HIGH ENERGY ELASTIC pp SCATTERING IN ADDITIVE QUARK MODEL

Yu.M. Shabelski and A.G. Shuvaev

Petersburg Nuclear Physics Institute, Kurchatov National Research Centre
Gatchina, St. Petersburg 188300, Russia

E-mail: shabelsk@thd.pnpi.spb.ru
E-mail: shuvaev@thd.pnpi.spb.ru

Abstract

High energy pp elastic scattering is treated in the framework of Additive Quark Model. The reasonable agreement with experimental data is achieved with natural parameters for the strong matter distribution inside proton.

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1 Introduction

Regge theory provides a useful tool for phenomenological description of high energy hadron collisions [1, 2, 3, 4]. The quantitative predictions of Regge calculus are essentially dependent on the assumed coupling of participating hadrons to Pomeron. In this paper high energy elastic pp scattering data including the recent LHC ones are analyzed in terms of simple Regge exchange approach in the framework of Additive Quark Model (AQM) [5, 6]. In AQM baryon is treated as a system of three spatially separated compact objects – constituent quarks. Each constituent quark is colored, has internal quark-gluon structure and finite radius that is much less than the radius of proton, \( r_q \ll r_p \). This picture is in good agreement both with \( SU(3) \) symmetry of strong interaction and the quark-gluon structure of proton [7, 8]

The three constituent quarks are assumed in AQM to be incident particles in pp scattering. Elastic amplitudes for the large energy \( s = (p_1 + p_2)^2 \) and small momentum transfer \( t \) are dominated by the Pomeron exchange. The single \( t \)-channel exchange
results into \( s \)-channel amplitude of constituent quarks scattering

\[
M_{qq}^{(1)}(s, t) = \gamma_{qq}(t) \cdot \left( \frac{s}{s_0} \right)^{\alpha_P(t) - 1} \cdot \eta_P,
\]

where \( \alpha_P(t) = \alpha_P(0) + \alpha_P' \cdot t \) is the Pomeron trajectory specified by the intercept, \( \alpha_P(0) \), and slope, \( \alpha_P' \), values, the (positive) signature factor,

\[
\eta_P = i - \tan^{-1}\left( \frac{\pi \alpha_P(t)}{2} \right),
\]
determines the complex structure of the amplitude. The factor \( \gamma_{qq}(t) = g_1(t) \cdot g_2(t) \) has the meaning of the Pomeron coupling to the beam and target particles, the functions \( g_1, g_2(t) \) being the vertices of constituent quark-Pomeron interaction (filled circles in Fig. 1). The one-Pomeron exchange between two protons includes the sum over all possible exchanges between the quark pairs [5, 6]. Each term in the sum has a form (1) with the functions \( g_1, g_2(t) \) attributed to the individual constituent quarks.

Due to factorization of longitudinal and transverse degrees of freedom the longitudinal momenta are integrated over separately in high energy limit. After this the transverse part of the quark distribution is actually relevant only. Writing the proton wavefunction in the transverse momentum space as \( \psi(k_1, k_2, k_3) \), where \( k_i \) are the quark transverse momenta,

\[
\int |\psi(k_1, k_2, k_3)|^2 \delta^{(2)}(k_1 + k_2 + k_3) d^2k_1 d^2k_2 d^2k_3 = 1,
\]

the proton-Pomeron vertex \( F_P(Q, 0, 0), t = -Q^2 \), is given by the overlap function

\[
F_P(Q_1, Q_2, Q_3) = \int \psi^*(k_1, k_2, k_3) \psi(k_1 + Q_1, k_2 + Q_2, k_3 + Q_3) \times \delta^{(2)}(k_1 + k_2 + k_3) d^2k_1 d^2k_2 d^2k_3.
\]

The function \( F_P(Q, 0, 0) \) plays a role of proton formfactor for strong interaction in AQM (see section 2).

In what follows we assume the Pomeron trajectory in the simplest form

\[
\left( \frac{s}{s_0} \right)^{\alpha_P(t) - 1} = e^{\Delta \xi} e^{-r_q^2 q^2}, \quad \xi \equiv \ln \frac{s}{s_0}, \quad r_q^2 \equiv \alpha' \cdot \xi.
\]

The value \( r_q^2 \) defines the radius of quark-quark interaction while \( s_0 = (9 \text{ GeV})^2 \) has the meaning of typical energy scale in Regge theory. Putting together all 9 equal quark-quark contributions (one of them is shown in Fig. 1a) we get the first order elastic \( pp \) amplitude

\[
M_{pp}^{(1)} = 9 \left( \gamma_{qq} \eta_P e^{\Delta \xi} \right) e^{-r_q^2 Q^2} F_P(Q, 0, 0)^2.
\]
Actually the formula (4) with a single Pomeron gives amplitude in the impulse approximation [5]. Similarly to light nuclear scattering the multipomeron exchange should be added (see Glauber theory [9, 10]) giving rise to the terms $M_{pp}^{(2)}$, $M_{pp}^{(3)}$ etc, so that the total amplitude

$$M_{pp} = \sum_n M_{pp}^{(n)}.$$  

If $r_q/r_p \to 0$ the multiple interactions become negligible leaving in the sum the first term only.

The optical theorem, that relates the total elastic cross section and imaginary part of the amplitude, in the normalization adopted here reads

$$\sigma_{pp}^{\text{tot}} = 8\pi \text{Im} M_{pp}(s, t = 0).$$

The differential cross section is evaluated in this normalization as

$$\frac{d\sigma}{dt} = 4\pi |M_{pp}(s, t = 0)|^2 = 4\pi \left[ (\text{Re} M_{pp}(s, t = 0))^2 + (\text{Im} M_{pp}(s, t = 0))^2 \right]. \quad (5)$$

Interference of contributions generated by various number of Pomerons leads to the occurrence of local minima in differential elastic cross section. Experimentally the minimum at $t \simeq 0.53 \text{ GeV}^2$ for the energy $\sqrt{s} = 7 \text{ TeV}$ is well observed at LHC [14, 15]. The minima at another $t$ are also possible. Basically there is interplay between the minima in real and imaginary parts of the amplitude in the expression (5) so that the minimum in the imaginary part could be filled by the large real part at the same $t$.

The present paper aims to give theoretical description of the experimental $pp$ elastic scattering data in the energy interval $546 \text{ GeV} \div 7 \text{ TeV}$ in the framework of AQM.

2 Elastic Scattering Amplitude in AQM

In AQM there are a total 9 orders of interactions. The first order is the sum of all interactions between single $qq$ pairs. It contains 9 terms. Similar to Glauber theory [9, 10] one has to rule out the multiple interactions between the same quark pair. AQM permits the Pomeron to connect any two quark lines only once. It crucially decreases the combinatorics leaving the diagrams with no more than 9 effective Pomerons. Examples of various order diagrams are shown in Fig [11, 12].

\footnote{Note that one $qq$ pair interaction in AQM may include the contributions of several Gribov’s Pomerons [16]. We assume that high energy $qq$ scattering is described by single effective Pomeron exchange between each $qq$ pair, the parameters of this effective Pomeron could be different from those of Gribov’s bare Pomerons.}
Let $q_i$ be the transverse momentum carried by the $i$-th effective Pomeron, $Q_k$ and $Q'_l$ denote the momenta transferred to the quark $k$ from the target proton or quark $l$ from the beam one during the scattering process. If no Pomerons are attached to the quark $j$, that is it does not interact, then $Q_j = 0$. If only one Pomeron carrying momentum $q_i$ is attached to the quark $j$, then $Q_j = q_i$. If two Pomerons with momenta $q_i$ and $q_k$ are coupled to it, then $Q_j = q_i + q_k$. In other words,

\[ Q_k = \sum q_i \quad \text{if Pomeron} \ i \ \text{is attached to quark} \ k, \]
\[ Q'_l = \sum q_{i'} \quad \text{if Pomeron} \ i' \ \text{is attached to quark} \ l. \]

With these notations the $n$ order amplitude is equal to

\[
M^{(n)} = i^{n-1} \left( \gamma_{qq} \eta_P e^{\Delta \cdot \xi} \right)^n \int \frac{d^2 q_1}{\pi} \ldots \frac{d^2 q_n}{\pi} \delta^{(2)}(q_1 + \ldots + q_n - Q) \times e^{-r^2(q_1^2 + \ldots + q_n^2)} \frac{1}{n!} \sum_{\text{n connections}} F_P(Q_1, Q_2, Q_3) F_P(Q'_1, Q'_2, Q'_3),
\]

where the sum in the last factor is taken over all distinct ways to connect the pairs of beam and target quarks by $n$ effective Pomerons, each pair being connected no more than once. The permutations of identical Pomerons in the integrals is compensated by $1/n!$ in front the sum.

There are two types of diagrams in the second order,

\[
\frac{1}{2!} \sum_{\text{2 connections}} F_P(Q_1, Q_2, Q_3) F_P(Q'_1, Q'_2, Q'_3)
= 18 F_P(Q, 0, 0) F_P(q_1, q_2, 0) + 18 F_P(q_1, q_2, 0) F_P(q_1, q_2, 0), \quad Q = q_1 + q_2,
\]

where the first term comes from the diagrams with both Pomerons coupled to the same quark line whereas in the second one they connect different lines (Fig. 1b,c).
The third order sum is

\[ \frac{1}{3!} \sum_{3 \text{ connections}} F_P(Q_1, Q_2, Q_3) F_P(Q'_1, Q'_2, Q'_3) \]

\[ = 6 F_P(Q, 0, 0) F_P(q_1, q_2, q_3) + 9 F_P(q_1 + q_2, 0, q_3) F_P(q_1 + q_3, 0, q_2) \]
\[ + 9 F_P(q_1 + q_2, 0, q_3) F_P(q_2 + q_3, 0, q_1) + 6 F_P(q_1 + q_2, 0, q_3) F_P(q_1, q_3, q_2) \]
\[ + 18 F_P(q_1 + q_3, 0, q_2) F_P(q_2 + q_3, 0, q_1) + 15 F_P(q_1 + q_3, 0, q_2) F_P(q_1, q_3, q_2) \]
\[ + 15 F_P(q_2 + q_3, 0, q_1) F_P(q_1, q_3, q_2) + 6 F_P(q_3, q_2, q_1) F_P(q_1, q_2, q_3), \]

\[ Q = q_1 + q_2 + q_3. \]

Here the first term arises when all three Pomerons is attached to the same line, in the last term they connect three different quarks, diagrams (a) and (d) in the Fig. 2. The second and the third terms correspond to diagrams (b) and (c). The rest terms are provided by various permutations of the Pomeron lines in these diagrams, in particular, by flipping a diagram as a whole. The numerical coefficients encounter the number of connections resulting into equal expressions after variables changing in the integrals (6).

In the highest order, containing 9 effective Pomerons,

\[ \frac{1}{9!} \sum_{9 \text{ connections}} F_P(Q_1, Q_2, Q_3) F_P(Q'_1, Q'_2, Q'_3) \]

\[ = F_P(q_1 + q_2 + q_3, q_4 + q_5 + q_6, q_7 + q_8 + q_9) F_P(q_1 + q_4 + q_7, q_2 + q_5 + q_8, q_3 + q_6 + q_9), \]

each quark from the beam proton interacts with the quark from the target one. All the other orders, \( 3 < n < 9 \), have similar but more lengthly structure due to large combinatorics to redistribute \( q_1, \ldots, q_n \) momenta among \( Q_i \) and \( Q'_i \) groups.\(^2\)

\(^2\)The full set of similar diagrams for \( pp \) and \( αα \) scattering can be found in (11, 12).
The function $F_P(q_1, q_2, q_3)$ is determined by the proton wavefunction in terms of the constituent quarks \[3\]. We model the transverse part of the wavefunction in a simplest form of two gaussian packets,

$$\psi(k_1, k_2, k_3) = N [e^{-a_1(k_1^2 + k_2^2 + k_3^2)} + C e^{-a_2(k_1^2 + k_2^2 + k_3^2)}],$$

normalized to unity \[2\]. One packet parametrization, $C = 0$, is insufficient to reproduce experimental data as imposing too strong mutual dependence between the total cross section, minimum position and the value of the slope at $t = 0$ of the differential cross section.

All parameters used in the calculation naturally fall into two different kinds: the parameters of the Pomeron and those specifying the structure of colliding particles. The former type, $\Delta, \alpha', \gamma_{qq}$, refers to the high energy scattering theory while the latter, $a_{1,2}$ and $C$, details the matter distribution inside the proton in low energy limit (similar to density distribution in atomic nuclei).

With the chosen values of $a_{1,2}$ and $C$ (see below) the wavefunction \[7\] results into the density shown in Fig. 3. It looks rather naturally having mean squared radius $\sqrt{<r^2>} = 0.68$ fm, that is close to the electromagnetic radius. Some discrepancy can be explained by the difference in the distributions of charged and strong interacting matter inside the proton.

![Figure 3: The radial density distribution in the proton calculated with the wavefunction (7) and parameters (8).](image)
3 Comparison with the experimental data

Here we present the numerical results obtained by summing over all 9 orders of interaction of the Additive Quark Model for the energies in the interval $\sqrt{s} = 546$ GeV ÷ 7 TeV. They are compared with the experimental data taken from [14, 15, 19, 21, 22, 23, 24]. The following set of parameters have been used

$$\Delta = 0.107 \quad \alpha' = 0.32 \text{ GeV}^{-2} \quad \gamma_{qq} = 0.44 \text{ GeV}^{-2} \quad C = 0.132$$

Figure 4: The differential cross section of elastic $pp$ scattering at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 1800$ GeV. The experimental points have been taken from [23, 21, 24, 13, 14, 15].

The calculated values of the total cross section $\sigma^{tot}$, of $d\sigma/dt(t = 0)$, of the slope of the elastic cross section $B (d\sigma/dt \sim \exp(-B \cdot t))$, and of the ratio $\rho = \Re \sigma/\Im \sigma(t = 0)$ are presented in the Table 1 along with experimental data. The theoretical slope is calculated within the interval $|t| = 0 - 0.1 \text{ GeV}^2$. 
Figure 5: The differential cross section of elastic $pp$ scattering at $\sqrt{s} = 546$ GeV and $\sqrt{s} = 62$ GeV. The experimental points have been taken from [19, 21, 22].

| $\sqrt{s}$   | $\sigma^{tot}$ (mb) | $d\sigma/dt(t = 0)$ (mb/GeV$^2$) | $B$ (GeV$^{-2}$) | $\text{Re } \sigma/\text{Im } \sigma(t = 0)$ |
|--------------|---------------------|---------------------------------|-----------------|----------------------------------|
| 7 TeV        | 98.54               | 500.32                          | 20.16           | 0.099                            |
| [13]         | 98.3 ± 2.8          | -                               | 20.1 ± 0.4      | -                                |
| 1.8 TeV      | 77.58               | 310.48                          | 17.33           | 0.104                            |
| [21]         | -                   | 334.6 ± 18.8                    | 16.98 ± 0.25    | -                                |
| 546 GeV      | 62.06               | 198.84                          | 15.09           | 0.11                             |
| [20]         | -                   | -                               | -               | 0.135 ± 0.015                    |
| [21]         | -                   | 196.1 ± 6                       | 15.35 ± 0.19    | -                                |
| [23]         | -                   | -                               | -               | -                                |
| 62 GeV       | 39.54               | 80.73                           | 11.46           | 0.11                             |
| [19]         | 43.55 ± 0.31        | -                               | 13.02 ± 0.27    | -                                |

Table 1. The comparison of the calculated values of total cross sections $\sigma^{tot}$, of $d\sigma/dt(t = 0)$, slope parameter $B$ and ratio $\text{Re } \sigma/\text{Im } \sigma(t = 0)$ with the available experimental data.

The values of total cross sections and the slopes are in reasonable agreement with the experimental data, the value $\rho = \text{Re } \sigma/\text{Im } \sigma(t = 0)$ is also well reproduced at $\sqrt{s} = 546$ GeV. The AQM assumption $r_q^2 \ll r_p^2$ continue to hold satisfactorily even at the LHC energy $\sqrt{s} = 7$ TeV for $r_q^2 \simeq 5$ GeV$^{-2}$, $r_p^2 \simeq 12$ GeV$^{-2}$. Probably it would be better fulfilled if the Pomeron slope $\alpha'$ would be lesser. The values obtained for $\sqrt{s} = 62$ GeV are smaller than the experimental data. The reason might be in non-Pomeron contributions, for example in $f$ Reggeon, whose effect could be significant for
low energies but disappears when the energy grows.

The position of the local minimum of the differential cross section at $\sqrt{s} = 7$ Tev is also reproduced although the theory predicts the more deep minimum here. As the transferred momentum increases, $|t| \geq 1$ GeV, the interaction becomes sensitive to the internal structure of constituent quarks that have a finite size. We have used a simple parametrization for their radius $r_q^2 = \alpha' \xi$, but there is a possibility for additional constants or more complicated functions.

![Figure 6: The differential cross section of elastic pp scattering at $\sqrt{s} = 7$ Tev. The dashed line ($n = 4$) presents the sum of first four orders of AQM calculated with the parameters $\Delta = 0.1$, $\alpha' = 0.38$ GeV$^{-2}$, $\gamma = 0.45$ GeV$^{-2}$, $a_1 = 6.5$ GeV$^{-2}$, $a_2 = 0.47$ GeV$^{-2}$, $C = 0.081$. The solid line ($n = 9$) presents the sum of all AQM orders for the parameters $\xi$. It exhibits the second local minimum. The experimental points have been taken from [13, 14, 15].](image)

It is important to note that complete 9 orders AQM calculation yields second local minimum of the differential elastic $pp$ cross section for $|t| \geq 1$ GeV unobservable experimentally. It may indicate invalidity of our description for $|t| > 1$ GeV where the internal structure of the constituent quarks become important. From another point of view there is uncontrolled contribution of multipomeron diagrams, (e.g. enhanced Pomeron diagrams [16]) rapidly growing with the number of Pomerons. Their numerical value is determined by unknown vertices of $m \rightarrow n$ Pomeron transitions, which
increases the uncertainty of the next orders. In this context, it is worth stressing that
the sum of only first 4 orders of AQM results (with a little modified parameters) into
theoretical curves that better fit the data at the more broad \(t\) interval and do not
exhibit an extra local minimum as is shown in Fig. 6.

4 Conclusion

We show that the simple AQM model gives reasonable description of the high energy \(pp\)
elastic scattering at the not large enough \(|t| \leq 1\) Gev), that is at the distances where
the internal structure of the constituent quarks probably do not manifest itself. Our
model contains 6 parameters, but only 3 of them, \((\Delta, \alpha', \gamma_{qq})\), are employed to describe
high energy scattering while 3 others \((a_{1,2}, C)\) determine the matter distribution in the
proton. The interesting fact is that the matter distribution cannot be parameterized
by one Gaussian packet (see also [3]).

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