Magnetic Monopoles and Massive Photons in a Weyl-Type Electrodynamics.

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Abstract

In a previous work the Weyl-Dirac framework was generalized in order to obtain a geometrically based general relativistic theory, possessing intrinsic electric and magnetic currents and admitting massive photons.

Some physical phenomena in that framework are considered. So it is shown that massive photons may exist only in presence of an intrinsic magnetic field. The role of massive photons is essential in order to get an interaction between magnetic currents. A static spherically symmetric solution is obtained. It may lead either to the Reissner-Nordstrøm metric, or to the metric created by a magnetic monopole.

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1 Introduction

Two fundamental electrodynamical phenomena are standing beyond the frames of Maxwell’s theory. The first ”outsider” being known as magnetic monopole was evoked by Dirac [1], [2], (cf. also an interesting review [3]), the second being the massive photon. The massless photon became a tacit axiom of physics own to the success of quantum electrodynamics in predicting experiments with enormously high exactness. But the same results would be obtained with photons having mass \( m \gamma < 10^{-48} \text{ g} \). (cf.[4], [5]). From the quantum-theoretical standpoint Dirac’s monopole and massive photons were discussed widely during the last decades ([3], [4], [6]). But a satisfactory classical framework including these two phenomena was absent until the last time. If one wants to discuss massive photons he has to consider Proca’s equations rather then the Maxwell equations. Further, if a magnetic charge (monopole) really exists, then Maxwell’s electrodynamics that suffers from an asymmetry, regarding to electric, and magnetic currents, must be replaced by a generalized theory, with the dual field tensor having a non-vanishing divergence. It would be desirable to build up the framework from geometrical reasons, starting from a generalization of Riemann’s geometry. Recently a massive electrodynamics, based on a space with non-metricity and torsion, was proposed [6]. Electric, and intrinsic magnetic currents, as well massive photons coexist within this framework. In the limiting case one obtains the ordinary Einstein-Maxwell theory.

In the present work that theory is developed, and some characteristic phenomena, and crucial problems are considered. The interaction between electric and magnetic currents and fields, as well the dynamical role of massive photons are considered. It is shown that in absence of fields, created by magnetic charges no massive photons are allowed. The energy-momentum conservation law is discussed, and the equation of motion of a charged (either magnetically, or electrically) test particle is derived from it. It is shown that two magnetic monopoles interact by means of massive photons. A static, spherically symmetric solution for vacuum is obtained. From it, by an appropriate choice of parameters, one obtains two alternative solutions, either the Reissner-Nordstrom one for an electric monopole, or the metric and magnetic field of a magnetic monopole. Thus one can not have both, an electric, and a magnetic charge located in one point. The magnetic monopole is found to be a massive entity.

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2 The Torsional Weyl-Dirac Electrodynamics

Let us consider in brief the Torsional Weyl-Dirac Electrodynamics. Details may be found in a previous work of the present writer [7]. We started from Weyl’s geometry [8], as modified by Dirac [9], and also a torsion tensor \( \Gamma^\lambda_{\mu \nu} \) in each point of the 4-dimensional space-time manifold. In this case the assymetric connection \( \Gamma^\lambda_{\mu \nu} \) may be written as (cf. [8])

\[
\Gamma^\lambda_{\mu \nu} = \{ \lambda_{\mu \nu} \} + g_{\mu \nu} w^\lambda - \delta^\lambda_{\nu} w_\mu - \delta^\lambda_{\mu} w_\nu + C^\lambda_{\mu \nu},
\]

where \( \{ \lambda_{\mu \nu} \} \) is the Christoffel symbol formed with \( g_{\mu \nu} \), and the contorsion is given in the torsion tensor as follows

\[
C^\lambda_{\mu \nu} = \Gamma^\lambda_{\mu \nu} - g^{\lambda \beta} g_{\sigma \mu} \Gamma^\sigma_{\nu \beta} - g^{\lambda \beta} g_{\sigma \nu} \Gamma^\sigma_{\mu \beta}.
\]

The Weylian character of the connection \( \Gamma \) causes a non-integrability of length, so that one is faced with local gauge transformations

\[
B \to \overline{B} = e^\lambda B ; \quad g_{\mu \nu} \to \overline{g}_{\mu \nu} = e^{2\lambda} g_{\mu \nu} ; \quad w_\mu \to \overline{w}_\mu = w_\mu + \lambda_{, \mu} ; \quad \beta \to \overline{\beta} = e^{-\lambda} \beta.
\]

where \( B^2 = g_{\mu \nu} B^\mu B^\nu \) is the length of a vector \( B^\mu \), and \( \lambda \) is an arbitrary function of the coordinates.

In this generalized torsional Weyl geometry it was assumed that the torsion tensor is gauge invariant, so that in addition to \( \overline{\Gamma} \) one has

\[
\Gamma^\lambda_{\mu \nu} \to \overline{\Gamma}^\lambda_{\mu \nu} = \Gamma^\lambda_{\mu \nu}.
\]

The equations of the theory were derived from a variational principle

\[
\delta I = 0 ,
\]

with the action \( I \), formed from curvature invariants of the torsional Weyl space (cf. [7]).

\[
I = \int \left( W^{\mu \nu} W_{\mu \nu} - \beta^2 R + \beta^2 (k - 6) w_\mu w^\mu + 2 (k - 6) \beta w^\mu \beta_\mu + k \beta \mu \beta, \mu + \frac{8}{3} \beta \Gamma^\alpha_{\lambda \alpha} \beta_\lambda \right) \overline{B^2} dx
\]

In the action \( I \) \( R \) is the Riemannian curvature scalar formed with the Christoffel symbols \( \{ \lambda_{\mu \nu} \} \), the Weyl curvature tensor is given by \( W_{\mu \nu} = w_{\mu \nu} - w_{\nu \mu} \), and \( L_{\text{matter}} \) is the Lagrangian density of matter. Further, \( k \) is an arbitrary parameter (cf. [7]), \( \Lambda \) is the cosmological constant, a underlined index is to be raised with the metric \( g^{\mu \nu} \), a comma stands for a partial derivative, and a semicolon (;) for a covariant derivative formed with \( \{ \lambda_{\mu \nu} \} \) the independent variables in \( I \) are : the metric tensor \( g_{\mu \nu} \), the torsion tensor \( \Gamma^\lambda_{\mu \nu} \), the Weyl connection vector \( w_\mu \), and the Dirac gauge function \( \beta \).

In the original Weyl-Dirac theory (cf. [8], [9]) the Weyl connection vector \( w_\mu \) was treated as the potential vector of the electromagnetic field, while the Weyl curvature tensor \( W_{\mu \nu} \) yielded the field tensor. Here, the divergence of the torsion enters into the field tensor, so that one has a dual field with non-vanishing divergence, and hence an intrinsic magnetic current is present.

From the variational principle \( I \), \( B \) one obtains the following equation for the electromagnetic field,

\[
\Phi^{\mu \nu} = (1/2)(k - 6) \beta^2 W^\mu + 4 \pi J^\mu ,
\]

where the electromagnetic field is introduced as

\[
\Phi_{\mu \nu} = W_{\mu \nu} - 2 \Gamma^\alpha_{\mu \nu} ; \alpha \equiv W_{\mu \nu} - W_{\nu \mu} - 2 \Gamma^\alpha_{\mu \nu} ; \alpha ,
\]

and the dual is defined in the usual manner

\[
\tilde{\Phi}^{\mu \nu} = - \frac{1}{2(-g)^{1/2}} \varepsilon^{\mu \nu \alpha \beta} \Phi_{\alpha \beta} ,
\]
with \( \varepsilon^{\mu\nu\alpha\beta} \) standing for the completely antisymmetric Levi-Civita symbol, and \( \varepsilon^{0123} = 1 \).

For the dual field one obtains
\[
\tilde{\Phi}^\mu_{\nu} = -2\pi L^\mu_{\nu} .
\]  
(10)

In equations (7), and (10) the following quantities are introduced: \( W_\mu \) stands for the gauge invariant Weyl vector
\[
W^\mu = w^\mu + (\ln \beta),_{\mu} ,
\]  
(11)

the electric current density is given by
\[
16\pi J^\mu = \frac{\delta L_{\text{matter}}}{\delta w^\mu} ,
\]  
(12)

and the magnetic current density vector \( L^\mu \) is introduced by
\[
L^\sigma = -\frac{1}{16}(\sqrt{-g})^{1/2} \varepsilon^{\sigma\mu\nu\lambda}(\Omega^\lambda_{\mu\nu} + \Omega^\lambda_{\nu\mu} + \Omega^\lambda_{\nu\mu}) ,
\]  
(13)

with a \( \Omega^\lambda_{\mu\nu} \equiv g_{\mu\alpha}g_{\nu\beta}\Omega^\lambda_{[\alpha\beta]} \), where the quantities \( \Omega^\lambda_{[\alpha\beta]} \) are defined as
\[
16\pi\Omega^\lambda_{[\mu\nu]} = \frac{\delta L_{\text{matter}}}{\delta \Gamma^\lambda_{[\mu\nu]}}. 
\]  
(14)

From the field equations (7), (10) one has the following current conservation laws:
\[
(k - 6)(\beta^2 W^\mu)_{,\mu} + 8\pi J^\mu_{,\mu} = 0 ,
\]  
(15)

and
\[
L^\mu_{,\mu} = 0 .
\]  
(16)

Generally the torsion can be broken into three irreducible parts (cf. e.g. [12], [13]): a trace part, a traceless one, and a totally antisymmetric part. It turns out that only the third part is relevant in our case, so that the totally antisymmetric torsion tensor \( \Gamma^\lambda_{[\mu\nu]} \) may be represented by a vector. If we introduce the auxiliary torsion tensors:
\[
\Gamma^\lambda_{[\mu\nu]} = g_{\sigma\lambda}\Gamma^\sigma_{[\mu\nu]} ;
\]  
\[
\Gamma^{\lambda[\mu\nu]} = g^{\alpha\mu}g^{\beta\nu}\Gamma^\lambda_{[\alpha\beta]} ,
\]  
(17)

we can express the torsion by means of a gauge invariant vector \( V^\mu \) (named below torsion vector) as follows
\[
\Gamma^\lambda_{[\mu\nu]} = (\sqrt{-g})^{1/2} \varepsilon^\lambda_{\mu\nu\sigma}V^\sigma ;
\]  
\[
\Gamma^{\lambda[\mu\nu]} = -(-g)^{-1/2} \varepsilon^{\lambda\mu\nu\sigma}V^\sigma .
\]  
(18)

This leads to
\[
\Gamma^\nu_{[\mu\nu]} = 0 .
\]  
(19)

From (18) one can also derive the following auxiliary formulae:
\[
\Gamma^{[\mu\nu]} = \frac{\varepsilon^{\mu\nu\alpha\sigma}}{2(-g)^{1/2}}(V_{\alpha ;\sigma} - V_{\sigma ;\alpha}) ;
\]  
\[
\Gamma^{[\lambda\nu\mu]} = 0 .
\]  
(20)

Further by the choice (18) one has from (8)
\[
\Phi^{\mu\nu} = (W^\mu_{\nu} - W^\nu_{\mu}) - \frac{\varepsilon^{\mu\nu\alpha\sigma}}{(-g)^{1/2}}(V_{\alpha ;\sigma} - V_{\sigma ;\alpha}) ,
\]  
(21)

and for the dual field (9)
\[
\tilde{\Phi}^{\mu\nu} = -2(V^\mu_{\nu} - V^\nu_{\mu}) - \frac{\varepsilon^{\mu\nu\alpha\sigma}}{2(-g)^{1/2}}(W_{\alpha ;\sigma} - W_{\sigma ;\alpha}) .
\]  
(22)

Inserting (21) into the field equation (8), and making use of (20), we obtain
\[
\Phi^{\mu\nu} = W^\mu_{\nu ;\mu} - W^\nu_{\mu ;\nu} = (1/2)(k - 6)(16\pi) + 4\pi J^\mu ,
\]  
(23)
while for the dual field we obtain from (20), and (22)

$$\Phi_{\mu\nu} = (V^\mu_{\;\nu} - V^\nu_{\;\mu}) = -2\pi L_{\mu} \ .$$

(24)

From (23) one sees that

$$\Phi_{\mu\nu} = W_{\mu\nu} .$$

(25)

Varying in (16) the metric tensor $g_{\mu\nu}$, one obtains the equation for the gravitational field

$$G_{\mu\nu} = -(8\pi/\beta^2)T_{\mu\nu} - (8\pi/\beta^2)(\bar{M}_{\mu\nu} - \bar{M}^{\mu\nu}) \ .$$

(26)

and

$$+(1/\beta^2)(4\beta_{\mu\nu}\beta_{\alpha\beta} - g_{\mu\nu}\beta_{\alpha\beta}) + (k - 6)(W_{\mu\nu} - \frac{1}{2}g_{\mu\nu}W_{\sigma\nu}) \ .$$

where $8\pi T_{\mu\nu} = \delta L_{\text{matter}}/\delta g_{\mu\nu}$, and the modified energy density tensors of the field are defined as follows

$$4\pi \bar{M}_{\mu\nu} = (1/4)g_{\mu\nu}\Phi_{\alpha\beta} - \Phi_{\mu\sigma}\Phi_{\alpha} \ ,$$

(27)

and

$$4\pi \bar{M}^{\mu\nu} = (1/4)g^{\mu\nu}(\Phi_{\alpha\beta} - W_{\alpha\beta})(\Phi_{\alpha\beta} - W_{\alpha\beta}) - (\Phi_{\mu\sigma} - W_{\mu\sigma})(\Phi_{\nu\alpha} - W_{\nu\alpha}) \ .$$

(28)

It is remarkable (cf. (23)) that the Weyl vector $W_\mu$ is created either by the electric currents $J_\mu$, or by a Proca-type self-inducing term. On the other hand (cf. (24)), the torsion vector $V_\mu$ is created by the magnetic current density vector $L^\mu$.

3 The Einstein Gauge

The torsionless Weyl-Dirac theory with $k = 6$ turns into the Einstein-Maxwell theory if one chooses the Einstein gauge $\beta = 1$. (cf. [9], [10], [14]).

Here we consider in that gauge the above treated generalized Weyl-Dirac theory, possessing torsion, and allowing arbitrary values for $k$. Turning to the Einstein gauge, we set

$$\beta = 1 .$$

(29)

Making use of (24), and replacing $W_\mu$ by $w_\mu$ (cf. (11)), we obtain from (24)

$$G_{\mu\nu} = -8\pi T_{\mu\nu} - 8\pi(\bar{M}_{\mu\nu} - \bar{M}^{\mu\nu}) - (k - 6)(w_\mu w_\nu - (1/2)g^{\mu\nu}w_\sigma w_\sigma) - 2V_\mu V_\nu - g^{\mu\nu}V_\sigma V_\sigma \ .$$

(30)

The fields in the Einstein gauge may be written as

$$\Phi_{\mu\nu} = (w^\mu_{\;\nu} - w^\nu_{\;\mu}) = \frac{\epsilon^{\mu\nu\alpha\sigma}}{(-g)^{1/2}}(V_{\alpha\sigma} - V_{\sigma\alpha}) \ ,$$

(31)

and

$$W_{\mu\nu} = w_{\mu ;\nu} - w_{\nu ;\mu} .$$

(32)

The energy conservation law can be obtained from (30) making use of the contracted Bianchi identities, so that (30) leads to

$$8\pi(T_{\mu ;\nu} + \bar{M}_{\mu ;\nu} - \bar{M}^{\mu ;\nu}) + (k - 6)(w_\mu w_\nu - (1/2)\delta_{\mu\nu}w_\sigma w_\sigma) + 2(V_\mu V_\nu);\nu + (V_\sigma V_\sigma)_{\nu} = 0 .$$

(33)

Making in (33) use of definitions (27), and (28), and of equations (24), (24), (23), one obtains

$$8\pi(T_{\mu ;\nu} + \Phi_{\mu\sigma}J^\sigma) + 4\pi\sqrt{-g}\epsilon_{\alpha\beta\mu\nu}W^{\alpha\beta}L^\sigma + (k - 6)(\Phi_{\mu\sigma} + W_{\mu\sigma})w^\sigma + (k - 6)w_\mu w_\nu + 2V_\mu V_\nu;\nu + 2V_\sigma V_\sigma;\mu = 0 .$$

(34)

For a moment let us go back to the field equations (23), and (24). Equation (24) remains unchanged, while (28), with a new parameter $\kappa^2 \equiv (1/2)(6 - k)$, takes on the form
$$\Phi_{\mu\nu} = u^\mu_{\omega\nu} - u^\nu_{\omega\mu} = -\kappa^2 w^\mu + 4\pi J^\mu.$$  \hspace{1cm} (35)

In absent of electric currents in a certain region we obtain from (15), and (29)

$$w_{\gamma\nu} = 0,$$  \hspace{1cm} (36)

so that equation (35) may be rewritten in the following form

$$w^\mu_{\Sigma,\nu} + w^\nu R^\mu_\nu + \kappa^2 w^\mu = 0;$$  \hspace{1cm} (37)

with $R^\mu_\nu$ being the Ricci tensor, formed from the usual Christoffel symbols. If the curvature in the current-free region is negligible, one obtains the Proca \[15\] equation for the vector field $w^\mu$

$$w^\mu_{\Sigma,\nu} + \kappa^2 w^\mu = 0.$$  \hspace{1cm} (38)

From the quantum mechanical standpoint this equation describes a particle having spin 1 and mass that in conventional units is given by

$$m_\gamma = (\hbar/c)\kappa = (\hbar/c)\sqrt{\frac{6 - k}{2}},$$  \hspace{1cm} (39)

thus, for $k < 6$ one obtains massive field particles, photons.

In the special case when, $V^\mu = 0$, and $k = 6$, one obtains from (24) $L^\mu = 0$, so that equations (30), and (35) turn into the equations of the Einstein-Maxwell theory, while (34) becomes the usual energy conservation law.

Let us go back to the conservation law (34), and consider the case of vacuum, so that

$$T^\mu_{\nu} = 0; \quad J^\sigma = 0; \quad L^\sigma = 0.$$  \hspace{1cm} (40)

in addition in vacuum condition (36) is holding, and hence one is left with

$$-2\kappa^2(\Phi_{\mu\sigma} + W_{\mu\sigma})w^\sigma + 2V^\nu(V_{\mu;\nu} + V_{\nu;\mu}) + 2V^\mu V^\nu_{\gamma\nu} = 0.$$  \hspace{1cm} (41)

One readily sees from (41) that the condition $V^\mu = 0$, leads to $\kappa = 0$, and hence in absence of magnetic fields massive photons do not exist, so that the classical Maxwell electromagnetism has only massless photons.

From the energy-momentum conservation law (34) taking into account (30), and (41), we can obtain the equation of motion of a test particle, having mass (rest energy) $m_0$ electric charge $\varepsilon_0$, and four-velocity $u_\mu$ in a given external field

$$\frac{du^\mu}{ds} + \{\mu_{\lambda\nu}\} u^\lambda u^\nu + (\varepsilon_0/m_0)u_\nu \Phi^{\mu\nu} = 0.$$  \hspace{1cm} (42)

For a test particle having a magnetic charge $\mu_0$ instead of an electric one we obtain

$$\frac{du^\mu}{ds} + \{\mu_{\lambda\nu}\} u^\lambda u^\nu + (1/2)(\mu_0/m_0)u^\sigma \sqrt{-g} \varepsilon_{\alpha\beta\lambda\sigma}g^{\lambda\mu}W^{\alpha\beta} = 0.$$  \hspace{1cm} (43)

One can imagine a charged test particle moving in the neighbourhood of electrically, and magnetically charged massive bodies. According to (24), (25), and (32) the torsion vector $V^\mu$ is created by magnetic charged bodies, while the Weyl vector $w^\mu$ may be created by bodies having electric charge, as well by a self-inducing Proca term. Now, the structure of the field strength tensors is given by (31), and (32).

Thus, according to (22), on an electrically charged test particle act both, electric, as well magnetic sources. These interactions are possible with both, massive or massless photons. On the other hand, according to (14), a magnetically charged test particle is accelerated by the field $W^{\mu\nu}$, so that it is affected by electrically charged bodies, and by massive photons. One might claim that there is no interaction between magnetic monopoles. However from (41) one sees that massive photons may accompany the magnetic torsion vector field. Thus, two magnetic monopoles interact by means of massive photons. The field of massive photons invoked by a magnetic monopole is considered in section 6.
4 Spherical Symmetry

Suposse there is a particle at rest in the origin, and we consider a region around the particle which is so small that we can neglect the cosmic curvature. The static spherically symmetric line-element may be written as

\[ ds^2 = e\nu' dt^2 - e\lambda' dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) , \]  

(44)

with \( \lambda \) and \( \nu \) being functions of \( r \). From symmetry reasons one can prove that there is only one non-vanishing component of the Weyl vector \( w_\mu \)

\[ w_0 \equiv w(r) , \]  

(45)

and also one non-vanishing component of the magnetic vector \( V_\mu \)

\[ V_0 \equiv V(r) . \]  

(46)

Let us write the Einstein equations (30) in brief as:

\[ G_{\mu\nu} = -8\pi E_\mu^\nu . \]  

(47)

Making use of (22), (27), (28), (31), as well as of (29), we obtain for the metric (44) the following non-zero components of \( E_\mu^\nu \)

\[ 8\pi E_0^0 = 8\pi T_0^0 + e^{-(\lambda + \nu)}(w')^2 - \kappa^2 e^{-\nu} w^2 + 3 e^{-\nu} V^2 ; \]  

(48)

\[ 8\pi E_1^1 = 8\pi T_1^1 + e^{-(\lambda + \nu)}(w')^2 + \kappa^2 e^{-\nu} w^2 + e^{-\nu} V^2 ; \]  

(49)

and

\[ 8\pi E_2^2 = 8\pi E_3^3 = 8\pi T_2^2 - e^{-(\lambda + \nu)}(w')^2 + \kappa^2 e^{-\nu} w^2 + e^{-\nu} V^2 . \]  

(50)

with \( f' \equiv df/dr \). If we substitute (48) - (50) into (47) we obtain the Einstein equations explicitely

\[ e^{-\lambda} \left( \frac{\lambda'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -8\pi T_0^0 - e^{-(\lambda + \nu)}(w')^2 + \kappa^2 e^{-\nu} w^2 - 3 e^{-\nu} V^2 ; \]  

(51)

\[ e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -8\pi T_1^1 - e^{-(\lambda + \nu)}(w')^2 - \kappa^2 e^{-\nu} w^2 - e^{-\nu} V^2 ; \]  

(52)

and

\[ e^{-\lambda} \left[ \nu' + \frac{1}{2}(\nu')^2 + \frac{1}{r}(\nu' - \lambda') - \frac{1}{2} \lambda' \nu' \right] = -8\pi T_2^2 + 2 e^{-(\lambda + \nu)}(w')^2 - 2 \kappa^2 e^{-\nu} w^2 - 2 e^{-\nu} V^2 . \]  

(53)

Taking into account (44), (45), and (46) one can rewrite (53), and (24) accordingly as follows

\[ w'' - \frac{1}{2}(\lambda' + \nu') w' + \frac{2}{r} w' = \kappa^2 e^\lambda w - 4\pi e^\lambda J_0 ; \]  

(54)

and

\[ V'' - \frac{1}{2}(\lambda' + \nu') V' + \frac{2}{r} V' = 2\pi e^\lambda L_0 . \]  

(55)

Integrating (54) one writes

\[ w' = e^{1/2(\lambda + \nu)} \frac{q(r) + \kappa^2 I(r) + Q}{r^2} , \]  

(56)

where the electric charge within a sphere of radius \( r \) is given by

\[ q(r) = -4\pi \int_0^r J_0 e^{1/2(\lambda - \nu)} r^2 dr = 4\pi \int_0^r \rho e^{1/2} r^2 dr , \]  

(57)

the Proca "charge" is given by

\[ I(r) = \int_0^r e^{1/2(\lambda - \nu)} w r^2 dr , \]  

(58)
and \( Q = \text{const} \) is the charge located in the origin. Similarly we obtain from (55)

\[
V' = \frac{e^{1/2(\lambda+\nu)}}{r^2} \left\{ l(r) + \tilde{M} \right\},
\]

where \( \tilde{M} = \text{const} \) stands for the magnetic charge located in the origin, and

\[
l(r) = 2\pi \int_0^r e^{1/2(\lambda-\nu)} L_0 r^2 dr.
\]

In addition, making use of (56), we can rewrite the energy condition (34), stemming from the Bianchi identity, as follows

\[
4\pi \left[ (T_1^1)' + \frac{1}{2} \nu' (T_1^1 - T_0^0) + \frac{2}{r} (T_1^1 - T_2^2) - J_0 w' e^{-\nu} \right] + \left[ 2 \kappa^2 w w' + V V' - V^2 \nu' \right] e^{-\nu} = 0.
\]

We can consider (56) and (59) together with the system of equations (51) - (53). Alternatively we can make use by (61), instead of (53).

5  A Simple Vacuum Solution

One can think about vacuum surrounding the particle, and about massless photons, so that.

\[
T_{\mu}^\nu = 0 \; ; \; J^\sigma = 0 \; ; \; L^\sigma = 0 \; ; \; \kappa = 0 \; ;
\]

and one is left with the following expressions for the fields (cf. (56), (59))

\[
w' = \frac{e^{1/2(\lambda+\nu)}}{r^2} Q,
\]

and

\[
V' = \frac{e^{1/2(\lambda+\nu)}}{r^2} \tilde{M}.
\]

Further, making use of (62), and of (63), one has from (51)

\[
e^{-\lambda} \left( -\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -\frac{Q^2}{r^4} - 3 e^{-\nu} V^2;
\]

On the other hand from (51), and (52) one obtains

\[
e^{-\lambda} \frac{\nu' + \nu}{r} = 2e^{-\nu} V^2,
\]

and by (61) one has from (51)

\[
VV' - V^2 \nu' = 0.
\]

Thus one can write

\[
V = Ke^\nu; \; (K = \text{const}).
\]

Inserting (68) into (66), and integrating we obtain

\[
e^{\lambda+\nu} = \frac{1}{B^2 - K^2 r^2},
\]

with \( B \) being an arbitrary constant.

Let us go back to eq.(55). With an auxiliary function

\[
y = e^{-\lambda};
\]

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and by (69) we can rewrite it as
\[ \frac{1}{r} y' + \frac{1}{r^2} y + \frac{3K^2 y}{B^2 - K^2 r^2} = \frac{1}{r^2} - \frac{Q^2}{r^4} . \] (71)

The solution may be written as
\[ y \equiv e^{-\lambda} = \frac{1}{B} \left( \frac{1}{r^2} - \frac{K^2}{B^2} \right) \left[ Br^2 + Q^2 (B^2 - 2K^2 r^2) \right] + \frac{K_1 (B^2 - K^2 r^2)^{3/2}}{B^3 r} ; \] (72)

and, making use of (69), we get
\[ e^{\nu} = \frac{1}{B^3 r^2} \left[ Br^2 + Q^2 (B^2 - 2K^2 r^2) \right] + \frac{K_1 (B^2 - K^2 r^2)^{1/2}}{B^3 r} , \] (73)

where \( K_1 \) is a constant. Let us take \( K_1 = -2m \) with \( m \) being the particle mass. The two other constants, \( Q \) and \( K \) represent the electric charge and the magnetic field (cf. (63), and (68)) respectively. In order to get for vanishing \( Q \), and \( K \) from (72), (73) the Schwarzschild metric we must set
\[ B = 1 . \] (74)

If magnetic monopoles are absent, we can set \( K = 0 \), and take into account (74), so that (72), (73) yield the Reissner-Nordstrom metric:
\[ e^{\nu} = e^{-\lambda} = 1 + \frac{Q^2}{r^2} - \frac{2m}{r} , \] (75)

Now, let us go back to (64), and (68). Taking into account (74) we have from (68):
\[ V' = \varepsilon K \left( e^{\nu} \right)' = \varepsilon K \left[ \frac{2Q^2}{r^3} + \frac{2m}{r^2(1 - K^2 r^2)^{1/2}} \right] . \] (76)

Where the unit charge (electric, as well as magnetic) \( \varepsilon \) is introduced in order to keep the dimensions identical with those of general relativity. From (76), and (69) one readily sees that (64) is satisfied only for \( Q = 0 \). In this case we have from (72), and (73)
\[ e^{-\lambda} = \left( 1 - K^2 r^2 \right) \left[ 1 - \frac{2m}{r} (1 - K^2 r^2)^{1/2} \right] , \] (77)

and
\[ e^{\nu} = 1 - \frac{2m}{r} (1 - K^2 r^2)^{1/2} ; \] (78)

while for the magnetic field strength we have from (73)
\[ \tilde{\Phi}_{01} = V' = \varepsilon K \left( e^{\nu} \right)' = \frac{2m\varepsilon K}{r^2(1 - K^2 r^2)^{1/2}} . \] (79)

Comparing this with (64), and making use of (69) we obtain
\[ K = \frac{\tilde{M}}{2\varepsilon m} . \] (80)

The solution (77) - (79) is defined for
\[ 0 < r < r_b = \frac{1}{K} = \frac{2\varepsilon m}{\tilde{M}} \] (81)

There exists also such a radius
\[ r = r_s = \frac{2m}{(1 + M^2/\varepsilon^2)^{1/2}} , \] (82)

that from (77), (78) one has
\[ e^{\nu}(r_s) = e^{-\lambda}(r_s) = 0 . \] (83)
Thus the particle is surrounded by a surface, on which the metric is singular. For \( \tilde{M} = 0 \), it turns into the Schwarzschild sphere.

One can imagine a particle having an elementary magnetic charge, given by the Dirac relation \( \tilde{M} = \frac{137}{2}e \) (with \( e \) the electron charge) and a Planckian mass \( m = m_{Pl} \). In general relativistic units one writes

\[
\tilde{M} \approx 9.1 \times 10^{-33}\text{cm} , \quad m \approx 1.608 \times 10^{-33}\text{cm} .
\]  

and this yield

\[
r_s = 3.216 \times 10^{-33}\text{cm} , \quad r_b = 3.53 \times 10^{-1}\text{cm} .
\]  

From the metric (cf. \((\ref{eq:metric1})\), \((\ref{eq:metric2})\)) discussed above

\[
ds^2 = \left(1 - \frac{2m}{r}(1 - K^2r^2)^{1/2}\right)dt^2 - \frac{1}{(1 - K^2r^2)^{1/2}} \left[1 - \frac{2m}{r}(1 - K^2r^2)^{1/2}\right]dr^2 - r^2d\Omega^2 ,
\]

we can turn to a new metric with the radial variable \( R \) by:

\[
r^2 = \frac{R^2}{1 + K^2R^2} ; \quad \frac{R^2}{1 - K^2r^2} ,
\]

one readily sees that the ranges are

\[
0 < R < \infty ,
\]

while for the old radial variable \( r \) we had \((\ref{eq:oldranges})\). For the line-element with the new variable we get

\[
ds^2 = \left(1 - \frac{2m}{R}\right)dt^2 - \frac{1}{(1 + K^2R^2)^2 (1 - 2m/R)}dR^2 - \frac{R^2}{1 + K^2R^2}d\Omega .
\]

and for the magnetic field strength

\[
\tilde{\Phi}_{01} = \frac{\tilde{M}}{R^2} . \quad (90)
\]

The metric \((\ref{eq:newmetric})\), and field strength \((\ref{eq:newfield})\), or alternativelly \((\ref{eq:fieldstrength})\), and \((\ref{eq:fieldstrength})\), may be treated as representing a magnetic monopole. It is worth noting that according to \((\ref{eq:massless})\) a massless electric monopole may exist. But, unlike it, magnetic monopoles have to be massive (cf. \((\ref{eq:massless})\) - \((\ref{eq:massless})\), and \((\ref{eq:massless})\), \((\ref{eq:massless})\)).

6 Proca Field Accompanying a Magnetic Monopole

In the previous section a magnetic monopole in vacuum was considered. We took \( \kappa = 0 \), so that massive photons were excluded from the scenario. But as mentioned above ( see the discussion after \((\ref{eq:monopole})\)) magnetic mopes interact just by means of these massive photons. For that reason it would be useful to consider the case of vacuum, but with \( \kappa \neq 0 \), at least in general. This situation is characterized by:

\[
T^{\mu\nu} = 0 , \quad J^{\mu} = 0 , \quad L^{\mu} = 0 , \quad Q = 0 , \quad \kappa \neq 0 .
\]  

so that the strength of the magnetic field is given as previous by equation \((\ref{eq:fieldstrength})\), while from \((\ref{eq:fieldstrength})\) - \((\ref{eq:fieldstrength})\) one obtains for the strength of the electric field

\[
u' = \kappa^2 \frac{e^{1/2(\lambda + \nu)}}{r^2} I(r) = \kappa^2 \frac{e^{1/2(\lambda + \nu)}}{r^2} \int_0^r e^{1/2(\lambda - \nu)w^2}dr .
\]

With \((\ref{eq:fieldstrength})\), and \((\ref{eq:fieldstrength})\), we can rewrite equation \((\ref{eq:fieldstrength})\) as

\[
e^{-\lambda} \left(-\frac{N}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = -\kappa^4 \frac{L^2}{r^4} + \kappa^2 e^{-\nu}w^2 - 3e^{-\nu}V^2 .
\]

Further we rewrite the difference between \((\ref{eq:fieldstrength})\), and \((\ref{eq:fieldstrength})\) as

\[
e^{-\lambda}(\lambda' + \nu') \frac{e^{-\nu}V^2}{r} = 2e^{-\nu} [V^2 - \kappa^2w^2]
\]

9
We have also the energy relation that follows by (91) from (61)

\[ 2\kappa^2 w_2 + VV' - V^2 \nu' = 0. \]  

(95)

Thus, as usually, we have two Einstein equations (64), and (92), two "equations of state" for the fields (64), and (92), and the energy condition (95), i.e. a complete system of equations.

One can integrate (95), rewriting this as

\[ \kappa^2 w^2 + \frac{1}{2} V^2 = 2 \int V^2 \nu' dr \]  

(96)

Recalling that \( \nu \) is an increasing function beyond the Schwarzschild surface we see that the signs in equation (96) are correct. By (96) we can express the \( w \)-field in terms of the \( V \)-field.

For a moment let us go back to (95). Introducing an auxiliary function

\[ z = V^2. \]  

(97)

one can rewrite (95) as

\[ \frac{1}{2} z' - z \nu' = -\kappa^2 (w^2)' \]  

(98)

and from this one obtains

\[ V^2 \equiv z = e^{2\nu} \left[ -2\kappa^2 \int e^{-2\nu} (w^2)' dr + K^2 \right]; \]  

(99)

where by comparison with (98) the integration constant is set equal to \( K^2 \). We can also rewrite:

\[ V = e^\nu \left[ -2\kappa^2 e^{-2\nu} w^2 - 4\kappa^2 \int e^{-2\nu} w^2 \nu' dr + K^2 \right]^{1/2}; \]  

(100)

Differentiating (100), comparing with (64), and making use of (92) we obtain the following relation:

\[ \left[ -2\kappa^2 e^{-2\nu} w^2 - 4\kappa^2 \int e^{-2\nu} w^2 \nu' dr + K^2 \right] e^{\nu} e^{1/2(\lambda-\nu)} \int \frac{1}{2} e^{1/2(\lambda-\nu)} w^2 dr = \frac{\tilde{M}}{\varepsilon} e^{1/2(\lambda+\nu)} \left[ -2\kappa^2 e^{-2\nu} w^2 - 4\kappa^2 \int e^{-2\nu} w^2 \nu' dr + K^2 \right]^{1/2} = 0; \]  

(101)

Equation (101) may be regarded as the compatibility condition between the energy relation (95), and the field equations (64), and (92). Thus one can choose an appropriate Weyl function \( w \), then from (100) one obtains \( V \), and after this one can solve (64), and (92). However, in order to consider the interaction between two resting magnetically charged bodies, one has to turn from spherical symmetry to cylindrical one. These procedures will be carried out in a subsequent paper.

7 Discussion

The Weyl geometry [8] is doubtless the most aesthetic generalization of the Riemannian geometry, the last being the framework of general relativity. Dirac [9] modified the Weyl theory. In order to build up an action integral, which is coordinate invariant, and gauge invariant, and which agrees with the general relativity theory, Dirac introduced a scalar gauging function, \( \beta \). The Weyl-Dirac theory offers a complete basis for deriving gravitation, and electromagnetism from geometry (cf. e.g. [14], [16]).

In a previous paper [7] it was shown, that extending the Weyl-Dirac theory to a framework including torsion, one can build up a geometrically based Torsional Weyl-Dirac Electrodynamics. This theory, possessing intrinsic electric and magnetic currents and admitting massive photons, is invariant under Weyl gauge transformations, and in the Einstein gauge (\( \beta = 1 \)) it takes a simple form. In the limiting case the theory turns into the Einstein-Maxwell theory.

In this work we have considered some crucial problems and investigated physical phenomena that occur in the discussed framework. In section 3. the energy-momentum conservation law (64) is treated.
It follows that in absence of fields, induced by magnetic charges, the photon mass must be set zero (A similar result, obtained by quite different methods, may be found in a recent paper by R. Kühne [18].) Thus in the limiting Einstein-Maxwell case the photon are massless in accordance with classical electrodynamics. From the same conservation law we have also derived the equation of motion of charged test particles (42), and (43). It turns out that the motion of an electrically charged particle is affected by the presence of both, electric sources and magnetic sources. This interaction may take place either with massless or with massive photons. On the other hand, on a magnetically charged particle the electric sources act in any case, while the bodies carrying magnetic charge can act only by means of massive photons. Thus, massive photons are responsible for the interaction between two magnetic monopoles. In sections 4., 5. several spherically symmetric solutions are considered. In vacuum a solution is obtained that gives either the Reissner-Nordstrøm solution describing an electric monopole, or the metric and field created by a magnetic monopole. An interesting consequence is the absence of massless magnetic monopoles, while in the framework of the Reissner-Nordstrøm solution we can imagine a massless charged particle.

In the theory discussed above magnetic interaction is transmitted by photons having a finite mass. The attractive idea of massive photons was discussed during the last 50 years by many physicists beginning with de Broglie [19] in 1934. Later, Bondi and Lyttleton [20] linked massive photons with the cosmological constant \( \Lambda \). In the modern interpretation one could claim that massive photons cause a dark matter effect in the universe. There exists also a speed-of-light catastrophe in nature, which was elegantly displayed by Ardavan [21]. Massive photons may help to avoid it. Not only in classical theories, but also in quantum field theories a zero-mass photon leads to difficulties and contradictions. Coleman and Weinberg [22] claim that the photon acquire a mass as a result of radiative corrections. Concerning the value of the photons mass, we see no plausible theoretical tools, which may enable fixing \( m_\gamma \) in our framework. Thus one has to consider experimental estimations. In a comprehensive review article by Golhaber, and Nieto, that appeared in 1971 [4] we find the value \( m_\gamma \leq 4 \times 10^{-48} \text{g} \approx 2.25 \times 10^{-15} \text{eV} \), while from a recent Review article [17] we have \( m_\gamma \leq 5.34 \times 10^{-60} \text{g} \approx 3 \times 10^{-27} \text{eV} \). If we adopt the latter value we get from (39) for our parameter \( \kappa \leq 1.52 \times 10^{-22} \text{(cm}^{-1}) \).

There are very few experimental data concerning properties of magnetic monopoles. If one believes that they were produced in the very early universe during the first \( 10^{-34} \text{seconds} \), their mass is \( m \approx 10^{19} \text{MeV} \approx 1.8 \times 10^{-8} \text{g} \). In interesting experiments during the last decade (cf. e.g. [23], [24], [25]) monopoles having mass \( 10^{6} \text{MeV} \leq m \leq 10^{18} \text{MeV} \) were recorded. One can hope that heavier monopoles will be recorded in the future.

If one would like to have a "standard" magnetic monopole, the Planckian mass \( m_{Pl} = 1.22 \times 10^{22} \text{MeV} \) would look very attractive. One could also imagine an elementary carrier of magnetic charge, as a non-singular entity, having charge, mass, and dimensions, and filled with matter possessing mass density, as well magnetic charge density. Building up a model of this particle would be an interesting challenge.

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