K-layer for Influencer Identification in Complex Networks

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Abstract. A small set of influential nodes, called influencers, spread information through a network faster and broader than other nodes. Identifying influencers has profound implications in various real-world spreading dynamics such as viral marketing, epidemic outbreaks and cascading failures. In this paper, we use and leverage the idea behind the widely-used k-shell index to introduce a new centrality index we call k-layer. The k-layer index, calculated through k-layer decomposition, quantifies the core of the network through the distance of nodes from the periphery of the network. Intuitively, the k-layer value of node i represents the depth of the node tree with node i as the root node within the scope of nodes that have been already removed. Our experimental results show that the proposed k-layer metric outperforms the k-shell index and the Mixed Degree Decomposition (MDD) in detecting the influencers of networks. After that, taking into account the node location characteristics in the network, an extended k-layer index, named KR-layer (KLR) is proposed and proved to have better ability to identify key nodes in complex networks. Our findings reveal the essential role of nodes' distance from the periphery, leading us closer to the optimal solution of the influencer identification problem.

1. Introduction

It is well known that in the process of information diffusion, influencers have a greater influence on the structure and function of the network[1]. Due to their network structural privileges, they make information spread faster and wider in the network. In recent years, this phenomenon has widely been used in viral marketing[2], computer virus spreading[3], social network analysing[4], power grid cascading failure prediction[5] and the like.

Therefore, identifying and locating influencers more accurately is critical to designing effective strategies in these different settings[6]. Various indices have been proposed for identifying influencers based on centrality-based heuristics which encode the rich structural information of nodes' location to distinguish influences of nodes in a network, such as degree[7], closeness[8] and PageRank[9], just to name a few.

K-shell[10,11] is one of the most propounding works proposed by Kitsak et al. It calculates the k-shell index (also called k-core) of each node by k-shell decomposition. K-shell decomposition partitions a network into k different clusters so the nodes in each cluster has the same k-shell value from 1 to k. Kitsak et al.[11] showed that k-shell decomposition performs better in identifying influencers in networks - although node's degree was once considered as the simplest and most effective index. The node with high k-shell value is usually at the core of the network which can initiate large-scale spreading in the network and therefore it has stronger spreading ability and is identified to be one of the most influential nodes.

Due to its low computational complexity[12], k-shell decomposition has been widely used to visualize large-scale networks[13], analyze the core structure of networks[14], and predict cascading
failure in a power grid, since "overload failures tend to occupy the network’s core"[5], to name but a few applications. However, k-shell index for influencer identification has several limitations[15]. For example, k-shell is not effective on networks without core structures such as the Barabasi-Albert network[6] or on star networks. Also, we found that k-shell index lacks good balance between using local and global structural attributes. For example, the highly connected nodes at the periphery of the network could have high k-shell value, suggesting that k-shell uses local structural information. It may be incapable of determining the real core of the network. The use of local structural attributes has much lower computational complexity at the cost of accuracy, such as degree. Global indices such as VitalRank[16] and betweenness[17] can achieve better performance in identifying influencers but at the price of computational cost.

The influential nodes identification problem is also called influencers identification since its accuracy is usually tested using epidemic spreading models. Epidemic spreading model is a theoretical modeling of how diseases spread in complex networks. It assumes that the propagation is driven by diffusion processes, because the transmission occurs from every infected through all its neighbors at each time step, producing a diffusion of the epidemics on the network, such as the SIR (Susceptible Infected Recovered) and SIS (Susceptible Infected Susceptible) models[18]. These models usually use given initially-infected nodes to initiate spreading in the network. The number of infectious nodes is used for identifying the spreading ability of them. Further it can be used for comparing the indices of influencers identification.

2. K-layer Decomposition
To overcome the limitations mentioned above, we propose a new centrality-based index for measuring the coreness of nodes in a network, referred to as k-layer, which is calculated by k-layer decomposition algorithm. K-layer index measures the comprehensive distances of nodes from the periphery of the network, taking the global network topology features and also the local features into consideration in order to measure the coreness of the nodes. The assumption behind the method is that nodes at the core of the whole network should be far from the periphery of the network which made them be able to reach to the other nodes much quicker.

Given an undirected and unweighted network \( G = (V, E) \), where \( V \) is the set of vertices, with \( |V| = n \), and \( E \) is the set of edges, with \( |E| = m \). K-layer is defined as equation(1),

\[
KL_i = \max(KL_{j_1}, KL_{j_2} + 1, KL_{j_3} + 1, ..., KL_{j_s} + 1),
\]

where \( KL_i \) is the k-layer of node \( i \), and \( j_1, j_2, ..., j_s \) are the adjacent nodes of node \( i \) that were previously removed from the network.

(i) (Initiation): Set \( KL_i = 0 \), where \( i = 1, 2, ..., n \) and \( k = 1 \).

(ii) (Removal): Remove node \( t \) if \( d_t \leq k \), and set \( KL_t = \max(KL_{j_1} + 1, KL_{j_2} + 1, ..., KL_{j_s} + 1) \), where \( j_1, j_2, ..., j_s \) are the adjacent nodes of node \( t \) that were previously removed from the network.

(iii) (Iteration): Repeat Step 2 until there is no node with a degree \( d_t \leq k \).

(iv) (Judgment): If the remaining subgraph is not empty, set \( k++ = 1 \) and go to Step 2. Otherwise, exit.

It is necessary to note that, in the k-layer decomposition process, when nodes with a residual degree less than \( k \) appear during the division of the k-layer, the nodes with a residual equal to \( k \) are first removed. Nodes with a residual degree less than \( k \) are then removed, in the order of their residual degree, beginning with those with the highest degree, because they may be located farthest from the periphery of the network.

The k-shell index, which is calculated by k-shell decomposition, can be considered as the residual degree of the node in the network decomposition process. In contrast, the k-layer index calculated by k-layer decomposition records the distance of the node from the periphery of the network. Intuitively, the k-layer value of node \( i \) represents the depth of the node tree with node \( i \) as the root node within the range of nodes that have been already removed.
Kitsak et al. [11] found that the infection capacities of nodes with the same degree but at different locations in a network may differ. This indicates that the location of nodes in the network can be better used to evaluate the dissemination importance of the nodes. This notion is illustrated by the example in Figure 1. The two nodes $a$ and $b$ in the graph have the same degree $d = 8$. By $k$-shell decomposition, node $a$ was determined to be located at the center of the network, with $KS = 3$ ($KS$ represents the $k$-shell value of the node); while node $b$ was determined to be located at the periphery of the network, with $KS = 1$. Obviously, during an epidemic spread, node $a$ is more important in the network than node $b$. It is therefore rational to conclude that the topological location of a node in a network better reflects its influence.

![Figure 1. A network that can be decomposed into 3 shells by k-shell decomposition.](image)

Some networks contain small groups with high-density connected nodes, as the example showed in Figure 2, which adds a small tightly connected community (Community 1) to that shown in Figure 1. The $k$-layer of nodes in this network is calculated as illustrated in Figure 3. The deeper the color of a node in the graph, the higher the $k$-layer value. The nodes in Community 1 is easily distinguishable from that of the nodes located at the real core of the network. This example shows that $k$-layer index uses the global structure of the network to better identify the influencers. By performing $k$-layer decomposition on the network, the $k$-layer value of the node records the farthest distance between the node and the periphery of the network, which solves the negative influence of tightly connected communities on the identification of influencers in the network. Compared with $k$-layer, $k$-shell index pays more attention to the local features of the network. The tightness of the nodes in the small group causes them to have a relatively high degree and makes them more difficult to remove early during $k$-shell decomposition.

![Figure 2. A network including a small group of tightly connected nodes.](image)
Figure 3. The network that decomposed by k-layer.

Furthermore, it has been importantly noted that a bona fide "global" core does not exist in a real network[19], because all real networks can be extended beyond their "ends". Unlike k-shell, k-layer index reflects the distance between the nodes and the periphery of the network. Geometrically, sorting of the nodes by the distance to the network periphery is considered by the present authors as being more effective for identifying the influencers, and it also reflects the global characteristics of the network. In the case of a circular network, the shorter its distance from the center, the farther away a node is from the periphery, resulting that k-shell and k-layer perform similarly here. However, in the case of an oval network, the fact that the distance between a node and the center (the midpoint of the line between the two foci of the oval) may not reflect whether the node is located at the core of the network. In the example shown in Figure 4, similar to an ellipse, node 1 and 3 can be considered as being at the two foci of the ellipse, and node 2 is at the core of the network. According to k-shell decomposition, the three nodes locate in the same shell. In contrast, node 1 has a higher k-layer value than node 2 and 3, which shows that k-layer is more accurate in identifying influencers.

Figure 4. Oval-shaped network.

To measure the performance of k-layer index, SIR is applied to the spreading dynamics to assess the effectiveness of k-layer, k-shell and a mixed degree decomposition (MDD)[15] in identifying influencers in networks. Our results show that k-layer index is a better way to determine the core of the whole network comparing with the other two. The proposed k-layer index is expected to open new research avenues, including toward improving k-layer decomposition and its application to different types of networks.

3. Results

This section presents the results of the evaluations of the different ranking indices based on the epidemic dynamics model and network robustness. The following eight real networks were considered in this study:

(v) Celegans[20]: a metabolic network of the nematode C.elegans;
(vi) Email-univ[21]: a email communication network at the University Rovira i Virgili in Spain;
(vii) Messages[22]: a Facebook-like Social Network originate from an online community for students at University of California, Irvine;
(viii) Hamster[23]: a friendship and family network at hamsterster.com;
(ix) Powergrid[24]: a network representing the Western States Power Grid of the United States;
(x) PGP[25]: a giant component of the network of users of the Pretty-Good-Privacy algorithm for secure information interchange;

(xi) Astro-ph[26]: a collaboration network of astrophysicists;

(xii) AS[27]: the Internet at an autonomous system level.

The network data used for the present experiments were those published online. The detailed information about the networks are presented in Table 1.

Table 1. Properties of the real-world networks considered in this work: number of nodes (N), number of edges (E), average degree (d̅), clustering coefficient (C), and epidemic threshold (βC).

| Network   | N   | E   | d̅   | C   | βC   |
|-----------|-----|-----|------|-----|------|
| Celegans  | 453 | 2040| 4.503| 0.327| 0.026|
| Email-univ| 1133| 5451| 4.811| 0.110| 0.057|
| Messages  | 1266| 6451| 5.096| 0.034| 0.038|
| Hamster   | 2426| 16631| 6.855| 0.269| 0.024|
| Powergrid | 4941| 6594| 1.335| 0.040| 0.348|
| PGP       | 10680| 24316| 2.277| 0.133| 0.056|
| Astro-ph  | 16706| 121251| 7.556| 0.333| 0.023|
| AS        | 22963| 48436| 2.109| 0.115| 0.004|

3.1. Results Based on an Epidemic Spreading Dynamics Model

To compare the effectiveness of the different ranking indices, their Kendall correlation coefficient τ values were calculated using the SIR model simulation propagation results. (The evaluation method is described in detail in Methods.) In the simulations, the infection coefficient β was set to slightly higher than the epidemic threshold βC, that is, β = 1.5βC[28]. The values of the epidemic threshold βC for the different considered networks are recorded in column 6 of Table 1. Their Kendall correlation coefficients are recorded in Table 2.

Table 2. A Kendall correlation coefficients τ, which show the correlation between the ordering of nodes obtained using different ranking metrics and the ordering of nodes obtained via SIR simulations.

| Network   | β = 1.5βC | τKS | τKL | τMDD |
|-----------|------------|-----|-----|------|
| Celegans  | 0.038      | 0.251| 0.255| 0.247|
| Email-univ| 0.085      | 0.386| 0.392| 0.391|
| Messages  | 0.057      | 0.406| 0.413| 0.407|
| Hamster   | 0.036      | 0.409| 0.413| 0.412|
| Powergrid | 0.522      | 0.276| 0.248| 0.305|
| PGP       | 0.084      | 0.402| 0.370| 0.399|
| Astro-ph  | 0.034      | 0.449| 0.370| 0.452|
| AS        | 0.006      | 0.123| 0.156| 0.124|

As indicated by the bold marks in Table 2, the k-layer index performed best in 5 of the 8 networks. In the other three networks, MDD performed better in two of them. It can be seen from this result that the k-layer index performs best in most networks based on the network epidemic spreading dynamics model.

In order to reduce the impact of SIR infection rate β on the experimental results, by setting β/βC to within 0.1 to 0.9, we further tested the effect of the different infection coefficients on the different ranking indices. The Kendall correlation coefficient τ values of the different indices with respect to the infection coefficient β and the network are presented in Figure 5. As can be observed from the figure, for most of the networks, the correlation coefficient τ of the k-layer index is always higher than those of the other two indices for the given , indicating that the node influence ranking sequence obtained by k-layer decomposition is the closest to that obtained by the SIR model simulation. This phenomenon is most evident in the largest network AS.
Figure 5. A network that can be decomposed into 3 shells by k-shell decomposition. Kendall correlation coefficient $\tau$ values of the different ranking indices with respect to the infection probability $\beta$ and network. (a) Celegans; (b) Email-univ; (c) Messages; (d) Hamster; (e) Powergrid; (f) PGP; (g) Astro-ph; (h) AS.

3.2. Results Based on Network Robustness

A node or group of nodes can be evaluated to determine whether it is critical to the network by observing the reduction of the giant component of the network caused by its removal. The importance of the core nodes of a network can also be quantified by the number of connected components that show up after their removal[29]. According to the node removal policy, nodes are removed in descending order of their ranking indices. In the present study, the focus was on the damage caused to the network stability or robustness. That is, a node or group of nodes is considered to be an influencer or influencers in the network if its or their removal significantly impairs the network robustness.

The above removal strategy was applied to the eight networks considered in this study to evaluate the abilities of k-layer and other indices to identify the influencers. The number of removed nodes was used as the abscissa, and the number of nodes in the giant component as the ordinate.

Figure 6. Number of nodes in the giant component when the nodes are removed in descending order of the ranking index with respect to the network type. (a) Celegans; (b) Email-univ; (c) Messages; (d) Hamster; (e) Powergrid; (f) PGP; (g) Astro-ph; (h) AS.

As can be observed from Figure 6, which shows the effect of the successive removal of nodes on the network structure, for most of the considered networks, the k-layer curve, which is represented by the
purple line, has a faster drop than the k-shell curve (the red line). Compared with MDD, the decline effect of the k-layer curve is similar, but most are better, which means the k-layer index best reflects the effect of the node removal on the network. In addition, as can be seen from the (g) of Figure 6, the k-layer curve drops the fastest and is very different from the k-shell, MDD and SIR curves. This also explains why the k-layer ranking sequence shown in (g) of Figure 5 is poorly correlated with the SIR’s.

4. Methods

4.1. Evaluation Methods

This section first reviews several centrality indices, which will be used for comparison with the proposed index. The evaluation criteria are then presented, consisting in two different evaluation methods that ensure that the different indices are compared in every possible way.

In a network $G = (V,E)$, where $V$ and $E$ represent the nodes and edges in the network, respectively, the degree of node $i$, denoted by $d_i$, is defined as the number of its directly connected neighbors. The k-shell coreness of node $i$ measured via k-shell decomposition is denoted by $K_S_i$. Coreness $K_S_i$ indicates that node $i$ belongs to a k-shell but not to any $(k+1)$-shell.

In the k-shell decomposition process, all nodes with degree $k \leq 1$ and their links are first removed. This step is repeated until there are no nodes with degree $k \leq 1$ in the network. All the nodes that have been removed at this point are those that constitute the 1-shell. The degree of the remaining nodes in the network is referred to as the residual degree.

The above process is repeated to further decompose the network into 2-shell, 3-shell, etc., until there are no more nodes in the network. The coreness $K_S$ of a node is thus equal to the $k$ of the k-shell that it belongs to and is used to describe the topological location of the node in the network.

The MDD method with a tunable parameter $\lambda$, also named mixed degree decomposition, proposed by Zeng et al. [15], is in order to identify the spreading abilities among nodes with the same k-shell value. It improves the k-shell decomposition by considering not only the residual degree, but also the exhausted degree. In our experiments, we set the parameter $\lambda = 0.7$, which is recommended by Zeng et al. [15].

When evaluating indices for ranking the importance of nodes in a network, the focus is usually placed on whether the index accurately reflects the position of the nodes in the network. However, it is difficult to objectively determine the position of a node in a large-scale network.

There are two common evaluation criteria. The first one is based on an epidemic spreading dynamics model, which is used to examine the infectious ability of a node on other nodes in the network. Infectious source nodes that are ranked higher in the infectious disease model are considered more susceptible to other nodes in the network.

The second evaluation criterion is based on network robustness and is used to sort nodes based on their ability to influence the network’s structures and functions. According to Albert et al. [30], if the removal of a node or group of nodes would significantly reduce the giant component of the network, that node or nodes can be considered to be important for the network. A ranking index is considered good when the sequence of nodes sorted by importance that is obtained using it can be used to attack the network more effectively.

4.2. Evaluation Criteria Based on an Epidemic Spreading Dynamics Model

In the present study, to evaluate the performance of different node importance ranking indices, an SIR model was used to simulate the diffusion effects of the nodes in different networks [31].

An initially infected node $i$ was set in the SIR model and all the other nodes were set to be susceptible at the initial stage. Each infected node then subsequently infected its susceptible neighbours with an infection probability $\beta$ and was restored to a steady state with a recovery probability $\mu$. This infection process was repeated until there were no infected nodes in the network. The number of nodes in the restored state at the end of the epidemic infection was the evaluation index value of the infection ability of the initially infected node $i$. 
In the employed SIR model, the recovery probability $\mu$ was set to 1, indicating that a node was restored to a steady state immediately after being infected. The infection probability $\beta$ should not be set too low nor too high. If $\beta$ were too low, it could prevent the epidemic from successfully propagating, and the communication capability of nodes would be impossible to measurable. Conversely, if $\beta$ were too high, all the nodes in the network could be infected and it would thus become difficult to distinguish between the diffusion capabilities of different nodes. According to Lin et al.\[32\], when $\beta$ approaches the epidemic threshold $\beta_C$, the central behavior of the eigenvector is enhanced. The epidemic threshold $\beta_C$ is given by equation (2):

$$\beta_C = \frac{d}{\bar{d}^2 - \bar{d}},$$

(2)

where $d$ is the degree of the node, $\bar{d}$ denotes the average degree of the nodes in the network, and $\bar{d}^2$ denotes the average of the squares of the degrees of all nodes.

To quantify the correctness of each of the ranking methods, Kendall’s rank correlation coefficient $\tau$ was used as an indicator\[33\]. For two sequences X and Y, if the elements in each sequence are unique, $\tau$ can be calculated using equation (3)\[34\]:

$$\tau = \frac{C - D}{\frac{1}{2}N(N - 1)},$$

(3)

where $C$ is the number of pairs of consistent elements in X and Y (two elements form a pair), $D$ is the number of pairs of inconsistent elements in X and Y, and $N$ is the sequence length. This indicator quantifies the similarity between the ordering of the elements in the two sequences. The higher the correlation coefficient $\tau$ is, the better the ranking index is.

### 4.3. Evaluation Criteria Based on Network Robustness

Intentional attacks can have devastating effects on the connectivity of a complex network. An evaluation criterion based on network robustness was used to assess the effects of removing a set of nodes on the network structure and functions. The greater the resulting impact is, the more important the removed node is.

A node or group of nodes can be evaluated to determine whether they are critical to the network by observing the reduction of the giant component of the network caused by their removal. The importance of the core nodes of a network can also be quantified by the number of connected components that remain after their removal. According to the node removal policy employed, nodes are generally removed in descending order according to their ranking indices. In the present study, our focus was on the damage caused to network stability or robustness. That is, a node or group of nodes were considered to be influencers in the network if their removal significantly reduced network robustness.

The robustness of a network can be characterized as $R$, which is calculated using equation (4):

$$R = \frac{1}{n} \sum_{i=1}^{n} \sigma\left(\frac{i}{n}\right),$$

(4)

where $\sigma\left(\frac{i}{n}\right)$ is the proportion of the number of nodes belonging to the giant component of the network after the removal of $\frac{i}{n}$ of the nodes and, $i$ is the number of nodes removed, and $n$ is the number of nodes in the network. In the evaluation experiment, we calculated the network robustness $R$ corresponding to different orders of removal obtained according to different node ranking indices. It is clear that the lower the calculated network robustness $R$ is (after the removal of nodes), the better the performance of the corresponding node importance index is.
5. KR-layer Index

The importance of nodes in complex networks is not only related to the topological features of nodes, but also affected by the influence of their neighbors. On the one hand, if a node’s neighbor node plays a key role in the network, the importance of the node will also increase. On the other hand, as the distance between two nodes increases, the influence correlation between the two nodes usually decreases. That is, the shorter the path length between nodes, the smaller the correlation. Thus, we can compare the relationship between k-layer index and the shortest distance between nodes as the universal gravitation formula proposed by Newton, that is, the relationship between object mass and distance. Here we can treat each node k-layer value as "node quality" and define the shortest path length between nodes as the distance between nodes. The proposed KR-layer index is obtained by the following formula (5), denoted as KLR, where $r_{ij}$ is the shortest path length between node $i$ and $j$; $\phi_i$ is the neighbourhood set with the shortest distance from node $i$ being less than or equal to the given value $R$.

$$KLR_i = \sum_{j \in \phi_i} \frac{KL_i \cdot KL_j}{r_{ij}^2}, \quad (5)$$

In this experiment, we set $R = 3$, that is, the neighbor node with the shortest distance from node $i$ is less than or equal to 3.

In order to evaluate KLR, three real-world undirected and unweighted networks, including Celegans, Hamster and Powergrid, are used for experiments, comparing with betweenness, closeness, k-shell, and k-layer index.

In order to compare the node importance ranking performance of KLR, we use the SIR infectious disease model to simulate the above three real networks, so as to obtain the order of importance of the nodes in the model. Due to the high simulation of the model's network information dissemination, in this evaluation method, we use the order of node importance obtained by SIR simulation as the benchmark for evaluating the importance index of other nodes. The Kendall correlation coefficient between the node sequence obtained by the SIR model and the node sequence calculated by each node importance index is shown in Table 3 below.

Observed in Table 3, for the three real networks, the Kendall correlation coefficient between the node importance ranking obtained by KLR and the SIR propagation model is larger than the other four indicators. This shows that the order of importance of nodes obtained by KLR is most similar to the order of importance of nodes obtained by SIR model. From the experimental results, we believe that the KLR index proposed in this section has further improved the ability of the k-layer index to identify key nodes in an undirected and unweighted network.

### Table 3. A Kendall correlation coefficients $\tau$, which show the correlation between the ordering of nodes obtained using different ranking metrics and the ordering of nodes obtained via SIR simulations.

| Network  | $\beta = 1.5\beta_C$ | $\tau_{betw}$ | $\tau_{clos}$ | $\tau_{KS}$ | $\tau_{KL}$ | $\tau_{KLR}$ |
|----------|----------------------|----------------|---------------|--------------|-------------|-------------|
| Hamster  | 0.036                | 0.368          | 0.376         | 0.438        | 0.421       | 0.448       |
| Celegans | 0.038                | 0.243          | 0.227         | 0.250        | 0.263       | 0.300       |
| Powergrid| 0.522                | 0.222          | 0.155         | 0.288        | 0.237       | 0.298       |

6. Conclusion

This paper proposes a new network node ranking index, k-layer, determined by k-layer decomposition on the network. The decomposition process, differing from the widely employed k-shell decomposition, which is used to determine how close a node is to the network core, determines how far a node is from the periphery of the network, taking into consideration the local and global features of the network. Nodes that are more centrally located in a network tend to play more important roles in the spread of infectious diseases.

Experiments were performed in which k-layer decomposition was applied to eight real networks. To compare the abilities of k-layer and other indices for the influence ranking of the network nodes, the
damages to the robustness of the network resulting from the removal of nodes in the order of their respecting ranking by three indices were observed. The damage is quantified by the number of nodes that remain in the giant component of the network. The values of the Kendall correlation coefficient of the different ranking indices relative to the SIR results were also calculated. It was found that the proposed k-layer index mostly produced better assessments of the impact of the nodes compared with other two indices. The experimental results thus confirmed that consideration of the distance between the nodes at the network periphery, which gives more attention to the global network topology, was more effective than the use of the distance between the nodes and the network core for identifying the most influential nodes. The proposed k-layer index is thus a better node ranking index than k-shell and MDD. In addition, the extended indicator KR-layer has also been proposed. The SIR experiment proves that the KR-layer index combined with the k-layer index and the shortest path length between nodes is better than the k-layer index in performance.

There is nevertheless the need for further study on k-layer. For example, k-layer decomposition can be extended to other types of networks such as directed networks and weighted networks.

References
[1] Kempe, D., Kleinberg, J., Tardos, A. (2003) Maximizing the spread of influence through a social network. ACM SIGKDD Int. Conf. on Knowl. Discov. and Data Min., 137–146.
[2] Leskovec, J., Adamic, L. A., Huberman, B. A. (2007) The dynamics of viral marketing. ACM Trans. Web 1.
[3] Pastor-Satorras, R., Vespignani, A. (2007) Evolution and structure of the Internet: A statistical physics approach. Cambridge University Press, London.
[4] Liu, S. et al. (2015) Identifying effective influencers based on trust for electronic word-of-mouth marketing: A domain-aware approach. Inf. Sci. 306, 34–52.
[5] Yang, Y., Nishikawa, T., Motter, A. E. (2017) Small vulnerable sets determine large network cascades in power grids. Science 358, eaan3184.
[6] Domingos, P., Richardson, M. (2001) Mining the network value of customers. In Proceedings of the Seventh ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, no.10 in KDD’01, 57–66.
[7] Freeman, L. C. (1978) Centrality in social networks conceptual clarification. Soc. Networks 1, 215–239.
[8] Sabidussi, G. (1966) The centrality index of a graph. Psychometrika 31, 581–603.
[9] Sehgal, U., Kaur, K., Kumar, P. (2009) The anatomy of a large-scale hyper textual web search engine. Int. Conf. on Comput. Electr. Eng. 2, 491–495.
[10] Liu, Y., Tang, M., Zhou, T., Do, Y. (2015) Core-like groups result in invalidation of identifying super-spreader by k-shell decomposition. Sci. Reports 5, 9602.
[11] Kitsak, M. et al. (2010) Identification of influential spreaders in complex networks. Nat. Phys. 6, 888–893.
[12] Batagelj, V., Zaversnik, M. (2011) Fast algorithms for determining (generalized) core groups in social networks. Adv. Data Analysis Classif. 5, 129–145.
[13] Carmi, S., Havlin, S., Kirkpatrick, S., Shavitt, Y., Shir, E. (2007) A model of internet topology using k-shell decomposition. Proc. Natl. Acad. Sci. 104, 11150–11154.
[14] Dorogovtsev, S., Goltsev, A., Mendes, J. F. (2006) k-core organization of complex networks. Phys. Rev. Lett. 96, 040601.
[15] Zeng, A., Zhang, C.-J. (2012) Ranking spreaders by decomposing complex networks. Phys. Lett. A 377, 1031–1035.
[16] Iannelli, F., Mariani, M. S., Sokolov, I. M. (2018) Influencers identification in complex networks through reaction-diffusion dynamics. Phys. review. E, Stat. nonlinear, soft matter physics 98, 062302.
[17] Freeman, L. C. (1977) A set of measures of centrality based on betweenness. Sociometry 40, 35–41.
[18] Barrat, A., Barthlemy, M., Vespignani, A. (2008) Dynamical Processes on Complex Networks (Cambridge University Press).

[19] Csermely, P., London, A., Wu, L.-Y., Uzzi, B. (2013) Structure and dynamics of core-periphery networks. J. Complex Networks 1, 93–123.

[20] Jordi, D., Alex, A. (2005) Community detection in complex networks using extremal optimization. Phys Rev E Stat Nonlin Soft Matter Phys 72, 027104.

[21] Guimerà, R., Danon, L., Díaz-Guilera, A., Giralt, F., Arenas, A. (2003) Self-similar community structure in a network of human interactions. Phys. review. E 68, 065103.

[22] Opsahl, T., Panzarasa, P. (2009) Clustering in weighted networks. Soc. networks 31, 155–163.

[23] Kunegis, J. (2013) KONECT – The Koblenz Network Collection. In Proc. Int. Conf. on World Wide Web Companion, 1343–1350.

[24] Watts, D., Strogatz, S. (1998) Collective dynamics of ‘small-world’ networks. Nature.

[25] Boguñá, M., Pastor-Satorras, R., Díaz-Guilera, A., Arenas, A. (2004) Models of social networks based on social distance attachment. Phys. review. E, Stat. nonlinear, soft matter physics 70, 056122.

[26] Newman, M. E. J. (2001) The structure of scientific collaboration networks. Proc. Natl. Acad. Sci. United States Am. 98, 404–409.

[27] Karrer, B., E. J. Newman, M., Zdeborova, L. (2014) Percolation on sparse networks. Phys. review letters 113, 208702.

[28] Castellano, C., Pastor-Satorras, R. (2010) Thresholds for epidemic spreading in networks. Phys. review letters 105, 218701.

[29] Yingluo, W., Jin, X., Youmin, X. (1993) The core and coritivity of a system (iv) — relations between a system and its complement. J. Syst. Eng. Electron. 4, 28–36.

[30] Albert, R., Jeong, H., Barabási, A. (2000) Error and attack tolerance of complex networks. Nature 340, 378—382.

[31] Newman, M. E. J. (2002) Spread of epidemic disease on networks. Phys. review. E, Stat. nonlinear, soft matter physics 66, 016128.

[32] Lin, J.-H., Guo, Q., Liu, J.-G., Zhou, T. (2015) Locating influential nodes via dynamics-sensitive centrality. Sci. Reports 6, 032812.

[33] G KENDALL, M. (1945) The treatment of ties in rank problems. Biometrika 33, 239–51.

[34] G KENDALL, M. (1938) A new measure of rank correlation. Biometrika 30, 81–93.

[35] Schneider, C., Moreira, A., S Andrade, J., Havlin, S., J Herrmann, H. (2011) Mitigation of malicious attacks on networks. Proc. Natl. Acad. Sci. United States Am. 108, 3838–41.