Universality of small black hole instability in AdS/CFT

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Abstract

AdS$_5$ type IIb supergravity compactifications on five-dimensional Einstein manifolds \( \mathcal{V}_5 \) realize holographic duals to four-dimensional conformal field theories. Black holes in such geometries are dual to thermal states in these CFTs. When black holes become sufficiently small in (global) \( \text{AdS}_5 \), they are expected to suffer Gregory-Laflamme instability with respect to localization on \( \mathcal{V}_5 \). Previously, the instability was demonstrated for gravitational dual of \( \mathcal{N} = 4 \) SYM, where \( \mathcal{V}_5 = S^5 \). We extend stability analysis to arbitrary \( \mathcal{V}_5 \). We point out that the quasinormal mode equation governing the instabilities is universal. The precise onset of the instability is \( \mathcal{V}_5 \)-sensitive, as it is governed by the lowest non-vanishing eigenvalue \( \lambda_{\text{min}} \) of its Laplacian.

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1 Introduction

Consider type IIb supergravity compactification on a five-dimensional Einstein manifold $\mathcal{V}_5$ with large five-form flux through it. The vacuum of the resulting five-dimensional effective gravitational action is $\mathcal{M}_5 = AdS_5$, dual to a strongly coupled four-dimensional conformal gauge theory living on the boundary $\partial \mathcal{M}_5$. The best studied realization of the holography is when $\mathcal{V}_5$ is a five-dimensional sphere, $S^5$, in which case the corresponding conformal gauge theory is $\mathcal{N} = 4$ supersymmetric Yang-Mills [1]. Other generalizations include the orbifolds $S^5/\mathbb{Z}_k$ [2], $T^{1,1} = (SU(2) \times SU(2))/U(1)$ [3] and $Y^{p,q}$ Sasaki-Einstein spaces [4]. While the details of the dual CFTs depend on what $\mathcal{V}_5$ is chosen (e.g., the central charge of the CFT is $\propto \frac{1}{\text{vol}(\mathcal{V}_5)}$), many aspects of the theories are in fact universal. The reason for this commonality stems from the fact that Kaluza-Klein reduction of type IIb supergravity on $\mathcal{V}_5$ contains a universal consistently truncated gravitational sector\(^1\)

$$S_5 = \int_{\mathcal{M}_5} d^5 \xi \sqrt{-g} (R_5 + 12) . \tag{1.1}$$

We consider the case when $\partial \mathcal{M}_5 = R \times S^3$. Besides (global) $AdS_5$ vacuum solution, (1.1) contains black holes solutions, dual to thermal states of the boundary CFT. In full ten-dimensional supergravity these black holes are “smeared” on $\mathcal{V}_5$. The size of the black hole $\rho_+$ (as measured by the radius of $S^3$ at the horizon\(^2\)) is related to the black hole mass $M$, compare to the vacuum energy $E_{\text{vacuum}}$ as

$$\rho_+^2 = \frac{1}{2} \left( \sqrt{1 + \epsilon} - 1 \right) , \quad \text{where} \quad \epsilon \equiv \frac{M}{E_{\text{vacuum}}} . \tag{1.2}$$

Notice that small black holes are light. It is expected that an $AdS_5$ black hole can not become arbitrarily small: it was proposed in [6, 7] that in the limit $\rho_+ \to 0$ it would suffer a Gregory-Laflamme (GL) instability [8], resulting in its localization on $\mathcal{V}_5$. The latter localization phenomenon was explicitly verified in [5, 9, 10] when $\mathcal{V}_5 = S^5$.

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\(^1\)Without loss of generality we set the asymptotic $AdS_5$ radius to unity.

\(^2\)See [5] for further details and conventions.
One might expect that small AdS$_5$ black hole localization would depend on details of $V_5$. The purpose of this paper is to explicitly demonstrate that this is not the case — all what matters for determining the onset of the instability is the smallest non-vanishing eigenvalue of the scalar Laplacian on $V_5$, feeding into the fluctuation equation originally constructed by Prestidge in [11], and later obtained for the case $V_5 = S^5$ in [9].

The rest of the paper is organized as follows. In section 2 we derive a single "master" second-order quasinormal mode equation in a radial AdS$_5$ coordinate. Besides $\rho_+$, this equation depends parametrically only on the quasinormal mode frequency $\omega$ and the scalar Laplacian on $V_5$ eigenvalue $\lambda$. It is precisely the quasinormal mode equation eq.(5.3) obtained in [5] for the case $V_5 = S^5$. For the quasinormal mode at the threshold of instability, i.e., for $\omega = 0$, this equation reduces to the threshold equation of Prestidge [11].

2 Stability of AdS$_5$ black holes smeared on $V_5$

Holographic dual to thermal states of a large class of conformal gauge theories on $R \times S^3$ is described by the following type IIb supergravity background

$$ds^2_{10} = (dM_5)^2_{BH} + (dV_5)^2, \quad F_5 = \text{vol}_{M_5} - \text{vol}_{V_5},$$
$$F_5 = \star_{10} F_5, \quad dF_5 = 0.$$

(2.1)

where $(dM_5)^2_{BH}$ is the global AdS$_5$ Schwarzschild black hole metric,

$$(dM_5)^2_{BH} = g_{\mu\nu}dx^\mu dx^\nu = -c_1(x)^2\, dt^2 + c_2(x)^2\, dx^2 + c_3(x)^2\, (dS^3)^2,$$

(2.2)

$$c_1 = \frac{\sqrt{a(x)}}{\sqrt{x}}, \quad c_2 = \frac{1}{2x\sqrt{1-x}\sqrt{a(x)}}, \quad c_3 = \frac{\sqrt{1-x}}{\sqrt{x}}, \quad a = \frac{(x_h + x(1-x_h))(x_h-x)}{x_h^2(1-x)}, \quad x_h = \frac{1}{1+\rho_+^2}, \quad x \in (0, x_h),$$

and

$$(dV_5)^2 = g_{\alpha\beta}dy^\alpha dy^\beta.$$

(2.3)

$(dV_5)^2$ is the metric on an Einstein manifold $V_5$, and $(dS^3)^2$ is a round metric on unit radius $S^3$. We use $\mu, \nu, \rho, \cdots$ indices on $M_5$, and $\alpha, \beta, \gamma, \cdots$ indices on $V_5$.

We are interested in SO(4)-invariant linearized fluctuations of (2.1) that carry an arbitrary angular momentum on $V_5$. Generically, metric fluctuations would couple with
the fluctuations of the 5-form $F_5$ [5]. Substantial simplification can be achieved with the judicious choice of the gauge. Let

$$
\delta g_{\mu\nu} = h_{\mu\nu}(t, x, y^\alpha) \equiv \left\{ \delta g_{tt}, \delta g_{xx}, \delta g_{tx}, \delta g_{ij} \equiv g_{ij}(x) \delta f(t, x, y^\alpha) \right\},
$$

(2.4)

$$
\delta g_{\mu\alpha} = h_{\mu\alpha}(t, x, y^\beta), \quad \delta g_{\alpha\beta} = h_{\alpha\beta}(t, x, y^\gamma),
$$

where we explicitly enumerated non-vanishing components of $\delta g_{\mu\nu}$ consistent with $SO(4)$ symmetry — $i, j$ are angles on $S^3$. To leading order in metric fluctuation, the linearized components of the ten-dimensional Ricci tensor on $V_5$ take form [12]

$$
R^{(1)}_{\alpha\beta} = -\frac{1}{2} \left[ \left( \square_{M_5} + \square_{V_5} \right) h_{(\alpha\beta)} - 2R^{\alpha\gamma\delta\beta} h^{(\gamma\delta)} - R^{\alpha\gamma} h_{(\gamma\beta)} - R^{\gamma\beta} h_{(\gamma\alpha)} 
+ \frac{1}{5} g_{\alpha\beta} \left( \square_{M_5} + \square_{V_5} \right) h^\gamma - \frac{16}{15} \nabla_\alpha \nabla_\beta h^\gamma + \nabla_\alpha \nabla_\beta \left( h^\mu + \frac{5}{3} h^\gamma \right) - \nabla_\alpha \nabla^\mu h_{\mu\beta} - \nabla_\beta \nabla^\mu h_{\mu\alpha} \right],
$$

(2.5)

where

$$
h_{\alpha\beta} = h_{(\alpha\beta)} + \frac{1}{5} g_{\alpha\beta} h^\gamma.
$$

(2.6)

Assuming that

$$
h^\mu = 0, \quad h_{\mu\alpha} = h_{\alpha\beta} = 0,
$$

(2.7)

we see that

$$
R^{(1)}_{\alpha\beta} = 0.
$$

(2.8)

Notice that with (2.7),

$$
\delta \text{vol}_{M_5} \propto O(h^2), \quad \delta \text{vol}_{V_5} = 0,
$$

(2.9)

which implies that it is consistent, at order $O(h)$, with the 5-form self-duality constraint and its Bianchi identity not to deform the background 5-form ansatz (2.1). The latter implies that the 5-form stress-energy tensor with components on $V_5$ is unchanged from its background value, i.e.,

$$
T^{(1)}_{\alpha\beta} = 0.
$$

(2.10)

Together, (2.8) and (2.10) imply that (2.7) solves all Einstein equations with indices on $V_5$. 

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For linearized components of the Ricci tensor with mixed indices we have \[12\]
\[\begin{align*}
R^{(1)}_{\mu\alpha} &= -\frac{1}{2} \left[ \Box h_{\mu\alpha} - \nabla_{\mu} \nabla^{\nu} h_{\nu\alpha} - R_{\mu}^{\ \nu} h_{\nu\alpha} + \Box h_{\mu\alpha} - R_{\alpha}^{\ \beta} h_{\beta\mu} \\
&- \nabla_{\alpha} \nabla^{\nu} h_{\nu\mu} + \nabla_{\mu} \nabla_{\alpha} \left( h_{\rho}^{\rho} + h_{\gamma}^{\gamma} \right) - \nabla_{\mu} \nabla_{\beta} h_{\beta\alpha} \right]. 
\end{align*}\] (2.11)

Since in a gauge (2.7) the mixed components of the 5-form stress-energy tensor vanish, \[T^{(1)}_{\mu\alpha} = 0, \] (2.12)
the Einstein equations with mixed indices reduce to
\[\nabla_{\alpha} \nabla^{\nu} h_{\nu\mu} = 0.\] (2.13)

Parameterizing the (traceless) fluctuations as
\[
\delta g_{tt} = -c_1(x)^2 e^{-i\omega t} f_1(x) Y_{V_5}(y^\alpha), \quad \delta g_{xx} = c_2(x)^2 e^{-i\omega t} f_2(x) Y_{V_5}(y^\alpha), \\
\delta g_{tx} = i e^{-i\omega t} f(x) Y_{V_5}(y^\alpha), \quad \delta g_{ij} = g_{ij}(x) e^{-i\omega t} \left( -\frac{1}{3} f_1(x) - \frac{1}{3} f_2(x) \right) Y_{V_5}(y^\alpha),
\]
(2.14)
equations (2.13) are equivalent to the following two equations
\[0 = \left( 2\omega f_1 c_2 - f \left( \ln \frac{f_1 c_3^3}{c_2} \right)' \right) \partial_{\alpha} Y_{V_5}(y^\gamma) e^{-i\omega t}, \]
\[0 = \left( f_2 \left( \ln f_2 c_4^4 \right)' + f_1 \left( \ln \frac{c_3^3}{c_1} \right)' - \frac{\omega f}{2c_1^2} \right) \partial_{\alpha} Y_{V_5}(y^\gamma) e^{-i\omega t}. \] (2.15)

Remaining Einstein equations\(^3\) involve components \(tt, tx, xx\) and a pair of the \(S^3\) components (by \(SO(4)\) symmetry):

- \(tt\) components:
\[0 = \left[ \left\{ \frac{c_1^2}{c_2^2} \left( f_1'' + f_1' \left( \ln \frac{c_1 c_3^3}{c_2} \right)' - 2f_2' \ln c_1 \right)' \right\} + \frac{\omega f}{c_2^2} \left( \ln \frac{f c_3^3}{c_2} \right)' - 8f_2 c_1^2 - \omega^2 f_1 \right] Y_{V_5} \\
+ f_1 c_1^2 \Box_{V_5} Y_{V_5} \] \(e^{-i\omega t}; \) (2.16)

- \(tx\) components:
\[0 = i \left[ \omega \left \{ f_1 \left( \ln \frac{c_1 c_3^3}{c_1} \right)' + f_2 \left( \ln \frac{f_2 c_3^4}{c_1} \right)' \right\} Y_{V_5} + \frac{1}{2} f \Box_{V_5} Y_{V_5} \] \(e^{-i\omega t}; \) (2.17)

\(^3\)We used here equations of motion for the background (2.1).
\[ \begin{align*}
\ \Box \ \nabla_5 Y_{\nu_5} &= -\lambda \ Y_{\nu_5}, \\
\ &= 0.
\end{align*} \]

The resulting system of ODEs is over-determined, which can be exploited to reduce it to the following equations:

\[ \begin{align*}
0 &= 2c_3\omega \left( c_2^2 c_1^2 \lambda + 4c_2^2 c_1^2 - c_2^2 \omega^2 \right) f_2 + 2\omega \left( 4c_2^2 c_1^2 c_3 - c_2^2 c_3^2 \omega^2 - 3c_1 c_3'\right) f_1 + 2c_3' \left( c_2^2 c_1^2 - c_1 c_3' \lambda + 3c_3' \omega^2 \right) f, \\
0 &= f' + f \left( \ln \frac{c_1 c_3'}{c_2} \right) - 2\omega c_2^2 f_1, \\
0 &= f_1' + \left( \frac{\omega}{c_1^2} - \frac{\lambda}{2\omega} \right) f - 2\left( \ln c_1 \right)' f_2.
\end{align*} \]
equation:

\[ 0 = f'' + f' \left( 3(\rho_+^2 + 1)^3 y^{14} - (5\rho_+^4 + 5\rho_+^2 + 9)(\rho_+^2 + 1)^2 y^{12} - \rho_+^2 (\rho_+^2 + 1)^2 (\lambda + 22) y^{10} \right. \\
- \rho_+^2 (\rho_+^2 + 1)(9\lambda \rho_+^4 + 61\rho_+^4 + 9\lambda \rho_+^2 + 3\omega^2 + 61\rho_+^2 + 2\lambda + 18) y^8 + \rho_+^2 (-11\omega^2 \rho_+^4 \\
+ 8\lambda \rho_+^4 - 11\omega^2 \rho_+^2 + 11\rho_+^4 + 8\lambda \rho_+^2 - 3\omega^2 + 11\rho_+^2 + 3\lambda + 12) y^6 + \rho_+^4 (10\lambda \rho_+^4 + 21\rho_+^4 \\
+ 10\lambda \rho_+^2 - 4\omega^2 + 21\rho_+^2 + 5\lambda + 19) y^4 + \rho_+^6 (-\omega^2 + \lambda + 4) y^2 - \rho_+^8 (\lambda + 3) \left( \\
y(y^2 - 1)(\rho_+^2 + y^2)((\rho_+^2 + 1)^2 y^2 - \rho_+^2 (\rho_+^2 + 1)(\lambda + 6) y^4 \\
+ \rho_+^2 (-\omega^2 + \lambda + 4) y^2 + \rho_+^4 (\lambda + 3)) \right)^{-1} + \left(4(\rho_+^2 + 1)^4 y^{20} \\
- (8(2\rho_+^4 + 2\rho_+^2 + 3))(\rho_+^2 + 1)^3 y^{18} - (\rho_+^2 + 1)^2(-4\rho_+^8 - 8\rho_+^6 + 7\lambda \rho_+^4 + 44\rho_+^4 + 7\lambda \rho_+^2 \\
+ 48\rho_+^2 - 24) y^{16} - \rho_+^2 (\rho_+^2 + 1)^2(22\lambda \rho_+^4 + 96\rho_+^4 + 22\lambda \rho_+^2 + \omega^2 + 96\rho_+^2 - 13\lambda - 112) y^{14} \\
+ \rho_+^2 (\rho_+^2 + 1)(9\lambda \rho_+^8 + 72\rho_+^8 + 18\lambda \rho_+^6 + \lambda^2 \rho_+^4 - 2\omega^2 \rho_+^2 + 144\rho_+^6 + 76\lambda \rho_+^4 + \lambda^2 \rho_+^2 \\
- 2\omega^2 \rho_+^2 + 416\rho_+^4 + 67\lambda \rho_+^2 + 6\omega^2 + 344\rho_+^2 - 6\lambda) y^{12} + \rho_+^4 (\rho_+^2 + 1)(2\lambda \rho_+^4 + 23\omega^2 \rho_+^4 \\
+ 39\lambda \rho_+^4 + 2\lambda^2 \rho_+^2 + 23\omega^2 \rho_+^2 + 184\rho_+^4 + 2\lambda \omega^2 + 39\lambda \rho_+^2 - 2\lambda^2 + 42\omega^2 + 184\rho_+^4 - 58\lambda \\
- 120) y^{10} + \rho_+^4 (\lambda \rho_+^8 - 15\lambda \rho_+^6 + 2\lambda^2 \rho_+^6 - 48\rho_+^8 + 4\lambda \omega^2 \rho_+^4 - 30\lambda \rho_+^6 - 5\lambda^2 \rho_+^4 + 42\omega^2 \rho_+^4 \\
- 96\rho_+^6 + 4\lambda \omega^2 \rho_+^2 - 114\lambda \rho_+^4 - 6\lambda^2 \rho_+^2 + \omega^4 + 42\omega^2 \rho_+^2 - 240\rho_+^4 - 2\lambda \omega^2 - 99\lambda \rho_+^2 + \lambda^2 \\
- 14\omega^2 - 192\rho_+^2 + 14\lambda + 40) y^8 - \rho_+^6 (-2\omega^2 \rho_+^4 + 6\lambda^2 \rho_+^4 - 6\omega^2 \rho_+^2 - 2\lambda \omega^2 \rho_+^2 + 48\lambda \rho_+^4 \\
+ 6\lambda^2 \rho_+^2 - 2\omega^4 - 6\omega^2 \rho_+^2 + 48\rho_+^4 + 6\lambda \omega^2 + 48\rho_+^2 - 4\lambda^2 + 37\omega^2 + 48\rho_+^2 - 128\lambda - 128) y^6 \\
- \rho_+^8 (2\lambda \rho_+^4 + \lambda \rho_+^4 + 2\lambda^2 \rho_+^2 - \omega^4 - 24\rho_+^4 + 6\lambda \omega^2 + \lambda \rho_+^4 - 6\lambda^2 + 32\omega^2 - 24\rho_+^4 - 63\lambda \\
- 148) y^4 + \rho_+^10(-2\lambda \omega^2 + 4\lambda^2 - 9\omega^2 + 35\lambda + 72) y^2 + \rho_+^12(\lambda + 4)(\lambda + 3) \right) \\
(y^2 - 1)(\rho_+^2 + 1)^2 y^8 + \rho_+^2 (\rho_+^2 + 1)(\lambda + 6) y^4 - \rho_+^2 (-\omega^2 + \lambda + 4) y^2 \\
- \rho_+^4 (\lambda + 3)((\rho_+^2 + 1)^2 y^2 + \rho_+^2)^2 (\rho_+^2 + y^2)^2 y^2 \right) \right)^{-1} f , \]

(2.22)

where we introduced a new radial coordinate \( y \), so that

\[ x \equiv \frac{y^2}{y^2 + \rho_+^2} , \quad x \in (0, 1) . \]

(2.23)

This is our universal quasinormal mode equation: the only information about \( \mathcal{V}_5 \) is in the choice of the scalar Laplacian eigenvalue \( \lambda \).
Figure 1: The dependence of $g = -\text{Im}(\omega)$ as a function of a black hole size $\rho_+$ and temperature $T = \frac{2\rho_+^2 + 1}{2\pi \rho_+}$ in KW model for $U(1)_R$ charged/neutral quasinormal modes (red/blue) with $T^{1,1}$ eigenvalues $\lambda_{j,\ell,r}$: $(j, \ell, r) = \{(\frac{1}{2}, \frac{1}{2}, 1), (1, 1, 2), (\frac{3}{2}, \frac{1}{2}, 1), (1, 0, 0), (1, 1, 0)\}$. $g$ increases with $\lambda_{j,\ell,r}$. Black holes with $g < 0$ are unstable with respect to condensation of these fluctuations.

Equation (2.22) is identical to eq.(5.3) derived in [5], provided we identify\(^4\)

$$f_{xy} \implies \frac{y}{y^2 + \rho_+^2} f, \quad s \implies \lambda.$$  \hfill (2.24)

Additionally, when $\omega = 0$ it reduces to the equation at the threshold of instability, originally derived in [11].

We now consider a simple application of (2.22) in the context of the holographic Klebanov-Witten (KW) model [3]. In this case $\mathcal{V}_5$ is $T^{1,1}$ coset manifold. Properties of the Laplacian on $T^{1,1}$ were extensively studied in [13–15]. The eigenvalues are completely determined by a pair of $SU(2)$ spins $\{j, \ell\} \in \frac{1}{2}\mathbb{Z}$ and a $U(1)_R$ $R$-symmetry charge $r \in \mathbb{Z}$ as follows

$$\lambda = \lambda_{j,\ell,r} = 6 \left( j(j + 1) + \ell(\ell + 1) - \frac{1}{8}r^2 \right).$$  \hfill (2.25)

A triplet $\{j, \ell, r\}$ is constraint so that both $2j$ and $2\ell$ have the same parity, and

$$r \leq \min\{2j, 2\ell\}.$$  \hfill (2.26)

\(^4\)We refer the reader to [5] for the details associated with solving (2.22).
Figure 2: Critical size of the $AdS_5$ black hole $\rho_{+,\text{crit}}^2$ below which a quasinormal mode with an eigenvalue $\lambda$ on $\mathcal{V}_5$ becomes unstable. The dashed red line is a large-$\lambda$ asymptotic (2.28). The black and the magenta vertical dotted lines correspond to the onset of the instability (the smallest non-vanishing value of $\lambda$ on $\mathcal{V}_5$) for $\mathcal{V}_5 = S^5$ and $\mathcal{V}_5 = T^{1,1}$ correspondingly.

Note that the lowest non-vanishing eigenvalue on $T^{1,1}$ is

$$\lambda_{\text{min}} = \frac{\lambda_{1,2,1}}{4} = \frac{33}{4}.$$  

(2.27)

The spectrum of the low-lying quasinormal modes of $AdS_5$ black holes in KW holography is shown on figure 1. The red curves correspond to states carrying $U(1)_R$ charge, and the blue curves represent neutral states.

The solid blue curve in figure 2 presents the threshold value $\rho_{+,\text{crit}}^2$ of the Gregory-Laflamme instability, corresponding to $\omega = 0$, for the $AdS_5 \times \mathcal{V}_5$ quasinormal mode with the $\mathcal{V}_5$ eigenvalue $\lambda$. The dashed red line is the large-$\lambda$ asymptotic, see [5],

$$\rho_{+,\text{crit}}^2 = \frac{1.61015}{\lambda} + \mathcal{O}(\lambda^{-2}).$$  

(2.28)

Notice that larger values of $\lambda$ result in smaller threshold values of $\rho_{+,\text{crit}}^2$. Thus, the onset of the instability of smeared $AdS_5 \times \mathcal{V}_5$ black holes is determined by the smallest non-vanishing eigenvalue $\lambda$ of the scalar Laplacian on $\mathcal{V}_5$. 

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