Far-infrared induced current in a ballistic channel – potential barrier structure.

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I. INTRODUCTION

Attempts have been made during the past few years to observe effects of the application of a far-infrared (FIR) field on electrons propagating coherently in narrow quantum channels. In focus of such investigations has been quantum point contacts (QPCs) defined by split-gate depletion of a two-dimensional electron gas in a GaAs/AlGaAs heterointerface. So far the experiments employing both transverse and parallel polarization have been explained by heating and rectification effects, while clear evidence of FIR-field influence inside the channel is still missing.

Considering the pronounced transverse quantization into waveguide modes it becomes particularly interesting to consider transverse polarization of the FIR field, since it allows strong mode coupling even for a homogeneous FIR field. For transverse polarization it is well known from theoretical considerations that transitions between modes take place mainly at points where the mode energy separation, $E_m - E_n$, equals the photon energy $\hbar \omega$. At such points momentum conservation is possible. In the case of a QPC the mode energy separation changes and resonance conditions can be fulfilled only in a small region, which reduces the sensitivity to the FIR field. This could explain why not even experiments using transverse polarization have succeeded in providing evidence of coherent FIR-field influenced transport.

The natural way to increase the sensitivity to a FIR field is to extend the region of resonance by considering a longer channel in which the width is constant. To consider long coherent channels has in the last few years become realistic due to progress in fabrication techniques. Since FIR-field induced transitions between propagating modes alone will not change the transmission probability, it is necessary to incorporate some detection mechanism that discriminates between modes. This idea has been developed in the special case when only one mode enters the channel and a QPC is used for detection, which leads to a reflection of excited electrons.

In this work we consider the complementary case of a multi-mode channel. A new feature is that there is a possibility for cancellation if one electron climbs the mode spectrum while another one descends. Since there is no lower limit to the mode energy separation when the restriction of only one propagating mode is relaxed, a wider range of frequencies is of interest. For detection we use a combination of an adiabatic widening and a line-gate barrier, which leads to an enhanced transmission for excited electrons. We find a general expression for the scattering states, that includes all relevant mode mixing, by solving an eigenvalue problem. An explicit analytical expression is derived which is valid in the limit of weak FIR-fields, where resonances between pairs of transverse modes will dominate. We also propose an experimental method for finding the mode spectrum using only fixed frequency FIR-sources.

II. MODEL SYSTEM

We consider a quasi-one-dimensional channel that smoothly connects two reservoirs. The channel consist of three regions, each with its own purpose (see Fig. 1). First, there is a resonance region featuring strong size quantization, in which an external high frequency electric field polarized across the channel can induce mode transitions. Second, there is an adiabatic widening in which there is a conversion of the energy stored in the transverse direction into kinetic energy for longitudinal motion. Finally, there is a barrier that blocks slow electrons.

Experimentally such a system can be realized in different ways, but in order to see resonances fully developed it takes a system that preserves coherence all along the channel. We primarily have a split-gate channel in a GaAs/AlGaAs-interface in mind. The barrier can then be created by a line gate across the channel. Unlike a split-gate barrier it gives the same barrier for all modes. The height of this barrier is adjusted so that no transport takes place in the absence of the FIR-field. Then only those electrons that have been excited can pass. Since
there is mode pumping on one side only a photocurrent will be generated even in absence of a driving voltage. We are interested in this zero bias photocurrent.

We now make a separation Ansatz for the wave function:

\[ \Psi(x, y, t) = \sum_m \Psi_m(x, y) \Phi_m(x, y) \]  

(3)

Here \( \Phi_m(x, y) \) are solutions to the transverse eigenvalue equation:

\[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} \Phi_m(x, y) + U(x, y) \Phi_m(x, y) = E_m(x) \Phi_m(x, y). \]  

(4)

If the channel geometry changes slowly along the channel we can neglect \( x \)-derivatives of \( \Phi_m(x, y) \) when inserting the Ansatz into the Schrödinger equation and find

\[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} \Psi_m(x, t) + E_m(x) \Psi_m(x, t) - i\hbar \frac{\partial}{\partial t} \Psi_m(x, t) = -2i \cos(\omega t) \sum_m M_{mm'}(x) \Psi_m(x, t), \]  

(5)

where the intermode transition elements \( M_{mm'}(x) = -M_{m'm}(x) \) are given by:

\[ M_{mm'}(x) = \frac{\hbar e E}{2m^* \omega} \int \Phi^*_m(x, y) \frac{\partial}{\partial y} \Phi_{m'}(x, y) dy. \]  

(6)

We now look for solutions to Eq. (5) in the resonance region where \( U(x, y) \) is assumed to be \( x \)-independent and use the following Ansatz:

\[ \Psi_m(x, t) = \sum_l C_{ml} e^{i[Px - (E + \hbar \omega) t]/\hbar}. \]  

(7)

When this Ansatz is inserted into Eq. (5) we get an infinite set of coupled equations for the coefficients \( C_{ml} \):

\[ (P^2/2m^* - K_{ml}(E)) C_{ml} = -\sum_m i M_{mm'}(C_{m'(l-1)} + C_{m'(l+1)}), \]  

(8)

where:

\[ K_{ml}(E) = E + \hbar \omega - E_m, \]  

(9)

Eq. (8) can be put in the form of an eigenvalue problem:

\[ \tilde{K}_{ml,m'l'}(E) C_{m'l'} = P^2/2m^* C_{ml}, \]  

(10)

where:

\[ \tilde{K}_{ml,m'l'}(E) = K_{ml}(E) \delta_{m'm'} \delta_{l'l'} - i M_{mm'}(\delta_{l'(l'+1)} + \delta_{l(l'-1)}), \]  

(11)

with \( \delta \) being the Kronecker delta.

We will now use the fact that the resonance region has a finite length \( L \), and that our solution inside the resonance region arise due to an incoming wave from the reservoir of the following form:

\[ \Psi^{INC}_{n,E}(x, y, t) = \Phi_n(x, y) e^{i[\int p_n(x)dx - Et]/\hbar} \sqrt{p_n(x)/p_n(0)}, \]  

(12)

where \( p_n(x) = \sqrt{2m^*(E - E_n(x))} \). This in combination with the restriction: \( |M| \ll \hbar \omega \), allows us to get a good approximation to the scattering state using a reduced number of states in the eigenvalue problem in Eq. (10).
We pay attention to resonance effects only. For this reason we keep only one coefficient $C_{ml}$ for each mode, i.e. the one that gives the best kinetic energy match with the incoming state. Explicitly we define $t_{mn}$ as the integer that minimizes $|E_m - E_n + t_{mn} h\omega|$, and keep only coefficients $C_{mln}$. 

By this approach we miss the possibility of indirect resonant coupling via non-resonant states. On the other hand all ignored states necessarily give a kinetic energy mismatch of at least $h\omega/2$. This leads to a reduction of the effective coupling, via one or several such states, by at least a factor of the order $|M|/h\omega$. The characteristic length scale for population oscillation is at resonance: $\lambda_R = \lambda / |M|$, where $\lambda$ is the de Broglie wavelength and $K$ is the kinetic energy. Obviously this length scale is a factor of $h\omega/|M|$ larger for the indirect resonant coupling than for the direct one. Therefore there is a regime of weak FIR fields in which $\lambda_R^{ND} \gg L$, where it suffices to keep only the “most resonant” coefficients.

We consider a situation in which the transport is adiabatic outside the resonance region. This is realistic if the FIR field is sufficiently weak. Far from the resonance region, FIR field induced transitions are suppressed by a factor of the order $|M|/h\omega$, because of a kinetic mismatch of the order $\hbar \omega$. Near the resonance region the mode potentials are taken to be rapidly varying on the scale of $\lambda_R$ (but still slowly on the scale of $\lambda$). In this case there is no room, for a resonance to develop a significant change in the mode population. One can think of the FIR field as being suddenly switched on at $x = 0$ and suddenly switched off at $x = L$. In particular we can assume that the incident mode, $n$, is still fully occupied at $x = 0$ (where the resonance region begins) and that the mode population at $x = L$ (where the resonance region ends) will remain for $x > L$.

In the resonance region we get, using Eq. (3), the following expression for the scattering wave function:

$$
\Psi_{n,E}^{RES}(x, y, t) = \sum_{rm} a^{(r)}_{n,E} \Phi_{m}(y) e^{i p_{r,E}^{(r)} x / \hbar - (E + \hbar \omega t_{mn}) t / \hbar},
$$

$$
= \sum_{m} \delta_{mn} C_{n,E,m} e^{i p_{n,E}^{(r)} x / \hbar - (E + \hbar \omega t_{mn}) t / \hbar},
$$

where $p_{n,E}^{(r)} = \sqrt{2m^*K_{n,E}^{(r)}}$, and $C_{n,E,m}$ are the eigenvalues and the orthonormal eigenvectors of the following reduced eigenvalue equation:

$$
\sum_{m} \left( K_{ml,mn} - M_{mn} \right) C_{m'} = K C_m,
$$

and the coefficients $a^{(r)}_{n,E}$ are chosen such that only mode $n$ is populated at $x = 0$:

$$
\delta_{mn} = \sum_{r} a^{(r)}_{n,E} C^{(r)}_{n,E,m}.
$$

In the widening region ($x > L$) we get, using the assumption of adiabaticity, the following expression for the scattering wave function:

$$
\Psi_{n,E}^{WID}(x, y, t) = \sum_{m} t_{mn}(E)
\times \Phi_{m}(x, y) e^{i \int_{p_{n,E,m}(x)}^{p_{n,E,m}(L)} dx - (E + \hbar \omega t_{mn}) t / \hbar} \sqrt{p_{n,E,m}(x) / p_{n,E,m}(L)}
$$

(16)

where $p_{n,E,m}(x) = \sqrt{2m^*(E + \hbar \omega t_{mn} - E_m(x))}$, and the transition amplitudes $t_{mn}$ are given by:

$$
t_{mn}(E) = \sum_{r} a^{(r)}_{n,E} C^{(r)}_{n,E,m} e^{i p_{n,E}^{(r)} x / \hbar - (E + \hbar \omega t_{mn}) t / \hbar}.
$$

(17)

When we call $t_{mn}(E)$ transition amplitudes we implicitly assume that scattering takes place between states with the same momentum. The coupling is strong only between states whose difference in kinetic energy $|\Delta K|$ fulfills $\Delta K < |M|$. Thus for coupled states we have $|\Delta p| \approx \Delta K/2K < |M|/2K$, and assuming $|M| < K$ we can forget this difference in momentum.

Our scattering solution $\Psi_{n,E}(x, y, t)$ is valid for sufficiently weak FIR fields since it does not take reflections into account. In general there will be reflections both at the entrance and at the exit, but they will be suppressed if $|M| \ll K$. In principle we could handle more than the weak FIR field limit by using an exact matching condition that takes into account also the derivative of the wave function. However, in the strong field limit we would also have to account for multiple reflection from the entrance, the exit and from the barrier.

An electron near the band bottom will not fulfill the condition $|M| \ll K$. For such an electron we fail to make reliable predictions. On the other hand, since each energy interval is weighed equally in the Landauer formula we can ignore the contribution from slow electrons provided the energy interval of integration is much larger than $|M|$.

According to the Landauer approach we have, at zero temperature, the following expression for the current.

$$
I = \frac{2e}{\hbar} \int_{0}^{E_F} dE \sum_{n} T_{n}(E).
$$

(18)

For $T_{n}(E)$ which is the total transmission probability we have:

$$
T_{n}(E) = \sum_{m} W_{mn}(E) |t_{mn}(E)|^2 \Theta(E - E_n),
$$

(19)

where $W_{mn}(E)$ is the barrier transmission probability for an electron in mode $m$ at energy $E + t_{mn}\hbar \omega$. Assuming the barrier to be smooth we can ignore tunneling and use the following expression:

$$
W_{mn}(E) = \Theta|E + t_{mn}\hbar \omega - (E_b + E_{mn}(x_b))|
$$

(20)

Here $E_b$ is the potential energy due to the line-gate and $E_{mn}(x_b)$ is the residual effective mode potential induced by the transverse confinement in the widened region. We consider the case when $E_b$ is adjusted so that there is no
transmission in absence of the FIR field, and we ignore $E_m(x_b)$. The part that is reflected from the barrier need not be accounted for since it will travel in the reverse direction until it reaches the reservoir.

IV. PAIR-COUPLING APPROXIMATION

We now restrict our attention to situations in which:

$$|(E_m - E_m') - (E_m - E_n)| \gg |M|,$$  \hspace{1cm} (21)

for all sets of three different modes: $m'$, $m$ and $n$. In this case, only pair coupling is important. If there are some modes: $m'$, $m$ and $n$ that do not satisfy the condition in Eq. (21), a pair-coupling approach is invalid for frequencies that couple these levels.

Two different kinds of coupling can be distinguished in the pair-coupling approximation. There is an “absorption coupling” in which an electron entering in mode $n$ at an energy $E$ is coupled strongly to a higher mode $m$ at a higher energy $E + h\omega$. There is also an “emission coupling” in which the same electron is coupled strongly to a lower mode $m$ at a lower energy $E - h\omega$. These couplings are not active simultaneously with our restriction in Eq. (21) on the spectrum.

The “absorption coupling” and the “emission coupling” give the same eigenvalue equation by suitable substitutions. For the “absorption coupling” we have $l_{nn} = 0$ and $l_{nm} = 1$, and we introduce:

$$\tilde{K}^{ABS}_{mn}(E) = (K_{n1} + K_{n0})/2$$
$$\Delta K^{ABS}_{mn} = K_{m1} - K_{n0}$$
$$\bar{K}^{ABS} = K^{ABS} - \tilde{K}^{ABS}_{mn}(E),$$

while for the “emission coupling” we have $l_{nn} = 0$ and $l_{mn} = -1$ and we introduce:

$$\bar{K}^{EM}_{mn}(E) = (K_{m(-1)} + K_{n0})/2$$
$$\Delta K^{EM}_{mn} = K_{m(-1)} - K_{n0}$$
$$\bar{K}^{EM} = K^{EM} - \bar{K}^{EM}_{mn}(E).$$

Introducing $M = M_{nn} = -M_{mn}$ and dropping subscripts and superscripts for now, we get the following symmetric and E-independent form of Eq. (14):

$$\begin{pmatrix} -\Delta K/2 & -iM \\ iM & \Delta K/2 \end{pmatrix} \begin{pmatrix} C_n \\ C_m \end{pmatrix} = \tilde{K} \begin{pmatrix} C_n \\ C_m \end{pmatrix}.$$  \hspace{1cm} (24)

The eigenvalues are: $\tilde{K}^{(+)} = \pm \sqrt{(\Delta K/2)^2 + M^2}$ and the orthonormal eigenvectors are:

$$\begin{pmatrix} C_n \\ C_m \end{pmatrix}^{(+)} = \begin{pmatrix} -ia \\ b \end{pmatrix}, \quad \begin{pmatrix} C_n \\ C_m \end{pmatrix}^{(-)} = \begin{pmatrix} ib \\ a \end{pmatrix},$$

where:

$$a = \frac{1}{\sqrt{2}} \sqrt{1 - \sin \alpha}$$
$$b = \frac{1}{\sqrt{2}} \sqrt{1 + \sin \alpha}$$
$$\sin \alpha = \frac{\Delta K/2}{\sqrt{(\Delta K/2)^2 + M^2}}.$$  \hspace{1cm} (26)

Using the eigenvectors in Eq. (23) we find the expansion coefficients using the matching condition in Eq. (15):

$$a_{n}^{(+)} = ia, \quad a_{n}^{(-)} = -ib.$$  \hspace{1cm} (27)

Then we have for the transition probability in Eq. (13):

$$|t_{mn}(E)|^2 = \begin{cases} \gamma_{mn} \sin^2(q_{mn}(E)L), & \tilde{K}_{mn}(E) \geq 0 \\ 0, & \tilde{K}_{mn}(E) < 0 \end{cases}$$

$$\gamma_{mn} = \frac{M_{mn}^2}{(\Delta K_{mn/2})^2 + M_{mn}^2},$$

$$q_{mn}(E) = \frac{P_{n,E}^+ - P_{n,E}^-}{2\hbar} \approx \frac{\pi}{\tilde{K}^{EM}_{mn}(E)} \left(\frac{(\Delta K_{mn/2})^2 + M_{mn}^2}{\tilde{K}^{EM}_{mn}(E)}\right)^{1/2},$$

$$\tilde{\lambda} = \bar{\lambda}_{mn}(E) = \frac{\hbar}{2m^2\tilde{K}^{EM}_{mn}(E)}.$$  \hspace{1cm} (28)

We have already assumed a large kinetic energy: $K_{00} \gg |M|$, in the incident mode. Then, if $\tilde{K} < 0$ we know that the kinetic energy in the other mode must be negative. This implies a large kinetic energy mismatch: $|\Delta K| \gg |M|$, and thus corresponds to an off-resonant situation ($\gamma \ll 1$). Thus we are allowed to ignore the coupling and put $|t_{mn}(E)|^2 = 0$ when $\tilde{K} < 0$, instead of trying to find an exact expression.

The approximation for $q_{mn}(E)$ in Eq. (28) is good when $(\Delta K/2)^2 + M^2 \ll \tilde{K}^2$. However, since $K_{00} \gg |M|$ we find that the approximation for $q_{mn}(E)$ is poor only in off-resonant situations: $(\Delta K/2)^2 \gg M^2$. Therefore we can use this approximation generally.

From Eq. (28) we see that the population oscillates between the incident mode $n$ to the other mode $m$ along the channel. This is similar to Rabi oscillation in time of a two-level system. The wavelength of oscillation is given by $\pi/q$, and the resonance strength is given by $\gamma$. It is clear from the expression for $\gamma$ that the pair comes to resonance when: $|\Delta K| < 2M$.

The following relations are easily verified:

$$R^{ABS}_{mn}(E) = \bar{K}^{EM}_{mn}(E + h\omega)$$
$$\Delta K^{ABS} = \Delta K^{EM},$$

which leads to:

$$|t^{ABS}_{mn}(E)|^2 = |t^{EM}_{mn}(E + h\omega)|^2.$$  \hspace{1cm} (30)

Equation (30) states that as the electron from $|n,E\rangle$ is pumped up there is another electron from $|m,E + h\omega\rangle$,
which is pumped down, completely canceling the net change in the mode population. This is provided that both the scattering states are occupied. The cancelation which is a manifestation of the orthogonality between scattering states, appears when all coupled modes are occupied. For this reason only pumping between states that have different occupation factors will give a net oscillation along the channel. At zero temperature this happens for electrons that are closer than \( \hbar \omega \) to the Fermi level (see Fig. 3).

![Total energy vs Kinetic energy](image)

**FIG. 2.** In the limit of weak coupling and zero temperature, only pair transitions close to the Fermi level matters. In this case an analytical expression for the transition amplitudes can be found.

We consider the case: \( T = 0, \ E_b = E_F \) and \( E_m(x_b) = 0 \). In this case only electrons that have been excited into an empty state will be transmitted over the barrier. It is clear that only “absorption coupling” can lead to this and we find the following expression for the current:

\[
I = \sum_{n=1}^{N} \sum_{m>n} I_{mn},
\]

(31)

with the partial currents given by:

\[
I_{mn} = \frac{2e}{\hbar} \int_{E_F}^{E_F} \left| t_{mn}^{AB} (E) \right|^2 \Theta(E - E_n) dE,
\]

(32)

In the sums in Eq. (31), \( N \) is the number of propagating modes and \( M \) can be chosen to fulfill: \( E_M - E_N \geq \hbar \omega \). For a symmetric confining potential there are no transitions for which \( m + n = \) even. The transition probabilities \( \left| t_{mn}^{AB} (E) \right|^2 \) are found from Eqs. (22) and (1) where for the “absorption coupling” we have from Eqs. (22) and (1)

\[
\tilde{K}_{mn}(E) = E + \frac{\hbar \omega - (E_m + E_n)}{2},
\]

\[
\Delta K_{mn} = \hbar \omega - (E_m - E_n),
\]

(33)

From Eqs. (32), (23) and (33) it is clear that the pair resonances give rise to peaks in the current for certain frequencies \( f_{mn} = (E_m - E_n)/\hbar \). The width of these peaks is: \( \Delta f_{mn} = 2M_{mn}/\hbar \).

**V. RESULTS**

We will here consider the case of a square well confining potential with impenetrable walls at \( y = \pm d/2 \). In this case we have from Eq. (6) the following transition elements:

\[
M_{mn} = \frac{4\hbar e \hbar}{3\omega d n^\ast} \frac{3mn}{2(m^2 - n^2)} \delta_{m+n,odd}
\]

First we will demonstrate the frequency and field strength dependence of the current found within the pair-coupling approximation (see Eq. (31)). In Fig. (3) we plot the current for a channel for which \( L = 5 \mu m, N = 6 \) and \( E_F = 14 \)meV.

![Photocurrent vs Field Strength and Frequency](image)

**FIG. 3.** Plot of the current versus both \( \nu = \omega/2\pi \) and \( \hbar \) for a 5 \( \mu m \) long channel with six propagating modes. Peaks (light areas) correspond to pair resonances. The right column gives the scale for the current in microamps.

For weak FIR fields the pair-resonance peaks are well separated and the pair coupling approximation is good. However, in the upper-right part of the figure there is a slight overlap, and using the general scattering state in Eq. (13) would lead to a more accurate result in this case.

In the situation where the confining potential is tunable by means of gates, it is interesting to consider the effect of varying the gate voltage, \( V_g \). The confining potential can change in different ways when \( V_g \) is changed. As an example we will consider what happens if only the potential can change in different ways when \( V_g \) is changed. In the upper-right part of the figure there is a slight overlap, and using the general scattering state in Eq. (13) would lead to a more accurate result in this case.

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we change the mode energy separation $E_m - E_n$ and thus the resonance frequencies. When $E_m - E_n$ matches the applied frequency a pair will come to resonance. In Fig. we show the result of sweeping $V_g$ for a set of frequencies, when $\tilde{E} = 5 \text{ V/cm}$. In this case one can verify that the condition on the spectrum in Eq. (21) is clearly fulfilled in the considered frequency range.

![Graph showing mode spectrum](image)

**FIG. 4.** By sweeping $V_g$ for a set of frequencies and interpolating between the positions of resonance peaks in the current one can find out how the mode energy separations $\Delta E_n = E_{n+1} - E_n$ change with $V_g$.

Each frequency gives its own set of peaks. It is clear that for a high frequency it takes a more negative $V_g$ to separate the mode energies sufficiently. Therefore the peaks are shifted to the left with increasing frequency in the diagrams. By drawing a line that interpolates this shift of the peaks we get a complete picture of how the mode energy separations $\Delta E_n = E_{n+1} - E_n$ change with $V_g$. By adding these separations we get the full mode spectrum relative to the lowest mode, $E_n(V_g) - E_1(V_g)$.

**VI. DISCUSSION**

In order to have a coherent influence on the propagating electrons the coherence time in the far infrared source must exceed the passage time of electrons. A highly coherent source is typically not widely tunable. In experiments of the considered kind, fixed frequency FIR lasers are used. Therefore it is experimentally advantageous that we can reconstruct the mode spectrum using only fixed frequency sources, and also see how it changes with $V_g$. By the interpolation method one can gain information about all occupied modes for a given $V_g$, and not only about the one which is closest to pinch-off. Theoretical considerations suggest that there is a transition from a parabolic confinement towards a more square-well like, as the channel is made wider. Such detailed information is not found by measuring only the conductance versus $V_g$ since, due to a variation in the charge density, the spectrum changes with $V_g$. For channels with a parabolic confining potential, $V(y) = m^*\omega_0^2y^2/2$, magnetotransport experiments can be used to determine $\omega_n$.

From a physical point of view our system is interesting in that it enables a directed acceleration of electrons caused by a high frequency field, via a two step process. First energy is absorbed in standing wave excitations across the channel. In this process a strong influence is possible since both energy and momentum along the channel is conserved at resonance. Then the energy stored in the transverse direction is released in the widening and adiabatically converted into kinetic energy along the channel.

A nonzero temperature will destroy our predictions in two ways. First, the coherence length will be reduced because of enhanced phonon scattering. In experiments clear quantized conductance in a 5 µm channel has been demonstrated at 1.3 K. Since such temperatures are used also in FIR-transport experiments this seems within reach. The second degradation comes from thermal smearing. As a criterion we can take that the temperature must be much smaller then the inter mode separation, which is typically of the order 20 K.

We have ignored collective effects. In absorption experiments on wire arrays one finds a depolarization effect. As a result the mode spectrum found from absorption spectroscopy can differ from that found from magnetoresistance oscillations. This is not immediately generalized to a non-homogeneous system. A difference is that we consider coherent transport and that the inhomogeneity acts as a boundary condition for propagating electrons. We can not rule out a significant influence due to depolarization, but leave it for future investigations.

Another collective effect is the static spatial modulation of the potential along the channel because of changes in the mode population. However such an influence is suppressed in a multi mode channel since only a fraction of the electrons are resonantly coupled and electrons moving with different velocities give depopulation at different locations along the channel. In addition most charge comes from slow electrons, and for these strong cancellation can be expected because of a strong energy dependence.

In conclusion we consider the influence of a FIR field on coherent transport in a multi mode channel. We focus on a long channel in which resonance conditions prevail over a considerable distance in order to achieve a high sensitivity to an applied FIR field. We derive both a general scattering state and an approximation valid in the limit of weak fields, where pair resonance will dominate. In this limit the current will peak when $h\nu = E_m - E_n$. We also propose a way to analyze the spectrum, using fixed frequencies only.
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16 If none of the modes: $m'$, $m$ and $n$ enter the channel, the condition in Eq. 21 does not have to be fulfilled for these modes. For a hard-wall potential of width $d$ one can prove:

$$|E_{m'} - E_m - (E_m - E_n)| \geq \frac{\hbar^2\pi^2}{2m^*d^2}.$$