Correlation between the charged kaon ratio and the baryon phase-space density in heavy-ion collisions

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It is found that the average baryon phase-space density obtained from the ratio of deuteron to proton yields is nearly constant over centrality in Au+Au collisions at the AGS. The finding offers an explanation for the puzzling centrality independence of the ratio of charged kaon total yields. The correlation between the charged kaon ratio and the average baryon phase-space density is studied for central heavy-ion collisions of various systems over a wide range of beam energy. It is found that the charged kaon ratio and the average baryon phase-space density both increase with decreasing beam energy, and are strongly correlated. Such study may provide a new approach to search for medium effects on the kaon mass.

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Strangeness production has been extensively studied in heavy-ion collisions because enhanced strangeness production may signal the formation of Quark-Gluon Plasma. It has been observed that kaon production per participant nucleon increases with centrality in Si+Al, Si+Au, and Au+Au collisions at the Alternating Gradient Synchrotron (AGS), reaching a factor of 3–4 enhancement in central collisions with respect to p+p interactions. However, the ratio of charged kaon total yields (K+/K−) varies little with the collision centrality. Similar results have been also observed in Ni+Ni collisions at the Schwer Ionen Synchrotron (SIS) and Pb+Pb collisions at the Super Proton Synchrotron (SPS). The nearly constant K+/K− is puzzling because K+ and K− are thought to be produced by different mechanisms: K−’s are produced by pair production together with a K+, while K+’s can be produced, in addition, by associate production together with a hyperon. These different production mechanisms lead to different rapidity distributions which are observed, namely, the K+ rapidity distribution is broader than K−’s. One naively expects that in heavy-ion collisions the relative contribution of associate over pair production increases with centrality, because the associate production threshold for kaons is lower than the pair production threshold, and there are more particle re-interactions (at lower than the full energy) in central than peripheral collisions. Therefore, one expects an increasing K+/K− with centrality.

On the other hand, the constituent quark model has been successfully applied to describe particle ratios in heavy-ion collisions. In the constituent quark model, K+ = u and K− = d. Hence, K+/K− depends on the baryon (baryon−antibaryon) phase-space density established in the collision zone at chemical freeze-out (when particles cease to interact inelastically hence particle abundances and ratios are fixed). This picture, the observed constant K+/K− implies a constant baryon phase-space density over the collision centrality. The extraction of the baryon phase-space density at chemical freeze-out heavily relies on chemical equilibrium models; such model studies have been so far limited to central collisions. On the other hand, the baryon phase-space density at final kinetic freeze-out (when particles cease to interact both inelastically and elastically) can be readily extracted from deuteron coalescence data. This is because deuterons, due to its small binding energy, are mostly formed by a proton and a neutron overlapping in phase-space, and cannot subsequently survive collisions with other particles. In this Letter, we study the kinetic freeze-out average baryon phase-space density ⟨f⟩ as a function of centrality, especially for Au+Au collisions at the AGS. We show that the ⟨f⟩ value in Au+Au collisions is nearly constant over centrality. The result may indicate that the average baryon phase-space densities at kinetic and chemical freeze-out ⟨f⟩ are connected in a way that is independent of centrality. We further study the correlation between K+/K− and ⟨f⟩ in central heavy-ion collisions and demonstrate that K+/K− increases with ⟨f⟩ from SPS, to AGS, to SIS (decreasing beam energy).

However, there is a physics origin that may result in an opposite behavior, a decreasing K+/K− with increasing ⟨f⟩, and that is mass modification in nuclear medium. It is predicted that the K− effective mass is lower in nuclear medium than in free space, and the K+ effective mass slightly higher. A decreasing K− (and/or an increasing K+) effective mass would yield a reduced K+/K− ratio. In fact, the KaoS Collaboration has studied K+/K− in Ni+Ni collisions at the SIS and inferred a significant drop in the K− effective mass. On the theory side, however, different conclusions can be reached depending on model assumptions. In this Letter, we take a different approach, i.e., studying K+/K− as a function of ⟨f⟩. If the mass modification is large, one may observe a spectacular phenomenon: K+/K− increases and then decreases with ⟨f⟩. Such phenomenon is not observed, however, we argue that this new approach may provide a more direct way to search for kaon medium modifications.
The Letter is organized as follows. First we present the rapidity distributions of the average nucleon phase-space density at the AGS for collisions of various centralities. Then we extract \( \langle f \rangle^\text{Kin}_B \) for each centrality and show that the value of \( \langle f \rangle^\text{Kin}_B \) is nearly constant. Finally we demonstrate that \( K^+ / K^- \) is strongly correlated with \( f^\text{Kin}_B \) using the central collision data and discuss implications of the results on medium effects to the kaon mass.

In the coalescence model, the neutron phase-space density is related to the ratio of deuteron to proton yields \((d/p)_{dk}[1,16]\). Following Ref. [16], but not assuming identical neutron and proton yields, we obtain the spatial average nucleon phase-space density at kinetic freeze-out as

\[
\langle f(y) \rangle^\text{Kin}_N = \frac{1 + p/n}{2} \left( \frac{1}{2} \langle dN/dy \rangle_p \right) \left( \frac{1}{2} \langle dN/dy \rangle_n \right),
\]

where \( p/n = \langle dN/dy \rangle_p/\langle dN/dy \rangle_n \), and \( \langle dN/dy \rangle_{p,n,d} \) are the rapidity densities of proton, neutron, and deuteron, respectively. The first term arises from isospin asymmetry and will be referred to as

\[
\alpha_t = (1 + p/n)/2.
\]

The value of \( p/n \) depends on the combination of the projectile and target species, the collision centrality and nucleon rapidity. The value of \( p/n \) may vary little with nucleon transverse momentum because of the isospin symmetry of strong interaction.

The deuteron and proton data from Si+Al and Si+Au at 14.6 AGeV/c [17] and Au+Au at 11.6 AGeV/c [18] are used to extract \( \langle f(y) \rangle^\text{Kin}_N \). We assume \( p/n = 1 \) for the nearly isospin symmetric Si+Al collisions. For Si+Au and Au+Au, the values of \( p/n \) are estimated using the Relativistic Quantum Molecular Dynamics (RQMD) model [19]. For the rapidity ranges covered by the data, the model indicates \( 0.8 < p/n < 1 \), hence \( 0.9 < \alpha_t < 1 \) for both systems.

Figure 1 shows the obtained \( \langle f(y) \rangle^\text{Kin}_N \) as a function of the proton rapidity. \( \langle f(y) \rangle^\text{Kin}_N \) varies little with rapidity in central Si+Al and Au+Au collisions. In peripheral Si+Al and Au+Au and both central and peripheral Si+Au collisions, \( \langle f(y) \rangle^\text{Kin}_N \) is larger at target rapidity than mid-rapidity. We note that at target rapidity deuteron may be produced by mechanisms other than coalescence (e.g. target fragmentation) [15], resulting in an overestimate of \( \langle f(y) \rangle^\text{Kin}_N \) by Eq. (1). The magnitude of this possible effect is not investigated in the present work.

The average baryon phase-space density is the relevant quantity. Hyperons are the most significant contributors to the baryons besides nucleons. The abundance of antihyperons is negligible at AGS energies, and is small at SPS energies. Generally, the average baryon phase-space density is obtained from

\[
\langle f \rangle^\text{Kin}_B = \langle f \rangle^\text{Kin}_N \left( 1 - \langle \overline{N}/N \rangle \right) \left[ 1 + (Y - \overline{Y})/(N - \overline{N}) \right],
\]

where \( \langle f \rangle^\text{Kin}_B \) is the rapidity averaged nucleon phase-space density (see below), and \( N, \overline{N}, Y, \) and \( \overline{Y} \) are the nucleon, antinucleon, hyperon, and antihyperon total yields, respectively. It has been assumed that the phase-space distributions of hyperons and antihyperons are the same as of nucleons [20]. This assumption does not introduce significant errors on \( \langle f \rangle^\text{Kin}_B \) as the contributions of hyperons and antihyperons are generally small (see Table I).

We denote

\[
\alpha_N = 1 - \overline{N}/N \approx 1 - \overline{p}/p
\]

where \( \overline{N}/N \) has been approximated by the antiproton to proton ratio \((\overline{p}/p)\), and

\[
\alpha_Y = 1 + (Y - \overline{Y})/(N - \overline{N}).
\]

Hence,

\[
\langle f \rangle^\text{Kin}_B = \langle f \rangle^\text{Kin}_N \cdot \alpha_N \cdot \alpha_Y.
\]

The conservations of global baryon number and strangeness give, respectively,

\[
N - \overline{N} = N_{\text{part}} - (Y - \overline{Y})
\]

where \( N_{\text{part}} \) is the total number of participant nucleons, and

\[
Y - \overline{Y} \approx K - \overline{K} = (1 + \alpha_K) \cdot (K^+ - K^-)
\]

where \( \alpha_K = (K^0 - \overline{K}^0)/(K^+ - K^-) \). Therefore, Eq. (1) becomes

\[
\alpha_Y = [1 - (1 + \alpha_K) \cdot (K^+ - K^-)/N_{\text{part}}]^{-1}.
\]

The value of \( \alpha_K \) is not one in isospin asymmetric collisions, however, we will use \( \alpha_K \approx 1 \) because the effect of a non-unitary value [21] is reduced by \((K^+ - K^-)/N_{\text{part}}\) which is a small quantity for collisions we consider. It should be noted that only one unit of strangeness per multi-strange hyperon is counted in Eq. (3). This is safe because the production of multi-strange hyperons is relatively small even at the SPS energies [22].

The AGS E859/E866 proton and deuteron measurements cover a broad rapidity range exploiting the (near) symmetry of the Si+Al and Au+Au systems [23]. We take the \((dN/dy)_{p} \) weighted average of \( \langle f(y) \rangle^\text{Kin}_N \) in Eq. (1) over this rapidity range to obtain \( \langle f \rangle^\text{Kin}_N \). The unmeasured \( d/p \) ratio in the mid-rapidity region is approximated by the dotted lines in Fig. 1 connecting the last point (i.e., closest to mid-rapidity) with the reflected one. For Si+Au we assume that the unmeasured \( d/p \) ratio at the more forward rapidities is the same as the last data point. We assign a 8% systematic error on \( \langle f \rangle^\text{Kin}_N \) for Si+Au due to this extrapolation by examining a smoothly dropping \( d/p \) ratio from the low rapidity data points. From \( \langle f \rangle^\text{Kin}_N \), we extract \( \langle f \rangle^\text{Kin}_B \) using
Eqs. (1), (3), and (4). For all three systems \( \alpha_N \approx 1 \) and \( \alpha_T \) ranges from 1.03 in peripheral to 1.13 in central collisions. For Si+Al and Au+Au, we assign a 5% systematic error on \( \langle f \rangle_{B}^{\text{Kin}} \) due to the assumptions made to extract \( \langle f \rangle_{B}^{\text{Kin}} \) and errors on the correction factors. For Si+Au, we assign a 10% systematic error on \( \langle f \rangle_{B}^{\text{Kin}} \).

Figure 2 shows \( K^+/K^- \) and \( \langle f \rangle_{B}^{\text{Kin}} \) at the AGS as a function of \( N_{\text{part}} \). Both quantities are nearly constant. It should be noted that the constant values do not necessarily demonstrate that \( K^+/K^- \) and \( \langle f \rangle_{B}^{\text{Kin}} \) are correlated. However, it will be shown that they are correlated in central collisions at different energies. Therefore, it is fair to argue that they are correlated also in the same collision system and are independent of the collision centrality. Because \( K^+/K^- \) is fixed at chemical freeze-out, the results may imply that the \( \langle f \rangle_{B}^{\text{Kin}} \) values at kinetic freeze-out, which presumably happens later than chemical freeze-out in heavy-ion collisions, are connected to the chemical freeze-out \( \langle f \rangle_{B}^{\text{Ch}} \) values in a way that is independent of centrality.

Since the difference between the kaon yields \( (K^+ - K^-) \) enters into \( \langle f \rangle_{B}^{\text{Kin}} \) as a correction, one may be concerned about auto-correlation. However, such concern is unnecessary because (1) no correlation is present between \( K^+/K^- \) and \( K^+ - K^- \); and (2) the \( K^+ - K^- \) correction to \( \langle f \rangle_{B}^{\text{Kin}} \) is small (at most 13%). In fact, one may correlate \( K^+/K^- \) with the average nucleon phase-space density which is shown as the dashed curve in Fig. 2.

It should be noted that the correlation exists only between the rapidity integrated values of \( \langle f \rangle_{B}^{\text{Kin}} \) and \( K^+/K^- \); locally in rapidity, \( \langle f \rangle_{B}^{\text{Kin}} (y) \) and \( K^+/K^- (y) \) are not or much less correlated (especially in central Au+Au collisions). This may be due to longitudinal flow 

\[
\frac{K^+}{K^-} = 1 + \left( \frac{\langle f \rangle_{B}^{\text{Kin}}}{0.0076 \pm 0.0009} \right)^{1.20 \pm 0.10}.
\]

The fit result is plotted in Fig. 3 as the dashed curve. The KaoS Collaboration observed a \( K^+/K^- \) ratio in Ni+Ni collisions at the same equivalent (sub-threshold) beam energy that is about seven times smaller than in p+p near threshold. The KaoS Collaboration contributed this difference to a drop in the \( K^- \) mass in Ni+Ni collisions, and inferred a mass drop of \( 270^{+55}_{-80} \) MeV/c\(^2\) by using the measured kaon excitation function. Since \( \langle f \rangle_{B}^{\text{Kin}} \) is a measure of matter density, we argue that studying \( K^+/K^- \) against \( \langle f \rangle_{B}^{\text{Kin}} \) may provide a different and probably more direct approach to search for medium effect on kaon mass: If the \( K^- \) mass is modified by medium at low energy but not (or differently) at high energy, then the low energy \( K^+/K^- \) should deviate from extrapolation of the high energy data. This Letter is a first attempt of this new approach: The fit result in Fig. 3 seems to imply no difference between the SIS and AGS/SPS data; fitting the data excluding the Ni+Ni point to power-law function, yielding the result shown in the dotted curve, seems to give the same implication.

Moreover, as noted, the average baryon phase-space densities at chemical and kinetic freeze-out may be different; the more suitable variable to correlate with \( K^+/K^- \) is the chemical freeze-out one. It has been found in Ref. [14], under the assumption of chemical and thermal equilibrium, that the chemical freeze-out \( \langle f \rangle_{B}^{\text{Ch}} \) is twice the kinetic freeze-out \( \langle f \rangle_{B}^{\text{Kin}} \) at both the AGS and SPS. Using the thermal model parameters obtained from the Ni+Ni data [11], the \( \langle f \rangle_{B}^{\text{Ch}} \) value is found to be the same as the \( \langle f \rangle_{B}^{\text{Kin}} \) value. In Fig. 3, if the \( \langle f \rangle_{B}^{\text{Ch}} \) values were plotted, then the Ni+Ni data point would lie above a sensible extrapolation from the AGS and SPS data by about a factor of two. This is even more contradictory to \( K^- \) mass drop, as \( K^- \) mass drop would give a reduced \( K^+/K^- \) ratio.

However, one should be careful to conclude from the above results that there is no \( K^- \) mass drop in the Ni+Ni data. This is because:

1. Although the power law parameterization is motivated by equilibrium models, it is not a priori for the relation between \( K^+/K^- \) and \( \langle f \rangle_{B}^{\text{Kin}} \). In fact, data from C+C collisions at the SIS do not follow the parameterization shown in Fig. 3: there is no \( K^+/K^- \) ratio of 39 \( \pm 6 \) was observed; no d/p data is available, but it can be conjectured that the \( \langle f \rangle_{B}^{\text{Kin}} \) value is lower in C+C than in Ni+Ni. It is possible that the C+C data follow a different systematic consistent with the AGS/SPS data, while the Ni+Ni data lie below this systematic. This
would be an interesting result because it implies a $K^-$ mass drop in Ni+Ni collisions.

2. Since the leverage of the AGS and SPS data is small, the systematic errors on these data are critically important to the extrapolation to the SIS data.

In order to address these concerns, more data are needed to fill in the gap between AGS and SIS. These data are underway from the E895 Collaboration [33]. New data from the SPS at 40 AGeV would be also valuable as they fill in the gap between the full-energy AGS and SPS data.

In summary, we have extracted the average baryon phase-space density at kinetic freeze-out from the deuteron to proton ratio in Si+Al, Si+Au and Au+Au collisions at the AGS [17,18]. The average baryon phase-space density at kinetic freeze-out is found to be nearly constant over the collision centrality, which offers an explanation for the puzzling centrality independence of the charged kaon mass drop in Ni+Ni collisions.

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FIG. 1. The spatial average nucleon phase-space density at kinetic freeze-out, \( \langle f(y) \rangle_{\text{Kin}}^N \), as a function of the proton center-of-mass rapidity \( y \) in central and peripheral heavy-ion collisions at the AGS. Collision centralities are indicated by the cross section ranges. The data points at backward rapidities \( y < 0 \) are measured, and are reflected about mid-rapidity \( y = 0 \). Errors shown are statistical only. The experimental systematic effects largely cancel in the d/p ratio. The solid lines connecting the data points are to guide the eye. The dotted lines are an extrapolation to the unmeasured regions.

FIG. 2. The \( K^+ / K^- \) ratio (filled symbols, left scale) and the average baryon phase-space density at kinetic freeze-out \( \langle f \rangle_{\text{Kin}}^B \) (open symbols, right scale) as a function of the total number of participant nucleons \( N_{\text{part}} \) at the AGS. Errors shown are statistical for \( K^+ / K^- \) and statistical and systematic errors added in quadrature for \( \langle f \rangle_{\text{Kin}}^B \). The experimental systematic effects largely cancel in the \( K^+ / K^- \) and d/p ratios. Note that both quantities are nearly constant over centrality. The dashed curve shows the average nucleon phase-space density at kinetic freeze-out (right scale); the errors are comparable to the \( \langle f \rangle_{\text{Kin}}^B \) ones.

FIG. 3. The \( K^+ / K^- \) ratio as a function of the average baryon phase-space density at kinetic freeze-out \( \langle f \rangle_{\text{Kin}}^B \) in central heavy-ion collisions. Errors shown are statistical and systematic errors added in quadrature. The data are tabulated in Table I. The dashed curve is a fit to power-law function. The dotted curve is a similar fit but excluding the Ni+Ni data point.
TABLE I. The $K^+/K^-$ ratio and the average baryon phase-space density at kinetic freeze-out $\langle f \rangle_{B}^{\text{Kin}}$ (extracted from d/p ratios) in central heavy-ion collisions. The factors $\alpha_I$, $\alpha_N$, and $\alpha_Y$ take into account, respectively, the effects of isospin asymmetry, antinucleon and (anti)hyperon yields. For $K^+/K^-$ and $\langle f \rangle_{B}^{\text{Kin}}$ the first error is statistical. The systematic errors (in percentage) are introduced by the extrapolation in rapidity, the uncertainties in $\alpha_I$, $\alpha_N$, and $\alpha_Y$, and the preliminary nature of some of the data. The experimental systematic effects largely cancel in the $K^+/K^-$ and d/p ratios.

| Collision System | Beam Energy per Nucleon (mb) | $K^+/K^-$ | $\alpha_I$ | $\alpha_N$ | $\alpha_Y$ | $\langle f \rangle_{B}^{\text{Kin}}$ |
|------------------|-----------------------------|----------|----------|----------|----------|-----------------|
| S + S            | 200 GeV, 160                | 1.58 ± 0.09 ± 5% | 1 | 0.89 | 1.32 | 0.0039 ± 0.0007 ± 10% |
| Pb+Pb            | 158 GeV, 340                | 1.83 ± 0.02 ± 5% | 0.97 | 0.93 | 1.32 | 0.0069 ± 0.0010 ± 10% |
| Si+Al            | 14.6 GeV/c, 100             | 4.45 ± 0.32 | 1 | 1 | 1.11 | 0.0221 ± 0.0017 ± 5% |
| Si+Au            | 14.6 GeV/c, 260             | 5.25 ± 0.42 | 0.95 | 1 | 1.13 | 0.0269 ± 0.0006 ± 10% |
| Au+Au            | 11.1, 11.6 GeV/c, 420, 470a | 6.35 ± 0.48 | 0.96 | 1 | 1.12 | 0.0297 ± 0.0009 ± 5% |
| Ni+Ni            | 1.8, 1.93 GeV, 100          | 21.1 ± 3.4 ± 5% | 1 | 1 | 1 | 0.0879 ± 0.0021 ± 5% |

$^a$ The first number is for the $K^+/K^-$ data; the second is for the d/p data. Quoted for Ni+Ni are the kinetic beam energies. $^b$ The Pb+Pb $\alpha_Y$ value is used because the total yield $K^+ - K^-$ is not available for S+S.