Possible Anderson localization in a holographic superconductor

Hua Bi Zeng\textsuperscript{1,2}

\textsuperscript{1}School of Mathematics and Physics, Bohai University JinZhou 121000, China
\textsuperscript{2}CFIF, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

We study the effect of disorder in a holographic superconductor by introducing a quasi-periodic potential on the boundary field theory, the strength of disorder is controlled by a parameter \( \alpha \). By tuning \( \alpha \) we find that when the condensation is small, the weak disorder will destroy the superconductivity, clearly this is a holographic realization of Anderson localization in superconductors.

\textbf{Introduction.} - The effect of disorder in superconductor has intrigued scientists for several decades. Soon after the BCS theory \cite{1}, Anderson found that weak disorder cannot destroy the superconductivity \cite{2}. Until now, both theories and experiments have confirmed that a strong disorder will eventually destruct superconductivity, driving the system into an insulating state or a normal metal state \cite{3-10}. However, the effect of interactions in a disordered superconductors is still not well understood. As a natural way to study a strongly coupled quantum field theory systems, the AdS/CFT correspondence \cite{17} has been used to study the interplay of disorder and interaction \cite{11-16}. The holographic correspondence has also been proved to be successful to study various properties of superconductors\cite{18 19}. In reference \cite{20} the authors firstly studied a dirty holographic superconductor, the found that the disordered superconductor always has a larger critical temperature relative to the to the \( T_c \) for the uniform one. In this paper we focus on understanding another important issue, the possible Anderson localization in a holographic superconductor. Technically, the weak disorder effect is introduced by a quasi-periodic chemical potential on the boundary field theory, the strength of the disorder is controlled by a parameter \( \alpha \). By tuning \( \alpha \) we find that when the condensation is small, the weak disorder will destroy the superconductivity, clearly this is a holographic realization of Anderson localization in superconductors.

\textbf{Model and definition of disorder.} - The starting action in the usual gravity dual of a holographic superconductor is \cite{18} \( S = \int d^4x \sqrt{-g} R - 2\Lambda - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - |\nabla \psi - i A \psi|^2 - m^2 |\psi|^2| \) where \( \Lambda = -d(d - 1)/2L^2 \) is the cosmological constant, \( d \) is the dimension of the boundary, and \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \) is the strength of the gauge field. The metric is an AdS Schwarzschild black hole, \( ds^2 = -f(r) dt^2 + \frac{r^2}{f(r)} dr^2 + r^2(dx^2 + dy^2) \) with \( f(r) = r^2/L^2(1 - r_0^3/r^3) \), \( r \) being the bulk radial coordinate, \( r_0 \) the horizon position, and \( x, y \) the boundary coordinates. Without loss of generality, we set \( L = 1 \). The temperature of the black hole is \( T = \frac{3r_0}{4\pi} \).

We use the ansatz of \( \psi = \psi(r, x) \) and \( A = (A_t(r, x), 0, 0, 0) \), where \( x \) is the spatial coordinate of the boundary field theory, and choose \( m^2 = -2 \). In the probe limit, with the scaling of \( \psi \rightarrow \psi/r \) and working in the coordinates with \( z = 1/r \), we have the following equations of motion (EoMs):

\begin{align}
(1 - z^3) A_t^{(2,0)}(z, x) + A_t^{(0,2)}(z, x) - 2A_t(z, x)\psi(z, x)^2 &= 0, \\
\psi(z, x) (A_t(z, x)^2 + z^4 - z) &+ (1 - z^3) \psi^{(0,2)}(z, x) + (z^3 - 1)^2 \psi^{(2,0)}(z, x) \\
+ 3(z^3 - 1) z^2 \psi^{(1,0)}(z, x) &= 0.
\end{align}

The superscripts on the fields mean the derivative of \( z \) and \( x \), for example \( A_t^{(2,0)}(z, x) \) means \( \partial^2 z A_t(z, x) \) and \( A_t^{(0,2)}(z, x) \) means \( \partial^2 x A_t(z, x) \). The expansions of \( \psi \) and \( A_t \) near the infinite boundary are:

\begin{align}
\psi(r, x) &\sim \psi^{(0)}(x) + \psi^{(1)}(x)z + \ldots, \\
A_t(r, x) &\sim \mu(x) + \rho(x)z + \ldots.
\end{align}
Interplay of disorder effect and periodic effect.- Holographic superconductor with periodic chemical potential has been studied in [25,27]. In [26,27] the authors found that the superconductivity is enhanced by the presence of the periodic chemical potential. In Fig. 2 we plot the average value of the order parameter $\langle O^a \rangle$ as a function of $\alpha$ for various combinations of $k_1$ and $k_2$ with fixed $\mu'$ and $\mu_a$. The lowest pink lines in Fig. 2 are the homogeneous solutions with $\mu(x) = \mu_a = 4.05$ and 5. It is known that $\mu_c = 4.06$ is the critical value for the homogeneous configuration after $\mu_c$ we will see no superconductivity [18]. From the left plot it can be seen that $\langle O^a \rangle = 0$ when $\mu(x) = 4.05 < 4.06$, while for the periodic or quasi-periodic cases we have non-zero condensation for some regions of $\alpha$. Similar phenomena also happen for the case of $\mu_a = 5$. In all cases, both periodic and quasi-periodic chemical potential induce a larger value of order parameter compared to the homogeneous case.

When $\alpha = 0$ or $\alpha = 1$, we recover the cases of periodic chemical potentials: $\mu(x) = \mu_a + (\mu' - \mu_a) \cos(k_1 \pi x)$ and $\mu(x) = \mu_a + (\mu' - \mu_a) \cos(k_2 \pi x)$. From Fig. 2 we can see that $\langle O^a \rangle$ decreases with increasing $k$ in the periodic cases. As a check we see that when $k = k_2 = 7/2, 9/2$, which is greater than $k_1 = 2$, $\langle O^a \rangle$ for a periodic $\mu(x)$ with $k = k_2$ is smaller than that of $k = k_1$. If we keep increasing $k$ (the results are not include here), the condensation $\langle O^a \rangle$ asymptotes some constant value. These results have also been found in [25,27].

Looking at the two red lines with dots in the top of Fig. 2 we see the condensation does not monotonically increase with increasing $\alpha$. The condensation decreases first then increases when we increase the portion of the case of $k_2 = 3/2$ by tuning $\alpha$. This means that there is an interplay between the disorder effect and the periodic effect: periodic chemical potential favors an increasing condensation, while disorder favors a decreasing one. In the left plot of Fig. 2, we see the condensation decreases for a periodic $\mu(x)$ with $k = k_2 = 7/2, 9/2$ (blue lines) also confirm the existence of disorder effect. We can also see a phase transition from the superconducting phase to a normal phase at $\alpha_c \sim 0.8$ when $k_1 = 2, k_2 = 9/2$, but the main reason of the phase transition is the periodic effect since the transition happens at $\alpha_c > 0.5$ and the periodic case with $k = k_2 = 9/2$ is of a vanishing condensation.

We also studied how the condensation behaves when we tune both $\mu'$ and $\alpha$ with a fixed $\mu_a$. Figure 3 shows...
The average value of order parameter $\langle O_\alpha \rangle$ as a function of $\alpha$ for different $k_2$ with a fixed $k_1 = 2$. In the left plot $\mu' = 8.1, \mu_a = 4.05$, in the right plot $\mu' = 10, \mu_a = 5$. The lowest two pink dotted lines is the homogeneous case with $\mu(x) = 4.05$ and $\mu(x) = 5$ respectively. In all the plots we increase $\alpha$ from 0 to 1 with a step $\delta \alpha = 0.05$.

$\langle O_\alpha \rangle$ as a function of both $\alpha$ and $\mu'$, where $0 < \mu' < 2\mu_a$ is chosen in order to have positive chemical potentials. The important information from Fig. 6 is that when we reduce $\mu'$ (the oscillating amplitude) with fixed $\alpha, \mu_a, k_1$ and $k_2$, the condensation will be decreased.

The two parameters $\alpha$ and $\mu'$ control the properties of the disorder effect, and the quasi-periodic $\mu(x)$ effect the superconductor in a complex way. With a fixed $\alpha$, increasing the amplitude $2(\mu' - \mu_a)$ of $\mu(x)$ enhances the superconductivity, as shown in Fig. 3. When $\alpha = 0$ or 1 we reproduce the result that the superconductivity of a striped holographic superconductor will be enhanced 20 28.

However, with a fixed amplitude $2(\mu' - \mu_a)$, the disorder can always suppress the superconductivity when by turning $\alpha$ from zero to a finite value, as shown in Fig. 1, Fig. 2, Fig. 3 and Fig. 4.

The interplay between the disorder effect and the periodic effect with fixed $\mu'$ and $\mu_a$ will result in a phase transition from the superconducting state to a non-superconducting state in some regions of parameters as shown in Fig. 1 and Fig. 3. The DC conductivity along the $y$ direction of the non-superconducting state is finite, as shown by the inset in Fig. 1 ($\alpha = 0.25$), which means that the non-superconducting state is a normal metal state rather than an insulating state.

Discontinuous phase transition from superconducting to normal state.- With the results in the above section, we already see that there is a phase transition when the superconductor is close to $T_c$ ($\mu_a \approx \mu_c = 4.06$) by increasing $\alpha$ form zero to a finite value ($\alpha = 0.5$) for $\mu_a = 4.02$ and $\mu_a = 4.01$ as shown in Fig. 1 and Fig. 3. Figure 4 shows the critical value of $\alpha_c < 0.5$, at which a phase transition from the superconducting state to the normal state occurs when $\mu_a = 4.02$ and $\mu_a = 4.01$. We note that the value of $\alpha_c$ for the case of $\mu' = 8.02, \mu_a = 4.01, k_1 = 2, k_2 = 3/2$ is larger than that for the case of $\mu' = 8.02, \mu_a = 4.01, k_1 = 2, k_2 = 7/2, (\approx 0.2)$, which is a consequence of the interplay between the disorder effect and the periodic effect as studied above. From the four insets in Fig. 4 (blue lines), we see the order parameter goes discontinuous at $\alpha_c$, which indicates that the superconducting to normal phase transition is a discontinuous one. By computing many cases with other values of $\mu_a$ systematically, we find that when $\mu_a > 4.03$ there is no disorder driven phase transition anymore with $k_1 = 2$.

When the phase transition happens, the free energy of the superconductor is also obtained by computing the on-shell action according to the AdS/CFT dictionary. The results are shown in Fig. 5. It is clearly to see that the free energy also goes discontinuously at $\alpha_c$, which means that this is a zeroth order phase transition. More details
for the calculations of conductivity and free energy will be presented elsewhere [29].

Conclusion.- In this paper, we systematically studied the interplay of disorder effect and periodic effect in two dimensional $s$-wave holographic superconductors. We reproduced the results in condensed matter physics that the disorder will suppress superconductivity and finally result in a discontinuous superconducting to normal state phase transition when the gap is sufficiently small relative to the strength of disorder. It seems natural to interpret the phenomena as an Anderson localization in holographic superconductor, however we are working in the probe limit and there is no momentum dissipation, and we do not see the signal of a transition from the normal state to the insulating state when increasing the disorder for $T > T_c$. Then it is still need further studying to make sure if we can say this is a holographic realization in AdS/CFT.

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