A biologically-motivated system is poised at a critical state

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Abstract

We explore the critical behaviors in the dynamics of information transfer of a biologically-inspired system by an individual-based model. “Quorum response”, a type of social interaction which has been recognized taxonomically in animal groups, is applied as the sole interaction rule among particles. We assume a truncated Gaussian distribution to quantitatively depict the distribution of the particles’ vigilance level and find that by fine-tuning the parameters of the mean and the standard deviation of the Gaussian distribution, the system is poised at a critical state in the dynamics of information transfer. We present the phase diagrams to exhibit that the phase line divides the parameter space into a super-critical and a sub-critical zone, in which the dynamics of information transfer varies largely.

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I. INTRODUCTION

Complex system, which is composed of large numbers of components that interact locally, shares some universal features in a dynamic process. Self-organized critically (SOC), which was proposed by Bak, Tang and Wiesenfeld in 1987, is now a commonly accepted underlying mechanism to phenomena as earthquakes, solar quakes, and even dynamics in brain et al. It states that a complex system can organize itself to a critical state without tuning parameters from outside. The “fingerprint” of a system entering a critical state is a power law distribution of the size of the avalanches which is measured by counting the number of the affected individual components in the dynamic process. Recently, Cavagna et al. observed that in the airborne motion of large starling flocks, the length of correlation between two individuals’ state doesn’t depend on the size of the flock, the so-called scale-free correlation. This observation reveals that the starling flocks work at a critical state, in which one individual can effectively affect the state of any others’ no matter what the group size is, and vice versa. This property confers the group an ability to share information efficiently so that it can optimally respond to external perturbations. A pioneering study on how information transfer in a collective animal group was carried out in a fish school reacting to a risky perturbation in front. It was found that the fishes at the front made a quick rotation from the risk and their local neighbors behind imitated this behavior. The consecutive rotations of the fishes resulted in a rapidly traveling “information waves”, which rippled from the front to the rear at a speed much faster than individual fish’s speed. However, besides these experimental studies, the underlying micro-mechanism of information transfer is left largely ignored. There are some other types of collective systems, such as swarms of cancer cells, bacterial colonies and even human brains, share many similarities to the collective animal groups, and the working efficiency of which may depend on the underlying mechanism of information transfer.

In this paper, we study the dynamics of information transfer at critical points in a minimum individual-based model with the interaction among particles being quorum response. Each particle is assigned a “vigilance number” to quantitatively depict its vigilance level to respond to its local neighbors’ commitment. We assume the distribution of the “vigilance number” to be a truncated Gaussian distribution in the interval of (0, 1). By tapping information from boundaries into the system, we find that, by fine tuning parameters of
the Gaussian distribution, the system can be poised at a critical state in the dynamics of information transfer. We present phase diagrams to show that the critical points divide the parameter space into a sub-critical and a super-critical zone, in which the dynamics of information transfer is quite different.

II. QUORUM RESPONSE

Quorum response is a type of social interaction widely found during the process of collective decision-making in the bee and ant colonies\textsuperscript{12,13}, the cockroach aggregations\textsuperscript{14}, the broiler chicken crowds\textsuperscript{15} and the fish schools\textsuperscript{16}. It quantitatively states that an individual’s chance of making one option depends on the number of its local neighbors that committed to this option.

![Figure 1: Function of quorum response according to equation (1). The y axis, $p^i_+(t+1)$, is the probability for the particle $i$ at time step $t+1$ to choose the option “+” and the x axis, $n^i_+(t)$, is the number of its local neighbors that committed to the option “+” at time step $t$. The total number of local neighbors is set to be 10 and the quorum value $q_i$ is set to be 5 in the figure. The parameter of $k = 3, 8, \infty$, which determines the steepness of the curve, is randomly selected from the infinite series.](image)

Let’s consider the following simple example, suppose there are only two options, e.g. being at an alarmed state (“+” state) or a naive one (“-” state). The mathematical description of
the rule of “quorum response” is as follows\textsuperscript{17}:

\[
p_\pm^i(t+1) = \frac{(n^i_\pm(t))k}{1 + (n^i_\pm(t))k}, \quad p_\pm^i(t + 1) = 1 - p_\pm^i(t + 1)
\]

(1)

Where \(p_\pm^i(t + 1)\) is the probability of the individual \(i\) choosing to be at an alarmed or a naive state at time step \(t + 1\), respectively and \(n^i_\pm(t)\) is the number of local neighbors who have committed to the alarmed or the naive state at time \(t\), respectively. The term \(q_i\) is the quorum value for the individual \(i\), which is set to be 5 in Figure 1. We see that the probability is a monotonic increasing function. Near the quorum value, it has an inflection point with a rapid increase and the function is sigmoid. If \(k\) is bigger, the variation of the curve becomes steeper than the linear increase at the quorum value. Thus it can be expected that \(k \geq 2\) is a necessity in the framework of the interaction rule\textsuperscript{17}. In field experiments, it is found that the animal group adapt \(k\) around \(3\textsuperscript{14,16,17}\). When the parameter \(k\) approaches infinity, the plot is practically a step-like switch at the quorum value, jumping from zero to unity. Quorum response is essentially a distributed positive feedback process that enables information propagation and it is believed that this type of interaction can enhance decision speed and accuracy for a group to make a collective decision\textsuperscript{17}.

III. AN INDIVIDUAL-BASED MODEL

A 2D system is composed of a square of the dimension of 100 × 100 evenly spaced grid. Each particle is positioned in a grid, and the individual particle’s mobility is ignored because its speed is far slower than the speed of information propagation\textsuperscript{6}. We assign each particle a “vigilance number” \(\alpha_i\) \((i = 1, 2, \ldots)\) which measures how vigilant the particle \(i\) is responding to its local neighbor’s commitment in the framework of quorum response. The distribution of \(\alpha_i\) is assumed to obey a truncated Gaussian distribution in the interval \((0, 1)\), with the mean being \(\mu\) and the standard deviation being \(\Delta\).

The sole interaction rule among the particles in the model is the quorum response according to equation (1), with the quorum value of the particle \(i\) is defined as,

\[q_i \equiv n_0 \ast \alpha_i\]

where \(n_0 = 4\) is the constant number of the nearest neighbors to any particle not positioned at boundaries (if a particle lies at one of the four corners, \(n_0 = 2\), or else if it lies at one of
the four boundary lines, \( n_0 = 3 \). Each particle can either be in an alarmed state (“+” state) or in a naive state (“-” state) at the probability calculated according to equation (1). The probability is realized by Monte Carlo method at each time step in the dynamic process of the system, i.e. a random number which is evenly distributed in the interval of \((0,1)\) is sampled at the time step \( t \) and being compared to the probability \( p_i^+(t + 1) \) calculated according to equation (1). If the sampled random number is smaller, then the particle \( i \) will turn into the alarmed state at the next time step. Otherwise, it will stay in the naive state.

Following the general assumption that individuals at peripheries find the approaching risks in advance\(^{18}\), we tap information in the system by randomly picking a particle lying at one of the boundary lines and turn its state to the alarmed one. The local interactions may affect the state of its neighbor(s), and the affected neighbors continue to repeat the interaction which may cascade into an “information wave” eventually. If a particle turns into the alarmed state at time step \( t \), then it will stay at the alarmed state unchanged during the dynamics of information propagation. One run of information propagation is considered completed when all the alarmed particles are not capable to alter the state of its nearest neighbors anymore.

FIG. 2: Power law distribution of the “information waves”. The total population of the particles is \( 100 \times 100 \) and the data is averaged over \( 10^6 \) runs of simulations. Standard deviation of the Gaussian distribution is set \( \Delta = 0.15 \) for the plots. Different \( k \) values, paired with particular \( \mu \) values are applied: \( k = 3 \) and \( \mu = 0.307 \) for the red dots, \( k = 8 \) and \( \mu = 0.310 \) for the blue triangles, \( k = \infty \), \( \mu = 0.316 \) for the black squares. The green dotted straight line, with a slope being -1.20, is a guide for eyes.
We find that if the parameters of $\mu$ and $\Delta$ are fine tuned, a power law distribution of the size of the information waves is emerged. The total population of the particles in the system is $100 \times 100$ and the standard deviation of the Gaussian distribution is set to $\Delta = 0.15$ in Fig. 2. Each data point is averaged over $10^6$ runs of simulation. The fine tuned parameters of $\mu$ and $k$ are: $k = 3, \mu = 0.307; k = 8, \mu = 0.310; \text{ and } k = \infty, \mu = 0.316$. The size of the information waves is quantified by counting the number of the particles turned into the alarmed state. The data collapse on one straight line in a double logarithmic scale and it is linear for more than three decades with a slope of -1.20, indicating that the fine-tuned parameters have poised the system to a critical state.

It is interesting to compare the critical behaviors in our model with its counterpart in the theory of self-organized criticality. The power law distribution of the size of the information waves, the counterpart of the dynamical avalanches in SOC, is a similar “finger print” to indicate that the system is at a critical state. The difference is that SOC states that a complex system can organize itself to a critical state without any tuning of parameters. Yet the critical state of our model is reached by fine-tuning the parameters. It was observed recently that not only the bird flocks, but also some other types of biological systems, e.g. networks of neurons, even brains, operate near or at the critical points in parameter space. It is tempting to speculate that the fine tuned parameters is a result from a long adaptation process in the risky nature. When operating at the critical points, the system becomes more efficient in information transfer, which confers the system a stronger responsiveness to predatory attacks.

IV. PHASE DIAGRAMS

We explore the relationships of the parameters of $\mu$, $k$ and $\Delta$ at the critical points, with a population of particles being $100 \times 100$ and the data being obtained over $5 \times 10^5$ runs of simulations in Fig. 3.

Figure 3(a) shows the phase diagram in the parameter space of $\mu$ and $k$, with $\Delta = 0.15$ being applied. For every $k$ extending from the minimum (restricted by the quorum rules) to infinity, there is a paired $\mu$ to poise the system to a critical state. As $k$ becomes larger, $\mu$ increases fast initially until it saturates when $k$ approaches infinity. The phase line divides the parameter space into a super-critical zone, in which any small perturbations will very
FIG. 3: Phase diagrams. The population of particles is $100 \times 100$ and the data is averaged over $5 \times 10^5$ runs of simulations. The dotted line is a guide for eyes. (a) The $x$ axis is $k$ and $y$ axis is $\mu$, with $\Delta = 0.15$ being applied. The phase line extends to the infinity of $k$ and divides the parameter space into a super-critical and a sub-critical zone. (b) The $x$ axis is $\Delta$ and $y$ axis is $\mu$ with $k$ being set to infinity. The phase line starts from a minimum value of $\Delta = 0.13$ and ends at when $\mu$ is zero. If $\Delta < 0.13$, the system cannot assume a critical state any more. The phase line divides the parameter space into a super-critical and a sub-critical zone too.

likely cascade into big information waves that affect a lot of particles, and a sub-critical zone, in which the perturbations only affect in local areas and information waves are often blocked by some insensitive particles (with big “vigilance number”).

When the system operates at a super-critical state, the random perturbations of the environment may easily startled the system because there are so many sensitive particles (with small “vigilance number”), which will cost a lot of energy waist. On the contrary, if at a sub-critical state, the big information waves are damped which results in a low efficiency of information propagation.

Figure 3(b) shows the fine-tuned, paired parameters of $\mu$ and $\Delta$, which can poise the system at a critical state. Note that $k$ is set to infinity in the simulations. Along the phase line, when $\Delta$ becomes bigger, $\mu$ becomes smaller at a rate faster than the linear drop. There exist a minimum $\Delta = 0.13$, where the phase line starts from. When $\Delta$ is smaller than the minimum, the system can either be at a sub-critical or a super-critical state depending on the value of $\mu$, but it cannot reach a critical state. The phase line ends at $\mu = 0$ and
\( \Delta = 0.425 \). It is worth noting that the evenly random distribution of vigilance number (other than the Gaussian distribution) in the interval \((0, 1)\) results in a sub-critical state.

Since \( \Delta \) measures the diversity of the vigilance level of the group members, Figure 3(b) also shows that if a system is composed of too many conformists which makes the \( \Delta \) value too small, the system cannot reach a critical state. On the contrary, if the particle’s diversity is too wide, the system cannot reach a critical state, either.

It has been observed that ants tune the parameters when applying quorum rules during colony emigration. If the situation is at different urgent level, e.g. old nest in crisis or in a good condition, they varied the set of parameters in the situation to respond to the environment adaptively\(^{20}\).

V. CONCLUSION AND DISCUSSION

We proposed a minimum individual-based model with introduction of quorum response as the local interaction rule to study the critical behaviors of a biological system, particularly the efficiency of information transfer. We assumed that the particles’ “vigilance number” obeys a truncated Gaussian distribution. By tuning the parameters of the mean and the standard deviation of the Gaussian distribution, we found that the system could be poised to a critical state in a dynamical process of information transfer. We presented the phase diagrams to show that the parameter space is divided into a sub-critical and a super-critical zone in which the efficiency of information propagation vary largely.

In our model, it is assumed that the particle’s “vigilance number” obeys a same Gaussian distribution. This assumption needs feedback from experiments, yet the complexity to identify the interaction among individuals in experiments may stand in the way currently\(^{21-23}\). It was observed in field that individuals of a group of animals at periphery are more vigilant in average than their conspecifics in center, the so called “edge effects”\(^{18}\). Although to what degree this effect works is still under discussion\(^{24}\). This effect is equivalent to positioning particles with relative smaller \( q_i \) at the boundaries, which may result in more big information waves thus enhancing the efficiency of information propagation. The widely applied interaction rule of collective animal motion in the biology literatures currently is: “averaging among the velocity of its local neighbors”\(^{7,25,26}\). Compared with it, quorum response has the advantages of passing information on without deterioration. This may be the reason
that in some risky environments animal groups apply quorum response rules to make the movement decision\textsuperscript{16} and even tuning the parameters if in urgency\textsuperscript{20}.

Collective animal groups, e.g. starling flocks, fish schools, are recently being called “collective minds”\textsuperscript{27}, because the coherent movements and the efficient responses to environmental perturbations as if the whole group is in one mind. Obviously, the enhanced efficiency of information transfer in the group enables these abilities. Our model explores the dynamics of information transfer in a system at critical points. Hopefully, our work may shed some light on this mysterious and beautiful phenomenon.

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