Kinematic Structures for Processing of Surfaces with a Circle Directrix and a Straight Line Generatrix (Part IV)

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Abstract. A body with a given geometry can be obtained by various manufacturing procedures, entailing, however, different surface quality, dimensional precision and (most importantly) productivity. Each body is characterised by one or more surfaces, simple or complex, as the case may be. It is known that a surface is characterised by a directrix and a generatrix, as well as by the modality of its physical achievement by means of the machines and tools uses for manufacturing. The directrix is obtained as a result of a primary motion and possibly one or more secondary motions. The generatrix is obtained as a result of one secondary motion, rarely more. If the generatrix is materialized on a tool, then a secondary motion is not necessary. In addition one or more auxiliary motions are needed. Other generation modalities not excluded, from the theory of surface generation on machine-tools it is known that the directrix, as well as the generatrix can be: materialized; generated by copying; kinematically generated as the trajectory of a point; kinematically generated as the envelope of a family of curves; generated by rolling; or programmed. Typically in literature generation “by copying” and “programmed generation” of the directrix and generatrix are not addressed distinctively, both being assumed as of the same nature. The recent evolution of industrial electronics and implicitly of machine-tools has determined a clear differentiation between the kinematics and construction of machine-tools (still) using generation by copying from a master and the kinematics of NC machine-tools. The paper is one of kinematic synthesis, oriented towards innovation-invention, and presents by means of examples, not necessarily known or typical, all six cases of generation of surfaces characterised by a straight line generatrix and a circle directrix obtained kinematically as the envelope of a family of curves.

1. Introduction
It is the destination of a machine-tool, as asserted in various definitions [1] [2], to manufacture components of pre-defined future technical systems. Manufacturing a part entails deliberate action, consisting of a sequence of certain steps in order to obtain a body of the desired geometry and characteristics in accordance with a known model or a “mental” image, described or graphically represented. The geometry of the part corresponds to one or several of its functions and is obtained by manufacturing or fabrication deploying other dedicated bodies or assemblies of bodies (tools, instruments, devices, equipment, and machines). The characteristics of a part, other than its geometry, can be very diverse and relate to its (mechanical, corrosion, etc.) strength, hardness, aspect and so on.

A body with a given geometry can be obtained by various manufacturing procedures, entailing, however, different surface quality, dimensional precision and (most importantly) productivity. Among
other attributes, each of the manufactured parts is characterised by one or more surfaces, simple or
complex, as the case may be. It is known [3] [4] [5] that a surface is characterised by a directrix and a
generatrix, as well as by at least one modality of its physical achievement by means of the machines
and tools uses for manufacturing.

Other generation modalities not excluded, from the theory of surface generation [3] on machine-
tools it is known that the directrix (D), as well as the generatrix (G) can be: materialized (M; m);
generated by copying (Co; co); kinematically generated as the trajectory of a point (Ci\textsubscript{tp}; ci\textsubscript{tp});
kinematically generated as the envelope of a family of curves (Ci\textsubscript{fc}; ci\textsubscript{fc}); generated by rolling (R; r);
programmed (P; p). Consequently the set of possibilities of achieving a surface characterised by a
certain directrix and a certain generatrix is given by the set of possible combinations of the modalities
of obtaining the respective directrices and generatrices [5] [6]. For example, Ci\textsubscript{tp}&m means
D(Ci\textsubscript{tp})&G(co), that is a modality of surface generation where the directrix D is obtained kinematically
as the trajectory of a point (Ci\textsubscript{tp}), and the generatrix G is obtained by copying (co). Parts of these
combinations are found in practice and are described by literature, as illustrations of known modalities
of surface generation [1] [3] [4] [7]. While any directrix-generatrix combination is theoretically
possible, only relatively few are also practically efficient. The systematic approach explores the entire
set of generation possibilities, applied or not (yet) in practice, hence being oriented towards discovery
and even inventics [8] [9]. The paper is limited to case of generation with a circle directrix
kinematically generated as the envelope of a family of curves.

Typically in literature [1] [3] generation “by copying” and “programmed generation” of the directrix
and generatrix are not addressed distinctively, both being assumed as of the same nature. The recent
evolution of industrial electronics and implicitly of machine-tools has determined a clear differentiation
between the kinematics and construction of machine-tools (still) using generation by copying from a
master and the kinematics of NC machine-tools.

Each motion is the result of the action (of the final effector element) of a generation or auxiliary
linkage, as the as the case may be. Depending on the way they are connected, linkages can be
independent or partially overlapping. The directrix is obtained as a result of a primary motion and
possibly one or more secondary motions. The generatrix is obtained as a result of one secondary
motion, rarely more. If the generatrix is materialized on a tool, then a secondary motion is not
necessary. In addition one or more auxiliary motions are needed.

Over time three modalities of obtaining a material body have been identified and utilised: a) by
material removal (in order to obtain a certain body the excess matter is removed from a body of
initially greater volume; b) by material redistribution (the matter of a body of initially the same
volume as the targeted volume is redistributed; or the matter of a body of initially greater volume is
redistributed, thus obtaining the desired body plus an excess that is removed), and c) by adding
material (matter is added to a body of initially smaller volume; or matter is added to fill an empty
cavity the form and volume of that are identical to those of the desired body). Various processes of
cutting, plastic forming and non-conventional technologies are utilized [7].

The examples given in this paper refer exclusively to the modality of “material removal”.

2. Generation with a circle directrix obtained kinematically as the envelope of a family of curves
and a straight line materialised directrix

Milling with the active cylindrical or tilted part of various mills, typically slab mills or side-and-face
cutters, surfaces generation is conducted with a materialised generatrix and a directrix obtained as the
envelope of a family of curves (family of identical epicycloids, prolonged, described by the tips of all
tool teeth). The directrix can be a circle, when the presence of a circular feed is required, carried out
either by the workpiece, the necessary motions are illustrated in figure 1, or the tool (the case of
“planetary milling” [2]).

The main motion I, conducted by the tool, and the circular feed II, conducted by the workpiece, are
both rotations. The directions of motions I and II are recommended to be opposing, so that, inter alia
the relative speed in the tangent point (the cutting speed) is obtained as the sum of the peripheral
speeds of tool and workpiece. The generation feed motion II participates in the forming of the directrix. As the generatrix is materialized, this machining does not require a feed linkage for obtaining the generatrix. The translation motions III, IV and V are auxiliary positioning motions.

Figure 1. Machining schematic for milling with a materialised generatrix of a cylindrical surface (example for case Ci&m).

The generated surface can also be an “interior” one, not just “exterior”.

In the case of planetary milling, motion II – circular feed, as all other motions, is carried out by the tool. This is the case most often when the workpiece is large and cannot or is inefficient to be actuated to conduct any generating or auxiliary motion.

Kinematic structure of machines for such processing is known [2].

3. Generation with a circular directrix obtained kinematically as the envelope of a family of curves and a straight line generatrix obtained by copying

A kinematic directrix is obtained most often as the envelope of a family of curves by milling and grinding, with a variety of directrix curves. The “circular directrix” case is a usual one. In order to obtain a straight line directrix by copying required are a master Sb (a materialised „programme”), on that the surface to be copied is tilted in relation to the axis of the workpiece by the same inclination as that of the conical generatrix of workpiece P, and a feeler Pp of the same profile as the generatrix of the tool S. Figure 2 illustrates a machining schematic for milling a surface described by a circle directrix obtained kinematically as the envelope of a family of curves, and a straight line generatrix obtained by copying.

Motion II is a circular feed carried out by the workpiece in order to obtain the whole circular directrix. Together with the tool’s axial feed III, a continuous motion and of small value compared to motion II, also the translation III’ is carried out by feeler Pp. Motions III and III’ are simultaneous and identical. Motion III’ of the feeler determines motion IV’ of the feeler, a translation perpendicular to motion IV conducted in its plane. Motion IV’ is transmitted by the same ratio as motion IV towards the axis of the tool. In this case the simultaneity and proportionality of motions III and IV ensure the obtaining of the desired straight line generatrix.

The master Sb can be replaced by a ruler, in which case the feeler Pp is replaced by a shoe.

Figure 3 shows a kinematical diagram of principle compatible with the considered machining schematic. This reveals certain particularities worth mentioning:
- motion II, the workpiece’s circular feed can be carried out towards either the left or the right. This is ensured by direction inverter INV2;
- motion III (implicitly also motions III’, IV’ and IV) can be discrete, what is ensured by the presence of intermitter device INT3. In the absence of this, motion III is continuous;
- motion III (implicitly also motions III’, IV’ and IV) can be conducted in both directions, a possibility ensured by inverter INV3;
- even if motion III is carried out, feeler Pp could not be actuated, in which case clutch C3’ is disengaged. This facility is not offered by the machining schematic in figure 3;
- also clutches C4 and C4’ can be provided for the possible decoupling of motions IV’ and IV, or only of motion IV, respectively;
- motion IV can be identical to motion IV’ or can differ from this by a non-dimensional proportionality factor $i_{C4}$, ensured, for example, by a lever system. This possibility is not captured in the schematic in figure 2;
- the auxiliary motions (V and VI) can be actuated mechanically or (only) manually.

**Figure 2.** Machining schematic for milling a conical surface, the straight line generatrix being obtained by copying (example for case $C_{iC}$&co).

**Figure 3.** A kinematic diagram of principle for the case $C_{iC}$&co presented in figure 2.

4. **Generation with a circle directrix obtained kinematically as the envelope of a family of curves and a straight line generatrix obtained kinematically as the trajectory of a point**

A possible example for the considered case is the finishing of an exterior cylindrical surface by grinding with the peripheral part of a cylindrical grinding wheel, figure 4. The tool $S$, the grinding wheel, carries out the main motion I and the radial feed IV, a positioning feed needed to ensure grinding at the imposed dimension. The workpiece $P$ carries out the circular feed II, necessary for obtaining (together with motion I) of the circle directrix, and axial feed III, continuous and of reduced speed by the axis of the part, necessary for obtaining the straight line generatrix of the workpiece. Figure 5 shows the kinematic diagram of principle corresponding to the considered case, minimal, and of strictly mechanical structure.

In many classical circular grinding machines of general use, the adjustment mechanism $i_{iI}$ is absent from the main linkage, or is very simple. Typically there is also no clutch $C_1$.

In the circular feed linkage the adjustment function represented by adjustment mechanism $i_{i2}$ is typically transferred to the actuating electric motor $M_2$, which is a DC motor allowing continuous speed adjustment. The presence of clutch $C_2$ is not imperative.

In order to achieve the axial feed motion III, a “mechanical” actuation as well as a manual one was provided by means of the same transformation mechanism $MT_3$. It has to be mentioned though, that in circular grinding machines the “mechanical” axial feed linkage has most often a hydraulic structure, the final element being a linear hydraulic motor, while for the manual axial feed a pinion-rack transformation mechanism is used, the two actuation modes being thus kinematically disjunct.

In order to obtain radial positioning motion IV, in the schematic in figure 5 only manual actuation is provided. In practice, however, often a second linkage is encountered for this motion, independent from the first one and of hydraulic structure.
5. Generation with a circle directrix and a straight line generatrix, both obtained kinematically as the envelope of a family of curves

This case can be exemplified by the machining schematic in figure 6, namely the milling of a surface with a circle directrix by means of a slab mill or the cylindrical part of a side-and-face cutter; the tool \( S \) is positioned tangentially to the cylindrical surface of workpiece \( P \) and carries out main motion \( I \), a rotation, and axial feed \( III \) along the workpiece axis, the latter being the motion that determines the achievement of the generatrix. The workpiece \( P \) carries out the circular feed \( II \) that determines the achievement of the directrix. To be satisfied is the condition that the axial feed (given by motion \( III \)) of the tool is small in relation to the circular feed of the workpiece, thus that a sufficiently small translation (of the order of tenths of millimeter) of the tool \( S \) along the direction of motion \( III \) corresponds to one revolution of workpiece \( P \). The machining schematic does not necessarily require perpendicularity between the axes of the tool and the workpiece.

Figure 7 presents a kinematic diagram of principle that satisfies all requirements for achieving the machining schematic of figure 6. The main linkage for the rotation \( I \) of the tool in a single direction includes a clutch \( C_i \), a brake \( F_i \), a speed control mechanism of the final shaft (that achieves an
adjustable final transmission ratio $i_{f1}$, a flywheel $V_1$ and, most probably, a motion transmission with a constant total transmission ratio $i_{C1}$. The function of flywheel $V_1$ can be assumed by another part of the main linkage, for example by one of the gears.

The linkage for the circular feed motion II of the workpiece includes an inverter mechanism $INV_2$, so that the revolution of the workpiece can be carried out to either the left or the right. The output speed is small and consequently the presence of a brake in the structure of this linkage is not required. In figure 7 only a mechanical actuation is indicated, but the possibility of manual actuation is also provided frequently.

The linkage for the axial feed motion III is provided with both mechanical and manual actuation, and it is by means of a clutch $C_{3a}$ that the branch of the two is selected, by that at a given time motion is transmitted to the final effector $E_{III}$ (a translation slide). Motion III can be carried out by either the tool or the workpiece.

In figure 7 distinctive, manually actuated linkages of identical structure are provided for auxiliary motions IV and V.

6. Generation with a circle directrix obtained kinematically as the envelope of a family of curves and a straight line generatrix obtained by rolling

It is known that rolling entails a mobile curve (rolling curve) that rolls without slippage over a fixed curve, called base. If the rolling curve does not carry out all motions entailed by rolling, than both rolling curve and base curve are mobile. The base and the rolling curve need to be mutually envelopable.

The base and the rolling curve can be most diverse, some most difficult or even impossible to generate kinematically. In most technical applications, the base and the rolling curve are easily generated plane curves, typically a straight line or a circle. The base and the rolling curve cannot be simultaneously both straight lines, but can be simultaneously both circles. Consequently, most technical applications of machining by rolling a resumed to the rolling of a circle over a straight line, or of a circle over another circle (at its interior or exterior, as the case may be), or of a straight line over a circle. In the particular case where the generatrix to be obtained is a straight line, then it is taken as the base, and the rolling curve will be a circle (or arc of circle) materialised as a generatrix on the tool used for machining. This situation is illustrated in the machining schematic presented in figure 8. Figure 9 presents a kinematic diagram of principle that ensures the specified necessary motions.

Figure 8. Machining schematic for milling a conical surface, example for the case $Ci_6_{&r}$.

Figure 9. A kinematic diagram of principle for the case $Ci_{6_{&r}}$ presented in figure 8.
The tool S carries out both the main motion I, a rotation motion, and motions III and IV that together ensure the rolling without slipping of the tool’s circle profile (its materialised generatrix) over the straight line generatrix of the machined workpiece P. Workpiece P carries out the circular feed II either as climb or conventional milling (in the same or opposite direction as the feed). Consequently the structure of the linkage needs to include an inverter mechanism. The tool S or the workpiece P carry out all or only one/some of the auxiliary motions V, VI and VII, which are positioning motions by the directions of the coordinate axes.

The main motion I, which is the revolution of the tool by its own axis, and the circular feed II of the workpiece are independent. The rotation III and translation IV have a rigid kinematic link and contribute together to the generation by rolling of the workpiece generatrix.

For the auxiliary motions V, VI and VII that ensure positioning along the directions of the coordinate axes linkages of minimal kinematic structures are provided, manually actuated from handwheels \( R_{m5} \), \( R_{m6} \) and \( R_{m7} \).

7. Generation with a circle directrix obtained kinematically as the envelope of a family of curves and a programmed straight line generatrix

As already mentioned, the directrix of a surface generated by milling is typically obtained as the envelope of a family of curves, most often a family of significantly prolonged cycloids. Consequently a schematic of milling machining can be used as an example in this case too.

Any of the machining schematics with a circle directrix obtained kinematically as the envelope of a family of curves and a straight line generatrix achieved kinematically (by copying, as the trajectory of a point, as the envelope of a family of curves or by rolling) can be converted into a machining schematic with a “programmed” straight line generatrix. Figure 10, however, exemplifies a slightly different schematic from the in figure 6, the generated surface being conical.

![Figure 10. Machining schematic for milling a conical surface, example for the case Ci6&p.](image)

![Figure 11. A kinematic diagram of principle for the case Ci6&p presented in figure 10.](image)

Because the generatrix is “programmed”, only motions III and IV, translations, need to be numerically controlled, as shown in the kinematic diagram of principle in figure 11. The main motion I and the circular feed motion of the workpiece II contribute together to obtaining the (continuously variably diameter) circle directrix described as the envelope of a family of curves.

Outside the machining time, motions III and IV can also have the role of auxiliary positioning motions. Only motion V, the tangential feed of the tool is exclusively an auxiliary positioning motion.
8. Conclusions

A body with a given geometry can be obtained by various manufacturing procedures, entailing, however, different surface quality, dimensional precision and (most importantly) productivity. Each body is characterised by one or more surfaces, simple or complex, as the case may be. A surface is characterised by a directrix and a generatrix, as well as by the modality of its physical achievement by means of the machines and tools used for manufacturing.

At present six modalities of achieving the directrices and generatrices are known and deployed: materialized; generated by copying; kinematically generated as the trajectory of a point; kinematically generated as the envelope of a family of curves; generated by rolling; and programmed. Typically in literature generation “by copying” and “programmed generation” of the directrix and generatrix are not addressed distinctively, both being assumed as of the same nature. The recent evolution of industrial electronics and implicitly of machine-tools has determined a clear differentiation between the kinematics and construction of machine-tools (still) using generation by copying from a master and the kinematics of NC machine-tools.

The set of possibilities of achieving a surface characterised by a certain directrix and a certain generatrix is given by the set of possible combinations of the modalities of obtaining the respective directrices and generatrices. Parts of these combinations are found in practice and are described by literature, as illustrations of known modalities of surface generation. Any directrix-generatrix combination is theoretically possible, however only relatively few are also practically efficient. The systematic approach explores the entire set of generation possibilities, applied or not (yet) in practice, hence being oriented towards discovery and even inventics.

At least one machining schematic corresponds to each combination of directrix and generatrix, depending on their nature and on the modality of their obtaining. Many of these schematics are known and frequently deployed in practice, while others – possibly just as many or more, the reserve of creativity being difficult to assess – are unknown. Revealing and analysing as many as possible of these is a contribution to the development of knowledge and can lead towards identifying new efficient machining schematics. Several kinematic diagrams of principle correspond to each machining schematic, as well as an undefined number of details kinematic diagrams of machine-tools capable of generating surface according to the respective machining schematic.

This paper represents a kinematic synthesis, oriented towards innovation and invention, and presents by means of examples – not necessarily known or usual – all six cases of generation of surfaces characterised by a straight line generatrix and a circle directrix obtained kinematically as the envelope of a family of curves. The obvious connection is emphasized between the machining schematic and the kinematic diagram of principle of a machine-tool, as a starting point in the kinematic synthesis of machine-tools with special destinations, either particular or more general. The subject is too rarely encountered in the landscape of current scientific research, and any new information represents a contribution to the development of knowledge in the field of machine-tool kinematic synthesis and beyond. In fact, the evolution of human society is in essence based on knowledge and seems to be aimed at developing and expanding knowledge, including its treasuring.

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