Rapidity Gap in Jet Events at LEP200

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We analyze various perturbative mechanisms for the production of jet events containing rapidity gaps in $e^+e^-$ annihilation at LEP200 energies. We found that the processes $e^+e^- \rightarrow \gamma^*\gamma^*, \gamma^*Z, ZZ, WW \rightarrow 4\text{ jets}$ generate gap events at an observable rate. We point out that LEP200 offers a unique opportunity for the study of gap events due to a smaller background of “fake-gaps”.
I. INTRODUCTION

High-energy collisions involving hadronic final states typically create a large number of particles. A convenient kinematic quantity for the analysis of many-particle events is the rapidity variable

\[ y = \frac{1}{2} \ln \left( \frac{E + P_\parallel}{E - P_\parallel} \right), \quad (1.1) \]

where \( E \) is the energy of a final-state particle and \( P_\parallel \) its longitudinal momentum with respect to a reference axis. Rapidity is a natural longitudinal phase space variable. (See for instance [1,2].) In fact, in \( e^+e^- \) annihilation into jets, one observes a roughly uniform distribution of hadrons per unit rapidity as measured with respect to the jet thrust axis, up to a maximum value \( y_{\text{max}} \sim \ln(E_{\text{cm}}/m_\pi) \), where \( E_{\text{cm}} \) is the total center-of-mass energy. A recent review on the use of the rapidity variable as a kinematic analysis tool can be found in Ref. [2,3].

In hadron-hadron collisions, the distribution of particles in the rapidity variable often contains gaps. That is, absence of particles in a rapidity interval. The existence of a gap may reflect the underlying exchange mechanism. For instance, a gap in diffractive scattering experiments can be interpreted as the exchange of a pomeron [4]. The possibility of using rapidity gaps as triggering signals in high-mass scale physics has motivated recent interest in the study of jet events containing gaps [3,4,5].

In hadron-hadron collisions, the analysis of gap physics is complicated by the presence of spectator partons. These partons can interact with each other during the collision, creating additional particles which may contaminate and spoil a potential rapidity gap. The survival probability [3] of a rapidity gap is an important subject of study for unraveling of high-mass scale physics.

In comparison, lepton-initiated collisions offer a cleaner environment, free of the uncertainties of survival probability considerations. The study of rapidity-gap events in lepton colliders can thus offer important complementary information to the study of gap events in hadron-hadron machines.
Various perturbative mechanisms for generating events containing large rapidity gaps in $e^+e^-$ annihilation have been analyzed at low energies and at the $Z$-peak \cite{6,7,8}. Theoretical calculations indicate that these mechanisms generate gap events at an observable rate at the $Z$-peak. The perturbative QCD mechanisms are shown in Fig. 1. Two color-singlet parton-pairs are produced in the final state. As these parton-pairs move away from each other, the color exchange between the two dipole systems is suppressed, and hadronization is dominantly confined to the phase space near each jet-pair. Hence we expect few or no particles to be produced in the rapidity interval between the two jet-pair systems. An interesting QED mechanism is shown in Fig. 2, where a small-invariant mass photon decays into a high-rapidity hadron system, separated by a large rapidity gap from the other jet-pair. Although this QED mechanism is suppressed due to the smallness of $\alpha_{\text{em}}$, its contribution can become important at low photon invariant-mass. Among all the QCD and QED mechanisms in Fig. 1 and Fig. 2, the QCD $q\bar{q}q\bar{q}$ mechanism of Fig. 1 (a) is the dominant channel for producing gap events at the $Z$-peak. As opposed to typical two-jet events in $e^+e^-$ annihilation which are distributed as $1 + \cos^2 \theta$ in the scattering angle of the thrust axis, the jet-pairs produced by the QCD mechanisms in Fig. 1 follow a distinctive $\sin^2 \theta$ distribution.

Random fluctuations in the final hadron distribution can produce “fake gaps”, which need to be distinguished from the gap events generated by the partonic mechanisms. The undergoing Monte Carlo studies \cite{9} are an important ingredient in the analysis of gap events.

We encounter a unique experimental environment for studying gap events at LEP200, the future upgrade of the present $e^+e^-$ collider at CERN. The four-jet events from the decay of $\gamma^*\gamma^*, \gamma^*Z, ZZ$ and $WW$ boson pairs occur at a rate comparable to the two-jet events from of the decay of single $\gamma^*$ and $Z$ bosons. Hence the “fake gap” background events at LEP200 are expected to be reduced with respect to the situation at the $Z$-peak, where two- and three-jet events dominate over four-jet events.

The study of gap events at LEP200 will also help to elucidate the nature of color flow in quark fragmentation. For the decays of $WW$ into four jets, Gustafson, Peterson and Zerwas \cite{10} have proposed two possible scenarios. In the first scenario, the color flow is
confined to the quark-antiquark pairs of each gauge boson, and the two jet-pairs fragment independently. In the second scenario the string fragmentation occurs between the quark and antiquarks from opposite gauge bosons. A measurement of gap event cross section will therefore constitute an interesting test of the underlying fragmentation mechanism.

In this paper we study the $e^+e^- \rightarrow \gamma^*\gamma^*, \gamma^*Z, ZZ, WW \rightarrow 4 \text{ jets}$ mechanisms for producing gap events at LEP200 energies. Due to the very different kinematic dependence of these channels (different flavor combinations, different invariant-mass resonant regions), we can neglect interference effects to first approximation, and consider the four mechanisms separately. (See also Ref. [11] for a discussion of interference effects in the production of heavy unstable particles.) The previously studied $Z$-peak mechanisms of Fig. 1 and Fig. 2 are suppressed by various kinematic factors (smaller coupling constant or large off-shellness in the gauge boson propagators), and do not have observable effects at LEP200. Our calculations indicate that the dominant channels for producing gap events at LEP200 come from $\gamma^*\gamma^*$ and $\gamma^*Z$ mechanisms. The $WW$ mechanism has a smaller cross section, whereas the $ZZ$ mechanism has a negligible contribution.
II. $\gamma^*\gamma^*$ CASE

The relevant diagrams are shown in Fig. 3. The final color-singlet jet-pairs $(q_1\bar{q}_1)\ (q_2\bar{q}_2)$ are effectively the decay products of the virtual photons. The dominant contribution of these amplitudes comes from the phase space regions where the photon propagators acquire large values, that is, near the photon poles. In fact, in the small photon invariant-mass limit, the cross section can be interpreted as the product of $\sigma(e^+e^-\to\gamma\gamma)$ with the splitting functions of the two photons into respective $q\bar{q}$ pairs. Actually, color-singlet jet-pairs can also be formed by taking the quark from a virtual photon and the antiquark from the other virtual photon. That is, we can also form $(q_1\bar{q}_2)\ (\bar{q}_1q_2)$ color-singlet pairs. However, for gap events the contribution from these “exchanged pairs” are strongly suppressed by the high virtualities carried by the photons, and also suppressed by an additional color factor $1/9$. (This is analogous to the situation discussed in Ref. [8] for QED mechanisms.) Hence, we need only to consider the contributions from the diagrams in Fig. 3. The expression for the differential cross section in the small photon invariant-mass limit is

$$d\sigma = \frac{9\alpha_{em}^4}{4\pi s^2}Q_{q_1}^2 Q_{q_2}^2 \frac{dM_1^2}{M_1^2} \frac{dM_2^2}{M_2^2} \Delta(s, M_1^2, M_2^2)$$

$$dx_1 \left[ x_1^2 + (1 - x_1)^2 \right] dx_2 \left[ x_2^2 + (1 - x_2)^2 \right]$$

$$d\cos \theta \left[ \tan^2(\theta/2) + \cot^2(\theta/2) \right],$$

(2.1)

where $M_1$ is the invariant mass of the $(q_1\bar{q}_1)$ pair, $M_2$ the invariant mass of the $(q_2\bar{q}_2)$ pair, $x_1$ the longitudinal momentum fraction in the first jet-pair carried by $q_1$, $x_2$ the longitudinal momentum fraction in the second jet-pair carried by $q_2$, $\theta$ the scattering angle of the thrust axis, $\sqrt{s}$ the total center-of-mass energy, $Q_{q_1}$ and $Q_{q_2}$ the electric charge of $q_1$ and $q_2$, $\alpha_{em} = e^2/4\pi$, with $e = g\sin\theta_W$ the electromagnetic coupling constant, $g$ the weak coupling constant and $\theta_W$ the weak angle, and finally, the triangular function is defined by

$$\Delta(s, M_1^2, M_2^2) = \sqrt{s^2 + M_1^4 + M_2^4 - 2sM_1^2 - 2sM_2^2 - 2M_1^2M_2^2}.$$  

(2.2)

The differential cross section is sharply peaked in the forward and backward beam direction. The apparent singularities at $\theta \to 0, \pi$ in $\cot(\theta/2)$ and $\tan(\theta/2)$ can be traced back to
the electron propagator in the $t$ and $u$ channel. These singularities are actually cut off by the photon invariant masses $M_1$ and $M_2$. For instance, for $\theta \to 0$ one can show that $t \to -s\theta^2/4 - M_1^2 M_2^2/s$, hence an appropriate cut-off value of $\theta$ is

$$\theta_{\text{cut}} = \frac{2M_1 M_2}{s}. \quad (2.3)$$

An analogous analysis for the $u$ channel propagator leads to the upper limit $\pi - \theta_{\text{cut}}$. Therefore the angular integral should be limited to the interval $\theta \in [\theta_{\text{cut}}, \pi - \theta_{\text{cut}}]$. The final result is not very sensitive to the precise value of $\theta_{\text{cut}}$ because near the singularities the dependence of the angular integral on $\theta_{\text{cut}}$ is logarithmic:

$$d\sigma \sim \int_{\theta_{\text{cut}}} \frac{d\theta}{\theta}. \quad (2.4)$$

The cross section is also sharply peaked in the invariant mass variables $M_1^2$ and $M_2^2$. The apparent singularities in $M_1^2, M_2^2 \to 0$ in Eq. (2.1) are physically cut off by the mass threshold of the $q\bar{q}$ systems. As in Ref. [8], we shall use the values $M_\rho^2, M_\omega^2, M_\phi^2, M_{J/\psi}^2$ and $M_{T}^2$ as representative values for the invariant mass thresholds for the $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$ and $b\bar{b}$ systems. To obtain the total cross section, we integrate over all allowed flavor combinations, noting that allowed flavors depend on the integration variables $M_1^2$ and $M_2^2$. For higher values of $M_1^2$ and $M_2^2$ one has more allowed flavors. The upper bounds of $M_1^2$ and $M_2^2$ are given by the kinematic constraint $\Delta(s, M_1^2, M_2^2) > 0$ in Eq. (2.2) and also by the constraints requiring the existence of a rapidity gap. The integration limits for $x_1$ and $x_2$ are set by the gap condition, too. (See Ref. [7,8].)

In Fig. 4 we present the Lego plot of a typical $\gamma^* \gamma^*$ event. Notice that the two jet-pairs usually have a small invariant mass, generating thus a large rapidity gap in the central region. As in Ref. [7] and Ref. [8], we define $g$ as the rapidity difference between the nearest jet centers from opposite jet-pairs. Since jet fragments typically are scattered over a radius $\sim 0.7$ in the Lego plot, the effective gap is expected to be given by $g_{\text{eff}} \sim g - 1.4$. Virtual photons can easily be transformed into vector mesons like $\rho, \omega, \phi, J/\psi$ and $\Upsilon$. Hence, we also have the special cases where a vector meson recoils against a two-jet system or another vector
meson. In particular, the maximum value of rapidity gap is determined by events where we have two $\rho$ mesons in the final state, that is, $g = \ln(s/M^2_\rho)$. For a total center-of-mass energy of $E = \sqrt{s} = 180$ [GeV], the maximum allowed gap is about 11 units of rapidity. In Fig. 5 we present the integrated cross section of events having a value of nearest jet-center rapidity separation larger than $g$, for different energy levels. We have used $\alpha_{em} \sim 1/128$ in our calculation. In Fig. 6 we plot the integrated cross section as function of energy, for different values of the gap-cut $g$. Given a total integrated luminosity of 500 [pb$^{-1}$] per experiment expected for LEP200 [12], the $\gamma^*\gamma^*$ mechanism should therefore generate gap events at an observable rate.

Actually, the $\gamma^*\gamma^*$ mechanism also contributes at the $Z$-peak energy. In the small invariant mass limit, there is no interference effects between this mechanism and the resonant QCD and QED mechanisms of Fig. 1 and Fig. 2 in the calculation of the total cross sections. (The interference terms vanish upon integration of the azimuthal angles $\phi_1$ and $\phi_2$. See Appendix for the definition of $\phi_1$ and $\phi_2$. A similar cancellation was discussed in Ref. [8].) Therefore, each one of them can be obtained separately. In Fig. 7 we plot the integrated gap cross section for the QCD, QED and the $\gamma^*\gamma^*$ mechanisms at the $Z$ peak ($E = 91.17$ [GeV]). We have used the results from Ref. [7,8], and imposed a representative invariant-mass cut of 30 [GeV] for the QCD and QED jet-pairs. We have used a $Z$-peak total visible cross section of 31 [nb]. The strong coupling constant is taken to be $\alpha_s \sim 0.13$ in the calculations. As can be seen from the figure, the $\gamma^*\gamma^*$ mechanism has small contribution in the low-gap region, but becomes dominant in the large-gap region. Physically, the gap events generated by the $\gamma^*\gamma^*$ are distinct because the jet-pairs tend to be peaked towards the forward and backward directions, whereas in the cases of the resonant QCD and QED mechanisms, the scattering angle is distributed smoothly as $\sin^2 \theta$ and $1 + \cos^2 \theta$. 

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III. γ*Z CASE

The relevant diagrams are shown in Fig. 8. This mechanism is similar to the γ*γ* case, but with a virtual photon replaced by a Z boson. As in the γ*γ* case, we shall not consider (q₁ ¯q₂) and (q₂ q₂) color singlet pairs, since the contribution of these “exchanged pairs” to gap events is suppressed by a large photon virtuality and also by the color factor. Due to the small invariant mass of the virtual photon and the large mass of the Z boson, we expect the decay fragments of γ* to be distributed in the large rapidity region on one side and those of Z in the lower rapidity region on the opposite side. In general we will encounter a lopsided gap as depicted in Fig. 9. A special case would be a vector meson recoiling against the two-jet system from the Z boson. The maximum value of rapidity gap \( g = \ln\left(\frac{s - M_Z^2}{M_\rho M_Z}\right) \) is determined by events where we have a ρ meson recoiling against two jets that are distributed symmetrically with respect to the thrust axis. For a total center-of-mass energy of \( E = \sqrt{s} = 180 \text{ GeV} \), the maximum allowed value of \( g \) is about 5.8 units of rapidity.

The differential cross section in the limit of small photon invariant mass is given by

\[
\frac{d\sigma}{dM_1^2} = \frac{9\alpha_{\text{em}}^2\alpha_W^2}{8\pi} Q_{q_1}^2 \frac{dM_1^2}{M_1^2} \frac{dM_2^2}{M_2^2} \frac{1}{(s - M_2^2)^2 + (\Gamma_Z M_Z)^2} \Delta(s, M_1^2, M_2^2)
\]

\[
dx_1 \, dx_2 \, d\cos \theta \left\{ \left(1 + M_2^2/s^2\right) \left[x_1^2 + (1 - x_1)^2\right] \right. \\
\left[ \left( Q_{q_2}^R Q_{q_2}^R + Q_{q_2}^L Q_{q_2}^L \right) \left(x_2^2 \cot^2(\theta/2) + (1 - x_2)^2 \tan^2(\theta/2)\right) \\
+ \left( Q_{q_2}^R Q_{q_2}^R + Q_{q_2}^L Q_{q_2}^L \right) \left(x_2^2 \tan^2(\theta/2) + (1 - x_2)^2 \cot^2(\theta/2)\right) \right] \\
+ 4Q_{q_2}^2 Q_{q_2}^2 \frac{M_Z^2}{s} \left[x_1^2 + (1 - x_1)^2\right] x_2(1 - x_2) \right\}. \tag{3.1}
\]

In the above expression, \( \alpha_{\text{em}} = e^2/4\pi = g^2 \sin^2 \theta_W/4\pi \) is the electromagnetic coupling constant and \( \alpha_W = g^2/4\pi \) the weak coupling constant, \( M_1 \) is the invariant mass of the \( (q_1 \bar{q_1}) \) pair, \( M_2 \) the invariant mass of the \( (q_2 \bar{q_2}) \) pair, \( M_Z \) and \( \Gamma_Z \) the mass and width of Z boson, \( x_1 \) the longitudinal momentum fraction of the virtual photon carried by \( q_1 \) and \( x_2 \) the longitudinal momentum fraction of the Z boson carried by \( q_2 \) (see Appendix), \( \theta \) the scattering angle of the thrust axis, \( \sqrt{s} \) the total center-of-mass energy, \( Q_{q_1} \) the electric charge
of $q_1$, $Q^e_{q_1}$, $Q^L_{q_2}$, $Q^R_{q_2}$ and $Q^L_{q_2}$ the weak charges of $e$ and $q_2$, where for a fermion of isospin $I_f$ and electric charge $Q_f$ we have

$$Q_f = \begin{pmatrix} Q^L_f \\ Q^R_f \end{pmatrix} = \begin{pmatrix} \sec \theta_W I_f - \sin \theta_W Q_f \\ -\sin \theta_W \tan \theta_W Q_f \end{pmatrix},$$

$$Q_f^2 = Q_f^L + Q_f^R.$$  (3.2)

Due to the narrow width of the $Z$ boson, we can make the approximation

$$\int \frac{dM_Z^2}{(M_Z^2 - M^2)^2 + (\Gamma_M^2)^2} = \frac{\pi}{\Gamma_M M_Z},$$  (3.3)

and replace $M_Z^2$ by $M_Z^2$ elsewhere in Eq. (3.1).

The $\theta$ angle dependence in Eq. (3.1) implies the existence of a forward-backward asymmetry in the cross section, which is also clear from the explicit dependence on the left-handed and right-handed weak charges. The singularities at $\theta \to 0, \pi$ in $\cot(\theta/2)$ and $\tan(\theta/2)$ can be traced back to the electron propagator in the $t$ and $u$ channel. These singularities are physically cut off by the photon invariant mass $M_1$. For $\theta \to 0$ one can show that

$$t \to -(s - M_Z^2)\theta^2/4 - M_1^2 M_Z^2/(s - M_Z^2),$$

therefore an appropriate cut-off angle is

$$\theta_{\text{cut}} \sim \frac{2M_1 M_Z}{s - M_Z^2}. $$  (3.4)

Similarly, by analyzing the $u$ variable, one can show that the upper limit for the $\theta$ integration should be $\pi - \theta_{\text{cut}}$. Define

$$I_\theta(M_1^2) = \int_{\theta_{\text{cut}}}^{\pi - \theta_{\text{cut}}} d\cos \theta \tan^2(\theta/2) = \int_{\theta_{\text{cut}}}^{\pi - \theta_{\text{cut}}} d\cos \theta \cot^2(\theta/2),$$  (3.5)

the expression of the cross section in Eq. (3.1) simplifies to

$$d\sigma = \frac{9\alpha^2}{8} \frac{\alpha_W^2}{8} Q^e_{q_1} Q^2_{q_2} \frac{M_Z}{\Gamma_Z (s - M_Z^2)^2} \frac{dM_1^2}{M_1^2} \Delta(s, M_1^2, M_Z^2)$$

$$dx_1 \left[ x_1^2 + (1 - x_1)^2 \right]$$

$$dx_2 \left\{ (1 + M_Z^4/s^2) \left[ x_2^2 + (1 - x_2)^2 \right] I_\theta(M_1^2) + 8 \left( M_Z^2/s \right) x_2 (1 - x_2) \right\}. $$  (3.6)
To obtain the total cross section, we sum over all allowed flavor combinations. As in the $\gamma^*\gamma^*$ case, we use the values $M_{\rho}^2, M_{\omega}^2, M_{\phi}^2, M_{J/\psi}^2$ and $M_{\Upsilon}^2$ as representative values for the invariant mass thresholds for the $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$ and $b\bar{b}$ systems. The $x_1, x_2$ integrations, as well as the $M_1^2$ integration, are also limited by the gap condition [7,8]. In Fig. 10 we plot the integrated cross section of events having a value of nearest jet-center rapidity separation larger than $g$, for different energy levels. We have used $\alpha_W \sim 1/29.8$ and $\sin^2 \theta_W = 0.233$ in our calculation. In Fig. 11 we plot the integrated cross section as function of energy, for different values of the gap-cut $g$. For the projected integrated luminosity of 500 [pb$^{-1}$] per experiment at LEP200, the $\gamma^*Z$ mechanism therefore also generates an observable number of gap events. For smaller values of $g$ ($g < 4$) the $\gamma^*Z$ mechanism dominates over the $\gamma^*\gamma^*$ mechanism. As discussed previously, the $\gamma^*Z$ mechanism cannot not generate gap events with $g$ larger than $\sim 5.8$, hence for large gaps ($g > 5$) the $\gamma^*\gamma^*$ mechanism dominates.
IV. ZZ CASE

As opposed to the $\gamma^*\gamma^*$ and $\gamma^*Z$ mechanisms, the direct jet-pairs from the decay of $ZZ$ and $WW$ particles are not capable of generating events with large rapidity gaps. This is because at LEP200 the $ZZ$ and $WW$ pairs are produced right above the threshold, thus the two gauge bosons recoil slowly against each other. For instance, at a center-of-mass energy of 200 [GeV], we have only 1.4 units of rapidity separating two recoiling $W$ particles, which means in practice no observable rapidity gap. However, the “exchanged pairs” as those shown in Fig. 1 provide a feasible channel of producing gap events. It is interesting to recall that the exchanged-pair contributions are negligible in the $\gamma^*\gamma^*$ and $\gamma^*Z$ cases due to the large invariant mass carried by the virtual photons, therefore the direct pairs are responsible for gap events there. The situation is exactly reversed for the $ZZ$ and $WW$ cases, where the exchanged pairs are the responsible mechanism.

Let us consider first the case of a $ZZ$ pair produced exactly at threshold. In the center-of-mass frame, each of these static $Z$ particles decays into a back-to-back quark-antiquark pair. In the phase space where $q_1$ becomes collinear with $\bar{q}_2$ (hence $\bar{q}_1$ becomes also collinear with $q_2$), we can form small-invariant-mass color-singlet jet pairs and have a rapidity gap. Naturally, to have an observable cross section, the center-of-mass energy should be larger than the threshold value. As the center-of-mass energy becomes higher and higher, the crossing of a quark from the phase space of a $Z$ boson to the phase space of the opposite $Z$ boson becomes less and less likely. Therefore, we expect the gap event cross section to rise from zero to a maximum value somewhere above the threshold, and then to decrease with increasing center-of-mass energy.

The crossing of particles in the kinematic phase space substantially complicates the calculation of the gap event cross section. This is because the gap condition becomes a unnatural cut in the integration of the various kinematic variables. In particular, as opposed to the $\gamma^*\gamma^*$ case and the $\gamma^*Z$ case, the azimuthal angles $\phi_1$ and $\phi_2$ (see Appendix) can no longer be integrated out analytically.
We compute the helicity amplitudes in terms of spinor products defined in Ref. [13]. After using the narrow width approximation for the $Z$ boson propagators to integrated out the jet-pair invariant masses $M_1^2$ and $M_2^2$, we obtain

$$
d\sigma = \frac{\alpha_W^4}{\pi(32\Gamma_Z M_Z s)^2} \Delta(s, M_1^2, M_2^2) \ dx_1 dx_2 d\phi_1 d\phi_2 d\cos \theta \sum_{q_1,q_2} \sum_{H_{q_1},H_{q_2}} Q_e^H e^{-4} Q_{q_1}^{H_1} e^{-2} Q_{q_2}^{H_2} e^{-2} |T(H_e, H_{q_1}, H_{q_2})|^2, \tag{4.1}
$$

where the first sum is over all allowed quark flavors, and the second sum is over all helicities of the electron and the final quarks $q_1$ and $q_2$. We have included in the formula an $1/2$ factor to account for the double counting of flavors in the summation of $q_1$ and $q_2$. The various charges in Eq. (4.1) are the weak charges of the electron, $q_1$ and $q_2$, as defined in the previous section. (We use indifferently $(R, L)$ and $(+, -)$ to denote right and left helicity.)

There are eight helicity amplitudes, but half of them are needed since the other half can be obtained by simple helicity conjugation:

$$
T(+++) = \frac{4}{t} [k_1 q_1 \langle k_2 q_2 \rangle \left\{ -|k_1 q_2 \langle k_1 \bar{q}_1 \rangle + |q_1 q_2 \langle q_1 \bar{q}_1 \rangle \right\} 
+ \frac{4}{u} [k_1 q_2 \langle k_2 \bar{q}_1 \rangle \left\{ -|k_1 q_1 \langle k_1 \bar{q}_2 \rangle + |q_1 q_2 \langle q_1 \bar{q}_2 \rangle \right\}],
$$

$$
T(+-+) = \frac{4}{t} [k_1 q_1 \langle k_2 q_2 \rangle \left\{ -|k_1 \bar{q}_2 \langle k_1 \bar{q}_1 \rangle + |q_1 \bar{q}_2 \langle q_1 \bar{q}_1 \rangle \right\} 
- \frac{4}{u} [k_1 \bar{q}_2 \langle k_2 \bar{q}_1 \rangle \left\{ |k_1 q_1 \langle k_1 \bar{q}_2 \rangle + |q_1 q_2 \langle q_1 \bar{q}_2 \rangle \right\}],
$$

$$
T(+-+) = \frac{4}{t} [k_1 \bar{q}_1 \langle k_2 q_2 \rangle \left\{ |k_1 q_2 \langle k_1 \bar{q}_1 \rangle + |q_1 q_2 \langle q_1 \bar{q}_1 \rangle \right\} 
+ \frac{4}{u} [k_1 q_2 \langle k_2 \bar{q}_1 \rangle \left\{ -|k_1 \bar{q}_1 \langle k_1 \bar{q}_2 \rangle + |q_1 \bar{q}_2 \langle q_1 \bar{q}_2 \rangle \right\}],
$$

$$
T(+-+) = \frac{4}{t} [k_1 \bar{q}_1 \langle k_2 q_2 \rangle \left\{ -|k_1 \bar{q}_2 \langle k_1 \bar{q}_1 \rangle + |q_1 \bar{q}_2 \langle q_1 \bar{q}_1 \rangle \right\} 
+ \frac{4}{u} [k_1 q_2 \langle k_2 \bar{q}_1 \rangle \left\{ -|k_1 \bar{q}_1 \langle k_1 \bar{q}_2 \rangle + |q_1 \bar{q}_2 \langle q_1 \bar{q}_2 \rangle \right\}]. \tag{4.2}
$$

The spinor products in these amplitudes can be evaluated numerically with the formulas given in the Appendix.

Due to the crossing of quark pairs, the thrust axis in general is not aligned with the gauge boson direction. We will denote the scattering angle of the thrust axis by $\Theta$, which in general differs from the gauge boson $\theta$ angle that appears in the expression of the cross
section of Eq. (4.1). In the Monte Carlo integration of the cross section in Eq. (4.1), we only retain those events containing a rapidity gap larger than the gap cut \( g \). The result is plotted in Fig. [13]. We notice that large gap events from the ZZ mechanism have cross sections much smaller than 0.01 [pb]. Hence, this mechanism is not expected to have observable effects at the projected luminosity of LEP200. In Fig. [14] we plot the angular distribution of the cross section for \( E = 192 \) [GeV] and \( g = 3 \). We see that the cross section has two smooth humps, as opposed to the \( \gamma^*\gamma^* \) and \( \gamma^*Z \) cases where the cross section has two sharp peaks \( \sim 1/\theta \) in the forward and backward beam directions.

V. **WW Case**

The kinematics of the WW case is similar to the ZZ case. Diagrammatically, the production of WW boson pairs is interesting because it involves the triple-gauge-boson vertex of the electroweak theory. The Feynman diagrams of this mechanism is shown in Fig. [13]. The \( u_1, \bar{u}_2, \) and \( d_1, d_2 \) quarks represents any up-type \((u, c)\) and down-type \((d, s, b)\) quarks and antiquarks. As explained in the previous section, at LEP200 the direct-pair contributions are not capable of producing events with large rapidity gaps, therefore we need only to focus on the exchanged-pair contributions of Fig. [13]. Like in the ZZ case, we also expect the gap event cross section to reach a maximum somewhere above the threshold, since at higher energies there is less phase space for the formation of exchanged pairs.

The differential cross section for the WW case is given by

\[
d\sigma = \frac{\alpha_W^4}{512\pi(\Gamma_Z M_Z s)^2} \Delta(s, M_W^2, M_W^2) \ dx_1 dx_2 d\phi_1 d\phi_2 d\cos \theta \left\{ |T^+_1|^2 \left[ \frac{1}{s-M_Z^2+i\Gamma_Z M_Z} \right]^2 \sin^4 \theta_W \\
+ \left[ \left( \frac{\sin^2 \theta_W}{s} + \frac{1}{s-M_Z^2+i\Gamma_Z M_Z} \right) T^-_1 - \frac{1}{2} T^-_2 \right]^2 \right\},
\]

where \( M_W \) and \( \Gamma_Z \) are the mass and width of the W boson, \( \Delta \) as defined in Eq. (2.2). We have summed over all allowed quark flavors and made following approximation for the
summation over the Cabbibo-Kobayashi-Maskawa matrix elements (See for instance Ref. [14]):

$$\sum_{i=\mu,\tau,\ell} \sum_{j=d,s,b} |V_{ij}|^2 \sim 2. \tag{5.2}$$

The $T_1^+$ and $T_1^-$ amplitudes come from the $s$-channel diagram in Fig. 15 (a), and are given by

$$T_1^+ = 4 [\bar{d}_1 \bar{u}_2] \langle d_2 u_1 \rangle \left\{ [k_1 u_1] \langle u_1 k_2 \rangle + [k_1 \bar{d}_1] \langle \bar{d}_1 k_2 \rangle \right\}$$

$$+ 4 [k_1 \bar{u}_2] \langle d_2 k_2 \rangle \left\{ [\bar{d}_1 \bar{u}_2] \langle \bar{u}_2 u_1 \rangle + [\bar{d}_1 d_2] \langle d_2 u_1 \rangle \right\}$$

$$- 4 [k_1 \bar{d}_1] \langle u_1 k_2 \rangle \left\{ [\bar{u}_2 u_1] \langle u_1 d_2 \rangle + [\bar{u}_2 \bar{d}_1] \langle \bar{d}_1 d_2 \rangle \right\},$$

$$T_1^- = 4 [\bar{d}_1 \bar{u}_2] \langle d_2 u_1 \rangle \left\{ [k_2 u_1] \langle u_1 k_1 \rangle + [k_2 \bar{d}_1] \langle \bar{d}_1 k_1 \rangle \right\}$$

$$+ 4 [k_2 \bar{u}_2] \langle d_2 k_1 \rangle \left\{ [\bar{d}_1 \bar{u}_2] \langle \bar{u}_2 u_1 \rangle + [\bar{d}_1 d_2] \langle d_2 u_1 \rangle \right\}$$

$$- 4 [k_2 \bar{d}_1] \langle u_1 k_1 \rangle \left\{ [\bar{u}_2 u_1] \langle u_1 d_2 \rangle + [\bar{u}_2 \bar{d}_1] \langle \bar{d}_1 d_2 \rangle \right\}, \tag{5.3}$$

where we have used the symbol $u_1$, $\bar{d}_1$, $\bar{u}_2$ and $d_2$ to denote the final state quarks as well as their respective final momenta. The $T_2^-$ amplitude is given by

$$T_2^- = 4 [u_1 k_1] \langle k_2 \bar{u}_2 \rangle \left\{ [u_1 \bar{d}_1] \langle d_2 u_1 \rangle - [k_1 \bar{d}_1] \langle d_2 k_1 \rangle \right\}. \tag{5.4}$$

The spinor products in these expressions can be evaluated with the formulas given in the Appendix.

As in the ZZ case, the thrust axis angle $\Theta$ in general differs from the gauge boson angle $\theta$ in the expression of the cross section of Eq. (5.1). In the Monte Carlo integration of the cross section in Eq. (5.1) we only retain those events with a rapidity gap larger than the gap cut value $g$. The resulting cross section is given in Fig. 16. We see from the plot that the $WW$ mechanism can contribute somewhat to the production of gap events at LEP200. For gap cuts $g$ between 3 and 4 units of rapidity, the maximum cross section is reached around 5 to 10 [GeV] above the threshold. In Fig. 17 we plot the angular distribution of the cross section for $E = 170$ [GeV] and $g = 3$. As in the ZZ case, the cross section shows a
double-hump structure, with moderate depletion near the beam direction and the direction perpendicular to it.

VI. CONCLUSION

We have analyzed in this paper various perturbative mechanisms for the decay of electroweak gauge bosons into jet events containing a rapidity gap. At LEP200 energies the dominant contributions come from the $\gamma^*\gamma^*$ and $\gamma^*Z$ mechanisms. To fix an idea, for a gap cut $g = 4$, $\gamma^*Z$ is the dominant contribution with a cross section $\sigma \sim 0.04$ [pb], and the $\gamma^*\gamma^*$ mechanism contributes another $\sigma \sim 0.015$ [pb]. For smaller gap cuts the cross section is larger. However, if the gap cut is too small, the background “fake gap” events from random fluctuation of final-state hadron fragments can be significant. For the $\gamma^*\gamma^*$ events the jet fragment distribution typically is peaked towards the forward and backward beam direction. For the $\gamma^*Z$ process one would typically observe a forward or backward jet system recoiling against a low-rapidity two-jet system on the opposite side, hence have a lopsided rapidity gap. It is also likely to observe the special cases where the virtual photons are converted into a vector mesons instead of jet-pairs.

The $WW$ mechanism may also contribute to the production of gap events. Our analysis indicates that the maximum event cross section is reached 5 to 10 [GeV] above the threshold. These events have a relatively smooth distribution in the scattering angle. The measurement the experimental gap event cross section here will help to test the hypothesis that the color-flow is confined to the collinear jets. Finally, the $ZZ$ mechanism has a very small cross section at LEP200, and we do not expect it to contribute in practice.

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All the production mechanisms in this paper involve the decay of two gauge bosons into four final-state quarks. The graphic representation of the particle momenta is shown in Fig. 18. We denote the momenta of the incoming positron and electron by $k_1$ and $k_2$, respectively, and we will use $q_1, \bar{q}_1, q_2, \bar{q}_2$ to denote the final-state quarks as well as their momenta. The momenta carried by the first and second gauge bosons are denoted by $P_1$ and $P_2$, Therefore we have

$$P_1 = q_1 + \bar{q}_1,$$

$$P_2 = q_2 + \bar{q}_2.$$  \hspace{1cm} (A1)

That is, the ($q_1\bar{q}_1$) pair is the decay products of the first gauge boson, and the ($q_2\bar{q}_2$) pair is the decay products of the second gauge boson.

At the center-of-mass frame, the momenta of the four quark jets can be parametrized in terms of seven kinematic variables: $M^2_1, M^2_2, x_1, x_2, \theta, \phi_1, \phi_2$; where $M^2_1 = P^2_1, M^2_2 = P^2_2$ are the invariant masses of the ($q_1\bar{q}_1$) and ($q_2\bar{q}_2$) pairs, $x_1$ and $x_2$ the longitudinal momentum fractions of $q_1$ within the ($q_1\bar{q}_1$) pair and $q_2$ within the ($q_2\bar{q}_2$), $\theta$ is the gauge-boson scattering angle, and $\phi_1, \phi_2$ the azimuthal angles of $q_1$ and $q_2$ with respect to the gauge boson direction. Notice that $\theta$ coincides with the thrust axis scattering angle for the $\gamma^*\gamma^*$ and $\gamma^*Z$ mechanisms. However, for the $ZZ$ and $WW$ mechanisms the thrust angle $\Theta$ defined by the direction of $q_1 + \bar{q}_2$ is different from $\theta$ due to the crossing of jet-pairs. The following is the explicit representation of the initial and final four-vectors:

$$k_1 = \frac{\sqrt{s}}{2} \left[ 1; \hat{k} \right],$$

$$k_2 = \frac{\sqrt{s}}{2} \left[ 1; -\hat{k} \right],$$

$$q_1 = \frac{1}{2} \left[ x_1 P^+_1 + \bar{x}_1 P^-_1; \ 2 \sqrt{x_1 \bar{x}_1} M_1 \hat{p}_{1\perp} + (x_1 P^+_1 - \bar{x}_1 P^-_1) \hat{P} \right],$$

$$\bar{q}_1 = \frac{1}{2} \left[ \bar{x}_1 P^+_1 + x_1 P^-_1; \ -2 \sqrt{x_1 \bar{x}_1} M_1 \hat{p}_{1\perp} + (\bar{x}_1 P^+_1 - x_1 P^-_1) \hat{P} \right],$$

$$q_2 = \frac{1}{2} \left[ x_2 P^+_2 + \bar{x}_2 P^-_2; \ 2 \sqrt{x_2 \bar{x}_2} M_2 \hat{p}_{2\perp} - (x_2 P^+_2 - \bar{x}_2 P^-_2) \hat{P} \right].$$
\[ q_2 = \frac{1}{2} \left[ \bar{x}_2 P_2^+ + x_2 P_2^- - 2\sqrt{x_2 \bar{x}_2} M_2 \hat{p}_{2\perp} - (\bar{x}_2 P_2^+ - x_2 P_2^-) \hat{P} \right]. \quad (A2) \]

It is convenient to choose the \( z \) axis along the direction of the momentum of the first gauge boson \( P_1 \). This will facilitate the calculation of spinor products later. Using the coordinate system shown in Fig. 18, we have

\[
\hat{k} = \cos \theta \hat{z} - \sin \theta \hat{x},
\]

\[
\hat{p}_{1\perp} = \cos \phi_1 \hat{x} + \sin \phi_1 \hat{y},
\]

\[
\hat{p}_{2\perp} = \cos \phi_2 \hat{x} + \sin \phi_2 \hat{y},
\]

\[
\hat{P} = \hat{z}. \quad (A3)
\]

The quantities \( x_1, \bar{x}_1, x_2, \bar{x}_2, P_1^+, P_1^-, P_2^+, P_2^- \) satisfy the following constraints

\[
x_1 + \bar{x}_1 = 1,
\]

\[
x_2 + \bar{x}_2 = 1,
\]

\[
P_1^+ P_1^- = M_1^2,
\]

\[
P_2^+ P_2^- = M_2^2. \quad (A4)
\]

From conservation of energy and momentum, we can obtain \( P_1^+, P_1^-, P_2^+, P_2^- \) explicitly in terms of \( M_1^2, M_2^2 \) and \( s \)

\[
P_1^+ = \frac{1}{2\sqrt{s}} \left[ s + M_1^2 - M_2^2 + \Delta(s, M_1^2, M_2^2) \right],
\]

\[
P_1^- = \frac{1}{2\sqrt{s}} \left[ s + M_1^2 - M_2^2 - \Delta(s, M_1^2, M_2^2) \right],
\]

\[
P_2^+ = \frac{1}{2\sqrt{s}} \left[ s - M_1^2 + M_2^2 + \Delta(s, M_1^2, M_2^2) \right],
\]

\[
P_2^- = \frac{1}{2\sqrt{s}} \left[ s - M_1^2 + M_2^2 - \Delta(s, M_1^2, M_2^2) \right], \quad (A5)
\]

where \( \Delta(s, M_1^2, M_2^2) \) is the triangular function defined in Eq. (2.2).

The spinor products (see Ref. [13] for a review on the helicity method) between the various four-vectors can be computed explicitly. We have
All other spinor products can be obtained by the antisymmetry operation or by helicity conjugation:

\[
\langle ab \rangle = -\langle ba \rangle
\]

\[
[ab] = -[ba] = \langle ba \rangle^*
\]  

(A7)
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FIGURES

FIG. 1. Perturbative QCD mechanisms for generating rapidity-gap events at the Z peak. The dashed lines indicate that the produced partons are in color-singlet state. (a) Two final-state quark-antiquark pairs. (b) A quark-antiquark jet pair and a two-gluon jet pair.

FIG. 2. A QED mechanism for generating rapidity-gap events at the Z peak. The dashed lines indicate that the produced partons are in color-singlet state.

FIG. 3. The $e^+e^- \rightarrow \gamma^*\gamma^* \rightarrow q\bar{q}q\bar{q}$ mechanism for generating gap events.

FIG. 4. A typical Lego plot for a gap event from the $\gamma^*\gamma^*$ mechanism. The physically observed gap is expected to be $g_{\text{eff}} \simeq g - 1.4$ due to the spreading of hadron fragments around each jet.

FIG. 5. Gap event cross section for the $\gamma^*\gamma^*$ mechanism, as function of the gap-cut $g$ and for two different values of energies.

FIG. 6. Gap event cross section for the $\gamma^*\gamma^*$ mechanism, as function of the energy and for different values of the gap-cut $g$.

FIG. 7. The $\gamma^*\gamma^*$ contribution to the gap-event cross section at the Z-peak (solid line). Also plotted are the contributions from the QCD (dashed line) and QED (dotted line) mechanisms (see Fig. 1, Fig. 2 and also Ref. [8].)

FIG. 8. The $e^+e^- \rightarrow \gamma^*Z \rightarrow q\bar{q}q\bar{q}$ mechanism for generating gap events at LEP200.

FIG. 9. A typical Lego plot for a gap event from the $\gamma^*Z$ mechanism. Notice that the rapidity gap is shifted towards the virtual photon side.

FIG. 10. Gap event cross section for the $\gamma^*Z$ mechanism, as function of the gap-cut $g$ and for two different values of energies.
FIG. 11. Gap event cross section for the $\gamma^* Z$ mechanism, as function of the energy and for different values of the gap-cut $g$.

FIG. 12. The $e^+ e^- \rightarrow ZZ \rightarrow q\bar{q}q\bar{q}$ mechanism for generating gap events at LEP200. Notice the exchange of quarks and antiquarks from different gauge bosons to form color-singlet jet-pairs.

FIG. 13. Gap event cross section for the $ZZ$ mechanism, as function of the energy and for different values of the gap-cut $g$.

FIG. 14. Angular distribution of gap event cross section for the $ZZ$ mechanism, where $\Theta$ is the thrust axis scattering angle.

FIG. 15. The $e^+ e^- \rightarrow WW \rightarrow q\bar{q}q\bar{q}$ mechanism for generating gap events at LEP200. Notice the exchange of quarks and antiquarks from different gauge bosons to form color-singlet jet-pairs.

FIG. 16. Gap event cross section for the $WW$ mechanism, as function of the energy and for different values of the gap-cut $g$.

FIG. 17. Angular distribution of gap event cross section for the $WW$ mechanism, where $\Theta$ is the thrust axis scattering angle.

FIG. 18. Initial and final momenta involved in $e^+ e^- \rightarrow q\bar{q}q\bar{q}$.
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