Unparticle effect on $B_s - \bar{B}_s$ mixing and its implications for $B_s \rightarrow J/\psi \phi, \phi\phi$ decays

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Abstract

We study the effect of unparticle stuff on $B_s - \bar{B}_s$ mixing and consider possible implications of it for the decay modes $B_s \rightarrow J/\psi \phi$ and $\phi\phi$. We find that due to the new contributions from the unparticles the $B_s - \bar{B}_s$ mixing phase could be observable at the LHC along with the possible sizable CP asymmetry parameters $S_{\psi\phi(\phi\phi)}$ in $B_s \rightarrow J/\psi\phi(\phi\phi)$ decay modes.

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The standard model (SM) has been found to be very successful in explaining the data up to the electroweak scale but still we believe that it is the low energy manifestation of some beyond the standard model scenario, which exists at high energy, the form of which is not yet known. In the literature, there exist various beyond the standard model scenarios which will be tested in the upcoming experiments. Whereas, an interesting and very much appealing idea has been proposed recently by Georgi [1] regarding the existence of some non-trivial scale invariant hidden sector. Since the conventional particles are not described by the scale invariant theory Georgi termed the physics described by the scale invariant sector as the “unparticle physics”.

In reality, the SM is not scale invariant and contains mostly particles having nonzero mass but a scale invariant theory, if it exists, can only have massless particles. It could be possible that the SM fields at high energy might be scale invariant but the scale invariance has to be broken at least at or above the electroweak scale. Let us assume that the lack of scale invariance of the SM is retained up to the high energy scale and further imagine that there exist scale invariant fields at a higher scale above TeV with a nontrivial infrared fixed point, termed as Banks-Zaks ($\mathcal{BZ}$) fields. Thus, the high energy theory contains both the SM fields and the $\mathcal{BZ}$ fields. They interact via the exchange of particles of large mass $M_\mathcal{U}$ which can generically be written as

$$\frac{1}{M_\mathcal{U}} O_{\text{SM}} O_{\mathcal{BZ}}$$

where $O_{\text{SM}}$ is the operator of mass dimension $d_{\text{SM}}$ and $O_{\mathcal{BZ}}$ is the operator of mass dimension $d_{\mathcal{BZ}}$ made out of SM and $\mathcal{BZ}$ fields respectively. At some scale $\Lambda_\mathcal{U}$ the renormalizable couplings of the $\mathcal{BZ}$ fields cause dimensional transmutation. Below this scale $\mathcal{BZ}$ operators match onto unparticle operators leading to a new set of interactions

$$C_\mathcal{U} \frac{\Lambda_{\mathcal{BZ}}^{d_{\mathcal{BZ}}-d_\mathcal{U}}}{M_\mathcal{U}^k} O_{\text{SM}} O_{\mathcal{U}}$$

where $C_\mathcal{U}$ is a coefficient function in the low energy effective theory and $O_{\mathcal{U}}$ is the unparticle operator with scaling dimension $d_\mathcal{U}$. Furthermore, $M_\mathcal{U}$ should be large enough such that its coupling to the SM fields must be sufficiently weak, consistent with the current experimental data. The production of these unparticles might be detectable by measuring the missing energy and momentum distribution in various processes [1, 2], e.g., $t \to u + \mathcal{U}$, $e^- + e^+ \to \gamma + \mathcal{U}$, $Z \to q\bar{q} + \mathcal{U}$, etc.
Unparticle stuff with scale dimension $d_{\text{U}}$ looks like a non-integral number $d_{\text{U}}$ of invisible massless particles. Unparticle, if exists, could couple to the standard model fields and consequently affect the low energy dynamics. The effect of unparticle stuff on low energy phenomenology has been explored in Refs. [2, 3, 4]. One of the most interesting thing about unparticles is the existence of the peculiar CP conserving phases in their propagators in the time like region, which lead to interesting CP violation phenomena. For example, if nonzero direct CP asymmetry is found in the process $B^0 \rightarrow l^+l^-$, it could be a direct signal of unparticle effects [5].

In this paper, we would like to see the effect of unparticle stuff on the mass difference between the neutral $B_s$ meson mass eigenstates $(\Delta M_s)$ that characterizes the $B_s - \overline{B}_s$ mixing phenomena. It is well known that flavor changing $b \rightarrow s$ transitions are particularly interesting for new physics searches. Among these $B_s - \overline{B}_s$ mixing plays a special role. In the SM, $B_s - \overline{B}_s$ mixing occurs at the one-loop level by flavor-changing weak interaction box diagrams and hence is very sensitive to new physics effects. The effect of unparticle stuff in $B_s - \overline{B}_s$ mixing has also been recently investigated in Ref. [4] where it is observed that large mixing phase could be possible due to unparticle effects, in accordance with our findings.

In general $B_s - \overline{B}_s$ mass difference is defined as $\Delta M_{B_s} = 2|M_{12}^s| = |\langle B^0_s | H_{\Delta B=2}^{\text{eff}} | \overline{B}_s^0 \rangle|/M_{B_s}$, where $H_{\Delta B=2}^{\text{eff}}$ is the effective Hamiltonian responsible for the $\Delta B = 2$ transitions. In the SM the mass difference is given by

$$\Delta M_{B_s} = \frac{G_F^2 M_W^2}{6\pi^2} M_{B_s} \hat{\eta}_B \hat{B}_{B_s} f_{B_s}^2 |V_{tb} V_{ts}^*|^2 S_0(x_t),$$

(1)

where $\hat{\eta}_B$ is the QCD correction factor and $S_0(x_t)$ is the Inami-Lim function with $x_t = m_t^2/m_W^2$. In fact the estimation of the SM value for $\Delta M_{B_s}$ contains large hadronic uncertainties due to $\hat{B}_{B_s} f_{B_s}^2$. Combining the results of JL [8] and HPQCD [9] yields the $B_s$ mass difference as [10]

$$(\Delta M_{B_s})^{\text{SM}}|_{(\text{HP+JL})QCD} = (23.4 \pm 3.8) \text{ ps}^{-1}.$$  

(2)

Recently, Lenz and Nierste [11] updated the theoretical estimation of the $B_s$ mass difference with value $(\Delta M_{B_s})^{\text{SM}} = (19.30 \pm 6.68) \text{ ps}^{-1}$ (for Set-I parameters) and $(\Delta M_{B_s})^{\text{SM}} = (20.31 \pm 3.25) \text{ ps}^{-1}$ (Set-II).

Experimentally, the DØ [12] and CDF [13] collaborations have reported new results for
the $B_s - B_s$ mass difference

$$17 \text{ ps}^{-1} < \Delta M_{B_s} < 21 \text{ ps}^{-1} \quad 90\% \text{ C.L. (DO)}$$

$$\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1} \quad \text{(CDF)}.$$ (3)

Although the experimental results appear to be consistent with the standard model prediction, but they do not completely exclude the possible new physics effects in $\Delta B = 2$ transitions. In the literature, there have already been many discussions both in model independent [10, 11, 14] and model dependent way [15] regarding the implications of these new measurements. In this work we would like to see the effect of unparticle stuff on the mass difference of $B_s$ system and its possible implications for the mixing induced CP asymmetries in $B_s \to J/\psi \phi$ and $\phi \phi$ decay modes. In our analysis we use the central value of (JL+HP)QCD results as the SM contribution and the central value of CDF result as the experimental value for $\Delta M_{B_s}$.

New physics contribution to the mixing amplitude $M_{12}^s$ can be parameterized in the most general way as

$$M_{12}^s = M_{12}^{SM} + M_{12}^{NP} = M_{12}^{SM} (1 - Re^{i\phi}) , \quad (4)$$

where $M_{12}^{SM}$ and $M_{12}^{NP}$ are the SM and new physics (NP) contributions, $R = |M_{12}^{NP}/M_{12}^{SM}|$ and $\phi$ is the relative phase between them. It should be noted here that since the SM contribution to $\Delta M_{B_s}$ is above the present experimental value, we have explicitly made the NP contribution to be negative in the last term of Eq. (4) so that it will interfere destructively with the corresponding SM value for $\phi = 0$. Alternatively, one can also parameterize these contributions as

$$\sqrt{\frac{M_{12}^s}{M_{12}^{SM}}} = r_s e^{i\theta_s} , \quad (5)$$

which gives

$$M_{12}^s = r_s^2 e^{2i\theta_s} M_{12}^{SM} . \quad (6)$$

Values of $r_s^2 \neq 1$ and $2\theta_s \neq 0$ would signal new physics. These two sets of parametrization can be related to each other by

$$r_s^2 = \sqrt{1 + R^2 - 2R \cos \phi} , \quad \text{and} \quad \tan 2\theta_s = \frac{-R \sin \phi}{1 - R \cos \phi} .$$

Now we proceed to see how unparticle stuff will affect the mixing amplitude $M_{12}^s$. It should be noted that, depending on the nature of the original $BZ$ operator $O_{BZ}$ and the transmutation,
the resulting unparticle may have different Lorentz structure. In our analysis, we consider only two kinds of unparticles i.e., scalar type and vector type. Under the scenario that the unparticle stuff transforms as a singlet under the SM gauge group \[1\], the unparticles can couple to different flavors of quarks and induce flavor changing neutral current (FCNC) transitions even at the tree level. Thus, the coupling of these unparticles to quarks is given as

\[
\frac{c^q}{\Lambda_{dU}} \bar{q} \gamma_\mu (1 - \gamma_5) q \partial^\mu O U + \frac{c^{q'}}{\Lambda_{dU-1}} \bar{q}' \gamma_\mu (1 - \gamma_5) q' \ O' U + h.c.,
\]

(7)

where \(O_U\) and \(O'_U\) denote the scalar and vector unparticle fields and \(c^{q,q'}\) are the dimensionless coefficients which in general depend on different flavors. If both \(q\) and \(q'\) belong to up (down) quark sector, FCNC transitions can be induced by the above effective interactions. Thus, the unparticles mediate the \(b \rightarrow s\) transitions in the \(B_s - \overline{B}_s\) mixing where they appear only as propagators with momentum \(P\) and scale dimension \(d_U\).

The propagator for the scalar unparticle field is given as \[1, 2\]

\[
\int d^4xe^{iP.x} \langle 0 | T O_U(x) O_U(0)|0 \rangle = i \frac{A_{dU}}{2 \sin d_U \pi} \frac{1}{(P^2 + i\epsilon)^{2-d_U}} e^{-i\phi_U},
\]

(8)

where

\[
A_{dU} = \frac{16 \pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1) \Gamma(2d_U)}, \quad \text{and} \quad \phi_U = (d_U - 2) \pi.
\]

(9)

Similarly the propagator for the vector unparticle is given by

\[
\int d^4xe^{iP.x} \langle 0 | T O'_U(x) O'_U(0)|0 \rangle = i \frac{A_{dU}}{2 \sin d_U \pi} \frac{-g_{\mu\nu} + P^\mu P^\nu / P^2}{(P^2 + i\epsilon)^{2-d_U}} e^{-i\phi_U}.
\]

(10)

From Eq. (7), one can easily see that the new effective operators contributing to \(B_s - \overline{B}_s\) due to vector/scalar type unparticle exchange are given by

\[
Q_{V-A} = \bar{s} \gamma^\mu (1 - \gamma_5)b \ \bar{s} \gamma_\mu (1 - \gamma_5)b,
\]

\[
Q_{S+P} = \bar{s}(1 + \gamma_5)b \ \bar{s}(1 + \gamma_5)b.
\]

(11)

Using the vacuum insertion method, the matrix elements of these operators are given as

\[
\langle \overline{B}_s| \bar{s} \gamma^\mu (1 - \gamma_5)b \ \bar{s} \gamma_\mu (1 - \gamma_5)b|B_s \rangle = \frac{8}{3} f_{B_s}^2 \hat{B}_{B_s} m_{B_s}^2,
\]

\[
\langle \overline{B}_s| \bar{s}(1 + \gamma_5)b \ \bar{s}(1 + \gamma_5)b|B_s \rangle = -\frac{5}{3} f_{B_s}^2 \hat{B}_{B_s} m_{B_s}^2.
\]

(12)
Thus, we get the new contributions to $M_{12}$ due to the scalar/vector like unparticles as

$$|M_{12}^{\mu}|_{\text{scalar}} = \frac{5}{6} \frac{f_{B_s}^2 \bar{B}_{B_s}}{m_{B_s}} \frac{A_{dU}}{2 |\sin d_U \pi|} \left( \frac{m_{B_s}}{\Lambda_U} \right)^{2d_U} |c_{S}^{sb}|^2,$$

$$|M_{12}^{\mu}|_{\text{vector}} = \frac{1}{2} \frac{f_{B_s}^2 \bar{B}_{B_s}}{m_{B_s}} \frac{A_{dU}}{2 |\sin d_U \pi|} \left( \frac{m_{B_s}}{\Lambda_U} \right)^{2d_U-2} |c_{V}^{sb}|^2. \quad (13)$$

From the above equations one can see that the unparticle contributions depend on three unknown parameters, namely, the dimension of the unparticle fields $d_U$, the scale $\Lambda_U$ and the couplings $c_{S,V}^{sb}$. Therefore, it is not possible to constrain the new physics contributions unless we fix some of these parameters. Now to obtain the constraint on the coupling constants, we fix the energy scale $\Lambda_U=1$ TeV and the scale dimension $d_U=3/2$. We use the value of the decay constant $f_{B_s} \sqrt{\bar{B}_{B_s}} = 0.262$ GeV from Blanke et al in Ref. [14] alongwith the relationship between the bag parameters [11] as $\bar{B}_{B_s} = \left( \frac{m_{B_s}^2}{m_b + m_s} \right)^2 \tilde{B}_{B_s} \approx 1.55 \tilde{B}_{B_s}$. Assuming that only scalar/vector type unparticles contribute at a given time and the total contributions is given by the unparticles one can obtain the upper bound on $c_{S,V}$ as

$$|c_{S}^{sb}| \leq 0.12, \quad \text{and} \quad |c_{V}^{sb}| \leq 0.001. \quad (14)$$

Now to obtain the lower bound on $c_{S,V}^{sb}$, we assume that the minimum value of the unparticle contribution is such that it will just be sufficient to lower the SM contribution to the present experimental value. Thus, from Eqs. (4) and (13) we obtain the lower bounds as

$$|c_{S}^{sb}| \geq 6.75 \times 10^{-2}, \quad \text{and} \quad |c_{V}^{sb}| \geq 5.8 \times 10^{-4}. \quad (15)$$

Now in Figures-1 and 2, we plot $\Delta M_{B_s}$ versus $2\theta_s$ using Eq. (4) for some representative set of values of $|c_{S,V}^{sb}|$ from the above range, where we have varied the weak phase $\phi$ between 0 and $2\pi$. From the figures it can be seen that large value of mixing phase could be possible due to unparticle effects. Measurement of this phase in the upcoming experiments such as LHC could imply an indirect evidence for the existence of unparticles.

Since large mixing phase is indeed possible due to unparticle effect, now we would like to look into its possible implications in the mixing induced CP violation in $B_s \rightarrow J/\psi \phi$. The $B_s \rightarrow J/\psi \phi$ decay channel is accessible at hadron colliders where plenty of $B_s$ is expected to be produced. It is therefore considered as one of the benchmark channels to be studied at the LHCb experiment. This decay mode proceeds through the quark level transition $b \rightarrow c\bar{c}s$ which is analogous to $B_d \rightarrow J/\psi K_s$. However, the final state in $B_s \rightarrow J/\psi \phi$ is not
FIG. 1: Correlation plot between $\Delta M_{B_s}$ in ps$^{-1}$ and $2\theta_s$ in degree for a representative set of values of $|c_S^{sb}|$ as labelled in the plots.

FIG. 2: Same as Figure-1, with vector like unparticle contributions where the constants $|c_V^{sb}|$ are in units of $10^{-4}$.

a CP eigenstate but a superposition of CP odd and even states, which can be disentangled through an angular analysis of their decay products [16]. Therefore, the mixing induced CP asymmetry in this mode is expected to give

$$S_{J/\psi \phi} = -\sin 2\beta_s ,$$

where $\beta_s \equiv \arg(V_{tb}V_{ts}^*) \approx -1^\circ$. Since this decay mode receives dominant contribution from the color suppressed tree level transition $b \rightarrow c\bar{c}s$, it is unlikely that new physics contribution to the decay amplitude will significantly modify the SM amplitude. Therefore,

7
we will assume that the new physics contribution to this decay amplitude is negligible and hence the CP asymmetry will be modified because of the new contributions to the mixing. Thus, in the presence of NP the mixing-induced CP asymmetry can be obtained as follows.

Assuming that there is no direct CP violation in this mode one obtains

\[ S_{J/\psi\phi}\sin \Delta M_s t = \frac{\Gamma(B_s(t) \rightarrow J/\psi\phi) - \Gamma(B_s(t) \rightarrow J/\psi\phi)}{\Gamma(B_s(t) \rightarrow J/\psi\phi) + \Gamma(B_s(t) \rightarrow J/\psi\phi)} \]

\[ = \frac{D \text{ Im} \left( \frac{q}{p} \rho_{\text{odd}} \right) + \text{ Im} \left( \frac{q}{p} \rho_{\text{even}} \right)}{D F_{\text{odd}}(t) + F_{\text{even}}(t)} \sin \Delta M_s t \]  

(17)

where

\[ F_{\text{odd,even}}(t) = \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + \text{ Re} \left[ \frac{q}{p} \rho_{\text{odd,even}} \right] \sinh \left( \frac{\Delta \Gamma_s t}{2} \right), \]  

(18)

with

\[ \rho_{\text{odd,even}} = \frac{A(B_s \rightarrow J/\psi\phi)_{\text{odd,even}}}{A(B_s \rightarrow J/\psi\phi)_{\text{odd,even}}}, \quad \text{and} \quad D = \frac{|A_\perp|^2}{|A_\parallel|^2 + |A_0|^2}. \]  

(19)

\( \Delta \Gamma_s \) is the lifetime difference between heavy and light \( B_s \) eigen states. Thus, we get

\[ S_{J/\psi\phi} = \frac{(1 - D) \sin 2|\beta_s|}{(1 + D) \cosh(\Delta \Gamma_s t/2) + (1 - D) \cos 2\beta_s \sinh(\Delta \Gamma_s t/2)}. \]  

(20)

Taking the limit \( \Delta \Gamma_s \rightarrow 0 \) and scaling out the CP odd fraction we obtain

\[ S'_{J/\psi\phi} = \frac{S_{J/\psi\phi}}{1 - 2f_\perp} = \sin(2|\beta_s| - 2\theta_s), \]  

(21)

where \( f_\perp = |A_\perp|^2/(|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2) \). Now plotting \( S'_{J/\psi\phi} \) versus the new mixing phase \( \theta_s \) in figure-3, we see that large CP violation could be possible in this mode.

FIG. 3: The variation of \( S'_{J/\psi\phi} \) versus the mixing \( \theta_s \) in degree.
Thereafter, we consider another decay channel $B_s \to \phi \phi$ which is a pure penguin induced process and proceeds through the quark level transition $b \to s \bar{s}s$. Assuming the top quark dominance in the loop, the mixing induced CP asymmetry in the SM turns out to be identically zero because the weak phase in the mixing and in the ratio of decay amplitudes exactly cancel each other. Since the dominant SM contribution arises at the one-loop level it is expected that this decay channel may receive new contribution from NP in its decay amplitude, unlike the $B_s \to J/\psi \phi$ process. Therefore, we are interested to see how $S_{\phi \phi}$ will be modified due to the unpraticle contributions in its decay amplitude.

To see the effect of NP in the decay amplitude we first consider the SM amplitude. In general the decay mode $B_s \to \phi \phi$ can be described in the helicity basis, where the amplitude for the helicity matrix element can be parametrized as

$$H_\lambda = \langle \phi(\lambda)\phi(\lambda)|H_{eff}|B_s \rangle$$

$$= \varepsilon^*_1(\lambda)\varepsilon^*_2(\lambda) \left[ a g^{\mu\nu} + \frac{b}{m_\phi^2} p^\mu p^\nu + \frac{ic}{m_\phi^2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right] ,$$

where $p$ is the $B_s$ meson momentum and $\lambda = 0, \pm 1$ are the helicity of the $\phi$ mesons. In the above expression $p_i$ and $\varepsilon_i$ ($i = 1, 2$) stand for their momenta and polarization vectors of the two $\phi$ mesons respectively. Furthermore, the three invariant amplitudes $a$, $b$, and $c$ are related to the helicity amplitudes by

$$H_{\pm 1} = a \pm c \sqrt{x^2 - 1} , \quad H_0 = -ax - b(x^2 - 1) ,$$

where $x = (p_1 \cdot p_2)/m_\phi^2 = (m_B^2 - 2m_\phi^2)/2m_\phi^2$.

The corresponding decay rate using the helicity basis amplitudes can be given as

$$\Gamma = \frac{p_{cm}}{8\pi m_{B_s}^2} \left( |H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2 \right) ,$$

where $p_{cm}$ is the magnitude of c.o.m. momentum of the outgoing $\phi$ mesons.

The amplitudes in transversity and helicity basis are related to each other through the following relations

$$A_\perp = \frac{H_{+1} - H_{-1}}{\sqrt{2}} , \quad A_\parallel = \frac{H_{+1} + H_{-1}}{\sqrt{2}} , \quad A_0 = H_0 .$$

The SM amplitude for the process $\overline{B}_s \to \phi \phi$ can be represented in the factorization approach as

$$A(\overline{B}_s \to \phi \phi) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* 2 \left[ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right] X ,$$

where $a_i$ are the Wilson coefficients at the weak scale.
where

\[ X \equiv \langle \phi(\varepsilon_2, p_2) | \bar{s} \gamma_\mu (1 - \gamma_5) s | 0 \rangle \langle \phi(\varepsilon_1, p_1) | \bar{\phi} \gamma^\mu (1 - \gamma_5) b | \bar{B}_s(p) \rangle \]  

(27)
is the factorizable hadronic matrix element and \( a_i \) are the QCD coefficients. In the factorization approximation, the factorized matrix element \( X \) (Eq. (27)) can be written, in general, in terms of form factors and decay constants which are defined as

\[
\langle \phi(\varepsilon_2, p_2) | V_\mu | 0 \rangle = f_\phi m_\phi \varepsilon^\mu_{2\varepsilon}, \\
\langle \phi(\varepsilon_1, p_1) | V_\mu | B_s(p) \rangle = \frac{2}{m_\phi + m_{B_s}} \epsilon_{\mu\alpha\beta} \varepsilon^\alpha_{1\varepsilon} p^\beta q V(q^2), \\
\langle \phi(\varepsilon_1, p_1) | A_\mu | B_s(p) \rangle = -i \frac{2 m_\phi (\varepsilon^\mu_1 \cdot q)}{q^2} q_\mu A_0(q^2) - i (m_\phi + m_{B_s}) \left[ \varepsilon^\mu_{1\varepsilon} - \frac{(\varepsilon^\mu_1 \cdot q)}{q^2} q_\mu \right] A_1(q^2) + i \left[ (p + p_1)_\mu - \frac{(m_{B_s}^2 - m_\phi^2)}{q^2} q_\mu \right] \frac{(\varepsilon^\mu_1 \cdot q)}{m_\phi + m_{B_s}} A_2(q^2),
\]

(28)

where \( V_\mu \) and \( A_\mu \) are the corresponding vector and axial vector quark currents and \( q = p - p_1 \) as the momentum transfer. In this way the invariant amplitudes \( a, b, \) and \( c \) read as

\[
a = i C_{eff} f_\phi m_\phi (m_{B_s} + m_\phi) A_1^{B_s \rightarrow \phi}(m_\phi^2), \\
b = -i C_{eff} f_\phi m_\phi \left( \frac{2 m_\phi^2}{m_{B_s} + m_\phi} \right) A_2^{B_s \rightarrow \phi}(m_\phi^2), \\
c = -i C_{eff} f_\phi m_\phi \left( \frac{2 m_\phi^2}{m_{B_s} + m_\phi} \right) V^{B_s \rightarrow \phi}(m_\phi^2),
\]

(29)

where

\[
C_{eff} = - \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} 2 \left[ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right].
\]

(30)

The values of the QCD improved effective coefficients \( a_i \) can be found in Ref. [18]. Now substituting the values of \( a_i \) for \( N_C=3 \), from Ref. [18], the value of the form factor \( V^{B_s \rightarrow \phi}(m_\phi^2) = 0.461, A_1^{B_s \rightarrow \phi}(m_\phi^2) = 0.317, A_2^{B_s \rightarrow \phi}(m_\phi^2) = 0.245 \) are obtained using the LCSR approach [18], and using the \( \phi \) meson decay constant \( f_\phi = 0.231 \) GeV, \( |V_{tb} V_{ts}^*| = 41.3 \times 10^{-3} \) and \( \tau_{B_s} = 1.466 \times 10^{-12} \) sec [20], we obtain the branching ratio in the SM as

\[
BR^{SM}(B_s \rightarrow \phi \phi) = 10.4 \times 10^{-6}.
\]

(31)

which appears to be consistent with the experimental value \( BR(B_s \rightarrow \phi \phi) = (14^{+8}_{-7}) \times 10^{-6} \) [21]. But still one cannot rule out the possibility of NP in the decay amplitude as we need the measurement of CP violating parameters to support it.

Now let us consider the effect of new physics in the decay amplitude. Since it is possible to obtain the different helicity contributions by performing an angular analysis [16, 22],

10
from now onward we will concentrate on the longitudinal (i.e., $A_0$ component), which is the dominant one. In the presence of NP the amplitude can be modified to

$$A_0 = A_0^{SM} + A_0^{NP} = A_0^{SM}(1 + re^{i\phi_n}),$$

(32)

where $r = |A_0^{NP}/A_0^{SM}|$, and $\phi_n$ is the relative weak phase between them. For simplicity we set the relative strong phase between these two amplitudes to zero, which in general is expected to be small. Thus, in the presence of new physics both in mixing and decay amplitude the mixing induced CP asymmetry (due to longitudinal component) is given as

$$S_{\phi\phi} = 2 \frac{\text{Im}(e^{-i2(\beta_s + \theta_s)}A_0^*\bar{A}_0)}{|A_0|^2 + |\bar{A}_0|^2}$$

$$= -\frac{\sin(2\theta_s) + 2r\sin(2\theta_s + \phi_n) + r^2\sin(2\theta_s + 2\phi_n)}{1 + r^2 + 2r\cos\phi_n}.$$  

(33)

To find out the value of $r$ due to unparticle contribution, we now consider the effective coupling of unparticles to the quarks as represented in Eq. (7). Here we consider the effect of vector like unparticle to the $B_s \rightarrow \phi\phi$ decay amplitude. Thus, the transition amplitude due to vector like unparticle exchange is given as

$$A(B_s \rightarrow \phi\phi) = -e^{-i\phi_{ul}} \frac{A_{A_0}}{2\sin\frac{d_{ul}}{4\pi} \left(\frac{m_{B_s}}{A_{ul}}\right)^{2d_{ul}-2} \left(\frac{1}{2}\right)^{d_{ul}-2} \frac{c_{V}^{u,s}}{m_{B_s}^2} 2X},$$

(34)

where $X$ is the factorized amplitude given in Eq. (27). In the above equation we have taken the momentum transferred to the unparticle as $P^2 = m_{B_s}^2/2$. Now for numerical evaluation, we use a representative value for $c_{V}^{u}$, i.e., $c_{V}^{u} = 8 \times 10^{-4}$ from its allowed range, $c_{V}^{s}=0.01$ and the same values for other parameters as used in $\Delta M_{B_s}$ case. Thus, we obtain the the ratio of the NP and SM amplitudes as

$$r = 0.03.$$ 

It is found that the unparticle contribution to the decay amplitude is almost negligible.

Now in figure-4, we plot $S_{\phi\phi}$ versus $\phi_n$, the weak phase in decay amplitude keeping the new mixing phase $\theta_s = 20^\circ$ and $\theta_s = 0$ (i.e., with no NP contribution to mixing). From the figure one can see that the unparticle contributions to the decay amplitude does not have significant effect in $S_{\phi\phi}$.

Motivated by the recent proposition of scale invariant unparticle physics we looked into the effect of the same on the $B_s - \bar{B}_s$ mixing. In doing so, we included the new physics
FIG. 4: The variation of $S_{\phi\phi}$ versus $\phi_n$ in degree where the thick (dashed) curve is for $\theta_s = 20^\circ (0^\circ)$.

coupled contribution, in the form of scalar/vector unparticles, to the SM contribution and obtained the constraints on the couplings of unparticle stuff to the SM particles ($c_{S,V}$) from the data on $\Delta M_{B_s}$. We found that due to the effect of “unparticles” large new mixing phase could indeed be possible. It has also been observed in Ref. [4] that due to unparticle effects it is possible to have large mixing phase in agreement with our results.

Furthermore, we looked into the possibility of obtaining the mixing induced CP asymmetry parameters $S_{\psi\phi, \phi\phi}$ for the decay modes $B_s \to J/\psi \phi$ and $\phi\phi$, which can be induced by the new contribution of unparticle stuff. In the SM the value of $S_{\psi\phi}$ is very small and $S_{\phi\phi}$ is identically zero, therefore observation of non-zero values for these parameters would signal new physics. Incorporating the NP contribution from the unparticle sector and using the constraint on $c_{S,V}$ we obtained the values of $S_{\psi\phi, \phi\phi}$ for the above mentioned decay modes, to be nonzero. Search for “unparticle effect” will be vigorously taken up at the upcoming experiments and in this context the observation of possible large new mixing phase in $B_s - \bar{B}_s$ system and non-zero values of $S_{\psi\phi, \phi\phi}$ will be very much useful.

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