Topological Valley Photonics: Physics and Device Applications

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Topological photonics has emerged as a promising field in photonics that is able to shape the science and technology of light. As a significant degree of freedom, valley is introduced to design and construct photonic topological phases, with encouraging recent progress in applications ranging from on-chip communications to terahertz lasers. Herein, the development of topological valley photonics is reviewed, from both perspectives of fundamental physics and practical applications. The unique valley-contrasting physics determines that the bulk topology and the bulk-boundary correspondence in valley photonic topological phases exhibit different properties from other photonic topological phases. Valley conservation allows not only robust propagation of light through sharp corners, but also 100% out-coupling of topological states to the surrounding environment. Finally, robust valley transport requires no magnetic materials or the complex construction of photonic pseudospin and, thus, can be integrated on compact photonic platforms for future technologies.

1. Introduction

Since the pioneering works of Yablonovitch[1] and John[2] in 1987, it has been recognized that periodic photonic structures have the remarkable capability of manipulating the flow of light in much the same way, as semiconductors steer the passage of electrons in an electric current. As this idea was inspired by the concept of energy bandgap in semiconductors, Yablonovitch and John decided to name these periodic photonic structures as photonic crystals.[1,2] Extending their insight, studies in photonic crystals in the past three decades have remarkably changed the physics and applications in photonic communication and information processing with various photonic-crystal devices.[3] Traditional physics and device applications of photonic crystals are based on utilizing the limited photonic degrees of freedom (DOFs), such as frequency, polarization, and phase.

In fact, electrons can also have additional DOFs, such as spin and valley. The manipulation of spin DOF of electrons, rather than the charge current, for information storage, transmission, and processing, has led to the blooming field of spintronics.[4] A strong spin-orbital coupling, as in the quantum spin Hall (QSH) effect, can lead to a class of exotic materials called topological insulators[5,6] which are some narrow-gap semiconductors that are insulating in the bulk but conduct electricity along the surfaces via a class of topological edge states. These exciting discoveries in semiconductor electronics have also started a new chapter in photonics by stimulating the emerging field of topological photonics.[7–9] In particular, many “photonic topological insulators” have been proposed and demonstrated as analogues of topological insulators,[7–9] where the electronic spin DOF can be simulated by constructing photonic pseudospin. For example, in photonic topological insulators made up of bianisotropic metamaterials, photonic pseudospins can be constructed from superposition of different polarizations of light, and bianisotropy can play the role of spin-orbital coupling.[10]

A valley, which generically emerges at high-symmetry points of the Brillouin zone, refers to a local minimum in the conduction band or a local maximum in the valence band. In addition to charge and spin, an electron in a lattice is also endowed with a valley DOF, which specifies the valley that the electron occupies. As shown in Figure 1a–c, a 2D honeycomb lattice with broken inversion symmetry (F) contains K and K’ valleys whose low-energy excitations carry an angular momentum that produces a spin-like magnetic moment, thus the name of “valley pseudospin.” The possibility of using the valley DOF to store and carry information (similar to spin in spintronics) has led to conceptual electronic applications known as valleytronics.[11,12] The recent explosion in valleytronics has been largely enabled by the invention of graphene and, subsequently, many other 2D materials.
properties in photonic valley-Hall systems. Section 5 discusses the robustness of valley kink states. Section 6 introduces the phenomenon of topologically protected refraction of valley kink states. In Section 7, we discuss the physics induced by the interaction between spin DOF and valley DOF. Section 8 reviews the studies on photonic valley-Hall systems with broken time-reversal symmetry (T). In Section 9, we introduce more photonic valley-Hall systems on different platforms operating at different frequencies. Section 10–12 discuss the applications of topological valley photonics, including topological cavities and lasers (Section 10), robust on-chip communication (Section 11), and topological channel intersection and routing (Section 12). Finally, Section 13 concludes the review with brief discussions on future directions.

2. Valley Band Topology: Trivial or Nontrivial?

While we are interested in the topological features of valley physics, it is interesting that many early studies have judged the relevant valley systems as topologically trivial. This issue arises from the different characterizations of band topology, a major difference between valley topological systems and other topological systems. Note that the band topology classification itself is a major topic. Here, we only introduce the basics that are sufficient to understand topological valley photonics.

Let us consider a simple tight-binding lattice, as shown in Figure 1a. This model describes a graphene lattice with two sublattices (colored in red and blue in Figure 1a) having different on-site energies. The corresponding Hamiltonian is given by

\[ H = t \sum_{<i,j>} c_i^\dagger c_j + M \sum_{i} \varepsilon_i c_i^\dagger c_i \]  

Here, \( c_i^\dagger (c_i) \) is the creation (annihilation) operator, \( t \) is the nearest-neighbor coupling strength, and \( M \) is the on-site energy detuning. \( \sum_{<i,j>} \) sums over nearest-neighbor sites, and \( \varepsilon_i = \pm 1 \) when \( i \) is on different sublattices. Without on-site detuning (\( M = 0 \)), the lattice hosts massless Dirac cones at the corners of Brillouin zone. There are several approaches to lifting the degeneracy of the Dirac points, resulting in different topological phases. For example, one can introduce complex next-nearest-neighbor couplings that break T and open a bandgap. This procedure leads to the well-known quantum Hall phase. The quantum Hall phase is characterized by an integer-valued quantity called Chern number that captures the global topological properties of an energy band over the entire Brillouin zone. Alternatively, by including spin DOF and introducing spin-orbit coupling, a topologically nontrivial bandgap can also be induced. In this case, a topological insulator protected by T is created. Similar to quantum Hall phase, this QSH phase is also characterized by integer-valued global quantities, such as \( \mathbb{Z}_2 \) index or spin Chern number. The third method to gap out the Dirac points is using the on-site energy detuning (i.e., \( M \neq 0 \)). Now, the induced insulating phase is a valley-Hall phase, mediated by \( T \) breaking instead of \( T \) breaking or spin-orbit coupling. As we discuss in the following, this valley-Hall phase is characterized by a half-integer local topological index, not defined over the entire Brillouin zone, and exhibits so-called valley-contrasting physics.
The bulk energy dispersion near the two valleys (corners of the hexagonal Brillouin zone, labeled $K$ and $K'$ in Figure 1b) when $\mathcal{F}$ is broken is shown in Figure 1b, where two massive Dirac cones are presented. Although from energy dispersion the two valleys look similar, they, in fact, behave very differently. Let us first consider bulk properties. By inspecting the Bloch states near the two valleys, one can find that the Bloch states are self-rotating in phase. Moreover, this self-rotation at different valleys has opposite directions, as shown in Figure 1c. This behavior corresponds to valley-dependent orbital magnetic moments for electrons.\cite{46} In classical wave systems, similar vortex Bloch states with valley-dependent orbital angular moments can also be found.\cite{15,17,47}

In addition to orbital magnetic moment, Berry curvature also features valley-contrasting physics. In 2D, Berry curvature only has one component, which is defined by

$$\Omega_n(k) = \frac{\partial A_y}{\partial k_x} - \frac{\partial A_x}{\partial k_y}$$

(2)

where $A_n = -i\langle u_n | \partial_n | u_n \rangle$ is the Berry connection with $u_n$ being the periodic part of the Bloch wave function of the nth band. Chern number associated with an energy band is then given by the integration of Berry curvature over the Brillouin zone

$$C_n = \frac{1}{2\pi} \int_{BZ} \Omega_n d^2k$$

(3)

The value of Berry curvature is constrained by symmetries.\cite{48} In the presence of $\mathcal{T}$, the distribution of Berry curvature must satisfy: $\Omega_n(-k) = -\Omega_n(k)$. In addition, $\mathcal{F}$ requires $\Omega_n(-k) = \Omega_n(k)$. Thus, due to the presence of $\mathcal{T}$, Chern number is always zero for all bands. In that sense, this valley system is topologically trivial, carrying no topological charge.

However, the absence of $\mathcal{F}$ indicates that Berry curvature does not have to vanish everywhere in the Brillouin zone. In fact, Berry curvature has nonzero values around two valleys, taking opposite signs at opposite valleys, as shown in Figure 1d. To establish a bulk-boundary correspondence, one can define a bulk quantity called a valley-Chern number, whose definition is similar to Chern number except that the integration of Berry curvature is done only around the valleys. In the low-energy theory, a valley-Chern number takes the values of $\pm \frac{1}{2}$, with the sign determined by the sign of mass term induced by $\mathcal{F}$ breaking. In that sense, each valley takes a half-integer topological charge. The valley-polarized half-integer topological charge from the integration of Berry curvature not only induces an anomalous velocity for bulk wave packet,\cite{49} but also is responsible for the emergence of topological kink states as discussed in the following.

### 3. Edge States or Kink States?

In topological physics, we usually hear about topological edge states that arise at the edges of materials. Here, another potential confusion can occur in valley physics, because the topological states in valley systems typically do not arise at the edges.

Due to the fact that Chern number is zero, there are no protected gapless states at the edges (here, an edge refers to the interface between a valley-Hall lattice and vacuum). Figure 2a schematically shows the dispersion for a lattice with a slab geometry and zigzag terminations. As shown, there are no states inside the bandgap but only two flat bands connecting the band edges at different valleys. These flat bands are edge states (see Figure 2b), whose originality can be traced to the edge states in graphene ($M = 0$).\cite{49} However, due to the nonzero $M$, they are not pinned at zero energy by chiral symmetry anymore. It shall be pointed out that, although these edge states at a natural zigzag termination do not exhibit valley-contrasting physics, one can still locally modify the edges to tune the edge states from gapped flat bands to gapless valley-locked bands.\cite{50}

There is another type of boundary in valley systems: an interface between two lattices with opposite on-site detuning (i.e., opposite signs of $M$). The corresponding dispersion for this kind of setup is shown in Figure 2c, where two gapless states, one per valley, emerge. These gapless bands correspond to states localized at the interface between the two domains (Figure 2d).\cite{14,31,51–58} In this review, we refer to them as kink states to avoid potential confusion (note that in the literature, they are also referred to as “edge states”). Here, valley-contrasting physics arises due to the fact that the wavenumbers of these kink states are around the two valleys, and the kink states in opposite valleys have opposite group velocity (as can be judged from the slope of the dispersion). Thus, similar to the helical states in QSH phases, these kink states counter-propagate (Figure 2d), with the propagation direction locked to the valley DOF instead of spin DOF. The existence of the kink states is a consequence of nonzero Berry curvature around the two valleys and can be understood with the low-energy theory. In fact, the kink state solutions can be found in a domain-wall type Dirac equation where the mass term flips sign across the interface.\cite{59} As we have defined

\[
\begin{align*}
\Omega_n(k) &= \frac{\partial A_y}{\partial k_x} - \frac{\partial A_x}{\partial k_y} \\
C_n &= \frac{1}{2\pi} \int_{BZ} \Omega_n d^2k
\end{align*}
\]
the valley-Chern number for each valley, from the bulk-boundary correspondence, the difference between the valley-Chern numbers in the two domains then predicts the number of kink states arising at each valley.

As described earlier, a valley-Hall phase is quite different from other 2D topological phases such as quantum Hall phase and QSH phase in terms of topological characterization and boundary states. Some comparisons between these 2D phases are listed in Table 1. Apart from the aspects on topological invariant and inducing factors, which have been discussed earlier, here, we would like to point out the differences between QSH phase and valley-Hall phase. These two phases possess similar yet distinct counter-propagating boundary modes. One important difference is that while the propagation directions of helical edge states are locked to spin DOF, the propagation directions of valley kink states are locked to valley DOF. In addition, helical edge states in QSH phase are protected by $\mathcal{T}$, whereas valley kink states are only robust under certain situations (see Section 5 for more discussions on the robustness of valley kink states).

The valley band topology and valley-contrasting physics described earlier serve as the basis for various phenomena and applications in both electronic valley materials and photonic valley systems. These nice properties point to the fact that valley is a powerful DOF that can be used for the manipulation of electromagnetic waves with unconventional approaches, as we will review in the rest of this article.

4. Photonic Valley-Hall Systems

Topological valley photonics is part of the emerging field of topological photonics. Before the introduction of valley-Hall phases into photonics, many other topological phases have already been proposed and realized in photonics. In the following, we first briefly introduce other types of 2D photonic topological systems for comparison with photonic valley-Hall systems.

The first topological phase that was introduced in photonics is the quantum Hall phase. To realize a quantum Hall phase, one needs to break $\mathcal{T}$, which is, in general, a difficult task in photonics. One widely adopted approach is to use gyromagnetic materials. However, gyromagnetic effect becomes quite weak at optical frequencies, limiting the applications of this approach. Alternatively, Floquet engineering can also be used to generate photonic chiral edge states. However, again, the implementation, especially in a genuine 2D time-dependent system, is highly nontrivial. Despite the challenges in implementation, photonic analogs of quantum Hall phase have the best performance over all other photonic topological phases, as the chiral edge states in the quantum Hall phase are protected without any symmetries and are robust against any gap-preserving perturbations. Besides quantum Hall phases, another topological phase that has been widely studied in photonics is QSH phase. Photonic analogs of QSH phase are, in general, constructed by emulating QSH Hamiltonians (effective Hamiltonian or lattice Hamiltonian) with photonic pseudospins. Examples are coupled ring resonator lattices, bianisotropic metamaterials, and Kekulé-textured photonic lattices. In these systems, helical edge states formed by counter-propagating pseudospin states emerge at the boundaries. In the electronic case, these helical edge states are protected by $\mathcal{T}$ with $T^2 = -1$. In photonic systems, however, $\mathcal{T}$ alone cannot protect a QSH phase anymore, because now $T^2 = 1$. Instead, a pseudo-time-reversal symmetry that involves spatial symmetries is responsible for topological protection. Thus, one would expect that the photonic QSH phases are less robust than the quantum Hall ones. But, the implementation of course would be relatively easier. In addition, spin provides an extra DOF for wave manipulation.

Having introduced photonic quantum Hall and QSH phases, now, we turn our attention to photonic valley-Hall systems. In contrast to photonic quantum Hall and QSH phases, valley-Hall phases do not require breaking $\mathcal{T}$ or engineering pseudospins. To obtain a valley-Hall system, one simply needs to start with the photonic lattice hosting Dirac points such as a "photonic graphene" and then gap out the Dirac points through the reduction of $\mathcal{F}$. The first design of a valley photonic crystal (VPC) was proposed by Ma and Shvets. In their design, dielectric rods are arranged into a triangular lattice. When dielectric rods are in circular shape, the second and the third transverse electric (TE) bands form Dirac points at $K$ and $K'$ valleys. As the rods are deformed into triangular shape, as shown in the inset of Figure 3a, a bandgap opens, and a valley-Hall phase emerges (Figure 3a). Another popular VPC design is based on photonic honeycomb lattice. As shown by Chen et al., a VPC can be obtained by arranging dielectric rods with different radii into a honeycomb lattice (Figure 3b). In both of the designs, $\mathcal{F}$ is broken by tuning some geometrical parameters that are easy to control.
After these works and other early theoretical studies on VPCs,\textsuperscript{[16,18,19]} various photonic valley-Hall systems were soon realized in experiments.\textsuperscript{[20–26,28,30]} For example, using metallic tripods arranged in a triangle lattice, researchers demonstrated a VPC at microwave range (Figure 3c).\textsuperscript{[20]} In this design, the Dirac cones for TE and transverse magnetic (TM) modes can be made degenerated by careful design, thus allowing for studying the interplay between the valley DOF and a pseudospin DOF. VPCs can also be implemented in spoof plasmonic structures, as demonstrated in the previous studies\textsuperscript{[21,22]} (Figure 3d,e). Near-field mapping can be easily performed in these systems. These realizations, although demonstrated in microwave frequencies with metallic elements, can be easily extended to higher frequencies. This is because the basic design principle revealed in these studies is to tune structural parameters to break $\mathcal{I}$. Besides structural parameters, one can also tune material parameters, such as refractive index, to break $\mathcal{I}$ and obtain a photonic valley-Hall lattice. This approach was demonstrated by Noh et al. using a laser-written waveguide lattice.\textsuperscript{[23]} As shown in Figure 3f, this system consists of coupled optical waveguides, forming a honeycomb lattice. The refractive index of each waveguide can be controlled independently through the fabrication process. Thus, a refractive index detuning between the two sublattices can be introduced to create a valley-Hall lattice.

These systems introduced earlier reveal the simplicity in constructing photonic valley-Hall systems, and most subsequent works follow similar design principles. However, valley-Hall lattices are not limited to these designs and can be realized in other settings, such as the kagome lattice,\textsuperscript{[71,72]} the square lattice,\textsuperscript{[73–75]} and even amorphous systems.\textsuperscript{[76,77]} Due to this simplicity and flexibility in design, photonic valley-Hall systems have been widely studied, and various new phenomena and applications have been discovered. In the next four sections, we will review several important aspects of photonic valley-Hall systems, showing that they are simple but effective.

5. Robustness Too Weak?

In this section, we discuss an important issue in topological valley photonics: robustness of the kink states. Kink states living on the interface between two valley-Hall lattices with opposite settings are of particular interest in photonics due to their potential applications in robust waveguiding. Being inside a bandgap, the kink states are inherently protected from scattering into the bulk. In addition, kink states also exhibit a certain degree of robustness against path bends and weak disorders. Compared with chiral edge states in photonic quantum Hall phases, valley kink states are certainly less robust. However, again, the easy implementation of valley-Hall systems is an important advantage. The comparison between photonic QSH systems and photonic valley-Hall systems is a little bit subtle. As mentioned in the previous section, photonic QSH systems are not protected by $\mathcal{T}$. In fact, many photonic analogues of QSH phases have fragile topology.\textsuperscript{[78,79]} Thus, when comparing photonic valley-Hall systems with photonic QSH systems regarding robustness, the answer should be dependent on system details and disorder/perturbation types.

There are two typical types of interfaces: zigzag interface and armchair interface. While, for a zigzag interface, the
translational-symmetry breaking in the direction perpendicular to the interface does not mix the two valleys, it does for an armchair interface. As a result, the kink states on an armchair interface are gapped, with the gap size depending on system details. Now let us consider the effects of path bends. When the bend is 120°, this perturbation will not lead to intervalley scattering, as demonstrated in many studies. For example, in the previous study, researchers built a path with two 120° bends (see Figure 4a). The measured transmission for this path was found to be almost the same as the one for a straight path with the same length, as shown in Figure 4b. It is also possible to experimentally map out $E_z$ field distribution, as plotted in Figure 4d, which clearly shows the robust propagation of kink states around the shape corners. The experimental results are consistent with simulations shown in Figure 4c. Besides the 120° bends, bends with arbitrary angles can also be constructed, resulting in interfaces that can be

![Figure 4](image-url)
regarded as hybrid types of both zigzag and armchair interfaces. Researchers have also found that the kink states can even go through these interfaces that are incompatible with the lattice with little scattering. For instance, the previous study\(^\text{[21]}\) numerically investigated the propagation of kink states along paths with different bending angles. As shown in Figure 4e–h, no noticeable scattering losses are observed in all cases. In the previous study,\(^\text{[15]}\) this phenomenon is understood by considering the field overlaps of the modes belonging to different valleys. It is shown that for a zigzag-type perturbation, the overlap is zero. Even for a perturbation involving a random arrangement of unit cells, the intervalley scattering is only a higher-order term compared with the bandgap size. Thus, the kink states behave very robustly when going through path bends, not worse than QSH phases.

Besides the ability to pass through path bends, kink states have also shown good performance against disorders, although random spatial disorders will induce intervalley scattering. As studied numerically in the previous study,\(^\text{[80]}\) the kink states exhibit superior robustness against both position and frequency disorders when compared with a photonic QSH analog. More recently, researchers also numerically found that the kink states are quantitatively, by almost five times, more robust than standard conventional waveguides with small disorder levels.\(^\text{[81]}\) In fact, valley transport is even found to exist in systems without long-range order.\(^\text{[76,77]}\) Thus, as long as the short-range order around the interface approximately holds, the backscattering and localization induced by disorders would not be significant, and the robustness of kink states would be approximately valid. In addition, to engineer fully topologically protected valley transport, researchers have suggested using chiral vortex in a nonlinear polariton lattice.\(^\text{[82]}\)

6. Topologically Protected Refraction

Valley conservation not only leads to robust propagation across path bends inside the lattice, but also offers possibility of having perfect out-coupling of the kink states into the surrounding environment. One can imagine that when a kink state belonging to a certain valley couples to a vacuum region, in the absence of intervalley scattering, the kink state cannot get reflected to the other valley but can only couple to the vacuum space.

This unique phenomenon was first proposed in the previous study,\(^\text{[15]}\) where the authors studied the out-coupling of kink states through two different terminations: zigzag termination (see red-dashed line in Figure 5a) and armchair termination (see red-dashed line in Figure 5b). To determine the presence/absence of intervalley scattering in each case, the authors calculated the field overlap between the modes in two valleys and found that the intervalley scattering is absent for zigzag termination, but remains nonzero when the termination is armchair-type. The authors then performed full-wave simulations to demonstrate this phenomenon. In the simulations, a phased array of dipole sources was used to excite kink states at one specific valley. As shown in Figure 5a, the wave couples to the air region through a zigzag termination with nearly zero reflection. In contrast, for an armchair termination, noticeable reflection can be observed (Figure 5b).

The refraction of kink states is experimentally demonstrated in the previous study on the VPC shown in Figure 3c. In the experiments, the authors used a phased array of dipole antennas located at the center of the domain wall (see the white bars in Figure 5c–f for the positions of the source) to selectively excite kink states moving to the right. This system supports both TE and TM kink states at the same frequencies. The authors presented simulated and experimentally measured field distributions for both modes at 6.12 GHz to visualize this phenomenon (see Figure 5c–f). As shown, for both TE mode (Figure 5c) and TM mode (Figure 5e), the kink states can out-couple to the vacuum space through a zigzag termination without reflection.

Moreover, it is noticeable from Figure 5c–f that the kink state sometimes refracts into more than one beam. This can be understood through a phase matching process. As shown in Figure 5g,h, phase matching can be observed when the phase difference between the kink state and the vacuum field is zero. The phase matching phenomenon is consistent with the previous study.\(^\text{[76–78]}\)
in Figure 5g,h, the hexagon denotes the Brillouin zone of the VPC, with three dots $K_i$ ($i = 1, 2, 3$) referring to the $K$ valley. The red and blue circles denote the dispersions in the vacuum region for TE mode and TM mode, respectively. They have different radii due to the fact that the dispersions for TE mode and TM mode are different. For TE mode, the dispersion is given by $k_{\text{TE}} = \sqrt{(\omega/c)^2 - (\pi/d)^2}$, where $\omega$ is the angular frequency, and $d$ is the height of the VPC. While for TM mode, it is simply $k_{\text{TM}} = \omega/c$. Applying the phase-matching condition to the interface parallel to zigzag termination ($e_{\text{zig}}$) requires finding the empty-space wave vectors $k$ that satisfy $k \cdot e_{\text{zig}} = K_i \cdot e_{\text{zig}}$ and $|k| = k_{\text{TE,TM}}$. As graphically solved in Figure 5g,h, two solutions can be found for the TM mode but only one for the TE mode, consistent with experiment and simulation.

Topologically protected refraction of kink states provides an excellent solution to the problem of poor coupling between guided and free-space modes that generically arises due to impedance mismatch. Thus, this phenomenon can be useful in many devices across the electromagnetic spectrum, such as directional antennas and lasers.

7. Spin-Valley Physics

In addition to the physics arising solely due to valley DOF, the interaction between valley DOF and spin DOF can lead to many other interesting phenomena.\[1\] Although photons do not possess intrinsic electron-like spins, various pseudospins can be constructed in photonic systems. When these pseudospin DOFs couple with valley DOF, interesting spin-valley physics can emerge (hereafter, we do not distinguish between spin and pseudospin for brevity).

Before going to the specific designs, it would be helpful to illustrate the basic ideas with a simple Dirac Hamiltonian description. Consider the following effective Hamiltonian

$$H(\delta k) = H_0 + H_{\text{SOC}} + H_P$$

(4)

where

$$H_0 = v_D(\delta k_x \tau_z \sigma_x + \delta k_y \tau_0 s_0 \sigma_y)$$

$$H_{\text{SOC}} = \Delta_{\text{SOC}} \tau_z s_z \sigma_z$$

$$H_P = \Delta_P \tau_0 s_0 \sigma_z$$

(5)

Here, $\tau_x, \tau_y, \tau_z, s_x, s_y, s_z$, and $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices acting on valley, spin, and orbital DOFs, respectively. $\tau_0$ and $s_0$ are the corresponding identity matrices. $v_D$ denotes the group velocity, and $\delta k = (\delta k_x, \delta k_y)$ is the distance from the Dirac point. The term $H_0$ represents a massless Dirac Hamiltonian (see Figure 6a). The other two terms $H_{\text{SOC}}$ and $H_P$ are induced by spin–orbital coupling and $\mathcal{I}$ breaking, respectively, with $\Delta_{\text{SOC}}$ and $\Delta_P$ being the corresponding mass terms. When only one of the mass terms is

![Figure 6. Spin-valley physics from Dirac Hamiltonian perspective. a) Massless Dirac cones located at $K$ and $K'$ valleys in an unperturbed honeycomb system. b) When spin–orbit coupling is introduced, the massless Dirac cones transform into massive ones, exhibiting the QSH phase. c) When inversion symmetry breaking is introduced, the massless Dirac cones also transform into massive ones, but now exhibiting a valley-Hall phase. d) When both perturbations are present, spin-valley physics emerges. Now, both bandgap size and topology are controlled by the competition of the two perturbations. Adapted with permission.\[2\] Copyright 2018, The Authors.](https://www.advancedsciencenews.com/figure/6)
nonzero, the system becomes gapped with gap size proportional to the mass term and can host either a QSH phase (Figure 6b) or a valley-Hall phase (Figure 6c). When both mass terms are nonzeros (i.e., $\Delta_{SOC} \neq 0$ and $\Delta_P \neq 0$), spin-valley physics will emerge. First, the energy dispersion for the two spins will no longer be degenerated. According to Equation (4), the sizes of bandgap for two spins at two valleys are proportional to $|s\Delta_{SOC} + v\Delta_P|$, where $s = \pm 1$ stands for spin up/down states, and $v = \pm 1$ denotes $K$ and $K'$ valleys. Thus, the interplay between spin and valley DOFs will lead to spin-valley-locked bands, as shown in Figure 6d. Furthermore, now, the topological phases are determined by the relative size of the two mass terms. The valley Chern numbers for different spins are $C_{sp} = \frac{1}{2}\text{sgn}(s\Delta_{SOC} + v\Delta_P)$. When $|\Delta_{SOC}| > |\Delta_P|$, the spin Chern number, $C_i = C_{KK} + C_{KK'} = \text{sgn}(s\Delta_{SOC})$, is nonzero, resulting in a QSH phase. However, when $|\Delta_{SOC}| > |\Delta_P|$, $C_i$ vanishes, and a valley-Hall phase emerges. One way to induce spin-valley physics is to introduce valley DOF into a spinful photonic system (or equivalently, introduce a spin DOF into a photonic valley-Hall lattice).\[16,27–29,83–86\] Biaisotropic materials are commonly used to engineer pseudo-spins in photonic systems.\[10,16,27–29,36,87–92\] In these systems, TE modes and TM modes couple with each other, and their in-phase and out-of-phase superpositions form the two pseudospins. Using biaisotropic metamaterials, researchers have demonstrated several interesting spin-valley phenomena. For example, Dong et al.\[16\] designed a biaisotropic photonic crystal, as shown in the upper panel of Figure 7a. The rods colored in blue and purple have opposite biaisotropic coefficients, thus breaking $\mathbb{I}$ and introducing valley DOF into the system. Owing to spin-valley coupling, dispersions of different spin states will split in frequency level, leading to spin-valley-locked bulk bands (lower panel in Figure 7a). This unique dispersion leads to

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Spin-valley physics. a) Upper panel: schematic of a VPC with biaisotropy. Rods with different colors have opposite biaisotropy. Lower panel: the corresponding bulk band structure, where red (blue) curves denote spin-up (down) states. Adapted with permission.\[16\] Copyright 2017, Springer Nature. b) A photonic crystal with both spin and valley DOFs. Upper-left panel shows the unit cell, and upper-right panel displays the phase diagram when sweeping two geometry parameters. Here, the white dashed line corresponds to the gap closing points that separate two distinct topological phases. The lower two panels show the dispersions for two different heterostructures that exhibit QSH phase (left) and valley-Hall phase (right), respectively. Adapted with permission.\[28\] Copyright 2018, The Authors. c) Upper panel: design of a VPC with three layers. Lower panel: dispersions for two different types of topological phases that can emerge in this lattice. Adapted with permission.\[83\] Copyright 2019, John Wiley and Sons. d) Left panel: a heterostructure consisting of two domains, with the upper domain being a photonic QSH insulator, and the lower domain being a valley-Hall insulator. Right panel: the corresponding dispersion that shows the existence of spin-valley-locked interface states. Adapted under the terms of the CC BY 4.0 license.\[26\] Copyright 2018, The Authors, published by Springer Nature.}
\end{figure}
the phenomenon of valley-dependent splitting of two spin states in the bulk and allows for the manipulation of spin DOF with valley DOF (or vice versa).

The idea of studying spin-valley physics in bianisotropic metamaterials has also been realized in experiments. In the previous study,[28] the authors designed a photonic lattice made up of metallic tripods sandwiched by two metal plates, as shown in the upper-left panel of Figure 7b. The tripod naturally breaks the in-plane inversion symmetry and, thus, introduces the valley DOF. In addition, there is a small air gap between the tripod and the upper plate. This asymmetry along out-of-plane direction couples TE and TM modes and introduces spin DOF. Moreover, the strengths of the two symmetry breakings can be tuned by the tripod angle and the air-gap size, respectively. Thus, one can switch between a photonic QSH phase and a valley-Hall phase by tuning these geometry parameters (see the upper-right panel of Figure 7b). The authors then demonstrated that this structure can support both QSH-like interface states where the propagation direction is locked to spin DOF (see the lower-left panel of Figure 7b) and valley-Hall-like interface states where the propagation direction is locked to valley DOF (see the lower-right panel of Figure 7b).

Besides bianisotropic metamaterials, layered structures have also been adopted to study spin-valley physics.[83,84] In the previous study,[83] researchers developed a layered spoof surface plasmon lattice where layer pseudospin interacts with valley DOF. Their design consists of three layers, as shown in the upper panel of Figure 7c. The system’s topological phase is determined by two angles $\alpha$ and $\beta$ that control the geometry in lower and upper layers, as indicated in the upper panel of Figure 7c. The authors found two distinct topological phases, which are termed layered valley-Hall phase and conventional valley-Hall phase. In the layered valley-Hall phase, the interface states are layer-polarized, and the states in each layer form a valley-Hall-like dispersion (see the lower-left panel of Figure 7c). While in the conventional valley-Hall phase, the interface states are distributed in both layers, and the propagation direction is locked to valley DOF (see the lower-right panel of Figure 7c).

Instead of constructing a system with both spin DOF and valley DOF, one can also study the interplay between spin and valley using two lattices, one in the valley-Hall phase and the other one in the QSH phase.[18,20,25,26] In this case, only the interface between the two lattices can exhibit spin-valley physics. In the previous study,[26] researchers designed a heterostructure consisting of two domains. As shown in the left panel of Figure 7d, the upper domain is a photonic QSH lattice, and the lower domain is a photonic valley-Hall lattice. Boundary states localized at the interface between these two domains can be judged from the difference between the spin-valley Chern numbers,[93,94] as indicated in the right panel of Figure 7d. As a result of the interplay between spin and valley, spin-valley-locked interface states are found (see right panel of Figure 7d).

Aforementioned spin-valley physics is not only interesting from fundamental point of view, but also offers much more possibilities for wave control both in the bulk and on the boundaries. Systems with both spin DOF and valley DOF can be used to study fundamental physics that are hard to access in condensed matter systems, such as spin-valley-coupled Klein tunneling.[27] Moreover, these systems are also promising for applications in integrated photonics and information processing.

8. $T$-Broken Valley Physics

This section reviews the physics in photonic valley-Hall systems with broken $T$. In a system with $T$, the valley states must appear in pairs at the same frequency. For example, in graphene, a pair of valley states appear around two Brillouin zone corners ($K$ and $K’$). However, when $T$ is broken, these constraints are relaxed, which means the valley states can be unpaired. In photonic valley-Hall systems, the interplay between valley and $T$ breaking has been shown to lead to many interesting phenomena, such as topological phase transition between valley-Hall phase and quantum Hall phase and selective control of bulk valley states.[27,95,96] At the phase transition point, the system hosts a single Dirac cone, which exhibits unusual features, such as weak antilocalization,[97] one-way Klein tunneling,[27] and zero-index behavior (only when the Dirac cone sits at Brillouin zone center).[98]

$T$-broken VPCs can be realized using gyromagnetic photonic crystals. As proposed in the previous study,[27] a VPC with broken $T$ can be realized in a honeycomb lattice consisting of gyromagnetic rods. As shown in Figure 8a, the rods located at different sublattices have different rods, breaking $T$. Then, an external magnetic field applied to the VPC breaks $T$. These two symmetry breakings compete with each other. When their strengths are equal, the system hosts an unpaired Dirac cone at one of the valleys (see Figure 8b). In the previous study,[27] the authors showed that this system can be used to demonstrate novel phenomena, such as valley-polarized Klein tunneling and edge states in the continuum.

Liu et al. experimentally demonstrated such a gyromagnetic photonic crystal featuring unpaired valley state very recently.[99] In fact, this kind of unpaired valley state has been indicated in the famous Haldane model of a 2D honeycomb lattice, where the unpaired valley state appears near the transition point between a Chern insulator and a normal insulator.[13] Liu et al.‘s gyromagnetic photonic crystal is in a triangular lattice, and each unit cell comprises of a gyromagnetic rod surrounded by three dielectric triangular pillars (see Figure 8c,d). Tuning the rotation angles of the triangular pillars, the photonic crystal experiences a topological phase transition. By judiciously choosing the rotational angle, unpaired valley states appear near the corners of the Brillouin zone. An important experimental signature of the unpaired valley states is that when a beam of the electromagnetic wave is incident onto the photonic crystal with momentum matching with that of the unpaired valley state, the beam can pass through the photonic crystal (see Figure 8c). Otherwise, the beam will be totally reflected (see Figure 8d). Such a property could find applications in angular selective photonic devices and valley filters. We also note that the unpaired valley states were also realized in optical waveguide arrays, where the on-axis momentum plays the role of energy.[99]

$T$-broken valley-Hall systems can also be used to study the physics of topological Anderson insulator. The topological phase of a $T$-broken valley-Hall system is determined by the relative strength between $T$ breaking and $\mathcal{F}$ breaking. Thus, a disorder that effectively reduces $\mathcal{F}$-breaking strength can drive a topological phase transition.[100–102]
VPCs on Different Platforms

In the previous sections, we have discussed the basic physics of photonic valley-Hall systems. Now, we turn our attention to the related device applications. In this section, we first discuss the implementation of photonic valley-Hall lattices in different platforms at different frequencies. Then, in Section 10–12, we review the related applications. After the experimental verification of the existence of valley bulk and kink states in topological photonic valley-Hall lattices, the concept of topological valley photonics was transferred to many integrated platforms operating at different frequencies ranging from microwave, terahertz, and optical frequencies. At microwave frequencies, a substrate-integrated VPC was proposed and experimentally realized. Triangular scatterers are placed between two metallic plates, and there is a tiny gap between the scatterers and the top plates. The structure can be manufactured by the standard printed circuit board (PCB) technique and has the advantages of ultrathin thicknesses, excellent self-consistent electrical shielding, and compatibility with the conventional microwave substrate-integrated circuits. Recently, based on the substrate-integrated platform, a VPC featuring dual-band topological valley kink states was realized, which may find applications in multiband robust information communications and data processing. In addition, the substrate-integrated platform can also be utilized to devise photonic components with spin-Hall topological states. Another microwave VPC platform is based on dielectric materials. The dielectric scatterers are placed in a honeycomb lattice with broken inversion symmetry, which has an advantage of low absorption losses.

At higher frequencies, such as terahertz and optical frequencies, the realization of VPCs compatible with the existing on-chip photonic devices and standard fabrication techniques is crucial to push the photonic valley physics toward commercialization and everyday applications. There have been some attempts to realize the on-chip VPCs at terahertz and telecommunication frequencies. Two terahertz on-chip VPC platforms were experimentally demonstrated. One terahertz on-chip VPC is realized using all-dielectric materials. Triangular holes in a graphene-like lattice are patterned on high-resistivity silicon chips with extremely low metallic absorption. With compatibility with the silicon-on-insulator (SOI) platform and the silicon microelectromechanical systems (MEMS), the dielectric terahertz VPC has shown a promising future for information and communication technologies, including sixth-generation (6 G) communication, interconnects for intra- and interchip communication, and terahertz integrated circuits.

Another realization of the terahertz VPC was in a metal–semiconductor–metal structure. Quasi-hexagonal holes in a...
triangle lattice are patterned on the top metal and the semiconductor layer (see Figure 9b). Such an architecture is intentionally designed for an electrically pumped QCL, in which a voltage bias is applied between the bottom and top metallic layers.

At optical frequencies, suspended dielectric VPCs (see Figure 9c) and SOI-based VPCs (see Figure 9d) were experimentally realized. In comparison with the valley-Hall photonic topological insulators based on waveguide arrays, the VPC-based approach has many advantages, including ultralow metallic losses, large topological bandgap widths (relative band-widths up to 10%), in-plane propagation, very small footprints (with unit cells comparable to the operating wavelength), and compatibility with the standard complementary metal–oxide–semiconductor (CMOS) fabrication technique. These silicon-based VPCs with topological protection could provide an ideal integrated platform for the next generation of optical interconnects and communications.

Recently, topological valley plasmonic crystals for manipulating surface plasmons (including graphene plasmons) have also been theoretically studied. In addition, there have been some works discussing tunable VPCs, opening a door toward dynamically reconfigurable topological nanophotonic devices.

10. Topological Cavities and Lasers

In this section, we review the progress on constructing topological cavities and lasers using photonic valley-Hall systems. The photonic cavity is an essential component in modern photonic devices, ranging from lasers, interferometers, and sensors. However, the performance of the existing photonic cavities is sensitive to defects, disorder, and fabrication imperfections. The topologically robust cavities based on VPCs are, thus, highly desired. There have been several approaches to construct VPC-based cavities. One way is to create a topological interface in a closed form. This kind of cavities supports valley kink states running around the cavities with both clockwise (CW) and counter-clockwise (CCW) propagating modes. Due to the topological nature of the valley kink states, the cavity modes are robust against defects and sharp corners. An alternative approach to design the topological resonator is placing a judiciously oriented mirror at the termination of the topological interface (see Figure 10a). Due to the near conservation of the photonic valley pseudospins, the valley kinks states are reflected from the mirror, and their optical energy is localized at the mirror surface (see Figure 10b). Intuitively, this is because the valley flipping requires a considerable time delay. Such topologically controlled cavities are potentially realizable in metallic structures at microwave and terahertz frequencies and all-dielectric structures at optical frequencies.

The topological cavities based on the first approach have found applications in chiral quantum optics. Such topological cavities supporting two counter-propagating modes with opposite polarization can be strongly coupled to a solid-state quantum emitter. Due to the chiral coupling between the topological cavity and the quantum emitter, the emitter emits preferably into CW or CCW running modes depending on its spin (see Figure 10c). In Figure 10d,e, a quantum dot is placed at point A and excited by a pumping laser. The measured photoluminescence signal at the left grating clearly shows a single branch of the Zeeman split quantum dot spectrum, which is a hallmark of chiral coupling.
The topological cavities have also been proved to be very useful for topological lasers.\textsuperscript{[12,120–123]} In the previous study,\textsuperscript{[12]} Zeng et al. demonstrated a terahertz QCL based on the topological cavity with sharp corners (see Figure 11a). The QCLs are compact, electrically pumped semiconductor lasers operating at the mid-infrared and terahertz frequencies. The terahertz QCLs are one of the most important high-efficiency sources for terahertz radiation and have found a plethora of terahertz applications, ranging from communications, imaging, and sensing. The previous study\textsuperscript{[12]} showed that the lasing modes in the QCL based on valley-Hall kink states are running wave modes that are uniformly distributed around the interface. As shown in Figure 11b, the emission spectra from three different areas are almost the same, which indicates this running-wave feature of the lasing modes.

The topological valley-Hall lasers at optical frequencies have also been investigated experimentally in the previous studies.\textsuperscript{[120,122]} In the previous study,\textsuperscript{[120]} the authors realized a room-temperature laser from the triangle-shaped topological cavities formed by two domains of VPCs with opposite valley-Chern numbers (Figure 11c). Different from other works, the previous study\textsuperscript{[120]} identified a unique triad lasing mode whose emission exhibits hot spots at the three corners of the cavity (see Figure 11d). In the previous study,\textsuperscript{[122]} Noh et al. demonstrated a single-mode lasing action in a topological valley-Hall laser at telecommunication wavelength. The measured output power versus input power spectrum (Figure 11e) clearly shows a threshold behavior characteristic of lasing. Importantly, as shown in Figure 11f, there is only one single lasing peak above threshold (see curves “3” and “4” in Figure 11f), which indicates that this is a single-mode laser. Besides these experimental efforts, topological valley-Hall lasers have also been theoretically studied in nanoscale VPCs\textsuperscript{[122]} and optical waveguide arrays.\textsuperscript{[123]}

### 11. Robust On-Chip Communication

On-chip communication plays an extremely important role in modern information and communication technologies, such as photonic integrated circuitry and intrachip and interchip communication. The topological valley kink states hold great potentials for on-chip communication, which attributes to three key properties of valley kink states, including linear dispersion, single-mode propagation, and topological protection.\textsuperscript{[31]} The linear dispersion indicates a negligible signal delay at different frequencies, which enables a larger bandwidth; the single-mode property precludes mode competition (similar advantages can also be found in a single-mode optical fiber); the property of topological protection renders the valley kink states generally robust against fabrication imperfections, as long as these defects do not couple the states of different valleys.

Their potentials in terahertz on-chip communications were proved experimentally very recently (see Figure 12).\textsuperscript{[31]} Apart from the abovementioned three advantages of the valley kink states, the silicon-based design in the previous study\textsuperscript{[131]} has two additional merits, including high-density integration and low propagation losses. Due to the five advantages listed earlier, communication through a highly twisted topological interface in...
A large-scale terahertz photonic chip was experimentally demonstrated.\cite{31} The experimental setup is shown in Figure 12a. The experiments demonstrate error-free communication with a data transfer rate up to 11 Gbit/s around 0.33 THz (see Figure 12b). The clear eye diagram manifests the high-quality signal transmitted through the twisted topological interface (see the lower subfigure of Figure 12b). Such a high-speed error-free communication further enables real-time transmission of uncompressed 4K high-definition (HD) video (see Figure 12a).

Apart from waveguides, many other passive components can be realized in the terahertz on-chip VPCs, including topological channel intersections (acting as special beam splitters), reflectionless directional antennas based on the topological refraction, and topological cavities. Active components, such as compact electronic sources and detectors (e.g., terahertz resonant tunneling diodes), can also be integrated with the terahertz VPC platform. Therefore, the on-chip terahertz VPC could be an ideal integrated platform for front ends of the next-generation (6 G) communication and many other technologies. Finally, the valley kink states should also be excellent information carriers for optical communication, which requires further investigations in the future.

12. Topological Channel Intersection and Routing

In this section, we discuss the applications of valley kink states in routing, sorting, and splitting electromagnetic waves. Due to the valley chirality locking properties, the topological valley kink states can be used to devise some novel photonic devices that cannot be achieved using the conventional waveguide modes. An example is the wave routing, sorting, and splitting at...
topological channel intersection, where topological states coming from one channel couple to the topological states in other channels.\textsuperscript{[18,21,25,73,108,112,124,125]} Considering the lattice shown in Figure 13a, the orange and gray regions represent the VPCs with opposite valley Chern numbers.\textsuperscript{[103]} The cross-shaped channel intersection comprises both zigzag and armchair interfaces, where the valley kink states locked with different valleys are highlighted with red and blue arrows. In the channels, valley kink states travel only along the paths locked to the same valley. As shown in Figure 13b,c, the valley kink states launched

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure12.png}
\caption{Robust on-chip communication based on VPC. a) Demonstration of the uncompressed 4 K HD video transmission. b) Measured bit error rate as a function of data rate and eye diagram at 11 Gbit/s. Adapted with permission.\textsuperscript{[31]} Copyright 2020, Springer Nature.}
\end{figure}

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure13.png}
\caption{Topological channel intersection and topological routing. a) Schematic of the geometry-dependent topological channel intersections. The orange and gray regions represent the scatterers with opposite valley Chern numbers. b) Valley kink states excited at port 1 travel along paths 2 and 4 and are forbidden to enter path 3. c) Valley kink states excited at port 2 propagate only along paths 1 and 3. d) Schematic of the topological routing. e,f) Light coupled to WVG1/WVG2 will generate a CW/CCW phase vortex in the designed small disk and then be routed to the upper/lower domain wall. a–c) Adapted with permission.\textsuperscript{[103]} Copyright 2019, John Wiley and Sons. d–f) Adapted under the terms of the CC BY 4.0 license.\textsuperscript{[107]} Copyright 2019, The Authors, published by Springer Nature.}
\end{figure}
at port 1 (port 2) can only propagate along path 2 and path 4 (path 1 and path 3) and are forbidden to travel along path 3 (path 4). This phenomenon can be intuitively understood in the following way: when the valley kink states locked to K valley are excited from port 1, it cannot enter path 3, because the kink states at the two paths are locked to opposite valleys. However, they can enter path 2 and path 4, owing to the same valley.

Wave routing can also be realized at the intersection between topological waveguides and conventional waveguides[107] As shown in Figure 13d–f, when incident light couples to the WVG1/WVG2 input waveguide, it will generate CW/CCW phase vortex in the specially designed small disk. Due to valley-chirality locking of the valley kink states, the CW/CCW phase vortex can only couple to upper/lower topological domain wall. The valley kink states then couple back to the free space by the grating couplers.

13. Conclusion and Outlook

We have reviewed the development of topological valley photonics from both perspectives of fundamental physics and practical applications. Different from other photonic topological phases, such as the quantum Hall and QSH phases, the nontrivial topology in a valley-Hall topological phase is not defined globally over the entire Brillouin zone, but is valid only in the vicinity of a valley. This sharp difference determines that the bulk-boundary correspondence is applicable to topological states at the edges, in general, but at the domain walls between two valley photonic systems with opposite settings. Apart from the robust propagation through sharp corners, the valley conservation also allows 100% out-coupling of topological states to the surrounding environment, which can be useful for the enhancement of quantum efficiency in light-emitting devices. As valley transport does not rely on magnetic materials or the complex construction of photonic pseudospin, the valley structures can be integrated on compact photonic platforms for future technologies. On the fundamental level, although the physics of the valley-Hall phase is studied well in Hermitian and linear systems, there are still much to be explored in non-Hermitian and/or nonlinear ones.[32,120–123,126–132]

It might be true that the robustness of valley photonic topological states is weaker than that of some other topological states. However, there is always a trade-off between robustness and complexity. The valley structures are simple to design, compact in size, and compatible with modern photonics platforms. In that sense, topological valley photonics can still be considered as one of the most promising directions in topological physics where real technologies can arise in the near future. It is the purpose of this review to provide a brief outline of the main features of topological valley photonics, as a guidance and motivation for interested photonics engineers in related technologies.

Acknowledgements

The authors thank Qiang Wang for helpful discussions. This work was supported by the Ministry of Education, Singapore, under its Tier 3 Grant Award MOE2016-T3-1-006 and Tier 2 Grant Award MOE2018-T2-1-022 (S).

Conflict of Interest

The authors declare no conflict of interest.

Keywords

topological photonics, valley photonic crystals, valley-Hall phases

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