Infrared Confinement: Mass-Scale Dependence of the Strong Effective Coupling, Leptonic Decay Constants and Spectrum of Mesons, the Lowest Glueball Mass

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Abstract. Properties of two-particle bound states composed of spin-zero and spin-half constituents have been investigated within a relativistic field model based on new analytically (infrared) confined propagators. A new mass scale dependence of the strong effective coupling has been obtained. By using this dependence, new estimates on the mass spectrum and leptonic decay constants of mesons have been performed with reasonable accuracy. Independent and new estimates have been obtained for the mass and radius of the lowest (scalar) glueball.

1. Introduction
QCD predicts a dependence of the physical coupling $g$ under changes of energy scale $Q$ (or, distance $\sim 1/Q$). This dependence $\alpha_s(Q) = g^2/(4\pi)$ is described theoretically by the renormalization group equations and determined experimentally at relatively high energies $Q > 1$ GeV [1]. Meanwhile, understanding of a number of phenomena such as quark confinement, hadronization etc., requires a correct description of hadron dynamics in the infrared (IR) region below $Q \sim 1$ GeV. Particularly, many quantities in particle physics are affected by the IR behavior of $\alpha_s$. However, the long-distance behavior of $\alpha_s$ has not been well defined yet, it needs to be more specified. The correct description of QCD effective coupling in the IR regime remains one of the actual problems in particle physics.

2. Model
Let’s consider a QCD-inspired relativistic field model with Lagrangian [2]:

$$L = -\frac{1}{4} \left( \partial^\mu A^A_\mu - \partial^\nu A^A_\nu - gf^{ABC} A^B_\mu A^C_\nu \right)^2 + \left( \bar{q}_f^a [\gamma_\alpha \partial^\alpha - m_f] q^b_f \right) + g \left( \bar{q}_f^a \Gamma^C_{\alpha} A^C_\alpha q^b_f \right),$$

(1)

where $A^A_\mu$ is the gluon and $q^a_f$ is a quark field of flavor $f$ with mass $m_f = \{m_u, m_s, m_c, m_b\}$ and $\Gamma^C_\alpha = i\gamma_\alpha t^C$. For the spectra of quark-antiquark and di-gluon bound states we solve Bethe-Salpeter type equations obtained in [3, 4]. By omitting intermediate stages of calculation (see for details [5]) we rewrite the master equation determining the meson mass as follows:

$$1 = \alpha_s \cdot \lambda_{JJ}(M_{J1}^2, m_1, m_2) = \alpha_s \frac{16\pi C_f}{9} \int \frac{d^4k}{(2\pi)^4} \int dx dy \ e^{-ik(x-y)} U_N(x) \sqrt{D(x)D(y)} U_N(y)$$

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\[ \text{Tr} \left[ O_J \tilde{S}_{m_i}(\vec{k} + \xi \hat{p}) O_J \tilde{S}_{m_j}(\vec{k} - \xi \hat{p}) \right] \bigg|_{-p^2 = M_f^2}, \tag{2} \]

where \( C_J = \{1, 1/2, -1/2\}, \) \( \xi_i = m_i/(m_1 + m_2), O_J = \{I, i\gamma_5, i\gamma_\mu, \gamma_5\gamma_\mu\} \) and the gluon \((D(x))\) and quark propagator \((\tilde{S}_{m_i}(\hat{p}))\) are represented in Euclidean space.

Note, the polarization kernel \( \lambda_{J,j'}(-p^2) \) has to be diagonalized on a complete system of orthonormal functions \( \{U_N\} \), where \( N = \{n,l,\mu,...\} \) is a set of quantum numbers. The solution of Eq.(2) is nothing else but the solution of the corresponding ladder Bethe-Salpeter equation.

Ultraviolet divergences in the model have been removed by renormalization of wave function and charge, but infrared singularities remain in Eq.(2) because of integration over variable \( k \). To avoid the appearance of the singularities in the mass formula, we follow theoretical predictions in favor of an IR-finite behavior of the gluon propagator [6, 7] and introduce a new scheme of infrared cutoffs on the limits of scale integrations for the propagators as follows:

\[ D(x) = \frac{1}{4\pi^2 x^2} \rightarrow \frac{1}{\Lambda/2} \int ds \exp[-sx^2], \quad \tilde{S}_{m_f}(\hat{p}) = \frac{1}{-i\hat{p} + m_f} \rightarrow (i\hat{p} + m_f) \int dt \exp[-(p^2 + m_f^2)], \tag{3} \]

where \( \Lambda \) is the mass scale of the IR confinement domain. These propagators are entire analytic functions in the Euclidean space. Note, another type of IR confinement applied to whole 'quark-antiquark' loop was applied in [8, 9]. The analytic confinement disappears as \( \Lambda \rightarrow 0 \).

3. Effective Strong Coupling in the IR Region

The meson mass \( M_f \) is defined by Eq.(2) at given \( \alpha_s \), quantum numbers \( N \) and constituent quark masses \( \{m_1, m_2\} \). And vice versa, \( \alpha_s \) can be estimated for given mass.

The QCD coupling may feature an IR-finite behavior (e.g., in [10, 11]). To study this, we consider \( M_f \) as an appropriate energy-scale for \( \alpha_s \) and choose a special case \( m_1 = m_2 \sim M/2 \) in the ground-state. Then, a new effective (mass-dependent) strong coupling \( \alpha_s(M) \) in time-like domain may be defined by:

\[ \alpha_s(M) = -1/\lambda(M, M/2, M/2). \tag{4} \]

![Figure 1. Dependence of the effective strong coupling \( \alpha_s \) on relative mass scale \( M/\Lambda \).](image)

A new variational upper bound \( \alpha(M) \) to \( \alpha_s(M) \) is shown in Fig.1. The slope of \( \alpha(M/\Lambda) \) depends on \( \Lambda > 0 \), but the newly revealed origin \( \alpha(0) = 1.032 \) (or, \( \alpha(0)/\pi = 0.328 \)) remains unchanged. This upper bound to the IR-fixed value \( \alpha(0) \) is in a reasonable agreement with often quoted estimates [12]:

\[ \frac{\alpha_s^0}{\pi} \simeq 0.19 \div 0.25 \quad [13], \quad \frac{\alpha_s^0}{\pi} \simeq 0.265 \quad [14], \quad \frac{\alpha_s^0}{\pi} \simeq 0.26 \quad [15]. \]
4. Lowest Glueball State

The existence of glueballs, the bound states of gluons, is predicted by QCD because of the self-interaction of gluons. Most known experimental signatures for glueballs are an enhanced production in gluon-rich channels of radiative decays and some decay branching fractions incompatible with \((q\bar{q})\) states. There are predictions expecting glueball-like states in the mass range \(M_G \sim 1.5 \div 5.0 \text{ GeV}\) with spin \(J = 0, 1, 2, 3\) [16].

Below we consider a two-gluon scalar bound state with \(J^{PC} = 0^{++}\). By omitting details of intermediate calculations (similar to those represented in the previous section) we define the scalar glueball mass \(M_{0^{++}}\) from equation:

\[
1 - \frac{8 \alpha}{3\pi} \int dz e^{izp} \Pi_G(z) = 0, \quad p^2 = -M_{0^{++}}^2,
\]

where \(\Pi_G(z)\) is the self-energy (polarization) function of the scalar glueball.

Particularly, for \(\Lambda = 236 \text{ MeV}\) and \(\alpha(M_G)\) defined from Eq.(4) we obtain new estimates:

\[
M_{0^{++}} = 1739 \text{ MeV}, \quad \alpha(M_{0^{++}}) = 0.451.
\]

The new value of \(M_{0^{++}}\) is in reasonable agreement with other predictions [2, 16, 17, 18, 19].

We also estimate the scalar glueball 'radius' \(r_G\) which leads to a new value:

\[
r_{0^{++}} \cdot M_{0^{++}} = 4.41
\]

that is in reasonable agreement with data in [16].

5. Meson Spectrum

The dependence of meson mass on \(\alpha_s\) and other model parameters is defined by Eq.(2). Note, the kernel function \(\lambda_N\) is real and finite, that allows us to evaluate a variational solution to \(M_J\).

Particularly, for the obtained \(\Lambda = 236 \text{ MeV}\) and \(\alpha(M)\) defined in Eq.(4) we derive meson mass formula Eq.(2) by fitting the conventional meson masses with adjustable constituent quark masses \(\{m_{ud}, m_s, m_c, m_b\}\). We have fixed a new final set of model parameters (in units of MeV) as follows:

\[
\Lambda = 236, \quad m_{ud} = 227.6, \quad m_s = 420.1, \quad m_c = 1521.6, \quad m_b = 4757.2.
\]

and represent in Tab. 1 our new estimates on the pseudoscalar \((P)\) and vector \((V)\) meson masses.

| \(J^{PC}\) | \(M_P\) (in units of MeV) | Data | \(J^{PC}\) | \(M_V\) (in units of MeV) | Data |
|------------|-----------------|------|------------|-----------------|------|
| 0^{--}     | 1893.6          | 1869.62 | \(\rho\)   | 774.3           | 775.26 |
| 0^{++}     | 2003.7          | 1968.50 | \(K^*\)    | 892.9           | 891.66 |
| 0^{--}     | 3032.5          | 2983.70 | \(D^*\)    | 2003.8          | 2010.29 |
| 1^{--}     | 5215.2          | 5259.26 | \(D_s^*\)  | 2084.1          | 2112.3 |
| 0^{--}     | 5323.6          | 5366.77 | \(J/\Psi\)| 3077.6          | 3096.92 |
| 0^{--}     | 6297.0          | 6274.5 | \(B^*\)    | 5261.5          | 5325.2 |
| 0^{--}     | 9512.5          | 9398.0 | \(\Upsilon\)| 9526.4          | 9460.30 |

Our estimates represented in Tab. 1 are in reasonable agreement with experimental evidences with relative errors less than 1.8 per cent.
6. Leptonic Decay Constants
The leptonic decay constants $f_J$ are important quantities in meson physics. The precise knowledge of their values provides more improvement in our understanding of various processes convolving meson decays. To describe effectively the ‘sawtooth’-type unsmooth dependence of $f_J$ on meson masses (see Tab.3), we introduce additional parameters $R_J$ into the basis functions as follows: $\tilde{U}_J(k) = \int_0^1 ds \, h(s) \exp\left[-sk^2/R_J^2\right]$, where $h(s)$ is a smooth functions and $R_J$ characterizes the meson ‘size’ in units of mass scale. Then, by using our model parameters in Eq.(8) along with the optimal meson ‘sizes’ shown in Tab. 2 we estimate new results on the leptonic decay constants of conventional mesons shown in Tab. 3.

Table 2. Meson ‘size’ parameters $R_J$ (in units of GeV)

| $D$ | $D_s$ | $\eta_c$ | $B$ | $B_s$ | $B_c$ | $\eta_b$ | $\rho$ | $K^*$ | $D_*$ | $D_s^*$ | $J/\Psi$ | $B^*$ | $\Upsilon$ |
|-----|------|----------|-----|-------|-------|----------|-------|-------|-------|-------|--------|-------|-------|
| 0.93 | 1.08 | 1.83 | 1.73 | 2.18 | 3.34 | 3.80 | 0.33 | 0.38 | 0.78 | 0.90 | 2.40 | 3.34 | 2.80 |

Table 3. Estimated leptonic decay constants of conventional mesons $f_P$ and $f_V$ (in MeV) compared to experimental data in [20, 21, 22, 23].

| $0^{+}$ | $f_P$ | Data | Ref. | $1^{-}$ | $f_V$ | Data | Ref. |
|--------|------|------|------|--------|------|------|------|
| $f_D$  | 207  | 206.7 ± 8.9 | [20] | $f_\rho$ | 221 | 221 ± 1 | [20] |
| $f_{D_s}$ | 257 | 257.5 ± 6.1 | [20] | $f_{K^*}$ | 217 | 217 ± 7 | [20] |
| $f_{\eta_c}$ | 238 | 238 ± 8 | [22] | $f_{D^*}$ | 245 | 245 ± 20 | [23] |
| $f_B$ | 193 | 192.8 ± 9.9 | [21] | $f_{D_s^*}$ | 271 | 272 ± 26 | [23] |
| $f_{B_s}$ | 239 | 238.8 ± 9.5 | [21] | $f_{J/\Psi}$ | 416 | 415 ± 7 | [20] |
| $f_{B_c}$ | 488 | 489 ± 5 | [22] | $f_{B^*}$ | 196 | 196 ± 44 | [23] |
| $f_{\eta_b}$ | 800 | 801 ± 9 | [22] | $f_{\Upsilon}$ | 715 | 715 ± 5 | [20] |

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