Scaling Study of the Metal-Insulator Transition in one-Dimensional Fermion Systems

Shi-Jian Gu
Departamento de Física, Universidade de Évora, Rua Romão Ramalho, 59, P-7000-671 Évora, Portugal and Zhejiang Institute of Modern Physics, Zhejiang University, China.

Vitor M. Pereira
Department of Physics, Boston University, Boston, Massachusetts 02215, USA, and Centro de Física do Porto and Departamento de Física, Faculdade de Ciências, Universidade do Porto, Rua do Campo Alegre 687, P-4169-007 Porto, Portugal.

N. M. R. Peres
Departamento de Física and Centro de Física da Universidade do Minho, Campus Gualtar, P-4700-320 Braga, Portugal. (September 12, 2018)

We consider the Ising phase of the antiferromagnetic XXZ Heisenberg chain on a finite-size lattice with $N$ sites. We compute the large $N$ behavior of the spin stiffness, obtaining the correlation length $\xi$. We use our results to discuss the scaling behavior of metal-insulator transitions in 1D systems, taking into account the mapping between the XXZ Heisenberg chain and the spinless fermion model and known results for the Hubbard model. We study the scaling properties of both the Hubbard model and the XXZ Heisenberg chain, by solving numerically the Bethe Ansatz equations. We find that for some range of values of $\xi/N$ the scaling behavior may be observed for the Hubbard model but not for the XXZ Heisenberg chain. We show how $\xi$ can be obtained from the scaling properties of the spin stiffness for small system sizes. This method can be applied to models having not an exact solution, illuminating their transport properties.

PACS numbers: 71.27+a, 71.30.+h, 72.15.Nj

I. INTRODUCTION

Since Bethe’s solution$^1$ of the one-dimensional (1D) isotropic Heisenberg model, that the Bethe Ansatz (BA) method for solving and understanding 1D many-body integrable systems has been a valuable tool in condensed matter.

Many physical quantities can be computed from the BA equations alone. Among these are thermodynamic quantities, which can be computed using the thermodynamic Bethe ansatz method.$^2$ Nevertheless, the BA solution does not provide a way of computing all physical quantities of interest. For example, non-diagonal correlation functions, such as the spectral function, important to understand photoemission experiments, or the optical conductivity, important to understand the response of the system to an electromagnetic field have to determined using other methods.$^3$–$^5$

The study of transport properties in integrable systems is one of the most active fields of research in integrable models in these days. The two most studied models have been the Hubbard and the spinless fermion (or the spin 1/2 Heisenberg chain) models. The understanding of the transport properties of these systems is related to the calculation of the optical conductivity (or spin conductivity$^6$). This quantity is defined as

$$\sigma(\omega) = 2\pi D_c \delta(\omega) + \sigma_r(\omega),$$  \(1\)

where $D_c$ is the charge stiffness and $\sigma_r(\omega)$ is the regular part of the conductivity. This last quantity is associated with absorption of radiation at finite $\omega$ values. The charge stiffness is associated with the existence of a metal-insulator transition in the system, driven by the interaction (a Mott-Hubbard transition). If $D_c$ is finite the system presents infinite conductivity (free acceleration), but if $D_c = 0$ the system is an insulator. This simple picture holds at zero temperature.

W. Kohn showed that $D_c$ can be computed from the ground state energy, $E_0$, of the system, imposing twisted boundary conditions, as$^7$

$$D_c = \frac{N}{2} \frac{\delta^2 E}{\delta \phi^2} \bigg|_{\phi=0},$$  \(2\)

where $\phi$ is a twisting angle (or, in the 1D case, a flux piercing the ring).

At finite temperature, the picture above is slightly changed but Eq. (1) still holds.$^8$ The first question one can raise is whether at finite temperature the system still has a finite value of $D_c$ if $D_c$ is finite at zero temperature. The answer to this question was first conjectured by Zotos and Prelovšek using numerical methods$^9$, and put on firm grounds by Zotos$^{10}$ in the context of the XXZ Heisenberg model. These authors have shown that integrable systems present ballistic transport of charge and spin at finite temperature if the charge (or spin) stiffness is finite at zero temperature.

These theoretical questions about the effect of integrability on transport properties triggered experimental...
research on transport of energy and spin in quasi 1D materials, such as \( \text{SrCuO}_2 \) and \( \text{Sr}_2\text{CuO}_3 \). The results are controversial. The experimental study of heat transport by spin excitation of \( \text{SrCuO}_2 \) and \( \text{Sr}_2\text{CuO}_3 \) materials revealed an anomalous enhanced thermal conductivity for temperatures above 40 K,\(^{11} \) and the authors concluded for ballistic transport of energy by the spinons. The MNR study,\(^{12} \) for zero momentum transfer, in the \( \text{Sr}_2\text{CuO}_3 \) material was able to probe the uniform spin susceptibility separately from the staggered \( \pm \pi/a \) (\( a \) is the lattice constant) mode. The authors concluded for ballistic transport in the \( T = 0 \) limit and for diffusion-like transport at finite temperatures (even for \( T \ll J \), where \( J \) is the antiferromagnetic coupling). Both from an experimental and theoretical\(^{13} \) point of view, the study of transport properties in ladder spin systems is open to research, and the questions raised in this work should also be pursued in those systems.

Although the theoretical picture for the spin transport in the XXZ Heisenberg model is clear if we consider the system in the parameters region corresponding to the XY universality class \( [\Delta < 1, \text{see Eq.}(3)] \), a complete understanding of the spin stiffness (or charge stiffness, in terms of the spinless fermion model) in the parameters region corresponding to the Ising universality class \( (\Delta > 1) \) is lacking. In the XY universality class region the zero temperature spin stiffness is given by \( D_c = J\pi \sin(\mu)/(4\mu(\pi - \mu)) \), with \( 1 \leq \mu \leq \pi \), reaching the value \( J/4 \) at the isotropic point \( (\mu = \pi) \).\(^{14} \) At finite temperature, and for \( 1 < \mu < \pi \), the spin stiffness was computed by Zotos,\(^{10} \) although there is some controversy about its temperature dependence.\(^{15} \) In the Ising universality class region the zero temperature spin stiffness is zero, in the thermodynamic limit, and its value for finite temperatures has not been computed yet but it was conjectured it should also be zero.\(^{9,16} \) We stress here that the above conjecture is by no means obvious. For example, in the Hubbard model at half-filling, the zero temperature charge stiffness is zero, but it was found to be finite at finite temperatures,\(^{18} \) although this result has been criticized by computing the curvature of levels explicitly in the \( U \gg t \) limit.\(^{19} \)

Also the scaling properties of \( D_c \) in the region \( \Delta > 1 \) have not been studied as were in the \( \Delta < 1 \) region,\(^{17} \) and for the Hubbard model.\(^{20} \)

In the study of the scaling properties of the Mott-Hubbard insulator the existence of a scaling function \( Y(\xi/N) \) for the charge stiffness was shown,\(^{20} \) where \( \xi \) is the correlation length and \( N \) is the number of sites, and it was conjectured that \( Y(\xi/N) \) should be a universal function characterizing other metal-insulator transitions in 1D systems and, in particular, it should characterize that transition in the spinless fermion model.

The spinless fermion model can be mapped into the spin 1/2 Heisenberg chain by a Jordan-Wigner transformation.\(^{21} \) Therefore the transition between a spin conductor (spin currents propagating ballistically) and a spin insulator (existence of spin diffusion), occurring at the isotropic point, and characterized by the spin stiffness value can be mapped into a metal-insulator transition, driven by the Coulomb interaction among spinless fermions at adjacent sites. In this work we consider the spin language, but our results are equally valid for spinless fermions.

This work is organized as follows. In the section II we derive the asymptotic (large \( N \)) behavior for the spin stiffness and obtain the correlation length \( \xi \) characterizing the insulating state. In section III we present a numerical study of the spin stiffness of the XXZ Heisenberg chain and the Hubbard model. In section IV we summarize our results.

**II. ASYMPOTIC BEHAVIOR**

The Hamiltonian of XXZ model reads

\[
H = -J \sum_{l=1}^{N} \left[ \frac{1}{2} (S_l^+ S_{l+1}^- + S_l^- S_{l+1}^+) e^{-i\phi/N} + \text{H.c.} \right] + \Delta S_l^z S_{l+1}^z, \tag{3}
\]

where \( N \) is the number of sites, \( S_l^+, S_l^-, S_l^z \) are spin 1/2 operators at site \( l \), and \( \phi \) is a flux piercing the 1D ring. The connection with spinless fermion model is made by means of the Jordan-Wigner transformation. In terms of spinless fermion the Hamiltonian (3) can be written as

\[
\mathcal{H} = -\frac{J}{2} \sum_{(i,j)} c_i^\dagger c_j + V \sum_i (\hat{n}_i - \frac{1}{2})(\hat{n}_{i+1} - \frac{1}{2}), \tag{4}
\]

where the spinless fermion operators \( c_i^\dagger, c_i \) obey the usual anti-commutation relations, \( \langle i,j \rangle \) means summation over nearest neighbors, \( \hat{n}_i = c_i^\dagger c_i \) is the usual local number operator, \( \frac{1}{2} \) is the hopping integral (normally called \( t \) but here called \( \frac{1}{2} \) to stress the important exact correspondence between this model and the XXZ Heisenberg spin 1/2 chain), and \( V = J\Delta \) is the nearest-neighbor Coulomb repulsion.

The Hamiltonian (3) can be solved by the BA,\(^{2,22,23} \) and the energy eigenvalues are given, for \( \Delta = \cosh(\gamma) \) \( (\gamma > 0) \), by

\[
E = E_0 - J \sum_{j=1}^{N_\uparrow} \frac{\sinh^2 \gamma}{\cosh(\gamma) - \cos(\gamma x_j)}.
\]

where \( E_0 = J\Delta N/4, N_\sigma (\sigma = \uparrow, \downarrow) \) stands for the number of up and down spins, and the numbers \( x_j \) are discussed below. In the case \( N_\uparrow = N_\downarrow \) the system has an energy gap relatively to the ground state. In terms of spinless fermions the system is an insulator (in the thermodynamic limit). The energy gap is given by

\[
E_\Delta = J \sinh \gamma \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{\cosh(n\gamma)}.
\tag{6}
\]
The numbers \( x_j \) obey Bethe ansatz equations (with twisted boundary conditions)

\[
N\theta_1(x_j, \gamma) = 2\pi I_j + \phi + \sum_{l=1}^N \theta_2(x_j - x_l, \gamma). 
\]  
(7)

where

\[
\theta_n(x, \gamma) = 2 \tan^{-1} \left( \frac{\tan(\gamma x/2)}{\tanh(n\gamma/2)} \right). 
\]  
(8)

The ground state quantum number configuration is:

\[
I_j = \left[ \frac{N_1 - 1}{2}, \frac{N_1 - 3}{2}, \ldots, \frac{N_1 - 1}{2} \right]. 
\]  
(9)

In the absence of magnetic flux, the ground state density is given by

\[
\rho_\infty(x) = \frac{1}{2Q} \left( 1 + \sum_{n=1}^\infty \frac{\cos(n\gamma x)}{\cosh(n\gamma)} \right) = \frac{K}{2\pi Q} \text{dn} \left( \frac{Kx}{Q}, k \right). 
\]  
(10)

where \( K = K(k) \) is the complete elliptic integral of the first kind, \( \text{dn}(y, k) \) is a Jacobian elliptic function of modulus \( k \), and we have

\[
\frac{K(\sqrt{1-k^2})}{K(k)} = \frac{1}{Q} = \frac{\gamma}{\pi}. 
\]  
(11)

The ground state density (10) can be obtained from the momentum density \( p(x) = \text{am}(Kx/Q, k) \) as \( dp(x)/dx = 2\pi \rho(x) \), and where \( \text{am}(y, k) \) is the amplitude function.

In order to obtain the charge-stiffness large-\( N \) asymptotic behavior we write the energy (6) as

\[
E = E_0 - J \int \frac{\sinh^2 \gamma \ dx}{\cosh \gamma - \cos(\gamma x)} \sum_{j=1}^{N_1} \delta(x - x_j). 
\]  
(12)

Using the properties of the delta function and the Poisson summation formula, Eq. (12) can be written as

\[
E = E_0 - JN \sinh^2 \gamma \int \frac{\rho_N(x, \phi) \ dx}{\cosh \gamma - \cos(\gamma x)} \times \sum_{m=-\infty}^{\infty} \exp\{im[Np_N(x) - \phi]\}, 
\]  
(13)

where \( p_N(x_j) = (2\pi I_j + \phi)/N \). For large \( N \) the integral in Eq. (13) is dominated by the \( m = 0, \pm 1 \) terms. As a consequence the stiffness (2) is a sum of two terms and reads

\[
D_c = D_c^{(m=\pm1)} + D_c^{(m=0)}, 
\]  
(14)

where

\[
D_c^{(m=\pm1)} = \frac{N \delta^2 E^{(m=\pm1)}}{2} \bigg|_{\phi=0} = \frac{\gamma JN \sinh^2 \gamma}{2\pi} \int \frac{\sin[Np_N(x)] \sin(\gamma x)}{[\cosh \gamma - \cos(\gamma x)]^2} \ dx, 
\]  
(15)

and

\[
D_c^{(m=0)} = \frac{N \delta^2 E^{(m=0)}}{2} \bigg|_{\phi=0} = -JN^2 \int \frac{\sinh^2 \gamma}{\cosh \gamma - \cos(\gamma x)} \frac{\delta^2 p_N(x)}{\delta \phi^2} \bigg|_{\phi=0}. 
\]  
(16)

The first term dominates Eq. (14) for large values of \( \Delta \), since the second one is, in this case, exponentially small. Both Eq. (15) and Eq. (16) are evaluated using the steepest descent method. The final asymptotic result for \( D_c \) can be cast in the form

\[
D_c(N) \sim D(\gamma) \sqrt{N} e^{-N/\xi(\gamma)}, 
\]  
(17)

where \( \xi(\gamma) = ip(Q + i) - i\pi N/2 \) is the correlation length given by

\[
1/\xi(\gamma) = \frac{\gamma}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n \sinh(n\gamma)}{n \cosh(n\gamma)}. 
\]  
(18)

and \( D(\gamma) \) is given by

\[
D(\gamma) = \left[ \frac{\tanh^2 \gamma}{4} + \sum_{n=1}^{\infty} (-1)^{n+1} ne^{-2n\gamma} \tanh(n\gamma) \right] \times \frac{\gamma J \sinh \gamma}{|2\pi f''(\zeta_0)|^{1/2}}. 
\]  
(19)

The correlation length (18) agrees with the value obtained previously using a different method, and is represented in Fig. 1.

![FIG. 1. Representation of the correlation length as function of \( \Delta \). It is clear that \( \xi \) diverges as we approach the isotropic point \( \Delta = 1 \).](image-url)
The result (18) is quite similar to that obtained for the Hubbard model by Stafford, Millis, and Shastry.\textsuperscript{20}

III. NUMERICAL RESULTS

At $T = 0$ the XXZ Heisenberg model for $\Delta > 1$ presents non-ideal spin transport in the thermodynamic limit, because the spin stiffness is zero. In terms of spinless fermions one has an insulating behavior. For a finite-size system $D_c$ is finite even for large values of $N$ and for moderate values of $\Delta$, as can be seen in Fig. 2. In this figure we see that even for values of $N$ as large as 1000 sites there are values of $\Delta$ for which $D_c$ is of the order of its value at the isotropic point (represented by a filled circle). This shows that the behavior of $D_c$, when $\Delta > 1$, in a finite-size system, is dominated by the value of the $D_c$ for $\Delta = 1$. The value of $D_c$ at the isotropic point is by no means trivial, and the presence of this point has consequences for the scaling behavior of $D_c$ in the Ising region as we show below. We believe this difference is due to the presence of logarithmic corrections to the scaling function by marginal interactions at the isotropic point.

In Ref. 20 the charge stiffness of the Hubbard model, at half-filling, was shown to present a scaling behavior of the form $Y(\xi/N)$, for all values of the Hubbard interaction $U$, where $\xi$ is the correlation length and $N$ is the system size. In Figure 4 we show data from a numerical solution of the BA equations for the Hubbard model, which agrees with the results of Ref. 20. Looking more carefully to Figure 4 we see there is scaling behavior for all values of $N$ as we get closer to the gapless phase $U = 0$ ($\xi \to \infty$). On the other hand, as $U$ increases, $\xi \to 0$, and the scaling behavior can only be verified for larger system sizes.

The above discussion stresses that for mesoscopic systems the separation between an insulator and a metal is not clear. Only when the correlation length is smaller than the system size a clear separation between an insulator and a metal exist. The correlation length (18) is independent of the system size, and the dependence of $D_c$ on $\xi$ is represented in Fig. 3. This figure makes the above statement clear, for example, for values of $\xi$ bigger than $10^5$, $D_c$ is finite even for $N = 1002$, whereas for values of $\xi$ smaller than $10^5$ $D_c$ is zero for $N = 1002$ but not for the other (smaller) values of $N$.

In Ref. 20 the charge stiffness of the Hubbard model, at half-filling, was shown to present a scaling behavior of the form $Y(\xi/N)$, for all values of the Hubbard interaction $U$, where $\xi$ is the correlation length and $N$ is the system size. In Figure 4 we show data from a numerical solution of the BA equations for the Hubbard model, which agrees with the results of Ref. 20. Looking more carefully to Figure 4 we see there is scaling behavior for all values of $N$ as we get closer to the gapless phase $U = 0$ ($\xi \to \infty$). On the other hand, as $U$ increases, $\xi \to 0$, and the scaling behavior can only be verified for larger system sizes.

Based on the scaling behavior of the Hubbard model, a conjecture that the same scaling behavior would apply to other 1D systems (in particular for the spinless
fermion system) presenting metal-insulator transitions was drawn.

Following these ideas we have first derived an asymptotic expression for the stiffness of the XXZ Heisenberg model (or spinless fermions). This allowed us to determine the correlation length $\xi$ for this model. We have then solved numerically the BA equations for systems of different sizes and tried to see if some type of scaling was possible. From Figure 5 we see that the representation of $D_c$ as function of $\xi/N$ gives curves that do not collapse into each other unless $N > 202$. This larger value of $N$ need for scaling to be observed contrast with the case of the Hubbard model in Figure 4, although the values of $\xi/N$ are the same in both cases. Also, as we get closer and closer to $\Delta = 1$, where the system presents a gapless phase (in Figure 5 this corresponds to $\xi/N \rightarrow \infty$), the collapsing of the data takes place for larger and larger values of $N$. This behavior contrasts with that seen above for the Hubbard model.

We proceed by grouping the several values of $N$ for the Heisenberg model. The inset shows the same but for $N = 502$ and 1002, being clear that the collapsing of the data close to $\Delta = 1$ requires larger values of $N$.

Since the scaling as function of $\xi/N$ is not perfect for all values of $N$, let us assume that $D_c$ could follow a scaling behavior of the form $D_c = N^{-\eta} f(\xi/N)$ and let us find the value of the exponent $\eta$ that leads to the collapsing of the data. We proceed by grouping the several $N$ in groups of two as follows $\{22, 50\}$, $\{50, 102\}$, $\ldots$, $\{502, 1002\}$, and for each set $\eta$ is determined by a minimization procedure which leads to the best possible scaling. Our findings are listed in Table I, and show that the value of the exponent $\eta$ decreases as the $N$ increases, leading for very large values of $N$, to the predicted scaling behavior $D_c = f(\xi/N)$. On the other hand, for small values of $N$ the scaling behavior $D_c = N^{-\eta} f(\xi/N)$ works much better as we show in Figure 6. The fact that it was not possible to find single scaling behavior for all $N$ is an indication that either a different scaling law or corrections containing $N$ and associated with the presence of the critical point are needed.

Let us now show that scaling relation $D_c = N^{-\eta} f(\xi/N)$ can be used to determine $\xi$ directly from the solution of small systems. The scaling relation

$$g(N, \xi) = \frac{1}{N^\eta} f\left(\frac{\xi}{N}\right)$$  \hspace{1cm} (20)

has an interesting property which can be used in order to determine the correlation length $\xi$ in cases where this can not be done analytically. Supposing that (20) holds for a given quantity, $g$, and assuming the existence of a correlation length depending solely on other parameter of the problem, i.e. $\xi = \xi(\lambda)$ (which, in the case of our problem, is the anisotropy $\Delta$), the scaling relation can be expressed in a slightly different, but equivalent notation as

$$g_\lambda(N) \equiv g(N, \xi(\lambda)) = \frac{1}{N^\eta} f\left(\frac{\xi(\lambda)}{N}\right).$$  \hspace{1cm} (21)

We can now consider, as is frequently the case in numerical calculations, measuring $g$ for various sizes and different values of the parameter $\lambda$. Then, for each pair of sets $g_\lambda(N)$ and $g_{\lambda_j}(N)$ we can write

$$g_{\lambda_j}\left(N, \frac{\xi(\lambda_j)}{\xi(\lambda_i)}\right) = \frac{1}{N^\eta} f\left(\frac{\xi(\lambda_j)}{\xi(\lambda_i)}\right),$$  \hspace{1cm} (22)

or, setting $r_{ij} = \frac{\xi(\lambda_j)}{\xi(\lambda_i)}$,

$$g_{\lambda_j}(N) = g_{\lambda_i}(N r_{ij}) r_j^\eta. \hspace{1cm} (23)$$

This relation should hold for given $r_{ij}$ and $\eta$, that can be extracted by direct minimization\textsuperscript{20}. Proceeding in this way for the entire range $\{\lambda_1, \ldots, \lambda_M\}$ in which the numerical values of $g$ are calculated, the dependence of $\xi$ in the parameter $\lambda$ can be obtained, except for a global multiplicative constant. An example of the typical result obtained via this procedure can be seen in Figure 7 (we remark that for this case the value of $\eta$ is constant for all values of $N$ and $\Delta$ used). Here, numerical results for the Heisenberg model with chains of sizes 6 to 24 were used to calculate the charge stiffens $D_c(N, \Delta)$. We remark that the values of $N$ used here are easily accessible to exact diagonalization. Although the scaling relation (20) doesn’t seem to apply exactly to this problem, it was found, for $\Delta$ not too close to the critical point, that a good scaling can be obtained as shown in the left frame of Figure 7, allowing the numerical calculation of $\xi(\Delta)$, as shown in the right frame of Figure 7. As can be seen, we were able to calculate the correlation length without any further information other than the numerical results for $D_c(N, \Delta)$. This concept can be usefully applied to other cases where the analytical result for $\xi$ is unknown, at least to give a rough idea of the behavior of the correlation length or any other quantity involved in the scaling relation.

FIG. 5. Representation of the stiffness, $D_c$, as function of $\xi/N$, for the Heisenberg model. The inset shows the same but for $N = 502$ and 1002, being clear that the collapsing of the data close to $\Delta = 1$ requires larger values of $N$. 

5
We believe the difference between the Hubbard model and the XXZ Heisenberg chain is related to the presence of the non-trivial point $\Delta = 1$, for the Heisenberg chain as opposed to the trivial point $U = 0$ (independent electrons) for the Hubbard model. An interesting question which remains to be answered concerns the scaling properties and the correlation length of the metal-insulator transition in an extended Hubbard model, considering next-nearest neighbor interaction, since this model is no longer integrable. For small system sizes the above scaling function does not apply and a scaling of the form $D(N) = N^{-\eta}f(N/\xi)$ produces a much better scaling behavior. Using this scaling function the correlation length $\xi$ was obtained from the analysis of the small systems alone, and shown to agree well with the exact value. This method can be used to derive the correlation length for systems which do not have an exact solution.

This work is supported by the Portuguese Program PRAXIS XXI under grant number 2/2.1/FIS/302/94. V.M.P. is support by Fundação para a Ciência e Tecnologia grant No. BD/46/55/2001. We thank P.D. Sacramento for helpful comments on the manuscript. V.M.P. thanks J.M.B. Lopes dos Santos for valuable discussions about exact diagonalization methods. N.M.R.P. thanks X. Zotos for valuable comments on an early stage of this work.

IV. SUMMARY

We have studied the spin stiffness of the XXZ Heisenberg chain for $\Delta > 1$. We have shown that the in large $N/\xi$ behavior of the stiffness agrees with the asymptotic behavior $D(N) = Y(N/\xi)$, where $Y(x)$ is a universal scaling function, but not for all ranges of $N/\xi$, as in the Hubbard model. Contrary to the case of the Hubbard model and for a given value of $\xi/N$ the system sizes required to observe the scaling behavior are about an order of magnitude larger for the XXZ Heisenberg chain. Moreover as we get closer and closer to the isotropic ($\Delta = 1$) point, characterized by $\xi/N \to \infty$, larger and larger system sizes are required, indicating the scaling behavior does not apply in that region. We believe the difference

\[ \xi/N \to \infty \]

between the Hubbard model and the XXZ Heisenberg chain is related to the presence of the non-trivial point $\Delta = 1$, for the Heisenberg chain as opposed to the trivial point $U = 0$ (independent electrons) for the Hubbard model. An interesting question which remains to be answered concerns the scaling properties and the correlation length of the metal-insulator transition in an extended Hubbard model, considering next-nearest neighbor interaction, since this model is no longer integrable. For small system sizes the above scaling function does not apply and a scaling of the form $D(N) = N^{-\eta}f(N/\xi)$ produces a much better scaling behavior. Using this scaling function the correlation length $\xi$ was obtained from the analysis of the small systems alone, and shown to agree well with the exact value. This method can be used to derive the correlation length for systems which do not have an exact solution.

This work is supported by the Portuguese Program PRAXIS XXI under grant number 2/2.1/FIS/302/94. V.M.P. is support by Fundação para a Ciência e Tecnologia grant No. BD/46/55/2001. We thank P.D. Sacramento for helpful comments on the manuscript. V.M.P. thanks J.M.B. Lopes dos Santos for valuable discussions about exact diagonalization methods. N.M.R.P. thanks X. Zotos for valuable comments on an early stage of this work.

1. H. A. Bethe, Z. Phys. 71, 205 (1931). H. A. Bethe, reprinted (translation) paper by Daniel C. Mattis in The Many-Body Problem - An Encyclopedia of Exactly Solved Models in One Dimension, (World Scientific, Singapore, 1993), page 689.
2. Minorou Takahashi, Thermodynamics of one-dimensional Solvable Models, (Cambridge, 1999). (see also references therein.)
3. J.M.P. Carmelo, N.M.R. Peres, and P.D. Sacramento, Phys. Rev. Lett. 84, 4673 (2000).
4. D.N. Aristov, Vadim V. Cheianov, and A. Luther, cond-mat/0203273.
5. J.M.P. Carmelo, J.M.B. Lopes dos Santos, L.M. Martelo, and P.D. Sacramento, preprint.
6. J. M. P. Carmelo, P. Horsch, and A. A. Ovchinnikov, Phys. Rev. B 46, 14728 (1992).
7. Walter Kohn, Phys. Rev. 133, A171 (1964).
8. H. Castella, X. Zotos, and P. Prelovšek, Phys. Rev. Lett. 47, 972 (1995).
9. X. Zotos and P. Prelovšek, Phys. Rev. B 53, 983 (1996).
10. X. Zotos, Phys. Rev. Lett. 82, 1764 (1999).
11. A.V. Sologubenko, K. Giannò, H.R. Ott, A. Vietkine, and A. Revcolevschi, Phys. Rev. B 64, 054412 (2001).
12. K.R. Thurber, A. W. Hunt, T. Imai, and F.C. Chou, Phys. Rev. Lett. 87, 247202 (2001).
13. V.R. Vieira, N. Guihery, J.P. Rodriguez, and P.D. Sacramento, Phys. Rev. B 63, 224417 (2001); P.D. Sacramento and V.R. Vieira, J. Phys.: Condens. Matter 14, 591 (2002).
14. B. S. Shastry and Bill Sutherland, Phys. Rev. Lett. **65**, 1833, (1990).
15. J. V. Alvarez and Claudius Gros, Phys. Rev. Lett. **88**, 077203 (2002).
16. N.M.R. Peres, P.D. Sacramento, D.K. Campbell, and J.M.P. Carmelo, Phys. Rev. B. **59**, 7382 (1999).
17. Nicolas Laflorencie, Sylvain Capponi, and Erik S. Sorensen, cond-mat/0105462.
18. Satoshi Fujimoto and Norio Kawakami, J. Phys. A **31**, 465 (1998).
19. N.M.R. Peres, R.G. Dias, P.D. Sacramento, and J.M.P. Carmelo, Phys. Rev. B, **61**, 5169 (2000).
20. C.A. Stafford, A.J. Millis, B.S. Shastry, Phys. Rev. B **43**, 13660 (1991); C.A. Stafford and A.J. Millis, Phys. Rev. **48**, 1409 (1993).
21. D.C. Mattis, *The Theory of Magnetism I*, (Springer, 1998, 2nd printing), p.91.
22. Jacques Des Cloizeaux and J.J. Pearson, Phys. Rev. **128**, 2131 (1962); Jacques Des Cloizeaux and Michel Gaudin, J. Math. Phys. **7**, 1384 (1966).
23. M. Gaudin, Phys. Rev. Lett. **26**, 1301 (1971).
24. Milton Abramowitz and Irene A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, (Dover, New York, 1970) chap. 16.
25. R. J. Baxter, *Exactly Solved Models in Statistical Mechanics*, (Academic Press, 1982) p. 155.
26. The minimization studies and the calculation of correlation lengths and energy gaps from the analysis of small systems in the XXZ model was first done by Vitor M. Pereira in his degree thesis. Vitor M. Pereira and J. M. B. Lopes dos Santos (2000), unpublished.

### TABLE I. Value of the exponent \( \eta \) leading to the collapse of the data, for all values of \( \xi/N \), giving a pair of consecutive \( N \)’s.

| pairs of \( N \)   | \( \eta \) |
|-------------------|----------|
| 22 and 50         | 0.040    |
| 50 and 102        | 0.028    |
| 102 and 202       | 0.022    |
| 202 and 502       | 0.016    |
| 502 and 1002      | 0.010    |