Research of Airfoil Optimization Based on CST Method and Genetic Algorithm

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Abstract. Aiming at the aerodynamic optimization problem of guided ammunition airfoil, genetic algorithm is used to optimize the lift-drag ratio of the airfoil based CST geometric description method with given inflow velocity and fixed angle of attack. Firstly, the parameter space generated by CST parameterized airfoil description method is performed with binary code and the basic operations of genetic algorithm such as selection, crossover and mutation. Then, the external CFD solver is called to automatically divide the grid and calculate the lift-drag ratio through the control script. Finally, the target solution is obtained by searching. The calculation results show that this method can obtain a optimal target solution in relative less generation for a group of airfoils generated randomly in a certain search space.

1. Introduction

The control efficiency and control characteristics of an aircraft are directly related to the aerodynamic characteristics of its control airfoil. To design optimized control airfoil that meets the target aerodynamic characteristics is a fundamental problem in aircraft design. Since the equations used for aerodynamic calculations are highly nonlinear partial differential equations, it is difficult to solve by a deterministic numerical optimization algorithm. However, genetic algorithm[1] doesn’t depend on the gradient, the single-peak hypothesis of objective function, and the global searchability, which can well solve such problems, and thus has been widely used.

Whether the genetic algorithm or other optimization algorithms, it is necessary to limit a certain parameter space for searching optimization. The properties of parameter space have a direct impact on the computation time and the characteristic range of design space. The existing airfoil parametric description methods include Bessel or B-spline interpolation control method[2], cubic spline interpolation method of control points[3], linear perturbation method of the shape function (Hicks Henne function)[4], and CST[5-7] method, etc. However, the airfoil method such as polynomial or spline interpolation cannot directly reflect the geometry of the airfoil; the linear perturbation method of the shape function needs to superimpose part of the design variables on a given airfoil to generate a new airfoil, which limits the search space of parameters; the CST parametric airfoil description method uses the geometric parameters such as the leading edge radius of the airfoil as part of the control parameters, which can reflect the geometry of the airfoil more intuitively, with fewer
parameters and controllable fitting precision. Therefore, this paper chooses the CST method to generate the parameter space of the airfoil.

This paper uses the genetic algorithm to search the optimization, which is the maximum lift-drag ratio of the subsonic missile control airfoil with a given inflow velocity and a fixed angle of attack, with optimizing in the search space established by CST parametric airfoil description method. Through binary coding the airfoil description parameters, performing the basic operations of the genetic algorithm such as copying, crossover, and mutation, invoking the external meshing tool to automatically mesh the new airfoil and the external CFD solver to calculate the fitness, it finally gets the target solution. The calculation results show that the proposed method can get a more optimized target solution in less generation.

2. Formatting two-dimensional airfoil optimization model
For the aerodynamic optimization problem of genetic algorithm, the expression is as follows. The objective function is given by equation (1).

\[
R_{\text{at}}(i) = \text{Max} \left[ F(X_i) \right]
\]

where \( F(X_i) \) is the lift-drag ratio of the airfoil calculated by CFD solver with design variable \( X_i \), no explicit expression.

The design variables are given by equation (2).

\[
X_i = \{ R_{\text{le}}, \beta, b_1, b_2, b_3, b_4, Z_e \}
\]

where \( X_i \) is A set of design variables produced when the CST airfoil description method is used; \( R_{\text{le}} \) is the leading edge angle of the wing; \( Z_e \) is the trailing edge angle of the wing; and \( b_1, b_2, b_3 \) and \( b_4 \) are parameters necessary for the CST airfoil description method.

The constraint conditions are given by equation (3).

\[
\begin{align*}
R_{\text{le}} & \in [R_{\text{le}}_{\text{min}}, R_{\text{le}}_{\text{max}}] \\
\beta & \in [\beta_{\text{min}}, \beta_{\text{max}}] \\
b_1 & \in [b_{1\text{min}}, b_{1\text{max}}] \\
b_2 & \in [b_{2\text{min}}, b_{2\text{max}}] \\
b_3 & \in [b_{3\text{min}}, b_{3\text{max}}] \\
b_4 & \in [b_{4\text{min}}, b_{4\text{max}}] \\
Z_e & \in [Z_e_{\text{min}}, Z_e_{\text{max}}]
\end{align*}
\]

The generative rule of variables is genetic algorithm, which generates new variable spaces and new solutions by the treatment of design variables such as encoding, selecting, copying, crossing, mutating, etc.

Therefore, an optimization process is generated based on the above optimization model, as shown in figure 1.
3. Key technologies

3.1. Genetic algorithm
Genetic algorithm is a global probability search algorithm based on genetic mechanism. This algorithm replaces the parameter space of the problem with the coding space, uses the fitness function as the evaluation basis, uses the coding population as evolutionary basis, implements the selection and genetic mechanisms by the genetic manipulation of individual bit strings in the population, and establishes the iterative process. In this process, a bit or bits of the encoded bit string is randomly selected for cross recombination, and a new bit string combination is generated according to the fitness, so that the new generation coding combination is superior to the old generation coding combination, which is finally mapped to the parameter space of the problem, and the optimal solution is continuously approached to achieve the purpose of solving the problem.

The genetic algorithm consists of the following basic steps:

1. Determine the objective function, establish an optimization model, and select the variable set and its constraint conditions in the parameter space;
2. Determine the encoding strategy to determine a mapping relationship between the variable set X and the encoding space S in the parameter space;
3. Define the fitness function F(x);
4. Determine the genetic strategy, select the population size n and evolution generations, determine the selection crossover, mutation method and crossover probability Pc, mutation probability Pm and other parameters;
(5) randomly initialize the generated population p;
(6) decode the coded population and substitute into the optimization model to solve the fitness F(x);
(7) according to the established genetic strategy, the selection, hybridization and mutation operators are applied to the group to form the next generation group;
(8) determine whether the performance of the group satisfies a certain indicator and determine the conditions for termination of the algorithm.

3.2. CST Parameterized airfoil description method

The CST method uses a class function and a shape function to represent an airfoil with closed-trailing edge, representing functions for the upper and lower airfoils of which are given by equation (4).

\[
\begin{align*}
(\xi)_u & = C(\varphi) * S_u(\varphi) + \varphi * \Delta \xi_u \\
(\xi)_l & = C(\varphi) * S_l(\varphi) + \varphi * \Delta \xi_l
\end{align*}
\]  

(4)

where the upper and lower airfoils are selected as symmetrical airfoils due to the randomness of the aerodynamic surface of the guided munitions; \(\varphi = x/c\), is a dimensionless x-direction coordinate; \(\xi = z/c\), is a dimensionless z-direction coordinate; \(\Delta \xi = z_u/c\), is a dimensionless thickness-chord ratio of the trailing edge; \(C(\varphi)\) is the class function, \(S_u(\varphi)\) and \(S_l(\varphi)\) are shape functions of the upper and lower airfoils respectively, which are given by equation (5) and (6).

\[
C(\varphi) = \varphi^{N_{\text{a}}w} * (1 - \varphi)^{N_{\text{a}}t}
\]  

(5)

\[
S(\varphi) = \sum_{i=1}^{n} A_i * S_i(\varphi)
\]  

(6)

For the airfoil with rounded head and pointed tail, take \(N_{\text{a}}w = 0.5, N_{\text{a}}t = 1\).

There are many expressions of \(S(\varphi)\), including Bernstein polynomial weighted sum, B-spline basis function weighted sum, and so on. Since the CST method which defines the shape function based on the Bernstein polynomial has a high filtering ability for the abrupt oscillation of the design parameters, the Bernstein polynomial is selected as the shape function, as shown in the equation (7).

\[
\begin{align*}
S(\frac{X}{c}) & = \sum_{i=0}^{n} b_i * \left [ K_n^i(\frac{X}{c})(1 - \frac{X}{c})^{n-i} \right ] \\
K_n^i & = \frac{n!}{i!*(n-i)!}
\end{align*}
\]  

(7)

From equations (4), (5), (6) and (7), an obtained equation is given by equation (8).

\[
\begin{align*}
\left( \frac{z}{c} \right)_{\text{up}} & = \left( \frac{x}{c} \right)^{0.5} \left( 1 - \frac{x}{c} \right) * \sum_{i=0}^{n} b_i * \left [ \frac{n!}{i!*(n-i)!} * \left( \frac{x}{c} \right)^i * (1 - \frac{x}{c})^{n-i} \right ] + \frac{x}{c} * \frac{z_{\text{a}}}{c} \\
\left( \frac{z}{c} \right)_{\text{low}} & = - \left( \frac{z}{c} \right)_{\text{up}}
\end{align*}
\]  

(8)

In order to reduce the calculated amount without losing the accuracy of the airfoil representation, a 5th-order Bernstein polynomial is adopted with \(b_0 = (2R_{le}/c)^{0.5}\) and \(b_2 = \tan \beta + z_{le}/c\), where \(R_{le}\) is the leading edge radius, and \(\beta\) is the tangent angle of the upper airfoil at the trailing edge of the airfoil.

Therefore, the parameter search space consists of 7 variables: \(X = \{ R_{le}, b_0, b_2, b_3, b_4, \beta, z_{le} \}\).
3.3. Fitness solution

Objective function $F(x_i)$ is calculated by CFD software. Since the optimization process involves multiple cycles and iterations, the computational meshing and fluent calculations of the airfoil use script automatic loading technology, to automatically generate envelopes and divides the mesh for each set of formatted airfoil description data generated by the CST method, which are then imported into fluent for calculation, and the calculation results are imported into MATLAB as reference data for genetic algorithm evolution optimization.

The meshing of airfoil profile uses an O-shaped structured mesh, and the calculation area is a circular area with a radius of 15 m centered on the apex of the leading edge of the wing. The key parts are encrypted and a total of 19,680 computing nodes are divided. The meshing diagram is shown in figure 2. The meshing quality is shown in figure 3.

The CFD calculation uses a two-dimensional implicit solver based on pressure, and the turbulence model uses the Spalart-Allmaras model. The ideal gas model is used for flow field materials, the viscous model uses the Sutherland model, the far field condition is set to the inflow velocity of 0.8 Mach, the temperature of 300K and 4 degrees of attack angle, the solver selects the pressure-velocity coupling model. In addition to the discrete format of pressure adopts second-order mode, the discrete formats of the rest variables adopt second-order upwind mode. Use multi-grid technology to speed up calculation and convergence. The number of iterations is 500. After the calculation is completed, the calculation results of the lift coefficient and the drag coefficient of the airfoil are saved in a text file for the genetic algorithm to call.

![Figure 2. Schematic diagram of meshing.](image)

In order to verify the correctness of the CFD solving method, the calculated data is compared with the reference data[8] under the condition of 0.75 Mach and 2.57 degrees of attack angle. The results are shown in table 1. The results show that the CFD solving results are credible.

| Benchmark Airfoil | NACA0012 | C1     | Cd    |
|-------------------|----------|--------|-------|
| Reference         | 0.4363   | 0.0242 |       |
| Calculated        | 0.4395   | 0.0237 |       |
| Error             | 0.0032   | 0.0005 |       |

4. Examples and optimization results

In this example, the crossover probability $P_c$ is equal to 0.9, the mutation rate $P_m$ is equal to 0.1, the population number is 10, and the evolution algebra is 50. The first generation of airfoils was produced in a random manner. The constraint conditions of each variable are shown in table 2.
Table 2. List of variable constraint conditions.

| Variable | Lower limit | Upper limit |
|----------|-------------|-------------|
| $Rle$    | 0           | 0.1         |
| $\beta$  | 0           | 10          |
| $b_1$    | -0.5        | 0.5         |
| $b_2$    | -0.5        | 0.5         |
| $b_3$    | -0.2        | 0.2         |
| $b_4$    | -0.08       | 0.08        |
| $Ze$     | 0           | 0.003       |

The first generation of airfoils is shown in figure 3. After 50 generations of optimization, the last generation of airfoils is shown in figure 4. The separation point of the boundary layer in the first generation of optimal airfoil is about 0.3 times the chord length, and the separation point of the boundary layer in the last generation of optimal airfoil is moved back to about 0.55 times the chord length.

Figure 3. First generation of airfoil.

Figure 4. Last generation of airfoil.

The evolution diagram of lift-drag ratio is shown in figure 5.
In order to verify the stability of the algorithm, the algorithm is repeatedly run 10 times to generate the lift-drag ratio of the optimal airfoil as shown in figure 6. The computational results show that the algorithm has certain stability. After searching, the average lift-drag ratio of the generated airfoil is 18.2543, and the deviation is within 5%.

5. Conclusions
This paper uses a genetic algorithm to aerodynamically optimize the airfoil based on the CST geometric description method with the optimal lift-drag ratio at a given inflow velocity and fixed attack angle. The calculation results show that, for a set of airfoils randomly generated in a certain search space, this method can obtain a more optimized target solution in relative less generation.

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