The specific charged hadron multiplicity in $e^-+p$ and $e^-+D$ semi-inclusive deep-inelastic scattering in the PYTHIA and PACIAE models

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We employed the PYTHIA 6.4 model and the extended parton and hadron cascade model PACIAE 2.2 to comparatively investigate the DIS normalized specific charged hadron multiplicity in the 27.6 GeV electron semi-inclusive deep-inelastic scattering off proton and deuteron. The PYTHIA and PACIAE results calculated with default model parameters not well and fairly well reproduce the corresponding HERMES data, respectively. In addition, we have discussed the effects of the differences between the PYTHIA and PACIAE models.

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I. INTRODUCTION

Since the eighties of last century the lepton inclusive and semi-inclusive deep inelastic scattering (DIS and SIDIS) off nuclear target have become one of the most active frontiers between the nuclear and particle physics. They have greatly contributed to the partonic structure of hadron \cite{1}, the parametrization of parton distribution function (PDF) \cite{2}, and the nuclear medium effect on PDF (EMC effect) \cite{3}. They also play important role in the hadronization of initial partonic state, the space-time evolution of the fragmentation process, and the extraction of polarization-averaged fragmentation function (FF) \cite{4,5}.

The multiplicity data of specific charged hadron ($\pi^+$, $\pi^-$, $K^+$, $K^-$) in the unpolarized SIDIS are crucial for distinguishing the quark fragmentation function of $D_h^q$ from the antiquark one of $D_h^{\bar{q}}$. Thus those multiplicity data are important for a reliable extraction of the FF. Recently, the HERMES collaboration has measured the charged pions and kaons multiplicity in the 27.6 GeV electron SIDIS off proton and deuteron \cite{6}. Meanwhile, they have compared their DIS normalized data to the HERMES Lund Monte Carlo (HLMC) simulations with thirteen model parameters tuned to the multiplicity as a function of \( z \), \( p_T \) (hadron transverse momentum), and \( \eta \) (hadron pseudorapidity) of the $\pi^-$, $K^-$, and $\bar{p}$ \cite{6,7}. HLMC is a combination of the DIS event generator Lepto \cite{8} (based on JETSET 7.4 and PYTHIA 5.7 \cite{9}), the detector simulation program (based on GEANT \cite{10}), and the HERMES reconstruction program \cite{7}.

FIG. 1: (color online) Leading order (Born approximation) Feynman diagram of the neutral current (NC, left panel) and charged current (CC, right panel) $e^-+p$ DIS.

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In this paper the PYTHIA 6.4 \cite{11} model and on which based model of PACIAE 2.2 \cite{12} (simplified as PYTHIA and PACIAE, respectively, later) were employed to calculate the DIS normalized $\pi^+$, $\pi^-$, $K^+$, and $K^-$ multiplicities in the 27.6 GeV electron SIDIS off the proton and deuteron targets. The PACIAE 2.2 model is a new issue of the PACIAE model updated presently from PACIAE 2.1 \cite{13} in order to cover the lepton DIS (SIDIS) off the nuclear target. The DIS normalized specific charged hadron multiplicity as a function of $z$ in the $e^-+p$ and $e^-+D$ SIDIS at 27.6 GeV beam energy calculated by PYTHIA and PACIAE with default model parameters is not well and fairly well consistent with the HERMES data \cite{6}, respectively. The default PYTHIA results in $e^-+p$ and $e^-+n$ (neutron) SIDIS at the same beam energy.

II. MODELS

The PACIAE model \cite{13} is based on PYTHIA \cite{11}. However, the PYTHIA model is for high energy elementary collision ($e^+e^-$, lepton-hadron, and hadron-hadron ($hh$) collisions) but PACIAE is also for lepton-nuclear and nuclear-nuclear collisions.

In the PYTHIA model, a $hh$ ($pp$) collision for instance, is described in term of parton-parton collisions. The hard parton-parton collision is dealt by the LO-pQCD parton-parton cross section with the modification of parton distribution function in a hadron. The soft parton-parton collision, a non-perturbative process, is considered empirically. The initial- and final-state QCD radiations as well as the multiparton interactions are taken into account. So the consequence of a $hh$ collision is a partonic multijet state composed of the diquarks (anti-diquarks), quarks (anti-quarks), and the gluons, besides a few hadronic remnants. All of the above quarks (anti-quarks), diquarks (anti-diquarks), and gluons are constructed into strings which are then hadronized by Lund string fragmentation regime, thus a final hadronic state is obtained for a $hh$ ($pp$) collision eventually.

Correspondingly, in the PACIAE model a $hh$ ($pp$) collision is also described by PYTHIA as mentioned in last paragraph. However, in the PACIAE model, the above constructed strings are not hadronized immediately, but are split into quarks (anti-quarks), diquarks (anti-diquarks), and gluons by force. And the diquarks (anti-diquarks) are split further into quarks (anti-quarks). Thus one obtains a partonic initial state composed of quarks, and quarks, and gluons for a $hh$ ($pp$) collision. The partons then take part in the partonic rescattering. After partonic rescattering, the partons are constructed into strings. The strings are then hadronized by Lund string fragmentation model. After hadronization, the produced hadrons proceed hadronic rescattering. And the final hadronic state is reached for a $hh$ ($pp$) collision eventually.

In PACIAE model a nucleus-nucleus collision is described as follows: The nucleons in a colliding nucleus are first randomly distributed according to the Woods-Saxon distribution in the spatial phase space. The participant nucleons, resulted from Glauber model calculation, are required to be inside the overlap zone, formed when two colliding nuclei path through each other at a given impact parameter. The spectator nucleons are required to be outside the overlap zone but inside the nucleus-nucleus collision system. If the beam direction is set on $z$ axis, then $p_x = p_y = p_{beam}$ are set for nucleons in the projectile nucleus for both the fixed target and collider. $p_x = p_y = p_z = 0$ and $p_x = p_y = 0$ as well as $p_z = -p_{beam}$ are set for nucleons in the target nucleus for both the fixed target and collider, respectively. We then decompose a nucleus-nucleus collision into nucleon-nucleon ($NN$) collision pairs according to the nucleon straight-line trajectories and the $NN$ total cross section. Each $NN$ collision is dealt by PYTHIA with string fragmentation switched-off and diquarks (anti-diquarks) broken into quarks (anti-quarks) as mentioned in last paragraph. A partonic initial state, composed of the quarks, antiquarks, gluons, and a few hadronic remnants, is obtained for a nucleon-nucleus collision when $NN$ collision pairs were exhausted.

This partonic initial stage is followed by a parton evolution stage. In this stage, the parton rescattering is performed by the Monte Carlo method with $2 \rightarrow 2$ LO-pQCD parton-parton cross sections \cite{14}. The hadronization stage follows the parton evolution stage. The Lund string fragmentation model and a phenomenological coalescence model are provided for the hadronization. However, the string fragmentation model is selected in the present calculations. Then the rescattering among produced hadrons is dealt with usual two body collision model \cite{13}. In this hadronic evolution stage, only the rescatterings among $\pi$, $K$, $\rho(\omega)$, $\phi$, $p$, $n$, $\Delta$, $\Lambda$, $\Sigma$, $\Xi$, $\Omega$, and their antiparticles are considered for simplicity.

The $p+\Lambda$ ($A+p$) collisions are simulated similar to the nucleus-nucleus collisions but the overlap zone is not introduced presently. We deal with the $l+p$ ($l+n$) and $l+A$ DIS (SIDIS) like the $p+p$ and $p+A$ collisions, respectively. However, instead of the $NN$ total cross section, the $l+p$ DIS total cross section is used and the lepton is assumed not resolvable in the PYTHIA and PACIAE models.
FIG. 2: (color online) The total cross section of pp and γp collisions as well as leading order $e^-p$ DIS.

FIG. 3: (color online) Multiplicity of DIS normalized specific charged hadron as a function of $z$ in the $e^-+p$ SIDIS at 27.6 GeV beam energy.
Fig. 1 shows the leading order (Born approximation) Feynman diagram for the neutral current (NC, the exchange of $\gamma/Z$ boson, left panel) and charged current (CC, the exchange of $W^\pm$ boson, right panel) $e^- + p$ DIS. There are two vertices in the left panel of Fig. 1 for instance. At the upper boson vertex the initial state QED and weak radiations have to be considered. At the lower boson vertex, not only the leading order parton level process of $V^* g \rightarrow q$ ($V^*$ refers to $\gamma/Z/W$) but also the first order QCD radiation of $V^* g \rightarrow qg$ as well as the boson-gluon fusion process of $V^* g \rightarrow q\bar{q}$ have to be introduced. Furthermore, the parton shower approach has been introduced to take higher than first order QCD effects into account \cite{8}. Therefore the DIS cross section can be formally expressed as

$$\sigma_{NC(CC)} = \sigma^{Born}_{NC(CC)}(1 + \delta^{qcd}_{NC(CC)})(1 + \delta^{weak}_{NC(CC)})(1 + \delta^{qcd}_{NC(CC)})$$  \hspace{1cm} (1)

where $\sigma^{Born}_{NC(CC)}$ is the Born cross section, $\delta^{qcd}_{NC(CC)}$ and $\delta^{weak}_{NC(CC)}$ are, respectively, the QED and weak radiative corrections, the QCD radiative correction of $\delta^{qcd}_{NC(CC)}$ is formally introduced in this paper.

In the lowest-order perturbative QCD theory, the NC/CC DIS Born cross section of the unpolarized electron on an unpolarized nucleon can be expressed by the structure functions $F_1, F_2, F_3$ as follows \cite{10}

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2} I_f \left( 1 - y - \frac{x^2y^2M^2}{Q^2} \right) F_2^f + y^2xF_1^f = \left( y - \frac{y^2}{2} \right) xF_3^f,$$  \hspace{1cm} (2)

where the mass of the initial and scattered leptons are neglected. In the above equation, $I$ denotes NC or CC. $\alpha$ stands for the fine structure constant. $x \equiv x_B$ and $y$ are the Bjorken scaling variable and fraction energy of $\gamma/Z/W$ boson, respectively. $M$ refers to the mass of target nucleon. $\eta_{NC} = 1, \eta_{CC} = (1 \pm \lambda)^2\eta_W$, and

$$\eta_W = \frac{1}{2} \left( \frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right),$$

$$G_F = \frac{e^2}{4\sqrt{2}\sin^2\theta_W M_W^2},$$

where $M_W$ and $\theta_W$ are the mass of $W$ boson and Weinberg angle, respectively. $\lambda = \pm 1$ is the helicity of the incident lepton.

The structure functions above can be expressed by the parton distribution function of nucleon in the quark-parton model \cite{14}. Although the PDF can not be calculated by first principle, it can be extracted from the QCD fits by black curve \cite{18}. In the calculation the cuts are first set for $Q^2$ models \cite{18}. The measure of the agreement between the experimental data of lepton-nucleon DIS cross section and the theoretical model \cite{17}.

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The structure functions above can be expressed by the parton distribution function of nucleon in the quark-parton model \cite{14}. Although the PDF can not be calculated by first principle, it can be extracted from the QCD fits by a measure of the agreement between the experimental data of lepton-nucleon DIS cross section and the theoretical models \cite{15}. The $e^- + p$ DIS total cross section calculated with HERAPDF1.5 LO \cite{16} PDF set, is given in Fig. 2 by black curve \cite{16}. In the calculation the cuts are first set for $Q^2 > 1$ GeV and $W^2 > 1.96$ GeV and then the cuts on $x$ and $y$ are derived according to relationships of the kinematical variables and $\cos^2\theta < 1$. The red and blue circles in Fig. 2 are, respectively, the total cross section of pp and $\gamma p$ collisions copied from \cite{16}.

One knows well that the incident proton, in the $p+Au$ collisions at RHIC energies for instance, may collide with a few ($\sim$2-5) nucleons when it passes through the gold target. Since the $e^- + p$ DIS total cross section is a few order of magnitude smaller than pp collision at the $\sqrt{s}$ range of $\sqrt{s} < 1000$ GeV (cf. Fig. 2), one may expect that the incident electron, in this energy range, may suffer at most one DIS with the nucleon when it passes through the target nucleus. The struck nucleon is the one with lowest approaching distance from the incident electron. This is the same for other incident leptons because the DIS total cross section is not so much different among the different leptons \cite{20}.

Therefore in the pioneer studies \cite{21} for lepton-nucleus DIS by PYTHIA + BUU transport model, the FRITIOF 7.02 model \cite{22} or PYTHIA 6.2 \cite{22} was employed to generate lepton-nucleon DIS event. This generated hadronic final state was then input into the couple channel BUU (Boltzmann-Uehling-Uhlenbeck) equation \cite{24} to consider the final state hadronic interaction (hadronic rescattering). They (Giessen group) employed this PYTHIA + BUU transport model successfully described the HERMES data of ratio of the DIS normalized charged hadron multiplicity in the nucleus A to the one in the deuteron for the 27.5 GeV beam energy $e^+ + ^{14}N$ and $e^+ + ^{84}Kr$ SIDIS \cite{23}. Of course, in the calculations they have to introduce an assumed values for the lepton-hadron interaction cross section and the hadronic formation time.

III. RESULTS

As mentioned in \cite{1, 2}, the measured hadron multiplicity in the $e^- + p$ and $e^- + D$ SIDIS at 27.6 GeV beam energy has first to correct for the radiative effects, limitations in geometric acceptances, and the detector resolution. The
Born-level multiplicity is then obtained in order to benefit the PDF extraction, etc. Then they normalized this Born-level multiplicity by the total DIS yield to reduce the uncertainties in the corrections above. This is of benefit to the comparison among the different experimental measurements and between the experiment and theory. The Born-level multiplicity of the type $h$ hadrons as a function of $z$ in the lepton SIDIS off a nuclear target, for instance, can be expressed as

$$\frac{1}{N_{DIS}} \frac{dN^h}{dz} = \int \frac{d^5N^h(x_B, Q^2, z, P_{h\perp}, \phi_h)}{dQ^2 dP_{h\perp} d\phi_h} dx_B. \quad (5)$$

Therefore, we can compare the default PYTHIA and PACIAE results of $\frac{1}{N_{DIS}} \frac{dN^h}{dz}$ calculated in the full kinematic phase space to the DIS normalized HERMES data, like in [6, 7].

In the default PYTHIA and PACIAE model simulations, the model parameters are unchanged. The default PYTHIA (black dashed line) and PACIAE (blue open circles) results of $\frac{1}{N_{DIS}} \frac{dN^h}{dz}$ are compared with the HERMES data (black solid circles) as well as the results of HLMC (black line) in the Figure 3 for $\pi^+$ (upper left panel), $\pi^-$ (upper right), $K^+$ (lower left), and $K^-$ (lower right) in the $e^-+p$ SIDIS at the 27.6 GeV beam energy. One sees in this figure that the default PACIAE results reproduce HERMES data nearly as good as HLMC (with thirteen tuned model parameters). However, the default PYTHIA results disagree with HERMES data. Figure 4 is the same as Fig. 3 but for $e^-+D$ SIDIS at the same beam energy. For Fig. 4 one may draw a similar conclusion like Figure 3.

Table I lists the discrepancies between the PYTHIA and PACIAE models. In the event generation there is no extra requirement in the PYTHIA model but is requirement of having one parton (quark, antiquark, or gluon) at least in each event in the PACIAE model. In the Figs. 5 and 6, the black dashed line is the default PYTHIA results, the blue circles are the default PACIAE results, and the red dash-dotted line is the results of PACIAE without both the PRS and HRS rescatterings. The later two, i.e. blue circles and red dash-dotted line, are close to each other, which really proves the small effect of both the PRS and HRS in the $e^-+p$ and $e^-+D$ SIDIS, because the reaction systems here are quite small.

The black open triangles, in the figures 5 and 6, are the default PACIAE results calculated without the requirement of having one parton at least in each generated event. These results are not different so much from the blue open circles (default PACIAE results calculated with event requirement). Thus the initial partonic state, introduced in the PACIAE model but not in PYTHIA, has to be the main reason of the large discrepancy between the default
FIG. 5: (color online) Multiplicity of DIS normalized specific charged hadron as a function of $z$ in the $e^-+p$ SIDIS at 27.6 GeV beam energy.

FIG. 6: (color online) Multiplicity of DIS normalized specific charged hadron as a function of $z$ in the $e^-+D$ SIDIS at 27.6 GeV beam energy.
PYTHIA and PACIAE results. The introduction of partonic initial state causes the dynamical simulation processes, such as the partonic rescattering, the space and time evolutions of hadronization, the hadronic rescattering, etc., in the PACIAE model are quite different from the one in the PYTHIA model. The space and time evolutions of hadronization is especially to be investigated further.

In the Figures 5 and 6 the black solid lines are calculated by the default PYTHIA model with the requirement of having one pion or kaon at least in each generated event. One sees here that the default PYTHIA results with the requirement (black solid line) is considerably larger than the one without the requirement (black dashed line). It may mean that the event generated by the default PYTHIA is not completely DIS, and may confuse with diffractive processes etc., which has to be study further.

| TABLE I: Discrepancies between the PYTHIA and PACIAE models |
|-------------------------------------------------------------|
| item       | PYTHIA | PACIAE |
| Partonic initial state | not introduced | introduced |
| Initial state PRS     | no      | yes    |
| Final state HRS       | no      | yes    |
| Event requirement    | no      | yes    |

IV. CONCLUSIONS

In summary, we have employed the PYTHIA 6.4 and the extended parton and hadron cascade model PACIAE 2.2 to investigate the DIS normalized specific charged hadron multiplicity, \( \frac{1}{N_{DIS}} \frac{dN_h}{dz} \), measured by HERMES in the \( e^- + P \) and \( e^- + D \) SIDIS at 27.6 GeV beam energy. The PYTHIA and PACIAE results, calculated with default model parameters, not well and fairly well reproduce the HERMES data \( \frac{1}{N_{DIS}} \frac{dN_h}{dz} \), respectively.

Additionally, we have investigated the effects of the differences between the PYTHIA and PACIAE models, i.e the effect of both the initial state PRS and final state HRS as well as the event requirement. It turned out that the effect of both the PRS and HRS is weak because of the small reaction system of \( e^- + p \) and \( e^- + D \) SIDIS. The event requirement of having one parton (quark, antiquark, or gluon) at least in each generated initial partonic state introduced in the PACIAE model plays a visible role. However the effect of the requirement of having at least one pion or kaon in each event generated by the PYTHIA model is relatively strong. The main reason, which causes a discrepancy between the default PACIAE and PYTHIA results, should be attributed to the initial partonic state, introduced in the PACIAE model but not in PYTHIA. This discrepancy leads to the simulation processes, such as the partonic rescattering, the space and time evolutions of hadronization, the hadronic rescattering, etc., in the PACIAE model are quite different from the PYTHIA model. The more investigations are especially required for the space and time evolutions of hadronization.

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