The production of top-antitop pairs at the Fermilab Tevatron shows a forward-backward asymmetry in which the top quark tends to follow the proton direction, while the antitop tends to follow the antiproton direction. The effect grows with increasing effective mass $m_{t\bar{t}}$ of the top-antitop pair, and with increasing rapidity difference between the top and antitop. The observed effect is about three times as large as predicted by next-to-leading-order QCD, but with the same sign. An estimate of non-factorizable effects based on a QCD string picture finds they are negligible, but that small distortions of the $m_{t\bar{t}}$ spectrum are possible. Tests for such effects, both at and above the level of this estimate, are suggested.

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I INTRODUCTION

The top quark has been shown to be produced at the Fermilab Tevatron with a noticeable forward-backward asymmetry [1, 2, 3] in excess of QCD predictions [4]. This has spawned a number of possible interpretations in terms of new physics [5].

In the present paper we explore a different possibility. According to the factorization hypothesis, the subprocess $q\bar{q} \rightarrow t\bar{t}$ is assumed to take place in isolation from the remaining constituents of the incident proton and antiproton. We investigate using a QCD string picture whether the tendency for the top quark to follow the direction of the proton, and for the antitop to follow the antiproton, may be due to nonperturbative QCD effects violating factorization. We find an effect much too small to account for the observed asymmetry, but note the possibility of small distortions of the $m_{t\bar{t}}$ spectrum. Such distortions are less likely at the CERN Large Hadron Collider (LHC), where the dominant top pair subprocess is $gg \rightarrow t\bar{t}$. Other tests are suggested for non-factorizable effects.

We begin by reviewing the data and QCD predictions (Sec. II). We then examine in Sec. III a source of non-factorizable effects due to the formation of QCD strings between the produced top or antitop and remaining $p$ or $\bar{p}$ constituents. The effects are found to be very small as a result of the rapid top quark decay. Qualitative tests for other manifestations of the breakdown of factorization in hadronic $t\bar{t}$ production include distortions in top reconstruction (Sec. IV), enhanced multiparticle production between the $t$ and $p$ remnants or the $\bar{t}$ and $\bar{p}$ remnants (Sec. V) and a forward-backward or charge asymmetry in baryon/antibaryon production (Sec. VI). We conclude in Sec. VII.
II DATA AND QCD PREDICTION

A top or antitop quark produced by $p\bar{p}$ reactions at the Tevatron may be characterized by its rapidity $y \equiv (1/2) \ln[(E + P_z)/(E - P_z)]$, where $E$ is its energy and $P_z$ the projection of its momentum along the beam axis. One may define $\Delta y \equiv y_t - y_{\bar{t}}$; this variable is invariant under boosts along the $z$ axis and has the same sign as $\cos \theta^*$, where $\theta^*$ is the angle the top quark makes with the $+z$ direction (taken to be that of the proton) in the $t\bar{t}$ center of mass system (c.m.s.) The forward-backward asymmetry in top production at the parton level then may be defined as

$$A_{FB} \equiv \frac{N_{\Delta y>0} - N_{\Delta y<0}}{N_{\Delta y>0} + N_{\Delta y<0}}.$$  \hspace{1cm} (1)

For the Tevatron running at $\sqrt{s} = 1.96$ GeV, the theoretical prediction at non-leading QCD order and including electroweak corrections \[4\] is $A_{FB} = 6.6\%$. The most recent experimental data are:

$$A_{FB} = \begin{cases} 
(16.2 \pm 4.7\%) \hspace{1cm} \text{[2]} \\
(19.6 \pm 6.5\%) \hspace{1cm} \text{[3]}
\end{cases}$$  \hspace{1cm} (2)

A linear growth of the asymmetry as a function of $\Delta y$ or $M_{t\bar{t}}$ is observed. This behavior is also predicted by theory, but the slopes of the effect are observed to be about three times the prediction.

At the LHC, the incident symmetric $pp$ collisions imply no forward-backward asymmetry, but a charge asymmetry $A_C$ between $t$ and $\bar{t}$ is still possible at next-to-leading order (NLO) in QCD, where

$$A_C \equiv \frac{N_{\Delta |y|>0} - N_{\Delta |y|<0}}{N_{\Delta |y|>0} + N_{\Delta |y|<0}},$$  \hspace{1cm} (3)

where $\Delta |y| \equiv |y_t| - |y_{\bar{t}}|$. The observed values are $A_C = -0.018 \pm 0.028 \pm 0.023$ (ATLAS \[6\]), $A_C = -0.013 \pm 0.028^{+0.029}_{-0.031}$ (CMS \[7\]), and $A_C = 0.004 \pm 0.010 \pm 0.012$ (CMS \[8\]). The theoretical prediction \[9\] is $A_C = 0.0115 \pm 0.0006$ (ATLAS \[6\] quotes a slightly different value of $0.006 \pm 0.002$.)

III STRING-FRAGMENTATION PICTURE

The mirroring of the qualitative behavior of $A_{FB}$ for top pair production in $p\bar{p}$ collisions by that of perturbative QCD suggests a possible origin in nonperturbative effects. One such mechanism is illustrated in Fig. 1.

At the Tevatron, the dominant subprocess in top-antitop production is $q\bar{q} \rightarrow t\bar{t}$, with quarks $q$ dominantly coming from the proton and $\bar{q}$ from the antiproton. A proton giving up a quark is left as a color antitriplet remnant, while an antiproton giving up an antiquark is left as a color triplet remnant.

A top quark is expected to decay via $t \rightarrow bW^+$ with a lifetime of about $5 \times 10^{-25}$ s \[10\]. The top or its daughter $b$ can interact during the initial stages of the fragmentation process with the remnant of either the proton or the antiproton; either interaction is expected to affect its angular distribution. Within the factorization hypothesis, this interaction is expected to be small and to be successfully described by perturbative QCD corrections at next-to-leading order.
Figure 1: Formation of QCD strings (dashed lines) between $t$ or $b$ (3 of SU(3)$_C$) and proton remnant (3$^*$ of SU(3)$_C$), and between $\bar{t}$ or $\bar{b}$ (3$^*$) and antiproton remnant (3).

A one-gluon exchange potential has a definite form between two color triplets in a 6 or 3$^*$ representation of color SU(3), or a triplet and antitriplet in an octet or singlet [11, 12, 13]:

\[
\begin{align*}
V_{33;3^*} &= -\frac{2}{3} \frac{\alpha_S}{r}; \\
V_{33;6} &= \frac{1}{3} \frac{\alpha_S}{r}; \\
V_{33^*;1} &= -\frac{4}{3} \frac{\alpha_S}{r}; \\
V_{33^*;8} &= \frac{1}{6} \frac{\alpha_S}{r}.
\end{align*}
\]

Thus, the most attractive force occurs between a color triplet and an antitriplet in an overall singlet of color SU(3). This hierarchy also appears to hold for the longer-range force, described by a linear potential. The slope $k$ of such a potential $V(r) = kr$ between a triplet and an antitriplet in the color singlet of SU(3) is related to the (approximately universal) slope $\alpha'$ of Regge trajectories relating masses and spins of mesons or baryons:

\[
\alpha(M^2) = \alpha_0 + \alpha' M^2, \quad \alpha' \simeq 0.88 \text{ GeV}^{-2}, \quad (4)
\]

with $k = (2\pi\alpha')^{-1} \simeq 0.18 \text{ GeV}^2$.

A QCD string between a heavy quark $Q$ and antiquark $\bar{Q}$ has been shown to break above a separation of about 1.5 fm = 7.5 GeV$^{-1}$ [16] due to production of a light $q\bar{q}$ pair corresponding to flavor threshold:

\[
Q\bar{Q} \rightarrow (Q\bar{q}) + (\bar{Q}q). \quad (5)
\]

Let us assume that this string is imparting an impulse in the direction of the proton or antiproton remnant (i.e., along the beam axis) for the lifetime of the top quark ($\tau_t = 5 \times 10^{-25}$ s). This is about a tenth of the time it would take the string to break if its ends separated at the velocity of light to the critical distance of 1.5 fm:

\[
\frac{1.5 \times 10^{-15} \text{ m}}{3 \times 10^8 \text{ (m/s)}} = 5 \times 10^{-24} \text{ s}. \quad (6)
\]

The average impulse imparted to the top quark along the beam axis is then ($c = 1$)

\[
\Delta p_{tz} = k\tau_t = \frac{(0.18 \text{ GeV}^2)(5 \times 10^{-25} \text{ s})}{6.582 \times 10^{-25} \text{ GeV} \cdot \text{s}} \simeq 0.14 \text{ GeV}, \quad (7)
\]

far too small to cause any noticeable forward-backward asymmetry. However, the top quark decays to a bottom quark, whose motion may be affected enough to cause a noticeable systematic shift in the reconstructed top quark mass, as we show in the next Section.
IV EFFECTS ON TOP RECONSTRUCTION

At the Tevatron, the top quark is produced in a $t\bar{t}$ pair with a $m(tt)$ distribution peaked below 400 GeV \[1, 2, 3\]. We shall thus neglect its motion with respect to the $t\bar{t}$ center of mass in calculating the effect of “string drag” on its colored decay product, the bottom quark.

The top quark decays via $t \rightarrow bW^+$ with a mean proper lifetime of $5 \times 10^{-25}$ s. The ensuing $b$ quark carries its color and hence is subject to the same drag force from the initial proton’s remnant. It is acted upon by this force for the time it takes the QCD string to break (i.e., to elongate to its critical length of 1.5 fm), or an average of $5 \times 10^{-24}$ s as noted above. The momentum imparted to the $b$ quark is thus about $\Delta p_b \simeq 1.4$ GeV. A detailed calculation of the corresponding effect on reconstructed top quark mass would involve adding $\Delta p_b \simeq 1.4$ GeV to every $b$ quark produced in a Monte Carlo simulation of $t \rightarrow bW^+$. However, the following simplified estimate captures the essence of the effect.

Top quark pairs are produced in $\bar{p}p$ collisions at the Tevatron with a distribution in rapidity $y$ peaked around $y = 0$ with typical width $|\Delta y| \simeq 1$. We shall consider pairs produced with $y = 0$ and shall neglect the motion of each top quark with respect to the pair’s center of mass. We shall thus consider a top quark at rest decaying to $bW^+$, where the $b$ is emitted with an angle $\theta^*$ with respect to the beam axis. With

\[ m_t = 173 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad M_W = 80.385 \text{ GeV}, \quad (8) \]

we find the momenta and energies of the $b$ and $W$ in the top rest frame to be

\[ |\vec{p}_b| = |\vec{p}_W| = 67.7 \text{ GeV}, \quad E_b = 67.9 \text{ GeV}, \quad E_W = 105.1 \text{ GeV}. \quad (9) \]

An impulse $\Delta p_{zb}$ along the proton beam axis causes a change

\[ \Delta m_t = \frac{|\vec{p}_b|}{E_b} \Delta p_{zb} \cos \theta^* \quad (10) \]

in the reconstructed value of the top quark mass, to lowest order in $\Delta p_{zb}$. The shift in the reconstructed top quark mass is thus very close to the impulse imparted by the string drag, as the velocity $\beta_b = |\vec{p}_b|/E_b$ of the $b$ quark in the $t$ center of mass is very nearly 1.

Such an effect, while small, could lead to systematic shifts in reconstructed top quark masses, depending not only on the angle of emission of the $b$ quark but on whether the recoiling $W$ decays to two jets or a lepton pair. We urge trial analyses in which arbitrary small impulses $\Delta p_{zb}$ are added to the $b$ quarks in top quark decays, to see if such impulses lead to consistent top mass reconstruction. These effects should be less pronounced at the LHC, where top quark production occurs mainly through $gg \rightarrow t\bar{t}$, leaving proton remnants of zero triality and hence symmetric interaction with $t$ and $\bar{t}$.

V EFFECTS ON UNDERLYING EVENT

The strings in Fig. \[1\] connecting the $b$ quark with the proton remnant and the $\bar{b}$ with the antiproton remnant will undergo further fragmentation as they stretch, in the manner of a string between a quark and an antiquark in $e^+e^- \rightarrow q\bar{q}$ \[17, 18\]. The products of this fragmentation will have limited transverse momenta with respect to the string axis. The underlying event in top pair production thus may exhibit non-uniformity in the distribution
of hadrons in an azimuthal angle $\phi$ around the beam axis, where $\phi = 0$ is defined by the plane containing the beam axis and the $b$ quark’s direction. At the Tevatron, the fragments should be more concentrated in $\phi$ for $b$ quarks making smaller angles with the proton.

VI FORWARD BARYON PRODUCTION

If a heavy quark tends to be produced in the direction of the incident baryon, as suggested in Fig. 1, one might expect to see more $\Lambda_b$ than $\bar{\Lambda}_b$ in the direction of the proton beam, and more $\bar{\Lambda}_b$ than $\Lambda_b$ in the direction of the antiproton beam, at the Tevatron. At the LHC, one should see the ratio of $\bar{\Lambda}_b$ to $\Lambda_b$ production decrease for large $|y|$. The CMS Collaboration finds no significant deviation of this ratio from a constant in the central region $|y^{\Lambda_b}| < 2$, though the ratio for $1.5 \leq |y^{\Lambda_b}| \leq 2$ of 0.67$^{+0.16}_{-0.08}$ is consistent with a mild decrease in the largest $|y|$ bin 19. The LHCb Collaboration is in an ideal position to extend this measurement to larger $|y|$, where a string-drag picture would predict a growing predominance of $\Lambda_b$ over $\bar{\Lambda}_b$ with increasing $|y|$.

VII CONCLUSIONS

The forward-backward asymmetry of top production at the Fermilab Tevatron remains a mystery in the standard model. We have examined whether a breakdown of the factorization assumption could lead the $t$ to preferentially follow the incident proton, perhaps as a result of unexpectedly large nonperturbative QCD effects. In a model in which a QCD string preferentially connects the $t$ to the proton remnants, it was shown that only a very small momentum is imparted to the top quark before it decays, and even the momentum imparted to its daughter $b$ quark by the QCD string is only about 1.4 GeV/c before the string breaks. While this is unlikely to account for any notable forward-backward asymmetry in top production, it could lead to small systematic shifts in the top quark mass reconstructed by CDF and D0, depending on the angle between the $b$ quark and the proton beam. We urge a systematic search for such effects.

Other symptoms of a “string-drag” effects on heavy quark production include azimuthal asymmetries of the underlying event in top pair production and a tendency of heavy baryons to be favored over antibaryons in the proton direction at the Tevatron.

While top pair production is dominated by initial $q\bar{q}$ states at the Tevatron, it is mainly governed by gluon-gluon collisions at the LHC, so the picture of Fig. 1 does not apply there. Nonetheless, in the the very forward direction ($|y| > 1.5$) at the LHC, one also expects heavy baryon production to be favored over antibaryon production. A systematic study of how this effect varies from $\Lambda_c$ to $\Lambda_b$ could help shed light on whether a similar charge asymmetry arises for top production.

Two recent observations relate to the possible enhancement of $A_{FB}$ within QCD. (1) Appreciable forward-backward asymmetries in top production can arise when the leading-order prediction (with no asymmetry) is supplemented with fragmentation in several Monte Carlo approaches 20. (2) Setting a renormalization scale using a prescription known as the Principle of Maximal Conformality 21 improves agreement between experiment and QCD prediction for $A_{FB}$. 
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