Gravitational ringdown signal from coalescences of SMBH binaries - detection rates for LISA and ET

Ch. Filloux\textsuperscript{1}, J.A. de Freitas Pacheco\textsuperscript{1}, F. Durier\textsuperscript{2} and J.C.N. de Araujo\textsuperscript{3}

\textsuperscript{1}Observatoire de la Côte d’Azur, Université de Nice-Sophia Antipolis - France
\textsuperscript{2}Max Planck Institute for Extraterrestrial Physics - Germany
\textsuperscript{3}Instituto Nacional de Pesquisas Espaciais, Brazil

E-mail: jcarlos.dearaujo@inpe.br

Abstract. The coalescence history of massive black holes is derived from cosmological simulations, in which their evolution and that of the host galaxies are followed in a consistent way. With the coalescence rate per comoving volume and per mass interval derived from the simulations we estimate the expected detection rate distribution of “ring-down” gravitational wave signals along frequencies accessible by LISA and Einstein Telescope (ET). For LISA, a total detection rate of about 15 yr\textsuperscript{-1} is predicted for events having a signal-to-noise ratio equal to 10. For ET, one event each 14 months down to one event each 4 years is expected with a signal-to-noise ratio of 5. The detection of these gravitational signals and their distribution in frequency would be in the future an important tool able to discriminate among different scenarios for the origin of supermassive black holes.

1. Introduction

It seems to be a well established fact that the center of massive galaxies host SMBHs (see, e.g., Ref. [1]). Presently, the general belief is that these objects have been formed by the growth of primordial “seeds” via matter accretion and coalescences resulting from merger episodes involving their hosts. However, the detection of bright SLOAN quasars associated to SMBHs of $10^9 \, M_\odot$ at $z \approx 6.5$ [2], led some authors to propose an alternative evolutionary scenario, in which these objects would have been formed directly via the gravitational collapse. Processes leading to the formation of SMBHs release, in particular when coalescences occur, an important amount of gravitational radiation.

The evolution of primordial massive stars [3] may lead to the formation of BH “seeds” having masses in the range $100 – 500 \, M_\odot$. The newly formed BHs are likely to be “distorted” with respect to the quiescent Kerr geometry, emitting gravitational waves (GWs), the so-called “ringdown” radiation. More massive BHs in a process of merging emit GWs during the inspiral and the plunge phases and lately, when the two horizons merge. In this case, the GW signal has frequencies that are accessible by LISA.

Coalescences of SMBHs are certainly one of the primary sources of GWs for LISA and many investigations have been devoted to this subject aiming, in particular, to describe in detail the inspiral phase as well as the plunge in order to obtain adequate templates for the waveform [4] and to estimate the expected rate of events. These estimates require not only a previous knowledge of the merging rate at different redshifts, but also of the evolution of the BH mass.
distribution. Early studies considered that the coalescence rate of BH binaries is related to the overall merger rate of galaxies, displaying a broad maximum around $z = 3$. Crude estimates based on this picture indicate a total coalescence rate ranging from one up to a hundred events per year [5, 6].

In the present study, cosmological simulations were performed in which the growth of $100M_\odot$ BHs seeds as well as the evolution of the host galaxy were followed in a self-consistent way. Seeds grow mainly by accretion of baryonic matter and by coalescences occurring after merger events, forming as a result SMBHs of up $\sim 10^9 M_\odot$. We then focus essentially on the “ring-down” emission and estimate the expected detection rate for LISA and Einstein.

2. The simulations

General aspects of the code and results concerning the main correlations between the BH mass and the properties of the host galaxy were already reported in Ref. [7] (see, also Ref. [8]). Here we only give the main characteristics of the code.

Simulations were performed by using the parallel TreePM-SPM code GADGET-2 in a formulation, despite the use of fully adaptive smoothed particle hydrodynamics (SPM), that conserves energy and entropy [9]. Different physical mechanisms affecting the gas dynamics were introduced, such as cooling (free-free transitions, radiative recombinations, $H_2$ molecular transitions, atomic fine-structure level excitation, Compton interactions with CMB photons), local heating by the UV-radiation of newly formed stars, mechanical energy injected by type II and type Ia supernovae as well as by AGNs and diffusion of heavy elements.

All simulations were performed in a cube with a size of $50 h^{-1}$ Mpc with two different mass resolutions for the gas/stellar particles respectively equal to $5.35 \times 10^8 M_\odot$ and $3.09 \times 10^8 M_\odot$. In all runs a $\Lambda$CDM cosmology was adopted, characterized by a Hubble parameter $h = 0.7$ in units of $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$, by a “vacuum” energy density parameter $\Omega_{\Lambda} = 0.7$ and by a total matter energy density parameter $\Omega_m = 0.3$. The fraction of baryonic matter was taken as $h^2\Omega_b = 0.0224$ and the normalization of the matter density fluctuation spectrum was taken to be $\sigma_8 = 0.9$. Initial conditions were fixed according to the algorithm COSMICS and all simulations were performed in the redshift interval $60 \geq z \geq 0$.

3. Coalescence rate

This paper is essentially focused on the coalescence process involving black holes and, in particular, on the rate at which objects with a given mass $M_{bh}$ are being formed at a given redshift. As is well known, such a process is intimately related to mergers suffered by the host galaxies. However, other mechanisms may affect the resulting BH mass distribution and, consequently, the expected frequency of the “ring-down” signal.

In Fig. 1, the coalescence rate per unit of volume and per mass interval $\Psi(M, z)$ (in $Mpc^{-3}yr^{-1}M_\odot^{-1}$) is shown as a function of the resulting BH mass and for different redshifts. This function is defined such that the local rate of coalescences $R$ in a given redshift interval is given by

$$\frac{dR}{dz} = \int_{M_1}^{M_{max}} \Psi(M, z) \frac{dV}{dz} dM$$

(1)

where $M_1 = 200 M_\odot$ is the minimum BH mass resulting from a fusion of two primordial seeds and $M_{max}$ is the maximum BH mass resulting from the coalescences process, that in our simulations is of the order of $5 \times 10^9 M_\odot$. Fig. 1 clearly shows the appearance of very massive BHs for decreasing redshifts, consequence of the accretion process and coalescences. This is the main result of the present investigation since it permits to predict the expected event rate for ground based and space interferometers.
4. The expected detection rates for LISA and Einstein

The coalescence rate per unit of volume and per mass interval $\Psi(M, z)$ derived from our simulations permits to compute the expected event rates for LISA and Einstein Telescope. The coalescence rate in a given interval of mass and redshift seen by an observer at $z = 0$ is

$$dR = \frac{\Psi(M, z) dV}{(1 + z) dM dz},$$

where $dV$ is the comoving volume element.

Note that the factor $(1 + z)$ in the denominator takes into account the time dilation. The comoving volume element $dV$ for a flat cosmology is given by

$$dV = 4\pi \left( \frac{c}{H_0} \right) \frac{r^2(z) dz}{\sqrt{\Omega_\Lambda + \Omega_m (1 + z)^3}},$$

and the comoving distance by

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dx}{\sqrt{\Omega_\Lambda + \Omega_m (1 + x)^3}}.$$  

With a change of variable in Eq. (2), i.e., expressing the BH mass in terms of the observed “ring-down” frequency, we obtain the expected coalescence rate per logarithm interval of frequency, namely

$$\frac{dR}{d \ln(\nu)} = \int_{z_{\text{min}}(\nu)}^{z_{\text{max}}(\nu)} \frac{\Psi(\nu, z)}{(1 + z)} \left( \frac{dM}{d \ln(\nu)} \right) \frac{dV}{dz} dz,$$

where the relation between the black hole mass and the observed characteristic “ring-down” frequency is given by [10]

$$\nu = \frac{\nu_m}{(1 + z)} = \frac{1.2 \times 10^4}{(1 + z)} \left( \frac{M_\odot}{M} \right) F(a) \ Hz,$$
where \( a = Jc/GM^2 \) is the spin parameter and the function \( F(a) \) is given approximately by [10]

\[
F(a) = \left[ \frac{100}{37} - \frac{63}{37} (1 - a)^{0.30} \right].
\]

The lower limit \( z_{\text{min}}(\nu) \) appearing in the integral defined by Eq. (5) is fixed by the maximum BH mass \( M_{bh}(\text{max}) \) present in our simulations. For a given frequency, using Eq. (6), the minimum redshift is given by the condition

\[
z_{\text{min}}(\nu) = \text{Max} \left[ 0, \frac{1.2 \times 10^4 F(a)}{\nu M_{bh}(\text{max})} \right].
\]

On the other hand, the upper limit \( z_{\text{max}}(\nu) \) represents the redshift below which a gravitational signal can be detected at a given signal-to-noise ratio.

Recall that the optimal signal-to-noise ratio derived from a matched filtering technique is

\[
\left( \frac{S}{N} \right)^2 = 4 \int_0^{\infty} \left| \frac{\hat{h}(\nu)}{S_n(\nu)} \right|^2 d\nu,
\]

where \( \left| \hat{h}(\nu) \right|^2 \) is the spectral density of the signal averaged over both polarizations states and \( S_n(\nu) \) is the effective one-sided spectral density of the noise in the detector [11, 12].

We model the “ring-down” waveform by a simple damped sinusoidal, namely

\[
h(t) = h_0 e^{-t/\tau_m} \cos(\omega_m t),
\]

where the amplitude \( h_0 \) of the signal is related to the total energy \( E \) carried out from the source under the form of gravitational waves by

\[
h_0^2 = \frac{16GE}{c^4 d_L^2} \frac{\tau_m}{1 + 4 Q_m^2},
\]

where \( G \) is the gravitational constant, \( d_L \) is the luminosity distance and \( Q_m = \pi \nu_m \tau_m = \omega_m \tau_m/2 \) is the quality factor of the oscillation, given approximately by [10]

\[
Q_m = K(a) \simeq 2(1 - a)^{9/20}.
\]

It is worth noting that more recent and more accurate fits to the quasinormal modes of a Kerr black hole can be found in Ref. [13]. However, our simplifying modeling of the quasinormal mode signal is not invalidated by this more recent study.

The Fourier transform of the signal gives

\[
\hat{h}(\omega) = \frac{h_0 \tau_m}{1 + (\omega - \omega_m)^2 \tau_m^2} = h_0 \tau_m f(\omega_m, \tau_m).
\]

Note that \( \hat{h}(\nu) \) has a Lorentz profile, indicating that the spectral density is peaked around the characteristic frequency \( \omega_m \).

Using the above equations and assuming that the total energy released under the form of GWs is given by \( E = \varepsilon M_{bh} c^2 \), one obtains for the \( S/N \) ratio

\[
\left( \frac{S}{N} \right) = \frac{2.83 \times 10^{-17}}{d_L} \left( \frac{\varepsilon M}{M_\odot} \right)^{1/2} \frac{G(a)}{\sqrt{\nu^2 S_n(\nu)}},
\]
where

$$G(a) = \frac{Q_m(a)}{\sqrt{1 + 4Q_m^2(a)}}. \tag{15}$$

The above estimate for SNR considers in fact a delta-function approximation, which was originally introduced in Ref. [14] (see also Ref. [13]). In Eq. (14) the luminosity distance is given in Mpc and we have assumed that in the evaluation of the integral in Eq. (9), the noise spectral density does not vary considerably near the frequency defining the maximum of the spectral density of the signal. The “ring-down” efficiency $\varepsilon$ was estimated in Ref. [15] to be about $7.8 \times 10^{-4}$ from fully relativistic calculations of head-on collisions between a black hole and a neutron star. In our numerical estimates, a more conservative value equal to $\varepsilon = 10^{-4}$ was also considered.

In the next two subsections we apply the above prescriptions to evaluate $z_{max}(\nu)$, the expected coalescence rates per logarithm interval of frequency $dR/d\ln(\nu)$ and the total expected rates of events to LISA and Einstein.

4.1. LISA

In order to evaluate $z_{max}(\nu)$ from Eq. (14), we have computed the spectral noise density using an online simulator [16], where the confusing noise introduced by white dwarf binaries in our Galaxy is also taking into account. In Fig. 2 it is shown the maximum redshift as a function of the observed frequency of the signal for $S/N=10$ and for different values of the spin parameter $a$ and the “ring-down” efficiency. Note that for the most favorable situation, the “ring-down” signal can be seen up to $z \sim 22$ around $\nu \sim 4$ mHz. In practice, the situation is a little bit different. The thin line in Fig. 2 indicates, for a fixed frequency, the effective redshift above which coalescences do not contribute significantly to the signal. The reason is essentially due to the evolution of the BH mass spectrum. Once the frequency is fixed, the mass contributing to the signal is related to the redshift via Eq. (6) and the absence of objects with the required mass produces the aforementioned effect. This critical redshift increases with frequencies probing lower and lower mass BHs which are more numerous than supermassive objects.

With $z_{max}(\nu)$ at hand, the integral in Eq. (5) can be computed numerically using the results of our simulations. Fig. 3 shows the expected distribution of coalescence rates in the frequency domain of LISA for $S/N=10$. This plot shows that most of the coalescences will produce “ring-down” signals mainly the frequency range 4–9 $mHz$, probing BHs in the mass range $2.2 \times 10^5 M_\odot$ up to $2.6 \times 10^6 M_\odot$. The event rate distributions shown in Fig. 3 reflect the formation history of these SMBHs resulting from the present simulations. Had most of the SMBHs been formed early in the universe, say around $z \sim 6$, most of the coalescences would occur at frequencies of only few $\mu$Hz and not in the $mHz$ domain as found in the present investigation. This is an important aspect indicating that LISA in its original configuration would be able to contribute significantly to astrophysics, discriminating the different evolutionary scenarios leading to the formation of SMBHs.

Integration of the coalescence rate gives the total expected rate of events. Table 1 shows the results for different values of the signal-to-noise ratio, the spin parameter and the “ring-down” efficiency. Inspection of Table 1 indicates that the predicted rates are in a very narrow range of values despite the different values of the spin parameter, GW emission efficiency and S/N ratios. In fact, increasing the efficiency or decreasing the S/N rate permits to probe effectively a higher volume of the universe. However, since the integral “saturates” above a certain redshift, the predicted total event rate does not change considerably, excepting that the rate at the high frequency side of the maximum increases slightly.
Figure 2. Maximum redshift seen by LISA at a given frequency for S/N=10. Different curves correspond to the different values of the spin parameter and the gravitational wave emission efficiency. The thin curve indicates the critical redshift value above which the contribution of coalescences to the signal becomes negligible. (From Ref. [8].)

Figure 3. Expected detection rate per logarithm interval of frequency for LISA. The adopted signal-to-noise ratio is S/N=10. (From Ref. [8].)

4.2. Einstein
The third generation of ground based interferometers like ET [17] is expected to be much more sensitive in the frequency range 1-60 Hz than present antennas. In this range, these interferometers would be able to detect the “ring-down” signal originated from coalescences giving origin to BHs with masses below 500 $M_\odot$. These masses represent the very early stage of growth of “seeds”. The detection of a gravitational signal emanating from coalescences involving these low mass objects could shed some light on the initial mass of “seeds” as well as on the environment conditions in which they evolve.

In order to estimate the event rate, the same procedure described above was adopted. Here, the sensitivity curve for the planned ET, version B, was taken from Ref. [18]. In Fig. 4, the
Table 1 - Coalescence Rates: columns give respectively: (1) the $S/N$ ratio, (2) the spin parameter $a$, (3) the gravitational wave radiation efficiency $\varepsilon$ and (4) the coalescence rate $R$.

| $S/N$ | $a$     | $\varepsilon$ | $R$ (yr$^{-1}$) |
|-------|---------|---------------|-----------------|
| 5     | 0.50    | $1.0 \times 10^{-4}$ | 15.9            |
| 5     | 0.50    | $7.8 \times 10^{-4}$ | 18.0            |
| 5     | 0.01    | $1.0 \times 10^{-4}$ | 16.7            |
| 10    | 0.50    | $1.0 \times 10^{-4}$ | 14.4            |
| 10    | 0.01    | $1.0 \times 10^{-4}$ | 14.8            |
| 10    | 0.50    | $7.8 \times 10^{-4}$ | 16.3            |

value of $z_{\text{max}}(\nu)$ is shown as a function of the frequency for different values of the spin and the efficiency parameters. Within an optimistic perspective, ET could be able to detect coalescences up to $z \sim 4$ at frequencies of few Hz. At these redshifts, low mass BHs will be probed and, according to our simulations, they are not located in the center of galaxies but are wandering in their halos. They constitute a population of seeds having suffered a small number of coalescences, since they live in an environment in which the probability to merger or to accrete mass is quite small.

In Fig. 5 the expected event rate per logarithm interval of frequency for ET is plotted as a function of the frequency. Most of the events are expected to occur in the range 10-20 Hz and total rates for $S/N = 5$ are about one event each 14 months for $\varepsilon = 7.8 \times 10^{-4}$ and one event each 4 years for $\varepsilon = 1.0 \times 10^{-4}$. A previous investigation [20], using the procedure developed in Ref. [19] to obtain the merger history of seeds, derived an event rate of about one per year for $S/N=5$. This is comparable to the present results if BHs have an average spin parameter $a = 0.50$ and the efficiency for emitting GWs is that derived from head-on collisions. The coalescence rate derived in Ref. [20] corresponds to a scenario in which seeds have the same mass, as assumed in our own investigation, but with a slightly higher value, i.e., 150 $M_\odot$ instead of 100 $M_\odot$. If seeds have a mass distribution (log-normal in the range 10 – 600 $M_\odot$) a higher event rate is obtained for the same $S/N$ ratio, i.e., about 2 – 3 events per year [20].

Recall that Ref.[21] also considers a detailed and interesting discussion of ET event rates related to different models of formation and coalescence of intermediate massive BHs.

5. Conclusions
The coalescence history of massive black holes is derived from cosmological simulations, in which their evolution and that of the host galaxies are followed in a consistent way. With the coalescence rate per comoving volume and per mass interval we estimate the expected detection rate distribution of “ring-down” gravitational wave signals along frequencies accessible by LISA and Einstein Telescope. For LISA in its original configuration, our study predicts a total event rate of about 15 coalescences per year with $S/N=10$ and with most of the GW bursts occurring in the frequency range 4-9 mHz. For ET, the expected rates are in the range of one event per year down to one event every 4 years, occurring in the frequency range 10-20 Hz. These characteristics reflect the growth history of SMBHs resulting from our simulations and can be used, in the future, to discriminate among different scenarios proposed to explain the origin of these objects.

Acknowledgments
JCNA thanks CNPq and Fapesp for financial support.
Figure 4. Maximum redshift probed by the gravitational wave telescope Einstein for S/N=5. (From Ref. [8].)

Figure 5. Expected detection rate per logarithm interval of frequency for Einstein-B and for S/N=5. (From Ref. [8].)

References
[1] J. Kormendy and D. Richstone, ARA&A 33 (1995) 581.
[2] X. Fan, V.K. Narayanan, R.H. Lupton et al., AJ 122 (2001) 2833.
[3] V. Bromm, P.S. Coppi and R.B. Larson, ApJ 527 (1999) L5.
[4] M. Preto, I. Berentzen, P. Berczik, D. Merrit and R. Spurzen, JPhCS 154 (2009) 2049.
[5] M.G. Haehnelt, AIP Conference Proceedings 456 (1998) 45.
[6] D. Merritt and R.D. Elers, Science 297 (2002) 1310.
[7] Ch. Filloux, F. Durier, J.A. de Freitas Pacheco and J. Silk, IJMPD 19 (2010) 1233.
[8] Ch. Filloux, J.A. de Freitas Pacheco, F. Durier and J.C.N. de Araujo, IJMPD 20 (2011) 2399; ArXiv:1108.2638.
[9] V. Springel, MNRAS 364 (2005) 1105.
[10] F. Echeverria, Phys.Rev. D40 (1989) 3194.
[11] N. J. Cornish, *Phys. Rev.* **D65** (2001) 022004.
[12] J. D. E. Creighton, *Phys. Rev.* **D60** (1999) 022001.
[13] E. Berti, V. Cardoso and C.M. Will, *Phys. Rev.* **D73** (2006) 064030; ArXiv: gr-qc/0512160.
[14] E.E. Flanagan and S.A. Hughes, **57**, 4535 (1998); gr-qc/9701039
[15] F. Löffler, L. Rezzolla and M. Ansorg, *Phys. Rev.* **D74** (2006) 104018.
[16] S. L. Larson, W. A. Hiscock and R. W. Hellings, *Phys. Rev.* **D62** (2000) 062001; Online Sensitivity Curve Generator, "http://www.srl.caltech.edu/shane/sensitivity:"
[17] S. Hild, S. Chelkowski and A. Freise, [ArXiv:0810.0604v2].
[18] Gravitational Wave Einstein Telescope - “http://www.et.gw.eu”.
[19] M. Volonteri, F. Haardt and P. Madau, *ApJ* **582** (2003) 599.
[20] A. Sesana, J. Gair, I. Mandel and A. Vechio, *ApJ* **698** (2009) L129.
[21] J.R. Gair, I. Mandel, M. Coleman Miller and M. Volonteri, *Gen. Rel. Grav.* **43** (2011) 485; arXiv:0907.5450