Genetic Algorithms for multimodal optimization: A review

Noe Casas
Email: research@noecasas.com

Abstract—In this article we provide a comprehensive review of the different evolutionary algorithm techniques used to address multimodal optimization problems, classifying them according to the nature of their approach. On the one hand there are algorithms that address the issue of the early convergence to a local optimum by differentiating the individuals of the population into groups and limiting their interaction, hence having each group evolve with a high degree of independence. On the other hand other approaches are based on directly addressing the lack of genetic diversity of the population by introducing elements into the evolutionary dynamics that promote new niches of the genotypical space to be explored. Finally, we study multi-objective optimization genetic algorithms, that handle the situations where multiple criteria have to be satisfied with no penalty for any of them. Very rich literature has arised over the years on these topics, and we aim at offering an overview of the most important techniques of each branch of the field.

I. INTRODUCTION

Genetic Algorithms aim at exploring the genotypical space to find an individual whose associated phenotype optimizes a prefinned fitness function. When the fitness function presents multiple local optima, the problem is said to be multimodal optimization.

The desired behaviour of a GA when applied to such a type of problem is not to get stuck in local optima, but find the global optimum of the function. This very same concept was introduced in the context of biological evolution by Sewall Wright in *Fitness Landscapes* in 1932 in [1]:

In a rugged field of this character, selection will easily carry the species to the nearest peak, but there may be innumerable other peaks which are higher but which are separated by valleys. The problem of evolution as I see it is that of a mechanism by which the species may continually find its way from lower to higher peaks in such a field. In order for this to occur, there must be some trial and error mechanism on a grand scale by which the species may explore the region surrounding the small portion of the field which it occupies

This type of scenarios require the GA to develop strategies to cover all the genotypic space without converging to a local optimum. Throughout this article we explore the different techniques applied to genetic algorithms to improve their effectiveness in multimodal fitness problems.

Multimodal optimization problems should not be confused with multiobjective optimization: while multimodal optimization tries to find the optimum of a single fitness function that has multiple local optima, multi-objective optimization tries to find the balance at optimizing several fitness functions at the same time. Nevertheless, they have many commonalities, and concepts from each world can be applied to the other. For instance, current multi-objective genetic algorithms tend to make use of multimodal optimization mechanisms in one of their steps, normally *crowding or fitness sharing* (see section [II-B]), and also, recent reinterpretations of multi-objective genetic algorithms allow to explicitly hadle multimodal single-objective optimization problems.

In section [II] we explore the different groups of techniques, covering first the approaches that structures the population so the the individuals are assigned to subpopulations with limited cross-interation (section [II-A]) and then the algorithms that condition the evolutionary dynamics to promote diversity, aiming at avoiding early convergence to a local optimum. We also explore multi-objective optimization algorithms (II-C), focusing on their aspects related to multi-modality. After the glance at the available techniques, section [III] provide a brief discussion their pros and cons. Finally, our conclusions are presented in section [IV]

II. STATE OF THE ART

Throughout this section we explore the different genetic algorithm branches devoted to multimodal problem optimization, describing the most remarkable representatives of each of the approaches.

It should be noted that in the literature, many of the algorithms described in the following sections are labeled under the term *niching methods*. The uses of such a wording are diverse, normally comprising GAs based on the spatial distribution of the population (section [II-A]) and some GAs based on the explicit control of the diversity within the population (II-B). However, due to the broad use of the word, we shall not use it, in order to avoid misunderstandings.

A. Structured Population GAs (implicit diversity promotion)

Structured Population Genetic Algorithms do not explicitly measure and enforce diversity, but impose certain constraints to the population aiming at regulate its dynamics. The two subgroups among this type of algorithms are those explicitly partitioning the population and those inducing measures that lead individuals to cluster into subpopulations. Both families are studied in the following subsections.
1) Algorithms based on the Spatial Segregation of One Population: Evolutionary Algorithms relying on spacial segregation usually divide the population into completely segregated groups that at certain points in time retrofit their genetic material. This aims at avoiding the homogeneity of panmictic approaches by keeping several homogenous groups and mixing them at controlled intervals, hence profiting from the good local optima found by each subgroup.

- Island Model GAs (aka Parallel GAs, Coarse-grained GAs) manage subpopulations (islands, demes), each one evolving separately with its own dynamics (e.g. mutation rate, population size), but at certain points they exchange some individuals (i.e. the new genetic operator: migration). The algorithm can be subject of different design decisions ([2]): number of islands and migration topology among them (individuals from which islands can migrate to which islands), synchronism of the migration (asynchronous scales better and profits from underlying hardware parallelism, but is non-deterministic, non-repeatable and difficult to debug), migration frequency, whether the migration is initiated by the source or by the destination and the migration selection and migration replacement policies.

- Spatially-Dispersed GAs ([3]) associate a two dimensional coordinate to every individual (initial positions are assigned at random), having offspring placed randomly but close to the first of the parents. Mating is only allowed with individuals that are located within certain visibility radius. These dynamics lead to the progressive spread and accumulation of the originally randomly distributed individuals into clusters (demes) that resemble subpopulations. The value of the aforementioned visibility radius is not important, as the populations spread according to its scale without performance penalties.

2) Algorithms based on Spatial Distribution of One Population: Genetic Algorithms belonging to this family do not impose hard divisions among groups of individuals, but induce their clustering by means of constraints in their evolutive dynamics. Their most remarkable approaches are:

- Diffusion Model ([4], [5]) keeps two subpopulations, said to be of different species. The individuals from both populations are spread over the same two-dimensional toroidal grid, ensuring each cell contains only one member from each population. Mating is restricted to individuals from the same species within the neighbouring cells with a fitness-proportionate scheme. Replacement follows the opposite approach from mating, that is, offspring probabilistically replaces their parents in the neighbourhood. Both populations compete to be the fittest, hence they co-evolve but do not mix with the other species.

- Cellular GAs (cGA) ([6]) is the name of the family of GAs evolved from the Diffusion Model. They also adjust the selection pressure and have the concept of neighbourhoods, but improve the base idea on several different directions. Some of the remarkable contributions are:
  - Terrain-Based Genetic Algorithms (TBGA) ([7], [8]) is a self-tuning version of cGA, where each grid cell of the two-dimensional world is assigned a different combination of parameters. They then evolve separately, each cell mating with their up, down, left and right neighbours. This algorithm can be used not only to address the optimization problem itself, but to find a set of suitable parameter values to be used in a normal cGA. In fact, the authors admit that a normal cGA using the parameters found by their TBGA performs better than the TBGA itself.
  - Genetic and Artificial Life Environment (GALE) ([9]) offers the concept of empty cells, where neighbouring offspring are placed. If no empty cells are present after breeding a cell, new individuals replace worst performing individuals from their original neighbourhood. This algorithm also presents fitness sharing (see section II-B1).
  - Co-evolutionary approaches like in [10], an improvement over sorting networks, where two species (referred to as hosts-parasites or prey/predators). Hosts are meant to sort some input data, while parasites represent test data to be supplied to a host as input. The fitness of each group is opposed to the other group: the fitness of the hosts depends on how many test cases (i.e. parasites) an individual has succeeded in sorting, while the fitness of the parasites depends on how many times it made a host fail sorting.
  - Multi-objective variations of cGA, namely cMOGA and MOCell, which are addressed in section II-C.

3) Algorithms imposing other mating restrictions: These algorithms impose mating restrictions based on other criteria, normally mimicking the high level dynamics of existing real-world environments. The most remarkable ones present in the literature are:

- Multinational Evolutionary Algorithms ([11]): divide the world into nations and partition the population among them, also having different roles within each nation, namely politicians, police and normal people. Their interaction and mating dynamics are defined by pre-established social rules.

- Religion-Based Evolutionary Algorithms ([12]): assigns each individual to a different religion and defines genetic operators for converting between religions. Mating is hence restricted to individuals with the same beliefs.

- Age Structure GAs ([13]) define the lifecycle of individuals and constrain the mating to individuals in the same age group.
B. Diversity Enforcing Techniques

The main trait of this group of algorithms is that they define a measure of the population diversity distribution over the genotypical space and act upon local accumulations of individuals, favouring their migration to new niches.

1) Fitness sharing: Fitness Sharing GAs ([14]) are based on having individual’s fitness points shared with their neighbours. The *neighbourhood* is defined as the individuals within a certain radius \( r_{\text{share}} \) over an established distance metric (e.g. euclidean distance, Hamming distance). This way, the new fitness \( F' \) of an individual \( i \) is calculated based on its distance \( d \) to every neighbour \( j \) as:

\[
F'(i) = \frac{F(i)}{\sum_{j \neq i} \text{sharingfunction}(d(i,j))}
\]

where \( \text{sharingfunction} \) receives as input the distance between two individuals and is computed as:

\[
\text{sharingfunction}(d) = \begin{cases} 
1 - (d/r_{\text{share}})^\alpha & \text{if } d \leq r_{\text{share}} \\
0 & \text{otherwise}
\end{cases}
\]

Having \( \alpha \) define the shape of the sharing function (i.e. \( \alpha = 1 \) for lineal sharing).

This way, the convergente to a single area of the fitness landscape is discouraged by pretending there are limited resources there. The more individuals try to move in, the more neighbours the fitness have to be shared with. Hence, for individuals in crowded areas, eventually another region of the fitness space becomes more attractive. Ideally, the algorithm stabilizes at a point where an appropriate representation of each niche is maintained.

2) Clearing: Clearing GAs ([15]) divide the population in subpopulations according to a dissimilarity measure (e.g. Manhattan distance). For each subpopulation, in the selection phase the fittest individual is considered the winner (normally referred to as the dominant individual). Then the other members of the subpopulation have their dissimilarity to the winner calculated. If such the distance of an individual of the subpopulation to its winner is greater than a certain threshold (the clearing radius), it gets its fitness set to zero (i.e. it gets cleared). After the whole population has been processed, the subpopulations are recalculated again based on the very same clearing radius.

3) Crowding: Crowding GAs associate to every individual breded in the current generation with another individual from the parent generation (pairing phase) and only keep one of the two in the population (replacement phase). The association is established based on genotypical similarity criteria (e.g. Manhattan distance, Euclidean distance). This approach favors the growth of individuals around underpopulated regions of the solution space and penalizes overcrowded areas because only the similar individuals get replaced.

In the original algorithm formulation, De Jong used a Crowding Factor parameter (CF) (section 4.7 of [16]) to specify the size of the sample of individuals initially selected at random as candidates to be replaced by a particular offspring, among which only one shall finally be chosen based on fitness. He found problems in the original formulation of the algorithm, as it failed to prevent genetic drift in many cases.

Mahfoud improved on the algorithm by identifying several weak points, most remarkably focusing on maintaining global diversity ([17], [18], [19]) and addressing them by introducing a different diversity measure that favoured niching, namely the number of peaks maintained by the population. With the new measures, Mahfoud re-evaluated De Jong’s mislead conclusions (i.e. that CF higher than 1 led to genetic drift) and reformulated the algorithm to only use the individual’s parents as candidates for replacement (hence reducing drastically the computational complexity). This variation is called Deterministic Crowding.

Mengshoel ([20]) proposed a variation called Probabilistic Crowding in which the selection criteria for individuals to be replaced is not fitness-proportionate but random, hence favouring the conservation of low-fitness individuals and avoiding genetic drift toward the high-fitness niche.

C. Multi-objective evolutionary algorithms

Multi-Objective Optimization problems are characterized by the need to find proper trade-offs among different criteria, each of them quantified by means of an objective function, formally ([21]):

A general Multi-objective Optimization Problem (MOP) is defined as minimizing (or maximizing) \( F(x) = (f_1(x),...,f_k(x)) \) subject to \( g_i(x) \leq 0, i = 1,...,m \) and \( h_j(x) = 0, j = 1,...,p \). An MOP solution minimizes (or maximizes) the components of a vector \( F(x) \) where \( x \) is a \( n \)-dimensional decision variable vector \( x = (x_1,...,x_n) \) from some universe \( \Omega \). It is noted that \( g_i(x) \leq 0 \) and \( h_j(x) = 0 \) represent constraints that must be fulfilled while minimizing (or maximizing) \( F(x) \) and \( \Omega \) contains all possible \( x \) that can be used to satisfy an evaluation of \( F(x) \).

The evaluation function, \( F : \Omega \rightarrow \Lambda \), maps from the decision variable space \( \bar{x} = (x_1,...,x_n) \) to the objective function space \( y = (f_1(\bar{x}),...,f_k(\bar{x})) \).\(^1\)

The cathegorization Multi-objective optimization normally refers to approaches that are defined in terms of Pareto Optimality ([21]):

A solution \( x \in \Omega \) is said to be Pareto Optimal with respect to \( \Omega \) if and only if there is no \( x \in \Omega \) for which \( v = F(x) = (f_1(x),...,f_k(x)) \) dominates \( u = F(x) = (f_1(x),...,f_k(x)) \).

Where a vector \( u \) is said to dominate another vector \( v \) if there is a subset of \( f_i(x) \) for which \( u \) is (assuming

\(^1\)In this equation we use the \( \bar{x} \) notation to clarify the vectorial nature of the parameter of \( f_i \), but we will not use it in the rest of the report
minimization) partially less than $v$, that is $\exists i : u_i < v_i$.

This means that $x^*$ is Pareto optimal if there exists no vector which would decrease some criterion without causing a simultaneous increase in at least one other criterion (again assuming minimization).

When we plot all the objective function vectors that are nondominated, the obtained curve is usually referred to as the Pareto front.

A Multi-objective Optimization Evolutionary Algorithm (MOEA) consists of the application of GAs to a MOE. The mechanism of a MOEA is the same as a normal GA. Their only difference is that, instead of a single fitness function, MOEAs compute $k$ fitness function and then perform on them a transformation in order to obtain a single measure, which is then used as the fitness in normal GAs. At each generation MOEAs output is the current set of Pareto optimal solution (i.e. the Pareto front) assuming minimization).

There exist multiple variations of MOEAs, each of them differing in either the way they combine the individual objective functions into the fitness value (a priori techniques) or the post-processing they do to ensure Pareto optimality.

Deb recently proposed a MOEA ([22], [23]) for addressing single-objective multimodal optimization problems. This algorithm defined a suitable second objective and added it to the originally single objective multimodal optimization problem, so that the multiple solutions form a pareto-optimal front. This way, the single-objective multimodal optimization problem turned artificially into a MOOP can be solved for its multiple solutions using the MOEA.

III. DISCUSSION

One of the most attractive traits of spatial segregation GAs is that each of the population subgroups can be evolved in parallel, hence making them suitable to profit from parallel architectures such as multicore or supercomputing facilities.

Another potentially attractive characteristic of this type of algorithms is that they are to some degree independent on the optimization algorithm. This enables to use different optimization algorithms (not constraining to GAs) to each island ([21]).

However, a significant concern about them is that they need to be carefully tuned in order to perform well. For instance, Fitness sharing (section I-B1) needs to manually set the niche radius, and the algorithm is quite sensitive to this choice. Failure to properly tune the algorithm parameters normally implies performing significantly under that of an equivalent panmictic implementation (II-C). In this regard, Spatially-Dispersed GAs require less configuration tuning than Island Model GAs.

Some self-adapting options are interesting in that they do not need such configuration tuning at all. However, many times these algorithms perform worse than their equivalent fine tuned non-adaptive version ([27], [8]).

One of the most significant problems among the reviewed techniques is the one suffered by those that rely on structuring the population (section II-A), which cannot assure an improvement on the solution space covered because they are not based on a measure of the distribution of the diversity ([24]). This way, these algorithms offer good performance for some problems, but worse performance than panmictic approaches, with no apparent reason. Moreover, given their loose relation to diversity control, it is often impossible to diagnose or fix the root cause of the algorithm underperforming for certain problem.

Most of the sources in the literature of the last years tend to agree that Crowding (section II-B3) is effective for any multimodal optimization problem. Nevertheless, promising MOEAs ([22], [23]) may also play an important role in the upcoming years.

IV. CONCLUSIONS

In most real-world optimization problems, the fitness landscape is unknown to us. This means that there is a non-negligible chance that it is multimodal. Failing to acknowledge so -and act accordingly- may likely result in the optimization to converge too early to a local optimum.

From the reviewed techniques, the only one with quorum among the scientific community regarding general effectiveness is Crowding. All other options have proven valuable for many concrete problems, but they certainly exhibit suboptimal performance compared to panmictic approaches for some other problems.

This tells us that selecting the appropriate multimodal optimization genetic algorithm cannot be addressed a priori, but has to undergo a trial-error process, driven by the intuition of the researchers to choose an approach that has proven effective for seemingly analogous or similar problems.

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