Flavor Physics in SO(10) GUTs with Suppressed Proton decay Due to Gauged Discrete Symmetry

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Abstract. Generic SO(10) GUT models suffer from the problem that Planck scale induced non-renormalizable proton decay operators require extreme suppression of their couplings to be compatible with present experimental upper limits. One way to resolve this problem is to supplement SO(10) by simple gauged discrete symmetries which can also simultaneously suppress the renormalizable R-parity violating interactions when they occur and make the theory “more natural”. We then present an extended 16_H model, with three 10 and three 45-Higgs, which is free of this problem. We propose this as a realistic and “natural” model for fermion unification and discuss the phenomenology of this model e.g. its predictions for neutrino mixings and lepton flavor violation.

1. Introduction

The neutrino observations of the past decade have put the spotlight on gauged B-L symmetry as well as unification groups such as SO(10) and SU(2)_L × SU(2)_R × SU(4)_4 containing B-L as prime candidates for theory of matter, forces and flavor. While both these groups incorporate the seesaw mechanism for neutrino masses, SO(10) has the additional attractive feature that gauge couplings unify at high scale. It is however highly nontrivial to obtain a “truly natural” SO(10) model due to such issues as doublet triplet splitting, rapid proton decay etc. In this paper we discuss how one aspect of this naturalness can be addressed i.e. how one can naturally suppress proton decay in SO(10) models while preserving our understanding neutrino masses.

We first note that SO(10) models for neutrinos discussed in recent literature can be divided into two classes:

(i) One class which uses only renormalizable couplings involving the Higgs fields 10, 120 and 126 for fermion masses and the last multiplet for breaking B-L symmetry and multiplets such as 45 and/or 210 for gauge symmetry breaking[1]. This theory could be considered as an ultraviolet complete theory by itself.

(ii) The second class uses 10 plus 16 ⊕ ̅16 for fermion masses with the 16’s breaking the B-L symmetry. Here one generally uses 45 + 54 Higgs fields for SO(10) breaking. An important feature of this class is that it has to rely on nonrenormalizable couplings to understand fermion masses and therefore has to be viewed necessarily as an effective theory at the GUT scale[2].

The first class of models leads to automatic R-parity conservation when SO(10) breaks down to MSSM so that there is a natural candidate for dark matter whereas the second class of models

1 In collaboration with R.N.Mohapatra
suffers from R-parity breaking and hence has no stable dark matter in the absence of additional symmetries. So in principle one could argue that this class of models are not “pure” SO(10) models.

Both models have an additional naturalness problem arising from the fact that they allow R-parity conserving nonrenormalizable couplings of the form $\lambda 16^4_m/M_{Pl}$ which lead to rapid proton decay. Such interactions could be induced by non-perturbative Planck scale effects and it is therefore not safe to ignore them. Present proton life time limits constrain $\lambda$ to be $\lesssim 10^{-7}$. Such a small value of $\lambda$ would suggest that there is probably a symmetry responsible for its smallness. This question is particularly urgent for the class of SO(10) models with $SU(2)$ such as MSSM or left-right models as well as $SU(4)$ models- one with $SU(2)$ and another with $SU(4)$ models and not just GUT theories. One way to understand the suppression of such operators despite the presence of non-perturbative gravitational effects, is to have an additional gauge symmetry beyond SO(10) which can forbid these unwanted terms. The simplest possibility is to have a discrete gauge symmetry\cite{3}.

The discrete gauge symmetry supplemented SO(10) models that suppress proton decay were studied for a large class of models in a recent papers\cite{5, 6}. In particular two minimal SO(10) models- one with $16$-Higgs breaking the B-L symmetry and another with $216$ breaking B-L were shown to be free of both proton decay problem as well as R-parity problem if SO(10) was supplemented by a gauged $Z_6$ symmetry.

In this paper we will show that within extended $16$-based models , one can achieve the desired doublet-triplet splitting and fermion masses that can match observations. This model differs from other $16$-based models in that proton decay here arises only from the gauge boson exchanges unlike other models where Planck scale induced effects as well as Higgsino exchange ones play a role\cite{7}; (ii) we study the phenomenological implications of this model and isolate some of its tests e.g. in the domain of lepton flavor violation.

This paper is organized as follows: in sec. 2, we review the conditions discrete group has to satisfy to suppress proton decay; in sec.3 we discuss the three Higgs$16$-based model that fits fermion masses and mixings; in sec. 4, we discuss how large neutrino mixings and observed neutrino masses arise in this model; in sec. 5 we discuss bounds arising from Lepton Flavor Violating processes, and then we conclude by summarizing presented results.

2. The $SO(10) \times Z_6$ model for $16$-Higgs B-L breaking

The main features of generic SO(10) models with $16$-Higgs fields breaking B-L symmetry are the following: (i) the quarks and leptons are assigned to three $16$-dimensional spinors (denoted by $\psi_m$, $m=1,2,3$); (ii) the GUT symmetry is broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ by a $45 \oplus 54$ set of Higgs fields; (iii) $SU(2)_R \times U(1)_{B-L}$ symmetry is broken by the $16$-Higgs pair denoted by $\psi_H^\pm \bar{\psi}_H$ set to the standard model symmetry which is then broken by $SU(2)$ doublets that are linear combinations of the doublets in SO(10) $10$ and $16$-Higgs fields. The standard model symmetry along with supersymmetry emerge just below the GUT scale of $2 \times 10^{16}$ GeV.

This model has several naturalness problems: it not only allows the dangerous R-parity conserving $\frac{16_m}{M_{Pl}}$ terms but also terms such as $\frac{(16_m)^3}{M_{Pl}}$ terms which on B-L symmetry breaking lead to all three types of R-parity violating operators present in general MSSM i.e. $LLe^c$, $QLd^c$ and $u^c d^c c^c$ type. Thus this model for natural values of couplings will lead to extremely rapid proton decay which is unacceptable. The question addressed in Ref.\cite{5} is to search for gauged discrete symmetries that will keep the model phenomenologically viable while keeping them “proton decay safe” and it was shown that the minimal anomaly free discrete gauge symmetry is $Z_6$. Similar considerations for $216$ type models also led to the symmetry $Z_6$ and in both cases an extra $10$-Higgs field denoted by $H'$ in addition to those considered already.
To see the discrete symmetry charges for various fields that forbid both R-parity violating terms as well as R-parity conserving baryon number violating terms, while at the same time keeping the required terms responsible for good phenomenology, we divide the superpotential terms into two classes: type I terms that must be kept for phenomenology and type II terms that must be forbidden to suppress proton decay and R-parity violating terms. They are given below:

**Terms of type I:** They include $\psi_m \bar{\psi}_m H, (\psi_m \bar{\psi}_H)^2/M_P, \psi_H \bar{\psi}_H, A^2, S^{2,3}$ and $S A^2$, where $H, A, S$ are 10-, 45-, 54-plets, respectively. Taking the discrete gauge symmetry to be $Z_N$, we can write down the constraints on the $Z_N$ charges that are required by the type I terms:

\[
2q_{\psi_m} + q_H = 0 \text{ mod } N, \quad 2q_{\bar{\psi}_m} + 2q_{\bar{\psi}_H} = 0 \text{ mod } N
\]

\[
q_{\psi_H} + q_{\bar{\psi}_H} = 0 \text{ mod } N, \quad q_H + q_{H'} = 0 \text{ mod } N, \quad 2q_A = 0 \text{ mod } N,
\]

\[
2q_S = 0 \text{ mod } N, \quad 3q_S = 0 \text{ mod } N, \quad 2q_A + q_S = 0 \text{ mod } N.
\]

Here, we denote the $Z_N$ charge for a field $F$ by $q_F$.

**Type II terms:** These are the terms that must be forbidden from appearing in the superpotential and are $\psi_m \psi_m H', \psi_m^4, \psi_m \bar{\psi}_H$ and $\psi_m^3 \psi_H, \psi_m \psi_H H'$ and $\psi_m \bar{\psi}_H A$. We forbid the $\psi_m \psi_m H'$ in order to avoid large Higgsino mediated contribution to proton decay since this is the very problem we are trying to solve. The necessary constraints on the $Z_N$ charges have to be chosen such that they satisfy the inequalities

\[
q_{\psi_m} + q_{H'} \neq 0 \text{ mod } N, \quad q_{\bar{\psi}_m} \neq 0 \text{ mod } N,
\]

\[
q_{\psi_m} + q_{\bar{\psi}_H} \neq 0 \text{ mod } N, \quad q_{\bar{\psi}_m} + q_{\bar{\psi}_H} \neq 0 \text{ mod } N
\]

\[
q_{\psi_m} + q_{H',H} + q_{\bar{\psi}_H} \neq 0 \text{ mod } N, \quad q_{\psi_m} + q_{\bar{\psi}_H} + q_A \neq 0 \text{ mod } N.
\]

The last set of constraints come from the requirement that the discrete symmetry must be a gauge symmetry i.e. it must be anomaly free. The anomaly freedom constraints are:

\[
16(N_g q_{\psi_m} + q_{\psi_H} + q_{\bar{\psi}_H}) + 10(q_H + q_{H'}) + 45q_A + 54q_S = 0 \text{ mod } N'
\]

\[
2N_g q_{\psi_m} + 2q_{\psi_H} + 2q_{\bar{\psi}_H} + q_H + q_{H'} + 8q_A + 12q_S = 0 \text{ mod } N
\]

where $N' = \{N, \text{ odd } N \}

\}

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\}

\}

It was shown in [5] that the smallest symmetry allowing us to fulfill all criteria is $Z_6$ for number of generation $N_g = 3$. A possible charge assignments is $q_{\psi_m} = 1, q_{\psi_H} = -2, q_{\bar{\psi}_H} = +2, q_H = -2, q_{H'} = +2, q_{45,54} = 0$ (cf. tables 3). This charge assignment allows for seesaw couplings and the possibility of fermion masses from couplings of type $\psi_m \psi_m H$. The allowed operator $\psi_m \psi_m \bar{\psi}_H^2$ contributes to both the fermion masses as well as to the seesaw. The model also eliminates the dangerous proton decay operator $QQQL$ or operator of type $(\psi_m)^4/M_P$.

While the allowed set of operators provide a necessary condition for the model being phenomenologically viable, the final step where we judge whether it is acceptable first requires that we do doublet triplet splitting and see if the sub-GUT scale structure of the model can generate acceptable pattern of fermion masses or not. We address this question in the next sub-section.

**3. Extended 16-Higgs model**

Minimal 16 and 126 presented in [5] suffer from predicting viable phenomenology, but as was shown in [6] a simple extension of 16 can cure all these problems. The field content of the
extended $\mathbf{16}$ -Higgs model and $Z_6$ assignments are presented in Table 1. One can see that anomaly freedom and proton decay constraints are satisfied and yet the model can lead to viable phenomenology. The superpotential is given by:

$$W = MH_1H_2 + SH_1H_2 + A_1H_1H_2 + \psi_H\psi_HH_1 + \bar{\psi}_H\bar{\psi}_HH_2 + M_\psi\bar{\psi}_H\psi_H + M_3H_3^2 + H_2H_3A_2 + H_1H_3A_3 + \frac{H_3A_2\psi_H\bar{\psi}_H}{M_{Pl}} + \frac{H_3A_2\psi_H\bar{\psi}_H}{M_{Pl}}$$

One can see that the vevs of the $A_2,3,\psi_H\bar{\psi}_H$ will break $Z_6$ will break $Z_6$ symmetry down to $Z_2$, so naively we lose our protection of the proton decay, but let us look at each of these vevs separately.

- vevs of the $\mathbf{16}, \mathbf{\overline{16}}$ will preserve the following combination of $2(B - L) + Z_6$, so they cannot lead to the proton decay, because it also preserves $(B - L)$.
- Potential minimization can ensure that vev $\langle A_3 \rangle = 0$, so we will not have the dangerous operator $\frac{\psi^4A_3}{M_{Pl}}$ which might lead to the large contribution to the proton decay operator with effective coupling $\lambda \simeq \frac{M_3}{M_{Pl}} \ll 1$ -not suppressed enough to be acceptable.
- Potential minimization can ensure that vev $\langle A_2 \rangle_{\text{Triplet}} = 0$ in this case we will not have Higgsino mediated contribution to the four fermion proton decay operator, and the effective coupling of the $\frac{\psi^4}{M_{Pl}}$ will have a suppression $\left(\frac{\langle A_2 \rangle_{\text{Doublet}}}{M_{Pl}}\right)^2 \leq 10^{-4}$ which is naively $\sim 10^{-4}$ times bigger than the present upper limit but still is a considerable improvement in naturalness.

To study the doublet triplet splitting in this model, note that the mass matrix for the $\mathbf{5}$ and $\mathbf{\overline{5}}$ of SU(5) is given by

$$M - A_1 + S = 0 \quad \begin{pmatrix} 0 & M + A_1 + S & 0 & c \\ M - A_1 + S & 0 & A_2 & 0 \\ 0 & -A_2 & M_3 & 0 \\ 0 & c & \delta & M_\psi \end{pmatrix}$$

Where $\delta$ in the $(43)$ element of the matrix comes from $\frac{H_3A_2\psi_H\bar{\psi}_H}{M_{Pl}}$ coupling. As before, we want the determinant of this matrix to vanish. This leads to the following constraints

Case (i)

$$M - A_1 + S = 0$$

Case(ii):

$$M + A_1 + S = \frac{c^2M_3 + cA_2\delta}{M_3M_\psi}$$

### Table 1. $Z_6$ charge assignment of the fields

| $\psi_m$ | $\mathbf{16}$ | $\psi_H$ | $\mathbf{16\overline{2}}$ |
|----------|---------------|---------|----------------------|
| $H_1$    | $\mathbf{10\overline{2}}$ | $\bar{\psi}_H$ | $\mathbf{16}_2$ |
| $H_2$    | $\mathbf{10}_2$ | $A_1$ | $\mathbf{45}_0$ |
| $H_3$    | $\mathbf{10}_0$ | $A_2$ | $\mathbf{45\overline{2}}$ |

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Here we consider only the simpler of the two cases above i.e. case (i) to illustrate that our proposal leads to a realistic model. In the first case \( M - A_1 + S = 0 \), \( D = (1, 0, 0, 0) \) (implying that the \( h_d \) has non-zero component in the multiplet \( H_1 \)) in the same way as was in the minimal model, but now due to the presence of \( A_2 \) field all the \( U_1 \) is nonvanishing, so that the "up" quarks will get masses from the \( \psi^2_m H_1 \) operator. In the next sections we will discuss the detailed fit to fermion masses for this extended 16 model.

As we can see extended 16 model can solve doublet-triplet splitting problem as well as provide masses for all fermions, but now we have to check whether higgsino mediated proton decay operators are allowed. Even though quarks and leptons couple only to the \( H_1 \) field and there is no mass term \( \propto H \), mixing between \( H_1, H_2, H_3, \psi_H \) fields can lead to the nonvanishing diagrams with higgsino exchange. The contribution of these diagrams will vanish if only the \( (H_1 H_1) \) element of the inverse mass matrix (11) for the heavy triplets vanishes, thus the triplet part of the \( A_2 \) should be zero. One can show that the requirement of the \( < A_3 >= 0 \) combined with F flatness condition will lead to this condition (see for details [6]).

4. Fermion masses
The following couplings allowed by \( Z_6 \times SO(10) \) symmetries will lead to fermion masses after symmetry breaking.

\[
W = h^{10}\psi_m\psi_m H_1 + f^{10}\psi_m^2 \frac{\psi_H^2}{M_{Pl}} + f^{126}\psi_m^2 \frac{\psi_H^2}{5!M_{Pl}} + k^{120}\psi_m^2 A H_1 + g^{120}\psi_m^2 A^2 H_1 + g^{126}\psi_m^2 A^2 H_1 \tag{14}
\]

Where \( h^{10}, f^{10}, f^{126}, g^{126} \) are symmetric \( 3 \times 3 \) and \( k^{120}, g^{120} \) antisymmetric matrices. The upper index of \( h^{10}, f^{126}, 10, 120, 126 \) shows the SO(10) structure of the fermion couplings.

\[
M_u = v_u \left[ \alpha \left( h^{10} - k^{120} \frac{a - b}{M_{Pl}} + g^{120} \frac{b(a - b)}{M_{Pl}^2} + g^{126} \frac{ab + a^2}{M_{Pl}^2} \right) \right] + \xi \left( f^{10} \frac{4c}{M_{Pl}} - f^{126} \frac{8c}{M_{Pl}} \right)
\]

\[
M_u^D = v_u \left[ \alpha \left( h^{10} + k^{120} \frac{3a + b}{M_{Pl}} - g^{120} \frac{b(3a + b)}{M_{Pl}^2} - g^{126} \frac{3(ab + a^2)}{M_{Pl}^2} \right) \right] + \xi \left( f^{10} \frac{4c}{M_{Pl}} + f^{126} \frac{24c}{M_{Pl}} \right)
\]

\[
M_d = v_d \gamma \left[ h^{10} - k^{120} \frac{a + b}{M_{Pl}} - g^{120} \frac{b(a + b)}{M_{Pl}^2} - g^{126} \frac{-ab + a^2}{M_{Pl}^2} \right]
\]

\[
M_e = v_d \gamma \left[ h^{10} + k^{120} \frac{3a - b}{M_{Pl}} + g^{120} \frac{b(3a - b)}{M_{Pl}^2} - g^{126} \frac{3(-ab + a^2)}{M_{Pl}^2} \right]
\]

\[
M_{\nu}^M = 16f^{126}\frac{(\xi v_u)^2}{M_{Pl}} - (M_{\nu}^D)^T (16f^{126}\frac{c^2}{M_{Pl}})^{-1} M_{\nu}^D \tag{15}
\]

Where the vev of the fields \( H^1 \) and \( \psi_H \) are related to the vev of the MSSM doublets \( h_u \) and \( h_d \) in the following way

\[
\langle H_{1u} \rangle = \alpha \langle h_u \rangle
\]

\[
\langle H_{1d} \rangle = \gamma \langle h_d \rangle
\]

\[
\langle \psi_H \rangle = \xi \langle h_u \rangle \tag{16}
\]

Our claim is that these Yukawa coupling structure is rich enough to fit all the fermion masses. We give below an example of a scenario where correct fermion masses can arise.

We take the case where all antisymmetric couplings vanish and that down quarks and leptons are brought to the diagonal basis at the same time. Thus \( g^{126} \) and \( h^{10} \) are diagonal, and all the
mixing in the quark and lepton sector arise from the couplings $f^{10}$ and $f^{126}$. We now show that even under such limiting assumptions we can fit all the fermion masses. We know the quark masses at the GUT scale thus we can find corresponding $h^{10}, g^{126}$, but on the other hand in the case when all the quark mass matrices are symmetric the mass matrix for the up quarks is equal to

$$M_u = (CKM)^T M^{diag}_u CKM;$$

this leads to the constraint on the linear combination of $f^{10}$ and $f^{126}$. On the other hand we can find $f_{126}$ from the known neutrino masses and mixing angles [10], so this fixes $f^{10}$ and $f^{126}$.

5. Constraints from the lepton flavor violation

In this section, we discuss the predictions of this model for lepton flavor violation. As is well known[11, 12], even if the slepton mass matrices are diagonal at the GUT scale the RGE running down to the scale of the righthanded neutrino will lead to the mixing in the slepton sector, which via one loop diagrams leads to lepton violation. We will assume mSUGRA boundary condition for scalar partner masses and use the renormalization group equations to run them down to the seesaw scale when the right handed neutrinos decouple.

We will work in the basis with diagonal righthanded majorana neutrino matrix, then the slepton mixing will be approximately equal to

$$(\delta_{ij}^{LL}) = -\frac{3m_0^2 + A_0^2}{8\pi^2 m_0^2} \sum_{k=1}^{3} (Y_{\nu})_{ik} (Y_{\nu}^*)_{jk} \ln \left( \frac{M^{GUT}_{Rk}}{M_{Rk}} \right)$$

where $Y_{\nu}$ are the Yukawa couplings of the Dirac neutrino. These Yukawa couplings appear to be of roughly

$$Y_{\nu} \sim \begin{pmatrix} 10^{-5} & 5 \times 10^{-4} & 5 \times 10^{-3} \\ 10^{-5} & 5 \times 10^{-3} & 2 \times 10^{-2} \\ 10^{-5} & 3 \times 10^{-3} & 0.3 \end{pmatrix}$$

Here $Y_{\nu}$ is a linear combination of the Yukawa couplings $h_{10}, f_{10}$ and $f_{126}$ of the previous section. The slepton mixing leads to the lepton flavor violating processes $l_i \rightarrow l_j \gamma$ with the amplitude equal to

$$iM = em_i e^{-\delta_{ij}^{LL}} (iq^\mu \sigma_{\lambda\mu}(A_L P_L + A_R P_R)) l_i$$

Where the $q$ is the momentum of the photon and $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, the exact expression for the $A_{L,R}$ can be found in [12]. The branching ratio for this processes will be equal to

$$\frac{BR(l_i \rightarrow l_j \gamma)}{BR(l_i \rightarrow l_j \nu \bar{\nu})} = \frac{48\pi^3}{25 e^2} (|A_L^{ij}|^2)$$

$$A_L^{ij} \propto \frac{\delta_{ij}^{LL}}{4\pi m_0}$$

The present bounds on this processes are[13]

We will carry out our calculations for the branching ratio in the mSUGRA scenario, where there are only four parameters that will fix the low energy values of the slepton masses $M_{1/2}, m_0, A_0, tan\beta, sign(\mu)$ but our fit for the fermion masses was carried out for the $tan\beta = 55$ so we will stay with this value. In the FIG.1 one can see dependence of the branching ratios on the $M_{1/2}$ for the fixed values of $m_0, tan\beta, A_0, sign(\mu)$. We note that branching ratio for $\mu \rightarrow e + \gamma$ for almost the entire parameter range of our model is above $10^{-13}$ a value which is in the accessible range of the ongoing MEG experiment[13].
Table 2. Current bounds on $l_i \to l_j \gamma$ processes

| Process          | BR       |
|------------------|----------|
| $\mu \to e \gamma$ | $1.2 \cdot 10^{-11}$ |
| $\tau \to \mu \gamma$ | $6.8 \cdot 10^{-8}$ |

Figure 1. Branching ratios $BR(\mu \to e \gamma)$ for different values of $m_0$ (black 300 Gev, green 400 Gev, blue 500 Gev), $A_0 = 0$, $tan\beta = 55$, sign($\mu$)=1

6. Comments

We add a few comments on the model described before closing:

(i) In this model, the leading order proton decay operator is $\psi^4 A_2^2$. After GUT symmetry breaking this leads to the effective strength $\lambda \sim \frac{M_2^2}{M_{Pl}^2}$. Naively this is of order $2 \times 10^{-5}$, bigger than the present upper limit but is a considerable improvement in the naturalness. It could also be that the GUT vev could arise mainly from $A_1$ with $\langle A_2 \rangle$ being an order of magnitude smaller. This would then give the desired suppression to proton decay. In that case this will be the dominant graph for proton decay. Note that there are no Higgsino mediated diagrams for proton decay in this model. In addition, there is the gauge exchange diagram, present in all SO(10) GUT models.

(ii) The $\mu \to e + \gamma$ appears to be the only other low energy test of the model which is similar to such models.

(iii) For the choice of parameters used in fermion mass fitting the neutrino mixing angles and mass differences could have any values in the allowed region.

7. Conclusion

In conclusion, we have presented the minimal SO(10) $16$-Higgs model for fermion masses where the problem of extreme fine tuning of higher dimensional Planck scale induced proton decay operators has been considerably ameliorated by the presence of discrete symmetries so that in the end, we only need to tune down the coupling only by a factor of $10^{-2}$. In this sense it is a more natural model. We exhibited a fit to all fermion masses and mixings including neutrinos in this model to show that it can indeed be a realistic description of nature.
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