Review of Quantum Chromodynamics (QCD)

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Abstract

This is a review of the elementary particles, quantum chromodynamics (QCD), and strong interactions in QCD theory via gluon exchange between quarks-antiquarks-producing mesons. Some mesons consist of an active gluon in addition to a quark-antiquark. They are called hybrid mesons. We also review the possible detection of the quark-gluon plasma, the constituent of the universe until about $10^{-4}$ s after the Big Bang, via relativistic heavy ion collisions (RHIC) producing heavy quark hybrid mesons.

Keywords: quantum chromodynamics, elementary particles, quark-gluon plasma

1. Introduction

In this review of quantum chromodynamics (QCD), basic QCD particles and forces, the theory of hybrid mesons, the cosmological quantum chromodynamics phase transition (QCDPT), the quark-gluon plasma (QGP), theoretical studies of jet quenching due to the formation of the QGP, and the possible detection of the QGP via the production of mixed hybrid meson states produced by RHIC are reviewed.

First we review elementary particles, fermions and bosons, and standard QCD theory with the concept of color for the basic fermions (quarks) and bosons (gluons).

In the next section, the structure of standard mesons in terms of quarks and antiquarks is discussed.

Next we review the theory predicting that some mesons consisting of heavy charmonium quarks ($\Psi(nS)$, $n = 1, 2$) and bottomonium quarks ($\Upsilon(mS)$, $m = 1, 2, 3, 4$) are partially hybrids. Experiments test the theory.

The interactions in quantum chromodynamics are strong, so perturbation theory does not work. Therefore, Feynman diagrams used for quantum electrodynamics cannot be used for quantum chromodynamics [1].

One nonperturbative QCD method involves lattice gauge theory. The articles “Twenty-first Century Lattice Gauge Theory: Results from the QCD Lagrangian” by Kronfeld [2] and “Lattice gauge theory in the microcanonical ensemble” by Calloway and Rahman [3] give a detailed description of how lattice gauge theory can calculate QCD interactions using computers.

Another nonperturbative theory, which is used in our review of mixed hybrid heavy quark mesons, is the method of QCD sum rules. This method, developed by Shifman et al. [4], does not require large computers.
It was the method of QCD sum rules that showed [5] that the $\Psi(2S)$ charmonium quark meson and the $\Upsilon(3S)$ bottomonium quark mesons are mixed hybrid states, while all the other $\Psi(nS)$ and $\Upsilon(mS)$ are standard charmonium and bottomonium meson states.

In our final section, we review the early universe QCDPT, with the production of the QGP.

In our second subsection, theoretical studies of jet quenching due to the formation of the QGP and experiments with Pb-Pb collisions verifying the theory are discussed.

In our final section, the possible production of the QGP via relativistic heavy ion collisions (RHIC) with the possible detection of the QGP by the production of mixed heavy quark hybrid mesons is reviewed. That is, we consider the collision of gold (Au) atomic nuclei with the energy of the Au nuclei large enough that after the Au-Au collision the temperature of the overlapping material is $T \geq T_{QCDPT}^{c} \simeq 150$ MeV, where $T_{QCDPT}^{c}$ is the critical temperature for the QCDPT. That matter with $T \geq T_{QCDPT}^{c}$ is QGP is discussed.

The production of the mixed hybrid states $\Psi(2S)$ and $\Upsilon(3S)$ via Au-Au collisions could detect the production of the QGP in the overlapping material. We conclude that the detection of $\Psi(2S)$ and $\Upsilon(3S)$ produced via RHIC could be a test for the creation of the early universe QGP.

2. Elementary particles and basic forces

Among the elementary particles—fermions and bosons—fermions have quantum spin = 1/2.

The elementary fermions are leptons and quarks. There are three generations of leptons: electron, muon, and tau, with electric charge $-1$, and their neutrinos with no electric charge. There are three generations of quarks: (u, d); (c, s); and (t, b). The (u, c, t) quarks have electric charge 2/3 while the (d, s, b) quarks have electric charge $-1/3$.

Bosons have quantum spin = 1: photon, quantum of the electromagnetic field; gluon, quantum of the strong field; and W and Z, weak field quanta, which we do not need.
2.1 Quantum chromodynamics (QCD): strong interaction field theory

Strong interactions are produced by quarks exchanging gluons, as illustrated in the figure below.

Color: Quarks have three colors. Color is to the strong interaction as electric charge is to the electromagnetic interaction.

A quark and antiquark can form a gluon, which has color 8.

Note that particles with color, like gluon and quarks, cannot move freely in space. Particles that can move freely are baryons, like the proton and neutron, and mesons, which have no total color.

Note that in the figure above, the quark and antiquark have a different color, which is why the gluon has color = 8.

Antiparticles: All fermions have antiparticles. The antiparticle of the electron \( e^- \) is the positron \( e^+ \), and a photon can create a \( e^- e^+ \) pair. Similarly, each quark \( q \) has an antiquark \( \bar{q} \).

Standard mesons consist of a quark and an antiquark. A meson that is important for today’s discussion is the \( J/\Psi(1S) \).

\[
|J/\Psi(1S)\rangle = |c\bar{c}(1S)\rangle.
\]  

(1)

As we discuss below, the first excited \( c\bar{c} \) state, the \( \Psi(2S) \) has a hybrid component with an active gluon: \( |c\bar{c}g(2S)\rangle \).

3. QCD sum rules and mixed heavy quark hybrid meson states

The starting point of the method of QCD sum rules [4] is the correlator

\[
\Pi^A(x) = \langle [T[ J_A(x) J_A(0)]],
\]  

(2)
with $|\rangle$ the vacuum state and the current $J_A(x)$ creates the states with quantum numbers $A$. For the charmonium states, $J_c$ is

$$J_c = f J_{c\sigma} + \sqrt{1 - f^2} J_{c\bar{g}},$$  \hspace{1cm} (3)$$

where $J_{c\sigma}$ creates a normal charmonium state and $J_{c\bar{g}}$ creates a hybrid state with an active gluon. It was shown that $f \approx -\sqrt{2}$ for the $\Psi(2S)$ and $\Upsilon(3S)$ and $f \approx 1.0$ for the other charmonium and bottomonium states [5]. Therefore,

$$|J/\Psi(1S)> \approx |c\bar{c}(1S)>$$

$$|\Psi(2S)> \approx -\sqrt{2}|c\bar{c}(1S)> + \sqrt{2}|c\bar{g}(2S)>$$

$$|\Upsilon(3S)> \approx -\sqrt{2}|b\bar{b}(3S)> + \sqrt{2}|b\bar{g}(3S)>,$$  \hspace{1cm} (4)$$

That is $J/\Psi(1S)$ is a standard charmonium meson and all of the $\Upsilon(nS)$ mesons are standard bottomium mesons except for $n = 3$.

3.1 Experimental verification that the $\Psi(2S)$ and $\Upsilon(3S)$ are mixed hybrid heavy quark mesons

$\Psi$ and $\Upsilon$ production by Cu-Cu collisions for $E = 200$ GeV [6] are shown in Figures 1–4. The dashed curves are for the standard model.

Tests of the mixed hybrid theory for $\Psi(2S)$ and $\Upsilon(3S)$ states using ratios of cross sections for Cu-Cu collisions at $E = 200$ GeV: Since the absolute magnitude of $d\sigma/dy$ for production of $\Psi(2S)$ states via Cu-Cu collisions is not certain, due to uncertainty in the normalization of the states, the tests of the theory [6] were carried out using ratios of cross sections, which can be compared to experiments.

From Figures 1 and 2, the ratios of $\Psi(2S)$ to $J/\Psi(1S)$ for the standard model (st) and the mixed hybrid theory (hy) for A-A (including Cu-Cu) collisions are

$$\begin{array}{c}
\text{Figure 1.} \\
d\sigma/dy \text{ for } E = 200 \text{ GeV Cu-Cu collisions producing } J/\Psi.
\end{array}$$
Figure 2.
\( \frac{d\sigma}{dy} \) for \( E = 200 \) GeV Cu-Cu collisions producing \( \Psi(2S) \).

Figure 3.
\( \frac{d\sigma}{dy} \) for \( E = 200 \) GeV Cu-Cu collisions producing \( \Psi(1S) \).

Figure 4.
\( \frac{d\sigma}{dy} \) for Cu-Cu collisions producing \( \Psi(2S), \Psi(3S) \).
\[
\sigma(\Psi(2S))/\sigma(J/\Psi(1S))_{st-A-A} \approx 0.27 \\
\sigma(\Psi(2S))/\sigma(J/\Psi(1S))_{hy-A-A} \approx 0.52 \pm 0.05,
\]

while the experimental result [7] is

\[
\sigma(\Psi(2S))/\sigma(J/\Psi(1S)) \approx 0.59,
\]

which shows that the mixed hybrid theory for the \(\Psi(2S)\) state is consistent with experiment, while the standard \(|\Phi(2S)\rangle\) is not. From studies of heavy quark state production in p-p collisions, theoretical results for the nature of \(\Upsilon(3S)\) state found [8]

\[
\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S)) \approx 0.04 \text{ standard} \\
\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S)) \approx 0.147 - 0.22 \text{ hybrid}
\]

compared to the experimental result of about 0.12–0.16 [9]. Therefore, the \(\Upsilon(3S)\) state, as well as the \(\Psi(2S)\) state, has been shown to be a heavy quark mixed hybrid state.

4. The cosmological QCDPT and possible detection of the QGP

In this section we first review the temperature of the universe based on the time when the cosmological quantum chromodynamic phase transition (QCDPT) occurred. At that time, the universe consisted of the quark-gluon plasma (QGP). Then we discuss the creation of the QGP via RHIC Au-Au collisions with the possible detection of the QGP.

4.1 The quantum chromodynamic phase transition (QCDPT)

From astrophysical studies, it is known that the QCDPT occurred at a time \(t \approx 10^{-5} - 10^{-4} \text{ s}\); one can estimate the temperature of the universe at time \(T(t)\) using the equations from Einstein’s General Theory of Relativity.

A simpler form of Einstein’s equations are Friedman’s equations. From Friedman’s equations, one can find an approximation to \(T(t)\) [10].

\[
T(t) \approx \frac{1 \text{ MeV}}{\sqrt{t \text{ (in s)}}}.
\]

![Diagram of QGP and Hadron Phase](image)

Figure 5. 
Hadron phase forming within the QGP during the QCDPT.
From this, one finds, using $t \approx 5 \times 10^{-5} \text{s}$, at the time of the QCDPT the critical temperature for the QCDPT $T_{c}^{\text{QCDPT}} \approx 150 \text{MeV}$. During the time that $T = T_{c}^{\text{QCDPT}}$, bubbles of our universe nucleated within the QGP, as shown in Figure 5.

4.2 Theoretical studies and predictions for the detection of the QGP

Theoretical studies of jet quenching due to the formation of the QGP were carried out. Theoretical studies of jet quenching due to the formation of the QGP in high-energy Pb-Pb collisions [11] help motivate experimental studies. The theoretical equation predicting jet quenching [11] is

$$\Delta_{\text{QGP}} \approx 1 - e^{-\frac{\Theta_{\text{jet}}}{\Theta_{c}}},$$

where $\Delta_{\text{QGP}}$ is the magnitude of the jet quenching and $\Theta_{\text{jet}}$ and $\Theta_{c}$ are the aperture of the jet and emitted gluons, respectively.

Motivated in part by the theoretical study, the CMS collaboration carried out a study [12] of jet quenching via jet+$Z$ boson correlations in Pb-Pb collisions. The results are shown in Figure 6.

As can be seen from the figure and Eq. (9), the theoretical prediction of jet quenching due to the QGP has been verified by experiments.

4.3 Creation and detection of the QGP via RHIC

A main goal of the study of heavy quark state production in relativistic heavy ion collisions (RHIC) is the detection of the quark-gluon plasma [13]. The energy of the atomic nuclei must be large enough so just after the nuclei collide, the temperature is that of the universe about $10^{-5} \text{s}$ after the Big Bang, when the universe was too hot for protons or neutrons and consisted of quarks and gluons (the constituents of proton and nucleons)—the quark-gluon plasma (QGP).

As Figure 7 illustrates for Au-Au collisions with sufficient energy that the temperature $T(t) \geq 150 \text{MeV}$ where the Au-Au nuclei merge, the emission of mixed

![Figure 6.](image)

$x = (\text{jet}/Z) \text{ momentum vs. } p^Z = \text{the momentum of the Z boson for } p-p \text{ and Pb-Pb collisions.}$
hybrid mesons, the $\Psi(2S)$ and $\Upsilon(3S)$ as discussed above, with active gluons, could be a signal of the formation of the QGP.

5. Conclusions

The results from comparison of the production of $\Psi(2S)$ and $\Upsilon(3S)$ differential cross sections with experiment confirm the theoretical prediction that the $\Psi(2S)$ and $\Upsilon(3S)$ states are mixed hybrid states.

There are tests of the creation of the QGP from jet quenching via experiments using Z-jet correlations, as well as other tests suggested by theoretical studies.

There are also possible tests of the creation and detection of the QGP by the production of $\Psi(2S)$ and $\Upsilon(3S)$ via relativistic heavy ion collisions as shown in Figure 7.

From this, one can conclude that the production of these states via RHIC with sufficient energy that part of the matter during the collision has reached a temperature $T \geq T_{QGP}^{\text{crit}} \approx 150$ MeV, the temperature when the universe was a dense plasma (the QGP), can be a test of the creation of the quark-gluon plasma.

This would be an important result for particle theory as well as astrophysics.

Acknowledgements

Leonard S. Kisslinger acknowledges support in part as a visitor at the Los Alamos National Laboratory,
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