Impact of a radius and composition variation on the solar subsurface layers stratification

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**ABSTRACT**

Several works have reported changes of the Sun’s subsurface stratification inferred from $f$-mode or $p$-mode observations. Recently a non-homologous variation of the subsurface layers with depth and time has been deduced from $f$-modes. Progress on this important transition zone between the solar interior and the external part supposes a good understanding of the interplay between the different processes which contribute to this variation. This paper is the first of a series where we aim to study these layers from the theoretical point of view. For this first paper, we use solar models obtained with the CESAM code, in its classical form, and analyze the properties of the computed theoretical $f$-modes. We examine how a pure variation in the calibrated radius influences the subsurface structure and we show also the impact of an additional change of composition on the same layers. Then we use an inversion procedure to quantify the corresponding $f$-mode variation and their capacity to infer the radius variation. We deduce an estimate of the amplitude of the 11-year cyclic photospheric radius variation.

*Subject headings:* Sun: helioseismology — Sun: oscillations — Sun: activity — Sun: interior — Sun: model — Sun: radius — Sun: luminosity — Sun: variability

1. Introduction

Helioseismology has been extremely useful to probe the internal structure of the Sun but two crucial regions need to be improved for a good understanding of the time evolution of the solar activity: (1) the solar core for a proper description of the transport of momentum implied by rotation, gravity waves and magnetic field along the evolution and (2) the subsurface layers which correspond to the transition region between large and small scale dynamics evolution.
This transition zone is important to study because it couples the internal dynamics to the dynamics of the solar atmosphere and it plays a crucial role in the emergence of the space weather science (see for example Rozelot & Lefebvre (2006)). Figure I sketches a schematic view of these subsurface layers above 0.96 \( R_\odot \), which are called the \textit{leptoline} region (from the greek “leptos” = thin, “klino” = tilt). This term was proposed by Godier & Rozelot (2001), who computed the solar oblateness and showed a curvature change in this zone which was interpreted as the presence of a double layer. Just below the surface, there is a change in radial and latitudinal rotation (Basu, Antia & Tripathy 1999; Corbard & Thompson 2002) and the treatment of the superadiabatic region supposes a proper description of the convection (presently described by the mixing length parameter) and of detailed molecular and atomic opacity calculations. One needs also to add the turbulent pressure and the emergence of local magnetic field to these processes, then below, hydrogen and helium pass from neutral to partially ionized and then totally ionized in a region where the magnetic pressure cannot be ignored. The mean and varying magnetic field amplitudes have been tentatively extracted by Nghiem et al. (2006) from the analysis of the low degree acoustic modes. The proper interaction between these different physical processes is not yet included in stellar evolution models and could lead to some variation of the solar radius along the solar cycle.

The observation of these subsurface layers is not so easy. The difficulty comes from the fact that the high-degree \( p \)-modes (Korzennik, Rabello-Soares & Schou 2004) have a small lifetime and are largely perturbed by the turbulent motions and the emerging magnetic field. But important progress has appeared recently thanks to the analyses of long series of seismic data. Rabello-Soares, Korzennik & Schou (2008) estimate the variations of the high-degree \( p \)-modes frequencies over the solar cycle and using ring diagram analysis Basu, Antia & Bogart (2007) extract now latitudinal and temporal variations of the sound speed or the density along the Hale solar cycle (at least its last half cycle).
Fig. 1.— Schematic view of the leptocline (non-scale scheme).
In parallel, Lefebvre & Kosovichev (2005) and Lefebvre, Kosovichev & Rozelot (2007) reported changes with the solar cycle of the solar subsurface stratification, inferred from inversion of SOHO/MDI $f$-mode frequencies. They notice a different behavior for the layers around 0.99 $R_\odot$. Between 0.97 and 0.99 $R_\odot$, it seems that the position of the layers varies in phase with the solar cycle, whereas it appears opposite in the upper part, above 0.99 $R_\odot$, where the variability is in antiphase. So, it seems that the most external layers of the Sun shrink during the ascending phase of the solar cycle and relax after the maximum, but these changes are not uniform with depth. At the surface, this result is coherent with the observation of the photospheric radius variations. The observed series of ground-based measurements (Laclare et al. 1996; Reis Neto et al. 2003) and the measurements aboard stratospheric balloons (Sofia, Heaps & Twigg 1994) suggest a reduction of the solar photospheric radius at the maximum of the cycle. The balloon variation estimates are much smaller than the ground-based observations, polluted by the atmospheric turbulence. They are consistent with results from Kuhn et al. (2004) who reported no evidence of solar-cycle visible radius variations between 1996 and 2004 larger than 7 mas. But they disagree with computations of Sofia et al. (2005). These authors predict also non-homologous subsurface stratification changes with the 11 year solar cycle but with an amplification up to a factor 1000 from the depth at 5 Mm to the surface. It leads to a variation of the solar radius up to 600 km along the cycle as shown in Fig. 2 of Lefebvre, Kosovichev & Rozelot (2007). Such a variation seems extremely large.

So the study of the detected $f$-modes is very useful in the present context even the interpretation of $f$-modes has appeared puzzling (Dziembowski, Goode & Schou 2001; Antia 2003; Dziembowski & Goode 2004) due to some apparent lack of sensitivity of these modes at the real surface. We develop here a theoretical approach to validate the $f$-modes inversion procedure used previously and applied here to some specific cases that we know perfectly. This paper is the first of a series where we estimate the impact of a change of radius and composition on the subsurface layers and on the $f$-mode frequencies using classical solar models including a detailed microscopic description of these layers. In the present work, we do not try to justify the origin of the variation of the radius. The next step will be to develop more complex models including magnetic field and differential rotation. Our general aim is to properly qualify the variabilities of $f$-modes and solar radius that we will study with the SDO and PICARD missions (Kosovichev et al. 2007 and Thuillier, Dewitte & Schmutz 2006, respectively). We will also investigate the capability of the $f$-modes to estimate the solar photospheric radius variability. This study will contribute also to clarify the notion of “solar radius” to bridge two different communities and obtain without ambiguity a unique definition of this fundamental quantity (Haberreiter, Schmutz & Kosovichev 2008).

After a description of our study (Section 2), we describe in Section 3 the models used
and examine the subsurface layer changes on different variables produced by a change in the solar radius and composition and we calculate the corresponding \( f \)-mode frequencies. Section 4 is devoted to the use of these frequencies to infer the change in the position of the subsurface layers. The validation of the procedure, an estimate of the present solar cyclic radius variation and the perspectives of this work will conclude the paper in Section 5.

2. A new estimate of the solar cycle radius change using models and \( f \)-modes predictions

The present theoretical work focus on the solar layers located above 0.96 \( R_\odot \). The previous approach of Li, Basu & Sofia (2003) tried to interpret the behavior of the seismic acoustic observations by introducing some magnetic effect in solar models to predict the radius variation over the cycle. In the present work, we concentrate on the \( f \)-mode frequencies. We use known models and perform inversions that we can verify and compare with the present observational \( f \)-mode frequency variations. For this first paper, we stay in the classical approach and only focus on basic quantities which may vary along the solar cycle without introducing the physical process at the origin of the corresponding variations.

The next objectives will be to introduce the different magnetohydrodynamical actors (Mathis & Zahn 2005). Turbulence, rotation and the local effect of magnetic field with a decent topology (Duez et al. 2007) must be introduced together in a more sophisticated way than it is generally done. But we would like first to study the potential of the \( f \)-modes in some well known cases and then show that it is possible to deduce an order of magnitude of the solar radius variation from the present variation of the observed \( f \)-mode frequencies.

The general idea is to progress step by step on the sensitivity of the \( f \)-modes to the complex physics of the subsurface layers using the same formalism than the one used in Lefebvre & Kosovichev (2005). In the present paper we do not yet impose any kind of magnetic field like in Li, Basu & Sofia (2003) or Mullan, MacDonald & Townsend (2007) but we estimate the local changes to decouple the origin of the different effects. We examine the theoretical \( f \)-mode frequency changes coming from known solar models which mimic a simple expansion or a change in composition. We deduce, for the first time from \( f \)-modes, an estimate on the quality of extraction of the photospheric radius change and an order of magnitude of its Hale cycle change.
3. Model computations

We use the CESAM evolution code (Morel 1997) to calculate several models of chosen solar radius \( R \), luminosity \( L \) and surface abundance \( (Z/X) \). The different computed cases are organized in 3 sets, and listed in Table 1. Our reference model \( M_1 \) (Turck-Chièze, Couvidat & Kosovichev 2001; Couvidat, Turck-Chièze, S., & Kosovichev 2003) with \( R_\odot = 6.9599 \times 10^5 \) km, \( L_\odot = 3.8460 \times 10^{33} \) erg/s and \( Z/X_\odot = 0.02447 \) is the seismic solar model (SeSM) built to reproduce the observed sound speed profile.

Set 1 is composed of models \( M_2 \) and \( M_3 \) converged with a radius fixed respectively at \( \frac{R_2}{R_\odot} = 1.0002 \) and \( \frac{R_3}{R_\odot} = 0.9998 \) and the composition fixed at the value of the reference model, but where the mixing-length parameter \( \alpha \) is a free parameter (the luminosity is not constrained in models \( M_2 \) and \( M_3 \)). The radius difference of \( 2 \times 10^{-4} \) corresponds to a difference of about 140 km.

Set 2 is composed of six models \( M_i \), for \( i = 4...9 \) built with the two free parameters \( Y_i \) the initial composition in helium and \( \alpha \); in these models we impose a change in the radius and luminosity, and the same ratio \( Z/X_{surf} \) of the final superficial chemical composition than that of model \( M_1 \).

Finally set 3 consists of five models \( M_i \), for \( i = 10...14 \) which differ from set 2 by the value of \( Z/X_{surf} \), and includes a model where \( R = R_\odot \) and \( L = L_\odot \) (\( M_{14} \)). A change of composition is interesting to study because the presence of varying magnetic field in the subsurface layers may lead to an apparent change of hydrogen/helium thermodynamic conditions in solar models with some impact on the density and pressure of the subsurface layers.

The goal was to mimic variations of the radius and luminosity which could be extrapolated to variations during the 11-year solar activity cycle. For instance, \( M_4 \) and \( M_9 \) have a luminosity varying in the same way than the radius, whereas \( M_6 \) and \( M_7 \) present luminosity and radius varying in an opposite way. The relative variations \( 2 \times 10^{-4} \) for the radius are chosen larger than the supposed observed ones in order to avoid numerical accuracy problems and to better see their influence on the subsurface layers. In the different cases, the models converge to get the calibrated radius within a precision of \( 10^{-5} \) that means within 7 km. The relative change in the luminosity of \( 10^{-3} \) is similar to the observed solar irradiance variation during the 11-year cycle. The way we introduce it in solar models leads generally to a variation of luminosity on the production of energy instead on the superficial layers. So we see only an indirect effect of the luminosity change on the subsurface layers through changes in temperature and radius, their effect is not negligible for models 10 and 13.
3.1. Changes of the subsurface layers

We estimate how the subsurface layers react to the induced changes. For that, we estimate the differences of the following variables between the different models and the seismic model: the mass $m$, the temperature $T$, the density $\rho$, the pressure $p$, the sound speed $c$, the adiabatic exponent $\Gamma_1 = (\frac{d\ln p}{d\ln \rho})_{\text{adia}}$, the density scale height $H_\rho = -\left(\frac{d\ln \rho}{dr}\right)^{-1}$, the pressure scale height $H_p = -\left(\frac{d\ln p}{dr}\right)^{-1}$, the radiative temperature gradient $\nabla_{\text{rad}} = \frac{d\ln T}{d\ln p}$, the real temperature gradient $\nabla_{\text{real}} = \min(\nabla_{\text{rad}}, \nabla_{\text{adia}})$, the Rosseland opacity coefficient $\kappa$ and the gravitational energy $E_g$.

\footnote{Note that each fractional radius $x_i$ is reported to the reference radius, i.e. $x_i^{\text{new}} = x_i R_i/R_{\text{ref}}$ to get all models refered to the reference model.}
Table 1: List of the calculated models.

| Model | \(X_i\) | \(Y_i\) | \(\alpha\) | \(R/R_\odot\) | \(L/L_\odot\) | \((Z/X)_{\text{surf}}\) | \(T_{\text{eff}}[K]\) |
|-------|---------|---------|-----------|-------------|-------------|-----------------|------------------|
| **Seismic model** | | | | | | | |
| 1     | 0.70642 | 0.27468 | 2.03990   | 1.0000     | 1.0000      | 0.02447         | 5777.52          |
| 2     | 0.70642 | 0.27468 | 2.03751   | 1.0002     | 1.0000      | 0.02447         | 5776.97          |
| 3     | 0.70642 | 0.27468 | 2.04260   | 0.9998     | 1.0001      | 0.02447         | 5778.30          |
| **Set 1** | | | | | | | |
| 4     | 0.70632 | 0.27478 | 2.04033   | 1.0002     | 1.0010      | 0.02447         | 5778.40          |
| 5     | 0.70642 | 0.27468 | 2.03757   | 1.0002     | 1.0000      | 0.02447         | 5776.98          |
| 6     | 0.70651 | 0.27458 | 2.03463   | 1.0002     | 0.9990      | 0.02447         | 5775.52          |
| 7     | 0.70634 | 0.27476 | 2.04508   | 0.9998     | 1.0010      | 0.02447         | 5779.56          |
| 8     | 0.70644 | 0.27466 | 2.04214   | 0.9998     | 1.0000      | 0.02447         | 5778.10          |
| 9     | 0.70653 | 0.27457 | 2.03937   | 0.9998     | 0.9990      | 0.02447         | 5776.67          |
| **Set 2** | | | | | | | |
| 10    | 0.69772 | 0.28186 | 2.06148   | 1.0002     | 1.0010      | 0.02676         | 5778.42          |
| 11    | 0.69791 | 0.28166 | 2.05575   | 1.0002     | 0.9990      | 0.02676         | 5775.53          |
| 12    | 0.69774 | 0.28184 | 2.06624   | 0.9998     | 1.0010      | 0.02676         | 5779.57          |
| 13    | 0.69793 | 0.28164 | 2.06048   | 0.9998     | 0.9990      | 0.02676         | 5776.69          |
| 14    | 0.69783 | 0.28175 | 2.06098   | 1.0000     | 1.0000      | 0.02676         | 5777.00          |

Note. — Model 1 is the reference model (Turck-Chièze et al. 2001; Couvidat, Turck-Chièze and Kossivichev, 2003), \(X_i\) and \(Y_i\) are respectively the initial composition in hydrogen and helium, \(\alpha\) is the mixing-length parameter, \(R\) and \(L\) are the radius and luminosity, \((Z/X)_{\text{surf}}\) is the final superficial abundance and \(T_{\text{eff}}\) the effective temperature.
Fig. 2.— Difference for the physical variables between models of set 2 calibrated with different combinations of radius and luminosity (see text) and the seismic solar model. The curves of set 1 are exactly superimposed with those of set 2. \( M_2 \) (or \( M_3 \)) and \( M_5 \) (or \( M_8 \)) looks like \( M_4 \) and \( M_6 \) (or \( M_7 \) and \( M_9 \)).
Fig. 3.— Difference for the physical variables between models of set 3 and the seismic solar model (same principle than in Figure 2 but including an additional effect in the composition).
Figures 2 and 3 illustrate the changes with depth of these variables in comparison with the seismic one. We focus our study to the zone located above \(0.96 R_\odot\), that is the zone where changes in the subsurface stratification have been found by \cite{lefebvre_kosovichev_2005} along the solar cycle. The upper limit is fixed at \(0.998 R_\odot\), beyond which the superadiabatic zone extends, the turbulence acts strongly and the rotation changes quickly. We have chosen the zone where the \(f\)-modes have a good sensitivity. At a first glance, we notice that most of these variables present non negligible variations in the studied region.

We first comment on the common trends of both figures:

- The variations shown on the different panels are dominated by the change in radius imposed in the calibration and at the second order by the change of composition for set 3.

- The way we introduce the change in luminosity acts on the nuclear burning layers and has a negligible effect on the subsurface layers. So for clarity, we have not plotted the models 2 and 3 of set 1, nor 5 and 8 of set 2, in fact their curves are aligned with the models having the same radius in Figure 2. In contrast the observed luminosity variation of \(10^{-3}\) along the solar cycle is most likely coming from the very external layers. In fact, the variation of radius and of the photospheric temperature of our models produces a change of luminosity determined by the Stefan’s law. It is generally too small to exhibit any structural effect, except for models 10 and 13 where the variation of luminosity coming from the variation of radius and temperature is near from the imposed variation of luminosity.

- The behaviors of \(c\), \(H_p\) and \(H_\rho\) are very similar i.e. a bump around \(0.99 R_\odot\) with opposite variations between models of different radius; this bump exists also for the temperature. This position corresponds to the transition between neutral He and \(\text{He}^+\) as discussed in \cite{lopes_etal_1997}.

- The differences of \(\Gamma_1\) and \(\nabla_{\text{real}}\) have similar variations and present a double peak near \(0.99 R_\odot\) with almost equal amplitude. In addition to the bump at \(0.99 R_\odot\), the second bump could be due to the transition \(\text{H}^+ / \text{H}\). It is reasonable that in this convective region, the gradient of the structure follows the adiabatic exponent.

- The variations of \(\kappa\) and \(\nabla_{\text{rad}}\) present a sign change at \(0.99 R_\odot\), connected to the variation of the opacities in the region where the light elements are partially ionized. The variation of the pressure and density modifies the corresponding opacity coefficients.

Figure 3 differs nevertheless from Figure 2. Figure 2 shows two symmetrical groups of models depending on the value of the calibrated radius. In the set 3 another change comes
from the modification of the heavy elements contribution (about 10%) which induces a small
tchange in helium (about 3%) and hydrogen. In order to separate properly the effect of radius
from the effect of composition, we have calculated model \( M_{14} \) that includes only the change
in composition. We show in Figure 3 the changes induced by the composition effect. As
an evident consequence, we loose the symmetry between the group of models with a greater
radius compared to the models with a smaller one, this is mainly visible on the temperature
and \( \Gamma_1 \) differences.

These first behaviors indicate that the subsurface layers above \( 0.96 \, R_\odot \) are significantly
affected by a change in radius and composition. To go further, we shall estimate the differences in \( f \)-mode frequencies issued from these models.

3.2. Calculation of the theoretical \( f \)-modes for the different models

We compute for each model the theoretical \( f \)-mode frequencies using the oscillation
code ADIPLS (Christensen-Dalsgaard 1982) to see the impact of the different changes on
these quantities. Figure 4 shows the relative difference of frequencies between each model
and the reference model \( M_1 \) versus the corresponding absolute frequencies.

We note first that the \( f \)-mode frequencies of a model with a larger radius are smaller
than those of the reference model. The change of \( 2 \times 10^{-4} \) in radius leads to a relative change
of about \( 3 \times 10^{-4} \) on \( f \)-mode frequencies.

The left panel of Figure 4 corresponds to sets 1 and 2. The difference associated to
\( M_4 \) and \( M_6 \) is slightly bigger in absolute value from the difference associated to \( M_7 \) and
\( M_9 \) due to the value of the reached real radius: +135 km for \( M_4 \) and \( M_6 \) and -143 km
for \( M_7 \) and \( M_9 \) (this difference is within the precision imposed of the model radius). The
frequency dependence has almost a flat behavior, whereas this is not the case for set 3 in
the right panel where the relative difference has an extremum around \( \approx 1300 \mu Hz \) due to
the additional effect of the change in composition, which produces non-uniform changes as
already discussed.
Fig. 4.— Relative difference of $f$-modes frequencies between models of set 2 (left panel) or models of set 3 (right panel) and $M_1$ versus the absolute frequencies. We do not plot the difference between the two reference models $M_{14}$ and $M_1$ because this difference is of the order of $10^{-5}$.
4. Numerical inversion of theoretical $f$-mode frequency to infer subsurface stratification changes

In this section we do the inversion of the $f$-mode frequencies for degrees $l$ between 100 and 300, in a range slightly bigger than the one chosen previously with the corresponding solar observational quantities (Lefebvre & Kosovichev 2005) to see the reproducibility of the radial variation for known models, to qualify the procedure and to extrapolate a new estimate of the radius variation along the solar cycle.

We use the same formalism as in Lefebvre & Kosovichev (2005) to infer the changes in the position of subsurface layers from the $f$-mode frequency variations. A relation between the relative frequency variations $\delta \nu/\nu$ for $f$-modes and the associated Lagrangian perturbation of the radius $\delta r/r$ of the subsurface layers has been established by Dziembowski & Goode (2004):

$$
\left( \frac{\delta \nu}{\nu} \right)_l = -\frac{3l}{2\omega^2 I} \int dl \frac{g \delta r}{r \cdot r}
$$

(1)

where $l$ is the degree of the $f$-modes, $I$ is the moment of inertia as classically defined in Dziembowski & Goode (2004), $\omega$ is the angular frequency of the eigenpulsation ($\omega = 2\pi \nu$) and $g$ is the gravity acceleration. The validity of this equation is limited to the case where any magnetic field effect is explicitly introduced in the equations (Dziembowski & Goode 2004) which is the case in this work. It also supposes that, if $y$ and $z$ are respectively the vertical and horizontal eigenfunctions, the property of $f$-modes $y^2 = l(l + 1)z^2$ is satisfied.

This equation allows us to obtain $\delta r$ from $\delta \nu/\nu$. For these inversions, we used $M_1$ as the reference model and a standard Regularized Least-Square technique, since Equation (1) defines an ill-posed inverse problem (Tikhonov & Arsenin 1977). The validity of this equation for the different cases analyzed here will be discussed in the last section.
Fig. 5.— *Left panel:* Radial variation $\Delta r$ as a function of the fractional radius $x = r/R_{\text{ref}}$, obtained by inversion of Equation 1 with frequency variations represented in the left panel of Figure 4 for set 2. The error bars are the standard deviation after averaging over a set of random noise added to the relative frequencies reconstructed in the right panel. *Right panel:* For each model, relative variation of frequencies $\Delta \nu/\nu_{\text{ref}}$ as a function of the degree $l$. The points with error bars (that have been arbitrarily added to the data, such as $\sigma = 0.5 \times 10^{-6}$) represent the input data, and the solid curves are the result of direct integration of Eq. 1 providing the solutions in the left panel.
Fig. 6.— Idem than Figure 5 for models of set 3. Error bars have been arbitrarily added to the data, such as $\sigma = 1 \times 10^{-6}$. Note here the behavior of the curves near $x = 0.99$ reflecting a non-uniform change in the subsurface stratification. For clarity we do not plot the relative variation of frequencies for model $M_{14}$ that is 10 times smaller than for the other models.
Figures 5 and 6 show results from inverting Equation 1 for each computed model. These figures also show the frequencies computed by integrating the solutions using Equation 1, and how well they match the model frequencies within the error bars. The uncertainties were set arbitrarily to $0.5 \times 10^{-6}$ and $1 \times 10^{-6}$ respectively for the need of the inversion process and the will to mimic real variations. This fact guarantees the good quality of the inversion. The main characteristics of our solutions are:

- The inversion solutions plotted in Figure 5 for set 2 lead to an almost uniform radial variation. The amplitude of the radial variation is similar to the one imposed at the surface, as expected from the relative variations of frequencies in the left panel of Figure 4, i.e. about $\pm 140$ km with a sign respecting the nominal one. Nevertheless, there is a small difference near the surface (see also next section), which is due to the poor spatial resolution of the kernels at the surface. This problem has already been raised in Lefebvre & Kosovichev (2005) where the shape of the kernels is shown.

- The solutions for set 3 shown in Figure 6 are slightly different. Below $0.98 R_{\odot}$, the variation is constant but closer to the surface there are non-monotonic changes in the stratification with a bump centered at $0.99 R_{\odot}$. The uncertainty of the localization of this bump is governed by the characteristic width of our kernels that is about $0.005 R_{\odot}$.

- These variations are different in shape and in amplitude to those found by Lefebvre & Kosovichev (2005). This is not surprising because we do not yet introduce the dynamical processes which generate the solar cycle.

- Since the only difference between some models of set 2 and some others of set 3 is the different value of $(Z/X)_{sf}$, this bump is clearly a consequence of the introduction of this change in composition affecting the subsurface layers through the pressure and mainly the equation of state. We note that the value of $\Delta R$ is not any longer about 140 km but about 110 km (135-20 km) for models $M_{10}$ and $M_{11}$ and -160 km (142+20 km) for $M_{12}$ and $M_{13}$ at $0.96 R_{\odot}$, this is easily explained by looking at the $\Delta R$ between the two reference models of about 20 km at this depth. It is clear that a change of composition has an effect largely below the superficial layers.

In fact the inversion process implies that the solution is not unique. By adjusting the regularisation parameters, we can find another profile with two bumps for $\Delta r$ with also a very good fit to frequency. That solution has not been retained because (i) the variations at the surface are not in agreement with the input radius variation as cited in Table 1, (ii) the error bars are bigger and (iii) the curves are more oscillating. Moreover, we cannot presently give a physical interpretation to this solution. On the contrary, for the first solution, we discuss in section 5 how we understand the present inversions.
5. Discussion

Equation 8 of Dziembowski & Goode (2004) supposes that the radial position \( r \) used in Equation 1 is the lagrangian radius. \( f \)-modes are surface oscillation trapped waves, so each mode oscillates around an equilibrium radius. We relate in this section this radial position to that of the structure.

5.1. Validity of the procedure of inversion

The extraction of the solar subsurface structure and the photospheric radius variation along the solar cycle is not so easy to determine and several papers have been published with different conclusions (Dziembowski, Goode & Schou 2001; Antia 2003; Sofia et al. 2005). The most recent work of Dziembowski & Goode (2004) shows the difficulty to obtain a general expression for the inversion, so this work allows to test such a procedure. We verify in this section if the displacement \( \Delta r \) of Equation 1 corresponds to the radial displacement at a given mass for the different cases studied.
Fig. 7.— Left panel: Radial displacement $\Delta r$ of a given $m(r)$, $p(r)$, $c, H_p, \Gamma_1$ and $\kappa$ defined in the reference model ($M_1$) for models of set 2 (the curves of set 1 models superpose the previous ones). These curves have to be compared to the left panel of Figure 5. Right panel: Idem with models of set 3.
Fig. 8.— Radial displacement $\Delta r$ issued from the inversion (same curves than in the left panel of Figure 6 added with the associated displacement at $H_p$ constant as plotted in the right panel of Figure 7 (here dashed curves associated at the same color for the corresponding model). Note the good superposition of the different curves.
With this purpose, we calculate for each set (sets 1 and 2 lead to the same result) the radial displacement of a layer corresponding to a given value of the structural quantities, i.e.

\[ \Delta r = (r_N - r_1)_{qN=q1} \]

where \( q \) represents a model structure quantity like \( m, c, H_p \) or \( \Gamma_1 \), and \( N \) is the model number.

Figure 7 shows the panels representing \( \Delta r \) at constant \( m, c, H_p \) and \( \Gamma_1 \). For sets 1 or 2, we have the same behaviour for all the quantities and Equation 1 can be applied without any doubt. For the set 3, the radial difference \( \Delta r \), computed by the procedure described above, differs from one quantity to another. The only curve similar in amplitude and in shape with the one computed by inversion of \( f \)-mode frequencies in Figure 6 is the curve computed at fixed pressure height \( H_p \). Effectively Figure 8 shows an excellent superposition of the corresponding curve and the radial displacement \( \Delta r \) issued from the inversion. This agreement, in shape and amplitude, demonstrates that the validity of equation 1 in this case requires to associate the radial displacement to this quantity. It is in fact not surprising because the \( f \)-mode frequencies are naturally sensitive to the pressure scale height.

We note also that the inversion is not able to reproduce precisely the behavior very near the surface due to the lack of sensitivity of the \( f \)-modes. But we can deduce from this study an estimate of the uncertainty on the determination of the photospheric radius variation: we estimate it of the order of 15%.

5.2. A new prediction on the solar radius variation along the solar cycle

We show in this paper that a pure variation in the solar radius produces changes in the subsurface layers that are in the same zone than that studied with the observed \( f \)-modes by Lefebvre & Kosovichev (2005). Such a change is characterized by changes in the subsurface stratification and more exactly variations in the computed position of the layers. We show that the changes detected by \( f \)-modes are physically related to the variation of the pressure. In our modelling we are able to produce non-uniform changes by slightly modifying the chemical composition below the solar surface. This is not excluded if one introduces dynamical processes in keeping the same constraints on luminosity and radius. The two studies show different stratifications of the outlayers but with similar effects on the \( f \)-mode frequencies. Considering larger effects on the \( f \)-mode frequencies than in the solar case, we propose to deduce from this study an estimate of the variation of the solar radius along the cycle. From \( \Delta \nu/\nu \) of about \( 3 \times 10^{-4} \) for a \( \Delta r \) of about 140 km, we lead to a solar radius variation along the solar cycle of about \( 7 \pm 1 \) km for the observed variations in frequency of about \( 1.5 \times 10^{-5} \). This result is consistent with the radius extrapolated by Lefebvre & Kosovichev (2005) (see respectively Figures 1 and 3 of this article). It is interesting to note that this
radius variation estimate is also compatible (in order of magnitude) with the observed low degree acoustic mode variation of about 0.4 µHz along the solar cycle of the low degree $p$-modes (Fossat et al. 1987; Chaplin, Elsworth & Isaak 2001).

5.3. Perspectives

This study represents the first step on the way to explain the variations of the subsurface stratification along the solar cycle and reinforces the interest of the $f$-modes. The next step will be the development of models including magnetic field and other dynamical processes, which will permit a more realistic study. Nevertheless it will suppose also an improved expression for the inversion of $f$-modes which is not yet obtained. We believe that the introduction of a magnetic field will influence the stratification of the subsurface layers. The study by Nghiem et al. (2006) pointed out non-uniform radial changes, depending on the importance of the magnetic pressure. Moreover, it will be also interesting to take into account the differential rotation and consequently some asphericity to confront them to the subsurface latitudinal stratification over the 11-year cycle.

We would like to emphasize that a better knowledge of these subsurface layers will contribute to our understanding (i) of the dynamics of the 11-year solar cycle and (ii) of the Sun-Earth relationship for space climate. This supposes in parallel the simultaneous measurement of the radius and frequencies change. This study is in the framework of coming future space missions: SDO (see http://sdo.gsfc.nasa.gov/) and PICARD (see http://smsc.cnrs.fr/PICARD/ and Thuillier, Dewitte & Schmutz (2006)).

The DynaMICCS/HIRISE perspective (Turck-Chièze et al. 2006, 2008), proposed in the framework of the ESA Cosmic Vision 2015-2025, could bring even more data on all sources of internal variability by putting together instruments which will follow seismically all the layers down to the core together with instruments measuring the variability of the above atmosphere.

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