Mathematical modelling and analysis of solar water collector with flat and V-groove absorber plate with helical and plain water flow path tubes

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Abstract. A mathematical model for Solar Flat Plate Collector (SFPC) has been developed to compare the thermal performance between different geometrical profiles of the collector like Flat plate, V-groove absorber plate and with plain and helical strip inserted in the absorber tube. Water, used as working fluid, is passed through the tubes to investigate the thermal performance. Different geometry of the absorber plate and tube are used to increase the heat gain by the collector. The temperatures at inlet and outlet are measured and analysis of collector recovery factor, heat gain, losses like bottom loss, edge loss and efficiency of collector has been done. The comparative study on the thermal performance shows 4\% higher collector efficiency for helical strip absorber tube than the other geometric profiles. Also, the results of mathematical analysis closely agree with the experimental results.

1. Introduction
Today’s scenario sees a tremendous consumption of electricity which in turn has depleted fossil fuels. As their availability is limited in nature, current trends focus on utilising naturally available renewable resources. Our country lies in the tropical zone where solar radiation is available in abundance. Latest researches focus on harnessing the potential uses of solar energy which in turn will help conserve fossil fuels. Solar thermal water heating system use the sun’s radiation to heat water directly. It is mainly used in application that require low temperature (50°C-80°C). Credits of this system is that it occupies less space and is easy to maintain. The converse side is that it is less efficient and undergoes few losses like loss from top, bottom, side and others. Researchers are going on to improve efficiency and minimize losses.

This research paper focuses on modeling and study of flat plate solar collector having varied geometry and improving heat transfer coefficient. Experiments were conducted with V-groove and flat plate absorber plate with plain and helical water flow path and results were noted. The developed mathematical model compares thermal performance and heat transfer coefficient of flat plate, V-groove plate with plain and helical strips. Many literatures on performance analysis of solar collectors were studied. Prabhu et al. [1] have come up with a mathematical model using Java based simulators to study the performance of Flat Plate Solar Collector having helical and plain flow path. Results of experiment and simulation agree very closely. It is also noted that the simulation has the feasibility of predicting performance of Solar Collector with Flat Plate. Selvanayagi Nevitha et al. [2] have also
made a study on the performance of solar collector with V-grooved and flat absorber plate by Java based simulation. A comparison of simulation and experimental results show a 5% increase in efficiency of collector that used a flat absorber plate.

Ranjith et al. [3] presents a comparative experimental, simulation and computational study of the processes that occur in a liquid flat plate solar collector having propylene glycol as working fluid. Four varied concentrations of this working fluid were used to study the efficiency of the collector both experimentally and computationally. The measured and computed results show close convergence. The model was stated to be efficient for verifying the overall efficiency of the system.

A mathematical model and solution strategy were presented by Fudholi et al. [4] for predicting efficiency of double-pass solar air collector. It is stated that the efficiency of the collector is highly dependent on mass flow rate and increasing the same would result in better efficiency. Rangababu et al. [5] has presented a numerical and thermodynamic analysis to improve the performance of solar flat plate collector using nanofluids, since they possess higher heat transfer coefficient. A numerical analysis was done by developing a 3D model and it is stated that the model predicts heat losses and fluid temperature with good accuracy.

Mulugeta Tadessa [6] has done performance analysis and project modeling of single pass V-grooved solar collector and double pass solar air collector with series and parallel flow to show their efficiency relationship. Numerical simulation using Discrete Transfer Radiation Model was done by Karanth et al. [7] to understand heat transfer capabilities of solar collectors. Numerical results incurred through CFD show that heat transfer increases and temperature of absorber plate decrease with raise in mass flow rate. Investigation using three different fluid flow using artificial neural network on the operation of flat plate solar collector has been modeled by Diez et al.[8].

The artificial neural network model built at the entrance was observed to be accurate for prediction of the outlet temperature of the working fluids and easy to apply on any solar collector. A study on V-corrugated absorber having multi-channels, applied to liquid flat plate solar collector was made by Fan et. al [9]. Results presented under different operating conditions comparing new and conventional collector indicate that new collector had almost 14% more thermal efficiency than the other. Test to compare daily energy performance of glazed flat plate collector and an evacuated tube collector is presented by Zambolin et. al [10].

The evacuated tube collector displays higher efficiency in wide range of operating conditions when compared to flat plate collector. A mathematical model is aimed in this paper for solar collector with flat plate having different geometrical profiles and presents a comparative study on the thermal performance between them.

2. Experimental setup

![Figure 1. SFPC with plain collector plate](image1)

![Figure 2. SFPC with V-groove collector plate](image2)
The experimental setup shown in Figure 1 comprises of two types of absorber plates made of copper; a flat absorber plate and a V-groove absorber plate mounted on a steel frame that has a glass cover of 4mm thickness on top and proper insulations at the bottom and the sides to reduce conduction losses. The collector area of the flat absorber plate is 0.320m$^2$ and that of V-groove absorber plate is 0.560m$^2$. Both the absorber plates have a thickness of 3mm and the thickness of bottom insulation is 0.025m. The operating fluid, water, is passed through tubes made of copper placed at a distance of 0.1m from each other. The heat tubes have an inner diameter of 0.0115m and an outer diameter of 0.127m. The inlet and outlet temperatures of the fluid, absorber plate temperature and mass flow rate are measured using necessary instrumentations connected to a data logger. The detailed experimental specifications are given in Table 1.

### Table 1. Experimental specifications

| Items                                      | Details                   |
|--------------------------------------------|---------------------------|
| Thermal conductivity of bottom insulation  | 0.04 (W/mK)              |
| Collector area (flat) $A_C$                | 0.320 m$^2$              |
| Collector area (V groove) $A_C$            | 0.560 m$^2$              |
| Thickness of bottom insulation             | 0.025 m                  |
| Edge area                                  | 0.390 m$^2$              |
| No of glass covers                         | 1                         |
| $\varepsilon_P$                            | 0.12                      |
| $\varepsilon_g$                            | 0.88                      |
| Thermal conductivity of copper plate       | 390 W/mK                 |
| Absorber plate thickness, $\delta$         | 0.003 m                  |
| Specific heat of water $C_P$               | 4180 J/Kg K              |
| Inner tube diameter                        | 0.0115 m                 |
| Pitch distance between tubes               | 0.10 m                   |
| Bonding Resistance $C_b$                   | 58 W/m K                 |
| Outer diameter of tube                     | 0.0127 m                 |

### 3. Mathematical modelling

Experiments were conducted for two different mass flow rates (1 LPM, 2 LPM). Heat rate and thermal efficiency were calculated. Polynomial regression method was used for mathematical modeling and results were compared with experimental data. Multiple iteration method was used to obtain data using developed regression equations and were compared with experimental data.

Heat rate $Q_u = A_0 + A_1 \Delta T + A_2 \Delta T^2 \quad (1)$

For Mass flow rate 1 liters per minute (MFR 1 LPM)

- $A_0 = 0.9187$
- $A_1 = 69.32$
- $A_2 = 0.00125$

For Mass flow rate 2 liters per minute (MFR 2 LPM)

- $A_0 = 0.9437$
- $A_1 = 139.125$
- $A_2 = 0.00125$

Thermal performance analysis equations are

Over all energy gained by collector

$$Q_u = A_C * FR [ I (\tau a) - U_L (T_{fi} - T_a) ] \quad (2)$$
where

\[ Qu = \text{Overall Energy gained by collector} \]

\[ FR= \text{Heat removal factor} \]

\[ Ae= \text{Cross sectional Collector area (m}^2) \]

\[ U_L = \text{Overall heat loss coefficient (W/m}^2\text{K}) \]

\[ T_{fi} = \text{Fluid inlet Temperature(K)} \]

\[ T_s = \text{Ambient temperature(K)} \]

\[ I = \text{Total radiation incident on the absorber plate (W/m}^2\text{)} \]

\[ \tau \alpha = \text{product of transmissivity and absorptivity} \]

Heat transfer removal rate

\[ FR = m^*c_p^* [1 - e^{\frac{-U_L}{m^*c_p^*}}] \]

(3)

Where

\[ m = \text{water mass flow rate (kg/sec)} \]

\[ c_p = \text{Water Specific heat in (J/kg K)} \]

\[ F^I = \text{Collector efficiency factor} \]

\[ U_L = \text{Overall heat loss coefficient (W/m}^2\text{K)} \]

Collector efficiency factor

\[ F^I = \frac{1}{U_L} \left( \frac{1}{U_L + (D+(W-D)/2)(1-c)

(4)

Where

\[ C_b = \text{Bond resistance (W/m K)} \]

\[ D_i = \text{Inner Diameter of absorber tube (m)} \]

\[ D = \text{Diameter of absorber tube (m)} \]

\[ W = \text{Distance of the absorber tubes from center (m)} \]

\[ F = \text{Fin Efficiency} \]

\[ K_{water} = \text{Water Thermal Conductivity (W/m K)} \]

\[ h_{fi} = \text{Heat transfer coefficient in inner surface of tube (W/m}^2\text{K)} \]

\[ Nu = \text{Nusselt Number} \]

Heat transfer coefficient in inner surface of the tube

\[ h_{fi} = \frac{Nu*K_{water}}{D} \]

(5)

Fin efficiency

\[ F = \frac{\tanh\left(\frac{W-D}{2m}\right)}{m\left(\frac{W-D}{2}\right)} \]

(6)

Where

\[ m = \sqrt{\frac{U_L}{k_B}} \]

(7)

Collector heat loss coefficient

\[ U_L = U_t + U_e + U_b \]

(8)

Where

\[ U_t = \text{Top Loss Coefficient (W/m}^2\text{K)} \]

\[ U_e = \text{Collector Edge Loss (W/m}^2\text{K)} \]
$U_b = \text{Collector bottom loss (W/m}^2\text{ K)}$

Heat coefficient from bottom loss

$$U_b = \frac{K_b}{x_b}$$  \hspace{1cm} (9)

Heat coefficient from edge loss

$$U_e = U_b \frac{A_e}{A_g}$$  \hspace{1cm} (10)

Top loss coefficient

$$U_t = \left[\frac{N}{C} \left(\frac{T_{pm} + T_a}{T_{pm} + T_a} - 1 \right) + \frac{1}{h_{in}} \right]^{-1} + \left[\frac{\sigma (T_{pm} + T_a) (T_{pm} + T_a)}{(C + 0.0005N) h_{in} + (2N + 1)}\right]^{-1}$$

where

C = 520 * (1 - 0.000051\beta^2), 0 < \beta < 70, for 70 < \beta < 90 \hspace{1cm} \text{take } \beta = 70

F = (1 + 0.089 \times 0.1166 \times \varphi) * (1 + 0.786\varphi)

c = 0.43 * (1 - 100/T_{pm})

N = \text{number of glass covers}

\sigma = \text{Stephan Boltzmann Constant}

$U_b = \text{Bottom loss coefficient (W/m}^2\text{ K)}$

$A_e = \text{Area of edge (m}^2\text{)}$

$A_c = \text{Area of collector (m}^2\text{)}$

$K_b = \text{Bottom insulation thermal conductivity of (W/m K)}$

$X_b = \text{Bottom insulation thickness (m)}$

I = \text{Total radiation (W/m}^2\text{)}$

Thermal efficiency of collector

$$\eta = \frac{\text{Overall Energy gained by collector}}{\text{Total radiation-Collector Area}} = \frac{Qu}{A_c \times T_{in}}$$  \hspace{1cm} (12)

Fluid mean temperature

$$T_m = \frac{T_p + T_{in}}{2}$$  \hspace{1cm} (13)

where

$T_m = \text{fluid mean temperature}$

$T_p = \text{Plate mean temperature}$

$T_{in} = \text{Inlet temperature}$

For fully developed laminar flow in a tube for a constant surface (Re < 2300), Nusselt Number will be constant 4.36 and Reynolds, Prandtl numbers will vary.

4. Result and discussion

Experimental data has been compared with the mathematical model data for heat rate and thermal efficiency by varying mass flow rate (1 LPM & 2 LPM). Variation of heat rate is shown in Figure 3 for the mass flow rate of 1 LPM and 2 LPM for Solar Flat Plate Collector with plain tube. As time progresses towards midday, the incident radiation of the sun falling on the collector surface increases, which in turn increases the heat rate across the collector surface. But as the mass flow rate increases, the heat rate decreases because of high heat loss across the collector from inlet to outlet. At around 12
noon when the radiation is at its maximum and with increase in mass flow rate, the heat rate with 1 LPM is 167W more than the heat rate with 2 LPM. After 12 noon, heat rate decreases due decrease in solar radiation. The developed model regression equations closely agree with the experimental data.

![Figure 3](image3.png)

**Figure 3.** Variation of heat rate for SFPC with Plain absorber plate (plain heat tube)

MODMFR 1/2LPM – Mathematical modeling for mass flow rate 1 liters per minute and 2 liters per minute.  
EXPMFR 1/2LPM – Experimental data for mass flow rate 1 liters per minute and 2 liters per minute

Figure 4 shows variation of heat rate for the mass flow rate of 1 LPM and 2 LPM for Solar Collector with Flat Plate having plain absorber plate helical strip inserted. Around midday, the incident radiation of the sun falling on the collector surface increases and so the heat rate across the collector surface also increases. But as the mass flow rate increases, the heat rate decreases because of high heat loss across the collector from inlet to outlet. At around 12 noon when the radiation is at its maximum and with increase in mass flow rate, the heat rate with 1 LPM is 202W more than the heat rate with 2 LPM. After 12 noon, heat rate decreases due decrease in solar radiation. The helical tube absorber plate has 35W more losses compared to plain tube absorber plate. The developed model regression equations are very much in line with experimental data.

![Figure 4](image4.png)

**Figure 4.** Variation of heat rate for SFPC with Plain Absorber plate (Helical strip inserted)
Figure 5. Variation of thermal efficiency of SFPC with Plain absorber plate

Figure 5 presents thermal efficiency variation of flat absorber plate for the varying flow rate of 1 LPM and 2 LPM. Thermal efficiency increases with solar radiation and decreases with sun down. Maximum thermal efficiency was 47% at 12 noon. Mathematical modeling data is closely compatible with experimental data.

Figure 6. Variation of thermal Efficiency of SFPC with V-groove absorber plate.

Figure 6 presents thermal efficiency variation of V-groove absorber plate for the varying flow rate of 1 LPM and 2 LPM. Variation of thermal efficiency is proportional to solar radiation. Maximum thermal efficiency was 70% at 12 noon. Mathematical data modeling fits favorably with experimental data. It is seen that there is raise in thermal efficiency by 23% due V-groove absorber plate profile and this due to increase in outlet temperature and heat transfer rate.

5. Conclusion

The following conclusions are drawn
- The developed mathematical model closely converges with experimental data with 4% deviation in heat rate and thermal efficiency.
- Heat rate decreases by 35W for absorber plate with plain tube when compared to helical strip
inserted on the tube.

- In case of thermal efficiency V-groove plate has 23% more maximum efficiency than the plain absorber plate with tubes.
- The modeling and experimentation can be modified to implement baffles in the absorber plate.

6. References

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