Abstract
Explicit expressions for the parity-violating s-wave and the parity-conserving p-wave contributions to \( \pi^- \) weak-decay rates of \( \Lambda \) hypernuclei in the \( 1p \) shell are given in the weak-coupling limit, to update previous shell-model calculations and to compare with recent \( \pi^- \) spectra and total decay rates measured by the FINUDA Collaboration for \( ^{\Lambda}Li, ^{\Lambda}Be, ^{11}_{\Lambda}B \) and \( ^{15}_{\Lambda}N \). A useful sum rule for the summed strength of \( \Lambda_{1s} \to p_{1p} \) hypernuclear \( \pi^- \) weak decays is derived. Fair agreement between experiment and calculations is reached, using the primary s-wave amplitude and Cohen-Kurath nuclear wavefunctions. The role of the p-wave amplitude is studied in detail for \( ^{15}_{\Lambda}N \) and found to be secondary. Previous assignments of ground-state spin-parity values \( J^{\pi}(^{\Lambda}Li_{g.s.}) = \frac{1}{2}^+ \) and \( J^{\pi}(^{11}_{\Lambda}B_{g.s.}) = \frac{5}{2}^+ \) are confirmed, and a new assignment \( J^{\pi}(^{15}_{\Lambda}N_{g.s.}) = \frac{3}{2}^+ \) is made, based on the substantial suppression calculated here for the \( ^{15}_{\Lambda}N(\frac{3}{2}^+) \to \pi^-^{15}O_{g.s.} \) weak decay rate.

Key words: mesonic weak decays of hypernuclei, ground-state spins of hypernuclei
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1. Introduction
Mesonic weak decays of \( \Lambda \) hypernuclei studied in stopped-\( K^- \) reactions on nuclear emulsions were used by Dalitz to determine ground-state spins of the s-shell species \( ^3_{\Lambda}H \) and \( ^4_{\Lambda}H \) as soon as parity violation had been established in the weak interactions; for overview see Refs. [1,2]. The essence of these early calculations was the strong dependence that two-body \( \pi^- \) decay branching ratios exhibit often on the ground-state (g.s.) spin of the decaying hypernucleus. Later, by studying \( \pi^- \) angular distributions or \( \alpha \alpha \) final-state correlations, the ground-state spins of the \( p \)-shell hypernuclei \( ^{8}_{\Lambda}Li \) and \( ^{12}_{\Lambda}B \) were determined, again from emulsion data [2]. The advent of counter experiments using the \( (K^-, \pi^-) \) and \( (\pi^+, K^+) \) reactions allows a systematic study of weak decays for other

Email address: avragal@vms.huji.ac.il (Avraham Gal).

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species not readily or uniquely accessible in emulsion work. Thus, the $\pi^-$ weak decay rates measured for the $p$-shell hypernuclei $^{11}_\Lambda B$ and $^{12}_\Lambda C$, both produced on $^{12}C$ [3], suggest on comparing to the calculations by Motoba et al. [4,5,6] that $J^\pi(^{11}_\Lambda B_{g.s.}) = \frac{5}{2}^+$ and $J^\pi(^{12}_\Lambda C_{g.s.}) = 1^-$. Recently, the spin-parity value $J^\pi(^7_\Lambda Li_{g.s.}) = \frac{1}{2}^+$ was determined at KEK in a $^7Li(p^+,K^+\gamma)$ experiment [7], comparing the yield of $\gamma$ rays subsequent to the weak decay $^7_\Lambda Li_{g.s.} \rightarrow \pi^- \cdot ^7Be^*(429 \text{ keV})$ again with the calculations of Motoba et al. 

Ground-state spin values of light $\Lambda$ hypernuclei provide valuable information on the spin dependence of the $\Lambda N$ interaction and on $\Lambda N - \Sigma N$ coupling effects in hypernuclei [8]. For a recent review and compilation of hypernuclear data, see Ref. [9].

Very recently, at PANIC08, $\pi^-$ spectra of weakly decaying $\Lambda$ hypernuclei have been presented by the FINUDA Collaboration for $^7_\Lambda Li$, $^9_\Lambda Be$, $^{11}_\Lambda B$, $^{15}_\Lambda N$ [10]. Whereas individual final nuclear states cannot be resolved in these decay spectra, the $^{11}_\Lambda B$ spectrum shows evidence for two groups of states separated by about 6 MeV in the residual $^{11}C$ nucleus, and the $^{15}_\Lambda N$ spectrum is clearly dominated by $^{15}O_{g.s.}$. Besides these features that concern the shape of the $\pi^-$ spectrum, the most meaningful entity to compare at present between experiment and theory is the total rate $\Gamma_{\pi^-}(^{4}_A Z)$ for $\pi^-$ decay from the ground state of $^{4}_A Z$. In hypernuclei, owing to Pauli blocking for the low-momentum recoil proton (with $q \sim q_0 = 101 \text{ MeV}/c$, where the momentum $q_0$ holds for $\Lambda \rightarrow \pi^- p$), the $\pi^-$ decay rate $\Gamma_{\pi^-}(^{4}_A Z)$ drops steadily with $A$ away from the corresponding free-space decay rate $\Gamma_{\pi^-}^{\text{free}}$. The FINUDA data indicate a fall-off of $\Gamma_{\pi^-}(^{4}_A Z)/\Gamma_\Lambda$ by a factor of three throughout the $1p$ shell, from about 0.35 for $^7_\Lambda Li$ down to about 0.11 for $^{15}_\Lambda N$. The data bear statistical uncertainties between approximately 15% and 30% and, except for $^{11}_\Lambda B$, considerably smaller systematic uncertainties.

Pionic decays of light hypernuclei have been studied particularly for $^{4}_\Lambda He$ and $^{12}_\Lambda C$, focusing on the renormalization of the $\Lambda \rightarrow N\pi$ decay vertex in the nuclear medium; for comprehensive reviews, see Refs. [12][13]. The present work, however, is related closely to spectroscopic aspects of pionic decays as studied systematically by Motoba et al., with application in Ref. [4] particularly to decays of $p$-shell hypernuclei, and with update and extension in Refs. [5][6]. In these calculations, the pion-final-state interaction was incorporated by using pion-nuclear distorted waves, and the structure of the nuclear core was treated by using the Cohen-Kurath (CK) spectroscopic calculations [14][15]. For $p$-shell hypernuclei, it was found that the total $\pi^-$ decay rate is dominated by transitions $\Lambda_{1s} \rightarrow p_{1p}$, with little strength left for transitions to higher nuclear configurations dominated by $\Lambda_{1s} \rightarrow p_{2s,1d}$ transitions. Here we follow this approach, providing explicit expressions for $\Lambda_{1s} \rightarrow p_{1p}$ transitions from $p$-shell hypernuclear ground states in the weak-coupling limit to final nuclear states within the $1p$ shell. The nuclear states are given by the CK wavefunctions, specifically in terms of the published CK coefficients of fractional parentage [15]. We discuss the choice of nuclear form factors involved in these transitions, with a parametrization that fully accounts for the distortion of the outgoing pion as calculated by Motoba et al. [4][5][6]. Furthermore, we derive and demonstrate the use of a new sum rule which encapsulates the suppressive effect of the Pauli principle on the total $\pi^-$ weak decay rate. Our primary aim is to allow any concerned experimenter to check on his/her own the calculational state of the art in $\pi^-$ weak decays of light

\[ \Gamma_{\pi^-}^{\text{free}}/\Gamma_\Lambda = 0.639 \] in terms of the $\Lambda$ decay width $\Gamma_\Lambda = \hbar/\tau_\Lambda$, where $\tau_\Lambda = (2.631 \pm 0.020) \times 10^{-10} \text{ s}$ for the $\Lambda$ mean life [11].
In the calculations reported in the present work, we have checked that for the main \( \Lambda_{1s} \rightarrow p_{1p} \) transitions, as well as for the summed strength, it suffices to consider the leading s-wave amplitude. The p-wave amplitude produces an observable effect only for decays that are suppressed within the s-wave approximation, as is demonstrated here in detail for the decay of \( ^{15}\Lambda N(\frac{1}{2}^+) \). The neglect of the p-wave weak decay amplitude incurs errors of less than 10% when the main transitions are summed upon, consistently with the stated precision of our calculations. A comparison is then made between the calculated \( \pi^- \) decay rates and the rates measured by FINUDA, based on which the hypernuclear g.s. spin-parity assignments \( J^{\pi}(^{7}\Lambda Li_{g.s.}) = \frac{1}{2}^+ \) and \( J^{\pi}(^{11}\Lambda B_{g.s.}) = \frac{5}{2}^+ \) are confirmed, and a new assignment \( J^{\pi}(^{15}\Lambda N_{g.s.}) = \frac{3}{2}^+ \) is established.

2. Methodology and Input

2.1. \( \Lambda \rightarrow N \pi \) decay input

The free \( \Lambda \) weak decay rate is dominated to 99.6% by the nonleptonic, mesonic decays \( \Lambda \rightarrow \pi^- p, \pi^0 n \):

\[
\Gamma_{\Lambda} \approx \Gamma_{\pi^-}^{\text{free}} + \Gamma_{\pi^0}^{\text{free}} \quad (\Gamma_{\pi^-}^{\text{free}} : \Gamma_{\pi^0}^{\text{free}} \approx 2 : 1),
\]

where each one of these partial rates consists of parity-violating s-wave and parity-conserving p-wave terms:

\[
\Gamma_{\pi^-}^{\text{free}} = c_\alpha \frac{q_0}{1 + \omega_\pi(q_0)/E_N(q_0)} (|s_\pi|^2 + |p_\pi|^2 q_0^2/s_\pi^2), \quad \frac{|p_\pi|}{s_\pi} \approx 0.132,
\]

with \( \omega_\pi(q_0) \) and \( E_N(q_0) \) the energies of the free-space decay pion and the recoil nucleon, respectively. The ratio 2 : 1 in Eq. (1), \( c_-/c_0 \approx 2 \) in terms of the strength parameters \( c_\alpha \) in Eq. (2), is a consequence of the \( \Delta I = 1/2 \) rule, satisfied empirically by the weak interactions.

2.2. \( \pi^- \) decay rates for \( \Lambda_{1s} \rightarrow p_{1p} \) transitions

We consider \( \pi^- \) decay from an initial hypernuclear state \( |i > \equiv |A^4 Z; \alpha_i, T_i \tau_i, J_i M_i > \) to a final nuclear state \( |f > \equiv |A^4(Z + 1); \alpha_f, T_f \tau_f, J_f M_f > \) in the nuclear 1p shell, where \( J \) (\( T \)) and \( M \) (\( \tau \)) stand for the total angular momentum (isospin) and their z projections, and \( \alpha \) stands for any other quantum numbers providing spectroscopic labels. The \( \pi^- \) decay rate for a \( \Lambda_{1s} \rightarrow p_{1p} \) transition induced by the s-wave amplitude is given by

\[
\Gamma^{(s)}_{\pi^-}(i \rightarrow f) = c_- \frac{q}{1 + \omega_\pi^{-}(q)/E^-_{A}(q)} |s_\pi|^2 P^{(s)}_{i \rightarrow f},
\]

where the effective proton number \( P \) for this transition is defined by

\[
P^{(s)}_{i \rightarrow f} = \int \frac{d\Omega_{\pi^-}}{4\pi} \frac{1}{(2J_i + 1)} \sum_{M_i, M_f} |< f | \int d^3r \chi_{\pi^-}^{-}(r) \sum_{k=1}^{A} \delta(r - r_k)V_{\kappa+} |i > |^2.
\]
In Eq. (4), $\chi_4^{(-)}$ is an incoming pion distorted wave (DW), $\exp(-i\mathbf{q} \cdot \mathbf{r})$ in the plane-wave (PW) limit, and the $V$-spin raising operator $V_+ \Lambda$ transforms $\Lambda$ to protons: $V_+ \Lambda = |p >$. The summation on $k$ extends over the nucleons of the nuclear decay product. Eq. (4) reduces to $\mathcal{P}_{i \rightarrow f}^{(s)} = S_{i \rightarrow f}^{(s)} |F_{DW}^{(s)}(q)|^2$, where the spectroscopic factor for the transition $\Lambda_1s(i) \rightarrow p_{1p}(f)$ associated with the $\pi^-$ decay $s$-wave amplitude is given by

$$S_{i \rightarrow f}^{(s)} = N_f^{(1p)} \langle T_i \tau_i, \frac{1}{2}; T_f \tau_f \rangle^2 (2J_f + 1) \left( \sum_j [\alpha_i, T_i, J_c; \frac{1}{2}; j] \alpha_f, T_f, J_f \right)$$

$$\times (-1)^{J_c + j + J_f} \sqrt{(2j + 1)} \left\{ \begin{array}{ccc} j & J_f & J_c \\ J_i & \frac{1}{2} & 1 \end{array} \right\}^2. \tag{5}$$

Here $J_c$ is the total angular momentum of the initial core nucleus which couples with the $s$-shell $\Lambda$ spin $1/2$ to yield $J_i$ and with the $p$-shell proton angular momentum $j = 1/2$, $3/2$ to yield $J_f$. In Eq. (5), $N_f^{(1p)}$, which is the number of $p$-shell nucleons in the final state is followed by a squared isospin Clebsch-Gordan (CG) coefficient accounting for $\Lambda \rightarrow p$ transitions; the first symbol in the sum over $j$ is a fractional-parentage coefficient (CFP), and the curly bracket stands for a 6j symbol. The phase factor $(-1)^{J_c + j + J_f}$ is consistent with the CK CFPs. Eq. (5) corrects the corresponding expression Eq. (8a) used in Ref. [16] for the special case of $^{11}$B. We note that it is straightforward to transform $S_{i \rightarrow f}^{(s)}$, Eq. (5), from a $jj$-coupling representation to $LS$, a representation that is quite useful in the beginning of the $1p$ shell.

The DW form factor $F_{DW}^{(s)}$ is defined by

$$F_{DW}^{(s)}(q) = \int_0^\infty u_P^{(s)}(r) j_i(qr) u_\Lambda^{(s)}(r)^2 dr, \tag{6}$$

where $u_P^{(s)}$ and $u_\Lambda^{(s)}$ are the radial wavefunctions of the $1p$ nucleon and the $1s \Lambda$, respectively, and the pion radial DW $j_i(qr)$ goes over to the spherical Bessel function $j_1(qr)$ in the plane-wave (PW) limit. Our definition of form factor, Eq. (6), is related to the squared form factor $\eta_\Lambda(1p_j; q)$ in Eq. (4.6) of Ref. [16] by $|F_{DW}^{(s)}(q)|^2 = \eta_\Lambda(1p_j; q)$, where the dependence on $j$, for a given $l = 1$, is here suppressed. Rather, a state dependence of the form factor arises from the implicit dependence of $F_{DW}^{(s)}(q)$ on the $\Lambda$ and proton binding energies which vary along the $p$ shell, and will be considered in the next subsection.

2.2.1. Digression to p-wave amplitudes

For completeness, as well as for use in the case of $^{15}_\Lambda$N below, we record relevant expressions for p-wave amplitudes in analogy with the $s$-wave expressions listed above. Thus, in addition to $\Gamma_{\pi}^{(s)} (i \rightarrow f)$ of Eq. (3) we have

$$\Gamma_{\pi}^{(p)} (i \rightarrow f) = c \frac{q}{1 + \omega_{\pi^-}(q)/E_A(q)} |p_x|^2 \frac{q^2}{q_0^2} \mathcal{P}_{i \rightarrow f}^{(p)}, \tag{7}$$

where $\mathcal{P}_{i \rightarrow f}^{(p)} = S_{i \rightarrow f}^{(p)} |F_{DW}^{(p)}(q)|^2$, and the spectroscopic factor for the transition $i \rightarrow f$ associated with the $\pi^-$ decay $p$-wave amplitude is given by
\[ S_{i\rightarrow f}^{(p)} = N_f^{(1p)} \langle T_i \tau_i, \frac{1}{2} T_f \tau_f \rangle^2 (\delta_{J_i J_f} [\alpha_i, T_i, J_c; \frac{1}{2}, j = \frac{1}{2}] \alpha_f, T_f, J_f)^2 \]

\[ + 4 (2J_f + 1) \left\{ \frac{2}{3} \frac{1}{2} \frac{3}{2} \frac{1}{2} \right\}^2 \langle \alpha_i, T_i, J_c; \frac{1}{2}, j = \frac{3}{2} \rangle \alpha_f, T_f, J_f \rangle^2. \]  

(8)

We note the approximation adopted here of using a single \( p \)-wave form factor \( F_{\text{DW}}^{(q)}(q) \), following Ref. [4] which in the relevant range of \( q \) values, furthermore, suggests that \( |F_{\text{DW}}^{(q)}(q)|^2 \approx |F_{\text{PW}}^{(q)}(q)|^2 \) to better than 10%.

2.3. \( \Lambda_{1s} \rightarrow p_{1p} \) transition form factor

The DW transition form factor [6] is evaluated as follows. First, we compute the PW form factor using harmonic-oscillator shell-model wavefunctions, with appropriate parameters \( \nu = 0.41 \text{ fm}^{-2} \) for a \( p \)-shell nucleon and \( \lambda = 0.33 \text{ fm}^{-2} \) for a \( s \)-shell \( \Lambda \) hyperon [17]:

\[ |F_{\text{PW}}^{(s)}(q)|^2 = \frac{q^2}{6 \lambda} \left( \frac{2 \sqrt{\nu \lambda}}{\nu + \lambda} \right)^5 \exp \left( -\frac{q^2}{\nu + \lambda} \right), \]  

(9)

yielding \( |F_{\text{PW}}^{(s)}(q_0)|^2 = 0.0898 \) for the free-space \( \Lambda \rightarrow \pi^- p \) decay center-of-mass momentum \( q = q_0 = 100.7 \text{ MeV/c} \). A more refined evaluation using a two-term Gaussian \( \Lambda \) wavefunction fitted in Ref. [18] to \( \Lambda \) binding energies in the \( 1p \) shell, from \( ^7\text{Li} \) to \( ^{13}\text{C} \), suggests that \( |F_{\text{PW}}^{(s)}(q_0)|^2 \approx 0.085 \) for more realistic wavefunctions as estimated from Fig. 2 of Ref. [4]. We then estimate the ratio \( |F_{\text{DW}}^{(s)}(q)|^2/|F_{\text{PW}}^{(s)}(q)|^2 \) by comparing PW rates from Table 6, Ref. [4], with the corresponding DW rates from Table 1, Ref. [6] (FULL V.R. entry), providing an average value 1.47 ± 0.16 for this ratio. Since both \( |F_{\text{PW}}^{(s)}(q)|^2 \) and \( |F_{\text{DW}}^{(s)}(q)|^2 \) rise approximately linearly with \( q \) around \( q_0 \), with similar slopes, the deviations from the average ratio reflect primarily the variation of \( \Lambda \) and proton wavefunctions across the \( 1p \) shell, from \( ^7\text{Li} \) to \( ^{13}\text{C} \). This uncertainty, of order 10%, is likely to be the largest one in the present calculation of \( \pi^- \) decay rates. In the present update we employ the following average value for \( |F_{\text{DW}}^{(s)}(q)|^2 \):

\[ |F_{\text{DW}}^{(s)}(q)|^2 \approx 1.47 \times |F_{\text{PW}}^{(s)}(q)|^2 \approx 0.129 \times \left( 1 + 1.296 \times \frac{q - q_0}{q_0} \right), \]  

(10)

where the linear \( q \) dependence follows by expanding Eq. [3] for \( |F_{\text{PW}}^{(s)}(q)|^2 \) about \( q_0 \).

2.4. Sum rules for \( \Lambda_{1s} \rightarrow p_{1p} \) transitions

Defining \( s \)-wave and \( p \)-wave \( (2J_i + 1) \)-averaged spectroscopic strengths and \( (2J_i + 1) \)-averaged effective proton numbers in the nuclear core of the decaying hypernuclear ground state,
it becomes possible to derive a sum rule involving all final states within the 1p shell. The \((2J_i+1)\)-average in Eq. (11) is over the two initial \(J_i = J_c \pm 1/2\) values for a given \(J_c\) (except for \(J_c = 0\) which implies a unique \(J_i = 1/2\)). For \(T_i = T_c = 0\) which holds for the present application, \(T_f = 1/2\) and the CG coefficient in Eqs. (5) and (8) assumes the value 1. We demonstrate the sum-rule derivation for transitions associated with the \(\pi^-\) decay s-wave amplitude. Noting that by orthogonality,

\[
\sum_{J_i}(2J_i + 1) \left\{ \begin{array}{ccc} j & J_f & J_c \\ J_i & 1/2 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} j' & J_f & J_c \\ J_i & 1/2 & 1 \end{array} \right\} = \frac{1}{(2j + 1)} \delta_{jj'},
\]

a \((2J_i+1)\)-average application to the right-hand side of Eq. (5) yields:

\[
S^{(s)}_{i \rightarrow f} = \frac{1}{4} \sum_{J_f} (2J_f + 1)(2T_f + 1) \frac{N_f^{(1p)}[\alpha_f, T_f; 1/2, J_f]}{(2J_c + 1)(2T_c + 1)} N_f^{(1p)}[\alpha_i, T_i; 1/2, J_i] \alpha_f, T_f, J_f \right| \right|^2, \tag{13}
\]

where we used implicitly the isospin values \(T_i = T_c = 0\), \(T_f = 1/2\). We then sum over \(\alpha_f, T_f, J_f\) using a well known stripping sum rule (Ref. [15], Eq. (6)):

\[
\sum_{\alpha_f, T_f, J_f} \frac{(2J_f + 1)(2T_f + 1)}{(2J_c + 1)(2T_c + 1)} N_f^{(1p)}[\alpha_i, T_i; 1/2, J_i] \alpha_f, T_f, J_f \right| \right|^2
\]

\[
= 2(2j + 1) - N_c^{(1p)}, \tag{14}
\]

where \(N_c^{(1p)}\) is the number of 1p nucleons in the initial (nuclear core) state. Summing over \(j = l \pm 1/2\) as indicated on the right-hand side of Eq. (13), we obtain the sum rules:

\[
\sum_{\alpha_f, T_f, J_f} S_{i \rightarrow f}^{(s)} = 3(1 - \frac{1}{12} N_c^{(1p)}), \quad \sum_{\alpha_f, T_f, J_f} \mathcal{P}_{i \rightarrow f}^{(s)} \approx \left(1 - \frac{1}{12} N_c^{(1p)}\right) 3|F_{DW}^{(s)}(\bar{q})|^2, \tag{15}
\]

where \(N_c^{(1p)} = \sum_j N_c^{(1p)}\) is the number of p-shell nucleons in the nuclear core state, and \(\bar{q}\) is an appropriate value of the decaying pion momentum conforming with the closure approximation implied in deriving the sum rule for \(\mathcal{P}\). The right-hand sides in Eqs. (15) display the suppressive effect of the Pauli principle through the factor \((1 - N_c^{(1p)}/12)\) which decreases from 1 (no suppression) for the decay of \(^5\Lambda\) He into the fully available 1p proton shell \((N_c^{(1p)} = 0)\) to 0 (full suppression) for the decay of \(^{16}\Lambda\) O into the fully occupied 1p proton shell. And finally, the factor 3 in front of \(|F_{DW}^{(s)}(\bar{q})|^2\) stands for the \(2l+1 = 3\) m-states available to the final p-shell nucleon.

By following similar procedures, it can be shown that precisely the same structure of sum rule, Eq. (15), holds also for transitions associated with the \(\pi^-\) decay p-wave amplitude:

\[
\sum_{\alpha_f, T_f, J_f} \mathcal{P}_{i \rightarrow f}^{(p)} \approx \left(1 - \frac{1}{12} N_c^{(1p)}\right) 3|F_{DW}^{(p)}(\bar{q})|^2. \tag{16}
\]
2.5. Summary of formulae

For comparison with measured $\pi^-$ decay rates, our s-wave results are summarized below, assuming that the effect of the neglected p-wave largely cancels out, particularly for summed rates. Thus, the decay rate for a specific transition $i \rightarrow f$ is given by

$$\Gamma_{\pi^-}^{i \rightarrow f} \approx 0.639 \frac{q}{q_0} \frac{1 + \omega_{\pi^-}(q_0)/E_A(q_0)}{1 + \omega_{\pi^-}(q)/E_A(q)} \mathcal{P}^{(s)}_{i \rightarrow f} \approx 0.743 \frac{q}{q_0} \mathcal{P}^{(s)}_{i \rightarrow f},$$

(17)

where the kinematic recoil factor $(1 + \omega_{\pi^-}(q)/E_A(q))$ was approximated by its value for $q = 0$. The summed rate, from an appropriately averaged hypernuclear ground-state doublet to all final states in the 1p shell through the $\pi^-$ decay s-wave amplitude, is given by

$$\frac{\sum_i \Gamma_{\pi^-}^{i \rightarrow f}}{\Gamma_A} \approx 0.743 \frac{q}{q_0} (1 - \frac{1}{12} N_c^{(1p)}) 3|F^{(s)}_{DW}(q)|^2,$$

(18)

where $|F^{(s)}_{DW}(q)|^2$ is given by Eq. (10). We note that for hypernuclei based on $J_c = 0$ nuclear cores, such as $^3\text{He}$, $^6\text{Be}$ and $^{13}\text{C}$, Eq. (15) gives the summed decay rate for the uniquely assigned $J = 1/2$ hypernuclear ground state.

As a brief aside we demonstrate the application of Eq. (15) to $^4\text{He}$, the $\pi^-$ weak-decay rate of which was also determined by FINUDA [10]. Using the form-factor expression (10) with $q = 99.9 \text{ MeV}/c$, Eq. (15) gives a 1p contribution of 0.282 to the total decay rate, to which we add a $(2s-1d)$ contribution of 0.023 estimated from the calculation in Ref. [6] for $^7\text{Li}$. Altogether, our estimated $\pi^-$ decay rate is $\Gamma^{\text{tot}}_{\pi^-}(^4\text{He}) = 0.305$, compared to $0.306 \pm 0.060^{+0.025}_{-0.026}$ [10] and to the KEK recent value $0.340 \pm 0.016$ [21].

3. Results and Discussion

Using expressions (17) and (18), our final results are listed in Table 1, where we grouped the calculated $\pi^-$ decay rates into two partial sums: to the ground state and to nearby levels which share its underlying symmetry in the final nucleus, $\Gamma_{\pi^-}(^4\Lambda Z \rightarrow \Xi^-n)/\Gamma_A$, and to excited levels that provide substantial decay width, $\Gamma_{\pi^-}(^4\Lambda Z \rightarrow \text{exc.})/\Gamma_A$. For the total rate $\Gamma^{\text{tot}}_{\pi^-}(^4\Lambda Z)/\Gamma_A$ we added where necessary the residual $\Lambda_{1s} \rightarrow p_{1p}$ contributions, as well as the small $(2s,1d)$ contribution of order 0.023 ± 0.006 from Ref. [6]. The choice of $\Lambda$ hypernuclear g.s. spins is discussed below for each hypernucleus separately. Unless stated differently, the nuclear wavefunctions are due to CK [15] and the $\Lambda$ hypernuclear wavefunctions are considered in the weak coupling limit which provides a very reasonable approximation for summed rates, with estimated uncertainty of few percent. An educated guess for the overall theoretical uncertainty of our results is 10 – 15%.

3.1. $^7\Lambda\text{Li}$

Assuming $J^\pi(\Lambda\text{Li}_{s,s}) = \frac{1}{2}^+ \ [22]$ we find that the $^7\Lambda\text{Li} \rightarrow \pi^- \ ^7\text{Be}$ weak decay is dominated by transitions to the lowest levels $^7\text{Be}_{s,s}(\frac{3}{2}^-)$ and $^7\text{Be}(\frac{1}{2}^-; 0.43 \text{ MeV})$, in agreement with Ref. [4], with a summed strength $S^{(s)}(\Lambda\text{Li} \frac{1}{2}^+ \rightarrow \pi^- \ ^7\text{Be}(\frac{3}{2}^-; 0 \ & \ 0.43 \text{ MeV})) = 2.32$. In
the LS coupling limit, which provides a good approximation in the beginning of the 1p shell, this summed strength equals $S^{(s)}(2S_\frac{1}{2} \rightarrow \frac{1}{2} P_J) = 5/2$ and is distributed according to $(2J_f + 1)$ whereas for $J^\pi(^7\Lambda Li_{g.s.}) = \frac{3}{2}^+$, owing to the spin-nonflip nature of the dominant s-wave $\pi$ decay, the LS limit gives $S^{(s)}(4S_\frac{3}{2} \rightarrow 2P_J) = 0$. The prominence of the $^7$Be(0 & 0.43 MeV) final states in the $^7\Lambda Li \rightarrow \pi^- ^7$Be weak decay FINUDA spectrum in Ref. [10] clearly rules out assigning $J^\pi(^7\Lambda Li_{g.s.}) = \frac{3}{2}^+$. Additional strength for $J^\pi(^7\Lambda Li_{g.s.}) = \frac{1}{2}^+$, with $S^{(s)} = 0.98$ using CK wavefunctions, goes to $^7$Be levels around 11 MeV ($S^{(s)} = 1$ in the LS limit). Altogether, the total strength to $p$-shell nuclear states is $S^{(s)}_{tot} = 3.29$ ($7/2$ in the LS limit). The calculated decay rates listed in the table agree very well with the shape and the strength of the measured FINUDA spectrum, confirming the spin assignment $J^\pi(^7\Lambda Li_{g.s.}) = \frac{1}{2}^+$ [7]. Our calculated total rate is 17% higher than previously calculated [6].

### 3.2. $^9\Lambda Be$

For this hypernucleus, with nuclear core spin $J_\pi^c = 0^+$, we use the sum-rule expression [15] to obtain a transition strength $S^{(s)}_{tot} = 2$. The leading transitions are to $^9B_{g.s.}(\frac{3}{2}^-)$ and to the $\frac{7}{2}^-$ excited state at 2.75 MeV, both of which saturate the sum-rule strength in the LS limit. Our calculated total rate agrees well with the FINUDA measurement [10] within the experimental uncertainty, and is 8% higher than in previous calculations [6].

### 3.3. $^{11}\Lambda B$

Assuming $J^\pi(^{11}\Lambda B_{g.s.}) = \frac{5}{2}^+$ [16], the calculated summed strength is $S^{(s)}_{tot} = 2.04$, with the largest contributions due to $^{11}C_{g.s.}(\frac{7}{2}^-)$ ($S^{(s)} = 0.73$) and to the $\frac{7}{2}^-$ excited state at
6.48 MeV \((S^{(s)} = 0.94)\). The calculated total decay rate given in the table is lower by 8% than that calculated previously \([6]\), and by just 1\(\sigma_{\text{stat}}\) than the measured mean value. Given an estimated 10% error on our calculated rate, there is certainly no disagreement between the present calculation and experiment.

If the spin of \(^{11}\Lambda B\) were \(\frac{3}{2}^+\), there would have been no \(s\)-wave transition to \(^{11}\text{C}_{\text{g.s.}}\), the transition to the \(\frac{5}{2}^−\) excited state at 6.48 MeV would have been \(5–6\) times weaker than it is supposed to be for \(J^\pi(\Lambda_B^{11}) = \frac{3}{2}^+\), and the dominant transition would have been to the \(\frac{5}{2}^−\) excited state at 8.42 MeV. A comparison with the shape of the measured spectrum rules out this possibility. Furthermore, assigning to all the \(\Lambda\) value \(q\) the \(S\)-confirm the conclusion of Ref. \([4]\) that the BNLE930 experiment \([26]\). Our calculated \(\pi^−\) transition to the \(\frac{5}{2}^+\) as fixed value \(q_{\text{exc.}} = 91.6\) MeV/c appropriate to this dominant transition to the \(\frac{5}{2}^−\) level, and adding the \(\Lambda_{1s} \to p_{2s,1d}\) strength, one obtains \(\Delta_{\text{tot}}(11\Lambda_B^{11}) = \frac{3}{2}^+\) made \([16]\) on the basis of \(\pi^−\) decays observed in emulsion, not all of which uniquely associated with \(^{11}\Lambda B\) \([22]\). Although this assignment is in good accord with previously reported total \(\pi^−\) decay rates \([3]\) and with several theoretical interpretations of other data \((e.g.\) Refs. \([23,24,25]\)), the present FINUDA measurement of the complete decay spectrum provides the best experimental evidence for \(J^\pi(11\Lambda_B^{11}) = \frac{3}{2}^+\).

### 3.4. \(^{15}\Lambda N\)

The ground-state spin of \(^{15}\Lambda N\) has not been determined experimentally. The first realistic discussion of spin dependence in \(p\)-shell \(\Lambda\) hypernuclei placed the \(\frac{1}{2}^+\) g.s. doublet member barely above the \(\frac{1}{2}^+\) ground state, at 14 keV excitation, so both of these levels are practically degenerate and would decay weakly \([24]\). This near degeneracy reflects the competition between the \(\Lambda N\) spin-spin interaction (favoring \(J^\pi = \frac{3}{2}^+\)) and the \(\Lambda N\) tensor interaction (favoring \(J^\pi = \frac{1}{2}^+\)) in the \(p_+\) subshell. The most recent theoretical update \([25]\) places the \(\frac{3}{2}^+\) excited level about 90 keV above the \(\frac{1}{2}^+\) ground state. The spin ordering cannot be determined from the \(\gamma\)-ray spectrum of \(^{15}\Lambda N\) measured in the BNL-E930 experiment \([20]\). Our calculated \(\pi^−\) weak-decay rates to final states in \(^{15}\text{O}\) confirm the conclusion of Ref. \([4]\) that \(^{15}\text{O}_{\text{g.s.}}(\frac{3}{2}^+)\) nearly saturates the total \(\Lambda_{1s} \to p_{1p}\) spectroscopic strength. This near saturation follows from the calculated decay rates for \(^{15}\Lambda N \to \pi^− \text{O}_{\text{g.s.}}(\frac{3}{2}^+)\) listed in Table 2. Indeed, the saturation is complete in the \(j\) coupling limit, where \(S_{s(p)}^{(s(p))}(\frac{3}{2}^+ \to \text{O}_{\text{g.s.}}(\frac{1}{2}^+)) = 1/2\) for the \((2J_i + 1)\)-average of strengths \(S_{s(p)}\), agreeing with the sum-rule value \(\sum_{i \to f} S_{s(p)}(\frac{3}{2}^+) = 3(1 - \frac{1}{2\pi}N_{c(1p)}^2)\) given in Eqs. \([15]\) and \([16]\) for \(s\)-wave and \(p\)-wave amplitudes, respectively. The saturation holds approximately in the more realistic CK model, for which the strengths listed in the table give \(S_{\text{CK}}^{(s(p))}(\frac{3}{2}^+ \to \text{O}_{\text{g.s.}}(\frac{1}{2}^+)) = 0.486\) for both \(s\)-wave and \(p\)-wave amplitudes. Very little spectroscopic strength is therefore left to the \(p_+\) \(p_+\) excited state \(^{15}\text{O}(\frac{1}{2}^+, 18.6\) MeV) which may safely be neglected. We note that the \(s\)-wave strength for \(J^\pi(\frac{3}{2}^+)\) is four times weaker in the \(j\) coupling limit than for spin \(\frac{3}{2}^+\). Using realistic wavefunc-
Table 2

\( ^{15}_{\Lambda}N(J^{\pi} \rightarrow \pi^{-}^{15}O_{g.s.}(\frac{3}{2}^{-}) \) calculated s-wave and p-wave spectroscopic factors \( S \), and summed s-wave plus p-wave weak decay rates, compared with FINUDA's measured decay rates \(^{10}\) and with those calculated by Motoba et al. \(^{6}\) (T. Motoba, private communication.)

| \( ^{15}_{\Lambda}N \) | exp. \(^{10}\) calc. \(^{6}\) | jj | CK |
|-----------------|-----------------|-----|-----|
| \( J^{\pi} \) | \( \Gamma_{\Lambda}^{s} \) | \( \Gamma_{\Lambda}^{p} \) | \( S^{(s)} \) | \( S^{(p)} \) | \( S^{(s)} \) | \( S^{(p)} \) | \( F_{s} \) | \( F_{p} \) |
| \( \frac{1}{2}^{+} \) | 0.072 ± 0.024 | 0.065 | \( \frac{2}{3} \) | 0 | 0.053 | 0.053 | 1.426 | 0.019 |
| \( \frac{1}{2}^{+} \) | 0.072 ± 0.024 | 0.050 | \( \frac{2}{3} \) | 0 | 0.029 | 0.052 | 1.426 | 0.019 |

The suppression of \( ^{15}_{\Lambda}N(\frac{1}{2}^{+}) \rightarrow \pi^{-}^{15}O_{g.s.} \) with respect to \( ^{15}_{\Lambda}N(\frac{3}{2}^{+}) \rightarrow \pi^{-}^{15}O_{g.s.} \) was overlooked in Ref. \(^{4}\). The suppression of \( ^{15}_{\Lambda}N(\frac{3}{2}^{-}) \rightarrow \pi^{-}^{15}O_{g.s.} \) was also not considered explicitly in Ref. \(^{4}\). The measured decay rate for \( ^{15}_{\Lambda}N(\frac{3}{2}^{-}) \rightarrow \pi^{-}^{15}O_{g.s.} \) was 0.080 upon explicit consideration of p-waves, to 0.080 upon explicit inclusion of p-waves. This latter value is 11\% lower than the decay rate 0.090 calculated by Motoba et al. \(^{6}\) and is short of the FINUDA measured total rate by just 1\( \sigma_{\text{stat}} \). In contrast, the calculated decay rate for \( ^{15}_{\Lambda}N(\frac{3}{2}^{-}) \rightarrow \pi^{-}^{15}O_{g.s.} \) is 2\( \sigma_{\text{stat}} \) below the measured mean value. In this respect we disagree with the conclusion deduced from Ref. \(^{4}\) that the total \( \pi^{-} \) decay rate of \( ^{15}_{\Lambda}N \) depends weakly on the value of its ground-state spin. A similar disagreement holds with respect to the two values of calculated partial rates listed in Table 2 for \( ^{15}_{\Lambda}N(\frac{3}{2}^{+}) \rightarrow \pi^{-}^{15}O_{g.s.}(\frac{1}{2}^{-}) \), where our calculated value is by over 2\( \sigma \) lower than the measured partial rate. We consider our present calculation, in conjunction with the FINUDA measurement \(^{10}\), a determination of the ground-state spin of \( ^{15}_{\Lambda}N \), namely \( ^{15}_{\Lambda}N(\frac{3}{2}^{+}) \).
Table 3
Calculated \((2J_i + 1)\)-averaged \(\Lambda\) hypernuclear \(\pi^-\) weak decay rates divided by \((6 - \beta_i^{(1p)})/2\) and by the calculated \((2J_i + 1)\)-averaged \(\Lambda\) hypernuclear \(\pi^-\) weak decay rate, see text. Given in the column before last are the mean and standard deviation of the averaged rates for the four preceding hypernuclei.

| Ref. | \(^7\)Li | \(^9\)Be | \(^{11}\)B | \(^{13}\)C | mean \((\Lambda\) Li – \(\Lambda\) C) | \(^{15}\)N |
|------|---------|---------|---------|---------|----------------|--------|
| present | 0.635 0.577 0.748 0.633 | 0.648 ± 0.072 | 1 |
| 0.945 0.873 0.920 0.813 | 0.888 ± 0.058 | 1 |

4. Sum-rule applications

The sum rule Eq. (18) for the \(s\)-wave amplitude and a similar one for the \(p\)-wave amplitude involve a \((2J_i + 1)\)-average over the initial hypernuclear ground-state configuration, and as such cannot be applied to any of the total mesonic weak decay rates measured by FINUDA, which relate to specific g.s. values \(J_i\), except for the \(J_c = 0\) core \(^9\)Be hypernucleus. However, it may be applied to available theoretical evaluations in order to provide consistency checks. This assumes that the \(A\) dependence of the closure \(\bar{q}\) value is weak, typically 10% or less, and also that the DW integrals depend weakly on \(A\) apart from the dependence on \(\bar{q}\) in Eq. (10). Sum-rule values are shown in Table 3 for the present \(s\)-wave calculation as well as for the comprehensive calculations by Motoba et al. [6], where we used the \((1 - N_i^{(1p)}/12)\) \(A\)-dependence from Eq. (15) to normalize the calculated \((2J_i + 1)\)-averaged \(\Lambda\) hypernuclear \(\pi^-\) weak decay rates to that of \(^{15}\)N \((N_i^{(1p)} = 10)\), in units of the calculated \((2J_i + 1)\)-averaged \(\Lambda\) hypernuclear \(\pi^-\) weak decay rate. For completeness, we added \(^{13}\)C so as to present a full range of odd-\(A\) hypernuclei in the 1p shell.

Table 3 demonstrates that the sum-rule-normalized \(\Lambda\) hypernuclear \(\pi^-\) weak decay rates of \(^7\)Li, \(^9\)Be, \(^{11}\)B, and \(^{13}\)C are represented fairly well by a mean value, with only one of the species \(^{13}\)C in the present one) departing by somewhat over one standard deviation. Percentage-wise, it represents 8% in our calculation and 15% for Ref. [6]. However, the value 1 for \(^{15}\)N deviates from the corresponding mean by five standard deviations in the calculation of Ref. [6], whereas it deviates by ‘merely’ two standard deviations in our calculation. This provides a circumstantial argument in favor of the correctness of the present \(^{15}\)N calculation in which even the seemingly substantial 2\(\sigma\) deviation for \(^{15}\)N amounts only to 13% departure from the mean rate. In contrast, the 5\(\sigma\) deviation for \(^{15}\)N in the calculation of Ref. [6] amounts to a huge 54% departure from the mean which is hard to accept.

5. Summary

We have checked and updated some of the pioneering calculations of mesonic weak decay of hypernuclei by Motoba et al. [4,5,6]. Our added value is in providing an explicit spectroscopic expression Eq. (5) and a related sum rule Eq. (15) for \(\Lambda\) hypernuclear \(s\)-wave transitions. The neglect of \(p\)-wave transitions for summed spectra is justified to better than 10\%. By comparing the calculated rates with the FINUDA measured rates [10] we have pointed out that these recent data confirm the hypernuclear ground-state spin-parity assignments \(J^\pi(\Lambda\) \(g.s.) = \frac{1}{2}^+\) and \(J^\pi(\Lambda\) \(B\) \(g.s.) = \frac{5}{2}^+\), while providing a first experimental
Whereas good agreement between calculations and experiment for $^7\Lambda\text{Li}$ and $^9\Lambda\text{Be}$ has been demonstrated, the total $\pi^-$ decay rates measured by FINUDA for $^{11}\Lambda\text{B}$ and $^{15}\Lambda\text{N}$ are higher than our calculated total rates by about 1σ_{stat}, which might suggest the presence of other hypernuclear weak decays than the ones studied here. We note that in both of these cases the formation of $^A\Lambda Z$ proceeds by proton emission from $^{A+1}(Z+1)$ continuum where contamination by nearby thresholds ($n, \alpha, \cdots$) could introduce a bias. Such a possibility has been analyzed for $^{15}\Lambda\text{N}$ in detail by FINUDA \cite{10}, but without providing (yet) a quantitative measure of the additional hypernuclear components that are likely to contribute to the spectral shape of the measured weak decay. All in all, we have demonstrated a satisfactory level of agreement between calculations of mesonic weak-decay rates of $p$-shell $\Lambda$ hypernuclei and the recent FINUDA measurements.

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