Completing the proportional navigation rule with
optimizational association funtion based on particle swarm
optimization rule

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Abstract: The Proportional Navigation (PN) is one of the most successful and widely applied conductive rules in the field of guidance since from the initial days of its proposal. The success of PN comes from very early by the simplicity and ease of actualization and the especially important reason that leads to the success of the PN is permanent or poor maneuverability target. However, with the development in aerospace science, the maneuverability of flying vehicles is constantly improving. This causes PN to reveal its disadvantages, reduce its accuracy when sticking and attacking mobile targets. The paper presents a guidance rule based on fuzzy logic on basis of Particle Swarm Optimization (PSO) algorithm. The proposed guidance rule (FPSOG) is simulated according to PN to exploit the advantages of PN but still ensure the optimal and flexible design by fuzzy logic and PSO algorithm. The results of the completion process of FPSOG were investigated and compared with PN and pure Fuzzy Proportional Navigation (FPN) rule MATLAB software.

1. Introduction
The Proportional Navigation (PN) is one of the most successful and widely applied conductive rules in the field of guidance since from the initial days of its proposal. The success of PN comes from very early by the simplicity and ease of actualization and the especially important reason that leads to the success of the PN is permanent or poor maneuverability target. However, with the development in aerospace science that the variety of types along with the speed of aircraft is constantly being developed. This causes PN to reveal its disadvantages, reduce its accuracy when sticking and attacking mobile targets. Therefore, researching, improving and proposing new guidance rule to improve conductive quality has always received the attention of many researchers.

From the 1960s, with the rapid development of optimal control theory, the guidance rule based on this theory began to appear [1]. The first publications on optimal control theory in the field of guidance must be included Bryson and Baron [3]; Bryson [4, 5]. Since then, there have been hundreds of published articles about applying optimal control in the field of guidance. However, the biggest disadvantage of the optimal conduction rule is that the guided quality will seriously degrade if the dynamic model of the components in the control loop is incorrect [6]. In addition, the process of implementing the guidance rule requires a large amount of computation, so up to now, the guidance rule of missile based on optimal control theory have not been applied in practice.

Fuzzy logic-based controllers can completely overcome the aforementioned disadvantages of optimal control because, especially for complex systems because fuzzy controllers do not need exact
systematic mathematical models, does not need quantitative data related to input - output relationship, allow integrate the designer's knowledge and experience into the controller through the association function and fuzzy rule basis [7]. Until now, fuzzy logic has been applied successfully in many different fields, for example civil, industrial and military equipment [8, 9]. However, the design of the fuzzy controller is often handmade based on the experience of the designer [7]. In the field of guidance, fuzzy logic was used to synthesize the rule of missile guidance very early. However, the rule of fuzzy guidance in these 2 studies is designed based on the experiences of the authors. Recently, there have also been published studies about synthesis of automatic fuzzy guidance rule with the help of new tools and algorithms such as neural networks, genetic algorithms, ant colony optimization [12-15].

In the group of evolutionary algorithms, the PSO algorithm is a random optimized search algorithm, in which the individuals of the populations adjust their position in the search space according to the information collected by that individual, and information obtained from the vicinity individual. The PSO algorithm has an fast convergence speed to the full-pole optimization root, easy to implement and only a few parameters need to be adjusted, so this algorithm is suitable for nonlinear, multi-objective optimization problems [16]. Currently, researches on the application of PSO algorithm in the guidance problem are not really diverse. The authors of the documents [16-18] use directly the PSO algorithm to synthesize the guidance rule. In the study [19], the author has proposed the fuzzy proportional navigation rule with optimization association function base on PSO algorithm. However, the disadvantage of that study is that in order to optimize the association function of the fuzzy rule must be used a 63-element matrix (dimension) to describe the position of an individual with the number of loop up to 1000. All of these require mainframe resources, long computation times. This makes the fuzzy proportional navigation rule proposed in [19] only optimized offline, i.e. the PSO algorithm is only applied once to determine the optimal distribution for the association function. The optimum of that distribution can no longer be guaranteed when the target changes the speed and the rule of maneuverability.

In this paper, a fuzzy guidance rule based on the fuzzy proportional navigation rule with association function is optimized by the PSO algorithm will be proposed. Different from the research results in the document [19], the FPSOG rule will only consider the fuzzy guidance rule with one input as the angle speed of line of sight $\dot{\lambda}$ and the output as the command of missile’s normal acceleration. Before optimization is performed by the PSO algorithm, the positions of the association functions will be indirectly represented to minimize the number of dimensions of the individual’s position vector. In addition, the number of loops of the PSO algorithm is 10 times lower than the published results in the article [19], the target function of the PSO algorithm is added with the meeting time parameter - an important parameter to evaluate the quality of the guidance rule.

2. Experimental

2.1. The fuzzy proportional navigation
The geometry movement of the guidance problem in the vertical plane is shown in figure 1 with the following assumptions:

- Missiles and targets are considered particles;
- Ignoring the impact of environmental factors on the guidance process;
- Missiles and targets fly at a constant speed.

In which:

- $V_m$, $V_t$ are velocities of the missile and the target, respectively,
- $\vec{n}_t$, $\vec{n}_t^-$, $\vec{n}_t^+$ are missile’s normal acceleration (perpendicular to the velocity vector) and ingredient missile’s normal acceleration (elements perpendicular to the line of sight (LOS) and elements parallel to the LOS),
\( \tilde{n}_r \) is target’s normal acceleration (perpendicular to the target’s velocity vector),

\( R_{TM} \) is distance between missile and target,

\( \lambda \) is angle LOS,

\( \beta' \) is the angle combined by the target’s velocity vector and the positive direction \( OX \),

\( B = \pi - \beta' \),

\( L \) is pickup angle,

\( \varepsilon \) is error of the pickup angle.

In the general case, the missile’s normal acceleration \( n_c \) is always decomposed into two component vectors: the one perpendicular to the LOS is \( n_c^\perp \) and the one parallel with LOS is \( n_c^\parallel \). In two components: \( n_c^\perp \) and \( n_c^\parallel \) only the perpendicular component \( n_c^\perp \) is causing the rotation of LOS. Therefore, the LOS rotation effect of \( n_c \) and \( n_c^\perp \) are equivalent. Therefore, instead of analyzing the guidance problem with \( n_c \), we will analyze the guidance problem with \( n_c^\perp \).

Because the proposed guidance rule is built based on the proportional navigation rule (PN), so we need to clarify the mathematical expressions describing the PN guidance rule. Theoretically, the the proportional navigation rule produces a normal acceleration command perpendicular to the LOS, the magnitude of the acceleration command is proportional to the LOS’s rate of rotation and the approach velocity according to the following expression [19]:

\[
 n_{c_{PN}}^\perp = N V_c \dot{\lambda} \tag{1}
\]

In which:

\[
 V_c = -\dot{R}_{TM} = -\frac{R_{TM} V_{TMx} + R_{TM} V_{TMy}}{R_{TM}} \tag{2}
\]

\[
 \dot{\lambda} = \frac{R_{TM} V_{TMx} - R_{TM} V_{TMy}}{R_{TM}^2} \tag{3}
\]
With $R_{TMx}, R_{TMy}, V_{TMx}$ and $V_{TMy}$ parameters are components on the $X$ and $Y$ axes of $R_{TM}$ and $V_{TM}$, respectively. In which (1), $N'$ is guidance coefficient, normally $N'$ is chosen to have an integer value from 3 - 5.

In the equation (1), $N'$ is a variable parameter for the purpose of reducing required input information. In the FPN rule, multiplication of $N'V_c$ will be replaced by a variable $k$ parameter. This parameter is one of the parameters that will be selected for optimization by the PSO algorithm. Then, the expression determining the command of missile’s normal acceleration according to the FPN rule has the form:

$$n_{c, PN}^i = k_n f_{FC}(\lambda)$$

In which, $f_{FC}(\lambda)$ is function representing a fuzzy controller with input as $\lambda$ and output as a standardized acceleration command (ie the magnitude is referred to the segment [-1, 1]) $n_{c}^i$, after multiplying output $n_{c}^i$ by $k_n$, we will have missile’s normal acceleration.

The fuzzy controller will be designed based on the fuzzy controller according to the Mamdani model described in the document [20]. In which, the fuzzy controller has an input $\lambda$ and an output $n_{c}^i$. The controller has three main components: the association functions of linguistic variables, fuzzy rules, and ratio coefficients that convert the range of physical input values to the range from -1 to 1 of the association function.

**Fuzzy rule system**: Select 7 fuzzy sets (linguistic values) for input linguistic variable $\lambda$ and output linguistic variable $n_{c}^i$ with the meanings and symbols given in table 1 below:

**Table 1. Meanings and symbols of the fuzzy sets.**

| Symbols | BN | MN | SN | Z | SP | MP | BP |
|---------|----|----|----|---|----|----|----|
| Meanings | Big minus | Medium minus | Small minus | Zero | Big plus | Medium plus | Small plus |

Then the fuzzy rule system used for the fuzzy guidance rule is given in table 2:

**Table 2. The fuzzy rule system of the fuzzy proportional navigation rule.**

| $\lambda$ | BN | MN | SN | Z | SP | MP | BP |
|--------|----|----|----|---|----|----|----|
| $n_{c}^i$ | BN | MN | SN | Z | SP | MP | BP |

**Ratio coefficient**: Because input variable $n_{c}^i$ has been standardized to segment [-1, 1] so we are only interested in the ratio coefficient $k_\lambda$ of the input variable $\lambda$. This coefficient is determined through a PN rule. This paper chooses $k_\lambda = 0, 1$.

**Association fonction**: The next element of the fuzzy guidance rule need to determine is the association function. There are many different types of association functions, but triangular association functions will be selected in this paper because of the advantage of fast response [19], the number of chosen association functions is equal to the number of linguistic variables. Therefore, the input and output variables have 7 association functions. The distribution of these association functions will be optimized by the PSO algorithm.

As mentioned in the previous section of the paper, to minimize the number of required parameters for representing the distribution of the association functions was proposed in [21]. Accordingly, the parameters represent the distribution of the association function of the input $\lambda$ and output $n_{c}^i$ linguistic variable according to the following expression:

$$c_{i+4} = sgn(i) \cdot g_e \cdot |\frac{|i|}{p_e}|^{\gamma'}$$ with $i = -\frac{n-1}{2}, ..., 0, ..., \frac{n-1}{2}$

(5)
In which:

\[ n = 7, 0 < q_c < 4 \] are the number of association functions and the distribution coefficient of the association function, respectively:

\[ \text{sgn}(.) \] is sign function:

\[
\text{sgn}(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{if } x < 0 
\end{cases}
\]

\[ g_c \] is the standardized coefficient:

\[
n^+_c = k_n f_{c, n}(\lambda) 
\]

\[ c_i \] is the center coordinate of the association functions shown in figure 2.

**Figure 2.** The parameter representation defines the association function of linguistic variables \( \lambda \) and \( n'_c \).

In Figure 2, each set of numbers \((l_i, c_i, r_i)\) where \( i = 1...7 \) defines the shape and distribution of the corresponding \( i \) association function. So with the use of expression (5), we only need to use 1 variable \( q_c \) to define 7 association functions of a linguistic variable, it mean only 2 real variables \( q_{c, n'_c} \) to describe the 14 association functions of the input linguistic variable \( \lambda \) and the output linguistic variable \( n'_c \).

### 2.2. Optimization of fuzzy proportional navigation rule by the PSO algorithm

The PSO algorithm was first proposed in 1995, this is a random optimal search algorithm that simulates the behavior of species living in groups such as birds, fishes, bees. The PSO algorithm will use information about the best location of the individual up to now (represented by parameter \( p_{best} \)) and the best location of the populations up to now (represented by parameter \( g_{best} \)). The velocity of each individual for the next move will be the function of \( p_{best}, g_{best} \) and the initial velocity of that individual. Whenever moving to a new position, through the target function \( f(.) \), the parameters \( p_{best} \) and \( g_{best} \) will be updated. That process is repeated until the algorithm stops. The PSO algorithm is described mathematically as follows [22]:

\[
V_{i,k+1} = wV_{i,k} + c_1 \text{rand}_1 \left( p_{best}_{i,k} - X_{i,k} \right) + c_2 \text{rand}_2 \left( g_{best}_{k} - X_{i,k} \right) 
\]

\[
X_{i,k+1} = V_{i,k} + V_{i,k+1}
\]

In which, \( X_{i,k}, X_{i,k+1} \) are the positions of the \( i \) individual at the time \( k \) and \( k+1 \) respectively; \( V_{i,k+1} \) is the velocity of the \( i \) individual at the time \( k+1 \); \( p_{best}_{i,k} \) is the best position of the \( i \) individual to time \( k \); \( g_{best}_{k} \) is the best position of the \( i \) populations to time \( k \); \( w \) is inertia coefficient; \( \text{rand}_1 \) and \( \text{rand}_2 \) are random numbers in segment \([0, 1]\); \( c_1 \) is weight of \( p_{best} \); \( c_2 \) is weight of \( g_{best} \). \( p_{best} \) and \( g_{best} \) are determined as follows:
\[ p_{best_{i,k+1}} = \begin{cases} p_{best_{i,k}} & \text{if } f(p_{best_{i,k}}) \leq f(X_{i,k+1}) \\ X_{i,k+1} & \text{if } f(p_{best_{i,k}}) > f(X_{i,k+1}) \end{cases} \]  

Then, the \( g_{best_k} \) value is the minimum \( p_{best} \) value, that is:

\[ g_{best_k} = \min(p_{best_{1,k}}, p_{best_{2,k}}, \ldots, p_{best_{N,k}}) \]  

With \( N \) is the number of individuals in the populations.

Flowchart for optimal implementation of the fuzzy proportional navigation rule by the PSO algorithm is described in figure 3.

**Figure 3.** General flowchart for PID rule by PSO.

**Step 1:** Set values for the inertia coefficient \( w = 1, 2 \); coefficients \( c_1 = c_2 = 2 \); the number of maximum repetitions \( k_{max} = 100 \); the number of individuals \( N = 9 \); the best position of each individual \( p_{best_{i,0}} \).

**Step 2:** Initialize the position and velocity of individuals in the populations.

Because the purpose of this paper is to determine the values of the three coefficients \( k_n, q_{c,k} \) and \( q_{c_{-k}} \), so the position of the \( i \) individual in the populations is characterized by these three parameters.
This step will perform random initialization of 3 coefficients \( k_n, q_{e,k}, \) and \( q_{c,kq} \) in the range of corresponding values \( [k_{n_{\text{min}}}, k_{n_{\text{max}}}], [q_{e,k_{\text{min}}}, q_{e,k_{\text{max}}}], [q_{c,kq_{\text{min}}}, q_{c,kq_{\text{max}}}] \). The velocity of the individuals will be initially initialized to 0.

**Step 3:** For each individual, we have 1 set of 3 coefficients \( k_n, q_{e,k}, \) and \( q_{c,kq} \). The set of these three coefficients will define the specific structure of the fuzzy guidance rule. With the received fuzzy guidance rule and the target information, the missile control circle is done by solving the geometrical dynamic equations of the guidance problem. At the end of the simulation, we get parameters to evaluate the quality of the proposed guidance rule.

**Step 4:** For the guidance problem, two important parameters that directly affect the guidance quality are the missile’s slip level and the flight time until attacked the target. In addition, another parameter used to evaluate quality of the guidance rule is integral of magnitude of the normal acceleration. At the end of Step 3, we get 3 values for evaluating the quality of the guidance rule such as slip level \( d_f \), integral of magnitude of the normal acceleration \( n_{c,\text{FPNPSO}}^+ \), time to meet the target \( t_f \). The paper will use all of these three parameters as input to the target function. For the PSO algorithm, the populations will move in the direction of minimizing the target function, it mean the target function and the input parameters are in the same direction, so the target function corresponds to the \( i \) individual in the iteration step \( k \) is given as follows:

\[
J_{i,k} = w_d^i \cdot d_f^i + w_n^i \cdot \int_0^{t_f} n_{i,\text{FPNPSO}}^+ dt + w_t^i \cdot t_f^i
\]  

(11)

In which, \( w_d^i, w_n^i \) and \( w_t^i \) are the weights that determine the roles of slip level and normal acceleration and target meeting time in the optimization process of PSO algorithm respectively.

**Step 5:** Update \( \text{pbest}_{i,k} \) and \( \text{gbest}_{k} \) when we have information about \( J_{i,k} \).

**Step 6:** Update the position and velocity of individuals according to new information about \( \text{pbest}_{i,k} \) and \( \text{gbest}_{k} \).

**Step 7:** Check for stop conditions. PSO algorithm will stop when running out of loops \( k_{\text{max}} \).

3. Results and discussion

The research results are conducted on the basis of 3 guidance rule:

- Proportional navigation rule (PN): guidance coefficient \( N' = 3 \).
- Fuzzy proportional navigation rule (FPN): The fuzzy rule system is presented in table 2; The distributions of the associated functions of the input and output linguistic variables are determined in the case of the set of numbers \( (q_{e,k}, q_{c,k} ) = (1,1) \) (associated functions are evenly distributed over the segment \([-1,1])\); Ratio coefficient \( k_n q_{e,\text{max}} = 30g \), with \( q_{e,\text{max}} \) is the missile’s allowed overload and \( g = 9.8 \text{ m/s}^2 \) is gravity.
- Fuzzy proportional navigation Particle Swarm Optimization rule (FPNPSO): The fuzzy rule system is presented in table 2; The distributions of the associated function is optimized according to the PSO algorithm; Ratio coefficient \( k_n \) is optimized according to the PSO algorithm.

The parameters are initialized in the simulation process:

- Missile’s velocity: \( V_M = 1000 \text{ m/s} \).
- Missile’s coordinates: \( (R_{Mx}, R_{My}) = (0,0) \text{ m} \).
• Guidance coefficient (using for proportional navigation rule): $N' = 3$.
• There is no error of guidance angle: $\varepsilon = 10'$.
• Target’s velocity: $V_T = 400m/s$.
• Target’s coordinates: $(R_x, R_y) = (15000, 10000)m$.
• The target maneuver one-sided as a rule: $n_T = -5g$, với $g = 9.8 m/s^2$.

Simulation results

**Figure 4.** The populations are converging to optimal location according to the target function.

**Figure 5.** The distribution of associated function of the input linguistic variable.

**Figure 6.** The distribution of associated function of the output linguistic variable.

**Figure 7.** Normal acceleration of the 3 guidance rule.

**Figure 8.** Orbital of Missile – Target.
Table 3. Results of slip level and normal acceleration at the meeting time of the three guidance rules

|                                | PN       | FPN     | FPNPSO  |
|--------------------------------|----------|---------|---------|
| Slip level at the meeting time $d_f$, m | 8.1625   | 2.4697  | 0.3568  |
| Integral of Missile’s normal acceleration magnitude, g | 1204.5   | 1046.2  | 961.64  |
| Target’s meeting time, $s$       | 15.4130  | 14.7420 | 14.555  |

Figure 4 shows the distribution of the populations after performing the PSO algorithm, the results show that the populations converged towards one position in the search space. This proves that the algorithm has converged to the optimal position according to the target function with the number of selected iterations.

Figures 5 and 6 show the distribution of associated functions with input and output linguistic variable optimized by the PSO algorithm. After implementing the PSO algorithm, set of numbers $(a_{q_1},q_{s_{eq}}) = (1.3694,4.0063)$ and ratio coefficient $k_{q_1} = 898.7909$.

From figure 7 and table 3, we can see by giving all three parameters: Slip level; Integral of missile’s normal acceleration magnitude; The target function’s meeting time belong the PSO algorithm that the FPNPSO rule receives for the above 3 parameters is smaller than the default fuzzy proportional navigation rule approach and significantly smaller than the fuzzy proportional navigation rule. Although at the beginning of flight, the missile requires a great normal acceleration when applying the proposed guidance rule, however when considering the flight process, most of the represented normal acceleration curves of the FPNPSO rule is closest to zero position because of optimal pressure on this parameter when implementing the PSO algorithm. As a result, the orbital of the missile when applying the proposed guidance rule has the smallest curvature among the three surveyed guidance rule (figure 9).

4. Conclusion

Through the simulation results, the paper has verified the advantages of the fuzzy guidance rule over the classic proportional navigation rule in the case of attacking a mobile target. But the most important thing is that the paper has proven the quality of that fuzzy guidance rule is still improved by optimal search algorithms like the PSO algorithm. In addition, with the association function encoding method proposed in the paper, the number of vector elements representing the populations’ position in the PSO algorithm is significantly reduced compared to the previous study [19].

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