Supersymmetric Duality in Deformed Superloop Space

Mir Faizal\textsuperscript{1} and Tsou Sheung Tsun\textsuperscript{2}
\textsuperscript{1}Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
\textsuperscript{2}Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter, Woodstock Road, Oxford OX2 6GG, United Kingdom

Abstract

In this paper, we will analyse the superloop space formalism for a four dimensional supersymmetric Yang-Mills theory in deformed superspace. We will deform the $\mathcal{N} = 1$ superspace by imposing non-anticommutativity. This non-anticommutative deformation of the superspace will break half the supersymmetry of the original theory. So, this theory will have $\mathcal{N} = 1/2$ supersymmetry. We will analyse the superloop space duality for this deformed supersymmetric Yang-Mills theory using the $\mathcal{N} = 1/2$ superspace formalism. We will demonstrate that the sources in the original theory will become monopoles in the dual theory, and the monopoles in the original theory will become sources in the dual theory.

1 Introduction

It has been observed that certain string theory effects can lead to a noncommutative deformation of field theories \cite{1}-\cite{3}. Such noncommutative deformation of ordinary field theories has motivated the study of non-anticommutative deformation of supersymmetric field theories \cite{5}-\cite{6}. The non-anticommutative deformation of supersymmetric gauge theories has also been studied \cite{7}-\cite{8}. In this deformation, the Grassmann coordinate of a superspace are promoted to non-anticommutating coordinates. Thus, this deformation breaks the supersymmetry corresponding to those Grassmann variables which are promoted to non-anticommutating coordinates. It is possible to break half the supersymmetry of a four dimensional theory with $\mathcal{N} = 1$ supersymmetry. In fact, this deformation has been used for constructing a four dimensional theory with $\mathcal{N} = 1/2$ supersymmetry \cite{9}-\cite{10}. It is also possible to using this deformation to break the supersymmetry of a three dimensional theory. As a three dimensional theory with $\mathcal{N} = 1$ supersymmetry has only two Grassmann coordinates, this deformation will break all the supersymmetry of a three dimensional theory with $\mathcal{N} = 1$ supersymmetry. However, it is possible to retain some supersymmetry for a three dimensional theory with $\mathcal{N} = 2$. It has been demonstrated
that the non-anticommutativity can be used to break the supersymmetry of a three dimensional theory from $N = 2$ supersymmetry to $N = 1$ supersymmetry [11]-[12]. In this paper we will analyse the superloop space duality using this $N = 1/2$ superspace formalism. This duality is motivated by the loop space duality for ordinary gauge theories. The loop space duality is in turn motivated by the Hodge duality for abelian gauge theories.

There is a duality between the electric and magnetic fields that can be constructed using the Hodge star operation. This duality relies on the fact that the field equation for pure electrodynamics can be interpreted as the Bianchi identity for a dual tensor. This dual tensor can be constructed in terms of a dual potential. This duality has been used for analysing various topological concepts inherent in field theories [13]-[16]. This duality has also been used for analysing many interesting physical phenomena [17]-[24]. It is also known that the the existence of magnetic monopoles is equivalent to the quantization of the electric. This is in turn is related to the fact that the electromagnetic gauge group is a compact group [25]. It is not possible to directly generalize this duality to non-abelian gauge theories. This is because the field tensor for a non-abelian gauge theories is defined as $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + ig[A_\mu, A_\nu]$. Now using a covariant derivative, which is defined as $D_\mu = \partial_\mu - igA_\mu$, it is possible to write $D^\nu F_{\mu\nu} = 0$, and the Bianchi identity, $D^\nu * F_{\mu\nu} = 0$. However, unlike the abelian case, this does not imply the existence of a dual potential. This is because the covariant derivative in the Bianchi identity involves the potential $A_\mu$ and not some dual potential $\tilde{A}_\mu$ corresponding to $*F_{\mu\nu} = 0$. However, it is possible to generalize the Hodge duality to non-abelian gauge theories using the loop space formalism [26]. It has also been demonstrated that this loop space duality reduces to the Hodge duality for an abelian gauge theory [27]-[28].

This loop space duality has been used for analysing aspects of the ‘t Hooft’s order-disorder parameters [29]-[31]. Such a dual potential can be constructed using the loop space formalism. The existence of this dual potential has also motivated the construction of a Dualized Standard Model [32]-[36]. The model has been used for analyzing the off-diagonal elements of the CKM matrix [37], and the the difference of masses between different generations of fermions [38]-[39]. This model has also been used for studying the Neutrino oscillations [40], and the Lepton transmutations [41].

The loop space formalism is constructed using the Polyakov loops [42]. These loops are expressed as the holonomies of closed loops in space-time. They has also been called as the Dirac phase factors in the the physics literature, and they do not depend on the parameterization chosen. These Polyakov loops are gauge group-valued functions of the infinite-dimensional loop space. So, no trace is taken over the gauge group. This is what makes the Polyakov loops different from Wilson loops, as in the Wilson loop a trace is taken over the gauge group [42]. Thus, unlike the Wilsons loops, the Polyakov loops are elements of the gauge group. The Wilsons loops for super-Yang-Mills theory with $N = 4$ supersymmetry has been analysed using the superspace formalism [43]. Furthermore, the Polyakov loops for three and four dimensional supersymmetric Yang-Mills theories with $N = 1$ supersymmetry have also been studied [44]-[45]. The superloop space duality has also been studied in $N = 1$ superspace [46]. In this paper, we will construct such superloops for deformed superspace. Then we will analyse the superloop space duality for the gauge theories using this formalism.
2 Deformed Superloop Space

In this section, we will analyse a four dimensional gauge theory in $\mathcal{N} = 1/2$ superspace formalism. Let us start by defining the coordinates of the superspace as $(x^\mu, \theta^a, \bar{\theta}^\dot{a})$. Here $\mu = 0, 1, 2, 3$, and $a, \dot{a} = 1, 2$. The non-anticommutative deformation can be performed by promoting the Grassmann coordinate $\theta^a$ to a non-anticommutating variables, such that

$$\{ \theta^a, \theta^b \} = C_{ab}. \quad (1)$$

Here the product of superfields of $\theta^a$ is Weyl ordered by replacing the ordinary product of superfields on the deformed superspace by a star product. This star product is the fermionic version of the Moyal product. Thus, for two supervector product of superfields on the deformed superspace by a star product. This star deformation can be performed by promoting the Grassmann coordinate $\bar{\theta}^\dot{a}$ as

$$\{ \bar{\theta}^\dot{a}, \theta_a \} = 0, \quad \{ \bar{\theta}^\dot{a}, \bar{\theta}^{\dot{a}} \} = 0, \quad [\bar{\theta}^\dot{a}, x^\mu] = 0. \quad (3)$$

However, we have

$$[x^\mu, x^\nu] = \bar{\theta} \delta^{\mu\nu} C_{ab}, \quad [x^\mu, \theta^a] = i C_{ab} \sigma^\mu_{ab} \bar{\theta}^\dot{a}, \quad (4)$$

where $C_{\mu\nu} = C_{ab} \epsilon_{bd}(a^{\mu\nu})^d_a$. Now we can define $y^\mu \equiv x^\mu + i \theta^a \sigma^\mu_{ab} \bar{\theta}^\dot{a}$, and obtain

$$[\theta_a, y^\mu] = 0, \quad [\bar{\theta}^{\dot{a}}, y^\mu] = 0, \quad [y^\mu, y^\nu] = 0. \quad (5)$$

Thus, we can take the superfields to be functions of $(y^\mu, \theta^a, \bar{\theta}^\dot{a})$. Now we can write a supervector field $V(y, \theta, \bar{\theta})$ in the Wess-Zumino gauge as

$$V(y, \theta, \bar{\theta}) = -\theta \sigma^\mu \bar{\theta} A_\mu + i \theta \theta \bar{\bar{\theta}} \bar{\lambda} - i \bar{\theta} \theta \bar{\theta} \theta \left( A_\mu + \frac{1}{4} \epsilon_{ab} \sigma^\mu_{ab} [\bar{\lambda}^d, A_\mu] \right) + \frac{1}{2} \bar{\theta} \theta \bar{\theta} \bar{\theta} \theta (D - i \partial_a A^\mu). \quad (6)$$

Here we have defined $V^A(y, \theta, \bar{\theta})T_A = V(y, \theta, \bar{\theta})$, with $[T_A, T_B] = i f_{ABC}^T T_C$. The Chiral and anti-Chiral field strength for the gauge theory are defined to be $4W_a = -D \bar{D} e^{-V}_a \star D_a e^V$ and $4\bar{W}_a = D \bar{D} e^{-V}_a \star \bar{D}_a e^V$, respectively. Now the Lagrangian for the deformed gauge theory can be written as

$$\mathcal{L} = Tr \int d^2 \theta W^a \ast W_a + Tr \int d^2 \bar{\theta} \bar{W}^a \ast \bar{W}_a. \quad (7)$$

In component form this can be written as

$$\mathcal{L} = Tr \left( -4i \bar{\lambda} \sigma^\mu D_\mu \lambda - F^{\mu\nu} \bar{F}_{\mu\nu} + 2D^2 \right) + Tr \left( -2i C^{\mu\nu} \bar{F}_{\mu\nu} \lambda \bar{\lambda} + \frac{C^{\mu\nu} C_{\mu\nu}}{2} (\lambda \bar{\lambda})^2 \right). \quad (8)$$
It is also possible to express this deformed four dimensional supergauge theory using covariant derivative defined as \[ 47 \]

\[
\nabla_A = (-i[D_A, D_a], D_a, D_a), \nonumber
\]

\[
\exp(V)_* \star \nabla_A \star \exp(-V)_* = (-i[D_A, D_a], D_a, D_a), \quad (9)
\]

where \( D_a = \exp(-V)_* \star D_a \exp(V)_* \) and \( D_a = \exp(V)_* \star D_a \exp(-V)_* \). It is also possible to express this covariant derivative as \( \nabla_A = D_A - i\Gamma_A \). Here the superspace derivative \( D_A \) is defined by \( D_A = (\partial_{\theta a}, D_a, D_a) \) and the superspace connection \( \Gamma_A \) is defined by \( \Gamma_A = (\Gamma_{a\dot{a}}, \Gamma_a, \Gamma_a) \). We can define \( H_{AB} = [\nabla_A, \nabla_B]_* = T^{CB}_{AB}\nabla_C - iF_{AB} \), and then the Bianchi identity will be written as \( [\nabla_A, H_{BC}]_* = 0 \).

The covariant derivative transforms under gauge transformation as \( \nabla_A \rightarrow e^{i\Lambda}_* \star \nabla_A \star e^{-i\Lambda} \), and \( \bar{e_v} \star \nabla_A \star e^{-v} \rightarrow e^{i\Lambda}_* \star e_v \star \nabla_A \star e^{-v} \star e^{-i\Lambda} \). It is possible to use another representation in which the covariant derivatives transform under gauge transformations as \( \nabla_A \rightarrow u \star \nabla_A \star u^{-1} \) \[ 47 \]. Here we have defined \( u = e^K \), where parameter \( K = K^A T_A \) is a real superfield. Now the transformation of the spinor fields can be expressed as \( \Gamma_a \rightarrow iu \star \nabla_a \star u^{-1}, \Gamma_a \rightarrow iu \star \nabla_a \star u^{-1} \), and \( \Gamma_{a\dot{a}} \rightarrow iu \star \nabla_{a\dot{a}} \star u^{-1} \).

Now we can derive the duality for this deformed superspace. This can be done by first using the conventional constraints as \( F_{a\dot{a}} = F_{ab} = F_{a\dot{b}} = 0 \). As the super-connection is defined by \( \Gamma_A = (\Gamma_{a\dot{a}}, \Gamma_a, \Gamma_a) \), we can parameterize the superloop by \( \xi(s) = (\sigma^a \xi_a(s))^{a\dot{a}} \theta_{\dot{a}} a + \xi^a(s) \theta_{a} + \bar{\xi}^a(s) \theta_{\dot{a}} \), and so we can write \( \xi^A = (\xi^{a\dot{a}}, \xi^a, \bar{\xi}^a) \) \[ 47 \]. It may be noted that for higher dimensional theories, and for theories with higher amount of supersymmetry we will have to choose a different parameterization. Now we can parameterized the superloop along a curve \( C \) as

\[
C : \{ \xi^A(s) : s = 0 \rightarrow 2\pi, \xi^A(0) = \xi^A(2\pi) \}, \nonumber
\]

where \( \xi^A(0) = \xi^A(2\pi) \) is a fixed point on this curve \[ 45 \]. We can now define the superloop variable for the deformed superspace as

\[
\Phi[\xi] = P_* \exp i \int_0^{2\pi} \bigg[ \Gamma^{a\dot{a}}(\xi(s)) \frac{d\xi_{a\dot{a}}(s)}{ds} + \Gamma^{a}(\xi(s)) \frac{d\xi_{a}(s)}{ds} \bigg]_* + \Gamma^{\bar{a}}(\xi(s)) \frac{d\xi_{\bar{a}}(s)}{ds} \bigg]_* \nonumber
\]

\[
= P_* \exp i \int_0^{2\pi} \bigg[ \Gamma^{\bar{A}}(\xi(s)) \frac{d\xi_{\bar{A}}(s)}{ds} \bigg]_* . \quad (11)
\]

where all the products are taken as star products. Furthermore, \( P_* \) denotes ordering in \( s \). Here this ordering is increasing from right to left. The derivative in \( s \) is taken from below. This superloop space is a scale superfield on the deformed superspace from the supersymmetric point of view. Thus, it has \( N = 1/2 \) supersymmetry.

The parallel transport between two points, \( \xi(s_1) \) and \( \xi(s_2) \), can be defined as

\[
\Phi[\xi : s_1, s_2] = P_* \exp i \int_{s_1}^{s_2} \bigg[ \Gamma^{a\dot{a}}(\xi(s)) \frac{d\xi_{a\dot{a}}(s)}{ds} + \Gamma^{a}(\xi(s)) \frac{d\xi_{a}(s)}{ds} \bigg]_* + \Gamma^{\bar{a}}(\xi(s)) \frac{d\xi_{\bar{a}}(s)}{ds} \bigg]_* \nonumber
\]
as the superloop operations have been performed. Now we can define the loop on which they can operate, so this limit can only be taken after all the $\epsilon$ limit involving $E$.

Thus, we can construct $H$ a point and along a path till we reach the point $\xi$. We can define a functional curl and a functional divergence.

It is possible to write the duality using loop space formalism for ordinary non-abelian gauge theories [27]-[28]. Here we will generalize this duality to deformed superspace. In order to that we will first analyze function curl and divergence of a superloop variable. We can define a functional curl and a functional divergence as

$$
\text{curl } F[\xi(s)]_{AB} = \delta_A(s) F_B[\xi(s)] - \delta_B(s) F_A[\xi(s)],
$$

$$
\text{div } F[\xi(s)] = \delta_A(s) F_A[\xi(s)].
$$

As the superloop variables are highly redundant, we need to constrained them by an infinite set of conditions. These can be expressed by the vanishing of the superloop space curvature [15], $G_{AB}[\xi, s] = (\text{curl } F[\xi(s)]_{AB} + i[F_A[\xi(s)], F_B[\xi(s)]]_s) = 0$. We can also define $-iG_{AB}[\xi, s]$ as a commutator of two covariant superloop derivatives, $[\nabla_A[\xi(s)], \nabla_B[\xi(s)], \omega]$, where $\nabla_A[\xi(s)] = \delta_A(s) - iF_A[\xi(s)]$.

It is possible to construct $E_A[\xi(s)]$ from $F_A[\xi(s)]$, $E_A[\xi(s)] = \Phi[\xi : s, 0] \star F_A[\xi(s)] \star \Phi^{-1}[\xi : s, 0]$. Thus, we can construct $E_A[\xi(s)]$ from $F_A[\xi(s)]$ using parallel transport. Now as the $E_A[\xi(s)]$ only depends on a segment of the loop $\xi(s)$ around $s$, it is a segmental variable rather than a full superloop variable. Now as the integrals involving $E_C[\xi(s)]$ depends will depend on the a little segment from $s_-$ to $s_+$, so limit $\epsilon \to 0$ can only be taken only after integration. Here we have defined $\epsilon = s_+ - s_-$. As segment can shrinks to a point, and we can write $E_A[\xi(s)] \to H^{AB}(\xi(s)) \star dE_B(s)/ds$. In fact, all the loop operations require a segment of the loop on which they can operate, so this limit can only be taken after all the superloop operations have been performed. Now we can define

$$
\text{curl } E[\xi(s)]_{AB} = \delta_A(s) E_B[\xi(s)] - \delta_B(s) E_A[\xi(s)],
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3 Deformed Superloop Space Duality

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$$

$$
\text{div } E[\xi(s)] = \delta_A(s) E_A[\xi(s)].
$$
Thus, we obtain
\[
\delta_A(s')E_B[\xi|s] = \Phi[\xi : s, 0] \ast \left[ \delta_A(s') F_B[\xi|s] + i \Theta(s - s') [F_A[\xi|s, F_B[\xi|s|s]] \ast \Phi^{-1}[\xi : s, 0] \right],
\]
where \(i \Theta(s - s')\) is the Heaviside function. So, the superloop space curvature can now be written as \(G_{AB}[\xi, s] = \Phi[\xi : s, 0] \ast (\text{curle}[\xi|s])_{AB} \ast \Phi^{-1}[\xi : s, 0]\) and thus the constraints can be fixed as \((\text{curle}[\xi|s])_{AB} = 0\).

Now we can define a new variable \(\tilde{E}_A[\eta|t]\) which is dual to \(E_A[\xi|s]\). Now using \(\eta(t)\) as another parameter superloop, we can write
\[
\omega^{-1}[\eta(t)] \ast \tilde{E}^A[\eta|t] \ast \omega[\eta(t)] = -\frac{2}{N} e^{ABC} d\eta_B(t) \ast \int D\xi ds \delta(\xi(s) - \eta(t)) \ast E_C[\xi|s] \ast \frac{d\xi_D(s)}{ds} \ast \left[ \frac{d\xi_F(s)}{ds} \ast \frac{d\xi_F(s)}{ds} \right]^{-2},
\]
where \(N\) is a normalization constant. Here the local rotational matrix is denoted by \(\omega[\eta(t)]\). This corresponded to transforming the quantities from a direct frame to the dual frame. The loop derivative of \(\tilde{E}^A[\eta|t]\) can be calculated by using the fact that \(\tilde{E}^A[\eta|t]\) is a segmental quantity. It depends on a segment from \(t_{-}\) to \(t_{+}\). Here again we take the limit \(\epsilon' \to 0\) only after all the superloop operations have been performed. Here we have defined \(\epsilon' = t - t_{-}\). We also have \(\epsilon' < \epsilon\). The \(\delta\)-function ensures the variable \(\xi(s)\) coincides with \(\eta(t)\) from \(s = t_{-}\) to \(s = t_{+}\). As the segment shrinks to a point, and we obtain \(E^A[\eta|t] \to \tilde{H}^{AB}(\eta(t)) d\eta_B(t)/dt\).

It may be noted that we can define the gauge transformation of \(E_A[\xi|s]\) and \(\tilde{E}^A[\eta|t]\) as
\[
E_A[\xi|s] = [1 + i \Lambda[\xi|s]] \ast E_A[\xi|s] \ast [1 - i \bar{\Lambda}[\eta|t]],
\]
\[
\tilde{E}^A[\eta|t] = [1 + i \bar{\Lambda}[\eta|t]] \ast \tilde{E}^A[\eta|t] \ast [1 - i \Lambda[\xi|s]].
\]
Here the gauge parameters \(\Lambda[\xi|s]\) and \(\bar{\Lambda}[\eta|t]\) have zero loop derivatives. Here the dual quantity \(\tilde{H}^{AB}\) can be constructed from a direct potential, where \(\bar{\Gamma}_A = (\bar{\Gamma}_{a\bar{a}}, \bar{\Gamma}_{a\bar{B}}, \bar{\Gamma}_{a\bar{B}}) = \bar{H}_{AB}\). Here we have defined \(\bar{\nabla}_A = D_A - i \bar{\Gamma}_A\). The dual covariant derivative transforms under gauge transformation as \(\bar{\nabla}_A \to \hat{u} \ast \bar{\nabla}_A \ast \hat{u}^{-1}\). Here we have defined \(\hat{u} = e^{i K}\) where parameter \(K\) is a real superfield. Now the transformation of the spinor fields can be expressed as \(\bar{\Gamma}_A \to i \hat{u} \ast \bar{\nabla}_A \ast \hat{u}^{-1}\), \(\bar{\nabla}_a \to i \hat{u} \ast \bar{\nabla}_a \ast \hat{u}^{-1}\) and \(\bar{\Gamma}_{a\bar{a}} \to i \hat{u} \ast \bar{\nabla}_{a\bar{a}} \ast \hat{u}^{-1}\). It is also possible to use a different representation under which the dual covariant derivative transform as \(\bar{\nabla}_A \to e^{i \lambda} \ast \hat{\nabla}_A \ast e^{-i \lambda}\) and \(e^{i \lambda} \ast \hat{\nabla}_a \ast e^{-i \lambda} \to e^{i \lambda} \ast e^{i \lambda} \ast \hat{\nabla}_A \ast e^{-i \lambda} \ast e^{-i \lambda}\).

We can again define the dual covariant derivative in terms of a dual superloop field \(\hat{V}\)
\[
\hat{\nabla}_A = (-i \{\hat{D}_a, D_a\} \ast \hat{D}_a, D_a),
\]
\[
\exp(\hat{V}) \ast \hat{\nabla}_A \ast \exp(-\hat{V}) \ast = (-i \{D_a, \hat{D}_a\} \ast D_a, \hat{D}_a),
\]
where \(\hat{D}_a = \exp(-\hat{V}) \ast D_a \exp(\hat{V})\) and \(\hat{D}_a = \exp(\hat{V}) \ast D_a \exp(-\hat{V})\). Now the dual superloop field \(V(y, \theta, \bar{\theta})\) can be written as
\[
\hat{V}(y, \theta, \bar{\theta}) = -\theta \sigma^a \bar{\theta} \bar{\lambda}_a + i \theta \bar{\theta} \bar{\lambda}_a - i \bar{\theta} \theta \lambda_a + \frac{1}{4} \epsilon_{abcd} \sigma^a \sigma^b \lambda^c \lambda^d, \bar{A}_a\).
where \( \zeta \) superspace coordinates. Thus, for two dual supervector fields \( V(y, \theta, \bar{\theta}) \), and \( \bar{V}^\dagger(y, \theta, \bar{\theta}) \), we have

\[
\bar{V}(y, \theta, \bar{\theta}) \star \bar{V}^\dagger(y, \theta, \bar{\theta}) = V(y, \theta, \bar{\theta}) \exp \left( -\frac{C^{ab}}{2} \frac{\partial}{\partial \theta^a} \frac{\partial}{\partial \bar{\theta}^b} \right) \bar{V}^\dagger(y, \theta, \bar{\theta}).
\]

(22)

It may be noted that the dual supervector potential is also a function of deformed superspace coordinates. Thus, for two dual supervector fields \( \tilde{V} \), and \( \bar{V}'(y, \theta, \bar{\theta}) \), we have

\[
\tilde{V}(y, \theta, \bar{\theta}) \star \bar{V}'(y, \theta, \bar{\theta}) = \tilde{V}(y, \theta, \bar{\theta}) \exp \left( -\frac{C^{ab}}{2} \frac{\partial}{\partial \theta^a} \frac{\partial}{\partial \bar{\theta}^b} \right) \bar{V}'(y, \theta, \bar{\theta}).
\]

It is also possible to define the Chiral and anti-Chiral field strength for the dual theory as \( 4\tilde{W}_u = -DDe^{-\tilde{V}} \star D_a e^\nu_u \) and \( 4\bar{W}_u = DD_e^{-\bar{V}} \star D_a e^{\nu}_u \), respectively.

4 Application of Duality

In this section, we will demonstrate that the sources of the ordinary theory becomes monopoles in the dual theory, and the monopoles in the dual theory become sources in the ordinary theory. Before doing that we note that this duality reduces to an ordinary superloop space duality if we neglect the effect of noncommutativity \[46\]. Furthermore, for if for the non-supersymmetric case, this reduces to the ordinary loop space duality. Thus, if we use \( \Phi[\xi] = \phi[\xi] \) as the loop space variable, then we can obtain \( E_\mu[\xi] \) from \( F_\mu[\xi] \), where \( F_\mu[\xi] \) is the loop space connection corresponding to loop variable \( \Phi[\xi] \). Now in absence of non-anticommutative deformation, we can construct the dual variable to the usual loop space variable \( \bar{E}_\mu[\xi] \) as \( \bar{E}_\mu[\xi] = \frac{\delta}{\delta \xi^\mu}[\xi] \). Then in the limit in which the width of \( \bar{E}_\mu[t] \) going to zero, we can show that \( \bar{F}_{\mu\nu}[x] = -\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}[x]/2 \). Thus, the usual Hodge star operation can be obtained by identifying \( F_{\mu\nu} \) with \( \bar{F}_{\mu\nu} \). It may be noted that the usual loop space variable can be used for analysing the ’t Hooft’s order-disorder parameters \[42\]. These order-disorder parameters can be constructed by using two spatial loops \( C \) and \( C' \) with the linking number \( n \) between them. Here \( su(N) \) is the which is used for this analysis. The magnetic flux through \( C \) is measured by \( A(C) \), and the electric flux through \( C \) is measured by \( B(C) \). The order-disorder parameters are defined as \( A(C)B(C') = B(C')A(C)\exp(2\pi in/N) \). Now \( (C) \) is expressed using the potential \( \tilde{A}_\mu \) and \( B(C) \) is expressed using the dual potential \( \bar{A}_\mu \) \[30\] - \[31\].

Now we will demonstrate that the duality transformation is invertible. This can be done by writing a duality transformation for \( E^A[\zeta][u] \) as,

\[
\omega^{-1} \ast [\zeta(u)]E^A[\zeta][u] \ast \omega[\zeta(u)] = \frac{2}{N}e^{ABC\bar{D}} \frac{d\zeta_B(u)}{du} \ast \int D\eta dt \tilde{E}_C[\eta][t] \ast \frac{dn_D(t)}{dt}
\]

\[
\ast \left[ \frac{dn_F(t)}{dt} \ast \frac{dn_F(t)}{dt} \right]^{-2} \delta(\eta(t) - \zeta(u)),
\]

(23)

where \( \zeta_B(u) \) is a new loop parameterized by \( u \). So, we can write \( A^4[\zeta(u)] \) as

\[
A^4[\zeta(u)] = \frac{2}{N}e^{ABC\bar{D}} \frac{d\xi_B(u)}{du} \ast \int D\eta dt \omega^{-1}[\eta][t] \ast \tilde{E}_C[\eta][t] \ast \omega[\eta(t)]
\]

\[
\ast \frac{dn_D(t)}{dt} \ast \left[ \frac{dn_F(t)}{dt} \ast \frac{dn_F(t)}{dt} \right]^{-2} \delta(\eta(t) - \zeta(u))
\]

7
Now we can write

Thus, by identifying

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the original theory will appear as the magnetic monopole in the dual t heory, and (curl ˜

monopole in the original theory should appear as the source term in t he dual

too.

The source term in the deformed supersymmetric Yang-Mills theory can be defined as \( \nabla C \times H_{BC} \neq 0 \) and \( \text{div} F[\xi|[s] \neq 0 \). As we have \( \text{div} E[\xi|[s] = \Phi[\xi : s_1, 0] \times \text{div} F[\xi|[s] \neq 0 \), so the source term can also be defined as \( \text{div} E[\xi|[s] \neq 0 \). Similarly, as the monopole can be defined as \( G_{AB}[\xi|[s] \neq 0 \), and \( (\text{curl} E[\xi|[s])_{ab} \neq 0 \). Now the monopole in the dual theory is characterized by \( (\text{curl} E[\eta|[t])_{ab} \neq 0 \), and the source in the dual theory is characterized by \( \text{div} E[\eta|[t] \neq 0 \). We require that under the duality transformation, the source in the original theory will appear as the magnetic monopole in the dual theory, so \( \text{div} E[\xi|[s] \neq 0 \) should imply \( (\text{curl} E[\eta|[t])_{ab} \neq 0 \). We also require that the monopole in the original theory should appear as the source term in the dual theory, so \( (\text{curl} E[\xi|[s])_{ab} \neq 0 \) should imply \( \text{div} E[\eta|[t] \neq 0 \). We can use the fact that \( \eta(t) \) coincides with \( \xi(s) \) from \( s = t_- \) to \( s = t_+ \), and write

\[
\frac{\delta}{\delta \eta_M(t)} \times \left( \omega^{-1}[\eta(t)] \times E^A[\eta|[t] \times \omega[\eta|[t] \right) \epsilon_{MANP}
\]

\[
= - \frac{2}{N} \epsilon^{ABCD} \frac{d\eta_B}{dt} \times \int D\xi ds \delta E_C[\xi|[s] \times \frac{d\xi_D}{ds} \times \frac{d\xi_C}{ds} \times \frac{d\xi_B}{ds} \times \frac{d\eta_A}{dt} \times \delta(\xi(s) - \eta(t)) \epsilon_{MANP}. \tag{26}
\]

Thus, we obtain

\[
\left( \omega^{-1}[\eta|[t] \times (\text{curl} E[\eta|[t])_{AB} \times \omega[\eta|[t] \right)
\]

\[
= - \frac{1}{N} \int D\xi ds \left[ \frac{d\eta^C(t)}{dt} \times \frac{d\xi^D(s)}{ds} - \frac{d\eta^D(t)}{dt} \times \frac{d\xi^C(s)}{ds} \right] \epsilon_{ABCD} \times \text{div} E[\xi|[s] \times \left[ \frac{d\xi^F(s)}{ds} \times \frac{d\eta^F(s)}{ds} \right] \times \delta(\xi(s) - \eta(t)). \tag{27}
\]

Thus, if \( \text{div} E[\xi|[s] = 0 \), then \( (\text{curl} E[\eta|[t])_{ab} = 0 \). As the duality is invertible, we can also demonstrate that if \( \text{div} E[\xi|[s] = 0 \), then \( (\text{curl} E[\eta|[t])_{ab} = 0 \). Thus,
The sources in the original theory become monopoles in the dual theory, and the monopoles in the dual theory become sources in the original theory. It may be noted that we have analysed the sources and monopoles in both original and dual superloop theories. As both the supervector field and the dual supervector field are defined on the deformed superspace, both the original theory and the dual theory will have $\mathcal{N} = 1/2$ supersymmetry.

5 Conclusion

In this paper, we have analysed a deformed four dimensional supersymmetric Yang-Mills theory using superloop space. The deformation broke half the supersymmetry of the original theory. Thus, as the original theory had $\mathcal{N} = 1$ supersymmetry, the theory after the deformation only has $\mathcal{N} = 1/2$ supersymmetry. We obtained the loop space variables for this deformed super-Yang-Mills theory in this deformed superspace. Thus, we obtained a generalized of the ordinary superloop space in four dimensions. This deformed superloop space was used for constructing a duality which reduced to the ordinary loop space duality in absence of supersymmetry. Thus, for an abelian gauge theory without any supersymmetry, this duality reduced to Hodge duality. We demonstrated that under this duality the monopoles in the original theory became sources in the dual theory, and the sources in the original theory became monopoles in the dual theory.

The loop space duality for ordinary Yang-Mills theory has been used for studying various interesting physical phenomena [31]-[39]. It will be interesting to use the deformed superloop space duality constructed here, for analysing similar phenomena in the deformed supersymmetric theories. Thus, we can construct a deformed supersymmetric Dualized Standard Model. This deformed supersymmetric Dualized Standard Model will have $\mathcal{N} = 1/2$ supersymmetry. The phenomenological consequences of this model can also be studied. It will be interesting to analyse the ABJM theory using this deformed superloop formalism [48]. Furthermore, it will also be interesting to study the effect of monopoles in the ABJM theory using this formalism. It may be noted that it is expected that the supersymmetry of the ABJM will get enhanced due to monopole [49]-[50]. It may be noted that the loop space formalism for the ABJM theory has already been constructed [51]. It will be interesting to study these effects in the formalism developed in this paper.

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