1. Introduction

Ultrafine grinding in the submicron range has currently attracted attention in connection with the development of new ceramics and electronic materials, and quite a few investigators have reported experimental data of wet process milling using grinding media. As mills using grinding media, conventional ball mills, vibration mills, planetary mills, and stirred mills are typical machineries; special interests have been focused on ultrafine grinding using grinding balls smaller than 1 mm in diameter. Based on the experimental data presented recently in Japan including the classical data well known worldwide, a general form of the selection function applicable commonly to a wide range of particle sizes if possible for various kinds of grinding mills, and the optimum ball size to maximize the rate of grinding are first dealt with in this review.

The comminution kinetics is referred to in order for the design of the ultrafine grinding mechanism to be emphasized, in which the size distribution of the finished product can be discussed in relation to the particle size and the time required. The size distribution governs the properties of the ground product so that the size distribution should be adjusted to meet the requirement by setting up a closed circuit grinding system. Various modes of the closed circuit system are considered together with the basic characteristics of the resulting size distribution as well as the basic design procedures.

Furthermore, the improvement in the rate of grinding is considered from various points of view. One possibility involves composite balls of different sizes which have been declared in most text books on comminution to be ineffective. On the contrary, this review shows that a specific size distribution of the balls may lead to an remarkable improvement in the rate of milling. Finally, physicochemical consideration is also taken into account for the purpose of grinding rate promotion. Grinding aids can be used not only for improving the grinding rate but also for the modification of the finished surface of the ground products.

1.1 Rate of ultrafine grinding related to ball size

K. Tanaka et al. reported extensive experimental data of finely grinding BaTiO₃ to 1.9 micron by using a ball mill and a vibration mill, varying widely the ball diameter and the specific gravity, the ball filling degree, the rotating speed or vibrational conditions as well as the slurry concentration. Among them the effect of ball size on the rate of grinding is noteworthy. The rate of grinding calculated from the specific surface area increase per unit time measured with the BET method was expressed in the form of the following empirical equation.

For ball milling

\[ R_t \propto r^{-0.741} \exp\left\{ -\left(0.738/r\right) \right\} \]  (1)

and for vibration milling the following is obtained

\[ R_t \propto r^{-0.633} \exp\left\{ -\left(0.270/r\right) \right\} \]  (2)

where \( r \) is the radius of a ball ranging from 15 mm down to 0.2 mm.

On the other hand, Tashiro et al. conducted a ball milling speed of 220 rpm with zirconia balls ranging in three stages from 1 to 15 mm in diameter \( D \). They proposed the following empirical equation under some theoretical assumptions:

\[ R_t D^{0.89} \exp\left\{ -0.896(D/b)^{0.278} \right\} \]  (3)

wherein \( b \) is the size of particle and \( D \) is the ball diameter.

The formulae above are somewhat similar and it is of some interests to note that Eqs(1) through (3) were obtained by taking the rate of grinding as being the product of an increasing function and a decreasing function with increasing \( D \) (or \( r \)) although the resulting expressions are quite opposite. These works, however, are extremely important contributions because they derived a general form of the selection function applicable to grinding in the submicron range.
in which, if dry process had been performed, a grinding limit would have been probable.

From these studies it is suggested that the ultrafine grinding predominantly proceeds by compression and shearing at the contact points between colliding balls, not by the impact of balls dropping onto the powder bed. Thus, the rate of grinding or the selection function may be proportional to the following items: (1) total number of contact points between balls, (2) collision frequency of a ball with another, (3) volume of particle nipped at a contact point for grinding, (4) pressure acting on the particles nipped by balls, (5) probability that the stress applied on the particles exceeds the inherent strength for possible fracture. Each item can be formulated as follows (1) the total number of contact points is inversely proportional to the cube of a ball diameter under a constant ball filling degree, varying with $D^3$. (2) The collision frequency can be given as a function of $v/D$, where $v$ is the speed of a colliding and the mean free path varies with $D$. (3) The volume nipped by two balls is calculated to be proportional to $D^2$. (4) The stress created by hitting balls can be expressed by the force divided by the effective contact area between a ball and the nipped particle bed. The surface area is found to be proportional to $D^2$ and to the force $pD^2a$ where $p$ is the density of a ball and $a$ is the acceleration following a collision. Thus, the stress is proportional to $D^2pa/b$. (5) If the stress above is smaller than the strength required to crush the material then no grinding can be expected. For this purpose the following probability $P$ should be multiplied:

$$P = \exp \left\{ -\frac{b\sigma_D}{\rho D^2a} \right\} = \exp \left\{ -\frac{(D_m/D)^n}{} \right\} \quad (4)$$

$$D_m = a \text{ function of } (\sigma_D/b) \rho a \quad (5)$$

$\alpha$ is unknown but the highest acceleration of a ball dropping onto a powder bed of the tightest packing was reported to be proportional to the hitting velocity, $v$. The exponent $n$ in Eq (4) appears equal to 2 dimensionally but preferably can be adjusted from experimental data. $D_m$ is the representative ball diameter for a given material to be nipped for possible fracture by overcoming the strength. The probability $P$ is plotted against $D/D_m$ in Figure 1, where $D_m$ is $D$ corresponding Figure 1 to $P = 0.368$. Therefore, the rate of grinding can be expressed by the product of the items (1) through (5), divided by the total weight of the material in the mill, considering the frequency of the materials introduced into the comminution area. The total holdup $W$ is proportional to $JU$, where $U$ is the fractional filling of material per interstitial space among balls. Then we have

$$R(\psi/D)(D^3)/D^2(\rho D^2v/b)\exp \left\{ -(D_m/D)^n \right\}/(JU)$$

or

$$R(\psi^2J_p/D^2)\exp \left\{ -D_m/D \right\} \quad (6')$$

where $f$, is the fractional volume of the material inside the mill and equals $JU$. Compared with Eq(1) to (3), the derived formula(6') looks like Eqs(1), (2) rather than Eq(3), as far as the effect of ball diameter on the rate $R$ is concerned, the latter of which, however, involves $b$, being consistent with Eq(6').

Differentiating the above equation with respect to $D$ and putting the derivative equal to zero gives $D_{opt} = (1/n)^{1/n}D_m$, wherein $D_{opt}$ is the optimum diameter of the grinding balls and when $n = 1$, $D_{opt} = D_m$ and when $n = 2$, $D_{opt} = 1.4D_m$. Therefore, if Eq(6') is to be confirmed by experimental data, $D_m$ becomes very important for determining the most efficient grinding condition for a given material.

2. Confirmation of the derived equation from various aspects

Current experimental data including the classically well known grinding studies are listed in Table 1, in which the documentations are presented for ball mills, vibration mills, planetary mills and stirred mills. Most works here were carried out through a wet process and fortunately the selected data were obtained almost at the same hitting velocity, $v = 1$ m/s, even with different types of mills of different sizes,
Table. 1 Published experimental conditions of various milling mechanisms for fine and ultrafine powder, using balls as the grinding media.

| No | Authors and reference | Kind of mill, wet or dry | Operating conditions | Hitting velocity, v (m/s) | Material crushed and size, b |
|----|-----------------------|--------------------------|----------------------|--------------------------|----------------------------|
| 1  | Tanaka et al, 1  | Ball Mill, wet 20 – 50 % solid |  $D_m = 30 cm, N = 58 – 105 r.p.m.$  | 0.9 – 1.6 | BaTiO$_3$, 1.9 micron |
| 2  | ditto, 2  | Vibration, wet 20 – 50 % solid |  Amplitude = 3 – 7 mm, freq. = 23.5 – 28 Hz | 0.5 – 1.3 | BaTiO$_3$, 1.9 micron |
| 3  | Tashiro et al, 3  | Ball, wet, 4.3 vol % |  $D_m = 8 cm, N = 220 r.p.m.$  | 0.92 | Pb (Zn, Nb, Fe, W)O$_3$, 1.6 micron |
| 4  | Kelsall et al, 14  | Ball, continuous, wet 66.7% solid |  $D_m = 12 inch, 60 r.p.m.$  | 1 | Calcite, 20 – 28 mesh, 8 – 10 mesh |
| 5  | Coghill & Devaney, 12  | Ball, wet |  $D_m = 19 inch, 50 r.p.m.$  | 1.2 | Dolomite, 0.01 – 0.07 inch |
| 6  | Bradley, 5  | Planetary, wet 67% solid |  $D_m = 6.33 cm, 4.2 Hz, 8.3 Hz, 16.6 Hz$  | 0.84, 1.65, 3.3 | Quartz, 2mm |
| 7  | Mankosa et al, 6 | Stirred, wet, 43% solid |  $D_m = 10.2 cm, 200 – 350 r.p.m.$  | 1.06 – 1.76/1.6, 2.4, 3.2 mm, eq. wt. | Coal, 70micron |
| 8  | Hashi et al, 7 | Stirred, wet, 30% solid |  1.3 lit. Vessel  | 78 | Calcite, 11.3 micron |
| 9  | Stadler et al, 11 | Stirred, wet, Visco = 1 – 500 mPas |  $D_m = 0.3 – 10 mm steel, 75 glass, 2.7$  | 14.5 – 19 | Pigment, 15 micron |
| 10 | Kanda et al, 15 | Ball, wet, 200g/240cc water |  $D_m = 14 cm, 108 r.p.m.$  | 1.58 | Calcite, -20 mesh |
| 11 | Zhao, Jimbo, 10 | Planetary, dry. |  $D_m = 8.4 mm, 78.5 mm, 367.5 mm, 2.5$  | Quartz, 63 – 2000 micron |
| 12 | Kugimija, 8 | Stirred, wet, 40 vol% |  $Vol = 1400cc$  | 5 | Pb (Zn, Nb, Sn, Ti, Zr)O$_3$, 2.2 micron |

so that we can cancel $v$ temporarily in Eq(6'). First, Eq(6') will be verified using the 4 kinds of mills data by varying the ball size to obtain the maximum value of $R$, the other variables remaining unchanged. Assuming $D_m$ first then $R$ is calculated from Eq(6') as a function of D. Figure 2 shows the calculated curve as compared with K.Tanaka’s$^1$ wet ultrafine ballmilling data. A logarithmic plot reveals that a better coincidence can be brought to about $n=1$ rather than $n=2$, to be more parallel. $D_m$ is 2 mm. Likewise, vibration milling data by Tanaka confirmed Eq(6’) where $D_m=1$ mm as shown in Figure 3. A similar trend was also found with the data of Coghill’s ball mill fine grinding as shown in Figure 4 where $n=1$ is still better than $n=2$.$^{12}$

As in Table 1 the materials used are quartz, calcite, coal, dolomite and some electronic materials like BaTiO$_3$, of which the strengths are not always clear except for calcite and quartz. Thus, it is assumed here that the strength of materials as well as the size dependence except for quartzw are the same as those of calcite. The strength of some materials was reported in detail by Yashima$^9$ as a function of size in air and water, respectively. Evaluating $D_m$ in this way
for each experiment in Table 1, an important correlation is found as shown in Figure 5, from which we have, regardless of the type of mills,

$$D_m = 1.75 \times 10^{-1} (b/\rho) \sigma_{st}^{0.8}$$  \hspace{1cm} (7)

where SI units are used for $\rho$ (kg/m$^3$) and $\sigma_{st}$ (Pa) under $v = 1 - 2$ m/s. Since $\sigma_{st} \approx b^q$ where $q$ nearly equals 0.5, Eq(7) becomes $D_{opt} \approx b^{0.6}$, which agrees with the conventional optimum ball size in relation to particle size. 

Next, as an example for confirmation by varying $b$ in Eq(6'), Zhao and Jimbo's planetary milling data are studied. The ball size was 8 mm with $b$

Fig. 2 Comparing the rate of ball milling by K. Tanaka with Eq(6') in which $n=1$, 2 and $r_m=1$, 0.7 mm, respectively.

Fig. 3 Comparing the rate of vibration milling by K. Tanaka with Eq(6') in which $n=1$, $r_m=0.5$ mm.

Fig. 4 Coghill's ball milling data with Eq(6') in which abscissa $\theta$ is the ratio of $D$ to $D_{M1}$, the mill diameter.

Fig. 5 Correlation between $D_m$, the optimum ball diameter, and other variables concerned ranging from 60-2000 microns. Using an average value for $b$ of some fractions together with $\rho$ and $\sigma_{st}$ gives $D_m$ in Eq(7), which enables one to calculate $R$ as depicted in Figure 6. The dotted curve was drawn by using a complex probability function, which appears to be almost proportionally coincident with Eq(6')

The influence of the density of the balls on the rate of grinding should be almost linear in Eq(6'), whereas K.Tanaka's varying $\rho = (2.7-7.8) \times 10^3$ kg/m$^3$, supported this both for ball mills and vibration mills. Zhao et al also recognized this effect in the planetary mill experiment, and Mankosa pointed out that steel balls are clearly more efficient than glass balls with stirred mills at a lower speed of the impeller, $v = 1.7$ m/s. It is very interesting, however, to note the latest report of Kugimiya who stated that no effect of ball density had been recognized in the wet stirred milling with much higher rotating speed, e.g., $v = 14$ m/s. This means that the 4th term in Eq(6) can be replaced by a constant shearing stress arising from the rotating
impellers, resulting in $R_e(b^2/D^3)$. In fact, Kugimiya's and Hashi's data for high speeds seem to confirm it in Figure 7, and more improvement would be expected with stirred mills for ultrafine grinding by varying the speed, because other types are more or less restricted in increasing the hitting speed freely due to their constructions and mechanisms.

As to the effect of $f$, the mill filling degree, in Eq(6'), Rose stated that the rate of milling is approximately linear with $f$ up to $f=50-60\%$ for ball mills and $f=80-90\%$ for vibration mills. K.Tanaka indicated that at $f=40$ and $60\%$ the rate of grinding was the same for ball mills. Furthermore, they also did experiments of vibration milling, varying the slurry concentration by holding a constant volume of water. This corresponds to the relation between $R$ and $f_c$ in order to check Eq(6'), which is depicted in Figure 8. Therefore, the confirmation of Eq(6') is reasonably satisfactory so far for the various terms concerned.

3. Kinetics of comminution and the selection function

The rate of grinding based on the specific surface increase per unit time has been dealt with in comparison with fine and ultrafine grinding mechanisms using balls as grinding media. On the other side, the particle size distribution of the finished product is also important as it influences the properties of the product. To discuss the size distribution in the lapse of time the kinetics of comminution is significant. Defining the selection function, $S(x, t)$ and the breakage function, $B(\gamma, x)$, a dynamic mass-size

![Fig. 6 Comparing the rate of milling by dry planetary mill by Zhao with Eq(6') in terms of particle size crushed.](image)

![Fig. 7 Effect of colliding speed on the rate of wet stirred milling.](image)

![Fig. 8 Rate of vibration milling by K.Tanaka compared with Eq(6') in terms of the fractional powder filling which is proportional to the slurry concentration.](image)
\[ \frac{\partial D(x, t)}{\partial t} \frac{\partial x}{\partial x} = \left( \frac{\partial D(x, t)}{\partial y} \frac{\partial y}{\partial x} \right) S(x, t) \]

where \( D(x, t) \) is the cumulative undersize, \( t \) the time, \( x \) the particle size, and \( y \) is the size of a single particle to be broken. \( x_m \) is the maximum particle size present. Assuming \( B(y, x) = (x/y)^m \) and \( S(x, t) = Kx^n \), then Eq(8) can be analytically integrated into the following forms, \( R(x, t) \) being the cumulative oversize:

When \( m = n \) (mostly \( n = 1 \))

\[ R(x, t) = R(x, 0) \exp(-Kxnt) \]

When \( m \neq n \), then

\[ R(x, t) = R(x, 0) \exp(\{-(\mu Kx^nt)^{\gamma}\}) \]

where \( \mu \) and \( \nu \) are determined from \( m/n \), see Figure 9. Eq(9), (9') have been accepted for many years as the Rosin-Rammler size distribution as a function of time and if the selection function \( S(x, t) \) is replaced and is assumed to be proportional to \( R \) in Eq(6'), then we have from Eq(6')

\[ S(x, t) = \frac{aKx_{c} n}{(\gamma^2)\rho_{c}/D}\exp\{-D(x_{c}/D)\} \]

This suggests a possible contribution of the operating variables of a mill to the rate constant \( K \) which controls the size distribution of a product. Figure 10 represents the product of the experimental selection function of a ball mill by the fractional holdup \( f_{c} \) together with \( J \) and \( U \), proposed by Shoji et al. Eq(6') indicates that \( S_{c} \) varies with \( J \), but is independent of \( U \), whereas Figure 10 indicates a similar tendency.

4. Size distribution controlled by various modes of closed circuit system

Size distribution can be regulated by adopting the closed circuit grinding system composed of a mill and a classifier as schematically shown in Figure 11. The theory has been dealt with elsewhere regarding the case where both a clean cut classifier and a nonideal classifier are involved, and, a nomography was given for the former as in Figure 12, to readily obtain the necessary parameters for designing the circuit. The basic equations are

\[ R_{c}(x_{c}/x_{c}^{*}) = (1+C_{1})\exp\{-((x_{c}/x_{c}^{*})^{n})/(1+C_{1})\}-C_{1} \]

(11)

\[ (x_{c}/x_{c}^{*})^{n} = (1+C_{1})\ln\{1+(1/C_{1})\} \]

(12)

\[ x_{c}^{*n} = F/KW \]

(13)

where \( R_{c} \) is the cumulative oversize of the product, \( C_{1} \) is the circulating load ratio (=mill return/product), \( x_{c} \) the cutoff size of the classifier, and \( x_{c}^{*} \) is the characteristic parting size when \( C_{1} \) tends to infinity. \( F \) is the feed to the circuit, \( W \) the holdup of particles in the mill and \( K \) is the grinding rate constant mentioned above. A mill design should be based on \( W \) in Eq(13) using \( K \). Somewhat modified circuits have
been considered in industry as in Figures 13 and 14. The characteristics of them are that the feed materials are first introduced into a classifier before entering a mill. The latter uses two classifiers. As a result of the analysis of these circuits, the nomograph in Figure 12 can be used as it stands, except that $C_l = 1 + "C1"$ for Figure 13 and $C_l(1+C_l) = 1 + "C1"$ for Figure 14, where "C1" is the value of $C_l$ derived from Figure 12 to fulfill the same design requirement with the use of a standard scheme as shown in Figure 11. The suffixes 1, 2 denote the number of the classifiers indicated, resp. Generally, the product of $C_l$ added by 1 means an advantage in the grinding capacity. On the other hand, two other schemes of closed circuit grinding systems are introduced in Figures 15 and 16, both of which are featured by using two mills and one classifier. The analysis of those circuits provides another nomograph to find out the necessary parameters for the design of those schemes (Type 1 and Type 2, respectively) as shown in Figure 17. Figure 18 illustrates the calculated size distribution of the product with each scheme as well as the circulating load. It should be emphasized here that $C_l$ being infinity appears
impossible in a real closed circuit but this is practically realized in a mill of an air swept system, for example. This corresponds to the mechanism where only fine particles producing smaller than \( x_c \) should instantaneously be discharged from the mill, the theory of which was reported by Ouchiya at al.\(^{23}\)

5. Possibility for increasing the rate of grinding

Now let's come back to the rate of grinding affected by the size of the grinding media. In view of the most experimental data in the past we feel that the size of balls was uniform and very few studies dealt with composite balls. Some books on comminution\(^{15,36}\) stated that the exponent \( q \) should be nearly 3 in the Gaudin-Schuhmann expression, \( P(D)\propto D^q \), where \( P(D) \) is the cumulative undersize weight distribution with respect to ball size \( D \). Based on this equation, the number frequency \( f(D) \) becomes

\[
f(D) = D^{q-1}/\int_{D_1}^{D} D^{q-1}dD
\]

The average ball diameter \( \bar{D} \) is therefore given by

\[
\bar{D} = \int_{D_1}^{D} Df(D)dD = D_1 \{(q-3)/(1-kq-2)/(q-2)/(1-kq-3)\}'
\]

where \( k = D_2/D_1 \), the ratio of the smallest size, \( D_2 \), to the largest one, \( D_1 \); \( \bar{D} \) can be used in items (2) through (5) mentioned in the 1st section. Next, regarding item(1) we can use the Ouchiya et al's formula for the number of contact points of a random assemblage, \( C_T \),\(^{24}\)

\[
C_T = 16(1-\varepsilon)N \{3+(\bar{D}^2/\bar{D}_s^2)\}/4
\]

where \( \varepsilon \) is the porosity for a single size ball, \( N \) the total number of balls and \( \bar{D}_s^2 \) is the average square diameter. They are expressed by using \( K, q, D_1 \) and \( W' \), the total weight of balls, respectively, as

\[
N = (6W'/\pi \rho D_1^3)/\{q(1-kq-3)/(q-3)(1-kq)\}
\]

\[
\bar{D}^2 = \int_{D_1}^{D} D^2f(D)dD = D_1^2(q-3)/(1-kq-3)/(q-1)(1-kq-3)
\]

Substituting Eq(15) through(18) into Eq(6') we have the rate of grinding at constant \( b \) and \( f_c \) as well as

\[
P=1 \text{ for the case of a composite ball size.}
\]

\[
R = (W'/D_1)(q/kq-2)/\{q(3)/(1-kq-3)\} \cdot \{q-2\}^2 + (1-kq-1)/(q-1)/4
\]

Setting the ratio of \( W'/D_1 \) to be unity, \( R \) is computed as a function of \( q \) and \( k \) and given in Figure 19. It is at once noted that as \( q \) tends to be small \( R \) is likely to increase, so that the larger size distribution of the balls appears to be preferable.

Only three runs of experiments listed in table 1,
i.e. Kelsall (5 components), Mankosa (3 components) and Kanda (2 components) were compared with uniform sized media. Based on the documents $q$ and $k$ are available for all experiments to calculate $R$ from Eq (19). See Figure 20. The net increase of the grinding rate can be calculated from $R(P/P_1)^{-1}$, where $P$ and $P_1$ correspond to $D$ and $D_1$ resp., in Eq(4). The calculation resulted in 7% as compared with the 8% increase obtained with experimental data given by Kelsall, whereas 21% was obtained compared with 23% for the reported Mankosa’s experiment. Finally, Kanda reported ultrafine grinding of calcite in a ball mill with a mixture of steel balls of $D=1.2$ mm and alumina balls of $D=20$ mm at a number ratio of 1000:1, the total weight being 2 kg compared to the single size of alumina balls of $D=20$ mm, the total weight being 2 kg. From Figure 20, the necessary parameters are obtained as $q=0.5$ and $k=0.06$, which yield $R=3.2$ from Eq(19). The experimental data demonstrated that the grinding rate was 3 times that obtained with single sized balls, indicating an outstanding increase and a satisfactory agreement with the calculation. In Figure 19 we see that $R$ never change too much with $q$ greater than 3. This does not contradict the experience which has been stated in textbooks which indicate composite balls were not effective.

It can be concluded that for fine and ultrafine grinding considerable improvements may still be expected by selecting appropriate grinding mechanisms as well as operating variables as described above. Moreover, an emphasis should be put upon the physicochemical devises that make it possible to increase the rate of grinding. The strength of a material $\sigma_{st}$ is given by

$$\sigma_{st} = \sqrt{\frac{Y\lambda}{c}}$$  \hspace{1cm} (20)

where $Y$ is the Young’s modules, $\lambda$ the surface energy and $c$ is the crack length. Wet process and addition of some surfactants serve as reducing $\lambda$ by adsorption effect on the solid surface, leading to a decrease in the strength, $\sigma_{st}$. These additives are called grinding aids which have been noted for use to modify a fresh surface exposed to grinding, taking advantage of the reactivity. This assists fine particles to be well dispersed, resulting in better grinding efficiency. It has been reported that about a 20 time improvement was secured in the rate of grinding. Cryogenic grinding is also an application of physical chemistry, which reduces the toughness $\sqrt{\frac{Y\lambda}{c}}$ to assist crushing.

Conclusion

Contribution of various parameters regarding grinding mechanisms using balls was reviewed theoretically and compared with current experimental data of fine and ultrafine grinding in the submicron range. An important correlation was obtained between the optimum ball size and density, and the size and the strength of a material regardless of the kind of machinery. The comminution kinetics was discussed not only in relation to the rate constant as a function of these parameters for design procedure, but also for the adjustment of particle size of the product by varying the closed circuit grinding schemes. A new
A proposal for the size distribution of balls has been made for securing the improvement of the rate of grinding, although it appears generally pessimistic in textbooks. Other various trials are to be considered from different aspects of comminution techniques.

Nomenclature

\[ B(y, x) : \text{breakage function} \quad [-] \]
\[ b : \text{particle size of material} \quad [\text{mm}] \]
\[ C_r : \text{total number of contact points} \quad [\text{mm}] \]
\[ c : \text{crack length} \quad [\text{m}] \]
\[ Cl : \text{circulating load ratio} \quad [-] \]
\[ D : \text{ball diameter} \quad [\text{mm}] \]
\[ D : \text{average ball diameter} \quad [\text{mm}] \]
\[ D^2 : \text{average square diameter} \quad [\text{mm}] \]
\[ D_i : \text{impeller diameter} \quad [\text{mm}] \]
\[ D_H : \text{ball mill diameter} \quad [\text{mm}] \]
\[ D_{st} : \text{representative ball diameter} \quad [\text{mm}] \]
\[ D_{opt} : \text{optimum ball diameter} \quad [\text{mm}] \]
\[ D_{max}, D_{min} : \text{max. and min. ball diameter} \quad [\text{mm}] \]
\[ D(x, t) : \text{Cumulative undersize of particles} \quad [\text{mm}] \]
\[ D(x, t) : \text{Cumulative undersize of particles} \quad [-] \]
\[ F : \text{feed to the circuit} \quad [\text{kg/s}] \]
\[ f(D) : \text{frequency ball size distribution} \quad [\text{l/mm}] \]
\[ f : \text{fractional volume of holdup} \quad [-] \]
\[ J : \text{mill filling degree} \quad [-] \]
\[ K : \text{grinding rate constant} \quad [1/\text{s.mm}^4] \]
\[ p(D) : \text{cumulative undersize of ball size distribution} \quad [\text{wt}] \quad [-] \]
\[ q : \text{exponent} \]
\[ R(x, t) : \text{cumulative oversize of particles} \quad [-] \]
\[ R_p : \text{R(x) of product} \quad [-] \]
\[ R : \text{rate of grinding} \quad [\text{m}^2/\text{kg.s}, 1/\text{s}] \]
\[ r : \text{radius of ball} \quad [\text{mm}] \]
\[ S(x, t) : \text{selection function} \quad [1/\text{s}] \]
\[ S : \text{specific surface} \quad [\text{m}^2/\text{kg}] \]
\[ t : \text{time} \quad [\text{s}] \]
\[ U : \text{fractional holdup per interstice of balls} \quad [-] \]
\[ ν : \text{hitting speed} \quad [\text{m/s}] \]
\[ W : \text{holdup of materials in a mill} \quad [\text{kg}] \]
\[ W' : \text{total weight of balls} \quad [\text{kg}] \]
\[ x : \text{particle size} \quad [\text{mm}] \]
\[ x : \text{classifier cut size} \quad [\text{mm}] \]
\[ x^* : \text{characteristic cutsize} \quad [\text{mm}] \]
\[ Y : \text{Young's modulus} \quad [\text{Pa}] \]
\[ α : \text{acceleration due to collision} \quad [\text{m/s}^2] \]
\[ γ : \text{particle size to be broken} \quad [\text{mm}] \]
\[ k : \text{ratio of min. to max. diameter of ball} \quad [-] \]
\[ m : \text{exponent} \]
\[ N : \text{total number of balls} \quad [-] \]
\[ n : \text{exponent} \]
\[ P : \text{crushing probability} \quad [-] \]

References

1) Tanaka, K.et al: J.Soc.Mat.Sci. Japan 35, 54 (1986); 36, 29 (1987)
2) Tanaka, K. et al: J.Ceramic.Soc., Japan, 95, 579 (1987)
3) Tashiro, S.and H.Igarashi: ibid., 98, 1082 (1990)
4) Tanaka, T.: J.Soc.Powder Tech., Japan, 31, 25 (1994)
5) Bradley, A.A.: Dechem.Mono.Zerklein.Symp.Cannes, B69, 1292, S.781 (1971)
6) Mankosa, M.J.et al: Powder techn., 49, 75 (1986)
7) Hashi, Y. and M.Kusunoki: 24th Symp. Text 135 (1989)
8) Kugimiya, K.: Micromeritics, 36, 177 (1992)
9) Yashima, S. and F.Saitoh: J. Soc. Powder techn., Japan, 16, 74 (1979)
10) Zhao, Q. and G. Jimbo: ibid., 25, 603 (1988)
11) Stadler, R.et al: Chem. Ing. tech., 62, 907 (1990)
12) Coghill, W.H. and F.D.Devaney: U.S.Bur.Mines, Tech. Paper, 581 (1937)
13) Rose, H.E. and R.M.E.Sullivan: Ball, Tube, Rod Mills, P.158, Constable (1958)
14) Kelsall, D.F.et al: Powder Techn., 1, 291 (1967/8)
15) Slegten, J.A.: Sympo/Zerklein.Frankfurt a/M, S355 (1962)
16) Beke, B: Process of Fine Grinding. p89, Akademiai Kiado (1981)
17) Rose, H.E. and R.M.E.Sullivan: Vibration Mills, p.99, Constable (1961)
18) Nakajima, Y. and T.Tanaka: IEC, Proc.Des.Dev. 12, 23 (1973)
19) Kanda, Y. et al: J.Soc.Powder Tech. Japan, 26, 406 (1989)
20) Shoji, K. and L.G.Austin: Powder Techn., 31, 121 (1982)
21) Furuya, M.et al: IEC., Proc.Des. Dev., 10, 449 (1971); 12, 18 (1973)
22) Tanaka, T.: J.Soc.Powder Tech. Japan, 31, 333 (1994)
23) Ouchiyama, N.et al: IEC.Proc.Des. Dev., 15, 471 (1976)
24) Ouchiyama, N. and T.Tanaka: IEC, Fundamental, 19, 338 (1980)
Tatsuo Tanaka

Tatsuo Tanaka is a graduate in Chemical Engineering from Kyoto University, where he received Doctor's degree in 1957. He studied as a Fulbrighter in 1958 through 1960 at the University of Minnesota. After working at Kanazawa University he was appointed in 1962 to be Professor of Chemical Engineering at Hokkaido University where he taught and conducted research until his retirement in 1990. His research interests are mainly concerned with the kinetics theory of particulate processes, particularly in comminution, tumbling granulation, solid-solid reaction, spontaneous ignition as well as dust explosion. For those works he was awarded by the Japan Society for Chemical Engineers in 1988 and the Society of Powder Technology, Japan, in 1989. He experienced visiting professors in Taiwan and Australia in the past. Now he is assigned Professor Emeritus, Hokkaido University, and currently once in a while he enjoys giving lectures upon the domestic and international requests.