Investigations on Neutronic Decoupling Phenomenon in Large Nuclear Reactors

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Abstract

Characteristic size (size of core expressed in terms of neutron migration length) of a nuclear reactor has been used as a simple thumb rule to assess the degree of neutronic coupling the reactor core. In the present paper, neutronic decoupling phenomena of reactor cores is studied using a sophisticated technique, called eigenvalue separation (EVS). A large eigenvalue separation reflects a distant higher harmonic and ensures neutronic coupling in respective neutron flux mode of the core. The EVS analysis technique has been used to optimize core geometry of pressurized water reactors, i.e., Height to Diameter (H/D) ratio of the reactor has been optimized to minimize the deep influence of higher harmonics on the stability. The study brings out that a typical H/D ratio beyond 1.3 should be avoided in large sized cores to maximize core neutronic coupling and hence the stability of the reactor.

Keywords: Neutronic decoupling, eigen value separation, PWRs

1. Introduction

Large sized nuclear reactors provide the advantage of achieving economy of scale. However, these reactors could suffer from spatial instability problem due to their large size and require a dedicated spatial control system. A few early studies [1,2] attracted the researchers to investigate the basic cause of this spatial instability. Neutronic decoupling among various zones of the large core was found to be the fundamental cause of the spatial instability [3]. Commonly used methodology to understand the neutronic decoupling phenomena is by expressing the size of the reactor in terms of its characteristic size, i.e., size expressed in terms of its migration length [4]. Beyond certain threshold value of characteristic size, the core tends to become neutronically decoupled. A comparison of characteristic size of different types of reactors is shown in Table 1.
Table 1: Characteristic Sizes of Power Reactors

| Reactor Type | Core Diameter (cm) | Core Height (cm) | Migration Length, M (cm) | Characteristic Diameter, (M) | Characteristic Height, (M) |
|--------------|-------------------|-----------------|--------------------------|-----------------------------|---------------------------|
| PWR          | 370               | 415             | 6.6                      | 56                          | 63                        |
| BWR          | 365               | 435             | 7.3                      | 50                          | 60                        |
| PHWR         | 750               | 580             | 21                       | 36                          | 27                        |
| FBR          | 175               | 100             | 5.0                      | 35                          | 20                        |

The characteristic size method gives a gross idea of neutronic coupling/decoupling of the core but it does not reveal the details of decoupling and its consequent effect on stability in any off normal situation. To understand the phenomenon in detail, more sophisticated techniques have come into vogue. One of such technique is Eigenvalue Separation (EVS) technique. In this technique, higher harmonics of the neutron flux derived from diffusion equation are evaluated and their relative influence on neutron flux tilt forms the basis of degree of neutronic coupling/decoupling. In the present paper, the eigenvalue separation and its connection with flux tilt and reactivity perturbation has been presented. The emphasis is placed on gaining insight through derivation of simplified analytical expressions. The eigenvalue separation is calculated as a function of core size and shape and an optimized cylindrical core shape has been assessed for pressurized water reactors to minimize neutronic decoupling. Method of calculation of EVS for complex geometry is presented. The study, besides providing insight in the complex phenomena of neutronic coupling in large size reactors, will be beneficial in the study of the neutronic coupling and adequacy of the control system in handling neutronic decoupling effects, of large size (more than 1000 MWe size) reactors.

2. EVS and Neutronic Coupling

Time dependence of neutron population in a nuclear reactor core can be studied by using neutron diffusion equation, which in operator form, can be written as,

$$\frac{\partial \phi}{\partial t} = -A + B \phi + \sum_{s} \lambda_{s} C_{s} \psi_{n}(r)$$  \hspace{1cm} (1)

where, $A$ is destruction operator and $B$ is the production operator. Other symbols in the equations have their usual meanings. Time dependent neutron flux, $\phi(r, t)$ can be expressed [4] in terms of unperturbed neutron flux distribution, $\phi_{0}(r)$ and its eigenfunction, $\psi_{n}(r)$ as,

$$\phi(r, t) = \phi_{0}(r) + \Delta \phi(r, t)$$  \hspace{1cm} (2)

where

$$\Delta \phi(r, t) = \sum_{n=1}^{\infty} a_{n}(t) \psi_{n}(r)$$  \hspace{1cm} (3)

represents the deviation of neutron flux from steady state. $a_{n}$ represents magnitude part of the eigenfunction and its value depends upon the magnitude of reactivity perturbations. The functions, $\psi_{n}(r)$, depend on higher harmonics of neutron diffusion equation. They could be any combination of trigonometric and Bessel’s functions. For cylindrical shape, these harmonics could be axial or azimuthal harmonics or mixed (Fig. 1).
As seen above, neutron flux, at any instant is the sum of fundamental mode and higher harmonics with different weightage. That is,

$$\Phi(r,t) = \text{Steady state flux} + \text{Contribution of higher harmonics (Axial + Azimuthal)} \quad (4)$$

Ideally, contribution of these higher harmonics should be fixed under all operating conditions to maintain non-varying power distribution in course of time. This can be achieved in small cores to a great extent but it is not possible in large cores. This will be discussed in later section of this paper.

Therefore in large core design, efforts are made to minimize the contribution of higher harmonics to gain maximum inherent stability. In an operating reactor, neutron flux shape could get disturbed because of several reasons such as insertion/removal of reactivity devices, xenon effects and localized perturbations due to reactivity feedbacks etc. The effect of such perturbations on power transients is different for different size of reactors. This can be demonstrated with the help of few basic equations. For ease in understanding, simplest geometry of slab reactor is considered.

3. Eigenvalue Separation and Core Size

Higher harmonics eigenvalue ($\lambda_n$) of equation (1), can be written as,

$$\lambda_n = \frac{k_{\infty}}{1 + M^2 B_n^2} \quad (5)$$

where geometric buckling, $B_n = (n+1)\frac{\pi}{a}$ for a slab reactor of core thickness $a$. $k_{\infty}$ is infinite multiplication factor, $M^2$ is neutron migration area, $M^2 = L^2 + \tau$, where $L$ and $\tau$ are diffusion length and ‘age to thermal neutron’ respectively. Eigenvalue separation ($\epsilon_1$), between the first mode and fundamental mode is expressed as,

$$\epsilon_1 = \frac{\lambda_1}{\lambda_0} \quad (6)$$

Substituting the expressions of $\lambda_1$ and $\lambda_0$ from equation (5), one gets,

$$\epsilon_1 = \frac{1 + M^2 B_1^2}{k_{\infty}} - 1 \quad \frac{1 + M^2 B_0^2}{k_{\infty}}$$

or

$$\epsilon_1 = \frac{M^2 \left[ \left( \frac{2\pi}{a} \right)^2 - \left( \frac{\pi}{a} \right)^2 \right]}{k_{\infty}}$$

That is,

$$\epsilon_1 \sim \frac{3(M\pi/a)^2}{k_{\infty}} \quad (7)$$

Thus, eigenvalue separation is inversely proportional to the square of the size of the reactor core. This means, a large sized reactor core has small eigenvalue separation.
4. Eigenvalue Separation and Flux Tilt

Consider a case, in which core gets subjected to asymmetric reactivity perturbation. These perturbations will disturb the neutron flux distribution. A classical way to represent such spatial perturbations is to represent them by higher mode reactivity. First mode reactivity ($\rho_1$) can be written as,

$$\rho_1 = \frac{\langle \delta \varphi_1^* - \delta \varphi_0 \rangle}{\langle \delta \varphi_2^* - \delta \varphi_0 \rangle} \lambda_0$$  \hspace{1cm} (8)

where $\delta A$ and $\delta B$ represent changes in destruction and production operator respectively due to the reactivity perturbation, $\varphi_1^*$ and $\varphi_0^*$ are first mode eigenfunction and adjoint eigenfunction respectively and $\lambda_0$ is fundamental mode eigenvalue. The induced flux tilt ($\tau$) in the core due to this perturbation ($\rho_1$) can be written as,

$$\tau = \frac{\langle \phi(r,E) \rangle_T - \langle \phi(r,E) \rangle_B}{\langle \phi(r,E) \rangle}$$  \hspace{1cm} (9)

where $\langle \cdot \rangle_T$ and $\langle \cdot \rangle_B$ and $\langle \cdot \rangle$ are integrals respectively over the top half, the bottom half and the total core volume. Under asymmetric perturbation, the tilt equation (Eqn. 9) can be approximated in terms of reactivity perturbation and EVS as,

$$\tau = \frac{\rho_1 \langle |\psi_1(r,E)|^2 \rangle}{\varepsilon_1 \langle |\psi_0(r,E)|^2 \rangle}$$  \hspace{1cm} (10)

where, $\psi_0$ is fundamental mode eigenfunction and $\psi_1$ is first harmonic eigenfunction. As the eigenfunctions are normalized as $\langle \psi_1^*, B \psi_1 \rangle = \lambda_1$ and $\langle |\psi_1(r,E)|^2 \rangle = 1$, the equation (10) can be simplified as,

$$\tau = \frac{\rho_1}{\varepsilon_1}$$  \hspace{1cm} (11)

From equations (7) and (11), it can be said that a large sized reactor core will exhibit higher flux tilt for the same magnitude of asymmetric reactivity perturbation.

5. Eigenvalue Separation and Higher Harmonics

Time dependent part of equation (2) can be shown to be proportional to eigenvalue separation [5] as,

$$\Delta \phi(r,t) \propto \frac{\lambda_n}{\lambda_n - 1} \exp \left[ \frac{\lambda_n - 1}{\lambda_n} \frac{t}{\lambda_n - 1} \right]$$  \hspace{1cm} (12)

where $\lambda_0$ is the prompt neutron lifetime. It can be seen that larger is the eigenvalue separation, more rapidly the higher harmonics contribution to the flux will decrease and more quickly the fundamental mode flux get established. In other words, the local disturbance at one place will be realized more quickly in the entire core. Thus a large EVS can be considered to represent the degree of neutronic coupling of the core. This feature articulates the reason of higher susceptibility of large sized cores against spatial instability.

6. EVS Evaluation

Evaluation of EVS for real core geometries is relatively a difficult job. In the present analysis, ‘Method of Mode Subtraction’ (described below), has been used. Solution of higher order eigenvalue equation of neutron diffusion equation is required for EVS evaluation. The $n^{th}$ order of a lambda mode eigenvalue equation, in operator form can be written as follows:

$$A \phi_n = \frac{1}{\lambda_n} B \phi_n$$  \hspace{1cm} (n=0,1,2,3…)

$\phi_n$ is $n^{th}$ order eigen vector of higher flux mode, and $\lambda_n$ is the corresponding eigenvalue. One of the widely known method to evaluate higher flux modes is the method of mode subtraction which is also known as deflation method. In this method, higher mode flux is obtained through source iterations by removing already calculated lower order mode components from an initially assumed neutron flux distribution using simple power iteration method,
That is,
\[
A\phi_n^{(m)} = \frac{1}{\lambda_n^{(m-1)}} B\phi_n^{(m-1)} - \sum_{n=1}^{n-1} \frac{1}{\lambda_n^{(m-1)}} B\phi_n^{(m-1)} \frac{\langle \phi_n^* B\phi_n^{(m-1)} \rangle}{\langle \phi_n^* B\phi_n^* \rangle} \quad (n \geq 1)
\]
where $\phi_n^{(m)}$: n$^{th}$ order of higher mode flux in m$^{th}$ outer iteration
$\lambda_n^{(m-1)}$: n$^{th}$ order of eigenvalue in m$^{th}$ outer iteration
$\phi_n^+$: n$^{th}$ order adjoint function

Differential terms in the eigenvalue equation (14) can be treated by either simple finite difference method (FDM) or by more refined nodal method. In the present work, FDM has been used. The higher mode flux is calculated with a finer mesh width than the fundamental mode, as distribution of higher mode flux spatially oscillates more widely between positive and negative values than that of a fundamental mode. Though, this method is relatively tedious in terms of computational effort, but with present day computers, it is possible to evaluate the first few harmonics in a few minutes.

7. Size Effect

To understand the effect of core size, a case has been studied by keeping the shape of the reactor core constant, i.e., by keeping height to diameter ratio constant. Core power density of the reactor is also maintained constant (100 kW/lit) and core size is expressed in reactor power (MWe). Higher harmonics are calculated for each size. The results, in terms of eigenvalue separation as a function of core size are presented in Fig. 2. It can be seen that for reactor size increasing from 200 MWe to 1000 MWe, the EVS decreases from 2600 pcm to 200 pcm. This is because for very large core of about 1000 MWe, the fundamental mode and first harmonic are very close to each other and therefore contribution of first harmonic to the total neutron flux will be significant in such cores and such cores will be more susceptible to axial mode of oscillations. Such systems will need special control system to maintain the desired power distribution. For small cores, axial mode EVS is large enough and therefore excited axial mode will die out quickly. Such systems will not require any spatial control system to maintain the desired power distribution.

![Fig.2.Core Size Effect](image)

8. Shape Effect

To understand the effect of shape, a study is carried out by conserving the volume of core (1000 MWe PWR core) and changing H/D ratios. Axial and azimuthal eigenvalue separation has been calculated. The results, in terms of EVS as a function of H/D ratio are plotted in Fig. 3. It can be seen that axial harmonic EVS decreases and azimuthal EVS increases with increase in H/D ratio. The Fig 3 also shows that at some value of H/D ratio (H/D≈0.92), EVS of axial and azimuthal modes is same. Therefore a large H/D ratio (H/D>0.92) would be preferred if it is easy to correct axial modes of oscillations. In case of large PWRs, group of control rods or partial rods can be efficiently used to suppress axially excited power density oscillations [6] Thus a H/D ratio slightly larger than 1.0 is a common practice in large PWR core design.
To minimize multimode susceptibility of core against spatial oscillations, desired perturbed neutron flux for PWRs is

$$\Phi'(r,t) = \text{Steady state flux} + \text{first axial harmonic}(\varepsilon_1) + \text{First Azimuthal harmonic}(\varepsilon_2)$$

where axial harmonic, being the first close (in terms of EVS) harmonic makes axial mode susceptible to oscillate, keeping azimuthal mode self correcting due large EVS ($\varepsilon_2 >> \varepsilon_1$). For large H/D ratio, say 1.5 or more, perturbed neutron flux takes the form,

$$\Phi'(r,t) = \text{Steady state flux} + \text{first axial harmonic}(\varepsilon_1) + \text{second axial harmonic}(\varepsilon_2)$$

Thus, in such situations, first two (first and second) consecutive higher harmonics become the axial harmonic with very small EVS. Under such situation, more than one closer axial harmonic will be available and this situation makes axial mode highly susceptible to even minor perturbations. This feature articulates that beyond some value of H/D ratio, core will have two close axial harmonics, making axial mode highly unstable. The parametric study (Table 1) carried out for larger values of H/D ratios (keeping core volume and power to be constant) indicates that an upper limit of about 1.3 for H/D ratio can be devised for large PWRs to optimize core shape, size against neutronic decoupling. EVS of second harmonic is shown in Fig. 4. Present 900-1000 MWe large PWRs, are typically designed to have H/D ratio of 1.1. One of the main objectives of new generation reactor designs is to scale up power output from 1000 MWe to 1500 MWe. From the present analysis, it can be recommended that new large sized reactors can be designed with a larger height (maintaining 1.1<H/D<1.3 limit). Such a shape will not pose any problem in azimuthal mode stability of core and axial mode, if excited, can be corrected with available existing spatial control measures. This H/D limit has been determined specific to PWRs (or VVERs). For other systems (CANDUs/PHWRs, FBRs, AGRs) this value will be different due to differences in the core physics design.

9. Conclusion

EVS can be used as an index to assess neutronic coupling/decoupling of reactor cores. Core geometries can be optimized using EVS technique. Based on this study, an H/D ratio beyond about 1.3 is not recommended for conventional large PWRs. The insight gained and the methodology established to study the neutronic coupling/decoupling in large sized PWRs, would be very useful in evaluating the neutronic coupling and the evaluation of adequacy of reactor control system in handling decoupling.
effects in PWRs of size more than 1000 MWe. This has special significance in the wake of India’s nuclear power program expansion through induction of large size PWRs.

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