On the paper “Quantum theory cannot consistently describe the use of itself”

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Abstract

In the paper “Quantum theory cannot consistently describe the use of itself” by D. Frauchiger and R. Renner an attempt is made at proving a “no-go theorem” that states that either quantum theory cannot be universally applied, even to macroscopic systems, or some very intuitive properties concerning recursive reasoning and uniqueness of physical values must be false.

In this paper, we give a concise description of the paper’s result, and expose a detail in the proof.

Introduction

This document deals on the paper [4], by D. Frauchiger and R. Renner, on the limits of applicability of quantum mechanics.

The paper strives to prove a “no-go theorem” that states that either quantum theory cannot be applied to macroscopic or “reasoning” systems or some other very intuitive assumptions (to be described later) must be false.

To this end, the authors describe a “Gedankenexperiment” in which several agents interact with certain physical systems and apply quantum theory and ordinary reasoning, to obtain a contradictory result.

After the publication of [4], there has been a number of papers stating several observations on the interpretation of the paper, some of them stating that the result is plainly wrong. See, for example, [2, 3, 5, 6, 7, 8]. See also [1] for a paper aiming to clarify, and defend, the conclusions of [4].

The aim of this paper is to describe the reasoning of [4] as succinctly and tersely as possible, and to expose what the author thinks is a detail that renders the proof of the result of [4] incomplete.

The structure of this paper is as follows. In section 1 the framework and methods of [4] are explained concisely. In section 2 we give the details of the “no-go” theorem. In section 3 we point out where a problem in the proof lies. In section 4 a little interpretation is given.

1 The “Gedankenexperiment” and the “no-go” theorem

To simplify the notation, the unitary operators that describe the evolution of the state of an isolated system from one instant of time to another will be implied. For example, suppose that the quantum system $X$ is isolated during a time interval $I$, and that it is in the state $|\Phi\rangle$ in time $t_1 \in I$. Then, if $t_2 \neq t_1$ is also in $I$, we will say that $X$ is also in the state $|\Phi\rangle$ in time $t_2$, instead of saying that it is in the state $U_{t_1\rightarrow t_2} |\Phi\rangle$, or some similar notation.

In the initial setting, we consider four agents named $\bar{F}$, $\bar{W}$, $F$ and $W$. The notation $\bar{W}$ stands for Wigner and $F$ for Wigner’s friend, referring to the well known Wigner’s friend paradox [9].

All four agents are aware of the protocol to be described and are able to apply quantum theory and make logical deductions.

Agent $\bar{F}$ is inside a lab $\bar{L}$, which consists on $\bar{F}$ itself, several experimental devices $\bar{D}$ and a random generator $R$ with two outcomes: heads, with probability 1/3 and tails, with probability 2/3. We can think about $R$ as a quantum system with a twodimensional state space, with a basis $B = \{|\text{heads}\rangle_R, |\text{tails}\rangle_R\}$, which is in an initial state $|\text{init}\rangle_R = \frac{1}{\sqrt{3}} |\text{heads}\rangle_R + \frac{\sqrt{2}}{3} |\text{tails}\rangle_R$. (1)

Thus $\bar{L} = \bar{F} \otimes \bar{D} \otimes R$.

On the other hand, agent $F$, together with a set of devices $D$ form what we provisionally call lab $L_0$, $L_0 = F \otimes D$. Agents $W$ and $\bar{W}$ are outside of $\bar{L}$ and $L_0$ and have no inner information on the outputs of measurements and preparations that take place inside the labs. They are aware of the protocol that is being followed.

Time is labelled in rounds $n$ and a continuous number $x$, as in $t = n : x$.

In the beginning of round $n$, $t = n : 00$ a spin−1/2 particle $S$ enters in $\bar{L}$ and $\bar{F}$ triggers the random generator $R$. That is, $\bar{F}$ measures $R$ in the basis $B$. Let $r$ be its output. If $r = \text{heads}$, $\bar{F}$ prepares $S$ in the state $|\downarrow\rangle_S$ with the device $\bar{D}$, if $r = \text{tails}$, $\bar{F}$ prepares $S$ in $|\rightarrow\rangle_S = (1/\sqrt{2})(|\uparrow\rangle_S + |\downarrow\rangle_S)$, time is now
As the one I am using, is certain that $x = \xi$ at time $t$.”

Then, agent $A$ can conclude “I am certain that $x = \xi$ at time $t$.”

Assumption S. Suppose agent $A$ has established that “I am certain that $x = \xi$ at time $t’$. Then agent $A$ must necessarily deny that “I am certain that $x \neq \xi$ at time $t$.”

2 The proof

After having prepared the spin $S$, agent $\bar{F}$ sends it to $F$ without revealing to her the result of the random trial $r$ and the state she has prepared $S$ in. This happens in $t = n: 05$.

To agents $\bar{W}$ and $\bar{W}$, (and to $F$ just an instant prior to the measurement of $S$ in $t = n: 10$) the system $\bar{L} \otimes \bar{S}$ is in the state:

\[
|\Psi_1\rangle_{L\otimes S} = \frac{1}{\sqrt{3}} (|\bar{h}\rangle_L |\downarrow\rangle_S + \frac{1}{\sqrt{2}} |\bar{f}\rangle_L |\rightarrow\rangle_S)
\]

\[
= \frac{1}{\sqrt{3}} (|\bar{h}\rangle_L |\downarrow\rangle_S + \frac{1}{\sqrt{2}} |\bar{f}\rangle_L (|\uparrow\rangle_S + |\downarrow\rangle_S))
\]

\[
= \frac{1}{\sqrt{3}} (|\bar{h}\rangle_L |\downarrow\rangle_S + |\bar{f}\rangle_L |\uparrow\rangle_S + |\bar{f}\rangle_L |\downarrow\rangle_S).
\]

Just after $F$ made the measurement of $S$ with respect to the basis $\{ |\uparrow\rangle_S, |\downarrow\rangle_S \}$, the system $\bar{L} \otimes \bar{S}$ is, to $W$ and $\bar{W}$, in the state:

\[
|\Psi_2\rangle_{L\otimes L} = \frac{1}{\sqrt{3}} (|\bar{h}\rangle_L |\downarrow\rangle_S = -\frac{1}{2} L + |\bar{f}\rangle_L |\downarrow\rangle_S = +\frac{1}{2} L
\]

\[
+ |\bar{f}\rangle_L |\downarrow\rangle_S = -\frac{1}{2} L).
\]

It can be directly checked that

\[
|\bar{L}\langle \bar{ok}|\bar{L}\langle \bar{ok}|\bar{L}\otimes S\rangle^2_{\bar{L}\otimes S} = \frac{1}{12}.
\]

so we can be certain that there will be a round $n^*$ in which the experiment will halt at $t = n^*: 40$.

However, it is also direct that

\[
|\bar{L}\langle \bar{ok}|\bar{L}\langle \bar{ok}|\bar{L}\otimes L\rangle^2 = 0.
\]

By (4), all the agents are certain that there will come a round $n^*$, such that $\bar{W}$ will publish $\bar{w} = \bar{ok}$ at time $t = n^*: 01$ and $\bar{W}$ will publish $w = \bar{ok}$ at time $t = n^*: 30$.

So, we suppose that we are in such a round $n^*$.

Due to (5), since $\bar{w} = \bar{ok}$, it is impossible that $F$ measured $z = -1/2$ at time $t = n^*: 10$, so it measured $z = +1/2$.

Then, it is impossible that $F$ prepared $S$ in the state $|\rightarrow\rangle_S$, and that was the state in which $F$ received $S$ in time $t = n^*: 05$.

Since $W$ knew in $t = n^*: 20$ the publication $\bar{w} = \bar{ok}$ by $\bar{W}$, all the above reasoning is available to $W$ in $t = n^*: 20$, so $W$ knew that $S$ was in the state $|\rightarrow\rangle_S$ in time $t = n^*: 09$.

So, to $W$, the state of $L$ in time $t = n^*: 10$, after $F$ measured $S$ in the basis $\{ |\uparrow\rangle_S, |\downarrow\rangle_S \}$, was
\[
\frac{1}{\sqrt{2}} \left( \left| z = + \frac{1}{2} \right>_L + \left| z = - \frac{1}{2} \right>_L \right).
\]

Now let's point out a detail:

**Lemma 1.** The state of \( L \) to \( W \) is also \( \text{(6)} \) in time \( t = n^* : 30 \).

*Proof.* Since \( L \) is isolated from \( t = n^* : 05 \) onwards, until \( W \) makes the measurement of \( L \) in \( t = n^* : 30 \).

This state is orthogonal to \( |\text{ok}\rangle_L \). So we know that it is impossible for \( W \) to measure \( w = \text{ok} \) in \( t = n^* : 30 \). A contradiction.

### 3 The problem

However, the state of \( L \) for \( W \) in time \( t = n^* : 10 \) is not \( \text{(6)} \).

Agent \( W \) has arrived to the conclusion that the state of \( S \) to \( F \) is \( |\uparrow\rangle_S \) after the measure, so the state of \( L \) was to him \( \text{(0)} \) in \( t = n^* : 09 \), before the measurement, but it was

\[
|z = + \frac{1}{2}\rangle_L \tag{7}
\]

in time \( t = n^* : 10 \), after the measurement.

Since \( W \) published the result of his measure \( \tilde{w} = \text{ok} \) in \( t = n^* : 20 \), \( W \) has the same knowledge and, for him, the state of \( L \) in \( t = n^* : 10 \) is not \( \text{(6)} \), but \( \text{(7)} \). And the state \( \text{(7)} \) is not orthogonal to \( |\text{ok}\rangle_L \), so \( W \) can observe \( w = \text{ok} \) at \( t = n^* : 30 \). Thus, there are no contradictory conclusions.

The key of the argument lies in Lemma \( \text{1} \). To \( W \), the state of \( L \) was \( \text{(6)} \) in \( t = n^* : 09 \). \( L \) was isolated from then up to \( t = n^* : 30 \). How is it possible that the state is \( \text{(0)} \) in the beginning and, without any interaction, it is \( \text{(7)} \) in the end?

### 4 A basic observation

Let's consider a simple example.

Suppose that Alice is preparing a qubit in a lab in either the state \( |0\rangle \) or \( |1\rangle \) with equal probabilities. Bob is waiting outside the lab in a room without any knowledge of Alice’s preparation.

When she is finished, Alice places the qubit in a perfectly isolating container and handles it to Bob, without revealing the state in which the qubit has been prepared.

Bob is in his room with the container. To him, the state of the qubit is the maximally mixed state

\[
|\Phi\rangle = \frac{1}{2} |0\rangle.
\]

Now suppose that Alice goes from her lab to Bob’s room and tells Bob the state she has prepared the qubit in. Let’s suppose that she lets Bob know that she prepared \( |0\rangle \).

To Bob, the state of the qubit is now \( |0\rangle \). So the qubit has been isolated from the Universe and without any internal dynamics, but by the change in Bob’s knowledge, its state (to Bob) changed from \( |\Phi\rangle \) to \( |0\rangle \).

A way to interpret things could be that the perfect isolating container was not so isolating, after all. The qubit was entangled with Alice, and in talking with Alice, Bob made an indirect measurement of the qubit, so the evolution was not unitary.

Another way of seeing it could be that the fact that a system, like the qubit, was isolated from external influences and had no internal dynamics, didn’t make its state constant, since the state of a system depends, no only on the system, but also on the information that the observer has about it, and this information has changed.

In our case, the state of the system \( L \) was \( \text{(6)} \) in time \( t = n^* : 09 \), but in \( t = n^* : 20 \) \( W \) gets some information from the register of \( \tilde{W} \), which is akin to an indirect measurement of \( L \) by \( W \). The evolution of \( L \), therefore, ceases to be unitary and lemma \( \text{1} \) does not apply.

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