The pygmy dipole resonance in neutron-rich nuclei

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Abstract. The pygmy dipole resonance (PDR), which has been observed via the enhancement of the electric dipole strength $E1$ of atomic nuclei, is studied within a microscopic collective model. The latter employs the Hartree-Fock (HF) method with effective nucleon-nucleon interactions of the Skyrme types plus the random-phase approximation (RPA). The results of the calculations obtained for various even-even nuclei such as $^{16−28}$O, $^{40−58}$Ca, $^{100−120}$Sn, and $^{182−218}$Pb show that the PDR is significantly enhanced when the number of neutrons outside the stable core of the nucleus is increased, that is, in the neutron-rich nuclei. As the result, the relative ratio between the energy weighted sum of the strength of the PDR and that of the GDR (giant dipole resonance) does not exceed 4%. The collectivity of the PDR and GDR states will be also discussed.

1. Introduction
The pygmy dipole resonance (PDR) was firstly identified via the enhancement of the electric-dipole strength $E1$ at low energy ($E \leq 10$ MeV) around the particle emission threshold of neutron-rich nuclei [1]. The PDR is commonly represented as the dipole vibration of the excess neutrons against the isospin-symmetry core of protons and neutrons in the nucleus. Nature of the PDR and its properties have been therefore the subjects of various theoretical and experimental studies. From the experimental point of view, the PDR has been investigated systematically from the measurements of dipole response of a large number of stable and unstable nuclei via electromagnetic excitation in heavy-ion collisions [2], photon scattering [3], and Coulomb excitation reactions [4]. From the theoretical aspect, various theoretical models have been proposed to describe the nature of PDR. Some of them are the large-scale shell model calculations [5], relativistic random-phase approximation (RRPA) [6], RRPA plus phonon-coupling model [7], quasiparticle random-phase approximation (QRPA) with Skyrme interaction [8], phonon-damping model (PDM) [9], density functional theory (DFT) [10], ect. One of the major issue in the theoretical study of the PDR is the discrepancy in the predictions of different approaches regarding the strength and collectivity of the PDR. For instant, the RRPA predicts a prominent peak, which is identified as the collective PDR below 10 MeV in $^{120−132}$Sn and $^{122}$Zr [6, 7], whereas the calculations within the QRPA do not expose any collective states in the low-energy region of the $E1$ strength distribution for the same isotope chain [8]. In addition, there has been lack of a systematic calculation within a self-consistent microscopic approach for the PDR.
in nuclei with the mass numbers ranging from the light to the heavy ones. The aim of present study is to perform such a calculation in order to see the evolution of the PDR in neutron-rich nuclei ranging from the light systems such as $^{16-28}\text{O}$ to the heavy one such as $^{182-218}\text{Pb}$. For that purpose, we employ the self-consistent Hartree-Fock (HF) plus the RPA code with the use of realistic Skyrme-type interactions [11]. Though this code has been successfully used in the study of the nuclear giant resonances such as giant monopole resonance (GMR), giant dipole resonance (GDR), giant quadrupole resonance (GQR), etc., it has not been used to describe the PDR in neutron-rich nuclei.

2. The random-phase approximation
As mentioned in the Introduction, present work employs the self-consistent HF + RPA code with realistic Skyrme-type interactions. The formalism of the RPA is presented in details in Refs. [11, 12], so we report here only the main equations. Basically, the RPA equation is derived based on the creation (destruction) operator $B^\dagger_{ph}(JM)$ that creates (annihilates) the particle-hole (p-h) pairs coupled to total angular momentum ($JM$)

$$B^\dagger_{ph}(JM) = \sum_{m_p,m_h} (j_p m_p j_h - m_h |JM)a^\dagger_{j_p m_p}(-1)^{j_h - m_h} a_{j_h - m_h}, \quad B_{ph} = (B^\dagger_{ph})^\dagger,$$

(1)

where $a^\dagger_{j_p m_p}$ is an operator that creates the particles on the orbital having the angular momentum $j_p$ and projection $m_p$, whereas $a_{j_h - m_h}$ is an operator that annihilates the particles or creates the holes having the angular momentum $j_h$ and projection $-m_h$. Within the RPA, the excited states $|\nu\rangle$ are formed by applying the excitation operators of the p-h pairs $Q^\dagger_{\nu}$ on the correlated ground state $|\tilde{0}\rangle$, namely

$$|\nu\rangle = Q^\dagger_{\nu}|\tilde{0}\rangle,$$

(2)

where $Q^\dagger_{\nu}$ is expressed as a combination of the creation and destruction operators of the p-h pairs, namely

$$Q^\dagger_{\nu} = \sum_{ph} \left[ X^n_{ph} B^\dagger_{ph}(JM) - Y^n_{ph} B_{ph}(\tilde{JM}) \right],$$

(3)

with $\tilde{JM}$ being the time-reversed state of $JM$, i.e., a state with the same $J$ but opposite $z$-projection ($-M$). The amplitudes $X^n_{ph}$ and $Y^n_{ph}$ are the eigenvectors of the so-called RPA matrix having the form as

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = E_{\nu} \begin{pmatrix} X^n \\ Y^n \end{pmatrix},$$

(4)

where the submatrices $A$ and $B$ are derived based on the particle-hole configurations

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \left\langle p'h'|V_{res}|hh'\right\rangle,$$

(5)

$$B_{ph,p'h'} = \left\langle p'h'|V_{res}|hh'\right\rangle,$$

(6)

where $\epsilon_p$ and $\epsilon_h$ are respectively the energies of the particle and hole levels, whereas the residual interaction $V_{res}$, which is an anti-symmetrized p-h interaction, is obtained within the calculations.
of the Hartree-Fock plus Skyrme interactions \[12\]. The reduced transition probabilities from the ground state to the excited states are calculated based on the solutions of the RPA equations

\[
B(EJ, \tilde{0} \rightarrow \nu) = \left| \sum_{ph} (X_{\nu ph}^{\nu} + Y_{\nu ph}^{\nu}) \langle p|\tilde{F}_J||h\rangle \right|^2,
\]

where \(\tilde{F}_J\) is a \(J\)-multipole of the external field because of which the nucleus is excited. \(\tilde{F}_J\) consists of the isoscalar \(\tilde{F}_J^{IS}\) and isovector \(\tilde{F}_J^{IV}\) components, whose explicit form are given as

\[
\tilde{F}_J^{IS} = \sum_{i=1}^{A} f_J(r_i)Y_{JM}(\hat{r}_i),
\]

\[
\tilde{F}_J^{IV} = \sum_{i=1}^{A} f_J(r_i)Y_{JM}(\hat{r}_i)\tau_z(i),
\]

where \(\tau_z(i)\) is the \(z\)-component of the isospin operator, whereas \(Y_{JM}(\hat{r}_i)\) is the spherical harmonics functions. The function \(f_J(r)\) in Eqs. (8) and (9) is often given in term of a spherical Bessel function for the electromagnetic external fields or it is proportional to \(r^J\) in the long wavelength limit. In the case of dipole vibration \((J = 1)\), in order to remove the center of mass contributions, we employ the electromagnetic operator of the form

\[
\tilde{F}_{1M} = \frac{eN}{A} \sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A} \sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i),
\]

where \(\frac{eN}{A}\) and \(-\frac{eZ}{A}\) are the effective charge for protons \((Z)\) and neutrons \((N)\), respectively.

In principle, the distribution of the transition probabilities \(B(EJ)\) in Eq. (7) is discrete. To have a continuous distribution, one should use the smoothed strength function \(S(E)\), which can be obtained based on the \(\delta\)-function representation \(\delta(x) = \varepsilon/[(\pi x^2 + \varepsilon^2)]\) with \(\varepsilon\) being the smoothing parameter. The explicit expression of \(S(E)\) is given as

\[
S(E) = \frac{\varepsilon}{\pi} \sum_{\nu} \frac{B(EJ)}{(E - E_{\nu})^2 + \varepsilon^2}.
\]

3. Results

The numerical calculations are carried out within the self-consistent HF + RPA code with Skyrme-type interactions for the isovector (IV) dipole states \((J^\pi = 1^-)\) of several spherical nuclei ranging from the closed-shell to neutron-rich isotopes such as \(^{16,20,24,28,40,48,52,56}\)Ca, \(^{100,106,114,120}\)Sn, and \(^{182,194,208,218}\)Pb. As for the Skyrme interaction, we employ the realistic SLy5 interaction \[13\], whose parameter sets are specified and used in the HF calculation. The later is done in a spherical box with radius \(R = 24\) fm and the cut-off energy \(E_c = 60\) MeV for the unoccupied states. Shown in Figs. 1 and 2 are the IV reduced transition densities \(B(E1)\) as functions of excitation energy \(E\). It is clear to see from these figures that beside the GDR region, whose excitation energy \(E\) ranges from 10 MeV to 30 MeV, there appears a low energy region \((E \leq 10\) MeV\), which is called the PDR region, where the \(B(E1)\) values increase with increasing the neutron number, especially in the case of light and medium-mass isotopes such as \(^{28}\)O [Fig. 1 (d)] and \(^{58}\)Ca [Fig. 1 (h)]. The more neutron is added to the system, the more pygmy dipole strength is gained. To see the contribution of the pygmy dipole states to the total \(B(E1)\) strength, we plot in Fig. 3 the ratio between the energy weighted sum of strength (EWSS)
Figure 1. Distribution of the reduced transition probabilities $B(E1, \tilde{0} \to 1^-)$ for the isovector dipole states in Oxygen and Calcium isotopes. The dashed lines denote the strength function $S(E)$ with the smoothing parameter $\varepsilon = 0.4$ MeV.

of the PDR and that of the GDR as function of the mass number, namely $r = S_{PDR}/S_{GDR}(\%)$, where $S = \sum_{E\nu} B(EJ, E\nu)$. This Fig. 3 shows that $r$ increases from almost 0% in the closed shell nuclei such as $^{16}$O, $^{40,48}$Ca, $^{100}$Sn, and $^{182}$Pb to about 4% in very neutron-rich nuclei such as $^{28}$O and $^{58}$Ca, which agrees with the experimental investigations [14, 15] (See e.g., Fig. 2 of Ref. [15]).

Obviously, each of the PDR or GDR states is formed by the combinations of many p-h configurations. As the result, the collectivity of the PDR (GDR) state should be defined based on the number of p-h components, which contribute an appreciable weight to the RPA wave function. The latter is given as

$$A_{ph} = \left(X_{ph}^{\nu}\right)^2 - \left(Y_{ph}^{\nu}\right)^2.$$  \hspace{1cm} (12)

The normalization of the total weight leads to the condition $\sum_{ph} A_{ph} = 100\%$. The values of
$A_{\text{ph}}$ for the p-h configurations mostly contributed to two selected PDR ($E = 3.42$ MeV) and GDR ($E = 17.19$) states are listed in Table 1. One can see easily from this Table that for the GDR state there are 7 p-h configurations which have comparable contributions to it with the largest contribution of only $33.93\% \ [(4p_{3/2}, 2s_{1/2})_n]$. It means that this GDR state can be considered as a collective. Meanwhile, for the PDR state only one neutron p-h configuration $(2p_{3/2}, 1d_{3/2})_n$ already contributed about $99.2\%$ of its total strength. This PDR state therefore has no collective property, which agrees with the QRPA prediction in Ref. [8], whereas it is in contradiction to the RRPA results in Refs. [6] and [7].

4. Conclusions
This work studies the evolution of the pygmy dipole resonance in neutron-rich nuclei by employing the self-consistent HF + RPA code with Skyrme interactions. The results obtained show that the PDR states increase significantly with increasing the neutron number.
Figure 3. Ratio between the energy weighted sum of the strength of the PDR and that of the GDR obtained within the HF + RPA as functions of the mass number.

Table 1. The weight $A_{ph}$ of the contribution of the proton ($p$) and neutron ($n$) particle-hole configurations to the PDR and GDR states obtained within the HF + RPA calculations for $^{28}$O nucleus.

| PDR ($E = 3.42\text{MeV}$) | GDR ($E = 17.19\text{MeV}$) |
|-----------------------------|-----------------------------|
|                            |                             |
| p-h conf.                  | $E$ (MeV)                  | $A_{ph}$ (%) |
| (1d$_{5/2}$, 1p$_{3/2}$)$_p$ | 12.65                      | 0.10         |
| (1d$_{3/2}$, 1p$_{3/2}$)$_p$ | 14.92                      | 0.04         |
| (1f$_{7/2}$, 1d$_{5/2}$)$_n$ | 11.43                      | 0.40         |
| (2p$_{3/2}$, 2s$_{1/2}$)$_n$ | 7.31                       | 0.01         |
| (1p$_{1/2}$, 1d$_{3/2}$)$_n$ | 3.64                       | 0.01         |
| (2p$_{3/2}$, 1d$_{3/2}$)$_n$ | 3.35                       | 99.20        |
| (2f$_{5/2}$, 1d$_{3/2}$)$_n$ | 10.13                      | 0.01         |
|                            |                             |              |
| (2d$_{5/2}$, 1p$_{3/2}$)$_p$ | 12.65                      | 9.00         |
| (1d$_{3/2}$, 1p$_{1/2}$)$_p$ | 14.92                      | 7.96         |
| (2f$_{5/2}$, 1d$_{5/2}$)$_n$ | 16.69                      | 13.99        |
| (1f$_{7/2}$, 1d$_{5/2}$)$_n$ | 11.43                      | 5.55         |
| (2f$_{5/2}$, 1d$_{5/2}$)$_n$ | 13.40                      | 4.93         |
| (2p$_{3/2}$, 1d$_{3/2}$)$_n$ | 19.43                      | 8.63         |
| (3f$_{7/2}$, 1d$_{5/2}$)$_n$ | 16.33                      | 33.93        |

Consequently, the relative ratio between the energy weighted sum of the strength of the PDR and that of the GDR increases with increasing the mass number and reaches the maximum value of about 4%, in agreement with the experimental investigation. By analyzing the weight of the contribution of the p-h configurations to the PDR and GDR states, one can conclude that the GDR state can be considered as collective, whereas the PDR has no collectivity. It is worth mentioning that present approach does not take into account the pairing correlation, which is one of the important features of neutron-rich nuclei. In addition, this code also neglects the high-order excitation modes that come from the two particle - two hole (2p-2h) or three particle - three hole (3p-3h) excitations. As the result, the conclusions above are made only in the basis of the present RPA calculations. To have a fully self-consistent microscopic prediction of the PDR, this RPA code should be improved by including the pairing correlation and high-order excitation modes. That is the goal of our oncoming studies. The numerical calculations were carried out using the Integrated Cluster of Clusters (RICC) system at RIKEN. This work is supported by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam through Grant No.103.04-2013.08.
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