Integrated Optimization of Traffic Signals and Vehicle Trajectories at Intersection With the Consideration of Safety During Signal Change

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This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFB1600600, and in part by the State Key Program of National Nature Science of China under Grant 51238008.

ABSTRACT
This paper develops an integrated optimization of traffic signals and vehicle trajectories. The signal is optimized to improve the intersection efficiency and the calculation of intergreen interval (IGI) serves as constraints to guarantee vehicle safety during signal changing. Then the vehicle trajectory in the approach lane and inside intersection is optimized to increase fuel efficiency. The proposed method is evaluated by microscopic simulation, comparing with the actuated signal control (ASC) method and an ad hoc cooperation method between traffic signals and vehicles. Results indicate the proposed control algorithm is effective to prevent conflicts during signal changing periods. Operation Efficiency and fuel efficiency are improved. The benefit is 24.9% on vehicle delay, and is 5.5% on fuel efficiency. The proposed system can potentially be used in real-time.

INDEX TERMS
Connected and autonomous vehicle, intersection safety, signal optimization, trajectory optimization.

I. INTRODUCTION
Accidents at intersections have been a serious problem. Researches show 22% of total crashes happen at intersections in the United States [1] and around 30% of urban traffic accidents took place at or near signalized intersections in China [2]. Signalized intersections play a significant role in the urban roadway network [3], [4]. The present study avoids vehicle collisions at signalized intersections through precise signal control, most notably by optimizing intergreen interval (IGI) [5]. IGI is generally defined as the interval between the end of the green time of the last traffic stream and the beginning of the green time of the next [6]. Inadequate IGI leads to potential conflicts for the vehicles from the next phase may drive into the conflicting point before the vehicles from the previous phase leaves. Overlong IGI is a waste of time [7]. Therefore, the design of IGI should be optimized for efficiency and ensuring safety.

In a conventional traffic environment with human-driven vehicles, IGI equals the sum of the yellow signal and all-red signal (red clearance interval). There are two categories of design methods for IGI based on different critical conditions. The first category is based on the maximum safety condition, which is widely applied in America [8] and Japan [7]. In the first category, vehicles from the next phase are allowed to pass the stop line only after the last vehicle from the last phase passes through the whole intersection [9]. The second category is based on the maximum efficiency condition, which is applied in German [10]. In this category, the first vehicle of the next phase can arrive at the conflict area after the last vehicle of the last phase passes the conflicting area. In both two categories, IGI design depends on vehicle speed inside the intersection. Researches often calibrate vehicle speed using aggregated historical data. This simplification results in safety risks.

Thanks to the technologies of the connected and autonomous vehicle (CAV) which collect detailed vehicle information, a new source of data for traffic control can be obtained for signal control [11]. This data helps the design
of IGI. But current researches of signal timing under CAV technology ignore the design of IGI. They simply choose the suggested value in conventional traffic environment such as 3s [12], [13], 4s [14], [15] or 6s [16]. This simplification will lead to both safety and efficiency problems in practical application. If the last vehicle passes the intersection with low speed, the constant IGI is sometimes insufficient for its leaving. This may lead to lateral conflict between the last vehicle and vehicles from the next conflicting phase. In contrast, if vehicles can quickly pass the intersection than average, or there is not any traffic in one phase at all, the constant IGI is a waste of time.

To summarize, previous studies have not specifically focus on safety during signal change. IGI was designed without considering vehicle states and intersection characteristics in these autonomous intersection management methods. However, the design of IGI is crucial for it affects both safety and efficiency. Thus, the objective of this research is to establish an integrated optimization of traffic signals and vehicle trajectories at an intersection which ensures the safety of vehicle operation during signal changing. Besides, this method must be fast enough for potential real-time implementation.

The remainder of the paper is organized as follows: Section 2 presents the problem and the key notations. Section 3 describes the appliance of IGI calculation in the integrated optimization of traffic signals and vehicle trajectories. Section 4 conducts a simulation evaluation. Finally, conclusions are delivered in Section 5.

II. NOTATIONS AND PROBLEM DESCRIPTION

A. NOTATIONS

Notations used in the paper are summarized in Table 1.

| Symbol | Description | Unit |
|--------|-------------|------|
| w      | Vehicle index | |
| Ω      | Set of vehicles inside the control zone at time t | |
| k      | Approach lane index | |
| K      | Set of approach lanes | |
| l_c    | Length of the control zone | m |
| d_1,k  | The distance between the stop line of approach lane k and the conflict point of approach lane k_1 along the longitudinal vehicle trajectory | m |
| D_w    | Delay of vehicle w | s |
| θ_l,k  | The start time of the l^th green light for the approach lane k | s |
| θ_l    | The start time of the last green light in defined signal timing in approach lane k | s |
| G_l,k  | Duration of the l^th green light in approach lane k | s |
| G_l    | The duration of the last green light in defined signal timing in approach lane k | s |
| G_max  | The maximum green duration | s |
| G_min  | The minimum green duration | s |
| N      | The number of the green periods considered in the optimization model for an approach lane | |
| π_u,k  | The intergreen interval between approach lane k and k_1 when the green period of approach lane k_1 is after the green period of approach lane k | s |
| t_0    | Start time of simulation | s |
| t_p    | The time when the optimization starts | s |
| t_p^*  | The predicted time when vehicle w passes the stop line | s |
| t_l^*  | The predicted time when vehicle w leaves the intersection | s |
| t_e    | The earliest arrival time of vehicle w to the stop line without consideration of afore vehicle and signal | s |
| t_c    | The time required for a vehicle to cross another vehicle safely | s |
| Δ_k    | The time of vehicles from approach lane k driving from the stop line to the conliction point | s |
| ν_l    | Speed of vehicle w at time t | m/s |
| ν_l^*  | The possible arrival speed of vehicle w without consideration of afore vehicle and signal | m/s |
| ν_l^+  | The predicted speed when vehicle w pass the stop line | m/s |
| V_l    | Speed limit at the approach line | m/s |
| V_l^+  | Speed limit inside the intersection for approach lane k | m/s |
| a_l    | Accelerate of vehicle w at time t | m/s² |
| a_max  | Maximum comfortable acceleration rate | m/s² |
| a_min  | Maximum comfortable deceleration rate | m/s² |
| x_l    | Distance traveled by vehicle w inside the control zone at time t | m |
| h_l    | The minimum time headway of a CAV to its preceding vehicle | s |
| τ_l    | Time displacement of vehicles in Newell’s car-following model | s |
| d_l    | Space displacement of vehicles in Newell’s car-following model | m |
| Y_{l1} | Time displacement of vehicles in Newell’s car-following model | m |
| ϕ_{l1} | 1, if the movements of the approach lane k and k_1 are incompatible; 0, otherwise | |
| ϕ_{l1}^+ | 1, if the green light starts at Θ_{l1}^+ is after the incompatible green light starts at Θ_{l1}; 0, otherwise | |
| ϕ_{l1}^* | 1, if vehicle w pass the stop line during | |
| ρ_l^+  | [Θ_{l1}, Θ_{l1}^+]; 0, otherwise | |
| T_l    | Sampling time step | s |
| Γ     | Planning horizon length | s |
| Γ_c    | The shortest signal reserve time | s |

B. CONTROL STRUCTURE

In this paper, a typical signalized intersection is shown in Fig. 1. All vehicles are CAVs. Control zone refers to the area where signal timings and vehicle trajectories get optimized. It is assumed that vehicles and the Intersection Manager (IM) can communicate with each other inside the control zone. Each vehicle gets trajectory guidance from IM and follows it.

Signal control is adopted for intersection control in this paper. Signal control guarantees non-motorized vehicles and pedestrians crossing the intersection and facilitates the
application of this method in the mixed-traffic environment in the future.

As shown in Fig. 2, the proposed control structure is divided into two modules. Module 1 focuses on efficiency improvement. It completes the signal optimization. Module 2 optimizes the specific trajectory of each vehicle to reduce fuel consumption. The optimal control system is activated when a vehicle drives into the control zone.

Module 1: Signal optimization: Based on the current state of vehicles, this module aims at letting all vehicles pass the intersection as early as possible. Safety during signal change is guaranteed by detailing IGI design. This module aims at optimizing the efficiency of the intersection.

Module 2: Trajectory optimization: After the signal plan is determined, IM optimizes the trajectory of each vehicle based on the required arrival time and the trajectory plan of the preceding vehicle. The trajectory plan received by each vehicle guides them to safely cross the intersection during the green light. This module aims at reducing fuel consumption by optimized trajectories.

III. METHODOLOGY
In this section, the formulation of Module 1 is entailed in Section 3.1, and Module 2 is entailed in Section 3.2. The planning horizon procedure is presented in Section 3.3.

A. SIGNAL OPTIMIZATION
This study first optimizes the signal timing plan. The signal plan determines the time of each vehicle to cross the intersection and is therefore related to the efficiency. The input to this section is the current vehicle status, including position and speed. This section generates an optimal signal plan and the arrival time of vehicles in $\Omega_i$. The arrival time information is then sent to module 2.

1) MODEL OBJECTIVE
Previous studies of signal optimization usually adopt a phase-based signal structure. The movement of vehicles from several approach lane are combined into phases, such as the order-fixed four-phase structure [12], [17]–[19], order-unfixed four-phase structure [15], [20], and the standard North American NEMA dual-ring, eight-phase control system [21]–[23]. These signal structures limit the flexibility of phase sequence optimization.

This paper adopts a flexible signal structure [24]. A phase is defined as a green timing unit of one approach lane. If vehicles from lane $k_1$ have no conflict with the vehicle from lane $k_2$, the phase of lane $k_1$ can associate with the phase of lane $k_2$. Therefore, a wider variety of phase associations are possible [14], [25].

The model objective is minimizing total vehicle delay. The definition of vehicle delay $D_w$ is shown in Fig.3. The signal plan $S = \{\Theta_k^i, G_k^i\}$, $\forall k \in K, i = 1, \ldots, N$ and the arrival states of vehicles $A_w = \{t_w^a, v_w^a\}$, $\forall w \in \Omega$ are formulated as the solution to the following optimal control problem $P1$.

$$P1 : \min_{S} \sum_{w \in \Omega} D_w$$

$$D_w = t_w^a - t_w^e$$
2) SIGNAL SETTINGS
This section summarizes the basic settings for the signal optimization model. For each approach lane \( k \), (3) shows each green time meets the constraint of maximum/minimum green time.

\[
G_{\text{min}} \leq G_k^i \leq G_{\text{max}}, \quad \forall k \in K; \quad i = 1, \ldots, N
\]  

(3)

For different approach lanes \( k_1 \) and \( k_2 \), (4) and (5) show a certain interval lying between the green light of conflicting approach lanes, and there is no limit to the green light of non-conflicting approach lanes.

\[
\Theta_{k_2}^j \geq \Theta^{j}_{k_1} + G_{k_1}^i + \pi_{k_1,k_2} - M \left( 2 - \gamma_{k_1,k_2} - \phi_{k_1,k_2}^{i,j} \right), \\
\forall k_1, k_2 \in K; \quad i = 1, \ldots, N; \quad j = 1, \ldots, N
\]  

(4)

\[
\Theta_{k_2}^j + G_{k_2}^i \leq \Theta^{j}_{k_1} + G_{k_1}^i + \pi_{k_1,k_2} - M \left( 1 - \gamma_{k_1,k_2} + \phi_{k_1,k_2}^{i,j} \right), \\
\forall k_1, k_2 \in K; \quad i = 1, \ldots, N; \quad j = 1, \ldots, N
\]  

(5)

where \( M \) is a sufficiently big number. \( \gamma_{k_1,k_2} = 1 \), if the movements of the approach lane \( k_1 \) and \( k_2 \) are incompatible; \( \gamma_{k_1,k_2} = 0 \), otherwise. \( \phi_{k_1,k_2}^{i,j} = 1 \), if the green light starts at \( \Theta_{k_1}^j \) is after the incompatible green light starts at \( \Theta_{k_1}^{j-1} \); \( \phi_{k_1,k_2}^{i,j} = 0 \), otherwise.

Meanwhile, the optimization results cannot conflict with the previously determined signal plan.

\[
\Theta_{k_2}^j \geq \Theta^{j}_{k_1} + \pi_{k_1,k_2} - M (1 - \gamma_{k_1,k_2}), \\
\forall k_1, k_2 \in K; \quad i = 1, \ldots, N; \quad j = 1, \ldots, N
\]  

(6)

3) PREDICTION OF VEHICLE STATE IN THE INTERSECTION
This section first calculates the earliest time of each vehicle \( w \) to arrive at the stop line \( t_w^e \). Its calculation has two cases, depending on whether the vehicle can accelerate to the speed limit.

Case 1. the vehicle speed can reach \( V_{\text{max}} \) before entering the intersection.

\[
t_w^e = t_0 + \Delta t_{0-1} + \Delta t_{1-2} + \Delta t_{2-3}
\]  

(7)

\[
\Delta t_{0-1} = \frac{V_{\text{max}} - v_{t_0}^w}{a_{\text{max}}}
\]  

(8)

\[
\Delta t_{2-3} = \frac{V_{\text{max}} - a_{\text{max}}}{a_{\text{min}}}
\]  

(9)

\[
L_c = x_{t_0}^w + \int_{t_0}^{t_1} v_{t_0}^w + a_{\text{max}} t dt + \Delta t_{1-2} V_{\text{max}} + \int_{t_1}^{t_2} V_{\text{max}} + a_{\text{min}} t dt
\]  

(10)

Case 2. the vehicle speed can never reach \( V_{\text{max}} \) before entering the intersection.

\[
t_w^e = t_0 + \Delta t_{0-1} + \Delta t_{1-2}
\]  

(11)

\[
v_{t_0}^w + a_{\text{max}} \Delta t_{0-1} = V_{\text{max}}^k + a_{\text{min}} \Delta t_{1-2}
\]  

(12)

4) GREEN INTERVAL CALCULATION
IGI aims at ensuring the safety of conflicting vehicles from adjacent green periods. The calculation method in this paper set the critical condition based on the maximum efficiency condition. For example, vehicle \( w_1 \) from approach lane \( k_1 \) get right of way before vehicle \( w_2 \) from approach lane \( k_2 \) in Fig. 4. Thus, to avoid a crash at the conflict point, their departure time should meet:

\[
r_{w_1}^{G_1} \geq x_{t_0}^{w_1} + \int_{t_0}^{t_1} v_{t_0}^{w_1} + a_{\text{max}} t dt + \Delta t_{1-2} V_{\text{max}} + \int_{t_1}^{t_2} V_{\text{max}} + a_{\text{min}} t dt
\]  

(13)

\[
t_w^e \geq t_w^w + \Delta t_c
\]  

(14)

Then, take the preceding vehicle \( w' \) into consideration. Each CAV needs to keep a stationary time headway with its preceding vehicle.

\[
t_w^e \geq t_w^w + h_c
\]  

(15)

where \( h_c \) is the minimum time headway of a CAV to its preceding vehicle.

Finally, take the signal into consideration. Each vehicle only passes the stop line during the green light.

\[
\Theta_k^i + M \left( 1 - \beta_{w}^i \right) \leq t_w^e \leq \Theta_k^i + G_k^i + M \left( 1 - \beta_{w}^i \right), \\
\forall i = 1, \ldots, N; \quad w \in S_p
\]  

(16)

\[
\sum_{i=1}^{N} \beta_{w}^i = 1, \quad \forall w \in S_p
\]  

(17)

(16) and (17) indicate at which green light the vehicle \( w \) passes the stop line. \( \beta_{w}^i = 1 \) if vehicle \( w \) pass the stop line during \( \Theta_k^i, \Theta_k^{i+1} \); otherwise, \( \beta_{w}^i = 0 \).

The arrival of CAVs is free from the influence of afore vehicle and the signal for CAVs can adjust their speed in the control zone. Thus, the predicted speed when vehicle \( w \) passes the stop line \( v_w^e \) is:

\[
v_w^e = \min \left\{ v_{t_0}^w + a_{\text{max}} \left( t_w^e - t_0 \right), V_{\text{max}} \right\}
\]  

(18)
After vehicles pass the stop line, they speed up to the speed limit of intersection and then pass the intersection as quickly as possible. Therefore, $\Delta t_{k_1}$ and $\Delta t_{k_2}$ are calculated as:

$$
\Delta t_k = \frac{V_{k_{\text{max}}} - v^{w}_{t}}{a_{\text{max}}} + \frac{d_{k_1,k_2} - \frac{v^{w}_{\text{max}} - v^{2}_{\text{max}}}{2a_{\text{max}}}}{V^{k}_{\text{max}}}
$$

where $d_{k_1,k_2}$ is the distance between the stop line of approach lane $k_1$ and the conflict point of approach lane $k_1$ and $k_2$ along the longitudinal vehicle trajectory. The vehicle trajectory inside the intersection is determined using the method in [26].

B. TRAJECTORY OPTIMIZATION

In this section, a trajectory optimization model is formulated to reduce fuel consumption and improve smoothness.

The trajectory optimization model is responsive. This module is activated when a new vehicle enters the control zone or the signal plan updates. When a new vehicle $w$ enters the control zone, the model first predicts the time when vehicle $w$ passes the stop line $t^{w}_{s}$ based on the current signal plan, similar to the steps in Section 3.1.3. Then the trajectory plan with the lowest fuel consumption is optimized for vehicle $w$. When the signal plan is updated, the trajectory plans are also updated according to the new signal plan and the arrival state through the stop line of the vehicles optimized in $P_1$.

Previous studies optimized the trajectory of vehicles at the approach lanes whereas neglecting the trajectory inside the intersection [14], [27], [28]. Nonetheless, the trajectory inside the intersection is not only directly related to whether the vehicle will collide inside the intersection, but also affects the intersection efficiency [29]. Thus, the trajectory of CAVs in the approach lane and inside the intersection is optimized in this section.

The trajectory optimization is formulated as the solution to the following optimization problem $P_2$. The optimized trajectory is designed based on CAV’s current status $\alpha^w_t = \{[v_{t-1}^w, v_{t-1}^w, \alpha_{t-1}^w]\}$, $\forall t \in [t^w_{\text{op}}, t^w_{\text{f}}]$ and future arrival state.

$$
P_2 : \min_{\alpha^w_t \in \Omega^w_{\text{op}}} \sum_{w \in \Omega^w_{\text{op}}} \sum_{t \in [t^w_{\text{op}}, t^w_{\text{f}}]} |\alpha^w_t| \, dt,
$$

subject to

$$
0 \leq v^{w}_{t} \leq V^{w}_{\text{max}}, \forall t \in [t^w_{\text{op}}, t^w_{\text{f}}]
$$

$$
0 \leq a^{w}_{t} \leq a^{w}_{\text{max}}, \forall t \in [t^w_{\text{op}}, t^w_{\text{f}}]
$$

$$
\alpha^{w}_{\text{min}} \leq \alpha^{w}_{t} \leq \alpha^{w}_{\text{max}}, \forall t \in [t^w_{\text{op}}, t^w_{\text{f}}]
$$

$$
x^{w}_{t} + d \leq x^{r}_{t-1}, \forall t \in [t^w_{\text{op}}, t^w_{\text{f}}]
$$

$$
x^{w}_{t_{\text{f}}} = L_e
$$

$$
v^{w}_{t_{\text{f}}} = v^{w}_{a}
$$

The objective function (22) aims to minimize the fuel consumption of each CAV. It is a simplified representation of fuel consumption commonly applied in optimal control theory [14], [30]. (23) and (24) describe vehicle dynamics. (25) (26) and (27) define the thresholds of the driving speed and acceleration. (28) guarantees safe movements for vehicle $w$ and its predecessor vehicle $w'$.

C. ITERATIVE SOLVING PROCESS

The flow chart in Fig. 6 summarizes the overall procedure of the proposed integrated optimization.

As shown in Fig. 7, the signal optimization model starts every other period $\Gamma_w$. In each optimization, the planning horizon length is $\Gamma_{\text{op}}$. The green lights started before $t_{\text{op}} + \Gamma_{\text{op}}$ are reserved. All vehicles can pass the intersection within the planning horizon.
IV. SIMULATION EVALUATION

A. EXPERIMENTAL DESIGN

The intersection safety, efficiency, and fuel efficiency are evaluated to verify the improvement of the proposed method. The proposed integrated optimization (Proposed) is compared with two benchmark intersection control methodologies:

1) Actuated signal control (ASC). All the control structure and optimization parameters are the same as the proposed system, except for the IGI. Two widely adopted values of IGI are chosen. Benchmark 1: IGI = 3 s. Benchmark 2 IGI = 4 s.

2) Cooperation between traffic signals and vehicles (CTV). A control method proposed in [31]. When comparing with CTV, the experimental design keeps consistent with the settings in [31] such as vehicle input and speed limit.

To evaluate the proposed intersection management, a typical intersection with four approach lanes shown in Fig. 1 is applied. Only left-turning and through vehicles are considered, for right-turning vehicles are not controlled by the signal. The length of the control zone is set as 300 meters. The length is select to be consistent with the reliable communication range of Dedicated Short-Range Communications (DSRC). Vehicles are generated according to Poisson distribution. Speed when vehicles enter the control zone $v_0$ follows a normal distribution. $v_0 \sim N(15, 0.7)$. The constraint of turning velocity is set based on the intersection design manual [32], [33].

The proposed control system is written in Python. The experiment adopts Gurobi 9.0 [34] on a computer with Intel® Core™ i5 – 1.80GHz to solve the two models $P_1$ and $P_2$. The settings for all parameters adopted are presented in Table. 2.

B. SIMULATION RESULTS AND DISCUSSIONS

The simulation results confirm the proposed approach is superior to the ASC method in safety, efficiency, and fuel efficiency. Besides, the proposed approach has larger throughput and fuel efficiency than the CTV method.

The proposed system can potentially be used in real-time. The computational time of $P_1$ is 0.46 s, given a 15 s optimization interval and 0.1 s time step. Fig. 8 shows the computational time of $P_1$ can be further reduced with shorter optimization time horizon. The computational time of $P_2$ is 0.0020 s, given a 0.1 s time step. Fig. 9 shows the computational time of $P_2$ can be further reduced with greater time step size.

1) SAFETY

In safety measurement based on CAV trajectories, the post encroachment time (PET) is examined using the surrogate safety assessment model (SSAM) software [35]. PET denotes the time between the departure of the encroaching vehicle from the conflict point and the arrival of the vehicle with the
right-of-way at the conflict point. A lower PET indicates a higher probability of a collision [36]. The safety threshold of PET set in this paper is 1.5s considering vehicle speed inside the intersection [37]. A PET less than the threshold is identified as one critical conflict.

Fig. 10 shows the PET of the proposed method meets the safety requirement. Under the control of ASC (IGI = 3 s), sometimes PET between conflict vehicles is less than the safety threshold. Collision may happen in these situations. Under the control of ASC (IGI = 4 s), the PET keeps higher than the threshold. This means high safety but also leads to a waste of time.

Table 3 shows the optimized IGI in the proposed approach varies from 0.84 to 3.14. The variation is caused by different phase sequences. For example, the maximum IGI in the proposed evaluation is 3.14. It occurs when the green light of approach lane \( k_1 = 2 \) is right after the green light of approach lane \( k_2 = 5 \). It takes a long time for the left-turn vehicle from approach lane 5 to pass the conflict point between approach lane 5 and 2. Thus, the first vehicle from approach lane 2 needs to wait for a longer time, which means a larger IGI.

### TABLE 2. Attributes in the simulation.

| Parameter                               | Value | Unit |
|-----------------------------------------|-------|------|
| Width of the approach lane              | 3     | m    |
| Simulation time horizon                 | 2000  | s    |
| Warming-up time                         | 1000  | s    |
| Speed limit \( V_{\text{max}} \)        | 15    | m/s  |
| Speed limit inside the intersection \( V_{\text{max}}^l \) for left-turn | 8    | m/s  |
| Speed limit inside the intersection \( V_{\text{max}}^t \) for through | 14   | m/s  |
| Maximum acceleration \( a_{\text{max}} \) | 2    | m/s\(^t\) |
| Minimum deceleration \( a_{\text{min}} \) | -4   | m/s\(^t\) |
| Time displacement \( r \) in Newell car-following model | 2    | s    |
| Space displacement \( d \) in Newell car-following model | 4    | m    |
| The minimum time headway of a CAV \( h_{\text{cav}} \) | 1.5   | s    |
| Planning horizon length \( T_{\text{p}} \) | 60   | s    |
| Optimization interval \( T_{\text{o}} \) | 15   | s    |
| Sampling time step \( T_{\text{s}} \) | 0.1  | s    |

2) **EFFICIENCY**

The result shows the proposed approach has higher intersection efficiency than the benchmark. Fig. 11 and Fig. 12 show the proposed approach increase the capacity of the intersection and it can decrease the average vehicle delay to 24.9%
on average when compared with ASC. Fig. 13 shows the proposed approach results in a 10.1% larger throughput than CTV at the proposed situation in [31]. This can be explained as follows. First, the proposed approach has a larger solution space in signal timing results from the flexible signal structure. Besides, the efficiency is inversely proportional to IGI since IGI is a waste of effective green time. The average IGI of the proposed method is 1.79 s for the proposed optimization method prefers the phase sequence with smaller IGI freely. Presented in Fig. 14, the small IGI appears more frequently in the optimized signal timing than large IGI. A small IGI can advance the vehicle’s arrival time, thereby reducing delays.

3) FUEL EFFICIENCY

Fig. 15 shows the proposed approach increases fuel efficiency by 5.5% averagely. On the one hand, trajectory optimization allows each vehicle to pass through the intersection without braking and idling, which causes larger fuel consumption. On the other hand, the proposed method improves efficiency means a higher average vehicle speed. Therefore, fuel efficiency is increased.

V. CONCLUSION

This paper develops a signal-trajectory optimization method for the signalized intersection under the autonomous and connected vehicles. In the signal optimization module, the signal timing problem with high flexibility in signal change is first formulated. The passing state of each CAV and their behavior inside the intersection is predicted. A dynamic IGI calculation is added to signal optimization to guarantee vehicle safety during signal change. In the trajectory optimization module, the trajectory of the vehicle at the approach lane and inside the intersection is optimized. This system makes use of the connected vehicle technology and uses present information as optimization input, which includes vehicle speed, location, road status, and dynamic speed limit.

The evaluation result of the proposed approach is compared with the performance of two benchmarks: the actuated signal control (ASC) and a cooperation method between the

![FIGURE 12. Average delay benefit comparing with ASC at different intersection traffic demands.](image)

![FIGURE 13. Throughput benefit comparing with CTV.](image)

![FIGURE 14. The relationship between the IGI and the number of its occurrences in the simulation.](image)

![FIGURE 15. Fuel efficiency benefit comparing with ASC at different intersection traffic demands.](image)

| Average IGI (s) | Approach lane $k_1$ | Approach lane $k_2$ |
|----------------|---------------------|---------------------|
| 1              | 2.87                | 0.84 1.38 1.97      |
| 2              | 2.01 1.11           | 2.87                |
| 3              | 1.38 1.97 2.87      | 0.84                |
| 4              | 2.61 0.84 1.38 1.97 | 2.87 0.84            |
| 5              | 3.14 2.61 2.01 1.11 | 3.14 1.11 3.14      |
| 6              | 2.87 0.84 1.38 1.97 | 2.87                |
| 7              | 2.01 1.11 3.14 2.61 |                     |
| 8              |                     |                     |
traffic signal and vehicles (CTV) in ad hoc research. Results showed:

1. The proposed method meets the safety requirements by the dynamic design of IGI. The IGI varies from 0.84 to 3.14 to ensure the safety of conflicting vehicles during different signal changes.

2. The proposed method has a higher efficiency than ASC and CTV. The proposed approach decreases the average vehicle delay to 24.9% comparing with ASC and the proposed approach results in a 10.1% larger throughput than CTV.

3. The proposed approach increases fuel efficiency by 5.5% averagely.

4. The computational time of signal optimization is up to 0.46 s, given a 60 s optimization time horizon and 0.1 s time step. The computational time of trajectory optimization is up to 0.0020 s, given a 0.1 s time step. The proposed approach can potentially be used in real-time.

The on-going work of this study could investigate adopting the proposed approach under a partially CAV environment. Future study should focus on safety during the signal change to safeguard the implementation ability of intersection management.

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