Equation of state for neutron stars. Some recent developments

P Haensel and M Fortin
N. Copernicus Astronomical Center, Bartycka 18, 00-716 Warszawa, Poland
E-mail: haensel@camk.edu.pl

Abstract. Calculations using the chiral effective field theory (ChEFT) indicate that the four-body force contribution to the equation of state (EOS) of pure neutron matter (PNM) at the nuclear density $n_0$ is negligibly small. However, the overall uncertainty in the EOS of PNM at $n_0$ remains $\sim 20\%$. Relativistic mean field (RMF) calculations with in-medium scaling, and including hyperons and $\Delta$ resonances, can be made consistent with recent nuclear and astrophysical constraints. Dirac-Brueckner-Hartree-Fock calculations with some medium dependence of the nuclear interaction yield neutron star (NS) models with hyperonic cores consistent with $2M_\odot$ stars and agreeing with the saturation parameters of nuclear matter. Many unified EOS for the NS crust and core were calculated, and are reviewed here. The effect of the finite size of baryons on the EOS, its treatment via the excluded-volume approximation, and its relevance for the hypothetical hybrid-star twins at $\sim 2M_\odot$ are discussed.

1. Introduction
The equation of state is necessary for constructing NS models. In particular, the EOS determines the maximum allowable mass for NS, $M_{\text{max}}[\text{EOS}]$. The discovery of two radio pulsars with $M \approx 2M_\odot$ [1, 2] yields a very important constraint: $M_{\text{max}}[\text{EOS}] > 2M_\odot$. This means that the nuclear forces acting between nucleons (or more generally - hadrons) push up $M_{\text{max}}$ by a factor of at least $\sim 3$ compared to the maximum allowable mass of $0.7M_\odot$ for the EOS of a free Fermi gas of neutrons [3].

A convenient unit for the NS matter baryon (number) density $n_b$ is the standard nuclear density $n_0 = 0.16$ fm$^{-3}$, corresponding to the mass-energy density unit $\varepsilon_0/c^2 = 2.8 \times 10^{14}$ g cm$^{-3}$. Matter inside NS is strongly degenerate, and therefore the pressure depends only on the density, $P = P(\varepsilon)$ or equivalently $P = P(n_b)$.

The theoretical calculation of the $P(\varepsilon)$ relation for NS is a challenge, because one is dealing with a strongly interacting, dense many-particle system. For $\varepsilon \ll \frac{1}{2} \varepsilon_0$, corresponding to the NS crust, nuclear theory methods are still valid, but some uncertainties persist. For NS cores one expects $\frac{1}{2} \varepsilon_0 < \varepsilon < 10 \varepsilon_0$, with theoretical uncertainties growing rapidly with $\varepsilon$. Does the deconfinement of quarks occur inside the cores of massive NS? The answer cannot be given by the theory alone, because even at $10 \varepsilon_0$ we are dealing with a non-perturbative QCD regime. Fortunately, NS exist and are available for observations! For $\varepsilon \gtrsim 100 \varepsilon_0$ reliable perturbative QCD calculations are possible [4], but this is very far from the $\sim 10 \varepsilon_0$ expected at the center of the most massive NS.

In the present paper we review some calculations of the EOS of NS done in the last five years. In section 2 we review calculations based on ChEFT. Section 3 is devoted to RMF models, with
Figure 1. Energy per nucleon calculated from the nucleon $2N+3N$ force obtained using ChEFT in the $\nu = 4$ approximation. Two methods for the solution of the $A$-body problem were applied [7]. The blue band results from uncertainties in the experimental constants in the contact forces, the variation of the $\Lambda$-cutoff, and errors in the numerical solution of the many-body problem. Based on results of [7, 8].

a scaling of masses and/or coupling constants in the dense NS medium. EOS with hyperons obtained using the relativistic theory of baryon matter with a one-boson-exchange interaction, is described in section 4. In section 5 new unified EOS for the NS crust and core are reviewed. In section 6 we describe a model dealing with the effect of the finite size of baryons on the EOS and its relevance to hypothetical quark-core twins with $M \simeq 2M_\odot$. Conclusions are summarized in section 7.

2. ChEFT and the EOS

ChEFT is used since the 1990s to construct two-body and many-body nuclear forces, within a diagrammatic scheme, guided by chiral symmetry, ordered by a power counting, and with a regularization of the momentum integrations (review in [5]). It considers pion-exchange-mediated long and intermediate range components of the force, while short-range components are represented by contact (zero-range) interactions. The strengths of the contact forces are determined experimentally (fitting scattering data and few-nucleon bound states).

The nuclear interaction operator $\hat{V}$ acting within an $A$-nucleon system is the sum of contributions of diagrams ordered by powers of the parameter $Q/\Lambda$, where $Q$ is the nucleon ($N$) momentum scale within the system and $\Lambda$ is the cut-off in the momentum space needed to regularize the integrations over momenta. For a few-$N$ systems one may put $Q \simeq m_\pi c/\hbar = 0.7$ fm$^{-1}$ and $\Lambda \simeq 500$ MeV/$(\hbar c) = 2.5$ fm$^{-1}$. Then $Q/\Lambda \sim 1/3$ and ordering by $(Q/\Lambda)\nu$ looks promising. Let the nuclear force calculated at an assumed $\Lambda$ be $\hat{V}(\Lambda)$ and $Q/\Lambda$ be small. Then the sum of the $\nu = 0$ and $\nu = 2$ terms is a reasonable approximation; it reminds the effective range theory approximation for the $2N$ scattering (the $\nu = 1$ contribution vanishes due to parity conservation). Notice that $\hat{V}$ is not an observable, but it is used to calculate the observables. The $\nu = 0, 2, 3$ diagrams give a complete $2N$-force, but they yield an incomplete $3N$-force, while adding $(Q/\Lambda)^4$ diagrams completes the $3N$ force and the $4N$ force. In this way one obtains $\hat{V} \simeq \hat{V}_{2N}(\Lambda) + \hat{V}_{3N}(\Lambda) + \hat{V}_{4N}(\Lambda)$. One has still to calculate the ground state energy of the $A$-body system under consideration. The simplest case relevant for NS is pure neutron matter (PNM). At $n_b = n_0$ the $4N$ forces contribute only $-0.2$ MeV to the energy per nucleon $E$ (e.g., [6]), and therefore the $2N + 3N$ approximation seems excellent (this was already known since long time for $A = 4$). However, uncertainties in $\hat{V}(\Lambda)$ and in the $A$-body calculations of $E$ grow quite rapidly with increasing $n_b$ (figure 1). The best calculations for PNM give $E(n_0)$ and $P(n_0)$ with a 20% and 30% uncertainty, respectively. Viewing that $Q$ grows with density, and this is
Figure 2. EOS and $M(R)$ for the RMF models with in-medium scaling of the hadron coupling constants and/or masses. Dotted segments in the EOS (left panel) are above central density of stable NS. Dashed segments in the right panel connect NS models with same $n_c$, whose value is indicated. See text for details.

accompanied with a decreasing precision of the $A$-body calculation of $E$, the ChEFT does not seem promising for the precise calculation of the EOS of the NS core.

3. RMF models with scaled masses and couplings

Within the RMF model baryon ($B$) fields are coupled to meson ($M$) fields, and their coupling constants and masses $g_{BM}$, $m_B$, and $m_M$, respectively, enter the effective Lagrangian density $\mathcal{L}$. The EOS is then calculated using static solutions of the field equations, obtained in the mean-field approximation.

Two different approaches can be chosen to reach consistency of an RMF model with nuclear and astrophysical constraints. The first approach assumes a minimal structure of $\mathcal{L}$, with only quadratic terms in the vector meson fields $\omega^\mu, \phi^\mu, \rho^i_\mu$ (the index $i$ reflects the isovector nature of $\rho$), the equations of motion for the vector fields are linear in the corresponding source-densities (baryon density, isospin density, strangeness density). Therefore, what one really had to solve was a highly nonlinear equation of motion for $f$ only. In the static case under consideration it is $\partial E[n_n, n_p, f]/\partial f = 0$. The EOS and the resulting $M-R$ relation for a purely nucleonic core is plotted in figure 2 (label N). The extension to include the hyperons $H$ (label $H_0\sigma$ in figure 2) assumed also a scaling with $f$, using experimental potential wells $U_H$ for hyperons in symmetric nuclear matter at $n_0$ to get $g_{\sigma H}$ at $n_0$, and applying the SU(6) symmetry for the
vector-mesons coupling to $H$. The vector meson field $\phi^\mu$, coupled to the $H$ only, was added to get an additional repulsion between $H$. The radius of $1.0 - 2.0 \, M_\odot$ NS is $\sim 12 \, \text{km}$. Noticeable is also the very small decrease of $M_{\text{max}}$ due to the presence of a hyperon core at highest masses. All these features are due to the fact that hyperons are hardly present, even for $M > 2 \, M_\odot$, in NS cores.

It has been suggested that the presence of $\Delta$ particles (resonances) in NS cores could strongly soften the EOS, leading to the $M_{\text{max}} < 2 \, M_\odot$ ($\Delta$ paradox [11]). The extension of the medium-scaled RMF model to the NH$\Delta$ plasma has shown that for a sufficiently deep potential well for $\Delta$ in symmetric nuclear matter ($-100 < U_\Delta < -50 \, \text{MeV}$), the decrease of $M_{\text{max}}$ caused by the $\Delta$s is very small, albeit with $\Delta^-$ appearing already in $1.0 \, M_\odot$ NS (in figure 2 curves $\Delta$ and $H\phi\sigma\Delta$ when hyperons are included, obtained for $U_\Delta = -100 \, \text{MeV}$). On the other hand, the radius $R$ of $1.0 - 2.0 \, M_\odot$ NS shrinks due to the $\Delta$ softening of the EOS by some $0.5 \, \text{km}$. Summarizing, the EOS presented in [10] and references therein yield $M_{\text{max}} \approx 2.3 \, M_\odot$ with $H$ and $\Delta$ in NS cores, and produce $R \approx 12 \, \text{km}$ for $1.0 - 2.0 \, M_\odot$ NS.

It is worth to mention new extensions of older RMF models with density-dependent $g_{NM}$ [12] to NH cores (DD2 H version of the purely nucleonic DD2 model in figure 2). It is argued there that the Lorentz invariance of $\mathcal{L}$ was respected, because $n_\mu = \sqrt{j_b^\mu j_B^\mu}$, where $j_b^\mu$ is the total baryon four-current $j_b^\mu = \sum_B j_B^\mu$. However, this is true only if all baryon species have the same macroscopic (bulk) motion.

4. Hyperon cores in the DBHF theory

The relativistic extension of the Brueckner-Hartree-Fock (BHF) theory of nuclear matter (Dirac-BHF = DBHF) is applied in [13] to calculate the EOS, starting from one-boson-exchange (OBE) interactions known as Bonn OBE models. The relativistic Bonn OBE $NN$ interactions fit a few thousand of $NN$ data (i.e., phase shifts, $^2\text{H}$) equally well as the (non-relativistic) Argonne or Urbana NN potentials. The numerical calculations reported in [13] were a real tour de force: the complexity of the DBHF theory is much higher than that of the BHF one. To get agreement with saturation parameters of nuclear matter the Bonn OBE coupling constant $g_{N\sigma}$ was weakly scaled with the scalar component of the mass operator $g_{N\sigma} \rightarrow g_{N\sigma}(\Sigma_N)$ (this was an analogue of the $g_{N\sigma}$ scaling with $\sigma$ in [9, 10]). The required enhancement of $g_{N\sigma}$ is of $2\%$ at $n_0$. This minute scaling yields a Bonn $A^*$ interaction with the correct saturation parameters of nuclear matter. The $3N$ forces needed not to be added to the $2N$ ones because their repulsive effect was reproduced by the density dependence of the vector component of the mass operator, $\Sigma_N$).
This yielded also a universal repulsion between the baryons. The generalization of the Bonn $A^*$ to the NH matter was done using the SU(6) symmetry for the coupling of the vector mesons to hyperons while the fitting to $U_H$ was used to get $g_{H\sigma}$. The $M(R)$ curves for different versions of the Bonn $A^*$ interaction are shown in figure 3. Results are only weakly dependent on the number of hyperon species and meson species included. The central density at $M_{\text{max}}$ is only about $4n_0$, while $M_{\text{max}} \approx 2.03 - 2.08 \, M_\odot$ (see figure 3). The radius for $1.0 - 1.9 \, M_\odot$ is $R > 13.5 \, \text{km}$.

5. New unified EOS for crust and core

The new unified EOS are based on the same nuclear interaction model for both core and the inner crust. Therefore, these EOS give a physically meaningful transition between the crust and the liquid core. This is important for the correct determination of $R$ and the crustal mass and moment of inertia. The many-body problem for the inner crust is solved using the energy density functional (EDF) method (overview, e.g., in [14]). For the uniform n-p-e plasma of the outer core the energy per nucleon becomes a function of the nucleon densities $E(n_n, n_p) = E_{\text{bulk}}^N(n_n, n_p) + E_e(n_e)$. For the inner crust the EDF method allows for the calculation of the non-uniform nucleon structure that minimizes $E$. In this way, additional "finite size" terms resulting from the surface tension ($E_{\text{surf}}^N$), the spin-orbit coupling ($E_{\text{s-o}}^N$), and the Coulomb interaction ($E_{\text{Coul}}$), appear in the expression for $E$, apart from the bulk (size-independent) and the electron contributions: $E = E_{\text{bulk}}^N + E_{\text{surf}}^N + E_{\text{s-o}}^N + E_{\text{Coul}} + E_e(n_e)$.

The recent BCPM (Barcelona-Catania-Paris-Madrid) EDF [15] is based on a realistic potential $v_{12} A_{18}$ model supplemented with an effective density-dependent potential $\tilde{v}_{12}$ resulting from the averaging of the 3N UIX potential over the third nucleon coordinate. Similarly as the UIX 3N, $\tilde{v}_{12}$ contains two phenomenological parameters, $c_1$ and $c_2$, whose values are adjusted to get correct saturation parameters of symmetric nuclear matter: $E_s = -16.0 \, \text{MeV}$ and $n_s = 0.16 \, \text{fm}^{-3}$. Then three additional phenomenological parameters ($c_3, c_4, c_5$) appear in $E_{\text{surf}}^N$, and the sixth one, $c_6$, measures the strength of $E_{\text{s-o}}^N$. The fitting of 467 known spherical nuclei yielded an optimal set of $c_3, c_4, c_5, c_6$, with a r.m.s. deviation with respect to experimental binding energies $\sigma_{\text{Bind}} = 1.3 \, \text{MeV}$. The BCPM EOS is very similar to the DH one [16], yields a very similar $M(R)$ relation, but the $Z$ of the proton clusters in the inner crust is $\sim 30$ instead of $\sim 40$ for the DH model. Moreover, so-called pasta layers composed of nonspherical nuclear clusters (see e.g., [17]) are present in the bottom sector of the inner BCPM crust, while there...
Figure 5. Left panel: Nucleon (N) and quark (Q) EOS from [20]. $\Phi_{\text{NR}}$ makes DD2-EV extremely stiff. Right panel: Based on unpublished results of J.L. Zdunik (2015). See text for details.

were no pastas in the DH crust. These differences are most probably due to different $E_{\text{surf}}^N$, which were obtained using different methods in DH and BCPM models.

Nine new unified EOS based on the RMF models were derived in [18] and four of them shown to be consistent with a set of modern constraints both from nuclear experiments and theoretical calculations. The presence of pasta layers in the inner crust was conditioned by the model-dependent surface energy, $E_{\text{surf}}^N$, which should not be too high. Otherwise, the crust with spherical proton clusters undergoes direct phase transition to the uniform n-p-e plasma of the core (like for DH).

The $P(\varepsilon)$ and $M(R)$ relations for new unified EOS are shown in figure 4, where we limited ourselves to NS with nucleon cores. Actually, some new RMF EOS could be extended by including the hyperons, and keeping $M_{\text{max}} > 2 M_{\odot}$ [18]. We illustrate the still persisting uncertainties in the EOS of the inner crust and outer core ($n_b < 2 n_0$) in figure 4. The apparent convergence of the RMF EOS and non-relativistic EOS at $10\varepsilon_0$ results actually from the crossings of the RMF and non-relativistic EOS lines. The crossings are due to $(dP/d\varepsilon)_{\text{RMF}} \rightarrow 1$ (from below) for $\varepsilon \gg \varepsilon_0$, while no such asymptotic behaviour of $P(\varepsilon)$ is exhibited by the non-relativistic models.

6. Finite size of baryons and quark-core twins

In contrast to quarks and leptons, baryons have a finite size. In spite of this fact, they are usually treated in the many-body theories as point-like objects. Their finite size can be accounted for in the excluded volume (ExVol) approach, where they are treated as non-penetrable spheres of radius $r_N (N = n, p)$. The ExVol construction can be consistently implemented within the RMF model of baryons and mesons (for review, see [19]). Let us denote the volume fraction available for baryons by $\Phi$. Then the Fermi momenta of nucleons moving within the volume available to them are calculated from $k_{FN} = (3\pi^2 n_N/\Phi)^{1/3}$. Treating nucleons non-relativistically, we get $\Phi_{\text{NR}} = 1 - n_b v$, where $v$ is the excluded volume per nucleon. Problems appear when we require the ExVol construction to keep $L$ a Lorentz scalar. We define $v$, as well as $r_N$, in the rest-frame of nuclear matter. It is easy to show that the excluded volume per nucleon (measured in the matter rest-frame) is $v = \frac{4\pi^2}{3} (2r_N)^3 = 4V_N$. As nucleons are moving, the nucleon volume measured in the (macroscopic) matter rest-frame depends on its speed. This is why the correct Lorentz invariant available volume fraction is $\Phi = 1 - n_b^{(s)} v$, where $n_b^{(s)}$ is the scalar density of...
nucleons, the source for the scalar field.

Benić et al. [20] started with the RMF DD2 model with density-dependent coupling constants [12]. Then they implemented a non-relativistic ExVol scheme to get DD2-EV, still fitting the same nuclear matter saturation parameters. Their non-relativistic $\Phi_{NR}$ with large $v = 2.86 \text{ fm}^3$ corresponds to $r_N = 0.55 \text{ fm}$ and this means a hard-core radius of 1.1 fm. Consequently, $\Phi_{NR} = 0$ at $n_{\text{crit}} = 0.35 \text{ fm}^{-3}$ (see right panel of figure 5), $P(n_{\text{crit}}) \rightarrow +\infty$. Therefore in this model the quark deconfinement $N \rightarrow Q$ has to occur before the $n_{\text{crit}}$ singularity. This (not-so-fine) tuning resulted in the 1st order $N \rightarrow Q$ phase transition at $P_{\text{NQ}}$, with a large jump in $\varepsilon$, $\varepsilon_Q/\varepsilon_N > \frac{3}{2}(1 + P_{\text{NQ}}/\varepsilon_N)$. Consequently, an unstable segment of $M(R)$ separated N-stars from more compact NQ-stars, twins with $M \approx 2 \text{ M}_\odot$, see left panel of figure 5. It is the use of $\Phi_{NR}$ that makes DD2-EV extremely stiff. Using physical $\Phi$ would make the EOS softer and then the twin-NQ stars would most probably go away.

7. Conclusions

Progress in calculating the NS EOS is very slow. The precise EOS for PNM for $n_b < n_0$ has still not be obtained, except for $n_b \ll n_0$. On the positive side, ChEFT calculations show that the 4BF contribution at $n_0$ is negligibly small. There seems to be no hope to calculate precise EOS using ChEFT for $n_b > n_0$ because of the very slow convergence at supra-nuclear densities. Many new unified EOS are now available; some EOS-dispersion in the inner crust and in the outer NS core is present. The scaling of the in-medium effective nuclear Lagrangian with scalar quantities makes EOS more flexible, and allows for the H-puzzle and the $\Delta$-puzzle to be solved, keeping agreement with nuclear constraints. Complete DBHF calculations of the EOS including hyperons were performed, and $M_{\text{max}} > 2.0 \text{ M}_\odot$ was obtained. However, to reproduce the saturation parameters of nuclear matter, an in-medium scaling of the $\sigma$ meson coupling to nucleons had to be introduced. The relativistic version of the excluded-volume approximation of the baryon size effect on the EOS is still to be performed. This is in particular still needed in the context of the hypothetical hybrid-twins at $M \sim 2 \text{ M}_\odot$.

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