L'}^I \Phi^{J'K'L'} D_{\beta} \Phi^J D_{\beta} \Phi^K D_{\gamma} \Phi^L \right)$$

$$\supset \text{Tr} \left( C_{IJKL} C_{J'K'L'}^I \phi^{J'K'L'} \epsilon^{\mu\nu\sigma} \partial_{\mu} \phi^J \partial_{\sigma} \phi^K \partial_{\nu} \phi^L \right)$$

$$= \frac{1}{4} C_{IJKL} C_{J'K'L'}^I \phi_{a'b'c'}^J \phi_{a'b'c'}^J \epsilon^{\mu\nu\sigma}$$

$$\times \partial_{\mu} \phi_{a'}^J \partial_{\sigma} \phi_{c'}^J \delta_{ab} \delta_{cd}$$

(39)

Another term that produces the same topological term is given by

$$L_{b_1'} = \int d^2 \theta \text{Tr} \left( C_{IJKL} D^2 \Phi^I D_{\beta} \Phi^J D_{\beta} \Phi^K D_{\gamma} \Phi^L \right)$$

$$\supset \text{Tr} \left( C_{IJKL} F^I \epsilon^{\mu\nu\sigma} \partial_{\mu} \phi^J \partial_{\sigma} \phi^K \partial_{\nu} \phi^L \right).$$

(40)

The relation between this term and the previous one is clear upon using equation (38). This term has also been produced in earlier studies of the M2 brane HD terms [15]. We also find three different fermionic terms

$$L_{f_1} = \int d^2 \theta T_{\mu} \left( C_{IJKL} \alpha^I [D_\alpha \Phi^J, D^2 \Phi^K, D^2 \Phi^L] \right)$$

$$\supset C_{IJKL} f^{abcd} \epsilon^{\mu\nu\sigma} \psi_{a'}^I \cdot \psi_{a'}^I \partial_{\mu} \psi_{c'}^J \cdot \partial_{\sigma} \psi^L_d$$

$$= \int d^2 \theta T_{\mu} \left( D^\alpha \Phi^I D_\alpha \Phi^J D^2 \Phi^I D^2 \Phi^J \right)$$

$$L_{f_2} = \int d^2 \theta T_{\mu} \left( D^\alpha \Phi^I D_\alpha \Phi^J D^2 \Phi^I D^2 \Phi^J \right)$$

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where $\psi \cdot \xi = \frac{1}{2} \psi^a \xi_a$ and $\psi \cdot \gamma^\mu \cdot \xi = \psi_a (\gamma^\mu)^{a3} \xi_3$. This last term can be expressed in the notation of [15] as $Tr (\Psi \Phi^{\mu} \Phi \gamma^\nu D_\alpha \Psi )$. Finally, we find the following mixed terms,

$$\mathcal{L}_{m1} = \int d^2 \theta Tr \left( D_\gamma D_\alpha \Phi^I [\Phi^I, D_\beta \Phi^J, D_\delta \Phi^K] \right)$$
$$\supset \delta^{(a\delta cd)} \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^J$$

$$\mathcal{L}_{m2} = \int d^2 \theta Tr \left( C_{IJKL} D_\gamma D_\alpha \Phi^I \Phi^J D_\beta \Phi^K D_\delta \Phi^L \right)$$
$$\supset C_{IJKL} \delta^{(a\delta cd)} \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^L$$

$$\mathcal{L}_{m3} = \int d^2 \theta Tr \left( C_{IJKL} C_{I'J'K'L} \Phi^{I'J'K'L} \Phi^I \Phi^J \Phi^K \Phi^L \right)$$
$$\supset C_{IJKL} C_{I'J'K'L} F_d \epsilon^{abc_d} \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^K$$

The same topological term can be produced by

$$\mathcal{L}_{m5} = \int d^2 \theta Tr \left( C_{IJKL} D_2 \Phi^L [\Phi^I, D_\alpha \Phi^J, D_\beta D_\gamma \Phi^K] \right)$$
$$\supset C_{IJKL} D_2 F_d \epsilon^{abc_d} \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^K$$

The last mixed term can be written as

$$\mathcal{L}_{m4} = \int d^2 \theta Tr \left( C_{I',J'K'L} \Phi^{I'J'K'L} \Phi^I \Phi^J \Phi^K \Phi^L \right)$$
$$\supset C_{I',J'K'L} \phi_a^{I'J'K'} \phi_b^{I'J'K'} \epsilon^{abc} \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^K$$

Another term that produces the same topological term, after using Eq. (38), takes the form

$$\mathcal{L}_{m4'} = \int d^2 \theta Tr \left( D_2 \Phi^I \Phi^J \Phi^K \Phi^L \right)$$
$$\supset \delta^{(ab cd)} F_d \epsilon^{\mu \nu \sigma} \phi_a^I \phi_b^J \partial_\mu \psi_c^I \cdot \gamma^\nu \cdot \partial_\sigma \psi_d^J$$

It is worth mentioning here that these six-dimensional HD terms of the M2-branes effective action have been calculated in earlier studies [15]. This
was done by using a novel Higgs mechanism, and this reduced the M2-brane action to a matter-Yang-Mills theory describing the low energy effective action of multiple D2-branes. The HD terms obtained in this work are written in terms of on-shell fields, and this made several of these HD terms to vanish. In contrast to this, the HD terms in this work have been constructed using off-shell fields, and they have been written in terms of $\mathcal{N} = 1$ superfields. This explains why all these terms were not obtained in earlier studies [15]. In fact, in the earlier on-shell formalism only the pure bosonic topological term was obtained. Apart from this pure bosonic terms, all the HD terms produced here using the off-shell formalism, vanishes on-shell, since they contain fermions and can be written in terms of one or more factors of $\partial_{\alpha\beta}\psi^\alpha$. This explains why these terms were absent in earlier studies [15].

6 Conclusion

In this paper, we have analyzed the higher derivative terms for three dimensional supersymmetric theories with $\mathcal{N} = 1$ supersymmetry. We first analyzed the higher derivative terms for a general scalar superfield theory and argue that the auxiliary field will not acquire a kinetic term for all possible actions of the theory. Therefore, the theory is completely described in terms of its physical fields. We calculated all the four, five and six dimensional terms for such a theory demonstrating the existence of various interesting topological terms. Some of these terms mix scalars and fermions in addition to a pure fermionic topological term. We apply this procedure to BLG theory describing two M2-branes written in $\mathcal{N} = 1$ superspace language. We show the existence of several mixed and pure fermionic terms which vanishes on-shell. These terms were absent in the list of six-dimensional terms generated in earlier studies [15], as those results were expressed in terms of on-shell fields and the results in this paper are written in terms of $\mathcal{N} = 1$ off-shell superfields.

It will be interesting to generalize the results of this paper, for theories with higher amount of supersymmetry. Furthermore, it will be interesting to perform a similar analysis for matter fields coupled to gauge fields. The results thus obtained can be used for analyzing HD corrections to the M2-branes effective actions using the ABJM theory. It is possible to extend the work done on global supersymmetry with higher derivative terms to local supergravity theories. In fact, the supergravity extension of scalar field theories with higher derivative terms has also been already studied [38]. This analysis was done using supergravity in $\mathcal{N} = 1$ superspace formalism. The elimination of auxiliary fields modifies both the kinetic and potential terms in
this theory. In this case, it has been demonstrated that potential energy can be generated even if there was no original superpotential term in the action. It will be interesting to extend the results of this paper to local supergravity theories in three dimensions.

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