FALL-BACK DISKS IN LONG AND SHORT GAMMA-RAY BURSTS

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\textbf{ABSTRACT}

We present time-dependent numerical calculations for fall-back disks relevant to gamma-ray bursts (GRBs) in which the disk of material surrounding the black hole powering the GRB jet modulates the mass flow and hence the strength of the jet. Given the initial existence of a small mass $\lesssim 10^{-4} M_\odot$ near the progenitor with a circularization radius $\sim 10^{10} - 10^{11}$ cm, an unavoidable consequence will be the formation of an “external disk” whose outer edge continually moves to larger radii due to angular momentum transport and lack of a confining torque. For long GRBs, if the mass distribution in the initial fall-back disk traces the progenitor envelope, then a radius $\sim 10^{11}$ cm gives a timescale $\sim 10^4$ s for the X-ray plateau. For late times $t > 10^7$ s a steepening due to a cooling front in the disk may have observational support in GRB 060729. For short GRBs, one expects most of the mass initially to lie at small radii $< 10^5$ cm; however, the presence of even a trace amount $\sim 10^{-9} M_\odot$ of high angular material can give a brief plateau in the light curve. By studying the plateaus in the X-ray decay of GRBs, which can last up to $\sim 10^7$ s after the prompt emission, Dainotti et al. find an apparent inverse relation between the X-ray luminosity at the end of the plateau and the duration of the plateau. We show that this relation may simply represent the fact that one is biased against detecting faint plateaus and therefore preferentially sampling the more energetic GRBs. If, however, there were a standard reservoir in fall-back mass, our model could reproduce the inverse X-ray luminosity–duration relation. We emphasize that we do not address the very steep, initial decays immediately following the prompt emission, which have been modeled by Lindner et al. as fall back of the progenitor core, and may entail the accretion of $\gtrsim 1 M_\odot$.

\textbf{Key words:} accretion, accretion disks – gamma-ray burst: general – gamma-ray burst: individual (GRB 060729, GRB 051221A) – stars: interiors – X-rays: general

\textbf{Online-only material:} color figures

\section{1. INTRODUCTION}

The tentative pre-Swift hints that breaks occur in the long-term afterglow light curves simultaneously in different wavebands at $\sim 1$–10 days (Frail et al. 2001) after the gamma-ray bursts (GRBs) have not been borne out by numerous detailed observations of Swift GRB afterglows in recent years (Ghisellini et al. 2007; Oates et al. 2007; Racusin et al. 2008; Liang et al. 2008). With Swift (Gehrels et al. 2004), the long-term ($t \gtrsim 10^6$ s) X-ray behavior has turned out to be surprisingly complex, exhibiting alternating steep and shallow slopes when plotted as $\log F$ versus $\log(t - T_0)$, where $T_0$ is the trigger time for prompt Burst Alert Telescope emission (Zhang et al. 2006; Nousek et al. 2006).

The traditional viewpoint that long-term fading of the GRB afterglow represents the deceleration and spreading of a relativistic jet (e.g., Sari et al. 1999; Frail et al. 2001) has given way to a much broader effort involving a diverse variety of hypotheses by which a central engine can be powered for a long time, including such scenarios as magnetars (Zhang & Mészáros 2001; Fan & Xu 2006; Troja et al. 2007; Lyons et al. 2010), quark stars (Staff et al. 2008), and long-term accretion onto a black hole (BH) formed during the collapsar or NS–NS merger (where NS is a neutron star; Kumar et al. 2008a, 2008b; Metzger et al. 2008a, 2008b). The accretion hypothesis entails exploring the possibility that the long-term decay of the X-ray flux is not due to the deceleration of baryonic ejecta, but rather a secular decrease in the rate of accretion powering the central engine, and therefore indirectly the jet (Kumar et al. 2008a, 2008b; Metzger et al. 2008a, 2008b; Cannizzo & Gehrels 2009, hereafter CG09). For long GRBs (lGRBs), the early, steep rate of decay may be giving us information about the radial density distribution within the progenitor core (Kumar et al. 2008b), whereas the later decay may be governed by the outward expansion of the transient disk formed from the remnants of the progenitor (CG09). As regards to short GRBs (sGRBs), even if a small amount of material ($\sim 10^{-5}$ to $10^{-4} M_\odot$) is expelled during the NS–NS merger and later accreted in a disk, that would be sufficient to power a bright afterglow, which may also be strongly influenced by the effects of $r$-process nucleosynthetic heating in the neutron-rich material that becomes the disk (Metzger et al. 2010).

Zhang et al. (2006) present a schematic for the decaying GRB light curve as seen by the X-Ray Telescope (XRT) on Swift. The decay is traditionally shown in $\log F$ versus $\log t$. There are four basic power-law decay ($F \propto t^{-\alpha}$) regimes: (1) a steep decline following the prompt emission with $\alpha_1 \simeq 3$ out to $10^2 - 10^3$ s, (2) a plateau with $\alpha_{\text{flat}} \simeq 0.5$ out to $10^{3} - 10^{4}$ s, (3) a steepening with $\alpha_{\text{st}} \simeq 1.2$ out to $10^{4} - 10^{5}$ s, and (4) a further steepening at late times (not always seen) with $\alpha_{\text{late}} \simeq 2$.

CG09 present a general analytical formalism to explain the different power-law decays using a fall-back disk, where the variations in $\alpha$ could potentially be explained by different physics operating within the disk. The results of CG09 were purely analytical; in this work we present time-dependent numerical calculations in order to examine in more detail the
potential of the model, and we apply the results to XRT data for one IGRB and one sGRB, taking the best studied of each class. In Section 2 we review the Dainotti relation, an empirical relation involving the duration and luminosity of segment II; in Section 3 we present our detailed numerical model; in Section 4 we compare the model with observations for theots GRB 060729, the GRB with the longest observational time series in X-rays; in Section 5 we compare theory and data for the sGRB 051221A, the sGRB with the longest and most detailed XRT light curve; in Section 6 we revisit the Dainotti relation, in the context of our numerical results; and in Section 7 we discuss and summarize our results.

2. THE DAINOTTI RELATION

Dainotti et al. (2008, 2010) found an empirical relation between the duration of the X-ray plateau in the source frame \( t_{\text{II}} = t_{\text{I}}(1 + z)^{-1} \) and the X-ray luminosity at the end of the plateau, also corrected into the source frame, \( L_{\text{II}}^s \). Expressing their relation in the form \( \log_{10} L_{\text{II}}^s (\text{erg s}^{-1}) = a \cdot t_{\text{I}}^b \log_{10} t_{\text{I}}^a(s) \), Dainotti et al. (2010) find \( a = 51.1 \pm 1 \) and \( b = -1.1 \pm 0.3 \). Their sample of IGRBs with small errors is defined by \( u < 4 \), where \( u \equiv ([\log \delta L_{\text{II}}^s]^2 + [\log \delta t_{\text{II}}^a]^2])^{1/2} \), and \( \log \delta L_{\text{II}}^s \) and \( \log \delta t_{\text{II}}^a \) represent the logarithmic errors in \( L_{\text{II}}^s \) and \( t_{\text{I}}^a \) respectively. Using the same sample of 62 IGRBs with small errors taken from Dainotti et al. (2010), we find \( a = 50.5 \pm 0.4 \) and \( b = -0.92 \pm 0.1 \), close to their results.

If the inverse relation between \( L_{\text{II}}^s \) and \( t_{\text{II}}^a \) is physical, and if long-term accretion is the correct explanation for the long-term X-ray light curve of GRBs, that would have implications for the amount of mass in the initial fall-back disk. To first order, the fact that \( L_{\text{II}}^s = t_{\text{I}}^a \) is constant would imply that the accreted mass reservoir is constant. Willingale et al. (2007) present a simple formalism for integrating the X-ray flux on the plateau and subsequent \( \delta t_{\text{II}} \) decay to obtain a total energy for segments II and III. We can convert this into a mass by making a few plausible assumptions about the energetic efficiencies (Krolik et al. 2007), e.g., an X-ray afterglow beaming factor \( f \approx 10^{-3} \) to \( 10^{-5} \), and an efficiency \( \epsilon_{\text{net}} \approx 0.01 \)–0.1 with which accretion onto the inner engine powers the observed X-rays, presumed to be created within the beamed jet. This overall accretion efficiency has two components: (1) the efficiency \( \epsilon_{\text{acc}} \) with which accretion onto the BH powers the jet, where \( \epsilon_{\text{acc}} \approx 0.4 \) for the anticipated high spin BHs in GRBs, and (2) the efficiency \( \epsilon_{\text{X}} \) with which the jet power is converted into 3–10 keV X-rays that can be observed by XRT. Therefore \( \epsilon_{\text{net}} = \epsilon_{\text{acc}} \epsilon_{\text{X}} \). We use Equations (3) and (4) from Willingale et al. (2007) to obtain a total energy \( \Delta E_X \) and hence \( \delta M \) derived from the X-ray fluence, for segments II and III. The results are shown in Figure 1. The hatched area is bounded from above by an estimate of the effective limiting XRT flux \( f_{\text{II}} \) for being able to observe and characterize plateaus sufficiently well that they would satisfy \( u < 4 \). There appears to be a selection effect giving rise to the Dainotti relation, namely that the flux detection limit for XRT prevents one from clearly observing and parameterizing faint plateaus at high \( z \). The Dainotti relation, effectively equivalent to \( L_{\text{II}}^s = \text{constant} \), results from the sampling of GRBs primarily beyond \( z \simeq 1.5 \), for which faint plateaus are observationally biased against. Thus we are seeing basically a narrow strip corresponding to the upper end of a much broader distribution, which makes it appear that \( L_{\text{II}}^s = \text{constant} \). For \( z \lesssim 1.5 \) one sees curvature in the lower envelope of \( \Delta E_X \) values due to the strong dependence of the detection limit on \( z \). Nevertheless, the fact that there appears to be a well-defined upper limit \( \delta M \approx 10^{-4} \) to \( 10^{-3} M_\odot \) is interesting: if one starts with a \( \sim 10 M_\odot \) progenitor and if accretion governs the long-term X-ray light curve, this indicates that no more than a fraction of \( \sim 10^{-5} \) to \( 10^{-4} \) of the progenitor mass survives the hypernova to form a fall-back disk (excluding segment I). Given the potential for strong outflows during the accretion process, this \( \delta M \) should be viewed as a lower limit.

3. ACCRETION DISK PHYSICS

By writing the equations for mass continuity and angular momentum transport in cylindrical coordinates, assuming Keplerian rotation \( \Omega_K = GM_* r^{-3} \) and integrating over the
vertical thickness of the accretion disk, one arrives at an equation for the evolution of the surface density \( \Sigma = 2 \pi h \), where \( \rho \) is the density and \( h \) is the disk semi-thickness (actually pressure scale height):

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left( r^{1/2} \frac{\partial}{\partial r} (\Sigma r^{1/2}) \right).
\]

(1)

The kinematic viscosity coefficient

\[
v = \frac{2 \alpha_{SS} \rho}{3 \Omega_K \rho},
\]

(2)

where \( P \) is the pressure and \( \alpha_{SS} \) is the Shakura–Sunyaev parameterization of the angular momentum transport and heating (Shakura & Sunyaev 1973). Guided by the current concordance between inferred values of \( \alpha_{SS} \approx 0.1 \) from dwarf nova outburst decays, both fast, thermal decays (Smak 1984) and slow, viscous decays (Cannizzo et al. 2010), and global three-dimensional general relativistic magnetohydrodynamic (GRMHD) models of the magnetorotational instability (MRI), which also give \( \alpha_{SS} \approx 0.1 \) at large radii in the disk (McKinney & Narayan 2007a, 2007b), we set \( \alpha_{SS} = 0.1 \). Equation (1) is discretized following the method of Bath & Pringle (1981), in which grid points are distributed as \( r^{1/2} \). In addition, one has a thermal energy equation governing the temperature evolution,

\[
\frac{\partial T}{\partial t} = \frac{2 (A - B + C + D)}{c_p \Sigma} - \frac{RT}{\mu c_p} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial r} \left( r v_r \right),
\]

(3)

where the viscous heating \( A = (9/8) \nu \Omega^2 \Sigma \), the radiative cooling \( B = \sigma T^4 \), and \( C \) and \( D \) represent radial heat fluxes due to turbulent and radiative transport (Cannizzo et al. 2010). For the calculations we present in this work, the thermal equation only becomes of potential importance at very late times \( > 1 \) yr. In other words, \( A \) is a very good proxy for most of the evolution, due to the fact that \( \xi \equiv d \log T(\Sigma)/d \log \Sigma > 0 \). In other words, disks with \( \xi > 0 \) are viscously and thermally stable (Piran 1978), and therefore simple viscous evolution provides a satisfactory physical description.

After \( \sim 1 \) yr the surface density in the very outer disk drops below that associated with the hysteresis due to the transition between ionized and neutral gas, the source of the dwarf nova limit cycle mechanism. This launches a cooling front that propagates to smaller radii and rapidly shuts off the supply of gas to the inner disk, as the effective viscosity plummets. The dwarf nova limit cycle is an accretion disk instability in which the \( T(\Sigma) \) relation exhibits a hysteresis at a temperature corresponding roughly to the peak in the Rosseland opacity curve—effectively an “S”-curve when plotted as \( T \) versus \( \Sigma \). It accounts for the outbursts seen in dwarf novae (Lasota 2001). The global manifestation of the instability is that material accumulates in the disk during quiescence and accretes onto the central star during outburst. The transition between the two states is mediated by the action of heating and cooling fronts. The cooling front we see at late times in our calculations is a result of the disk mass becoming so small that the transition from ionized to neutral gas is instigated. (A detailed discussion of the effects associated with the heating and cooling fronts is given in Cannizzo 1993, 1998.)

At early times and small radii the local disk accretion can be significantly super-Eddington, and in the thin disk formalism one would have \( h/r \gg 1 \), an inconsistency. Section 2.3.2 of CG09 contains scalings for the thin disk branch of solutions (Abramowicz et al. 1988) which we employ when \( h/r > (3/4)^{1/2} \) locally. Combining Equations (24) and (26) of CG09 gives

\[
T(\Sigma) = 1.56 \times 10^5 K m_{BH,1}^{-1/2} \Sigma^{1/4},
\]

(4)

where \( m_{BH,1} = M_{BH}/(10 M_\odot) \) and \( r_{11} = r/(10^{11} \) cm). For completeness, we note that at early times and small radii, other types of disks are possible, such as neutrino-cooled accretion disks (e.g., Narayan et al. 2001; Kohri et al. 2005; Chen & Beloborodov 2007). The controlling dynamics of the long-term accretion rate is governed by the physical state of the outer disk, since that is where the slowest (i.e., controlling) timescale is. A complete physical modeling of the disk at small radii, including neutrino cooling and nucleosynthesis, is beyond the scope of this work, but also not necessary given our primary interest, the long-term accretion rate. At late times and large radii where \( h/r < (3/4)^{1/2} \), we utilize the thin disk scalings from CG09. Combining Equations (10) and (11) from CG09 gives

\[
T(\Sigma) = 84.1 K \alpha_{SS,1}^{1/3} m_{BH,1}^{1/6} \Sigma^{-1/2} \Sigma^{2/3},
\]

(5)

where \( \alpha_{SS,1} = \alpha_{SS}/0.1 \). From a pragmatic standpoint, even though the use of a thin disk at small radii would be inconsistent because one would find \( h/r \gg 1 \), the use of a slim disk at small radii versus a thin disk has minimal effect on the light curve. This comes about because the controlling timescale in the problem comes from the largest disk radius. In fact, one could probably model the entire disk as a one-zone model, taken at the outer disk edge as was done by Metzger et al. (2008a) for NS–NS merger disks, and still get a reliable result.

The composition of the accretion disk is substantially different from solar. The (\( T(\Sigma) \) relation) exhibits a weak inverse scaling with mean molecular weight \( \mu \) for gas pressure and electron scattering opacity, \( T \approx \mu^{-1/3} \) (Cannizzo & Reiff 1992). The expected mixture of alpha elements yields \( \mu \simeq 16 \), rather than the value \( \mu \simeq 0.65 \) more generally relevant for solar composition material. Thus \( T \approx \mu^{-1/3} \) brings about a decrease in \( T(\Sigma) \) by a factor of \( \sim 2.9 \) at given surface density, and therefore \( h/r \) by a factor of \( \sim 1.7 \), compared to solar composition material.

For completeness, we note that significant outflow may accompany accretion flows that are substantially super-Eddington. For computational expedience we only consider the slim disk in the super-Eddington limit, since the addition of outflows would introduce additional free parameters. However, given the strong potential for outflow, we stress that our accreted masses represent lower limits as to the amount of mass that may actually be accreted during segments II and III. Similarly, the inferred accreted masses shown in Figure 1 may significantly underestimate the true remnant masses left over after the collapsar event.

CG09 present analytical models for the afterglow light curves powered by fall-back disks. The main strength of model is the universal decay law \( d \log L_{\gamma}/d \log t \simeq -1.3 \) characterizing fall-back disks without the external, confining tidal torque of a companion star (Cannizzo et al. 1990). This law seems consistent with the late time decay of GRBs seen in X-rays. The early time decay, in particular the steep decay for \( t \lesssim 10^3 \) s, following the prompt emission, is difficult to explain within the fall-back disk scenario and may be giving us information about the radial density profile within the progenitor (Kumar et al. 2008b). In this work we present results of time-dependent numerical calculations using a general accretion disk code.
IGRBs are thought to be associated with the explosions of massive stars (MacFadyen & Woosley 1999; Woosley & Heger 2006). The initial failure of the supernova is the reason for the GRB in the collapsar model. The hosts for IGRBs tend to be subluminous, irregular galaxies rich in star formation (Fruchter et al. 2006). If the long-term X-ray light curves of GRBs are indicative of feeding from a fall-back accretion disk, then from Figure 1 we see that no more than $\sim 10^{-4}$ to $10^{-3} M_\odot$ of material in the fall-back disk is needed, for nominal assumptions about the efficiencies. In other words, for a $\sim 10 M_\odot$ progenitor, a mass fraction only $\sim 10^{-5}$ to $10^{-4}$ (excluding the $\sim 10 M_\odot$ which ends up within $t \lesssim 10^{-10}$ s in the BH) is required to survive and persist in the vicinity of the progenitor to power the long-term light curve. If a significant fraction of the mass that tries to accrete onto the BH is ejected, the remnant mass around the BH could be substantially larger. We stress that in this discussion we only include mass accretion beginning in segment II, since segment I is too steep to be accounted for easily in the accretion disk scenario.

Figure 2 shows a light curve for $M_{\text{BH}} = 5 M_\odot$ and $M_{\text{disk}} = 10^{-4} M_\odot$, overlaid with XRT data from GRB 060729. The initial $\Sigma(r)$ profile is taken from Heger progenitor 12SE (Woosley & Heger 2006) in the following way: the progenitor is collapsed in cylindrical coordinates along the $z$-axis to obtain $\Sigma_{\text{initial}}(r)$, and then scaled down in mass with a radial factor $\alpha (r/R_\odot)$ so that $\sqrt{2 \pi r \Sigma_{\text{initial}}(r)} \equiv 10^{-4} M_\odot$. A radial scaling is also applied so that the integrated specific angular momentum in the final state, taken to represent the disk and therefore to be Keplerian, equals that in the progenitor. This effectively introduces a radial scaling $\sqrt{\text{H}/f_{\text{crit}}}$ to the $\Sigma_{\text{initial}}(r)$ distribution. Due to the very long plateau of GRB 060729, we adopted a progenitor model with a large radius. (The value log $t_H^*(s) \approx 4.7$ for GRB 060729 is at the upper end of the distribution of log $t_H^*$ values given in Dainotti et al. 2010.) The lower panel of Figure 2 indicates the local values of the temporal decay index $\alpha$ associated with the model. There are two power-law segments evident in the model decay light curve: a brief one $\alpha \simeq 0.4$ between $10^3$ and $10^4$ s, and a longer one $\alpha \simeq 1.2$ between $10^5$ and $10^7$ s. The latter value $\alpha \simeq 1.2$ is close to that expected from analytical models, 19/16 (Cannizzo et al. 1990). Figure 3 gives the evolution of surface density $\Sigma(r,t)$ and local aspect ratio $h/r$, and Figure 4 indicates the transition point between the slim disk and thin disk states, i.e., $h/r$ dropping below $(3/4)^{1/2}$.

The steep decay associated with segment I is not accounted for in our model and may well be due to a separate physical process, such as the accretion of the progenitor core (Kumar et al. 2008b; Lindner et al. 2010, see their Figure 2). The timescale $t_H$ for the plateau associated with segment II comes out naturally in the models, given the $\sim 10^{13}$ cm radius for the progenitor—assuming that the initial fall-back mass distribution roughly traces the envelope of the progenitor (after taking into account the radial scaling factor $\sqrt{\text{H}/f_{\text{crit}}}$). The observed X-ray decay for GRB 060729, the GRB which has been observed the longest time in X-rays (Grupe et al. 2007, 2009), is at $t \sim 642$ days. The initial profile is determined from Heger progenitor model 12SE as explained in the text. The flat decay portion in the model light curve corresponds to the time during which the initial $\Sigma(r)$ is being redistributed into an accretion disk with a self-consistent radial profile. The lower panel shows the evolution of the locally defined rate of decay $\alpha = -\log L_x/d\log t$. The two quasi-constant portions, $\alpha \simeq 0.4$ and 1.2, are indicated by the heavy dashed red lines. The long-term decay rate $\sim 1.2$ is characterized well by the analytical solution for electron scattering, gas pressure disks, $\alpha = 19/16$ (Cannizzo et al. 1990).
at which the dwarf nova limit cycle becomes active (indicated by the parallel dashed lines in Figure 3), and a cooling front is launched from the outer edge that instigates a transition from ionized to neutral gas. This quenches the source of accretion onto the central engine. The late time Chandra observations of GRB 060729 (Grupe et al. 2010) appear to coincide with the timing of this drop-off, which may therefore represent a cooling transition front in the outer disk which diminishes the rate of accretion onto the central engine, rather than a jet break.

5. sGRBs

sGRBs are thought to be caused by the merger of two neutron stars (Eichler et al. 1989; Paczyński 1991; Narayan et al. 2001; Rosswog & Ramirez-Ruiz 2002; Rezzolla et al. 2011). The hosts for sGRBs are indicative of an older population with less active star formation (Fox et al. 2005; Gehrels et al. 2005; Villasenor et al. 2005; Bloom et al. 2006; Prochaska et al. 2006), and also tend to be ∼5 times more spatially extended than those of lGRBs, matching the ∼5 times wider projected spatial distribution of sGRBs versus lGRBs within their hosts (Troja et al. 2008; Fong et al. 2010; Church et al. 2011). Due to their faintness relative to lGRBs, sGRBs are much less well studied in X-rays. Indeed, there is only one certifiable example of an sGRB which has a long and well-sampled X-ray light curve, to the extent that a statement can be made as to the presence of a plateau—GRB 051221A (Soderberg et al. 2006; Burrows et al. 2006). In addition, there are currently only two secure cases of an association of sGRBs with an old population: GRB 050724 (Berger et al. 2005; Barthelmy et al. 2005) and 100117A (Fong et al. 2011).

The importance of the outward spreading of the disk formed from the NS–NS merger was clearly demonstrated by Metzger et al. (2008a, see their Figure 1). The initial radial density profile, and hence Σ(r) profile after angular momentum conservation has vertically compacted the merger remnants into their accretion plane, is of course much more radially condensed than that expected for lGRBs. Theoretical guidance on a potential Σ(r) profile to begin the calculations, particularly at large radii, is scarce, yet there are indications of small amounts of matter ∼10^{-6} to 10^{-4} M⊙ ejected at early times that may have high angular momentum, and therefore circularize at large radii (Rosswog 2007). One potentially important effect we cannot model is the disruption of a portion of the disk by strong nucleosynthesis in the NS supplied material as it expands rapidly to subnuclear densities (Metzger et al. 2008a, 2008b, 2010). Lee et al. (2009) find that strong winds can be launched from the surface of post-merger NS–NS disks, powered by the recombination of free nucleons into α-particles.

Figure 4 shows the evolution of a fall-back disk of potential relevance for the aftermath of a NS–NS merger. We take M_{BH} = 3 M⊙ and M_{disk} = 10^{-5} M⊙. Unlike the much longer evolution shown in Figure 2 for lGRBs, in this case the outer edge of the disk is still freely expanding to larger radii by the end of the run (indicated by the narrow spike in Σ(r) between 10^{11} cm and 3 × 10^{11} cm shown in the second lower panel). The brief plateau at ∼10^{4} s results from the ad hoc introduction of the small amount ∼10^{-9} M⊙ of high angular momentum material at the large circularization radius 10^{10} cm. The high efficiencies associated with BH accretion show the potential for a small amount of material ∼1 M⊙ to have a dramatic effect on the long-term light curves as regards to brief plateaus or inflection points.

6. POTENTIAL APPLICATION TO A DAINOTTI-LIKE RELATION

In Section 2 we show that the Dainotti relation L_{II} ∝ t_{II}^{-1} as originally envisioned may be due to the observational bias against detecting and characterizing faint plateaus. Nevertheless, the accreted mass estimates δM inferred using the L_{II} and t_{II} values from Dainotti et al. (2010) are interesting in the context of this work, and one might legitimately ask what theoretical prediction for the L_{II}(t_{II}) relation the fall-back accretion hypothesis would make, given a hypothetical physical constraint.
in which $\delta M$ were held constant and the initial radius of the fall-back disk were allowed to vary. Figure 6 shows the results of seven runs in which we fix $\delta M = 10^{-3} M_\odot$ and vary the initial outer radius of the fall-back disk. One can see a clear inverse relation between the duration of the plateau $t^*$ and the luminosity at the end of the plateau $L^*$. The low-$z$ behavior of $\delta M$ seen in Figure 1, however, indicates that a spread in $\delta M$ may be more realistic, in which case one would not expect to be able to use the theoretical prediction of the $L^*(t^*)$ relation as a useful discriminant for the theory.

7. DISCUSSION AND CONCLUSION

We have presented time-dependent calculations of the fall-back disk scenario to account for the long-term X-ray light curves for GRBs. For IGRBs, an initial radial scale of a stellar radius $\sim 10^{11}$ cm gives a natural viscous evolution time of $\sim 10^4$ s to redistribute the matter into a quasi-steady disk, roughly consistent with the observed plateau duration. GRB 060729 which we model in the context of an IGRB had one of the longest plateaus ever observed, thus more generally we anticipate progenitor radii, or more specifically, circularization radii, given by $\sim \sqrt{f_{\text{hor}}/\dot{J}_p}$, to lie in the range $10^{10}$–$10^{11}$ cm.

The rate of decay for GRB 060729 is close but does not match the models precisely, which may hint at time-variable emission processes that would affect our adopted efficiencies $\epsilon_{\text{net}}$ and $f$. It is interesting to note that our effective values of the efficiencies, accretion plus beaming, required to match the observed X-ray flux levels are comparable between IGRBs and sGRBs: $f\epsilon_{\text{net}}^{\text{eff}} \approx 10^{-2}$ for GRB 060729 and $f\epsilon_{\text{net}}^{\text{eff}} \approx 3 \times 10^{-2}$ for GRB 051221A. If the accretion efficiency $\epsilon_{\text{net}}$ is about the same for IGRBs and sGRBs, this may indicate that sGRBs are less beamed by a factor of $\sim 3$ compared to IGRBs, roughly in line with previous results (Watson et al. 2006; Grupe et al. 2006). This similarity between IGRBs and sGRBs afterglows is not entirely unexpected, given the general similarities in their afterglow properties (Gehrels et al. 2008; Nysewander et al. 2009).

The concept of powering the long-term X-ray light curve of GRBs by accretion onto the BH represents a departure from the standard model in which the fading corresponds to the deceleration of a baryonic jet. In the accretion model, the jet itself would be very light, perhaps composed almost exclusively of Poynting flux, and the long-term fading would not be due to variations in the Lorentz factor and beaming factor. Recent high fidelity GRMHD calculations of BH accretion support the idea of a high Lorentz factor jet with minimal baryon loading ( McKinney & Narayan 2007a, 2007b). The accretion scenario involving fall back of the progenitor core has been examined in detail by Kumar et al. (2008a, 2008b) and Lindner et al. (2010). Kumar et al. (2008b) argue that constraints may be placed on the density profiles and radii of the progenitor core and envelope, as well as their rotation rates relative to breakup. Lindner et al. (2010) examined the progenitor core fall-back scenario in much greater detail with the adaptive mesh refinement FLASH code, where their calculations are done in cylindrical coordinates. Starting with a Heger progenitor, they follow the fall-back evolution of the progenitor. Their Figure 2 shows the potential
for the model to obtain a steeply decaying light curve as is seen in X-rays for segment I, where $\alpha \simeq 3$ in the Zhang et al. (2006) schematic. In their discussion, Lindner et al. mention several caveats, such as the lack of nuclear physics in the inner disk, the neglect of the MRI, and the lack of modeling the axial relativistic jet. A more basic concern is simply the range of rate of accretion in comparison to that inferred from observations. Figure 2 of Lindner et al. shows that if progenitor core fall back is the correct explanation for segment I in the X-ray light curve, the rate of accretion onto the BH during this phase varies between about 0.1 and $10^{-5} M_{\odot} \text{yr}^{-1}$, whereas from Figure 2 of this work we see that, for nominal assumptions about the accretion efficiency, the rate inferred from observations on segment I for GRB 060729 varies between about 10 and 3 $\times 10^{-3} M_{\odot} \text{yr}^{-1}$, lower by a factor $\sim 10^5$--$10^6$ than in Lindner et al. (2010). One does in fact expect for the theoretical accretion rate in this context to be an underestimate, given the potential for outflow, but the discrepancy here seems extreme. There are several possibilities for this discrepancy. The vertical scale shown in Figure 2 of this work is already super-Eddington by between about one and eight orders of magnitude, and therefore the range in the Lindner et al. calculations would be correspondingly greater. It may be that even highly advective disks cannot accommodate that much accretion onto the BH and that a significant fraction of the material is blown away before it can accrete. This would have to occur, however, in such a way that it did not interfere with the propagation of the jet. Also, for the shape of the decay light curve calculated by Lindner et al. to correspond to segment I, the ratio of accreted to ejected gas would have to remain constant. If it varied significantly, the accretion-derived luminosity would have a different decay power law. Another possibility is that the Lindner et al. calculations greatly overestimate the fall back, in which case a much greater fraction of the progenitor would be ejected on a timescale $\lesssim 10^3$ s. Lindner et al. note that they do not find evidence for the thin disk hypothesized by CG09 as characterizing the fate of the progenitor envelope fall back. Given the apparent mismatch in accretion rates between their theory and the observations, and the findings of this work, their criticism could have at least two mitigating factors. (1) In Figure 4 we see that for the period covered by the Lindner et al. calculations, namely $t < 10^3$ s, a significant fraction of the disk lies within $r_{\text{trans}}$ i.e., the more spatially extended slim disk rather than the thin disk. (2) Given the apparent mismatch between theory and observation for the rates of accretion, the actual densities within the volume formerly occupied by the progenitor may be much less than calculated by Lindner et al. (2010), which would interfere far less with an accretion disk formed from fall-back debris. At late times Chandra observations indicate a steepening in the rate of decay (Grupe et al. 2010), which appears to be consistent with the onset of a cooling front in the disk. This would be an alternative to the standard jet-break interpretation discussed by Grupe et al. (2010). For sGRBs, the picture is less clear, given that we only have a single well-studied example. We have shown that the presence of even a very small amount of high angular momentum gas $\sim 10^{-9} M_{\odot}$ can give a slight inflection to the X-ray decay, as was observed in GRB 051221A. As for the overall X-ray light curve, segments II and III, if only $\sim 10^{-5}$ to $10^{-4} M_{\odot}$ of gas survives either the hypernova (iGRBs) or NS–NS merger (sGRBs), then the accretion resulting from the ensuing fall-back disk should power a long-term jet. Lastly, we have shown that the Dainotti relation $L_{\text{II}} \propto t^{-1}$ may be due to an observational bias against detecting and characterizing faint plateaus: the relation is governed by GRBs at $z \gtrsim 1.5$ for which we only detect the upper envelope of a broad distribution. Nevertheless, the existence of an apparent upper limit to the total X-ray energies inferred from the X-ray fluences, and therefore the accreted masses if one assumes accretion onto the central engine as the long-term powerhouse for the X-ray flux, is extremely interesting. For nominal values of the accretion efficiency $\epsilon_{\text{conv}}$ and the beaming factor $f$, we find an upper limit $\lesssim 10^{-4}$ to $3 \times 10^{-3} M_{\odot}$ for the accreted mass during segments II and III. (The lower end of the distribution of accreted masses, which is partially revealed for GRBs at $z \lesssim 1.5$, may extend down to $\lesssim 10^{-8}$ to $10^{-7} M_{\odot}$.) This means that for a progenitor mass $\sim 10 M_{\odot}$ only a maximum mass fraction $\sim 10^{-5}$ to $10^{-4}$ of the progenitor survives in the vicinity of the progenitor to be accreted as a fall-back disk (excluding the $\sim 10 M_{\odot}$ that ends up in the BH during the prompt emission and subsequent segment I consisting of the steep-decay). This has important ramifications for the energetics associated with the hypernova explosion and subsequent removal of most of the progenitor envelope.

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