Development of a Computer simulation approach for honeycomb constructions for aerospace application

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Abstract. An approach to definition of a homogeneous simulation model for honeycomb structures has been developed and verified for specimens containing a finite number of cells. The elastic characteristics of the model were evaluated basing on the results of tensile and shear numerical tests of honeycomb specimen. This is an extension of earlier work related with spatially reinforced composites. The simulation model was validated for specimens comprised of different numbers of cells in the specimen to expose the scale effect influence. As the number of cells was increased, the calculated values of the moduli $E_x$ and $E_y$ converged, confirming the theoretical result that the appropriate model is transversely isotropic rather than orthotropic for the honeycomb specimen investigated. Elastic properties obtained from the numerical test of the honeycomb structure were then applied in the characterization of continuous medium. The examination was carried out using criteria expressing basic features of homogeneous body. The case of a honeycomb integrated with composite plates as a sandwich structure was analysed for a complex loading case as well as buckling and eigen-frequency analysis.

1. Introduction

There are two prevailing approaches to the analysis of constructions containing honeycomb panels. According to one the real geometrical configuration of a honeycomb is considered, i.e. a honeycomb structure is presented as a set of hexagonal cells [1-3]. That type of a model can be applied to the simulation of a construction of a simple configuration. In dealing with aerospace parts, it is often necessary to take into account an ensemble of several elements made of different types of materials. In this case the consideration of each individual cell in the analysis of a complex construction containing honeycomb panels becomes difficult and impractical.

In such cases, a honeycomb structure can be simulated by an equivalent continuous medium, with adequate elastic characteristics. In accordance with such an approach to the simulation of a honeycomb structure, formulas based on the standard beam theory are usually used for averaging the elastic characteristic [4]. It should be noted that the beam approach gives the transversal isotropic material for modeling of a honeycomb. Such an approach is based on boundary symmetry conditions and therefore assumes an infinite number of regular cells: it does not duly take into account certain geometrical and mechanical features of a honeycomb construction, for instance irregularities in the hexagonal geometry, tolerance variations in shape and wall thickness, and the presence of construction features such as weld lines or material crimping, nor does it take in account with the finite size ratio of a honeycomb construction and a cell, or anisotropy of mechanical properties of a honeycomb core.
material. All of this has led to the development of the numerical model that would adequately simulate the honeycomb structure. The proposed approach is an extension of a previously work related with the modelling of spatially reinforced composites [5].

2. Homogeneous model of honeycomb

In problems of mechanics concerning materials containing regularly distributed heterogeneities, for instance, composites, porous materials or honeycombs the homogeneous continuous model is to be chosen. Mechanical properties of such a model are determined either by means of mechanical tests or using appropriate averaging methods. Such a model has to meet three criteria:

1. A model and a prototype must have the same type of symmetry concerning mechanical properties.
2. Potential energy of the model and prototype must be equal under the same loading condition.
3. Mass of the representative volume of the model and prototype must be equal.

In a regular hexagonal cell honeycomb structure elastic symmetry axes of order 6 are found passing through the finite number of points, exactly through the center of each cell (Figure 1).

It is known that a material through each point of which there passes an axis of elastic symmetry of the 6th order is a transversely isotropic one [6]. The number of such axes in a honeycomb is limited by the number of cells in the structure. That means that a transversal isotropic material can be considered as a honeycomb homogeneous model in the cases when the characteristic dimension of a cell much lesser compeering with the characteristic dimension of a construction. For transversely isotropic materials there are 5 independent elastic constants. Where there are only a small number of honeycomb cells, where the shape is not a regular hexagon, or where geometry variation has to be taken into account, the symmetry required for transverse isotropy breaks down. In this case the elastic properties of a honeycomb model are fully determined by 9 independent elastic constants,

- Elastic moduli – $E_x, E_y, E_z$,
- Shear moduli – $G_{xy}, G_{xz}, G_{yz}$,
- Poisson's ratios – $\nu_{xy}, \nu_{xz}, \nu_{yz}$.

but it might be anticipated that for honeycombs displaying only minor symmetry violations, some of independent constants would have very similar values.
The second criterion for a homogeneous model can be presented by a relation that establishes the equality of potential elastic energy of a honeycomb and that of a model

\[ \iint_{S_h} p_i u_i ds = \iiint_{V_m} \sigma_{ij} \varepsilon_{ij} dv. \quad (1) \]

In (1), the left side expresses the work of the surface load \( p_i \) applying on the boundary surface \( S_h \) of a honeycomb when the displacement is \( u_i \). The right side presents the elastic energy of the volume \( V_m \) of the model.

For the determination of elastic characteristics of the honeycomb two types of numerical tests based on solid FE models were carried out. Elastic moduli and Poisson's ratios were calculated using data of honeycomb tension test simulation (Figure 2). For instance, the \( E_x \) modulus was evaluated as

\[ E_x = \sigma_x \varepsilon_x = \frac{F_x}{S_x \tan \theta}, \quad (2) \]

where \( \sigma_x = F_x/S_x \) is the normal stress, \( F_x \) the resultant tensile force, \( S_x \) the effective area of honeycomb end face, strain \( \varepsilon_x = \partial u_x/\partial x = \tan \theta \) is the strain, and \( \theta \) the angle between straight line representing the relationship between displacement \( u_x \) and the \( x \) coordinate (Figure 3).

**Figure 2.** Finite element model (solid elements) of honeycomb under tension. Deformed style.

**Figure 3.** Displacement \( u_x \) vs \( x \) coordinate.

**Figure 4.** Finite element (solid elements) evaluation of shear modulus \( G_{xy} \). Deformed style.

**Figure 5.** Displacement \( u_y \) vs \( y \) coordinate.
Shear moduli were evaluated basing on the data obtained from the simulation of pure shear test (Figure 4). For example, shear modulus $G_{xy}$ was calculated as

$$G_{xy} = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{T_y}{S_x \tan \omega}$$  (3)

where $\tau_{xy} = T_y/S_x$ is the shear stress, $T_y$ the resultant tangential force applied on the $S_x$ face of the honeycomb specimen, $\gamma_{xy} = \partial u_x \partial y = \tan \omega$ the shear strain and $\omega$ the angle between the straight line representing the relationship between displacement $u_x$ and $y$ coordinate (Figure 4).

It should be noted that relations (2) and (3) follow from (1) under the assumption that the loads are uniformly distributed over the surfaces of a honeycomb.

3. Honeycomb model

As an example, an aluminum honeycomb was considered. Table 1 contains the specifications defining the mechanical properties of the honeycomb core material and cell dimensions.

| Table 1. Honeycomb specifications |
|----------------------------------|
| **Young’s modulus (MPa)**         | 70000  |
| **Poisson’s ratio**               | 0.3    |
| **Cell wall thickness (mm)**      | 0.3    |
| **Cell wall height (mm)**         | 5      |
| **Cell wall length (mm)**         | 3.5    |

The development of a computational model included the choice of the number of cells in the specimen. The graph shown in Figure 6 exhibits the scale effect in the determining of honeycomb elastic moduli.

**Figure 6.** Scale effect in the determining of elastic moduli $E_x$, $E_y$ and $G_{xy}$

The observed variation of the difference between moduli $E_x$ and $E_y$ shows the tendency of conversion from orthotropic to transversely isotropic model as number of cells increases. The influence of scale factor that can be determined as ratio of characteristic specimen size $L$ to characteristic cell size $l$ is illustrated by data presented in the Table 2.
Table 2. Elastic moduli vs scale factor $L/l$

| $N$  | $L/l$ | $E_x$ | $E_y$ | $E_x = E_y$ | $G_{xy}$ | $G_{xy}$ |
|------|-------|-------|-------|--------------|----------|----------|
| 32   | 9.5   | 88.0  | 111.0 | 101.8        | 35.1     | 25.5     |
| 59   | 13.0  | 92.8  | 109.6 | 101.8        | 31.9     | 25.5     |
| 94   | 16.4  | 95.7  | 108.9 | 101.8        | 30.1     | 25.5     |
| 314  | 30.4  | 99.5  | 107.4 | 101.8        | 26.5     | 25.5     |

Elastic properties of homogeneous model which have been obtained using a set of numerical tension and shear tests of the honeycomb specimens are presented in Table 3.

Table 3. Elastic properties of aluminum honeycomb

| $E_x$ | $E_y$ | $E_z$ | $G_{xy}$ | $G_{xz}$ | $G_{yz}$ | $\nu_{xy}$ | $\nu_{xz}$ | $\nu_{yz}$ |
|-------|-------|-------|----------|----------|----------|------------|------------|------------|
| 99.5  | 107.4 | 6446.4| 26.5     | 55.9     | 50.1     | 0.92       | 0.005      | 0.300      |

It should be noted that all of these data were obtained from tests that were carried out with a heterogeneous honeycomb structure, however, they were used as characteristics of a continuous medium, that is, a homogeneous model of honeycomb. Therefore, it was necessary to verify their suitability for the characterization of the continuous medium. The verification of the homogeneous model developed was performed using criteria expressing well-known features of homogeneous orthotropic and transversely isotropic media [6]. The verification data are presented in Table 4. This analysis thereby establishes the level of orthotropy that arises as a result of the finite size of the specimen, so that where more realistic honeycomb geometries are analysed using the same methodology, it will be possible to differentiate between scale effects and asymmetry effects.

Table 4. Verification of model

| Quantity               | Value        | Criterion            |
|------------------------|--------------|----------------------|
| $\nu_{xy}/E_y$         | 0.00929      | $\nu_{xy}/E_y = \nu_{xy}/E_x$ |
| $\nu_{xy}/E_x$         | 0.00923      |                      |
| $\nu_{xy} + \nu_{yz} + \nu_{xz}$ | 1.272 | $<1.5$ |
| $1 - \nu_{xy}\nu_{yx} - \nu_{yz}\nu_{zy} - \nu_{xz}\nu_{zx} - 2\nu_{yx}\nu_{zy}\nu_{xz}$ | 0.031 | $>0$ |

The results confirm the adequacy of the homogeneous model of a honeycomb in the case of simple loading. It was of interest to verify the adequacy of this model for a honeycomb integrated with other construction elements as well as for the case of complex loading, in problems of buckling and dynamics.
4. Honeycomb sandwich structures

Honeycomb sandwich structure is a composition of two skins, made of laminate composites, separated by a honeycomb. These structures are widely used in aerospace items such as wings and fuselage parts, interior elements, etc. due to their high specific stiffness. There are three main types of problem in the design of honeycomb sandwich structures for aerospace application:

1. The bending stresses analysis of the sandwich structure.
2. The buckling analysis of honeycomb sandwich structure.
3. The eigenfrequencies spectrum analysis of the sandwich structure.

In all three cases, it must be understood that where the overall sandwich structure is being considered, and therefore where a homogenized material modelling approach is adopted, the detail of the deformation of the honeycomb and skin is not apparent. In this work both analysis approaches were adopted, and the results compared.

4.1. Bending

An idealized sandwich panel consisting of two carbon skins and aluminum honeycomb, subjected to bending loading was considered. The honeycomb properties were assumed to be the same as in the Table 1. The specifications of carbon skins are presented in Table 5. The scale factor of the sandwich specimen was taken as 10.9.

| Table 5. Carbon skins specifications |
|-------------------------------------|
| $E_x$ (GPa) | 150 |
| $E_y$ (GPa) | 10.7 |
| $G_{xy}$ (GPa) | 5.7 |
| $\nu_{xy}$ | 0.24 |
| Skin thickness (mm) | 1.0 |
| Skin length (mm) | 88 |
| Skin width (mm) | 38 |

The FE model of the sandwich panel contained solid elements for modelling of the honeycomb and laminate plate elements for modelling of the skins. The ends of the skins were constrained. The bending load was applied as shown in Figure 7. Taking into account the conditions of symmetry, the numerical model was chosen as $\frac{1}{4}$ part of the structure (Figure 8). Analysis was carried out for two models of the sandwich. In the first one, the honeycomb structure was considered. In the second, the honeycomb was substituted for by a homogeneous model. The skins were modelled explicitly in both cases.

![Figure 7. Loading of a honeycomb sandwich.](image1)

![Figure 8. Bending of a honeycomb sandwich plate ($\frac{1}{4}$ part of the heterogeneous model).](image2)
The results of the calculation are presented in Figure 9. There are two curves in the graph that show the distribution of the panel deflections for the two models. Both sets of results coincided to within a tolerance of 2.6%.

![Figure 9. Distribution of deflection along sandwich plate](image)

4.2. Buckling

The same sandwich panel was considered in the buckling problem. On one end of the sandwich both skin plates was loaded by compressive force distributed along skin edges as shown in Figure 10. The other end was constrained. In Table 5, the coefficients of the critical load for both simulation models are presented. Figures 10 and 11 illustrate the 4th buckling mode corresponding to the honeycomb and homogeneous model.

It should be noted that the results in the coefficients and modes match to a sufficient degree.

| Table 6. Critical compressive force coefficient |
|-----------------------------------------------|
| **Honeycomb sandwich**                       | **Sandwich with homogeneous core** |
| 9.716                                        | 9.469                             |
| 39.107                                       | 39.029                            |
| 51.864                                       | 52.887                            |
| 60.958                                       | 61.337                            |
| 70.346                                       | 68.213                            |

![Figure 10. Honeycomb sandwich buckling](image)

![Figure 11. Sandwich model with homogeneous core buckling](image)
4.3. Eigen-frequency

In the eigen-frequency analysis, a sufficient match of the result was reached as in previous cases. In Table 7 the eigen-frequencies corresponding to the first 5 modes are presented.

| Mode | Honeycomb sandwich | Sandwich with homogenous core |
|------|--------------------|-----------------------------|
| 1    | 2.174              | 2.156                       |
| 2    | 13.210             | 13.104                      |
| 3    | 19.772             | 19.906                      |
| 4    | 25.617             | 25.549                      |
| 5    | 26.040             | 25.971                      |

5. Conclusions

In the case of real engineering materials, there would be significant departure from idealized material properties. The proposed approach allows creating simulation models for static and dynamic analysis of constructions containing honeycomb structures, taking into account the main features of the real materials. It can be applied for honeycomb structures and cellular materials with different type of a core material and cells with a regular and an irregular shape.

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