Quantum speed limit of a photon under non-Markovian dynamics

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Quantum speed limit (QSL) under noise has drawn considerable attention in real quantum computational processes and quantum communication. Though non-Markovian noise is proven to be able to accelerate quantum evolution for a damped Jaynes-Cummings model, in this work we show that non-Markovianity may even slow down the quantum evolution of an experimentally controllable photon system. As an important application, QSL time of a photon can be well controlled by regulating the relevant environment parameter properly, which is close to reach the currently available photonic experimental technology.

I. INTRODUCTION

A quantum version of brachistochrone problem that how fast a quantum system can evolve between two distinguishable states is of paramount importance in quantum information processing, for a transition from a state to its orthogonal one is regarded as the elementary step of a computational process [1, 2]. During the past decades, the study on the minimum time a quantum state required for reaching its orthogonal one, i.e., the quantum speed limit (QSL) time, has been mainly focused on closed quantum systems with unitary evolution, and a unified lower bound of QSL was obtained [3–8]: $\tau_{\text{QSL}} = \max \left\{ \pi h / (2\Delta E), \pi h / (2E) \right\}$, where the first quantity in braces is known as Mandelstam-Tamm (MT) type bound with $(\Delta E)^2 = \langle H^2 \rangle - \langle H \rangle^2$ and $H$ is the Hamiltonian of the quantum system [3–5] while the second one is referred to as Margolus-Levitin (ML) type bound with $E = \langle H \rangle$ [7]. These bounds, providing a fundamental limit of the operation rate, are applicable to considerable quantum tasks such as quantum state transfer [9], quantum optimal control [10], and quantum metrology [11] and have been extended to nonorthogonal state cases [12–16] and derived from geometric aspects [17, 18].

In realistic physical processes, however, the quantum systems are open, and the environmental influence must be taken into account [19]. Recently, the QSL time has been extended to nonunitary evolution of open systems [20–22]. Two MT type bounds of QSL, based on the variance of the generator of the dynamics, were derived and applied to several typical noisy channels [20] and estimate the speed limits for quantum metrology under noise [21]. Importantly, a unified QSL bound including both MT and ML types for non-Markovian processes has been introduced in Ref. [22], where the ML bound is also proven to be sharper than the MT bound. Interestingly, it is discovered that the non-Markovian effect can speed up the quantum evolution with a damped Jaynes-Cummings (JC) model [22].

All-optical system has been regarded as an excellent test bed to explore the foundations of quantum physics as well as quantum information processing [23–24]. In this paper, with a photon in a simulated non-Markovian environment, we show that non-Markovian effect can slow down the quantum evolution which is contrary to former situation that non-Markovianity will lead to smaller QSL time for JC model [22]. In addition, we illustrate that the QSL time of a photon can be well controlled by adjusting the environment parameter. The above phenomena can be immediately tested with the experimental setups in Refs. [22–27].

II. NON-MARKOVION MODEL

The open system we consider in this paper is the polarization degree of a photon with its frequency functioning as the environment. To simulate the non-Markovian dynamics of the photon, we employ an experimental setup containing a rotatable Fabry-Pérot (FP) cavity followed by an interference filter and a quartz plate [23–27] (see in Fig. 1). A frequency comb of the photon is generated by a FP cavity and then two peaks are filtered out through an interference filter. The filtered frequency distribution $f(\omega)$, representing the probability density of finding photon in a mode with frequency $\omega$ in this letter, is set to be a two-peaked Gaussian distribution [28]

$$f(\omega) = \frac{\cos^2 \xi \sqrt{2\pi\sigma}}{\sqrt{2\pi\sigma}} e^{-\frac{(\omega-\omega_1)^2}{2\sigma^2}} + \frac{\sin^2 \xi \sqrt{2\pi\sigma}}{\sqrt{2\pi\sigma}} e^{-\frac{(\omega-\omega_2)^2}{2\sigma^2}},$$

where $\omega_1$ and $\omega_2$ are the centers of the Gaussian distribution and $\sigma$ is the width. The parameter $\xi$ is

![FIG. 1: A schematic diagram (simplified) of the photonic experimental setup for testing the non-Markovian effect on quantum speed limit.](image-url)

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controls the relative weight of the two peaks, which can be adjusted by changing the tilted angle of the FP cavity [20]. Then the non-Markovian dephasing process of the polarization degree of the photon can occur in the interaction with its frequency degree in a quartz plate with the following Hamiltonian [29]

$$H^{ac} = -\hbar \int (n_H |H \rangle_s \langle H| + n_V |V \rangle_s \langle V|) \otimes \omega |\omega \rangle_e \langle \omega| \ \text{d}\omega,$$

where $n_{H(V)}$ is the refraction index of photon in the quartz plate, $|H(V)\rangle_s$ and $|\omega\rangle_e$ are the horizontal(terminal) polarization and frequency states of the photon.

Provided that an initial product photon state is of the form $\rho^s \otimes \rho^f$, where $\rho^s = (\rho_{jk})_{2 \times 2}$ ($j, k = V, H$) denotes the polarization state serving as the open system, and $\rho^f = \int \text{d}\omega \omega' F(\omega) \rho^f(\omega') \langle \omega'| \langle \omega|$ is the environmental state with $f(\omega) = |F(\omega)|^2$. The photon polarization state at time $t$ reads

$$\rho_t^s = \Lambda_t \rho^s \text{ with } U_t = \text{tr}_e \left\{ U_t \rho^s \otimes \rho^f U_t^\dagger \right\},$$

with $\Lambda_t$, the quantum map and $U_t = \exp[-(i/\hbar) \int_0^t \text{d}t' H^{ac}]$. The density matrix of the polarization degree is then explicitly given by

$$\rho_t^f = \left( \begin{array}{cc} \rho_{VV} & \rho_{VH} \kappa_t \\ \rho_{HV} \kappa_t^* & \rho_{HH} \end{array} \right),$$

where $\kappa_t = \int f(\omega) e^{i\omega \Delta t} \text{d}\omega$ is the dephasing rate with $\Delta t = n_V - n_H$. By the frequency distribution in Eq. (1), it takes the form

$$\kappa_t = e^{-\frac{\pi}{2} \Delta t} \left( e^{i\omega_1 \Delta t} \cos^2 \xi + e^{i\omega_2 \Delta t} \sin^2 \xi \right).$$

### III. QUANTUM SPEED LIMIT FOR A PHOTON

A unified lower bound including both MT and ML types for the minimal evolution time between an initial open system state $\rho = |\psi_0\rangle \langle \psi_0|$ and its target state $\rho_{\tau}$, governed by the master equation $\dot{\rho}_t = \mathcal{L}_t \rho_t$ with $\mathcal{L}_t$ the positive generator of the dynamical semigroup $\Lambda_t = \exp(\mathcal{L}_t t)$, has been derived [22]:

$$\tau_{\text{QSL}} = \max \{ \tau_1, \tau_2, \tau_\infty \},$$

with

$$\tau_p = \frac{1}{\Gamma_p^2} \sin^2 \left[ L(\rho, \rho_\tau) \right], \ (p = 1, 2, \infty)$$

where $\Gamma_p^2 = (1/\tau) \int_0^\tau \| \mathcal{L}_t \rho_t \|_p$, and $\| A \|_p = (a_1^p + \cdots + a_n^p)^{1/p}$ denotes the $p$-norm of operator $A$, with $a_1, \cdots, a_n$ the singular values of operator $A$. $L(\rho, \rho_\tau) = \text{arccos} \frac{\langle \psi_0 | \rho_\tau | \psi_0 \rangle}{\sqrt{\langle \psi_0 | \rho_\tau | \psi_0 \rangle^2 + \langle \psi_0 | \rho | \psi_0 \rangle^2}}$ represents the Bures angle between initial pure state $\rho = |\psi_0\rangle \langle \psi_0|$ and the target state $\rho_\tau$. Note that $\tau_1$ and $\tau_\infty$ are bounds of ML type derived by von Neumann trace inequality while the $\tau_p$ is a bound of MT type deduced according to Cauchy-Schwarz inequality.

With above model, for instance, we will evaluate the minimal evolution time between states $\rho^s$ and $\rho^s_\tau$, where $\rho^s$ and $\rho^s_\tau$ denote the states of photon entering and leaving the quartz plate respectively with $\tau$ the actual driving time when the photon under non-Markovian dephasing. For simplicity, the initial state is set to be pure of the form $\rho^s = |\psi_0\rangle \langle \psi_0|$ with $|\psi_0\rangle = \sin \alpha |H \rangle + \cos \alpha |V \rangle$. It is convenient to check that the maximum in Eq. (1) is $\frac{\tau_\infty}{2}$, for $\Gamma_{\infty} = \Gamma_1^2/2 = \Gamma_2/\sqrt{2}$. With Eq. (10), the QSL time of a photon can be written as

$$\tau_{\text{QSL}} = \frac{2 \tau \sin^2 \theta}{\sin 2\alpha / \int_0^\tau \text{d}t |\kappa_t|},$$

with

$$\theta = \arccos \sqrt{1 - \frac{1}{2} (1 - \text{Re} \kappa_t) \sin^2 2\alpha}.$$

### IV. NON-MARKOVIAN EFFECT ON QUANTUM SPEED LIMIT

In order to study the non-Markovian effect on the QSL time [Eq. (10)], we will employ two popular measures for non-Markovianity [10]: the divisibility of quantum maps [30] and the information flow [31, 32] based methods.

A quantum map $\Lambda = \{ \Lambda_t \}_{t \in [0, \tau]}$ is divisible if $\Lambda_t = \Lambda_{t-r} \Lambda_r$ for all $0 \leq r \leq t$ with $\Lambda_{t-t}$ completely positive. In Ref. [30], $\Lambda$ is regarded as Markovian if it is divisible, which implies that $\left\| (\Lambda_{t+\epsilon, t} \otimes 1) \rho^{ss'} \right\|_1 = 1$, $\epsilon \geq 0$, where $\rho^{ss'} = |\Psi\rangle \langle \Psi|$ with $|\Psi\rangle = (1/\sqrt{d}) \sum_{l=1}^d |l\rangle \langle l|$, a maximally correlated pure state of the $d$-dimensional open system $s$ and an ancillary system $s'$. The non-Markovianity is then defined as [31],

$$N_{\text{RHP}}(\Lambda) = \int_{h_t > 0} h_t dt,$$

with $h_t = \lim_{\epsilon \to 0^+} \frac{\left\| (\Lambda_{t+\epsilon, t} \otimes 1) \rho^{ss'} \right\|_1 - 1}{\epsilon}$.

The quantum dynamics $\Lambda$ we consider here is given by $\rho_t^s = \Lambda_t \rho^s$ [Eq. (3)]. Note that $\rho^{ss'} = |\Psi\rangle \langle \Psi|$ with $|\Psi\rangle = (|H\rangle \langle H| + |V\rangle \langle V|)/\sqrt{2}$. After simple calculations, we find that for small $\epsilon$, the non-zero eigenvalues of $\left( \Lambda_{t+\epsilon, t} \otimes 1 \right) \rho^{ss'}$ are

$$\frac{1}{2} \pm \frac{1}{2} \sqrt{1 + \frac{2}{\Gamma_t^2 \kappa_t} \epsilon + \frac{\Gamma_t^2 \kappa_t}{\kappa_t^2} \epsilon + o(\epsilon)}.$$

We then get

$$h_t = \left\{ \begin{array}{ll} \partial_t \ln(|\kappa_t|), & \text{if } \partial_t |\kappa_t| > 0, \\
0, & \text{if } \partial_t |\kappa_t| \leq 0. \end{array} \right.$$
According to Eq. (10), we have

$$N_{RHP}(\Lambda) = \int_{\partial|\xi|>0} \partial_t \ln(|\xi|) dt. \quad (13)$$

The second measure for the non-Markovianity is based on the total amount of information, characterized by

$$N_{BLP}(\Lambda) = \max_{\rho_1, \rho_2} \int_{g_t>0} g_t dt, \quad (14)$$

where the maximization is over all initial state pairs. For single qubit, the optimal problem is easy to solve [20, 22, 43] and the optimal trace distance of the evolved states is found to be $D(\Lambda_{1}\rho_1^{t}, \Lambda_{1}\rho_2^{t}) = |\xi_t|$ [24]. Therefore,

$$N_{BLP}(\Lambda) = \int_{\partial|\xi|>0} \partial_t |\xi_t| dt. \quad (15)$$

Since $\ln(|\xi|)$ in Eq. (13) owns the same monotonicity as $|\xi|$ in Eq. (14), the divisibility of quantum maps and the information flow based methods is equivalent in this model. Due to the simplicity form of $|\xi|$, in the following, we will focus on the information flow measure (we also drop the subscript index $BLP$ of $N_{BLP}(\Lambda)$ for convenience).

The non-Markovianity $N(\Lambda)$ is dependent on the dephasing duration $\tau$, which is related to the thickness of the quartz plate. As an illustration, we consider $\tau \in [\pi/(\Delta\omega\Delta n), 2\pi/(\Delta\omega\Delta n)]$, and the non-Markovianity reads

$$N(\Lambda) = |\xi| - |\cos 2\pi e^{-\frac{t}{2}\left(\frac{\Delta\omega}{\Delta n}\right)^2}. \quad (16)$$

For a fixed time $\tau$, the non-Markovianity can be adjusted by the parameter $\xi$ and two critical points of sudden transition between Markovian and non-Markovian regions are found to be

$$\xi_1 = \frac{1}{2} \arccos(-q) \quad \text{and} \quad \xi_2 = \frac{1}{2} \arccos(q), \quad (17)$$

where $q = \sqrt{v} |\cos \delta|/\sqrt{v-u-v\sin^2 \delta}, \quad u = e^{2\pi \Delta n^2 t^2}, \quad v = e^{2\pi \Delta n^2 t^2}, \quad \delta = \Delta\omega\Delta n t/2$.

In Fig. 2, the QSL time $\tau_{QSL}(\tau_n)$ (blue solid curve) together with $\tau_2$ (red dot-dashed curve) and $\tau_1$ (green dashed curve) and non-Markovianity $N(\Lambda)$ (black dotted curve) are compared to parameter $\xi$ in the case $\alpha = \pi/4$ and $\tau = 2\pi/(\Delta\omega\Delta n)$ with $\Delta\omega = \omega_2 - \omega_1$, where the related parameters are all selected according to experimental data with $\Delta n = 0.01$, $\sigma = 1.8$ THz, $\omega_1 = 2.676$ PHz ($\approx 704.5$ nm), and $\omega_2 = 2.692$ PHz ($\approx 700.3$ nm) [26].

![FIG. 2: Non-Markovian effect on quantum speed limit (QSL) of a photon under dephasing noise. QSL time $\tau_n$ (blue solid curve), $\tau_2$ (red dot-dashed curve), $\tau_1$ (green dashed curve), and $N(\Lambda)$ (black dotted curve) as a function of parameter $\xi$ controlling the relative height of two peaks of frequency distribution. The initial state with $\alpha = \pi/4$ evolves during an actual driving time $\tau = 2\pi/(\Delta\omega\Delta n) \approx 0.39$ ps with $\Delta n = 0.01$, $\sigma = 1.8$ THz, $\omega_1 = 2.676$ PHz ($\approx 704.5$ nm), and $\omega_2 = 2.692$ PHz ($\approx 700.3$ nm) [26].](image)

The most remarkable feature appeared in Fig. 2 is that the non-Markovian effect will slow down the quantum evolution, for the monotonicity of $N(\Lambda)$ is in agreement with $\tau_{QSL}$ in the non-Markovian region $\xi \in [\xi_1, \xi_2]$. By controlling the environment parameter $\xi$ (related to the tilted angle of the FP cavity), QSL time can be well controlled. The above phenomenon that the stronger the non-Markovianity, the longer time required to reach the target state is just the opposite side illustrated in Ref. [22], where the non-Markovian effect will speed up the evolution for corresponding model. To our common wisdom, the non-Markovianity reflects the memory effect of the environment, which is usually thought as beneficial in quantum tasks [44].

V. CONCLUSION

In this paper, with a photonic non-Markovian dephasing model, we illustrate that the non-Markovian effect can slow down the quantum speed limit, which presents an opposite effect of non-Markovianity that can speed up the quantum evolution for a JC model. The phenomenon we illustrated in this work is analyzed by real experimental data and can be tested immediately by all-optical setups in Refs. [23–27]. A strict theorem whenever the non-Markovian effect can speed up or slow down the quantum evolution is still not clear, however, the answer to this question is of great importance, especially in...
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There have been introduced several other non-Markovianity measures based on Fisher information [33], fidelity [34], mutual information [35] etc. They do not coincide in general [36–40] except for a few cases [35, 41].