Reciprocity-protected Anomalous Brewster Effect
for Ultra-broadband Reflection-free Manipulation of Waves

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The Brewster’s law predicts the maximum polarization of light for incidence on a dielectric surface at a particular angle. However, when loss is introduced into the permittivity of the dielectric, the impedance matching condition breaks down, unavoidably invalidating the Brewster effect. In this work, we have found an exception to this centuries-old dilemma by applying the reciprocity principle to nonmagnetic anisotropic metamaterials. Nontrivial anomalous Brewster effect, which exhibits independently tunable absorption and refraction via the extra degrees of freedoms introduced by the anisotropy of metamaterials, is found to be strictly protected by the reciprocity principle. The bandwidth of the anomalous Brewster effect can cover an extremely wide spectrum from the quasi-static limit to optical frequencies. Brewster-effect absorbers are demonstrated by proof-of-principle experiments at microwave frequencies. Our finding reveals the anomalous Brewster effect that bestows reflection-free manipulation of waves as a universal principle for reciprocal materials.

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In the early 1810’s, Sir David Brewster experimentally showed that when unpolarized light impinges on a dielectric interface, the reflected light can be linearly polarized if the reflected beam is perpendicular to the refracted one [1, 2]. The origin of this Brewster effect lies in the perfect impedance matching (PIM) that occurs for the transverse-magnetic (TM)-polarized light at Brewster’s angle. However, the impedance matching condition imposes severe limits on the material parameters. For instance, when loss is introduced to the permittivity of dielectrics, the impedance matching condition breaks down and the Brewster effect can no longer be maintained, rendering substantial reflection and imperfectly polarized light [3, 4]. This also explains why the Brewster effect, which has an inherently ultra-broad bandwidth, was never adopted for wide-band absorption applications. Instead, complex engineering on the material permeability and dispersions [5, 6] has become the common procedure for the bandwidth widening of absorption. Recently, with the rise of metamaterials that go beyond natural materials in many aspects, the phenomenon of the Brewster effect has also be reexamined [7-15]. Generalized Brewster effect has been realized for enhanced transmission at selected angles [11, 12]. Wide-angle PIM has also been realized [13-15]. Nevertheless, due to the resonant nature of metamaterials, most of the observed phenomena were limited in relatively small bandwidth.

In this work, by applying the reciprocity principle to anisotropic metamaterials, we propose to realize ultra-broadband anomalous Brewster effect allowing independently tunable absorption and refraction, which are not supported by the traditional Brewster effect. The schematic graph of this idea is shown in Fig. 1(a). The reciprocity principle requires that the response of a transmission channel is symmetric when the source and detection points are interchanged. For a wave incident and reflected on the surface of a reciprocal material (symmetric permittivity, permeability and conductivity tensors) [16], the change of incident angle between \( \theta_i \) and \(-\theta_i\) is equivalent to the exchange between the source and detection. Therefore, according to the reciprocity principle, the reflection coefficients are exactly the same for incident angles \( \pm \theta_i \), i.e. \( r(-\theta_i) = r(\theta_i) \) [16, 17]. As a result, when the Brewster effect emerges, it is always simultaneously satisfied for \( \pm \theta_i \). Contrarily, the transmission is not
protected by reciprocity. By introducing anisotropy properly in the material, the transmission can be tuned at will, rendering the anomalous Brewster effect. When the material is lossless, the angle of refraction can be tuned flexibly from positive to negative. When the material is lossy, ultra-broadband and controllable absorption can be achieved. Mathematically, we have obtained two nontrivial solutions for such an anomalous Brewster effect, both of which can realize large absorption without breaking the impedance matching condition, bestowing Brewster-effect absorbers. Non-resonant metamaterials can be utilized to realize such Brewster-effect absorbers and the working bandwidth can cover an extremely wide spectrum from the quasi-static limit to the optical regime. Proof-of-principle experiments have been performed in the microwave regime. The damping can be sufficiently large to achieve almost total absorption within a thickness of 1~2 wavelengths. In this way, the centuries-old dilemma between the Brewster effect and absorption is solved.

Without loss of generality, we consider a TM-polarized wave incident on a nonmagnetic tilted anisotropic medium (TAM) under the incident angle of \( \theta_i \), as illustrated in Fig. 1(a). The background is a dielectric medium with isotropic permittivity \( \varepsilon_b \). The optical axis (along with the permittivity component \( \varepsilon_x \)) of the TAM has a tilt angle \( \alpha \) with respect to the \( x \) axis. As depicted in Fig. 1(a), we assume that \( \theta_i > 0 \) (\( \theta_i < 0 \)) for incidence from the left (right). For \( \theta_i > 0 \), we find that there are two trivial solutions for realizing the Brewster effect between the background and TAM which is independent of \( \varepsilon_y \). One solution for \( \varepsilon_x \neq \varepsilon_b \) requires a condition of \( \theta_i = \pi/2 - \alpha \) (\( \alpha = \arctan (\sqrt{\varepsilon_x}/\sqrt{\varepsilon_y}) \)), which is exactly the condition of the traditional Brewster effect. The other solution for \( \varepsilon_x \neq \varepsilon_b \) requires a condition of \( \theta_i = \alpha \) (\( \alpha \) is arbitrary), which is also easily understandable [18]. Intriguingly, in both cases, the waves incident at \( \theta_i > 0 \) cannot “see” \( \varepsilon_y \). This inspires us to utilize the new parameter \( \varepsilon_y \) as a control parameter without affecting the impedance matching condition. In the following, we provide a more comprehensive picture based on the equal-frequency contours (EFCs) of the metamaterial.
Figures 1(b) and 1(e) show the EFCs of the TAM with fixed $\varepsilon_x$ and $\alpha$ for different $\varepsilon_y$ under the conditions of $\varepsilon_y \neq \varepsilon_o$ and $\varepsilon_y = \varepsilon_\infty$, respectively. It is seen that in both figures, all the EFCs of the TAM cross a fixed point in the $k_y < 0$ space, which is marked as the point I (at $k_y = \sqrt{\varepsilon_x k_0 \sin \alpha}$, $k_z = -\sqrt{\varepsilon_x k_0 \cos \alpha}$, where $k_0$ is the wave number in free space). When the propagating state at the point I is excited in the TAM (by a proper incident angle $\theta_i > 0$), the electric fields of the transmitted waves are polarized along the direction of $\varepsilon_x$, indicating that the waves cannot be influenced by $\varepsilon_y$. Here, the wave impedance $Z$ is defined as the ratio of $E_x/H_z$, where $E_x$ and $H_z$ are, respectively, the $x$-component of electric fields and the $z$-component of magnetic fields at the interface (normal to $y$). Through some derivation, we obtain $|Z| = Z_0 \cos \alpha \sqrt{\varepsilon_x}$ at the point I for the TAM, which confirms that $|Z|$ is independent of $\varepsilon_y$. Here, $Z_0$ is the impedance of vacuum (~377\,Ω). More importantly, the same impedance $|Z| = Z_0 \cos \alpha \sqrt{\varepsilon_x}$ is invariant along a vertical line of $k_y = \sqrt{\varepsilon_x k_0 \sin \alpha}$ for various EFCs which goes through point I, when we vary $\varepsilon_y$. This effect can be clearly seen in Figs. 1(b) and 1(e) where $|Z|$ is plotted in colors in the EFCs. Therefore, this vertical line is denoted as an equal-$|Z|$ line (gray lines) here. The strict proof is summarized in and Supplemental Materials [18].

Here, we would like to emphasize that despite that the two trivial solutions of the Brewster effect for $\theta_i > 0$ are both independent of $\varepsilon_y$, they cannot provide the flexible manipulation of refraction or absorption yet as we mentioned in the introduction. This is because refracted waves cannot “see” $\varepsilon_y$ and therefore, cannot be influenced by $\varepsilon_y$ in any way. However, interesting things happen when considering the reciprocity principle. Now, we consider flipping the sign of the incident angle from $\theta_i > 0$ to $\theta_i < 0$ with the magnitude unchanged. According to the reciprocity principle, the Brewster effect is guaranteed and independent of $\varepsilon_y$. Therefore,
we have another equal-\(|Z|\) line at \(k_z = -\sqrt{\varepsilon_r k_0 \sin \alpha}\), as marked by gray lines in Figs. 1(b) and 1(e). In the case of \(\theta_i < 0\), the propagating states at point II are excited in the TAM, where the equal-\(|Z|\) line and EFC intersect. Obviously, the EFC and the refraction angle both vary significantly with \(\varepsilon_r\), bestowing tunable refractive behaviors by changing \(\varepsilon_r\), while the Brewster effect is guaranteed by reciprocity at the same time. When \(\varepsilon_r\) has an imaginary part, the absorption can be introduced. Therefore, we have obtained two nontrivial solutions of the anomalous Brewster effect with tunable refraction and absorption.

In the following, we perform numerical simulations to prove the reciprocity-protected Brewster effect with tunable refraction in the background of air (i.e. \(\varepsilon_a = 1\)). The simulations were performed by the commercial finite-element-method software COMSOL Multi-physics. Here, we consider TM-polarized waves incident on the TAM. First, the TAM is chosen to be \(\varepsilon_r = 2\) and \(\alpha = \arctan\left(\sqrt{\varepsilon_r}/\sqrt{\varepsilon_t}\right) = 35.3^\circ\). The reflectance on the air-TAM interface as a function of the \(\theta_i\) and \(\varepsilon_r\) is plotted in Fig. 1(c). Clearly, near-zero reflection is observed around \(\theta_i = \pm 54.7^\circ = \pm (90^\circ - \alpha)\), irrespective of \(\varepsilon_r\). We also notice blue lines related to omnidirectional PIM, which can be understood through coordinate transformation [15, 19]. In Fig. 1(d), we plot the distributions of magnetic fields (color) and group velocity (arrows) for the cases of \(\theta_i = 54.7^\circ\) (left), \(\theta_i = -54.7^\circ\) and \(\varepsilon_r = 3\) (middle), and \(\theta_i = -54.7^\circ\) and \(\varepsilon_r = -3\) (right), respectively. In all three cases, the reflection clearly disappears. By tuning \(\varepsilon_r\) from 3 to -3, the angle of refraction is evidently changed from positive to negative, which confirms the controllability of refraction by tuning \(\varepsilon_r\). Second, we consider a TAM with \(\varepsilon_r = 1\). In this situation, \(\alpha\) can be arbitrary. Here, we take \(\alpha = 30^\circ\) as an example. The reflectance on the air-TAM interface is plotted in Fig. 1(f), which clearly shows \(\varepsilon_r\)-independent near-zero reflection under \(\theta_i = \pm 30^\circ\), i.e. \(\theta_i = \pm \alpha\). The reflection-less refraction for \(\theta_i = \pm 30^\circ\), as well as the controllable refraction for \(\theta_i = -30^\circ\) has also been well confirmed
by simulations, as shown in Fig. 1(g).

It is worth noting that when $\varepsilon_r$ approaches infinity, the EFCs of the TAM turn into two parallel lines with constant wave impedance $|Z| = Z_0 \cos \alpha / \sqrt{\varepsilon_r}$, as are also shown as cyan color in Figs. 1(b) and 1(e). Such a case may be realized by using a tilted aluminum film array, which leads to ultra-broadband reflection-less negative refraction [18]. As an important consequence of the above reciprocity-protected Brewster effect, we can further let $\varepsilon_r$ be a complex number and introduce loss into the system. Such reciprocity consideration allows us to realize perfect-impedance-matched absorbing materials with unprecedented wide bandwidth, which is extremely difficult via other approaches, if not impossible. Such absorbers are denoted as Brewster-effect absorbers here. In the following, we demonstrate the design and realization of this ultra-broadband Brewster-effect absorbers by using tilted conductive film (CF) arrays.

The tilted CF array is embedded in a dielectric host of $\varepsilon_d$ under a tilt angle of $\alpha$, as illustrated by the upper inset graph of Fig. 2(a). The separation distance between two adjacent CFs is $a$, which is much larger than the thickness of the CFs $t$ (i.e. $a >> t$), but smaller than the wavelength $\lambda$ in the dielectric host (i.e. $a < \lambda$). Under such circumstances, the tilted CF array can be approximately homogenized as an effective TAM (the lower inset) with $\varepsilon_{r, \text{eff}} = \varepsilon_d$ and $\varepsilon_{r, \text{eff}} = \varepsilon_d + i\gamma(\omega)$, where $\gamma(\omega) = (Z_0 / R_y) / (k_0 \alpha \cos \alpha)$ is a function of angular frequency $\omega$. Here, $R_y$ is the sheet resistance of the CFs. Figure 2(b) presents the equal-frequency surface of the effective TAM in the three-dimensional $k$ space composed of the $k_x$, $\text{Re}(k_y)$, and $\text{Im}(k_y)$ coordinates, which is obtained by adopting different values of $\gamma(\omega)$. The EFCs regarding the particular cases $\gamma(\omega) = 0$ and $\gamma(\omega) = \varepsilon_d$ are plotted as black and green lines, respectively. In the absence of material loss ($\gamma(\omega) = 0$), the EFC is a circle on the $k_x$-$\text{Re}(k_y)$ plane. When loss is introduced, the EFC separates into two curves extending into the $\text{Im}(k_y)$ direction. Interestingly, we find that the EFCs of the TAM with any $\gamma(\omega)$ always pass through a fixed point I located at the coordinates $k_x = \sqrt{\varepsilon_d} k_0 \sin \alpha$, $\text{Re}(k_y) = -\sqrt{\varepsilon_d} k_0 \cos \alpha$, and $\text{Im}(k_y)$.
\( \text{Im}(k_y) = 0 \) (see the enlarged inset graph), indicating that it is independent of \( \gamma(\omega) \). Similar to the lossless case, we have an equal-\( |Z| \) surface at \( k_\parallel = \sqrt{\varepsilon_\parallel k_y \sin \alpha} \), which crosses the point I, and another equal-\( |Z| \) surface at \( k_\parallel = -\sqrt{\varepsilon_\parallel k_y \sin \alpha} \), whose intersection with the equal-frequency surface turns into the ring II, as shown in Fig. 2(b). At the point I and on the ring II, the wave impedance is always a constant \( Z = \frac{\varepsilon_\parallel}{\sin \theta_1} \), and another equal-\( Z \) surface at \( k_\parallel = \frac{\varepsilon_\parallel}{\sin \theta_1} \), whose intersection with the equal-frequency surface turns into the ring II, as shown in Fig. 2(b).

Interestingly, on the ring II, the induced \( \text{Im}(k_y) \neq 0 \) by \( \gamma(\omega) \neq 0 \) indicates the intriguing possibility to achieve large absorption without breaking the impedance matching condition.

We have also performed numerical simulations to verify the reciprocity-protected anomalous Brewster effect with large absorption. Here, we consider TM-polarized waves incident on the TAM from an air background. First, the TAM is set as \( \varepsilon_d = 2 \) and \( \alpha = 35.3^\circ \). In Fig. 2(c), we plot the reflectance on the air-TAM interface as a function of \( \theta_\parallel \) and \( \gamma(\omega) \), which clearly shows a regime of near-zero reflection around \( \theta_\parallel = \pm 54.7^\circ \), irrespective of \( \gamma(\omega) \).

In Fig. 2(d), we plot the simulated magnetic field distributions for Gaussian beams incident on a TAM with \( \gamma(\omega) = 3 \) at \( \theta_\parallel = \pm 54.7^\circ \). Clearly, for \( \theta_\parallel = 54.7^\circ \) (the upper inset), the Gaussian beam transmits through the TAM, with almost no reflection or absorption. While for \( \theta_\parallel = -54.7^\circ \) (the lower inset), the Gaussian beam is almost totally absorbed by the TAM, with no reflection or transmission. Second, the TAM is set as \( \varepsilon_d = 1 \). The reflectance on the air-TAM interface is plotted in Fig. 2(e), in which the TAM has \( \alpha = 30^\circ \). Clearly, near-zero reflection emerges around \( \theta_\parallel = \pm 30^\circ \), irrespective of \( \gamma(\omega) \). Interestingly, in this case, \( \alpha \) is arbitrary and PIM is realized as long as \( \theta_\parallel = \pm \alpha \). This characteristic could be valuable for the absorption of the near field. For a demonstration, in Fig. 2(f), we demonstrate the simulated magnetic field distributions when a dipole source is placed above two inhomogeneous TAMs with variant \( \alpha \) along the surface. The condition of \( \theta_\parallel = \alpha \) (upper inset) or \( \theta_\parallel = -\alpha \) (lower inset) is satisfied everywhere on the surface of the TAMs, where the \( \gamma(\omega) \) is set as 2 and the orientation of the
axis $e_y$ is denoted by arrows. Interestingly, in the case of $\theta = \alpha$, the radiation waves of the dipole source are almost totally transmitting through the TAM, with no reflection. While in the case of $\theta = -\alpha$, almost all the radiated waves from the dipole source are absorbed, with no reflection. This indicates a transition from total transparency to total absorption, simply by varying the orientation of the axis of the TAM. In other words, we first insert the parallel plates in a way that they do not perturb the fields as all these parallel plates act as waveguides without cut-off to allow unit transmittance. Then, all these conducting plates have the orientation flipped horizontally, allowing reciprocity-protected zero reflection while locally the E-fields have components parallel to the plates to generate huge absorption.

Since the reciprocity-protected anomalous Brewster effect is irrespective of $\gamma(\omega)$, ultra-broadband reflection-less absorbers of electromagnetic waves can be realized. In the following, by using the tilted CF arrays, we experimentally verify this phenomenon in microwave frequencies. A sample of the CF array is fabricated, which consists of alternative polymethyl methacrylate (PMMA, relative permittivity ~2.6) and indium tin oxide (ITO) films (thickness~100μm, as CFs) with a separation distance of $a=5$mm. The sample has a tilt angle of $\alpha = 31.8^\circ$, a thickness of 30mm, and a height of 200mm, as shown by the photo shown in Fig. 3(a). Ultra-broadband Brewster effect appears at $\theta = \pm 58.2^\circ$ for such a TAM, while absorption only emerges at $\theta = -58.2^\circ$. In Fig. 3(a), we plot the $|\text{Im}(k)|$ in the CF array (red dots) and the corresponding effective medium (black lines) as the function of the sheet resistance $R_s$ of the CFs at 10GHz. Here, $|\text{Im}(k)|$ determines the attenuation rate, and a larger $|\text{Im}(k)|$ implies a thinner thickness requirement for the absorber. In Fig. 3(a), the maximal $|\text{Im}(k)|$ is found to be around the optimal sheet resistance $\sim 0.35Z_0=132\Omega$ for the CF array, which coincides well with the effective medium calculation, indicating the validity of effective medium here. In the experiments, each piece of ITO film has a sheet resistance of $\sim 370\Omega$. Therefore, we stacked two ITO films into one, so that the composite film has a sheet resistance of $\sim 185\Omega$ (blue dashed lines in Fig. 3(a)), and relatively good performance of absorption can be attained. Figures 3(b) and 3(c) show, respectively, the simulated reflectance...
and absorbance of the sample. In Fig. 3(b), near-zero reflection emerges around $\theta_i = \pm 58.2^\circ$ for all frequencies below 16GHz. Above 16GHz, the non-reflection effect gradually deteriorates because the effective medium turns inaccurate in the short wavelength regime. Nevertheless, we note that the upper working frequency can be significantly raised, even to the optical frequency, by simply decreasing the scale of the structures, i.e. the separation distance. While the absorbance in Fig. 3(c) shows a distinct asymmetric behavior. Zero absorbance occurs around $\theta_i = 58.2^\circ$ and high absorbance ($>0.9$) occurs around $\theta_i = -58.2^\circ$ in the frequency regime from 7~15GHz. Here, we note that the relative low absorbance below 7GHz is mainly induced by transmission since the reflectance is almost zero. Therefore, by increasing the sample thickness, the absorption at low frequencies can be easily increased.

The experimental results are shown in Fig. 3(d). In the experiment, an emitting horn antenna is placed ~0.8 meters away from the sample to generate the incident waves. We measured the near-field electric fields before and after the sample by using a probing antenna. The scanning areas are 700×150mm$^2$ each, located on the central plane of the sample. In the following, we demonstrate the measured electric field distribution at 7GHz, 10GHz and 15GHz (marked by stars in Figs. 3(b) and 3(c)). The upper (lower) inset graph of Fig. 3(d) presents the measured electric fields at $\theta_i = 58.2^\circ$ ($\theta_i = -58.2^\circ$). Clearly, there is no reflection in all cases, demonstrating the ultra-broadband anomalous Brewster effect with absorption. Interestingly, near-total transmission is observed at $\theta_i = 58.2^\circ$, while large absorption is clearly observed at $\theta_i = -58.2^\circ$, for all three selected frequencies. In experiments, we evaluate the absorbance through integrating far-field power for all directions, finding absorbance as high as 0.99, 0.98, and 0.95 at 7GHz, 10GHz, and 15GHz, respectively.

In the second experiment, we verify the other nontrivial solution of the ultra-broadband anomalous Brewster effect with absorption for the TAM designed with $\varepsilon_r = 1$. In this case, $\alpha$ is arbitrary and PIM is realized as long as $\theta_i = \pm \alpha$. The TAM sample is composed of alternative foam (relative permittivity ~1) and ITO films (thickness~100μm, as CFs) with a separation distance of $a = 10$mm. The sample has a tilt angle of $\alpha = 30^\circ$, a thickness of 30mm, and a
height of 200mm, as shown by the photo shown in Fig. 4(a). In Fig. 4(a), we plot the in the sample (red dots) and the corresponding effective medium (black lines) as the function of the sheet resistance $R_s$ of the CFs at 10GHz. The maximal $\text{Im}(k_y)$ is observed around the optimal sheet resistance $Z_0 \cos \alpha/(k_0 a) = 155\Omega$. Theoretically, the maximal value $\text{Im}(k_y)$ is derived to be $k_0 \sin \alpha \tan \alpha$. In the experiment, we have exploited a composite film with $R_s \sim 185\Omega$ consisting of two ITO films as adopted in the previous experiment (see blue dashed lines in Fig. 4(a)). Figures 4(b) and 5(c) show, respectively, the simulated reflectance and absorbance of the sample. Clearly, ultra-low reflection is achieved around $\theta \approx \pm 30^\circ$ for all frequencies below 16GHz. Near-perfect absorption is achieved around $\theta \approx 30^\circ$ in the frequency regime from 7~15GHz, while there is almost no absorption around $\theta \approx 30^\circ$ . These results were also confirmed by experimental measurement. The measured electric field distributions are plotted in Fig. 4(d) for $\theta = 30^\circ$ (upper inset) and $\theta = -30^\circ$ (lower inset), respectively, at 7GHz, 10GHz and 15GHz. Indeed, near-total transmission is observed at $\theta = 30^\circ$, while large absorption is clearly observed at $\theta = -30^\circ$, for all three selected frequencies. The inset graphs of Fig. 4(d) show the measured far-field radiation patterns (green and red lines) of the sample. The black dashed lines denote the reference patterns in the absence of the sample. Clearly, ultra-broadband anomalous Brewster effect with near-perfect absorption (>0.99 at 7GHz, >0.95 at 10GHz, >0.87 at 10GHz) is obtained.

There are several ways to further improve the absorption performance for the TAM demonstrated above. The simplest method is to increase the thickness of the samples since the metamaterial absorber is perfectly impedance-matched to free space. By adding a reflector behind the CF arrays, the angular-asymmetric performance of absorption would be changed into angular-symmetric large absorption, which may be important in practical applications [18]. In the case of $\varepsilon_{r} = \varepsilon_{r}$, the rotation angle $\alpha$ can be arbitrary. Interestingly, the maximal value $\text{Im}(k_y)$ is derived as $\text{Im}(k_y) = k_0 \sin \alpha \tan \alpha$, which tends to infinity as $\alpha$ goes to $90^\circ$. This means that ultra-broadband perfect absorption can be realized even when $\alpha \to 90^\circ$. 
Experimentally, we have also fabricated another sample with $\varepsilon_d = 1$ and $\alpha = 45^\circ$, showing higher absorption [18]. Ultrathin perfect absorbers with extremely thin thickness can also be obtained based on the TAM with hyperbolic dispersions [18].

The hallmark advantage of the Brewster-effect absorbers is that they are based on inherently non-resonant metamaterials, and thus the bandwidth of impedance-matched absorption can cover the ultra-broad spectrum from the quasi-static regime to optical frequencies, which is far beyond those of other absorber techniques [5, 6, 20-35]. Traditional metamaterial absorbers usually rely on localized resonances and multiple interferences, which significantly limit the bandwidth of impedance matching. Here, impedance matching is guaranteed by applying the reciprocity principle to a non-resonant effective medium, thus the bandwidth is inherently ultra-broadband. Since the theory is based on an effective medium, there is no lower limit for the working frequency, while the upper limit is determined by the validity of the effective medium approximation. Through the reduction of the microstructure unit (e.g. the thickness and the separation distance of the CFs), the working frequency range of perfect-impedance-matched absorption can, in principle, be further widened to cover an unprecedented broad regime from the quasi-static limit to THz, infrared and even optical regimes [18]. Another fundamental advantage of the proposed Brewster-effect absorbers lies in the tunability of the magnitude of material loss as well as the refractive behaviors, which offers significant flexibility.

In conclusion, in this work, we reveal an anomalous Brewster effect for reflection-free manipulation of waves, including tunable absorption and refraction. The anomalous Brewster effect, as protected by the reciprocity principle, guarantees zero reflection for one polarization at a particular incident angle. At the same time, the refraction and absorption are flexibly tunable via the extra degrees of freedoms introduced by the anisotropy of metamaterials. The anomalous Brewster effect bestows Brewster-effect absorbers with an unprecedented wide bandwidth of impedance-matched absorption. While conventional wisdom tells us that addition of loss will destroy the Brewster effect, we have demonstrated the mechanism of reciprocity protection escapes from such a deficiency. This principle of anomalous Brewster effect is universal for any reciprocal materials for general classical waves.
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Author contributions

Y. L. and J. Luo conceived the idea. J. Luo formulated the theory and carried out the numerical simulations. J. Luo and J. Li established the reciprocity argument. H. C. carried out the experiments. All authors were involved in the discussion and analysis of data. Y. L., J. Luo and J. Li supervised the project and prepared the manuscript.

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FIG. 1. Simultaneous realization of $\varepsilon_y$-independent Brewster effect and $\varepsilon_y$-controlled transmission in TAMs. (a) Illustration of the Brewster effect between a background dielectric $\varepsilon_b$ and a nonmagnetic TAM. Due to the protection of reciprocity, the wave impedance in the left case $\theta_i > 0$ is exactly the same as that in the right case $\theta_i < 0$. The transmission in the right (left) case is dependent (independent) on $\varepsilon_y$. [(b) and (e)] The EFCs of the background dielectric (dashed lines) and the TAM when $\varepsilon_y$ and $\alpha$ are fixed but $\varepsilon_y$ is varied (solid lines), regarding the cases of (b) $\varepsilon_y \neq \varepsilon_b$ and (e) $\varepsilon_y = \varepsilon_b$, respectively. The colors denote the magnitude of wave impedance $|Z|$. The gray vertical lines are the equal-$|Z|$ lines, which intersect with the EFCs at points I and II, respectively. The gray arrows denote the wave vectors of the incident and reflected beams, and the red and green arrows denote the refracted beam.
[(c) and (f)] Calculated reflectance as functions of $\theta_i$ and $\varepsilon_y$ for TM-polarized waves incident from free space onto the TAM with (c) $\varepsilon_y = 2$ and $\alpha = 35.3^\circ$, or (f) $\varepsilon_y = 1$ and $\alpha = 30^\circ$, respectively. [(d) and (g)] Simulated distributions of magnetic fields (color) and group velocity (arrows) for the TAM with (d) $\varepsilon_y = 2$ and $\alpha = 35.3^\circ$ under $\theta_i = 54.7^\circ$ (left) or $\theta_i = -54.7^\circ$ (middle and right), or (g) $\varepsilon_y = 1$ and $\alpha = 30^\circ$ under $\theta_i = 30^\circ$ (left) or $\theta_i = -30^\circ$ (middle and right), respectively. We set $\varepsilon_y = 3$ (left and middle) or $\varepsilon_y = -3$ (right).
FIG. 2. Ultra-broadband Brewster effect with controllable loss by using tilted CF arrays. (a) Illustrations of a tilted CF array embedded in a dielectric of $\varepsilon_d$ (upper) as the effective TAM (lower). (b) The equal-frequency surface of the effective TAM that is formed by varying $\gamma(\omega)$.

The black and green lines denote the EFCs regarding the cases of $\gamma(\omega)=0$ and $\gamma(\omega)=\varepsilon_d$, respectively. The gray surfaces denote the equal-$|Z|$ surfaces, which intersect with the equal-frequency surface at point I and the ring II (enlarge insets in the right). [(c) and (e)] Calculated reflectance as functions of the $\theta_i$ and $\gamma(\omega)$ for TM-polarized waves incident onto the effective TAM with (c) $\varepsilon_d=2$ and $\alpha=35.3^\circ$, or (e) $\varepsilon_d=1$ and $\alpha=30^\circ$, respectively. (d) Simulated magnetic field-distributions when a TM-polarized Gaussian beam is incident onto
the effective TAM with $\varepsilon_d = 2$, $\alpha = 35.3^\circ$ and $\gamma(\omega) = 3$ for $\theta_I = 54.7^\circ$ (upper) or $\theta_I = -54.7^\circ$ (lower). (f) Simulated magnetic field-distributions for a TM-polarized dipole source above an inhomogeneous TAM with fixed $\varepsilon_d = 1$ and $\gamma(\omega) = 3$ but varied $\alpha$. The $\alpha$ is adjusted along the surface so that the condition of $\theta_I = \alpha$ (upper) or $\theta_I = -\alpha$ (lower) is satisfied everywhere. The arrows in (d) and (f) denote the orientation of the axis $\varepsilon_y$. 
FIG. 3. Experimental observation of ultra-broadband Brewster-effect absorbers in the case of \( \varepsilon_d \neq \varepsilon_a \). (a) The \( \text{Im}(k_y) \) in the CF array (red dots) and the corresponding effective medium (black solid lines) as the function of the \( R_s \) at 10GHz. The inset shows the picture of the fabricated sample consisting of alternative PMMA and ITO films \( \alpha = 31.8^\circ \). The thickness and height of the sample are 30mm and 200mm, respectively. [(b) and (c)] Simulated (b) reflectance and (c) absorbance on the fabricated sample as functions of the \( \theta_i \) and the working frequency, respectively. The stars denote the measured cases in experiments. (d) Measured electric field-distributions under \( \theta_i = 58.2^\circ \) (upper) and \( \theta_i = -58.2^\circ \) (lower) at 7GHz (left), 10GHz (middle) and 15GHz (right). The black thick lines near the sample denote the absorptive foam.
FIG. 4. Experimental observation of ultra-broadband Brewster-effect absorbers in the case of $\varepsilon_x = \varepsilon_y$. (a) The $|\text{Im}(k)|$ in the CF array (red dots) and the corresponding effective medium (black solid lines) as the function of the $R_z$ at 10GHz. The inset shows the picture of the fabricated sample consisting of an alternative foam and ITO films. [(b) and (c)] Simulated reflectance and absorbance on the fabricated sample as functions of the $\theta_i$ and the working frequency, respectively. The stars denote the measured cases in experiments. (d) Measured electric field-distributions under $\theta = 30^\circ$ (upper) and $\theta = -30^\circ$ (lower) at 7GHz (left), 10GHz (middle) and 15GHz (right). The insets show the measured far-field radiation patterns with (green and red solid lines) or without (black dashed lines) samples.