Abstract. For a sample of 51 blazars with extensive optical polarization data, we used circular statistics to calculate the scatter among the polarization position angles for each object. We found that this scatter is correlated with the radio core dominance. We compared this relationship with the predictions of a simple transverse shock model. The result suggests that blazar jets are likely to cover a wide range of speeds consistent with those derived from observations of superluminal motion.
the range $0^\circ$ to $180^\circ$. However, the $2\theta_i$ are truly circular data. Therefore, we first calculate $c(2\theta)$, then divide by 2 to get $c(\theta)$:

$$c(\theta) = \left[ -\ln\left( \frac{\sqrt{(\sum_{i=1}^{n} \cos 2\theta_i)^2 + (\sum_{i=1}^{n} \sin 2\theta_i)^2}}{n} \right) \right]^{\frac{1}{2}}$$

where $n$ is the total number of measurements. If the sample has a large intrinsic scatter, the determination of the real scatter depends on the accurate shape of the wrapped distribution between $-\pi/2$ and $\pi/2$. If the sampling of data is not enough to reveal the detailed shape of the distribution, the $c(\theta)$ calculation algorithm gives a maximum $c(\theta) \sim 66^\circ$.

2.2. Radio Core Dominance

In the unification scheme of powerful radio sources, the core-dominated sources are the Doppler-boosted jet, viewed end-on. At larger angles the core becomes much weaker and collimated jets link the well resolved double lobes. Doppler boosting in core-dominant sources is supported by their very high brightness temperatures and by superluminal motions. Statistical analyses of radio source samples support the unified scheme (M. Chiaberge, this conference). The ratio of core-to-lobe luminosity, or the core dominance, $R$, is therefore a good indicator of viewing angle. However, in blazar samples, viewing angle is not the only parameter affecting Doppler boosting. From work of other authors (Orr and Browne 1982), we know that $R$ depends on three physical parameters and can be expressed as

$$\log R = \log(R_T \times (1 - \beta \times \cos(\phi))^{-3})$$

where $\phi$ is the viewing angle of the jet, $\beta$ is the bulk speed of the emitting electrons in the jet and $R_T$ is the tangential value of $R$ measured for an average edge-on jet in radio galaxies. We used $R_T = 0.0024$ at 5GHz rest frequency (e.g. Figure 1 of Hoekstra, Barthel & Hes 1997). While $R$ is of course sensitive to $\phi$, for smaller $\phi$ (e.g. $< 20^\circ$), it is increasingly sensitive to $\beta$.

For each object we calculated $R$ at 5 GHz rest frequency, using core and lobe flux densities from literature. In Section 4, we will calculate $\log R$ from models using the formula above and compare the models with observed values.

3. Correlation

Figure 1 (a) shows that there is a correlation between $c(\theta)$ and $\log R$. The correlation is significant at the 99.9% level. However, we note that the correlation is not linear. For the $\log R < 2$ region, which includes most blazars, $c(\theta)$ increases with $\log R$, but for $\log R > 2$, there is a lot of scatter. No object has $c(\theta)$ larger than $66^\circ$, as expected from the algorithms for large scatter and small sample size.
Figure 1. (a) – Correlation between log R and the scatter of optical polarization position angle. Triangles are BL Lacs and squares are QSOs. The two thin curves represent $\beta = 0.93, 0.99$ models. (b) – Model simulations. Each represents a model with fixed $\beta$ and variable $\phi$. The $\beta$ for each model is noted on top of each curve. The markers along each curve denote the corresponding $\phi$ in the observer’s frame.

4. Model

The model we use makes the following assumptions:

1. Optical emission comes from a transverse shock propagating along the jet.

2. The shock compresses the magnetic field so much that the magnetic component along the jet direction is negligible.

3. In the plane of the shock, there is a dominant magnetic field direction that changes randomly with time. This magnetic field direction corresponds to the emission region whose radiation happens to be boosted towards us. Different ejection directions observed for emerging blobs on VLBI scales and complex VLBI structure suggests that these dominant regions change rapidly.

4. The shock is optically thin.

When the shock is viewed face on, the magnetic field direction projected on the sky will vary randomly with time. As the viewing angle is increased, the projected magnetic field direction will lie increasingly perpendicular to the projected jet direction.

A simplification is afforded by the Lorentz invariance of the ratios of Stokes parameters, so our calculations of projected magnetic field direction in the co-moving frame (viewing angle $\phi'$) can be referred to the observer’s frame using the relation (Bjornsson, C.-I. 1982):
\[
\sin(\phi') = \frac{\sin(\phi)}{1 - \beta \cos(\phi)} \cdot \sqrt{1 - \beta^2}
\]

For a given set of \(\beta\) and \(\phi\), we generate 100 randomly oriented magnetic field directions to simulate the random changes over time inside the shock plane and project them onto the sky in the co-moving frame. The circular standard deviation of those angles can then be compared with real observations, using the relation between \(R\) and \(\phi\) given earlier. \(R_T\) is fixed by observations, so the only free parameter in this model is \(\beta\).

The results are shown in Figure 1 (b). Each curve represents a model with fixed \(\beta\), with \(\phi\) decreasing from bottom left to top right. The values of \(\phi\) are marked along the curve. From left to right, \(\beta = 0.93, 0.95, 0.97, 0.99\). The upper limit on \(c(\theta)\) is the result of the computational limit. The lower limits are caused by the fact that we only have 100 simulated vectors to do statistics and any one vector that happens to be parallel to the jet direction contributes a lot to the scatter. In principle, if we had an increasing number of vectors, the lower limit for \(c(\theta)\) would approach 0. On our model, the \(c(\theta)\) vs. \(\log R\) correlation arises because \(\beta \sim 0.95\) for most blazars, and so the viewing angle dependence dominates for \(\log R < 2\). For \(\log R > 2\), the scatter is the result of a tail to higher jet speeds.

Our simple model with only one free parameter, \(\beta\), accounts for the correlation we find between the scatter in optical polarization angles, \(c(\theta)\), and the core-dominance, \(\log R\). Figure 1 shows that the average jet speed differs significantly amongst the powerful blazars, with a range of values comparable with those derived from superluminal motion and from the statistics of core-dominance in radio-source surveys (Urry & Padovani 1995). In Unified Schemes it is important to take into account that \(\log R\) is not simply a measure of orientation. Jet speed introduces considerable scatter, especially at \(\log R > 2\).

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