Effect of deformation twinning on forming limit analysis of polycrystalline magnesium

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Abstract. A formability assessment using the crystal plasticity model is conducted to investigate the effect of deformation twinning on the formability of polycrystalline magnesium sheet. For presenting the polycrystalline behaviors of magnesium, the homogenization-based finite element method is introduced, and the Marciniak-Kuczyński type sheet necking analysis is demonstrated. Two kinds of deformation twinning model, i.e., the asymmetric slip type and van Houtte’s models are adopted, and the effect of those models on the forming limit analysis is numerically discussed. It is revealed that the lattice reorientation due to deformation twinning plays an important role in plastic flow localization of magnesium, while shear deformation caused by deformation twinning has an insignificant effect.

1. Introduction
Recently, an attention has been drawn to metallic materials with a hexagonal close packed structure in a wide range of engineering fields. Magnesium, which is the lightest of the practical metals, is a typical example of hexagonal metals and has excellent specific rigidity and specific strength. On the other hand, the hexagonal metals generally have poor formability, especially at room temperature because of strong anisotropy at the crystalline scale and deformation twinning. Therefore, it is essential to understand the theoretical characteristics of hexagonal metals to improve their formability. The mechanical properties of magnesium, which is a hexagonal metal at room temperature, are considerably different from those of cubic metals because of two main reasons: the strong anisotropy at the crystalline scale due to the several types of non-identical slip systems, and deformation twinning. Therefore, the effect of these two factors should be considered. As for magnesium, along with slip deformation, deformation twinning is an important deformation mechanism, because magnesium has a limited number of dominant slip systems, i.e., basal slip is the most easily activated and others are hard to be activated. As a result, deformation twinning may strongly affect the formability of magnesium.

In this study, a forming limit assessment using the crystal plasticity model is conducted to investigate the effect of deformation twinning on the formability of polycrystalline magnesium sheet. In the previous study, several numerical assessments of the formability of magnesium alloy using the crystal plasticity were reported [1,2]. A forming limit analysis of pure magnesium with several rolling textures was also demonstrated using the homogenization-based crystal plasticity model [3-5]. In the studies, it was shown that the effect of deformation twinning on the formability of magnesium cannot be negligible. In the present study, the effect of deformation twinning on the forming limit of magnesium sheet is numerically discussed.
2. Numerical procedure

2.1. Constitutive model

The constitutive model adopted in this study is based on the rate-dependent crystal plasticity [6]. In this model, the evolution equation of the slip rate given by

\[ \dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \text{sgn} \left( \tau^{(\alpha)} \right) \frac{1}{g^{(\alpha)}} \]

where \( \tau^{(\alpha)} \) is the resolved shear stress given by \( \tau^{(\alpha)} = \mathbf{s}^{(\alpha)} \cdot (\mathbf{\sigma} \cdot \mathbf{m}^{(\alpha)}) \), and \( \text{sgn}(x) = 1 \) if \( x \geq 0 \) and \( \text{sgn}(x) = -1 \) if \( x < 0 \). \( \dot{\gamma}_0 \) is a material constant and \( m \) characterizes the rate sensitivity of material. \( g^{(\alpha)} \) is a hardening function of each slip system. The evolution equation of hardening function presented by Graff et al. is used [7].

\[ g^{(\alpha)} = \sum_{\beta} h_{\alpha\beta} \dot{\gamma}^{(\beta)} \]  
(2)

\[ h_{\alpha\beta} = \begin{cases} h(\gamma_a) & (\alpha = \beta) \\ q_{\alpha\beta} h(\gamma_a) & (\alpha \neq \beta) \end{cases} \]  
(3)

\[ \gamma_a = \sum_{\alpha} \int_{0}^{\tau} \dot{\gamma}^{(\alpha)} d\tau \]  
(4)

The matrices \( q_{\alpha\beta} \) describe the latent hardening, and the subscripts A and B indicate slip or twinning system families of the systems \( \alpha \) and \( \beta \), respectively, i.e., one of basal, prismatic, pyramidal slips, and twinning. The strain hardening functions are as follows:

\[ h(\gamma_a) = h_0 \]  
(5)

\[ h(\gamma_a) = h_0 \left( 1 - \frac{\tau_a}{\tau_\infty} \right) \exp \left( -\frac{h_0 \gamma_a}{\tau_\infty} \right) \]  
(6)

Equations (5) and (6) are used for basal and non-basal slips, respectively. For deformation twinning, Eq. (5) is adopted. The detailed parameters for the hardening functions are identical to Tadano [5].

If the deformation twinning occurs on a specified twinning plane, the crystal lattice in the twinned region takes the mirrored configuration of the original crystal lattice. The geometrical relation between twinned and untwinned regions is expressed with an orthogonal tensor \( T^{\text{twin}} \):

\[ \mathbf{m}^{(\alpha)} = T^{\text{twin}} \cdot \mathbf{m}^{(\alpha)} \]  
(7)

\[ \mathbf{s}^{(\alpha)} = T^{\text{twin}} \cdot \mathbf{s}^{(\alpha)} \]  
(8)

\[ T^{\text{twin}} = \mathbf{I} - 2 \left( \mathbf{m}^{\text{twin}} \otimes \mathbf{m}^{\text{twin}} \right) \]  
(9)

\( \mathbf{m}^{\text{twin}} \) is the vector normal to the twinning plane, and \( \mathbf{m}^{(\alpha)} \) and \( \mathbf{s}^{(\alpha)} \) are the normal to the slip plane and the slip direction vectors in the untwinned region, and \( \mathbf{m}^{(\alpha)} \) and \( \mathbf{s}^{(\alpha)} \) are those in the twinned region. The norm of each vector is unity. \( \mathbf{I} \) is the second order unit tensor. In this study, \{10\bar{1}2\} \{\bar{1}1\bar{2}0\} tensile twinning, which is the most typical twinning in magnesium, is introduced. Although \{10\bar{1}1\} \{10\bar{1}2\} compressive twinning may also be an important deformation mechanism in magnesium, the present study does not introduce it according to the previous literature [5,6].

For the criterion of the onset of reorientation due to deformation twinning, the method proposed by van Houtte [8] is adopted. For comparison, the asymmetric slip type deformation twinning model [7], in which the lattice reorientation expressed by Eq. (7)-(9) is not introduced, is also adopted.
To extend the above model for representing polycrystalline behaviors, the homogenization-based finite element method [3,4] is used.

2.2. Sheet necking formulation
In this study, the Marciniak-Kuczyński (M-K) type sheet necking problem [9] is considered. A bulk specimen involving a thin band with an initial inhomogeneity with unit normal \( n \) is introduced, and the strain fields outside the band are assumed to always be uniform. The quantities inside and outside the band are denoted by the superscripts \( b \) and \( o \), respectively. The compatibility and equilibrium conditions at the band interface are written as

\[
L^b = L^o + \dot{c} \otimes n \tag{10}
\]

\[
n \cdot \sigma^b = n \cdot \sigma^o \tag{11}
\]

where \( L \) is the velocity gradient tensor and \( \sigma \) is the homogenized Cauchy stress tensor. \( \dot{c} \) is a vector parameter to be determined as a solution of this problem. Introducing the general constitutive equation for rate-dependent materials given as \( \sigma = \tilde{C} : \dot{L} - \tilde{P} \), we finally obtain the following equilibrium equation in the present framework:

\[
\left[ n \cdot \tilde{C}^b \cdot n \right] \cdot \dot{c} = \tilde{n} \cdot (\sigma^o - \sigma^b) + n \left( \tilde{\sigma}^o - \tilde{C}^b : \dot{L}^o + \tilde{P}^b \right) \tag{12}
\]

The combination of the M-K type formulation and the homogenization-based finite element method is found in the previous literatures [10].

3. Numerical results
In the present analysis, a typical rolling textured magnesium sheet, in which the c-axes of most crystal grains are almost perpendicular to the sheet specimen, is considered. The number of crystal grains is set to 512. As mentioned in the previous section, two kinds of deformation twinning model, i.e., the asymmetric slip type model and the van Houtte’s model, are introduced. The forming limit analysis without deformation twinning is also conducted.

Forming limit diagrams (FLDs) for the assumed conditions are shown in Fig. 1. The FLDs without
twinning and with the asymmetric slip model are almost identical, although they are slightly different under uniaxial tension. In contrast, the FLD obtained by the van Houtte’s model obviously differs from others. The histories of the relative activity of each slip and twinning system under uniaxial and equibiaxial tensions are illustrated in Fig. 2. The relative activities seem independent from the deformation twinning model, meaning deformation twinning has an insignificant effect on relative activities of slip systems. This is a reason why the asymmetric slip model provides almost same results as that without twinning. An essential difference between the van Houtte’s and the asymmetric slip

![Figure 2](image_url)

**Figure 2.** Histories of relative activity of each slip system: (a) uniaxial tension without twinning, (b) equibiaxial tension without twinning, (c) uniaxial tension with asymmetric slip model, (d) equibiaxial tension with asymmetric slip model, (e) uniaxial tension with van Houtte’s model, and (f) equibiaxial tension with van Houtte’s model. Blue, red, purple and green lines denote basal, prismatic and pyramidal slips and deformation twinning.
models is the presence or absence of lattice reorientation due to deformation twinning. Therefore, the lattice reorientation may play an important role in plastic flow localization of magnesium, and the contribution of shear deformation caused by deformation twinning may be negligible.

4. Conclusion
The plastic flow localization analysis for polycrystalline magnesium using the homogenization-based finite element method is conducted, and the effect of deformation twinning is discussed. The asymmetric slip type twinning model provides the FLD almost identical to that without deformation twinning, when the van Houtte’s model gives an obvious different FLD. The present result suggests that the lattice reorientation due to deformation twinning may play an important role in plastic flow localization of magnesium and shear deformation caused by deformation twinning has an insignificant effect.

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