Analysis of drift correction in different simulated weighing schemes

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Abstract. In the calibration of high accuracy mass standards some weighing schemes are used to reduce or eliminate the zero drift effects in mass comparators. There are different sources for the drift and different methods for its treatment. By using numerical methods, drift functions were simulated and a random term was included in each function. The comparison between the results obtained from ABABAB and ABBA weighing series was carried out. The results show a better efficacy of ABABAB method for drift with smooth variation and small randomness.

1. Introduction

When the accuracy limit of a weighing equipment is reached, for example in highest accuracy mass standards calibration, mass comparators (high resolution short range electronic balances) often exhibit drift of zero (nonzero indication when there is no load on the weighing plate) which adds up to weighing indications. This drift is caused by influence of several factors such as self-heating and environmental conditions changes. This zero drift can by far outweigh the comparator’s last significant digit. In an ultramicro mass comparator with 0.1 microgram resolution may add up to tens of micrograms in balance indication during a calibration weighing series taking typically about 20 to 30 minutes long. Thus, drift would become an important source of uncertainty if not properly treated.

Calibration of high accuracy mass standards is based on the determination of the mass difference between a test standard (object) and a reference standard. In the zero drift treatment comparative weighing cycles are widely used, as an example we can mention the ABA and ABBA (A = reference standard weighing, B = test standard weighing). These weighing schemes assume that within a relatively short time interval the drift has a linear shape, and the average of the differences between the final and initial weighings within each cycle realized at equal time intervals (or symmetrical), can eliminate the zero drift in the balance indicator. Experimentally the drift shows numerous behaviors and are best defined as a sum of some functions, but in a limited period of time we can recognize linear, power, or periodic (i.e. sine, cosine) behavior and even of random nature.

In this work we apply simulated drifts using functions of some nature: linear, quadratic, cubic, sine and to all of them it was added a random term in order to compare their performance (effectiveness) in the reduction or elimination of the drift effects arising from weighing equipment. We also present results from real measurements obtained by applying the weighing schemes in weighing series using the mass comparator Mettler-Toledo AT1006.

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2. Drift in weighing cycles

In order to determine the conventional mass of standard weights [1], calibration laboratories perform comparative weighing cycles of test standard against a reference standard [2]. Such weighing cycles are intended to reduce or eliminate errors due to systematic effects not correctable, usually environmental, which act during individual weighings of the standards. These errors can be considered as an additive term in the mathematical model for the indication of the scale \( I \) [3] according to equation (1).

\[
I = k \cdot g \cdot \left\{ m_c \cdot \left[ \frac{\left(1 - \frac{\rho_0}{\rho_c}\right)}{\left(1 - \frac{\rho_0}{\rho(T)}\right)} \right] \right\} + \delta_T + \delta_M + \delta_E + \delta_S + \delta_X + \delta_R + \delta_N + \delta_A + f(t) \tag{1}
\]

Where:
- \( k \) \quad \text{adjust constant;}
- \( g \) \quad \text{local gravity acceleration;}
- \( m_c \) \quad \text{conventional mass of the standard weight;}
- \( \rho_0 \) \quad \text{conventional air density (1.2 kg/m}^3\text{);} 
- \( \rho_c \) \quad \text{conventional density of the standard weight (8000 kg/m}^3\text{);} 
- \( \rho_a \) \quad \text{air density during weighing;}
- \( \rho(T) \) \quad \text{density of standard weight, at temperature } T \text{ of the standard weight;}
- \( \rho(20°C) \) \quad \text{density of standard weight at reference temperature 20 °C;}
- \( \delta_T \) \quad \text{error due to thermal effects;}
- \( \delta_M \) \quad \text{error due to magnetic effects;}
- \( \delta_E \) \quad \text{error due to electrical effects;}
- \( \delta_S \) \quad \text{error due to surface effects;}
- \( \delta_X \) \quad \text{eccentricity error due to the standard weight on the weighing plate;}
- \( \delta_R \) \quad \text{resolution error of balance indication;}
- \( \delta_N \) \quad \text{nonlinearity error of the balance;}
- \( \delta_A \) \quad \text{random error;}
- \( f(t) \) \quad \text{zero drift of balance indication at weighing time } t.

The mathematical model for the indication difference \( \Delta I \) of weighing of a reference standard and a test standard with similar densities, under stable environmental conditions, can be determined from equation (2).

\[
\Delta I = k \cdot g \cdot \left\{ m_{cob} - m_{cp} \right\} \cdot \left[ \frac{\left(1 - \frac{\rho_0}{\rho_c}\right)}{\left(1 - \frac{\rho_0}{\rho(20°C)}\right)} \right] \right\} + \epsilon_{T,M,E,S,X,R,N,A} + f(t_2) - f(t_1) \tag{2}
\]

Where:
- \( \epsilon_{T,M,E,S,X,R,N,A} \) \quad \text{difference between the errors } \delta \text{ in the indications difference of the object and the standard;}
- \( f(t_2) \) \quad \text{balance drift in the object weighing at } t_2;
- \( f(t_1) \) \quad \text{balance drift in the object weighing at } t_1.

While the effects due to systematic errors would be constant and with null true value [4], the difference between the drift in weighings could be non-zero. In fact, considering that the balance indications, even in unloaded condition, vary due to environmental effects and the heating of the balance in different times, the drift value would not be the same. With the purpose of eliminating the
drift effect on the indication differences, several weighing cycles types can be applied [5]. At Inmetro's Mass Laboratory weighing cycles ABBA and ABABAB are routinely used, both are effective when the drift is linear.

The condition of equal time intervals between weighings applies to both weighing cycles. Typical results obtained in ultramicro comparator, with the application weighing series (cycles) ABABAB and ABBA are shown respectively in figures 1 and 2. The assumption of linear drift in equal time intervals might not be true for any type of curve drift.

Thus, in order to evaluate the efficacy in reducing or eliminating drift of the weighing in ABBA and ABABAB sequences, we used simulated drift curves with linear, quadratic, cubic, sine shapes with and without random term. The indication differences being determined for each weighing series in accordance with equations (3) and (4).

**Figure 1.** Upper: Test and Reference weighings. Lower: drift corrected differences B – A for weighing cycles ABABAB
3. Drift simulation method
Two mass standards, A and B, were used to perform the zero drift simulation. The mass difference was set as being 0.2 (in mass arbitrary units). The reference standard mass value, $M_A$, was assigned equal to zero. The test standard, B, has been assigned the mass value, $M_B$, equal to $M_A$ plus the amount of 0.2 mass units. Thus when there exists an exact cancellation of the balance drift, then the comparative weighing series will result in the mass difference value equal to 0.2.

The simulations are divided into two groups. In the first, simulated drift curves are continuous functions [6], where the dependent variable is the balance indication and the independent variable is time, the latter represented by the ordered number of the respective weighing. Regarding that each

\[
\Delta I_{ABBA} = \frac{(I_{b2} - I_{a2}) + (I_{b1} - I_{a1})}{2}
\]

(3)

\[
\Delta I_{ABABAB} = \begin{cases} 
I_{b1} - \frac{(I_{a3} + I_{a2})}{2} & \text{para a ABA} \\
\frac{(I_{b2} + I_{b3})}{2} - I_{a3} & \text{para a BAB} 
\end{cases}
\]

(4)

Where:

$I_b$ balance indication for Test standard weighing $i$;

$I_a$ balance indication for Reference standard weighing $i$;

$i$ weighing order, corresponds to a constant time interval.
weighing takes place at equal time intervals. In the second group it was added a random term $A(t)$ to the drift functions.

The random term has a rectangular probability distribution of width 1, which can be modulated by a multiplier factor. From these drift curves the weighing points are obtained in each of the ABBA and ABABAB weighing schemes for direct comparison of simulated results. In order to study the drift correction of random nature only, simulations were carried out with null drift, just acting the random term. A rectangular distribution and a Gaussian distribution for a series of up to one million repetitions were simulated.

### 3.1 Drift functions

To simulate in a more comprehensive way the types of drift that may arise during the mass comparison process we used the drift functions shown in table 1, and the graphs of these functions are shown in figures 3 and 4.

| Function | Drift equation | Parameters |
|----------|----------------|------------|
| Linear   | $I = K \cdot N + C + M_{A,B} + A(t)$ | $K = 1.6 \quad C = -1.6$ |
| Quadratic| $I = K \cdot N^2 + C + M_{A,B} + A(t)$ | $K = 0.06 \quad C = -0.06$ |
| Cubic    | $I = K \cdot N^3 + C + M_{A,B} + A(t)$ | $K = 0.002 \quad C = -0.002$ |
| Sine     | $I = K \cdot \sin(\pi \cdot C \cdot (N - 1)) + M_{A,B} + A(t)$ | $K = 30 \quad C = 0.05$ |

Where,

- $I$ balance indication;
- $K$ adjust coefficient;
- $C$ adjust constant;
- $A(t)$ random variable with adjustable amplitude;
- $N$ weighing order (time of weighing);
- $M_{A,B}$ mass value. Zero mass units for standard $M_A$ and 0.2 mass units for object mass $M_B$.

The simulations were performed in an Excel spreadsheet. As these are generic functions, the factors $K$, $C$ and $M$ were chosen to reproduce behaviors close to those observed in real measurements. But even with the simulation including other factors relative behaviors remain stable. Only for values $A(t)$ greater than the mass difference ($B - A$) the drift functions are outweighed by the random term making equivalent the ABBA and ABABAB procedures.

In figure 3 it can be seen that the maximum drift range was chosen to approximately 40 mass units. Even in this drift range both weighing schemes can reduce the zero drift effects of the mass comparator. In figure 4 are shown the same functions as in figure 3, but with the addition of a random term (in figure 4 the oscillations due to random factor were amplified by a factor 5 in order to be more easily seen).
**4. Results**

The weighing simulation was performed for 24 virtual weighings which correspond to a typical measurement of E₁ accuracy class standards. This number corresponds to 6 weighing cycles in ABBA scheme and to 8 cycles ABA and BAB. The simulations results of the weighing sequences are shown in table 2.
Table 2. Simulations results

| Drift      | $\Delta m_{\text{ABBA}}$ | $\Delta m_{\text{ABABAB}}$ | $\sigma_{\text{max}}$ | $\Delta m_{\text{ref}}$ |
|------------|---------------------------|----------------------------|-----------------------|------------------------|
| Linear     | 0.200                     | 0.200                      | 0.000                 | 0.200                  |
| Quadratic  | 0.008                     | 0.200                      | 0.064                 | 0.200                  |
| Cubic      | 0.050                     | 0.209                      | 0.091                 | 0.200                  |
| Sine       | 0.567                     | 0.211                      | 0.339                 | 0.200                  |

The $\Delta m$ values correspond to the mass difference of each weighting scheme, $\Delta m_{\text{ref}}$ is the theoretical reference value and $\sigma_{\text{max}}$ is the maximum standard deviation obtained from two weighing schemes. Results indicated that both methods are suitable to remove the purely linear drift. The ABABAB scheme is still accurate for the elimination of quadratic drift. In the case of cubic and sine drifts both schemes do not completely eliminate the drift effects but ABABAB scheme was more effective in reducing the drift effects.

These results remain the same even if the random term is included, since the variation is typically less than the mass difference to be measured. Thus for a mass difference of 0.2 arbitrary mass units, a random variation in the interval $[-0.1, 0.1]$ was chosen and the obtained results are shown in table 3.

Table 3. Simulations results with random term

| Drift + $A(t)$ | $\Delta m_{\text{ABBA}}$ | $\Delta m_{\text{ABABAB}}$ | $\sigma_{\text{max}}$ | $\Delta m_{\text{ref}}$ |
|----------------|---------------------------|----------------------------|-----------------------|------------------------|
| Linear         | 0.214                     | 0.191                      | 0.008                 | 0.200                  |
| Quadratic      | 0.069                     | 0.199                      | 0.063                 | 0.200                  |
| Cubic          | 0.019                     | 0.182                      | 0.091                 | 0.200                  |
| Sine           | 0.602                     | 0.199                      | 0.339                 | 0.200                  |

In a linear drift with random term acting simultaneously there is an important difference between the analyzed weighing schemes. Again the ABABAB scheme was superior to ABBA scheme for reducing the drift effects, even considering the maximum standard deviation $\sigma_{\text{max}}$, the mass differences values quadratic drift, cubic and sine, obtained by ABBA scheme does not reach the 0.2 difference value. Even the ABBA scheme providing lower standard deviations than the ABABAB scheme, the final results were out of the theoretical value to be obtained, 0.200 mass unit.

4.1 Isolated random term effect
The simulations with random term and without drift, have inconclusive results for 24 weighing series.

Table 4. Random drift effect (arbitrary units)

| Distribution | $\Delta m$ | $\sigma$ | ABBA     | ABABAB    | Average   |
|--------------|------------|----------|----------|-----------|-----------|
| Retangular   | -0.0001    | 0.0482   | -0.0002  | 0.0510    | 0.0240    |
| Gaussian     | 0.0108     | 0.2142   | 0.0052   | 0.2151    | 0.1127    |
Thus, series of $1 \times 10^6$ repetitions of 24 weighings were performed with a rectangular distribution and a Gaussian distribution with adjustable width (range). In both cases the results indicate that there is no difference in applying the ABBA and ABABAB schemes because indication differences always remain one to two orders of magnitude smaller than the standard deviations involved in the simulation.

As an example, some results of simulations with $1 \times 10^6$ random repetitions are shown in table 4, where the results correspond to width range equal to 1.

### 4.2 Experimental data analysis

For a comparative analysis with real measurements three sets of display readings were performed using mass comparator Mettler-Toledo AT1006. The reading of the comparator indications are given without the use of mass standards (empty plate), with the mass comparator only performing weighing operation (with internal weights) and under the influence of the equipment drift and the environment conditions.

The series of 180 weighings are shown in figures 5, 6 and 7. In the first sequence, it was obtained an average value of $0.0002 \text{ mg}$ from the 180 weighing indications. By applying the ABBA weighing scheme the result of the elimination of drift is $0.0000 \text{ mg}$, meanwhile the ABABAB scheme resulted in $0.0002 \text{ mg}$ with their respective standard deviations of $0.00035$ and $0.00044 \text{ mg}$.

![Figure 5. AT1006 weighing indications](image)

The balance resolution is 1 microgram, but by performing the integration of the last digit for 10 seconds it gets an effective resolution of 0.1 micrograms.

![Figure 6. AT1006 weighing indications](image)
In the second sequence, figure 6, the average measures resulted in 0.0029 mg. In ABBA scheme it was obtained 0.0001 mg and in ABABAB scheme 0.0007 mg, with standard deviations, respectively, 0.00039 and 0.00039 mg. In the third sequence of 180 weighings, figure 7, it was obtained an average of 0.0014 mg, the ABBA scheme results 0.0001 mg and 0.0007 mg in ABABAB, with standard deviations respectively 0.00035 and 0.00037 mg.

In this case, unlike the simulation, do not have an exact value to be achieved. Because it is a zero drift measurement it is expected that, in a sufficiently large time interval (infinite measurements), a null average value will be obtained, but being an empty plate measurement thus zero is the value which is to be expected as mass difference. In a short time, the average value of the measurements cannot be taken as the expected value. The drift may occur in such a way that the average value does not correspond to the null value as would be expected in a system with random zero drift behavior. ABBA and ABABAB schemes are designed in order to, under the specified conditions, reduce the drift effects. So by weighing without a test object on the balance weighing plate, both weighing schemes should get null value for the mass difference.

In the first weighing, both weighing schemes were equivalent because the null value obtained was within the standard deviation. In the second and third sequences the ABABAB remained more than one standard deviation away from one standard deviation, while the ABBA scheme achieved a result within the standard deviation.

5. Conclusion
According to the results, both cycles eliminated the drifts of linear nature, when a random nature variation is added to the linear behavior of both weighing schemes are still equivalent in treatment. In the drifts of quadratic nature the elimination of effects is achieved only in the ABABAB cycle.

In cubic and sinusoidal drifts the ABABAB advantage occurs at the same rate, with or without the addition of the random term, provided that the last one have amplitude lesser than the mass differences. Despite the ABABAB cycle has a disadvantage for providing results with slightly larger standard deviations [7, 8, 9] than the ABBA cycle, this disadvantage is offset by better results from differences in indication for the studied theoretical cases.

The theoretical identification of this advantage in mass difference results, in case of real weighings, given the small number of comparison cycles, the drift magnitude and its variations are more complex functions than the ones studied here, also in tests with real weighings (just a few cycles) the differences of the achieved results in both weighing schemes are very close to each other, making it inconclusive the assessment of the effectiveness of both treatments for real measurements.

Figure 7. AT1006 weighing indications
The random drift analysis shows that there is no advantage for any of the studied comparative weighing schemes. In tests with one million simulated weighing results both approaches eliminated the effects of the random nature drift. When the effects of random term are amplified, the behavior between the ABBA and ABABAB schemes approaches the purely random behavior, that is because the random fluctuations dominate the character of the chosen drift function.

In this work it was identified differences in the zero drift elimination characteristics and the calculated standard deviations between comparison weighing schemes, ABBA and ABABAB, but there is still room to study these schemes with more complex functions, function combinations and real measurements.

6. References

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