Measurements of Earthquake Moment Magnitude (Mw) to Estimate Detonation Energy of a Nuclear Bomb as Plutonium-240

A Arimuko¹, E Sujarwanto²,³, and A Marsono¹

¹Department of Geophysics, College of Meteorology Climatology and Geophysics, Indonesia
²Department of Physics Education, Siliwangi University, Indonesia
³Doctor of Science Education Program, State University of Surabaya, Indonesia

*E-mail: abrahamarimuko98@gmail.com

Abstract. Observation of nuclear tests carried out by CTBTO utilizes seismic method. Seismic methods can be used to estimate the strength of seismic sources and seismic radiation energy emitted by seismic sources. Seismic waves originating from explosions can be found in the amount of radioactive material using atomic fission equation. From 2006 to 2017, seismic monitoring noted that North Korea conducted a nuclear test six times, namely on 9 October 2006 (mb 4.3), 25 May 2009 (mb 4.7), 12 February 2013 (mb 5.1), 6 January 2016 (mb 5.1), September 9, 2016 (mb 5.3), and September 3, 2017 (mb 6.3). The purpose of this study is to find a comparative value between seismic radiation energy and the energy of radioactive material Plutonium-240. This study used earthquakes originating from the September 3, 2017 nuclear explosion in North Korea. The waveforms used from INCN Station (Incheon, South Korea) were downloaded from IRIS. The processing of earthquake waveforms used Seismic Analysis Code (SAC) utilizing seismic radiation energy of primary wave approximately 30 seconds after the origin time. The results obtained include get moment magnitude (Mw) 6.4, seismic moment (Mo) = 6 x 10¹⁸ N.m, and seismic radiation energy (Es) = 3 x 10¹⁴ Joule. If energy Q = 4.18 x 10⁶ Joules is energy that can be produced by 1 kg of TNT then the energy released by 72 kt TNT is equal to 72 times the energy released by a nuclear explosion on spontaneous fission of Plutonium-240.

1. Introduction
One method of monitoring nuclear tests conducted at a ground level by CTBTO is using seismic methods. Seismic methods can be used to analyse seismic source locations, the origin of the time of the explosion, the magnitude of seismic events, and the seismic radiation energy emitted by seismic sources. If seismic waves originate from an explosion of nuclear tests, a comparison of the seismic radiation energy can be found. The energy of radioactive materials used by fusion or atomic fission equations generally uses the value of kilotons of TNT. Information about seismic sources can be analysed based on the recorded seismic waveforms. In special cases, such as explosions, to calculate the magnitude of an earthquake event cannot use the fault area approach because the source parameters are unknown. Waveform recordings are used to calculate how many seismic moments result from an explosion. The seismic moment is used to determine the moment magnitude (Mw).
During the period 2006 to 2019, CTBTO has detected six nuclear tests in North Korea. The results of these nuclear tests are also recorded in seismic networks in neighbor country of North Korea, such as South Korea and China. If you see the value of the magnitude of the recorded earthquake, North Korea every time you conduct a nuclear test the following year uses more radioactive substances. In 2006, North Korea conducted the first nuclear test detected after CTBTO was formed. The earthquake magnitude recorded from this first trial is mb 4.3. In 2009, North Korea conducted a second trial which produced earthquake magnitude mb 4.7. In 2013, North Korea conducted its third nuclear test by producing an earthquake magnitude of 5.1. The third trial magnitude has the same value as the fourth trial in January 6, 2016. Eight months later in the same year, North Korea conducted its fifth trial in September 9, 2016 which produced earthquake magnitude mb 5.3. Whereas in the last trial in 2017, North Korea carried out rapid nuclear development when viewed from earthquake magnitude mb 6.3 [1]. In this latest trial, it was suspected that North Korea had developed H-Bomb.

**Figure 1.** Earthquake epicenter allegedly due to North Korean nuclear tests from 2006 to 2017. This map was modified from IRIS.

2. Method
Waveform can be analyzed using the Seismic Analysis Code to obtain a moment magnitude. The factors that influence the calculation of magnitude are the distance between the hypocenter to the
station and the average density value of the medium through which seismic waves pass. The distance between the hypocenter to the station affects the average speed. The seismic moment ($M_o$) is obtained from the largest of the first peak ($P_1$) of the P-wave portion of the seismogram, or the difference between $P_1$ and the amplitude ($P_2$) of $pP$ or $sP$ ($P_1-P_2$) [2, 3]. Equation 1 become:

$$M_o = Max(|P_1|, |P_1 - P_2|) \frac{4\pi \rho \alpha^3}{F_p}$$  \hspace{1cm} (1)

where ($M_o$) is the seismic moment, $\rho$ is mean density, $\alpha$ is P-wave velocity along the propagation path, $d$ is hypocenter distance, and $F_p$ is the P-wave radiation pattern.

In the $M_{wp}$ calculation there are some variables used. CR1 and CR2 are the signal limits used in processing. $\Delta$ is the distance the hypocenter-station used to get $APV_3$, that is Average P-wave Velocity power 3.[9] All variables are used in estimating the value of using seismic moments in Seismic Code Analysis (SAC). $APV^3$ can be determined using equation 2.[9]

$$APV^3 = 0.16 \times \Delta + 7.9$$  \hspace{1cm} (2)

| Table 1. The parameters value used in the $M_{wp}$ calculation.[9] |
|---|
| **Stacion** | **Magnitude (mb)** | **Date-Month-Year** | **CR1 (s)** | **CR2 (s)** | **$\Delta$ (m)** | **$\Delta$ (deg)** | **$APV^3 (10^{11} m/s)$** |
| INCN | 6.3 | 03-09-2017 | 60 | 85 | 475,645.70 | 4.27 | 6.32438 |
| INCN | 5.3 | 09-09-2016 | 60 | 76 | 472,761.60 | 4.25 | 6.31522 |
| MDJ | 5.1 | 06-01-2016 | 60 | 70 | 371,294.70 | 3.34 | 5.99861 |
| MDJ | 5.1 | 12-02-2013 | 60 | 71.5 | 369,141.20 | 3.32 | 5.99201 |
| INCN | 4.7 | 25-05-2009 | 60 | 68 | 473,136.20 | 4.25 | 6.3164 |
| INCN | 4.3 | 09-10-2006 | 60 | 66.5 | 474,256.00 | 4.26 | 6.31996 |

Based on Kanamori, 1977, the moment magnitude ($M_w$) can be calculated if the seismic moment ($M_o$) is known [4]:

$$M_w = \frac{\log M_o - 9.1}{1.5}$$  \hspace{1cm} (3)

The moment magnitude that has been obtained is correlated with the magnitude value found in the IRIS Catalog Special Event. After the magnitude of the waveform processing approaches or equals the reference magnitude, the moment magnitude ($M_w$) is returned back to the seismic moment ($M_o$). This is done to determine the waveform time range (cutting waveform) that is processed to determine $sP$ is very difficult. This is because the source of the seismic wave is from an explosion. On the basis of reasonable average assumptions about rigidity ($\mu$) and the stress drop ($\Delta \sigma$) , (i.e., with $\Delta \sigma/\mu = $ constant), Kanamori (1977) derives the relationship in Bormann (2010).[5] So, seismic radiation energy can be determined using equation 4.

$$Es = 5 \times 10^{-5} M_o (Joule)$$  \hspace{1cm} (4)
Earthquake event indicated originating from North Korea's nuclear test [1] on September 3, 2017 shows the magnitude mb 6.3, depth 0 km, latitude 41.343 °N, longitude 129.036 °E, origin time in 03:30:01 UTC, and 22km ENE of Sungjibaegam, North Korea. The moment magnitude is used in waveform processing so it is necessary to change the magnitude body wave (mb) to the moment magnitude (Mw) using empirical equations [6]:

\[ M_w = 1.0107 \times m_b + 0.0801 \]  \hspace{1cm} (5)

Moment magnitude (Mw) is obtained 6.4 and seismic moment value is \(6 \times 10^{18}\text{N.m}\) based on processing from INCN station waveform data (Incheon, South Korea). This difference is possible because of differences in the review of seismic moment values originating from the source area (in the general fault area) and seismic wave radiation. The source of those seismic events was not a fault area. This method can be used to find seismic moments and seismic radiation energy. From fig. 3 obtained the value of seismic radiation energy is \(3 \times 10^{14}\text{Joule}\). By knowing the value of energy, we can find the comparative value between energy and the amount of radioactive substances.

Energy Q = \(4.18 \times 10^6\text{Joule}\) is energy that can be produced by 1 kg of TNT [7]. Equality is related to the average energy of 0.1 eV / atom or \(\sim 1000\text{K}\) at the core (the nuclear reactor). Mixed Plutonium up to Plutonium-240 15% can be compared to Plutonium at the weapon level [7]. It is known that Plutonium at the weapon level can reach energy equivalent to 100 kt TNT so that the results can be obtained that Mixed Plutonium with \(^{239}\text{Pu}\) 15% can be equal to 1 kt TNT.

3. Results and Discussion
Waveform processing uses the recording of INCN stations on the Global Network with IU code and waveform data taken from http://ds.iris.edu/wilber3/find_event. Data waveforms are used only on
vertical component on broadband seismograph. The processing uses a Seismic Analysis Code with scripts from the International Institute of Seismology and Earthquake Engineering[9]. The following is an example of a waveform data that was processed in the $M_w$ 6.3 September 6, 2017.

Figure 3. Waveform from INCN (Incheon, South Korea) Station is cut in the range 60 s to 85 s.

Figure 4. Moment magnitude obtained from wave processing is $M_w$ 6.4. It is according to the reference catalogue.
Figure 5. Seismic moment \((M_o)\) obtained is \(6 \times 10^{18}\) N.m from returning the moment magnitude to the seismic moment.

Figure 6. Seismic radiation energy is obtained from the comparison of the moment seismic values. It is using eq. 4. Seismic radiation energy \((E_s)\) obtained is \(3 \times 10^{14}\) Joule.
The energy measured by INCN.00 station is $3 \times 10^{14}$ Joule. If 1 kg TNT is equivalent to $4.18 \times 10^6$ joule then we can know that the estimated energy at INCN.00 station has an equivalent to 71.77 ktTNT.

In six earthquakes, the moment magnitude obtained from the P wave radiation. It is shown in the following table.

| Station | Date-Month-Year | Magnitude $(M_{wp})$ | Seismic Moment $(M_o)$ in $10^{15}$ N.m | Seismic Radiation Energy $(E_s)$ in $10^{11}$ Joule | Ratio Detonation Energy as Plutonium-240 |
|---------|-----------------|----------------------|-----------------------------------------|--------------------------------------------------|------------------------------------------|
| INCN    | 03-09-2017      | 6.4                  | 6000                                    | 3000                                             | 71.77                                    |
| INCN    | 09-09-2016      | 5.4                  | 175                                     | 87.5                                             | 2.09                                     |
| MDJ     | 06-01-2016      | 5.2                  | 120                                     | 60                                               | 1.44                                     |
| MDJ     | 12-02-2013      | 5.2                  | 120                                     | 60                                               | 1.44                                     |
| INCN    | 25-05-2009      | 4.8                  | 10.5                                    | 5.5                                              | 0.13                                     |
| INCN    | 09-10-2006      | 4.4                  | 6                                       | 3                                                | 0.07                                     |

This result was compared with the H-Bomb trial conducted by the United States on October 31, 1952. The device detonated in the Ivy Mike ("m" for "megaton") test, called the Sausage, was the first "true" H-Bomb ever tested and equivalent used 10.4 Mt TNT to mean Ivy Mike’s explosion yield is $10,400$ kt TNT.[8] So, earthquake energy $= 0.0069$ times H-Bomb Ivy Mike

The energy from Ivy Mike is not the initial energy when an explosion occurs, but rather the total energy from a nuclear explosion which is calculated based on the diameter and the time difference after the explosion. Mathematically can be written [10]

$$W = 1.294 \times 10^{-8} \rho \phi^5 \quad (6)$$

Where $W$ is total energy release in kilotons, $\rho$ is air density at burst in grams per liter, and $\phi$ is average value of diameter (meter)/time $2/5$ ($10^{-3}$ second) for all films (averaged over the region of 0.4 slope).

4. Conclusion

The energy released by 72 kt TNT is equal to 72 times the energy released by a nuclear explosion on spontaneous fission of Plutonium-240. Thus, the energy measured by the INCN station is equivalent to 72 times the energy released by a nuclear explosion on the spontaneous side of Plutonium-240. The H-Bomb trial apparently still had more energy used by the United States than North Korea, but according to records the earthquake due to the Ivy Mike trial had not been found or even none. This shows that the trials carried out by North Korea are more efficient based on energy on the effects caused, such as earthquakes.

The availability of waveform data due to a nuclear explosion at IDC can be processed as in the result above so that it can be seen how much radioactive material used is dangerous for the survival of humanity on earth or monitoring nuclear power plant uses seismograph.

Author Contributions

All authors contributed equally to this work. All authors have reviewed the final version of the manuscript and approved it for publication.
Acknowledgements
Thank for Kenji Kanjo and Kuninori Okamoto of the International Institute of Seismology and Earthquake Engineering for distributing digital signal processing materials using Seismic Code Analysis and for IRIS who provide the data.

References
[1] IRIS 2017 Special Event: 2017 North Korean nuclear test. 
https://ds.iris.edu/ds/nodes/dmc/specialevents/2017/09/03/2017-north-korean-nuclear-test/
[2] Tsuboi S, Abe K, Takano K, and Yamanaka Y 1995 Bull. Seismol. Soc. Am.85 606.
[3] Tsuboi S, WhitmoreP M, and SokolowskiT J 1999 Bull. Seismol. Soc. Am.89 1345
[4] Kanamori H 1977 J. Geophys. Res.82 2981.
[5] Bormann P and Giacomo D D 2011 J. Seismol.15 411.
[6] Pusat Studi Gempa Nasional 2017 “Konversi Magnitudo” in Peta Sumber dan Bahaya Gempa Indonesia Tahun 2017 1 Ed. (Jakarta: Puslitbang PUPR) 90
[7] Şahin S and Ligou J 1980 Nucl. Technol.50 88
[8] Anonim 1999 Ivy Mike http://nuclearweaponarchive.org/Usa/Tests/Ivy.html
[9] Kanjo K & Okamoto K 2008 Practice of Seismic Analysis Code ISEE Lecture Note (Iran: International Institute of Seismology and Earthquake Engineering) 8
[10] Anomin 1954 Operation Ivy: Pacific Proving Grounds (Washington DC: Defense Atomic Support Agency) 20301