Modeling the crystallization process of an eutectic solution in cold accumulators of a refrigerated truck

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Abstract. The duration of crystallization process of an eutectic solution of an infinite plate may be presented as a sum of three components: duration of the cooling process of the eutectic solution of liquid phase \( \tau_1 \) from initial temperature \( t_I \) to crystallization beginning; duration of crystallization process \( \tau_2 \) and duration of the precooling process of solid phase \( \tau_3 \) to certain final temperature \( t_f \) in the plate center.

1. Introduction
At present the use of refrigerated trucks with cold accumulators designed for interurban transportation is a perspective mode of cooling with no moving parts. The melting heat of the eutectic solution after freezing is used in accumulator systems of refrigeration. The appliance of cold accumulators in refrigerated trucks ensures maintaining environment safety.

Abroad such a kind of cooling is in widespread use. In particular Carrier Transicold (France) offers a series of “Vanta” units with machine and accumulator refrigeration for isothermal motor vans at capacity from 4 to 23 m³ at minus 32°C eutectic solution freezing temperature. The units are intended for transportation of ice cream and quick frozen products. They are designed for 14 hours of continuous operation at 30°C ambient air temperature.

The world leader in eutectic plate production FIT s.p.a. from Italy offers a series of EBS and EFR models of eutectic plates in Russian market.

2. The solution of the problem
Below a mathematical model to define the duration of the process of eutectic solution freezing in accumulator cooling devices is offered (Fig.1). An approximate definition of duration of the process of eutectic solution crystallization of the infinite plate is accepted under boundary conditions of the third kind.

The duration of crystallization process of an eutectic solution of an infinite plate may be presented as a sum of three components: duration of the cooling process of the eutectic solution of liquid phase \( \tau_1 \) from initial temperature \( t_1 \) to crystallization beginning; duration of crystallization process \( \tau_2 \) and
duration of the process of solid phase \( \tau_3 \) precooling to a certain final temperature \( t_f \) in the infinite plate center.

\[ \tau_3 = \text{const} \]

**Figure 1.** The temperature field of the infinite flat plate at cooling

Neglecting convective heat exchange, the duration of the process of liquid phase cooling \( \tau_1 \) from initial temperature \( t_I \) to crystallization temperature \( t_C \) of the eutectic solution on the outer surface can be obtained by solution of the one-dimensional non-stationary equation of heat conductivity:

\[
\frac{\partial t(x, \tau)}{\partial \tau} = a_L \frac{\partial^2 t(x, \tau)}{\partial x^2}, \quad (\tau > 0; -R \leq x \leq R)
\]  

(1)

Under initial boundary conditions:

\[
q = -\lambda_L \frac{\partial t(R, \tau)}{\partial x} = \alpha [t(R, \tau) - t_i]
\]

\[
\frac{\partial t(o, \tau)}{\partial x} = 0
\]

(2)

where \( a_L, \lambda_L \) are the coefficients of temperature conductivity and heat conductivity of liquid phase respectively; \( \alpha \) is the coefficient of heat emission on outer surfaces of the infinite plate; \( t_{a.m.} \) is the temperature of ambient medium, °C; \( q \) is the heat flow, W/m².

The solution of equation (1) under boundary conditions (2) is obtained by:

\[
\theta = \frac{t_I - t(x, \tau)}{t_I - t_i} = 1 - \sum_{n=1}^{\infty} A_n \cos \frac{\mu_n x}{R} \exp \left( -\mu_n^2 F_{o_L} \right),
\]

(3)

where \( A_n = \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \) and \( \mu_n = \left( \frac{2 B_{iL}(B_{iL}^2 + \mu_n^2)^{1/2}}{\mu_n (B_{iL}^2 + B_{iL} + \mu_n^2)} \right)^{1/2} \); \( \mu_n \) is the shadow of the characteristic equation root, \( \cot \mu = \frac{1}{B_{iL}} \mu \); \( F_{o_L} = \frac{t_i}{R^2} \) is the Fourier number; \( B_{iL} = \frac{aL}{\lambda_L} \) is a Bio criterion for the eutectic solution liquid phase.

For large Fourier numbers it is possible to be limited to the first term in series (3), and then, the duration of the process of liquid phase cooling can be approximately obtained by the next dependence:

\[
\tau_1 = \frac{R^2}{a_L \mu_1^2} \ln \left[ \frac{A_1 \mu_1}{B_{iL}^2 + \mu_1^2 \left(1 - \theta_3 \right)^{1/2}} \right], \quad \text{where} \quad \theta_3 = \frac{t_I + t_C}{t_I - t_{a.m.}}
\]
The accurate value $\tau_1$ can be found using a numerical solution of equation (3) relative to the Fourier number $F_{OL}$ at

$$\theta_3 = \frac{t_1 + t_C}{t_1 - t_{a.m.}} \text{ and } x = R$$

The definition of the duration of the process $\tau_2$ of the eutectic solution crystallization is the most difficult.

To determine approximately the duration of the eutectic solution crystallization process we use the L.S. Leibenzone second method.

It is assumed that at each time point the temperature distribution in the solid phase is determined according to the linear law (quasistationary model):

$$t = t_C - \frac{t_C - t_{a.m.}}{\frac{1}{Bt_S} + 1 - \frac{\xi}{R}} x - \frac{\xi}{R}$$

where $\xi$ is the coordinate of the boundary of phase separation.

The current value of the enthalpy of solid phase is defined using the next dependence:

$$W_S = W_{SC} + c_S (t_C - t)$$

where $W_{SC}$ is the enthalpy of the eutectic solution solid phase at crystallization temperature; $c_S$ is the heat capacity of solid phase.

Likewise, we find the current value of liquid phase enthalpy:

$$w_L = w_{LC} + c_L (t - t_C)$$

where $w_{LC}$ is the enthalpy of eutectic solution liquid phase at crystallization temperature; $c_L$ is the specific heat of liquid phase.

The equation of heat balance for each time point is written as

$$\frac{d}{dt} \left[ \int_0^t \rho_L w_L dx + \int_{-R}^{R} \rho_S w_S dx - \lambda_S \int_0^R \frac{\partial t}{\partial x} \bigg|_{x=R} dx \right] = 0$$

In equation (6) the first term specifies the change of heat content of liquid phase in time; the second one – the change of heat content of solid phase in time; the third one - the change of heat flow in time on external surfaces of the plate.

For simplicity the initial moment of time is accepted as equal to zero.
The current value of the first and second integrals is defined by equation (6):
\[
\int_0^\xi \rho_L w_L \, dx = \rho_L \int_0^\xi \left( w_{LC} + \bar{n}_{dL}(t - t_c) \right) \, dx = \rho_L w_L \xi + \int_0^\xi \rho_L \bar{n}_{dL}(t - t_c) \, dx.
\]
(7)
The value of the second term is defined approximately as:
\[
\int_0^\xi \rho_L \bar{n}_{dL}(t - t_c) \, dx = \frac{\xi}{R} Q_L
\]
where \( Q_L \) is a total heat quantity to be removed from the liquid phase to cool it to the crystallization temperature, i.e. the liquid phase should be in a state as at the moment of the crystallization beginning:
\[
Q_L = \int_0^R \rho_L \bar{n}_{dL}(t_L - t_c) \, dx
\]
where \( t_L \) is the space temperature distribution in time point \( t_\tau \) of the crystallization process beginning that can be approximately defined by the formula:
\[
t_L = t_1 - (t_1 - t_i) \left[ 1 - A_1 \cos \left( \frac{\mu}{R} x \right) \exp(-\mu^2 F_{\tau 1}) \right]
\]
Therefore,
\[
Q_L = \bar{n}_{dL} \rho_L R \left\{ (t_1 - t_c) - (t_1 - t_i) \left[ 1 - \frac{A_1}{\mu^2} \sin \mu \exp(-\mu^2 F_{\tau 1}) \right] \right\}
\]
So, the first term of equation (6) is approximately expressed by:
\[
\int_0^\xi \rho_L w_L \, dx = \rho_L w_L \xi + \frac{\xi}{R} Q_L
\]
(8)
The second term of equation (6) is defined by the liquid phase thickness at linear law of change of temperature distribution:
\[
\int_\xi^R \rho_S w_S \, dx = \int_\xi^R \rho_S \left( w_{SC} + \bar{n}_{dC}(t_c - t) \right) \, dx = \rho_S w_{SC}(R - \xi) + \rho_S C_s \int_\xi^R \frac{t_c - t_{a.m.} x - \xi}{R} \, dx = \rho_S w_{SC}(R - \xi) + \frac{1}{2} \rho_S C_s R \left[ \frac{t_c - t_{a.m.}}{R} \right] \left( 1 - \frac{\xi}{R} \right)^2
\]
(9)
After substituting expressions (8) and (9) for the first and second terms into equation (10) and making differentiation we get the next equation determining the speed of moving the boundary of phase separation:
\[
\left( \rho_S H + \frac{1}{R} \rho_L w_L \right) \frac{d\xi}{dt} + \frac{1}{2} \rho_S C_s R \left( t_c - t_{a.m.} \right) \frac{d}{d\xi} \left[ \frac{1 - \xi}{R} \right] \frac{d\xi}{dt} + \lambda_s \frac{1}{R} \frac{t_c - t_{a.m.}}{R} \frac{1 - \xi}{R} = 0
\]
(10)
Equation (10) specifies latent heat \( H \) of phase transition (crystallization) that is defined as:
\[
H = \frac{1}{\rho_S} \left( \rho_L w_{LC} - \rho_L w_{HC} \right)
\]
Integrating equation (10) we get the formula to define the duration of the eutectic solution crystallization process:
\[
\tau_2 = \frac{1}{2} \frac{R \rho_S H + Q_L}{A_1} \left( 1 + \frac{2}{B_i} \right) + \frac{1}{2} \frac{\rho_S C_s R^2}{\lambda_s} \frac{2}{B_i Z_s} \ln \left( 1 + B_i S \right) - 1 - \frac{2}{B_i}
\]
or in dimensionless form:
\[
F_{\tau 2} = \frac{\tau_2 \rho_S}{R^2} = \frac{1}{2} \frac{\rho_S H + Q_L}{\rho_S S \left( t_c - t_{a.m.} \right)} \left( 1 + \frac{2}{B_i} \right) + \frac{1}{2} \frac{\rho_S C_s R^2}{\lambda_s} \frac{2}{B_i Z_s} \ln \left( 1 + B_i S \right) - 1 - \frac{2}{B_i}
\]
(11)
The first term of equation (11) defines the duration of the process that is needed for crystallization properly and liquid phase cooling to the crystallization temperature; the second one defines the duration of the process needed to the solid phase precooling.
According to allowance admitted, at the end of the crystallization process the temperature distribution along the solid phase thickness is defined by the linear dependence:
\[ t = t_C - \frac{t_C - t_{\text{a.m.}} x}{\frac{1}{Bi} + 1} \frac{x}{R} \]

To determine the duration of the process of liquid phase precooling \( \tau_3 \) it is necessary to solve equation of heat conductivity (5) under next boundary and initial conditions:

\[
\lambda_S \frac{\partial t(R, \tau)}{\partial x} = a[t(R, \tau) - t_{\text{a.m.}}] \\
\frac{\partial t(0, \tau)}{\partial x} = 0, \\
t(x, 0) = t_C - \frac{t_C - t_{\text{a.m.}} x}{\frac{1}{Bi} + 1} \frac{x}{R}
\]

(12)

Herewith, the coefficient of liquid phase \( a_L \) in equation of heat conductivity (1) should be substituted for coefficient of temperature conductivity of the eutectic solution solid phase \( a_S \).

This expression is written in the next for:

\[ t(x, \tau) = t_l + t_C \sum_{n=1}^{\infty} D_n \cos \left( \frac{v_n x}{R} \right) \exp \left[-v_n^2 \text{Fo}_3 \right] \]

(13)

Decomposition values \( D_n \) of the initial distribution of temperature are defined by the formula:

\[ D_n = \frac{\frac{v_n}{v_n + \sin v_n \cos v_n}}{2} \int_0^R \left[ 1 - \frac{t_C - t_{\text{a.m.}} x}{\frac{1}{Bi} + 1} \frac{x}{R} \right]^3 \cos \left( \frac{v_n x}{R} \right) dx = \]

\[ = \frac{2}{v_n + \sin v_n \cos v_n} \left\{ \sin v_n - \frac{t_C - t_{\text{a.m.}}}{\frac{1}{Bi} + 1} \frac{1}{v_n^3} \left[ 1 - \cos v_n - v_n \sin v_n \right] \left[ 1 - \cos v_n - v_n \sin v_n \right] \right\} \]

where \( v_n \) is the coefficient of kinematic viscosity.

For large Fourier numbers the duration of the process of precooling of liquid phase \( \tau_3 \) can be determined taking into account only the first term of the Fourier series (13):

\[ \tau_3 = \frac{R^2}{\alpha_{5L} v_1^2} \ln \frac{d_1 t_C}{t_C - t_{\text{a.m.}}} \quad \text{or} \quad \text{Fo}_{T3} = \frac{1}{v_1^3} \ln \frac{d_1 t_C}{t_C - t_{\text{a.m.}}} \]

An accurate value of the duration of the process of liquid phase precooling \( \tau_3 \) can be defined using a numerical solution of equation (13) relative to the Fourier number \( \text{Fo}_{T3} \) at \( t(0, \tau) = t_l \).

3. Conclusion

Therefore, the duration of all process of crystallization of the eutectic solution is equal to:

\[ \tau_0 = \tau_1 + \tau_2 + \tau_3 \]

or in dimensionless form:

\[ \text{Fo}_0 = \text{Fo}_{T1} \frac{a_L}{a_0} + \text{Fo}_{T2} + \text{Fo}_{T3} \]

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