Super-resolution imaging of negative-refractive graded-index photonic crystal flat lens

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Abstract

Photonic crystal (PC) not only breaks through the diffraction limit of traditional lenses but also can realize super-resolution imaging. Improving the resolution is the key task of PC imaging. The main work of this paper is to use a graded-index Photonic crystal (GPC) flat lens to improve the image resolution. An air-hole type two-dimensional (2D) GPC structure based on silicon medium is proposed in this paper. Numerical simulations through RSoft reveal that when the medium in the imaging area is air, the full width at half maximum (FWHM) value of a single image reaches 0.362λ. According to the Rayleigh criterion, the images of two point sources 0.57λ apart can also be distinguished. In the imaging system composed of cedar oil and GPC flat lens, the FWHM value of a single image reaches 0.34λ. In addition, the images of multiple point sources 0.49λ apart can still be distinguished.

1. Introduction

In 1968, Veselago first proposed negative refractive index materials (NIMs) with both the dielectric constant \(\varepsilon\) and the magnetic permeability \(\mu\) negative simultaneously [1]. Pendry theoretically predicted that NIMs can break through the diffraction limit of traditional lenses and become ‘perfect’ lenses [2]. This kind of material does not exist in nature, so man-made materials have come out one after another. The first structure is a metamaterial consisting of a periodic arrangement of metal split ring resonators and wires [3, 4]. Although this structure has many advantages, the narrow working frequency band and absorption loss in metal limits its potential applications in the optical field. Another structure is the photonic crystal (PC) proposed by John [5] and Yablonovitch [6] in 1987. PC have been proved to behave similarly to negative refractive materials both theoretically [7] and experimentally [8]. Notomi found that PC can adjust the effective refractive index (ERI) by controlling the band structure, which makes PC a negative refractive index material with interesting optical properties [7]. Luo et al proved that PC with circular pores have all-angle negative refraction behavior [9]. Parimi et al used the negative refraction effect in the PC material to study the imaging properties of the PC flat lens [10]. Wang et al proposed a triangle 2D PC which composed of periodic air holes embedded in a dielectric matrix. This PC structure is isotropic, and the effective refractive index approaches \(-1\), which can realize non-near-field imaging [11]. In addition, other different PC flat lens structures with ERI of \(-1\) have also been studied [12, 13]. In 2005, David Cassagne and Emmanuel Centeno jointly present a concept a graded-index photonic crystal (GPC). This PC makes it possible to bend light at the micrometer scale by gradually changing the lattice periodicity [14]. Later, more and more studies pointed out that the PC can gradually adjust the radius of the air holes [15, 16], lattice constant [14, 17, 18] and material parameters [19] to achieve the gradual change of the internal effective refractive index. And GPC have been studied extensively for focusing [15–17, 20–22], self-collimation [19], superbending [23] and waveguide [24, 25]. Cen et al proposed a GPC flat lens which can realize the imaging of the point source and sub-wavelength focusing of the plane wave in the first, second and fifth bands [26]. At the moment, technologies such as real-time biological display, high-density optical storage, and microelectronic lithography require that super-resolution imaging needs to evolve toward higher resolution and
more applicable bands. The PC flat lens has the characteristics of negative refraction and low loss, and is an ideal structure for super-resolution imaging device. Researches on super-resolution imaging of PC flat lens have been published in recent two years [27–29]. In addition, Gharraati et al investigated the coupling efficiency of optical coupler based on PC flat lens [30]. The two-dimensional PC superlens proposed by E. Cubukcu et al can distinguished two images separated by a distance of 0.3 λ, but the distance between the image plane and PC emitting surface is too short, which limits its application in practice [31]. Therefore, it is necessary to design a PC structure that is far from the emitting surface and has better imaging quality and resolution. And the method of improving the imaging resolution of PCs needs more exploration.

The non-uniform refractive index inside the GPC flat lens will leads to focal shift. In this paper, we continuously adjust and optimize the internal structure of the GPC flat lens to change this focal shift and compensate the focal shift in the small pupil imaging system. This method is used to optimize the imaging quality and improve the resolution. The distance between the image and the emitting surface can be increased simultaneously, which is convenient for practical application. This paper also presents a system composed of cedar oil and GPC flat lens. In this system, the FWHM value of image can be reduced and the resolution can be improved.

2. Model and theory

This paper proposes a 2D PC structure based on silicon (refractive index n = 3.45) with hexagonal arrangement of air holes. In this structure, the lattice constant a = 0.482 μm, and the point source wavelength λ0 = 1.55 μm. Figure 1(a) are equifrequency surface (EFS) calculated by the plane wave expansion (PWE) method in the first TE-polarized band of the PC when the air hole cylinders diameter D0 = 0.4 μm. According to the PWE method, the group velocity and phase velocity of the electromagnetic wave in the frequency range of the first photonic band of PCs are opposite, and the point multiplicity of Poynting vector and wave vector K of PCs is negative, so its ERI is negative. When the angular frequency ω0 = 2πc/a = 0.311, the value of K is 4.6062 (while c is the speed of light in vacuum). According to the formula neff = Kλ/2π, the ERI under the diameter (D0) of the air-hole cylinders can be obtained as −1.137. Constantly changing the diameter of the air-hole cylinders can obtain the corresponding ERI. As shown in figure 1(b), when the diameter of the air holes changes within the range of 0.336 to 0.443, the corresponding ERI is −0.661 ∼ −1.78.

Figure 2(a) shows the schematic diagram of GPC flat lens sub-wavelength imaging. The ERI is adjusted by changing the diameter of air-hole cylinders in the Z-axis direction, but remains unchanged in the X-axis direction. The point source is placed under the GPC flat lens and continuously moved in the Z-axis direction. Monitors are setting in the imaging area to detect the FWHM value of the images. By adjusting the diameter of the air-hole cylinders, the internal refractive index of the GPC flat lens can be modified according to several refractive index distribution patterns in figure 2(b). These ERI distribution patterns have turning points at different situations. As shown in figure 2(c), assume that the size of the GPC flat lens on the X-axis and Z-axis are 12 μm and 4.6 μm (11 layers in total), respectively. And the point source is placed 0.2 μm below the GPC flat lens. The deflection of light with the numerical aperture angle of 53° (blue line) and 80° (black line) is calculated, when the ERI of the GPC flat lens is −1. It can be seen from the figure that when the numerical aperture is greater than 53°, light cannot reach emitting surface, resulting in a reduction in image quality. Figure 2(c) also shows when the aperture angle is 80°, the deflection of light in ERI distribution patterns L5 (red line) and L9 (purple
line), where compared with the ERI − 1, GPC flat lens can converge lights with larger numerical aperture to imaging. In the process of multiple refraction, different refractive index distribution modes will produce diverse focal shifts, which can also be resulted in small pupil system for diffraction \[32\]. Next, this paper will explore the compensating effect between the focal shift caused by different ERI distribution patterns and the focal shift of small pupil imaging system, so as to optimize the imaging quality.

3. Results and discussion

In order to verify the above design, this paper uses the finite time domain difference method (FDTD) to simulate the propagation of electromagnetic waves in the GPC flat lens. The structure in this paper can theoretically extend in the X-axis direction. The perfect matching layer is set to 0.5 μm and the grid size is set to 0.03 μm. Figure 3(a) suggests that \( u \) is the object distance and \( v \) is the image distance. The point source moves in the opposite direction of the Z-axis, and the moving range of \( u \) is \( 0.1 \sim 2 \) μm. With the increase of \( u \), the image will be close to the emitting surface of the GPC flat lens. The near-field effect will lead to the FWHM value of the image, but it will increase the difficulty of image observation and limit its application in practice. It can be seen from figure 3(b) that the FWHM value of image corresponding to different ERI distribution patterns is indeed different. Under the different object distance \( u \), the FWHM of image with the internal ERI distribution pattern L9 is smaller. When the \( u \) is 0.1 and 0.2 μm, the FWHM value of image is 0.3718 \( \lambda \) and 0.3791 \( \lambda \) respectively, breaking the diffraction limit. And the image distance \( v \) is about 3 \( \lambda \). The following optimized imaging and super-resolution effects are based on the ERI distribution pattern L9, and the value of \( u \) is 0.2 μm. What is worth

![Figure 2](image-url)

**Figure 2.** (a) Schematic diagram of GPC flat lens. (b) The effective refractive index distribution patterns corresponding to each layer of air-hole cylinders in the GPC flat lens. (c) Deflections of light under serval internal refractive index distribution patterns. Numerical aperture angle \( \theta_1 \) and \( \theta_2 \) are 53° and 80°.
mentioning is that in other GPC flat lens of different thicknesses, the ERI distribution pattern like L9 can also be found, which can reach the best FWHM value.

The simulation results of Ma et al [13] and Sun et al [33] show that the optimized surface termination can excite two kinds of surface mode which can couple with the Bloch wave in the PCs. With the help of these surface modes, the super resolution capacity for the PC flat lens can be improved greatly. Figure 4(a) indicates that we define a variable distance ‘d’ to find out what value of ‘t’ can reach the best image, where t = (1 – d/r) × 100% denotes the cutting ratio of the air holes on the upper and lower surface termination of GPC flat lens. Figure 4(b) demonstrates that different cutting ratios will affect the FWHM value of the image. When t = 15%, the FWHM value of the image can be further optimized to 0.362\(\lambda\). It can be seen from figure 4(b) that different cutting ratios will affect the FWHM value of the image point. The FWHM value of the image point can be further optimized to 0.362\(\lambda\).

We place two point sources at 0.2 \(\mu m\) below the GPC flat lens, and constantly reduce the distance between the two point sources and detect the value of the peak and trough of the images to evaluate the super-resolution capability of the GPC flat lens. According to the Rayleigh criterion, while the ratio of the trough value to the peak value is 0.81, the two point sources can be distinguished. Figure 5(a) indicates that when the distance between two point sources is 0.54\(\lambda\), the peak value and trough value are equal, which means that the two point sources are completely indistinguishable. As the distance between the point sources increases, the trough value drops while the peak value rises. When the distance stands at 0.57\(\lambda\), the images of two point sources are exactly distinguished. Once the distance of the point light source increases to 0.71\(\lambda\), the value of the trough of the two image is 0, and the two image points can be entirely differentiated at this time. Figures 5(b) and (c) are the simulated propagation paths of single source and two point sources 0.2 \(\mu m\) below the GPC flat lens. And figures 5(e) and (f) shows the intensity distribution diagram of the corresponding images.
What is acknowledged is that the propagation wavelength and refractive index of electromagnetic waves in different media are different. Hence, different refractive materials (such as water, cedar oil) can be utilized to enhance the resolution. This article replaces the medium in the imaging area of the GPC flat lens system with cedar oil (refractive index 1.515). The wavelength in the cedar oil can be rewritten as $\lambda' = \frac{\lambda}{1.515}$. The wavelength of the electromagnetic wave in the liquid is equal to the original wavelength divided by the refractive index. The shrinking of the wavelength will lead to the minimum FWHM value of single point source to 0.515 $\lambda'$ (0.34 $\lambda$). And the limited distance of the distinguishable two point sources can be reduced to 0.49 $\lambda$. As shown in figure 6(a), two pairs of point sources with an interval of 0.49 $\lambda$ separated by $L = 20 \, \mu m$ are placed at the object plane, and the corresponding image is finally obtained at the image plane. Figure 6(b) is the intensity distribution diagram of the image, which indicates that the ratio of the trough value to the peak value of the image point can still reach 0.81. According to the Rayleigh criterion, the GPC flat lens achieves the super-resolution of several sources. The lens will not be influenced by the off-axis effect for the lack of main optical axis.

![Figure 5](image_url)
4. Conclusion

In this paper, the focal shift caused by the non-uniform refractive index in the GPC flat lens compensate with the focal shift in the small pupil imaging system to optimize the FWHM value of the imaging and improve the effect of super-resolution. Unlike the traditional optical lens, the proposed GPC flat lens does not have the main optical axis, so it can realize large-scale off-axis imaging. We cut the PC and change the medium in the imaging area to make the FWHM value of image smaller. In the imaging system composed of cedar oil and GPC flat lens, two point sources 0.49 λ apart can also be distinguished. Moreover, the image distance is about 3 λ, which is more convenient for application. At the same time, it also provides a method for improving the imaging and super-resolution effect of GPC flat lens.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Conflict of interest

The authors declare no conflict of interest.

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References

[1] Veselago V G 1968 Soviet Physics Uspekhi 10 509–14
[2] Pendry J B 2000 Phys. Rev. Lett. 85 3966–9
[3] Shelby R A, Smith D R and Schultz S 2001 Science 292 77–9
[4] Algarin J M, Freire M J, Lopez M A, Lapine M, Jakob P M, Behr V C and Marqués R 2011 Appl. Phys. Lett. 98 014105
[5] John S 1987 Phys. Rev. Lett. 58 2486–9
[6] Yablonovitch E 1987 Phys. Rev. Lett. 58 2959–62
[7] Notomi M 2000 Phys. Rev. B 62 10696–705
[8] Fabre N, Lalouat L, Cluzel B, Mélique X, Lippens D, Fornel F D E and Vanbésien O 2008 Phys. Rev. Lett. 101 073901
[9] Luo C, Johnson S G, Joannopoulos J D and Pendry J B 2002 Phys. Rev. B 65 201104
[10] Parimi P V, Lu W T, Vodo P and Sridhar S 2003 Nature 426 404–404
[11] Wang X, Ren Z F and Kempa K 2004 Opt. Express 12 2919–24
[12] Gajić R, Meisels R, Kuchar F and Hingerl K 2006 Phys. Rev. B 73 165310
[13] Ma H, Liang B, Zhuang S, Chen J, Jiang Q and Ding J 2016 Opt. Lett. 41 3833–5
[14] Centeno E and Cassagne D 2005 Opt. Lett. 30 2278–80
[15] Chien H-T and Chen C-C 2006 Opt. Express 14 10759–64
[16] Wu Q, Gibbons J M and Park W 2008 Opt. Express 16 16941–9
[17] Kurt H, Colak E, Cakmak O, Caglayan H and Ozbay E 2008 Appl. Phys. Lett. 93 171108
[18] Centeno E, Cassagne D and Albert J-P 2006 Phys. Rev. B 73 235119
[19] Kurt H and Citrin D S 2007 Opt. Express 15 1240–53
[20] Li Y H, Fu Y Q, Minin O V and Minin I V 2016 Optical Materials Express 6 2628–36
[21] Liang S, Xie J, Tang P and Liu J 2019 Opt. Express 27 9601–9
[22] Rezaei B, Giden I H and Kurt H 2017 Opt. Commun. 382 28–35
[23] Schonbrun E, Wu Q, Park W, Yamashita T, Summers C J, Abashin M and Fainman Y 2007 Appl. Phys. Lett. 90 041113
[24] Wang H-W and Chen L-W 2011 J. Opt. Soc. Am. B 28 2098–104
[25] Hadi Badri S and Gilarlue M M 2019 J. Opt. Soc. Am. B 36 1288–93
[26] Cen Y, Xie J and Liu J 2019 Chinese Optics Letters 17 080501
[27] Lu J and Zhang Y R 2020 J. Russ. Laser Res. 41 406–12
[28] Sheng J, Xie J L and Liu J J 2020 Chinese Optics Letters 18 120510
[29] Zhao H X, Xie J L and Liu J J 2021 Results Phys. 27 104537
[30] Gharaati A, Zareian Z and Khalkhali T F 2021 Pramana-J. Phys. 95 23
[31] Cubukcu E, Aydin K, Ozbay E, Foteinopoulou S and Soukoulis C M 2003 Phys. Rev. Lett. 91 207401
[32] Li Y 2005 J. Opt. Soc. Am. A 22 68–76
[33] Sun J, Shen Y F, Chen J, Wang L G, Sun L L, Wang J, Han K and Tang G 2010 Photon. Nanostruct.: Fundam. Appl. 8 163–71