Dark Energy from Strings

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Abstract. A long-standing problem of theoretical physics is the exceptionally small value of the cosmological constant $\Lambda \sim 10^{-120}$ measured in natural Planckian units. Here we derive this tiny number from a toroidal string cosmology based on closed strings. In this picture the dark energy arises from the correlation between momentum and winding modes that for short distances has an exponential fall-off with increasing values of the momenta. The freeze-out by the expansion of the background universe for these transplanckian modes may be interpreted as a frozen condensate of the closed-string modes in the three non-compactified spatial dimensions.

INTRODUCTION

This talk is based on a paper with Bastero-Gil and Mersini, necessarily shortened to fit the space available, so I refer to it for more detail.

In this work we will attempt to make a quantitative argument about the origin of dark energy from string theory. The transition from string theory to conventional cosmology is of importance not only to theoretical physics in general but to inflationary cosmology in particular. Corrections to short distance physics due to the nonlocal nature of strings contribute to dark energy. The possibility to detect their signature observationally is very intriguing. In Ref. [2] it was shown that a nonlinear dispersion function modifying the frequency of the transplanckian perturbation modes [3] can produce the right contribution to the dark energy of the universe [4]. The physics mechanism that gave rise to dark energy was the freeze-out of these ultralow frequency modes by the expansion of the background universe. Superstring duality [5] was invoked to justify the dispersion function. This work attempts to carry out this derivation.

In Section 2 we review some preliminaries of the Friedman-Robertson-Walker (FRW) cosmological solutions found for string theory in a D-dimensional torus [3, 2, 8, 1]. The quantum hamiltonian from closed string theory obtained in [10] by using the correspondence principle between string and quantum operators, is reviewed in Section 3. Although the background is an FRW universe, it is globally nontrivial and thus it allows two types of quantum string field configurations, twisted and untwisted fields.

Based on the equivalence between Euclidean path integral and statistical partition functions, we perform in Section 4 the calculation of a coarse-grained effective action [11, 12] for the momentum and winding modes of the system described in Section 2 for the case of 3 expanding spatial dimensions $R$ in the $T^D$ toroidal topology. The string scale is taken as the natural UV lattice cutoff scale of the theory. The renormalization group equations (RGE) of the coupling constants for the winding and momentum modes describe the evolution from early to late times of their entanglement. Based on T-
duality the whole spectrum is obtained by exchanging momentum to winding modes and \( R \rightarrow R^{-1} \). Their coupling is strong when the radius of the torus \( R \) is of the same order as the string scale \( \sqrt{\alpha'} \), i.e. during the phase transition from a winding dominated universe to a momentum mode dominated universe. Due to the expanding background, we have a non-equilibrium dynamics and calculate the effective action by splitting our modes into the open system degrees of freedom (low energy modes, mainly momentum modes) and the environment degrees of freedom (high energy modes, mainly winding modes). The coarse-graining is performed by integrating out the environmental degrees of freedom. The scale factor \( a(t) = R(t) \) serves as the collective coordinate that describes the order parameter for the environment degrees of freedom. The effective action calculated in this way contains the influence of the environment at all times in a systematic way and the coarse graining process encodes the dispersion function and corrections to short distance physics due to the correlation between the two types of modes in the system and environment. This procedure results in the RGEs for the coupling constants that offers information about their running to trivial and nontrivial fixed points at early and late times, therefore the flow of one family of lagrangians (string theory phase) to another family of lagrangians at late times (conventional 3+1 quantum theory). Results of this non-equilibrium phase transition are summarised in Section 5 with a discussion about the possibility of their observational signatures through the equation of state of the frozen short distance modes. In this section we also briefly touch upon the issue of the two field configurations in a globally nontrivial topology and the instabilities in the theory arising from their interaction. A detailed summary of the main coarse-graining formulas and procedure [11] needed in Section 4, are attached in the Appendix. In essence, the dark energy arises from the study of the UV behavior of the correlations with environmental modes.

**TOROIDAL STRING COSMOLOGY.**

We consider the string cosmological scenario proposed by Brandenberger and Vafa[6, 8, 13]. Strings propagate in compact space, a box with D spatial dimensions and periodic boundary conditions, the \( T^D \) torus. It was argued that [6] a thermodynamic description of the strings with positive specific heat, is well defined only when all the spatial dimensions are compact.

Let us begin with the Universe placed in a \( T^D \) box with a size of the order of the string scale, that we are taking to be the Planck scale. In such a space, string states also contain winding modes, which are characteristic of having an extended object like a string, “winding” around the compact spatial dimension, besides the usual momentum modes, and oscillator modes with energy independent of the size of the box. The energy of the winding modes increases with the size of the box as \( wR \), while the energy of the momentum modes decreases as \( m/R \). The spectrum is symmetric under the exchange \( R \leftrightarrow 1/R \) and \( m \leftrightarrow w \). This symmetry known as T-duality [5] is not only a symmetry of

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1 Herein referred to as the BV model.
the spectrum but of the theory.

The BV model [6] argues that if the Universe expands adiabatically in more than 3 spatial dimensions, it would not be possible to maintain the winding modes in thermal equilibrium. As their energy density grows with the radius, their number would have to decrease, for example through annihilation processes. But typically strings do not meet in more than 3 spatial dimensions and do not interact with each other; therefore the winding modes fall out of equilibrium [14]. In summary, their growing energy density will tend to slow down the expansion of the universe and eventually stop it. But if the Universe starts to contract, the dual scenario of the momentum modes opposing contraction would take place and the Universe may oscillate between expanding/contracting eras. In what follows we use this argument of [6] to justify the assumption that only \( D = 3 \) dimensions of the \( T^D \) torus will expand to create an FRW universe.

Cosmological solutions for an arbitrary number of anisotropic toroidal spatial dimensions \( T^D \) were found by Mueller in [7]. He studied the cosmology of bosonic strings propagating in the background defined by a time-dependent dilaton field, \( \Phi(t) \), and space-time metric

\[
ds^2_d = G_{\mu\nu}(X)dX^\mu dX^\nu = -dt^2 + \sum_{i=1}^{D} 4\pi R_i^2(t)dX_i^2,
\]

The radii of the torus, \( R_i(t) \), become the time-dependent scale-factors, and the spacetime dimensions is \( d = 1 + D \). The equations of motion of the bosonic string in background fields are obtained from the following action\(^2\) [15]

\[
I = \frac{1}{4\pi\alpha'} \int d^2\sqrt{g} \left[ g^{mn}G_{\mu\nu}(X)\partial_mX^\mu\partial_nX^\nu + \frac{1}{2}\alpha'\Phi R^{(2)} \right],
\]

where \( g_{mn} \) is the two-dimensional world-sheet metric, and \( R^{(2)} \) the world-sheet scalar curvature. The background field equations are obtained by imposing the condition that the theory be free from Weyl anomalies. To lowest order in perturbation theory this leads to the equations:

\[
\beta G_{\mu\nu} = R_{\mu\nu} + \nabla_{\mu}\nabla_{\nu}\Phi = 0,
\]

\[
\beta^\Phi = \frac{d-26}{3\alpha'} - R + (\nabla\Phi)^2 - 2\nabla^2\Phi = 0,
\]

Using the metric given in Eq. (1), they reduce to:

\[
\frac{\ddot{\Phi} - \sum_i \frac{\ddot{R}_i}{R_i}}{R_i} = 0,
\]

\[
\frac{\ddot{R}_i}{R_i} + \sum_{j\neq i} \frac{\dot{R}_i\dot{R}_j}{R_i R_j} - \Phi \frac{\dot{R}_i}{R_i} = 0,
\]

\(^2\) The antisymmetric tensor field is taken to be zero.
\[ \Phi - \frac{1}{2} \Phi^2 + \sum_{i<j} \frac{\dot{R}_i \dot{R}_j}{R_i R_j} = \frac{d-26}{3 \alpha'} . \]  

When \( D = 25 \), the solutions obtained in \([7]\) are:

\[ e^{-\Phi(t)} \propto t^p, \]  

\[ R_i(t) \propto t^{p_i}, \]  

with the constraints,

\[ \sum_{i=1}^{D} p_i^2 = 1, \quad \sum_{i=1}^{D} p_i = 1 - p . \]  

Note that these solutions are found in the absence of matter sources. In general the back-reaction of the matter action of the strings in \( T^D \) alters the solutions for the background geometry.\(^3\) It is clear that we can have an arbitrary number of compact spatial dimensions \( D_c \) with \( p_i < 0 \), that are decreasing with time \([1, 4]\), and \( D - D_c \) expanding spatial dimensions with \( p_i > 0 \). Among the many solutions found in \([7]\) we select the solution \( D - D_C = 3 \) that although it is not unique it is justified by the BV argument. The assumption that our Universe is expanding in only 3 spatial dimensions, with the remaining \( D - 3 \) being small and compact, as well as considering a constant dilaton field \([8, 9]\) \((p = 0)\), are consistent with Mueller’s solutions Eqs. \( (10) \). The issue of stabilising the dilaton is beyond the scope of this paper, and we assume that the dilaton has acquired a mass and become stable at some fixed value. It is also assumed that the backreaction of the matter string sources on the background geometry is small enough such that the deviations from the FRW metric, Eq. \( (10) \), can be neglected.

Due to the toroidal string cosmology, the three expanding dimensions contain both types of modes: momentum and winding, propagating in the 3+1 FRW space-time. The number of winding modes at each stage of the evolution of the Universe is determined by the dynamics of the background. In the next section, we touch base with quantum field theory through correspondence principle between string and quantum operators, in order to use coarse graining techniques for studying the influence of the winding modes on the momentum modes as the Universe expands.

**QUANTUM HAMILTONIAN FROM CLOSED STRING THEORY.**

Let us consider BV model \([6]\) of a D-dimensional anisotropic torus with radius \( \bar{R}_i \), by including the dynamics of both modes: momentum modes, \( p_{1,i} = m/\bar{R}_i \) (where \( m \) is

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\(^3\) See \([8, 9]\) and references therein for the geometry solutions in the presence of a matter action. Inclusion of matter sources alters the solutions of \([7]\) due to the backreaction of the winding modes, such that the scale factor approaches asymptotically a constant value at late times .

\(^4\) We do not address the concern that the time dependence of the compactified \( R_i \) endangers the constancy of the dimensionless parameters in the \( D = 3 \) theory.

\(^5\) The authors of \([8]\) argued that a constant dilaton background may not be consistent with a high temperature phase of strings thermodynamics.
the wavenumber), and winding modes with momenta \( p_{2,i} = wR_i/\alpha' \). The dimensionless quantity for the radius is \( R_i = \tilde{R}_i/\sqrt{\alpha'} \), where \( \alpha' \) is the string scale. Based on the arguments reviewed in Section 2, we choose a cosmology with three toroidal radii equal and large \( R \gg 1 \) in units of the string or Planckian scale, with the other \( (D-3) \) toroidal radii equal and small \( R_C \ll 1 \). Here the subscript \( C \) refers to compactified dimensions. Then, \( R(t) \) becomes the scale factor for the 3+1 metric in conventional FRW (Friedman Robertson Walker) cosmology \( R(t) = a(t) \), while \( R_C \) corresponds to the radius, in this factorizable metric, of the \( D-3 \) compact dimensions \( z_j \) that decrease with time,

\[
ds_D^2 = -dt^2 + 4\pi R^2(t)dx_i^2 + 4\pi R^2_C(t)dz_j^2 = a(\eta)^2[-d\eta^2 + dy^2] + ds_{D-3}^2. \tag{11}\]

Using the string toroidal solution of [7] the time-dependence of these radii is:

\[
R(t) = \alpha_U t^{p_U} \tag{12}
\]

\[
R_C(t) = \alpha_C t^{p_C} \tag{13}\]

The solutions in Ref. [7] show that \( p_U \) and \( p_C \) depend on the dimensionality \( D \) in an interesting way. There is a plethora of possible solutions but if we assume, for example, that the dilaton is time-independent and the compactification is isotropic we find that for \( 4 \leq D < \infty \), then \( 0.5 \leq p_U < 1/\sqrt{3} \simeq 0.577 \). Let us take \( D = 4 \) where the scale factor behaves as a radiation-dominated universe; if, in fact, \( D \geq 5 \) we can assume that the \( D-4 \) additional dimensions have \( p_C \ll p_C \) to achieve the same result. In this case, \( p_C = -0.5 \). Here we do not, however, need to specialise to a particular solution.

What we have in mind for the dark energy is the correlation of momentum to winding string modes. The question is, given the well-known form for the kinetic energy of these strings, e.g. [16], how to describe best the interaction between the winding and momentum modes. Some aspects are addressed in [16] which focuses on the smallness of temperature \( (T/T_H) \). For temperature \( T \) very much below the Hagedorn or string temperature \( T_H \) we expect that only very small winding numbers \( w_i = 0 \) or 1 in the compact dimensions are of any significance [16]. Similar arguments apply to the momentum modes \( m_i \) for the time-reversed case.

Let us consider the small parameter \( \delta(t) \), taken to be:

\[
\delta = \frac{R_C}{R} \sim t^{p_C-p_U} \tag{14}\]

For the case \( D = 3 \) (d=4), for example \( \delta \sim t^{-1} \sim (T/T_H)^2 \) and is an extremely small number \( (\sim 10^{-60}) \) at present. The point is that in the \( \delta \rightarrow 0 \) limit these modes are in separate spaces and for very small \( \delta \) are therefore expected to be highly restricted. The compactified dimensions can be integrated out, and we are left with the momentum and winding modes in the remaining \( D = 3 \) spatial dimensions.

The partition function for this system was calculated, from first principles, by summing up over their momenta in [16]:

\[
Z = \sum_\sigma e^{-n_\sigma \xi_\sigma}, \tag{15}\]
where \( n_\sigma \) is the number of strings in state \( \sigma \) with energy \( \varepsilon_\sigma \)

\[
\varepsilon_\sigma = p_0 = \sqrt{\left(\frac{m}{R}\right)^2 + (wR)^2 + N + \tilde{N} - 2},
\]

(16)

and \( \sigma \) counts over \((m, w)\), with the constraint \( N - \tilde{N} = mw \) for closed strings where \( N \) and \( \tilde{N} \) are the sums over the left- and right- mover string excitations, respectively. By now, in Eq. (16), we are considering only the large 3 spatial dimensions. The string state for left and right modes can be expanded in terms of the creation and annihilation operators \( \alpha_m, \alpha_n \), with higher excitation string states given by \( N = \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n \) (similarly for \( \tilde{N} \)), and string energy \( L_0 + \tilde{L}_0 = p_1^2 + p_2^2 + (N + \tilde{N} - 2)/\alpha' \).

We would like to write the path integral for this configuration in terms of quantum fields\(^6\). The path integral is calculated from the hamiltonian density. In order to use the correspondence between the Euclidean path integral of the persistence vacuum amplitude \(|\langle \text{in}|\text{out}\rangle|^2\) and the partition function \( Z \), we need to write a hamiltonian density over the fields in configuration space in such a way that its Fourier transform in \( k \)-space corresponds to the string energy expression Eq. (16).

Thus in writing a Coarse-Grained Effective Action (CGEA), the kinetic terms are unambiguous while for the interaction terms we must appeal to simplicity and the requirement of T-duality. Closed-string field theory provides guidance, since in e.g. \([17]\) truncation at a quartic coupling can be sensible, and this will lead to a CGEA which is renormalisable and satisfies useful RG equations.

Generally, closed string field theory contains couplings of all non-polynomial orders. In a semi-classical approximation we may restrict to genus \( g = 0 \) since the genus \( g \) contribution is proportional to \( h^g \) \([18]\).

The quantum hamiltonian is in any case known for the classical string in axi-symmetric or toroidal backgrounds \([19]\). They explicitly calculated the quantum hamiltonian and demonstrated the correspondence principle between the string operators \( L_0, \tilde{L}_0 \) and quantum field operators in the form (in the notation of \([14]\))

\[
\hat{H} = \hat{L}_0 + \hat{\tilde{L}}_0 = \frac{1}{2}\alpha' \left(-E^2 + p_2^2 + \frac{1}{2}(Q_+^2 + Q_-^2)\right) + N + \tilde{N} - 2c_0
\]

\[
-\alpha' [(q + \beta)Q_+ + \beta E] - \alpha' [(q - \alpha)Q_- + \alpha E]J_L
\]

\[
\frac{1}{2} \alpha' q \left[(q + 2\beta)J_R^2 + (q - 2\alpha)J_L^2 + 2(q + \beta - \alpha)JJ_L\right]
\]

(17)

\[
\hat{L}_0 - \hat{\tilde{L}}_0 = N - \tilde{N} - mw
\]

(18)

where \( J_{R,L} \) are bilinear quadratic operators in terms of creation and annihilation operators and the higher string oscillators \( N, \tilde{N} \) contribute the string mass. Therefore the \( J_{\tilde{R}} \) term would be a quartic interaction in terms of creation and annihilation operators.

This particular solution is for a cylindrical topology where the uncompactified \( x_1 \) and \( x_2 \) are written in polar coordinates \( x_1 + i x_2 = r e^{i\theta} \) and \( x_3 \) is also uncompactified (but

\(^6\) Below we use quantum string equations under the assumption that the dilaton is massive and stable.)
could be compactified along with additional similar coordinates), together with time
and one additional compactified dimension $y \subset (0, 2\pi R)$. Although an exact solution
for the hamiltonian of the string matter in a toroidal background is not yet known, a
quartic potential energy was advocated and found in [8] by arguments similar to those
of Eq. (17), for the classical string and the three string coupling level. We take this
as an indication, in the subsequent section (if the exact solution were known to all
orders), that an quantum hamiltonian analogous to Eq.(17) for closed strings on a torus,
similarly containing only quartic terms as suggested by [10], exists for our present case
of $(T_3) \times (T_{D-3}) \times (time)$ and focus on the uncompactified 3 spatial dimensions.

The hamiltonian depicted in Eq.(17) is for a static background, i.e a constant scale
factor $R(t)$. In the next section, we base our calculation in the coarse-grained effective
action (CGEA) formalism where the dynamics of an expanding background is replaced
by scaling on a static background.

Thus Eq.(17) which applies to a static background (as in Eq.(16)) can be generalized
to a cosmologically-expanding background as in Eq.(11) by using this technique of re-
scaling, for details of which see [11]. The result is a dispersion formula characterized by
a dispersed frequency with short distance modifications contained in a $\tilde{m}_0^2$ term:

$$
\tilde{m}_0^2 \to \frac{m_0^2}{2 \cosh^2 p \sqrt{\alpha'}} \approx \frac{1}{2} m_0^2 e^{-2\sqrt{\alpha'} p}. \quad (19)
$$

**DARK ENERGY FROM CLOSED STRING THEORY.**

**DISCUSSION**

We argue that closed strings on a toroidal cosmology lead to a plausible explanation of
the dark energy phenomenon. Although bosonic strings have been used, it is expected
that superstrings will lead to a similar conclusion. Certainly it is crucial that closed
strings are involved because open strings do not have the same aspect of winding around
the torus.

The scale factor of the universe $a(\eta)$ has been used as a collective coordinate for the
environment degrees of freedom, and as the fundamental scaling parameter in the coarse-
graining. The choice of a $D = 3$ expanding cosmology was chosen phenomenologically.
An argument for this choice in the BV model was presented in [6], and we believe this
argument does provide a possible justification. It is encouraging that inclusion of branes
gives a similar result [13]. It has further been assumed that the mass gap $\Delta p$ can be safely
assumed to be slowly-varying during our coarse-graining procedure.

We would like to make the reader aware of another subtlety related to the torus
topology of our background. A globally nontrivial topology like $T^3 \times R^1$ admits two
types of quantum field configurations, twisted and untwisted fields, due to the periodic
and anti-periodic boundary conditions imposed on the fibre bundle of the manifold. This
is a long-standing problem [19] that does not have a definite remedy. The problem is the
following: twisted fields can have a negative two-point function. These fields interact
with each other while preserving the symmetries of the hamiltonian. Their interaction
thus contributes a negative mass squared term to the effective mass of the untwisted
field due to the negative two-point function of the twisted field and render the untwisted
field unstable. It is often assumed that Nature simple chose to preserve the untwisted configuration only or forbids their interaction due to some, as yet unknown, symmetry [20].

String theory preserves Lorentz invariance. This symmetry has been broken for the open system of our low energy string modes due to the backreaction from the coarse grained environment. Their correlation results in our dispersion relation. If a specific frame must be chosen, it could be e.g. the rest frame of the CMB. The initial condition is a vacuum state conformally equivalent to the Minkowski spacetime - the so-called Bunch-Davis vacuum[21]. Finally, before summarising we should note that if there are other modes without the exponential suppression at high \(k\), all that we need is one such mode to lead to the frozen tail comprising the dark energy.

The high wave number behaviour \(e^{-ak}\) of the dispersion relation \(\omega(k)\) leads again to the correct estimate for the dark energy as a fraction \(\sim 10^{-120}\) of the total energy during inflation. This dark energy is certainly completely stringy because our derivation depends on the existence of winding modes, as seen by the role of the generalised level-matching condition

\[
N - \tilde{N} = \sum m_i w_i
\]

This correlation between momentum and winding modes leads to the quantum hamiltonian and hence to the interpretation of the dark energy as the weak correlation with the winding mode energy at short distances. The excitation modes of these correlations with energy less than the current expansion rate are currently frozen by the expanding background.

Within string cosmology there has always been the question of the fate of the winding modes in the uncompactified three spatial dimensions, whether they combine to a single string per horizon which wraps around the universe. Our remedy is intuitively appealing that while the momentum modes are in evidence as quarks, leptons, gauge bosons, etc. the winding modes are now condensed uniformly in the environmental background, hence with a weak correlation at short distances to the momentum modes, frozen by the expansion of the FRW universe in the form of the dark energy.

The observed small value \(\Lambda \sim 10^{-120}\) in natural units observed small value \(\Lambda \sim 10^{-120}\) in natural units has an explanation in the toroidal cosmology of closed strings and thus the dark energy provides an exciting opportunity to connect string theory to precision cosmology. We may argue that numerically the size of the cosmological constant in the present approach is a combination of the string scale and the Hubble expansion rate in the sense that \(\Lambda/M_{\text{Planck}}^2 \sim 10^{-120} \sim (H_0/M_{\text{Planck}})^2\). Therefore the correct amount of dark energy obtained by this frequency dispersion function does not require any fine tuning and relies, besides a physical mechanism (such as freeze-out), only on the string scale as the parameter of the theory. However, our approach does not solve the second puzzle about the dark energy namely, the coincidence problem for the following reason: The expansion rate of the universe is determined by the total energy density in the universe by the relation given in the Friedman equation. As can be seen from our dispersion function which approaches conventional cosmology in the subplanckian regime \((k \leq M_{pl})\), most of the other contributions to the energy density are not frozen modes. Therefore the Hubble rate \(H^2\) is not always proportional to the dark energy of the frozen modes due to the contributions in \(H^2\) from other forms of energy densitites.\(H^2\) is dominated by frozen
modes (and thus proportional to the dark energy $\rho_{DE}$) only at some late times $t \geq t_E$ when all other energy contributions $\rho_{other}$ have diluted enough below $\rho_{DE}$ due to their redshift.

The quantitative effort we have made in this work suggests that an interpretation of the dark energy in terms of string theory is more convincing than either a simple cosmological constant or the use of a slowly-varying scalar field with fine tuned parameters.

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