Real time ultrasoft fermion self energy at next to-leading order in hot QED

Karima Bouakaz¹* and Abdessamad Abada²
¹Department of Physics, Laboratoire de Physique des Particules et de Physique Statistique, Ecole Normale Supérieure, BP 92 Vieux-Kouba, Algiers.
²Department of Physics, United Arab Emirates University, P. O. B. 17551, Al Ain, United Arab Emirates.

* kbouakaz@yahoo.fr

Abstract. Subsequent studies of the behavior of the gluon and quark damping rates in the imaginary-time formalism have indicated that there are difficulties in the infrared sector [1, 2, 3, 4, 5, 6, 7]. To look further into the infrared behavior, we propose to calculate the next-to-leading order dispersion relations for slow-moving Fermions at high-temperature quantum electrodynamics (QED) in real-time formalism. We determine a compact analytic expression for the complete next-to-leading contribution to the retarded fermion self-energy with ultrasoft momentum in the framework of hard-thermal-loop (HTL) -summed perturbation of massless QED at high temperature. The calculation is done using real-time formalism. The next-to-leading order fermion self-energy is written in terms of three and four HTL-dressed vertex functions. The real part and the opposite of the imaginary part of the retarded fermion self-energy are related to the next-to-leading order contributions of energy and damping rate respectively.

1. Introduction
One runs into difficulties when one applies standard perturbation theory to gauge theories at high temperature: physical quantities become gauge dependent [8]. The problem was resolved in [9], which showed that in order to calculate consistently at high temperature, we have to use an effective perturbation that sums the so-called hard thermal loops (HTL) into dressed propagators and vertices [9, 10]. In this framework, the zero-momentum transverse gluon damping rate was shown to be finite and positive [11]. However, subsequent studies of the behavior of the gluon and quark damping rates in the imaginary-time formalism have indicated that there are difficulties in the infrared sector [1-7]. A similar observation has been done in the context of scalar electrodynamics [12]. Following this logic, the natural step forward is to try to calculate the NLO energies of the quasiparticles using the real-time formalism. With this in mind, we have determined the next-to-leading contributions to the longitudinal gluon and retarded quark self-energy in the context of HTL-summed perturbation of massless QCD at high temperature using real time formalism [13,14,15].
In the present communication, we show the main steps leading to the determination of an analytic expression for the next-to-leading contribution to the retarded fermion self-energy in massless QED at high temperature using the real-time formalism of finite-temperature quantum field theory [16, 17].

2. Effective expansion

The HTL-summed fermion propagator is defined as:

$$\Delta_f(P) = i(P - \partial\Sigma_{\text{HTL}}(P))^{-1},$$

where $\partial\Sigma_{\text{HTL}}(P)$, the HTL contribution to the fermion self-energy, is obtained by summing the HTL fermion self-energy diagrams, which are given by:

$$\Sigma_{\text{HTL}}(P, Q) = \frac{-m_f^2}{4\pi} \int d\Omega \frac{K}{QK + i\epsilon},$$

$$\Sigma_{\text{HTL}}(P, Q) = \frac{-m_f^2}{4\pi} \int d\Omega \frac{K}{QK - i\epsilon}. $$

(2)

In these two expressions $m_f^2 = \frac{1}{8}e^2T^2$ is the square fermion thermal mass, $T$ is the temperature and $\epsilon$ is the small coupling constant.

In the Landau gauge, the HTL-summed photon propagator is given by the following expression:

$$\Delta_{\gamma}^\text{HTL}(k) = P_{\gamma}^{-1} \frac{1}{\partial_{\gamma}^\text{HTL} - k^\mu + i\text{sgn}(\eta_k)k^\mu} + P_{\gamma}^+ \frac{1}{\partial_{\gamma}^\text{HTL} + k^\mu + i\text{sgn}(\eta_k)k^\mu}$$

(3)

where $P_\gamma^- : P_\gamma^+$ are the usual transverse and longitudinal projectors respectively, and we have:

$$\partial_{\gamma}^\text{HTL}^- = -3m_f^2 \left[ \frac{1}{2k} \left( \ln \frac{k + k\pm i\epsilon}{k - k\pm i\epsilon} \right) \right]$$

$$\partial_{\gamma}^\text{HTL}^+ = \frac{3}{2}m_f^2 k \left[ \ln \frac{k + k\pm i\epsilon}{k - k\pm i\epsilon} \right]$$

with $m_f^2 = \frac{1}{6}e^2T^2$ the square photon thermal mass.

Finally, the dressed vertices are the sum of the tree amplitudes and the hard-thermal loops. The HTL-dressed fermion-photon vertex in the Keldysh formalism is defined with the following expressions:

$$\Gamma_{\mu\nu}^{\text{HTL}}(P, Q) = \gamma^{\mu} + I^{\mu}(P, Q);$$

$$\Gamma_{\mu,\alpha}^{\text{HTL}}(P, Q) = \gamma^{\mu} + I^{\mu,\alpha}(P, Q);$$

$$\Gamma_{\mu}^{\text{HTL}}(P, Q) = \Gamma_{\mu,\alpha}^{\text{HTL}}(P, Q) = 0$$

(5)

and the HTL-dressed two-fermion-two-photon vertex is given by these expressions:

$$\Gamma_{\mu\nu,\alpha\beta}^{\text{HTL}}(P, K) = I^{\nu\beta}(P, K);$$

$$\Gamma_{\mu\nu,\alpha\beta}^{\text{HTL}}(P, K) = 0$$

(6)

with the quantities $I$'s being the hard-thermal loops that given by these solid-angle integrals:

$$I^{\mu}_{\alpha\beta}(P, Q) = \int \frac{d\Omega_s}{4\pi} \frac{S}{(P.S + i\eta_s)(Q.S + i\eta_s)}$$

$$I^{\mu\nu}_{\alpha\beta}(P, Q) = \int \frac{d\Omega_s}{4\pi} \frac{-S^\mu S^\nu}{(P.S + i\eta_s)(Q.S + i\eta_s)}$$

$$\times \frac{1}{(P + K).S + i\eta_s} + \frac{1}{(P - K).S + i\eta_s}.$$ 

(7)

3. Damping rate and energy for Fermions in hot QED

The fermion dispersion relations are defined by this expression:

$$\det(P - \Sigma_{\text{HTL}}(P)) = 0$$

(8)
In this expression, \( p = (p_\nu, \delta) \) is the fermion external soft four-momentum and \( \Sigma_{\mu\nu}(p) \) is the retarded fermion self-energy. The next-to-leading order fermion energy and damping rates are given by these expressions:

\[
\Omega_\pm = \frac{\operatorname{Re} \Sigma_\pm(\omega_\pm, p)}{\partial \omega_\pm \Sigma_\pm(\omega_\pm, p)} \quad \gamma_\pm(p) = -\frac{\operatorname{Im} \Sigma_\pm(\omega_\pm, p)}{\partial \omega_\pm \Sigma_\pm(\omega_\pm, p)}
\]

In these expressions, \( \omega_\pm \) is the leading-order fermion energy. So, to obtain the next-to-leading order dispersion relations for slow-moving fermions, we have to determine the next-to-leading order fermion self-energy. The diagrams that contribute to next-to-leading order fermion self-energy are the following two diagrams, in which the internal momenta are soft.

![Figure 1: NLO HTL-summed fermion self-energy](image)

The contribution of the first diagram in the Keldysh basis is given by:

\[
\Sigma_1 \! \left( P \right) = -ie^2 \operatorname{Tr} \left[ \Gamma^\mu \left( P, Q \right) \Delta(Q) \Gamma^\nu \left( Q, P \right) \Delta^\mu\nu \left( K \right) \right]
\]

and the contribution of the second diagram is:

\[
\Sigma_2 \! \left( P \right) = -\frac{ie^2}{2} \operatorname{Tr} \left[ \Gamma^\mu \left( P, K \right) \Delta^\mu \left( K \right) \right]
\]

Doing the sum over the indices in the Keldysh basis, we obtain the following results for the first contribution:

\[
\Sigma_1 \! \left( P \right) = -ie^2 \int \frac{d^4 K}{(2\pi)^4} \gamma_\pm \gamma_\mp \left[ \Gamma^\mu \left( P, Q \right) \Delta^R \left( Q, P \right) \Delta^S \left( K \right) \right]
\]

and for the second contribution the following expressions:

\[
\Sigma_2 \! \left( P \right) = -\frac{ie^2}{16} \int \frac{d^4 K}{(2\pi)^4} \gamma_\pm \gamma_\mp \left[ \Gamma^\mu \left( P, K \right) \Delta^S \left( K \right) \right]
\]

In the above expressions, the retarded (R), advanced (A), and symmetric (S) propagators are given by:

\[
\Delta^R_{\mu\nu}(K) = \Delta^A_{\mu\nu}(K) = \Delta_{\mu\nu} \left( \vec{k} + ie \vec{\epsilon}, \vec{k} \right);
\]

\[
\Delta^S_{\mu\nu}(K) = \Delta^2_{\mu\nu} \left( K \right) = \Delta_{\mu\nu} \left( \vec{k} - ie \vec{\epsilon}, \vec{k} \right);
\]

\[
\Delta^A_{\mu\nu}(K) = \Delta^2_{\mu\nu} \left( K \right) = \left( 1 \pm 2 \Delta_{\mu\nu} \left( k_0 \right) \right) \text{sign}(k_0) \left[ \Delta_{\mu\nu} \left( K \right) - \Delta^2_{\mu\nu} \left( K \right) \right].
\]

We find the following expression for the NLO HTL-dressed fermion self-energy, written in a compact form:

\[
\Sigma \! \left( P \right) = -\frac{ie^2}{2} \int \frac{d^4 k}{(2\pi)^4} \left[ \Delta^R_{\mu\nu} \left( P, K \right) + \Delta^A_{\mu\nu} \left( P, K \right) + 2 \Delta^S_{\mu\nu} \left( P, K \right) + F^R_{\mu\nu} \left( P, K \right) + F^A_{\mu\nu} \left( P, K \right) + F^S_{\mu\nu} \left( P, K \right) + G^R_{\mu\nu} \left( P, K \right) \right]
\]
The contributions that do not include a hard-thermal loop are as follows:

\[ F^{XY}_{\gamma,\gamma'}(P, K) = \text{tr} \langle \gamma \gamma' \rangle \mathcal{X}^X_{\gamma',\gamma}(K) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \]

\[ = -2i \left( \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^X_{\gamma',\gamma}(Q) - \left[ k^2 \left( 1 + \hat{p}_e \hat{q}_e \right) \right] \mathcal{X}^Y_{\gamma,\gamma'}(Q) - 2k \cdot k \]

\[ \times \left( \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \]

(16)

The contributions that involve one hard thermal loop vertex function are given as follows:

\[ F^{XY}_{e,\gamma,\gamma',\nu,\eta,\gamma'}(P, K) = \text{tr} \langle \gamma \gamma' \rangle \mathcal{X}^X_{\gamma',\gamma}(K) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \]

\[ = m_f^2 \int \frac{d\Omega}{4\pi} \left[ \frac{1}{(PS + i\eta\epsilon)(QS + i\eta\epsilon)} \right] \]

\[ \left[ \left( \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^X_{\gamma',\gamma}(Q) - \left( \mathcal{K}^{XY}_{\gamma,\gamma'}(Q) - k^2 \left( 1 + \hat{p}_e \hat{q}_e \right) \right) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \right] \]

(17)

Finally, the contributions involving two hard-thermal-loop vertex functions are as follows:

\[ F^{XY}_{e,\gamma,\gamma',\nu,\eta,\gamma'}(P, K) = \text{tr} \langle \gamma \gamma' \rangle \mathcal{X}^X_{\gamma',\gamma}(K) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \]

\[ = m_f^2 \int \frac{d\Omega}{4\pi} \left[ \frac{1}{(PS' + i\eta\epsilon)(QS' + i\eta\epsilon)} \right] \left[ \left( \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^X_{\gamma',\gamma}(Q) \right. \]

\[ \left. - \left( \mathcal{K}^{XY}_{\gamma,\gamma'}(Q) - k^2 \left( 1 + \hat{p}_e \hat{q}_e \right) \right) \mathcal{X}^Y_{\gamma,\gamma'}(Q) \right] \]

(18)

In the second diagram, the contribution is as follows:

\[ G^{X}_{\nu,\eta,\gamma}(P, K) = \text{tr} \langle \gamma' \rangle \mathcal{X}^X_{\gamma,\gamma'}(K) \mathcal{X}^X_{\gamma,\gamma'}(Q) = m_f^2 \frac{d\Omega}{4\pi} \frac{1}{(PS + i\eta\epsilon)(PS + i\eta\epsilon)} \]

\[ \times \left[ \frac{1}{(P + K)S + i\eta\epsilon} \left( P - K \right) \frac{1}{(P - K)S + i\eta\epsilon} \right] \]

\[ \left[ \left( \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^X_{\gamma,\gamma'}(Q) \right. \]

\[ \left. + \left( k^2 \left( 1 + \hat{p}_e \hat{q}_e \right) - 2k \cdot k \hat{p}_e \hat{q}_e \right) \mathcal{S}(K) \mathcal{X}^X_{\gamma,\gamma'}(Q) \right] \]

(19)
4. Conclusion

In this communication, we have reported on the progress in our determination of the HTL next-to-leading fermion dispersion relation. We have derived a compact analytic expression for the complete next-to-leading contribution to the retarded fermion self-energy in the context of hard-thermal-loop summed perturbation of QED at high temperature using real time formalism. We have also determined the compact expressions for the hard-thermal-loop contributions to the vertices. These expressions need to be manipulated, mainly numerically, to determine the next-to-leading order fermion dispersion relations. This work is in progress.

REFERENCES

[1] Abada A, Azi O and Benchallal K 1998 Phys. Lett. B 425 158-164.
[2] Abada A and Azi O 1999 Phys. Lett. B 463 117-25.
[3] Abada A, Bouakaz K and Azi O 2006 Phys. Scr. 74 77-103.
[4] Abada A, Bouakaz K and Daira Aifa N 2001 Eur. Phys. J. C 18 765-77.
[5] Abada A, Daira Aifa N and Bouakaz K 2006 Int. J. Mod. Phys. A 21 5317-32.
[6] Bouakaz K and Abada A 2008 AIP Conf. Proc. 1006 150-53.
[7] Abada A, Bouakaz K and Deghiche D 2007 Mod. Phys. Lett. A 22 903-14.
[8] Silin V P 1960 Zh. Eksp. Teor. Fiz. 38 1577 [1960 Sov. Phys. JETP 11 1136]; Tsytovich V N 1961 Zh. Eksp. Teor. Fiz. 40, 1775 [1961 Sov. Phys. JETP 13 1249]; Kalashnikov O and Klimov V V 1980 Yad. Fiz. 31 1357 [1980 Sov. J. Nucl. Phys. 31 699]; Klimov V V 1981 Yad. Fiz. 33 1734 [1981 Sov. J. Nucl. Phys. 33 934]; Klimov V V 1982 Zh. Eksp. Teor. Fiz. 82 336 [1982 Sov. Phys. JETP 55 199].
[9] Braaten E and Pisarski R D 1990 Nucl. Phys. B 337 569-634.
[10] Le Bellac M 1996 Thermal Field Theory ( U.K: Cambridge University Press).
[11] Braaten E and Pisarski R D 1990 Phys. Rev. D 42 2156-60.
[12] Abada A and Bouakaz K 2006 J. High Energy Phys. JHEP 01.
[13] Benchallal K, Abada A and Bouakaz K 2015 Acta Phys. Pol. 128 290.
[14] Bouakaz K, Abada A and Benchallal K 2015 Int. J. of Computational and Experimental Science and Engineering Vol. 1-No.1 pp. 1-5
[15] Abada A, Benchallal K and Bouakaz K 2015 J. High Energy Phys. JHEP 03.
[16] Fueki Y, Nakkagawa H, Yokota H and Yoshida K 2002 Prog. Theor. Phys. 107 759-84.
[17] Carrington M E, Fugleberg T, Irvine D S and Pickering D 2007 Eur. Phys. J. C 50 711-27.