Nonequilibrium Fluctuations and Decoherence in Nanomechanical Devices Coupled to the Tunnel Junction.

Anatoly Yu. Smirnov
D-Wave Systems Inc., 320-1985 W. Broadway
Vancouver, British Columbia Canada V6J 4Y3

Lev G. Mourokh and Norman J.M. Horing
Department of Physics and Engineering Physics,
Stevens Institute of Technology, Hoboken, NJ 07030
(March 22, 2022)

Abstract

We analyze the dynamics of a nanomechanical oscillator coupled to an electrical tunnel junction with an arbitrary voltage applied to the junction and arbitrary temperature of electrons in leads. We obtain the explicit expressions for the fluctuations of oscillator position, its damping/decoherence rate, and the current through the structure. It is shown that quantum heating of the oscillator results in nonlinearity of the current-voltage characteristics. The effects of mechanical vacuum fluctuations are also discussed.

85.85.+j, 73.63.-b, 03.65.Yz
I. INTRODUCTION

The rapid development of nanotechnology in recent years has ushered in a new generation of quantum electronic devices incorporating the mechanical degrees of freedom [1,2], so-called nanoelectromechanical systems (NEMS). A particular example of such a device is the tunnel junction having its transition matrix element modulated by a vibrational motion. This modulation can be achieved by the introduction of small grains (shuttles) embedded in the elastic medium between the leads [3,4,5,6,7], or by coupling the tunnel junction to a mechanical oscillator (cantilever) [8,9]. These two cases can be described by similar formulations but the oscillator masses are very different (for example, \( m \sim 10^{-20}\) g for the \( C_{60} \) molecule used as a shuttle and \( m \sim 10^{-12}\) g for the cantilever). Furthermore, in addition to nanomechanical applications, such systems have a direct relation to the fundamental quantum theory of measurements [11,12]. In this context, the tunnel junction can be considered as a device measuring the position of the oscillator. As was shown in Ref. [10] for the case of a large voltage applied to the junction, this measurement process induces decoherence and dephasing of the oscillator, even at zero temperature. From another point of view, the nanomechanical oscillator can serve as a measurement device for the electronic subsystem. A similar situation takes place in magnetic resonance force microscopy [13], where the magnetic moment of electron or nuclear spins in the sample (attached to the cantilever) is measured by means of optical interferometry of the cantilever motion.

There are two manifestations of the effect of mechanical oscillations on system behavior and, consequently, there are two characteristic scales. The first phenomenology occurs when the uncertainty of the oscillator position is of the order of the tunneling length. This is always the case for the shuttle system, but for the cantilever it only exists at high bias accompanied by oscillator heating. (In the present paper we simplify the model used in Refs. [3,4,5], such that our shuttle system has a permanent electrical connection to one lead, while the connection to the other lead is mediated by the tunnel junction.)

The second class of phenomena is associated with resonance between the oscillator frequency and the bias voltage applied to the tunnel junction. The characteristic frequencies of the oscillator are of order of 1GHz [1,14] and this resonance occurs at sufficiently low bias. It should be noted that the theoretical approach used by Mozyrsky and Martin in Ref. [10] fails at low bias voltage and requires improvement.

NEMS can be used as an ultrasensitive device for magnetic and biological applications and, therefore, its own fluctuation level - determining the signal/noise ratio - is of crucial importance. Moreover, quantum measurement procedures also require a high level of sensitivity. The mechanical oscillator can be considered as an open quantum system interacting with two electron reservoirs (leads) serving as an effective heat bath. In the present paper we employ the general theory of open quantum systems developed in Refs. [15,16,17] to determine the nonequilibrium fluctuations of the mechanical oscillator for arbitrary bias voltage and temperature. Current noise created by the junction is assumed to be the main mechanism of decoherence of the oscillator.

We derive general formulas and employ them in the two cases described above, showing that at low temperature the bias applied to the tunnel junction leads to oscillator heating at voltages larger than the critical value associated with the characteristic frequency of the oscillator. With increasing temperature this resonant picture is smoothed and, moreover,
for higher temperature the fluctuation level approaches that of an uncoupled oscillator. Furthermore, we show that in the shuttle case, a nonlinearity of the oscillator-junction interaction makes a pronounced contribution to the level of mechanical vacuum fluctuations even for zero bias and weak coupling between the above-mentioned subsystems.

II. GENERAL FORMALISM

The Hamiltonian of the system \( \text{tunnel junction and mechanical oscillator} \) is given by

\[
H = H_L + H_R + H_{\text{tun}} + H_0. \tag{1}
\]

Here,

\[
H_\alpha = \sum_k E_{k\alpha} c_{k\alpha}^+ c_{k\alpha}
\]

is the Hamiltonian of the left, right leads (\( \alpha = L, R \), respectively)

\[
H_0 = \frac{p^2}{2m} + \frac{m\omega_0^2 x^2}{2} \tag{3}
\]

is the Hamiltonian of the mechanical oscillator with a mass \( m \) and the resonant frequency \( \omega_0 \), and

\[
H_{\text{tun}} = -\sum_{kq} (T_{kq} c_{kL}^+ c_{qR} + h.c.) e^{-x/\lambda} \tag{4}
\]

is a tunneling term depending on a position of the oscillator, where \( \lambda \) is the characteristic tunneling length and \( h.c. \) is the Hermitian conjugate. The electron gas in the leads plays the role of a heat bath for the oscillator and their nonlinear interaction given in Eq.(4) can be written in the form

\[
H_{\text{tun}} = -Q e^{-x/\lambda} \tag{5}
\]

where \( Q(t) \) is interpreted as an effective heat bath variable given by

\[
Q(t) = \sum_{kq} (T_{kq} c_{kL}^+ c_{qR} + h.c.). \tag{6}
\]

The heat bath thus defined is characterized by a response function \( \varphi(t, t_1) \) and a symmetrized correlation function \( M(t, t_1) \) of the unperturbed variables \( Q^{(0)}(t) \) taken in the absence of interaction (\( \hbar = 1, k_B = 1 \)), as

\[
\varphi(t, t_1) = i\langle [Q^{(0)}(t), Q^{(0)}(t_1)]_- \rangle \theta(t - t_1),
\]

\[
M(t, t_1) = \frac{1}{2} \langle [Q^{(0)}(t), Q^{(0)}(t_1)]_+ \rangle, \tag{7}
\]

where \( \theta(\tau) \) is the unit Heaviside step function and \([..., ...]_+ \) and \([..., ...]_- \) are the anticommutator and the commutator, respectively. The angle brackets refer to averaging over the equilibrium states of both the left and right leads at the same information. The chemical
potentials of the leads can be different with $\mu_R - \mu_L = eV$, where $V$ is a voltage applied to the tunnel junction. For weak coupling between the mechanical oscillator and the electronic bath (weak tunneling) the action of the oscillator on the dissipative environment is described by the formula \[15\]

$$Q(t) = Q^{(0)}(t) + \int dt_1 \varphi(t, t_1)e^{-x(t_1)/\lambda}. \tag{8}$$

In turn, the effect of the dissipative environment on the mechanical oscillator is determined by substituting Eq.(8) into the Heisenberg equation of motion for the position operator of the oscillator given by

$$\ddot{x} + \omega_0^2 x = -\frac{1}{m\lambda}Q(t)e^{-x(t)/\lambda}. \tag{9}$$

To eliminate the unperturbed heat bath variables $Q^{(0)}(t)$ from Eq.(9), we apply the quantum Furutsu-Novikov theorem \[15\]

$$\langle Q^{(0)}(t)e^{-x(t)/\lambda} \rangle = \int dt_1 \langle Q^{(0)}(t), Q^{(0)}(t_1) \rangle \langle \frac{\delta}{\delta Q^{(0)}(t_1)}e^{-x(t_1)/\lambda} \rangle, \tag{10}$$

where $\delta/\delta Q^{(0)}(t_1)$ is the functional derivative with respect to the free heat bath variable $Q^{(0)}(t_1)$. The functional derivative of an arbitrary operator $A(t)$ of the dynamical system is proportional to the commutator \[15\] in the form

$$\frac{\delta A(t)}{\delta Q^{(0)}(t_1)} = \frac{i}{\hbar} \left[ A(t), e^{-x(t_1)/\lambda} \right] \theta(t - t_1). \tag{11}$$

As the result, we obtain the non-Markovian stochastic equation of motion for the position operator of the oscillator as given by

$$\ddot{x} + \omega_0^2 x = \xi(t)$$

$$-\frac{1}{\lambda m} \int dt_1 \left( \tilde{M}(t, t_1)i \left[ e^{-x(t_1)/\lambda}, e^{-x(t_1)/\lambda} \right] \right) \varphi(t, t_1) \frac{1}{2} \left[ e^{-x(t)/\lambda}, e^{-x(t_1)/\lambda} \right] \theta(t - t_1), \tag{12}$$

where $\tilde{M}(t, t_1) = M(t, t_1)\theta(t - t_1)$ is the causal correlation function of the heat bath and $\xi(t)$ is the fluctuation source,

$$\xi(t) = -\frac{1}{m\lambda} \left( Q^{(0)}(t)e^{-x(t)/\lambda} - \int dt_1 \langle Q^{(0)}(t), Q^{(0)}(t_1) \rangle i \left[ e^{-x(t)/\lambda}, e^{-x(t_1)/\lambda} \right] \theta(t - t_1) \right), \tag{13}$$

having zero mean value, $\langle \xi \rangle = 0$, according to Eq.(10). The Langevin-like equation, Eq.(12), as well as the whole method of Refs. \[15,16,17\], takes into account a nonlinearity of coupling between the subsystems and, in addition, incorporates the nonlocal character of heat bath fluctuations. In these respect this treatment goes beyond the well-known Caldeira-Leggett approach \[15\].

In the case of weak coupling, the correlation function of fluctuation sources is given by

$$\langle \frac{1}{2} [\xi(t), \xi(t')]_+ \rangle = \frac{1}{4m^2\lambda^2} \left\{ \langle [Q^{(0)}(t), Q^{(0)}(t')]_+ \rangle \langle \left[ e^{-x(t)/\lambda}, e^{-x(t')/\lambda} \right]_+ \rangle + \langle [Q^{(0)}(t), Q^{(0)}(t')]_- \rangle \langle \left[ e^{-x(t)/\lambda}, e^{-x(t')/\lambda} \right]_- \rangle \right\}. \tag{14}$$
It should be mentioned that the fluctuations of electronic variables \( \{Q^{(0)}(t)\} \) are non-Gaussian. However, as indicated above, Eqs.(8),(10),(14) are valid for weak coupling between the dynamical system (mechanical oscillator) and the dissipative environment (electrons in the leads). In this case we can calculate the (anti)commutators in Eqs.(12),(14) using a free evolution approximation

\[
x(t) = x(t_1) \cos \omega_0(t - t_1) + \frac{p(t_1)}{m \omega_0} \sin \omega_0(t - t_1),
\]

and employing the Baker-Hausdorff theorem, we obtain

\[
\frac{1}{2} \left[ e^{-x(t)/\lambda}, e^{-x(t_1)/\lambda} \right]_+ = \cos(\nu_0 \sin \omega_0 \tau) \exp \left( -\frac{x(t) + x(t_1)}{\lambda} \right),
\]

\[
i \left[ e^{-x(t)/\lambda}, e^{-x(t_1)/\lambda} \right]_- = 2 \sin(\nu_0 \sin \omega_0 \tau) \exp \left( -\frac{x(t) + x(t_1)}{\lambda} \right).
\]

Here,

\[
\nu_0 = \frac{\hbar}{2m \omega_0 \lambda^2}
\]

is the square of the ratio of the unperturbed oscillator position uncertainty at zero temperature and the tunneling length \( \lambda \). The operator \( x(t) \) can be written as a sum of the mean and fluctuating parts, \( x(t) = \bar{x}(t) + \tilde{x}(t) \). Even if the averaged position of the oscillator is small compared to the tunneling length, \( \bar{x}(t) \ll \lambda \), the fluctuating amplitude can be of the order of the tunneling length causing nonlinear effects. To examine this nonlinearity, we assume that oscillator position fluctuations are approximately described by Gaussian statistics with a dispersion \( \langle \tilde{x}^2 \rangle = \langle \tilde{x}(t)\tilde{x}(t) \rangle \). In this approximation, we obtain the exponent involved in Eq.(16) as

\[
\exp \left( -\frac{x(t) + x(t_1)}{\lambda} \right) = \left[ 1 - \frac{\bar{x}(t) + \bar{x}(t_1)}{\lambda} - \frac{\bar{x}(t) + \bar{x}(t_1)}{\lambda} \right] \exp \left( \frac{\langle \tilde{x}^2 \rangle + \langle (1/2)[\tilde{x}(t), \tilde{x}(t_1)]_+ \rangle}{\lambda^2} \right).
\]

Eqs.(16) and (18) provide the expressions required on the right side of the non-Markovian stochastic equation, Eq.(12). The dispersion \( \langle \tilde{x}^2 \rangle \) and the correlator \( \langle (1/2)[\tilde{x}(t), \tilde{x}(t_1)]_+ \rangle \) have to be determined self-consistently. To accomplish this, we suppose that the operator \( \tilde{x} \) of the oscillator position obeys the equation

\[
\ddot{\tilde{x}} + \gamma \dot{\tilde{x}} + \omega_0^2 \tilde{x} = \xi,
\]

where the damping rate \( \gamma \) can be calculated from the collision term on the right of Eq.(12). Using Eq.(19), we obtain correlator and the dispersion of the oscillator position as

\[
\langle \tilde{x}^2 \rangle = \frac{K(\omega_0)}{2 \omega_0 \gamma},
\]

\[
\frac{1}{2} \langle [\tilde{x}(t), \tilde{x}(t_1)]_+ \rangle = \langle \tilde{x}^2 \rangle e^{-\gamma |t-t_1|/2} \cos \omega_0(t - t_1).
\]
Here,

\[ K(\omega) = \int d\tau e^{i\omega\tau}\langle \frac{1}{2}[\xi(\tau), \xi(0)]_+ \rangle \tag{21} \]

is the spectral function of the fluctuation forces, Eq.(14). Assuming that the averaged deviation of the oscillator position is much less than the tunneling length \( \lambda \), \( \bar{x} \ll \lambda \) (the root-mean-square amplitude of mechanical oscillations can be of the order of \( \lambda \)), we obtain a simplified equation for the total coordinate \( x = \bar{x} + \tilde{x} \) as

\[ \ddot{x} + \omega_0^2x + \frac{1}{m\lambda^2}\int dt_1L(t-t_1)[x(t) + x(t_1)] = \xi(t) - \frac{1}{m}F_0, \tag{22} \]

where the collision kernel \( L(\tau) \) is given by the expression

\[ L(\tau) = \left[ 2\tilde{M}(\tau)\sin(\nu_0\sin\omega_0\tau) + \varphi(\tau)\cos(\nu_0\sin\omega_0\tau) \right]\exp[\nu_c(1 + \cos\omega_0\tau)] \tag{23} \]

with the parameter

\[ \nu_c = \frac{\langle \tilde{x}^2 \rangle}{\lambda^2}, \tag{24} \]

describing nonequilibrium fluctuations of the oscillator position. The deterministic force \( F_0 \) on the right side of Eq.(22), which arises from interaction of the oscillator with the tunnel junction, is given by \( F_0 = (1/\lambda)^{\Lambda}(\omega = 0) \), where \( \Lambda(\omega) \) is the Fourier transform of the kernel \( L(\tau) \), Eq.(23). It follows from Eq.(22), that coupling to the tunnel junction also results in a shift of the oscillator frequency, \( \omega_0^2 \rightarrow \omega_0^2 - (1/m\lambda^2)\Lambda(\omega = 0) \), jointly with damping and decoherence described by \( Im\Lambda(\omega) \equiv \Lambda''(\omega) \).

The relaxation rate of mechanical oscillations due to coupling to the tunnel junction is given by

\[ \gamma = \frac{\hbar}{m\lambda^2} \frac{\Lambda''(\omega_0)}{\omega_0}, \tag{25} \]

where

\[ \Lambda''(\omega) = \frac{1}{2}e^{\nu_c} \sum_{l=\pm\infty} I_l(\nu_c)J_0(\nu_0)\{\chi''(\omega - l\omega_0) + \chi''(\omega + l\omega_0)\} \]

\[ + \frac{1}{2}e^{\nu_c} \sum_{l=-\infty}^{l=+\infty} \sum_{n=\infty} \sum_{n=1} I_l(\nu_c)J_{2n}(\nu_0)\{\chi''(\omega + 2n\omega_0 - l\omega_0) + \chi''(\omega - 2n\omega_0 + l\omega_0) \}
\]

\[ + \chi''(\omega + 2n\omega_0 + l\omega_0) + \chi''(\omega - 2n\omega_0 - l\omega_0) \}
\]

\[ + \frac{1}{2}e^{\nu_c} \sum_{l=-\infty}^{l=+\infty} \sum_{n=0} I_l(\nu_c)J_{2n+1}(\nu_0) \]

\[ \times [S[\omega - (2n + 1)\omega_0 - l\omega_0] + S[\omega - (2n + 1)\omega_0 + l\omega_0] - S[\omega + (2n + 1)\omega_0 - l\omega_0] - S[\omega + (2n + 1)\omega_0 + l\omega_0]]. \tag{26} \]

Here, \( J_n(\nu) \) is the Bessel function of n-th order, and \( I_l(\nu) \) is the modified Bessel function of l-th order. The spectral function \( S(\omega) \) and the imaginary part of the susceptibility \( \chi''(\omega) \) are
the Fourier transforms of the correlator and the response function of the heat bath, Eq.(7), respectively,

\[ S(\omega) = \int d\tau e^{i\omega \tau} M(\tau), \]

\[ \chi''(\omega) = \int d\tau \sin(\omega \tau) \varphi(\tau). \]  

(27)

For a harmonic oscillator, the inverse relaxation rate \( \gamma^{-1} \) given by Eq.(25) also represents a characteristic time scale for the coherence decay, \( \tau_d = \gamma^{-1} \).

The parameter \( \nu_c \) of Eq.(24) describing the level of mechanical fluctuations can be found from the self-consistent equation

\[ \nu_c = \frac{K(\omega_0)}{2\omega_0^2 \lambda^2 \gamma}, \]  

(28)

where \( K(\omega) \) is given by

\[ K(\omega) = \frac{\hbar^2}{2m^2 \lambda^2} \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_l(\nu_c) J_n(\nu_0) \{ S(\omega - l\omega_0) + S(\omega + l\omega_0) \} + \frac{1}{2} e^{\nu_c} \sum_{l=-\infty}^{\infty} \sum_{n=1}^{\infty} I_l(\nu_c) J_{2n}(\nu_0) \{ S(\omega + 2n\omega_0 - l\omega_0) + S(\omega - 2n\omega_0 + l\omega_0) + S(\omega + 2n\omega_0 + l\omega_0) + S(\omega - 2n\omega_0 - l\omega_0) \} \]

\[ - \frac{1}{2} e^{\nu_c} \sum_{l=-\infty}^{\infty} \sum_{n=0}^{\infty} I_l(\nu_c) J_{2n+1}(\nu_0) \{ \chi''[\omega + (2n + 1)\omega_0 - l\omega_0] - \chi''[\omega - (2n + 1)\omega_0 - l\omega_0] + \chi''[\omega + (2n + 1)\omega_0 + l\omega_0] - \chi''[\omega - (2n + 1)\omega_0 + l\omega_0] \}. \]  

(29)

### III. RESERVOIR CORRELATION FUNCTIONS AND TUNNEL CURRENT

To determine the oscillator damping rate \( \gamma \) of Eq.(25) and the level of fluctuations \( \nu_c \) of Eq.(24), we need to determine the spectral function \( S(\omega) \) and the susceptibility \( \chi''(\omega) \) of the electronic heat bath, Eq.(27), for the case with no interaction with the oscillator. Assuming that the variables \( \{ c_{kL}^{(0)}, c_{qR}^{(0)}, \ldots \} \) of the free electron gases in the leads obey the Wick theorem (with uncorrelated electron systems in the different leads), we can express the correlator of unperturbed heat bath variables in the form

\[ \langle Q^{(0)}(t)Q^{(0)}(t_1) \rangle = \sum_{kq} |T_{kq}|^2 \{ \langle c_{kL}^{(0)}(t)c_{kL}^{(0)}(t_1) \rangle \langle c_{qR}^{(0)}(t)c_{qR}^{(0)}(t_1) \rangle + \langle c_{kL}^{(0)}(t)c_{kL}^{(0)}(t_1) \rangle \langle c_{qR}^{(0)}(t)c_{qR}^{(0)}(t_1) \rangle \}. \]  

(30)

Introducing retarded, advanced and "lesser" Green’s functions of electrons in the leads as

\[ g_{k\alpha}(t, t_1) = \langle (-i)[c_{k\alpha}(t), c_{k\alpha}^+(t_1)]\rangle \theta(t - t_1) - i e^{-iE_{k\alpha}(t-t_1)} \theta(t - t_1), \]

\[ g_{k\alpha}^a(t, t_1) = \langle i[c_{k\alpha}(t), c_{k\alpha}^+(t_1)]\rangle \theta(t_1 - t) = i e^{-iE_{k\alpha}(t_1-t)} \theta(t_1 - t), \]

\[ g_{k\alpha}^<(t, t_1) = i \langle c_{k\alpha}^+(t), c_{k\alpha}(t_1) \rangle \theta(t - t_1) = i f_{\alpha}(E_{k\alpha}) e^{-iE_{k\alpha}(t-t_1)}, \]  

(31)
where

\[ f_\alpha(E) = f(E - \mu_\alpha) = \left[ \exp \left( \frac{E - \mu_\alpha}{T} \right) + 1 \right]^{-1} \] (32)

is the Fermi distribution in the \( \alpha \)-lead (\( \alpha = L, R \)) with the chemical potential \( \mu_\alpha \) and temperature \( T \), we obtain

\[
M(t, t_1) = \frac{1}{2} \sum_{kq} |T_{kq}|^2 \{ g_{qR}^>(t, t_1) [g_{kL}^r(t_1, t) - g_{kL}^a(t_1, t)] \\
+ [g_{qR}^r(t, t_1) - g_{qR}^a(t, t_1)][g_{kL}^>(t_1, t) + g_{kL}^< (t, t_1)] [g_{qR}^r(t_1, t) - g_{qR}^a(t_1, t)] \\
+ [g_{kL}^r(t, t_1) - g_{kL}^a(t, t_1)][g_{qR}^>(t_1, t) + 2g_{qR}^< (t, t_1)] g_{kL}^< (t, t_1) + 2g_{kL}^< (t, t_1) g_{qR}^> (t_1, t)] \} \\
\phi(t, t_1) = i \theta (t - t_1) \sum_{kq} |T_{kq}|^2 \{ [g_{qR}^r(t, t_1) - g_{qR}^a(t, t_1)][g_{kL}^> (t_1, t) - g_{kL}^< (t, t_1)] [g_{qR}^r (t_1, t) - g_{qR}^a(t_1, t)] \\
+ [g_{kL}^r(t, t_1) - g_{kL}^a(t, t_1)][g_{qR}^> (t_1, t) - g_{kL}^< (t, t_1)] [g_{qR}^r (t_1, t) - g_{qR}^a(t_1, t)] \}. \] (33)

As a result, the formulas for the spectrum and the imaginary part of the susceptibility of the dissipative environment are given by

\[
S(\omega) = \pi \sum_{kq} |T_{kq}|^2 [\delta(\omega - E_{kL} + E_{qR}) + \delta(\omega + E_{kL} - E_{qR})] \\
\times [f_R(E_{qR}) + f_L(E_{kL}) - 2f_L(E_{kL})f_R(E_{qR})], \\
\chi''(\omega) = \pi \sum_{kq} |T_{kq}|^2 [\delta(\omega + E_{kL} - E_{qR}) - \delta(\omega - E_{kL} + E_{qR})] [f_L(E_{kL}) - f_R(E_{qR})]. \] (34)

It should be mentioned that when the leads are in the full thermodynamic equilibrium state, \( \mu_L = \mu_R \), the spectral function \( S(\omega) \) and the function \( \chi''(\omega) \) are related by means of the fluctuation-dissipation theorem \( S(\omega) = \chi''(\omega) \coth(\omega/2T) \).

The electric current through the junction is defined as \( I = I_L = -I_R \), where \( I_\alpha = e \langle \dot{N}_\alpha \rangle \) with \( N_\alpha = \sum_k c^+_\alpha c^{}_{\alpha k} \) being an electron number in the \( \alpha \)-lead. It follows from the equations of motion for the electron operators of the leads,

\[
i \dot{c}_{pL} = E_{pL} c_{pL} - \sum_{q'} T_{pq'} c_{q'R} e^{-x/\lambda}, \\
i \dot{c}_{qR} = E_{qR} c_{qR} - \sum_{k'} T_{k'q} c_{k'L} e^{-x/\lambda}, \] (35)

that the electric current depends on the oscillator position \( x \) as

\[
I = -i \sum_{pq} \langle \{ T^*_{pq} c^+_{q'R} c_{pL} - h.c. \} e^{-x/\lambda} \rangle. \] (36)

In the case of weak tunneling between leads, the electron operators of the leads can be represented as

\[
c_{pL}(t) = c^{(0)}_{pL}(t) - \sum_{q'} T_{pq'} \int dt_1 g_{q'pL}(t, t_1) c^{(0)}_{q'R}(t_1) e^{-x(t_1)/\lambda}, \\
c_{qR}(t) = c^{(0)}_{qR}(t) - \sum_{p'} T^*_{p'q} \int dt_1 g_{q'R}(t, t_1) c^{(0)}_{p'L}(t_1) e^{-x(t_1)/\lambda}. \] (37)
Therefore, the electric current is given by

\[ I = \sum_{pq} |T_{pq}|^2 \int dt_1 \{ g^\alpha_{pL}(t, t_1) g^\omega_{qR}(t_1, t) 
- g^\omega_{qR}(t, t_1) g^\alpha_{pL}(t_1, t) \} \left\{ \frac{1}{2} [e^{-x(t)/\lambda}, e^{-x(t_1)/\lambda}]_+ \right\} + h.c. \]  \hspace{1cm} (38)

Substituting the expressions for the anticommutator, Eq.(16), we obtain

\[ I = \frac{\pi e^{\nu_c}}{2} \sum_{l=-\infty}^{l=+\infty} \sum_{n=-\infty}^{n=+\infty} I_1(\nu_c) J_{2n}(\nu_0) \sum_{pq} |T_{pq}|^2 [f_R(E_{qR}) - f_L(E_{kL})] \times \delta[E_p - E_{qR} + (l - 2n)\omega_0] + \delta[E_p - E_{qR} - (l - 2n)\omega_0] \]
\[ + \delta[E_p - E_{qR} + (l + 2n)\omega_0] + \delta[E_p - E_{qR} - (l + 2n)\omega_0]. \] \hspace{1cm} (39)

We can determine the spectral function \( S(\omega) \) and the imaginary part of the susceptibility \( \chi''(\omega) \) of the electron reservoir, Eq.(34), by introducing the densities-of-states \( D_\alpha(E) \) of the leads and replacing sums over \( k, q, \ldots \) by integrations over the corresponding energies as

\[ \sum_{kq}(\ldots) \rightarrow \int dE_L dE_R D_L(E_L) D_R(E_R)(\ldots). \]

We assume that the tunneling elements \( |T_{kq}|^2 \) do not depend on energies, \( |T_{kq}|^2 = |T_0|^2 \), and the densities-of-states near the Fermi surface in the leads are also energy-independent, \( D_\alpha(E) \simeq D_\alpha(\mu) = D_\alpha \) with \( \mu_R = \mu + eV/2, \mu_L = \mu - eV/2, \alpha = L, R \). Furthermore, we assume that all energy parameters of our problem \( (eV, T, \omega_0, \ldots) \) are much less than the basic chemical potential \( \mu \) of the electron gas in the leads. In this case, we obtain the imaginary part of the susceptibility as

\[ \chi''(\omega) = \pi D_L D_R |T_0|^2 \int_{-\infty}^{+\infty} dE [f(E - \omega) - f(E + \omega)], \] \hspace{1cm} (40)

whereas the spectral function \( S(\omega) \) has the form

\[ S(\omega) = \pi D_L D_R |T_0|^2 \int_{-\infty}^{+\infty} dE \{ [1 - f(E - \mu_L)] [f(E - \mu_R + \omega) + f(E - \mu_R - \omega)] + f(E - \mu_L) [2 - f(E - \mu_R + \omega) - f(E - \mu_R - \omega)] \}. \] \hspace{1cm} (41)

As a result, the electrons in the leads represent an Ohmic heat bath with respect to the mechanical oscillations of the cantilever with the imaginary part of susceptibility given by

\[ \chi''(\omega) = \alpha \omega, \] \hspace{1cm} (42)

where \( \alpha = 2\pi D_L D_R |T_0|^2 \), and the frequency-dependent spectral function has the form

\[ S(\omega) = \frac{\alpha}{2} \left[ (\omega - eV) \coth \left( \frac{\omega - eV}{2T} \right) + (\omega + eV) \coth \left( \frac{\omega + eV}{2T} \right) \right]. \] \hspace{1cm} (43)

At zero temperature \( S(\omega) \) is given by
\[ S(\omega) = \alpha [\omega \theta(\omega - eV) + eV \theta(eV - \omega)], \quad (44) \]

where \( \theta(\omega) \) is the Heaviside step function.

For the electric current, we finally obtain
\[ I = G(V)V = \alpha e^{2\nu_c} eV, \quad (45) \]

where \( G(V) = e\alpha \exp(2\nu_c) \) is the nonlinear conductance of the tunnel junction, which depends on the fluctuation level of the mechanical oscillator.

**IV. RESULTS AND DISCUSSION**

Although the general formalism developed in the previous sections makes it possible to perform comprehensive analyses of the rich phenomenology occurring in a wide variety of nanoelectromechanical systems, we have restricted ourselves in the present paper to the examination of nonequilibrium fluctuations of a mechanical oscillator coupled to a biased tunnel junction, along with the study of quantum heating effects on the decoherence rate (Eq.(25)) of the oscillator and on the current-voltage characteristics (Eq.(45)) of the junction.

To start, we demonstrate analytical results related to the case of weak nonlinearity of the cantilever-junction coupling, Eq.(5), when \( \nu_c \ll 1 \). In this case, the contribution of vacuum fluctuations to the collision term \( \Lambda''(\omega) \), Eq.(26), and to the correlator \( K(\omega) \), Eq.(29), can be neglected. Therefore, the dissipative kernel is proportional to \( \omega \), \( \Lambda''(\omega) = \alpha \exp(2\nu_c) \omega \).

This corresponds to Eq.(19) for the oscillator position with the damping rate \( \gamma \), Eq.(25), and the spectrum of the fluctuation forces \( K(\omega) \), Eq.(29), given by
\[ \gamma = \alpha \frac{\hbar}{m\lambda^2} e^{2\nu_c}, \]
\[ K(\omega) = \frac{\hbar^2}{2m^2\lambda^2} e^{2\nu_c} \sum_{l=-\infty}^{l=+\infty} I_l(\nu_c)[S(\omega - l\omega_0) + S(\omega + l\omega_0)]. \quad (46) \]

In contrast to the approach of Ref. [10], the fluctuations of the random forces \( \xi \) involved in the Langevin equation, Eq.(19), are not white noise because of the frequency dispersion of the correlator \( K(\omega) \), Eq.(46). Even for the case of weak heating, when the root-mean-square amplitude of the cantilever fluctuations is much less than the tunneling length \( \lambda, \sqrt{\langle \tilde{x}^2 \rangle} \ll \lambda \), and \( \nu_c \ll 1 \), the spectrum \( K(\omega) \) of the fluctuation sources,
\[ K(\omega) = \alpha \frac{\hbar^2}{2m^2\lambda^2} e^{2\nu_c} \left[ (\omega - eV) \coth \left( \frac{\omega - eV}{2T} \right) + (\omega + eV) \coth \left( \frac{\omega + eV}{2T} \right) \right], \quad (47) \]
demonstrates a non-white character, especially at zero temperatures of the electron reservoirs.

On the basis of the above-mentioned approximations and with Eqs. (28), (46), (47) we have derived the dispersion of the cantilever fluctuations \( \langle \tilde{x}^2 \rangle = \lambda^2 \nu_c \) as a function of voltage \( V \) applied to the junction and of temperature \( T \) of the electron reservoirs, as
\[ \langle \tilde{x}^2 \rangle = \frac{\hbar}{4m\omega_0} \left[ \frac{\omega_0 - eV}{\omega_0} \coth \left( \frac{\omega_0 - eV}{2T} \right) + \frac{\omega_0 + eV}{\omega_0} \coth \left( \frac{\omega_0 + eV}{2T} \right) \right]. \quad (48) \]
At low temperatures, $T \ll \omega_0 \pm eV$, and at low voltages, $eV < \omega_0$, when the current in the tunnel junction can not stimulate an excitation of the mechanical system, the dispersion of the oscillator fluctuations remains on the vacuum level, $\langle \tilde{x}^2 \rangle = (\hbar/2m\omega_0)$. However, at the higher voltage, $eV > \omega_0$, the level of fluctuations increases linearly with the voltage, as $\langle \tilde{x}^2 \rangle = (\hbar/2m\omega_0)(eV/\omega_0)$, and we find the dimensionless parameter $\nu_c$ as $\nu_c = \nu_0(eV/\omega_0)$. This linear increase of the fluctuation level may be interpreted as an increase of an effective temperature of the oscillator: $T_{\text{eff}} = eV/2\hbar$, where $T_{\text{eff}}$ is proportional to the dispersion of mechanical fluctuations, $\langle \tilde{x}^2 \rangle = T_{\text{eff}}/m\omega_0^2$. This confirms the result obtained by Mozyrsky and Martin \[10\] for the Caldeira-Leggett model.

The heating process affects also the bias dependence of the damping rate $\gamma$, Eq.(46), and the behavior of the conductance $G(V)$, Eq.(45). At low voltages, $eV < \hbar\omega_0$, both of these characteristics do not depend on $V$, $[G = e\alpha(1 + 2\nu_0), \gamma = (\hbar\alpha/m\lambda^2)(1 + 2\nu_0)]$, whereas above the threshold of the excitation of cantilever oscillations, $eV > \hbar\omega_0$, the heating process makes a linear contribution to conductance and to the relaxation rate:

$$G(V) = e\alpha \left(1 + 2\nu_0 \frac{eV}{\hbar\omega_0}\right), \gamma = \frac{\hbar\alpha}{m\lambda^2} \left(1 + 2\nu_0 \frac{eV}{\hbar\omega_0}\right). \tag{49}$$

We consider here two models of an oscillator having different masses. For the first model, the shuttle, the mass is $10^{-20} g$, and this model describes the molecules embedded in an elastic medium between the leads. The molecule is taken to be electrically connected to one lead, whereas a connection to the other lead is realized by a tunnel junction.

The second model is associated with the cantilever having a mass $10^{-12} g$ which is placed in the neighborhood of the electrical contact; in this case variations of the cantilever position modulate the tunneling matrix element of the junction. The oscillator frequencies are the same for both cases and are equal to $1 GHz$, $\hbar\omega_0 = 10^{-18} erg$. The tunneling lengths are also the same both for the shuttle and the cantilever and are equal to $10^{-8} cm$. As a result, the parameter $\nu_0$ is much less than one ($\sim 5 \cdot 10^{-7}$) for the cantilever and is of the order of one ($\sim 0.5$) for the shuttle.

To characterize the nonequilibrium fluctuations of the systems, we plot the bias and temperature dependencies of the parameter

$$\bar{\nu}_c = \frac{\nu_c}{\nu_0 \coth(\hbar\omega_0/2T)} = \frac{2m\omega_0 \langle \tilde{x}^2 \rangle}{\hbar \coth(\hbar\omega_0/2T)},$$

i.e. the ratio of the dispersion fluctuations of the oscillator coupled to the tunnel junction to that of the uncoupled oscillator at temperature $T$.

The bias dependencies at various temperatures are presented in Fig.1(a) for the cantilever and in Fig.1(b) for the shuttle. One can see that at extremely low temperature there is the change of the curve shape at the critical voltage corresponding to the characteristic frequency of oscillator. For the cantilever, there is not any heating below this bias and there is linear growth of fluctuations thereafter, as indicated by the analytical results above. As temperature increases the curve becomes smoother and heating decreases, and, finally, there is almost no heating at $T = 0.1 K$. For the shuttle case, there is some heating even below the critical voltage and, moreover, the equilibrium value (at $V = 0$) of $\bar{\nu}_c$ is not one. The reason for this is that at $\nu_0 \sim 1$, vacuum fluctuations in the leads (back and forth tunneling events) give rise to oscillator anharmonicity and the steady-state fluctuation level
of the oscillator nonlinearly coupled to the electron reservoirs is different from that of the uncoupled oscillator.

The temperature dependencies are exhibited in Figs.2, 3(a,b) and 4(a,b) for the cantilever, and similar features have been found for the shuttle. The bias voltages are chosen to be below (Fig.2), above (Fig.3(a,b)) and far above (Fig.4(a,b)) that of the excitation threshold $\bar{h}\omega_0/e = 0.625 \times 10^{-6} V$, respectively. The “b” parts of the figures magnify the low temperature behavior. It is evident from these figures that as temperature increases, $\tilde{\nu}_c$ approaches one for any bias voltage. However, for low bias there is a peak at low temperature which becomes more pronounced below the critical bias voltage. This peak occurs when temperature is such that $T \sim \omega_0 - eV$; the peak is absent at high voltage. This phenomenon occurs for both the cantilever and the shuttle.

V. SUMMARY

In summary, we have applied the theory of open quantum system \cite{15,16,17} to the nanoelectromechanical system consisting of an oscillator coupled to a tunnel junction. We have obtained explicit expressions for the nonequilibrium fluctuation level of the oscillator (Eqs.(28),(48)) as well as for the tunnel current through the structure (Eq.(45)) and for the decoherence rate of the oscillator (Eqs.(25),(46)). The bias and temperature dependencies of the oscillator fluctuation level have been determined. Considering two specific models with different oscillator masses, we have shown that, for small mass, the level of mechanical vacuum fluctuations in this system differs significantly from that of a linear harmonic oscillator with the same frequency $\omega_0$ because of an anharmonicity induced by the weak nonlinear oscillator-current coupling. This effect takes place even at zero bias voltage, but vanishes at higher temperatures. We have also found that for voltages below the excitation threshold, $\bar{h}\omega_0/e$, the relative level of oscillator fluctuations (normalized to the equilibrium value) peaks at temperatures of the order of the difference between the characteristic frequency of the oscillator and the applied bias.

Acknowledgement

We are thankful to Dima Mozyrsky for valuable discussions which brought our interest to this problem. L.G.M. and N.J.M.H. gratefully acknowledges support from the Department of Defense, DAAD 19-01-1-0592.
FIGURES

FIG. 1. Voltage dependence of the nonequilibrium fluctuations of oscillator position (normalized to those of the uncoupled oscillator) at various temperatures; (a) for oscillator mass $m = 10^{-12}g$, (b) for oscillator mass $m = 10^{-20}g$.

FIG. 2. Temperature dependence of the nonequilibrium fluctuations of oscillator position (normalized to those of the uncoupled oscillator) for bias voltage $10^{-7}V$.

FIG. 3. (a) Temperature dependence of the nonequilibrium fluctuations of oscillator position (normalized to those of the uncoupled oscillator) for bias voltage $10^{-6}V$; (b) Magnified low temperature part of this dependence.

FIG. 4. (a) Temperature dependence of the nonequilibrium fluctuations of oscillator position (normalized to those of the uncoupled oscillator) for bias voltage $10^{-4}V$; (b) Magnified low temperature part of this dependence.
REFERENCES

[1] M.L. Roukes, Technical Digest of the 2000 Solid-State Sensor and Actuator Workshop.
[2] H.G. Craighead, Science 290, 1532 (2000).
[3] L.Y. Gorelik, A. Isacsson, M.V. Voinova, B. Kasemo, R.I. Shekhter, and M. Jonson, Phys. Rev. Lett. 80, 4526 (1998).
[4] A. Isacsson, L.Y. Gorelik, M.V. Voinova, B. Kasemo, R.I. Shekhter, and M. Jonson, Physica B 255, 150 (1998).
[5] H. Park, J. Park, A.K.L. Lim, A.H. Anderson, A.P. Alivisatos, and P.L. McEuen, Nature 407, 58 (2000).
[6] D. Fedorets, L.Y. Gorelik, R.I. Shekhter, and M. Jonson, Europhys. Lett. 58, 99 (2002).
[7] A.D. Armour and A. MacKinnon, Phys. Rev. B 66, 035333 (2002).
[8] N.F. Schwabe, A.N. Cleland, M.C. Gross, and M.L. Roukes, Phys. Rev. B 52, 12911 (1995).
[9] M.P. Blencowe and M.N. Wybourne, Appl. Phys. Lett. 77, 3845 (2000).
[10] D. Mozyrsky and I. Martin, Phys. Rev. Lett. 89, 018301 (2002).
[11] S.A. Gurvitz, Phys. Rev. B 56, 15215 (1997).
[12] B. Elattari and S.A. Gurvitz, Phys. Rev. Lett. 84, 2047 (2000).
[13] J.A. Sidles, J.L. Garbini, K.J. Bruland, D. Rugar, O. Züger, S. Hoen, and C.S. Yannoni, Rev. Mod. Phys. 67, 249 (1995).
[14] A.N. Cleland and M.L. Roukes, in Proceedings of ICPS-24, edited by D. Gershoni (World Scientific, Singapore, 1999).
[15] G.F. Efremov and A.Yu. Smirnov, Zh. Eksp. Teor. Fiz. 80, 1071 (1981). [Sov. Phys. JETP 53, 547 (1981)].
[16] G.F. Efremov, L.G. Mourokh, and A.Yu. Smirnov, Phys.Lett. A, 175, 89 (1993).
[17] A.Yu. Smirnov, N.J.M. Horing, and L.G. Mourokh, Appl. Phys. Lett. 77, 2578 (2000).
[18] A.O. Caldeira and A.J. Leggett, Physica (Amsterdam) 121A, 587 (1983); A.O. Caldeira and A.J. Leggett, Ann.Phys. (N.Y.) 149, 374 (1983).
A.Yu. Smirnov, et al., Figure 1(a) of 4.
A.Yu. Smirnov, et al., Figure 1(b) of 4.
A.Yu. Smirnov, et al., Figure 2 of 4.
A.Yu. Smirnov, et al., Figure 3(a) of 4.
A.Yu. Smirnov, et al., Figure 3(b) of 4.
A.Yu. Smirnov, et al., Figure 4(a) of 4.
A.Yu. Smirnov, et al., Figure 4(b) of 4.