Improved performance of quantum annealing by a diabatic pulse application

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(Dated: June 25, 2018)

The success probability to find the ground state of an Ising Hamiltonian in the quantum annealing or adiabatic quantum computing is reduced by unwanted nonadiabatic processes such as the Landau-Zener transition. In this paper, we propose a protocol to modulate the success probability by applying a diabatic pulse during quantum annealing. By optimizing the parameters of a diabatic pulse, the success probability can be enhanced, compared to the conventional quantum annealing in the presence of the Landau-Zener transitions due to fixed annealing times. Our results shows that non-adiabatic processes can be used to improve the efficiency of quantum annealing machines, breaking away from the usual adiabaticity requirement.

PACS numbers:

I. INTRODUCTION

The quantum annealing (QA) has been introduced as an alternative to the simulated annealing for efficiently solving combinatorial optimization problems [1, 2]. The main advantage of QA over classical simulated annealing [3] is that during the annealing the search for the optimum state can stuck to a local minima, then quantum fluctuations can help the system to tunnel from the local minima to the global one [4]. The research on QA was intensified in recent years, after the commercialization of QA machines by D-Wave Systems Inc. using superconducting flux qubits [5–7]. The QA machines from D-Wave Inc. have been used in a number of diverse hard optimization problems, to name a few: global warming [8], traffic control [9] and analyzing data regarding the Higgs boson discovery from Large Hadron Collider [10]. In addition, new superconducting quantum annealing machines are now developed by several groups [11–13].

Adiabaticity guarantees that if the annealing process is slow enough the final state will be the ground state of the problem Hamiltonian, thus a solution of the optimization problems. The annealing time is inversely proportional to energy gap between the ground and the first excited states [14]. Hence, optimization problems with extremely small energy gap demand unrealistically long annealing times. However, in reality, longer annealing than the relaxation and coherence time, can reduce the success probability (SP) due to coupling with the environment and decoherence [15]. As SP we define the probability of the instantaneous ground state of the Hamiltonian to the system state at the end of the annealing, SP measures how close we are to the optimum solution. Furthermore, for spin glass problems, for a fixed annealing time, it has been found that decreasing the annealing time can be beneficial [16, 17]. The physical explanation is that for fixed annealing times the problem can stuck in a local minima, by reducing the annealing time non-adiabatic processes are introduced that can kick the system from the false minima to somewhere closer to the ground state. Thus, an induced nonadiabatic process can be used to increase the SP. In addition, reducing the annealing times is also appreciated by D-Wave Systems Inc. and future QA machines will have reduced annealing times [18]. These facts strongly indicate that the nonadiabatic protocol will be important for enhancing SP for fixed annealing time in an actual QA machines.

On the other hand, in order to reduce the influence of decoherence and dissipation in gate-type quantum computers several error correction schemes have been proposed [19]. One powerful method for fighting such effects is the bang-bang control, i.e. the application of a pulse train [20–22]. Recently the bang-bang protocol has been applied for QA as an alternative to the adiabatic one [23]. The authors consider bang-bang annealing protocols for multiqubit instances, up to 10 qubits, and they found an enhancement of the SP, compared to the conventional QA which follows a linear ramp scheduling. Of course these results focus on fixed annealing times, which are inherently short, thus the adiabaticity is already broken, while decoherence and dissipation effects are negligible. Reducing the annealing time and simultaneously increasing the SP is of absolute importance for getting full advantage of a practical QA machine[24].

On the same time, QA process is connected with the Landau-Zener physics [25]. The way the adiabatic QA algorithm is implemented makes important the transition probability at avoided energy crossings between the ground and first excited energy states [26]. The influence of a diabatic pulsed quantum fluctuation has been investigated, where a number of analytical expressions extracted, and analyzed in terms of the Landau-Zener physics [27]. These results for large pulse durations and strong overlap with the avoided crossing have been used in the context of the adiabatic quantum dynamics [28]. Although Ref.[27] discussed on the sudden change of the single spin system undergoes, because of the diabatic pulsed quantum fluctuations that causes an oscillatory behavior of the SP, they do not account on the exact physical mechanism. Furthermore, the bang-bang protocol, i.e. diabatic pulse train application, for QA has been used and shows that holds an advantages over the conventional QA, there is no physical explanation behind this mechanism as well [23].

In this paper we shed light on the physical explanation behind the applicability of such diabatic pulse protocols in order

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to enhance SP. Starting from the single qubit case, we show that varying the pulse parameters we can modulate the SP due to destructive and constructive interference caused during the pulse application. We use the transfer matrix approach, a semi-analytical method \cite{29}, which compares well to the full numerical method. The transfer matrix approach has been used successfully to explain experimental results concerning Rabi oscillations in diamond NV-centers and Stuckelberg interference in superconducting qubits \cite{30, 31}.

The paper is organized as follow. In Sec.II we consider the case of a single qubit where the conventional QA plus the diabatic pulse application are considered. We present the transfer matrix method, using the sudden approximation, and compare it with full numerical results. The interference patterns of the SP, by varying the pulse parameters, are explained by the Mach-Zehnder interferometer analogy. In Sec.III we consider the multiqubit case, where again the SP can be increased, compared to the case the conventional QA is considered. We show that for any instance generated, for certain pulse pulse parameters, the SP is increased, compared to conventional QA. In Sec.IV we give the concluding remarks and discuss for possible future directions. In the Appendix A we present in more details the transfer matrix method using the sudden approximation.

II. PULSED QUANTUM ANNEALING FOR A SINGLE QUBIT

The Hamiltonian of a single qubit, including the applied pulse, has the form,

\[
H(t) = \left(\frac{t}{t_f} \varepsilon + C \Lambda(t)\right) \sigma_z + \left(1 - \frac{t}{t_f}\right) \Delta \sigma_z,
\]

where \(\sigma_z\) and \(\sigma_x\) are the Pauli matrices, \(t_f\) is the annealing time, \(\Delta\) gives the strength of the quantum fluctuations, \(\varepsilon\) is the energy difference between the |0\> and |1\> states in the diagonal term and \(C\) is the strength of the applied diabatic pulse. \(\Lambda(t) = \Theta(t - t_C + t_D/2) \Theta(t_C + t_D/2 - t)\) is the pulse shape, in the diagonal part, which is characterized by the pulse center \(t_C\) and the pulse duration \(t_D\). Through out this paper we use energy units expressed through \(\Delta\), thus the time scales are expressed in \(h/\Delta\) units. When there is no applied pulse we have the conventional linear ramp QA, which is characterized by the usual Landau-Zener physics at the avoided crossing, where the minimum energy gap between the ground and excited states is \(E_{\min}^G = 2\sqrt{2} \varepsilon \Delta/\sqrt{\varepsilon^2 + \Delta^2}\) at the time \(t_{\min} = t_f \Delta^2/ (\varepsilon^2 + \Delta^2)\). Moreover, in this paper we focus on cases where the temperature is well below the minimum energy gap, \(E_{\min}^G \gg k_B T\), thus we take the zero temperature limit, \(T = 0K\).

Our numerical approach consists of numerical solution of the time-dependent Schrödinger equation

\[
i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle,
\]

which is used to simulate the QA process, with and without the diabatic pulse. Also, the exact diagonalization of the Hamiltonian \(H(t)\) (1), without the applied pulse, is used to calculate its instantaneous eigenstates \(|\psi_i(t)\rangle\), for \(i = 0,1, \ldots, 2^n - 1\), for \(n\) qubit systems. At the end of the annealing, \(t = t_f\), we define the SP as the projection of the instantaneous ground state \(|\psi_0(t_f)\rangle\) to the system state \(|\psi(t_f)\rangle\)

\[
P = \left|\langle \psi_0(t_f) | \psi(t_f) \rangle \right|^2.
\]

In this section we discuss the single spin system and compare fully numerical results with semi-analytical ones. In the next section we discuss \(n\) qubit systems, for \(n = 3\) to 10.

The conventional adiabatic QA protocol has to fulfill the adiabatic condition \cite{14}

\[
t_f \gg \max_{0 \leq s \leq 1} \left[ \frac{\langle \psi_0(s) | \frac{dH(t)}{dt} | \psi_1(s) \rangle}{\min_{0 \leq s \leq 1} |E_0(s)|^2} \right],
\]

for completely reducing unwanted Landau-Zener transitions, where \(s = t/t_f, |\psi_i(s)\rangle\) describe the ground, \(i = 0\), and first, \(i = 1\), excited instantaneous eigenstates of the Hamiltonian Eq. (1), without the pulse term, and \(E_0(s)\) the energy difference between them. The essential message of Eq. (4) is that the annealing process should be long enough so as to slowly pass through an avoided crossing, suppressing undesired Landau-Zener transitions.

The energy difference between the ground and the excited states of Eq. (1) is given by \(E_G(t) = 2 \sqrt{(1 - t/t_f)^2 \Delta^2 + (t/t_f \varepsilon + C \Lambda(t))^2}\) and is plotted in Fig. 1(a) for the case of \(t_f = 10 h/\Delta\) and \(\varepsilon = 1\Delta\) with the pulse, continuous line, and without the pulse, dashed line. The pulse parameters are \(t_C = 5h/\Delta\), \(t_D = 5h/\Delta\) and \(C = 1\Delta\). We observe that the pulse application has split the time evolution path that the qubit’s state follows in three parts, denoted by the times \(t_1 = t_C - t_D/2\), at which the pulse starts, and \(t_2 = t_C + t_D/2\), at which the pulse terminates. The adiabatic condition states that, if the system under consideration is evolved under a process that changes gradually, then allows it to stay at its initial state at the end. The pulse application is a fast diabatic process, which violates the adiabatic condition at the pulse application times \(t_1\) and \(t_2\). Away from \(t_1\) and \(t_2\), in the regions \(j = A, B, C\), qubit follows essentially an adiabatic evolution path as long as Eq. (4) is satisfied, there in no obstacle to disturb the time evolution of the qubit’s state nonadiabatically. Then, the time dependent state of the single qubit, described by the state vector \(|\psi(t)\rangle\) (see Appendix A), during these regions can be found by \(|\psi(t)\rangle = U_j(t_q; t_s) |\psi_i(t_q)\rangle\), where \(j = A, B, C\). The unitary evolution matrix is defined as

\[
U_j(t_q; t_s) = \begin{pmatrix}
e^{-i\zeta_j(t_q; t_s)} & 0 \\
0 & e^{i\zeta_j(t_q; t_s)}
\end{pmatrix} = e^{-i\zeta_j(t_q; t_s)} |\sigma_z\rangle,
\]

where

\[
\zeta_j(t_q; t_s) = \int_{t_q}^{t_s} E_G(t) dt,
\]

is the phase acquired during the adiabatic evolution in regions \(j = A, B, C\). The acquired phases \(\zeta_j(t_q; t_s)\) have a geometrical interpretation, they are connected with the area below the
we present the SP, where we observe the construction evolution of the quantum system is considered, except at the times $t_1$ and $t_2$ where a state mixing, between the ground and the excited states, is described in the sudden approximation regime, instead of using the Landau-Zener transitions. The sudden approximation has been used to explain the Stuckelberg interference in superconducting qubits caused by periodic latching modulation [31]. To our knowledge, it is the first time such physical explanation applied in the context of QA.

In Fig. 2 we present the SP, $P_{SP}$, for fixed annealing time, $t_f = 10\hbar/\Delta$, for which the adiabatic condition holds only for the conventional QA ($C = 0$), and fixed $\epsilon = 1\Delta$. In panel 2(a) we vary the pulse amplitude, $C$, for fixed value pulse center, $t_c = 1\hbar/\Delta$, and pulse duration, $t_p = 1\hbar/\Delta$, while in Fig. 2(b) we vary the pulse duration, $t_p$, for fixed value of the pulse center, $t_c = 5\hbar/\Delta$, and the pulse amplitude, $C = 1\Delta$. The blue dots give the SP for the case the Schrödinger equation is solved numerically, using the Hamiltonian of Eq. (1), and the red continuous line gives the results using the transfer matrix approach described earlier. We observe a good agreement between the two methods. The observed oscillations in the SP, $P_{SP}$, are due to destructive and constructive interference caused by the accumulative phase the qubit acquires during the pulse application. In Fig. 1(b) a direct analogy can be drown with a Mach-Zehnder interferometer [36], which is composed from two beam splitters. The first one divides the optical beam into two coherent beams that can follow different paths. The second beam splitter recombines and superimpose these beams, the different paths followed give interference fringes. To sum up, the pulse application due to interference effect, caused by the pulse application, can increase the SP for varying the pulse parameters. If we consider the case that $\zeta(t_0, t_1) \approx \zeta(t_2, t_f)$ then the SP can be approximated by the expression $P_{SP, appr} = 1 - 4p_s(1 - p_s)\sin^2(\zeta(t_1, t_2))$. This approximate expression is plotted in Fig. 2 where we observe a reasonable agreement for the position of the maxima and minima.
Figure 2: (Color online) The success probability (SP) by varying the pulse parameters when the annealing time is $t_f = 10\hbar/\Delta$ and $\varepsilon = 1\Delta$, for which the adiabatic condition holds for $C = 0$. (a) The pulse amplitude $C$ is varied for fixed pulse center $t_c = 1\hbar/\Delta$ and pulse duration $t_D = 1\hbar/\Delta$. (b) the pulse duration $t_D$ is varied for fixed value of the pulse center $t_c = 5\hbar/\Delta$ and pulse amplitude $C = 1\Delta$. The blue dashed line gives the full numerical results, $P_{SP,num}$, the red continuous line the transfer matrix approach, $P_{SP,tm}$, and the green dots the approximate expression $P_{SP,appr}$.

Figure 3: (Color online) Contour plot of the success probability (SP), $P_{SP}$, of a single qubit for fixed annealing time, $t_f = 5\hbar/\Delta$, and pulse center, $t_c = 2.5\hbar/\Delta$, for varying the pulse amplitude, $C$, and the pulse duration, $t_D$. The energy difference between the ground and the excited states is $\varepsilon = 0.5\Delta$, at $C = 0$. The green line encloses the area where $P_{SP} > 0.98$. In the case of $C = 0$, the adiabatic condition does not hold.

III. PULSED QUANTUM ANNEALING FOR MULTIQUBIT SYSTEMS

The Hamiltonian describing the QA process for the multiqubit case is:

\[ H = \frac{t}{t_f} H_t + C A_P(t) \sum_{i=1}^{n} \sigma_i^+ + \left( 1 - \frac{t}{t_f} \right) \Delta \sum_{i=1}^{n} \sigma_i^-, \]  

(9)

In Fig. 2 we considered the case where the SP for the conventional QA is close to 1, which means that the adiabatic condition holds for $C = 0$. So now we will focus on the case where we decrease the annealing time, $t_f = 5\hbar/\Delta$, and reduce to $\varepsilon = 0.5\Delta$, then the SP, for the case without the pulse application, is diminished, to the value 0.89, due to the Landau-Zener transition. The minimum energy gap is $E_{min} = 0.22\Delta$ at $t_{min} = 4\hbar/\Delta$. Using Eq. (4) we find that the right hand side has the value 3.6$\hbar/\Delta$ which is close to the annealing time $t_f$, thus the adiabatic condition breaks down. In Fig. 3 we present a contour plot of the SP, $P_{SP}$, for varying the values of the pulse amplitude, $C$, and the pulse duration, $t_D$, by keeping fixed the value of pulse center, $t_c = 2.5\hbar/\Delta$. We choose the pulse center value, $t_c$, so as to be at the center of the annealing time. The SP, $P_{SP}$, is increased, compared to its value for the conventional QA, up to values of 1. Furthermore, we observe that this increase is persisted for a wide range of parameters values implying the robustness of the proposed protocol. We also observe that for wider pulse durations, $t_D$, we need smaller values of the pulse amplitude in order to have an enhancement of the SP, compared to the $C = 0$ case. The position of the maxima of the SP are given by considering the maxima of the $P_{SP,appr}$, which are connected with the points at which $\sum_{i=1}^{N} \sigma_i^+ = N\pi$, for $N = 0, 1, 2, \ldots$. The full expression for the $\sum_{i=1}^{N} \sigma_i^+$, which can be analytically extracted from Eq. (6), has a complicate form and does not add anything to the discussion. In Fig. 3 we focus on the $N = 0$ case for later reference. Similar interference patterns have been observed for driving a superconducting qubit multiple times at an avoided energy level crossing [36–38], although the fringes of Fig. 3 have a different physical explanation. The physical reason behind such result is the constructive interference effect due to the pulse application, which can be used for increasing the SP for the multiqubit case as we see in the next section.
the third term represents the quantum fluctuation part, with an amplitude $\Delta$, the second term is the diagonally applied diabatic pulse, where the $\lambda(t)$ scheduling is introduced in Eq. (1), with a strength $C$, and $n$ is the number of qubits involved. The quantum fluctuation $\sigma_i$ part gives a superposition state in the computational basis ($|0\rangle$ and $|1\rangle$), thus helps on exploring the energy landscape in order to find the ground state. The first term is the target or problem (spin-glass) Hamiltonian:

$$H_f = \sum_{i=1}^{n} \epsilon_i \sigma^z_i + \sum_{i,j=1}^{n} J_{ij} \sigma^z_i \sigma^z_j,$$

where $\epsilon_i$ and $J_{ij}$ are independent Gaussian random numbers with zero mean and variance $\langle \epsilon_i^2 \rangle / \Delta^2 = \langle J_{ij}^2 \rangle / \Delta^2 = 1$.

Finding the ground state of $H_f$ can be connected with minimizing a cost function for an encoded optimization problem. Thus, following the QA protocol with long enough annealing time, $t_f$, so as to guaranty adiabaticity, we will end up to the ground state of Eq. (10). However, in reality, long annealing times cause unwanted decoherence and dissipation effects, while short ones also induce nonadiabatic Landau-Zener transitions which reduce SP. Applying a diabatic pulse, like in the single qubit case, we can modulate the SP, of a multi-qubit system and, for specific pulse parameters, enhance it, for a fixed annealing time. We proceed by solving numerically the time-depended Schrödinger equation Eq. (2) using the Hamiltonian Eq. (9) and its instantaneous eigenstates to calculate the SP, Eq. (3).

As we mention in the introduction, for certain difficult optimization problem, fast diabatic processes can increase the SP, compared to the conventional QA. The diabatic transitions can kick the system out of a local minima. Furthermore, our approach is to perturb the system using an extra time-depended term in the Hamiltonian and subsequently try to optimize the parameters of this pulse term, in order to increase the SP. For these types of methods to have a sizable speed-up of the annealing process we need to decrease the annealing time, $t_f$, at the point where, running the pulse-parameter optimization protocol multiple times, the total computational time can be shorter than the total annealing time needed to obey the adiabatic condition. This type of parameter optimization will require a classical data accumulation which controls the parameters of the diabatic pulse term and store the annealing results with the higher SP. In ref.[39] for obtaining a high value of the SP, above 0.95, while being in the coherent regime where the theoretical results regarding the SP for close and open systems to be similar, an annealing time of the order of $t_f \sim 10$ns is required. For our method using $t_f = 10h/\Delta = 0.03$ns, for $\Delta \sim 5GHz$, which differs 3 orders of magnitude. Thus, multiple runs, in the order of $10^5$, can be performed in the same time span.

Moreover, for the annealing times considered here the decoherence and dissipation effects caused by coupling with the environment are negligible. The coherence times of the superconducting qubits composing the QA machines provided by D-Wave Systems Inc. are of the order of 100ns [6], the annealing times $t_f$ considered in this paper are $10^4$ times shorter. For a real QA machine, in order to have an acceptable value for the SP, above 0.6, annealing times of the order of 10ms are required [7].

We start by studying the SP of a 5 qubit spin-glass instance, where for the conventional QA, the SP is $P_{30} = 0.47$ for an annealing time of $t_f = 10h/\Delta$, in which the adiabatic condition does not hold. We name this instance as instance A (iA). In Fig. 4(a) we present the energy spectrum of iA for varying time $t$, we observe that the ground state, thick blue dashed line, is very close to the first excited state towards the end of the QA process, thus Landau-Zener transitions induce a reduction of the SP. In order to increase the SP we consider the effect of a single diabatic pulse application during the QA to the 5 qubit iA instance. In Fig. 4(b) the pulse parameters are

\[ t_f = 10h/\Delta, \quad t_c = 5h/\Delta, \quad t_0 = 5h/\Delta. \]
$t_C = 5h/\Delta$ and $t_D = 5h/\Delta$, for the diabatic pulse center and duration respectively, while we vary the pulse strength $C$. We observe that due to interference effects, which are caused during the diabatic pulse application, the SP oscillates for varying the pulse amplitude $C$. For small pulse amplitudes, compared to the energy scale defined by the transition amplitude $\Delta$, we observe that the SP increases compared to the conventional QA. Furthermore, the diagonally applied diabatic pulse increases the energy gap between the ground and the first excited states. Thus, the pulse application can be used to enhance the SP of a multiqubit system to higher values than the conventional QA, for fixed annealing times. The oscillations observed in Fig. 4(a) are caused due to interference effects during the pulse application, although the fact that we have a 5 qubit system causes the oscillation’s amplitude to reduce while increasing the pulse strength, unlike the single qubit case in Fig. 2(a). High pulse strength, $C$, values causes transitions to the higher excited states, thus reducing the SP.

In order to validate the importance of the pulse application for each of the generated instances, one needs to find the appropriate pulse parameters which will increase the SP. Let us define the SP when the pulse is applied as $P_{SP}$ and without the pulse $P_{S0}$. In Fig. 5(a) we present a plot of $P_{SP}$ to $P_{S0}$ for 200 instances for the 5 qubit case, for fixed annealing time $t_f = 10h/\Delta$. For each instance we run a pulse-parameter optimization routine which samples 2706 repetitions, we define the maximum attained value of the SP as $P_{SP,max}$ and the averaged SP over all samplings per instance as $P_{SP,av}$. With blue dots we present the $P_{SP,av}$ versus $P_{S0}$ for each instance. The thick red line shows the $P_{SP} = P_{S0}$ diagonal line and is used as a guide for the reader, for all instances that are above this line there is an increase in the SP for the pulsed QA. We observe that for all 200 instances exist a set of pulse parameters that can increase the attained value of the SP, even for the cases that the initial SP is $P_{S0} > 0.9$. The physical reason behind such a remarkable behavior is connected with the constructive interference during the diabatic pulse application for the specific set of parameters per instance. In Fig. 5(a) the green dots present $P_{SP,av}$ versus $P_{S0}$ pairs for each instance, where we also show the standard deviation over the sampling for each instance as error bars. We observe that the pulse application on average reduces the possible SP, thus for fabricating a QA machine following the pulse QA scheduling one would need to run it $O(10^5)$ times, then record the minimum energy and at the end store the desired arrangement, for the pulse parameters, for obtaining the ground state. As we have already mentioned the annealing time considered in the proposed pulsed QA protocol are $O(10^3)$ shorter than the conventional one, for having a decent value of the SP, so a reduction in the sampling number is desirable.

In Fig. 5(b) we present the averaged pulse parameters per instance for each sampling that we have $P_{SP} > P_{S0}$. We observe that the averaged pulse center has the value $t_C \approx t_f/2 = 5h/\Delta$, which means that the optimum pulse center $t_C$ is close to the middle of the annealing, something natural if we remember that for the linear ramping followed by the conventional annealing, the highest probability for existing an avoided energy level crossing is around, and after, the $t/t_f \gtrsim 0.5$. Thus, applying a pulse where the avoided crossing exists is beneficial for creating a constructive interference and also increasing the gap energy between the ground and the first excited states.

Secondly, we notice that the optimum amplitude parameter, $|C^{\text{opt}}|$, is smaller than the energy scale defined by the tunneling amplitude $\Delta$. Hence, the pulse amplitude modulation enhances the SP and can have a peak for small $C$ value. Lastly, we observe that half of the generated instances are enhanced for the case we have positive pulse strength, $C$, while
Figure 6: (Color online) (a-b) The three lowest energy states and (c-d) the projection probability of the ground and excited instantaneous eigenstates to the system state $P_i(t) = |\langle \psi_i(t) | \psi(t) \rangle|^2$, for $i = 0, 1$ respectively. We consider the 5 qubit instances which are optimized for positive, iPC, and negative, iNC, pulse strengths. Panels (a) and (c) are presented for the instance iPC and the panels (b) and (d) for the iNC instance. $P_{i,\text{con}}(t)$ and $P_{i,\text{pul}}(t)$ are the state probability overlap for the conventional and pulsed quantum annealing, respectively. The annealing time is $t_f = 10h/\Delta$.

The other half of the instances for negative. Therefore, the enhancement of the SP is not due to the energy gap opening but due to the constructive interference during the pulse application. Based on the above remarks we can reduce the sampling parameter space considerably, considering mainly pulses of small strength $C$ with a pulse center close to $t_f/2$, although we make a broader sampling for the pulse center. In Fig. 5 the sampling space is $O(10^3)$ after following the above remarks the sampling space reduced to $O(10^1)$.

In order to further analyze the behavior of the positive and negative applied pulse strength $C$ we present in Fig. 6 two instances for the 5 qubit system. For the two generated instances the pulse parameter optimization gives the optimized values $t_C = 4h/\Delta$, $t_D = 4h/\Delta$ and $C = 0.3\Delta$, for panels Fig. 6(a) and 6(c), and $t_C = 5h/\Delta$, $t_D = 6.5h/\Delta$ and $C = -0.2\Delta$, for Fig. 6(b) and Fig. 6(d). We name the instance with positive pulse strength iPC and the one with negative iNC. In Fig. 6(a) and 6(b) we present the three lowest energy states of the iPC and iNC, while in Fig. 6(c) and 6(d) we present the probability overlap of the ground, $i = 0$, and first excited, $i = 1$, instantaneous eigenstates to the system state evolution, $P_i(t) = |\langle \psi_i(t) | \psi(t) \rangle|^2$.

For the instance iPC, we observe that there is a steep avoided crossing around $t/t_f = 0.4$, which cause a Landau-Zener transition that reduces the SP at the end of the annealing. The optimized pulse center, $t_C$, for the iPC is at the avoided crossing and the applied diabatic pulse increases the energy gap between the ground and the excited states. In Fig. 6(c) we observe that initially the ground state for the conventional QA follows the ground state until approaching the avoided crossing which causes a state mixing, between the ground and first excited states, which leads to a reduced SP at the end of the annealing. The pulse application causes a state mixing, due to the pulse application at earlier time, $t/t_f = 0.2$, which has the effect to reduce the overlap with the ground
state initially, the pulse end leads to a different path followed by the system thus the SP is enhanced compared to the conventional QA.

For the instance iNC we observe in Fig. 6(b) that the ground and the first excited energy states are very close toward the end of the annealing, at these time we observe that following the conventional QA the SP is reduced. The pulse application induces an initial increase in the probability for the system to be at the excited state but subsequently relax to the ground state at the second state mixing, at time $t/t_f = 0.82$, caused by the pulse. These two instances iPC and iNC show how the diabatic processes can be used to effectively increase the SP. The positive and negative strength, $C$, of the applied pulse, to have $P_5 > P_{30}$, follows a bimodal distribution over the number of generated instances on how increase the SP. Thus, we need to sample both cases and keep the one that is showing the largest SP.

The diabatic processes triggered by the pulse application, in both instances iPC and iNC, can cause counterintuitive transitions from the higher energy states to the ground state [40]. These transitions are counterintuitive because the involved energy avoided crossings are at later times from the avoided crossing between the ground and the first excited states, i.e. Fig. 6(a) the avoided crossing for the first excited to the second excited states is at $t/t_f = 0.73$. These type of processes have been considered for three level systems [40].

The next step is to reduce the annealing time, $t_f$, in order to further reduce the sampling space and, simultaneously, to reduce the SP for the conventional QA, thus allowing further space for enhancement for the pulsed QA and further present the significance of the introduced pulsed QA protocol.

In Fig. 7 we present the SP for the QA, of 1000 instances of 5 qubit systems, with the diabatic pulse application, $P_{SP}$, compared without the pulse, $P_{30}$, for annealing times of $t_f = 5h/\Delta$, Fig. 7(a), and $t_f = 3h/\Delta$, Fig. 7(b) in which the adiabatic condition is violated largely even in the absence of the diabatic pulse. For each instance we consider two samplings with positive and negative pulse strengths $C$, we compare the averaged sampling value per instance and we keep the largest one. The blue circles give the maximum attained value of the SP, $P_{SP,max}$, and the green circles give the averaged SP for all trial samplings per instance, $P_{SP,av}$, for annealing time of $t_f = 5h/\Delta$ the sampling number is 90 and for $t_f = 3h/\Delta$ the sampling number is 60. For the case of annealing time $t_f = 5h/\Delta$, Fig. 7(a), for 37% of instances $P_{SP,av} > P_{30}$. For the case of annealing time of $t_f = 3h/\Delta$ (Fig. 7(b)) where for 50% of the instances $P_{SP,av} > P_{30}$. Thus, reducing the annealing time, $t_f$, and the sampling space the overall averaged sampling per instance that $P_{SP,av} > P_{30}$ increases. On the same time, while the annealing time, $t_f$, is reduced, the instances that $P_{SP,max} > P_{30}$ is 100%, where on the same time for 80% of the instances we have $P_{SP,max} > P_{30} + 0.05P_{30}$. The above results support that for fixed annealing times we can enhance the SP for specific set of pulse parameters.

Moreover, in Fig. 7 we split the $P_{30}$ over finite steps, at each step we average over the $P_{SP,av}$ of the instances, where we introduce the averaged SP of the instances $P_{SP,av}$. In Fig. 7 the black dots present $P_{SP,av}$ vs $P_{30}$, which show how much on average can be enhanced the SP for pulsed QA, compared to the SP of the conventional QA. The averaged SP $P_{SP,av}$ is close to the linear fit of $P_{SP,av}$ to $P_{30}$, we observe that the standard deviation is above the line defining the $P_{SP} = P_{30}$, this means that for the generated sampling instances have a high probability to increase the SP for the pulsed over the conventional QA.

In Fig. 7 we present a linear fit, of the form $Ax + B$, of the

![Figure 7](image-url)
We present the guide red line $f$ for the longest annealing time, the linear fit parameters are $A = 0.72$ and $B = 0.13$, and for $t_f = 3h/\Delta$ the linear fit parameters are $A = 0.80$ and $B = 0.07$. These fitting parameters show that as $P_{30} \to 0$ the SP is higher for the longest annealing time, $t_f = 5h/\Delta$, on the other hand the rate of increase of the SP for the pulsed QA is larger for the shorter annealing time, $t_f = 3h/\Delta$.

In Fig. 7 we present the guide red line $P_{SP} = P_{30}$ and we observe that there is a crossover between this line and the fitting of $P_{SP,av}$ vs $P_{30}$, for $t_f = 5h/\Delta$ the crossover is at $P_{30}^c = 0.45$ and for $t_f = 3h/\Delta$ is at $P_{30}^c = 0.37$. For $P_{30} < P_{30}^c$ the averaged SP, $P_{SP,av}$, is increased for the case of pulsed over the conventional QA. Furthermore we observe that decreasing the annealing time, $t_f$, the crossover SP drops, this is due to decreasing the annealing time, $t_f$, the overall SP for the conventional QA also decreases. The parameter space can be reduced and for 1/2 of instances, for $t_f = 3h/\Delta$, we have an overall enhancement of the averaged SP, compared to conventional QA. The crossover SP drops as the number $n$ of the multiqubit system is increased as expected.

Up to now we focus on the case of 5 qubit to analyze and understand what is the influence of a diabatic pulse to the SP of a multiqubit system, it is desirable to investigate how the proposed diabatic pulse QA scales as the number of qubits $n$ increases. We again consider the cases of annealing time of $t_f = 5h/\Delta$, with a sampling of 90 per instance, and $t_f = 3h/\Delta$, with a 60 sampling per instance, both positive and negative values of the pulse strength $C$ are considered and the highest value for $P_{SP,av}$ per instance is recorded. In order to check this behavior we introduce the relative success probability per multiqubit system

$$R_{SP,i} = \frac{\bar{P}_{SP,i} - \bar{P}_{30}}{\bar{P}_{30}},$$

where $\bar{P}$ is the averaged SP over all instances and $C$. $R_{SP,i}$ is a measure of how much the SP increases or decreases for the pulsed compared to the conventional QA. For $i = max$, we compare with the attained maximum SP $P_{SP,max}$ while for $i = av$ we compare with the mean success probability $P_{SP,av}$.

In Figs. 8(a) and (b) we present the value of $R_{SP,max}$. blue dots connected by blue lines, for increasing number of qubit $n$, for annealing times $t_f = 5h/\Delta$ and $t_f = 3h/\Delta$, respectively. We observe that for increasing $n$ the averaged $R_{SP,av}$ fluctuates around a value of 0.3. We know that the energy gap between the ground and the first excited state decreases as the number of qubits is increased, thus making it harder for the conventional quantum annealing to find the ground state, for fixed annealing time. The fact that the averaged SP $R_{SP,av}$ is positive shows that the pulsed QA outperforms the conventional one. For an annealing time of $t_f = 5h/\Delta$ the maximum relative SP, $R_{SP,av,max}$, shows a slight steady increase as the qubit number $n$ increases. Of course the averaged relative SP, $R_{SP,av,max}$ (blue), suffers from high value of the standard deviation, which is plotted as error bars in Fig. 8, multiplied by 0.5. For an annealing time of $t_f = 3h/\Delta$ the maximum relative success probability seems to saturate as $n$ is increased. Generally, averaging over randomly generated instances is the main source of the large standard deviation.

The averaged relative SP $R_{SP,av}$, green dots connected by green lines, for the averaged values of SP per sampling is presented for increasing $n$ in Figs. 8(a) and (b), for $t_f = 5h/\Delta$ and $t_f = 3h/\Delta$, respectively. We observe that the averaged relative SP $R_{SP,av}$ drops slightly as $n$ increases, for both annealing times. For the case of $t_f = 5h/\Delta$ the drop is slower than the case of $t_f = 5h/\Delta$. This effect is due to the fact that for $t_f = 5h/\Delta$ the conventional QA can reach higher values than the case of $t_f = 3h/\Delta$, larger annealing times reduce Landau-Zener transitions, thus for $t_f = 3h/\Delta$ there is a larger space of improvement when the pulsed QA is applied. The enhancement of the SP is expected to saturate with increasing $n$, for constant success probability. Concretely, increasing $n$ we increase the difficulty of finding the ground state of an
optimization problem for the standard QA, the pulse application can increase the maximum attained value of the SP, after optimizing the pulse parameters.

The data presented in this paper are for up to \( n = 10 \) qubit systems, due to the fact that we use a sampling to find the appropriate pulse parameters that provide an enhancement of the SP. We expect the scaling to sustain for increasing \( n \), due to the fact that we consider a closed system at zero temperature. This approximation is justified due to the short annealing times considered, future QA machines can be used to experimentally test the pulsed QA protocol. The diabatic pulse is introduced in the diagonal part of the Hamiltonian, which determines the energy difference between the ground and the excited states of a superconducting flux qubit, used for a D-Wave like machine. Hence, decreasing the annealing time and a better control of the energy difference parameter, \( \epsilon \), for each qubit can be used to apply our pulsed QA protocol, in the near future.

**IV. SUMMARY AND CONCLUSIONS**

In this paper we start by investigating the success probability (SP) of a single qubit for the case of the conventional quantum annealing (QA) plus a diabatic pulse application. The diabatic pulse application can modulate the SP, by varying the pulse parameters, and enhance it for specific set of parameters. The constructive and destructive interference effect due to the acquired phase during the pulse application is the physical reason for such behavior of the system. Using the transfer matrix method, combined with the sudden approximation, we were able to present such an effect in a semi-analytical manner.

For the multiqubit case, concentrating in the 5 qubit instance iA, we were able to show that the diabatic pulse application can also modulate the SP, by varying the pulse parameters. For specific set of parameters we can increase the SP, compared to the conventional QA. When a number of random (spin-glass) 5 qubit instances is considered, for each one of them we were able to increase the SP, for specific set of pulse parameters. Furthermore, reducing the annealing time, \( t_f \), and based on the physical observations, that the avoided crossing emerges at times around the middle of the annealing time, and the need not to put too much energy on the system, we are able to reduce the sampling space. Furthermore, reducing the annealing time we showed that the averaged SP \( P_{SP,av} > P_{SP} \) for \( 1/2 \) of instances, for \( t_f = 3\hbar/\Delta \).

As the number of qubits, \( n \), is increased we have an enhancement of the SP. This behavior is justified by the fact that increasing \( n \) the energy gap between the ground and excited states decreases, thus reducing the SP for the conventional QA, and giving space for larger enhancement for the SP. This behavior is studied by introducing the relative SP, \( R_{SP} \), and investigate how this quantity behaves with increasing \( n \). The averaged maximum attained SP, per sampling over all instances, per number of qubits shows better performance for the case of pulsed over the conventional QA after optimizing the pulse parameters.

The significance of these results is connected with the fact that breaking away from adiabaticity we are able to present an increase in the SP in shorter annealing times by applying a pulse. We have two sources for this increment. Reduction of the annealing time reduces the interaction of the system with the environment, thus reducing decoherence and dissipation effects. On the same time, reducing the annealing time significantly we can run the appropriate number of samplings, while increasing the SP, within a reasonable time frame. Driving the system to high SP regardless of its difficulty paves the way for considering and understanding the bang-bang protocol for realizing real quantum annealing machines [23].

We use the transfer matrix approach in the framework of the QA for the single qubit and showed good agreement with full numerical results. The next step on similar line of research will be to consider the multi-pulse scenario and extent the transfer matrix approach to multiqubit systems. The main goal of this paper is to present the physics behind the diabatic pulse protocols and show that breaking away from adiabaticity we can increase the SP. Of course, the current technological obstacles for creating a reliable quantum annealing machines needs to be addressed, nevertheless the design of future application based on such protocols will give a significant improvement.

An other route of research is to investigate the introduced diabatic pulse annealing protocol for non-stoquastic Hamiltonians which may accelerate the performance of QA [41–45]. Moreover, a hybrid thermal and quantum optimization annealing scheme has been used to find the ground state of large spin glass systems, the hybrid scheme shows better performance from each scheme individually [46]. An application of a diabatic pulse to both temperature and the transverse field, of the \( \sigma_z \), form, is used to further increase the performance of the QA. These scenarios will be examined in the future.

Recently, D-Wave Systems Inc. announced the implementation of the reverse QA protocol [47], where the initial state is not the usual ground state of the quantum fluctuations, but a specific state which might be closer to the ground state of the problem Hamiltonian. In Ref.[48] it is proposed a hybrid computing method where initially a classical algorithm, like simulated annealing, can be used to find a solution close to the real ground state of a complicated problem. Then, this state can be used as the initial state for applying the reverse QA protocol for a local search in the phase space. The pulsed QA protocol can also be used in a similar manner, to provide if not the real ground state, a state very close to it. This is a different research path to follow in the future for a pulsed QA machine application.

**Acknowledgments**

The authors would like to thank S. Tanaka, M. Maezawa, and K. Imafuku for useful discussions. This work is based on results obtained from a project commissioned by the New Energy and Industrial Technology Development Organization (NEDO). 
Appendix A: Sudden approximation

Following the refs. [31, 32] let’s consider the case where we have the diabatic pulse application at the time \( t_1 \), for times \( t < t_1 \) the Hamiltonian has the form

\[
H_0(t) = \left( \begin{array}{cc} t/t_f & \Delta \\
\Delta & -t/t_f \end{array} \right) = E_G^0(t) \left( \begin{array}{cc} \cos(\theta_0(t)) & \sin(\theta_0(t)) \\
-\sin(\theta_0(t)) & \cos(\theta_0(t)) \end{array} \right),
\]

where we define \( \cos(\theta_0(t)) = (t/t_f) / E_G^0(t) \) and \( \sin(\theta_0(t)) = (1-t/t_f) / E_G^0(t) \). \( E_G^0(t) = \sqrt{(t/t_f)^2 \Delta^2 + (1-t/t_f)^2 \Delta^2} \) is the energy gap between the ground and the excited states. The relevant eigenstates are

\[
|\psi_G^-(t)\rangle = \left( \begin{array}{c} -\sin(\theta_0(t)/2) \\
\cos(\theta_0(t)/2) \end{array} \right), \quad |\psi_G^+(t)\rangle = \left( \begin{array}{c} \cos(\theta_0(t)/2) \\
\sin(\theta_0(t)/2) \end{array} \right),
\]

where \( |\psi_G^-(t)\rangle \) is the ground and \( |\psi_G^+(t)\rangle \) the excited states. Similarly for \( t > t_1 \) we have the Hamiltonian

\[
H_C(t) = \left( \begin{array}{cc} t/t_f & +C \Delta \\
\Delta & -t/t_f \end{array} \right) = E_G^C(t) \left( \begin{array}{cc} \cos(\theta_C(t)) & \sin(\theta_C(t)) \\
-\sin(\theta_C(t)) & \cos(\theta_C(t)) \end{array} \right),
\]

where we defined \( \cos(\theta_C(t)) = (t/t_f + C) / E_G^C(t) \) and \( \sin(\theta_C(t)) = (1-t/t_f) / E_G^C(t) \). \( E_G^C(t) = \sqrt{(t/t_f + C)^2 + (1-t/t_f)^2 \Delta^2} \) is the energy gap between the ground and the excited states in the presence of the pulse of amplitude \( C \). The relevant eigenvectors are

\[
|\psi_C^-(t)\rangle = \left( \begin{array}{c} -\sin(\theta_C(t)/2) \\
\cos(\theta_C(t)/2) \end{array} \right), \quad |\psi_C^+(t)\rangle = \left( \begin{array}{c} \cos(\theta_C(t)/2) \\
\sin(\theta_C(t)/2) \end{array} \right),
\]

where \( |\psi_C^-(t)\rangle \) is the ground and \( |\psi_C^+(t)\rangle \) the excited states.

At the time \( t = t_1 \) due to the sudden transition of the Hamiltonian from the form \( H_0(t) \) to \( H_C(t) \) the system stays in the same state, then the matrix \( N_1 \) can be used to describe the transition from the basis \( |\psi_G^-(t = t_1)\rangle \) to the basis \( |\psi_C^-(t = t_1)\rangle \), thus the relevant state mixing between the ground and the excited states. The elements of the matrix \( N_1 \) are \( \sqrt{p_s} = \sin(\theta_C(t_1) - \theta_0(t_1)) \)

\[
\left( \begin{array}{c} b_0(t_1^-) \\
b_1(t_1^-) \end{array} \right) = \left( \begin{array}{cc} \sqrt{1-p_s} \\
\sqrt{p_s} \end{array} \right) \left( \begin{array}{c} b_0(t_1^-) \\
b_1(t_1^-) \end{array} \right).
\]

Most of the readers would anticipate the physical evolution of the system under the QA protocol to be determined by the Landau-Zener transitions. Then, at the diabatic pulse application times the faulty impression is that we would had a transition to the excited state with probability \( 1 \). This is wrong as we can see from the full numerical data and the explanation given from the transfer matrix approach, using the sudden approximation. The pulse application discussed in this paper is a diabatic process thus the effect of the Landau-Zener physics is irrelevant at \( t_1 \) and \( t_2 \).

[1] T. Kadowaki and H. Nishimori, Phys. Rev. E 58, 5355 (1998).
[2] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, Science 292, 472 (2001).
[3] S. Kirkpatrick, C. D. Gelatt, and M.P. Vecchi, Science 220, 671 (1983).
[4] A. Das and B.K. Chakrabarti, Rev. Mod. Phys.80, 1061 (2008).
[5] R. Harris, M.W. Johnson, T. Lanting, A.J. Berkley, J. Johansson, P. Bunyk, E. Tolkacheva, E. Ladizinsky, N. Ladizinsky, T. Oh, F. Cioca, I. Perminov, P. Spear, C. Enderud, C. Rich, S. Uchaikin, M. C. Thom, E. M. Chapple, J. Wang, B. Wilson, M. H. S. Amin, N. Dickson, K. Karimi, B. Macready, C.J.S. Truncik and G. Rose, Phys. Rev. B 82, 024511 (2010).
[6] M. W. Johnson, M. H. S. Amin, S. Gildert, T. Lanting, F. Hamze, N. Dickson, R. Harris, A. J. Berkley, J. Johansson, P. Bunyk, E. M. Chapple, C. Enderud, J. P. Hilton, K. Karimi, E. Ladizinsky, N. Ladizinsky, T. Oh, I. Perminov, C. Rich, M. C.
