Measurement of the $\eta$-Meson Mass using $\psi(2S) \to \eta J/\psi$

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Abstract

We measure the mass of the $\eta$ meson using $\psi(2S)\rightarrow\eta J/\psi$ events acquired with the CLEO-c detector operating at the CESR $e^+e^-$ collider. Using the four decay modes $\eta\rightarrow\gamma\gamma$, $3\pi^0$, $\pi^+\pi^-\pi^0$, and $\pi^+\pi^-\gamma$, we find $M_\eta=547.785\pm0.017\pm0.057$ MeV, in which the first uncertainty is statistical and the second systematic. This result has an uncertainty comparable to the two most precise previous measurements and is consistent with that of NA48, but is inconsistent at the level of 6.5$\sigma$ with the much smaller mass obtained by GEM.
The $\eta$ meson, the second-lightest pseudoscalar, is commonly understood as being predominantly in the SU(3)-flavor octet with a small singlet admixture, so that it has comparable $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ content and virtually no gluonium component $[1,2]$. Its mass is of fundamental importance to understanding the octet-singlet mixing as well as the gluonium content of both $\eta$ and $\eta'$ $[3]$, although theoretical and phenomenological precision on related predictions $[1,3]$ has not yet matched that of experiment. On the experimental side, there has long been a situation of conflicting $M_\eta$ measurements that improvements in precision have been unable to resolve. Indeed, the current status is the worst it has ever been, with a confidence level (C.L.) of 0.1% that the measurements are consistent $[6]$. In 2002 the world average was $M_\eta=547.30\pm0.12$ MeV $[7]$ and results included in it were generally consistent with one another, but only because 1960's-era bubble chamber experiments, all of which favored a larger $M_\eta$, had been dropped $[8]$. Then NA48, based on exclusively reconstructed $\eta\rightarrow3\pi^0$ decays, reported $M_\eta=547.843\pm0.030\pm0.041$ MeV $[9]$, which appeared to vindicate the dropped experiments. In 2005 GEM reported $M_\eta=547.311\pm0.028\pm0.032$ MeV $[10]$ using the $^3\text{He}$ recoil mass in $p+d\rightarrow^3\text{He}+X$. The GEM result was consistent with less precise results made during the period 1974-1995, but also was eight standard deviations below that of NA48. More measurements with sub-100 keV precision are needed to clarify the matter.

This work presents a new measurement of $M_\eta$ using $\psi(2S)\rightarrow\eta J/\psi$. Events were acquired with the CLEO-c detector at the CESR (Cornell Electron Storage Ring) symmetric $e^+e^-$ collider. The data sample corresponds to $\sim27$ million produced $\psi(2S)$ mesons, of which about $0.8\times10^6$ decay to $\eta J/\psi$. We measure the mass by exploiting kinematic constraints in the decay chain $\psi(2S)\rightarrow\eta J/\psi$, $J/\psi\rightarrow\ell^+\ell^-$ ($\ell^\pm=e^\pm$ or $\mu^\pm$), and $\eta\rightarrow\gamma\gamma$, $3\pi^0$, $\pi^+\pi^-\pi^0$, or $\pi^+\pi^-\gamma$. Because both $\psi(2S)$ and $J/\psi$ are very narrow resonances with precisely known masses, the constraints enable a significant improvement in $\eta$ mass resolution over that achieved by the detector alone. This is the first $M_\eta$ measurement to use $\psi(2S)\rightarrow\eta J/\psi$.

The CLEO-c detector is described in detail elsewhere $[11]$; it offers 93% solid angle coverage of precision charged particle tracking and an electromagnetic calorimeter comprised of CsI(Tl) crystals. The tracking system enables momentum measurements for particles with momentum transverse to the beam exceeding 50 MeV/c, and achieves resolution $\sigma_p/p \simeq 0.6\%$ at $p=1$ GeV/c. The barrel calorimeter reliably measures photon showers down to $E_\gamma=30$ MeV and has a resolution of $\sigma_E/E \simeq 5\%$ at 100 MeV and 2.2% at 1 GeV.

Event selection begins with that described in Ref. $[12]$ for the four $\eta$ decay modes used here. We also accumulate samples of $\psi(2S)\rightarrow\pi^+\pi^- J/\psi$, $\pi^0\pi^0 J/\psi$, and $\pi^0 J/\psi$ for studies of systematic uncertainties. Every particle in the decay chain is sought, and events are separated into those with $J/\psi\rightarrow\mu^+\mu^-$ and $J/\psi\rightarrow e^+e^-$. Leptons are loosely identified and restricted to $|\cos \theta_{\ell\bar{\ell}}| < 0.83$, where $\theta$ is the angle of the track with respect to the incoming positron beam. Lepton momenta are augmented with calorimeter showers found within a 100 mrad cone of the initial track direction, under the assumption that they are produced by bremsstrahlung. All photon candidates are required to be located in the central portion of the barrel calorimeter where the amount of material traversed is smallest and therefore energy resolution is best: $|\cos \theta_\gamma| < 0.75$. Backgrounds of 1-4% are present in each $\eta J/\psi$ subsample $[12]$, consisting of cross-feed from other $\eta$ decays as well as other $\psi(2S)\rightarrow X J/\psi$ decays: $\pi^+\pi^- J/\psi$, $\pi^0\pi^0 J/\psi$, and $\gamma\chi_cJ$, $\chi_cJ\rightarrow\gamma J/\psi$.

Kinematic constraints are applied in two two-step fits: first, the lepton tracks are constrained to a common origination point (vertex) and thence to the $J/\psi$ mass, $M_{J/\psi}^{\text{PDG}}$ $[6]$; second, the constrained $J/\psi$, the beam spot and the $\eta$ decay products are constrained to a common vertex and then to the $\psi(2S)$ mass, $M_{\psi(2S)}^{\text{PDG}}$ $[6]$. Separate fit quality restrictions are
applied to vertex (χ^2) and mass (χ^2_m) constraints. For η→π^+π^−κ^0, the κ^0→γγ candidate is constrained to the κ^0 mass prior to the fits described above. The decay η→3κ^0 is treated as η→6γ because reliably making a unique set of correct photon-κ^0 assignments is not possible; typically several such assignments per event of comparable probability exist and are indistinguishable. To ensure that only the best measured events survive into the final sample, the χ^2 restrictions from Ref. [12] are tightened to χ^2/ν<10 and χ^2_m/ν<5 for both the J/ψ and ψ(2S) constrained fits.

Alternative event topologies are used to compare measurements of the κ^0 mass to its established value, M_{κ^0}^{PDG}=134.9766±0.0006 MeV [3], for two different κ^0 momentum ranges. The first (κ^0) is κ→π^0J/ψ, π^0→γγ, which features a monochromatic κ^0 with p≃500 MeV/c, and the second (κ^0) κ→ηJ/ψ, η→γγ, which contains κ^0's with p≃0-250 MeV/c. For these tests, the individual photons (instead of a constrained κ^0) are used in the M_{ψ(2S)} constraint on all final state four-momenta.

Each event yields an invariant mass M of the kinematically-constrained decay products; a single mass value is extracted for each decay mode i by fitting a Gaussian shape to the distribution of δ_i≡M_i−M_0, where M_0 is simply a reference value, either the current Particle Data Group world-average M_{κ^0}^{PDG}=547.51±0.18 MeV [6], or, in the case of the κ^0 cross-check modes, M_{κ^0}^{PDG} as given above. The fits are restricted to the central portion of each δ distribution because the tails outside this region are not represented well by a single Gaussian form. The fits span ±1.6σ to ±2.0σ about the peak (δ), where σ is the fitted Gaussian width, and in all cases the resulting fit has a C.L. exceeding 1%. The distributions of δ_i for κ^0 and η decay with fits are shown in Figs. [1] and [2] respectively. Other shapes that might fit the tails, such as a double Gaussian, have been found to yield unstable fits and/or do not improve precision of finding the peak.

There is an unavoidable low-side tail in any monochromatic photon energy distribution from the CLEO calorimeter. It originates from losses sustained in interactions prior to impinging upon the calorimeter and from leakage outside those crystals used in the shower reconstruction. This asymmetric photon energy resolution function also results in a small but significant systematic bias in ⟨δ⟩: for simplicity of the kinematic fitting formalism, input uncertainties are assumed to be symmetric, and a bias occurs if they are not. This bias in fitted Gaussian mean is mode-dependent because each presents a different mix of charged and neutral particles.

The biases β_i are estimated by following the above-described procedure on MC signal samples. Each β_i is the difference between the Gaussian peak value of the M_η distribution and the input M_{η}^{MC}. We define the bias as β_i≡⟨δ_i⟩_{MC}, in which we use the MC input M_{η}^{MC} instead of M_{η}^{PDG} for M_0. A non-zero value of β_i means that, for mode i, the Gaussian peak mass ⟨δ_i⟩ is offset from the true mass and must be corrected. We evaluate the four β_i for trial values of M_{η}^{MC} (547.0, 547.3, 547.8, and 548.2 MeV) that cover the spread of previous measurements. The biases extracted for these M_{η}^{MC} inputs are consistent, and the final bias for each mode, shown in Table [3] is taken as their average. Bias values for the κ^0_a and κ^0_b cross-checks are determined similarly.

Table [3] summarizes results by decay mode. Both κ^0_a and κ^0_b mass values are consistent with expectations within their respective statistical uncertainties. The total number of reconstructed events involved in the determination of M_η is 16325. The four values of ⟨δ_i⟩−β_i have an average, weighted by statistical errors only, of ⟨(δ−β)⟩≡277±17 keV with a χ^2=4.8 for three degrees of freedom (C.L.~20%).

Systematic errors are summarized in Table [3]. Uncertainties that are uncorrelated mode-
to-mode, including statistical, are used to determine the weights ($w_i=0.19, 0.03, 0.60, 0.18$ for $\gamma\gamma, 3\pi^0, \pi^+\pi^-\pi^0,$ and $\pi^+\pi^-\gamma,$ respectively) applied to combine values from the four modes into the weighted sum $\left(\langle \delta \rangle - \beta \right)_w = \sum_{i=1}^{4} w_i \times (\langle \delta_i \rangle - \beta_i) = 272\pm 17$ keV.

As the mass distributions are not perfectly Gaussian, there is some systematic variation of the peak value with the choice of mass limits for each fit. However, as long as the limits chosen do not result in a confidence level below 1% and remain roughly symmetric about the peak, such variation is observed to be bounded by approximately half of a statistical standard deviation. Hence this value was assigned as a conservative estimate of the systematic uncertainty attributable to the fit limits.

Uncertainties attributable to imprecision in the masses of the $J/\psi$ (11 keV) and $\psi(2S)$ (34 keV) mesons [6] are directly calculated by repeating the analysis using an altered $\psi(2S)$ or $J/\psi$ mass and the deviation in $\langle \delta \rangle$ per “1σ” change from nominal taken as the error.

The bias $\beta_i$ from kinematic fitting can be attributed to the effect of the asymmetric resolution function of photons; the bias for the four modes varies by an order of magnitude, with larger values corresponding to modes with more photons. The $\pi^0$ cross-check indicates, within its 47 keV statistical precision on $\langle \delta \rangle - \beta$, that the bias is indeed accurately estimated. A more sensitive cross-check comes from comparing the $M_\eta$ shifts in data and MC for $\eta \rightarrow \pi^+\pi^-\pi^0$ when the $\pi^0$ mass constraint is removed: the MC predicts an increase in bias of $76\pm 6$ keV compared to an observed shift in the data of $55\pm 24$ keV (i.e. the shift in bias of 76 keV is verified in the data within the 24 keV statistical error, which amounts to about a third of the MC shift itself). Based on these comparisons and the generally favorable agreement [12] between data and MC characteristics, we take one third of the bias central value $(\beta_i/3)$ as our estimate of the systematic uncertainty.

By performing mass fits on MC signal samples with and without simulated backgrounds, it is determined that for mode $i$, modeled backgrounds reduce the bias by the amount $B_i$: $21\pm 19$, $-3\pm 67$, $2\pm 17$, and $-13\pm 27$ keV for the $\gamma\gamma$, $3\pi^0$, $\pi^+\pi^-\pi^0$, and $\pi^+\pi^-\gamma$ channels, respectively, where uncertainties listed are statistical. The unmodeled background in the $\pi^+\pi^-\gamma$ sample [12] is estimated to have an effect on $M_\eta$ that is negligible compared to the 27 keV uncertainty on the modeled background for this mode. After weights are applied, the net effect is a positive offset to $M_\eta$, of $B_w = 3\pm 12$ keV.

Uncertainties in charged particle momentum and calorimeter energy scale are evaluated by shifting those scales by the appropriate amount and repeating the analysis. The charged particle momentum scale is confirmed at high momentum ($\sim 1.5$ GeV/$c$) using unconstrained $J/\psi\rightarrow\mu^+\mu^-$ decays from $\psi(2S)\rightarrow\pi^+\pi^-J/\psi$ and $\pi^0\pi^0J/\psi$ events. A low-momentum (75-500 MeV/$c$) calibration, which is more relevant to our measurement of $M_\eta$, can be made by comparing the mass of $\psi(2S)\rightarrow\pi^+\pi^-J/\psi$ to $M_{\psi(2S)}^{PDG}$ with no kinematic constraints on the $\pi^+\pi^-$ but with a mass-constrained $J/\psi\rightarrow\ell^+\ell^-; this checks the \pi^+\pi^-$ momentum scale because the accurately known $J/\psi$ mass takes up $\sim 84\%$ of the available energy. Events of both types are subjected to Gaussian fits to mass difference variables similar to $\delta$, in the first case to $\langle \delta_{\mu^+\mu^-} \rangle \equiv M(\mu^+\mu^-) - M_{\psi(2S)}^{PDG}$, and in the second to $\langle \delta_{\pi^+\pi^-} \rangle \equiv M(\pi^+\pi^-J/\psi) - M_{\psi(2S)}^{PDG}$. For MC simulation, where we can employ our perfect knowledge of the magnetic field for the momentum scale, the means and statistical errors of these measures are $\langle \delta_{\mu^+\mu^-} \rangle = -90\pm 22$ keV and $\langle \delta_{\pi^+\pi^-} \rangle = 2\pm 3$ keV. This demonstrates that this technique is accurate to $|\langle \delta_{\mu^+\mu^-} \rangle|/M_{J/\psi} \approx 3 \times 10^{-5}$ and $|\langle \delta_{\pi^+\pi^-} \rangle|/(M_{\psi(2S)} - M_{J/\psi}) \approx 1 \times 10^{-5}$. The magnetic field scale in data is tuned to that value which keeps both of these means close to zero: with this setting, measurements yield $\langle \delta_{\mu^+\mu^-} \rangle = 7\pm 46$ keV and $\langle \delta_{\pi^+\pi^-} \rangle = -23\pm 6$ keV, indicating a similar level of sensitivity as the MC samples. Therefore we quote a relative
momentum scale accuracy of 0.01% and use this value for our 1σ systematic variation.

Several processes are used for the calorimeter calibration: inclusive π⁰ decays to γγ (where we can constrain M(γγ) to a known mass), e⁺e⁻ → ℓ⁺ℓ⁻γ (in which energy-momentum conservation and well-measured track momenta allow constraint of the photon energy), and ψ(2S) → γχcJ (where the transition photon energies are known well). The photon energy scale is calibrated to give the correct peak, i.e. the most probable value, for any monochromatic photon energy distribution. These calibrations are combined and result in an overall energy scale known to 0.6% or better over the energy range (30-400 MeV) relevant for photons from the slow η-mesons produced in ψ(2S)→ηJ/ψ.

Any deviation from ideal in momentum or energy scale is substantially damped by the mass constraints, as is evident from Table III: the relative momentum (energy) scale uncertainty of 0.01% (0.6%) induces, at most, ∼ 1 (≈ 5) parts in 10⁵ shift in η-mass scale.

We have also computed ⟨(δ) − β⟩, when the decays occur in combination with either J/ψ→μ⁺μ⁻ or J/ψ→e⁺e⁻ separately; its value for J/ψ→e⁺e⁻ events is higher than that for J/ψ→μ⁺μ⁻ by 98±34 keV (+2.9σ), where the error is statistical only. Broken down by mode, this difference is 71±57 keV (+1.2σ) for γγ, −82±208 keV (−0.4σ) for 3π⁰, 119±49 keV (+2.4σ) for π⁺π⁻π⁰, and 141±91 keV (+1.5σ) for π⁺π⁻γ. We do not observe such an effect in MC simulations. Further investigations, detailed below, reveal no firm explanation. To allow for a hidden systematic effect, we add an MC modeling uncertainty of 46 keV; it is the dominant uncertainty in this analysis.

In order to investigate the possibility that there could be an unmodeled systematic pull of J/ψ→e⁺e⁻ or J/ψ→μ⁺μ⁻ decays in the kinematic fitting process, we examine ψ(2S)→π⁺π⁻J/ψ decays. If such an effect existed, we would expect to observe a shift between the dipion mass computed with the constrained pion (π⁺) momenta relative to the unconstrained (π⁺) values. However, we find no evidence for such a pull: defining f⁺⁻(M(π⁺π⁻)−M(π⁺π⁻))_e⁺e⁻, the difference f⁺⁻−f⁺⁻=13±11 keV in data and 7±9 keV in MC simulations (errors shown are statistical).

To investigate the effect of less-well-measured events upon the analysis in general and the e⁺e⁻−μ⁺μ⁻ discrepancy in particular, we have repeated the analysis after tightening the kinematic fitting restrictions from χ²/d.o.f.<10 and χ²/d.o.f.<5 on J/ψ and ψ(2S) constrained fits to 5 and 2, respectively, losing about 40% of the original events. The net offset from backgrounds changes from ±3±12 keV to −8±14 keV. The overall final η mass, including the background offset, changes by −16±17 keV, demonstrating stability of the measured mass with respect to the kinematic fit quality. For this restrictive selection, the M⁺diff between e⁺e⁻ and μ⁺μ⁻ events in data goes down to 48±44 keV, whereas the MC difference remains near zero. It appears that the e⁺e⁻−μ⁺μ⁻ discrepancy moderates for this class of events, but the statistical precision is not conclusive.

In order to study dependence of Mη upon the time of data collection, we divide the data into nine contiguous and consecutive data-taking periods. One mass from each period is obtained by averaging the results obtained from the four modes with statistical weights. The χ² for the nine values to be consistent with their statistically-weighted average is 9.5 for 8 degrees of freedom (C.L.=25%), demonstrating the absence of any time-dependent systematics.

After combining the ⟨(δ)−β⟩ values in Table II using the quoted weights, including the aforementioned net effect of backgrounds Bw, and adding the MPDG offset, our result is Mη=⟨(δ)−β⟩+Bw+MηPDG = 547.785±0.017±0.057 MeV, where the first error is statistical and the second systematic. This result has comparable precision to both NA48 and GEM.
measurements, but is consistent with the former and 6.5 standard deviations larger than the latter. All four prominent $\eta$ decay modes contribute to this result, and each independently verifies a significantly larger $M_\eta$ than obtained by GEM ($\sim 2.0\sigma$ in $3\pi^0$, more for each of the other three).

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TABLE I: For each π⁰ or η decay mode, the number of events $N$, the Gaussian width on the mass distribution of those data events, $\sigma$, the values of $\delta$, $\beta$ (from MC), and the difference $\langle \delta \rangle - \beta$, (see text). Uncertainties shown are statistical.

| Channel   | $N$  | $\sigma$ (MeV) | $\langle \delta \rangle$ (keV) | $\beta$ (keV) | $\langle \delta \rangle - \beta$ (keV) |
|-----------|------|----------------|-------------------------------|---------------|----------------------------------|
| $\gamma\gamma$ ($\pi^0_a$) | 420  | 3.51           | $-285\pm195$                | $-51\pm21$    | $-234\pm196$                    |
| $\gamma\gamma$ ($\pi^0_b$)   | 4692 | 1.94           | $74\pm46$                   | $128\pm8$     | $-54\pm47$                      |
| $\gamma\gamma$                | 11140| 1.96           | $419\pm27$                  | $126\pm5$     | $293\pm27$                      |
| $3\pi^0$                        | 1278 | 2.83           | $384\pm102$                 | $233\pm18$    | $151\pm104$                     |
| $\pi^+\pi^-\pi^0$             | 3137 | 1.12           | $257\pm24$                  | $5\pm4$       | $252\pm24$                      |
| $\pi^+\pi^-\gamma$            | 770  | 0.91           | $377\pm44$                  | $38\pm6$      | $339\pm44$                      |

TABLE II: For each η channel, systematic uncertainties in $M_\eta$ (in keV) from the listed sources (see text); where applicable the degree of variation of the source level is given (“Var”). The sources marked with an asterisk (*) are assumed to be fully correlated across all modes; others are assumed to be uncorrelated. The final column combines the uncertainties across all modes with the weights given in the text.

| Source                | Var | $\gamma\gamma$ | $3\pi^0$ | $\pi^+\pi^-\pi^0$ | $\pi^+\pi^-\gamma$ | All |
|-----------------------|-----|----------------|----------|--------------------|--------------------|-----|
| Fit Window            |     | $14$           | $52$     | $12$               | $22$               | $9$ |
| $M_{\psi(2S)}$ *      |     | $34$ keV       | $27$     | $32$               | $25$               | $27$ |
| $M_{J/\psi}$ *        |     | $11$ keV       | $9$      | $16$               | $9$                | $9$ |
| Bias                  |     | $\beta_i/3$    | $42$     | $78$               | $2$                | $13$ |
| Backgrounds           |     | $19$           | $67$     | $17$               | $27$               | $12$ |
| $p_{\pi\pm}$ scale    |     | $0.01\%$       | $1$      | $4$                | $5$                | $1$ |
| $E_\gamma$ scale      |     | $0.6\%$        | $13$     | $26$               | $3$                | $7$ |
| MC Modeling *         |     | $46$           | $46$     | $46$               | $46$               | $46$ |
| Sum                   |     | $74$           | $132$    | $57$               | $68$               | $57$ |
FIG. 1: Distributions of $\delta$ for two evaluations of $M_{\pi^0}$ using $\pi^0 \rightarrow \gamma \gamma$ decay, with the data represented by the points with error bars and the Gaussian fit overlaid. The solid line portion of the fit indicates the window used for the fit and the dashed portions its extension. The solid line histogram represents MC simulation of signal and backgrounds with $M_{\pi^0}^{MC} = M_{\pi^0}^{PDG}$ and $M_{\eta}^{MC} = 547.78$ MeV.
FIG. 2: Distributions of $\delta$ for $\eta$ decay modes as shown. Symbols are as defined in Fig. 1.