Chirally Extended QCD or An Asymptotically Free Chiral Linear Sigma Model With Quarks Coupled to Gluons

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Abstract

We show that the quark (not nucleon) based chiral linear $\sigma$ model which receives strong support as a mean field theory for the strong interactions can actually be made asymptotically free by coupling to gluons. It has the further bonus of providing confinement. Alternatively, this theory is QCD that is chirally extended by the addition of a $(\sigma, \pi)$ chiral multiplet. It could therefore be a candidate theory for the strong interactions which is somewhat different from QCD.
I. INTRODUCTION

Quantum Chromodynamics (QCD) is an acceptable starting point for a theory of strong interactions. The main experimental support for this comes from the well established phenomenon of asymptotic freedom which gives a qualitatively correct description of deep-inelastic scattering (DIS) and scaling. However, the increase of the coupling at low energy renders non-perturbative and difficult the explicit low energy calculations.

The aspects of QCD that preserve at low energy are the symmetries. It is well known that chiral symmetry is almost exact and furthermore, spontaneously broken at low energy, a fact that actually precedes QCD. This has been given a field theory realization via effective, but, renormalizable field theories like the Gell-Mann Levy chiral $\sigma$ model with the pion as the goldstone boson. Via PCAC this model provides us with various tested low energy theorems, for example, the Goldberger-Treimann relation, the Adler-Weisberger relation etc.

The linear sigma model being a Yukawa theory is not asymptotically free, and therefore, inspite of being pertubatively renormalizable, is usually considered to be a low energy effective theory. Nevertheless, more recently, the same chiral linear $\sigma$ model, with quarks substituted for the nucleons, has been found to provide a very reasonable description of the nucleon and strong interaction properties at finite temperature (hadronic screening masses) and baryon density, even at scales well beyond chiral restoration (see also Ref. [3]).

This is unexpected from a theory that is only an effective theory. We shall show in what follows that, surprisingly, the chiral linear $\sigma$ model with quarks, when coupled to gluons can be asymptotically free.

II. THE CHIRAL QUARK SIGMA MODEL (WITH GLUONS)

The lagrangian is

\[ \mathcal{L} = -\frac{1}{2} (\partial_\mu \sigma)^2 - \frac{1}{2} (\partial_\mu \vec{\pi})^2 - \lambda^2 (\sigma^2 + \vec{\pi}^2 - f^2)^2 \]
\[ -\bar{\Psi}_q [D_\mu + g_3 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})] \Psi_q - \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a + \mathcal{O}(m\pi) \]  

where \( D_\mu = \partial_\mu - ig_3 A_\mu^a T^a \) and \( G^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_3 f^{abc} A^b_\mu A^c_\nu \). 

\( A_\mu^a \) is the color gluon field and \( T^a \) is the SU(3) color matrix in the fundamental representation. \( g_y, g_3 \) and \( \lambda \) are the Yukawa, QCD, and Higgs couplings respectively. \( \bar{\Psi}_q \) is the quark field.

Apart from the quarks and the chiral fields \((\sigma, \pi)\) we have additionally included the gluon interaction with the quarks. Note that the chiral fields \((\sigma, \pi)\) are color singlets and do not interact with the gluons. The relevance of these gluons will be clarified later. Note that this lagrangian is alternatively nothing but QCD extended by the addition of a \((\sigma, \pi)\) chiral multiplet. First let us examine the support this lagrangian receives at the mean field level from very different quarters, even in the neglect of gluons.

### III. MEAN FIELD RESULTS

#### A. Lattice Results

Recently, lattice QCD results \cite{3,4} which look at the chiral transition at finite T, in the presence of dynamical quarks, give an interesting picture which we shall briefly recount.

We focus on the screening mass spectrum for the hadrons as the chiral restoration temperature, \( T_\chi \), is approached. The menagerie of hadrons (like the nucleon, the \( \rho \), the \( A_1 \) etc.) begin to scale as their free constituent quarks and antiquarks. For example, for \( T > T_\chi \) the nucleon screening mass is just three times the screening mass of the free quark, whereas the \( \rho \) and \( A_1 \) screening masses are two times the free quark(antiquark) screening mass.

\[
\frac{M_N}{M_\rho} \sim \frac{3}{2} \quad \text{and} \quad \frac{M_{A_1}}{M_\rho} \sim 1
\]  

However, the pion and sigma screening masses, though they are degenerate, at \( T_\chi \) (chiral restoration), stay well below the free \( q\bar{q} \) threshold all the way till about \( 3T_\chi \), the maximum temperature achieved by the lattice calculations

\[
\frac{M_{\pi,\sigma}}{M_{\rho,A_1}} < 1
\]
Thus, the pion and sigma continue to be bound states all the way till the maximum temperature reached on the lattice. Calculations for larger $T$ will yield even more information.

Interestingly enough, as indicated by Goksch [3], a finite temperature version of the chiral $\sigma$ model with quarks instead of nucleons can reproduce the lattice results rather well. Of course, in this instance the pions (sigma) are elementary and will continue to exist for all $T > T_\chi$, except not as Goldstone bosons. Whether such a scenario will match lattice results at higher temperature is not known. Nevertheless it shows that the chiral $\sigma$ model with quarks is an effective lagrangian well above the chiral restoration temperature. Gluons may be included perturbatively but have been neglected in Ref. [3].

**B. The Nucleon**

In Ref. [1] it was found that in the spontaneously broken phase this lagrangian yields the nucleon (ground state in the $B = 1$ sector) in a Skyrme background with bound state quarks. Nucleon static properties are reasonably described at the mean-field level [2].

Further, the nucleon realised (as above) as a chiral quark soliton in a Skyrme background is (like the Skyrmion) roughly consistent with the EMC result, that the axial vector direct form factor (which is twice the total ‘spin’ of the quarks) is consistent with zero for the proton state [3,7,8]. This is in contrast to the non-relativistic quark model for which the axial vector direct form factor is obviously close to unity.

**C. Equation of State**

A very interesting support for the lagrangian over the entire range of bayron density was demonstrated by one of us [5] in Mean Field Theory (MFT).

It was shown in Ref. [5] that the model lagrangian has two phases at the MFT level. Starting from zero density we get an equation of state where the ground state occurs in phase 1 – solitonic nucleons composed of bound quarks in a skyrme background, with a qualitatively correct binding energy and an absolute minimum in $E_B$ at around, $\rho_{nuc}$. The
phase 2 in this model is the so called Abnormal or Lee-Wick phase in which the condensate (VEV) is space uniform. This provides a Yukawa or spontaneous mass for th quark. The VEV is a function of baryon density going to zero at some critical density. At density higher than this we go into a chirally restored phase. The transition from phase 1 to phase 2 occurs at a density when the latter is well into chiral restoation and the ground state is described by a free fermi gas of quarks. The two phases thus conspire to recover the expected features of the strong interaction equation equation of state at all densities.

The results quoted here are indicative of the fact that Eq. (1) is a good effective lagrangian at scales even well above chiral restoration. Is it possible, then, that this theory may be more than just an effective theory. In this case, it would need to be asymptotically free, and to this we now now turn.

**IV. BEYOND MEAN FIELD THEORY, GLUONS AND ASYMPTOTIC FREEDOM**

We now come to a feature which has to do with the lack of asymptotic freedom in simple yukawa theories like the chiral σ model. In the attempt to go beyond MFT to 1-loop order one encounters the vacuum instability to small length scale fluctuations that occurs for all simple yukawa theories.

This is related to the fact that due to the lack of asymptotic freedom the yukawa coupling has a Landau singularity. This fact does not permit us to go beyond the MFT level in an otherwise (perturbatively) renormalizable theory. It has been observed that the wave function renormalization for the scalar(pseudo scalar) is inversely proportional to the running yukawa coupling. This implies that when the yukawa coupling blows up, the wavefunction renormalization, \( Z \), goes to zero. The scale at which this happens is often interpreted as a compositeness scale for the scalar i.e., at and after this scale the scalar

\[ \text{We shall prove this shortly via the RNG} \]
ceases to be (elementary).

In an attempt to circumvent this problem we had proposed the inclusion of gluons that interact directly only with the quarks \([6]\). Being vector bosons they clearly stem the rise of the yukawa coupling as can be seen from the following calculation. Further, the gluons are important for confinement, a feature that is absent in the chiral quark \(\sigma\) model.

The significant question then is: Can the theory described by Eq.(1) possibly be asymptotically free. We now turn to this.

This question can be answered by looking at the \(\beta\)-function for the yukawa coupling and the QCD coupling for our lagrangian given in Eq. (1).

The \(\beta\) function for the QCD coupling, \(\alpha\), is

\[
\frac{\partial \alpha}{\partial t} = -\left(\frac{33}{3} - 2N_F\right) \frac{\alpha^2}{8\pi^2} \quad \left(g_3^2 = \alpha\right)
\]

(4)

This is for \(m_q = 0\) and \(t = \ln(p/\mu)\).

Clearly this is valid for small \(\alpha\), or only in the ultraviolet when we can take \(N_G = 3\) or \(N_F = 6\) (Remember all quarks couple to the gluons). Note, that to one-loop order the \(\beta\)-function for the QCD coupling does not receive any contribution from the yukawa coupling, \(g_y\), or the scalar self coupling \(\lambda\), as the scalar (pseudoscalars) do not couple directly to the gluons.

The yukawa coupling \(g_y\) for the pion and sigma to the quarks has the following \(\beta\) function (Remember the pion and sigma couple to only one, the \([u,d]\), generation. We have assumed that the \(\pi\) and \(\sigma\) coupling to the other generations is absent).

\[
\frac{\partial g_y^2}{\partial t} = \frac{g_y^2}{8\pi^2} \left[12g_y^2 - 8\alpha\right]
\]

(5)

One can see here explicitly how the QCD coupling, \(\alpha\), slows down the increase of the yukawa coupling, \(g_y^2\), with \(t\).

We can now define the ratio \(\rho = g_y^2/\alpha\) and write the following equation for \(\rho\) using Eqs (4) and (5). This has the advantage that \(\rho\) can be expressed as a function of a single variable \(\alpha\)

\[
\frac{\partial \rho}{\partial \alpha} = -\frac{\rho}{\alpha A} \left[12\rho - 8 + A\right]
\]

(6)
where $A = (33 - 2N_F)/3$ and $N_F$ is the number of flavours that effectively couple to the gluons. For the case of 3 generations or $N_F = 6$, (This will correspond to the high energy behaviour of this theory).

$$\frac{\partial \rho}{\partial \alpha} = -\frac{\rho}{i\alpha}[12\rho - 1]$$

(7)

Here we observe that there are two regimes.

(i) $0 < \rho < 1/12$

In this case $\partial \rho/\partial \alpha > 0$. This implies that $\rho$ decreases as $\alpha$ decreases, that is, $\rho$ will decrease with increasing momentum scale. We can integrate the $\rho$ equation to get

$$\rho = \frac{\alpha^{1/7}K}{(1 + 12\alpha^{1/7}K)}$$

(8)

where $K$ is a positive integration constant that is set by initial data on $\alpha$ and $g_y^2$.

This equation is interesting. It shows that in the ultraviolet when $\alpha \to 0$

$$\rho \sim K\alpha^{1/7}$$

This further implies that in the ultra violet

$$g_y^2 \sim K\alpha^{8/7}$$

which shows that $g_y^2$ is asymptotically free and goes to zero faster than $\alpha$. Therefore, the leading behaviour of this theory in the ultraviolet is given by the QCD coupling with the yukawa coupling contributing only in sub leading order.

On the other hand if we go to $\alpha \to \infty$ we find $\rho \to 1/12$. Since, in the 1-loop approximation, $\alpha \to \infty$ at some length scale, $\Lambda$, we may want to think of $\rho = 1/12$ as a quasi fixed point [14][15] (fixed point in the ratio of the coupling) in the infrared. Evidently, such an interpretation is fraught with uncertainty as the 1-loop analysis breaks down well before $\alpha \to \infty$. Thus the quasi fixed point
\[ \rho = \frac{1}{12}, \text{ cannot be taken seriously unlike the Ross-Pendelton fixed point \cite{14} for the standard electroweak model where both } \alpha \text{ and } g_y^2 \text{ are small in the infrared (250 GeV) and the quasi fixed point is meaningful.} \]

(ii) \( \rho > \frac{1}{12} \)

Here \( \partial \rho / \partial \alpha < 0 \) and therefore \( \rho \) increases as \( \alpha \) decreases that is \( \rho \) increases with the momentum scale. The integration of the \( \rho \) equation now gives

\[ \rho = \frac{\alpha^{1/7}}{(12\alpha^{1/7} - C)} \]

where \( C \) is a positive constant set by data on \( \alpha \) and \( g_y^2 \).

However, this equation is very different. Naively, when \( \alpha \to \infty \) we recover the so-called quasi fixed point, \( \rho = 1/12 \). But as we have pointed out earlier this quasi fixed point is not really meaningful. On the other hand as we move towards the ultraviolet and \( \alpha \) comes down, there is a pole in the denominator at \( C = 12\alpha^{1/7} \) indicating that \( \rho \) blows up, which in turn means that the yukawa coupling \( g_y^2 \) blows up even as \( \alpha \) is finite. The scale at which this happens, as we shall see shortly, can be identified with the vanishing of the wavefunction renormalization for the scalar(pseudoscalar) where the scalar(pseudoscalar) is no longer an elementary particle.

The regime can therefore be thought of as a phase in which the scalar(pseudoscalar) is composite. (A complete investigation of these results which includes the scalar self coupling will be reported separately \cite{16,17}).

V. THE WAVEFUNCTION RENORMALIZATION FOR THE
SCALAR(PSEUDOSCALAR)

The wavefunction renormalization for the scalar(pseudoscalar) going to zero has the interpretation of the particle being composite i.e., not elementary, whereas if it does not go to zero the particle is elementary. We shall now calculate the renormalization group (RNG)
improved wavefunction renormalization to show its relation to the yukawa coupling in both the regimes (i) $0 < \rho < 1/12$ and (ii) $\rho > 1/12$.

Let us now briefly consider the wavefunction renormalization for the scalar (psuedoscalar) fields to see how it relates to the yukawa and QCD couplings. To bring out the right connection we must carry out a RNG improvement of the two point function. The function can be calculated in perturbation theory.

$$Z^{(2)}_{\text{loop}} = 1 + 2Bg_y^2t \quad (m_q = 0) \quad (10)$$

where $B = -4N_c/16\pi^2$ and $t = \log(p/\mu)$.

The $\gamma$ function is $\gamma = Bg_y^2$. The RNG improvement proceeds as follows

$$\left( -\frac{\partial}{\partial t} + \beta \frac{\partial}{\partial g_y} + 2\gamma_0 \right) Z^{(2)} = 0 \quad \text{where} \quad \gamma_0 = \gamma(t = 0) \quad (11)$$

The RNG improved $Z^{(2)}$ is then

$$Z^{(2)} = \exp\left(2 \int_0^t \gamma(t') dt' \right) \quad (12)$$

Using the two equations of the $\beta$ functions for the QCD and yukawa coupling, we can cast the $\gamma$ function

$$\gamma(g_y^2(t')) = Bg_y^2(t') = \frac{B16\pi^2}{4N_c} \frac{\partial \log(g_y)}{\partial t'} - \frac{8B}{4N_c} \frac{16\pi^2}{A} \frac{\partial \log(g_3)}{\partial t'} \quad (13)$$

It follows

$$Z^{(2)} = \left( \frac{g_y(t)}{g_y(t = 0)} \right)^{-2} \left( \frac{g_3(t)}{g_3(t = 0)} \right)^{16/7} = \left( \frac{g_y(t)}{g_y(t = 0)} \right)^{-2} \left( \frac{\alpha(t)}{\alpha(t = 0)} \right)^{8/7} \quad (14)$$

This is a proof for the inverse relation between the yukawa coupling and the wavefunction renormalization $Z^{(2)}$.

Furthermore, it is interesting to see what happens to $Z^{(2)}$ in the ultraviolet for

(i) $0 < \rho < 1/12$:

Here we know $g_y^2 \approx K(\alpha)^{8/7}$ in the ultraviolet. This gives $Z^{(2)} = 1$ a rather interesting result.
(ii) $\rho > 1/12$:

$$g_y(t)|_{uv} \to \infty \text{ with } Z^{(2)} \to 0 \text{ in the ultraviolet}$$

If we find ourselves in the regime $\rho < 1/12$ where the yukawa coupling as asymptotically free there is a rather unorthodox consequence. The pion continues to be elementary to all ultraviolet scales. This is strange but lattice gauge theories do indicate an elementary non-dissociated-pion (sigma) till $T \approx 3T_\chi$. Better lattice gauge theory calculations could test this conjecture, which will have interesting physical consequences.

**VI. REMARKS**

1. We would like to emphasize that our results depend importantly on our taking the number of flavours, $N_F = 6$ (or number of generations=3) for QCD $\beta$ function. This is true for the scales in the ultraviolet ($q^2 \gg m_t^2$) but not in the infrared. Anyhow, we do not trust the 1-loop results in the infrared.

   However if we do take the number of flavours to be $N_F = 2$ (just u and d quarks) then the value of $A$ in the Eq. (6) becomes $29/3$ and

   $$\frac{\partial \rho}{\partial \alpha} = \rho \frac{3}{29} \left[ \frac{1}{12\rho} + \frac{5}{3} \right]$$

   (15)

   This moves the asymptotically free regime to negative $\rho$ and there is no ready interpretation for this.

2. We have not addressed the question of the asymptotic freedom of the scalar(pseudoscalar) self coupling, $\lambda$. However, there is an analysis by Schrempp and Schrempp [15] for the standard model which deals with the two ratios of the coupling, $\rho = g_y^2/\alpha$ and $R = \lambda/g_y^2$ that can be adapted to our model. This analysis is carried out in Refs [16,17]. We make only some brief observations from these results here. The $\beta$-function for $\lambda$ in our model is

   $$\frac{\partial \lambda}{\partial t} = \frac{1}{8\pi^2} \left[ 2\lambda^2 - 144g_y^4 + 24g_y^2 \lambda \right]$$

   (16)
Again, by defining the ratio $R = \lambda/g_y^2$ we can convert to an equation for $R$ that depends on the single variable $\rho$ and find that

\[
\frac{\partial R}{\partial \rho} = \frac{1}{[12\rho - 8 + A]} \left[ 2R^2 + R \left( 12 + \frac{8}{\rho} \right) - 144 \right]
\]

(17)

It is found that at least on a single trajectory in the $[R, \rho]$ parameter space, that is the invariant line \cite{15,16,17} the ultra violet behaviour of $R$ for the regime $\rho < 1/12$ is such that $R \to 0$ in the ultraviolet. This would indicate that our results are also intact insofar as the coupling $\lambda$ is concerned. $\lambda \to 0$ in the extreme ultraviolet even faster than $g_y^2$. As we have indicated these results will be reported separately where we shall present a complete analysis of such field theories \cite{16,17} and where we will also look at tests to decide on any of these theories can compete with QCD as far as the physics of the ultraviolet (DIS) is concerned \cite{17}

\section{VII. CONCLUSION}

- We have shown the existence of an asymptotically free chiral theory of quarks mesons and gluons with yukawa and gluon couplings such that the leading ultraviolet behaviour, like QCD, comes from the gluon coupling and only subleading effects arise from the yukawa coupling. The 1-loop calculation indicates this is true for the ‘phase’ of the theory that has $0 < \rho < 1/12$. In other words it depends on a set of initial data on the QCD and yukawa couplings of the theory. Note, also that this initial data must be gleaned (experimentally) from the ultraviolet. Data for the infrared cannot really be used as our 1-loop calculation breaks down for large coupling.

- This theory implies an elementary pion for all scales

- For initial data, when $\rho > 1/12$, in our 1-loop approximation (note, this number will change if we go beyond 1-loop ! ) ‘there is another phase’ of the theory. In this
‘composite’ phase, the yukawa coupling acquires a Landau singularity and the theory will lose its elementary scalar (pseudoscalar).

- Such a theory has infrared quasi fixed points (fixed point in the ratio of the couplings).

We have shown that our theory is asymptotically free and thus a candidate for a consistent theory of the strong interactions valid at all scales. This begs a new question how is this theory different from QCD?

The quark based chiral $\sigma$ model receives strong support at the mean field level as a candidate to describe the strong interaction. We have shown that by coupling to gluons it can be made asymptotically free depending on initial data for the QCD, yukawa and meson self coupling. A bonus is that it can then also provide confinement. We strongly believe it needs further study.

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