Mathematical model of a heat transducer with a cylindrical heat pipeline and with a focused heat source

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Abstract. The mathematical model of heat converters with a homogeneous extended cylindrical heat pipe is considered. Based on the developed mathematical models, the main characteristics of thermal converters are analyzed.

Currently, heat converters are widely used in various systems to control and control the speed and flow rate of gas flows. According to the principle of construction, thermal converters are divided into converters of calorimetric, thermal boundary layer and hot-wire type. At the present time, monitoring and control systems place high demands on the measuring transducer of gas flow parameters for accuracy, sensitivity, reliability, cost and ease of manufacture. In addition, a number of control and management systems require that the gas flow parameter converters be multifunctional and allow receiving output signals both about the speed and flow rate, and about the flow temperature, direction, and also in some cases about the presence of a gas flow.

The basic physical model of this converter for analysis can be represented in the form shown in Fig. 1.

For the considered thermal converter of humidity of liquid materials, the main conditions for the design are:
- the heating element should be low inertia, provide a sufficient heat flow, which propagates along the heat conduit and enters heat exchange with the flow of liquid material.

Fig. 1. Physical model of a heat converter with a cylindrical heat pipe 1 and with a concentrated heat source 2: 1 - heat pipe; 2 - thermosensitive element; 3 - heating element
- the heating element must be suitable for stationary and non-stationary modes of operation of the heat converter.
- the thermosensitive element should also be low inertia, or small in size and high sensitivity.
- a cylindrical electric wire must also have small geometric dimensions and must be made of a material with high heat conductivity and low heat capacity.

A feature of the considered heat converters is that the temperature is distributed along the heat conduit and depends both on the type of heating element (heat source) and on the controlled humidity of the liquid material, which in the form of a flow describes the heat conduit of the transducer during heat transfer at a constant flow rate \( V = \text{const} \). The above is required to analyze the thermal system of the conversion under consideration for such a research area. It is most effective for analyzing the basic characteristics of thermal converters of humidity of liquid materials using the matrix method on theory and thermal stability.

The matrix equations for determining \( T(x) \) and \( \Phi(x) \) along the heat pipe are

\[
\begin{bmatrix} T(x, p) \\ \Phi(x, p) \end{bmatrix} = \begin{bmatrix} A(x, p) & B(x, p) \\ C(x, p) & D(x, p) \end{bmatrix} \begin{bmatrix} T(0, p) \\ \Phi(0, p) \end{bmatrix},
\]

(1)

\[
A(x, p) = ch[(p)x]; B(x, p) = Z(p)sh[\gamma(p)x]; C(x, p) = \frac{1}{Z(p)}[sh[\gamma(p)x];
\]

\[
D(x, p) = ch[\gamma(p)x].
\]

\[
\gamma(p) = \sqrt{r(cp + g)}; Z(p) = \frac{r}{\sqrt{(cp+g)}}.
\]

Thermal parameters per unit length:
1) Conductivity:

\[
g = \alpha \pi d,
\]

(2)

where: \( \alpha \) – heat transfer coefficient from heat conduit to flow;
\( \pi = 3,14; \)
\d – diameter of cylindrical heat conduit;

1) Heat Resistance:

\[
r = \frac{1}{\lambda \pi d},
\]

(3)

where: \( \lambda \pi d \) – thermal conductivity of the heat transfer material;
\( F \pi \) – area of material heat conduit;

Heat capacity:

\[
C = \rho C_p F,
\]

(4)

where: \( \rho \) – heat conduit material density;
\( C_p \) – enhanced heat capacity of the material of the heat conduit;
\( F \) – cross-sectional area of the heat conduit.

Based on the equations for determining \( T(x) \) and \( \Phi(0) \) will receive

\[
T(x, p) = T(0, p)ch[\sqrt{r(cp + g)x}] - \Phi(0, p) \frac{r}{\sqrt{(cp+g)}} sh[\sqrt{r(cp + g)x}]
\]

(5)

and

\[
\Phi(x, p) = -\frac{r(0, p)}{r}sh[\sqrt{r(cp + g)x}] + \Phi(0, p)ch[\sqrt{r(cp + g)x}].
\]

(6)

Because
$T(0, p) = Z_c(p) \Phi(0, p)$. 

And in the thermal circuit the attenuation is large, you can equate the thermal resistance $Z_c(p)$ in which the resistance $Z_{\text{in}}(p)$ 

$$Z_{\text{in}}(p) = Z_c(p) \sqrt{\frac{r}{cp+g}} \tag{7}$$

Based on the foregoing, you can write

$$T(x, p) = \Phi(0, x) \sqrt{\frac{r}{cp+g}} \cdot ch \left[ \sqrt{r(cp+g)x} \right] - sh \left[ \sqrt{r(cp+g)x} \right] = \Phi(0, x) \sqrt{\frac{r}{cp+g}} \cdot \exp \left[ -\sqrt{r(cp+g)x} \right] \tag{8}$$

The transition from the image according to Laplace (2.10) to the original, you can get a formula for assessing the dynamics of measuring the distribution of temperature over time.

$$T(x, \tau) - T(0) = P_H \sqrt{\frac{r}{cp+g}} \left[ e^{\sqrt{g}r x} \text{erfc} \left( \frac{x}{2 \sqrt{\tau} - \sqrt{g} \sqrt{\tau}} \right) - e^{\sqrt{g}r x} \text{erfc} \left( \frac{x}{2 \sqrt{\tau} + \sqrt{g} \sqrt{\tau}} \right) \right] \tag{9}$$

For the stationary mode, the distributed $T(x)$ has the form as $\tau \to \infty$

$$T(x) - T(0) = P_H \sqrt{\frac{r}{g}} \exp(-\sqrt{g}x) \tag{10}$$

Studies of the moisture content of various liquid materials show that most of them (such as aqueous solutions of glycerol, acetone and others) have an almost linear dependence of the thermal conductivity coefficient of the liquid material $\lambda_{\text{jm}}$ on the degree of moisture content. The increase in the thermal conductivity of a liquid material can be expressed through the following linear formula:

$$\lambda_{\text{jm}} = \lambda_0 (1 + K_w W) \tag{11}$$

where: $\lambda_0$ - thermal conductivity of a liquid material without moisture [W・m / deg]
$K_w$ - the coefficient reflecting the increase in the coefficient of thermal conductivity of the liquid material with increasing degree of moisture, %;
$W$ - moisture (or moisture content) of the liquid material, %.

Given the above values, the values of the values and the parameter of their input and in the formulas for calculating the increase in thermal conductivity $g$. It is known that the Nussert number ($N_u$) is determined from the expression

$$N_u = \alpha \cdot \frac{\lambda_{\text{jm}}}{d} \tag{12}$$

The foundation

$$\alpha = \frac{\lambda_{\text{jm}}}{\nu} N_u \tag{13}$$

then

$$g = \lambda_{\text{jm}} \cdot \frac{N_u}{d} \cdot \pi \tag{14}$$

The heat transfer criterion $N_u$ for the operation mode of the considered heat conversion with a cylindrical heat conduit across the flow can be determined by the following criteria formula: for Reynold numbers (Re) less than 10,3, calculations by the formula are recommended

$$N_u = 0,56Re^{0,5}Pr^{0,36} (Pr_{\text{in}}/Pr_{\text{c}}) \tag{15}$$
Fig. 2. The dependence of the thermal conductivity of an aqueous solution of glycerin $\lambda_{m}$ from the moisture content of W,\%.

Fig. 3. The dependence of the thermal conductivity of an aqueous solution of acetone $\lambda_{m}$ from the moisture content of W,\%.

Taking into account formula (12), (15) at the flow rate of liquid material $V = 0.05 \, \text{m/s}$, the values of the dependence $\alpha = t\,(W)$ were determined for the liquid material in the form of an aqueous solution of glycerol.
Fig. 4. The dependence of the heat transfer coefficient $\alpha$ on the heat conduit, where the transducer is $c_d=4\cdot10^{-3}$ of the moisture content of the aqueous glycerol solution. Changes in the distributed thermal conductivity $g$ with a copper heat conductor with $c_d=4\cdot10^{-3}m$ gave the following value.

Fig. 5. Change in the distributed thermal conductivity $g$ of the thermal moisture converter of the aqueous glycerol solution $W,\%$.

Based on formula (14), the temperature distribution was determined for the smallest mode for a physical model with concentrated mesh bodies with a thermal conductor diameter $d=4\cdot10^{-3}$, $r_t = 206.5$ deg / $W \cdot m$ for various heating powers.
Fig. 6. Distribution graph temperature $T(x)$ along a cylindrical copper heat conductor with a diameter of $d = 4 \times 10^{-3} m$ with heat sources concentrated at the lower points of the heat conduit at power: 1-1.6W; 2-3w; 3-5w and 4-10w. And when controlling the humidity of a mixture of glycerol with water at $W = 20\%$ water.

According to expression (9) in dynamic mode, the process of heat flux transfer $f(x, \tau)$ and distribution, the temperature $T(x, \tau)$ in the heat conductor is determined by guineas $\operatorname{erfc}\left(\frac{x}{\sqrt{2\pi}} \frac{c}{\tau}\right)$ and $\operatorname{erfc}\left(\frac{x}{\sqrt{2\pi}} \frac{g}{\tau}\right)$.

For a heat pipe from a copper pipe with diameters $d=4\times10^{-3} m$, the heat rate at $= 0.5 \times 10^{-3} m$, the time constant $T_{\text{vpr}}$ was estimated by rejecting the distributed thermal parameters $C$ and $g$.

$$\tau_{\text{vpr}} = \frac{C}{g}$$

The value of the parameters is: $C=18.3\frac{Bm \cdot m^{-1}}{m^2 \cdot \text{C}}; g=12.2\frac{Bm \cdot m^{-1}}{m^2 \cdot \text{C}}$.

With humidity $W = 10\%$, the time constant is equal.

$$\tau_{\text{vpr1}} = 1.5 \text{ sec}$$

The experimental value of the time constant is $\tau_{\text{vpr1 exp}} = 1.8 \text{ sec}$.

In this paper, the principle of constructing thermal converters is analyzed and it is shown that thermal converters are the most promising for monitoring various parameters of fluids of liquids and gases. The mathematical models of heat converters with a homogeneous extended cylindrical heat pipe are considered in detail. Based on the developed mathematical models, the main characteristics of thermal converters are analyzed.

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