Simple basic model for concrete and its application
Part 3. Factors affecting consistency, material balance equations and mix design

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Abstract
During the plant observations a number of interesting questions arose concerning consistency. The manufacturing plants (or the sites where pre-mixed concrete is transported from) are the ideal place for carrying out investigations to gather a wealth of valuable data. Nevertheless, the statistical characteristics for consistency in no way demonstrate the same kind of close correlation as those for compressive strength or deformation, so there is justification in being cautious, even though being cautious should never prevent us from drawing certain conclusions. When designing the composition of fresh concrete mixes, it may be useful to incorporate the (structural) content indicators of the concrete composition, provided that we know (or conduct experiments to find out) the correlations between the concrete composition content indicators and the properties of the fresh and hardened concrete, and apply the material balance equations of concrete mixes. The final part in this series of papers focuses on these issues.

1. A brief review of the literature

A knowledge of the factors that affect the consistency of fresh concrete is important for conscious planning of workability, and research has therefore long been conducted into this area. The work of Alexanderson is especially interesting because he has examined the different composition conditions under which constant consistency (consistency of the same class) can be provided, across a very broad range of mixes, from paste (mortar) mixes to concrete mixes [1]. The diagrams he published (see Figs. 27 and 28) express the principle that consistency could only remain constant when dry aggregates are added to a given cement paste of a known consistency if the water-cement ratio is simultaneously increased. The volumetric ratio of the aggregates in the concrete, however, can only be increased to a maximum limit, which also depends on consistency, roughly in line with Fig. 28. Alexanderson is remarkable in that he not only considers the air content of pastes (admixtures which form bubbles of air) and the impact they have on consistency, but he also calculates with it.

Fig. 27. Relationship between the percentage of aggregate by volume and the water/cement ratio at constant consistency, after Alexanderson [1]

In Hungary, the work of Ujhelyi in mix design accounting for consistency is outstanding: his DSc thesis of 1989 [2] not only gives a thorough summary of the methods for estimating water demands, but also describes a new method for calculating consistency-dependent water demands. The method is suitable for the design of consistency of mixes without admixtures. In the proposed formula the water demands of aggregates depend on the fineness modulus, and the water demands of cement depend on the specific surface area. The latter predicts the so-called surface aspect: it is noticed that not only the masses and volumes of the set of solid phase components are included in concrete mixes, but also their surface areas, and the surface areas may have particular influences on the properties of the concrete.

Design of consistency is complicated due to the presence of cements and additions which have high specific surface areas and by the effects of high range water reducing admixtures - which can produce special rheological phenomena. This is the reason behind the ever increasing number of articles published in the last decade that deal with observations of the rheological behaviour of cementitious mixes. In Hungary, Spránitz and his associates have examined the flow time and flow value of pastes made by different
cements and admixtures, and have found numerical correlations between the rheological properties and the composition properties [3,4]. Their work was consciously directed towards the investigation of pastes, since (as Alexander also recognised) the rheological behaviour of concrete is determined to a great extent by the properties of the paste.

A source of inspiration for future research in Hungary could be provided by the report of Toutou and Roussel [5]. The authors examined the rheological behaviour of concrete constituents in four cumulative stages: firstly in suspensions (water + silica fume), secondly in pastes (water + silica fume + cement), thirdly in mortars (water + silica fume + cement + sand), and fourthly in concretes (water + silica fume + cement + sand + gravel). The rheological properties of all four types of mixes were investigated, and they reported interesting data concerning the correlations between shear stress and shear rate, and between yield stress and combined volumetric ratios of solid phase constituents. It would be worth pursuing this direction of research in Hungary too; it would not require unaffordable costs, but mainly systematic and perseverant work. The outcome would be more reliable mix design for concrete workability, particularly in the case of high performance concretes, which are more sensitive from a rheological point of view.

2. Factors influencing consistency according to the simple basic model for concrete

In the simple basic model, the influence of concrete composition content indicators is also a subject for analysis with regard to consistency and the rheological properties of fresh concrete (p: ratio of paste in the concrete, x: volumetric ratio of liquid to paste powder, Χ: volumetric ratio of cement in the paste powder, λ_AD: volumetric ratio of admixtures to paste powder, l: volumetric ratio of air in the concrete), although the physical properties of concrete constituents must be considered at all times. For example, one such important influencing factor is the volumetric specific surface area of the combined solid phase matter (paste powder and aggregate particles) making up the concrete, which is calculated by the method described in [6]; which is essentially the procedure proposed by Kausay [7], with the difference that as standard the particles are considered to be not spherical, but ellipsoid in shape. The role of the surface area of materials included in concrete mixes is obvious, because during the flow of the concrete, the combined solid particles have a mutual effect on each other through their surface areas, and also have an effect on the fluid (water, which may also contain admixtures) that plays the role of dispersing agent.

Concrete is considered from a rheological point of view to be a self-affine (fractal-like) composite system, which manifests itself in the combined solid phase of constituents in dimensions of different scales showing a repeating structure. The cement + addition powder as dispersed phase plays the same role in paste as it is played by aggregate in concrete. Also, the fluid (mostly water), which is the dispersing agent in the paste plays the same role as it is played by the paste in concrete. Paste acts as the dispersing agent for the aggregate as the dispersed phase. The only substantial difference between the aforementioned dispersed phases lies in the 2 order of magnitude difference between their particle sizes, but the structure in the different magnitude scales is continuously repeated, which can presumably be traced back to the exponential distribution of the particle sizes of the combined solid phase constituents of concrete.

If the surface of the powder particles of the paste are covered in a thick layer of fluid (but naturally not so thick that they can bleed) then the paste can be expected to flow better than if the layer of fluid covering the particles is thinner. The same large-scale range of dimensions is still valid: if the surface of the aggregate is covered by a thick layer of paste (acting almost as a fluid) then the fresh concrete mix can be expected to flow better than in the case where the aggregate gets a thin layer of paste. From the point of view of fluidity, the consistency of the neat paste serves as the maximum threshold value (in this case the paste thickness is infinite), since when combined solid phase matter is added to the paste (either aggregate or paste powder) then the added surplus surface area dilutes the thickness of the dispersing agent covering the dispersed particles, causing a decreased fluidity of the mix.

The influence of water-reducing admixtures (λ_AD > 0) – following the above idea – can also be interpreted as diluting the thicknesses of the dispersing agent which would be necessary to achieve a given consistency in mixes without admixtures in every range of dimensions of the dispersed phase particles. In the case of the 132 batches of plastic consistency concrete mixes investigated during the plant observations detailed in [8] (of which 72 were made without admixtures, and 60 were made by water-reducing admixture), the average thickness of the paste (as dispersing agent) reaching the aggregates within the range of dimensions 0.063–32 mm was, significantly, several μm thinner for mixes with admixtures than for mixes without admixtures. This paste diluting effect can actually represent savings in paste between 10 to 30 litres (depending on the type and characteristics of the aggregate) when applied to each cubic meter of mix.

If the mix composition of the concrete is known, then the thickness of a dispersing agent covering a d sized particle in the dispersed phase can be calculated simply by Eqs. (37) and (38):

\[ t_s = \frac{1}{2} \cdot d_s \left( 1 + \frac{1}{a} \right) \left( 1 - \frac{1}{1 + \frac{a}{x}} \right) \]  

(37)

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(38)

where:

- \( t_s \) is the thickness [mm] of the layer of paste covering the aggregate particles of diameter \( d_s \) [mm],
- \( t_s \) is the thickness [μm] of the layer of fluid (water) covering the paste powder particles of diameter \( d_s \) [μm],
- \( p \) is the volumetric ratio of the paste in the concrete (concrete composition content indicator),
- \( a \) is the volumetric ratio of aggregate in the concrete (a>0), and
- \( x \) is the volumetric ratio of fluid-paste powder.
Table 11. Concrete composition content indicators and other data for mixes subjected to consistency examinations during plant observations.

| Admixture code name | Number of observations | \( p \) | \( x \) | \( \chi_p \) | \( \lambda_{\text{avg}} \) | \( \lambda_{\text{max}} \) | Measured flow values [mm] | Factors for Eq. (39) |
|---------------------|-------------------------|--------|--------|---------|---------|---------|--------------------------|---------------------|
| without admixture   | 113                     | 0.153  | 0.302  | 1.254   | 3.381   | 0.000   | 300 470                  | 1 1                 |
| HRWR                | 49                      | 0.220  | 0.267  | 1.287   | 1.675   | 0.010   | 390 550                  | 14.256 1.004        |
| MRWR                | 10                      | 0.212  | 0.314  | 0.924   | 1.872   | 0.008   | 340 590                  | 8.868 1.012         |
| WR-1                | 28                      | 0.214  | 0.285  | 1.076   | 1.975   | 0.012   | 380 480                  | 4.529 1.012         |
| WR-2                | 31                      | 0.217  | 0.290  | 1.297   | 1.876   | 0.006   | 350 480                  | 4.846 1.017         |
| WR-3                | 11                      | 0.240  | 0.263  | 1.215   | 1.850   | 0.012   | 390 480                  | 4.042 1.004         |

Table 12. Data concerning the concrete composition content indicators and consistency values for mixes made with and without different types of admixture, and the factors \( f_{1,\text{AD}} \) and \( f_{2,\text{AD}} \) of Eq. (39).

| Admixture code name | p - paste volumetric ratio \( (\%\) | x - fluid-powder volumetric ratio \( (\%\) | \( \lambda_{\text{min}} \) | \( \lambda_{\text{avg}} \) | \( \lambda_{\text{max}} \) | Measured flow values [mm] | Factors for Eq. (39) |
|---------------------|-------------------------------------|-------------------------------------|---------------------|---------------------|---------------------|--------------------------|---------------------|
| High range water reducer (HRWR) | 0.335 | 0.652 | 4 | | |
| Mid range water reducer (MRWR) | 1.218 | 0.790 | 32 | | |
| 3 types of water reducers (WR) | 0.685 | 0.728 | 24 | | |

3. Results and evaluation of plant observations

3.1. General data

Consistency was measured by the flow table test acc. to EN 12350-5 within 15±5 minutes of the start of mixing the concrete. The compositional properties of the mixes examined were hardly altered from those described in Table 5 of [6], although the database had been updated with the data from new mixes. The current data is summarised in Table 11. More detailed data on the concrete composition content indicators of the mixes prepared with admixtures are summarised in Table 12. It can be realised from the data in Table 12 that the mixing plant produced an overwhelming majority of concrete mixes of F2 and F3 consistency according to EN 206-1, according to its customer orders.

A general presentation of the data would be incomplete without showing the range limits of the variations in particle distributions of the mixes of sand and gravel fractions. The distribution curves of all aggregate mixes have been condensed into the single diagram in Fig. 29, which shows the distribution according to the surface area pass calculated for the unit of volume rather than the volume or mass percentage pass. It can be seen that there are considerable differences in the volumetric specific surface areas over 0.5 mm particle size, changing between 4000-6000 m²/m³. The influence of the volumetric specific surface area is, therefore, expected to be a major factor when considering the consistency of the mixes.
3.2. Relationship between consistency and concrete composition content indicators

In processing the results of the observations, correlations were initially sought in the data of mixes made without admixtures, and the basic equations derived from the analysis were later used for compositions containing admixtures to calculate factors that expressed the effects of the admixtures. The equation that was found to match most closely the observed measured results is shown in Eq. (39).

\[
\text{Flow} = \left(1 + f_{1,\text{AD}} \cdot \lambda_{\text{AD}}\right) \cdot f_{2,\text{AD}} \cdot A \cdot \left(1 + \frac{a \cdot f_z}{z \cdot f_a} \right)^{n_x} \left(1 + \frac{a \cdot f_z}{z \cdot f_a} \right)^{n_f} \left(1 + \frac{a \cdot f_z}{z \cdot f_a} \right)^{n_a} \quad (39)
\]

where:
- Flow is the estimated flow value [mm] of the concrete mix by flow table,
- \(\lambda_{\text{AD}}\) is the dose of admixture in the volumetric ratio of paste powder (concrete composition content indicator),
- \(f_{1,\text{AD}}\) is the dose factor of the admixture (for specific values of the observed admixtures, see Table 12),
- \(f_{2,\text{AD}}\) is the factor typical of the material of the admixture (for specific values of the observed admixtures, see Table 12),
- \(A\) is the experimental constant, in this case \(A=39386.8\),
- \(n_x, n_f, n_a\) are exponents, with observed values as follows: \(n_x=0.288, n_f=1.208\) and \(n_a=0.319\),
- \(f_z\) is the volumetric specific surface area of the paste powder \(\text{[m}^2/\text{m}^3]\),
- \(f_a\) is the volumetric specific surface area of the aggregate \(\text{[m}^2/\text{m}^3]\),
- \(z\) is the volumetric ratio of paste powder in the concrete,
- \(a\) is the volumetric ratio of aggregate in the concrete, and
- the other notations are the same as for Eqs. (37) and (38).

The correlation is quite weak \((R^2 \approx 0.29)\), which is not surprising in the case of consistency, but the significance of the equation’s multiplication factor was proved with a probability of \(p=0.06\). The confidence interval for the flow values obtained from estimates was ± 88 mm at a probability level of 95% and ± 36 mm at a probability level of 67%. An interesting feature of the relationship is that when \(a=0\) (that is, for pure paste) it assumes the maximum value, which complies with the principles defined by Alexanderson. What is new is that it also takes into account the influence of the dose of admixture \(\lambda_{\text{AD}}\), as well as its effectiveness \((f_{1,\text{AD}})\) and the influence of its type \((f_{2,\text{AD}})\). For the latter, a value that very closely approaches unity was obtained during the plant observations (see Table 12).

Where mixes without admixture are concerned, the equation gives the estimated flow values for mixes with only water added, since \(f_{1,\text{AD}} = f_{2,\text{AD}} = 1\) if \(\lambda_{\text{AD}}=0\).

The flow values estimated from Eq. (39) must be regarded as expected values, which can assume any value within the confidence interval given above, but the calculated values are the most probable (expected) values. A few illustrations from processing the results are shown in Figs. 30 to 33. It can be realised from Fig. 33 that for a given w/c range and paste...
content, a high dosage of the medium range admixture may perform almost identically to a medium dosage of high range admixture. Increasing the dosage of admixture within a certain limit is a more significant influence than increasing the w/c ratio.

4. Material balance equations of concrete mixes

According to the definition given in the EN 206-1 European Standard concrete is a “material formed by mixing cement, coarse and fine aggregate and water, with or without the incorporation of admixtures, additions or fibres, which develops its properties by hydration” [9]. In addition to this definition – taking account of recent developments in the field of concrete engineering – the simple basic model for concrete mixes also regards concrete in its fresh state as a macroscopically heterogeneous composite system consisting of solid phase (aggregates), liquid phase (paste) and gaseous phase (air), in which the paste is itself a heterogeneous system, likewise consisting of three separate phases, which are:

- solid phase paste powder (cement, as a hydraulic bonding agent, perhaps together with (a mixture of) fine-grain additions, which are inert or have pozollanic reactivity, latent hydraulic reactivity, etc.),
- liquid phase water (which may also contain one or more dissolved admixtures) and
gaseous phase air (which may be intentionally added air bubbles by air entraining admixture, or just randomly remaining voids due to incomplete compaction).

\[ A \ \text{V}_{\text{concrete}} \ (\text{m}^3) \] volume of (fresh) concrete mix generally consists of five main concrete constituent components, while certain concrete constituents may themselves be made up of mixtures of sub-constituents (in known proportions):

- \( K_{\text{virt},i} \ [\text{kg}] \): total additions; \( K_{\text{virt},i} = \sum (\alpha_{M,AGvirt,j}K_{i,j}) \), where \( \alpha_{M,AGvirt,j} \) is the mass ratio of the \( j \)-th AGvirt component, and where \( 0 \leq \alpha_{M,AGvirt,j} \leq 1 \) and \( \sum \alpha_{M,AGvirt,j} = 1 \),
- \( c \ [\text{kg}] \): quantity of cement added,
- \( \alpha_{M,AGvirt,j} \ [\text{kg}] \): the volume of water added when blending the concrete mix, \( AD \ [\text{kg}] \): total admixtures; \( AD = \sum(\alpha_{M,AD,k}AD) \), where \( \alpha_{M,AD,k} \) is the mass ratio of the \( k \)-th AD component, and where \( 0 \leq \alpha_{M,AD,k} \leq 1 \) and \( \sum \alpha_{M,AD,k} = 1 \).

\[ \begin{align*}
\sum \alpha_{M,AGvirt,j} & = 1, \\
\sum \alpha_{M,AD,k} & = 1, \\
\sum \alpha_{M,AD,k} & = 1.
\end{align*} \]

Note: the “virt” (virtual) symbol used above expresses that parts belonging to the different phases of the concrete are (or may be) present in the concrete constituent components. For example, the part of the addition material which is larger than 0.063 mm is included in the solid phase of the aggregate, while the part of the aggregate which is smaller than 0.063 mm is included in the fluid paste as paste powder. The water which adheres to the surface of the aggregate, and which in many cases is not negligible, is also included in the paste – as a liquid – so in addition to the \( W_{\text{virt}} \) added water, this must also be taken into consideration in the liquid phase of the concrete.

The structural composition of concrete mixes described by the mass data above may also be basically described using five dimensionless ratios:

- \( p \): the volumetric ratio of paste in the concrete \((0 < p \leq 1)\),
- \( x \): the volumetric ratio of the (free) liquid and the paste powder in the paste \((x > 0)\), but generally in practice: \(-0.6 \leq x \leq -3.6)\),
- \( k \): the volumetric ratio of air (void) in the concrete \((0 \leq k \leq 1)\), generally in practice: \(-0.010 \leq k \leq -0.060 \ldots -0.120\ldots)\),
- \( \chi_c \): the volumetric ratio of cement in the paste powder \((0 < \chi_c \leq 1)\), if the paste is pure cement \( \chi_c = 1 \),
- \( \lambda_{AD} \): the combined volumetric ratio of the admixture compared with the paste powder, (where the \( \lambda_{AD} \) ratios of individual – different effect – admixture components are \( \lambda_{AD} \geq 0 \), where the concrete mix contains no admixture \( \lambda_{AD} = 0 \); in general, however, \( \lambda_{AD} = -0.005 \ldots -0.050 \), and is therefore a small value).

The correlations between the concrete composition content indicators and the amounts of the given concrete constituents with known physical properties are defined by the system of linear equations in the matrix equation shown in Eq. (40), where:

- \( \psi_{AD,j} \): the part of the \( j \)-th addition component which is smaller than 0.063 mm, as mass ratio,
- \( \rho_{\text{AD,j}} [\text{kg/m}^3] \): the particle density of the \( j \)-th AD component,
- \( \varphi_{AD,j} \): the part of the \( j \)-th aggregate component which is smaller than 0.063 mm, as mass ratio,
- \( W_{\text{virt}} \) the moisture content of the \( j \)-th aggregate component as mass ratio,
of components. The matrix factor that may be regarded as known in the product on the left hand side of the equation (materials property matrix) contains the physical properties and compositional proportions (e.g.: fraction ratios) of the concrete constituent materials, while the vector factor of the product (mix composition vector) expresses the masses of the concrete constituents which are to be added - and which may be regarded as unknown during mix design. The so-called structural vector on the right hand side of the equation contains the volumes of paste powder, paste fluid and aggregates in a designed volume of concrete, as well as the volume of cement and the volume of the admixtures, respectively. The structural vector directly depends on the targeted concrete composition content indicators, which are either already known during mix design, or are determined on the basis of prior knowledge of particular correlations.

The mix composition vector and the structural vector mutually define each other in a clear way, so the design of the composition of concrete mixes is possible by solving the matrix equation of Eq. (40). During concrete mix design for a volume $V_{\text{concrete}}$, the first step is to determine the concrete composition content indicators, from which the structural vector on the right hand side of the matrix equation of Eq. (40) can be obtained. The matrix equation of Eq. (40) can be set up by the materials property matrix that contains the material properties and particular component proportions. The solution of the system of linear equations provides the mix composition vector, which produces, as a direct result, the quantities of the concrete constituents that are to be added to the targeted concrete mix with a volume $V_{\text{concrete}}$.

It should be noted that Eq. (40) (and therefore the concrete mix) may only be clearly defined if all the concrete composition content indicators are known. It also follows that fully comprehensive consequences can in theory only be drawn from observations related to concrete mixes, if the influence of each

5. Using the simple basic model to design the composition of concrete mixes to meet given criteria

Several concrete mix design methods are available which meet pre-defined criteria. The common principle behind the different methods is to search for compositions which meet the criteria by counting back from the design criteria (e.g. compressive strength and consistency), which assumes that accepted correlations are available, as is the case with the Palotás-Bolomey [10,11] and Ujhelyi [2] methods used widespread in Hungary for concretes without admixtures.

New correlations are necessary for concrete mix design with admixtures. The simple basic model for concrete mixes provides this opportunity: the use of material balance equations is included, and greater flexibility in applying correlations with a restricted range of validity in connection with the influences of factors is offered; that are developed through experiments.

In the followings, a specific example illustrates the essence of the method.

6. Example for concrete mix design

The present example details the steps of concrete mix design by the simple basic model for a concrete mix with designation C30/37- XC3-16-F4-MSZ 4798-1:2004.

6.1 Identifying and determining requirements

Compressive strength: The concrete in the example needs to have a characteristic compressive strength of $f_{\text{ck, cube}}$ according to MSZ EN 206-1 standard that lies between the minimum values for compressive strength prescribed for compressive strength classes C30/37 and C35/45, i.e. $37 \text{ N/mm}^2 \leq f_{\text{ck, cube}} <45 \text{ N/mm}^2$. It is possible to ensure the characteristic compressive strength in continuous production if the mean compressive strength of the concrete meets the condition of $f_{\text{m, mean}} \geq f_{\text{ck, cube}} +1.485\sigma$, where $\sigma \geq 3 \text{ N/mm}^2$ is acceptable in the example. The estimate for the standard deviation of a population is recorded at 10% of the expected mean compressive strength of the concrete in the example, and the range of values for characteristic compressive strength is narrowed by 10% at both the upper and lower limits for safety reasons (which results $37.8 \text{ N/mm}^2 \leq f_{\text{ck, cube}} <44.2 \text{ N/mm}^2$). The achieved result safely fulfil the given conditions for compressive strength if the mean compressive strength lies in the range $44.4 \text{ N/mm}^2 \leq f_{\text{m, mean}} <51.9 \text{ N/mm}^2$.

Consistency: The flow value [mm] prescribed for consistency class F4 according to MSZ EN 206-1 standard lies in the range $490 \text{ mm} \leq \text{Flow} \leq 550 \text{ mm}$. In view of the uncertainties in estimating consistency, the range midpoint of Flow=520 mm has been taken as the criterion.

Composition criteria: The recommendations according to MSZ EN 206-1 standard for exposure class XC3 are w/c:0.55 and $c \geq 280 \text{ kg/m}^3$. Further requirements in Hungary are the minimum body density of fresh concrete of 2380 kg/m$^3$ and the minimum body density of hardened concrete of 2310 kg/m$^3$. 

ρ_{AG,j, fine}[kg/m$^3$]: the dried particle density of the part of the $j$th aggregate component which is smaller than 0.063 mm,

ρ_{AG,j}[kg/m$^3$]: the density of the $k$th admixture component,

$\Delta_{i}$: the dry content of the $k$th addition component, as mass ratio,

$\rho_{w}$[kg/m$^3$]: water density,

sw_{k,j}; (short-term) water absorption of the $i$th addition component, as mass, ratio,

sw_{AD,j}; (short-term) water absorption of cement, as mass ratio,

$\lambda_{AD,j}$: the ratio of the volume of the $k$th aggregate component which is larger than 0.063 mm, compared with the volume of the paste powder, the other notations are the same as previously explained.

The system of linear equations presented in the matrix equation of Eq. (40) is fundamental in the mix design, or are only clarified if it is worked in in the product on the left hand side of the equation (materials property matrix) contains the physical properties and compositional proportions (e.g.: fraction ratios) of the concrete constituent materials, while the vector factor of the product (mix composition vector) expresses the masses of the concrete constituents which are to be added - and which may be regarded as unknown during mix design. The so-called structural vector on the right hand side of the equation contains the volumes of paste powder, paste fluid and aggregates in a designed volume of concrete, as well as the volume of cement and the volume of the admixtures, respectively. The structural vector directly depends on the targeted concrete composition content indicators, which are either already known during mix design, or are determined on the basis of prior knowledge of particular correlations.

The mix composition vector and the structural vector mutually define each other in a clear way, so the design of the composition of concrete mixes is possible by solving the matrix equation of Eq. (40). During concrete mix design for a volume $V_{\text{concrete}}$, the first step is to determine the concrete composition content indicators, from which the structural vector on the right hand side of the matrix equation of Eq. (40) can be obtained. The matrix equation of Eq. (40) can be set up by the materials property matrix that contains the material properties and particular component proportions. The solution of the system of linear equations provides the mix composition vector, which produces, as a direct result, the quantities of the concrete constituents that are to be added to the targeted concrete mix with a volume $V_{\text{concrete}}$.

It should be noted that Eq. (40) (and therefore the concrete mix) may only be clearly defined if all the concrete composition content indicators are known. It also follows that fully comprehensive consequences can in theory only be drawn from observations related to concrete mixes, if the influence of each concrete composition content indicator has been taken into account and evaluated.
Table 13. Data of the concrete constituents that are available for concrete with designation C30/37-XC3-16-F4-MSZ 4798-1:2004, which is to be designed in the example
(note: sz refers to dry content of the MRWR admixture)
13. táblázat  A példa szerint tervezendő C30/37-XC3-16-F4-MSZ 4798-1:2004 jelű betonhoz rendelkezésre álló betonalkotók adatai

Table 14. Concrete composition content indicators and other data of CEM II/A-M (V-LL) 42.5 N mixes evaluated during industrial observations (a* : volumetric ratio of the aggregate in the concrete mix)
14. táblázat  Az üzemi megfigyelések során értékelésbe vont CEM II/A-M (V-LL) 42.5 N keverékek betonösszetételi állapotjelzői és néhány más adata
(a*: volumetric ratio of the aggregate in the concrete mix)

Table 15. Data on the concrete composition content indicators and consistency measurements of mixes made by mid-range water-reducing MRWR admixture
15. táblázat  Az erős hatási adalékkéről (MRWR) készített keverékek betonösszetételi állapotjelzői és konzisztencia mértékzámainak adatai, valamint a konszisztenciabeclő (42) képlet f1,AD és f2,AD faktorai

Table 16. The composition calculations performed on the basis of the example criteria, where f... are the assumed design criteria (target parameters), and x, λ are calculated content indicators dependent on the preceding. The Table contains the recipes derived from solving the matrix equation of Eq. (40), mixing instructions (for wet aggregate components) and a control column for the exposure requirements
16. táblázat  A példa szerinti kritériumok alapján végzett összetétel-számítások, ahol az f... és Flow... feltéve tevékenység kritériumok (előírások) és az x és λ kalkulált tartalék jelölési határok. A táblázat tartalmazza az (40) egyenlőtrendező megoldásából kapott receptúrákat, keverési utasításokat (nadrág alakulási-komponensek) és egy ellenőrző blokkat a készítés követelményekre

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13. táblázat  A példa szerint tervezendő C30/37-XC3-16-F4-MSZ 4798-1:2004 jelű betonhoz rendelkezésre álló betonalkotók adatai

| marking | name            | marking value | mass ratios in constituents | volumetric specific surface [m³/m³] | fine-content (<0.063 mm) marking | moisture content | absorption of water marking | table 16. the composition calculations performed on the basis of the example criteria, where fcm,28,exp and flowexp are the assumed design criteria (target parameters), and x, λ are calculated content indicators dependent on the preceding. the table contains the recipes derived from solving the matrix equation of eq. (40), mixing instructions (for wet aggregate components) and a control column for the exposure requirements
|标记 | 名称 | 标记值 | 成分的比例 | 体积特定表面积 [m³/m³] | <0.063 mm 标记 | 水分含量 | 吸水性 | 16. 的计算方法根据例子标准，其中 fcm,28,exp 和 flowexp 是假设设计标准（目标参数），x, λ 是计算成分指标依赖于前一个。表包含由解矩阵方程（40）得出的配方，湿成分的混合指令和一个控制列用于暴露要求。
6.2 Identifying the concrete constituent materials and determining their properties

The example does not take the quantity of evaporating water into consideration, so \( \Delta W = 0 \), and the other available materials and their properties are listed and presented in Table 13 and in Fig. 34.

For the experiential correlations used for consistence Eq. (39) applies, but replacing Flow with \( \text{Flow}_{\text{exp}} \) which is the expected mean flow value of the fresh concrete mix, see Eq. (42).

\[
\text{Flow}_{\text{exp}}[\text{mm}] = \left(1 + f_{1,\text{AD}} \lambda_{\text{AD}} \right) f_{2,\text{AD}} A \left( \frac{x^{n_x}}{f_r} \right)^{\left(1 + \frac{a}{z} \frac{f_r}{f_p}\right)} \left(1 + a \frac{f_r}{f_p}\right)^{n_x} \left(1 + \frac{a}{z} \frac{f_r}{f_p}\right)^{n_f} \left(1 + \frac{a}{z} \frac{f_r}{f_p}\right)^{n_a}.
\]  

(42)

where:

\( \text{Flow}_{\text{exp}} \) is the expected mean flow value [mm] of the concrete mix by flow table,

\( \lambda_{\text{AD}} \) is the dose of admixture in the volumetric ratio of paste powder (concrete composition content indicator),

\( f_{1,\text{AD}} \) is the dose factor of the admixture (for specific values of MRWR see Table 15),

\( f_{2,\text{AD}} \) is the factor typical of the material of the admixture (for specific values of MRWR see Table 15),

\( A \) is the experimental constant, in this case \( A = 39386.8 \),

\( n_x, n_f \) and \( n_a \) are exponents, with observed values as follows:

\( n_x = 0.288 \), \( n_f = 1.208 \) and \( n_a = 0.319 \).

\( f_r \) is the volumetric specific surface area of the paste powder \([\text{m}^2/\text{m}^3]\), in this case \( f_r = 1.711 \),

\( f_p \) is the volumetric specific surface area of the aggregate \([\text{m}^2/\text{m}^3]\), in this case \( f_p = 1.6047 \cdot \chi_c \).

6.3 Identifying the correlations related to the design criteria as target parameters

Instead of searching for new, universally valid correlations, the simple basic model for concrete mixes preferably relies on local correlations, obtained through experience from controlled experimental data, whose validity is restricted to the materials used in the given experiment and their range of interpretation. Such experimental correlations may be obtained at any concrete mixing plant where industrial production is controlled under supervision by trained professional concrete technicians.

The correlations obtained through experience are used here that relate to compressive strength and to consistency. Validity extends to the concrete constituents identified in paragraph 6.2. 46 observations are available for the compressive strength of mixes made from CEM II/A-M (V-LL) 42.5 N cement, and 10 observations are available for the consistence of mixes with MRWR admixture (see Tables 14 and 15).

For the experiential correlations used for compressive strength Eq. (33) applies, but replacing \( f_{cm,28} \) with \( f_{cm,28,\text{exp}} \) which is the expected mean compressive strength of concrete at the age of 28 days, see Eq. (41).

\[
f_{cm,28,\text{exp}} = A \cdot \chi_c^{n_x} \cdot p^{n_p} \cdot (1 + x)^{n_x} \cdot (1 + l)^{n_l}.
\]  

(41)

where:

\( f_{cm,28,\text{exp}} \) [N/mm\(^2\)] is the target mean compressive strength measured on cube specimens after 28 days standard curing,

\( A \) is the experiment constant, for CEM II/A-M (V-LL) \( A = 342.302 \),

\( \chi_c \) is the cement volumetric ratio in the paste powder, \( n_x \) is the exponent of \( \chi_c \), for CEM II/A-M (V-LL) \( n_x = 1.711 \),

\( p \) is the paste volumetric ratio in the concrete, \( n_x \) is the exponent of \( p \), for CEM II/A-M (V-LL) \( n_x = 0.240 \),

\( x \) is the liquid-paste volume factor in the paste, \( n_x \) is the exponent of \( (1+x) \), for CEM II/A-M (V-LL) \( n_x = 2.355 \).

6.4 Determining the concrete composition content indicators required to fulfil the criteria, and determining the possible mix compositions by solving the system of material balance equations

Concrete mixes are determined by five linearly independent content indicators, but there are two estimate formulae, Eqs. (41) and (42), and furthermore, one of the content indicators is assumed to have a value of l=0.020 (therefore with an air content of 20 litres/m\(^3\)), so the number of concrete composition content indicators that may be freely selected is reduced to two, and even between \( x \) and \( \chi_c \) there is the restriction given by Eq. (44). It is practical to freely select the values of \( p \) and \( \chi_c \), and to calculate \( x \) from the interactions of Eqs. (41), (42) and (44) – the possible content indicators \( x \) and \( \chi_c \). After the five content indicators have been determined in this way, they also provide the structural vector.
on the right hand side of Eq. (40), from which the matrix equation can be solved to find the mix composition vector. The calculations in the example have been performed for paste contents of 280 and 300 l/m³ (p=0.280 and p=0.300) and for addition contents of almost zero and 15 % (\(\chi_p=0.850\) and \(\chi_p=0.972\ldots0.976\)) and the results of the calculations are presented in Table 16.

6.5 Discussion

Of the mix compositions presented in Table 16, mixes no. 1, 6 and 7 fulfil all the requirements, including those for body density – which are too strict in the opinion of the author of the present paper. The data in Table 16 show that there is not much to be gained by increasing the paste content in a given concrete and by targeting the upper limit within the strength class, as this involves an increase in the dose of cement, and furthermore the dose of water reducing admixture also needs to be increased. The mix that best meets the requirements is no. 1, in the opinion of the author of the present paper, even though mix no. 1 also has the blemish in that according to Table 15, the industrial observations were in the range \(\lambda_{AD}<0.030\), while here one can see \(\lambda_{AD}=0.034\), which is an extrapolated value. It would be possible to make adjustments by amending the flow value criterion, but it is not worthwhile, as even if \(\lambda_{AD}=0.030\), the estimated flow value is \(\pm 508\) mm, which is a negligible difference to the designed 520 mm.

7. Present and future tasks in concrete engineering

Concrete engineering in the last few decades has witnessed the appearance of newer and newer types of cements and admixtures, and it is impossible for standard methods to keep up with technical developments when it comes to describing the effects of the new materials. This necessitated a new systematic and harmonised examination of the effects of concrete constituents, in order to gain a deeper insight into their effects. This is not just in the interest of the cement producers or the companies that distribute the admixtures, but also in the interest of concrete mixing plants. Research is already underway in many institutes separately. Author of the present paper has no doubt about the commitment of ÉMI Nonprofit Ltd in this sense: the research is ongoing in cooperation with scientific institutions and with smaller and larger concrete mixing plants, which have the objective of optimising concrete compositions and of improving the advancement of the technical culture in concrete mixing plants.

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Betonkeverékek egyszerűsített alapmodellje és alkalmazása

3. rész: Konzisztcenciát befolyásoló tényezők, anyagmérleg egyenletek, összetételek tervezése

A gyártóüzemi megfigyelések során a konzisztcenciára vonatkozóan is számos érdekes kérdés merült fel. A szerző véleménye szerint igaúz értékelt adathalmaz éppen a gyártóüzemekben (illetve a transzportbeton kiszállítási helyszínén) végzett vizsgálatokkal nyerhetünk. Mindazonál- tak és a friss és megszilárdult betonkeverékek összetételének ter- vezése során is, amennyiben ismerjük, vagy kísérleti úton meghatározók a betonkeverékek összetételein ter- vezése során is, amennyiben ismerjük, vagy kísérleti úton meghatározók a betonkeverékek összetételein ter- vezése során is, amennyiben ismerjük, vagy kísérleti úton meghatározók a betonkeverékek összetételein ter- vezése során is, amennyiben ismerjük, vagy kísérleti úton meghatározók a betonkeverékek összetételein ter-