Theory of pairing symmetry inside the Abrikosov vortex core

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We show that the Cooper pair wave function at the center of an Abrikosov vortex with vorticity $m$ has different parity with respect to frequency from that in the bulk if $m$ is an odd number and has the same parity if $m$ is an even number. As a result, in a conventional vortex with $m = 1$, the local density of states at the Fermi energy has a maximum (minimum) at the center of the vortex core in even(odd)-frequency superconductor. We propose a scanning tunneling microscope experiment using a superconducting tip to explore odd-frequency superconductivity.

The study of the mixed state in type-II superconductors has a long history and revealed a variety of physical phenomena.\textsuperscript{1} In the clean limit, low-energy bound states (the Andreev bound states) are generated in the vortex core due to the spatial structure of the superconducting pair potential.\textsuperscript{2} One of the manifestations of the bound states is the enhancement of zero-energy quasiparticle density of states (DOS) locally in the core, observable as a zero-bias conductance peak by scanning tunneling microscope (STM).\textsuperscript{3, 4} However, despite extensive studies of the vortex core, the issue of pairing symmetry in the core remains unexplored.

Generally, superconducting pairing is classified into even-frequency or odd-frequency state according to a symmetry with respect to time. Due to the Fermi statistics, even-frequency superconductors belong to the symmetry class of spin-singlet even-parity (ESE) or spin-triplet odd-parity (ETO) pairing state, while odd-frequency superconductors belong to the spin-singlet odd-parity (OSO) or spin-triplet even-parity (OTE) pairing state. Although vortex core state of even-frequency superconductors has been well studied, that in odd-frequency superconductors has not been clarified yet.

The possibility of the odd-frequency pairing state in various kinds of uniform systems was discussed in Refs.\textsuperscript{5, 6}, albeit its realization in bulk materials is still controversial. On the other hand, the realization of the odd-frequency pairing state in inhomogeneous even-frequency superconducting systems has recently been proposed. It is established that odd-frequency pairing is induced due to symmetry breaking in such systems. In ferromagnet/superconductor junctions, odd-frequency pairing emerges due to the broken symmetry in a spin space.\textsuperscript{7} It was recently realized that proximity-induced odd-frequency pairing may be generated near normal metal/superconductor interfaces due to the breakdown of translational symmetry\textsuperscript{8} or in a diffusive normal metal attached to a spin-triplet superconductor.\textsuperscript{9}

Since an Abrikosov vortex breaks translational symmetry in a superconductor, one may expect the emergence of an odd-frequency pairing state around the vortex core even in a conventional spin-singlet $s$-wave superconductor. On the other hand, one may imagine that even-frequency pairing state may be generated around the vortex core in odd-frequency superconductor.

In this Letter, based on the quasiclassical theory of superconductivity, we develop a general theory of pairing symmetry in an Abrikosov vortex core in clean superconductors, including odd-frequency superconductivity. We show that for a vortex with vorticity $m$ in superconductor, pairing function of the Cooper pair at the vortex center has the opposite (same) symmetry with respect to frequency to (as) that of the bulk if $m$ is an odd (even) integer. For a conventional vortex with $m = 1$, we show that zero energy local DOS is enhanced (suppressed) at the center of the vortex core in even (odd)-frequency superconductor. We further reveal that OSO $p$-wave pairing is generated at the center of the core of ESE $s$-wave superconductor. On the other hand, in OSO $p$-wave superconductor, ESE $s$-wave pairing state emerges at the center of the vortex core. Based on these results, we propose an experimental setup to explore odd-frequency superconductivity by probing a local Josephson coupling by STM with superconducting tip.

The electronic structure of the vortex core in a single Abrikosov vortex in a clean superconductor is described by the quasiclassical Eilenberger equations\textsuperscript{10, 11} based on the Riccati parametrization of the quasiclassical propagator\textsuperscript{12}. Along a trajectory $r(x') = r_0 + x' \hat{v}_F$ with unit vector $\hat{v}_F$ parallel to $v_F$, the Eilenberger equations are generally represented in $4 \times 4$ matrix form.\textsuperscript{13} For a singlet (triplet) superconductor with $\Delta = \Delta \sigma_y (\sigma_x)$ ($\sigma_x$ and $\sigma_y$ are Pauli’s matrices in spin space)\textsuperscript{14}, these equations are reduced to the set of two decoupled differential equations of the Riccati type for the functions $a(x')$ and $b(x')$,

\begin{align}
\hbar v_F \partial_x a(x') + [2 \epsilon_n + |\Delta|^2 a(x')] a(x') - \Delta &= 0, \\
\hbar v_F \partial_x b(x') - [2 \epsilon_n + \Delta^* b(x')] b(x') + \Delta^* &= 0
\end{align}

where $\epsilon_n$ are the Matsubara frequencies, and $\Delta^* = (-\Delta^*)^*$ for even (odd)-frequency superconductor. For simple case of a cylindrical Fermi surface, the Fermi velocity can be written as $v_F = v_F (e_1 \cos \theta + e_2 \sin \theta)$.

We choose the following form of the pair potential:

\begin{align}
\Delta(r, \theta, E) = \Delta_0 \chi(\theta, E) F(r) \exp(i m \varphi)
\end{align}
with \( r = \sqrt{x^2 + y^2} \) and \( \exp(i\varphi) = (x + iy)/\sqrt{x^2 + y^2} \). Here, \( F(r) \) denotes the spatial profile of the gap, \( m \) is the vorticity, and \( \chi(\theta, E) \) is the symmetry function. Also, we introduce the coherence length \( \xi = \hbar v_F/\Delta_0 \), the center of a vortex is situated at \( x = y = 0 \), and \( \exp(im\varphi) \) is the phase factor which originates from the vortex.

We obtain pairing function of the Cooper pair (anomalous Green’s function) \( f \) as \( f = -2a/(1 + ab) \). For the calculation of the local DOS normalized by its value in the normal state, the quasiclassical propagator has to be integrated over the angle \( \theta \) which defines the direction of the Fermi velocity. The normalized local DOS in terms of functions \( a \) and \( b \) is given by

\[
N(r_0, E) = \int_0^{2\pi} \frac{d\theta}{2\pi} \text{Re} \left[ \frac{1 - ab}{1 + ab} \right] \left. \right|_{\text{even}} \text{vortex}, \quad \text{N}\left( r_0, E \right) = \int_0^{2\pi} \frac{d\theta}{2\pi} \text{Re} \left[ \frac{1 - ab}{1 + ab} \right] \left. \right|_{\text{odd}} \text{vortex},
\]

where \( E \) denotes the quasiparticle energy with respect to the Fermi level and \( \delta \) is an effective scattering parameter that corresponds to an inverse mean free path. In numerical calculations throughout this paper, we will fix this value as \( \delta = 0.1\Delta_0 \).

First, we discuss the general property of the symmetry at the vortex center. Vorticity and the symmetry of superconductor with respect to frequency crucially affect the symmetry of the Cooper pair at the core center. Consider a trajectory passing through the center of the vortex. By setting \( x' = 0 \) at the vortex center, we get \( b(x', \epsilon_n) = -1/a(-x', -\epsilon_n) \) from the Eilenberger equations for even-frequency superconductor with odd integer \( m \) or odd-frequency superconductor with even integer \( m \). Similarly, we obtain \( b(x', \epsilon_n) = 1/a(-x', -\epsilon_n) \) for odd-frequency superconductor with odd integer \( m \) or even-frequency superconductor with even integer \( m \). Thus, at the vortex center \( x' = 0 \), we get \( f(\epsilon_n) = -f(-\epsilon_n) \) in the former case, while \( f(\epsilon_n) = f(-\epsilon_n) \) in the latter. Note that spin is conserved in the vortex state considered. Therefore, quite generally, for an odd integer \( m \) the induced pairing at the vortex center has different symmetry with respect to frequency from that in the bulk superconductor. On the other hand, for an even integer \( m \), the induced pairing at the vortex center has the same symmetry as that of the bulk. We summarize pairing symmetry at the vortex center in Table I.

For conventional \( s \)-wave case, there have been several studies of multi-vortex state with \( m \geq 1 \). \( \text{[13, 16]} \). It was shown that zero energy peak in the DOS only appears for odd number \( m \) at the vortex center \( \text{[16]} \). This statement is consistent with our result for the conventional \( s \)-wave case of \( \chi(\theta, E) = 1 \) because odd-frequency pairing state is generated only for odd integer \( m \). The relation between zero energy peak in DOS and odd-frequency pairing state will be discussed later.

In general, the most realizable vorticity is \( m = 1 \). Thus, in the following, we will study in detail two typical cases at \( m = 1 \) with ESE \( s \)-wave and OSO \( p_x \)-wave superconductors where we choose \( \chi(\theta, E) = 1 \) and \( \chi(\theta, E) = (C \cos \theta E/\Delta_0)/[1 + (E/\Delta_0)^2] \) with \( C = 0.8, \text{[9]} \) respectively. Also, spatial dependence of the gap is chosen as \( F(r) = \text{tanh}(r/\xi) \).

| Table I: Pairing symmetry in the vortex state. |
| --- |
| \( \text{bulk state} \) | \( \text{vorticity} \) | \( \text{symmetry at the center} \) |
| \( 1 \) | ESE | odd | OSO |
| \( 2 \) | ESE | even | ESE |
| \( 3 \) | ETO | odd | OTE |
| \( 4 \) | ETO | even | ETO |
| \( 5 \) | OSO | odd | ESE |
| \( 6 \) | OSO | even | OSO |
| \( 7 \) | OTE | odd | ETO |
| \( 8 \) | OTE | even | OTE |
With the increase of the distance from the core center, the magnitudes of $f$ decrease rapidly, except for $s$-wave one. Note that other angular momentum components not shown in this figure are negligibly small. We also find that the following representation of anomalous Green’s function $f$ holds at the center of the core:

$$f = \left(\text{Re}f_{px} + i\text{Im}f_{py}\right)\left(\cos \theta + i \sin \theta\right) = f_{px} \exp(i\theta). \quad (5)$$

Thus, we see that anomalous Green’s function at the core center has chiral $p$-wave symmetry.

The enhancement of the local DOS in the presence of odd-frequency pairing can be understood, irrespective of the detailed shape of $\Delta$ by using the normalization condition for the quasiclassical Green’s functions, $g^2 + ff = 1$. Indeed, since for odd-frequency pairing state, the anomalous Green’s function $f = -2b/(1 + ab)$ at $E = 0$ is given by $f(\theta) = -f^*(\theta)$ (see Ref. [13]) and local DOS is given by $N(E) = -\text{Reg}$, one can show that generally $N(E = 0) > 1$ since $g^2 = 1 + |f|^2 > 1$. This means that the emergence of the odd-frequency pairing is a physical reason of zero energy peak of the local DOS inside the core. The manifestation of the odd-frequency chiral $p$-wave pairing state at the center of the vortex core is also consistent with the experimental fact that the observed zero-bias conductance peak by STM at a vortex center is very sensitive to disorder [17], since $p$-wave pairing is fragile to impurity scattering.

Figure 2 depicts the results for OSO $p_x$-wave superconductor. The local DOS around the vortex at $E = 0$ is shown in Fig. 2(a). In dramatic contrast to the result for the ESE $s$-wave, zero energy DOS is suppressed at the core. The spatial dependencies of decomposed anomalous Green’s function at $E = 0$ are shown in Figs. 2(b) and (c). As is seen, only ESE pairing components exist at the center of the core. For even-frequency pairing state, $\hat{f}(\theta) = f^*(\theta)$ is satisfied at $E = 0$ and hence we get $N(E = 0) < 1$ which is consistent with Fig. 2(a). By comparing Figs.1 and 2, it is clear that zero energy local DOS $N(0)$ has a maximum at the center of the vortex core in even-frequency superconductor, while it has a minimum at the core center in odd-frequency superconductor. This difference can be detected by STM.

As regards the candidate for the odd-frequency superconductor, CeCu$_2$Si$_2$ and CeRhIn$_5$ are possible materials [6, 18]. In these systems, OSO state with $p$-wave symmetry is considered to be promising [6]. In the light of the present theory, ESE $s$-wave pairing is expected to appear inside the vortex core. Based on this idea, we propose an experimental setup to verify the existence of odd-frequency pairing in bulk materials by using superconducting STM where we use conventional $s$-wave superconductor as a STM tip [13] as shown in Fig. 3. Local Josephson current measured in STM experiment with su-
We have also shown that zero energy local density of states is enhanced (suppressed) at the center of the vortex core for even(odd)-frequency superconductor. Based on the obtained results, we proposed a scanning tunneling microscope experiment using a superconducting tip to detect local Josephson coupling in order to explore and identify odd-frequency superconductor.

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In summary, we have developed a general theory of pairing symmetry inside the Abrikosov vortex core in superconductors, including odd-frequency superconductivity. We have found that for a vortex with vorticity $m$ in a superconductor, anomalous Green’s function at the vortex center has the opposite (same) symmetry with respect to frequency to (as) that of the bulk if $m$ is an odd (even) integer.
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