An Empirical Study on the “Usage of Not” in Real-World JSON Schema Documents
(Long Version)

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Abstract. In this paper, we study the usage of negation in JSON Schema data modelling. Negation is a logical operator that is rarely present in type systems and schema description languages, since it complicates decision problems. As a consequence, many software tools, but also formal frameworks for working with JSON Schema, do not fully support negation. As of today, the question whether covering negation is practically relevant, or a mainly theoretical exercise (albeit challenging), is open. This motivates us to study whether negation is really used in practice, for which aims, and whether it could be — in principle — replaced by simpler operators. We have collected the most diverse corpus of JSON Schema documents analyzed so far, based on a crawl of 90k open source schemas hosted on GitHub. We perform a systematic analysis, quantify usage patterns of negation, and also qualitatively analyze schemas. We show that negation is indeed used, following a stable set of patterns, with the potential to mature into design patterns.

Keywords: Empirical Study of Conceptual Modeling · JSON Schema.

1 Introduction

Negation is a logical operator that is rarely present in type systems and schema description languages, since it complicates decision problems. JSON Schema introduced negation, together with the And, Or, and XOr operators, in Draft-04 (2013). We study here whether negation is really used by JSON Schema users, for which aims, and whether it could be in principle replaced by simpler operators.

For this aim, we collected a big set of JSON Schema files out of GitHub and started a systematic analysis, based on numerical quantification of usage patterns of negation and direct analysis of schemas with negation.

We discovered that negation is indeed used by JSON Schema users, according to a set of usage patterns that we are going to describe. We also discovered some peculiar uses, that we will describe later.
2 Method

2.1 Schema collection and duplicate elimination

In order to collect the schemas to analyse, we proceeded as follows.

We used the cloud-service Google BigQuery to identify all JSON Schema documents with an open source license (excluding the specification of JSON Schema drafts) on GitHub. Our query was executed in July 2020 and identified 91,600 URLs. Of these, we could successfully retrieve 85,600 (using `wget`).

We searched this collection for all files ending in `.json`, containing a property `$schema` that identifies it as a schema, but not being the JSON Schema declaration itself. Having discarded all files with invalid JSON syntax, we are left with 82,000 files, which became 24,000 after the elimination of perfect copies of the same schema. To our knowledge, this is the largest collection of real-world JSON Schema documents ever analyzed so far.

Analysing these files, we realized that, still, many of them were just different versions of essentially the same file: among the 24,000 different schemas, we found for example 14 different versions of a schema with title “A JSON Schema for Swagger 2.0 API” (in the original set of 82,000 schemas these 14 versions were replicated over 1,147 files).

The presence of almost-duplicates is a problem, for some different reasons. A first reason is that duplication creates skews in our numeric statistics. This is not very important, since these statistics are interesting as they indicate trends, but the actual numbers are not crucial, as will be discussed again in the paper.

The most important reason is the very practical fact that we stumbled many times across the same pattern during direct exploration, which is very time-consuming, and moreover it was not easy to understand whether the identical repetition of a pattern indicated that many designers were following the same approach or just the presence of many versions of the same schema.

A third problem is that duplication makes our dataset uselessly big, hence our analysis queries take more time to run. For these reasons we tried to reduce this form of redundancy.

Hence, in order to reduce the number of similar files, we computed for each file how many times each of the JSON Schema keywords appears in that file and, for each equivalence class with respect to this measure, we selected the longest file. After this step we remained with 13,700 files.

We then performed a further reduction step, where we regarded all files having the same (id/$id, title, description) triple as versions of the same schema, provided that at least one of these members was present in the schema, and we only selected the longest of version out of each equivalence class. This brought the number down to 11.5K files.

At this point we started looking for **not** assertions, but we soon realized that an important fraction of these have the form

```json
not: { $ref: "reference" }
```
$\textit{ref}$ is an operator that refers to another JSON Schema object that may be in the same file or in a different file. The presence of $\textit{ref}$ pointers prevents us from analysing the intended use of the $\textit{not}$ keyword, hence we prepared a ref-expanded version of our 82,000 files, where we tried to substitute any instance of "$\textit{ref}$": "reference" with an expanded instance "$\textit{eref}$": referred object. This is not always possible, since we were not always able to retrieve the file that is referenced, in case of non-local references, and since some references were not correct, but we have been able to expand the vast majority of references (the 83%). The referenced object may contain other references, and references may be recursive, but this is not a problem since, for our aims, we only need to substitute every reference with the referenced object, with no further expansion. Nevertheless, this one-level expansion has been sufficient to increase the average size of our schemas by a factor of almost three.

We finally looked for instances of the $\textit{not}$ keywords in these 11.5K distinct schemas using a combination of automated counting techniques, to get statistics about pattern frequencies, and direct exploration, where we looked at how programmers use negation and what are they trying to express.

2.2 Pattern quantification

In order to quantify the relevance of each usage pattern, we proceed as follows.

For each usage pattern we define a set of pattern queries, that characterize the usage pattern, such as, for example, the query

$\textit{..not.required}$

that characterizes situations where we have a $\textit{not.required}$ path inside the schema, that is, where $\textit{required}$ is a field of the object associated to a $\textit{not}$ keyword. For each query, we count the number of documents that contain at least one instance, and the total number of instances in the document collection.

Patterns are expressed using the following language:

\[
\begin{align*}
  p & ::= \text{step} | \text{step } p \\
  \text{step} & ::= .\text{key} | .* | [*] | .. \\
  \text{filtered } p & ::= p ? p
\end{align*}
\]

Pattern matching is defined as in JSONPath [7]: the step $\.\text{key}$, applied to an object, retrieves the member value whose name is $\text{key}$. The step $.*$ retrieves all member values of an object, $[*]$ retrieves all items of an array, and $..$ is the reflexive and transitive closure of the union of $.*$ and $[*]$, hence it navigates to all nodes of the JSON tree to which it is applied.

Finally, we will use the conditional form $p1 ? p2$ to denote those nodes $n$ that are reached by a path $p1$ and such that if we follow a path $p2$ starting from $n$ we will arrive to some node, so that, for example the query:

$\textit{..any0f..not}$

indicate all subtrees reached by a path $\textit{..any0f}$ that include an instance of $\textit{not}$. 


2.3 Working with our data

One important result of our research has been the construction of our dataset, that allows researchers to analyse any kinds of aspects that relate to the real-world use of JSON Schema, not only to use of negation.

This dataset is available to every researcher. In [] we describe how to rebuild our dataset on a local Postgres installation, how to check all numbers that we publish here, and how to build different queries related to other aspects of JSON Schema usage.

3 Description of the retrieved schemas

3.1 Versions of JSON Schema

JSON Schema exists essentially in five versions: Draft-03 of November 2010 [4], Draft-04 of February 2013 [1], Draft-06 of April 2917 [1], Draft-07 of March 2018 [2], Draft 2019-09 of September 2019 [3]; Draft-05 was essentially a cleanup of Draft-04 and used the same meta-schema, while versions before Draft-03 have been absorbed by that one.

For each retrieved file, we analysed the $schema declarations that it contains in order to see the version of JSON Schema to which it adheres. The distribution of the collected schemas across different versions of JSON Schema is reported in Table 3.1; Draft 2019-09 (also known as Draft-08) is still quite new, hence is not really represented. Draft-04 is declared in the vast majority of the files, while Draft-07, Draft-06, and the old Draft-03, are declared in comparable quantities. It is worth adding that an analysis of files content showed us that the actual version that a schema follows is often different from the version declared.

| Version       | Amount of files | Percentage |
|---------------|-----------------|------------|
| draft-01      | 1               | 0.00       |
| draft-03      | 3106            | 3.78       |
| draft-04      | 64732           | 78.85      |
| draft-05      | 21              | 0.03       |
| draft-06      | 6787            | 8.27       |
| draft-07      | 2195            | 2.67       |
| no version specified | 5252     | 6.40       |
| total         | 82094           | 100.00     |

Table 1. Distribution of files according to the version referred in their $schema attribute

For our study, it is worth recalling that the boolean operators not, oneOf, anyOf, and allOf, have been added with Draft-04. The possibility of using false and true wherever an assertion is expected has been introduced with Draft-06, together with the operator contains, while the if-then-else operators have been introduced with Draft-07.
3.2 Frequency of different keywords

We counted the number of occurrences of keywords, and the number of files where each keyword appears. The result is presented in table 3.2.

A precise count of keyword occurrences presents some difficulties. Consider for example the following snippet (schema 37687 in our corpus):

```json
"properties": {
  "not": { "title": "...",
  "description": "...",
  "not": {"type": "null"}
}, ...
}
```

The outermost `not` is not an occurrence of the `not` keyword, but is just a property name, while the internal `not` is an actual keyword occurrence. Of course, we cannot just avoid to count occurrences that are below `properties`, since `properties` itself may be a property name, as in the following snippet of code (schema 69248 in our corpus):

```json
"not": { "anyOf": [
  "properties": {
    ...
  },
  "properties": { "not": { "properties": { "name": { }
```

Here, both occurrences of `not` are occurrences of the operator, since the `properties` that surrounds the internal `not` is not a keyword but is a property name, since it is contained into a `properties` keyword.

In order to solve this problem, we rewrote all of our files and we prepended every occurrence of a member name that is in a position where a property name, rather than a keyword, is expected, with “___”, and we did this through a complete parsing of the document, guided by JSON Schema conventions.

Unfortunately, this is not yet enough. The problem is that programmers are allowed to extend JSON Schema as they like hence, when we meet a user-defined keyword, we have no way to know which fields to rename, as happens in the following snippet (schema 32451 in our corpus).

```json
{ "description": "...",
  "@errorMessages": { "not": "Invalid target: ...",
  "not": { "pattern": "..." }
... }
```

Here, “@errorMessages” is a user-defined keyword whose value is an object that describes the error, and not a JSON Schema assertion, hence the contained `not` is not an occurrence of the JSON Schema keyword but is just a member
| Keyword       | Occurrences | Distinct files | Draft- |
|---------------|-------------|----------------|--------|
| Total         | 1347003     | 11508          | 3      |
| type          | 420780      | 11419          | 3      |
| description   | 222091      | 8022           | 3      |
| $ref          | 179811      | 6288           | 3      |
| properties    | 87747       | 11110          | 3      |
| required      | 51476       | 8706           | 3      |
| oneOf         | 46786       | 2274           | 4      |
| enum          | 44383       | 5134           | 3      |
| items         | 44380       | 6345           | 3      |
| title         | 40903       | 6254           | 3      |
| additionalProperties | 37111   | 5738           | 3      |
| id            | 22063       | 4794           | 4      |
| default       | 21298       | 2494           | 3      |
| pattern       | 18345       | 3016           | 3      |
| maxLength     | 15406       | 1472           | 3      |
| minLength     | 15324       | 1673           | 3      |
| $schema       | 13681       | 11508          | 3      |
| minimum       | 11222       | 2015           | 3      |
| anyOf         | 8843        | 1504           | 4      |
| minItems      | 6830        | 1582           | 3      |
| maximum       | 5488        | 1275           | 3      |
| maxItems      | 4460        | 648            | 3      |
| uniqueItems   | 4390        | 1269           | 3      |
| definitions   | 4197        | 3750           | 4      |
| allOf         | 3813        | 1397           | 4      |
| patternProperties | 2823   | 914            | 3      |
| examples      | 2765        | 356            | 3      |
| $id           | 2116        | 748            | 6      |
| additionalItems | 1276     | 401            | 3      |
| readOnly      | 1244        | 107            | 7      |
| not           | 787         | 298            | 4      |
| const         | 648         | 137            | 6      |
| exclusiveMinimum | 496      | 143            | 3      |
| minProperties | 465         | 230            | 4      |
| dependencies  | 416         | 221            | 3      |
| multipleOf    | 350         | 83             | 4      |
| $comment      | 200         | 58             | 7      |
| exclusiveMaximum | 196     | 60             | 3      |
| if            | 166         | 29             | 7      |
| then          | 166         | 29             | 7      |
| maxProperties | 141         | 78             | 4      |
| deprecated    | 78          | 18             | 3      |
| propertyNames | 54          | 26             | 6      |
| contains      | 45          | 10             | 6      |
| else          | 25          | 12             | 7      |
| writeOnly     | 5           | 7              | 7      |
| $defs         | 2           | 2              | 8      |

Table 2. Occurrences of each keyword and number of files where it appears
name (the not in the next line, on the contrary, is an occurrence of the keyword). Unfortunately, there are other user-defined keyword whose value must be, on the contrary, interpreted as an assertion, and we cannot know which user-defined keyword belongs to which category.

We tried two approaches. In the “strict” approach we renamed everything that was inside a user-defined keyword, hence making it inaccessible by the analysis, and in the “lax” approach we kept the content of any user-defined keyword with no renaming, so that both instances of not in the previous example were counted as keywords. With the strict approach, some interesting usage patterns are lost, and keyword usage is under-estimated. With the lax approach, we get some “false positive” as in the example above, hence some over-estimation. One could also try a more sophisticated approach, where one tries to guess the best way of performing renaming for each different user-defined keyword. For our aims, we decided that the over-estimation of the lax approach was better than the under-estimation of the strict one, and that a more sophisticated approach was not worth the extra effort, hence we adopted the lax approach; the reader should keep into account this detail while reading the tables. The presence of user-defined keywords is far from negligible — at a first analysis, we have more than 1 user-defined keyword occurrence for any 10 standard keyword occurrences.

For our purposes, the most interesting outcome of this first analysis is the fact that not appears in the 3% of all schemas, and it occupies the 30-th position in our table, out of 46 keywords analysed. This indicates that not, although it clearly belongs to the bottom half of JSON Schema operators in terms of usage, is indeed used by a non-negligible portion of JSON Schema designers.

If we analyse the use of boolean operators in general, we notice that the most common boolean operator is, by far, oneOf, whose occurrences are five times more common than anyOf. We found that quite surprising, since the exclusive-disjunctive semantics of oneOf is, in some sense, more complicated than the purely disjunctive semantics of anyOf. A possible explanation of this preference is the fact that the description of a class as a oneOf combination of a set of “subclasses” is a close relative of the exclusive-subclassing mechanism of object-oriented languages, which is a well-known jargon. allOf is even less common. For any single occurrence of not, we have 5 of allOf, 11 of anyOf, and 59 of oneOf.

The last boolean operator, if-then-else, is even less common than not, but it has only been added with Draft-07.

At this point, we know that not appears 787 times in 298 different files. This is a quite small fraction of the initial set of files, but the numbers are still big enough to deserve a systematic study.

4 Counting negated keyword

4.1 Counting not.k paths

In order to analyse how users use negation, we first examined which are the JSON Schema operators that are typically negated by a schema designer. The
result is in table 4.1, where you find, for each keyword $k$, the frequency (number of occurrences) of the \texttt{not.$k$} pattern, and the number of files where that pattern appears.

| path           | occ. | files path    | occ. | files |
|----------------|------|---------------|------|-------|
| not            | 787  | 298 not.$eref | 93   | 28    |
| not."          | 840  | 289 not.$eref." | 338  | 28    |
| not.required   | 240  | 84 not.$eref.required | 36   | 15    |
| not.items      | 126  | 27            |      |       |
| not.type       | 62   | 51 not.$eref.type | 51   | 20    |
| not.properties | 71   | 47 not.$eref.properties | 40   | 18    |
| not.$eref      | 93   | 28            |      |       |
| not.enum       | 61   | 52 not.$eref.enum | 12   | 8     |
| not.allOf      | 23   | 23 not.$eref.allOf | 38   | 5     |
| not.pattern    | 47   | 28            |      |       |
| not.anyOf      | 45   | 36 not.$eref.anyOf | 2    | 2     |
| not.description| 4    | 4 not.$eref.description | 41   | 7     |
| not.title      | 2    | 2 not.$eref.title | 39   | 7     |
|                |      |               |      |       |
|                |      | not.$eref.$schema | 41   | 9     |
| not.$fref      | 27   | 14            |      |       |
| not.oneOf      | 6    | 4 not.$eref.oneOf | 18   | 3     |
| not.additionalProperties | 11  | 11 not.$eref.additionalProperties | 9    | 7     |
| not.patternProperties | 15 | 15            |      |       |
| not.const      | 6    | 1             |      |       |
|                |      | not.$eref.definitions | 3    | 3     |
|                |      | not.$eref.id   | 2    | 2     |
|                |      | not.$eref.dependencies | 2    | 2     |
|                |      | not.$eref.not  | 2    | 2     |
|                |      | not.$eref.$ref | 2    | 2     |
| not.$comment   | 1    | 1             |      |       |

Occurrences of \texttt{not} without a \texttt{not."} path

\begin{tabular}{c|c}
'tot': {} & 16 & 8 \\
'tot': '...' & 1 & 1 \\
\end{tabular}

| Table 3. Occurrences of not.$K$ paths |

At the left-hand-side of the table we analyse occurrences of \texttt{not.$K$} for different keywords. As we anticipated in the introduction, \texttt{not} is often followed by \$ref. In 93 cases out of 93+27 we have been able to expand that \$ref: 'reference' into a structure \$eref: referenced schema, and the result is analysed in the right-hand side of the table.

Observe that Table 4.1 indicates a total of 840 occurrences of \texttt{not.$K$}, but Table 3.2 has a total of 787 occurrences of \texttt{not}. The two numbers differ since a single \texttt{not} occurrence may generate two or more \texttt{not.$K$} paths, or may generate none. It generates more paths when its argument contains many fields, as in the following example.
"not" : {"enum": ["generic-linux"], "type": "string"}

Here, we say that not has a complex argument since its argument contains more than one keyword, and we will say that the two keywords co-occur in the negated schema; otherwise we say that the schema is simple.

While most sub-schemas inside a schema are actually complex, most instances of not have a simple argument, that is, an argument with exactly one field. We only found 55 instances out of 787 of not with a complex argument, most of them with just two keywords, but some with three or even four keywords in the negated schema, as in {'type': 'object', 'title': ..., 'required': ..., 'properties': ...}.

On the other side, a not may not correspond to any not.k pattern, when not is followed by { }. We found 16 occurrences of 'not': { }, which are used to express the schema false that is not satisfied by any instance. This use of not is a consequence of the fact that the general use of false has only been introduced with Draft-06.

The situation is very different with $eref: in this case we have that 93 occurrences of not.$eref correspond to 338 occurrences of not.$eref.$k. In this case, thanks to the mediation of $eref, the schema designer is implicitly applying negation to a complex argument, with an average number of 3-4 members.

We sorted the table on the total number of not.k+not.$eref.$k occurrences, and it is interesting to compare the weight of different keywords in the two table.

A surprising difference is the fact that the operator items never appears under not.$eref but it appears under not with a high frequency, and we will discuss this later.

The most common argument of negation is clearly required, which we will discuss later. The operators type and properties follow required, with similar frequency. Here, one notices that while not.required dominates the not.k case, the two most common cases of the not.$eref group are not.$eref.type, whose value is object in the 80% of the cases, and not.$eref.properties, which indicates that not.$eref is mostly used to negate complex object definitions.

Another obvious difference is the much higher occurrence of descriptive keywords, such as description, $schema, and title, inside referenced arguments with respect to the direct arguments of not.

The last two lines of the table describe cases where not is not followed by any keyword. The case 'not': {} has been already discussed. The case 'not': '...' has been described in Section 3.2 and is an artefact of a user-defined keyword "@errorMessage" that uses not as the message name.

We now move to a detailed analysis of each keyword.

Keywords found below not can be divided in three categories, that are called, in JSON Schema jargon, assertions, applicators, and annotations.

Assertions include required, enum, const, pattern and type, and indicate a test that is performed on the corresponding instance.

Applicators include the boolean operators anyOf, allOf, oneOf, not, the object operators properties, patternProperties, additionalProperties, the
array operator items, and the reference operators $ref, and they indicate a re-
quest to apply a different operator to the same instance or to a component of
the current instance.

Annotation operators include title, description, and $comment, they do
not affect validation but they indicate an annotation that should be associated
to the instance. Since we are mostly interested in validation, and since, moreover,
annotations are removed by the not operator, we will ignore them.

In Section 4.2 we will describe the assertion keywords that are arguments
of not, and in Section 4.3 we will describe the applicators. In Section 5 we will
look in the opposite direction, and will analyse which applicators have not as
an argument.

We now discuss assertions and applicators separately, but the separation is
not total, since in a complex schema assertions and applicators may co-occur, as
in the following example.

"not": { "type": "object",
    "properties": {"a": { ... } }
}

4.2 Assertions: required, enum, const, pattern, type

not.required: field exclusion The most common operator that appears below
not is required.

The simplest, and most common case, of not.required path is when the
argument of not is simple(hence no other keyword co-occurs with required),
and moreover the argument of not is a one-string array, as in the following
example.

"not": { "required": ["DisplaceModules"] }

Out of 240 occurrences of required, 224 of them have this simple shape,
which hence constitutes alone the 27% of all the 840 occurrences of not.k (Table
4.2).

While “not required” may sound like “optional”, it actually means that the
object must violate the 'required': ["DisplaceModules"] specification, that is,
that the field 'DisplaceModules' must be absent.

Remark 1. . One may imagine to add an operator 'excluded': "k" to JSON
Schema, that specifies that a field k cannot be present, with the idea that 'not':
{ 'required': ['k'] } would then be the same as 'excluded': "k", but we must
be careful about the precise meanings of JSON Schema assertions. By definition,
'required': ["k"] means:

if the instance is an object, then it contains the field "k"
in other words, a string or an integer satisfies "required": ["k"). Assume that we
added an operator 'excluded': "k" with the usual implicative meaning:
if the instance is an object, then it does not contain the field "k".

In this case, we would have the following equivalences:

'not': { "required": ["k"] } ⇔ {"type": "object", "excluded": "k"}

and

'not': { "type": "object", 'required': ["k"] } ⇔ { 'excluded': 'k' }

Of course, in a context that forces the instance to be an object, the two operators would be one the opposite of the other, hence we would have the following equivalences:

{ 'type': 'object', 'not': { 'required': ["k"] } }
⇔ { 'type': 'object', { 'excluded': 'k' } } 

{ 'type': 'object', 'not': { 'excluded': ["k"] } }
⇔ { 'type': 'object', { 'required': 'k' } } 

To sum up, not.required is the same as the hypothetical excluded when we know that the instance is an object. Otherwise, the story is slightly more complex.

While required usually occurs as a the only argument of not, we found 16 cases (Table 4.2, complex cases) where it co-occurs with other keywords, most typically with properties as in the following examples.

"not": {"required": ["bundleDependencies"],
"properties": {"bundleDependencies": {}}} 

"not": {"required": ["objectType"],
"properties": {"objectType": {"enum": ["SubStatement"]}}} 

The first example, characterized by the use of the trivial type {}, deserves closer inspection. The semantics of 'properties':{"k":{}} is:
if the instance is an object then: if the instance has the $k$ property then: 
the value of $k$ satisfies {} 
Since every instance satisfies {}, an assertion 'properties': {'$k$': {}} is always true, hence 

"not": {"required": ["bundleDependencies"],
          "properties": {"bundleDependencies": {}}}

is just a verbose way of writing 

"not": {"required": ["bundleDependencies"] }

The second case is subtler. Consider the following schema, where $S$ is not trivial. 

'not': {['required': ['$k'], 'properties': {'$k': $S }} – (notReqProS)
We first observe that if the instance is not an object then the body 
{ 'required' ...'properties' ...} 
is trivially satisfied, hence the entire (notReqProS) fails. Hence the semantics of (notReqProS) is 'type': 'object' AND .... 

We then observe that if the analysed object does not contain $k$, then the body fails, hence the entire (notReqProS) holds, hence its semantics is something like: 

'type': 'object' AND (if $k$ is present then ...).

Finally, for an object that contains the $k$ field, the only way to fail $k: S$ is to have the $k$ field bound to an instance that does not satisfy $S$, hence the specification above is equivalent to: 

'type': 'object', 'properties': {'$k': { 'not': $S } }

To sum up, we have the following equivalences:

(1) 

'not': { ['required': ['$k'], 'properties': { '$k': { } } } 
\Leftrightarrow 'not': { 'required': ['$k'] } 

and 

(2) 

'not': { 'required': ['$k'], 'properties': { '$k': $S } } 
\Leftrightarrow 'type': 'object', 'properties': {'$k': { 'not': $S } }

In case (1), the properties specification is useless. The jargon of case (2) seems, to our eyes, a slightly more complicated way of saying, at the same time, that $k$ should not satisfy $S$ and the instance should really be an object.

We suspect that the reason why the first jargon is relatively common is because schema designers find it strange to have a required assertion without a related properties assertion nearby. For the second jargon, we have no intuition.

Besides the 173 cases where required is bound to a unary array, we have found 64 situations (Table 4.2, case "*/twofields") where not.required is followed by an array of two keys, as in the following example. 

"not": {"required": ["result", "error"],
          "description": "cannot have result and error at the same time"}
This schema requires the instance to violate 'required': ["result", "error"], hence the instance must be an object and must miss one of the two fields, or both of them. In other words, this jargon specifies a mutual exclusion constraint: if result is present, then error is absent, and vice versa.

Observe that “not”: {“required”: ["a1", ..., "an"]} would not imply mutual exclusion among the n fields, but the fact that at most n-1 of them must be present, which seems somehow less useful. Indeed, while positive instances of required are followed by arrays of any length (we found one example with 411 entries), all instances of not.required that we found are followed by either one key (field exclusion), two keys (mutual exclusion), or three keys (max two keys out of three), but three keys only appear in three instances.

While the typical use of the not.required combination is the most elementary one in the direct case (simple, one field), the opposite happens when we consider the not.$eref.required path, that is, the situation where the negation of a required assertion is mediated by a reference. In this case, we have the results illustrated in Table 4.2: we found no instance where the negated referred object was simple, but in all 36 different instances is was complex, with a number of assertions ranging from 3 to 7. In all 36 cases, required was accompanied by a properties assertion and, in 35 cases out of 36, also by a "type": 'object' assertion. In the same way, the number of fields whose simultaneous presence is excluded, in this case, is higher than one in 10 cases out of 36 (Table 4.2, cases two/manyfields), and in one case it arrived up to 7.

| Simple/complex | excluded fields | occurrences | files |
|----------------|-----------------|-------------|------|
| complex        |                 | 36          | 15   |
| complex        | onefield        | 26          | 7    |
| complex        | manyfields      | 7           | 5    |
| complex        | twofields       | 3           | 3    |

Table 5. The structure of arguments of not.$eref that contain required.

The negation of a required assertion with a list of seven fields is quite a strange requirement, when considered in isolation: it is satisfied whenever the instance presents any of the $2^7 - 1$ strict subsets of the seven field names, but is violated if all the seven are present. In practice, we have observed that this kind of negative specifications are not used in isolation, but rather in conjunctive contexts with the following structure, where "RefOne" establish a context that helps understanding the real meaning of 'not': { "$ref": "RefTwo" }:

"allOf" : [ { "$ref" : "RefOne"}, {"not" : { "$ref" : "RefTwo"} } ]

This analysis confirms what we had already noticed from Table 4.1: in our analysis we cannot just substitute '$ref': 'x' with the schema that is referred by 'x'. This substitution is correct from a semantic point of view, but is misleading in the context of usage analysis. The data that we collected clearly indicate that the shape of the argument of negation is very different when the argument is
explicit and when the argument is mediated by a $\text{ref}$: in the first case the structure of the argument is usually elementary, in the second case it is typically much more complex, in the first case what is negated is typically an assertion, in the second case is most commonly the definition of a class of objects. In a sense, it seems that definitions are not used like macros that may be used in order to factorize any commonly used combination of assertions, but rather to give a name to a set of homogeneous objects, like classes in an object-oriented language. For this reason, we will always distinguish here between the immediate arguments of negation and those that are mediated by $\text{ref}$.

**not.enum and not.const: value exclusion** The path not.enum appears 61 times and the path not.const appears 6 times. In the enum+const case, the negated schema is usually simple (44+6 cases out of 61+6, see Table 4.2). In the remaining 17 complex cases, enum is always paired with a 'type': 'string' assertion, which is redundant since all values listed by enum in these cases are strings. This co-occurrence has little to do with negation, since also in the positive schemas enum is paired with a redundant type assertion in the vast majority of cases.

The list of excluded values is typically quite short, and 27 out of 61 negated occurrences of enum only exclude one value.

| not.enum         | occurrences | files |
|------------------|-------------|-------|
| *                | 61          | 52    |
| simple           | 44          | 36    |
| complex          | 17          | 17    |
| * manyValues     | 29          | 24    |
| * oneValue       | 27          | 25    |
| * twoValues      | 5           | 5     |
| simple manyValues| 25          | 20    |
| simple oneValue  | 14          | 12    |
| complex oneValue | 13          | 13    |
| simple twoValues | 5           | 5     |
| complex manyValues| 4          | 4     |

| not.const        | occurrences | files |
|------------------|-------------|-------|
| complex oneValue | 3           | 1     |
| simple oneValue  | 3           | 1     |

Table 6. Occurrences of not.enum and not.const

Hence, the following are the three typical cases (schema \text{89480} in our corpus), (schema \text{5887} in our corpus), (schema \text{3458} in our corpus).

"not": { "enum" : [ "markdown","code","raw" ] }

"not": { "const": "DateTime" }
not: { "enum": ["generic-linux"], "type": "string" }

Such schemas have an obvious interpretation: the instance may have any type and must be different from the string or strings that are listed.

By expanding references, we get 12 more cases, where, as in the previous section, complex schemas are more common than simple ones. In this case the numbers are too small to deserve a table.

not.pattern  The path not.pattern appears 47 times, in 28 files. The pattern schema is simple in 47 cases over 48, which is surprising since, in the positive cases, pattern is almost invariably coupled with "type":"string" (in approximately 17,900 cases out of 18,300). Under negation, however, schema designers prefer the structure that we exemplify below, where type co-occurs with not rather than with pattern. Observe that, differently from the enum case, here 'type':"string" is not redundant, since pattern does not imply that the instance is a string, but is, on the contrary, satisfied by any instance that is not a string (and the same is true for format). Hence, the choice of the presence, and the position, of "type":"string" is not stylistic but semantic. The following is a typical example of negated pattern: the pattern schema is simple, but the not is in a context that forces the type to string, and that often presents other assertions (schema 57662 in our corpus).

```json
{ "format": "uri",
  "not": { "pattern": "^(\$)" },
  "type": "string"
}
```

Apart from the slightly different use of type, the typical use of not.pattern is very similar to the typical use of not.enum: a general string type is defined, and some exceptions are noted. In the following specific example, the not.pattern assertion is indeed equivalent to a not.enum assertion (thanks to the presence of the outer 'type':"string") (schema 51181 in our corpus).

```json
{"type": "string",
 "description": "The definition_type property identifies ...",
 "not": { "pattern": "^(\(\$)\|tlp)$" }
}
```

The path not.$ref.pattern — that is, a pattern not.$ref where the reference denotes a type with a pattern member — is not present in our collection.

not.type: lot of redundant use  The assertions that we have seen up to now are used to exclude some specific keys from an object type or some specific values from a string type. The use of not.type is somehow different, as we will see now.

When type appears in a negated schema, we have three cases that appear with similar frequency: negation of a simple schema (21 occurrences), of a complex schema declaring a string type (22 occurrences), of a complex schema declaring an object type (19 occurrences).
simple schema  When the negated schema is simple, we have both single-type instances as the following one:

```
"not": { "type": ["number"] }
```

and many-types instances, as in (schema 12174 in our corpus):

```
"not": { "type": ["array","object","null"] }
```

At a careful analysis, negated simple schemas of the single-type family are used in a redundant way in a surprisingly high number of schemas. Consider for example the following schema (schema 35140 in our corpus).

```
(\text{anyOfNotType})
```

```
"anyOf": [ { "not": { "type": "array" } },
{ "items": { "pattern": ..... } }
]
```

By classical logical equivalences, it specifies that, if the instance is an array, then it must satisfy:

```
(\text{oneLine})
```

```
"items ": { "pattern": ..... }
```

However, the implication if the instance is an array is already part of the semantics of \text{items}, hence all the \text{anyOf-not-type} part is redundant, and the schema (\text{oneLine}) is equivalent to the longer schema (\text{anyOfNotType}).

Another redundant use is the following one (schema 2142 in our corpus), where the final \text{not-type-object} line could just be removed since it is implied by the rest of the schema.

```json
{ "anyOf": [ { "type": "string",
  "pattern": "Annotation$"
 },
  { "type": "array",
    "anyOf": [ { "type": "string",
      "pattern": "Annotation$"
    } ]
  }
],
  "not": { "type": "object" }
}
```
We also found cases where the pattern is not redundant since it is used just to specify: “this field cannot be a number”, but the redundant cases seem as common as the non-redundant ones, if not more.

The simple-schema many-types cases are rare but quite interesting. Consider, for example, the following schema (schema 4179 in our corpus).

```
"not":{"type":["string","number","array","object","boolean","null"]}
```

It may be surprising, since the authors are excluding all JSON Schema types. Actually, they are requiring the use of a non-standard type.

**Complex schemas with 'type': 'string'**

Out of 22 complex schemas that declare 'type': 'string', 21 combine that declaration with either enum or const, hence making it redundant, as in the following example.

```
"not": { "enum": ["generic-linux"],
    "type": "string"
}
```

This redundancy is not related to negation: in the set of JSON Schemas that we analyzed, half of the positive enum assertions are coupled with a redundant 'type': 'string' assertion. Hence, the fact that 17 out of the 61 not.enum paths are coupled with a redundant type is just an instance of this general phenomenon, that we already discussed in Section 4.2.

Only one schema (schema 65826 in our corpus) combines 'type': 'string' with pattern. Due to the implicative semantics of pattern, adding a 'type': 'string' to pattern is not redundant, and it implies that everything that is not a string satisfies the negation. However, the assertion "type": 'string" that we find in the same schema at the outer level (see the schema below) forces the instance to be a string, hence, also in this case, the 'type': 'string' assertion that is under negation (line 5) is redundant.

```
"items": {
    "type": "string",
    "minLength": 1,
    "not": { 
        "type": "string",
        "description": "...string that contain two or more (*)",
        "pattern": ".*\*.*\*.*" 
    }
}
```

**Complex schemas with 'type': 'object'**

We now analyze the 19 negated complex schemas that combine 'type': 'object' with object assertions. While 'type': 'string' is redundant when combined with an enum assertion that only enumerates strings, 'type': 'object' would not be redundant, in general, near to an object assertion, due to the hypothetical semantics of JSON Schema. While the following assertion 'required': [k]

means: if the instance is an object, then k is required, the combination:
"type": 'object', 'required': [ k ]
means: the instance is an object and k is required, and this distinction is reversed under negation, as discussed in Remark 1.

However, like in the above pattern example, the 'type': 'object' part becomes redundant if it is already asserted in the positive part of the schema, as in the following example (schema 24167 in our corpus).

```json
{ "not": { "type": "object",
  "properties": { "scheme": { ... } }
},
  "type": "object"
}
```

The structure above is found in 13 out of 19 negated complex schemas that contain 'type': 'object'.

**not.$eref.type** The situation is totally different with the path not.$eref.type, that is, when we have not.$eref that refers to a schema that contains type. We have 51 instances, all of them complex and all of them with just one type, and only in 5 cases out of 51 the negation co-occurs with a type as in the example above. In 10 case out of 51, the path not.$eref arrives at a combination 'type': 'string' plus enum, which makes 'type': 'string' redundant, but in the other 41 cases not.$eref arrives at a combination 'type': 'object' plus some object keywords, usually properties and required, so that finally 'type': 'object' is not redundant: thanks to its presence, an instance that is not an object satisfies this specification.

To sum up, under negation, type is used either as 'type': 'object' in a complex object schema, or as 'type': 'string' in combination with enum, or as a simple schema by itself. While in the enum case it is always redundant, in the other two cases we have found a mixture of redundant and non-redundant use - typically non redundant when mediated by $eref and when type is followed by an array, typically redundant in the other cases. Of course, redundancy is not necessarily a problem - redundant assertions may be added on purpose for reasons of homogeneity or for readability.

### 4.3 Negation of applicators: boolean operators, object operators, array operators

**Negation of boolean operators: general introduction** The three boolean combinations not.anyOf, not.allOf and not.oneOf all appear with a non-negligible frequency. The first thing we measure is the relative frequency, and it is interesting to observe that here oneOf is not any more the most used combinator, but becomes the less used one (Table 4.3). This makes sense: when the aim is reasoning on the negation of boolean operators, not.anyOf is easier to understand than not.oneOf. While most negated schemas are simple, we have 55 of them that are complex. A complex schema { 'a' : S, 'b' : T } is equivalent to a conjunction of simple schemas "allOf"[ { 'a' : S } , { 'b' : T } ], hence we will
analyse complex schemas together with boolean conjunctions, although there is an important difference: in a complex schema, all fields have a different keyword, while in a allOf the same keyword may appear in many different arguments. As we will see, this difference is very important in practice.

In presence of a boolean combinator, the next natural question is which keyword do we typically find below. In Table 4.3 we count the instances of not.(anyOf/oneOf/allOf)[*].k for the different keywords.

| op     | anyOf | $ref  | properties | pattern | description |
|--------|-------|-------|------------|---------|-------------|
| not    | required |    |    |    | 23 | 46 | 23 |
| not    | anyOf | $ref  |    |    | 5 | 13 | 4 |
| not    | anyOf | $ref  |    |    | 4 | 10 | 3 |
| not    | anyOf | pattern | 1 | 2 | 1 |
| not    | anyOf | description | 1 | 2 | 1 |
| not    | allOf | required | 27 | 71 | 22 |
| not    | oneOf | $ref  | 2 | 2 | 1 |
| not    | oneOf | properties | 2 | 2 | 1 |
| not    | oneOf | required | 2 | 2 | 1 |
| not    | oneOf | $ref  | 2 | 2 | 1 |

Table 8. Number of occurrences of not.anyOf/oneOf/allOf[*].k for the different keywords k.

As expected, required is still the dominating case. This will be discussed below.

**Negation of anyOf** The most striking feature of the occurrences of not.anyOf is the extreme homogeneity and simplicity of the arguments of the disjunction. We found 46 instances of this pattern, and in each of them all the arguments of the disjunction have exactly the same set of operators, and that set is always a singleton, apart from one case where the arguments present two keys (description and required). This is illustrated in Table 4.3: in 26 cases, every argument of not.anyOf is a simple schema that only contain a required keyword. In 13 cases, every argument is a simple schema that only contain a $ref keyword. In 4 cases, every argument is a simple schema that only contain a properties keyword. In 1 case, each of the two arguments contains both description and required. In 1 case, each of the two arguments contains pattern.

For each keyword k, the first column in the table reports the occurrences of not.anyOf that are followed by k, and the second column the number of occurrences of not.anyOf[*].k. We see for example that 26 occurrences of not.anyOf followed by [*].required correspond to 69 occurrences of not.anyOf[*].required, which means that, on average, every anyOf has 69/26 arguments, which means that, in practice, most disjunctions have just 2 arguments.
Hence, the \texttt{not.anyOf[\*].required} path constitutes the vast majority of the uses of \texttt{not.anyOf}, and they are typically very simple, as in the following example, where \texttt{anyOf} has two arguments and each \texttt{required} has one.

\begin{verbatim}
"not": {
  "anyOf": [
    { "required": ["constructor"] },
    { "required": ["statics"] }
  ]
}
\end{verbatim}

This case $2 \times 1$ case is the most common one. The number of branches of \texttt{anyOf} varies actually from 1 to 10. As for the number of arguments of \texttt{required}, it is either 1 when we want to express field exclusion or 2 when we want to express mutual exclusion, as in Section 4.2. In both cases, the number of arguments of \texttt{required} is always homogeneous at the internal of the same \texttt{anyOf}, and we have 17 cases with 1 argument in every branch (field exclusion) and 10 cases with two arguments (mutual exclusion) in every branch.

In the first group, composed of 17 negated disjunctions of unary \texttt{required}, the designer uses this combination to exclude from the instance all keywords in a set. In the second group, of 10 negated disjunctions of binary \texttt{required}, the \texttt{not.anyOf} construct is used to collect a set of $n$ mutual exclusion statements. By direct analysis, we have seen that, in many cases, these $n$ statements describe one field that is exclusive wrt to $n$ other fields, in other cases they collect $n$ pairs regarding $2^n$ distinct fields, but there are rare cases that describe exclusions graphs that are more complex than these.

The \texttt{not.anyOf.$\mathrm{Seref}$} case is similar to the previous one in the sense that, if one analyses the set of references collected below the same \texttt{anyOf}, they are always extremely homogeneous. They are all complex schemas that either combine \texttt{"type":'string'} with \texttt{pattern} or \texttt{"type":'object'} with \texttt{properties} and \texttt{required}.

The other cases, \texttt{not.anyOf} followed by \texttt{properties} and \texttt{pattern} are exceptional, and we will not discuss them.

To sum up, \texttt{not.anyOf} is mostly used to combine fields exclusion statements, and is characterized by a short list of simple and homogeneous arguments.

Negation of \texttt{allOf} The next most common case is \texttt{not.allOf[\*].required}. We found 23 occurrences. All occurrences of \texttt{not.allOf} only present simple
required schemas as arguments. They all follow the same binary pattern below, the only variation being in the X and Y names (schema 3212 in our corpus), (schema 9186 in our corpus), (schema 3020 in our corpus) . . .

"not" : { "allOf" : [{"required": ["X"]}, {"required": ["Y"]}] }

By and/or duality, we are here requesting to satisfy any of the two exclusion schemas, or both (and, as usual, the instance must be an object). That is, the only combination that is forbidden is when both X and Y are present, any other combination is good. Hence the schema above is equivalent to the following schema (discussed in Section 4.2).

"not" : { "required": ["X", "Y"] }

We feel that both specifications are not easy to read, but the second one is at least more compact, hence we wonder why the first one has been preferred in these 23 occurrences.

Negation of complex schemas A complex schema { "a" : S, "b" : T } is equivalent to a conjunction of simple schemas "allOf"[ { "a" : S } , { "b" : T } ], but is used in a very different way.

Negated complex schemas belong, essentially, to three different categories. They either combine enum, const, or pattern with 'type': 'string' (22 occurrences). Or, they combine a subset of properties, required, 'type':'object'(21 occurrences). Or, they combine additionalProperties, patternProperties and, sometimes, also 'type':'object'(11 occurrences). All these three combinations have been already discussed.

We only have one exception, where the negated combined schema also includes a anyOf operator, which changes completely the picture. This happens in schema (schema 37789 in our corpus), and will be discussed in Section 6.4.

| Structure                                      | occurrences |
|------------------------------------------------|--------------|
| enum, const, pattern, type:string              | 22           |
| properties, required, type:object               | 21           |
| additionalProperties, patternProperties, type:object | 11           |
| anyOf, properties, required                     | 1            |

Table 10. Combinations of keywords in

Negation of oneOf The last boolean negation to consider is not.oneOf.

This case is rare, since we only have six instances. Moreover, in four cases the pattern not.oneOf is found at the end of a not.items not.oneOf pattern, and we believe, as discussed in Section 4.3, that they must be understood as
(not.items.not).oneOf. Hence, we only have two real instances, but we will reserve them some space nevertheless, since they give us an excellent example of how the interpretation of a JSON Schema may be complicated by the interplay between negation, oneOf, and implicative semantics (that is, the fact every number assertion is trivially satisfied by anything that is not a number, and the same for strings, arrays, and objects). Both instances are binary, as in 'not': { "oneOf": [ S1, S2 ] } (as are the four not.items.not.oneOf instances that we discarded).

We first observe that the semantics of 'not': { "oneOf": [ S1...Sn ] } is: "either all of S1...Sn are not satisfied, or exist i, j with i ≠ j such that Si and Sj are both satisfied". This is quite complicated to understand, but in practice we noticed that it is often the case that all arguments [ S1...Sn ] of any oneOf operator, negated or not, are mutually exclusive. In this case, oneOf is equivalent to anyOf, and not.oneOf is equivalent to not.anyOf, which makes it much easier to understand.

However, deciding whether the different branches of oneOf are exclusive is not obvious, since it often depends on the context.

Consider the following example (schema 4896 in our corpus)

"oneOf": [ 
  { 
    "properties": { 
      "status": { "enum": ["content error" ] } 
    },
    "required": [ "content_error_detail" ]
  },
  { "not": { 
    "oneOf": [ 
      { "properties": { 
        "status": { "enum": ["uploading", "released" ] } 
      },
      "required": [ "content_error_detail"]
    },
    { "properties": { 
      "status": { "enum": [ "content error"] } 
    }
  ]
  ]
}
]

Let us analyse the internal not.oneOf. Here, the two branches are not mutually exclusive. They are both satisfied by anything that is not an object, and also by any object that has the required field and does not have the status field, hence this not.oneOf specification can be satisfied either by violating both constraints (as for not.anyOf) but also by satisfying both constraints, which is quite bizarre.

But let us look now at the context around the not.oneOf. The not.oneOf is combined here, through an external oneOf, with a specification that is satisfied, because of the implicative semantics of properties and of required, by anything that is not an object, and by any object that has the content_error_detail
and does not have a status field. Hence, the exclusive semantics of the external oneOf together with the first branch prevents exactly those cases that would make the internal not.oneOf different from not.anyOf. For example, a number would satisfy both arguments of the internal oneOf, hence it would violate the oneOf itself, hence it would satisfy the not.oneOf. But a number would also satisfy the first branch of the external oneOf, hence it would fail the entire specification. In the same way, an object with the content_error_detail field (hereafter, c_e_d) and without a status field would satisfy the three branches, hence would fail the entire specification.

So the internal not.oneOf is equivalent to not.anyOf because of the exclusive semantics of the external oneOf. But if we zoom out a bit in the original schema (not reported here), we discover that the entire construct is inside a dependencies: status declaration, and is combined with a allOf/oneOf/anyOf combination that forces the value of status to be one of the three value listed (actually they are much more than three, we are here simplifying the things a little bit), so that the semantics of the oneOf schema is relevant only for situations where the instance is actually an object that contains the status field, with a value chosen from those listed. So, in order to decode this specification, we can restrict ourselved to this class of instances.

In practice, we find this reasoning technique extremely complex, because of the combination between implicative semantics and exclusive semantics. We have the impression that the simplest way of decoding this specific schema is just a model-checking approach, where we check which combinations of values for the status field and presence/absence for the c_e_d field satisfy the specification. We propose the following alternative way of expressing the entire specification, and leave to the reader the task to verify its equivalence with the original one, under the assumption that the instance is an object with a status property and whose value is included in the list.

"oneOf": [  
  {   "required": ["content_error_detail"],  
      "properties": {  
          "status": {"enum": ["content error"]}  
      }  
  },  
  {   "not": { "required": ["content_error_detail"]},  
      "properties": {  
          "status": { "enum": ["uploading", "released"]}  
      }  
  }  
]

The aim of this long analysis was to illustrate the complexity of the interplay between not, oneOf, and implicative semantics, which we believe explains why oneOf, which is the most common boolean operator in positive position, becomes the less frequent one when we move to the negated occurrences.
not.properties The path not.properties appears 71 times. A negated schema that contains properties is simple in 54 cases over 71, and the schema associated to properties contains only one property in 62 cases over 71, hence the typical case is the simple properties schema with one property only, as in the following example.

"not": { "properties": { "status": { "enum": ["released","revoked"] } } }

The meanings of properties is implicative: if the instance is an object, if the field is present, then it must satisfy the indicated subschema. Hence, its negation is conjunctive: the instance is an object, the property status is present, and it must violate the subschema. Hence, this simple specification combines the effect of 'type', 'object', 'required': ["k"], and a negated type assertion for the property. While this fact is important in theory, in practice we have seen this pattern mostly used in situations where the context already implies that the instance is an object with that field. In those situations, the position of the negation is not relevant, and we have the following equivalence:

'type': 'object', 'required': ["p"], 'not': { "properties": {"p": S } } ⇐⇒
"type": 'object', 'required': ["p"], 'properties': {"p": { 'not': { S } } }

Hence, not.properties is mostly used to impose that a member has a type not S. While negation is, in general, mostly used in front a 'required', in the not.properties case the type operator that is most often negated is enum, as in the example above: not.properties is mostly used to exclude some values from the domain of the property. In the other few cases, the schema associated with S features properties again, type, some user-defined keywords, and also a maxItems = 0, whose negation looks like a more complex way of expressing minItems = 1.

In the 17 complex cases, properties is mostly combined with required (11 cases over 17), as in the following example (schema 76666 in our corpus).

"not": { "properties": { "objectType": {"enum": ["SubStatement"]} }, "required": ["objectType"] }

In the other cases, it is combined with 'type', 'object', or with both required and 'type': 'object'.

Each of these forms has a different meaning, for the reasons that we discussed many times, since we have the two following equivalences, that specify that whenever one side forces the instance to be an object (is assertive on that) its negation is satisfied by any instance that is not an object (is implicative on that feature), and the same with required: if a side is assertive on the field presence, the negation is implicative.

not: { 'type': 'object', 'required': ["p" ], 'properties': {"p": S } } ⇐⇒ { 'properties': { 'not': {"p": S } } }

not: { 'required': ["p" ], 'properties': {"p": S } }    
⇔  { 'type': 'object', 'properties': { 'not': {"p": S } } }
Hence, the fact that we find all different combinations may give the impression that JSON Schema users have a sophisticated usage of the assertive/hypothetical distinction. However, from our direct analysis we got rather the opposite impression: in most situations, some contextual information forces the type to be an object, and sometimes forces the property to be present, hence making all the variations above mutually equivalent.

We conclude with yet another example that shows how negated schemas can be tricky to interpret, even when short; we leave this one to the reader with no comment (schema 21864 in our corpus). 6

```json
"not": { "properties": {
  "FirstName": {"type": "string"},
  "LastName": {"type": "string"}
},
  "required": ["FirstName","LastName"]
}
```

The path not.patternProperties is present in 15 schemas, and it follows two different usage patterns. In 9 schemas, most of which seem to have been produced by the same group, we find the following structure (schema 11675 in our corpus).

```json
"type": "object",
"patternProperties": {
  "^[0-9]{3}$|^(default)$": {"$ref": "/definitions/responseValue"},
  "x-": { "$ref": "/definitions/vendorExtension" }
},
"not": {"type": "object",
  "additionalProperties": false,
  "patternProperties": {
    "x-": { "$ref": "/definitions/vendorExtension" }
  }
} "additionalProperties": false,
"minProperties": 1
```

This structure is interesting to analyse, to see how programmers use JSON Schema. We first recall that, in JSON Schema, additionalProperties receives its meaning by the properties and patternProperties that co-occur as members of the same object, since it specifies a schema for everything that does not match the cooccurring properties and patternProperties specifications. The 'type': 'object' inside the not is redundant, because of the external 'type': 'object', as we have discussed many times. The reference "/definitions/vendorExtension" refers to a trivial schema that is satisfied by any instance, hence the patternProperties under negation is not used to limit the structure.

6 The schema is satisfied by any instance that is an object and either misses one of the two members, or it has both members but at least one is not of type string.
of the members whose name starts with \(x\). Hence, \texttt{patternProperties} is just used to specify that the co-occurring \texttt{additionalProperties} refers to any member that does not start with \(x\). The negated schema describes an object where every field starts with \(x\). Hence, since the external schema forces the instance to be an object, the only way of failing this negated schema is by presenting a member that does not start with \(x\), and is associated to any value. If we combine this with the pair \texttt{patternProperties-additionalProperties} at the outer level that specifies that every field must match either the \(\^\text{[0-9]}\{\text{3}\}$\texttt{default}\$ (\texttt{responseValue}) or the \(\sim\text{-}x\) (\texttt{vendorExtension}) specification, then this is a way to require that at least one member matches \texttt{responseValue}. By the way, observe that this makes the \texttt{minProperties}= 1 specification redundant.

We find this structure extremely interesting: JSON Schema has a \texttt{required} specification that allows the programmer to require the presence of a member with a given name, but has no \texttt{patternRequired} operator to require the presence of member whose name matches a specific pattern, hence the need of the complex construction above.

Other 2 schemas, which seems to be different versions of the same schema, present instead the following structure (schema 64748 in our corpus) and (schema 51189 in our corpus).

\begin{verbatim}
"type": "object",
"description": "Specifies any other header fields (except for date, received_lines, ..., and subject) found in the...",
"not": { "additionalProperties": false,
  "patternProperties":
    { "^date|received_lines|subject$":
      { "description": "Invalid additional header field types" 
    }
  }
}, ...
\end{verbatim}

As before, since the schema associated to the internal pattern is trivially satisfied, the argument of negation is satisfied by any object that does not have any other field besides those listed, hence the \texttt{not}: \{ \} schema enforces the presence of at least one field that does not match the listed patterns. However, the internal description \texttt{Invalid additional header field types} seems to suggest that the intention was rather that of forbidding the presence of the listed fields, which would have been accomplished by the following schema (written by us) - where we wrote \texttt{not}: \{ \texttt{description}'...\}, instead of just \texttt{false}, in order to show that this second schema can be obtained by the first one by changing the position of the negation, and hence its meaning. Of course we do not know whether our interpretation of the \texttt{description} field is correct.

\begin{verbatim}
"type": "object",
"patternProperties": {
  "^date|received_lines|...|subject$":
    { "not": { "description": "Invalid additional header field types" } }
},
\end{verbatim}
Finally, the last 4 schemas present variations of the following schema (schema 51196 in our corpus).

```
"anyOf": [ 
{ "patternProperties": { "^windows-process-ext$": { 
"type": "object", ...} } }, 
{ "patternProperties": { "^windows-service-ext$": { 
"type": "object", ...} } }, 
{ "not": { "patternProperties": { 
"^windows-service-ext|windows-process-ext$": { 
"description": "Invalid custom file extension" 
} 
} 
} 
] 
```

Again, 'patternProperties': { "p": { 'description': ... } } is a trivial statement since it says: if the instance in an object, then if it has a field that matches the name p then its value must match { }, that is, it can be any value. This statement is satisfied by anything that is not an object, that is an object without a p member, and that is an object with a p member. Hence, its negation is unsatisfiable. Since falsehood is a neutral element of disjunction, the entire unsatisfiable { 'not': { } } is just redundant. It is difficult to guess the intention of the authors of this schema, but the impression is that this redundancy was not their aim. We have also the impression that the outermost anyOf was rather intended to be an allOf, but this would have made the entire schema unsatisfiable.

If the intention was that of forbidding

```
"windows-service-ext|windows-process-ext$,
```

this effect can be obtained either with the use of additionalProperties, as previously discussed, or with a code like the following one, where "***pattern-Not...***" is a pattern that matches the complement of the pattern

```
"windows-service-ext|windows-process-ext$
```

(observe that not is inside patternProperties and not outside). Unfortunately, the computation of the complement of a pattern is a very complex operation and is not natively supported by JSON Schema, hence the additionalProperties approach is the only practical one.

```
"patternProperties": { 
"***patternNot('windows-service-ext|windows-process-ext$')***": 
{ "not": { "description": "Invalid custom file extension" } } 
not.additionalProperties  
```

The path not.additionalProperties appears exactly 11 times in our collections, it always co-occurs with patternProperties and with a false argument, and these 11 cases are the first 11 cases that we described in the previous section.
not.items The pattern not.items, with 126 occurrences in 27 files, is the second most common not-path, which we found quite surprising.

All not.items schemas have one of the two structures exemplified below (schema 88916 in our corpus, (schema 21120 in our corpus).

```
"not": {"items": {"not": {"$ref": 
"not": {"items": {"enum": ["ansible",...,"terraform"] } }

Hence, all negated schemas with items are simple schemas that only contain the items keyword, where the argument of items is itself a simple schema, which is always either a simple not schema or a simple enum schema. The not.items not form is the most common, with 84 occurrences in 22 files, while not.items enum occurs 42 times in 5 files.

The items assertion is verified by any instance that is not an array, or that is an empty array, or that is an array where every element satisfies the schema associated with items. Hence, it is only violated by instances that are arrays, and which contain at least one element that violates the schema. Hence, while items specifies a universally quantified properties, not.items can be used to specify an existentially qualified property, as does the contains keyword, according to the following equivalences.

```

Hence, the not.items.enum jargon specifies that the array must contain at least one value that is not one of those listed in the argument of enum.

The not.items.not jargon specifies that the instance is an array that contains at least one value that satisfies S, according to the following equivalence.

```

The two examples we gave exhaust, with minimal variations, the shapes of the 126 occurrences of not.items. Their extreme homogeneity prompted us to check the nature of the schemas where we found them, and we realized that the occurrences are spread in 27 schemas, that belong to just three groups: (a) “Annotation”, which is a group of 102 schemas that collectively formalize the Web Annotation Data Model (https://www.w3.org/TR/annotation-model/), 21 of which use the not.items not structure; (b) “G-Cloud”, which is a group of 10 files whose title is a variation of “G-Cloud XX Cloud YY Product Schema”, 5 of which use the not.items.enum structure; (c) “Other”, which collects all other schemas, where we find one instance of the not.items.not structure.

To sum up, not.items is used as an alternative way to express contains constraints. It appears very often, but this is mostly due to two specific groups that do a massive use of this jargon.

This case is quite instructive: it shows that the numbers that we collect are submitted to a bias that depends of the productivity of some specific groups,
and this bias can be very strong. This should never be forgotten when these numbers are examined.

5 The context around \texttt{not}

5.1 Counting the contexts

JSON Schema is a compositional language, where a negative schema ’\texttt{not}’: \( S \) may either be used to specify the structure of a property, or may be bound to a name through the use of \texttt{definitions}, or may be the argument of another combinator, such as \texttt{oneOf, anyOf, allOf, if-then-else, dependencies}.

Table 5.1 shows the context of 774 occurrences of \texttt{not}. This number is lower than the total of 787 of Table 4.1, since we did not count the cases where the context is a user-defined keyword.\footnote{As discussed in Section 3.2, inside a user-defined keyword we are not even sure that \texttt{not} is used as a JSON Schema keyword or just as a member name} The most represented group of contexts is that of boolean operator: if we consider \texttt{oneOf, allOf, anyOf, dependencies, if and then together}, we have a total of 400 occurrences. The structural operators, such as \texttt{properties} and \texttt{items}, and the top-level contexts, such as \texttt{definitions} and the root of the schema, constitute the other half, of 373 contexts. This is already an informative observation: the frequency of boolean contexts is in the 10-15\% range for the generic nodes of the schema, but is 52\% for the occurrences of \texttt{not}.

In order to measure the correlation between the different contexts and \texttt{not}, we computed the ratio between the occurrences of \texttt{not} in a given position, for example at the end of a path \texttt{oneOf[\*].not}, and the total number of nodes at the end of a path \texttt{oneOf[\*].k}, and we put it in the table. Observe that the global ratio 3/10,000 in the first line is not the average of the other lines, since it includes all the contexts where \texttt{not} does not appear at all, that is, all lines such that, if they were in the table, then their frequency would have a value of 0. This ratio shows that there is a strong general correlation between \texttt{not} and the boolean operators, and it is very strong when the context is \texttt{allOf}, while more generic contexts, such as \texttt{properties, definitions}, and the root, do not attract negation in any special way. The assertion \texttt{dependencies} has a very high frequency of negation, which we will examine later. We ignore the four contexts after \texttt{dependencies} since none of them appears in more than 3 distinct schemas.

We now examine all the different contexts in the table up to \texttt{dependencies}.
Table 12. Occurrences of a $k$.not path for different keywords, frequency of not for every 10,000 nodes in the same position, and occurrences of the context (e.g., occurrences of properties followed by *.not)

| path                      | occ. of the path | freq. of not $\times$ 10,000 | occ. of distinct contexts | files |
|---------------------------|------------------|-------------------------------|----------------------------|-------|
| total                     | 773              | 3                             | 624                        | 295   |
| properties+.not           | 194              | 2                             | 187                        | 101   |
| oneOf[*].not              | 162              | 11                            | 126                        | 78    |
| allOf[*].not              | 132              | 130                           | 74                         | 47    |
| items.not                 | 96               | 13                            | 96                         | 30    |
| anyOf[*].not              | 65               | 20                            | 38                         | 32    |
| definitions.*.not         | 43               | 2                             | 35                         | 35    |
| .not (root position)      | 35               | 5                             | 35                         | 35    |
| dependencies.*.not        | 24               | 191                           | 11                         | 8     |
| then.not                  | 14               | 385                           | 14                         | 3     |
| if.not                    | 3                | 169                           | 3                          | 1     |
| patternProperties*.not    | 3                | 6                             | 3                          | 2     |
| additionalProperties.not  | 2                | 3                             | 2                          | 2     |

5.2 Boolean contexts

While the occurrences of not.boolOp are characterized by an extreme homogeneity of the argument of boolOp, (see Section 4.3 and following sections), the pattern boolOp.not has the opposite behaviour.

Out of 74 instances of allOf.not, 25 are not-homogeneous, that is, all arguments are negated, and 49 are mixed: some of the arguments are positive and some are negative, with a great heterogeneity of situations.

After direct analysis of the 49 instances, the impression is that schema designers are typically describing an instance that satisfies a schema S but should not present a specific field, or a specific value, or a specific pattern, hence what they are describing is, essentially, a subtraction, as in the following examples (schema 89142 in our corpus), (schema 4198 in our corpus): an “amountbase” but without a value field, or a string that is not a “namespace”.

"allOf":
  ["$ref": "amountbase"],
  "not": {
    "required": ["value"]
  }]
"allOf":
  ["type": "string"],
  "not": {
    "$ref": "/core/namespace"
  }]

While the 49 mixed instances are very different, the 25 pure-not instances all have the following shape (schema 17072 in our corpus): the 25 occurrences have a total of 70 arguments, most of them with shape not: { required: [k] }. This is just a different way of expressing the not.anyOf[*].required pattern that we described in Section 4.3.

"allOf":
  ["not": {
    "required": ["XAngle"]
  },
  "not": {
    "required": ["YAngle"]
  },
  "not": {
    "required": ["ZAngle"]
  }]

The instances of oneOf followed by some not are typically quite complex, as well. We have 126 such instances, with a total of 320 arguments, 162 of them containing a not. Hence, also in this case, there is a heavy mixture of negated arguments and positive arguments. Moreover, half of the 162 arguments containing a not are complex.

A typical example of negation in a oneOf context is the following one (schema 79022 in our corpus).

```
"type": "string",
"oneOf": [ { "pattern": "^#/components/schemas/" },
           { "not": { "pattern": "^#/" } }
        ]
```

The two cases of oneOf are mutually exclusive; they would actually be both satisfied by any instance that is not a string, but the external 'type': 'string' assertion excludes these cases. Hence, the designer is not using oneOf in order to force the instance not to satisfy both branches, but rather to stress the fact that the two branches are incompatible. In other terms, here oneOf could be substituted by anyOf without changing the semantics, but oneOf may convey the designer intention in a way that is clearer. Hence, this is really a “not A or B” pattern, hence an implication: if the string matches the pattern "^#/", then it must match "^#/components/schemas/".

Another typical example is the following one.

```
"dependencies": {
 "construct_type": {
  "oneOf": [
    { "properties": {
      "construct_type": { "enum": ["fusion protein"] },
      "tags": { "minItems": 1 }
     },
    "required": [ "tags" ]
   },
   { "not": {
    "properties": {
      "construct_type": { "enum": ["fusion protein"] }
     }
   }
  ]
 }
}
```

Again, the dependencies context ensures that oneOf will only be relevant in situations where the instance is an object with a construct_type field, hence the two branches can be regarded as mutually exclusive, hence the semantics is an implication: if the value of construct_type is "fusion protein", then the tags field is required, and it must satisfy 'minItems': 1. We observe that the dependencies assertion of JSON Schema allows one to specify that if a field is present then the instance should satisfy some specific properties. In this case we
need to specify that if a field is present and its value satisfies a given type, then the instance should satisfy some specific properties, but this form of dependency is not supported, hence the use of disjunction-negation. A slightly simpler way to express this form of dependency would be by using the if-then construct as follows, but this construct has only been introduced with Draft-07, hence its use is very rare.

```
"if": {
  "required": ["construct_type"],
  "properties": { "construct_type": { "enum": ["fusion protein"] } }
},
"then": {
  "required": ["tags"],
  "properties": { "tags": { "minItems": 1 } }
}
```

These two examples do not exhaust the uses of oneOf followed by not. On the contrary, out of 126 instance of the patter, we counted 41 different structures of use, differing either in the number of arguments of oneOf or in the set of keywords in this set of arguments. This variety is quite interesting, but we do not think it is worth to pursue this specific pattern any more, since the different cases are too many, and many of them are quite complex.

As far as anyOf is concerned, we have a similar situation: out of 38 occurrences of anyOf where some branch is a negation, 23 mix some arguments that contain negation with arguments that have no negation, hence encoding an implication. Very few of them are homogeneous, like the following one where all three arguments contain the same pair of operators not-required (schema 4139 in our corpus):

```
"anyOf": [{
  "not": {
    "required": ["image"]
  },
  "required": ["build"]
},
  { "not": {
    "anyOf": [{
      "required": ["build"]
    },
    { "required": ["dockerfile"]
    }
  },
  "required": ["image"]
},
  { "not": {
    "required": ["build","image"]
  },
  "required": ["extends"]
}
```

This specification is quite interesting. One notices that the second and the third branch are not mutually exclusive - they are both satisfied by an instance that has both image and extends provided it does not contain neither build nor dockerfile, and similarly for the first and the third branch. One also notices that the second branch suggests that image and dockerfile are not compatible, but
the third branch allows the presence of both, when a field extends is also present. We conclude that, while oneOf is quite hard to decode because of the mutual exclusion problem, the readability of schemas with anyOf may be complicated by the lack of a mutual exclusion property. Finally, this example also shows that real-world schemas do present a rich variety of many-levels nesting of boolean operators, that are often simple to read but sometimes require a bit of effort in order to be decoded.

5.3 Contexts properties and definitions

Negation is used at the top level of the definition of a schema for a property in 194 schemas in 101 different files, and in the top level of a schema in the definitions section in 43 schemas in 35 distinct files (Table 5.1).

The 194 occurrences found belows a properties assertions are uniformly distributed across the range of complexity of the schema where not appears, and of complexity and operators of the argument of not. The 43 occurrences in the definitions section are, on the contrary, characterized by the fact that the schema where not appears is typically complex, that is, it combines negation with positive specifications in 39 cases out of 43 as in the following example (schema 90953 in our corpus). By contrast, in the properties context the complex schemas are one out of two (96/194), and, in the general case, the frequency of complex schemas over the totality of schemas with not is even lower (341 / 787).

```
"definitions": {
  "common": {
    "type": "object",
    "not": { "description": "cannot have result and error at the same time",
      "required": ["result","error"] },
    "properties": { 
      "id": { "type": ["string","integer","null"],
        "note": [ ... ] }
    },
    "jsonrpc": { "enum": ["2.0"] }
  },
  "required": ["id","jsonrpc"]
},
```

The more complex nature of schemas with negation found below definitions is not surprising: we have already seen that definitions are mostly used to define complex object structures once for all, while properties may admit a description that is either simple or complex, with similar frequency.

5.4 Context root

Negation is used 35 times at the root of the schema, and in 33 cases it respects one of the following shapes (schema 64729 in our corpus), (schema 13020 in our corpus), (schema 91408 in our corpus):
The first form lists a set of prohibited member names, while the second (more common) and the third (quite rare) express mutual exclusion, hence we find, at the root level, the use of negation that is the most common of all.

The only real surprise with the root context was the following schema (schema 90941 in our corpus) that specifies, in a sense, that the root implies its own negations, since \{"$ref": 
"#"\} is a recursive reference to the root of the schema. This form is actually illegal in JSON Schema, since it produces an infinite loop.

\{
  "not": \{"$ref": 
  "#"\},
  ...
\}

Naturally this is just a “test” schema, as testified by its url:

.../json/tests/testData/jsonSchema/highlighting/cycledWithRootRefInNotSchema.json

5.5 Context dependencies

Negations is used after a path dependencies.k in 24 occasions. In the vast majority of the cases, it is yet another way to specify field exclusion, as in (schema 58369 in our corpus):

"dependencies":
  \{"page_action": \{"not": \{"required": \{"browser_action": \}\}\}\},
  "browser_action": \{"not": \{"required": \{"page_action": \}\}\},
  ...

We have found also a couple of situations where negation is used to express more complex dependencies, as in (schema 4912 in our corpus), which specifies that, when a member age_units is present, a member age must be present with a value different from "unknown".

"dependencies":
  \{"age_units": \{"not": \{"properties": \{"age": \{"enum": \{"unknown"\}\}\}\}\}\},
  "required": \{"age","life_stage":
    "comment": "Age units is required if age is specified as anything but unknown."
  },
  ...
}
6 Some interesting examples of complex usage

6.1 Using negation to express discriminated union

This piece of code comes from file (schema\_6924 in our corpus). We have reduced it a little bit by removing all descriptive fields and we have simplified the type associated with "href" at line 21.

```json
{
  "not": {
    "anyOf": [
      {
        "properties": {
          "type": {"enum": ["name"]},
          "not": {
            "properties": {
              "name": {"type": "string",
                        "minLength": 1
            }
            },
            "required": ["name"]
        }
      },
      {
        "properties": {
          "type": {"enum": ["link"]},
          "not": {
            "type": "object",
            "properties": {
              "href": {"type": "string"},
              "type": {
                "type": "string"
            }
            },
            "required": ["href"]
        }
      }
    ],
    "oneOf": [
      {
        "type": "null" },
      {
        "type": "object",
        "properties": {
          "type": {"type": "string",
                    "minLength": 1 }
        },
        "properties": { "type": "object" }
      },
      "required": ["type","properties"]
    ]
  }
}
```
This schema is quite hard to read, first of all since it is a bit long and since it uses the name "type" and 'properties' as property names. Hence, we first rewrite it using the notation of the tool we developed to reason about JSON Schema[]. This notation is a bit more compact and uses quotes only for user-level names but not for JSON Schema keywords, and this should help a bit.

\[
\begin{align*}
&\{ \text{not(anyOf[props["type": const("name"),}
\hspace{1em}
&\hspace{1em} "props":not({props["name": {type[\text{str}], length(1,\infty)]};]
\hspace{1em}
&\hspace{1em} req["name"]})
\hspace{1em}
&\hspace{1em})],
\hspace{1em}
&\hspace{1em} props["type":const("link"),
\hspace{1em}
&\hspace{1em} "props":not({type[\text{obj}],
\hspace{1em}
&\hspace{1em} props["href":type[\text{str}],"type":type[\text{str}];],
\hspace{1em}
&\hspace{1em} req["href"]})
\hspace{1em}
&\hspace{1em}]}
\hspace{1em}
&\hspace{1em};
\hspace{1em}
&\hspace{1em}\}
\hspace{1em}
&\hspace{1em},
\hspace{1em}
&\hspace{1em} oneOf[
\hspace{1em}
&\hspace{1em} type[\text{null}],
\hspace{1em}
&\hspace{1em} {type[\text{obj}],
\hspace{1em}
&\hspace{1em} props["type":{type[\text{str}], length(1,\infty)},"props":type[\text{obj}];],
\hspace{1em}
&\hspace{1em} req["type","props"]}
\hspace{1em}
&\hspace{1em}]}
\end{align*}
\]

In this notations properties is abbreviated to props (and we did the same with the property name), required to req, and every operator that may have a variable number of fields, such as props, uses \([]\) for its arguments.

We observe immediately the we have a conjunction \{"\text{not, oneOf }\}" where the not part implies that the instance is an object: only an object may violate a properties constraints. Hence, any instance that validates the schema will never validate the branch type[null] of oneOf, hence we can substitute oneOf with the second branch:

\[
\begin{align*}
&\{ \text{not(anyOf[...}
\hspace{1em}
&\hspace{1em})},
\hspace{1em}
&\hspace{1em} {type[\text{obj}],
\hspace{1em}
&\hspace{1em} props["type":{type[\text{str}], length(1,\infty)},"props":type[\text{obj}];],
\hspace{1em}
&\hspace{1em} req["type","props"]}
\hspace{1em}
&\hspace{1em}]
\end{align*}
\]

Now, we rewrite the not(anyOf[properties.properties]) part as allOf[not(properties).not(properties)].

\[
\begin{align*}
&\{ \text{allOf[not(props["type": const("name"),}
\hspace{1em}
&\hspace{1em} ]}
\end{align*}
\]
We have the following equivalences, where $\text{ifThen}(A;B)$ is implication:
\[
\begin{align*}
\text{not}(\text{props}) & \iff \text{not}(\text{allOf}(\text{props}));
\text{anyOf}(\text{not}(\text{props}); \text{not}(\text{props}))) \\
\text{ifThen}(\text{props}; \text{props}) & \iff \text{ifThen}(\text{props}; \text{props})))) \end{align*}
\]
We use it in order to rewrite the two $\text{not}(\text{props})$ in the expression above. In the process, we transform the nestes conjunction “{ allOf[A,B,C] }” into a flat conjunction “{ A', B', C }.”

Now, the statements $\text{type}[\text{obj}]$ and $\text{req}[	ext{props}]$ in the then branch of the first $\text{ifThen}$ are redundant and can be removed, since they are implied by the top-level $\text{type}[\text{obj}]$ and $\text{req}[	ext{type}, \text{props}]$ found at the end of the schema, and the same for the second $\text{ifThen}$. And we also eliminate the double negation in front of the schema for the “props” property.
Now the specification is clear: the instance is an object, and it must contain at least fields "type", whose value is a non-empty string, and "props", whose value is an object. If the value of "type" is "name", then the value of "props" must contain a "name" member whose value is a non-empty string. If the value of "link" is "name", then the value of "props" must contain an "href" member whose value is a string and, if it contains a nested "type" member, its value must be a string.

This example is quite interesting, since it shows three different things:

1. In order to understand a schema that contains negation, one absolutely needs a reasonable notation, a clear semantics, and some rewrite rules.
2. Programmers are really using negation with some sophistication. Here, in a nutshell, the designer has used:
   
   ```json
   not(props["discr": const("caseOne"),
   "body": not({ req["oneField"], props["oneField": { T }]; } ) })
   ```

   in order to say that, when "discr": "caseOne", then the member "body" must contain the "oneField" member, with type T. This is not an unreasonable way to express that specification, but is very difficult to decode.
3. It also shows that JSON Schema would need a syntax to express value-to-type dependencies or, at least, discriminated unions. That would have solved this example in a totally natural way.

### 6.2 Field exclusion

In file (schema 58639 in our corpus) we have found a use of negation that is quite surprising.

```json
"dependencies": {
  "name": { "not": { "required": ["name"] } },
  "icons": { "not": { "required": ["icons"] } },
  "popup": { "not": { "required": ["popup"] } }
}
```
At first sight, this may look like a contradiction: if "name" is present, then it must be absent. Actually, the meaning is quite simple: the three listed fields cannot appear in the instance. This could also be expressed in many simpler ways, using any of the four following forms. All these expressions are equivalent if the context implies that the instance is an object, otherwise the third and fourth ones are stronger, since they force the instance to actually be an object.

"dependencies": { "name": { "not": { } },
  "icons": { "not": { } },
  "popup": { "not": { } } }

"properties": { "name": { "not": { } },
  "icons": { "not": { } },
  "popup": { "not": { } } }

"allOf": [{ "not": { "required": ["name"] }},
  { "not": { "required": ["icons"] }},
  { "not": { "required": ["popup"] } }]

"not": { "anyOf": [ { "required": ["name"] },
  { "required": ["icons"] },
  { "required": ["popup"] } ] }

6.3 Discriminated unions, again

The following code comes from (schema 90970 in our corpus) and is quite puzzling because of the the use of oneOf.

{ "type" : "object",
  "oneOf": [
    { "properties": {"when": {"enum": ["delayed"]}},
     "required": ["when","start_in"]
    },
    { "properties": {"when": { "not": {"enum": ["delayed"]}}} }
  ]
}

We first verify that the two branches of oneOf are mutually exclusive. If "when" is absent, then only the second branch holds. If it is present, then is associated to complementary types in the two branches. Hence, oneOf is really anyOf, hence it can be rewritten as an implication: if the second branch is false then the first one holds.

{ "type" : "object",
  "if": { "not" : { "properties": {"when": { "not": {"enum": ["delayed"]}}} }
  }
  "then" : { "properties": {"when": {"enum": ["delayed"]}},
     "required": ["when","start_in"]
  }
}
We rewrite the negation in the usual way.

```json
{
   "type": "object",
   "if": { "required": ["when"],
            "properties": {"when": {"enum": ["delayed"]}} },
   "then": { "properties": {"when": {"enum": ["delayed"]},
                         "required": ["when", "start_in"]
                   }
   }
}
```

And now the specification is clear: if "when" has the value "delayed", then "start_in" is required. If "when" is missing, or has any other value, then the specification is satisfied.

6.4 Discriminated unions, again

File (schema 37789 in our corpus).

6.5 Negation and recursions

The interaction between negation and recursion is very problematic. Prolog and Datalog research produced a great wealth of techniques in order to define the semantics of this difficult combination, while the $\mu$-calculus, which is the logic languages with greater similarity, just forbids recursion under negation.

JSON Schema allows recursive negation, provided that it does not create an infinite loop in the checking process. The no-loops rule is actually restraining the use of recursion, and is unrelated with the presence of negation in the loop. For example, the following two schemas are both ok — "$\text{ref}$: '#' is a recursive reference to the current schema.

```json
{
   "type": "object",
   "properties": {"foo": {"$ref": "#"}}
}
```

```json
{
   "type": "object",
   "properties": {"foo": {"not": {"$ref": "#"}}}
}
```

While the following two schemas are not legal, since they would both cause an infinite loop.
Recursion may be direct, with a schema referring to itself, as in the two examples above, or indirect, with two different definitions referring the one to the other, as in the following example.

```
{"$ref": "#/definitions/d1",
"definitions": {
  "d1": {"$ref": "#/definitions/d2"},
  "d2": { "not": { "$ref": "#/definitions/d1" } }
}
```

We checked whether we could find examples of recursion with negation, and indeed we found one example of direct recursion with usage of not. Consider the following snippet (schema 1520 in our corpus), of which we found 4 versions, all very similar.

```
{"oneOf": [
  { "$ref": "#/definitions/MSC.Section-common" },
  ...,
],
"definitions": {
  "MSC.Section-common": {
    "type": "object",
    "properties": {
      ...
    }
  }
}
```

It specifies that a Control (this is the name of the schema) may be a Section, which is an object with an array of elements each of them being itself a Control ("$ref": '#') but not a Section.
This is another example (schema 12177 in our corpus), this time of indirect recursion, of which we found two copies.

```
{  "type": "object",
    "required": [ ... , "properties" ],
    "properties": {
      ...
        "properties": {
          "type": "object",
          "additionalProperties": {
            "$ref": "#/definitions/propertyObject"
          }
        },
      "definitions": {
        "propertyObject": {
          "allOf": [
            { "not": { "$ref": "#" } },
            { "$ref": "dataTypleBase.json#" }
          ]
        }
      }
    }
  }
```

Here, we have a modelObject (the name of the schema file) with an mandatory properties member, whose members may have any name, since their schema is specified using additionalProperties, but whose schema must obey propertyObject. A propertyObject must satisfy dataTypleBase but must not be a modelObject itself.

Finally we found a file (schema 90941 in our corpus) which exhibits the forbidded not.$ref: # pattern, but the name of the file indicates that this is just a test.

```
{  ...
    "not": {"$ref": "#"}
  }
```

7 Conclusions

The lessons drawn from the analysis of negation usage are numerous.

We first learned that negation is used in many different ways, many more than we would have imagined, some of which are extremely creative, although some typical cases are very common: field exclusion, first of all, value exclusion from sets of strings, and field mutual exclusion. Field exclusion is so common that one may imagine to add an ad-hoc operator to JSON Schema.

Another general lesson that we learned is that JSON Schema can be extremely tricky to decode, and the fact that JSON notation is used does not help at all.
We observed that negation is also often used in order to express, in a cumbersome way, the \textit{discriminated union} pattern, where the value of one field determines the presence/absence and the type of the others. This fact may also trigger some reflections about schema design.

We also observed the use of negation in order to force the presence of a member whose name satisfies a pattern. We already noticed that JSON Schema is, in some sense, missing this operator, and we found here a confirmation of that.

Finally, we think that our effort proved that automatic tools to assist the task of decoding JSON Schema specification could be quite useful, which is a motivation to expand our research in that direction.

References

1. Francis Galiegue and Kris Zyp. \textit{Json schema: interactive and non interactive validation - draft-fge-json-schema-validation-00}. Technical report, Internet Engineering Task Force, feb 2013.
2. A. Wright, G. Luff, and H. Andrews. \textit{Json schema validation: A vocabulary for structural validation of json - draft-wright-json-schema-validation-00}. Technical report, Internet Engineering Task Force, mar 2018.
3. A. Wright, G. Luff, and H. Andrews. \textit{Json schema: A media type for describing json documents draft-handrews-json-schema-02}. Technical report, Internet Engineering Task Force, sep 2019.
4. Kris Zyp and G. Court. \textit{A json media type for describing the structure and meaning of json documents - draft-zyp-json-schema-03}. Technical report, Internet Engineering Task Force, nov 2010.