σ Meson In $J/\Psi$ Decays

Wujun Huo, Xinmin Zhang, and Tao Huang

CCAST (World Lab.), P.O. Box 8730, Beijing 100080

and

Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4),
Beijing 100039, P.R. China

Abstract

Recently BES at BEPC found evidence for the existence of the $\sigma$ meson in the process of $J/\Psi \rightarrow \sigma \omega \rightarrow \pi \pi \omega$. In this paper we firstly discuss the relevant coupling $g_{\sigma \pi \pi}$ and show that the linear $\sigma$ model gives rise to a reasonable description of the $\sigma$ decay into $\pi$’s, then we calculate the coupling constant $g_{J/\Psi \sigma \omega}^{th}$ using the perturbative QCD technique and the light-cone wave functions of the $\sigma$ and $\omega$ mesons. The results show that the theoretical value of $g_{J/\Psi \sigma \omega}^{th}$ is within the range of experimental value $g_{J/\Psi \sigma \omega}$. 

1 Introduction

Beijing Spectrometer (BES) at Beijing Electron-Positron Collider (BEPC) recently reports an evidence for the existence of the $\sigma$ particle in $J/\Psi$ decays. In the $\pi^+\pi^-$ invariant mass spectrum in the process of $J/\Psi \rightarrow \pi^+\pi^-\omega$ they found a low mass enhancement and the detailed analysis strongly favors that the spin-parity is $O^{++}$ and the statistical significance for the existence of the $\sigma$ particle is about 18 $\sigma$ [1]. The BES measured values of the $\sigma$ mass and width are:

$$m_\sigma = 390^{+60}_{-36}\text{MeV},$$
$$\Gamma_\sigma = 282^{+77}_{-50}\text{MeV},$$

(1)

and the branching ratio is

$$\text{Br}(J/\Psi \rightarrow \sigma\omega \rightarrow \pi^+\pi^-\omega) = (1.71 \pm 3.4 \pm 4.3) \times 10^{-3}.$$  \hspace{1cm} (2)

The $\sigma$ particle has been absent for many years in the Review of Particle Physics [2] by the Particle Data Group (PDG), however in the recent years there has been a revival of interest in studying the light scalar-isoscalar meson, the $\sigma$ particle, as a broad resonance [3] experimentally and theoretically. A direct experimental evidence for the $\sigma$ meson is reported recently by the Fermilab E791 collaboration [4] in the D-meson decay process, $D^+ \rightarrow \sigma\pi^+ \rightarrow 3\pi$. Theoretically, $\sigma$ meson can play important roles in some problems. In ref. [5], $\sigma$ can be regarded as the dominant contribution to the $u\bar{u} + d\bar{d}$ current. Moreover, using $sigma$ mass and width from E791, some people [6] investage $\sigma$ contribution to $B \rightarrow 3\pi$ and find it can explain the recent data from CLEO and BABAR collaboration.

In this paper, we study phenomenologically the decay process $J/\Psi \rightarrow \sigma\omega \rightarrow \pi^+\pi^-\omega$ and discuss the couplings $g_{\sigma\pi\pi}$ and $g_{J/\Psi\sigma\omega}$. By taking the meson wavefunctions of $\sigma$ and $\omega$ to be similar to that of $\pi$ and $\rho$, and using the perturbative QCD technique, we calculate the decay constant $g_{J/\Psi\sigma\omega}^0$. Our theoretical prediction is shown to be within the range of experimental value $g_{J/\Psi\sigma\omega}$. 

2
2 Coupling constants $g_{\sigma\pi\pi}$ and $g_{J/\Psi\sigma\omega}$

Given the data on the $\sigma$ mass and width measured by BES in Eqs.(1) and (2) we study the coupling constants $g_{\sigma\pi\pi}$ and $g_{J/\Psi\sigma\omega}$. For a two-body decay of particle $X$ into final states $X_1$ and $X_2$ the decay width is given by

$$\Gamma(X \to X_1X_2) = \frac{|p|}{32\pi^2 M_X^2} \int |M|^2 d\Omega = \frac{|p|}{8\pi M_X^2} |M|^2,$$

where $|p|$ is 3-momentum of the final state in the center-of-mass (c.m.). For $\sigma \to \pi^+\pi^-$, the $\sigma$ meson decays 100% into $\pi\pi$. Furthermore isospin conservation requires for two thirds of the time the final states be the charged pions. So we have:

$$\frac{2}{3} \Gamma_0^0 = g_{\sigma\pi\pi}^2 \frac{1}{8\pi m_\sigma^2} \sqrt{\frac{m_\pi^2}{4} - m_\pi^2}.$$

Using BES data on the $\sigma$’s mass, width, and the experimental value of the $\pi$ mass, we obtain:

$$g_{\sigma\pi\pi} = 2.0^{+0.30}_{-0.19} \text{GeV}. \quad (5)$$

This number is surprisingly consistent with the theoretical value of the linear sigma model\[7\]

$$g_{\sigma\pi\pi}^{linear-sigma} = \frac{\sqrt{2} m_\sigma}{f_\pi} = 1.80^{+0.50}_{-0.30} \text{GeV}, \quad (6)$$

where $f_\pi$ is the pion decay constant. In Eq.(6) the $\sigma$ mass is taken from the BES measurement in Eq.(1).

To get the phenomenological value of $g_{J/\Psi\sigma\omega}$, we use the full three-body decay width (for example, see [7]):

$$\Gamma(J/\Psi \to \sigma\omega \to \pi^+\pi^-\omega) = \frac{1}{2} \frac{1}{2m_{J/\Psi}} g_{J/\Psi\sigma\omega}^2 g_{\sigma\pi\pi}^2 \int_{4m_\omega^2}^{(m_{J/\Psi}-m_\omega)^2} \frac{d\chi^2}{2\pi} \frac{1}{8\pi} \chi^{1/2} \left(1, \frac{\chi^2}{m_{J/\Psi}^2}, \frac{m_\omega^2}{m_{J/\Psi}^2} \right)$$

$$\times \frac{1}{8\pi} \chi^{1/2} \left(1, \frac{m_\omega^2}{\chi^2}, \frac{m_\omega^2}{\chi^2} \right) \frac{1}{(\chi^2 - m_\sigma^2)^2 + \Gamma_0^0 (\chi)^2 m_\sigma^2}. \quad (7)$$


where factor $1/(8\pi) \times \lambda^{1/2}$ is the phase space integral of the corresponding two-body decay subprocess and
\[
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca. \tag{8}
\]
And in Eq. (7)
\[
\Gamma_\sigma(\chi) \equiv \Gamma_0^0 \times (m_\sigma/\chi) (p^*(\chi)/p^*(m_\sigma)) \tag{9}
\]
is the co-moving resonance width where $p^*(\chi) = \sqrt{\chi^2/4 - m_\sigma^2}$ and $\Gamma_0^0 = (370^{+60}_{-30})$ MeV the experimental value in Eq.(1).

With the experimental values of $g_{\sigma\pi\pi}$ in Eq.(5) and the branching ratio in Eq.(2), solving Eq. (7) gives rise to
\[
g_{J/\Psi \sigma\omega}^{exp} = 7.3^{+2.6}_{-1.9} \text{ MeV}. \tag{10}
\]

### 3 $g_{J/\Psi \sigma\omega}$ calculated by the perturbative QCD

In this section we follow closely the calculation of the exclusive decays of $\Upsilon$ in ref. [8] to study the decay of $J/\Psi \rightarrow \sigma\omega$. The decay amplitude consists of two parts. One is the hard decay amplitude of the three-gluon modes and another is the bound-state matrix elements of outgoing mesons.

In general the decay amplitude of $J/\Psi \rightarrow \sigma\omega$ can be written as
\[
\mathcal{M} = \Psi_{J/\Psi}(0) \int_{-1}^{1} dx \int_{-1}^{1} dy \phi_{\sigma}^*(x, M_c^2) \phi_{\omega}^*(y, M_c^2) T_h(x, y, M_c^2), \tag{11}
\]
where $x = x_1 - x_2$, $y = y_1 - y_2$ and $x_i$, $y_i$ are the constituent’s fractional longitudinal momenta which satisfy $\Sigma x_i = 1$ and $\Sigma y_i = 1$. In Eq.(11) $\Psi_{J/\Psi}(0)$ is a non-relativistic approximate wave function of the $J/\Psi$ meson, $\phi_{\sigma}(x, M_c^2)$ and $\phi_{\omega}(y, M_c^2)$ are the distribution amplitude of the $\sigma$ and $\omega$ wave functions respectively and $M_c$ is the charm quark mass.

$T_h(x, y, M_c^2)$ is the hard decay amplitude of the charm quark pairs into two light quark-antiquark pairs, which is defined by
\[
T_h(x, y, M_c^2) = \int \frac{d^4l}{(2\pi)^4} T_h(x, y, l, M_c^2). \tag{12}
\]
There are twelve Feynmann diagrams shown in Fig. 1 contributing to $T_h$, which are all explicitly shown in Ref. [8]. Using the Landau rules, it is found that both infrared and collinear divergences exist in every diagram. Fortunately, through a use of color neutrality and collinear Ward identities, one can prove that when summing up all of the diagrams, the divergences cancel [8].

For quarkonia $J/\Psi$, the non-relativistic approximate wave function $\Psi_{J/\Psi}(0)$ can be determined by the decay $J/\Psi \rightarrow e^+e^-$, 

$$\Gamma(J/\Psi \rightarrow e^+e^-) = \frac{16\pi\alpha_s^2 e^2}{M^2_{J/\Psi}}|\Psi(0)|^2.$$  \hspace{1cm} (13)

According to the Brodsky-Huang-Lepage [9], the light-cone wavefunction of a hadron is essentially determined by the off-shell energy variable. Thus, for the light scalar meson, $\sigma$, we assume the light-cone wave function to be the same as that of the pion [10]

$$\psi_\sigma(x_i, k^\perp) = A_\sigma \exp[-b_\sigma^2(\frac{k_{1\perp}^2 + m_u^2}{x_1} + \frac{k_{2\perp}^2 + m_u^2}{x_2})],$$ \hspace{1cm} (14)

where $k^\perp$ is the relative transverse momentum of the final meson, $m_u$ is $u$ quark mass and $x_i$ are the constituent’s fractional longitudinal momenta. $A_\sigma$ and $b_\sigma$ are two free parameters which are taken to be $A_\sigma = A_\pi \approx 32\text{GeV}^{-1}$ and $b_\sigma^2 = b_\pi^2 \approx 0.84\text{GeV}^{-2}$ [10]. For the wave function of the vector meson, $\omega$, we assume it to be the same as that of the $\rho$ meson [11]

$$\psi_\omega(y_i, k^\perp) = A_\omega \exp[-b_\omega^2(\frac{k_{1\perp}^2 + m_u^2}{y_1} + \frac{k_{2\perp}^2 + m_u^2}{y_2})],$$ \hspace{1cm} (15)

where $A_\omega$ and $b_\omega$ are taken to be $A_\omega = A_\rho \approx 30\text{GeV}^{-1}$ and $b_\omega^2 = b_\rho^2 \approx 0.55\text{GeV}^{-2}$, which can be determined from two constraints [10, 11].

The distribution amplitude $\phi$ in Eq.(11) is defined as

$$\phi(x_1, x_2, M^2_c) = \int_0^{M^2_c} \frac{dk_{2\perp}^2}{16\pi^2} \psi(x_1, x_2, k_{2\perp}^2).$$ \hspace{1cm} (16)

Making use of the wave function of $\sigma$ and $\omega$ in Eqs.(14) and (15) we have:

$$\phi_\sigma(x, M^2_c) = \frac{A_\sigma(1-x^2)}{64\pi^2 b_\sigma^2} [\exp(-4b_\sigma^2\frac{m_u^2}{1-x^2}) - \exp(-4b_\sigma^2\frac{M^2_c + m_u^2}{1-x^2})].$$ \hspace{1cm} (17)
\[ \phi_\omega(y, M_c^2) = \frac{A_\omega(1-y^2)}{64\pi^2b_\omega^2} \left[ \exp(-4b_\omega^2\frac{m_u^2}{1-y^2}) - \exp(-4b_\omega^2\frac{M_c^2 + m_u^2}{1-y^2}) \right], \]

where we have used \( x = x_1 - x_2, y = y_1 - y_2, \Sigma x_i = 1 \) and \( \Sigma y_i = 1. \) In deriving the integration of Eq.(16), we have ignored the QCD evolution on the distribution amplitude since the charm quark \( M_c \) is not large and the evolution effects are not significant in this energy range. Thus the distribution amplitude is essentially determined by the non-perturbative model[10]. In Fig.2 we show the distribution amplitudes of \( \sigma, \omega. \)

With the values of the parameters \( A_\pi, b_\pi^2, A_\rho, b_\rho^2 \) given above, we obtain the coupling constant \( g^{th}_{J/\Psi \sigma \omega} \) responsible for the decay \( J/\Psi \rightarrow \sigma \omega, \)

\[ g^{th}_{J/\Psi \sigma \omega} = 10.7 \text{MeV}. \] (19)

One can see that our theoretical prediction on \( g^{th}_{J/\Psi \sigma \omega} \) above agrees within the errors with the experimental value \( g^{exp}_{J/\Psi \sigma \omega} \) in Eq.(10).

### 4 Conclusion

In this paper we have used the new BES experimental evidence for the \( \sigma \) particle and studied its properties in the process of \( J/\Psi \rightarrow \sigma \omega \rightarrow \pi \pi \omega. \) We have obtained the phenomelogical values of the coupling constants \( g_{\sigma \pi \pi} \) and \( g_{J/\Psi \sigma \omega}, \) then the theoretical prediction on the \( g^{linear-\sigma}_{\sigma \pi \pi} \) in the linear \( \sigma \) model. We show that linear \( \sigma \) model gives rise to a reasonable description of the \( \sigma \) decay into \( \pi \)'s. We have calculated \( g_{J/\Psi \sigma \omega} \) by the perturbative QCD and in this approach we take the wave functions of the \( \sigma \) and \( \omega \) to be similar to that of \( \pi \) and \( \rho. \) Our theoretical predictions and the experimental values of the \( g_{J/\Psi \sigma \omega} \) are shown to be consistent.

### Acknowledgments

One of the authors (W. J. Huo) acknowledges support from the Chinese Postdoctoral Science Foundation and CAS K.C. Wong Postdoctoral Research Award Fund. We thank
Zhi-Peng Zheng and N. Wu for useful discussions. This work is supported in part by the NSF of China.

References

[1] N. Wu, hep-ex/0104050.

[2] Particle Data Group, C. Caso et al., Eur. Phys. J. C3, 1 (1998).

[3] S. Ishida, M. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 95, 745 (1996); N.A. Törnqvist, hep-ph/0008133; V. E. Markushin, Z. Xiao and H. Q. Zheng, Nucl. Phys. A695, 273 (2001); H. Jin and X. Zhang, hep-ph/9805412; N.A. Törnqvist, A. D. Polasa, Frascati Phys. Ser. 20, 385 (2000); M. Ishida, hep-ph/0012325.

[4] E. M. Aitala, et al. [E791 Collaboration], Phys. Rev. Lett. 86, 770 (2001); R. Gatto, G. Nardulli, A.D. Polosa and N.A. Törnqvist, Phys. Lett. B494, 168 (2000); C. Gobel [E791 Collaboration], hep-ex/0012009.

[5] S.N. Cherry and M. R. Pennington, hep-ph/0111158.

[6] A. Deandrea and A. Polosa, Phys. Rev. Lett. 86, 216 (2001); S. Garder and U. Meissner, hep-ph/0112281.

[7] C. Dib and R. Rosenfeld, Phys. Rev. D63, 11750 (2001).

[8] S.C. Chao, Nucl. Phys. B195, 381 (1982).

[9] S.J. Brodsky, T. Huang and G. P. Lepage, Particle and Fields, ed. by A.Z. Capri and A.N. Kamal V2, P.143 (1983); T. Huang, Proceedings of the XXth International Conference on High Energy Physics, Madison (1980), p.1000.

[10] X.-H. Guo, T. Huang, Phys. Rev. D43, 2931 (1991); T. Huang, in Proceedings of the International Symposium on Particles and Nuclear Physics, Beijing, China, 1985, edited by N. Hu and C. S. Wu (World Scientific, Singapore, 1987); in Second Asica
Pacific Physics Conference, proceedings, Bangalore, India, 1986, edited by S. Chandrasekhar (World Scientific, Singapore, 1987); F.G. Cao and T. Huang, Commun. Theor. Phys. 27, 217 (1997).

[11] V.L. Chernyak and A.R. Zhitnisky, Phys. Rep., 112, 173 (1984); X.D. Xang, X.N. Wang and T. Huang, Phy. Rev. D35, 1013 (1987).
Figure 1: Feynmann diagram for the hard three-gluon decay. There are 12 diagrams in total, however shown is just one of them.

Figure 2: Distribution amplitudes $\phi_\sigma(x)$ and $\phi_\omega(x)$ of the $\sigma$ and $\omega$ mesons. The solid curve $\phi_\sigma(x)$ is determined by Eq. (17) with $A_\sigma = 32\text{GeV}^{-1}$ and $b_{\pi}^2 = 0.84\text{GeV}^{-2}$ and the dashed curve $\phi_\omega(x)$ is determined by Eq. (18) with $A_\omega = 30\text{GeV}^{-1}$ and $b_\rho^2 = 0.55\text{GeV}^{-2}$. 