On the equivalence principle and gravitational and inertial mass relation of classical charged particles

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Received 25 August 2009, in final form 2 December 2009
Published 23 December 2009
Online at stacks.iop.org/CQG/27/025005

Abstract

We show that the locally constant force necessary to get a stable hyperbolic motion regime for classical charged point particles, actually, is a combination of an applied external force and of the electromagnetic radiation reaction force. It implies, as the strong equivalence principle is valid, that the passive gravitational mass of a charged point particle should be slightly greater than its inertial mass. An interesting new feature that emerges from the unexpected behavior of the gravitational and inertial mass relation, for classical charged particles, at a very strong gravitational field, is the existence of a critical, particle-dependent, gravitational field value that signs the validity domain of the strong equivalence principle. For electrons and protons, these critical field values are \(g_c \approx 4.8 \times 10^{31}\text{m s}^{-2}\) and \(g_c \approx 8.8 \times 10^{34}\text{m s}^{-2}\), respectively.

PACS numbers: 04.20.Cv, 03.50.De

1. Introduction

The problem of the electromagnetic radiation reaction force on the charged particle dynamics, as given by the Lorentz–Abraham–Dirac (LAD) equation [1–11], has been a subject of active investigation. There are a lot of works about this subject accumulated since the first attempt was made by Dirac [1].

There is now a renewed interest on this subject, with works pointing to something new, which should affect the validity of the weak equivalence principle at some circumstances [12–21]. Perhaps because the main experimental justification that led Einstein to formulate the equivalence principle (EP), which is one of the foundations of his general theory of relativity, is the numerical equality between inertial and gravitational mass, nowadays they are taken
quite as synonymous, so we have to be aware to avoid misleading conclusions. According to Weinberg [5], we distinguish the weak equivalence principle (WEP) of the strong equivalence principle (SEP). The strong equivalence principle postulates that at every spacetime point in a arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that, within a sufficiently small region of the point in question, the laws of the nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation. On the other hand, the weak equivalence principle is nothing but a restatement of the observed equality of gravitational and inertial mass.

About the verification of the WEP, there is a surprising richness in the variety of experimental techniques and choice of the test bodies which have been used so far. The equality of gravitational and inertial mass is in fact what the experiments, since the famous Eötvös balance until recent experiments, actually measure. We show a brief review. The most obvious way to proof the WEP is to compare the motion of two bodies during free fall. These experiments, limited by rather short free falling periods, reached an accuracy of about 1 part in $10^{-10}$ [22]. The Bremen drop tower experiments, using SQUID displacement sensors, will provide a much longer time for free fall, allowing to reach an accuracy of about $10^{-12}$–$10^{-13}$ [23, 24]. Torsion balance experiments have reached an accuracy of few parts in $10^{-13}$ [25]. A planned experiment using a cryogenic balance claims an accuracy of $10^{-14}$ [26]. The most sensitive long-range measurements have used the Sun as the source and Earth and Moon type test bodies. The lunar laser ranging (LLR) techniques reach an accuracy of $5 \times 10^{-13}$ [27, 28]. On the other hand, future space experiments promise much better precision in this measurement. The MICROSCOPE mission [29] aims to test, on a microsatellite of the MYRIADE series developed by CNES/FRANCE, the WEP with a $10^{-15}$ accuracy. Galileo Galilei (GG) is a proposed experiment in low orbit around the Earth aiming to test the WEP to the level of 1 part in $10^{-17}$ [30]. STEP, using pairs of concentric free-falling proof-mass, will be able to test the WEP to a sensitivity at 1 part in $10^{-18}$ [31].

On the other hand, note that these mentioned experiments do not investigate the WEP in the case of charged particles. The reason is that electromagnetic fields influence gravitation experiments with charged particles and must be shielded carefully. The experiments for freely falling electrons carried out by Witteborn and Fairbank [32], with an accuracy of $10^{-1}$, is the only one cited in the literature. Nowadays, Dittus and Lämmerzahl [33] showed that an experiment in space with the Witteborn–Fairbank set-up may be well suited to test the WEP and to improve the results for free fall test with charged particles by orders of magnitude.

Some important comments should be made on these two formulations of the equivalence principle (EP). The SEP is valid only in a static and homogeneous gravitational field, but it is always possible to choose a sufficiently small spacetime region where the gravitational field can be locally approximated by a static homogeneous field, so that the SEP is valid locally. For a scalar particle, the Pauli formulation of EP proposes that a homogeneous gravitational field can always be transformed away globally so that in a suitable reference frame there is only a Minkowski space—no gravitational field. On the other hand, Audretsch [34] observes that if one takes a particle with spin, the equation of motion for such a particle will inevitably involve the curvature tensor, which cannot be eliminated by any transformation of coordinates. Some authors ignore the influence of curvature (second derivates) or tidal effects, but this means that they get rid of gravitational field. Finally, some results have been obtained for an infinite homogeneous gravitational field (in the entire space) or for an uniformly accelerated boundless reference frame. These gravitational fields are not a true gravitational fields [14].

About the WEP, the equations of motion of a point mass in a curved background spacetime were investigated by Mino, Sasaki and Tanaka [15]. The same equations of motion were later obtained by Quinn and Wald [16, 17] from an axiomatic approach. Following Mino, Quinn
and Wald, Haas and Poisson [18] calculate the self-force acting on a point scalar charge in a wide class of cosmological spacetimes. The self-force produces two effects: a time-changing inertial mass and a deviation relative to geodesic motion. The work of Dewitt and Brehme [19], corrected by Hobbs [20], showed that a point charge in a true gravitational field not follows a geodesic, so that WEP is violated for a charged particle. Using the techniques of finite-temperature field theory, Donoghue et al. [21] showed that the equality of inertial mass and gravitational mass, for charged spin-\(\frac{1}{2}\) or spin-zero particles, is not valid in the context of quantum field theory at finite temperature. Higuchi [35] calculated the position shift of the final-state wave packet of the charged particle due the radiation and showed that it disagrees with the result obtained using the Lorentz–Abraham–Dirac equation for the radiation–reaction force. In an alternative approach, Spohn [36] and other authors [37–39] changed the Lorentz–Abraham–Dirac equation for the force on an accelerating charge, which avoids the pathologies of preacceleration and runaway solutions. Yaghjian [40] suggested that these problems will be absent once the finite-size effects are properly taken into account. Finally, the validity of the WEP is very well tested for macroscopic bodies to a sensitivity of few parts in \(10^{-13}\), but this does not necessarily imply that such a principle continues to hold at a microscopic scale and in the quantum regime.

The goal of this work is not to discuss these papers, but, instead, to add the possibility of analyzing the problem of local motion of the classical charged point particles in a different perspective, with emphasis in the EP, which validity is used as a good starting point. As the subject of this work is about the conditions of validity or not of the EP, it is just to note that results from general gelativity are not used at any moment, in order to avoid any possibility of falling in a vicious causal recurrence.

This text is a review of an old work of Goto [41]. What we have to do is to figure out the condition that we have to provide such that a classical charged point particle can reach a locally stable hyperbolic motion. We show that it is necessary to furnish a balance between an applied external force and the electromagnetic radiation reaction force to get a hyperbolic motion regime. An important consequence is that the strong EP, in this case, implies a gravitational mass slightly greater than the inertial mass. From this result, we show that what seems uncomfortable, as the presence of radiation for the charged particle in a hyperbolic motion and its absence for the particle at rest in a uniform gravitational field [42, 43], both equivalent situations taking into account the strong EP, leads to a new physical feature performed by charged particles. As a consequence of the unexpected behavior of the passive gravitational and inertial mass relation, at a very strong gravitational field, we find the presence of a divergence that indicates a critical field value that signs the validity domain of the SEP.

This paper is organized as follows. In section 2, we show that the locally external force necessary to produce a hyperbolic motion in neutral particles is smaller than the locally external force necessary to give the same hyperbolic motion in classical charged particles. In section 3, we figure out that, to the SEP to be valid, the WEP is violated for classical charged point particles in a stable local hyperbolic motion regime. Moreover, we show that there exists a critical, particle-dependent, gravitational field value that signs the validity domain of the SEP. Section 4 is devoted to a final discussion and conclusions.

### 2. Local hyperbolic motion of charged particles

A hyperbolic motion is the natural generalization of the concept of the Newtonian uniformly accelerated motion due to a constant force applied to a particle, which might be due to a uniform gravitational field. At a relativistic level, as the velocity is upper limited by the
light velocity, constant force does not imply a constant acceleration; instead, it results in the above-mentioned hyperbolic motion, which denomination comes from the hyperbola that is drawn in the $ct$-plane by this kind of motion.

A one-dimensional hyperbolic motion of a particle of mass $m$ occurs as a solution of the relativistic equation of motion [6, 7]:

$$m \frac{d^2 x^\mu}{d\tau^2} = f^\mu (\tau),$$

when the external force $F$ is parallel to the velocity $v$ and it is locally constant in the proper referential frame. In (1) $f^\mu$ is the relativistic force defined as

$$f^\mu = \gamma \left( \frac{v \cdot F}{c^2}, F \right)$$

with $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ and $\beta = v/c$, (2)

where $c$ is the velocity of light.

Supposing the motion along the $z$-axis, the trajectory of a hyperbolic motion is given by

$$(z^0, z) = c^2 \frac{a}{\lambda} \left( \sinh \lambda \tau, \cosh \lambda \tau \right),$$

where $a = F/m$ is a constant proper acceleration and $\lambda = a/c$. From (3) the velocity and acceleration are given by

$$(\dot{z}^0, \dot{z}) = c \left( \cosh \lambda \tau, \sinh \lambda \tau \right) = \gamma c \left( 1, \beta \right)$$

and

$$(\ddot{z}^0, \ddot{z}) = a \left( \sinh \lambda \tau, \cosh \lambda \tau \right),$$

respectively, so that the relativistic force responsible for the hyperbolic motion is

$$f^\mu (\tau) = m(\dot{z}^0, \dot{z}) = ma \left( \sinh \lambda \tau, \cosh \lambda \tau \right).$$

The choice of the metric tensor $g^{\mu \nu}$ is such that $v^\mu v^\mu = -c^2$ for four velocity $v^\mu = \dot{x}^\mu$ and, at non-relativistic limit, $a^\mu a^\mu = a^2$ for four acceleration $a^\mu = \ddot{x}^\mu$.

The equation of motion of a classical charged point particle, including electromagnetic radiation reaction force, is given by the well-known Lorentz–Abraham–Dirac equation [1–11, 36–39],

$$m a^\mu (\tau) = f^\mu_{\text{ext}} (\tau) + f^\mu_{\text{rad}} (\tau),$$

where $f^\mu_{\text{ext}} (\tau)$ is the external four-force and

$$f^\mu_{\text{rad}} (\tau) = m \tau_0 \left( \dot{a}^\mu - \frac{1}{c^2} a^\nu a_\nu v^\mu \right),$$

with

$$\tau_0 = \frac{2}{3} \frac{e^2}{mc^3},$$

is the Lorentz–Abraham–Dirac relativistic electromagnetic radiation reaction force. The first term in (8) is known as the Schott term [4] and it is responsible for the well-known non-physical runaway solutions. The second is the Rohrlich term, related to the power radiated:

$$R = \frac{dW_{\text{rad}}}{dt} = m \tau_0 a^\nu a_\nu.$$  

A well-known condition for the hyperbolic motion, which satisfies (3)–(6), is

$$\dot{a}^\mu = \frac{1}{c^2} a^\nu a_\nu v^\mu = 0,$$  

(11)
which also implies in \( f_{\text{rad}}^\mu (\tau) = 0 \), so it seems to be easy to produce the hyperbolic motion of a charged particle imposing a locally constant external force, as in the uncharged particle case, but it could induce to a misunderstanding. A hyperbolic motion is an ideal concept that implies an eternal constant local acceleration, not existing in a real world, and what happens immediately before reaching this regime will be freezed in the final hyperbolic motion. The important result that we are going to show is that while the final force that supports the hyperbolic motion for uncharged and charged particles is equal, the composition of such forces is different. For uncharged particles it is just the external force, but for charged particles, it is composed of the sum of external force and the Rohrlich electromagnetic radiation reaction force. In other words, to have a stable hyperbolic motion for charged particles we have to get a very sensible balance between external and electromagnetic radiation reaction force, and before it condition (11) is not true. It means that what happens before is very important to get a stable hyperbolic motion regime and, although we have the same equation (1) after that, the force \( f^\mu (\tau) \) is not just \( f^\mu_{\text{ext}} (\tau) \) anymore. To figure out why, let us consider the Lorentz–Abraham–Dirac equation (7) written as [11]

\[
m \left( 1 - \tau_0 \frac{d}{d\tau} \right) a^\mu = f^\mu_{\text{ext}} (\tau) - \frac{1}{c^2} R v^\mu = K^\mu (\tau) .
\]

Formal expansion like

\[
\left( 1 - \tau_0 \frac{d}{d\tau} \right)^{-1} = 1 + \tau_0 \frac{d}{d\tau} + \tau_0^2 \frac{d^2}{d\tau^2} + \cdots
\]

enables us to get a formal solution of the Lorentz–Abraham–Dirac equation as

\[
ma^\mu (\tau) = \sum_{n=0}^{\infty} \tau_0^n \frac{d^n}{d\tau^n} K^\mu (\tau) .
\]

We can insert the mathematical identity

\[
\frac{1}{n!} \int_0^\infty s^n e^{-s} ds = 1
\]

to transform (14) into a second-order integro-differential equation

\[
ma^\mu (\tau) = \int_0^\infty e^{-s} K^\mu (\tau + \tau_0 s) ds ,
\]

which shows a possible non-causal behavior. In an explicit form, we have

\[
ma^\mu (\tau) = \int_0^\infty \left( f^\mu_{\text{ext}} - \frac{1}{c^2} R v^\mu \right) \bigg|_{\tau + \tau_0 s} e^{-s} ds .
\]

Equation (7), or the equivalent equation (17), is valid during the hyperbolic motion. On the other hand, while the motion is approaching the hyperbolic regime, as discussed above, we have the limiting process

\[
\dot{a}^\mu = \frac{1}{c^2} a^\nu a^\rho v_\rho \rightarrow 0 \Rightarrow f^\mu_{\text{rad}} (\tau) \rightarrow 0 ,
\]

such that the total force behaves like

\[
f^\mu_{\text{ext}} (\tau) + f^\mu_{\text{rad}} (\tau) \rightarrow f^\mu (\tau) ,
\]
or \( f^\mu_{\text{ext}} \rightarrow f^\mu \), so that equation (17), in this regime, can be rewritten as

\[
\int_0^\infty \left( f^\mu - \frac{1}{c^2} R v^\mu \right) \bigg|_{\tau + \tau_0 s} e^{-s} ds = f^\mu (\tau) ,
\]

recovering equation (1), in accordance with equation (7) and condition (11).
From (20), to have a hyperbolic motion, it is necessary that the applied external force goes to
\[ f_{\text{ext}}^\mu(\tau) \rightarrow f_{\text{ext}}^\mu(\tau) = \int_0^\infty f^\mu(\tau + \tau_0 s) e^{-s} ds \, , \] (21)
as well as the electromagnetic radiation reaction force goes to
\[ f_{\text{rad}}^\mu(\tau) \rightarrow f_{\text{Roh}}^\mu(\tau) = \int_0^\infty \frac{1}{e^2} \mathcal{R} v^\mu_{\tau + \tau_0 s} e^{-s} ds \, , \] (22)
such that, from (19)–(22),
\[ f_\mu(\tau) = f_{\text{ext}}^\mu(\tau) + f_{\text{Roh}}^\mu(\tau) \, , \] (23)
where \( f_\mu(\tau) \) is given by (6).

Using (6), the spatial component of equation (21) becomes
\[ f_{\text{ext}}^0(\tau) = ma^2 \left( e^{\lambda \tau} \left( \frac{1}{1 - \lambda \tau_0} \right) + e^{-\lambda \tau} \left( \frac{1}{1 + \lambda \tau_0} \right) \right) \, . \] (24)

Analogously, from equations (4)–(6) and (10), the spatial component of equation (22) becomes
\[ f_{\text{Roh}}^0(\tau) = -ma^2 \lambda \tau_0 \left( e^{\lambda \tau} \left( \frac{1}{1 - \lambda \tau_0} \right) - e^{-\lambda \tau} \left( \frac{1}{1 + \lambda \tau_0} \right) \right) \, . \] (25)

The same way, time components become
\[ f_{\text{ext}}^0(\tau) = ma^2 \left( e^{\lambda \tau} \left( \frac{1}{1 - \lambda \tau_0} \right) - e^{-\lambda \tau} \left( \frac{1}{1 + \lambda \tau_0} \right) \right) \, , \] (26)
and
\[ f_{\text{Roh}}^0(\tau) = -ma^2 \lambda \tau_0 \left( e^{\lambda \tau} \left( \frac{1}{1 - \lambda \tau_0} \right) + e^{-\lambda \tau} \left( \frac{1}{1 + \lambda \tau_0} \right) \right) \, , \] (27)
where \((1 - \lambda \tau_0) > 0\) in (24)–(27). For \((1 - \lambda \tau_0) < 0\) integrals (21)–(22) are divergent. In section 3, this condition will be discussed.

From equations (24)–(27), it is easy to see that the total force (23) satisfies (6), a condition necessary to have a hyperbolic motion. It shows that the external force necessary to produce a hyperbolic motion in neutral particles,
\[ f_{\text{ext}}(\text{neutral})^\mu(\tau) = ma \cosh \lambda \tau \, , \] (28)
is smaller than the external force (24) necessary to give the same hyperbolic motion in charged particles. All external force applied to a neutral particle is used to increase its kinetic energy,
\[ \frac{dW}{dt} = v F = ma v = ma c \sinh \lambda \tau = m c^2 \frac{d\gamma}{d\tau} \, , \] (29)
where \(\gamma = \cosh \lambda \tau\) from (4) and \(\lambda = a/c\). On the other hand, for charged particles, the external force (24), that can be written as
\[ f_{\text{ext}}(\text{charged})^\mu(\tau) = f_{\text{ext}}(\text{neutral})^\mu(\tau) - f_{\text{Roh}}^\mu(\tau) \, , \] (30)
provides the increase of kinetic energy in the same amount as given in (29) and supplies, through \(f_{\text{Roh}}(\tau)\), the energy lost carried by electromagnetic radiation.
3. Classical charged particles in a local uniform gravitational field

We saw in the previous section that a classical charged particle performing a hyperbolic motion has to be submitted to an external force given by

\[ F_{\text{ext}} = \frac{ma}{2} \left( \frac{(1 + \beta)}{(1 - \lambda \tau_0)} + \frac{(1 - \beta)}{(1 + \lambda \tau_0)} \right) \]  

(31)

where \( F_{\text{ext}} \) is the spatial component of the measurable force related to the relativistic force by \( f_{\text{ext}}(\tau) = \gamma F_{\text{ext}}(\tau) \) (see (2)). To obtain (31) observe, from (4), that \( \gamma (1 + \beta) = e^{\lambda \tau_0} \) and \( \gamma (1 - \beta) = e^{-\lambda \tau_0} \).

The SEP [10, 44] says that a particle at rest in the laboratory frame \( R_{\text{lab}} \) immersed in a uniform gravitational field \( g \) is seen by an observer in a free falling inertial frame \( R_{\text{in}} \) as performing a hyperbolic motion with local constant acceleration \( a = g \). The local constant force responsible for its hyperbolic motion is the normal force \( F_n \) that supports the particle against the gravitational force \( F_g \), so that, in absolute value, it is equal to \( mg \) for an uncharged particle. But, for a charged particle, the normal force \( F_n \) must be equal to the external force \( F_{\text{ext}} \) of equation (31) for \( \beta = 0 \):

\[ F_{\text{ext}} \rightarrow F_n = \frac{mg}{1 - \lambda^2 \tau_0^2} \]  

(32)

This result suggests that the observer in the laboratory frame \( R_{\text{lab}} \) measures the gravitational force acting on a charged particle as

\[ F_g = -\frac{mg}{1 - \lambda^2 \tau_0^2} \]  

(33)

such that

\[ m^* = \frac{m}{1 - \lambda^2 \tau_0^2} \]  

(34)

should define the passive gravitational mass \( m^* \) of a charged particle with inertial mass \( m \). Relation (34) shows that, to the SEP to be valid, the WEP is violated for classical charged particles in a stable hyperbolic motion regime.

In other words, the external force applied on uncharged particles must be slightly lower than external force applied on charged particles. The first one disposes all the external force to increase its kinetic energy, while the charged one needs an external force to supply the same kinetic energy plus the energy lost by electromagnetic radiation. As the strong version of the EP is invoked, the equivalence between the proper uniformly accelerated hyperbolic motion referential and the referential supported in the presence of a uniform gravitational field implies that the charged particle gravitational force, as well as the gravitational potential energy, must be slightly greater than for the uncharged one. This difference should be interpreted as due to their different gravitational mass and the same inertial mass. It implies that the gravitational and inertial masses of charged particles are different.

For typical charged particles, as electrons or protons, \( \tau_0 \approx 6.3 \times 10^{-24} \text{s} \) and \( \tau_0 \approx 3.4 \times 10^{-22} \text{s} \), respectively. In a field magnitude typical for a terrestrial gravitational field \( g \approx 10 \text{ m s}^{-2} \), we have \( \lambda = g/c \approx 3.3 \times 10^{-8} \text{s}^{-1} \). So, we can see that \( \lambda^2 \tau_0^2 \approx 4.3 \times 10^{-62} \) and \( \lambda^2 \tau_0^2 \approx 1.3 \times 10^{-68} \), respectively for electrons and protons, very small numbers, such that \( 1 - \lambda^2 \tau_0^2 \approx 1 \). As a consequence, the passive gravitational mass is just slightly greater than the inertial mass, \( m^* \gtrsim m \), and the gravitational and inertial mass relation defined by equation (34) is much as close to unit, \( r = m^*/m \approx 1 \). It means that there is no consequence, for practical purpose, due to this slight up deviation of passive gravitational mass in relation to the inertial mass, at least in a region with gravitational field of magnitude as considered
Figure 1. Electron gravitational and inertial mass relation, \( r = m^*/m \), as a function of the gravitational field \( g \). There is a critical point defined by \( g_c = c/\tau_0 \), which is particle dependent, and has the value \( g_c \approx 4.8 \times 10^{31} \text{m s}^{-2} \) for an electron.

above. In fact, it is true for a very long field interval, starting with \( g = 0 \) and going until it reach a very strong gravitational field of the order \( g \sim 10^{30} \text{m s}^{-2} \).

In such a strong gravitational field region, the field dependence of \( r = m^*/m \), see equation (34), starts to manifest, as we can see in figure 1. It increases very slowly and remains very close to unit, starting with \( g = 0 \) until it reaches the very strong field magnitude of the order \( g \sim 10^{30} \text{m s}^{-2} \), approaching the divergence point given by the condition \( 1 - \lambda^2 \tau_0^2 = 0 \). Then \( r \) increases fast to the infinite as the gravitational field goes to its critical value \( g_c = c/\tau_0 \). This critical field value is mass dependent, with \( g_c \approx 4.8 \times 10^{31} \text{m s}^{-2} \) and \( g_c \approx 8.8 \times 10^{34} \text{m s}^{-2} \) for electrons and protons, respectively.

The divergence of the gravitational and inertial mass relation at the critical field value signs that the SEP should not be valid in this situation. Consistently, the condition \((1 - \lambda \tau_0) > 0\) for which integrals (21)–(22) are finites implies that \((1 - \lambda \tau_0)(1 + \lambda \tau_0) = (1 - \lambda^2 \tau_0^2) > 0\), so that the investigation of the values above the critical point \( g_c \), as the mass relation \( r \) turns to be negative, is nonsense.

Where is it possible to find such a strong gravitational field? Astrophysical compact objects are the natural place to search for more intense gravitational fields. For example, white dwarf like Sirius B, with mass close to the solar mass, \( 1.0 \times M_\odot \), and radius near \( 5.5 \times 10^6 \text{m} \simeq 0.008R_\odot \), is a very compact object with a surface gravitational acceleration \( g \simeq 4.6 \times 10^9 \text{m s}^{-2} [45] \). (The solar mass and radius are \( M_\odot \approx 1.98844 \times 10^{30} \text{kg} \) and \( R_\odot \approx 6.961 \times 10^8 \text{m} \), respectively.)

Neutron stars are more compact than white dwarfs. For a mass of order \( 1.4 \times M_\odot \) there corresponds a radius of order \( R \simeq 4.4 \times 10^8 \text{m} \) with a surface gravitational acceleration \( g \simeq 2 \times 10^{12} \text{m s}^{-2} [46] \).

Extreme compact objects are of course the black holes. Caution is needed to deal with such objects, because there is no back information access beyond their event horizon, defined by the spherical surface at the Schwarzschild radius given by [46]

\[
R_S = \frac{2MG}{c^2} \simeq 2.95 \times \frac{M}{M_\odot} \text{ km}.
\]

The gravitational acceleration at the Schwarzschild surface is [46]

\[
g = \frac{MG}{R_S^2} = \frac{c^4}{4MG} \simeq 1.5 \times 10^{13} \frac{M_\odot}{M} \text{ m s}^{-2}.
\]
For a typical black hole with about ten times solar mass, $M \simeq 10 \times M_\odot$, $R_\text{S} \simeq 30\text{ km}$ and the gravitational acceleration at its Schwarzschild surface is $g \simeq 1.5 \times 10^{12}\text{ m s}^{-2}$. It is believed that galaxies’ central region shelters very massive black holes, with a mass of order $10^5 \times M_\odot$ to $10^9 \times M_\odot$. Such a black hole, with mass $M \simeq 10^5 \times M_\odot$, has $R_\text{S} \simeq 2.95 \times 10^5\text{ km}$ and gravitational acceleration at its Schwarzschild surface $g \simeq 1.5 \times 10^3\text{ m s}^{-2}$. If the mass is $M \simeq 10^9 \times M_\odot$, the Schwarzschild radius is $R_\text{S} \simeq 2.95 \times 10^9\text{ km}$ and $g \simeq 1.5 \times 10^4\text{ m s}^{-2}$. Increasing the black hole mass does not imply increasing the gravitational acceleration, because the Schwarzschild radius increases together. It shows that the astrophysical environment hardly provides us a gravitational acceleration stronger than about $g \sim 10^{13}\text{ m s}^{-2}$, which implies $\lambda^2 \tau^2_0 \sim 10^{-36}$, nothing to worry about. Only gravitational accelerations as strong as $g \sim 10^{16}\text{ m s}^{-2}$ are going to be sensible in equation (34), close to an infinite singularity. Such a too strong gravitational field is very unlikely.

On the other hand, collision of very energetic particles could take place with instantaneous acceleration that might be of such an order, simulating a local and instantly gravitation, as that needed to test the equivalence principle. Nevertheless, it seems to be more comfortable to abdicate of the principle of equivalence in such an extreme regime, where quantum effects certainly are dominant, and the gravitation, as it is a belief, is going to be unified with the other interactions.

4. Conclusion

Although the final equation of motion of classical neutral and charged particles in a hyperbolic motion regime seems to be identical, we realize that there is a fundamental difference between them. For classical charged particles, in fact, the total locally constant force is the sum of an applied force and the radiation reaction force in such a combination that leads to the same results obtained for neutral particles. An interesting implication, as the SEP is taken into account, is that the passive gravitational mass of the charged particle must be greater than its inertial mass in a very small amount. It is small enough to not be detected by any experimental or practical devices, but it helps us to figure out the condition necessary to get a hyperbolic motion regime and to understand the meaning of its equivalence with a charged particle supported at rest in a uniform gravitational field.

Until now the equality between gravitational and inertial mass was understood as the essence of the EP, a condition that we realized not to be true for charged particles, since, due to the presence of electromagnetic radiation reaction, a slight deviation of gravitational mass compared to inertial one is necessary to hold the SEP. However, a new feature comes from the gravitational and inertial mass relation behavior for a very strong gravitational field. There exists a critical, particle-dependent, gravitational field value that signifies the validity domain of the SEP. For electrons and protons these critical field values are $g_e \simeq 4.8 \times 10^{13}\text{ m s}^{-2}$ and $g_e \simeq 8.8 \times 10^{14}\text{ m s}^{-2}$, respectively. They possibly coincide where the quantum effects turn out to be relevant.

References

[1] Dirac P A M 1938 Classical theory of radiating electrons Proc. R. Soc. A 167 148–68
[2] Wheeler J A and Feynman R P 1945 Interaction with the absorber as the mechanism of radiation Rev. Mod. Phys. 17 157–81
[3] Wheeler J A and Feynman R P 1949 Classical electrodynamics in terms of direct interparticle action Rev. Mod. Phys. 21 425–33
[4] Rohrlich F 1961 The equations of motion of classical charges Ann. Phys. NY 13 93–109
[5] Weinberg S 1972 Gravitation and Cosmology—Principles and Applications of the General Theory of Relativity (New York: Wiley)
[6] Møller C 1972 The Theory of Relativity 2nd edn (Oxford: Clarendon Press)
[7] Landau L and Lifshitz E L 1975 The Classical Theory of Fields (New York: Pergamon Press)
[8] Tietelboim C, Villarroel D and Weert Ch G Van 1980 Classical electrodynamics of retarded fields and point particles Rev. Nuovo Cimento 3 1–64
[9] Birrell N D and Davies P C W 1984 Quantum Fields in Curved Space (Cambridge: Cambridge University Press)
[10] Rohrlich F 1990 Classical Charged Particles (New York: Addison-Wesley)
[11] Jackson J D 1998 Classical Electrodynamics 3rd edn (New York: Wiley)
[12] Ginzburg V L and Eroshenko Yu N 1996 Comments on the paper by A A Logunov et al. Phys. Usp. 39 81–2
[13] Denef F, Raeymaekers J, Studer U M and Troost W 1997 Classical tunneling as a consequence of radiation reaction forces Phys. Rev. E 56 3624–7
[14] Rohrlich F 2000 The equivalence principle revised Found. Phys. 30 621–30
[15] Mino Y, Sasaki M and Tanaka T 1997 Gravitational radiation reaction to a particle motion Phys. Rev. D 55 3457–76
[16] Quinn T C and Wald R M 1997 Axiomatic approach to electromagnetic and gravitational radiation reaction of particles in curved spacetime Phys. Rev. D 56 3381–94
[17] Quinn T C 2000 Axiomatic approach to radiation reaction of scalar point particles in curved spacetime Phys. Rev. D 62 064029 1–9
[18] Burko L M, Harte A I and Poisson E 2002 Mass loss by a scalar charge in an expanding universe Phys. Rev. D 65 124006 1–11
[19] Haas R and Poisson E 2005 Mass change and motion of a scalar charge in cosmological spacetimes Class. Quantum Grav. 22 739–52
[20] Dewitt B S and Brehme R W 1960 Radiation damping in a gravitational field Ann. Phys. NY 9 220–59
[21] Vodel W, Nietzche S, Neubert R and Dittus H 2002 Application of LTS-SQUIDs for testing the weak equivalence principle at the Drop Tower Bremen Physica C 372:376 154–7
[22] Mehls C, Dittus H and Lochmann S 2001 SQUID-based accelerometer and final concept for testing the equivalence principle at the Drop Tower Microgravity Sci. Technol. 13 24–8
[23] Adelberger E G, Guittach J H, Heckel B R, Hoedl S and Schlamminger S 2009 Torsion balance experiments: a low-energy frontier of particle physics Prog. Part. Nucl. Phys. 62 102–34
[24] Bantel M K and Newman R D 2000 A cryogenic torsion pendulum: progress report Class. Quantum Grav. 17 2313–8
[25] Adelberger E G 2001 New tests of Einstein’s equivalence principle and Newton’s inverse-square law Class. Quantum Grav. 18 2397–405
[26] Nordtvedt K 2003 Testing the equivalence principle with laser ranging to the Moon Adv. Space Res. 32 1311–20
[27] Tourbou P and Rodrigues M 2001 The MICROSCOPE space mission Class. Quantum Grav. 18 2487–98
[28] Nobili A M, Bramanti D, Polacco E, Rovburch I W, Comandi G L and Catalani G 2000 Galileo Galilei (GG) small-satellite projet: an alternative to the torsion balance for testing the equivalence principle on Earth and in space Class. Quantum Grav. 17 2347–9
[29] Comandi G L, Nobili A M, Bramanti D, Toncelli R, Polacco E and Chiofalo M L 2006 Dynamical response of the Galileo Galilei on the ground rotor to test the equivalence principle: theory, simulation, and experiment: I. The normal modes Rev. Sci. Instrum. 77 034501 1–15
[30] Wortten P, Torri R, Mester I C and Everitt C F 2000 The STEP payload and experiment Adv. Space Res. 25 1205–8
[31] Overduin J, Everitt F, Mester J and Worden P 2009 The science case for STEP Adv. Space Res. 43 1532–7
[32] Witteborn F C and Fairbank W M 1967 Experimental comparison of the gravitational force on freely falling electrons and metallic electrons Phys. Rev. Lett. 19 1049–52
[33] Dittus H and Lämmerzahl C 2007 Test of the equivalence principle for charged particles in space Adv. Space Res. 39 244–8
[34] Audretsch J 1981 Dirac electron in space-times with torsion: spinor propagation, spin precession, and nongeodesic orbits Phys. Rev. D 24 1470–7
[35] Higuchi A 2002 Radiation reaction in quantum field theory Phys. Rev. D 66 105004 1–12
[36] Spohn H 2000 The critical manifold of the Lorentz–Dirac equation Europhys. Lett. 50 287–92
[37] Blinder S M 2001 Classical electrodynamics with vacuum polarization: electron self-energy and radiation reaction Rep. Math. Phys. 47 269–77
[38] Rohrlich F 2001 The correct equation of motion of a classical point charge Phys. Lett. A 283 276–8
[39] Rohrlich F 2001 Why the principles of inertia and of equivalence hold despite self-interaction Phys. Rev. D 63 127701 1–3
[40] Yaghjian A D 1992 Relativistic dynamics of a charged sphere (Berlin: Springer)
[41] Goto M 2002 Equivalence principle and gravitational and inertial mass relation of classical charged particles arXiv:physics/010402v2
[42] Fulton T and Rohrlich F 1960 Classical radiation from a uniformly accelerated charge Ann. Phys. NY 9 499–517
[43] Boulware D G 1980 Radiation from a uniformly accelerated charge Ann. Phys. NY 124 169–88
[44] Rohrlich F 1963 The equivalence principle Ann. Phys. NY 22 169–91
[45] Barstow M A, Bond H E, Holberg J B, Burleigh M R, Hubeny I and Koester D 2005 Hubble Space telescope spectroscopy of the Balmer lines in Sirius B Mon. Not. R. Astron. Soc. 362 1134–42
[46] Carrol B W and Ostlie D A 2007 An Introduction to Modern Astrophysics 2nd edn (San Francisco: Addison-Wesley)