Extracting a value of the slope of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ by reanalysis of the experimental data

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Abstract

A new value $b = 0.0205 \pm 0.0017$ fm$^2$ of the slope of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ compatible with deuteron properties has been extracted by using a linear relation between $b$ and $A^2_s(1 + \eta^2)$ we found for a class of nonlocal potential models having the experimental values of both $r_D$ and $Q$. Another model dependent value $b_{\text{MHKZ}} = 0.0206 \pm 0.0014$ fm$^2$, which is also compatible with deuteron properties, has been determined by applying "constrained" unitary transformations to the local MHKZ potential model. The sensitivity of a small changes in the experimental values used for $r_D$ and $Q$ on the value obtained for $b$ is also investigated.

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1 Introduction

Measurements of the elastic scattering of electrons by deuterons lead in a direct way to the deuteron form factors as functions of the momentum transfer $q^2$. These form factors are of interest in connection with the study of the structure of the deuteron and the determination of the electromagnetic form factors of the neutron [1].

Correlations between the slope $b$ of the neutron electric form factor $G_{En}(q^2)$ at $q^2 = 0$ and the deuteron root mean square (rms) matter radius $r_D$ for different NN-potential models have been used to determine $r_D$ [2,3,4]. Berard et al. [2] found a linear relation with a positive slope between $b$ and $r_D^2$ for a class of potential models. They used their experimental data for the ratio $R(q^2)$ of
the deuteron to the proton electric form factors in the range of momentum transfers $0.05 \leq q^2 \leq 0.50 \text{ fm}^{-2}$ and the experimental value $b = 0.0189 \pm 0.0004 \text{ fm}^2$ given by Krohn and Ringo [5] to extract the value $r_D = 1.9635 \pm 0.0045 \text{ fm}$. Allen et al. [3] found a linear relation with a positive slope between $b$ and $r_D^2$ for the radial wave functions for a single local potential model and its family of the wave functions produced by unitary transformations. The experimental data of $R(q^2)$ of Berard et al. [2] and the experimental value $b = 0.0199 \pm 0.0003 \text{ fm}^2$ given by Koester et al. [6] have been used to determine the model dependent value $r_D = 1.952 \pm 0.004 \text{ fm}$ and the model independent one $r_D = 1.948 \pm 0.023 \text{ fm}$ indirectly from the plotted straight line. They used also the experimental data of $R(q^2)$ of Akimov et al. [1] in the range $0.05 \leq q^2 \leq 0.50 \text{ fm}^{-2}$ to determine $r_D = 2.005 \pm 0.118 \text{ fm}$. In an analogous procedure Mustafa et al. [4] found a linear relation between $b$ and $r_D^2$ for the radial wave functions for the MHKZ local potential model [4] and its family of the wave functions produced by unitary transformations. They used also the experimental data of $R(q^2)$ of Berard et al. [2] and the experimental value $b = 0.0199 \pm 0.0003 \text{ fm}^2$ given by Koester et al. [6] to determine $r_D = 1.9546 \pm 0.0021 \text{ fm}$.

In this work, correlations between $b$ and deuteron properties, i.e. the asymptotic $S$-state amplitude $A_S$, the asymptotic $D/S$ ratio $\eta$, the rms radius $r_D$, the quadrupole moment $Q$ and the binding energy $E_b$, have been used to determine $b$. Two linear relations between $b$ and $A_S^2(1 + \eta^2)$ have been found for deuteron potential models, one with a positive slope and the other with a negative one.

At large distances, outside the nuclear potential range, the $^3S_1$- and the $^3D_1$-state radial wave functions ($u$ and $w$, respectively) behave asymptotically as [7]

$$u(r) = A_S e^{-\gamma r},$$

$$w(r) = A_D \left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2}\right) e^{-\gamma r}, \quad (1)$$

where $\gamma^2 = -\frac{2m}{\hbar^2} E_b$ [8] and $m$ is the reduced neutron-proton mass. If the proton and the neutron are at a distance $r$ apart, the deuteron rms radius $r_D$ may be written as [7]

$$r_D^2 = \frac{1}{4} \int_0^\infty r^2 \left(u^2 + w^2\right) dr. \quad (2)$$

The deuteron quadrupole moment $Q$ may be also given by [7]
\[ Q = \frac{1}{\sqrt{50}} \int_0^\infty r^2 u w dr - \frac{1}{20} \int_0^\infty r^2 w^2 dr. \]  

(3)

The correlation with \( A_S^2(1 + \eta^2) \) has been used because the term \((u^2 + w^2)\) which has been involved in the definition of \( r_D^2 \) proportional to \( A_S^2(1 + \eta^2) \) if we use the asymptotic forms of the radial wave functions \( u \) and \( w \) of Eq. (1) and ignore the centrifugal term \([1 + 3/(\gamma r) + 3/(\gamma^2 r^2)]\).

The aim of this work is to extract a new value of \( b \) which is compatible with deuteron properties. This value is extracted indirectly by using the linear correlation between \( b \) and \( A_S^2(1 + \eta^2) \) of nonlocal potential models (having the experimental values of both \( r_D \) and \( Q \)) produced by applying unitary transformations to standard nonrelativistic potential models.

In Sect. 2 we will outline the method which has been used to calculate a model value of the slope \( b \) of the neutron form factor \( G_{En}(q^2) \) at \( q^2 = 0 \). The new determinations of this parameter using the local and nonlocal potential models will be distinguished in Sect. 3. In Sect. 4 a simple method to calculate the parameter \( b \) using one single potential model will be obtained. Finally, the sensitivity of a small changes in the experimental values used for \( r_D \) and \( Q \) on the extracted value of \( b \), i.e. \( \frac{\delta b}{\delta r_D} \) and \( \frac{\delta b}{\delta Q} \), will be given in Sect. 5.

2 Calculating model values of \( b \)

In order to extract the slope \( b \) of the neutron form factor \( G_{En}(q^2) \) at \( q^2 = 0 \) from elastic electron-deuteron scattering, we start from the expression of the deuteron electric form factor in the nonrelativistic impulse approximation, neglecting two-body contributions [7]

\[
G_{ED}(q^2) = \left[ G_{Ep}(q^2) + G_{En}(q^2) \right] \left[ C_E^2(q^2) + C_Q^2(q^2) \right]^{1/2} \left( 1 + \tau \right)^{-1/2}
\]

\[
\simeq \left[ G_{Ep}(q^2) + G_{En}(q^2) \right] C_E(q^2) \left( 1 + \tau \right)^{-1/2}, \tag{4}
\]

where \( G_{Ep}(q^2) \) is the proton electric form factor, \( C_E(q^2) \) and \( C_Q(q^2) \) are electric charge and quadrupole form factors, respectively, which are given in terms of the nonrelativistic S- and D-wave functions by [7]

\[
C_E(q^2) = \int_0^\infty (u^2 + w^2) \ h \left( \frac{qr}{2} \right) \ dr
\]

\[
\tag{5}
\]
and

\[
C_Q(q^2) = 2 \int_0^\infty \left( uw - \frac{w^2}{\sqrt{8}} \right) j_2 \left( \frac{qr}{2} \right) dr ,
\]

(6)

where \(j_0\) and \(j_2\) are the spherical Bessel functions of order zero and two, respectively. The factor \(\tau = (q^2/4m_p^2)\) is called the Darwin-Foldy correction to the nucleon form factors [9], where \(m_p = 4.5749098\) fm\(^{-1}\) is the proton mass. Note that, Eq. (4) is usually used because \(C_Q \ll C_E\) for small \(q^2\).

The value of \(b\) for a particular potential model is insensitive to the choice of the proton electric form factor \(G_{Ep}(q^2)\). In this work we take the parameterization of Simon et al. [10] for \(q^2\) in fm\(^{-2}\)

\[
G_{Ep}(q^2) = \frac{0.312}{1 + \frac{q^2}{6}} + \frac{1.312}{1 + \frac{q^2}{15.02}} - \frac{0.709}{1 + \frac{q^2}{44.08}} + \frac{0.085}{1 + \frac{q^2}{154.2}}.
\]

(7)

For the evaluation of the charge form factor \(C_E(q^2)\) given in Eq. (5) we split the integration according to

\[
C_E(q^2) = \int_0^R \left( u^2 + w^2 \right) j_0 \left( \frac{qr}{2} \right) dr + \Delta C_E(q^2) ,
\]

(8)

where \(\Delta C_E(q^2)\) is the asymptotic analytic correction to \(C_E(q^2)\) given by

\[
\Delta C_E(q^2) = \int_R^\infty \left( u^2 + w^2 \right) j_0 \left( \frac{qr}{2} \right) dr .
\]

(9)

The first integration of Eq. (8) is calculated numerically from \(r = r_c\) to \(r = R\), where \(r_c\) is the hard-core radius - if any - and \(R\) is chosen to be \(16+r_c\) fm which is beyond the range of the nuclear potential. The integral in Eq. (9) is carried out analytically from \(r = R\) to \(r = \infty\) by using the asymptotic forms of the radial wave functions \(u\) and \(w\) of Eq. (1). The analytic formula of the \(\Delta C_E(q^2)\) is given in Ref.[4], for which the values of \(C_E(q^2)\) are correct in the momentum transferred range \(0 \leq q^2 \leq 25\) fm\(^{-2}\). In an analogous procedure the deuteron rms radius \(r_D\) given in Eq. (2) and the deuteron quadrupole moment \(Q\) given in Eq. (3) are evaluated.

Given \(C_E(q^2)\) for a particular potential model, then the experimental values of the ratio \(R(q^2)\) of the deuteron form factor \(G_{ED}(q^2)\) to the proton form factor \(G_{Ep}(q^2)\) can be used to determine the neutron form factor [2] from
\begin{equation}
G_{En}(q^2) = \left[ \frac{R(q^2)\sqrt{1+\tau}}{C_E(q^2)} - 1 \right] G_{Ep}(q^2)
\end{equation}
(10)

and hence, the slope $b$ of $G_{En}(q^2)$ at $q^2 = 0$,

\begin{equation}
b = \frac{dG_{En}(q^2)}{dq^2} \bigg|_{q^2=0}.
\end{equation}
(11)

The experimental values of $R(q^2)$ of Simon et al. [11] are used in this work. This set of data covers the range of values $0.044 \leq q^2 \leq 4$ fm$^{-2}$.

Various parameterizations have been used for $G_{En}(q^2)$ [2,12,13,14,15,16,17,18]. In order to calculate $b$ in this work we parameterize $G_{En}(q^2)$ by a polynomial of order three in $q^2$ as

\begin{equation}
G_{En}(q^2) = bq^2 + cq^4 + dq^6
\end{equation}
(12)

which agrees well for $0.044 \leq q^2 \leq 4$ fm$^{-2}$. The model dependency of $b$, here, is a result of the model dependence of the charge form factor $C_E(q^2)$ via the radial wave functions.

3 The new determinations of $b$

In order to extract a new value of the slope $b$ of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$, a linear relation between $b$ and $A_S^2(1+\eta^2)$ with a positive slope has been found for standard nonrelativistic potential models as shown in Fig. 1. These models and the "names" given to them are the potentials of Glendenning and Kramer "GK1, ......, GK9" [19], Lacombe et al. "PARIS" [20], Mustafa et al. "MHKZ" [4], Reid "RHC, RSC, RSCA" [21], Machledit et al. "MACH-A, -B, -C" [22], Machledit et al. "Bonn-F, -Q" [23], de Tourreil and Sprung "TS-A, -B, -C" [24], de Tourreil et al. "TRS" [25], Hamada and Johnston "HJ" [26], Mustafa and Zahran "MZ" [27], Mustafa "A, B" [28], Mustafa et al. "r1, r3, ......, r7" [29], Mustafa "L1, L2, 1, 2, ......, 6" [33] and Mustafa "a, b, ......, i" [30]. The value

\begin{equation}
b = 0.0296 \pm 0.0118 \text{ fm}^2
\end{equation}
(13)

which is compatible with the experimental values of $A_S$, $\eta$ and $E_b$ but not with the experimental values of $r_D$ and $Q$ is extracted from this correlation. It is the value of $b$ corresponding to the experimental value $A_S^2(1+\eta^2) = 0.7817$ fm$^{-1}$. This value of $b$ is greater than the previously extracted value.
$b = 0.0199 \pm 0.0003$ fm$^2$ by Koester et al. [6] and its error $\Delta b$ is relatively large. Therefore, the result obtained for $b$ using the local potential models is not very reliable.

Moreover, it is desirable to obtain a value of $b$ which is not only compatible with the experimental values of $A_S$, $\eta$ and $E_b$, but also with the experimental values of $Q$ and $r_D$, which have dependencies on the interior parts of the radial wave functions. The experimental values of these quantities are listed in Table 1. Contributions to the value of $r_D$ and $Q$ from meson exchange currents and other relativistic corrections $\Delta r_D = 0.0034$ fm and $\Delta Q = 0.0063$ fm$^2$ of Kohno [35] are neglected in the present work. Kermode et al. [36] and Mustafa and Hassan [37] have proven that $Q$ and $r_D$ are not purely asymptotic quantities by showing that phase-equivalent potentials can have different quadrupole moments and radii. Therefore, the linear relation between $b$ and $A_S^2(1 + \eta^2)$ obtained in Fig. 1 has been re-drawn using only potential models which reproduce the experimental values of both $Q$ and $r_D$. These phase-equivalent potential models are produced by using "constrained" short-range unitary transformations like those used previously by Kermode et al. [36] for the radial wave functions $(u_i, w_i)$ of a local potential model. The transformed wave functions $(\overline{u}_i, \overline{w}_i)$ of a nonlocal potential model are given by

$$
\overline{u}_i = u_i - 2g(r) \int_0^\infty g(s) u_i(s) ds ,
$$

$$
\overline{w}_i = w_i - 2g(r) \int_0^\infty g(s) w_i(s) ds .
$$

(14)

Various parameterizations have been used for the function $g(s)$ [37,38,39,40,41]. We have chosen the parameterization $g(s) = Cs(1 - \beta s)e^{-\alpha s}$, where $s = r - r_c$ and $C = [4\alpha^5/((\alpha^2 - 3\beta\alpha + 3\beta^2))^{1/2}$ is a normalizing factor such that $\int_0^\infty g^2(r) dr = 1$. The parameters $\alpha$ and $\beta$ are adjustable parameters. The parameter $\alpha$ can be regarded as the "range" and $\beta$ as the "strength" of the unitary transformation. The values of $\alpha < 1.5$ fm$^{-1}$ produce wave functions of different asymptotic behaviour [37]. In this work, the parameter $\alpha$ is assumed to changed from $\alpha = 1.5$ fm$^{-1}$ to $\alpha = 7$ fm$^{-1}$ in steps of 0.0001 fm$^{-1}$. For each value of $\alpha$, the parameter $\beta$ is changed from $\beta = 0$ fm$^{-1}$ to $\beta = 4$ fm$^{-1}$ in steps of 0.0001 fm$^{-1}$. The function $g(r)$ is of short range, hence, the wave functions $u$ and $w$ of a local potential model and $\overline{u}$ and $\overline{w}$ of the corresponding nonlocal potential model are the same in the asymptotic region. This implies that the asymptotic quantities $A_S$ and $A_D$ and hence, $\eta = A_D/A_S$, are the same for both the local and nonlocal potential models, but $r_D$ and $Q$ could be different.

The values of the non-locality parameters $\alpha$ and $\beta$ are adjusted to produce
transformed wave functions having the experimental values of \( r_D \) and \( Q \). This could not be achieved for all the local potentials considered. It is always possible to find for a given value of \( \alpha \) a value of \( \beta \) which corresponds either to the experimental value of \( r_D \) or to the experimental value of \( Q \). In fact, for a given \( \alpha \) one often finds two solutions for \( \beta \) as is shown by the two branches in Fig. 2. But it was not always possible for some potential models to find pairs \((\alpha, \beta)\) such that both \( r_D \) and \( Q \) could be reproduced. In this case we fix the non-locality parameters \((\alpha, \beta)\) which give the experimental value of \( r_D \) by searching for the closest value of \( Q \) with respect to the experimental value or vice versa. If both values would be fitted, the two type of curves possess an intersection point as is demonstrated in Fig. 2 for selected potential models. The transformed wave functions having the experimental values of both \( r_D \) and \( Q \) are compared to the radial wave functions in Fig. 3 for selected potential models. Altogether, we have studied forty-nine potential models. Out of these, we only found seventeen transformed models giving the experimental values for both \( r_D \) and \( Q \). The pairs of the non-locality parameters \((\alpha, \beta)\) having the experimental values of both \( r_D \) and \( Q \) are listed in Table 2.

In this case of applying “constrained” unitary transformations, a linear relation with a negative slope between \( b \) and \( A_S^2(1+\eta^2) \) is found as shown in Fig. 4. The points representing the transformed potential models lie on or closely scattered around the straight line. We would like to mention that the five points representing the nonlocal potentials which are phase equivalent to the family of the Glendenning and Kramer potential models (GK2, GK3, GK5, GK7 and GK8) [19] lie on a separate line with a different negative slope (see also, Fig.1 in Klarsfeld et al. [42] in which they used deuteron potential models in an empirical relation and found that the point representing the Glendenning and Kramer potentials also not lie on the empirical line, but lie on a separate line with a different slope). However, these old models predict a relatively larger deuteron binding energy \( E_b \) in agreement with the then experimental value \( E_b = -2.226 \pm 0.003 \text{ MeV} \) [43]. Therefore, these five potential models are not considered in Fig. 4. The new value

\[
b = 0.0205 \pm 0.0017 \text{ fm}^2
\]  

(15)

is extracted from the straight line of Fig. 4 using only the remaining twelve nonlocal potential models. This value of \( b \) is compatible with the experimental values of \( A_S, \eta, r_D, Q \) and \( E_b \). It is more consistent with the previously published values \( b = 0.0189 \pm 0.0004 \text{ fm}^2 \) of Krohn and Ringo [5] and, \( b = 0.0199 \pm 0.0003 \text{ fm}^2 \) of Koester et al. [6]. The standard error in Eq. (15) is much less than that of Eq. (13), but is greater than those of the previous measurements.
4 Determination of $b$ using MHKZ potential model

So far, we have used two methods to extract the slope parameter $b$. The first uses the linear relation between $b$ and $A_S^2(1 + \eta^2)$ of the local potential models as shown in Fig. 1. The second is based on Fig. 4 using the nonlocal potential models having the experimental values of both $r_D$ and $Q$. The two extracted values in Eqs.(13) and (15) differ substantially.

A third method to extract $b$ does not use the linear relation between $b$ and $A_S^2(1 + \eta^2)$ and a large number of potential models. It uses only one single potential model which is the potential model of Mustafa et al. [4]. The importance of the MHKZ potential model comes from the fact that it reproduces the experimental values of $A_S$ and $\eta$. The unitary transformation applied to the MHKZ wave functions are constrained to produce a phase-equivalent transformed wave functions having the experimental values of both $r_D$ and $Q$. The model value,

$$b_{\text{MHKZ}} = 0.0206 \pm 0.0014 \text{ fm}^2$$

(16)

is extracted by using the phase-equivalent potential of the MHKZ potential model [4]. It is also compatible with the experimental values of $A_S$, $\eta$, $E_b$, $r_D$ and $Q$. This value is in very good agreement with the value obtained in Eq. (15) of this work.

5 The sensitivity of a small changes in the experimental values used for $r_D$ and $Q$ on the extracted value of $b$

The dependence of the determined value of $b$ on the experimental values used for $r_D$ and $Q$ has been investigated. The whole procedure has been repeated twice, first by allowing for a small change $\delta r_D$ in the value assumed as an experimental value of $r_D$ and fixing the experimental values of $A_S$, $\eta$, $E_b$ and $Q$ to find the corresponding change $\delta b$. The published experimental values of $r_D$ are in the range between $r_D = 1.947 \pm 0.029$ fm of Akimov et al.[1] and $r_D = 1.9635 \pm 0.0045$ fm of Berard et al.[2]. The value of $r_D$ of Akimov et al.[1], which is the lower limit of the experimental determination of $r_D$, has been used in this work to study the correlation between the determined value of $b$, Eq. (15), and the value used for $r_D$ as an experimental value. The value

$$b = 0.0176 \pm 0.0013 \text{ fm}^2$$

(17)
is extracted in this case by using the transformed wave functions of potential models having the values of both \( r_D \) and \( Q \). We show that the decrease in the value of \( r_D \) leads to a corresponding decrease in the value of \( b \). Therefore, we find

\[
\frac{\delta b}{\delta r_D} = 0.6744 \text{ fm}. \tag{18}
\]

In the second case, only the experimental value used for \( Q \) is changed to obtain \( \delta b/\delta Q \). The latest two published measurements of the experimental value of the deuteron quadrupole moment \( Q \) are the value of Reid and Vaida [44], \( Q = 0.2860 \pm 0.0015 \text{ fm}^2 \), and the value of Bishop and Cheung [31], \( Q = 0.2859 \pm 0.0003 \text{ fm}^2 \). To study the correlation between the extracted value of \( b \) and \( Q \) we used the experimental value \( Q = 0.2860 \pm 0.0015 \text{ fm}^2 \) [44]. The result of the analysis is

\[
b = 0.0205 \pm 0.0011 \text{ fm}^2. \tag{19}
\]

From these calculations we show that the increase in the value of \( Q \) leads to a small decrease in the value of \( b \) resulting in

\[
\frac{\delta b}{\delta Q} = -0.0052. \tag{20}
\]

Similarly, we found

\[
\frac{\delta b}{\delta A_S} = -0.098 \text{ fm}^{3/2}, \tag{21}
\]

and

\[
\frac{\delta b}{\delta \eta} = -0.002 \text{ fm}^2. \tag{22}
\]

From these results we conclude that the correlation between \( b \) and \( r_D \) is the dominant one.

Now, we would like to point out the reason for the changing slope of the correlation between \( b \) and \( A_S^2(1 + \eta^2) \) from being positive in the case of the local potential models of Fig. 1 to a negative value in the case of the nonlocal potential models of Fig. 4. Since \( b \) is proportional to \( r_D \) and \( r_D \) in turn proportional to \( A_S^2(1 + \eta^2) \), the value of \( b \) is proportional to \( A_S^2(1 + \eta^2) \). The values of \( A_S \)
and $\eta$ and then $A_S^2(1 + \eta^2)$ are the same for the local and the corresponding nonlocal model, but $b$ and $r_D$ are different. We see that the value of $b$ in the case of using the radial wave functions of the local model is considerably greater than that in the case of using the transformed wave functions of the corresponding nonlocal potential model. Therefore, the slope will change sign in the case of using the transformed wave functions of the nonlocal potential models.

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Table 1
The experimental values used in the analysis of this work.

| Value                        | Ref. |
|------------------------------|------|
| $r_D = 1.9547 \pm 0.0019$ fm | [9]  |
| $Q = 0.2859 \pm 0.0003$ fm$^2$ | [31] |
| $A_S = 0.8838 \pm 0.0004$ fm$^{-1/2}$ | [32] |
| $\eta = 0.02713 \pm 0.00006$ | [33] |
| $E_b = -2.224575 \pm 0.000009$ MeV | [34] |

Table 2
The pairs of the nonlocality parameters $(\alpha, \beta)$ of the unitary transformations producing transformed wave functions having the experimental values of both $r_D$ and $Q$.

| local potentials | Ref. | $\alpha$ | $\beta$ |
|------------------|------|----------|---------|
| MHKZ             | [4]  | 5.70890  | 2.9672  |
| GK2              | [19] | 1.5818   | 0.3966  |
| GK3              | [19] | 2.0516   | 0.9938  |
| GK5              | [19] | 1.6421   | 0.4143  |
| GK7              | [19] | 2.0455   | 1.0127  |
| GK8              | [19] | 2.0894   | 1.0066  |
| RSC              | [21] | 2.0650   | 0.7787  |
| RHC              | [21] | 2.7131   | 1.2650  |
| RSCA             | [21] | 1.8211   | 0.7141  |
| MACH-C           | [22] | 1.5363   | 0.6680  |
| r1               | [29] | 2.2160   | 0.9669  |
| r3               | [29] | 2.3759   | 1.1125  |
| r5               | [29] | 2.9344   | 1.4908  |
| r6               | [29] | 3.5364   | 1.8926  |
| r7               | [29] | 3.0106   | 1.6228  |
| TRS              | [25] | 1.4821   | 0.6472  |
| HJ               | [26] | 3.0968   | 1.4556  |
Figure 1. The variation of the slope $b$ versus $A_S^2(1 + \eta^2)$ of standard local potential models. The middle (lower) part of the graph is magnified in the upper (lower) inner frame. The value of $b$ of Eq. (13) is extracted from this straight line corresponding to $A_S^2(1 + \eta^2) = 0.7817$ fm$^{-1}$. 
Figure 2. The variation of the "strength" parameter $\beta$ versus the "range" parameter $\alpha$ for some local potential models, indicated on the graphs, which have intersection points, i.e., have a pair $(\alpha, \beta)$ which give the experimental values of both $Q$ and $r_D$. The solid (dotted) lines represent the values of $\alpha$ and $\beta$ which give the experimental values of $Q$ ($r_D$).
Figure 3. Transformed wave functions having the experimental values of both $r_D$ and $Q$ produced by the unitary transformation (solid line) are compared to the local potentials which are represented by the dotted line. The upper (lower) curves are the $u$ ($w$) wave functions.
Figure 4. The correlation of the slope $b$ versus $A_S^2(1 + \eta^2)$ of the twelve nonlocal "transformed" potential models having the experimental values of $r_D$ and $Q$. The new value of $b$ of Eq. (15) is extracted from this line. It is the value of $b$ corresponding to the experimental value of $A_S^2(1 + \eta^2) = 0.7817$ fm$^{-1}$. 