Information Systems Self-description and Quantum Measurement Problem

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Abstract

Information-Theoretical restrictions on the systems self-description and the information acquisition are applied to Quantum Measurements Theory. For the measurement of quantum object \( S \) by the information system \( O \) such restrictions are described by restricted states \( R_O \) formalism. \( R_O \) ansatz can be introduced phenomenologically from the agreement with Schrödinger dynamics and measurement statistics. The analogous restrictions obtained in Algebraic QM from the consideration of Segal algebra \( U_O \) of \( O \) observables; the resulting \( O \) restricted states \( \{ \xi^O_i \} \) set is defined as \( U_O \) dual space. From Segal theorem for associative (sub)algebras it’s shown that \( \xi^O_i \) describes the random 'pointer' outcomes \( q_j \) observed by \( O \) in the individual events.

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1 Introduction

There are several fundamental problems concerning the interpretation of Quantum Mechanics (QM), mainly involving the measurement process (Jauch, 1968; Aharonov, 1981). The oldest and most prominent of them is probably the State Collapse or Quantum Measurement Problem (D’Espagnat, 1990; Busch, 1996). In this paper we shall analyze the quantum measurements within the Information-Theoretical framework and demonstrate the importance of such consideration. Indeed, the measurement of any kind results in the reception of data about the observed system \( S \) parameters by another system \( O \) (Observer) (Guilini, 1996; Duvenhage, 2002). Therefore the studies of Information-Theoretical restrictions on the information which can be transferred from \( S \) to \( O \) can be important in the Measurement Theory (Breuer, 1996). In the model regarded here \( O \) is the information gaining and utilizing system (IGUS); it processes and memorizes the information acquired as the result of \( S \) interactions with the measuring system (MS) which element is \( O \). We assume that QM description is applicable both for a microscopic and macroscopic objects, this is the standard approach in Quantum Measurements Theory.
In particular, $O$ state supposedly is described by the quantum statistical state $\rho$ relative to another observer $O'$ (Rovelli, 1995; Bene, 2000). In principle, in our approach $O$ can be either a human brain or some automatic device, in all cases it’s the system which final state correlates with the input data.

$S$ measurement by $O$ can be described by MS state $\rho_{MS}$ evolution relative to external $O'$, yet in our approach we regard $S$ information recognition and memorization performed by $O$ itself, not by $O'$. Therefore the reception of information by $O$ during the measurement should be analyzed within the self-description framework (Svozil, 1993). The systems self-description was studied extensively in Self-reference Problem context (Finkelstein, 1988; Mittelstaedt, 1998). It was shown that the self-description of an arbitrary system is always incomplete; this result often interpreted as the analog of Gödel Theorem for Information theory (Svozil, 1993). The self-description in the measurement process that is called also the measurement from inside was considered in the formalism of inference maps between $MS$ and $O$ (Breuer, 1996). It leads to some general results for the measurement properties but doesn’t permit to derive the internal (restricted) $O$ states $R_O$ from the first principles of the theory. Following this approach, we propose here the novel formalism which under simple assumptions describes MS states mapping to $O$ states. In this formalism MS quantum state is represented by the doublet $\Phi = \{\phi^D, \phi^I\}$, where $\phi^D = \rho_{MS}$ is MS density matrix, $\phi^I(n)$ is $O$ restricted state which describes $O$ subjective information in the given individual event $n$ (Mayburov, 2001). It follows that $\phi^I$ is the stochastic state which describes the random outcomes in the measurement of $S$ states superpositions and this effect can be interpreted as the subjective state collapse observed by $O$. It will be shown that such formalism corresponds to the well-known generalization of standard QM - algebraic QM based on Segal and $C^*$- algebras of observables. (Emch, 1972). In its framework $\phi^D$ is defined on MS observables algebra $\mathcal{U}$, $\phi^I$ corresponds to the state defined on $O$ observables subalgebra $\mathcal{U}_O$.

We should stress that the observer’s consciousness doesn’t play any role in our theory and isn’t referred to directly anywhere (London, 1939). The terms 'perceptions', 'impressions' used here are defined in strictly physical terms. In our model the perception is the acquisition of some information by $O$, i.e. the change of $O$ state; a different $O$ impressions are associated with a different $O$ states. The systems states are defined in $O$ reference frame (RF) (or other $O'$ RF) and are referred to also as ‘$S$ state for $O’$.

## 2 Measurements and Quantum States Restrictions

Our formalism exploits both the quantum states in the individual events - i.e. the individual states and the statistical states which describe the quantum ensembles properties (Mittelstaedt, 1998). In QM the individual states are the pure states represented by Dirac vectors $|\Psi_i\rangle$ in $\mathcal{H}$ ; the statistical states are described by the normalized, positive operators of trace 1 - density matrixes $\rho$ on $\mathcal{H}$. If $|\Psi_i\rangle$ composition is known for the given ensemble, its state can be described in more detail by the ensemble state (Gemenge) represented by the table $W^e = \{\Psi_i; P_i\}$, where $P_i$ are the corresponding probabilities (Busch, 1996). Algebraic QM states will be considered in chap. 3.

We shall start from the analysis of simple MS model which includes the measured state $S$ and IGUS $O$ storing the incoming $S$ information. $S$ represented by two-dimensional state vector $\psi_s$, whereas $O$ is described by three-dimensional Hilbert space $\mathcal{H}_O$. Its basis consists of the orthogonal states $|O_{0,1,2}\rangle$, which are the eigenstates of $Q_O$ ‘internal pointer’ observable with eigenvalues $q^O_{0,1,2}$. In our model the detector $D$ is omitted in MS chain, the role of $O$ decoherence
effects will be discussed below. Let us consider the measurement of S observable \( \hat{Q} \) for MS initial state:

\[
\Psi_{MS}^0 = \psi_s|O_0) = (a_1|s_1) + a_2|s_2)\langle O_o | ,
\]

where \( |s_{1,2}⟩ \) are \( Q \) eigenstates with eigenvalues \( q_{1,2} \). S.\( O \) interaction \( \hat{H}_I \) starts at \( t_0 \) and finished effectively at some finite \( t_1 \), in that case Schrödinger equation for \( \Psi_{MS}^0 \) supposedly results in MS final state \( \rho_{MS}^f \):

\[
\Psi_{MS} = \sum \Psi_j^{MS} = \sum a_i|s_i)\langle O_i |.
\]

It turns out that \( \hat{Q}_O = \hat{Q} \), so \( O \) performs the unbiased \( Q \) measurement (Von Neuman,1932). Meanwhile for any \( O \) observable \( \hat{Q}_O \neq F(Q_O) \); \( Q_O = 0 \) independently of \( \psi_s \). Concerning the information recognition by IGUS \( O \), we assume that \( |O_{1,2,0}⟩ \) correspond to the certain information patterns - an impressions percepted by \( O \) as \( Q_O \) values \( q_{1,2,0}^O \) (Guilini,1996). For the external observer \( O' \) at \( t > t_1 \), MS is in the pure state \( \Psi_{MS} \) of (2); whereas from \( O \) 'point of view' \( \Psi_{MS} \) describes the simultaneous superposition (coexistence) of two contradictory impressions: \( Q_O = q_1^O \) and \( Q_O = q_2^O \) percepted by \( O \) simultaneously. However, it’s well known that experimentally the macroscopic \( O \) observes at random one of \( Q_O \) values \( q_{1,2}^O \) in any individual event. It means that S final state is \( |s_1⟩ \) or \( |s_2⟩ \) and S state collapse occurs. S final state is described by the density matrix of mixed state:

\[
\rho_s^m = \sum_i |a_i|^2 |s_i⟩⟨ s_i |.
\]

In accordance with it one can ascribe to MS the mixed state :

\[
\rho_{MS}^m = \sum_i |a_i|^2 |s_i⟩⟨ s_i ||O_i⟩⟨ O_i|
\]

which differs principally from \( \rho_{MS}^p \) of (2). This discrepancy constitutes the basis of Wigner 'Friend Paradox' for \( O, O' \) (Wigner,1961). We shall propose here the formalism which incorporate consistently this two MS descriptions 'from outside' by \( O' \) and 'from inside' by \( O \).

Generally, the measurement of an arbitrary system \( S' \) is the mapping of \( S' \) states set \( N_S \) to the given IGUS \( O^S \) states set \( N_O \). If the final \( O^S \) and \( S' \) states can’t be factorized, then \( O^S \) should be considered as the subsystem of the large system \( S_T = S' + O^S \) with the states set \( N_T \) (Mittelstaedt,1998). In this situation - 'measurement from inside', \( N_O \) is \( N_T \) subset and the inference map of \( S_T \) state to \( N_O \) defines \( O^S \) state called also the restricted state \( R_O \). The important property of \( S_T → O^S \) inference map is formulated by Breuer theorem: if for two arbitrary \( S_T \) states \( \Lambda_S, \Lambda'_S \) their restricted states \( R_O, R'_O \) coincide, then for \( O^S \) this \( S_T \) states are indistinguishable (Breuer,1996). \( O^S \) has less degrees of freedom than \( S_T \) and can’t discriminate all possible \( S_T \) states, due to it some number of \( S_T \) indistinguishable states should exist for any \( S_T \) and \( O^S \) (Svozil, 1993). In quantum case the observables noncommutativity and nonlocality introduce some novel features regarded below. Despite that \( R_O \) are incomplete \( S_T \) states, they are the real physical states for \( O^S \) observer - 'the states in their own right’, as Breuer characterizes them.

The obtained \( S', O^S, S_T \) relations are applicable to our MS model which can be treated as MS measurement from inside. Breuer’s results leave the considerable freedom for the choice of the inference map and don’t permit to derive the restricted states ansatz directly. It was proposed phenomenologically (Breuer,1996) that \( R_O \) coincides with MS partial trace which for MS final state (2) is equal:

\[
R_O = Tr_s\rho_{MS}^p = \sum |a_i|^2 |O_i⟩⟨ O_i|
\]
For MS mixed state $\rho_{MS}^{m}$ of (4) the corresponding restricted state is the same $R_{0}^{mix} = R_{0}$. This equality doesn’t mean the collapse of MS pure state $\Psi_{MS}$ because the collapse appearance should be verified also for MS individual states. For the pure case MS individual state is $\Psi_{MS}$ of (2), yet for the incoming S statistical mixture - Gemenge (3) it differs from event to event:

$$\varsigma(n) = \rho_{l}^{I} = |O_{l}\rangle\langle O_{l}|s_{l}\rangle\langle s_{l}|$$

(6)

where the random $l(n)$ frequency is stipulated by the probabilistic distribution $P_{l} = |a_{l}|^{2}$. In any event $\varsigma(n)$ differs from MS state (2) and its restricted state $\varsigma^{O}(n) = |O_{l}\rangle\langle O_{l}|$ also differs from $R_{0}$ of (5) in any event. Because of it for the restricted individual states the main condition of Breuer Theorem is violated and $O$ can differentiate pure/mixed states 'from inside' in the individual events (Breuer,1996). Therefore the proposed formalism doesn’t result in the state collapse for the measurement from inside in standard QM. Note also that the formulae (5) describes also the restricted $O$ statistical state $R_{0}^{it} = R_{0}$ for the pure and mixed MS state $\rho_{MS}^{m}$ and this ansatz is correct both in $O$ and $O'$ RFs.

Note that in Breuer theory $O$ can’t observe the difference between MS states with different $D_{12} = a_{1}^{*}a_{2} + a_{1}a_{2}^{*}$. Such difference revealed by MS interference term (IT) observable:

$$B = |O_{1}\rangle\langle O_{2}|s_{1}\rangle\langle s_{2}| + j.c.$$  

(7)

which characterizes $O,S$ quantum correlations. Being measured by external $O'$ on $S,O$, it gives $B = 0$ for the mixed MS state (4), but $B \neq 0$ for the pure MS states (2). Yet $B$ value can’t be measured by $O$ 'from inside', so $O,S$ correlations are unavailable for $O$ directly. In standard QM $R_{0}' = R_{0}$ is regarded as $O$ individual (partial) state in $O'$ RF (Lahti,1990). Such $R_{0}'$ choice is advocated first of all by the correct $Q_{O},\bar{Q}_{O}$ values obtained for that state. Let’s consider as the example MS state of (2) with $a_{1,2} = \frac{1}{\sqrt{2}}$, in this state $B$ has the eigenvalue $\bar{b} = 1$. It means that in $R_{0}'$ state $Q_{O}$ is principally uncertain for $O'$: $q_{O}^{l} \leq q_{O}^{O} \leq q_{O}^{2}$. In particular, such $B$ value excludes the ignorance interpretation of $Q_{O}$ uncertainty for $O'$, which assumes that in that state $Q_{O}$ in any event is sharp and is equal either to $q_{O}^{l}$ or $q_{O}^{2}$ (Busch,1996). However, such reasoning fails for $O$ observer because $B$ value is unobservable for him, and therefore it’s impossible to conclude in that theory whether $Q_{O}$ is uncertain or sharp in $O$ RF. Hence $R_{0}$ of (5) is just the phenomenological choice which should be advocated additionally, and other solutions for $O$ individual state, as we argue below, are possible. In particular, in this case the 'subjective' ignorance interpretation, in which $Q_{O}$ value is sharp for $O$ but is simultaneously uncertain for $O'$, can’t be excluded.

MS individual state for $O$ can be rewritten in doublet form $\Phi^{B}(n) = |\phi^{D},\phi^{I}\rangle$ where $\phi^{D} = \rho_{MS}$ is the dynamical state component, the information component $\phi^{I}$ describes $O$ subjective information in the given event $n$. In Breuer theory $\phi^{I}$ is just $\phi^{D}$ trace, however, in the alternative self-description formalism described below it represents the novel $O$ state features. As the example of the simple $O$ In this case the state collapse appears in MS measurement from inside and is described by the information component $\phi^{I}$ (Mayburov, 2001). To agree with the quantum Schrödinger dynamics (SD), this doublet state formalism (DSF) should satisfy to two operational conditions:

i) if an arbitrary system $S'$ doesn’t interact with IGUS $O^{S}$, then for $O^{S}$ this system evolves according to Schrödinger-Liouville equation (SLE)

ii) If $S'$ interacts with $O^{S}$ and the measurement of some $S'$ observable occurs, then SD can be violated for $O^{S}$ but, as follows from condition i), $S',O^{S}$ evolution for the external observer $O'$ should obey SLE.

Below it will be shown that DSF corresponds to the measurements description in Algebraic QM.
For DSF state \( \Phi = |\phi^D, \phi^I \rangle \gg \) the dynamical component is also equal to QM density matrix \( \phi^D = \rho \) and obeys SLE:

\[
\frac{\partial \phi^D}{\partial t} = [\phi^D, \hat{H}]
\] (8)

For our MS model the initial \( \phi^D \) of (1) evolves at \( t > t_1 \) to \( \phi^D(t) = \rho_{MS}^q \) of (2). For \( t \leq t_0 \), the initial \( \phi^I = |O_i\rangle\langle O_i| \); after S measurement finished at \( t > t_1 \), its \( \phi^I \) outcome is supposed to be stochastic: \( \phi^I(n) = \phi^I_i \), where \( \phi^I_i = |O_i\rangle\langle O_i| \); here \( i(n) \) described by the probabilistic distribution with \( P_i = |a_i|^2 \). It turns out that such doublet individual state \( \Phi(n) \) can change from event to event, and \( \phi^I(n) \) is partly independent of \( \phi^D \) being correlated with it only statistically. 

Therefore the conditions of Breuer theorem are fulfilled and the subjective state collapse is observed by \( O \).

An arbitrary ensemble evolution can be described via the doublet statistical states \( |\Theta \rangle \gg |\eta_D, \eta_I \rangle \gg \), where \( \eta_D = \phi^D, \eta_I(t) \) describes the probabilistic distribution \( \{P^I_j(t)\} \) of \( \phi^I \) observations by \( O \) at given \( t \):

\[
P^I_j(t) = \text{tr}(\hat{P}_j^D \eta_D(t))
\] (9)

where \( \hat{P}_j^O \) - the projector on the given \( \phi^I_j \). For MS in particular, if S don’t interact with \( O \) (no measurement), then \( \eta_I \) is time invariant and MS obeys the standard QM SLE evolution for the dynamical component \( \eta_D \). Therefore our doublet states are important only for the measurement-like processes with direct \( S - O \) interactions. \( \eta_I(t) \) is defined by \( \eta_D(t) \) which obeys SLE, because of it \( \Theta \) evolution is reversible and the acquired \( O \) information can be erased completely (Mayburov,2002).

Plainly, in this theory the quantum states for external \( O' \) (and other observers) also has the same doublet form \( \Phi^I \). In the regarded situation \( O' \) doesn’t interact with MS and so \( O' \) information \( \phi^I \) doesn’t change during S measurement. Consequently, MS state evolution for \( O' \) described by \( \phi^D \), which obeys SLE. Because of it MS state collapse isn’t observed by \( O' \) in agreement with the conditions i, ii. As the example of the simple \( O \) toy-model can be regarded the hydrogen-like atom \( A_H \) for which \( O_0 \) is its ground state and \( O_i \) are the metastable levels excited by \( s_i \), resulting so into the final \( S - O \) entangled state. In our model 'internal pointer' \( O_i \) and \( O \) memory which normally differ supposed to be the same object. Witnessing QM Interpretation proposed by Kochen is quite close to DSF but doesn’t exploits the self-description approach (Kochen,1985; Lahti,1990).

In DSF \( |O_i\rangle \) constitutes the preferred basis (PB) in \( H_O \) and its appearance should be explained in the consistent theory; this problem called also the basis degeneracy is well-known in Quantum Measurement Theory (Lahti,1990; Elby,1994). In its essence, the theory consistency demands that the final MS states decomposition should be unique, but this isn’t the case for \( \Psi_{MS} \) of (2). In DSF PB problem acquires the additional aspects related to the information recognition by \( O \). The plausible explanation prompts \( O \) decoherence - i.e. \( O \) interaction with environment \( E \) (Zurek,1982). Such interaction results in the final entangled S,O,E state which decomposed on some orthogonal \( O \) basis \( \{O^E_i\} \) (Guilini,1996). Tuning the \( H_{O,E} \) interaction parameters, \( \{O^E_i\} \) basis can be made equivalent to \( |O_i\rangle \) basis. For the initial MS state \( \Psi_{MS}^{in} \) of (1) it results in the final MS-E state:

\[
\Psi_{MS+E} = \sum a_i |s_i\rangle |O_i\rangle |E_i\rangle
\] (10)

where \( |E_i\rangle \) are final \( E \) states. It was proved that such triple decomposition is unique, even if \( |E_i\rangle \) aren’t orthogonal (Elby,1994). It isn’t necessary in our case to use \( N \rightarrow \infty \) limit, \( E \) can be also a finite system.
In addition, the decoherence results in the important consequences for the information storage by \( \phi^j \). Really, for the practical IGUS \( O \) the memorized states \( |O^f_i\rangle \) excited by \( |s_i\rangle \) signals must be stable or at least long-living. But as follows from eq. (10), any state \( |O^f_i\rangle \) different from one of \( |O_i\rangle \) in the short time would split into \( |O_i\rangle \) combinations - entangled \( O,E \) states superpositions, so that \( |O^f_i\rangle \neq |O_i\rangle \) are in fact a virtual states, Consequently, in this model \( O,E \) decoherence selects the basis of long-living \( O \) eigenstates which supposedly describes \( O \) events perception and memorization. It means that, if our \( S \) signal is \( Q \) eigenstate which induces \( Q_O \) eigenstate \( |O_i\rangle \), then it’s memorized by \( O \) for the long time. It makes also \( Q_O \) the preferable observable for \( O \) observables set, since for any \( Q'_O \neq F(Q_O) \) it gives \( \hat{Q}'_O = \hat{0} \) for \( \rho_{MS+E} \) resulting from the arbitrary \( \rho_{MS} \) (even if it’s not the entangled state of (2)). This measurement scheme denoted as \( MS+E \) model will be considered below, together with \( MS \) model in which PB settled a priori. In other aspects decoherence doesn’t change our selfmeasurement model; its most important role is the unambiguous definition of \( O \) PB. In fact, \( \mathcal{H}_O \) symmetry broken dynamically by \( \mathcal{H}_{O,E} \) interaction which makes majority of \( O \) states unstable. Despite that \( O,E \) decoherence clearly indicates PB existence, it doesn’t mean that our DSF follows from first QM principles. Our theory is still phenomenological and decoherent PB in \( MS+E \) model only reveals how \( \phi^j \) basis should be chosen if such theory is correct.

### 3 Quantum Measurements in Algebraic QM

Now the quantum measurements and \( O \) selfdescription for the finite quantum systems will be regarded in Algebraic QM formalism (Bratelli,1981). Besides the standard quantum effects, Algebraic QM describes successfully the phase transitions and other nonperturbative phenomena which standard QM fails to incorporate (Emch,1972). Consequently, there are the serious reasons to regard Algebraic QM as the consistent generalization of standard QM with the fixed Hilbert space. Algebraic QM was applied extensively to the superselection models of quantum measurements, in which the detector \( D \) or environment \( E \) are regarded as the infinite systems with \( m,V \to \infty \) (Primas,1990; Guilini,1996). The algebraic formalism of nonperturbative QFT was used also in the study of measurement dynamics in some realistic systems (Mayburov,1998).

In standard QM the fundamental structure is the states set - Hilbert space \( \mathcal{H} \) on which an observables - Hermitian operators are defined. It can be shown that for some nonperturbative systems the structure of states set differs principally from \( \mathcal{H} \), and the standard QM axiomatics becomes preposterous. In distinction, the fundamental structure of Algebraic QM is Segal algebra \( \mathcal{U} \) of observables which incorporates the main properties of regarded system \( S_f \) (Emch, 1972). It’s more convenient technically to deal with \( C^* \)-algebra \( \mathcal{C} \) for which \( \mathcal{U} \) is the subset. Roughly speaking, \( \mathcal{C} \) is an algebra of complex elements for which \( \mathcal{U} \) is the subset of its real (Hermitian) elements. This elements in Algebraic QM are the linear operators for which the sum \( A+B \) and product \( A \cdot B \) defined. For our problems \( \mathcal{C}, \mathcal{U} \) are in the unambiguous correspondence: \( \mathcal{C} \leftrightarrow \mathcal{U} \), and below their use is equivalent in this sense. \( S_f \) states set \( \Omega \) defined by \( \mathcal{U} \) via the notorious GNS construction; it proves that \( \Omega \) is the vector space dual to the corresponding \( \mathcal{C} \) (Bratelli,1981). Such states are called here the algebraic states \( \varphi \) and are the normalized, positive, linear functionals on \( \mathcal{U} \): for any observable \( A \in \mathcal{U}, \forall \varphi \in \Omega; A = \langle \varphi; A \rangle \). The pure states - i.e. \( \Omega \) extremal points are regarded as the algebraic individual states (AIS) \( \xi \); their set denoted \( \Omega^p \) (Emch,1972; Primas,1990). The algebraic mixed states \( \varphi_{mix} \) can be constructed as \( \xi_i \) ensembles, the ensemble states \( W^A \) are defined analogously to their QM ansatz. Here only a finite-dimensional \( S_f \) will be considered, for them \( \varphi \) states set is isomorphic to some QM density matrices \( \rho \) set. In that case we regard the corresponding \( \varphi \) and \( \rho \) as equivalent, despite that,
strictly speaking, they are the different objects (Segal, 1947).

In many practical situations the only some restricted linear subspace $M_R$ or subalgebra $U_R$ of $S_f$ algebra $U$ is available for the observation. For such subsystems the restricted algebraic states $\varphi_R$ and their set $\Omega_R$ can be defined consistently via the expectation values of $A_R \in U_R$:

$$\bar{A}_R = \langle \varphi; A_R \rangle = \langle \varphi_R; A_R \rangle$$

(11)

$\varphi_R$ doesn’t depend on any $A' \notin U_R$, thereby $\forall \varphi_R$, $\langle \varphi_R; A' \rangle = 0$. Remind that any classical system $S^c$ can be described by some associative Segal algebra $U^c$ of $S^c$ observables $\{A\}$; in algebraic QM $U$ associativity corresponds to its observables commutativity (Emch, 1972). The theorem by Segal proves that any associative Segal (sub)algebra $U'$ is isomorphic to some algebra $U^c$ of classical observables, its $\varphi^c$ states set $\Omega^c$ is isomorphic to the set $\Omega^c$ of the classical statistical states $\varphi^c$ (Segal, 1947). The corresponding AIS - i.e. the pure states correspond to the classical individual states $\xi_i^c$ - points in $S^c$ phase space. For the systems self-description the most important is the case when $U'$ is elementary, i.e. includes only $I$ - unit operator and the single $A \neq I$, then $\xi_i^c = \delta(q^A - q_i^A)$ corresponds to $A$ eigenvalues $q_i^A$ spectra. Consequently, even if quantum $S_f$ is described by nonassociative $U$, it should contain the subalgebras $U' \subset U$ for which the restricted states are classical.

For the classical observing system $S^c_f = S^c + O^c$ described by some $U^c$, IGUS $O^c$ self-description restrictions are simple and straightforward - the restricted $S^c_f$ states depend only on those $S^c_f$ coordinates $\{x_j^O\}$, which are $O$ internal degrees of freedom (Breuer, 1996). They constitute the subalgebra $U^c_R \subset U^c$. In practice $O^c$ effective subalgebra $U^c_O \subset U^c_R$ which really defines the measurements can be even smaller, because some $x_j^O$ can be uninvolved directly into the measurement process. QM Correspondence principle prompts that for the quantum IGUS $O$ the restricted $O$ subalgebra $U_R$ also should include only $O$ internal observables. In this case any effective self-description subalgebra $U_O$ belongs to $U_R$, their states sets are denoted $\Omega_O, \Omega_R$ correspondingly. The main assumption of our theory is as follows: given the subalgebra $U_A \in U$, in any individual event $n$ an arbitrary MS AIS $\xi^{MS}(n)$ induces some restricted AIS $\xi^A(n)$ spanned on $U_A$. This hypothesis seems to us quite natural and below the additional arguments in its favor will be presented for the particular $U_A$. For illustration we regard it first for $U_R$, and accept ad hoc that, according to the formal definition of the individual states, such states are $\Omega_R$ extremal points.

MS is described by $U$ Segal algebra of MS observables which defines $\varphi^{MS} \in \Omega$ properties, $MS + E$ model involves $U_{MS,E}$ algebra correspondingly. $O$ subalgebra $U_R$ includes $I$ and all $O$ internal observables, so it means that $\Omega_R$ is isomorphic to $O$ statistical states $\rho^O$ set $\Gamma_O$. Consequently, $O$ AIS set $\Omega^p_O$ is isomorphic to $\mathcal{H}_O$ and any $O$ AIS $\xi^R_i$ corresponds to some state vector $|O^R_i\rangle \in \mathcal{H}_O$. We don’t study here $\xi^R$ states in detail because they are unimportant for our aims, note only that Breuer state (5 ) for $O R_O \notin \Omega^p_O$, and can’t be $O$ AIS on $U_R$ for an arbitrary $a_{1,2}$. To define $U_O$, remind that for the regarded MS dynamics $O$ can measure only the observable $Q_O$. For any other $Q'_O \neq F(Q_O)$ one obtains $Q'_O = 0$ for any $\Psi^{p}_{MS}$, therefore for any $\varphi^O \in \Omega_O$ it follows that $\langle \varphi^O; Q'_O \rangle = 0$. It means that $O$ effective subalgebra $U_O$ is equal to the elementary $U^c_O$, which includes only $Q_O$ and $I$. Really, only in this case $\langle \varphi'; Q'_O \rangle = 0$ for all $\varphi' \in \Omega^c_R$ defined on $U^c_O$; each $\varphi'$ corresponds to $\varphi^O$ with the same $Q_O$ and vice versa; so $\varphi^O$ set $\Omega_O$ is isomorphic to $\Omega^c_R$. There is no other $U_R$ subalgebras with such properties and that settles $U_O$ finally. It turns out that the obtained $\varphi^O$ are equivalent to $R^O_{MS}$ of (5) which are MS statistical states $\rho_{MS}$ restriction to $O$.

From Segal theorem the restricted algebraic $O$ states $\varphi^O \in \Omega_O$ are isomorphic to the classical $q_i^O$ distributions; meanwhile $O$ AIS $\xi^O_i$ - $\Omega_O$ extremal points are the positive states:

$$\xi^O_i = \delta(q^O_i - q_i^O)$$

(12)
which correspond to the classical pointlike states. In particular, such \( \xi_i^O \) can appear in \( \psi_s = |s_i\rangle \) measurement by \( O \) as the restriction of MS final state \( \xi_i^{MS} = |s_i\rangle\langle O| \). As was supposed above, the formal individual state \( \xi_i^O \) perceived by \( O \) as the definite \( Q_O \) value \( q_i^O \). Such information pattern permits to reveal \( \xi_{ij}^O \) distinction in the single event per each state which is important for our theory interpretation. Note also that Algebraic formalism, in fact, extracts PB \( |O_i\rangle \sim \xi^O \) in \( \mathcal{H}_O \) even without the account of E decoherent interaction. The decoherence, as argued below, only duplicates this effect.

The incoming S mixture - \( |s_i\rangle \) Gemenge results in MS algebraic final state \( \varphi_{mix}^O \) which is equivalent to \( \rho_{MS}^m \) of (4); O restricted state \( \varphi_{mix}^O \) is defined from the relation for \( Q_O \):

\[
\tilde{Q}_O = \langle \varphi_{mix}^O; Q_O \rangle = \langle \varphi_{mix}; Q_O \rangle = \sum |a_i|^2 q_i^O
\]

which results in the solution \( \varphi_{mix}^O = \sum |a_i|^2 \varphi_i^O \), where \( \varphi_i^O = \xi_i^O \) of (12). From the correspondence of MS state \( \xi_i^{MS} \) and O state \( \xi_i^O \) in the individual events, the restricted algebraic state \( \varphi_{mix}^O \) represents the statistical mixture of AIS \( \xi_i^O \) described by O ensemble state \( W_{mix}^O = \{\xi_i^O; P_i = |a_i|^2; i = 1,2\} \). If MS final state is the pure state \( \Psi_{MS} \) of (2) and corresponding MS algebraic state is \( \varphi^{MS} = \xi^{MS} \), it results in the same \( Q_O \) value. Therefore its O restricted algebraic state coincides with the mixed one: \( \varphi^O = \varphi_{mix}^O \). From that one can define what are O individual states induced by MS pure states \( \xi^{MS} \), in particular, whether they differ from the obtained \( \Omega_O \) extremal points \( \xi_i^O \) - the formal individual states. Remind that any physically different states can be operationally discriminated by the particular observation procedure, which puts in correspondence to this states some parameters values. For the statistical states it can be their probabilistic distributions parameters. The individual states \( \psi_{a,b} \) are the eigenstates of some \( A, B \) observables and their difference is revealed by their eigenvalues \( q_{i,j}^{A,B} \) which exist objectively. In principle, this values can be extracted from the single event per each state (Mittelstaedt,1998).

If to suppose that such correspondence maintained also for the restricted states, then in our case the only \( \mathcal{U}_O \) observable is \( Q_O \) and \( \xi_{ij}^O \) difference reflected by \( q_{i,j}^O \) values. If to suppose that some other \( \xi_i^O \neq \xi_i^O \) exists, then some other observable \( Q^e \in \mathcal{U}_O \), for which \( \xi_i^O \) is the eigenstate, should exist, such that \( Q^e \neq F(Q^O) \). However this assumption is inconsistent with the obtained \( \Omega_O \) structure. In particular, Breuer state \( R_O \) analog \( \xi_R^O = \sum |a_i|^2 \xi_i^O \) can’t be O individual state on \( \mathcal{U}_O \), because it isn’t \( Q_O \) eigenstate and can’t be \( \Omega_O \) extremal point.

To illustrate this reasoning, consider the superposition \( \xi_{s}^{MS} = \Psi_{MS} \) of (2) with \( a_{1,2} = \frac{1}{\sqrt{2}} \); we denote the (fuzzy) values of \( Q^O \) and \( B \) of (7) as \( \tilde{q}^O, \tilde{b} \). Let’s define the conditions to which the restricted \( \xi_s^{MS} \) state - \( \xi_s^O \) should satisfy. In \( O’ \) RF \( \xi_s^{MS} \) is B eigenstate with the eigenvalue \( \tilde{b} = 1 \) which represents IT condition, and \( q_i^O \leq \tilde{q}_1^O \leq \tilde{q}_2^O \) is the spectral condition. Taken together this conditions indicate that \( \tilde{q}^O \) is located within the interval \([q_1^O, q_2^O]\) and is principally uncertain inside it, as \( \tilde{b} \) value evidences. \( \xi_s^{MS} \) restricted state \( \xi_s^O \) is defined on \( \mathcal{U}_O = \{ I, Q_O \} \), therefore the spectral condition also holds for \( O \). Now because \( B \notin \mathcal{U}_O \) and unavailable for \( O \), IT condition can be dropped. Without it the spectral condition alone means that \( \tilde{q}^O \) is localized in \([q_1^O, q_2^O]\) interval, but can be either uncertain or sharp. Hence \( \xi_{1,2}^O \) states with the sharp \( \tilde{q}^O = q_{1,2}^O \) satisfy to that only necessary condition for the restricted states. Meanwhile there are no state \( \xi_i^O \) in \( \Omega_O \) with \( \tilde{q}^O \) uncertain. The reason of it, as was explained above, is that such \( \xi_i^O \) should be an eigenstate of some IT observable which value, alike \( \tilde{b} \), can demonstrate \( \tilde{q}^O \) uncertainty for \( O \). Yet \( \mathcal{U}_O \) doesn’t include any such observable. Consequently, \( \xi_{1,2}^O \) are the only suitable candidates and the formal solution for \( \xi_s^{MS} \rightarrow \xi_s^O \) restriction in the individual event n is:

\[
\xi_s^O(n) = \xi_1^O \text{ or } \xi_2^O
\]
and it can be shown to be the same for any $a_{1,2} \neq 0$. It means that $MS \rightarrow O$ map is ambiguous and breaks $Q_O$ symmetry of $\xi^{MS}$. Note also that an arbitrary $\varphi^O$ admits the unique decomposition into $\xi^O$ set and so can be interpreted as $\xi^O$ ensemble with the given probabilities $P'_i$. Thus, in each event $\xi^O_1$ or $\xi^O_2$ can appear at random. For the pure MS ensemble regarded above the only solution which gives the correct $\hat{Q}_O$ is $P'_i = |a_i|^2$, and so $W^O = W^O_{mix}$. The obtained results demonstrate that for MS ensemble $\xi^{MS} \rightarrow \xi^O$ restriction map is stochastic and results in the subjective state collapse observed by $O$ analogously to DSF state collapse described above. Since $\xi^O_i \in \Omega^O_R$, $\xi^O_s$ can be taken also as the possible ansatz for individual MS states restriction on $U^O_R$. DSF doublet state $\Phi$ components $\phi^D, \phi^I$ correspond to $\xi^{MS}, \xi^O$; the algebraic states $\varphi(t), \varphi^O(t)$ describe $\eta^O_i$ statistical distributions analogously to DSF $\eta_D, \eta_I$.

Note that MS individual states $\xi^{MS}$ symmetry is larger than the symmetry of the restricted $O$ states. In Algebraic QM such symmetry reduction results in the phenomena of Spontaneous Symmetry Breaking which leads to the randomness of outcomes for the models of measurements in the infinite systems (Guilini,1996; Mayburov,1998). The algebraic self-description permits to extend such randomness mechanism on the finite systems. From the mathematical point of view the Algebraic QM formalism contains the generic structure - the restricted AIS set $\Omega^I_R$ defined on the elementary subalgebra $U^I_R$, $U^I_R = U$ ($U^O_R = U^I_O$ in our model). $\Omega^I_R$ extremal points, being treated as the individual states, describe the collapse of the pure states defined on $U$. Hence in this theory the quantum state reduction results from the reduction of a system algebra to its associative subalgebra.

If to analyze this results in the Information-Theoretical framework, remind that the difference between the pure and mixed MS states reflected by $B$ of (7) expectation values (and other IT observables). Therefore $O$ possible observation of $S$ pure/mixed states difference means that $O$ can acquire the information on $B$ expectation value. But $B \notin U_R$ and isn’t correlated with $Q_O$ via $S,O$ interaction alike $Q$ of $S$, so $S,O$ correlations supposedly are unobservable from inside by $O$ (Mittelstaedt,1998). It corresponds to Wigner conclusion that the perception by $O$ of the superposition of two contradictory impressions is nonsense and should be excluded in the consistent theory (Wigner, 1961). Consequently, it’s possible that Information Theory rules by itself can lead to the subjective state collapse in the quantum measurements.

In practice it’s possible that $O$ effective subalgebra is larger than $U_O$, but this case demands more complicated calculations which we plan to present in the forthcoming paper. In Algebraic QM the only important condition for the classicality appearance is $U_O$ associativity but it feasible, in principle, also for the complex IGUS structures. If to consider MS+E system and its $U_{MS,E}$ algebra, the effective $O$ information subalgebra will be the same $U_O$ considered above. Therefore $O$ subalgebra and its states set properties can’t depend directly on the surrounding $E$ properties. Despite of the acknowledged Algebraic QM achievements, its foundations are still discussed and aren’t settled finally. In particular, it’s still unclear whether all the algebraic states $\varphi$ correspond to the physical states (Primas,1983). This question is important by itself and can be essential for our formalism feasibility. We admitted also that for MS arbitrary $\xi^{MS}$ some $O$ restricted AIS responds in any event. It agrees with the consideration of the restricted states as the real physical states, however, this assumption needs further clarification. For the regarded simple MS model Algebraic QM formalism in many aspects is analogous to Orthomodular Algebra of propositions or Quantum Logics (Jauch, 1968; Emch, 1972). The possibility of its application to the systems self-description and the measurement from inside demands the additional investigation.
4 Discussion

in this report the information-theoretical restrictions on the quantum measurements were studied in the simple selfdescription model of IGUS O. Breuer self-description theory shows that by itself O inclusion as the quantum object into the measurement scheme doesn’t result in the state collapse appearance (Breuer,1996). Our considerations indicates that to describe the state collapse and in the same time to conserve Schrödinger linear evolution, it’s necessary to extend the quantum states set over standard QM Hilbert space. Such modification proposed in DSF leads to the doublet states Φ ansatz, where one of its components φI corresponds to O subjective information - i.e. O selfdescription. Algebraic QM presents the additional arguments in favor of this approach, in its formalism O structure described by O observables algebra UO which defines the multiplet states set analogs to Φ. In Algebraic formalism the stochastic events appearance stipulated by MS individual states restriction to O. Algebraic QM formalism - Segal and C*-algebras of operators is acknowledged generalization of standard QM (Emch, 1972). In addition, from the mathematical point of view the duality of the operators algebra and the states set is better founded, than the states set priority postulated in standard QM (Bratelli, 1981). In this paper it was applied for the simplest measurement model but if this formalism universality will be proved, it would mean that the proposed measurement theory follows from the established Quantum Physics realm (Emch,1972). Algebraic formalism permits to calculate the restricted individual O states ξO ansatz without any phenomenological assumptions. From the formal point of view the only novel feature of our approach is the use of Segal algebra for the individual restricted states RO(n) calculations. We don’t see any compelling mathematical arguments why for the individual states RO ansatz should be chosen the same QM formulae (5) which used for the statistical restricted states Rst (Lahti,1990).

Our theory demonstrates that the probabilistic realization is generic and unavoidable for QM and without it QM supposedly can’t acquire any operational meaning. Wave-particle dualism was always regarded as characteristic QM feature but in our theory it has straightforward correspondence in our DSF. Our approach stresses also the dual character of quantum measurement : this is the interaction of studied S with IGUS O and in the same time the information acquisition and recognition by IGUS. Note that Self-reference problem avoided in this case by use of the natural assumption that all observers are similar in relation to their information acquisition properties (Svozil,1993).

Under the realistic conditions, the rate of E atoms interactions with macroscopic detector D is very high, and due to it in a very short time t_d S,D partial state \( \rho_p = Tr_{EPSDE} \) becomes approximately equal to the mixed one, as \( \rho_p \) nondiagonal elements become very small. This fact induced the claim that the objective state collapse can be completely explained by detector state decoherence without the observer’s inclusion, but it was proved to be incorrect (D’Espagnat,1990). The development of decoherence approach proposed in Zurek ‘Existential interpretation (Zurek,1998). IGUS O regarded as the quantum object and included in the measurement chain; memorization of input S signal occurs in several binary memory cells \( |n_1,2\rangle \) (chain) which are the analog of the brain neurons. O memory state suffers the decoherence from surrounding E ‘atoms’ which results in the system state analogous to (10). Under practical conditions, the decoherence time t_d is also small and for \( t \gg t_d \) S,O partial state \( \rho_p \) differs from the mixture very little. From that Zurek concludes that O percepts input pure S signal as the random measurement outcomes. Yet the system S,O,E is still in the pure state even at \( t \gg t_d \) and IT observable B analogous to (7) which proves it exists. Therefore in standard QM framework it’s incorrect to claim that IGUS percepts random events. The regarded IGUS model doesn’t differs principally from our MS scheme; in Algebraic formalism IGUS subjective perception is
described by $O$ restricted state $\xi^O$ defined on $\mathcal{U}_O$ which describes the random outcomes for the input pure $S$ state. Consequently, Algebraic QM application to Zurek IGUS model supports Existential Interpretation.

It’s important to stress that all the experiments in Physics at the final stage include the human subjective perception which simulated by $O$ state in our model. The possible importance of observer in the measurement process was discussed first by London and Bauer (London,1939). They supposed that Observer Consciousness (OC) due to ‘introspection action’ violates in fact Schrodinger equation and results in the state reduction. In distinction, in our theory $O$ perception doesn’t violate MS Schrodinger evolution from $O'$ point of view. In principle, Self-description Theory permits to regard the relation between MS, IGUS $O$ states and $O$ subjective information (impression), and one can try to extend it on the human perception. This is the separate, important problem which is beyond our scope and here we consider briefly only some its principal points which formulated here as the simple Impression Model (IM). Following our approach, $O$ perception affected only by $O$ internal states defined on $O$ observables subalgebra $\mathcal{U}_O$. For $O$ perception the following, calibration assumption introduced: for any $Q$ eigenstate $|s_i\rangle$ after S measurement finished at $t > t_1$ and $O$ 'internal pointer'state is $|O_i\rangle$ observer $O$ have the definite impression $I^O$ corresponding to $I^O = q^O_i$ eigenvalue - the information pattern percepted by $O$. It settles the hypothetical correspondence between MS quantum dynamics model and human perception. Impression $I^O$ is $O$ subjective information which isn’t dynamical parameter and its introduction can’t have any influence on the theory dynamics. Furthermore, we assume that if $S$ state is the superposition $\psi_s$ then its measurement by $O$ also results in appearance for each individual event $n$ of some definite and unambiguous $O$ impression $I^O = q^{sup}(n)$. In Algebraic formalism the corresponding $O$ subjective information - impression in the individual event is equal to $I^O(n) = q^O_i$ and can be consistently defined in this ansatz for our IM as the stochastic state appearing with probability $|a_i|^2$. In algebraic QM framework they correspond to the restricted $O$ AIS $\xi^O$ set $\Omega^O$. The obtained picture is the analog of Von Neuman psychophysical parallelism hypothesis. Yet we must stress that, on the whole, our theory doesn’t need any addressing to human OC. Rather, in this model IGUS $O$ is active RF which internal state excited by the interaction with the studied object.

To conclude, the quantum measurements were studied within the Information-Theoretical framework and self-description restrictions on the information acquisition are shown to be important in the Measurement Theory. Algebraic QM represents the appropriate formalism of systems self-description, in particular, IGUS $O$ observable algebra $\mathcal{U}_O$ defines $O$ restricted states $\xi^O$ set $\Omega^O$. The appearance of stochastic events stipulated by MS individual states restriction to $\xi^O$ states and results in the state collapse observation by $O$. The regarded IGUS model is quite simple and on the whole doesn’t permit us to make any final conclusions at this stage. Yet the obtained results evidence that the IGUS information restrictions and its interactions with the observed system should be accounted in Quantum Measurement Problem analysis (Zurek,1998).

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