Extended object model for grinding operation

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Abstract. The article discusses the issues of building a dynamic model of a flat grinding operation, taking into account the statistical characteristics of the disturbances. Deviations of the shape of the grinding wheel in a static mode are random in nature and, when analyzing the processing process, can be taken into account by introducing an additional link into the system - the corresponding shaping filter. Such a filter can be built on the basis of profile analysis for any particular grinding wheel with a representation of the effect on the system in the state space as equations of state and observation. On the basis of the developed model for grinding operations in floating workshops, it is possible to use high-precision CNC machines equipped with adaptive control systems based on optimal systems with a stochastic process controller.

1. Introduction
The increase in strength, hardness and wear resistance of structural materials has determined a general tendency to reduce material processing, which leads to increased tool wear, an increase in cutting forces, deformations, and consequently, a decrease in machining accuracy and quality of machined surfaces. Under these conditions, the task of creating and improving high-performance finishing methods for the material processing, on which the specified quality parameters are formed, becomes particularly relevant.

2. Modeling process dynamics
One of the most common methods for materials processing is flat grinding. Achieving the specified quality parameters during processing is possible by taking into account the dynamics of the technological system.

Based on the analysis of the interaction of the elements of the dynamic system, taking into account the presentation of the grinding process in the form [1], it is possible to construct a description of the dynamics of the machining process in the form:

\[
\begin{align*}
& m_1 \ddot{x}_1 + h_i \dot{x}_1 + c_i (x_{10} + x_1) + h_3 (\dot{x}_1 + \Delta \dot{R}) + c_3 (x_{10} + x_1 + R + \Delta R) - h_3 (\Delta \dot{x}_1 - \dot{R}) - c_3 (x_{20} + x_2 - k - \Delta k) = 0, \\
& m_2 \ddot{x}_2 + h_2 \dot{x}_2 + c_2 (x_{20} + x_2) + h_3 (\dot{x}_2 - \Delta \dot{R}) + c_3 (x_{20} + x_2 - k - \Delta k) - h_3 (\dot{x}_1 + \Delta \dot{R}) - c_3 (x_{10} + x_1 + R + \Delta R) - h_2 \dot{L} - c_3 (\dot{L} + \Delta L) = 0,
\end{align*}
\]

where \( m_1, m_2 \) – reduced mass of the workpiece with the device and a circle with a spindle; \( h_i \) – \( i \)-link resistance coefficient; \( C_i \) – stiffness coefficient of the \( i \)-th link; \( x_{10}, x_{20}, x_1, x_2 \) – coordinates of the
center of rotation of the circle and the base surface of the part and their increments, respectively; \( R \) – circle radius; \( k \) – corresponding linear part size; \( \Delta R, \Delta k \) – variations of the radius of the circle and the linear size of the workpiece, respectively; \( L, \Delta L \) – the distance from the base surface of the part to the center of rotation of the circle and its change along the limb of the machine.

Taking into account the conditions of the power circuit of the technological system under the nominal processing conditions, it is possible to construct relations characterizing the process dynamics in variations:

\[
\begin{aligned}
&\begin{cases}
  m_1 \ddot{x}_1 + h_1 \dot{x}_1 + c_1 x_1 + h_3 (\dot{x}_1 + \Delta \dot{R}) + c_3 (x_1 + \Delta R) - h_3 (\dot{x}_2 - \Delta \dot{k}) - c_3 (x_2 - \Delta k) = 0, \\
  m_2 \ddot{x}_2 + h_2 \dot{x}_2 + c_2 x_2 + h_3 (x_2 - \Delta k) + c_3 (x_2 - \Delta k) - h_3 (x_1 + \Delta \dot{R}) - c_3 (x_1 + \Delta R) - h_2 \dot{L} - c_2 \Delta L = 0.
\end{cases}
\end{aligned}
\]

It is expedient to present the system (2) in the form of the Frobenius state-space:

\[
Z_0 = A_0 Z_0 + B_0 W_0 + C_0 U_0
\]

\[
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \dot{x}_3 \\
  \dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  -\frac{a_0}{a_4} & -\frac{a_1}{a_4} & -\frac{a_2}{a_4} & -\frac{a_3}{a_4} \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
+ (R_1 + R_2)
\begin{bmatrix}
  \lambda_1 \\
  \lambda_2 \\
  \lambda_3 \\
  \lambda_4
\end{bmatrix}
S,
\]

where \( Z_1 = x_1, Z_2 = z_1 p = \dot{x}_1, Z_3 = z_2 p = \dot{x}_1, Z_4 = z_3 p = \ddot{x}_1, Z_5 = z_4 p = x_1; \)

\( R = \Delta R; R_2 = \Delta k; \Delta L = S; \lambda_1 = \beta_1; \lambda_2 = \beta_2 = \beta_1 - \alpha_1 \lambda_1; \lambda_3 = \beta_0 - \alpha_1 \lambda_2 - \alpha_2 \lambda_1; \)

\( \alpha_4 = (m_1 m_2); \alpha_5 = (h_1 m_2 + h_2 m_1 + m_1 m_2 + h_3 m_1); \alpha_2 = (h_1 h_2 + h_1 h_3 + h_2 h_3 + q_1 m_2 + q_1 m_1 + q_2 m_2 + q_2 m_1); \)

\( \beta_2 = h_2 h_3; \beta_3 = (h_2 h_3 + h_2 h_4); \beta_0 = q_2 c; \)

\( \lambda_4 = h_2 h_3 = \beta_2; \)

\( \lambda_0 = (c_1 h_2 + c_2 h_3) - (h_1 m_2 + h_2 m_1 + h_3 m_1) h_2 h_3 = \beta_1 - \alpha_3 \lambda_1; \)

\( \lambda_3 = c_2 c_3 - (h_3 m_2 + h_3 m_1 + h_3 m_1) h_2 h_3 \times [(c_2 h_2 + c_2 h_3) - (h_3 m_2 + h_3 m_2 + h_3 m_1 + h_3 m_1) h_2 h_3] = \beta_0 - \alpha_3 \lambda_2 - \alpha_2 \lambda_1; \)

\( \gamma_1 = -h_3 m_2 = -\chi_3; \)

\( \gamma_2 = -(c_2 m_2 + h_3 h_2) - (h_3 m_2 + h_3 m_2 + h_3 m_1 + h_3 m_1)(-h_2 m_2) = -\chi_3 - \alpha_2 \gamma_1; \)

\( \gamma_3 = -(c_2 h_2 + c_2 h_3) - (h_3 m_2 + h_3 m_2 + h_3 m_1 + h_3 m_1) \times [(c_2 h_2 + c_2 h_3) - (h_3 m_2 + h_3 m_2 + h_3 m_2 + h_3 m_1) h_2 h_3] = -\chi_3 - \alpha_2 \gamma_2 - \alpha_2 \chi_1; \)

\( \gamma_4 = -c_2 c_3 - (h_3 m_2 + h_3 m_2 + h_3 m_1 + h_3 m_1) \times (-c_2 h_2 + c_2 h_3) - (h_3 m_2 + h_3 m_2 + h_3 m_1 + h_3 m_1)(-h_3 m_2) \times [(c_2 h_2 + c_2 h_3) - (h_3 m_2 + h_3 m_2 + h_3 m_2 + h_3 m_1) h_2 h_3] = -\chi_3 - \alpha_3 \gamma_3 - \alpha_2 \gamma_2 - \alpha_2 \chi_1. \)

The Kalman controllability of the system (3) can be analyzed directly using the numerical values of the above parameters for the object matrices \( A_0 \) and operation \( C_0. \)
The analysis of the work of the field of study of the dynamics of grinding processes [2-5] shows that the most unstable link in the dynamic system under consideration is the grinding wheel. In accordance with model (2), the main parameters of the grinding wheel, which influence the dynamics of the machining process, are static and dynamic variations of its profile. Static variations are deviations of the circle shape from the ideal in the static mode, and dynamic variations are deviations of the shape caused directly by the processing. The latter include the deviations of the trajectory of the circle surface from the given trajectories due to its imbalance. These deviations are deterministic in nature and can be significantly reduced by known technological methods, such as pre-balancing a circle. Deviations of the grinding wheel shape in a static mode are random in nature and, when analyzing the processing process, they can be taken into account by introducing into the system (3) an additional link - the corresponding shaping filter [6-8]. Such a filter can be built, for example, on the basis of a profile analysis for any particular grinding wheel with a representation of the impact on the system (3) in the state-space as equations of state (4) and observation (5) [9-11]:

\[
\begin{align*}
\dot{G}_1 &= A_1 G_1 + B_1 W_1, \\
\dot{y}_1 &= D_1 G_1 + E_2 v_2
\end{align*}
\]

where \( G_1 = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \ A_1 = \begin{bmatrix} 0 & 1 \\ -\frac{1}{T_1^2} & -\frac{1}{T_2^2} \end{bmatrix}, \ B_1 = \begin{bmatrix} K T_3 \\ -1 - K T_2 T_3 \end{bmatrix}, \ D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ E_2 = \rho, \)

\( W_1, v_2 - \) uncorrelated single generating Gaussian white noises,

\( T_1 = \frac{1}{\sqrt{\alpha^2 + \beta^2}}, \ T_2 = \frac{2\alpha}{\sqrt{\alpha^2 + \beta^2}}, \ T_T = \frac{1}{\sqrt{\alpha^2 + \beta^2}}, \ K = \sqrt{\frac{D\alpha}{\sqrt{\alpha^2 + \beta^2}}}, \)

\( D - \) dispersion of the heights of the unevenness of the surface of the grinding wheel, \( v - \) its speed, \( \alpha, \beta - \) corresponding correlation coefficients.

Taking into account \( y_0 = y_1 \) with the introduction of the shaping filter (4), the system (3) takes the form:

\[
\begin{align*}
\dot{z}_o &= A_o z_o + B_o y_1 + C_o u_o \\
\dot{G}_1 &= A_1 G_1 + B_1 W_1 \\
\dot{y}_1 &= D_1 G_1 + E_2 v_2 \\
y_o &= D_o z_o + E_o v_o
\end{align*}
\]

which leads to:

\[
\begin{align*}
\dot{x}_o &= A_o x_o + B_o D_1 G_1 + B_o E_2 v_2 + C_o u_o, \\
\dot{G}_1 &= A_1 G_1 + B_1 W_1
\end{align*}
\]

that allows us to represent the equation of state of the grinding process, taking into account the disturbances determined by the statistical characteristics of the circle as a state of the expanded object:

\[
\begin{bmatrix} \dot{z}_o \\ \dot{G}_1 \end{bmatrix} = \begin{bmatrix} A_o & B_o D_1 \\ 0 & A_1 \end{bmatrix} \times \begin{bmatrix} x_o \\ G_1 \end{bmatrix} + \begin{bmatrix} B_o E_2 & 0 \\ 0 & B_1 \end{bmatrix} \times \begin{bmatrix} v_2 \\ W_1 \end{bmatrix} \times \begin{bmatrix} C_o \\ 0 \end{bmatrix} \times u_o
\]

or
System (7) represents the standard form for describing a dynamic system in terms of the state-space theory, which allows using it to study the characteristics and behavior of the grinding process, as well as to synthesize control systems for this process.

3. Perspectives of use

On the basis of the developed model for grinding operations in floating workshops, it is possible to use high-precision CNC machines equipped with adaptive control systems, for example, on the basis of optimal systems with a stochastic process controller.

To synthesize an optimal deterministic controller, we use the method of synthesizing a linear regulator, which was first proposed by Kalman [12]. At the same time, an optimality criterion is necessary. Such a criterion for linear systems with Gaussian disturbances such as white noise can be a functional of the form [8]:

$$I = M \left[ \frac{1}{2} \gamma^T(t)P_1\gamma(t) + \frac{1}{2} \int_0^t \left( \chi^T(t)R_1\chi(t) + u^T(t)R_1^{-1}u(t) \right) dt \right],$$

where $Q_1$, $P_1$, $R_1^{-1}$—corresponding weight matrices of control quality.

Because $w(t)$, $v(t)$—are by [8] independent white noises with intensities $Q_1$ or $R_1$ accordingly, the control is unbiased and optimal for the stochastic formulation [13], therefore, the following structure is valid for the control algorithm:

$$u^*(t) = -K_{y_0}\gamma(t); \quad K_{y_0} = R_1^{-1}E_iP_1,$$

and the matrix, where $P_1$ satisfies the equation:

$$\dot{P}_1 = \tilde{P}_1 A + A^T\tilde{P}_1 + P_1BR_1^{-1}B^T\tilde{P}_1 - Q_1, \quad P_1(0) = P_{10}.$$

The system with the regulator takes the form:

$$\dot{\gamma} = A\gamma + B\gamma(t) + K_{y_0}[\gamma(t) - C\gamma(t)]; \quad \gamma(0) = \tilde{y}(0)$$

due to the fact that the initial state $z(0)$ uncorrelated with $w(t)$, $v(t)$ and distributed according to the normal law with the expectation $M(\gamma_0) = \tilde{y}(0)$ and covariance $M[(\gamma - \tilde{y}_0)(\gamma - \tilde{y}_0)^T] = P_{20}.$

To get ratings $\gamma$ built Kalman filter with coefficient $K = \tilde{V}\cdot C^T \cdot \Psi_{y_0}^{-1}$. Redefine $K_{y_0} = P_2C^TR_2^{-1}$, then the Riccati equation is written:

$$\dot{P}_2 = AP_2 + P_2A^T - P_2C^T R_2^{-1}CP_2 + Q_2, \quad P_2(0) = P_{20}.$$

$$\begin{bmatrix}
    \dot{z}_1 \\
    \dot{z}_2 \\
    \dot{z}_3 \\
    \dot{z}_4 \\
    \dot{\psi}_1 \\
    \dot{\psi}_2
\end{bmatrix} = 
\begin{bmatrix}
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 \\
    -\frac{\alpha_2}{\alpha_4} & \frac{-\alpha_2}{\alpha_4} & \frac{-\alpha_2}{\alpha_4} & \frac{\alpha_2}{\alpha_4} & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4 \\
    \psi_1 \\
    \psi_2
\end{bmatrix} + 
\begin{bmatrix}
    \rho_1 \\
    \rho_2 \\
    \rho_3 \\
    \rho_4 \\
    0 \\
    0
\end{bmatrix} 
\begin{bmatrix}
    V_0 \\
    W_0
\end{bmatrix} + 
\begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix} \times S \cdot (7)
Based on (7), a closed control loop consisting of an object, a filter and a regulator for a technological grinding system (Figure 1) is described by a generalized system of differential equations [14]:

\[
\begin{bmatrix} y' \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A & -BK_g \\ K_iC & A-BK_g-K_iC \end{bmatrix} \times \begin{bmatrix} y' \\ \dot{y} \end{bmatrix} + \begin{bmatrix} E' \\ 0 \end{bmatrix} \times w + \begin{bmatrix} 0 \\ RK_i \end{bmatrix} \times v. \tag{11}
\]

![Block diagram of the grinding process control system.](image)

**Figure 1.** Block diagram of the grinding process control system.

### 4. Conclusions

On the basis of the models obtained [15], it is possible to create an automatic process control system of technological process, the use of which will make it possible to effectively use software control cycles without losing stability in the product quality indicators with external influences on the machine technological system.

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