Renormalization of quark axial current in the chiral potential model

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Non-conserved composite operators like the quark axial current have divergent matrix elements therefore must be renormalized. We explore how this can be done in quark model calculations where the systematic procedure of dimensional regularization and minimal subtraction is not applicable.

We propose a most natural and convenient regularization scheme of cutting the intermediate quark states over which we sum in loop diagram calculations at a certain energy. We show that this scheme works perfectly for the quark axial current and we obtain the quark spin contribution to the proton spin: \( \Delta_u = 0.82, \Delta_d = -0.43, \Delta_s = -0.10 \), which is in excellent agreement with experiments.

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The expression nucleon spin “crises” denotes the findings of the European Muon Collaboration (EMC)\textsuperscript{[1]} that a small proportion of the nucleon spin is carried by the quark spin and that strange quark polarizes significantly in the nucleon. This has been under hot debate for over ten years but one obtained not yet a fully satisfactory description (for a review of the nucleon spin problem, see, e.g.,\textsuperscript{[2]}). It should be emphasized that the “crises” is not for the fundamental theory of quantum chromodynamics (QCD), in the viewpoint of which the nucleon is a complicated object of quarks and gluons and quarks do not necessarily carry most of the nucleon spin. The “crises” is, however, for the naive SU(6) quark model which is quite successful in many aspects but nevertheless attributes all the nucleon spin to constituent quarks. To explore whether this “crises” is real, i.e., whether the SU(6) model could be taken as a good lowest order approximation for the nucleon, a natural way is to start from the SU(6) wavefunction and study whether we can explain the experimental result of the nucleon spin content by going to higher orders.

In the past years there has been countless work using quark models along this direction (for references see\textsuperscript{[3]}), but the problem is not really solved. The obstacle is that the quark axial current, which is the operator for defining the quark spin contribution to nucleon spin, is a non-conserved composite operator. Therefore when we go to higher orders divergent matrix element will be encountered and the quark axial current must be renormalized. But unfortunately, the usual renormalization schemes are not applicable in quark model calculations, where instead of divergent loop integrations over the continuous momentum, we encounter divergent summations over the discrete quark excited states whose wavefunctions are obtained numerically. To explore how one can renormalize a composite operator in quark models and what would be the result of the renormalized quark axial charge (i.e., the quark spin contribution to nucleon spin) are the aims of this paper.

In the following we will first construct the bare matrix element of the quark axial current and demonstrate its divergence, then we explore how we can renormalize the bare quantity by systematically subtracting a divergent part and obtain a (finite) physical result.

The quark spin contribution to the nucleon spin is defined as the quark axial charge of the nucleon:

\[
\langle ps|\bar{u}_q\gamma^5\bar{u}_q|ps\rangle \equiv \bar{u}_ps\gamma^5u_{ps} \cdot \Delta_q,
\]

where \( q = u, d, s \) and \( u_{ps} \) is the nucleon spinor. An equivalent but more suitable expression for model calculation is

\[
\Delta_q = \frac{\langle p + |d^3x\bar{q}_q\gamma^5q_q|p+\rangle}{\langle p + |p+\rangle}.
\]

Here \( |p+\rangle \) is a nucleon state with positive momentum and polarization along the third direction. We are going to adopt Eq.\textsuperscript{[4]} to pursue a perturbative calculation of \( \Delta_q \) in a chiral potential model. Our model Lagrangian is

\[
\mathcal{L} = \bar{\psi}(i\not\!\partial - S(r) - \gamma^0V(r))\psi - \frac{1}{2F_\pi}\bar{\psi}[S(r)(\sigma + i\gamma^5\lambda^i\phi_i) + (\sigma + i\gamma^5\lambda^i\phi_i)S(r)]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}m_{\phi_i}^2\phi_i^2.
\]

The model Lagrangian is derived from the \( \sigma \) model in which meson fields are introduced to restore chiral symmetry\textsuperscript{[5]}. The flavor and color indices for the quark field \( \psi \) are suppressed; the scalar term \( S(r) = cr + m \) represents the linear scalar confinement potential \( cr \) and the quark mass matrix \( m \); \( V(r) = -\alpha/r \) is the Coulomb type vector potential and \( F_\pi = 93\text{MeV} \) is the pion decay constant. \( \sigma \) and \( \phi_i \) (\( i \) runs from 1 to 8) are the scalar and pseudoscalar meson fields, respectively and \( \lambda_i \) are the Gell-Mann matrices. The quark-meson interaction term of Eq.\textsuperscript{[4]} is symmetrized since the mass matrix \( m \) does not commute with all \( \lambda_i \) for different quark masses.

At zeroth order the nucleon is taken as the usual SU(6) three-quark ground state of the Hamiltonian
\[ H_q = \int d^3x \psi^\dagger [\vec{\sigma} \cdot \frac{1}{i} \partial + \beta S(r) + V(r)]\psi. \]  

The diagrams for the numerator and denominator of Eq. (3) up to second order are shown in Figs. 1 and 2 respectively.

\[ \begin{array}{c}
\begin{array}{c}
A \hspace{1cm} B \\
\hline
\hline
\hline
\end{array}
\end{array} \]  

\[ \begin{array}{c}
\begin{array}{c}
C \hspace{1cm} D \\
\hline
\hline
\hline
\end{array}
\end{array} \]  

FIG. 1. Feynman diagrams for the matrix element \( \langle N | f d^3x \bar{\psi}^\gamma \psi | N \rangle \) up to second order; a cross on the quark line denotes the quark axial vertex; A is of the zeroth order, B is the renormalization counter term, C and D are vertex and exchange diagrams respectively. The meson line in C can be a \( \pi, \eta \), or \( \sigma \) (while the intermediate quark is \( u \) or \( d \)), or a \( K \) (while the intermediate quark is \( s \)); the meson line in D can only be a \( \pi, \eta \) or \( \sigma \).

FIG. 2. Feynman diagrams for the normalization \( \langle N | N \rangle \) up to second order; A is of the zeroth order which is simply unity, B is the meson exchange diagram. The meson line in B is a \( \pi, \eta \), or \( \sigma \).

We first discuss how to determine the renormalization constant \( Z_2 \). The mass-shell renormalization scheme is not applicable here, since our unperturbed quark basis are confined wavefunctions. But we can still use the charge renormalization condition. The conserved electromagnetic current of the Lagrangian of Eq. (3) is \( j^\mu = \sum_q \bar{q} j^\mu_q + q j^\mu_\phi \), where \( j^\mu_q \) and \( j^\mu_\phi \) are the quark and meson current respectively:

\[ j^\mu_q = Q_q \bar{\psi}_q \gamma^\mu \psi_q, \]
\[ j^\mu_\phi = \epsilon(\phi_1 \partial^\mu \phi_2 - \phi_2 \partial^\mu \phi_1 + \phi_4 \partial^\mu \phi_5 - \phi_5 \partial^\mu \phi_4). \]  

The charge renormalization condition is to require that for a quark state

\[ \langle q | \int d^3x j^0(x) | q \rangle = Q_q. \]  

Up to second order this is shown in Fig. 3.

\[ Q_q = \]  

\[ \begin{array}{c}
\begin{array}{c}
A \hspace{1cm} (Z_2 - 1) \\
\hline
\hline
\hline
\end{array}
\end{array} \]  

\[ \begin{array}{c}
\begin{array}{c}
B \hspace{1cm} C \hspace{1cm} D \\
\hline
\hline
\hline
\end{array}
\end{array} \]  

FIG. 3. Charge renormalization condition; a cross on the quark or meson line denotes the zeroth component of the vector current vertex.

By computing Figs. 3C and 3D we can determine the renormalization constant \( Z_2 \), which is then to be used in Fig. 1B.

Next we remark that at zero momentum transfer, the exchange diagram Fig. 1D is actually the product of Figs. 1A and 2B, therefore the sum of Figs. 1A and 1D over the normalization Figs. 2A and 2B is just Fig. 1A. (However this is not true when we calculate the axial form factor at finite momentum transfer.) Thus only Fig. 1C is left to be evaluated together with Fig. 3C and 3D.

The essential ingredients needed for calculating these diagrams are the quark and meson propagators. The meson propagator given by the Lagrangian of Eq. (3) is the free propagator:

\[ \Delta_{ij}(x_1, x_2) = \frac{i}{(2\pi)^4} \int d^4q \frac{\delta_{ij} e^{-iq(x_1-x_2)}}{q^2 - m_i^2 + i\epsilon}. \]  

Since the non-perturbative confinement is included in \( H_q \) the quark propagator has to be obtained numerically, and in practise we have to work with time-ordered perturbation theory. We write the solution of \( H_q \) as

\[ \psi(x) = \sum_\alpha u_\alpha(x) a_\alpha + \sum_\beta v_\beta(x) b_\beta^\dagger, \]  

where \( u_\alpha(x) = e^{-iE_\tau_\alpha} u_\alpha(\bar{x}) \tau_\alpha, v_\beta(x) = e^{iE_\tau_\beta} v_\beta(\bar{x}) \tau_\beta; \tau \) is the flavor wavefunction and \( u_\alpha(\bar{x}) \) and \( v_\beta(\bar{x}) \) are the spatial wavefunctions. The quark propagator is then

\[ D(x_1, x_2) = \theta(t_1 - t_2) \sum_\alpha u_\alpha(x_1) \bar{u}_\alpha(x_2) - \theta(t_2 - t_1) \sum_\beta v_\beta(x_1) \bar{v}_\beta(x_2). \]  

It can be shown by carrying out the time and energy integration that apart from an isospin factor, Figs. 3C and 3D yield the same expression. Therefore we define for Figs. 3C and 3D the pure space-time amplitudes:

\[ B_{\phi} \equiv \frac{1}{F_\pi^2} \int d^3x d^4x_1 d^4x_2 \Delta(x_2, x_1) \times \int d^4x \bar{u}_f(x_2) \Gamma_\phi D(x_2, x) \gamma^\mu D(x, x_1) \Gamma_\phi u_i(x_1), \]  

where \( u_i \) and \( u_f \) are the initial and final quark state respectively, the vertex function \( \Gamma_{\pi,K,\eta} = S(r) \gamma^5 \) and \( \Gamma_\sigma = -iS(r) \). All flavor wavefunctions are dropped out.
and it should be understood that for $\phi = K$ the intermediate quark is the $s$ quark and otherwise is the $u$ or $d$ quark. Accordingly for Fig. 1C we define

$$A_\phi \equiv \frac{1}{F^2} \int dx^4 \bar{x}_1 \gamma^5 D(x_1) \Lambda(x_1) \times \bar{u}_f(x_2)\Gamma_\phi D(x_2, x)\gamma^5 D(x, x_1)\Gamma_\phi u_1.$$  \hspace{1cm} (11)

Now we can express the renormalization constant $Z_2$ and the axial charge $\Delta_\eta$ in terms of $A_\phi$ and $B_\phi$ multiplied by spin and isospin factors which are calculated straightforwardly:

$$Z_2^{u,d} = 1 - (3B_u + 2B_K + \frac{1}{3}B_\eta + B_\sigma),$$

$$Z_2^s = 1 - (B_K + \frac{4}{3}B_\eta + B_\sigma),$$

$$\Delta_u = \frac{4}{3}fRZ_2^{u} + \frac{2}{3}A_u + \frac{4}{3}A_\eta + \frac{4}{3}A_\sigma,$$

$$\Delta_d = \frac{1}{3}fRZ_2^{u} + \frac{7}{3}A_u - \frac{1}{9}A_\eta - \frac{1}{3}A_\sigma,$$

$$\Delta_s = 2A_K.$$  \hspace{1cm} (12)

Another useful relation is for the nucleon axial charge

$$g_A = \Delta_u - \Delta_d = \frac{5}{3}fRZ_2^{u} - \frac{5}{3}A_u + \frac{5}{9}A_\eta + \frac{5}{3}A_\sigma.$$  \hspace{1cm} (13)

In Eqs. (12)-(14) we have assumed equal masses for $u, d$ and for $\pi^0, \pi^+$, therefore $Z_2^s = Z_2^{u,d}$ which is just a statement of SU(2) symmetry (If SU(3) symmetry is unbroken $Z_2^s$ would also be the same as $Z_2^{u,d}$; 5/3$fR$ is the zeroth order values of $g_A$ and $fR$ is a relativistic reduction factor.

Since $A_\phi$ and $B_\phi$ correspond to loop diagrams, they would naturally be divergent. If the quark axial current were conserved, then the divergence of $A_\phi$ and $B_\phi$ would automatically cancel in Eq. (13) and we get a finite result for $\Delta_\eta$. This is the phenomenon that conserved operators do not need extra renormalization besides the usual renormalization of mass, charge and wavefunction. However the quark axial current is not conserved, the divergences of $A_\phi$ and $B_\phi$ will not cancel and the naively obtained $\Delta_\eta$ in Eq. (13) is divergent, which is just the general case that a composite operator has divergent matrix element and needs extra renormalization, i.e., we are going to subtract a divergent piece from $A_\phi$ and $B_\phi$ simultaneously and leave a finite part in Eq. (13). This finite leftover would depend on how we regularize $A_\phi$ and $B_\phi$ and how much is to be subtracted as the divergent part. This is the renormalization scheme dependence.

In the usual plane-wave perturbation theories, we have a most powerful and systematic renormalization scheme of dimensional regularization and minimal subtraction (MS) or modified minimal subtraction (MS), but it is evidently not applicable here. We must find a proper renormalization scheme for the quark model calculations. The divergent integration over the (continuous) momentum in the plane-wave perturbation theories is here contained in the summation over the (discrete) quark intermediate states. Thus we propose to regularize by cutting the summation at a certain energy, which is analogous to the lattice regularization by a finite lattice spacing. In the following we explore how this scheme works in practice.

Our model parameters are listed in Table I. As we demonstrated in a recent paper, $A_\phi$ is actually very insensitive to the model parameters in the above described energy-cutoff regularization scheme. This property is also found for $B_\phi$. Therefore we do not take too much effort in choosing the parameters except for a fit to the nucleon and $\Delta$ masses with meson exchange potentials.

Figs. 4 and 5 give the numerical results of $B_\phi$ and $A_\phi$ as a function of the maximum energy up to which one sums the intermediate quark states. The divergences are clearly seen. The problem is now how to determine the cutoff. In general the cutoff might vary from one diagram to another, without any guidance we would be at lost. Fortunately, besides $\Delta_s = 2A_K$, Eq. (13) provides another clean relation:

$$\Delta_u + 4\Delta_d = 10A_\pi.$$  \hspace{1cm} (15)

Thus with the experimental results:

$$\Delta_u = 0.80(6), \Delta_d = -0.46(6), \Delta_s = -0.12(4), \Delta_s = 0.82(6), \Delta_d = -0.44(6), \Delta_s = -0.10(4),$$

we can determine the cutoff for $A_K$ and $A_\pi$ using Fig. 5. It is amazing to notice that the cutoffs needed for $A_K$ and $A_\pi$ are roughly the same, i.e., independent of whether the intermediate quark is $u, d$ (for $A_\pi$) or $s$ (for $A_K$) and also independent of the meson masses. This reminds us that in the MS or $\overline{\text{MS}}$ scheme, the subtraction is independent of the mass parameters. We will therefore choose a $\phi$-independent cutoff. It is also very interesting to notice that the cutoff value needed here roughly equals the inverse of the lattice spacing $a^{-1}$ in lattice QCD calculation of $\Delta_\eta$. To reduce the number of parameters, we choose the cutoff for $A_\phi$ to be the same as $a^{-1} = 1.74\text{GeV}$ in $\overline{\text{MS}}$. But there remains still one important question to be asked: should the cutoff for $A_\phi$ and $B_\phi$ be the same?

**TABLE I.** Model parameters and basic model predictions; the units for mass, $\alpha$, and $c$ are MeV, MeV/fm, and MeV/fm, respectively; center of mass corrections are made for the quark core contribution using the Peierls-Thouless method [5]. $g_A^{(0)}$ is the zeroth order nucleon axial charge, the full $g_A$ is obtained later.

| $m_{u,d}$ | $m_s$ | $m_\pi$ | $m_K$ | $m_Q$ | $m_\Delta$ |
|----------|------|--------|-------|-------|-----------|
| 70       | 250  | 138    | 495   | 547   | 675       |
| $\alpha$ | $c$  | $m_N$  | $m_\Delta$ | $g_A^{(0)}$ | $g_A$ |
| -31.35   | 820  | 939    | 1232  | 1.41  | 1.26      |
It might be taken for granted that they are the same, (the simplest example is that we use the charge renormalization condition to determine the renormalization constant and then calculate the matrix element of the charge operator itself). However, we call special attention here that this is not necessarily the case. The problem is the lack of Lorentz covariance in the model.

It is actually not difficult to understand this point: If a theory does not respect Lorentz covariance, then the renormalization constant determined with the time and spatial component of the electromagnetic currents are possibly different. Therefore if we calculate the matrix element of the charge operator but use the renormalization constant determined through the renormalization condition for the spatial component of the vector current, then we might not obtain the physical charge (in some cases the result may still be finite, but this is not enough).

After describing the formalism, we are now in the position to see how this renormalization scheme can explain the experimental results. As we explained the cutoff for $A_\phi$ is chosen as the inverse of the lattice spacing $a^{-1} = 1.74$ GeV [6]. Using Eq. (14) and Figs. 4, 5 and requiring $g_A = 1.257$, the cutoff for $B_\phi$ is determined to be 0.734 GeV. Then combining Figs. 4, 5 and using Eq. (13), we finally obtain:

$$\Delta_u = 0.823, \quad \Delta_d = -0.432, \quad \Delta_s = -0.104.$$  (17)

By adjusting only one free parameter (the cutoff for $B_\phi$) to fit $g_A$, we reproduced the experimental results of $\Delta_{u,d,s}$ perfectly. This can be regarded as a success of our model and renormalization scheme. Since we have started from the SU(6) wavefunction as the zeroth order approximation, we could say that the spin "crisis" for the naive SU(6) model is not real.

We end our discussions by emphasizing three points: (1) Non-conserved operators have divergent matrix elements and even in model calculations they must be consistently renormalized. (2) A natural and convenient renormalization scheme in quark model calculations is to cut the summation over the intermediate quark states at a certain energy; a fit to the experimental data reveals that this approach shares the same advantage of mass-parameter-independence as in the MS or $\overline{\text{MS}}$ scheme. (3) However, due to the lack of Lorentz covariance in quark models, the cutoffs for operators of different Lorentz type are not necessarily the same.

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