Testing spontaneous wavefunction collapse with quantum electromechanics

Germain Tobar, Stefan Forstner, Arkady Fedorov and Warwick P Bowen

1 Australian Research Council Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, St. Lucia, QLD 4072, Australia
2 Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
3 ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, Castelldefels (Barcelona) 08860, Spain

* Author to whom any correspondence should be addressed.
E-mail: g.tobar@uq.net.au

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Abstract
Theories of spontaneous wavefunction collapse offer an explanation of the possible breakdown of quantum mechanics for macroscopic systems. However, the challenge of resolving predicted collapse signatures above background noise has precluded conclusive tests. Here, we propose to overcome this challenge using quantum control and measurement of a superconducting qubit coupled to a macroscopic mechanical resonator. We show that this can amplify the weak signals from collapse-induced heating and simultaneously suppress qubit noise, initializing the qubit close to its ground state. Combined, this could enable a conclusive test of collapse models. The ability to quantum control macroscopic mechanical resonators and resolve extremely small signals from them could further other fundamental research beyond collapse models, such as laboratory-based dark matter searches and the reconciliation of quantum mechanics with gravity.

1. Introduction

Despite much recent experimental progress on exploring the quantum-to-classical transition [1–3], there is still no widely accepted explanation as to how classical realism emerges for macroscopic systems. Collapse models provide an experimentally testable solution [4], postulating that objective collapse of the wavefunction occurs for sufficiently large quantum systems. The most widely studied of these is Continuous Spontaneous Localization (CSL) [4].

Thought experiments provide lower bounds on the rate of CSL-induced collapse $\lambda_c$ at a given correlation length $r_c$ [5, 6]. Upper bounds are set through experimental tests that search for deviations from quantum theory. These tests fall into two categories: interferometric tests, which aim to produce macroscopic quantum superpositions in order to probe the breakdown of the quantum superposition principle [2, 4, 7–12]; and non-interferometric tests, which aim to observe spontaneous heating as an unavoidable side effect of the collapse process [13, 14]. Given the challenge of isolating macroscopic quantum superpositions from environmental decoherence, the strictest tests have so far been performed non-interferometrically using mechanical resonators [15–18]. However, the inability to resolve collapse-induced heating above thermal noise has prevented any conclusive test of CSL up to Bassi’s predicted lower bound of $\lambda_c = 10^{-12}$ s$^{-1}$ at an expected correlation length of $r_c = 10^{-7}$ m [5, 6, 19]. While optomechanical systems operating at low temperatures and high frequencies have been proposed as a means to circumvent thermal noise, currently available optomechanical coupling strengths are too low for CSL heating to be resolved [19].

In this work, we propose the use of electromechanical systems for non-interferometric tests of CSL. In our approach a macroscopic mechanical resonator is coupled to a superconducting qubit. This brings new quantum control and measurement capabilities to non-interferometric tests. We exploit these capabilities to
both amplify the collapse signal and suppress measurement noise. The CSL heating of the mechanical resonator is read out by swapping its state onto the qubit and performing a quantum non-demolition (QND) measurement of the qubit [20–23]. We propose a pulsed scheme, where the mechanical resonator and qubit are strongly coupled to perform the swap, but only for short periods. This allows CSL heating to build up in the resonator prior to the swap operation, amplifying the CSL signal. Somewhat remarkably, an appropriate sequence of swaps with an array of mechanical resonances can, in parallel, greatly suppress qubit noise, cooling the qubit to close to its ground state prior to each swap.

Our proposal unlocks the capability measure extremely low phonon numbers in heavy mechanical resonators. While we benchmark our proposal on CSL, it could allow to probe a range of collapse models [4] and be applicable to laboratory-based searches for dark matter and other exotic particles [24–26]. Such experiments could have implications on fundamental questions such as whether quantum mechanics applies at all scales, whether gravity is quantized [27–29], and how the structure of the observable Universe is formed [30].

As a specific example implementation of a spontaneous collapse test, we consider a charge qubit coupled to an array of internal bulk acoustic wave (BAW) resonances [20–22]. We perform an extensive noise analysis, finding that quasiparticle heating of the qubit [31–33] is likely to be the dominant noise channel for this system. We predict that our protocol can suppress this noise by three orders of magnitude. Moreover, we find that the ultra-low dissipation achievable in BAW resonators significantly amplifies the expected CSL occupancy. The combination of noise suppression and signal enhancement is sufficient to close the gap between the measured upper and calculated lower bounds on the collapse rate and therefore conclusively test the CSL model. Furthermore, the access to natural arrays of mechanical resonators in a BAW resonator allows this to be achieved in a matter days, rather than the months or years of most previous proposals [19].

2. Scheme

Our scheme utilises Jaynes–Cummings coupling to swap excitations between a mechanical resonator and a superconducting qubit (figure 1) described by the standard Jaynes–Cummings Hamiltonian [20]

\[ H = \sum_{l,m} g_{l,m} \left( \sigma_+ b_{l,m} + \sigma_- b_{l,m}^\dagger \right), \]

where \(g_{l,m}\) is the coupling strength to the \(l\)th longitudinal and \(m\)th transverse mechanical mode, \(\sigma_+\) is the raising operator for the qubit, and \(b_{l,m}^\dagger\) is the creation operator for the \(l, m\) mechanical mode.

This serves the dual purposes of allowing the qubit to be used to detect the collapse-induced excitations of the mechanical resonator, and cooling it towards the typically lower temperature of the mechanical resonator. To increase the collapse signal relative to qubit noise, we consider a pulsed scheme in which collapse-induced excitations are allowed to build up in the resonator while it is off-resonance with the qubit. The qubit and the mechanical resonator are decoupled until the population of the mechanical resonator reaches equilibrium. The qubit is then tuned on-resonance with the first mechanical resonator, for instance, by flux tuning its frequency [34].

Maintaining resonance for only the duration of a single swap operation transfers qubit excitation that would otherwise be a significant source of noise into the resonator while minimising the time over which qubit-related decoherence can occur. After this swap process, the state of the qubit is measured via a dispersive QND measurement. For instance, let us envisage that a collapse induced phonon exists within the resonator at the same time as a thermal excitation of the qubit. The operation swaps these, so that in the ideal limit the thermal excitation is dumped in the mechanical resonator, to eventually decay into environment, while the CSL phonon is perfectly converted into a qubit excitation to be measured.

Using an array of frequency non-degenerate resonators, our scheme can be repeated in rapid succession, cooling the qubit beyond what can be achieved with a single swap operation and multiplying the rate of phonons transferred to the qubit by the number of mechanical resonators. Ultimately, a given CSL collapse rate can be ruled out if it predicts a higher CSL phonon flux than is observed for the relevant correlation length.

While our protocol could be applied using a variety of electromechanical systems [20, 21, 35], we consider electromechanical coupling between resonant BAW and a superconducting qubit. We choose this example because strong Jaynes–Cummings coupling has been demonstrated in such a system using a piezoelectric interaction [20, 21], BAW resonances can easily reach frequencies at which thermal noise is frozen out at
cryogenic temperatures, the high number of mechanical modes within the tunable frequency range of the qubit provides a natural array of mechanical resonators, and because BAW resonances also offer the potential for very high collapse induced heating rates at relevant collapse correlations lengths $r_c$. As we will show in what follows, this arises due to the close matching of their wavelength (at typical GHz frequencies) to the expected collapse correlation length ($r_c = 10^{-7}$ m), and to their ultra-low acoustic dissipation.

As a specific example, we consider a qubit-coupled mechanical resonator based on [20]. We proceed by calculating the specifications for a Silicon Carbide (SiC) BAW resonator, due to their ultra-high Q factors and suitability for interfacing with qubits [36]. We choose a resonator thickness of 30 $\mu$m, for which the BAW mode with longitudinal and transverse mode numbers $l = 40$ and $m = 0$, respectively, has a frequency $\omega/2\pi = 6.33$ GHz, within the 4–9 GHz tunable frequency range of the qubit [37]. This corresponds to a phonon wavelength of $\lambda \approx 15 \times 10^{-7}$ m, which is sufficiently close to the expected correlation length to maximise the CSL heating rate of the resonator at this correlation length (see appendix A).

3. CSL phonon flux

For a single resonance, the phonon flux predicted by CSL is quantified as $\dot{n}_c = \lambda_c D$ [19], where $D$ is the geometry dependent CSL cross-section. Previous calculations of the CSL cross-section have mainly focussed on resonators for which the centre-of-mass motion is dominant [15–17, 38]. A fundamental breathing mode has also been considered, taking the approximation of a uniform linear expansion [19]. To allow accurate predictions for BAW resonances, we go beyond these approximations, modelling CSL heating up to quadratic expansions of the acoustic modes shape and for higher order longitudinal modes. We find that at a fixed frequency, lower longitudinal mode numbers imply a higher sensitivity to the collapse-induced heating. This is because the CSL phonon flux decreases with the size of the BAW resonator at a correlation length that matches the BAW wavelength. The reason is that as originally shown in [13], for optomechanical systems in the presence of noise, the CSL signal-to-noise ratio surprisingly does not increase with the mass of the system, despite the CSL-Linblad term’s dependence on mass. This is because that while the CSL diffusion rate scales at most linearly with mass, the noise also scales linearly with mass [13]. Therefore, thinner BAW resonators are more optimal for tests of CSL.

We find the CSL cross-section at the expected $r_c$ to be $D \approx 5.5 \times 10^5$ (see appendix A). For the lower bounds to the collapse rate postulated by Adler [5] and Bassi et al [6], it gives collapse-induced heating rates of $\dot{n}_c \approx 5.5 \times 10^{-3}$ s$^{-1}$ and $\dot{n}_c \approx 5.5 \times 10^{-5}$ s$^{-1}$, respectively. The lowest of these rates equates to the generation of one phonon in under 22 days. Other BAW modes with similar $l$ have comparable CSL cross-sections and heating rates, enabling experiments with arrays of resonances.

With the qubit off-resonance, the steady-state flux of phonons leaving the mechanical resonator is $P\gamma_r$, where $\gamma_r$ is the mechanical decay rate and $P$ is the steady-state occupancy of the BAW resonator due to CSL heating. In the absence of other forms of heating, this outwards phonon flux must equal the inwards flux from CSL, so that $P \approx \frac{2\pi \dot{n}_c}{\gamma_r}$. The highest Q-factor measured experimentally in a BAW resonator is $Q \sim 10^9$.
Quality factors as high as \( Q = 10^7 \) have been achieved in SiC BAWs at a temperature of 7 K \[36\]. We find that \( Q = 10^7 \) would be insufficient to fully exclude CSL. However, the quality factor of BAWs scales as \( T^{-4} \) at low temperatures \[36\]. Conservatively assuming only a \( T^{-1} \) scaling, it was predicted in \[36\] that values as high as \( Q = 10^{10} \) will be possible at the mK temperatures required to suppress thermal noise in our system. We choose this higher \( Q \) value for our analysis. Using \( n_c = 5.5 \times 10^{-7} \) s\(^{-1} \) for the full exclusion of Bassi \textit{et al}’s lower bounds and \( \gamma_r/2\pi \approx 1 \text{ Hz} \) \( (Q = 10^{10} \) for GHz frequency modes) for the decay rate of a SiC BAW resonator, we find that full exclusion at \( r_c = 10^{-7} \) m requires the ability to resolve occupancy’s as low as \( P \leq 5.5 \times 10^{-7} \).

To determine how efficiently the phonon occupancy can be transferred from the mechanical resonator into the qubit we model the coupling using a phenomenological Linblad master equation in appendix B. We choose parameters consistent with the experiment in \[20\], apart from the piezoelectric coupling which can be made an order of magnitude higher through the use of alternative materials (see supplementary information). One might expect that, with an appropriate choice of interaction time, a swap efficiency approaching 100% would be possible. However, we find that off-resonance coupling due to residual piezoelectric interaction with adjacent BAW modes (in combination with qubit decay and dephasing) limits the efficiency to \( \eta_{\text{swap}} \approx 0.85 \). To fully exclude Bassi \textit{et al}’s lower bound, all spurious sources of heating in the qubit must therefore be suppressed below the probability \( P_{\text{CSL}} = \eta_{\text{swap}} P \approx 4.7 \times 10^{-7} \).

### 4. Noise sources

The expected noise sources in our protocol are measurement induced heating, state discrimination error, quasiparticle poisoning and the Purcell effect. We find thermal phonons to be negligible when using BAW modes due to their high frequency (see appendix D).

#### 4.1. Measurement induced heating

We propose to measure the state of the qubit through coupling to a microwave readout cavity, as is conventional for the measurement of superconducting qubits \[41\–45\]. In the dispersive regime, where the number of photons in the resonator is much less than the critical number \( n_{\text{crit}} = \Delta^2/4g^2 \), with \( \Delta \) and \( g \) the qubit-cavity detuning and coupling strength respectively, the coupling approximates an ideal QND measurement \[45\]. In standard qubit operation, measurement-induced heating due to non-QND effects is considered significant only for photon numbers comparable to the critical number \[41\–43\]. However, for the ultra-low rate of false positives required to carry out our proposed CSL tests, even minute non-QND effects become significant. The primary non-QND effect is the measurement induced heating that occurs due to higher order terms in \( g/\Delta \) in the dispersive approximation \[41\]. Taking these higher order terms into account, we find that for experimental parameters achieved in recent experiments \[46\] it is possible to suppress measurement induced heating to below \( P_{IJ} = 8 \times 10^{-8} \) (see appendix E).

#### 4.2. State discrimination error

Dispersive readout encodes information about the state of the qubit on the phase of the scattered microwave signal. A phase threshold is then used to discriminate whether the qubit was in its excited or ground state. The probability of misidentifying the qubit state is known as the state discrimination error \( \epsilon_{\text{disc}} \) \[47\]. Unlike quantum computing, where errors in assigning the state of the qubit are generally equally important, for the purpose of distinguishing collapse-induced phonons it is much more important to suppress false positives than false negatives. This motivates the choice of a low discrimination threshold (which we denote \( a \) (see supplementary information)), so that noise alone is unlikely to cause a false positive. However, too high a threshold will also reduce the rate of correctly identifying the qubit in the excited state (true positives), increasing the required duration of the experiment. We find that with an appropriate choice of discrimination threshold it is possible to exponentially suppress the error in false positives to well below \( P_{\text{CSL}} \approx 10^{-7} \), while maintaining a reasonable probability of detecting true positives (see supplementary information). For example, the error in false positives can be suppressed to \( 10^{-7} \) while correctly identifying CSL-heating events with probability \( \eta_{\text{disp}} = 0.1 \), so long as the microwave resonator can distinguish the excited state of the qubit from its ground state with a signal-to-noise (SNR) ratio of at least 8.
4.3. Purcell and quasiparticle heating

The Purcell effect arises from the enhancement of the qubit’s spontaneous excitation rate due to its coupling to the readout cavity, which is in turn coupled to a transmission line. It has been shown experimentally to produce qubit excitations at a rate of around $\Gamma_p/2\pi \approx 0.5$ Hz. Quasiparticle heating occurs due to hot out-of-equilibrium quasiparticles tunnelling across the qubit Josephson junctions into the superconducting island. This causes the qubit to transition to its excited state with a phenomenological excitation rate $\Gamma_{QP}$ [33, 44]. Recent measurements have found that $\Gamma_{QP}/2\pi \approx 300$ Hz [44], while quasiparticle traps have been shown to suppress quasiparticle excitations by more than an order of magnitude [48–50]. Therefore, we take the lowest decoherence rate achievable with current technology to be $\Gamma_{QP}/2\pi \approx 30$ Hz.

Taking $\Gamma_{QP}/2\pi = 30$ Hz and $\Gamma_p/2\pi = 0.5$ Hz, the equilibrium population of the qubit due to quasiparticle and Purcell heating is $P_s \approx \Gamma_{QP} + \Gamma_p \approx 10^{-3}$, where $\gamma_q/2\pi = 27$ kHz is the qubit dissipation rate [20]. This population is orders-of-magnitude higher than the steady-state CSL occupancy, suggesting that conclusive tests of CSL may not be possible. However, this is not the case in our proposal due to the build-up of the collapse-induced heating within the mechanical resonator, and because each interaction between the qubit and a mechanical resonance transfers heat out of the qubit, to be dissipated into the mechanical bath.

In the experimentally relevant regime where the mechanical resonator has a high Q-factor ($\omega_{l,m} \gg \gamma_r, \gamma_q$), the dynamics of the system are described with the phenomenological Lindblad master equation [51]

$$
\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \gamma_q D[\sigma_-] \rho + (\Gamma_{QP} + \Gamma_p) D[\sigma_+] \rho + \frac{\gamma_c}{2} D[\sigma_z] \rho + \sum_{l,m} \left[ \gamma_l (1 + \bar{n}_l) D[b_{l,m}] \rho + (\gamma_r \bar{n}_l + \bar{n}_l) D[b_{l,m}^\dagger] \rho \right],
$$

which accounts for all known sources of decoherence. $H = \sum_{l,m} g_{l,m} \left( \sigma_+ b_{l,m} + \sigma_- b_{l,m}^\dagger \right)$ is the standard Jaynes–Cummings Hamiltonian (see supplementary information), $\rho$ is the density matrix, $\bar{n}_l = \left( e^{\hbar \omega_{l,m} / k_B T} - 1 \right)^{-1}$ is the mechanical mean thermal occupancy (where $k_B$ is the Boltzmann constant and $T$ is the temperature of the surrounding thermal environment), $D$ is the dissipating superoperator $D[L] \dot{\rho} = L \rho L^\dagger - \frac{1}{2} (L^\dagger L \dot{\rho} + \dot{\rho} L^\dagger L)$.

The subscripts $l,m$ refer to the $l,m$ mechanical modes, which are for simplicity, assumed to have equal dissipation rates $\gamma_r$, $\Gamma_{QP}/2\pi = 30$ Hz and $\Gamma_p/2\pi = 0.5$ Hz are the heating rates in the qubit due to quasiparticle poisoning and the Purcell effect respectively, while $\gamma_q/2\pi = 0.3$ MHz is the qubit dephasing rate [42] and $\gamma_q/2\pi = 27$ kHz is the qubit dissipation rate [20].

In order to calculate how effectively our protocol can suppress the Purcell and quasiparticle heating, we solve the qubit-resonator Lindblad master equation in equation (2) to simulate a series of swap operations between a qubit with initial state

$$
\rho(0) = \frac{\Gamma_{QP} + \Gamma_p}{\gamma_q} \left| e \right> \left< e \right| + \left( 1 - \frac{\Gamma_{QP} + \Gamma_p}{\gamma_q} \right) \left| g \right> \left< g \right|
$$

and an array of mechanical resonances. In equation (3), $\left| g \right>$ denotes the ground state of the qubit, and $\left| e \right>$ denotes its excited state. As further discussed in the supplementary information, we find that significant cooling is possible, consistent with recent experiments [20, 21]. In the strong-coupling regime, for which $g_{l,m} > \gamma_q$ (where $g_{l,m}$ is the coupling strength to the $l,m$ mechanical mode), we find that the temperature can be reduced by a factor of $\frac{\Gamma_{QP} + \Gamma_p}{\gamma_q}$ in the limit of a large number of operations.

While one would naively expect a single swap operation to make the qubit occupancy equal to that of the last resonator, it is important to account for additional processes which reduce the fidelity of the swap operation. An example of such a process is qubit dephasing. As shown in figure 2, qubit-dephasing contributes to an imperfect swap operation, preventing all of the noise in the qubit from being transferred to the mechanical resonator in a single swap operation. Therefore, multiple swap operations will be required to sufficiently cool the qubit to allow a subsequent swap operation to transfer any CSL signal in the mechanical resonator into the qubit occupancy.

The qubit in [20] is well within the strong-coupling regime with $g_{l,m}/\gamma_q = 10$. Using their parameters, our previously assumed order of magnitude in coupling strength, and $(\Gamma_{QP} + \Gamma_p)/2\pi = 30$ Hz, we find the qubit can be cooled by a factor of $\sim 10^3$, to $P_s \approx 7 \times 10^{-7}$ (see supplementary information). As shown in figure 2, our simulations suggest that only one or two swap operations will be required to approach this temperature, depending on the qubit dephasing rate. Together with the build-up of CSL heating within the mechanical resonator, this provides a three order of magnitude total suppression of the Purcell and quasiparticle heating.
Table 1 displays a quantitative summary of all noise sources in the qubit-resonator system. Their sum produces a spurious qubit excited state population of $P(\{\bar{e}\}) \approx 9 \times 10^{-7}$. Equating this to the CSL-induced excitation probability of $P_{\text{CSL}} \approx 2\pi \lambda_c D\eta/\gamma_r$, yields a minimum testable collapse rate of $\lambda_{c,\text{min}} \approx 1.9 \times 10^{-12} \text{s}^{-1}$ at the expected correlation length ($r_c = 10^{-7} \text{m}$). Our analysis predicts that the dominant noise source is likely to arise from quasiparticle poisoning. While still in its infancy, techniques to suppress this are under active study since it is a key noise source in superconducting quantum computing. Early techniques have already proven to be highly effective [52], and variety of recent proposals have been made to achieve further suppression [48, 53–55]. Suppression of one order of magnitude beyond the current state of the art would reduce the quasiparticle heating to $\Gamma_{\text{QP}}/2\pi \approx 3 \text{Hz}$, corresponding to $\lambda_{c,\text{min}} \approx 5.3 \times 10^{-13} \text{s}^{-1}$.

Figure 3 shows the CSL exclusion region as a function of correlation length. It includes the upper bounds on the collapse rate that our proposal would achieve with existing technology (solid orange line), as well as the upper bounds from previous experiments and Bassi et al’s and Adler’s lower bounds. As can be seen, with the realistic parameters chosen, our proposal has the potential to almost fully close the gap allowing a conclusive test of Bassi et al’s lower bound [18, 19]. With further suppression of quasiparticle poisoning, or improved BAW quality factors, it could extend the upper bounds beyond Bassi et al’s lower bound [18, 19].

6. Measurement time

Bassi et al’s lower bound corresponds to the minimum proposed level of CSL heating that is consistent with the observed classical behaviour of macroscopic systems [6]. For it, the average time until one collapse-induced excitation is observed from a single BAW resonance would be $T_{\text{meas}} = (\hbar/\eta_{\text{swap}} \cdot \eta_{\text{disp}})^{-1} \approx 2.1 \times 10^9 \text{s}$. Therefore, to conclusively rule out collapse models our experiment would need to be conducted for approximately 247 days. The use of parallel measurements on an array of $N$ mechanical resonances (in our protocol these are different modes of the same mechanical resonator) would further
reduce the required measurement time by a factor of $N^{-1}$. This scaling applies in the limit that the measurement time is shorter than the average time for one noise phonon to enter the system. The BAW resonator we model has around 35 distinct longitudinal modes in the tunable frequency range of the qubit (4–9 GHz (see supplementary information)) which if used would reduce the experiment time to about seven days. The access to modes of different frequencies may, further, allow identification of the physical origin of the collapse process and unambiguous differentiation of CSL-induced heating from noise (see appendix C). This could be achieved by measuring the expected frequency dependent CSL-heating rate predicted by coloured models [56].

7. Conclusion

We propose a test of spontaneous wavefunction collapse models using qubit-coupled mechanical resonators. The proposal offers the advantage of strong suppression of qubit noise, exquisitely precise readout enabled by the use of a qubit, and access to ultra-high quality BAW resonances that enhance the CSL signal at the expected correlation length. A significant innovation is the use of strong qubit-resonator coupling to cool the qubit. This system may allow for a conclusive test of the CSL model.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Appendix A. CSL heating rate

Here we present the expression for the CSL cross-section of a BAW resonator. For a cylindrical BAW resonator of radius $R$, density $\rho_0$, with phonon wavelength $\lambda$, longitudinal mode number $l$, zero-point motion $x_0$, after a lengthy derivation (see supplementary information), we find the CSL cross-section for a
BAW resonator to be

\[
D = \frac{2\gamma^2 v_0^2}{\alpha m u^2} \left( 64\pi^{1/2} \frac{\lambda}{\lambda} \right) \cdot \frac{\exp \left( \frac{\pi^2}{\lambda^2} \right) - 1}{2 \exp \left( \frac{\pi^2}{\lambda^2} \right)} \cdot \int_{-\infty}^{\infty} \exp \left( -r^2 \left( \frac{2\pi a}{\lambda} \right)^2 \right) \left( -8 \left( 8 + a^2 \pi^2 \right) \cos \left( \frac{\alpha \pi}{2} \right) \right)^2 \frac{1 - \cos(\pi a)}{\cos(\pi a) + 1} da. \tag{A1}
\]

Here, \(I_n\) are modified Bessel functions of the first kind.

For SiC with a density of \(\rho_0 \approx 3210\ \text{kg m}^{-3}\), and a cylindrical resonator of radius \(R \approx 35\ \mu\text{m}\) with a phonon wavelength of \(\lambda \approx 15 \times 10^{-7}\ \text{m}\), which corresponds to a mode frequency of \(\omega/2\pi \approx 6.33\ \text{GHz}\) and assuming the expected collapse noise correlation length to be \(\tau_c \approx 10^{-7}\ \text{m}\), we obtain

\[
D \approx 5.5 \times 10^5 \tag{A2}
\]

for the \(l = 40\) longitudinal mode.

It can be checked that equation \((A1)\) shows that the CSL cross-section is maximised for phonon wavelengths \(\lambda \approx 4\tau_c\). Therefore, a phonon wavelength of \(\lambda \approx 15 \times 10^{-7}\ \text{m}\) is chosen to enhance the sensitivity to collapse-induced heating, while ensuring that the mode frequency is within the tunable frequency range of the qubit.

**Appendix B. Swap operation efficiency**

In the experimentally relevant regime where the mechanical resonator has a high Q-factor \((\omega_{l,m} \gg \gamma_r, \gamma_q)\), the dynamics of the system are described with the phenomenological Linblad master equation \([51]\)

\[
\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \gamma_4 D [\sigma_-] \rho + (\Gamma_{QP} + \Gamma_{P}) D [\sigma_+] \rho + \frac{\gamma_2}{2} D [\sigma_2] \rho
+ \sum_{l,m} \left[ \gamma_r (1 + n_{h_b}) D[n_{l,m}] \rho + (\gamma_q n_{h_b} + n_{c}) D[n^1_{l,m}] \rho \right], \tag{B1}
\]

which accounts for all known sources of decoherence. \(H = \sum_{l,m} \omega_{l,m} (\sigma^+ b_{l,m} + \sigma^- b_{l,m}^\dagger)\) is the standard Jaynes–Cummings Hamiltonian (see supplementary information), \(\rho\) is the density matrix, \(n_{h_b} = (e^{\omega_{l,m}/k_B T} - 1)^{-1}\) is the mechanical mean thermal occupancy (where \(k_B\) is the Boltzmann constant and \(T\) is the temperature of the surrounding thermal environment), \(D\) is the dissipating superoperator \(D[L] = L_\rho L^\dagger - \frac{1}{2} \{L^\dagger L_\rho + \rho L^\dagger A\}\). The subscripts \(l,m\) refer to the \(l,m\) mechanical modes, which are for simplicity, assumed to have equal dissipation rates \(\gamma_{r}\), \(\Gamma_{QP}/2\pi = 30\ \text{Hz}\) and \(\Gamma_{P}/2\pi = 0.5\ \text{Hz}\) are the heating rates in the qubit due to quasiparticle poisoning and the Purcell effect respectively, while \(\gamma_{q}/2\pi = 0.3\ \text{MHz}\) is the qubit dephasing rate \([42]\) and \(\gamma_{q}/2\pi = 27\ \text{kHz}\) is the qubit dissipation rate \([20]\).

In order to solve this master equation, we assume a qubit decay rate of \(\gamma_{q}/2\pi \approx 27\ \text{kHz}\), a mechanical decay rate of \(\gamma_{r}/2\pi \approx 1\ \text{Hz}\) and a dephasing rate of \(\gamma_{q}/2\pi = 0.3\ \text{MHz}\), consistent with existing experiments \([42]\). We assume the joint system is initially in the state \(|g\rangle |n_{l,m=0} = 1\rangle\), where \(|n_{l,m} = 1\rangle\) denotes the \(l,m\) mode in the \(n = 1\) Fock state and all other modes in the ground state, and \(|g\rangle\) denotes the ground state of the qubit. Using the qubit-BAW coupling strength of \(g \approx 2\pi \times 28.9\ \text{MHz}\) (see supplementary information), we find the system has an efficiency of \(\eta_{\text{swap}} \approx 0.85\) after a single swap operation, with evidence of minor off-resonance coupling to adjacent modes shown in figure 4. The green line in figure 4 quantifies how much energy leaks into the adjacent modes during the swap operation.

One might expect that due to the pulse shaping of the qubit flux bias necessary to avoid leakage to higher qubit levels, that the timescale of the swap operation should be longer than \(g\). However, the required pulse shaping for removing the unwanted frequency components can be achieved without requiring longer interaction times. We refer to \([45]\) for a review of such techniques for fast and high fidelity qubit operations restricted to the two level subspace.
Figure 4. Occupancy probabilities for the qubit (red), BAW resonator $l = 40$, $m = 0$ mode (blue), and off-resonant BAW modes (green). The x-axis is normalised to the coupling strength $g$.

Appendix C. Coloured CSL

For coloured CSL, the CSL-induced heating rate is

$$\dot{n}_c = \lambda_c D \frac{\Omega_c^2}{\Omega_c^2 + \Omega^2}, \quad (C1)$$

where $\Omega_c$ is the high-frequency cut-off and $\Omega$ is the frequency of the mechanical resonator [6]. This equation has two unknowns: $D$ and $\lambda_c$, both of which are independent of the frequency of the mechanical resonator. Therefore, if measurements of the heating rate are made for two mechanical resonances of different frequencies, equation (C1) can be used to determine $\Omega_c$ and therefore whether a coloured CSL mode is needed. Specifically,

$$\Omega_c = \sqrt{\frac{\dot{n}_c^{(1)} - \dot{n}_c^{(2)}}{\Omega_1^2 - \Omega_2^2}}, \quad (C2)$$

where $\Omega_1$ and $\Omega_2$ are the frequencies of the two resonators and $\dot{n}_c^{(1)}$ and $\dot{n}_c^{(2)}$ are their respective measured CSL heating rates.

If the collapse noise field has a cosmological origin, it is expected that the high frequency cut-off will lie in the range $\Omega_c \sim 10^{10} - 10^{11}$ Hz [6]. Assuming $\Omega_c = 10^{10}$ Hz, if the qubit’s frequency is tuned on-resonance to a 4 GHz frequency mechanical mode, then the coloured CSL heating rate is reduced by a factor of 0.86 compared to the heating rate for white noise CSL (assuming a fixed $\lambda_c, \Omega_c, r_c$). If the qubit’s frequency is tuned on-resonance to a 9 GHz mechanical mode, then the coloured CSL heating rate is reduced by a factor of 0.55 compared to the heating rate for white noise CSL (again assuming a fixed $\lambda_c, \Omega_c, r_c$). This frequency-dependent reduction in the CSL heating rate is sufficiently large to be resolvable if sufficient statistics are obtained. Therefore, our proposed experiment has the capability to test coloured CSL models and to determine the high-frequency cut-off.

Appendix D. Thermal noise

A large thermal noise background has precluded previous experiments from resolving collapse-induced heating within the expected CSL parameter range [17, 18]. This is because these experiments have operated with low frequencies in the high temperature limit ($k_B T \gg \hbar \omega$), requiring them to resolve CSL-induced heating above this thermal noise background. This necessitates a precise knowledge of the thermal noise level, and can lead to inconsistent interpretations [17]. Here this problem is solved by accessing the regime where $k_B T \ll \hbar \omega$ [19]. Taking $\omega = \omega_q = \omega_r$ and $k_B T \ll \hbar \omega$, the thermal heating rates are $\dot{n}_{th,r} \approx \frac{\gamma_r}{2\pi} \exp\left(-\frac{\hbar \omega}{k_B T}\right)$ and $\dot{n}_{th,q} \approx \frac{\gamma_q}{2\pi} \exp\left(-\frac{\hbar \omega}{k_B T}\right)$ in the mechanical resonator and the qubit respectively. An upper bound on the minimum testable collapse rate limited by thermal noise is therefore
\[ \lambda_{\text{min}} = \left( \bar{n}_{\text{th}} + \bar{n}_{\text{q,eq}} \right) / D. \] Assuming the system is thermalised to the base-plate of a dilution fridge at \( T = 10 \text{ mK} \), while taking \( \omega / 2\pi = 6.33 \text{ Hz} \), we obtain a minimum testable collapse rate of \( \lambda_{\text{min}} \sim 10^{-15} \text{ s}^{-1} \), many orders of magnitude below both Adler and Bassi’s predicted lower bounds.

The above analysis assumes perfect thermalisation with the mixing chamber plate. However, in practice there will be imperfections in the thermalisation. The lowest BAW resonator temperature ever measured was 18 mK [57]. Assuming our system is instead thermalised to 18 mK, due to imperfect thermalisation with the mixing chamber plate, the thermal occupation would be \( \sim 10^{-8} \), an order of magnitude below the signal needed for a conclusive test of CSL given in section 3. Therefore, our proposal can be still be implemented given such imperfect thermalisation, as long as the system thermalises to a temperature no greater than a factor of two larger than the base plate temperature.

During review of our work, a manuscript performing an experimental test of quantum macroscopicity with a qubit-coupled BAW was reported. The superposition state achieved in that experiment set a new record in the quantum macroscopicity [58] of superpositions of mechanical resonators, improving it by more than two orders of magnitude compared to previous experiments [59]. It does however not extend the currently excluded CSL parameter space, as the lifetime of the achieved superposition is limited by noise, despite the expectation that thermal noise should be suppressed at mK temperatures. This further motivates a thorough analysis of possible heating sources involved in experiments involving BAW resonators.

Appendix E. Measurement induced heating

Non-QND effects arise due to higher order terms in \( g / \Delta \) in the dispersive approximation. Most significantly, the drive Hamiltonian acts as creation and annihilation operators on eigenstates of the dispersive Hamiltonian, while the eigenstates of the system in the lab frame are instead eigenstates of the full interaction Hamiltonian \( H = \hbar \omega_{\text{q}} a^\dagger a - \hbar \omega_{\text{q}} \bar{a}^\dagger \bar{a} + \hbar g (a^\dagger \sigma_- + a \sigma_+) \). Mismatch between these dressed eigenstates and the bare eigenstates of the dispersive Hamiltonian causes the drive Hamiltonian (\( H_d \)) to induce qubit transitions in the dressed basis:

\[ H_d = \epsilon^* a + \epsilon a^\dagger \approx \left( \epsilon^* \bar{a} + \epsilon \bar{a}^\dagger \right) - \frac{\hbar}{\Delta} \left( \epsilon^* \bar{\sigma}_- + \epsilon \bar{\sigma}_+ \right), \] (E1)

where \( \epsilon \) and \( \bar{\epsilon} \) are the bare and dressed creation operators respectively. This mismatch between the bare and dressed creation and annihilation operators in the readout Hamiltonian generates qubit excited state population in the dressed basis given by \( P_{\text{th}} \approx \frac{|\bar{\epsilon}|^2}{\Delta^2} \) [41]. For a measurement drive of \( \epsilon / 2\pi = 25 \text{ MHz} \) (allowing the readout resonator to be populated with 2.5 photons within the first 10 ns being driven), a coupling strength and detuning of \( g / 2\pi = 100 \text{ MHz} \) and \( \Delta / 2\pi = 3 \text{ GHz} \) respectively we obtain \( P_{\text{th}} \approx 8 \times 10^{-8} \). This model for measurement-induced-heating can also be used to model any heating produced by microwave pulses used to flux-tune the qubit’s frequency [34].

ORCID iDs
Germain Tobar @ https://orcid.org/0000-0002-4605-9716
Stefan Forstner @ https://orcid.org/0000-0002-2747-9838

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