Reconstruction of a Deceleration Parameter from the Latest Type Ia Supernovae Gold Dataset

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In this paper, a parameterized deceleration parameter \( q(z) = 1/2 - a/(1 + z)^b \) is reconstructed from the latest type Ia supernovae gold dataset. It is found out that the transition redshift from decelerated expansion to accelerated expansion is at \( z_T = 0.35^{+0.14}_{-0.07} \) with 1σ confidence level in this parameterized deceleration parameter. And, the best fit values of parameters in 1σ errors are \( a = 1.56^{+0.99}_{-0.55} \) and \( b = 3.82^{+3.70}_{-2.27} \).

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I. INTRODUCTION

Recent observations of high redshift Type Ia Supernova indicate that our universe is undergoing accelerated expansion which is one of the biggest challenges in present cosmological research, now [1, 2, 3, 4, 5, 6]. Meanwhile, this suggestion is strongly confirmed by the observations from WMAP [7, 8, 9, 10] and Large Scale Structure survey [11]. To understand the late-time accelerated expansion of the universe, a large part of models are proposed by assuming the existence of an extra energy component with negative pressure and dominated at late time pushing the universe to accelerated expansion. In principle, a natural candidate for dark energy could be a small cosmological constant \( \Lambda \) which has the constant equation of state (EOS) \( w_\Lambda = -1 \). However, there exist serious theoretical problems: fine tuning and coincidence problems. To overcome the coincidence problem, the dynamic dark energy models are proposed, such as quintessence [12], phantom [13], quintom [14], k-essence [15], Chaplygin gas [16], holographic dark energy [17], etc., as alternative candidates.

Another approach to study the dark energy is by an almost model-independent way, i.e., the parameterized equation state of dark energy which is implemented by giving the concrete form of the equation of state of dark energy directly, such as \( w(z) = w_0 + w_1 z \) [18], \( w(z) = w_0 + w_1 (1 + z)^{-1} \) [19, 20], \( w(z) = w_0 + w_1 \ln(1 + z) \) [21], etc.. By this method, the evolution of dark energy with respect to the redshift \( z \) is explored, and it is found that the current constraints favors a dynamical dark energy, though the cosmological constant is not ruled out in 1σ region. But, the rapid changed dark energy, \( \left| \frac{dw}{dz} \right| \ll 1 \), is ruled out [22]. Also, the dark energy favors a quintom-like dark energy, i.e. crossing the cosmological constant boundary. In all, it is an effective method to rule out the dark energy models. As known, now the universe is dominated by dark energy and is undergoing accelerated expansion. However, in the past, the universe was dominated by dark matter and underwent a decelerated epoch. So, inspired by this idea, the parameterized decelerated parameter is present in almost model independent way by giving a concrete form of decelerated parameters which is positive in the past and changes into negative recently [22, 24, 25]. Moreover, it is interesting and important to know what is the transition time \( z_T \) from decelerated expansion to accelerated expansion. This is the main point of the paper to be explored.

The structure of this paper is as follows. In section II, a parameterized decelerated parameter is constrained by latest 182 Sne Ia data points compiled by Riess [23]. Section III is the conclusion.

II. RECONSTRUCTION OF DECELERATION PARAMETER

We consider a flat FRW cosmological model containing dark matter and dark energy with the metric

\[
    ds^2 = -dt^2 + a^2(t)dx^2.
\]
The Friedmann equation of the flat universe is written as

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{de}), \quad (2)$$

where, $H \equiv \dot{a}/a$ is the Hubble parameter, and its derivative with respect to $t$ is

$$\dot{H} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2, \quad (3)$$

which combined with the definition of the deceleration parameter

$$q(t) = -\frac{\ddot{a}}{aH^2}, \quad (4)$$

gives

$$\dot{H} = -(1 + q) H^2. \quad (5)$$

By using the relation $a_0/a = 1 + z$, the relation of $H$ and $q$, i.e., Eq. (5) can be written in its integration form

$$H(z) = H_0 \exp \left[ \int_0^z [1 + q(u)] d\ln(1 + u) \right], \quad (6)$$

where the subscript "0" denotes the current values of the variables. If the function of $q(z)$ is given, the evolution of the Hubble parameter is obtained. In this paper, we consider a parameterized deceleration parameter [22],

$$q(z) = 1/2 - a/(1 + z)^b, \quad (7)$$

where, $a, b$ are constants which can be determined from the current observational constraints. From Eq. (7), it can be seen that at the limit of $z \to \infty$, the decelerated parameter $q \to 1/2$ which is the value of decelerated parameter at dark matter dominated epoch. And, the current value of decelerated parameter is determined by $q_0 = 1/2 - a$. In the Eq. (7) form of decelerated parameter, the Hubble parameter is written in the form

$$H(z) = H_0 (1 + z)^{3/2} \exp \left[ a \left( (1 + z)^{-b} - 1 \right) /b \right] \quad (8)$$

From the explicit expression of Hubble parameter, it can be seen that this mechanism can also be tried as parametrization of Hubble parameter.

Now, we can constrain the model by the supernovae observations. We will use the latest released supernovae datasets to constrain the parameterized deceleration parameter Eq. (7). The Gold dataset contains 182 Sne Ia data [23] by discarding all Sne Ia with $z < 0.0233$ and all Sne Ia with quality='Silver'. The 182 datasets points are used to constrain our model. Constraint from Sne Ia can be obtained by fitting the distance modulus $\mu(z)$

$$\mu_{th}(z) = 5 \log_{10}(D_L(z)) + \mathcal{M}, \quad (9)$$

where, $D_L(z)$ is the Hubble free luminosity distance $H_0d_L(z)$ and

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')} \quad (10)$$

$$\mathcal{M} = M + 5 \log_{10} \left( \frac{H_0^{-1}}{Mpc} \right) + 25$$

$$= M - 5 \log_{10} h + 42.38, \quad (11)$$

where, $M$ is the absolute magnitude of the object (Sne Ia). With Sne Ia datasets, the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi^2_{\text{SneIa}}(p_s) = \sum_{i=1}^{N} \frac{(\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i))^2}{\sigma_i^2}, \quad (12)$$

where $N = 182$ for Gold dataset, $\mu_{\text{obs}}(z_i)$s are the modulus obtained from observations, $\sigma_i$ are the total uncertainty of the Sne Ia data.
TABLE I: The Fitting Result from 182 Gold Sne Ia, the best fit parameters $a$, $b$ and $z_T$ with 1σ errors.

| $\chi^2$ with 180 dof | $a$       | $b$       | $z_T$     |
|------------------------|-----------|-----------|-----------|
| 156.66                 | 1.56$^{+0.99}_{-0.53}$ | 3.82$^{+3.70}_{-2.27}$ | 0.35$^{+0.14}_{-0.07}$ |

FIG. 1: The contour plot of the parameter $a$ and $b$ with 1σ and 2σ confidence level.

In our reconstruction, measurement errors are considered by using the well-known error propagation equation for any $y(x_1, x_2, ..., x_n)$,

$$
\sigma^2(y) = \sum_{i=1}^{n} \left( \frac{\partial y}{\partial x_i} \right)_{x=\bar{x}}^2 \text{Cov}(x_i, x_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left( \frac{\partial y}{\partial x_i} \right)_{x=\bar{x}} \left( \frac{\partial y}{\partial x_j} \right)_{x=\bar{x}} \text{Cov}(x_i, x_j)
$$

(13)

is used extensively (see Ref. [26] for instance). For Ansatz Eq. (7), we obtain errors of the parameterized decelerated parameter. The evolution of the decelerated parameters $q(z)$ with 1σ error are plotted in Fig. 2.

### III. CONCLUSION

In this paper, by an almost model-independent way, we have used a parameterized decelerated parameter to obtain the transition time or redshift $z_T$ from decelerated expansion to accelerated expansion. It is found out that the best fit transition redshift $z_T$ is about $z_T = 0.35^{+0.14}_{-0.07}$ with 1σ error in this parameterized equation which is compatible with the result of Ref. [23]. Though, we also can derive the transition redshift from a giving equation of state of dark energy and an concrete dark energy models, they are much model dependent. So, we advocate the almost model-independent way to test and rule out some existent dark energy models.
FIG. 2: The evolution of decelerated parameter with respect to the redshift \( z \). The center solid lines is plotted with the best fit value, where the shadows denote the 1\( \sigma \) region.

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