Galactic Sources of High-Energy Neutrinos: Highlights

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Abstract

We overview high-energy neutrinos from galactic sources, transparent to their gamma-ray emission. We focus on young supernova remnants and in particular on RX J1713.7-3946, discussing expectations and upper bounds. We also consider the possibility to detect neutrinos from other strong galactic gamma-ray sources as Vela Junior, the Cygnus Region and the recently discovered Fermi Bubbles. We quantify the impact of the recent hint for a large value of $\theta_{13}$ on high-energy neutrino oscillations.

Key words: Neutrinos and gamma rays; galactic sources of high-energy radiation; supernova remnants and cosmic rays

1. Context, motivations and assumptions

The successes of low energy neutrino astronomy and the discovery of neutrino oscillations added momentum to the search for high energy neutrinos from cosmic sources, initiated long ago with the theoretical proposals of Zheleznykh, Markov and Greisen [1]; see Fig. 1. The first km$^3$-class detector, IceCUBE and the smaller ANTARES (following MACRO, BAIKAL, AMANDA) have not yet revealed the first signal, yet the excitement remains high. However, nowadays it becomes clear that the search for high energy neutrino sources is difficult. In this paper, we attempt to examine certain generic and specific expectations for such a search, in the hope to contribute to the scientific planning of the future experimental activities. We are interested in the possibility to proceeding further, by building a new telescope of km$^3$-class in the Northern Hemisphere--as K3NET in the Mediterranean Sea or GVD in Lake Baikal.

The fact that the first explorations of the high-energy neutrino sky, that did not reveal any signal, contradicted the over-optimistic expectations may lead one to doubt that we can reach reliable predictions. But, even if some degree of diffidence toward theory is healthy, we are convinced that the process of formulating expectations is an important step toward understanding. We would like to clarify our view by proposing some issues: Do we really need to have high expectations for high energy neutrinos? Perhaps not, although one should keep in mind that the surprises often happen in astronomy. As recent example in this regard are the discovery of the Fermi Bubbles and the variability of the Crab Nebula. Are expectations useful? Of course, yes; good predictions are precious for experiments, but also reasonable expectations eventually contradicted are not useless. Do we have some relevant and succesful precedent? Yes, many: High-energy $\gamma$-ray sources have been predicted in the fifties (Morrison). For solar neutrinos, we had predictions with errorbars since the sixties (Bahcall). For supernova neutrinos, quantitative expectations were available even before SN1987A (Nadyozhin).

We adopt a rather common astrophysical attitude: the observation of high-energy neutrinos is very demanding but perhaps possible and would amount to an unambiguous sig-

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1 We thank ML.Costantini, N.Sahakyan and F.Villante for collaboration, P.Blasi and P.Lipari for pleasant and important discussions, T.Schwetz for an explanation on $\theta_{13}$ and F.Halzen for an interesting public discussion on Fig. 7 at NUSKY meeting (ICTP).

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Fig. 1. Energy ranges of various types of neutrino telescopes, in $\log_{10}(E/eV)$; note that they span more than 10 decades in energy. In this paper, we are concerned with the third item, that after IceCUBE we identify with the technology ‘ice Cherenkov’.
nal of cosmic ray collisions—hopefully, those in their source. We simplify our task by restricting our attention on the specific but important class of sources, those transparent to their gamma rays. We will show how to proceed towards precise expectations and eventually observations of neutrinos, limiting the use of theory inputs and just by the help of gamma-ray observations. This aspect is important enough to be emphasized, as we do with Fig. 2. In the next section, we quantify the connection between gamma and neutrinos.

2. Expected neutrino intensity and signal

If we measure the very high gamma-ray emission from a cosmic source, and if we attribute it to cosmic ray colliding with other hadrons, it is straightforward to derive the muon neutrino flux. In fact, when both the neutrinos and unmodified, hadronic gamma-rays are linear functions of the cosmic ray intensity, they are linked by a linear relation [2]:

\[ I_{\nu_{\mu}}(E) = 0.380 I_{\gamma} \left( \frac{E}{1 - r_{\gamma}} \right) + 0.013 I_{\gamma} \left( \frac{E}{1 - r_{K}} \right) + \int_{0}^{1} \frac{dx}{x} K_{\mu}(x) I_{\gamma} \left( \frac{E}{x} \right) \]

(1)

where the coefficients are determined by the amount of mesons produced and by their decay kinematics. The first and second contributions are due to direct decays into neutrinos and the third one to mu decay, the kernel \( K_{\mu}(x) \) being

\[
\begin{align*}
& 0 < x < r_{K} \\
& 0.0165 + 0.1193x + 3.747x^2 - 3.981x^3 r_{K} < x < r_{\pi} \\
& (1 - x)^2(-0.6698 + 6.588x) r_{\pi} < x < 1
\end{align*}
\]

(2)

where \( r_{x} = (m_{\mu}/m_{x})^2 \) with \( x = \pi, K \). Similar formulas describe antineutrino production; the effects of the oscillations are included [2], compare also with [3] and see the Appendix for an updated discussion.

At this point, calculating the muon signal is just applying a standard procedure. First, one evaluates the probability of converting muon neutrinos into muons

\[ P_{\nu_{\mu} \rightarrow \mu} = \int_{E_{th}}^{E} dE_{\mu} \frac{d\sigma_{cc}}{dE_{\mu}} R_{\mu}/m_{n} \]

(3)

where \( E_{th} = \) [say, \( 10^{-35} \text{cm}^2 / \text{s} \times m_{n} \beta \sim 10^{-6} \)].

Fig. 2. We can exploit the strict connection of secondary muon neutrinos and gamma-rays produced by collisions of cosmic rays above 10 TeV, assuming that the gamma-rays are not absorbed or strongly modified.

Fig. 3. Distribution of \( \nu_{\mu} \) leading to muons, assuming \( E^{-2} \) primary spectrum (upper curve), then, including Earth absorption, for a source at \( \delta = -39^\circ \) as seen from ANTARES (middle curve); then with a spectrum \( E^{-2} \exp(-\sqrt{E/150 \text{ TeV}}) \) (lower curve), i.e., with primaries cutoffed at \( \sim 3 \text{ PeV} \).

Unless stated otherwise, we will use \( E_{th} = 1 \text{ TeV} \) in order to cope with atmospheric neutrino background; see [4] and Fig. 1. Then, one should calculate the effective neutrino detector area, proportional to the previous probability, to the physical area of the muon detector (that in general depends on the zenith angle \( \theta \)), and to the probability that the Earth absorbs neutrinos:

\[ A_{\nu_{\mu}} = A_{\mu}(\theta) \times P_{\nu_{\mu} \rightarrow \mu}(E, \theta) \times e^{-\alpha z/m_{n}} \]

(4)

[say, \( 1 \text{ km}^2 \times 10^{-6} \sim 1 \text{ m}^2 \)]

When \( E \sim 10 \text{ TeV}, s \sim 2m_{\nu}E \sim Q^2 > M_{\odot}^{2} \), thus this is the energy where the cross section begins to grow more slowly. The Earth absorption begins at \( E \sim \text{few} \cdot 100 \text{ TeV} \), when \( \sigma(E) \sim m_{n}/(R_{\odot}^{2} \rho_{\odot}) \sim 5 \cdot 10^{-34} \text{ cm}^2 \), see Fig. 3.

3. Neutrinos and gamma rays

If one assumes a power-law distribution with an exponential cutoff for the primary cosmic rays, then the spectrum of gamma-rays has the same power-law index but drops slower [5]

\[ I_{\gamma} \propto E_{\gamma}^{-\alpha} \cdot \exp(-\sqrt{E_{\gamma}/E_{c}}). \]

(5)

Typically, in gamma-ray sources \( \alpha \) varies between 1.8 – 2.2 and \( E_{c} \) varies from 1 TeV to 100 TeV. Assuming gamma-ray transparency we can calculate the critical (minimal) intensity that corresponds 1 muon/km\(^2\) year above 1 TeV; this is given in Fig. 4, where we consider \( \alpha = 1.8, 2.2 \) and \( E_{c} = 1, 30, 100 \text{ TeV} \). Interestingly, the integrated flux above 10 TeV is almost the same in all these cases

\[ I_{\gamma}(> 10 \text{ TeV}) = (1 - 2) \times 10^{-13} / (\text{cm}^2 \text{s}) \]

(6)

Note that the present Cherenkov telescopes are less sensitive above 10 TeV, due to the small collection area. Thus we ask how to detect these fluxes. In order to collect \( \geq 100 \text{ } \gamma \)'s in a reasonable time, we need a \( \text{km}^2 \) area:

\[ \text{Exposure} = L^2 \times T \sim 2 \times \text{km}^2 \times 10 \text{h} \]

(7)

e.g., a \( 10 \times 10 \text{ Cherenkov telescopes array, or one dedicated EAS array. A large area gamma apparatus, such as CTA or even a custom instrument, could be then worthwhile even for}
neutrino community, and should cost much less than a neutrino-telescope.

Now that we have compared the energy-response of gamma- and neutrino-telescopes, we would like to emphasize the fact that γ and νµ views are complementary, as suggested by Fig. 5. The maximal complementarity would be obtained with two detectors in antipodal locations. Stated in more quantitative terms, a steady source at declination δ is seen from a detector at latitude φ for a fraction of time:

\[ f_\gamma = \frac{\cos^{-1}(-\tan \delta \tan \phi)}{\pi}; \tag{8} \]

the fraction of time for neutrinos is just \[ f_\nu = 1 - f_\gamma. \]

A concrete way to illustrate this feature is to note that a hypothetical νµ (resp., γ) emission from Galactic Center, \( \delta \approx -30° \), is visible from North (resp., South) Pole.

This argument can be generalized. Suppose that the galactic sources of TeV neutrinos trace the mass distribution, and calculate the relevance of a detector as a function of its latitude. The matter of the Galaxy is mostly located in the region \( \delta < 0 \), i.e., below the celestial equator. Thus, a telescope at the latitude of NEMO has a priori 2.9 better chances (or 1.4, weighting the mass distribution with \( 1/r^2 \)) to see galactic neutrino sources than IceCUBE as shown in Fig. 6 and argued in [6]. These remarks show why we think that a large, high-energy neutrino detectors in the Northern Hemisphere, not only would complement the IceCUBE field of view, but would also be important to probe the existence of hypothetical galactic neutrino sources.

However these considerations, as general as they are, leave something to be desired. For instance, the use the average galactic matter distribution to infer the source neutrino distribution is doubtful, also because HESS sky-maps show only few intense γ-ray sources. In this regard it is more appropriate to study individual sources.

In order to proceed, it is useful to formulate explicitly some pending question and doubt: (i) Is γ-transparency a reliable hypothesis? (ii) Are we sure of the ‘point-source’ hypothesis? Similar as asking whether \( \ll 1° \) pointing is really important for very-high-energy γ and/or ν telescopes? (iii) γ above 10 TeV can help ν astronomy, and this would be a natural direction of progress anyway, but do we have any guaranteed aim for such a search? (iv) Eventually, the true question is: How to separate leptonic from hadronic gamma’s? Neutrino identify CR collisions, but is this the only way to proceed?

All these questions suggest to define more precisely which (type of) source we want to investigate.

4. Supernova remnants+molecular clouds

The hypothesis that supernova remnants (SNR) are main sites of the acceleration of the galactic cosmic rays (CR) can and should to be tested.\(^2\) As indicated in ref. [7], the associations between SNR and molecular clouds (MC) provide the optimal conditions to produce intense neutrino- and γ-ray fluxes, that thereby can be used as a tool to test the SNR paradigm.

The best hope to detect ‘hadronic’ neutrinos is connected then to young SNR, i.e., of age less than 2000 yr. In fact, these are expected to contain the highest energy cosmic rays, and can thus produce energetic neutrinos that stand over the background of atmospheric neutrinos. In other words, we are interested in special types of SNR’s as poten-

\(^2\) Let us recall the main steps that consolidated this hypothesis: 1) Baade & Zwicky’s propose that supernova explosions and CR are connected; 2) Fermi argues that the kinetic energy of gas can convert into CR acceleration via magneto-hydrodynamical processes; 3) Ginzburg & Syrovatskii note that the kinetic energy injected by SNR is 1-2 orders of magnitudes greater than the galactic energy losses of CR; 4) several authors note that the shock wave present in astrophysical environments as SNR offers the conditions for efficient CR acceleration; 5) Bell & Lucek et al. remark that the CR lead to an amplification of the relevant magnetic fields in the region of the shock wave, as observed; 6) the present efforts are directed to develop a full non-linear theory of CR and SNR possibly including MC.
tional neutrino sources: the closest and the youngest, among those associated to molecular clouds. It should however be observed that core collapse supernovae, being connected to short living stars, are pretty naturally associated with sites of intense stars formation—i.e., molecular clouds.

Before analyzing a specific and important case of such a young SNR, let us discuss in some generality the pros & cons of the SNR+molecular clouds paradigm, outlined above. Some support comes from the detected GeV γ’s from relatively old SNR, as W28 and W44. Moreover, gamma transparency usually holds. One can expect, a priori, that the closest SNR should be at about 1 kpc; in fact, we have 1 new SN each many tens or years and the size of the Galaxy is several tens of kpc. Also, it is true that the CR acceleration above 100 TeV is still an open theoretical problem but, on the other hand, it is a problem that can be approached observationally measuring gammas above 10 TeV.

In any case, we should expect that in order to describe any individual SNR we need theoretical modeling anyway (and the presence of MC will not make the matter simpler) plus as many multiwavelength observations as possible.

### 5. The SNR RX J1713.7-3946: a case study

In this section, we will discuss the object RX J1713.7-3946, in particular, in connection with the possibility that this is an intense source of neutrinos above TeV. The nature of the object, its age and its distance are still debated issues; however, the identification of the location of this source of non-thermal X-ray emission [9] with the position of a historical supernova [10] supports the idea that this is a SNR about 1600 years old. The distance is estimated around 1 kpc [11]; thus, it is pretty close. Moreover, the observations indicate an existence of dense condensations inside the shell of RX J1713.7-3946 [11]. The most prominent one has \( \sim 400 M_\odot \), and is 1/10 of the SNR angular size. Thus, it should have a volume of \((1 \text{ pc})^3\) and therefore a column density of 20 µm of lead, that cannot produce a significant attenuation of the gamma rays. It has been argued that the structure of this system of molecular clouds and the SNR shock waves are intimately connected [12].

Another feature of this SNR is that γ-rays up to 30 TeV or perhaps even more have been measured by HESS [13].

#### Table 1

| Muon threshold | Expected signal | 1 sigma statistical error | Atmospheric background |
|----------------|------------------|--------------------------|-----------------------|
| 50 GeV         | 5.7              | 6%                       | 21                    |
| 200 GeV        | 4.7              | 7%                       | 7                     |
| 1 TeV          | 2.4              | 10%                      | 1                     |
| 5 TeV          | 0.6              | 30%                      | 0.1                   |
| 20 TeV         | 0.1              | 100%                     | 0.0                   |

Dependence on the threshold of the number of signal muons per km² per year from RX J1713.7-3946 and assuming the hadronic hypothesis. Also quoted the estimated error from HESS statistics and the estimated background. From [2].

This is a pretty unique circumstance, in view of the limited area of the existing γ-ray telescopes that explored the TeV region but barely began the exploration of the sky above few 10 TeV. The spectrum is non-trivial: it is not described by a simple power law, but rather by a broken-power-law or by a modified-exponential-cut as in Eq. 5 and with [14]

\[
\gamma = 1.79 \pm 0.06 \quad \text{and} \quad E_c = 3.7 \pm 1 \text{ TeV} \tag{9}
\]

this does not seem to contradict the expectations, but the cutoff suggests that the cosmic rays with energies above 150 TeV have already abandoned this SNR.

#### 5.1. Expectations within the hadronic hypothesis

Thanks to HESS measurements, the upper bound on neutrino signal is now quite precise. The most detailed recent calculation [2] deduced the \( \nu_\mu \) and \( \bar{\nu}_\mu \) from these data, finding for a threshold of 1 TeV an induced muon flux of

\[
I_{\mu + \bar{\mu}} = 2.4 \pm 0.3 \pm 0.5/\text{km}^2 \times \text{yr} \tag{10}
\]

In other words, the HESS data, with the hadronic hypothesis, permit us a reliable evaluation of the expected fluxes—or more plausibly, the upper bounds—on high energy neutrinos. It is not possible to lower much the detection threshold since the background due to atmospheric muons increases rapidly [4]; in fact, the flux of muon neutrinos decreases as \( \sim E^{-3.7} \) in the relevant region, as we see from Tab. 1.

The expectations changed in the course of the time as shown in Fig. 7; let us recall the reasons. Ref. [15]: The first work on high energy neutrinos from RX J1713.7-3946 used a bold extrapolation of Cangaroo data, and adopted a simplified approach, subsequently improved. Ref. [16]: the inclusion of oscillations, Earth absorption, and livetime caused a factor of 4 decrease; moreover, the hypothesis of an early cutoff in the spectrum was remarked, estimating the maximum decrease in a factor of 3. Ref. [3]: the results of HESS, which first revealed a cutoff in the spectrum, permitted us to quantify the decrease in about a factor of 2. Ref. [2]: None of the subsequent theoretical and observational improvements considered introduced large changes, and the prediction became stable.

Two remarks are in order: (i) the last three predictions (or better, it is good to repeat it, upper bounds) agree
within errors; (ii) for a fair comparison, the threshold has been set to the common value of $E_{th} = 50$ GeV, but this value is too low to permit background rejection. As shown in Tab. 1, this implies that the expected signal should be more than halved.

5.2. Fermi-LAT view at GeV and above

However, the true point is whether the hadronic hypothesis holds true (i.e., whether the observed $\gamma$-rays come predominantly from CR collisions): crucial tests of this hypothesis (that are of course relevant to the matter of predicting the high energy neutrino signal) include the observations of $\gamma$-rays in various regions of energy.

Thus it is very interesting that, recently, Fermi-LAT released their first results on RX J1713.7-3946 [17], that revealed a rather complicated environment: 1) there is a wide source approximatively in the SNR location with spectrum

$$I_{\gamma} \propto E_{\gamma}^{-1.5\pm0.1}$$

that results from measurements above 5 GeV and from an upper bound on the spectrum at 0.5-5 GeV; 2) there are several point sources in the surrounding, including one sloping as $\approx E_{\gamma}^{-2.45}$, outshining the wide source at GeV energies; 3) finally, there is the diffuse background from the Milky Way.

On the one hand, the location of the extended source does not disagree with the location of the dense condensations as shown in Fig. 8; however, what seems most crucial for the interpretation is the slope of the spectrum, that is different from the one given in Eq. 9 and also harder than theoretically expected one, assuming that the observed emission is due to CR collisions.

Let us pause here to recollect. From a purely observational point of view, it seems very important to understand better the emission below 5 GeV, and one should note that a spectrum like $E_{\gamma}^{-1.7}$ is not yet excluded firmly—which could be reconciled more easily, perhaps, with hadronic emission and a very efficient acceleration. From a theoretical point of view, the spectrum could be leptonic ($E_{\gamma}^{-\gamma} \Rightarrow E_{\gamma}^{\gamma+(\gamma+1)/2}$) or hadronic, with an energy-dependent penetration factor ($E_{\gamma}^{-\gamma} \Rightarrow E_{\gamma}^{\gamma+1/2}$), if primaries—electrons or protons, respectively—have $\gamma \approx 2$ [12]. Moreover, the lack of thermal X-rays could imply that the emission is leptonic if the medium is uniform, or that non-uniformity plays a major role [18,12].

Since we expect progress from the GeV region in the close future, we believe that the right attitude is: Wait and see.

5.3. Next steps?

The possibility that the spectrum of RX J1713.7-3946 is composite was discussed in a concrete theoretical model [19], see Fig. 9 (note however that the need to disentangle hadronic and leptonic emission was pointed out immediately, see e.g., the statement of Aschenbach in [20]). This shifts the discussion on the need to quantify hadronic and leptonic $\gamma$ emission, rather than excluding one model in favor of the other one. Consider the possibility that the spectrum is composite. If we want to check thoroughly the SNR+MC paradigm, thus improving the expectations on high-energy neutrinos, we should not study only the spectrum: we should also measure precisely the arrival directions of the high-energy $\gamma$’s, testing correlations with molecular clouds. This is especially important for $\gamma$’s at 10 TeV and above, that are unlikely to be leptonic. Is it possible to use the existing data for this purpose? HESS has about 500 events above 30 TeV, but 70% of them is expected to be cosmic ray background (i.e., noise) [13]. Then, in order to perform this check, we should wait for CTA or for an equivalent km$^2$ class $\gamma$-ray telescope, sensitive above 10 TeV, and with good angular pointing.

6. Other possible sources of high energy neutrinos

Vela Junior: Above the bound necessary to have more than $1 \mu/(\text{km}^2 \cdot \text{yr})$ [6], namely,

$$I_{\gamma}(20 \text{ TeV}) = (2 - 6) \times 10^{-15}/\text{TeV cm}^2 \text{ s}$$

there are 2 more young SNR, Vela Jr and Vela X, observed by HESS. The first one, also known as RX J0852-4622, is a young shell-type SNR, just as RX J1713.7-3946, with angular size $2^\circ$. The specific parameters are however quite
unknown; the most common values of the estimated age are of 660-1400 yr, implying a distance of 0.26-0.50 kpc. Actually, this source is even more intense than RX J1713.7-3946 in γ-rays:

\[ I_\gamma (20 \text{ TeV}) = (1 - 3) \times 10^{-14} / \text{TeV cm}^2 \text{ s} \]  
(13)

but 20 TeV is the last point presently measured by HESS. HESS spectrum \( \propto E^{-2.1} \), whereas Fermi-LAT’s preliminary measurement gives \( \propto E^{-1.9} \) [21], which could indicate a spectral break.

**Cygnus Region:** At 1.7 kpc from us, there is a large star forming region of 100,000\( M_\odot \) mass in the direction of Cygnus constellation. This is remarkable target for cosmic ray collisions, even if we do not know for sure which should be the associated source of cosmic rays. It includes several sources of TeV γ-rays, seen by Milagro [22], which could also radiate neutrinos. These are potentially visible from IceCube location and two of them deserve a special mention:

- MGRO 2019+37 is still unidentified and there is no visible correlation with matter excess. ARGO & Veritas do not see it. Fitting

\[ I_\gamma = 10^{-11} \times E^{-2.2} \times e^{-\sqrt{E/E_c}} / \text{TeV cm}^2 \text{ s} \]  
(14)

with \( E_c = 45 \text{ TeV} \), it could imply up to 1.5 muon events per km² year above 1 TeV.

- MGRO 1908+06 is seen also by ARGO≈Milagro>HESS; a pulsar has been found by Fermi-GLAST there. Using

\[ I_\gamma = 2 \times 10^{-11} \times E^{-2.3} \times e^{-\sqrt{E/E_c}} / \text{TeV cm}^2 \text{ s} \]  
(15)

with \( E_c = 30 \text{ TeV} \), it could imply up to 2.5 muon events per km² year above 1 TeV.

A few remarks: 1) The cutoffs \( E_c \) have been introduced to account for the CASA-MIA bounds at 100 TeV. 2) There is another source, MGRO 2032+41, but weaker. 3) Let us repeat that these objects are weaker theoretical cases than SNR’s, but at least, they include a large target material.

**Fermi Bubbles:** Possible cases of diffuse/wide sources could be given by the recently discovered ‘Fermi bubbles’. Are they a reservoir of galactic cosmic rays? If so, they could be also promising neutrino sources [23]. By extrapolating the flux to

\[ I_\gamma (E) = \Omega \times 10^{-9} e^{-\sqrt{E/E_c}/E^2} / \text{TeV cm}^2 \text{ s} \]  
(16)

with \( E_c = 100 \text{ TeV} \) (meaning a cut at 1 PeV in CR spectrum) and using an angular size of \( \Omega = 0.2 \text{ sr} \approx \pi \times (15^\circ)^2 \), we get a signal of about 100 muons a year for 1 km² detector area. This could be observable in Km3NET as a wide neutrino source over the background.

### 7. Summary and outlook

The promising galactic neutrino sources are intimately tied to γ’s above 10 TeV. The main points regarding the predictions of the high-energy neutrino fluxes from galactic sources are simply two:

- There are only few bright γ-ray sources, that correspond to detectable neutrino sources.
- For these sources, the expected neutrino signal is modest, even assuming that the γ-ray emission is fully contributed by hadronic processes.

The studies of γ-rays from RX J1713.7-3946 prove that it is possible to proceed toward reliable expectations for high-energy vs. The future years will offer us good occasions of progress in this sense:

(i) Multiwavelength observations and theory will help us to understand better the young SNR’s and to quantify their hadronic and leptonic emissions.

(ii) Sub-degree pointing with very high energy γ-rays could permit us to further test the paradigm of SNR+MC association for RX J1713.7-3946.

These considerations emphasize the relevance of certain lines of progress, and suggest to increase synergies and collaboration between neutrino and gamma ray search.

### Appendix A. Neutrino oscillations

**Averaged oscillation probabilities:** Three flavor oscillations are well-understood and relevant. For the transparent sources we are discussing, the simplest regime [24] applies and the relevant probabilities are just constant:

\[ P_{\ell \ell'} = \sum_{i=1}^{3} |U_{\ell i}^2| |U_{\ell' i}^2| \quad \ell, \ell' = e, \mu, \tau \]  
(A.1)

In order to provide explicit expressions of the averaged probabilities, we adopt the standard parameterization of the leptonic mixing matrix to solve the unitarity constraints. Then, defining

\[ \epsilon = \sin^2 \theta_{13} \]  
(A.2)

\[ \alpha = \sin \theta_{13} \cos \delta \sin 2\theta_{12} \sin 2\theta_{23} \]  
(A.3)

\[ \beta = (1 - \epsilon) \cos 2\theta_{12} / 2 + \gamma \]  
(A.4)

\[ \gamma = (1 + \epsilon) \cos 2\theta_{12} \cos 2\theta_{23} / 2 \]  
(A.5)

we obtain relatively simple expressions for the averaged probabilities:

\[ P_{ee} = (1 - \epsilon)^2 \left(1 - \frac{1}{2} \sin^2 2\theta_{12}\right) + \epsilon^2 \]  
(A.6)

\[ P_{e\mu} = \frac{1 - \epsilon}{2} \left[\cos^2 \theta_{23} \sin^2 2\theta_{12} + 4\epsilon \sin^2 \theta_{23} \times \left(1 - \frac{\sin^2 2\theta_{12}}{4}\right) + \alpha \cos 2\theta_{12}\right] \]  
(A.7)

\[ P_{\mu\mu} = \frac{1}{3} + 3 \left[(1 - \epsilon) \sin^2 \theta_{23} - \frac{1}{3}\right]^2 + \frac{1}{2} (\alpha - \beta)^2 \]  
(A.8)

\[ P_{\mu\tau} = \frac{1}{8} \left[(1 + \epsilon)^2 + (1 - \epsilon)^2 (3 \sin^2 2\theta_{23} - \sin^2 2\theta_{12})\right] + \frac{1}{2} (\alpha - \gamma)^2 \]  
(A.9)
One can check explicitly that the diagonal probabilities obey $1 \geq P_{\ell \ell} \geq 1/3$ and the out-of-diagonal ones $1/2 \geq P_{\ell' \ell} \geq 0$. The expressions of the remaining probabilities $P_{\ell \tau}$ and $P_{\tau \ell}$ are obtained from these expressions, mapping $\delta \rightarrow \delta + \pi$ and $\theta_{23} \rightarrow \pi/2 - \theta_{23}$. Note that these formulae depend on $\epsilon$, on the 'doubled angles' $2\theta_{12}$ and $2\theta_{23}$, and on the combination $\sin \theta_{13} \cos \delta$.

The muon neutrino/antineutrino flux after oscillation $F_\mu$, as a function of the fluxes before oscillation, $F_{\epsilon, \mu}$, is:

$$F_\mu = P_{\mu \mu} F_{\mu}^0 + P_{\epsilon \mu} F_{\epsilon}^0 = P_{\epsilon, \mu} F_{\epsilon}^0$$  \hspace{1cm} (A.10)

Two important remarks on $P_{\epsilon, \mu}$ are in order:

1) $P_{\epsilon, \mu}$ depends on the energy through the flux ratio $F_\epsilon^0 / F_\mu^0$. A typical value for CR collisions is 1/2, for when we consider an almost equal amount of charged pions $\pi^\pm$, 1 electron (anti)neutrino each 2 muon (anti)neutrinos are produced, all with similar energies (in [2] one finds more precise statements for power law distributions). However, other cases are possible at least at a speculative level: e.g., if muons are absorbed before decaying (which could happen, e.g., in microquasars) the initial beam is purely made of muon neutrinos, $F_\epsilon^0 / F_\mu^0 = 0$; or, if the neutrino originate from neutron decays, only electron antineutrinos are produced $F_\epsilon^0 / F_\mu^0 = \infty$.

2) The probability $P_{\epsilon, \mu}$ depends also, to some extent, on the poorly known oscillation-parameters. Fig. A.1 shows the variation of $P_{\epsilon, \mu}$ with the CP-violating phase $\delta$, where $\theta_{12} = 34^\circ$ is well-known and thus fixed, whereas the other angles are moved in the wide and conservative ranges $\theta_{23} = (40^\circ \rightarrow 50^\circ)$ and $\theta_{13} = (6^\circ \rightarrow 12^\circ)$—wiggly yellow band—and we set $F_\epsilon^0 / F_\mu^0 = 1/2$. For comparison, we show also the curves obtained by setting $\theta_{23} = 50^\circ$ and $\theta_{13} = 6^\circ$ (upper, red) and $\theta_{23} = 40^\circ$ and $\theta_{13} = 12^\circ$ (lower, blue); the lower part, grey region, is forbidden. $\theta_{13}$ is a relevant variable, thus future precise measurements will impact somewhat on oscillation probabilities.

**Numerical values**: The values of $P_{\ell \ell'}$ change a bit with the new best fit values of $\theta_{23} = 40^\circ$ and $\theta_{13} = 9^\circ$ of [25] (assuming the new reactor fluxes) along with [26] $\delta = -90^\circ$, that replace the previous reference (or extremal) values $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$, now disfavored; $\theta_{12}$ changed slightly $35^\circ \rightarrow 34^\circ$. The new values imply that the symmetric matrix with elements $P_{\ell \ell'}$ changes as follows:

![Fig. A.1. Dependence of the effective oscillation probability on the CP-violating phase and other oscillation parameters; see the text for the meaning of the various lines.](image)

![Fig. A.2. Values of $P_{\ell \mu}$ and $P_{\mu \mu}$, compatible at 1$\sigma$, 90% and 2$\sigma \approx$ 95% with the present information on the mixing angles, as described in Eq. A.12.](image)

The changes are not very large, though.

The present ranges of parameters, approximatively 95%, are $\theta_{12} = 32^\circ - 36^\circ$, $\theta_{13} = 4^\circ - 12^\circ$ and $\theta_{23} = 37^\circ - 51^\circ$, compare [25] and [26]. The precise values of the best fit and of the ranges depend on the details of the analysis,\(^3\) for illustration, we adopt in the following the comparably larger value of $\theta_{13}$ and smaller central value of $\theta_{23}$ of [25], and take into account also the existing (feeble) indication for $\delta < 0$ [25,26], assuming:

$$\begin{align*}
\theta_{12} &= 34^\circ \pm 1^\circ, \quad \sin^2 \theta_{13} = 0.025 \pm 0.007, \\
\theta_{23} &= 42^\circ \pm 3.5^\circ, \quad \delta = -\frac{\pi}{2} \pm \frac{3\pi}{4}
\end{align*}$$  \hspace{1cm} (A.12)

that allow us to perform simple statistical analyses. More precisely, we can consider these parameters as independent Gaussian variables, and constrain the averaged probabilities as described in [20].

Now, let us consider the allowed ranges for the two most important averaged probabilities, $P_{\epsilon \mu}$ and $P_{\mu \mu}$. From Eq. A.12, we derive at $1\sigma$ the values $P_{\mu \mu} = 0.37_{-0.05}^{+0.03}$ and $P_{\epsilon \mu} = 0.25_{-0.04}^{+0.04}$. Fig. A.2 shows that these two quantities are strongly correlated; thus, it is not immediate to derive the expected range of $P_{\epsilon, \mu}$. This can be done however propagating the error directly; the result is shown in Tab. A.1. For the important case $F_\epsilon^0 / F_\mu^0 = 1/2$, we find the $2\sigma$ range $P_{\epsilon, \mu} = 0.32 - 0.36$, which is wider but does not disagree severely with the result in [2], $P_{\epsilon, \mu} = 0.33 - 0.35$.

All in all, the new results may eventually lead to a small increase of $P_{\epsilon, \mu}$, and thus of the signal: about 5%. In view of the larger particle physics uncertainties, and much larger astrophysical ones, we maintain our position, expressed in 2004 [16], that the small uncertainties in the oscillation parameters imply that “there is little hope to learn anything\(^3\)
Expansion in small parameters: It is also possible to expand the averaged probabilities $P_{ell'}$ in sin $\theta_{13}$ and cos(2$\theta_{23}$) that are small parameters and, due to the present uncertainties, vary in a similar range; in fact, $\sin(10^\circ) = \cos(2 \times 40^\circ) = -\cos(2 \times 50^\circ) \sim 1/6$. Thus, we expand the averaged probabilities around the point $\theta_{13} = 0$ and $\theta_{23} = 45^\circ$ (namely, where the two small parameters are zero) neglecting cubic terms and finding:

$$P_{ee} = 1 - \frac{x}{2} - 2w, \quad P_{\mu\tau} = \frac{1}{2} - \frac{x}{8} - z$$

$$P_{\mu\mu} = \frac{x}{4} + y + w, \quad P_{ee} = \frac{1}{2} - \frac{x}{8} - y - w + z$$

(A.13)

$$P_{\tau\tau} = \frac{x}{4} - y + w, \quad P_{\tau\tau} = \frac{1}{2} - \frac{x}{8} + y - w + z$$

We introduced for convenience various quantities of different size,

order 1: $x \equiv \sin^2(2\theta_{13})$  \hspace{2cm} (A.14)

linear: $y \equiv \frac{1}{4} \sin \theta_{13} \cos \delta \sqrt{x(1-x)} + \cos(2\theta_{23})x$  \hspace{2cm} (A.15)

quadratic: $w \equiv \sin^2 \theta_{13} (1-x/2)$  \hspace{2cm} (A.16)

quadratic: $z \equiv \frac{1}{2} \sin^2 \theta_{13} (1+\cos(2\delta)x/2) + \cos^2(2\theta_{23})x$  

$(1-x/4) - \sin \theta_{13} \cos(2\theta_{23}) \cos \delta \sqrt{x(1-x)}$  \hspace{2cm} (A.17)

the linear expansion has been obtained already in [16]. Several remarks are in order:

(i) By summing $x$, $y$, $w$ or $z$ in any row or column of the symmetric matrix with elements $P_{ell'}$ one gets zero; this is a consequence of unitarity.

(ii) The unique linear term $y$ involves a weighted sum of $\sin \theta_{13}$ and $\cos(2\theta_{23})$; this implies that there is a strong correlation between the linear variation of $P_{\mu\mu}$ and $P_{\mu\tau}$, while $P_{ee}$ and $P_{\tau\tau}$ do not vary at this order. Note that due to the coefficients, the impact of $\cos(2\theta_{23})$ is larger than the one of $\sin \theta_{13}$.

(iii) The relative size of $y$ and $z$ depends on $\delta$; the latter can be larger than the former (when $\delta \sim \pi$) even if these two terms are formally of different orders.

These expressions can be useful for some purposes, e.g., to understand the correlations among the probabilities, however they are not much simpler than the full expressions. Moreover, the a priori limits $1 \geq P_{ee} \geq 1/3$ and $1/2 \geq P_{\tau\tau} \geq 0$ (when $\ell \neq \ell'$) and the specific values of the parameters of oscillations imply that $P_{\mu\tau}$ is close to the physical boundary [2]; in other terms, the non-linear terms have a certain role, as consistent with the previous last considerations. For these reasons, we decided to use the full expressions of $P_{ell'}$ for the above analysis.

| $\ell$ | $P_{\mu\tau}$ | $P_{ee}$ | $P_{\mu\mu}$ | $P_{\tau\tau}$ |
|-------|---------------|----------|-------------|--------------|
| $P_{\mu\tau}$ | $0.33^{+0.02}_{-0.01}$ | $0.33^{+0.01}_{-0.00}$ | $0.29^{+0.03}_{-0.02}$ | $0.25^{+0.05}_{-0.05}$ |

| $\ell$ | $\delta P_{\mu\tau}/P_{\mu\tau}$ |
|-------|-------------------------------|
| $P_{\mu\tau}$ | $11\%$ | $4\%$ | $3\%$ | $6\%$ | $18\%$ |

Table A.1

Value of the effective probability of oscillation, Eq. A.10, and of its error, for various values of the flavor ratio $P_{\mu\tau}/P_{ee}$.

References

[1] For the references to the original literature, see the book of M.A. Markov, The Neutrino, 1963, Dubna preprint D-1269, chapt. 6. The first work on the subject is attributed to the master thesis of his student, I. Zheleznykh (1957).

[2] F.L. Villante, F. Vissani, Phys. Rev. D 78 (2008) 103007.

[3] F. Vissani, Astropart. Phys. 26 (2006) 310.

[4] P. Lipari, Nucl. Instrum. Meth. A567 (2006) 405. M.D. Kistler, J.F. Beacom, Phys. Rev. D74 (2006) 063007.

[5] S.R. Kelner, F.A. Aharonian, V.V. Bugayov, Phys. Rev. D74 (2006) 034018.

[6] F. Vissani, F. Aharonian, N. Sakayan, Astropart. Phys. 34 (2011) 778.

[7] F. Aharonian, L. O'Drury, H.J. Völk, Astron. Astroph. 285 (1994) 645; L. O'Drury, F. Aharonian, H.J. Völk, Astron. Astroph. 287 (1994) 959.

[8] F. Vissani, G. Pagliaroli, F.L. Villante, N. Cim. C32 (2009) 353.

[9] E. Pfeffermann and B. Aschenbach, in Röntgenstrahlung from the Universe, Intern. Conf. of X-ray A&A, MPE Report 263 (Zimmermann, Trumper & Yorke ed.) p.267

[10] Z.-R. Wang, Q.-Y. Qu, Y. Chen, A&A 318 (1996) L59.

[11] H. Sano et al., Astrophys. J. 724 (2010) 59.

[12] Y. Fukui et al., arXiv:1107.6508.

[13] F. Aharonian et al. [HESS Collaboration], Astron. Astrophys. 464 (2007) 235.

[14] F.L. Villante, F. Vissani, Phys. Rev. D 76 (2007) 125019.

[15] J. Alvarez-Muniz, F. Halzen, Astrophys. J. 576 (2002) L33.

[16] M.L. Costantini, F. Vissani, Astropart. Phys. 23 (2005) 477.

[17] A.A. Abdo et al., ApJ 734 (2011) 28.

[18] D.C. Ellison, D.J. Patnaude, P. Slane, J. Raymond, ApJ, 712 (2010) 287.

[19] V.N. Zirakashvili and F.A. Aharonian, Ap. J. 708 (2010) 965.

[20] F. Vissani, Proc. of the Vulcano Workshop 2006, edited by F. Giovannelli and G. Mannocchi (SIF, Bologna), Vol. 93, p. 599.

[21] S. Razzzaque, talk at NuSKY workshop, July 2011, ICTP, Trieste.

[22] L. Anchordoqui, F. Halzen, T. Montaruli, A. O’Murchadha, Phys. Rev. D76 (2007) 067301.

[23] R.M. Crocker, F. Aharonian, Phys.Rev.Lett. 106 (2011) 101102.

[24] V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493.

[25] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, arXiv:1106.6028 [hep-ph].

[26] T. Schwetz, M. Tortola, J.W.F. Valle, arXiv:1108.1376 [hep-ph].

[27] P. D. Serpico and M. Kachelriess, Phys. Rev. Lett. 94, 211102 (2005). Compare also with next references.

[28] P. Bhattacharjee and N. Gupta, hep-ph/0501191. W. Winter, Phys. Rev. D 74, 033015 (2006). P.D. Serpico, Phys. Rev. D 73, 047301 (2006). W. Rodejohann, JCAP 0701, 029 (2007).