Strings in meson and baryon physics

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Abstract

The relativistic string with massive ends (the meson model) and four various string baryon models: $q$-$qq$, $q$-$q$-$q$, $Y$ and $\Delta$ are considered. In particular, the rotational motions for all these systems are applied to describing the leading Regge trajectories, and also the classical dynamics beyond the usual rectilinear string rotations is studied. For the string meson model the two types of quasirotational motions (disturbances of the planar uniform rotations) are obtained. They are oscillatory motions in the form of stationary waves in the rotational plane and in the orthogonal direction. The analysis of the stability problem for the rotational motions for all mentioned string configurations shows that these motions for the $Y$ and $q$-$q$-$q$ models are unstable on the classical level.

In various string models of hadrons the relativistic string simulates the strong interaction between quarks at large distances and the QCD confinement mechanism \cite{1}. One of the starting points for strings in particle physics was the natural explanation of linearly growing Regge trajectories for orbitally excited hadron states, where the Regge slope $\alpha' = 1/(2\pi\gamma)$ is simply connected with the string tension $\gamma$.

The string model of the meson is geometrically obvious, on the classic level it is the relativistic string with massive ends shown in Fig. 1 (a) \cite{1}. But for the baryon we have four string models with different topology \cite{2}: (b) the meson-like quark-diquark model $q$-$qq$ \cite{3}, (c) the linear configuration $q$-$q$-$q$ \cite{4}, (d) the “three-string” model or $Y$ configuration \cite{5}, and (e) the “triangle” model or $\Delta$ configuration \cite{6}.

![Figure 1: String models of the meson (a) and the baryon (b) – (e).](image-url)
linear $q$-$q$-$q$ configuration was less popular because it was supposed to be unstable with respect to transformation into the quark-diquark one [3]. However this problem was studied quantitatively only recently in Ref. [4], where we showed that the classic rotational motions of this model are unstable indeed, but the system doesn’t transform into the $q$-$qq$ one. The stability problem of the classic rotations for all other baryon model was studied in Ref. [7]. This was surprising, but the $Y$ configuration appeared to be unstable too.

For the three-string model or $Y$ configuration the massless variant was better investigated [2, 5] in comparison with $q$-$q$-$q$ or $\Delta$ ones (for other models the massless case is trivial non-baryon open or closed string), but even for the massless three-string the dynamics remains too complicated. The delay with study of the last baryon model “triangle” was also connected with some difficulties in its dynamics.

The problem of choosing the most adequate string baryon model among the four mentioned ones has not been solved yet, because All these models can work in the particle physics under certain assumptions [8]. All models have a some degree of the QCD motivation but without explicit preferences. In particular, different authors in the frameworks the QCD-based baryon Wilson loop operator approach give some arguments in favour of the $Y$ or the $\Delta$ configuration [9].

For all mentioned string hadron models the dynamics may be described by the action

$$S = -\gamma \int_\Omega \sqrt{-g} \, d\tau \, d\sigma - \sum_{i=1}^N m_i \int \sqrt{\dot{x}_i^2(\tau)} \, d\tau,$$

where $\gamma$ is the string tension, $m_i$ are masses of $N$ material points (modelling quarks, anti-quarks or diquarks), $N = 2$ for the meson-like models (a), (b) in Fig. 1 and $N = 3$ for others, $X^\mu(\tau, \sigma)$ are coordinates of a string point in Minkowski space with signature $+, -, -, \ldots$, $g = X^2 X'^2 - (\dot{X}, X')^2$, $\dot{X}^\mu = \partial_\tau X^\mu$, $X'^\mu = \partial_\sigma X^\mu$, the domain $\Omega$ mapping into the world surface is bounded by the inner lines $\sigma = \sigma_i(\tau)$ of the material points, $\Omega$ is different for different configurations [4, 5]. $\dot{x}_i^\mu = \frac{d}{d\tau} X^\mu(\tau, \sigma_i(\tau))$, $c = 1$.

For the triangle configuration the string is closed (but it is not smooth) so we use the closure condition [6]

$$X^\mu(\tau, \sigma_0(\tau)) = X^\mu(\tau^*, \sigma_3(\tau^*)).$$

Here $\sigma = \sigma_0(\tau)$ and $\sigma = \sigma_3(\tau)$ describe the trajectory of the same quark.

For the three-string baryon model in three parametrizations $X^\mu_i(\tau_i, \sigma)$ of the three world sheets the different “time-like” parameters $\tau_i$ [6] are connected at the junction world line $\tau_2 = \tau_2(\tau)$, $\tau_3 = \tau_3(\tau)$, $\tau_1 = \tau$. So the general form of the junction condition is

$$X^\mu_i(\tau, 0) = X^\sigma_2(\tau_2(\tau), 0) = X^\sigma_3(\tau_3(\tau), 0).$$

The equations of motion and the boundary conditions at the trajectories of massive points for all the models result from action [4]. If we choose coordinates $\tau, \sigma$ (it is possible for all the models [1, 4, 5]) satisfying the orthonormality conditions or conditions of conformally flat induced metric

$$\dot{X}^2 + X'^2 = 0, \quad (\dot{X}, X') = 0,$$

the equations of motion become linear

$$\ddot{X}^\mu - X''^\mu = 0,$$
but the boundary conditions for the massive point at an end

\[ m_i \frac{d}{d\tau} U_i^\mu(\tau) + \gamma [X'^\mu + \sigma'_i(\tau) X'^\mu] \bigg|_{\sigma=\sigma_i} = 0, \quad U_i^\mu(\tau) = \frac{\dot{X}_i^\mu + \sigma'_i X'^\mu}{|X + \sigma'_i X'|} \bigg|_{\sigma=\sigma_i} \]

or in the middle point (for the models \( q-q-q \) or \( \Delta \))

\[ m_i \frac{d}{d\tau} U_i^\mu(\tau) - \gamma [X'^\mu + \sigma'_i(\tau) X'^\mu] \bigg|_{\sigma=\sigma_i+0} + \gamma [X'^\mu + \sigma'_i(\tau) X'^\mu] \bigg|_{\sigma=\sigma_i-0} = 0, \]

remain essentially nonlinear. These massive points make the models much more realistic but they bring additional nonlinearity and (hence) a lot of problems with quantization of these models.

The well known exact solution of Eq. (3) satisfying orthonormality (4) and all the boundary conditions describes the rotational motions of the meson-like, \( q-q-q \) and \( Y \) string configurations (flat uniform rotations of the rectilinear string segments) and may be represented in the form [3, 4, 8]:

\[ X^0 \equiv t = \tau, \quad X^1 + iX^2 = \omega^{-1} \sin \omega \sigma \cdot e^{i\omega \tau}. \]  

Here \( \omega \) is the angular velocity, \( \sigma_i = \text{const.} \)

For the linear \( q-q-q \) configuration the middle quark is at rest at \( \sigma = \sigma_2 = 0 \) and for the three-string model 3 rotating rectilinear string segments joined in the rotational plane at the angles \( 120^\circ \) [2, 3].

Rotational motions for the baryon model “triangle” have the form [2, 3, 8]

\[ X^0 = \tau - \frac{T}{2} \sigma, \quad X^1 + iX^2 = u(\sigma) \cdot e^{i\omega \tau}. \]

This exact solution of Eq. (3) satisfies the orthonormality (4), closure (2), boundary (6) conditions and describes an uniformly rotating closed string (curvilinear triangle) composed of three segments of a hypocycloid. In Eq. (3) \( u(\sigma) = A_1 \cos \omega \sigma + B_1 \sin \omega \sigma, \sigma \in [\sigma_i, \sigma_{i+1}] \), the six complex constants \( A_i, B_i \) and the real constants \( \sigma_i, D = \sigma_3 - \sigma_0, V_i^2, \tau^* - \tau = T \) are connected by the set of relations [3, 8, 10].

The energy \( E \) and angular momentum \( J \) of the states (8), (9) are [3, 8]

\[ E = E_{st} + \sum_{i=1}^{N} \frac{m_i}{\sqrt{1 - v_i^2}} + \Delta E, \quad J = \frac{1}{2\omega} \left[ E_{st} + \sum_{i=1}^{N} \frac{m_i v_i^2}{\sqrt{1 - v_i^2}} \right] + S, \]

where \( v_i \) are the quark velocities, \( E_{st} = \gamma \omega^{-1} \arcsin v_i \) for the motions (8) and \( E_{st} = \gamma D(1 - T^2/D^2) \) for the triangle states (9). The quark spins with projections \( s_i \) (\( S = \sum_{i=1}^{N} s_i \)) are taken into account, in particular, as the spin-orbit correction \( \Delta E = \Delta E_{sl} = \sum_i \beta(v_i)(\bar{s}_i) \) to the energy of the classic motion. Here we use \( \beta(v_i) = 1 - (1 - v_i^2)^{1/2} \) for this correction (8, 10).

The expression (10) for all string hadron models in Fig. 1 describes quasilinear Regge trajectories with the similar ultrarelativistic behavior \( (v_i \to 1) \):

\[ J \approx \alpha' E^2 - \nu E^{1/2} \sum_{i=1}^{N} m_i^{3/2} + \sum_{i=1}^{N} s_i [1 - \beta(v_i)]. \]

Here the slopes are different: \( \alpha' = (2\pi \gamma)^{-1} \) for the meson-like models, \( \alpha' = \frac{2}{3}(2\pi \gamma)^{-1} \) for the \( Y \) and \( \alpha' = n(n^2 - k^2)^{-1}(2\pi \gamma)^{-1} \) for the “triangle” [3, 10].

The parent Regge trajectories for the \( N, \Delta \) and strange baryons may be described with using all string baryon models under following assumptions: \( \gamma = \gamma_{qq} = 0.175 \text{ GeV}^2 \) the
effective tension for the Y and “triangle” is to be different $\gamma_Y = 2\gamma$, $\gamma_\Delta = 3\gamma$. Under these assumptions and the effective quark masses $m_u = m_d = 130$ MeV, $m_s = 270$ MeV the mentioned baryonic trajectories and also the Regge trajectories for the light and strange mesons are well described \[8, 10\].

The model “triangle” has a set of topologically different solutions \[9\] numerated by the integer numbers $n = \lim_{m_i \to 0} D / (\sigma_1 - \sigma_0)$, $k = n \lim_{m_i \to 0} T / D$. The states with $n = 3$, $k = 1$ (“simple states” \[8\]) are stable \[9\] and they are used here for describing the Regge trajectories. Other values $n$, $k$ correspond to the exotic states with some string points moving at the speed of light \[9\]. These states are unlikely to be physical (except for possible describing glueballs and hybrids) but they may act as physical excitations when we consider small disturbances of the simple rotational states.

Let us clarify this point on the example of the meson-like string with massive ends. For this model exotic states are rotations of $n$ times folded rectilinear string. It was proved in Ref. \[11\] that in 3 + 1-dimensional Minkowski space only these states

$$X^\mu(\tau, \sigma) = x^\mu_0 + p^\mu \tau + \alpha_n \cos(\tilde{\omega}_n \sigma + \phi_n)(e^\mu_1 \cos \tilde{\omega}_n \tau + e^\mu_2 \sin \tilde{\omega}_n \tau), \quad \sigma \in [0, \pi]$$

are motions of this string system with linearizable boundary conditions \[8\], which take the linear form \[\tilde{X}^\mu + (-1)^i Q_i X^\mu]\big|_{\sigma = \sigma_i} = 0 under restrictions $X^2|_{\sigma = \sigma_i} = C_i^2 = \text{const.}$ Here $Q_i = C_i \gamma / m_i$ and $\tilde{\omega}_n$ are roots of the equation

$$(\tilde{\omega}^2 - Q_1 Q_2)/[(Q_1 + Q_2) \tilde{\omega}] = \cot \pi \tilde{\omega}. \quad (11)$$

It is interesting that the same equation \[11\] describes physical states — excitations of the string with massive ends when we consider small disturbances of its rotational motions \[8\]. These disturbances were studied in Refs. \[12\] but the results were not correct. However in Ref. \[11\] these disturbances in the linear approximation with using the conditions \[8\] were obtained in the form

$$X^\mu(\tau, \sigma) = X^\mu_{\text{rot}}(\tau, \sigma) + \sum_{n=-\infty}^{\infty} \left\{ e_3^\mu \alpha_n \cos(\tilde{\omega}_n \sigma + \phi_n) \exp(-i \tilde{\omega}_n \tau) + \beta_n [e_\parallel^\mu f_\parallel(\sigma) + i(e_0^\mu f_0(\sigma) + e_2^\mu f_2(\sigma))] \exp(-i \Omega_n \tau) \right\}. \quad (12)$$

Here $X^\mu_{\text{rot}}$ is the pure rotational motion \[8\], $e_\parallel^\mu(\tau) = e_3^\mu \cos \tilde{\omega}_1 \tau + e_2^\mu \sin \tilde{\omega}_1 \tau$, $e_\perp^\mu = \tilde{\omega}_1^{-1} \frac{\partial}{\partial \tau} e^\mu(\tau)$; $e_0$, $e_1$, $e_2$, $e_3$ is the orthonormal tetrad. Each term in Eq. \(12\) describes the string oscillation that looks like the stationary wave with $n$ nodes. There are two types of these stationary waves: (a) orthogonal oscillations along $z$ or $e_3$-axis at the frequencies proportional to the roots $\tilde{\omega}_n$ of Eq. \(11\), and (b) planar oscillations (in the rotational plane). In the latter case the eigenfrequencies $\Omega_n$ result from the equation

$$\frac{(\Omega^2 - \tilde{Q}_1^2)(\Omega^2 - \tilde{Q}_2^2) - 4 Q_1 Q_2 \Omega^2}{2 \Omega[Q_1(\Omega^2 - \tilde{Q}_1^2) + Q_2(\Omega^2 - \tilde{Q}_2^2)]} = \cot \pi \Omega,$$

$\tilde{Q}_i^2 = Q_i^2 (1 + v_i^{-2})$. The frequencies $\Omega_n$ and $\tilde{\omega}_n$ are real numbers so the rotations \(8\) of the string with massive ends are stable in the linear approximation.

Let us make some conclusive remarks:

- Quasirotational motions of the string with massive ends in the form of Fourier series \(12\) may be used as the basis of quantization in the linear vicinity of the stable solution \(8\). Expression \(12\) satisfies the constraint \(11\) so we have no an analog of the Virasoro conditions in this case.
• Progress in quantization of the considered nonlinear meson and baryon string models is necessary for describing not only orbital but also radial and other excitations of hadrons known from potential models [10].

• Quantization in the linear vicinity is possible only for stable solutions, but we show that the rotational motions (8) for the linear $q$-$q$-$q$ and three-string models are unstable on the classic level. However this doesn’t mean that these models are finally closed for further applications.

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