Stability of one-dimensional relativistic laser plasma envelope solitons

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Abstract. Stability of one-dimensional relativistic laser plasma envelope solitons formed by the nonlinear coupling of relativistically intense electromagnetic waves with plasma waves in a cold plasma to has been examined using fluid simulations. It is observed that the solutions for which scalar potential is weaker than the vector potential are stable whereas solutions with comparatively much higher scalar potential amplitudes are unstable. For unstable solutions, the instability mechanism has been identified as forward stimulated Raman scattering and the subsequent density bursts observed in the wake of the light field is understood in terms of phase mixing of nonlinear oscillations. For stable solutions, an interesting possibility opens up in their potential use as energy carriers through overdense plasma regions, which is an issue of crucial relevance to the fast ignition scheme of laser fusion.

1. Introduction
A special class of exact one dimensional solutions for modulated light pulses coupled to electron plasma waves in a relativistic cold plasma has received considerable numerical and analytic attention in the past few years (see [1] and references cited therein) due to the intrinsic interest in their nonlinear properties as well as for their potential applications in areas like particle and photon acceleration or in the fast ignition scheme of laser fusion. Physically these solutions represent a propagating nonlinear stationary state in which the light pulse is trapped in a plasma wave that it itself generates. With the availability of ultraintense lasers there exists a possibility of their experimental realization. This opens up their utility as energy carriers through overdense plasma regions which is of paramount importance for fast ignition concept of laser fusion. Thus the questions of accessibility and stability of such structures are of overriding practical importance. The objective of this work is to investigate these questions in detail.

In this paper, we have examined the stability of these soliton solutions using relativistic fluid simulations. In section 2, we present the coupled set of nonlinear equations which govern the evolution of these structures and briefly describe both the stable and unstable solutions. Section 3 contains a physical interpretation of the instability in terms of forward stimulated Raman scattering (FSRS) process and a physical explanation of the occurrence of the density bursts observed in the wake of the light pulse (for unstable cases) in terms of phase mixing of nonlinear oscillations. Finally section 4 contains the summary.
Figure 1. Subplots (a) and (b) show the profiles of the vector potential $R$ (——), scalar potential $\phi$ (- - - -) and perturbed electron density $n_e - 1$ (· · · · ·) for single peak and the multipeak solutions respectively. The evolution $\phi$ and $R$ is shown for the multipeak solution at two later times $v_i.e. \omega_{pe0}t = 100$ and $120$ respectively in subplots (c) and (d).

2. Governing equations and solutions

The basic set of nonlinear equations derived from the relativistic cold fluid equations for the electrons, immobile ions and Maxwell equations are [2]

$$R'' + \frac{R}{1-\beta^2} \left[ \frac{\lambda^2}{1-\beta^2} - \frac{\beta}{\beta-u} \right] = 0 \quad (1)$$

$$\phi'' = \frac{u}{(\beta-u)} \quad (2)$$

$$u = \frac{\beta(1+R^2) - (1+\phi)(1+\phi^2 - (1-\beta^2)(1+R^2))^{1/2}}{(1+\phi)^2 + \beta^2(1+R^2)} \quad (3)$$

where $R$ is the envelope of a circularly polarized laser wave whose normalized vector potential is taken to be of the form, $\vec{A} = (R(\xi)/2)[\hat{y} + i\hat{z}] \exp(-i\lambda \tau + i\lambda \beta \xi/(1-\beta^2)) + c.c.$ and $\phi$ is the electrostatic potential for the plasma wave. The primes represent derivatives in the traveling wave variable $\xi = x - \beta t$ and $\tau = t$. $\beta$ is the normalized group velocity and $\lambda$ is a wave frequency parameter. The length $x$ is normalized by the skin depth $c/\omega_{pe0}$ (where $\omega_{pe0}$ is the plasma frequency) and time by the inverse of the plasma frequency. The scalar and vector potentials are normalized by $mc^2/e$. A numerical solution of eqns. (1-3) for isolated pulse like structures yields two types of solutions with different stability properties as observed through direct fluid simulation of these structures in $(x,t)$ frame [1]. It is observed that those solutions for which typically the scalar potential amplitude is weaker than the vector potential ($\phi_{max} \ll R_{max}$, single hump solution, Fig. 1(a)) are stable, whereas solutions with comparatively higher scalar potential amplitudes ($\phi_{max} \gg R_{max}$, multi-hump solutions, Fig. 1(b)) are unstable to electromagnetic disturbances trailing the original soliton. Fig. 1(c) and (d) show snapshots of the pulse break up and wave emission at the trailing edge as well as the subsequent emergence of density bursts in the wake region of the pulse at different time instants. Below we identify the instability mechanism as FSRS process and provide physical explanation for the occurrence of density bursts through a model calculation based on phase mixing arguments.
3. Instability of Multi-hump Solutions

We made a quantitative assessment of the growth of the various perturbed variables as the pulse begins to break up. The growth rate of the instability $\Gamma_{\text{sim}}$ has been estimated by observing the growth of the perturbed fields $d\phi$ and/or $dR$ defined as the difference between the numerically observed value of the scalar potential $\phi$ (and/or $R$) at a certain point in space and time with that of the exact equilibrium solution. It is observed that small amplitude perturbations (in both $\phi$ and $R$) start at the front edge of the pulse and experience a continuous amplification as they keep slipping towards the rear edge. As the perturbation is observed to grow only within the pulse region, the amplification factor $\alpha$ which is the ratio of final to initial perturbation amplitudes, is related to $\Gamma_{\text{sim}}$ through $\alpha = \exp(\Gamma_{\text{sim}} L / (V_1 V_2^{1/2}))$ [3], where $L$ is the interaction length and $V_1$, $V_2$ are the relative group speeds of the daughter (scattered) waves measured with respect to the pump wave. In our simulations we observe that both the scattered waves almost remain stationary in the lab frame so that $V_1 \approx V_2 \approx \beta$ and the expression for the amplification factor reduces to $\alpha \approx \exp(\Gamma_{\text{sim}} L / \beta)$. Measuring the amplification factor $\alpha$ gives an estimate of $\Gamma_{\text{sim}}$ which we compare with the theoretical expression of $\Gamma_{\text{rf}}$ given by $\Gamma_{\text{rf}} = \frac{1}{2} \sqrt{2} \omega A_0 (1 + A_0^2 / 2)$ [4, 5, 6, 7]. Fig. 2 shows the result of such a comparison, made for a large number of multi-hump solutions which differ from each other in the number of light peaks, the magnitudes of $R_{\text{max}}$ and the values of $\lambda$. It is clear from this figure that over a wide range of multi-hump solutions, the growth rate of the observed instability agrees closely with the analytical value of FSRS. One further evidence of FSRS is that for finite pulses, the temporal growth rate is slower viz. $\exp(at^{1/2})$ than linear exponential growth rate as shown in Fig. 3. Here $a$ is fitting parameter whose value is consistent with forward Raman scattering theory of finite pulses [4, 6].

We also note here that the observed stability of single hump solutions in our fluid simulations is due to the fact that they do not satisfy the density threshold criterion for FSRS which is $n_e \leq n_{\text{th}}$ where $n_{\text{th}} = \gamma \omega^2 / 4$, $\omega$ and $\gamma$ being respectively the laser frequency and relativistic factor.

Finally we present an explanation for the "bursts" observed in the plasma waves excited and left behind by the propagating light pulse in terms of phase mixing effects. Phase mixing happens because of spatial dependence of plasma frequency which arises here due to relativistic variation of electron mass [8]. Due to phase mixing, the plasma wave slowly loses its coherence eventually leading to crossing of electron orbits which generates these bursts in density. In order to further illustrate the mechanism of generation of these bursts, we have carried out a model calculation where we treat the evolution of two relativistically intense waves whose wave
numbers differ by an amount $\Delta k$. It can be shown that in the weakly relativistic limit the resulting electron density evolves as

$$n_e(x,t) \approx \frac{n_0\{1 + \Delta \cos(\frac{\Delta k}{2}x) \cos kx\}}{1 + \Delta \cos(\frac{\Delta k}{2}x)[\cos kx + \cos(\omega_p \tau - kx)\{\frac{\partial \omega_p}{\partial x} - 1\}]}$$

(4)

where $\omega_p \approx \omega_p \left[1 - \frac{3}{16} \frac{\omega_p^2 k^2}{c^2} \cos^2(\frac{\Delta k}{2}x)\right]$. The presence of a secular term in the denominator clearly shows that the electron density will eventually explode in a time scale $\omega_p \tau_{mix} \sim \frac{1}{\frac{3}{16} \frac{\omega_p^2 k^2}{c^2} (\Delta k)}$. (The secular term goes to zero as $\Delta k \to 0$). This time scale depends on the level of density fluctuation $\Delta$ and the spread ”$\Delta k$” of the plasma waves. Using $\Delta \sim 4.0$, $k \sim 1.3\omega_p/c$ and $\Delta k \sim 0.2\omega_p/c$ from our simulation (fig. 4), gives an estimate of time between two bursts as $\omega_p \tau_{mix} \sim 2$ which matches very well with our observations (fig. 5).

4. Summary
To summarize, we have investigated and identified the forward SRS to be the primary instability responsible for the breakup of the multi-hump nonlinear 1d solutions representing modulated light pulses coupled to electron plasma waves in a relativistic cold plasma. The numerical growth rates and threshold conditions are found to be in good agreement with known theoretical estimates. Furthermore the density bursts observed in the wake of the moving multi-hump solutions are explained by a model calculation based on the relativistic wave breaking phenomenon.

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Figure 4. Expanded view of the k-spectrum of relaxed state at $\omega_{pe}t = 112.1$.

Figure 5. Two consecutive density bursts at $\omega_{pe}t = 110.9 (---)$ and $\omega_{pe}t = 113.5 (- - -)$.