An Analysis of Phase Synchronization Mismatch Sensitivity for Coherent MIMO Radar Systems

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Abstract

In this study, the hybrid Cramer-Rao bound (CRB) is developed for target localization, to establish the sensitivity of the estimation mean-square error (MSE) to the level of phase synchronization mismatch in coherent Multiple-Input Multiple-Output (MIMO) radar systems with widely distributed antennas. The lower bound on the MSE is derived for the joint estimation of the vector of unknown parameters, consisting of the target location and the mismatch of the allegedly known system parameters, i.e., phase offsets at the radars. Synchronization errors are modeled as being random and Gaussian. A closed-form expression for the hybrid CRB is derived for the case of orthogonal waveforms. The bound on the target localization MSE is expressed as the sum of two terms; the first represents the CRB with no phase mismatch, and the second captures the mismatch effect. The latter is shown to depend on the phase error variance, the number of mismatched transmitting and receiving sensors and the system geometry. For a given phase synchronization error variance, this expression offers the means to analyze the achievable localization accuracy. Alternatively, for a predetermined localization MSE target value, the derived expression may be used to determine the necessary phase synchronization level in the distributed system.

Index Terms

MIMO radars, Hybrid CRB, mismatch parameters, localization.

I. INTRODUCTION

Improvement in target parameter estimation capabilities is a primary advantage of MIMO radar systems [1]-[5]. In particular, target localization with coherent MIMO radar systems, utilizing widely distributed antennas, offers significant advantages [5]. Typically, performance analysis of system parameter estimation problems is based on the derivation of the Cramer-Rao bound (CRB), which sets a lower bound on the estimation MSE for unbiased estimators [6]. Such an evaluation is provided in [2], [5] for coherent MIMO radar systems, demonstrating a localization

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accuracy advantage, inversely proportional to the signal carrier frequency. In addition, a spatial advantage of the
order of the product of the number of transmit and receive radars is also incorporated in the CRB.

This performance gain comes with the challenge of attaining phase synchronization in a distributed system. Errors introduced to the system parameters by phase synchronization mismatch, will result in parameter estimation
mean-square error (MSE) degradation and bias. In this work, the hybrid CRB (HCRB) is used to test the sensitivity
of the target localization MSE to phase errors. The HCRB takes into account deterministic unknown parameters,
such as the target location, as well as random parameters, phase calibration errors, in this case. This method has
been applied to passive source localization [8], [9] for the problem of source bearing and range estimation with,
uncertainty in the sensors’ locations or phase synchronization errors.

In this work, the HCRB is derived for coherent MIMO radars, with phase synchronization errors. A closed-form
expression for the HCRB for the target’s location \((x, y)\) is derived, providing the means to assess the effects of
phase errors on the localization accuracy. The effect of the number of radars, their geometric layout, and the phase
mismatch MSE is incorporated in the HCRB terms.

The paper is organized as follows: brief theoretical background is provided in Section II. The system model is
introduced in Section III, and the HCRB on the targets localization estimation errors is derived. Numerical examples
are presented in Section IV. Finally, Section V concludes the paper.

II. BACKGROUND

The hybrid CRB provides a low bound on the MSE of any unbiased estimator for an unknown parameter(s),
where the parameters are partially deterministic and partially random [7]. Given a vector parameter \(\theta = [\theta_{nr}, \theta_r]^T\),
where \(\theta_{nr}\) stands for the nonrandom parameter vector and \(\theta_r\) for a random parameter vector, its unbiased estimate
\(\hat{\theta}\) satisfies the following inequality [7]:

\[
E_{\theta_{nr}, \theta_r} \left\{ (\hat{\theta}_i - \theta_i) (\hat{\theta}_i - \theta_i)^T \right\} \geq \left[ J_H^{-1}(\theta_{nr}, \theta_r) \right]_{i,i},
\]

where \(J_H(\theta)\) is the hybrid Fisher Information matrix (HFIM) expressed as

\[
J_H(\theta_{nr}, \theta_r) = J_D + J_P.
\]

The elements of the matrices \(J_D\) and \(J_P\) given by

\[
[J_D]_{i,j} = -E_{\theta_{nr}|\theta_r} \left\{ E_{r|\theta_{nr}, \theta_r} \left\{ \frac{\partial \ln p(r|\theta_{nr}, \theta_r)}{\partial \theta_i \partial \theta_j} \right\} \right\},
\]

and

\[
[J_P]_{i,j} = -E_{\theta_r|\theta_{nr}} \left\{ \frac{\partial^2 \ln p(\theta_r|\theta_{nr})}{\partial \theta_i \partial \theta_j} \right\},
\]

where \(p(r|\theta_{nr}, \theta_r)\) is the conditional, joint probability density function (pdf) of the observations and \(p(\theta_r|\theta_{nr})\) the
conditional joint pdf of \(\theta_r\). The matrix \(J_D\) represents the contribution of the data and the matrix \(J_P\) represents the
contribution of prior information.

The HCRB matrix is defined as

\[
\text{HCRB} = \left[ J_H(\theta_{nr}, \theta_r) \right]^{-1}.
\]
In cases in which the observation statistic is expressed in terms of \( p(\mathbf{r}|\kappa_{nr}, \kappa_r) \), and the relationship between the unknown parameters \( \theta_{nr}, \theta_r \) and \( \kappa_{nr}, \kappa_r \) is given by \( \kappa_j = f_j(\theta) \), the chain rule, can be used to express \( \mathbf{J}_H(\theta_{nr}, \theta_r) \) in an alternative form [11]:

\[
\mathbf{J}_H(\theta_{nr}, \theta_r) = \mathbf{P} \left( \mathbf{J}_H(\kappa_{nr}, \kappa_r) \right) \mathbf{P}^T,
\]

where the elements of the matrix \( \mathbf{P} \) are given by \( [\mathbf{P}]_{i,j} = \frac{\partial \kappa_j}{\partial \theta_i} \).

### III. HCRB WITH PHASE MISMATCH

In this section, the HCRB is developed for target localization. A point target is assumed with complex reflectivity \( \vartheta = \vartheta_{Re} + j \vartheta_{Im} \), located in a two dimensional plane at coordinates \( X_o = (x_o, y_o) \). Consider a set of \( M \) transmitting stations and \( N \) receiving stations, widely distributed over a given geographical area, and time and phase synchronized. A set of orthogonal waveforms is transmitted, with the lowpass equivalents \( s_k(t), k = 1, \ldots, M \), and effective bandwidths \( \beta \) [10]. The signals are narrowband in the sense that for a carrier frequency of \( f_c \), the narrowband signal assumption implies \( \beta^2 / f_c^2 \ll 1 \).

In [5], perfect phase synchronization was assumed. In practice, synchronization errors exists, modeled here as zero mean Gaussian random variables with standard deviation \( \sigma_\Delta^2 \) and denoted by \( \Delta \phi = [\Delta \phi_{t_1}, \Delta \phi_{t_2}, \ldots, \Delta \phi_{t_M}, \Delta \phi_{r_1}, \Delta \phi_{r_2}, \ldots, \Delta \phi_{r_N}]^T \), where \( \Delta \phi_{t_k} \) and \( \Delta \phi_{r_\ell} \) are phase errors at transmitting radar \( k \) and receiving radar \( \ell \), respectively. The phase errors introduced by the different stations are assumed to be statistically independent. The vector of unknown parameters is defined by

\[
\theta = [\theta_{nr}, \theta_r]^T,
\]

where \( \theta_{nr} = [x_o, y_o, \vartheta_{Re}, \vartheta_{Im}] \) denotes the deterministic unknowns and \( \theta_r = \Delta \phi^T \) denotes the random unknowns.

The estimation process is based on the signals observed at the receiving sensors. The signal received at sensor \( \ell \) is a superposition of the transmitted signals, reflected from the target, and given by:

\[
r_\ell(t) = \sum_{k=1}^{M} \vartheta s_k(t - \tau_{tk}) \eta_{tk} + n_\ell(t),
\]

where \( \eta_{tk} \) accounts for the phase information and has the value of \( \eta_{tk} = \exp(-j2\pi f_c \tau_{tk}) \exp(-j(\Delta \phi_{tk} + \Delta \phi_{r_\ell})) \).

The noise \( n_\ell(t) \) is assumed to be circularly symmetric, zero-mean, complex Gaussian, spatially and temporally white with autocorrelation function \( \sigma_n^2 \delta(\tau) \). The propagation time, \( \tau_{tk} \), is a sum of the time delays from station \( k \) to the target and from the target to station \( \ell \), and may be expressed as

\[
\tau_{tk} = \frac{1}{c} \left( \sqrt{(x_{tk} - x_o)^2 + (y_{tk} - y_o)^2} + \sqrt{(x_{r_\ell} - x_o)^2 + (y_{r_\ell} - y_o)^2} \right),
\]

where \( c \) denotes the speed of light, \( (x_{tk}, y_{tk}) \) denotes the location of transmitting radar \( k \) and \( (x_{r_\ell}, y_{r_\ell}) \) denotes the location of receiving radar \( \ell \). The following vector notation is introduced: \( \tau = [\tau_{t_1}, \tau_{t_2}, \ldots, \tau_{tk}, \ldots, \tau_{N,M}]^T \).

The received signals are separated at the receiver by exploiting the orthogonality between the transmitted waveforms. The signal in (7) is defined as a function of the time of arrival, \( \tau_{tk} \), the reflectivity value \( \vartheta \), and...
the phase mismatch \( \Delta \phi \). The vector of unknown parameters for the observations \( r_\ell(t) \) is expressed as a function of the time delays \( \tau \) rather than a function of the unknown location \((x_o, y_o)\) (as seen in (8)); i.e., the vector of unknown parameters is denoted by \( \kappa = [\kappa_{nr}, \kappa_r]^T \), with \( \kappa_{nr} = [\tau^T, \vartheta_{Re}, \vartheta_{Im}] \) and \( \kappa_r = \Delta \phi^T \). The following notation is defined for later use: \( r = [r_1(t), \ldots, r_N(t)]^T \), \( Q = MN, L = M + N \).

In order to derive the HFIM given in (2) and (3), the conditional joint pdf \( p(r|\kappa) \) is required. For the signal model given in (7), the conditional joint pdf of the observations (time samples at multiple receive antennas) parametrized by the unknown parameters vector \( \kappa \), is then

\[
p(r|\kappa) \propto \exp \left\{ -\frac{1}{\sigma_n^2} \sum_{\ell = 1}^{N} \int_{T} \left| r_\ell(t) - \sum_{k=1}^{M} \vartheta_{s_k}(t - \tau_k t_s) \eta_{\ell k} \right|^2 dt \right\}.
\]

The observation is given as a function of \( \kappa \). Therefore, the matrix \( P \), defined following (5), needs to be derived. The relation given in (8) is used, resulting in

\[
P = \begin{bmatrix} D^{T}_{2 \times Q} & 0 \\ 0 & I_{(2+L) \times (2+L)} \end{bmatrix},
\]

with

\[
D = -\frac{1}{c} \begin{bmatrix} \cos \alpha_1 + \cos \gamma_2 & \sin \alpha_2 + \sin \gamma_2 \\ \vdots & \vdots \\ \cos \alpha_M + \cos \gamma_N & \sin \alpha_M + \sin \gamma_N \end{bmatrix}^T,
\]

where \( \alpha_k \) is the bearing angle of the transmitting sensor \( k \) to the target, measured with respect to the \( x \) axis, and \( \gamma_\ell \) is the bearing angle of the receiving radar \( \ell \) to the target, measured with respect to the \( x \) axis.

Using the conditional pdf \( p(r|\kappa) \) in (9) and the Gaussian distribution of the phase errors, the HFIM \( J_H(\kappa) \), defined by (2) and (3), is derived in Appendix I, resulting in

\[
J_H(\kappa_{nr}, \kappa_r) = \begin{bmatrix} R_\tau & G \\ G^T & H \end{bmatrix},
\]

where matrices \( G \) and \( H \) are defined by

\[
G = \begin{bmatrix} F_{\tau \vartheta} & F_{\tau \Delta} \end{bmatrix}_{Q \times (2+L)},
\]

and

\[
H = \begin{bmatrix} \Sigma_\vartheta & F_{\vartheta \Delta} \\ F_{\Delta \vartheta} & \Sigma_\Delta + \frac{1}{\tau_\Delta^2} I \end{bmatrix}_{(2+L) \times (2+L)},
\]

and the other submatrices in (12), (13) and (14) are defined and derived in Appendix I (see (27), (29), (30) and (31)). Applying (10) and (12) in (5) yields

\[
J_H(\theta) = \begin{bmatrix} DR_\tau D^T & DG \\ G^T D^T & H \end{bmatrix}.
\]
The HCRB for the unknown parameters \((x_o, y_o)\) may be derived from (15), applying the relation given in (4) [5]:

\[
HCRB(x_o, y_o) = \left[ DR_xD^T - DGH^{-1}G^TD^T \right]_{2\times2}^{-1}.
\]  

(16)

To find the closed-form solution to \(HCRB(x_o, y_o)\), the matrix \(H^{-1}\) is expressed using the formula for the inverse of a partitioned matrix [12]:

\[
H^{-1} = \left[ \begin{array}{c}
\Sigma_{\theta} - F_{\theta\Delta}A_{\Delta}^{-1}F_{\theta\Delta}^T \\
P_{\Delta} - F_{\theta\Delta}S_{\theta}^{-1}F_{\theta\Delta}^T - A_{\Delta}
\end{array} \right]^{-1}
\]

(17)

\[
H^{-1} = \left[ \begin{array}{c}
\Sigma_{\theta} - F_{\theta\Delta}A_{\Delta}^{-1}F_{\theta\Delta}^T \\
P_{\Delta} - F_{\theta\Delta}S_{\theta}^{-1}F_{\theta\Delta}^T - A_{\Delta}
\end{array} \right]^{-1}
\]

(18)

where \(A_{\Delta} = (\Sigma_{\Delta} + \frac{1}{\Delta^2}I)\). The term \([\Sigma_{\theta} - F_{\theta\Delta}A_{\Delta}^{-1}F_{\theta\Delta}^T]^{-1}\) in (17), is transformed based on the formula for the inverse of a matrix \(B\) of the form \(B = A + XRY\), given in [12]. Following some additional matrix manipulations, the HCRB for the location MSE can be expressed as

\[
HCRB(x_o, y_o) = J_F^{-1} + \left[ J_F - J_F P_{\Delta}^{-1} J_F \right]^{-1}
\]

(20)

\[
= CRB_o(x_o, y_o) + \Delta CRB,
\]

where \(CRB_o(x_o, y_o) = J_F^{-1}\) is the CRB with no phase mismatch, and \(\Delta CRB = \left[ J_F - J_F P_{\Delta}^{-1} J_F \right]^{-1}\) represents the increment in the bound due to phase synchronization errors. The matrices \(J_F\) and \(P_{\Delta}\) are defined by

\[
J_F = DR_xD^T - DF_{\theta\Delta}S_{\theta}^{-1}F_{\theta\Delta}D^T,
\]

and

\[
P_{\Delta} = DF_{\theta\Delta}S_{\theta}^{-1}F_{\theta\Delta}R_{\Delta}^{-1}F_{\theta\Delta}S_{\theta}^{-1}F_{\theta\Delta}^T + \frac{2 Re \left\{ DF_{\theta\Delta}R_{\Delta}^{-1}F_{\theta\Delta}^T \Sigma_{\theta}^{-1} F_{\theta\Delta} D^T \right\}}{D F_{\theta\Delta}R_{\Delta}^{-1} F_{\theta\Delta}^T D^T},
\]

resulting in

\[
R_{\Delta}^{-1} = \begin{bmatrix}
\lambda_1 I_{M \times M} + \frac{N \lambda_2^2}{M (1 - N \lambda_2)} 11^T & 0 \\
0 & \lambda_2 I_{N \times N} + \frac{M \lambda_2^2}{N (1 - N \lambda_2)} 11^T
\end{bmatrix},
\]

(22)

where \(1 = [1, 1, \ldots, 1]^T\) and the terms \(\lambda_1\) and \(\lambda_2\) are

\[
\lambda_1 = \left( N + \frac{1}{2 \text{snr} \sigma_\Delta} \right)^{-1} \quad \text{and} \quad \lambda_2 = \left( M + \frac{1}{2 \text{snr} \sigma_\Delta} \right)^{-1}.
\]

(23)

Calculating the explicit value of \(\Delta CRB\), we get

\[
\Delta CRB = \left[ J_F - J_F \left( \mu_0 \sum_{m=1}^{3} k_m B_m \right)^{-1} \right]_{2 \times 2}^{-1}.
\]

(24)
where the constants $\mu_m, m = 0, \ldots, 3$ are functions of the phase synchronization error variance $\sigma_{\Delta}^2$ (through $\lambda_1$ and $\lambda_2$, defined in (23)) and the number of transmitting and receiving radars $M$ and $N$, as follows:

$$\mu_0 = \frac{8\pi^2 (f_c^2 + \beta^2) \text{snr}}{c^2}, \quad (25)$$

$$\mu_1 = \frac{\lambda_1}{M} + \frac{\lambda_2}{N},$$

$$\mu_2 = \frac{\lambda_1 (N^2/M)}{M} \quad \text{and} \quad k_3 = \frac{\lambda_2 (M^2/N)}{N}.$$

The matrices $B_m, m = 1, 2, 3$ depend on the geographical layout of the radars with respect to the target location:

$$B_m = \begin{bmatrix} [D_m 1]_{1,1}^2 & [D_m 1]_{1,1} [D_m 1]_{2,1}^2 & [D_m 1]_{2,1}^2 \end{bmatrix},$$

using the following $D_m$ matrices:

$$D_1 = \varepsilon \mathbf{D} \quad (26)$$

$$D_2 = \begin{bmatrix} \cos \alpha_1 & \sin \alpha_2 \\ \vdots & \vdots \\ \cos \alpha_M & \sin \alpha_M \end{bmatrix}^{T}_{M \times 2},$$

and

$$D_3 = \begin{bmatrix} \cos \gamma_2 & \sin \gamma_2 \\ \vdots & \vdots \\ \cos \gamma_N & \sin \gamma_N \end{bmatrix}^{T}_{N \times 2}.$$

The expression for the HCRB as given in (20), offers an interesting observation on the effects of phase errors on the target localization MSE. First, it is apparent that the HCRB may be expressed as the sum of the CRB with no phase error and a term dependent on the statistics of the phase errors. This term is a function of the sensors location with respect to the target, through the matrices $B_m$, and the system parameters (SNR, phase errors variance $\sigma_{\Delta}^2$ and the number of mismatched transmitting and receiving radars) through the coefficients $\mu_m$. The manner in which the number of radars, their spread and the phase synchronization error variance affect the performance is not readily understood from (24). For this reason, numerical examples are employed in the next section to gain some insight into the relationships between system parameters and performance degradation.

IV. NUMERICAL ANALYSIS

We have evaluated the HCRB expression given in (20) numerically using the following example: $M = 11, N = 9$ and $\sigma_{\Delta}^2 = [0, 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05]$, where $\sigma_{\Delta}^2$ is expressed in $(\text{rad})^2$. The HCRB $(x_o, y_o)$ is drawn in Figure 1. As $\sigma_{\Delta}^2$ increases beyond a specific value, the additional CRB term $\Delta \text{CRB}$ dominates the performance and the curve. For high phase error levels, the performance degradation starts at lower SNRs. For small phase errors, localization accuracy is not undermined by the phase mismatch, and the HCRB $(x_o, y_o)$ curve follows the $\text{CRB}_o (x_o, y_o)$ closely.
Fig. 1. HCRB for M=11 and N=9. The blue line represent the CRB value with no phase errors. $\sigma_\Delta^2$ range from 0 to 0.05.

For a given system, the tolerated $[\sigma_\Delta^2]_{\text{max}}$ may be determined by solving $\Delta\text{CRB} \left( [\sigma_\Delta^2]_{\text{max}} \right) \preceq \text{CRB}_o (x_o, y_o)$. This value can serve as a design goal in the system phase calibration. For a given phase synchronization error variance $\sigma_\Delta^2$, the expression $\Delta\text{CRB} \left( \sigma_\Delta^2 \right)$ gives the localization accuracy penalty.

V. CONCLUSIONS

MIMO radar with coherent processing exploits the signal phase measured at the receive antennas to generate high resolution target location estimation. To take advantage of this scheme, full phase synchronization is required among all participating radars. In practice, inevitable phase synchronization errors reflect on the system localization performance. In this paper, a closed-form expression of the HCRB of target localization has been derived, capturing the impact of the phase synchronization errors on the achievable target localization accuracy. In particular, it has been shown that the HCRB can be expressed as a sum of the CRB with no phase error and a term that represents the phase error penalty. The latter has been shown to be a function of the sensors geometry, SNR, and the number of transmitting and receiving radars in addition to the phase error MSE. As phase synchronization over distributed platform is a complex operation and phase errors are unavoidable, the HCRB offers valuable information at the system design level. For a given phase error MSE, the HCRB may be used to derive the attainable target localization.
accuracy. Otherwise, for a given system performance goal on localization accuracy, the HCRB provides with an upper bound on the necessary phase error MSE values.

**Appendix I**

**Derivation of the J(D) matrix**

In this appendix, we develop the elements of the matrix $J(D(\kappa))$, i.e., $J_{D(\kappa)}$, based on the conditional pdf in (9). The diagonal submatrix $R_\tau$ is derived as follows:

$$
[R_\tau]_{i,j} = -E_{\kappa|\kappa_{nr}} \left\{ \frac{\partial^2 \ln(p(r|\kappa))}{\partial \tau_\ell \partial \tau_{\ell'}} \right\},
$$

and

$$
R_\tau = 8\pi^2 (f_c^2 + \beta^2) \text{snr} I_{Q \times Q},
$$

where $\text{snr} = |\theta|^2 / \sigma_n^2$, and the following notation is used:

$$
i = [(\ell - 1) * M + k], \quad j = [(\ell' - 1) * M + k'],
$$

$$
k, k' = 1, \ldots, M, \quad \ell, \ell' = 1, \ldots, N.
$$

The elements of the matrix $\Sigma_\vartheta$ are given by

$$
[\Sigma_\vartheta]_{1,1} = \frac{2MN \text{snr}}{|\theta|^2} = [\Sigma_\vartheta]_{2,2},
$$

and

$$
[\Sigma_\vartheta]_{1,2} = 0 = [\Sigma]_{2,1},
$$

and the elements of the matrix $\Sigma_\Delta$ are given by

$$
\Sigma_\Delta = 2 \text{snr} \begin{bmatrix}
N I_{M \times M} & (11^T)_{M \times N} \\
(11^T)_{N \times M} & M I_{N \times N}
\end{bmatrix}
$$

The off-diagonal submatrices are as follows:

$$
F_{\tau\vartheta_{Q \times 2}} = \frac{4\pi f_c}{\sigma_n^2} \begin{bmatrix}
\vartheta_{\text{Im}} 1_{Q \times 1} & -\vartheta_{\text{Re}} 1_{Q \times 1}
\end{bmatrix},
$$

$$
F_{\tau\Delta_{Q \times L}} = 4\pi f_c \text{snr} \begin{bmatrix}
I_{M \times M} & \Pi(1) \\
I_{N \times M} & \Pi(N)
\end{bmatrix}
$$

and

$$
\Pi(\ell) = \begin{bmatrix}
0_{N \times (\ell-1)} & 1_{N \times 1} & 0_{N \times (N-\ell-1)}
\end{bmatrix}_{N \times N}
$$

$$
F_{\vartheta\Delta_{2 \times L}} = \frac{2 \text{snr}}{|\theta|^2} \begin{bmatrix}
\vartheta_{\text{Im}} N 1_{1 \times M}^T & \vartheta_{\text{Im}} M 1_{1 \times N}^T \\
-\vartheta_{\text{Re}} N 1_{1 \times M}^T & -\vartheta_{\text{Re}} M 1_{1 \times N}^T
\end{bmatrix}.
$$
REFERENCES

[1] E. Fishler, A. M. Haimovich, R. S. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, “MIMO radar: An idea whose time has come,” in Proc. of the 2004 IEEE Int. Conf. on Radar, Philadelphia, April 2004, pp. 71–78.

[2] H. Godrich, A. M. Haimovich and R. S. Blum, “Concepts and Applications of a MIMO Radar System with Widely Separated Antennas,” book chapter in MIMO Radar Signal Processing, John Wiley 2008.

[3] A. Haimovich, R. Blum, and L. Cimini, “MIMO radar with widely separated antennas,” IEEE Signal Proc. Magazine, Vol. 25, January 2008, pp. 116 - 129.

[4] J. Li, and P. Stoica, "MIMO radar with colocated antennas," IEEE Signal Proc. Magazine, Vol. 24, September 2007, pp. 106–114.

[5] H. Godrich, A. M. Haimovich, and R. S. Blum, "Target localization accuracy gain in MIMO radar based system," submitted to IEEE Trans. on Information Theory.

[6] H. V. Poor, An Introduction to Signal Detection and Estimation, New York; Springer, 2nd ed, 1994.

[7] H. L. Van Trees, and K. L. Bell, Bayesian Bounds for Parameter Estimation and Nonlinear Filtering/Tracking, New York; Wiley-Interscience, 2007.

[8] Y. Rockah, H. Messer, and P. M. Schultheiss, "Localization performance of arrays subject to phase errors," IEEE Trans. Acoust., Speech, Signal Proc., Vol. ASSP-35, March 1987, pp. 286-299.

[9] Y. Rockah, and P. M. Schultheiss, "Array shape calibration using sources in unknown locations - part I: Far-field sources," IEEE Trans. Acoust., Speech, Signal Proc., Vol. ASSP-35, March 1987, pp. 286-299.

[10] M. Skolnik, Introduction to Radar Systems, New York: McGraw-Hill, 3rd, 2002.

[11] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, vol. 1, Upper Saddle River, NJ: Parentice Hall PTR, 1st ed., 1993.

[12] R. A. Horn, and C. R. Johnson, Matrix Analysis, Cambridge, UK:Cambridge University Press, 1990.

[13] K. S. Miller, “On the inverse of the sum of matrices,” Mathematics Magazine, Vol. 54, No. 2, 1981, pp. 67-72.