ANGLES OF THE CKM UNITARITY TRIANGLE
MEASURED AT BELLE

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Abstract

The Belle experiment has used several methods to measure or constrain the angles $\phi_1$, $\phi_2$, and $\phi_3$ (or $\beta$, $\alpha$, and $\gamma$) of the CKM unitarity triangle. The results are $\sin 2\phi_1 = 0.728 \pm 0.056 \text{ (stat)} \pm 0.023 \text{ (syst)}$ or $\phi_1 = (23.4_{-2.4}^{+2.7})^\circ$ from $B^0 \rightarrow J/\psi K^0$ decays (140 fb$^{-1}$); $\phi_2 = (0-19)^\circ$ or $(71-180)^\circ$ at 95.4$\%$ CL from $B^0 \rightarrow \pi^+ \pi^-$ decays (253 fb$^{-1}$); and $\phi_3 = [68_{-15}^{+14} \text{ (stat)} \pm 13 \text{ (syst)} \pm 11 \text{ (model)}]^\circ$ from $B^\pm \rightarrow (D^0, \overline{D}^0)K^\pm$, $(D^0, \overline{D}^0) \rightarrow K_S^0 \pi^+ \pi^-$ decays (253 fb$^{-1}$). These values satisfy the triangle relation $\phi_1 + \phi_2 + \phi_3 = 180^\circ$ within their uncertainties. The angle $\phi_1$ is also determined from several $b \rightarrow s\bar{q}q$ penguin-dominated decay modes; the value obtained by taking a weighted average of the individual results differs from the $B^0 \rightarrow J/\psi K^0$ result by more than two standard deviations. The angle $\phi_2$ is constrained by measuring a $CP$ asymmetry in the decay time distribution; the asymmetry observed is large, and the difference in the yields of $B^0, \overline{B}^0 \rightarrow \pi^+ \pi^-$ decays constitutes the first evidence for direct $CP$ violation in the $B$ system.
1 Introduction

The Standard Model predicts $CP$ violation to occur in $B^0$ meson decays owing to a complex phase in the $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix $^1)$. This phase is illustrated by plotting the unitarity condition $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ as vectors in the complex plane: the phase results in a triangle of nonzero height. Various measurements in the $B$ system are sensitive to the internal angles $\phi_1$, $\phi_2$, and $\phi_3$ (also known as $\beta$, $\alpha$, and $\gamma$, respectively); these measurements allow us to determine the angles and check whether the triangle closes. Non-closure would indicate physics beyond the Standard Model. Here we present measurements of $\phi_1$ and $\phi_2$ obtained by measuring time-dependent $CP$ asymmetries, and a measurement of $\phi_3$ obtained by measuring an asymmetry in the Dalitz plot distribution of three-body decays. The results presented are from the Belle experiment $^2$), which runs at the KEKB asymmetric-energy $e^+e^-$ collider $^3$ operating at the $\Upsilon(4S)$ resonance.

In Belle, pion and kaon tracks are identified using information from time-of-flight counters, aerogel Čerenkov counters, and $dE/dx$ information from the central tracker $^4)$. $B$ decays are identified using the “beam-constrained” mass $M_{bc} = \sqrt{E_{\text{beam}}^2 - p_B^2}$ and the energy difference $\Delta E = E_B - E_{\text{beam}}$, where $p_B$ is the reconstructed $B$ momentum, $E_B$ is the reconstructed $B$ energy, and $E_{\text{beam}}$ is the beam energy, all evaluated in the $e^+e^-$ center-of-mass (CM) frame. A tagging algorithm $^5$) is used to identify the flavor at production of the decaying $B$, i.e., whether it is $B^0$ or $\bar{B}^0$. This algorithm examines tracks not associated with the signal decay to identify the flavor of the non-signal $B$. The signal-side tracks are fit for a decay vertex, and the tag-side tracks are fit for a separate decay vertex; the distance $\Delta z$ between vertices is to a very good approximation proportional to the time difference $\Delta t$ between the $B$ decays: $\Delta z \approx (\beta \gamma c) \Delta t$, where $\beta \gamma$ is the Lorentz boost of the CM system.

The dominant background is typically $e^+e^-\rightarrow q\bar{q}$ continuum events, where $q = u, d, s, c$ in the CM frame such events tend to be jet-like, whereas $B\bar{B}$ events tend to be spherical. The sphericity of an event is usually quantified via Fox-Wolfram moments $^6$ of the form $h_\ell = \sum_{i,j} p_i p_j P_\ell(\cos \theta_{ij})$, where $i$ runs over all tracks on the tagging side and $j$ runs over all tracks on either the tagging side or the signal side $^7)$. The function $P_\ell$ is the $\ell$th Legendre polynomial and $\theta_{ij}$ is the angle between momenta $p_i$ and $p_j$ in the CM frame. These moments are combined into a Fisher discriminant, and this is combined with the probability density function (PDF) for $\cos \theta_B$, where $\theta_B$ is the polar angle in the CM frame between the $B$ direction and the $z$ axis (nearly along the $e^-$ beam direction). $B\bar{B}$ events are produced with a $1 - \cos^2 \theta_B$ distribution while $q\bar{q}$ events are produced uniformly in $\cos \theta_B$. The PDFs for signal and $q\bar{q}$ background are obtained using MC simulation and $M_{bc}\Delta E$ sidebands in data,
respectively. We use the products of the PDFs to calculate a signal likelihood $L_s$ and a continuum likelihood $L_{q\bar{q}}$ and require that $L_s/(L_s+L_{q\bar{q}})$ be above a threshold.

The angles $\phi_1$ and $\phi_2$ are determined by measuring the time dependence of decays to $CP$-eigenstates. This distribution is given by

$$\frac{dN}{d\Delta t} \propto e^{-\Delta t/\tau} \left[ 1 - q\Delta \omega + q(1 - 2\omega) [A \cos(\Delta m \Delta t) + S \sin(\Delta m \Delta t)] \right], \tag{1}$$

where $q = +1$ ($-1$) corresponds to $B^0$ ($\bar{B}^0$) tags, $\omega$ is the mistag probability, $\Delta \omega$ is a possible difference in $\omega$ between $B^0$ and $\bar{B}^0$ tags, and $\Delta m$ is the $B^0-\bar{B}^0$ mass difference. The $CP$-violating coefficients $A$ and $S$ are functions of the parameter $\lambda$: $A = (|\lambda|^2 - 1)/(|\lambda|^2 + 1)$ and $S = 2 \text{Im}(\lambda)/(|\lambda|^2 + 1)$, where

$$\lambda = \frac{q}{p} \frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} \approx \sqrt{\frac{M_{12}^2}{M_{12}^2}} \left( \frac{V_{td} V_{tb}^*}{V_{td} V_{tb}} \right) = \left( \frac{V_{td} V_{tb}^*}{V_{td} V_{tb}} \right) A(\bar{B}^0 \to f)/A(B^0 \to f). \tag{2}$$

In this expression, $q$ and $p$ are the complex coefficients relating the flavor eigenstates $B^0$ and $\bar{B}^0$ to the mass eigenstates, $M_{12}$ is the off-diagonal element of the $B^0-\bar{B}^0$ mass matrix, and we assume that the off-diagonal element of the decay matrix is much smaller: $\Gamma_{12} \ll M_{12}$. If only one weak phase enters the decay amplitude $A(\bar{B}^0 \to f)$, then $|A(\bar{B}^0 \to f)/A(B^0 \to f)| = 1$ and $\lambda = \eta_f e^{i 2\phi}$, where $\eta_f = \pm 1$ is the $CP$ of the final state $f$. For the final states discussed here, $|\theta| = \phi_1$ or $\phi_2$.

2 The angle $\phi_1$

This angle is most accurately measured using $B^0 \to J/\psi K^0$ decays\(^1\). The decay is dominated by a $b \to c \bar{c} s$ tree amplitude and a $b \to s \bar{c} v$ penguin amplitude. The latter can be divided into two pieces: a piece with $c$ and $t$ in the loop that has the same weak phase as the tree amplitude, and a piece with $u$ and $t$ in the loop that has a different weak phase but is suppressed by $\sin^2 \theta_c$, relative to the first piece. Due to this suppression, $A(\bar{B}^0 \to f)$ is governed by a single weak phase: $\text{Arg}(V_{cb} V_{cs}^*)$. The ratio $A(\bar{B}^0 \to J/\psi K^0_S)/A(B^0 \to J/\psi K^0_S)$ includes an extra factor $(p/q)_{K^0_S} = V_{cd} V_{cs}/(V_{cd} V_{cs})$ to account for the $K^0_S$ oscillating to a $K^0_L$, and thus $\lambda = -[V_{td} V_{tb}^*/(V_{td} V_{tb})] [V_{cb} V_{cs}^*/(V_{cb} V_{cs})] [V_{cd} V_{cs}/(V_{cd} V_{cs})] = -e^{-i 2\phi_1}$. The $CP$ asymmetry parameters are therefore $S = \sin 2\phi_1, A = 0$. To determine $\phi_1$, we fit the $\Delta t$ distribution for $S$; the result is $2\phi_1 = 0.728 \pm 0.056 \text{ (stat)} + 0.007 \text{ (syst)} \text{ (stat)}$.

\(^1\)This measurement includes $B^0 \to J/\psi K^0_S, J/\psi K^0_L, \psi(2S)K^0_S, \chi^0_c K^0_S, \eta_K K^0_S$, and $J/\psi K^{*0} (K^{*0} \to K^0_S \pi^0)$; we use "$B^0 \to J/\psi K^0$" to denote all six modes.
Table 1: Decay modes used to measure $\sin 2\phi_1$, the number of candidate events, the value of $\sin 2\phi_1$ obtained, and the parameter $A$ obtained [see Eq. (1)]. The $B^0 \rightarrow J/\psi K^0$ result corresponds to 140 $fb^{-1}$ of data; the other results correspond to 253 $fb^{-1}$ of data.

| (CP) Mode | Candidates | $\sin 2\phi_1$ | $A$ |
|-----------|------------|----------------|-----|
| $(\rightarrow) J/\psi K_S^0$ | 2285 | $0.728 \pm 0.056 \pm 0.023$ | $-$ |
| $(\rightarrow) J/\psi K_L^0$ | 2332 | $-$ | $-$ |
| $(\rightarrow) \phi K_S^0$ | $139 \pm 14$ | $0.08 \pm 0.33 \pm 0.09$ | $0.08 \pm 0.22 \pm 0.09$ |
| $(\rightarrow) \phi K_L^0$ | $36 \pm 15$ | $-$ | $-$ |
| $(\rightarrow) K^+ K^- K_S^0$ | $398 \pm 28$ | $0.74 \pm 0.27 \pm 0.09$ | $-0.09 \pm 0.12 \pm 0.07$ |
| $(+ = 83\%) (\rightarrow) f_0(980) K_S^0$ | $94 \pm 14$ | $-0.47 \pm 0.41 \pm 0.08$ | $-0.39 \pm 0.27 \pm 0.09$ |
| $(\rightarrow) K^0_s K^0_s K^0_S$ | $88 \pm 13$ | $-1.26 \pm 0.68 \pm 0.20$ | $0.54 \pm 0.34 \pm 0.09$ |
| $(\rightarrow) \eta' K_S^0$ | $512 \pm 27$ | $0.65 \pm 0.18 \pm 0.04$ | $-0.19 \pm 0.11 \pm 0.05$ |
| $(\rightarrow) \pi^0 K_S^0$ | $247 \pm 25$ | $0.32 \pm 0.61 \pm 0.13$ | $-0.11 \pm 0.20 \pm 0.09$ |
| $(\rightarrow) \omega K_S^0$ | $31 \pm 7$ | $0.76 \pm 0.65 \pm 0.13$ | $0.27 \pm 0.48 \pm 0.15$ |

0.023 (syst), or $\phi_1 = (23.4^{+2.7}_{-2.4})^\circ$ (the smaller of the two solutions for $\phi_1$). The fit result for $A$ yields $|\lambda| = 1.007 \pm 0.041$ (stat) $\pm 0.033$ (syst), in agreement with the theoretical expectation. These results correspond to 140 $fb^{-1}$ of data.

There are several decay modes that proceed exclusively via penguin amplitudes (e.g., $B^0 \rightarrow \phi K^0$ proceeding via $b \rightarrow s \bar{s}s$) or else are dominated by penguin amplitudes (e.g., $B^0 \rightarrow (\eta'/\omega/\pi^0) K^0$ proceeding via $b \rightarrow s d d$) but have the same weak phase as the $b \rightarrow c \bar{c}s s$ tree amplitude. This is because the penguin loop factorizes into a $c,t$ loop with the same weak phase and a $u,t$ loop with a different weak phase; the latter, however, is suppressed by $\sin^2 \theta_C$ relative to the former and plays a negligible role. We thus expect these decays to also have $S = \sin 2\phi_1$, $A = 0$. There are small mode-dependent corrections ($|\Delta S| \leq 0.10$) to this prediction due to final-state rescattering. Table 1 lists these modes and the corresponding values of $\sin 2\phi_1$ obtained from fitting the $\Delta t$ distributions; Fig. 1 shows these results in graphical form. Neglecting the small rescattering corrections and simply averaging the penguin-dominated values gives $\sin 2\phi_1 = 0.40 \pm 0.13$. This value differs from the $B^0 \rightarrow J/\psi K^0$ world average value by 2.4 standard deviations, which may be a statistical fluctuation or may indicate new physics.
Figure 1: Values of $\sin^2 \phi_1$ measured in decay modes dominated by $b \to s\bar{q}q$ penguin amplitudes, for 253 fb$^{-1}$ of data. The average value differs from the world average (WA) value measured in $B^0 \to J/\psi K^0$ decays.

### 3 The angle $\phi_2$

This angle is measured by fitting the $\Delta t$ distribution of $B^0 \to \pi^+ \pi^-$ decays. The rate is dominated by a $b \to u \bar{d}d$ tree amplitude with a weak phase $\arg(V_{ub}V_{ud}^*)$. If only this phase were present, then $\lambda = [V_{td}/(V_{td}^*)] [V_{ub}/(V_{ub}^*)] = e^{i2\phi_2}$, and $S = \sin 2\phi_2$, $A = 0$. However, a $b \to d\bar{u}u$ penguin amplitude also contributes, and, unlike the penguin in $B^0 \to J/\psi K^0_S$ decays, the piece with a different weak phase is not CKM-suppressed relative to the piece with the same weak phase. The $CP$ asymmetry parameters are therefore more complicated $^{11}$:

$$A_{\pi\pi} = - \frac{1}{R} \left( 2 \left| \frac{P}{T} \right| \sin(\phi_1 + \phi_2) \sin \delta \right)$$

$$S_{\pi\pi} = \frac{1}{R} \left( 2 \left| \frac{P}{T} \right| \sin(\phi_1 - \phi_2) \cos \delta + \sin 2\phi_2 - \left| \frac{P}{T} \right|^2 \sin 2\phi_1 \right)$$

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\[ R = 1 - 2 \left| \frac{P}{T} \right| \cos(\phi_1 + \phi_2) \cos \delta + \left| \frac{P}{T} \right|^2, \]  

(5)

where \( |P/T| \) is the magnitude of the penguin amplitude relative to that of the tree amplitude, \( \delta \) is the strong phase difference between the two amplitudes, and \( \phi_1 \) is known from \( B^0 \to J/\psi K^0 \) decays. Since Eqs. (3) and (4) have three unknown parameters, measuring \( A_{\pi \pi} \) and \( S_{\pi \pi} \) determines a volume in \( \delta - |P/T| - \phi_2 \) space.

The most recent Belle measurement uses 253 fb\(^{-1} \) of data; the event sample consists of 666 ± 43 \( B^0 \to \pi^+ \pi^- \) candidates after background subtraction. These events are subjected to an unbinned maximum likelihood (ML) fit for \( \Delta t \); the results are \( A_{\pi \pi} = 0.56 \pm 0.12 \) (stat) ± 0.06 (syst) and \( S_{\pi \pi} = -0.67 \pm 0.16 \) (stat) ± 0.06 (syst), which together indicate large CP violation. The nonzero value for \( A_{\pi \pi} \) indicates direct CP violation. Fig. 2 shows the \( \Delta t \) distributions for \( q = \pm 1 \) tagged events along with projections of the ML fit; a clear difference is seen between the fit results.

The values of \( A_{\pi \pi} \) and \( S_{\pi \pi} \) determine a 95.4% CL (2\( \sigma \)) volume in \( \delta - |P/T| - \phi_2 \) space. Projecting this volume onto the \( \delta - |P/T| \) axes gives the region shown in Fig. 3; from this region we obtain the constraints \( |P/T| > 0.17 \) for any value of \( \delta \), and \( -180^\circ < \delta < -4^\circ \) for any value of \( |P/T| \).

The dependence upon \( \delta \) and \( |P/T| \) can be removed by performing an isospin analysis of \( B \to \pi \pi \) decays. This method uses the measured branching fractions for \( B \to \pi^+ \pi^- \), \( \pi^0 \pi^0 \), and the \( CP \) asymmetry parameters \( A_{\pi^+ \pi^-} \), \( S_{\pi^+ \pi^-} \), and \( A_{\pi^0 \pi^0} \). We scan values of \( \phi_2 \) from 0° to 180° and for each value construct a \( \chi^2 \) based on the difference between the predicted values for the six observables and the measured values. We convert this \( \chi^2 \) into a confidence level (CL) by subtracting off the minimum \( \chi^2 \) value and inserting the result into the cumulative distribution function for the \( \chi^2 \) distribution for one degree of freedom. The resulting function \( 1 - CL \) is plotted in Fig. 4. From this plot we read off a 95.4% CL interval \( \phi_2 = (0 - 19)^\circ \) or \( (71 - 180)^\circ \), i.e., we exclude the range \( 20^\circ - 70^\circ \).

4 The angle \( \phi_3 \)

The angle \( \phi_3 \) is challenging to measure by fitting the \( \Delta t \) distribution, as the two requisite interfering amplitudes have very different magnitudes, and the small ratio of magnitudes multiplies the \( \phi_3 \)-dependent term \( \sin(2\phi_1 + \phi_3 + \delta) \)\(^{14} \). As an alternative, one can probe \( \phi_3 \) via interference in the Dalitz plot distribution of \( B^\pm \to (D^0, \bar{D}^0)K^\pm \) decays: the additional phase \( \phi_3 \) causes a difference between the interference pattern for \( B^+ \) decays and that for \( B^- \) decays\(^{15} \).
Figure 2: The $\Delta t$ distribution of background-subtracted $B^0, \bar{B}^0 \rightarrow \pi^+\pi^-$ candidates (top), and the resulting $CP$ asymmetry $[N(\bar{B}^0)-N(B^0)])/[N(\bar{B}^0)+N(B^0)]$ (bottom). The smooth curves are projections of the unbinned ML fit.

We study this asymmetry by reconstructing $B^\pm \rightarrow (D^0, \bar{D}^0)K^\pm$ decays in which the $D^0$ or $\bar{D}^0$ decays to the common final state $K_S^0 \pi^+\pi^-$. Denoting $m(K_S^0, \pi^+) \equiv m_+, m(K_S^0, \pi^-) \equiv m_-$, $A(D^0 \rightarrow K_S^0 \pi^+\pi^-) \equiv A(m^+, m^-)$, and $A(\bar{D}^0 \rightarrow K_S^0 \pi^+\pi^-) \equiv A(m^-, m^+) \equiv A(m^-, m^+)$ (i.e., assuming $CP$ conservation in $D^0$ decays), we have

$$A(B^+ \rightarrow \bar{D}^0 K^+, \bar{D}^0 \rightarrow K_S^0 \pi^+\pi^-) = A(m_+^2, m_-^2) + re^{i(\delta + \phi_3)}A(m_+^2, m_-^2)$$

(6)

$$A(B^- \rightarrow \bar{D}^0 K^-, \bar{D}^0 \rightarrow K_S^0 \pi^+\pi^-) = A(m_-^2, m_+^2) + re^{i(\delta - \phi_3)}A(m_-^2, m_+^2).$$

(7)

where $\bar{D}^0$ denotes $(D^0 + \bar{D}^0)$, $r$ is the ratio of magnitudes of the two amplitudes $|A(B^+ \rightarrow D^0 K^+)/A(B^+ \rightarrow \bar{D}^0 K^+)|$, and $\delta$ is the strong phase difference.
Figure 3: Projection of the 68.3% CL (dashed) and 95.4% CL (solid) volumes in $\delta$-$|P/T|$-\phi_2 space onto the $\delta$-$|P/T|$ axes. From the solid contour we obtain the constraints $|P/T| > 0.17$ and $-180^\circ < \delta < -4^\circ$ (95.4% CL).

between the amplitudes. The decay rates are given by

$$|A(B^+ \to \bar{D}^0 K^+ \to (K_S^0 \pi^+ \pi^-)K^+)|^2 = |A(m_{+}^{2}, m_{-}^{2})|^2 + r^2 |A(m_{-}^{2}, m_{+}^{2})|^2 + 2r |A(m_{+}^{2}, m_{-}^{2})||A(m_{-}^{2}, m_{+}^{2})| \cos(\delta + \phi_3 + \theta)$$

(8)

$$|A(B^- \to \bar{D}^0 K^- \to (K_S^0 \pi^+ \pi^-)K^-)|^2 = r^2 |A(m_{+}^{2}, m_{-}^{2})|^2 + |A(m_{-}^{2}, m_{+}^{2})|^2 + 2r |A(m_{+}^{2}, m_{-}^{2})||A(m_{-}^{2}, m_{+}^{2})| \cos(\delta - \phi_3 + \theta)$$

(9)

where $\theta$ is the phase difference between $A(m_{+}^{2}, m_{-}^{2})$ and $A(m_{-}^{2}, m_{+}^{2})$ and varies over the Dalitz plot. Thus, given a $D^0 \to K_S^0 \pi^+ \pi^-$ decay model $A(m_{+}^{2}, m_{-}^{2})$, one can fit the $B^\pm$ Dalitz plots to Eqs. (8) and (9) to determine the parameters $r$, $\delta$, and $\phi_3$. The decay model is determined from data, i.e., $D^0 \to K_S^0 \pi^+ \pi^-$ decays produced via $e^+e^- \to c\bar{c}$.

The data sample used consists of 253 fb$^{-1}$; there are $209 \pm 16$ $B^\pm \to \bar{D}^0 K^\pm$ candidates with 75% purity, and an additional $58 \pm 8$ $B^\pm \to D^{0*} K^\pm$ ($D^{0*} \to \bar{D}^0 \pi^0$) candidates with 87% purity 16). The background is dominated by $q\bar{q}$ continuum events in which a real $D^0$ is combined with a random kaon, and random combinations of tracks in continuum events. The Dalitz plots for the final samples are shown in Fig. 5.
Figure 4: The result of fitting the branching fractions for $B \to \pi^+\pi^-$, $\pi^\pm\pi^0$, $\pi^0\pi^0$ and the CP asymmetry parameters $A_{\pi^+\pi^-}$, $S_{\pi^+\pi^-}$, and $A_{\pi^0\pi^0}$, as a function of $\phi_2$ (see text). The vertical axis is one minus the confidence level. The horizontal line at $1-CL = 0.046$ corresponds to a 95.4% CL interval for $\phi_2$.

The events are subjected to an unbinned ML fit for $r$, $\delta$, and $\phi_3$. The decay model is a coherent sum of two-body amplitudes and a constant term for the nonresonant contribution:

$$A(m_+^2, m_-^2) = \sum_{j=1}^{N} a_j e^{i\alpha_j} A_j(m_+^2, m_-^2) + a_{\text{nonres}} e^{i\alpha_{\text{nonres}}}, \quad (10)$$

where $a_j$, $\alpha_j$, and $A_j$ are the magnitude, phase, and matrix element, respectively, of resonance $j$; and $N = 18$ resonances are considered. The parameters $a_j$ and $\alpha_j$ are determined by fitting a large sample of continuum $D^0 \to K_S^0 \pi^+\pi^-$ decays. The dominant intermediate modes as determined from the fraction $\int |a_j A_j|^2 \, dm_+^2 \, dm_-^2 / \int |A(m_+^2, m_-^2)|^2 \, dm_+^2 \, dm_-^2$ are $K^*(892)^+\pi^-$ (61.2%), $K_S^0 \rho^0$ (21.6%), nonresonant $K_S^0 \pi^+\pi^-$ (9.7%), and $K_0^* (1430)^+\pi^-$ (7.4%).

The central values obtained by the fit are $r = 0.25$, $\delta = 157^\circ$, and $\phi_3 = 64^\circ$ for $B^+ \to D^0 K^+$; and $r = 0.25$, $\delta = 321^\circ$, and $\phi_3 = 75^\circ$ for $B^+ \to D^{*0} K^+$. The errors obtained by the fit correspond to Gaussian-shaped likelihood distributions, and for this analysis the distributions are non-Gaussian. We therefore use a frequentist MC method to evaluate the statistical errors. We first obtain
choose the one that satisfies $\delta, \phi$ taken as the statistical errors; the values that maximize the PDF are taken as the central values, and the $3.7\%$ confidence level for the Dalitz plot analysis for $r, \delta, \phi_3$. All results are listed in Table 2. The second error listed is systematic but does not include uncertainty from the decay model, e.g., from the choice of form factors used for the intermediate resonances and the $q^2$ dependence of the resonance widths.

We combine the $B^+ \to \bar{D}^0 K^+$ and $B^+ \to \bar{D}^{*0} K^+$ results by multiplying together their respective PDF’s, taking the parameter $\phi_3$ to be common between them. This gives a PDF for the six measured parameters $r_1, \delta_1, \phi_3(1)$, $r_2, \delta_2, \phi_3(2)$ in terms of the five true parameters $\bar{r}_1, \bar{\delta}_1, \bar{r}_2, \bar{\delta}_2, \bar{\phi}_3$. The value of $\bar{\phi}_3$ that maximizes the PDF is taken as the central value, and the $3.7\%$ confidence interval prescribed by the PDF (corresponding to $1\sigma$ for a five-dimensional Gaussian distribution) is taken as the statistical error. The systematic error provides an estimate of the overall uncertainty in the results.

Table 2: Results of the Dalitz plot analysis for $r, \delta, \phi_3$. The first error listed is statistical and is obtained from a frequentist MC method (see text); the second error listed is systematic but does not include uncertainty from the $\bar{D}^0 \to K^0_S \pi^+ \pi^-$ decay model; the third error listed is due to the decay model.

| Parameter | $B^+ \to \bar{D}^0 K^+$ | $B^+ \to \bar{D}^{*0} K^+$ |
|-----------|--------------------------|--------------------------|
| $r$       | $0.21 \pm 0.08 \pm 0.03 \pm 0.04$ | $0.12 \pm 0.16 \pm 0.02 \pm 0.04$ |
| $\delta$  | $157^\circ \pm 19^\circ \pm 11^\circ \pm 21^\circ$ | $321^\circ \pm 57^\circ \pm 11^\circ \pm 21^\circ$ |
| $\phi_3$  | $64^\circ \pm 19^\circ \pm 13^\circ \pm 11^\circ$ | $75^\circ \pm 57^\circ \pm 0^\circ \pm 11^\circ$ |
is taken from the $B^+ \to \bar{D}^0 K^+$ measurement, as this sample dominates the combined measurement. The overall result is

$$\phi_3 = \left[ 68^{+14}_{-15} \text{ (stat)} \pm 13 \text{ (syst)} \pm 11 \text{ (decay model)} \right] \circ.$$

The 2σ confidence interval including the systematic error and decay model error is $22^\circ < \phi_3 < 113^\circ$.

In summary, the Belle experiment has measured or constrained the angles $\phi_1$, $\phi_2$, and $\phi_3$ of the CKM unitarity triangle. We obtain $\sin 2\phi_1 = 0.728 \pm 0.056 \text{ (stat)} \pm 0.023 \text{ (syst)}$ or $\phi_1 = (23.4^{+2.7}_{-2.4})^\circ$ with 140 fb$^{-1}$ of data; $\phi_2 = (0-19)^\circ$ or $(71-180)^\circ$ at 95.4% CL with 253 fb$^{-1}$ of data; and $\phi_3 = \left[ 68^{+14}_{-15} \text{ (stat)} \pm 13 \text{ (syst)} \pm 11 \text{ (decay model)} \right] ^\circ$ with 253 fb$^{-1}$ of data. Within their uncertainties, these values satisfy the triangle relation $\phi_1 + \phi_2 + \phi_3 = 180^\circ$.

The angle $\phi_1$ is measured from $B^0 \to J/\psi K^0$ decays and also from several $b \to s\bar{q}q$ penguin-dominated decay modes; the value obtained from the penguin modes differs from the $B^0 \to J/\psi K^0$ result by 2.4σ. The $\phi_2$ constraint results from measuring the $CP$ asymmetry coefficients $A_{\pi\pi}$ and $S_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ decays; the results are $A_{\pi\pi} = 0.56 \pm 0.12 \text{ (stat)} \pm 0.06 \text{ (syst)}$ and $S_{\pi\pi} = -0.67 \pm 0.16 \text{ (stat)} \pm 0.06 \text{ (syst)}$, which together indicate large $CP$ violation. The nonzero value for $A_{\pi\pi}$ indicates direct $CP$ violation; the statistical significance (including systematic uncertainty) is 4.0σ. These values also imply that the magnitude of the penguin amplitude relative to that of the tree amplitude ($|P/T|$) is greater than 0.17 at 95.4% CL, and that the strong phase difference ($\delta$) lies in the range $(-180^\circ, -4^\circ)$ at 95.4% CL. The $\phi_3$ measurement is obtained from a Dalitz plot analysis of $B^+ \to \bar{D}^{(*)0} K^+$, $\bar{D}^0 \to K_S^0 \pi^+\pi^-$ decays; the statistical significance of the observed (direct) $CP$ violation is 98%.

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Figure 5: Dalitz plots of $\tilde{D}^0 \to K^0_S \pi^+ \pi^-$ decays obtained from samples of $B^+ \to \tilde{D}^0 K^+$ (top left), $B^- \to \tilde{D}^0 K^-$ (top right), $B^+ \to \tilde{D}^*0 K^+$ (bottom left), and $B^- \to \tilde{D}^*0 K^-$ (bottom right).
Figure 6: Confidence regions for pairs of parameters: the left-most plots correspond to $r$-$\phi_3$ and the right-most plots to $\delta$-$\phi_3$. The top row corresponds to $B^{\pm} \to D^0 K^{\pm}$ decays and the bottom row to $B^{\pm} \to D^* K^{\pm}$ decays.