Cosmological inflation with orbifold moduli as inflatons

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ABSTRACT

Cosmological inflation is studied in the case where the inflaton is the overall modulus $T$ for an orbifold. General forms of the (non-perturbative) superpotential are considered to ensure that $G = K + \ln |W|^2$ is modular invariant. We find generically that these models do not produce a potential flat enough for slow roll to a supersymmetric minimum, although we do find a model which produces up to 20 e-folds of inflation to a non-supersymmetric minimum.

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The need for an inflationary stage \[1\] in the early universe is well known. (For reviews see ref.\[2\]) Cosmological inflation can solve the flatness and horizon problems and, because of de Sitter fluctuations during inflation, may also account for the observed density and temperature fluctuations. In the new inflationary universe scenario \[3, 4\] the expectation value of a scalar field (the inflaton) rolls slowly in a region of the effective potential which is extremely flat with a positive vacuum energy during inflation. Eventually, it evolves more rapidly and oscillates about the minimum of the effective potential converting the vacuum energy into radiation and reheating the universe.

Good candidates \[5, 9\] for the role of inflaton are the dilaton and moduli fields in orbifold or Calabi-Yau compactifications of string theory. To all orders in string perturbation theory their potential is completely flat \[10\]. Non-perturbative effects, such as gaugino condensation, can provide a non-trivial effective potential with variation on the Planck scale as the dilaton or modulus expectation value varies. Even then, the generic result to be expected \[6\] is that the number of e-folds of inflation \(N_e\) will be of order 1 rather than \(N_e \sim 60\) as required to succeed in solving the various cosmological problems.

There have been some attempts to employ the dilaton \(S\) as inflaton \[5, 9\]. Unfortunately, dilaton dynamics is little understood and, consequently, the conclusions reached have been largely qualitative. However, if the relevant dynamics is a multiple gaugino condensate \[5\] that fixes the dilaton expectation value at the minimum at a realistic value with regard to the strength of the gauge coupling constant, the effective potential may be sufficiently flat in the \(\text{Im}S\) direction. On the other hand, the potential would be expected to be very steep in the \(\text{Re}S\) direction \[11\].

Instead, we shall focus here on orbifold moduli as candidate inflatons. The reason for doing this is that moduli dynamics is more constrained because it is subject to modular symmetries which are believed to be symmetries of the theory not only to all orders in string perturbation theory but also when non-perturbative effects are included (see, for example, ref.\[12\] and references therein.) We study orbifolds rather than Calabi-Yau manifolds because the form of the moduli Kähler potential is not known in the Calabi-Yau case, except for large values of the moduli \(T\), because is difficult to calculate world-sheet instanton contributions \[13\]. We shall focus in the case of a single overall modulus \(T_1 = T_2 = T_3 = T\). Unlike some other authors \[14\] we treat the modulus field itself as the inflaton rather than some combination that involves a matter field scalar and also work in the context of the standard new inflationary scenario rather than in the context of hybrid inflation \[15, 16\].

It will be assumed that the effective superpotential is the sum of two components \[7\]. One of these components has a large scale, and gives an effective potential with unbroken supersymmetry and zero cosmological constant at the minimum when the other component is neglected. The large scale component is responsible for driving inflation when the modulus expectation value is in a flat region away from the minimum. The other component has a much smaller scale and is responsible for the supersymmetry breaking in the low energy world. It is the former component of the superpotential that we are interested in here. Neglecting the low energy component
of the superpotential, it is convenient to write the effective potential \( V \) in the form

\[
V = \mu^4 F(T/\tilde{M}_p)
\]

where \( F \) is of order 1,

\[
\tilde{M}_p \approx 2.44 \times 10^{18} \text{GeV}
\]

and \( \mu \sim 10^{16} \text{GeV} \)

in order to obtain density perturbations \( \delta \rho / \rho \sim 5 \times 10^{-5} \) as required by COBE data. This contrasts with a scale of \( 10^{10} - 10^{11} \text{GeV} \) if the component of the effective potential were due to the component of the \( W \) responsible for low energy supersymmetry breaking with soft supersymmetry breaking masses in the scale of \( 10^2 - 10^3 \text{GeV} \). At the end of inflation, we need the modulus \( T \) to have reached a supersymmetric minimum of the effective potential to avoid large supersymmetry breaking in the low energy theory and to avoid a large cosmological constant in the present universe. Thus, we require a model in which there is at least a local supersymmetric minimum of the effective potential with zero cosmological constant. We also require that the effective potential should have a maximum or saddle point in the vicinity of which the potential is sufficiently flat for 60 e-folds of inflation to occur. We shall come back to possible models after we have set out the conditions for slow roll and an estimate for \( N_e \) in terms of the effective potential and its derivatives.

The discussion of slow roll for the modulus field \( T \) differs from the standard case \([3, 4, 17]\) in two respects. First, \( T \) is a complex scalar field rather than a real scalar and rolls in a 2-dimensional space. Second, \( T \) has a non-minimal Kähler potential and corresponding non-minimal kinetic terms. (For reviews of supergravity and superstrings see ref. \([18]\).)

The tree level Kähler potential for the overall orbifold modulus \( T \) takes the form

\[
K = -3\ln(T + \bar{T})
\]

We shall allow for the possibility of stringy non-perturbative corrections \([19]\) and write

\[
K = Q(T + \bar{T})
\]

Then, the Lagrangian for \( T \) is

\[
\mathcal{L} = \frac{\partial^2 Q}{\partial T \partial \bar{T}} \partial_\mu T \partial^\mu \bar{T} - V
\]

Assuming a homogeneous field \( T \) with zero spatial gradients, the covariantized field equations are

\[
\frac{\partial^3 Q}{\partial T^2 \partial \bar{T}} T^2 + \frac{\partial V}{\partial T} + \frac{\partial^2 Q}{\partial T \partial \bar{T}} (\dot{T} + 3HT) = 0
\]

where \( H \) is the Hubble constant given by

\[
H^2 = \frac{\rho}{3} = \frac{1}{3} \frac{\partial^2 Q}{\partial T \partial \bar{T}} \dot{T} \dot{\bar{T}} + \frac{V}{3}
\]
in units where
\[ \kappa^2 = 8\pi G_N = \tilde{M}_p^{-2} \]  
has been taken to be 1, and neglecting the curvature of the universe. Writing \( T \) in terms of its real and imaginary parts
\[ T = T_1 + i T_2 \]
the slow roll equations are
\[ 6H \frac{\partial^2 Q}{\partial T \partial \bar{T}} \dot{T}_1 = -\frac{\partial V}{\partial T_1} \]  
and
\[ 6H \frac{\partial^2 Q}{\partial T \partial \bar{T}} \dot{T}_2 = -\frac{\partial V}{\partial T_2} \]  
If the energy density is dominated by the potential energy, so that \( H^2 \approx \frac{V}{3} \), as we shall assume in what follows, then we require
\[ \left( \frac{\partial^2 Q}{\partial T \partial \bar{T}} \right)^{-1} \left( \frac{\partial V}{\partial T_1} \right)^2 + \left( \frac{\partial V}{\partial T_2} \right)^2 \leq 1 \] (13)
For the slow roll approximation to be valid it is necessary to have
\[ \left| \frac{\dot{T}_1}{3HT_1} \right|, \left| \frac{\dot{T}_2}{3HT_2} \right| \ll 1 \] (14)
which can be cast as the sufficient conditions
\[ \frac{1}{6} |q|^{-2} \left| V^{-1} \frac{\partial V}{\partial T_1} \left( \frac{\partial q}{\partial T} + \frac{\partial q}{\partial \bar{T}} \right) \right| \ll 1, \]
\[ \frac{1}{6} |q|^{-2} \left| V^{-1} \frac{\partial V}{\partial T_2} \left( \frac{\partial q}{\partial T} - \frac{\partial q}{\partial \bar{T}} \right) \right| \ll 1, \]
\[ \frac{1}{6} |q|^{-1} \left| (V \frac{\partial V}{\partial T_1})^{-1} \left( \frac{\partial^2 V}{\partial T_1^2} \frac{\partial V}{\partial \bar{T}_1} + \frac{\partial V}{\partial T_1} \frac{\partial^2 V}{\partial \bar{T}_1 \partial T_2} \right) \right| \ll 1, \]
\[ \frac{1}{6} |q|^{-1} \left| (V \frac{\partial V}{\partial T_2})^{-1} \left( \frac{\partial^2 V}{\partial T_2^2} \frac{\partial V}{\partial \bar{T}_2} + \frac{\partial V}{\partial T_2} \frac{\partial^2 V}{\partial \bar{T}_2 \partial T_1} \right) \right| \ll 1 \] (15)
where we have written
\[ q \equiv \frac{\partial^2 Q}{\partial T \partial \bar{T}} \] (16)
and have dropped a condition identical to \( (13) \) It is also necessary to have
\[ \frac{1}{3H} \left| \frac{\partial q}{\partial T} \dot{T}_1 \right|, \frac{1}{3H} \left| \frac{\partial q}{\partial T} \dot{T}_2 \right| \ll 1 \] (17)
which when the slow roll is consistent can be cast as
\[
\frac{1}{6} |q^{-2} \frac{\partial q}{\partial T} V^{-1} \frac{\partial V}{\partial T}| , \quad \frac{1}{6} |q^{-2} \frac{\partial q}{\partial T} V^{-1} \frac{\partial V}{\partial T}| \ll 1
\]
(18)

If the slow roll starts from a point \( T_1 = (T_1)_0 , T_2 = (T_2)_0 \) close to a saddle point or maximum of the effective potential then we may write
\[
\frac{\partial V}{\partial T_1} \approx \left( \frac{\partial V}{\partial T_1} \right)_0 + \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0 (T_1 - (T_1)_0) + \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0 (T_2 - (T_2)_0), \]
\[
\frac{\partial V}{\partial T_2} \approx \left( \frac{\partial V}{\partial T_2} \right)_0 + \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0 (T_1 - (T_1)_0) + \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0 (T_2 - (T_2)_0)
\]
(19)

Writting
\[
X_\alpha = T_\alpha - (T_\alpha)_0 , \quad \alpha = 1, 2
\]
then, correct to linear order in the \( X_\alpha \), the slow roll equations may be written as
\[
\dot{X} = A - MX
\]
(21)
with
\[
X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \quad A = -\frac{q_0^{-1}}{6H} \begin{pmatrix} \left( \frac{\partial V}{\partial T_1} \right)_0 \\ \left( \frac{\partial V}{\partial T_2} \right)_0 \end{pmatrix}
\]
(22)
\[
M = \frac{q_0^{-1}}{6H} \begin{pmatrix} \left( \frac{\partial^2 V}{\partial T_1 \partial T_1} \right)_0 & \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0 \\ \left( \frac{\partial^2 V}{\partial T_2 \partial T_1} \right)_0 & \left( \frac{\partial^2 V}{\partial T_2 \partial T_2} \right)_0 \end{pmatrix}
\]
(23)

After we diagonalizing \( M \) we find that the displacements \( X_\alpha \) are superpositions of eigensolutions with time dependence \( e^{-\lambda_i t} \) where
\[
\lambda_i = \frac{q_0^{-1}}{6H} \mu_i, \quad i = 1, 2
\]
(24)
\[
2\mu_{1,2} = (N_{11} + N_{22}) \pm \sqrt{(N_{11} - N_{22})^2 + 4N_{12}^2}
\]
(25)
\[
N_{11} = \left( \frac{\partial^2 V}{\partial T_1^2} \right)_0, \quad N_{12} = \left( \frac{\partial^2 V}{\partial T_1 \partial T_2} \right)_0, \quad N_{22} = \left( \frac{\partial^2 V}{\partial T_2^2} \right)_0
\]
(26)

The number of \( e \)-folds of inflation may then be written as
\[
N_e = 2V_0 q_0 \min \left\{ -\mu_1^{-1}, -\mu_2^{-1} \right\}
\]
(27)

if \( \mu_1 \) and \( \mu_2 \) are both negative. Otherwise, \( N_e \) is controlled by the negative \( \mu_i \). (Notice that \( q_0 > 0 \) for correct sign kinetic terms.)

To avoid de Sitter fluctuations driving \( T \) across the slow roll region more rapidly than the semi-classical motion we require [20, 17]
\[
\left| \frac{\Delta T_1}{(T_1)_0} \right|, \left| \frac{\Delta T_2}{(T_1)_0} \right| \geq \frac{(2q_0)^{-1/2} N_e^{1/2} V_0^{1/2} \bar{M}_P^{-2}}{2\pi (T_1)_0}
\]
(28)
where $\Delta T_1, \Delta T_2$ quantify the width of the slow roll region. Putting $N_e \sim 60$ and demanding $\tilde{M}_P^{-2} V^{1/2}_0 \sim 10^{-4}$ for consistency with $\frac{\delta \rho}{\rho} \sim 5 \times 10^5$, (28) requires
\[
\left| \frac{\Delta T_1}{(T_1)_0} \right|, \left| \frac{\Delta T_2}{(T_1)_0} \right| \geq \frac{(2q_0)^{-1/2}}{(T_1)_0} 10^{-4} \tag{29}
\]

It is convenient to focus on the value of $N_e$ in what follows. When a small value of $N_e$ is obtained we would not expect the slow roll conditions to be satisfied and the small value of $N_e$ is a symptom of this rather than a reliable determination of $N_e$. The outcome for the number of e-folds of inflation depends on the choice of the (non-perturbative) superpotential and of the modulus Kähler potential. We shall take the superpotential to be of the form
\[
W = \Omega(S) \tilde{H}(T) \eta^{-6}(T) \tag{30}
\]
where
\[
\tilde{H}(T) = \left( j(T) - 1728 \right)^{m/2} j^{n/3}(T) P(j(T)) \tag{31}
\]
In (30), $\eta(T)$ is the Dedekind eta function and we do not specify $\Omega(S)$ which depends on the little understood dilaton dynamics. In (31), $j(T)$ is the absolute modular invariant, $m$ and $n$ are positive integers and $P$ is a polynomial. This form of $\tilde{H}(T)$ is the most general form consistent with $PSL(2, Z)$ modular invariance and the absence of singularities in the fundamental domain of the modular group [12] when the modulus Kähler potential given by (4) or transforms in the same way. Taking the dilaton Kähler potential to have the standard form
\[
K = -\ln(S + \bar{S}) \tag{32}
\]
(neglecting the effects of the small Green-Schwarz coefficient) the corresponding effective potential $V$ is given by
\[
V = \left( S + \bar{S} \right)^{-1} |\Omega(S)|^2 e^{Q(T; \bar{T})} |\eta(T)|^{-12} |\tilde{H}(T)|^2 \times \left\{ |F_S|^2 - 3 + \left( \frac{\partial^2 Q}{\partial T \partial \bar{T}} \right)^{-1} \left[ \frac{d \ln \tilde{H}}{dT} + \frac{3 \tilde{G}_2}{2\pi} + \frac{\partial Q}{\partial T} + 3(T + \bar{T})^{-1} \right]^2 \right\} \tag{33}
\]
where
\[
\tilde{G}_2(T, \bar{T}) = -2\pi(T + \bar{T})^{-1} - 4\pi \eta^{-1} \frac{d\eta}{dT} \tag{34}
\]
and
\[
F_S = 1 - (S + \bar{S}) \frac{d \ln \Omega}{dS} \tag{35}
\]
We shall take $F_S = 0$ so that the dilaton auxiliary field is zero and there is a possibility of finding a minimum with unbroken supersymmetry.
The case where the modulus Kähler potential is given by the tree level form (4) will be considered first. When the non-perturbative superpotential is of the minimal form given by (30) with $m = n = 0$, $P(j) = 1$, it is well known [21] that, $V$ has a single minimum with negative vacuum energy and broken supersymmetry (see Fig.1). As discussed earlier, both of the features are undesirable for the high energy component of the effective potential. Moreover, $V$ is not even positive at the maximum and there is no positive vacuum energy to derive inflation. However, the potential is flat enough for slow roll to occur. An expression of this flatness is that values of $N_e$ as large as $\sim 2.5$ can be obtained by rolling from close to the maximum if (unjustifiably) we replace $V_0$ by $|V_0|$ in (27).

The situation is different if at least one of $m$ and $n$ is positive in (31), but keeping $P(j) = 1$. A superpotential of this form may perhaps arise in gaugino condensate models with certain gauge non-singlet states becoming massless at some special value of $T$ [12] or through some other non-perturbative mechanism. Then, it is often possible for the effective potential to possess at least one minimum with zero vacuum energy and unbroken supersymmetry, though such minima are never the absolute minimum. Since we have no reason to exclude the possibility that we are in a long-lived metastable vacuum, we study the scenario in which the expectation value of $T$ rolls from near a saddle point or maximum of the potential to such a minimum. It is known [12] that, there is a local minimum at $T = e^{i\pi/6}$ with $V = 0$ and unbroken supersymmetry whenever $n \geq 2$. It is also known that there is such a minimum at $T = 1$ when $m \geq 2$. We have investigated the saddle points and maxima of the effective potential (with $P(j) = 1$) in case where there is a supersymmetric minimum with $V = 0$, see Figs 2,3,4. In all of these cases there are saddle points of the potential at which $V$ has very different (positive) values. Allowing the expectation value of $T$ to start rolling from close to a saddle point or maximum, we can use (27) to calculate the number of $e$-folds of inflation (assuming that the slow roll conditions are satisfied.) The maximum values of $N_e$ obtained are summarized in tables 1-2. It can be seen that except for $m = 0, n = 1$ in no case is $N_e$ greater than $10^{-2}$. In these circumstances, the value of $N_e$ is a symptom that the potential is not sufficiently flat for slow roll to occur rather than a reliable determination of the number of $e$-folds of inflation. As discussed earlier, the generic result for an effective potential with variation on the Planck scale with respect to the inflaton expectation value would be expected to be inflation with values of $N_e$ of order 1. The outcome here is non-generic but in the direction of smaller amounts of inflation (or no inflation at all) rather than in the direction of enhanced inflation. This is essentially a consequence of $\frac{j''}{j}$ and

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3 Note though that in the case $m = 2, n = 0$ (see Tables 1-2), besides the zero energy supersymmetric minimum we have a global negative energy minimum with SUSY broken. Also there is a negative energy local maximum around the vicinity of such minima at the fixed point of moduli space, $T = \sqrt{3} + \frac{1}{2} i$. Interestingly, if we allow $T$ to roll from the neighbourhood of such a point the potential is flat enough for slow roll to occur and we obtain a number of $e$-folds $N_e$, of order 1 to 10 if we again (unjustifiably) replace $V_0$ by $|V_0|$ in (27). The same discussion applies to case $m = 1, n = 0$ although in the latter case we do not have a supersymmetric minimum at all.
$(\frac{\lambda'}{\lambda})^2$ being in the range $10^2 - 10^3$ close to a saddle point or maximum. However, for $m = 0, n = 1$, i.e $W_{np} = \eta^{-6} j^{1/3}$ we obtain a much flatter potential with negative non-supersymmetric minimum at $T = 1.0$. In this case, the saddle point is at positive energy (fig.5) and we obtain a number of e-folds of order 1 to 10. In particular, if we start rolling from $T \sim 0.862 + 0.505$ we obtain $N_e \sim 20$. This is an interesting example even though it does not satisfy all of our constraints.

The Kähler potential may be modified by stringy non-perturbative effects [13]. In that case, we have considered some illustrative examples consistent with the Kähler potential continuing to transform in the same way under modular transformations as (4). This is required if

$$G = K + \ln|W|^2$$

(36)

with $W$ given by (30) and (31), is to remain modular invariant. The specific examples we have considered are

$$K = Q(T + \bar{T}) = -3\ln(T + \bar{T}) + a \ln (j(T) + \bar{j}(\bar{T}))$$

(37)

$$K = Q(T\bar{T}) = -3\ln(T + \bar{T}) + \tilde{a} (j(T) + \bar{j}(\bar{T}))$$

(38)

$$K = Q(T + \bar{T}) = -3\ln(T + \bar{T}) + \tilde{a} (T + \bar{T})|\eta(T)|^4$$

(39)

Similar results were obtained as in the case of minimal Kähler potential.

In conclusion, typically models with overall orbifold modulus as the inflaton do not produce a potential flat enough for slow roll. In the case $W_{np} = \eta^{-6} j^{1/3}$ slow roll does occur with as much as 20 e-folds of inflation. However, it does not appear possible to obtain a model which produces a minimum with $V = 0$ and unbroken supersymmetry when slow-roll occurs. Thus we are unable to explain the scale of the observed density fluctuations by cosmological inflation which preserves supersymmetry. However, it has been observed that inflation by about 12 e-folds can be sufficient to solve the moduli problem, namely the domination of the energy density by massive scalar fields which do not decay before nucleosynthesis; in the present context the $T$-moduli acquire masses $m_T \sim m_{3/2} \sim 1$ TeV when supersymmetry is broken. To solve the moduli problem [22] the potential $V_0$ at the end of inflation must satisfy $V_0^{1/4} < 10^7 - 10^8$ GeV. In our case, the potential at the supersymmetry-breaking minimum satisfies $V_0^{1/4} \sim \sqrt{m_{3/2} m_P} \sim 10^{11}$ GeV, so the moduli problem too remains unsolved.

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Table 1: Extrema of $V_{\text{eff}}$ for different choices of the integers $m, n$

| $m$ | $n$ | $V_{\text{saddle/max}}$ | $V_{\text{min}}$ | $T_{\text{max}}/T_{\text{saddle}}$ |
|-----|-----|-------------------------|------------------|----------------------------------|
| 0   | 0   | -8.41                   | -8.93            | $1 + \frac{1}{2} i/1 + n i, \frac{\sqrt{3}}{2} + \frac{1}{2} i$ | $\sim 1 + 0.3 i$ |
| 1   | 3   | $O(10^{11})$            | 0                | $\frac{\sqrt{3}}{2} + \frac{1}{2} i/ \sim 1 + 0.2 i$ |
| 2   | 0   | $O(10^8) / - O(10^7)$   | 0 / $- O(10^7)$  | $1.0 + 0.1 i, 1.0 + 0.3 i$ |
| 2   | 3   | $O(10^{14})$            | 0 / $- O(10^{12})$ | $\sim O(1) + \frac{1}{2} i$ |
| 0   | 1   | 3550.97                 | -1277.99         | $O(1)$ to $O(10)$ |

Table 2: $N_e$ for different choices of the integers $m, n$

| $m$ | $n$ | range of $N_e$             |
|-----|-----|-----------------------------|
| 0   | 0   | $\sim 2.5$                 |
| 1   | 3   | $2 \times 10^{-3} - 5 \times 10^{-3}$ |
| 2   | 0   | $\sim 10^{-2} s_{\text{saddle}} / O(1) - O(10)_{\text{max}}$ |
| 2   | 3   | $\sim 10^{-3}$             |
| 0   | 1   | $O(1) to O(10)$            |
Figure 1: $V_{eff1}$ for $m = n = 0$. 
Figure 2: $V_{eff}$ for $m = 1, n = 3$. 
Figure 3: $V_{\text{eff}}$ for $m = 2, n = 0$
Figure 4: $V_{eff}$ for $m = 2, n = 3$. 
Figure 5: $V_{eff}$ for $m = 0, n = 1$