Validation of wind turbine wake environment modelling

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Abstract. The widely used wake environment models in engineering are validated according to wind tunnel experiment in this paper, namely the Jensen wake model, Frandsen wake model, the Larsen wake model and simplified vortex wake model. From the validation results, it can be found that the wake expansion are overestimated, namely the expansion efficient still needs to be further investigated. In Jensen and Frandsen models, the assumption of uniform distribution of wake speeds along radius differs from the experimental data, with low precision. The wake speed with an approximate Gaussian distribution adopted by Larsen model improves the accuracy. But for all these wake models, due to the overestimation of wake expansion, the wake profiles are definitely calculated with error, thereby lowering the precision of power prediction of wind farms. It is suggested that these models needs to be improved based on more measurement/experimental data.

1. Introduction

The arrangement of wind turbines in a wind farm (namely, microscopic site selection) is closely related to the power generation efficiency of the wind farm. The reason is that when the incoming wind flow passes through the wind turbine upstream, the energy of the wind turbine downstream is weakened, which affects the output of the wind turbine in the rear row, which is called the wake effect. According to the research, the influence of wake on the output of wind farm can be as high as 10-20%[1].

In order to estimate the influence of wake, some scholars put forward wake environment modelling, but the calculation results of these wake models are quite different, so it is urgent to verify and analyze the accuracy of wind farm wake effect prediction, providing references for practical engineering application. In this paper, four wake models commonly used in engineering are compared and evaluated through wind tunnel experimental data.

2 Wake models

At present, the wake models widely used in engineering include Jensen wake model[2, 3], Frandsen wake model[4], Larsen wake model[5] and simplified vortex wake model[6].

(1) Jensen wake model[2, 3]

Jensen's model was proposed by Røs of the national laboratory of Denmark in 1983, and was adopted by the wind resource assessment software WASP. This model assumes that the expansion of wake presents a linear change, and the expression of wake influence radius $R$ and wake velocity $u_w$ is shown in equation (1) and (2):

$$R = R_0 + k_x x$$

(1)
In the above formula, $R_0$ is the radius of the wind turbine, expansion coefficient $k_1=0.075$, $x$ is the distance from the downstream of the wind turbine, $u_\infty$ is the free flow velocity, and $C_T$ is the thrust coefficient of the wind turbine.

(2) Frandsen model\[4\]
Frandsen wake model is proposed based on momentum theory, and wake velocity $U_w$ is expressed as

$$U_w = \frac{U_\infty}{2} \left(1 + \left(1 - C_T\right)^{1/2} \left(\frac{R_0}{R_0 + k_1 x}\right)^2\right)$$

The model defines wake diameter $D(x)$ as

$$D(x) = \left(\beta^{1/2} + \alpha x\right)^{1/2} D_0$$

where $D_0$ is the diameter of wind turbine, $A_0 = \pi D_0^2/4$, $A = \pi D^2/4$, $s = x/D_0$, $k_2=3$, $\alpha = 0.5$, $\beta = 0.5\left(1 + \sqrt{1-C_T}\right)(1-C_T)^{-1/2}$.

(3) Larsen wake model\[5\]
Larsen wake model is a semi-analytical model, which is derived based on the turbulence boundary layer theory of Prandtl and includes first-order and second-order models\[5,6\].

(4) Simplified vortex wake model\[7\]
The vortex model is based on the research on the wake vortex structure of wind turbines. However, considering that the calculation of wake evolution process is complex, it is difficult to be directly applied to engineering practice. Therefore, simplified vortex model is often adopted, as follows:

$$U_w = U_\infty \left(1 - a - af \left(1 + f^2\right)^{-1/2}\right)$$

$$R = R_0 \left(1 - Ef \left(1 + f^2\right)^{-3/2}\right)^{-1/2}$$

where $f = x/R_0$, $a$ is axial induction factor, $E = a/(1-a)$.

3. Experimental setup
The experimental measurement of wind turbine wake was carried out in an open wind tunnel of Delft University of Technology in the Netherlands. In the experiment, a two-blade horizontal axis wind turbine model (with a diameter of 60cm) was adopted, and Stereoscopic Particle Image Velocimetry (SPIV) was used to collect the images of wake flow field\[8,9\]. The experimental data were collected randomly when the wind turbine blade rotates based on the SPIV system and then averaged. The experimental tip speed ratios were 4.8 and 6.0 respectively, and the wake flow data of 2.7 times of wind turbine diameter range along the downstream direction of the wind turbine were collected.

4. Analysis and validation
The wake models are analyzed based on wake boundary and wake velocity profile. The prediction of the radial influence range of wake is an important prerequisite for wind turbine layout in wind farms, and the wake velocity profile is also an important index for evaluating the attenuation and recovery of the wake flow. The combination of the two aspects can provide a direct basis for the micro-site selection of wind turbines and an important content of the wake calculation model.

In table 1, the similarities and differences of each wake model referring to wake radial boundary and wake velocity profile are summarized. It can be seen from the table that Jensen model only assumes that the wake boundary expands linearly along the downstream direction, while Jensen model and Frandsen model both assume that the wake velocity does not change along the radius.
Table 1. Comparisons between wind turbine wake analytical models

| Wake model       | Wake radial boundary                                   | Wake velocity profile                      |
|------------------|--------------------------------------------------------|--------------------------------------------|
| **Jensen model** | Linear expansion along the downstream                 | Distributed uniformly along the radial direction |
| **Frandsen model** | Nonlinear expansion along the downstream with initial expansion at wind turbine location | Distributed uniformly along the radial direction |
| **Larsen model** | Nonlinear expansion along the downstream with initial expansion at wind turbine location | Distributed approximately Gaussian along the radial direction |
| **Simplified vortex wake model** | Nonlinear expansion along the downstream | Only the velocity distribution on the wake center axis is considered |

Based on the experimental data, the calculation accuracy of wake models are compared and verified.

4.1 Wake expansion

Figure 1 compares the predicted wake radial boundary distribution with the experimental values by Jensen model, Frandsen model, Larsen model and simplified vortex model with the tip-speed-ratio TSR of 4.8 and 6.0. In figure 1, the x-axis represents the distance from the position of the wind turbine to the downstream (in dimensionless form x/D), and the y-axis represents the radial distance from the hub of the wind turbine (in dimensionless form y/R). The wake boundary in the experiment is determined at the location when the wake speed reaches 99% u∞.

When TSR=4.8 (see figure 1(a)), the wind turbine wake measured in the experiment gradually expands outward downstream from y/R=1, that is, the blade tip, and reaches the maximum radius (y/R is about 1.2) at x/D=2, then roughly remaining unchanged. It can be seen from figure 1(a) that the boundary of the simplified vortex model also shows a trend that the boundary expands radially and then remains stable, but its calculated value is about 8.2% higher than the experimental value. Jensen model assumes that the wake boundary continues to expand linearly downstream from y/R=1 (at x/D=0). At x/D=2.7, it expands to y/R=1.4, about 16.7% higher than the experimental value. As shown in table 1, Jensen model is based on the premise that the wake boundary expands linearly. Both Frandsen model and Larsen model set the wake boundary where the wind turbine is located to have initial expansion, and the influence range y/R is around 1.3 (at x/D=0). As can be seen from figure 1, both of them are greatly deviated from the experimental value, while the continuous expansions along the downstream with linear rule also deviate from the experimental data which brings significant errors to the horizontal spacing arrangement of wind turbines in the actual wind farm.

When TSR=6.0 (see Figure 1(b)), the measured wake boundary from the experiment is more significant than that of TSR=4.8, and the wake radius y/R at the position x/D=2.5 reaches 1.3. It can be seen that the radial range of the wake flow increases with the increase of the tip-speed-ratio. The wake boundary calculated by Jensen model does not take into account the influence of the tip speed ratio. The predicted result of Jensen model is the same as that of TSR=4.8. The other three models all take the influence of the tip speed ratio into consideration. It can be obtained from figure 1(b) that at x/D=2.5, the calculation result of the simplified vortex model is about 7.7% higher than the experimental value, the calculated results of Frandsen model and Larsen model are about 23% and 29% separately higher than the experimental value.
4.2 wake velocity profiles

In the micro-site selection of wind farm, in addition to the prediction of radial influence range of wake, the velocity field in the downstream development process of wake is also one of the key bases for wind turbine layout.

Figure 2 shows the radial distribution of the velocity at different downstream positions ($x/D=0.5, 1, 1.5, 2, 2.5$) (represented by dimensionless form $u_w/u_{\infty}$). Near the wake center axis, the velocity attenuation is significant, and $u_w/u_{\infty}$ is about 35%, which is consistent with the momentum theory of blade element[1]. In the range near the blade tip ($0.6<y/R<1.2$), with the development of the wake to the downstream (from $x/D=0.5$ to 2.5), the wake speed gradually recovered. However, the experimental data collected by the time-averaged technique are different from the phase-locked technique, so the data of the two are slightly different. However, it can be clearly observed that the velocity near the blade tip increases as the wake extrapolates downstream.

When TSR=4.8 (see figure 2(a)), first of all, Jensen model and Frandsen model assume that wake velocity $u_w$ does not change in the radial direction and becomes $u_{\infty}$ abruptly at the wake boundary. Both these two models overestimate the speed recovery, Jensen model results are closer to the experimental value than that of Frandsen model but still about 25% higher than the experimental value, namely underestimated the wake effect. Second, the velocity Gaussian distribution law of Larsen model (first and second order) is close to the experimental data. In particular, Larsen model presents a relatively ideal prediction result at $x/D=2.5$ (in the region of $y/R<1.2$), and Larsen's first-order model is more consistent with the experimental value than the second-order model in the region of $y/R>1.2$. Finally, from the calculation results of simplified vortex model on the wake center axis velocity, the predicted value is slightly higher than the experimental value about 5%~10%. According to the velocity data on the wake center axis, only simple prediction can be made for the downstream wake flow field.

When TSR=6.0 (see figure 2(b)), the predicted accuracy of the other three models is not ideal, except that the prediction of the velocity at the wake center axis by the simplified vortex model is in good agreement with the experimental value by comparing the calculated value with the experimental value of each model. First of all, at the downstream positions of $x/D=0.5$ and 1, although Jensen model can
better reflect the wake velocity distribution, but due to the interference between the consideration of wind turbines, wind turbines with engineering in the front row layout of the longitudinal spacing will not less than twice the diameter, and with the increase of the downstream distance $x$, prediction accuracy of Jensen model is not better than TSR=4.8 condition. In addition, the calculation results of Frandsen model deviate from the experimental data to a large extent, and the prediction accuracy of Larsen model (first and second order) is lower than that of TSR=4.8 condition.

Figure 2. Validations of wake speed profiles $u_w/u_\infty$ (a) TSR=4.8 (b) TSR=6.0

5 Conclusion
In this paper, the wake models commonly used in current engineering are verified and analyzed based on experimental data. The conclusions can be drawn as follows:
(1) The wake boundary expansion rate behind wind turbine varies along downstream, while Jensen model, Frandsen model and Larsen model do not make corresponding adjustment and overestimate the boundary of wake. The simplified vortex model considers the variation trend of wake expansion along the downstream, but the predicted wake expansion range is larger than the experimental data.
(2) The assumption of uniform distribution of wake velocity by Jensen model and Frandsen model is inconsistent with the actual situation. In comparison, the result of Jensen model is close to the test data within the radius of the wind turbine, while Frandsen model overestimates the speed recovery of the wake, namely underestimating the influence of wake effect. Larsen model adopts Gaussian
distribution which improves the prediction accuracy of wake velocity profile, however due to the prediction error on wake boundary, the velocity distribution presents a large deviation outside the radius of the wind turbine.

To sum up, there are still many shortcomings in the wake models applied in engineering. In order to improve the accuracy of engineering application, the calculation of wake influence boundary and wake velocity profile should be optimized. In addition, the wake experiment data used for verification also need to be further supplemented, so as to better verify and improve the wake model.

6. References
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