Quantum ground-state computation with static gates

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Abstract

We develop a computation model for solving Boolean networks that implements wires through quantum ground-state computation and implements gates through identities following from angular momentum algebra and statistics. The gates are static in the sense that they contribute Hamiltonian 0 and hold as constants of the motion; only the wires are dynamic. Just as a spin 1/2 makes an ideal 1-bit memory element, a spin 1 makes an ideal 3-bit gate. Such gates cost no computation time: relaxing the wires alone solves the network. We compare computation time with that of an easier Boolean network where all the gate constraints are simply removed. This computation model is robust with respect to decoherence and yields a generalized quantum speed-up for all NP problems.

1 Introduction

The prevailing approach to quantum computation evolved from classical reversible algorithmic computation (Bennett 1979, Fredkin and Toffoli 1982), where a stored program drives a sequence of elementary logically reversible transformations. In reversible-algorithmic computation a time-varying Hamiltonian drives a sequence of unitary transformations (Benioff 1982, Feynman 1985). It was then found (first by Deutsch 1985) that entanglement, interference and measurement yield in principle dramatic speed-ups over the corresponding classical algorithms in solving some problems.

In spite of this important result, this form of computation faces two possibly basic difficulties. Its speed-ups rely on quantum interference, which requires computation reversibility. Decoherence may then limit computation size below practical interest. Only two speed-ups of practical interest have been found so far (factoring and database search), and none since 1996.

Reversible-algorithmic computation is not the most general form of quantum computation. Its limitations justify reconsidering quantum ground-state computation (Castagnoli 1998, Farhi et al. 2001, Kadowaki 2002, among others),

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a formerly neglected approach still believed to be mathematically intractable. Quantum ground-state computation evolved from classical ground-state computation (Kirkpatrick & Selman 1994, among others), a well-developed approach competitive with algorithmic computation for solving Boolean networks. A Boolean network is a set of nodes (Boolean variables) variously connected by gates and wires that impose relations on the variables they connect (Fig. 1). A Boolean assignment satisfying all gates and wires is a network solution. Roughly speaking, all NP problems can readily be converted to the problem of solving a Boolean network.

In quantum ground-state computation, one sets up a quantum network whose energy is minimum when all gates and wires are satisfied. In quantum annealing, one form of ground-state computation, coupling the network with a heat-bath of suitably decreasing temperature relaxes the network to its ground state, a mixture of solutions (we assume with no significant restriction that there is at least one). Measuring the node variables (Hermitian operators with eigenvalues 0 and 1) yields a solution.

It is believed that quantum annealing yields a (still ill-defined) speed-up over its classical counterpart. Quantum tunneling reduces the risk that the network, in its way toward the absolute energy minimum, remains trapped in local minima (e.g. Kadowaki 2002). However, long simulation times seriously limit research on this approach.

Here we develop a hybrid mode of computation. We implement wires by ground-state computation. We implement gates as algebraic identities resulting from quantum symmetries and statistics.

We show that relaxation-computation time is comparable with that of an easier (more loosely constrained) logical network where all the gate constraints implemented by quantum symmetries are removed. The comparison is based on a special projection method. We show that the relaxation of the actual network can be obtained as a special projection of the relaxation of the easier comparison network. This projection method shorts mathematical complexity and sheds light on the nature of this form of computation.

We conjecture that for this computation mode all hard-to-solve (NP) networks become easy (P) and support this conjecture with plausible estimates. Decoherence is not expected to be as serious a problem for this computation model as for algorithmic computation since the network state is intentionally a thermal mixture during most of the computation.

This discussion of quantum computation still belongs to the realm of principles, like other literature on quantum ground-state computation, while algorithmic-reversible computation is now almost a technology. Nevertheless it is worth starting over with a new approach that might overcome fundamental limitations of algorithmic computation.
2 Computation model

We use a network normal form composed just of wires and triodes (Fig. 1). Each triode $\tau$ — properly a partial gate — connects three nodes labeled $\tau_x$, $\tau_y$, $\tau_z$ (replaced by collective indices in Fig. 1) with the sum-2 relation

$$q_{\tau x} + q_{\tau y} + q_{\tau z} = 2,$$

where $q$’s are Boolean variables and $+$ denotes arithmetical sum. The three solutions are the rows of Table I.

Each wire $w(i, j) = w(j, i)$ is an equality relation $q_i = q_j$ between two nodes $i, j$ (Table II).

The example in Fig. 1, with $Q = 6$ nodes, $W = 4$ wires (lines), and $T = 2$ triodes (dashed triangles), has just one solution: $q_3 = q_5 = 0$, $q_1 = q_2 = q_4 = q_6 = 1$.

![Fig. 1. A network](image)

In the following we give idealized physical models of the various network elements.

2.1 Nodes and triodes

Network nodes represent qubits, which here we consider in a most general way as commuting Hermitian operators with eigenvalues 0 and 1. In most computation, the relations to be satisfied are achieved by a dynamical development. In principle we may model any network relation by any valid physical relation, however. Here we model the triode relation by a spin identity. In the present computation model, each node belongs to a triode $\tau$, a spin 1 system, which might be two spins $1/2$, $\sigma_{r1}, \sigma_{r2}$ in units $\hbar = 1$, in a triplet state, with total spin vector $s_r = \frac{1}{2}(\sigma_{r1} + \sigma_{r2})$, $s_{rz} = \pm 1, 0$. For each spin 1 we define three qubits,

$$q_{\tau x} = s_{\tau x}^2, \quad q_{\tau y} = s_{\tau y}^2, \quad q_{\tau z} = s_{\tau z}^2,$$

(2) each representing a node of triode $\tau$.

By the composition of angular momentum, the three qubits of each proton pair satisfy the XOR gate equation (Table III). The four rows of Table III correspond to the singlet and the three triplet states of proton pair $\tau$, spanning the Hilbert space $\mathcal{H}_\tau^{(1)}$. We use $\mathcal{H}_N^{(4)} = \bigotimes_{\tau=1}^T \mathcal{H}_\tau^{(4)}$ as the network space.
Note that network nodes are not divided into inputs and outputs of the computation process as in algorithmic computation. They are all simultaneously present in the network as commuting Hermitian operators, related by time-independent identities that we use as gates.

To simplify the physical model, we assume that the spatial wave function of each proton pair $\tau$ is frozen throughout the computation in a stable antisymmetric ground state, like that of the protons of an orthohydrogen molecule. In the triplet state the three qubits of any triode $\tau$ obey the triode sum-2 relation of (1) and Table I. This follows from angular momentum composition and triplet symmetry, both extradynamical relations; the triode Hamiltonians are zero in $H^{(4)}_N$.

Let $H^{(3)}_\tau$ be the space spanned by the three triplet states of triode $\tau$. $H^{(3)}_\tau = \otimes^{T}_{\tau=1} H^{(3)}_\tau \subset H^{(4)}_N$ is the network subspace with all the triodes satisfied.

### 2.2 Wires

We define the frustration Hamiltonian of wire $w(i,j)$ in $H^{(4)}_N$ by $H^{(4)}_{i,j} = g (q_i - q_j)^2$, where $g$ is a coefficient to provide the dimension of energy. The eigenvalues of $H^{(4)}_{i,j}$ are zero when the wire is satisfied, $g$ when it is not (the wire is then “frustrated”). All $H^{(4)}_{i,j}$ commute. Therefore the network frustration Hamiltonian is

$$H^{(4)}_N = g \sum_{\{w\}} (q_i - q_j)^2,$$

where $\{w\}$ is the set of all wires in the network.

Let $X_\tau$ be the exchange operator for the two protons of triode $\tau$. $H^{(4)}_N$ is symmetric under all the $X_\tau$ ($X_\tau H^{(4)}_N X_\tau = H^{(4)}_N$), since the $q$’s are. Therefore the triplet symmetry projection operator $T_\tau$ of the triode $\tau$ is a constant of motion of $H^{(4)}_N$. If the initial network state is in $H^{(3)}_N$, under $H^{(4)}_N$ it remains in it. The ground state of $H^{(4)}_N$ (in $H^{(3)}_N$) hosts a mixture of network solutions, since all wires and triodes are satisfied.

### 2.3 Ising model

$H^{(4)}_N$ is quadrilinear in the spin components $s_{\tau x,y,z}$. As a step toward implementation, we show that $H^{(3)}_N$ can be represented by pairwise spin-spin interactions by adjoining two “idlers,” auxiliary spin-1/2 variables $\sigma_i, \sigma_j$, to the spin-1 variables $s_i, s_j$ already defined for each wire $w(i,j)$. For convenience we normalize the idler variables to eigenvalues $\sigma_0 = 0, 1$. One of many suitable frustration Hamiltonians for wire $w(1,2)$, connecting the nodes $q_1 = s_1^z$ and $q_2 = s_2^z$ (using collective indices 1, 2) is the bilinear form:

$$H_{w(1,2)} = g \left[ (s_1 + s_2)^2 + 5 (s_1 + s_2) (-\sigma_1 + \sigma_2) + 6 (\sigma_1 + \sigma_2) \right].$$
This is chosen so that (as is readily checked) the ground-state projection operator of the wire (= nodes + idlers) \( \rho_0(s_1, s_2, \sigma_1, \sigma_2) \) has energy eigenvalue 0 and includes eigenvectors \(|s_1, s_2, \sigma_1, \sigma_2\rangle\) with all five combinations of spin-1 eigenvalues that satisfy the wire, namely \((s_1 = \pm 1, s_2 = \pm 1)\) and \((s_1 = 0, s_2 = 0)\), with correlated idler eigenvalues \(s_1, s_2\). All the states where the wire is frustrated \((s_1 = \pm 1, s_2 = 0; s_1 = 0, s_2 = \pm 1)\) have energy \(\geq g\) for any values of \(\sigma_1, \sigma_2\).

The reduced state of the nodes in the ground state is the trace over the idlers,

\[
\rho_0(s_1, s_2) = \text{tr}_{\sigma_1, \sigma_2} \rho_0(s_1, s_2, \sigma_1, \sigma_2)
\]

(5)

Since all the wires are satisfied in the network state \(\rho_0(s_1, s_2, \sigma_1, \sigma_2)\), they are satisfied in the reduced node state \(\rho_0(s_1, s_2)\). After the network relaxes to the ground state, we can find a solution to the network problem by simultaneously measuring both node bits \(q_1, q_2\), ignoring the idlers.

It is then straightforward to construct the network purely out of spins 1/2 with pairwise coupling, as in the Ising model. We leave the idlers alone but replace each spin 1 by the sum of two spins 1/2 with a coupling that favors the triplet (parallel) state over the singlet (antiparallel) overwhelmingly.

2.4 Heat-bath and coupling

It is convenient to use heat-bath quanta that are distinguishable from the network quanta. We use a photon-filled cavity with Hilbert space \(\mathcal{H}_B\). \(\mathcal{H}^{(4)}_B := \mathcal{H}^{(4)}_N \otimes \mathcal{H}_B\) is the “system” (=network+bath) space, \(\mathcal{H}^{(3)} := \mathcal{H}^{(3)}_N \otimes \mathcal{H}_B\) is the subspace with triplet symmetry, all triodes satisfied.

We denote by \(H^{(4)}_B(t)\) the heat-bath Hamiltonian and define the network-bath coupling in \(\mathcal{H}^{(4)}\) by

\[
H^{(4)}_I(t) = g \sum_{\tau} \left[ \vec{B}_\tau(t) \cdot \vec{\sigma}_{\tau 1} + \vec{B}_\tau(t) \cdot \vec{\sigma}_{\tau 2} \right].
\]

(6)

This couples each proton spin to the small random Gaussian time-varying magnetic field \(\vec{B}_\tau(t)\) of the photon field at the site of the spin. To maintain the symmetry that we have assumed, we assume that the two protons of the same triode \(\tau\) experience the same magnetic field and that the spatial wave functions of different proton pairs do not overlap.

Therefore triplet symmetry (satisfaction of all triodes) is a constant of motion of \(H^{(4)}_I(t)\), thus also of the system Hamiltonian \(H^{(4)}(t) = H^{(4)}_N + H^{(4)}_B(t) + H^{(4)}_I(t)\); in fact \(H^{(4)}_N\) is already symmetric.

2.5 Network relaxation process

With a suitable time-variation of \(\vec{B}_\tau(t)\), \(H^{(4)}(t)\) relaxes the network to its zero point.

Let \(|\psi, t\rangle\) be the state of the system at time \(t\). The development of \(|\psi, t\rangle\) is generated by \(H^{(4)}(t)\) according to the Schrödinger equation. The relaxation of the network state is described by the statistical operator \(\rho_N(t) :=\)
tr$_B$ (|ψ, t⟩ ⟨ψ, t|), where tr$_B$ means trace over the heat-bath degrees of freedom. If ρ$_N$(t) starts in $H^{(3)}_N$ it remains in it, since both the wire frustration Hamiltonian and the heat-bath coupling are symmetric.

A direct estimate of relaxation time is likely mathematically intractable, and a simulation is very long. We take a shortcut that also sheds light on the nature of this hybrid computation.

We compare the network relaxation time with that of an easier network obtained by replacing all triodes (Table I) by XOR gates (Table III): as if proton indistinguishability were suspended – each proton pair were replaced by a deuteron.

The restriction to $H^{(3)}$ vanishes: a network of XOR gates and wires is loosely constrained and easy to solve. In particular $q_i = 0$ for all $i$ is always a solution. A XOR network is solvable in poly($Q$) time in classical computation and, one reasonably supposes, in the present hybrid computation also. Note that the XOR gates of the comparison network are also extra-dynamical; they represent a physical law, namely the composition of angular momentum.

That the relaxation of the comparison network is quick can be plausibly seen as follows.

We first replace $H^{(4)}_N$ by the new Hamiltonian

$$H^{(4)}_N = H^{(4)}_N \left[1 + \frac{g'}{g} \sum_\tau (q_{\tau x}^2 + q_{\tau y}^2 + q_{\tau z}^2) \right].$$

Since each triode has exactly two nodes equal to 1 (Table I), we have $H^{(4)}_N = H^{(4)}_N \left(1 + 2T\frac{g'}{g} \right)$. In the case of the actual network we have merely multiplied $H^{(4)}_N$ by a constant factor and the replacement is inessential. The ground states of $H^{(4)}_N$ and $H^{(4)}_N$ are the same, and the respective energy landscapes are proportional.

Not so for the comparison network, no longer restricted to $H^{(3)}_N$. If $g' \gg g$, the energy landscape of $H^{(4)}_N$ has a gradient everywhere toward the solution $q_i = 0$ allowed by Table III. There are no local minima that can trap the comparison network on its way toward the absolute minimum; thus the relaxation time of the comparison network is reasonably poly($Q$).

We conjecture that introducing $H^{(4)}_N$ is unnecessary. The XOR gates, being extra-dynamical, would not affect relaxation time; they could be removed, which would leave us with a set of independently relaxing wires, and no local minima.

### 2.6 Comparison system

The asymmetric Hamiltonian of the comparison system in $H^{(4)}$ is $H^A(t) = H^{(4)}_N + H^{(4)}_B(t) + H^I_A(t)$, where

$$H^I_A(t) = g \sum_\tau \left[ \vec{B}_\tau(t) \cdot \vec{\sigma}_\tau + \vec{B}_{\tau 2}(t) \cdot \vec{\sigma}_{\tau 2} \right]$$

(8)
is the asymmetric coupling. Now we have two independent random Gaussian
time-varying magnetic fields at each proton site, such that
\[
\vec{B}_\tau(t) = \left[ \vec{B}_{\tau 1}(t) + \vec{B}_{\tau 2}(t) \right] / 2
\]  
(9)
is the actual heat-bath. This is always possible since the sum of two Gaussian
distributions is also Gaussian.

Let \( |\varphi, t\rangle \) be the state of the comparison system, whose development is generated by \( H^A(t) \). The comparison network relaxation is described by the statistical operator \( \rho^c_N(t) := \text{tr}_B (|\varphi, t\rangle \langle \varphi, t|) \).

### 2.7 Continuous projection method

The symmetrization operator for all the proton pairs is
\[
P := \prod_{\tau=1}^{T} \frac{1 + X_\tau}{2}.
\]  
(10)
It projects \( \mathcal{H}^{(4)} \) on \( \mathcal{H}^{(3)} \). Clearly \( PH^A(t)P = H^{(4)}_I(t) \), and so
\[
PH^A(t)P = H^{(4)}(t),
\]  
(11)
given that \( H^{(4)}_N \) is symmetric and \( P \) is the identity in \( \mathcal{H}_B \).

The development of the actual system (hard triode network and bath) in \( \mathcal{H}^{(3)} \) is driven by \( H^{(4)}_I(t) \); that of the comparison system (easy XOR network and bath) in \( \mathcal{H}^{(4)} \) is driven by \( H^A(t) \). We show that, given (11), the continuous projection on \( \mathcal{H}^{(3)} \) of the development of the comparison system yields the development of the actual system.

Let \( |\psi, t\rangle \) be an initial state of the actual system in \( \mathcal{H}^{(3)} \), therefore \( P|\psi, t\rangle = |\psi, t\rangle \). Under \( H^{(3)}_I(t) \), it develops into
\[
|\psi, t + dt\rangle = \left( 1 - iH^{(4)}_I(t)dt \right) |\psi, t\rangle .
\]  
(12)
Under \( H^A(t) \), it develops into
\[
|\varphi, t + dt\rangle := \left( 1 - iH^A(t)dt \right) |\psi, t\rangle ,
\]  
(13)
in general non-symmetric. We restore particle indistinguishability by projecting \( |\varphi, t + dt\rangle \) on \( \mathcal{H}^{(3)} \), symmetrizing it:
\[
P|\varphi, t + dt\rangle = \left( P^2 - iP H^A(t)P \right) |\psi, t\rangle = \left( 1 - iH^{(4)}_I(t)dt \right) |\psi, t\rangle = |\psi, t + dt\rangle .
\]  
(14)
We can see that the continuous projection of the comparison development yields the actual development.
2.8 Comparing computation times

Computation time is by assumption poly(Q) for the comparison easy XOR network. To estimate that of the actual hard triode network, we decompose a $\Delta T$ into $N = \Delta T / \Delta t$ consecutive time slices $\Delta t_i \equiv [t_i, t_{i+1}]$ of equal length $\Delta t$. Within each $\Delta t_i$, we consider the relaxation of the comparison XOR network in $\mathcal{H}^{(4)}$, described by $\rho_N^A(t)$. At the end of each $\Delta t_i$, we project $\rho_N^A(t)$ on $\mathcal{H}^{(3)}$, then take the limit $\Delta t \to 0$. This yields the actual network relaxation $\rho_N(t)$.

Within each $\Delta t_i$ we consider the decomposition

$$\rho_N^A(t) := \rho_0(t) + \rho_F(t) + \rho_V(t). \quad (15)$$

- $\rho_0(t)$ describes networks with satisfied triodes and wires, namely solutions of the actual network; its probability is $p_0(t) := \text{tr}\rho_0(t)$.
- $\rho_F(t)$ describes networks with satisfied triodes and at least one frustrated wire; $p_F(t) := \text{tr}\rho_F(t)$.
- $\rho_V(t)$ describes networks with at least one violated triode, wires are either satisfied or frustrated; $p_V(t) := \text{tr}\rho_V(t)$.

We have considered all the possible states of the comparison network. Therefore $p_0(t) + p_F(t) + p_V(t) = 1$. $p_V(t)$ goes to zero with $\Delta t$ and is annihilated by each projection.

The actual network-bath interaction soon randomly generates a $\rho_0(t_h)$, a mixture of solutions of the actual network, with an extremely small probability $p_0(t_h) = O(1/2^Q)$. For a given confidence level, $t_h$ does not depend on $Q$.

For $t > t_h$ we apply the projection method. $p_0(t_h) = O(1/2^Q)$ becomes the nucleus of condensation of the network solutions.

Within each and every $\Delta t_i$, we take a constant-average logarithmic rate of decrease $k$ of the frustration energy of the comparison network:

$$E_N(t_{i+1}) = (1 - k\Delta t) E_N(t_i). \quad (16)$$

We will show later that there is no error in taking a constant-average rate. The relaxation time constant $1/k$ is by assumption poly(Q).

We have $E_N(t) := \text{tr} \rho_N^A(t) H_N^{(4)} = \text{tr} \rho_F(t) H_N^{(4)}$. In fact there is no contribution from $\rho_0(t)$, which is the ground state of $H_N^{(4)}$, and a possible contribution from $\rho_V(t)$ would anyhow be second order infinitesimal. Let $p_F(j)(t)$ be the $j$-th (population) element of the diagonal of $\rho_F(t)$. Of course $\sum_j p_F(j)(t) = \text{tr} \rho_F(t) = p_F(t)$. $H_N^{(4)}$ is diagonal, thus we have $E_N(t) = \sum_j p_F(j)(t) E(j)$, where $E(j)$ is the $j$-th diagonal element of $H_N^{(4)}$. Therefore $E_N(t)$ and $p_F(t)$ go to zero together. Thus on average:

$$p_F(t_{i+1}) = (1 - k\Delta t) p_F(t_i). \quad (17)$$
The decrease of $p_F(t)$ implies an equal increase of $p_0(t) + p_V(t)$. It is reasonable and conservative to consider the increase of $p_V(t)$ dominant. In fact the relaxation of the comparison network is quicker because triodes can be violated.

Note that we compare relaxation rates, not directions: the comparison network can head toward $H^{(4)}_N \sim H^{(3)}_N$, the actual network remains in $H^{(3)}_N$. It is like comparing the speed of the keel and the wind on a broad reach. Speeds are proportional, while keel and wind go to different places.

Furthermore, $H^{(4)}_N$ does not couple $\rho_0(t)$ with $\rho_E(t)$ or $\rho_V(t)$. In fact $H^{(4)}_N \rho_0(t) = \rho_0(t) H^{(4)}_N = 0$. Therefore $\rho_0(t)$ neither decreases nor increases on average. Since $p_F(t)$ decreases and $p_0(t)$ does not, the ratio $p_F(t)/p_0(t)$ decreases. When we project on $H^{(3)}_N$ at the end of $\Delta t_i$, we remain with a smaller $p_F(t)$ and a larger $p_0(t)$ (probability of solutions of the actual network).

We can focus on the “take-off” of the probability of solution from the extremely small value $p_0(t_h) = O(1/2^N)$ to $p_0(t)$ close to 1, say $p_0(t) = 1/10$.

During take-off and within each $\Delta t_i$ we have $p_F(t) \approx 1$; $p_V(t)$ grows from $p_V(t_i) = 0$ to $p_V(t_i + \Delta t) = k\Delta t p_F(t_i) \approx k\Delta t$, because of (17) and the assumption that $p_0(t)$ remains unaltered. The projection at the end of $\Delta t_i$ annihilates $p_V(t_i + \Delta t)$ reducing $\rho^k_N(t)$ by about $k\Delta t$. Renormalizing $\rho^k_N(t)$ then multiplies $p_0(t)$ by about $(1 - k\Delta t)^{-1} \approx 1 + k\Delta t$ at each $\Delta t_i$. After a time $\Delta T = N\Delta t$ and in the limit $\Delta t \to 0$, we obtain for the actual network:

$$p_0(t_h + \Delta T) \approx p_0(t_h) \lim_{\Delta t \to 0} (1 + k\Delta t)^{\frac{\Delta T}{\Delta t}} = p_0(t_h) e^{k\Delta T} \approx \frac{1}{2^Q} e^{k\Delta T}. \quad (18)$$

The probability of having solutions of the actual network becomes $O(1)$ in a time $\Delta T \approx Q/k = Q\text{poly}(Q) = \text{poly}(Q)$.

Using a different $k_i$ for each $\Delta t_i$, with average value $k = \sum_i k_i \Delta t_i / \Delta T$, yields the same result: $e^{k\Delta T}$ in (18) should be replaced by $\prod_i e^{k_i \Delta t_i} = e^{k\Delta T}$.

### 3 Conclusions

The extradynamical algebraic relations expressing particle statistics and angular momentum composition can replace the dynamical algebraic relations following from equations of motion as computational gates. In this new form of quantum computation, the gates of a Boolean network are always satisfied as constants of the motion, leaving only equality relations (wires) to be implemented dynamically. This form of quantum computation is expected to be robust, since it relies on thermal mixtures, not pure states, and is plausibly conjectured to be fast, turning all NP problems in principle into P. As in quantum algorithmic computation, the speed-up is due to the extradynamical character of the computation (Castagnoli & Finkelstein 2001, 2002).

This model of computation highlights the conceptual difference between how structures can be assembled in the classical and quantum domain. Quantally it is as though one could assemble a jigsaw puzzle simply by piling the pieces up and letting gravity lower them into mutual positions that solve the puzzle,
analogously to quantum wire relaxation. Classically this way of assembling the pieces would be plagued by local energy minima. One may wonder whether the assembly of biological molecules under hydrophobic pressure draws on similar quantum effects.

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Fig. 1. A network