We present a model-independent analysis of the mass spectrum of nonstrange $\ell = 1$ baryons in large $N_c$ QCD. The $1/N_c$ expansion is used to select and order a basis of effective operators that spans the nine observables (seven masses and two mixing angles). Comparison to the data provides support for the validity of the $1/N_c$ expansion, but also reveals that only a few nontrivial operators are strongly preferred. We show that our results have a consistent interpretation in a constituent quark model with pseudoscalar meson exchange interactions.

1 Introduction

It has been known for some time that QCD admits a useful and elegant expansion in powers of $1/N_c$, where $N_c$ is the number of colors. Given this expansion, it is possible to determine the order in $N_c$ of any Feynman diagram or matrix element. The $1/N_c$ expansion has been utilized successfully in baryon effective field theories to isolate the leading and subleading contributions to a variety of physical observables.

Here we study the mass spectrum of the nonstrange, $\ell = 1$ baryons (associated with the SU(6) $70$-plet for $N_c = 3$) in a large-$N_c$ effective theory. We describe the states as a symmetrized “core” of $(N_c - 1)$ quarks in the ground state plus one excited quark in a relative $P$ state. “Quarks” in the effective theory refer to eigenstates of the spin-flavor-orbit group, SU(6) $\times$ O(3), such that an appropriately symmetrized collection of $N_c$ of them have the quantum numbers of the physical baryons. Baryon wave functions are antisymmetric in color and symmetric in the spin-flavor-orbit indices of the quark fields. While this construction assures that we obtain states with the correct total quantum numbers, we do not assume that SU(6) is an approximate symmetry of the effective Lagrangian. Rather, we parameterize the most general way in which spin and flavor symmetries are broken by introducing a complete set of quark operators that act on the baryon states. Matrix elements of these operators are hierarchical in $1/N_c$, so that predictivity can be obtained without recourse to ad hoc phenomenological assumptions.

The nonstrange $70$-plet states which we consider in this analysis consist of two isospin-$3/2$ states, $\Delta_{1/2}$ and $\Delta_{3/2}$, and five isospin-$1/2$ states, $N_{1/2}$, $N'_{1/2}$, $N_{3/2}$, $N'_{3/2}$, and $N'_{5/2}$. The subscript indicates total baryon spin; unprimed states have quark spin $1/2$ and primed states have quark spin $3/2$. These quantum numbers imply that two mixing angles, $\theta_{N_{1}}$ and $\theta_{N_{3}}$, are necessary to specify the total
angular momentum 1/2 and 3/2 nucleon mass eigenstates, respectively. Thus we may write

\[
\begin{bmatrix}
N(1535) \\
N(1650)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{N1} & \sin \theta_{N1} \\
-\sin \theta_{N1} & \cos \theta_{N1}
\end{bmatrix} \begin{bmatrix}
N_{1/2} \\
N'_{1/2}
\end{bmatrix}
\]

(1)

and

\[
\begin{bmatrix}
N(1520) \\
N(1700)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{N3} & \sin \theta_{N3} \\
-\sin \theta_{N3} & \cos \theta_{N3}
\end{bmatrix} \begin{bmatrix}
N_{3/2} \\
N'_{3/2}
\end{bmatrix}
\]

(2)

where the \(N(1535), N(1650), N(1520)\) and \(N(1700)\) are the appropriate mass eigenstates observed in experiment.

2 Operators Analysis

To parameterize the complete breaking of SU(6)×O(3), it is natural to write all possible mass operators in terms of the generators of this group. The generators of orbital angular momentum are denoted by \(\ell^i\), while \(S^i, T^a, \) and \(G^{ia}\) represent the spin, flavor, and spin-flavor generators of SU(6), respectively. The generators \(S^c_i, T^a_c, G^{ia}_c\) refer to those acting upon the \(N_c - 1\) core quarks, while separate SU(6) generators \(s^i, t^a, \) and \(g^{ia}\) act only on the single excited quark. Factors of \(N_c\) originate either as coefficients of operators in the Hamiltonian, or through matrix elements of those operators. An \(n\)-body operator, which acts on \(n\) quarks in a baryon state, has a coefficient of order \(1/N_c^{n-1}\), reflecting the minimum number of gluon exchanges required to generate the operator in QCD. Compensating factors of \(N_c\) arise in matrix elements if sums over quark lines are coherent. For example, the unit operator contributes at \(O(N_c^1)\), since each core quark contributes equally in the matrix element. The core spin of the baryon \(S^c_i\) contributes to the masses at \(O(1/N_c)\), because the matrix elements of \(S^c_i\) are of \(O(N_c^0)\) for baryons that have spins of order unity as \(N_c \to \infty\). Similarly, matrix elements of \(T^a_c\) are \(O(N_c^0)\) in the two-flavor case since the baryons considered have isospin of \(O(N_c^0)\), but the operator \(G^{ia}_c\) has matrix elements on this subset of states of \(O(N_c^1)\). This means that the contributions of the \(O(N_c)\) quarks add incoherently in matrix elements of the operator \(S^c_i\) or \(T^a_c\) but coherently for \(G^{ia}_c\). Thus, the full large \(N_c\) counting of the matrix element is \(O(N_c^{1-n+m})\), where \(m\) is the number of coherent core quark generators.

A complete operator basis for the nonstrange 70-plet masses is shown in Table 1. Index contractions are left implicit wherever they are unambiguous, and the \(c_i\) are operator coefficients. The tensor \(\ell^{(2)}_{ij}\) represents the rank two tensor combination of \(\ell^i\) and \(\ell^j\) given by \(\ell^{(2)}_{ij} = \frac{1}{2}(\ell_i, \ell_j) - \frac{1}{3} \delta_{ij}\). Note in Table 1 that operators 1, 2–3, and 4–9 have matrix elements of order \(N_c^1, N_c^0\), and \(N_c^{-1}\), respectively.

3 Results

Since the operator basis in Table 1 completely spans the 9-dimensional space of observables, we can solve for the \(c_i\) given the experimental data. For each baryon mass, we assume that the central value corresponds to the midpoint of the mass

Some of these operators have been studied previously.
Table 1: A complete operator basis, $O_i$, $i = 1 \ldots 9$, for the nonstrange 70-plet masses.

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|-------|-------|-------|-------|-------|
| 1     | $\ell s + \frac{4}{N_c} \ell t G_c$ | $\frac{1}{N_c} \ell S_c$ | $\frac{1}{N_c} \ell^{(2)} g G_c$ | $\frac{1}{N_c} S_c^2$ |

Table 2: Operator coefficients in GeV, assuming the complete set of Table 1.

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|-------|-------|-------|-------|-------|
| +0.470 | -0.036 | +0.369 | +0.089 | +0.087 |
| ±0.017 | ±0.041 | ±0.208 | ±0.203 | ±0.157 |
| $c_6$ | $c_7$ | $c_8$ | $c_9$ |
| +0.418 | +0.040 | +0.048 | +0.012 |
| ±0.085 | ±0.074 | ±0.172 | ±0.673 |

Naively, one expects the $c_i$ to be of comparable size. Using the value of $c_1$ as a point of comparison, it is clear that there are no operators with anomalously large coefficients. Thus, we find no conflict with the naive $1/N_c$ power counting rules. However, only three operators of the nine, $O_1$, $O_3$, and $O_6$, have coefficients that are statistically distinguishable from zero! A fit including those three operators alone is shown in Table 3 and has a $\chi^2$ per degree of freedom is 1.87. Fits involving other operator combinations are studied in Refs. 3, 4. Clearly, large $N_c$ power counting is not sufficient by itself to explain the $\ell = 1$ baryon masses—the underlying dynamics plays a crucial role.

4 Interpretation and Conclusions

We will now show that the preference in Table 2 for two nontrivial operators, $\frac{1}{N_c} \ell^{(2)} g G_c$ and $\frac{1}{N_c} S_c^2$, can be understood in a constituent quark model with a single pseudoscalar meson exchange, up to corrections of order $1/N_c^2$. The argument goes as follows:

The pion couples to the quark axial-vector current so that the $\overline{q}q \pi$ coupling introduces the spin-flavor structure $\sigma^a \tau^a$ on a given quark line. In addition, pion exchange respects the large $N_c$ counting rules given in Section 2. A single pion exchange between the excited quark and a core quark is mapped to the operators...
Table 3: Three parameter fit using operators $O_1$, $O_3$, and $O_6$, giving $\chi^2$/d.o.f. = 11.19/6 = 1.87. Masses are given in MeV, angles in radians.

| Mass   | Fit $\Delta(1700)$ | Exp. $\Delta(1700)$ | Fit $\Delta(1620)$ | Exp. $\Delta(1620)$ | Fit $N(1675)$ | Exp. $N(1675)$ | Fit $N(1700)$ | Exp. $N(1700)$ |
|--------|------------------|------------------|------------------|------------------|--------------|--------------|--------------|--------------|
| $\Delta(1700)$ | 1683 | 1720 ± 50 | $\Delta(1620)$ | 1645 ± 30 | $N(1520)$ | 1530 | 1523 ± 8 | $N(1535)$ | 1503 | 1538 ± 18 |
| $N(1675)$ | 1673 | 1678 ± 8 | $N(1700)$ | 1725 | 1700 ± 50 | $N(1650)$ | 1663 | 1660 ± 20 | $\theta_{N_1}$ | 0.45 | 0.61 ± 0.09 |
| $N(1675)$ | 1673 | 1678 ± 8 | $N(1700)$ | 1725 | 1700 ± 50 | $N(1650)$ | 1663 | 1660 ± 20 | $\theta_{N_3}$ | 3.04 | 3.04 ± 0.15 |

Parameters (GeV): $c_1 = 0.461 ± 0.005$, $c_3 = 0.360 ± 0.059$, $c_6 = 0.453 ± 0.030$

$g^{ia}G^{ja}\ell^{(2)}_{ij}$ and $g^{ia}G^{ia}$, while pion exchange between two core quarks yields $G^{ia}G^{ia}$. These exhaust the possible two-body operators that have the desired spin-flavor structure. The first operator is one of the two in our preferred set. The third operator may be rewritten

$$2G^{ia}G^{ia} = C_2 \cdot 1 - \frac{1}{2} T^{ia}T^{ia} - \frac{1}{2} S_c^2$$

where $C_2$ is the SU(4) quadratic Casimir for the totally symmetric core representation (the 10 of SU(4) for $N_c = 3$). Since the core wavefunction involves two spin and two flavor degrees of freedom, and is totally symmetric, it is straightforward to show that $T^{ia}_c = S^2_c$. Then Eq. (3) implies that one may exchange $G^{ia}G^{ia}$ in favor of the identity operator and $S^2_c$, the second of the two operators suggested by our fits.

The remaining operator, $g^{ia}G^{ia}$, is peculiar in that its matrix element between two nonstrange, mixed symmetry states is given by

$$\frac{1}{N_c} \langle gG \rangle = -\frac{N_c + 1}{16N_c} + \delta_{S,L} \frac{I(I + 1)}{2N_c^2},$$

which differs from the identity only at order $1/N_c^2$. Thus to order $1/N_c$, one may make the replacements

$$\{1, g^{ia}G^{ia} \ell^{(2)}_{ij}, g^{ia}G^{ia}, G^{ia}G^{ia}\} \Rightarrow \{1, g^{ia}G^{ia} \ell^{(2)}_{ij}, S_c^2\}.$$  

We conclude that the operator set suggested by the data may be understood in terms of single pion exchange between quark lines. This is consistent with the interpretation of the mass spectrum advocated by Glozman and Riska. Other simple models, such as single gluon exchange, do not directly select the operators suggested by our analysis and may require others that are disfavored by the data.

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