Bulk torsion fields in theories with large extra dimensions

Biswarup Mukhopadhyaya\textsuperscript{1}, Somasri Sen\textsuperscript{2}
Harish-Chandra Research Institute,
Chhatnag Road, Jhusi, Allahabad - 211 019, India
Soumitra SenGupta\textsuperscript{3}
Physics Department, Jadavpur University,
Kolkata - 700 032, India

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Abstract
We study the consequences of spacetime torsion coexisting with gravity in the bulk in scenarios with large extra dimensions. Having linked torsion with the Kalb-Ramond antisymmetric tensor field arising in string theories, we examine its artifacts on the visible 3-brane when the extra dimensions are compactified. It is found that while torsion would have led to parity violation in a 4-dimensional framework, all parity violating effects disappear on the visible brane when the torsion originates in the bulk. However, such a scenario is found to have characteristics of its own, some of which can be phenomenologically significant.

\textsuperscript{1}E-mail: biswarup@mri.ernet.in
\textsuperscript{2}E-mail: somasri@mri.ernet.in
\textsuperscript{3}E-mail: soumitra@juphys.ernet.in
1 Introduction

Although we apparently live in four-dimensional spacetime, some recent theoretical developments confront us with the tantalizing possibility of extra spacelike compactified dimensions holding the key to the fundamental laws of physics. This, it is further argued, can happen even at energy scales about as low as those probed by experiments so far. Among the variants of such theories, two principal streams can be identified. These are the scenarios proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] on one hand, and by Randall and Sundrum (RS) [2] on the other. Both of these approaches have sprung from a bid to solve the naturalness problem of the standard electroweak theory by recognizing scales arising out of compactified dimensions as the natural cut-offs on the standard model.

Some features are common to the two types of approach. The gravitational field tensor propagates in the higher dimensional manifold (called the ‘bulk’) in each case. While the standard model (SM) fields are generally assumed to be confined to a 3-dimensional ‘brane’, some models with additional fields (for example, right-handed neutrinos) in the bulk have also been proposed [3]. In both cases [1, 2] compactification of the extra spacelike dimensions creates a tower of massive fields on the 3-brane, out of erstwhile massless ones propagating in the bulk. The interaction of these towers of states with the SM fields provides the ‘new physics’ inputs from a phenomenological viewpoint. Such interactions, having manifold enhancement over the usual gravitational effects via either the summation over a densely packed tower of states (ADD) or a boost in the coupling by an exponential factor (RS), make such theories distinguishable in experiments.

In this paper, we examine the above scenarios in the context of spacetime with torsion. First introduced into the fold of general relativity in the Einstein-Cartan (EC) framework, torsion has come to be linked with matter fields having spin [4], just as curvature is connected with mass. A crucial difference brought about by torsion is the existence of an antisymmetric tensor part in the affine connection [5]. Some effects of torsion on theories with large extra dimensions can be found, for example, in [6]. Here we take a more specific approach where torsion is linked with the rank-2 antisymmetric tensor field $B_{\mu\nu}$, known as the Kalb-Ramond (KR) field, which occurs in the spectrum of heterotic string theories. The fact that most of the models under investigation are string-inspired lends legitimacy to such an approach and also makes it possible to use certain properties of the KR field in making our predictions.

It has been demonstrated earlier [7, 8] that torsion (occurring in the Lagrangian as an auxiliary field) gets equated to the KR field strength $H$ on using the equations of motion. Under such circumstances, the antisymmetric part in the affine connection is provided by the tensor $H_{\mu\nu\lambda}$, with

$$H_{\mu\nu\lambda} = \partial_{[\mu}B_{\nu\lambda]}$$  \hspace{1cm} (1)

Since both the symmetric tensor field of the graviton and the antisymmetric tensor field $B_{\mu\nu}$
control the geometry of spacetime, we feel that it is natural to place them on identical footing by having both of them exist in the bulk. In that case, the modification of the affine connection takes place in \((4 + n)\) dimensions itself.

It has been shown, in the context of 4-dimensional spacetime, that torsion destroys the cyclic property of the Riemann-Christoffel tensor \(R_{\mu\nu\alpha\beta}\), as a result of which a term of the form \(\epsilon_{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}\) can be added to the scalar curvature \(\mathcal{R}\) in the gravitational action, causing the latter to be parity-violating. A consistent formalism was developed in [10] for incorporating the ensuing parity-violating effects in the interaction of different spin fields with spacetime curvature containing torsion. This was done by realizing that the modified connection in an EC theory can have a pseudo-tensorial part as well:

\[
\bar{\Gamma}^\kappa_{\nu\lambda} = \Gamma^\kappa_{\nu\lambda} - \frac{1}{M_P} \left[ H_{\nu\lambda}^\kappa - q \left( \epsilon_{\nu\lambda}^{\gamma\delta} H_{\gamma\delta}^\kappa - \epsilon_{\beta\lambda}^{\kappa\alpha} H_{\nu\alpha}^\beta + \epsilon_{\beta\nu}^{\kappa\alpha} H_{\lambda\alpha}^\beta \right) \right] 
\]

(2)

where \(\Gamma^\kappa_{\nu\lambda}\) is the Christoffel symbol of Einstein gravity, symmetric in the two lower indices. The coefficient \(1/M_P\) arises from dimensional requirements. Such a generalization of the covariant derivative can be regarded as the most general way of incorporating parity violation in the presence of torsion, since it not only yields the added term \(\epsilon_{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}\) in the scalar curvature but also leads to parity-violating effects for all matter fields with spin. Here \(q\) is a parameter determining the degree of parity violation. Further implications of such parity violation has been explored in a number of recent works [8, 11, 12, 18].

We now ask the question: what if the KR field coexists with gravity in the bulk, leading to a tower of antisymmetric tensor fields upon compactification? More importantly, since parity-violation through the modified connection may now arise in higher dimensions, will its effects survive in 4-dimensions as well? These and a few related questions are discussed in what follows. We shall outline our main arguments in the context of an ADD scenario, although we shall also comment upon their validity for RS-type theories.

In section 2 we outline the basic features of such a theory in higher dimensions, with special reference to a 6-dimensional framework. The results of compactification of the extra dimensions and the new interactions obtained on the visible brane are discussed in section 3. We summarise and conclude in section 4.

## 2 Torsion in higher dimensional theories

We begin by recalling the most important contentions of the ADD and RS types of models. In ADD-type models [1], the compact and Lorentz degrees of freedom can be factorized. The string scale \(M_s\) (which can be as low as tens of TeV) controls the strength of gravity in \((4 + n)\) dimensions, and is related to the 4-dimensional Planck scale \(M_P\) by
\[ \frac{R^n}{M_P^2} = (4\pi)^{n/2} \Gamma(n/2) M_s^{-(n+2)} \]  

where \( R \) is the compactification radius. The current limits on the departure from Newton’s law of gravity at small distances are consistent with \( R \) within a mm, for \( n \geq 2 \). Compactification of the extra dimensions leads to a tower of Kaluza-Klein (KK) modes on the brane where we reside. Thus a massless field in the bulk in general gives rise to a massive spectrum, the density of states being given by

\[ \rho(m_{\vec{n}}) = \frac{R^n m_{\vec{n}}^{(n-2)}}{(4\pi)^{n/2} \Gamma(n/2)} \]

where \( m_{\vec{n}} = \left( \frac{4\pi^2 \vec{n}^2}{k^2} \right)^{1/2} \) is the mass of a KK state with \( \vec{n} = (n_1, n_2, \ldots, n_n) \) \[13\].

Consequently, in any process (involving the graviton, for example) where a cumulative contribution from the tower is possible, a summation over the tower of fields, convoluted by the density, causes an enhancement, in spite of the suppression of individual couplings by \( M_P \). One thus expects appreciable contributions to various processes at energies close to \( M_s \). However, though it provides a stabilization of the electroweak scale, this type of a model cannot avoid a still unexplained hierarchy between \( 1/R \) and \( M_s \).

In the RS framework \[2\], the last problem is ameliorated by introducing a non-factorizable geometry. The metric contains a ‘warp factor’ which is an exponential function of the compact dimension \( \phi \):

\[ ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 \]  

where \( r_c \) is the compactification radius on a \( Z_2 \) orbifold, and \( k \) is of the order of the higher dimensional Planck mass \( M_5 \). The standard model fields are at \( \phi = \pi \) whereas gravity propagating in the bulk peaks at \( \phi = 0 \). It turns out that the 4-dimensional Planck mass \( M_P \) in this case is related to \( M_5 \) by

\[ M_P^2 = \frac{M_5^3}{k} \left[ 1 - e^{-2kr_c\pi} \right] \]  

Furthermore, for \( kr_c \simeq 12 \), the exponential factor generates a mass scale of about a TeV from the Planck scale without requiring the postulate of an inordinately large compactification radius. The finite renormalization of the tower coming from any bulk field generates an additional factor of \( e^{kr_c} \) in the coupling of any massive member of the tower to the SM fields, although the tower itself remains rather sparse, having mass separations on the order of TeVs.

Let us now consider a scenario with torsion in bulk spacetime. Following our earlier philosophy, we wish to retain the possibility of parity violation in \( (4+n) \) dimensions. Such a goal is attained via the completely antisymmetric tensor density only if \( n \) is even. Let us further assume that torsion enters into the geometry only through a ‘minimal coupling’ scheme, being added linearly to the covariant derivative.
Our next observation is that a minimally coupled torsion can contribute a pseudo-tensorial component to the affine connection only in 6-dimensions (i.e. for $n = 2$), in the following way

$$\tilde{\Gamma}^\kappa_{\nu\lambda} = \Gamma^\kappa_{\nu\lambda} - \frac{1}{M_s} [H^\kappa_{\nu\lambda} - q\epsilon^\kappa_{\nu\lambda\rho\beta} H^\rho_{\beta\alpha}]$$ (7)

The reason is obvious from the expression for the connection itself; the parity violating (pseudo-tensorial) part must be of rank 3, and, with a rank 3 torsion field strength available to us, the completely antisymmetric tensor density that one has to use here must be of rank 6. This constrains one to a specific dimensionality, namely, $n = 2$. For $n > 2$, one has to introduce terms in higher powers of $H_{\mu\nu\lambda}$ in the modified connection in order to make it parity violating.

With the modified affine connection defined in the above manner, it is straightforward to calculate the scalar curvature in 6-dimensions:

$$R(g, H) = R(g) - \frac{1}{M_s^2} [H_{\mu\nu\lambda} H^{\mu\nu\lambda} - 2q\epsilon^\alpha_{\mu\nu\lambda} H_{\mu\nu\lambda} H^\alpha_{\beta\gamma} + q^2 \epsilon^\alpha_{\mu\nu\lambda} \epsilon^\beta\gamma_{\delta\rho} H_{\mu\nu\lambda} H^{\alpha_{\beta\gamma}}]$$ (8)

where the first term is the scalar curvature in 6-dimensions in the absence of torsion. The second term is the extra piece arising in a Einstein-Cartan picture. The third and fourth terms are the artifacts of the pseudo-tensorial extension of the affine connection.

However, the relation $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$ immediately implies that the third term in equation(8) is nothing but a surface term. Thus we are led to conclude that $R(g, H)$ is invariant under parity in spite of the pseudo-tensorial extension. This can be attributed both to the way in which the KR field finds its way into the Lagrangian and to the restricted manner in which a rank-3 tensor can be combined with the rank-6 Levi Civita tensor density to produce a modification to the connection.

Let us now specify the form of the 6-dimensional Lagrangian so as to allow the interaction of the torsion field with matter fields on the visible brane. If the $U(1)_{em}$ gauge field is to couple to torsion, a consistent method is to extend $H_{\mu\nu\lambda}$ by a Chern-Simons term:

$$H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]} + \frac{1}{M_s} A_{[\mu} F_{\nu\lambda]}$$ (9)

Here $F_{\nu\lambda}$ is the usual electromagnetic field tensor given as $F_{\nu\lambda} = \partial_{[\nu} A_{\lambda]}$ and the corresponding Lagrangian density for the electromagnetic field $A_{\lambda}$ is given in the usual way in terms of this electromagnetic field tensor. Such a Chern-Simons term, invoked to achieve gauge anomaly cancellation in heterotic String theory, produces a gauge invariant interaction term between torsion and the gauge fields $[3]$. The implication of this term in 4-dimensional torsioned gravity has been examined in recent works on a number of issues, ranging from parity violation $[7]$ to the rotation of the plane of polarization of light $[11, 12]$.

With all the standard model fields confined to the 3-brane, the Chern-Simons term will contribute only when the indices attached to $H_{\mu\nu\lambda}$ correspond to the noncompact dimensions. There is a potential source of parity violation by virtue of the Chern-Simons term. The only addition in
$R(g,H)$ which can have this effect has to be linear in the completely antisymmetric tensor density, and is of the form

$$
\epsilon^{\mu\nu\lambda\beta\gamma}(\partial_{[\mu} B_{\nu\lambda]} A_{[\alpha} F_{\beta\gamma]} + \partial_{[\alpha} B_{\beta\gamma]} A_{[\mu} F_{\nu\lambda]})
$$

(10)

This, however, vanishes as a result of the antisymmetry of $\epsilon^{\mu\nu\lambda\beta\gamma}$. Thus, unlike in the 4-dimensional case, the coupling of gauge fields to torsion again turns out to be parity-conserving despite the pseudo-tensorial part in the modified connection. However, there are additional contributions to this interaction from the last term in equation (8). We shall present these contributions in the next section where interactions in terms of the KK modes are listed.

Next, let us examine how a spin-1/2 field couples to torsion in this kind of a scenario. For this we first need to write the free fermion Lagrangian in terms of the 6-dimensional Christoffel symbols and the torsion tensor:

$$
\mathcal{L}_D = \bar{\psi} i \gamma^\mu (\partial_\mu - i \frac{\alpha}{2} g_{\lambda\nu} \sigma^{ab} v_\alpha^a \partial_\mu v_\beta^b - \frac{\alpha}{2} \delta^{ab} \sigma \delta^{\lambda\nu} v_\alpha^a \partial_\mu v_\beta^b) \psi
$$

where the $v_\alpha^a$ are tetrads (here the Latin indices correspond to directions in the tangent space). It should be noted that the confinement of fermions to the brane requires the indices answering to the Dirac matrices to always correspond to the Lorentz (i.e. non-compact) dimensions.

From above, one obtains

$$
\mathcal{L}_D = \mathcal{L}_E + \frac{1}{M_s} \bar{\psi} [i \gamma^\mu \sigma^{ab}] \psi H_{cab} - \frac{1}{M_s} \bar{\psi} [i q \gamma^\mu \sigma^{ab}] \psi \epsilon_{cab}^{\mu\nu\lambda} H_{\mu\nu\lambda}
$$

(12)

As opposed to the cases with the scalar curvature and gauge field interaction, the fermion coupling to bulk torsion can thus violate parity as defined in 6-dimensions. This can be linked with the fact that the relevant terms involve a single power of the torsion field in this case.

To see whether the above Lagrangian entails parity violation in 4-dimensions as well, one has to look at the KK towers of states on the 3-brane. We investigate them in the next section.

3 Interactions on the 3-brane

In principle, compactification in an ADD scenario can give rise to a set of tensor fields $\tilde{B}^{\mu\nu}_{\mu}$, vector fields $B^n_{\mu}$ and scalar fields $\chi^n$ in 4-dimensions. However, the bulk $B_{\mu\nu}$ can be assumed to be block-diagonal in the compact and noncompact dimensions without any loss of generality. Besides, apart from sparing us the embarrassing predicament of having massive vector fields in the low-energy spectrum, this assumption is also consistent with the $SU(3)$ holonomy of the Calabi-Yau manifold on which the process of compactification is performed. Therefore, we shall consider only the tensor and scalar fields on the visible brane. As regards $\epsilon^{\mu\nu\lambda\beta\gamma}$, two of its six indices have to correspond to the compactified dimensions, reducing it to its counterpart in 4-dimensions.
The first term in equation (8) reproduces Einstein gravity on the brane together with the modifications caused by the tower of gravitons [13]. The remaining part of the modified scalar curvature in 6-dimensions yields the following kinetic and mass terms, corresponding to the tensor and scalar fields resulting from $B_{\mu \nu}$, in the Lagrangian $L_{\text{tor}}$:

$$L_{\text{tor}} = \sum_{\vec{n}, \vec{n}'} \left\{ \partial_{[\mu} \bar{\tilde{B}}_{\vec{n} \nu \lambda]} \partial_{\nu} \tilde{B}_{\vec{n} \mu \lambda]} - 6q^2 (\det g) \partial_{[\mu} \bar{\tilde{B}}_{\vec{n} \nu \lambda]} \partial_{\nu} \tilde{B}_{\vec{n} \mu \lambda]} - 3 \left( \frac{4\pi^2 \vec{n}^2}{R^2} \right) \bar{\tilde{B}}_{\mu \nu} \tilde{B}_{\mu \nu} \right\}$$

where $\eta (\eta')$ runs over $\{ \mu \nu \lambda (\alpha \beta \gamma) \}$.

Thus we have a tower of tensor fields, whose kinetic and mass terms can be obtained in standard forms after proper rescaling and basis redefinition (assuming a small $q^2$). However, an important point to note here is that the antisymmetry of $H_{\mu \nu \lambda}$ forbids any scalar mass term. One, therefore, is left with just one massless scalar $\chi$ in $(3 + 1)$ dimensions. For $q^2 = 1$, the kinetic energy term vanishes, leaving $\chi$ with no dynamical content.

As for the KK modes coupling to the $U(1)$ gauge field, both the second and fourth terms in $R(g, H)$ become instrumental. After integrating out the compact dimensions, we thus obtain the following interactions:

$$L_{\text{tor-em}} = \frac{2}{M_P} \sum_{\vec{n}} \frac{i}{M_P} \bar{\psi} \gamma^c \sigma^{ab} \psi \partial_{[\nu} \bar{B}_{\vec{n} \mu a b]} - \frac{144q m}{M_P} \bar{\psi} \gamma_5 \psi \chi$$

where $\eta (\eta')$ runs over $\{ \mu \nu \lambda (\alpha \beta \gamma) \}$. Therefore, only the tensor tower couples with the gauge field via the Chern-Simons term.

The only place where the massless scalar $\chi$ exhibits some coupling in 4-dimensions is in the interactions with a fermion. On reduction of the higher-dimensional Lagrangian given in equation (12), we have

$$L_D = L_E + \sum_{\vec{n}} i \frac{1}{M_P} \bar{\psi} \gamma^c \sigma^{ab} \psi \partial_{[\nu} \bar{B}_{\vec{n} \mu a b]} - \frac{144q m}{M_P} \bar{\psi} \gamma_5 \psi \chi$$

$m$ being the mass of the fermion. Here the first term corresponds $\psi$ coupling to the KK tensor tower, and the second, to the fermionic coupling of the massless scalar. The second term arises purely due to the pseudo-tensorial extension in the 6-dimensional Lagrangian. The fact that the fermionic current is confined to the 3-brane constrains all indices of $H_{\mu \nu \lambda}$ to be Lorenzian if the latter is directly contracted with the fermionic current. However, from the viewpoint of parity transformation in 4-dimensions, the above Lagrangian is again invariant, as one can always use the phase freedom of the fields $\tilde{B}_{\mu \nu}$ and $\chi$ independently on the 3-brane.

Thus we come to an important conclusion: even though the covariant derivative in 6-dimensions can always be augmented with a pseudo-tensorial part in presence of torsion, thereby causing parity violation in the bulk, the ensuing theory in 4-dimensions turns out to be parity-conserving in every sector.

Several comments are in order here:
• Although we have performed our analysis in the context of (4 + 2) dimensions, the above conclusion has wider applicability. The chain of arguments followed in this section and the previous one tells us that parity violation in 4-dimensions can arise from torsion field only when the KR field propagates in the 3-brane where all the matter and gauge fields are confined. An exception to this is possible only when the affine connection is extended by a pseudo-tensorial part consisting of higher powers of the KR field strength tensor.

• In spite of the fact that parity violation disappears in 4-dimensions, the pseudo-tensorial extension in equation (7) has non-trivial consequences in this scenario. The fermionic coupling of the massless scalar field arises exclusively from this extension. Moreover, the pseudo-tensor added in the bulk also modifies the gauge interaction of the tower resulting from $B_{\mu\nu}$ on the visible brane. The possibility of the massless scalar $\chi$ losing its dynamical content in the special case of $q^2 = 1$ is also a consequence of parity violation in 6-dimensions.

• A string-inspired scenario also suggests the modification of the third rank antisymmetric field strength $H$ by a gravitational Chern-Simons term in addition to the gauge Chern-Simons term [14]:

$$H = dB - c_1 \omega_Y - c_2 \omega_L$$

(16)

where $\omega_Y$ and $\omega_L$ are respectively the gauge and the gravitational Chern-Simons terms required for the gauge and gravitational anomaly cancellation and $c_1, c_2$ are constants. The gravitational Chern-Simons term however has not been considered in the foregoing discussion, since it is a higher derivative effect and in fact contains three derivatives where the others contain one. This term is therefore much more suppressed than the other terms and has been ignored in our analysis.

• Although the above conclusions are derived in reference to an ADD-type model, they are mostly true in the 6-dimensional analogue of an RS framework [15] as well. This is because no recourse has been taken to any particular metric in the reasoning leading to the disappearance of parity violation in the modified scalar curvature and in gauge interaction of torsion fields. For fermion-torsion interactions, on the other hand, we have had to consider the Lagrangian after compactification. There our arguments centrally depend on (a) properties of the completely antisymmetric tensor density, and (b) the fact that the indices answering to the fermion current must be Lorenzian. All of the above points hold in an RS scenario, too, with appropriate effects coming from the warp factor multiplied with the Minkowski part of the metric. A detailed investigation on an RS scenario in this context will be reported later [16].

• The presence of an additional weakly coupled massless (pseudo)scalar may in general affect Big Bang Nucleosynthesis (BBN). An analysis of this problem has been performed in the second reference of [1] in the context of a torsion-free ADD model where, again, a massless scalar
arises from a bulk graviton as a result of compactification. There it has been demonstrated
dissipation of the accumulated energy from the light scalar(s) through Hubble expansion
as well as freezing out of extra dimensions well before the onset of the BBN era can be
consistently accommodated. This, it has been further shown, is possible with string scales
of the order of $1 \sim 10$ TeV. Very similar considerations apply to the light scalar in our case,
ensuring that it is safe from a BBN point of view. This is because the coupling of both
types of scalars (i.e. those arising out of graviton and torsion) to matter is the same, being
suppressed by the 4-dimensional Planck mass. At the same time, one can avoid overclosing
the universe if the vacuum expectation value of each light scalar is small compared to the
inverse of its decay constant [17].

We end this section by mentioning a few phenomenological consequences of a bulk torsion field.
The first of these is the possibility of helicity flip of a fermion via scattering with the torsion field(s).
This possibility has already been studied in 4-dimensions where, however, the coupling to torsion
is suppressed by a factor of $\frac{1}{M_P}$ at each vertex [18]. With a KK tower of tensor fields $\tilde{B}_{\mu \nu}$,
the effective cross-section gets boosted upon integration over the entire tower of tensors. Thus, using
equation (15) the forward scattering cross-section for helicity flip of a fermion $f$ in the process
$f(p_1) \tilde{B} \rightarrow f(p_2) \tilde{B}^\dagger$ with $n = 2$ is given by

$$\frac{d\sigma_{\text{tot}}}{d\Omega_{\beta=0}} = \frac{1}{32 \pi S} \int \int \left| \mathcal{M} \right|^2 \rho(m_\chi^2) \rho(m_{\chi'}^2) dm_\chi^2 dm_{\chi'}^2$$

(17)

where

$$\mathcal{M} = \frac{1}{M_P^2} \left[ \tilde{u}^+ (p_2) \gamma_\alpha \gamma_5 \left( \frac{\gamma_1 + \gamma_1 + m_f}{\gamma_1 \gamma_5} \right) \gamma_\beta \gamma_5 u^- (p_1) \left( \frac{-k_{1\delta} k_{2\lambda}}{(p_1 - k_1)^2 - m_f^2} \right) \epsilon^{\mu \nu} \epsilon^\alpha \epsilon^\beta \epsilon_{\mu \nu} (k_2) \epsilon_{\mu \nu} (k_1) \right]$$

(18)

is the amplitude for helicity flip. Here $k_1^2 = m_\chi^2$ and $k_2^2 = m_{\chi'}^2$, giving the masses of the initial
and final tensor states. The density of states $\rho$ can be expressed in terms of $M_s$ and $M_P$ using
equations (3) and (4), for $n = 2$. The convolution with this density of KK states effectively replaces
the suppression factor $M_P$ by the string scale $M_s$ in the torsion-fermion coupling. Thus the cross-
section picks up an extra enhancement factor of $(\frac{M_P}{M_s})^4$ (leaving out the $m_{\chi, \chi'}^2$-dependence of the
amplitude), so long as the integration over the tower is carried out up to a mass scale on the order
of $M_s$.

In a similar way, a boost can be expected in the forward scattering amplitude for flip from
negative to positive helicity for a neutrino when it is propagating against a background of a tower of
KK torsion states. This essentially means that an off-diagonal element can arise in the Hamiltonian
of a two-level system consisting of an active and a sterile neutrino, thus resulting in active-sterile
oscillation if the neutrino has a Dirac mass. The signature of such a phenomenon can be in the
form of a depletion in high-energy neutrino flux of cosmological origin. Further details of the effect of a torsion tower on both neutrino oscillation and the active-to-sterile scattering cross-section will be presented in a subsequent paper.

4 Summary and Conclusions

We have performed a systematic analysis of the consequences of spacetime torsion in a scenario with large extra dimensions. Torsion has been regarded here as arising from a massless Kalb-Ramond field existing in the bulk. Working in an ADD scenario, we have included a pseudo-tensorial extension to the affine connection in the bulk, which is linear in the KR field strength. In a 4-dimensional framework, such an extension would have led to parity violation both in the modified scalar curvature and in the coupling of torsion to matter fields with spin. However, when one starts from higher dimensions and looks at the resulting 4-dimensional action involving the KK towers of states, one gets back a parity-conserving theory.

The pseudo-tensorial part in higher dimensions is nonetheless found to be of non-trivial consequence, since it gives rise to additional interaction terms involving the KK tower of tensor states as well as the massless scalar obtained from the bulk KR field. We have also indicated that cumulative contribution from the tower can enhance helicity flip of a fermion when it is propagating in space-time with torsion. And finally, the conclusions drawn here are found to be by and large applicable to an RS scenario as well, thus causing bulk torsion to stand out as a distinctive phenomenon as far as low-energy gravitational effects are concerned.

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