Macroscopic quantum superpositions in highly–excited strongly–interacting many–body systems

S.Yu. Kun,\textsuperscript{1} L. Benet,\textsuperscript{2} L.T. Chadderton,\textsuperscript{3} W. Greiner,\textsuperscript{4} and F. Haas\textsuperscript{5}

\textsuperscript{1}Department of Theoretical Physics, RSPhysSE, IAS, The Australian National University, Canberra ACT 0200, Australia
\textsuperscript{2}Centro de Ciencias Físicas, National University of Mexico (UNAM), Campus Morelos, 62210–Cuernavaca, Mexico
\textsuperscript{3}Atomic and Molecular Physics Laboratories, RSPhysSE, IAS, The Australian National University, Canberra ACT 0200, Australia
\textsuperscript{4}Institut für Theoretische Physik, Johann Wolfgang Goethe Universität, D–60054 Frankfurt am Main, Germany
\textsuperscript{5}Institut de Recherches Subatomiques, CNRS–IN2P3 et Université Louis Pasteur, BP 28, F–67037 Strasbourg Cedex 2, France

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We demonstrate a break-down in the macroscopic (classical–like) dynamics of wave–packets in complex microscopic and mesoscopic collisions. This break-down manifests itself in coherent superpositions of the rotating clockwise and anticlockwise wave–packets in the regime of strongly overlapping many–body resonances of the highly–excited intermediate complex. These superpositions involve $\sim 10^4$ many–body configurations so that their internal interactive complexity dramatically exceeds all of those previously discussed and experimentally realized. The interference fringes persist over a time–interval much longer than the energy relaxation–redistribution time due to the anomalously slow phase randomization (dephasing). Experimental verification of the effect is proposed.

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Understanding the boundary between microscopic (quantum) and macroscopic (classical) worlds has been a longstanding and key problem of modern science. In quantum mechanics a single object can be represented by different quasiclassical wave–packets (WP) simultaneously localised in two different locations. When these WP spatially overlap this produces interference fringes, thereby demonstrating coherent superposition of the two distinct locations of the same object. Such a superposition is generally referred to as “Schrödinger cat state” (SCS) following Schrödinger’s discussion of quantum superposition of “live cat” and “dead cat” states \cite{1}, stressing a sharp contrast between the quantum world and our everyday macroscopic experience. The quantum–classical transition, driven by decoherence, occurs ever quicker with increasing size of the system. SCS’s have been realized, e.g., for the Rydberg atomic electron \cite{2,3,4}, photons in a microwave cavity \cite{5}, and laser–cooled trapped ions \cite{6}. Quantum superposition of distinct macroscopic states, involving $\sim 10^9$ Cooper pairs, is also reported \cite{6}. Even though these examples represent quite different systems they have one common feature, namely that SCS’s \cite{2,3,4,6} are quantum superpositions of a relatively small number ($\leq 10$) of single-particle (Fock) states reflecting either the single-particle nature of the system \cite{2} or absence of interaction between different degrees of freedom \cite{3,4}. In particular, the data \cite{6} can be understood in terms of quantum superposition of just two \textit{single} quasi–particle (Fock) states of the superconducting condensate, each of these states being occupied by a macroscopically large number of \textit{noninteracting} Cooper pairs. Yet the \textit{internal interactive complexity} and the \textit{high excitation energy} are also characteristic properties of macroscopic objects with extremely small energy level spacings. This motivates a search for SCS’s involving coherent superpositions of a very large number of highly–excited strongly–mixed many–body configurations.

It was shown \cite{2,3} that spontaneous off–diagonal spin correlations between strongly overlapping resonances of the deformed highly–excited intermediate complex (IC), which can be formed in nuclear, molecular and atomic cluster collisions, produce rotational WP. These spatially localised WP rotate in opposite, clockwise and anticlockwise, directions. The period of rotation is much shorter than the inverse average level spacing of the deformed IC, reflecting correlations between the very large number of complex spatially extended many–body configurations with different total spins. It should be noted that relatively stable WP in highly–excited many–body systems have been identified and associated with the existence of invariant manifolds in classical phase space of the system \cite{4}.

In this Letter we demonstrate a break–down of the simple WP classical–like dynamics \cite{7} by revealing the interference between the two WP. This is a manifestation of the coherent quantum, rather than statistical classical, nature of superposition of the two WP representing the same system. The quantum superpositions discussed here involve $\sim 10^4$ highly–excited individually ergodic many–body configurations, each of these being a superposition of $\sim 10^5$ single–particle states (e.g. Slater determinants). In this sense, and to the best of our knowledge, \textit{internal interactive complexity} of such SCS’s \textit{dramatically exceeds} all of those previously discussed and experimentally realized. Yet, in spite of the extreme complexity, it could be the smallest Schrödinger “kitten”, if created in heavy–ion collisions, in all SCS sys-
tems previously identified. Alternatively it might also be
of nanometer size if created in atomic cluster collisions
[10]. We propose an experimental test of the effect.

Consider a peripheral collision of heavy–ions, or poly-
atomic molecules, or atomic clusters. Then suppose that
the collision proceeds through formation of a deformed
relatively long–living highly–excited IC. This is a likely
scenario provided that the double–folding potential be-
tween the collision partners has a pocket. The calcu-
lations indicate the existence of such pockets, e.g., for
heavy–ion [11] and atomic cluster [12] systems. [It should
be noted that the unambiguous experimental demon-
stration of quantum coherence for hot fullerenes [13]
strongly calls for a fully quantum–mechanical treatment of atomic
cluster (i.e. mesoscopic) collisions.] As the relative ra-
dial kinetic energy of the colliding partners transfers into
intrinsic excitation they drop into this pocket, forming
a highly–excited deformed IC. We concentrate on that
regime of the highly–dense spectrum, where \( D \ll 1 \) with
\( D \) being an average level spacing, and \( \tilde{h} / \Gamma \) the average
life–time of the IC. In this domain of strongly overlapping
resonances the energy spectrum is not resolved, and the
dynamics of the IC is dominated by a very large number
of simultaneously excited many–body quasi–
bound configurations. This regime is opposite in the ex-
treme to that of Bose–Einstein condensation [14].

Let us consider the spinless collision partners in the
entrance \( a \) and exit \( b \) channels. The time \( t \) and scattering angle \( \theta \) dependence intensity of the decay of IC is given by
\[
P(t, \theta) \propto \lim_{t \to -\infty} \left(1 / \tilde{T}_t \right) \int_{-\tilde{T}_t / 2}^{\tilde{T}_t / 2} \exp \left(-\text{i} \eta t / \tilde{h} \right) \rho(\eta, \theta) \, d \eta,
\]
where \( \rho(\eta, \theta) = \langle \delta f(E + \epsilon) \delta f(E)^\ast \rangle \) is the amplitude
energy autocorrelation function and brackets \( \langle \ldots \rangle \) de-
ote the energy averaging. Here \( \delta f(E) = \sum_J (2 J + 1) \exp(i J \Phi) \delta S^J(E) P_J(\theta) \) is the oscillating zero
value \( (\langle \delta f(E) \rangle = 0) \) collision amplitude reflecting a time-
delayed reaction mechanism [7]: \( \delta S^J(E) = S^J(E) –
\langle S^J(E) \rangle \) is the fluctuating \( \langle \delta S^J(E) \rangle = 0 \) com-
ponent of the \( S \)–matrix \( S^J(E) \) with total spin \( J \) and \( \langle S^J(E) \rangle \) is
its energy averaged component. \( \Phi \) is the deflection angle
due to the \( J \)–dependence of the potential phase shifts, and
the \( P_J(\theta) \) are Legendre polynomials. We obtain
\[
P(t, \theta) \propto H(t) \exp(-\tilde{\Gamma} t / \tilde{h}) \sum_{\mu \nu} \sum_J \left[ W(J) W(J') \right]^{1/2} \exp(i \Phi(J – J') – i (E^J_\mu - E^{J'}_{\mu'}) t / \tilde{h}) \rho(\eta, \theta) P_J(\theta) P_{J'}(\theta),
\]
where \( \gamma^{J}_{\mu}(b) \) and \( \gamma^{J'}_{\mu'}(a) \) are the partial width amplitudes, \( E^J_\mu \) and \( E^{J'}_{\mu'} \) are the resonance energies, and \( W(J) = \langle \delta S^J(E) \delta S^J(E)^\ast \rangle \propto \left( c^{J}_{\mu} \right)^2 \) is the average partial reaction probability.
The overbars stand for the averaging over ensemble of \( \gamma^{J}_{\mu}(b) \) and \( \gamma^{J'}_{\mu'}(a) \) which are considered as Gaussian
random variables. The Heaviside step function \( H(t) \)
signifies that the IC cannot decay before it is formed at
t = 0. We take into account the spin–off–diagonal cor-
relation [6, 11, 16]: \( \tilde{c}^{J}_{\mu} \tilde{c}^{J'}_{\mu'} = (1 / \pi) D \beta |J – J'| / \{|E^J_\mu - E^{J'}_{\mu'} -
\tilde{h} \omega (J – J')^2 + \beta^2 (J – J')^2 | \}, \) while \( \tilde{c}^{J}_{\mu} \tilde{c}^{J'}_{\mu'} = \delta_{\mu \mu'} \), the latter being a conventional assumption of random matrix
theory [12, 17, 18]. In the correlator, \( \beta \gg D \) is the spin
dehasing width and \( \omega \gg D / \tilde{h} \) is the angular
velocity of the coherent rotation of the highly–excited IC.
Changing from the \( \mu \)–summation to the integration over
\( (E^J_\mu, E^{J'}_{\mu'}) \), which is an accurate approximation for
\( D \ll \beta \) and \( D \ll \Gamma \), i.e. for \( \Gamma \ll \hbar / D \), when the spectrum
is not resolved, we obtain
\[
P(t, \theta) \propto \frac{1}{H(t)} \exp(-\tilde{\Gamma} t / \tilde{h}) \sum_J \left[ W(J) W(J') \right]^{1/2} \exp[i (\Phi - \omega t)(J – J') – \beta |J – J'| t / \tilde{h}] \rho(\eta, \theta) P_J(\theta) P_{J'}(\theta). \tag{2}
\]
Therefore although Eq. [10] involves \( \mu \)–sums over \( \sim \beta^2 / D^2 \gg 1 \) strongly overlapping \( (\Gamma \gg D) \) resonances,
\( P(t, \theta) \) reduces to a sum over \( (J, J') \).

We take the partial average reaction probability in the
J–window form, \( W(J) = \langle |\delta S^J+E|^2 \rangle \propto \exp(-\tilde{J}^2 / d^2) \), where \( I \) is the average spin and \( d \) is the J–window width. We use the asymptotic form of \( P_J(\theta) \)
for \( \theta, \pi - \theta \geq 1 / I \) and employ the Poisson summation
formula. For relatively slow dephasing, \( d / \tilde{h} \ll 1 \), and,
for \( d \ll 1 \), we obtain
\[
P(t, \theta) \sim H(t) (1 / \sin \theta) \exp(-\tilde{\Gamma} t / \tilde{h}) \times
\sum_{k = 0}^{\infty} \left\{ \mathcal{P}^{+}_{k}(t, \theta)^2 + \mathcal{P}^{-}_{k+1}(t, \theta)^2 +
2 \cos[(2 I + 1) \theta - \pi / 2] \times \mathcal{P}^{+}_{k}(t, \theta) \mathcal{P}^{-}_{k+1}(t, \theta) \right\}, \tag{3}
\]
where \( \mathcal{P}^{+}_{k}(t, \theta) = \Delta^{-1}(t) \exp[-(\Phi \pm \theta + 2 \pi k - \omega t)^2 / 2 \Delta^2(t)] \), and \( \Delta^2(t) = (d^2 - \beta^2 t^2 / \tilde{h}^2) \). We observe
that \( \mathcal{P}^{+}_{k}(t, \theta) \) and \( \mathcal{P}^{-}_{k+1}(t, \theta) \) have the meaning of time
dependent amplitudes for the decay into the angle \( \theta \), after
\( k \) consecutive revolutions of the IC rotating in opposite
directions. Due to the azimuthal symmetry of the problem
the emission directions of the collision fragments, at
\( t = 0 \), are concentrated along a cone of angle \( \Phi \). If we
intersect this cone by the reaction plane at \( t = 0 \) we
obtain two spatially localised WP, with angular disper-
sion \( \sim 1 / d \), located symmetrically around the direction
of the initial beam at \( \theta = |\Phi| \) and \( \theta = 2 \pi - |\Phi| \). As
time proceeds the WP rotate with angular velocity \( \omega \),
in opposite directions. This reflects an opening or closing
(depending on the value of \( \Phi \) of the cone. The spin off–
diagonal \( \delta S \)–matrix correlation is a necessary condition
for formation of the WP. The WP meet and overlap in a
vicinity of forward and backward angles producing inter-
ference fringes due to the interference term between the
amplitudes \( \mathcal{P}^{+}_{k}(t, \theta) \) and \( \mathcal{P}^{-}_{k+1}(t, \theta) \) in Eq. [4].

As an example, we illustrate the dynamics and coher-
ent superpositions of WP for \( ^{12} \text{C} + ^{24} \text{Mg} \) elastic scattering
[19]. For this system analysis of the cross section en-
ergy autocorrelation function, obtained from the excita-
FIG. 1: Time and angle dependence of intensity of decay of the highly–excited intermediate molecule created in a $^{12}$C+$^{24}$Mg collision. Panel (a) corresponds to $t = 0$; (b) $t = T/16$; (c) $t = T/8$; (d) $t = 5T/16$; (e) $t = 3T/8$; (f) $t = 7T/16$; (g) $t = T/2$; (h) $t = 3T/4$; (i) $t = T$. Period of one complete revolution of the molecule is $T = 3.06 \times 10^{-21}$ sec (see text).

tion function measured over the c.m. energy range 12.27–22.8 MeV at $\theta = \pi$, reveals rotational WP in spite of strong overlap of resonance levels in the highly–excited intermediate molecule. We calculate $P(t, \theta)$ with a set of parameters evaluated from the fit of $P(t, \theta)$: $\Phi = 0$, $d = 3$, $I = 14$, $\beta = 0.01$ MeV, $\hbar \omega = 1.35$ MeV, $\Gamma = 0.3$ MeV. The quantity presented in Fig. 1 is $AP(t, \theta)/\langle \sigma(E, \theta) \rangle$, where $\langle \sigma(E, \theta) \rangle \propto \int_0^\infty dt P(t, \theta)$ is the energy average differential cross section for the time–delayed collision. The reason for this is that amplitudes of individual WP are strongly and abruptly enhanced in the close vicinity of $\theta = 0, \pi$. This is because, in contrast to an intermediate angular range $\pi/I \leq \theta \leq \pi - \pi/I$, for these forward and backward angles the reaction plane is ill defined, and the cone degenerates into a line so that all azimuthal angles ($0 \leq \phi \leq 2\pi$) contribute to the decay. However, this also results in a similar enhancement of $\langle \sigma(E, \theta) \rangle$ for $\theta \sim 0, \pi$, so that the quantity $P(t, \theta)/\langle \sigma(E, \theta) \rangle$ permits us to probe interference between the two WP, amplitudes of which depend smoothly on $\theta$. The constant $A$ is derived from the condition $AP(t = 0, \theta = 0)/\langle \sigma(E, \theta = 0) \rangle = 1$.

In Fig. 1 at the initial moment of time, $t = 0$, the forward-oriented ($\theta \sim 0, 2\pi$) WP completely overlap producing strong interference fringes. As the WP move apart the amplitude of the fringes is reduced. Thus, in panel (d), Fig. 1 the fringes are absent since the WP are far apart and do not overlap. As the WP begin to overlap around $\theta \sim \pi$, the interference fringes reappear and are most pronounced at $t = T/2$ ($T = 3.06 \times 10^{-21}$ sec is the rotation period) when the backward–oriented WP completely overlap (panel (g)). Then the WP pass each other and move apart again, first becoming isolated (Fig. panel (h)), and finally, at $t = T$ (panel (i)), completely overlap around $\theta = 0, 2\pi$. As time proceeds the overall intensity of the decay decreases exponentially due to the $\exp(-\Gamma t/\hbar)$ factor in $P(t, \theta)$. Note that the interference fringes at $t = T$ are not as pronounced as at $t = 0$ due to a relatively small, but finite dephasing width.

It has been shown that rotational coherence effects in the molecular decay are manifestations of the temporal evolution of the angular orientation of molecules.
Therefore we interpret the interference fringes in $P(t, \theta)$ in Fig. II in terms of a quantum mechanical superposition of distinct orientations of a highly-excited molecule.

Values of $P(t, \theta)$ in Fig. II are obtained for the high intrinsic excitation energy ($\sim 15$ MeV) of the deformed intermediate molecule with average level spacing $D \sim 10^{-5}$ MeV. Consequently, the energy spectrum of the molecule is not resolved during its lifetime, $\hbar/\tau \ll \hbar/D$. The time for formation of ergodic many-body configurations, i.e. the time it takes the two-body interaction to redistribute the energy between the particles of the system, is $\tau_{\text{erg}} = \hbar/\Gamma_{\text{spr}}$ with $\Gamma_{\text{spr}}$ being the spreading width of strongly overlapping many-body states. Therefore the interference of the SCs in Fig. II persist for $t \gg \tau_{\text{erg}}$ clearly demonstrates their many-body nature, i.e. they originate from the interference of very large number ($\Gamma_{\text{spr}} \sim 10^4$) of strongly overlapping many-body states in Fig. II. The internal many-body aspect of the problem on the relationship between the energy redistribution and dephasing rates could not be addressed in the previous studies of the SCs created in the single–particle non–interactive systems.

The SCs in Fig. II is a manifestation of the anomalously long spin dephasing time $\hbar/\beta \sim 10^{-19}$ sec, as compared with $\tau_{\text{erg}} \sim 10^{-22}$ sec. In the limit of short dephasing time, $\beta \sim \Gamma_{\text{spr}} \gg \Gamma$, we would have $P(t, \theta) \propto \exp(-\Gamma t/\hbar)\langle\sigma(E, \theta)\rangle$, where $\langle\sigma(E, \theta)\rangle \propto \sum_{j}(2J+1)^2W(J/P)(\theta)^2$ is the energy averaged cross section obtained in the limit of absence of spin–off–diagonal correlations, $\beta \sim \Gamma_{\text{spr}}$. This corresponds to the limit of random matrix theory, where macroscopically distinct quantum superpositions exist over the time interval $\Gamma_{\text{spr}}$, which, for $\Gamma \ll \Gamma_{\text{spr}}$, yields $AP(t, \theta)/\langle\sigma(E, \theta)\rangle = \exp(-\Gamma t/\hbar)$, i.e. angle–independent straight horizontal lines in Fig. II (not shown), completely washing out the WP and their interference. Another source for the WP spreading and suppression of their interference is a possible $J$–dependence of $\omega$. Then the WP and SCs are washed out for $d\omega \tau \geq \pi$. For the $J$–independent moment of inertia $J$ of the IC this condition reads $(d/J)^2 \omega t \geq \pi$. For $d/J = \text{const}$, $\omega = \pi$ and the interference fringes persist for any $d \geq 1$ provided $\beta t/\hbar < \pi$.

The bimolecular type of collision considered here cannot be experimentally studied by the methods of femtochemistry used to monitor unimolecular reactions. However, it has been demonstrated that measurements of collision cross sections with pure energy and angle resolutions allow one to extract $P(t, \theta)$, i.e. to obtain information concerning the underlying reaction dynamics equivalent to that obtained using real–time methods in femtochemistry. It follows that the SCs predicted here (Fig. II) can be tested experimentally.

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