Anisotropy of the sky distribution of gamma-ray bursts

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Abstract. The isotropy of gamma-ray bursts collected in Current BATSE catalog is studied. It is shown that the quadrupole term being proportional to \( \sim \sin^2 \theta \sin \lambda \) is non-zero with a probability 99.9%. The occurrence of this anisotropy term is then confirmed by the binomial test even with the probability 99.97 %. Hence, the sky distribution of all known gamma-ray bursts is anisotropic. It is also argued that this anisotropy cannot be caused exclusively by instrumental effects due to the nonuniform sky exposure of BATSE instrument. Separating the GRBs into short and long subclasses, it is shown that the short ones are distributed anisotropically, but the long ones seem to be distributed still isotropically. The character of anisotropy suggests that the cosmological origin of short GRBs further holds, and there is no evidence for their Galactical origin.

Key words: cosmology: large-scale structure of Universe - gamma rays: bursts

1. Introduction

The true physical nature of gamma-ray bursts (GRBs) is one of the tantalizing enigmas of the recent astrophysics. Although since their first detection (Klebesadel et al. 1973) there were several suggestions trying to give a clear explanation for their origin, no definite answer has been given yet (cf. Paczyński 1995). Recently, the successful identifications made by the Beppo-SAX satellite, followed by the detection of optical counterparts (van Paradijs et al. 1997), seem to give a firm support for the models aiming to explain the bursts by merging neutron stars (Usov & Chibisov 1975; Rees & Mészáros P. 1994; Mészáros P. & Rees 1997) and seem to put them definitely into the cosmological distances. The alternative Galactical origin seems to be ruled out (a survey of the question of distances may be found, e.g., in Paczyński 1995). However, the small number of optically identified events is far from being enough to characterize the properties of the whole burst population. On the other hand, the existence of cosmological distances of GRBs seems to be definite.

In addition, even before this identification, indirect observational evidences were known for the cosmological origin. These evidences were based mainly on the modifications of \(< V/V_{\text{max}} >\) test (cf. Norris et al. 1994; Mészáros P. & Mészáros A. 1995; Norris et al. 1995; Nemiroff 1995; Horváth et al. 1996), and on the study of the time dilatation (cf. Norris et al. 1995; Mészáros A. & Mészáros P. 1996; Stern 1996; Mészáros A. et al. 1996; Che et al. 1997). A further important indirect support of cosmological origin is based on the observed isotropy on the sky (Briggs 1993; Syer & Saha 1994; Briggs 1995; Tegmark et al. 1996a,b; Briggs et al. 1996). All these papers suggest that the angular distribution is isotropic, because there are no statistically significant departures from the isotropy. Also the separations of GRBs into the different subclasses either due to the duration (Kouveliotou et al. 1993; Belli 1995; Dezalay et al. 1996) or due to the fluence on channel above the 300 keV (Pendleton et al. 1997) do not change the situation; the proposed subclasses alone also seem to be distributed isotropically. Hence, it can well occur that the total number of observed GRBs is a mixture of a wide variety of physically different objects, but all GRBs should be at cosmological distances due to their isotropic angular distribution on sky, and due to other direct and indirect supports.

In this paper we find a clear anisotropy of all GRBs, and then separately of the short GRBs, too. On the other hand, the long GRBs seem to be still distributed isotropically. To do this we will use both the standard analysis based on the spherical harmonics and also the so called binomial test. It may seem that all this is an argument against the cosmological origin of short GRBs. It is shown that this is not the case.
The paper is organized as follows. In Section 2 the key ideas of the used statistical tests are recapitulated. In Section 3 the anisotropy of all GRBs is demonstrated. In Section 4 the anisotropy of short and the isotropy of long GRBs is shown. Section 5 discusses the results, and argues for the cosmological origin of short GRBs. Finally, in Section 6 there is a summation of results.

2. Mathematical skeleton of the problem

Testing the isotropy on the celestial sphere one may use several methods. Nevertheless, strictly from the mathematical point of view, the necessary condition for the isotropy is the stochastic independency of the sky distribution of the bursts on their observed physical properties. It means that, if \( f(b, l, x_1, ..., x_n) dF dx_1 ... dx_2 \) is the probability of finding an object in the \( dF = \cos b \, dl \, db \) infinitesimal solid angle and in the \((x_1, x_1 + dx_1, ..., x_n, x_n + dx_n)\) interval, one must have

\[
f(l, b, x_1, ..., x_n) = \omega(l, b) g(x_1, ..., x_n). \tag{1}
\]

Here \(0 \leq l \leq 360^\circ, -90^\circ \leq b \leq 90^\circ\) give the celestial positions in Galactic coordinates, \(x_n (n \geq 1)\) measure the physical properties (peak fluxes, fluences, durations, etc.) of GRBs and \(g\) is their probability density. It may well be assumed that in the case of isotropy the distribution of GRBs fulfills this equation (cf. Briggs et al. 1996; Tegmark et al. 1996a,b).

However, statement (1) is only a necessary but not a sufficient condition for isotropy. Isotropy means that also \(\omega(l, b) = 1/(4\pi)\). Hence, in this case, for \(N\) observed GRBs the events \(dN = N \omega dF\), i.e. the expected number of GRBs in an infinitesimal solid angle, is not depending on \([l, b]\). In other words, the isotropy means that the probability of observation of a burst in a solid angle \(0 \leq \Omega \leq 4\pi\) (\(\Omega\) is in steradians) is given by \(\Omega/(4\pi)\) and is independent on its location on the celestial sphere. This follows immediately from (1), if one does integration over \(l\) and \(b\) to obtain, first, the solid angle \(\Omega\), and, second, the whole sky. Then the ratio of two results gives \(\Omega/(4\pi)\), and the concrete form of \(g\) is unimportant.

The most frequently used procedure to test the isotropy of GRBs is based on the spherical harmonics (Briggs 1993, 1995; Briggs et al. 1996; Tegmark et al. 1996a,b). The key idea is the following. In general case one may decompose the function \(\omega(b, l)\) into the well-known spherical harmonics. One has:

\[
\omega(b, l) = (4\pi)^{-1/2}\omega_0 - (3/(4\pi))^{1/2}(\omega_{1, -1} \cos b \sin l - \omega_{1, 1} \cos b \cos l + \omega_{1, 0} \sin b) + (15/(16\pi))^{1/2}(\omega_{2, -2} \cos^2 b \sin 2l + \omega_{2, 2} \cos^2 b \cos 2l - \omega_{2, -1} \sin 2b \sin l - \omega_{2, 1} \sin 2b \cos l) + (5/(16\pi))^{1/2}\omega_{2, 0}(3\sin^2 b - 1) + \text{higher order harmonics}. \tag{2}
\]

The first term on the right-hand side is the monopole term, the following three ones are the dipole terms, the following five ones are the quadrupole terms (cf. Press et al. 1992; Chapt. 6.8). Nevertheless, \(\omega\) is constant for isotropic distribution, and hence on the right-hand side any terms, except for \(\omega_0\), should be identically zeros. To test this hypothesis one may proceed, e.g., as follows. Let there are observed \(N\) GRBs with their measured positions \([b_j, l_j]\) \((j = 1, 2, ..., N)\). In this case \(\omega\) is given as a set of points on the celestial sphere. Because the spherical harmonics are orthogonal functions, to calculate the \(\omega_1\) coefficients one has to compute the functional scalar products. For example, \(\omega_{2, -1}\) is given by

\[
\omega_{2, -1} = -(15/(16\pi))^{1/2} N^{-1} \sum_{j=1}^{N} \sin 2b_j \sin l_j. \tag{3}
\]

Because \(\omega\) is given only in discrete points, the integral is transformed into an ordinary summation (cf. Kendall & Stuart 1969, p. 16). In the case of isotropy one has \(\omega_{2, -1} = 0\), and hence \(N^{-1} \sum_{j=1}^{N} \sin 2b_j \sin l_j = 0\). Therefore, the expected mean of \(\sin 2b_j \sin l_j\) values is zero. One has to proceed similarly to any other \(\omega_1\) coefficient.

In order to test the zero value of, e.g., \(\omega_{2, -1}\) one has to calculate, first, \(\sin 2b_j \sin l_j\) for any \(j = 1, 2, ..., N\), and, second, the mean, standard deviation and Student’s \(t\) variable (cf. Press et al. 1992, Chapt. 14). Finally, third, one has to ensure the validity of zero mean from Student test. As far as it is known, no statistically significant anisotropies of GRBs were detected yet by this procedure (cf. Briggs et al. 1996; Tegmark et al. 1996a,b).

Nevertheless, there are also other ways to test the isotropy. An extremely simple method uses the binomial distribution. In the remaining part of this section we explain this test (see also Mészáros A. 1997).

In order to test the anisotropy by this method one may proceed as follows. Let us given an area on the sky defined by a solid angle \(0 < \Omega < 4\pi\) (\(\Omega\) is in steradians). In the case of isotropy the \(p\) probability to observe a burst within this area is \(p = \Omega/(4\pi)\). Then, obviously, \(q = 1 - p\) is the probability to have it outside. Observing \(N > 0\) bursts on the whole sky the probability to have \(k\) bursts (it may be \(k = 0, 1, 2, ..., N\)) within \(\Omega\) is given by the binomial (Bernoulli) distribution taking the form

\[
P_p(N, k) = \frac{N!}{k!(N-k)!} p^k q^{N-k}. \tag{4}
\]

This distribution is one of the standard probability distributions discussed widely in statistical text-books (e.g.
Trumpler & Weaver 1953; Kendall & Stuart 1969, p.120; about its use in astronomy see, e.g., Mészáros 1997). The expected mean is $Np$ and the expected variance is $Npq$. One may also calculate the integral (full) probability, too, by a simple summation.

In our case we will consider $N$ GRBs, and we will test the hypothesis whether they are distributed isotropically on the sky. Assume that $k_{obs}$ is the observed number of GRBs at the solid angle $\Omega$. If the apriori assumption is the isotropy, i.e. $p = \Omega/(4\pi)$, then one may test whether the observed number $k_{obs}$ is compatible with this apriori assumption. Of course, any $0 \leq k_{obs} \leq N$ can occur with a certain probability given by the binomial distribution. But, if this probability is too small, one hesitates seriously to accept the apriori assumption.

Consider the value $|k_{obs} - Np| = k_o$. The value $k_o$ characterizes the departure of $k_{obs}$ from the mean $Np$.

Then one may introduce the probability

$$P(N,k_{obs}) =$$

$$1 - P_{p,tot}(N,(Np + k_o)) + P_{p,tot}(N,(Np - k_o)).$$

$P(N,k_{obs})$ is the probability that the departure of $k_{obs}$ from the $Np$ mean is still given by a chance.

In order to test the isotropy of the GRBs celestial distribution we will divide the sky into two equal areas, i.e. we will choose $p = 0.5$. It is essential to note here that neither of these two parts must be simply connected compact regions.

### 3. Anisotropy of all GRBs

In order to test the isotropy of GRBs we will test the three dipole and five quadrupole terms in accordance with the method described in the previous Section. We will consider all GRBs that have well-defined angular positions. Up to the end of Year 1997 there were 2025 such objects at Current BATSE catalog (Meegan et al. 1997; Paciesas et al. 1998). The results are summarized in Table 1.

We see that, except for the terms defined by $\omega_{1,-1}$ and $\omega_{2,-2}$, the remaining six terms may still be taken to be zero. This means that there is a clear anisotropy defined by term $\sim \sin 2b \sin l$. The probability that this term is zero is smaller than 0.27%. It is practically sure that the second quadrupole term being proportional to $\sim \cos^2 b \sin 2l$ is non-zero, too. Nevertheless, we will not deal with this second term in this paper, because the purpose of this paper is to demonstrate only qualitatively the existence of anisotropies.

The anisotropy defined by $\omega_{2,-2}$ may be defined in another form, too. This quadrupole term has a positive sign, when both $b$ and $l$ have the same signs, and has a negative sign, when $l$ and $b$ have opposite signs. Therefore, let us define two parts of the sky having the same sizes ($2\pi$ steradians). The first one is defined by Galactical coordinates $b > 0, \ l > 180^\circ$ and $b < 0, \ 0 < l < 180^\circ$. This means that this first part is in fact composed from two separated “sky-quarters”. The second part is then given by $b > 0, \ 0 < l < 180^\circ$ and $b < 0, \ l > 180^\circ$. This means that this second part is also given by two separated “sky-quarters”. Then the detected quadrupole anisotropy suggests that there should be an essential difference, e.g., in the number of GRBs in these two parts.

In order to test again this expectation we will do the test based on Bernoulli distribution. We divide the whole sky into these two parts, and hence we expect a Bernoulli distribution with $p = 0.5$ for $N = 2025$. As it is noted in Section 2, it is certainly allowable that these parts are composed from several subregions.

A straightforward counting of GRBs in these regions shows that 930 GRBs are in the first one and 1095 are in the second. (Note that no GRBs had coordinates exactly either $b = 0$, or $b = \pm 90^\circ$, or $l = 0$, or $l = 180^\circ$. This means that in the paper no problems have arisen from the fact that the boundaries of “sky-quarters” were not taken into account.) Assuming $p = 0.5$ the binomial (Bernoulli) test gives a 0.03% probability that this distribution is caused only by a chance. Hence, the relatively smaller number in the first region compared with the second one is not a chance, and the distribution of all GRBs is anisotropic with a certainty.

Clearly, concerning the consequences of the intrinsic anisotropy of GRBs, one must be still careful. Instrumental effects of the BATSE experiment may also play a role, and in principle it can also occur that the detected anisotropy is caused exclusively by instrumental effects. To be as correct as possible, one may claim that, in essence, there can be three different causes of this observed anisotropy: a. The anisotropy is purely caused by the BATSE’s nonuniform sky exposure (in other words, the intrinsic angular distribution of GRBs is still isotropic, and the observed anisotropy is a pure instrumental effect); b. The anisotropy is purely given by the instrin-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$t$ & $\%$ \\
\hline
$\omega_{1,-1}$ & 1.51 & 13.4 \\
$\omega_{1,1}$ & 1.77 & 7.7 \\
$\omega_{1,0}$ & 0.71 & 47.7 \\
$\omega_{2,-2}$ & 2.76 & 0.6 \\
$\omega_{2,2}$ & 1.54 & 12.1 \\
$\omega_{2,-1}$ & 3.26 & 0.1 \\
$\omega_{2,1}$ & 0.98 & 33.3 \\
$\omega_{2,0}$ & 0.36 & 71.9 \\
\hline
\end{tabular}
\caption{Student test of the dipole and quadrupole terms of 2025 GRBs. In the first column the coefficients defined in Eq.2 are given. In the second column the Student $t$ is provided. The third column shows the probability that the considered terms are still zeros.}
\end{table}
sotropic anisotropy of GRBs, and the instrumental effects are unimportant; c. The anisotropy is given both by instrumental effects and also by the intrinsic anisotropy. To be sure that there is also an intrinsic anisotropy of GRBs, one must be sure that the possibility a, does not occur. In what follows, when we speak about "the possibility a," we will consider this one.

It is well-known that the sky exposure of the BATSE instrument is nonuniform. This question is described and discussed in several papers (cf. Briggs 1993; Fishman et al. 1994; Tegmark et al. 1996a,b; Briggs et al. 1996). The BATSE sky coverage depends on the declination only in the equatorial coordinate system (Tegmark et al. 1996b) in such a manner that the probability of detection is about the equatorial coordinate system (Tegmark et al. 1996b) in such a manner that the probability of detection is about

The results are shown in Table 2.

| sample | N | k_{obs} | (N - k_{obs}) | % |
|--------|---|---------|---------------|---|
| all GRBs | 932 | 430 | 502 | 2.0 |
| T_{90} < 1 s | 206 | 82 | 124 | 0.43 |
| T_{90} < 2 s | 251 | 103 | 148 | 0.55 |
| T_{90} < 10 s | 372 | 154 | 218 | 0.11 |
| T_{90} > 2 s | 681 | 327 | 354 | 0.52 |
| T_{90} > 10 s | 560 | 276 | 284 | 0.77 |
| T_{90} > 15 s | 507 | 247 | 260 | 0.59 |

Table 2. Results of the binomial test of subsamples of GRBs with different durations. N is the number of GRBs at the given subsample, k_{obs} is the observed number GRBs at the first part in this subsample, and % is the probability in percentages that the assumption of isotropy is still valid.

Table 2 shows, e.g., that there are 251 GRBs with T_{90} < 2 s, and 681 GRBs with T_{90} > 2 s. Then, from the short GRBs 103 are at the first part of sky and 148 at the second one. This gives X = 103/148 = 0.70. It seems that at the first part there is even a smaller portion of...
shorter GRBs than that of all GRBs. The probability that this is a chance is given by 0.55%. On the other hand, from the long GRBs 327 are at the first part and 354 at the second one. This gives \( X = 327/354 = 0.92 \). Hence, it seems immediately that for the long GRBs the isotropy is still an acceptable assumption. The binomial test quantifies: there is a 32\% probability that this distribution is given by a chance. Doubtlessly, the long GRBs are distributed more isotropically than the short ones; there is no statistically significant departure from isotropy for long GRBs. The subsamples \( T_{90} < 1 \) s and \( T_{90} > 15 \) s confirm this expectation; the boundary at \( T_{90} = 10 \) s also does not change the conclusion. One may claim that the anisotropy of short GRBs is statistically significant, but for long GRBs it is not.

Doubtlessly, the short and long subclasses are distributed differently. This also excludes the possibility a.; the observed anisotropy of all GRBs cannot be caused exclusively by instrumental effects. It is difficult to imagine an instrumental effect which leads to isotropy of long GRBs and to anisotropy of short GRBs.

Pendleton et al. (1997) introduces the subclasses of HE and NHE bursts. The criterion depends on the ratio \( f_4/f_3 \), where \( f_3 \) is the fluence on energy channel \([100, 300]\) keV and \( f_4 \) is the fluence on the energy channel \( >300 \) keV. (From this it is also clear, why we needed non-zero \( f_3 \). On the other hand, \( f_4 \) can be vanishing; these GRBs are simply NHE bursts.) Application of this criterion is not so simple, because for a great portion of GRBs there are large uncertainties of the values of \( f_4 \) due to their errors. Concretely, for 693 GRBs the value of \( f_4 \) is bigger than the corresponding error of this \( f_4 \); for 131 GRBs there is no \( f_4 \); for the remaining 108 GRBs there are some values of \( f_4 \) at the Current BATSE catalog (Meegan et al. 1997), but they are smaller than their errors. Hence, 693 GRBs can be taken as HE bursts (HE1 subsample). 131 GRBs having no \( f_4 \) may be taken as NHE bursts (NHE1 subsample). We did binomial tests for these two subclasses. Separation of the remaining 108 GRBs into the HE and NHE subclasses is not so clear. We consider artificially the boundary as follows: If the value of \( f_4 \) is bigger than the half of error, then we have HE; otherwise NHE. Applying this criterion we will have 168 NHE (NHE2 subsample) and 764 HE bursts (HE2 subsample). For them the binomial tests were also done. The results are collected in Table 3.

Table 3 gives an ambiguous result. Due to the smaller number in subclasses no anisotropy is confirmed yet on a satisfactorily high level of significance. In addition, contrary to the short-long separation, there is no obvious difference between HE and NHE classes. Hence, there is no obvious and unambiguous result here.

### 5. Discussion

The quadrupole anisotropy reported in this paper is an unexpected and new result. As far as it is known no anisotropy terms were detected yet (cf. Tegmark et al. 1996a,b). Probably this situation was given by the fact that the majority of these isotropy studies concentrated the effort into dipole and \( \omega_{2,0} \) quadrupole terms, which are expected to differ from zero, if the GRBs are arisen in the Galaxy.

The essentially different angular distribution of short and long GRBs suggest that their separation into these two subclasses has a deeper cause. It is well-known that in average the short bursts have higher hardnesses (hardness \( \approx f_3/f_2 \), where \( f_2 \) is the fluence on energy channel \([50, 100]\) keV). Katz & Canel (1996) have also shown that the \( < V/V_{\text{max}} > \) values are different: the smaller value for longer GRBs suggests that they are on average at bigger cosmological distances. Keeping all this in mind it seems to be definite that these two types are physically different objects at different cosmological scales.

Contrary to this, we did not find any significant difference in the angular distribution of HE and NHE subclasses. This is an unclear result, because in Pendleton et al. (1997) it is clearly stated that the \( < V/V_{\text{max}} > \) values are different for the HE and NHE subclasses, and hence they should also be at different distances. The isotropy tests do not confirm this expectation. This also means that the separation based on the most energetic channel remains unclear. In fact, the question of fourth channel is highly topical recently, because Bagoly et al. (1998) shows - independently on Pendleton et al. (1997) - that \( f_4 \) alone is an important quantity. The question of most energetic channel trivially needs further study, and is planned to be done.

At the end of Section 3 we pointed out that the dependence of the BATSE detection probability on the declination may mimic some sort of anisotropy. Therefore, without a detailed correction in accordance with the sky exposure function one may state only the presence of an intrinsic anisotropy from the different behaviour of short and long bursts. The requirement of the study of this correction for both subclasses is a triviality, and is planned to be done in the near future.
One may also speculate that the short GRBs can arise in the Galaxy and the long ones at cosmological distances. (About the cosmological origin of long GRBs there seems to exist no doubt; see, cf., Mészáros A. et al. 1996). The existence of non-zero $\omega_{2-1}$ and probably also of $\omega_{2-2}$ quadrupole terms with the simultaneous zeros for other dipole and quadrupole terms is a strange behavior for any objects arising in the Galaxy. E.g., it is highly complicated to have an $\omega_{2-1}$ term, and simultaneously not to have the $\omega_{2,0}$ term, if the sources have arisen in the Galaxy. Simply, any objects in the Galaxy should have fully different anisotropy terms (for further details and for the survey of earlier studies of isotropy see, e.g., Briggs 1993, 1995; Briggs et al. 1996; Tegmark et al. 1996a,b; Meegan et al. 1996).

Remark also the following. The so called transition scale to homogeneity (cf. Mészáros A. 1997) is minimally of size $\simeq 300h^{-1}$ Mpc ($h$ is the Hubble constant in units $100$ km/(s Mpc)). This means that up to this distance an inhomogeneous and anisotropic spatial distribution is not only possible but it is even expected. In addition, at the last time several observational implications both from the distribution of galaxies and from the anisotropy of cosmic microwave background radiation highly query the fulfilment of homogeneity and isotropy even up to the the Hubble scale (Lauer & Postman 1994; Slehta & Mészáros A. 1997; Mészáros A. & Ványsek 1997; Coles 1998; Sylos-Labini et al. 1998). Hence, as anisotropy concerns we think that the different distribution of short and long GRBs and their cosmological origin are not in contradiction, and we further mean that all GRBs are at cosmological distances.

6. Conclusion

The results of paper may be collected as follows:

A. The distribution of 2025 GRBs is anisotropic.

B. This anisotropy is not caused exclusively by instrumental effects.

C. Separating GRBs into the short and long subclasses it is shown that the short ones are distributed anisotropically, but the long ones can still be distributed isotropically.

D. No such difference was found for the HE-NHE separation.

E. We conjectured that the anisotropic distribution of short GRBs does not query their cosmological origin.

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References

Bagoly Z., Mészáros A., Horváth I., Balázs L.G., Mészáros P., 1998, ApJ in press
Bell i B.M., 1995, Ap&SS 231, 43
Briggs M.S., 1993, ApJ 407, 126
Briggs M.S., 1995, Ap&SS 231, 3
Briggs M.S., Paciesas W.S., Pendleton G.F., et al., 1996, ApJ 459, 40
Che H., Yang Y., Wu. M., Li T.P., 1997, ApJ 477, L69
Coles P., 1998, Nat 391, 120
Dezalay J.P., Lestrade J.P., Barat C., et al., 1996, ApJ 471, L27
Fishman G.J., Meegan C.A., Wilson R.B., et al., 1994, ApJS 92, 229
Horváth I., Mészáros P., Mészáros A., 1996, ApJ 470, 56
Katz J.I., Canal L.M., 1996, ApJ 471, 915
Kendall M.G., Stuart A., 1969, The Advanced Theory of Statistics, Vol. 1, London: Griffin
Klebesadel R.W., Strong I.B., Olson R.A., 1973, ApJ 182, L85
Kouveliotou C., Meegan C.A., Fishman G.J., et al., 1993, ApJ 413, L101
Lauer T.R., Postman M., 1994, ApJ 425, 418
Meegan C.A., Pendleton G.N., Briggs M.S., et al., 1996, ApJS 106, 65 (The Third BATSE Gamma-Ray Burst Catalog)
Meegan C.A., Pendleton G.N., Briggs M.S., 1997, Current BATSE Gamma-Ray Burst Catalog, http://www.batse.msfc.nasa.gov/data
Mészáros A., 1997, A&A 328, 1
Mészáros A., Mészáros P., 1995, ApJ 449, 9
Mészáros P., Rees M.J., 1997, ApJ 482, L29
Nemiroff R.J., 1995, PASP 10, 1131
Norris J.P., Nemiroff R.J., Scargle J.D., et al., 1994, ApJ 424, 540
Norris J.P., Bonnell J.T., Nemiroff R.J., et al., 1995, ApJ 439, 542
Paciesas W. S., Meegan C.A., Pendleton G.N., et al., 1998, ApJS in preparation (The Fourth BATSE Gamma-Ray Burst Catalog)
Paczynski B., 1995, PASP 107, 1167
Pendleton C.N., Paciesas W.S., Briggs M.S., et al., 1997, ApJ 489, 175
Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1992, Numerical Recipes, Cambridge University Press, Cambridge
Rees M.J., Mészáros P., 1994, ApJ 430, L93
Slehta M., Mészáros A., 1997, Ap&SS 249, 1
Stern B.E., 1996, ApJ 464, L111
Syer D., Saha P., 1994, ApJ 427, 714
Sylos-Labini F., Montuori M., Pietronero L., 1998, Phys. Rep. in press
Tegmark M., Hartmann D.H., Briggs M.S., Hakkila J., Meegan C.A., 1996a, ApJ 466, 757
Tegmark M., Hartmann D.H., Briggs M.S., Meegan C.A., 1996b, ApJ 468, 214
Trumpler R.L., Weaver H.F., 1953, Statistical Astronomy, University of California Press, Berkeley
Usov V., Chibisov G., 1975, SvA 19, 115

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