Tracking Quintessence, WIMP Relic Density, PAMELA and Fermi LAT

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The generation of an early *kination* dominated (KD) era within a tracking quintessential model is investigated, the relic density of the Weakly Interacting Massive Particles (WIMPs) is calculated and we show that it can be enhanced *with respect to* (w.r.t) its value in Standard Cosmology (SC). By adjusting the parameters of the quintessential scenario, the cold dark matter abundance in the universe can become compatible with large values for the annihilation cross section times the velocity of the WIMPs. Using these values and assuming that the WIMPs annihilate predominantly to $\mu^+\mu^-$, we calculate the induced fluxes of $e^\pm$ cosmic rays and fit the current PAMELA and Fermi-LAT data. We achieve rather good fits in conjunction with a marginal fulfillment of the restriction arisen from the Cosmic Microwave Background (CMB).

1 Introduction

A plethora of recent data [1] indicates that the two major components of the universe are the *Cold Dark Matter* (CDM) and *Dark Energy* (DE). The DE component can be explained with the introduction of a slowly evolving today scalar field, $\chi$, called quintessence whereas WIMPs, $\chi$, are the most natural candidates to account for CDM. In this talk, which is based on Ref. [2], we reconsider (Sec 2) the creation of $\chi$'s annihilation in the galaxy (Sec. 4), the reported [4, 5] excess on the positron ($e^+$) cosmic-ray (CR) flux under the assumption that $\chi$'s annihilate predominantly into $\mu^+\mu^-$ (Sec. 5). Throughout, the subscript $h$ is referred to present-day values [to values at the onset of our scenario] and $\rho_i = \rho_i/\rho_{0i}$ ($i = q, R$ and M) where $\rho_{0i} = 8.1 \times 10^{-47} h^2$ GeV$^4$ with $h = 0.72$.

2 The Tracking Quintessential Model

The quintessence field, $q$, of our *quintessential scenario* (QS) satisfies the equation:

$$\ddot{q} + 3H\dot{q} + V'/q = 0,$$

with $V = M^{4+q}/q^4 + b H^2 q^2/2$ and $H/H_0 = \dot{H} = \sqrt{\rho_q + \rho_R + \rho_M}.$ (1)

Here $H$ is the Hubble parameter, dot denotes derivative w.r.t the cosmic time $t$, $\rho_q = \dot{q}^2/2 + V$, $\rho_R \simeq \rho_{R0} \exp(-4\tau)$ and $\rho_M = \rho_{M0} \exp(-3\tau)$ is the $q$, radiation and matter energy density respectively, $\tau = \ln(R/R_0)$ is the logarithmic time and $R$ is the scale factor of the universe.

We impose on our QS the following constraints:

(a) *Initial Domination of Kination.* We focus our attention in the range of parameters with $\Omega_{qI} = \Omega_q(\tau_1) = 1$ where $\Omega_q \simeq \rho_q/(\rho_q + \rho_R + \rho_M)$ is the quintessential energy-density parameter.

(b) *Inflationary Constraint.* Assuming that the power spectrum of the curvature perturbations is generated by an early inflationary stage, we impose the bound $\dot{H}_I \lesssim 1.72 \cdot 10^{56}$.

(c) *Nucleosynthesis* (BBN) *Constraint.* At the onset of BBN, $\tau_{BBN} = -22.3$, $\rho_q$ is to be sufficiently suppressed compared to $\rho_R$, i.e., [6] $\Omega_q(\tau_{BBN}) \leq 0.21$ at 95% confidence level (c.l.).
the universe and initially by \( \tau \).

\[ \tau \text{ between } \chi \text{ and } \Delta \Omega \text{ is the Boltzmann equation which governs the evolution of the section times the velocity of } \rho_w \text{ w.r.t its value in the SC, } \rho_w > 0 \text{.} \]

Solving Eq. (1) we find that \( \tau \text{ is set in harmonic oscillations for } b > 0 \text{. In particular, } \dot{q} \text{ develops extrema at } \]

\[ \tau_{\text{ext}} \simeq (2k + 1) \sqrt{\frac{1}{b} \frac{\pi}{2}} + \tau_i, \text{ with } k = 0, 1, 2, \ldots \quad (2) \]

For \( \tau > \tau_{\text{KR}} \) the universe becomes initially radiation and then matter dominated whereas \( \rho_q \) is dominated initially by \( \dot{q}/2 \) and then by \( V \). As \( \tau \) approaches 0 the system in Eq. (1) admits a tracking solution since the energy density of the attractor:

\[ \dot{\rho}_A \propto \exp \left(-3(1 + u_{\text{eq}}^0)\tau\right) \text{ with } u_{\text{eq}}^0 = -2/(a + 2) \quad (3) \]

tracks \( \dot{\rho}_A \) until it outstrips and dominates the current expansion of the universe. It can be shown that \( b > 0 \) ensures the coexistence of an early KD phase with the achievement of the tracking solution in time. Moreover, for \( a < 0.6 \) the requirement 2-(e) can be marginally fulfilled, too.

3 The WIMP Relic Density

The relic density of a WIMP \( \chi \), \( \Omega_\chi h^2 \), with mass \( m_\chi \) is calculated using the formula:

\[ \Omega_\chi h^2 = 2.748 \cdot 10^8 \left( n_\chi 0 / s_0 \right) \left( m_\chi / \text{GeV} \right), \text{ where } \dot{n}_\chi + 3Hn_\chi + \langle \sigma v \rangle \left( n_\chi^2 - n_\chi^{\text{eq}}^2 \right) = 0 \quad (4) \]

is the Boltzmann equation which governs the evolution of the \( \chi \text{’s number density, } n_\chi \). Also, \( n_\chi^{\text{eq}} \propto x^{3/2} e^{-1/x} \) with \( x = T/m_\chi \) is the equilibrium configuration of \( n_\chi \). \( \langle \sigma v \rangle \) is the thermal-averaged cross section times the velocity of \( \chi \text{’s and } s \propto T^3 \) is the entropy density with \( T \) the temperature.

The decoupling of \( \chi \) from the cosmic fluid during the QS and SC is visualized in Fig. 2-(a). We observe that, in both cases, the current \( \rho_\chi / s \) follows \( \rho_\chi^{\text{eq}} / s \) and at some \( \tau = \tau_F \), \( \rho_\chi / s \) dominates over \( \rho_\chi^{\text{eq}} / s \) and remains constant until today. For the selected \( m_\chi \) and \( \langle \sigma v \rangle \) we obtain an enhancement of \( \Omega_\chi h^2 \) within QS w.r.t its value in the SC, \( \Omega_\chi h^2|_{\text{SC}} \), since \( \Omega_\chi h^2 = 0.11 \) whereas \( \Omega_\chi h^2|_{\text{SC}} = 0.0045 \). This enhancement can be further analyzed, by defining \( \Delta \Omega_\chi = \Omega_\chi h^2 / \Omega_\chi h^2|_{\text{SC}} - 1 \). The behavior of \( \Delta \Omega_\chi \) as a function of the free parameters of the QS can be inferred from Fig. 2-(b). For \( b = 0 \) we obtain a pure KD era and \( \Delta \Omega_\chi \) increases when \( m_\chi \) increases or \( \langle \sigma v \rangle \) decreases. For \( b \neq 0 \), \( \Delta \Omega_\chi \) depends crucially on the hierarchy between \( \tau_F \) and \( \tau_{\text{ext}} \). As \( m_\chi \) increases above 0.1 TeV, \( \tau_F \) decreases and moves closer to \( \tau_{\text{ext}} \) and \( \Delta \Omega_\chi \) decreases with its minimum \( \Delta \Omega_\chi|_{\min} \) occurring at \( m_\chi = m_\chi|_{\min} \) which corresponds to \( \tau_F|_{\min} \simeq \tau_{\text{ext}} \).
galaxy is fully acceptable. All in all, we impose the following constraints: Residual annihilation products and the astrophysical backgrounds. We adopt encodes the galactic astrophysics and can be read off from Ref. [8].

\[ \chi \]

where \( \chi \) troubles with observations on combinations of the two data-sets; and the Fermi-LAT data. Adding the latter contributions to the one in Eq. (5) we get the total fluxes, \( \langle \sigma v \rangle \approx 10^{-6} \text{ GeV}^{-2} \) [\( \langle \sigma v \rangle = 10^{-7} \text{ GeV}^{-2} \) ] (gray [light gray] lines) and \( b = 0 \) (solid lines), \( b = 0.15 \) (dashed lines) and \( b = 0.32 \) (dotted lines).

4 \( e^\pm\)-CRs From WIMP Annihilation

Residual \( \chi \)'s annihilations in the galaxy induce a \( e^+ \) flux per energy at Earth which is given by

\[
\Phi^{\chi^+}(E) = \frac{1}{2} \frac{v_{e^+}}{4\pi b(E)} \left( \frac{\rho_\odot}{m_\chi} \right)^2 \langle \sigma v \rangle \int E dE' I(\lambda_D(E, E')) \frac{dN_{e^+}}{dE_{e^+}}, \tag{5}
\]

\( v_{e^+} \) is the velocity of \( e^+ \), \( \rho_\odot = 0.3 \text{ GeV/cm}^3 \) is the local CDM density, \( b(E) = E^2/(\text{GeV} t_E) \) with \( t_E = 10^{16} \text{ s} \) is the energy loss rate function and \( dN_{e^+}/dE_{e^+} \) denotes the energy distribution of \( e^+ \)'s per \( \chi \) annihilation and can be found in Ref. [7]. Also, \( I(\lambda_D) \) is the dimensionless halo function which fully encodes the galactic astrophysics and can be read off from Ref. [8].

There are three sources of uncertainty in our computation: the CDM distribution, the propagation of \( \chi \) annihilation products and the astrophysical backgrounds. We adopt (a) the isothermal halo profile, to avoid troubles with observations on \( \gamma \)-CRs; (b) the MED propagation model, which provides the best fits to the combinations of the two data-sets; and (c) commonly assumed background \( e^\pm \) fluxes normalized with the Fermi-LAT data. Adding the latter contributions to the one in Eq. (5) we get the total fluxes, \( \Phi_{e^\pm} \).

In order to qualify our fittings to the experimental data, we define the \( \chi^2 \) variables as follows:

\[
\chi^2_A = \sum_{i=1}^{N_A} \left( \frac{F_{A_i}^{\text{obs}} - F_{A_i}^{\text{th}}}{\Delta F_{A_i}^{\text{obs}}} \right)^2, \quad \text{with} \quad F_A = \left\{ \begin{array}{ll} \Phi_{e^+} / (\Phi_{e^+} + \Phi_{e^-}) & \text{for PAMELA,} \\ \Phi_{e^+} / (\Phi_{e^+} + \Phi_{e^-}) & \text{for Fermi LAT,} \end{array} \right. \tag{6}
\]

where \( i \) runs over the data points of each experiment \( A \), “obs” [“th”] stands for measured [theoretically predicted] values. The best fits to the combined experimental data can be achieved with \( m_\chi \approx 1.28 \text{ TeV} \) and \( \langle \sigma v \rangle \approx 1.95 \cdot 10^{-6} \text{ GeV}^{-2} \) resulting to \( (\chi^2_1 + \chi^2_2)/\text{d.o.f} = 24/31 \).

5 Results

To systematize our approach, we can define regions in the \( m_\chi - \langle \sigma v \rangle \) plane which are favored at 95% c.l. [99% c.l.] by the various experimental data on the \( e^\pm\)-CRs demanding

\[
\chi^2 \lesssim \chi^2_{\text{min}} + 6 \left( \chi^2 \lesssim \chi^2_{\text{min}} + 9.2 \right) \quad \text{with} \quad \chi^2 = \left\{ \begin{array}{ll} \chi^2_1 & \text{for PAMELA,} \\ \chi^2_1 + \chi^2_2 & \text{for PAMELA and Fermi LAT,} \end{array} \right. \tag{7}
\]

where \( \chi^2_{\text{min}} \) can be extracted numerically by minimization of \( \chi^2 \) w.r.t \( m_\chi \) and \( \langle \sigma v \rangle \).

The large \( \langle \sigma v \rangle \)'s which are required in order to fit the experimental data on \( e^\pm\)-CRs are to be consistent with a number of requirements so as the interpretation of the data on \( e^\pm\)-CRs via CDM annihilation in the galaxy is fully acceptable. All in all, we impose the following constraints:
PAMELA and Fermi-LAT data. Regions above the black solid, dashed, dot-dashed and dotted lines are ruled out by the upper bounds on $\sigma v$ to the conventional RD era. It remains the construction of a particle model with the appropriate couplings so the graph. The light gray shaded areas are allowed the constraint 5-95% c.l. by the PAMELA data and the dense black [red] hatched areas are preferred at 4\%.

Table of Fig. 3. We remark that the requirement 5-parameters of the QKS. In all cases we obtain a combination of PAMELA and Fermi-LAT data which is consistent with all the constraints is given in the table. Shown are also $\Omega_\chi h^2|_{\text{SC}}$ and several $b$'s and $H_1$'s and the resulting $T_{KR}$'s leading to $\Omega_\chi h^2 \simeq 0.11$ in our QS.

(a) **Constraint from the CDM abundance [11]:** $0.097 \lesssim \Omega_\chi h^2 \lesssim 0.12$.

(b) **BBN constraint [9]:** $\langle \sigma v \rangle \leq 8.6 \cdot 10^{-5}$ GeV$^{-2}$ $(m_\chi/$ TeV)$.$

(c) **CMB constraint [10]:** $\langle \sigma v \rangle \leq 1.3 \cdot 10^{-6}$ GeV$^{-2}$ $(m_\chi/$ TeV)$.$

(d) **Constraint from the $\gamma$-CRs [11]:** $\langle \sigma v \rangle \lesssim 4 \cdot 10^{-6}$ GeV$^{-2}$ for the isothermal halo profile.

(e) **Unitarity constraint:** $\langle \sigma v \rangle \leq 8\pi$ GeV$^{-2}$ $(m_\chi/$ GeV)$^{-2}.$

Imposing all the constraints above we can delineate our findings in the $m_\chi - \langle \sigma v \rangle$ plane as in Fig. 3. A simultaneous interpretation of the $e^\pm$ CR anomalies consistently with the various constraints can be achieved in the regions where the gray shaded areas overlap the lined ones below the dashed lines. We observe that part of the region favored at 99% c.l. by PAMELA and Fermi-LAT is allowed. The best-fit $(m_\chi, \langle \sigma v \rangle)$ – with $\chi^2$/d.o.f. $= 33/31$ – which saturates the most stringent (CMB) bound is arranged in the Table of Fig. 3. We remark that the requirement 5-(a) is violated within SC but can be met by adjusting the parameters of the QKS. In all cases we obtain $T_{KR} < 0.02$ GeV with $T_{KR}$ being the transition temperature to the conventional RD era. It remains the construction of a particle model with the appropriate couplings so that $\chi$'s annihilate into $\mu^+ \mu^-$ with the desired $\langle \sigma v \rangle$ derived self-consistently with the (s)particle spectrum.

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**References**

[1] E. Komatsu et al. [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 330 (2009).
[2] S. Lola, C. Pallis and E. Tzelati, *J. Cosmol. Astropart. Phys.* **11**, 017 (2009); C. Pallis, arXiv:0909.5026 (to appear in Nucl. Phys. B).
[3] A. Masiero, M. Pietroni and F. Rosati, *Phys. Rev. D* **61**, 023504 (2000); F. Rosati, *Phys. Lett. B* **570**, 5 (2003).
[4] O. Adriani et al. [PAMELA Collaboration], *Nature* **458**, 607 (2009).
[5] A.A. Abdo et al. [The Fermi-LAT Collaboration], *Phys. Rev. Lett.* **102**, 181101 (2009).
[6] R.H. Cyburt et al., *Astropart. Phys.* **23**, 313 (2005).
[7] I.Z. Rothstein, T. Schwetz and J. Zupan, *J. Cosmol. Astropart. Phys.* **07**, 018 (2009).
[8] E.A. Baltz and J. Edsjo, *Phys. Rev. D* **59**, 023511 (1999); T. Delahaye et al., *Phys. Rev. D* **77**, 063527 (2008); M. Cirelli, R. Franceschini and A. Strumia, *Nucl. Phys.* **B800**, 204 (2008).
[9] J. Hisano, M. Kawasaki, K. Kohri, T. Moroi and K. Nakayama, *Phys. Rev. D* **79**, 083522 (2009).
[10] T.R. Slatyer, N. Padmanabhan and D.P. Finkbeiner, *Phys. Rev. D* **80**, 043526 (2009).
[11] G. Bertone, M. Cirelli, A. Strumia and M. Taoso, *J. Cosmol. Astropart. Phys.* **03**, 009 (2009).