SPIN PUZZLE IN NUCLEON

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ABSTRACT

The object of this brief review is to reconcile different points of view on how the spin of proton is made up from its constituents. On the basis of naive quark model with flavour symmetry such as isospin or SU(3) one finds a static description. On the contrary the local SU(3) colour symmetry gives a dynamical view. Both these views are contrasted and the role of U(1) axial anomaly and the ambiguity for the measurable spin content is discussed.
Introduction

Matter in the universe is mostly made up of nucleons (and as many electrons as there are protons). It is important that the nucleons are spin 1/2 particles and the concomitant Fermi statistics is responsible for the fact that many hard objects can be made of these constituents. When it became clear that the nucleons are not elementary and they are presumably made up of other spin 1/2 species known as quarks, it is useful to understand how the nucleon spin is distributed in its constituents.

There are two different SU(3) groups associated with the strong nuclear interactions. The flavour SU(3) group is the improvement on the older isospin symmetry indicated by the near equality of masses of proton and neutron. Adding strangeness as an additional flavour, in the early sixties, the underlying symmetry governing the properties of nucleons was established to be SU(3). Soon it became clear that the neutron and proton are two members of an octet of baryons (other members of the family being an isosinglet Λ, isodoublet Ξ, and the triplet of pions are accompanied by the doublets of K(0, -1) mesons and an isosinglet η meson, thus making up an octet of spin zero odd parity (pseudo scalar) mesons. That neither the set of baryons nor mesons have obtained by the simple substitution of the energy-momentum operator of electric charge as the underlying principle. The minimal electromagnetic coupling is described by the local interactions that has gauge invariance and the related conservation interaction dynamics is similar to the more familiar quantum electrodynamics (QED) SU(3), believed to be exact, governs the dynamics of the strong interactions. The strong symmetry is ensured by endowing now the matrix valued gauge field A\(\mu\)(x), elements of unitary matrices of unit dimension – just the phases of a unimodular complex number – an abelian group of transformations and is related is the conservation of the electromagnetic charge. The strongly interacting matter carries analogous, but non-abelian, charge characterised by the unitary unimodular group SU(3), the generators of the group being exp(iα\(a\)Q\(a\))/\[a = 1, 2, \ldots 8\]. The symmetry transformation will be an operator U(x) = exp[iα\(a\)Q\(a\)]; ψ(x) → U(x)ψ(x). The local symmetry is ensured by endowing now the matrix valued gauge field A\(\mu\)(x)\(\equiv A^\mu_a(x)Q^a\) the property that it transforms as A\(\mu\)(x) → UA\(\mu\)U\(^{-1}\) − U∂\(\mu\)U\(^{-1}\). Like QED, the quantum chromodynamics (QCD) is a renormalizable field theory that lets us obtain finite regularized matrix elements for all physically relevant amplitudes.

It is not surprising that kinematic and dynamic SU(3) groups are both relevant in the description of nucleons (and other baryons and mesons) as composite states of quarks, which possess “charges” that are consequences of both symmetry groups. The spin content of the nucleon, I will argue, arises out of differing roles of the kinematic SU(3), that specifies the flavour quantum numbers of hadronic matter and the dynamic SU(3), related
to the “colour” of the strongly interacting matter.

Nucleon and other baryons are bound states of three quarks; however unlike the atomic states, there is no concept of “ionisation” or “dissociation” of quarks, since the states carrying colour quantum numbers are not found as proper asymptotic states. This is referred to as the property of confinement of colour—a property of the QCD, the precise mechanism for which is yet to be clarified. While it is impossible to pull quarks out of the nucleon, at very short distances probed by high momentum-transfer photons, the quarks appear to be “weakly” interacting, thereby justifying the use of perturbative quantum chromodynamics for the description of deep inelastic scattering of leptons by nucleons. We will contrast the spin structure of the nucleon revealed in such a study with what one may expect on the basis of the wavefunction of a static nucleon in terms of its flavour and spin content.

Flavour SU(3) and Static Spin Structure of Nucleons

To begin with, it has been popular in the sixties to view nucleons as a three body bound state [1], while two up quarks and one down quark make up a proton, two and one u will constitute a neutron. It was possible to accommodate an octet of baryons and a decimet of excited states in a 56 dimensional representation of SU(6) symmetry group[2]. The three-quark symmetric state spans a 56 dimensional representation, made up of J = 1/2 doublets of baryon octet and J = 3/2 spin quartet of excited baryon decimet. The latter is made up of the prominent p-wave meson-baryon resonances: Δ(1232 MeV, I = 1/2, S = 0), Σ*(1395 MeV, I = 1, S = −1) and Ξ*(1530 MeV, I = 1/2, S = −2) and the stable (with respect to strong interactions) state Ω−(1672 MeV, I = 0, S = −3).

The proton wave function (with spin sz = 1/2) is given in terms of the quarks through [2]: (for neutron wave function interchange u and d quarks)

\[
\psi_p = \left\{ \frac{1}{\sqrt{18}} \{ 2|u_1^+d_2^-u_3^+\rangle + 2|u_1^+u_2^+d_3^-\rangle + 2|d_1^-u_2^+u_3^-\rangle - |u_1^+u_2^-d_3^+\rangle - |u_1^-d_2^+u_3^+\rangle - |u_1^-u_2^-d_3^+\rangle \} \right\}
\]

(1)

where the letters u and d refers to up and down quarks ± to spin sz = ±1/2 and the subscripts 1, 2 and 3 to label the position (space-time χμi; i = 1, 2, 3) of the quarks. In such a quark model all relative orbital angular momenta vanish in the ground state and thus the proton spin is carried symmetrically in terms of the constituent quark spin. Further the static magnetic dipole moments of the states are given by the operator

\[
\bar{\mu}_B = \sum_q \mu_q \vec{\sigma}_q
\]

(2)

We may read off the expectation value for μp and μn:

\[
\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d \quad (3a)
\]

2The up, down and strange quarks with ±1/2 spin states each makes up the six dimensional fundamental representation of SU(6) symmetry.
\[ \mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u \quad (3b) \]

Since the magnetic dipole moment and angular momentum are related through

\[ \vec{\mu} = g \mu_0 \vec{J}; \quad \mu_0 = \frac{e \hbar}{2mc} \text{ and } g = -2 \text{ for Dirac fermion} \]

We expect for each quark

\[ \mu_q = Q_q \left( \frac{e \hbar}{2m_q c} \right) \quad (4) \]

where \( Q_q \) is the fractional unit of charge carried by the quark \( q \) and \( m_q \) is the “constituent” quark mass.

Thus \( \mu_n = \frac{2}{3} \left( \frac{m_u}{m_n} \right) \mu_N, \quad \mu_d = -\frac{1}{3} \left( \frac{m_u}{m_d} \right) \mu_N \) and \( \mu_s = -\frac{1}{3} \left( \frac{m_u}{m_s} \right) \mu_N \), where \( \mu_N \) is the Nucleon Bohr magneton. Isospin symmetry will lead to \( m_u = m_d = \bar{m} \) so that

\[ \mu_p = \frac{m_p}{\bar{m}} \mu_N = 2.79 \mu_N \Rightarrow \bar{m} = 336 \text{ MeV} \quad (5) \]

With this, we should get

\[ \mu_n = -\frac{2}{3} \frac{m_p}{\bar{m}} \mu_N = -1.86 \mu_N \quad (6) \]

as against the experimental value of \( \mu_n = -1.91 \mu_N \). Indeed the magnetic moments of all baryonic states (\( p, n, \Lambda, \Sigma, \Xi \) as well as \( \Delta, \Sigma^*, \Xi^* \) and \( \Omega^- \)) are given in terms of just two parameters \( (\bar{m} = m_u = m_d \text{ and } m_s) \); say \( \mu_p \) and \( \mu_A \). Experimental numbers match the static quark model prediction to a large extent.

| Magnetic moment | Quark model values | Experimental values (in \( \mu_N \)) |
|-----------------|--------------------|-------------------------------------|
| \( \mu_p \)     | \( \frac{1}{3} \mu_u - \frac{1}{3} \mu_d \) (input) | 2.793 |
| \( \mu_n \)     | \( \frac{1}{3} \mu_d - \frac{1}{3} \mu_u = -1.86 \mu_N \) | −1.913 |
| \( \mu_A \)     | \( \mu_s \) (input) | −0.613 ± 0.004 |
| \( \mu_{\Sigma^+} \) | \( \frac{1}{3} \mu_u - \frac{1}{3} \mu_s = 2.69 \mu_N \) | 2.458 ± 0.010 |
| \( \mu_{\Sigma^0 - \Lambda} \) | \( \frac{1}{3} (\mu_u - \mu_d) = 1.65 \mu_N \) | 1.61 ± 0.08 |
| \( \mu_{\Sigma^-} \) | \( \frac{1}{3} \mu_d - \frac{1}{3} \mu_s = -1.04 \mu_N \) | −1.160 ± 0.025 |
| \( \mu_\Xi^- \) | \( \frac{1}{3} \mu_s - \frac{1}{3} \mu_u = -1.44 \mu_N \) | −1.250 ± 0.014 |
| \( \mu_{\Omega^-} \) | \( \frac{1}{3} \mu_s = -0.51 \mu_N \) | −0.679 ± 0.031 |
| \( \mu_{\Xi^0} \) | \( 3 \mu_s = -1.84 \mu_N \) | −1.94 ± 0.22 |

**Table 1. Magnetic Moments of Baryons**

We may conclude that the naive quark model describes adequately the static spin structure of the nucleon.

**Colour SU(3) and dynamical spin content**

The spin content of the nucleon is probed dynamically by the longitudinal polarization asymmetries in the deep inelastic scattering by leptons off the nucleon targets [3]. The matrix elements of both electromagnetic and weak currents provide valuable information as the structure of the hadrons. Typically for \( \ell(k) + p(p) \rightarrow \ell' (k') + X \), we have, in the
leading order, the contribution for the inclusive cross section is given by the product of the leptononic tensor \(\ell_{\mu\nu} = k_{\mu} k_{\nu} + k_{\nu} k_{\mu} - g_{\mu\nu} (k \cdot k') + \epsilon_{\mu\nu\alpha\beta} m s^\alpha k^\beta\) and the hadronic tensor

\[
W^{\mu\nu} = \frac{1}{2\pi} \int d^4 x \exp(i q \cdot x) \langle p, s | [j^\mu(x), j^\nu(0)] | p, s \rangle
\]

and the \(W^\pm, Z\) or \(\gamma\) propagator.

While the symmetric part of \(W_{\mu\nu}\) measures the unpolarized hadronic structure, the spin content is coded in the antisymmetric part. The Lorentz covariance of the hadronic tensor together with the constraint arising from the conservation of electromagnetic current will imply for \(e + N \rightarrow e + X\), two structure functions \(F_1\) and \(F_2\) for the symmetric part (and these are analogous to the electric and magnetic form factors of the elastic scattering) and two more \(g_1\) and \(g_2\) for the antisymmetric part:

\[
\frac{M}{2\pi} W^{\mu\nu} = (g^{\mu\nu} - q^\mu q^\nu/q^2) F_1(q^2, \nu \equiv p \cdot q/2M)
+ \frac{1}{M\nu} \left( p^\mu - \frac{\nu}{q^2} (p \cdot q) q^\mu \right) \left( p^\nu - \frac{\nu}{q^2} (p \cdot q) q^\nu \right) F_2(q^2, \nu)
- \frac{1}{\nu} \epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta g_1(q^2, \nu) - \frac{1}{\nu} \epsilon^{\mu\nu\alpha\beta} q_\alpha (s_\beta - \frac{q \cdot s_p}{p \cdot q}) g_2(\nu, q^2)
\]  

where \(q^2(-Q^2)\) and \(\nu\), the energy transfer in the lab frame are related to the two independent kinematic variables (energy transfer and the scattering angle of the lepton) that can be measured in an all inclusive deep inelastic scattering process. It is instructive to use instead, the variables \(Q^2(\equiv -q^2 = 4k_0 k_0 \sin^2 \theta/2)\) and \(x(\equiv Q^2/(2p \cdot q = Q^2/(2M\nu))\) and find that the structure function have only a very weak (logarithmic) dependence on \(Q^2\) and is mostly expressible in terms of the scaling variable \(x\). In the quark-parton model, one views the process of deep inelastic scattering as an incoherent sum of the scattering of the lepton by the quark that carries a momentum fraction \(X\) of the parent nucleon and the scattering event with the variable \(X\) is correlated with the quark carrying corresponding fractional momentum \(X = \bar{x}\). Thus the measured structure functions are directly translated as the probabilities of quark distributions in a hadron. What is more, the slow logarithmic \(Q^2\) dependence – the so called scaling violation – can be computed in the perturbative quantum chromodynamics and turns out to be the important evidence for QCD as the experimentally verifiable strong interaction theory [4].

It is easy to derive in the leading order,

\[
2x F_1(x) = F_2(x) = x [\sum \epsilon_i^2 \{ q_i^+(x, Q^2) + q_i^-(x, Q^2) \} + (q_i^+(x, Q^2) + q_i^-(x, Q^2))]
\equiv \sum_i x \epsilon_i^2 [ q_i(x, Q^2) + q_i(x, Q^2) ]
\]

where \(q_i(x, Q^2) = q_i^+ + q_i^-\) is the net probability of finding a quark of flavour \(i\), carrying a momentum of fraction \(x\) of the parent proton. The summation will run over all quark and antiquark flavours. The superscript \(\pm\) refers to the helicity of the quark inside, say, a positive helicity proton. Perturbative QCD gives the \(Q^2\) evolution of the various moments of the quark densities and the experimental verification of the expected scaling violation (i.e. \(\ell n Q^2\) dependence of the \(q_i^n(Q^2) = \int_0^1 dx x^{n-1} q_i(x, Q^2)\)) is hailed as the triumph of QCD. The first part of Eq.(8), relating \(F_1(x)\) and \(F_2(x)\) is a consequence of the spin 1/2 nature of quarks.
The spin structure is similarly revealed in the functions \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \). For the longitudinal asymmetries \( g_2(x, Q^2) \) does not contribute – since \( s^\sigma \) is parallel to \( p^\sigma \). In the leading order,

\[
\begin{align*}
# g_1(x, Q^2) &= \frac{1}{2} \sum_i e_i^2 [ (q_i^+ - q_i^-)(x, Q^2) - (q_i^+ - q_i^-)(x, Q^2) ] \\
&= \frac{1}{2} \sum_i e_i^2 [ \Delta q_i(x, Q^2) + \Delta q_i(x, Q^2) ]
\end{align*}
\]

The \( Q^2 \) dependence is governed by the Altarelli-Parisi equation [5], which has a very transparent interpretation as a process of finding within a parton, (quark of flavour \( i \) or gluon \( G \)) another parton (quark of flavour \( j \) or gluon \( G \)) and is expressed as a convolution of a splitting function \( p(z) \), \( z \) being the momentum fraction. The splitting functions \( p(z) \) are given in terms of the basic vertices of QCD and the resultant integro-differential equation can be solved to give ordinary algebraic differential equations for the moments of quark and gluon densities. For non-singlet flavour (all valence quarks), we get

\[
\begin{align*}
\frac{d}{dt} \Delta q_{i \text{valence},n}(t) &= \frac{\alpha_s(t)}{2\pi} \tilde{A}^n_{qq} \Delta q_{i \text{valence},n}(t)
\end{align*}
\]

where \( t = \ell n\frac{Q^2}{Q_0^2} \); \( Q_0^2 \) being a reference scale, \( \alpha_s(t) (= g_s^2/4\pi) \) is the running coupling parameter in QCD (analogue of \( \alpha_{em} \)) with \( Q^2 \) dependence given by

\[
\alpha_s(t) = \alpha_s(0)/(1 + b\alpha_s(0)t); \quad b = (33 - 2f)/12\pi .
\]

for the colour group SU(3) with \( f \) flavours. \( \tilde{A}^n_{qq} \) are the anomalous dimensions of the relevant operator and can be read off the QCD splitting functions. \( \alpha_s(t) \) decreases as \( t \) increases and signifies the asymptotic freedom of quark gluon coupling at short distances, permitting therefore the perturbative QCD analysis. We find:

\[
\Delta q_{i \text{valence},n}(t) = [\alpha_s(0)/\alpha_s(t)]^{2\pi b/\tilde{A}^n_{qq}} \Delta q_{i \text{valence},n}(0)
\]

For \( \Delta q_{i \text{valence},1} \), which measures the spin structure in terms of the difference of number of quarks with + and – helicities, the solution is particularly simple, since \( \tilde{A}^1_{qq} = 0 \) and so is \( t \)-independent. If we identify these valence quark densities with the SU(6) wave function (Eq. 1 and 3a,b), we have [8]

\[
\Delta q_{u \text{valence},1} = 4/3, \quad \Delta q_{d \text{valence},1} = -1/3 .
\]

As an alternate to the pure SU(6) description it is also possible to use the experimental value of

\[
G_A/G_V \bigg|_{p \rightarrow n} = 1.25 = \Delta q_{u \text{valence},1} - \Delta q_{d \text{valence},1}
\]

and a similar relation for

\[
G_A/G_V \bigg|_{\Xi^- \rightarrow \Xi^0} = -0.25 = \Delta q_{d \text{valence},1}
\]
to give [7]

$$\Delta q^u_1 = 1 \text{ and } \Delta q^d_1 = -0.25$$  \hspace{1cm} (15)

Yet another option is to use the model of Carlitz and Kaur, who propose that the valence quark hypothesis and most of the momentum and helicity is carried by the "leading quark" in the hadron [8]. Accordingly

$$\Delta q^u_v(x) = \cos 2\Theta[q^u_v(x) - \frac{2}{3}q^d_v(x)]$$

$$\Delta q^d_v(x) = -\frac{1}{3} \cos^2 \Theta(x)q^u_v(x)$$  \hspace{1cm} (16)

with \(\cos 2\Theta(x) = [1 + 0.052(1 - x^2)/\sqrt{x}]^{-1}\).

Since \(\cos 2\Theta(x)\), called spin-dilution factor, approaches unity as \(x \to 1\) and since \(q^u_v(x)\) dominates over \(q^d_v(x)\) at large \(x\), the valence \(u\)-quarks in protons have their spins aligned fully along proton spin in this kinematical region. The first moment – the net number of valence spin – is remarkably close to the values in Eq. (15), since the integral of Eq. (16) yields:

$$\Delta q^u_1 = 1.01 \text{ and } \Delta q^d_1 = -0.25$$  \hspace{1cm} (17)

### Flavour Singlet Spin Puzzle

The nucleon has, in addition to the valence quarks, \(q\bar{q}\) pairs referred to as the sea and gluons\(^3\). It was generally believed that both the sea and gluons are unpolarized and the spin is carried entirely by the valence quarks. It is remarkable that this is far from the truth and there is a strong tendency for the quarks and gluons to be polarized. This was conjectured by me through an analysis of the \(Q^2\) evolution in the early eighties [9] and subsequent experimental measurement [10] of EMC (European Muon Collaboration) gave values for longitudinal spin asymmetry that implied that the bulk of the spin is resident elsewhere. Closely related to this feature is the notion that QCD has \(U(1)\) axial anomaly and as a consequence what is being perceived as the quark spin (in the flavour singlet part) contains a strong component of gluon spin, which will normally be ignored as being higher order in \(\alpha_s(Q^2)\). Denote \(G^+(x, Q^2) - G^-(x, Q^2) = \Delta G(x, Q^2)\), where \(\pm\) refer to gluon helicities. The \(U(1)\) axial anomaly causes the term \(\alpha_s(Q^2)\Delta G^1(Q^2)\) to be \(Q^2\) independent [11] and it gets added to the flavour singlet quark spin \(\Sigma_i\Delta q^1_i(Q^2)\). Thus the flavour singlet spin content of the hadron is measured to be \(\Sigma_i[\Delta q^1_i(Q^2) - \frac{\alpha_s(Q^2)}{2\pi}\Delta G^1(Q^2)]\).

The QCD evolution equation for the singlet sector is given by [12]

$$\frac{d}{dt} \left( \begin{array}{c} \Sigma_i \Delta q^u_i(t) \\ \Delta G^u(t) \end{array} \right) = (\alpha_s(t)/2\pi) \left( \begin{array}{cc} \tilde{A}^n_{qq} & 2f \tilde{A}^n_{qG} \\ \tilde{A}^n_{Gq} & \tilde{A}^n_{GG} \end{array} \right) \left( \begin{array}{c} \Sigma_i \Delta q^u_i \\ \Delta G^u \end{array} \right)$$  \hspace{1cm} (18)

\(\tilde{A}^n\) are known constants related to the anomalous dimensions of relevant operators, obtainable from the basic QCD couplings. For \(n = 1\), they are rather special with \(\tilde{A}_{qq}^1 = 0 = \tilde{A}_{qG}^1\), \(\tilde{A}_{Gq}^1 = 3c_F\) and \(\tilde{A}_{GG}^1 = \frac{11c_A - 4c_F}{6} = 2\pi b\) where \(c_F\) (Casimir invariant for the fundamental

\(^3\)We ignore the possibility that it may also have a very small component of heavy quarks charm, bottom or top.
representation), \(c_A\) (Casimir invariant for the adjoint representations) have values, for 4/3 and 3, respectively when the gauge group is \(SU(3)\) and \(T = f/2\).

\[
\frac{d}{dt} \Sigma \Delta q_i^1(t) = 0 \tag{19a}
\]

and

\[
\frac{d}{dt} \Delta G^1(t) = (\alpha_s(t)/2\pi)(2\Sigma_i \Delta q_i^1 + 2\pi b \Delta G^1) \tag{19b}
\]

To this, we must add the helicity sum rule

\[
\frac{1}{2} \Sigma_i \Delta q_i^1 + \Delta G(t) + L_z(t) = \frac{1}{2} \tag{20}
\]

Since the renormalized gauge coupling constant satisfies \(\frac{d}{dt} \alpha_s(t) = -b \alpha_s^2(t)\), it is easily observed that

\[
\frac{d^2}{dt^2} \Delta G^1 = 0 \text{ upto } O(\alpha_s^2) \nonumber
\]

Thus, the right hand side of Eq. (19b) gives

\[
\frac{\alpha_s(t)}{\pi} (\Sigma_i \Delta q_i^1 + \pi b \Delta G^1) = C \tag{21}
\]

where \(C\) is a constant (\(t\) independent). Indeed the helicity sum rule (Eq.20) calls for

\[
\langle L_z(t) \rangle = \langle L_z(0) \rangle - Ct \tag{22}
\]

with the same constant \(C\). All the above equations are consistent with the following values for the first moments:

\[
\Sigma \Delta q^1 = \left( \frac{33 - 2f}{9 - 2t} \right) (1 - 2\langle L_z(0) \rangle) \tag{23a}
\]

and

\[
\Delta G^1 = \left( \frac{-12}{9 - 2f} \right) (1 - 2\langle L_z(0) \rangle) + Ct \tag{23b}
\]

Equation (23a,b) indicate non-trivial magnitude of the proton spin to be found in the favour singlet part. It may also be observed, as per the helicity sum rule, while \(\Sigma \Delta q^1\) is conserved quantity \(\Delta G^1\) and \(L_z\) are not. This is easily understood as a general property that the fermion helicity is conserved in the QCD process, leaving the emitted gluon to have either helicity and the total angular momentum conservation will so adjust to have the net orbital angular momentum \(L_z\) and \(\Delta G^1\) together remain conserved. These features are reflected in Eqs. (22) and (23a,b).

The most significant feature is that \(\alpha_s(Q^2) \Delta G^1(Q^2)\) is the conserved quantity and in an experiment that involves measuring the singlet spin, this quantity is not distinguished from the net quark spin. Most straightforward way to see this is to look at the axial vector \(U(1)\) current and observe that it is not conserved due to the presence of the anomaly.

\[
j_5^i = \Sigma_i \bar{q}_i \gamma_5 q_i \tag{24}
\]
In the massless (for quarks) limit, the axial anomaly gives

\[ \partial_\mu j_\mu^5 = f \frac{\alpha_s}{2\pi} T_r(F_{\mu\nu}\tilde{F}^{\mu\nu}) \]  

(25)

where \( \tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \), \( F_{\mu\nu} = \sum_{a=1}^{8} F_{\mu\nu}^a Q^a \); \( Q^a \): generators of \( SU(3) \) and \( trQ^aQ^b = \frac{1}{2}\delta^{ab} \). The right hand side is expressible as a four-divergence of a current \( k_\mu \):

\[ k_\mu = \left( \frac{\alpha_s}{2\pi} \right) \varepsilon_{\mu\nu\lambda\sigma} Tr[A^\nu(F^{\lambda\sigma} - \frac{2}{3} A^\lambda A^\sigma)] \]

so that now the Eq. 25 is reexpressed as

\[ \partial_\mu (j_\mu^5 - f k_\mu) = 0. \]  

(26)

The conserved quantity \( (\Sigma \Delta q_1^i - \frac{\alpha_s}{2\pi} \Delta G^i) \) may be regarded as a consequence of the above conservation law. Strictly speaking \( k_\mu \), being gauge dependent, should not occur as a physically measurable quantity. However it turns out that the diagonal matrix elements of this operator is gauge independent and in a “parton model” like analysis one deals only with diagonal matrix elements.

Altarelli and Lampe [14] observe that it will be appropriate to develop the \( t \)-evolution equation for the two combination of densities. Denoting \( \Delta \Gamma = \frac{\alpha_s}{2\pi} \Delta G^1 \) and \( \Delta \Sigma = \Sigma_i \Delta q_1^i \), upto two loop level, we have

\[ \frac{d}{dt} \Delta \Gamma = 2 \left( \frac{\alpha_s}{2\pi} \right)^2 (\Delta \Gamma) \]  

(27)

and

\[ \frac{d}{dt} \Delta \Sigma = 0. \]  

(28)

It is seen that the quantity \( \Delta \Sigma' \equiv (\Delta \Sigma - f \Delta \Gamma) \) satisfies the two loop evolution equation:

\[ \frac{d}{dt} \Delta \Sigma' = -2f \left( \frac{\alpha_s}{2\pi} \right)^2 \Delta \Sigma' \]  

(29)

and can be solved to give

\[ \Delta \Sigma'(t) = \Delta \Sigma' (\mu^2) \exp -2f \int \left( \frac{\alpha_s}{2\pi} \right)^2 dt \]  

(30)

\[ \approx \Delta \Sigma' (\mu^2) \exp \left( \frac{f \alpha_s(t)}{\pi b} \right) \]

if we neglect higher order terms in \( \alpha_s(t)^4 \). It is seen that \( Q^2 \) dependence is rather weak and may be neglected to begin with in any phenomenological analysis.

\[ ^4 \text{We should have used } \alpha_s(t) \approx \frac{1}{bt} + \frac{b_1}{2bt^2} \text{ with } b_1 = \frac{306-38f}{48\pi^2}. \]
Phenomenological Consequences

The EMC measurement at CERN of the longitudinal asymmetry in the deep inelastic muon-proton scattering yielded directly $g_{1}^{p}(x, Q^{2})$. At $Q^{2} = 10.7 GeV^{2}$, it was found to give the first moment, with suitable extrapolation, yielding [10]

$$\int_{0}^{1} g_{1}^{p}(x, Q^{2}) = 0.126 \pm 0.010 + 0.015$$

(31)

Since this is the measure of $\frac{1}{2} \Sigma e_{i}^{2} \Delta q_{1}^{i}(Q^{2})$, it is seen to be the combination $\frac{1}{12} (\Delta q_{u}^{1} - \Delta q_{d}^{1}) + \frac{1}{36} (\Delta q_{u}^{1} + \Delta q_{d}^{1} - 2\Delta q_{s}^{1}) + \frac{1}{8} (\Delta q_{u}^{1} + \Delta q_{d}^{1} + \Delta q_{s}^{1})$. Using the information on $g_{A}/g_{V} (= \Delta q_{u}^{1} - \Delta q_{d}^{1})$ and the $D/F$ ratio for octet baryon matrix element, extract the singlet part to give

$$\Sigma \Delta q_{1}^{i} = 0.00 \pm 0.24$$

(32)

This result is indeed the puzzle. This gave rise to the notion that the flavour singlet spin structure is vanishingly small and needed an explanation. The analysis made by Altarelli and Stirling [11] makes use of the EMC result, $G_{A}/G_{V} \bigg|_{p \rightarrow n}$, and the $D/F$ ratios as determined by the weak interaction of the octet of baryon $\beta$-decays to give ($\Delta q_{u}^{1} - \Delta q_{d}^{1} = G_{A}/G_{V} = 1.25$; with $D = 0.79$ and $F = 0.46 \Delta q_{u}^{1} + \Delta q_{d}^{1} - 2\Delta q_{s}^{1} = 3F - D = 0.59$),

$$\Delta q_{u}^{1} - \Delta \Gamma = 0.74 \pm 0.08$$

$$\Delta q_{d}^{1} - \Delta \Gamma = -0.51 \pm 0.08$$

$$\Delta q_{s}^{1} - \Delta \Gamma = -0.23 \pm 0.08$$

(33)

and rewrite the Eq.32 as

$$\Sigma i \Delta q_{1}^{i} - 3\Delta \Gamma = 0.00 \pm 0.24$$

(32a)

$\Delta \Gamma = \alpha_{s}/2\pi \Delta G^{1}$ is the anomaly contribution to quark spin in proton. If we take $\Delta q_{s}^{1} = 0$, then $\Delta \Gamma = 0.23$ and $\Sigma \Delta q_{1}^{i} = 3\Delta \Gamma = 0.69$, which implies 30% of the proton spin is due to the gluon. The fraction of spin carried by quarks decreases rapidly for negative $\Delta q_{s}^{1}$. For $\Delta q_{s}^{1} = -0.10$, we will have $\Delta \Gamma = 0.13 \Rightarrow \Sigma \Delta q_{1}^{i} = 0.39$, which then means 60% of the proton spin is carried by the gluon component. These numbers are indicative of a not too dramatic depletion of quark spin content in proton.

Subsequent experiments have improved the quality of the data and the spin Muon Collaboration (SMC) now reports data that are much less dramatic. The results reported at the Glasgow conference [15] suggests

$$\Sigma \Delta q_{1}^{i} \cong 0.31 \pm 0.07$$

and the violation of Ellis-Jaffe sum rule, interpreted as the strangeness content of proton spin gives

$$\Delta q_{s}^{1} \cong -0.10 \pm 0.04$$

Notwithstanding the absence of dramatically puzzling experimental result, the spin puzzle cannot be regarded as a solved problem. What we have seen in this note is the inherent ambiguity in what is theoretically analyzed and what is amenable to measurement. The theoretical analysis makes it clear that what is measured as the singlet component of the spin and for that matter any flavoured quark spin is the combination consisting of
the naive parton model quark spin and \(-\frac{\alpha_s}{2\pi} \Delta G\) for each flavour. Since this combination occurs in the (only) conserved current of the theory, it is quite clearly what is measured. This is the reason, we believe, following Altarelli and Lampe, that what is perceived as quark spin in experiments is the combination \((\Delta q_1^i - \alpha_s(Q^2)\Delta G^1(Q^2)/2\pi)\), which show \(Q^2\) evolution with terms of order \(\mathcal{O}(\alpha_s^2)\) and higher.

Gluon polarization can be probed by studying large \(p_T\) \(pp\) scattering process in which a prompt photon is observed [16]. The dominant hard scattering process, that gives rise to prompt photon is the compton analogue; a valence quark of one proton scatters off a polarized gluon \(g + q \rightarrow \gamma + q\). To probe the proton spin content in gluon, we may either use single polarized proton to observe circular polarization of the prompt photons \((p + p^\uparrow \rightarrow \gamma^\uparrow \downarrow X)\) or study the longitudinal polarization asymmetry in the scattering of polarized proton on a polarized target \((p^\uparrow + p^\uparrow \downarrow \rightarrow \gamma + X)\). It is found that in both these cases, the dominant contribution comes from the “compton” subprocess and hence can be used a clear probe for the gluonic content of the proton spin.

Before we conclude it may be worthwhile reminding us that the structure functions are intrinsically non-perturbative inputs in QCD. The analysis, we have presented largely banks on the one hand on the perturbative QCD and the effect it has on the evolution of the structure functions and on the other hand as the consistency conditions that arise from the underlying symmetries and conservation principles. It will be desirable to see what one can say about the spin content from an ab-initio non-perturbative model, say through a lattice computation. In view of the reported success of the technology of lattice gauge computation in being able to get low energy parameters, it should be possible to arrive at reliable results on the spin structure as well.

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Figure Captions

Figure 1 - Deep inelastic scattering to probe the dynamical spin content of nucleon.