Modeling and simulation of ocean wave propagation using lattice Boltzmann method

Dian Nuraiman
Graduate School of Natural Sciences and Technology, Kanazawa University, Japan
Department of Mathematics, UIN Sunan Gunung Djati Bandung, Indonesia
E-mail: dianuraiman@uinsgd.ac.id

Abstract. In this paper, we present on modeling and simulation of ocean wave propagation from the deep sea to the shoreline. This requires high computational cost for simulation with large domain. We propose to couple a 1D shallow water equations (SWE) model with a 2D incompressible Navier-Stokes equations (NSE) model in order to reduce the computational cost. The coupled model is solved using the lattice Boltzmann method (LBM) with the lattice Bhatnagar-Gross-Krook (BGK) scheme. Additionally, a special method is implemented to treat the complex behavior of free surface close to the shoreline. The result shows the coupled model can reduce computational cost significantly compared to the full NSE model.

1. Introduction
Observation of behavior of ocean wave propagation is one of the most important issues on ocean and coastal engineering research field. The big problem arise when considering large domain. The larger computational domain, the higher computational cost.

In order to reduce computational cost for large domain simulation, it is necessary to find a coupling strategy between a cheap model with a precise model. Some coupling strategies have been proposed, such as coupling a Boundary Element Model (BEM) for the potential flow equations with a Volume of Fluid (VOF) model for the NSE [1], a 1D model of Boussinesq equations with a 2D SPH model for the NSE [2], [3], and a 2D SWE model with a 3D NSE model using LBM [4].

This paper presents on modeling and simulation of ocean wave propagation from the deep sea to the shoreline. In order to save the computational cost, we propose to couple a 1D SWE model for the deep sea with a 2D incompressible NSE model close to the shoreline. We need a cheap model for the deep sea as it represents the largest part of computational domain and a precise model close to the shoreline where waves break and overturn. The coupled model is then compared to the full NSE model.

2. Models
2.1. Incompressible Navier-Stokes Equations (NSE)
The NSE are the governing equation in describing general fluid flows. The incompressible NSE consist of the continuity and momentum equation which can be written in tensor form as,
respectively,
\[
\frac{\partial u_j}{\partial x_j} = 0
\]  
(1)
and
\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i
\]  
(2)

The Enstein summation convention is used in the above equations.

2.2. Shallow Water Equations (SWE)

The SWE can be derived from incompressible NSE by taking depth-integration under assumption the wavelength is much larger than the corresponding depth. The SWE can be written in tensor form for the continuity equation
\[
\frac{\partial h}{\partial t} + \frac{\partial (hu_j)}{\partial x_j} = 0
\]  
(3)
and the momentum equation
\[
\frac{\partial (hu_i)}{\partial t} + \frac{\partial (hu_i u_j)}{\partial x_j} = - \frac{\partial}{\partial x_i} \left( \frac{gh^2}{2} \right) + \nu \frac{\partial^2 (hu_i)}{\partial x_j^2} + F_i
\]  
(4)

3. Methods

The lattice Boltzmann method (LBM) was originated from Boltzmann’s kinetic theory of gases. The basic idea is the fluid can be imagined containing a lot of particles. The LBM does not consider behavior of each particle but behavior of a collection of particles as a unit represented by a distribution function. The LBM can be used to solve both the SWE model [5] and the NSE model [4], [6].

The LBM can be derived from Boltzmann equation by taking first order discretization in discrete phase-space. Basic formulation of the lattice Boltzmann equation in discrete form with the lattice BGK model is given by
\[
f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) - \frac{1}{\tau} (f_i(x, t) - f_i^{eq}(x, t))
\]  
(5)
where \( f \) is distribution functions (DFs), \( f^{eq} \) is local equilibrium distribution function, \( i \) is direction indice, and \( \tau \) is relaxation parameter.

The LBM is a mesh-based method. The computational domain is divided into Cartesian grid. There are two main steps in the LBM namely streaming and collision step. In the streaming step, the particles move to neighboring cells in direction of their lattice velocities. The streaming step can be formulated as
\[
f^*_i(x, t) = f_i(x + e_i \Delta t, t)
\]  
(6)
where \( f^*_i \) denote DFs after streaming step.

In the collision step, the arriving particles at the cells collide with one another and change their velocity directions toward their equilibrium state. The collision step can be written as
\[
f_i(x, t + \Delta t) = f^*_i(x, t) - \frac{1}{\tau} (f^*_i - f_i^{eq})
\]  
(7)
where \( \tau \) represents the fluid viscosity with \( \tau = 3\nu + \frac{1}{2} \).
3.1. LBM for a 2D Incompressible NSE Model

The most commonly used lattice pattern in 2D simulation is D2Q9 model which has nine lattice directions as shown in the left part of Figure 1. Macroscopic density and velocity of fluid can be written as, respectively,

\[ \rho(x, t) = \sum_{i=0}^{8} f_i(x, t) \]  
\[ u(x, t) = \sum_{i=0}^{8} e_i f_i(x, t) \]

where \( f_i^{eq} \) is given by

\[ f_i^{eq} = w_i \left[ \rho + 3 e_i \cdot u - \frac{3}{2} u^2 + \frac{9}{2} (e_i \cdot u)^2 \right] \]  

3.2. LBM for a 1D SWE Model

For SWE model, we use a 1D lattice pattern of D1Q3 which has three lattice directions as shown in the right part of Figure 1. Macroscopic water depth and velocity are given by, respectively,

\[ h(x, t) = \sum_{i=0}^{2} g_i(x, t) \]  
\[ v(x, t) = \frac{1}{h(x, t)} \sum_{i=0}^{2} e_i g_i(x, t) \]

Local equilibrium distribution function \( g_i^{eq} \) is given by

\[ g_i^{eq} = \begin{cases} h + w_i h(-\frac{3gh}{2c^2} - \frac{3v^2}{2c^2}) & i = 0 \\ w_i h(\frac{3gh}{2c^2} + \frac{3e_i u^2}{c^2} + \frac{3e_i^2 v^2}{c^4}) & i = 1, 2 \end{cases} \]  

3.3. Free Surface Treatment

The free surface treatment method is only used for the NSE model. The method was proposed by Thürey [7]. This requires two additional values to be stored for each cell, the mass \( m \) and the fluid fraction \( \epsilon \), where

\[ \epsilon(x, t) = \frac{m(x, t)}{\rho(x, t)} \]

This method does not consider the gas phase as a separate fluid under assumption the viscosity difference between gas and fluid is high enough to approximate gas velocity near the surface with the surface velocity [8].
3.4. Parametrization
Given $S$ as domain length $[m]$, $r$ as grid resolution, and $g_c$ as compressibility constant, simulation parameters are set as shown in Table 1.

| Parameters          | Real Unit                  | Lattice Unit                  |
|---------------------|----------------------------|-------------------------------|
| Cell size $\Delta x'$ | $\frac{S}{r} [m]$         | $\Delta x = 1$                |
| Time step $\Delta t'$ | $\sqrt{\frac{g_c \Delta x'}{g'}} [s]$ | $\Delta t = 1$                |
| Gravitational       | $\frac{g'}{\Delta t'} [m^2 s^2]$ | $g = g' \frac{\Delta t'^2}{\Delta x'^2}$ |
| Kinematic viscosity | $\nu' [\frac{m^2}{s}]$    | $\nu = \nu' \frac{\Delta t'^2}{\Delta x'^2}$ |

Table 1: Simulation Parameters

4. Coupled Model
The two models are coupled by constructing a buffer zone to transfer information between two models. The coupling design as shown in Figure 2 where $N_{sw}$ and $N_{ns}$ are number of grid points in length of the SWE region and the NSE region, respectively, and $N_b$ is number of grid points in the buffer zone.

Both models have different characteristics. There is no depth in NSE model. Furthermore, SWE model has no velocity in vertical direction. There are two coupling procedures, SWE to NSE transfer and vice versa.

4.1. SWE to NSE transfer
The main idea is depth information from the SWE region used to define fluid cells in the NSE region by adding or removing some fluid or interface cells at the leftmost two column of NSE region. The DFs of fluid or interface cells are initialized by the SWE velocity

$$f_k = f_k^{eq}(\rho, u_{ij})$$

where $k = 0, 1, ..., 8$ and

$$u_{ij} = (s_v^v u_{N_{sw} - N_b + i}, s_v w_i)$$

where $s_v$ is scaling factor that was determined based on numerical experiments. Moreover, velocity in vertical direction is calculated by change of the SWE depths of the current and previous time step over time.
4.2. NSE to SWE transfer
Depth of SWE region at the right boundary is determined by depth of fluid cells of NSE region. The DFs are then initialized by taking the average velocity \( u_{\text{avg}} \) in horizontal direction over the right column given by

\[
g_i = g^\text{eq}_i (H_{N_b-1}, u_{\text{avg}})
\]

where \( i = 0, 1, 2 \).

5. Results
Length of computational domain is 26 meters with time step for SWE model \( \Delta t_{sw} = 0.001 \), time step for NSE model \( \Delta t_{ns} = 0.0002 \), and the scaling factor for SWE velocity \( s_v = 0.015 \). A wall boundary condition is implemented at left and right boundary. The initial condition of wave at the first half domain with zero initial velocity as shown in figure below

![Figure 3: Initial condition of wave](image)

Figure 4 shows the result at the second half domain plotted based on velocity at several times with two black vertical lines as boundaries of buffer zone. We can see how the wave is coming from the left domain, passing the buffer zone, approaching the shoreline, and going back to the deep sea. By this coupling strategy, ocean wave smoothly propagated from the deep sea to the shoreline without any reflected parasitic wave when passing the buffer zone.

The computation time for the coupled model was 5 minutes. It was compared with the full NSE model that requires 34 minutes to solve the same domain. Hence, the coupled model can reduce the computational cost significantly up to 85% in case of the setting in Figure 4 when compared to the full NSE model.
6. Conclusion
We have presented a coupling strategy between the SWE model and the NSE model for ocean wave propagation using the LBM. However, two points that remains are to provide a theoretical justification for the velocity scaling used in the SWE to NSE transfer and to verify the coupled model on benchmark problems.

Acknowledgment
I would like to thank Assoc. Prof. Karel Svadlenka for his help, support and suggestions in this work.

References
[1] Lachaume C, Biausser B, Grilli S T, Fraunie P and Guignard S 2003 Modeling of breaking and post-breaking waves on slopes by coupling of BEM and VOF methods Proc. 13th Int. Offshore Polar Eng. Conf. pp 353–359
[2] Narayanaswamy M S 2009 A hybrid Boussinesq-SPH wave propagation model with applications to forced waves in rectangular tanks PhD Dissertation (Maryland: The Johns Hopkins University)
[3] Kassiotis C, Ferrand M, Violeau M, Rogers B D, Standsby P K and Benoit M 2011 Coupling SPH with a 1-D Boussinesq-type wave model 6th International SPHERIC Workshop pp 241–247
[4] Thürey N, Rüde U and Stamminger M 2006 Animation of open water phenomena with coupled shallow water and free surface simulations Proc. of the 2006 ACM SIGGRAPH/Eurographics Symposium on Computer Animation pp 157–164
[5] Koshimura S and Murakami K 2009 Applicability of the lattice Boltzmann method for tsunami modeling J. Japan Soc. Civil Engrs. 65 pp 256–260
[6] Kwak Y, Kuo J C C and Nakano A 2009 Hybrid lattice-Boltzmann/level-set method for liquid simulation and visualization, Int. J. Comput. Sci. 3 pp 579–592
[7] Thürey N 2007 Physically based animation of free surface flows with the lattice Boltzmann method PhD Dissertation (Nürnberg: University of Erlangen)
[8] Thürey N and Rüde U 2009 Stable free surfaces flows with the lattice Boltzmann method on adaptively coarsened grids Comput. Vis. Sci. 12 pp 247–263