Electromagnetic Moments of the Baryon Decuplet

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Abstract

We compute the leading contributions to the magnetic dipole and electric quadrupole moments of the baryon decuplet in chiral perturbation theory. The measured value for the magnetic moment of the $\Omega^-$ is used to determine the local counterterm for the magnetic moments. We compare the chiral perturbation theory predictions for the magnetic moments of the decuplet with those of the baryon octet and find reasonable agreement with the predictions of the large–$N_c$ limit of QCD. The leading contribution to the quadrupole moment of the $\Delta$ and other members of the decuplet comes from one–loop graphs. The pionic contribution is shown to be proportional to $I_z$ (and so will not contribute to the quadrupole moment of $I = 0$ nuclei), while the contribution from kaons has both isovector and isoscalar components. The chiral logarithmic enhancement of both pion and kaon loops has a coefficient that vanishes in the $SU(6)$ limit. The third allowed moment, the magnetic octupole, is shown to be dominated by a local counterterm with corrections arising at two loops. We briefly mention the strange counterparts of these moments.

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Static electromagnetic moments are a valuable tool for understanding internal structure. In nuclear physics, static moments played a crucial role in understanding the strong tensor interaction arising from one-pion exchange which lead to significant deviations from spherical symmetry in simple nuclei like the deuteron. In the same way, hadronic structure can be investigated using static moments. The magnetic moments of the octet baryons have been understood in the context of $SU(3)$ for many years \cite{1}. Leading, model independent corrections to these $SU(3)$ relations have been computed in chiral perturbation theory \cite{2}–\cite{3} and have been found to improve agreement with experimental data.

The electrostatic properties of the $\Delta$ and other members of the baryon decuplet have received little theoretical attention since experimental data has been scarce until recently. The $\Omega^-$ magnetic moment was found to be $\mu_{\Omega} = -1.94 \pm 0.17 \pm 0.14 \mu_N$ \cite{4}, and a recent measurement for the $\Delta^{++}$ moment using pion bremsstrahlung (a model dependent extraction) found $\mu_{\Delta^{++}} = 4.5 \pm 0.5 \mu_N$ \cite{5} (we note that this result is not used by the PDG in their estimate \cite{4}). The magnetic moments have been examined in the cloudy-bag model \cite{6}, in quark models \cite{4}–\cite{9}, in a Bethe-Salpeter model \cite{10}, in the Skyrme model \cite{11}–\cite{13}, using QCD sum rules \cite{14}, and also recently in quenched lattice gauge theory \cite{15}–\cite{16}. Our goal here is to understand the magnetic moments in a model-independent, systematic way, using chiral perturbation theory.

The decuplet baryons also have electric quadrupole and magnetic octupole moments. These moments have been studied recently using quenched lattice QCD \cite{15}–\cite{16}. In chiral perturbation theory, the pion one–loop graphs tend to dominate over either kaon one–loop graphs or the local counterterm because of the presence of the (calculable) chiral logarithm, $\log(M_\pi^2/\Lambda^2_\chi)$. This was seen in the calculation of the electric quadrupole matrix element for the decay $\Delta \to N\gamma$ \cite{17}–\cite{18}. For the decuplet quadrupole moments, however, we find that the pion one–loop contributions are proportional to the third component of isospin, with the result that baryons with $I_z = 0$ receive no contribution from the lowest order pion loop, and $I_z = \frac{1}{2}$ baryons receive approximately equal contributions from kaon and pion loops at lowest order. These are still formally dominant over the dimension six local counterterm for the quadrupole moment. We show that the octupole moment is dominated by a dimension seven local counterterm with corrections occurring first at two loops.

Heavy baryon chiral perturbation theory uses the chiral symmetry of QCD to construct an effective low–energy theory to describe the dynamics of the goldstone bosons associated with the spontaneous breaking of chiral symmetry. The baryons can be included in a consistent manner, as shown in Ref. \cite{19}. For a review of the simplifications available
in calculating with this formalism see Ref. [20]. The decuplet of resonances as explicit degrees of freedom has been shown to be important for most physical observables, and for consistency of the perturbative expansion [19][21]. Further, Ref. [22] shows that including the decuplet (in fact, the entire tower of $I = J$ baryons) is required in order for a low energy theory of pions and nucleons to be unitary in the large–$N_c$ limit of QCD (where $N_c$ is the number of colours). The decuplet appears with couplings to the pions satisfying a contracted $SU(2N_f)$ algebra (where $N_f$ is the number of light flavours in the theory). Ref. [23] shows that the corrections to the relations arising from this $SU(2N_f)$ symmetry occur first at order $1/N_c^2$.

The leading $SU(3)$ invariant local counterterm for the decuplet magnetic moment is given by a dimension five operator [3]

$$L_{M1}^{CT} = -i \frac{e}{M_N} \mu_c q_k T^\mu_{vk} T^\nu_{vk} F_{\mu\nu},$$

where $T^\mu$ is the decuplet field and $q_k$ is the charge of the $k$th baryon of the decuplet. We have normalised the coefficient of the operator so that the magnetic moment of the $k$th baryon is $q_k \mu_c$ nuclear magnetons. A simple tree-level fit to the magnetic moment of the $\Omega^-$ hyperon gives $\mu_c = 1.94 \pm 0.22 \mu_N$ (where we have added the systematic and statistical errors of $\mu_{\Omega}$ in quadrature). The leading corrections to the magnetic moments arise from the one-loop diagrams shown in fig. 1. A computation gives the following matrix elements:

$$\mathcal{M}^{TTT} = i \frac{e}{16\pi^2} \mathcal{H}^2 (\overline{T}_v \cdot k T_{v\mu} - \overline{T}_{v\mu} T_v \cdot k) A^\mu \frac{2}{3} \sum_i \frac{\alpha_i}{f_{M_i}^2} F(\Delta m_i, M_i)$$

and

$$\mathcal{M}^{TBT} = i \frac{e}{16\pi^2} \mathcal{C}^2 (\overline{T}_v \cdot k T_{v\mu} - \overline{T}_{v\mu} T_v \cdot k) A^\mu \sum_i \frac{\beta_i}{f_{M_i}^2} F(\Delta m_i, M_i),$$

where

$$F(\Delta m, M) = \Delta m \log \left( \frac{M^2}{\Lambda^2} \right) + \sqrt{\Delta m^2 - M^2} \log \left( \frac{\Delta m + \sqrt{\Delta m^2 - M^2 + i\epsilon}}{\Delta m - \sqrt{\Delta m^2 - M^2 + i\epsilon}} \right),$$

and $A^\mu$ is the electromagnetic gauge field. The superscripts $TTT$ and $TBT$ denote the contribution from graphs with intermediate decuplet and octet baryons respectively. The mass splitting between the external baryon and the baryon in the loop is $\Delta m_i$, the mass of the relevant pseudogoldstone boson is $M_i$ ($i = \pi$ or $K$), the chiral symmetry breaking scale is $\Lambda_\chi$, and the decay constant of the meson in the loop is $f_{M_i}$ ($f_\pi = 132\text{MeV}$ and
$f_K = 1.22 f_\pi$). The decuplet-octet-meson coupling constant is $C$ and the decuplet-decuplet-meson coupling constant is $H$. The constants $\alpha^i$ and $\beta^i$ are the product of the electric charge of octet meson $i$ and SU(3) Clebsch–Gordan coefficients (explicit values are given in the appendix). The one–loop corrections to the multipole moments depend only on the coupling constants $C$ and $H$. Using $C = -1.2 \pm 0.1$, $H = -2.2 \pm 0.6$ [17] and the measured value of the $\Omega^-$ magnetic moment to fix $\mu_c$, we predict the magnetic moments of the other members of the baryon decuplet. These results are shown in table 1 and also graphically in fig. 2. It is clear from fig. 2 that the SU(3) violating corrections induced by the one-loop graphs (dominated by the contribution from kaons) is small and that the tree level relation (where the magnetic moment is proportional to the electric charge of the baryon) is not badly broken. The one–loop chiral perturbation theory prediction for the $\Delta^{++}$ magnetic moment of $\mu_{\Delta^{++}} = 4.0 \pm 0.4$ (the tree-level result is $\mu_{\Delta^{++}} = 5.8 \pm 0.7$) agrees within errors with the recent measurement (but model dependent extraction) of $\mu_{\Delta^{++}} = 4.5 \pm 0.5$ [3], and the results from quenched lattice QCD $\mu_{\Delta^{++}} = 4.9 \pm 0.6$ [15], but is significantly smaller than the prediction of the naive quark model $\mu_{\Delta^{++}} \sim 5.6$ [9]. For the charged members of the decuplet we agree with the quenched lattice computations [15] but differ in the predictions for the magnetic moments of the neutral baryons. Note that the neutral baryon magnetic moments do not depend on the local leading counterterm that appears for the charged baryons, making these predictions independent of the measured value of $\mu_{\Omega^-}$.

As mentioned earlier, the axial matrix elements in $I = J$ baryons such as $N$ and $\Delta$ must obey a contracted $SU(2N_f)$ algebra in the large $N_c$ limit of QCD [22]. This results from the need for the low energy theory of baryons and goldstone bosons to be unitary in the large $N_c$ limit. Further, it was shown that this requires the $1/N_c$ correction to the axial matrix elements be proportional to the leading term. Therefore, relationships between axial matrix elements have vanishing $1/N_c$ corrections, but are corrected at order $1/N_c^2$ and higher [23]. A similar argument can be constructed for the matrix elements of the isovector magnetic moment operator. We expect that they satisfy the relations of a contracted $SU(2N_f)$ algebra up to corrections arising from terms $1/N_c^2$ and higher in the $1/N_c$ expansion. In this limit the isovector magnetic moments satisfy

$$\frac{\mu_{\Delta^{++}} - \mu_{\Delta^-}}{\mu_p - \mu_n} = \frac{9}{5} + \mathcal{O}(\frac{1}{N_c^2})$$  \hspace{1cm} (5)$$

and

$$\frac{\mu_{\Delta^+} - \mu_{\Delta^0}}{\mu_p - \mu_n} = \frac{3}{5} + \mathcal{O}(\frac{1}{N_c^2})$$  \hspace{1cm} (6).$$
Our analysis of magnetic moments is a non-trivial test of these relations. The local counterterm given in (1) has both isoscalar and isovector components, since it is proportional to the electric charge operator. At tree–level

\[
\left( \frac{\mu_{\Delta^+} - \mu_{\Delta^-}}{\mu_p - \mu_n} \right)_{\text{tree}} = -\frac{3\mu_{\Omega}}{\mu_p - \mu_n} \sim 1.2
\]  

(7)

and

\[
\left( \frac{\mu_{\Delta^0} - \mu_{\Delta^0}}{\mu_p - \mu_n} \right)_{\text{tree}} = -\frac{\mu_{\Omega}}{\mu_p - \mu_n} \sim 0.4 ,
\]  

which are about 2/3 the values expected in the large $N_c$ limit. Including the one–loop graphs improves the situation somewhat and we find that

\[
\left( \frac{\mu_{\Delta^+} - \mu_{\Delta^-}}{\mu_p - \mu_n} \right)_{\text{one–loop}} = 1.35 \pm 0.15
\]  

(9)

and

\[
\left( \frac{\mu_{\Delta^0} - \mu_{\Delta^0}}{\mu_p - \mu_n} \right)_{\text{one–loop}} = 0.45 \pm 0.05 .
\]  

(10)

Despite the fact that both quantities are still smaller than the numbers expected from large-$N_c$ QCD, the one-loop corrections tend to reduce the discrepancy in each case. There are modifications to the large $N_c$ relations from terms subleading in the $1/N_c$ expansion and also corrections at the 25% level from terms higher order in the chiral expansion that may improve the agreement.

The quadrupole moment for each of the decuplet baryons receives a contribution from both long–distance physics in the form of pion and kaon loops, and from short distance physics in the form of a local counterterm with an unknown coefficient. This dimension six counterterm has the form

\[
\mathcal{L}^{CT}_{E2} = Q_{CT} \frac{e}{\Lambda^2} q_i (\bar{T}_i \gamma^\mu T^\mu_i + \bar{T}_i T^{\mu}_i T^\mu_i) - \frac{1}{2} g^\mu\nu \bar{T}_i T^{\mu\nu}_i ) \nu^\alpha \partial_\mu F_{\nu\alpha} .
\]  

(11)

The contribution to the quadrupole moment from the diagrams in fig. [1] are formally enhanced over the naive contribution from the local counterterm by a chiral logarithm, \( \log(M^2/\Lambda^2) \), and we will neglect the contributions from the local counterterm, taking \( Q_{CT} \sim 0 \) for the rest of this discussion. The explicit contributions from the graphs in fig. [1] are

\[
Q^{TTT} = -i \frac{e}{16\pi^2} \frac{2}{9} \mathcal{H}^2 \omega (\bar{T}_v \cdot kT_{\mu\nu} + \bar{T}_{\mu\nu} T_{\nu\mu} \cdot k - \frac{1}{2} k_{\mu} \bar{T}_v \cdot T_{\nu\mu} ) A^\mu \sum_i \frac{\alpha_i}{f_{M_i}^2} G(\Delta m_i, M_i) \]  

(12)
and

\[ Q^{TBT} = i \frac{e}{16\pi^2} G^2 \omega' \left( T_v \cdot k T_{v\mu} + T_{v\mu} T_v \cdot k - \frac{1}{2} k_\mu T_v \cdot T_v \right) A^\mu \sum_i \frac{\beta_i}{f_{M_i}^2} G(\Delta m_i, M_i), \]  

(13)

where

\[ G(\Delta m, M) = \log \left( \frac{M^2}{\Lambda^2} \right) + \frac{\Delta m}{\sqrt{\Delta m^2 - M^2}} \log \left( \frac{\Delta m + \sqrt{\Delta m^2 - M^2 + i\epsilon}}{\Delta m - \sqrt{\Delta m^2 - M^2 + i\epsilon}} \right). \]  

(14)

As before, \( \Delta m_i \) is the mass splitting between the external and loop baryon, \( M_i \) is the mass of the goldstone boson in the loop, and \( f_{M_i} \) is the meson decay constant. The notation for \( \alpha \) and \( \beta \) is the same as for the magnetic moment equations. We can extract quadrupole moments from this calculation by using the definition of the quadrupole interaction energy,

\[ H^Q = -\frac{1}{6} \sum_{ij} Q_{ij} \frac{\partial E_i}{\partial x^j}, \]  

(15)

where \( E \) is the electric field and \( Q_{ij} \) is the quadrupole tensor which is symmetric and traceless. The quadrupole moment is defined to be \( Q_{zz} \), and can be extracted from (12) and (13). The results for the various members of the decuplet, neglecting the formally subdominant counterterm of (11), are shown in Table 2 and graphically in fig. 3. These moments are large, and comparable to the moments of light nuclei such as the deuteron \((Q_D = 2.8 \times 10^{-27} e\text{ cm}^2)\). With such large moments, the presence of constituent \( \Delta \)'s in nuclei might have a significant effect on nuclear quadrupole moments. Naively, the pion loop graphs should be logarithmically enhanced over the kaon loop graphs. Yet, the Clebsch-Gordan coefficients (given in the appendix) are such that the quadrupole moment generated by the pion loops depend only upon the \( I_z \) quantum number of the baryon (the kaon loops have both isovector and isoscalar dependence). This is distinctly different from the dependence of the local counterterm which depends on the charge of the baryon. The importance of this result becomes apparent when considering the quadrupole moment of a nucleus, in particular an \( I = 0 \) nucleus such as the deuteron. One might imagine that the intrinsic quadrupole moment of the \( \Delta \) would contribute to the quadrupole moment of a nucleus through virtual \( \Delta \) states. However, as most of the intrinsic quadrupole moment of the \( \Delta \) depends on \( I_z \), this contribution to the quadrupole moment of an \( I = 0 \) nucleus vanishes. Hence, the \( \Delta \) contribution to the quadrupole moment of the deuteron is greatly suppressed over naive expectations, appearing first from the kaon loop contribution.
Our values for the quadrupole moments are not always consistent with the values found in quenched lattice computations [15], though all but the neutral baryons agree within errors. In particular, where we find that the dominant component behaves as \( I_z \), lattice computations find behaviour more consistent with dependence upon the baryon charge.

Another interesting, perhaps more mysterious result that can be found by examining the Clebsch-Gordan coefficients in the appendix is that the coefficient of both the \( \log(M_\pi^2/\Lambda^2) \) and \( \log(M_K^2/\Lambda^2) \) terms are proportional to \( \frac{4}{9}H^2 - C^2 \). This vanishes when \( H/C = 3/2 \), which is exactly the relationship satisfied in the \( SU(6) \) limit. In this \( SU(6) \) limit, the contribution to the quadrupole moment from these one-loop graphs arises entirely from the mass splittings amongst the baryons. We can reconcile our results with that of quenched lattice QCD if indeed the axial couplings are very close to their \( SU(6) \) values. The quadrupole moments would then receive a non-negligible, and possibly dominant, contribution from the incalculable local counterterm (which we have neglected for our discussions), giving the characteristic dependence on the baryon charge that the lattice calculations find. Our central value predictions would then be substantially smaller in magnitude than those obtained using the experimentally fit values of \( H \) and \( C \).

In dealing with the magnetic moments of the decuplet, we saw that the large \( N_c \) limit of QCD gave results consistent with those of chiral perturbation theory calculations. For the quadrupole moments, the large \( N_c \) limit of the one-loop contribution approaches a constant value. This is because the relationships between axial coupling constants \( F, D, C, \) and \( H \) approach their \( SU(6) \) values [22] [23], and the hyperfine mass splittings between the baryons vanish as \( 1/N_c \) [24]. The coefficient of the quadrupole counterterm and the \( 1/N_c^2 \) corrections to the hyperfine mass splittings are needed in order to make a more explicit comparison between the large \( N_c \) predictions and chiral perturbation theory results for the quadrupole moments.

Finally, the decuplet baryons could also have a magnetic octupole moment. We can construct a dimension seven local counterterm for this moment, of the form

\[
\mathcal{L}_{M3}^{CT} = e \frac{\Theta}{\Lambda^3} q_i \left( T_{\alpha v i}^\mu S_{\nu v}^\alpha T_{\nu v i}^\alpha + T_{\nu v i}^\alpha S_{\nu v}^\alpha T_{\nu v i}^\mu + T_{\nu v i}^\mu S_{\nu v}^\mu T_{\nu v i}^\nu \right) \epsilon_{\alpha \beta \lambda \sigma} v^{\beta} \partial_{\mu} \partial_{\nu} F_{\lambda \sigma},
\]

where \( \Theta \) is an unknown coefficient. This tensor structure, in particular the three derivatives of the electromagnetic field, does not appear in the one–loop graphs shown in fig. [4]. Therefore, the magnetic octupole moment will be dominated by the local counterterm and corrections can first occur from two–loop diagrams.
In addition to the electrostatic moments of these baryons we can examine their strange moments. Strange moments of the nucleons as suggested in [25] have been the subject of an immense amount of both theoretical and experimental interest. Estimates of the size of these moments have been made for the octet baryons in the context of different hadronic schemes [24][27]. The strange moments of the decuplet baryons may never be measured, yet we are able to see what form they will have in the language of chiral perturbation theory. The strange magnetic and quadrupole moments could be substantially different from their electromagnetic counterparts. Since the strange charge operator has both flavour octet and singlet components, there are two unknown counterterms for each strange moment, with $SU(3)$ structure

$$L \sim S \mathcal{T}^{abc} Q_c^{(s)} d T_{abd} + \sigma \mathcal{T}^{abc} T_{abc} Q^{(s)} \alpha,$$

where the strange charge matrix is $Q^{(s)} = \text{diag}(0,0,1)$ and $S$ and $\sigma$ are unknown coefficients. For investigating the baryon sea, however, we are most interested in looking at the strange moments of the non-strange baryons, namely the $\Delta$'s. The first term, $S$, does not contribute, which leaves one unknown counterterm, $\sigma$, that contributes equally to all baryons in the decuplet, yet is unclear how to determine experimentally. We can compute corrections to the strange magnetic moments and also the dominant contribution to the strange quadrupole moment just as we did in the electrostatic case. For these strange moments the pion loops do not contribute (they do not carry strange charge) but both charged and neutral kaons will contribute. Therefore, we expect the strange quadrupole moment to be much smaller than the electrostatic counterpart for large $I_z$ baryons, with the other quadrupole moments perhaps comparable in size to the electrostatic ones. The strange magnetic moment may be the same size as the electrostatic magnetic moment. Since the strange charge is an isoscalar, the moments of each of the $\Delta$'s are identical. The one–loop induced quadrupole moments are proportional to $\frac{4}{3} \mathcal{H}^2 - C^2$ (up to isospin breaking mass differences), a quantity that vanishes in the $SU(6)$ limit. (Unlike the case for the electromagnetic quadrupole moment, the intermediate baryons contributing to these quadrupole moments are all isospin degenerate.) If the axial couplings are near the $SU(6)$ point, as there is strong evidence to suggest, then the strange moments of the $\Delta$’s are each dominated by one incalculable local counterterm. These quantities are of theoretical interest as the appearance of possibly another non-zero strange matrix element in non-strange hadrons.
In conclusion, we have discussed the electrostatic properties of the Δ and other members of the baryon decuplet. Using chiral perturbation theory we have computed the leading non-analytic contributions to the magnetic dipole and electric quadrupole moments, and shown that the leading contribution to the octupole moment is from a local dimension seven counterterm with corrections arising at two-loops. We have compared our prediction for the $\Delta^{++}$ magnetic moment with its recent model dependent extraction from pion bremsstrahlung data and found it to be in good agreement. The one-loop computation moves the isovector magnetic moments into better agreement with the predictions of large-$N_c$ QCD compared to the tree-level results. Although the quadrupole moments of the decuplet have not been measured yet, there may be some hope for such measurements at CEBAF. We computed the leading contribution to the quadrupole moments from long-distance pion and kaon loops, which are formally dominant over the dimension six local counterterm. The pion contribution depends only on $I_z$ and hence the contribution from the intrinsic quadrupole moment of Δ’s to that of an $I = 0$ nucleus from the Δ components in the nuclear wavefunction is suppressed. This is an important result particularly for the deuteron since the magnitude of the Δ quadrupole moments are comparable to that of the deuteron. Further, the formally dominant terms of the form $\log(M^2_\pi/\Lambda^2)\chi$ and $\log(M^2_K/\Lambda^2\chi)$ vanish when the axial couplings approach their $SU(6)$ limit.

We have compared our results to those obtained in quenched lattice QCD [15] and find that the magnetic moments of the charged baryons agree well. This is not unexpected since they are dominated by the local counterterm that is fixed experimentally. This agreement does not exist for the neutral baryons, which have no counterterm dependence. The predictions for the charged baryon electric quadrupole moments also agree within errors, yet have a different dependence on the baryon isospin. Again, there is not agreement for the neutral baryons. Our leading contribution for the quadrupole moment arises from pion loops and depends on $I_z$ only. The lattice computation indicates that the quadrupole moment depends on the charge of the baryon. These two results can be reconciled if the axial coupling constants $C$ and $H$ satisfy $SU(6)$ relations as required in the large $N_c$ limit and also approximately found experimentally [17][20]. In this scenario the quadrupole moments receive a non-negligible and potentially dominant contribution from the local (incalculable) counterterm.

We expect that some of our predictions will be tested at CEBAF, and that measurements of the quadrupole moments in particular may help test the validity of the heavy
baryon chiral perturbation theory approach in understanding low–energy QCD. In addition, we expect the comparison to help determine if the contracted $SU(2N_f)$ algebra is a useful symmetry for describing low–energy hadronic properties.

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Appendix

SU(3) Clebsch-Gordan coefficients $\alpha$ and $\beta$ used for the evaluation of decuplet electromagnetic moments. For brevity, only the intermediate baryon state is given as a label. The boson index is implicit.

$\Delta^{++}$

\[
\begin{align*}
\alpha_{\Delta^+} &= \frac{1}{3} & \beta_p &= 1 \\
\alpha_{\Sigma^{++}} &= \frac{1}{3} & \beta_{\Sigma^+} &= 1 \\
\end{align*}
\]

$\Delta^+$

\[
\begin{align*}
\alpha_{\Delta^0} &= \frac{4}{9} & \beta_n &= \frac{1}{3} \\
\alpha_{\Delta^{++}} &= -\frac{1}{3} & \beta_p &= -\frac{1}{3} \\
\alpha_{\Sigma^{*0}} &= \frac{2}{9} & \beta_{\Sigma^0} &= \frac{2}{3} \\
\end{align*}
\]

$\Delta^0$

\[
\begin{align*}
\alpha_{\Delta^-} &= \frac{1}{3} & \beta_p &= \frac{2}{9} \\
\alpha_{\Delta^+} &= -\frac{4}{9} & \beta_{\Sigma^0} &= \frac{1}{9} \\
\alpha_{\Sigma^{*0}} &= \frac{1}{9} & \beta_{\Sigma^-} &= \frac{1}{3} \\
\end{align*}
\]

$\Delta^-$

\[
\begin{align*}
\alpha_{\Delta^0} &= -\frac{1}{3} & \beta_n &= -1 \\
\end{align*}
\]

$\Sigma^{*+}$

\[
\begin{align*}
\alpha_{\Sigma^{*0}} &= \frac{2}{9} & \beta_{\Sigma^0} &= \frac{1}{6} \\
\alpha_{\Xi^*} &= \frac{1}{9} & \beta_{\Xi^0} &= \frac{1}{3} \\
\alpha_{\Delta^{++}} &= -\frac{1}{3} & \beta_p &= \frac{2}{9} \\
\end{align*}
\]

$\Sigma^{*0}$

\[
\begin{align*}
\alpha_{\Sigma^-} &= \frac{2}{9} & \beta_{\Sigma^*} &= \frac{1}{6} \\
\alpha_{\Sigma^{*+}} &= -\frac{4}{9} & \beta_{\Sigma^+} &= -\frac{1}{6} \\
\alpha_{\Xi^*} &= \frac{2}{9} & \beta_{\Xi^-} &= \frac{1}{6} \\
\alpha_{\Delta^+} &= -\frac{2}{9} & \beta_p &= -\frac{1}{6} \\
\end{align*}
\]

$\Sigma^{*-}$

\[
\begin{align*}
\alpha_{\Sigma^*} &= -\frac{4}{9} & \beta_{\Sigma^0} &= -\frac{1}{6} \\
\alpha_{\Lambda} &= -\frac{1}{2} \\
\alpha_{\Delta^0} &= -\frac{1}{9} & \beta_n &= -\frac{1}{3} \\
\end{align*}
\]
\[ \Xi^{*0} \]

\[ \alpha_{\Xi^{*-}} = \frac{1}{9} \quad \beta_{\Xi^{-}} = \frac{1}{3} \]

\[ \alpha_{\Omega^{-}} = \frac{1}{3} \]

\[ \alpha_{\Omega^{*+}} = -\frac{1}{3} \quad \beta_{\Omega^{*+}} = -\frac{1}{3} \]

\[ \Xi^{*-} \]

\[ \alpha_{\Xi^{*0}} = -\frac{1}{9} \quad \beta_{\Xi^{0}} = -\frac{1}{9} \]

\[ \beta_{\Lambda} = -\frac{1}{2} \]

\[ \alpha_{\Omega^{-0}} = -\frac{2}{9} \]

\[ \beta_{\Omega^{-0}} = -\frac{1}{6} \]

\[ \Omega^{-} \]

\[ \alpha_{\Xi^{*0}} = -\frac{1}{3} \]

\[ \beta_{\Xi^{0}} = -1 \]
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Table 1
Magnetic moments of the baryon decuplet from chiral perturbation theory. Uncertainties reflect uncertainties in the couplings $\mathcal{C}$ and $\mathcal{H}$, and in the magnetic moment of the $\Omega^-$ used to constrain the local counterterm.

|       | $\mu (\mu_N)$ |       | $\mu (\mu_N)$ |
|-------|---------------|-------|---------------|
| $\Delta^{++}$ | 4.0 ± 0.4     | $\Sigma^{++}$ | 2.0 ± 0.2     |
| $\Delta^{+}$   | 2.1 ± 0.2     | $\Sigma^{*0}$  | −0.07 ± 0.02  |
| $\Delta^{0}$   | −0.17 ± 0.04  | $\Sigma^{*-}$  | −2.2 ± 0.2    |
| $\Delta^{-}$   | −2.25 ± 0.25  | $\Xi^{*0}$     | 0.10 ± 0.04   |
| $\Omega^{-}$   | −1.94 ± 0.22  | $\Xi^{*^{-}}$  | −2.0 ± 0.2    |

Table 2
Quadrupole moments of the baryon decuplet arising from one-loop graphs in chiral perturbation theory. Uncertainties reflect uncertainties in the couplings $\mathcal{C}$ and $\mathcal{H}$.

|       | $Q (10^{-27} e - cm^2)$ |       | $Q (10^{-27} e - cm^2)$ |
|-------|------------------------|-------|------------------------|
| $\Delta^{++}$ | −0.8 ± 0.5           | $\Sigma^{++}$ | −0.7 ± 0.3           |
| $\Delta^{+}$   | −0.3 ± 0.2            | $\Sigma^{*0}$  | −0.13 ± 0.07         |
| $\Delta^{0}$   | 0.12 ± 0.05           | $\Sigma^{*-}$  | 0.4 ± 0.2             |
| $\Delta^{-}$   | 0.6 ± 0.3             | $\Xi^{*0}$     | −0.35 ± 0.2           |
| $\Omega^{-}$   | 0.09 ± 0.05           | $\Xi^{*^{-}}$  | 0.2 ± 0.1             |
Figure Captions

Fig. 1. Leading one-loop graphs contributing to the multipole moments of the decuplet baryons. The dashed lines correspond to charged goldstone bosons and the wiggly line to photons. \( T \) is a decuplet baryon and \( B \) is an octet baryon.

Fig. 2. The magnetic moments of the decuplet baryons, in units of nuclear magnetons. The dark points are the moments derived from the central values of \( C \) and \( H \) and the lighter lines are the associated uncertainties.

Fig. 3. The quadrupole moments of the decuplet baryons. The contribution from the local counterterm is subleading and we have set it to zero. The quadrupole moments here come from one-loop graphs only. The dark points are the moments derived from the central values of \( C \) and \( H \) and the lighter lines are the associated uncertainties.
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