EARLY JAINA GEOMETRY IN INDIA AND ITS RELEVANCE IN LATER INDIAN GEOMETRY

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This paper attempts to highlight the relevance of the early Jaina geometry in the field of mathematical or rather geometrical works of Āryabhaṭa I, Brahmagupta, Mahāvīraśārya, Āryabhaṭa II, Nemicandra, Bhāskara II and other Indian mathematicians. We shall at first present some geometrical formulae (including mensuration) contained in the Jaina religious and semi-religious works of Umāsvāti in the Tattvārthadhiṣṭhāna-sūtra-bhāṣya and Jambhāvīpasamāsā. Thereafter we will examine the validity of these formulae with the help of plane geometry. Finally, we would like to make an effort to find out the similarity (or dissimilarity) of these formulae with the results (or rules) given by the later Indian mathematicians like the ones mentioned above.

Introduction

India had a glorious past in the field of mathematics and the branch of it, which received the attention earliest was geometry. There is no doubt that the beginning of geometric way of thinking occurred firstly in two main cities, Harappa and Mohenjodaro, in the Indus Valley during 3250–2750 B.C.E.1 As this civilization having grown on the basis of development of cities, obviously, for city planning and construction of big mansions, public baths etc., a knowledge of practical geometry was presumably a must for its dwellers. But they did not leave behind any sign of their geometrical knowledge in the form of books or manuscripts. This fully developed and highly cultured civilization was discovered in the north-western region of undivided India during archaeological excavations in the first half of the 20th century C.E.2 Sir John Hubert Marshall (1876–1958 C.E.), the then Director-General of the Archaeological Survey of India, oversaw the excavations of the said two main cities. After the end of that glorious era, at about 1500 B.C.E., a new civilization known as the Vedic Civilization began to emerge in India.3 The most noteworthy feature of this civilization is the construction of (ritual) altars which are necessary in connection with the sacrifices of the Vedic Hindus.4 For which pristine knowledge of geometry is indispensable for its dwellers and this knowledge, although meagre, is contained in the Rg-veda and the Śatapatha Brāhmaṇa.5,6

After a long period since the time of the Brahmanic literature the most remarkable advancement in the field of geometry is found in the texts, the Śulbas or the Śulbasūtras, evolved in the late Vedic period. There are at least eight extant Śulbasūtras among which Baudhāyana-śulbasūtra is the oldest one, written in about 800 B.C.E.7 and the others during c. 800–500 B.C.E. Now, it is to be mentioned that these texts give us a picture of development of early geometry and mathematics in India before the advent and rise of Jaina sect during c. 500–300 B.C.E.8 Let us glance at the traces of geometrical knowledge contained in the Jaina religious and semi-religious works of Umāsvāti (c. 1st century C.E.), Jaina Acharya and great author on the Jaina doctrines.

Early Jaina Mathematics

The early Jainas had a far-reaching influence in Indian mathematics including geometry after the end of Śulba period, just before the pre-Āryabhaṭīyan age i.e., up to 5th...
century C.E. It will be beside the mark that there were at least four mathematicians, namely, Maskarı, Pūraṇa, Mudgala and Pūtana, prior to Āryabhata I. But their works are not available till now either in book or manuscript form. However, the Jainas had given rise to spread out the study of geometry mainly for meeting their religious needs like the Hindus in the Vedic age. The Hindus used to erect altars of varied geometrical shapes for different deities for practising their religious rites. Amongst the variously named altars, the Mahāvēdi (The Great Altar) is significantly in the shape of an isosceles trapezium. Almost, for adoption of the same practice, the discussion about the informations and theories of geometry have been done in various religious and semi-religious books of the Jainas (in Prakrit and Sanskrit). Again, they had taken an utmost effort to establish several mathematical or rather geometrical results relating to the circle and its different parts with respect to their discussion on the appearance of the universe i.e., on their theory of Cosmography by pondering the shape of the Jambūdvīpa (the earth) like a circle of diameter 100,000 yojanas.

The religious literature of the Jainas is generally classified into four groups among which Gaṇitānyuyoga is one of them. In this book, various informations and principles of mathematics have been narrated. Amongst the various religious books of the Jainas, which are important contain the whole range of numbers and theories of geometry have been done in various religious and semi-religious books of the Jainas (in Prakrit and Sanskrit). The historians of Indian mathematics, these books may be written in about 500 to 300 B.C.E. Now, it should be mentioned that most of the original mathematical works of the Jainas of that period have not come to light yet. Anyway, our present knowledge on their contributions on mathematics is acquired mostly from the available contemporary commentaries.

Needful to say that, early Jaina mathematics was rich in Theory of Numbers. Big number of the order of Nījuta could have been written in Jaina Mathematics. Jaina scholars were acquainted with Decimal Place Value System (Sthāna). In Sūryaprajñāpti (c. 400 B.C.E.) the whole range of numbers was classified into three sets: Sankheya (enumerable), Asankheya (innumerable) and Ananta (infinite).

Geometry or Mensuration was named as Rajju in Jaina Mathematics. Jaina scholars mostly followed Vedic Sulavādis in geometrical calculations.

A Few Geometrical Formulae

Amongst the mathematical results contained in the Tattvārthādhigama-sūtra-bhāṣya of Umāsvāti, the following formulæ related to geometry and also mensuration are found. If C be the circumference of a circle of diameter d and area A, then

\[ C = \sqrt{10d^2} \]

Further, if a be the length of the arc of a segment of the circle less than a semi-circle, c be the length of the chord of it and h be the height or arrow (śāra) of it, then

\[ c = \sqrt{4(h-d)} \]

\[ h = \frac{1}{2}(d - \sqrt{d^2 - c^2}) \]

\[ d = \left( h^2 + \frac{c^2}{4} \right) / h \]

\[ a = \sqrt{6h^2 + c^2} \]

Incidentally, the name of Umāsvāti has come down to us as a great author on the Jaina doctrines, but not as an author on mathematics. It is doubtful whether he was a mathematician or not. So, it would not be irrelevant to conclude that the mathematical formulæ which he had quoted in his work Tattvārthādhigama-sūtra-bhāṣya or Jambūdvīpasamāsa were taken from some other contemporary or earlier treatises on mathematics known at his time. It is to be mentioned here that Umāsvāti was also the author of the work Kṣetrasamāsa which dealt with geography and mensuration.

Kusumapura School of Mathematics

It is said that the naming of Umāsvāti is a combination of his mother’s name Umā and father’s name Svāti. He resided at Kusumapura – near ancient Pātaliputra, now known as Patna in Bihar. A number of mathematicians of Bihar believe that present day ‘Phulwari Sharīr’ (Garden of Flowers) near Patna is ancient Kusumapura. The existence of a school of mathematics at Kusumapura was first found out in his work. The historians of Indian mathematics are of opinion that this school originated long before the time of famous Jaina saint Bhadrabāhu (300 B.C.E.) of Kusumapura. It is to be noted here that both Bhadrabāhu and Umāsvāti hailed from Kusumapura. However, the study of mathematics and astronomy continued in Kusumapura School for many centuries, and Āryabhata I (c. 476 – 550 C.E.), an eminent Hindu mathematician and astronomer, perhaps learnt his lessons from this school in 5th century C.E.
Validity of the Geometrical Formulae

Now, an effort can be made to justify the precision of the above mentioned formulae with the help of plane geometry.

It is evident that formulae in SL. Nos. (1) and (2) indicate the approximate value of the circumference and the area of a circle respectively as because the early Jainas have usually used the value of the ratio of the circumference to the diameter as \(\frac{10}{\pi}\) in place of \(\pi\) (pāi). It is needless to say that the value of this ratio known as \(\pi\), was first calculated by Greek mathematician Archimedes (in c. 3rd century B.C.E.) and independently by Indian mathematician Āryabhaṭa I (in c. 5th century C.E.).

Further, formulae under SL. Nos.(3), (4) and (5) represent respectively the accurate value of the length of the chord, the height or arrow (śara) of a segment (less than a semi-circle) and diameter of the circle. These three formulae may be determined with the help of plane geometry as follows:

Proof of formula (3) \(c = \sqrt{4h(d-h)}\)

Using the Pythagoras’ theorem (now it is known as the Baudhāyana-Pythagoras’ theorem), we have from the right-angled triangle COE (vide Figure -1),

\[CE^2 = OC^2 - OE^2 \quad \text{or}, \quad \left(\frac{c}{2}\right)^2 = \left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - h\right)^2\]

\[\therefore CE = \frac{1}{2} CD = \frac{c}{2}, \quad EB = h \quad \text{and} \quad OE = OB - EB = \frac{d}{2} - h\]

or, \(c^2 = 4dh - 4h^2\), \(c^2 = 4h(d-h)\)

\[\therefore c = \sqrt{4h(d-h)} \quad \text{(proved)}\]

Proof of formula (4) \(h = \frac{1}{2}\left(d - \sqrt{d^2 - c^2}\right)\)

According to the Figure -1 we get,

\[h = EB = OB - OE\]

\[= OB - \sqrt{OC^2 - CE^2} = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{c}{2}\right)^2}\]

\[\therefore \quad \text{OC} = \frac{1}{2} AB = \frac{d}{2} \quad \text{and} \quad CE = \frac{1}{2} CD = \frac{c}{2}\]

\[= \frac{d}{2} - \frac{1}{2}\sqrt{d^2 - c^2}\]

\[\therefore h = \frac{1}{2}\left(d - \sqrt{d^2 - c^2}\right) \quad \text{(proved)}\]

Proof of formula (5) \(d = \left(h^2 + \frac{c^2}{4}\right)/h\)

Formula (5) is the alternative form of formula (3). Because, from formula (3) i.e., from \(c = \sqrt{4h(d-h)}\) we get formula (5) as follows:

Here, \(\frac{c^2}{4} = dh - h^2\) \(\text{or}, \quad dh = h^2 + \frac{c^2}{4}\)

\[\therefore d = \left(h^2 + \frac{c^2}{4}\right)/h \quad \text{(proved)}\]

Proof of formulae (6) and (7)

Left out two formulae mentioned under SL. Nos. (6) and (7) are alternative forms of each other. These two formulae imply respectively the approximate value of the length of the arc and the height or arrow (śara) of a segment of the circle less than a semi-circle. Because, these can be derived roughly from the formula (1) i.e., from \(C\) (circumference) = \(\sqrt{10d^2}\) by considering the segment as a semi-circle.

If the segment of a circle is a semi-circle then \(c\) will be equal to \(d\), and \(h\) will be equal to \(\frac{d}{2}\). So, we can write the formula (1) as follows:
\[
\frac{1}{2} \text{(circumference)} = \frac{1}{2} \left(\sqrt{10d^2}\right)
\]

or, semi-circumference = \[
\frac{\sqrt{6d^2 + 4d^2}}{4}
\]

= \[
\frac{\sqrt{6d^2}}{2} + d^2
\]

= \[
\sqrt{6h^2 + c^2}
\]

Thus, when the segment of a circle is equal to a semi-circle we may write roughly

\[
a = \sqrt{6h^2 + c^2},\]

which itself the formula (6).

The early Jainas most probably admit this formula as true for all segments of a circle. However, formula (7) i.e.,

\[
h = \frac{\sqrt{a^2 - c^2}}{6}
\]

can be derived very easily from formula (6), which need not be stated.

Now, we would like to make an attempt to find out the similarity (or dissimilarity) of all these geometrical formulae (including mensuration) with the results (or rules) given by the later Indian mathematicians like Āryabhata I, Brahmagupta, Mahāvīrācārya, Āryabhata II, Nemicandra, Bhāskara II and others. Chronological account of their contributions related to the above discussed matters are given below in a nutshell.

**In the contributions of Āryabhata I (c. 476 – 550 C.E.)**

We have already cited that the early Jainas have usually used the value of the ratio of the circumference (of a circle) to the diameter as \(\sqrt{10}\) in place of \(\pi\) (pāi). In this connection it should also be remembered that Āryabhata I's value of this ratio (=3.1416) is far more precise than that of the Jaina’s. In the 10\(^{th}\) verse of chapter ‘Gaṇitapāda’ of the book “Āryabhaṭiya” (c. 499 C.E.), Āryabhata I enunciates the rule (regarding this value) as follows:

\[
caturadhikaṣatamaṣṭaganuṁ dvāṣaṣṭistathā sahasrānāṁ
\]

\[
ayutadvayavīskambhāyāsanno vṛttapariṇāhah || 10 ||
\]

**English translation**: “A hundred increased by four, multiplied by eight, and added to sixty-two thousand (62,000), will be the nearly approximate (āsama) value of the circumference of a circle of diameter two ayuta (20,000).”

This rule gives

\[
\frac{\text{circumference}}{\text{diameter}} = \pi = \frac{(100 + 4) \times 8 + 62000}{20000}
\]

\[
= \frac{62832}{20000} = \frac{3927}{1250} = 3.1416
\]

(8)

Here it is to be specially noted that mention has been made of the nearly approximate value. Naturally, a question may arise; was the author, long ago from this date (c. 499 C.E.), could comprehend that the ratio \((\pi)\) was a transcendental number? However, this value of the ratio \((\pi = 3.1416)\) does not occur in any earlier work on Indian mathematics, and forms an important contribution of Āryabhaṭa I.

We have previously mentioned the Umāsvāti’s formula (2), \(A = \frac{C \cdot d}{4}\) i.e., area of a circle = circumference × \(\frac{\text{diameter}}{4}\). This formula may be written easily in the form

\[
\text{area of a circle} = \frac{\text{circumference} \times \text{diameter}}{2}
\]

In this context it should be stated that this formula is found also in the said treatise “Āryabhaṭiya” of Āryabhata I. In the 1\(^{st}\) half of the 7\(^{th}\) verse of chapter ‘Gaṇitapāda’ of this book, he says:

\[
samaparināhaysāyārdham viśkambhārdhahatameva vṛttaphalam ||
\]

**English Translation**: “Half the circumference, multiplied by half the diameter is the area of a circle.”

That is, area of a circle = \[
\frac{\text{circumference} \times \text{diameter}}{2}
\]

or, \(A = \left(\frac{C}{2}\right) \left(\frac{d}{2}\right)\) (9)

Next, we may come to discuss about the Umāsvāti’s formula (3), \(c = \sqrt{4h(d - h)}\), cited earlier. This formula can be easily written in the form \((d - h)h = \left(\frac{c}{2}\right)^2\), which may be written again, in accordance with the Figure - 1, as

\[
AE.EB = \left(\frac{1}{2}CD\right)^2 = CE^2
\]
This relation attracts our attention to a geometric theorem related to the chord of a circle. The theorem is—

In any circle, the product of the two intercepted parts of a diameter perpendicular to the chord i.e., the product of the two arrows ( śāras), is equal to the square of half the chord.

Here it is to be specially highlighted that this theorem has been noticed also in the aforenamed book “Āryabhaṭīya” of Āryabhata I. In the 2nd half of the 17th verse of chapter ‘Gaṇitapāda’ of this treatise, Āryabhata I has mentioned the formula for determination of the length of the chord of a circle. In this verse he writes:

ttṛte śarasaṃvargo ’rdhajyāvargah ya khalu dhanusōḥ ||

**English Translation**: “In a circle, when a chord divides it into two arcs, the product of the arrows of the two arcs is certainly equal to the square of half the chord.”

This formula states that if in a circle, a chord CD and a diameter AB intersect each other at right-angle at E, then

\[
\text{AE.EB} = \left(\frac{1}{2} \text{CD}\right)^2 = \text{CE}^2 \text{ (vide Figure - 1)}
\]

Interestingly, Āryabhata I, as an expert in the contemporary trigonometry, could easily see, from the identity \((\text{R Sin}\theta)^2 + (\text{R Cos}\theta)^2 = \text{R}^2\) that

\[
(\text{R Sin}\theta)^2 = (\text{R} + \text{R Cos}\theta)(\text{R} - \text{R Cos}\theta), \tag{10}
\]

where R is the radius of the circle and \(\theta\) the angle subtended at the centre O by the arc BC (vide Figure - 1).

Geometrically (10) is equivalent to

\[
\text{CE}^2 = (\text{AO} + \text{OE})(\text{OB} - \text{OE}) = \text{AE}. \text{EB} \tag{11}
\]

\(\therefore \text{OA} = \text{OB} = \text{OC} = \text{R}, \text{ CE} = \text{R Sin}\theta \text{ and OE} = \text{R Cos}\theta\)

But contrary to Umāsvāti’s formula (3), no square-root appears in the Āryabhata I’s result (11).

**In the contributions of Brahmagupta (598 – c. 668 C.E.)**

Here it is worth knowing that Brahmagupta, the most prominent Hindu mathematician and astronomer, pronounces the rules to find the diameter of a circle (d), length of a chord of it (c) and height or arrow (sara) of a segment (less than a semi-circle) of it (h) in his treatise “Brāhma-Sphuṭa Siddhānta” (628 C.E.). In the verse 41, chapter – XII, of the said treatise he writes about these (rules) which are given below:

\[
vṛtta śarona guṇitād vyāsāccaturāhātāt padam jīvā | \]

\[
jyāvargasācaturāhātasaśrabhaktāḥ šarayuto vyāsāḥ || 41 ||
\]

**English Translation**: “In a circle, the diameter should be diminished and then multiplied by the arrow; then the result is multiplied by four; the square-root of the product is the chord. Divide the square of the chord by four times the arrow and then add the arrow to the quotient: the result is the diameter. Half the difference of diameter and the square-root of the difference between the squares of the diameter and chord, is the smaller arrow.”

Thus the rules give the formulae:

\[
c = \sqrt{4h(d - h)}; \tag{12}
\]

\[
d = \frac{c^2}{4h} + h; \tag{13}
\]

\[
h = \frac{1}{2} \left( d - \sqrt{d^2 - c^2} \right). \tag{14}
\]

Clearly, all these formulae (12), (13) and (14) are same to the previously cited Umāsvāti’s formulae (3), (5) and (4) respectively.

**In the work of Mahāvīrācārya (c. 800?– 875 C.E.)**

Meanwhile we are familiar with the Umāsvāti’s formula (6) i.e., \(a = \sqrt{6h^2 + c^2}\). This formula occurred also in the later mathematical work of Mahāvīrācārya, a Jaina mathematician who belonged to Digambara sect. In the verse 73 \(\frac{1}{2}\), chapter – VII, of his book “Gaṇitasāra-Samgraha”, Mahāvīrācārya (c. 850 C.E.) gives the rule in the following words:

\[
\text{śaravargaḥ ṣadgunito jyāvargasamanvantastu yastasya } \]

\[
mālaṃ dhanurguṇeṣuprasādhanē tatra viparitam||73\frac{1}{2}||
\]

**English Translation**: “The square of the arrow measure is multiplied by six. To this is added the square of the string measure. The square-root of that (which happens to be the resulting sum here) gives rise to the measure of the (bent) bow-stick. ….”

That is: the square of the arrow is multiplied by six and then added by the square of the chord; the square-root of the result is the arc.

Here if we denote the arrow measure (i.e., height or sara of a segment less than a semi-circle), string measure (i.e., length of the chord of it) and measure of the bent
bow-stick (i.e., length of the arc of it) by h, c and a respectively, then this rule gives

$$a = \sqrt{6h^2 + c^2}$$

(15)

which is same to the Umāsvāti’s formula (6).

**In the work of Āryabhaṭa II (c. 920 – 1000 C.E.)**

It is well known to the scholars that the Greek value of π, which is equal to \(\frac{22}{7}\), is found in India first in the work of Āryabhaṭa II,31 a Hindu mathematician and astronomer. In the 1st half of the verse 92, chapter–XIV, of his treatise “Mahāsiddhānta” (c. 950 C.E.), Āryabhaṭa II describes the rule for finding the value as follows:\[32\]

vyāśā ’kṛtīgho’t’śairvihṛtāḥ sūkṣmo bhavet paridhiḥ |

**English translation** : “The diameter multiplied by 22 and divided by 7 will become nearly equal to the circumference.”33

This rule gives

$$\frac{\text{diameter} \times 22}{7} = \text{circumference}$$

That is, $$\frac{\text{circumference}}{\text{diameter}} = \pi = \frac{22}{7},$$

(16)

which is usually used as a rough approximation suitable for practical purposes.

Further, Umāsvāti’s formulae (6) and (7) i.e., $$a = \sqrt{6h^2 + c^2}$$ and $$h = \frac{\sqrt{(a^2 - c^2)}}{6}$$ came into view also in the said treatise “Mahāsiddhānta” of Āryabhaṭa II. In the 90th verse, chapter–XIV, of this book he enunciates the rules as conferred below:\[34\]

saravargāt ṣaḍguntiājyākṛtyayuktāt padam cāpam |

jyācāpakṛtiviyogāt ṣaḍbhaktiṣyayat padam sa sarah || 90 ||

**English Translation** : “The square-root of the square of the chord multiplied by six and added by the square of the chord is the arc. The square-root of the difference of the square of the arc and chord as divided by six, is the arc.”35

This is to say, the rough formulae are:

$$a = \sqrt{6h^2 + c^2},$$

(17)

$$h = \frac{\sqrt{(a^2 - c^2)}}{6}.$$

(18)

Evidently, the above formulae (17) and (18) are similar to the Umāsvāti’s formulae (6) and (7) respectively.

**In the contributions of Nemicandra (fl. 10th century C.E.)**

Here it is worth mentioning that Nemicandra (c. 975 C.E.), most distinguished Jaina Acharya and author, belonged to Digambara sect, writes in gâtā 760 (second half) of his treatise “Tiloyasāra” (Sanskrit, Trilokasāra) the following Prakrit words:\[36,37\]

bānakādim chahi gunide tattha jude dhanukadī hodi ||760||

The Sanskrit rendering of the rule is:

bānakṛtim ṣāḍbhīḥ gunite tatra yute dhanuḥ kṛtiḥ bhavati|

**English Translation** : “Six times the square of the bāṇa (height of the segment) added there (to the square of the chord stated in the first half of the gâtā) becomes the square of the arc (of the segment).”38

That is, $$a^2 = 6h^2 + c^2$$

(19)

which is equivalent to the square of the relation between a, h and c given by $$a = \sqrt{6h^2 + c^2}$$, Umāsvāti’s formula (6).

**In the works of Bhāskara II (1114 – 1185 C.E.)**

We have already discussed that Āryabhaṭa I’s value of π, viz. \(\frac{3927}{1250}\) (=3.1416), is far more accurate than that of the Jaina’s (cf. $$\pi = \sqrt{10}$$), and the rough approximate Greek value, $$\pi = \frac{22}{7}$$, is found in India first in the work of Āryabhaṭa II. Now, we would like to remember that Bhāskarācārya or Bhāskara II, prominent Hindu mathematician and astronomer, gives two values for π, the aforesaid nearly accurate, \(\frac{3927}{1250}\) and the rough approximate Greek value, \(\frac{22}{7}\).

Bhāskara II states the rule in the verse 199 of vṛttavahārahā (वृत्तवहार्य : of his mathematical treatise “Līlāvatī” (c. 1150 C.E.), as written below:\[39\]

vyāśe bhanandagnihate vibhakte |

khabāṇasārīreyeh paridhiḥ susūkṣmah |

dvāvinīśatīghne vihṛte’ha śailaiḥ |

śhūlt’tavā svād vyavahārayogyah || 199 ||

**English Translation** : “When the diameter is multiplied by 3927 and divided by 1250, the result is the nearly
precise value of the circumference; but when multiplied by 22 and divided by 7, it is the gross circumference which can be adopted for practical purposes.\(^{40}\)

That is, this rule gives

\[
\pi = \frac{3927}{1250} \quad \text{(nearly precise value)} \quad \text{and} \quad \pi = \frac{22}{7} \quad \text{(gross value)} \tag{20}
\]

Further, the Umāsvāti’s formula (2) i.e., \(A = \frac{d}{4}\). Here it should be referred that this formula appears also in the aforesaid book “Līlāvatī” of Bhāskara II. In the 1\(^{st}\) line of the verse 201 of kṣetraphala ghanaphalānayanam (क्षेत्रफल घनफलानयनम्) of this book, he pronounces the rule:\(^{41}\)

\[
वर्तक्षेत्रे परिधिगुनितयासपादः पहलम्, तत \parallel 201½\parallel
\]

**English Translation**: “In a circle, the one-fourth of the diameter multiplied by the circumference gives the area.”\(^{42}\)

That is, area of a circle = \(\frac{\text{diameter}}{4} \times \text{circumference} \)

or, \(A = \frac{d}{4} \cdot C\) \tag{21}

Needful to say, the formula (21) is the identical form of the Umāsvāti’s formula (2).

Lastly it must be mentioned that Umāsvāti’s formulae (3), (4) and (5) have been noticed also in the said treatise “Līlāvatī”. Bhāskara II narrates the rules, concerning these formulae, in the verse 204 of sarajīvānayanam (सराजीवानयनम्) of this text as follows.\(^{43}\)

\[
jyāvyāsayogāntarāghātāmulaṃ vyāsastadūno dalitāḥ sarāḥ syāt ||
\]

\[
yāsāccharonāccharasangunācca mulaṃ dvijnignam bhavatī và ||
\]

\[
jīvārdhavarge sarabhaktayukte vyāsapramāṇaṃ pravadantī vrte || 204 ||
\]

**English Translation**: “Find the square-root of the product of the sum and difference of the diameter and chord, and subtract it from the diameter: half the remainder is the arrow. The diameter being diminished and then multiplied by the arrow; twice the square-root of the result is the chord. In a circle, the square of the semi-chord being divided and increased by the arrow, the result is stated to be the diameter.”\(^{44}\)

Hence, the rules give the following formulae:

\[
h = \frac{1}{2}\left(d - \sqrt{d^2 - c^2}\right)\; ; \tag{22}
\]

\[
c = \sqrt{4h(d - h)}\; ; \tag{23}
\]

\[
d = \frac{c^2}{4h}\; . \tag{24}
\]

Clearly, the above formulae (22), (23) and (24) are same to the Umāsvāti’s formulae (4), (3) and (5) respectively.

**Conclusion**

It can be easily inferred from the above discussion that the early Jaina geometry had some relevance, may be meagre, in the works of a few later Indian mathematicians like Āryabhata I, Brahmagupta, Mahāvīrācārya, Āryabhata II, Nemicandra, Bhāskara II and others.

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