Perturbation Theory for the Rosenzweig-Porter Matrix Model

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We study an ensemble of random matrices (the Rosenzweig-Porter model) which, in contrast to the standard Gaussian ensemble, is not invariant under changes of basis. We show that a rather complete understanding of its level correlations can be obtained within the standard framework of diagrammatic perturbation theory. The structure of the perturbation expansion allows for an interpretation of the level structure on simple physical grounds, an aspect that is missing in the exact analysis (T. Guhr, Phys. Rev. Lett. 76, 2258 (1996), T. Guhr and A. Müller-Groeling, cond-mat/9702113).

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I. INTRODUCTION

Random matrix ensembles were introduced into physics by Wigner, Dyson and others as phenomenological models of complex quantum systems. Such ensembles are defined as to obey certain symmetries and constraints but are otherwise “as random as possible”. For instance, the Gaussian unitary ensemble (GUE) consists of all $N \times N$ Hermitian matrices $H$, the only constraint being that, on the average, $\text{tr} H^2$ is a given constant. This leads to a probability density in the matrix space, $P_0 \sim \exp \left( -\text{tr} H^2 \right)$, which is invariant under unitary transformations.

Recently there has been some interest in various generalizations of the GUE and its orthogonal and symplectic counterparts. One possible generalization amounts to breaking the $U(N)$ symmetry of the GUE by introducing a parameter $\mu$ into the probability density function

$$P(H) dH = N \exp \left( -F(H) \right) dH,$$

$$F(H) = \sum_{i=1}^{N} H_{ii}^2 + 2 (1 + \mu) \sum_{i<j} \left[ (\text{Re} H_{ij})^2 + (\text{Im} H_{ij})^2 \right].$$

Here $H_{ij}$, with $i \leq j$, designate the independent matrix elements of a $N \times N$ Hermitian matrix, $dH = \prod_{i=1}^{N} dH_{ii} \prod_{i<j} d(\text{Re} H_{ij}) d(\text{Im} H_{ij})$ is the volume element in the matrix space and $N$ is a normalization constant. For $\mu = 0$ the expression in the curly brackets is equal to $(-\text{tr} H^2)$, so that the GUE is recovered. The parameter $\mu$ breaks the $U(N)$ symmetry and introduces a preferential basis. When $\mu \to \infty$, for fixed $N$, all matrices become diagonal in that basis. The ensemble, thus, exhibits a crossover from the Wigner-Dyson statistics of the standard random matrix theory ($\mu = 0$) to the Poisson statistics of uncorrelated levels ($\mu = \infty$). Such an ensemble (for real symmetric matrices) was introduced by Rosenzweig and Porter in their studies of complex atomic spectra, and more recently appeared in the field of quantum chaos and localization.

We shall be interested in the behavior of this ensemble in the $N \to \infty$ limit. In this limit, significant deviations from the GUE-behavior can only occur if $\mu$ increases with $N$ sufficiently fast. The local level statistics is controlled by the parameter $\mu/N^2$. Only when this parameter approaches infinity, the Wigner-Dyson statistics becomes completely obliterated and the Poisson limit of uncorrelated levels is reached. In the opposite case, i.e. when $(\mu/N^2) \to 0$, an arbitrary large sequence of levels will obey the Wigner-Dyson statistics of the GUE. The “critical” case corresponds to $\mu = cN^2$, with $c = \text{const}$. The situation resembles the one which occurs in disordered electronic systems where, in the thermodynamic limit, three distinct types of statistics corresponding to insulator, metal and a mobility-edge system exist.

In the present paper we shall take a closer look at the eigenvalue statistics, with an emphasis on the “critical” case $\mu = cN^2$. We will show that a rather complete picture emerges already from diagrammatic perturbation calculations, along the lines of Refs. 10,11. In this case the two-point correlation function $R(s)$ (smoothed out over few level spacings) differs substantially from both, the GUE and the Poisson correlation functions. Here $s$ denotes the energy difference in units of the average level spacing. For small $c$, $R(s)$ is approximately given by its Wigner-Dyson value, $-1/(2\pi^2 s^2)$, as long as $s \ll 1/\sqrt{c}$ (12). For larger $s$, however, $R(s)$ changes sign, reaches a maximum and eventually decreases as $1/(\pi cs^2)$.

II. DIAGRAMMATIC ANALYSIS

To begin with, let us introduce a definition of the Rosenzweig-Porter (RP) model which is equivalent to Eq. (1) but more convenient for diagrammatic computation. We define...
where \( \varepsilon_i \) are independent real random numbers with Gaussian distribution
\[
p(\varepsilon) = \pi^{-1/2} e^{-\varepsilon^2}.
\]
The matrix elements \( V_{ij} \) of the hermitean matrix \( V \) are independent complex random numbers with Gaussian distributed real and imaginary parts. The distribution is determined by
\[
\langle V_{ij} \rangle = 0, \quad \langle |V_{ij}|^2 \rangle = \frac{1}{2(1+\mu)}.
\]
It is easy to see that the probability density function of the thus defined Hamiltonian is just Eq. (4).

The diagrammatic analysis amounts to a locator expansion of the full single particle Green’s function
\[
G^\pm = (E^\pm - H)^{-1}, \quad E^\pm = E \pm i\delta \quad \delta \gg 0,
\]
with respect to the off-diagonal \( V \). The unperturbed (bare) Green’s function \( G_0 = G^i |_{\delta = 0} \) is called the locator.

We consider the density of states \( \rho(E) = \text{tr} \delta(E - H) \), its average value
\[
\nu(E) = \langle (\rho(E))_V \rangle_\varepsilon
\]
and its correlation function
\[
R(E, E') = \langle (\rho(E)\rho(E'))_V \rangle_\varepsilon - \nu(E)\nu(E').
\]
Here \( \langle \ldots \rangle_\varepsilon \) stands for averaging with respect to the diagonal elements \( V_{ij} \) (the diagonal elements \( \varepsilon_i \)).

We concentrate on energy separations \( \omega = E' - E \) for which both energies are close to the middle of the band, i.e. close to \( E = 0 \) where \( \nu(E) \) is maximal. In this region, \( \nu(E) \) is approximately constant, \( \nu(E) = \nu(E') \) up to \( \mathcal{O}(1/N) \) relative corrections. The density-density correlator \( R(s) \equiv R(E, E')/\nu^2 \) will then be a function of the dimensionless level separation \( s = \omega/\Delta \) where the average level spacing is \( \Delta = 1/\nu(E = 0) \).

We next analyze the spectral correlation function \( R(s) \) in the regime \( s > 1 \) where perturbative methods are applicable. To begin with, we decompose \( R(s) \) according to
\[
R(s) = R_1(s) + R_2(s)
\]
where
\[
R_1(s) = \frac{1}{\nu^2} \left\{ \langle (\rho(E + \omega))_V \langle \rho(E')_V \rangle_\varepsilon - \langle (\rho(E + \omega))_V \rangle_\varepsilon \langle (\rho(E')_V \rangle_\varepsilon \right\},
\]
\[
R_2(s) = \frac{1}{\nu^2} \left\{ \langle (\rho(E + \omega)\rho(E'))_V - \langle (\rho(E + \omega))_V \rangle_\varepsilon \langle (\rho(E')_V \rangle_\varepsilon \right\}.
\]

Note that the decomposition \( R = R_1 + R_2 \) is exact. The physical significance of the two functions \( R_{1/2} \) will be discussed below. Here we merely note that \( R_1 \) measures correlations remaining in the GUE-averaged density of states whereas \( R_2 \) focuses on the GUE-correlations as such.

Representing the density of states in terms of the single particle Green’s function
\[
\rho(E) = -\pi^{-1} \text{Im} \text{tr} G^+(E),
\]
and making use of the fact that correlations (of any type) between products of purely retarded/advanced Green’s functions vanish for \( N \to \infty \): \( \langle G^{\mp^N} \rangle = \langle G^\pm \rangle^N \), we obtain
\[
R_1 = \frac{\Delta^2}{2\pi^2} \text{Re} \left\{ \langle \text{tr} G^+(E + \omega) \rangle_\varepsilon \langle \text{tr} G^-(E) \rangle_\varepsilon \right\},
\]
\[
R_2 = \frac{\Delta^2}{2\pi^2} \text{Re} \left\{ \langle \text{tr} G^+(E + \omega) \text{tr} G^-(E) \rangle_\varepsilon \right\},
\]
where \( \langle \ldots \rangle_\varepsilon \) denotes the connected average, \( \langle XY \rangle_\varepsilon = \langle XY \rangle - \langle X \rangle \langle Y \rangle \). Before turning to the actual calculation of these functions let us make a few methodological remarks and introduce some building blocks that will be of importance throughout. The whole approach will be based on a perturbative expansion of the Green’s functions in powers of \( V \). It is instructive to visualize the structure of the expansion scheme graphically. To this end we introduce the notation of Fig. 1(a) where \( i \) and \( j \) represent matrix indices (which will not be indicated explicitly unless necessary). As a first step of our perturbative analysis (cf. the definition of the correlation function \( R_1 \) above) we have to calculate the \( V \)-average of the Green’s function \( G \). In a diagrammatic language, the expansion of the Green’s function can be visualized as is shown in Fig. 1(b). The subsequent diagrammatic analysis of this equation is simplified drastically by two observations that hold to leading order in \( N^{-1} \).

i) Contributions with ‘crossed GUE-lines’ (see Fig. 1(c)) are negligible \[10\].

ii) Diagram segments which are separated from each other by vertices \( V_{ij} \) are statistically independent with respect to the average over the on-site distribution functions \( \rho(\varepsilon) \).

The second statement is based on the fact that the indices \( i \) and \( j \) in a diagram like the one shown in Fig. 1(f) are eventually summed over independently of each other. (All contributions where one of the summations is constrained will be of higher order in \( N^{-1} \).) On the other hand, the variables \( \varepsilon_i \) at different sites are statistically independent. Put together, these two facts imply that ii) holds to leading order in \( N^{-1} \).

The diagrammatic expression for the \( V \)-averaged Green’s function \( \langle G \rangle_\varepsilon \equiv \langle G_{ij} \rangle_\varepsilon \), subject to rule i), is displayed in Fig. 1(c). The statement ii) implies that to leading order in \( N^{-1} \), the self energy part \( \Sigma^\pm \) (as shown in Fig. 1(d)) can be replaced by the \( \varepsilon_i \) averaged one. We thus obtain

\[
H = H_0 + V, \quad (H_0)_{ij} = \varepsilon_i \delta_{ij}, \quad V_{ii} = 0
\]
In order to solve this equation self consistently, we have to compute the energy average of \( \mathcal{G} \). Anticipating that (a) the self energy will be largely imaginary \( \Sigma \), and (b) \( \Gamma \ll 1 \), we obtain

\[
\langle \mathcal{G}^\pm \rangle_\varepsilon = \frac{1}{\pi i/2} \int d\varepsilon \frac{e^{-\varepsilon^2}}{E - \varepsilon \pm i\Gamma} \approx \mp \frac{1}{\pi i/2} \int d\varepsilon \frac{e^{-\varepsilon^2} i\Gamma}{(E - \varepsilon + i\Gamma)(E - \varepsilon - i\Gamma)} = \mp i\sqrt{\pi}.
\]

The second equality is based on the assumption that the energy argument \( E \ll 1 \) is close to the middle of the band. As a consequence the real part of the integral is of \( O(E) \) and negligible in comparison with the imaginary part \( O(1) \). This justifies the assumption a) above. Collecting everything so far we obtain the \( V \)-averaged Green’s function

\[
\mathcal{G}_i^\pm \approx \frac{1}{E - \varepsilon_i \pm i\Gamma}, \quad \Gamma = \frac{N\pi^{1/2}}{2(1 + \mu)}, \quad (11)
\]

(Note that assumption b) above holds for all \( \mu \approx N \pi^2 \), \( x \gg 1 \), i.e. Eq. (11) indeed represents the self consistent solution of the Dyson equation in Fig. 1(c).) We next insert this expression into the defining equation of the correlation function \( R_1 \) and obtain

\[
R_1 = \frac{\Delta^2 N}{2\pi^2} \text{Re} \left\{ \frac{1}{\pi i/2} \int d\varepsilon \frac{e^{-\varepsilon^2}}{E + \omega - \varepsilon + i\Gamma} \frac{1}{E - \varepsilon - i\Gamma} \right\} \approx \frac{\Delta^2 N}{\pi^3/2 \omega^2 + 4\Gamma^2}.
\]

Noting that the level spacing \( \Delta = \pi^{1/2}/N \), we arrive at the final result

\[
R_1(s) = \frac{1}{\pi c} \frac{1}{s^2 + e^{-2}}, \quad c = \frac{\Delta}{2\Gamma}, \quad (12)
\]

for the first of the above introduced correlation functions. We postpone the discussion of this equation until the complementary correlation function \( R_2 \) has been calculated. In principle one might compute \( R_2 \) via a straightforward perturbative expansion of the Green’s function \( G \). However, experience gained from the analysis of similar correlation functions (11) has shown that it is advantageous to represent the Green’s functions according to

\[
G^\pm(E) = \partial_E \ln(E^\pm - H)
\]

prior to the perturbative expansion. In this way we are led to consider

\[
R_2 = \text{Re} \left\{ \frac{\Delta^2}{2\pi^2} \partial^2_E, E \right|_{E' = E + \omega} \left\langle \left( \text{tr} \ln(E'^\pm - H) \right) \text{tr} \ln(E^\mp - H) \right\rangle_{V, \varepsilon} \right|_{E' = E + \omega}, \quad (13)
\]

Expanding the logarithms in powers of \( V \) and applying the non-crossing rule i) we obtain

\[
R_2 = \frac{\Delta^2}{2\pi^2} \partial^2_E, E \right|_{E' = E + \omega} \text{Re} \sum_{n=2}^\infty \frac{1}{n} \langle S_n(E, E') \rangle_\varepsilon, \quad (13)
\]

where the diagrammatic representation of \( S_n(E, E') \) is shown in Fig. 2. There the segments on the outer (inner) ring correspond to the Green’s function \( G^+(E') (G^-(E)) \) and the two rings are connected by \( n \) \( V \)-lines. (Note that a \( n = 1 \) contribution is absent because the potential \( V \) is off-diagonal in the site indices.) The rule ii) implies that each segment of the ‘wheel’ above can be averaged individually over the on-site energies \( \varepsilon_i \). As a result, the diagram \( S_n \) factorizes, \( S_n = \gamma^n, \gamma = N/(2(1 + \mu)) (G^+(E') G^-(E))_\varepsilon \) and \( \sum_{n=2}^\infty \frac{1}{n} \langle S_n \rangle_\varepsilon = -\ln(1 - \gamma) - \gamma \). Computing the energy average (cf. the computation of the correlation function \( R_1 \) above)

\[
\langle G^+(E') G^-(E) \rangle_\varepsilon = \frac{2\pi^{1/2} i}{E' - E + 2i\Gamma},
\]

and collecting constants we obtain

\[
R_2(s) = \frac{1}{2\pi^2} \partial^2_E \text{Re} \left[ \ln \left( \frac{s}{s - ic^{-1}} \right) - \frac{ic^{-1}}{s - ic^{-1}} \right]. \quad (14)
\]

We finally carry out the differentiation and add the \( R_1 \) contribution (12) to arrive at the final result

\[
R(s) = \frac{1}{c\pi} \frac{1}{s^2 + e^{-2}} + \frac{1}{2\pi^2} \left\{ \frac{1}{s^2} + \frac{s^2 - 3e^{-2}}{(s^2 + e^{-2})^2} + \frac{8s^2e^{-2}}{(s^2 + e^{-2})^3} \right\}. \quad (15)
\]

Eq. (15) is applicable when the energy \( s \gg 1 \) and fine structures on scales \( s \approx 1 \) are inessential. Let us conclude this section with a brief discussion of this result.

The contribution \( R_1 \) (cf. Eq. (12)) has the following interpretation: The \( V \)-averaged Green’s function \( \mathcal{G} \) is similar to \( G_0 \) except for the fact that a finite width \( \Gamma \) has been attached to each of the uncorrelated levels \( \varepsilon_i \). This ‘smearing’ implies that the corresponding correlation function \( R_1 \) is Lorentzian, i.e. it is a broadened version of the \( \delta \)-function that would be obtained for sharply defined autocorrelated levels. The complementary term \( R_2 \) describes correlations between the \( V \) degrees of freedom. After combining the two contributions three qualitatively different regimes can be identified:

- For \( s \gg c^{-1} \) the dominant contribution comes from \( R_1 \) and we obtain \( R(s) \approx (\pi cs^2)^{-1} \).
- \( c^{-1/2} \ll s \ll c^{-1} \): Still \( R_1 \) dominates but now \( R_1 \approx c/\pi \).
\[ s \ll c^{-1/2} \]: The \( R_2 \) contribution becomes the dominant one and we obtain the GUE-result \( R_2 \approx -(2\pi^2 s^2)^{-1} \) corrected by a small term \( R_1 \approx c/\pi \).

In summary, Eq. (13) essentially represents a superposition of a GUE correlation function and a smeared Poissonian auto-correlation function.

III. NON-PERTURBATIVE RESULTS

The diagrammatic treatment is incapable of describing structures on the energy scale of \( O(\Delta) \). For \( c \gg 1 \) an alternative perturbation technique, applicable over the whole energy axis, can be used. Within this approach spectral correlations are described in terms of stochastic evolution equations (see also Ref. [14]). In this way one obtains a spectral correlation function \( R(s) \) that depends only on the combination \( s\sqrt{c} \). For large energies, \( s \gg 1/\sqrt{c} \), the result coincides with ours, i.e. \( R(s) \approx (\pi cs^2)^{-1} \), and for small energies, \( s \ll 1/\sqrt{c} \) level repulsion sets in, i.e. \( R(s) + 1 \propto cs^2 \).

The complementary regime of \( c \ll 1 \) can be treated by Efetov’s non-perturbative supersymmetry technique [13] (for a recent review see [15]), where averages of Green’s functions are obtained from a generating functional. The generating functional corresponding to the RP model is similar to the one described in [14]. In these works the problem of a random banded matrix with additional diagonal disorder was addressed. Taking the band width equal to the matrix size \( N \) leads to the RP model. From the generating functional one can obtain the correlation function of retarded and advanced Green’s functions \( K^{12} = \langle \text{tr} G^+(E + \omega) \text{tr} G^-(E) \rangle \) where the average is taken with respect to Eq. (1). The final integrations can be carried out within a saddle-point expansion the validity of which is controlled by \( N^2/\mu \gg 1 \) or equivalently by \( c \ll 1 \), and by \( \omega \ll N/\mu \) or equivalently by \( s \ll c^{-1} \). In the present work we skip the technical details and concentrate on the discussion of the results.

In the limit \( N \rightarrow \infty \) the function \( K^{12} \) is given by

\[
K^{12}(E, \omega) = \left(1 + \frac{c}{\pi}\right) \left(-\frac{2i}{s^2 \Delta^2} e^{-i\pi s} \sin(\pi s) + \frac{2ic}{\Delta^2} + \frac{2\pi c}{\Delta^2}\right) \quad \mathrm{(16)}
\]

The first term is the entirely disconnected part and the terms of \( O(c) \) describe deviations from a pure GUE behavior. These terms represent the analogue of the contribution \( R_1 \) that appeared in the diagrammatic analysis. They result from the correlation between the on-site energies \( \epsilon_i \). (Note that in principle correlations of this type exist in the pure GUE as well. In that case, however, they represent negligible \( O(1/N) \)-effect.) From Eq. (16) we obtain the correlation function

\[
R(s) = \left(1 + \frac{c}{\pi}\right) \left(\frac{\sin(\pi s)}{\pi s} \right)^2 \delta(s) + \frac{c}{\pi} \quad \mathrm{(17)}
\]

describing the spectral behavior in the regime \( s \ll c^{-1} \), \( c \ll 1 \). We next turn to the discussion of this result. We first observe that the term \( c/\pi \) equals the leading contribution of the smeared auto-correlation \( R_1(s) \) for \( s \ll c^{-1} \). For very small level separations \( R(s) \) behaves as \(-1 + (1 + c/\pi)(\pi s)^2/3 \), i.e. apart from a slightly modified prefactor we obtain generic GUE-behavior. For larger values \( 1 \ll s \ll c^{-1} \) the leading terms are identical with those obtained in the diagrammatic treatment, as expected. (By ‘leading’ we mean the first order terms of an expansion in the parameter \( 1/s \ll 1 \) after the oscillatory structure in \( G \) has been averaged out.) In particular, the GUE behavior is only valid up to \( s \ll 1/\sqrt{c} \). Thus, the non-perturbative results underline the conclusion drawn from the diagrammatic analysis: \( R(s) \) is essentially a superposition of a GUE correlation with a smeared Poissonian auto-correlation. A conclusion to be drawn from this observation is that the analogy between the Wigner-Dyson/Poisson transition in the RP model and disordered electron systems, respectively, is not complete. In the latter case the critical correlation function can hardly be interpreted as a simple superposition of two terms. This qualitative difference manifests e.g. in the behavior of the level compressibility, \( \chi = \lim_{N \rightarrow \infty} \int_{-S}^{S} ds R(s) \) (where it is essential that the limit \( N \rightarrow \infty \) is taken first). The two extremes GUE (Poisson) correspond to values \( \chi = 0 \) (\( \chi = 1 \)). In the case of a disordered metal at criticality the compressibility takes an intermediate value \( 0 < \chi < 1 \) [17]. In the critical RP model, however, \( \chi = 1 \), i.e. perturbing a Poisson ensemble by a GUE ensemble does not change the level compressibility [13].

Finally, we would like to comment on the analysis [18][19]. In these references, the RP model was solved exactly for arbitrary values of the parameters \( \mu \) and \( s \). As a result of a sophisticated combination of supersymmetry and group-theoretical concepts Guhr [13] obtained non-trivial double integral representations for the correlation functions which turned out to be difficult to evaluate. In order to derive closed expressions for \( R(s) \) the integral was analyzed in the two limiting cases \( c \gg 1 \) [13] and \( c \ll 1 \) [19] by means of asymptotic expansion schemes. The price to pay for the mathematical rigor of Guhr’s approach is that the physical origin of the various ingredients to \( R(s) \) is hard to identify. For this reason we believe, that a more conventional analysis like the one discussed above was calling for.

IV. CONCLUSIONS

We have studied the density of states correlation function \( R(s) \) (\( s \) measures the energy difference in units of the average level spacing) of the Rosenzweig-Porter model. This random matrix model contains a parameter \( \mu \) which
allows to interpolate between GUE ($\mu = 0$) and Poisson statistics ($\mu = \infty$). In the thermodynamic limit $N \to \infty$ the model shows three different types of universal functions $R(s)$ depending on how $\mu$ scales with $N$. From a diagrammatic analysis (locater expansion) assisted by non-perturbative methods we draw the following conclusions: parameter values scaling like $\mu(N)/N^2 \to 0$ ($\mu(N)/N^2 \to \infty$) lead to GUE (Poisson) statistics. In the borderline case $\mu(N)/N^2 \equiv c$, however, a novel universal type of spectral behavior is observed. The corresponding correlation function $R(s)$ has the following features: Like in the GUE-case levels repel each other, i.e. $R(s) \to -1$ for $s \to 0$. At some $c$-dependent value $s_0$, $R(s)$ changes sign, then reaches a maximum and decreases as $(\pi \cos^2)^{-1}$ for large $s$. For $c \ll 1$ we find that the spectrum shows GUE-type statistics up to values $s \sim 1/\sqrt{c}$. For larger values of $s$, a different type of statistics is observed, this being a consequence of the non-GUE-correlation of the diagonal matrix elements in the RP model. These large energy correlations can be interpreted as the tails of a widely 'smeared' energy autocorrelation of Poissonian type. We thus conclude that the RP model in the critical state of Wigner-Dyson and Poissonian behavior. Let us finally comment on the aspect of symmetries. In this paper we have considered the Rosenzweig-Porter model in its unitary version. It is a straightforward matter to extend both the diagrammatic and the non-perturbative analysis to the case of orthogonal respectively symplectic symmetry. On the other hand, none of our main conclusions on the structure of the models eigenvalue statistics did depend in a conceptual way on symmetry aspects. We thus expect the level statistics of the models of higher symmetry to be qualitatively similar but did not embark on any kind of quantitative analysis.

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Figure 1: (a) Graphical representation of the Green's function $G_0$, the perturbation matrix elements $V_{ij}$ and its correlations. (b),(c) The Dyson equation for the full and the averaged Green's function, respectively. (d) Graphical representation of the self-energy. (e) A diagram with crossed GUE lines. (f) Explanation see text.

Figure 2: Graphical representation of $S_0(E, E')$ appearing in Eq. (13)
\[ G_{0,i} = \begin{array}{c} i \end{array} \]

\[ V_{ij} = \begin{array}{c} i \end{array} \begin{array}{c} j \end{array} \quad \langle V_{ij} V_{ji} \rangle_V = \begin{array}{c} j \end{array} \begin{array}{c} i \end{array} \]

(a)

\[ G = \begin{array}{c} \end{array} + \begin{array}{c} \end{array} \]

(b)

\[ G = \begin{array}{c} \end{array} + \begin{array}{c} \end{array} \]

(c)

\[ \Sigma = \begin{array}{c} \end{array} \]

(d)

\[ \begin{array}{c} \end{array} \]

(e)

\[ \begin{array}{c} \end{array} \]

(f)