Parametric optimization and modelling of rough cut WEDM operation of pure titanium using grey-fuzzy logic and dimensional analysis

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Parametric optimization and modelling of rough cut WEDM operation of pure titanium using grey-fuzzy logic and dimensional analysis

Rupesh Chalisgaonkar¹ and Jatinder Kumar²

Abstract: In this study, effects of input parameters such as pulse on time ($T_{ON}$), pulse off time ($T_{OFF}$), peak current (IP), wire feed (WF), wire tension (WT) and servo voltage (SV) on machining characteristics such as material removal rate (MRR), surface roughness (SR) and wire weight consumption (WWC) were investigated in wire electric discharge machining (WEDM) process using commercially pure titanium as work material. The consumption of wire and its correlation with wire wear was also investigated. The responses were optimized simultaneously using grey-fuzzy logic approach. Surface integrity aspects such as microstructure analysis (including recast layer thickness, debris and cracks, crater size and shape, etc.) of the selected machined titanium samples have also been investigated to evaluate the suitability of WEDM for machining titanium. The material transfer mechanism between the zinc-coated wire electrode and the work surface has been studied using energy dispersive X-ray analysis. A mechanistic model has also been developed and validated for prediction of the response parameter (MRR) over a wide range of input variables.

Subjects: Industrial Engineering & Manufacturing, Manufacturing Engineering, Production Engineering, Technology

Keywords: WEDM, material removal rate, wire consumption, surface roughness, grey-fuzzy logic, microstructure, commercially pure titanium

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PUBLIC INTEREST STATEMENT
Titanium and its alloys are finding a widespread use in many challenging fields such as biomedical, defence, aerospace and offshore due to their unique characteristics such as low density, high tenacity and exceptional corrosion resistance. However, the poor machinability of titanium hinders its manufacturability and poses a threat against the ever increasing usage of recently developed materials such as advanced ceramics and nickel-based alloys. Wire electric discharge machining (WEDM) could be a viable alternative for commercial machining of titanium as the process is based on the use of thermal energy for machining purpose and can machine any material which is electrically conductive irrespective of its mechanical and physical properties. The outcome of this investigation is likely to shed some light on various issues related to the productivity and surface quality in WEDM of titanium.
1. Introduction

Wire electric discharge machining (WEDM) is found to be an extremely potential thermal-based non-traditional machining process in which the spark is generated between workpiece and conductive wire (usually master brass wire CuZn37) flushed with de-ionized water. The material removal takes place due to rapid and repetitive spark discharges between workpiece and tool electrode connected in an electrical circuit. The gap suitably ranges between 0.025 and 0.075 mm across the wire and workpiece. A liquid dielectric medium is continuously passed in the gap provided between the wire and workpiece. A collection spool, which is located at the bottom, is utilized to collect the used wire. WEDM process is generally used in tool and dies industry where accuracy and surface finish is of great importance. WEDM has the capability to impart production accuracy in the range of ±2.54 μm. WEDM is used for machining of newer and difficult to machine materials, such as hardened steel, high-strength temperature-resistant alloys, fibre-reinforced composites and ceramics in aerospace, nuclear, missile, turbine, automobile, and tool and die making industries. This process enables machining of any type of feature such as deep, blind, inclined and micro holes and complicated profiles.

Titanium and its alloys are used extensively in aerospace because of its high specific strength (strength-to-weight ratio) maintained at higher temperature. These are having exceptional corrosion resistance and exclusively used in aerospace, defence, biomedical and offshore applications. Titanium finds some applications in potential areas such as petroleum refining, chemical processing, surgical implantation, pulp and paper, pollution control, nuclear waste storage, food processing, gas turbines and marine application. (Ezugwu & Wang, 1997). The high cost involved in machining restricts the widespread applications of titanium worldwide. Conventional machining of titanium and its alloys is difficult due to the following reasons: titanium is chemically very reactive and has a tendency to weld to the cutting tool during machining; the poor Young’s modulus attributes to excessive chatter and deflection. Poor thermal conductivity leads to severe heat concentration at the cutting edge and higher mechanical stresses are concentrated due to small chip–tool contact area on the rake face of the tool (Ezugwu, Bonney, & Yamane, 2003; Ezugwu & Wang, 1997). As a result, tool encounters notching, flank wear, crater wear, chipping and catastrophic failure due to the combination of high temperature, high cutting stresses and strong chemical reactivity during machining. In the recent past, many approaches have been tried for improvement of machinability of titanium. Few of them are the use of coated carbide tools, development of effective cutting fluids, cryogenic treatment of work material and the use of innovative systems for discharge of cutting fluids (Ezugwu et al., 2003; Ezugwu & Wang, 1997). However, even these approaches have their own limitations and the poor machinability of titanium is still a cause of concern for the machinists.

Hence, there is a critical need for development of cost-effective machining methods for machining of titanium, so that its applicability is not restricted. WEDM is a potential candidate for machining of titanium and its application in this regard needs proper investigation. The poor thermal conductivity (16.3 W/m K) of titanium is expected to yield a higher amount of heat concentration in machining zone and result in higher cutting speed of workpiece during wire electrical discharge machining (WEDM) as compared to other materials such as steel and Ni-based alloys. Moreover, many of the problems encountered in machining of titanium have been linked to the physical interaction of the tool and work materials. As WEDM is a non-contact process, almost all of these problems may be addressed. Also, the process capabilities are not restricted by the mechanical or physical properties of the work material, which could be an added advantage.

2. Literature review

Liao, Huang, and Su (1997) reported that with increased value of pulse on time, material removal rate (MRR), surface roughness (SR) and gap width increase while machining SKD11 alloy steel using WEDM process. Prohaszka, Mamalis, and Vaevekanidis (1997) demonstrated the effect of wire electrode coating such as Sn, Zn and Mg on machinability of annealed non-alloyed steel in WEDM process. It was found that coating of copper, brass, steel and Molybdenum wires by a layer of a material possessing a small work function such as Mg and alkaline metals may increase the cutting
efficiency during WEDM. Tosun and Cogun (2003) experimentally found that wire wear ratio (WWR) increases with increased value of pulse duration and open circuit voltage. WWR decreases with increased value of wire speed and dielectric fluid pressure while machining AISI 4140 steel. Lee and Li (2003) presented the study of the surface integrity of the machined workpiece in the EDM of tungsten carbide. It was found that EDMed surfaces exhibited cracks, especially at high peak current (IP) and pulse duration. Hascalyk and Caydas (2004) investigated the machining characteristics of AISI D5 tool steel in WEDM process. Parameters such as open-circuit voltage, pulse duration, wire speed and dielectric fluid pressure were manipulated to explore their effects on SR and metallurgical structure. It was found in this investigation that intensity of process energy does affect the amount of recast layer and SR, as well as microcracking, whereas the wire speed and dielectric fluid pressure do not have much influence. Huang, Hsu, and Yao (2004) investigated microstructure analysis of martensitic stainless steel surface fine cut by WEDM process. It was concluded after SEM examination that on the brim of the crater, presence of spherical particles, with composition similar to workpiece, was observed. Tosun, Cogun, and Tosun (2004) concluded that open circuit voltage and pulse duration are more significant parameters than wire speed and dielectric flushing pressure for their effects on both MRR and SR while machining AISI 4140 using WEDM. Mahapatra and Potnaik (2006) investigated that discharge current, pulse duration, dielectric flow rate and interaction of discharge current with pulse duration and dielectric flow rate are highly significant for both MRR and SR while machining D2 tool steel with WEDM. Ramakrishnan and Karunamoorthy (2006) optimized simultaneously three responses such as MRR, SR and WWR with the help of Taguchi’s Multi response S/N (MRSN) ratio for heat-treated tool steel using WEDM process

Ramakrishnan and Karunamoorthy (2008) reported that during WEDM of Inconel 718 alloy, MRR and SR increase when pulse on time is increased and delay time is decreased. Jangra, Jain, and Grover (2010) presented optimization of performance characteristics such as cutting speed, SR and dimensional lag using Taguchi method and grey relational analysis in WEDM process. Garg, Jain, and Bhushan (2012) investigated the WEDM of titanium alloy 6-2-4-2, to study their effect on dimensional deviation. The experiments were conducted using Box–Behnken designs to study the effect on dimensional deviation. Lahane, Rodge, and Sharma (2012) optimized multi-performance characteristics such as MRR and WWR using principal component analysis method. Kumar, Kumar, and Kumar (2013a) investigated the effect of input process parameters on wire breakage frequency and the surface integrity of wire wear during machining of pure titanium. Wire rupture was observed at higher values of IP and spark frequency. Kumar, Kumar, and Kumar (2013b) investigated multiple response variables, i.e. machining rate, SR and dimensional deviation in WEDM process using response surface methodology for pure titanium (grade 2). Pulse IP and pulse on time were discovered as the most significant factors for SR and WWR (Kumar, Kumar, & Kumar, 2013c). Chalisgaonkar and Kumar (2013) optimized WEDM process parameters for performance characteristics, i.e. cutting speed and SR using utility concept, while machining pure titanium. Singh and Sharma (2014) obtained the optimal sets of process parameters for MRR and surface finish during WEDM of P20 tool steel. Lodhi and Agarwal (2014) optimized process parameters for SR in WEDM process for AISI D3 steel. Rao and Krishna (2014) developed the empirical model between input parameters (SiC particulate size, volume percentages, pulse on time, pulse off time and wire tension (WT)) and response parameters (SR, metal removal rate and WWR) using response surface methodology. Khan, Khan, Siddiquee, and Chandra (2014) investigated the effect of WEDM process parameters on SR and micro-hardness using grey relation analysis for machining of high-strength low-alloy steel (ASTM A572 grade 50).

The novel contribution of this paper is mainly focused on the following issues.

(1) Consideration of wire weight consumption instead of WWR as considered in previous studied reported—In the available literature, wire wear phenomenon has been investigated by calculating the term WWR, which has been computed as the ratio of the loss of wire weight to the original wire weight. It is pertinent to mention that the eroded wire collected after machining with WEDM is not reusable as it could affect the dimensional accuracy and the machining
efficiency. Hence, the focus of this experimental study is on minimizing the wire consumption from the economical considerations. No investigation for wire consumption has been reported in the previous studies as observed from the literature review.

(2) Correlation of wire wear phenomena with wire weight consumption—Wire wear phenomena has been correlated with consumption pattern of wire weight after each experimental run. This has been done with an objective to identify the process settings that involve higher wire wear and least weight consumption.

(3) Multi-response optimization using grey-fuzzy logic methodology—The concept of grey-fuzzy logic methodology for multi-response optimization of response parameters such as MRR, SR and wire weight consumption (WWC) in WEDM process for pure titanium has not been used in past studies. It has been recognized since long that grey-fuzzy method has the capability to yield better results in comparison with other optimization algorithms.

(4) Development of mechanistic model for MRR—It has also been observed that the relevant machining guidelines for pure titanium are not available in most of the manufacturer’s catalogues of WEDM. This necessitates the development of appropriate mechanistic models for prediction of the machining speed (MRR) for WEDM of titanium. This issue has been ignored by almost all of the studies reported previously on WEDM of titanium and its alloys.

(5) Microstructure aspects—Surface integrity aspects such as microstructure analysis (including recast layer, debris and cracks, crater size and shape, etc.) of the machined titanium samples under different process conditions have also been investigated to evaluate the suitability of WEDM for machining titanium. The analysis has been extended by conducting energy dispersive X-ray analysis (EDX) of the machined titanium samples as well as the wire electrode to investigate and correlate the material transfer during machining with the discharge energy.

3. Materials and methods

3.1. Material
In this research, intricate machining of pure titanium was performed as shown in Figure 1(B). Commercially, pure titanium was taken as a work material in the form of rectangular block of 24.25 mm thickness. The chemical composition of material and mechanical properties are given in Tables 1a and 1b, respectively.

3.2. Experimental procedures
The experiments were performed on sprintcut (ELPULS-40A DLX) wire-EDM manufactured by Electronica Machine Tool Limited, India (Figure 1(A)). Zinc-coated brass wire having 0.25 mm dia was used in these experiments.
Six process parameters namely pulse on time ($T_{ON}$), pulse off time ($T_{OFF}$), IP, wire feed (WF), wire tension WT, servo voltage (SV) and three one-way interactions, viz. $T_{ON} \times T_{OFF}$, $T_{ON} \times IP$ and $T_{OFF} \times IP$, were selected as input variables during intricate cutting of pure titanium with WEDM. The selection of above interactions is based on the review of past literature. All six variables were assigned three levels. The values for these levels were fixed on the basis of a pilot experimentation, which was conducted using “One factor at a time” (OFAT) approach, to recognize the trends of influence for the machining variables. Table 2 depicts the levels of the selected process variables.

The orthogonal array forms the basis for the experimental analysis in the Taguchi method. The selection of orthogonal array is concerned with the total degree of freedom (DOF) of process parameters. Total DOF associated with six parameters and three one-way interactions is equal to 24 ($4 \times 3 + 6 \times 2$). The DOF for the orthogonal array should be greater than or at least equal to that of the process parameters. Thereby, a L27 orthogonal array having degrees of freedom equal to 26 has been considered in present case. Based on the experimental layout depicted in Table 3, the experiments were performed in random order and each specific experiment was repeated three times to have an estimate of the experimental error.

### 3.3. Measurement methodology

Three machining characteristics, namely MRR, WWC and SR, were measured. Cutting speed was measured by CNC WEDM monitor and subsequently, the MRR was calculated by using the formula:

$$MRR \left( \text{mm}^2/\text{min} \right) = \text{Cutting speed} \left( \text{mm/min} \right) \times \text{Thickness of material} \left( \text{mm} \right)$$

A roughness tester (Mitutoyo make) was used for the measurement of average SR ($R_a$) of the workpiece. The cut-off length ($l_c$) and the sampling number were chosen as 0.8 mm and 5, respectively. Three independent readings were taken on each surface of machined surface and the average of these was taken. Eroded wire after completion of each experiment was collected from collection spool and weighted by weighing machine (SHIMADZU electronic balance with 0.01 g accuracy) to compute the WWC. The experimental layout along with the mean values of the responses is shown in Table 3.

### Table 1a. Chemical composition of commercially pure titanium

| Element | C  | Fe  | O   | Ti  |
|---------|----|-----|-----|-----|
| N       | 0.001 | 0.06 | 0.10 | 0.002 | 99.82 |

### Table 1b. Mechanical properties of pure titanium

| Property                  | Value       |
|---------------------------|-------------|
| Young’s modulus           | 116 GPa     |
| Shear modulus             | 44 GPa      |
| Bulk modulus              | 110 GPa     |
| Poisson ratio             | 0.32        |
| Vickers hardness          | 970 MPa     |
| Brinell hardness          | 716 MPa     |

### Table 2. Process parameters with their levels

| Process parameters (unit) | Parameter designation | Level 1 | Level 2 | Level 3 |
|---------------------------|-----------------------|--------|--------|--------|
| Pulse on time (μs)        | A                     | 0.5    | 0.7    | 0.9    |
| Pulse off time (μs)       | B                     | 7      | 9.5    | 14     |
| Peak current (A)          | C                     | 80     | 140    | 200    |
| Wire feed (m/min)         | D                     | 6      | 8      | 10     |
| Wire tension (g)          | E                     | 850    | 1,200  | 1,600  |
| Servo voltage (V)         | F                     | 30     | 50     | 70     |

### Table 3

The experimental layout along with the mean values of the responses is shown in Table 3.
4. Results and discussion

4.1. Effect of process parameters on MRR

Figure 2(A) shows that the MRR increases with the increase in pulse on time and IP and decreases with the increase in pulse off time and spark gap set voltage. This response of MRR is linked to pulse discharge energy in the spark gap, which increases with the higher pulse on time, IP and lower values of pulse off time and spark gap set voltage. The concentration of high discharge energy in the spark gap leads to rapid melting and vaporization of molten metal in the spark gap leading to increased MRR. The decrement of MRR with higher pulse off time is related with reduced number of discharges within a given period (i.e. the discharge frequency). MRR decreases with increased spark gap set voltage due to widening of average discharge gap between wire and workpiece. The above facts have been discussed in microstructure study section of this paper. The effects of WF and WT on MRR were not found to be as significant as for other parameters. All the interactions considered were found to be statistically significant at 95% confidence level (Figure 3(A) and Table 4). Figure 4(A) shows the normal probability plot of the residuals for MRR. It shows that most of the residuals are clustered around a straight line, which indicates that errors are normally distributed and model assumptions are valid.

| Table 3. L27 orthogonal array and the experimental results |
|----------------------------------------------------------|
| Run | A | B | A × B | A × B | A × C | A × C | B × C | D | E | B × C | F | MRR (mm²/min) | WWC (g) | SR (μm) |
|-----|---|---|-------|-------|-------|-------|-------|---|---|-------|---|----------------|--------|--------|
| 1   | 1 | 1 | 1     | 1     | 1     | 1     | 1     | 1 | 1 | 1     | 1 | 51.286        | 37.61  | 2.084  |
| 2   | 1 | 1 | 1     | 2     | 2     | 2     | 2     | 2 | 2 | 2     | 1 | 49.33         | 43.84  | 2.141  |
| 3   | 1 | 1 | 1     | 3     | 3     | 3     | 3     | 3 | 3 | 3     | 1 | 33.619        | 82.227 | 2.063  |
| 4   | 1 | 2 | 2     | 1     | 1     | 1     | 2     | 2 | 2 | 2     | 3 | 18.799        | 115.55 | 2.014  |
| 5   | 1 | 2 | 2     | 2     | 2     | 2     | 3     | 3 | 3 | 1     | 1 | 48.17         | 48.113 | 2.136  |
| 6   | 1 | 2 | 2     | 3     | 3     | 3     | 3     | 3 | 3 | 3     | 2 | 39.022        | 46.81  | 2.016  |
| 7   | 1 | 3 | 3     | 3     | 3     | 3     | 1     | 1 | 1 | 1     | 3 | 19.68         | 118.02 | 2.021  |
| 8   | 1 | 3 | 3     | 3     | 2     | 2     | 2     | 1 | 1 | 1     | 3 | 16.843        | 116.49 | 1.961  |
| 9   | 1 | 3 | 3     | 3     | 3     | 3     | 3     | 2 | 2 | 2     | 1 | 39.599        | 58.16  | 2.05   |
| 10  | 2 | 1 | 2     | 3     | 3     | 3     | 3     | 2 | 2 | 2     | 1 | 57.494        | 33.66  | 2.333  |
| 11  | 2 | 1 | 2     | 3     | 3     | 3     | 1     | 2 | 3 | 1     | 2 | 47.533        | 53.47  | 2.247  |
| 12  | 2 | 1 | 2     | 3     | 3     | 2     | 3     | 3 | 1 | 2     | 3 | 59.72         | 28.877 | 2.382  |
| 13  | 2 | 2 | 3     | 1     | 1     | 2     | 3     | 2 | 3 | 1     | 3 | 58.321        | 44.213 | 2.456  |
| 14  | 2 | 2 | 3     | 1     | 2     | 3     | 1     | 3 | 1 | 2     | 1 | 56.425        | 29.533 | 2.283  |
| 15  | 2 | 2 | 3     | 1     | 3     | 1     | 2     | 1 | 2 | 3     | 2 | 44.348        | 48.017 | 2.263  |
| 16  | 2 | 3 | 1     | 2     | 3     | 1     | 2     | 3 | 1 | 2     | 2 | 23.239        | 80.05  | 2.265  |
| 17  | 2 | 3 | 1     | 2     | 3     | 1     | 2     | 3 | 1 | 2     | 3 | 61.589        | 32.543 | 2.416  |
| 18  | 2 | 3 | 1     | 2     | 3     | 1     | 2     | 3 | 1 | 2     | 1 | 51.749        | 56.637 | 2.411  |
| 19  | 3 | 1 | 3     | 2     | 3     | 2     | 1     | 3 | 2 | 1     | 3 | 53.281        | 47.98  | 2.261  |
| 20  | 3 | 1 | 3     | 2     | 3     | 2     | 1     | 3 | 2 | 1     | 2 | 69.996        | 25.367 | 2.882  |
| 21  | 3 | 1 | 3     | 2     | 3     | 2     | 1     | 3 | 2 | 1     | 3 | 93.338        | 24.13  | 2.774  |
| 22  | 3 | 2 | 1     | 3     | 3     | 2     | 2     | 1 | 3 | 3     | 2 | 60.525        | 24.753 | 2.683  |
| 23  | 3 | 2 | 1     | 3     | 3     | 2     | 1     | 3 | 3 | 2     | 1 | 54.23         | 40.87  | 2.483  |
| 24  | 3 | 2 | 1     | 3     | 3     | 2     | 1     | 3 | 3 | 2     | 2 | 75.981        | 35.35  | 2.338  |
| 25  | 3 | 3 | 2     | 1     | 3     | 3     | 2     | 3 | 2 | 1     | 2 | 62.16         | 35.05  | 2.44   |
| 26  | 3 | 3 | 2     | 1     | 3     | 3     | 2     | 3 | 2 | 1     | 3 | 61.65         | 41.627 | 2.365  |
| 27  | 3 | 3 | 2     | 1     | 3     | 3     | 2     | 3 | 2 | 1     | 3 | 45.074        | 37.163 | 2.434  |
4.2. Effect of process parameters on SR
Main effects plot of means for SR (Figure 2(B)) shows that the SR increases with the increase in pulse on time and IP and decreases with the increase in pulse off time and spark gap set voltage. The above trend can be attributed to the fact that higher discharge energy produces an impact on workpiece surface in terms of large size craters formation, which attributes towards higher SR. High discharge energy coupled with low pulse off time results in poor flushing of molten material and produces thicker layer of debris, resulting in increased SR. These facts have been confirmed through microstructure analysis, which is described in the later section of this paper. SR decreases with increased spark gap set voltage due to widening of average discharge gap resulting in lower pulse discharge energy. All interactions considered for the above study were found to be significant at 95% confidence level (Figure 3(B) and Table 5). Figure 4(B) shows the normal probability plot of the residuals for SR. It shows that most of the residuals are clustered around a straight line, which indicates that errors are normally distributed. The mean effect plot of means, interaction plot and normal probability plot of raw data of SR was plotted with the help of MINITAB 16 software.

4.3. Effect of process parameters on WWC
WWC is the weight of the eroded wire collected in collection spool after completion of each experimental run. It has been investigated and shown in main effects plot of means for WWC (Figure 2(C)) that if pulse on time and IP is increased (high pulse discharge energy), less amount of eroded wire is collected, which indicates less wire consumption. Similarly if pulse on time and IP is decreased (lower pulse discharge energy), then higher amount of eroded wire is collected, which means a higher wire consumption. These findings could be explained on the basis of the fact that if \( T_{\text{on}} \) or IP is increased, then higher amount of heat is available for machining the same cross section of specimen, which does not necessitate a large amount of wire. Consequently, less amount of wire is sufficient to cut the material. The same fact could be applied in either way, for understanding the higher wire consumption observed with lower values of pulse duration and IP. It can be concluded from the above fact that if pulse discharge energy in the form of \( T_{\text{on}} \) and IP is increased, then WWC decreases and vice versa. Figure 2(C) also shows that WWC increases as pulse off time is increased. Higher pulse off time yields reduced the number of discharges within a given period resulting in increased machining time (for a given volume of metal to be machined) which results in higher wire consumption. Higher spark gap set voltage also leads to extended machining time and therefore higher wires consumption. It is also observed from Figure 2(C) that higher feed rate of wire results in increased wire consumption. The effect of WT on WWC was not found to be significant. All interactions considered for the above study were found to be significant (Figure 3(C) and Table 6). Figure 4(C) shows the normal probability plot of the residuals for WWC. It shows that most of the residuals are
Figure 3. (A) Interaction plot for means of metal removal rate, (B) interaction plot for means of SR and (C) interaction plot for means of WWC.
clustered around a straight line, which indicates that errors are normally distributed. WWC has been considered as “lower is better” option for economical aspect of WEDM process under single response optimization study in the later section of this paper.

4.4. Interpretations of wire wear phenomena with WWC
From the above discussion, it is clear that if pulse discharge energy is increased, the WWC decreases. Now, a little consideration will show that with decreased amount of WWC, lesser amount of wire will undergo the machining of same cross-sectional area of workpiece, which may erode the wire.

Table 4. ANOVA of material removal rate

| Factor         | DOF  | Seq. SS  | Adj. SS  | Adj. MS  | F ratio | p Value | % Contribution |
|----------------|------|----------|----------|----------|---------|---------|----------------|
| T<sub>ON</sub> | 2    | 11,301.27| 11,301.27| 5,650.64 | 416.29  | 0.000*  | 45.70%         |
| T<sub>OFF</sub>| 2    | 3,004.86  | 3,004.86 | 1,502.43 | 110.69  | 0.000*  | 12.15%         |
| IP             | 2    | 1,114.34  | 1,114.34 | 557.17   | 41.05   | 0.000*  | 4.50%          |
| WF             | 2    | 575.92    | 575.92   | 287.96   | 21.21   | 0.000*  | 2.32%          |
| WT             | 2    | 276.71    | 276.71   | 138.36   | 10.19   | 0.000*  | 1.11%          |
| SV             | 2    | 6,737.37  | 6,737.37 | 3,368.86 | 248.19  | 0.000*  | 27.24%         |
| T<sub>ON</sub> × T<sub>OFF</sub> | 4    | 270.13    | 270.13   | 67.53    | 4.98    | 0.000*  | 1.09%          |
| T<sub>ON</sub> × IP   | 4    | 412.09    | 412.09   | 103.02   | 7.59    | 0.002*  | 1.66%          |
| T<sub>OFF</sub> × IP  | 4    | 273.26    | 273.26   | 68.32    | 5.03    | 0.000*  | 1.10%          |
| Error          | 56   | 760.13    | 760.13   | 13.57    |         |         | 3.07%          |
| Total          | 80   | 24,726.44 |         |         |         |         | 100%           |

*aDegree of freedom.
*bSequential sums of squares.
*cAdjusted sums of square.
*dAdjusted mean of square
*eSignificant at 95% level.

Figure 4. (A) Normal probability plot of the residuals for material removal rate, (B) normal probability plot of the residuals SR and (C) normal probability plot of the residuals for WWC.
excessively. On the counterpart, if WWC increases, more amount of wire will be available for sustaining the impact of pulse energy and will result in less wire erosion. These findings have been confirmed through comparison of SEM graph of eroded wire in experimental run No. 8 (lower discharge energy setting, WWC = 116.49 g) and experimental run No. 21 (higher discharge energy setting, WWC = 24.13 g). The SEM graphs of eroded wire has been discussed in microstructure study section. Hence, it can be concluded that higher wire consumption leads to less wire wear and vice versa.

5. Analysis of variance of experimental results
ANOVA (analysis of variance) is a statistically based, objective decision-making tool for detecting any differences in the response that might be attributed to the variation in the control parameters considered in the study. ANOVA helps in formally testing the significance of all main factors and their

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### Table 5. ANOVA of surface roughness

| Factor   | DOFa | Seq. SSb | Adj. SSc | Adj. MSd | F ratio | p Value | % Contribution |
|----------|------|----------|----------|----------|---------|---------|----------------|
| T_on     | 2    | 2.95648  | 2.95648  | 1.47824  | 131.31  | 0.000*  | 62.76          |
| T_off    | 2    | 0.10974  | 0.10974  | 0.05487  | 4.87    | 0.011*  | 2.31           |
| IP       | 2    | 0.07146  | 0.07146  | 0.03573  | 3.17    | 0.049*  | 1.50           |
| WF       | 2    | 0.03041  | 0.03041  | 0.01520  | 1.35    | 0.267   | 0.63           |
| WT       | 2    | 0.09816  | 0.09816  | 0.04908  | 4.36    | 0.017*  | 2.08           |
| SV       | 2    | 0.32741  | 0.32741  | 0.16370  | 14.54   | 0.000*  | 6.94           |
| T_on × T_off | 4    | 0.16560  | 0.16560  | 0.04140  | 3.68    | 0.010*  | 3.50           |
| T_on × IP  | 4    | 0.15055  | 0.15055  | 0.03764  | 3.34    | 0.016*  | 3.18           |
| T_off × IP  | 4    | 0.17048  | 0.17048  | 0.04262  | 3.79    | 0.009*  | 3.60           |
| Error    | 56   | 0.63044  | 0.63044  | 0.1126   |         |         |                |
| Total    | 80   | 4.71074  |          |          |         |         |                |

*aDegree of freedom.
*bSequential sums of squares.
*cAdjusted sums of square.
*dAdjusted mean of square.
*Significant at 95% level.

### Table 6. ANOVA of WWC

| Factor   | DOFa | Seq. SSb | Adj. SSc | Adj. MSd | F ratio | p Value | % Contribution |
|----------|------|----------|----------|----------|---------|---------|----------------|
| T_on     | 2    | 22,465.8 | 22,465.8 | 11,232.9 | 114.22  | 0.000*  | 36.09          |
| T_off    | 2    | 6,988.7  | 6,988.7  | 3,494.35 | 35.53   | 0.000*  | 11.23          |
| IP       | 2    | 2,836.8  | 2,836.8  | 1,418.4  | 14.42   | 0.000*  | 4.56           |
| WF       | 2    | 2,156.1  | 2,156.1  | 1,078.05 | 10.96   | 0.000*  | 3.46           |
| WT       | 2    | 184.1    | 184.1    | 92.05    | 0.94    | 0.398   | 0.30           |
| SV       | 2    | 13,672   | 1,3672   | 6,836    | 69.51   | 0.000*  | 21.96          |
| T_on × T_off | 4    | 3,396.3  | 3,396.3  | 849.075  | 8.63    | 0.000*  | 5.46           |
| T_on × IP  | 4    | 1,965.9  | 1,965.9  | 491.475  | 5       | 0.002*  | 3.16           |
| T_off × IP  | 4    | 3,083.4  | 3,083.4  | 770.85   | 7.84    | 0.000*  | 4.95           |
| Error    | 56   | 5,507.2  | 5,507.2  | 98.34286 |         |         |                |
| Total    | 80   | 62,256.3 |          |          |         |         |                |

*aDegree of freedom.
*bSequential sums of squares.
*cAdjusted sums of square.
*dAdjusted mean of square.
*Significant at 95% level.
interactions by comparing the mean square (MS) against an estimate of the experimental error at a specific confidence level. First, the total sum of squared deviations (SS) is determined which is measured by subtracting the mean from each score in a distribution of scores, squaring this difference and then adding all of the differences to produce a total. Further MS are the estimates of variance across the treatments (i.e. factors), which are calculated as the sum of squares divided by its appropriate degrees of freedom.

The relative significance of machining parameters for their effects on machining characteristics is determined by applying ANOVA test to the experimental results obtained. F-test gives the information of decision at a predefined confidence level, whether the parameters selected under study are statistically significant. Larger F value indicates a strong significance level of input parameter for its effect on output response under consideration. If the computed F value of the selected parameter is larger than \( F_{\alpha, v_1, v_2} \) value (confidence table value) where \( \alpha \) is risk, \( v_1 \) and \( v_2 \) are DOF associated with the factor and error terms, the input parameter has a significant effect on machining characteristics under question. The \( p \) value tests the null hypothesis that data from all groups are drawn from populations with identical means. The \( p \) value less than 0.05 indicates the statistical significance of selected parameter under 95% confidence levels.

ANOVA was performed on experimental results of each quality characteristics as obtained in Table 3. MINITAB 16 software was used to perform ANOVA on the above experimental data.

5.1. Material removal rate

From Table 4, \( T_{\text{ON}} (45.70\%) \), \( SV (27.24\%) \) and \( T_{\text{OFF}} (12.15\%) \) factors have a strong influence on MRR. All the factors and interactions under study are found to be significant at 95% confidence level. It has been found that \( T_{\text{ON}} \) and \( SV \) are the most effective parameters at 95% confidence level for MRR.

5.2. Surface roughness

\( T_{\text{ON}} (62.76\%), SV (6.94\%) \) and \( T_{\text{OFF}} \times IP (3.6\%) \) have a strong influence on SR. All the factors and interactions under study are found to be significant at 95% confidence level except WF (see Table 5). It has been found that \( T_{\text{ON}} \) and \( SV \) are the most effective parameters at 95% confidence level for SR.

5.3. Wire weight consumption

\( T_{\text{ON}} (36.09\%), SV (21.96\%) \) and \( T_{\text{OFF}} (11.23\%) \) have a strong influence on WWC. All the factors and interactions under study are found to be significant at 95% confidence level except WT (see Table 6). It has been found that \( T_{\text{ON}} \) and \( SV \) are the most effective parameters at 95% confidence level for WWC.

6. Microstructure aspects of commercially pure titanium processed by WEDM

After WEDM operations, surface integrity of the machined workpiece surface was examined using a scanned electron microscope SEM (Make Zeiss EV040). This investigation helped to know the structural and composition changes during the WEDM processes. The microstructures of machined surfaces of experimental run No. 4th (lower discharge energy level: \( T_{\text{ON}} = 0.5 \mu s, T_{\text{OFF}} = 9.5 \mu s, IP = 80A, WF = 8 \text{ m/min}, WT = 1,200 \text{ g}, SV = 70 \text{ V} \) and 21st (higher discharge energy level: \( T_{\text{ON}} = 0.9 \mu s, T_{\text{OFF}} = 7 \mu s, IP = 200A, WF = 8 \text{ m/min}, WT = 850 \text{ g}, SV = 50 \text{ V} \)) was investigated using 1,000x, 2,000x and 3,000x magnification level as shown in Figures 5 and 6.

Figure 5(A) and (B) shows the SEM graph of experimental run No. 4th and 21st at 1,000x magnification level. The SEM graph shows that machined surface texture is irregularly fashioned (Figure 5(A)) as compared to layered structure (Figure 5(B)). The detailed surface morphology of SEM graph for the above experimental settings is shown in Figure 6 at 2,000x and 3,000x. SEM graph of 4th experimental run shows shallow craters (A), higher density of gas holes (B), thinner layers of global appendages or debris (C) and small-sized spherical nodules (D) (Figure 6(A)). The shallow craters were formed by rapid electric discharging with lower energy intensity. Some of the molten material during WEDM was flushed out by the forced dielectric circulation while remaining melt resolidifies to
form an undulating terrain or layer of debris. The spherical nodules were resulted from the solidification of molten or vaporized workpiece material and wire material during machining. Spherical shape of the nodules indicated that the surface energy is minimized during solidification (Huang et al., 2004), while gas holes in the surface were typically developed from the melting and recast process.

It is observed from SEM graph (Figure 6(B)) that the machined surface texture consists of higher amount of micro-cracks (E), bigger spherical sized nodules (F), thicker layers of overlapping debris (G) and overlapped craters (H) for the sample machined at process setting corresponding to 21st experimental run. The higher values of pulse duration and IP coupled with lower off time and SV, imparted higher input energy rate (discharge) to the work surface in this case. The cracks were formed due to the development of high thermal stresses exceeding the fracture strength of the material, as well as with plastic deformation (Lee & Li, 2003). These thermal stresses were prevailed at the surface due to rapid cooling after the discharge. The small amount of gas holes on the surface of high discharge energy setting sample may be due to faster cooling rate. It has been found from figure that the deposits and spherical nodules were bigger in size as compared to Figure 6(A). It is also revealed from Figure 6 that these spherical nodules were evenly covered with white cover which may be due to the oxide formation (Huang et al., 2004).

6.1. Recast layer aspect in rough cut WEDM

Recast layer is formed on the machined surface after rough cut WEDM operation, which is undesirable from metallurgical point of view as it may lead to reduction of fatigue strength and formation of surface defects.
of micro-cracks in the outer surface of the machined component. Recast layer is developed due to the re-solidification of melted material which was not completely ejected or removed during the WEDM process. It has been reported that heat-affected zone is below recast layer. The region of the workpiece near the molten zone, which get affected due to pulse discharge energy, is termed as the heat-affected zone.

SEM micrograph was taken at the cross section of the sample for recast layer analysis. Recast layer was measured at three positions using IMAGE J software and the average value was calculated (Figure 7). Figure 7(A) and (B) shows the SEM graph of recast layer formed for the WEDMed surface of experiment No. 8 (lower discharge energy parametric setting) and experiment No. 21st (higher discharge energy parametric setting), respectively. It is clear from Figure 7(B) that thicker recast layer (average 24.26μ) is formed if pulse on time, IP is increased with decreasing level of pulse off time and SV. Due to higher pulse discharge energy, more material is melted/evaporated during pulse on time and subsequently, some of the molten material could not get flushed out due to surface tension and cooling effects. Figure 7(A) shows the thinner recast layer (average 10.21μ) formed on the machined surface due to lower pulse discharge energy.

6.2. EDX analysis of machined samples (Ti)

The EDX analysis of sample workpiece WEDMed by lower (Exp. No. 8th) and higher discharge energy setting (Exp. No. 21st) was performed with SEM (Make Zeiss EV040) with energy-dispersive X-ray spectrometer (EDX) (Figure 8). It was revealed (Table 7) from comparative EDX analysis between Exp. No. 8th and Exp. No. 21st that proportions of the parental workpiece elements such as titanium, oxygen and nitrogen reduce, while carbon and iron increase. Migrated tool elements such as copper were found to be increased from 1.41 to 8.59%, while Zn reduced from 1.74% to a negligible amount. The detection of copper and zinc on machined surface is due to partly melting or vaporization and subsequently deposition of wire electrode elements on the machined surface during WEDM process. Zn might have been evaporated due to higher discharge energy produced during 21st experimental run. The reason for zinc elimination in 21st experiment is mainly due to lower melting and evaporation temperature of Zn (419.5°C) as compared to Cu (1,084.62°C).

The presence of oxygen reveals oxide formation of titanium which was present in white colour shade in SEM graph. Oxide formation might be due to the fact that de-ionized water contains chemical compound of hydrogen and oxygen and it decomposes during sparking and forms TiO₂ on workpiece after machining.

The reason for reduction of oxygen from Exp. No. 8th to 21st is due to the fact that Exp. No. 8 has been performed at lower cutting speed with respect to Exp. No. 21. Lower cutting speed in Exp. No.
8 necessitates higher machining time due to which more machining time is available to react titanium with oxygen decomposed from de-ionized water during discharge interval, and therefore higher concentration of oxides are formed on machined surface.

7. Microstructure study of zinc-coated wire

Figure 9(A) represents SEM of unused, fresh wire. Figure 9(B) represents SEM graph of wire used in 21st experimental run ($T_{\text{on}} = 0.9 \mu s$, $T_{\text{off}} = 7 \mu s$, IP = 200 A, WF = 8 m/min, WT = 850 g, SV = 50 V) which
shows large amount of debris, craters and significant number of micro-cracks on wire electrode surface. Debris is deposited due to melting and vaporizing of wire electrode surface and titanium during machining. The traces of white colour on eroded surface of wire electrode may indicate the oxide formation during machining. The formation of craters on wire surface could be linked to higher amount of pulse on time and IP used in 21st experimental setting. The higher thermal stresses developed at the wire electrode surface, which is indicated by the higher density of micro-cracks, might be due to rapid cooling (due to low pulse off time) after the intense heat discharge (higher pulse duration and IP).

Figure 9(C) represents SEM view of eroded wire used in machining 8th experimental run (\(T_{\text{ON}} = 0.5 \ \mu\text{s}, \ T_{\text{OFF}} = 14 \ \mu\text{s}, \ IP = 140 \ \text{A}, \ WF = 6 \ \text{m/min}, WT = 850 \ \text{g}, SV = 70 \ \text{V}\)). It is shown from Figure 9(C) that comparatively smaller amount of debris, craters and no cracks were found on wire

Table 7. Elemental analysis by EDX for workpiece machined under 21st experimental setting

| Element     | Exp. No. 8th (Weight %) | Exp. No. 21st (Weight %) |
|-------------|-------------------------|--------------------------|
| Titanium    | 82.92                   | 78.15                    |
| Copper      | 1.41                    | 8.59                     |
| Carbon      | 0.53                    | 0.62                     |
| Iron        | 0.08                    | 0.15                     |
| Oxygen      | 12.40                   | 10.89                    |
| Nitrogen    | 0.92                    | 0.83                     |
| Zinc        | 1.74                    | 0                        |

Figure 9. SEM micrograph of zinc-coated wire (A) Unused wire at 500x, (B) Eroded wire at 21st run (\(T_{\text{ON}} = 0.9 \ \mu\text{s}, \ T_{\text{OFF}} = 7 \ \mu\text{s}, IP = 200 \ \text{A}, WF = 8 \ \text{m/min}, WT = 850 \ \text{g}, SV = 50 \ \text{V}\)) at 1,500x and (C) eroded wire at 8th run (\(T_{\text{ON}} = 0.5 \ \mu\text{s}, T_{\text{OFF}} = 14 \ \mu\text{s}, IP = 140 \ \text{A}, WF = 6 \ \text{m/min}, WT = 850 \ \text{g}, SV = 70 \ \text{V}\)) at 1,500x.
surface due to lower discharge energy. It may be concluded from comparative SEM graph of Figure 9 that wire surface experiences severe degradation when pulse discharge energy is increased. As discussed earlier, it may be correlated with the WWC.

8. Multi-machining characteristics optimization using grey-fuzzy logic methodology

8.1. Determination of grey relation coefficient

In this methodology, experimental data (MRR, SR and WWC) has been normalized in the range between 0 and 1. Then, the grey relational coefficient is calculated from the normalized experimental data to express the relationship between the desired and actual experimental data. The MRR has been selected as “larger the better”, while wire wear consumption (WWC) and SR have been considered as “lower the better type” characteristics.

The experimental data have been normalized for MRR (“larger the better” characteristic) as follows:

\[ x^*_i(k) = \frac{y^*_i(k) - \min y^0_i(k)}{\max y^0_i(k) - \min y^0_i(k)} \]  

while SR and WWC (“Lower is better” characteristics) have been normalized as follows:

\[ x^*_i(k) = \frac{\max y^0_i(k) - y^*_i(k)}{\max y^0_i(k) - \min y^0_i(k)} \]  

where \( y^*_i(k) \) is the experiment result in \( i \)th experiment, \( x^*_i(k) \) is the normalized result in \( i \)th experiment and \([\max y^0_i(k) - \min y^0_i(k)]\) is the difference between maximum and minimum value of the experimental results.

After normalizing of the experimental results, the deviation sequence is found out. The deviation sequence \( \Delta_0^1(k) = |x^*_0(k) - x^*_i(k)| \) is calculated, where \( x^*_0(k) \) is the reference sequence which is generally set to be equal to 1 and \( x^*_i(k) \) is the normalized results of first experiment. Table 8 shows normalized experimental results.

8.2. Grey relational coefficient and grey relational grade

Grey relation coefficient is calculated for each of the performance characteristics, which expresses the relationship between ideal and actual normalized experimental results.

The grey relational coefficient is calculated in the following manner:

\[ \xi_i(k) = \frac{\Delta_{\min} + \tau \Delta_{\max}}{\Delta_{0i} + \tau \Delta_{\max}} \]  

where \( \Delta_{0i}(k) \) is the deviation sequence of the reference sequence \( x^*_0(k) \) and the comparability sequence \( x^*_i(k) \).

\[ \Delta_{0i}(k) = \|x^*_0(k) - x^*_i(k)\| \]  

\[ \Delta_{\max} = \max_{v_j} \max_{v_k} \|x^*_0(k) - x^*_i(k)\| \]  

\[ \Delta_{\min} = \min_{v_j} \min_{v_k} \|x^*_0(k) - x^*_i(k)\| \]  

where \( \tau \) is distinguishing coefficient and its value is generally taken as 0.5. Table 9 shows the deviation sequence and grey relational coefficient for each experiment using the L27 orthogonal array.
8.3. Fuzzy logic optimization

Fuzzy logic employs linguistic terms to form profound relationship between input and output parameters. Fuzzy logic involves a fuzzy inference engine and a fuzzification–defuzzification module. A fuzzy-rule based inference engine comprises three basic components: fuzzifier, inference engine and defuzzifier. The important function of the fuzzy logic system is to establish mapping between input and output parameters. The mapping mechanism is based on the concept of human knowledge, experience and logics represented in linguistic terms (if–then rules).

In this research work, ‘grey relation fuzzy grade’ has been computed to optimize the input parameters considered in this study. The output parameter ‘grey relation fuzzy grade’ has been defined in the range between 0 and 1. The multi-response optimization of the parameters (mapping of input and output parameter) is done using fuzzy logic technique. The desired output is targeted on maximizing grey relation fuzzy grade. In this research work, grey relation grade (Table 9) of MRR, SR and WWC has been taken as fuzzy input and grey relation fuzzy grade as output for finding out optimum parametric setting. Fuzzy logic inference system has been developed using MathWorks MATLAB 7.8.0 (Release 2009a), Fuzzy Logic Toolbox. The fuzzy logic scheme has been represented in Figure 10.

| Exp. No. | MRR  | SR  | WWC |
|---------|------|-----|-----|
| 1       | 0.45 | 0.87| 0.86|
| 2       | 0.42 | 0.80| 0.79|
| 3       | 0.22 | 0.89| 0.38|
| 4       | 0.03 | 0.94| 0.03|
| 5       | 0.41 | 0.81| 0.74|
| 6       | 0.29 | 0.94| 0.76|
| 7       | 0.04 | 0.93| 0.00|
| 8       | 0.00 | 1.00| 0.02|
| 9       | 0.30 | 0.90| 0.64|
| 10      | 0.53 | 0.60| 0.90|
| 11      | 0.40 | 0.69| 0.69|
| 12      | 0.56 | 0.54| 0.95|
| 13      | 0.54 | 0.46| 0.79|
| 14      | 0.52 | 0.65| 0.94|
| 15      | 0.36 | 0.67| 0.75|
| 16      | 0.08 | 0.67| 0.40|
| 17      | 0.59 | 0.51| 0.91|
| 18      | 0.46 | 0.51| 0.65|
| 19      | 0.48 | 0.67| 0.75|
| 20      | 0.69 | 0.00| 0.99|
| 21      | 1.00 | 0.12| 1.00|
| 22      | 0.57 | 0.43| 0.99|
| 23      | 0.49 | 0.43| 0.82|
| 24      | 0.77 | 0.37| 0.88|
| 25      | 0.59 | 0.48| 0.88|
| 26      | 0.59 | 0.56| 0.81|
| 27      | 0.37 | 0.49| 0.9 |
The input variables such as grey relation grade of MRR, SR and WWC have been represented by membership functions having three levels, namely low (L), middle (M) and high (H) as shown in Figure 11. The output variable (grey-fuzzy grade) is represented by membership functions having nine levels, namely extremely small (ES), very small (VS), small (S), small medium (SM), medium (M), large medium (LM), large (L), very large (VL) and extremely large (EL) (Figure 12). The trapezoidal membership function was selected for both input and output parameters. The fuzzy rule base consists of a group of if-then control rules with the three grey relational coefficients of MRR, SR and WWC and one multi-response output of grey-fuzzy grade. Twenty-seven fuzzy rules are directly derived based on the fact that the larger grey relational fuzzy grade is the better process response.

### Table 9. Deviation sequence, grey relational coefficient and grey relational fuzzy grade

| Exp. No. | $\Delta_0$ (MRR) | $\Delta_0$ (SR) | $\Delta_0$ (WWC) | $\xi_{MRR}$ | $\xi_{ISR}$ | $\xi_{WWC}$ | GRFG  |
|----------|-----------------|-----------------|-----------------|-------------|-------------|-------------|-------|
| 1        | 0.55            | 0.13            | 0.14            | 0.48        | 0.79        | 0.78        | 0.639 |
| 2        | 0.58            | 0.20            | 0.21            | 0.67        | 0.72        | 0.70        | 0.607 |
| 3        | 0.78            | 0.11            | 0.62            | 0.39        | 0.82        | 0.45        | 0.519 |
| 4        | 0.97            | 0.06            | 0.97            | 0.34        | 0.90        | 0.34        | 0.464 |
| 5        | 0.59            | 0.19            | 0.26            | 0.46        | 0.72        | 0.66        | 0.596 |
| 6        | 0.71            | 0.06            | 0.24            | 0.61        | 0.89        | 0.67        | 0.591 |
| 7        | 0.96            | 0.07            | 1.00            | 0.34        | 0.88        | 0.33        | 0.452 |
| 8        | 1.00            | 0.00            | 0.98            | 0.33        | 1.00        | 0.34        | 0.485 |
| 9        | 0.70            | 0.10            | 0.36            | 0.42        | 0.84        | 0.58        | 0.56  |
| 10       | 0.47            | 0.40            | 0.10            | 0.52        | 0.55        | 0.83        | 0.583 |
| 11       | 0.60            | 0.31            | 0.31            | 0.46        | 0.62        | 0.62        | 0.558 |
| 12       | 0.44            | 0.46            | 0.05            | 0.53        | 0.52        | 0.91        | 0.607 |
| 13       | 0.46            | 0.54            | 0.21            | 0.52        | 0.48        | 0.70        | 0.548 |
| 14       | 0.48            | 0.35            | 0.06            | 0.51        | 0.59        | 0.90        | 0.618 |
| 15       | 0.64            | 0.33            | 0.25            | 0.44        | 0.60        | 0.66        | 0.546 |
| 16       | 0.92            | 0.33            | 0.60            | 0.35        | 0.60        | 0.46        | 0.443 |
| 17       | 0.41            | 0.49            | 0.09            | 0.55        | 0.50        | 0.85        | 0.575 |
| 18       | 0.54            | 0.49            | 0.35            | 0.48        | 0.51        | 0.59        | 0.514 |
| 19       | 0.52            | 0.33            | 0.25            | 0.49        | 0.61        | 0.66        | 0.565 |
| 20       | 0.31            | 1.00            | 0.01            | 0.62        | 0.33        | 0.97        | 0.597 |
| 21       | 0.00            | 0.88            | 0.00            | 1.00        | 0.36        | 1.00        | 0.875 |
| 22       | 0.43            | 0.57            | 0.01            | 0.54        | 0.47        | 0.99        | 0.625 |
| 23       | 0.51            | 0.57            | 0.18            | 0.49        | 0.47        | 0.74        | 0.559 |
| 24       | 0.23            | 0.63            | 0.12            | 0.69        | 0.44        | 0.81        | 0.675 |
| 25       | 0.41            | 0.520           | 0.12            | 0.55        | 0.49        | 0.81        | 0.578 |
| 26       | 0.41            | 0.439           | 0.19            | 0.55        | 0.53        | 0.73        | 0.556 |
| 27       | 0.63            | 0.514           | 0.1            | 0.44        | 0.49        | 0.78        | 0.56  |

The input variables such as grey relation grade of MRR, SR and WWC have been represented by membership functions having three levels, namely low (L), middle (M) and high (H) as shown in Figure 11. The output variable (grey-fuzzy grade) is represented by membership functions having nine levels, namely extremely small (ES), very small (VS), small (S), small medium (SM), medium (M), large medium (LM), large (L), very large (VL) and extremely large (EL) (Figure 12). The trapezoidal membership function was selected for both input and output parameters. The fuzzy rule base consists of a group of if-then control rules with the three grey relational coefficients of MRR, SR and WWC and one multi-response output of grey-fuzzy grade. Twenty-seven fuzzy rules are directly derived based on the fact that the larger grey relational fuzzy grade is the better process response.
Figure 11. Membership functions for input variables (MRR, SR and WWC).
Rule 1: If grey relational coefficient (MRR) is L and grey relational coefficient (SR) is L and grey relational coefficient (WWC) is L, then grey-fuzzy grade is L.

else

Rule 2: If grey relational coefficient (MRR) is L and grey relational coefficient (SR) is L and grey relational coefficient (WWC) is M, then grey-fuzzy grade is VS.

...

Rule 27: If grey relational coefficient (MRR) is H and grey relational coefficient (SR) is H and grey relational coefficient (WWC) is H, then grey-fuzzy grade is EL.

Furthermore, the grey relational coefficient of each output parameter of MRR, SR and WWC was used as an input in the rule editor of fuzzy inference engine (Figure 13). The de-fuzzified values of grey-fuzzy grade obtained for each experimental run are tabulated in Table 9. Since the experimental layout is orthogonal, it is possible to separate out the effect of each process parameter at selected levels of this experimental study. The response table for mean of the output parameter (grey relation fuzzy grade) for each level of the input parameters is shown in Table 10. Figure 14 shows the main effect plot of grey-fuzzy grade. The optimal parametric setting for multi-response output parameter was found out as follows:

Pulse on time ($T_{on}$)—0.9 μs, Pulse off time ($T_{off}$)—7 μs, IP—200 A, WF rate—8 mm/min, WT—850 g and Servo-voltage (SV)—50 V.

9. Confirmation experiments

The Taguchi approach for predicting the mean performance characteristics and determination of confidence intervals for the predicted mean has been applied. Three confirmation experiments for each of the performance characteristics have been performed at optimal settings of the process parameters and the average value has been reported. The average values of the performance characteristics obtained through the confirmation experiments (three runs) must be within the 95% confidence interval, $C_{1\alpha}$ (fixed number of confirmation experiments).

The optimum values of response parameters (μ) have been predicted for the optimized setting identified though the grey-fuzzy technique, i.e. Pulse on time ($T_{on}$)—0.9 μs, Pulse off time ($T_{off}$)—7 μs, IP—200 A, WF rate—8 m/min, WT—850 g and Servo-voltage (SV)—50 V.
CI is given by the following equation.

\[
CI_{CE} = \sqrt{F_{\alpha}(1, f_e) \left( \frac{1}{n_{eff}} + \frac{1}{R} \right) V_e}
\]  

(8)

where \(F_{\alpha}(1, f_e)\) is the F-ratio at a confidence level of \((1 - \alpha)\) against DOF 1 and error degrees of freedom and \(V_e\) is the error variance (\(V_e = 0.0046\), from ANOVA of GFG).
where \( N \) stands for total number of experiments \((27 \times 3 = 81)\) and \( R \) is the sample size for confirmatory experiments \((R = 3)\).

Predicted confidence interval for confirmation experiment is given below:

\[
\mu_{\text{mean}} - C_{\text{CI}} < \mu_{\text{mean}} < \mu_{\text{mean}} + C_{\text{CI}}
\]

By putting the values of \( \mu_{\text{mean}} \) and \( C_{\text{CI}} \) in the above equation, we get:

\[
0.54 < \mu_{\text{mean}} < 1.00
\]

Three confirmation experiments were conducted at optimal parametric setting. The average values of quality characteristics are reported in Table 11.

The experimental value of mean value of grey-fuzzy grade was calculated by using the above values of quality characteristics and was found 0.87. Since this value falls within 95% CI limit (Equation 10), the optimal results are validated. Furthermore, the above validated value of mean value of grey-fuzzy grade (0.87) was also found better than predicted value (0.772) at the above parametric setting.

10. Micro-model for prediction of MRR

It was considered to develop a model for quality characteristics (MRR) of the rough cut WEDM operation, which includes the critical process parameters such as pulse on time \( T_{\text{ON}} \), pulse off time \( T_{\text{OFF}} \), IP, WF, WT and SV. All of the input parameters selected for developing the model have been found to be significant for MRR, as observed from the results of ANOVA test. The present work uses the technique of dimensional analysis for modelling. The theory of dimensional analysis is the mathematical theory which is purely algebraic. The applicability of dimensional analysis to a certain situation is based on the hypothesis that the solution of the problem is expressible by means of a dimensionally homogeneous equation in terms of specified variables.

Buckingham’s \( \pi \) theorem states that if there are \( n \) variables in a problem and these variables contain \( m \) primary dimensions (e.g. M, L, T and I), the equation relating all the variables will have \( (n - m) \) dimensionless groups. Buckingham referred to these groups as \( \pi \) groups. The final equation obtained is in the form of following equation:

\[
\pi_1 = f(\pi_1, \pi_2, \ldots, \pi_{n-m})
\]

This method offers the advantage of being simpler than the method of solving simultaneous equations for obtaining the values of the indices (the exponent values of the variables). In this method of solving the equation, there are two conditions:

(1) Each of the fundamental dimensions must appear in at least one of the \( m \) variables.

(2) It must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

| Quality characteristics | Confirmatory experimental value |
|-------------------------|--------------------------------|
| MRR                     | 93.33 mm²/min                 |
| SR                      | 2.77 μm                       |
| WWC                     | 24.13 g                       |
After applying the dimensional analysis, MRR can be given by the following equation.

\[
MRR = f \left( T_{\text{ON}}, T_{\text{OFF}}, \text{IP}, \text{WF}, \text{WT}, \text{SV} \right)
\]

where MRR is the material removal rate, \( T_{\text{ON}} \) is the pulse on time, \( T_{\text{OFF}} \) is the pulse off time, IP is the peak current, WF is the wire feed, WT is the wire tension and SV is the servo-voltage.

The dimensions and units for process variables and quality characteristics are given in Table 12.

In this case, number of variables \( n = 7 \) (i.e. MRR, \( T_{\text{ON}}, T_{\text{OFF}}, \text{IP}, \text{WF}, \text{WT}, \text{SV} \))

Number of fundamental dimensions \( m = 4 \) (i.e. \([M],[L],[T],[I]\))

By Buckingham's theorem,

number of dimensionless groups \( = n - m = 7 - 4 = 3 \)

The dimensionless term can be expressed in the following way:

\[
[M^4L^2T^{-1}]a[M^0L^0T^1]b[M^0L^0T^1]c[M^0L^0T^{-1}]d[M^1L^2T^{-3}I^{-1}]g = [M^0L^0T^0I^0]
\]

The set of simultaneous equations is as follows:

\[
f + g = 0 \tag{15}
\]

\[
2a + e + 2g = 0 \tag{16}
\]

\[
-a + b + c - e - 3g = 0 \tag{17}
\]

\[
d - g = 0 \tag{18}
\]

The simultaneous equation can be converted into matrix form of \([A] \times [X] = [B]\).

where

\[
[A] = \begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 2 \\
0 & -1 & 0 & -3 \\
1 & 0 & 0 & -1
\end{bmatrix}
\]

\[
[X] = \begin{bmatrix}
d \\
e \\
f \\
g
\end{bmatrix}
\]

By putting \( a = 1, \ b = 0 \) and \( c = 0 \) in the above simultaneous Equations 15–18, we get:

\[
B = \begin{bmatrix}
0 & -2 & 1 & 0
\end{bmatrix}^T
\]

But \( X = [A]^{-1} \times [B] \)

### Table 12. Variables, units and dimensions used in model

| Variable               | Units     | Symbol | Dimensions   |
|------------------------|-----------|--------|--------------|
| Material removal rate  | mm²/min   | MRR    | \( M^0L^2T^{-1} \) |
| Pulse on time          | \( \mu s \) | \( T_{\text{ON}} \) | \( M^0L^0T \) |
| Pulse off time         | \( \mu s \) | \( T_{\text{OFF}} \) | \( M^0L^0T \) |
| Peak current           | A         | IP     | \( M^0L^0T^0I \) |
| Wire feed              | mm/min    | WF     | \( M^0L^0T^1 \) |
| Wire tension           | gram      | WT     | \( M^0L^0T^0 \) |
| Servo-voltage          | V         | SV     | \( M^0L^0T^{-1}I \) |
Subsequently, we get:

\[ X = \begin{bmatrix} 1 & -4 & -1 & 1 \end{bmatrix}^T \] (20)

Putting the values from the above equation in dimensional analysis equation

\[ a = 1, b = 0, c = 0, d = 1, e = -4, f = -1, g = 1 \]

Now dimensionless term can be written as:

\[ \pi_1 = [\text{MRR}]^1[\text{IP}]^{-4}[\text{WF}]^{-1}[\text{WT}]^{-1}[\text{SV}] \] (21)

Similarly, by putting \( b = 1, a = 0 \) and \( c = 0 \), we get:

\[ d = 1, e = -2, f = -1, g = 1 \]

Dimensionless term can be written as:

\[ \pi_2 = [\text{T}_{\text{ON}}][\text{I}]^{-2}[\text{SV}] \] (22)

Similarly, by putting \( c = 1, a = 0 \) and \( b = 0 \), we get:

\[ d = 1, e = -2, f = -1, g = 1 \]

Dimensionless term can be written as:

\[ \pi_3 = [\text{T}_{\text{OFF}}][\text{I}]^{-1}[\text{SV}] \] (23)

The finalized mathematical model for MRR can be represented in the following way using Equations 21–23.

\[ \text{MRR} = \frac{C \times \text{T}_{\text{ON}} \times \text{T}_{\text{OFF}} \times \text{I} \times \text{SV}}{\text{WT}} \] (24)

where \( C \) is a constant of proportionality. The value of “\( C \)” was determined by performing experiments by “changing OFAT approach”, i.e. by varying pulse on time (\( T_{\text{ON}} \)) and keeping remaining parameters fixed at baseline level. MRR was computed experimentally for various values of pulse on time. The experimental data are presented in Table 13.

The experimentally obtained values of MRR were put in Equation 24, along with the values of input parameters, to compute corresponding values of constant \( C \). Afterwards, equation for \( C \) and fitted line plot was determined and shown in Figure 15. Regression equation was developed to predict the value of \( C \) for a given value of \( T_{\text{ON}} \).

Thus, the regression equation for MRR in this case is:

\[ C = (0.002840 + 0.04080T_{\text{ON}} - 0.01607T_{\text{ON}}^2 - 0.00208T_{\text{ON}}^3) \] (25)

| Parameter       | Level value (μs) | Experimental MRR (mm³/min) |
|-----------------|------------------|-----------------------------|
| Pulse on time (\( T_{\text{ON}} \)) | 0.4              | 19.6245                     |
|                 | 0.6              | 37.345                      |
|                 | 0.8              | 58.2                        |
|                 | 1                | 75.175                      |
|                 | 1.2              | 89.725                      |

Table 13. Experimental values of MRR. (\( T_{\text{OFF}} = 10 \) μs, IP = 110 A, SV = 45 V)
Hence, the final equation for MRR is:

\[
MRR = \left(0.002840 + 0.04080T_{ON} - 0.01607T_{ON}^2 - 0.00208T_{ON}^3\right) \frac{T_{ON} \times T_{OFF} \times I \times SV}{WT} 
\]

(26)

Furthermore, to predict the MRR over a range of process parameters, Equation 26 was used. The different values of \(T_{ON}\) (as mentioned in Table 13) were used along with the fixed values of other parameters \((T_{OFF} = 10\, \mu s, IP = 110\, A, SV = 45\, V)\). Comparison between predicted and experimental MRR is depicted in the form of bar chart as shown in Figure 16, where \(R\) (Pearson correlation coefficient) is 0.997 and \(p\) value is 0.000, which indicates a very good agreement of the predictions of the model with the experimental results.

Thus, the developed model is validated and could be very useful on the shop floor for predicting the value of MRR over a range of input parameters.

11. Conclusions

In this paper, Taguchi’s design of experiments approach has been used in conjunction with the grey-fuzzy logic technique to optimize the rough cut WEDM of pure titanium for multiple performance characteristics. Surface integrity aspects of the selected machined titanium samples such as recast layer, cracks, debris, crater size and material transfer between the electrode and work surface have also been evaluated. The following conclusions were drawn:
Pulse on time ($T_{ON}$) and SV were found to be most influential parameters for MRR, SR and WWC, as verified through statistical testing.

The Multi-response optimization using grey-fuzzy logic optimization process yielded a single optimum solution for simultaneous optimization of all the three responses as: $T_{ON}$—0.9 μm, $T_{OFF}$—7 μm, IP—200 A, WF—8 m/min, WT—850 g and SV—50 V. The results obtained from the confirmation experiments validated the predicted optimal results.

A mechanistic model was developed and validated for the prediction of MRR over a wide range of input parameters. The model is capable of predicting MRR as a function of input parameters and therefore could be highly useful for the machinists for WEDM of titanium.

Higher wire consumption (due to lower pulse discharge energy) leads to lesser wire wear (wire surface degradation), and lesser wire consumption (due to higher pulse discharge energy) is related with the high wire wear. Hence, to obtain economy in machining, it is critical to set the process parameters at those levels that contribute to higher wire wear, so that the wire consumption could be controlled.

Microstructure study of WEDMed titanium samples machined under lower discharge energy setting revealed shallow craters, less density of global appendages or debris, smaller size nodules and higher density of gasholes as compared to the samples machined under higher discharge energy setting. A higher amount of micro-cracks were observed for machining under higher discharge energy conditions. It was also observed that spherical nodules were evenly covered with white cover which may be the oxide layer.

Thicker recast layer (average 24.26μ) was found on workpiece, while machined under higher pulse discharge energy condition as compared to lower pulse discharge energy conditions.

The amount of copper was found to be significant (8.59%), while Zn was not detected at the machined surface under higher pulse discharge energy. The reason for the above might be due to melting and evaporation of Zn. The higher amount of the presence of oxygen (10.89%) reveals oxide formation of titanium which was present in white colour in the microstructure.

Large amount of debris, craters and significant number of micro-cracks were observed on eroded surface of wire used under higher pulse discharge energy settings, while wire eroded less for machining at low pulse discharge energy conditions.

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