Dieker, A. B.; Saliola, F. V.
Spectral analysis of random-to-random Markov chains. (English) Zbl 1405.60113
Adv. Math. 323, 427-485 (2018).

Summary: We compute the eigenvalues and eigenspaces of random-to-random Markov chains. We use a
family of maps which reveal a remarkable recursive structure of the eigenspaces, yielding an explicit and
effective construction of all eigenbases starting from bases of the kernels.

MSC:
60J10 Markov chains (discrete-time Markov processes on discrete state
spaces)
60B15 Probability measures on groups or semigroups, Fourier transforms,
factorization

Keywords:
random-to-random shuffling; spectrum; discrete Markov chain; representation theory; symmetric group

Software:
Sage-Combinat; SageMath

Full Text: DOI arXiv

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