SO(3) Gauge Symmetry and Nearly Tri-bimaximal Neutrino Mixing

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In this note I mainly focus on the neutrino physics part in my talk and report the most recent progress made in [1]. It is seen that the Majorana features of neutrinos and SO(3) gauge flavor symmetry can simultaneously explain the smallness of neutrino masses and nearly tri-bimaximal neutrino mixing when combining together with the mechanism of approximate global U(1) family symmetry. The mixing angle \( \theta_{13} \) and CP-violating phase are in general nonzero and testable experimentally at the allowed sensitivity. The model also predicts the existence of vector-like Majorana neutrinos and charged leptons as well as new Higgs bosons, some of them can be light and explored at the LHC and ILC.

Keywords: Gauge Flavor Symmetry; Neutrino Mixing; Vector-like Fermions.

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The observed massive neutrinos and dark matter in our universe challenge the standard models of both particle physics and cosmology. So both flavor physics and cosmology may tell us fundamental physics [2]. In this note I will concentrate on discussing the neutrino physics.

Various neutrino experiments [3-11] provide more and more stringent constraints on the three mixing angles and mass squared differences for three neutrinos [12]

\[
\begin{align*}
30^\circ < \theta_{12} < 38^\circ, & \quad 36^\circ < \theta_{23} < 54^\circ, \quad \theta_{13} < 10^\circ \\
7.2 \times 10^{-5} \text{ eV}^2 < \Delta m^2_{21} = m_{\nu_{\mu}}^2 - m_{\nu_e}^2 < 8.9 \times 10^{-5} \text{ eV}^2, & \\
2.1 \times 10^{-3} \text{ eV}^2 < \Delta m^2_{32} = m_{\nu_{\tau}}^2 - m_{\nu_{\mu}}^2 < 3.1 \times 10^{-3} \text{ eV}^2
\end{align*}
\]

at the 99% confidence level [12]. In comparison with the quark sector, it raises a puzzle that why neutrino masses are so tiny, but their mixing angles are so large [15].

The only peculiar property for neutrinos is that they could be Majorana fermions, so a natural solution to the puzzle is most likely attributed to the Majorana features. Thus revealing the origin of large mixing angles and small masses of neutrinos is important not only for understanding neutrino physics, but also for exploring new physics beyond the standard model [17].
The current data on the neutrino mixing angles are consistent with the so-called tri-bimaximal mixing with \( \theta_{12} = \sin^{-1}(1/\sqrt{3}) = 35^\circ, \theta_{23} = \sin^{-1}(1/\sqrt{2}) = 45^\circ \) and \( \theta_{13} = 0 \), which was first proposed by Harrison, Perkins and Scott \(^{13}\) and investigated by various groups \(^{19,20,21,22}\). The mixing angle \( \theta_{13} \) is the most unclear parameter and expected to be measured in near future. Such a tri-bimaximal mixing matrix has been found to be yielded by considering some interesting symmetries, especially the discrete symmetries \(^{23}\). In general, it is shown that the discrete symmetries lead to \( \theta_{13} = 0 \) \(^{24}\). In such a case, it is hard to be directly tested experimentally. Alternatively, it is interesting to consider a non-abelian gauge family symmetry SO(3) \(^{1}\) instead of discrete symmetries discussed widely in literature. In this case, the tri-bimaximal neutrino mixing matrix is generally obtained as the lowest order approximation from diagonalizing a special symmetric mass matrix \(^{11}\). In fact, the greatest success of the standard model (SM) is the gauge symmetry structure \( SU(3)_c \times SU(2)_L \times U(1)_Y \) which has been tested by more and more precise experiments. SO(3) gauge family symmetry can be regarded as a simple extension of the standard model with three families and Majorana neutrinos. It is noted that only SO(3) rather than SU(3) is allowed due to the Majorana feature of neutrinos.

The \( SO(3) \times SU(2)_L \times U(1)_Y \) invariant Lagrangian for Yukawa interactions of leptons with Majorana neutrinos can be constructed as follows \(^{11}\)

\[
\mathcal{L}_Y = y_{\nu_1} \bar{H} \nu_R + y_N \bar{H} N + \frac{1}{2} \xi_N \bar{N} \Phi_\nu N + \frac{1}{2} M_R \bar{\nu}_R \nu_R^c + y_{\nu_2} \bar{H} E + \xi_\nu \bar{E} \Phi_\nu E + \text{H.c.} \tag{3}
\]

where \( y_{\nu_1}, y_{\nu_2}, y_N, \xi_\nu, \xi_N \) and \( \xi_E \) are all real Yukawa coupling constants and \( M_R \) is the mass of right-handed Majorana neutrinos. All the fermions \( \nu_{Li}, \nu_{Ri}, e_{Li}, e_{Ri}, E_i \) and \( N_i \) \((i = 1, 2, 3)\) belong to SO(3) triplets in family space. Where \( \bar{L}_i = (\bar{\nu}_{Li}, \bar{e}_{Li}) \) denote \( SU(2)_L \) doublet leptons, \( H \) and \( H_N \) are \( SU(2)_L \) doublet Higgs bosons with \( H = \tau_2 H^*, \nu_{Ri} \) are the right-handed neutrinos with \( \nu_{Ri}^c = c_i \nu_{Ri}^T \) the charge conjugated ones. \( E_i \) are \( SU(2)_L \) singlet vector-like charged leptons and the \( N_i \) are \( SU(2)_L \) singlet vector-like Majorana neutrinos with \( N_i^c = N_i \), \( \phi_s \) is a singlet Higgs boson. The scalar fields \( \Phi_\nu \) and \( \Phi_s \) are SO(3) tri-ttriplets Higgs bosons satisfying \( \Phi_\nu = \Phi_\nu^c \), \( \Phi_\nu = \Phi_\nu^T \) and \( \Phi_s = \Phi_s^T \) which is required by the hermiticity condition of Lagrangian and the Majorana condition of vector-like neutrinos. The above Lagrangian is solely ensured by the following discrete symmetry \((Z_2 \text{ and } Z_4)\)

\[
N \rightarrow i\gamma_5 N, \quad \Phi_\nu \rightarrow -\Phi_\nu, \quad H_N \rightarrow -i H_N, \quad \phi_s \rightarrow -\phi_s, \quad e_R \rightarrow -e_R \tag{4}
\]

In terms of SO(3) representation, one can reexpress the real symmetric tri-triplet Higgs boson into the following general form

\[
\Phi_\nu = O_\nu \phi_\nu O_\nu^T, \quad O_\nu(x) = e^{i\lambda^a T_\alpha^a(x)}, \quad \phi_\nu(x) = \begin{pmatrix}
\phi_{1\nu}^c & \phi_{2\nu}^c & \phi_{3\nu}^c \\
\phi_{2\nu}^c & \phi_{3\nu}^c & \phi_{1\nu}^c \\
\phi_{3\nu}^c & \phi_{1\nu}^c & \phi_{2\nu}^c
\end{pmatrix} \tag{5}
\]
with $\lambda^i$ ($i = 1, 2, 3$) being the generators of SO(3). Where $\Theta^i_r(x)$ ($i = 1, 2, 3$) may be regarded as three rotational scalar fields of SO(3), and $\phi^i_r(x)$ ($i = 1, 2, 3$) are three dilatation scalar fields.

SO(3) gauge invariance allows us to fix the gauge by making SO(3) gauge transformation $g(x)$, so that $g(x) \equiv O_\nu(x) \in SO(3)$, we then arrive at the following Yukawa interactions

$$\mathcal{L}_Y = y_e \bar{\nu} R H_R + y_N \bar{N} H_N + \frac{1}{2} \xi_N \nu^c \nu + \frac{1}{2} M_R \nu^c \nu_R$$

$$+ y_e \bar{\nu} R H + \xi_e \nu^c \bar{R} + \frac{1}{2} \xi_e \nu^c \bar{E} \nu + H.c.$$  \hfill (6)

which is invariant under $Z_3$ transformation. Where $\hat{\Phi}_e = O^T \Phi \nu \nu_e$ remains Hermitian and contains nine independent scalar fields, which can generally be reexpressed in terms of SO(3) representation as the following form

$$\hat{\Phi}_e \equiv U_e \nu \nu_e U_e^T, \quad U_e(x) \equiv P_e O_e, \quad O_e(x) = e^{i\lambda^i(x)}$$  \hfill (7)

and

$$P_e(x) = \begin{pmatrix} e^{i\eta_1(x)} & 0 & 0 \\ 0 & e^{i\eta_2(x)} & 0 \\ 0 & 0 & e^{i\eta_3(x)} \end{pmatrix}, \quad \phi_e(x) = \begin{pmatrix} \phi^1_e(x) \\ \phi^2_e(x) \\ \phi^3_e(x) \end{pmatrix}$$  \hfill (8)

where $\chi^i_e(x)$ ($i = 1, 2, 3$) are regarded as three rotational scalar fields of SO(3), $\eta^i_e(x)$ ($i = 1, 2, 3$) denote three phase scalar fields and $\phi^i_e(x)$ ($i = 1, 2, 3$) are three dilatation scalar fields.

We now consider the following general vacuum structure of scalar fields under the above gauge fixing condition

$$\langle H(x) \rangle = v, \quad \langle H_N(x) \rangle = v_N$$

$$\langle \phi_s(x) \rangle = v_s, \quad \langle \phi^c_s(x) \rangle = v^c_s$$

$$\langle \phi^c_2(x) \rangle = v^c_2, \quad \langle \chi^c_3(x) \rangle = \theta^c_3$$  \hfill (9)

namely $\langle P_e \rangle = \text{diag}(e^{i\delta^c_1}, e^{i\delta^c_2}, e^{i\delta^c_3})$ and $\langle O_e \rangle = e^{i\lambda^c_\nu(x)}$. Here $\delta^c_i$ ($i=1,2,3$) are CP phases arising from spontaneous symmetry breaking and $\theta^c_3$ are three rotational angles of SO(3).

Such a vacuum structure after spontaneous symmetry breaking leads to the following mass matrices for neutrinos with a type II like see-saw mechanism and for charged leptons with a generalized see-saw mechanism

$$M_\nu = m_\nu^D M_R^{-1} m_\nu^D + m_N^D M_N^{-1} m_N^D$$  \hfill (10)

$$M_e = V_e m_E^E M_E^{-1} m_E^E V_e^T$$  \hfill (11)
with $m^D_\nu = y_\nu v$, $m^D_N = y_N v_N$, $m^D_E = \sqrt{y_\nu v} v_s$, $V_e = U_e = P_\nu e^{i\lambda^i \theta^i}$, and

$$M_N = \xi_N \begin{pmatrix} v_1' & v_2' & v_3' \\ v_2' & v_2' & v_2' \\ v_3' & v_1' & v_2' \end{pmatrix}, \quad M_E = \xi_E \begin{pmatrix} v_1' & 0 & 0 \\ 0 & v_2' & 0 \\ 0 & 0 & v_3' \end{pmatrix} \quad (12)$$

with $c^i_{ij} \equiv \cos \theta^i_{ij}$ and $s^i_{ij} \equiv \sin \theta^i_{ij}$, and $\theta^i_{ij}$ are given as functions of $\theta^i$ ($i=1,2,3$)

A similar special symmetric neutrino mass matrix like $M_N$ was also resulted for the Dirac-type neutrinos with a new symmetry [35] and the Majorana-type neutrinos with the $Z_3$ group [36].

When taking the Majorana neutrino masses $M_R$ and $M_N$ to be infinity large, the interactions with Majorana neutrinos decouple from the theory,

$$\frac{y_\nu^2}{M_R} i\vec{H} \tilde{H}^T l^c, \quad \frac{y_\nu^2}{M_N} iH_N \tilde{H}^T l^c \rightarrow 0, \quad \text{for } M_R, M_N \rightarrow \infty \quad (14)$$

which implies that the resulting Yukawa interactions in this limit generate additional global U(1) family symmetries for the charged lepton sector. Namely, once the Majorana neutrinos become very heavy, the Yukawa interactions possess approximate global U(1) family symmetries. Thus when applying the mechanism of approximate global U(1) family symmetries [37] to the Yukawa interactions after SO(3) symmetry is broken down spontaneously, we have $M_R \gg m^D_\nu$, $M_N \gg m^D_N$ and $\theta^i_{ij} \ll 1$, namely

$$M_\nu \ll 1, \quad \theta^i_{ij} \ll 1 \quad (15)$$

which provides a possible explanation why the observed left-handed neutrinos are so light and meanwhile the charged lepton mixing angles must be small. Thus the neutrino mass matrix is given by a type II like see-saw mechanism and the charged lepton mass matrix is presented by a generalized see-saw mechanism. By diagonalizing the mass matrices, we obtain

$$V^T_\nu M_\nu V_\nu = \text{diag.}(m_\nu, m_\nu, m_\nu), \quad V^T_e M_e V_e = \text{diag.}(m_e, m_\mu, m_\tau) \quad (16)$$

where

$$V_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} c_\nu & \frac{1}{\sqrt{3}} s_\nu & \frac{1}{\sqrt{2}} s_\nu \\ \frac{1}{\sqrt{6}} c_\nu - \frac{1}{\sqrt{2}} s_\nu & \frac{1}{\sqrt{3}} c_\nu - \frac{1}{\sqrt{2}} s_\nu & \frac{1}{2} s_\nu \\ -\frac{2}{\sqrt{6}} c_\nu + \frac{1}{\sqrt{2}} s_\nu & \frac{1}{\sqrt{3}} c_\nu - \frac{1}{\sqrt{2}} s_\nu & -\frac{1}{2} s_\nu \end{pmatrix} \equiv V_0 V_1 \quad (17)$$

with

$$V_0 = \begin{pmatrix} \frac{2}{\sqrt{6}} c_\nu & 1 & 0 \\ \frac{1}{\sqrt{3}} c_\nu & \frac{1}{\sqrt{3}} s_\nu & \frac{1}{\sqrt{2}} s_\nu \\ -\frac{1}{\sqrt{6}} c_\nu & -\frac{1}{\sqrt{3}} s_\nu & -\frac{1}{\sqrt{2}} s_\nu \end{pmatrix}, \quad V_1 = \begin{pmatrix} c_\nu & 0 & s_\nu \\ 0 & 1 & 0 \\ -s_\nu & 0 & c_\nu \end{pmatrix} \quad (18)$$
Here $V_0$ is the so-called tri-bimaximal mixing matrix. For short, we have introduced the notations $c_\nu \equiv \cos \theta_\nu$ and $s_\nu \equiv \sin \theta_\nu$ with

$$\tan 2\theta_\nu = \frac{\sqrt{3}(v_2^\nu - v_3^\nu)}{v_{21}^\nu + v_{31}^\nu}, \quad v_{21}^\nu \equiv v_2^\nu - v_1^\nu, \quad v_{31}^\nu \equiv v_3^\nu - v_1^\nu$$  \hspace{1cm} (19)

As the smallness of charged lepton mixing angles $\theta_{ij}^e$ ($i < j$) can be attributed to the mechanism of approximate global U(1) family symmetries in the Yukawa sector, in a good approximation up to the first order of $s_{ij}^e$, the leptonic CKM-type mixing matrix in the mass eigenstates of leptons may be expressed as the following simplified form

$$V = V_\nu^\dagger V_\nu \approx \begin{pmatrix} 1 & -s_{12} & -s_{13} \\ s_{12} & 1 & -s_{23} \\ s_{13} & s_{23} & 1 \end{pmatrix} P_3 \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} c_\nu & s_\nu \\ 0 & -s_\nu \end{pmatrix}$$  \hspace{1cm} (20)

The three vector-like heavy Majorana neutrino masses are obtained via diagonalizing the mass matrix $M_N$, i.e., $V_\nu^\dagger M_N V_\nu = \text{diag.}(m_{N_1}, m_{N_2}, m_{N_3})$

$$m_{N_1} = -m_N \sqrt{1 - \Delta}, \quad m_{N_2} = m_N, \quad m_{N_3} = m_N \sqrt{1 - \Delta}$$  \hspace{1cm} (21)

with

$$m_N \equiv \xi_N(v_1^\nu + v_2^\nu + v_3^\nu), \quad \Delta = 3(v_1^\nu v_2^\nu + v_2^\nu v_3^\nu + v_3^\nu v_1^\nu)/(v_1^\nu + v_2^\nu + v_3^\nu)^2$$  \hspace{1cm} (22)

The masses of three left-handed light Majorana neutrinos are given in the physics basis as follows

$$m_{\nu_e} = \tilde{m}_0 - m_1(2 + \tilde{\Delta}), \quad m_{\nu_\mu} = \tilde{m}_0, \quad m_{\nu_\tau} = \tilde{m}_0 + m_1 \tilde{\Delta}$$  \hspace{1cm} (23)

with $\tilde{m}_0 \equiv m_0 + m_1$, $\tilde{\Delta} = 1/\sqrt{1 - \Delta} - 1$, $m_0 = (m_P^D)^2/M_R$ and $m_1 = (m_P^D)^2/m_N$. It then enables us from the experimentally measured neutrino mass square differences to extract two mass parameters $m_0$ and $m_1$ for a given value of parameter $\Delta$ with $\Delta \neq 1$.

The numerical results are presented in table 1 by taking the central values of the mass square differences $\Delta m_{21}^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2 = 8 \times 10^{-5} \text{eV}^2$ and $\Delta m_{32}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 = 2.5 \times 10^{-3} \text{eV}^2$.

| $\Delta$ (input) | $m_0(10^{-3}\text{eV})$ | $m_1(10^{-3}\text{eV})$ | $m_{\nu_e}(10^{-3}\text{eV})$ | $m_{\nu_\mu}(10^{-3}\text{eV})$ | $m_{\nu_\tau}(10^{-3}\text{eV})$ |
|------------------|------------------------|------------------------|-----------------|-----------------|-----------------|
| 0.75             | 1.297                  | 2.487                  | -3.677          | 3.784           | 6.271           |
| 0.73             | 1.271                  | 2.637                  | -4.003          | 3.908           | 6.346           |
| 0.71             | 1.245                  | 2.789                  | -4.333          | 4.034           | 6.424           |
| 0.69             | 1.220                  | 2.943                  | -4.666          | 4.163           | 6.506           |
Without losing generality, considering the case \( v_1' \ll v_2', v_3' \), we then have the following approximate relations

\[
\Delta \simeq \frac{3r}{(1 + r)^2}, \quad \tan 2\theta_\nu \simeq \frac{\sqrt{3}(1 - r)}{1 + r}, \quad r \equiv v_3'/v_2'
\]

which shows that in this case both the mixing angle \( \theta_\nu \) and the ratio \( r \) can be determined for the given values of \( \Delta \), an interesting solution is

\[
\Delta = r = \sqrt{3} - 1, \quad \tan 2\theta_\nu = 2 - \sqrt{3}, \quad \theta_\nu = 7.5^\circ
\]

For a numerical estimation, it is useful to investigate the following two interesting cases

Case I : \( s_{13}^c \simeq 0, \quad s_{12}^c \simeq 0 \)

Case II : \( s_{13}^c \ll s_{12}^c \sim \sqrt{m_e/m_\mu} \simeq 0.07, \quad \delta_1^c - \delta_2^c = \pi/2 \)

which allows us to present a reasonable estimation for \( \theta_{13} \) and \( \delta_\nu \) (see table 2).

| \( \Delta \) (input) | \( r = v_3'/v_2' \) | \( \theta_\nu \) | \( \theta_{13} \) (Case I) | \( \delta_\nu \) (Case I) | \( \theta_{13} \) (Case II) | \( \delta_\nu \) (Case II) |
|---------------------|---------------------|-----------------|---------------------|---------------------|---------------------|---------------------|
| 0.75                | 1.0                 | 0.0             | 0                  | 2.8^\circ           | 90^\circ            |
| 0.748               | 0.90                | 2.6^\circ       | 2.1^\circ          | 0                  | 3.5^\circ           | 54^\circ            |
| 0.745               | 0.85                | 4.0^\circ       | 3.3^\circ          | 0                  | 4.4^\circ           | 41^\circ            |
| 0.74                | 0.79                | 5.8^\circ       | 4.7^\circ          | 0                  | 5.5^\circ           | 31^\circ            |
| 0.73                | 0.72                | 7.9^\circ       | 6.4^\circ          | 0                  | 7.0^\circ           | 24^\circ            |
| 0.71                | 0.63                | 10.7^\circ      | 8.7^\circ          | 0                  | 9.2^\circ           | 18^\circ            |
| 0.69                | 0.56                | 13.0^\circ      | 10.6^\circ         | 0                  | 11.0^\circ          | 15^\circ            |

where \( v_3' \neq v_2' \) (i.e., \( r \neq 1 \)) should be a more general and reasonable case when no symmetry is imposed, the resulting mixing angle \( \theta_{13} \) can be large enough to be detected. For the case II, both mixing angle \( \theta_{13} \) and CP-violating phase \( \delta_\nu \) are in general testable by the future neutrino experiments. For the typical range \( \Delta \simeq 0.72 \pm 0.028 \), we are led to the most optimistic predictions for the mixing angle \( \theta_{13} \) and CP-violating phase \( \delta_\nu \)

\[
\theta_{13} \simeq 7^\circ \pm 4^\circ, \quad \delta_\nu \simeq 35^\circ \pm 20^\circ, \quad r = v_3'/v_2' \simeq 0.73 \pm 0.17
\]

which can be tested in the future experiments.

When taking the Dirac type neutrino masses \( m_\nu^D \) and \( m_N^D \) to be at the order of \((0.1 \sim 1.0) \text{ MeV}\) (i.e., at the same order of electron mass), and the mass parameter \( m_D^E \simeq (15 \sim 25) \text{ GeV} \), we have the lightest vector-like Majorana neutrino masses and charged lepton mass to be

\[
m_{N_1} = m_{N_3} \simeq O(250) \text{ GeV} \sim O(25) \text{ TeV}
\]

\[
m_{E_3} \simeq (127 \sim 352) \text{ GeV}
\]

which is at the electroweak scale and can be explored at LHC and ILC.
In conclusion, we have shown how the puzzles on the smallness of neutrino masses and the nearly tri-bimaximal neutrino mixing may simultaneously be understood in the flavor SO(3) gauge symmetry model with the mechanism of approximate global U(1) family symmetry. The vacuum structure of SO(3) symmetry breaking for the SO(3) tri-triplet Higgs bosons plays an important role. The mixing angle $\theta_{13}$ is in general nonzero and its typical values range from the experimentally allowed sensitivity to the current experimental bound. CP violation in the lepton sector is caused by a spontaneous symmetry breaking and can be significantly large for exploring via a long baseline neutrino experiment. A similar consideration can be extended to the quark sector, unlike the lepton sector with the features of Majorana neutrinos, the mechanism of approximate global U(1) family symmetry can be applied to understand the smallness of quark mixing angles.

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