Charge Density Wave Ratchet

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Abstract

We propose to operate a locally-gated charge density wave as an electron pump. Applying an oscillating gate potential with frequency $f$ causes equally spaced plateaux in the sliding charge density wave current separated by $\Delta I = 2eNf$, where $N$ is the number of parallel chains. The effects of thermal noise are investigated.
A metallic gate electrode on the surface of charge density wave (CDW) compounds can cause transistor action by modulating the threshold field for depinning of the collective CDW mode [1]. Several suggested microscopic mechanisms fail to explain the observed large asymmetric gate modulation. Nevertheless, we may expect that the sliding mode can be manipulated even more effectively on thin-films of charge density wave materials which have been grown recently [2]. Here we propose an electron-pump based on structured CDW films, which we call a “CDW ratchet”.

In the early nineties the “single-electron turnstile” was realized in both metals [3] and semiconductors [4]. In these structures electrons are transferred one by one by using time-dependent gate voltages to modulate the Coulomb blockade or by alternatively raise and lower tunnel barrier heights. The dc current through such a device scales linearly with the applied frequency \( I = ef \), and the current-(bias)voltage characteristics shows equally spaced current plateaux. The single electron pump may find an application as a current standard, since the oscillation frequency \( f \) can be controlled to high precision. The accuracy of the current is affected by numerous error mechanisms as offset charges, cycle missing, thermal fluctuations, or co-tunneling [5,6]. Some of these errors may be suppressed in linear arrays of tunnel junctions [8] or in periodically gated quantum wires [7], but devices with a charge density wave ground state should not suffer from these drawbacks at all.

Charge density waves occur in quasi one-dimensional metals, like NbSe\(_3\) or K\(_{0.3}\)MnO\(_3\) [10]. Below a critical temperature \( T_c \) (183 K for K\(_{0.3}\)MnO\(_3\)), the ground state consists of a lattice distortion coupled to an electron density modulation \( n_{CDW} \sim |\Delta(x,t)| \cos [2k_Fx + \chi(x,t)] \), where \( 2|\Delta| \) is the gap in the quasi-particle spectrum and the phase \( \chi \) denotes the position of the CDW relative to the crystal lattice. Incommensurate CDW’s support a unique sliding mode of transport above a weak threshold field. This collective motion of the CDW carries electrical current proportional to \( \partial_t \chi \) and is the source of narrow-band noise and non-linear conductance characteristics. The threshold field arises from the interaction of the CDW with defects or impurities in the system, which can, as mentioned above, be manipulated by external gate electrodes [11].

As experimental setup for the CDW electron pump we envisage a thin strip of CDW material consisting of \( N \) chains with dimensions of the order of the Fukuyama-Lee-Rice coherence lengths (\( \xi_\parallel \) is typically micrometers and \( \xi_\parallel/\xi_\perp \sim 10 - 100 \) [11]), such that the CDW is characterized by a single degree of freedom. A thin metallic gate electrode separated by an insulating layer is placed on top, perpendicular to the CDW chains, and connected to an oscillating voltage. Alternatively, one could also think of the tip of an STM as gate electrode. The dynamics of the CDW with an oscillating time-dependent gate potential can be described by the classical equation of motion for the phase \( \chi(t) \) in the single particle model [5]

\[
\eta \frac{\partial \chi}{\partial t} + V_p(\chi; t) = V_b + V_n(t)
\]

where \( \eta = \hbar R_c/eR_Q \), \( R_c \) is a damping resistance and \( R_Q = \hbar/2Ne^2 \) is the \( N \)-mode quantum resistance. The pinning potential \( eV_p \) is periodic in the phase and contains an explicit time dependence \( V_p(\chi+2\pi;t+2\pi/\omega) = V_p(\chi;t) \), where \( \omega = 2\pi f \) is the frequency of the oscillating gate potential. The driving term consists of the bias voltage \( V_b \) and a thermal (Nyquist) noise term \( V_n(t) \) with \( \langle V_n(t) \rangle = 0 \) and \( \langle V_n(t)V_n(t') \rangle = 2\eta k_B T \delta(t-t')/e \). We will
disregard the inertia since CDW’s are in general strongly overdamped due to their large effective mass. Note that there is no oscillating drive term, which is known to lead to phase-locking and Shapiro steps in the current–voltage characteristic. The interpretation of Eq. (1) is straightforward: a thermally activated classical particle moving in a tilted washboard potential, of which the amplitude changes periodically in time. The applied electric field directs the motion of the CDW, and the oscillating gate guides the CDW downwards, thus causing an electric current.

In the presence of thermal noise, the dynamics of the CDW can be described by a Fokker-Planck equation for the probability density \( P(\chi, \tau) \) of finding the phase \( \chi \) in the interval \( \chi + d\chi \) at time \( \tau \) \[1\]. In the following we will assume that the pinning potential may be approximated by its lowest harmonics:

\[
V_p(\chi; t) = (V_T + \alpha V_g \sin \omega t) \sin \chi, \tag{2}
\]

where \( V_T \) is a constant threshold potential, which is modulated by the fraction \( \alpha \) of the oscillating gate potential \( V_g \). In this case the Fokker-Planck equation reads

\[
\frac{\partial P}{\partial \tau} = D \frac{\partial^2 P}{\partial \chi^2} + (1 + \tilde{V}_g \sin \tilde{\omega} \tau) \frac{\partial}{\partial \chi} (\sin \chi P) - \tilde{V}_b \frac{\partial P}{\partial \chi}, \tag{3}
\]

where we introduced the dimensionless parameters \( \tilde{V}_g = \alpha V_g / V_T \), \( \tilde{V}_b = V_b / V_T \), \( D = k_B T / e V_T \), and \( \tilde{\omega} = \eta \omega / V_T \). The initial condition at time \( \tau = \tau_0 \) is given by \( P(\chi, \tau_0) = \delta(\chi - \chi_0) \). The problem is now reduced to the diffusion of a classical particle in a time dependent periodic potential. By substituting the Fourier series

\[
P(\chi, \tau) = \sum_{n=-\infty}^{\infty} P_n(\tau) e^{-in\chi} \tag{4}
\]

into Eq. (3) we obtain the equation for the Fourier components \( P_n(\tau) \)

\[
\frac{\partial P_n}{\partial \tau} = (-Dn^2 + in\tilde{V}_b)P_n - (1 + \tilde{V}_g \sin \tilde{\omega} \tau) \frac{n}{2} [P_{n+1} - P_{n-1}]. \tag{5}
\]

The total dc current \( I \) through the system is defined as

\[
I = \frac{V_b}{R_c} - \frac{1}{2i R_c} < (V_T + V_g \sin \tilde{\omega} \tau) [P_1(\tau) - P_{-1}(\tau)] >, \tag{6}
\]

where the brackets denote time averaging.

We first consider the \( T = 0 \) (noiseless) limit (Eq. (1)). Without gate voltage the current is obviously given by \( I = 0 \) for \( V_b < V_T \) and \( I = \sqrt{V_b^2 - V_T^2} / R_c \) for \( V_b \geq V_T \). Figure [3] shows the \( I - V_b \) characteristic for different values of the external frequency \( f \). Distinct current plateaux appear in the \( I - V_b \) below the normal threshold potential. Each plateau corresponds to the displacement of a quantized number of wave fronts in one cycle. The steps are equally separated by \( \Delta I = 2eNf \), where the factor 2 reflects spin-degeneracy. Far above threshold the differential resistance approaches its normal value \( R_c \). Furthermore, the
current-frequency relation has a fan structure, as is shown in Fig. 2. The current scales linearly with frequency

$$I = 2emNf,$$  \hspace{1cm} (7)

where the integer $m$ is the number of displaced wavelengths in one cycle. In the limit $\omega \to 0$ the current approaches the constant value as

$$I(\omega \to 0) = \frac{1}{2\pi R_c} \text{Re} \int_0^{2\pi} d\tau \sqrt{V_b^2 - (V_T + V_g \sin \tau)^2}. \hspace{1cm} (8)$$

The current drops to zero at the frequency which corresponds to the time to displace the CDW by one wavelength.

Next we investigate the effects of the thermal noise at finite temperatures. In the case where the external oscillating gate potential is absent, the stationary state solution to Eq. (5) is easily calculated as

$$P_n(\tau \to \infty) = \frac{I_{n-i\omega_0}(z)}{I_{-i\omega_0}(z)}, \hspace{1cm} (9)$$

where $I_q(z)$ is a modified Bessel function with imaginary argument $q$, $z_0 = \tilde{V}_b/D$ and $z = 1/D$. Using the relation $P_n^* = P_{-n}$, the CDW current is obtained from Eq. (8)

$$I = \frac{V_b}{R_c} - \frac{V_T}{R_c} \text{Im} \frac{I_{1-i\omega_0}(z)}{I_{-i\omega_0}(z)}. \hspace{1cm} (10)$$

This static result is exactly analogous to the case of strongly overdamped Josephson junctions with an external noise current [13,14]. The main effect of finite temperatures is a smoothening of the square-root threshold singularity near $V_T$ and an exponentially small (for $1/D \gg 1$) but nonzero conductance as $V_b \to 0$. In the presence of the oscillating gate we solve Eq. (3) numerically. In Fig. 3 the $I - V_b$ curves at external frequency $\tilde{\omega} = 0$ and $\tilde{\omega} = 0.4$ are shown for different temperatures $D = k_B T/eV_T$. As expected, finite temperatures smear out the sharp transitions between the current plateaux. This is more pronounced at larger bias, since the escape rate due to the thermal fluctuations increases. At even higher temperatures $1/D \lesssim 5$ the plateaux disappear and the resistance becomes linear $V_b = IR_c$.

For operation as a current standard, an individual CDW ratchet must first be gauged to determine the number of parallel chains $N$. Once known, the linear dependence of the current on $N$ in principle improves the accuracy of the current quantization as compared to single electron pumps. The bandwidth of single-electron turnstile devices is limited by the competition between large detection currents (large frequencies) and low noise levels (low frequencies). The present CDW device, however, allows for large currents at low frequencies, and is robust to the error sources of the single-electron pump.

Since CDW’s have a large single-particle energy gap $2|\Delta|$, the quasi-particle contribution to the current is of the order $\exp(-2|\Delta|/k_BT)$ and can be neglected for sufficiently low temperatures $T \ll T_c$. The gap also makes the system robust to static disorder and single-particle quantum fluctuations in the total charge per unit cell. Phase-slip processes at the current contacts can be avoided in a four-terminal measurement, where the voltage probes
are located far from the current source and drain. Higher harmonics of the pinning potential Eq. (2) will give corrections only near the thresholds. The elasticity of the CDW can phenomenologically be described by geometrical capacitances $C$ of the leads, which define the typical Coulomb energy for phase deformations. The assumption of a rigid CDW is then justified in the limit of weak pinning and low temperatures, such that $|\Delta| \gg e^2/C \gg eV_T \gg k_B T$. We disregarded the role of macroscopic quantum tunneling of the CDW, which we consider less important than coherent co-tunneling in single-electron devices because of the high effective mass.

We conclude by summarizing our results. We present the idea of a locally gated CDW as an electronic ratchet. An oscillating gate potential causes equidistant current plateaux. The current scales linearly with the external frequency and the number of chains. The coherent electronic ground state of CDW’s and the intrinsic property of parallel conducting chains makes this device a serious candidate for the current standard with possible higher accuracy than single electron devices, even in the presence of thermal noise.

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FIGURES

FIG. 1. Current-voltage characteristic of the CDW ratchet at $T = 0$ for external frequencies $\tilde{\omega} = 0; 0.1; 0.5$. The current plateaux are equally spaced by $\Delta I = 2eNf$.

FIG. 2. Current-frequency fan at bias voltages $\tilde{V}_b = 0.4$ and 0.5.

FIG. 3. Current-voltage characteristic of the CDW ratchet including thermal noise for different temperatures, $eV_T/k_B T = \infty, 100, 20, 10$. 