Abstract

We construct $R$-invariant unification models where a pair of massless Higgs doublets is naturally obtained. The masslessness of the Higgs doublets is guaranteed by the unbroken $R$ symmetry. Mass generation for the Higgs doublets is considered from various viewpoints.
1 Introduction

Nonvanishing superpotential gives a negative cosmological constant in supergravity. $R$ symmetry is a unique symmetry that forbids a constant term in superpotential and thus it may play a fundamental role for understanding a vanishing cosmological constant in supergravity. The $R$ symmetry has been widely considered in phenomenology of supersymmetric (SUSY) gauge theories or supergravity, since it (or its discrete subgroup) can avoid too rapid proton decays \cite{1,2} and provide a candidate for cold dark matter in our universe \cite{3}. In a recent article \cite{4}, it has been pointed out that the spontaneous breakdown of the $R$ symmetry $U(1)_R$ to its discrete subgroup $Z_{2^nR}$ \cite{5} produces a flat potential for a new inflation model.

Motivated by the above theoretical and phenomenological arguments, we construct, in this paper, $R$-invariant unification models.

In the next section, we show that the minimal SUSY grand unified theory (GUT) is easily extended to an $R$-invariant one. However, we stress that this model has a serious doublet-triplet splitting problem as all the SUSY-GUT’s do. In section three, we construct an $R$-invariant extension of recently proposed natural unification theories \cite{6,7}, where the doublet-triplet splitting problem is solved. Namely, masslessness of the Higgs doublets is guaranteed by the $R$ symmetry, while the Higgs triplets have $R$-invariant masses at the unification scale. In this model, however, a pair of Higgs doublets is completely massless as long as the $R$ symmetry is unbroken. In section four, we discuss how to generate a mass for the Higgs doublets at the electroweak scale by modifying the above model. The final section is devoted to a discussion on low-energy predictions of the $R$-invariant natural unification model. A possible connection to the superstring theory is also briefly noted.
2 A SUSY-GUT with a $U(1)_R$ symmetry

In this section, we consider the minimal GUT based on a gauge group $SU(5)$ \[8\]. The Higgs sector in the SUSY $SU(5)$ GUT consists of a pair of Higgs chiral multiplets $H_i$ and $\bar{H}^i$ transforming as $5$ and $5^*$ of $SU(5)_{GUT}$ and a $24$ Higgs chiral multiplet $\Sigma^i_j$ ($i, j = 1, \cdots, 5$). The vacuum expectation value of the adjoint Higgs $\Sigma$ is supposed to break the $SU(5)_{GUT}$ group down to the standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\langle \Sigma \rangle = \begin{pmatrix} 2 & 2 \\ 2 & -3 \\ -3 & -3 \end{pmatrix} V, \quad (1)$$

where $V$ is at the GUT scale of order $10^{16}$ GeV.

An important point is that $R$ charge of $\Sigma$ must be vanishing, otherwise the $R$ symmetry is spontaneously broken at the GUT scale to produce a negative cosmological constant of order the GUT scale in supergravity. Provided that the negative vacuum energy is canceled out by condensation energy of SUSY-breaking in the hidden sector, we obtain too large SUSY-breaking scale of order $10^{14}$ GeV in the visible sector \[4\]. Therefore we assume that the $\Sigma$ multiplet transforms trivially under the $U(1)_R$ symmetry.

In order to construct a superpotential with $R$ charge two, we introduce another adjoint Higgs $\Sigma'^i_j$ whose $R$ charge is two. Then the renormalizable superpotential for the adjoint Higgs is given by

$$W_\Sigma = m \text{Tr}(\Sigma'^i \Sigma) + \lambda \text{Tr}(\Sigma'^i \Sigma^2). \quad (2)$$

We have a desired SUSY vacuum:

$$\langle \Sigma' \rangle = 0 \quad (3)$$
with Eq. (1) and $V = m/\lambda$. In this vacuum, the GUT gauge group is broken down to the standard-model one, while the superpotential is kept vanishing: $\langle W \rangle = 0$. This results from the fact that the $R$ symmetry $U(1)_R$ is unbroken under Eqs. (3) and (1).

Let us turn to the mass term for the Higgs $H$ and $\bar{H}$. We assume that the product $H \bar{H}$ has $R$ charge two to make the color triplets in $H$ and $\bar{H}$ heavy. Then their superpotential is given by

$$W_H = m' H \bar{H} + \lambda' H \Sigma \bar{H}. \quad (4)$$

In the broken phase Eq. (1), the color-triplet and weak-doublet components of $H$ and $\bar{H}$ have different masses as

$$m_{HC} = m' + \frac{2\lambda'}{\lambda} m, \quad m_{Hf} = m' - \frac{3\lambda'}{\lambda} m, \quad (5)$$

respectively. As pointed out in Ref. [1, 10], we need an extreme fine-tuning even in the SUSY extension of GUT’s to obtain a pair of Higgs doublets with $m_{Hf}$ of order $10^2$ GeV. To avoid this unnatural tuning, we proceed to consider $R$ invariance in recently proposed natural unification scheme [6].

### 3 An $R$-invariant natural unification model

The natural unification models [6, 7] are based on a product of two distinct gauge groups, $G_{GUT}$ and $G_H$. The group $G_{GUT}$ is the usual GUT gauge group and its coupling constant is in a perturbative regime, while the group $G_H$ is a hypercolor gauge group whose coupling is strong at the unification scale. The color gauge group SU(3)$_C$ (or SU(3)$_C \times U(1)_Y$) at low energies is a linear combination of SU(3) (or SU(3) $\times U(1)$) subgroups of $G_{GUT}$ and $G_H$, and the weak gauge group SU(2)$_L$ is a subgroup of $G_{GUT}$. The strong

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1The converse choice is also possible.
coupling for the hypercolor group $G_H$ is necessary to achieve an approximate unification of three gauge coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$.

In this paper, we consider an $SU(5)_{GUT} \times U(3)_H$ model. The quarks and leptons obey the usual transform law under the GUT group $SU(5)_{GUT}$, while they are all singlets of the hypercolor group $U(3)_H$. We introduce a pair of Higgs $H_i$ and $\bar{H}_i$ ($i = 1, \cdots, 5$) which transform as 5 and $5^*$ under the $SU(5)_{GUT}$ and as singlets under the $U(3)_H$. So far all matter multiplets are the same as in the minimal SUSY $SU(5)_{GUT}$. This guarantees the electric charge quantization for the ordinary sector and the $m_b = m_\tau$ unification.

We introduce six pairs of hyperquarks $Q^I_\alpha$ and $\bar{Q}^\alpha_I$ ($\alpha = 1, \cdots, 3; I = 1, \cdots, 6$) which transform as 3 and $3^*$ under the hypercolor $SU(3)_H$ and have $U(1)_H$ charges 1 and $-1$, respectively. The first five pairs $Q^i_\alpha$ and $\bar{Q}^\alpha_i$ belong to $5^*$ and 5 of $SU(5)_{GUT}$, respectively, and the last pair $Q^6_\alpha$ and $\bar{Q}^\alpha_6$ are singlets of $SU(5)_{GUT}$.

Since $Q^i_\alpha$ and $\bar{Q}^\alpha_i$ are supposed to have vacuum expectation values of order the unification scale, they must be trivial representations of the $R$ symmetry $U(1)_R$ as explained in the previous section. To cause a desired breaking of the total gauge group $SU(5)_{GUT} \times U(3)_H$, we are led to introduce chiral multiple with $R$ charges two which couple to $Q^I_\alpha$ and $\bar{Q}^\alpha_I$. We take an adjoint representation $X^\alpha_\beta$ of $U(3)_H$ in this paper. The renormalizable superpotential for $Q^I_\alpha, \bar{Q}^\alpha_I, X^\alpha_\beta, H_i$, and $\bar{H}^i$ is given by

$$W = \lambda Q^I_\alpha \bar{Q}^\alpha_I X^\alpha_\beta + \lambda' Q^6_\alpha \bar{Q}^\alpha_6 X^\alpha_\beta + hQ^I_\alpha \bar{Q}^\alpha_6 H_i + h'Q^6_\alpha \bar{Q}^\alpha_6 \bar{H}^i,$$

where we have assumed the $R$ charges of $H_i$ and $\bar{H}^i$ to be two and those of $Q^6_\alpha$ and $\bar{Q}^\alpha_6$ vanishing. In addition to the $U(1)_R$, this superpotential possesses another axial symmetry $U(1)_\chi$ given by

$$Q^I_\alpha \rightarrow e^{i\xi}Q^I_\alpha, \quad \bar{Q}^\alpha_I \rightarrow e^{i\xi}\bar{Q}^\alpha_I, \quad X^\alpha_\beta \rightarrow e^{-2i\xi}X^\alpha_\beta, \quad H_i \rightarrow e^{-2i\xi}H_i, \quad \bar{H}^i \rightarrow e^{-2i\xi}\bar{H}^i.$$

(7)
We can also impose this axial symmetry to avoid nonrenormalizable terms in the superpotential. Note that the global symmetries $U(1)_R$ and $U(1)_\chi$ have no $SU(3)_H$ anomaly.

The regular terms in the effective superpotential allowed by the symmetries of the model is exactly the tree-level ones in Eq. (6). Thus we consider the tree-level $D$ and $F$ term flatness conditions to obtain our quantum vacua.

The $F$-flatness conditions are given by

$\{ Q_\alpha^i Q_\beta^i - \frac{1}{3} \delta_\alpha^\beta \text{Tr}(Q^i Q^i) \} + \{ Q^6_\alpha Q^6_\alpha - \frac{1}{3} \delta_\alpha^\alpha \text{Tr}(Q^6 Q^6) \} = 0,$

$Q^i_\alpha Q^6_\alpha = 0, \quad Q^6_\alpha Q^i_\alpha = 0, \quad \lambda Q^i_\alpha X^\alpha_\beta + h Q^i_\alpha H_i = 0,$

$\lambda Q^i_\alpha X^\alpha_\beta + h' Q^6_\alpha \tilde{H}^i = 0, \quad \lambda' Q^i_\alpha X^\alpha_\beta + h' Q^6_\alpha \tilde{H}^i = 0, \quad \lambda' Q^6_\alpha X^\alpha_\beta + h Q^i_\alpha H_i = 0.$

Together with the $D$-flatness conditions for $U(3)_H$, we obtain desired vacua as follows:

$\langle X^\alpha_\beta \rangle = \langle H_i \rangle = \langle \tilde{H}^i \rangle = 0, \quad \langle Q^i_\alpha \rangle = \langle \bar{Q}^i_\alpha \rangle = 0, \quad \langle Q^6_\alpha \rangle = v \delta_\alpha^i, \quad \langle \bar{Q}^6_\alpha \rangle = v \delta_\alpha^i.$

For $v \neq 0$, the total gauge group $SU(5)_{GUT} \times U(3)_H$ is broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$.

As noted in the beginning of this section, the color $SU(3)_C$ and the $U(1)_Y$ are, respectively, a linear combination of an $SU(3)$ subgroup of the $SU(5)_{GUT}$ and the hypercolor $SU(3)_H$ and that of a $U(1)$ subgroup of the $SU(5)_{GUT}$ and the strong $U(1)_H$. Thus the gauge coupling constants $\alpha_C$, $\alpha_2$, and $\alpha_1$ for $SU(3)_C \times SU(2)_L \times U(1)_Y$ are given by

$\alpha_C \simeq \frac{\alpha_{GUT}}{1 + \alpha_{GUT}/\alpha_{3H}}, \quad \alpha_2 = \alpha_{GUT}, \quad \alpha_1 \simeq \frac{\alpha_{GUT}}{1 + \frac{1}{15} \alpha_{GUT}/\alpha_{1H}}, \quad (10)$

$^2$An invariant term with $R$ charge two given by $\{ \text{det}(Q^i_\alpha X^\beta_\beta \bar{Q}^i_\beta) \}^{1/6}$ is singular at the origin $Q^i_\alpha = \bar{Q}^i_\alpha = X^\alpha_\beta = 0$. 

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where \(\alpha_{3H}\) and \(\alpha_{1H}\) denote gauge coupling constants \(\alpha_3\) for the hypercolor \(SU(3)_H\) and \(U(1)_H\), respectively. We see from Eq.\((\Phi)\) that the unification of three gauge coupling constants \(\alpha_C, \alpha_2, \alpha_1\) is achieved in the strong coupling limit of the hypercolor gauge interaction: \(\alpha_{3H} \rightarrow \alpha_{1H} \rightarrow \infty\). \(^4\)

In the vacuum Eq.\((\Phi)\), the color triplets \(H_a\) and \(\bar{H}^a\) \((a = 1, \cdots, 3)\) gain masses of order \(v\) together with the sixth hyperquarks \(\bar{Q}^6_\alpha\) and \(Q^6_\alpha\) (see Eq.\((\Phi)\)). On the other hand, the weak doublets \(H_i\) and \(\bar{H}^i\) \((i = 4, 5)\) remain massless since there are no partners for them. The masslessness for these doublets is guaranteed by the unbroken \(R\) symmetry.

Notice that the scale \(v\) is undetermined so far. This implies the presence of a flat direction in the present vacuum. The imaginary part of the scalar component of the corresponding massless chiral multiplet is a Nambu-Goldstone mode related to the breaking of the axial \(U(1)\chi\) symmetry.

Thus we should break the \(U(1)\chi\) explicitly to fix the unification scale \(v\) and eliminate the flat direction. We introduce a singlet chiral multiplet \(\phi\) whose \(U(1)_R\) and \(U(1)\chi\) charges are 2 and \(-2\) to have a soft breaking of the axial \(U(1)\chi\). Then we have a superpotential for \(\phi\):

\[
W_\phi = k Q^i_\alpha \bar{Q}^\alpha_i \phi + k' Q^6_\alpha \bar{Q}^\alpha_6 \phi + M^2_\phi \phi. \tag{11}
\]

The \(M^2_\phi\) term is a \(U(1)\chi\) breaking one. With this superpotential, we obtain \(3k v^2 = M^2_\phi\).

The \(U(1)\chi\)-breaking mass \(M_\phi\) may stem from a condensation of some other hypercolor quarks. For example, we consider an \(SU(2)_{H'}\) strong gauge theory with four hyperquarks \(Q^{i}_{\alpha}\) where \(\alpha = 1, 2\) and \(i = 1, \cdots, 4\). Provided that the hyperquarks \(Q^{i}_{\alpha}\) have vanishing \(R\) charges and their \(U(1)\chi\) charges

\(^3\)See Ref.\((\Phi)\) for the normalization of \(\alpha_{1H}\).

\(^4\)Precisely speaking, in the strong coupling region, \(\alpha_{3H}, \alpha_{1H} \gg 1\), the threshold corrections may yield substantial effects on the relations in Eq.\((\Phi)\). However, it has been pointed out \((\Phi)\) that \(\alpha_{3H}, \alpha_{1H} \sim \mathcal{O}(1)\) is sufficient to get the observed values of \(\alpha_C, \alpha_2, \) and \(\alpha_1\).
are one, the singlet $\phi$ can couple to $Q'^i_\alpha$ as
\[
W'_\phi = k''_{ij} \epsilon^{\alpha\beta} Q'^i_\alpha Q'^j_\beta \phi.
\] (12)

Nonperturbative effects of the strong $SU(2)_{H'}$ cause the $\epsilon^{\alpha\beta} Q'^i_\alpha Q'^j_\beta$ condensation [14], which eventually induces the $M^2_\phi$ term in Eq.(11).

Since the $U(1)_\chi$ has the strong $SU(2)_{H'}$ anomaly, there arises no massless Nambu-Goldstone multiplet associated with the spontaneous breaking of $U(1)_\chi$. Note that the $R$ symmetry $U(1)_R$ has no strong $SU(2)_{H'}$ anomaly and hence it is still an exact symmetry.

We finally stress that there is no massless multiplet in the present vacuum except for the pair of Higgs doublets $H_i$ and $\bar{H}^i$ ($i = 4, 5$) and three families of quark-lepton chiral multiplets. Our low-energy spectrum is nothing other than that of the SUSY standard model.

4 Mass generation for the Higgs doublets

The Higgs doublets are kept massless by the $U(1)_R$ symmetry. All the quark-lepton chiral multiplets have vanishing $U(1)_R$ charges so that they have Yukawa couplings to the Higgs doublets $H_i$ and $\bar{H}^i$. It is remarkable that the $R$ symmetry has a QCD anomaly and thus plays a role of the Peccei-Quinn symmetry [15] to suppress the strong CP violation.

The breaking scale $v_{PQ}$ of the Peccei-Quinn symmetry is subject to astrophysical and cosmological constraints [16] as
\[
10^{10}\text{GeV} \leq v_{PQ} \leq 10^{12}\text{GeV}.
\] (13)

Thus we assume that the breaking scale $v_R$ of the $U(1)_R$ symmetry satisfies the same constraints:
\[
10^{10}\text{GeV} \leq v_R \leq 10^{12}\text{GeV}.
\] (14)
In general, the Higgs doublets receive masses from the U(1)\(_R\) breaking sector. The value of their masses depends on the \(R\) charge of the Higgs field \(\eta\) which breaks the \(R\) symmetry.

When the \(\eta\) has \(R\) charge \(-1\), a nonrenormalizable term

\[
W_\theta = \frac{g}{M} H_i \bar{H}^i \eta^2
\]

induces a Higgs mass

\[
\mu \simeq g \langle \eta \rangle^2 M \simeq g v^2 R M ,
\]

where \(M\) denotes the gravitational scale \(M \simeq 2.4 \times 10^{18}\) GeV. Eqs.\((14)\) and \((16)\) lead to a mass of the electroweak scale for the Higgs doublets with \(g\) of order one. This may be an encouraging result in the present model, though it seems accidental that the Higgs mass \(\mu\) comes out to be of the same order as the SUSY-breaking scale.

In the rest of this section, we consider another model so modified that the Giudice-Masiero term \([17]\) induces the Higgs-doublet mass of the electroweak scale through SUSY breaking in the hidden sector. \(^5\) The difference between the previous and modified models amounts to the global U(1) charge assignments. The charges in the two models are given in Table \([\text{I}]\). In the modified model, the renormalizable superpotential is given by

\[
W = \lambda Q_i^\alpha \bar{Q}_i^\beta X^\alpha + k Q_i^\alpha \bar{Q}_i^\alpha \phi + h Q_i^\alpha \bar{Q}_6^\alpha H_i + h' Q_6^\alpha \bar{Q}_i^\alpha \bar{H}^i + M_\phi^2 \phi ,
\]

where the \(M_\phi^2\) term is the soft breaking one of the U(1)\(_\chi\) symmetry. We get the desired vacuum as in the previous model. \(^6\)

\(^5\)One may keep only a discrete subgroup of U(1)\(_R\) in the previous model to allow the Giudice-Masiero term.

\(^6\)The superpotential Eq.\((17)\) possesses another global U(1). Two linear combinations of the three global U(1)'s are the same as the U(1)\(_R\) and U(1)\(_\chi\) in the previous model. One must introduce nonrenormalizable superpotentials to eliminate the extra U(1), which however does not affect our conclusion.
The Higgs doublets $H_i$ and $\bar{H}^i$ ($i = 4, 5$) never acquire masses as long as $\langle X \rangle = \langle \phi \rangle = \langle Q^6_\alpha \rangle = \langle \bar{Q}^\alpha_6 \rangle = 0$. In contrast to the previous model, however, the Higgs multiplets $H_i$ and $\bar{H}^i$ have vanishing charges for $U(1)_R$ and $U(1)_\chi$, which allows their coupling to the Polonyi field $Z$ in the Kähler potential as

$$K = \frac{g'}{M} Z^* H_i \bar{H}^i + h.c. \quad (18)$$

Since the Polonyi field $Z$ is supposed to have a nonvanishing $F$ term $\langle F_Z \rangle \simeq (10^{11}\text{GeV})^2$ to break the SUSY, the Kähler potential (18) gives rise to the Higgs mass \[ \mu \simeq g' \langle F_Z \rangle M. \quad (19) \]

With $g'$ of order one, the mass $\mu$ turns out to be of the SUSY-breaking scale or the electroweak scale as desired.

## 5 Conclusion

We have constructed $R$-invariant unification models where a pair of massless Higgs doublets is naturally obtained.\footnote{We have used $U(1)_R$ in this paper, though a discrete $R$ symmetry $Z_{nR}$ with large $n$ is sufficient for our purpose.} The masslessness of the Higgs doublets is guaranteed by the unbroken $R$ symmetry. These natural unification models are based on the gauge group $SU(5)_{GUT} \times U(3)_H$. All of the quark-lepton multiplets and the Higgs $H_i$ and $\bar{H}^i$ ($i = 1, \cdots, 5$) are singlets of the $U(3)_H$ and they belong to the standard representations of the $SU(5)_{GUT}$ as in the minimal SUSY-GUT. Therefore some predictions on the quark-lepton sector are intact in our models with a non-simple gauge group. The $m_b = m_\tau$ unification is an example. Another example is the unification of soft SUSY-breaking masses for scalar quarks and leptons. Namely, scalar components of the chiral multiplets which belong to to the same multiplets of the $SU(5)_{GUT}$ have the same SUSY-breaking masses at the unification scale.
On the contrary, the gauge sector in the natural unification models is different from that in the usual SUSY-GUT’s. It is a crucial difference that the color SU(3) is a diagonal subgroup of an SU(3) subgroup of the SU(5)_{GUT} and the hypercolor SU(3)_H. As a consequence, we have a smaller value of \( \alpha_c \) than the prediction of the usual SUSY-GUT’s. We note that the recent experimental values of \( \alpha_c \) [18] seem to support the present models. As for the gaugino masses, we have no prediction without an extra assumption.

An important ingredient in the present models is the presence of unbroken \( R \) symmetry. As a direct consequence of the symmetry, the dimension-five operators for nucleon decays [2] are suppressed. Thus the observation of dimension-five nucleon decays would exclude the present type of natural unification models.

All of the natural unification models are based on products of two distinct gauge groups, one of whose coupling constants is in a perturbative regime and the other in a strong coupling region. This basic structure of the gauge group might be realized in some superstring theories. In fact, a recent development on string theory indicates that nonperturbative dynamics in string theory may produce some extra gauge symmetries beside the perturbative one [19], though compactifications to four-dimensional spacetime is not yet thoroughly understood. We hope that an extensive study on the superstring compactification to realistic models reveals the structure of unification in elementary particle physics.

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8The natural unification model in Ref.[7] allows the dimension-five operator for nucleon decays.
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Table 1: Charge assignments in the two models.

|            | First model | Second model |
|------------|-------------|--------------|
|            | $U(1)_R$   | $U(1)_X$    | $U(1)_R$   | $U(1)_X$    |
| $Q_i^\alpha, \bar{Q}_i^\alpha$ | 0           | 1            | 0           | 1            |
| $Q_6^\alpha, \bar{Q}_6^\alpha$ | 0           | 1            | 2           | -1           |
| $X_\beta^\alpha, \phi$         | 2           | -2           | 2           | -2           |
| $H_i, H^i$          | 2           | -2           | 0           | 0            |