1. introduction

Transport in two-dimensional (2D) electronic systems has been a subject of intense research for several decades. One of the reasons for the attractiveness of this field is that quantum interference is strong in 2D and interesting phenomena exist, such as the quantum Hall effect. Despite its long history, some aspects of electronic transport are still puzzling. For some time there was a consensus about the role disorder plays in transport processes, dominated by Anderson localization of electronic wave functions for conventional 2D systems [1–3]. A first hint, though, for an unconventional behavior was the transition between Hall plateaux in quantum Hall systems, which indicated the existence of a metallic state in a 2D electronic system under special conditions [4]. Even more important was the discovery of metallic states in graphene [5–7] and in a number of chemical compounds, commonly referred to as topological insulators [8–11], where the band structure has nodes and the dispersion of electronic quasiparticles is linear in the vicinity of these nodes. Although these compounds represent pristine 2D systems, they reveal a finite DC conductivity which is very robust against thermal fluctuations and disorder. In the course of subsequent years these systems underwent careful studies from both the experimental and the theoretical point of view, which clearly indicate that the finite DC value is a robust property. In the experiments the following features have been observed for the conductivity: (1) It decreases with increasing sample size to the DC value, starting from some value considerably higher than this [12]; (2) For a finite sample it exhibits a pronounced temperature dependence, decreasing logarithmically with decreasing temperature to a plateau at low temperatures [13–20]. The plateau value varies only slightly from sample to sample. However, the latter effect is also observable in conventional 2D metals where this saturation is usually linked to the presence of magnetic impurities [21]. The same argumentation is sometimes used in the context with the thermal conductivity saturation in 2D electron gases with nodal points [20]. This argument gives rise to the question of what would happen if we remove these impurities: would the conductivity decrease to negative values logarithmically without limit?

In the field–theoretical approach it is natural to use the density–density Kubo formula, which predicts a nonzero plateau [22–26], whose values are always non-negative. The current–current Kubo formula, on the other hand, leads to an infinite negative conductivity in the weak–localization approach. This unphysical result is usually avoided by a phenomenological inelastic scattering cut-off for small momentum transfer...
2. The Kubo–Greenwood conductivity formula

Motivated by diffusion, we consider the field theoretical version of the Kubo–Greenwood conductivity formula \[ \sigma_{\mu \nu}(\omega) = \frac{e^2}{h} \int dE \Gamma_\mu(E, \omega) \frac{f(E) - f(E + \omega)}{\omega}, \] where \( \omega \) denotes the physical frequency, \( f(E) \) the Fermi function, and the disorder averaged kernel
\[ \Gamma_\mu(E, \omega) = -\omega^2 T \sum_r r_r^2 \langle (G^+(r, 0) G^-(0, r)) \rangle. \]

3. Weak random scattering approach

We approach the DC conductivity within a weak scattering approach, in which the disorder average is performed perturbatively. At frequencies small as compared to the typical band width of the clean system and below room temperature we can employ the usual approximation \( f(E + \omega) \sim f(E) + \omega \delta f(E) \), which trivializes the energy integral in equation (1). Then the conductivity formula becomes
\[ \sigma_{\mu \nu}(\omega) = \frac{e^2}{h} \lim_{\epsilon \to 0} \epsilon^2 \left( \frac{\partial^2}{\partial q_\mu^2} \right) \sum_r \sum_{q=0}^\infty e^{i q r} \langle G_{\mu q}^+(r, 0) G_{\nu q}^-(0, r) \rangle. \]
where we have used the Fourier representation of the position operator and the summation convention for matrix elements with respect to the spinor index. The averaged two-particle Green’s function $\langle G_{ii}(r,0)G_{ji}(0,r) \rangle$ can be treated within a perturbation expansion in powers of a weak scattering rate $\eta$. Using a summation of ladder and maximally crossed diagrams leads to

$$\tilde{\delta}_{ij} = \frac{e^2}{h} \lim_{\epsilon \to 0} \epsilon \left( -\frac{\partial^2}{\partial q^2} \right)^2 \sum_{ij} \int d^2p \left[ \tilde{G}^+(p) \Sigma \tilde{G}^-(p) \right]_{ii} \times \left[ \tilde{G}^+(p) \Sigma \tilde{G}^-(p) \right]_{jj} \,,
$$

where the full transposition operator $\mathbf{T}$ applies to all degrees of freedom, i.e. to the spatial ones as well. The renormalized one-particle Green’s functions read

$$\tilde{G}^\pm(r,r') = \langle r | \pm \pm H^{-1}| r' \rangle \,,
$$

with $z = \epsilon + \eta$. The scattering rate $\eta$ is determined self-consistently from

$$\pm \frac{\eta}{g} = -\langle r | \pm \pm H^{-1}| r \rangle = \int \frac{d^2p}{(2\pi)^2} \frac{\pm \pm \pm}{p^2 + z^2} \,.
$$

In the field theoretic language this condition defines the saddle point of the functional integral, around which an expansion in powers of a small $\eta$ can be performed. For $\epsilon \sim 0$ we get

$$1 - g \int \frac{d^2p}{(2\pi)^2} \frac{1}{p^2 + z^2} \sim \frac{\epsilon}{\eta} \,.
$$

We introduce for the terms in the second line of equation (8)

$$\begin{align*}
(a) \ t &= g[\tilde{G}^+ \Sigma] [\tilde{G}^\Sigma], \\
(b) \ \tau &= g[\tilde{G}^+ \Sigma] [\tilde{G}^- \Sigma]^T
\end{align*}
$$

the ladder equation (12a) and the maximally crossed equation (12b) channel matrices respectively [40]. The matrices $t$ and $\tau$ read in terms of their Fourier components:

$$\begin{align*}
(t_{r'=\epsilon q APC})_{abcd} &= g \int \frac{d^2q}{(2\pi)^2} \ e^{-iq \cdot (r'-r)} \int \frac{d^2p}{(2\pi)^2} \ [\tilde{G}^+(p) \Sigma]_{ac} \\
&\quad \times [\tilde{G}^-(q+p) \Sigma]_{bd} \,,
\end{align*}
$$

$$\begin{align*}
(\tau_{r'=\epsilon q APC})_{abcd} &= g \int \frac{d^2q}{(2\pi)^2} \ e^{-iq \cdot (r'-r)} \int \frac{d^2p}{(2\pi)^2} \ [\tilde{G}^+(p) \Sigma]_{ac} \\
&\quad \times [\tilde{G}^-(q-p) \Sigma]_{bd} \,.
\end{align*}
$$

The different signs in the argument of $t$ and $\tau$ are a consequence of the transposition on the position space in the MC-channel. The corresponding Fourier transformed matrices $1 - \tilde{l}_q$ and $1 - \tilde{\tau}_q$ for $\epsilon = 0$ and $q = 0$ read

$$\begin{align*}
M_{abcd}^{LC} &= \delta_{ac}\delta_{bd} - g \int \frac{d^2p}{(2\pi)^2} \ [\tilde{G}^+(p) \Sigma]_{ac} [\tilde{G}^-(p) \Sigma]_{bd} \bigg|_{\epsilon = 0} \,,
\end{align*}
$$

$$\begin{align*}
M_{abcd}^{MC} &= \delta_{ac}\delta_{bd} - g \int \frac{d^2p}{(2\pi)^2} \ [\tilde{G}^+(p) \Sigma]_{ac} [\tilde{G}^-(p) \Sigma]_{bd} \bigg|_{\epsilon = 0} \,.
\end{align*}
$$

The eigenvalues of matrices $M$ provide a decay length for the full matrices in equation (12). In particular, a vanishing (e.g. gapless) eigenvalue gives a long-range diffusion-like behavior, which gives a non-vanishing contribution to the conductivity. Massive modes, on the other hand, do not contribute to the conductivity in the limit $\epsilon \to 0$ due to the prefactor $\epsilon^2$ in equation (7). Therefore, massive modes can be neglected subsequently. Depending on the type of disorder there may or may not be gapless modes. If gapless modes exist, then for small momenta and frequencies we get in the ladder-channel in diagonal representation (in the channel of maximally crossed diagrams analogously)

$$1 - \tilde{l}_q \sim \left( \frac{\epsilon}{\eta} + gD_0\eta^2 \right)_{1_N} \,.
$$

for $N$ massless channels. $D_0$ is the expansion coefficient:

$$D_0 = \frac{1}{2} \int d^2p \left( \frac{1}{p^2 + \eta^2} \right)^2 \,.
$$

Thus, the conductivity becomes

$$\tilde{\delta}_{ij} = \frac{e^2}{h} \int d^2p \left( \frac{1}{p^2 + \eta^2} \right)^2 \bigg[ (1 - \tilde{l}_q)_{\eta,ik} + (1 - \tilde{\tau}_q)_{\eta,ik} \bigg] \bigg|_{\epsilon = 0} \,.
$$

4. Conductivity for particular disorder types

Below we investigate the effect of the intra-node scattering. For this discussion it is useful to introduce the matrix notation

$$A = \begin{pmatrix}
A_{11,11} & A_{11,12} & A_{11,21} & A_{11,22} \\
A_{12,11} & A_{12,12} & A_{12,21} & A_{12,22} \\
A_{21,11} & A_{21,12} & A_{21,21} & A_{21,22} \\
A_{22,11} & A_{22,12} & A_{22,21} & A_{22,22}
\end{pmatrix} \,.
$$

In the simplest case, with diagonal disorder and in the absence of the inter-node scattering the calculation reduces to a single cone only. Then the Dirac propagators $\tilde{G}^\pm(p)$ read in Fourier representation

$$\tilde{G}^\pm(p) = \frac{1}{p^2 + z^2} \begin{pmatrix}
\mp iz & p_1 - ip_2 \\
p_1 + ip_2 & \mp iz
\end{pmatrix} \,.
$$

where $z = \epsilon + \eta$. The limit $\epsilon \to 0$ yields for the mass matrices in both channels:

$$M^{LC} = \begin{pmatrix}
\alpha & 0 & 0 & 0 \\
0 & -\alpha & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha
\end{pmatrix} \,; \quad M^{MC} = \begin{pmatrix}
\alpha & 0 & 0 & 0 \\
0 & -\alpha & 0 & 0 \\
\alpha & 0 & 0 & 0 \\
0 & 0 & 0 & -\alpha
\end{pmatrix} \,.
$$

i.e. each of them has a single zero eigenvalue. Moreover, for $q \sim 0$ we get
\[ \begin{align*}
1 - \tilde{i}_q \sim & \left( \alpha + \frac{\epsilon}{\eta} + D'q^2, \quad i\gamma q e^{-i\nu\tilde{q}}, \quad i\gamma q e^{-i\nu\tilde{q}}, \quad \zeta q^2 e^{-i\nu\tilde{q}} \right) \\
& \left\{ \begin{array}{ll}
i\gamma q e^{i\nu\tilde{q}} & \alpha + \frac{\epsilon}{\eta} + D'q^2 \quad -\alpha + D'q^2 \quad i\gamma q e^{-i\nu\tilde{q}} \\
-i\gamma q e^{i\nu\tilde{q}} & -\alpha + D'q^2 \quad \alpha + \frac{\epsilon}{\eta} + D'q^2 \quad i\gamma q e^{-i\nu\tilde{q}} \\
\zeta q^2 e^{i\nu\tilde{q}} & i\gamma q e^{i\nu\tilde{q}} \quad i\gamma q e^{i\nu\tilde{q}} \quad \alpha + \frac{\epsilon}{\eta} + D'q^2 \end{array} \right. \\
\end{align*} \]

(23)

and

\[ \begin{align*}
1 - \tilde{\eta}_q \sim & \left( \alpha + \frac{\epsilon}{\eta} + D'q^2, \quad i\gamma q e^{i\nu\tilde{q}}, \quad -i\gamma q e^{-i\nu\tilde{q}}, \quad \alpha - D'q^2 \right) \\
& \left\{ \begin{array}{ll}
i\gamma q e^{i\nu\tilde{q}} & \alpha + \frac{\epsilon}{\eta} + D'q^2 \quad -\zeta q^2 e^{-i\nu\tilde{q}} \quad -i\gamma q e^{i\nu\tilde{q}} \\
-i\gamma q e^{i\nu\tilde{q}} & -\zeta q^2 e^{i\nu\tilde{q}} \quad \alpha + \frac{\epsilon}{\eta} + D'q^2 \quad i\gamma q e^{i\nu\tilde{q}} \\
\alpha - D'q^2 & -i\gamma q e^{i\nu\tilde{q}} \quad i\gamma q e^{i\nu\tilde{q}} \quad \alpha + \frac{\epsilon}{\eta} + D'q^2 \end{array} \right. \\
\end{align*} \]

(24)

where

\[ D' = \frac{g}{6} \int \frac{d^2p}{(2\pi)^2} \frac{2\eta^2}{[p^2 + \eta^2]^3}, \quad D'' = \frac{g}{6} \int \frac{d^2p}{(2\pi)^2} \frac{\eta^2 + 3p^2}{[p^2 + \eta^2]^3}, \quad \varphi_q = \arctan \frac{q_y}{q_x}, \quad \gamma = g \int \frac{d^2p}{(2\pi)^2} \frac{\eta}{[p^2 + \eta^2]^2}, \quad \zeta = \frac{g}{6} \int \frac{d^2p}{(2\pi)^2} \frac{1}{[p^2 + \eta^2]^2}. \]

Inserting these expressions into our general conductivity expression (19) gives eventually

\[ \begin{align*}
\sigma_{\mu\nu}^{MC} & \sim 4\eta^2 D_0 \frac{e^2}{h}, \\
\sigma_{\mu\nu}^{MC} & \sim 0. \\
\end{align*} \]

(25)

(26)

In equation (25) terms of the order \(1/\alpha\), \(\alpha \sim \log p\), which arise from the linear order of gradient expansion in equation (23) are neglected due to their smallness. Equation (26) reveals the total suppression of the conductivity contribution (except for possible higher order contributions) from the backward scattering processes, with which maximally crossed diagrams are usually associated [34, 35].

The most striking feature of the obtained results is the absence of any logarithmic divergences in the conductivities. However, the question remains how the logarithmic behavior, observed in a number of experiments for positive values of the conductivity, can be explained within our approach? In the next section this will be described within a finite-size scaling approach, following closely the argumentation of our previous work [39, 41].

5. Scaling behavior of transport properties

The evaluation of equation (25) for the infinite-size limit gives for the DC conductivity \(\sigma_0\), which was predicted theoretically [22, 23, 25] and measured experimentally [6, 7]. In order to compare with experimental measurements we must also consider finite-size effects. In graphene, for instance, a typical sample size is of the order of several micrometers. To include the finite size of the sample in our theory, we must introduce a discretization of the momentum integral in the diffusion coefficient for a square of size \(2L \times 2L\):

\[ D_0 = \frac{1}{2} \int \frac{d^2p}{(2\pi)^2} \frac{1}{[p^2 + \eta^2]^2} \rightarrow \frac{1}{2} \left(\frac{1}{2L} \right)^2 \sum_{n=0}^{L} \frac{1}{k_n^2 + k_m^2 + \eta^2}. \]

with the scattering rate \(\eta\) evaluated from the saddle-point equation

\[ \frac{1}{g} = \int \frac{d^2p}{(2\pi)^2} \frac{1}{p^2 + \eta^2} \rightarrow \frac{1}{(2L)^2} \sum_{n,m=-L}^{L} \frac{1}{k_n^2 + k_m^2 + \eta^2}. \]

The wave numbers are defined as

\[ k_n = \frac{\pi n}{L}, \]

(29)

where \(n \in \mathbb{Z}\), which corresponds to the periodic boundary conditions. However, using periodic boundary conditions is by no means obvious nor obligatory. Other boundary conditions may well be employed, which would lead to the different scaling behavior (see [41]). The relation between the dimensionless quantity \(L\) and physical length \(\ell\) is established via

\[ \ell = \frac{h\nu_F}{E_b} L, \]

(30)

where \(E_b\) is the band width. In particular, for \(E_b = 0.75\) eV, which we use below for fitting purposes, we get

\[ \frac{h\nu_F}{E_b} \sim 2.9 \, \text{Å}, \]

(31)

which is slightly larger than the carbon lattice spacing \(a \sim 2.4\) Å. The disorder strength is measured in units of \((h\nu_F)^2\), therefore the quantity \(s = (h\nu_F)^2\) represents the appropriate reference scale.
Then the main goal is to determine the $\beta$-function

$$\beta = \frac{d\log \bar{\sigma}}{d\log L}. \quad (32)$$

a function which describes the behavior of $\sigma$ under a change of the linear system size $L$. Here we use a regularization scheme which fixes the maximum number of accounted modes to $(2L + 1)^2$. The continuous limit of the momentum integral is recovered for $L \rightarrow \infty$. If periodic boundary conditions are used, then the main contribution to the conductivity at small scales comes from the zero modes with $n = 0$ and $m = 0$. In this case the conductivity approaches the plateau of the infinite sample from above. This scenario was recently realized experimentally in [12].

At first we study the case of a finite size sample which confines a 2D Dirac electron gas subject to random potentials of different disorder strength. Choosing $L = 120$ we solve equations (27) and (28) for $g/L = 0.1...2$. This gives the scaling plot of the $\beta$-function depicted in figure 1. Every piece of this graph corresponds to the particular disorder strength $g$. One recognizes a wide area where the $\beta$-function decreases logarithmically and, therefore, obeys the one-parameter scaling behavior predicted by the famous scaling theory of Abrahams et al in [1]. For a sufficiently small conductivity, though, the flow turns up towards the fixed point which corresponds to the finite conductivity plateau. When extracted from the scaling plot, the behavior of the conductivity as a function of $s/g$ exhibits two distinct regimes (see right-hand side of figure 1): the linearly decreasing regime at disorder strength smaller than the hopping amplitude, i.e. $g < s$ and a plateau-like regime of nearly constant conductivity at disorder strengths above $s$. Even for disorder as strong as $g = 2s$, it still can be considered as a weakly disordered electron gas. However, the validity of our approach for larger values of $g$ is not guaranteed. Interestingly, the conductivity reveals a similar behavior if plotted versus the ratio $1/(\eta L)^2$. The effect of increasing $L$ reveals a broadening of the plateau region down to smaller values of $g$, while leaving the slope of the linear part at the same value. It is expected that in the limit $L \rightarrow \infty$ it must stretch over the whole range of disorder strengths, but even for the largest numerically accessible lattice sizes the crossover value is found at $g \sim 0.8s$, i.e. it does not change significantly.

In the second scenario the disorder strength is kept constant and the number of modes is gradually increased. For the case of weak disorder (e.g. for $g = 0.3s$), the plateau cannot be reached for a moderate sample size, and the conductivity reveals the steady logarithmic decline, which is characteristic for the weak localization regime. For moderate disorder (e.g. for $g = 0.9s$), the crossover into the plateau regime is reached roughly at $L \sim 10^3$, revealing a broad area of logarithmic decrease which can be fitted with the formula

$$\bar{\sigma}/\sigma_0 \sim C - a \log(2L), \quad (33)$$

where the slope $a \sim 2$, specific for the case of orthogonal ensemble [3], is roughly the same for not too large values of $g$, while the constant $C$ depends strongly on $g$, taking for instance $C \sim 42$ for $g = 0.3s$ and $C \sim 12.7$ for $g = 0.9s$. A temperature dependence of the conductivity can be included as a finite cutoff for the sample size, using the substitution

$$L = \frac{E_b}{k_B T}. \quad (34)$$

This behavior can be seen on the right-hand side of figure 2, where the band width is chosen to be $E_b = 0.75$ eV. There is a crossover temperature of roughly 15 K and an increase of the conductivity up to 4 times at room temperature. The overall shape of the temperature dependent conductivity, the value of the crossover temperature, as well as the prefactor of the logarithmic regime are in good agreement with recent experimental observations, performed on graphene with different degrees of disorder [13–20].

![Figure 1](image-url)
6. Discussion and conclusions

A big challenge in the experimental investigation of transport in graphene and other realizations of a 2D Dirac electron gas is controlling the amount of disorder. A possible way to create stronger disorder in the samples is by a bombardment with fast ions. We can use the results of such experiments and compare them with our theoretical results. A considerable amount of published data can be fitted with the expressions in equations (27), (28) and (34), assuming a moderate disorder strength \( g \) and band width \( E_b = k_B T L \) (see equation (34)). However, these parameters cannot be measured independently, since they enter the scattering rate via

\[
\eta \sim E_b e^{-T/g},
\]

which is directly accessible experimentally via the scattering time measurements: \( \tau = h \eta / \Gamma \). From this point of view, our results represent in fact one-parameter fits.

The second group of reported experimental data (e.g. [14, 20] and some curves from [18]) cannot be fitted under assumption of weak or moderate disorder (i.e. for \( g \ll 2s \)). In fact, they still can be fitted reliably well for much larger values of \( g \) and much smaller bandwidths (i.e. \( E_b \ll t \)) but the validity of the weak scattering assumption in this regime is not given anymore.

In conclusion, we have performed a thorough investigation of the DC conductivity of a weakly disordered 2D Dirac
electron gas, using perturbative ensemble averaging technique. Our results confirm that the temperature dependence of the conductivity of disordered 2D Dirac electron gases in finite size samples is characterized by two regimes in which it behaves distinctly differently. These are the regime of a logarithmic decay with sample size or decreasing temperature, observable at higher temperatures, and the regime of a nearly constant conductivity at lower temperatures. The plateau value as well as the crossover temperature between the two regimes are not universal but change from one sample to another, thus suggesting a strong dependence on the disorder, sample size and perhaps chemical doping. The obtained analytical expressions are very simple and do not reveal any logarithmic divergences in the conductivity. Our results reflect correctly the shape and all features of the conductivity scaling behavior.

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