In section A.3.4 (appendix A), part (a), subpart (i), there is an error in the first unnumbered equation

\[ H \left( \alpha_{\pi_1}, \alpha_{\pi_2}^*, \ldots, \alpha_{\pi_N}^* \right) = H^* \left( \alpha_{\pi_1}^*, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N} \right). \]  

The equation should read

\[ H \left( \alpha_{\pi_1}', \alpha_{\pi_2}', \ldots, \alpha_{\pi_N}' \right) = H^* \left( \alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N} \right), \]  

where \( \pi' \) denotes the inverse permutation to \( \pi \), that is \( \pi \pi' = \text{id} \). The reason is that, under reversal, the permutation \( \pi \) with amplitude \( \alpha_{\pi} \) yields the amplitude \( \alpha_{\pi'} \), for the permutation \( \pi' \) (see figure 1). Consequently, the unnumbered equation that follows

\[ Q(\pi') = Q^*(\pi), \]  

and equation (19),

\[ Q(\pi) = \pm 1, \]  

only hold for permutations \( \pi \) that are self-inverses, \( \pi^{-1} = \pi \), not for all \( \pi \) as stated.

As it turns out, the remainder of the argument, contained in subpart (ii) and continuing thereafter, can be minimally modified in such a way that it does not require making use of equation (2) at all. We now give this modification. For the sake of clarity, we give the modified argument, culminating in the final expression for \( H \left( \alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N}, \right) \), in full.

(ii) Establishing \( Q(\pi') = Q(\pi) \) whenever \( \pi \) and \( \pi' \) are both odd or both even.

In equation (A11), let \( \alpha_{\pi} = k (\delta_{\pi,\pi} + \delta_{\pi',\pi}) \), where \( k \) is some constant. Then, using equations (A18), namely

\[ H \left( \alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N} \right) = \sum_{\pi \in S_N} \frac{Q(\pi) \alpha_{\pi}}{\sum_{\pi \in S_N} Q(\pi) \alpha_{\pi}}, \]  

equation (A11) becomes

\[ \left| Q(\pi) + Q(\pi') \right| = \left| 1 + (-1)^{\sigma} \right|, \]  

with \( \sigma = 0 \) or \( 1 \), where \( \pi \) is any permutation and \( \pi' = \tau \pi \) where \( \tau \) is any transposition. Now, equation (A9) implies that \( |Q(\tau)| = 1 \) for all \( \tau \). Therefore

\[ Q(\pi') = (-1)^{\sigma} Q(\pi). \]  

Now, let \( \pi'' = \tau' \tau \), where \( \tau' \) is some transposition. Then, equation (3) implies that

\[ Q(\pi'') = (-1)^{\sigma} Q(\pi'). \]  

Combining this with \( Q(\pi') = (-1)^{\sigma} Q(\pi) \), we obtain

\[ Q(\pi'') = Q(\pi). \]  

That is, any two permutations, \( \pi, \pi'' \), that are connected by a pair of transpositions have the same \( Q \)-value.

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1 Due to an error in the proof, these equations were not labelled in the published version of the paper.
But every even permutation can be written as a product of an even number of transpositions and the identity permutation. Therefore, all even permutations have the same $Q$-value, $Q_e$. Similarly, every odd permutation can be written as a product of an even number of transpositions and a given odd permutation. Therefore, all odd permutations have the same $Q$-value, $Q_o$. Finally, equation (3) implies that $Q_o = (-1)^p Q_e$.

With these results for $Q(\pi)$, equations (A18) become

\[
H(\alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N}) = Q(\pi) \sum_{\pi \in S_N} \left( \text{sgn}(\pi) \right)^p \alpha_{\pi} \quad (5a)
\]

\[
H(\alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N}) = Q(\pi) \sum_{\pi \in S_N} \left( \text{sgn}(\pi) \right)^o \alpha_{\pi}^o \quad (5b)
\]

where $\text{sgn}(\pi)$ takes the value $+1$ or $-1$ according to whether $\pi$ is even or odd.

Insofar as the probability of the transition of the system of $N$ indistinguishable particles is concerned, the multiplicative phase factor $Q(\pi)$ and the complex conjugation are irrelevant. More generally, consider a system, $S$, that consists of subsystems $S_1$ and $S_2$, where $S_i$ consists of $N_i$ indistinguishable particles of one type (say, electrons). Suppose, first, that $S_1$ only contains particles that can be distinguished from those in $S_2$. As described above, let measurements $\mathbf{L}_{1,m}, \mathbf{L}_{2,n}$ and $\mathbf{M}_{1,i}, \mathbf{M}_{2,j}$ be performed on $S_1$ at times $t_1$ and $t_2$. Additionally, let measurements $\mathbf{U}$ and $\mathbf{V}$ be performed on $S_2$ at times $t_1$ and $t_2$, respectively, yielding outcomes $u$ and $v$. Let $\alpha_{\pi}$ be the amplitude of the transition of $S$ from $(\ell': u)$ to $(m: v)$ in which the particles in $S_2$ are treated as distinguishable and make the transition described by $\pi$. Then, by the same argument as described above, the amplitude of the process from $(\ell': u)$ to $(m: v)$ where the particles in $S_1$ are treated as indistinguishable is given by $H(\alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N})$, and the transition probability is again unaffected by the overall sign or complex conjugation of $H$. In the case that $S_1$ does contain, say, $M$ particles that are indistinguishable from those in $S_1$, the boundaries of $S_1$ must be redrawn to encompass them. The resulting situation, namely a system composed of subsystem $S'_1$, containing $N + M$ indistinguishable particles, and subsystem $S'_2$, containing only particles that are distinguishable from those in $S_1$, is of the same type as the one previously considered.

Therefore, in general, the overall multiplicative factor $Q(\pi)$, which has modulus unity\(^2\), and the complex conjugation in equation (5b), are irrelevant insofar as predictions are concerned, and can be discarded without any loss of generality. Hence, without loss of generality, we can take equation (5a) with $Q(\pi) = 1$, namely

\(^2\)Parenthetically, equation (2) implies that $Q^4(\pi) = Q(\pi)$. Therefore, $Q(\pi)$ is in fact $\pm 1$. However, we do not need to make use of this fact in the present argument.
\[ H(\alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N}) = \sum_{\pi \in S_N} (\text{sgn}(\pi))^{\sigma} \alpha_{\pi}, \] \hspace{1cm} (6)

where \( \sigma = 0 \) or \( \sigma = 1 \) is the only remaining degree of freedom, corresponding respectively to bosons and fermions.

The above result for the amplitude holds for a particular labelling of the outcomes at times \( t_1 \) and \( t_2 \). However, the corresponding transition probability is invariant under relabelling of these outcomes. To see this, suppose that the outcomes at \( t_1 \) and \( t_2 \) are relabelled such that \( \ell_i \rightarrow \ell'_{\pi(i)} \) and \( m_j \rightarrow m_{\pi(j)} \) for \( i, j \in \{1, 2, \ldots, N\} \), where \( \pi \) and \( \pi' \) are each permutations of \( N \) elements. Then the transition \( \pi \rightarrow \pi' \) is relabelled \( \pi \rightarrow \pi' \pi \tau \). Now, \( \text{sgn}(\tau' \pi \tau) = \text{sgn}(\tau' \pi) \text{sgn}(\pi) \). Therefore, as a result of the outcome relabelling, either all even transitions go to even transitions, and odd transitions go to odd transitions; or all even transitions go to odd transitions, and all odd transitions go to even transitions. Now, note that equation (6) can be written as

\[ H(\alpha_{\pi_1}, \alpha_{\pi_2}, \ldots, \alpha_{\pi_N}) = \sum_{\pi \text{ even}} \alpha_{\pi} + (-1)^{\sigma} \sum_{\pi \text{ odd}} \alpha_{\pi}. \] \hspace{1cm} (7)

Thus, the effect of relabelling of outcomes is at most a change of sign of \( H \). Hence, the corresponding transition probability is invariant under outcome relabelling, and there is no loss of generality in taking equation (6) for any particular labelling.

**Acknowledgment**

I thank Klil Neori for bringing my attention to the error in equation (1).

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3 As originally given, the second summation in the equation below was erroneously given the multiplicative factor \( \sigma \) instead of the correct factor \((-1)^\sigma\).