Effects of Flavor-dependent $q\bar{q}$ Annihilation on the Mixing Angle of the Isoscalar Octet-Singlet and Schwinger’s Nonet Mass Formula

De-Min Li$^a$, Hong Yu$^{a,b}$ and Qi-Xing Shen$^{a,b}$

$^a$Institute of High Energy Physics, Chinese Academy of Sciences,
P.O.Box 918 (4), Beijing 100039, China

$^b$Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

By incorporating the flavor-dependent quark-antiquark annihilation amplitude into the mass-squared matrix describing the mixing of the isoscalar states of a meson nonet, the new version of Schwinger’s nonet mass formula which holds with a high accuracy for the $0^{-+}$, $1^{--}$, $2^{++}$, $2^{-+}$ and $3^{--}$ nonets is derived and the mixing angle of isoscalar octet-singlet for these nonets is obtained. In particular, the mixing angle of isoscalar octet-singlet for pseudoscalar nonet is determined to take the value of $-12.92^\circ$, which is in agreement with the value of $-13^\circ \sim -17^\circ$ deduced from a rather exhaustive and up-to-date analysis of data. It is also pointed out that the omission of the flavor-dependent $q\bar{q}$ annihilation effect might be a factor resulting in the invalidity of Schwinger’s original nonet mass formula for pseudoscalar nonet.

**PACS numbers:** 11.30.Hv, 12.40.Yx, 14.40-n

---

*Supported by the National Natural Science Foundation of China under Grant Nos. 19991487, 19677205 and 19835060, and the Foundation of the Chinese Academy of Sciences under Grant No. LWTZ-1298.

†E-mail:lidm@hptc5.ihep.ac.cn
According to the quark model, the $q\bar{q}$ mesons containing $u$, $d$ and $s$ quarks correspond to an octet and a singlet of the $SU(3)$ flavor group:

$$3 \otimes \bar{3} = 8 \oplus 1. \quad (1)$$

In general, states with the same isospin-spin-parity $IJ^{PC}$ and additive quantum numbers can mix. Thus, the $I = 0$ member of ground state octet $\eta_8$ mixes with the corresponding singlet $\eta_1$ to produce the two physical states $\eta$ and $\eta'$. Here, we assume that the possibility of the mixing of $\eta_8$, $\eta_1$ and other isosinglets such as glueball and hidden flavor heavy quark meson can be ignored.

In the $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ and $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ basis, the mass-squared matrix $M^2$ describing the mixing of $\eta_8 - \eta_1$ can be described as

$$M^2 = \begin{pmatrix} M^2_{\eta_8} & M^2_{\eta_8} \\ M^2_{\eta_8} & M^2_{\eta_1} \end{pmatrix}, \quad (2)$$

where $M^2_{\eta_8}$ is the sum of the mass-squared of $\eta_8$ and the transition amplitude of $\eta_8 \leftrightarrow gg...g \leftrightarrow \eta_8$, $M^2_{\eta_1}$ is the sum of the mass-squared of $\eta_1$ and the transition amplitude of $\eta_1 \leftrightarrow gg...g \leftrightarrow \eta_1$, and $M_{\eta_8}$ is the transition amplitude of $\eta_8 \leftrightarrow gg...g \leftrightarrow \eta_1$ (g denotes gluon). $M^2$ satisfies

$$UM^2U^{-1} = \begin{pmatrix} M^2_\eta & 0 \\ 0 & M^2_{\eta'} \end{pmatrix}, \quad (3)$$

where

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad (4)$$

$\theta$ is the mixing angle of $\eta_8 - \eta_1$, $M_\eta$ and $M_{\eta'}$ are the masses of the physical states $\eta$ and $\eta'$, respectively. Equation (3) reads

$$M^2 = U^{-1} \begin{pmatrix} M^2_\eta & 0 \\ 0 & M^2_{\eta'} \end{pmatrix} U = \begin{pmatrix} \cos^2 \theta M^2_\eta + \sin^2 \theta M^2_{\eta'} & \sin \theta \cos \theta (M^2_{\eta'} - M^2_\eta) \\ \sin \theta \cos \theta (M^2_{\eta'} - M^2_\eta) & \sin^2 \theta M^2_\eta + \cos^2 \theta M^2_{\eta'} \end{pmatrix}. \quad (5)$$

Comparing Eq. (2) with Eq. (5), one can get the following relations:

$$\tan^2 \theta = \frac{M^2_{\eta_8} - M^2_{\eta}}{M^2_{\eta'} - M^2_{\eta_8}}. \quad (6)$$
\[
\sin 2\theta = \frac{2M_{18}^2}{M_{\eta'}^2 - M_\eta^2},
\]
(7)
\[
\tan 2\theta = \frac{2M_{18}^2}{M_{11}^2 - M_{88}^2},
\]
(8)
\[
\tan \theta = \frac{M_{88}^2 - M_\eta^2}{M_{18}^2}.
\]
(9)

The mixing angle \( \theta \) can be determined by any of the above four relations.

In the presence of the flavor-independent \( q \bar{q} \) annihilation, the matrix elements of \( M^2 \) can be given by Refs.\[2, 3, 4, 5\], i.e.,

\[
M_{88}^2 = \frac{1}{3}(4M_K^2 - M_{\pi^0}^2),
\]
(10)
\[
M_{18}^2 = -\frac{2\sqrt{2}}{3}(M_K^2 - M_{\pi^0}^2),
\]
(11)
\[
M_{11}^2 = \frac{1}{3}(2M_K^2 + M_{\pi^0}^2 + 9A),
\]
(12)

where \( A = (M_\eta^2 + M_{\eta'}^2 - 2M_K^2)/3 \), \( M_{\pi^0} \) and \( M_K \) are the masses of the isovector \( \pi^0 \) and isodoublet \( K \), respectively; \( M_K^2 = (M_{K^+}^2 + M_{K^0}^2)/2 \). From Eqs. (2) and (5), one can get

\[
M_{88}^2 + M_{11}^2 = M_\eta^2 + M_{\eta'}^2,
\]
(13)
\[
M_{88}^2 M_{11}^2 - M_{18}^2 M_{18}^2 = M_\eta^2 M_{\eta'}^2.
\]
(14)

By eliminating \( A \) from the two relations, one can get Schwinger’s original nonet mass formula\[6\]

\[
(4M_K^2 - 3M_\eta^2 - M_{\pi^0}^2)(3M_{\eta'}^2 + M_{\pi^0}^2 - 4M_K^2) = 8(M_K^2 - M_{\pi^0}^2)^2.
\]
(15)

However, Eqs.(10)~(12) are not self-consistent. First, for the \( 0^{-+} (\pi, K, \eta, \eta') \), \( 1^{--} (\rho, K^*, \omega, \phi) \), \( 2^{++} (a_2(1320), K_2^*(1430), f_2(1270), f_2'(1525)) \), \( 2^{-+} (\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870)) \) and \( 3^{--} (\rho_3(1690), K_3^*(1780), \omega_3(1670), \phi_3(1850)) \) nonets (All the masses of the physical states are taken from Particle Data Group 98\[1\]), the left-hand side and right-hand side of Eq. (15) are not balance, especially for pseudoscalar nonet Eq. (15) is obviously invalid (see the columns II, III and IV of Table 1). Second, for a meson nonet, the values of the mixing angle derived from different relations (\( \theta_6 \sim \theta_9 \)) are different (see Table 2). Therefore, the matrix elements of \( M^2 \) (Eqs. (10)~(12) ) should be modified.
Recently, Burakovsky et al.\[4, 5\] discussed this problem by incorporating the pseudoscalar decay constants into the matrix elements of $M^2$, however, which is valid only for pseudoscalar nonet. In this letter, we shall discuss the same issue by incorporating the effect of the flavor-dependent $q\bar{q}$ annihilation into the matrix elements of $M^2$, which is valid for all above nonets.

We assume that the transition between $q\bar{q}$ and $q'\bar{q}'$ is flavor-dependent\[7\], i.e., the transition between different flavor quarkonia is not flavor blind, taking into account the possibility that the nonstrange quarkonia and strange quarkonia system have the different wave functions at the origin as the result of the different mass. In the $N = (u\bar{u} + d\bar{d})/\sqrt{2}$, $S = s\bar{s}$ basis, from Refs.\[8, 9, 10\], the mass-squared matrix $M^2$ can be replaced by

$$M'^2 = \begin{pmatrix} M^2_N + r^2A' & rA' \\ rA' & M^2_S + A' \end{pmatrix},$$  \hspace{1cm} (16)

where $r$ describes the effect of the flavor-dependent $q\bar{q}$ annihilation, $r = \sqrt{2}$ means $q\bar{q}$ annihilation is flavor-independent; $A'$ is the transition amplitude of $S \leftrightarrow gg...g \leftrightarrow S$. Owing to

$$(N, S) = (\eta_8, \eta_1)R = (\eta_8, \eta_1) \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix},$$  \hspace{1cm} (17)

the mass-squared matrixes $M^2$ and $M'^2$ can be connected by

$$M^2 = RM'^2R^{-1}. \hspace{1cm} (18)$$

If we assume $M_N = M_{\pi^0}$ and $M^2_S = 2M^2_K - M^2_N$ \[4, 5, 11\], the matrix elements of $M^2$ can now be replaced by

$$M^2_{88} = \frac{1}{3}(4M^2_K - M^2_{\pi^0}) + \frac{1}{3}A'r^2 - \frac{2\sqrt{2}}{3}A'r + \frac{2}{3}A',$$  \hspace{1cm} (19)

$$M^2_{18} = -\frac{2\sqrt{2}}{3}(M^2_K - M^2_{\pi^0}) + \frac{\sqrt{2}}{3}A'r^2 - \frac{1}{3}A'r - \frac{\sqrt{2}}{3}A',$$  \hspace{1cm} (20)

$$M^2_{11} = \frac{1}{3}(2M^2_K + M^2_{\pi^0}) + \frac{2}{3}A'r^2 + \frac{2\sqrt{2}}{3}A'r + \frac{1}{3}A',$$  \hspace{1cm} (21)

where

$$A' = \frac{(M^2_{\eta'} - 2M^2_K + M^2_{\pi^0})(M^2_{\eta} - 2M^2_K + M^2_{\pi^0})}{2(M^2_{\pi^0} - M^2_K)},$$  \hspace{1cm} (22)
\[ r^2 = \frac{(M_{\eta}^2 - M_{\pi^0}^2)(M_{\pi^0}^2 - M_{\eta'}^2)}{(M_{\eta'}^2 - 2M_K^2 + M_{\pi^0}^2)(M_{\eta}^2 - 2M_K^2 + M_{\pi^0}^2)}. \] (23)

Based on Eqs. (13) and (14), the new version of Schwinger’s nonet mass formula including the effect of the flavor-dependent \( q\bar{q} \) annihilation can be derived as

\[ [2r^2M_K^2 - (1+r^2)M_{\eta}^2 - (r^2-1)M_{\pi^0}^2][(1+r^2)M_{\eta'}^2 + (r^2-1)M_{\pi^0}^2 - 2r^2M_K^2] = 4r^2(M_K^2 - M_{\pi^0}^2)^2. \] (24)

If \( r = \sqrt{2} \), Eq. (24) can be reduced to Eq. (15). From Eqs. (19)\~(24), both sides of Eq. (24) are given in the columns V and VI of Table 1, and the values of mixing angle determined from different relations (\( \theta_6 \sim \theta_9 \)) are shown in Table 3.

The columns V and VI of Table 1 show that the new version of Schwinger’s nonet mass formula holds with a high accuracy for all above nonets. At the same time, Table 3 indicates that for a meson nonet, the values of the mixing angle derived from different relations (\( \theta_6 \sim \theta_9 \)) are exactly equal. Furthermore, comparing Table 2 with Table 3, one can clearly conclude that the effect of the flavor-dependent \( q\bar{q} \) annihilation on the mixing angle of \( \eta_8 - \eta_1 \) for pseudoscalar nonet is quite significant while relatively weak for the \( 1^{--}, 2^{++}, 2^{--} \) and \( 3^{--} \) nonets. The mixing angle of \( \eta_8 - \eta_1 \) for pseudoscalar nonet, \( \theta = -12.92^\circ \), is also consistent with the value of \(-13^\circ \sim -17^\circ \) deduced from a rather exhaustive and up-to-date analysis of data including strong decays of tensor and higher spin mesons, electromagnetic decays of vector and pseudoscalar mesons, and the decays of \( J/\psi \).[12]

It should be emphasized that the only difference between Eqs. (15) and (24) is that Eq. (24) contains \( r \), the term describing the effect of the flavor-dependent \( q\bar{q} \) annihilation. However, for pseudoscalar nonet Eq. (24) holds with a high accuracy while Eq. (15) is obviously invalid, which implies that the omission of the flavor-dependent \( q\bar{q} \) annihilation effect might be a factor resulting in the failure of Schwinger’s original nonet mass formula for pseudoscalar nonet.

In conclusion, by investigating the effect of the flavor-dependent \( q\bar{q} \) annihilation on the mixing angle of isoscalar octet-singlet and Schwinger’s nonet mass formula for the \( 0^{-+}, 1^{--}, 2^{++}, 2^{--} \) and \( 3^{--} \) nonets\footnote{The related discussions have been done in Ref.[13] for \( 1^{++} \) nonet}, we find that the effect of the flavor-dependent \( q\bar{q} \) annihilation should
be considered when we discuss the mixing of isoscalar octet-singlet of a meson nonet, especially for pseudoscalar nonet. We believe that the omission of flavor-dependent $q\bar{q}$ annihilation effect might be a factor resulting in the invalidity of Schwinger’s original nonet mass formula for pseudoscalar nonet.

References

[1] Particle Data Group, (C. Caso et al.), Eur. Phys. J. C3 (1998) 1.

[2] F.E. Close, An Introduction to Quarks and Partons, (Academic Press, London, 1979 ), Chapter 17.

[3] V. Dmitrasinovic, Phys. Rev. D56 (1997) 247.

[4] L. Burakovsky and T. Goldman, [hep-ph/9709461].

[5] L. Burakovsky and T. Goldman, Phys. Lett. B427 (1998) 361.

[6] J. Schwinger, Phys. Rev. Lett. 12 (1964) 237.

[7] A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12 (1975) 147.

[8] N.H. Fuchs, Phys. Rev. D14 (1976) 1192.

[9] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D58 (1998) 114006.

[10] E. Kawai, Phys. Lett. B124 (1983) 262.

[11] S. Okuba, Prog. Theor. Phys. 27 (1962) 949.

[12] A. Bramon et al., Eur. Phys. J. C7 (1999) 271.

[13] De-Min Li, Hong Yu and Qi-xing Shen, [hep-ph/0001011], Chin. Phys. lett. 17 (2000) 558.
Table 1. Values of \( l_{15} \) and \( r_{15} \) (\( l_{24} \) and \( r_{24} \)) denoting the left hand side and right hand side of Eq. (15) (Eq. (24)), respectively.

| Nonet | \( l_{15} \) | \( r_{15} \) | \( |l_{15} - r_{15}| \) | \( l_{24} \) | \( r_{24} \) |
|-------|-------------|-------------|----------------|-------------|-------------|
| 0\(^{+}\) | 0.1178 | 0.4140 | 251\% | 0.6788 | 0.6788 |
| 1\(^{-}\) | 0.3956 | 0.3399 | 14.1\% | 0.1045 | 0.1045 |
| 2\(^{++}\) | 0.8485 | 0.7426 | 12.5\% | 1.6049 | 1.6049 |
| 2\(^{-}\) | 0.9455 | 1.006 | 6.4\% | 0.8059 | 0.8059 |
| 3\(^{-}\) | 0.7815 | 0.7303 | 6.6\% | 1.2153 | 1.2153 |

Table 2. Values of the mixing angles (\( \theta_6 \), \( \theta_7 \), \( \theta_8 \) and \( \theta_9 \) are respectively derived from Eqs. (6), (7), (8) and (9) in the presence of the flavor-independent \( q \bar{q} \) annihilation).

| Nonet | \( \theta_6 \) | \( \theta_7 \) | \( \theta_8 \) | \( \theta_9 \) |
|-------|-------------|-------------|-------------|-------------|
| 0\(^{+}\) | \(-10.87^\circ\) | \(-21.98^\circ\) | \(-18.39^\circ\) | \(-5.85^\circ\) |
| 1\(^{-}\) | \(-50.73^\circ\) | \(-57.35^\circ\) | \(-51.16^\circ\) | \(-52.84^\circ\) |
| 2\(^{++}\) | \(-59.34^\circ\) | \(-62.42^\circ\) | \(-60.16^\circ\) | \(-60.99^\circ\) |
| 2\(^{-}\) | \(-61.56^\circ\) | \(-60.11^\circ\) | \(-61.15^\circ\) | \(-60.80^\circ\) |
| 3\(^{-}\) | \(-58.24^\circ\) | \(-60.04^\circ\) | \(-58.63^\circ\) | \(-59.10^\circ\) |

Table 3. Values of the mixing angles (\( \theta_6 \), \( \theta_7 \), \( \theta_8 \) and \( \theta_9 \) are respectively derived from Eqs. (6), (7), (8) and (9) in the presence of the flavor-dependent \( q \bar{q} \) annihilation).

| Nonet | \( \theta_6 \) | \( \theta_7 \) | \( \theta_8 \) | \( \theta_9 \) |
|-------|-------------|-------------|-------------|-------------|
| 0\(^{+}\) | \(-12.92^\circ\) | \(-12.92^\circ\) | \(-12.92^\circ\) | \(-12.92^\circ\) |
| 1\(^{-}\) | \(-51.31^\circ\) | \(-51.31^\circ\) | \(-51.31^\circ\) | \(-51.31^\circ\) |
| 2\(^{++}\) | \(-59.00^\circ\) | \(-59.00^\circ\) | \(-59.00^\circ\) | \(-59.00^\circ\) |
| 2\(^{-}\) | \(-61.51^\circ\) | \(-61.51^\circ\) | \(-61.51^\circ\) | \(-61.51^\circ\) |
| 3\(^{-}\) | \(-58.12^\circ\) | \(-58.12^\circ\) | \(-58.12^\circ\) | \(-58.12^\circ\) |