I. INTRODUCTION

When Press-Schechter theory is used to study the number density of galaxy clusters and its evolutions, there are two key parameters \( w_0 \) and \( w_1 \) needing to be extracted from the spherical collapse models. \[14, 15\] are two early works studying this model. They considered it in a background universe without dark energies. \[12, 21, 23, 25, 26, 27\] are some recent works containing discussions about this model in ΛCDM or QCDM cosmologies. In these existing works, when dark energy is included, it is assumed that it does not cluster on the scale of galaxy clusters.

However, if dark energy does not cluster on this scale, then when we write down the Einstein equations to describe the evolution of an over-dense region which will develop into the future galaxy clusters, we should include a dark energy current in the energy momentum tensor to describe the dark energy component flowing outside the over-dense region. But, in all these existing literatures, such a current was ignored. To see the effects of such a current, there are two methods. The first is, adding it in the energy momentum tensor and solving the resulting Einstein equations. The second is, assuming that such a current does not exist at all and solving the appropriate Einstein equations. The first is, adding it in the energy momentum tensor and solving the resulting Einstein equations. The second is, assuming that such a current does not exist at all and solving the appropriate Einstein equations to see the effects of dark energy’s clustering on the scale of galaxy clusters.

The purpose of this paper is to study this problem by the second method. We leave the study of this first way as the topic of another paper where similar conclusions will be obtained also.

II. PRESS-SCHETCHEER THEORY

Press-Schechter Theory \[14\] predicts that the fraction of volume which has collapsed at a certain red-shift \( z \) is

\[
\frac{f_{\text{coll}}(M(R), z)}{2} \int_{\sigma_c}^{\infty} d\sigma e^{-\delta^2/2\sigma^2(R,z)}.
\]

Here, \( R \) is the radius over which the density field has been smoothed, \( \sigma(R,z) \) is the rms of the smoothed density field \( \delta \), \( \delta_c \) is the threshold of density contrast at time \( z \) (count stopping time) beyond which objects collapse.

By the notations of \[13\], chapter 7, \[2\],

\[
\sigma^2(R, z) = \frac{\left[ \int k^{n_s+2} T^2(k) W^2(k \cdot R) dk \right] D_f^2([1+z]^{-1})}{\left[ \int k^{n_s+2} T^2(k) W^2(k \cdot 8 h^{-1} \text{Mpc}) dk \right] D_f^2(1)}
\]
where \( n_s \) is the spectral index of perturbation powers, \( n_s = 1 \) corresponds to scale invariant power; \( T(k) \) is the transfer function, in this paper we will use the BBKS fitting formulae for it \( \text{[12 13 23]} \); \( W(k \cdot R) \) is the smoothing window function, we will use the top hat window in this paper; \( D_1(1 + z)^{-1} \) is the growth function of linear perturbation theory,

\[
D_1(a) = \frac{5\Omega m_0 H_0^2}{2} H(a) \int_0^a da' [a' H(a')]^{-3}. \tag{3}
\]

Operationally \( \text{[21]} \), \( \delta_c \) is defined as the extrapolation of primordial perturbations to the collapse epoch using the growth law of linear perturbation theory, i.e.,

\[
\delta_c = \left[ \frac{\rho_{mc}(a)}{\rho_{mb}(a)} - 1 \right] \frac{1}{D_1(a)} \bigg|_{a \to 0} D_1(a_c), \tag{4}
\]

where \( \rho_{mc} \) and \( \rho_{mb} \) are the matter densities of cluster and background respectively. It can be shown that \( D_1(1 + z)^{-1} \to a \). Using the method of \( \text{[16]} \) we can show that

\[
\left[ \frac{\rho_p}{a} \right]_{a \to 0} = (1 - \alpha \cdot a), \tag{5}
\]

where \( \alpha \) is the scale factor of the cluster, we use the normalization for \( \alpha \) so that when \( a \to 0 \), \( \alpha \approx a \), see section \( \text{III A} \). Substituting eq(\ref{eq:5}) into eq(\ref{eq:4}) we get

\[
\delta_c = 3\alpha \cdot D_1(a_c). \tag{6}
\]

By studying spherical collapse model we will give an analytical formulae for the growth of \( \alpha \).

We comment here that, because \( D_1(1 + z)^{-1} \to a \), as long as the definition eq(\ref{eq:5}) is to be of any sense, we must have \( \left[ \frac{\rho_{mc}}{\rho_{mb}} - 1 \right]_{a \to 0} \sim a \). So eq(\ref{eq:5}) holds regardless whatever the background cosmology model is. Physically, this is because, any realistic cosmological model behaves like a flat matter dominated cosmology at early times, so they have the same limit behavior as \( a \to 0 \). Mathematically, eq(\ref{eq:5}) is only derived out explicitly in the OCDM cosmologies \( \text{[16]} \) and ΛCDM cosmologies \( \text{[22] and 24]}. \)

It should be noted besides the partition of the cosmological components and the observational epoch, the value of \( \delta_c \) is also quite dependent on the choice of smoothing window used to obtain the dispersion \( \sigma(R, z) \) \( \text{[10]} \). We will not consider this effect in this paper.

III. SPHERICAL COLLAPSE MODEL IN QCDM COSMOLOGIES

A. Einstein Equations and Energy Conservation

In \( \text{[22] and 24]} \), it is stated that the energy hold by the dark energy-Quintessence inside the the over-dense region varies independently of the region’s radius, and the curvature parameter of the over-dense region is time dependent, so we have no Friedman-like equation, with a constant curvature parameter to describe the evolution of the over-dense region. However, if this is the case, then when we write down the Einstein equation \( G_{\mu \nu} = -8\pi G T_{\mu \nu} \), we should include a dark energy current \( -Q \) current- in the energy momentum tensor to describe the flowing of dark energy outside the over-dense region. But \( \text{[22] and 24] did not do so.} \)

To see the effects of such a current, we have two choices. The first is directly including such a current in the energy momentum tensor and solving the resulting Einstein equations, we will do this way in \( \text{[28]} \). The second is assuming that such a Q-current does not exist and solving the resulting Einstein equations to see the effects of Quintessence’s clustering on the properties of galaxy clusters. In this method, Quintessence will cluster or expand synchronously with matters in the over-dense region, just as we usually do in the case of universes. This is an indirect but easier method. We will take this way in this paper.

Considering an uniform over-dense region in the back ground of a flat cosmology containing general Quintessence component, the background satisfies

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_{mb} + \rho_{Qb}). \tag{7}
\]

For the over-dense region, starting from the metric ansatz

\[
d s^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2 \theta d\phi^2), \tag{8}
\]

using Einstein equations, we can prove that

\[
U(t, r) = \frac{a_p^2(t)}{1 - kr^2}, V(r) = a_p^2(t)r^2, \tag{9}
\]

where \( k \) is a constant independent of time while \( a_p \) satisfy:

\[
a_p \ddot{a}_p = -\frac{4\pi G}{3} [\rho_{mc} + \rho_{Qc} + 3p_{Qc}]a_p^3. \tag{10}
\]

\[
a_p \ddot{a}_p + 2a_p \dot{a}_p + 2k = 4\pi G [\rho_{mc} + \rho_{Qc} - p_{Qc}]a_p^2. \tag{11}
\]

The difference of eq(\ref{eq:10}) and \( \text{(11)} \) yields

\[
\frac{\dot{a}_p}{a_p}^2 + \frac{k}{a_p^2} = \frac{8\pi G}{3} U(t). \tag{12}
\]

In eq(\ref{eq:11}), \( a \) denotes the scale factor of the background universe, \( \rho_{mc} \) and \( \rho_{Qb} \) denote the density of matter and Quintessence in it respectively. While in eq(\ref{eq:10}), \( a_p \) denotes the scale factor of the over-dense region, the future galaxy clusters. \( \rho_{mc}, \rho_{Qc} \) denote the density of matter and quintessence inside the over-dense region respectively, while \( p_{Qc} \) denotes the pressure produced by the Quintessence. By the statement of \( \text{[22] and 24]} \), the curvature parameter \( k \) of the over-dense region is time dependent. This is equivalent to say, the metric function \( U(t, r) \) cannot be factorized as eq(\ref{eq:9}). If this is the
case, then not only Friedmann equation \(^{[12]}\), but also Raychaudhuri-equation \(^{[10]}\), or eq(A2) of \(^{[23]}\) will follow the usual way.

In practice, we need taking a normalization for the definition of scale factor \(a_p\) of the over-dense region. We choose the convention so that \(a_p \approx a\) when \(a \to 0\). Under this convention, the value of \(k\) in eq(9) is not 1 definitively, its value depends on the total energy density inside the over-dense region. But, it is a constant independent of time.

Using the metric ansaltz eq(5) and (9), the energy conservation equation \(T_{\mu \nu}^c = 0\) can be integrated to give
\[
\rho_m a_p^3 + \rho_Q a_p^{3(1+w)} = \text{a constant},
\]
where \(w\) is the equation of state coefficients of the Quintessence\((-1 < w < 0)\) or phantoms\((w < -1)\). In this paper, we only consider Quintessence/phantoms with time-independent \(w\) models. During the formation of galaxy clusters, the mass of the system is conserved at least approximately. This directly leads to the conclusion that, the density of Quintessence inside the over-dense region does not vary independently of the radius of the region. If the equation of state coefficients satisfy \(-1 < w < 0\), then as \(a_p\) decreases, \(\rho_Q\) increases, Quintessence clusters. If \(w < -1\), then as \(a_p\) decreases, \(\rho_Q\) also decreases, i.e., phantom is repulsed outside the over-dense region, it anti-clusters. Contrary to our reasonsing here, in \(^{[28]}\), the energy held by Quintessence in the over-dense region does not conserve, see eq(14) of it.

In \(^{[29, 30, 31]}\), it is pointed out that on cosmological scales, a smoothly distributed, time-varying component violates equivalence principle, so is un-physical. Here, on the scale of galaxy clusters, we see that to preserve Einstein equations, we should either include a Q-current in the energy momentum tensor, or assume that Quintessence (or phantom, even time independent vacuum energy) clusters (anti-clusters) just like matters. As is well known, when the total universe is looked as a cluster of size Hubble scale, such a current which describes Q-component flowing outside the Hubble horizon is usually assumed not exist. Galaxy clusters are a class of objects whose formation and evolution is still in the linear perturbation theory applicable area and in the energy composition of these objects, dark energy occupies a rather big part, just like our biggest cluster - our total universe. So the assumption that Quintessence will cluster on the scale of galaxy clusters like ordinary matters is at least partly the case. We will also see in \(^{[28]}\) that, if dark energy is assumed not to cluster on the this scale, then all the effects displayed in this paper will be enforced instead of weakened.

B. The Key Parameters of the Spherical Collapse Model of Galaxy Clusters Formation

In the ideal model, if there is an over-dense region in a flat background universe, then at very early times, this region will expand as the background universe expands; but because this region’s over-dense, its expanding rate will decrease and stop doing so at some middle times; then it starts to shrink because of self-gravitating, the final fate of this over-dense region is a singular point. But in practice, when this region shrinks to some degree, the pressures originate from the random moving of particles inside the over-dense region will balance the self-gravitating and the system enters the virialization period. In theoretical studies, it is usually assumed that the virialization point is coincident with the collapse point of the ideal model on the time axis.

According to Press-Shectner theory eq(11), if an over-dense region is to be virialized at some time \(a_c\), its density-contrast should be no less than \(\delta_c(w, \Omega_{m0}, a_c)\). While to relate the mass of a galaxy cluster with its characteristic X-ray temperature, the ratio of cluster/background matter densities at the virialization point is a very important parameter,
\[
\Delta_c(w, \Omega_{m0}, a_c) = \rho_{mc,c}/\rho_{mb,c}
\]
To see the effects of Quintessence/Phantom’s clustering/anti-clustering on the formation and properties of the galaxy clusters, we would like to calculate this two important parameters in this section. To calculate \(\delta_c(w, \Omega_{m0}, a_c)\), starting from eq(14) and (12), let
\[
\zeta = \frac{\rho_{mc,ta}}{\rho_{mb,ta}}, x = \frac{a}{a_{ta}}, y = \frac{a_p}{a_{p,ta}}, \nu = \frac{\rho_Q b, ta}{\rho_{mb,ta}}
\]
and using the fact that \([\dot{a}_p]_{ta} = 0\), we can get
\[
(\frac{dy}{dx})^2 = \frac{\zeta y^{-1} + \nu c^{1+w} y^{1-3w} - (\zeta + \nu c^{1+w})}{x^{-1} + \nu x^{-1-3w}}.
\]
Then using eq(15) and the approximate mass conserving condition we can prove
\[
\frac{y}{x} |_{x \to 0} = \frac{a_p a_{ta}^{-1}}{a_{ta}} |_{a \to 0} = 1 - \frac{\zeta}{\alpha}
\]
and
\[
\frac{d}{dx} \left[ y_{x \to a, x \neq 0} \right] = \frac{\zeta}{\alpha} (1 - 2a \cdot a_{ta} x).
\]
Substituting eqs(5), (10) and (11) into eq(17), taking limit \(x \to 0\) and preserving only the first order small quantities in \(x\), we can get
\[
\alpha = \frac{1}{3} a_{ta}^{-1} [5 \cdot \frac{\zeta}{\alpha} + \nu \cdot \frac{\zeta}{\alpha} c^{1+w}]
\]
Substituting this result into eq(10) we can get
\[
\delta_c(w, \Omega_{m0}, a_c) = \frac{3}{5} a_{ta}^{-1} [5 \cdot \frac{\zeta}{\alpha} + \nu \cdot \frac{\zeta}{\alpha} c^{1+w}] D_1(a_c)
\]
Now let us come to \( \Delta_c \)'s calculation. According to the definition eq. (14),

\[
\Delta_c(w, \Omega_{m0}, a_c) = \frac{\rho_{mc, c}}{\rho_{mb, c}} = a_{p, ta}^3 \frac{a_{p, c}^3}{a_{p, ta}^3} = \frac{a_{p, ta}^3}{a_{p, c}^3} = \frac{\rho_{mb, ta}}{\rho_{mc, ta}} a_{c}^3 a_{ta}^3
\]

(22)

To calculate the second factor of the above equations' right-most part, we can use energy conserving condition and virial theorem. Assuming that at the collapse point, the system virialized fully, then we have the following relations:

\[
E_{\text{kinetic}, c} = -\frac{1}{2} U_{mm, c} + U_{mc, Q} c - \frac{1}{2} U_{QQ, c}
\]

\[
\frac{1}{2} U_{mm, c} + 2 U_{mc, Q} + \frac{1}{2} U_{QQ, c}
\]

(23)

where \( U_{mm}, U_{mc, Q} \) and \( U_{QQ, c} \) denote the matter-matter, matter-Quintessence and Quintessence-Quintessence gravitation potentials respectively. The subscripts \( c \) and \( ta \) mean that the quantities carrying them take values at the collapse and turn around time respectively. Explicitly, eq (24) can be written out as:

\[
(\rho_{mc, c}^2 + 4 \rho_{mc, c} \rho_{c, c} + \rho_{c, c}^2) a_{p, c}^5
\]

\[
= 2(\rho_{mc, ta}^2 + \rho_{mc, ta} \rho_{Qc, ta} + \rho_{Qc, ta}^2) a_{p, ta}^5
\]

(25)

\[
\frac{a_{p, ta}^5}{a_{p, c}^5}(1 + 4 \rho_{Qc, ta} \rho_{mc, c} + \rho_{mc, c}^2) = 2(1 + \rho_{mc, ta} \rho_{Qc, ta} + \rho_{Qc, ta}^2)
\]

(26)

Using energy conservation law and the approximate mass conserving condition, we can change eq (26) into the following form:

\[
\frac{a_{p, ta}^5}{a_{p, c}^5}(1 + 4 w \rho_{Qc, ta} a_{p, ta}^3 + \xi^2 a_{p, ta}^3 a_{p, c}^6 w) = 2(1 + \xi + \xi^2)
\]

(27)

where

\[
\xi = \nu \zeta^w
\]

(28)

Numerical studying shows that the third term on both sides of eq (27) can be neglected, just as eq (28) did. But, for a general valued \( w \), even neglecting this two terms, this equation cannot be solved analytically. So we preserve this two terms in the equation and resort to numerical methods to calculate \( a_{p, ta} \).

From eqs (21) and (22), we see that, given \( w, \Omega_{m0} \) and \( a_c \), to calculate \( \Delta_c \) and \( \Delta_v \), we need to express \( a_{ta}, \nu \) and \( \zeta \) as functions of \( w, \Omega_{m0} \) and \( a_c \). For \( \nu \)

\[
\nu = \frac{1 - \Omega_{mb, ta}}{\Omega_{mb, ta}}
\]

(29)

\[
\Omega_{mb, ta} = \frac{1}{1 + \nu_0 a_{ta}^{1 - 3w}}, \quad \nu_0 = \frac{1 - \Omega_{m0}}{\Omega_{m0}}
\]

(30)

While for \( a_{ta} \), we can use the fact that \( t_c = 2 t_{ta} \) and eq (7) to set up an integration equation

\[
\int_0^{a_{ta}} da' \sqrt{1 + \nu_0 a'^{-3w}} = 2 \int_0^{a_{ta}} da' \sqrt{1 + \nu_0 a'^{-3w}}
\]

(31)

In the case of \( w = -1 \), eq (31) can be solved analytically:

\[
a_{ta} = \left[ \frac{\sqrt{1 + \nu_0 a_{c}^3} - 1}{2 \nu_0} \right]^{1/3}
\]

(32)

While in the case of general valued \( w \), eq (31) should be solved numerically to get the function \( a_{ta}(a_c) \).

For \( \zeta \)'s calculation, there is two method. The simplest is directly integrate eq (17) to set up an equation:

\[
\int_0^1 dx\left[ \frac{x}{1 + \nu x^{-3w}} \right]^{1/2} = \int_0^1 dy\left[ \frac{\zeta + \nu \zeta^w y^{-3w} - (\zeta + \nu \zeta^w y)^{3w}}{y^2} \right]^{1/2}
\]

(33)

Solving this equation will tell us \( \zeta \)'s dependence on \( w \) and \( \nu \). Combining the results with eq (29) will gives us \( \zeta \)'s dependence on \( w, \Omega_{m0} \) and \( a_c \). The second method is to use eqs (17), (10) and the notations in eqs (15)-(16) to set up an eigen-value problem

\[
\frac{y}{x} \bigg|_{x \rightarrow 0} = 1 \cdot \zeta^w
\]

(34)

\[
y \bigg|_{x = 1} = 1, \quad y' \bigg|_{x = 1} = 0.
\]

\[
d^2 y \frac{d y}{d x} \frac{1}{2 x} + \frac{d \Omega_{mb}}{d x} \frac{\Omega_{mb}}{\Omega_{mb}}
\]

\[
+ \frac{1}{2} \left[ 1 + 3 w \right] \left[ \frac{y}{x} \right]^{-3(1 + w)} \zeta^{1 + w} y(1 - \Omega_{mb}) x^2 y^2 \zeta x \Omega_{mb} = 0
\]

(35)

Solving this eigen-value problem can also tell us \( \zeta \)'s dependence on \( w \) and \( \nu \). For a given value of \( w \), we can get \( \zeta \)'s dependence on \( \Omega_{mb, ta} \) by solving eq (35) or eq (31) and fitting the results as \( \zeta = \zeta(\Omega_{mb, ta}) \). For example, in the case of \( w = -1 \),

\[
\zeta = \frac{3 \pi}{4} \Omega_{mb, ta}^{0.7384 + 0.2451 \Omega_{mb, ta}}
\]

(36)

But for the general cases when \( \Omega_{mb, ta} \) and \( w \) both vary, we cannot fit the results into a simple function, see Fig 1. It should be noted that this kind of fitting formula eq (36) only coincide with numerical result when \( \Omega_{mb, ta} \) is not too small, for example, \( \Omega_{mb, ta} > 0.05 \).

C. Numerical Results, Effects of \( w \) on the Key Parameters of the Model

We provide our numerical results in FIG 1. From FIG 1 we see that, when the dark energy's clustering is
FIG. 2: $\Delta_c$’s dependence on $\Omega_m, w$ and $a_c$. The upper right part is solved from eq(5)-(6) of [23], where Quintessence is assumed not to cluster on the scale of galaxy clusters but neglect the Q-current flowing outside the clusters, solved from eq(A9) of [23].

FIG. 3: $\delta_c$’s dependence on $w, \Omega_m, w$ and $a_c$. Quintessence’s clustering effects are considered.

In FIG 2, we display $\Delta_c$’s dependence on $w, \Omega_m, w$ and $a_c$. Unlike in FIG 1, we did not compare $\delta_c$’s dependence on $w, \Omega_m$ in this figure between the cases where Quintessence is assumed to cluster and the contrary. By considered, $\zeta$ depends on the value of $w$ remarkably. For a fixed $\Omega_{mb,ta}$, $\zeta$ first increases then decreases as $w$ decreases in the range $[-1.7, -0.4]$, this forms a strong contrast with the result of [23], which is also depicted in the figure. In FIG 2, besides similar dependence of $\Delta_c$ on $w$ and $\Omega_m$, we can still see that for a given $\Omega_m$ and $w$, $\Delta_c$ decreases as $a_c$ decreases, i.e. the early an over-dense region virialized, the less is its matter density over that of the background. This is easy to understand, because the earlier an over-dense region virializes, the less dark energy is contained in the background cosmology, so the weaker the anti-cluster effects will be and the less dense a region is required to keep balance by self-gravitations.

In FIG 3, we see that, if $a_c = 1$, $\delta_c$ may be as large as 2.1 when $w \approx -1.7$. As $a_c$ decreases, $\delta_c$ also decreases and goes to the limit value 1.686, what ever value $w$ and $\Omega_m$ takes. This is trivial, because the earlier an over-dense region virializes, the more likely is the back ground universe a totaly matter dominated one.

IV. THE NUMBER DENSITY OF GALAXY CLUSTERS AND ITS EVOLUTIONS

A. Theoretical Formulae

According to Press-Schechter theory, the comoving number density of clusters which have collapsed (i.e., virialized) at certain red-shift $z$ and have masses in the range $M \sim M + dM$ could be calculated:

$$n(M,z)dB = -\frac{\sqrt{2}}{\pi} \frac{\rho_{tot}}{M} \sqrt{\frac{\kappa^2}{3}} \frac{d\sigma}{dR} \exp[-\frac{\kappa^2}{2\sigma^2}][dM]$$

in this equation; $\rho_{tot}$ is the energy density of background universe, $M = \frac{4\pi}{3} R^3 \rho_{tot}$, so the factor $\frac{\rho_{tot}}{M}$ denotes the average number density of clusters with mass $M$. The other factors in this equations are just obtained by differentiating eq(1) with respect to $M$. In the original paper of Press and Schechter [14], the quantity $\rho$ and $M$
appearing in eq(37) only refer to density and mass of matters. But under our assumptions, Quintessence and matters cluster synchronously so we have to understand them as the total density and mass.

To relate the mass of a cluster with its characteristic X-ray temperature, consider a virialized over-dense spherical region in the background universe containing quintessences, according to virial theorem, we have

\[
E_{\text{kinetic,vir}} = \left[ -\frac{1}{2} U_{mm} + U_{mq} - \frac{1}{2} U_{QQ} \right] \text{vir}
\]

i.e.

\[
(\rho_{mc} + \rho_{Qc})_{\text{vir}} \bar{V}^2_{\text{vir}} = \frac{4\pi G}{5} \left[ a^2_p (\rho_{mc} - \rho_{Qc})^2 \right]_{\text{vir}}
\]

where \(\bar{V}^2_{\text{vir}}\) is the mean square velocity of particles in the cluster when the system is fully virialized and \(a_p\) is the physical radius of the cluster. From this equation, we have

\[
\bar{V}^2_{\text{vir}} = \frac{3}{5} \left[ (GMH)^2 \left( \frac{1}{2} \rho_{mc} + \rho_{Qc} \right) \cdot (1 - \frac{2}{\rho_{mc} + \rho_{Qc}}) \right]\]

Using relation:

\[
k_B T = \frac{\mu_m p}{\beta} \cdot \bar{V}^2_{\text{vir}}
\]

where \(k_B\) is the Boltzmann constant, \(m_p\) is the mass of proton, while \(\mu_m\) is the average mass of particles, \(\beta\) is the ratio of kinetic energy to temperature. So we have mass-temperature relation:

\[
M = \frac{1}{GH(z)} \left[ \frac{5 \beta k_B T \cdot 1}{\mu_m p \cdot f(z)} \right]^2
\]

or

\[
R = \left[ \frac{2GM}{H^2} \right]^4 = 1 \cdot \left[ \frac{5 \cdot 2^2 \beta k_B T \cdot 1}{5 \cdot \mu_m p \cdot f(z)} \right]^2
\]

with

\[
f(z) = \left( \frac{1}{2} \Omega_{mb,c} + \Omega_{Qb,c} \Delta_c w^c \right)^{\frac{1}{2}} \times
\]

\[
\left( 1 - \frac{2 \Omega_{Qb,c} \Delta_c w^c}{\Omega_{mb,c} + \Omega_{Qb,c} \Delta_c w^c} \right)^2
\]

\[
H(z) = H_0 [\Omega_{m0} (1 + z)^3 + (1 - \Omega_{m0}) (1 + z)^3]^{\frac{1}{2}}
\]

and \(\Delta_c\) given by eq(22).

Just as [23] pointed out, since the mass-temperature relation is red-shift dependent, simply substituting eq(11) into eq(37) cannot give us correct number density of clusters in a given temperature range today. Instead, we should first find out the virialization rate and multiply it by the mass-temperature relation then integrate over red-shift

\[
n(T, z) dT = -\frac{1}{\sqrt{2\pi}} \int_z^\infty \rho_{\text{tot}} d\ln \sigma d\ln \sigma \delta_c \frac{\delta^2}{\sigma^2} - 1) \exp[\frac{-\delta^2}{2\sigma^2}] dz dT
\]

From eqs(13), (11), (21) and (2) we can see that besides the constant \(\frac{\mu}{\beta}\), \(n(T, z)\) will also depend on the cosmological parameters \(w, \Omega_{m0}, h, n_s\), and the normalization \(\sigma_8\) of the cosmic density fluctuations. In principle, if we can measure the number density v.s. temperature relation precisely enough, by numerical fittings, we can determine all these parameters or some of their special combinations simultaneously from observations.

**B. Numerical Results, Effects of \(w\) on the Number Density of Galaxy Clusters and Its Evolutions**

In FIG 4 we display the effects of \(w\) on the galaxy clusters number density v.s. temperature relation when \(z = 0\) for two values of \(\frac{\mu}{\beta}\). From this figure we can see that the value of \(w\) affects the number density of galaxies exponentially. Just as we explained in section A since the author of [28] and [29] did not include the Q-current in their energy moment tensor at the right hand side of Einstein equations, which should appear under their assumptions, the conclusions there about the effects of \(w\) on the number density v.s. temperature relation is worth...
further explorations. Although we do not directly correct their analysis by including such a current, we only assume that Quintessence will cluster on the scale of galaxy clusters so that such a current do not exist at all, our results here may from the contrary tell us the importance of such a current on the scale of galaxy clusters.

Comparing the number density v.s. temperature relations between the two values of $\beta$ case, we can see that this parameter also affects the relation strongly. This parameter $\beta$ has physical meaning of efficiency of energy transformation from thermal-dynamic to x-ray. So the lower is this efficiency, the less possibly a galaxy cluster will be observed if its mass is not big enough. While the larger is the clusters, the less the number density of them will be. In [21] and [22], the authors fit the number density v.s. temperature relations to numerical simulations to normalize this parameter and get that $\beta \approx 1.3 \sim 1.4$. By the notations of these works, the symbol $f_\beta$ is used to denote the same relevant quantity as our $\beta$, our $f_\beta = 1$, see the notations under eq(4) of [22].

Considering the time dependence of mass-temperature relation explained by [22], the value of this parameter is about 0.944 according to [22]. Since we assume that Quintessence clusters synchronously with ordinary matters, while the Q-component could not make contributions to the x-ray emission processes. We should expect an even lower value of $\beta$ than 0.944. In the numerical analysis in the next subsection, we will set $\beta = 0.75$, but will point out qualitatively the way a larger or smaller value of $\beta$ affects our conclusions.

In FIG.5 we displayed the effects of $w$ on the evolution of number density of massive galaxy clusters for one value of $M_0$. Note that

$$n_{M>M_0} = \int_{M_0}^{\infty} n(M', z) dM'$$

From the figure we can also see that the clustering of Quintessence affects the evolution of the number density of massive galaxy clusters exponentially, and that the larger is $\sigma_8$, the more remarkable the effects will be.

In this figure, what should interests us mostly may be, if $w$ is too greater than $-1$, the number density of galaxy clusters would increase as we look back to the past, while if $w$ is too less than $-1$, the contrary is the trend. And, the larger is $\sigma_8$, the strong this trend will be. Physically, this trend can be easily to understand. Because, the more $w \to 0$, the more weakly Quintessence anti-clusters, so the more usually emerging of galaxy clusters could take place which will lead to the decreasing of number of galaxy clusters. On the contrary, if $w < -1$, then the more $w \to -\infty$, the more strongly Quintessence anti-clusters, so the more often splitting of massive galaxy clusters will take place which will lead to the increasing of the number of galaxy clusters. According to the current observations [23]: as we look back to past, we see smaller and smaller number density of galaxy clusters. This may implies that the equation of state coefficients of dark energies should not be greater than $-1$ too much.

Note that, we do not directly consider the emerging and splitting of galaxy clusters. However, our explanations about the two contrary evolution trends of the number density of massive clusters above is still valid. Because, there are such configurations where initially ”two” clusters are closely located, if $w < -1$, then as time goes on, these two clusters go far and far and finally really become two clusters. However, if $-1 < w$, then as time goes on, these ”two” clusters will not separate until today and have to be counted as one. This difference of ”2”→2 and ”2”→1 processes can be sensed by Press-Schechter theory itself through the effects of $w$ on the key parameter $\delta_c$ displayed in FIG.3 with no use of direct consideration of galaxy clusters emerging and splitting processes [10, 17, 18].

C. Constraints From Observational Results

Although the clustering of dark energy on the scale of galaxy clusters is only a logically possible assumption. Since this assumption predicts that the number density of galaxy clusters and its evolutions are affected exponentially by the equation of state coefficient $w$ of dark energy. If theoretical predictions are to be coincide with observations, we should have constraints on $w, \Omega_{m0}$ and $\sigma_8$ which may be rather different from that comes from other observation such as CMB [10] or large scale matter power spectrum [11]. Does these observation leave spaces for our assumption?

Motivated by this fact, we try our best to fit the observation results of [6] (with a factor of 2 corrected by [7]) and [8] into theoretical formulaes to find constrains on $w, \Omega_{m0}$ and $\sigma_8$. Our fitting method is minimizing $\chi^2$, for details please refer to [4]. However, constrained by our computation powers, we do not vary all the parameters involved in this problem and look for the best composition. Instead, we choose to fit the measurements into theoretical formulaes in three cases. In the first case, we focus our attentions on $\sigma_8$ and $\Omega_{m0}$ but fix the other parameters. In the second case we focus our attention on $\sigma_8$.
and \( w \), while in the third case we focus our attentions on \( \Omega_{m0} \) and \( w \). We also tried best to fit the four parameters \( \frac{\beta}{s} \), \( w \), \( \Omega_{m0} \) and \( \sigma_8 \) simultaneously, but the convergence of the fitting result is so poor that we can not get any meaningful conclusions so we will not provide numerical results in this case.

In the data source we used in this paper, \( \frac{\beta}{s} \) is the observed number density v.s. temperature relation at \( z \approx 0.05 \). The data from this source only gives errors in the \( y \)(number density v.s. temperature function values) axis. We will fit it with our theoretical formulae eq(13). \( \frac{\beta}{s} \) is the observed number density v.s. red-shift relation of galaxy clusters whose comoving \( 1.5h^{-1}\text{Mpc} \) radius inside mass is greater than \( 8 \times 10^{14}h^{-1}M_\odot \). The data from this source have large errors in both the \( x \)(red-shift) and \( y \)(number density) axis. We will neglect its errors on the \( x \) axis although it is very large. We also neglecte the fact that the number density provided by this source is only that of the clusters whose comoving \( 1.5h^{-1}\text{Mpc} \) radius inside mass is greater than \( 8 \times 10^{14}h^{-1}M_\odot \) and think that is the number density of clusters whose total mass is greater than \( 8 \times 10^{14}h^{-1}M_\odot \). This approximation will make our fitted \( \sigma_8 \) greater than the real value. For other observational results in this area, please refer to [8] and the references therein.

For reasons explained in the above two paragraphs, we should not look at the numerical results of this subsection too seriously. What should be emphasized is the consisteny between our assumption that dark energy will cluster synchronously with ordinary matters and the observational results of WMAP and SDSS.

We display our confidence level analysis of the best fitting in the first case in FIG.6 where except \( \sigma_8 \) and \( \Omega_{m0} \), the other parameters are fixed as \( w = -1 \), \( h = 0.71 \), \( n_s = 1.0 \) and \( \frac{\beta}{s} = 0.75 \). In this figure there are two points which is worth noticing or could be criticized by peoples. The first is, even use only the observed number density v.s. temperature relation of [6] we determined the cosmological parameters \( \sigma_8 \) and \( \Omega_{m0} \) to some degree. This forms a strong contrast with the conclusion of [21] and [23], which says that only using this observational data we cannot determine any one of this two parameters but only a special composition of them. We find if we neglect the normalization factor of the growth factor \( D_1(1) \) in eq(2), we will qualitatively reproduce the results of [21] and [23], please see our FIG.7. We also find that to get the results of [21], Figure 1, we need to artificially set the initial value of either of the two parameters \( \delta_0 \) or \( \kappa \), eq(A16) and (A21) of [21]. The second important point which could be criticized by peoples is, our results indicate that \( \sigma_8 \approx 0.5 \), which is exceptional low comparing with the results of WMAP[10] and SDSS[11]. We think this is because when WMAP or SDSS make their best fittings, they did not consider the perturbation of dark energies. If the perturbation of dark energies are considered, we expect WMAP and SDSS would also predict a lower \( \sigma_8 \) than their current reported value. We note that the current version (V4.5.1) of the program CMBFast have been able to calculate the Cosmological Microwave Background Anisotropy (CMB) spectrums when dark energies are perturbed, but WMAP and SDSS did not use this version of CMBFast to fit their results.

As is well known, an exceptional low \( \sigma_8 \) implies a strongly biased universe, while \( \sigma_8 = 1 \) implies a non-biased universe, which has been observed by the galaxy astronomers. So our result indicate that, on the scale of galaxy clusters, the mass distribution is rather different from that on the scale of galaxies. This is just what we should expect, because we assume that on the scale of galaxy clusters, Quintessence clusters like ordinary matters, while on the scale of galaxies, the clustering effects of Quintessence are usually neglected by peoples. Physically this is because on the scale of galaxy clusters, Quintessence contribute a rather large part to the total energy of our studying objects, while on the scale of galaxies, the contributions of Quintessence to the total energy of the studying objects is really negligible. It should be noted that our conclusion \( \sigma_8 \approx 0.5 \) is obtained by fitting the observational result of the number density v.s. red-shift relation into theoretical formulaes and has no dependence on our choice of \( \frac{\beta}{s} \).

In FIG.8 we display the confidence level analysis of our best fitting in the second case where except \( \sigma_8 \) and \( w \) all the other parameters are fixed as \( \Omega_{m0} = 0.27 \), \( h = 0.71 \), \( n_s = 1.0 \) and \( \frac{\beta}{s} = 0.75 \). About this figure we would like give two comments. The first is, our results give an exceptional low value of \( \sigma_8 \), the physical reason of which we have explained in the previous paragraph. The second point is, although \( w \) affects the number density v.s.
FIG. 7: Still ΛCDM cosmology best fitting of galaxy clusters’ observation, but ignored the normalization denominator $D_1(1)$ of the growth factor appearing in eq(2). The meaning of different symbols appearing in the figure is the same as that of FIG. 6.

FIG. 8: QCDM cosmology best fitting of the galaxy clusters’ observation, except $w$ and $\sigma_8$, all the other parameters are fixed as $\Omega_{m0} = 0.27$, $\frac{\Omega_{k0}}{\Omega_{m0}} = 0.75$, $h = 0.71$ and $n_s = 1.0$. The +ed region denote the 68.3% confidence region of best fitting the number density v.s. red-shift relation reported by $\sigma_8$, while the ×ed region, the 99% confidence. If $\frac{\Omega_{k0}}{\Omega_{m0}}$ takes values greater than 0.75, the black boxed and +ed region moves down left.

V. CONCLUSIONS

We studied the top-hat spherical collapse model of galaxy clusters formation in the flat QCDM or Phantom-CDM cosmologies under the assumption that Quintessence or Phantom clusters or anti-clusters like ordinary matters. We found that under this assumption, the key parameters of the model exhibit rather non-trivial and remarkable dependence on the equation of state coefficients $w$ of Quintessence or Phantoms. We then applied the results in Press-Schecter theory and calculated the number density v.s. temperature function and the evolution of the number density of massive galaxy clusters and found that these two Quantities are both affected by $w$ exponentially. For the number density v.s. temperature function of galaxy clusters, we found that the nearer $w \to 0$, the larger the function value will be, the more $w \to -\infty$, the smaller the function value. While for the evolution of the number density of massive galaxy clusters, we found that if $w$ is too less than $-1$, the number density de-
creases as we look back to the past. On the contrary, if \( w \) is too greater than \(-1\), the number density of massive galaxy clusters increases as we look back. Using this fact, we studied the possibility of determining \( w \) by the observational results of galaxy clusters. Constrained by our computation powers and the big errors of the observational data, we cannot determine all the parameters involved in this problem independently. However, on the basis some priors, we get that to 99% confidence level, 
\[
 w = -1.08 \pm 0.09 .
\]
On the other hand, if we fix \( w = -1 \) as priors, we find that \( \sigma_8 \approx 0.5 \) which is exceptional low comparing with the reported value of WMAP and SDSS. This is because WMAP and SDSS both did not consider the perturbation of dark energies.

Since from the beginning, we assume that dark energy will cluster synchronously with ordinary matters, we have no dark energy current flowing outside over-dense region galaxy clusters. Although this is too strong an assumption, this assumption simplifies our discussions and makes it a self-consistent one. As a comparison, we point out that the discussions in the previous literatures which assume that Quintessence does not cluster on the scale of galaxy clusters but neglect the Q-current flowing outside the over-dense region is not self-consistent and the conclusions of these literatures are worth further explorations. Our studies do not correct this problem directly, but may from the contrary indicate the importance of such a current’s effects on the number density of galaxy clusters and its evolutions.

In Ref. 28, we will dispose the assumption that dark energy cluster synchronously with ordinary matters and will find that, in that case, the equation of state coefficients of dark energy affects the number-density of galaxy cluster even more strongly!

As a discussion, we would like to point out that the actual case should be, dark energy should cluster on the scale of galaxy clusters but not do so synchronously with ordinary matters. So that is a case lies between the synchronous clustering with ordinary matter and not clustering on the scale of galaxy clusters at all. Whatever the actual way is, the equation of state coefficients should affect the number density of galaxy clusters remarkably. So measuring the number density of galaxy clusters and its evolutions is a potential effective way to determine the equation of state coefficients.

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