On physical reliability of constitutive relations in plasticity for plane processes with bend point

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Abstract. We focus our attention on the physical reliability of plasticity at complex loading. It is important for reliable solution of boundary-value problems. The results of experiments on complex loading in the form of trajectory in the form of two-element trajectories for weakly hardened material are considered in this issue. An analysis of physical reliability of the simplest versions of plasticity theories for describing active processes of complex loading with bend point is given here. We provide analysis and recommendations as to preferable applications of classical theories for plane complex loading modes. It is shown that neglecting the loading process geometry lead to considerable errors in theoretical calculations.

1. Introduction
The essence of method for evaluating of physical reliability is in the fact that conventional structural calculations based on chosen variant of constitutive relations are supplemented by the construction of theoretical trajectories of stresses \( \bar{\sigma} = \bar{\sigma}(x_k, \lambda) \) and deformations \( \bar{R} = \bar{R}(x_k, \lambda) \) and a set of trajectories constructed in experiments on complex loading according to the program \( \bar{R} = \bar{R}(x_k, \lambda) \) or \( \bar{\sigma} = \bar{\sigma}(x_k, \lambda) \). Using these data, degree of accuracy of theoretical solution is estimated (\( \lambda \)-parameter of sequence of states, monotonically increasing in \( t, x_k \) is an arbitrarily given point inside body of the structure). The classification of plane complex loading processes can be carried out according to the well-known technique [1].

2. Theoretical approach
Let us consider plane stress state. We will proceed from the condition of vectors coplanarity \( \bar{\sigma}, d\bar{\sigma} \) and \( d\bar{R} \) [2, 3], which is written as:

\[
\begin{align*}
d\bar{\sigma} &= N d\bar{R} - (N - P) \frac{\bar{\sigma} d\bar{R}}{\sigma^2} \bar{\sigma} \\
d\bar{R} &= \frac{1}{N} d\bar{\sigma} + \left(1 - \frac{1}{N}\right) \frac{\sigma d\bar{\sigma}}{\sigma^2} 
\end{align*}
\]

(1)

(2)

here \( N \) and \( P \) are functionals along the arc length from curvature of the trajectory, \( \bar{\sigma} \) stress vector, \( \bar{R} \) strain vector.

It is important to note that in differential-linear theories of flow \( N \) and \( P \) are considered independent of angle of inclination \( \theta \) of stress vector to the tangent to the deformation trajectory, which, as calculations show, leads to a significant (about 30-40%) deviation of theoretical solutions.
from experiment. Considering that when solving plane problems (plane stress state, plane formation), loading or deformation trajectories are spatial, an algorithm for flattening these trajectories has been developed [1]. This algorithm in general can be used for processes of arbitrary curvature and small torsion.

Let us consider two-link trajectories of an active process of complex loading. P-M experiments (tension-torsion) were carried out on CL test facility machine of kinematic type. Material was steel-3. The specimens taken for experiments have inner diameter \( d = 24^{+0.07} \) mm, outer one \( D = 26^{+0.07} \) mm and length \( l = 216 \) mm. To eliminate the initial anisotropy, specimens were annealed in furnace up to 880\(^\circ\)C for 2 hours and then cooled with furnace to 680\(^\circ\)C. Further cooling was carried out in air to room temperature. The elastic constants are determined from a series of tensile and pure shear experiments on tubular specimens. Let's assume: \( E = 203067 \) MPa, \( G = 77499 \) MPa, \( \sigma_s = 239 \) MPa. Poisson's ratio in the elastic region is taken as \( \mu = 0.31 \), and in the plastic zone it takes on the value \( \mu = 0.5 \). The purpose of steel-3 is load-bearing and non-load-bearing elements of welded structures and parts operating at positive temperatures and variable loads.

The following diagram is adopted for weakly hardening steel-3 (vector space):

\[
\bar{\sigma} = \begin{cases} 2G \cdot s & \text{at } s < R_s \\ 2G \cdot s \cdot (1-\omega) & \text{at } s \geq R_s, \quad \omega = \lambda(1-R_s/l), \end{cases}
\]

Here \( \lambda = 0.97 \), \( R_s = 12.59 \times 10^{-4} \).

Four experiments were carried out - loading in the form of two-link broken lines with a break at an angle \( \theta = 90^\circ \) (deformation space). Before trajectory’s bend, the specimen was loaded with tensile force up to the value \( S_0 = 3.39R_s; 5.6R_s; 6.04R_s \) (preloading). Then, at fixed value of tensile force, torsion torque was applied. The experiments were carried out until normal voltage dropped to zero. As a result of experiment, character of local material softening and its vector properties are considered depending on loading history.

Theoretical calculations were carried out according to the theory of small elastic-plastic deformations of Il' yushin, the theory of Kadashevich-Novozhilov and theory of two-link broken lines. For the theory of small elasto-plastic deformations written in differential form in equations (1) or (2) we have [3]:

\[
N = \frac{\sigma}{R} = \frac{F(R)}{R} = 2G, \quad P = \frac{d\sigma}{dR} = \frac{dF(R)}{dR} = 2G_k
\]

For Kadashevich-Novozhilov theory with an ideal Bauschinger effect:

\[
N = 2G, \quad P = 6G \cdot g^2 \left[ 2G \cdot g + 3G \cdot g^2 - \sqrt{6G(\sigma - \sigma_f)}g' \right]
\]

Here \( g = \frac{G(\sigma - \sigma_f)}{2GR - \sigma} \).

The relationship between stresses and strains for two-link deformation processes is as follows [4]:

\[
d\bar{\sigma} = NdR - \left( N - P \right) dR \cdot \bar{R}_k^0 \bar{R}_k^0,
\]
\[
\begin{align*}
N &= \frac{d[\varphi]}{d\xi} \sin(\theta - \varphi) - |\sigma| \cos(\theta - \varphi) \frac{d\varphi}{d\xi} \\
\Phi &= \frac{\sigma R}{|\varphi|} \\
P &= \frac{d[\varphi]}{d\xi} \cos(\theta - \varphi) + |\sigma| \sin(\theta - \varphi) \frac{d\varphi}{d\xi}
\end{align*}
\]

(7)

here \(\theta\) - bend angle of strain trajectory, \(R^0_k\) - unit vectors of strain trajectory before the path bend.

For a long time, theory of two-link broken lines did not find practical application. When solving bifurcation stability problems, V.G. Zubchaninov succeeded in proposing successful approximations for determining the functions \(N\), \(\dot{\sigma}\) and in solving a number of specific stability problems, taking into account influence of complex loading. Backpressure method widely used for alloys processing [5] also, where complex loading effects are applied. The idea of ways to mitigate cutting-induced plasticity by controlling stress redistribution through optimisation of the contour cutting configuration is introduced in [6]. However, it should be noted that taking into account complex loading in problems today remains limited. For example, for multi-parametric loads that are often encountered in practice, construction of constitutive relations for multi-link processes requires additional studies of scalar and vector properties of materials. The issue of generalized Bauschinger effect, which plays an essential role when some link of trajectory corresponds to unloading with a subsequent transition to the region of plastic deformations, has also been little studied.

3. Experimental part

From the plotted graphs (Figures 1-6), it can be seen that no theoretical calculation takes into account local softening - a dive in the \(\sigma-\Delta s\) diagram after a bend point. This can be very important at calculations of material’s strength. These effects were studied in [7-13]

Figure 1. Characteristics of scalar properties of steel-3 at \(S_0=3.39R_s\).
On graphs (Figures 1, 3, 5) scalar properties of a material are shown. On graphs (Figures 2, 4, 6) vector properties of a material can be observed. In graphs obviously clear lack of “classic” theories.

Figure 2. Characteristics of vector properties of steel-3 at $S_0=3.39R_s$.

Figure 3. Characteristics of scalar properties of steel-3 at $S_0=5.6R_s$. 
Figure 4. Characteristics of vector properties of steel-3 at $S_0=5.6R_s$.

Figure 5. Characteristics of scalar properties of steel-3 at $S_0=6.04R_s$. 
The theories cannot reflect local softening of material (scalar properties) at bend point on strain trajectory. In some cases it can lead to considerable errors at stress strain state estimation. The same situation arises at description of vector properties of a material.

Earlier in experiments, it was noted [1] that at bend angles up to \( \theta = \pi/6 \), this does not significantly affect the calculations. But at \( \theta = \pi/2 \) there are significant discrepancies between calculations and experiment. As can be seen from (Fig. 1), vector properties of the material are better described by Kadashevich-Novozhilov theory and theory of two-link broken lines, which cannot be said in relation to the deformation theory. The maximum deviation of the calculated curves \( \sigma \sim s \) from the experimental ones increases with increasing \( s \). The trace of the retardation of the vector properties of steel-3 is determined in experiments from which we have:

\[
\begin{align*}
\lambda &= 3.39 \text{ at } s_0 = 3.8 \text{ } R_s, \\
\lambda &= 2.8 \text{ at } s_0 = 4.28 \text{ } R_s.
\end{align*}
\]

4. Conclusion
Since classical theories take into account the "dive" after the bend in the trajectory, the value of calculation error for these theories reaches 20% or more. When such loading processes occur in a body, it is preferable to use the theory of flow. In this case, the maximum deviation of the results of calculations according to various theories of plasticity from experiments occurs in the vicinity of the bend point of the trajectory.

Description within the framework of classical approach is probably possible with the imposition of certain restrictions. This concerns both development of plastic deformations and form of hardening of the material itself. More promising is the use of theory of Il’yushin processes, which is associated with the definition of plasticity functionals, which require a series of additional experiments.
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