Special features of short-duration processes in condensed media

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Abstract. The problem of the short-duration processes is considered on the base of the non-local theory of non-equilibrium transport, taking into account inertial effects. The system temporal evolution out of equilibrium connected to the dynamic structure transition described by the Speed-Gradient principle (SG-principle or SGP) developed in control theory and cybernetic physics. In the manuscript, we show that retardation of the system response to the short-duration loading due to inertial effects influences on the system evolution and can change its direction. The response to the shock loading of condensed matter is compared to quasi-stationary loading in a wide range of conditions. The short duration loading can lead the system into the structure unstable state and even give rise to self-organization of turbulent structures in the medium. The use of SGP for the modelling of such processes opens new possibilities to control them.

1. Introduction

Unlike long-duration processes, fast, highly non-equilibrium processes cannot be described by models of continuum mechanics. Therefore, the responses of the medium to shock and long-term loading, determined by different stages of relaxation, differ from each other [1]. At the first stage, elastic waves induced by an external action interfere and scatter on inhomogeneities of the real medium, forming mesoscopic wave packets [2]. At the second stage, a transition to plastic deformation occurs, which is accompanied by the self-organization of various turbulent effects. The last stage is a hydrodynamic flow. The initial state of the medium determines the stage at which relaxation begins, and the final state depends on the energy received by the system from the outside.

The mathematical model of the formation and propagation of elastic-plastic waves [3, 4], developed using the nonlocal theory of non-equilibrium transport processes [5], allowed us to explicitly demonstrate the effect of relaxation and inertia in a wide range of loading conditions for condensed media. The obtained relationships between stress and deformation, taking into account the duration and the strain-rate of loading, showed that relaxation and inertia play a very important role in fast processes. It has been found that in fast processes, the effects of self-organization of turbulent motions are possible, as a result of which the irreversible part of the mesoscopic structures can remain in the material after shock loading [3, 4, 6, 7, 8].

2. The integral model of the short-duration processes with retardation effects

A problem on the shock-induced elastic-plastic wave propagating in condensed matter has been solved by Meshcheryakov and Khantuleva [3, 4]. The conception of the shock-induced elastic-
plastic transition based on the nonlocal theory of nonequilibrium transport processes [5] radically differs from the conventional one. Within the nonlocal theory based on nonequilibrium statistical mechanics [9] and cybernetical physics [10], in the case of the plane shock loading when the induced wave propagates along the x-axis, the integral relationship between longitudinal stress component \( J(x, t) \) and the strain-rate \( \dot{\varepsilon} = -\partial v/\partial x \) (\( v \) is the mass velocity) takes a form

\[
J(x, t) = -\rho_0 C^2 t_r \int_0^{\omega(t)} \frac{dt'}{t_r} \int_0^{\Omega(t)} \frac{dx'}{l_r} R \frac{\partial v}{\partial x'}
\]

(1)

Here \( \rho_0 \) is initial unperturbed density, \( C \) is longitudinal sound velocity defined by the relationship \( \rho_0 C^2 = K + 4G/3 \), where \( K, G \) are volume compression and shear elastic moduli. \( t_R \) is typical loading time (force acts over a period \( t_R \)) not included in conventional deformation models, \( L \) is a typical distance from the impact surface (target thickness). The correlation function \( R(t, t'; x, x'; t_r, l_r) \) describes the impulse relaxation, inertial and collective effects. \( R \) depends on the parameters: typical shear relaxation time \( t_r \) and length \( l_r = C t_r \), and on retardation time \( t_m \).

Relaxation of shear degrees of freedom considers being much faster than volume relaxation. For the short duration processes, we can assume the volume relaxation frozen. Unlike differential wave models, the integral relationship (1) determines the stress component \( J(x, t) \) by the strain-rate history all over the wave period. Due to the inertial and relaxation effects in a dense medium, the wave duration can essentially exceed the loading time \( t_R \). During the loading, impulse accumulates in the medium and relaxes after the loading.

In the reference connected to the elastic precursor running at the constant longitudinal sound velocity \( C, \xi = (t - x/C)/t_R, \xi = x/L \) an essential simplification of the non-local model for the impulse transport is gained on the condition \( \tau \partial/\partial \xi \gg \epsilon \partial/\partial \xi \) resulted from the experimentally tested evaluation \( \epsilon/\tau = C t_R/L \ll 1 \). It must be noticed that the division of the typical parameters into fast and slow ones is the necessary condition for self-organization effects. Here \( \tau = t_r/t_R, \theta = t_m/t_R, \epsilon = C t_r/L \) are normalized parameters of relaxation, retardation and nonlocality. Due to the variables division we can consider the wave propagation quasi-stationary and depending on the distance \( x \) only by means of the parameters.

The momentum transport equation with the terms of the order \( \epsilon/\tau \ll 1 \) neglected results in an equation for the mass velocity in the wave with the model integral kernel depending on the parameters \( \tau, \theta \).

\[
v = \int_0^{\omega} d\zeta' \exp \left\{ -\frac{\pi(\zeta - \zeta' - \theta)^2}{\tau^2} \right\} \frac{\partial v}{\partial \zeta'}
\]

(2)

Here the velocity \( v \) is normalized by the impact velocity \( V_0 \). The stress component \( J(\zeta, t) = \rho_0 C V_0 v = \rho_0 C^2 \epsilon \) corresponds to the elastic stress with the mass velocity satisfying to (2). However, unlike the elastic stress, \( v \) in equation (2) depends on the relaxation and delay effects. The equation (2) describes the waveform evolution accompanied by relaxation of the elastic precursor and retardation of the plastic front in terms of the parameters \( \tau, \theta \) depending on the distance \( x/L \) traveled by the wave. In the elastic limit \( \tau \to \infty, \zeta < 1 \) (2) becomes an identity.
Due to the parameters $\tau$, $\theta$ the velocity $v(\zeta, \tau, \theta)$ in the left side of (2) differs from that under the integral in the right side $v(\zeta_0, \tau_0, \theta_0)$, where parameters $\tau_0, \theta_0$ correspond to an initial waveform. It means that stress and strain relate to different spatial points due to the nonlocal and delay effects. So, the integral operator in (2) plays a role of an evolution operator that describes the waveform evolution during its propagation. Substituting an acceleration induced by the loading under the integral in eq. (2), we can get the medium response at a distance from the loading surface.

3. The shock-induced and quasi-stationary loading of condensed matter

An approximate solution to (2) during the loading interval $t_R$ at the constant acceleration $\partial v/\partial \zeta = 1$ has an explicit form [3, 4]

$$v(\zeta, \tau, \theta) = \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}\theta}{\tau} \right), \zeta \leq 1. \quad (3)$$

The solution (3) describes the waveform evolution during the loading.

At the initial stage when $t \leq t_R \ll t_r$ the parameter $\tau = t_r/t_R \gg 1$ is large. Taking into account $\partial v/\partial \zeta = 1$, in the limit $\tau \to \infty$ we get the initial wave front $v(\zeta) = \zeta$ without relaxation. It means that at small times during the loading, the strain $\varepsilon = (V_0/C)v(\zeta)$ and the stress component $J_1 = \rho_0 CV_0 v(\zeta)$ are linearly connected in the same spatiotemporal point as in the elastic case. The relaxation has not begun and considered frozen. The smaller duration of the shock, the closer is the medium response of solids to the elastic one. With large $\tau$ the medium response is always elastic.

During long loading $t_r \ll t \leq t_R$ with small $\tau$, the shear relaxation generates a viscous flow of the Newtonian fluid. In the limit $\tau \to 0$, the shear relaxation is entirely completed

$$J_1 = \rho_0 CV_0 v(\zeta) \longrightarrow \rho_0 CV_0 \left( \frac{C_0^2}{C^2} + \frac{C_0^2}{C^2} \frac{\partial v}{\partial \zeta} \right) \rightarrow \rho_0 CV_0 \frac{C_0^2}{C^2} v$$

During the transition region between the two limits the shear elastic module is changing due to the shear relaxation. In the general case when the loading is more intensive, both modules can change due to beginning of the volume relaxation.

After the short-duration loading $t_R \leq t \ll t_r$ the solution to the equation (2) is different

$$v(\zeta, \tau, \theta) = \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}(1 - \zeta + \theta)}{\tau} \right), \zeta > 1. \quad (4)$$

It describes the shear relaxation as post-shock effect that accompanies the wave propagation. The post-shock effect can be considered as a non-monotone relaxation or as an inertial effect of a dense medium. In the limit $\tau \to \infty$, the waveform and the stress preserve their amplitude.

$$J_1 = \rho_0 CV_0 v(\zeta) \longrightarrow \rho_0 CV_0 \left( \frac{C_0^2}{C^2} + \frac{C_0^2}{C^2} \right) \rightarrow \rho_0 CV_0$$

Without the memory effects after the loading the shear stress disappears

$$J_1 = \rho_0 CV_0 v(\zeta) \longrightarrow \rho_0 CV_0 \left( \frac{C_0^2}{C^2} + \frac{C_0^2}{C^2} \right) \rightarrow \rho_0 CV_0 \frac{C_0^2}{C^2} = \rho_0 C_0^2 \varepsilon.$$

Let us consider as an example the behavior of the stress in a wave induced in a condensed medium by loading and subsequent unloading under various conditions

$$\frac{J_1}{\rho_0 CV_0} = \frac{C_0^2}{C^2} v(\zeta) + \frac{C_0^2}{C^2} \Pi(\zeta; \tau, \theta).$$
Here we take into account only the effects of shear relaxation and assume that the averaged loading force causes a constant acceleration $\frac{\partial v}{\partial \zeta} = 1$, and during unloading, respectively, acceleration in the opposite direction $\frac{\partial v}{\partial \zeta} = -1$.

$$
\Pi_{g1}(\zeta, \tau, \theta) = \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}\theta}{\tau} \right), \; \zeta \leq 1.
$$

$$
\Pi_{g2}(\zeta, \tau, \theta) = \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}(1 - \zeta + \theta)}{\tau} \right) - \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - 1 - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}\theta}{\tau} \right), \; 1 < \zeta \leq 2.
$$

$$
\Pi_{g3}(\zeta, \tau, \theta) = \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}(1 - \zeta + \theta)}{\tau} \right) - \frac{\tau}{2} \left( \text{erf} \frac{\sqrt{\pi}(\zeta - 1 - \theta)}{\tau} + \text{erf} \frac{\sqrt{\pi}(2 - \zeta + \theta)}{\tau} \right), \; \zeta > 2.
$$

**Figure 1.** Behavior of the shear stress during relaxation at the retardation parameter $\theta = 0.1$

It can be seen from the Figures 1, 2 that with an increase in the loading duration, which corresponds to a decrease in the parameters $\tau$, plastic effects arise, which eventually turn into hydrodynamic flows. In this case, with an increase in the delay $\theta$, which can be caused by the inertia of the condensed medium, the maximum stresses are reached already during unloading, after which areas of significant negative stresses are formed. We see that the model of elastic solid is valid at large $\tau$ and low $\theta$ whereas the Newtonian model of liquid is valid at low values...
Figure 2. Behavior of the shear stress during relaxation at the retardation parameter $\theta = 1$ of both $\tau$ and $\theta$. The most deviation of the system response to loading is observed in the intermediate region $\tau \sim \theta \sim 1$ with inertial effects increasing. This is highly non-equilibrium region where the models of continuum mechanics become unsuitable [8, 11, 12].

Just the same value of the stress $J_1 = \rho_0 CV_0$ can be achieved by two different ways. One way is stationary long-duration loading at low strain-rate, the other is the shock loading when during the very short-duration loading, the medium acquires a large impulse inducing an elastic-plastic wave. The retardation effects play an important role in the elastic-plastic waveform.

During the time interval $\theta$, the shear relaxation is already completed, the wave amplitude reaches its maximal value and remains constant on the waveform plateau as far as the volume relaxation is frozen. The waveform amplitude is defined by the shock velocity but not by the plastic medium flow. Unlike the conventional approaches to the shock-induced processes, the plastic front is forming by the shear relaxation after the force of the shock proportional to the medium acceleration (or strain-rate) stops acting [3, 4]. So, the plastic front should be considered as the relaxation front resulted from the retarding response of condensed matter to the high-rate loading. First, this point of view has been presented in the paper [13]. As it was shown in [3, 4], the shock-induced waveforms given by the solutions to the integral equation (2) adequately describe all experimentally obtained dependencies beyond the concepts of continuum mechanics.

Figure 3 shows the fundamental difference between quasistatic and shock loading. According to the solution (3)-(4), during the shock loading, the elastic precursor reaches its the maximal amplitude at the time $t = t_R$ and after that, the shear relaxation forms the plastic front. During the slow continuous loading at the small constant strain-rate no two-wave front forms, one front arises and beyond the elastic limit becomes plastic. Comparison of the two type loading at the same maximal amplitude shows that the strain-rate during the continuous loading should be approximately 14 times lower than during the shock.
Figure 3. Stress growth under shock and continuous loading at $\tau = 15$, $\theta = 10$ and maximal amplitude

4. Temporal evolution of the traveling waveform via SG-principle

High-rate processes out of equilibrium are followed by multi-scale and multi-stage effects that make the system response nonlinear, probabilistic and introduce a feedback between the system structure and its response to an external loading [3, 5, 7, 8]. The system becomes unstable and evolves in the direction of the more equilibrium states under the imposed conditions [14]. The temporal evolution of the system is governed by an internal control with thermodynamically determined goal function and internal control parameters. Maximum entropy principle by Jaynes [14] defines the goal function indicating the direction of the system evolution. Speed gradient principle developed in control theory designed for the control problems of continuous-time systems [10] can be applied to construct an algorithm describing the fastest way to reach the goal.

Unlike conventional continuum mechanics, far from equilibrium, as it was derived in non-equilibrium statistical mechanics, only total entropy generation [15] in the system meets the second law of thermodynamics [9]. Therefore, we choose the total entropy generation as a control goal function that reaches its maximal value in a steady state under the imposed conditions. In the paper, we describe the temporal evolution of the condensed medium response to various loading in a wide range conditions.

The local entropy production $\sigma$, the total entropy generation $S$ and the rate of the entropy generation $dS/d\xi$ (integral entropy production) are defined as follows [16]

$$\sigma(\zeta, \tau, \theta) = J_1 \frac{\partial v}{\partial \zeta}, \quad S(\xi) = \int_0^\xi \int_0^p d\zeta \sigma(\zeta, \tau, \theta), \quad \Sigma = \frac{dS}{d\xi}(\tau(\xi), \theta(\xi)) = \int_0^p d\xi J_1 \frac{\partial v}{\partial \zeta}, \quad (5)$$

where $p$ is duration of a loading process. According to the SG-principle, the integral entropy
production tends to its minimum [1, 16].

\[ \Sigma(\xi) = \int_0^P d\xi \frac{\partial v}{\partial \xi} \int_0^\omega d\xi' \exp \left\{ \frac{-\pi(\xi - \xi' - \theta(\xi))^2}{\tau^2(\xi)} \right\} \frac{\partial v}{\partial \xi'} \]  

(6)

For the processes presented in Figure 1 and Figure 2 the integral entropy production takes a form

\[ \Sigma(\tau(\xi), \theta(\xi)) = \int_0^1 d\zeta \Pi_1(\zeta; \tau(\xi), \theta(\xi)) - \int_1^2 d\zeta \Pi_2(\zeta; \tau(\xi), \theta(\xi)) \]

The surface \( \Sigma \) constructed over the plane of the parameters \( \tau(\xi), \theta(\xi) \) is presented in Figure 4.

In Figure 4 we can see an elevation and a recess on the surface of the integral entropy production. The elevation corresponds to the zone of dissipation located close to local equilibrium. In the recess zone the values of the integral entropy production become negative. This part of the surface corresponds to the case in Figure 1 — 2 when the positive stress values retain during unloading due to the inertial effects. In non-equilibrium statistical mechanics, it was shown that such case is possible far from equilibrium [9].

In high-rate processes induced by short-duration loading when the wave propagation is followed by self-organization of new internal structures, the generalized integral entropy production becomes negative due to the energy consumption for the formation of internal structures [6, 7]. For example, in experimental research on shock loading of solid materials growth of the mass velocity dispersion indicating to the turbulent velocity pulsations were observed inside the waveform in real-time. After the wave propagation, an irreversible part of the pulsations was found out in a material in the form of mesoscopic rotational structures. As it was revealed the turbulent structures may change macroscopic properties of the material, including its strength.

According to the SG-principle [10] the slow temporal evolution of the control parameters \( \tau, \sigma \), connected to the relaxation and retardation effects is a gradient descent on the surface. The finite form of the SG algorithm in this case coincides to the gradient descent algorithm with the gain parameter \( g > 0 \).

\[ \frac{d\tau}{d\xi} = -g_\tau \frac{\partial \Sigma}{\partial \tau}, \quad \frac{d\theta}{d\xi} = -g_\theta \frac{\partial \Sigma}{\partial \theta}. \]  

(7)

Coefficients \( g_\tau > 0, g_\theta > 0 \) are empiric constants characterizing inertial properties of the system internal structure.

Depending on the initial state of the medium structure, the descent path along the surface can lead to quite different final states. The descent from the hill to the origin means the full system relaxation to equilibrium. The descent in the opposite direction can lead to the recess and irreversible self-organization of shear and rotational structures in the medium. Otherwise, the medium response becomes reversible elastic waves. On the top of the hill there is a meta-stable point. Even a slight influence can direct the system evolution to the desired final state.

The control parameters during the descent change, the stress due to the feedback changes too. Through the entropy production rate in the right parts of the (7) when the surface evolves, a feedback between the medium response and turbulent structure evolution is introduced. The formulation of the control problem to describe the system evolution becomes closed and self-consistent. The control close-loops are known to make systems more stable than rigid program control.

The constructed integral mathematical model indicates that only with a short loading regime such structurally unstable states of the system are achievable. The medium properties can
change because of the internal feedback between the stress in the medium and its deformation, which, in turn, can lead to changes of the deformation mechanisms and the medium internal structures. The controlled external influence at the system in the structure unstable state can be used to construct materials with given properties.
5. Conclusion
The proposed approach to modeling high-strain-rate processes based on the nonlocal theory of non-equilibrium transport [5] and cybernetic physics [10] allows an adequate description in a wide range of the loading conditions including transitions between different modes of deformation. Unlike conventional models within continuous mechanics, the integral model relationships between stress, strain and strain-rate depend on the loading conditions and take into account aftereffects arising due to inertial effects of condensed matter. The integral models do not require the separation of stress and strain components into elastic and plastic parts a priori.

Methods of cybernetic physics describe the time evolution of non-equilibrium systems under imposed conditions. The comparison of the shock-induced waveform evolution during its propagation along condensed matter in the framework of SG-principle with the results obtained in experimental study of shock loading materials [3, 4, 7] for different target widths shows that all experimental points on the plane of the control parameters lay down the theoretical straight lines derived for the integral model [17].

The evolution paths allow us to predict the macroscopic properties of the system after loading in the future. For the short-duration loading, such structure unstable states are available in which even a small influence can lead to a radical change of the system properties and behavior. So, the use of the SG-principle for modeling the short-duration processes opens new possibilities to control them.

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