Neutrino-Induced Coherent Pion Production

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\textbf{Introduction}

- \(\nu\)-induced \textbf{coherent} \(\pi\) production reactions:
  - \textbf{Charged Current} \(\nu_l A \rightarrow l^- \pi^+ A\)
  - \textbf{Neutral Current} \(\nu_l A \rightarrow \nu_l \pi^0 A\)

- Important for oscillation experiments: systematic uncertainties
  - Example: \(\nu_l A \rightarrow \nu_l \pi^0 X\) ← background for \(\nu_e\) appearance

- Also interesting for hadronic and nuclear physics:
  - N, N-R axial form factors
  - Nuclear correlations
  - \(\pi\) in the nuclear medium

- Measured at \textbf{high energies} \(E_\nu > 2\) GeV (FNAL, GGM, SKAT, BEBC, ...)

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Introduction

- G. Zeller, hep-ex/0312061 (NUINT 02)
- Data scaled to $^{12}\text{C}$ assuming $A^{1/3}$ dependence
- $\sigma(\text{CC})=2 \cdot \sigma(\text{NC})$

Measured at high energies $E_\nu > 2$ GeV (FNAL, GGM, SKAT, BEBC, ...)

CC Coherent Pion Production Cross Section

- $\nu^- + A \rightarrow \mu^- + \pi^0 + A$

$\sigma(10^{-40} \text{cm}^2)^{12}\text{C}$ NUCLEUS

$\sigma (E_\nu (\text{GeV}))$

$\sigma (\text{cm}^2)/12$ C

NUANCE

NEUGEN

 FNAL (CC), Wilocz, Phys. Rev. D47, 2661 (1993)
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 GGM (NC), Isosol, Phys. Rev. Lett. 52, 1096 (1984)
 SKAT (CC), Grabosch, Z. Phys. C31, 203 (1986)
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 BEBC (CC), Marage, Z. Phys. C43, 523 (1989)
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Introduction

- Measured at high energies $E_\nu > 2$ GeV (FNAL, GGM, SKAT, BEBC, ...)
- These data are well described by models based on PCAC
  Rhein & Sehgal, NPB 223 (83)
- However, at low energies ...

- Data scaled to $^{12}$C assuming $A^{1/3}$ dependence
- $\sigma(\text{CC}) = 2 \sigma(\text{NC})$

G. Zeller, hep-ex/0312061 (NUINT 02)
Introduction

- K2K ($<E_\nu>=1.3$ GeV) finds a significant deficit of $\mu^-$ at forward angles
- Upper bound for CC Coh. $\pi^+$ production below theoretical expectations
- MiniBooNE ($<E_\nu>=0.75$ GeV) NC $\pi^0$ data set is under analysis

- **Our goal**: theoretical study of CC and NC Coherent $\pi$ production at intermediate energies ($E_\nu \sim 1$ GeV) improving the calculations of: Kelkar et al, PRC 55 (97), Singh et al, PRL 96 (06)
  - Complete relativistic elementary amplitude
  - Hadronic degrees of freedom: $\pi$, $N$, $\Delta(1232)$
  - Renormalization of the $\Delta$ properties in the nuclear medium
  - Realistic treatment of $\pi$ distortion
The model

- Elementary mechanisms ([Hernandez, Nieves & Valverde, hep-ph/071149](http://example.com)): 

![Diagram of elementary mechanisms](attachment:image.png)

- Other contributions

![Diagram of other contributions](attachment:image.png)

cancel for isospin symmetric nuclei
The model

Elementary mechanisms (Hernandez, Nieves & Valverde, hep-ph/071149):

\[ W, Z \quad \pi \quad W, Z \quad \pi \quad W, Z \quad \pi \quad W, Z \quad \pi \]

\[ N \quad N \quad N \quad N \quad N \quad \Delta \quad N \quad \Delta \quad N \]
The model

Elementary mechanisms (Hernandez, Nieves & Valverde, hep-ph/071149):

large cancellation
The model

Elementary mechanisms (Hernandez, Nieves & Valverde, hep-ph/071149):

- Large cancellation
- Small
The model

- Elementary mechanisms (Hernandez, Nieves & Valverde, hep-ph/071149):

- Large cancellation

- Dominant

- Small
The amplitude for $\text{CC } \pi^+$ production: $\mathcal{M}_C = \frac{G}{\sqrt{2}} \cos \theta_C l_\mu J^\mu$

$J^\mu \leftarrow \text{Nuclear current} \leftrightarrow \text{sum over all nucleons}$

For the dominant direct $\Delta$ mechanism:

$J^{\mu}_{IA} = -\frac{\sqrt{3}}{2} i \int d\vec{r} e^{i(\vec{q} - \vec{p}_\pi) \cdot \vec{r}} \left[ \rho_p(r) + \frac{\rho_n(r)}{3} \right] \frac{f^*}{m_\pi} D_\Delta p_\pi^\alpha \text{ Tr} \left\{ \bar{u} \Lambda_\alpha \beta \mathcal{A}^\beta \mu u \right\}$

$D_\Delta \leftarrow \text{propagator} \quad \Lambda_\alpha \beta \leftarrow \text{spin 3/2 projection operator}$

$\mathcal{A}^\beta \mu = \left( \frac{C^V_3}{M} (g^{\beta \mu} q - q^\beta \gamma^\mu) + \frac{C^V_4}{M^2} (g^{\beta \mu} q \cdot p' - q^\beta p'^\mu) + \frac{C^V_5}{M^2} (g^{\beta \mu} q \cdot p - q^\beta p^\mu) + g^{\beta \mu} C^V_6 \right) \gamma_5$

$+ \frac{C^A_3}{M} (g^{\beta \mu} q - q^\beta \gamma^\mu) + \frac{C^A_4}{M^2} (g^{\beta \mu} q \cdot p' - q^\beta p'^\mu) + C^A_5 g^{\beta \mu} + \frac{C^A_6}{M^2} q^\beta q^\mu$

Form factors:

$C^V_{3,4,5} \leftarrow \text{e N scattering} \quad C^V_6 = 0 \leftarrow \text{CVC}$

$C^A_6 = C^A_5 \frac{M^2}{m_\pi^2 - q^2} \quad C^A_5(0) = \frac{g_{\Delta N \pi} f_{\pi}}{\sqrt{6} M} \approx 1.2 \leftarrow \text{PCAC}$

$C^A_4 = -\frac{1}{4} C^A_5 \quad C^A_3 = 0 \leftarrow \text{Adler model}$
Formalism

Delta in the medium:

\[
D_{\Delta} \Rightarrow \tilde{D}_{\Delta}(r) = \frac{1}{(W + M_{\Delta})(W - M_{\Delta} - \text{Re}\Sigma_{\Delta}(\rho) + i\tilde{\Gamma}_{\Delta}/2 - i\text{Im}\Sigma_{\Delta}(\rho))}
\]

\[\tilde{\Gamma}_{\Delta} \leftarrow \text{Free width } \Delta \rightarrow N \pi \text{ modified by Pauli blocking}\]

\[\text{Re}\Sigma_{\Delta}(\rho) \approx 40 \text{MeV} \frac{\rho}{\rho_0}\]

\[\text{Im}\Sigma_{\Delta}(\rho) \leftarrow \text{many-body processes: }\]

- \(\Delta N \rightarrow N N\)
- \(\Delta N \rightarrow N N \pi\)
- \(\Delta N N \rightarrow N N N\)
Pion distortion:

\[ e^{-i \vec{p}_\pi \cdot \vec{r}} \to \phi_{\text{out}}^*(\vec{p}_\pi, \vec{r}) \quad \vec{p}_\pi e^{-i \vec{p}_\pi \cdot \vec{r}} \to i \nabla \phi_{\text{out}}^*(\vec{p}_\pi, \vec{r}) \]

\[ \phi_{\text{out}}^*(\vec{p}_\pi, \vec{r}) \leftarrow \text{solution of the Klein-Gordon equation} \]

\[ \left( -\nabla^2 - \frac{\vec{p}_\pi^2}{2} + 2\omega_{\pi} \hat{V}_{\text{opt}} \right) \phi_{\text{out}} = 0 \]

\[ \hat{V}_{\text{opt}}(r) \leftarrow \text{optical potential in the } \Delta\text{-hole model:} \]

\[ Nieves, Oset & Garcia Recio NPA 554 (93) \]

\[ 2\omega_{\pi} \hat{V}_{\text{opt}}(r^*) = 4\pi \frac{M^2}{s} \left[ \nabla \cdot \frac{\mathcal{P}(r)}{1 + 4\pi g' \mathcal{P}(r)} \nabla - \frac{1}{2} \frac{\omega}{M} \Delta \frac{\mathcal{P}(r)}{1 + 4\pi g' \mathcal{P}(r)} \right] \]

\[ g' = 0.63 \leftarrow \text{Landau-Migdal parameter} \]

\[ \mathcal{P} = -\frac{1}{6\pi} \left( \frac{f^*}{m_\pi} \right)^2 \left\{ \frac{\rho_p + \rho_n/3}{\sqrt{s} - M_\Delta - \text{Re}\Sigma_\Delta + i \Gamma_\Delta / 2 - i \text{Im}\Sigma_\Delta} + \frac{\rho_n + \rho_p/3}{-\sqrt{s} - M_\Delta + 2M - \text{Re}\Sigma_\Delta} \right\} \]

Direct \quad \text{Crossed} \\
\Delta\text{-hole excitations}
Results

Medium effects reduce considerably the cross section

Pion distortion shifts down the peak

Eikonal fails for $p_\pi < 400$ MeV
Results

- Dependence on the effective number of participants $P = Z + N/3$

\[
\sigma \sim \frac{1}{P^2}
\]

- Strong pion absorption forces de reaction to be peripheral
- Effect of the nuclear form factor on heavier nuclei
Results

- **CC Coherent $\pi$ production at K2K:**
  - No evidence for coherent $\pi$ production:
    \[
    < \sigma_{coh} >_{p_\mu > 450 \text{ MeV} / c} < 7.7 \times 10^{-40} \text{ cm}^2
    \]
  - Our result:
    \[
    < \sigma_{coh} >_{p_\mu > 450 \text{ MeV} / c} = 11 \times 10^{-40} \text{ cm}^2
    \]
- **Reasons for the discrepancy:**
  - Axial N-$\Delta$ not sufficiently constrained (more data needed)
  - More complete theoretical description of the elementary amplitude (heavier resonances) required
  - Optical potential at lower and higher energies can be improved
  - Difficulties in the experimental separation of coherent and incoherent processes:
The coherent cross section might be considerably smaller than the Impulse Approximation prediction but still bigger than the K2K upper limit.
Results

NC Coherent Pion Production: \( \nu_l A \rightarrow \nu_l \pi^0 A \)

\[
\sigma_{NC} \neq \frac{\sigma_{CC}}{2}
\]

Phase Space: \( m_\mu \neq 0 \)

Interference terms: \( q^2 \neq 0 \) contributions
Results

- **Average over the MiniBooNE flux:**
  
  \[ < \sigma_{coh} > = 8.2 \times 10^{-40} \text{ cm}^2 \]

- **Preliminary experimental result (J. Raaf, Thesis):**
  
  \[ < \sigma_{coh} > = (7.7 \pm 1.6(stat) \pm 3.6(syst)) \times 10^{-40} \text{ cm}^2 \]
Results

**NC Coherent Pion Production:**

\[ \sigma(\nu) \neq \sigma(\bar{\nu}) \quad \text{for } q^2 \neq 0 \text{ contributions} \]

The difference is larger for lighter nuclei.
Conclusions

- Theoretical study of CC & NC coherent pion production:
  - Complete relativistic elementary amplitude in terms of $\pi$, $N$, $\Delta(1232)$
  - Nuclear form factor in the Impulse Approximation
  - Renormalization of the $\Delta$ properties in the nuclear medium
  - $\pi$ distortion $\leftrightarrow$ KG equation with a realistic optical potential
- Nuclear effects significantly reduce the coherent cross section
- The experimental separation between coherent and incoherent processes is model dependent and should be handled with care
- Good agreement with preliminary NC MiniBooNE results