Solar wind and motion of dust grains

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Abstract. Action of solar wind on arbitrarily shaped interplanetary dust particle is investigated. The final relativistically covariant equation of motion of the particle contains both orbital evolution and change of particle’s mass. Non-radial solar wind velocity vector is also included. The covariant equation of motion reduces to the Poynting-Robertson effect in the limiting case when spherical particle is treated, the speed of the incident solar wind corpuscles tends to the speed of light and the corpuscles spread radially from the Sun. The results of quantum mechanics have to be incorporated into the physical considerations, in order to obtain the limiting case. The condition for the solar wind effect on motion of spherical interplanetary dust particle is

\[ p'_{\text{out}} = (1 - \sigma'_{\text{pr}}/\sigma'_{\text{tot}}) p'_{\text{in}}, \]

where \( p'_{\text{in}} \) and \( p'_{\text{out}} \) are incoming and outgoing radiation momenta (per unit time) measured in the proper frame of reference of the particle; \( \sigma'_{\text{pr}} \) and \( \sigma'_{\text{tot}} \) are solar wind pressure and total scattering cross sections.

Analytical solution of the derived equation of motion yields qualitative behaviour consistent with numerical calculation. This holds also if decrease of particle’s mass is important, when also outspiralling from the Sun may occur.

Real flux density of radial solar wind energy yields that the time of spiralling toward the Sun may differ in about 10 % from the case of constant flux for heliocentric distances smaller than about 10 AU (if radius of the particle is unchanged). The differences between the real solar wind effect (when also non-radial solar wind velocity vector is taken into account) and the standard approach may be even more significant for heliocentric distances greater than 10 AU: solar wind can cause outspiralling from the Sun, even if radius of the particle is constant.

Real flux density of solar wind energy produces shift of perihelion of interplanetary dust particles. This result significantly differs from the standard treatment of the action of the solar wind on dust particles, when analogy with the Poynting-Robertson effect is stressed. Moreover, the evolution of the shift of perihelion depends on orbital position of the parent body at the time of ejection of the particle.

Key words. cosmic dust, electromagnetic radiation, solar wind, relativity theory, quantum theory, equation of motion, orbital evolution

1. Introduction

The Poynting-Robertson effect is used in modelling of orbital evolution of dust grains under the action of electromagnetic radiation (of the central star), for many decades (e. g., Poynting 1903, Robertson 1937, Wyatt and Whipple 1950, Dohnanyi 1978, Kapišinský 1984, Jackson and Zook 1989, Leinert and Grün 1990, Gustafson 1994, Dermott et al. 1994, Reach et al. 1995). It was presented that solar wind operates in a similar way. The action of the solar wind on motion of interplanetary dust particle was discussed, in a heuristic way, e. g., by Whipple (1955). He has mentioned also the results of laboratory experiments: intense bombardment of a material by energetic corpuscles destructs the material and this effect is known as a "sputtering". Current opinion is that we have two different effects of the solar wind: i) motion of dust particle is influenced by the incident solar wind, and, ii) the corpuscular sputtering decreases mass of the particle (e. g., Whipple 1955, Dohnanyi 1978, Kapišinský 1984, Leinert and Grün 1990). There has been an attempt to better understand physics of the action of the solar wind on the motion of dust particle. As a first attempt, we can mention Robertson and Noonan (1968, pp. 122-123). The authors formulated relativistically covariant equation of motion of the particle under the action of the solar wind. However, their result does not admit any destruction of
the particle. A more realistic view was presented in Klačka and Saniga (1993), where also space-time formulation of the problem was suggested. As a result, corpuscular sputtering is an indispensable part of the equation of motion for the action of solar wind on interplanetary dust particle.

Our paper presents space-time formulation of the action of solar wind on arbitrarily shaped interplanetary dust particle. Equation of motion in a relativistically covariant form is derived. Moreover, in order to be physically correct, results of quantum theory are also taken into account. The results of the paper are consistent with the results of the papers by Klačka (2008a, 2008b) for the electromagnetic radiation. Our theoretical derivations hold for any solar wind velocity vector and the result can be easily applied to other stars with stellar winds.

Application of the derived equation of motion to spherical interplanetary dust particle is presented in the form of orbital evolution of the particle. While submicron dust particles are driven mainly by Lorentz force (motion of charged particles in the interplanetary magnetic field, Dohnanyi 1978, Leinert and Grün 1990, Dermott et al. 2001), collisions among particles are important for particles of radii larger than hundred of micrometres, approximately (Grün et al. 1985, Dermott et al. 2001). This paper deals with orbital evolution of micron-sized spherical interplanetary dust particles, when effects of solar gravity, solar electromagnetic radiation and solar wind (solar corpuscular radiation) are relevant. Radial solar wind is conventionally used. However, the newest observations (Bruno et al. 2003) show that velocity vector of the solar wind corpuscles is non-radial and the angle between the velocity vector and radial direction is practically independent on heliocentric distance. Our paper compares orbital evolution of spherical interplanetary dust particle for standardly used approach of time independent radial solar wind and more real solar wind model. Also the action of mass decrease of the interplanetary dust particle is taken into account.

Section 2 derives relativistically covariant equation of motion of an arbitrarily shaped particle under the action of solar wind (including non-radial component of the solar wind velocity). Sec. 3 summarizes important equations for the Poynting-Robertson effect. Equation of motion of the spherical particle under the action of solar corpuscular and electromagnetic radiation, and, solar gravity, is given in Sec. 4. Sec. 5 deals with secular evolution of particle’s orbital elements under the action of solar radiation (electromagnetic and corpuscular – solar wind), in an analytical way. The following section Sec. 6 concentrates on detail treatment of the numerical results and compares the results for conventional time independent radial solar wind with those obtained for more real solar wind model.

2. Equation of motion - solar wind effect

Relativistically covariant equation of motion of interplanetary dust particle is derived in this section. Our derivation enables understand physics of the action of the solar wind on the motion of dust particle (compare heuristic explanation by Whipple 1955 and space-time formulations presented by Robertson and Noonan 1968, pp. 122-123, and, Klačka and Saniga 1993).

We will show that the corpuscular sputtering is an indispensable part of the action of the solar wind on the interplanetary dust particle, so the sputtering cannot be considered as an another effect of the solar wind. Moreover, non-radial solar wind velocity vector can be easily incorporated into the final equation of motion. Finally, the covariant formulation yields the Poynting-Robertson effect in the limiting case when speed of the solar wind corpuscles tends to the speed of light. The limiting case is fulfilled under the assumption that the total cross section of the interaction between the solar wind corpuscles and the interplanetary dust particle is given by the results of quantum theory and not by the classical non-quantum physics.

This section presents also results with an accuracy to the order \((v/u)^2\), where \(v\) is orbital velocity of the particle with respect to the Sun and \(u\) is the solar wind speed. These results will be used in practical modelling of orbital evolution of interplanetary dust grains in Secs. 4 and 5.
2.1. Incident radiation

Let us introduce two inertial reference frames. The first is the proper reference frame of a particle moving with velocity \( \mathbf{v} \) around the Sun. The particle is at rest in its proper frame of reference. Quantities measured in this frame will be primed. The second frame is associated with the Sun. This frame is "stationary reference frame".

We will suppose that all corpuscles of the solar wind are of the same mass \( m_1 \) and of the same velocity \( \mathbf{u} \) (or \( \mathbf{u}' \) in the proper reference frame of the particle). Thus, each of the corpuscles has the following four-momentum

\[
\mathbf{p}'_1 = \left( E'_1/c ; \mathbf{p}'_1 \right) = m_1 \gamma (u') (c ; \mathbf{u}')
\]

in the proper reference frame of the interplanetary dust particle, or

\[
\mathbf{p}'_1 = \left( E_1/c ; \mathbf{p}_1 \right) = m_1 \gamma (u) (c ; \mathbf{u})
\]

in the stationary reference frame; \( c \) is the speed of light.

Let a beam of such solar wind corpuscles hits the dust grain. Energy and momentum incident on the particle per unit time in its proper frame are

\[
E'_{in} = \sigma'_{tot} n' u' E'_1 ,
\]

\[
\mathbf{p}'_{in} = \sigma'_{tot} n' u' \mathbf{p}'_1 ,
\]

where \( n' \) is the concentration of the solar wind corpuscles and \( \sigma'_{tot} \) is the total scattering cross section of the interplanetary dust particle. Using Eq. (1), we can rewrite Eqs. (3) into the form of the incident four-momentum per unit time

\[
\mathbf{p}'_{in} = \sigma'_{tot} n' u' \left( \frac{E'_1}{c} ; \mathbf{p}'_1 \right)
\]

\[
E'_{in} = \frac{1}{c} \sigma'_{tot} n' u' E'_1 \left( 1 ; \frac{u'}{c} \right) .
\]

Introducing the flux density of the incident energy (energy flow per unit area perpendicular to the beam of solar wind corpuscles per unit time)

\[
S' = n' u' E'_1 ,
\]

Eq. (4) can be rewritten to the form

\[
\mathbf{p}'_{in} = \left( \frac{E'_1}{c} ; \mathbf{p}'_{in} \right),
\]

\[
\mathbf{p}'_{in} = \frac{1}{c} S' \sigma'_{tot} \left( 1 ; \frac{u'}{c} \right) .
\]

Having a four-vector \( B'^\mu = (B'^0 ; \mathbf{B}') \) in the proper reference frame, the components of the four-vector in the stationary reference frame are given by generalized special Lorentz transformation:

\[
B'^0 = \gamma (v) \left( B'^0 + \frac{v \cdot \mathbf{B}'}{c} \right),
\]

\[
\mathbf{B} = \mathbf{B}' + \left\{ \frac{\gamma (v) - 1}{v^2} \frac{v \cdot \mathbf{B}'}{c} + \frac{\gamma (v)}{c} B'^0 \right\} v,
\]

or inverse
\[ B'^0 = \gamma(v) \left( B^0 - \frac{v \cdot B}{c} \right), \]

\[ B' = B + \left\{ \gamma(v) - 1 \right\} \frac{v \cdot B}{v^2} - \gamma(v) \frac{B^0}{c} \right\} v. \quad (8) \]

Now, using Eqs. (6) and (7) we get

\[ p_0^{in} = \frac{1}{c} S' \sigma'_\text{tot} \gamma(v) \left( 1 \frac{v \cdot u'}{e^2} \right), \]

\[ p_{in} = \frac{1}{c^2} S' \sigma'_\text{tot} \left\{ u' + \left[ \frac{\gamma(v) - 1}{v^2} \frac{v \cdot u'}{e^2} + \gamma(v) \right] v \right\}. \quad (9) \]

We have to express the primed quantities (except of \( \sigma'_\text{tot} \)) on the right-hand sides of Eqs. (9), i.e. \( S' = n'u'E'_1 \) and \( u' \), through the unprimed quantities measured in the stationary reference frame of the Sun. The energy \( E'_1 \) we obtain from Lorentz transformation of \( p_1^\mu \) to the proper reference frame of the interplanetary dust particle. It holds

\[ E'_1 = \gamma(v) (E_1 - v \cdot p_1) = \gamma(v) \left( 1 \frac{v \cdot u}{e^2} \right) E_1 = \omega E_1, \quad (10) \]

where we defined the quantity

\[ \omega \equiv \gamma(v) \left( 1 \frac{v \cdot u}{e^2} \right). \quad (11) \]

Other quantities we get from transformation of the four-vector of the current density \( j'^\mu = (nc; nu) \) to the corresponding four-vector \( j'^\mu = (n'c; n'u') \). The transformation yields

\[ n' = \omega n, \]

\[ u' = \frac{1}{\omega} \alpha, \quad (12) \]

where the vector

\[ \alpha \equiv u + \left[ (\gamma(v) - 1) \frac{v \cdot u}{e^2} - \gamma(v) \right] v, \quad (13) \]

has magnitude

\[ \alpha = \left\{ u^2 + \gamma^2(v) v^2 - 2\gamma^2(v) v \cdot u + \gamma^2(v) \left( \frac{v \cdot u}{e^2} \right)^2 \right\}^{1/2}. \quad (14) \]

Thus, \( u' = \alpha/\omega \) and the flux density of energy is

\[ S' = \frac{\alpha \omega}{u} S, \]

\[ S \equiv nuE_1, \quad (15) \]

according to Eqs. (5), (10) and (12).

Finally, using Eqs. (9), (11), (12), (13) and (15), one obtains

\[ p_{in}^0 = \frac{1}{c} \sigma'_\text{tot} S \frac{\alpha \omega}{u} \frac{1}{\omega_1}, \]

\[ p_{in} = \frac{1}{c^2} \sigma'_\text{tot} S \frac{\alpha \omega}{u} \frac{1}{\omega} \frac{u}{c}. \quad (16) \]

The incident four-momentum of solar wind per unit time is

\[ p_{in}^\mu = \frac{1}{c} \sigma'_\text{tot} S \frac{\omega}{u} \xi^\mu, \]

\[ \xi^\mu \equiv \left( \frac{1}{\omega}; \frac{1}{\omega} \frac{u}{c} \right). \quad (17) \]
2.2. The reaction of spherical dust particle on the incident solar wind

The incident solar wind corpuscle may be reflected from the surface of the interplanetary dust particle (IDP) or may cause its erosion/destruction and decrease the mass of the IDP, or, in general, similar processes as reflection, absorption and diffraction may occur. Let the particle’s loss of energy (per unit time) in the proper reference frame of the IDP, be $E'_{\text{out}} - E'_{\text{in}}$. $E'_{\text{out}}$ can be written as an $x'$-part of the incident energy per unit time. The relation

$$E'_{\text{out}} = x' E'_{\text{in}}$$  \hspace{1cm} (18)

holds for the outgoing energy. In order to express the outgoing momentum we declare the orthonormal vector basis $\{f'_{j}; j = 1, 2, 3\}$ in the proper reference frame of the particle and three velocity vectors $\{u'_{j} = u' f'_{j}; j = 1, 2, 3\}$ corresponding to these unit vectors. We suppose that $u'_1 \equiv u'$. Now, the outgoing momentum per unit time is

$$p'_{\text{out}} = \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \right) p'_{\text{in}} - \sigma'_{\text{tot}} \frac{S'}{c} \sum_{j=2}^{3} \frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}} u'_{j},$$  \hspace{1cm} (19)

where $\sigma'_{\text{pr},j} (j = 1, 2, 3; \sigma'_{\text{pr},1} \equiv \sigma'_{\text{pr}})$ are pressure cross sections (analogy with optics – see Klaćka 2008a, 2008b).

The outgoing four-momentum per unit time is given by Eqs. (6), and (18)-(19):

$$p''_{\mu\text{out}} = \left( \frac{E'_{\text{out}}}{c}; p'_{\text{out}} \right),$$

$$p''_{\mu\text{out}} = \left\{ \frac{1}{c} \sigma'_{\text{tot}} S' x'; \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \right) p'_{\text{in}} - \sigma'_{\text{tot}} \frac{S'}{c} \sum_{j=2}^{3} \frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}} u'_{j} \right\}.$$  \hspace{1cm} (20)

Generalized special Lorentz transformation of $p''_{\mu\text{out}}$, using Eqs. (15), gives for the outgoing four-momentum per unit time in the stationary reference frame

$$p''_{\mu\text{out}} = \frac{1}{c} \sigma'_{\text{tot}} S \frac{\alpha}{u} \frac{U'_{\mu}}{c}$$

$$+ \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \right) \frac{1}{c} \sigma'_{\text{tot}} S \frac{\alpha}{u} \left( \xi'_{\mu} - \frac{U'_{\mu}}{c} \right)$$

$$- \sigma'_{\text{tot}} \frac{3}{c} \sum_{j=2}^{3} \frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}} \left( \xi'_{\mu} - \frac{U'_{\mu}}{c} \right),$$  \hspace{1cm} (21)

where $U'_{\mu} = (\gamma(v) c; \gamma(v) v)$

is four-velocity of the IDP. The other four-vectors are

$$\xi'_{\mu} = \left( \frac{1}{\omega_{j}}; \frac{1}{\omega_{j}} \frac{u_{j}}{c} \right),$$

$$\omega_{j} \equiv \gamma(v) \left(1 - \frac{v \cdot u_{j}}{c^2} \right),$$

$$u_{j} = \left[ \gamma(v) \left(1 + \frac{v \cdot u'_{j}}{c^2} \right) \right]^{-1} \left\{ u'_{j} + \left[ (\gamma(v) - 1) \frac{v \cdot u'_{j}}{v^2} + \gamma(v) \right] v \right\},$$  \hspace{1cm} (23)

and $\omega_{1} \equiv \omega$, $\xi'_{1} \equiv \xi'_{\mu}$, $u_{1} \equiv u$. 

2.3. Equation of motion

Now we can write equation of motion of the IDP under the action of the solar wind, in a relativistically covariant form:

$$\frac{dp^\mu}{d\tau} = p^\mu_{\text{in}} - p^\mu_{\text{out}}.$$  \hspace{1cm} (24)

Eq. (24) yields, using Eqs. (17) and (21),

$$\frac{dp^\mu}{d\tau} = \frac{1}{c} \sigma'_{\text{tot}} S \frac{\alpha \omega}{u} \nonumber \times \left\{ \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \xi^\mu - \left[ x' - \left( 1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \right) \frac{U^\mu}{c} \right] \right\} \nonumber + \sigma'_{\text{tot}} S \frac{\alpha \omega}{u} \sum_{j=2}^{3} \frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}} \left( \xi^\mu_j - \frac{U^\mu}{c} \right),$$ \hspace{1cm} (25)

where $p^\mu = m U^\mu$ is four-momentum of the IDP of mass $m$ and $\tau$ is the proper time of the particle.

Using

$$\frac{dp^\mu}{d\tau} = \frac{d}{d\tau} (m U^\mu) = \frac{dm}{d\tau} U^\mu + m \frac{dU^\mu}{d\tau},$$ \hspace{1cm} (26)

Eq. (25) yields not only acceleration of the particle, but also change of the particle’s (rest) mass, due to the interaction of the IDP with the solar wind. The change of the mass is given, on the basis of Eqs. (11), (17), (22), (25) and (26), by the expression ($U_\mu U^\mu = c^2$, $U_\mu dU^\mu/d\tau = 0$)

$$\frac{dm}{d\tau} = - \frac{1}{c^2} \sigma'_{\text{tot}} S \frac{\alpha \omega}{u} (x' - 1).$$ \hspace{1cm} (27)

One can easily verify that Eq. (27) corresponds to the famous Einstein’s equation $dm/d\tau = (E_{\text{in}} - E_{\text{out}})/c^2$, if also Eqs. (3) and (18) are used.

Eqs. (25)-(27) yield for the four-acceleration of the IDP

$$\frac{dU^\mu}{d\tau} = \sigma'_{\text{pr}} S \frac{mc}{u} \frac{\alpha \omega}{u} \left( \xi^\mu - \frac{U^\mu}{c} \right) + \sigma'_{\text{tot}} S \frac{mc}{u} \frac{\alpha \omega}{u} \sum_{j=2}^{3} \frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}} \left( \xi^\mu_j - \frac{U^\mu}{c} \right).$$ \hspace{1cm} (28)

In the further treatment we will consider the case $\sigma'_{\text{pr},j} \equiv 0$ for $j = 1, 2$. As a consequence, the equation of motion will be of the form

$$\frac{dp^\mu}{d\tau} = \frac{1}{c} \sigma'_{\text{tot}} S \frac{\alpha \omega}{u} \nonumber \times \left\{ \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \xi^\mu - \left[ x' - \left( 1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}} \right) \frac{U^\mu}{c} \right] \right\} \hspace{1cm} (29)$$

and the four-acceleration will be

$$\frac{dU^\mu}{d\tau} = \sigma'_{\text{pr}} S \frac{mc}{u} \frac{\alpha \omega}{u} \left( \xi^\mu - \frac{U^\mu}{c} \right).$$ \hspace{1cm} (30)

The change of particle’s mass is given by Eq. (27). Conventional approach is that the force due to the solar wind bombardment considers fixed mass of the particle (see, e. g., Mukai and Yamamoto 1982).
2.4. Total scattering cross section

We have used the total scattering and pressure cross sections, in the previous theoretical parts. We need to determine them. We will consider spherical particle.

We will use a some sort of approximation to the hard-core scattering problem, which corresponds to the limiting case of a short-range potential $V(r) = \infty$ for $r < R$, $V(r) = 0$ for $r > R$ (hard-core potential, see, e. g., Iro 2002, p. 158). We will consider the scattering of point solar wind corpuscles from an almost hard sphere of radius $R$. In the case of the infinitely hard sphere of radius $R$, "the dynamics reduces to the laws of reflection at the surface of the sphere" (Iro 2002, p. 158). The result of classical physics is following: "In the case of a finite-range potential, the total [scattering] cross section is finite and gives the effective area of the potential. (This is actually the definition of a finite-range potential.) For example, when point masses are incident onto a hard sphere, $\sigma'_\text{tot}$ is the cross section of the sphere – only particles incident within that area are deflected." (Iro 2002, p. 161).

However, correct physics for the incident electromagnetic radiation suggests that geometric cross section may not lead to correct results for the incident solar wind corpuscles (Klačka 2008a, 2008b). Inspiring by de Broglie’s idea about wave character of massive particles, one can come to the conclusion that scattering by a hard sphere at very high energies leads to the total scattering cross section

$$\sigma'_\text{tot} = 2\pi R^2$$

and "the classical total cross section is just half of the quantum-mechanical result in the limit of very short wavelength" (Messiah 1999, pp. 393-395).

If we use a some sort of approximation to the hard sphere, we can use the total scattering cross section given by Eq. (31), in our paper.

As for the comparison of the results obtained by quantum and non-quantum physics, we will use Eq. (29) or Eq. (25). The non-quantum approach uses $\sigma'_\text{tot} = \sigma'_\text{pr} = A' = \pi R^2$:

$$\left(\frac{dp^\mu}{d\tau}\right)_{\text{non-quantum}} = \frac{1}{c} A' S \frac{\alpha \omega}{u} \left(\xi^\mu - x' \frac{U^\mu}{c}\right).$$

Comparison between Eqs. (29) and (32) yields (compare coefficients at $\xi^\mu$ and $U^\mu$):

$$\sigma'_\text{pr} = A',$$

$$x'(\text{quantum}) = [x'(\text{non-quantum}) - 1] \frac{\sigma'_\text{pr}}{\sigma'_\text{tot}} + 1.$$

Using also Eq. (31), one obtains

$$x'(\text{quantum}) = \frac{1}{2} [x'(\text{non-quantum}) + 1].$$

The case $\{\sigma'_\text{pr} = A', \sigma'_\text{tot} = 2A'\}$ is analogous to the cases of perfectly absorbing or reflecting spheres within geometrical optics approximation for electromagnetic radiation (Klačka 2008b). This analogy explains also the importance of quantum physics in our derivations – non-quantum physics would not yield correct results in the limit $u \to c$.

2.5. Equation of motion to the second order in $v/u$

In the approximation to the first order in $v/c$, we can replace the spacelike part of the four-acceleration of the IDP by acceleration $dv/dt$, where $t$ is time measured in the stationary reference frame (associated with the Sun). Further, using Eqs. (2), (11), (14), (15), (17) and (22), we can express the right-hand
side of Eq. (30) in the approximation to the second order in \(v/u\). We get

\[
\frac{dv}{dt} = A'n'nm_1u^2 \left\{ \left[ 1 - \frac{v \cdot \dot{u}}{u} \right] \frac{\dot{u}}{u} - \frac{v}{u} \right\} + \frac{1}{2} \left( \frac{v^2}{u^2} \right)^2 \frac{\dot{u}}{u},
\]

where \(\dot{u} \equiv u/u\) is the unit vector in direction of the solar wind.

Let us introduce the cylindrical coordinate system associated with the orbital plane of the IDP and determined by unit vectors \(e_R\) (radial vector), \(e_T\) (transversal vector) and \(e_N = e_R \times e_T\) (normal vector). We can write (Klačka 1994)

\[
\dot{u} = \gamma_R e_R + \gamma_T \hat{u}_T,
\]

where

\[
\gamma_R = \cos \varepsilon, \quad \gamma_T = \sin \varepsilon
\]

and

\[
\hat{u}_T = \frac{1}{N} \mathbf{k} \times e_R = \frac{1}{N} \left( e_T \cos i - e_N \cos \Theta \sin i \right),
\]

\[
N = \sqrt{(\cos i)^2 + (\cos \Theta)^2 (\sin i)^2}.
\]

The quantity \(\varepsilon\) is an angle between the radial direction and the real direction of the solar wind. The unit vector \(\mathbf{k}\) corresponds to the vector of angular velocity of solar rotation. The inclination of the orbital plane of the IDP with respect to the solar equatorial plane is \(i\). Finally, \(\Theta\) is a position angle of the IDP (an angle measured from the ascending node of the orbit of the IDP to its actual position).

Inserting Eqs. (36) and (38) to Eq. (35), and using the decomposition of the velocity vector into its radial and transversal components, \(v = v_R e_R + v_T e_T\), one obtains

\[
\frac{dv}{dt} = A'n'nm_1u^2 \left\{ X_R e_R + X_T e_T - X_N e_N \right. \]

\[
\left. + \gamma_R \frac{v_R}{u} \frac{v}{u} + \gamma_T \frac{\cos i}{N} \frac{v_T}{u} \frac{v}{u} \right\},
\]

\[
X_R = \gamma_R - \left( 1 + \frac{\gamma_T}{R} \right) \frac{v_R}{u} - \gamma_R \gamma_T \frac{\cos i}{N} \frac{v_T}{u} + \gamma_R \frac{1}{2} \frac{v^2}{u^2},
\]

\[
X_T = \left( 1 - \gamma_R \frac{v_R}{u} - \gamma_T \frac{\cos i}{N} \frac{v_T}{u} + \frac{1}{2} \frac{v^2}{u^2} \right) \gamma_T \frac{\cos i}{N} \frac{v_T}{u},
\]

\[
X_N = \left( 1 - \gamma_R \frac{v_R}{u} - \gamma_T \frac{\cos i}{N} \frac{v_T}{u} + \frac{1}{2} \frac{v^2}{u^2} \right) \frac{\gamma_T \cos \Theta \sin i}{N}.
\]

The angle \(\varepsilon\) is small, its value lies between 2° - 3° (Bruno et al. 2003). Thus, we can neglect terms proportional to \(\gamma_T^2\) and \(\gamma_T (v/u)^2\). Similarly, we put \(\gamma_R \approx 1\). Then

\[
\frac{dv}{dt} = A'n'nm_1u^2 \left\{ \left( 1 - \frac{2 v_R}{u} - \gamma_T \frac{\cos i}{N} \frac{v_T}{u} + \frac{1}{2} \frac{v^2}{u^2} \right) e_R \right. \]

\[
+ \left( 1 - \frac{v_T}{u} \right) \gamma_T \frac{\cos i}{N} \frac{v_T}{u} \frac{v}{u} \left. \right\} e_T \]

\[
- \left( 1 - \frac{v_R}{u} \right) \gamma_T \frac{\cos \Theta \sin i}{N} e_N + \frac{v_R}{u} \frac{v}{u}\right\}. \quad (40)
\]
3. Equation of motion - electromagnetic radiation effect

Up to now, we dealt with the action of solar wind on the motion of an IDP. The role of solar electromagnetic radiation cannot be neglected in the motion of the IDP in the Solar System. Relativistically covariant equation of motion for an arbitrarily shaped dust particle under the action of parallel beam of photons (Klačka 2008a, 2008b):

$$\frac{dp^\mu}{d\tau} = \sum_{j=1}^{3} \left( \frac{w_j^2}{c^2} S_{elm} \frac{C'_{pr,j}}{c} + \frac{1}{c} F'_{e,j} \right) \left( c b_j^\mu - U^\mu \right), \quad (41)$$

where $p^\mu$ is four-vector of the particle of mass $m$, four-vector of the world-velocity of the particle is given by Eq. (21) and four-vectors $b_j^\mu$, $j = 1, 2, 3$ are given as:

$$b_j^\mu = \left( \frac{1}{w_j}; e_j \right), \quad w_j = \gamma(v) \left( 1 - \frac{v \cdot e_j}{c} \right),$$

$$e_j = \left[ \gamma(v) \left( 1 + \frac{v \cdot e'_j}{c} \right) \right]^{-1} \left\{ e'_j + \left( \frac{\gamma(v) - 1}{v^2} \frac{v \cdot e'_j}{c} + \gamma(v) \right) v \right\}, \quad j = 1, 2, 3, \quad (42)$$

where $\{ e'_j; j = 1, 2, 3 \}$ is orthonormal vector basis in the proper reference frame of the particle and $\{ e_j; j = 1, 2, 3 \}$ is corresponding vector basis in the stationary frame; $e_1$ corresponds to the radial direction (i.e. the Sun - particle direction). $S_{elm}$ is flux density of the electromagnetic radiation and $C'_{pr,j}$ ($j = 1, 2, 3$) are spectrally averaged cross sections of radiation pressure

$$C'_{pr,j} = \frac{\int_0^\infty I(\lambda) C'_{pr,j}(\lambda) \, d\lambda}{\int_0^\infty I(\lambda) \, d\lambda}, \quad j = 1, 2, 3, \quad (43)$$

where $I(\lambda)$ is the flux of monochromatic radiation energy. If $C'_{pr,2} = C'_{pr,3} = 0$, then Eq. (41) reduces to the Poynting-Robertson effect (Poynting 1903, Robertson 1937, Klačka 2008a, 2008b, Klačka et al. 2009), since also components of the thermal emission force $F'_{e,j}$ ($j = 1, 2, 3$) are equal zero, in that case (Mishchenko 2001, Mishchenko et al. 2002). It can be easily verified that Eq. (41) yields $dm/d\tau = 0$, i.e., mass of the particle is conserved, under the action of electromagnetic radiation.

To the first order in $v/c$, Eq. (41) yields

$$\frac{dv}{dt} = \frac{S_{elm}}{mc} \sum_{j=1}^{3} C'_{pr,j} \left[ (1 - 2v \cdot e_1/c + v \cdot e_j/c) e_j - v/c \right]$$

$$+ \frac{1}{m} \sum_{j=1}^{3} F'_{e,j} \left[ (1 + \frac{v \cdot e'_j}{c}) e_j - \frac{v}{c} \right],$$

$$e_j = \left( 1 - \frac{v \cdot e'_j}{c} \right) e'_j + v/c, \quad j = 1, 2, 3, \quad (44)$$

It is worth mentioning to stress that the values of radiation pressure cross sections $C'_{pr,j}$, $j = 1, 2, 3$, depend on particle’s orientation with respect to the incident radiation – their values are time dependent, in general. General equation of motion, represented by Eq. (41) or Eq. (44) differs from the Poynting-Robertson effect. Eqs. (41)-(44) hold for arbitrarily shaped particles. Experimental evidence that nonspherical dust grains move in a different way than spherical particles was given by Krauss and Wurm (2004).
As it is conventionally used in Solar System studies, we will restrict ourselves to the Poynting-Robertson effect, as for the effect of solar electromagnetic radiation. Thus, instead of Eq. (44), we will use
\[
\frac{dv}{dt} = \frac{S_{elmg} A' \bar{Q}'_{pr,1}}{mc} \left\{ \left( 1 - \frac{v \cdot e_1}{c} \right) e_1 - \frac{v}{c} \right\},
\]
where a dimensionless efficiency factor of radiation pressure $\bar{Q}'_{pr,1}$ is defined by relation $\bar{Q}'_{pr,1} = \bar{Q}'_{pr,1}^0$. The values of $\bar{Q}'_{pr,1}$ can be calculated according to Mie (Mie 1908; see also van de Hulst 1981, Bohren and Huffman 1983).

3.1. Electromagnetic radiation effect as a special case of the solar wind effect

The transformations
i) $u \rightarrow c e_1$,
ii) $S' = S \alpha \omega / u \rightarrow S'_{elmg} = S_{elmg} \omega_0^2$ (Klačka 2008b),
iii) $\sigma'_{pr,j} \rightarrow \bar{C}'_{pr,j}$ ($j = 1, 2, 3$), and,
iv) $x' = 1$ (Klačka 2008b)
reduce Eq. (25) into Eq. (41) without thermal emission terms ($F'_{e,j} \equiv 0$ for $j = 1, 2, 3$). This means that our theory for electromagnetic radiation effect without thermal emission is consistent with the theory for corpuscular radiation effect.

4. Equation of motion – solar radiation and solar gravity

Let us consider spherical body orbiting the Sun under the action of solar radiation, i.e. solar corpuscular (solar wind) and electromagnetic radiation. The effect of the solar electromagnetic radiation on the motion of spherical particle corresponds to the Poynting-Robertson effect (P-R effect).

4.1. Solar electromagnetic radiation effect

It is useful to introduce a $\beta$-parameter defined as the ratio of radial component of the radiation force and the gravitational force between the Sun and the particle with zero velocity:
\[
\beta = \frac{L_\odot A' \bar{Q}'_{pr}}{4 \pi c m \mu},
\]
\[
\mu \equiv G (M_\odot + m) = GM_\odot.
\]
$L_\odot = 3.842 \times 10^{26}$ W (Bahcall 2002) is luminosity of the Sun, $\bar{Q}'_{pr} \equiv \bar{Q}'_{pr,1}$, $G$ is the gravitational constant and $M_\odot$ is mass of the Sun. For homogeneous spherical particle we can write
\[
\beta = 5.760 \times 10^2 \frac{\bar{Q}'_{pr}}{R[\mu m] \rho[kg/m^3]},
\]
where $R$ is radius of the particle and $\rho$ is mass density of the particle. Conventionally it is assumed that $\beta = \text{const}$: neither optical properties nor mass of the IDP change. We do not restrict ourselves to the validity of this assumption.

Now, using Eq. (46), the relation $S_{elmg} = L_\odot / (4\pi r^2)$ holds, where $r$ is a heliocentric distance of the IDP. On the basis of the decomposition of the velocity vector $v = v_R e_R + v_T e_T$, we can rewrite Eq. (45) into the form
\[
\left( \frac{dv}{dt} \right)_{P-R} = \beta \frac{\mu}{r^2} \left[ \left( 1 - \frac{v_R}{c} \right) e_R - \frac{v_T}{c} e_T \right],
\]
This is the acceleration of the IDP under the action of the Poynting-Robertson effect.
4.2. Solar wind effect

Let us replace the fraction behind the curly braces in Eq. (40) by new quantities

\[
\frac{A' \eta \bar{Q} \rho \mu}{m} = \frac{\eta \bar{Q} \rho \mu}{c^2 r^2}.
\]

(49)

\(\eta\) is conventionally a constant of the value: 0.22 (Whipple 1967, Dohnanyi 1978) or 0.30 (Gustafson 1994, Abe 2009). The solar wind speed \(u\) values of 350 km/s (Dohnanyi 1978) or 400 km/s (Gustafson 1994) are used in modelling of orbital evolution under the action of solar wind.

Let us look on the numerical values of \(\eta\) and \(u\) on the basis of the solar physics data. In order to calculate the value of \(\eta\), we need the values of \(n, u\) and \(E_1\), on the basis of Eq. (15). The average values near the orbit of the Earth (1 AU) are (Hundhausen 1997, p. 92): proton density \(n_1 = 6.6 \text{ cm}^{-3}\), electron density \(n_2 = 7.1 \text{ cm}^{-3}\), He\(^{2+}\) density \(n_3 = 0.25 \text{ cm}^{-3}\), flow speed \(u = 450 \text{ km s}^{-1}\). Eq. (15) yields for the average value \(S(\text{solar wind; 1 AU}) = u \sum_{i=1}^{3} n_i E_1 = 515.642 \text{ kg s}^{-3}\). Moreover, we will take into account that \(n_i u(1 \text{ AU}) = \langle n_i \rangle \langle u \rangle (1 - 0.15 \cos \varphi)^2, \varphi = 2 \pi [t - t(max)] / T, T = 11.1 \text{ years and } t(max)\) is the time of the solar cycle maximum (Svalgaard 1977 – chapter 13). This result, together with Eqs. (11) (\(\omega = 1\)), (14), (17) and (30), yields \(S(\text{solar wind}) = S_{\text{elmg}}(A'/\sigma_{pr}) \eta\) and

\[
\eta = \eta_0 (1 - \delta \cos \varphi)^2,
\]

(50)

\[
u = u_0 (1 - \delta \cos \varphi),
\]

where \(\varphi = 2 \pi \left(\frac{t - t_{\text{retard}} - t(max)}{T}\right), T = 11.1 \text{ years},\)

if we put \(\sigma_{pr} = A'\) and \(S_{\text{elmg}}(1 \text{AU}) = L_{\odot} / \left(4 \pi [r(=1 \text{AU})]^2 \right), L_{\odot} = 3.842 \times 10^{26} \text{ W}.\) The value of \(T\) represents the average value of the solar cycle period (see, e. g., Foukal 2004, p. 366). The retarded time \(t_{\text{retard}}\) is of the order \(r/u_0\) and it is only a better approximation to reality than the omission of this term.

4.2.1. More exact solution

In reality, one needs to know concentration \(n(r,t)\) and solar wind velocity \(u(r,t)\), when the dust grain is situated at a position of heliocentric distance \(r\) at the time \(t\). More precise information can be obtained from continuity equation (radial component of the velocity \(u\) is approximated by the magnitude \(u\))

\[
\frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 nu)}{\partial r} = 0
\]

if \(r \neq 0\). Using the observational fact \(n = \text{const} \ u / r^2\) (Svalgaard 1977 – chapter 13), one obtains

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial r} = 0
\]

if \(r \neq 0\). Using the boundary condition

\[
\lim_{r \to 0} u(r,t) = u_0 \left(1 - \delta \cos \left(2 \pi \left(\frac{t - t(max)}{T}\right)\right)\right)
\]

(53)
the quasi-linear partial differential equation Eq.(52) can be solved.

Eq. (52) is known as Burgers equation:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 \tag{52}
\]

see, e.g., Ševčovič (2008, pp. 40 - 42).

If the boundary condition

\[ u(x = 0, t) = \chi(t) \]

is given, then the solution of the Burgers equation is

\[ u(x, t) = \chi(t - x/u(x, t)) \]

The last nonlinear algebraic equation can be solved by an iteration method, e.g.:

\[ u = \chi(t - x/u_0) + \lim_{k \to \infty} v_k, \]
\[ v_{k+1} = \chi\{t - x/[\chi(t - x/u_0) + v_k]\} - \chi(t - x/u_0), \]
\[ v_1 = 0. \]

On the basis of the known solution of the Burgers equation, Eqs. (52)-(53) yield

\[ u(r, t) = u_0 \left\{ 1 - \delta \cos \left[ 2\pi \frac{t - r/(2u(r, t)) - t(max)}{T} \right] \right\}. \tag{54} \]

Comparison with Eq. (50) shows that the retarded time is

\[ t_{retard} = r/(2u) \]

The nonlinear algebraic Eq. (54) can be solved by the iteration method presented above, or by the following iteration:

\[ u_{k+1}(r, t) = u_0 \left\{ 1 - \delta \cos \left[ 2\pi \frac{t - r/(2u_k(r, t)) - t(max)}{T} \right] \right\}, \tag{55} \]

since the right-hand side of Eq. (54) is a contractive/contraction function for the case \( u^2 > \pi u_0 r \delta / T \) and heliosphere is characterized by condition \( r < 150 \text{ AU} \) (approximately). We can put \( u_1(r, t) = u_0 \).

4.2.2. Summary

Using definition by Eq. (49), we can summarize, on the basis of Eqs. (50) and (54):

\[
\frac{A' \eta m u^2}{m} \equiv \frac{\eta}{Q_{pr}} \beta \frac{u}{c} \frac{\mu}{r^2},
\]

\[ u(r, t) = u_0 \left\{ 1 - \delta \cos \left[ 2\pi \frac{t - r/(2u(r, t)) - t(max)}{T} \right] \right\}, \]
\[ \eta(r, t) = \eta_0[u(r, t)/u_0]^2, \]
\[ \eta_0 = 0.38, \]
\[ \delta = 0.15, \]
\[ u_0 = 450 \text{ km/s}, \]
\[ T = 11.1 \text{ years}. \tag{56} \]

One can use also \( n \equiv n(r, t) = n_0 [u(r, t)/u_0] (1 \text{ AU } / r [\text{AU}])^2 \). These results represent a more realistic model than the model conventionally used. It takes into account more observational facts (Svalgaard 1977, Hundhausen 1997).

A more simple accesses use Eq. (50) with \( t_{retard} = r/(2u_0) \) or \( t_{retard} = 0 \). Shock waves do not exist in these simple cases. The shock waves are generated by solution of the Burgers equation and this is considered in Eq. (56). As was pointed out in the comment to Eq. (55), the shock waves are not realized in the Solar System.
We put \( i = 0 \) in Eq. (40). Then, the acceleration of the IDP caused by the solar wind has the form

\[
\left( \frac{dv}{dt} \right)_{SW} = \frac{\eta}{Q_{pr}^{'}} \frac{\mu}{r^2} \beta \left\{ \left( \frac{u}{c} - 2 \frac{v_R}{c} \right) e_R - \frac{v_T}{c} e_T \right. \\
- \gamma_T \left[ \frac{v_T}{c} e_R - \left( \frac{u}{c} - \frac{v_R}{c} \right) e_T \right] \\
+ \left. \frac{1}{2} \frac{u^2}{uc} e_R + \frac{v_R}{c} \frac{v}{u} \right\}.
\]

\[(57)\]

### 4.3. Equation of motion

Gravitational acceleration from the Sun is \( -(\mu/r^2)e_R \). Neglecting the solar wind pressure term \( \beta(\eta/Q_{pr}^{'})/(\mu/r^2)(u/c) \) in Eq. (57), we can write the final equation of motion of the IDP in the form

\[
\frac{dv}{dt} = -\frac{\mu}{r^2} (1 - \beta) e_R - \left( 1 + \frac{\eta}{Q_{pr}^{'}} \right) \beta \frac{\mu}{r^2} \left( 2 \frac{v_R}{c} e_R + \frac{v_T}{c} e_T \right) \\
+ \frac{\eta}{Q_{pr}^{'}} \beta \frac{\mu}{r^2} \left\{ -\gamma_T \frac{v_T}{c} e_R + \gamma_T \left( \frac{u}{c} - \frac{v_R}{c} \right) e_T \right. \\
+ \left. \frac{1}{2} \frac{u^2}{uc} e_R + \frac{v_R}{c} \frac{v}{u} \right\}.
\]

\[(58)\]

Eqs. (48) and (57) were used. We introduced a new central acceleration \( -\mu(1 - \beta)e_R/r^2 \), i.e., the gravitational acceleration from the Sun reduced by the solar electromagnetic radiation pressure. Other terms on the right-hand side of Eq. (58) constitute the nongravitational disturbing acceleration. Eq. (50) has to be taken into account.

Moreover, Eq. (27) can be rewritten to the form describing decrease of particle’s radius \( R \):

\[
\frac{dR}{dt} = -\frac{K}{r^2} \frac{|u - v|}{u} (1 - \delta \cos \varphi)^2,
\]

\[u = u_0 \left( 1 - \delta \cos \varphi \right),\]

\[\delta = 0.15,\]

\[u_0 = 450 \text{ km/s},\]

\[\varphi = 2\pi \frac{t - t_{\text{retard}} - t_{\text{max}}}{T},\]

\[T = 11.1 \text{ years},\]

\[(59)\]

where \( K \) is a constant characterizing decrease of the radius of the particle; Eq. (50) was used, also (see also text between Eqs. 50 and 57). Eq. (37) has to be used, too: \( u = u \dot{u} \). One can use Eq. (59) as an approximation to the process of erosion of the particle due to the solar wind corpuscles. Since \( \beta \) is a function of \( R \) (also \( Q_{pr}^{'}, \beta \)), Eqs. (50), (58)-(59) have to be solved simultaneously, together with the Mie’s calculations yielding \( Q_{pr}^{'}, \beta \) for a given \( R \).

The real flux density of solar wind energy and the approximation of a constant flux \( (\eta = \eta_0) \) for the radial solar wind, can be described by the approximation

\[\eta \approx \eta_0 = 0.38.\]

\[(60)\]

Analytical approach to solution of Eqs. (50), (58)-(59) is presented in Sec. 5. Detail numerical solutions are given in Sec. 6.
5. Secular evolution of particle’s orbital elements under the action of solar radiation – analytical approach

We have obtained complete equation of motion in the preceding section. We want to obtain qualitative understanding of the orbital evolution of IDP. This task is the main subject of this section. Trend is that $\beta$ is a decreasing function of $R$. Thus, the term $-\left[\mu(1-\beta)/r^2\right] e_R$, in Eq. (58), does not correspond to Keplerian acceleration, if Eq. (59) is taken into account. As a consequence, osculating orbital elements have to be calculated for Keplerian acceleration and it is given by the term $-(\mu/r^2) e_R$.

We will calculate the secular evolution of semimajor axis $a$, eccentricity $e$ of the particle’s orbit under this nongravitational perturbation. We have to use Eqs. (100) or (103) in Klačka (2004) and Eqs. (32), (34) and (37) in Klačka (1993b), for the time evolution of the osculating elements.

Secular values of semimajor axis and eccentricity, when central acceleration is given by solar gravity term $-\left[\mu(1-\beta)/r^2\right] e_R$, are

$$a = a_\beta \left(1-e^{2}_{\beta}\right)^{3/2} \frac{1}{2\pi} \int_{0}^{2\pi} \frac{\left[1 + \beta \left(1 + e^{2}_{\beta} + 2 e_{\beta} \cos x\right) / \left(1 - e^{2}_{\beta}\right)\right]^{-1}}{(1 + e_{\beta} \cos x)^2} \, dx$$

$$e = (1-e^{2}_{\beta})^{3/2} \frac{1}{2\pi} \int_{0}^{2\pi} \frac{\sqrt{(1-\beta)^2 e^{2}_{\beta} + \beta^2 - 2\beta (1-\beta) e_{\beta} \cos x}}{(1 + e_{\beta} \cos x)^2} \, dx . \quad (61)$$

The quantities denoted by index $\beta$ hold for central acceleration $-\left[\mu(1-\beta)/r^2\right] e_R$ and it is supposed that their values do not significantly change during particle’s revolution around the Sun. Analytical approach for secular evolution of orbital elements is possible under such an assumption, otherwise detail numerical integration of Eq. (58) has to be done. The quantities denoted by index $\beta$ can be calculated from Eq. (67) below.

5.1. Calculation of quantities for Eq. (61)

Let us use perturbation equations of celestial mechanics in the following form:

$$\frac{da_\beta}{dt} = \frac{a_\beta}{1-e^{2}_{\beta}} \left\{ 2 \sqrt{\frac{p_{\beta}}{\mu(1-\beta)}} \left[ a_R e_{\beta} \sin f_\beta + a_T (1 + e_{\beta} \cos f_\beta) \right] \right.$$  

$$+ \frac{\dot{\beta}}{1-\beta} \left(1+e^{2}_{\beta} + 2 e_{\beta} \cos f_\beta\right) \right\} ,$$

$$\frac{de_\beta}{dt} = \sqrt{\frac{p_{\beta}}{\mu(1-\beta)}} \left[ a_R \sin f_\beta + a_T \left(\cos f_\beta + \frac{e_{\beta} + \cos f_\beta}{1+e_{\beta} \cos f_\beta}\right) \right]$$

$$+ \frac{\dot{\beta}}{1-\beta} \left( e_{\beta} + \cos f_\beta \right) ,$$

$$\frac{d\omega_\beta}{dt} = -\sqrt{\frac{p_{\beta}}{\mu(1-\beta)}} \frac{1}{e_{\beta}} \left[ a_R \cos f_\beta - a_T \sin f_\beta \frac{2 + e_{\beta} \cos f_\beta}{1+e_{\beta} \cos f_\beta} \right]$$

$$+ \frac{\dot{\beta}}{e_{\beta}} \frac{1}{1-\beta} \sin f_\beta , \quad (62)$$

where $p_{\beta} = a_\beta (1-e^{2}_{\beta})$, $f_\beta$ is true anomaly of the IDP and $\omega_\beta$ is argument of perihelion of the particle’s orbit; the dot over $\beta$ denotes differentiation with respect to time. It is assumed that the longitude of the ascending node is time independent. Moreover, $a_R$ and $a_T$ are radial and transversal components.
of the disturbing acceleration. Eqs. (62) are consistent with Klačka (1993a – Eqs. 12, 14) and Klačka (1993b – Eqs. 32, 34, 37 if $M = M_\odot(1 - \beta)$). Using Eq. (58) and expressions

$$v_R = \sqrt{\frac{\mu}{p\beta}} e_\beta \sin f_\beta, \quad v_T = \sqrt{\frac{\mu}{p\beta}} (1 + e_\beta \cos f_\beta),$$

we obtain

$$a_R = -2 \left(1 + \frac{\eta}{Q'_{pr}}\right) \beta \frac{\mu}{r^2} \frac{\sqrt{\mu(1 - \beta)/p\beta}}{c} e_\beta \sin f_\beta$$

$$+ \frac{\eta}{Q'_{pr}} \beta \frac{1}{r^2} \frac{1}{c} \left\{ - \gamma_T \sqrt{\frac{\mu(1 - \beta)}{p\beta}} (1 + e_\beta \cos f_\beta) 
+ \frac{\mu(1 - \beta)/p\beta}{u} \left[ \frac{1}{2} \left( 1 + e_\beta^2 + 2e_\beta \cos f_\beta \right) + e_\beta^2 \sin^2 f_\beta \right]\right\},$$

$$a_T = - \left(1 + \frac{\eta}{Q'_{pr}}\right) \beta \frac{\mu}{r^2} \frac{\sqrt{\mu(1 - \beta)/p\beta}}{c} (1 + e_\beta \cos f_\beta)$$

$$+ \frac{\eta}{Q'_{pr}} \beta \frac{1}{r^2} \frac{1}{c} \left\{ \gamma_T \frac{u}{c} = \gamma_T \sqrt{\frac{\mu(1 - \beta)}{p\beta}} e_\beta \sin f_\beta 
+ \frac{\mu(1 - \beta)/p\beta}{u} e_\beta (1 + e_\beta \cos f_\beta) \sin f_\beta \right\}.$$  

(64)

Secular evolution of the orbital element $g$ we get by the time averaging of $dg/dt$ over one orbital period $P$, i.e.

$$\langle \frac{dg}{dt} \rangle \equiv \frac{1}{P} \int_0^P \frac{dg}{dt} \, dt = \frac{1}{a_\beta^2 \sqrt{1 - e_\beta^2}} \frac{1}{2\pi} \int_0^{2\pi} r^2 \frac{dg}{dt} (f_\beta) \, df_\beta.$$

(65)

We have used the second and the third Kepler’s laws: $r^2 d\beta/dt = \sqrt{\mu(1 - \beta)p\beta} - d\omega/\beta - (d\Omega/\beta) \cos i_\beta = \sqrt{\mu(1 - \beta)p\beta}$ and $a_\beta^3 / P^2 = \mu(1 - \beta)/(4\pi^2)$; $p\beta = a_\beta(1 - e_\beta^2)$, $\omega_\beta$ is argument of perihelion and $\Omega_\beta$ is longitude of the ascending node.

As for the quantity $\dot{\beta}$, it is a function of radius $R$ of the particle. Using an approximation $\beta = A/R + B$, where $A$ and $B$ are constants for a given particle, we can write

$$\beta = \frac{A}{R} + B,$$

$$\frac{d\beta}{dt} = - \frac{A}{R^2} \frac{dR}{dt} = \frac{A}{R^2} \frac{K}{r^2},$$

(66)

if also dominant part of Eq. (59) is used ($\delta = 0, \abs{u - \nu} = u$).

Application of Eq. (65) to Eqs. (62) and (66), using the assumption that orbital elements and $\beta$-parameter do not significantly change during particle’s revolution around the Sun, yields ($\delta = 0$ is assumed)

$$\frac{da_\beta}{dt} = - \beta \frac{\mu}{c} \frac{2 + 3e_\beta^2}{a_\beta(1 - e_\beta^2)^{3/2}}$$

$$\times \left\{ 1 + \frac{\eta_0}{Q'_{pr}} \left[ 1 - 2\gamma_T \frac{1}{2 + 3e_\beta^2} \frac{u_0}{\sqrt{\mu(1 - \beta)/p\beta}} \right] \right\}.$$
\[
\frac{de_\beta}{dt} = -\beta \frac{\mu}{c} \frac{5e_\beta/2}{a_\beta^3 \sqrt{1 - e_\beta^2}} \times \left\{ 1 + \frac{\eta_0}{Q_{pr}' \gamma T} \left[ 1 - 2 \frac{1 - \sqrt{1 - e_\beta^2}}{e_\beta^2} \frac{u_0}{\sqrt{\mu (1 - \beta) / p_\beta}} \right] \right\}
\]

\[
\frac{d\omega_\beta}{dt} = -\frac{\eta_0}{Q_{pr}' \gamma T} \beta \frac{\mu}{c} \frac{1}{a_\beta^2 \sqrt{1 - e_\beta^2}} \times \left\{ \gamma T \frac{1 - \sqrt{1 - e_\beta^2}}{e_\beta^2} - \frac{1}{2} \frac{\sqrt{\mu (1 - \beta) / p_\beta}}{u_0} \right\},
\]

where the symbol \(\langle \rangle\) was omitted on the left-hand sides. The quantities with index \(\beta\) in Eq. (67) are the quantities which have to be used in Eq. (61). Eqs. (67) hold under the assumption \(\eta \equiv \eta_0\).

5.2. Complete set of differential equations for secular evolution of particle’s orbital elements

We have obtained the following set of differential equations for secular evolution of orbital elements: Eqs. (61), (66)-(67). This set has to be completed by equation for secular evolution of radius of the particle. Using Eqs. (59) and (65), we obtain \(\delta = 0, |u - v| \approx u\)

\[
\frac{dR}{dt} = -\frac{K}{a_\beta^2 \sqrt{1 - e_\beta^2}}.
\]

Initial conditions must be added to the set of differential equations for orbital elements. If the particle is ejected from a parent body of known orbital elements, then the particle’s initial orbital elements have to be calculated from Eqs. (60)-(61) in Kláčka (2004).

If one would take into account also thermal change of optical properties of the spherical dust particle, then also further changes of orbital elements exist (secular changes of semimajor axis and eccentricity, perihelion motion). This would correspond to the change of parameter \(A\) and \(B\) in Eq. (66). We do not deal with this case (see Kláčka et al. 2007, Pástor et al. 2009).

5.3. Discussion

Let us look on secular evolution of semimajor axis \(a_\beta\) of the particle’s orbit. If the P-R effect and radial velocity component of the solar wind are considered alone (i.e. \(\gamma T \equiv 0\) and \(\dot{\beta} \equiv 0\)), then Eqs. (61) and (66)-(68) show that \(a\) is a decreasing function of time. If one takes into account also the non-radial velocity component of the solar wind, or, \(\dot{\beta} > 0\), the situation may be different. The secular value of \(a\) can be an increasing function of time. Thus, the effect of real solar wind may cause particle’s spiralling outward from the Sun. (Similarly, also the secular value of \(e\) can be an increasing function of time.)
5.4. Radial solar wind and decrease of particle’s radius

According to Eqs. (27) and (59), the mass of the particle may decrease. Eq. (68) holds under assumption that the decrease of the particle’s radius is small enough during particle’s revolution around the Sun.

When we consider only the P-R effect and radial solar wind effect ($\gamma_T = 0$), we are able to calculate the radius of IDP as a function of its orbital eccentricity. Using Eqs. (67) and (68), one can immediately write

$$\frac{de_\beta}{dR} = e_\beta \left\{ \frac{5}{2} \left( 1 + \frac{\eta_0}{Q_{pr}} \right) \beta \frac{\mu}{c} - \frac{1}{1 - \beta} \frac{A}{R^2} K \right\}. \tag{69}$$

Using also Eqs. (47) and (66), Eq. (69) can be integrated:

$$e_\beta = e_{\beta \text{ in}} \left\{ \frac{A}{A - (1 - B)R_{\text{in}}} \left( \frac{R}{R_{\text{in}}} \right)^{1+k_2} \right\} \times \exp \left\{ k_1 (R - R_{\text{in}}) \right\},$$

$$k_1 = \frac{5}{2} \frac{\mu}{c} \frac{B}{K[\text{cm} \ AU^2 \ \text{yr}^{-1}]},$$

$$k_2 = \frac{5}{2} \frac{\mu}{c} \frac{1}{K[\text{cm} \ AU^2 \ \text{yr}^{-1}]} \left\{ A + \eta \times 5.760 \times 10^2 \right\},$$

$$\beta(R) = \frac{A}{R[\mu \text{m}]} + B ,$$

$$\mu = 4\pi^2 \text{AU}^3 \ \text{yr}^{-2},$$

$$c = 6.3114 \times 10^4 \text{AU} \ \text{yr}^{-1},$$

$$\eta \equiv \eta_0 = 0.38 . \tag{70}$$

and the quantities $A$ and $R$ are given in microns; the subscript "in" denotes initial values.

In order to find secular evolution of semimajor axis $a$ and eccentricity $e$, according to Eq. (61), we have to solve the following set of equations: Eq. (66) for a given values of $A$ and $B$, equation for $a_\beta$ in Eq. (67), Eq. (68) and Eq. (70) (or, equation for $e_\beta$ in Eq. 67 instead of Eq. 70).

5.5. Semi-latus rectum, time of spiralling for radial solar wind and $K \equiv 0$

From Eqs. (67) we can determine also the secular evolution of the semi-latus rectum $p_\beta$ of the particle’s orbit. Whence $p_\beta = a_\beta(1 - e_\beta^2)$ we can write

$$\langle \frac{dp_\beta}{dt} \rangle = (1 - e_\beta^2) \left\{ \frac{da_\beta}{dt} \right\} - 2 a_\beta e_\beta \langle \frac{de_\beta}{dt} \rangle , \tag{71}$$

what using Eqs. (67) yields (the symbol $\langle \rangle$ is omitted)

$$\frac{dp_\beta}{dt} = \left( 1 - e_\beta^2 \right)^{3/2} \frac{p_\beta}{\langle \rangle}$$

$$\times \left\{ - 2 \left( 1 + \frac{\eta_0}{Q_{pr}} \right) \beta \frac{\mu}{c} + \frac{1}{1 - \beta} \frac{A}{R^2} K \right\}$$

$$+ 2 \gamma_T \frac{\eta_0}{Q_{pr}} \beta \frac{\mu}{c} \frac{u_0}{\sqrt{\mu (1 - \beta) / p_\beta}} \frac{1 - e_\beta^2}{p_\beta} . \tag{72}$$
Let us rewrite equation for \( \langle \frac{de_\beta}{dt} \rangle \) into the form (the symbol \( \langle \rangle \) is omitted)

\[
\frac{de_\beta}{dt} = \frac{e_\beta \left(1 - e^2_\beta\right)^{3/2}}{p^2_\beta} \times \left\{-\frac{5}{2} \left(1 + \frac{\eta_0}{Q'_{pr}}\right) \frac{\mu}{c} + \frac{1}{1 - \beta} \frac{A}{R^2} K\right\} \\
+ \gamma_T \frac{\eta_0}{Q'_{pr}} \frac{\beta}{c} \frac{\mu}{\sqrt{\mu (1 - \beta) / p_\beta}} \times \frac{1 - \sqrt{1 - e^2_\beta}}{p^2_\beta \sqrt{e_\beta}}.
\]

(73)

Now, let us consider only the P-R effect and the radial solar wind effect. We put \( \gamma_T = 0 \) in Eqs. (72)-(73). In this case, we obtain the following equation

\[
\frac{dp_\beta}{de_\beta} = p_\beta \frac{\beta}{e_\beta} \left\{-2 \left(1 + \frac{\eta_0}{Q'_{pr}}\right) \frac{\mu}{c} + \frac{1}{1 - \beta} \frac{A}{R^2} K\right\}^{-1}
\]

\[
\times \left\{-\frac{5}{2} \left(1 + \frac{\eta_0}{Q'_{pr}}\right) \frac{\beta}{c} + \frac{1}{1 - \beta} \frac{A}{R^2} K\right\} \times \left(1 - \sqrt{1 - e^2_\beta}\right)^{3/2} / p_\beta e_\beta.
\]

(74)

from Eqs. (72) and (73). Eq. (74) yields the relation

\[
p_\beta = p_{\beta \text{ in}} \left(\frac{e_\beta}{e_{\beta \text{ in}}}\right)^{4/5}, \quad K \equiv 0,
\]

(75)

where \( p_{\beta \text{ in}} \) and \( e_{\beta \text{ in}} \) are initial values of semi-latus rectum and eccentricity of the particle’s orbit. Eq. (75) can be considered as a generalization of the result obtained by Wyatt and Whipple (1950): we have taken into account not only the P-R effect, but also the radial solar wind effect. Eq. (75) allows us to write equation for secular evolution of eccentricity in the form

\[
\langle \frac{de_\beta}{dt} \rangle = -\frac{5}{2} \left(1 + \frac{\eta_0}{Q'_{pr}}\right) \frac{\mu}{c} \frac{e_\beta^{8/5}}{p^2_\beta e_{\beta \text{ in}}} \frac{1 - e^2_\beta}{e_{\beta \text{ in}}^{3/5}} \left(1 - e^2_\beta\right)^{3/2}, \quad K \equiv 0.
\]

(76)

It is evident from Eqs. (74) that due to the P-R effect and radial solar wind the particle is spiralling inward to the Sun, for \( K \equiv 0 \). Semimajor axis \( a_\beta \) and eccentricity \( e_\beta \) of the particle’s orbit converge to 0 (see Eqs. 75, 76 and relation among \( a_\beta, p_\beta \) and \( e_\beta \)). Eq. (76) can offer the time of spiralling of the particle with initial orbital elements \( a_{\beta \text{ in}}, e_{\beta \text{ in}} \) into the orbit with osculating elements \( a_\beta, e_\beta \). This time is given by relation

\[
\tau(e_{\beta \text{ in}}, e_\beta) = -\frac{2}{5} \left[ \left(1 + \frac{\eta_0}{Q'_{pr}}\right) \frac{\beta}{c} \right]^{-1} \frac{a^2_{\beta \text{ in}}}{e_{\beta \text{ in}}^{8/5}} \left(1 - e^2_{\beta \text{ in}}\right)^2 I(e_{\beta \text{ in}}, e_\beta),
\]

\[
I(e_{\beta \text{ in}}, e_\beta) = \int_{e_{\beta \text{ in}}}^{e_\beta} \frac{x^{3/5}}{(1 - x^2)^{3/2}} \, dx, \quad K \equiv 0.
\]

(77)
The time of spiralling of the particle into the Sun is then
\[
\tau_{(e_{\beta \text{ in}},0)} = -\frac{2}{5} \left[ 1 + \frac{\eta}{Q'_{pr}} \right] \beta \frac{\mu}{c} \left[ \frac{a_{\text{in}}^2}{1 - e_{\beta \text{ in}}^2} \right] I(e_{\beta \text{ in}},0),
\]
\[K \equiv 0.\]  
(78)

Let us consider two particles characterized by the values \(\beta_1\) and \(\beta_2\). Moreover, let the particles have the same value of \(\bar{Q}'_{pr}\). If we are interested in times of stay of the particles within an interval of semimajor axes \((a_{\text{lower}}, a_{\text{upper}})\), then Eq. (77) yields: \(\tau_1/\tau_2 = \beta_2/\beta_1\), if the initial values of semimajor axes and eccentricities are equal for both particles. On the basis of Eqs. (72)-(73) \((K \equiv 0)\), this result can be approximately generalized also to the case of real solar wind effect, under the assumption \(\beta_1, \beta_2 \ll 1\).

As for the secular evolution, under the assumption that particle’s radius does not decrease, we have to solve only one differential equation Eq. (76); Eq. (75) immediately yields semi-latus rectum and one can easily obtain semimajor axis \(a_\beta = p_\beta / (1 - e_\beta^2)\).

5.6. Summary

Our analytical approach shows that decrease of particle radius due to the solar wind abrasion (corpuscular sputtering) generates increase of particle’s semimajor axis and eccentricity. This result is consistent with detailed numerical calculations presented by Kocifaj and Klačka (2008).

6. Numerical results

In this section we will concentrate on orbital evolution of interplanetary dust particle under the action of solar electromagnetic and corpuscular radiation when solar wind erosion is neglected. The results based on our new approach presented in Secs. 2-4 will be compared with the standard approach when only radial solar wind with constant \(\eta\) is taken into account.

6.1. Radial solar wind

Real flux density of solar wind energy and the approximation of a constant flux \((\eta = 1/3)\) are compared, for radial solar wind, in Fig. 1. Eqs. (50) and (58) were numerically calculated for a particle of fixed radius: \(\beta = 0.01, Q'_{pr} = 1\). The orbital elements were calculated for the central acceleration \(-\left(\frac{GM\odot}{r^2}\right) e_R\).

The variable solar wind properties cause that semimajor axis exhibits a little faster decrease than the standard approach when \(\eta = 1/3\) is used. The same holds also for eccentricity of the particle. The time of spiralling toward the Sun for variable \(\eta\) is about 10 % smaller than for the case of \(\eta = 1/3\). The most significant difference between variable and constant \(\eta\) exists for secular evolution of argument of perihelion. While the constant \(\eta\) produces no shift of perihelion, the more realistic approach of variable \(\eta\) produces nonnegligible shift of perihelion. Moreover, the shift of perihelion depends on the initial position of the particle in its orbit. Fig. 1 shows evolution of the shifts of perihelia for two cases, for aphelion and perihelion ejections of the particle from a parent body (zero ejection velocity). Semimajor axis and eccentricity of the parent body were \(a_P = 5\) AU, \(e_P = 0.9\).

6.2. Real solar wind

If we take into account even more realistic description of the solar wind, when its nonradial velocity component is considered, the resulting orbital evolution differs from the cases discussed in Sec. 6.1.
Fig. 1. Orbital evolution of spherical dust particle ($\bar{Q}^{pr}_p = 1$, $\beta = 0.01$) for radial solar wind. Results for variable $\eta$ and standard value $\eta = 1/3$ are compared. Secular evolution of semimajor axis and eccentricity and also shift of perihelion are depicted. More realistic case yields that the evolution of the shift of perihelion depends on the position (A - aphelion, P - perihelion) of the parent body at the time of the particle's ejection. Effect of solar wind erosion is of negligible importance.

Fig. 2 compares the standard approach (radial solar wind, $\eta = 1/3$) with the variable $\eta$ and $\gamma_T \neq 0$. Again, Eqs. (50) and (58) were numerically calculated for the particle of fixed radius: $\beta = 0.01$, $\bar{Q}^{pr}_p = 1$. The orbital elements were calculated for the central acceleration $-\left(GM_\odot/r^2\right) e_R$.

The real solar wind causes that semimajor axis exhibits a little slower decrease than the standard approach (radial wind, $\eta = 1/3$) is used. The same holds also for eccentricity of the particle. The time of spiralling toward the Sun for real solar wind action is about 15% greater than for the case of $\eta = 1/3$ (one must be aware that this value holds for the case presented in Fig. 2 – the real percentage may be greater/less for larger/smaller initial values of semimajor axis than for the initial 5 AU). The most significant difference between the usage of the real solar wind and the standard approach exists for secular evolution of argument of perihelion. While the constant $\eta$ produces no shift of perihelion, the realistic approach produces nonnegligible shift of perihelion. Moreover, the shift of perihelion depends on the initial position of the particle in its orbit. Fig. 2 shows evolution of the shifts of perihelia for two cases, the aphelion and perihelion ejections of the particle from a parent body (zero ejection velocity). Semimajor axis and eccentricity of the parent body were $a_P = 5$ AU, $e_P = 0.9$.

7. Discussion

We have derived relativistically covariant equation of motion for the action of solar wind corpuscles on motion of interplanetary dust particles. As for spherical shape of the particles, the equation of motion
Fig. 2. Orbital evolution of spherical dust particle, $\bar{Q}_{pr}' = 1, \beta = 0.01$. Secular evolution of semimajor axis and eccentricity and also shift of perihelion are depicted. Results for real solar wind (variable $\eta$ and non-radial component of solar wind velocity vector $\varepsilon = 2.9^\circ$ are included) and radial solar wind (standard value $\eta = 1/3$) are compared. More realistic case yields that the evolution of the shift of perihelion depends on the position (A - aphelion, P - perihelion) of the parent body at the time of the particle’s ejection. Effect of solar wind erosion is of negligible importance.

is represented by Eq. (29). It differs from the force conventionally presented in literature (although only to the first order in $v/u$):

$$ F_{sw} = F_{sw}' \left[ \left( 1 - \frac{2}{x} \right) e_R - \left( \frac{r}{u} \hat{\theta} \right) e_T \right] $$

where $v$ is the velocity of the grain $v = \dot{r} e_R + r \dot{\theta} e_T$, $u$ is the heliocentric solar-wind speed and $F_{sw}'$ is the force on the dust for $v = 0$ (Minato et al. 2004 – Sec. 2.1; equivalent force is presented in Eq. (7.10) by Mann 2009; see also p. 12 in Burns et al. 1979).

We have to stress that the standard form corresponds to Eq. (32) if $x' = 1$ (reality is: $1 < x' < 3$, approximately) and not to physically correct form given by Eq. (29). Eq. (29) yields the correct limiting result $u \to c$ equivalent to the P-R effect.

As for practical application of the above discussed physical results, Eqs. (56)-(59) are astronomically relevant. Decrease of mass of the particle may cause its spiralling outward from the Sun and not toward the Sun, as it is commonly accepted for the action of solar wind on interplanetary dust particles. This result is evident from analytical equations presented in Sec. 5 (see, e. g., Eqs. 61-62) and detailed numerical calculations confirming this result can be found in Kocifaj & Klačka (2008).

Sec. 6 concentrates on the action of solar wind on motion of interplanetary dust particles for the cases when variable flux of solar wind energy and non-radial solar wind velocity are considered. The most evident difference between the standard and more realistic approaches is represented by the shift of perihelion (secular evolution of argument of perihelion). It is generally believed that the shift of
perihelion does not exist, both for the solar wind effect and the P-R effect. However, the real action of the solar wind differs from the action of the P-R effect – the P-R effect really produces no shift of perihelion. The real shift of perihelion depends on the initial orbital position of the particle. The non-radial solar wind velocity can lead to outspiralling from the Sun, in the region of outer planets. The particle may or may not spiral toward the Sun due to the simultaneous action of the P-R effect and solar wind effect (see also Klačka et al. 2008).

8. Conclusion

Relativistically covariant equation of motion for arbitrarily shaped dust particle under the action of solar wind is derived. Change of the particle's mass is an indispensable part of the space-time formulation of the equation of motion for the action of the solar wind. The solar wind effect would reduce to the Poynting-Robertson effect, in the limiting case when: i) solar wind speed would tend to the speed of light, ii) no decrease of mass of the interplanetary dust particle would exist, and, iii) the velocity of the solar wind would be radial. However, the solar wind may have qualitatively different effect on orbital evolution of interplanetary dust particle, since the points ii) and iii) are not fulfilled, in general. The decrease of mass of the interplanetary dust particle and non-radial component of the solar wind velocity may cause outspiralling of the particle from the Sun. Time variable solar wind leads to the shift of perihelion of the particle. The found results may have important consequences for evolution of dust disks in the vicinity of stars with stellar winds.

Appendix A: Emission from the particle

The other possible force influencing dynamics of dust particle may originate from an emission, e.g., radioactive decay. Let the particle emits an energy $E_{em}'$ per unit time due to the emission in the proper reference frame. We will suppose that this emission is represented by the flux of particles with a speed $u_{em}'$. Further, we declare the orthonormal vector basis $\{f'_j; j = 1, 2, 3\}$, as it was used in Sec. 2.2. The corresponding velocities are: $u'_{em,j} = u_{em}' f'_j, j = 1, 2, 3$.

The outgoing four-momentum of the emission per unit time, in the proper reference frame of the particle, is

$$p_{em}^\mu = \left( \frac{1}{c} E_{em}' \right) + \frac{1}{c} E_{em}' \sum_{j=1}^{3} r'_j \left( \frac{u_{em}' f'_j}{c} \right),$$  \hspace{2cm} (A.1)

where $r'_j (j = 1, 2, 3)$ are dimensionless coefficients expressing the part of the total flux of radiation which is emitted in the corresponding directions.

Lorentz transformations of (A.1) yield the outgoing four-momentum per unit time in stationary reference frame:

$$p_{em}^\mu = \frac{1}{c} E_{em}' \frac{U^\mu}{c} + \frac{1}{c} E_{em}' \sum_{j=1}^{3} r'_j \left( \frac{\xi_{em,j}^\mu}{c} - \frac{U^\mu}{c} \right),$$  \hspace{2cm} (A.2)

where

$$\xi_{em,j}^\mu = \left( \frac{1}{\omega_{em,j}} \right) = \left( \frac{1}{\omega_{em,j}} \frac{u_{em,j}}{c} \right),$$

$$\omega_{em,j} = \gamma (v) \left( 1 - \frac{v \cdot u_{em,j}}{c^2} \right),$$

$$u_{em,j} = \left[ \gamma (v) \left( 1 + \frac{v \cdot u'_{em,j}}{c^2} \right) \right]^{-1} \left\{ u_{em,j}' + \left[ \gamma (v) - 1 \right] \frac{v \cdot u'_{em,j}}{v^2} + \gamma (v) \right\} v, \hspace{2cm} j = 1, 2, 3.$$  \hspace{2cm} (A.3)
Thus, expression on the right-hand side of Eq. (A.2) should be added to the right-hand side of Eq. (21) when we would like to take into account also the effect of the emission of dust particle.

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