Entanglement redistribution in the Schwarzschild spacetime

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Abstract

The effect of Hawking radiation on the redistribution of the entanglement and mutual information in the Schwarzschild spacetime is investigated. Our analysis shows that the physically accessible correlations degrade while the unaccessible correlations increase as the Hawking temperature increases because the initial correlations described by inertial observers are redistributed between all the bipartite modes. It is interesting to note that, in the limit case that the temperature tends to infinity, the accessible mutual information equals to just half of its initial value, and the unaccessible mutual information between mode $A$ and $I\!I$ also equals to the same value.

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I. INTRODUCTION

Entanglement plays a pivotal role in quantum information — it is a resource for various computational tasks such as quantum communication and teleportation. It is believed that investigation of the entanglement in a relativistic framework is not only helpful in understanding some of the key questions in quantum information theory, but also plays an important role in the study of entropy and the information paradox of black holes [1–3]. Thus, much attention has been focused on the relativistic effects in the context of the quantum information theory [4–18]. Recently, we investigated the effect of the Hawking radiation [19] on the entanglement and teleportation in a general static and asymptotically flat black hole with spherical symmetry [20], and found that the entanglement degraded as the increase of the Hawking temperature both for the scalar and Dirac fields. However, we now face a intriguing question: where has the lost entanglement gone?

In this paper we will investigate the redistribution of entanglement for the Dirac fields in the background of a Schwarzschild black hole. The loss of entanglement will be explained by the redistribution of the entanglement among all accessible and unaccessible modes. We use some methods of the quantum information to quantify and identify the property of the correlations (both quantum and classical) from the perspective of physical observers who can access field modes only outside the event horizon. Our scheme proposes that two observers, Alice and Bob, share a generically entangled state at the same initial point in flat region. After their coincidence, Alice remains at the asymptotically flat region but Bob freely falls in toward the black hole and locates near the event horizon. Due to the presence of a horizon, an observer in each side of the horizon has no access to field modes in the causally disconnected region. Therefore, the observer must trace over the inaccessible region and lose some information about the state. Thus we must calculate the entanglement in all possible bipartite divisions of the system: (i) the mode A described by Alice, (ii) the mode I in exterior region of the black hole (described by Bob), and (iii) the complimentary mode II in the interior region of the black hole.

The outline of the paper is as follows. In Sec. II we recall the vacuum structure and Hawking radiation for the Dirac fields. In Sec. III we discuss the essential features of the background spacetime and the redistribution of entanglement. We will summarize and discuss our conclusions in the last section.
II. VACUUM STRUCTURE AND HAWKING RADIATION FOR DIRAC FIELDS

The line element for the Schwarzschild spacetime is
\[ ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \] (1)
where the parameter \( M \) represents the mass of the black hole.

Solving the Dirac equation [21] near the event horizon, we obtain the positive (fermions) frequency outgoing solutions outside and inside regions of the event horizon \( r_+ \) \[22, 23\]
\[
\Psi_k(r > r_+) = G e^{-i\omega u},
\]
\[
\Psi_k(r < r_+) = G e^{i\omega u},
\]
where \( G \) is a 4-component Dirac spinor [17], \( u = t - r^* \) and \( r^* = r + 2M \ln \frac{r - 2M}{2M} \) is the tortoise coordinate. Hereafter we will use the wavevector \( k \) labels the modes. Particles and antiparticles will be classified with respect to the future-directed timelike Killing vector in each region.

Making a analytic continuation for Eqs. (2) and (3), we find a complete basis for positive energy modes according to Domour-Ruffini’s suggestion [24]. Then we can quantize the Dirac fields in the Schwarzschild and Kruskal modes respectively, from which we can easily get the Bogoliubov transformations [25] between the creation and annihilation operators in the Schwarzschild and Kruskal coordinates. After properly normalizing the state vector, the vacuum state of the Kruskal particle for mode \( k \) is found to be
\[
|0\>_K = (e^{-\omega k/T} + 1)\frac{1}{2} \exp \left[ e^{-\omega k/T} a_k b_{-k}^\dagger \right] |0_k\>_I^+ |0_{-k}\>_II

= \left[ (e^{-\omega k/T} + 1)\frac{1}{2} + (e^{\omega k/T} + 1)\frac{1}{2} a_k b_{-k}^\dagger \right] |0_k\>_I^+ |0_{-k}\>_II

= (e^{-\omega k/T} + 1)\frac{1}{2} |0_k\>_I^+ |0_{-k}\>_II + (e^{\omega k/T} + 1)\frac{1}{2} |1_k\>_I^+ |1_{-k}\>_II.
\] (4)
where \( \{|n_{-k}\>_II\} \) and \( \{|n_k\>_I\} \) are the orthonormal bases for the inside and outside regions of the event horizon respectively, the superscript \( \{+, -\} \) on the kets is used to indicate the particle and antiparticle vacua, and \( T = \frac{1}{8\pi M} \) is the Hawking temperature [26]. The only excited state is
\[
|1\>_K = |1_k\>_I^+ |0_{-k}\>_II.
\] (5)
Hereafter we will refer to the particle mode \( \{|n_k\>_I\} \) simply as \( \{|n\>_I\} \), and the antiparticle mode \( \{|n_{-k}\>_II\} \) as \( \{|n\>_II\} \).
III. ENTANGLEMENT REDISTRIBUTION

We assume that Alice has a detector which only detects mode $|n\rangle_A$ and Bob has a detector sensitive only to mode $|n\rangle_B$, and they share a generically entangled state at the same initial point in flat Minkowski spacetime. The generically entangled initial state is

$$|\Psi\rangle_{AB} = \alpha |0\rangle_A |0\rangle_B + \sqrt{1 - \alpha^2} |1\rangle_A |1\rangle_B,$$

where $\alpha$ is a state parameter which satisfies $|\alpha| \in (0, 1)$. After the coincidence of Alice and Bob, Alice stays stationary at the asymptotically flat region, while Bob falls toward the black hole and hovers outside the event horizon. Using Eqs. (4) and (5), we can rewrite Eq. (6) in terms of Minkowski modes for Alice and black hole modes for Bob

$$|\Psi\rangle_{A,I,II} = \alpha |0\rangle_A \left[ (e^{-\omega_k/T} + 2)^{-\frac{1}{2}} |0\rangle_I |0\rangle_{II} + (e^{\omega_k/T} + 2)^{-\frac{1}{2}} |1\rangle_I |1\rangle_{II} \right]$$

$$+ \sqrt{1 - \alpha^2} |1\rangle_A |1\rangle_I |0\rangle_{II}.$$

(7)

A. The physically accessible correlations

Since the exterior region is causally disconnected from the interior region of the black hole, the only entanglement which is physically accessible to the observers is encoded in the mode $A$ described by Alice and the mode $I$ in exterior region of the black hole described by Bob. Thus, when observers describe the state they find that some of the correlations are lost [20]. Taking the trace over the state of the interior region we obtain

$$\rho_{A,I} = \alpha^2 (e^{-\omega_k/T} + 1)^{-1} |00\rangle\langle 00| + \alpha \sqrt{1 - \alpha^2} (e^{-\omega_k/T} + 1)^{-\frac{1}{2}} (|00\rangle\langle 11| + |11\rangle\langle 00|)$$

$$+ \alpha^2 (e^{\omega_k/T} + 1)^{-1} |01\rangle\langle 01| + (1 - \alpha^2) |11\rangle\langle 11|.$$

(8)

where $|mn\rangle = |m\rangle_A |n\rangle_{B,I}$. The partial transpose criterion provides a necessary and sufficient condition for the entanglement in a mixed state of two qubits [4]: if at least one eigenvalue of the partial transpose is negative, the density matrix is entangled. The partial transpose $\rho_{AB}^T$ is obtained by interchanging Alice’s qubits, which yields a negative eigenvalue

$$\lambda_- = \frac{1}{2} \left[ \alpha^2 (e^{\omega/T} + 1)^{-1} - \sqrt{\alpha^4 (e^{\omega/T} + 1)^{-2} + 4 \alpha^2 (1 - \alpha^2) (e^{-\omega/T} + 1)^{-1}} \right].$$

Thus, the state is always entangled for any Hawking temperature $T$. To quantify the entanglement of $\rho_{A,I}$ in Eq. (8) we compute the spin-flip matrix $\tilde{\rho}_{A,I}$, and find that the eigenvalues
of the matrix $\varrho_{A,I}\varrho_{A,I}^\dagger$ are \( 4\alpha^2(1-\alpha^2)(e^{-\omega k/T}+1)^{-1},0,0,0 \). Then we find the concurrence \[27, 28\] of this state

\[ C(\varrho_{A,I}) = 2\alpha \sqrt{1 - \alpha^2 (e^{-\omega k/T}+1)^{-\frac{1}{2}}}, \tag{9} \]

which is \( 2\alpha \sqrt{1 - \alpha^2} \) at zero Hawking temperature, i.e., the case of supermassive or an almost extreme black hole, as expected. And approaches the value \( C_f(\varrho_{A,I}) = \alpha \sqrt{2(1-\alpha^2)} \) for infinite Hawking temperature \( T \to \infty \), i.e., the black hole evaporates completely. The entanglement of formation \[28\] is

\[ E_F(\varrho_{A,I}) = \mathcal{H} \left[ \frac{1 + \sqrt{1 - 4\alpha^2(1-\alpha^2)(e^{-\omega k/T}+1)^{-1}}}{2} \right], \]

where \( \mathcal{H}(x) = -x \log_2 x - (1-x) \log_2(1-x) \).

The mutual information \[29\], which can be used to estimate the total (classical and quantum) amount of correlations between any two subsystems of the overall system, is found to be

\[ I(\varrho_{A,I}) = \mathcal{F}[1 - \alpha^2(e^{\omega/T}+1)^{-1}] + \mathcal{F}[\alpha^2(e^{\omega/T}+1)^{-1}] - \mathcal{F}(1-\alpha^2) \]

\[ -\mathcal{F}[1 - \alpha^2(e^{-\omega/T}+1)^{-1}] - \mathcal{F}[\alpha^2(e^{-\omega/T}+1)^{-1}] - \mathcal{F}(\alpha^2), \tag{10} \]

where \( \mathcal{F}(x) = x \log(x) \). Note that the initial mutual information is \( I_i(\varrho_{A,I}) = -2[\mathcal{F}(\alpha^2) + \mathcal{F}(1-\alpha^2)] \) for vanishing Hawking temperature. In the infinite Hawking temperature limit \( T \to \infty \), the mutual information converges to \( I_f(\varrho_{A,I}) = -[\mathcal{F}(\alpha^2) + \mathcal{F}(1-\alpha^2)] \), which is just half of \( I_i \).

**B. The physically unaccessible correlations**

To explore entanglement in this system in more detail we consider the tripartite system consisting of the modes \( A, I, \) and \( II \). In an inertial frame the system is bipartite, but from a non-inertial perspective an extra set of modes in region \( II \) becomes relevant. We therefore calculate the entanglement in all possible bipartite divisions of the system. Let us first comment on the quantum correlations created between the mode \( A \) and mode \( II \), tracing over the mode in region \( I \), we obtain the density matrix

\[ \varrho_{A,II} = \alpha^2(e^{-\omega k/T}+1)^{-1}|00\rangle\langle 00| + \alpha \sqrt{1 - \alpha^2 (e^{\omega k/T}+1)^{-\frac{1}{2}}}(|10\rangle\langle 01| + |01\rangle\langle 10|) \\
+ \alpha^2(e^{\omega k/T}+1)^{-1}|01\rangle\langle 01| + (1-\alpha^2)|10\rangle\langle 10|, \tag{11} \]
where $|mn\rangle = |m\rangle_A |n\rangle_{B,II}$. The partial transpose of $\varrho_{A,II}$ has an eigenvalue

$$\lambda_- = \frac{1}{2} \left[ \alpha^2 (e^{-\omega/T} + 1)^{-1} - \sqrt{\alpha^4 (e^{-\omega/T} + 1)^{-2} + 4\alpha^2 (1 - \alpha^2)(e^{\omega/T} + 1)^{-1}} \right],$$

which is less than or equal to zero. At $T = 0$ the eigenvalue is zero, which means that there is no entanglement at this point. However, for $T > 0$ entanglement does exist between these two modes.

Calculating the spin-flip of $\varrho_{A,II}$

$$\tilde{\varrho}_{A,II} = \alpha^2 (e^{-\omega_k/T} + 1)^{-1}|11\rangle\langle 11| + \alpha \sqrt{1 - \alpha^2} (e^{\omega_k/T} + 1)^{-\frac{1}{2}} (|01\rangle \langle 10| + |10\rangle \langle 01|)$$

$$+ \alpha^2 (e^{\omega_k/T} + 1)^{-1}|10\rangle \langle 10| + (1 - \alpha^2)|01\rangle \langle 01|,$$  \hspace{1cm} (12)

we find that

$$\varrho_{A,II} \tilde{\varrho}_{A,II} = 2\alpha^2 (1 - \alpha^2)(e^{\omega_k/T} + 1)^{-1}(|01\rangle \langle 01| + |10\rangle \langle 10|) + 2\sqrt{\alpha^6 (1 - \alpha^2)}$$

$$\times (e^{\omega_k/T} + 1)^{-\frac{3}{2}}|01\rangle \langle 10| + 2\sqrt{\alpha^2 (1 - \alpha^2)^3 (e^{\omega_k/T} + 1)^{-\frac{1}{2}}}|10\rangle \langle 01|,$$  \hspace{1cm} (13)

has eigenvalues $\left[ 4\alpha^2 (1 - \alpha^2)(e^{\omega_k/T} + 1)^{-1}, 0, 0, 0 \right]$. Thus, the concurrence is given by

$$C(\varrho_{A,II}) = 2\alpha \sqrt{1 - \alpha^2}(e^{\omega_k/T} + 1)^{-\frac{1}{2}},$$  \hspace{1cm} (14)

which is zero at zero Hawking temperature as expected, and approaches the value $C_f(\varrho_{A,II}) = \alpha \sqrt{2(1 - \alpha^2)}$ for infinite Hawking temperature, which is just equal to $C_f(\varrho_{A,I})$ in this limit. The entanglement of formation between mode $A$ and mode $II$ is

$$E_F(\varrho_{A,II}) = \mathcal{H} \left[ \frac{1 + \sqrt{1 - 4\alpha^2 (1 - \alpha^2)(e^{\omega_k/T} + 1)^{-1}}}{2} \right]$$

and the mutual information is

$$I(\varrho_{A,II}) = -\mathcal{F}[1 - \alpha^2(e^{-\omega/T} + 1)^{-1}] - \mathcal{F}[\alpha^2(e^{-\omega/T} + 1)^{-1}] + \mathcal{F}(1 - \alpha^2)$$

$$- \mathcal{F}[1 - \alpha^2(e^{\omega/T} + 1)^{-1}] - \mathcal{F}[\alpha^2(e^{\omega/T} + 1)^{-1}] + \mathcal{F}(\alpha^2).$$  \hspace{1cm} (15)

At $T = 0$ the mutual information is zero, while in the infinite Hawking temperature limit $T \to \infty$ the mutual information becomes to $I_f(\varrho_{A,II}) = -[\mathcal{F}(\alpha^2) + \mathcal{F}(1 - \alpha^2)]$, which is equal to $I_f(\varrho_{A,I})$ in this limit.

We now study the entanglement between mode $I$ and mode $II$. Tracing over the modes in $A$, we obtain the density matrix

$$\varrho_{I,II} = \alpha^2 (e^{-\omega_k/T} + 1)^{-1}|00\rangle \langle 00| + \alpha^2 (e^{\omega_k/T} + e^{-\omega_k/T} + 2)^{-\frac{1}{2}} (|00\rangle \langle 11| + |11\rangle \langle 00|)$$

$$+ (1 - \alpha^2)|10\rangle \langle 10| + \alpha^2 (e^{\omega_k/T} + 1)^{-1}|11\rangle \langle 11|,$$  \hspace{1cm} (16)
where $|mn⟩ = |m⟩_I |n⟩_{II}$.

FIG. 1: The concurrence $C(\rho_{a,b})$ as a function of the Hawking temperature $T$ with the fixed $\omega$ and $\alpha$.

The partial transpose of $\rho_{I,II}$ has an eigenvalue

$$\lambda_- = -\frac{1}{2}[1 - \alpha^2 - \sqrt{1 - 2\alpha^2 + \alpha^4 + \alpha^4(e^{\omega k/T} + e^{-\omega k/T} + 2)^{-1}}];$$

which is less than or equal to zero. Again, similar to the last case, the entanglement does exist between these two modes according to the partial transpose criterion. The matrix $\rho_{I,II} \tilde{\rho}_{I,II}$ has eigenvalues $[\alpha^4(e^{\omega k/T} + e^{-\omega k/T} + 2)^{-1}, 0, 0, 0]$. Thus, the concurrence is given by

$$C(\rho_{I,II}) = \alpha^2(e^{\omega k/T} + e^{-\omega k/T} + 2)^{-\frac{1}{2}},$$

which is zero at zero Hawking temperature, and approaches the value $\alpha^2/2$ for infinite Hawking temperature.

The entanglement of formation in this case is

$$E_F(\rho_{I,II}) = \mathcal{H}\left[\frac{1 + \sqrt{1 - \alpha^4(e^{\omega k/T} + e^{-\omega k/T} + 2)^{-1}}}{2}\right],$$

and the mutual information is

$$I(\rho_{I,II}) = \mathcal{F}[1 - \alpha^2(e^{\omega/T} + 1)^{-1}] + \mathcal{F}[\alpha^2(e^{\omega/T} + 1)^{-1}] - \mathcal{F}(1 - \alpha^2) - \mathcal{F}[1 - \alpha^2(e^{-\omega/T} + 1)^{-1}] - \mathcal{F}[\alpha^2(e^{-\omega/T} + 1)^{-1}] - \mathcal{F}(\alpha^2).$$
Again, we find that the mutual information vanishes for zero Hawking temperature, and increases to a finite value as the Hawking temperature goes to infinity.

In Figs. (1) and (2) we plot the behavior of the concurrence and the entanglement of formation with the fixed $\omega$ and $\alpha$ which show how the Hawking temperature would change the properties of all the bipartite entanglement. When the Hawking temperature is lower, modes $A$ and $I$ remain almost maximally entangled while there is little entanglement between modes $I$ and $II$ and between modes $A$ and $II$. As the Hawking temperature grows, the unaccessible entanglement between modes $I$ and $II$ and between modes $A$ and $II$ increases, while the accessible entanglement between modes $A$ and $I$ degrades. We arrive at the conclusion that the original entanglement in the state Eq. (6) which is described by the inertial observers is now redistributed among the mode $A$ described by Alice, the mode $I$ in exterior region of the black hole described by Bob, and the complimentary mode $II$ in the interior region of the black hole. Therefore, as a consequence of the conservation of entanglement, the physically accessible entanglement between the two modes described by Alice and Bob is degraded.

The properties of the mutual information are shown in Fig. (3). It demonstrates that the mutual information of $\rho_{A,I}$ decreases while the mutual information of $\rho_{A,II}$ and $\rho_{I,II}$ increases as the Hawking temperature increases. It is interesting to note that when black hole evaporates completely, the mutual information between modes $A$ and $I$ equals to just half of the its initial value and $I(\rho_{A,I}) = I(\rho_{A,II})$ in this limit, which are independent of the
state parameter $\alpha$.

![Graph of mutual information $I(\varrho_{a,b})$ of the Dirac modes versus Hawking temperature $T$ with the fixed $\omega$ and $\alpha$.]

FIG. 3: Mutual information $I(\varrho_{a,b})$ of the Dirac modes versus Hawking temperature $T$ with the fixed $\omega$ and $\alpha$.

In Ref. [20] we found that the entanglement and mutual information of the mode $\varrho_{A,I}$ is degraded as the increase of the Hawking temperature (or acceleration [9]). The open question should be addressed in this paper is whether the lost correlations are destroyed or transferred to somewhere. Here we presented a complete description of the information behavior across an event horizon by discussing the redistribution of the entanglement and mutual information. We find that the correlations lost from the mode described by Alice and the field mode outside the event horizon is gained by other subsystems, especially between mode A and the mode $II$ inside the event horizon. The results obtained here not only interpreted the lose of entanglement and mutual information in the presence of a horizon but also gave a better insight into the entanglement entropy and information paradox of the black holes.

IV. SUMMARY

The effect of Hawking radiation on the redistribution of entanglement in the Schwarzschild Spacetime is investigated. It is shown that the entanglement between modes $I$ and $II$ and between modes $A$ and $II$ increases, while the entanglement between modes $A$ and $I$ is degraded as the Hawking temperature grows. The original two-mode entanglement, which is described by Alice and Bob from an inertial perspective, is now redistributed among the
mode A described by Alice, the mode I in exterior region of the event horizon described by Bob, and the complimentary mode II in the interior region of the horizon. This is a good explanation of physically accessible entanglement between the two modes described by Alice and Bob is degraded as the Hawking temperature grows. It is also found that the mutual information of $\rho_{A,I}$ decreases while mutual information of $\rho_{A,II}$ and $\rho_{I,II}$ increases as the Hawking temperature increases. It is interesting to note that, in limit case that the temperature tends to infinity, the accessible mutual information equals to just half of its initial value, and the unaccessible mutual information between mode A and II also equals to the same value. The results obtained here not only interpreted the lose of entanglement and mutual information in the presence of a horizon but also gave a better insight into the entanglement entropy and information paradox of the black holes.

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