Q-ball Instability due to $U(1)$ Breaking

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Q-ball is a non-topological soliton whose stability is ensured by global $U(1)$ symmetry. We study a Q-ball which arises in the Affleck-Dine mechanism for baryogenesis and consider its possible instability due to $U(1)$ breaking term ($A$-term) indispensable for successful baryogenesis. It is found that the instability destroys the Q-ball if its growth rate exceeds inverse of the typical relaxation time scale of the Q-ball. However, the instability is not so strong as it obstructs the cosmological formation of the Q-balls.

I. INTRODUCTION

The Q-ball is a non-topological soliton that arises in scalar field theory with some global $U(1)$ symmetry. The Q-ball solution naturally exists in the minimal supersymmetric (SUSY) standard model (MSSM), especially in the context of the Affleck-Dine (AD) baryogenesis where the MSSM flat directions play important roles in baryon number generation. In this case the Q-ball consists of squarks and sleptons, therefore carrying baryonic and/or leptonic charges. It was shown that Q-balls with large baryon number are actually produced in the early universe in Refs. 6, 8. Furthermore, a Q-ball with large baryon number is stable against decay into protons in theories based on the gauge-mediated SUSY breaking. Therefore, such stable Q-ball can be a promising candidate for dark matter as well as the source of the baryon asymmetry of the universe, which makes the Q-balls interesting candidate for dark matter (=LSPs) and the baryon number of the universe, making the Q-balls the lightest SUSY particles (LSPs). Then it is possible that the Q-balls account for the dark matter (=LSPs) and the baryon asymmetry of the universe simultaneously. For further applications of Q-balls and their variants, see e.g. Refs. 13-17.

In the AD mechanism we need the $U(1)_{B(L)}$ breaking terms for successful generation of the baryon (lepton) number. Here $U(1)_{B(L)}$ is the global symmetry associated with baryon (lepton) number, and in the following we drop the subscript $B(L)$, since the distinction makes no difference to the following discussion. If Q-balls are not formed, the $U(1)$ breaking terms can be neglected soon after the baryon number is generated. This is because the cosmic expansion decreases the amplitude of the AD field and the $U(1)$ breaking terms become much smaller than the $U(1)$ conserving ones. However, once the Q-balls are formed, since the amplitude of the AD field inside the Q-ball is fixed it becomes nontrivial whether the $U(1)$ breaking terms are truly negligible or not. Therefore, in the present paper, we study the possible instability due to the $U(1)$ breaking $A$-terms and their effect on the Q-ball stability. It is found that the $A$-term induces instability similar to that of the parametric resonance and it upsets the stability of the Q-ball if the growth rate of the induced instability is larger than the inverse of the relaxation time of the Q-ball configuration. However, in the realistic cosmological situation, the $A$-term inside the Q-ball is not so strong to cause the strong instability, hence the previous studies which assumed the Q-ball stability remain valid.

II. LINEAR ANALYSIS ON INSTABILITIES

First let us consider the instabilities due to $U(1)$ breaking term in the homogeneous background. To this end we perform linear analysis assuming small perturbations. The potential of the AD field $\Phi$ is written as

$$V(\Phi) = m_\Phi^2 \left( 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right) |\Phi|^2 + Am_{3/2} \left( \frac{\Phi^d}{dM_{3/2}^d + h.c.} \right) + \frac{|\Phi|^{2d-2}}{M_{2d-6}^d},$$ (1)

where $m_\Phi$ is the mass of the AD field, $K$ a numerical coefficient of the one-loop correction, $m_{3/2}$ the gravitino mass, $M$ the renormalization scale, and $M_{d}$ some cut-off scale for the nonrenormalizable operator. Also $A$ is assumed to be a $O(1)$ real parameter, and $K$ is assumed to be negative. One can see that the second term (called $A$-term) breaks the $U(1)$ symmetry. In the following argument we can safely neglect the last term since the amplitude of the AD field is relatively small.

In order to study the instabilities of the AD field we first divide the AD field into homogeneous part and fluctuation: $\Phi = \Phi + \delta \Phi$. The equations of motion are

$$\ddot{\Phi} + 3H \dot{\Phi} + m_\Phi^2 \left( 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right) \Phi + Am_{3/2} \frac{\Phi^{d-1}}{M_{2d-6}^d} = 0,$$ (2)
for the homogeneous mode, and
\[
\delta \ddot{\Phi} + 3 H \delta \dot{\Phi} + \frac{k^2}{a^2} \delta \Phi + K m_\Phi^2 \Phi \left( \frac{\delta \Phi}{\Phi} + \frac{\delta \Phi^*}{\Phi^*} \right) + m_\Phi^2 \left( 1 + K + K \log \left( \frac{\Phi^2}{M^2} \right) \right) \delta \Phi + A (d - 1) m_\Phi^2 \frac{\delta \Phi^* - 2}{M_*^2 - 3} = 0, \tag{3}
\]
for the fluctuation with the wavenumber \( k \) in the momentum space. Although we have introduced the Hubble parameter \( H \) and the scale factor \( a \) in the above equations, we will neglect the cosmic expansion for the moment.

In order to parametrize the strength of the instability, let us define \( \xi \) as the ratio of the \( A \)-term to the mass term:
\[
\xi \equiv 2 \left| \frac{A m_\Phi^2 \Phi_0^{d-2}}{0 m_\Phi^2 M_*^{d-3}} \right|. \tag{4}
\]
where \( \Phi_0 \) is the maximal magnitude of \( \Phi \) during the course of the oscillation in the limit of \( \xi \to 0 \) and we will set \( M = \Phi_0 \). Note that the amplitude of the oscillation in the limit of \( \xi \to 0 \) is constant if we neglect the cosmic expansion. We have numerically solved Eqs. 2 and 3 by decomposing \( \Phi \) and \( \delta \Phi \) into their real and imaginary components:
\[
\Phi = \phi_1 + i \phi_2, \quad \delta \Phi = \delta \phi_1 + i \delta \phi_2. \tag{5}
\]
For convenience we also define the polar decomposition:
\[
\Phi = \phi e^{i \theta} / \sqrt{2}. \quad \Phi = \phi e^{i \theta} / \sqrt{2}.
\]

Before closing this section, let us consider the effect of the cosmic expansion on the instabilities. When the oscillation of the AD field starts, the Hubble parameter \( H \) is comparable to \( m_\Phi \). Hence the growth rate of the \( A \)-term instability is comparable to \( H \) only if \( \xi \sim O(1) \). However, since the amplitude of the AD field decreases as \( a^{-3/2} \), \( \xi \) quickly becomes smaller (\( \propto a^{-3(d-2)/2} \)) and the growth rate is soon overcome by the cosmic expansion.
Therefore, it seems that the A-term instability does not play an important role at least in the Q-ball formation. However, it is still nontrivial how the instability affects the evolution of Q-balls after they are formed, since the amplitude inside them is fixed.

III. INSTABILITIES INSIDE Q-BALL

In the previous section we have found that the A-term causes the instabilities in the homogeneous motion of the AD field. Next let us investigate the instabilities due to the A-term inside the Q-ball. Without the A-term, the Q-ball solution for the potential \([\Phi]\) is obtained with use of the Gaussian ansatz [4]:

\[
\Phi(t, r) = \frac{1}{\sqrt{2}} e^{i \omega t} \phi(r),
\]

\[
\phi(r) = \phi(0) e^{-r^2 / 2R^2},
\]

\[
(\Phi) \]

where the Q-ball radius \(R\) and angular velocity \(\omega\) are given by

\[
R^2 = \frac{2}{m^2[K]},
\]

\[
\omega^2 = m^2(1 + 2|K|).
\]

To study the instabilities due to the A-term inside the Q-ball, we have numerically solved the evolution of the AD field on one and two dimensional lattices. The equation of motion for the potential \([\Phi]\) is

\[
\ddot{\Phi} - \nabla^2 \Phi + m^2 \Phi \left( 1 + K + K \log \left( \frac{\Phi^2}{M^2} \right) \right) \Phi + Am^{d-1} \frac{\Phi^{d-1}}{M^{d-3}} = 0.
\]

We have neglected the expansion of the universe since the Q-balls decouple from the cosmic expansion once they are formed.

In the numerical calculations we have set \(M = \phi(0) / \sqrt{2}\) and investigated several values of \(K\) between \(-0.01\) and \(-0.3\) for both \(d = 4\) and \(d = 6\), varying \(\xi\). For the Q-ball configuration, \(\xi\) is evaluated at the center of the Q-ball; \(\Phi_0\) in Eq. \([\Phi]\) should be interpreted as \(\Phi_c\), the field value at the center. Also we have added small initial seed for fluctuations of \(O(10^{-5})\). The size of the lattices is 2048 and 200 \(\times\) 200 for 1D and 2D, respectively. A number of simulations with varying values of the lattice spacing and box size have been done to ensure that the exact values of the lattice parameters do not affect the results.

Then we have found that the Q-ball breaks up for \(\xi\) larger than \(O(10^{-2})\) (see Tables \([\Phi]\) and \([\Phi]\) for the precise values). Figs. \([\Phi]\) \([\Phi]\) and \([\Phi]\) show how the Q-balls are dispersed by the A-term instabilities. It is worth noting that
FIG. 6: The typical breakdown of the Q-ball due to the A-term for $d = 4$ on $1+1$ lattices. $\tau = 0$, $10^3$ and $10^4$ from top to bottom. The adopted parameters are $K = -0.01$ and $\xi = 8 \times 10^{-3}$.

TABLE I: The critical values $\xi_c$ for $d = 4$

| Criterion | $K = -0.01$ | $K = -0.03$ | $K = -0.1$ | $K = -0.3$ |
|-----------|-------------|-------------|------------|------------|
| A         | $4 \times 10^{-3}$ | $9 \times 10^{-3}$ | $2.8 \times 10^{-2}$ | $6.4 \times 10^{-2}$ |
| B         | $3 \times 10^{-3}$ | $9 \times 10^{-3}$ | $2.6 \times 10^{-2}$ | $5.5 \times 10^{-2}$ |

TABLE II: The critical values $\xi_c$ for $d = 6$

| Criterion | $K = -0.01$ | $K = -0.03$ | $K = -0.1$ | $K = -0.3$ |
|-----------|-------------|-------------|------------|------------|
| A         | $2.2 \times 10^{-2}$ | $3.5 \times 10^{-2}$ | $2.3 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |
| B         | $2.1 \times 10^{-2}$ | $3.5 \times 10^{-2}$ | $2.3 \times 10^{-2}$ | $1.2 \times 10^{-2}$ |

The Q-ball is actually stable for small enough values of $\xi$. The critical values ($\xi_c$) above which the Q-ball becomes unstable due to the A-term instabilities for several values of $K$ are shown in Tables I and II. We have followed the evolution until $\tau = 10^4$ and then decided whether the Q-ball configuration is lost or not on the basis of the following criterion; if the total charge of the Q-ball is less than 10% (50%) of the initial value, we judge that the Q-ball is dispersed. We call this criterion A (B).

The critical value $\xi_c$ is found to depend on the values of both $d$ and $K$ to a certain degree. The dependence of $\xi_c$ on $d$ comes from the fact the instability bands have different structures between the cases of $d = 4$ and $d = 6$ (see Figs. 1 and 2). The dependence on the other variable, $K$, possibly comes from the following two effects; (i) the Q-ball is more stable for larger $|K|$; (ii) the Q-ball is more tolerant to perturbations if its charge is larger, and the charge becomes larger for smaller $|K|$ with the amplitude of the AD field $\phi(0)$ fixed because of the dependence of the radius on $|K|$ (see Eq. (11)). Such a naive reasoning might explain the behavior of $\xi_c$ especially in Table II.

Apart from the detailed dependence, can we tell a rough value of $\xi_c$? Put another way, is $\xi_c \sim O(10^{-2})$ reasonable? It is conceivable that $\xi_c$ is determined by the competition between the growth of the instabilities and the relaxation of the Q-ball configuration. Since the Q-ball configuration minimizes the energy of the system in the limit of $\xi \to 0$, the Q-ball tends to keep its configuration for a certain range of $\xi$. It is expected that the relaxation time scale of the Q-ball is set by the mass of the AD field, $m_{\Phi}$, possibly multiplied by some powers of $|K|$. On the other hand, the growth rate calculated from Eq. (6) is about $0.1m_{\Phi}$ for the critical values of $\xi$. So, the relaxation time scale of the Q-ball should be around...
10 m_0^{-1}. This relaxation time scale is in an agreement with Ref. 18 where the excited Q-balls were studied by numerical simulations. Therefore it is probable that the instability due to the A-term destroy the Q-ball if its growth rate exceeds the inverse of the typical relaxation rate of the Q-balls. Lastly let us comment on the realistic value of $\xi$ at the Q-ball formation. When the AD field starts oscillating, $\xi$ should be order unity if $m_{3/2} \simeq m_{\Phi}$ and $A \simeq O(1)$. Then the typical value of $\xi$ is given by $\xi \sim (\Phi_c/\Phi_{osc})^{d/2}$, where $\Phi_{osc}$ is the amplitude of the AD field at the onset of oscillation. According to the numerical calculations (see e.g. Ref. [11]), $\Phi_c/\Phi_{osc} \simeq 6 \times 10^{-3}(|K|/0.1)^{3/4}$. Thus the realistic value of $\xi$ is much smaller than $\xi_c$:

$$ \xi \sim \begin{cases} 3 \times 10^{-5} \left( \frac{|K|}{0.1} \right)^{\frac{3}{2}} & \text{for } d = 4 \\ 10^{-9} \left( \frac{|K|}{0.1} \right)^{\frac{3}{2}} & \text{for } d = 6 \end{cases} $$

Note that $\xi$ becomes smaller while $d$ increases.

IV. CONCLUSION

In this paper we have investigated how the A-term affects the evolution of the AD field, especially paying attention on the stability of the Q-balls. In the linear analysis we first have found that there exist instability bands similar to those of parametric resonance. The growth rate of the instabilities, however, is not so large after the baryon asymmetry is generated. Thus the Q-ball formation in the expanding universe would not be disturbed by the presence of the instabilities. Next we have studied the stability of Q-balls and estimated the critical value of $\xi_c$, above which the Q-balls cannot stay stable, for several values of $K$. From our result it is conceivable that $\xi_c$ is determined by the competition between the growth rate and the relaxation rate of the Q-balls. It should be also noted that the obtained $\xi_c$ is rather large in the realistic cosmological situations. Therefore the extensive studies on Q-balls thus far should remain valid, since the realistic value of $\xi$ should be smaller than $\xi_c$. On the other hand, the instability found in the present paper may be important when Q-balls grow up by absorbing $U(1)$ charge such as solitonsynthesis [13].

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