Numerical study on the flow and heat transfer characteristics of slush nitrogen in a corrugated pipe

Y J Li, S Q Wu, T Jin
Institute of Refrigeration and Cryogenics/Key Laboratory of Refrigeration and Cryogenic Technology of Zhejiang Province, Zhejiang University, Hangzhou 310027, China
Email: jintao@zju.edu.cn

Abstract. Slush nitrogen has lower temperature, higher density and higher heat capacity than that of liquid nitrogen at normal boiling point. It is considered to be a potential coolant for high-temperature superconductive cables (HTS) that would decrease nitrogen consumption and storage cost. The corrugated pipe can help with the enhancement of heat transfer and flexibility of the coolants for HTS cables. In this paper, a 3-D Euler-Euler two-fluid model has been developed to study the flow and heat transfer characteristics of slush nitrogen in a horizontal helically corrugated pipe. By comparing with the empirical formula for pressure drop, the numerical model is confirmed to be effective for the prediction of slush nitrogen flow in corrugated pipes. The flow and heat transfer characteristics of slush nitrogen in a horizontal pipe at various working conditions (inlet solid fraction of 0-20%, inlet velocity of 0-3 m/s, heat flux of 0-12 kW/m²) have been analyzed. The friction factor of slush nitrogen is lower than that of subcooled liquid nitrogen when the slush Reynolds number is higher than 4.2 x 10⁴. Moreover, the heat transfer coefficient of slush nitrogen flow in the corrugated pipe is higher than that of subcooled liquid nitrogen at velocities which is higher than that 1.76 m/s, 0.91 m/s and 0.55 m/s for slush nitrogen with solid fraction of 5%, 10% and 20%, respectively. The slush nitrogen has been confirmed to have better heat transfer performance and lower pressure drop instead of using liquid nitrogen flowing through a helically corrugated pipe.

1. Introduction
Slush nitrogen, the cryogenic suspension of solid nitrogen particles and subcooled liquid nitrogen, is considered to be a potential coolant for high temperature superconductive cables [1-3]. Superior to the normal boiling temperature of liquid nitrogen, which is generally used to cool the superconductive materials with a transition temperature above 77 K, slush nitrogen can help to reduce the risk of quenching, to decrease the coolant cost of transport and storage attributed to the lower temperature, higher density and heat capacity [4]. The corrugated pipes have been introduced to improve the overall thermal-hydraulic performance and flexibility of cooling system of HTS cables, which could help reduce the heat exchanger size and the cost of operation. To design and optimize the cooling system for HTS cables, the flow and heat transfer characteristics of slush nitrogen in corrugated pipes have been focused on. Ohira et al. [5] studied the slush nitrogen flow and heat transfer in annular corrugated pipes, and found that the pressure drop reduction and heat transfer deterioration phenomena occur for slush nitrogen with high Reynolds number and light particle concentration. CFD analysis has also been adopted for the flow and heat transfer process of slush nitrogen. Jiang and Zhang [6] built the
Euler-Euler approach coupled with the granular kinetic theory to consider the particle pulse behavior and inelastic collisions. Jin et al. [7, 8] built a two-fluid numerical model coupled with population balance equations for slush nitrogen in circular pipes, in which the particle size distribution is calculated by the population balance equations to account for the influence of particle size on the interfacial interactions.

However, due to the complexity of corrugated wall, modelling the flow in corrugated pipe is different from that in a circular pipe. Currently, the flow and heat transfer characteristics of slush nitrogen in corrugated pipes have not been numerically studied yet. In this paper, a 3-D Euler-Euler two-fluid model has been built to simulate the flow and heat transfer characteristics in a horizontal helically corrugated pipe. Based on the earlier studies [7, 8], the Cheng-Law empirical formula has been introduced to calculate the effective viscosity of slush nitrogen and to modify the drag law and the interphase heat transfer model, so as to account for the influence of solid fraction on the momentum and energy exchange. The standard k-ω model with low-Re corrections has been adopted for numerical analysis for turbulence. The model has then been applied to calculate the pressure drop and heat transfer coefficient of slush nitrogen flow at various velocity and solid fraction in a helically corrugated pipe.

2. Numerical method
In the Euler-Euler two-fluid model, the solid and liquid phases of slush nitrogen are mathematically treated as interpenetrating continua. The equations for the mathematical model are presented in Table 1, where the subscripts $q$ and $p$ represent either $s$ (solid phase) or $l$ (liquid phase), and $p$ is the opposite phase of $q$. In the conservation equations, $\vec{R}_{pq}$ is the interphase momentum exchange, composed by the drag, lift and virtual mass forces. The lift coefficient and virtual mass coefficient are given by $C_L' = 6.46$ [9] and $C_{VM} = 0.5$ [10]. The drag coefficient is modified by the effective viscosity of solid-liquid mixture, as given by Eq. (9), and $\beta$ is the sole empirical parameter to account for the shape, size and type of solid particle [11], taken the value of 1.5 for slush nitrogen [8].

![Figure 1. Numerical domain and grids for the slush nitrogen flow in a helically corrugated pipe](image)

The solid phase is treated as the granular phase, and the transport equation for granular phase is derived from the granular kinetic theory, as given by Eq. (13). $\Theta_s$ is the granular temperature, defined by $\Theta_s = 1/3 u_{s,i} u_{s,i}$, representing the ensemble average of the particles’ random velocity within a finite volume and time period. $e_{ss}$ is the restitution coefficient for collisions between particles, taken the value of 0.99 for slush nitrogen. $\phi$ is the specularity coefficient between the particle and the wall, set to 0.01 for slush nitrogen in the present calculation, and $e_{sw}$ is the restitution coefficient for the collisions between the particle and the wall, which is set to 0.95 for slush nitrogen [7]. The three empirical coefficients help account for the interaction between the particles and the particle and the
wall [12]. The standard k-ω model is used for the prediction of turbulence. The empirical constants for the model are \( \alpha_\omega^* = 1, \ \alpha_\omega = 0.52, \ \alpha_y = 0.1111, \ \beta_\omega^* = 0.09, \ \beta_\omega = 0.072 \) and \( R_i = 6 \) [13].

### Table 1. Mathematical model

#### Conservation equations:

\[
\frac{\partial}{\partial t}(\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = \dot{m}_{pq} - \dot{m}_{pq} \tag{1}
\]

\[
\frac{\partial}{\partial t}(\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = -\alpha_q \nabla p + \nabla \cdot \vec{F}_q + \alpha_q \rho_q \vec{g} + \vec{R}_{pq} + \left( \dot{m}_{pq} \vec{v}_p - \dot{m}_{pq} \vec{v}_q \right) \tag{2}
\]

\[
\nabla \cdot (\alpha_q \rho_q h_q) - \nabla \cdot (\alpha_q \rho_q \vec{u}_q h_q) = \nabla \cdot \vec{u}_q - \nabla \cdot \vec{q}_q + h_{pq} (T_p - T_q) + \dot{m}_{pq} h_{melt} \tag{3}
\]

#### Solid-liquid interphase momentum exchange:

\[
\vec{F}_{ls} = -C_i \alpha \rho_l(\vec{v}_l - \vec{v}_s) \times (\nabla \times \vec{v}_l) \tag{4}
\]

\[
\vec{F}_{g,vm} = C_{vm,\alpha \rho_l}(dv_l / dt - dv_s / dt) \tag{5}
\]

\[
\vec{F}_{\rho,q} = K_{\rho} |\vec{v}_l - \vec{v}_s| \tag{6}
\]

\[
C_{\rho} = \max \left[ \frac{24}{Re_p^{0.697}} (1 + 0.15Re_p^{0.697}), 0.44 \right] \tag{7}
\]

\[
Re_p = \rho_j d \left| \vec{v}_s - \vec{v}_l \right| / \mu_m \tag{8}
\]

\[
\mu_m = \mu_f \exp \left[ \frac{2.5}{\beta} \left( 1 - \frac{1}{(1 - \alpha_l)^\beta} \right) \right] \tag{9}
\]

#### Solid-liquid mass and heat transfer:

\[
\dot{m}_{pq} = h_{pq} (T_p - T_q) / h_{melt} \tag{10}
\]

\[
h_{pq} = 6 \lambda_\alpha \nu_{\max} / d_p^2 \tag{11}
\]

\[
Nu_{\alpha} = (7 - 10\alpha_f + 5\alpha_f^2)(1 + 0.7Re_{\alpha}^{0.2}Pr^{1/3}) + (1.33 - 2.4\alpha_f + 1.2\alpha_f^2)Re_{\alpha}^{0.7}Pr^{1/3} \tag{12}
\]

#### Granular temperature:

\[
\frac{3}{21} \frac{\partial}{\partial t} \left( \alpha \rho \Theta_s \right) + \nabla \cdot (\alpha \rho \vec{v}_s \Theta_s) = \left( -p_s \nabla + \vec{F}_s \right) \cdot \nabla \vec{v}_s + \nabla \cdot (k_{\Theta_s} \nabla \Theta_s) - \gamma_{\Theta_s} + \phi_{\Theta_s} \tag{13}
\]

\[
p_s = \alpha \rho \Theta_s + 2 \alpha \rho \vec{v}_s \Theta_s + \rho_s \vec{v}_s \Theta_s d_{\max} (1 + e_m) \tag{14}
\]

\[
\vec{F}_{s} = \frac{\pi}{6} \sqrt{\tilde{F} \rho_s \Theta_{\max}} \rho_s \Theta_{\max} \vec{U}_{s,m} \vec{U}_{s,m} - \frac{\pi}{4} \sqrt{3} \frac{\alpha_s}{\alpha_{s,max}} \left( 1 - e_m \right) \rho_s \Theta_{s,\max} \Theta_{s,\max} \tag{15}
\]

\[
q_s = \frac{\pi}{6} \sqrt{\tilde{F} \rho_s \Theta_{\max}} \rho_s \Theta_{\max} \vec{U}_{s,m} \vec{U}_{s,m} - \frac{\pi}{4} \sqrt{3} \frac{\alpha_s}{\alpha_{s,max}} (1 - e_m) \rho_s \Theta_{s,\max} \Theta_{s,\max} \tag{16}
\]

#### Standard k-\omega turbulent model:

\[
\frac{\partial}{\partial t}(\rho \vec{u}) + \frac{\partial}{\partial x_i}(\rho \vec{u} \vec{u}_i) = \frac{\partial}{\partial x_j} \left[ \mu + \frac{\mu_\omega}{\sigma_k} \frac{\partial k}{\partial x_j} \right] - \rho \vec{u}_i \vec{u}_j \frac{\partial \vec{u}_i}{\partial x_j} - \rho \beta \vec{u}_i \vec{u}_j \omega \tag{17}
\]

\[
\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_i}(\rho \omega \vec{u}_i) = \frac{\partial}{\partial x_j} \left[ \mu + \frac{\mu_\omega}{\sigma_\omega} \frac{\partial \omega}{\partial x_j} \right] + \alpha_o \frac{\omega}{k} G_k - \rho \beta \vec{u}_i \vec{u}_j \omega^2 \tag{18}
\]

\[
\alpha_\omega = \alpha_\omega^* \left( \frac{\alpha_\omega^* + \rho \omega / \mu_\omega R_k}{1 + \rho \omega / \mu_\omega R_k} \right) \tag{19}
\]
The horizontal helically corrugated pipe used in the numerical analysis is presented in Figure 1, with the internal diameter of \( d=10 \) mm, the outer diameter of \( D=14 \) mm, a pitch of \( s=2.8 \) mm, and the length of \( L=100 \) mm. A circular pipe with the length of 100 mm is added to the inlet of corrugated pipe so as to reduce the influence of the turbulence of flow development. The computational grids have a cell quantity of 1,275,000. The grids of the circular pipe are structured while the grids of the corrugated part are unstructured. To comprehensively analyse the flow and heat transfer characteristics of slush nitrogen, the boundary conditions are given as follows: the flow velocity of 0.5-3 m/s, the inlet solid fraction of 0-20\%, the ambient pressure outlet, and the wall heat flux of 0-12 kW/m\(^2\). The calculations are completed by ANSYS-Fluent 14.5, where the user defined functions (UDFs) are adopted to include the interphase mass and moment transfer source terms and the drag law.

3. Results and discussion

3.1. Pressure drop

The numerical results for the pressure drop of subcooled liquid nitrogen (at 63.15 K) with the flow velocity between 0-3 m/s are given in Figure 2. Figure 2-a illustrates the comparison between the numerical and experimental results, which shows a good agreement with a fluctuation of ±15\%. Details about the experimental setup were given in our earlier work [2]. Figure 2-b gives the comparison between the numerical results and the empirical correlation given by Hawthorne and Von Helms [14]

\[
\Delta P = \frac{1}{2} f \rho v^2 \frac{L}{d} = \frac{d}{s} \left[ 1 - \left( \frac{d}{d + \gamma s} \right)^2 \right] \frac{1}{2} \rho v^2 \frac{L}{d}
\]

(20)

where \( \Delta P \) is the pressure drop, and \( f \) is the friction factor. \( \rho \) and \( v \) are the density and the mean velocity of fluid, respectively. \( d \) is the inner diameter of corrugated pipe, and \( L \) is the pipe length, and \( s \) is the pitch of the wall. \( \gamma \) is the experimental empirical coefficient, depending on the structure of the corrugated wall and taken as 0.438, which is derived from experimental results for water and air.

The results of Eq.(20) illustrated in Figure 2-b are for \( \gamma=0.438, \gamma=0.49 \) and \( \gamma=0.6 \), respectively, and the pressure drops for \( \gamma=0.49 \) have the best agreement with the CFD results. Thus, \( \gamma=0.49 \) is adopted for theoretical calculation of the pressure drop of slush nitrogen flow.

![Figure 2. Pressure drop of subcooled liquid nitrogen](image-url)

Figure 3 shows the numerical results for the pressure drop of subcooled liquid nitrogen and slush nitrogen at various flow velocity and solid volumetric fractions. The pressure drop increases with the increasing flow velocity. On the other hand, the pressure drop varies little with the solid fraction,
which indicates that the fluidity of slush nitrogen is roughly equivalent with that of the subcooled liquid nitrogen. In fact, as illustrated in the enlarged section of Figure 3, the pressure drop of slush nitrogen with particle volumetric concentration of 5-20% can be lower than that of the subcooled liquid nitrogen, and decreases with the increasing solid fraction.

Figure 4 shows the friction factor of slush nitrogen with the slush Reynolds number. The slush Reynolds number is defined as \( Re_s = \frac{\rho_s v D}{\mu_s} \), where \( \rho_s \) is the density of slush nitrogen, calculated by \( \rho_s = \alpha \rho_l + \alpha_p \rho_p \), and \( \mu_s \) is the slush effective viscosity to better characterize the flow condition of slush nitrogen. The results for friction factor of subcooled liquid nitrogen in Figure 4 is calculated by Eq. (20), which are constant of 0.1836. As illustrated in Figure 4, the friction factor of slush nitrogen is lower than that of subcooled liquid nitrogen when the slush Reynolds number is higher than \( 4.2 \times 10^4 \). One reason for the reduction of friction factor for the slush nitrogen case could be that the particles suppress the turbulence of liquid phase when the flow velocity is high.

Figure 3. Pressure drop of slush nitrogen

Figure 4. Friction factor of slush nitrogen

3.2. Heat transfer coefficient

Figure 5 shows the variations of heat transfer coefficient with inlet flow velocity at various solid volumetric fractions for the wall heat flux of 12 kW/m². The heat transfer coefficient is the average heat transfer coefficient of the helically corrugated pipe, as given by

\[
\bar{h} = \frac{q}{\Delta T}
\]

where \( q \) is the heat flux, and \( \Delta T \) is the logarithmic mean temperature difference. \( \Delta T \) can be calculated by logarithmic mean temperature difference

\[
\Delta T = \frac{(T_w - T_{out}) - (T_in - T_{out})}{\ln(T_w - T_{out}) - \ln(T_in - T_{out})}
\]

where \( T_w \) is the mean wall temperature, \( T_{in} \) and \( T_{out} \) are the bulk fluid temperature at the inlet and the outlet of the corrugated pipe, respectively.

As shown in Figure 5, the heat transfer coefficient of slush nitrogen increases with the increasing flow velocity and solid faction. Figure 6 presents the heat transfer coefficient ratios \( h_{sl}/h_l \) of slush nitrogen and subcooled liquid nitrogen at the same flow velocity. The heat transfer coefficient ratio rises with the increasing flow velocity, and then gradually tends to become stable. The heat transfer performance of slush nitrogen could be better than that of subcooled liquid nitrogen at high velocity, with a critical velocity of 1.76 m/s, 0.91 m/s and 0.55 m/s for slush nitrogen with particle volumetric concentration of 5%, 10% and 20%, respectively. The results for Nusselt number with slush Reynolds number are given in Figure 7, comparing with the Seider-Tate equation, which is the empirical correlation for calculating the Nusselt number of subcooled liquid nitrogen in a horizontal circular
smooth pipe. As indicated in the figure, the Nusselt number in the corrugated pipe is higher compared to a smooth pipe. In addition, the corrugated pipe has better heat transfer performance for the slush nitrogen flows than the circular pipe.

![Figure 5. Heat transfer coefficient of slush nitrogen](image1)

![Figure 6. Variation of $h_s/h_l$ with flow velocity and solid fraction](image2)

![Figure 7. Nusselt number of slush nitrogen](image3)

**4. Conclusions**

In the present study, a 3-D Euler-Euler two-fluid model has been developed to study the flow and heat transfer characteristics of slush nitrogen in a horizontal helically corrugated pipe. Slush nitrogen flow in the corrugated pipe under various working conditions (inlet solid fraction of 0-20%, inlet velocity of 0-3 m/s, heat flux of 0-12 kW/m$^2$) are calculated to analyze the pressure drops and heat transfer coefficients. The numerical model is verified to be effective to predict the slush nitrogen flow by comparing with the empirical formula. The experimental coefficient $\gamma=0.49$ is applicable for the theoretical calculation of pressure drop of slush nitrogen flow.

The pressure drop of slush nitrogen with solid volumetric fraction within 5-20% is roughly equivalent with that of the subcooled liquid nitrogen. Moreover, the friction factor of slush nitrogen is lower than that of subcooled liquid nitrogen when the slush Reynolds number is higher than $4.2\times10^4$. For the slush nitrogen with high slush Reynolds number, the particles help suppress the turbulence of liquid phase, which could contribute to less wall friction.

The heat transfer coefficient of slush nitrogen increases with the increasing flow velocity and solid faction. The heat transfer performance of slush nitrogen could be better than that of subcooled liquid nitrogen at high velocities, and the critical velocity is 1.76 m/s, 0.91 m/s and 0.55 m/s for slush...
nitrogen with particle volumetric concentration of 5%, 10% and 20%, respectively. Moreover, the corrugated pipe is confirmed to have better heat transfer performance for the slush nitrogen flows than that of the circular pipe.

**NOMENCLATURE**

- $C_D$: Drag coefficient
- $C_L$: Lift coefficient
- $C_{VM}$: Virtual mass coefficient
- $D$: Pipe diameter, mm
- $d_i$: Particle diameter, mm
- $e_s$: Restitution coefficient for particles
- $e_{sw}$: Restitution coefficient for particles-wall
- $F_{D,p}$: Drag force
- $F_{L,q}$: Lift force
- $F_{VM,q}$: Virtual mass force
- $G_k$: Generation of turbulence kinetic energy
- $g$: Gravitational acceleration, m/s²

| Subscripts | Description |
|-------------|-------------|
| $l$ | liquid phase |
| $s$ | solid phase |
| $sl$ | slush |
| $p$ | either solid phase or liquid phase |
| $q$ | the opposite solid phase of $p$ |

**Greek Letters**

- $\alpha$: Volume fraction, %
- $\alpha_{s,max}$: The packing limit, %
- $\beta$: Effective viscosity parameter
- $\omega$: Specific dissipation rate
- $\phi$: Specularity coefficient
- $\phi_p$: Interphase energy exchange
- $\gamma_{\tau}$: Collisonal dissipation of energy

- $\eta$: Intrinsic viscosity, Pa·s
- $\mu$: Fluid viscosity, Pa·s
- $\sigma_1, \sigma_2$: Turbulent Prandtl number
- $\Theta_s$: Granular temperature, m²/s²
- $\rho$: Density, kg/m³
- $\bar{\tau}$: Shear stress

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Acknowledgement
This work is financially supported by the Zhejiang Provincial Natural Science Foundation, China (LZ14E060001) and the National Key Basic Research Program of China (No. 613322).