Fleet Size from Split Delivery in Vehicle Routing Problems

H S Hasibuan\textsuperscript{1}, S Suwilo\textsuperscript{2}, O S Sitompul\textsuperscript{3}

\textsuperscript{1,2} Department of Mathematics, Universitas Sumatera Utara, Medan, 20155, Indonesia
\textsuperscript{3} Department of Computer Science, Universitas Sumatera Utara, Medan, 20155 Indonesia

* hafnisarihasibuan24@gmail.com

Abstract. Vehicle Routing Problem (VRP) is a problem of determining the route by minimizing the number of vehicles used to minimize the total mileage of a vehicle to deliver an item to a customer that starts from a depot and ends at the depot. VRP relates to the distribution of goods, people, information, vehicles and roads. All customers must be visited only once and by one vehicle must not exceed the capacity of the vehicle. VRP varies due to real life constraints related to the type of vehicle, number of depots, conditions, transportation, and time period. Among them the problem of a heterogeneous vehicle fleet is a type of VRP whose vehicles have different capacities and costs, besides different vehicles also have a Split Delivery variation where customers can be visited by one, two, three or even four vehicles. This is due to the capacity constraints possessed by each courier, as a result if the first courier is unable to complete all orders from the customer then the order delivery can be continued by a second courier or other courier who also crosses the route from the customer. The aim is to minimize vehicle fixed costs and transportation costs. Where the problem is, each customer will get at least one delivery, capacity limit for each vehicle and determine the number of vehicles to be used in the delivery of goods. To find a solution to this problem, the Integer Linear Programming (ILP) model is used. The solution obtained is an optimal solution that minimizes the objective function and fulfills all constraints or constraints made. The results obtained show that the model created produces a distribution route with a minimum total distance.

1. Introduction

The shipping business in the country is increasing with the rapid growth of online businesses today. In fact, with the development of the internet, buying and selling activities via online is even more prevalent and will become a practical lifestyle trend in the future. Online business even requires shipping / courier services as an inseparable link. So the need for this shipping service covers all of Indonesia. The development of an online shop business opens a great opportunity that is very promising for shipping service companies.

In sending goods, the company must be able to determine the distribution path configuration correctly so that shipping becomes fast and does not cost much. Determination of this configuration must consider distribution strategies that are in accordance with the characteristics of the company. The problem of distribution systems of a company is an important factor that involves several considerations, namely the selection of vehicle routes, vehicle capacity to vehicle scheduling. The main consideration is what is known as the Vehicle Routing Problem (VRP).
VRP is a matter of determining the route of a vehicle that serves several consumers with a homogeneous vehicle. Each vehicle has a transport capacity and every customer has demand. Each customer is only visited once by vehicle and must not exceed the capacity of the vehicle. The more customer demand, the more the carrying capacity will increase, while the carrying capacity must not exceed. With existing problems, Split Delivery Vehicle Routing Problem (SDVRP) appears. Where SDVRP is a variant of VRP that allows customers to be served more than one vehicle with heterogeneous vehicles.

Hiermann et al. [5] introduce the size of electric vehicles and mix VRP with time windows and recharge stations. The model decisions that will be made relate to vehicle computing and actual vehicle routes including choice of charging time and location. While Ahkamiraad and Wang [1] formulated a mixed integer linear programming model of a special type of cross-doctor VRP with capacity and multiple with pickup, delivery and time windows. From some studies above some use homogeneous vehicle sizes such as Wilk and Nathalie.

Recently research on VRP with heterogeneous vehicle size has been carried out by Tellez et al. [7] by introducing vehicle sizes and mixed travel problems with several types of heterogeneous passengers and vehicles. The problem is modeled as a mixed-mix program that comes from a heterogeneous dial a ride problem model. Koc et al. [7] Presenting hybrid evolution algorithms (HEA) to solve heterogeneous vehicle route problems with time windows. In the following year Koc et al. (2016) offers a number of new efficient procedures that are tailored to handle heterogeneous vehicles and location decisions. Drexl et al. [4] describe limited vehicles with trail and transshipments. Continued with Wang et al. [1] studied low carbon for location routing problems with heterogeneous vehicles, simultaneous pickup and time windows. The goal to be achieved is also to find out the optimal number of vehicles to be used in the distribution process. Heterogeneous vehicles have different specifications and types, so the capacity is also different. In addition, each vehicle also has a limited fleet.

Based on this background, according to the survey results of the research literature on the heterogeneous size of vehicles that have been carried out, this research will add a variant that uses Split delivery vehicle routing problem (SDVRP) with heterogeneous fleet. And the final results of this study are expected to increase vehicle knowledge from split delivery in vehicle routing problems so that it can apply to represent a problem that exists in real life.

2. Research methods

2.1. Description and formulation of problems
Description and Problem Formulation The first stage in modeling is determining the purpose of determining the vehicle route in distributing goods. In general the purpose of the problem of determining the vehicle route in distributing goods is to minimize the total mileage of the vehicle by considering the capacity of the vehicle, the number of requests per customer and the distance between customers.

In the matter of determining the route of this vehicle, the decision variable will be limited by a number of restrictions. The limitation consists of several general limitations that cover the problem of determining the vehicle route in distributing goods from the depot to each of its customers.

2.2. Modeling
After the problem formulation stage, then the problem formulation is presented in a mathematical model. The problem of determining vehicle routes can be modeled as an Integer Linear Programming (ILP) model.

2.3 Split delivery vehicle routing problem (SDVRP)
Split Delivery Vehicle Routing Problem (SDVRP) is VRP expansion if each customer can be served with a different vehicle, one customer can be served by more than one vehicle. This expansion needs
to be done if the number of customer requests is as large as the capacity of the vehicle. SDVRP is described as follows:

- **Purpose:** Minimize the number of vehicles and total travel time for services.
- **Feasibility:** A solution is considered feasible if each route meets the VRP standard limit. The solution is deemed feasible if it meets the VRP standard limitation plus each customer can be served by more than one vehicle.

Suppose there is one depot of 11 customers and 3 vehicles. The first vehicle in sequence visits consumers 4, 8, 1, 6 then returns to the depot. The second vehicle visited consumers 3, 5, 11, 9 then returned to the depot. The third vehicle departed from the depot then visited customers 7, 9, 10, 2 and then returned to the depot. It can be seen that customer 9 is visited twice by two and three vehicles so Split Delivery is done to the customer 9.

![SDVRP Illustration](image)

**Figure 1.** SDVRP illustration

The SDVRP can now be formulated as follows [2]:

**Objective function:**

\[
\text{Minimize } \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{v=1}^{NV} c_{ij} x_{ij}^{p}
\]

**Subject to:**

\[
\sum_{i=1}^{n} \sum_{j=0}^{NV} x_{ij}^{p} \geq 1 \quad j = 0, 1, \ldots, n \quad (1)
\]

\[
\sum_{i=0}^{n} x_{ip}^{p} - \sum_{j=0}^{n} x_{pj}^{p} = 0 \quad p = 0, 1, \ldots, n; v: 1, 2, \ldots, NV \quad (2)
\]

\[
\sum_{i \in S} \sum_{j \in S} x_{ij}^{p} \leq |S| - 1 \quad v = 1, \ldots, m; S \subseteq V - \{0\} \quad (3)
\]

\[
\sum_{v=1}^{NV} y_{i}^{p} = d_{i} \quad i = 1, \ldots, n \quad (4)
\]
4

\[ \sum_{i=1}^{n} y_{i}^{v} \leq Q \text{ untuk } v = 1, \ldots, NV \]  

Constraints (1) ensure that each customer will get at least one shipment, (2) ensure that vehicles that visit consumers after serving will leave the customer, (3) are the subtours elimination constraints, (4) ensure that each customer will receive the shipment in full, (5) ensure that shipments for each vehicle do not exceed capacity.

2.4 Fleet Size Vehicle Routing Problem

Fleet Size (FS) is a big case of optimization problems. In general, fleets or vehicles used for distribution have heterogeneous fleet sizes. Customers are served from one depot with a heterogeneous and unlimited fleet. Vehicles depart from and return to the depot. In heterogeneous problems, we define a set of \( T: \{1, \ldots, K\} \) from different types of vehicles. Vehicles of type \( k \in T \) have \( a_{k} \) capacity, fixed costs \( f_{k} \), and unit costs per variable distance \( g_{k} \). The cost of a vehicle of type \( k \in T \) across the pair \((i, j)\) is denoted by \( c_{ij}^{k} \), which is obtained by multiplying the distance \( d_{ij} \) and the variable cost. The number of \( n_{k} \) type \( k \) vehicles is assumed to be unlimited (\( n_{k}: \infty, k \in T \)).

We suppose \( R_{i}: \{r_{i}(1), \ldots, r_{i}(n_{i})\} \) shows the route for vehicle \( i \), where \( r_{i}(j) \) is the customer index visited and this is the number of customers on the route. We assume that each route is completed at depot, that is, \( r_{i}(n_{i} + 1): 0 \).

The FSVRP can now be formulated as follows:

\[ \min \sum_{v=1}^{NV} \sum_{j=1}^{n} f_{v} x_{oj}^{v} + \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{v=1}^{NV} c_{ij}^{v} x_{ij}^{v} \]  

Subject to:

\[ \sum_{i=1}^{n} \sum_{v=1}^{NV} x_{ij}^{v} = 1 \quad j = 0, 1 \ldots, n \]  

\[ \sum_{i=0}^{n} x_{ip}^{v} - \sum_{j=0}^{n} x_{pj}^{v} = 0 \quad p = 0, 1 \ldots, n; v: 1, 2 \ldots, NV \]  

\[ \sum_{v=1}^{NV} y_{i}^{v} = d_{i} \quad i = 1, \ldots, n \]  

\[ \sum_{i=1}^{n} y_{i}^{v} \leq Q_{v} \text{ untuk } v = 1, \ldots, NV \]  

Constraints (2) ensure that each customer is only passed by one vehicle, constraint (3) guarantees that vehicles leaving customers \( i \) have served these customers, constraints (4) ensure that each customer will receive the shipment in full (5) ensure that shipments for each vehicle do not exceed capacity.

3. Results

3.1. Fleet size split delivery vehicle routing problem (FSSDVRP) mathematical model
The purpose of the mathematical model of determining the vehicle route made is to minimize the total mileage of the vehicle in distributing goods from the depot to a number of customers scattered in a number of places.

**Indexed sets:**
i = customer 𝑖; \{1, 2, 3, ..., 𝑛\}  
j = customer 𝑗; \{1, 2, 3, ..., 𝑛\}  
v = vehicle  
0 = depot

**Parameter:**
m = the number of vehicle routes  
n = the number of customer  
Q = the vehicle capacity  
c\textsubscript{ij} = the cost or distance from node 𝑖 to node 𝑗  
d\textsubscript{i} = the demand of customer 𝑖

**Decision variables:**
\[x\textsubscript{ijv} = 1\] when arc (𝑖, 𝑗) is traversed on route 𝑣; 0 otherwise  
\[y\textsubscript{iv} = 1\] when node 𝑖 is visited on route 𝑣; 0 otherwise  
\(F_v = \text{fixed cost}\)

so that the FSSDVRP can be formulated as follows:

\[\min \sum_{v=1}^{m} f_v \sum_{j=1}^{n} x_{0j}^v + \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{v=1}^{m} c_{ij}^v x_{ij}^v \]

Subject to

\[\sum_{i=1}^{n} \sum_{v=0}^{m} x_{ij}^v \geq 1 \quad j = 1, \ldots, n \]

Constraints (2) impose that each customer is visited at least once

\[\sum_{i=0}^{n} x_{ip}^v - \sum_{j=0}^{n} x_{jp}^v = 0, \quad p = 0, \ldots, n; v = 1, \ldots, m \]

Constraints (3) every vehicle that visits consumers, after finishing serving will leave the customer

\[\sum_{i \in S} \sum_{j \in S} x_{ij}^v \leq |S| - 1 \quad v = 1, \ldots, m; \quad S \subseteq v - \{0\} \]

Constraints (4) are the subtours eliminations constraints

\[\sum_{v=1}^{m} y_{iv}^v = d_i \quad i = 1, \ldots, n \]

Constraints (5) all requests from each customer are met

\[\sum_{i=1}^{n} y_{iv}^v \leq Q_v \quad v = 1, \ldots, m \]

Constraints (6) the total number of consumer requests does not exceed the capacity of the vehicle
4. Conclusion

Based on the results of the discussion obtained in this study the conclusions obtained are: The problem of distribution of goods can be formulated in the Integer Linear Programming model in order to obtain the minimum total distance.

References
[1] Ahkamiraad A and Wany, Y. 2018 Computer & Industrial Engineering, 119 76
[2] Archetti, C. and Speranza, G.M. 2008 Operation Research/Computer Science Interfaces, 43 1
[3] Belfiore J M and Yoshizaki H T Y 2013 Computers & industrial engineering, 64 589
[4] Drexl M 2014 Branch and Cut Algorithm for the Vehicle Routing Problem with Trailer and Transshipments. Volume 63, no.1, 119-133.
[5] Hiermann G, Puchinger J, Ropke S and Harti F R 2016 European Journal of Operational Research, 252 995
[6] Koc C, Bektas T, Jabali O and Laporte G 2016 European Journal of operation Research. 248 11
[7] Tellez O, Vercaene S, Lehuede F and Peton O 2018 Transportation Research. 91 99