Interplay of magnetization dynamics with a microwave waveguide at cryogenic temperatures

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In this work, magnetization dynamics is studied at low temperatures in a hybrid system that consists of a thin epitaxial magnetic film coupled with a superconducting planar microwave waveguide. The resonance spectrum was observed over a wide magnetic field range, including low fields below the saturation magnetization and both polarities. Analysis of the spectrum via a developed fitting routine allowed for the derivation of all magnetic parameters of the film at cryogenic temperatures, the detection of waveguide-induced uniaxial magnetic anisotropies of the first and the second order, and the uncovering of a minor misalignment of magnetic field. A substantial influence of the superconducting critical state on resonance spectrum is observed and discussed.

I. INTRODUCTION

The field of magnonics studies the application of magnetization oscillations and waves in ferromagnetic structures. The following benefits make magnonics promising for application in processing of microwave signals: tunability of the magnon dispersion with applied magnetic field and the geometry of the medium, low dissipation and power consumption, high operational frequencies, convenient micron and sub-micron scales of spin wavelength at microwave frequencies, and, finally, absence of parasitic coupling of spin waves with non-magnetic environments. Conventionally, magnonics is a room-temperature research discipline.

Currently a sub-discipline is emerging that deals with magnetization dynamics at cryogenic temperatures and can be referred to as “cryogenic magnonics”. Indeed, quantum magnonics is of high current interest. Microwave experiments in quantum magnonics are typically performed at milli-kelvin temperatures, often using setups equipped with superconducting quantum circuits. On the other hand, a development of various hybrid devices is taking place based on superconducting resonators and Josephson junctions. Also, it was shown that hybridization of a magnon medium with superconducting structures results in substantial modification of dispersion properties, as well as in the formation of magnonic band structures. Lastly, metamaterial properties have been reported for superconductor/ferromagnet superlattices. More generally and beyond superconductor-induced phenomena, the magnetic properties at low temperatures are probed in absence or only with minor thermal excitations. Typical thermal effects for standard magnonics, such as reduced saturation magnetization or thermally activated domain wall motion, are lessened for cryo-magnonics, leading to new phenomena in ferromagnetic resonance (FMR).

In this regard, investigation of magnetic properties of ferromagnetic films at low temperature as well as of their interaction with superconducting circuits is imperative. This report addresses both problems. We focus on the ferromagnetic resonance in a thin Yttrium Iron Garnet (YIG) film coupled to a superconducting Nb planar waveguide in out-of-plane magnetic field. We obtain the FMR spectrum at low temperature in a wide field range. The spectrum shows linear magnetic resonance versus field dependence for high fields and a range of nonlinear dependence of FMR frequency at low magnetic fields where the Kittel formulas are inapplicable. Developing a fitting routine, we derive all magnetic parameters of the YIG film. Our analysis shows that the waveguide itself induces substantial uniaxial magnetic anisotropy. Next, we study the FMR spectrum at temperatures below the superconducting critical temperature of the waveguide and observe an influence of the superconducting critical state of Nb on the resonance spectrum.

We note that while YIG is probably the most popular magnetic material for magnonic applications, owing to
its low damping, the damping in YIG and its temperature dependence are not addressed in this paper and can be found elsewhere.\textsuperscript{23,24} In this report, YIG is selected as a model magnetic single-crystalline thin film with distinct magnetocrystalline anisotropy and sufficiently low saturation magnetization, which is convenient for out-of-plane measurements.

II. EXPERIMENTAL DETAILS

The FMR absorption measurements were performed using the so-called VNA-FMR approach\textsuperscript{25–27} (VNA stands for the vector network analyzer). A schematic illustration of the investigated system is shown in Fig. 1. The single-crystalline epitaxial YIG film of thickness \( d = 51 \text{ nm} \) was deposited on single-crystal [111]-oriented Gadolinium Gallium Garnet (GGG) substrate by means of LPE. A Nb CPW is fabricated directly on top of YIG film. The main direction of the CPW is indicated with the axis. Magnetic field \( H \) is applied out-of-plane, along [111] orientation of YIG/GGG (i.e., along \( z \)-direction).

![Figure 1](Image)

**FIG. 1.** Schematic illustration of the investigated system. YIG epitaxial film is grown on [111]-oriented single-crystal GGG substrate by means of LPE. A Nb CPW is fabricated directly on top of YIG film. The main direction of the CPW is indicated with the axis. Magnetic field \( H \) is applied out-of-plane, along [111] orientation of YIG/GGG (i.e., along \( z \)-direction).

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III. RESULTS AND DISCUSSION

Figures 2a,b show transmission spectra of the studied sample at \( T = 10 \text{ K} > T_c \) of Nb and at 2 K < \( T_c \). Spectra have been normalized with \( S_{21}(f) \) at \( \mu_0 H = 0.5 \text{ T} \). Figures 2c,d show a set of normalized absorption curves \( S_{21}(f) \) of the sample at the same temperatures and several magnetic fields. Field dependent spectral lines in Figs. 2a,b with the minimum transmission correspond to FMR curves \( f_\pi(\mu_0 H) \). Both spectra show linear FMR response at \( [\mu_0 H] > 0.2 \text{ T} \), which is typical for the Kittel-FMR mode of a thin film in out-of-plane magnetic field. The resonance frequency with out-of-plane field is \( f_\pi \propto (\mu_0 H - 4\pi M_{\text{eff}}) \), which indicates the value of the effective saturation magnetization 4\( \pi M_{\text{eff}} \approx 2000 \text{ Oe} \) at \( f_\pi \rightarrow 0 \). Upon decreasing \( [\mu_0 H] \), the linear resonance line is terminated with a kink at \( [\mu_0 H] \sim 4\pi M_{\text{eff}} \) and transforms into two FMR branches with nonlinear dependence of resonance frequency versus field for \( [\mu_0 H] < 4\pi M_{\text{eff}} \). We refer to the higher-frequency FMR branch with stronger absorption as C-line and to the lower-frequency FMR branch with weaker absorption as G-line. Note that in general, observation of FMR in thin films with out-of-plane geometry at \( [\mu_0 H] < 4\pi M_{\text{eff}} \) might be challenging due to formation of nonuniform magnetization configurations. With out-of-plane field \( [\mu_0 H] < 4\pi M_{\text{eff}} \), ferromagnetic films are not magnetized to saturation, and Kittel formulas for FMR are not applicable. Splitting of the FMR response into several spectral lines at \( [\mu_0 H] < 4\pi M_{\text{eff}} \) can be caused by various factors, including standing spin wave resonance, phase separation, or magnetic phase separation.

After transition of the Nb CPW into the superconducting state, the transmission spectrum changes (compare Figs. 2a and b, Figs. 2c and d). While the spectrum at \( T < T_c \) consists of the same resonance lines as at \( T > T_c \), superconductivity manifests itself in hysteresis of FMR peak absorption at \( [\mu_0 H] < 0.2 \text{ T} \), which is best visible for the C-line (compare \( S_{21}(f) \) at \( \mu_0 H = 0.1 \text{ T} \) and \( \mu_0 H = -0.1 \text{ T} \) in Figs. 2a,b): FMR absorption at negatively swept magnetic field (positive \( H \) in Figs. 2c,d) is substantially stronger than at positively swept magnetic field (negative \( H \) in Figs. 2c,d). In addition, at \( T < T_c \) a suppression of FMR response is observed at the low field region \( [\mu_0 H] < 0.02 \text{ T} \). Below we will discuss the FMR response of YIG in absence of superconductivity, establish causes for the split of FMR at \( [\mu_0 H] < 4\pi M_{\text{eff}} \), and define the contribution of superconductivity to the FMR
FIG. 2. a) and b) Gray-scale-coded transmission spectra $|S_{21}(\mu_0 H, f)/S_{21}(\mu_0 H = 0.5 \text{T}, f)|$ measured at $T = 10 \text{ K}$ above $T_c$ (a) and $T = 2 \text{ K}$ below $T_c$ (b). c) and d) Corresponding frequency dependencies of the normalized transmission $|S_{21}(f)|$ at several magnetic fields at $T = 10 \text{ K}$ (c) and $T = 2 \text{ K}$ (d). For curves in (c) and (d) the background was subtracted. At $T < T_c$ the spectrum shows hysteresis of absorption. Magnetic field was swept negatively from $+0.5$ to $-0.5$ T (indicated with arrows), and, therefore, the part of spectra in (b) at positive fields provides the “up-field-sweep” data. Labels C and G indicate higher- and lower-frequency spectral lines, respectively.

spectrum.

A. FMR at $T > T_c$. Magnetic properties of YIG film at cryogenic temperatures.

Having analysed possible origins for the split of the FMR into the C-line and G-line in Fig. 2 we can state that neither domain structure nor spin waves can contribute to the FMR spectrum for our particular study. For instance, nucleation of magnetic domains upon demagnetization at $\mu_0 H < 4\pi M_{\text{eff}}$ occurs for thin films with strong perpendicular anisotropy in comparison with the demagnetizing energy $36,37$, i.e. when the magnetic quality parameter $Q = K_u/2\pi M_s^2 > 1$, where $K_u$ is the out-of-plane uniaxial anisotropy, and $M_s$ is the saturation magnetization. However, a typical field of uniaxial anisotropy $\mu_0 H_{K_u} = 2K_u/M_s$ in LPE-grown YIG thin films ranges up to $\sim 200\text{ Oe}$, ensuring $Q < 1$. The highest values of uniaxial anisotropy in YIG films $\mu_0 H_{K_u} \sim 10^3\text{ Oe}$ that can be obtained in pulsed-laser-deposited films still ensure $Q < 1$. As an additional test, we have performed a magnetic force microscopy study of magnetic flux structure at the surface of the YIG film at $4 \text{ K}$ using attocube attoDRY 1000 closed-cycle cryogenic microphone, supplied with a superconducting solenoid, and found no traces of domains or any field-dependent magnetic structure. Therefore, we confirm that formation of the domain structure does not occur. The magnetic state of the YIG film is single-domain, and variation of the out-of-plane component of magnetization at $\mu_0 H < 4\pi M_{\text{eff}}$ occurs via rotation of the magnetization vector from out-of-plane orientation to in-plane.

The absence of contribution of standing spin-wave resonances to the FMR spectrum can be illustrated in the following way. At $H = 0$ the magnetization vector of...
a single-domain film is aligned in-plane. Therefore, the Kittel formula for FMR and dispersion relations for any spin-wave mode at \( H = 0 \) become applicable. When several resonances are observed, a contributing spin-wave mode can be identified by estimating a resonance frequency difference \( \Delta f_r \) between the Kittel mode and any standing spin-wave resonance mode. The latter appears due to quantization of the wavelength with geometrical parameters of a sample. The difference \( \Delta f_r \) is then compared with the experimentally observed one \( \sim 1.3 \) GHz at \( H = 0 \) (Fig. 2). If an in-plane magnetostatic standing spin-wave mode \( \phi_h \) is assumed, e.g., the backward volume mode or the magnetostatic surface mode, its wavelength \( \lambda/2 \) for the standing mode should be quantized with dimensions of the CPW, i.e., \( \lambda/2 \sim 20 - 40 \) μm. Such a standing spin-wave mode provides only marginal difference \( \Delta f_r \lesssim 10 \) MHz due to a small ratio \( d/\lambda \sim 10^{-3} \). Alternatively, if exchange-dominated perpendicular standing spin-wave resonance \( \phi_m \) is assumed the typical exchange constant in YIG films \( \sim 4 \times 10^{-12} \) J/m provides \( \Delta f_r = 2.5 \) GHz for \( d = \lambda/4 \) and \( \Delta f_r \sim 7.5 \) GHz for \( d = \lambda/2 \). Thus, none of the possible standing spin-wave modes can provide \( \Delta f_r \sim 1.3 \) GHz. Overall, when a standing spin-wave resonance is excited multiple consequential spectral lines are expected. FMR absorption for each line should decrease progressively with the mode number (see, for instance Ref. [38]). Such a picture is not observed in our experiment. Therefore, we confirm that several spectral lines in Fig. 2 at \( |\mu_0 H| < 4\pi M_s \) are not caused by standing spin-wave resonances.

The remaining explanation for two FMR lines requires the existence of two resonating areas with different magnetic properties in the vicinity of the CPW. The magnetic structure is essentially single-domain in each area. The resonating areas can be identified by the coupling strength of microwaves to precessing magnetization that is proportional to the FMR amplitude and correlates directly with the amplitude of excitation AC magnetic fields. In CPW geometry, AC magnetic fields are mainly focused in the vicinity of the central transmission line [13,25]. Therefore, geometry of the experiment (Fig. 1) suggests that the lower-frequency, weaker G-line originates from YIG at gap areas of CPW where the coupling is weaker, while higher-frequency, stronger C-line appears due to FMR response of YIG area under the central conducting line of the CPW. The accuracy of that explanation is strengthened by additional features, as discussed below.

For the case of the single-domain single-crystalline YIG film, the analytical resonance curve \( f_r(\mu_0 H) \) can be obtained in the entire \( H \)-range following Refs. [38,39,47] (we keep the notations given in Ref. [38]). The orientation of magnetization of a single-domain film at arbitrarily oriented magnetic field is defined by the minimum of free magnetostatic energy \( g = g(M_s, h, k_1, k_2, k_u, \theta, \psi, \phi_h, \phi_m) \), where \( k_1, k_2 \) and \( k_u \) are unitless parameters of cubic magnetocrystalline anisotropy and out-of-plane uniaxial anisotropy, respectively, \( h = \mu_0 H/4\pi M_s \) is the normalized external magnetic field, and \( \theta, \psi, \phi_h, \phi_m \) define the orientations of \( H \) and \( M_s \) with respect to principle crystallographic axes of YIG in spherical coordinates (see Fig. 3). In addition, the system in Fig. 1 has a distinct directionality along the orientation of the CPW. This directionality may contribute to the orientation of magnetization. We account for its possible contribution by an additional energy term \( g_a \) added to the free magnetostatic energy \( g \) that provides a phenomenological in-plane uniaxial anisotropy of the first order. The term of the in-plane uniaxial anisotropy of the first order in the coordinates of Fig. 3 is

\[
g_a = -k_a \sin(\psi)^3 \cos(\phi_h - \alpha)^2. \tag{1}\]

FMR frequency is defined by derivatives at position of minimum energy [38,44] as

\[
f_r \sim \gamma (g_{\phi_h} g_{\phi_m} - g_{\psi}^2)^{1/2}/\sin(\psi), \tag{2}\]

where \( \gamma \) is the gyromagnetic ratio. See Refs. [38,45,47] for details.

Dotted data in Fig. 4a show the experimental \( f_r(\mu_0 H) \) resonance curves extracted from Fig. 2. First, we focus on the G-line of the FMR spectrum. In order to fit the data, we have developed the following routine, which allowed us to obtain all magnetic parameters of \( g \) and \( f_r \), despite a large number of parameters and their partial interdependence. First, we note that when misalignment \( \theta \) of orientation of magnetic field with the \( z \)-axis is small, and in-plane CPW uniaxial anisotropy \( k_a \) is negligible, the linear part at \( \mu_0 H \gg 4\pi M_{sat} \) can be fitted with the simplified expression [38]

\[
f_r = 2\gamma M_s (h - (1 - k_u + 2k_1/3 + 2k_2/9)). \tag{3}\]

Fitting the data in the field range from 0.3 T to 0.4 T with Eq. 3, we obtain the gyromagnetic ratio \( \gamma/2\pi = \)
Next we note that (i) the position of the kink at

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tion magnetization in relation with anisotropy parame-

er $k_1$ and $k_2$ are by a factor of 2-3 higher than typical values at room temperature.\textsuperscript{23,15a}

This trend correlates well with the temperature dependence of the cubic magnetocrystalline anisotropy in YIG bulk single crystal.\textsuperscript{15}

After fitting the G-line, which corresponds to the FMR response of YIG areas at the CPW gaps, the only option to fit the C-line, which corresponds to the FMR response of YIG areas under the central CPW line, is to introduce an additional term into the energy $g$ that represents the second order uniaxial anisotropy induced by the CPW. The term of the in-plane uniaxial anisotropy of the sec-

ond order in the coordinates of Fig. 3 is

\begin{equation}
  g_a = -(k_{a1} + 2k_{a2}) \sin(\psi)^2 \cos(\phi_a - \alpha)^2 + \\
  + k_{a2} \sin(\psi)^4 \cos(\phi_a - \alpha)^4
\end{equation}

Using magnetic parameters obtained for the G-line, the fitting procedure for the C-line with the anisotropy given by Eq. [4] provides $k_{a1} = 0.121$ and $k_{a2} = -0.048$. This fit is shown in Fig. [4b] with the blue curve.

Possible origins of the CPW-induced anisotropy include a distinct directionality of microwave currents. Also, directionality of the surface stress can be consid-
ered that appears from differences in thermal expansion of narrow central transmission line of metal CPW and YIG/GGG oxides. The surface stress may appear either due to deposition of Nb film at elevated temperature or due to performance of experiments at cryogenic tempera-

ures. For instance, the difference in thermal expansions between Nb and garnets can enable a strain in YIG at 2 K of up to $\epsilon \approx +6 \times 10^{-4}$ along the CPW in case of absence of mechanical relaxation in Nb. In contrast, if a complete relaxation of tensions occurs in Nb at room tem-

perature, the difference in thermal expansions enables the opposite-sign strain in YIG of $\epsilon \approx -4 \times 10^{-4}$. Both values of the strain are well comparable with the growth induced tensions provided by the lattice misfit between the GGG substrate and YIG film that induces the uniaxial anisotropy in LPE-grown\textsuperscript{[25]} and PLD-grown film.\textsuperscript{[29,30]}

See the Appendix section for details. Importantly, pres-

ence of both first- and second-order anisotropies suggests different mechanisms for their induction.

Figure [4] shows dependencies of orientations of mag-

netization $\psi(\mu_0 H)$ and $\phi_a(\mu_0 H)$ for C- and G-FMR lines on magnetic field. A marginal difference between C- and G-curves in the entire field range indicates co-alignment of magnetization orientations at both gap and center areas of the CPW, implying that the entire volume of YIG that is subjected to the FMR remains in the single-domain state throughout the experiment.

Our experimental setup does not allow us to study microwave transmission at higher temperatures $T \gtrsim 15$ K. Therefore, temperature dependence of magnetic param-

eters of YIG is not addressed in this report and can be found elsewhere.\textsuperscript{[22,13,15]}
B. FMR at \( T < T_c \): Impact of the superconducting critical state.

At \( T < T_c \) of Nb, in presence of superconductivity, the FMR absorption spectrum changes (see Fig. 2b). Since the Nb CPW is placed directly on top of the YIG film, all changes in absorptions in the C-branch can be attributed to the magnetization state under the Nb line. Therefore, the effect of superconductivity on the FMR can be tracked by analyzing the superconducting critical state of Nb film and its variation with applied magnetic field.

Figure 5a shows the zero-field-cooled (ZFC) transmission spectra that is acquired when the sample is cooled down to 2 K at zero magnetic field, and afterward \( S_{21} \) measurements were performed while sweeping magnetic field from 0 to 0.11 T. Figure 5b shows the field-cooled (FC) transmission spectra that is acquired when the sample was cooled down to 2 K at \( \mu_0 H = 0.25 \) T, and afterward \( S_{21} \) measurements were performed while sweeping magnetic field back from 0.11 T down to 0. The hysteresis in peak absorption can be tracked by fitting \( S_{21}(f) \) curves at each value of \( H \) and plotting dependencies of FMR amplitude \( I(H) \) on magnetic field \( H \) (Fig. 5c). \( I(\mu_0 H) \) dependency is caused by variation of the CPW-FMR coupling strength with magnetic field, i.e., by variation of magnetization and magnetic flux inhomogeneity in the YIG induced by the Nb superconducting critical state. Note that no hysteresis in peak absorption is observed at \( T > T_c \) (Figs. 2a and 5c), where the transmission spectra is fully reversible and independent of the ZFC/FC initial state.

First we discuss the ZFC curve in Fig. 5c, where three intervals in \( I(\mu_0 H) \) can be distinguished. At low fields, the strongest FMR absorption is observed with \( I \sim 0.1 \) at \( \mu_0 H \) up to \( 2 \times 10^{-3} \) T (highlighted with the red circle in Fig. 5c). This corresponds to the Meissner state of the Nb line when the Meissner screening currents circulate at the edges of the Nb film and exclude magnetic flux from its cross-section. In the Meissner state, DC magnetic flux remains homogeneous across the Nb line and ensures a strong coupling of the CPW to the YIG at FMR. At intermediate fields \( 2 \times 10^{-3} < \mu_0 H < 10^{-2} \) T, the FMR absorption drops rapidly from \( I \sim 0.1 \) to the minimum \( I \sim 0.02 \), caused by the partially penetrated superconducting critical state where superconducting vortices start to penetrate Nb film. The magnetic flux profile in partially penetrated superconducting films is the most inhomogeneous, which causes a weak coupling of the FMR to the CPW and low absorption intensity. The partially penetrated state commences at the flux-focus enhanced first critical field of the superconducting film \( \mu_0 H_{c1} \sim 2 \times 10^{-3} \) T, where the first Abrikosov vortices start to penetrate into the film, and terminates at the magnetic field of full penetration \( 10^{-2} \) T. At high fields \( \mu_0 H > 0.01 \) T, after full penetration is reached, magnetic flux in the superconducting film forms a constant gradient that can be depicted by the Bean critical state model. The gradient is formed due to pinning of vortices and induces a homogeneous circulating

FIG. 5. Gray-scale-coded transmission spectra \( |S_{21}(\mu_0 H, f)/S_{21}(\mu_0 H = 0.5 \) T, f) \) measured at 2 K starting from ZFC state (a) and FC state (b). C and G spectral lines are indicated. Red circle in (a) highlights FMR absorption in the Meissner state of the CPW. c) Dependencies of resonance peak absorption for C-lines on magnetic field \( I(\mu_0 H) \) obtained at 2 K and 10 K. Direction of magnetic field sweep is indicated with arrows. The \( \mu_0 H \) axis is given on a log scale. Field regions for three superconducting states of Nb at ZFC curve in (c) are separated with blue dashed lines.
critical currents. Upon increasing magnetic field, both the pinning of vortices and the slope of magnetic flux reduces, making magnetic flux in YIG more homogeneous. A smaller gradient of the magnetic flux in the superconductor increases the coupling that we observe in gradual increase of the FMR peak absorption upon increasing magnetic field from 0.01 T to higher fields. Note that such nonmonotonic behavior of \( I(\mu_0 H) \) is not observed for the G-line (Fig. 5), which indicates additionally that the absorption at G-line is caused by the FMR at the gap areas of the CPW, where the influence of the superconducting state of Nb is marginal.

Increasing magnetic field further beyond the field range in Fig. 5, the ZFC curve should coincide with the FC curve at the so-called irreversibility field \( \mu_0 H_i \) where pinning of vortices becomes negligible. The FC curve in Fig. 5 consists of two parts. For \( \mu_0 H > 0.03 \) T the coupling remains by a factor of \( \sim 2 \) higher than one for the ZFC curve. This difference is attributed to the fact that upon decreasing magnetic field, the Bean critical currents counteract the Meissner currents, diamagnetic response of the superconducting film is reduced as compared to the ZFC measurement, and the influence of YIG at the FMR decreases. Below 0.03 T, \( I \) drops rapidly, which can be explained by a gradual formation of a complex remanent critical state at \( H = 0 \) with highly nonuniformly distributed frozen magnetic flux. Also at low magnetic fields, magnetization of individual Abrikosov vortices may contribute to YIG inhomogeneity by inducing substantial local magnetic fields of up to \( \mu_0 H_i \sim \Phi_0/\pi \lambda_L^2 \sim 0.06 \) T, where \( \Phi_0 \) is the magnetic flux quantum and \( \lambda_L \sim 10^{-7} \) m is the typical London penetration depth in Nb films.

Overall, the influence of the superconducting critical state in our geometry on the FMR appears to be destructive. The FMR intensity for both ZFC and FC curves remains below values of \( I \) at \( T > T_c \) (Fig. 5). However, magnetic hysteresis often is employed in magnetic logic devices. Also, in vicinity to \( H = 0 \), FMR is substantially stronger when superconductor is in the Meissner state than for normal metal CPW. This effect may be a result of interaction of magnetic moments in YIG with Meissner screening currents in the ideal diamagnet.

IV. CONCLUSION

In conclusion, ferromagnetic resonance of YIG film is studied in out-of-plane magnetic fields and cryogenic temperatures using a superconducting coplanar waveguide that is fabricated directly on top of the magnetic film (see Fig. 1). FMR absorption spectra are obtained in a wide field range. Nonlinear dependence of the FMR frequency on magnetic field at low field values, below the field of saturation magnetization, showed a split of resonance into two spectral lines, which were identified as the FMR response of YIG at gap areas of the CPW and of YIG located directly under the central conducting line of the CPW.

A routine was developed for fitting the FMR lines. This routine allowed us to obtain all magnetic parameters of YIG, i.e., the saturation magnetization, the gyromagnetic ratio, and parameters of magnetocrystalline and out-of-plane uniaxial anisotropies. In addition, the fitting routine has issued the misalignment angle of 1.4° between magnetic field and the out-of-plane orientation, as well as parameters of in-plane magnetic anisotropy of the first and the second order, which are induced by the CPW.

The FMR spectrum at temperatures below the superconducting critical temperature of the waveguide showed a hysteresis in FMR peak absorption. The hysteresis is explained by influence of magnetization of the Nb transmission line in the superconducting critical state. Tracking the dependence of the intensity of the FMR on magnetic field allowed us to identify all fundamental states of a superconducting film in out-of-plane magnetic field, i.e., the Meissner state, the partially penetrated state, and the fully penetrated Bean critical state. Also, it allowed explanation of the hysteresis by the pinning of magnetic vortices, which induces the gradient of magnetic flux in superconducting films. The gradient is controlled by direction of the magnetic field sweep.

In general, this report suggests that development of magnonics at cryogenic temperatures may be beneficial due to: (i) substantially different properties of magnetic materials, including magneto-crystalline anisotropy, (ii) the possibility to engineer additional anisotropies with metal structures, and (iii) the potential to affect the spectra by hybridization of a magnonic media with superconductors. As a final remark we would like to point out a related work by Jeon et al. on the effect of the superconducting critical state on magnetization dynamics in thick superconductor/ferromagnet/superconductor trilayer.

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VI. APPENDIX: STRESS INDUCED IN YIG BY NB CPW

One possible cause of the CPW induced anisotropy that is derived in Sec. IIIA is the stress in YIG that is forced by differences in thermal expansion of the narrow extended central transmission line of the metal CPW and YIG/GGG oxides. Assuming that an unstressed continuous interface is formed between Nb and YIG during deposition of Nb at the deposition temperature \(T_d\) ≈ 600 K, the stress at the interface at the measurement temperature \(T_m = 2\) K can be estimated with the following expression

\[
\sigma \approx \frac{E}{1 - \nu} \epsilon = \frac{E}{1 - \nu} \int_{T_d}^{T_m} [\alpha_G(T) - \alpha_{Nb}(T)]dT, \tag{5}
\]

where \(\sigma\) is the stress in YIG, \(E = 2 \times 10^{12}\) dyne/cm\(^2\) is the Young’s modulus of YIG at the temperature range from 0 to 300 K\(^{62}\), \(\nu = 0.29\) is the Poisson’s ratio, \(\epsilon\) is the strain at the interface at \(T_m\) due to the difference in thermal expansion, \(\alpha_G(T)\) and \(\alpha_{Nb}(T)\) are temperature dependencies of the linear thermal expansion of the garnet and Nb, respectively. Importantly, the stress in Eq. 5 implies absence of mechanical relaxation.

However, estimation of the stress at the Nb/YIG interface using Eq. 5 is impeded. While thermo-mechanical properties of Nb are well studied in a wide temperature range\(^{64,65}\) from \(\approx 0\) K up to about the melting point, a consistent study of thermo-mechanical properties of YIG is not available for the required temperature range. The coefficient \(\alpha_G(T)\) for YIG is available piecewise and can be obtained by interpolation of \(\alpha_G(T)\) at temperatures above\(^{62,65}\) and below\(^{63}\) the room temperature. On the other side, the coefficient \(\alpha_{Nb}(T)\) for YIG can be substituted with one for GGG since their thermo-mechanical properties are almost identical\(^{64,66}\). The coefficient \(\alpha_G(T)\) for GGG is reported for several temperature ranges separately: room temperature and higher temperature data is available in Refs.\(^{64,65}\), \(\alpha_G(T)\) at low temperatures is reported in Ref.\(^{67}\) for the range from 6 K to 300 K and in Ref.\(^{65}\) for the range from 80 K to 330 K.

Figure 6 shows dependencies of the thermal expansion coefficient on temperature \(\alpha(T)\) for Nb, YIG and GGG.

![Figure 6](image_url)

FIG. 6. Dependencies of the thermal expansion coefficient on temperature \(\alpha(T)\) for Nb, YIG and GGG.

Calculations with Eq. 5 and coefficients \(\alpha(T)\) in Fig. 6 provide the strain at YIG/Nb interface \(\epsilon \approx +6.4 \times 10^{-3}\) that produces a compressive stress \(\sigma \sim 10^9\) dyne/cm\(^2\). Note however, that if the room-temperature deposition of Nb takes place, or the strain in Nb relaxes at room temperature, according to Eq. 5 and Fig. 6 an opposite-sign strain \(\epsilon \approx -4 \times 10^{-5}\) emerges at cryogenic temperature \(T_m\). If the data for GGG is used instead of YIG, the integral in Eq. 5 provides approximately the same strain \(\epsilon \approx +5.6 \times 10^{-4}\) at the interface with unrelaxed Nb, and a larger opposite-sign strain \(\epsilon \approx -4 \times 10^{-4}\) at the interface with the room-\(T\) deposited or relaxed Nb. These values are well comparable with the growth induced tensions provided by the lattice misfit between the GGG substrate and the YIG film that induces the uniaxial anisotropy in LPE-grown\(^{23}\) and PLD-grown\(^{19,20}\) films.

1. B. Lenk, H. Ulrichs, F. Garbs, and M. Münzenberg, “The building blocks of magnonics,” Phys. Rep. 507, 107 (2011).
2. A. V. Chumak, V. I. Vasyuchka, A. A. Serga, and B. Hilbrands, “Magnon spintronics,” Nat. Phys. 11, 453 (2015).
3. Y. Kajiwara, K. Harii, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takanashi, S. Maekawa, and E. Saitoh, “Transmission of electrical signals by spin-wave interconversion in a magnetic insulator,” Nature 464, 262 (2010).
4. Special Issue on Magnonics, J. Phys. D: Appl. Phys. 50 (2017).
5. S. O. Demokritov and A. N. Slavin, eds., Magnonics: From Fundamentals to Applications (Springer-Verlag Berlin Heidelberg, 2013).
10. R. G. E. Morris, A. F. van Loo, S. Kosen, and A. D. I. A. Golovchanskiy, N. N. Abramov, V. S. Stolyarov, S. E. Barnes, M. Aprili, I. Petkovic, and S. Maekawa, “High cooperativity in coupled microwave resonator ferromagnetic-insulator hybrids,” Phys. Rev. Lett. 111, 127003 (2013).

11. Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, “Hybridizing ferromagnetic magnons and microwave photons in the quantum limit,” Phys. Rev. Lett. 113, 083603 (2014).

12. X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, “Strongly coupled magnons and cavity microwave photons,” Phys. Rev. Lett. 113, 156401 (2014).

13. R. G. E. Morris, A. F. van Loo, S. Kosen, and A. D. Karenzowska, “Strong coupling of magnons in a YIG sphere to photons in a planar superconducting resonator in the quantum limit,” Sci. Rep. 7, 11511 (2017).

14. Marco Pfirrmann, Isabella Boventer, Andre Schneider, Tim Wozwand Mathias Kläui, Alexey V. Ustinov, and Martin Weides, “Magnons at low excitations: Observation of incoherent coupling to a bath of two-level-systems,” arXiv:1903.03983 (2019).

15. I. A. Golovchanskiy, N. N. Abramov, V. S. Stolyarov, I. V. Shchetinin, P. S. Dzhumaev, A. S. Averkin, S. N. Kozlov, A. A. Golubov, V. V. Ryazanov, and A. V. Ustinov, “Probing dynamics of micro-magnets with multimode superconducting resonator,” J. Appl. Phys. 123, 173904 (2018).

16. S. E. Barnes, M. Aprili, I. Petkovic, and S. Maekawa, “Ferromagnetic resonance with a magnetic josephson junction,” Supercond. Sci. Technol. 24, 024020 (2011).

17. S. Mai, E. Kandelaki, A. F. Volkov, and K. B. Efetov, “Interaction of josephson and magnetic oscillations in josephson tunnel junctions with a ferromagnetic layer,” Phys. Rev. B 84, 144519 (2011).

18. I. A. Golovchanskiy, N. N. Abramov, V. S. Stolyarov, O. V. Emelyanova, A. A. Golubov, A. V. Ustinov, and V. V. Ryazanov, “Ferromagnetic resonance with long josephson junction,” Supercond. Sci. Technol. 30, 054005 (2017).

19. B.M. Lebed and S.V. Yykovlev, “Dispersion of surface spin waves in a layered superconductor-ferrite structure,” Pis’ma v ZhTF (in Russian) 15(14), 27 (1989).

20. V. B. Anfinogenov, Y. V. Gulyaev, P. E. Zilberman, I. M. Kotelyanskiy, N. I. Polzikova, and A. A. Suhanov, “Observation of the electronic absorption of magnetostatic waves in a ferrite - high temperature superconductor structure,” Pis’ma v ZhTF (in Russian) 15(19), 24 (1989).

21. I. A. Golovchanskiy, N. N. Abramov, V. S. Stolyarov, V. V. Bolginov, V. V. Ryazanov, A. A. Golubov, and A. V. Ustinov, “Ferromagnet/superconductor hybridization for magnonic applications,” Adv. Func. Mater. 28, 1802375 (2018).

22. I. A. Golovchanskiy, N. N. Abramov, V. S. Stolyarov, V. V. Ryazanov, A. A. Golubov, and A. V. Ustinov, “Modified dispersion law for spin waves coupled to a superconductor,” J. Appl. Phys. 124, 233903 (2018).

23. O. V. Dobrovolskii, R. Sachser, T. Brächer, T. Böttcher, V. V. Kruglyak, R. V. Vovk, V. A. Shklovskij, M. Huth, B. Hillebrands, and A. V. Chunak, “Magnon-fluxon interaction in a ferromagnet/superconductor heterostructure,” Nature Physics, doi/10.1038/s41567-019-0428-5 (2019).

24. A. Pimenov, A. Loidl, P. Przybysz, and B. Dabrowski, “Negative refraction in ferromagnet-superconductor superlattices,” Phys. Rev. Lett. 95, 247009 (2005).

25. M. Haidar, M. Ranjarb, M. Balinsky, R. K. Du- mas, S. Khartsev, and J. Akerman, “Thickness- and temperature-dependent magnetodynamic properties of yttrium iron garnet thin films,” J. Appl. Phys. 117, 17D119 (2015).

26. N. Beaulieu, N. Kervarec, N. Thiery, O. Klein, V. Nale- tov, H. Hurdequint, G. de Loubens, J. B. Youssef, and N. Vukadinovic, “Temperature dependence of magnetic properties of a ultrathin yttrium-iron garnet film grown by liquid phase epitaxy: Effect of a Pt overlayer,” IEEE Magnetics Letters 9, 3706005 (2018).

27. I. Boventer, M. Pfirrmann, J. Krause, Y. Schön, M. Kläu, and M. Weides, “Complex temperature dependence of coupling and dissipation of cavity magnon polaritons from mil- likin to room temperature,” Phys. Rev. B 97, 184420 (2018).

28. I. Neudecker, G. Woltersdorf, B. Heinrich, T. Okuno, G. Gubbioiti, and C.H. Back, “Comparison of frequency, field, and time domain ferromagnetic resonance methods,” J. Magn. Magn. Mat. 307, 148 (2006).

29. S. S. Kalarickal, P. Krivosik, M. Wu, C. E. Patton, M. L. Schneider, P. Kabos, T. J. Silva, and J. P. Nibarger, “Ferromagnetic resonance linewidth in metallic thin films: Comparison of measurement methods,” J. Appl. Phys. 99, 093909 (2006).

30. Y.-C. Chen, D.-S. Hung, Y.-D. Yao, S.-F. Lee, H.-P. Ji, and C. Yu, “Ferromagnetic resonance study of thickness-dependent magnetization precession in Ni80Fe20 films,” J. Appl. Phys. 101, 09C104 (2007).

31. C. Dubs, O. Surzhenko, R. Linke, A. Danilewsky, U. Brückner, and Jan Dellith, “Sub-micrometer yttrium iron garnet LPE films with low ferromagnetic resonance losses,” J. Phys.: D Appl. Phys. 50, 204005 (2017).

32. I. A. Golovchanskiy, V. V. Bolginov, N. N. Abramov, V. S. Stolyarov, A. Ben Hamida, V. I. Chichkov, D. Roditchev, and V. V. Ryazanov, “Magnetization dynamics in dilute iron garnet LPE films with low ferromagnetic resonance losses,” J. Phys: Appl. Phys. 50, 204005 (2017).

33. A. Conca, “Measurements of the exchange stiffness of Fe3O4 thin films and patterned microstructures considered for superconducting electronics,” J. Appl. Phys. 120, 163902 (2016).

34. C. Kittel, “On the theory of ferromagnetic resonance absorption,” Phys. Rev 73, 155 (1948).

35. Y. V. Khivintsev, L. Reisman, J. Lovejoy, R. Adam, C. M. Schneider, R. E. Camley, and Z. J. Celinski, “Spin wave resonance excitation in ferromagnetic films using planar waveguide structures,” J. Appl. Phys. 108, 023907 (2010).

36. S. Klingler, A. V. Chunak, T. Mewes, B. Khodadadi, C. Mewes, C. Dubs, O. Surzhenko, B. Hillebrands, and A. Conca, “Measurements of the exchange stiffness of YIG films using broadband ferromagnetic resonance techniques,” J. Phys. D: Appl. Phys. 48, 015001 (2015).

37. J. O. Artman and S. H. Charap, “Ferromagnetic resonance in periodic domain structures,” J. Appl. Phys. 49, 1587 (1978).

38. M. Ramesh and P. E. Wigen, “Ferromagnetodynamics of parallel stripe domains - domain walls system,” J. Mag. Magn. Mater. 74, 123 (1988).

39. I. S. Camara, S. Tacchi, L.-C. Garnier, M. Edruff, F. For- tuna, G. Carlotti, and M. Marangolo, “Magnetization dy-
nematics of weak stripe domains in Fe-N thin films: a multi-
technique complementary approach,” J. Phys.: Condens.
Matter 29, 465803 (2017).
36 T. G. W. Blake, C.-C. Shir, Y.-0 Tu, and E. D. Torre,
“Effects of finite anisotropy parameter Q in the deter-
mination of magnetic bubble material parameters,” IEEE
Trans. Magn. 18, 985 (1982).
37 F. Viot, L. Favre, R. Hayn, and M. D. Kuz’min, “Theory
of magnetic domains in uniaxial thin films,” J. Phys. D:
Appl. Phys. 45, 405003 (2012).
38 S. Lee, S. Grudichak, J. Sklenar, C. C. Tsai, M. Jang,
Q. Yang, H. Zhang, and J. B. Ketterson, “Ferromagnetic
resonance of a YIG film in the low frequency regime,” J.
Appl. Phys. 120, 033905 (2016).
39 S. A. Manuilov, S. I. Khartsev, and A. M. Grishin,
“Pulsed laser deposited Y3Fe5O12 films: Nature of mag-
netic anisotropy I,” J. Appl. Phys. 106, 123917 (2009).
40 D. Stancil, Theory of Magnetostatic Waves (Springer-
Verlag New York, Inc., 1993).
41 A. A. Serga, A. V. Chumak, and B. Hillebrands, “YIG
magnonics,” J. Phys. D: Appl. Phys. 43, 264002 (2010).
42 C. Kittel, “Excitation of spin waves in a ferromagnet by
a uniform rf field,” Phys. Rev. 100, 1295 (1955).
43 M. H. Seavey and P. E. Tannenwald, “Direct observation
of spin wave resonance,” J. Appl. Phys. 90, S227 (1996).
44 S. A. Bunyaev, V. O. Golub, O. Yu. Salyuk, E. V.
Tartakovskaya, N. M. Santos, A. A. Timopheev, N. A.
Sobolev, A. A. Serga, A. V. Chumak, B. Hillebrands, and
G. N. Kakazei, “Splitting of standing spin-wave modes in
regular submicron ferromagnetic dot under axial symme-
try violation,” Sci. Rep. 5, 18480 (2015).
45 J. Smit and H. G. Beljers, “Ferromagnetic resonance ab-
sorption in BaFe12O19 highly anisotropic crystal,” Philips
Res. Rep. 10, 113 (1955).
46 H. Suhl, “Ferromagnetic resonance in nickel ferrite between
one and two kilomegcycles,” Phys. Rev. 97, 555 (1955).
47 S. M. Rezende, J. A. S. Moura, F. M. de Aguiar, and W. H.
Schreiner, “Ferromagnetic resonance of Fe(111) thin films
and Fe(111)/Cu(111) multilayers,” Phys. Rev. B 49, 15105
(1994).
48 P. Hansen, “Anisotropy and magnetostriiction of gallium-
substituted yttrium iron garnet,” J. Appl. Phys. 45, 3638
(1974).
49 Brandon M. Howe, Satoru Emori, Hyung-Min Jeon,
Trevor M. Oxholm, John G. Jones, Krishnamurthy Ma-
halingam, Yan Zhuang, Nian X. Sun, and Gail J. Brown,
“Pseudomorphic yttrium iron garnet thin films with high
and inhomogeneous linewidth broadening,” IEEE
Magnetics Letters 6, 3500504 (2015).
50 Biswanath Bhoi, Bosphung Kim, Yongsub Kim, Min-Kwan
Kim, Jae-Hyeok Lee, and Sang-Koo Ko, “Stress-
induced magnetic properties of PLD-grown high-quality
ultrathin YIG films,” J. Appl. Phys. 123, 203902 (2018).
51 H. Maijer-Flaig, S. Klingler, C. Dubs, O. Surzhenko,
R. Gross, M. Weiler, H. Huebl, and S. T. B. Goen-
nwein, “Temperature-dependent magnetic damping of
yttrium iron garnet spheres,” Phys. Rev. B 95, 214423
(2017).
52 Ch. Jooss, J. Albrecht, H. Kuhn, S. Leonhardt, and
H. Krommüller, “Magnetooptical studies of current dis-
tributions in high-Tc superconductors,” Rep. Prog. Phys.
65, 651 (2001).
53 F. S. Wells, A. V. Pan, S. Wilson, I. A. Golovchanskiy,
S. A. Fedoseev, and A. Rozenfeld, “Dynamic magneto-
optical imaging of superconducting thin films,” Supercond.
Sci. Technol. 29, 035014 (2016).
54 F. S. Wells, A. V. Pan, I. A. Golovchanskiy, S. A. Fedoseev,
and A. Rozenfeld, “Observation of transient overcritical
currents in YBCO thin films using high-speed magneto-
optical imaging and dynamic current mapping,” Sci. Rep.
7, 40235 (2017).
55 C. P. Bean, “Magnetization of high-field superconductors,”
Rev. Mod. Phys. 36, 31 (1964).
56 W. T. Norris, “Calculation of hysteresis losses in hard su-
perconductors carrying ac: isolated conductors and edges
of thin sheets,” J. Phys. D: Appl. Phys. 3, 489 (1969).
57 D.-X. Chen and R. B. Goldfarb, “Kim model for magne-
tization of type-II superconductors,” J. Appl. Phys. 66,
2489 (1989).
58 I. A. Golovchanskiy, A. V. Pan, O. V. Shcherbakova, and
S. A. Fedoseev, “Rectifying differences in transport, dy-
namic, and quasi-equilibrium measurements of critical cur-
rent density,” J. Appl. Phys. 114, 163910 (2013).
59 M. Tinkham, “Resistive transition of high-temperature su-
perconductors,” Phys. Rev. Lett. 61, 1658 (1988).
60 I. A. Golovchanskiy, A. V. Pan, J. George, F. S. Wells,
S. A. Fedoseev, and A. Rozenfeld, “Vibration effect on
magnetization and critical current density of superconduc-
tors,” Supercond. Sci. Technol. 29, 075002 (2016).
61 K. Jeon, C. Ciccarelli, H. Kurebayashi, L. F. Cohen,
X. Montiel, M. Eschrig, T. Wagner, S. Komori, A. Sri-
vastava, J. W. A. Robinson, and M. G. Blamire, “Effect
of meissner screening and trapped magnetic flux on mag-
ettization dynamics in thick Nb/Ni80Fe20/Nb trilayers,”
Phys. Rev. Appl. 11, 014061 (2019).
62 D. F. Gibbons and V. G. Chirba, “Acoustical loss and young’s modulus of yttrium iron garnet,” Phys. Rev. 110,
770 (1958).
63 Kai Wang and Robert R. Reeber, “The role of defects on
thermophysical properties: Thermal expansion of V, Nb,
Ta, Mo and W,” Materials Science and Engineering R23,
101 (1998).
64 S. Geller, G. P. Espinosa, and P. B. Crandall, “Thermal
expansion of yttrium and gadolinium iron, gallium and al-
uminum garnets,” Jr. Appl. Cryst. 2, 86 (1969).
65 Rui sheng Liang and Feng chao Liu, “Measurement of ther-
mal expansion coefficient of substrate GGG and its epi-
taxial layer YIG,” Powder Diffraction 14, 2 (1999).
66 H. J. Levinstein, E. M. Gyorgy, and R. C. LeCraw, “Thermal
expansion of YIG and YIG with Mn and Si additions,”
J. Appl. Phys. 37, 2197 (1966).
67 A. M. Antyukhov, A. A. Sidorov, I. A. Ivanov, and A. V.
Antonov, “Thermal expansion coefficients of crystals of cer-
tain garnets over the range 6-310 K,” Inorg. Mater.
(Translated from Izv. Akad. Nauk SSSR, Neorg. Mater.)
23, 702 (1987).
68 Tso Yee Fan, Daniel J. Ripin, Roshan L. Aggar-
wal, Juan R. Ochoa, Bien Chann, Michael Tillement, and
Joshua Spitzberg, “Cryogenic Yb3+-doped solid-state las-
ers,” IEEE Journal of Selected Topics in Quantum Elec-
tronics 13, 448 (2007).