Single-mode realizations of the Higgs algebra

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Abstract

In this paper we obtained for the Higgs algebra three kinds of single-mode realizations such as the unitary Holstein-Primakoff-like realization, the non-unitary Dyson-like realization and the unitary realization based upon Villain-like realization. The corresponding similarity transformations between the Holstein-Primakoff-like realizations and the Dyson-like realizations are revealed.

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In recent years, polynomial angular momentum algebra (PAMA) and its increasing applications in quantum problems have been the focus of very active research. This kind of PAMA, spanned by three elements $J_\mu$ ($\mu = +, -, 3$), has a coset structure $h + v$, where $h$ is an ordinary Lie algebra $U(1)$ generated by $J_3$; the remaining two elements $J_+, J_- \in v$ transform according to a representation of $U(1)$, and their commutator yields a polynomial function of $J_3 \in U(1)$. Hence, PAMA can be viewed as a deformation of an ordinary angular momentum algebra $SU(2)$.

In fact, the first special case of PAMA is the so-called Higgs algebra, here denoted by $\mathcal{H}$. In 1979, Higgs [3] found that there exists a kind of PAMA with an extra cubic term in the isotropic oscillator and Kepler potentials in a two-dimensional curved space. Zhedanov [4] presented a connection between the Higgs algebra $\mathcal{H}$ and the quantum group $SU_q(2)$ [5]. Daskaloyannis [6] and Bonatsos [7, 8] discussed the PAMA by means of generalized deformed oscillator respectively, and Quesne [9] related it to generalized deformed parafermion. Junker et al. [10] constructed (nonlinear) coherent states of $\mathcal{H}$ for the conditionally exactly solvable model with the radial potential of harmonic oscillator, and Sunilkumar et al. [11] did for the quantum optical model of four-photon process governed by quadrilinear boson Hamiltonian. Recently, Beckers et al. [12] discussed single-variable differential realizations of $\mathcal{H}$ and gave a unitary two-boson realization. Ruan et al. [13] studied indecomposable representations of the PAMA with a quadratic term, and then from these representations obtained its inhomogeneous one-, two- and three-boson realizations. In the present work we shall study in detail for $\mathcal{H}$ various single-mode realizations of importance in physical applications, which are generalizations of Holstein-Primakoff realization, [14] Dyson realization [15] and Villain realization [16] for $SU(2)$.

Let us begin with reviewing briefly the elementary results of boson operators [2]: $a^+$ and $a$ ($a$ is adjoint to $a^+$, i.e., $a = (a^+)^\dagger$, $a^+ = (a)^\dagger$) are creation and annihilation boson operators respectively, which, together with particle number operator $\hat{n} \equiv a^+a$, satisfy commutation relations

$$[a, a^+] = 1, \quad [\hat{n}, a^+] = a^+, \quad [\hat{n}, a] = -a.$$  \hspace{1cm} (1)

Furthermore, the complete set of basis vectors of Fock space, $\{|n\}; n = 0, 1, 2, \ldots\}$, may be constructed from $|0\rangle$ by using the definition

$$|n\rangle = \frac{(a^+)^n}{\sqrt{n!}} |0\rangle.$$  \hspace{1cm} (2)

In fact, these vectors are normalized eigenvectors of $\hat{n}$ belonging to eigenvalue $n$

$$\hat{n} |n\rangle = n |n\rangle,$$  \hspace{1cm} (3)

and satisfy

$$a^+ |n\rangle = \sqrt{n + 1} |n + 1\rangle, \quad a |n\rangle = \sqrt{n} |n - 1\rangle.$$  \hspace{1cm} (4)
Now turn to the Higgs algebra $\mathcal{H}$, whose three elements $\{J_+, J_-, J_3\}$ satisfy the following commutation relations

$$[J_+, J_-] = C_1 J_3 + C_3 J_3^3, \quad [J_3, J_\pm] = \pm J_\pm,$$

(5)

where $C_1$ and $C_3$ are arbitrary real numbers. When $C_1 = 2$ (or $-2$) and $C_3 = 0$, $\mathcal{H}$ defined by Eq. (3) goes back to SU(2) (or its non-compact type SU(1,1)). Similar to the Casimir commutation relations where $C_j$ is the constant eigenvectors of the matrix elements of $H$.

Making use of the very parallel treatment of angular momentum in quantum mechanics, we have adopted the similar phase factor as the Condon-Shortley rule of SU(2) so that $J_\pm \to \sqrt{C_1} J_\pm$.

It is worthy of reminding the readers that the constant $C_1$ in Eq. (3) is remained for convenience though it may become some fixed real number, say $q$, by rescaling the elements $J_\pm$: $J_\pm \to \sqrt{C_1} J_\pm$.

Mapping the sequence of half-integers onto a set of non-negative integers $n$, by the displacement

$$m = j - n,$$

(9)

it follows that Eq. (8) becomes, in a convenient notation with omitting the eigenvalue $j$,

$$\langle m+1 | J_+ | jm \rangle = \frac{1}{2} \sqrt{\frac{1}{C_1} [j(j+1) - m(m+1)] + \frac{1}{C_3} [j^2(j+1)^2 - m^2(m+1)^2]},$$

$$\langle m-1 | J_- | jm \rangle = \frac{1}{2} \sqrt{\frac{1}{C_1} [j(j+1) - m(m-1)] + \frac{1}{C_3} [j^2(j+1)^2 - m^2(m-1)^2]},$$

(8)

$$\langle jm | J_3 | jm \rangle = m,$$

$$\langle jm | C | jm \rangle = C_1 j(j+1) + \frac{1}{2} C_3 j^2(j+1)^2.$$

Here we have adopted the similar phase factor as the Condon-Shortley rule of SU(2) so that the matrix elements of $J_\pm$ given by Eq. (3) are real. In Eq. (3), $j$ may take half-integers, 0, 1/2, 1, 3/2,..., and for the finite dimensional representation with a fixed $j$, the values that $m$ may take, being a part of $\{-j, -j+1, ..., j\}$, are different for the different $C_1$ and $C_3$.
Comparing Eq. (10) with Eq. (4), we notice that the behavior of $J_± (J_-)$ acting on the basis vector $|n⟩$ to produce another basis vector $|n - 1⟩ (|n + 1⟩)$ is similar to that of the boson operator $a (a^+)$. Using Eqs. (11)-(13), we can obtain from Eq. (10) a single-mode realization of $\mathcal{H}$

\[
B^\text{HP}_1(J_+) = \frac{1}{2} \sqrt{(2j - \hat{n}) \{2C_1 + C_3[2j^2 - (2j - \hat{n} - 1)\hat{n}]\}}a,
\]

\[
B^\text{HP}_1(J_-) = \frac{1}{2} a^+ \sqrt{(2j - \hat{n}) \{2C_1 + C_3[2j^2 - (2j - \hat{n} - 1)\hat{n}]\}},
\]

\[
B^\text{HP}_1(J_3) = j - \hat{n}.
\]

This realization (11) preserves the unitary relation $B^\text{HP}_1(J_+) = (B^\text{HP}_1(J_-))^\dagger$. Taking $C_1 = 2$ and $C_3 = 0$ in Eq. (11) leads to the standard Holstein-Primakoff realization of SU(2), (14) which, together with the Villain realization to be discussed later, is frequently applied to solving various antiferromagnetic and ferromagnetic models. It can be seen from Eq. (10) that the space that the operators $B^\text{HP}_1(J_\mu) (\mu = ±, 3)$ act on is a subspace of the boson Fock space $\{|n⟩\}$ with $n$ limited in order that the value of the equation in the square-root operator $HP$ is Hermitian. Here for $j = 1, 2, 3, \ldots$, we call $B_k(J_\mu) (\mu = ±, 3)$ the realizations of simple type, quadratic type, cubic type and so on respectively owing to the fact that the action of $B_k(J_\pm)$ on the basis vector $|n⟩$ of the boson Fock space will give another basis vector $|n'⟩$ with $n' = n ± k$. The first commutation relation of Eq. (13) requires that $f_1(\hat{n})$ and $f_2(\hat{n})$ satisfy the following difference equation

\[
(\hat{n} + 1) \prod_{i=2}^k (\hat{n} + 2^{i-2} + 1)f_1(\hat{n})f_2(\hat{n}) - \prod_{i=1}^k (\hat{n} - i + 1)f_1(\hat{n} - k)f_2(\hat{n} - k) = C_1(j - \hat{n}) + C_3(j - \hat{n})^3,
\]

with the help of the relations

\[
(a^+)^k f_i(\hat{n}) = f_i(\hat{n} - k)(a^+)^k,
\]

\[
a^k f_i(\hat{n}) = f_i(\hat{n} + k)a^k, \quad i = 1, 2.
\]
For a given \( k \), the solution of Eq. (14) with \( k \) initial conditions

\[
\begin{align*}
\{ f_1(l) f_2(l) &= C_1(j - l) + C_3(j - l)^3; \quad l = 0, 1, \ldots, k - 1 \}
\end{align*}
\]  

is unique, for example,

\[
f_1(\hat{n}) f_2(\hat{n}) = \frac{1}{4} (2 j - \hat{n}) \left\{ 2 C_1 + C_3 [2 j^2 - (2 j - \hat{n} - 1) \hat{n}] \right\}
\]  

for \( k = 1 \), and

\[
f_1(\hat{n}) f_2(\hat{n}) = \frac{1}{16(n+1)(\hat{n}+2)} \left\{ 2 C_1 \left[ (-1)^{\hat{n}} (2 j + 1) - (\hat{n} + 1) \\ + (2 j - \hat{n}) (2 \hat{n} + 3) \right] + C_3 [1 + 6 j^2 (2 j - 1) \\ + (-1)^{\hat{n}} (2 j + 1) (2 j^2 + 2 j - 1) \\ + 2 \hat{n} (2 j - \hat{n} - 2) (2 j^2 - (2 j - \hat{n}) (\hat{n} + 2))] \right\}^{1/2} a^2,
\]  

for \( k = 2 \).

The above solutions show that we may have some freedom in the choice of the functions \( f_1(\hat{n}) \) and \( f_2(\hat{n}) \).

(1) If the unitary relation \( B_k(J+) = (B_k(J-))^\dagger \) need satisfying, that is, \( f_1(\hat{n}) = f_2(\hat{n}) \), then solving Eqs. (17) and (18) and substituting them into Eq. (13) respectively, we may regain \( B^\text{HP}_1(J_\mu) \) (see Eq. (11)) for \( k = 1 \), and obtain

\[
\begin{align*}
B^\text{HP}_2(J_+) &= \frac{1}{4} [(\hat{n} + 1) (\hat{n} + 2)]^{-1/2} \left\{ 2 C_1 \left[ (-1)^{\hat{n}} (2 j + 1) - (\hat{n} + 1) \\ + (2 j - \hat{n}) (2 \hat{n} + 3) \right] + C_3 [1 + 6 j^2 (2 j - 1) \\ + (-1)^{\hat{n}} (2 j + 1) (2 j^2 + 2 j - 1) \\ + 2 \hat{n} (2 j - \hat{n} - 2) (2 j^2 - (2 j - \hat{n}) (\hat{n} + 2))] \right\}^{1/2} a^2, \\
B^\text{HP}_2(J_-) &= \frac{1}{4} (a^+)^2 [(\hat{n} + 1) (\hat{n} + 2)]^{-1/2} \left\{ 2 C_1 \left[ (-1)^{\hat{n}} (2 j + 1) - (\hat{n} + 1) \\ + (2 j - \hat{n}) (2 \hat{n} + 3) \right] + C_3 [1 + 6 j^2 (2 j - 1) \\ + (-1)^{\hat{n}} (2 j + 1) (2 j^2 + 2 j - 1) \\ + 2 \hat{n} (2 j - \hat{n} - 2) (2 j^2 - (2 j - \hat{n}) (\hat{n} + 2))] \right\}^{1/2} a^+, \\
B^\text{HP}_2(J_3) &= j - \hat{n}
\end{align*}
\]

for \( k = 2 \). Similar to \( B^\text{HP}_1(J_\mu) \), the values of \( n \) in the matrix elements of \( B^\text{HP}_2(J_\mu) \) in the Fock space need limiting as well. We call Eqs. (17) and (18) the Holstein-Primakoff-like realizations of simple type and quadratic type of \( \mathcal{H} \) respectively.

(2) If the unitary relation need not satisfying, it follows from Eqs. (17) and (18) that the conventional choice \( f_2(\hat{n}) = 1 \) (or \( f_1(\hat{n}) = 1 \)) may immediately give rise to another kind of single-mode realization of importance

\[
\begin{align*}
B^\text{D}_1(J_+) &= \frac{1}{4} (2 j - \hat{n}) \{ 2 C_1 + C_3 [2 j^2 - (2 j - \hat{n} - 1) \hat{n}] \} a, \\
B^\text{D}_1(J_-) &= a^+, \\
B^\text{D}_1(J_3) &= j - \hat{n}
\end{align*}
\]
for $k = 1$, which may also be obtained by means of the approach adopted in Ref. [13], and

$$B_2^D(J_+^i) = \frac{1}{16(n+1)(n+2)} \{2C_1[(-1)^\hat{n}(2j + 1) - (\hat{n} + 1) + (2j - \hat{n})(2\hat{n} + 3)]$$

$$+ C_3[1 + 6j^2(2j - 1) + (-1)^\hat{n}(2j + 1)(2j^2 + 2j - 1)$$

$$+ 2\hat{n}(2j - \hat{n} - 2)(2j^2 - (2j - \hat{n})(\hat{n} + 2))]\} a^2;$$

$$B_2^D(J_-^i) = (a^+)^2,$$

$$B_2^D(J_3^i) = j - \hat{n}$$

for $k = 2$. When $C_1 = 2$ and $C_3 = 0$, Eq. (24) becomes the standard Dyson realization of SU(2) introduced originally by Dyson [15] in his study of spin-wave interactions, hence, we call Eqs. (24) and (25) the Dyson-like realizations of simple type and quadratic type of $\mathcal{H}$ respectively. Different from the Holstein-Primakoff-like realizations, (11) and (19), no square-root operator appearing in the Dyson-like realizations, (24) and (25), may not only avoid the convergence questions associated with the expansion of square-root operator but also make the value of $\hat{n}$ in $\{n\}$ unlimited, i.e., $n = 0, 1, 2,...$

Furthermore, it is not difficult to find that the non-unitary Dyson-like realizations $B_i^D(J_\mu^i)$ ($i = 1, 2$) may be related to the unitary Holstein-Primakoff-like realizations $B_i^{\text{HP}}(J_\mu^i)$ by their corresponding similarity transformations $S_i$:

$$S_iB_i^D(J_3^i)S_i^{-1} = B_i^{\text{HP}}(J_3^i) = B_i^D(J_3^i),$$

$$S_iB_i^D(J_\pm^i)S_i^{-1} = B_i^{\text{HP}}(J_\pm^i), \quad i = 1, 2. \quad (22)$$

Considering the unitary relations $B_i^{\text{HP}}(J_+^i) = \left(B_i^{\text{HP}}(J_-^i)\right)^\dagger$, we obtain from the second equation of Eq. (22)

$$U_i^{-1} \left(B_i^D(J_-^i)\right)^\dagger U_i = B_i^D(J_+^i), \quad i = 1, 2, \quad (23)$$

where $U_i = S_i^\dagger S_i$ are Hermitian operators. Note that $B_i^D(J_3^i)$ are already Hermitian. The equations (23) are called unitarization of the Dyson-like realizations. As an example, let us calculate concretely the explicit expression of $S_1$. The first equation of Eq. (22) implies that $S_1$ commutes with $J_3$ and is at most the function of $\hat{n}$, thus, calculating the matrix element of $\hat{n}$ between the basis vectors $\langle n - 1 |$ and $| n \rangle$ and using the former two equations of Eq. (20), we derive

$$\langle n | S_1 | n \rangle^2 = \frac{1}{4}(2j - n + 1)\{2C_1 + C_3[2j^2 - (2j - n)(n - 1)]\} \langle n - 1 | S_1 | n - 1 \rangle^2. \quad (24)$$

Solving Eq. (24) with the initial condition $\langle 0 | S_1 | 0 \rangle = q$ ($q$ is a real number), and then using Eq. (8), we obtain

$$S_1 = \pm \sqrt{q(\frac{-C_3}{4}\hat{n}(-2j)(-Z_\pm)\hat{n}(-Z_-)\hat{n}}.$$

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**Note:** The above text is a continuation of a mathematical derivation involving quantum mechanics and algebra, particularly focusing on the Dyson and Holstein-Primakoff realizations of SU(2). It includes equations and operations involving operators $B_i^D$, $S_i$, $C_1$, and $C_3$, and discusses the unitarity and similarity transformations between these realizations.
where \( Z_\pm \) have been given by Eq. (12), and \((q)_n\) is a operator function of \( \hat{n} \) for a fixed real number \( q \)

\[
(q)_n = q(q + 1)\ldots(q + n - 1),
\]

whose expectation value in the boson Fock space \(|n\rangle\) is in fact the usual Pochhammer symbol \((q)_n\) for a positive integer \( n \), i.e., \([n] (q)_n [n] \equiv (q)_n = q(q+1)\ldots(q+n-1)\), with setting \( \langle 0 | (q) \hat{n} | 0 \rangle = 1 \).

Finally consider the third kind of single-mode realization, which is based upon the Villain-like realizations of \( \mathcal{H} \) in terms of a coordinate \( X \) and the corresponding momentum operator \( P = -\frac{i}{\sqrt{2}} \frac{\partial}{\partial X} \). However, the Villain-like realization of \( \mathcal{H} \) is not unique, here we give two different realizations

\[
V_1(J_+) = \exp(iX)\sqrt{g_1^2 - \frac{1}{4}C_3[P(P+1)]^2 - \frac{1}{2}C_1(P + \frac{1}{2})^2},
\]

\[
V_1(J_-) = \sqrt{g_1^2 - \frac{1}{4}C_3[P(P+1)]^2 - \frac{1}{2}C_1(P + \frac{1}{2})^2} \exp(-iX),
\]

\[
V_1(J_3) = P,
\]

and

\[
V_2(J_+) = \frac{1}{2\sqrt{c_3}} \exp(iX)\sqrt{g_2^2 - (C_3P^2 + C_3P + C_1)^2},
\]

\[
V_2(J_-) = \frac{1}{2\sqrt{c_3}} \sqrt{g_2^2 - (C_3P^2 + C_3P + C_1)^2} \exp(-iX),
\]

\[
V_2(J_3) = P,
\]

with \( C_3 \neq 0 \), where \( g_1 \) and \( g_2 \) are the functions of the eigenvalue \( j \), their explicit expressions may be respectively chosen as

\[
g_1 = \pm \sqrt{\frac{3}{2}}C_1(j + \frac{1}{2})^2 + \frac{1}{4}C_3J^2(j + 1)^2,
\]

\[
g_2 = \pm \sqrt{\frac{3}{2}}C_2^2 + C_1C_3j(j + 1) + \frac{1}{4}C_3J^2(j + 1)^2
\]

in order that the corresponding Casimir operators, \( V_1(\mathcal{C}) \) and \( V_2(\mathcal{C}) \), have the same value as \( \langle jm|\mathcal{C}|jm\rangle \), i.e.,

\[
V_1(\mathcal{C}) = V_2(\mathcal{C}) = C_1j(j + 1) + \frac{1}{2}C_3J^2(j + 1)^2.
\]

By means of the equation

\[
\exp(i\theta X)P^k \exp(-i\theta X) = (P - \theta)^k,
\]

where \( \theta \) is a formal parameter, it is easy to check out the validity of the commutation relations (3) for the Villain-like realizations (27) and (28). Taking \( C_1 = 2 \) and \( C_3 = 0 \), the first Villain-like realization, Eq. (27), becomes the standard Villain realization of SU(2) \footnote{16} with the correspondence \( X \leftrightarrow \varphi, \quad P \leftrightarrow S_{X}^R \) and \( g_1 = s + \frac{1}{2} \).

Thus, substituting

\[
X = \frac{1}{\sqrt{2}}(a + a^+), \quad P = -\frac{i}{\sqrt{2}}(a - a^+),
\]
into Eqs. (27) and (28) respectively, we may obtain the following two single-mode realizations of $H$

\begin{align}
B_1^V(J_+) &= \exp\left[\frac{i}{\sqrt{2}}(a + a^+)\right]\sqrt{g_1^2 - \frac{1}{2}C_3\left[\frac{1}{2}(a - a^+)^2 + \frac{1}{\sqrt{2}}(a - a^+)\right]^2 - \frac{1}{2}}C_1\left[\frac{1}{2}(a - a^+) - \frac{1}{2}\right]^2, \\
B_1^V(J_-) &= \sqrt{g_1^2 - \frac{1}{2}C_3\left[\frac{1}{2}(a - a^+)^2 + \frac{1}{\sqrt{2}}(a - a^+)\right]^2 - \frac{1}{2}}C_1\left[\frac{1}{2}(a - a^+) - \frac{1}{2}\right]^2 \exp[-\frac{i}{\sqrt{2}}(a + a^+)], \\
B_1^V(J_3) &= -\frac{i}{\sqrt{2}}(a - a^+),
\end{align}

and

\begin{align}
B_2^V(J_+) &= \frac{1}{2\sqrt{3}}\exp\left[-\frac{i}{\sqrt{2}}(a + a^+)\right]\sqrt{g_2^2 - \left[\frac{1}{2}C_3(a - a^+)\right]^2 + \frac{1}{\sqrt{2}}C_3(a - a^+) - C_1]^2, \\
B_2^V(J_-) &= \frac{1}{2\sqrt{3}}\sqrt{g_2^2 - \frac{1}{2}C_3(a - a^+)^2 + \frac{1}{\sqrt{2}}C_3(a - a^+) - C_1]^2 \exp[-\frac{i}{\sqrt{2}}(a + a^+)], \\
B_2^V(J_3) &= -\frac{i}{\sqrt{2}}(a - a^+).
\end{align}

Obviously, the above two realizations are unitary.

In summary, we have obtained for the Higgs algebra the explicit expressions for the unitary Holstein-Primakoff-like realizations of simple and quadratic types, the non-unitary Dyson-like realizations of simple and quadratic types, and two unitary realizations based upon the Villain-like realizations, some of which can be found their prototypes for SU(2). It can be checked that all these single-mode realizations, (11), (19), (20), (21), (33) and (34), satisfy the commutation relations (5) of the Higgs algebra. Furthermore, we have revealed the fact that the Holstein-Primakoff-like realizations and the Dyson-like realizations of the Higgs algebra may be related by the corresponding similarity transformations. Due to the tight relations between boson operators and differential operators (4), for another example except for Eq. (32), $a \leftrightarrow \frac{d}{dX}$ and $a^+ \leftrightarrow X$, the differential realizations of the Higgs algebra may be obtained directly from the above various single-mode realizations. They are not given here. These results may be applied to some typical quantum mechanical systems characterized by the Higgs algebra to obtain the corresponding energy spectra, (nonlinear) coherent states and so on. The method adopted in this paper may be used to discuss multi-mode realizations of the Higgs algebra and to treat the more general PAMA, for example, with the commutator $[J_+, J_-] = \sum_{i=0}^n J_3^i$. These studies are now under way.

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