Comment on “Quasiparticle Anisotropy and Pseudogap Formation from the Weak-Coupling Renormalization Group Point of View”

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In their recent Letter, Katanin and Kampf (KK) reported numerical results for the self-energy $\Sigma(k; \varepsilon)$, $\varepsilon \in \mathbb{R}$, of the single-band Hubbard Hamiltonian (hereafter ‘single-band’ will be implicit) in two space dimensions (i.e. $d = 2$), obtained through employing the functional renormalization-group (fRG) formalism (see references in [4, and 5]) at the one-loop level. Several of the results by KK are in full conformity with the exact formal results reported earlier in [3, 4, 5]. This, as we shall elaborate below, strengthens one’s confidence in the reliability of the fRG in dealing with models of strongly-correlated fermions.

(i) The quasi-particle (qp) weight $Z_\varepsilon$ at the Fermi surface. — Amongst other things, KK showed that [1] “At van Hove (vH) band fillings and at low temperatures, the quasiparticle weight along the Fermi surface (FS) continuously vanishes on approaching the ($\pi, 0$) point . . . .” In [3] we have obtained a general expression for the momentum-distribution function $n(k)$ at $k = k^\perp_\varepsilon$, i.e. infinitesimally in- and outside Fermi sea (here we suppress the spin indices encountered in [3, 4, 5]), which on the basis of the available numerical results at the time we have shown to reduce to (Eq. (99) in [3]): $n(k^\perp_\varepsilon) = (a + Ub^\perp)/(a + 2Ub^\perp)$, where $a$ and $b$ are components of vectors $a(k_\varepsilon) \equiv \nabla_{k^\perp_\varepsilon} |k^\perp_\varepsilon|$, and $b(k^\perp_\varepsilon)$ along the outward unit vector normal to Fermi surface $S_\varepsilon$ at $k = k^\perp_\varepsilon$; here $\varepsilon \perp$ is the non-interacting energy dispersion and $b(k)$ the gradient of a well-defined scalar ground-state (GS) correlation function. Stability of the latter GS is tantamount to the satisfaction of $b^\perp > a/(U\Lambda^-)$, $b^\perp < -a/U$, where $\Lambda^- \equiv n(k^\perp_\varepsilon)/[1 - n(k^\perp_\varepsilon)] > 0$. Our above expression for $n(k^\perp_\varepsilon)$ makes explicit that (a) $n(k^\perp_0) \equiv 1/2$, (b) $n(k_\varepsilon^\perp) \rightarrow 1/2$, i.e. $Z_\varepsilon \equiv n(k^\perp_\varepsilon) - n(k^\perp_0) \rightarrow 0$, for $U b^\perp \rightarrow \infty$, and (c) $n(k^\perp_\varepsilon) = 1/2$, i.e. $Z_\varepsilon = 0$, for $a = 0$, that is for $k_\varepsilon$ at vH points. In Fig. 1 we compare our results for $Z_\varepsilon$ with those determined by KK [1], disregarding the dependence of $b^\perp$ on the direction of $k_\varepsilon$.

(ii) The single-particle spectral function $A(k; \varepsilon)$ for $k$ in the pseudogap (PG) region. — KK observed that [1] “The qp weight suppression [for $k_\varepsilon \rightarrow (\pi, 0)$] is accompanied by the growth of two additional incoherent peaks in the spectral function, from which an anisotropic pseudogap originates.” On general theoretical grounds, in [3, 4, 5] we have shown that the singular nature of $n(k)$ at all $k \in S^{(0)}_\varepsilon$ (see (iii) below) implies that (see Sec. 10 in [5]) $Z_\varepsilon \rightarrow 0$, for $k \rightarrow \text{PG}$, must necessarily be accompanied by at least two resonant peaks (to be distinguished from qp peaks) in $A(k; \varepsilon)$, one strictly below and one strictly above the Fermi energy $\varepsilon_F$. Here $S^{(0)}_\varepsilon$ is the Fermi surface associated with $\varepsilon_F$ (see (iii) below).

(iii) Fermi surface non-deformation. — For models involving solely contact-type interaction, we have shown that $S_\varepsilon \subseteq S^{(0)}_\varepsilon$; PG consists of those points of $S^{(0)}_\varepsilon$, if any, which do not belong to $S_\varepsilon$. Interestingly, $S_\varepsilon \subseteq S^{(0)}_\varepsilon$ turns out to be the working hypothesis for many calculations, amongst which those by KK [1].

We should like to emphasize that the above-mentioned results, cited from [3, 4, 5], are not restricted to the weak-coupling limit; the only constraint for the validity of these results is the uniformity of the underlying GSs. We further point out that, in principle, depending on the values of $d, U/t, t'/t$, etc., some other ‘universality classes’ for the uniform metallic GSs of the Hubbard Hamiltonian (explicitly considered in [3, 5]) may/do become viable than that dealt with in this Comment.

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