Electroweak nuclear response at moderate momentum transfer

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We discuss the convergence of the expansion of the nuclear electroweak current in powers of \( |k|/M \), where \( M \) is the nucleon mass and \( k \) denotes either the momentum transfer or the momentum of the struck nucleon. We have computed the electron and neutrino scattering cross sections off uniform nuclear matter at equilibrium density using correlated wave functions and the cluster expansion formalism. The results of our work suggest that the proposed approach provides accurate estimates of the measured electron scattering cross sections. On the other hand, the description of the current based on the widely used leading-order approximation does not appear to be adequate, even at momentum transfer as low as 300 MeV.

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I. INTRODUCTION

Many aspects of the recent neutrino data remain puzzling [1–4]. While unconventional explanations have been proposed [5, 6], before advocating exotics, more conventional effects should be analyzed within realistic models.

One of the main sources of uncertainty in the interpretation of the available data is the treatment of nuclear effects, which are often described using oversimplified models of both nuclear dynamics and the reaction mechanism.

Most neutrino experiments take data at beam energy \( E_\nu \lesssim 1 \) GeV, where quasielastic scattering dominates. In this channel the elementary neutrino-nucleon cross section is determined by the form factors \( F_1, F_2, \) and \( F_A \), where \( N = p \) and \( n \) for protons and neutrons, respectively. The vector form factors have been measured to high accuracy in electron-proton and electron-deuteron scattering experiments, whereas the \( Q^2 \) dependence of the axial form factor \( F_A \) is still controversial, as different collaborations reported significantly different values of the axial mass \( M_A \) [8, 9].

Many studies of neutrino-nucleus scattering are based on the impulse approximation (IA), the underlying assumptions of which are that (i) the momentum transfer is large enough for the probe to interact with only one nucleon, and (ii) final-state interactions, of both statistical and dynamical origin, between the struck particle and the spectators can be neglected.

In a previous paper [10], we investigated the limits of applicability of the IA scheme in the case of the linear response of infinite nuclear matter to a scalar probe. The results of that analysis suggest that, at momentum transfer below about twice the Fermi momentum, corresponding to \( \sim 500 \) MeV at equilibrium density, the IA breaks down, mostly due to the oversimplified description of the final state.

In this paper, we develop a formalism suitable to obtain realistic estimates of the lepton-nucleus cross section in the region of moderate momentum transfer. As an illustrative example, we consider uniform nuclear matter, consisting of equal numbers of protons and neutrons, at equilibrium density \( \rho = 0.16 \) fm\(^{-3} \), corresponding to Fermi momentum \( k_F = 1.33 \) fm\(^{-1} \).

II. ELECTROWEAK CROSS SECTION

The scattering cross section can be written as

\[
\frac{d\sigma}{d\omega} = \propto L_{\mu\nu} W^{\mu\nu}, \tag{1}
\]

where \( L_{\mu\nu} \) is determined by the lepton kinematics and

\[
W^{\mu\nu} = \sum_n |n\rangle \langle j^\nu| n\rangle \langle j^\mu| 0\rangle \delta(\omega + E_0 - E_n). \tag{2}
\]

In the above equation, \( \omega \) is the energy transfer, \( |0\rangle \) and \( |n\rangle \) are the initial and final states of the target, and \( J^\mu \) is the current operator accounting for the lepton-nucleus interaction. Within our approach, nuclear matter is described using \textit{correlated} states, including the effects of the nonperturbative components of nucleon-nucleon (NN) forces [11, 12].

We have used Eqs. (1) and (2) to obtain the electron and neutrino scattering cross sections, including the contributions arising from one-particle–one-hole correlated states. In our calculation, antisymmetrization of the final state and dynamical final-state interactions are both taken into account.

At moderate momentum transfer \( |q| \lesssim 500 \) MeV, the nuclear current operator

\[
J_\mu = \sum_i j^i_\mu, \tag{3}
\]

where \( j_\mu \) is the nucleon current, is usually treated expanding in powers of the ratio \( |k|/M \) and keeping only the leading terms. Here, \( M \) is the nucleon mass, whereas \( k \) denotes either the momentum transfer or the momentum of the struck nucleon.

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In this paper, we present the results of a systematic analysis of the convergence of this expansion, showing that contributions beyond leading order may be important and cannot be neglected. A similar analysis, carried out within the context of the mean-field approach using the formalism reviewed in Ref. [13], is discussed in Ref. [14].

III. NUCLEAR HAMILTONIAN AND NUCLEAR WAVE FUNCTIONS

The correlated states are defined as

\[ |n\rangle = \frac{F|n\rangle}{\langle n|F^\dagger F|n\rangle^{1/2}}, \]

where \(|n\rangle\) is a Fermi gas (FG) state and the operator \(F\), embodying the correlation structure induced by the NN interaction, is written in the form

\[ F(1, \ldots, N) = S \prod_{j>i=1}^{N} f_{ij}, \]

where \(S\) is the symmetrization operator, accounting for the fact that, in general, \([f_{ij}, f_{ik}] \neq 0\).

The two-body correlation function \(f_{ij}\) features an operatorial structure reflecting the complexity of the NN potential:

\[ f_{ij} = \sum_{TS} [f_{TS}(r_{ij}) + \delta_{iS} f_{Ir}(r_{ij})S_{i}] P_{TS}. \]

In the above equation, \(r_{ij} = |r_{ij}|\) with \(r_{ij} = r_i - r_j\). \(P_{TS}\) is the operator projecting onto two-nucleus states of total spin and isospin \(S\) and \(T\), respectively, and

\[ S_{i} = 3(\sigma_{i} \cdot r_{ij})(\sigma_{j} \cdot r_{ij})/r_{ij}^2 - (\sigma_{i} \cdot \sigma_{j}). \]

The shapes of the radial functions \(f_{TS}(r_{ij})\) and \(f_{Ir}(r_{ij})\), are determined through functional minimization of the expectation value of the nuclear Hamiltonian in the correlated ground state. The assumption that the correlation operator obtained using this procedure can be used to generate the whole basis of correlated states is one of the main tenets of correlated-basis function (CBF) perturbation theory. It relies on the premise that the mean field and the correlation structure induced by the nucleon-nucleon interaction are largely decoupled from one another. The validity of this assumptions is supported by the fact that the results of CBF calculation of a variety of nuclear matter properties are in quantitative agreement with the results of Monte Carlo calculations, as well as with the available empirical data.

The calculations of matrix elements of both the Hamiltonian and the nuclear current between correlated states involve severe difficulties. To overcome this problem, in our work, we use the cluster expansion formalism [12], which amounts to writing the matrix element as a sum of contributions arising from subsystems involving an increasing number of nucleons.

The results discussed in the present paper have been obtained at lowest order, i.e., including two-body cluster contributions only. Using this approximation appears to be justified in the context of a calculation of the nuclear matter response at equilibrium density [13, 14].

The Hamiltonian employed to determine the correlation functions includes the kinetic energy term and the Argonne \(v_6\) NN potential [17]. The resulting correlation functions are then used to construct the effective interaction discussed in Ref. [18] needed to obtain the nuclear matter single-particle spectrum [16]. Note that the effective interaction of Ref. [18] includes the effects of three-nucleon forces, which are known to be important.

IV. CURRENT OPERATOR

The matrix representation of the weak current is the sum of the vector part,

\[ \Gamma^\mu_V = \gamma^\mu(F^1 + F^2) - \frac{(p + p')^\mu F^2}{2M}, \]

and the axial one,

\[ \Gamma^\mu_A = \gamma^\mu \gamma_5 F^A + \gamma_5 \frac{\lambda^\mu F p}{M}. \]

In general, the upper \(\lambda\) and lower Pauli spinor \(\phi_\sigma\) describing an on-shell Dirac particle of mass \(M\) and momentum \(p\) are related through

\[ (\chi_\sigma, \phi_\sigma) (\Gamma^\mu_V + \Gamma^\mu_A) (\chi_\sigma) = \chi_\sigma^T (V^\mu + A^\mu) \chi_\sigma, \]

with

\[ V^0_{\lambda p, \lambda' p'} = (F^1 + F^2) [1 + \hat{p}' \cdot \hat{p} + i\sigma \cdot (\hat{p}' \times \hat{p})] \]

\[ - F^2 \frac{E_{p'} + E_p}{2M} [1 - \hat{p}' \cdot \hat{p} - i\sigma \cdot (\hat{p}' \times \hat{p})], \]

\[ V^k_{\lambda p, \lambda' p'} = (F^1 + F^2) [(\hat{p}' + i\sigma \times \hat{p}')^k + (\hat{p} - i\sigma \times \hat{p})^k] \]

\[ - F^2 \frac{p + p'}{2M} [1 - \hat{p}' \cdot \hat{p} - i\sigma \cdot (\hat{p}' \times \hat{p})], \]

\[ A^0_{\lambda p, \lambda' p'} = F_A \sigma \cdot (\hat{p}' + \hat{p}) - F_p \frac{E_{p'} - E_p}{M} \sigma \cdot (\hat{p}' - \hat{p}), \]

\[ A^k_{\lambda p, \lambda' p'} = F_A \sigma \cdot (\hat{p}' + \hat{p}) + p^k (\hat{p}' \cdot \hat{p}) - (\hat{p}' \cdot \hat{p}) \sigma^k \]

\[ - i F_A (\hat{p}' \times \hat{p})^k - F_p \frac{(\hat{p}' - \hat{p})^k}{M} \sigma \cdot (\hat{p}' - \hat{p}), \]

(12)
V. RESULTS

Figure 1(a) shows a comparison between our results and the inclusive electron scattering cross section of nuclear matter at $|q| = 537$ MeV, obtained in Ref. 19, by extrapolating the available data for finite nuclei. The curve labeled LO corresponds to nonrelativistic kinematics and nonrelativistic single-particle spectrum, while the one labeled NLO has been obtained using relativistic kinematics and kinetic energies, and including next-to-leading-order terms in the expansion of the current. The exact calculation includes all terms. Note that most existing studies of the nuclear matter response carried out within nuclear many-body theory using CBFs are based on the LO approximation 20, 21. The results of Fig. 1(a) show that, even at momentum transfer as low as $\sim$500 MeV, the contribution of higher-order terms is not negligible, and must be taken into account. It clearly appears that the NLO and exact calculations accurately reproduce the data in the region of the quasielastic peak. For comparison, we also show the corresponding results obtained using the FG model with pure kinetic energy spectrum. Note that, while the wrong position of the maximum of the FG cross section might be fixed introducing an average nucleon separation energy, the failure to reproduce height and width of the measured cross section cannot be cured without an ad hoc modification of the Fermi momentum.

To illustrate the importance of finite size effects, in Fig. 1(b) we compare our nuclear matter results, in the same kinematical setup as in Fig. 1(a), to the cross section measured using an iron target 22. The theoretical results have been corrected to take into account neutron excess in iron, the influence of which turns out to be negligibly small. It clearly appears that finite size effects are large and that a meaningful comparison can only be done using extrapolated data. Unfortunately, however, the extrapolation procedure developed in Ref. 19 is based on the IA scheme, and cannot be employed for momentum transfer below $\sim$500 MeV 23. In the following, we only discuss the results of nuclear matter calculations, for both electron and neutrino scattering, to illustrate the convergence of the expansion of the current operator in different kinematical conditions.

Figure 2 shows the electron scattering cross sections at fixed scattering angle, 30 degrees, and beam energies ranging between 600 and 1000 MeV, corresponding to...
momentum transfer $300 \lesssim |q| \lesssim 500$ MeV. Comparison between the dashed line, corresponding to the LO calculation, and the solid line, representing the exact result, shows that even at momentum transfer as low as 300 MeV, the effect of using relativistic kinematics is sizable. Contrary to the nonrelativistic case, when relativistic kinematics is used, the width of the nuclear matter response becomes independent of $|q|$ at large momentum transfer, being determined by the Fermi momentum only. A suppression of the cross section is also clearly visible over the whole energy loss range.

To make a connection with an observable that is often analyzed in neutrino experiments, in Fig. 3 we show the $Q^2$ distribution at fixed beam energies $200 \leq E_\nu \leq 600$ MeV. The LO approximation appears to be quite accurate at the lowest neutrino energy, where the cross section picks up contributions at momentum transfer $170 \lesssim |q| \lesssim 300$ MeV. As $E_\nu$ increases, inclusion of higher-order terms leads to the appearance of sizable corrections, affecting both the magnitude and the shape of the $Q^2$ distributions.

VI. CONCLUSIONS

We have developed a theoretical approach allowing for a consistent treatment of electron- and neutrino-nucleus interactions in the region of moderate momentum transfer, in which the IA is expected to break down. Our results suggest that the widely used LO approximation for the nonrelativistic reduction of the nuclear current conspicuously fails at momenta $\gtrsim 300$ MeV. On the other hand, inclusion of higher-order terms leads to a remarkably good agreement between theory and extrapolated electron scattering data at $|q| \sim 500$ MeV.

The main feature of our work is the inclusion of nucleon-nucleon correlations, the effects of which are long known to be important in electron-nucleus scattering. The use of nonrelativistic correlation functions in the final state may be questionable at large momen-
tum transfer. However, as \(|\mathbf{q}|\) increases the role of correlations between the struck nucleon, carrying a large momentum, and the spectators becomes less and less important, and the impulse approximation limits is smoothly recovered \(10\).

A systematic comparison with the available data will require the extension of our formalism to the case of finite nuclei. In addition, the contribution of processes involving meson-exchange currents, which is known to be significant in the transverse electromagnetic response \(25\), needs to be included.

The approach based on correlated basis states, discussed in this paper, can be readily generalized to describe finite systems. It has been already applied to obtain a variety of properties of medium-heavy nuclei \((^{12}\text{C}, \; ^{16}\text{O}, \; ^{40}\text{Ca}, \; \text{and} \; ^{48}\text{Ca})\), such as ground-state energies, charge-density and momentum distributions, natural orbits, occupation numbers, quasi-hole wave functions, and spectroscopic factors \(24\). The extension of the techniques developed in Ref. \(26\) to the calculation of transition matrix elements of the electroweak current does not involve any additional conceptual problems.

The role of meson-exchange currents in neutrino-nucleus interactions has long been recognized \(27\). Their effects must be included in a consistent fashion, as the longitudinal components are related to the nuclear Hamiltonian, determining the correlation functions, through the continuity equation. The formalism to obtain the model-independent part of the currents within nonrelativistic many-body theory is discussed in Ref. \(28\), while the relativistic expressions, as well as their reduction, are analyzed in Ref. \(13\). It has been recently suggested \(10, \; 29–31\) that the effects of meson-exchange currents may explain at least part of the discrepancies between the results of theoretical calculations and the double-differential charged-current quasielastic cross section measured by the MiniBooNE Collaboration \(9\).

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