D-Brane Actions with Local Kappa Symmetry

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Abstract

We formulate world-volume actions that describe the dynamics of Dirichlet $p$-branes in a flat 10d background. The fields in these theories consist of the 10d superspace coordinates $(X^m, \theta)$ and an abelian world-volume gauge field $A_\mu$. The global symmetries are given by the N=2A or N=2B super-Poincaré group, according to whether $p$ is even or odd. The local symmetries in the $(p + 1)$-dimensional world volume are general coordinate invariance and a fermionic kappa symmetry.

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Introduction

During the past year (following the contribution of Polchinski [1]) the important role played by D-branes in non-perturbative superstring physics has become apparent. Many of their remarkable properties have been elucidated [2, 3, 4]. In particular, they have provided a powerful tool for the study of black holes in string theory [5]. Very recently an interesting proposal for understanding non-perturbative 11d (M theory) physics in terms of ensembles of D 0-branes has been put forward [6]. For all these reasons it is desirable to achieve as thorough an understanding of D-branes as possible. One issue that has not been explored as thoroughly for D-branes as it has been for more traditional super p-branes (without world-volume gauge fields) is the covariant formulation of the world-volume action. A crucial ingredient in such actions is a local fermionic symmetry of the world volume theory called “kappa symmetry.” It was first identified by Siegel [7] for the superparticle [8], and subsequently applied to the superstring [9]. Next it was simplified (to eliminate an unnecessary vector index) and applied to a super 3-brane in 6d [10]. Then came the super 2-brane in eleven dimensions [11], followed by all super p-branes (without world-volume gauge fields) [12].

In the case of D-branes, most studies have focused on their bosonic degrees of freedom and the coupling to bosonic background fields [13, 14]. Also, some studies have worked in a physical gauge without describing the more symmetrical gauge-invariant formulas from which they arise. Interesting as all of these studies are, there is a subtle and beautiful structure in the fermionic sector that they do not address.

The main distinction between D-branes and the previously studied super p-branes is that the field content of the world-volume theory includes an abelian vector gauge field \( A_\mu \) in addition to the superspace coordinates \( (X^m, \theta) \) of the ambient \( d \)-dimensional space-time. In the case of super p-branes whose only degrees of freedom are \( (X^m, \theta) \), \( (p+1) \)-dimensional actions have been formulated that have super-Poincaré symmetry in \( d \) dimensions realized as a global symmetry. In addition they have world-volume general coordinate invariance, which ensures that only the transverse components of \( X^m \) are physical, and a local fermionic kappa symmetry, which effectively eliminates half of the components of \( \theta \). This symmetry reflects the fact that the presence of the brane breaks half of the supersymmetry in \( d \) dimensions, so that half of it is realized linearly and half of it nonlinearly in the world-volume theory. The physical fermions of the world-volume theory correspond to the Goldstinos associated
to the broken supersymmetries.

The purpose of this paper is to present formulas for D-brane actions with local kappa symmetry analogous to those of the super $p$-branes. For this purpose the ambient space-time dimension is restricted to $d = 10$ throughout. Explicit Dirichlet $p$-brane actions, with all the appropriate symmetries, will be presented for all values of $p$ ($p = 0, 1, \ldots, 9$). We know from the rank of RR gauge fields that when $p$ is even the supersymmetry should be IIA and when $p$ is odd it should be IIB. In a physical gauge $X^m$ gives rise to $9 - p$ degrees of freedom and $A_\mu$ gives $p - 1$ of them, for a total of 8 bosonic modes. The 32 $\theta$ coordinates are cut in half by kappa symmetry and in half again by the equation of motion, so they give rise to 8 fermionic degrees of freedom.

One case, namely $p = 2$, has been studied previously. As noted in [13], the super 2-brane action in 11d can be converted to the D 2-brane in 10d by performing a duality transformation in the world volume theory that replaces one of the $X$ coordinates by a world-volume vector. This has been worked out in detail by Townsend [16], and some of his formulas have given us guidance in generalizing to all $p$. (See also [17].) One technical detail that aids the analysis is the following: We do not introduce an auxiliary world-volume metric field in the formulas. They have been included in most studies of super $p$-branes, though this was not necessary. If one attempted to incorporate them in the D-brane formulas, this would create considerable algebraic complications.

Conventions

Our conventions are the following. $X^m, m = 0, 1, \ldots, 9$, denotes the 10d space-time coordinates and $\Gamma^m$ are $32 \times 32$ Dirac matrices appropriate to 10d with

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn}, \text{ where } \eta = (- + \ldots +).$$

(1)

(These $\Gamma$’s differ by a factor of $i$ from those of ref. [18].) The Grassmann coordinates $\theta$ are space-time spinors and world-volume scalars. When $p$ is even $\theta$ is Majorana but not Weyl. It can be decomposed as $\theta = \theta_1 + \theta_2$, where

$$\theta_1 = \frac{1}{2}(1 + \Gamma_{11})\theta, \quad \theta_2 = \frac{1}{2}(1 - \Gamma_{11})\theta.$$  

(2)

These are Majorana–Weyl spinors of opposite chirality. When $p$ is odd there are two Majorana–Weyl spinors $\theta_\alpha (\alpha = 1, 2)$ of the same chirality. The index $\alpha$ will not be displayed explicitly. The group that naturally acts on it is $\text{SL}(2,\mathbb{R})$, whose generators we denote by
Pauli matrices $\tau_1, \tau_3$. (We will mostly avoid using $i\tau_2$, which corresponds to the compact generator.) With these conventions the supersymmetry transformations (for all $p$) are given by

$$\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon X^m = \epsilon \Gamma^m \theta. \quad (3)$$

World-volume coordinates are denoted $\sigma^\mu$, $\mu = 0, 1, \ldots, p$. The world-volume theory is supposed to have global IIA or IIB super-Poincaré symmetry. This is achieved by constructing it out of three supersymmetric quantities. Besides $\partial_\mu \theta$, they are

$$\Pi^m_\mu = \partial_\mu X^m - \bar{\theta} \Gamma^m \partial_\mu \theta, \quad (4)$$

and

$$F_{\mu\nu} = F_{\mu\nu} - b_{\mu\nu}, \quad (5)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $b_{\mu\nu}$ will be defined later. Another useful quantity is the induced world-volume metric

$$G_{\mu\nu} = \eta_{mn} \Pi^m_\mu \Pi^n_\nu. \quad (6)$$

The Action

As in the case of super $p$-branes, the world-volume theory of a D-brane is given by a sum of two terms $S = S_1 + S_2$. The first term

$$S_1 = - \int d^{p+1} \sigma \sqrt{-\det(G_{\mu\nu} + F_{\mu\nu})} \quad (7)$$

is essentially an amalgam of the Born–Infeld and Nambu–Goto formulas. The second term

$$S_2 = \int \Omega_{p+1}, \quad (8)$$

where $\Omega_{p+1}$ is a $(p+1)$-form, is a Wess–Zumino-type term. $S_1$ and $S_2$ are separately invariant under the global IIA or IIB super-Poincaré group as well as under $(p+1)$-dimensional general coordinate transformations. However, local kappa symmetry will be achieved by a subtle conspiracy between them, just as in the case of super $p$-branes.

Under local kappa symmetry the variation $\delta \theta$ will be restricted in a way that will be determined later. In addition, we require that (whatever $\delta \theta$ is)

$$\delta X^m = \bar{\theta} \Gamma^m \delta \theta, \quad (9)$$

just as for super $p$-branes. It follows that

$$\delta \Pi^m_\mu = -2 \bar{\theta} \Gamma^m \partial_\mu \theta. \quad (10)$$
Another useful definition is the “induced $\gamma$ matrix”

$$\gamma_\mu \equiv \Pi^m_\mu \Gamma_m.$$  \hfill (11)

Note that $\{\gamma_\mu, \gamma_\nu\} = 2G_{\mu\nu}$. These formulas imply that

$$\delta G_{\mu\nu} = -2\delta \bar{\theta}(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu)\theta.$$  \hfill (12)

The structure of $F_{\mu\nu}$ is most easily described in terms of the 2-form $F = \frac{1}{2} F_{\mu\nu} d\sigma^\mu d\sigma^\nu$. Then $F = F - b$, and the appropriate choice of $b$ turns out to be

$$b = -\bar{\theta} \Gamma_{11} \Gamma_m d\theta \left( dX^m + \frac{1}{2} \bar{\theta} \Gamma^m d\theta \right).$$  \hfill (13)

This is the formula for $p$ even. When $p$ is odd, $\Gamma_{11}$ is replaced by $\tau_3$. For this choice

$$\delta \epsilon b = -\bar{\epsilon} \Gamma_{11} \Gamma_m d\theta \left( dX^m + \frac{1}{2} \bar{\theta} \Gamma^m d\theta \right) + \frac{1}{2} \bar{\theta} \Gamma_{11} \Gamma_m d\theta \epsilon \Gamma^m d\theta.$$  \hfill (14)

Then $\delta \epsilon F = 0$ if we take

$$\delta \epsilon A = \bar{\epsilon} \Gamma_{11} \Gamma_m d\theta dX^m + \frac{1}{6}(\bar{\epsilon} \Gamma_{11} \Gamma_m \bar{\theta} \Gamma^m d\theta + \bar{\epsilon} \Gamma_m \theta \Gamma_{11} \Gamma^m d\theta).$$  \hfill (15)

The fundamental identity used to prove this, valid for any three Majorana–Weyl spinors $\lambda_1, \lambda_2, \lambda_3$ of the same chirality, is

$$\Gamma_m \lambda_1 \bar{\lambda}_2 \Gamma^m \lambda_3 + \Gamma_m \lambda_2 \bar{\lambda}_3 \Gamma^m \lambda_1 + \Gamma_m \lambda_3 \bar{\lambda}_1 \Gamma^m \lambda_2 = 0.$$  \hfill (16)

This formula is valid regardless of whether each of the $\lambda$’s is an even element or an odd element of the Grassmann algebra. (Note that $\theta$ is odd and $d\theta = d\sigma^\mu \partial_\mu \theta = -\partial_\mu \theta d\sigma^\mu$ is even.) The variation of $F$ under a kappa transformation is

$$\delta F = 2\delta \bar{\theta} \Gamma_{11} \Gamma_m d\theta \Pi^m,$$  \hfill (17)

for $p$ even (and $\Gamma_{11} \to \tau_3$ for $p$ odd) provided that we decree

$$\delta A = -\delta \bar{\theta} \Gamma_{11} \Gamma_m d\theta \Pi^m + \frac{1}{2} \delta \bar{\theta} \Gamma_{11} \Gamma_m \bar{\theta} \Gamma^m d\theta - \frac{1}{2} \delta \bar{\theta} \Gamma^m \theta \bar{\theta} \Gamma_{11} \Gamma_m d\theta.$$  \hfill (18)

**Determination of $S_2$**

Now let’s consider a kappa transformation of $S_1$. Inserting the variations $\delta G_{\mu\nu}$ and $\delta F_{\mu\nu}$ given above

$$\delta \left( -\sqrt{-\det(G + F)} \right) = -\frac{1}{2} \sqrt{-\det(G + F)} \text{tr}[(G + F)^{-1} (\delta G + \delta F)].$$

4
\[
= 2\sqrt{-\det(G + \mathcal{F})}\delta \bar{\theta} \gamma_\mu \{(G - \mathcal{F} \Gamma_{11})^{-1}\}^{\mu \nu} \partial_\nu \theta.
\] (19)

For \( p \) odd \( \Gamma_{11} \) is replaced this time by \( -\tau_3 \) (since it has been moved past \( \gamma_\mu \)). Now the key step is to rewrite this in the form

\[
\delta L_1 = 2\delta \bar{\theta} \gamma^{(p)} T^\nu_{(p)} \partial_\nu \theta,
\] (20)

where

\[
(\gamma^{(p)})^2 = 1.
\] (21)

It is not at all obvious that this is possible. The proof that it is, and the simultaneous determination of \( \gamma^{(p)} \) and \( T^\nu_{(p)} \) is the key to the whole problem. (The details of the proof will be given elsewhere [19].) Assuming that this is okay, we require that

\[
\delta L_2 = 2\delta \bar{\theta} T^\nu_{(p)} \partial_\nu \theta,
\] (22)

so that

\[
\delta (L_1 + L_2) = 2\delta \bar{\theta} (1 + \gamma^{(p)}) T^\nu_{(p)} \partial_\nu \theta.
\] (23)

Then \( \delta \bar{\theta} = \bar{\kappa} \), where \( \bar{\kappa}(1 + \gamma^{(p)}) = 0 \), gives the desired symmetry.

It is very useful to define

\[
\rho^{(p)} = \sqrt{-\det(G + \mathcal{F})}\gamma^{(p)},
\] (24)

which satisfies

\[
(\rho^{(p)})^2 = -\det(G + \mathcal{F}),
\] (25)

and to represent it by

\[
\rho^{(p)} = \epsilon^{\mu_1 \mu_2 \ldots \mu_{p+1}} \rho_{\mu_1 \mu_2 \ldots \mu_{p+1}};
\] (26)

or by a \((p + 1)\)-form

\[
\rho_{p+1} = \rho_{\mu_1 \mu_2 \ldots \mu_{p+1}} (p + 1)! \sigma^{\mu_1} \sigma^{\mu_2} \ldots \sigma^{\mu_{p+1}}.
\] (27)

The requirement

\[
\sqrt{-\det(G + \mathcal{F})} \gamma_\mu \{(G - \mathcal{F} \Gamma_{11})^{-1}\}^{\mu \nu} = \gamma^{(p)} T^\nu_{(p)}
\] (28)

can then be recast in the more convenient form

\[
\rho^{(p)} \gamma_\mu = T^\nu_{(p)}(G - \mathcal{F} \Gamma_{11})_{\nu \mu}.
\] (29)

Writing

\[
T^\nu_{(p)} = \frac{\epsilon^{\nu_1 \nu_2 \ldots \nu_p \nu}}{p!} T_{\nu_1 \nu_2 \ldots \nu_p},
\] (30)
In this notation, the kappa variation of $S_2$ takes the form
\[ \delta S_2 = 2(-1)^{p+1} \int \delta \bar{\theta} T_p d\theta = \delta \int \Omega_{p+1}. \] (32)

It is convenient to characterize $S_2$ by a $(p+2)$-form $I_{p+2} = d\Omega_{p+1}$. The preceding formula implies that
\[ I_{p+2} = (-1)^{p+1} d\bar{\theta} T_p d\theta, \] (33)
provided that we can show that
\[ d\bar{\theta} \delta T_p d\theta + 2\delta \bar{\theta} dT_p d\theta = 0. \] (34)

A corollary of this identity is the closure condition $dI_{p+2} = d\bar{\theta} T_p d\theta = 0$.

Let us now present the solution of eqs. (25) and (29) first for the case of $p$ even. For this purpose we define the matrix-valued one-form
\[ \psi = \gamma_\mu d\sigma^\mu = \Pi^m \Gamma_m, \] (35)
and introduce the following formal sums of differential forms (the subscript $A$ denotes IIA)
\[ \rho_A = \sum_{p=even} \rho_{p+1} \quad \text{and} \quad T_A = \sum_{p=even} T_p. \] (36)

Then the solution of eqs. (25) and (29) is described by the formulas
\[ \rho_A = e^F S_A(\psi) \quad \text{and} \quad T_A = e^F C_A(\psi) \] (37)
where
\[ S_A(\psi) = \Gamma_{11} \psi + \frac{1}{3!} \psi^3 + \frac{1}{5!} \Gamma_{11} \psi^5 + \frac{1}{7!} \psi^7 + \ldots \] (38)
\[ C_A(\psi) = \Gamma_{11} + \frac{1}{2!} \psi^2 + \frac{1}{4!} \Gamma_{11} \psi^4 + \frac{1}{6!} \psi^6 + \ldots \] (39)

Thus, $\rho_1 = \Gamma_{11} \psi$, $\rho_3 = \frac{1}{6} \psi^3 + \mathcal{F} \Gamma_{11} \psi$, etc., and $T_0 = \Gamma_{11}$, $T_2 = \frac{1}{2} \psi^2 + \mathcal{F} \Gamma_{11}$, etc. The fact that $T_0 \neq 0$ means that $S_2 \neq 0$ for the D 0-brane. The significant difference from the superparticle of [8] is that the D 0-brane is massive, whereas the superparticle was massless.

The proof that these expressions for $\rho_A$ and $T_A$ satisfy eq. (25) uses the fact that the two terms on the right-hand side of the equation correspond to $\frac{1}{2} \{ \rho^{(p)}, \gamma_\mu \}$ and $\frac{1}{2} [\rho^{(p)}, \gamma_\mu]$. Using eq. (16) one can also show that this expression for $T_A$ satisfies eq. (34).
The solution for $p$ odd is very similar. In this case we define (the subscript $B$ denotes IIB)

$$\rho_B = \sum_{p=\text{odd}} \rho_{p+1} \quad \text{and} \quad T_B = \sum_{p=\text{odd}} T_p. \quad (40)$$

The solution is given by

$$\rho_B = e^F C_B(\psi) \tau_1 \quad \text{and} \quad T_B = e^F S_B(\psi) \tau_1, \quad (41)$$

where

$$S_B(\psi) = \psi + \frac{1}{3!} \tau_3 \psi^3 + \frac{1}{5!} \psi^5 + \frac{1}{7!} \tau_3 \psi^7 + \ldots \quad (42)$$

$$C_B(\psi) = \tau_3 + \frac{1}{2!} \psi^2 + \frac{1}{4!} \tau_3 \psi^4 + \frac{1}{6!} \psi^6 + \ldots \quad (43)$$

Thus $\rho_2 = \frac{1}{2} \tau_1 \psi^2 + i \tau_2 F$, $\rho_4 = \frac{1}{2} i \tau_2 \psi^4 + \frac{1}{2} \tau_1 F \psi^2 + \frac{1}{2} i \tau_2 F^2$, etc., and $T_1 = \tau_1 \psi$, $T_3 = \frac{1}{6} i \tau_2 \psi^3 + \tau_1 F \psi$, etc. The quantity $\rho_0 = i \tau_2$ may be relevant to the D-instanton, which we are not considering here.

**Conclusion**

This brief letter has not presented proofs of three key identities — eqs. (25), (29) and (34). We plan to write a longer paper that includes the details of those proofs, as well as some additional results [19]. Other issues to explore include the extension of the formulas to non-trivial space-time backgrounds and the fixing of a physical gauge. A more challenging problem is the formulation of generalizations appropriate to the description of multiple D-branes.

As this work was nearing completion, a paper was posted that gives the kappa invariant action for the D 3-brane [20]. In fact, it also describes the coupling to background fields.

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