BETTI NUMBERS AND TORSIONS IN HOMOLOGY GROUPS OF DOUBLE COVERINGS

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ABSTRACT. Papadima and Suciu proved an inequality between the ranks of the cohomology groups of the Aomoto complex with finite field coefficients and the twisted cohomology groups, and conjectured that they are actually equal for certain cases associated with the Milnor fiber of the arrangement. Recently, an arrangement (the icosidodecahedral arrangement) with the following two peculiar properties was found: (i) the strict version of Papadima-Suciu’s inequality holds, and (ii) the first integral homology of the Milnor fiber has a non-trivial 2-torsion. In this paper, we investigate the relationship between these two properties for double covering spaces. We prove that (i) and (ii) are actually equivalent.

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1. INTRODUCTION

Double coverings of CW-complexes are well-studied subject in topology. We first recall several classical constructions related to double coverings [10]. Let $X$ be a connected CW-complex. Let $p : Y \to X$ be an unbranched double covering with connected $Y$. Fix a base point $x_0 \in X$. By definition, the fiber $p^{-1}(x_0)$ consists of two points. Let $\gamma : [0,1] \to X$ be a closed path with $\gamma(0) = \gamma(1) = x_0$. The local triviality of the covering enables us to construct parallel transport of the fiber along $\gamma$ and a bijection $p^{-1}(x_0) \to p^{-1}(x_0)$. Because of the homotopy lifting property, this bijection depends only on the homotopy type of $\gamma$. Hence, the group

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homomorphism $\pi_1(X, x_0) \to \text{Aut}(p^{-1}(x_0))$ is well-defined, and is called the characteristic map, or the monodromy, of the double covering. Since $\text{Aut}(p^{-1}(x_0)) \simeq \{\pm 1\} \simeq \mathbb{Z}_2$, the double covering assigns an element of $\mathbb{Z}_2$ to each closed path. Thus an element of $H^1(X, \mathbb{Z}_2)$ is attached to a double covering, which is called the characteristic class of the double covering $p : Y \to X$. Conversely, any nonzero element $\omega \in H^1(X, \mathbb{Z}_2)$ determines a double covering $p : X^\omega \to X$.

Although the double covering $p : X^\omega \to X$ is determined by $\omega \in H^1(X, \mathbb{Z}_2)$, the topology of $X^\omega$ is not so simple. In fact, double coverings have attracted a lot of attention recently in the topological study of hyperplane arrangements [16, 18, 19, 21]. A hyperplane arrangement is a finite collection of hyperplanes in a linear (affine, or projective) space. The relationship between the topological and combinatorial structures of arrangements has been much studied. [4, 5, 6, 7, 8, 11, 12, 14, 15, 17, 18, 19, 20, 21].

Hyperplane arrangements are also important as hypersurfaces with non-isolated singularities. The topology of the Milnor fiber of a central hyperplane arrangement is one of the central topics in the theory of arrangements [1, 2, 3, 4, 17, 20]. However, even the first Betti number of the Milnor fiber of hyperplane arrangements have not yet been understood well.

The basic strategy to study the Milnor fiber $F_A$ of a (central) hyperplane arrangement $A$ is to use the monodromy action and eigenspace decomposition

\[
H^k(F, \mathbb{C}) \simeq \bigoplus \lambda^{|A|} H^k(F, \mathbb{C})_{\lambda}
\]

where $\lambda$ runs complex numbers satisfying $\lambda^{|A|} = 1$ and $H^k(F, \mathbb{C})_{\lambda}$ is the $\lambda$-eigenspace of the monodromy action.

Papadima and Suciu studied the relationship between the monodromy eigenspace and the so-called Aomoto complex with finite field coefficients [13, 14]. They proved an inequality between the dimension $\dim_{\mathbb{C}} H^k(F, \mathbb{C})_{\lambda}$ and the cohomology of the Aomoto complex (see (2.5) below). Furthermore, they conjectured that the equality holds, which enables us to formulate a combinatorial procedure computing $\dim_{\mathbb{C}} H^k(F, \mathbb{C})_{\lambda}$ purely combinatorial way using Aomoto complex.

However, recently, [21] exhibits an example of an arrangement of 16 planes in $\mathbb{C}^3$ (the icosidodecahedral arrangement $A_{TD}$, see Example 3.1 for details) for which

(i) Papadima-Suciu’s conjectural equality breaks and the strict inequality holds.

(ii) the integral homology of the Milnor fiber $H_1(F, \mathbb{Z})$ has a non-trivial 2-torsion.
Examples of torsion in the first homology of Milnor fibers of arrangements with multiplicities \([1]\) and for higher degree homology groups \([4]\) have been known.

In \([21]\), the study of double coverings of the complement to affine hyperplane arrangements plays a crucial role. The key idea is to use the transfer long exact sequence for double coverings and \(\mathbb{Z}_2\) cohomology groups \([9, 10]\) in connection with the Aomoto complex. Recently \([18]\) applies the idea to more general setting and problems.

In this paper, we will focus on the two peculiar properties (i) and (ii) that the icosidodecahedral arrangement possesses. They seem to be independent. However, the main result of this paper shows that they are actually equivalent for double coverings.

In the next §2, we will introduce several invariants related to double coverings of CW-complexes, and present the main theorem. Then in §3 we recall the icosidodecahedral arrangement. We will also exhibit a simpler example of 10 lines for which the integral homology of a double covering has a 2-torsion.

### 2. Main result

Let \(X\) be a connected CW-complex. Let \(\omega \in H^1(X, \mathbb{Z}_2)\). For simplicity, we assume \(\omega \neq 0\). Since \(H^1(X, \mathbb{Z}_2) \cong \text{Hom}(\pi_1(X), \mathbb{Z}_2) \cong \text{Hom}(\pi_1(X), \mathbb{Z}_2)\), \(\omega\) determines a homomorphism \(\pi_1(X) \to \mathbb{Z}_2 \cong \{\pm 1\}\). Since \(\omega \neq 0\), the group homomorphism \(\pi_1(X) \to \{\pm 1\}\) is surjective. Then the \(\text{Ker}(\pi_1(X) \to \{\pm 1\})\) is a subgroup of \(\pi_1(X)\) of index 2, which determines the associated double covering \(p_\omega : X^\omega \to X\). The group homomorphism \(\pi_1(X) \to \{\pm 1\} = \mathbb{Z}^\times\) also induces a local system of rank one over \(\mathbb{Z}\) which we denote by \(L_\omega\).

**Definition 2.1.** (The rank of \(\omega\)-twisted local system homology group.) Denote the rank of the local system homology group with coefficients in \(L_\omega\) by

\[
\rho_k(\omega) := \text{rank}_\mathbb{Z} H_k(X, L_\omega) = \dim \mathbb{C} H_k(X, L_\omega \otimes \mathbb{C}),
\]

From now on, we assume that the element \(\omega \in H^1(X, \mathbb{Z}_2)\) satisfies \(\omega \wedge \omega = 0\). Then, the multiplication map

\[
\omega \wedge : H^\bullet(X, \mathbb{Z}_2) \to H^{\bullet+1}(X, \mathbb{Z}_2)
\]

induces a cochain complex which we call the mod 2 Aomoto complex \((H^\bullet(X, \mathbb{Z}_2), \omega \wedge)\).

**Definition 2.2.** (The rank of the cohomology of the mod 2 Aomoto complex.) Denote the rank of the cohomology of the Aomoto complex by

\[
\alpha_k(\omega) := \text{rank}_\mathbb{Z}_2 H^k(H^\bullet(X, \mathbb{Z}_2), \omega \wedge).
\]
These invariants are related to integral and mod 2 Betti numbers of the double covering \( X^\omega \). By Leray spectral sequence, the complex cohomology group of \( X^\omega \) decomposes into a direct sum as \( H^k(X^\omega, \mathbb{C}) \simeq H^k(X, \mathbb{C}) \oplus H^k(X, L_\omega \otimes \mathbb{Z} \mathbb{C}) \). Therefore, we have

\[
b_k(X^\omega) = b_k(X) + \rho_k(\omega).
\]

In [13], Papadima and Suciu proved the inequality

\[
\rho_k(\omega) \leq \alpha_k(\omega).
\]

Therefore, the \( k \)-th Betti number of the double covering \( X^\omega \) is bounded by the sum

\[
b_k(X^\omega) \leq b_k(X) + \alpha_k(\omega).
\]

In [14], they conjectured the equality “\( \rho_k(\omega) = \alpha_k(\omega) \)” holds for \( X = M(\mathcal{A}) \) the complement to a complex hyperplane arrangement and the covering \( F \rightarrow M(\mathcal{A}) \) which is the Milnor fiber of the cone of \( \mathcal{A} \).

Recently, a counterexample to the equality was found. Let \( \mathcal{A}_{TD} = \{H_1, \ldots, H_{15}\} \) be as in Figure 1 (see Example 3.1 for details). Let \( e_1, \ldots, e_{15} \in H^1(M(\mathcal{A}_{TD}), \mathbb{Z}_2) \) be the dual basis to the basis of \( H_1(M(\mathcal{A}_{TD}), \mathbb{Z}_2) \) determined by the meridians of each line. Consider \( \omega = e_1 + e_2 + \cdots + e_{15} \). Then, \( \rho_1(\omega) = 0 \) and \( \alpha_1(\omega) = 1 \) ([21]). Thus we have the strict inequality

\[
\rho_1(\omega) < \alpha_1(\omega).
\]

In [21] it was also proved that \( H_1(M(\mathcal{A}_{TD})^\omega, \mathbb{Z}) \) has a 2-torsion.

The main result of this paper is a refinement of Papadima-Suciu’s inequality (2.5). Actually, the gap between \( \rho_k(\omega) \) and \( \alpha_k(\omega) \) can be precisely measured by 2-torsions. To state the main result, we need the following.

**Definition 2.3.** Let

\[
H_k(X^\omega, \mathbb{Z})[2] := \{ \alpha \in H_k(X^\omega, \mathbb{Z}) \mid 2\alpha = 0 \}
\]

be the 2-torsion part of the \( k \)-th homology group of \( X^\omega \). Note that the abelian group \( H_k(X^\omega, \mathbb{Z})[2] \) can be considered as a vector space over \( \mathbb{Z}_2 \). We denote its rank by

\[
\tau_k(X^\omega) := \text{rank}_{\mathbb{Z}_2} H_k(X^\omega, \mathbb{Z})[2].
\]

Note that \( \tau_k(X^\omega) \neq 0 \) if and only if \( H_k(X^\omega, \mathbb{Z}) \) has a non-trivial 2-torsion element. More precisely, \( \tau_k(X^\omega) \) is the number of even order summands when we express the torsion part of \( H_k(X^\omega, \mathbb{Z}) \) as a direct sum of finite cyclic groups.

**Theorem 2.4.** Let \( X \) be a connected CW-complex and \( \omega \in H^1(X, \mathbb{Z}_2) \) with \( \omega \wedge \omega = 0 \). Then,

\[
\alpha_k(\omega) = \rho_k(\omega) + \tau_k(X^\omega) + \tau_{k-1}(X^\omega).
\]
In particular, the equality $\alpha_k(\omega) = \rho_k(\omega)$ holds if and only if $H_k(X^\omega, \mathbb{Z})$ and $H_{k-1}(X^\omega, \mathbb{Z})$ do not have non-trivial 2-torsion elements.

Proof. We compute the rank of the mod 2 cohomology group $\text{rank}_{\mathbb{Z}_2} H^k(X^\omega, \mathbb{Z}_2)$ in two ways. First we apply the Universal coefficient theorem for cohomology (see e.g. [9, Theorem 3.2]). We have

\begin{equation} H^k(X^\omega, \mathbb{Z}_2) \simeq \text{Hom}(H_k(X^\omega, \mathbb{Z}), \mathbb{Z}_2) \oplus \text{Ext}(H_{k-1}(X^\omega, \mathbb{Z}), \mathbb{Z}_2). \end{equation}

Using the equality (2.4) and definitions, it is easily seen that the $\mathbb{Z}_2$-rank of the right-hand side is equal to $b_k(X) + \rho_k(\omega) + \tau_k(X^\omega) + \tau_{k-1}(X^\omega)$. Secondly, using the formula [21, Theorem 3.7], we obtain the following.

\begin{equation} \text{rank}_{\mathbb{Z}_2} H^k(X^\omega, \mathbb{Z}_2) = b_k(X) + \alpha_k(\omega). \end{equation}

Thus we have the formula (2.9). \hfill \square

As a special case of $k = 1$, we have the following.

Corollary 2.5.

\begin{equation} \alpha_1(\omega) = \rho_1(\omega) + \tau_1(X^\omega). \end{equation}

Thus the strict inequality $\alpha_1(\omega) > \rho_1(\omega)$ holds if and only if $H_1(X^\omega, \mathbb{Z})$ has non-trivial 2-torsion elements.

Proof. Since $H_0(X^\omega, \mathbb{Z})$ is torsion free, we have $\tau_0(X^\omega) = 0$. \hfill \square

Combining (2.4) and Theorem 2.4, we have the following.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(A deconing of the icosidodecahedral arrangement) $A_{ID} = \{H_1, \ldots, H_{15}\}$}
\end{figure}
Corollary 2.6. If \( \omega \wedge \omega = 0 \), the Betti number of the double cover is
\[
(2.13) \quad b_k(X^\omega) = b_k(X) + \alpha_k(\omega) - \tau_k(X^\omega) - \tau_{k-1}(X^\omega).
\]

In particular, \( b_1(X^\omega) = b_1(X) + \alpha_1(\omega) - \tau_1(X^\omega) \).

Remark 2.7. Let us consider the eigenspace decomposition of the Milnor fiber \( F_A \) for a central arrangement \( A \) with \( |A| \) even. In this case, the \((-1)\)-eigenspace \( H_k(F_A, \mathbb{C}) \) appears as a direct summand of \( H^k(F_A, \mathbb{C}) \). Recall that \( H_k(F_A, \mathbb{C}) \) is isomorphic to the local system cohomology group \( H^k(M(A), \mathcal{L}_\omega \otimes \mathbb{C}) \) with \( \omega = \sum_i e_i \). Therefore,
\[
(2.14) \quad \dim H_k(F_A, \mathbb{C}) = \rho_k(\omega) = \alpha_k(\omega) - \tau_k(X^\omega) - \tau_{k-1}(X^\omega),
\]
where \( X = M(dA) \) is the complement of the deconing \( dA \) (or equivalently, projectivized complement \( X = M(A)/\mathbb{C}^\ast \)). In particular, \( \dim H_1(F_A, \mathbb{C}) = \alpha_1(\omega) - \tau_1(X^\omega) \). Thus, to compute the dimension of the \((-1)\)-eigenspace \( \dim H_k(F_A, \mathbb{C}) \), the rank of 2-torsion part \( \tau_k(X^\omega) \) of the homology of the double covering \( H_k(X^\omega, \mathbb{Z}) \) is unavoidable. A combinatorial description of \( \dim H_k(F_A, \mathbb{C}) \) must involve a combinatorial description of \( \tau_k(X^\omega) \).

3. Examples related to the icosidodecahedral arrangement

Example 3.1. ((The deconing of the) icosidodecahedral arrangement [21])

Let \( \mathcal{A}_{TD} = \{ H_1, \ldots, H_{15} \} \) be the arrangement of affine 15 lines as in Figure 1. The first homology \( H_1(M(\mathcal{A}_{TD}), \mathbb{Z}) \) of the complement \( M(\mathcal{A}_{TD}) = \mathbb{C}^2 \setminus \bigcup_{i=1}^{15} H_i \otimes \mathbb{C} \) is generated by meridians of each \( H_i \). Let \( e_1, \ldots, e_{15} \) be the dual basis to meridians.

1. Let \( \omega = e_1 + e_2 + \cdots + e_{15} \) be the sum of all \( e_i \)'s. Let \( \eta = e_{11} + \cdots + e_{15} \). Then \( \omega \wedge \eta = 0 \). We can show that the cohomology of the Aomoto complex \( H^1(H^\bullet(\mathcal{A}_{TD}), \mathbb{Z}_2), \omega \wedge \) \) is rank one generated by \( \eta \). Thus we have \( \alpha_1(\omega) = 1 \). We also have \( \rho_1(\omega) = 0 \) (see [21] Theorem 4.3) for details). Hence \( \tau_1(X^\omega) = 1 \) and \( H_1(X^\omega, \mathbb{Z}) \) has a non-trivial 2-torsion element.

2. There are many other choices of \( \omega \) such that \( \alpha_1(\omega) = 1 \) and \( \rho_1(\omega) = 0 \). For example, \( \omega = e_5 + e_6 + e_9 + e_{10} + e_{12} + e_{13} + e_{15} \). Hence, there exist many double coverings of \( M(\mathcal{A}_{TD})^\omega \) of \( M(\mathcal{A}_{TD}) \) with torsion in \( H_1(M(\mathcal{A}_{TD})^\omega, \mathbb{Z}) \). As far as the authors checked by computer, for every \( \omega \) with \( \alpha_1(\omega) = 1 \) and \( \rho_1(\omega) = 0 \), the first homology of the double covering and the associated local system homology group are
\[
(3.1) \quad H_1(M(\mathcal{A}_{TD})^\omega, \mathbb{Z}) \simeq \mathbb{Z}^\oplus_{15} \oplus \mathbb{Z}_2,
\]
\[
H_1(M(\mathcal{A}_{TD}), \mathcal{L}_\omega) \simeq \mathbb{Z}^\oplus_{13} \oplus \mathbb{Z}_4.
\]
Another interesting element is $\omega = e_{11} + e_{12} + e_{13} + e_{14} + e_{15}$. Then the rank of the cohomology of the Aomoto complex is $\alpha_1(\omega) = 6$. The rank of the local system cohomology is also $\rho_1(\omega) = 6$. Hence $\tau_1(M(A_{TD})^\omega) = 0$.

The above example shows that several double coverings of $M(A_{TD})$ have non-trivial $2$-torsions in the integral homology groups of double coverings. The above example $A_{TD}$ is not the smallest example with non-trivial $2$-torsions. One of subarrangements of $A_{TD}$ also have $2$-torsions as follows.

**Example 3.2.** (Double star arrangement) Let $A_{DS}$ be the subarrangement of $A_{TD}$ consisting of $10$ lines $\{H_6, \ldots, H_{10}, H_{11}, \ldots, H_{15}\}$ (Figure 2).

![Figure 2. (The double star arrangement) $A_{DS} = \{H_6, \ldots, H_{15}\}$](image)

Let $\omega = e_6 + e_7 + \cdots + e_{15}$ be the sum of all $e_i$’s. As Example 3.1, $\alpha_1(\omega) = 1$ and $\rho_1(\omega) = 0$. Hence $\tau_1(X^\omega) = 1$ and $H_1(X^\omega, \mathbb{Z})$ has a non-trivial $2$-torsion element. As far as the authors checked by computer, it is the unique $\omega$ with $\tau_1(X^\omega) = 1$. The first homology of the double covering and the associated local system homology group are

$$H_1(M(A_{DS})^\omega, \mathbb{Z}) \simeq \mathbb{Z}^{\oplus 10} \oplus \mathbb{Z}_2,$$

$$H_1(M(A_{DS}), \mathcal{L}_\omega) \simeq \mathbb{Z}_2^{\oplus 8} \oplus \mathbb{Z}_4.$$

The computations in Example 3.1 and Example 3.2 suggest that $2$-torsions are closely related to the local system (co)homology groups with rank one local systems over $\mathbb{Z}$, which have been recently studied in [19] for complexified real arrangements. We pose the following conjecture.
Conjecture 3.3. Let $\mathcal{A}$ be a complex hyperplane arrangement. Let $\omega \in H^1(M(\mathcal{A}), \mathbb{Z}_2)$. The integral homology $H_1(M(\mathcal{A})^\omega, \mathbb{Z})$ has a non-trivial 2-torsion if and only if the local system homology $H_1(M(\mathcal{A}), L_\omega)$ has a 4-torsion ($\mathbb{Z}_4$-summand).

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