Demonstration of dynamical control of three-level open systems with a superconducting qutrit

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Abstract

We propose a method for the dynamical control in three-level open systems and realize it in the experiment with a superconducting qutrit. Our work demonstrates that in the Markovian environment for a relatively long time (3 μs), the systemic populations or coherence can still strictly follow the preset evolution paths. This is the first experiment for precisely controlling the Markovian dynamics of three-level open systems, providing a solid foundation for the future realization of dynamical control in multiple open systems. An instant application of the technique demonstrated in this experiment is to stabilize the energy of quantum batteries.

1. Introduction

The control of quantum systems is always an extremely interesting and essential topic among diverse types of quantum information tasks [1, 2]. With the gradual development of experimental techniques, decoherence, mainly resulted from energy relaxation and dephasing, has become a primary barrier lying on the way to experimentally implement the quantum control. To clear this barrier, researchers choose to take measures to shorten the evolution time and have achieved remarkable successes in the experiments, such as, preparing 20-qubit entanglement [3, 4] and simulating quantum walk on 62-qubit processor [5]. Meanwhile, some researchers attempt on constructing qubits that are insensitive to decoherence [5–8]. So far, the relaxation time $T_1$ and dephasing time $T_2$ of superconducting qubit have both been extended to 200 μs [8]. Indeed, the methods of shortening the evolution time and extending the decoherence time of qubits naturally reduce the impact of decoherence rather than avoiding it, which still process quantified errors. For realizing the leap of quantum error rates from $10^{-3}$ to $10^{-15}$ [9, 10] and further accomplishing near-term quantum works, it is necessary to explore the control of open quantum systems, tailoring the decoherence as one of the effective elements involved in the quantum control.

The earlier open-system quantum control theory draws supports from a kind of nuclear magnetic resonance technology, called spin-echo [11], in which a π pulse is inserted midway in the evolution time to increase $T_2$. However, this inserted π pulse will change the systemic dynamics. Several spin-echo-like techniques, such as bang–bang control [12], dynamical decoupling technique [13], and parity-kicks [14], process the same problems with the destruction of dynamics. So these works generally served as techniques for storing coherence of quantum systems [15–20]. Researchers alternatively develop an open-system adiabatic theorem [21] to fully control the dynamics. This method utilizes super operators to describe the open systems and can approximately predict the density matrix at each moment. Since the adiabatic theorem needs a long evolution time to guarantee the adiabatic approximation, researchers further try to build a shortcut to adiabaticity (STA) (similar to that in the closed systems [22–35]).

By virtue of super operators, Sarandy et al [36] have proposed a concept about open-system dynamical invariants to build a STA in open systems. They constructed some four-dimensional invariants for engineering two-level open systems, nevertheless, with several simplifications, such as, either considering
energy relaxation or dephasing. Finding proper invariants will be complicated when someone simultaneously takes energy relaxation and dephasing into account, let alone expansion to three-level open systems with nine-dimensional invariants. Therefore other control methods are eagerly in need of discovery.

The open-system populations control with a superconducting qubit has been put forward [37]. Recently, Medina and Semião [38] have directly set the density operator with time-dependent parameters and then submitted them to the Markovian master equation [39], which gives appropriate functions of pulses to control the populations in two-level open systems. The error rates of populations control in reference [38] under serious decoherence are almost zero, an exciting result. Some follow-up studies [40, 41] have made a lot of additions to the application scenarios, but still remaining in two-level open systems. With the general trend of quantum tasks toward multiplication [42], it is of great significance to explore [43] the dynamics of three (or more)-level open systems.

In this letter, we propose a method for the dynamical control in three-level open systems with considering energy relaxation and dephasing, and demonstrate it with a superconducting circuit platform [44–50]. This method is essentially one of the kinds of inverse engineering [24]. The inverse engineering method originated from the Lewis–Riesenfeld dynamics invariant [51], whose basic idea is to preset an interesting invariant for constructing pulses to induce desired dynamics. There also be some earlier studies that skip the process of preset invariants and reversely design pulses directly [26, 38, 40, 41, 52, 53]. Several of these studies are generally conducted in closed [26] systems or two-level open systems [38, 40, 41, 53], and others focused on fighting energy relaxation [54–57] or controlling population [52]. The present research carries out both population and coherence control, with considering energy relaxation and dephasing, in three-level open systems.

In our method, two time-dependent parameters are presupposed in the Markovian master equation to singly control the populations or coherence, and then the analytical functions of driving pulses are given to complete desired dynamical control. For proving the correctness of this method, we apply the designed driving pulses to a superconducting Xmon qutrit and measure populations of excited states 1 and 2, or coherence between ground and excited states. The experimental results are in good agreement with the theoretical design for the dynamical control. This is the first experiment to precisely control the Markovian dynamics of three-level open systems, where the decoherence is effectively dominated. In addition, an interesting freezing phenomenon of populations is observed. That is, in the late stages of evolutions, the populations stabilize at specific values for a relatively long time (>1.2 μs) in the presence of decoherence. An instant application scenario of this freezing phenomenon is the quantum battery (QB) [58–72]. One can steadily lock the energy of QBs for a comparatively long time (>1.2 μs), resisting the relaxation of excitation, and therefore building prototypes of QBs with more stable energy.

2. Method

Assume that a three-level system consists of bases |0⟩, |1⟩, and |2⟩. The systemic Hamiltonian in the interaction picture is \((\hbar = 1 \text{ hereafter}) H_{I}(t) = \Omega_{01}(t)|0⟩⟨1| + \Omega_{12}(t)|1⟩⟨2| + \text{H.c.}\), where \(\Omega_{01}(t)\) and \(\Omega_{12}(t)\) respectively represent the Rabi frequencies of the designed driving pulses for controlling transitions \(|0⟩ \leftrightarrow |1⟩\) and \(|1⟩ \leftrightarrow |2⟩\). We assume the density matrix of this three-level system being (basis order \{|2⟩, |1⟩, |0⟩\}) and dropping \((t)\) henceforth

\[
\rho = \begin{pmatrix}
    f_2 & -i h_1 & h_2 \\
    i h_1 & f_1 & -i h_3 \\
    h_2 & i h_3 & 1 - f_1 - f_2
\end{pmatrix},
\]

in which \(f_1, f_2, h_1, h_2, \) and \(h_3\) are all time-dependent real functions. Here \(f_1\) and \(f_2\) describe the populations of states \(|1⟩\) and \(|2⟩\), respectively. Additionally, \(h_1, h_2,\) and \(h_3\) describe the coherence of this three-level system. Equation (1) (5 degrees of freedom) is not the most generalized case for a three-level density operator (8 degrees of freedom), however sufficient as a demonstration.

We assume that the system–environment process is Markovian, with several consistency properties, such as being trace-preserving and satisfying complete positivity [73–76]. The deriving of the Markovian master equation by projection operators is initialed by Nakajima [77] and Zwanzig [78], also shown in [75, 76]. The specific Lindblad-form master equation here is [39]

\[
\dot{\rho} = -i[H_{I}+\rho] + \sum_{j=1}^{\mu} \left[ L_j \rho L_j^\dagger - \frac{1}{2}(L_j^\dagger L_j \rho + \rho L_j^\dagger L_j) \right],
\]

where Lindblad operators are \(L_k = \sqrt{\gamma_k} |k⟩⟨k|\) and \(L_{k+2} = \sqrt{\Gamma_k} |k-1⟩⟨k|\), with \(\Gamma_k\) and \(\gamma_k\) being the rates of energy relaxation and dephasing of state \(|k⟩\), respectively \((k = 1, 2)\). By utilizing equations (1) and (2), we
can derive the Rabi frequencies to control the dynamics of this three-level system, yielding

\[ \Omega_{01} = \frac{f_1 \Gamma_1 + f_1 + f_2}{2h_3}, \]

\[ \Omega_{12} = \frac{f_2 \Gamma_2 + f_2}{2h_1}. \] (3)

It is noteworthy that there are three constraint equations

\[ \hat{h}_1 = -\frac{1}{2}(\gamma_1 + \gamma_2 + \Gamma_1 + \Gamma_2)h_1 + (f_1 - f_2)\Omega_{12} - h_2\Omega_{01}, \]

\[ \hat{h}_2 = -\frac{1}{2}(\gamma_2 + \Gamma_2)h_2 - h_3\Omega_{12} + h_1\Omega_{01}, \] (4)

\[ \hat{h}_3 = -\frac{1}{2}(\gamma_1 + \Gamma_1)h_3 + h_2\Omega_{12} + (-1 + f_2 + f_1)\Omega_{01}. \]

Up to now, the populations or coherence of the three-level system can be effectively controlled by the interaction Hamiltonian \( H_i \), i.e., by Rabi frequencies \( \Omega_{01} \) and \( \Omega_{12} \). A specific example is, an evolution from the ground state \( |0\rangle \) to a final state with populations respectively being \( P_0(t_f) \), \( P_1(t_f) \), and \( P_2(t_f) \) in states \( |0\rangle \), \( |1\rangle \), and \( |2\rangle \), can be achieved by adjusting \( f_1 = f_{p_1}(t_f) \) and \( f_2 = f_{p_2}(t_f) \). The intermediate function \( f \) changing from 0 to 1 in the time interval \([0, t_f]\) can be set as \( f = [1 + e^{-a(t-t_f)}]^{-1} \) \([26, 27, 38]\), with \( a = 50/t_f \) determining the gradient of the transformation. By submitting \( f_1 \) and \( f_2 \) into equations (3) and (4), one can obtain Rabi frequencies \( \Omega_{01} \) and \( \Omega_{12} \) to accomplish the desired transformation of populations.

3. Device

Here we use a frequency-tunable superconducting Xmon qutrit \([44–50]\) to test the above theory. The original Hamiltonian reads \( H = \Omega_{01} e^{-i\omega_{12}t} |0\rangle\langle 1| + \Omega_{12} e^{-i\omega_{12}t} |1\rangle\langle 2| + \text{H.c.} + \sum_{i=0,1,2} \omega_i |i\rangle\langle i| \), where \( \omega_i \) and \( \omega_{01(12)} \) are the angular frequencies of energy level \( |i\rangle \) and microwave pulses coupling \( |0\rangle \leftrightarrow |1\rangle(1\rangle2\rangle \), respectively. We adjust \( \omega_{01}/2\pi = (\omega_1 - \omega_0)/2\pi = 5.9600 \) GHz and \( \omega_{12}/2\pi = (\omega_2 - \omega_1)/2\pi = 5.7208 \) GHz (in this experiment, frequency accuracy to four decimal places is mandatory) to ensure that the pulses can resonantly drive the transitions between adjacent energy levels. Note the fixed frequency of the resonator (not used here) is 5.584 GHz \([46–50]\), dynamically decoupled with the above system.

An essential process for this experiment is to precisely measure the coefficients of decoherence, \( \gamma_1 \) and \( \Gamma_1 \). According to the above form of Lindblad operators, one can deduce \( \Gamma_1 = 1/T_{11}^{01} \), \( \gamma_1 = 2/T_{11}^{01} - \Gamma_1 \), \( \Gamma_2 = 1/T_{12}^{12} \), and \( \gamma_2 = 2/T_{12}^{12} - \Gamma_2 - \Gamma_1 - \gamma_1 \). Here \( T_{11}^{01(12)} \) and \( T_{12}^{12(12)} \) are the energy relaxation and dephasing time measured between \( |0\rangle \) and \( |1\rangle(1\rangle2\rangle \), shown in figures 1(a) and (b), respectively. From figure 1, we find \( T_1 \) and \( T_2 \) fluctuate a lot during the measured period of 20 h. In addition, several works \([15–20, 79]\) support that \( T_2 \) will change with the applying of microwave pulses (the spin-echo technique \([11]\) is a strong proof). Therefore the values of \( T_1 \) and \( T_2 \) we utilized to design pulses are a little different from those measured in figure 1, specifically, \( [T_{11}^{01}, T_{11}^{01}, T_{12}^{12}, T_{12}^{12}] = [9.5, 4.6, 6, 1.9] \) \( \mu s \), which are fixed and utilized for all the experimental control (figures 3–5).

4. Results of the populations control

Due to the choice of \( f_1 \) and \( f_2 \), there are infeasible zones of the populations transformation, shrinking as \( t_f \) extending, shown in figure 2(a), ranging \( P_1(t_f) \) as \( x \)-axis and \( P_2(t_f) \) as \( y \)-axis. In these infeasible zones, the
Figure 2. (a) Feasible area (FA, surrounded by curves and x-axis) and infeasible area (IFA, the outside zone) of the populations control in the experiment when the evolution time is 3, 5, and 10 μs. Condition $P_1(t_f) + P_2(t_f) \leq 1$ triangulates the boundary. (b) A sample point marked as purple plus sign in (a): Rabi frequencies for the control of populations $P_1(0) = 0 \rightarrow P_1(t_f) = 0.3$ and $P_2(0) = 0 \rightarrow P_2(t_f) = 0.2$. The used parameters of decoherence are $[T_{01}^1, T_{12}^1, T_{01}^2, T_{12}^2] = [9.5, 4.6, 6.19]$ μs and the inset shows the magnification picture of the Rabi frequencies.

Figure 3. Experimental results of the control of populations $P_1(0) = 0 \rightarrow P_1(t_f) = 0.3$ and $P_2(0) = 0 \rightarrow P_2(t_f) = 0.2$. The experimental results of applying pulses designed by the open-system and closed-system master equations are shown (with circles, triangles, and pentagrams) in the outer and inner layers, respectively. The numerically simulated results of $P_0(t)$, $P_1(t)$, and $P_2(t)$ are indicated by solid, dashed and dotted dashed curves, respectively.

designed pulses $\Omega_{01}(t)$ and $\Omega_{12}(t)$ are singular, and therefore hard to realize in the experiments. So we carry out the experimental dynamical control in the feasible zones. While the feasible areas still takes a large part. As an example, we utilize equations (3) and (4) to design $\Omega_{01}$ and $\Omega_{12}$ (see figure 2(b)) for the transformation of populations $P_1(0) = 0 \rightarrow P_1(t_f) = 0.3$ and $P_2(0) = 0 \rightarrow P_2(t_f) = 0.2$. For the evolution time, we choose $t_f = 3$ μs to accumulate enough impact of the decoherence. The corresponding experimental results are shown in figure 3, with outer and inner layers indicating the results of applying pulses designed by the open-system master equation (see equations (3) and (4)) and the closed-system master equation (see equations (3) and (4), preset $\gamma_k = \Gamma_k = 0$) pulses, respectively. Intuitively, the populations are controlled more precisely in the outer layer, as compared to the inner one. More rigorously, we define a standard deviation to describe the error rate of the population control

$$\text{error} = \sqrt{\sum_{i=0,1,2} [P_i(t_f)^{\text{sim.}} - P_i(t_f)^{\text{exp.}}]^2} / 3,$$

where $P_i(t_f)^{\text{sim.}}$ and $P_i(t_f)^{\text{exp.}}$ are the numerically simulated and experimental values of the population in $|i\rangle$, respectively. In the outer layer of figure 3, the populations control achieves an error rate of 1.02%, not small [80] because we intentionally extended the evolution time (3 μs). In contrast, if the control is performed for such a long time by applying pulses designed by the closed-system master equation, the error rate reaches 7.49%. This stark difference in error demonstrates the effectiveness of the present control method.

We also measure the results of 29 different controls of populations, 6 of them shown in figure 4 and all displayed in appendix F. The error rates of the controls in figure 4 are around 1%, which we believe can be further improved with more stable superconducting qutrits [8].
5. Results of the coherence control

Since only two free variables, $\Omega_{01}$ and $\Omega_{12}$ in equations (3) and (4) can be controlled in the experiment, we can only control two parts of coherence. Here we choose $h_3$ and $h_2$ as an example for illuminating the coherence control. Corresponding variations of equations (3) and (4) and the forbidden zone of coherence control are shown in appendix D. According to the same preset of parameters and intermediate function $f$, we show 6 groups of results of the coherence control in figure 5 (all the 36 groups of results in appendix F). The error rate here is

$$\text{error}' = \sqrt{\sum_{m=2,3} [h_m(t_f)_{\text{sim}} - h_m(t_f)_{\text{exp}}]^2}/2, \quad (6)$$

where $h_m(t_f)_{\text{sim}}$ and $h_m(t_f)_{\text{exp}}$ are the numerically simulated and experimental values of coherence $h_m$, respectively. Figure 5 shows that $h_2$ and $h_3$ are controlled well and the values of $h_1$ are accurately predicted. Similarly, the microwaves protect the coherence (lasting 1.2 $\mu$s) from the influence of dephasing. If there are no microwaves applied, roughly, after the same 1.2 $\mu$s, $h_3(1.2 \mu s)/h_3(0) \sim \exp(-1.2 \mu s/T_{01}^2) \sim 0.8$ and $h_2(1.2 \mu s)/h_2(0) \sim \exp(-1.2 \mu s/T_{12}^2 + 1.2 \mu s/T_{01}^2) \sim 0.65$, the coherence will be seriously damaged. In contrast, the error rates are all within 2% when we apply microwaves for the coherence control.

6. An interesting freezing phenomenon

Figures 3–5 exhibit an interesting phenomenon that the populations (or coherence) seem to be frozen in the time interval 1.8–3 $\mu$s. This freezing phenomenon is similar to the dark-state phenomenon [81, 82], in which the systems stay in dark states. The difference is that in our freezing phenomenon, the states in the
Figure 6. Experimental verification of the QB. We plot the experimental and numerically simulated results by diamond points and solid curves, respectively. The inserted circle depicts the Bloch sphere of the numerically simulated evolution and the density of points represents the speed of evolution, one point per 15 ns. The parameters of decoherence are $[T_{01}^1, T_{01}^2] = [9.5, 6] \mu s$.

freezing stage are not the eigenstate states (with zero eigenvalues) of the Hamiltonian and the Lindblad superoperator. While one of the population and the coherence is essentially immobile. This process can be clearly seen in figure 6, inset Bloch sphere (charging stage with blue circles parallel to the $xy$ plane), where the state nonadiabatically moves along the vertical $z$-axis surface, ensuring population stability.

In more depth, this freezing phenomenon of populations (or coherence) does not mean that the driving pulses $\Omega_{01}$ and $\Omega_{12}$ have stopped. On the contrary, it is the driving pulses (see the insets of figure 2(b)) we applied that caused this freezing phenomenon to occur. Specifically, such a freezing phenomenon arises from the interplay between the dynamics induced by the continuous microwave drives and the two decoherence channels characterized by $T_1$ and $T_2$ (both available for $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions), similar to the cases for the generation of steady states in most open systems [36, 83–91]. But here it differs significantly in that both the energy relaxation and the dephasing are involved in the nonequilibrium dynamical processes and together help freeze the populations (or coherence) of three-level systems, as compared to the previous ones which generally consider only one decoherence channel [36, 85, 90, 91].

It is worth mentioning that every experimental point is a certain time evolutionary process. So we can directly cut the 1.8–3 $\mu s$ part of experiments to achieve a dynamical control without the freezing phenomenon. Alternatively, we can reduce $a$ for control function $f = [1 + e^{-a(t-t_f)/2}]^{-1}$, such as $a = 10/t_f$, to alleviate the freezing phenomenon.

7. Application

The current scheme can be applied to the QB [58–72]. We notice there is a recent study for stabilizing the energy of the QB by dark states [92], and realized in an experimental system including a microcavity enclosing a molecular dye [93]. Additionally, there is a recent relevant experiment optimizing the compromise between charging time and stored energy of quantum batteries [94]. The freezing phenomenon seems helpful to stabilize the energy of the QB for a fairly long time, especially in the energy-storing stage of the QB.

For proving this, we design a pulse for a qutrit in the experiment to simulate the charging, storing, discharging processes of the QB (see figure 6). A considerably long period, about 1.2 $\mu s$, of energy $[\epsilon = (\omega_1 - \omega_0)P_1]$ stability was observed. This stability is hard to achieve by applying a pulse designed without considering decoherence because energy relaxation of the superconducting qutrit will exponentially drop the excitation down. Therefore, the present method may contribute to the construction of more stable QBS.

8. Improvement for the control areas

Referring to the numerical discussion of parameter choices and relations in [38, 52–57, 95], we here give an improvement for the control areas of the dynamical control.

Hereafter the ratio of the feasible area of populations (coherence) control to the total area is defined as $\eta_p(\eta_c)$. The way to increase the areas of the feasible region for control areas is to increase $T_1$ and $T_2$ as well as to reduce $t_f$. Due to the limitations of the current experimental parameters, we plot the feasible areas of
the dynamical control in figures 2(a) and D1 based on parameter choices \([T_{01}, T_{12}, T_{01}, T_{12}]\) = [9.5, 4.6, 6, 1.9] µs, which is the real parameter situation of our experimental device. Under these parameter choices, \(\eta_p = (\eta_c =) 55.69\% (51.05\%), 42.07\% (41.85\%), 24.76\% (29.95\%),\) and 18.95\% (20.70\%) for \(t_f = 3, 5, 10,\) and 20 µs, respectively. If improve the parameter choices to \([T_{01}, T_{12}, T_{01}, T_{12}] = [200, 200, 50, 50]\) µs, \(\eta_p = (\eta_c =) 72.65\% (77.7\%), 72.62\% (76.85\%), 71.71\% (74.74\%),\) and 70.13\% (72.93\%) for \(t_f = 3, 5, 10,\) and 20 µs, respectively. Such parameter choices of decoherence have been realized in [8]. Additionally, reducing of \(a\) for the control function \(f(t)\) will also lead to the increase in feasible areas.

9. Conclusion

We have proposed a method to control the dynamics of three-level open systems and realized it in the experiment with a platform of a superconducting Xmon qutrit [44–50]. The populations and coherence could be singly controlled with error rates around 1% under the influence of decoherence for a relatively long time, 3 µs, close to the dephasing time \(T_{12}^2\) of the qutrit. In some situations where the control is more successful, the error rates can even be less than 0.3%. We believe these error rates can be further reduced with more stable superconducting qutrits [8].

Additionally, an interesting freezing phenomenon of populations was observed. The designed microwave pulses just offset the impact of decoherence and visually freeze the populations. We then applied this phenomenon to make more stable prototypes of QBs, whose energy can hold 1.2 µs with only one charging. Moreover, this freezing phenomenon strongly proves that the Markovian master equation precisely describes the dynamics of three-level open systems, as demonstrated here with a superconducting Xmon qutrit that possesses the specially intrinsic decoherence rates. The present work provides a positive prospect of accurately realizing the dynamical control of three (or more)-level open systems by using the adjustable driving pulses in the Markovian environment.

The present method is one of the kinds of inverse engineering methods, which can roughly be seen as a shortcut to the stimulated Raman adiabatic passage (STIRAP). Thereby the present work perhaps has application for the open STIRAP [96]. Recently, a proposal demonstrates that a time-dependent three-level system can be used to detect the timelike Unruh effect with current technology [97]. Although the three-level model of our Xmon qutrit is ladder-type (transition construction |0⟩ ↔ |1⟩ ↔ |2⟩), we believe the \(\Lambda\)-system qutrit sample is not hard to build. Additionally, our theory can also apply to the \(\Lambda\)-system model (\(\Delta\)-system as well). Therefore, this work may have potential applications for measuring the Unruh effect with \(\Lambda\)-type system [97].

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Experimental setup

The entire control and read-out layout of the experiment is shown in figure A1. Electronic devices for qutrit read-out, Josephson parametric amplifier (JPA) control, and qutrit control are displayed from top to bottom. The read-in and \(XY\) control signals are the mixtures of low-frequency signals (from two independent digital-to-analog converter (DAC) channels I and Q) and high-frequency signals (from microwave sources) to achieve nanosecond fast tuning. On the other hand, the qutrit \(Z\) control signal is sent directly from the DAC without mixing, whose frequency can also be slowly tuned by the direct current (DC) bias line. Additionally, the output signal of the read-out feeder is amplified by the JPA, high electron mobility transistor (HEMT), and room temperature amplifier, and then demodulated by the analog-to-digital converter (ADC). Four cryogenic unidirectional circulators are inserted between the JPA
and the 4K HEMT to block the reflection and noise from outside. As for the JPA, it is pumped by an independent microwave signal source and a DC bias, which is converged by the bias tee. At different temperature stages of the dilution refrigerator, each control line is balanced with some attenuators and filters (intuitively shown in figure A1) to prevent unwanted noise from affecting the equipment.

Appendix B. Qutrit read-out and calibration

Here we show the results of the qutrit read-out in figure B1, where blue, orange, and yellow points represent the results of applying $I, X_{01},$ and $X_{01}X_{12}$ to the qutrit, i.e., depicting the results of preparing $|0\rangle, |1\rangle,$ and $|2\rangle$, respectively. The read-out pulses are 1 $\mu$s-long and the repetition is 3000. It can be seen that these three states are clearly separated, though with several error transitions caused by the read-out errors. Like that in references [47, 48], the calibration matrix can be defined as

$$F = \begin{pmatrix} F_{00} & F_{01} & F_{02} \\ F_{10} & F_{11} & F_{12} \\ F_{20} & F_{21} & F_{22} \end{pmatrix},$$  \hspace{1cm} (B.1)

where $F_{ii'} (i, i' = 0, 1, 2)$ is the probability of measuring the qutrit in $|i\rangle$ when it is prepared in $|i'\rangle$. Such that the measured probability is given by $P_m^{|i\rangle} = \sum_{i'} F_{ii'} P_{i'}$, with $P_{i'}$ being the calibrated probability of $|i\rangle$, viz,

$$\begin{pmatrix} P_0^i \\ P_1^i \\ P_2^i \end{pmatrix} = F^{-1} \begin{pmatrix} P_0^{i'} \\ P_1^{i'} \\ P_2^{i'} \end{pmatrix}. \hspace{1cm} (B.2)$$

We can derive, for instance, $F_{01} = N_{OL}/3000$, in which $N_{[B,O,Y],[L,R,T]}$ is the number of [blue, orange, yellow] points in the [left, right, top] zones in figure B1. In addition, we repeat the measurement for 10 times to take the average values, yielding

$$F = \begin{pmatrix} 0.974 & 0.102 & 0.041 \\ 0.017 & 0.885 & 0.141 \\ 0.009 & 0.013 & 0.818 \end{pmatrix}. \hspace{1cm} (B.3)$$

Note that all the experimental data have been calibrated by the matrix in equation (B.3).
Appendix C. Variation of equations (3) and (4) in the main text for the coherence control

Equations (3) and (4) in the main text are used to control the populations, with designable $f_k (k = 1, 2)$ and iterative $h_{k+1}$ ($i = 0, 1, 2$). If we alternatively aim to control two parts of the coherence (e.g., $h_2$ and $h_3$) of systems, we need to alter equations (3) and (4) in the main text as

$$
\Omega_{01} = \frac{h_3[(\gamma + \Gamma)h_3 + 2h_3] + h_2[(\gamma_2 + \Gamma_2) + 2\hat{h}_2]}{2h_1h_2 - 2h_3(2f_1 + f_2 - 1)},
\Omega_{12} = \frac{(2f_1 + f_2 - 1)[h_2(\gamma_2 + \Gamma_2) + 2\hat{h}_2] + h_1[\gamma + \Gamma + 2\hat{h}_1]}{2h_1h_2 - 2h_3(2f_1 + f_2 - 1)},
\dot{h}_1 = \Omega_{12}(f_1 - f_2) - \frac{1}{2}h_1(\gamma + \gamma_2 + \Gamma_2) - h_2\Omega_{01},
\dot{\gamma}_1 = -\Gamma f_1 + \Gamma_2 f_2 + 2h_3\Omega_{01} - 2h_1\Omega_{12},
\dot{f}_1 = -\gamma f_1 + \Gamma_2 f_2 + 2h_3\Omega_{01} - 2h_1\Omega_{12},
$$

where

in which $h_{k+1}$ is designable and $f_k$ is iterative. Therefore one can design suitable $h_2$ and $h_3$ to accomplish desired coherence control.

Appendix D. Feasible and infeasible areas of the coherence control

For the control of $h_2$ and $h_3$, similarly, there are feasible area (FA) and infeasible area (IFA), which mainly depend on the intermediate function $f$ and decoherence. For $f = [1 + e^{-a(t-t_0/2)}]^{-1}$, we plot FA and IFA of the coherence control in figure D1. The total area is constructed by conditions $[h_2, h_3] \leq 0.5$ and $h_1^2 + h_2^2 \leq [1 - f_1^2 - f_2^2 - (1 - f_1 - f_2)^2 - 2h_1^2]/2 \leq 1/3$.

Appendix E. Tomography of the coherence control

The populations of three-level systems can be directly measured by the read-out cavity. (In the experiments, only diagonal elements of the density matrix can be directly measured.)

While the coherence can be indirectly measured by a simplified tomography, including four steps with tomography procedure errors about 0.16%, 0.5%, 0.6%, and 0.9%, respectively. These errors are not calibrated and therefore be accumulated to the overall errors. These four steps are shown as follows,

(a) $U_1 = I$,

$$
\rho_i = U_1 \rho U_1^\dagger = \begin{pmatrix}
    f_2 & -ih_1 & h_2 \\
    ih_1 & f_1 & -ih_3 \\
    h_2 & ih_3 & 1 - f_1 - f_2
\end{pmatrix},
$$

(E.1)
Here we show 30 and 36 groups of experimental data for the populations and coherence control in Figure D1. Figure D1. Feasible area (surrounded by solid, dashed, and dotted dashed curves) and infeasible area (the outside zone surrounded by dotted curves) of the coherence control in the experiment when the evolution time is 3, 5, and 10 μs.

\[
\begin{align*}
(b) & \ U_2 = \langle X/2 \rangle_{01}, \\
\rho_2 &= U_2 \rho U_2^\dagger = \begin{pmatrix}
\frac{f_2}{\sqrt{2}} & -\frac{i}{\sqrt{2}}(h_1 - h_2) & \frac{h_1 + h_2}{\sqrt{2}} \\
-\frac{i}{\sqrt{2}}(h_1 - h_2) & \frac{-f_2}{2} + h_3 + \frac{1}{2} & \frac{1}{2}(2f_1 + f_2 - 1) \\
\frac{h_1 + h_2}{\sqrt{2}} & \frac{1}{2}(2f_1 + f_2 - 1) & \frac{-f_2 - 2h_3 + 1}{2}
\end{pmatrix}, \\
\text{(E.2)}
\end{align*}
\]

(c) \( U_3 = \langle X/2 \rangle_{12}, \)
\[
\rho_3 = U_3 \rho U_3^\dagger = \begin{pmatrix}
\frac{i}{\sqrt{2}}(f_1 - f_2) & \frac{1}{2}(f_1 + f_2 - 2h_1) & \frac{1}{\sqrt{2}}(h_2 - h_3) \\
\frac{1}{\sqrt{2}}(h_2 - h_3) & \frac{i}{\sqrt{2}}(h_2 + h_3) & 1 - f_1 - f_2 \\
\frac{1}{\sqrt{2}}(h_2 - h_3) & \frac{i}{\sqrt{2}}(h_2 + h_3) & 1 - f_1 - f_2
\end{pmatrix}, \\
\text{(E.3)}
\]

(d) \( U_4 = U_3 U_2, \)
\[
\rho_4 = U_4 \rho U_4^\dagger = \begin{pmatrix}
\frac{1}{4}(f_1 + 2\sqrt{3}h_1 - 2\sqrt{3}h_2 + 2h_3 + 1) & \frac{1}{2}(3f_1 - 2h_3 + 1) & \frac{2f_1 + f_2 + \sqrt{3}h_1 - \sqrt{3}h_2 - 1}{2\sqrt{2}} \\
-\frac{1}{4}(3f_1 + 2\sqrt{3}h_1 + 2\sqrt{3}h_2 + 2h_3 - 1) & \frac{1}{2}(-2f_1 + 2\sqrt{3}h_1 + 2\sqrt{3}h_2 + 1) & \frac{2f_1 + f_2 + \sqrt{3}h_1 + \sqrt{3}h_2 - 1}{2\sqrt{2}} \\
\frac{1}{2}(2f_1 + f_2 - 2h_3 - 1) & \frac{1}{2}(-2f_1 - f_2 + 2\sqrt{3}h_1 + \sqrt{3}h_2 + 1) & \frac{-2f_1 - f_2 + \sqrt{3}h_1 + \sqrt{3}h_2 - 1}{2\sqrt{2}}
\end{pmatrix}, \\
\text{(E.4)}
\]

where \( I \) represents identity gate and \( \langle X/2 \rangle_{0112} \) denotes a \( \pi/2 \) rotation over the \( X \) axis of Bloch sphere in basis \( \{\{0\},\{1\}\} \) (\( \{\{1\},\{2\}\} \)). For all the experimental data points of the coherence (i.e., \( h_{i+1} \)), we measure three diagonal elements of \( \rho_p \) (\( p = 1, 2, 3, 4 \)), signed as \( \rho_p^{(i)} \) (\( i = 0, 1, 2 \)), and further deduce \( h_{i+1} \), i.e.,
\[
\begin{align*}
h_1 &= \rho_3^{(1)} - \frac{\rho_4^{(1)}}{2} = \frac{\rho_3^{(2)} + \rho_1^{(22)}}{2}, \\
h_3 &= \rho_3^{(2)} - \frac{1 - \rho_1^{(11)}}{2}, \\
h_2 &= -\frac{\rho_1^{(11)} - 2\sqrt{2}h_1 + 2h_3 - 4\rho_4^{(22)} + 1}{2\sqrt{2}}.
\end{align*}
\]

By now, a simplified tomography for reading out coherence \( h_{i+1} \) of three-level systems is completed.

**Appendix F. Supplementary experimental data for the dynamical control**

Here we show 30 and 36 groups of experimental data for the populations and coherence control in figures F1 and F2, respectively, whose average error rates are 1.02% and 1.39%, respectively, demonstrating the feasibility of the dynamical control in three-level open systems.
Figure F1. Experimental results (error bars) of the controls of populations. The numerically simulated results of $P_0(t)$, $P_1(t)$, and $P_2(t)$ are represented by solid, dashed and dotted dashed curves, respectively. The error rates are shown in corresponding subgraphs.

Figure F2. Experimental results (error bars) of the controls of coherence. The numerically simulated results of $h_0(t)$, $h_1(t)$, and $h_2(t)$ are represented by solid, dashed and dotted dashed curves, respectively. The error rates are shown in corresponding subfigures.

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