The cosmological effect of $N$-entangled state in multiverse

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Abstract. We investigate the effect of quantum entanglement between $N$-causally disconnected universes by calculated the power spectrum of quantum fluctuation. To investigate it the general form of $N$-quantum state and specific $N$-entangled state (GHZ and W state) are used. We found that the power spectrum has the maximum value when the state is near to pure one particle state. For the specific $N$-entangled state, the power spectrum of W state is equal to pure vacuum state when the amount of $N$-causally disconnected universes is near to infinite and for GHZ state the power spectrum has same value for every $N$-causally disconnected universes.

1. Introduction
Quantum entanglement is one of the interesting objects in quantum physics. The quantum entanglement idea was firstly noted by Albert Einstein and his colleagues [1]. The interesting about quantum entanglement is when there are two systems or more are said to be entangled, one can get the information about it just by measure one of that systems. There are a lot of experiments that proved quantum entanglement [2][3][4]. The first experiment was done by John Bell [2], in his experiment, they introduced bell’s inequality to approved that quantum entanglement can happen.

The research area of quantum entanglement now usually talk about its application. Quantum entanglement can be applied in many ways, like in quantum teleportation, quantum cryptography, and many more. In this paper, we will focus on the application of quantum entanglement in the early universe. In cosmology perturbation theory, is said that the early universe wasn't perfectly homogeneous. There were small inhomogeneities that lead the inflation has fluctuations. The natural consequence of these fluctuations is at the initial condition of the inflation, it has to have quantum characteristics which it can be found by quantizing the Mukhanov-Sasaki equation [5]. The vacuum solution of this equation is called bunch davies vacuum. By taking the idea that our universe it’s not the only universe that exists [6][7][8], one can make the assumption that the bunch davies vacuum and one particle state of our universe are entangled with another universe. This idea of research has been done before by Sugumi Kanno [9]. In his work to explore the cosmological effect, the calculation of power spectrum are used, but the quantum entanglement states are only the state of two causally disconnected universe. This paper will be explored the cosmological effect of $N$-causally disconnected universe.
The $N$-causally disconnected universe is chosen to be the general form of $N$-quantum state and $N$-entangled specific state. For the specific $N$-entangled state, will be used generalized of Greenberger-Horne-Zeilinger (GHZ) and W state[10][11]. Like Sugumi Kanno’s work[9], will be calculated the power spectrum by using the reduced density matrix of our universe.

This paper will be divided into five-part. The first part is the introduction part. In the second part, the $N$-quantum state of $N$-causally disconnected universe are introduced, and by using the partial trace the reduced matrix density of our universe are found, which can be used to calculate the power spectrum in the next section. For the general form of $N$-quantum state, the power spectrum is calculated in section 3 and for the specific state is in section 4. In section 3, the Von Neumann entropy [12] is calculated to, as a comparison to investigate the effect of the entanglement, because the general form of $N$-quantum state is a state that can be entangled or non-entangled. In the last section will be the conclusion.

2. Reduced density matrix of the quantum state
As it mentioned before, in this section the $N$-quantum state of $N$-causally disconnected universe are introduced, and the reduced density matrix of our universe are derived by using the partial trace. Started with the general form of $N$-quantum state

$$|\psi\rangle = C_1 |0_10_2...0_{N-1}1_N\rangle + C_2 |0_10_2...1_{N-1}0_N\rangle + ... + C_{2^N} |1_11_2...1_{N-1}1_N\rangle$$  \hspace{1cm} (1)

Which the Hilbert space is $\mathcal{H} = \mathcal{H}_{BD1} \otimes \mathcal{H}_{BD2} \otimes ... \otimes \mathcal{H}_{BD2}$, $C_1, C_2, ..., C_{2^N}$ are constant with $\sum_{n=1}^{2^N} |C_n|^2 = 1$ and the notation in the kets are depend on it’s bunch davies space. Take the assumption that our universe is in the bunch davies two. The reduced density matrix of our universe can be obtained easily using partial trace, $\rho_{BD2} = \sum_{n_1,n_2,...,n_N=0}^{2^N} \left( \langle n_1 | \otimes \langle n_2 | \otimes ... \otimes \langle n_N | \right) |\psi\rangle \langle \psi| \left( |n_1\rangle \otimes |n_2\rangle \otimes ... \otimes |n_N\rangle \right)$ as

$$\rho_{BD2} = \alpha |0\rangle \langle 0| + \beta |0\rangle \langle 1| + \gamma |1\rangle \langle 0| + \delta |1\rangle \langle 1|$$  \hspace{1cm} (2)

with $\alpha, \beta, \gamma, and \delta$ are

$$\alpha = \sum_{n=1}^{2^{N-2}} \left( |C_n|^2 + |C_{2N-1}|^2 \right)$$  \hspace{1cm} (3)

$$\beta = \sum_{n=1}^{2^{N-2}} \left( (C_n)(C_{2^{N-2}+n})^* + (C_{2^{N-1}+n})(C_{3^{N-1}+n})^* \right)$$  \hspace{1cm} (4)

$$\gamma = \sum_{n=1}^{2^{N-2}} \left( (C_n)^*(C_{2^{N-2}+n}) + (C_{2^{N-1}+n})^*(C_{3^{N-1}+n}) \right)$$  \hspace{1cm} (5)

$$\delta = \sum_{n=1}^{2^{N-2}} \left( |C_{2^{N-2}+n}|^2 + |C_{3^{N-1}+n}|^2 \right)$$  \hspace{1cm} (6)

From equation (2) can be seen that the matrix density for the general state has four degrees of freedom. Assume that the power spectrum is gaussian, the density matrix has to be in zero average therefore $\beta$ and $\gamma$ are chosen to be zero. So the matrix density will possess two degrees of freedom. To simplify the investigation it will be better if the matrix density possesses only one degree of freedom. To do so let’s try to add $\alpha$ with $\beta$

$$\alpha + \beta = \sum_{n=1}^{2^{N-2}} \left( |C_n|^2 + |C_{2N-1}|^2 + |C_{2^{N-2}+n}|^2 + |C_{3^{N-1}+n}|^2 \right)$$
\[ n = 1 \]

One can see that \( \alpha + \beta \) is equal to one. It’s mean that the matrix density in equation (2) can be written in from that possess only one degree of freedom as

\[ \rho_{BD2} = \alpha |0\rangle \langle 0| + (1 - \alpha) |1\rangle \langle 1| \] (7)

For the \( N \)-entangled specific state, two kinds of entangled states which are generalized of Greenberger-Horne-Zeilinger (GHZ) and W state are used. GHZ and W state are the states that have 3 partite entanglements, in this form GHZ and W state are said to be a maximally entangled state. Those two states can be generalized into a multipartite system

\[ |W\rangle = \frac{1}{\sqrt{N}} \left( |110\ldots0\rangle + |011\ldots0\rangle + \ldots + |010\ldots1\rangle \right) \] (8)

\[ |GHZ\rangle = \frac{1}{\sqrt{2}} \left( |001\ldots0\rangle + |111\ldots0\rangle \right) \] (9)

By using the partial trace like before, for the W state is obtained the reduced density matrix

\[ \rho_{BD2} = \frac{1}{N} \left( (N - 1) |0\rangle \langle 0| + |1\rangle \langle 1| \right) \] (10)

and for GHZ state

\[ \rho_{BD2} = \frac{1}{2} \left( |0\rangle \langle 0| + |1\rangle \langle 1| \right) \] (11)

3. Power spectrum for general state

In this section will be calculated the power spectrum for the general state. Before doing so, will be introduced the mode function in our universe. In the cosmological perturbation theory, the scalar field is written as

\[ \Phi(\tau, x) = \phi(\tau) + \delta \phi(\tau, x) \] (12)

where \( \tau \) is the conformal time. The second term is the perturbation filed or the field that leads to inhomogeneities in the early universe. This field is defined as

\[ \delta \phi(\tau, x) \equiv \frac{f(\tau, x)}{a(\tau)} \] (13)

with \( a(\tau) \) is the scale factor that characterizes the relative size of a spacelike hypersurface in different times and \( f(\tau, x) \) is a mode function that has to satisfy the Mukhanov-Sasaki equation.

In the Fourier mode, the Mukhanov-Sasaki equation can be written

\[ f''_k + \left( k^2 - \frac{a''}{a'} \right) f_k = 0 \] (14)

the vacuum solution for this equation with positive frequency \( \tau \to -\infty \) is

\[ f_k = \frac{e^{-ik\tau}}{\sqrt{2k}} \alpha \mathcal{H} \left( i - k\tau \right) \] (15)
The quantum state in the early universe can be found by quantizing equation (14). Which means the mode function can be expanded as an operator

$$\hat{f}_k(\tau) = f_k(\tau)\hat{a}_k + f^*_k(\tau)\hat{a}^\dagger_k$$

with \(\hat{a}_k\) and \(\hat{a}^\dagger_k\) are annihilation and creation operator. The quantum states in Hilbert space are constructed by defining vacuum state

$$\hat{a}_k |0\rangle = 0$$

To calculate the power spectrum, starting with calculating the variation of the perturbation field in equation (13). To do so, we need to expand the perturbation field as a quantum operator

$$\delta \hat{\phi} = \frac{1}{a(\tau)} \int \frac{d^3k}{(2\pi)^3} \left( f_k \hat{a}_k e^{ikx} + f^*_k \hat{a}^\dagger_k e^{-ikx} \right)$$

So the variation of the perturbation field can be written

$$\langle \Delta \delta \hat{\phi} \rangle^2 = \frac{1}{a^2} \left( \langle |\hat{f}|^2 \rangle - \langle \hat{f} \rangle \langle \hat{f}^\dagger \rangle \right)$$

Because the power spectrum that will be calculated is only in our universe, the trace rule of the density matrix are used to calculate \(\langle |\hat{f}|^2 \rangle, \langle \hat{f} \rangle\) and \(\langle \hat{f}^\dagger \rangle\). Take the assumption that our universe are in bunch davies two, equation (20) become

$$\langle \Delta \delta \hat{\phi} \rangle^2 = \frac{1}{a^2} \left( tr (\rho_{BD2} |\hat{f}|^2) - tr (\rho_{BD2} \hat{f}) tr (\rho_{BD2} \hat{f}^\dagger) \right)$$

By using the density matrix in equation (7), the variation become

$$\langle \Delta \delta \hat{\phi} \rangle^2 = \int d\ln k k^3 \frac{1}{2\pi^2} \frac{1}{a^2} \left( 3(1 - 2\alpha) |f_k|^2 \right)$$

So the power spectrum is obtained as

$$P = \frac{k^3}{2\pi^2} \frac{1}{a^2} \left( 3(1 - 2\alpha) |f_k|^2 \right)$$

In the last step equation (23) was found by substituting equation (15). In the horizon exit \((\tau \to 0)\) the power spectrum can be written as

$$P = (3 - 2\alpha) \left( \frac{\mathcal{H}}{2\pi} \right)^2$$

Back again into equation (1). The general form of \(N\)-quantum state is a state that can be entangled or non-entangled depends on the quantity of the constant \(C_1, C_2, \ldots C_2^N\). To investigate the effect of \(N\)-entanglement from this power spectrum will be used the measure of entanglement.
which is called by Von Neumann entropy as a comparison. The Von-Neumann entropy of the reduced density matrix in bunch davies two is written as

$$S = -\text{tr}(\rho_{BD2} \log \rho_{BD2})$$

(25)

by substituting equation (7), the Von-Neumann entropy become

$$S = -\alpha \log \alpha - (1 - \alpha) \log (1 - \alpha)$$

(26)

to get more imagination about the result of the power spectrum and the Von Neumann entropy, equations (24) and (25) are plotted into a graph that depends on $\alpha$. From figures 1. and 2., the power spectrum and the Von Neumann entropy have a different graph. The power spectrum has a linear decreased graphic which means when $\alpha$ is equal to zero the power spectrum has the maximum value. For the Von Neumann entropy, the graph result is a negative quadratic curve with the maximum value when $\alpha$ is equal to 1/2.

4. Power Spectrum for specific state

In this section will be calculated the power for $N$-entangled specific state. As it’s mentioned before the $N$-entangled specific state are generalized of W state and GHZ state. Let’s begin with the calculation for the W state. By using the reduced density matrix in equation (10) the variational from perturbation field can be written

$$\left(\Delta \delta \phi\right)^2 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^2} \left(\frac{N-1}{N}|f_k|^2 + 3|f_k|^2\right)$$

$$= \int dlnk \frac{k^3}{2\pi^2} \frac{1}{a^2} \left(\frac{2}{N} + 1\right)|f_k|^2$$

(27)

So the power spectrum is obtained

$$\mathcal{P}_W = \frac{k^3}{2\pi^2} \frac{1}{a^2} \left(\frac{2}{N} + 1\right)|f_k|^2$$

$$= \left(\frac{2}{N} + 1\right)\left(\frac{\mathcal{H}}{2\pi}\right)^2(1 + k^2\tau^2)$$

(28)

In the horizon exit ($\tau \to 0$) the power spectrum can be written

$$\mathcal{P}_W = \left(\frac{2}{N} + 1\right)\left(\frac{\mathcal{H}}{2\pi}\right)^2$$

(29)
For generalized GHZ state, by substituting the reduced density matrix in equation (11) to the variational form of perturbation field in equation (21) is obtained

\[
(\Delta \delta \phi)^2 = \int d\ln k \frac{k^3}{2\pi^2} \frac{1}{a^3} \left(2|f_k|^2\right) \tag{30}
\]

which mean the power spectrum is evaluated as

\[
P_{\text{GHZ}} = 2\left(\frac{\mathcal{H}}{2\pi}\right)^2 (1 + k^2\tau^2) \tag{31}
\]

In the horizon exit (\(\tau \to 0\)) the power spectrum can be written

\[
P_{\text{GHZ}} = 2\left(\frac{\mathcal{H}}{2\pi}\right)^2 \tag{32}
\]

From these result, the power spectrum for GHZ state does not depend on the numbers of \(N\)-causally disconnected universe are used. It’s quite different with W state, in W state the quantity of power spectrum is inversely proportional to the amount of \(N\)-causally disconnected universes. If the numbers of \(N\)-causally disconnected universes are near to infinite the power spectrum for W state is equal to pure vacuum state or the half of power spectrum for GHZ state.

5. Conclusion

In this work, we investigated the effect of quantum entanglement between \(N\)-causally disconnected universes. Those effects can be investigated by chose the general from of \(N\)-quantum state and \(N\)-entangled specific state as the \(N\)-causally disconnected universes. To do so, we calculate the power spectrum for \(N\)-causally disconnected universes. For the general form of \(N\)-quantum state, the Von Neumann entropy was calculated to. So the effect of the entanglement for the general form of \(N\)-quantum state can be analyzed.

For the general form of \(N\)-quantum state, we got the result that the power spectrum and Von Neumann entropy have a different graph. The power spectrum has a linear decreased graphic with the maximum value at \(\alpha\) equal to zero and for the Von Neumann entropy, the graph result is a negative quadratic curve with the maximum value when \(\alpha\) is equal to 1/2. Because the Von Neumann entropy is a quantity that is used to measure the entanglement, it’s mean in generally the entanglement has no effect on the power spectrum of \(N\)-quantum state. But for the specific \(N\)-entangled state the entanglement still has an impact, because the power spectrum of W state is equal to pure vacuum state when the amount of \(N\)-causally disconnected universes is near to infinite and for GHZ state the power spectrum has same value for every \(N\)-causally disconnected universes.

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