Phase diagram of hot quark matter under magnetic field

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In this talk, I review the computation of the phase diagram of hot quark matter in strong magnetic field, at zero baryon density, within an effective model of Quantum Chromodynamics.

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I. INTRODUCTION

The modification of the QCD vacuum, and of its thermal excitations as well, under the influence of external fields, is an attractive topic. Firstly, it is extremely interesting to understand how an external field can modify the main characteristics of confinement and spontaneous chiral symmetry breaking. Secondly, strong magnetic fields, with order of magnitude between $eB \approx m_\pi^2$ and $eB \approx 15m_\pi^2$, might be produced in the very first moments of the non-central heavy ion collisions [1, 2]. In this case, it has been argued that the non-trivial topological structure of thermal QCD gives rise to Chiral Magnetic Effect (CME) [1, 3, 4].

An useful approach to the physics of strong interactions in external magnetic fields is the use of some model. Among them, the Nambu-Jona Lasinio (NJL) model [5] is quite popular, see Refs. [6] for reviews. In this model, the QCD gluon-mediated interactions are replaced by effective interactions among quarks, which are built in order to respect the global symmetries of QCD. On the other hand, the NJL model lacks confinement of color. It is well known that color confinement can be described in terms of the center symmetry of the color gauge group and of the Polyakov loop [6], which is an order parameter for the center symmetry. Motivated by this property, the Polyakov loop extended Nambu-Jona Lasinio model (P-NJL model) has been introduced [6, 7], in which the concept of statistical confinement replaces that of the true confinement of QCD, and an effective interaction among the chiral condensate and the Polyakov loop is achieved by a covariant coupling of quarks with a background temporal gluon field. In the literature, there are several studies about various aspects of the P-NJL model [10–15, 17–26]. Lattice studies on the response of the QCD ground state to external magnetic and chromomagnetic fields can be found in [27–31]. Previous studies of QCD in magnetic fields, and of QCD-like theories as well, can be found in Refs. [32–38].

Beside the Polyakov loop, it has been suggested [39] that another observable which is an order parameter for the center symmetry, hence for confinement, is the dressed Polyakov loop. The dressed Polyakov loop has been computed in Refs. [40] within the scheme of truncated Schwinger-Dyson equations; within the Nambu-Jona Lasinio model, $\Sigma_1$ has been computed at finite temperature and chemical potential in [41]. Finally, the dressed Polyakov loop has been computed within the PNJL model in [21] at finite temperature, and in [26] at finite temperature with strong magnetic field.

In this talk, I present results obtained in Ref. [26] about the phase structure and the dressed Polyakov loop of hot two massive flavor quark matter at zero chemical potential, in an external magnetic field. To compute the effective potential, I rely on the PNJL model of strongly interacting quarks.

II. THEORETICAL FRAMEWORK

I consider here two flavor quark matter whose Lagrangian density is specified as [26]

$$\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m_0)q + g_a [(\bar{q}q)^2 + (\bar{q}i\gamma_\tau q)^2]$$

$$+ g_a [(\bar{q}q)^2 + (\bar{q}i\gamma_\tau q)^2]^2.$$  

(1)

The covariant derivative embeds the quark coupling to the external magnetic field and to the background gluon field as well, see below. In Eq. (1), $q$ represents a quark field in the fundamental representation of color and flavor (indices are suppressed for notational simplicity); $\tau$ is a vector of Pauli matrices in flavor space; $m_0$ is the bare quark mass,
which is fixed to reproduce the pion mass in the vacuum, \( m_\pi = 139 \text{ MeV} \). The model at hand is called Polyakov loop extended Nambu-Jona Lasinio model (PNJL in the following), since a coupling of the chiral condensate and the Polyakov loop is introduced via the covariant derivative in Eq. (1), see below.

In this study, I limit myself to the one-loop approximation for the partition function. In order to couple the Polyakov loop to the quark fields, it is customary, in the PNJL model, to introduce a background temporal, static and homogeneous Euclidean gluon field, \( A_4 \), in terms of which the Polyakov loop is given by \( P = \text{Tr}[\exp(i\beta A_4)] / 3 \). \( A_4 \) is coupled to the quarks via the covariant derivative, see Eq. (1); as a consequence, a coupling among the quark fields and the Polyakov loop arises naturally when the integration over fermion fields in the partition function is performed.

I work in the Landau gauge, and take the magnetic field homogeneous, static and aligned with the positive \( z \)-axis. Moreover, I take twisted fermion boundary conditions along the compact temporal direction, while for spatial directions the usual periodic boundary condition is taken. The one-loop thermodynamic potential in the general case of twisted boundary conditions is given by \[\Omega = U(P, \bar{P}, T) + \frac{\sigma^2}{g_\sigma} + \frac{3\sigma^4 g_k}{g_\sigma^4} - \sum_{f=u,d} \frac{|q f e B|}{2\pi} \sum_k \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} g_A(p_z, k) \omega_k(p_z) \]

\[ - T \sum_{f=u,d} \frac{|q f e B|}{2\pi} \sum_k \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} (1 + 3 Pe^{-\beta \varepsilon_-} + 3 \bar{P} e^{-2\beta \varepsilon_-} + e^{-3\beta \varepsilon_-}) \]

\[ - T \sum_{f=u,d} \frac{|q f e B|}{2\pi} \sum_k \alpha_k \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} (1 + 3 Pe^{-\beta \varepsilon_+} + 3 \bar{P} e^{-2\beta \varepsilon_+} + e^{-3\beta \varepsilon_+}) \].

In the previous equation, \( \sigma = g_\sigma \langle \bar{q} q \rangle = 2g_\sigma \langle \bar{u} u \rangle \); \( k \) is a non-negative integer which labels the Landau level; \( \alpha_k = \delta_k 0 + 2(1 - \delta_k 0) \) counts the degeneracy of the \( k \)-th Landau level. Moreover, \( \omega_k(p_z) = p_z^2 + 2 |q f e B| k + M^2 \), with \( M = m_0 - 2\sigma - 4\sigma^2 g_k / g_\sigma^2 \). The arguments of the thermal exponentials are defined as \( \varepsilon_\pm = \omega_k(p_z) \pm i(\phi - \pi) / \beta \), with \( \phi \) defined in Eq. (2). The vacuum part of the thermodynamic potential, \( \Omega(T = 0) \), is ultraviolet divergent. In this study, I use a smooth regularization procedure by introducing a form factor \( g_A(p) \) in the diverging zero-point energy. The potential term \( U(P, \bar{P}, T) \) in Eq. (3) is built by hand in order to reproduce the pure gluonic lattice data [10]. Among several different potential choices I adopt the logarithmic form of [10].

Following [39], I introduce the dual quark condensate,

\[\bar{\Sigma}_n (m, V) = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \bar{q} q \rangle \overline{V} \]

where \( n \) is an integer. The expectation value \( \langle \cdot \rangle_G \) denotes the path integral over gauge field configurations. The case \( n = 1 \) is called the dressed Polyakov loop. For my later convenience, I scale the definition of the dressed Polyakov loop in Eq. (4), and introduce

\[\Sigma_1 = -2\pi g_\sigma \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{q} q \rangle \overline{V} \]

\[ = - \int_0^{2\pi} d\varphi \, e^{-i\varphi} \sigma(\varphi) \],

where \( \sigma(\varphi) \) corresponds to the expectation value of the \( \sigma \) field computed keeping twisted boundary conditions for fermions.

III. RESULTS

In the left panel of Fig. [4] I collect the results for the dressed Polyakov loop as a function of temperature and magnetic field strength. In the right panel of the same figure, I plot the data of the effective susceptibility, \( d\Sigma_1 / dT \). The bifurcation of the effective susceptibility at large value of the magnetic field strength arises because the dressed Polyakov loop is capable to feel both the deconfinement and the chiral crossovers, which are split of almost 15% if the magnetic field strength is large enough.

In Figure [5] I collect the results on the pseudo-critical temperatures for chiral and Polyakov loop crossovers, in the form of phase diagrams in the \( eB - T \) plane. The dashed line denotes the Polyakov loop crossover, and the dot-dashed line
FIG. 1. Dressed Polyakov loop (left panel) and its effective susceptibility (right panel) as a function of temperature and magnetic field strength.

FIG. 2. Phase diagram of the PNJL model in magnetic field. Dashed line denotes the Polyakov loop crossover; dot-dashed line corresponds to the chiral crossover. The shaded area is the region, in the $eb - T$ plane, in which quark matter is not statistically confined, but chiral symmetry is still broken by the chiral condensate. Temperatures on the vertical axes are measured in units of the pseudo-critical temperature at zero field, which is $T_0 = 175$ MeV.

It is instructive to compare this result with those obtained in a different model. The shape of the phase diagram drawn in Fig. 2 is similar to that drawn by the Polyakov extended quark-meson model, see e.g. Fig. 13 of Ref. 37. In that reference, an interpretation of the split in terms of the interplay among vacuum and thermal contribution, is given. I totally agree with those arguments, which are reproduced within the PNJL model as well, as the results on critical temperatures show.

IV. CONCLUSIONS

In this talk, I have reported about the computation of the dressed Polyakov loop and of the phase diagram of hot quark matter under the influence of a strong external magnetic field. The results on the dressed Polyakov loop, $\Sigma_1$, ...
in magnetic field show that this quantity is capable to describe both Polyakov loop and chiral crossovers. This is resumed in the double peak structure of its effective susceptibility. We measure an increase of both deconfinement and chiral crossovers; the tiny split of the two critical temperatures is of the order of 10% for the largest value of the magnetic field strength considered here.

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