Dense QCD in a Finite Volume

Naoki Yamamoto and Takuya Kanazawa

Department of Physics, The University of Tokyo,
Tokyo 113-0033, Japan

We study the properties of QCD at high baryon density in a finite volume where color superconductivity occurs. We derive exact sum rules for complex eigenvalues of the Dirac operator at finite chemical potential, and show that the Dirac spectrum is directly related to the color superconducting gap \( \Delta \). Also, we find a characteristic signature of color superconductivity: an X-shaped spectrum of partition function zeros in the complex quark mass plane near the origin, reflecting the \( Z(2)_L \times Z(2)_R \) symmetry of the diquark pairing. Our results are universal in the domain \( \Delta^{-1} \ll L \ll m_\pi^{-1} \) where \( L \) is the linear size of the system and \( m_\pi \) is the pion mass at high density.

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Revealing Quantum Chromodynamics (QCD) in the regime of finite temperature \( (T) \) and chemical potential \( (\mu) \) is important for understanding a wide range of phenomena from ultrarelativistic heavy ion collisions, the early Universe, and neutron stars to possible quark stars \[1\]. A lot of theoretical progress has been made by the first-principles lattice QCD Monte Carlo simulations in the study of the finite-\( T \) regime \[2\]. However, the application of the lattice technique to QCD at finite \( \mu \) is still hampered by the notorious fermion sign problem: calculation of the QCD partition function requires dealing with a path integral with a measure including a complex fermion determinant. This is the main reason why our understanding of the properties of QCD at finite \( \mu \) is still immature, except at asymptotic high \( \mu \) where the ground state is shown to be the most symmetric three-flavor \( (N_f = 3) \) color superconductivity (CSC), i.e., the color-flavor locked (CFL) phase \[3, 4\] by using the weak coupling calculations.

In this Letter, we demonstrate exact analytical results for QCD at high \( \mu \) specific for a large but finite volume. By matching the partition function of QCD against that of the effective theory of CSC, we derive exact sum rules for the Dirac eigenvalues (Dirac spectrum) as well as the spectrum of partition function zeros (Lee-Yang zeros) in the complex quark mass \( m \). As is well known at \( \mu = 0 \), the Dirac spectrum and the Lee-Yang zeros spectrum of QCD in a finite volume are very closely related to the chiral symmetry breaking: the chiral condensate \( \langle \bar{q}q \rangle \) is directly connected to the Dirac spectrum by the exact relations such as the Banks-Casher relation \[2\] and the Leutwyler-Smilga sum rules \[7\]. Also, a nonzero \( \langle \bar{q}q \rangle \) implies the existence of a line of the Lee-Yang zeros going through \( m = 0 \) in the complex \( m \)-plane independent of \( \mu \) \[8\]. Nevertheless, such exact relations at finite \( \mu \) and the relevance of the Lee-Yang zeros spectrum to the CSC have not been fully understood.

As we shall show below, the Dirac spectrum at high \( \mu \) is intimately related to the CSC gap \( \Delta \), rather than to \( \langle \bar{q}q \rangle \), through our spectral sum rules. Also, a nonzero gap \( \Delta \) necessitates an X-shaped cut of Lee-Yang zeros near \( m = 0 \). In particular, the \( Z(2)_L \times Z(2)_R \) symmetry of the diquark pairing plays a crucial role on both Dirac and Lee-Yang zeros spectra. Together with the exact results at \( \mu = 0 \) \[2\], we expect that our results impose strong constraints on their possible spectra and provide important insights to the properties of QCD at finite \( \mu \).

In the following, we will focus on QCD with \( N_f = 3 \) (light up, down and strange quarks) at finite quark chemical potential \( \mu \) living on the four-dimensional torus \( V_4 = L \times L \times L \times \beta \) with \( \beta = 1/T \sim L \). Let us consider the Euclidean QCD Lagrangian defined as \( Z_{QCD} = Z_{QCD}^{(f)}(\bar{q}Dq)q + \mathcal{L}_q \) with \( \mathcal{L}_q = \frac{1}{2} F_{\mu \nu} F^{\mu \nu} \) and the Dirac operator \( \hat{D} = \gamma^\mu (\partial_\mu + i g A_\mu) + \mu \gamma_0 \). Here \( q \) is the quark field and \( A_\mu = A_\mu^a t^a \) is the gluon field with the color \( SU(3)_C \) generators \( t^a \) \((a = 1, 2, \cdots, 8)\). \( M \) is the complex quark mass matrix, \( g \) is the QCD coupling constant and \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu] \). Since \( \hat{D} \) is not anti-Hermite with \( \mu > 0 \), its eigenvalues \( i \lambda_n \) are generally complex values, whereas \( i \lambda_n \) are pure imaginary at \( \mu = 0 \). Even so, \( \hat{D} \) preserves the chirality, \( \{\gamma_\mu, \hat{D}\} = 0 \). The chirality ensures that if \( i \lambda_n \) is the eigenvalue of \( \hat{D} \), then so is \( -i \lambda_n \).

The QCD partition function \( Z_{QCD} \) involves a sum over the different topological sectors of the gauge-field configurations characterized by the integer topological charge \( \nu \) as \( Z_{QCD} = \sum_{\nu} e^{i\nu \theta} Z_{\nu} \). At \( \mu \gg \Lambda_{QCD} \) \( (\Lambda_{QCD} \text{ the typical scale of QCD}) \), however, the topological susceptibility is highly suppressed as \( \langle \nu^2 \rangle \propto (\Lambda_{QCD}/\mu)^8 \) \[9\] owing to the screening of instantons in the medium together with the asymptotic freedom of QCD. Thus, we can focus on the topological sector \( \nu = 0 \) alone.

The QCD partition function with \( \nu = 0 \) can be written in the functional integral using the symmetry \( i \lambda_n \leftrightarrow -i \lambda_n \).

\[
Z_{QCD} = \left\langle \prod_{\text{Rel}(\lambda_n) > 0} \det \left( 1 + \frac{M^T M}{\lambda_n^2} \right) \right\rangle, \tag{1}
\]

where \( \langle O \rangle = \int [dA] O e^{-S_T(\prod_n \lambda_n^2)^N_f} / \int [dA] e^{-S_T(\prod_n \lambda_n^2)^N_f} \).
is the average of $\mathcal{O}$ over all gauge configurations with $S_g$ being the classical action of the gluon field. $Z_{QCD}$ is normalized so that $Z_{QCD} = 1$ when quark masses are turned off.

We shall give the partition function $Z_{\text{EFT}}$ from the effective theory of the color superconductivity (CSC). For definiteness, we consider the most predominant diquark pairing, the color-flavor locked (CFL) phase \cite{3}: \( \langle (ql)C(qL) \rangle \sim \epsilon_{abc}c_{ijk}[d^a_i]_{ai} \) and \( \langle (qq)C(qR) \rangle \sim \epsilon_{abc}c_{ijk}[d^a_i]_{ai} \) where \( i, j, k (a, b, c) \) are the flavor (color) indices, and \( C \) is the charge conjugation matrix. The remarkable feature here is that chiral symmetry is dynamically broken by the diquark condensate: the symmetry breaking pattern of the CFL phase at asymptotic high \( m \) is \( SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A \rightarrow SU(3)_{C+L+R} \times Z(2)_L \times Z(2)_R \) \cite{3}. The \( Z(2)_L \times Z(2)_R \) symmetry left reflects the fact that we can change the sign of \( c \), \( s \) in the \( SU(3)_C \) symmetry left.

The first condition in Eq. (2) follows by comparing the contribution to $Z_{\text{EFT}}$ of the pions, \( e^{-m_{\pi}L} \), to that of the other heavier particles, \( e^{-\Delta L} \). This condition allows us only to deal with the pions described by the CFL effective Lagrangian. On the other hand, the second condition in Eq. (2) means that the Compton wavelength of the pions is much larger than the linear size of the box, so that the CFL effective Lagrangian can be truncated to its zero momentum sector. Note that the second condition is automatically satisfied at sufficiently high \( m \) with $L$ and quark mass $m$ fixed, since \( m_{\pi} \sim \Delta m/\mu \) (see Eq. (3) below) together with the relation \( \Delta \sim \mu \exp \left(-\frac{m_\pi^2}{\Lambda_{1/4}^2}\right) \). Note also that, in the domain $\Delta \ll T_c$, temperature \( T \) is low enough for the CSC to be realized since $T \ll \Delta \sim T_c$ with $T_c$ being the critical temperature of the CSC.

We give the parameter $Z_{\text{EFT}}$ from the effective theory of the color superconductivity (CSC). For definiteness, we consider the most predominant diquark pairing, the color-flavor locked (CFL) phase \cite{3}: \( \langle (ql)C(qL) \rangle \sim \epsilon_{abc}c_{ijk}[d^a_i]_{ai} \) and \( \langle (qq)C(qR) \rangle \sim \epsilon_{abc}c_{ijk}[d^a_i]_{ai} \) where \( i, j, k (a, b, c) \) are the flavor (color) indices, and \( C \) is the charge conjugation matrix. The remarkable feature here is that chiral symmetry is dynamically broken by the diquark condensate: the symmetry breaking pattern of the CFL phase at asymptotic high \( m \) is \( SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A \rightarrow SU(3)_{C+L+R} \times Z(2)_L \times Z(2)_R \) \cite{3}. The \( Z(2)_L \times Z(2)_R \) symmetry left reflects the fact that we can change the sign of \( c \), \( s \) in the \( SU(3)_C \) symmetry left.

The CFL effective Lagrangian in the Minkowski spacetime up to the leading order $O(M^2)$ is given by \cite{14}:

\[
\mathcal{L}_{\text{EFT}} = \frac{f^2}{4} \text{Tr}[\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2_\pi \partial_i \Sigma \partial^i \Sigma^\dagger]
\]

\[
+ \frac{3f^2}{4} \left[ \partial_0 V \partial_0 V^* - v^2_{\pi} \partial_0 V \partial_0 V^* \right]
\]

\[
+ \frac{3\Delta^2}{4\pi^2} \left[ V(T \mathcal{M} \Sigma^\dagger)^2 - V(T \mathcal{M} \Sigma^\dagger)^2 + \text{H.c.} \right],
\]

where $\Sigma = \exp(i \pi^a/\lambda_F)$ and $V = \exp(2\pi m'/\sqrt{6} f_{\pi})$ are the pion and $\eta'$ fields respectively, $f_\pi$ ($f_{\pi'}$) is the pion ($\eta'$) decay constant, $\pi_\tau$ ($\eta_{\pi'}$) is the pion ($\eta'$) velocity, $\lambda^a$ ($a = 1, 2, \ldots, 8$) are the Gell-Mann matrices, and the covariant derivative including the effective chemical potential (Bedaque-Schafer term \cite{15}) is given by $\nabla_0 \Sigma = \partial_0 \Sigma + iq \left( \frac{M_\Sigma}{2p_F} \right) \Sigma - i\Sigma \left( \frac{M_\Sigma}{2p_F} \right)$ with the Fermi momentum $p_F$. The quantities $f_\pi$ and $f_{\pi'}$ can be perturbatively computed at $\mu \gg \Lambda_{QCD}$ as $f^2_{\pi}/p_F^2 = \frac{21-8\log 2}{36\pi^2}$ and $f^2_{\pi'}/p_F^2 = \frac{3}{8\pi^2}$ \cite{14}.

In Eq. (3), we have neglected the mass term of order $O(M)$, since this term originates from the instanton contribution and is suppressed at asymptotic high $\mu$ \cite{10}. Thus, the leading mass term in the CFL effective Lagrangian is $O(M^2)$, unlike the $O(M)$ term in the usual chiral Lagrangian at low $\mu$. A more intuitive explanation for this fact is that $Mqq$ is prohibited by the $Z(2)_L \times Z(2)_R$ symmetry, but $(Mqq)$ is not.

In the domain $L \ll m_{\pi}^{-1}$, one can neglect the contribution of the kinetic term. Then the partition function for the CFL effective Lagrangian reads:

\[
Z_{\text{EFT}} = \int d\Sigma \exp \left( \frac{3\Delta^2}{4\pi^2} \left[ (\text{Tr}(M \Sigma^\dagger)^2 - \text{Tr}(M \Sigma^\dagger)^2) \right] \det \Sigma + \text{H.c.} \right),
\]

where the integral is over $\Sigma \equiv \Sigma \psi \in U(3)$ and $Z_{\text{EFT}}$ is normalized so that $Z_{\text{EFT}} = Z_{QCD} = 1$ in the chiral limit. In Eq. (4), we have also neglected the effect of the effective chemical potential. [If one includes it, Eq. (4) can be expanded in terms of not only $(V/D^2)^2O(M^4)$ but also $VO(M^4)$. In the domain $\Delta^{-1} \ll L$, however, the latter is negligible.]

Owing to the property of $\Sigma \in U(3)$, \( [(\text{Tr}(M \Sigma^\dagger)^2 - \text{Tr}(M \Sigma^\dagger)^2)] \det \Sigma = 2\det M \det \text{Tr}(M^{-1} \Sigma) \), Eq. (4) can be expressed analytically as shown in Ref. \cite{10}. In particular, in the flavor symmetric case $M = m_1$, one can evaluate $Z_{\text{EFT}}$ in a simpler form using Weyl’s formula \cite{7,17}:

\[
Z_{\text{EFT}} = \det_{0 \leq i, j \leq 2} \left[ I_{j-i}(x) \right],
\]

where $I_j(x)$ is the modified Bessel function and $x = 3V_\mu m^2 \Delta^2/\pi^2$. The combination of $m^2 \Delta^2$ is expected, since $m^2$ acts as a source for $\Delta^2$. It should be remarked that the expression (5) is exactly the same form as the
partition function $Z_0$ with $\nu = 0$ at $\mu = 0$ which is given by Eq. (5) with the replacement of the argument: $x \to x' = V_4 \rho_i(\bar{q}q)$. This is a novel correspondence between the CSC phase and the hadronic phase, and may have relevance to the idea of their continuity [15]. In Table. I, we summarize our main results at $\mu \gg \Lambda_{\text{QCD}}$ below compared with the results at $\mu = 0$ in Ref. [7].

Expanding in terms of quark mass $M$ and performing the group integral over $\Sigma \in U(3)$ order by order, Eq. (1) reduces to the following form up to $\mathcal{O}(M^0)$:

$$Z_{\text{EFT}} \sim 1 + \frac{3}{8} \left( V_4 \frac{\Delta^2}{\pi^2} \right)^{2} \left[ (\text{Tr} M^4)^{2} - \text{Tr} (M^4 M M^4 M) \right]. \tag{6}$$

Using the relation, $\text{det}[1+e] = 1 + \text{Tr} e + \frac{1}{2}[(\text{Tr} e)^2 - \text{Tr} e^2] + \mathcal{O}(e^3)$, one can expand the QCD partition function (1) in terms of the quark mass matrix $M$. Then one obtains the spectral density for the Dirac eigenvalues $i\lambda_n$ by matching this expansion against Eq. (4). By rescaling $z_n = \sqrt{V_4} \Delta \lambda_n$, the results read

$$\left\langle \sum_n \left( \frac{1}{z_n} \right) \right\rangle = \left\langle \left( \sum_n \frac{1}{z_n^2} \right) \right\rangle = \frac{3}{4 \pi^2}, \tag{7}$$

$$\left\langle \sum_n \left( \frac{1}{z_n^2} \right) \right\rangle = \left\langle \left( \sum_n \frac{1}{z_n^2} \right) \right\rangle = \left\langle \left( \sum_n \frac{1}{z_n^2} \right) \left( \sum_n \frac{1}{z_n^4} \right) \right\rangle = 0, \tag{8}$$

where the summation $\sum'$ is taken over $z_n$ satisfying $\Re(z_n) > 0$ and $|z_n| \lesssim \sqrt{V_4} \Delta^2$ ($|\lambda_n| \lesssim \Delta$). These relations are highly nontrivial, since sums of inverse powers of complex $\lambda_n$ with the average taken over the gauge configurations give the real value involved with the CSC gap $\Delta$. In particular, the sums in Eq. (8) are identically zero, which is a direct consequence of the $Z(2)_L \times Z(2)_R$ symmetry of the quarks at high $\mu$. This situation should be compared with QCD at $\mu = 0$ [21] (at low $\mu$ [19]), where sums of inverse powers of real $\lambda_n$ (complex $\lambda_n$) take positive values involved with the chiral condensate $\langle \bar{q}q \rangle$. As in the case of $\mu = 0$ [20, 21], our spectral sum rules must be universal (i.e., independent of microscopic details) in the domain (2).

By using the spectral density $\rho(\lambda) = \langle \sum_n \delta^2 (\lambda - \lambda_n) \rangle$, the first sum rule in Eq. (7) reduces to the relation:

$$\int_{C_+} \frac{dz}{\sqrt{V_4} \Delta^2} \rho \left( \frac{z}{\sqrt{V_4} \Delta^2} \right) = \frac{3}{4 \pi^2}, \tag{9}$$

where $C_+ = \{ z : \Re(z) > 0 \}$ and $d^2 z = d(\Re z) d(\Im z)$. This implies the existence of the microscopic limit of the spectral density defined as

$$\rho_\rho(z) = \lim_{V_4 \to \infty} \frac{1}{V_4 \Delta^2} \rho \left( \frac{z}{\sqrt{V_4} \Delta^2} \right). \tag{10}$$

From Eq. (9), we find that the microscopic spectral density at high $\mu$ is governed by the CSC gap $\Delta$. Also Eq. (9) shows that the linear spacing of eigenvalues $\delta \lambda_n$ in the complex $\lambda$-plane satisfies $\delta \lambda_n \propto 1/\sqrt{V_4}$. Since $\delta \lambda_n \propto 1/V_4$ at low $\mu$ with $\langle \bar{q}q \rangle \neq 0$ [7], and $\delta \lambda_n \propto 1/V_4^{1/4}$ in a free theory, our result indicates a sizable deformation of the Dirac spectrum due to the dynamics of the CSC. We expect that the random matrix theory (RMT) [21] or the supersymmetric approach [22] incorporating the CSC and the symmetries of the CFL not only reproduce the above results, but also clarify the concrete form of the microscopic spectral density $\rho_\rho$.

Let us consider the partition function zeros (Lee-Yang zeros) in the complex $m$-plane in the flavor-symmetric case. Using the asymptotic form of $I_\nu(x)$, we find the partition function (11) for $|x| \gg 1$ as

$$Z_{\text{EFT}}(x = -iz) \sim z^{-5/2} \cos \left( 4z - \frac{3 \pi}{4} \right). \tag{11}$$

Then, the Lee-Yang zeros for $|x| \gg 1$ are given by $x = -i(n + \frac{1}{2})$ (n $\in \mathbb{Z}$). Remembering $x = 3V_4 m^2 \Delta^2/\pi^2$, the zeros are spaced along the lines

$$\text{Re}(m) = \pm \text{Im}(m), \tag{12}$$

at $\mu \gg \Lambda_{\text{QCD}}$. In the thermodynamic limit $V_4 \to \infty$, the density of the zeros increases and they join into a cut in the vicinity of massless limit $m = 0$. However, the cut does not go through $m = 0$, since $dx/dm = 6V_4 m \Delta^2/\pi^2$ and the density of the zeros vanishes at $m = 0$.

In Fig. 1(a), we draw spectra of the Lee-Yang zeros in the complex $m$-plane near $m = 0$: the spectrum (a) at $\mu \gg \Lambda_{\text{QCD}}$ is the exact result obtained above. For comparison, we show the spectra (b) at $\mu = 0$ previously obtained exactly [22], and (c) for $\mu > \mu_c$ obtained from the RMT [3], where $\mu_c$ is the critical chemical potential of chiral symmetry restoration. In case (b), the density of the zeros at $m = 0$ is finite as $dx/dm = V_4/|\bar{q}q|$, and the chiral condensate $\langle \bar{q}q \rangle$ takes nonzero value. As $\mu$ increases, the result (c) shows that the zeros will move away from the origin and chiral symmetry restores [9].

Our exact result, however, demonstrates that the scenario (c) of the RMT should suffer from dramatic modifications if the effects of the CSC is taken into account: even at $\mu \gg \Lambda_{\text{QCD}}$, there exists an X-shaped cut (11) near $m = 0$ as shown in Fig. 1(a), reflecting the $Z(2)_L \times Z(2)_R$ symmetry of the diquark pairing. The distinctive feature of the cut (11) is that there is a discontinuity in the $m^2$-plane at $m = 0$ along the real axis.

| $\mu$ | $\delta \lambda_n$ | Lee-Yang zeros |
|-------|------------------|---------------|
| $\gg \Lambda_{\text{QCD}}$ | $\Delta^2 \propto |d_{\gamma}|^2/\mu^4$ | $m^2 \propto 1/\sqrt{V_4}$, $\text{Re}(m) = \pm \text{Im}(m)$ |
| $0$ | $|\langle \bar{q}q \rangle|$ | $m \propto 1/\sqrt{V_4}$, $\text{Re}(m) = 0$ |

TABLE I: The summary of our main results at $\mu \gg \Lambda_{\text{QCD}}$ compared with the results at $\mu = 0$ in Ref. [7]: order parameters, the corresponding source terms, the linear spacing of eigenvalues $\delta \lambda_n$, and the spectra of partition function zeros (Lee-Yang zeros) in the complex $m$-plane, respectively.
but not in the $m$-plane, which is related to the fact that $\Delta \neq 0$ and $\langle \bar{q}q \rangle = 0$ at $\mu \gg \Lambda_{\text{QCD}}$. How the spectrum of the Lee-Yang zeros evolve from (b) to (a) as $\mu$ increases is beyond the scope of this paper and needs further investigations. Note that summing over the topological sector $\nu$ would also change the scenario (c) where $\nu = 0$ alone is considered.

![Graph](image)

**FIG. 1:** The spectra of the Lee-Yang zeros in the complex $m$-plane in the vicinity of $V_4 \to \infty$: (a) $\mu \gg \Lambda_{\text{QCD}}$ (exactly obtained in our analysis), (b) $\mu = 0$ [23], and (c) $\mu > \mu_c$ with the critical $\mu_c$ of chiral symmetry restoration obtained from the random matrix theory [9]. The density of the zeros fades away as $m \to 0$ in case (a), while that remains constant in case (b).

It is important to generalize our spectral sum rules or to directly investigate the distributions of the partition function zeros at lower baryon densities. One can, e.g., match the QCD partition function at finite density against the effective theory of the generalized pions [24] in the entire span of the density where the microscopic regime can be defined as $m_{\pi}^{-1} \ll L \ll m_{\rho}^{-1}$ according to Ref. [12]. Also the generalization of our spectral sum rules to QCD-like theories, such as the two-color QCD at high density, would be an interesting problem to be investigated [25], which can be tested on the lattice QCD simulation.

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