Shape-dependent Depinning of a Domain Wall by a Magnetic Field and a Spin-Polarized Current

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The effect of sample shape on the depinning of the domain wall (DW) driven by an applied magnetic field or a spin-polarized current is studied theoretically. The shape effect resulting from the modulation of the sample width (geometric pinning) can essentially affect the DW depinning. We found a good agreement between the ratios of the critical values of the magnetic field and the spin-polarized current predicted by the theory and measured in the experiment.

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I. INTRODUCTION

During the last years an immense interest in current-driven domain wall (DW) motion in thin magnetic films, nanotubes and point contacts has been initialized by possible applications in spintronic device technology (see, e.g. [1, 2] and references therein). These devices are expected to be highly efficient, fast and consuming less energy. They possess such important features as non-volatility, portability and capability of simultaneous data storage and processing. A manipulation of magnetization by spin-polarized current predicted by Berger for non-uniform ferromagnets [3] and Slonczewski for multi-layered ferromagnetic structures [4] has attracted the attention of physical community during last decade and gained further development in [5]-[12].

The problem of DW motion under another driving force, applied magnetic field, was addressed in late 70s in connection to possible application in memory storage devices (see, e.g. [13]). In pioneering work of Schryer and Walker [14] it was shown the existence of an instability in the laminar movement of the DW. These early results can actually serve as a reference for the studies of spin-polarized current-driven DW motion. In particular, it is well established now that for DW driven by magnetic field (or spin-polarized current, or both) there exists a critical value of driving parameter(s) which corresponds to maximum velocity of the DW (Walker velocity [14]). Thus, the DW dynamics includes two distinct regimes, so-called subcritical (below the Walker breakdown) and supercritical (above the Walker breakdown) ones [1, 7]). Below the Walker limit the overdamped transient response of the system to applied magnetic field follows by a steady state response, while above the Walker limit the DW dynamics is oscillatory with non-zero average velocity.

In case of field-driven DW motion the exact stationary wave solution in subcritical regime was obtained more than half-century ago by Walker [15]. Analytical results of Bourne [16] reproduced a velocity profile in full range of magnetic field confirming numerical simulations of Slonczewski in supercritical regime [17].

Modern fabricated logic elements based on manipulation of the DWs are represented by a complex networks of nanowires, which can form three-dimensional memory storage structure, e.g., a racetrack device that comprises an array of magnetic nanowires arranged horizontally or vertically on a silicon chip [18]. The spacing between consecutive DWs is controlled by pinning sites fabricated by patterning notches along the edges of the track or modulating the track’s size and material properties. Being a necessary component of the logic elements, the pinning sites define the bit length and provide the DWs stability against external perturbations, such as thermal fluctuations or stray magnetic fields from nearby racetracks. The variation of the nanowire geometry creates the pinning potential for the DW. The knowledge of the behavior of the DW in artificially created structural defects and constrictions is extremely important for producing reliable memory devices. In spite of a growing number of experimental studies which gain insight into the properties of the DWs pinned by artificial defects (see, e.g. [11], [19]-[22]), a detailed understanding of the role of the shape effects on the dynamics of the DW driven by magnetic field and (or) spin-polarized current is still lacking.

In this paper we study shape-dependent effects on the properties of DW confined in potential wells created by the defects in the bulk and by the variation of the sample shape (geometrical pinning). We analyze the difference in the
the stiffness of the spin system. The choice of the sign $q > 0$ orthorhombic anisotropy and the magnetostatic energy, and $\Delta L \text{DW}$ width, $\Delta$, is much less than the width of the plate, is the parameter of the easy-axis crystallographic anisotropy, the ratio $0 \ll \Delta L \text{DW}$. The DW displacement under driving magnetic field and spin-polarized current can be adequately described by Landau-Lifschitz-Gilbert (LLG) equations complemented by a spin-transfer torque, $\tau_J$:

$$\partial_t \mathbf{m} = -\gamma [\mathbf{m} \times \mathbf{H}_e] + \alpha [\mathbf{m} \times \partial_t \mathbf{m}] + \tau_J,$$

where $\gamma$ is the gyromagnetic ratio, $\alpha$ is the dissipation parameter, $\mathbf{H}_e = -\delta F/\delta \mathbf{M}$ is effective magnetic field, $F$ is free energy density of the ferromagnet, and $\mathbf{m} = \mathbf{M}/M_s$ is a unit vector in the direction of the magnetization $\mathbf{M}$ ($M_s$ is the saturation magnetization).

Despite different approaches in calculation of the spin-transfer torque $\tau_J$, there is a consensus about the existence of adiabatic and non-adiabatic terms (or $\beta$-term) that contribute to the spin-transfer torque

$$\tau_J = -(\mathbf{U} \cdot \nabla)\mathbf{m} + \beta \mathbf{m} \times (\mathbf{U} \cdot \nabla)\mathbf{m},$$

where $\mathbf{U} = U \mathbf{n}$, $U = \mu_B J P/\epsilon M_s$ ($\mu_B$ is Bohr magneton, $\epsilon$ is the elementary charge, $J = \mathcal{I}/A$ is the current density, $\mathcal{I}$ is a value of the current, $A$ is a cross-section area of the sample and $P$ is the spin polarization), $\mathbf{n}$ is unit vector in the direction of the current. Parameter $\beta$ is a ratio between the values of non-adiabatic and adiabatic torques.

Let us consider the $180^\circ$ DW of Bloch type in a constricted plate-like sample with variable size $L_x = L_x(y) \equiv L_0[1 + G(y)]$ along $x$-direction and constant dimensions $L_y$ and $L_z$ along $y$- and $z$-axis as shown in Fig. 1. The function $G = G(y)$ describes the change of the sample shape. In absence of constriction $G = 0$ and $L_x = L_0$. The DW width, $\Delta$, is much less than the width of the plate, $L_z$, i.e. $\Delta \ll L_z$. The surface of the sample is parallel to the $xy$-plane, and the domain wall, being parallel to the $xz$-plane, separates two domains with magnetization $\mathbf{M}(y)$ along the +$z$ or -$z$ direction and is located initially in the constriction at $y = 0$.

The free energy functional of the ferromagnet is $F = \int_V d \mathbf{r} F$ where the free energy density $F = F[\mathbf{m}(\mathbf{r}), \partial \mathbf{m}(\mathbf{r})/\partial x_i]$ is defined by

$$F = K [-m_z^2 + q m_y^2 + \Delta_0^2 (\partial_i m_j)(\partial_i m_j)] - H M_s m_z .$$

where $K > 0$ is the parameter of the easy-axis crystallographic anisotropy, the ratio $q = K_L/2K > 0$ determines a joint effect of orthorhombic anisotropy and the magnetostatic energy, and $\Delta_0$ is a half-width of the DW at rest, which characterizes the stiffness of the spin system. The choice of the sign $q > 0$ implies alignment of the magnetization $\mathbf{M}$ in the $xz$-plane. Thus, in the absence of the driving force the formation of DW of Bloch type with the rotation of the magnetization in $xz$-plane is energetically more favorable.

The magnetization $\mathbf{m}$ can be expressed in polar coordinates, $\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. In the absence of driving forces ($\mathbf{H}, \mathbf{U} = 0$) the ground state of the system is defined by minimization of the free energy with respect

\[ \text{FIG. 1: (color online) } 180^\circ \text{ Bloch domain wall. The magnetization in the domains are parallel and antiparallel along the } z \text{ axis (the easy axis). Inside the domain wall the magnetization rotates in the } xz \text{ plane. The low panel shows an example of constricted sample with the DW located in the constriction.} \]
to azimuthal, $\phi$, and polar, $\theta$, angles, i.e., $\delta F/\delta\phi = 0$ and $\delta F/\delta\theta = 0$. This yields the well-known structure of Bloch DW located at the center of the sample constriction ($y = 0$): $\theta_0 = 2\tan^{-1}\exp(y/\Delta_0)$ and $\phi_0 = 0$.

The standard approach in statics and dynamics of the DW is to use the Slonczewski equations for two canonically conjugated variables, the coordinate of the center of the DW $\eta$, and azimuthal angle $\phi$ (see, e. g. [12]), which are independent of the coordinate $y$. For a sample with variable cross-section area $A(y) = A_0[1 + G(y)]$ ($A_0 = L_zL_0$ is the area of the cross-section at $y = 0$, and $G(\eta)$ is a shape function dependent on the geometry of the sample) these equations are:

$$
\left(\frac{d\phi}{dt} + \alpha \frac{d\eta}{dt}\right)[1 + G(\eta)] = -\frac{\gamma}{2M_s\delta\eta} + \beta\frac{U_0}{\Delta}, \quad \left(\frac{d\eta}{dt} - \alpha \frac{d\phi}{dt}\right)[1 + G(\eta)] = \frac{\gamma}{2M_s\delta\phi} + U_0,
$$

where $U_0 = [1 + G(y)]U(y)$, $\Delta = \Delta_0(1 + q\sin^2\phi)^{-1/2}$ is the effective DW width, and $\sigma(\eta, \phi)$ is the surface energy of the DW determined by the integral across the whole DW width:

$$
\sigma = A_0 \int dy [1 + G(y)] F[\theta_0(y - \eta), \phi]. \tag{5}
$$

The goal of the present work is not investigation of the DW motion but determination of the depinning threshold, when such motion becomes possible. Therefore, one may ignore the time dependence of the variables. The second term $\propto \beta$ in the spin torque (nonadiabatic torque), Eq. (2), is most relevant for depinning. This term is related to the momentum transfer between the polarized current and the DW. The adiabatic torque [the first term in Eq. (2)] causes the rotation of the DW plane around $z$–axis relative to its equilibrium position ($xz$–plane), but does not affect the DW depinning directly.

### III. DEPINNING OF DOMAIN WALLS

Usually, the pinning (coercivity) force on a DW originates from randomly located defects, which create potential wells for the DW in the sample bulk. However, key features of the bulk pinning phenomenon can be investigated using a simpler model of a DW in a periodic potential. The latter enters the DW surface energy, i.e., $\int dy \sigma(\eta, \phi)$, which can be expanded in series on DW displacement $\eta$. Within the period $(-\xi \leq \eta \leq \xi)$ this function can be approximated by a parabolic function $f(\eta) = (1/2)(\eta/\xi)^2$. In addition to the pinning on the defects, there is another type of pinning (geometrical pinning) related to the change of the sample shape. Eventually neglecting the structure of the DW, i.e., assuming that $\sin^2\theta(y) = 2\Delta\delta(y)$, we obtain

$$
\beta \frac{U_0}{1 + G(\eta)} + \gamma H \Delta = \gamma H_0 \Delta_0 \frac{\partial^2 G(\eta)}{1 + G(\eta)} + \gamma H_p \xi \partial_\eta f(\eta) \Delta, \quad \frac{U_0}{1 + G(\eta)} = -\frac{q}{4} H_0 \Delta \sin 2\phi, \tag{6}
$$

where $H_0 = 2K/M_s$ is a magnetic field corresponding to the easy-axis magnetic crystallographic anisotropy of a sample. According to Eq. (6), the modulation of the sample width gives rise to an effective geometrical pinning of the DW. It follows from Eqs. (6) that DW can be depinned by action of magnetic field, $H$, or the nonadiabatic contribution of the spin-polarized current, $\beta \neq 0$.

In the following, we consider two particular cases, $H \neq 0$, $U = 0$ and $U \neq 0$, $H = 0$.

#### A. Depinning by applied magnetic field: $H \neq 0$, $U = 0$

In case of $H \neq 0$ and $U = 0$, it follows from Eqs. (6), that $\phi = 0$, $\Delta = \Delta_0$, and the depinning of the DW by magnetic field is not affected by the presence of the orthorhombic anisotropy. Thus, instead of the first equation in (6) we have

$$
[H - H_p \xi \partial_\eta f(\eta)][1 + G(\eta)] - H_0 \Delta_0 \partial_\eta G(\eta) = 0, \tag{7}
$$

Equation (7) manifests the absence of a total force experienced by DW. The driving force from the magnetic field (the term $\propto H$) and the bulking pinning force confining the DW in a potential well (the term $\propto H_0\Delta$) are proportional to the total DW area, $\sim [1 + G(\eta)]$, while the force experienced by the DW in a shape-dependent pinning potential is determined by the derivative of the shape function $\partial_\eta G(\eta)$ [the last term in Eq. (7)].

For simple shape potential, which can be expanded in series on DW displacement $\eta$, the function $\partial_\eta G(\eta)/[1 + G(\eta)]$ reaches a maximum at some value of the parameter $\eta = \zeta$. Thus, it is insightful to characterize the geometric pinning
potential, which is responsible for the shape effect, by the strength of the potential $H_\zeta$ and the characteristic distance $\zeta$, which are analogous to the parameters $H_p$ and $\xi$ of the pinning potential due to the defects. The value of the critical magnetic field $H_\zeta$ is defined by

$$H_\zeta = \max \left\{ H_a \Delta_0 \frac{\partial \delta G(\eta)}{1 + G(\eta)} \right\} = H_a \Delta_0 \frac{\partial \zeta G(\zeta)}{1 + G(\zeta)},$$

while $\zeta$ is obtained from the condition of a potential extremum $(1 + G)\partial^2_{\zeta_\zeta} G - (\partial_{\zeta} G)^2 = 0$.

Thus, the presence of a constriction results in the change of the DW area and appearance of the geometrical pinning, which is independent of the distribution of defects. In absence of pinning on defects ($H_p = 0$), the increase in the applied magnetic field below the critical field ($H < H_\zeta$) causes the DW displacement $\eta(H)$ (which is not a linear function of a magnetic field in general) until its depinning at $H = H_\zeta$.

With use of $H_\zeta$ and $\zeta$ both terms that contribute to the DW pinning [see Eq. (7)] can be rewritten in a similar way:

$$H - H_p \frac{\eta}{\xi} \Theta(\xi - |\eta|) - H_\zeta \psi(\eta) \Theta(\zeta - |\eta|) = 0,$$

where $\Theta(\xi)$ is a step-function, and the function $\psi(\eta)$ is defined according to

$$\psi(\eta) = \frac{\partial \delta G(\eta)[1 + G(\zeta)]}{\partial \zeta G(\zeta)[1 + G(\eta)]}.$$

Equation (9) defines a function $\eta = \eta(H)$ and can be solved for given geometry of the constriction. The results of numerical calculation of the function $\eta = \eta(H)$ in the particular case of a parabolic potential created by the constriction when $G = (\eta/\zeta)^2$ are shown in Fig. 2 and reveal the general features of the DW depinning by the applied magnetic field $H$. Figure 2 illustrates the possibility of four different scenarios of DW depinning dependent on the relation between $\xi$ and $\zeta$, from one side, and $H_p$ and $H_\zeta$, from another. The analysis shows that these scenarios can

FIG. 2: (Color online) The plots $\eta = \eta(H)$, which illustrate four different scenarios for the depinning of the DW subjected to an applied magnetic field $H$, for $\zeta > \xi$ [(a) and (b)] and $\zeta \leq \xi$ [(c) and (d)], as described in the text. The solid curves 1 show the DW displacement $\eta$ for the joint effect of bulk pinning and geometrical pinning potentials. The DW displacement in the presence only of bulk pinning or geometric pinning is shown by dashed curves 2 and 3 respectively. The open circle corresponds to critical point $A(\eta_c, H_c)$, where eventually depinning takes place. The critical magnetic field $H_c$ and the critical displacement of the DW $\eta_c$ are defined by Eqs. (11) and (12).
be classified according to the values of two critical parameters, namely, the depinning magnetic field $H_c$ and critical value of the DW displacement $\eta_c$. The value of $H_c$ and $\eta_c$ are defined according to

$$H_c = \begin{cases} \max\{H_p, H_\xi + (\zeta/\xi)H_p\}, & \text{if } \zeta \leq \xi \\ \max\{H_\xi; H_p + H_\Delta \partial_\xi \mathcal{G}(\xi)/[1 + \mathcal{G}(\xi)]\}, & \text{if } \zeta > \xi \end{cases} \quad (11)$$

and

$$\eta_c = \begin{cases} \zeta, & \text{if } H_c = H_p, H_p + H_\Delta \partial_\xi \mathcal{G}(\xi)/[1 + \mathcal{G}(\xi)] \\ \zeta, & \text{if } H_c = H_\xi, H_\xi + (\zeta/\xi)H_p \end{cases} \quad (12)$$

where we assume that the maximum width of the constriction along $y$-axis exceeds the critical distance $\eta_c$. For $H \geq H_c$ ($\eta \geq \eta_c$) the pinning potential cannot stop motion of the DW.

**B. Depinning by a spin-polarized current: $U \neq 0$, $H = 0$**

In the case of $U \neq 0$, $H = 0$, instead of Eqs. (6) we have another pair of equations:

$$\beta U_0 - \gamma H_a \Delta_0^2 \partial_\eta \mathcal{G}(\eta) - [1 + \mathcal{G}(\eta)]\gamma H_a \Delta \partial_\eta f(\eta) = 0 \quad (13)$$

$$\frac{U_0}{1 + \mathcal{G}(\eta)} = -\frac{q}{2} \gamma H_a \Delta \sin 2\phi \quad (14)$$

Equation (13) manifests the balance of forces on the DW: the forces from the spin-polarized current ($\sim \beta$) and the geometrical pinning ($\sim H_a$), and the bulk pinning force ($\sim H_p$). Equation (14) defines the function $\phi = \phi(\eta, U_0)$:

$$\sin \phi = -\mathcal{U}\left\{\frac{1}{2}(1 - q \mathcal{U}^2) + \frac{1}{4}(1 - q \mathcal{U}^2)^2 - \mathcal{U}^2\right\}^{-1/2}, \quad (15)$$

where $\mathcal{U} = U_0/2q\gamma H_a \Delta_0/[1 + \mathcal{G}(\eta)]$. Substituting Eq. (15) into Eq. (13) one can calculate the function $\eta = \eta(U_0)$. For the sake of simplicity we neglect the change of the DW width assuming that $\Delta \approx \Delta_0$, which is true if $\phi \ll 1$ ($\mathcal{U} \ll 1$). In this case Eq. (14) can be rewritten in a following way:

$$\frac{\beta U_0}{\gamma \Delta_0} - H_a \Delta_0 \partial_\eta \mathcal{G}(\eta) - [1 + \mathcal{G}(\eta)]H_p \partial_\eta f(\eta) = 0 \quad (16)$$

It follows from Eq. (16), that in an unconstricted sample ($\mathcal{G} = 0$) a DW can be depinned and participate in a steady-state motion when the the current exceeds the critical value defined by the equation $\beta U_0^c/\gamma \Delta_0 = \max H_p \partial_\eta f(\eta)$.

![Diagram](image)

**FIG. 3**: (Color online) The plots $\eta = \eta(\beta U_0)$ of the DW confined in potential wells created by artificially fabricated constriction and bulk pinning centers. The jumps of the function $\eta = \eta(\beta U_0)$ are the result of the DW depinning from local volume defects within periods of the potential landscape. The values of the current-dependent parameter $\beta U_0$ required to depin the wall from the local potential are shown by the vertical dashed lines. The increase in the driving force results in a step-by-step drift of the DW. In calculations we use a parabolic shape function $\mathcal{G} = (\eta/\xi)^2$. Open circle corresponds to the critical parameters for a sample without constriction. Note the absence of the depinning critical point for the constricted sample.
The condition for DW geometric depinning by the current is more severe than by the magnetic field. This because the pressure on the DW from the constant magnetic field (the force per unit area of the DW) does not depend on the DW area growing with the DW displacement, whereas the pressure from the constant current is determined by the current density, which is inversely proportional to the DW area. According to Eq. \ref{eq:16} at \(H_p = 0\), the critical value of a current is determined by the threshold \(\beta U_0^{(s)} = \gamma H_a \Delta^2 \max \{ \partial \eta G(\eta) \} \) above which the DW can overcome the geometrical pinning. For a parabolic shape function \( G = (\eta/\zeta)^2 \), the function \( \partial \eta G(\eta) \) is unbound, and DW depinning from the geometrical pinning potential is impossible. Let us consider the DW behavior in this case.

We assume that a single DW is initially located in the constricted area \( \eta = 0 \). The increase of the current density above its critical value at a given position of the DW results in the drift of the DW from the constriction into expansion part of the sample. Such a drift is accompanied by the increase in the DW area followed by the decrease of the current density (the current does not depend on the location of the DW) below its critical value. As a result, the DW will be eventually pinned in a new position by an array of the nearest defects. The further increase in the current results in increase in a driving force and a displacement of the DW into a new position where the driving force is balanced by the increase in the pinning force. At this new position the DW is stuck again till the next increase of the current. Actually, in the presence of bulk pinning centers the displacement of the DW under spin-polarized current is characterized by a step-by-step drift over an array of bulk defects.

Equation \ref{eq:16} can be solved numerically for given functions \( G(\eta) \) and \( f(\eta) \). The results of numerical calculation of the function \( \eta = \eta(\beta U_0) \) for a parabolic shape function \( G = (\eta/\zeta)^2 \) are present in Fig.\ref{fig:3} which illustrates the absence of the critical point on the curve \( \eta = \eta(\beta U_0) \) \((\eta \to \infty \text{ when } \beta U_0 \to \infty)\), contrary to the case of depinning of a DW driven by a magnetic field \(H\) (see Fig.\ref{fig:2}).

C. Domain wall in a constricted sample of SrRuO\textsubscript{3} and comparison with experiment

The developed theory can be exploited to understand the data on depinning of the DW from double \(V\)-shape constriction in submicroscopic patterns of SrRuO\textsubscript{3} \cite{11}. SrRuO\textsubscript{3} is a metallic perovskite with orthorhombic structure \((a = 5.53, b = 5.57, c = 7.82 \text{ Å})\) and an itinerant ferromagnet with Curie temperature of \(150 \text{ K} \) and a saturated magnetization of \(1.4 \mu_B\) per ruthenium. It shows so-called bad metal behavior at high temperatures, but is a Fermi liquid at low temperatures. SrRuO\textsubscript{3} exhibits a positive Seebeck coefficient in the wide range of the temperature from \(0\) K till \(1000 \text{ K} \) \cite{23, 24}, manifesting the hole-like character of the charge carriers. The samples are high-quality epitaxial thin films of SrRuO\textsubscript{3} grown by reactive electron beam coevaporation on slightly miscut \((\sim 0.2^\circ)\) SrTiO\textsubscript{3} substrates with the [001] and [110] axes in the film plane. These films exhibit a large uniaxial magnetocrystalline anisotropy (anisotropy field \(H_a \approx 10 \text{ T} \) at \(T = 0 \text{ K}\)) with the easy axis tilted out of the film \(\sim 45^\circ\) and with an in-plane projection along [110]. Consequently, the domain magnetization is out of film plane, the Bloch DWs are parallel to [110] axis and the orthorhombic anisotropy (including the magnetostatic energy related to the shape anisotropy of the sample) contributes to the structure of the DW. Due to the large uniaxial magnetic anisotropy the DWs are relatively narrow with temperature independent width of \(\sim 3 \text{ nm}\).

The experimental setup is shown in Fig.\ref{fig:1}(a). The measurements on the displacement of the DW driven by magnetic field and spin-polarized current were performed on a high-quality 375 Å thick film of SrRuO\textsubscript{3} with the resistivity ratio of 20 \((\sim 10\mu\Omega \text{ cm} \) at \(4 \text{ K}\)). The DW initially located at the constriction (see, Fig.\ref{fig:1}(a)) was unpinned under the action of a magnetic field or a spin-polarized current and moved in the positive direction of \(y\)-axis (parallel to [001]) towards the pair of leads EF. The magnetic state of the sample in the region of contacts EF was monitored by measuring of extraordinary Hall effect (EHE) proportional to the average component of magnetization \(M_z\) perpendicular to the film plane \((xy\)-plane\). The final location of the DW at the leads EF was deduced by the change of the sign of EHE followed by the change of the magnetic state at EF. The experiment shows that the DW displacement into a final position at the leads EF is achieved only with a value of the magnetic field \((\text{current density})\) above a certain threshold for the magnetic field or the current.

The shape of the sample [see Fig.\ref{fig:4}(b)] can be approximated by a function

\[
G(\eta) = \frac{(\eta/\zeta_1)^2}{1 + \eta/\zeta_2} + \frac{(\eta/\zeta_3)^2}{1 + \eta/\zeta_4},
\]

where \(\zeta_1 = 1.138 \mu m, \zeta_2 = 2.6 \mu m, \zeta_3 = 0.561 \mu m, \) and \(\zeta_4 = 0.52 \mu m.\) The results of numerical calculation show that a function \(G(\eta) = \partial \eta G(\eta)/[1 + G(\eta)]\) has a maximum at \(\eta = \zeta = 0.44 \mu m\) with \(G(\zeta) = 1.1675 \mu m^{-1}.\) The assumption \(\zeta \ll \zeta_\parallel,\) which is true assuming that \(\xi \sim \Delta \sim 1.5 \text{ nm}\), leaves two possibilities for the critical value of magnetic field \(H_c\) [see Eq. \ref{eq:11}]:

\[
H_c = \max \left\{ H_c : H_p + \frac{1}{2} H_a \xi \Delta \left( \frac{1}{\xi_1^2} + \frac{1}{\xi_3^2} \right) \right\},
\]

where \(\xi_1 = 5.53 \text{ nm, } \xi_2 = 5.57 \text{ nm, } \xi_3 = 1.138 \mu m, \) and \(\xi_4 = 1.5 \text{ nm}.\)
where we use $G(\eta) \approx \eta^2 (\zeta_1)^{-2} + \zeta_1^{-2}$ for $\eta \sim \xi$. The choice of $H_c$ in Eq. (13) depends on the critical value of magnetic field corresponding to the geometrical pinning $H_\xi$ given by Eq. (5). Using the values of $H_a \approx 10$ T and $\Delta_0 \approx 1.5$ nm \cite{11}, we obtain $H_\xi \approx 160$ Oe, which is less than the highest value of the depinning field $H_c = 500$ Oe measured at the temperature $T = 40$ K \cite{11}. Therefore, we conclude that the critical magnetic field measured in Ref. \cite{11} is dominated by the contribution of bulk pinning on defects, i.e., $H_c \approx H_p$, in accordance with the conclusions of \cite{11}. Evaluating the critical field $H_c$ \cite{18}, we have neglected the small contribution of $(\xi \Delta / 2 \zeta^2)H_\xi$, which is on the order of several Oe. It follows from Eq. (12) that the critical value of the DW displacement $\eta$ is $\xi \sim \Delta_0 = 1.5$ nm $\ll \zeta = 0.44$ µm, therefore the scenario illustrated in Fig. 2(b) is realized. After depinning when $H \geq H_\xi$ and $\eta \geq \xi \sim \Delta_0 = 1.5$ nm the DW moves freely (see Ref. \cite{25}) till it reaches the leads EF (Fig. 3(a)).

In case of current driven DW motion the measured value of a spin-polarized current corresponds to the arriving of the DW at the leads EF (see Fig. 4(a)). Both contributions to the DW pinning resulting from the distribution of the defects and the change of the sample shape can be evaluated from the data \cite{11} by use of Eq. (16). To calculate the value of the current predicted by the theory we replace $\xi \partial_\eta f(\eta)$ in Eq. (16) by its maximum value: $\max\{\xi \partial_\eta f(\eta)\} = 1$,

$$\frac{\beta U_0}{\gamma \Delta_0} - H_a \Delta_0 \partial_\eta G(\eta) - [1 + G(\eta)] H_p = 0 .$$

Furthermore, we assume that the DW moves at the distance $\eta \approx 1.5$ µm till it is registered by observation of the change of the sign of extraordinary Hall effect at the leads EF (see, Fig. 4). With use of $H_a \approx 10$ T, $\Delta_0 = 1.5$ nm and the measured value of the depinning field $H_p = 571$ Oe at $T = 40$ K \cite{11} one can show that $H_a \Delta_0 \partial_\eta G(\eta) \approx 411$ Oe and $[1 + G(\eta)] H_p \approx 2250$ Oe which gives a relative contribution of the geometrical pinning $\sim 20\%$. The measurement of the corresponding current density allows to evaluate the parameter of the non-adiabaticity $\beta$. For SrRuO$_3$, the current density $J$ translates into the velocity $U_0$ according to $U_0[\text{m/s}] = 3.64 \times 10^{-10} P J[\text{A/m}]$. Substituting the measured value of the current density $J = 5.8 \times 10^{10}$ A/m$^2$ into Eq. (19) and using the value of spin polarization $P \approx 0.5$ (see Ref. \cite{26}) we obtain $\beta \approx 6$. Being in accord with the conclusions of the high efficiency of monitoring of the DW by spin-polarized current in SrRuO$_3$ \cite{11}, such large value of $\beta$ cannot be explained by the contribution of the spin-relaxation process, which gives the value of $\beta \sim \alpha \ll 1$ (see Refs. \cite{7,8}), but can be understood due to the dominant role of the reflection of the charge carriers from thin DWs in the framework of the theory developed in Ref. \cite{3}.

Since bulk pinning on defects varies from a sample to sample, it is unpractical to look for quantitative comparison of the theory and the experiment in the case of predominantly bulk pinning. However, if there are data for the critical magnetic field and the critical current for the same sample, one may easily find from the theory their ratio and compare it with the experimental values. As it was shown by Tatara and Kohno \cite{6} the dynamics of the abrupt DW in ideal plate-parallel sample is dominated by the momentum transfer from the charge current to the DW via reflection of charge carriers from the DW. Since this momentum transfer determines also the DW contribution to the resistance, in accordance with Ref. \cite{6} the parameter $\beta$ can be found from the relation

$$\frac{\beta U_0}{\gamma \Delta_0} = \frac{enR_wA}{2M_s} J ,$$

(20)
where \( n \) is a total charge carrier density and \( R_w \) is the DW contribution to the resistance. Substituting Eq. (20) into Eq. (16), we arrive at the equation that defines the value of the current density \( J \) as a function of the displacement of the DW in the presence of bulk and geometric pinning. As was pointed out in Sec. III B, complete depinning of the DW from the potential produced by growing cross-section of the sample is impossible. However, one may introduce the critical current density \( J_c \) (determined at the constriction), at which the DW reaches the position where the DW is detected (the leads EF in Fig. 4). Then

\[
J_c = J_{c0} [1 + G(\eta)] \left[ 1 + \frac{H_a \Delta_0 \partial_\eta G(\eta)}{H_c} \right],
\]

where \( \eta \) is the distance between the sample constriction and the place of DW detection, and

\[
J_{c0} = \frac{2M_s H_c}{enA_0 R_w}
\]

is the critical current density of depinning for the DW in an unconstricted sample.

The temperature dependent saturation magnetization \( M_s = M_s(T) \) and the DW resistance \( R_w \) were measured in Refs. 27 and 11 respectively. According to Ref. 11 the carrier density \( n \) is about \( 1.6 \times 10^{28} \) [1/m\(^3\)]. This offers an opportunity to compare the developed theory with the data [11]. Calculating the critical current density \( J_c \) with help of Eq. (21), we assume that the relative contribution of the geometric pinning [the term \( \propto H_a \) in Eq. (21)] is temperature independent and equals to the value of \( \sim 20 \% \) calculated at \( T = 40 \) K. Then Eq. (21) yields \( J_c \approx 5J_{c0} \).

Figure 5 shows the experimentally determined critical current density (circles) together with the prediction of the theory using the experimentally found critical magnetic fields and taking into account the shape dependent effect (geometric pinning). The figure illustrates a satisfactory agreement between the experiment and the theory.

It is important to note that the sign of the ratio between the current and the DW displacement depends on the sign of charge carriers. The relative sign of the current and the displacement in the experiment gives evidence that charge carriers are holes. This agrees with the experiment on the Seebeck effect [23].

IV. CONCLUSIONS

We investigated pinning of a domain wall by potentials produced by bulk defects and the sample shape. The process of depinning by an external magnetic field and by a spin-polarized current was analyzed. The shape-dependent pinning potential (geometric pinning) can essentially affect the process of depinning and may even make complete depinning by the spin-polarized current impossible. Though the absolute values of the critical magnetic fields and the critical currents, at which depinning occurs, are sample dependent and difficult for theoretical prediction, their ratio must be sample independent [6] and allows reliable comparison of the theory and the experiment. We performed this comparison and found a satisfactory agreement.
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