A data-driven minimum stiffness prediction method for machining regions of aircraft structural parts

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Abstract

Large thin-walled structural parts have been widely used in aircrafts for the purpose of weight reduction. These parts usually contain various thin-walled complex structures with weak local stiffness, which are easy to deform during machining if improper machining parameters are selected. Thus, local stiffness has to be seriously considered during machining parameter planning. Existing stiffness calculation methods including mechanical methods, empirical formula methods, finite element methods, and surrogate-based methods are either inaccurate or time consuming for complex structures. To address this issue, this paper proposes a data-driven method for predicting local stiffness of aircraft structural parts. First, machining regions of aircraft structural part finishing are classified into bottom, sidewall, rib, and corner to further define the minimum stiffness of machining regions. By representing the part geometry with attribute graph as the input feature, while computing the minimum stiffness using FEM as the output label, stiffness prediction is turned to a graph learning task. Then, graph neural network (GNN) is designed and trained to map the attribute graph of a machining region to its minimum stiffness. In the case study, a dataset of aircraft structural parts is used to train four GNN models to predict the minimum stiffness of the defined four types of machining regions. Compared with FEM results, the average errors on the test set are 6.717%, 7.367%, 7.432%, and 5.962% respectively. In addition, the data driven model once trained, can greatly reduce the time in predicting the stiffness of a new part compared with FEM, which indicates that the proposed method can meet the engineering requirements in both accuracy and computational efficiency.

Keywords Aircraft structural parts · Stiffness · Surrogate models · Graph neural networks

1 Introduction

To improve flight performance, large thin-walled structural parts have been widely used in various aircrafts [1]. These parts are of lightweight but usually complex structures as well as weak local stiffness. During the finishing process of this kind of parts, elastic deformation and dimensional error appear easily in the regions with weak stiffness due to cutting forces caused by improper machining parameters [2], as shown in Fig. 1. Therefore, local stiffness should be seriously considered during machining parameter planning.

Traditional stiffness calculation methods mainly include mechanical methods [3], empirical formula methods [4], finite element methods (FEM) [5] and surrogate-based methods [6]. However, due to the structural complexity, these methods are either inaccurate or time consuming, which is far away from the requirements of real manufacturing. As a consequence, machining parameters now are still mainly determined by technologists, and to avoid elastic deformation and dimensional error, conservative machining parameters are usually selected which will greatly reduce the production efficiency [2].

This paper proposes graph neural network (GNN)-based surrogate models, which can learn the maps between part geometry and minimum stiffness from the high-fidelity finite element simulation data to support automatic machining parameter planning. First, based on the geometric features of such parts, machining regions are classified into bottom, sidewall, rib and corner, and the minimum stiffness of these regions is further defined. Then, the part attribute graph is
constructed as the input, while the output is the minimum stiffness of machining regions which is calculated based on FEM. At last, the minimum stiffness prediction of a machining region is converted to a graph learning task based on GNN model. Compared with FEM, once these GNN models are trained, they can efficiently calculate the minimum stiffness of machining regions for other similar parts, which can greatly improve the computational efficiency when the accuracy meets the engineering requirements. Therefore, the proposed surrogate-based method can significantly reduce the process planning cycle for aircraft structural parts.

2 Related works

The stiffness describes the ability of a structure to resist elastic deformation. As the structures in different regions of a part usually have different stiffness, it is of great significance to calculate the stiffness in different regions for selecting proper machining parameters. Existing stiffness calculation methods can be generally classified into mechanical methods, empirical formula methods, FEM, and surrogate-based methods.

2.1 Mechanical methods

The mechanical methods approximate the structure of the part as thin-walled plate or beam, and calculate their local stiffness based on the thin-walled plate theory or beam theory, which can be used to further analyze the part deformation. For the use of thin-walled plate theory, Tang and Liu [7] calculated the stiffness of parts for constructing a static deformation prediction model under different linear loads and thicknesses. Wu et al. [8] integrated the finite difference method into the calculation process of thin-walled plate theory to establish a machining deformation prediction model which is suitable for different machining parameters. Gao et al. [9] built a semi-analytical model of thin-walled parts to analyze the influence of initial residual stress and stiffness on machining deformation. For the beam theory, Liu et al. [10] and Jia et al. [3] simplified the sidewall of parts to cantilever beam, simply supported beam, etc., which realized the machining deformation prediction under different cutting forces.

The above mechanical calculation methods have a mature theoretical system and can obtain accurate results under ideal circumstances. However, to consider a real part with complex structures, these methods have to induce a lot of simplifications and approximations, which will make the inaccurate stiffness prediction result.

2.2 Empirical formula methods

The empirical formula methods first choose the most sensitive factors of stiffness to establish empirical models, and then carry out a parameter identification operation using experimental data to obtain the determinate models. Zhang and Chen [11] built an empirical model which considered elastic modulus, thickness and outer diameter to calculate the stiffness of any point on the thin-walled cylindrical part. On this basis, Wang [12] used this model to calculate the stiffness of the working cylinder in the pump-controlled hydraulic press system. Ma et al. [4] constructed a cutting unit based on cutter contact point with various cutting widths, and then established a calculation model to analyze the stiffness variation at different cutter contact points in NC machining.

The empirical formula methods directly calculate the stiffness of parts, and is fast enough in real manufacturing. However, empirical models are usually only applicable for a specific set of parts. Moreover, it is really difficult to build an empirical model for calculating the stiffness of aircraft structural parts with complex structures.

2.3 Finite element methods

FEM establishes finite element model of the part and use professional CAE software to obtain the stiffness
information. Wan et al. [13] obtained the element stiffness matrices of a part based on finite element simulation (FEM) for predicting the dimensional error in machining the part surface. Wang et al. [5] calculated the local stiffness of large thin-walled parts based on FES for proposing a real-time compensation method for machining errors. Smith et al. [14] introduced a force placed at different points in the local region of a part and calculated the displacement of these points based on FES, so as to obtain the minimum stiffness of this region. Huang et al. [15] proposed an equivalent bending strain energy method based on FES, which was used to analyze the stiffness variations in NC machining.

The results of FEM are more accurate than other methods when the mesh is fine. However, its computational cost is extremely high when the mesh is fine, e.g., it takes several hours for the stiffness calculation of an aircraft structural part with small size, while for the large parts, it usually takes more than one day which is unacceptable in real industry.

2.4 Surrogate-based methods

Since FEM are computational expensive, some scholars use the data of FEM to build a surrogate model to perform approximate stiffness calculation [16, 17]. This class of existing methods mainly choose neural networks as the surrogate models to predict the element stiffness matrices of a part, which is used to analyze and optimize the part structure based on the loading condition. Sun et al. [18] and Jia et al. [19] took the relative coordinates of grid nodes as the sample features, and then built a three-layered neural network and a convolutional neural network respectively to predict the element stiffness matrices of a part. White et al. [20] established a two-layered neural network model to map the geometrical parameters of microscale metamaterials to the element stiffness matrix. Qian and Ye [6] improved the above method and built a dual-model neural network to improve the accuracy of topology optimization.

Existing surrogate models are mainly oriented to the structural design of the part, where the stiffness matrix and stress matrix are used to solve the strain of a part, which can’t be directly used to calculate the local stiffness. In addition, since it is difficult to directly vectorize the local region for machine learning models, existing studies only select the size, the type of grid elements, and the coordinates of grid nodes as the sample features, which could be regarded as a big simplification to the prediction task.

Although the above researches have achieved great progress for the stiffness calculation, there are still some shortcomings about the accuracy or computational efficiency for aircraft structural parts. Therefore, this paper proposes a data driven method for predicting the minimum stiffness of a part. The prediction is converted to a graph learning task by representing the part geometry as an attribute graph. The minimum stiffness calculated by FEM is used as the labels and a graph neural network (GNN) is designed to establish the map between the attribute graph of a machining region and its minimum stiffness. The rest of this paper is organized as follows. Section 3 will calculate the minimum stiffness of machining regions based on FEM and the proposed method for predicting it quickly and approximately. Section 4 will establish a dataset of aircraft structural parts for training and testing the proposed models. Section 5 will describe the conclusion and future work.

3 Method

For the proposed data-driven stiffness prediction method, machining regions of aircraft structural part finishing is first classified into bottom, sidewall, rib, and corner for further definition of minimum stiffness. Then, with the minimum stiffness calculated by FEM as the output label, and the part model represented by attribute graph as the input feature, the minimum stiffness prediction task is turned to a graph learning task. Next, a graph neural network (GNN) is designed and trained to map the attribute graph of a machining region to its minimum stiffness. The workflow of the proposed method is shown in Fig. 2.

3.1 FEM-based minimum stiffness calculation of machining regions

The NC process planning of aircraft structural parts is composed of a large number of ordered machining operations, and each machining operation corresponds to a machining region, whose minimum stiffness is significant to machining parameter selection of the machining operation. To calculate the minimum stiffness of machining regions, the machining regions in aircraft structural part finishing are first classified into bottom, sidewall, rib and corner. Next, the minimum stiffness of each type of machining region is defined, and then calculated by FEM, which will be taken as the output label of the data-driven model.

3.1.1 The definition of minimum stiffness

In the NC process planning of aircraft structural parts finishing, technologists usually group the machining regions in bottom, sidewall, rib and corner, which is based on the structural and positional similarity. Different types of machining regions vary in cutting position, cutting force, and constraint condition. For each type of machining region, the minimum stiffness direction corresponding to a point can be intuitively determined as the thinnest material thickness direction of the part, which is further validated by FEM. Thus, the minimum stiffness direction
in the bottom region is perpendicular to the bottom face, the minimum stiffness direction in the sidewall region, rib region, and corner region is perpendicular to the face of the side wall, as shown in Fig. 3.

After the minimum stiffness direction is determined for each type of machining region, the minimum stiffness can be further defined. For a machining region, it is first divided into multiple discrete grid elements after meshing, including $p$ nodes. Then, the stiffness of each node in the minimum stiffness direction is calculated and represented as $k_1, k_2, \ldots, k_p$. Next, the minimum stiffness $k_{\min}$ of the region is defined as the minimum value of $k_1, k_2, \ldots, k_p$, i.e., $k_{\min} := \min(k_1, k_2, \ldots, k_p)$. In addition, for the corner region, the edge points with thinnest material thickness always reach the minimum stiffness, thus to improve the efficiency, only the points at the corner edge is considered in the minimum stiffness calculation.

### 3.1.2 The calculation of minimum stiffness

The minimum stiffness of each machining region can be calculated based on FEM. First, the part is divided into grid elements with a large quantity of nodes after meshing. The minimum stiffness direction of all nodes is calculated according to the associated machining regions. Then, based on the calculated minimum stiffness directions, multiple coordinate systems, which have the same origin but differ in coordinate directions, are built to divide the nodes into multiple groups. Each group corresponds to a coordinate system whose coordinate directions are parallel to the minimum stiffness directions corresponding to the nodes in the group. Since the stiffness matrix in FEM only records the information in coordinate directions, it is required to construct new coordinate systems according to the required directions, i.e., minimum stiffness directions.
For nodes of each group, FEM is used to calculate the stiffness in the minimum stiffness direction, which contains five steps. First, all the element stiffness matrices in the global coordinate system are obtained. Secondly, coordinate system transformation is used to obtain the element stiffness matrices in the coordinate system corresponding to the group. Thirdly, the global stiffness matrix is combined based on the element stiffness matrices, where the large values are used to deal with boundary conditions. Fourthly, the global compliance matrix is calculated based on the global stiffness matrix, and the stiffness of all nodes is further calculated. Fifthly, the stiffness of the nodes in the group is screened out from the stiffness of all nodes. After obtaining the stiffness of all nodes in their respective directions, the minimum stiffness of each machining region is searched based on node-region affiliations. The process is shown in Fig. 4, and the detailed calculation process of node stiffness in each group is as follows:

For each group, there is a local coordinate system whose coordinate directions are parallel to the minimum stiffness directions corresponding to the nodes in the group. It is first required to obtain all element stiffness matrices in global coordinate system, whose generalized definition is expressed as follows:

\[
\mathbf{F}_e = \mathbf{D}_e \mathbf{U}_e
\]  

where \( \mathbf{D}_e \in \mathbb{R}^{3n' \times 3n'} \), \( \mathbf{F}_e \in \mathbb{R}^{3n' \times 1} \), and \( \mathbf{U}_e \in \mathbb{R}^{3n' \times 1} \) are the element stiffness matrix, element load matrix and element displacement matrix, respectively, \( n' \) represents the number of nodes contained in the grid element.

The coordinate directions of the global coordinate system may not be parallel to the minimum stiffness directions of the nodes in the group, yet the stiffness matrix only records the information in the coordinate directions. Thus, it is required to transform the global coordinate system to the local coordinate system whose coordinate directions are parallel to the minimum stiffness directions that corresponds to the nodes in the group. Let \( \mathbf{R}_e \in \mathbb{R}^{3n' \times 3n'} \) be the rotation matrix from global coordinate system to the new local coordinate system, based on the theory of structural mechanics, the coordinate transformation of \( \mathbf{D}_e \) is realized by first transforming the

![Fig. 4 The calculation process for the minimum stiffness of machining regions](image-url)
element load matrix $\mathbf{F}'$ and the element displacement matrix $\mathbf{U}'$, i.e., $\mathbf{F}' = \mathbf{R}_e \mathbf{F}$ and $\mathbf{U}' = \mathbf{R}_e \mathbf{U}$. Then the new element stiffness matrix $\mathbf{D}'_e$ can be calculated based on $\mathbf{F}'$ and $\mathbf{U}'$, i.e., $\mathbf{F}' = \mathbf{D}'_e \mathbf{U}'$. Thus, the relation between $\mathbf{D}'_e$ and $\mathbf{D}_e$ is as follow:

$$\mathbf{D}'_e = \mathbf{R}_e \mathbf{D}_e \mathbf{R}_e^{-1}$$

where matrix $\mathbf{R}_e$ is an orthogonal matrix which satisfies $\mathbf{R}_e^{-1} = \mathbf{R}_e^T$. $\mathbf{R}_e^{-1}$ represents matrix transposition, thus $\mathbf{D}'_e$ can be efficiently calculated by the following formula:

$$\mathbf{D}'_e = \mathbf{R}_e \mathbf{D}_e \mathbf{R}_e^T$$

Then, the global stiffness matrix $\mathbf{D}' \in \mathbb{R}^{3n \times 3n}$ in the coordinate system corresponding to the group is combined based on all element stiffness matrices, where $n$ is the total number of nodes. To deal with the boundary conditions, the elements in the global stiffness matrix, which corresponds to the nodes at the constraint end, are substituted by the large values. Next, the compliance method is used to calculate node stiffness. Through the inverse of $\mathbf{D}'$, the global compliance matrix $\mathbf{C}' \in \mathbb{R}^{3n \times 3n}$ is obtained, whose diagonal elements are used to calculate the node stiffness in the coordinate directions. More detailed descriptions are as follow:

$\mathbf{C}'$ is the inverse matrix of $\mathbf{D}'$ in the coordinate system corresponding to the group, it satisfies $\mathbf{U}' = \mathbf{C}' \mathbf{F}'$, which can be expanded as:

$$
\begin{bmatrix}
\mathbf{u}_{1x} \\
\mathbf{u}_{1y} \\
\mathbf{u}_{1z} \\
\vdots \\
\mathbf{u}_{nx} \\
\mathbf{u}_{ny} \\
\mathbf{u}_{nz}
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & \cdots & c_{1,3n-3} & c_{1,3n-2} & c_{1,3n-1} & c_{1,3n} \\
c_{21} & c_{22} & c_{23} & \cdots & c_{2,3n-3} & c_{2,3n-2} & c_{2,3n-1} & c_{2,3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
c_{3n-3,1} & c_{3n-3,2} & c_{3n-3,3} & \cdots & c_{3n-3,3n-3} & c_{3n-3,3n-2} & c_{3n-3,3n-1} & c_{3n-3,3n} \\
c_{3n-2,1} & c_{3n-2,2} & c_{3n-2,3} & \cdots & c_{3n-2,3n-3} & c_{3n-2,3n-2} & c_{3n-2,3n-1} & c_{3n-2,3n} \\
c_{3n-1,1} & c_{3n-1,2} & c_{3n-1,3} & \cdots & c_{3n-1,3n-3} & c_{3n-1,3n-2} & c_{3n-1,3n-1} & c_{3n-1,3n} \\
c_{3n,1} & c_{3n,2} & c_{3n,3} & \cdots & c_{3n,3n-3} & c_{3n,3n-2} & c_{3n,3n-1} & c_{3n,3n}
\end{bmatrix}
\begin{bmatrix}
\mathbf{f}_{1x} \\
\mathbf{f}_{1y} \\
\mathbf{f}_{1z} \\
\vdots \\
\mathbf{f}_{nx} \\
\mathbf{f}_{ny} \\
\mathbf{f}_{nz}
\end{bmatrix}
$$

Taking the first element of $\mathbf{U}'$ as an example, $\mathbf{u}_{1x}$ is the displacement of node 1 in the X direction, the calculation is as follow:

$$\mathbf{u}_{1x} = c_{11} \mathbf{f}_{1x} + c_{12} \mathbf{f}_{1y} + \cdots + c_{1,3n-1} \mathbf{f}_{ny} + c_{1,3n} \mathbf{f}_{nz}$$

When node 1 is forced only in the X direction, i.e., $\mathbf{u}_{1x} = c_{11} \mathbf{f}_{1x}$, the stiffness of node 1 in the X direction is $k_{1x} = 1/c_{11}$. Similarly, the stiffness of node 1 in the Y and Z direction is $k_{1y} = 1/c_{12}$ and $k_{1z} = 1/c_{13}$, respectively. Thus, the diagonal elements of the compliance matrix are used to calculate the node stiffness in the coordinate directions:

$$
\begin{align*}
k_{nx} &= 1/c_{3n-2,3n-2} \\
k_{ny} &= 1/c_{3n-1,3n-1} \\
k_{nz} &= 1/c_{3n,3n}
\end{align*}
$$

where $k_{nx}$, $k_{ny}$, and $k_{nz}$ are the stiffness of node $n$ in the X, Y, and Z axis, respectively.

Then, the stiffness of the nodes in the group is screened out from the stiffness of all nodes. Finally, after obtaining the stiffness of all nodes in their respective directions, the minimum stiffness of each machining region can be searched based on node-region affiliations.

### 3.2 GNN-based minimum stiffness prediction of machining regions

Graph neural network (GNN) is an artificial neural network that operates on graph domain. It has achieved remarkable results in various graph learning tasks in recent years [21]. Since the part model is represented as an attribute graph, GNN is the suitable model to predict the minimum stiffness of machining regions. The model input contains the part attribute graph and non-geometrical influence factors, including material properties, thickness, etc., while the model output is the minimum stiffness of all machining regions on the part.

#### 3.2.1 Sample features

The considered sample features are composed of the influence factors of stiffness, including the non-geometrical factors and geometrical factors of machining regions. The former contains material properties, area of constraint end, and minimum thickness of machining regions, etc., which can be directly vectorized as one-hot vectors. The latter includes the geometrical elements of the machining region and their topology information, i.e., the faces that make up the machining region and the connections among these faces, yet it is hard to directly represent them with vectors. For this problem, this paper represents the part model as an attribute graph and associates each machining region with a subgraph, which is based on our team’s previous research work [22, 23]. Then, the vectorized representation of geometrical information will be obtained based on the GNN models during representation learning.
1. The construction of part attribute graph

Gird nodes and their connections in finite element model can naturally be represented as a graph to describe the geometry, yet the large number of nodes make it difficult to deal with the graph efficiently. Thus, another graph is considered in this research, i.e., the graph constructed by the faces and edges of the part model. The 3D model of a part can be regarded as a closed entity surrounded by multiple faces, where the topology is the connections among those faces. Then, the part can be expressed as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertexes, and $\mathcal{E}$ is the set of edges, corresponding to the faces and edges on the part model respectively, as shown in Fig. 5. In addition, considering that each face has geometrical attributes including normal vector, area, perimeter, etc., 11 typical geometrical attributes are selected according to the correlation to the local stiffness, as listed in Table 1. These geometrical attributes compose the attribute vector $\mathbf{h}$, and all the attribute vectors form an attribute matrix $\mathbf{H}$. Therefore, the part attribute graph is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{H})$.

After the part model is represented as an attribute graph, the machining regions correspond to the subgraphs in the graph, and relation matrix $\mathbf{R} \in \mathbb{R}^{m \times t}$ is constructed to describe the correlation between each vertex and each subgraph, where $m$ is the number of vertexes, and $t$ is the number of machining regions. Let $f_j$ be a machining region, whose vertexes are represented as the set $\mathcal{F}_j$, then, the relation between vertex $v_i$ in $\mathcal{F}_j$ and the matrix element $r_{ij}$ in $\mathbf{R}$ is shown as follows:

$$r_{ij} = \begin{cases} 1 & v_i \in \mathcal{F}_j, 1 \leq i \leq m, 1 \leq j \leq t \\ 0 & v_i \notin \mathcal{F}_j \end{cases}$$

(7)

2. Other influence factors

In addition to geometrical factors, some other non-geometrical influence factors are also important to the stiffness, including material properties (e.g., elastic modulus, Poisson's ratio and density), area of constraint end, and minimum thickness of machining region, etc., as listed in Table 2. These factors can be directly vectorized.

| No. | Attributes                              | Data type | Dimension |
|-----|----------------------------------------|-----------|-----------|
| 1   | Normal vector                          | Double    | 3         |
| 2   | Face type                              | Integer   | 3         |
| 3   | Whether has inner loop                 | Integer   | 2         |
| 4   | Area                                   | Double    | 1         |
| 5   | Perimeter                              | Double    | 1         |
| 6   | The ratio of area to perimeter          | Double    | 1         |
| 7   | Min, max height from the bottom face   | Double    | 2         |
| 8   | The length of the shortest, longest edge| Double    | 2         |
| 9   | Number of the edges                    | Integer   | 1         |
| 10  | Number of convex, concave connections  | Integer   | 2         |
| 11  | Number of tangent, non-tangent connections | Integer | 2         |

3.2.2 GNN architecture

In this paper, the defined four types of machining regions have different geometric structures, which leads to the difference in their mapping from geometric structure to the minimum stiffness. If only one GNN model is used, it will require much training data whose acquisition is time consuming and of high cost. Thus, to reduce the dependence on training data, it is better to build GNN models for each type of the machining region, rather than building a unified model. The four GNN models have similar network architecture, but not the same. The number of layers and the number of units at each layer are different. Furthermore, each GNN model is trained with its own data. The input contains the part attribute graph and the encoding vectors of non-geometrical influence factors, while
the output is the minimum stiffness of machining regions. First, in graph convolution layers, vertexes of the attribute graph are embedded as representation vectors to represent the feature of each vertex. Then, the machining region representation layer is used to combine the feature of vertexes belonging to the same machining region to represent the geometrical information, and further concatenate with the vectorized non-geometrical influence factors to represent the complete sample feature. Finally, in fully connected prediction layers, further feature extraction of machining regions is performed and processed to generate the prediction. The architecture of the GNN model is shown in Fig. 6.

1. Graph convolution layers

For GNN, graph convolution is the main way to learn vertex representations. Existing graph convolution mainly follows the aggregation of vertex attributes, where each vertex absorbs the attributes of its neighbor vertexes. After multiple iterations of aggregation, the representation vector of each vertex contains both geometrical and topological information, as shown in Fig. 7. In this paper, graph attention mechanism is used to realize the aggregation of vertex attributes [24], as shown in formula (8):

$$h^{(l)}_i = A\left(\sum_{j \in \mathcal{N}_i \cup \{v_i\}} a^{(l-1)}_{ij} W^{(l-1)} h^{(l-1)}_j\right)$$  \hspace{1cm} (8)

where $A(\cdot)$ is the activation function, $\mathcal{N}_i$ is the set of neighbor vertexes of vertex $v_i$, $a_{ij}$ is the attention weight coefficient which is based on node attributes, $W \in \mathbb{R}^{F \times F}$ is a learnable weight matrix, and $h^{(l)}_i \in \mathbb{R}^F (1 \leq i \leq m)$ is the representation vector of vertex $v_i$ at the $l$ th layer, where $m$ is the number of vertexes, $F$ the dimension of the attribute vector.

To make the learning of GNN more stable, multi-head is used, which performs $K$ aggregations simultaneously, and the results are combined together, as shown below:

$$h^{(l)}_i = \bigcup_{k=1}^{K} h^{(l)}_i$$  \hspace{1cm} (9)

where $\bigcup$ represents the concatenation operation except for the last aggregation, in which $\bigcup$ represents the summation operation.

Considering the size of the subgraph in this paper, it requires $2 \sim 3$ convolution layers to achieve proper representation learning.

2. Machining region representation layer

Table 2 The other influence factors of stiffness in machining regions

| No. | Description                          | Data type | Dimension |
|-----|-------------------------------------|-----------|-----------|
| 1   | Elastic modulus                     | Double    | 1         |
| 2   | Poisson’s ratio                     | Double    | 1         |
| 3   | Density                             | Double    | 1         |
| 4   | The bottom thickness of the part    | Integer   | 1         |
| 5   | Area of constraint end of the part  | Double    | 1         |
| 6   | The volume of the part              | Double    | 1         |
| 7   | Max distance between machining region and the constraint end | Double | 1 |
| 8   | Min distance between machining region and the bottom face | Double | 1 |
| 9   | Min thickness of machining region    | Double    | 1         |
| 10  | Constraint condition of machining region | Integer | 3 |
The machining region representation layer is a single layer to represent the complete sample feature of machining regions, which contains two steps: The first step is to represent the geometrical information of the machining region, and the second step is to incorporate non-geometrical influence factors of stiffness.

For the first step, since each machining region is correlated to a subgraph, summation is used to combine the representation of the vertexes contained in the subgraph to represent the geometrical information, as shown below:

\[
f^r = \sum_{v_j \in G_{sub}} h^j
\]

where \(f^r\) is the geometrical information representation vector of the machining region, \(G_{sub}\) is a subgraph corresponding to the machining region, and \(h^j\) is the representation vector of vertex \(v_j\) which is the output of graph convolution layers.

For the second step, the representation of geometrical information is concatenated with the vectorized non-geometrical influence factors to completely represent a machining region:

\[
f = f^r \parallel f_{non}
\]

where \(f\) and \(f_{non}\) are the complete representation vector and the vectorized non-geometrical influence factors of the machining region, and \(\parallel\) is the concatenation operation.

3. Fully connected prediction layers

The fully connected prediction layers are used to further extract features from the representation of machining regions, and then generate the minimum stiffness prediction. In the constructed GNN models, there are 3 ~ 4 fully connected layers, each of which consists of a linear combination and a nonlinear activation except for the last prediction layer. Considering that the output minimum stiffness is continuous value, mean square error (MSE) is used as the loss function, as shown below:

\[
\text{loss}_{\text{MSE}} = \frac{1}{t} \sum_{i=1}^{t} (\hat{y}_i - y_i)^2
\]

where \(\hat{y}_i\) and \(y_i\) are the predicted value and label value respectively, \(t\) is the total number of machining regions.

4 Case study

To test the proposed GNN-based surrogate models, the dataset is first built based on 36 aircraft structural parts including large, medium and small frame and beam parts, which are made from aluminum alloy. The dataset contains 36 attribute graphs, 3899 machining operations corresponding to 3899 machining regions, and the machining regions of bottom, sidewall, rib, and corner are 1094, 1072, 716, and 1017 respectively, as listed in Table 3. Then, four GNN models are established to predict the minimum stiffness of each type of machining region. To validate the model performance, the predicted results on test set are compared with the calculation results from FEM to validate the predicting performance. The total computation time of both methods is compared to validate the computational efficiency.

4.1 Data preparation

The data preparation contains three aspects: the selection and acquisition of sample data, the selection and acquisition of label data, and data splitting. For the proposed models, the sample features include the geometrical information of the machining region and non-geometrical influence factors. The former is represented based on the part attribute graph, where each vertex contains 11 types of geometrical attributes that may affect the stiffness, as listed in Table 1. The later contains material properties, area of constraint end, and constraint condition of machining regions, etc., as listed in Table 2. Since the part material in the dataset is only aluminum alloy, the factors related to material properties are not considered. For data extraction, a data extraction tool was developed in CATIA based on Microsoft Visual Studio 2015 to effectively extract sample data.

In terms of label data, the minimum stiffness of all machining regions on the part is calculated in ABAQUS, a widely used FEM software. The elastic modulus is
72000 Mpa, the Poisson’s ratio is 0.33, the density is $2.82 \times 10^{-9}$ tonne/mm$^3$, and the grid element type is C3D10.

For data splitting, 36 parts are divided into the training set with 34 parts, the validation set with 1 part and the test set with 1 part, the number of each type of machining region in each dataset is listed in Table 4. Some parts models are shown in Fig. 8, they large, medium and small frame and beam parts models, i.e., no. 7 large beam, no. 12 large frame, no. 15 large beam, no. 20 medium frame, no. 26 medium beam, and no. 28 small beam.

4.2 GNN development

To predict the minimum stiffness of four types of machining regions, including bottom, sidewall, rib, and corner, four GNN models are developed based on TensorFlow 1.13.1 [25], a widely used deep learning framework proposed by Google. The input of these GNN models includes the part attribute graph and vectorized non-geometrical influence factors, while the output is the minimum stiffness of all machining regions on the part. The hyper-parameters of the GNN models are listed in Table 5.

4.3 Validation

4.3.1 Predicting performance

Before analyzing the predicting performance, loss curves are used to observe the learning process of these four GNN models, as shown in Fig. 9. The overall trends of these four loss curves are very similar. The loss curve of each GNN model drops rapidly at the beginning, then the decreasing trend gradually slows down, and after 1200 training steps, the loss curve tends to be stable. Thus, the overall training trend is regular, each model is trained rapidly first, then slowly, and finally the model is stable.

To analyze the predicting performance, the model prediction on validation set and test set is compared with the FEM

### Table 3 Details of the built dataset in this paper

| Part no. | Number of faces | Number of edges | Number of machining regions |
|----------|----------------|----------------|----------------------------|
| 1        | 324            | 940            | 137                        |
| 2        | 219            | 629            | 99                         |
| 3        | 357            | 1033           | 154                        |
| 4        | 192            | 559            | 66                         |
| 5        | 227            | 660            | 86                         |
| 6        | 233            | 675            | 100                        |
| 7        | 386            | 1128           | 188                        |
| 8        | 461            | 1294           | 185                        |
| 9        | 340            | 1003           | 131                        |
| 10       | 443            | 1237           | 192                        |
| 11       | 293            | 828            | 110                        |
| 12       | 481            | 1369           | 197                        |
| 13       | 338            | 969            | 127                        |
| 14       | 416            | 1218           | 184                        |
| 15       | 449            | 1310           | 187                        |
| 16       | 293            | 836            | 119                        |
| 17       | 343            | 996            | 142                        |
| 18       | 308            | 906            | 125                        |

### Table 4 Details of the number of each type of machining region on different dataset

|                | GNN-bottom | GNN-sidewall | GNN-rib | GNN-corner | Total  |
|----------------|------------|--------------|---------|------------|--------|
| Training set   | 1071       | 1049         | 694     | 1012       | 3816   |
| Validation set | 15         | 15           | 10      | 8          | 48     |
| Test set       | 8          | 8            | 12      | 7          | 35     |
calculation results, where the FEM calculation results are taken as the baseline, and the average percentage errors (APE) is used as the metric, as follows:

\[
APE = \frac{1}{t} \sum_{i=1}^{t} \frac{|\hat{k}_i - k_i|}{k_i} \times 100\%
\]  

(13)

where \(\hat{k}_i\) and \(k_i\) are the predicted value of the proposed model and calculation value of FEM respectively, \(t\) is the total number of machining regions. The results are shown in Table 6. The model performance on validation set and test set is very close, which indicates that the generalization performance of the model is stable. Furthermore, each metric is less than 10%. According to the engineering experience, the predicting result of the GNN-based surrogate model can be used in planning machining parameters.

### 4.3.2 Computational efficiency

To validate the computational efficiency of the proposed method, taking a test part as an example, the total computation time of the proposed method and FEM is compared. The part dimension is 1534 mm \(\times\) 208 mm, the number of grid elements in FEM is 12920, and the number of nodes is 26602.

![Fig. 8 Some part models on the dataset](image)

**Table 5** The hyper-parameters of four GNN models

| Parameters                                      | GNN-bottom | GNN-sidewall | GNN-rib   | GNN-corner |
|------------------------------------------------|------------|--------------|-----------|-----------|
| Number of graph convolution layers             | 2          | 3            | 3         | 3         |
| Number of units at the graph convolution layers| 30, 30     | 20, 20, 20   | 20, 20, 20| 20, 20, 20|
| Number of attention heads                      | 25, 25     | 20, 20, 20   | 20, 20, 20| 20, 20, 20|
| Number of machining region representation layer| 1          | 1            | 1         | 1         |
| Number of fully connected prediction layers     | 3          | 4            | 4         | 3         |
| Number of units at the fully connected prediction layers| 25, 15, 8 | 20, 15, 10, 5| 20, 15, 10, 5| 15, 10, 5|
| Learning rate                                  | 0.0008     | 0.0006       | 0.0018    | 0.0015    |
The steps of FEM and the time spent at each step are as follows. First, preparations were made. The minimum stiffness directions of all nodes were calculated, and 5 coordinate systems were built to divide all nodes into 5 groups based on the calculated directions, which took 4 min. Then, for nodes of each group, the minimum stiffness calculation contained 5 steps: (1) All element stiffness matrices in the global coordinate system were calculated once in total, which took 3.5 min. (2) Coordinate system transformation was used to obtain the element stiffness matrices in the new coordinate system corresponding to each group. Since there was a group whose coordinate system was the global coordinate system, it required a total of 4 calculations, which took $4 \times 0.5$ min. (3) The global stiffness matrix was combined based on the calculated element stiffness matrices, which took $5 \times 2.5$ min. (4) The global compliance matrix was calculated by reversing the global stiffness matrix, and the stiffness of all nodes was further obtained, which took $5 \times 33$ min. (5) The stiffness of the nodes in each group was screened out, because each group took less than 0.1 s, the time spent can be ignored. After obtaining the stiffness of all nodes in the respective directions, the minimum stiffness of each machining region was searched in 0.1 s, which can also be ignored. Thus, the FEM calculation took 187 min in total.

The proposed data-driven method contained 3 steps: (1) Data preparation, including sample features and label data

![Fig. 9 The loss curves of the proposed models on training set](image)

**Table 6** The average percentage errors of four GNN models on validation set and test set

| Prediction model | Validation set | Test set  |
|------------------|----------------|-----------|
| GNN-bottom       | 8.693%         | 6.717%    |
| GNN-sidewall     | 6.314%         | 7.367%    |
| GNN-rib          | 6.689%         | 7.432%    |
| GNN-corner       | 7.595%         | 5.962%    |
acquisition, the former was fast, yet the latter was time-consuming, which required 34 FEM calculation corresponding to 34 parts on the training set. (2) Model training, which spent 10.5 h, 12 h, 11.5 h, and 9.5 h for four GNN models, respectively. (3) The application of the trained models. The four trained GNN models took 23 s, 28 s, 25 s, and 26 s in predicting, respectively, i.e., 102 s in total. Although the data acquisition and the model training require much time, once the training is completed, the proposed model can efficiently calculate the minimum stiffness of machining regions of other similar parts, which can greatly reduce the time compared with FEM.

4.4 An application case

The preceding validation shows the proposed surrogate-based method can learn the maps between part geometry and stiffness from the high-fidelity finite element simulation data. The application procedures of these GNN-based surrogate models in practical process planning are illustrated in this Section.

To obtain satisfactory results, one should select the parts whose material and structure are included in the training set. Thus, a new part with similar material and structure is selected, the minimum stiffness of the machining regions of the part is generated by these trained GNN-based surrogate models, as shown in Fig. 10. First, the attribute graph is constructed based on the part 3D model, and the machining regions of the part are divided into bottom, sideline, rib, and corner for further extraction of the subgraph information and non-geometrical influence factors of these regions. Next, the part attribute graph, the subgraph information and non-geometrical influence factors of machining regions are input to the trained GNN models corresponding to the four types of machining regions, which can generate the minimum stiffness of all machining regions. Finally, the generated stiffness data can be applied to the process planning for the part.

To further validate that the proposed method can meet the engineering requirements in both accuracy and computational efficiency, the test part is used to validate the effectiveness of the proposed method. In terms of accuracy, the average errors of minimum stiffness prediction to four types of machining regions are less than 10% compared with FEM. More specifically, among the 76 machining regions, there are only 4 of them where the prediction errors of the minimum stiffness exceed 10%, as shown in Fig. 11. For computational efficiency, the GNN-based surrogate models only take 104 s to obtain the minimum stiffness of all machining regions, while FEM takes 259 min, which indicates the computational efficiency of the surrogate model is greatly improved compared with FEM when the accuracy meets the engineering requirements. Therefore, the proposed method can be applied in practical process planning of parts.

Fig. 10 The application procedures of the GNN-based surrogate models
parameters planning according to the engineering experience. Besides, in terms of computational efficiency, once the model training is completed, the proposed models can efficiently calculate the minimum stiffness of machining region for any other similar parts, which can greatly reduce the time compared with FEM. In the future work, the proposed method will be combined with the optimization of machining parameters for aircraft structural parts to improve the machining efficiency while ensuring the machining quality.

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Declarations

Ethics approval and consent to participate Not applicable.

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Fig. 11 The comparison of the proposed method and FEM

5 Conclusion and future work

Large thin-walled structural parts are easy to deform during machining if improper cutting parameters are selected, thus local stiffness has to be seriously considered during machining parameter planning. However, existing methods for calculating the stiffness of the part are either inaccurate or time-consuming. To address this issue, this paper proposes a data-driven method for minimum stiffness prediction of aircraft structural parts. The main contributions of this paper are as follows:

1. The minimum stiffness of the machining region is defined after classifying the machining regions of aircraft structural part finishing into bottom, sidewall, rib, and corner.
2. The minimum stiffness prediction of machining regions is converted to a graph learning task by representing the 3D part model as an attribute graph which can be used as the input feature of GNN.
3. Four GNN models are designed and trained to effectively predict the minimum stiffness of the four types of machining regions.

The proposed data-driven method can efficiently predict the minimum stiffness of all machining regions for aircraft structural parts, and the model training is end-to-end, which is convenient once the data is sufficient. In terms of model performance, the predicted results are compared with the calculation results from FEM in case study, and taking FEM results as the baseline, the average percentage errors of the predicting minimum stiffness corresponding to four types of machining regions are 6.717%, 7.367%, 7.432%, and 5.962% respectively, which can meet the requirements in machining.
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