Constraining the Phase of $B_s - \bar{B}_s$ Mixing

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New physics contributions to $B_s - \bar{B}_s$ mixing can be parametrized by the size ($r_s^2$) and the phase ($2\theta_s$) of the total mixing amplitude relative to the Standard Model amplitude. The phase has so far been unconstrained. We first use the $D_0$ measurement of the semileptonic CP asymmetry $A_{SL}$ to obtain the first constraint on the semileptonic CP asymmetry in $B_s$ decays, $A_{SL}^s = -0.008 \pm 0.011$. Then we combine recent measurements by the CDF and $D_0$ collaborations - the mass difference ($\Delta M_s$), the width difference ($\Delta \Gamma_s$) and $A_{SL}$ - to constrain $2\theta_s$. The errors on $\Delta \Gamma_s$ and $A_{SL}^s$ should still be reduced to have a sensitive probe of the phase, yet the central values are such that the regions around $2\theta_s \sim 3\pi/2$ and, in particular, $2\theta_s \sim \pi/2$, are disfavored.

Introduction. Flavor changing $b \to s$ transitions are a particularly sensitive probe of new physics. Among these, $B_s - \bar{B}_s$ mixing occupies a special place. New physics contributions to the mixing amplitude $M_{12}^s$ can be parametrized in the most general way as follows:

$$M_{12}^s = r_s^2 e^{2i\theta_s} (M_{12}^{SM})^s,$$

where $(M_{12}^{SM})^s$ is the Standard Model (SM) contribution to the mixing amplitude. Values of $r_s^2 \neq 1$ and/or $2\theta_s \neq 0$ would signal new physics. Assuming that the new physics can affect any loop processes but is negligible for tree level processes, and that the 3 × 3 CKM matrix is unitary (i.e. no quarks beyond the known three generations), we can use various experimental measurements to constrain the new physics parameters $r_s^2$ and $2\theta_s$:

1. The mass difference between the neutral $B_s$ states:

$$\Delta M_s = (\Delta M_s)^{SM} r_s^2.$$

2. The width difference between the neutral $B_s$ states:

$$\Delta \Gamma_s^{CP} = \Delta \Gamma_s \cos 2\theta_s = (\Delta \Gamma_s)^{SM} \cos 2\theta_s.$$

3. The semileptonic asymmetry in $B_s$ decays:

$$A_{SL}^s = -\text{Re} \left( \frac{\Gamma_{sH}}{M_{12}^{SM}} \right) \sin 2\theta_s. \tag{4}$$

4. The CP asymmetry in $B_s$ decays into final CP eigenstates such as $\psi \phi$:

$$S_{\psi \phi (CP=+)} = -\sin 2\theta_s. \tag{5}$$

Our convention here is defined by $\Delta M_s \equiv M_{sH} - M_{sL}$ and $\Delta \Gamma_s \equiv \Gamma_{sH} - \Gamma_{sL}$. The observable $\Delta \Gamma_s^{CP}$ is defined by $\Delta \Gamma_s^{CP} \equiv \Gamma_- - \Gamma_+$, where $\Gamma_- (\Gamma_+)$ is deduced from fitting the decay rate into a final CP-odd (even) state assuming that it is described by a single exponential. This assumption introduces an error of $\mathcal{O}(y_s^2) = 0.01 [y_s \equiv \Delta \Gamma_s/(2\Gamma_s)]$. In the expressions for $\Delta \Gamma_s$ and $S_{\psi \phi}$ we neglect terms of $\mathcal{O}(\sin 2\beta_s) = 0.04$ [where $\beta_s = \arg \{(V_{ts}V_{tb})/(V_{cs}V_{cb})\}$], while the approximation for $A_{SL}^s$ is good to $\mathcal{O}(m_c^2/m_t^2) \sin 2\beta_s = 0.004$.

Until very recently, experiments gave only a lower bound on $\Delta M_s$, a large error on $\Delta \Gamma_s$, and no meaningful information on the CP asymmetries. Under these circumstances, there has been only a lower bound on $r_s^2$ and no constraint at all on $2\theta_s$.

Recently, three important experimental developments took place in this context:

- The CDF collaboration measured $\Delta M_s$:

$$\Delta M_s = 17.33^{+0.42}_{-0.21} \pm 0.07 \text{ ps}^{-1}. \tag{6}$$

(The $D_0$ collaboration provided a milder two-sided bound 4.)

- The $D_0$ collaboration measured $\Delta \Gamma_s^{CP}$:

$$\Delta \Gamma_s^{CP} = -0.15 \pm 0.07 \pm 0.003 \text{ ps}^{-1}. \tag{7}$$

Averaging this result with the earlier measurements by CDF 4 and ALEPH 7, we obtain

$$\Delta \Gamma_s^{CP} = -0.22 \pm 0.08 \text{ ps}^{-1}. \tag{8}$$

- The $D_0$ collaboration searched for the semileptonic CP asymmetry $A_{SL}^s$:

$$A_{SL}^s = -0.0026 \pm 0.0024 \pm 0.0017. \tag{9}$$

As obvious from eq. (5), the main implication for new physics of the new result for $\Delta M_s$, eq. (6), is a range for $r_s^2$ which can be further translated into constraints on parameters of specific models 10–12, 14, 15, 17. Here, we would like to focus instead on the phase of the mixing amplitude $2\theta_s$. In order that a measurement of $\Delta \Gamma_s^{CP}$ can be used to constrain $\cos^2 2\theta_s$, the experimental error should be at or below the level of $(\Delta \Gamma_s)^{SM}$. The new $D_0$ measurement of $\Delta \Gamma_s^{CP}$ is the first to reach the
required level. There are three necessary conditions in order that a measurement of $A_{\text{SL}}$ can be used to constrain $2\theta$:

1. The experimental error on $A_{\text{SL}}$ should be at or below the level of $|\Gamma_{12}^d/M_{12}^d|^{\text{SM}}$;

2. An upper bound on $r^+_{\tau}$ should be available;

3. An independent upper bound on $A_{\text{SL}}^d$ (the semileptonic asymmetry in $B_d$ decays) should be available.

Both the DØ measurement of $A_{\text{SL}}$ and the CDF measurement of $\Delta M_s$ are thus crucial for our purposes, because they satisfy, for the first time, the first and second condition, respectively.

Relating $A_{\text{SL}}$ to $A_{\text{SL}}^d$. The semileptonic asymmetry measured at the Tevatron,

$$A_{\text{SL}} = \frac{\Gamma(b \to \mu^+\mu^-X) - \Gamma(b \to \mu^-\mu^+X) + \Gamma(b \to \mu^-\mu^-X)}{\Gamma(b \to \mu^+\mu^+X) + \Gamma(b \to \mu^-\mu^-X)}$$

where

$$T_q^\pm = T(B_q \to B_q) \pm T(B_q \to \bar{B}_q).$$

The relevant time integrated transition probabilities are as follows:

$$T(B_q \to \bar{B}_q) = \left(1 - \frac{\delta_q}{1 + \delta_q}\right) \frac{Z_q}{2T_q},$$

$$T(B_q \to B_q) = \left(1 + \frac{\delta_q}{1 - \delta_q}\right) \frac{Z_q}{2T_q}.$$

where $(y_q = \Delta \Gamma_q/(2T_q), x_q = \Delta M_q/T_q)$

$$Z_q \equiv \frac{1}{1 - y^2_q - 1/1 + x^2_q}.\quad (15)$$

The quantity $\delta_q$ characterizes CP violation in mixing [$\delta_q = (1 - |q/p|^2)/(1 + |q/p|^2)$]. Given that it is small, one can write to leading order $\delta_q = \frac{A_{\text{SL}}^d}{2}, T_q = \frac{A_{\text{SL}}^d}{2} Z_q/T$ and $T_q^+ = Z_q/T$. Taking again the SU(3) limit, $\Gamma_d = \Gamma_s$ (the equality is violated at high order in $1/m_b$; experimentally $^{13}\frac{\tau_s}{\tau_d} \sim 0.96 \pm 0.04$), we obtain

$$A_{\text{SL}} = \frac{f_d Z_d A_{\text{SL}}^d + f_s Z_s A_{\text{SL}}^s}{f_d Z_d + f_s Z_s}. \quad (16)$$

Given the experimental ranges $^{23}|y_d| = 0.004 \pm 0.019$ and $|y_s| = 0.16 \pm 0.06$ we can safely neglect $y_d^2$ and $y_s^2$. (Within our framework, we expect $^{22}y_d^2 \sim 0.01$.)

Using the experimental values $^{18}$ $f_d = 0.4, f_s = 0.1, x_d = 0.78$ and $x_s = 25.3$, we obtain

$$A_{\text{SL}} \simeq 0.6 A_{\text{SL}}^d + 0.4 A_{\text{SL}}^s. \quad (17)$$

There are two sets of measurements that, in combination, allow us to extract a range for $A_{\text{SL}}^d$. First, we have the DØ measurement of $A_{\text{SL}}$ (eq. \[5\]), which we can average together with previous measurements by the LEP experiments OPAL $^{24}$ and ALEPH $^{22}$ (we neglect here the small difference between LEP and the Tevatron regarding the measured values of $f_{d,s}$). We find

$$A_{\text{SL}} = -0.0027 \pm 0.0029. \quad (18)$$

Second, we have measurements of $A_{\text{SL}}^d$ at the $\Upsilon(4S)$ energy by Babar $^{26}$, Belle $^{27}$ and CLEO $^{28}$. We find

$$A_{\text{SL}}^d = +0.0011 \pm 0.0055. \quad (19)$$

Thus, we obtain

$$A_{\text{SL}}^d = -0.008 \pm 0.011. \quad (20)$$

(One could include also the Babar measurement from hadronic modes $^{29}$. While this is not, strictly speaking, a measurement of $A_{\text{SL}}^d$, it does give $1 - |q/p|$.)
change the average to $A_{SL}^E = -0.0004 \pm 0.0055$ and, consequently, $A_{SL}^O = -0.006 \pm 0.011$. Our conclusions would remain unchanged.

**Constraining $2\theta_s$.** Our constraints on $2\theta_s$ involve eqs. 43 and 41. As concerns $(\Gamma_{12}/M_{12})^{SM}$, we use 22 (see also 24 for a different calculation with similar results)

\[
\mathcal{R}e \left( \frac{\Gamma_{12}}{M_{12}} \right)^{SM} = -0.0040 \pm 0.0016. \tag{21}
\]

As concerns $(\Delta M_s)^{SM}$, we use 11

\[
(\Delta M_s)^{SM} = \frac{G_F^2}{6\pi^2} \eta_B m_B \bar{B}_B F_B S(x_1) |V_{tb}V_{ts}|^2 = 17.8 \pm 4.8 \text{ ps}^{-1}. \tag{22}
\]

It is important to note that the range for $|V_{ts}V_{tb}|$ is derived using true level processes and CKM unitarity. The combination of 21 and 22 gives

\[
(\Delta \Gamma_s)^{SM} = -0.07 \pm 0.03 \text{ ps}^{-1}. \tag{23}
\]

We can now fit the new physics parameters $r_s^2$ and $2\theta_s$ to the experimental values of eqs. 6, 7, and 20 via eqs. 2, 3, and 4. To do so, we use the SM estimates of eqs. 21, 22, and 23.

It is easy to understand the constraint on $r_s^2$ by simply using eq. 2:

\[
r_s^2 = \frac{(\Delta M_s)^{exp}}{(\Delta M_s)^{SM}} = 0.97 \pm 0.26, \tag{24}
\]

To get a feeling for the situation concerning $2\theta_s$, we first use eqs. 3 and 4 separately. The $\Delta \Gamma_s$ measurement gives

\[
\cos^2 2\theta_s = \frac{(\Delta \Gamma_s)^{CP}}{(\Delta \Gamma_s)^{SM}} = 3.1 \pm 1.7. \tag{25}
\]

This range disfavors (at the 1.8$\sigma$ level) small $\cos^2 2\theta_s$ values, that is $2\theta_s \sim \pi/2, 3\pi/2$. The $A_{SL}^o$ measurement gives

\[
\sin 2\theta_s = -\frac{A_{SL}^o}{\mathcal{R}e(\Gamma_{12}^2/M_{12}^2)^{SM}} \frac{(\Delta M_s)^{exp}}{(\Delta M_s)^{SM}} = -1.9 \pm 2.8. \tag{26}
\]

This range disfavors large positive $\sin 2\theta_s$ values, that is $2\theta_s \sim \pi/2$. The combination of the two sources of constraints should therefore disfavor the regions around $2\theta_s \sim \pi/2, 3\pi/2$, with stronger significance for the first. This can be seen in Fig. 1 where we present the constraints in the $\cos 2\theta_s - \sin 2\theta_s$ plane. In Fig. 2 we present the constraints in the $r_s^2 - 2\theta_s$ plane. Note that eqs. 23 and 26 and Fig. 1 do not take into account the correlations between the contributions to the various observables, since they are meant to emphasize the impact of each measurement separately. The correlations are, however, fully taken into account in Fig. 2.

We note that the $O(30\%)$ error on $r_s^2$ is mainly **theoretical**: it reflects the theoretical uncertainty in $(\Delta M_s)^{SM}$. In contrast, the $O(100\%)$ error on $\sin 2\theta_s$ is mainly **experimental**: it comes from the error in the determination of $A_{SL}^o$. The $O(50\%)$ error on $\cos^2 2\theta_s$ has both experimental and theoretical aspects.

We learn that the constraints on $2\theta_s$ are still rather
weak. In principle, the error on $A_{SL}^b$ is still a factor of three larger than what is needed to have sensitivity to $\sin 2\theta_s$. However, since the central value for $\sin 2\theta_s$ happens – presumably due to statistical fluctuations – to lie below the physical region, large positive values of $\sin 2\theta_s$ are disfavored (at the 1$\sigma$ level). The error on $\Delta G_{CP}$ is closer to what is needed to be sensitive to $2\theta_s$ and, indeed, the resulting constraint is more significant.

We also consider a subclass of our framework, where new physics contributions are significant only in $\bar{b} \to s$ transitions. This modifies the analysis in three ways:

1. We can now extract a narrower range for $(\Delta M_s)^{SM}$ by using, in addition to the direct calculation of eq. (22), an indirect calculation [30, 31] that makes use of experimental measurements of $b \to d$ (and $s \to d$) processes and, in particular, identify $\Delta M_d^{exp} = \Delta M_d^{SM}$: $(\Delta M_s)^{SM} = 21.7^{+5.5}_{-4.2}$ ps$^{-1}$ [32]. The direct calculation of eq. (22) and the indirect one quoted here are essentially independent of each other. Therefore, we average over these two results and get

$$(\Delta M_s)^{SM} = 19.7 \pm 3.5 \text{ ps}^{-1}. \quad (27)$$

2. We can set $A_{SL}^d = 0$ and then

$$A_{SL}^b \approx 2.5 A_{SL} = -0.007 \pm 0.007. \quad (28)$$

3. We can now use (27) to obtain a more precise estimate of $(\Delta G_s)^{SM}$:

$$(\Delta G_s)^{SM} = -0.08 \pm 0.03 \text{ ps}^{-1}. \quad (29)$$

Now we get

$$r_s^2 = 0.88 \pm 0.16, \quad (30)$$

$$\cos^2 2\theta_s = 2.8 \pm 1.6, \quad (31)$$

$$\sin 2\theta_s = -1.4 \pm 1.6. \quad (32)$$

The situation is then quite similar to the first scenario. The smaller central value and smaller error on $r_s^2$ and on $\cos^2 2\theta_s$, compared to eqs. (24) and (25), respectively, correspond to the larger central value and smaller theoretical error in eq. (27) compared to eq. (22). In contrast, the higher central value and smaller error on $\sin 2\theta_s$, compared to eq. (26), are both mainly a result of the shift in the central value of $r_s^2$ and, in particular, little affected by the smaller error on $(\Delta M_s)^{SM}$.

As can be seen in the Figure, $2\theta_s = \pi/2$ is disfavored at the $2\sigma$ level.

**Conclusions.** The measurement of $A_{SL}$ by D$\phi$ probes CP violation in $B_s - \bar{B}_s$ mixing, $A_{SL}^b = -0.008 \pm 0.011$. In combination with the measurement of $\Delta M_s$ by CDF, and the measurements of $\Delta G_{CP}$ by D$\phi$ and CDF, the CP violating phase of the mixing amplitude is constrained for the first time. The constraints are still weak. Since experiments favor large values of $\Delta G_s$ compared to the SM value, small values of $\cos^2 2\theta_s$ (i.e., $2\theta_s \sim \pi/2, 3\pi/2$) are disfavored. Furthermore, since experiments favor a negative $A_{SL}$ (see eqs. (20) and (28)) and $Re(\Gamma_{12}/M_{12})^{SM}$ is negative, large positive values of $\sin 2\theta_s$ (i.e., $2\theta_s \sim \pi/2$) are disfavored even more strongly.

To improve the constraint, smaller experimental errors on $\Delta G_s$ and on $A_{SL}$ are welcome. Note however that a similar improvement in the measurement of $A_{SL}$ (see eq. (14)) is also required. Thus, the accuracy in determining $A_{SL}$ depends on both high energy hadron machines and $\Upsilon$(4S)-energy B factories.

In principle, $A_{SL}$ could also be extracted from measurements at hadron colliders only. To do this one needs, in addition to the measurement of $A_{SL}$, another measurement of a CP asymmetry in semileptonic decays, with a different weight of $B_d$ and $B_s$ in the sample. (For example, requiring at least one kaon in the final state would enhance the fraction of $B_s$.)

Of course, the phase $2\theta_s$ will be strongly constrained once $S_{\phi\phi}$ is measured. Then the combination of the four measurements – $\Delta M_s$, $\Delta G_s$, $A_{SL}$ and $S_{\phi\phi}$ – will provide a test of the assumption that new physics affects only loop processes [14, 11, 33]. The strength of this
test will, however, be limited by theoretical uncertainties, particularly by the calculation of $\Gamma_{12}^{SM}$.

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