Fractal grid-induced turbulence strength characterization via piezoelectric thin-film flapping velocimetry

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The centerline streamwise and cross-sectional (x/Dh = 0.425) turbulence characteristics of a 2D planar space-filling square-fractal-grid (SFG) composed of self-similar patterns superimposed at multiple length-scales is experimentally unveiled via piezoelectric thin-film flapping velocimetry (PTFV). The fluid-structure-interaction between a flexible piezoelectric thin-film and SFG-generated turbulent flow at ReDh = 4.1 × 10^4 is investigated by analysis of the thin-film's mechanical response. Measurements of the thin-film-tip deflection δ and induced voltage V demonstrate increasing flow fluctuation strength in the turbulence generation region, followed by rapid decay further downstream of the SFG. Interestingly, SFG-induced turbulence enables the generation of maximum centerline thin-film’s response (Vrms, δrms) and millinewton turbulence-forcing (turbulence-induced excitation force Frms) which are respectively, 7× and 2× larger than the classical square-regular-grid of similar blockage ratio. The low frequency, large-scale energy-containing eddies at SFG’s central opening plays a critical role in driving the thin-film vibration. Most importantly, the SFG-generated turbulence at (y/T = 0.106, z/T = 0.125) away from the centerline allows equivalent mechanical characteristics of turbulence generation and decay, with peak of 1.9× nearer from grid. In short, PTFV provides a unique expression of the SFG-generated turbulence, of which, the equivalent turbulence length-scale and induced-forcing deduced could aid in deciphering the flow dynamics for effective turbulence management.

Fractal grid has received an increasing amount of interest recently as a turbulence generator to enhance fluid mixing and convective heat transfer1–4. Fractal is a geometrical structure that repeats itself and diminishes in size, forming a complex pattern of several iterations5,6. A square fractal grid (SFG) restructures the upstream flow to generate more intermittent vorticity field, higher vorticities and higher turbulence intensities when compared to classical regular grid (RG) of similar or smaller blockage ratio7. A piezoelectric transducer converts mechanical vibration into AC electrical signal through piezoelectric effect. Despite possessing high mechanical strength, a piezoelectric thin-film has a wide frequency range and is highly responsive to turbulent flow8. When immersed in a fractal grid-induced flow domain, the random vortical structures shed from the grid bars will impart a mechanical strain on the piezoelectric to generate electrical charges as a function of fluid fluctuating properties. Significant work has been carried out lately to investigate the interplays between fluid flow properties and random excitation force against piezoelectric’s structural and electrical responses. Akaydin et al.9 theoretically estimated the piezoelectric tip deflection from open-circuit voltage measurement to characterize the interaction between piezoelectric beam and turbulent eddies in the wake of cylinder based on the statistical properties of the voltage and deflection’s local extremes. The piezoelectric tip deflection at various free stream velocities and distances from the test section wall was measured using a strain-gage to study the fluid-structure interaction in turbulent boundary layers10. Forces measurement and theoretical analysis demonstrated the dependence of turbulence-induced excitation force upon the local mean velocity, turbulence intensity and relative size of the beam length to the integral length scale of turbulence. Experimental work11 showed that the force acting on the piezoelectric beam depends on the velocity and length scales of turbulence. The average power harvested can be modelled as a power-law with respect to the piezoelectric’s distance from the turbulator which closely follows the decay rate of turbulence kinetic energy. A frequency response model12 was established to compute the drag force applied to a piezoelectric vibration sensor from the experimentally recorded voltage signal. The force
amplitude was reported to be linearly correlated with the Laser Doppler Anemometry-measured kinetic energy fluctuation intensity rather than the velocity fluctuation. In spite of the extensive research conducted over the past decade to study the behaviour of piezoelectric thin-film in highly unsteady flow, the majority of them focused on evaluating the piezoelectric performance in turbulent boundary layers and passive rectangular grid-induced flow for energy harvesting purposes. To date, the characterization of fractal grid-induced turbulence based upon the direct interaction of the flow with a piezoelectric thin-film has not been given much consideration in literature. Although recent experimental studies were conducted to investigate the effects of grid-beam distance on the electrical power extracted by a single and two side-by-side piezoelectric beams from fractal grid-induced turbulence, the piezoelectric output profile over the grid’s cross-section remains obscure. Moreover, the fractal grid-generated turbulence force acting on a flexible piezoelectric thin-film has yet to be unravelled.

Hence, the current study aims to experimentally unveil the interaction between a flexible piezoelectric thin-film and fractal grid-induced fluid flow by means of analysing the thin-film's physical response towards turbulent flow dynamic. In particular, the expression of local equivalent fluid flow undulation and the corresponding turbulence forcing (turbulence-induced excitation force acting on the thin-film) via an in-house established system, namely, the piezoelectric thin-film flapping velocimetry (PTFV) to comprehend the fundamental turbulence and the strength of grid-generated flow. In the present work, a polyvinylidene fluoride (PVDF) piezoelectric cantilever beam is placed at different lateral positions and centerline streamwise distances leeward of SFG and RG to measure the film's tip deflection and voltage response. A detailed exploration of the various equivalent turbulence statistics determined from the film undulation allows the mechanical characteristics of the grid-induced cross-sectional and centerline streamwise turbulent flow structures to be effectively expressed.

Methods

Experimental setup. A schematic of the experimental setup is presented in Fig. 1. The channel is 0.16 m wide and 4.41 m long square wind tunnel is constructed from 10 mm thick transparent acrylic sheets with a bellmouth attached to the inlet. Air is drawn into the channel by an axial fan (Kruger MTD200, SG) at a fixed centerline inlet velocity $v_n = 4 \text{ m/s}$ with hydraulic diameter-based flow Reynolds number $Re_{Dh} = 4.1 \times 10^4$. Flow straighteners are placed at the inlet right after the bellmouth and in front of the fan at the outlet. A 10 mm thick 2D planar space-filling SFG with fractal iteration $N = 3$ is inserted at 2.02 m from the inlet. The geometry of the fractal insert is the same as that numerically optimized by Hoi et al. in their numerical study, with thickness ratio $t_r = 9.76$ ($t_r = t_0/t_2$, see Fig. 2a) and blockage ratio (ratio of area covered by grid over channel cross-section): $\sigma = 0.493$. The RG employed in this study has equal thickness and $\sigma$ as the SFG. Figure 1a,b show the schematic of the inserts with their geometrical parameters listed in Table 1. Note that $x_s$ denotes the wake-interaction length scale representing the location on the centerline where the wakes resulting from the largest grid bars interact, and can be expressed as

![Figure 1. Schematic of experimental setup: (a) Side-view and (b) top-view.](image-url)
where $L_0$ and $t_0$ are the length and width of the largest grid bar, respectively.

The unimorph piezoelectric thin-film (TE Connectivity LDT1-028 K, CH) is composed of a PVDF film element with silver ink screen printed electrodes laminated to a Mylar sheet (see Fig. 3). The properties of the thin-film are summarized in Table 2. The thin-film is clamped onto a 3D printed holder and placed parallel to the flow with its output connected to a 10 MΩ external electrical load. The thin-film edge is fluorescently labelled and illuminated by an LED light source. The piezoelectric voltage signal is sampled and digitized at a sampling rate of 10 kHz using a data acquisition device (LabJack U3-HV, US) with a voltage resolution of 4.88 mV/bit. Note that the voltage recorded has a high signal-to-electrical noise ratio of 47.3 even when the thin-film is vibrating at small amplitude. The voltage signal is logged for a period of 60 s to ensure the recording contains large enough samples to achieve stochastic convergence despite the random and unpredictable characteristic of turbulent flow. The lateral fluctuations of the thin-film are simultaneously captured for 10 s using an industrial

![Figure 2](image-url)

**Figure 2.** Schematic of (a) ($N=3$, $t_0=9.77$) SFG and (b) RG. Positions $P_0$ to $P_{22}$ indicate the individual allocation of piezoelectric thin-film in the lee of (c) SFG and (d) RG.

| Parameter | SFG | RG |
|-----------|-----|----|
| $\sigma$  | 0.493 | 0.492 |
| $L_0$ (mm) | 77.0 | 20.0 |
| $L_1$ (mm) | 45.7 | |
| $L_2$ (mm) | 20.4 | |
| $t_0$ (mm) | 24.4 | 5.75 |
| $t_1$ (mm) | 5.00 | |
| $t_2$ (mm) | 2.50 | |
| $x^*$ (mm) | 243.0 | 69.6 |

**Table 1.** Geometrical parameters of SFG and RG.

\[ x_n = \frac{L_0^2}{t_0} \] (1)
high-speed camera (fps4000-720, UK) at 10³ fps. The recorded images of pixel resolution 1280 × 720 are stored in the camera frame buffer memory before being transferred to the PC for processing. To ensure synchronized measurement of voltage signal and piezoelectric undulation, a simple light-emitting diode (LED) circuit as illustrated in Fig. 1b is employed. Where the start of both recordings is marked by a sharp increase in voltage across the LED just as the LED lights up the moment when circuit is closed. For each set of the experiment, six repeated recordings are performed.

The first series of wind tunnel experiments is conducted to examine the direct fluid–structure interaction of insert-induced turbulent flow and piezoelectric thin-film on the centerline of SFG and RG along different grid-film distances: $5 \leq x \leq 400$ mm i.e. $0.03 \leq x/D_h \leq 2.50$, where $D_h = 160$ mm is the hydraulic diameter of the wind tunnel. Following that, a preliminary study is performed in the turbulence generation region at $x/D_h = 0.43$ to confirm the symmetrical behaviour of the flow field leeward of SFG on positions $P_2$, $P_{2a}$ and $P_{13}$, $P_{13a}$ (see Fig. 2c). In the second series of experiments, the thin-film is positioned in the lower quarter region of SFG and RG (i.e. $- 68.5 \leq y \leq 0$ mm and $- 67 \leq z \leq 0$ mm corresponding to $- 0.43 \leq y/T \leq 0$ and $- 0.42 \leq z/T \leq 0$, respectively) at $x/D_h = 0.43$ as indicated in Fig. 2c,d. It is important to note that all the piezoelectric placements are outside...
the turbulent boundary layers created by the walls of test section, of which, the boundary layer displacement thickness is estimated to be 3.6 mm at $x/D_h = 0.43$ based on the standard turbulent boundary layer thickness equation in literature. This is to ensure the thin-film is completely vibrating in the grid-induced flow dynamic throughout the entire experiment. Lastly, the thin-film is positioned in the lee of SFG on $P_5$ where turbulence forcing is the maximum, with $x/D_h$ varying from 0.25 to 0.81 to compare its $x_{peak}$ against the one obtained on the centerline ($P_0$). For each of the positions investigated, the local mean velocities in $x, y$ and $z$-directions are measured using a hotwire anemometer (Testo 405i, DE) of accuracy $\pm 0.3$ m/s.

**Image-detection algorithm.** An in-house thin-film fluctuating image-detection algorithm is developed using MATLAB (version 9.7, USA) to identify the turbulence-induced piezoelectric thin-film tip deflection $\delta$. During data post-processing, the camera recorded RGB images are first converted into binary series. Image pixels with luminance greater than certain threshold level are replaced with white pixels, while black pixels are used to substitute the remaining. Noise present in the output images is removed through Area Opening Operation. In addition, boundaries of the thin-film are computed and plotted followed by polynomial curve fitting of the film's immediate edge to justify the pixel length. Once the ratio of the thin-film physical length scale to pixel dimension, i.e. $r$ has been secured, the $\delta$ can then be determined using,

$$\delta = \left( \sum_{n} Y_{ins} - Y_{ins} \right) \times r$$  \hspace{1cm} (2)

where $Y_{ins}$ is the instantaneous tip pixel coordinate and $n$ is the total number of images processed. To ensure accurate $\delta$ detection, the luminance threshold during RGB to binary image conversion is set based on the following conditions. (1) Before the fan is turned on, the coordinate of the film at clamped end must match the coordinate obtained via zero-crossing edge-detection algorithm, and (2) the difference between the thin-film's pixel length for film fluctuation and initial film position are to be less than $\pm 1\%$. As a consequence of the current camera resolution and lens limitation, the $\delta$ detected has an average uncertainty of 0.02 mm.

**Expression of turbulence forcing as a function of thin-film flapping.** The electromechanical equation describing the $\delta$ and voltage $V$ generated by a single degree-of-freedom (SOF) piezoelectric thin-film cantilever beam subjected to an arbitrary force $F$ (see Fig. 3) has been previously derived by. With the assumptions that the shear forces acting on the beam are very small and that the normal pressure forces are the predominant load, the time-dependent normal flow excitation force acting on the piezoelectric thin-film $F_y$ can be expressed as,

$$m\ddot{\delta} + c\dot{\delta} + k\delta - \theta V = F_y$$  \hspace{1cm} (3)

where $m, c$ and $k$ are respectively, the mass, damping and stiffness terms for a vibrating piezoelectric beam, while $\theta$ denotes the electromechanical coupling coefficient. For a forced vibration of a SDOF mass-spring-damper system, $k$ and $c$ are defined as follow,

$$k = (2\pi f_n)^2 m$$  \hspace{1cm} (4)

$$c = 2\zeta \sqrt{km}$$  \hspace{1cm} (5)

where $f_n$ is the natural frequency and $\zeta$ the damping ratio. $f_n$ of the piezoelectric thin-film is experimentally determined by lightly flicking the film tip to obtain the dominant frequency of the measured decaying voltage signal (see Fig. 4a) through Fast Fourier Transform (FFT). $\zeta$ on the other hand can be estimated from the FFT of experimentally detected $\delta$ using the half-quadratic gain method, namely,

![Figure 4](https://www.nature.com/scientificreports/)
we stress that the EI L 7.69 = 2 = ϑ (d =

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velocity fluctuation v′ from the time derivative of experimentally detected δ, related with the local flow undulation. Hence in the present study, we calculate the local equivalent lateral flow which is given by,

\[ \beta_1 L = \frac{\sinh \beta_1 L - \sin \beta_1 L}{\cosh \beta_1 L + \cos \beta_1 L} = 0.7341 \] (9)

From Elvin and Elvin29, the ϑ in Eq. (3) can be deduced as,

\[ \vartheta = d_{31} E_p b_p (2 h_a - h_p) \sigma_1 \beta_1 \] (10)

where \( h_a \) denotes the distance from the top of the PVDF layer to the neutral axis of composite beam. The calculated SDOF piezoelectric beam parameters are tabulated in Table 3. Once \( F_p \) in Eq. (3) are secured, the local turbulence forcing of grid-induced flow at various lateral positions and distances streamwise from the grids can be effectively unravelled.

Expression of local equivalent fluid flow undulation via PTFV. The \( V \) generated by the piezoelectric thin-film in the absence and presence of air flow are plotted in Fig. 4a,b, respectively to demonstrate the role of flow fluctuations in piezoelectric beam vibration response. With a light flick of the piezoelectric thin-film’s tip in the absence of air flow, the \( V \) amplitude in Fig. 4a oscillates about the equilibrium and decays exponentially to zero in merely 0.3 s. This denotes that the thin-film is undergoing an underdamped vibration as a consequence of mechanical damping. Conversely, when air flow is present, the vortical structures shed from the grid bars induce piezoelectric thin-film vibration to generate a continuous \( V \) as a function of fluid fluctuating properties (see Fig. 4b). Since piezoelectric transducer is an AC-coupled device where electrical power is virtually generated from the turbulence fluctuations in the flow, and that the average power harvested exhibits trend and behaviour that is similar to that of the turbulence kinetic energy11, we could therefore infer that \( V \) and δ are interrelated with the local flow undulation. Hence in the present study, we calculate the local equivalent lateral flow velocity fluctuation \( v' \) from the time derivative of experimentally detected \( \delta \), i.e., \( \frac{\partial }{\partial t} \). We stress that the \( v' \) calculated represents the velocity fluctuation that is qualitatively, but not quantitatively equivalent to the actual fluid fluctuating velocity obtained using direct hot-wire measurements. The various equivalent turbulence statistics deduced from the present \( v' \) will be discussed in the following section to provide insights into the mechanical characteristics of grid-induced turbulent flow.

Results and discussion

Effects of grid-film distance on insert-induced flow characteristics. Figure 5a presents the RMS voltage \( V_{rm} \) and RMS tip deflection \( \delta_{rm} \) of the piezoelectric thin-film recorded at various streamwise distances \( x/D_h \) along SFG(P0, P5) and RG centerline, with their corresponding dominant frequencies \( f_V \) and \( f_δ \) shown in

| Parameter | Symbol | Value |
|-----------|--------|-------|
| Mass (g)  | \( m \) | 0.17 [36] |
| Stiffness (N/m) | \( k \) | 17.31 |
| Damping (10^-1 Ns/m) | \( c \) | 1.30–7.77 |
| Electromechanical coupling coefficient (10^-6 C/m) | \( \vartheta \) | 7.69 |
| Natural frequency (Hz) | \( f_\delta \) | 50.78 [36] |
| Damping ratio (10^-2) | \( \zeta \) | 1.20–7.16 |

Table 3. SDOF piezoelectric beam parameters. Superscript [m] denotes experimentally measured values.

where \( f_p \) is the frequency of peak amplitude in the FFT spectrum. \( f_p \) and \( f_l \) are respectively the frequencies obtained at half-quadratic gain level above and below \( f_p \), whereby the amplitudes at half-quadratic gain level are 1/√2 of the peak amplitude. It is crucial to mention that the maximum δ recorded for the present piezoelectric thin-film is only 20% of its film length, and that the film length is 145 × larger than the film thickness. Based on the classical Euler–Bernoulli cantilevered beam undergoing transverse vibration in \( y \)-direction \( y(x, t) \) can be related to the bending moment \( M(x, t) \) as28,

\[ M(x, t) = EI(x) \frac{\partial^2 y(x, t)}{\partial x^2} \] (7)

where \( x \) is the position along the beam length \( L \). For a clamped-free beam with first mode vibration, the mode shape \( \phi(x) \) at \( x=L \) is defined as28,

\[ \vartheta(L) = \cosh \beta_1 L - \cos \beta_1 L - \sigma_1 [\sinh \beta_1 L - \sin \beta_1 L] \] (8)

The weighted frequency \( \beta_1 L = 1.87528 \) while the mode shape coefficients \( \sigma_1 \) is given by,

\[ \sigma_1 = \frac{\sinh \beta_1 L - \sin \beta_1 L}{\cosh \beta_1 L + \cos \beta_1 L} = 0.7341 \] (9)

\[ \vartheta = d_{31} E_p b_p (2 h_a - h_p) \sigma_1 \beta_1 \] (10)
Fig. 5b. The local equivalent turbulence intensity $I_y$ in $y$-direction, defined as the ratio of RMS $v'$ to the inlet velocity ($v'_r$/$v_{in}$) is plotted against $x/D_h$ in Fig. 5c. The autocorrelation function of $v'$ is obtained to determine its internal correlation within a time series by comparing $v'$ at time $t$ with that at time lag $k$. The first zero-crossing $\tau_{zc}$ of the autocorrelation plot denotes the period at which $v'$ decorrelates from the primarily induced turbulence. By integrating the autocorrelation function of $v'$, the size of the large energy-containing eddies, viz. the equiva-
lent lateral integral length scale $L_v$ is calculated as per Eq. (11). A comparison between the $L_v$ for SFG$_0$, SFG$_5$, and RG$_0$ are depicted in Fig. 5d, where $L_v$ is normalized against the tunnel width $T$.

$$L_v = \frac{\tau_0}{v_m} \frac{\langle v'_v v'_{z+k} \rangle}{\langle v'^2 \rangle} \, dk$$  \hspace{1cm} (11)

Figure 5a,c reveal the existence of two turbulent regions, viz. the turbulence generation and decay regions as described in literature$^{7,21,30}$. The $V_{rms}$, $\delta_{rms}$, and $I_y$ induced by SFG-generated turbulence along $P_0$ and $P_5$ increase, respectively in the generation region to a peak at $x_{peak}/D_h = 0.813$ and 0.425, which are then decrease exponentially further downstream in the decay region. For RG, the values reach a peak at $x_{peak}/D_h = 0.125$ which are 5 to 7× lower than SFG before experiencing a power-law decay with respect to $x/D_h$. The overall 5× higher $f_v$ flow generated by the SFG along the centerline as compared to RG of similar $\sigma$ is in line with the previous studies$^{7,20}$. Essentially, the vorticity of vortical structures (eddies) shed from the grid bars induces fluctuations in velocity perpendicular to the piezoelectric thin-film surface which brings fluid up and down towards the thin-film$^{7}$.

Hence, higher $L_v$ promotes higher conversion of induced fluid dynamic perturbation into vibrational energy, giving rise to a 6× higher average $V_{rms}$ and $\delta_{rms}$ attained downstream of SFG$_0$ in comparison to RG$_0$. It is also important to note that although the signal-to-noise ratio of $\delta$ for small thin-film undulation is 9.3× lower than the voltage signal-to-electrical noise ratio, the primary signal, or the large-scale fluctuations of the $\delta$ and $V$ responses are rather well fitted to each other as depicted in Fig. 12b. Therefore, the statistical results deduced from the camera recorded $\delta$ are deemed reliable as evidenced in Fig. 5a,c where similar profiles are obtained for $V_{rms}$, $\delta_{rms}$, and $I_y$.

The further peak point observed for SFG$_0$ as compared to SFG$_5$ in Fig. 5a,c is due to the wake interactions of different-size bars at different $x/D_h$. The typical wake width $w$ at a streamwise distance $x$ based on a wake-generating bar of thickness $t$ is approximated as $w \approx \sqrt{xt}$ $^{21}$. Figure 5e illustrates the possible wake interactions originating from the bars of SFG at $y/D_h = -0.106$. Along position $P_5$ (shaded in blue), the wakes generated by the largest bar and smallest bar nearest to the midplane interact at a distance $x/D_h = 0.4$. This location is considerably closer to the grid as compared to the interaction between the wakes of largest grid bars on the centerline which occurs at $x/D_h = 1.5$. On the contrary, the peak location for RG$_0$ occurs closest to the grid since RG has a smaller $x_c$ as compared to SFG. In the present study, $x_{peak}/x_c = 0.54$, 0.28 and 0.29 for SFG$_0$, SFG$_5$, and RG$_0$, respectively. The larger $x_{peak}/x_c$ value attained for SFG$_0$ as compared to the 0.45 constant determined by Mazellier and Vassilicos$^{31}$ is due to our current SFG having a $\sigma$ which is almost double than that of their grids. Such high $\sigma$ is a direct result of the large $t/L_v$ ratio (0.32) which enlarges the size of the recirculation regions immediately leeward the largest grid bars, causing the streamlines of the flow to diverge towards the tunnel walls$^{7}$. This leads to a delay in the wake-interaction from the largest bars and prompts the wakes to meet further downstream.

As seen in Fig. 5b, although $f_v$ and $f_z$ in the lee of SFG have variations of less than 15% from the $f_v$ of the piezoelectric beam, its vibration frequency is affected to some extent, by the characteristic frequency of the vortices shed from the grid bars. The overall dominant frequencies $f$ of SFG$_0$ are the lowest, followed by SFG$_5$ and lastly RG$_0$. This can be explained by the larger overall $L_v/T$ of SFG$_0$ evident in Fig. 5d, of which large, slowly rotating eddies possess lower frequency while small eddies experience rapid fluctuations. As a result, the $L_v/T$ profiles are a complete reverse of the $f$ profiles. For the case of SFG$_0$, the $L_v/T$ calculated are the same order of magnitude as those determined by Hurst and Vassilicos$^{80}$. The $L_v$ for SFG$_0$ ranges from 11.5 to 13.7 mm, which on average is 1.1 to 1.3× larger than SFG$_5$ and RG$_0$. In grid-generated turbulence, the size of the largest eddies is set by the characteristic geometric length scale of the mean flow. Since $L_v$ is smaller for RG, its $L_v/T$ are considerably lower than SFG, whereas the difference in $L_v/T$ for SFG$_0$ and SFG$_5$ are due to the multi-scale vortices shed from the multiple-length bars of SFG.

From Fig. 5b,d, we further verify the existence of two distinct flow dynamic regimes downstream of the SFG. The highly irregular $f$ and $L_v/T$ observed near the grid corresponds to the turbulence generation region where the flow is inhomogeneous and anisotropic, while the relative uniform $f$ and $L_v/T$ obtained further downstream signifies the homogeneous and isotropic flow in the decay region. Additionally, it is observed that there is a drop in $f$ when $V_{rms}$ and $\delta_{rms}$ are the maximum for SFG$_0$ and SFG$_5$. Low $f$ reflects large-scale energy-containing turbulent eddies with significant amplitude of velocity fluctuations, acting as the primary components of the driving mechanism for thin-film flapping. Hence, a peak in film bending and voltage generation is generally accompanied by an increase in $L_v/T$ and a drop in $f$. This behaviour however, is not observable for the case of RG. Interestingly, even though the eddies for SFG$_5$ at $x_{peak}/D_h = 0.425$ have a smaller $L_v/T$ and higher $f$ compared to SFG$_0$ at $x_{peak}/D_h = 0.813$, their $V_{rms}$ and $\delta_{rms}$ are almost equivalent, with the $I_y$ of SFG$_0$ to be 1.1× higher than SFG$_0$. The wake-interaction phenomenon illustrated in Fig. 5e could provide a possible explanation, of which, $x_{peak}$ of SFG$_5$ is in close proximity to the location where the wake from the largest bar encounters the wake from the smallest bar nearest to the midplane. This implies that wake interactions could further intensify the local turbulence strength generated by the SFG which adds to the higher-than-expected peak responses obtained for SFG$_5$. Conversely, whilst the peak point on the centerline is bounded by the location where the inward spreading wakes from the largest bars interact with each other, its peaks are mainly attributable to the higher energy containing large-scale effective flow structures shed from the grid bars.

To comprehend the Gaussianity of $v'$, we observe the skewness $S_v$ (Fig. 6a) and flatness $F_v$ (Fig. 6b) of $v'$ against $x/D_h$ for SFG$_0$, SFG$_5$ and RG$_0$, where $S_v$ and $F_v$ are computed as,

$$S_v = \frac{\langle v'^3 \rangle}{\langle v'^2 \rangle^{3/2}}$$

$$F_v = \frac{\langle v'^4 \rangle}{\langle v'^2 \rangle^{2}}$$
strong suppression of the non-Gaussian behaviour of mechanical properties’ piezoelectric thin-film to examine how the inherited strength may be responsible for the flow. However, more research is needed to confirm this hypothesis, such as the use of different geometrical and in which PTFV may have potentially filtered out information relating to the higher order statistics of turbulent complex interaction between the flow and the thin-film structure as compared to direct hot-wire measurements, events either do not occur for a lateral flow or, the inherited strength is rather inferior. This could be due to the pronounced, with the major peak $Ev$ voltage output. On the other hand, minor peaks with relatively smaller shedding gives rise to higher turbulence intensity flow that yields larger film bending and consequently higher consistent with the trend previously observed in Fig. 5a,c. One can therefore deduce that more energetic vortex free decay homogenous isotropic turbulence. The variability of $Ev$ with their respective vortex shedding frequencies. In addition, the streamwise evolution of the maximum mined earlier in Fig. 5b, of which, they are associated to the first resonant mode of piezoelectric beam amalgamate variations in $Sv$ generally close to zero yet prone to extreme fluctuations sporadically. Furthermore, SFG demonstrates higher inhomogeneity in fractal grid-generated turbulence particularly in the turbulence generation region. Nevertheless, the extraordinary large longitudinal velocity flatness $Fv$ and negative velocity skewness $Su$ observed by along the centerline at $x/x_∗=0.2$ are not evident in Fig. 6a,b. We postulate that these intense decelerating flow events either do not occur for a lateral flow or, the inherited strength is rather inferior. This could be due to the complex interaction between the flow and the thin-film structure as compared to direct hot-wire measurements, in which PTFV may have potentially filtered out information relating to the higher order statistics of turbulent flow. However, more research is needed to confirm this hypothesis, such as the use of different geometrical and mechanical properties’ piezoelectric thin-film to examine how the inherited strength may be responsible for the strong suppression of the non-Gaussian behaviour of $v'$. Figure 7a–c present the streamwise energy spectra $E_v$ for SFG0, SFG5, and RG0 with the left column of figures showing the 3D downstream flow dynamic’s energy spectra, and top view along the right. The energy spectra are obtained from the Fourier transform of the autocovariance function, i.e., $v'$. As shown in Fig. 7, there appears to be multiple peaks in the spectrum for individual streamwise distance, indicating that energy is distributed over a multitude of vortices with a wide range of spatial scales and characteristic frequencies. The major peaks occurring near frequency $f=49$ Hz, 55 Hz and 62 Hz, respectively for SFG0, SFG5 and RG0 are the results of vortex shedding from the grid bars. One can infer that the vortex shedding effect from the SFG bars is more pronounced, with the major peak $Ev$ at $x_{peak}/D_h$ leeward of SFG to be approximately 30 $\times$ higher in comparison to RG. Such intense vortex shedding observed for the case of SFG0 could also have masked the non-Gaussian behaviour of $v'$ in the turbulence generation region, causing the corresponding $Sv$ and $Ev$ values in Fig. 6a,b to be close to 0 and 3, respectively. Moreover, more energetic fluctuations ($E_v>10^{-5}$) are detected for a broader range of $f$ in the turbulence generation region leeward of SFG0 (22 Hz < $f$ < 98 Hz) and SFG5 (29 Hz < $f$ < 85 Hz) as compared to RG0 (53 Hz < $f$ < 69 Hz), suggesting that SFG-induced turbulence contains more effective flow structures contributing to the thin-film undulation.

As shown in Fig. 6a,b, both grids reveal $Sv$ and $Ev$ values correspond to Gaussian distribution ($Sv=0, Ev=3$) except for SFG0 at $x/D_h=0.43$, where $Ev$ has a relatively larger magnitude ($Ev=3.5$), suggesting that the $v'$ are generally close to zero yet prone to extreme fluctuations sporadically. Furthermore, SFG demonstrates higher variations in $Sv$ and $Ev$ compared to the rather uniform values obtained for RG at all $x/D_h$ as a consequence of free decay homogeneous isotropic turbulence. The variability of $Sv$ and $Ev$ with $x/D_h$ could be an indication of the inhomogeneity in fractal grid-generated turbulence particularly in the turbulence generation region. Nevertheless, the extraordinary large longitudinal velocity flatness $Fv$ and negative velocity skewness $Su$ observed by along the centerline at $x/x_∗=0.2$ are not evident in Fig. 6a,b. We postulate that these intense decelerating flow events either do not occur for a lateral flow or, the inherited strength is rather inferior. This could be due to the complex interaction between the flow and the thin-film structure as compared to direct hot-wire measurements, in which PTFV may have potentially filtered out information relating to the higher order statistics of turbulent flow. However, more research is needed to confirm this hypothesis, such as the use of different geometrical and mechanical properties’ piezoelectric thin-film to examine how the inherited strength may be responsible for the strong suppression of the non-Gaussian behaviour of $v'$.

Figure 7a–c present the streamwise energy spectra $E_v$ for SFG0, SFG5, and RG0 with the left column of figures showing the 3D downstream flow dynamic’s energy spectra, and top view along the right. The energy spectra are obtained from the Fourier transform of the autocovariance function, i.e., $v'$. As shown in Fig. 7, there appears to be multiple peaks in the spectrum for individual streamwise distance, indicating that energy is distributed over a multitude of vortices with a wide range of spatial scales and characteristic frequencies. The major peaks occurring near frequency $f=49$ Hz, 55 Hz and 62 Hz, respectively for SFG0, SFG5 and RG0 are the results of vortex shedding from the grid bars. One can infer that the vortex shedding effect from the SFG bars is more pronounced, with the major peak $Ev$ at $x_{peak}/D_h$ leeward of SFG to be approximately 30 $\times$ higher in comparison to RG. Such intense vortex shedding observed for the case of SFG0 could also have masked the non-Gaussian behaviour of $v'$ in the turbulence generation region, causing the corresponding $Sv$ and $Ev$ values in Fig. 6a,b to be close to 0 and 3, respectively. Moreover, more energetic fluctuations ($E_v>10^{-5}$) are detected for a broader range of $f$ in the turbulence generation region leeward of SFG0 (22 Hz < $f$ < 98 Hz) and SFG5 (29 Hz < $f$ < 85 Hz) as compared to RG0 (53 Hz < $f$ < 69 Hz), suggesting that SFG-induced turbulence contains more effective flow structures contributing to the thin-film undulation.

The $f$ corresponding to the major peaks for all three cases are in excellent agreement with the $f_v$ and $f_δ$ determined earlier in Fig. 5b, of which, they are associated to the first resonant mode of piezoelectric beam amalgamate with their respective vortex shedding frequencies. In addition, the streamwise evolution of the maximum $E_v$ are consistent with the trend previously observed in Fig. 5a,c. One can therefore deduce that more energetic vortex shedding gives rise to higher turbulence intensity flow that yields larger film bending and consequently higher voltage output. On the other hand, minor peaks with relatively smaller $Ev$ tend to appear at higher $f$ close to the second vibration mode of the piezoelectric beam. These peaks are more marked in RG in comparison with SFG, especially in the domain proximate to grid, denoting that higher frequency small-scale eddies possess more significant contribution to RG-induced turbulence. As one proceeds downstream in the decay region, both the mechanically deduced major and minor peaks diminish due to viscous dissipation, with the minor peak intensity of RG attenuating up to 3 $\times$ faster than its major peak, since smaller eddies of higher characteristic frequencies decay faster.

Figure 6. (a) Thin-film undulation skewness $S_v$ and (b) flapping flatness $F_v$ against $x/D_h$.

$$F_v = \frac{\langle v'^4 \rangle}{\langle v'^2 \rangle^2}$$ (13)
Streamwise evolution of grid-induced turbulence forcing. To demonstrate the turbulence forcing of grid-induced flow at various distances leeward from SFG₀, SFG₅ and RG₀, the RMS of the normal flow excitation force $F_{rms}$ acting on the thin-film was computed as per Eq. (3) and plotted in Fig. 8a. The $F_{rms}$ calculated in the present study are in the range of 2 to 14 mN, which are consonant with the millinewton excitation force dynamically measured by Goushcha et al.¹⁰ in their turbulent boundary layer study with momentum thickness-based Reynolds number: $2.0 \times 10^3 \leq Re_\theta \leq 7.5 \times 10^3$.

As seen in Fig. 8a, the $F_{rms}$ for all three cases display a similar streamwise flow dynamic evolution as the results in Fig. 5a,c. The turbulence generated by SFG₀ generally possesses larger forcing compared to RG₀ except for $x/D_h \leq 0.25$. At $x_{peak}$, the turbulence forcing of fractal grid-induced flow on the centerline is more than twice of that induced by RG. Perhaps more interestingly may be the observation that the $F_{rms}$ for SFG₅ has a lower peak
than SFG0 despite both having almost the same peak $V_{rms}$ and $\delta_{rms}$ (see Fig. 5a). This could be explained by the higher equivalent vortex shedding energy of SFG0 in comparison to SFG5 as represented by the wider $f$ range of high energy fluctuations in Fig. 7. The same explanation applies to the case of RG0 and SFG0 at $x/D_h = 0.13$, where the former has an equivalent vortex shedding energy distributed among a range of frequencies that is $1.3 \times$ broader than the latter. Moreover, the small-scale eddies of RG0 possess more energy in exciting the thin-film considering that the $E_v$ of its minor peak is an order of magnitude larger than SFG0. From these findings, we can deduce that the experimentally recorded $V_{rms}$ and $\delta_{rms}$ reflect only the vortex shedding intensity of dominant large eddies. Conversely, the calculated $F_{rms}$ takes into consideration the broad-band random forcing across various eddy sizes including small-scale eddies of larger characteristic frequencies.

Figure 8b shows the damping ratio $\zeta$ of the thin-film vibration computed from Eq. (6). The film oscillations for all three cases are underdamped, with $\zeta < 0.1$. From the $\zeta$ evolution seen in Fig. 8b, the average $\zeta$ for SFG0 is $1.5 \times$ higher than RG0, signifying a higher thin-film damping in fractal grid-generated turbulence. This might be due to the complicated multilength-scale flow structures of SFG that flow through the thin-film over time, making the fluid domain around the thin-film becomes rather "crowded" yet highly mechanical effective.

The forced vibration of piezoelectric thin-film is the result of the interplays between fluid excitation force, inertia force, viscous effect and oscillating piezoelectric structure. Here we normalize the RMS of acceleration $ma_{rms}$, velocity $cv'_{rms}$, deflection $k\delta_{rms}$ and voltage $\theta V'_{rms}$ terms in Eq. (3) by their respective maximum over the streamwise distances. The downstream profile of the normalized terms ($ma_{rms}/ma_{max}$, $cv'_{rms}/cv'_{max}$, $k\delta_{rms}/k\delta_{max}$ and $\theta V'_{rms}/\theta V'_{max}$) along SFG0 and RG0 are depicted in Fig. 8c with their respective maximum terms listed in the side table. Clearly, the downstream evolution of all the four normalized terms for both grids are almost similar to the $F_{rms}$ profiles in Fig. 8a with the exception of the velocity term. Where $cv'_{rms}/cv'_{max}$ peaks at $x/D_h = 1.0$ for SFG0 and $x/D_h = 0.03$ for RG0, considering that the $\zeta$ at these two locations are the highest, respectively. One can also see that all the four terms are an order of magnitude higher for SFG0 when compared against RG0.

From the table in Fig. 8c, one can infer that the force acting on the piezoelectric cantilever beam is generally dominated by the inertia ($ma$) and bending forces ($k\delta$) followed by the mechanical damping force ($cv'$) of an order of magnitude smaller. The momentum exchange between turbulent fluctuations generated from the
grids contributes predominantly towards the beam forcing in the form of inertia force. The splashing of high momentum fluid transported by the moving vortical structures on the thin-film surface creates a considerable amount of bending force that causes the beam to deflect. At the same time, energy is lost through mechanical damping, viz. viscous air damping and structural damping. The dissipation of the electrical charges generated by piezoelectric through the circuit will also act to dampen out the beam vibration. In view of the low viscosity of air and high elastic compliance of current piezoelectric thin-film employed, mechanical damping force has a relatively minor effect on the beam forcing. Hence, SFG has an overall greater $F_{rms}$ than RG in spite of the higher thin-film damping imposed by fractal grid-generated turbulence. On the other hand, the force due to electromechanical coupling ($\theta V$) has an order of magnitude between $10^{-6}$ and $10^{-5}$ which is almost negligible, signifying that the electrical damping imposed by the present PVDF film is insignificant.

**Cross-sectional turbulence mechanical characteristics in the lee of SFG and RG.** In order to have a better comprehension of the grid’s cross-sectional turbulence mechanical characteristics in the turbulence generation region, the $V$ and $\delta$ of piezoelectric thin-film at $x/D_h = 0.425$ downstream of SFG and RG were recorded by placing the thin-film at the lateral positions indicated in Fig. 2c,d. Table 4 compares the results obtained for SFG($P_2$, $P_2a$) and SFG($P_{13}$, $P_{13a}$). All their results are seen to be respectively close to each other, which proves that the flow field leeward of SFG are symmetrical and therefore, the present study focuses mainly on the lower quarter grid. Figure 9a–d illustrate the 2D contours of $L/T$, $f_V$, $V_{rms}$ and local mean velocity $U$ for SFG, of which, $U$ is defined as,

$$U = \sqrt{U_x^2 + U_y^2 + U_z^2}$$

where $U_x$, $U_y$ and $U_z$ are respectively the local mean velocities in x, y and z-directions measured using hotwire anemometer. The corresponding results obtained for RG at positions $P_0$, $P_3$, $P_8$, $P_{10}$, $P_{19}$ and $P_{22}$ are plotted against SFG in Fig. 9e–h. Note that $\delta_{rms}$ and $I_y$ are omitted owing to the similarity with respect to $V_{rms}$ in Fig. 9c,g.

| Position | $V_{rms}$ (V) | $f_V$ (Hz) | $F_{rms}$ (mN) | $\xi$ | $L/T$ | $S_\theta$ | $S_\xi$ | $F_e$ | $U$ (m/s) |
|----------|---------------|------------|----------------|-------|-------|----------|--------|-------|----------|
| $P_2$    | 0.8           | 54.0       | 4.9            | 0.050 | 0.068 | 0.025    | 3.5    | 2.6   |
| $P_{2a}$ | 0.7           | 54.3       | 4.2            | 0.072 | 0.069 | 0.021    | 3.4    | 3.2   |
| $P_{13}$ | 1.1           | 68.5       | 9.2            | 0.032 | 0.055 | 0.003    | 3.1    | 4.8   |
| $P_{13a}$| 1.1           | 67.7       | 9.8            | 0.034 | 0.055 | 0.004    | 3.1    | 4.4   |

Figure 9. 2D contours of (a) $L/T$, (b) $f_V$, (c) $V_{rms}$ and (d) local mean velocity $U$ at $x/D_h = 0.425$ leeward of SFG, and (e–h) the corresponding results for RG at positions $P_0$, $P_3$, $P_8$, $P_{10}$, $P_{19}$ and $P_{22}$. 

Table 4. Results obtained for SFG at positions $P_2$ vs. $P_{2a}$ and $P_{13}$ vs. $P_{13a}$. 

$\delta_{rms}$ and $I_y$ are omitted owing to the similarity with respect to $V_{rms}$ in Fig. 9c,g.
However, due to the unique geometry of SFG, the small-scale eddies on P19 possess higher energy than P0 as wake region in Fig. 9d. Low ζ at the rear side of the largest grid bar due to the presence of recirculating flow as represented by the low velocity ζ grid-generated turbulence as supported by the high Vrms of the contour. This observation agrees well with the Vrms result in Fig. 9c, of which, large energy-containing eddies induce larger thin-film bending, giving rise to a higher voltage output. On the other hand, energy dissipates faster in smaller size eddies, leading to lower voltage generated. We hypothesize that the small-scale eddies present at bottom left of the contour is the result of vortices shed from smaller size grid bars near the corner of SFG, as well as the breaking up of large-scale eddies into smaller ones when flow strikes the side and bottom walls of test section. This could also explain the slight decrease in L/T and the increase in fV for both grids as one approaches the bottom of the test section (see Fig. 9e,f). At each of the lateral positions investigated, RG has a smaller L/T but higher fV compared to SFG.

In Fig. 9c, high Vrms is observed in the region between the largest grid bar and centerline of SFG with P12 having the greatest Vrms. This can be explained by the high velocity jet flowing through the grid opening (see Fig. 9d) as well as the wake-interaction occurring at P9 as discussed previously (see Fig. 5c). On the contrary, Vrms is low on the rear side of the largest grid bar due to the presence of recirculating flow as represented by the low velocity wake region in Fig. 9d. Low Vrms is also detected at the bottom left corner of the contour albeit the high U, with P22 having the lowest Vrms. This could be a consequence of the chaotic flow stirred up by the aforementioned small-scale eddies in the bottom left region of the test section. The force acting on the thin-film provided by the highly disordered eddies would most likely counteract each other, imposing thin-film damping upon fractal grid-generated turbulence as supported by the high ζ in Fig. 10b. On average, the cross-sectional Vrms in the lee of SFG are 5× higher than the RG in Fig. 9g. At P9, SFG has a lower U but higher Vrms than RG (see Fig. 9g,h) owing to the 3× higher localized Iy flow generated by SFG behind the largest grid bar.

The 2D contours of Vrms, ζ, S, and F, for SFG are presented in Fig. 10a–d. The results are compared against RG(P10, P11, P12, P13, P14, P19, P22) and plotted in Fig. 10e–h. With the exclusion of SFG(P12, P13, P19), the Frms in Fig. 10a generally displays a similar profile as the Vrms in Fig. 9c. By taking P9 as the baseline, one can see that both P10 and P11 have comparable Vrms due to the same intensity of vortex shedding, i.e., same maximum E5 (see Fig. 11a). However, due to the unique geometry of SFG, the small-scale eddies on P10 possess higher energy than P9 as depicted by the higher E5 of the minor peak near f = 240 Hz, giving rise to a larger Frms upon thin-film flapping. Applying a similar analysis, P13 has a lower Vrms than P9 due to the lower shedding intensity of faster rotating yet smaller L, vortices from the second iteration grid bar. Nevertheless, the additional forcing provided by the higher E5 small eddies on P13 gives rise to a larger Frms as compared to P9. Moving on to P12, the flow recirculation behind the largest grid bar leads to a less pronounced vortex shedding effect but with no significant impact on the energy content of small-scale eddies, thus the discrepancy in the Frms and Vrms on P12. As seen in Fig. 10c, the cross-sectional turbulence forcing of the RG-generated flow is comparatively uniform, which on average is 2× lower than the SFG-induced turbulence. The present findings further justify our speculations made earlier.
Figure 11. Lateral variations in energy spectrum (left) along $z/T = -0.419$ and $z/T = 0$ at $x/D_h = 0.425$ leeward of (a) SFG and (b) RG; top view of the respective spectrum (right).
that $F_{rms}$ expresses the force acting on the thin-film contributed by various scales flow structures, including the additional forcing provided by small-scale turbulence with less significant amplitude of velocity fluctuation.

From Fig. 10b,f, we once again observe that the film oscillations at all the lateral positions for both grids are underdamped, with SFG having an average $\zeta$ that is $1.5 \times$ higher than RG. As discussed previously, the high $\zeta$ in the bottom left region leeward of SFG is due to the highly chaotic fluid domain at the corner of the test section, whereas the high $\zeta$ behind the largest grid bar is caused by the recirculating flow in the wake region. On the other hand, the jet-like behaviour through the opening is the strongest on the centerline of SFG. When this strong central jet flows over the film surface, it may possibly hinder the upward and downward movement of thin-film, causing a less effective thin-film flapping as supported by the relatively higher $\zeta$ on $P_0$ than $P_1$ and $P_a$. Hence, lower $F_{rms}$ and $V_{rms}$ is found to be $7 \times 37 \times$ higher than RG with the exception of $P_{22}$ owing to the lower shedding intensity of high right side of Fig. 11. Hence, more intense vortex shedding gives rise to more effective flow structures responsible for the bottom wall of test section are identical to the $V_{rms}$ trends in Fig. 9c.g. It can also be observed that an increase in the $E_x$ of major peak is generally accompanied by a wider range of high velocity fluctuations as shown in the right side of Fig. 11. Hence, more intense vortex shedding gives rise to more effective flow structures responsible for the higher voltage generated through piezoelectric thin-film flapping. The vortex shedding intensity of SFG is found to be $7 \times 37 \times$ higher than RG with the exception of $P_{22}$ owing to the lower shedding intensity of high frequency, low energy containing vortices from the local higher iteration grid bars. Moreover, the breaking up of vortical structures by the corner walls as previously discussed could also have possibly weakened the vortex shedding effect. The vortex shedding phenomenon is also observed to be less pronounced on SFG($P_0, P_5, P_6$) since they are both located in the wake of the largest grid bar where flow recirculation occurs. As one advances towards the bottom wall, the $f$ corresponding to the major peak increases from 48 to 62 Hz for SFG and 61 to 74 Hz for RG, which agrees well with the $f_s$ results in Fig. 9b.f. For all the lateral positions investigated, we once again secure a more prominent minor peak for RG with respect to SFG except for $P_{22}$, signifying that the contribution of small-scale eddies towards $v'$ is very significant for SFG near the corner walls.

A probabilistic description of piezoelectric voltage response towards grid-induced turbulence. Figure 12a,b show a segment of the recorded time-history for $V$ and $\delta$ at $x/D_h = 0.425$ leeward of SFG and RG centerlines. The intermittent behavior observed for both grids are mainly due to the random nature of grid-induced turbulence impingement on the thin-film, of which, the $V$ and $\delta$ for SFG are about 6× greater than that of RG. Although at first glance the $V$ and $\delta$ responses for both grids seem well fitted, the small-scale fluctuations are not well matched to each other particularly for the RG in Fig. 12b. These small-scale fluctuations are the result of minute eddies impinging on the thin-film which may either intensify or offset the primary film oscillation induced by the large eddies. Nonetheless, if the large eddies contain high enough energy to cause observable film bending as seen for the case of SFG in Fig. 12a, it will mask the relatively insignificant fluctuations caused by the small eddies. These results are in good agreement with the energy spectra in Fig. 11, where a much prominent minor peak is observed for RG as compared to SFG. Furthermore, the small-scale fluctuations are mostly visible in the $V$ response since the output voltage is based on the overall piezoelectric deformation induced by the multilengh-scale eddies along the thin-film surface, instead of merely the flow structures revolving around the film tip. This demonstrates that $V$ not only possesses higher signal-to-noise ratio but also comprises of more comprehensive information of the flow dynamic, and is more responsive towards small-scale turbulence in comparison with camera recorded $\delta$.

Considering that $V$ better characterizes grid-induced turbulence than the camera recorded $\delta$, we attempt to examine the probability distribution of a random piezoelectric voltage response towards the turbulent flows generated by SFG and RG. On the left side of Fig. 13, we present the cumulative multiplicity of $V^2$ at different $x/D_h$ for SFG, SFG$_b$ and RG. It can be observed that the variations in the probability distribution with $x/D_h$ for all three cases follow the streamwise profiles in Fig. 5a.c. For SFG$_b$, at $x/D_h = 0.13$ where $V_{rms}$, $\delta_{rms}$ and $I_f$ are the lowest, the probability in obtaining large $V^2$ is also the lowest, of which, 99% are less than 3 $V^2$. At $x_{peak}/D_h = 0.81$, 95% of $V^2$ are below 68 $V^2$ while the top 1 percentile has $V^2 > 120 V^2$, which is highly similar to the probability distribution at $x_{peak}/D_h = 0.43$ of SFG. On the contrary, RG has much smaller $V^2$ as compared to SFG($P_0, P_5$). For the former, 99% of the $V^2$ at $x_{peak}/D_h = 0.13$ do not even exceed 3 $V^2$ and this is further reduced to 0.2 $V^2$ at $x/D_h = 2.50$.

On the right of Fig. 13, we plot the cumulative probability of $V^2$ at different lateral positions of SFG and RG where $x/D_h = 0.425$. It is immediately apparent that the variations in the distribution plots from middle to the bottom wall of test section for both grids are once again identical to the $V_{rms}$ profile in Fig. 9c.g, with RG having a narrower range of variation than the SFG. For the latter, the probability in obtaining large $V^2$ is high on $P_a$ and...
P4, where top 5 percentile have $V^2 > 16$ and 47 $V^2$ while top 1 percentile have $V^2 > 30$ and 88 $V^2$, respectively. In the wake of the largest grid bar (P8 and P11), 99% of $V^2$ are not more than 6 $V^2$ on average. $V^2$ is observed to be the lowest on P22 where 99% are below 0.5 $V^2$, which is similar to P22 of RG. For the rest of the positions, lower $V^2$ are attained for RG in comparison with SFG. It is also more likely to obtain small $V^2$ closer to the bottom wall for instance, top 1 percentile of RG (P0, P3) has $V^2$ above 0.8 $V^2$ but is narrowed down to 0.3 $V^2$ on P19.

In short, Fig. 13 not only demonstrates the dependency of voltage response on the grid-film distance and film lateral placement in the lee of grid, but also the unique advantages of SFG-induced turbulence over the turbulence generated by RG. Hence, this crucial finding presents opportunity for future study on correlating the piezoelectric output voltage with the various turbulence statistics and forcing of fractal grid-generated turbulence to unveil the multi-scale turbulence-mechanical interplays of fractal grid.

Furthermore, the overall high voltage response secured from the SFG-induced turbulence presents potential future energy harvesting opportunities. Piezoelectric thin-films could be placed at the regions where local turbulence strength is high to maximize the amount of electrical power harvested, particularly at the four P4 locations leeward of the SFG. Further research may be undertaken to scale-up the energy harvesting capability of the system, which include but not limited to optimising the geometrical parameters of the SFG and materials properties of the piezoelectric thin-films. Such energy harvesting module which consists of a SFG and an array of piezoelectric cantilever beams could be installed in various sections of the heating, ventilation and air conditioning (HVAC) ducts. The micro to milliwatt-scale electrical output harnessed may be used for self-powering low-power temperature and humidity sensors to monitor, control, and manage the HVAC systems. At the same time, the piezoelectric thin-films could also act as a flow sensing device to characterise the flow in the air ducts which could not be achieved by small wind turbines or solar panels.

**Conclusion**

The present experimental study was set out to explore the mechanical characteristics of fractal grid-generated turbulence based upon the direct fluid-structure interaction between the flow and a flexible piezoelectric thin-film. The film undulation $\delta$ and voltage response $V$ at different grid-film distances uniquely revealed the strength and the corresponding coverage of turbulence generation and decay regimes. Interestingly, the wake-interaction at P5 not only shifts the peak location $x_{\text{peak}}$ upstream but also enhances the local turbulence strength generated using SFG. Centerline results at $x_{\text{peak}}$ showed that the $V_{\text{rms}}$ and $\delta_{\text{rms}}$ in the lee of SFG are 7× larger than that of the RG of equivalent blockage ratio $\sigma$, with the former having a millinewton turbulence forcing $F_{\text{rms}}$ that is more than twice of RG.

The cross-sectional profiles of the thin-film’s physical response at $x/D_h = 0.425$ disclosed the inhomogeneity of fractal grid-generated flow in the turbulence generation region as compared to the relatively uniform flow induced by RG. The equivalent lateral integral length scale $L_v$ of the vortices shed from the multi-scale SFG bars ranges from 8.7 to 12.8 mm which on average is 1.3× larger than RG. Low frequency, large-scale energy-containing eddies at the central opening of SFG are primarily responsible for the high voltage generation, of which, the average cross-sectional $V_{\text{rms}}$ and $F_{\text{rms}}$ are respectively, 5× and 2× higher than RG, with SFG5 having the highest values. Nevertheless, the recirculating flow in the wake of the largest grid bar, in addition to the highly chaotic
flow stirred up by the small-scale eddies broken up by the corner walls of test section impose thin-film damping upon fractal grid-generated turbulence, which in turn weaken the local vortex shedding effect.

Our findings demonstrate the unique expression of insert-induced turbulence mechanical characteristics via PTVF, along with the characterization of the large-size flow structures' turbulence length scale desired for effective thermal dissipation and energy harvesting. Such system could potentially be employed in real-world applications for the purpose of evaluating the flow's local turbulence strength to gain insights into the broad-band random forcing across turbulence of various scales.

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References
1. Verbeek, A. A., Bouten, T. W. F. M., Stoffels, G. G. M., Geurts, B. J. & van der Meer, T. H. Fractal turbulence enhancing low-swirl combustion. Combust. Flame 162, 129–143 (2015).
2. Teh, A. L. et al. Thermal mixing enhancement of a free-cooling system with a fractal orifice plate. Chem. Eng. Res. Des. 100, 57–71 (2015).
3. Skanthan, S., Yeoh, C. V., Chin, W. M. & Foo, J. J. Forced convective heat transfer and flow characteristics of fractal grid heat sinks. Int. J. Therm. Sci. 125, 176–184 (2018).

Figure 13. Cumulative probability of $V^2$ at different streamwise locations along SFG($P_{0}$, $P_{5}$) and RG centreline (left), and at $x/D_h = 0.425$ downstream from the SFG and RG (right).
4. Hoi, S. M., Teh, A. L., Ooi, E. H., Chew, I. M. L. & Foo, J. J. Forced convective heat transfer optimization of plate-fin heat sink with insert-induced turbulence. Appl. Therm. Eng. 160, 114066 (2018).
5. Mandelbrot, B. B. The Fractal Geometry of Nature (W.H. Freeman and Company, 1983).
6. Kochergin, V. & Kearney, M. Existing biorefinery operations that benefit from fractal-based process intensification. Appl. Biochem. Biotechnol. 130, 349–360 (2006).
7. Laizet, S. & Vassilicos, J. C. DNS of fractal-generated turbulence. Flow Turbul. Combust. 87, 673–705 (2011).

8. Picozzi, F. & Mazzacurati, B. P. Field sensors technical manual. 2-3 (Measurement Specialties, Inc., 1999).
9. Akaydin, H. D., Elvin, N. & Andreopoulos, Y. Wake of a cylinder: A paradigm for energy harvesting with piezoelectric materials. Exp. Fluids 49, 291–304 (2010).
10. Goushcha, O., Akaydin, H. D., Elvin, N. & Andreopoulos, Y. Energy harvesting prospects in turbulent boundary layers by using piezoelectric transduction. J. Fluids Struct. 54, 823–847 (2015).
11. Danesh-Yazdi, A. H., Goushcha, O., Elvin, N. & Andreopoulos, Y. Fluidic energy harvesting beams in grid turbulence. Exp. Fluids 56, 161 (2015).
12. Meng, J., Xie, W., Brennan, M., Runge, K. & Bradshaw, D. Measuring turbulence in a flotation cell using the piezoelectric sensor. Miner. Eng. 66–68, 84–93 (2014).
13. Tabosa, E., Runge, K. & Holtham, P.N. Development and application of a technique for evaluating turbulence in a flotation cell. In XXVI International Mineral Processing Congress-IMPC 2012 5377–5390 (Technowrites, 2012).
14. Akaydin, H. D., Elvin, N. & Andreopoulos, Y. Energy harvesting from highly unsteady fluid flows using piezoelectric materials. J. Intell. Mater. Syst. Struct. 21, 1263–1278 (2010).
15. Akaydin, H. D., Elvin, N. & Andreopoulos, Y. The performance of a self-excited fluidic energy harvester. Smart Mater. Struct. 21, 025007 (2012).
16. Danesh-Yazdi, A. H., Elvin, N. & Andreopoulos, Y. Parametric analysis of fluidic energy harvesters in grid turbulence. J. Intell. Mater. Syst. Struct. 27, 2757–2773 (2016).
17. Ferko, K., Lachendorf, D., Bradley, A. & Danesh-Yazdi, A.H. Feasibility study of interacting side-by-side piezoelectric harvesters in low-intensity grid-generated turbulence. In Active and Passive Smart Structures and Integrated Systems XII (ed. Erturk, A.) 105950Q (SPIE, 2018).
18. Ferko, K., Chiappazzi, N., Gong, J. & Danesh-Yazdi, A.H. Average power output and the power law: Identifying trends in the behavior of fluidic energy harvesters in grid turbulence. In Active and Passive Smart Structures and Integrated Systems XIII (ed. Erturk, A.) 109670Z (SPIE, 2019).
19. Ferko, K., Lachendorf, D., Chiappazzi, N. & Danesh-Yazdi, A.H. Interaction of side-by-side fluidic harvesters in fractal grid-generated turbulence. In Active and Passive Smart Structures and Integrated Systems XII (ed. Erturk, A.) 105951E (SPIE, 2018).
20. Ferko, K., Chiappazzi, N., Gong, J. & Danesh-Yazdi, A.H. Power output comparison of side-by-side fluidic harvesters in different types of fractal grid-generated turbulence. In Active and Passive Smart Structures and Integrated Systems XIII (ed. Erturk, A.) 109670P (SPIE, 2019).

21. Mazellier, N. & Vassilicos, J. C. Turbulence without Richardson-Kolmogorov cascade. Phys. Fluids 22, 075101 (2010).
22. LDF1-028K piezo sensor w/ leads attached (Measurement Specialties, Inc., 2015).
23. Munson, B. R., Young, D. F., Okiishi, T. H. & Huebsch, W. W. Fundamentals of Fluid Mechanics Vol. 486 (Wiley, 2009).
24. Sodano, H. A., Park, G. & Inman, D. Estimation of electric charge output for piezoelectric energy harvesting. Strain 40, 49–58 (2004).
25. Elvin, N. G., Lajnef, N. & Elvin, A. A. Feasibility of structural monitoring with vibration powered sensors. Smart Mater. Struct. 15, 977–986 (2006).
26. Castro, M.J. Extracting damping ratio from dynamic data and numerical solutions. 2–4 (2016). At https://ntrs.nasa.gov/citations/2017005173
27. Hobeck, J. D. & Inman, D. J. Artificial piezoelectric grass for energy harvesting from turbulence-induced vibration. Smart Mater. Struct. 21, 105024 (2012).
28. Inman, D. J. Engineering Vibration PIE 545–552 (Pearson Australia Pty Ltd, 2013).
29. Elvin, N. G. & Elvin, A. A. A coupled finite element—circuit simulation model for analyzing piezoelectric energy generators. J. Intell. Mater. Syst. Struct. 20, 587–595 (2008).
30. Hurst, D. & Vassilicos, J. C. Scalings and decay of fractal-generated turbulence. Phys. Fluids 19, 035103 (2007).
31. Maxey, M. R. The velocity skewness measured in grid turbulence. Phys. Fluids 30, 935 (1987).
32. Zhou, Y. et al. Relevance of turbulence behind the single square grid to turbulence generated by regular- and multiscale-grids. Phys. Fluids 26, 075105 (2014).
33. Melina, G., Bruce, P. J. & Vassilicos, J. C. Vortex shedding effects in grid-generated turbulence. Phys. Rev. Fluids 1, 044402 (2016).
34. Gomes-Fernandes, R., Ganapathisubramani, B. & Vassilicos, J. C. The energy cascade in near-field non-homogeneous non-isotropic turbulence. J. Fluid Mech. 771, 676–705 (2015).
35. Nagata, K., Saiki, T., Sakai, Y., Ito, Y. & Iwano, K. Effects of grid geometry on non-equilibrium dissipation in grid turbulence. Phys. Fluids 29, 015102 (2017).
36. Nagata, K. et al. Turbulence structure and turbulence kinetic energy transport in multiscale/fractal-generated turbulence. Phys. Fluids 25, 065102 (2013).
37. Blevins, R. D. Flow-Induced Vibration 241–243 (Krieger Pub. Co., 2001).

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T.S.L. conducted the experiments and prepared the manuscript draft. E.H.O., W.S.C. and J.J.F. supervised the project, revised and edited the manuscript. All authors reviewed the manuscript.

Competing interests
The authors declare no competing interests.

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