Bearng-Only-Based Formation Circumnavigation Guided by Multiple Unknown Targets

SHAOHENG CHUN
School of Automation, Southeast University, Nanjing 210096, China
e-mail: chunsh123@126.com
This work was supported by the National Natural Science Foundation of China under Grant 61573105.

ABSTRACT In this paper, we consider the problem of the formation circumnavigation on an ellipse guided by multiple unknown targets in two-dimensional space, where each agent only employ bearing measurement without knowing any target’s position and velocity. A geometric center estimator is first established, only needing to use local information in gain design rather than any specific initialization. In the gain design, neither is the global information of some variables from all agents needed, nor is the gain needed to be designed large enough. Instead of designing corresponding controllers respectively, a unified circumnavigation controller is designed for the two cases of the stationary targets and the dynamic targets. The exponential stability of the whole algorithm is proved for the two cases. The conditions of some parameters in the circumnavigation controller are strictly derived. These conditions are very weak, and there exist a weak condition in which the controller can be simultaneously applied to the two cases. Finally, a series of simulations are conducted to verify the correctness of the algorithm.

INDEX TERMS Formation, unknown targets, bearing measurements, multi-agents system, elliptical circumnavigation.

I. INTRODUCTION

With the rapid development of electronic technology, 5G communication technology and sensor technology, intelligent robots used in the fields of reconnaissance, monitoring, management and search and rescue have been continuously proposed and iteratively designed. Since this kind of robot has huge application prospect and development space in both the civilian and military areas, it has been widely concerned by many people from different fields [1]–[4].

For these robots, they all have an obvious commonality that they have to locate the target, and then circumnavigate it to collect some information. Many existing results assume that the position of the target is known, and then the control algorithm only needs to force each robot to move to the vicinity of the target and circumnavigate it [5]–[8]. However, for some scenarios, such as military reconnaissance, the position of the unknown target is difficult for our robots to obtain. To solve this problem, some algorithms for simultaneous localization and circumnavigation has been designed. While some algorithms use distance measurements [9]–[11], the algorithm only using bearing measurements has drawn much attention because this passive measurement technique can make the algorithm more useful in many scenarios [12]. Deghat, Shames, Anderson and Yu studied an algorithm which is suitable for an agent only using the bearing measurements to locate and circumnavigate a single target in 2D space [13], [14]. When the algorithm works, the position of the target is accurately estimated by the estimator of the agent, and a controller is used to force the agent to circumnavigate the target with the desired radius. On this basis, with a geometric center estimator of multi-targets devised, Deghat, Xia, Anderson and Hong further proposed an algorithm which is suitable for an agent to locate and circumnavigate multi-targets [15], [16]. Based on the bearing angles and the relative heading angles, Yu, Liu and Feng proposed a distributed circular formation and circumnavigation algorithm for a stationary target [17]. The algorithm does not require any communication, and the position of the target is assumed to be only known to some agents. Bras, Hamel, Mahony and Samson studied an algorithm for a dynamic agent to locate and circumnavigate multiple stationary targets on an arbitrary trajectory [18]. It assumes that the agent can know its own velocity, the bearing angles to all targets and the position of one of the targets. When the algorithm works, the dynamic agent first estimates its own position through a position estimator which
is designed based on the bearing angle, its own velocity and the known target position. Then the dynamic agent estimates the positions of other targets according to the bearing angles and the estimate of its own position. Shao and Tian considered a general situation: multi-targets are located and circumnavigated by multi-agents [19]. In the algorithm, the multiple agents cooperatively estimate the geometric center of multi-targets, and then circumnavigate each target on a circle centered at the geometric center. Chun and Tian proposed a more universal algorithm in which multi-agents are used for multi-target’s localization and elliptical circumnavigation by designing a new circumnavigation controller [20]. Dou, Cheng, Wang, Liu and Feng considered the problem of how a dynamic target is located and circumnavigated in formation by multi-agents [21]. Li, Shi and Song considered the problem of how a stationary target is located and circumnavigated by multi-agents in 3D space and studied different circumnavigation types by designing a velocity coordinator [22].

Most of the above algorithms can only make agent(s) circumnavigate the target(s) on a circle. Even though [18] considers the circumnavigation of an arbitrary trajectory, it only focuses on the special situation in which a single agent is used for multiple stationary target’s localization and circumnavigation. Only [19] and [20] have considered the more general situation in which multiple dynamic targets are located and circumnavigated by multi-agents. Compared to [19], elliptical circumnavigation is firstly designed in [20]. Obviously, elliptical trajectory has many advantages. First of all, the elliptical trajectory, by setting the long and short axis, can make the agents, which is outside the detection range of the hostile targets, circumnavigate the hostile targets in a long and narrow space. Then, elliptical trajectory is more advanced than circular trajectory in terms of universality, energy consumption and circumnavigation efficiency. However, it is found that in order to avoid collision, the algorithm in [20] has to allow multi-agents to circumnavigate on different elliptical trajectory. Fig. 1 shows the trajectory planning when the hostile targets are surrounded by some obstacles, where the red circle is the safe boundary of the agents or the detection boundary of the hostile targets. If the agents are still distributed on different trajectories to perform task, the trajectory planning is very difficult or inexisten under the interference of obstacles and hostile targets. When the agents are to avoid being captured by the hostile targets, their trajectories have to be very close, which greatly increases the probability of collision between the agents. To avoid collision, the agents have to increase the distance between the trajectories, but this method increases the probability of being captured by the hostile targets. However, if circumnavigation in formation is used, the above problems can be well solved. In recent years, formation control of multi-agents system based on bearing measurements has attracted tremendous research interests [17], [21]–[24]. However, it is still a relatively new field to force multi-agents to circumnavigate multi-targets in formation on an elliptical trajectory without knowing the position and velocity of any target. Due to the potential to be applied to both the civilian and military areas, studying this kind of problem has strong practical significance.

When multiple agents locate and circumnavigate multiple unknown targets in formation on an elliptical trajectory, there will be many unexpected problems in the geometric center estimator design, controller design and stability analysis. For the geometric center estimator, the existing estimator is proposed by Shao and Tian [19], which is motivated by Chen et al. [25]. But the estimator requires the specific initialization, i.e., the sum of an intermediate state variable from each agent needs to be 0. This indicates that when an agent is suddenly dead or joined, the entire network must be re-initialized, which greatly decreases the efficiency of the network. In the gain design of the estimator, the existing method either need to know the global information of some variables or design the gain large enough. Obtaining global information can increase the communication resource overhead. And when the gain is set large enough, for the dynamic targets, the sign function can cause the system chattering significantly [20]. This means that we have to design a new estimator to solve the above problems. Then, the design of the circumnavigation controller is a cumbersome and complicated process. Unlike the formation on a circle, when the formation is completed on an ellipse, the design of tangential velocity becomes more complex because the angle between the axial velocity and the tangential velocity, as well as the trajectory radius (the distance between the agent on the desired elliptical trajectory and the targets center) is time-varying. Since the design of the tangential velocity involves the bearing information of itself and all neighbors, it is difficult to obtain the range of its own the velocity and the angular velocity. At the same time, the axial velocity and tangential velocity of the elliptical motion are not orthogonal, which indicates that the tangential velocity can affect the motion in the axial direction. As a result, the agents may always move toward or hit the targets or the geometric center of the targets. Therefore, in the design of the controller, we must consider how to solve these problems. Moreover, when the targets are dynamic, the geometric center which is just known as an estimate value in the controller design no longer converges to a constant, but moves with the targets. Therefore, when the targets are dynamic, apart from the influence of some time-varying variables and the design of the formation...
algorithm, the interference of the geometric center motion must also be considered. All these not only increase the design difficulty of the estimator and controller, but also make the stability analysis for the closed-loop system much more complicated.

This paper is an improvement and perfection of [20]. The main contribution of this paper is that it proposes an estimator and controller design framework for the multi-agents to collaboratively locate and circumnavigate multiple unknown targets in an elliptical formation by using bearing measurements. No similar existing results have been found. The specific contributions are listed as follows. (1) A new geometric center estimator is firstly established. When the estimator works, it does not require the specific initialization, which can effectively solve the problem that the system has to be re-initialized when an agent is suddenly dead or joined. In the gain design, the estimator introduces maximum consensus to calculate gain. So, each agent only needs to know some local information in the gain design without relying on the global information of some variables or designing the gain large enough, which further enhances the robustness of the algorithm. (2) Instead of designing corresponding controllers respectively, a unified circumnavigation controller is designed for the two cases of the stationary targets and the dynamic targets, which increases the practicality of this controller. And it is proved that even for the dynamic targets, the controller can force each agent to circumnavigate all targets in formation on the desired elliptical trajectory. (3) We prove that the whole algorithm is exponentially stable. The conditions of some gains in the circumnavigation controller are strictly derived. These conditions are very weak, and there exist a weak condition in which the controller can be simultaneously applied to both the stationary targets and the dynamic targets.

The rest of the paper is organized as follows. Section II is the problem statement. Section III presents the algorithm composed of a position estimator, a geometric center estimator and a circumnavigation controller. Section IV gives the stability analysis of the closed-loop system. A series of numerical simulations is presented in Section V. Section VI is the conclusion.

Notations of this paper are listed here. $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}_{\geq 0}$ and $\mathbb{R}^m$ respectively denote the set of integers, the set of real numbers, the set of non-negative real numbers and $m$-dimensional real Euclidean space. $\mathbf{0}$ denotes $[0, \cdots, 0]^T$ with compatible dimensions. $I$ is an identity matrix. For a matrix $X$, $X^T$ is the transpose of $X$. $\parallel \cdot \parallel$ represents the absolute value, and $\parallel \cdot \parallel$ represents the Euclidean norm. An undirected graph $\mathcal{G}=(\mathcal{V}, \mathcal{E})$ is used to denote the communication topology of the agents, where $\mathcal{V}=\{1, 2, \cdots, n\}$ is the set of agents, and $\mathcal{E}$ is the edge set which defines all the available communication links. We assume that no self-loop is contained in $\mathcal{G}$. Under $\mathcal{G}$, if $(i, j) \in \mathcal{E}$, it means that agent $i$ and agent $j$ can communicate with each other, and if $(i, j) \notin \mathcal{E}$, then they cannot communicate with each other. $\mathcal{N}_i$ is the set of neighbors of agent $i$. $\mathcal{O} = \{1, 2, \cdots, m\}$ is the set of targets. $N_i^O$, $i \in \mathcal{V}$ is the set of targets which can be measured the bearing information by agent $i$. $N_i^O, k \in \mathcal{O}$ is the set of agents which can measure the bearing information of target $k$. The Laplace matrix of graph $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{n \times n}$, where $l_{ij} = -1$ when $i \neq j$ and $(i, j) \in \mathcal{E}$, $l_{ij} = 0$ when $i \neq j$ and $(i, j) \notin \mathcal{E}$, and $l_{ij} = -\sum_{j=1,j \neq i}^{n} l_{ij}$ when $i=j$. $\mathcal{L} \in \mathbb{R}^{(n-1) \times (n-1)}$ is defined as

$$
\mathcal{L} = \begin{bmatrix}
l_{22} - l_{12} & \cdots & l_{2n} - l_{1n} \\
\vdots & \ddots & \vdots \\
l_{n2} - l_{12} & \cdots & l_{nn} - l_{1n}
\end{bmatrix}.
$$

The edge set $\mathcal{E}$ can also be defined as $\mathcal{E} = \{e_1, \cdots, e_\ell\}$, where the edges are labeled according to the incoming links of agents 1 to $n$. The incidence matrix of the graph is defined as $C = [c_{ij}] \in \mathbb{R}^{n \times \ell}$, where $c_{ij} = -1$ when $e_j$ leaves agent $i$, $c_{ij} = 1$ when $e_j$ enters agent $i$, and $c_{ij} = 0$ otherwise. For the undirected graph $\mathcal{G}=(\mathcal{V}, \mathcal{E})$, each undirected edge is considered as two distinct directed edges.

II. PROBLEM STATEMENT

There are $n$ agents, whose dynamics are defined by

$$
\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V},
$$

where $x_i(t)$ is the position of agent $i$, and $u_i(t)$ is the velocity of agent $i$, used as a control input. Assume that each agent knows its position $x_i(t) \in \mathbb{R}^2$, $i \in \mathcal{V}$. And there are $m$ targets whose positions $\xi_k(t) \in \mathbb{R}^2$, $k \in \mathcal{O}$ are unknown to any agent. But we assume that each target can be measured bearing information by at least one agent.

We use $\xi^*(t)$ to denote the geometric center of the multi-targets which is defined as

$$
\xi^*(t) = \frac{1}{m} \sum_{k=1}^{m} \xi_k(t).
$$

Fig. 2 shows the relationship between $x_i(t)$, $\xi_k(t)$, $r_i(t)$ and $\xi^*(t)$ in 2D space. The counterclockwise direction is assumed to be positive. $\theta_i^k(t)$ is the bearing angle of target $k$ measured by agent $i$. $\varphi_i^k(t)$ is the unit vector from $x_i(t)$ to $\xi_k(t)$.

![Figure 2. Relationship between $x_i(t)$, $\xi_k(t)$, $r_i(t)$ and $\xi^*(t)$](image-url)
i.e., \( \phi_i^k(t) = \begin{bmatrix} \cos \theta^k_i(t) \\ \sin \theta^k_i(t) \end{bmatrix} \). \( \phi_i^k(t) \) is obtained by \( \frac{3\pi}{2} \) counter-clockwise rotation of \( \phi_i^k(t) \). \( r_i(t) \) is used to denote the estimate of \( \xi^k(t) \) by agent \( i \). \( \psi_i(t) \) is the bearing angle of the vector from \( x_i(t) \) to \( r_i(t) \), and \( \phi_1(t) \) is the unit vector in this direction, i.e., \( \phi_1(t) = \begin{bmatrix} \cos \psi_1(t) \\ \sin \psi_1(t) \end{bmatrix} \). \( \psi_1(t) \) is the bearing angle of the vector onto the tangential velocity of agent \( i \), and \( \phi_1(t) \) is the unit vector in this direction, i.e., \( \phi_1(t) = \begin{bmatrix} \cos \psi_1(t) \\ \sin \psi_1(t) \end{bmatrix} \).

\( \rho_i(\psi_i(t)) \) is the distance from agent \( i \) to \( r_i(t) \) when agent \( i \) moves to the desired elliptical trajectory at \( \psi_i(t) \). \( a, b \) and \( \alpha \) are respectively the long semi-axes, the short semi-axes and the counter-clockwise rotation angle of the desired elliptical trajectory. And they are also referred to as the parameters of the elliptical trajectory. Note that these parameters are selected in advance according to the distribution of all targets.

Remark 1. For \( \theta_i^k(t) \) and \( \psi_i(t) \), we need to make a detailed explanation here. Our explanation contains two aspects: (1) First of all, since the bearing measurement technology is adopted in this paper, \( \theta_i^k(t) \) and \( \psi_i(t) \) are always well-defined when agent \( i \) can obtain the bearing information relative to target \( k \in N_i^O \) and the bearing information relative to \( r_i(t) \). In fact, although most of the above algorithms based on bearing measurement are not explicitly discussed, this point is also implicitly included in them. (2) Furthermore, when agent \( i \) is close enough to target \( k \in N_i^O \) or \( r_i(t) \) so that agent \( i \) cannot obtain the bearing information at instant \( t \), we can simply let the algorithm composed of the position estimator (3), the geometric center estimator (4) and the circumnavigation controller (5) not update. And at the next instant, agent \( i \) can leave target \( k \) or \( r_i(t) \) due to the rapid movement of agent \( i \). After leaving, the algorithm can continue to work normally.

Our goal is to design a control strategy so that all agents can circumnavigate each unknown target in a desired formation on a desired elliptical trajectory centered at \( \xi^k(t) \). Since the positions of \( \xi^k(t) \) and \( \xi^k(t) \) are unknown, we need to complete this goal by the following three steps. The first step is to call a bearing-only based position estimator so that \( \hat{\xi}_i(t) \), \( k \in N_i^O \) can be estimated by agent \( i \). The second step is to design a estimator to collaboratively estimate the geometric center \( \xi^k(t) \). The third step is to design a controller so that all agents can circumnavigate each target in desired formation on a desired elliptical trajectory centered at \( \xi^k(t) \). For the third step, it consists of the following three sub-goals:

1. \( \lim_{t \to \infty} \hat{\psi}_i(t) - \psi_i(t) = 0, i \in V \);  
2. \( \lim_{t \to \infty} \hat{\psi}_i(t) = c, i \in V \), where \( c \) is a constant that needs to be designed;  
3. \( \lim_{t \to \infty} \hat{\xi}_i(t) = \frac{2\pi}{n}, i \in V \), where \( \hat{\psi}_i(t), i = 1, 2, \ldots, n-1 \) is the angle between \( \phi_i^k(t) \) and \( \phi_{i+1}^k(t) \). Here \( \phi_i^k(t) \) is defined as \( \phi_i^k(t) = \frac{x_i(t) - x_o(t)}{\|x_i(t) - x_o(t)\|}, i \in V \).

The above three sub-goals are very necessary. The first sub-goal ensures that when the system is stable, all agents can circumnavigate all targets at the same angular velocity. The second sub-goal ensures that the desired angular velocity can be designed. The third sub-goal ensures that all agents can circumnavigate all targets in desired formation on the desired elliptical trajectory.

In fact, the problem considered in this paper can also be regarded as a tracking control problem. For this type of problem, some results have been achieved [26]. But compared with this paper, there are still some differences. The main difference is that in [26], some agents can directly obtain position of the targets in real time. Then, the multi-agents system can track the targets through the distributed control protocol. However, in this paper, the position and velocity information of each target cannot be obtained by any agent, and only some measurement (bearing measurement) can be used to estimate the position and velocity of each target to further perform the tracking task.

Next, we will show the algorithm composed of the position estimator, the geometric center estimator and the circumnavigation controller in detail.

### III. FORMATION CIRCUMNAVIGATION

To estimate target \( k \)'s position based on the bearing measurements \( \theta_i^k(t) \) and agent \( i \)'s location \( x_i(t) \), the following position estimator which was firstly designed by Deghat et al. [13], [14] needs to be used.

\[
\hat{\xi}_k^i(t) = k_i(I - \phi_i^k(t)\phi_i^k(t)^T)(x_i(t) - \hat{\xi}_k^i(t)),
\]

where \( \hat{\xi}_k^i(t) \) is the estimate of \( \xi_k(t) \) by agent \( i \), \( k_i \) is a positive constant gain that needs to be designed, \( \phi_i^k(t)\phi_i^k(t)^T \) is a projection matrix onto the vector \( \phi_i^k(t) \). Further, \( I - \phi_i^k(t)\phi_i^k(t)^T \) is a projection matrix onto the vector \( \hat{\phi}^k_i(t) \).

Taking the stationary targets as an example, Fig. 3 shows the geometric illustration of the estimator (3). According to (3) and Fig. 3, \( \hat{\xi}_k^i(t) \) moves perpendicularly to the line passing through \( x_i(t) \) and \( \hat{\xi}_k^i(t) \) at time \( t \). At the next time \( t_2 > t_1 \), \( \hat{\xi}_k^i(t_2) \) moves perpendicularly to the line passing through \( x_i(t_2) \) and \( \hat{\xi}_k^i(t_2) \). Let \( \hat{\xi}_k^i(t_2) = \hat{\xi}_k^i(t(t(t_2)) \hat{\xi}_k^i(t_2) \). The goal of the estimator (3) is that \( \hat{\xi}_k^i(t) \) converges to \( \theta \). From Fig. 3, we can get \( \|\hat{\xi}_k^i(t_2)\| < \|\xi_k^i(t_1)\| \). Therefore, as long as the trajectory of the agent fulfills a certain condition, \( \hat{\xi}_k^i(t) \) can converge to \( \hat{\xi}_k^i(t) \) under the control of the estimator. In fact, the condition is that the unit
vector \( \hat{\rho}_i(t) \) is persistently exciting, which will be discussed in detail below.

Considering the estimated position \( \hat{\xi}_i(t) \), the following multi-targets geometric center estimator which is motivated by George and Freeman [27] is designed to collaboratively estimate the geometric center \( \xi^*(t) \).

\[
\dot{\xi}_i(t) = \text{sgn}_+(\sum_{j \in \mathcal{N}_i} (e_j(t) - e_i(t)))
\]

\[
\dot{\omega}_i(t) = -e_i(t)\text{sgn}\left(\sum_{j=1}^{n} c_{ij}r_j(t)\right),
\]

\[
r_j(t) = \sum_{j=1}^{\ell} c_{ij}\omega_j(t) + \frac{n}{m} \sum_{k \in \mathcal{N}_i^O} \frac{1}{|\mathcal{N}_k^V|} \hat{\xi}_k(t),
\]

where \( e_i(t) \) is a positive gain; \( \text{sgn}_+(x) = 1 \) when \( x > 0 \), and 0 otherwise; \( \omega_j(t) \) is an intermediate state variable; and \( \text{sgn}(\cdot) \) is a standard sign function. Note that the sign functions can ensure the estimator converges in a finite time, but they can also cause the system chattering.

**Remark 2.** For the stationary targets, the problem of estimating the positions of the targets and their geometric center in this paper is a special case of the network localization problem considered in [28]. However, for the dynamic targets, we need to use the geometric center estimator (4) which is based on dynamic average consensus (DAC) and is also applicable to the stationary targets to estimate the geometric center of the targets.

**Remark 3.** Compared with [20], the estimator (4) proposed in this paper does not require any specific initialization, i.e., \( \sum_{j=1}^{n} \omega_j(t_0) = 0 \), where \( \omega_j(t) \) is an intermediate state variable similar to \( \omega_j(t) \). This design can effectively solve the problem that the system needs to be re-initialized when an agent is suddenly dead or joined, which improves the work efficiency of the system. As shown in Lemma 4, in the design of the gain in (4), each agent only needs to know some local information without relying on the global information of some variables or designing the gain large enough, which further enhances the robustness of the estimator.

Then, to force all agents to circumnavigate all targets in desired formation on the desired elliptical trajectory, the following distributed controller is designed.

\[
u_i(t) = \eta_i(\hat{\rho}_i(t) - \rho_d(\psi_i(t)))\phi_i(t) - \dot{\phi}_i(t) + \frac{\eta_i(\hat{\rho}_i(t) - \rho_d(\psi_i(t)))}{\sin(\psi_i(t) - \phi_i(t))}(\omega^*)
\]

\[
+ \sum_{j \in \mathcal{N}_i} (\psi_j(t) - \psi_i(t) - (j - i)\frac{2\pi}{n}),
\]

where \( \rho^*_i(t) = \|r_i(t) - x_i(t)\| \) is the distance from \( x_i(t) \) to \( r_i(t) \), which can be obtained based on the estimate of the target’s position; \( v_i(t) \) is the linear velocity of agent \( i \); both \( \eta_i \) and \( \eta \) are positive constant gains that need to be designed; \( \omega^* \) is a positive constant that also needs to be designed; and the calculation formulas of \( \phi_i(t) \) and \( \rho_d(\psi_i(t)) \) based on the measurements \( \psi_i(t) \) are given in [20]. In fact, the values of \( \eta \) and \( \omega^* \) determine the angular velocity of each agent.

**Remark 4.** The distributed circumnavigation controller (5) consists of two parts. The first part \( \eta_i(\rho_i(t) - \rho_d(\psi_i(t)))\phi_i(t) \) can force agent \( i \) to move to the elliptical trajectory centered at \( r_i(t) \). \( \eta_i \) is used to adjust the velocity of agent \( i \) when the agent moves to the geometric center of the targets and ensure that the agent does not always move towards or hit the geometric center. When \( \|\hat{\rho}_i(t) - \rho_d(\psi_i(t))\| = 0 \), the first part can be eliminated, which indicates that the agent is on the desired elliptical trajectory. The remaining part \( v_i(t)\phi_i(t) \) can force agent \( i \) to circumnavigate each target in desired formation on the elliptical trajectory. \( \eta_i \) is used to adjust the tangential velocity of the agent. There is a denominator term in \( v_i(t) \), which is caused by forming a formation on the ellipse. In the following, it will be given that this denominator term is never equal to 0.

**IV. STABILITY ANALYSIS**

To ensure that all agents can circumnavigate all targets in formation on the desired elliptical trajectory, the following two assumptions which are also included in [19], [20] are necessary.

**Assumption 1:** The graph \( \mathcal{G} \) is undirected and connected.

**Assumption 2:** The distance between any two targets satisfies

\[
\max_{i,j \in \mathcal{G}, i \neq j} \|\xi_i(t) - \xi_j(t)\| < 2b.
\]

These two assumptions are very reasonable. Assumption 1 is a common assumption for a multi-agents system and the DAC problem. Note that for the directed and connected graph \( \mathcal{G} \), different situations need to be discussed further. When the topology is a spanning tree, the estimator (4) can also work, but it obviously cannot converge to the geometric center of the targets. When the topology is a directed and balanced graph, asymptotic convergence may be available. But faster convergence, such as exponential convergence and finite time convergence, may be difficult to obtain or need to further research. Assumption 2 ensures that the distance between all targets is bounded, and all targets can be elliptically surrounded by each agent, making the circumnavigation have a certain physical meaning. Note that in practical applications, some rough prior information is needed for the determination of \( b \) in the assumption. For example, in some military reconnaissance, our satellites can roughly determine the range of activities of all hostile targets in advance. Then, in order to accurately reconnoitre this area, we only need to set \( b \) larger than the maximum radius of this area. Actually, our algorithm can also cope with the case which the distance between the targets is large enough or divergent due to the velocity difference. We can first group the targets so that the distance between each group of targets is bounded and small. Then, some prior information is used to determine \( b \) of each group. Finally, the agents are divided into the same number of groups, and the agents between different groups are set to
no longer communicate with each other. At this time, each group of the agents can use the algorithm to complete the task of position estimation and reconnoissance.

\[
\acute{\xi}(t) = \frac{1}{n} \sum_{i=1}^{n} \sum_{m} \sum_{k \in N_i} \frac{1}{|N_k|} \xi_k(t).
\]

It is obvious that \(\acute{\xi}(t) \rightarrow \frac{1}{m} \sum_{k=1}^{m} \xi_k(t) = \xi^*(t)\) as \(\acute{\xi}_k(t) \rightarrow \xi_k(t)\). Before analyzing the stability of the algorithm, we need the following lemmas.

**Lemma 1 [29]:** All eigenvalues of matrix \(\tilde{L}\) have positive real parts if and only if a graph is connected.

**Lemma 2 (30), (31):** Let \(V(\cdot) : \mathbb{R}^m \rightarrow \mathbb{R}^m\). Then \(\dot{x} = -V^T \dot{x}\) is exponentially asymptotically stable if and only if there exist some positive constants \(\sigma_1, \sigma_2, T\) such that for any \(t_0 \in \mathbb{R}^m\)

\[
\sigma_1 \leq \int_{t_0}^{t_0+T} (U^T V(t))^2 dt \leq \sigma_2,
\]

is satisfied, where \(U \in \mathbb{R}^m\) is an any constant unit vector. And \(V(\cdot)\) is persistently exciting (PE) if (7) is satisfied.

**Lemma 3 [32]:** If the coefficient matrix \(A(t)\) is continuous for all \(t \in [0, \infty)\) and constants \(\bar{a} > 0, \bar{b} > 0\) exist such that for every solution of the homogeneous differential equation \(\dot{x}(t) = A(t)x(t)\), one has \(\|x(t)\| \leq \bar{b}\|x(t_0)\|e^{-\bar{a}(t-t_0)}\), \(0 \leq t_0 \leq t < \infty\), then for each \(f(t)\) bounded and continuous on \([0, \infty)\), every solution of the nonhomogeneous equation \(\dot{x}(t) = A(t)x(t) + f(t), x(t_0) = 0\) is also bounded for \(t \in [0, \infty)\). In particular, if \(\|f(t)\| \leq K_f < \infty\) then the solution of the perturbed system satisfies \(\|x(t)\| \leq \bar{b}\|x(t_0)\|e^{-\bar{a}(t-t_0)} + \frac{bK_f}{1 - e^{-\bar{a}(t-t_0)}}\).

**Lemma 4:** Consider the geometric center estimator (4). Let \(f_i(t) = \frac{1}{m} \sum_{k \in N_i} \frac{1}{|N_k|} \xi_k(t).
\]

The undirected connected graph \(G\), if \(\xi_k(t), i \in V, k \in N_i\) is bounded, and \(\varepsilon_i(t_0) > \sup_{t \in [0, \infty)} \|f_i(t)\|\), then \(\|r_i(t) - \xi_k(t)\|\) converges to 0 in a finite time, i.e., there exists a positive constant \(M\) such that \(\lim_{t \to M} \|r_i(t) - \xi_k(t)\| = 0\).

**Proof:** The proof contains two steps. Firstly, we prove that for each agent, \(\varepsilon_i(t) \in \dot{\xi}(t) = \frac{\sin \psi_i(t)}{\cos \psi_i(t)} \sum_{j \in N_i} (\xi_j(t) - r_i(t))\) reach consensus on \(\varepsilon_{\max} = \max \{\varepsilon_1(t_0), \ldots, \varepsilon_n(t_0)\}\) in a finite time. The proof of this step can be found in [33]. Then, we prove that \(\|r_i(t) - \xi_k(t)\|\) converges to 0 in a finite time. The proof of this step can be found in [27].

**Remark 5:** When \(\varepsilon_i(t_0)\) is initialized, only local \(f_i(t)\) need to be considered. This is weaker than the condition of the gain in [19], [20], which requires that each agent knows the upper bounds of all \(f_i(t)\).

To facilitate stability analysis below, we list the calculation formulas of \(\psi_1(t)\) and \(\rho_2(\psi_1(t))\) based on the measurements \(\psi_i(t)\), which are given in [20].

\[
\cos \psi_1(t) = \frac{a^2 \sin \psi_i(t) \cos \alpha + b^2 \cos \psi_i(t) \sin \alpha}{\sqrt{a^4 \sin^2 \psi_i(t) + b^2 \cos^2 \psi_i(t) - \alpha}},
\]

(8)

\[
\sin \psi_1(t) = \frac{a^2 \sin \psi_i(t) \sin \alpha - b^2 \cos \psi_i(t) \cos \alpha}{\sqrt{a^4 \sin^2 \psi_i(t) + b^2 \cos^2 \psi_i(t) - \alpha}},
\]

(9)

\[
\rho_2(\psi_1(t)) = \frac{ab}{\sqrt{a^2 \sin^2 \psi_i(t) + b^2 \cos^2 \psi_i(t) - \alpha}}.
\]

(10)

Next, we will study the stability of the algorithm in two cases: the stationary targets and the dynamic targets.

**A. STATIONARY TARGETS**

Let \(\rho_1(t) = \|\xi_k(t) - \xi_k(t_0)\|, \rho_2(t) = \|\xi^*(t) - \xi_k(t_0)\|\) and \(\tilde{r}_i(t) = r_i(t) - \xi_k(t)\). Then, under the above four lemmas, we will prove that \(\tilde{r}_i(t), \rho_1(t)\) and \(\rho_2(t)\) are all exponentially convergent, and \(\tilde{\xi}_k(t)\) and \(\tilde{r}_i(t)\) exponentially converge to 0.

**Proposition 1:** Consider the position estimator (3), the geometric center estimator (4) and the circumnavigation controller (5). Under Assumptions 1-2, there exist \(\varepsilon_i(t_0)\) in (4) and \(\eta_i > 0, \eta > 0\) and \(\omega > 0\) in (5) such that

(1) \(\rho_1(t), \rho_2(t)\) and \(\tilde{r}_i(t)\) are bounded;

(2) \(\tilde{\xi}_k(t)\) and \(\tilde{r}_i(t)\) exponentially converge to 0.

**Proof:** (1) The proof contains two steps. Firstly, we prove the convergence of \(\tilde{r}_i(t)\). Then, we prove the boundedness of some variables.

Now, we prove the first step. By considering \(\varphi_i^k(t)^T \varphi_i^k(t) = 1\) and by combining (3), we have that

\[
\tilde{\xi}_k(t) = k_i (\rho_1(t) - \varphi_i^k(t) \varphi_i^k(t)^T \rho_2(t) - \tilde{\xi}_k(t))
\]

\[
= -k_i (\rho_1(t) \varphi_i^k(t) \varphi_i^k(t)^T) \tilde{\xi}_k(t)
\]

\[
= -k_i \varphi_i^k(t) \varphi_i^k(t)^T \tilde{\xi}_k(t).
\]

(11)

Since \(\varphi_i^k(t)\) is a vector obtained by \(\frac{3\pi}{2}\) counterclockwise rotation of \(\varphi_i^k(t)\), and

\[
\varphi_i^k(t) = \left[ \begin{array}{c} \cos \theta_i^k(t) \\ \sin \theta_i^k(t) \end{array} \right],
\]

we have that

\[
\varphi_i^k(t) = \left[ \begin{array}{c} \cos \frac{3\pi}{2} \\ -\sin \frac{3\pi}{2} \end{array} \right] = \left[ \begin{array}{c} \sin \theta_i^k(t) \\ \cos \theta_i^k(t) \end{array} \right] = \left[ \begin{array}{c} \cos \theta_i^k(t) \\ -\sin \theta_i^k(t) \end{array} \right].
\]

By choosing a Lyapunov function \(V_i^k(t) = \frac{1}{2} \tilde{\xi}_k(t)^T \tilde{\xi}_k(t)\) and by combining (11), we can get

\[
\dot{V}_i^k(t) = -k_i \|\varphi_i^k(t) \tilde{\xi}_k(t)\|^2 \leq 0.
\]

(12)

So, (11) is uniformly stable, and \(\tilde{\xi}_k(t)\) eventually converges to a constant vector but not necessarily 0.

Since \(\tilde{\xi}_k(t) = \tilde{\xi}_k(t) - \xi_k(t)\) and \(\xi_k(t)\) is stationary, \(\tilde{\xi}_k(t)\) eventually converges to a constant vector. Then, \(\tilde{\xi}_k(t)\) eventually converges to 0. Obviously, there exists \(\varepsilon_i(t_0)\) such that \(\varepsilon_i(t_0) > \sup_{t \in [0, \infty)} \|f_i(t)\|\). According to Lemma 4, we have that there exists a positive constant \(M > 0\) such that \(\lim_{t \to M} \|r_i(t) - \xi_k(t)\| = 0\). Since \(\tilde{\xi}_k(t)\) eventually converges
to \(\mathbf{0}\), \(\xi(t)\) eventually converges to \(\mathbf{0}\) according to (6). Thus, \(\dot{r}_i(t)\) eventually converges to \(\mathbf{0}\).

Then, we consider that \(\dot{\rho}^*_{i}(t)\) does not tend to 0. For the problem, we only need to prove that when \(\dot{\rho}^*_{i}(t) = \rho^*_i(\psi_i(t))\), the velocity in the axial direction of agent \(i\) relative to \(r_i(t)\) does not tend to 0. The velocity is \(\eta(\dot{\rho}^*_i(\psi_i(t)) - \dot{\rho}^*_i(t)) - v_i(t)\cos(\psi_i(t) - \psi_{11}(t)) + \dot{r}_i(t)^2 \phi_i(t)\). Since \(\dot{r}_i(t)\) eventually converges to \(\mathbf{0}\), we only need consider that \(\eta(\dot{\rho}^*_i(\psi_i(t)) - \dot{\rho}^*_i(t)) - v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) does not tend to 0. For \(v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\), it is either bounded or divergent. When \(v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) is bounded, it is obvious that there exists \(\eta_i > 0\) such that \(\eta(\dot{\rho}^*_i(\psi_i(t)) - \dot{\rho}^*_i(t)) - v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) does not tend to 0. When \(v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) is divergent, since \(\eta(\dot{\rho}^*_i(\psi_i(t)) - \dot{\rho}^*_i(t)) - v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) is bounded, there exists \(\eta_i > 0\) such that \(\eta(\dot{\rho}^*_i(\psi_i(t)) - \dot{\rho}^*_i(t)) - v_i(t)\cos(\psi_i(t) - \psi_{11}(t))\) does not tend to 0. Thus, there exists \(\eta_i > 0\) such that \(\dot{\rho}^*_i(t)\) does not tend to 0.

To make (5) meaningful, we need to consider that \(\sin(\psi_i(t) - \psi_{11}(t))\) does not tend to 0. According to (8) and (9), we have

\[
\sin(\psi_i(t) - \psi_{11}(t)) = \frac{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}{\sqrt{a^4 \sin^2(\psi_i(t) - \alpha) + b^4 \cos^2(\psi_i(t) - \alpha)}}. \tag{13}
\]

Obviously, \(\sin(\psi_i(t) - \psi_{11}(t)) \in [\frac{2ab}{a^2 + b^2}, 1]\). Thus, \(\sin(\psi_i(t) - \psi_{11}(t))\) does not tend to 0.

Since \(\dot{\rho}^*_i(t)\) and \(\sin(\psi_i(t) - \psi_{11}(t))\) do not tend to 0, according to the relationship between linear velocity \(v_i(t)\) and angular velocity \(\dot{\psi}_i(t)\), we have

\[
\dot{\psi}_i(t) = \frac{v_i(t) \sin(\psi_i(t) - \psi_{11}(t)) - v_{11}(t)}{\dot{\rho}^*_i(t)} = \eta \sum_{j \in N_i} (\psi_i(t) - \psi_j(t) - (i - j) \frac{2\pi}{n}) + \eta \alpha^* - \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}, \tag{14}
\]

where \(v_{11}(t)\) is the velocity component of \(r_i(t)\) onto \(\phi_i(t)\), which is obtained by \(\frac{2\pi}{n}\) counterclockwise rotation of \(\phi_i(t)\). Since \(\dot{r}_i(t)\) eventually converges to \(\mathbf{0}\), both \(v_{11}(t)\) and \(\frac{v_{11}(t)}{\dot{\rho}^*_i(t)}\) eventually converge to 0.

Let \(\dot{\psi}_i(t) = \psi_i(t) - \psi_{11}(t) - (i - 1) \frac{2\pi}{n}, i = 1, 2, 3, \ldots, n\). Note that \(\dot{\psi}_i(t) = 0\). Then, considering (14), we have

\[
\dot{\psi}_i(t) = \dot{\psi}_i(t) - \psi_{11}(t) = \eta \sum_{j \in N_i} (\psi_j(t) - \psi_i(t) - (i - j) \frac{2\pi}{n}) - \eta \sum_{j \in N_i} (\psi_j(t) - \psi_{11}(t) - (i - 1) \frac{2\pi}{n}) - v_{11}(t) - \frac{v_{11}(t)}{\dot{\rho}^*_i(t)} = \eta \sum_{j \in N_i} (\dot{\psi}_j(t) - \dot{\psi}_i(t) - \frac{2\pi}{n}) = \eta \sum_{j \in N_i} \psi_j(t) - \dot{\psi}_i(t) - \frac{2\pi}{n} = \eta \sum_{j \in N_i} \frac{v_{11}(t)}{\dot{\rho}^*_i(t)} + \frac{v_{11}(t)}{\dot{\rho}^*_i(t)} = \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}.
\]

Since \(\dot{\psi}_i(t) - \dot{\psi}_{11}(t) = 0\) when \(j = i\) and \(\sum_{j=1}^{n} l_{ij} = 0\), equation (15) can be written as

\[
\dot{\psi}_i(t) = -\eta \sum_{j=1}^{n} l_{ij} (\dot{\psi}_j(t) - \dot{\psi}_i(t)) + \eta \sum_{j=1}^{n} l_{ij} \dot{\psi}_j(t) + \frac{v_{11}(t)}{\dot{\rho}^*_i(t)} - \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}.
\]

When \(i = 1\), \(\dot{\psi}_i(t) = 0\). This is a very obvious result. So, we only need to consider the case of \(i = 2, 3, \ldots, n\). When \(i = 2, 3, \ldots, n\), equation (15) can be written as the following matrix.

\[
\dot{\psi}_i(t) = -\eta L \psi_i(t) + A(t), \tag{16}
\]

where \(\dot{\psi}_i(t) = [\dot{\psi}_2(t), \dot{\psi}_3(t), \ldots, \dot{\psi}_n(t)]^T\), and \(A(t) = [-\frac{v_{21}(t)}{\dot{\rho}^*_i(t)} + \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}, -\frac{v_{31}(t)}{\dot{\rho}^*_i(t)} + \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}, \ldots, -\frac{v_{n1}(t)}{\dot{\rho}^*_i(t)} + \frac{v_{11}(t)}{\dot{\rho}^*_i(t)}]^T.\)

The solution of (16) is

\[
\psi(t) = Q(t, t_0) \psi(t_0) + \int_{t_0}^{t} Q(t, \tau) A(\tau) d\tau, t > t_0, \tag{17}
\]

where \(Q(t, t_0)\) is the state-transition matrix.

According to Lemma 1, we have that the solution of the homogeneous differential equation \(\dot{\psi}_i(t) = -\eta L \psi_i(t)\) exponentially converges to \(\mathbf{0}\), i.e., there exist constants \(\tilde{a} > 0, \tilde{b} > 0\) such that \(\|\dot{\psi}_i(t)\| \leq \|\psi(t_0)\| \|Q(t, t_0)\| \leq \tilde{a} \|\dot{\psi}(t_0)\| e^{-\tilde{a}(t-t_0)}\).
Then, we have $\|Q(t,t_0)\| \leq \bar{b} e^{-\bar{a}(t-t_0)}$. Thus, considering (17), we can get
\[
\|\hat{\psi}(\tau)\| \leq \bar{b}\|\hat{\psi}(t_0)\|e^{-\bar{a}(t-t_0)}
+ \bar{b} \int_{t_0}^{t} e^{-\bar{a}(\tau-t)}\|A(\tau)\|d\tau, \quad t > t_0.
\] (18)

When $t_1 > t_0$, we use $\tilde{A}^{f}_{t_0}$ to denote the maximum value of $\|A(\tau)\|$ in $[t_0, t_1]$. Considering (18), we have
\[
\|\hat{\psi}(\tau)\| \leq \bar{b}\|\hat{\psi}(t_0)\|e^{-\bar{a}(t-t_0)}
+ \bar{b} \int_{t_0}^{t_1} e^{-\bar{a}(\tau-t)}\|A(\tau)\|d\tau
\leq \bar{b}\|\hat{\psi}(t_0)\|e^{-\bar{a}(t-t_0)}
+ \bar{b}\tilde{A}^{f}_{t_0} e^{-\bar{a}(t-t_0)}.
\] (19)

Since $\|\tilde{A}^{f}_{t_0}\|$ eventually converge to 0, $-\frac{\tilde{A}^{f}_{t_0}}{\rho_i^{(k)}}$ eventually converge to 0. Then, $\tilde{A}^{f}_{t_0} \to 0$ as $t_1 \to \infty$. Thus, when $t \to \infty$, $t-t_1 \to \infty$ and $t_1 \to \infty$, $\hat{\psi}(\tau)$ eventually converge to 0. That is, $\psi_i(t)$ eventually converge to $\psi_i(t) + (i-1)\frac{\pi}{2}$.

Then, $v_i(t)$ in (5) can be written as $v_i(t) = \frac{\eta(\omega^* + \zeta_i(t))\hat{\psi}_i(t)}{\sin(\hat{\psi}_i(t) - \psi_i(t))}$, where $\zeta_i(t)$ eventually converges to 0. That is, $v_i(t)$ eventually converge to $\frac{\eta(\omega^* + \zeta_i(t))\hat{\psi}_i(t)}{\sin(\hat{\psi}_i(t) - \psi_i(t))}$.

Next, we prove the boundedness of some variables. Since $\hat{\psi}_i(t) = \|x_i(t) - r_i(t)\| = ((x_i(t) - r_i(t))^T(x_i(t) - r_i(t)))^{1/2}$, we have that
\[
\hat{\psi}_i(t) = \frac{1}{2}((x_i(t) - r_i(t))^T(x_i(t) - r_i(t)))^{-1/2} \times ((x_i(t) - r_i(t))^T(x_i(t) - r_i(t)))'
= -\hat{\psi}_i(t)^T \hat{\psi}_i(t)
= -\eta_i \hat{\psi}_i(t) + \eta_i \rho_d(\psi_i(t)) - v_i(t)\phi_1(t)\hat{\psi}_i(t)
+ \hat{\psi}_i(t)\phi_1(t).
\] (20)

According to (8) and (9), we have
\[
\phi_1(t)^T \phi_1(t) = \frac{(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}.
\] (21)

By combining (20) and (21), we have
\[
\hat{\psi}_i(t) = \hat{\psi}_i(t) + \frac{\eta_i \rho_d(\psi_i(t)) + \hat{\psi}_i(t)^T \phi_1(t)}{\eta_i \rho_d(\psi_i(t)) + \hat{\psi}_i(t)^T \phi_1(t)}
- \frac{\eta(\omega^* + \zeta_i(t))\hat{\psi}_i(t)(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}.
\] (22)

Define $\delta_1(t)$ as
\[
\delta_1(t) = \frac{\eta_i \rho_d(\psi_i(t)) + \hat{\psi}_i(t)^T \phi_1(t)}{\eta_i \rho_d(\psi_i(t)) + \hat{\psi}_i(t)^T \phi_1(t)}
- \frac{\eta(\omega^* + \zeta_i(t))\hat{\psi}_i(t)(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}.
\] (23)

and define $\delta_2(t)$ as
\[
\delta_2(t) = \eta_i \rho_d(\psi_i(t)) + \hat{\psi}_i(t)^T \phi_1(t).
\] (24)

Since $\frac{(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}$ is bounded obviously and $\zeta_i(t)$ eventually converges to 0, (22) can be written as
\[
\hat{\psi}_i(t) = \eta_i \delta_1(t) + \zeta_i(t) + \delta_2(t).
\] (25)

Since $-\frac{\eta^*}{\rho_i} \leq \frac{(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)} \leq \frac{a^2 - b^2}{2ab},
\eta_i - \frac{\eta^*}{\rho_i} \leq \frac{(a^2 - b^2)\sin(\psi_i(t) - \alpha)\cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)} \leq \frac{\eta^*}{\rho_i} + \frac{a^2 - b^2}{2ab} + \zeta_i(t).$ Then, when the values of $\eta_i$ and $\omega^*$ are set to satisfy inequality $\eta_i > \frac{\eta^*}{\rho_i} + \frac{a^2 - b^2}{2ab}$, there always exist a constant $\lambda_1$ and a sufficiently large time $t_1 > t_0$ such that $\eta_i + \delta_1(t) + \zeta_i(t) > \lambda_1, \quad t > t_1.$ (26)

So, we have that the solution of the homogeneous differential equation $\hat{\psi}_i(t) = \eta_i \delta_1(t) + \zeta_i(t)$ exponentially converges to 0. According to (10), $\rho_d(\psi_i(t))$ is bounded. Since $\hat{\psi}_i(t)$ eventually converges to 0, $\delta_2(t)$ is bounded obviously. Then, according to Lemma 3, $\hat{\psi}_i(t)$ is bounded.

Let $\hat{l}_k(t) = \|\tilde{e}_k(t) - \hat{\xi}_k(t)\|$. The relationship between $\hat{\psi}_i(t)$, $\|\tilde{e}_k(t)\|$, $\hat{\psi}_i(t)$, $\hat{l}_k(t)$, $\hat{\psi}_i(t)$, $\hat{l}_k(t)$, $\hat{\psi}_i(t)$, $\|\tilde{e}_k(t)\|$ and $\hat{\psi}_i(t)$ is shown in Fig. 4.

Then, by triangle inequality, we have that
\[
\rho_i(t) \leq \rho_i(t) + \hat{l}_k(t),
\] (27)

\[
\rho_i(t) \leq \rho_i(t) + \hat{l}_k(t).
\] (28)

Since $\hat{\psi}_i(t)$ eventually converges to a constant vector, $\hat{\psi}_i(t)$ is bounded according to (6). Considering $\lim_{t \to M} \|r_i(t) - \hat{\psi}_i(t)\| = 0$, we have that $r_i(t)$ is bounded. Since all targets are stationary, $\|\tilde{e}_k(t)\|$ is bounded. According to Assumption 2, $l_k(t)$ is bounded obviously. Since $\hat{\psi}_i(t)$ is bounded, $\rho_i(t)$ and $\rho_i(t)$ are both bounded. Since $\hat{\psi}_i(t)$ eventually converges to a constant vector, $\rho_i(t)$ is bounded.

(2) We prove it by contradiction. Assume that at least one $\hat{\psi}_i(t), i \in \mathcal{V}, k \in \mathcal{N}_k^{(i)}$ is not PE. Since $\psi_i(t)$ converge to $\psi_i(t) + (1-i)\frac{\pi}{2}$ and $\hat{\psi}_i(t)$ converge to 0, by considering (14), we have that for an arbitrarily small positive constant $\zeta_2$, there exists a sufficiently large time $t_2 > t_1$ such that
\[
\psi_i(t) > \eta \omega^* - \zeta_2, \quad t > t_2.
\] (29)
Fig. 5 shows the relationship between $\psi_i(t)$, $\theta_i(t)$, and $\beta_i(t)$, where $\gamma_i(t)$ is the angle between $U$ and $\phi_i(t)$, $\theta_i(t)$ is the bearing angle of $\phi_i(t)$, and $\beta_i(t)$ is the bearing angle of $\psi_i(t)$. Since $U$ is a constant unit vector, $\theta_i(t) - \gamma_i(t)$ is a constant. Since $\phi_i(t)$ is obtained by $3\pi T \over 2$ counterclockwise rotation of $\psi_i(t)$, $\beta_i(t) - \theta_i(t) = 3\pi T \over 2$. Then, we have

$$
\frac{d\psi_i(t)}{dt} = \frac{d\beta_i(t)}{dt} = \frac{d\theta_i(t)}{dt}. \tag{30}
$$

Since $U$ and $\phi_i(t)$ are both unit vectors, (7) can be written as

$$
\sigma_1 \leq \frac{1}{T} \int_{t_0}^{t_0+T} \cos^2 \gamma_i(t) dt \leq \sigma_2. \tag{31}
$$

Since $\cos(\cdot) \leq 1$, the right side of (31) is true obviously. So, only the left side of (31) need to be proved.

Considering (29), we have that $\psi_i(t)$ is a strict monotone function when $t > t_2$. Then, $\psi_i(t)$ does not eventually converge to a constant. Since $\rho_i^\ast(t)$ and $\rho_i(t)$ are bounded, $\theta_i(t)$ does not eventually converge to a constant, i.e., $\frac{d\theta_i(t)}{dt}$ does not eventually converge to 0. By (30), $\frac{d\psi_i(t)}{dt}$ also does not eventually converge to 0. Then, $\psi_i(t)$ does not eventually converge to a constant. Thus, when $t > t_3$, we can always to find some positive constants $\sigma_1 > 0$ and $T > 0$ to make the left side of (31) true. So, $\psi_i(t)$ is PE.

This result contradicts the assumption, and the assumption is not true. Therefore, $\phi_i(t)$, $\forall i \in \mathcal{V}$, $k \in \mathbb{N}_0^C$ is PE and $\xi_i(t)$ exponentially converges to 0. Thus, $\xi_i(t)$ exponentially converges to $\xi_i(t)$. Further, $\xi_i(t)$ exponentially converges to $\xi_i(t)$ in a finite time, $r_i(t)$ exponentially converges to $\xi_i(t)$ obviously. Thus, $\tilde{r}_i(t) = r_i(t) - \xi_i(t)$ exponentially converges to 0. The proof is completed. □

Remark 6: $\omega_\ast$ can also take a negative constant. When $\omega_\ast < 0$, it means that each agent $i$ circumnavigates these targets clockwise. And the stability analysis process of this case is the same as the process when $\omega_\ast > 0$.

Theorem 1: Consider the position estimator (3), the geometric center estimator (4) and the circumnavigation controller (5). Under Assumptions 1-2, there exist $\tilde{r}_i(t_0)$ in (4) and $\eta_i > 0$, $\eta > 0$ and $\omega_\ast > 0$ in (5) such that

1. $\psi_i(t) - \psi_{j,k}(t)$, $\forall i,j \in \mathcal{V}$ exponentially converge to 0.
2. $\tilde{r}_i(t)$ exponentially converge to $\eta \omega_\ast$, and $\psi_i(t)$ exponentially converge to $\frac{2\pi}{n}$;
3. $\rho_i^\ast(t)$ exponentially converges to $\rho_i(\psi_i(t))$.

Proof: (1) Considering (6) and $\tilde{\xi}_i(t) = \epsilon_i(t) - \xi_i(t)$, we have that

$$
\tilde{\xi}_i(t) = \frac{1}{m} \sum_{k=1}^{m} \xi_k(t) + \frac{1}{n} \sum_{i=1}^{n} \sum_{k \in \mathbb{N}_0^C} \frac{1}{|\mathbb{N}_0^C|} \epsilon_i(t). \tag{32}
$$

Since all targets are stationary, we have that $\tilde{\xi}_i(t)$ exponentially converges to 0. By combining (11), we have that $\tilde{\xi}_i(t)$ exponentially converges to 0. Then, $\tilde{\xi}_i(t)$ exponentially converges to 0. Since $\lim_{t \to \infty} \|\tilde{r}_i(t) - \tilde{\xi}_i(t)\| = 0$, $\tilde{r}_i(t)$ exponentially converges to 0 obviously. Since $\psi_{i1}(t)$ is a component of $\tilde{r}_i(t)$, $\psi_{i1}(t)$ also exponentially converges to 0.

Considering (16), we have that $\psi_1(t)$ exponentially converges to $\psi_1(t) + (i-1) \frac{2\pi}{n}$. Considering (14), we have that $\psi_1(t)$ exponentially converges to $\eta \omega_\ast$. Thus, $\psi_1(t) - \psi_1(t) = \eta \omega_\ast$, $\forall i,j \in \mathcal{V}$ exponentially converge to 0. According to (2) of Proposition 1, we have that $\tilde{r}_i(t)$ exponentially converges to $\xi_i(t)$. Then, $\psi_i(t)$ exponentially converges to $\phi_i(t)$. Thus, $\psi_i(t)$ exponentially converges to $\phi_i(t)$. Since $\psi_i(t) - \psi_i(t)$ exponentially converges to $\frac{2\pi}{n}$, $\psi_i(t)$ exponentially converges to $\frac{2\pi}{n}$.

(2) Define $\Delta_i(t)$ as $\Delta_i(t) = \tilde{r}_i(t) - \rho_i(\psi_i(t))$. Considering (10), we have that

$$
\rho_i(\psi_i(t)) = -\frac{a^2 - b^2}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}. \tag{33}
$$

Since $\dot{\psi}_i(t)$ exponentially converges to $\eta \omega_\ast$, (33) can be written as

$$
\dot{\rho}_i(\psi_i(t)) = -(\eta \omega_\ast + \zeta_3(t)) \frac{\rho_i(\psi_i(t))(a^2 - b^2) \sin(\psi_i(t)) \cos(\psi_i(t) - \alpha)}{a^2 \sin^2(\psi_i(t) - \alpha) + b^2 \cos^2(\psi_i(t) - \alpha)}. \tag{34}
$$

where $\zeta_3(t)$ exponentially converges to 0.

Combining (22), (23) and (34), we have

$$
\dot{\Delta_i}(t) = -\left(\eta_t + \delta_{\tilde{r}_1}(t)\right) \Delta_i(t) + \tilde{r}_i(t)^T \phi_i(t) - \xi_1(t) \rho_i^\ast(t) - \zeta_3(t) \rho_i(\psi_i(t)). \tag{35}
$$

Obviously, there always exist positive constants $\eta_t$, $\eta$, $\omega_\ast$, $\lambda_2$ and a sufficiently large time $t_3$ such that

$$
\eta_t + \delta_{\tilde{r}_1}(t) > \lambda_2, \quad t > t_3. \tag{36}
$$

Since $\psi_i(t)$ exponentially converges to $\psi_1(t) + (i-1) \frac{2\pi}{n}$, according to (5), we have that $\xi_i(t)$ exponentially converges to 0. Since $\tilde{r}_i(t)$ and $\xi_i(t)$ exponentially converge to 0, and $\tilde{r}_i(t)$ and $\rho_i(\psi_i(t))$ is bounded, according to (35) and (36), we have that $\Delta_i(t) = \tilde{r}_i(t) - \rho_i(\psi_i(t))$ exponentially converges to 0.
converges to 0. Therefore, \( \hat{\rho}_i^*(t) \) exponentially converges to \( \rho_i(\psi_i(t)) \).

According to Fig. 4, we can get the triangle inequality

\[
|\rho_i^*(t) - \hat{\rho}_i^*(t)| \leq \|\tilde{r}_i(t)\|. 
\]  
(37)

Since \( \tilde{r}_i(t) \) exponentially converges to 0, \( \rho_i^*(t) \) exponentially converges to \( \hat{\rho}_i^*(t) \). Therefore, \( \rho_i^*(t) \) exponentially converges to \( \rho_i(\psi_i(t)) \). The proof is completed. \( \square \)

### B. DYNAMIC TARGETS

When target \( k \in \mathcal{O} \) is dynamic, the estimate error \( \tilde{\xi}_k(t) \) can be rewritten as

\[
\dot{\tilde{\xi}}_k(t) = -k_i \tilde{\psi}_i(t) \tilde{\rho}_i(t) \tau \tilde{\xi}_k(t) - \tilde{\xi}_k(t). 
\]  
(38)

To ensure that each agent can catch up with and circumnavigate all targets during movement, we need the following assumption. Note that the similar assumption is also included in [13]–[16], [19]–[21].

**Assumption 3:** There exists a sufficiently small constant \( \tilde{v} > 0 \) such that \( \|\tilde{\xi}_k(t)\| < \tilde{v} \) for \( \forall k \in \mathcal{O} \).

Then, under the above assumptions, we will prove that even if the targets are dynamic, the algorithm is also effective.

**Proposition 2:** Consider the position estimator (3), the geometric center estimator (4) and the circumnavigation controller (5). Under Assumptions 1–3, there exist \( e_i(t_0) \) and \( \eta_i > 0 \) such that \( e_i(t_0) \) and \( \eta_i > 0 \) in (5) such that

1. \( \dot{\tilde{\xi}}_k(t) \) in \( \tilde{\xi}_k(t) \) exponentially converges to 0;
2. \( \tilde{\xi}_k(t) \) in (38) exponentially converges to a neighborhood of \( 0 \), and \( \tilde{r}_i(t) \) exponentially converges to a neighborhood of \( 0 \).

**Proof:** (1) Firstly, we need to prove that \( \rho_i^*(t) \), \( \rho_i^*(t) \) and \( \rho_i^*(t) \) are bounded. Since \( \tilde{\xi}_k(t) = -k_i \tilde{\psi}_i(t) \tilde{\rho}_i(t) \tau \tilde{\xi}_k(t) + \tilde{\xi}_k(t) \), (12) is valid for the dynamic targets. Therefore, \( \dot{\tilde{\xi}}_k(t) \) eventually converges to a constant vector but not necessarily \( 0 \). Then, \( \dot{\tilde{\xi}}_k(t) \) eventually converges to 0. Since \( \tilde{\xi}_k(t) = \dot{\tilde{\xi}}_k(t) - \dot{\tilde{\xi}}_k(t) = \dot{\tilde{\xi}}_k(t) + \dot{\tilde{\xi}}_k(t) \), then \( \dot{\tilde{\xi}}_k(t) \) eventually converges to \( 0 \). Considering Assumption 3, \( \dot{\tilde{\xi}}_k(t) \) is bounded. Thus, \( \dot{\tilde{\xi}}_k(t) \) is bounded. So, there exists \( \tilde{\epsilon}_i(t_0) \) such that \( e_i(t_0) = \max_{t \in [0, \infty)} \|\tilde{\xi}_k(t)\| \). According to Lemma 4, we have that there exists a positive constant \( M \) such that \( \lim_{t \to M} \|\tilde{r}_i(t) - \tilde{\xi}_k(t)\| = 0 \). Considering (32), we have

\[
\dot{\tilde{\xi}}_k(t) = \frac{1}{m} \sum_{k=1}^m \tilde{\xi}_k(t) - \frac{1}{n} \sum_{i=1}^n \sum_{k \in \mathcal{O}_i} \frac{1}{|\mathcal{N}_k^{\tilde{\xi}_k}|} \dot{\tilde{\xi}}_k(t). 
\]  
(39)

Then, \( \dot{\tilde{r}}_i(t) \) eventually converges to 0. Therefore, \( \|\tilde{r}_i(t)\| \leq \tilde{v} \). The targets are dynamic, (14) is still valid. Then, (19) is valid. Since \( \tilde{r}_i(t) \) is the component of \( \tilde{r}_i(t) \) onto \( \phi_i(t) \), \( \|\tilde{r}_i(t)\| \leq \tilde{v} \). Thus, when \( t \to \infty \), \( t \to t_1 \to \infty \) and \( t \to t_2 \to \infty \), \( \psi_i(t) \) exponentially converges to a neighborhood of 0. Then, \( \psi_i(t) \) exponentially converges to a neighborhood of \( \psi_i(t) + (i - 1) \frac{\pi}{\alpha} \). Considering (5), we have that \( \tilde{v}_i(t) \) exponentially converges to a neighborhood of \( \frac{n \rho_i}{\sin^2(\psi_i(t) - \psi_i(t))} \).

Further, we can use \( \tilde{v}_i(t) = \frac{n \rho_i}{\sin^2(\psi_i(t) - \psi_i(t))} \) to denote the true value of \( \psi_i(t) \), where \( \psi_i(t) \) is bounded.

When the targets are dynamic, (22) need to be rewritten as

\[
\hat{\rho}_i^*(t) = -\eta_i \hat{\rho}_i^*(t) - \eta_i \rho_i(\psi_i(t)) \tilde{r}_i(t) + \tilde{\xi}_k(t) 
\]  
(40)

Since (23) and (24) are valid, by combining (23), (24) and (40), we have that

\[
\hat{\rho}_i^*(t) = -(\eta_i + \delta_i(t) + \psi_i(t)) \tilde{r}_i(t) + \tilde{\xi}_k(t) 
\]  
(41)

Since \( -\frac{a^2 - b^2}{2ab} < \frac{(a - b^2)^2}{a^2 + b^2} \), \( \tilde{v}_i = \frac{n \rho_i}{\sin^2(\psi_i(t) - \psi_i(t))} \leq \frac{a^2 - b^2}{2ab} \), \( \tilde{v}_i \leq \tilde{v}_i < \tilde{v}_i \). So, we have that the solution of the homogeneous differential equation \( \tilde{\xi}_k(t) = -\eta \hat{\rho}_i^*(t) \alpha(t) \tilde{r}_i(t) \) exponentially converges to 0. According to (10), \( \rho_i(\psi_i(t)) \) is bounded. Since \( \|\tilde{r}_i(t)\| \leq \tilde{v} \), \( \tilde{\xi}_k(t) \) is bounded. Then, according to Lemma 3, \( \hat{\rho}_i^*(t) \) is bounded.

Considering (22) and (32), we have

\[
\lim_{t \to M} \tilde{r}_i(t) = \lim_{t \to M} \tilde{\xi}_k(t) + \lim_{t \to M} \frac{1}{n} \sum_{i=1}^n \sum_{k \in \mathcal{O}_i} \frac{1}{|\mathcal{N}_k^{\tilde{\xi}_k}|} \tilde{\xi}_k(t). 
\]  
(43)

By (43), we have

\[
\lim_{t \to M} \tilde{r}_i(t) = \lim_{t \to M} \frac{1}{n} \sum_{i=1}^n \sum_{k \in \mathcal{O}_i} \frac{1}{|\mathcal{N}_k^{\tilde{\xi}_k}|} \tilde{\xi}_k(t). 
\]  
(44)

So, \( \tilde{r}_i(t) \) is bounded. The triangle inequalities (27) and (28) are still valid for the dynamic targets. \( \tilde{r}_i(t) \) is bounded obviously according to Assumption 2. Therefore, \( \rho_i^*(t) \) and \( \rho_i^*(t) \) are bounded.
Next, we will prove that $\bar{q}_i^k(t)$ is PE when $\check{q}_i^k(t) = -k_2 \bar{q}_i^k(t) \check{q}_i^k(t)^T \check{x}_i^k(t)$.

We prove it by contradiction. Assume that at least one $\bar{q}_i^k(t)$, $i \in V$, $k \in \mathcal{N}_i^k$ is not PE.

Let $\xi_i(t) = n \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$. Since $\psi_i(t)$ exponentially converges to a neighborhood of $\psi_i(t) + (i-1)2\pi$, $\xi_i(t)$ is bounded. According to (14), we have $\bar{q}_i^k(t) = \xi_i(t) + \eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$. Since $\bar{q}_i^k(t)$ is bounded, then, we have that $\check{x}_i^k(t)$ exponentially converges to a neighborhood of $\check{x}_i^k(t)$. It is obvious that $\|\xi_i^*(t)\|_2 < \check{v}$. Then, $\|\check{x}_i^k(t)\|$ is bounded.

Then, we consider that $\bar{q}_i^k(t)$ does not tend to 0. According to the proof steps of (1) of Proposition 2, it is obvious that $\bar{q}_i^k(t)$ does not tend to 0.

Considering (19), we have that $\|\hat{r}_i(t) - \psi_i(t)\|_2 \leq \|\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)\|_2$. Since $\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$ is bounded.

Then, $\|\hat{r}_i(t)\|$ exponentially converges to a neighborhood of $\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$.

When the targets are dynamic, (33) is still valid. Since $\psi_i(t)$ exponentially converges to a neighborhood of $\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$, can be written as

$$\tilde{r}_d(\psi_i(t)) = \eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi).$$

Combining (41) and (46), we have

$$\Delta_i(t) = -\eta_i + \delta_i(t) \Delta_i(t) + \rho_i(t)^T \phi_i(t)$$

$$- \xi_i(t) \rho_i(t) - \xi_i^*(t).$$

Obviously, there always exist positive constants $\eta_i, \eta, \omega^* \lambda_5$, and a sufficiently large time $t_5 > t_5$ such that $\eta_i + \delta_i(t) > \lambda_5$, $\eta_i > 0$. Since $\Delta_i(t)$ is bounded, then $\rho_i(t)$ exponentially converges to a neighborhood of $\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$. The proof is completed.

**Theorem 2:** Consider the position estimator (3), the geometric center estimator (4) and the circumnavigation controller (5). Under Assumptions 1-3, there exist $\varepsilon_i(t_0)$ such that $\varepsilon_i(t_0) > \sup_{t \in [0,\infty)} \|\bar{r}_i(t)\|_2$. According to (48), we have that $\|\bar{r}_i(t)\|_2$ exponentially converges to a neighborhood of $\eta_\omega \sum_{j \in \mathcal{N}_i^k} (\psi_i(t) - \psi_j(t) - (j-i)2\pi)$. The proof is completed.

**V. NUMERICAL SIMULATIONS**

In this section, a series of numerical simulations for different cases are given to verify the effectiveness of the theoretical results. Fig. 6 shows the communication topology composed of 5 agents and 6 targets.
Firstly, we simulate the case that the targets are stationary. We assume that
\[ a = 10 \text{ (m)}, \ b = 6 \text{ (m)}, \ \alpha = 0, \]
\[
\begin{bmatrix}
    x_1(0), x_2(0), x_3(0), x_4(0), x_5(0) \\
    \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), \xi_5(t), \xi_6(t)
\end{bmatrix}
\]
\[
= \begin{bmatrix}
    -20 & 15 & 20 & 0 & -10 \\
    -15 & -15 & 10 & 20 & 15
\end{bmatrix},
\]
\[
\begin{bmatrix}
    \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), \xi_5(t), \xi_6(t) \\
    k_i = 3, \eta_i = 1, \eta = 1, \text{ and } \omega^* = 1.8. \text{ And } \xi(t_0) \text{ are randomly}
\]
selected in \([3, 5], \omega(t) \text{ are randomly selected in } [2, 3]. \text{ Fig. 7 shows the}

circumnavigation trajectories, where "\( + \)" and "\( \ast \)" respectively denote the target, the

geometric center of the targets and the agent. Fig. 8 shows the convergence of \( \| \hat{\xi}_i(t) - \xi_i(t) \|, \| r_i(t) - \xi^*(t) \| \text{ and } \| x(t) - \xi^*(t) \| - \rho_d(\psi(t)) \). \text{ For Fig. 8, when the targets are stationary, the algorithm composed of (3), (4) and (5) can make the}

three errors exponentially converge to 0, which is consistent with Proposition 1 and (2) of Theorem 1. \text{ Fig. 9 shows the convergence of } \psi_i(t) \text{ and } \hat{\psi}_i(t). \text{ From this figure, we have that under the algorithm, each agents can circumnavigate all targets in desired formation, which is consistent with (1) of Theorem 1. In summary, when the targets are stationary, the algorithm composed of (3), (4) and (5) can accurately estimate the positions of the targets and the geometric center of the targets, and it can force each agent to circumnavigate all targets in desired formation on the desired elliptical trajectory centered at } \xi^*(t). \text{ And this algorithm is exponentially stable.}

Fig. 10 shows the circumnavigation trajectories when \[
\begin{bmatrix}
    \xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t), \xi_5(t), \xi_6(t)
\end{bmatrix}
\]
is respectively rotated counterclockwise by \( \frac{\pi}{4} \) and \( \frac{3\pi}{4} \). According the figure, we have that the algorithm can adapt to more complex environments by setting the value of } \( \alpha \).
each agent catch up with the targets, but it cannot accurately estimate the positions of the targets and the geometric center of the targets, but it can estimate the positions within a small error range. As a result, each of the agents can circumnavigate all targets in desired formation within a error range, which is consistent with (1) of Theorem 2. In summary, when the targets are dynamic, the algorithm composed of (3), (4) and (5) can make each agent catch up with the targets, but it cannot accurately estimate the velocities of the targets. Further, the algorithm is designed to be robust against perturbations that may affect the targets’ positions and velocities.

In practice, the algorithm is applied in various scenarios where multiple targets need to be navigated by agents. The algorithm’s ability to deal with dynamic targets and errors makes it suitable for applications such as patrolling, search and rescue missions, and other tasks involving the coordination of multiple agents.

Theoretical analysis and simulations have confirmed the effectiveness of the algorithm. The convergence of the algorithm is shown in the figures, which demonstrate how the agents navigate towards their targets with high accuracy. The algorithm’s performance under different conditions is also verified through simulations, further validating its applicability in real-world scenarios.

In conclusion, the algorithm presented in this paper is a significant contribution to the field of multi-agent systems, particularly in the context of formation circumnavigation guided by multiple unknown targets. Its robustness, efficiency, and accuracy make it a valuable tool for researchers and practitioners working on similar problems.
VI. CONCLUSION

This paper considers the problem of the formation circumnavigation on an ellipse guided by multiple unknown targets in two-dimensional space, where each agent only employs bearing measurement without knowing any target’s position and velocity. A new geometric center estimator is established, only needing to use local information in gain design rather than any specific initialization. In the gain design, neither is the global information of some variables from all agents needed, nor is the gain needed to be designed large enough. This further enhances the robustness of the entire algorithm. For the two cases of the stationary targets and the dynamic targets, we design a unified circumnavigation controller. And we prove that when some parameters are set reasonably, even for the dynamic targets, the agents can also circumnavigate the targets in formation on a desired ellipse, and the entire system is exponentially stable. Then, we derive the conditions of some parameters in the circumnavigation controller. And there exist weak conditions such that the controller can be simultaneously applied to the two cases. Finally, some simulations verify the correctness of the algorithm.

However, although the elliptical trajectory has a certain universality, its practicality is still limited in some extreme environments. So, in the future work, we will study the simultaneous localization and formation circumnavigation of arbitrary trajectories based on bearing-only measurements. Moreover, to ensure that the whole algorithm is exponentially stable, we assume that the communication topology of the agents is an undirected and connected graph. But for the directed and connected graph, we do not consider. So, in the future work, we will study the stability of the system under a directed and connected graph.

ACKNOWLEDGMENT

The authors would like to thank associate editor and anonymous reviewers for their insightful comments that have improved the presentation of the article.
REFERENCES

[1] I. Shames, S. Dasgupta, B. Fidan, and B. D. O. Anderson, “Circumnavigation using distance measurements under slow drift,” IEEE Trans. Autom. Control, vol. 57, no. 4, pp. 889–903, Apr. 2012.

[2] Y. Cao, “UAV circumnavigating an unknown target under a GPS-denied environment with range-only measurements,” Automatica, vol. 55, pp. 150–158, May 2015.

[3] A. S. Matveev, A. A. Semakova, and A. V. Savkin, “‘Tight circumnavigation of multiple moving targets based on a new method of tracking environmental boundaries,” Automatica, vol. 79, pp. 52–60, May 2017.

[4] Z. Miao, Y. Wang, and R. Fierro, “Cooperative circumnavigation of a moving target with multiple nonholonomic robots using backstepping design,” Syst. Control Lett., vol. 103, pp. 38–65, May 2017.

[5] N. Csecseri, M. Di Marco, A. Garulli, and G. Giannitrapani, “Collective circular motion of multi-vehicle systems,” Automatica, vol. 44, no. 12, pp. 3025–3035, Dec. 2008.

[6] F. Chen, W. Ren, and Y. Cao, “Surrounding control in cooperative agent networks,” Syst. Control Lett., vol. 59, no. 11, pp. 704–712, Nov. 2010.

[7] T.-H. Kim and T. Sugie, “Cooperative control for target-capturing task based on a cyclic pursuit strategy,” Automatica, vol. 43, no. 8, pp. 1426–1431, Aug. 2007.

[8] I. Shames, B. Fidan, and B. D. O. Anderson, “Close target reconnaissance with guaranteed collision avoidance,” Int. J. Robust Nonlinear Control, vol. 21, no. 16, pp. 1823–1840, Nov. 2011.

[9] A. A. Matveev, A. A. Semakova, and A. V. Savkin, “Range-only based circumnavigation of a group of moving targets by a non-holonomic mobile robot,” Automatica, vol. 65, pp. 76–89, Mar. 2016.

[10] S. H. Dandach, B. Fidan, S. Dasgupta, and B. D. O. Anderson, “A continuous time linear adaptive source localization algorithm, robust to persistent drift,” Syst. Control Lett., vol. 58, no. 1, pp. 7–16, Jan. 2009.

[11] I. Shames, S. Dasgupta, B. Fidan, and B. D. O. Anderson, “Circumnavigation using distance measurements,” in Proc. Eur. Control Conf. (ECC), Aug. 2009, pp. 2444–2449.

[12] R. Zheng, Y. Liu, and D. Sun, “Enclosing a target by nonholonomic mobile robots with bearing-only measurements,” Automatica, vol. 53, pp. 400–407, Mar. 2015.

[13] M. Deghat, I. Shames, B. D. O. Anderson, and C. Yu, “Target localization and circumnavigation using bearing measurements in 2D,” in Proc. 49th IEEE Conf. Decis. Control (CDC), Dec. 2010, pp. 334–339.

[14] M. Deghat, I. Shames, B. D. O. Anderson, and C. Yu, "Localization and circumnavigation of a slowly moving target using bearing measurements," IEEE Trans. Autom. Control, vol. 59, no. 8, pp. 2182–2188, Aug. 2014.

[15] L. Xia, M. Deghat, B. D. O. Anderson, and Y. Hong, “Localization and circumnavigation of a group of targets by a single agent using bearing measurements,” in Proc. Austrlial. Control Conf., 2012, pp. 253–258.

[16] M. Deghat, L. Xia, B. D. O. Anderson, and Y. Hong, “Multi-target localization and circumnavigation by a single agent using bearing measurements,” Int. J. Robust Nonlinear Control, vol. 25, no. 14, pp. 2362–2374, Sep. 2015.

[17] X. Yu, L. Liu, and G. Feng, “Distributed circular formation control of nonholonomic vehicles without direct distance measurements,” IEEE Trans. Autom. Control, vol. 63, no. 8, pp. 2730–2737, Aug. 2018.

[18] F. L. Bras, T. Hanel, R. Mahony, and C. Samson, “Observers for position estimation using bearing and biased velocity information,” in Sensing and Control for Autonomous Vehicles, Cham, Switzerland: Springer, 2017, pp. 3–23.

[19] J. Shao and Y.-P. Tian, “Multi-target localisation and circumnavigation by a multi-agent system with bearing measurements in 2D space,” Int. J. Syst. Sci., vol. 49, no. 1, pp. 15–26, Jan. 2018.

[20] S. Chun and Y. Tian, “Multi-targets localization and elliptical circumnavigation by multi-agents using bearing-only measurements in two-dimensional space,” Int. J. Robust Nonlinear Control, vol. 30, no. 8, pp. 3250–3268, May 2020.

[21] L. Dou, C. Song, X. Wang, L. Liu, and G. Feng, “Target localization and enclosing control for networked mobile agents with bearing measurements,” Automatica, vol. 118, pp. 1–12, Aug. 2020.

[22] R. Li, Y. Shi, and Y. Song, “Localization and circumnavigation of multiple agents along an unknown target based on bearing-only measurement: A three dimensional solution,” Automatica, vol. 94, pp. 18–25, Aug. 2018.

[23] C. Wang, G. Xie, and M. Cao, “Forming circle formations of anonymous mobile agents with order preservation,” IEEE Trans. Autom. Control, vol. 58, no. 12, pp. 3248–3254, Dec. 2013.

[24] M. Ye, B. D. O. Anderson, and C. Yu, “Multiagent self-localization using bearing only measurements,” in Proc. 52nd IEEE Conf. Decis. Control, Dec. 2013, pp. 2157–2162.

[25] F. Chen, Y. Cao, and W. Ren, “Distributed average tracking of multiple time-varying reference signals with bounded derivatives,” IEEE Trans. Autom. Control, vol. 57, no. 12, pp. 3169–3174, Dec. 2012.

[26] Y. Shang and Y. Ye, “Fixed-time group tracking control with unknown inherent nonlinear dynamics,” IEEE Access, vol. 5, pp. 12833–12842, 2017.

[27] J. George and R. A. Freeman, “Robust dynamic average consensus algorithms,” IEEE Trans. Autom. Control, vol. 64, no. 11, pp. 4615–4622, Nov. 2019.

[28] M. H. Trinh, D. Zelazo, and H.-S. Ahn, “Pointing consensus and bearing-based solutions to the fermat–weber location problem,” IEEE Trans. Autom. Control, vol. 65, no. 6, pp. 2339–2354, Jun. 2020.

[29] Y. Zhang and Y.-P. Tian, “Consentability and protocol design of multi-agent systems with stochastic switching topology,” Automatica, vol. 45, no. 5, pp. 1195–1201, May 2009.

[30] B. Anderson, “Exponential stability of linear equations arising in adaptive identification,” IEEE Trans. Autom. Control, vol. 22, no. 1, pp. 83–88, Feb. 1977.

[31] J. S. Ashty and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness. Upper Saddle River, NJ, USA: Prentice-Hall, 1994.

[32] H. D’Angelo, Linear Time-Varying Systems: Analysis and Synthesis. Boston, MA, USA: Allyn Bacon, 1970, pp. 225–226.

[33] J. Cortés, “Distributed algorithms for reaching consensus on general functions,” Automatica, vol. 44, no. 3, pp. 726–737, Mar. 2008.