Magnetic Response of Majorana Kramers Pairs Protected by $\mathbb{Z}_2$ Invariants

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On the surface of time-reversal-invariant topological superconductors, Kramers pairs of Majorana fermions with chiral and crystalline symmetries exhibit completely uniaxial or octupole anisotropic magnetic response. This paper reports possible types of magnetic responses of Majorana Kramers pairs with one-dimensional $\mathbb{Z}_2$ invariants defined by crystalline symmetry. In particular, the general theory predicts a new type of magnetic response where two Majorana Kramers pairs associated with the $\mathbb{Z}_2$ invariant show biaxially (quadrupolar) anisotropic magnetic response, which is a novel type of response that is rarely observed in conventional and Majorana fermions.

The Majorana fermion is a long-sought particle that is its own antiparticle. Some unconventional superconductors with topological numbers called topological superconductors (TSCs) host Majorana fermions on their surfaces as gapless Andreev bound states. The emergent Majorana fermion on the surface follows non-Abelian statistics and is completely stable as long as the superconducting gap remains in the bulk. These novel properties allow us to apply TSCs to external field. For instance, a Majorana Kramers pair exhibits additional response that is rarely observed in conventional and Majorana fermions.

Many classes of TSCs were discovered based on the concept of symmetry. Time-reversal symmetry (TRS) protects degenerate gapless states that form a Kramers pair, which is called a Majorana Kramers pair, on the surface of time-reversal-invariant TSCs; examples include superconducting states in doped topological insulators and Dirac semimetals. In particular, crystalline symmetries define a new type of topological crystalline superconductor (TCSC). One-dimensional time-reversal-invariant superconductors (class DIII) without crystalline symmetry are classified by the $\mathbb{Z}_2$ topological invariant. In addition to the above $\mathbb{Z}_2$ phase, topological phases in the presence of crystalline symmetry have been thoroughly explored for reflection, all order-2, nonsymmetric, and rotational symmetries. To study the fundamental nature and possible applications of these TCSCs, it is necessary to understand how Majorana fermions on TCSCs respond to an external field. For instance, a Majorana Kramers pair exhibits a completely anisotropic (Ising) magnetic response, which is distinct from the response of conventional (complex) spin-1/2 fermions owing to TRS reinforced by crystalline symmetries, i.e., magnetic symmetry.

Previously, we revealed the relation among the magnetic-dipole and magnetic-octupole responses of a Majorana Kramers pair; magnetic winding number $\mathbb{Z}$, which coincides with the number of Majorana fermions; and irreducible representation of the superconducting pair potential. By applying this result, one can easily determine which multipole magnetic response occurs in a TCSC associated with the $\mathbb{Z}$ invariant. This method, however, is incomplete: it does not include a Majorana Kramers pair associated with $\mathbb{Z}_2$ topological invariants protected by crystalline symmetry. Recent studies have reported results useful for determining the topological invariants of TCSCs from the symmetry indicators in the normal state. In the present paper, we extend these studies by systematically elucidating the magnetic response of Majorana Kramers pairs with one-dimensional $\mathbb{Z}_2$ invariants protected by an order-2 symmetric magnetic field breaks the symmetry $U$. Surprisingly, two Majorana Kramers pairs protected by nonsymmetric magnetic response show a highly anisotropic magnetic response, which is expressed by a quadrupolar-shaped energy gap depending on the magnetic field.

Magnetic response depending on the number of Majorana Kramers pairs. Firstly, we reveal the relationship between the magnetic responses of Majorana Kramers pairs and the number of pairs. $N$ Majorana Kramers pairs are described by Majorana operators $\gamma_1, \gamma_2, \ldots, \gamma_{2N}$ satisfying $\gamma_i^\dagger = \gamma_i$ and $[\gamma_i, \gamma_j] = 2\delta_{ij}$. Two Majorana Kramers pairs form a Majorana Kramers pair. The time reversals are given by $\gamma_{2n-1} \rightarrow -\gamma_{2n-1}$ and $\gamma_{2n} \rightarrow -\gamma_{2n}$. Hermitian operators from $N$ Majorana Kramers pairs are represented by antisymmetric matrices as

$$J = \frac{i}{2} \mathbf{\gamma}^T A \mathbf{\gamma}, \quad A^T = -A, \quad A^* = -A. \quad (1)$$

A single Majorana Kramers pair $\mathbf{\gamma} = (\gamma_1, \gamma_2)^T$ hosts only one operator $A = s_z$, where $s_i (i = 0, x, y, z)$ denotes the $i$th Pauli matrix. The operator $J$ is time-reversal-odd (magnetic) and coupled to a magnetic field $\mathbf{B}$. Thus, the Hamiltonian for a single Majorana Kramers pair under a magnetic field $\mathbf{B}$ has the form $H_{MF} = f(\mathbf{B}) s_z$ and the gapped energy spectrum $E_M = f(\mathbf{B})$, where $f(\mathbf{B})$ is an analytic odd function:

$$f(\mathbf{B}) = \sum_i \rho_i B_i + \sum_{i,j,k} \rho_{ijk} B_i B_j B_k + O(B^5). \quad (2)$$

Here, the coefficients $\rho_i$ and $\rho_{ijk}$ depend on the system parameters.

Two Majorana Kramers pairs, on the other hand, can form four magnetic ($A_1, A_2, A_3,$ and $A_4$) and two electric ($B_1$ and $B_2$) operators. They are represented by $A_1 = s_z \tau_0, A_2 = s_z \tau_3,$
The effective Hamiltonian is given by $H_{\text{MF}} = \sum_{i=1}^{4} A_i f_i(B) + \sum_{i=1}^{2} B_i g_i(B)$, where $f_i(B)$ is an analytic odd function that has the same form as Eq. (2) and $g_i(B)$ is an analytic even function of magnetic fields. By diagonalizing the above matrix, the energy gap is obtained as

$$E_M = \sqrt{f_1^2 + f_2^2 + f_3^2} - \sqrt{f_1^2 + g_1^2 + g_2^2} = \left( \sum_{ij} \rho_{ij}^R B_i B_j - \sum_{ij} \rho_{ij}^L B_i B_j \right).$$

Consequently, two Majorana Kramers pairs vanish on applying a magnetic field along any direction. However, the induced gap shows a highly anisotropic magnetic response, as demonstrated later.

**Symmetry operation.** Next, we introduce the Hamiltonian and symmetry operation, following which we classify the possible magnetic responses of Majorana Kramers pairs on TCSCs. The BdG Hamiltonian for time-reversal-invariant three-dimensional superconductors has the form

$$H(k) = \left( \begin{array}{cc} h(k) - \mu & \Delta(k) \\ \Delta^*(k) & -h(k) + \mu \end{array} \right) = [h(k) - \mu] \tau_z + \Delta(k) \tau_x,$$

in the basis of $(c_{\uparrow} c_{\downarrow}, c_{\downarrow}^T c_{\uparrow}^T, -c_{\downarrow}^T c_{\uparrow}^T)$, where $\uparrow$ and $\downarrow$ denote the up and down spins, respectively, and the indices for the orbital and sublattice degrees of freedom are implicit. The Hamiltonian satisfies TRS, which is expressed as $\Theta H(k) \Theta^{-1} = H(-k)$; particle-hole symmetry (PHS), expressed as $CH(k)^{-1} = -H(-k)$, $C = \tau_z \Theta$; and chiral $\Gamma$ symmetry, expressed as $[\Gamma, H(k)] = 0$, $\Theta C = \tau_y$. When the system is invariant against a symmetry operation $g = [R_k |_g ]_g$ of a space group, which consists of a rotation/screw axis or reflection/glide plane $R_k$ followed by the translation $\tau_g$, the Hamiltonian $h(k)$ in the normal state satisfies $D_{\text{ht}}(g) h(k) D_{\text{ht}}(g) = h(gk)$, where $D_{\text{ht}}(g)$ is the representation matrix of $g$ and the momentum $k$ is transformed to $gk$. The pair potential $\Delta(k)$, on the other hand, satisfies $D_{\text{ht}}^{\dagger}(g) \Delta(k) D_{\text{ht}}(g) = \chi(g) \Delta(g)$, where $\chi(g)$ is the character of $g$ for the one-dimensional representation of the pair potential.30 Then, the BdG Hamiltonian is invariant, $D_{\text{ht}}^{\dagger}(g) H(k) D_{\text{ht}}(g) = H(gk)$, for $D_{\text{ht}}(g) = \text{diag}[D_k(g), \chi(g) D_k(g)]$. The particle-hole and chiral transformations of the representation matrices depend on the character:

$$D_{\text{ht}}^{\dagger}(g) C D_{\text{ht}}(g) = \chi(g) C,$$

$$D_{\text{ht}}^{\dagger}(g) \Gamma D_{\text{ht}}(g) = \chi(g) \Gamma,$$

where we use $\Theta D_{\text{ht}}(g) \Theta^{-1} = \tilde{D}_{\text{ht}}(g)$. The square of the representation matrix is given by

$$D_{\text{ht}}^{\dagger}(g) = -e^{-ik \cdot (R_k \tau_x + \tau_y)}$$

and classified into $D_{\text{ht}}^{\dagger}(g) = -1$ and $1$ for time-reversal-invariant momenta (TRIMs). $D_{\text{ht}}^{\dagger}(g) = -1$ holds when $g$ represents twofold rotation and reflection. The twofold screw axis and glide plane also satisfy $D_{\text{ht}}^{\dagger}(g) = -1$ for $k \cdot 2 \tau_y = 0$. On the one hand, $D_{\text{ht}}^{\dagger}(g) = 1$ holds when $g$ represents the screw axis and glide plane on the zone boundary with $k \cdot 2 \tau_y = \pi$ because $\tau_x$ can be a half translation vector. We call $D_{\text{ht}}^{\dagger}(g)$ $-1$ and $1$ symmetric and nonsymmetric symmetry, respectively.

Symmetry that preserves the surface, which is referred to as surface symmetry, can protect Majorana Kramers pairs. To formulate the surface symmetry, we denote the momentum by $(k^x, k^y)$, which are components perpendicular and parallel to the surface, respectively. Next, $k^z$ is fixed to a TRIM, where Majorana Kramers pairs can appear with zero energy, and omitted. The surface symmetry $U$ satisfies

$$[D_{\text{ht}}(U), h(k)] = 0.$$ (8)

The symmetry can divide the BdG Hamiltonian $H$ into parts in the eigenspaces of $D_{\text{ht}}(U)$ as $H = H_+ \oplus H_-$, where the subscript $\pm$ denotes the eigenvalue of $D_{\text{ht}}(U)$. Majorana fermions in $H_+$ and $H_-$ do not hybridize with each other. In other words, they are protected by the symmetry $U$. Note that the representation matrix $D_{\text{ht}}(U)$ must be independent of $k^z$ in order to protect Majorana Kramers pairs, i.e., $U$ is the reflection perpendicular, glide plane consisting of the reflection perpendicular and translation parallel, or twofold rotation perpendicular to the surface. Hereafter, the subscript $k^z$ is omitted.

**Crystalline $Z_2$ topological phase and magnetic response.** Here, we derive a condition on the representation matrix and character to protect the $Z_2$ topological phase with Majorana Kramers pairs and relate it to the magnetic responses. According to Eq. (5), $H_+$ has particle-hole symmetry for $\chi(U) = D^2(U) = \pm 1$ because the eigenvalues of $D(U)$ are $\pm 1$ and $\pm i$ for $D^2(U) = 1$ and $-1$, respectively. The magnetic responses are classified into three types, (A)–(C), as summarized in Table I, which is the main result of this paper.

Type (A). For $\chi(U) = -1$ and $D^2(U) = -1$, $H_+$ breaks TRS (class D). A single Majorana Kramers pair in $H = H_+ \oplus H_-$ is divided into Majorana fermions in $H_+$ and $H_-$, which are associated with the $Z_2$ invariant:

$$v_{\pm}[U] = \int_{-\pi}^{\pi} dk \frac{a_{\pm}(k)}{\pi} \mod 2,$$ (9)

$$a_{\pm}(k) = -i \sum_n (k, n \pm \hbar \delta_k [k, n \pm \hbar].$$ (10)

In terms of the negative-energy states, $H_+(k) |k\pm\rangle = -E_{\pm}(k) |k\pm\rangle$ for $E_{\pm}(k) > 0$. Because $H_+$ and $H_-$ are switched by time reversal, $v[U] = v_+[U] = v_-[U]$ holds. A magnetic field along $n$ is perpendicular to the mirror when $U$ is a reflection/glide plane or is parallel to the rotational axis when $U$ is a twofold rotation because the applied field $B_n$ keeps the symmetry $U$. Then, the $Z_2$-invariant $v[U]$ remains well-defined under the magnetic field $B_n$ and has the same value as that under zero field. Therefore, we find that the energy gap $E_M$ is expressed by Eq. (2) as $E_M = f(B)$, satisfying $f(B_n) = 0$.

For $\chi(U) = 1$ and $D^2(U) = 1$, on the other hand, $H_+$ respects TRS (class DIII) and can have a single Majorana Kramers pair, which is associated with

$$v_{\pm}[U] = \int_{-\pi}^{\pi} dk \frac{a_{\pm}(k)}{2\pi} \mod 2,$$ (11)

with the gauge fixed to $\Theta(k, 2n = -1z) = [-k, 2n\pm]$. The whole system can be classified into types (B) and (C) as follows.

Type (B). This type corresponds to a single Majorana Kramers pair with $v_+[U], v_-[U]) = (1, 0)$ or $(0, 1)$, which
is protected by the nonsymmetric symmetry $U$ for $\chi(U) = D^2(U) = 1$. In reality, it is protected by the magnetic glide-plane symmetry $\Theta(U) = \hat{D}(U) \Theta$ with $\Theta^2(U) = -1$, which is regarded as TRS in the whole system with $H = H_+ + H_-$. An energy gap occurs when the direction $\mathbf{n}$ is perpendicular to the glide plane because the applied magnetic field $Bn$ breaks the magnetic glide-plane symmetry. The resulting energy gap is proportional to $E_M \propto n \cdot B$.

Type (C). This type corresponds to two Majorana Kramers pairs with $(\nu_4[U], \nu_5[U]) = (1, 1)$, which is protected by the nonsymmetric symmetry $U$ for $\chi(U) = D^2(U) = 1$. The energy gap of the Majorana Kramers pairs is given by Eq. (3).

**Magnetic response of Majorana Kramers pairs for Pmma.** Here, we show an example of the magnetic response of two Majorana Kramers pairs on the $(xz)$ surface for the space group $Pmma$ (No. 51). The primitive translation vector $\mathbf{a}$ is set in the $x$ direction. Three symmetry surface operations

$$U_1 = [C_2(y)][0], U_2 = [\sigma(xy)][a/2], U_3 = [\sigma(yz)][a/2],$$

(12) (and identity) out of 8 symmetry operations $g$ of $Pmma$ preserve the $(xz)$ surface, and four symmetry operations

$$P_1 = [\sigma(xz)][0], P_2 = [C_2(z)][a/2], P_3 = [C_2(x)][a/2],$$

(13) and inversion $I = U_0 P_1$ invert the $(xz)$ surface, where $C_2(i)$ and $\sigma(ij)$ denote the twofold rotation along the $i$th axis and the reflection with respect to the $(ij)$ plane, respectively.

We focus on the $B_{1u}$ pairing for $k_z = \pi$, which is defined by $\chi(U_1) = -1$, $\chi(U_2) = 1$, $\chi(U_3) = -1$, $\chi(P_1) = 1$, $\chi(P_2) = -1$, $\chi(P_3) = 1$, and $\chi(I) = -1$. This results in a magnetic response of type (C), as shown below. Since the representation matrices obey $\chi(U_1) = \chi(U_5) = D^2(U_1) = D^2(U_5) = -1$ and $\chi(U_2) = D^2(U_3) = 1$, the possible magnetic responses are type (A) for $U_1$ and $U_3$ and type (C) for $U_2$. The number of the Fermi surfaces on $k_z = \pi$ is a multiple of 4 because all the energy bands are fourfold degenerate \[^{10,52,53}\] at $k = (\pi, 0, 0)$ and $(\pi, \pi, 0)$. The subsystems $H_+$ and $H_-$ have the same number of Fermi surfaces owing to TRS. Therefore, no Majorana Kramers pair protected by $U_1$ or $U_3$ appears, because $\nu[U_1] = \nu[U_3] = \#FS_{zz} = 2n = 0 \mod 2$, where $\#FS_{zz}$ denotes the number of Fermi surfaces of $H_\pm$ between $k_\perp = 0$ and $k_\perp = \pi$. Here, we assume that the system is in the weak coupling regime, where the $Z_2$ invariant is determined by the parity of the Fermi surfaces.\[^{10,52,53}\]

The glide plane $U_2$ satisfies $\chi(U_2) = D^2(U_2) = 1 + i$ and anticommutes with inversion symmetry $D(I)$. Therefore, the classification $Z_2 \otimes Z_2$ reduces to $Z_2$ because $\nu[U_2] = \nu[U_3] = \nu[U_3]$.\[^{10,52,53}\] Then, two Majorana Kramers pairs appear when the $Z_2$ invariant $\nu[U_2]$ is nontrivial. All the surface symmetries satisfy $\chi(U_i) = D^2(U_i)$ and $[D(U_i), D(U_j)] = 0$ for $k_z = \pi$.

Thus, the Hamiltonian is decomposed into four subsystems $H_+, H_-, H_+,$ and $H_-$, where the subscripts denote the eigenvalues of $\hat{D}(U_1)$ and $\hat{D}(U_2)$. The sub Hamiltonian $H_{zz}$ is characterized by the $Z_2$ invariant or the parity of the Fermi surfaces of $H_{zz}$, which is equivalent to $\nu[U_2]$. Hence, two Majorana Kramers pairs emerge for

$$\nu[U_2] = \#FS_{zz} = (\#FS)/4 = 1 \mod 2.$$

Consequently, we find that two Majorana Kramers pairs possibly exist for the $B_{3u}$-pairing case. The magnetic response is of type (C), i.e., an energy gap $E_M \sim \sqrt{\sum_{jj'} \rho_{jj'} B_i B_j}$ is induced by a magnetic field.

**Model Hamiltonian and numerical results.** We finally verify the above general result by examining a toy model on a layered 2D lattice with two sublattices, which has a glide plane,\[^{54}\] as shown in Fig. 1. We set the lattice constants as 1. The normal part $h(k)$ of the BdG Hamiltonian is

$$h(k) = c(k)[\sigma_0 s_0 + t_3 \cos(k_z/2)\sigma_1(k_z) s_0 + (\lambda_1 s_x \sin k_y + \lambda_2 s_y \sin k_x)\sigma_3],$$

(15)

$$c(k) = m_0 + t_1 \cos k_x + t_2 \cos k_y,$$

(16)

and the pair potential $\Delta(k)$ for the $B_{3u}$ pairing is

$$\Delta(k) = \Delta \sigma_0 s_z \sin k_z + \Delta' \sigma_2(k_z) s_x \sin(k_z/2) s_z,$$

(17)

where $s$ and $\sigma$ denote the Pauli matrices representing the spin and layer degrees of freedom (A and B), respectively. Here, we introduce the modified Pauli matrices

$$\sigma_1(k_z) = \begin{pmatrix} 0 & e^{ik_z/2} \\ 0 & 0 \end{pmatrix}, \sigma_2(k_z) = \begin{pmatrix} 0 & -ie^{ik_z/2} \\ ie^{-ik_z/2} & 0 \end{pmatrix},$$

(18)

and $\sigma_3 = \text{diag}(1, -1)$. When the pair potential is smaller than spin-orbit coupling, i.e., $\Delta' > \Delta^2 + \Delta'^2$, nodes exist at $k_z = \pi$. Hereafter, we assume the gapped case $\Delta' < \Delta^2 + \Delta'^2$, where Majorana Kramers pairs appear. The band structure of the nor-
Table II. Parameters of the numerical model.

| $m_0$ | $t_1$ | $t_2$ | $t_1$ | $\Lambda_1$ | $\Lambda_2$ | $\Delta$ | $\Delta'$ | $|B|$ |
|-------|-------|-------|-------|-------------|-------------|---------|---------|-------|
| 2.0   | 0.5   | -3.5  | 0.5   | 0.7         | -0.5        | 1.2     | 0.6     | 0.1   |

Fig. 2. Energy dispersion (left) and Fermi surface (right) of the normal state $h(k)$.

Fig. 3. (a) Energy spectrum of Eq. (19) without a magnetic field. Two Majorana Kramers pairs are located at $k_x = \pi$. They are gapped by an external magnetic field. (b) The polar plot of the energy gap $|E_M|$ of $H + H_Z$ as a function of $B$.

The energy spectrum of $H(k)$ is shown in Fig. 2. The parameters are set to those listed in Table II. All the bands are twofold degenerate at any momentum owing to inversion and time-reversal symmetries. Particularly, the bands are fourfold degenerate at the $X$ and $S$ points because of those symmetries and the glide-plane symmetry.

The Hamiltonian with the $(xz)$ surfaces has the form

$$H(k) = \sum_{n=1}^{N_x} \epsilon_n(k_x) c_n^\dagger(k_x) c_n(k_x) + \sum_{n=1}^{N_y} \left[ c_n^\dagger(k_x) t_y(k_x) c_{n+1}(k_x) + \text{h.c.} \right], \quad (19)$$

where $N_y$ denotes the number of sites along the $y$ direction. The Fermi energy is set to 0. The system has four Fermi surfaces between the $X$ and $S$ points. The onsite energy $\epsilon(k_x)$ and hopping $t_y(k_x)$ are defined by $\epsilon(k_x) = (m_0+t_1 \cos k_x) \cos \tau_x + t_2 \cos (k_x/2) \cos \tau_y + \Lambda_1 \sin k_x \cos \tau_y + \Lambda_2 \cos k_x \sin \tau_y$ and $t_y(k_x) = (t_2/2) \cos \tau_x \cos \tau_y - i(\Lambda_1/2) \sin k_x \sin \tau_y - i(\Lambda_2/2) \cos k_x \sin \tau_y$, respectively. A magnetic field induces the Zeeman term

$$H_Z(k_x) = \sum_{n=1}^{N_y} c_n^\dagger(k_x) B \cdot s \sigma_0 \tau_y c_n(k_x). \quad (20)$$

The energy spectrum of $H(k_x) + H_Z(k_x)$ is shown in Fig. 3. Without a magnetic field, i.e., with $|B| = 0$ [Fig. 3(a)], the superconducting gap in the bulk is of the order of $\sim 1.4$. At $k_x = \pi$, two Majorana Kramers pairs exist with zero energy on the $(xz)$ surface. Figure 3(b) shows a polar plot of the energy gap $E_M(B)$. Magnetic fields along any direction destroy the two Majorana Kramers pairs and yield a gap of the order of $\sim 0.12$ at maximum. Interestingly, $E_M(B)$ mimics the anisotropy of a quadrupole. Such a biaxial anisotropy is allowed within the form $\sqrt{g_1(B)} - \sqrt{g_2(B)}$, where $g_1(B)$ is an analytic even function. This behavior is entirely different from those of other Majorana and complex fermions.

Discussion. We found three types of the magnetic responses in Majorana Kramers pairs with the $\mathbb{Z}_2$ invariants, which depend on the surface symmetry and the number of Majorana Kramers pairs. In type (A), i.e., the symmorphic case, there exists only a single Majorana Kramers pair, in which an external magnetic field creates a uniaxially anisotropic gap. On the other hand, in types (B) and (C), the case of nonsymmorphic symmetry, the magnetic response depends on the number of Majorana Kramers pairs: (B) a single Majorana Kramers pair behaves as an Ising spin, which is the same as the response associated with the $\mathbb{Z}$ invariant. (C) two Majorana Kramers pairs show a biaxially (quadrupolar) anisotropic magnetic response, which is a novel type of response rarely observed in conventional and Majorana fermions.

This prediction was verified in a bilayer model with a glide plane ($Pnma$). In fact, our result can be applied to materials with any nonsymmorphic space group, such as UC$\text{CoGe}$, which has the crystalline symmetry of the space group $Pnma$. Previous studies have strongly suggested that this material is a ferromagnetic superconductor at ambient pressure and a time-reversal-invariant one at high pressure. A theoretical study reported that, based on an experimental result, the ferromagnetic superconducting state has $A_u$ symmetry of $C_{4h}$ and the state can deform into either $A_u$ or $B_{1u}$ symmetry of $D_{3d}$ at high pressure. The $B_{1u}$-pairing (glide-even $\chi(G_n) = 1$) state hosts two Majorana Kramers pairs on the $(011)$ surface, while the $A_u$-pairing (glide-odd $\chi(G_n) = -1$) hosts no Majorana Kramers pairs because the surface has only $G_n$ symmetry satisfying $D_{3d}^{A_u}(0,0,0) (G_n) \neq \chi(G_n) = -1$ for the $B_{1u}$ pairing. Thus, the magnetic response of Majorana Kramers pairs on UC$\text{CoGe}$ is predicted to be of type (C). The gap induced in the Majorana Kramers pairs may be observed through surface tunneling spectroscopy under a magnetic field or with a magnet attached.

Fig. 3. (a) Energy spectrum of Eq. (19) without a magnetic field. Two Majorana Kramers pairs are located at $k_x = \pi$. They are gapped by an external magnetic field. (b) The polar plot of the energy gap $|E_M|$ of $H + H_Z$ as a function of $B$.

C$\alpha$, C$\beta$, and C$\delta$ symmetries, which are beyond the scope of this paper, might realize new types of magnetic responses. The discussions in this paper suggest that multiple Majorana Kramers pairs can be active against electric perturbations. Therefore, we also need to clarify the $\mathbb{Z}$ and $\mathbb{Z}_2$ topological invariants and the electric responses of multiple Majorana Kramers pairs. These issues will be addressed in a future paper.

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