Galactic modulation of extragalactic cosmic rays
— Alternative scenario of the origin of the knee —

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Abstract. Recently we have shown that the observed knee structure is reproduced well by a
superposition of the hypothetical extragalactic cosmic rays modulated by the galactic wind and
the galactic cosmic rays originating in supernova remnants. We reexamine this model assuming
the chemical composition of the hypothetical extragalactic cosmic rays is the same as that of
the galactic cosmic rays of supernova origin. The knee structure is reproduced well again.
The energy dependence of mean mass of cosmic rays above the knee energy is presented to be
compared with observations.

1. Introduction
Cosmic rays (CRs) with energies below the knee have been believed to be originated in supernova
remnants (SNRs) in our galaxy from general arguments about energetics and the diffusive
particle acceleration mechanism in shocks [1, 2]. This argument of the CR origin is corroborated
by recent observations of X-rays and TeV gamma-rays from SNRs [3, 4, 5, 6, 7, 8, 9, 10]. Yet
the origin of CRs above the knee is still not settled [11, 12, 13, 14, 15].

Recently, the existence of diffuse CR electrons in the intergalactic space has been suggested
from the results of extreme-ultraviolet and high energy X-ray observation of clusters of galaxies
[16] and from the observation of the diffuse cosmic gamma-ray background [17, 18]. If nuclear
components with energies extending well above \( \sim 1 \) PeV also exist together with diffuse electrons
near our galaxy, these components modulated by the galactic wind might be directly observable
on Earth. This modulation phenomena of the extragalactic CRs is essentially similar to the
solar modulation of the galactic cosmic rays (GCRs) by the solar wind except it occurs in more
large scale.

We have examined such a possibility by solving numerically a coupled stochastic differential
equations (SDEs) equivalent to the Fokker-Planck equation for the spherical symmetric case
[19]. We have shown that the hypothetical extragalactic CR spectrum which is proportional to
\( E^{-3} \), where \( E \) is the total energy of a particle, breaks around the knee energy at Earth. We also
indicated that the all-particle spectrum of CRs around the knee can be accounted for by the
superposition of CRs originating from SNRs in our galaxy and the hypothetical extragalactic
CRs modulated by the galactic wind. We present a composite model spectrum around the knee
assuming the elemental abundance ratios are the same as those of observed GCRs around 100
GeV/particle.

2. Numerical Simulations and Results
In our model, we postulate the existence of hypothetical CRs just outside of the “galactic sphere”
where the galactic wind [20] terminates. The energy spectrum of these CRs is assumed to be the
same as the spectrum of the CRs observed on Earth with energies higher than the knee region
but extrapolated to a much lower energy range. More precisely, the spectrum is proportional to
$E^{-3}$, where $E$ is the total energy of a particle. These CRs may diffuse into inner region of the
galactic sphere, moving against the expanding galactic wind. We examine how the spectrum
of these CRs should be modulated during this propagation process. The transport of CRs is
described by the Fokker-Planck equation for the spherical symmetric case [21, 22, 23]

$$
\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa \frac{\partial f}{\partial r} \right) - V \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 V p \frac{\partial f}{\partial p} \right),
$$

(1)

where $f$ is the phase space distribution function, $t$ is the time, $r$ is the radial distance, $V$ is the
speed of galactic wind, $p$ is the particle momentum, and $\kappa$ is the diffusion coefficient for
radial propagation. Here we ignore processes that change the energy, except for those inducing
adiabatic losses.

It is known that Eq.(1) is equivalent to a set of coupled SDEs. We write the SDEs equivalent
to Eq.(1) in terms of the new quantity $u = \ln\left(\frac{p}{mc}\right)$ (where $m$ is the particle mass and $c$ is the
speed of light) as

$$
\begin{align*}
\frac{dr}{dt} &= (V + \frac{2\kappa}{r}) dt + \sqrt{2\kappa} dW_r, \\
\frac{du}{dt} &= -\frac{2V}{3r} dt,
\end{align*}
$$

(2)

(3)

where $dW_r$ is a Wiener process given by the Gaussian distribution, $P(dW_r) = (2\pi dt)^{-1/2}
\exp(-dW_r^2/2dt)$. Here we assume that $V$ does not depend on $r$. The modulated spectrum
can easily be obtained by solving the set of SDEs (Eq.(2) and Eq.(3)) numerically, “backward
in time” [24, 25] starting from the boundary of the galactic sphere to Earth (at 8.5 kpc from the
galactic center). We numerically integrated Eqs.(2) and (3) assuming the diffusion coefficient $\kappa$
as

$$
\kappa = \eta \kappa_B \sim 3.3 \times 10^{28} \eta Z^{-1} \left(\frac{E}{1\text{PeV}}\right) \left(\frac{B}{1\mu\text{G}}\right)^{-1} \text{cm}^2\text{sec}^{-1},
$$

(4)

where $\eta$ is the ratio of the mean free path of the particle to the Larmor radius and $\kappa_B$ is the
Bohm diffusion coefficient, $\kappa_B = Ec/(3ZeB)$ (where $E$, $Z$, and $B$ are the total energy of particle,
the atomic number and the magnetic field intensity, respectively).

Fig. 1 shows the calculated differential energy spectra of hypothetical nuclear components
labeled in the figure at Earth ($r_0 = 8.5$ kpc) as a function of total energy $E$ [19]. Here we assume
$V = 300 \text{ km sec}^{-1}$, $R = 100$ kpc, $B = 1\mu\text{G}$ and $\eta = 1000$ as an example [26]. The straight
line indicates the assumed unmodulated spectrum at the boundary of the galactic sphere. We
find that the spectrum breaks around the knee energy. It is also worth noting how the break
point is shifted when the other parameters, $B$, $R$, and $V$ are changed. Here, if we define the
breaking energy $E_{\text{break}}$ as the energy at which the modulated spectrum shown in Fig. 1 becomes
maximal, it is approximately given by

$$
E_{\text{break}}(Z, \eta, B, R, V) \sim 6 \times 10^{15} Z \eta^{-1} B R V \text{ eV},
$$

(5)
Figure 1. Galactic modulated energy spectra of the hypothetical CRs with five values of $Z$ for $\eta = 1000$, $R = 100$ kpc, $V = 300$ km sec$^{-1}$, and $B = 1\mu$G [19]. The straight line indicates the differential energy spectrum of the hypothetical CRs with $Z$ at the boundary of the galactic sphere $R$ which is a power law in total energy with a spectral index of $-3.0$.

where $\eta_*=\eta/(100)$, $B_*=B/(1\mu$G$)$, $R_*=R/(100$ kpc$)$, and $V_*=V/(300$ km s$^{-1}$), respectively [19]. This result indicates clearly that we can reproduce the knee by choosing appropriate values for these parameters.

In order to demonstrate how well our model reproduces the observed all-particle spectrum near the knee region, we consider a simple model introduced below. We assume that the all-particle spectrum observed on Earth, $F_{\text{total}}(E)$, is a superposition of two components, namely, the modulated extragalactic component, $F_{\text{modul}}(E)$, and the component originating in SNRs in our galaxy, $F_{\text{SNR}}(E)$:

$$F_{\text{total}}(E) = F_{\text{SNR}}(E) + F_{\text{modul}}(E).$$  \hspace{1cm} (6)

Here, $F_{\text{SNR}}(E)$ and $F_{\text{modul}}(E)$ are given by

$$F_{\text{SNR}}(E) = \sum_Z f^Z_{\text{SNR}}(E),$$ \hspace{1cm} (7)

$$F_{\text{modul}}(E) = \sum_Z f^Z_{\text{modul}}(E),$$ \hspace{1cm} (8)

where $f^Z_{\text{SNR}}(E)$ and $f^Z_{\text{modul}}(E)$ are each nuclear components. Now we consider the case in which $F_{\text{SNR}}(E)$ consists of the sum of components after propagating in interstellar space from each of the SNRs in our galaxy, in which particles are accelerated according to a power law by shock acceleration. Thus we simply assume that each nuclear component observed on Earth, $f^Z_{\text{SNR}}(E)$, is represented by a power law with each spectral index of $\alpha$ as

$$f^Z_{\text{SNR}}(E) \propto E^{-\alpha}\exp(-E/(ZE_{\text{max}})),$$ \hspace{1cm} (9)

where $E_{\text{max}}$ is the maximum energy of protons, which may come from the maximum energy attained by protons accelerated in SNRs. Here we fitted the published data of CR proton, helium, CNO-group, NeMgSi-group and iron in subTeV − TeV energy region which are believed to originate in SNRs in our galaxy [27, 28, 29, 30, 31, 32]. We assumed in this fitting $E_{\text{max}} = 500$ TeV, for example, and also assumed that $(Z, A)$ are $(7, 14)$ for the CNO-group and $(12, 24)$ for the NeMgSi-group. We define the sum of these components as the SNR component, $F_{\text{SNR}}(E)$.

In previous work, we have shown the model spectrum $F_{\text{total}}(E)$ around the knee for a two example cases [19]; one is a single component model for $F_{\text{modul}}(E)$ (e.g. $F_{\text{modul}}(E) = f^p_{\text{modul}}(E)$
or $f_{\text{Fe modul}}(E)$), and the other is a composite model for $F_{\text{modul}}(E)$ which consists of the same spectrum of $f_{p \text{ modul}}(E)$, $f_{\text{He modul}}(E)$, $f_{\text{CNO modul}}(E)$, $f_{\text{NeMgSi modul}}(E)$, and $f_{\text{Fe modul}}(E)$ at the boundary. We found that our model can reproduce the observed spectrum around the knee fairly well. However, we don’t know the chemical composition of the hypothetical extragalactic CRs. We simply assume the chemical composition of the extragalactic CRs is the same as that of observed CRs around 100 GeV/particle on Earth [33, 34].

The relative abundance of proton, helium, CNO-group, NeMgSi-group, and iron is 1, 0.58, 0.26, 0.014, and 0.0036 respectively, when we adopt the model stated above for the chemical abundance for the extragalactic CRs. The expected composite model all-particle spectrum is shown by the thick solid curve in Fig. 2 when we adopted the same value for the relevant parameters, $\eta = 250$, $R = 100$ kpc, $V = 300$ km sec$^{-1}$, and $B = 1 \mu$G, as adopted in our previous paper [19]. Our model spectrum reproduces well again the observed all-particle spectrum.

By using the results of our model spectrum shown in Fig. 2, mean logarithmic mass of CRs which is compared with observations is calculated. The result is shown in Fig. 3 by the thick solid curve as a function of total energy $E$ together with the mean mass determined by direct measurements in energy region below the knee and the mean mass expected for the extreme case for the chemical composition investigated by our previous work [19]. Experimental
determinations of the mean logarithmic mass in the region above the knee are, however, scattered over a rather wide range due to various causes, depending on the methods in the ground-based measurements and also on the interaction models of the hadrons used in the data analysis [38, 39, 40]. Our model predicts that the mean logarithmic mass above \( \sim 50 \text{ PeV} \) will tend to that of the hypothetical extragalactic CRs, as inferred from Figs. 2 and 3. Our model should be testified by direct observations near the knee region by more sophisticated methods and future indirect observations in the energy range much higher than the knee.

3. Summary
We show that all-particle spectrum of CRs around the knee region is reproduced well by a superposition of the two components; the GCRs of SNR origin and, the extragalactic CRs modulated by the galactic wind. Future observations of CRs above the knee region will tell us the chemical composition of the extragalactic CRs. Simulation experiments in more realistic setting for galactic structure using three dimensional code are needed.

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Figure 3. Mean logarithmic mass of CRs around the knee as a function of total energy of particle. Symbols indicate the values obtained by direct measurements [31, 32]. Thick solid curve represents \( F_{\text{total}}(E) = F_{\text{SNR}}(E) + f_{\text{He}}(E) + f_{\text{CNO}}(E) + f_{\text{NeMgSi}}(E) + f_{\text{Fe}}(E) \) (shown in Fig. 2). Dashed, thin solid and dotted curves represent the results of our previous paper [19].
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