Scale and chiral symmetry in an effective lagrangian for nuclear physics

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Abstract. We study the restoration of chiral symmetry and scale invariance at high baryon density and/or energy-density through an effective chiral lagrangian for nuclear physics. In this lagrangian the breaking of scale invariance is regulated by the expectation value of a scalar field, called dilaton. A peculiar feature of this model is that the masses of the $\omega$ and $\rho$ mesons initially slightly decrease with the density but then they increase again for densities larger than $\sim 3\rho_0$.

1. Introduction

In the next years several Heavy Ion Collisions (HICs) experiments at energies of the order of a few ten A GeV (as e.g. the ones proposed at facility FAIR at GSI [1], at RHIC (Brookhaven) and at the Nuclatron in Dubna) will probably start their activity. In these experiments the Equation of State (EOS) of matter will be tested at large density and/or temperature. It is therefore very important to provide, through theoretical investigations, a map of the “new” regions which will likely be explored by the new experiments.

Lattice QCD calculations have been crucial in providing a rather precise hint about what can happen at large temperature [2, 3]. One important result of lattice calculations is that, at least at zero density, chiral symmetry restoration and deconfinement take place at the same temperature. An important question is if the two symmetries are restored together also when working at finite density. It is therefore interesting to explore models in which chiral symmetry is embedded in the hadronic lagrangian, so that one can study chiral symmetry restoration independently of quark deconfinement.

In this work, based on our recent paper [4], we use an effective model based on a chiral invariant lagrangian for nuclear physics. In that model chiral fields are present together with a dilaton field which reproduces the breaking of scale invariance in QCD, but at a mean field level. This feature is by itself extremely interesting because it allows us to explore, within this effective model, the interplay between chiral symmetry and scale invariance.

2. The Chiral Dilaton Model

Spontaneously chiral symmetry breaking has long been studied in several microscopic models because chiral symmetry is a fundamental feature of low-energy effective lagrangians. At finite temperature the restoration of chiral symmetry in the linear and non-linear sigma model has been discussed but the attempt to describe nuclear dynamics fails due to the impossibility to reproduce basic properties of nuclei [5]. More sophisticated approaches have been proposed...
in the literature, both within a SU(2) chiral symmetric models [6, 7] and also extending the symmetry to the strange sector [8, 9, 10, 11, 12]. Here we use the model introduced by the Minnesota group [13, 14, 15]. In that model chiral fields are present together with a dilaton field which reproduces at a mean field level the breaking of scale invariance which takes place in QCD. In [13, 14, 15] it has been developed a formalism (which we adopt) allowing resummations of the thermal contributions. This is important when studying a strongly non-perturbative problem as the restoration of chiral symmetry. The lagrangian of the model reads:

$$\mathcal{L} = \frac{i}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{i}{2} \partial_\mu \pi \cdot \partial^\mu \pi + \frac{i}{2} \partial_\mu \phi \partial^\mu \phi - 4 \omega_{\mu \nu} \omega^{\mu \nu}$$
$$- \frac{i}{2} B_{\mu \nu} \cdot B_{\mu \nu} + \frac{1}{2} G_{\omega \phi} \phi^2 \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} G_{b \phi} \phi^2 b_{\mu} \cdot b^{\mu}$$
$$+ \left( (G_A)^2 \omega_{\mu \nu} \omega^{\mu \nu} \right)^2 - \mathcal{V}$$

where the potential reads as the following:

$$\mathcal{V} = B \phi^4 \left( \ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) - \frac{i}{2} B \delta \phi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2}$$
$$\quad + \frac{1}{2} B \delta \sigma^2 \phi^2 \left[ \sigma^2 + \pi^2 - \frac{\phi^2}{2 \delta^2} \right] - \frac{3}{4} \tau'$$

(2)

Here $\sigma$ and $\pi$ are the chiral fields, $\phi$ the dilaton field, $\omega_\mu$ the vector meson field and $b_\mu$ the vector-isovector meson field, introduced in order to study asymmetric nuclear matter. The field strength tensors are defined by $F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, B_{\mu \nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$. In the vacuum $\phi = \phi_0, \sigma = \sigma_0$ and $\pi = 0$. The $\omega$ and $\rho$ vacuum masses are generated by their couplings with the dilaton field so that $m_\omega = G_{\omega \phi}^2 \phi_0$ and $m_\rho = G_{b \phi}^2 \phi_0$. Moreover $\zeta = \phi_0 / \sigma_0, B$ and $\delta$ are constants and $\tau'$ is a term that breaks explicitly the chiral invariance of the lagrangian.

The form of the potential $\mathcal{V}$ in eq. (2) comes from the necessity to reproduce the same divergence of the scale current as in QCD [4]. That divergence does not vanish due to the presence of the dimensional parameters $\phi_0$ and $\sigma_0$ [4]. Moreover the potential $\mathcal{V}$ plays the role of the well known “Mexican hat” potential in generating the spontaneous breaking of the chiral symmetry. Notice also that when the mean value of the dilaton field drops to zero also the radius of the chiral circle drops to zero (see figure 1). In other words, when the scale invariance is restored also the chiral symmetry is automatically restored. This result is crucial since the restoration of scale invariance means the vanishing of all the condensates, including the chiral one. Finally, the values of the parameters used in calculations (listed in table 1 of Ref. [4]) were determined in Ref. [14] by fitting the properties of nuclear matter and finite nuclei.

3. Phase diagrams

The advantage of using a sophisticated model which includes both the chiral dynamics and the gluon condensate dynamics, is that one can make previsions on the restorations of both the chiral and the scale symmetry. In the following we study the restoration of both these symmetries.

3.1. Chiral symmetry restoration

Concerning the restoration of chiral symmetry, the Chiral Dilaton Model (CDM) gives different previsions depending on the presence or the absence of a symmetry breaking term into the lagrangian.
Figure 1. Dilaton potential (in GeV$^4$) as a function of the mean values of $\sigma$ and $\pi$ (in GeV) plotted for different values of the dilaton mean field. Here we consider $\epsilon_1' = 0$.

Figure 2. Mean value of the sigma field as a function of the baryon density for different values of the temperature. Here we are in the case where $\epsilon_1' = 0$. 
Figure 3. Chiral phases diagram for symmetric nuclear matter on the temperature versus baryon density plane. Here the explicit symmetry breaking term $\epsilon_1' = 0$.

In particular the restoration of chiral symmetry at finite density and temperature occurs only when the chiral invariance of the laagrangian is exact, or equivalently, only when the explicit symmetry breaking term $\epsilon_1' = 0$. In this case, as shown in figure 2, at a given temperature we observe that the mean field value of $\sigma$ drops to zero after a critical density is reached. The transition turns out to be first order, due to the discontinuous behavior of the sigma field. The chiral phase diagram for isospin symmetric nuclear matter is shown in figure 3 in the temperature versus baryon density plane. Notice that the critical temperature at zero baryon density is $T_C = 165$ MeV, which is rather close to the QCD critical temperature estimated by lattice QCD [2, 3]. Obviously, in the case of QCD the restoration of chiral symmetry comes together with the appearance of the quark-gluon plasma phase.

As it can be observed, the critical temperature decreases with increasing density, but it never reaches zero, not even at very large densities. It can be interesting to notice that some (very preliminary) lattice calculations at finite density seem to support this result [16].

The chiral phase restoration in the chiral limit is also represented in figure 5, on the energy-density vs. baryon-density plane, by two bold lines, very close one to the other. These lines are separated by a small energy gap, which takes place since the chiral transition is first order. Let us concentrate now in the more realistic case where $\epsilon_1' \neq 0$ and the pion has a finite mass. As shown in figure 4, in this case the $\sigma$ mean field is continuous everywhere and it never drops to zero but decreases continuously with the density. This behavior, called “cross over”, is predicted by the QCD lattice calculations with two flavors. Although a phase transition does not occur, there is a region where the $\sigma$ mean field drops rapidly with the temperature, reaching very small values. This region, ranging from $T=150$ MeV to $T\sim 190$ MeV, is indicated by a dashed
Figure 4. Same as in figure 2, but for the case where the chiral symmetry is explicitly broken ($\epsilon'_1 \neq 0$).

curve in figure 5, where a sort of “phases map” for symmetric nuclear matter is shown on the energy-density versus baryon-density ($\rho_B, \epsilon$) plane. In the same figure we have indicated with asterisks a rough estimates of the maximum energies and densities reached at the moment of maximum compression during a HIC experiment (the numbers above the marks indicate the beam energy in units of A GeV). These estimates, for a given beam energy, are extracted by two simple formulae provided in Ref.[17]:

$$\epsilon = 2\gamma_{cm}^2 m_N \rho_0; \quad \rho = 2\gamma_{cm} \rho_0,$$

where $m_N$ is the nucleon mass, $\rho_0 = 0.15 fm^{-3}$ is the saturation density and $\gamma_{cm}^2 = 1 + E_0/2m_N$, with $E_0$ the beam energy. These formulae are based on the idea that the maximum baryon density can be estimated by considering the overlapping of two Lorentz-contracted nuclei. Notice that when the energy is increased from $E_0 = 2$ A GeV to $E_0 = 10$ A GeV the system crosses the dashed line and therefore the effect of the (at least partial) restoration of the chiral symmetry should be observable. In Ref. [18] we discussed the softening of the EOS due to chiral symmetry restoration and an attempt was made to connect our results with a analysis of the experimental data [19, 20].

3.2. Scale invariance restoration

Let us first recall that the order of a phase transition cannot be computed at a mean field level. Therefore our result, indicating a first order transition for scale invariance restoration, probably just indicates a rapid drop of the dilaton condensate at large temperatures [4]. As shown in figure 5, a large energy gap ($\sim$ 1 GeV) appears between the broken symmetry phase and the restored symmetry phase (the energy gap is indicated in figure 5 by a dark-shaded region). As previously discussed, the scale invariance restoration induces the restoration of the chiral symmetry. At zero baryon density, we obtain a critical energy density $\epsilon_c(0) = 523$ MeV/fm$^3$, which corresponds to $T \sim$ 200 MeV. Notice also that the critical energy density $\epsilon_c(\rho_B)$ increases.
Figure 5. Phase diagram for symmetric nuclear matter on the energy-density versus baryon-density plane. Bold lines separate the (low-lying) region where chiral symmetry is broken from the (upper-lying) region where the chiral symmetry is restored in the chiral limit case $\epsilon'_1 = 0$. All other graphical signs refer to the broken symmetry case $\epsilon'_1 \neq 0$. The lower shaded area indicates the forbidden region under the $T=0$ EOS. The upper shaded area separates the (low-lying) region where scale symmetry is broken from the region where scale symmetry is restored. The dashed line indicates the maximum (negative) variation of the sigma mean value with the temperature (see text). The asterisks indicate the maximum energies and densities reached in the initial phase of a HICs experiment. The numbers above the marks indicate the beam energy in units of A GeV.

with the baryon density (it is possible to check that also the critical temperature for scale invariance restoration increases with the baryon density). This interesting effect arises from the repulsive contribution of the $\omega$ vector meson, which contributes to the energy density as $E_\omega = (1/2)(g_\omega/m^*_\omega)^2 \rho^2_B$ (here for simplicity we have assumed $G_4 = 0$). Since the $\omega$ meson mass scales with the dilaton field, $E_\omega$ increases when the dilaton field decreases. Therefore at large densities it is less convenient to restore scale symmetry.

4. Vector meson masses
The experimental determination of the in-medium modification of the light vector mesons ($\omega$, $\rho$ and $\Phi$) is still uncertain. Recent experiments, based on the dilepton spectroscopy, have advanced to a level that in-medium spectral information on vector mesons can be deduced. Within the CDM the vector meson masses are generated by coupling the vector mesons to the dilaton field (see the lagrangian density), so that the in the medium:

$$m^*_\omega = m_\omega \left( \frac{\phi}{\phi_0} \right)$$

(4)
In figure 6 the peculiar behavior of $m^*_\rho$ and $m^*_\omega$ with the density is shown at $T=0$ and for different proton ratios: at low densities the masses initially drop, reaching a minimum in the neighborhood of $\rho = 3.5 \rho_0$, while for higher values of $\rho$, the masses increase again. This effect, due to the difficulty in restoring the scale invariance at large densities (see previous section), seems to be in agreement with several experimental analysis indicating a moderate reduction of the masses of the vector mesons at densities of the order of $\rho_0$ and no significant reduction when the masses are tested at very large densities [21, 22, 23, 24, 25, 26]. These experimental results presumably discards the scenario predicted by Brown and Rho [23, 27], where the vector meson masses drop continuously with the density [28].

Notice also that the effect of a finite isospin density is to increase the vector meson masses as it can be observed in figure 6, although at $\rho = 9 \rho_0$ the increment is only of about 1%.

5. Conclusions
The main aim of our work was to investigate the behavior of matter at large densities and temperatures by using a effective lagrangian with broken scale invariance and chiral symmetry. In this contribution, extracted from a recent work [4], we presented two important results:

– we have provided a phase diagram mapping the regions in which chiral symmetry and/or scale invariance are restored. We have shown that scale invariance restoration is more difficult at large densities, because the repulsive effect of the exchange of vector mesons is enhanced if their mass is reduced;
– the masses of the vector mesons first reduce at finite density, but at larger densities they even increase.
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