Isoscalar-vector interaction and hybrid quark core in massive neutron stars

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The hadron-quark phase transition in the core of massive neutron stars is studied with a newly constructed two-phase model. For nuclear matter, a nonlinear Walecka type model with general nucleon-meson and meson-meson couplings, recently calibrated by Steiner, Hemper and Fischer, is taken. For quark matter, a modified Polyakov-Nambu-Jona-Lasinio (mPNJL) model, which gives consistent results with lattice QCD data, is used. Most importantly, we introduce an isoscalar-vector interaction interaction in the description of quark matter, and we study its influence on the hadron-quark phase transition in the interior of massive neutron stars. With the constraints of neutron star observations, our calculation shows that the isoscalar-vector interaction between quarks is indispensable if massive hybrids star exist in the universe, and its strength determines the onset density of quark matter, as well as the mass-radius relations of hybrid stars. Furthermore, as a connection with heavy-ion-collision experiments we give some discussions about the strength of isoscalar-vector interaction and its effect on the signals of hadron-quark phase transition in heavy-ion collisions, in the energy range of the NICA at JINR-Dubna and FAIR at GSI-Darmstadt facilities.

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I. INTRODUCTION

The Equation of State (EoS) of neutron star matter is closely associated with particles that appear in neutron stars. Since the matter in the core of neutron stars is possibly compressed to several times of the saturation nuclear density, new particles and even hadron-quark phase transitions may appear in the interior of these compact objects [1–13]. On the other hand, the EoS of neutron star matter is crucial for the macroscopic features of neutron stars. Each EoS corresponds to one unique mass-radius relation of neutron stars by solving the Tolman-Oppenheimer-Volkoff (TOV) equation [14].

Interestingly, more and more neutron star observations, especially, the accurate measurement of the pulsar J1614-2230 with the mass 1.97 ± 0.04 M⊙ [15], the astrophysical observations of X-ray bursts [16–19] and thermal emissions from quiescent low-mass X-ray binaries (LMXBs) in the globular clusters [20–22], gradually provide a reliable constraint on the mass-radius relations which is tightly connected to the EoS of neutron star matter. The analysis of these astrophysical observations shows that the radius of a 1.4 solar mass neutron star lies between 10.4 and 12.9 km, independent of assumptions about the composition of the core [23, 24]. The relatively small radius of 1.4 solar mass neutron stars means the EoS near the saturation density is soft, and the discovery of massive neutron star J1614-2230 requires the EoS is stiff at high densities. The combination of these constraints rules out many EoSs of hadron models. Besides, experimental information from heavy-ion collisions (HIC) [25, 26] and lattice QCD simulations [27–33] are also available to put some constraints on the EoSs of nuclear and quark matter.

All these progresses on astrophysical observations and laboratory nuclear experiments promote scientists to explore the relevant physics behind. The hadron-quark phase transition is one of the most concerned topics, and so far it is still controversial whether quarks can appear in cold neutron star [34–38]. It is also an important topic in heavy-ion collisions, and related experiments at medium and high densities will be performed in the near future on the updated facilities of NICA at JINR-Dubna and FAIR at GSI-Darmstadt.

The hybrid neutron star picture, with direct quark contributions in the inner core, seems to have problems in describing large mass neutron-stars. All that is due to a lack of repulsion at high baryon densities of the present quark matter effective interactions [39]. This is in fact the main point which has motivated the present paper.

Hadron models based on the relativistic mean-field (RMF) theory are usually used to study the properties of finite nuclei and nuclear matter. But in literature with the same parameter sets, the obtained mass-radius (M – R) relations with or without a hadron-quark phase transition (e.g., [11, 40–43]) are not supported by the recent analysis of neutron star observations [23, 24]. To solve the problem of inconsistency between neutron star observations and nuclear matter EoS in RMF theory, in this study we will investigate...
the hadron-quark phase transition with a newly constructed two-phase model. To describe nuclear matter, we take the extended Walecka model with general nucleon-meson and meson-meson couplings recently calibrated by Steiner, Hemper and Fischer [44], which describes well the properties of nuclei and nuclear matter, and supports recent simulations of the supernovae dynamics. For quark matter, we take the mPNJL quark model [41, 42, 45] which shares with QCD global symmetries and the phenomenon of chiral symmetry breaking as well as an effective (de)confinement at finite densities and temperatures.

Most importantly, in the description of quark matter, we focus on the inclusion of the isoscalar-vector interaction and its influence on the hadron-quark phase transition in the interior of massive neutron stars. With the constraints of neutron star observations, our calculation shows that the isoscalar-vector channel interaction is needed if massive hybrid stars exist in the universe, and its strength determines the onset density of quark matter, as well as the mass-radius relation of hybrid stars. Our previous study [46] also illustrates that this channel interaction affects the hadron-quark phase transition in heavy-ion collisions at finite temperatures and moderate densities. Therefore, as a connection with heavy-ion collision experiments, we further give some discussions about the strength of isoscalar-vector interaction and its influence on the possible phase transition signals from asymmetric nuclear matter to quark matter in heavy-ion collision experiments.

The paper is organized as follows. In Section II, we describe briefly the two-phase model and give the relevant formulas of the extended nonlinear Walecka model and the mPNJL model. In Section III, we present the numerical results, and give some discussions about the phase transition in massive neutron stars, as well as the connection with the phase transition in heavy-ion collisions. Finally, a summary is given in Section IV.

II. THE MODELS

In the two-phase model, the pure hadronic phase and quark phase are described by the nonlinear Walecka type model and the mPNJL model, respectively. As for the coexisted phase between the pure hadronic phase and quark phase, the two phases are connected through the Gibbs conditions. Based on the thermal, chemical and mechanical equilibriums, as well as the global charge neutrality condition [47],

A. The hadronic model

Recently one new equation of state of nuclear matter, labeled SFHO, was constructed based on the extended non-linear Walecka model in RMF theory, and it was taken to simulate core-collapse supernova [44]. The obtained results satisfy the requirements of nuclear physics and match well the astrophysical observations. The Lagrangian of this model is written as

\[
\mathcal{L}^H = \sum_N \bar{\psi}_N \left[ i \gamma_\mu \partial^\mu - M + g_\sigma \sigma - \bar{g}_\omega \gamma_\mu \omega^\mu - \bar{g}_\rho \gamma_\mu \rho^\mu - \rho^\mu \right] \psi_N + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho^\mu \rho_\mu - \frac{1}{4} \rho_{\mu \nu} \rho^{\mu \nu} + \frac{\zeta}{24} \bar{g}_\omega^4 (\omega^\mu \omega_\mu)^2 + \frac{\xi}{24} \bar{g}_\rho^4 (\rho^\mu \rho_\mu)^2 + \bar{g}_\rho^2 f(\sigma, \omega^\mu \omega_\mu) \rho^\mu \rho_\mu + \sum_l \bar{\psi}_l (i \gamma_\mu \partial^\mu - m_l) \psi_l,
\]

where $\omega_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\rho_{\mu \nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$, and the scalar meson potential $V(\sigma) = \frac{4}{3} b (g_\sigma \sigma)^3 + \frac{1}{4} c (g_\sigma \sigma)^4$. $f(\sigma, \omega^\mu \omega_\mu)$ takes the general form as

\[
f = \sum_{i=1}^6 a_i \sigma^i + \sum_{j=1}^3 b_j (\omega^\mu \omega_\mu)^j.
\]

It was first introduced in [48] to provide additional freedom in varying the symmetry energy, and the parameters have been recalibrated recently to fulfill the constraints of nuclear physics and astrophysical observations. Due to

the uncertainties of hyperon-meson couplings, only protons and neutrons are considered in this study. Such point will be further discussed in Sect.III. Electrons and muons, last term of Eq.(1), are included in the calculation in order to keep the charge neutrality of neutron star matter.

Under the mean field approximation, the meson field equations can be obtained as

\[
m_\sigma^2 \sigma - g_\sigma \rho_S + \bar{b} g_\sigma^3 \sigma^2 + c g_\sigma^4 \sigma^3 - \bar{g}_\rho^2 \rho^2 \sigma \partial f \partial \sigma = 0
\]
\[ m^2_{\omega} - g_\omega \rho_B + \frac{\zeta}{6} g^3_{\omega} \rho^3 + g_\rho^2 \rho^3 \frac{\partial f}{\partial \omega} = 0 \quad (4) \]
\[ m^2_{\rho} + \frac{1}{2} g_\rho \sum \tau_{3i} \rho_i + 2 g_\rho^2 \rho f + \frac{\xi}{6} g^4_{\rho} \rho^3 = 0 \quad (5) \]

In Eqs. (3-4),
\[ \rho_S = \sum_{i=p,n} \int \frac{d^3k}{(2\pi)^3} \frac{M^*}{E^*} [f_i(k) - \bar{f}_i(k)] \quad (6) \]
\[ \Omega^H = -\beta^{-1} \sum_{i=N,l} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \left[1 + e^{-\beta(E^*_i(k) - \mu^*_f)} \right] + \ln \left[1 + e^{-\beta(E^*_i(k) + \mu^*_f)} \right] \right\} + \frac{1}{2} m^2_\sigma \sigma^2 - \frac{1}{2} m^2_{\omega} \omega^2 - \frac{1}{2} m^2_{\rho} \rho^2 \quad (8) \]

where \( E^*_i(k) = \sqrt{k^2 + (M_i - g_\omega \sigma)^2} \), \( \mu^*_i = \mu_i - g_\omega \omega - g_\rho \tau_{3i} \rho \) for nucleons; \( E^*_i(k) = \sqrt{k^2 + m^2_i} \), \( \mu^*_i = \mu_i \) for leptons.

The corresponding energy density and pressure of nuclear matter can be derived as
\[ \varepsilon^H = \sum_{i=N,l} \int \frac{d^3k}{(2\pi)^3} \left[ \ln \left[1 + e^{-\beta(E^*_i(k) - \mu^*_f)} \right] + \ln \left[1 + e^{-\beta(E^*_i(k) + \mu^*_f)} \right] \right] + \frac{1}{2} m^2_\omega \omega^2 - \frac{1}{2} m^2_\rho \rho^2 \quad (9) \]
\[ P^H = \sum_{i=N,l} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{E^*_i(k)} [f_i(k) + \bar{f}_i(k)] \left[ \ln \left[1 + e^{-\beta(E^*_i(k) - \mu^*_f)} \right] + \ln \left[1 + e^{-\beta(E^*_i(k) + \mu^*_f)} \right] \right] + \frac{1}{2} m^2_\omega \omega^2 + \frac{1}{2} m^2_\rho \rho^2 \quad (10) \]

where \( f_i(k) \) and \( \bar{f}_i(k) \) are the fermion and antifermion distribution functions:
\[ f_i(k) = \frac{1}{1 + \exp[(E^*_i(k) - \mu^*_f)/T]} \quad (11) \]
\[ \bar{f}_i(k) = \frac{1}{1 + \exp[(E^*_i(k) + \mu^*_f)/T]} \quad (12) \]

The model parameter set labeled SFHO is used in the calculation, which can be fixed by fitting the properties of symmetric nuclear matter at saturation nuclear density. For the details of model parameters, on can refer [44]

\[ \rho_B = \sum_{i=p,n} \int \frac{d^3k}{(2\pi)^3} [f_i(k) - \bar{f}_i(k)] \quad (7) \]

are the scalar and baryon densities, respectively.

The thermodynamical potential of the nucleon-meson system is

\[ \mathcal{L}^Q = \bar{q}(i\gamma^\mu D_\mu - m_0)q + G \sum_{k=0}^8 \left[ \bar{q}\lambda_k q \right]^2 + \bar{q} \sigma \gamma_5 q \right] + \mathcal{L}^f = \mathcal{L}^f \left[ \bar{q}(1 + \gamma_5)q \right] \left[ \bar{q}(1 - \gamma_5)q \right] \quad (13) \]

where \( q \) denotes the quark fields with three flavors, \( u, d, \) and \( s, \) and three colors; \( m_0 = \text{diag}(m_u, m_d, m_s) \) in flavor space; \( G \) and \( K \) are the four-point and six-point interacting constants, respectively. The isoscalar-vector interaction channel is also included. As shown by Eq. (13), the \( \mu = 0 \) component of the isoscalar vector interaction corresponds to the density operator \( (\bar{q}\gamma^\mu q)^2 \), therefore it is conceivable to expect that the finite-density environment brings a significant contribution to this channel. Moreover, it is also well known that standard (P)NJL models lead to a chiral restoration transition at unphysically baryon densities just above the saturation point [49], easily reached in heavy ion collisions, without any evidence of such effect. The presence of a repulsive vector field, as considered in the present work, moves the transition to more realistic, higher densities.

This channel interaction reduces the effective quark chemical potential, \( \tilde{\mu} = \mu - 2G_V n_q. \) In this study we will focus on its influence on the hadron-quark phase transition in dense neutron star matter.

The covariant derivative in the Lagrangian is defined as \( D_\mu = \partial_\mu - iA_\mu \). The gluon background field \( A_\mu = \delta_\mu^0 A_0 \) is supposed to be homogeneous and static, with \( A_0 = gA_0^a \frac{\lambda^a}{2} \), where \( \frac{\lambda^a}{2} \) is \( SU(3) \) color generators. The
effective potential $\mathcal{U}(\Phi[A], \tilde{\Phi}[A], T)$ is expressed in terms of the traced Polyakov loop $\Phi = (\text{Tr}_c L)/N_c$ and its conjugate $\tilde{\Phi} = (\text{Tr}_c L^\dagger)/N_c$. The Polyakov loop $L$ is a matrix in color space

$$L(\vec{x}) = \mathcal{P}\exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right],$$

where $\beta = 1/T$ is the inverse of temperature and $A_4 = iA_0$.

Different effective potentials were adopted in literatures $[41, 50-52]$. The modified chemical dependent one

$$\mathcal{U} = (a_0 T^4 + a_1 \mu^4 + a_2 T^2 \mu^2)^2$$
$$a_3 T_0^4 \ln(1 - 6 \Phi^2 + 8 \Phi^3 - 3 \Phi^4)$$

(15)

was used in $[41, 45]$ which is a simplification of

$$\mathcal{U} = (a_0 T^4 + a_1 \mu^4 + a_2 T^2 \mu^2)^2 \tilde{\Phi} \Phi$$
$$a_3 T_0^4 \ln \left[ 1 - 6 \Phi \Phi + 4(\Phi^3 + \Phi^3) - 3(\Phi \Phi)^2 \right]$$

(16)

because the difference between $\tilde{\Phi}$ and $\Phi$ is smaller at finite chemical potential, and $\tilde{\Phi} = \Phi$ at $\mu = 0$. In the calculation we will take the form given in equation. (16), as used in $[42]$. The related parameters, $a_0 = -1.85$, $a_1 = -1.44 \times 10^{-3}$, $a_2 = -0.08$, $a_3 = -0.4$, are still taken from $[41]$, which can reproduce well the data obtained in lattice QCD calculation.

In the mean-field approximation, quarks can be taken as free quasiparticles with constituent masses $M_i$, and the dynamical quark masses (gap equations) are obtained as

$$M_i = m_i - 4G\phi_i + 2K\phi_j \phi_k \quad (i \neq j \neq k),$$

(17)

where $\phi_i$ stands for quark condensate.

The thermodynamic potential of quark matter in the mean-field level can be derived as

$$\Omega^Q = \mathcal{U}(\tilde{\Phi}, \Phi, T) + 2G \left( \phi_u^2 + \phi_d^2 + \phi_s^2 \right) - G_V(n_u + n_d + n_s)^2 - 4K \phi_u \phi_d \phi_s - 2 \int_\Lambda (2\pi)^3 d^3p \ln \left[ A(\tilde{\Phi}, \Phi, E_i - \mu_i, T) \right] - 2T \int_\Lambda (2\pi)^3 d^3p \ln \left[ A(\tilde{\Phi}, \Phi, E_i + \mu_i, T) \right],$$

(18)

where $A(\tilde{\Phi}, \Phi, E_i - \mu_i, T) = 1 + 3\Phi e^{-2(E_i - \mu_i)/T} + 3\Phi e^{-3(E_i - \mu_i)/T}$ and $A(\tilde{\Phi}, \Phi, E_i + \mu_i, T) = 1 + 3\Phi e^{-(E_i + \mu_i)/T} + 3\Phi e^{-2(E_i + \mu_i)/T} + e^{-3(E_i + \mu_i)/T}$.

The values of $\phi_u, \phi_d, \phi_s, \Phi$, and $\tilde{\Phi}$ are determined by minimizing the thermodynamical potential

$$\frac{\partial \Omega^Q}{\partial \phi_u} = \frac{\partial \Omega^Q}{\partial \phi_d} = \frac{\partial \Omega^Q}{\partial \phi_s} = \frac{\partial \Omega^Q}{\partial \Phi} = \frac{\partial \Omega^Q}{\partial \tilde{\Phi}} = 0.$$ 

(19)

All the thermodynamic quantities relevant to the bulk properties of quark matter can be obtained from $\Omega^Q$. Particularly, the pressure can be derived with $P = -(\Omega^Q(T, \mu) - \Omega^Q(0, 0))$. From Eq.(18) it is clear that the introduction of the isoscalar vector interaction in the PNJL model may give important contributions to the pressure of quark matter. Indeed the EOS may become much stiffer at high densities for large $G_V$ values. A similar behavior can be obtained in the MIT bag model if such term is included.

As an effective model, the (P)NJL model is not renormalizable, so a cut-off $\Lambda$ is implemented in 3-momentum space for divergent integrations. The model parameters: $\Lambda = 603.2$ MeV, $GA^2 = 1.835$, $K\Lambda^5 = 12.36$, $m_{u,d} = 5.5$ and $m_s = 140.7$ MeV, determined by fitting $f_s$, $M_a$, $m_K$ and $m_{\eta}$ to their experimental values $[53]$, are used in the calculation.

Concerning the strength of the isoscalar-vector coupling $G_V$, there are no explicit constraints at finite density. Some efforts to estimate a possible range of values for this coupling are briefly described below. For the convenience to compare $G_V$ with the strength of the isoscalar-scalar interaction $G$, and for later discussion, we define $R_V = G_V/G$

A very naive estimation, based on the value taken by this ratio in the hadronic sector, would give $R_V$ around 0.6-0.7 $[54]$. In Ref.[46] a vector/scalar coupling ratio around $R_V = 0.2$ was obtained from an evaluation of only the Fock contributions of the scalar channels. Values in the range $0.25 < R_V < 0.5$ are derived by a Fierz transformation of an effective one-gluon exchange interaction, with $G_V$ depending on the strength of the $U_A(1)$ anomaly in the two-flavor model $[55, 56]$. However, the point is that the coupling strength of the direct term cannot be fixed, so the total effect of the isoscalar vector interaction is still unknown. Other attempts to estimate $G_V$ are based on the fit of the vector meson spectrum $[57]$. However, the relation between the vector coupling in dense quark matter and the meson spectrum in vacuum is expected to be strongly modified by in-medium effects $[52, 58]$.

Because of the uncertainties discussed above, we will treat $R_V$ as a free parameter. Aim of the present work
is just to get some relevant information from neutron star properties. We notice that suggestions to catch information on the isoscalar vector interaction may also come from heavy ion reactions at next-generation colliders, such as NICA and FAIR [46]. A combined study of the two aspects appears as a promising tool to get hints on the strength of $R_V$.

C. The hadron-quark phase coexistence

The Gibbs criteria are usually implemented for the phase equilibrium of a complex system with more than one conservation charge. The Gibbs conditions for the phase coexistence within a hadron-quark transition in compact star are

$$
\mu^H_i = \mu^Q_i, \quad T^H = T^Q, \quad P^H = P^Q,
$$

(20)

where $\mu^H_i$ are usually chosen with $\mu_n$ and $\mu_e$. Under the $\beta$ equilibrium without trapped neutrino, the chemical potential of other particles including all baryons, quarks, and leptons can be derived by

$$
\mu_i = g_i \mu_n - q_i \mu_e,
$$

(21)

where $g_i$ and $q_i$ are the baryon number and electric charge number of particle species $i$, respectively.

The baryon number density and energy density in the mixed phase are composed of two parts with the following combinations

$$
\rho = (1 - \chi) \rho^H_B + \chi \rho^Q_B,
$$

(22)

and

$$
\varepsilon = (1 - \chi) \varepsilon^H + \chi \varepsilon^Q,
$$

(23)

where $\chi$ is the volume fraction of quark matter. For the coexisted phase, the electric neutrality is fulfilled globally with

$$
q_{\text{total}} = (1 - \chi) \sum_{i=B,l} q_i \rho_i + \chi \sum_{i=q,l} q_i \rho_i = 0.
$$

III. NUMERICAL RESULTS AND DISCUSSIONS

We present in Fig. 1 the EoSs of neutron star matter without and with the hadron-quark phase transition for different strength of the isoscalar-vector interaction. For each value of $R_V = G/V/G$, the two solid dots with the same color indicate the range of the coexisted phase, and the cycle marks the largest pressure that can be reached in the core of neutron star by solving the TOV equation. With increasing vector strength in the quark sector the onset of the transition is moving to higher densities since the quark pressure is also increasing.

This figure demonstrates that the isoscalar-vector interaction of quark matter plays an important role in the hadron-quark phase transition. In particular the value of $R_V$ is crucial for the EoS of neutron star matter at high densities. Besides, it shows that the mixed range can be reached only inside massive neutron stars. In fact, for the case $R_V$ larger than 0.484, the calculation shows that quark matter does not appear in the neutron star core.

To compare with the data from heavy-ion-collision experiments and neutron star observations, we re-scaled in Fig. 2 the EoSs displayed in Fig. 1. It shows all the
EoSs can fulfill these constraints at low and moderate densities. However, for the case of hybrid star at high densities, \( R_V > 0.1 \) appears to be favored by recent neutron star observations of the mass-radius correlation in 1\( \sigma \) contours. In the following we will constrain the value of \( R_V \) using the \( M - R \) relations and the accurately measured mass of Pulsar J1614-2230.

As already stated before, in the present study hyperons are not included in the hadron sector mostly because of the uncertainties on the hyperon-meson couplings in the nuclear medium. When hyperons appear in the nuclear matter, just from a degree of freedom counting we can expect a sudden softening of the nuclear EoS. Such effect can be largely modified from the hyperon interactions. Indeed this seems to be the indication emerging from Heavy Ion data of Fig. 2: the hadronic EoS appears to remain rather stiff at densities between 3 and 5 \( \rho_0 \), where hyperons would appear in the medium just from chemical potential arguments. However we must also note that the high density matter formed in HIC will certainly differ from the one of neutron-stars, in particular for the lack of chemical equilibrium. In any case, mass-radius calculations for hybrid neutron stars seem to be more sensitive to the model adopted for the quark phase than to the hadronic EoS [39, 59].

As a further theoretical study, to investigate the properties of massive neutron stars with both hyperons and quarks, one can fine-tune the meson-hyperon couplings and/or introduce the strange meson mediated interaction between hyperons in the hadron sector, to adjust the stiffness of the hadronic EOS. This will be the object of a forthcoming analysis.

In order to see how the isoscalar-vector interaction affects the threshold of the hadron-quark phase transition, we display in Fig. 3 the relative fractions of different species as functions of baryon density for \( R_V = 0, 0.2, 0.4 \), respectively. This figure shows that a stronger isoscalar-vector interaction postpones the onset density of quark matter, and the central density of the corresponding hybrid star moves to a higher value. However a larger \( R_V \) also means that the fraction of quark matter is smaller in the core of neutron star. Particularly, if \( R_V \) is large enough, the onset density of quark matter, in fact in the mixed phase, will be larger than the central density of the neutron star. In this case, no quarks can appear in the core of neutron stars. This clearly demonstrates the crucial role that the isoscalar-vector interaction plays on the hadron-quark phase transition in massive neutron stars.

In Fig. 4 we plot the mass-radius relations of hybrid stars with different \( R_V \). The inner (outer) two contours represent the 1\( \sigma \) and 2\( \sigma \) confidence ranges of the \( M - R \) relations given in Ref. [23] (Ref. [24]), based on six (eight) neutron star observations of the X-ray bursts [16–19] and thermal emissions from quiescent low-mass X-ray binaries (LMXBs) in the globular clusters [20–22]. This figure shows that the EoS of neutron star matter with the parameter set of SFHO fulfills well the constraints of neutron star observations. This figure also gives us an explicit picture of how the strength of the isoscalar-vector interaction influences the macroscopical properties of massive hybrid stars. The radio timing observations of the binary millisecond pulsar J1614-2230 with a strong general relativistic Shapiro delay signature, implies that the pulsar mass is 1.97 \( \pm 0.04 \) M\(_\odot\) [15]. The discovery of this massive pulsar rules out many soft EoSs. If the isoscalar-vector interaction of quark matter is not included, the maximum mass of hybrid stars is 1.88 M\(_\odot\), less than the known maximum neutron star mass. However, with the inclusion of this channel interaction and taking \( R_V = 0.055 \), the obtained maximum mass of a hybrid star can reach the lower mass limit of the pulsar J1614-2230. Therefore, \( R_V \geq 0.055 \) is required for the existence of massive hybrid stars in the universe, and no quarks appear for \( R_V > 0.484 \) as shown in Fig. 1.

Finally, we like to present some further discussions about the isoscalar-vector interaction in quark matter. Apart the general argument about the relevance of this term in hadronic matter at low temperature and increasing baryon density, we have some specific points from results of effective non-perturbative QCD models.

When including this channel interaction in quark model, the value of \( R_V \) affects the location and emergence of critical points of chiral symmetry restoration [56, 60–63]. We note that in all the present effective quark models with chiral restoration, like NJL or PNJL, show, at zero temperature, the chiral transition takes place at unphysical low baryon densities, just above the saturation value. The inclusion of the isoscalar-vector term, which increases pressure and kinetic chemical potential, will shift the transition to more realistic density regions.

Moreover this channel in the effective quark Lagrangian also influences the onset densities and the expected phase-transition signals from charge asymmetric
nuclear matter to quark matter in the two-phase model related to experiments in heavy-ion collisions [46]. New data about properties of a mixed phase eventually probed in high density regions will provide important information on the strength of the isoscalar-vector term in the quark interaction.

In conclusion although this coupling presently cannot be determined from experiments and lattice QCD simulations, there are some good hints about its existence:

- Compared with the hadron Walecka model, the isoscalar-vector interaction of quark matter plays a repulsive role similar to the $\omega$ meson, important for the properties of nuclear matter in Quantum Hadron Dynamics model, in particular at finite densities and low temperatures.

- This channel interaction can be derived from higher order Fock (exchange) terms or Fierz transformations or fitting the vector meson spectrum [46].

- The existence of massive hybrid neutron stars as shown in this study.

IV. SUMMARY

We have studied the hadron-quark phase transition in dense neutron star matter with an improved two-phase model. The calculations show that massive hybrid stars possibly exist in the universe. In this respect the isoscalar-vector interaction between quarks is crucial for the hadron-quark phase transition. Its strength determines whether quarks can appear in the interior of neutron stars.

Although the accurate value of $R_V$ is still not known by far, neutron star observations can gradually provide some constraints on it. In our previous study about the hadron-quark phase transition in heavy-ion collisions, we have demonstrated that the inclusion of this channel interaction postpones the onset density of quark matter. The corresponding phase-transition signals, in particular on properties of the mixed phase, in the case of charge asymmetric nuclear matter to quark matter will be strengthened. Then, based on the phase transition features of asymmetric strongly interacting matter, we also proposed some suggestions to probe the phase transition signals in relevant experiments at the FAIR and NICA facilities [46]. The promising conclusion is that in a near future the combination of neutron star observations and the energy scan of the phase-transition signals at FAIR/NICA may provide us some hints on the value of $R_V$, which is helpful for the understanding of quark matter interactions and neutron star structure.

Acknowledgments

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