Properties of charmed and bottom hadrons in nuclear medium:
Results for $\Lambda_c^+$ and $\Lambda_b$ hypernuclei

K. Tsushima$^*$, F.C. Khanna

1Department of Physics and Astronomy, University of Georgia, Athens, GA 30602, USA
2Department of Physics, University of Alberta, Edmonton, Canada, T6G 2J1, and TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., V6T 2A3, Canada

This reports our recent studies on changes in properties of heavy hadrons containing at least a charm or a bottom quark in nuclear matter, and that the results for the $\Lambda_c^+$ and $\Lambda_b$ hypernuclei are studied quantitatively. Comparisons are made with the results for the $\Lambda$ hypernuclei studied previously in the same approach. It is shown that although the scalar and vector potentials for the $\Lambda$, $\Lambda_c^+$ and $\Lambda_b$ in the hypernuclei multiplet with the same baryon numbers are quite similar, the wave functions obtained, e.g., for $1s_{1/2}$ state, are very different. The $\Lambda_c^+$ probability density distribution in $^{209}\text{Pb}$ is much more pushed away from the center than that for the $\Lambda$ in $^{209}\text{Pb}$ due to the Coulomb force. On the contrary, the $\Lambda_b$ probability density distributions in $\Lambda_b$ hypernuclei are much larger near the origin than those for the $\Lambda$ in the $\Lambda$ hypernuclei due to its heavy mass. A possibility of $B^-$ nuclear bound (atomic) states is also briefly discussed.

§1. Introduction

Partial restoration of chiral symmetry in nuclear medium is now one of the most important and interesting issues in nuclear and hadronic physics. There is no doubt that it plays a crucial role in understanding numerous phenomenon involving many particles, such as relativistic heavy ion collisions, structure and properties of neutron stars, and those of heavy nuclei, and so on. Coverage of the issue is too vast to cite complete references. This workshop is also one of the activities to understand and study the role of chiral restoration in nuclear medium.

Our focus is on the consequences of partial restoration of chiral symmetry in nuclear medium, on the properties of heavy hadrons containing at least a charm or a bottom quark. We report the results on the changes in properties of heavy hadrons in nuclear matter, and results for the $\Lambda_c^+$ and $\Lambda_b$ hypernuclei studied quantitatively.

In spite of the importance, there are studied for heavy mesons with charm only a limited number ($J/\Psi$ and $D(D\bar{D})$) in nuclear matter using the QCD sum rule. However, there seem to exist no studies for heavy baryons with a charm or a bottom quark, except for the studies made recently. Furthermore, in considering recent experiments on high energy heavy ion collisions, to study general properties of heavy hadrons in nuclear medium is essential, because elementary hadronic reactions occur in high nuclear density zone of the collisions, and many hadrons produced there are under the influence of a surrounding nuclear medium.

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Although the baryons with a charm or a bottom quark have a typical mean life of the order $10^{-12}$ seconds (magnitude is shorter than hyperons), we would like to gain an understanding of the movement of such a hadron in its nucleonic environment. The light quark in the hadron (and nucleons) would change its property in nuclear medium in a self-consistent manner, and will thus affect the overall interaction with nucleons. With this understanding we will be in a better position to learn about the hadron properties with the presence of heavy quarks, or those for the hadrons containing heavy quarks in nuclear matter (in finite nuclei).

The approved construction of the Japan Hadron Facility (JHF), with a beam energy of 50 GeV, will produce charmed hadrons profusely and bottom hadrons in lesser numbers but still with an intensity that is comparable to the present hyperon production rates. The production of charmonium ($\bar{c}c$), mesons with charm, and baryons with charm quarks will be sufficiently large to make it possible to study charmed hypernuclei. In mid 70’s, a possibility of such charmed hypernuclei was predicted theoretically. There were also studies of possible experimental searches at the ARES facility and at the $\pi\tau$-factory. It is clear that the situation for the experiments to search for such charmed and bottom hypernuclei is now becoming realistic and would be realized at JHF.

At JHF, in addition to charmed and bottom hyperons, mesons with open charm (bottom) like $D^-(\bar{c}d)$ ($B^-\bar{u}b$) will be produced. Such mesons like $K^-\bar{u}s$ can form mesic atoms around finite nuclei. The atomic orbits will be very small and will thus probe the surface of light nuclei and will be within the charge radii for heavier nuclei. Thus, at least for light nuclei they will give a precise information about the charge density.

To perform theoretical studies, at present we need to resort to a model which can describe the properties of finite nuclei as well as hadron properties in nuclear medium based on the quark degrees of freedom. With its simplicity and applicability, we use quark-meson coupling (QMC) model which has been extended and successfully applied to many problems in nuclear physics, hypernuclei, and properties of hadrons in nuclear medium. In particular, recent measurements of polarization transfer performed at MAMI and Jlab support the medium modification of the proton electromagnetic form factors calculated by the QMC model. The final analysis seems to become more in favor of QMC, although still error bars may be too large to draw a definite conclusion. This gives us some confidence in using QMC, and we hope it will provide us with a valuable glimpse into the properties of charmed and bottom hypernuclei.

\section*{2. Charmed and Bottom hypernuclei}

\subsection*{2.1. Mean-field equations of motion}

We consider static, (approximately) spherically symmetric charmed and bottom hypernuclei (closed shell plus one heavy baryon configuration) ignoring small nonspherical effects due to the embedded heavy baryon. We adopt Hartree, mean-field approximation. In this approximation, $\rho NN$ tensor coupling gives a spin-orbit force...
for a nucleon bound in a static spherical nucleus, although in Hartree-Fock it can give a central force which contributes to the bulk symmetry energy. Furthermore, it gives no contribution for nuclear matter since the meson fields are independent of position and time. Thus, we ignore the \(\rho N\) tensor coupling as usually adopted in the Hartree treatment of quantum hadrodynamics (QHD).

Using the Born-Oppenheimer approximation, mean-field equations of motion are derived for a charmed (bottom) hypernucleus in which the quasi-particles moving in single-particle orbits are three-quark clusters with the quantum numbers of a charmed (bottom) baryon or a nucleon. Then a relativistic Lagrangian density at the hadronic level can be constructed, similar to that obtained in QHD, which produces the same equations of motion when expanded to the same order in velocity:

\[
\mathcal{L}_{\text{QMC}}^{\text{CHY}} = \mathcal{L}_{\text{QMC}} + \mathcal{L}_{\text{QMC}}^C,
\]

\[
\mathcal{L}_{\text{QMC}} = \bar{\psi}_N(\vec{r}) \left[ i \gamma \cdot \partial - M_N^*(\sigma) - (g_\omega \omega(\vec{r}) + g_\rho \frac{\tau_3^N}{2} b(\vec{r}) + \frac{e}{2} (1 + \tau_3^N) A(\vec{r}) ) \gamma_0 \right] \psi_N(\vec{r})
- \frac{1}{2} \left[ (\nabla \sigma(\vec{r}))^2 + m_\sigma^2 \sigma(\vec{r})^2 \right] + \frac{1}{2} \left[ (\nabla \omega(\vec{r}))^2 + m_\omega^2 \omega(\vec{r})^2 \right]
+ \frac{1}{2} \left[ (\nabla b(\vec{r}))^2 + m_b^2 b(\vec{r})^2 \right] + \frac{1}{2} (\nabla A(\vec{r}))^2,
\]

\[
\mathcal{L}_{\text{QMC}}^C = \sum_{C=A_c^+, A_b} \bar{\psi}_C(\vec{r}) \left[ i \gamma \cdot \partial - M_C^*(\sigma) - (g_\omega^C \omega(\vec{r}) + g_\rho^C \frac{\tau_3^C}{2} b(\vec{r}) + eQ_C A(\vec{r}) ) \gamma_0 \right] \psi_C(\vec{r}),
\]

where \(\psi_N(\vec{r})\) \((\psi_C(\vec{r}))\) and \(b(\vec{r})\) are respectively the nucleon (charmed and bottom baryon) and the \(\rho\) meson (the time component in the third direction of isospin) fields, while \(m_\sigma, m_\omega\) and \(m_\rho\) are the masses of the \(\sigma, \omega\) and \(\rho\) meson fields. \(g_\omega\) and \(g_\rho\) are the \(\omega\)-\(N\) and \(\rho\)-\(N\) coupling constants which are related to the corresponding \((u, d)\)-quark-\(\omega\), \(g_\omega^q\), and \((u, d)\)-quark-\(\rho\), \(g_\rho^q\), coupling constants as \(g_\omega = 3g_\omega^q\) and \(g_\rho = g_\rho^q\). Hereafter, we will use notations for the quark flavors, \(q \equiv u, d\) and \(Q \equiv s, c, b\). Note that in usual QMC (QMC-I) the meson fields appearing in Eqs. \((2.2)\) and \((2.3)\) represent the quantum numbers and Lorentz structure as those in QHD corresponding, \(\sigma \leftrightarrow \phi_0\), \(\omega \leftrightarrow V_0\) and \(b \leftrightarrow b_0\), and they are not directly connected with the physical particles, nor quark model states. Their masses in nuclear medium do not vary in the present treatment. For the other version of QMC (QMC-II), where masses of the meson fields are also subject to the medium modification in a self-consistent manner, see Ref. However, for a proper parameter set (set B) the typical results obtained in QMC-II are very similar to those of QMC-I. The difference is \(\sim 16\%\) for the largest case, but typically \(\sim 10\%\) or less. (For the effective masses of the hyperons, it is less than \(\sim 8\%\).) In an approximation where the \(\sigma, \omega\) and \(\rho\) fields couple only to the \(u\) and \(d\) quarks, the coupling strengths in the charmed (bottom) baryon are obtained as \(g_\omega^C = (n_q/3) g_\omega\), and \(g_\rho^C = g_\rho = g_\rho^q\), with \(n_q\) being the total number of valence \(u\) and \(d\) (light) quarks in the baryon. \(I_C^S\) and \(Q_C\) are the third component of the baryon isospin operator and its electric charge in units of the proton charge, \(e\), respectively. The field dependent \(\sigma\)-\(N\) and \(\sigma\)-\(C\) coupling strengths predicted by the QMC model, \(g_\sigma(\sigma)\) and \(g_\sigma^C(\sigma)\), related to the Lagrangian...
densities, Eqs. (2.2) and (2.3), at the hadronic level are defined by:

\[
\begin{align*}
M_N^* (\sigma) & \equiv M_N - g_\sigma (\sigma) \sigma (\vec{r}), \\
M_C^* (\sigma) & \equiv M_C - g_C^* (\sigma) \sigma (\vec{r}),
\end{align*}
\]

(2.4)  \hspace{1cm} (2.5)

where \( M_N (M_C) \) is the free nucleon (charmed and bottom baryon) mass (masses). Note that the dependence of these coupling strengths on the applied scalar field may be the case in nuclear matter. More explicit expressions for \( g_\sigma (\sigma) \) and \( g_C^* (\sigma) \) will be given later. From the Lagrangian density, Eq. (2.1), a set of equations of motion for the charm or bottom hypernuclear system is obtained,

\[
\begin{align*}
[i \gamma \cdot \partial - M_N^* (\sigma) - (g_\omega \omega (\vec{r}) + g_\rho \tau_3^N b(\vec{r}) + \frac{e}{2} (1 + \tau_3^N) A(\vec{r})) \gamma_0] \psi_N (\vec{r}) & = 0, \\
[i \gamma \cdot \partial - M_C^* (\sigma) - (g_C^* \omega (\vec{r}) + g_\rho \tau_3^C b(\vec{r}) + e Q C A(\vec{r}) \gamma_0) \psi_C (\vec{r}) & = 0, \\
( - \nabla^2 + m_\sigma^2 ) \sigma (\vec{r}) & = - \frac{\partial M_N^* (\sigma)}{\partial \sigma} \rho_s (\vec{r}) - \frac{\partial M_C^* (\sigma)}{\partial \sigma} \rho_C^* (\vec{r}), \\
( - \nabla^2 + m_\omega^2 ) \omega (\vec{r}) & = g_\omega \rho_B (\vec{r}) + g_C^* \rho_C^* (\vec{r}), \\
( - \nabla^2 + m_\rho^2 ) b(\vec{r}) & = \frac{g_\rho}{2} \rho_3 (\vec{r}) + g_C^* \rho_C^* (\vec{r}), \\
( - \nabla^2 ) A(\vec{r}) & = e \rho_p (\vec{r}) + e Q C \rho_B (\vec{r}),
\end{align*}
\]

(2.6)  \hspace{1cm} (2.15)

where, \( \rho_s(\vec{r}) \) (\( \rho_C^* (\vec{r}) \)), \( \rho_B(\vec{r}) \) (\( \rho_C^* (\vec{r}) \)), \( \rho_3(\vec{r}) \) and \( \rho_p(\vec{r}) \) are the scalar, baryon, third component of isovector, and proton densities at the position \( \vec{r} \) in charmed or bottom hypernucleus. On the right hand side of Eq. (2.8), \(- \frac{\partial M_N^* (\sigma)}{\partial \sigma} \equiv g_\sigma C_N (\sigma)\) and \(- \frac{\partial M_C^* (\sigma)}{\partial \sigma} \equiv g_C^* C_C (\sigma)\), where \( g_\sigma \equiv g_\sigma (\sigma = 0) \) and \( g_C^* \equiv g_C^* (\sigma = 0) \), are a new, and characteristic feature of QMC beyond QHD. The effective mass for the charmed or bottom baryon \( C \) is defined by,

\[
\frac{\partial M_C^* (\sigma)}{\partial \sigma} = - n_q g_\sigma^3 \int_{\text{bag}} d\vec{x} \bar{\psi}_q (\vec{x}) \psi_q (\vec{x}) \equiv - n_q g_\sigma^3 S_C (\sigma) = - \frac{\partial}{\partial \sigma} [g_\sigma^C (\sigma) \sigma],
\]

(2.12)

with the MIT bag model quantities, and the in-medium bag radius satisfying the mass stability condition,

\[
M_C^* (\sigma) = \sum_{j=q,Q} \frac{n_j \Omega_j^R}{R_C^*} - z_C + \frac{4}{3} \pi (R_C^*)^3 B,
\]

(2.13)

\[
S_C (\sigma) = [\Omega_q^R / 2 + m_q^* R_C^* (\Omega_q^R - 1)] / [\Omega_q^R (\Omega_q^R - 1) + m_q^* R_C^* / 2],
\]

\[
\Omega_q^R = \sqrt{x_0^2 + (R_C^* m_q^*)^2}, \quad \Omega^*_Q = \sqrt{x_0^2 + (R_C^* m_Q^*)^2}, \quad m_q^* = m_q - g_\sigma^3 \sigma (\vec{r}),
\]

(2.14)  \hspace{1cm} (2.15)

\(^*\) Strictly, this is true only when the bag radii of nucleon and baryon \( C \) are exactly the same in the present model. See Eq. (2.16).
$$C_C(\sigma) = \frac{S_C(\sigma)}{S_C(0)}, \quad g^F_\sigma \equiv n_q g^F_\sigma S_C(0) = \frac{n_q}{3} g^F_\sigma S_C(0) = \frac{n_q}{3} g^F_\sigma \Gamma_{C/N},$$  
$$dM^*_C/dR_C|_{R_C=R_C} = 0. \tag{2.17}$$

Quantities for the nucleon are similarly obtained by replacing the indices, $C \to N$. Here, the MIT bag model quantities are calculated in a local density approximation using the spin and spatial part of the wave functions, $\psi_f(x) = N_F e^{-iN_f/\hbar \omega}\psi_f(\vec{r})$ ($N_F$: the normalization factor), where the wave functions, $\psi_f(x)$, satisfy the Dirac equations for the flavor $f$ quarks (and antiquarks) in the hadron bag centered at a position $\vec{r}$ of the nucleus, approximating the constant, mean, meson fields within the bag (and neglecting the Coulomb force) ($|\vec{x} - \vec{r}| \leq \text{bag radius}$) [39–42]:

$$\begin{align*}
[i\gamma \cdot \partial_x - (m_q - V^q_{\sigma}(\vec{r})) + \gamma^0 \left(V^q_{\omega}(\vec{r}) + \frac{1}{2} V^q_{\rho}(\vec{r})\right)] & \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} = 0, \tag{2.18}
[i\gamma \cdot \partial_x - (m_q - V^q_{\rho}(\vec{r})) - \gamma^0 \left(V^q_{\omega}(\vec{r}) - \frac{1}{2} V^q_{\rho}(\vec{r})\right)] & \begin{pmatrix} \psi_d(x) \\ \psi_q(x) \end{pmatrix} = 0, \tag{2.19}
[i\gamma \cdot \partial_x - m_Q] \psi_Q(x) \quad (\text{or } \psi_Q^c(x)) = 0. \tag{2.20}
\end{align*}$$

The (constant) mean-field potentials within the bag centered at the position $\vec{r}$ of the nucleus, are defined by $V^q_{\omega}(\vec{r}) \equiv g^2_\omega \sigma(\vec{r})$, $V^q_{\rho}(\vec{r}) \equiv g^2_\rho \omega(\vec{r})$ and $V^q_{\rho}(\vec{r}) \equiv g^2_\rho b(\vec{r})$, with $g^2_\omega$, $g^2_\rho$ and $g^2_\sigma$ the corresponding quark-meson coupling constants. In Eqs. (2.13) - (2.17) $z_C, B, x_q,Q, \text{ and } m_{q,Q}$ are the parameters for the sum of the c.m. and gluon fluctuation effects, bag pressure, lowest eigenvalues for the quarks, $q$ or $Q$, respectively, and the corresponding current quark masses. $z_N$ and $B$ ($z_C$) are fixed by fitting the nucleon (charmed or bottom baryon) mass in free space. The current quark masses for the quarks, we use, $(m_{u,d}, m_s, m_c, m_b) = (5, 250, 1300, 4200) \text{ MeV}$, and $B = (170.0 \text{ MeV})^4$ is obtained by choosing the bag radius for the nucleon in free space, $R_N = 0.8$ fm. Calculated bag radii in free space, and the bag parameters are $(R_A, R_{A^+}, R_{A_b}) = (0.806, 0.846, 0.930) \text{ fm}$, and $(z_N, z_A, z_{A^+}, z_{A_b}) = (3.295, 3.131, 1.766, -0.643)$, respectively. The parameters associated with the $u, d$ and $s$ quarks are the same as in the previous studies [3]. The parameters at the hadron level, which are already fixed by the study of nuclear matter and finite nuclei [12] are as follows: $m_\omega = 783 \text{ MeV}$, $m_\rho = 770 \text{ MeV}$, $m_\sigma = 418 \text{ MeV}$, $e^2/4\pi = 1/137.036$, $g^2_\sigma/4\pi = 3.12$, $g^2_\omega/4\pi = 5.31$ and $g^2_\rho/4\pi = 6.93$. Concerning the sign of $m^*_q$ in (hyper)nucleus in Eq. (2.17), it reflects nothing but the strength of the attractive (negative) scalar potential, and thus naive interpretation of the mass for a physical particle, which is positive, should not be applied.

At the hadronic level, the entire information on the quark dynamics is condensed into the effective coupling $C_{N,C}(\sigma)$ of Eq. (2.8). Furthermore, when $C_{N,C}(\sigma) = 1$, which corresponds to a structureless nucleon or heavy baryon $C$, the equations of motion given by Eqs. (2.6)-(2.11) can be identified with those derived from QHD [34–39] except for the terms arising from the tensor coupling and the non-linear scalar field interaction introduced beyond naive QHD.
2.2. Nuclear matter limit

Here, we consider a charmed or a bottom hadron in nuclear matter. In this limit the meson fields become constant, and we denote the mean-value of the $\sigma$ field as $\langle \sigma \rangle$. Furthermore, under this limit, we can also generally consider a hadron, $h$, embedded in the nuclear matter in the same way as that for the charmed (bottom) baryon. (For example, a Lagrangian density for a meson-nuclear system can be also written in a similar way, if $L_{QMC}^C$ is replaced by the corresponding meson Lagrangian density in Eq. (2.3).) The self-consistency condition for the $\sigma$ field, $\langle \sigma \rangle$, is given by

$$\langle \sigma \rangle = \frac{g_{\sigma}}{m_{\sigma}^2} C_N(\sigma) \int d\vec{k} \theta(k_F - k) \frac{M_N^2(\sigma)}{\sqrt{M_N^2(\sigma) + \vec{k}^2}},$$

(2.21)

where $g_{\sigma} = (3g_{\sigma}^2S_N(0))$, $k_F$ is the Fermi momentum, and $C_N(\sigma)$ is now the constant value of $C_N$ in the scalar field. Note that $M_N^2(\sigma)$, in Eq. (2.21), must be calculated self-consistently by the MIT bag model, through Eqs. (2.12) - (2.17). This self-consistency equation for $\sigma$ is the same as that in QHD, except that in the latter model one has $C_N(\sigma) = 1$.

Using the obtained mean field value, $\langle \sigma \rangle$, the corresponding quantity for the hadron $h$ in nuclear matter can be also calculated using Eqs. (2.12) - (2.17), where the effect of a single hadron on the mean field value, $\langle \sigma \rangle$, in nuclear matter can be neglected.

In Figs. 1 and 2 we show the calculated ratios of effective masses versus those of the free. With increasing density the ratios decrease as usually expected, but decrease in magnitude is from larger to smaller: hadrons with only light quarks, with one strange quark, with one charm quark, and with one bottom quark. This is because their masses in free space are in the order from light to heavy. Thus, the net ratios for the decrease in masses (developing of scalar masses) compared to that of the free masses becomes smaller. This may be regarded as a measure of the role of light quarks in each hadron system in nuclear matter, in a sense by how much do they lead to a partial restoration of the chiral symmetry.
We next compare the scalar potentials of hadrons in nuclear matter. The scalar, $V^s_h$, and vector, $V^v_h$, potentials for the hadrons $h$, in nuclear matter are given by,

$$V^s_h = m^*_h - m_h,$$

$$V^v_h = (n_q - n_{\bar{q}})V^q_\rho + I^3_h V^q_\omega, \quad (V^q_\omega \rightarrow 1.4^2 V^q_\omega \text{ for } K, K^-, D, \bar{D}, B, \bar{B}),$$

where $I^3_h$ is the third component of isospin projection of the hadron $h$, and $n_q (n_{\bar{q}})$ is the number of light quarks (antiquarks) in the hadron $h$. Thus, the vector potential for a heavy baryon containing at least a charm or bottom quark, is equal to that of the hyperon with the same light quark configuration in QMC. Note that, in studies of the kaon system, we found that it was phenomenologically necessary to increase the strength of the vector coupling to the non-strange quarks in the $K^+$, i.e., $g^q_{K^+} \equiv 1.4^2 g^q_{\omega}$, to reproduce the empirically extracted $K^+$-nucleus interaction. This may be related to the fact that kaon is a pseudo-Goldstone boson, where treatment of the Goldstone bosons in a naive quark model is usually unsatisfactory. We assume this, $g^q_{\omega} \rightarrow 1.4^2 g^q_{\omega}$, also for the $D$, $\bar{D}$, $B$ and $\bar{B}$ to allow an upper limit situation. Calculated scalar potentials are shown in Fig. 3.

From the results it is confirmed that the scalar potential for the hadron $h$, $V^s_h$, follows a simple light quark number scaling rule:

$$V^s_h \simeq [(n_q + n_{\bar{q}})V^N_s]/3,$$

where $V^N_s$ is the scalar potential for the nucleon. It is interesting to notice that, the baryons with charm and bottom quarks ($\Xi_c = (qsc)$), show very similar features to those of the corresponding hyperons with strange quarks. Then, we can naively expect at this stage that these heavy baryons will also form charmed (bottom) hypernuclei, as the hyperons with strangeness do. (Recall that the repulsive, vector potentials are the same for the corresponding hyperons with the same light quark configurations.)

In addition, $B^-$ meson will also certainly form meson-nuclear bound states, because $B^-$ meson is $\bar{u}b$ and feels a strong attractive vector potential in addition to the attractive Coulomb force. This makes it much easier to be bound in a nucleus compared to the $D^0$, which is $\bar{u}u$ and blind to the Coulomb force. This reminds us of a situation for the kaonic ($K^-(\bar{u}s)$) atom. Such atoms like $B^- (\bar{u}b)$ atoms will have the meson much closer to the nucleus and will thus probe even smaller changes in the nuclear density. This will be a complementary information to the $D^-(\bar{c}d)$ nuclear bound states, which would provide us details of the vector potential in a nucleus.
§3. Results for $A^+_c$ and $A_b$ hypernuclei

Here, we present results for the $A^+_c$ and $A_b$ hypernuclei, and compare them with those for the $\Lambda$ hypernuclei studied previously\textsuperscript{18} in QMC.

We briefly discuss the spin-orbit force in QMC.\textsuperscript{31, 32} (See Refs.\textsuperscript{11, 18} for detail.) To include the spin-orbit potential approximately correctly, e.g., for the $A^+_c$, we add perturbatively the correction, $-\frac{2}{2M^2_{\Lambda_c}} \frac{d}{dr} \left[ M^*_{A^+_c}(\vec{r}) + g^\omega \omega(\vec{r}) \right] \vec{I} \cdot \vec{s}$, to the single-particle energies obtained with the Dirac equation.\textsuperscript{33} This may correspond to a correct spin-orbit force which is calculated by the underlying quark model.\textsuperscript{31, 32, 33}

\begin{equation}
V_{S.O.}\left(\vec{r}\right) = -\frac{1}{2M^2_{\Lambda_c}} \left( \frac{d}{dr} \left[ M^*_{A^+_c}(\vec{r}) + g^\omega \omega(\vec{r}) \right] \right) \vec{I} \cdot \vec{s}, \tag{3.1}
\end{equation}

since the Dirac equation at the hadronic level in usual QHD-type models leads to:

\begin{equation}
V_{S.O.}\left(\vec{r}\right) = -\frac{1}{2M^2_{\Lambda_c}} \left( \frac{d}{dr} \left[ M^*_{A^+_c}(\vec{r}) - g^\omega \omega(\vec{r}) \right] \right) \vec{I} \cdot \vec{s}, \tag{3.2}
\end{equation}

which has the opposite sign for the vector potential, $g^\omega \omega(\vec{r})$. The correction to the spin-orbit force, which appears naturally in the QMC model, may also be modeled at the hadronic level of the Dirac equation by adding a tensor interaction, motivated by the quark model.\textsuperscript{31, 32, 33}

Here, we should make a comment that, as was discussed by Dover and Gal\textsuperscript{32} in detail, one boson exchange model with underlying (approximate) SU(3) symmetry in strong interaction, also leads to the weaker spin-orbit forces for the (strange) hyperon-nucleon (YN) than that for the nucleon-nucleon (NN).

However, in practice, because of its heavy mass ($M^*_{A^+_c}$), contribution to the single-particle energies from the spin-orbit potential, with or without including the correction term, turned out to be even smaller than that for the $\Lambda$ hypernuclei, and further smaller for the $A_b$ hypernuclei\textsuperscript{32}.

First, we show in Fig. 4 the total baryon density distributions in $^41\text{Ca}$ and $^{200}\text{Pb}$ ($j = \Lambda, A^+_c, A_b$), for $1s_{1/2}$ state in each hypernucleus. Note that because of the self-consistency, the total baryon density distributions are dependent on the state of the embedded particles. The total baryon density distributions obtained are quite similar for the $\Lambda, A^+_c$ and $A_b$ hypernuclei multiplet with the same baryon numbers, $A$, since the effect of $\Lambda, A^+_c$ and $A_b$ is $\approx 1/A$.

![](image.png)

**Fig. 4.** Total baryon density distributions in $^41\text{Ca}$ and $^{200}\text{Pb}$ ($j = \Lambda, A^+_c, A_b$), for $1s_{1/2}$ state for the $\Lambda, A^+_c$ and $A_b$.}
Next, in Figs. 5 and 6, we show the scalar and vector potential strengths for the $\Lambda$, $\Lambda_c^+$, and $\Lambda_b$ for $1s_{1/2}$ state in $^{41}$Ca and $^{209}$Pb ($j = \Lambda, \Lambda_c^+, \Lambda_b$), and the corresponding probability density distributions in Fig. 7.

In Figs. 5 and 6, "Pauli" stands for the effective, repulsive potential representing the Pauli blocking at the quark level, plus the $\Sigma_{e,b}N - A_{e,b}N$ channel coupling, introduced at the hadronic level phenomenologically. For the $\Lambda_c^+$, the Coulomb potentials are also shown. The scalar and vector potentials for these particles in hypernuclei multiplet with the same baryon numbers are quite similar. Thus, as far as the total baryon density distributions and the scalar and vector potentials are concerned, $\Lambda, \Lambda_c^+$ and $\Lambda_b$ hypernuclei show quite similar features within the multiplet.

However, as shown in Fig. 7, the wave functions obtained for $1s_{1/2}$ state are very different. The $\Lambda_c^+$ probability density distribution in $^{209}$Pb is much more pushed away from the center than that for the $\Lambda$ in $^{209}$Pb due to the Coulomb force. On the contrary, the $\Lambda_b$ probability density distributions in $\Lambda_b$ hypernuclei are much larger near the origin than those for the $\Lambda$ in the corresponding $\Lambda$ hypernuclei due to its heavy mass.

Finally, we show the calculated single-particle energies in Tables I and II. Results for the $\Lambda$ hypernuclei are from Ref. 18. Recall that since the mass difference for $\Lambda_c^+$ and $\Sigma_c$, and probably for $\Lambda_b$ and $\Sigma_b$, are larger than that for $\Lambda$ and $\Sigma$, we expect the
Table I. Single-particle energies (in MeV) for $^{17}$O, $^{41}$Ca and $^{49}$Ca ($j = \Lambda, \Lambda^\pm, \Lambda_b$). Results for the hypernuclei are taken from Ref. Spin-orbit splittings for $\Lambda$ hypernuclei are not well determined by the experiments.

|         | $^\Lambda_1$O | $^{17}O$ | $^{17}_\Lambda^\pm$O | $^{17}_\Lambda_b$O | $^\Lambda_1$Ca | $^{41}Ca$ | $^{41}_\Lambda^\pm$Ca | $^{41}_\Lambda_b$Ca | $^\Lambda_1$Ca | $^{49}Ca$ | $^{49}_\Lambda^\pm$Ca | $^{49}_\Lambda_b$Ca |
|---------|----------------|---------|----------------------|-------------------|-------------|---------|----------------------|-------------------|-------------|---------|----------------------|-------------------|
| 1s$\frac{1}{2}$ | -12.5          | -14.1   | -12.8                | -19.6             | -20.0       | -19.5   | -12.8                | -23.0             | -21.0       | -14.3   | -24.4                |                     |
| 1p$\frac{1}{2}$ | -2.5           | -5.1    | -7.3                 | -16.5             | -12.0       | -12.3   | -9.2                 | -20.9             | -13.9       | -10.6   | -22.2                |                     |
| 1p$\frac{1}{2}$ | (1p$\frac{1}{2}$) | -5.0    | -7.3                 | -16.5             | (1p$\frac{1}{2}$) | -12.3   | -9.1                 | -20.9             | -13.8       | -10.6   | -22.2                |                     |
| 1d$\frac{5}{2}$ | -4.7           | -4.8    | -18.4                |                   | -6.5        | -6.5    | -19.5                |                   |                     |
| 2s$\frac{1}{2}$ | -3.5           | -3.4    | -17.4                |                   | -5.4        | -5.3    | -18.8                |                   |                     |
| 1d$\frac{3}{2}$ | -4.6           | -4.8    | -18.4                |                   | -6.4        | -6.4    | -19.5                |                   |                     |
| 1f$\frac{7}{2}$ | -2.0           |         |                      |                   | -2.0        |         |                      |                   |                     |

Table II. Single-particle energies (in MeV) for $^{91}$Zr and $^{208}$Pb ($j = \Lambda, \Lambda^\pm, \Lambda_b$).

|         | $^\Lambda_9$Yb | $^{91}Zr$ | $^{91}_\Lambda^\pm$Zr | $^{91}_\Lambda_b$Zr | $^\Lambda_9$Pb | $^{208}Pb$ | $^{208}_\Lambda^\pm$Pb | $^{208}_\Lambda_b$Pb |
|---------|----------------|---------|----------------------|-------------------|-------------|---------|----------------------|-------------------|
| 1s$\frac{1}{2}$ | -22.5          | -23.9   | -10.8                | -25.7             | -27.0       | -27.0   | -5.2                 | -27.4             |
| 1p$\frac{1}{2}$ | -16.0          | -18.4   | -8.7                 | -24.2             | -22.0       | -23.4   | -4.1                 | -26.6             |
| 1p$\frac{1}{2}$ | (1p$\frac{1}{2}$) | -18.4   | -8.7                 | -24.2             | (1p$\frac{1}{2}$) | -23.4   | -4.0                 | -26.6             |
| 1d$\frac{5}{2}$ | -9.0           | -12.3   | -5.8                 | -22.4             | -17.0       | -19.1   | -2.4                 | -25.4             |
| 2s$\frac{1}{2}$ | -10.8          | -3.9    | -21.6                |                   | -17.6       |         | -24.7                |                   |
| 1d$\frac{3}{2}$ | (1d$\frac{5}{2}$) | -12.3   | -5.8                 | -22.4             | (1d$\frac{5}{2}$) | -19.1   | -2.4                 | -25.4             |
| 1f$\frac{7}{2}$ | -2.0           | -5.9    | -2.4                 | -20.4             | -12.0       | -14.4   |         | -24.1             |
| 2p$\frac{3}{2}$ | -4.2           | -19.5   | -12.4                | -23.2             |            |         |                  |                  |
| 1f$\frac{5}{2}$ | (1f$\frac{7}{2}$) | -5.8    | -2.4                 | -20.4             | (1f$\frac{7}{2}$) | -14.3   | -         | -24.1             |
| 2p$\frac{1}{2}$ | -4.1           | -19.5   | -12.4                | -23.2             |            |         |                  |                  |
| 1g$\frac{9}{2}$ | -18.1          | -7.0    | -9.3                 | -22.6             |            |         |                  |                  |
| 1g$\frac{7}{2}$ | (1g$\frac{9}{2}$) | -9.2    | -22.6                |                   |            |         |                  |                  |
| 1h$\frac{11}{2}$ | -3.9           | -21.0   | -21.0                |                   |            |         |                  |                  |
| 2d$\frac{3}{2}$ | -1.7           | -20.1   | -20.1                |                   |            |         |                  |                  |
| 2p$\frac{1}{2}$ | -1.0           | -19.6   | -19.6                |                   |            |         |                  |                  |
| 3p$\frac{1}{2}$ | -1.0           | -19.6   | -19.6                |                   |            |         |                  |                  |
| 3p$\frac{1}{2}$ | -1.0           | -19.6   | -19.6                |                   |            |         |                  |                  |
| 1f$\frac{13}{2}$ | -19.3          |         |                      |                   |            |         |                  |                  |

The effect of the channel coupling for the charmed and bottom hypernuclei to be smaller than those for the strange hypernuclei, although the same parameters are used. In addition, we searched for the single-particle states up to the same highest state as that of the core neutrons in each hypernucleus, since the deeper levels are usually easier to observe in experiment.

First, it is clear that the $\Lambda^\pm$ single-particle energy levels are higher than the corresponding levels for the $\Lambda$ and $\Lambda_b$. This is because of the Coulomb force. This
feature becomes stronger as the proton number increases.

Second, the level spacing for the $\Lambda_b$ single-particle energies is much smaller than that for the $\Lambda$ and $\Lambda_c^+$. This is due to the heavy mass of $\Lambda_b$ (or $M_b^*$). In the Dirac equation for the $\Lambda_b$, the mass term dominates more than that of the term proportional to Dirac’s $\kappa$, which classifies the states, or single-particle wave functions. (See Refs. 12, 13 for detail.) This small level spacing would make it very difficult to distinguish the states in experiment, or to achieve such high resolution. On the other hand, this may imply also many new phenomena. It will have a large probability to trap a $\Lambda_b$ among one of those many states, especially in heavy nuclei. What are the consequences? May be the $\Lambda_b$ weak decay happens inside a heavy nucleus with a very low probability? Does it emit many photons when the $\Lambda_b$ gradually makes transitions from a deeper state to a shallower state? All these questions raise a flood of speculations.

Finally, it should be emphasized that we have used the values for the coupling constants of $\sigma$ (or $\sigma$-field dependent strength), $\omega$ and $\rho$ to $\Lambda, \Lambda_c^+$ and $\Lambda_b$ to be determined automatically based on the underlying quark model, as for the nucleon and other baryons. (Recall that the values for the vector $\omega$ fields to any baryons can be obtained by the number of light quarks in a baryon, but those for the $\sigma$ are different as shown in Eqs. (2.12)-(2.17).) Phenomenology would determine ultimately if the coupling constants (strengths) determined by the underlying quark model actually work for $\Lambda_c^+$ and $\Lambda_b$ or not. Although implications of the present results can be speculated a great deal, we would like to emphasize that, what we showed is that the $\Lambda_c^+$ and $\Lambda_b$ hypernuclei would exist in realistic experimental conditions. Experiments at facilities like JHF would provide further inputs to gain a better understanding of the interaction of $\Lambda_c^+$ and $\Lambda_b$ with the nuclear matter. Additional studies are needed to investigate the semi-leptonic weak decay of $\Lambda_c^+$ and $\Lambda_b$. The role of Pauli blocking and density in influencing the decay rates as compared to those for the free hyperons would be highly useful. Will the high density lead to a slower decay and that a higher probability to survive its passage through the material? At present the study of the presence of $\Lambda_c^+$ and $\Lambda_b$ in finite nuclei is its infancy. Careful investigations, both theoretical and experimental, would lead to a much better understanding of the role of heavy quarks in finite nuclei, and the role of partial restoration of chiral symmetry in nuclear medium.

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