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Numerical simulation of anomalous wave phenomena in hot nuclear matter

A V Konyukhov and A P Likhachev
Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13 Bldg 2, Moscow 125412, Russia
E-mail: konyukhov_av@mail.ru

Abstract. The collective dynamic phenomena accompanying the collision of high-energy heavy ions are suggested to be approximately described in the framework of ideal relativistic hydrodynamics. If the transition from hadron state to quark-gluon plasma is the first-order phase transition (presently this view is prevailing), the hydrodynamic description of the nuclear matter must demonstrate several anomalous wave phenomena—such as the shock splitting and the formation of rarefaction shock and composite waves, which may be indicative of this transition. The present work is devoted to numerical study of these phenomena.

1. Introduction
The hot quantum chromodynamic (QCD) matter arising in collisions of ultrarelativistic heavy ions demonstrates a number of collective phenomena, which can be described in terms of an almost ideal (with very small viscosity) relativistic hydrodynamics [1, 2]. Now two approaches are used to construct the phase diagram of nuclear matter: (1) lattice QCD calculations (see [3], a review of subsequent modifications is given in [4]) and (2) phenomenological models of the MIT-bag type (see detailed comparison of such models in [5]). The first approach meets serious difficulties in the case of calculations with nonzero chemical potential that makes the use of bag models in hydrodynamic calculations more preferable [6].

If the hadron to quark-gluon plasma (QGP) transition is the first-order phase transition (presently this view is prevailing), the hydrodynamic description of the nuclear matter must demonstrate several anomalous wave phenomena (in particular, the shock splitting and the formation of rarefaction shock and composite waves), which may be indicative of this transition. The shock splitting is the phenomenon closely allied with shock wave stability problem, the formation of rarefaction shock and composite waves is due to abnormal thermodynamic properties of the medium [7, 8].

In the present work two problems are considered. The first one is the two-dimensional shock wave splitting with formation of transverse shock waves. The possibility of such shock wave behaviour was discussed earlier in connection with the shock wave neutral stability [9]. The principal difference between the case under consideration and one-dimensional shock wave splitting is the presence of transverse momentum which can affect the angular distribution of reaction products after collision. The second problem is the development of a composite rarefaction wave which can be formed at the stage of the fireball expansion. It needs to note...
that the stability of shock waves in nuclear matter was considered earlier in a number of works (see, e.g., [10–12]).

2. Equation of state and shock wave splitting

In the present paper the variant of the bag equation of state [13] is used to model anomalous wave effects caused by the quark-hadron phase transition. In the framework of this model, the pressure, energy density and baryon number density of quark gluon plasma, which is considered as a gas of quarks and gluons with the number of flavors $N_f = 2$, number of colors $N_c = 3$ and the number of gluon color states $N_g = 8$, have the following form:

$$P_p = \frac{1}{3}(\varepsilon_p - 4B) = \frac{37}{90}\pi^2 T^4 + \mu_q^2 T^2 + \frac{\mu_q^4}{2\pi^2} - B,$$

$$n_p = \frac{2}{3}\left(\mu_q T^2 + \frac{\mu_q^3}{\pi^2}\right),$$

where $\mu_q$ is quark chemical potential, $T$ is the temperature, $n_p$ is the baryon number density, $B$ is the parameter of the MIT-bag model. Hadronic phase is considered as an ideal gas of pions, nucleons and antinucleons. Let $P_h$, $\varepsilon_h$ and $n_h$ are the pressure, the energy density and the baryon number density of hadronic matter being the functions of the baryon chemical potential $\mu$ and the temperature $T$ (see [13]). The phase equilibrium is considered according to [13]: $P_p = P_h$, $T_p = T_h$, $\mu = 3\mu$. The baryon number density and the energy density in the two-phase state are given by

$$\varepsilon = \lambda\varepsilon_p(T, \mu_c) + (1 - \lambda)\varepsilon_h(T, \mu_c), \quad n = \lambda n_p(T, \mu_c) + (1 - \lambda)n_h(T, \mu_c),$$

(1)

where $\lambda$ is the depth of the phase transition, $T_c$, $\mu_c$ are the temperature and the baryonic chemical potential under the equilibrium.

The corresponding phase diagram in terms of pressure, baryon density and temperature is presented in figure 1. A shock adiabat with the initial state in hadronic phase ($H$) and two isentropes ($A_1$, $A_2$) passing through the two-phase region are shown.

Let us consider fulfillment of the neutral stability conditions for the shock waves in nuclear matter. The importance of the analysis of neutral stability of shock waves in quark-gluon plasma is caused by the following circumstance. It was shown earlier [9] that the condition of neutral stability (spontaneous sound emission) coincides with the condition for the existence of triple shock-wave configurations with outgoing wave of finite intensity. In the presence of finite perturbations two-dimensional splitting can occur with formation of oblique (or transverse) shock waves. On the other hand, the formation of oblique shock during the impact of relativistic nuclei considered in the hydrodynamic approximation is suggested to affect the direction of emission of the particles after the collision. In particular, the formation of an oblique shock wave (Mach cone) can explain the experimental data on the collisions of nuclei of different sizes [1]. Two-dimensional shock wave splitting provides theoretically an alternative mechanism for the formation of oblique shock waves associated with implementation of the neutral stability condition for the shock wave in presence of perturbations. The possibility of implementation of such a scenario exists in the case of relativistic shock waves with the initial state in hadron phase and the final state in the two-phase region. A necessary condition for neutral stability (spontaneous emission of sound) of the relativistic shock wave has the form [14]

$$1 - v^2 - (v_0/v - 1)M \left(1 + M \frac{\partial p}{\partial \varepsilon} \bigg|_n \right) < 0,$$

(2)
Figure 1. Pressure versus baryon number density and temperature $p(n, T)$ for the MIT-bag equation of state [13] at $B^{1/4} = 153.8 \text{MeV}$.

where $v_0$, $v$ are the pre- and post-shock velocity in the coordinate system attached to the shock front, $M = |v|/c_s$ is the speed of sound given by

$$c_s^2 = \frac{\partial p}{\partial \varepsilon}_{s/n} = \frac{\partial p}{\partial n}_{n} + \frac{n}{p + \varepsilon} \left[ \frac{\partial p}{\partial \varepsilon}_{\varepsilon} \right]_n.$$

This condition admits an alternative formulation in terms of thermodynamical functions of the post-shock state and the shock wave intensity $p/p_0$:

$$\frac{n}{p} \left[ \frac{\partial p}{\partial n}_{\varepsilon} \right]_n + \frac{\varepsilon}{p} \left[ \frac{\partial p}{\partial \varepsilon}_{\varepsilon} \right]_n - 1 + \frac{p_0}{p} \left( 1 + \left[ \frac{\partial p}{\partial \varepsilon}_{\varepsilon} \right]_n \right) < 0.$$  (4)

The check of the fulfillment of this condition for the equation of state [13] has shown that it can not be satisfied for the shock waves with the post-shock state neither in pure hadronic phase nor in the quark-gluon plasma. This can be explained by the fact that hadronic phase is considered as an ideal gas according to [13]; on the other hand, the relation between the pressure and the energy density in QGP, which is given by $p = \frac{4}{3}(\varepsilon - 4B)$, ensures positivity of the left-hand side of the inequality for an arbitrary initial pressure $p_0$. For shock waves with the post-shock state in the two-phase region the term in brackets is found to be positive

$$1 + \left[ \frac{\partial p}{\partial \varepsilon}_{\varepsilon} \right]_n > 0.$$  (5)

At the same time, the rest part of the expression in the left-hand side is negative

$$\frac{n}{p} \left[ \frac{\partial p}{\partial n}_{\varepsilon} \right]_n + \frac{\varepsilon}{p} \left[ \frac{\partial p}{\partial \varepsilon}_{\varepsilon} \right]_n - 1 < 0.$$  (6)

Thus necessary condition for the neutral stability can be fulfilled for such shock waves. However, as is known, these shock waves can be unstable with respect to splitting. The splitting can be one-dimensional or two-dimensional. In the last case the aforementioned triple configurations with
outgoing shock waves are formed. The effect of the two-dimensional splitting is of importance because it leads to the formation of the transverse momentum. Let consider an evolution of such a shock wave in the presence of initial perturbation introduced by the displacement of the shock front position (coordinate). The initial pre- and post-shock states correspond to unperturbed shock wave. The periodicity conditions are defined at $y = \text{const}$ boundaries.

The calculations have been performed on the basis of relativistic hydrodynamics equations, which describe conservation of the baryon number

$$\nabla_{\beta} (n u^\beta) = 0,$$

and conservation of energy and momentum

$$\nabla_{\beta} T^{\alpha\beta} = 0,$$
Figure 4. Two-dimensional expansion of the lens-shaped fireball: \( \ln(p) \) is shown, where \( p \) is the pressure measured in MeV\(^4\).

where energy-momentum tensor has the form

\[
T^{\alpha\beta} = (\varepsilon + p)u^\alpha u^\beta - g^{\alpha\beta}p.
\]  

(9)
Here, \( n \) is the baryon density measured in the rest coordinate system, \( \varepsilon \) is the energy density, \( p \) is the pressure, \( u \) is the 4-velocity, \( g^{\alpha\beta} \) is the metric tensor \([15]\). The system of the relativistic hydrodynamics equations is closed by the MIT-bag equation of state \([13]\).

The shock wave splitting is illustrated in figure 3, where the pressure distributions presented at different points in time. The calculations have shown that at the initial stage the splitting has essentially two-dimensional character, triple configurations with outgoing shocks are really formed, but transverse waves are quickly damped and two-wave structure becomes practically one-dimensional.

3. Composite rarefaction waves
The example of isentropes passing through the region of the quark-hadron phase transition is shown in figure 2. The adiabats have two inflection points at the boundaries of the two-phase region. Since the adiabats are not convex in \((p, \tau)\)-plane, where \( \tau \) is the generalized volume \((\tau = h/n, h \) is the enthalpy), the composite rarefaction wave consisting of isentropic rarefaction waves, rarefaction shock and region of constant solution is formed instead isentropic rarefaction wave in the Riemann problem.

This type of solution is analogous to those in the case of non-relativistic hydrodynamics with the phase transition of the first order \([7]\).

Consider two-dimensional expansion of QGP initially concluded in the lens-shaped region, which is intersection of two circular disks. The distance between the centers of the discs is \( b = 1.5R \), where \( R \) is the radius. The initial pressure distribution is presented in the first panel in figure (4). The composite rarefaction wave is formed after the break-up of the initial discontinuity. The composite wave includes the isentropic rarefaction wave in the quark-gluon phase (the pressure interval, which corresponds to this wave is denoted by \( \delta_1 \)), the rarefaction shock, which is the wave of phase transition (\( \delta_2 \)) and isentropic rarefaction wave in hadronic matter (\( \delta_3 \)). The converging rarefaction wave in quark-gluon plasma reflects from the plane of symmetry with formation of rarefaction wave, which includes rarefaction shock. The difference between the case under consideration and the one-dimensional solution is presence of the reflection point in which the secondary rarefaction shocks meet each other. The calculations have shown formation of plateau-like region of relatively small solution gradients, which is seen in pressure distribution (figure 4, where the pressure distribution in the flow field is shown). This region corresponds to the mixture of the hadronic and the QGP phases, being in the phase equilibrium. The temperature variation in this region is relatively small because of high specific heat of the mixed phase.

The interaction of the converging and reflected rarefaction shocks determines the life time of the plateau. In accordance with our calculations its value in the laboratory coordinate system is approximately \( 1.1R/c \approx 10 \text{ fm/c} \). On the other hand, the time of termalisation of quarks and gluons have been estimated as \( \tau_{eq} < 1 \text{ fm/c} \) (see \([1]\)). This means that the anomalous wave phenomena in the presence of the quark-hadron phase transition can affect theoretically the observables connected with the collective phenomena in the expanding fireball. Since the formation of the plateau-like regions in the solution has clear physical ground, one can expect analogous behavior in the case when the initial distribution of thermodynamic parameters is smooth and the velocity field is not zero.

4. Conclusion
Wave phenomena occurring at direct and reverse hadron-QGP phase transition have been studied numerically. The behavior of the preliminary perturbed shock wave having an ambiguous representation has been simulated. It is shown that at the initial stage it has essentially two-dimensional character. The triple configurations with outgoing shocks are formed on the
shock front, but transverse waves are quickly damped and two-wave structure becomes one-dimensional. It is also shown that the two-dimensional fireball expansion is characterized by the composite rarefaction wave arising, which includes isentropic rarefaction wave in the quark-gluon phase, the rarefaction shock (wave of phase transition) and isentropic rarefaction wave in hadronic matter. The plateau-like region of relatively small pressure and temperature gradients has been detected, which corresponds to the two-phase states. The lifetime of the plateau in the laboratory coordinate system is approximately $1.1R/c$ that makes this phenomenon theoretically observable.

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