Analyzing the Effect of Consistency Violation Faults in Self-Stabilizing Programs

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Abstract. Consistency violation faults \( cvf \)s refer to faults that occur due to inconsistent reads in a shared memory program. In the execution of shared-memory, interleaving, self-stabilizing programs, \( cvf \)s offer a trade-off. Specifically, preventing \( cvf \)s requires processes to coordinate with each other thereby slow ing the execution of each action. On the other hand, permitting \( cvf \)s requires less coordination and faster execution of each action. However, when a \( cvf \) occurs, it can disrupt the convergence of self-stabilizing programs. Thus, a computation of a program in the presence of \( cvf \)s can be thought of as a contest where program actions attempt to take the program to a legitimate state whereas \( cvf \)s could potentially disrupt this convergence. We analyze three self-stabilizing programs (token ring, coloring and maximal matching) to evaluate this contest between program transitions and \( cvf \)s. We find that the relative cost of \( cvf \)s is generally small, i.e., as long as a few program transitions can execute between \( cvf \)s, the program (probabilistically) converges. We also find that the cost-distribution of \( cvf \)s is exponential in nature in that the fraction of \( cvf \)s with cost \( c \) exponentially decreases with \( c \). We validate these results via simulation where we control the rate of \( cvf \)s.

Keywords: Consistency violating faults · Stabilization · Fault-tolerance.

1 Introduction

A self-stabilizing algorithm ensures that starting from an arbitrary state, it recovers to a legitimate state. And, subsequent computation remains in legitimate states forever. Our focus is on self-stabilizing shared memory programs. While we define the precise nature of these programs in Section 2, these programs consist of actions, and the property of self-stabilization is guaranteed under the constraint that actions execute in an interleaving manner. In other words, the computation is of the form \( \langle s_0, s_1, s_2, \ldots \rangle \) where each successive state is obtained by executing one of the actions of the program.

To ensure that actions are executed in an interleaving manner, in practice, we need to impose additional constraints. For example, we need to use some type of local mutual exclusion [1] [2] to ensure that action execution can be serialized.

Without properly coordinated execution among actions, the execution may not correspond to an interleaved execution. In [5], such violations were introduced as consistency violation faults \( (cvf) \). While a \( cvf \) has the potential to disrupt the recovery of self-stabilizing programs, by the nature of self-stabilization,
the program is guaranteed to recover from states reached by a $cvf$. Thus, recovery to legitimate states can be viewed as a contest between $cvfs$ that (potentially) perturb the recovery and program transitions that help the recovery.

In [6], it is shown that this contest is won by program transitions in several self-stabilizing algorithms. Specifically, in [6], the authors show that the speedup obtained by permitting $cvfs$ by removing the requirement of local mutual exclusion is $2–15$ times when compared with eliminating $cvfs$. However, the reasoning behind the reduction in time is unclear.

The goal of this paper is to discern the reasons behind such speedup by understanding the contest between $cvfs$ and program transitions. With this in mind, we evaluate several self-stabilizing programs to determine the constraints under which the property of self-stabilization is (probabilistically) preserved.

The ability to converge in the presence of $cvfs$ depends upon the relative cost of $cvfs$. If the relative cost of $cvfs$ is high (i.e., they perturb the recovery substantially) then it would imply that a small frequency of $cvfs$ would prevent the recovery to legitimate states. By contrast, a low relative cost would imply convergence to legitimate states even with frequent $cvfs$. We find that the relative cost of $cvfs$ is generally small. Specifically, the contributions of the paper are as follows:

**Contributions of the paper.**

- We define the notion of $cvf_{max}$ which is a set of transitions that perturb the state of one of the program’s processes. $cvf_{max}$ is a superset of $cvfs$ that could occur in the program. We show that the average cost of $cvf_{max}$ transitions is 0, i.e., if all $cvf_{max}$ transitions could occur during the execution then they are unlikely to affect the recovery to legitimate states.
- Since the exact $cvfs$ cannot be easily determined for a given program, we focus on $cvfs$ that perturb the recovery (i.e., we ignore $cvfs$ that help the recovery). We analyze three classic self-stabilizing programs, namely token ring [7], maximal matching [8], and coloring [9], for their relative cost of $cvfs$.
- Our analysis of those case studies shows that the relative cost of $cvfs$ is often small, roughly $2–4$. In other words, as long as $2–4$ program transitions can execute between $cvfs$, recovery to legitimate states will not be prevented.
- We also analyze the case studies to determine the ratio of the number of steps required to converge in the presence of $cvf$ and in the absence of $cvf$. We observe that this ratio is small (close to 1) even when only 4–8 program transitions execute between $cvfs$. Moreover, since permitting $cvfs$ does not require the use of local mutual exclusion, each step is significantly faster than when $cvf$ is eliminated by using local mutual exclusion. This analysis explains why tolerating $cvfs$ provides better performance than eliminating them, thus providing a foundation to the results in [6].
- We analyze the cost of individual $cvfs$ and find that the number of $cvfs$ with a given cost $c$ decreases exponentially with $c$.
- The tool used in this analysis is developed as a general-purpose toolset that allows one to analyze other programs with minimal implementation effort. The source code and evaluation results are available at [10].
Organization of the paper. In Section 2, we define program model and self-stabilization. In Section 3, we identify the need for local mutual exclusion in refining self-stabilizing programs. Section 4 defines cvfs and show how they can disrupt or help the recovery process. Section 5 identifies how the cost of cvfs and the benefit of program transitions can be computed. In Section 6, we analyze our case studies to evaluate the effect of cvfs on recovery. We discuss implications of our results in implementing self-stabilizing programs in Section 7. Section 8 discusses related work. Finally, Section 9 presents concluding remarks and discuss the future work.

2 Preliminaries
A distributed program consists of a set of n processes. Each process j has a set of neighboring processes, denoted as Nb.j. Each process j is associated with a set of variables and a set of actions. Each action of process j is of the form g → st, where g is a Boolean expression involving variables of j and its neighbors (Nb.j), and st is the statement that updates the variables of j.

Variables of program p are the union of the variables of its processes. A state of program p is obtained by assigning each variable of p a value from its domain. The state space of program p, denoted by Sp, is the set of all possible states of p.

We say that an action g → st is enabled in state s iff g evaluates to true in state s. Transitions of process j in program p is the set \{(s0, s1) | s0, s1 ∈ Sp and s1 is obtained from s0 by executing one of the enabled actions of j\}. Transitions of program p are the union of the transitions of its processes.

A computation of program p is of the form ⟨s0, s1, ..., sl⟩, where si, i ≥ 0 is a state of the program and si+1 is obtained from si by executing one of the enabled actions of the program. For sake of simplicity, we assume that each state has an outgoing transition, i.e., for any state s, there exists a state s′ such that (s, s′) is a transition of p. To achieve this, if no transition is enabled in state s then we add (s, s) as a transition of p.

A state predicate S is a Boolean expression involving variables of p. We also overload S to denote a subset of the state space where predicate S is true. In other words, we use the Boolean expression x = 0 and the set of all states where the x value is 0 interchangeably.

A program is self-stabilizing (respectively, stabilizing) with invariant inv iff

Any program computation includes a state where inv is true. In other words, for any computation ⟨s0, s1, ..., sl⟩, there exists a state s1 such that s1 ∈ inv

If (s0, s1) is a transition of p and s0 ∈ inv then s1 ∈ inv.

3 Examples of Self-Stabilizing Programs and Local Mutual Exclusion
In this section, we discuss the implementation issues associated with two distributed programs. First, consider a simple graph coloring program where a node changes its color to another available color if it finds that its color matches with that of its neighbor (see Appendix B for detailed descriptions of the program).
Further consider the state \(\langle c.0 = 0, c.1 = 0 \rangle\) of this program where processes 0 and 1 are neighbors. If process 0 (respectively 1) executes then the resulting state would be \(\langle c.0 = 1, c.1 = 0 \rangle\) (respectively, \(\langle c.0 = 0, c.1 = 1 \rangle\)). On the other hand, if both processes execute simultaneously then the resulting state would be \(\langle c.0 = 1, c.1 = 1 \rangle\). (Note that simultaneous execution is not allowed in the interleaving model of program computation defined in Section 2.) Clearly, this is undesirable and the colors may remain invalid forever.

This problem can be addressed via local mutual exclusion \[1\] that guarantees that two neighboring processes do not execute at the same time. Clearly, local mutual exclusion introduces an overhead as it requires extra work to coordinate among the processes and block the waiting processes.

Next, we consider the (3-state) token ring program introduced by Dijkstra \[7\] (cf. Appendix B for complete details of the algorithm) and its state \(\langle x.2 = 1, x.3 = 2, x.4 = 0 \rangle\). (The \(x\) values of other processes are omitted since they are not relevant for this discussion.) In this state, actions of process 2 and 3 are enabled. If process 2 (respectively 3) executes its action, the resulting state would be \(\langle x.2 = 2, x.3 = 2, x.4 = 0 \rangle\) (respectively \(\langle x.2 = 1, x.3 = 0, x.4 = 0 \rangle\)). If both processes execute simultaneously then the resulting state would be \(\langle x.2 = 2, x.3 = 0, x.4 = 0 \rangle\). Observe that the program does not include a transition from the original state \(\langle x.2 = 1, x.3 = 2, x.4 = 0 \rangle\) to the state \(\langle x.2 = 2, x.3 = 0, x.4 = 0 \rangle\). However, since this execution is equivalent to execution of both actions one after another, (it appears that) this specific cvf would not affect the convergence. (We would like to remind the reader that this claim should not be generalized to the whole program in \[7\] since we have not evaluated all cvfs of this program.)

4 Consistency Violation Faults.

While the above discussion focused on simultaneous execution of the program actions, such execution is an instance of consistency violation faults where the process executing the action reads an incorrect value for one of its neighbors’ variables. In the above example, transition from \(\langle c.0 = 0, c.1 = 0 \rangle\) to \(\langle c.1 = 1, c.1 = 1 \rangle\) can be thought of as a transition from \(\langle c.0 = 0, c.1 = 0 \rangle\) to \(\langle c.0 = 1, c.1 = 0 \rangle\) by process 0 and then transition from \(\langle c.0 = 1, c.1 = 0 \rangle\) to \(\langle c.0 = 1, c.1 = 1 \rangle\) by process 1 where it reads an old value \(\langle c.0 = 0 \rangle\) of \(c.0\).

More generally, if \(g \rightarrow st\) is an action of process \(j\) then reading old values in \(g\) or \(st\) could cause process \(j\) to change its state incorrectly. Such transitions are denoted as consistency violation faults (cvfs). While determining the cvfs for a given process/program is potentially a challenging task, we can see that cvfs at \(j\), namely \(cvf.j\) is a subset of \(cvf_{max}.j\), where

\[
(cvf_{max}.j) = \{ (s,s')|s' \text{ is obtained by changing variables of } j \text{ in } s \}\]

Similar to transitions of a program, we can define cvfs for a program. In other words, \(cvf.p\) \((cvf_{max}.p)\) for program \(p\) is the union of \(cvf.j\) \((cvf_{max}.j)\) where \(j\) is a process in \(p\).

A sequence \(\langle s_0, s_1, s_2, \cdots \rangle\) is a computation of program \(p\) in the presence of \(cvf.p\) iff for each \(i\), \(\langle s_i, s_{i+1} \rangle\) is a transition of \(p\) or is in \(cvf.p\). (Note that cvfs are not required to be terminating.) Thus, we can view the execution of a stabilizing program as shown in Figure 1, where the rank of each state is depicted by how far
the state is from a legitimate state (a state in the invariant). Examples of such rank functions include (minimum, average, or maximum) steps/time required to reach a state in $inv$. Specifically, the rank of states in $inv$ is 0 and a sufficiently long program computation decreases the rank of the program state.

![State space diagram]

**Fig. 1.** Computation in the Presence of $cvs$

Note that execution of $cvs$ may cause the rank value to increase thereby increasing steps for recovery (or preventing the recovery). However, the time required to execute one step is lower if $cvs$ are permitted to occur, as local mutual exclusion is no longer required and each process could execute at its own speed without having to coordinate with other processes.

To guarantee convergence in spite of $cvs$, we need to ensure that the rank decreases in a computation over time even if it does not decrease in every step. In other words, consider a sufficiently long computation in the presence of $cvs$: $\langle s_0, s_1, \ldots, s_l, \ldots \rangle$, where all states in the computation are outside legitimate states. In this case, we need to ensure that $\text{rank}(s_0) > \text{rank}(s_l)$. If this is assured then eventually the program will reach a state whose rank is 0, thereby guaranteeing the recovery of the program. Additionally, $cvs$ are not malicious in nature since they are caused when a node reads an older value for one of its neighbors. In other words, $cvs$ are random and unintentional. Hence, we can focus on average effect of $cv$ instead of the worst-case effect.

The above discussion suggests that if some condition that involves (1) frequency of $cvs$, (2) average increase in rank by $cvs$ and (3) average decrease in the rank by program transitions is satisfied then permitting $cvs$ would be beneficial for convergence. The goal of this paper is to evaluate the average change in rank by a $cv$ and average decrease in rank by a program transition in a given program to determine the frequency of $cvs$ that would still preserve the convergence property.

**Average rank change by $cvs$.** Since identifying precise $cv$ transitions for a given program is a challenging task, we can approximate it with $cvf_{max}$. We have various choices for defining the rank of a state. For example, we could use the average/minimum/maximum number of transitions required to reach a legitimate state. One observation in this context is that the increase (respectively, decrease) in the rank from $s_0$ to $s_1$ ($\text{rank}(s_1) - \text{rank}(s_0)$) is the same as the decrease (respectively, increase) in the rank from $s_1$ to $s_0$. Moreover, if $(s_0, s_1) \in cvf_{max}$ then $(s_1, s_0) \in cvf_{max}$. Thus, if we average the change in rank of all transitions in $cvf_{max}$ then the average is 0.

**Observation 1** The average change in rank by transitions in $cvf_{max}$ is 0.
What the above result shows is interesting and misleading. It is interesting because it shows that if all \( cvf \)s were equally likely then the overall benefit and loss caused by \( cvf \)s would be 0. At the same time, this result is misleading because the actual \( cvf \)s that could occur are a subset of those in \( cvf_{max} \). Since our goal is to determine the frequency under which convergence property is not affected, we only focus on \( cvf \)s that increase the rank.

We also introduce the notion of \( cvf_{feasible} \). This is also a superset of actual \( cvf \)s that could occur in the program but a subset of \( cvf_{max} \). To understand the notion of \( cvf_{feasible} \), observe that, in action \( g \rightarrow st \), the process may read arbitrary values of its neighbors but it will change its state by executing \( st \). For example, if \( st \) was of the form \( x = x + 1 \) where \( x \) is a local variable of the process then a \( cvf \) cannot perturb the program from a state where \( x = 1 \) to \( x = 0 \). Since \( cvf \)s of a given program can be very difficult to compute, we use \( cvf_{feasible} \) as an approximation of \( cvf \)s. Note that the average rank of transitions in \( cvf_{feasible} \) need not be 0. Furthermore, we only consider those \( cvf \)s that increase the rank.

5 Computing the Benefit of Program Transitions and Cost of \( Cvfs \)

As discussed earlier, to compute the benefit of a program transition (respectively, cost of \( cvf \)) \( (s_0, s_1) \), we need to find the ranks of \( s_0 \) and \( s_1 \) and the benefit of the program transition (respectively, cost of \( cvf \)) is \( rank(s_1) - rank(s_0) \). There are several choices to define the rank subject to the constraint that the rank of legitimate states should be 0 and the rank of illegitimate states should be positive. We consider two options

- \( max-rank \), where \( max-rank(s_0) \) is the length of the maximum path to reach a legitimate state.
- \( average-rank \), where \( average-rank(s_0) \) is the average length of all the paths from \( s_0 \) to reach a state in the invariant.

Since computation of \( max-rank \) and \( average-rank \) is achieved using existing approaches, we provide it in Appendix. Note that for a stabilizing program, the rank will always be finite since each computation is eventually guaranteed to reach the invariant. Once the rank of every state is computed, we compute the benefit of a program transition \( (s_0, s_1) \) to be \( rank(s_1) - rank(s_0) \). Then, we compute the average benefit of all program transitions. For \( cvf \)s, as discussed above, we only consider \( cvf \)s that disturb the convergence, i.e \( cvf = (s_0, s_1) \) where \( rank(s_0) < rank(s_1) \). We compute the average cost of those \( cvf \)s.

Another issue in determining the frequency of \( cvf \)s that can be tolerated is that it is quite difficult to determine the rank of a given state. Specifically, determining the rank requires one to analyze the entire state space of the program to determine the number of steps required for convergence from a given state. Hence, we use both full and partial analysis to determine the ranks of states in the state space as well as the benefit of program transitions and cost of \( cvf \)s.
6 Analysis and Simulation Results

6.1 Methodology

As discussed in Section 5, each program state has a rank (either max-rank or average-rank). The rank effect of a state change \((s_0, s_1)\) (which could be a program transition or a cvf) is the difference \(\text{rank}(s_1) - \text{rank}(s_0)\). Observe that if \((s_0, s_1)\) is a program transition and \(s_0 \in \text{inv}\) then \(s_1 \in \text{inv}\) and the rank effect is 0. Hence for a given program, we evaluate \(\text{effect}_{\text{prog}}\), the average of the rank effects of all program transitions that originate outside the invariant (i.e. \(s_0 \notin \text{inv}\)). Note that \(\text{effect}_{\text{prog}} < 0\) since program transitions reduce the rank overall. Similarly, we compute \(\text{effect}_{\text{cvf}}\) for cvf transitions that begin outside the invariant. As mentioned in Section 4, we only focus on cvf transitions that increase the rank thereby making the analysis conservative in nature. Therefore, \(\text{effect}_{\text{cvf}} > 0\).

The relative recovery cost of cvfs is defined as \(\text{rel}_{\text{cvf}} = -\frac{\text{effect}_{\text{cvf}}}{\text{effect}_{\text{prog}}}\). The relative recovery cost \(\text{rel}_{\text{cvf}}\) measures how many program transitions, on average, are needed to equalize the perturbation effect of a cvf.

We evaluate the cost of cvfs and the benefits of program transitions on three self-stabilizing programs: token ring [7], graph coloring [9], and maximal matching [8] (see Appendix B for description of these programs). For graph coloring and maximal matching, we used three types of input graph (topology) in our evaluation: ring, power-law graph, and random regular graph. In a power-law graph, node degrees (number of process’ neighbors) follow the power-law distribution and nodes form clusters within the graph. In a random regular graph, nodes have the same degree and are randomly connected. Ring is the special case of random regular graph where every node has degree 2. We used the tool networkx [11] to generate those graphs.

6.2 Evaluation Results

We consider the analysis with both max-rank and average-rank. Observe that analysis with max-rank is focused on the worst-case convergence time whereas analysis with average-rank is focused on the average-case convergence time. We anticipate that the analysis with average-rank is more valuable in practice.

We analyze the effect of cvfs in three dimensions. First, we evaluate the effective rate of cvfs that can be tolerated. Towards this end, we compute the cvf relative cost \(\text{rel}_{\text{cvf}} = -\frac{\text{effect}_{\text{cvf}}}{\text{effect}_{\text{prog}}}\) for the three case studies. Second, we examine the distribution of cvfs in them. Finally, we simulate the execution of the given program in the presence of cvfs and observe its convergence behavior when the frequency of cvfs is varied.

Computing \(\text{rel}_{\text{cvf}}\) From Figure 2, we observe that the cvf relative recovery cost \(\text{rel}_{\text{cvf}}\) does not significantly differ between max-rank and average-rank in full analysis. In other words, when we utilize full analysis, the number of program transitions required to neutralize a cvf remains the same whether we use max-rank or average-rank.
Comparing cvf relative recovery cost of case study programs

Figure 2. The relative recovery cost of cvfs for different case study programs, program sizes, and topologies.

Figure 2 also considers the analysis via partial analysis and full analysis. When full analysis is feasible, we conducted both full and partial analysis. Otherwise, we only considered partial analysis. When both are possible, they are consistent with each other. This indicates that partial analysis provides reasonably good estimates of full analysis. We also find that partial analysis via average-rank is more stable and follows the trend of full analysis better than the analysis of max-rank. This is expected because the sampling method generally provides a good estimate of the average value but not the extremum. This is also beneficial since the average-case cost of cvfs is more important for the sake of convergence to legitimate states. It also implies that we can get a good estimate of the relative cost of cvfs when the state space is large.

Although the specific pattern and values of rel cvf depend on the specific program, we observe that the value of rel cvf is small (roughly 2-4) indicating that the stabilization property is likely to be (probabilistically) preserved even if cvfs are frequent and only a small number (2-4) of transitions execute between cvfs. Also, the value of rel cvf does not vary significantly with the number of processes indicating that analysis of a problem with a small number of processes could generalize to a large number of processes.

Distribution of Cost of cvfs. Figure 3 considers the histogram of the rank effect of program transitions and cvfs using max-rank and average-rank. As an illustration of these results in Figure 3b, we consider the token ring program...
Fig. 3. Distribution of program transitions and cvfs by their rank effect.

with 9 processes. We find that 9% of the cvf transitions have no effect on the rank. 27% of cvf transitions change the rank by at most 1. In other words, they undo the effect of at most one program transition. 50% of the cvf transitions change the rank by at most 5. However, 0.6% of cvfs change the rank by 50 or more, i.e., to undo their effect, 50 or more program transitions are needed.

Yet another observation from Figure 3 is that with max-rank, the rank effect of program transitions is always negative whereas with average-rank, the effect of program transitions is occasionally positive. (This is expected since average-rank may not decrease with every program transition.)

From Figure 3 we find that the distribution of cvfs is exponential in nature in that the fraction of cvfs with cost c decreases exponentially with c.
Analyzing the Effect of $rel_{cvf}$ on Convergence. Next, we evaluate the observation about $rel_{cvf}$ to determine the ability of a program to converge in the presence of $cvf$s. If $rel_{cvf} = 3$, this indicates that a $cvf$ undoes the progress achieved via $3$ program transitions. Hence, if more than $3$ program transitions are executed between $cvf$s, it is anticipated that the recovery to legitimate states would still occur. Of course, if the number of program transitions between $cvf$s is just above $3$ then there may be a substantial increase in the number of steps required for convergence. We evaluate this hypothesis with simulations of the given programs in the presence of $cvf$s.

We choose simulations to validate the hypothesis because it allows us to control the rate of $cvf$s. By contrast, in an actual experiment on real networked systems, we cannot easily change the rate of $cvf$s. Furthermore, in an actual system, methods used to detect $cvf$s interfere with the actual computation. In simulations, we introduce the parameter $cvf\text{-interval}$ to denote the average number of program transitions between $cvf$s. For example, $cvf\text{-interval} = 4$ means (on average) $4$ program transitions execute between consecutive $cvf$s.

We conduct our simulations as follows: We randomly select an initial state, say $s_0$, and consider a computation of the given program from that initial state. To obtain such a computation, in each iteration, we execute a random enabled action. Furthermore, in each iteration, we randomly choose a process and perturb the state of that process in such a way that mimics the effect of a $cvf$. The parameter $cvf\text{-interval}$ determines how frequently $cvf$s are introduced. We terminate the execution when the program state is inside the invariant or when the number of iterations exceeds a threshold. In the latter case, we treat it as if the program has failed to converge in the presence of $cvf$s. For each initial state ($s_0$), we consider five computations and take the average of convergence steps. Subsequently, for the same initial state, we compute convergence steps for different values of $cvf\text{-interval}$.

We present the data from our simulations in Figure 4 as a scatter plot and in Figure 5 as the relative increase in the number of steps for convergence due to $cvf$s. In Figure 4, a point with coordinate $(X,Y)$ indicates that for one of the states considered, $X$ was the number of steps to converge in the absence of a $cvf$ and $Y$ is the number of steps to converge in the presence of $cvf$s with the given value of $cvf\text{-interval}$. To present the data in an easy-to-read fashion, we remove certain details when $Y$ coordinate is large. For example, in Figure 4a, we group the points when $Y$ coordinate is between $500 – 10000$ (threshold) to indicate that convergence was achieved but it took too long.

Figure 5 presents the simulation data to compare the number of steps for convergence in the absence of $cvf$s and in the presence of $cvf$s. If $cvf\text{-interval}$ is small then the number of steps to converge could be very large (or infinite). This is observed in Figure 5a where the number of steps increases by a factor of $100$ or more if $cvf\text{-interval} = 1$. However, if $cvf\text{-interval} = 8$, the number of steps for convergence in the presence of $cvf$s is about $1.2 – 2.6$ times more than the number of steps in the absence of $cvf$s. This is consistent with the analysis in Figure 2 for the token ring program where the ratio $rel_{cvf}$ is about $3$. That
means that a cvf negates the benefit of (approximately) 3 program transitions. Hence, the benefit of 8 program transitions and 1 cvf is (approximately) the same as 5 program transitions, thereby increasing the number of steps to be \( \frac{8+1}{5} \) (i.e., 1.8 times).

Figure 5b analyzes the coloring program. Here, from Figure 2, the program is expected to converge even if a cvf occurs between two consecutive program transitions. Figure 5b confirms this anticipation and finds that the increase in the number of steps is approximately 3 times if cvf\_interval = 1. If cvf frequency is reduced then the increase in the number of steps drops quickly. For example, if cvf\_interval = 8 then there is virtually no increase in the number of steps for convergence. Figure 5c considers the same question for the maximum match problem. As expected, when cvfs are too frequent (cvf\_interval = 1), the increased steps is very high. But when frequency is reduced (cvf\_interval \geq 8), the increase is only 0 – 20%.

Note that it is anticipated that convergence\_steps will increase in the presence of cvfs. However, each step is expected to be faster, as processes do not need to synchronize with each other.

7 Interpreting the Results of Case Studies and Implications for Implementing Stabilizing Programs

Self-stabilizing algorithms are often thought of as if the programs that start in an arbitrary state but then recover to legitimate states. In practice, however,
the ability to start from an arbitrary state is not as essential. Rather, practical systems need to recover from faults such as node crashes, message losses, reconfiguration of the network, etc. Self-stabilization, however, provides an inherent benefit that even if some unexpected (temporary) faults occur, the program is still guaranteed to recover.

For example, consider a spanning tree algorithm that was designed to tolerate node crashes. If this program suffers from an unanticipated message loss during recovery, it is possible that the algorithm fails completely thereby not forming the spanning tree. However, if that algorithm were designed to be a self-stabilizing algorithm then this fault may cause recovery to get longer but eventually the program will recover. Likewise, if a (non-stabilizing) program was designed with the assumption that nodes do not crash during recovery, then such a crash could cause the program to fail. By contrast, a self-stabilizing algorithm would eventually recover from such a crash.

With this intuition, the execution can be partitioned into two phases (1) quiescent phase where the algorithm has recovered to legitimate state(s) and no recovery actions are needed, (2) recovery phase, which is initiated by some node due to an observed inconsistency.

As an example, in the spanning tree algorithm discussed above, the recovery phase begins when some node notices an error (parent of the node has failed, node detects/suspects a cycle, etc.). Nodes continue to execute their actions until the quiescent phase is reached where the spanning tree is completed. Each node may not be aware of whether the system is in the recovery phase or the quiescent phase. But it can do an estimation (that is of course prone to errors). For example, if a node has not observed any inconsistency for a while, it can interpret it to mean that the system is in the quiescent phase. By contrast, if a node detects that the recovery phase began a certain time $\tau$ ago then it can conclude that there is a very good chance that the system would soon be in a quiescent phase in the near future; the value of $\tau$ would depend upon the expected recovery time of the algorithm.

For any self-stabilizing algorithm, it is desirable that the time spent in the recovery phase is as minimal as possible. The results in this paper allow us to reduce the time spent in the recovery phase. This can be achieved as follows:

If a node enters the recovery phase, it should transition itself to quiescent phase after a certain time, say $\tau_1$. When a node enters the quiescent phase, it should remain there for a certain duration, $\tau_2$, to prevent constant transition from recovery to quiescent phase (and vice versa). Furthermore, in recovery phase, the execution would be aggressive, i.e., a node will execute its actions without local mutual exclusion thereby permitting cvfs. By contrast, in quiescent phase, the execution would be conservative, i.e., it will ensure local mutual exclusion to eliminate cvfs. (Note that cvfs are possible even if a single node is in the recovery phase.)

Observe that as long as $\tau_2$ is large enough, recovery to legitimate states is guaranteed. Furthermore, as long as the program is in a legitimate state and each process is in the quiescent phase, the program would remain in a legitimate
state. Additionally, as long as CVFs are rare, during the recovery phase, faster execution to the invariant would be possible. From the analysis, we observe that the actual perturbation caused by a CVF is typically very small, just a handful of program transitions are sufficient to neutralize its effect. In turn, this implies that the time spent by the program outside legitimate states can be reduced if the program permits CVFs when the state is perceived to be outside legitimate states and eliminates CVFs when the state is perceived to be inside legitimate states.

8 Related Work

Self-stabilization [7] is a design principle that allows a distributed program to recover from an arbitrary state perturbation. This principle has been used in designing robust algorithms for spanning trees [12], leader election [13], matching [14], dominating set [15], clustering [16], etc. The design of those algorithms typically assumes some mechanism of local mutual exclusion so that the program actions during recovery are executed in an interleaving manner. However, such a mechanism has an expensive overhead. (We note that there are programs that do not use interleaving semantics [17] but they are significantly harder to design or analyze.) The benefits of removing local mutual exclusion to improve the performance of distributed computation have been observed in the literature for applications in machine learning [18] and graph computation [6]. In [6], the concept of consistency violation fault was introduced as a formalization of the program perturbations that occur when local mutual exclusion is removed. While the experimental results of [6] demonstrated an improvement in performance, the reasoning behind it was unclear. To the best of our knowledge, this paper is the first work that formally analyzes the effect of CVFs during the recovery phase of self-stabilizing programs to show that the effect of CVFs is small and can be computed for a given program. It also shows the rate of CVFs for which overall recovery would be faster. In turn, it can assist designers to determine the best approach for reducing convergence time for a given self-stabilizing program.

9 Conclusion

In this paper, we focused on the problem of implementing shared memory self-stabilizing programs. Self-stabilizing programs are often written in shared memory models where in one (atomic) action, the process reads the state of its neighbors and updates its own state. To implement this in a practical system, we need to translate these programs into read/write atomicity or message passing programs. While several such transformations have been proposed in the literature [1–4], they rely on the principle of local mutual exclusion that ensures that actions of multiple processes can be serialized.

In this paper, we considered the problem of what would happen if we do not guarantee that the execution of the actions is serializable. In this case, it is possible that the overall execution may not deterministically converge to the legitimate states. However, if these consistency violations are rare, we may be still
guaranteed the convergence to legitimate states. For example, it is well-known that if the number of consistency violations is finite then convergence to legitimate states is guaranteed. This is due to the fact that a self-stabilizing program converges from any state (including the one reached after the last consistency violation fault). However, if the consistency violations occur forever then whether recovery to a legitimate state occurs would depend upon (1) frequency of such consistency violations \(cvfs\) and (2) the cost of such \(cvfs\) compared with the benefit of program transitions.

When faced with permitting \(cvfs\) or eliminating \(cvfs\), we find that the former allows more efficient implementation as long as \(cvfs\) are not too frequent. Improvement in efficiency occurs due to the fact that there is less coordination required among processes to serialize their action execution. This suggests that for a given program, we should identify the frequency of \(cvfs\) where the convergence to legitimate states is still (probabilistically) guaranteed.

We analyzed three case studies to compute the average benefit of program transitions to get closer to the invariant and average cost of \(cvfs\) to move away from legitimate states. We analyzed the ratio \(rel_{cvf}\) that determines how many program transitions need to execute between \(cvfs\) to preserve the convergence property. For the three case studies, we find that this ratio is often small (less than 4). Thus, we anticipate that if (on average) 4 program transitions execute between \(cvfs\), convergence property is preserved. While the occurrence of \(cvfs\) does increase the total number of steps required to reach the legitimate state, each step is expected to be significantly faster, as local mutual exclusion is no longer required.

We analyzed our case studies with simulations of \(cvfs\). Here, we find that as long as the \(cvf\) frequency is not too high (e.g., only one program transition executes between \(cvfs\)), the increase in the steps required for convergence is low.

We analyzed the relative cost of \(cvfs\) to the benefit of program transitions using two characteristics: (1) max-rank and average-rank where we consider the worst-case convergence and average-case convergence, and (2) full analysis of the state space or partial analysis of the state space. For the case where full analysis is feasible, relative cost of \(cvfs\) under max-rank and average-rank is close. Furthermore, when both full and partial analysis is feasible, we find that the relative cost of \(cvfs\) with average-rank is close. This indicates that for the case where full analysis is not possible (e.g., large state space), partial analysis provides sufficient information about the relative cost of \(cvfs\).

We have developed a toolset that is generic so that by only coding the specific algorithm, one can obtain the effect of \(cvf\) on that program. Specifically, our analysis of max-rank and average-rank uses program transitions as a parameter. Thus, simply specifying the new program transitions, one can analyze the effect of \(cvfs\) for the given program. We intend to provide this as a toolset for the community to use. We are also working to extend these results with Markov chain model that accounts for the exponential distribution of \(cvf\) cost.
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A Defining max-rank and average-rank

The max-rank of each state is computed as follows: First, we set the rank of all legitimate states to 0. Subsequently, we find a state \( s_0 \) such that \( \text{rank}(s_0) \) is still unknown but the rank of all successors of \( s_0 \) is known. Note that successors of \( s_0 \) are \( \{ s_1 | (s_0, s_1) \text{ is a transition of the given program} \} \). Thus, \( \text{rank}(s_0) \) is set to \( \text{max}(\text{rank}(s_1) + 1 | (s_0, s_1) \text{ is a transition of the given program}) \).

To compute average rank, we use a similar approach. Each state \( s \) is associated with \( \text{pathCount} \) and \( \text{totalPathLength} \) where \( \text{pathCount} \) is the number of paths from \( s \) to the first state in the invariant and \( \text{totalPathLength} \) is the sum of the lengths of those paths. The average rank is computed as \( \text{average-rank} = \frac{\text{totalPathLength}}{\text{pathCount}} \). For states in the invariant, we set \( \text{pathCount} = 1 \) and \( \text{totalPathLength} = 0 \). Subsequently, we find a state \( s_0 \) whose rank is unknown but the ranks of all its successors are known. Then we update \( \text{pathCount} \) and \( \text{totalPathLength} \) for \( s_0 \). Specifically, let \( \text{succ} \) denote the set of all successors of \( s_0 \) by program transitions and \( |\text{succ}| \) is the number of successors of \( s_0 \). Then \( \text{pathCount}(s_0) = \sum_{s_1 \in \text{succ}} \text{pathCount}(s_1) \) and \( \text{totalPathLength}(s_0) = |\text{succ}| + \sum_{s_1 \in \text{succ}} \text{totalPathLength}(s_1) \).

B Case study programs

In this section, we present three case study programs used in our evaluation: token ring, graph coloring, and maximal matching.

Token ring program: The 3-state token ring program is one of the classic self-stabilizing program introduced by Dijkstra [7]. In this program, we have \( n \) processes, 0, 1, \ldots, \( n-1 \), organized in a ring. Each process \( j \) has a variable \( x.j \) in the domain \( \{0, 1, 2\} \). Process 0 checks if \( x.0 + 1 \) equals \( x.1 \). If so, process 0 decrements the value of \( x.0 \). (All operations of this token ring program are in modulo 3 arithmetic). Process \( n - 1 \) checks if \( x.(n-2) = x.0 \) and \( x.(n-2)+1 \neq x.(n-1) \). If so, process \( n - 1 \) copies the value of \( x.(n-2) \). Other processes check if their \( x \) value plus 1 is equals to their left (or right, respectively) neighbor. If that is true, it copies the \( x \) value of the left (or right, respectively) neighbor.

The actions of the processes are as follows:

At 0: \( (x.0 + 1) \mod 3 == x.1 \rightarrow x.0 = (x.0 - 1) \mod 3; \)
At \( n - 1 \): \( (x.0 == x.(n-2)) \land ((x.(n-2) + 1) \mod 3 \neq x.(n-1)) \rightarrow x.(n-1) = x.(n-2); \)
At \( j \) where \( 1 \leq j \leq n - 2 : \)
At \( j \) where \( 1 \leq j \leq n - 2 : (x.j+1) \mod 3 == x.(j-1) \rightarrow x.j = x.(j-1); \)
At \( j \) where \( 1 \leq j \leq n - 2 : (x.j+1) \mod 3 == x.(j+1) \rightarrow x.j = x.(j+1); \)

Graph coloring program: We use the self-stabilizing graph coloring program by Gradinariu and Tixeuil [9]. This program guarantees to use no more than \( \Delta + 1 \) colors where \( \Delta \) is the maximum degree of the graph. In this program, each process \( j \) is associated with a color \( c.j \). Each process has one action. It checks the color of its neighbors. If \( c.j \) is the same as \( c.k \) for some neighbor \( k \).
then $c_j$ is changed to a color not present in the neighborhood. Thus, the action at process $j$ is as follows.

$$c_j \in \{ c_k | k \in Nb_j \} \rightarrow c_j = c,$$

where $c = \min\{ l \in \mathbb{N} | l \notin \{ c_k | k \in Nb_j \} \}$.

**Maximal-matching program:** We use the self-stabilizing maximal matching program by Manne et al. [8]. In this program, each process $j$ is associated with an integer $p_j$. The value of $p_j$ indicates the process with which $j$ proposes to be matched. If $p_j == \text{null}$ then $j$ currently does not propose to any process. If two processes $i$ and $j$ propose with each other, i.e. $p_i == j$ and $p_j == i$, then we say the two processes are married. The marital status of a process $j$ is determined by evaluating the predicate $PR\text{married}(j) \equiv \exists i \in Nb_j : (p_i = j) \land (p_j = i)$. A free process $j$ will not try to get matched with a married neighbor. However, $j$ is not able to evaluate the predicate $PR\text{married}()$ of its neighbors because the evaluation requires information about the neighbors of $j$’s neighbors. To help processes communicate their marital status with their neighbors, each process $j$ is also associated with a Boolean variable $m_j$ indicating whether process $j$ has been married or not. It is possible that the value of $m_j$ is not consistent with $PR\text{married}(j)$, in this case, $m_j$ has to be updated to be consistent.

When the value of $m_j$ is consistent with $PR\text{married}(j)$, the maximal matching algorithm at $j$ works as follows. If $j$ currently does not propose to anyone and there is a neighbor who proposes to $j$, then $j$ proposes back to that neighbor. This action will match 2 processes together. If $j$ currently does not propose to anyone and none of the neighbors proposes to $j$ then $j$ will propose to a neighbor who has free marital status and smaller ID than $j$ and currently does not propose to anyone. If there are multiple such neighbors, $j$ will choose the one with the smallest ID. This action will let free processes find a new partner. If $j$ proposes to a neighbor $i$ but $i$ does not propose to $j$ and $i$ is married then $j$ will cancel its proposal. This action will protect existing matches. If $j$ proposes to a neighbor $i$ but $i$ does not propose to $j$ and the ID of $i$ is greater than the ID of $j$ then $j$ will cancel its proposal. This action is to break the symmetry. The actions at process $j$ are as follows.

$$m_j \neq PR\text{married}(j) \rightarrow m_j = PR\text{married}(j);$$

$$m_j = PR\text{married}(j) \land p_j = \text{null} \land \exists i \in Nb_j : (p_i = j) \rightarrow p_j = i;$$

$$m_j = PR\text{married}(j) \land p_j = \text{null} \land \forall i \in Nb_j : (p_i \neq j \land \exists k \in Nb_j : (p_k = \text{null} \land k < j \land \neg m_k));$$

$$m_j = PR\text{married}(j) \land p_j = \text{null} \land \forall i \in Nb_j : (p_i \neq j \land \exists k \in Nb_j : (p_k = \text{null} \land k < j \land \neg m_k));$$

$$m_j = PR\text{married}(j) \land p_j = \text{null} \land \exists i \in Nb_j : (p_i \neq \text{null} \land i \neq j \land (m_i \lor j \leq i)) \rightarrow p_j = \text{null};$$