Standardizing Dainotti-correlated gamma-ray bursts, and using them with standardized Amati-correlated gamma-ray bursts to constrain cosmological model parameters

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ABSTRACT

We show that each of the three Dainotti-correlated gamma-ray burst (GRB) data sets recently compiled by Wang et al. and Hu et al., that together probe the redshift range $0.35 \leq z \leq 5.91$, obey cosmological-model-independent Dainotti correlations and so are standardizable. We use these GRB data in conjunction with the best currently-available Amati-correlated GRB data, that probe $0.3399 \leq z \leq 8.2$, to constrain cosmological model parameters. The resulting cosmological constraints are weak, providing lower limits on the non-relativistic matter density parameter, mildly favoring non-zero spatial curvature, and largely consistent with currently accelerated cosmological expansion as well as with constraints determined from better-established data.

Key words: cosmological parameters – dark energy – cosmology: observations – gamma-ray bursts

1 INTRODUCTION

The observed currently accelerated cosmological expansion indicates that — if general relativity provides an accurate description of gravitation on cosmological scales — dark energy must contribute significantly to the current cosmological energy budget. The simpler spatially-flat $\Lambda$CDM model (Peebles 1984) is consistent with this and other observations. Fits of this model to most better-established cosmological data suggest that a time-independent cosmological constant ($\Lambda$) provides $\sim 70\%$ of the current cosmological energy budget, non-relativistic cold dark matter (CDM) provides $\sim 25\%$, and non-relativistic baryonic matter provides most of the remaining $\sim 5\%$ (see, e.g. Farooq et al. 2017; Scolnic et al. 2018; Planck Collaboration 2020; eBOSS Collaboration 2021). While the spatially-flat $\Lambda$CDM model is consistent with most observations (see, e.g., Di Valentino et al. 2021b; Perivolaropoulos & Skara 2021), observational data do not strongly rule out a little spatial curvature or dynamical dark energy. In this paper, in addition to the spatially-flat $\Lambda$CDM model, we also study spatially non-flat and dynamical dark energy models.

Observational astronomy now provides many measurements that can be used to test cosmological models. Largely, these data are either at low or at high redshift. So cosmological models are mostly tested at low and high redshifts, remaining poorly tested in the intermediate redshift regime. The highest redshift of the better-established low-redshift data, $\sim 2.3$, is reached through baryon acoustic oscillation (BAO) observations; the high redshift region, $z \sim 1100$, is probed by better-established cosmic microwave background anisotropy data. Fits of these better-established data to cosmological models provide mostly mutually consistent results. However, for a better understanding of our Universe, it is necessary to also test cosmological models in the intermediate redshift range of $2.3 \lesssim z \lesssim 1100$. Some progress has been achieved: methods that test cosmological models in the intermediate redshift region include the use of H II starburst galaxy measurements which reach to $z \sim 2.4$ (Mania & Ratra 2012; Chávez et al. 2014; González-Morán et al. 2019, 2021; Cao et al. 2020, 2021b; Johnson et al. 2021), quasar angular size measurements which reach to $\sim 2.7$ (Cao et al. 2017; Ryan et al. 2019; Cao et al. 2020, 2021b; Zheng et al. 2021; Lian et al. 2021), and quasar flux measurements which reach to $\sim 7.5$ (Risaliti & Lusso 2015, 2019; Khadka & Ratra 2020a,b, 2021a,b; Yang et al. 2020; Lusso et al. 2020; Zhao & Xia 2021; Li et al. 2021; Lian et al. 2021; Rezaei et al. 2021; Luongo et al. 2021).}
Gamma-ray burst (GRB) measurements are another high redshift probe and reach to $z \sim 8.2$ (Amati et al. 2008, 2019; Salvaterra et al. 2009; Tanvir et al. 2009; Samushia & Ratra 2010; Cardone et al. 2010; Dainotti et al. 2013a; Wang et al. 2015, 2016; Dainotti & Del Vecchio 2017; Fana Dirrisi et al. 2019; Khadka & Ratra 2020c; Demianski et al. 2021; Khadka et al. 2021b; Luongo et al. 2021; Luongo & Muccino 2021). While there are quite a few Amati correlation long GRBs that have been used to constrain cosmological parameters, currently only a smaller fraction of 118 such GRBs (hereafter A118) that cover the redshift range $0.3 < z < 8.2$ (Khadka & Ratra 2020c; Khadka et al. 2021a) are reliable enough to be used to constrain cosmological parameters. To date, this is the lower-$z$ data set used to constrain cosmological parameters that spans the widest range of redshifts. These A118 data provide cosmological constraints which are consistent with those obtained from the better-established cosmological probes but the GRB constraints are significantly less restrictive. To obtain tighter cosmological constraints using GRB data, we need to make use of more GRBs.

Recently Wang et al. (2021) and Hu et al. (2021) have compiled smaller GRB data sets that together probe the redshift range $0.35 < z < 5.91$. These are GRBs whose plateau phase luminosity $L_0$ and spin-down characteristic time $t_s$ are correlated through the Dainotti ($L_0 - t_s$) correlation (Dainotti et al. 2013b, 2017). This correlation between $L_0$ and $t_s$ allows one to use these GRBs for cosmological purposes. These GRBs can be classified in two categories depending on whether the plateau phase is dominated by magnetic dipole (MD) radiation or gravitational wave (GW) emission (Wang et al. 2021; Hu et al. 2021). In this paper we use long and short GRBs whose plateau phase is dominated by MD radiation (hereafter MD-LGRBs and MD-SGRBs) and long GRBs whose plateau phase is dominated by GW emission (hereafter GW-LGRBs). All three sets of GRBs obey the Dainotti correlation but each set can have different correlation parameters. We use the three individual GRB data sets, as well as some combinations of them, to constrain cosmological model parameters and Dainotti correlation parameters simultaneously. We find that these GRBs are standardizable, as was assumed in Wang et al. (2021) and Hu et al. (2021). However, cosmological constraints obtained from these Dainotti correlation GRB data sets are very weak.

When we combine the MD-LGRB or GW-LGRB data sets with the 115 non-overlapping Amati correlation GRBs from the A118 data set, they slightly tighten the constraints from the 115 Amati correlation GRBs, but not significantly so. Each of the individual Amati or Dainotti correlation GRB data sets, as well as combinations of these GRB data sets, mostly provide only lower limits on the current value of the non-relativistic matter energy density parameter $\Omega_{m0}$ and the resulting cosmological parameter constraints are mostly consistent with those obtained from better-established cosmological data.

In this paper, we use a combination of Hubble parameter ($H(z)$) and BAO data, $H(z) + \text{BAO}$, results as a proxy for better-established data results, to compare with our GRB data results. Qualitatively, results from the individual GRB data sets, as well as those from combinations of GRB data sets, are consistent with those from the $H(z) + \text{BAO}$ data which favor $\Omega_{m0} \sim 0.3$, but there are a few combinations of GRB data sets with constraints on $\Omega_{m0}$ being more than 2σ away from 0.3 in the $\Lambda$CDM models.

This paper is structured as follows. In Sec. 2 we summarize the cosmological models we use. In Sec. 3 we describe the data sets we analyze. In Sec. 4 we summarize our analyses techniques. In Sec. 5 we present our results. We conclude in Sec. 6.

## 2 COSMOLOGICAL MODELS

In this paper we derive cosmological parameter constraints in six different general-relativity cosmological dark energy models. Three of them assume spatially-flat geometry while the other three allow non-flat spatial geometry. These models are used to predict the luminosity distance for a GRB at a given redshift. For this purpose the fundamental quantity is the expansion rate of the Universe, or the Hubble parameter, $H(z)$, a function of cosmological parameters and redshift.

The Hubble parameter in all six models we use can be written as

$$H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_{DE}(z)},$$

(1)

where $H_0$ is the Hubble constant and $\Omega_{k0}$ is the current value of the spatial curvature energy density parameter. In analyses of the $H(z) + \text{BAO}$ data set we express $\Omega_{k0}$ in terms of the current value of the baryonic matter energy density parameter ($\Omega_b$) and the current value of the cold dark matter energy density parameter ($\Omega_c$) through the equation $\Omega_{k0} = \Omega_b + \Omega_c$. In four of the six models, the dark energy density parameter term is expressed as $\Omega_{DE}(z) = \Omega_{DE0}(1+z)^{3w_{DE}}$, where $\Omega_{DE0}$ is the current value of the dark energy density parameter and $w_{DE}$ is the dark energy equation of state parameter.

In the $\Lambda$CDM models $w_{DE} = -1$ and $\Omega_{DE0} = \Omega_\Lambda$ is the cosmological constant dark energy density parameter and is time-independent. The current values of the three energy density parameters are related by the energy budget equation, $\Omega_{m0} + \Omega_{k0} + \Omega_\Lambda = 1$. In the spatially-flat $\Lambda$CDM model we choose to constrain $\Omega_{m0}$ and $H_0$ while in the spatially non-flat $\Lambda$CDM model we constrain $\Omega_{m0}$, $\Omega_{k0}$, and $H_0$. For analyses which involve $H(z) + \text{BAO}$ data, instead

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3 For recent discussions of constraints on non-flat models see Chen et al. (2016), Rana et al. (2017), Ooba et al. (2018a,c), Yu et al. (2018), Park & Ratra (2019a,c), Wei (2018), DES Collaboration (2019), Li et al. (2020), Handley (2019), Efstathiou & Gratton (2020), Di Valentino et al. (2021a), Velasco-Toribio & Fabris (2020), Vagnozzi et al. (2021a,b), KiDS Collaboration (2021), Arjona & Nesseris (2021), Dhawan et al. (2021), and references therein.
of $\Omega_{\text{m0}}$ we constrain $\Omega h^2$ and $\Omega_b h^2$; here $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$.

In the XCDM parametrizations, the equation of state for the dynamical dark energy X-fluid is $P_X = w_X \rho_X$, where $P_X$, $\rho_X$, and $\omega_X$ are the pressure, energy density, and equation of state parameter for the dynamical dark energy X-fluid, and $\Omega_{\text{X0}} = \Omega_X$ is the current value of the X-fluid dynamical dark energy density parameter. In this case, the current values of the three energy density parameters are related by $\Omega_{\text{m0}} + \Omega_{\text{X0}} + \Omega_{\Lambda} = 1$. The X-fluid energy density decreases with time when $w_X$ satisfies the conditions $-1 < w_X < 0$.

In the spatially-flat XCDM parametrization, we choose to constrain $\Omega_{\text{m0}}$, $\omega_X$, and $H_0$ while in the non-flat XCDM parametrization, we constrain $\Omega_{\text{m0}}$, $\Omega_{\Lambda}$, $\omega_X$, and $H_0$. For analyses which involve $H(z) + \Lambda$.GRB cosmological parameter constraints

\begin{equation}
V(\phi) = \frac{1}{2} \kappa m_\phi^2 \phi^{-\kappa}.
\end{equation}

Here $m_\phi$ is the Planck mass, $\alpha$ is a positive parameter, and $\kappa$ is a constant whose value is determined by using the shooting method to ensure that the current energy budget equation $\Omega_{\text{m0}} + \Omega_{\Lambda} + \Omega_X(z = 0,\alpha) = 1$ is satisfied.

In the $\phi$CDM models, the dynamics of a spatially homogeneous scalar field is governed by two coupled non-linear differential equations. The first is the dark energy scalar field equation of motion

\begin{equation}
\ddot{\phi} + 3 \left( \frac{\dot{a}}{a} \right) \dot{\phi} - \frac{k}{a^2} \phi^{-\alpha - 1} = 0,
\end{equation}

and the second is the Friedmann equation

\begin{equation}
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3m_\phi^2} (\rho_m + \rho_\phi) - \frac{k}{a^2},
\end{equation}

where $a$ is the scale factor and an overdot denotes a time derivative. In eq. (4), $-k/a^2$ is the spatial curvature term with $\Omega_{\Lambda} = 0$, $> 0$, $< 0$ corresponding to $k = 0$, $-1$, $+1$, respectively, and $\rho_m$ and $\rho_\phi$ are the non-relativistic matter and scalar field energy densities where

\begin{equation}
\rho_\phi = \frac{m_\phi^2}{32\pi} \left( \dot{\phi}^2 + \kappa m_\phi^2 \phi^{-\alpha} \right).
\end{equation}

By solving eqs. (3) and (4) numerically, we can compute $\rho_\phi$ and then compute $\Omega_\phi(z,\alpha)$ by using the expression

\begin{equation}
\Omega_\phi(z,\alpha) = \frac{8\pi \rho_\phi}{3m_\phi^2 H_0^2}.
\end{equation}

In the spatially-flat $\phi$CDM model, we choose to constrain $\Omega_{\text{m0}}$, $\alpha$, and $H_0$ while in the non-flat $\phi$CDM model we constrain $\Omega_{\text{m0}}$, $\Omega_{\Lambda}$, $\alpha$, and $H_0$. For analyses which involve $H(z) + \Lambda$.GRB, we use $(\sigma_\Lambda^2 + \sigma_\phi^2)/2$, where $\sigma_\Lambda$ and $\sigma_\phi$ are the asymmetric upper and lower error bars.

3 DATA

In this paper, we analyze four different $\Lambda$.GRB data sets as well as some combinations of these data sets. We also use a joint $H(z) + \Lambda$.GRB data set. These data sets are summarized in Table 1 and described in what follows.

\begin{itemize}
\item \textbf{MD-LGRB sample.} This includes 31 long GRBs, with burst duration longer than 2 seconds, listed in Table 1 of Wang et al. (2021). For this data set, measured quantities for a $\Lambda$.CDM model are the redshift $z$, X-ray flux $F_\text{X}$, characteristic time scale $t_b$, and spectral index during the plateau phase $\beta'$. This sample probes the redshift range $1.45 \leq z \leq 5.91$.

\item \textbf{MD-SGRB sample.} This includes 5 short GRBs, with burst duration shorter than 2 seconds, listed in Table 1 of Hu et al. (2021). For this data set, measured quantities for a $\Lambda$.CDM model are the redshift range $0.35 \leq z \leq 2.6$.

\item \textbf{GW-LGRB sample.} This includes 24 long GRBs listed in Table 1 of Hu et al. (2021). For this data set, measured quantities for a $\Lambda$.CDM model are the redshift range $0.55 \leq z \leq 4.81$.

\item \textbf{A118 sample.} This sample include 118 long GRBs listed in Table 7 of Khadka et al. (2021a). For this data set, measured quantities for a $\Lambda$.CDM model are the redshift range $0.3399 \leq z \leq 8.2$.

\item \textbf{A118 sample.} This sample include 118 long GRBs listed in Table 7 of Khadka et al. (2021a). For this data set, measured quantities for a $\Lambda$.CDM model are the redshift range $0.3399 \leq z \leq 8.2$.
\end{itemize}

5 In this table and elsewhere, for compactness, we sometimes use ML, MS, and GL as abbreviations for the MD-LGRB, MD-SGRB, and GW-LGRB data sets compiled by Wang et al. (2021) and Hu et al. (2021).

6 ML, MS, and GL data error bars on $F_\text{X}$ and $t_b$ are mostly asymmetric. We symmetrize these error bars using the method applied in Wang et al. (2021) and Hu et al. (2021), with the symmetrized error bar $\sigma = \sqrt{(\sigma^2 + \sigma'^2)/2}$, where $\sigma_\alpha$ and $\sigma_\beta$ are the asymmetric upper and lower error bars.
Table 1. Summary of data sets used.

| Data set | N (Number of points) | Redshift range |
|----------|----------------------|----------------|
| ML       | 31                   | 1.45 ≤ z ≤ 5.91 |
| MS       | 5                    | 0.35 ≤ z ≤ 2.6  |
| GL       | 24                   | 0.55 ≤ z ≤ 4.81 |
| MS + GL  | 29                   | 0.35 ≤ z ≤ 4.81 |
| A118     | 118                  | 0.3399 ≤ z ≤ 8.2 |
| A115\(^a\) | 115                 | 0.3399 ≤ z ≤ 8.2 |
| A115\(^b\) | 115                | 0.3399 ≤ z ≤ 8.2 |
| H(z)     | 31                   | 0.670 ≤ z ≤ 1.965 |
| BAO      | 11                   | 0.38 ≤ z ≤ 2.334 |

\(^a\) Excluding from A118 those GRBs in common with MD-LGRB (GRB060526, GRB081008, and GRB090516).

\(^b\) Excluding from A118 those GRBs in common with GW-LGRB (GRB060206, GRB091029, and GRB131105A).

Table 2. Flat priors of the constrained parameters.

| Parameter | Prior |
|-----------|-------|
| \(H_0\) \(^a\) | [None, None] |
| \(\Omega_b h^2\) \(^b\) | [0, 1] |
| \(\Omega_c \Omega_k\) | [0, 1] |
| \(\Omega_k\) | [-2, 2] |
| \(\alpha\) | [0, 10] |
| \(w_X\) | [-5, 0.33] |
| \(k\) | [-10, 10] |
| \(b\) | [0, 1] |
| \(\sigma_{\text{int}}\) | [0, 5] |
| \(\beta\) | [0, 5] |
| \(\gamma\) | [0, 300] |

\(^a\) \(k\) km s\(^{-1}\) Mpc\(^{-1}\). In the GRB alone cases, \(H_0\) is set to be 70 km s\(^{-1}\) Mpc\(^{-1}\), while in the \(H(z) +\) BAO case, the prior range is irrelevant (unbounded).

\(^b\) In the GRB alone cases, \(\Omega_b h^2\) is set to be 0.0245, i.e. \(\Omega_b = 0.05\).

\(^c\) In the GRB alone cases, \(\Omega_c \in [-0.05, 0.95]\) to ensure \(\Omega_{\text{tot}} \in [0, 1]\).

\(^d\) Note that \(k\), \(b\), and \(\sigma_{\text{int}}\) of MD-SGRBs are different from those of MD-LGRBs/GW-LGRBs, but with the same prior ranges.

\(^e\) \(b < 0\) values are possible for MD-SGRBs (due to fewer data points) but, as discussed below, requiring \(b \geq 0\) does not have significant consequences.

\(\Omega_0\) is set to be 0.245, i.e. \(\Omega_b = 0.05\).

\(\sigma_{\text{int}}\) is the intrinsic scatter parameter (which also contains the known systematic uncertainty).

For GRBs which obey the Dainotti correlation the luminosity of the plateau phase is (Dainotti et al. 2008, 2010, 2011)

\[ L_0 = \frac{4 \pi D_L^2 F_0}{(1 + z)^{1 - \beta}}. \]

where \(F_0\) is the GRB X-ray flux, \(\beta\) is the spectral index in the plateau phase, and \(D_L\) is the luminosity distance.

\(D_L\), as a function of redshift \(z\) and cosmological parameters \(p\), is given by

\[ H_0 \sqrt{\Omega_{\text{tot}}} D_L(z, p) = \frac{c(1 + z)}{\gamma} \left( \begin{array}{l} \sinh \left[ g(z, p) \right] \quad \text{if } \Omega_{\text{tot}} > 0, \\ g(z, p) \quad \text{if } \Omega_{\text{tot}} = 0, \\ \sin \left[ g(z, p) \right] \quad \text{if } \Omega_{\text{tot}} < 0, \end{array} \right. \]

where

\[ g(z, p) = H_0 \sqrt{\Omega_{\text{tot}}} \int_0^z \frac{dz'}{H(z', p)}, \]

\(c\) is the speed of light, and \(H(z, p)\) is the Hubble parameter that is described in Sec. 2 for each cosmological model.

For these GRBs the luminosity of the plateau phase \(L_0\) and the characteristic time scale \(t_\sigma\) are correlated through the Dainotti or luminosity-time relation

\[ y \equiv \log \left( \frac{L_0}{10^{44} \text{ erg s}^{-1}} \right) = k \log \frac{t_b}{10^7 (1 + z)^2} + b \equiv k x + b, \]

where \(\log L_{10}\) and the slope \(k\) and the intercept \(b\) are free parameters to be determined from the data.

We predict \(L_0\) as a function of cosmological parameters \(p\) at the redshift of each GRB by using eqs. (7), (8), and (10). We then compare predicted and measured values of \(L_0\) by using the natural log of the likelihood function (D’Agostini 2005)

\[ \ln L_{\text{GRB}} = -\frac{1}{2} \sum_{i=1}^N \ln \left( 2 \pi (\sigma_{\text{int}}^2 + \sigma_y^2 + k^2 \sigma_x^2) \right), \]

where

\[ \chi^2_{\text{GRB}} = \sum_{i=1}^N \left( \frac{y_i - k x_i - b}{\sigma_{\text{int}}^2 + \sigma_y^2 + k^2 \sigma_x^2} \right)^2. \]

Here \(N\) is the number of data points (e.g., for MD-LGRB \(N = 31\)), and \(\sigma_{\text{int}}\) is the intrinsic scatter parameter (which also contains the known systematic uncertainty).

For GRBs which obey the Amati correlation the rest frame isotropic radiated energy \(E_{\text{iso}}\) is

\[ E_{\text{iso}} = \frac{4 \pi D_L^2}{1 + z} S_{\text{bol}, o}, \]

where \(S_{\text{bol}, o}\) is the bolometric fluence.

For these GRBs the rest frame peak photon energy \(E_p\) and \(E_{\text{iso}}\) are correlated through the Amati (or \(E_p - E_{\text{iso}}\)) relation (Amati et al. 2008, 2009)

\[ \log E_{\text{iso}} = \gamma + \beta \log E_p, \]

where the intercept \(\gamma\) and the slope \(\beta\) are free parameters to be determined from the data. Note that the peak energy \(E_p = (1 + z)E_{p, \text{obs}}\) where \(E_{p, \text{obs}}\) is the observed peak energy.

We predict \(E_{\text{iso}}\) as a function of cosmological parameters \(p\) at the redshift of each GRB by using the natural log of the likelihood function (D’Agostini 2005)

\[ \ln L_{\text{A118}} = -\frac{1}{2} \sum_{i=1}^N \ln \left( 2 \pi (\sigma_{\text{int}}^2 + \sigma_y^2 + \beta^2 \sigma_x^2) \right). \]
where

\[ \chi^2_{\text{1D}} = \sum_{i=1}^{N} \left( \frac{(y_i' - \beta x_i' - \gamma)^2}{\sigma^2_{\text{int}} + \sigma^2_{\text{p}} + \beta^2 \sigma^2_{x_i'}} \right). \]

Here \( x' = \log(E_p/\text{keV}), \sigma_{y_i'} = \sigma_{E_p}/(E_{\text{p}} \ln 10), \) \( y' = \log(E_{\text{int}}/\text{erg}), \) and \( \sigma_{\text{int}} \) is the intrinsic scatter parameter, which also contains the unknown systematic uncertainty.

The \( H(z) + \text{BAO} \) data analyses follow the method described in Sec. 4 of Khadka & Ratra (2021a).

We maximize the likelihood function using the Markov chain Monte Carlo (MCMC) method as implemented in the MontePython code (Brinckmann & Lesgourgues 2019) and determine the best-fitting posterior mean values and the corresponding uncertainties for all free parameters. We assure convergence of the MCMC chains for each free parameter from the Gelman-Rubin criterion \((R - 1 < 0.05)\). Flat priors used for the free parameters are given in Table 2.

The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are used to compare the goodness of fit of models with different numbers of parameters. These are

\[ AIC = -2 \ln \mathcal{L}_{\text{max}} + 2n, \]

(17)

and

\[ BIC = -2 \ln \mathcal{L}_{\text{max}} + n \ln N. \]

(18)

In these equations, \( \mathcal{L}_{\text{max}} \) is the maximum value of the relevant likelihood function and \( n \) is the number of free parameters of the model under consideration.

5 RESULTS

5.1 Constraints from ML, MS, and GL data

In Table 3 we list Dainotti correlation parameters computed using the ML, GL, and MS data sets. These are computed in the flat \( \Lambda \)CDM model with \( \Omega_{\text{m}_0} = 0.3 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), the same model and parameter values used in Wang et al. (2021) and Hu et al. (2021). The first line of parameter values in each of the three subpanels of Table 3 is taken from these papers.\(^7\) To compare to these results, we used emcee (Foreman-Mackey et al. 2013) to compute the values listed in the second and third lines of each subpanel. Comparing the first and second lines in each subpanel, we find that they are consistent, except: i) for the GL case our \( b \) uncertainties are larger than those of Hu et al. (2021); and ii) for the MS case we have larger \( b \) and \( k \) error bars and a larger central value of \( \sigma_{\text{int}} \) than those of Hu et al. (2021), but they agree within 1\( \sigma \). In the third line of each subpanel we list results obtained assuming wider prior ranges of the parameters. We find that the ML results do not change, the GL results are shifted closer to those of Hu et al. (2021), except for the values of \( \sigma_{\text{int}} \), and the MS results are shifted away from those of Hu et al. (2021) with larger error bars, especially for \( \sigma_{\text{int}} \).

We now record and discuss results when these data sets are used to jointly constrain the Dainotti parameters and the cosmological parameters of the six spatially-flat and non-flat dark energy cosmological models. Figure 1 shows the flat \( \Lambda \)CDM Dainotti correlations for the ML, MS, GL, and MS + GL data sets. The unnormalized best-fitting results and the one-dimensional (1D) posterior mean values and uncertainties are reported in Tables 4 and 5, respectively. The corresponding posterior 1D probability distributions and two-dimensional (2D) confidence regions of these parameters are shown in Figs. 2–4, in blue (ML), gray (MS), green (GL), pink (ML + MS), violet (ML + GL), orange (MS + GL), and red \((H(z) + \text{BAO}, \text{as a baseline})\). Note that \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \Omega_{\text{m}_0} = 0.3 \) are applied in the GRB cases. ML, MS, and GL GRB data have almost cosmological-model independent Dainotti parameters. This means that it is not unreasonable to treat the ML, MS, and GL GRBs as standardizable candles, as was assumed in Wang et al. (2021) and Hu et al. (2021).

In the ML case (with subscript “ML” in the first line of Tables 4, 5, 7, and 8), the slope \( k \) ranges from a high of \(-0.996 \pm 0.097 \) (non-flat XCMD) to a low of \(-1.017 \pm 0.090 \) (flat XCMD), the intercept \( b \) ranges from a high of \(1.611^{+0.113}_{-0.277} \) (flat XCMD) to a low of \(1.448^{+0.129}_{-0.165} \) (non-flat \( \phi \)CDM), and the intrinsic scatter \( \sigma_{\text{int}} \) ranges from a high of \(0.306^{+0.036}_{-0.054} \) (flat XCMD) to a low of \(0.303^{+0.035}_{-0.052} \) (non-flat \( \phi \)CDM), with central values of each pair being 0.16\( \sigma \), 0.54\( \sigma \), and 0.05\( \sigma \) away from each other, respectively.

In the MS case (with subscript “MS” in the first line of Tables 4 and 5) with prior range of \( b \) in [0, 10], the slope \( k \) ranges from a high of \(-1.425^{+0.311}_{-0.222} \) (flat \( \phi \)CDM) to a low of \(-1.450^{+0.362}_{-0.258} \) (flat XCMD), the intercept \( b \) ranges from a high of \(0.497 \pm 0.458 \) (flat XCMD) to a low of \(0.352^{+0.331}_{-0.300} \) (non-

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Table 3. One-dimensional marginalized posterior means and 68.27% limits of the Dainotti correlation parameters for the ML, GL, and MS data sets using the flat XCMD model with \( \Omega_{\text{m}_0} = 0.3 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), and comparison with the results given in Wang et al. (2021) and Hu et al. (2021).

| Data set | Source | \( k \) | \( b \) | \( \sigma_{\text{int}} \) |
|----------|--------|--------|--------|------------------|
| ML       | a      | \(-1.022^{+0.009}_{-0.008} \) | \(1.73^{+0.07}_{-0.07} \) | 0.303^{+0.032}_{-0.050} |
|          | b      | \(-1.026 \pm 0.005 \) | \(1.726 \pm 0.074 \) | 0.303^{+0.032}_{-0.050} |
|          | c      | \(-1.026 \pm 0.006 \) | \(1.726 \pm 0.074 \) | 0.303^{+0.032}_{-0.050} |
| GL       | d      | \(-1.77^{+0.20}_{-0.30} \) | \(0.66^{+0.01}_{-0.01} \) | 0.42^{+0.08}_{-0.06} |
|          | b      | \(-1.75^{+0.20}_{-0.26} \) | \(0.64^{+0.01}_{-0.01} \) | 0.42^{+0.08}_{-0.06} |
|          | c      | \(-1.76^{+0.20}_{-0.25} \) | \(0.65^{+0.01}_{-0.01} \) | 0.43^{+0.08}_{-0.06} |
| MS       | d      | \(-1.38^{+0.15}_{-0.19} \) | \(0.33^{+0.17}_{-0.16} \) | 0.35^{+0.12}_{-0.10} |
|          | b      | \(-1.38^{+0.20}_{-0.19} \) | \(0.32^{+0.16}_{-0.15} \) | 0.42^{+0.08}_{-0.06} |
|          | c      | \(-1.39^{+0.24}_{-0.21} \) | \(0.35^{+0.15}_{-0.15} \) | 0.52^{+0.04}_{-0.04} |

\(^7\) Wang et al. (2021) do not list a value for \( \sigma_{\text{int}} \) in the ML case.
Table 4: Unmarginalized best-fitting parameter values for all models from various combinations of data.

| Model               | Data set | \( \Omega_c h^2 \) | \( \Omega_b h^2 \) | \( \Omega_m h^2 \) | \( \sigma_8 \) | \( h \) | \( m_0 \) | \( \alpha \) | \( \Delta AIC \) | \( \Delta BIC \) | \( -2 \ln \mathcal{L}_{\text{min}} \) | \( AIC \) | \( BIC \) | \( \Delta AIC \) | \( \Delta BIC \) |
|---------------------|----------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Flat CDM            |          |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| ML                 | 0.0245   | 0.1197               | 0.298                | –                     | –                     | –                     | 69.13                | –                     | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| MS                 | 0.0245   | 0.1197               | 0.298                | –                     | –                     | –                     | 69.13                | –                     | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| Flat CDM + GL      |          |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| ML                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| MS                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| ML + MS            | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| Non-flat CDM       |          |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| ML                 | 0.0245   | 0.2000               | 0.180                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| MS                 | 0.0245   | 0.2000               | 0.180                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| ML + MS            | 0.0245   | 0.2000               | 0.180                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| H(z) + BAO         | 0.0245   | 0.1094               | 0.209                | –                     | –                     | –                     | 66.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| Flat CDM + GL      |          |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| ML                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| MS                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| FLat CDM + GL      |          |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| ML                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| MS                 | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |
| ML + MS            | 0.0245   | 0.4505               | 0.898                | –                     | –                     | –                     | 68.8                | 0.075                | –                     | –                     | –                     | –                     | –                     | –                     | –                     |

*a km s^{-1} Mpc^{-1}. In the GRB only cases, \( H_0 \) is set to be 70 km s^{-1} Mpc^{-1}.**
| Model   | Data set | $\Omega_m$ $^2$ | $\Omega_{\Lambda}$ $^2$ | $\Omega_\nu$ | $H_0$ $^\dagger$ | $\sigma_{H_0}$ | $h$ | $\sigma_h$ | $\sigma_{\Omega_m}$ | $\sigma_{\Omega_\Lambda}$ | $\sigma_{\Omega_\nu}$ | $\sigma_{\sigma_{H_0}}$ | $\sigma_{\sigma_h}$ | $\sigma_{\sigma_{\Omega_m}}}$ | $\sigma_{\sigma_{\Omega_\Lambda}}}$ | $\sigma_{\sigma_{\Omega_\nu}}$ | $\sigma_{\sigma_{\sigma_{H_0}}}$ | $\sigma_{\sigma_{h}}$ | $\sigma_{\sigma_{\sigma_{\Omega_m}}}$ | $\sigma_{\sigma_{\sigma_{\Omega_\Lambda}}}$ | $\sigma_{\sigma_{\sigma_{\Omega_\nu}}}$ |
|---------|----------|----------------|-----------------|----------|----------------|----------------|----|-----------|------------------|------------------|-------------------|------------------|----------------|------------------|------------------|------------------|----------------|-----------------|----------------|-----------------|-----------------|
| H(z) + BAO | 0.0231 ± 0.0029 | 0.1883 ± 0.0014 | 0.297 ± 0.03 | 0.227 ± 0.012 | > 0.108 | 0.030 ± 0.003 | 1.52 ± 0.08 | 0.07 ± 0.05 | 0.067 ± 0.001 | 1.97 ± 0.008 | 1.97 ± 0.008 | 0.055 ± 0.001 | 0.04 ± 0.08 | 1.45 ± 0.06 | – | – | – | – | – | – | – |
| ML + MS | 0.520 ± 0.170 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | – | – | – |
| Flat ΩCDM | 0.155 ± 0.06 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | > 0.020 | – | – | – |
| α H | 0.0029 ± 0.0001 | 1.04 ± 0.01 | 1.06 ± 0.01 | 1.07 ± 0.01 | 1.08 ± 0.01 | 1.09 ± 0.01 | 1.10 ± 0.01 | 1.11 ± 0.01 | 1.12 ± 0.01 | 1.13 ± 0.01 | 1.14 ± 0.01 | 1.15 ± 0.01 | 1.16 ± 0.01 | 1.17 ± 0.01 | 1.18 ± 0.01 | 1.19 ± 0.01 | 1.20 ± 0.01 | 1.21 ± 0.01 | 1.22 ± 0.01 | 1.23 ± 0.01 | 1.24 ± 0.01 | 1.25 ± 0.01 | 1.26 ± 0.01 | 1.27 ± 0.01 |
| 0.95 ± 0.01 | 0.96 ± 0.01 | 0.97 ± 0.01 | 0.98 ± 0.01 | 0.99 ± 0.01 | 1.00 ± 0.01 |

$^\dagger$ In the GRBs only cases, $H_0$ is set to be 70 km s$^{-1}$ Mpc$^{-1}$. 

Table 5: One-dimensional marginalized posterior mean values and uncertainties (1σ error bars or 2σ limits) of the parameters for all models from various combinations of data.
flat $\phi$CDM), and the intrinsic scatter $\sigma_{\text{int}}$ ranges from a high of $0.73^{+0.031}_{-0.056}$ (non-flat XCDM) to a low of $0.589^{+0.023}_{-0.045}$ (non-flat $\phi$CDM), with central values of each pair being $0.66\sigma$, $0.5\sigma$, and $0.05\sigma$ away from each other, respectively.  

In the GL case (with subscript “GL” in the first line of Tables 4, 5, 7, and 8), the slope $k$ ranges from a high of $-1.53^{+0.259}_{-0.060}$ (non-flat XCDM) to a low of $-1.720^{+0.000}_{-0.260}$.  

Figure 1, panel (c), shows that the GL and MS GRBs obey the same Dainotti correlation in the flat $\Lambda$CDM model.  

Table 6 shows that the differences between the GL and MS Dainotti parameters in all six cosmological models are within $1\sigma$. GL and MS GRBs however follow a different Dainotti correlation than the ML GRBs. Given the similarity of the GL and MS Dainotti correlations, it is unclear if this is more than just a coincidence, as the plateau phases in the two cases are dominated by GW emission (GL) and MD radiation (MS), respectively.

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8 Note, however, that the lower error bars of $b$ and $\sigma_{\text{int}}$ are considerably larger than the upper ones due to cut-off prior ranges of the former and skewed distributions of the latter. Therefore here we also consider the MS case with wider prior range of $b \in [-10, 10]$, which are not listed in the tables due to their insignificant differences. Because the lowest and highest values of $k, b,$ and $\sigma_{\text{int}}$ from these two MS cases differ from each other at only $0.22\sigma, 0.56\sigma$, and $0.37\sigma$, respectively, and the constraints of the cosmological parameters are also within $1\sigma$ range, the prior range of $b \in [0, 10]$ is an acceptable choice.

9 It is unclear if this is more than just a coincidence, as the plateau phases in the two cases are dominated by GW emission (GL) and MD radiation (MS), respectively.
correlation parameters, it is not unreasonable to use just three (not six) correlation parameters in joint analyses of MS and GL data (with subscript “MS+GL” in the first line of Tables 4 and 5). In this case, the slope \( k \) ranges from a high of \(-1.363 \pm 0.175 \) (non-flat XCDM) to a low of \(-1.577 \pm 0.155 \) (flat ΛCDM), the intercept \( b \) ranges from a high of \(0.470_{-0.206}^{+0.108} \) (flat XCDM) to a low of \(0.337_{-0.127}^{+0.110} \) (non-flat ΛCDM), and the intrinsic scatter \( σ_{\text{int}} \) ranges from a high of \(0.412_{-0.079}^{+0.052} \) (flat ΛCDM) to a low of \(0.357_{-0.075}^{+0.040} \) (non-flat ΛCDM), with central values of each pair being 0.92σ, 0.57σ, and 0.60σ away from each other, respectively. In contrast to the GL case, the MS + GL case tightens the constraints a little bit, with smaller error bars, and prefers lower values of \( b \) and \( σ_{\text{int}} \), and higher values of \( k \). When we jointly analyze ML + GL and ML + MS, the constraints on the Dainotti parameters follow the same pattern as that of MS + GL against GL.

The constraints on the cosmological parameters are very loose for all of these cases. In the flat ΛCDM model, the highest 2σ lower limit of \( Ω_{m0} \) among these cases is \( Ω_{m0} > 0.294 \) of the ML + GL data. In the non-flat ΛCDM model, the highest 2σ lower limit of \( Ω_{m0} \) is \( Ω_{m0} > 0.391 \) of the MS + GL case, which is inconsistent with that of the \( H(z) + BAO \) case. The MS data favor open hypersurfaces while all other cases favor closed hypersurfaces, with the favored spatial geometries for GL, ML + GL, and MS + GL data being more than 1σ (or even 2σ) away from flat geometry. In the flat XCDM parametrization, the highest 2σ lower limit of \( Ω_{m0} \) is \( Ω_{m0} > 0.192 \) for the MS + GL data, and the constraints on the X-equation of state parameter \( w_X \) are very loose, with the highest 1σ upper limit being 0.111 for the ML case. In the non-flat XCDM parametrization, the highest 2σ lower limit of \( Ω_{m0} \) is \( Ω_{m0} > 0.268 \) for the MS + GL data, and the constraints on \( w_X \) are very loose, with the highest 1σ upper limit being 0.238 for the MS + GL data. The favored spatial geometries for these cases follow the same pattern as that for non-flat ΛCDM, but with larger upper limits of \( Ω_{m0} \) except for the MS data. In the flat φCDM model, the highest 2σ lower limit of \( Ω_{m0} \) is \( Ω_{m0} > 0.235 \) for the ML + GL data. In the non-flat φCDM model, the highest 2σ lower limit of \( Ω_{m0} \) is \( Ω_{m0} > 0.340 \) for the ML + GL case, which is inconsistent with that of the \( H(z) + BAO \) data. Except for the MS case, closed spatial hypersurfaces are favored, but only in the ML + GL case is flat geometry slightly more than 1σ away. There are no constraints on \( α \) from these GRB data.

In the ΛCDM and XCDM cases, all GRB data combinations more favor currently accelerating cosmological expansion. They however more favor currently decelerating cosmological expansion in the φCDM models, in the \( Ω_{m0} - α \) and \( Ω_{m0} - Ω_0 \) parameter subspaces.

From the \( AIC \) and \( BIC \) values we compute \( ΔAIC \) and \( ΔBIC \) values with respect to the flat ΛCDM model. These are listed in the last two columns of Table 4. In the ML case, flat ΛCDM is the most favored model but there is only weak or positive evidence against any other model. In the MS case non-flat ΛCDM model is the most favored model and, except for non-flat XCDM (with positive evidence against it), the other models are very strongly disfavored. In the GL case, non-flat ΛCDM is again the most favored model, while the evidence against the others are mostly positive, except for non-flat φCDM (with strong \( BIC \) evidence against it). In the MS + GL case, similar to the MS case, non-flat ΛCDM is the most favored model and, except for non-flat XCDM (with weak \( AIC \) and positive \( BIC \) evidence against it), the others are strongly disfavored. In the ML + GL case, non-flat ΛCDM is the most favored model but, except for flat ΛCDM (with strong \( BIC \) evidence against it), the evidence against the other models is either weak or positive. In the ML + MS case, the best candidates are non-flat XCDM based on \( AIC \) and flat ΛCDM based on \( BIC \), while the evidence against the other models is either weak or positive.

### Table 6. MD-SGRB and GW-LGRB data \( L_{\text{b}} - t_{\text{b}} \) correlation parameters and \( σ_{\text{int}} \) differences.

| Model           | \( Δσ_{\text{int}} \) | \( Δk \) | \( Δb \) |
|-----------------|----------------------|--------|--------|
| Flat ΛCDM       | 0.48σ                | 0.31σ  | 0.80σ  |
| Non-flat ΛCDM   | 0.50σ                | 0.31σ  | 0.01σ  |
| Flat XCDM       | 0.49σ                | 0.79σ  | 0.22σ  |
| Non-flat XCDM   | 0.55σ                | 0.26σ  | 0.07σ  |
| Flat φCDM       | 0.37σ                | 0.92σ  | 0.58σ  |
| Non-flat φCDM   | 0.36σ                | 0.48σ  | 0.36σ  |

5.2 Constraints from A118, A115 (and jointly with ML), and A115’ (and jointly with GL) data

The A118 data set was previously studied by Khadka et al. (2021b). Here we analyze it along with the truncated A115 and A115’ data sets, which are also used in joint analyses with the ML and GL data sets. The constraints from these data sets on the GRB correlation parameters and on the cosmological model parameters are presented in Tables 7 and 8. The corresponding posterior 1D probability distributions and 2D confidence regions of these parameters are shown in Figs. 5 and 6, in gray (A118), red (A115 and A115’), green (ML and GL), and purple (ML + A115 and GL + A115’). Note that these analyses assume \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( Ω_b = 0.05 \).

The constraints from A115 data and from ML data, and from A115’ data and from GL data, are not mutually inconsistent, so it is not unreasonable to examine joint ML + A115 and GL + A115’ constraints. The ML data have the smallest intrinsic dispersion, \( ∼ 0.30 – 0.31 \), with A115, A115’, and GL having larger intrinsic dispersion, \( ∼ 0.40 – 0.45 \).

The constraints on the Amati parameters are quite cosmological-model-independent for these GRB data sets. In the A118 case, the slope \( β \) ranges from a high of 1.121±0.091 (non-flat XCDM) to a low of 1.106±0.080 (flat XCDM), the intercept \( γ \) ranges from a high of 50.15±0.26 (flat XCDM) to a low of 50.01±0.26 (non-flat ΛCDM), and the intrinsic scatter \( σ_{\text{int}} \) ranges from a high of 0.412±0.028 (flat XCDM) to a low of 0.411±0.027 (flat φCDM), with central values of each pair being 0.12σ, 0.35σ, and 0.02σ away from each other, respectively.

In the A115 case, the slope \( β \) ranges from a high of 1.124±0.092 (non-flat ΛCDM) to a low of 1.109±0.092 (flat XCDM), the intercept \( γ \) ranges from a high of 50.14±0.27 (flat XCDM) to a low of 50.00±0.27 (non-flat ΛCDM), and the intrinsic scatter \( σ_{\text{int}} \) ranges from a high of 0.418±0.028 (non-flat ΛCDM) to a low of 0.416±0.034 (flat φCDM), with...
Figure 2. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours from MD-LGRB (blue), MD-SGRB (gray), MD-LGRB + MD-SGRB (pink), and $H(z) + \text{BAO}$ (red) data for all six models. The zero-acceleration lines are shown as black dashed lines, which divide the parameter space into regions associated with currently-accelerating and currently-decelerating cosmological expansion. In the non-flat XCDM and non-flat $\phi$CDM cases, the zero-acceleration lines are computed for the third cosmological parameter set to the $H(z) + \text{BAO}$ data best-fitting values listed in Table 4. The crimson dash-dot lines represent flat hypersurfaces, with closed spatial hypersurfaces either below or to the left. The magenta lines represent $w_x = -1$, i.e. flat or non-flat XCDM models. The $\alpha = 0$ axes correspond to flat and non-flat $\Lambda$CDM models in panels (e) and (f), respectively.
Figure 3. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours from MD-LGRB (blue), GW-LGRB (green), MD-LGRB + GW-LGRB (violet), and $H(z)$ + BAO (red) data for all six models. The zero-acceleration lines are shown as black dashed lines, which divide the parameter space into regions associated with currently-accelerating and currently-decelerating cosmological expansion. In the non-flat XCDM and non-flat $\phi$CDM cases, the zero-acceleration lines are computed for the third cosmological parameter set to the $H(z)$ + BAO data best-fitting values listed in Table 4. The crimson dash-dot lines represent flat hypersurfaces, with closed spatial hypersurfaces either below or to the left. The magenta lines represent $w_\infty = -1$, i.e. flat or non-flat $\Lambda$CDM models. The $\alpha = 0$ axes correspond to flat and non-flat $\Lambda$CDM models in panels (e) and (f), respectively.
Figure 4. One-dimensional likelihoods and $1\sigma$, $2\sigma$, and $3\sigma$ two-dimensional likelihood confidence contours from MD-SGRB (gray), GW-LGRB (green), MD-SGRB + GW-LGRB (orange), and $H(z) + $ BAO (red) data for all six models, without subscripts on $\sigma_{\text{flat}}$, $k$, and $b$. The zero-acceleration lines are shown as black dashed lines, which divide the parameter space into regions associated with currently-accelerating and currently-decelerating cosmological expansion. In the non-flat XCDM and non-flat $\phi$CDM cases, the zero-acceleration lines are computed for the third cosmological parameter set to the $H(z) + $ BAO data best-fitting values listed in Table 4. The crimson dash-dot lines represent flat hypersurfaces, with closed spatial hypersurfaces either below or to the left. The magenta lines represent $w_X = -1$, i.e. flat or non-flat $\Lambda$CDM models. The $\alpha = 0$ axes correspond to flat and non-flat $\Lambda$CDM models in panels (e) and (f), respectively.
| Model                  | Data set  | GL + A115 | GL + A118 | ML + A118 | GL  | ML  | Flat XCDM | GRB cosmological parameter constraints |
|------------------------|-----------|-----------|-----------|-----------|-----|-----|-----------|---------------------------------------|
|                        |           |           |           |           |     |     |           |                                       |
| A118                   | 0.4089    | -        | 0.4172    | 0.4550    | 0.961 | 0.97 | 0.4529    | 0.4622                               |
|                        | 0.884     | -        | 0.904     | 0.97    | -    | -   | 0.962     | 0.996                               |
| A115                   | 0.4647    | -        | 0.962     | 0.996    | 0.962 | 0.97 | 0.4641    | 0.997                               |
|                        | 0.97    | -        | 0.97    | -       | -    | -   | 0.97    | -                                   |
| GL + A115              | 0.4089    | -        | 0.4172    | 0.4550    | 0.961 | 0.97 | 0.4529    | 0.4622                               |
|                        | 0.884     | -        | 0.904     | 0.97    | -    | -   | 0.962     | 0.996                               |
| GL + A118              | 0.4647    | -        | 0.962     | 0.996    | 0.962 | 0.97 | 0.4641    | 0.997                               |
|                        | 0.97    | -        | 0.97    | -       | -    | -   | 0.97    | -                                   |
| ML + A115              | 0.4089    | -        | 0.4172    | 0.4550    | 0.961 | 0.97 | 0.4529    | 0.4622                               |
|                        | 0.884     | -        | 0.904     | 0.97    | -    | -   | 0.962     | 0.996                               |
| GL  | 0.4647    | -        | 0.962     | 0.996    | 0.962 | 0.97 | 0.4641    | 0.997                               |
|                        | 0.97    | -        | 0.97    | -       | -    | -   | 0.97    | -                                   |
| ML                   | 0.4089    | -        | 0.4172    | 0.4550    | 0.961 | 0.97 | 0.4529    | 0.4622                               |
|                        | 0.884     | -        | 0.904     | 0.97    | -    | -   | 0.962     | 0.996                               |
| Flat XCDM             | 0.4089    | -        | 0.4172    | 0.4550    | 0.961 | 0.97 | 0.4529    | 0.4622                               |
|                        | 0.884     | -        | 0.904     | 0.97    | -    | -   | 0.962     | 0.996                               |
| GRB cosmological parameter constraints |           |           |           |           |     |     |           |                                       |
|                       |           |           |           |           |     |     |           |                                       |
|                        |           |           |           |           |     |     |           |                                       |

Table 7: Unmarginalized best-fitting parameter values for all models from various combinations of data.

*In these GRB cases, Ω and H0 are set to 0.05 and 70 km s⁻¹ Mpc⁻¹, respectively.
Table 8: One-dimensional marginalized posterior mean values and uncertainties ($\pm 1\sigma$ error bars or $2\sigma$ limits) of the parameters for all models from various combinations of data.

| Model       | Data set       | $\Omega_{b}$ | $\Omega_{c}$ | $w_X$ | $\alpha$ | $\sigma_{\alpha,b}$ | $h_X$ | $\sigma_{h_X,b}$ | $\gamma$ | $\sigma_{\gamma}$ | $\beta$ | $\sigma_{\beta}$ |
|-------------|----------------|---------------|---------------|-------|-----------|----------------------|-------|------------------|-----------|------------------|--------|-----------------|
| Flat CDM    | ML + A115      | $0.287$       | $0.964^{+0.026}_{-0.025}$ |       | $0.301^{+0.025}_{-0.023}$ | $1.478^{+0.121}_{-0.146}$ | $-1.000^{+0.006}_{-0.006}$ | $0.094^{+0.027}_{-0.032}$ | $50.01^{+0.26}_{-0.24}$ | $1.16^{+0.09}_{-0.09}$ |       | $0.034^{+0.004}_{-0.004}$ | $0.556^{+0.127}_{-0.296}$ | $-1.706^{+0.215}_{-0.260}$ |
|             | GL             | $0.290$       | $0.909^{+0.014}_{-0.015}$   |       | $0.304^{+0.025}_{-0.024}$ | $1.494^{+0.130}_{-0.153}$ | $-1.012^{+0.008}_{-0.008}$ | $0.074^{+0.027}_{-0.032}$ | $50.02^{+0.25}_{-0.25}$ | $1.13^{+0.08}_{-0.09}$ |       | $0.033^{+0.004}_{-0.004}$ | $0.512^{+0.127}_{-0.250}$ | $-1.698^{+0.208}_{-0.239}$ |
|             | GL + A115      | $0.381$       | $0.214^{+0.014}_{-0.015}$   |       | $0.304^{+0.025}_{-0.024}$ | $1.494^{+0.130}_{-0.153}$ | $-1.012^{+0.008}_{-0.008}$ | $0.074^{+0.027}_{-0.032}$ | $50.02^{+0.25}_{-0.25}$ | $1.13^{+0.08}_{-0.09}$ |       | $0.033^{+0.004}_{-0.004}$ | $0.512^{+0.127}_{-0.250}$ | $-1.698^{+0.208}_{-0.239}$ |
| Non-flat CDM| ML + A115      | $0.346$       | $0.352^{+0.046}_{-0.045}$   |       | $0.304^{+0.025}_{-0.024}$ | $1.494^{+0.130}_{-0.153}$ | $-1.012^{+0.008}_{-0.008}$ | $0.074^{+0.027}_{-0.032}$ | $50.02^{+0.25}_{-0.25}$ | $1.13^{+0.08}_{-0.09}$ |       | $0.033^{+0.004}_{-0.004}$ | $0.512^{+0.127}_{-0.250}$ | $-1.698^{+0.208}_{-0.239}$ |
|             | GL             | $0.290$       | $0.909^{+0.014}_{-0.015}$   |       | $0.304^{+0.025}_{-0.024}$ | $1.494^{+0.130}_{-0.153}$ | $-1.012^{+0.008}_{-0.008}$ | $0.074^{+0.027}_{-0.032}$ | $50.02^{+0.25}_{-0.25}$ | $1.13^{+0.08}_{-0.09}$ |       | $0.033^{+0.004}_{-0.004}$ | $0.512^{+0.127}_{-0.250}$ | $-1.698^{+0.208}_{-0.239}$ |
|             | GL + A115      | $0.381$       | $0.214^{+0.014}_{-0.015}$   |       | $0.304^{+0.025}_{-0.024}$ | $1.494^{+0.130}_{-0.153}$ | $-1.012^{+0.008}_{-0.008}$ | $0.074^{+0.027}_{-0.032}$ | $50.02^{+0.25}_{-0.25}$ | $1.13^{+0.08}_{-0.09}$ |       | $0.033^{+0.004}_{-0.004}$ | $0.512^{+0.127}_{-0.250}$ | $-1.698^{+0.208}_{-0.239}$ |
| A118        | $> 0.247$      | $> 0.188$     | $> 0.181$      |       | $> 0.030^{+0.013}_{-0.010}$ | $1.552^{+0.118}_{-0.109}$ | $-0.017^{+0.007}_{-0.010}$ | $0.143^{+0.027}_{-0.031}$ | $50.09^{+0.26}_{-0.24}$ | $1.11^{+0.09}_{-0.09}$ |       | $0.034^{+0.004}_{-0.004}$ | $0.556^{+0.127}_{-0.296}$ | $-1.720^{+0.219}_{-0.173}$ |
| A115        | $0.630^{+0.315}_{-0.155}$ | $> 0.188$ | $> 0.181$ |       | $> 0.030^{+0.013}_{-0.010}$ | $1.552^{+0.118}_{-0.109}$ | $-0.017^{+0.007}_{-0.010}$ | $0.143^{+0.027}_{-0.031}$ | $50.09^{+0.26}_{-0.24}$ | $1.11^{+0.09}_{-0.09}$ |       | $0.034^{+0.004}_{-0.004}$ | $0.556^{+0.127}_{-0.296}$ | $-1.720^{+0.219}_{-0.173}$ |

* In these GRB cases, $\Omega_b$ and $H_0$ are set to be 0.05 and 70 km s$^{-1}$ Mpc$^{-1}$, respectively.
central values of each pair being 
0.12σ, 0.34σ, and 0.05σ away from each other, respectively.

In the A115 case, the slope β ranges from a high of 1.117 ± 0.090 (non-flat ΛCDM) to a low of 1.103 ± 0.092 (flat XCDM), the intercept γ ranges from a high of 50.15 ± 0.27 (flat XCDM) to a low of 50.02 ± 0.26 (non-flat ΛCDM), and the intrinsic scatter σint ranges from a high of 0.415 ± 0.034 (non-flat XCDM) to a low of 0.414 ± 0.034 (the others), with central values of each pair being 0.11σ, 0.33σ, and 0.02σ away from each other, respectively.

The lowest and highest values of β, γ, and σint from the A118, A115, and A115+ cases differ from each other at 0.16σ, 0.37σ, and 0.16σ, respectively. This implies that excluding three GRBs from A118 does not significantly affect the constraints on the Amati parameters.

In the joint analysis of ML and A115 (ML + A115) data, β ranges from a high of 1.113 ± 0.091 (non-flat ΛCDM) to a low of 1.106 ± 0.090 (non-flat φCDM), γ ranges from a high of 50.13 ± 0.30 (flat XCDM) to a low of 50.03 ± 0.25 (flat and non-flat φCDM), and σint ranges from a high of 0.417 ± 0.034 (flat XCDM) to a low of 0.416 ± 0.034 (flat ΛCDM) and flat and non-flat φCDM), with central values of each pair being 0.05σ, 0.26σ, and 0.02σ away from each other, respectively; also k ranges from a high of −1.012 ± 0.088 (flat XCDM) to a low of −1.019 ± 0.091 (non-flat ΛCDM), b ranges from a high of 1.503 ± 0.226 (flat XCDM) to a low of 1.453 ± 0.135 (non-flat φCDM), and σint ranges from a high of 0.304 ± 0.052 (non-flat ΛCDM) to a low of 0.300 ± 0.051 (flat XCDM), with central values of each pair being 0.06σ, 0.44σ, and 0.06σ away from each other, respectively. The lowest and highest values of β, γ, and σint from the A115 and ML + A115 cases differ from each other at 0.14σ, 0.32σ, and 0.05σ, respectively; also those of k, b, and σint differ from each other at 0.17σ, 0.53σ, and 0.09σ, respectively.

In the joint analysis of GL and A115+ (GL + A115+) data, β ranges from a high of 1.104 ± 0.089 (flat ΛCDM) to a low of 1.096 ± 0.089 (non-flat φCDM), γ ranges from a high of 50.14 ± 0.26 (flat XCDM) to a low of 50.04 ± 0.25 (non-flat φCDM), and σint ranges from a high of 0.415 ± 0.027 (non-flat ΛCDM) to a low of 0.413 ± 0.027 (flat XCDM), with central values of each pair being 0.06σ, 0.26σ, and 0.05σ away from each other, respectively; also k ranges from a high of −1.682 ± 0.208 (non-flat XCDM) to a low of −1.705 ± 0.210 (flat ΛCDM), b ranges from a high of 0.51 ± 0.113 (flat XCDM) to a low of 0.401 ± 0.125 (non-flat φCDM), and σint ranges from a high of 0.423 ± 0.056 (flat XCDM) to a low of 0.416 ± 0.058 (non-flat ΛCDM), with central values of each pair being 0.08σ, 0.46σ, and 0.07σ away from each other, respectively. The lowest and highest values of β, γ, and σint from the A115+ and GL + A115+ cases differ from each other at 0.17σ, 0.30σ, and 0.05σ, respectively; also those of k, b, and σint differ from each other at 0.52σ, 0.47σ, and 0.20σ, respectively.

Judging from the constraints on the parameters of the Amati and Dainotti correlations, those from the joint analyses do not deviate much from the individual cases. We next focus on constraints on cosmological model parameters.

Similar to GRB data from Sec. 5.1, these data more favor currently accelerating cosmological expansion in the ΛCDM and XCDM cases, but also more favor currently de-
Figure 5. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours from MD-LGRB (green), A118 (gray), A115 (red), and MD-LGRB + A115 (purple) data for all six models. The zero-acceleration lines are shown as black dashed lines, which divide the parameter space into regions associated with currently-accelerating and currently-decelerating cosmological expansion. In the non-flat XCDM and non-flat φCDM cases, the zero-acceleration lines are computed for the third cosmological parameter set to 0.5, i.e. flat or non-flat ΛCDM models.
Figure 6. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours from GW-LGRB (green), A118 (gray), A115 (red), and GW-LGRB + A115′ (purple) data for all six models. The zero-acceleration lines are shown as black dashed lines, which divide the parameter space into regions associated with currently-accelerating and currently-decelerating cosmological expansion. In the non-flat XCDM and non-flat φCDM cases, the zero-acceleration lines are computed for the third cosmological parameter set to the $H(z) = BAO$ data best-fitting values listed in Table 7. The crimson dash-dot lines represent flat hypersurfaces, with closed spatial hypersurfaces either below or to the left. The magenta lines represent $w_X = -1$, i.e. flat or non-flat ΛCDM models. The $\alpha = 0$ axes correspond to flat and non-flat ΛCDM models in panels (e) and (f), respectively.
6 CONCLUSION

We have used six different cosmological models in analyses of the three (ML, MS, and GL) Dainotti \((L_\text{iso} - t_\text{iso})\) correlation GRB data sets compiled by Wang et al. (2021) and Hu et al. (2021). We find for each data set, as well as the MS + GL, ML + GL, and ML + MS combinations, that the GRB correlation parameters are independent of cosmological model. Our results thus indicate that these GRBs are standardizable through the Dainotti correlation and so can be used to constrain cosmological parameters, justifying the assumption made by Wang et al. (2021) and Hu et al. (2021). These results also mean that the circularity problem does not affect cosmological parameter constraints derived from these GRB data.

In contrast to Wang et al. (2021) and Hu et al. (2021) we do not use \(H(z)\) data to calibrate these GRB data, instead we use these data to derive GRB only cosmological constraints. We find that ML, MS, GL, MS + GL, ML + GL, and ML + MS GRBs provide only weak restrictions on cosmological parameters.

We have also used the more-restrictive ML and GL Dainotti data sets in joint analyses with the largest available reliable compilation of Amati \((E_\text{p} - E_{\text{iso}})\) correlation A118 GRB data (Khadka et al. 2021b), but excluding three overlapping GRBs from the A118 data in the joint analyses. While the joint analyses do result in slightly tighter constraints, typically with larger lower limits on \(\Omega_{\text{tot}}\) than those from the ML, GL, or A118 data alone, the improvements relative to the A118 data constraints are not significant.

Current GRB data provide quite weak constraints on cosmological parameters but do favor currently accelerated cosmological expansion in the \(\Lambda\)CDM models and the XCDM parametrizations. We hope that in the near future there will be more and better-quality GRB measurements that will result in more restrictive GRB cosmological constraints. GRBs probe a very wide range of cosmological redshift space, a significant part of which is as yet unprobed, so it is worth putting effort into further developing GRB cosmological constraints.

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DATA AVAILABILITY

MD-LGRB data are available in Wang et al. (2021) and MD-SGRB and GW-LGRB data are available in Hu et al. (2021).

REFERENCES

Amati L., Guidorzi C., Frontera F., Delia Valle M., Finelli F., Landi R., Montanari E., 2008, MNRAS, 391, 577
Amati L., Frontera F., Guidorzi C., 2009, A&A, 508, 173
Amati L., D’Agostino R., Luongo O., Muccino M., Tantalo M., 2019, MNRAS, 486, L46
Arjona R., Nesseris S., 2021, Phys. Rev. D, 103, 103539
Brinckmann T., Lengourgues J., 2019, Physics of the Dark Universe, 24, 100260
Cao S., Biesiada M., Jackson J., Zheng X., Zhao Y., Zhu Z.-H., 2017, J. Cosmology Astropart. Phys., 2, 012
Cao S., Ryan J., Ratra B., 2020, MNRAS, 497, 3191
Cao S., Ryan J., Ratra B., 2021a, MNRAS, 505, 1520
Cao S., Ryan J., Ratra N., Ratra B., 2021b, MNRAS, 501, 1520
Cao S., Ryan J., Ratra B., 2021c, MNRAS, 504, 300
Cardone V. F., Dainotti M. G., Capozziello S., Willingale R., 2010, MNRAS, 408, 1181
Chávez R., Terlevich R., Terlevich E., Bresolin F., Melnick J., Pilionis M., Basialkos S., 2014, MNRAS, 442, 3565
Chen Y., Ratra B., Biesiada M., Li S., Zhu Z.-H., 2016, ApJ, 829, 61
Chen Y., Kumar S., Ratra B., 2017, ApJ, 835, 86
D’Agostino G., 2005, preprint, (arXiv:physics/0511182)
DES Collaboration 2019, Phys. Rev. D, 99, 123505
Dainotti M. G., Del Vecchio R., 2017, New Astron. Rev., 77, 23
Dainotti M. G., Cardone V. F., Capozziello S., 2008, Monthly Notices of the Royal Astronomical Society: Letters, 391, L79
Dainotti M. G., Willingale R., Capozziello S., Cardone V. F., Ostrowski M., 2010, The Astrophysical Journal, 722, L215
Dainotti M. G., Cardone V. F., Capozziello S., Ostrowski M., Willingale R., 2011, The Astrophysical Journal, 730, 135
Dainotti M. G., Cardone V. F., Piedipalumbo E., Capozziello S., 2013a, MNRAS, 436, 82
Dainotti M. G., Petrovian S., Singal J., Ostrowski M., 2013b, ApJ, 774, 157
Dainotti M. G., Nagataki S., Maeda K., Postnikov S., Pian E., 2017, A&A, 600, A98
de Cruz Perez J., Sola Peracaula J., Gomez-Valent A., Moreno-Pulido C., 2021, preprint, (arXiv:2110.07569)
Demianski M., Piedipalumbo E., Sawant D., Amati L., 2021, MNRAS, 506, 903
Dhawan S., Alsing J., Vagnozzi S., 2021, MNRAS, 506, L1
Di Valentino E., et al., 2021a, Classical and Quantum Gravity, 38, 153001
Di Valentino E., Melchiorri A., Silk J., 2021b, ApJ, 908, L9
eBOSS Collaboration 2021, Phys. Rev. D, 103, 083533
Efstathiou G., Gratton S., 2020, MNRAS, 496, L91
Fana Dirirsa F., et al., 2019, ApJ, 887, 13
Farooq O., Ranjeet Madiyar F., Crandall S., Ratra B., 2017, ApJ, 835, 26
Foreman-Mackey D., Hogg D. W., Lang D., Goodman J., 2013, PASP, 125, 306
González-Morán A. L., et al., 2019, MNRAS, 487, 4669
González-Morán A. L., et al., 2021, MNRAS, 506, 1181
Handley W., 2019, Phys. Rev. D, 100, 123517
Khadka N., Arjona R., Nesseris S., 2021, Phys. Rev. D, 103, 123517
Khadka N., Biesiada M., Jackson J., Zheng X., Zhao Y., Zhu Z.-H., 2017, J. Cosmology Astropart. Phys., 2, 012
Khadka N., Zajaček M., Martinez-Aldama M. L., Czerny B., Ratra B., 2021a, MNRAS, 509, 4722
Khadka N., Luongo O., Muccino M., Ratra B., 2021b, J. Cosmology Astropart. Phys., 2021, 042
KiDS Collaboration 2021, A&A, 649, A88
Li E.-K., Du M., Xu L., 2020, MNRAS, 491, 4960
Li X., Keeley R. E., Shafieloo A., Zheng X., Cao S., Biesiada M., Zhu Z.-H., 2021, MNRAS, 507, 919
