Benchmarking models of breakup reactions

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Abstract. The continuum discretised couple-channel model (CDCC) and the dynamical eikonal approximation (DEA) are two state-of-the-art models used to study the breakup of halo nuclei. In this work, we compare them for the breakup of $^{15}$C on Pb at 68.4 MeV. Excellent agreement is obtained for both energy and angular distributions. Slight discrepancy is observed in the contributions of some partial waves, which might affect other breakup observables, like momentum distributions. The use of relativistic velocity has little effect on the energy distribution, but affects more significantly the angular distribution.

1. Introduction

The development of radioactive-ion beams in the mid 80s has enabled the study of nuclei far from stability. This technical breakthrough lead to the discovery of halo nuclei [1]. These light, neutron-rich nuclei exhibit a matter radius significantly larger than their isobars. This peculiarity is due to their small one- or two-neutron separation energy. The small binding of these valence neutrons enables them to tunnel far outside the classically allowed region and form a sort of halo around the other nucleons. This halo is responsible for the large matter radius of these nuclei [2, 3]. Examples of halo nuclei are $^{11}$Be and $^{15}$C, with a one-neutron halo, and $^6$He and $^{11}$Li, with a two-neutron halo. Proton haloes, though less probable, are also possible. These fascinating objects are thus the subject of many theoretical and experimental studies [2, 3]. Being short lived, halo nuclei cannot be studied through usual spectroscopic techniques, and one must resort to indirect methods, like reactions, to obtain information about their structure. In breakup reaction, the halo dissociates from the core through the interaction with a target. In order to get valuable information from breakup measurements, the reaction mechanism has to be well understood. Several models have been used to describe accurately the breakup of one-neutron halo nuclei: time-dependent models [4, 5, 6], eikonal approximations [7, 8], and the continuum discretised coupled-channel method (CDCC) [9, 10].

All these models rely on the same assumptions that the projectile exhibits a two-cluster structure and that the interactions of these clusters with the target can be simulated by optical potentials. They only differ in the approximation they use to solve the subsequent three-body Schrödinger equation. Their results are usually in fair agreement with experimental data, but they have never been closely compared to one another. The aim of this work is to perform such a comparison between the CDCC model [9, 10] and the dynamical eikonal approximation (DEA) [8, 11]. The reaction chosen is the breakup of $^{15}$C on Pb at 68.4 MeV, which corresponds
to the conditions of an experiment performed at Riken [12]. The ultimate goal of this study is the evaluation of the approximations performed in each model, and to deduce the domain of validity of these models. It could also help replacing CDCC by DEA, which is much less computationally expensive. We also investigate the influence on these calculations of using a relativistic velocity.

After briefly describing both reaction models, we present the results of the comparison for two breakup observables: the energy distribution, which corresponds to the breakup cross section as a function of the relative energy between the core and the halo neutron after dissociation, and the angular distribution, which is the breakup cross section as a function of the scattering angle between the projectile center of mass and the target. In Sec. 4, the influence of the relativistic velocity upon breakup calculations is reported. The conclusion and perspectives of this work are drawn in the last section.

2. Breakup models
2.1. Theoretical framework

As mentioned above, both CDCC and DEA share a common set of approximations. In both cases, the projectile $P$ is described as a core $c$, of mass $m_c$ and charge $Z_c e$, to which a fragment $f$, of mass $m_f$ and charge $Z_f e$ is loosely bound. This two-body structure is modelled by the Hamiltonian $H_0$

$$H_0 = -\frac{\hbar^2}{2\mu_{cf}} \Delta + V_{cf}(r),$$

(1)

where $\mu_{cf}$ is the core-fragment reduced mass, $r$ is the coordinate of the fragment to the core, and $V_{cf}$(r) is the potential that simulates the interaction between the two components of the projectile. This potential contains a Woods-Saxon central part plus a spin-orbit coupling term. In partial wave $l_j$, the eigenstates of $H_0$ are defined by

$$H_0 \phi_{ljm}(E, r) = E \phi_{ljm}(E, r),$$

(2)

where $E$ is the $c$-$f$ relative energy. The negative-energy solutions of Eq. (2) correspond to the bound states of the projectile. The positive-energy states describe the continuum of the projectile, i.e. when the fragment is dissociated from the core.

The target $T$ is assumed to be a structureless particle, and its interaction with the projectile is simulated by optical potentials

$$V_{PT}(r, R) = V_{cT}\left(\frac{m_f}{m_c + m_f} r\right) + V_{fT}\left(\frac{m_f}{m_c + m_f} r\right),$$

(3)
where $R$ is the coordinate of the projectile relative to the target. The set of coordinates $(r$ and $R)$ is sketched in Fig. 1.

Within this framework, the study of breakup reduces to the resolution of the three-body scattering problem

$$
\left( -\frac{\hbar^2}{2\mu_{PT}}\Delta + H_0 + V_{PT} \right) \Psi(r, R) = E\Psi(r, R),
$$

with the initial condition that the projectile, in its ground state $\phi_{l_0j_0m_0}$, is impinging on the target

$$
\Psi^{(m_0)}(r, R) \xrightarrow{Z \to -\infty} e^{iKZ}... \phi_{l_0j_0m_0}^0(E_0, r).
$$

In Eq. (4) $\mu_{PT}$ is the projectile-target reduced mass, and $E$ is the total energy of the system in the $P$-$T$ center of mass rest frame. The wavenumber $K$ appearing in Eq. (5) is obtained from the total energy $E = \frac{\hbar^2K^2}{2\mu_{cf}} + E_0$. Since Eq. (4) cannot be solved analytically, one must resort to numerical methods. The main idea of this study is to compare two reaction models based on different approximations. The chosen test case for this comparison is the breakup of $^{15}$C on Pb at 68A MeV. The potentials used to simulate the interactions between the three bodies, $c$, $f$, and $T$, are inspired from Ref. [13].

2.2. Continuum discretised coupled-channel model (CDCC)

In CDCC [9, 10], the three-body wave function $\Psi$ is expanded over the eigenstates of $H_0$

$$
\Psi(r, R) = \sum_i \tilde{\phi}_i(r) \chi_i(R),
$$

where $\tilde{\phi}_i(r) = \phi_{l_i,j_i,m_i}(E_i, r)$, for bound states. The description of the broken-up projectile in $\Psi$, however, requires the discretisation of the continuum of $H_0$ [9]. We use the binning procedure, in which discretised continuum states are constructed as an average of eigenstates of $H_0$ (2).

Although other techniques exist, we perform the averaging in the momentum space

$$
\tilde{\phi}_i(r) = \frac{1}{\sqrt{N_i}} \int_{k_{i-1}}^{k_i} w(k) \phi_{l_i,j_i,m_i}(E, r) dk,
$$

with $E = \hbar^2k^2/2\mu_{cf}$. In Eq. (7), $w$ is a weight function, and $N_i$ a normalisation factor [9, 10]. This leads to a set of coupled equations

$$
\left( -\frac{\hbar^2}{2\mu_{PT}}\Delta + E_i \right) \chi_i(R) + \sum_j \langle \tilde{\phi}_i | V_{PT} | \tilde{\phi}_j \rangle \chi_j(R) = E \chi_i(R),
$$

which are solved under condition (5) using the code FRESCO [14].

2.3. Dynamical eikonal approximation (DEA)

The DEA is obtained by using the eikonal ansatz for the three-body wave function $\Psi$ [7]

$$
\Psi(r, R) = e^{iKZ} \hat{\Psi}(r, R).
$$

At high incident energy, most of the rapid variations in $R$ is contained in the plane wave $e^{iKZ}$. Therefore $\hat{\Psi}$ is smoothly varying with $R$ and its second-order derivative can be neglected in comparison with its first-order derivative. This leads to the DEA equation [8, 11]

$$
i\hbar v \frac{\partial}{\partial Z} \hat{\Psi}(r, R) = (H_0 + V_{PT} - E_0)\hat{\Psi}(r, R),
$$
where \( v = \frac{\hbar K}{\mu p_T} \) is the initial \( P-T \) relative velocity. Eq. (10) must be solved for each value of \( b \), the transverse part of the \( P-T \) relative coordinate \( R \) (see Fig. 1), with the initial condition

\[
\hat{\Psi}^{(m_0)}(r, b, Z) \rightarrow \mathcal{Z}^{-\infty} \phi_{l_0 m_0}(E_0, r).
\] (11)

Interestingly, Eq. (10) is mathematically equivalent to a time-dependent Schrödinger equation, although no semiclassical approximation has been made. It can thus be solved numerically with any code used by time-dependent reaction models. We use the algorithm detailed in Ref. [6].

3. Comparison

The results of the calculations are illustrated in Fig. 2. Fig. 2(a) shows the breakup cross section as a function of the relative energy \( E \) between the \(^{15}\text{C}\) and the neutron after dissociation, the contributions of the dominant \( p \) partial waves are also shown. Fig. 2(b) displays the breakup cross section as a function of the scattering angle \( \theta \) between the projectile center of mass and the target. The CDCC results are shown in blue, with different line styles depending on the model space considered for the calculation. The DEA results are displayed in red full lines.

Both CDCC and DEA lead to very similar cross sections, indicating their equivalence at these intermediate energies. For the energy distribution, this agreement is obtained only for a sufficiently large CDCC model space: all multipoles of \( V_{PT} \) up to \( \lambda_{\text{max}} = 4 \) must be included in order to reach convergence. This is larger than what is usually considered in most of the CDCC calculations. Interestingly, the good agreement between the two reaction models is also observed in the dominant \( p \) contributions. Only in the less populated partial waves, like \( s \) and \( d \) waves, do we observe some discrepancy between the two reaction models (not shown in Fig. 2). Being predominantly populated through higher-order effects, these partial waves are also those for which the slowest convergence is observed within the CDCC framework. Albeit negligible in the total energy distribution, this discrepancy may be more significant in other breakup observables, like parallel-momentum distribution.

In the angular distribution (Fig. 2(b)), we observe a small discrepancy between CDCC and DEA: the latter slightly underestimates the former. This appears mainly at large angles. This might be the signature of the absence of couplings between different \( b \)s in the DEA model. However, this difference is limited and probably will not affect the analysis of experimental data. Note that the angular distribution is less sensitive to the CDCC model space than the energy distribution: convergence is already reached at \( \lambda_{\text{max}} = 3 \).
4. Influence of relativistic velocity
The effects of relativity are usually neglected at these intermediate energies. However, they may affect breakup calculations [15] and hence values extracted from the comparison with data, like ANC [13]. It is therefore useful to evaluate their influence.

In time-dependent [4, 5, 6, 16] and DEA [8, 11] calculations, relativistic effects are usually accounted for by using a relativistic formula to derive the $P-T$ relative velocity from the incident energy. In order to evaluate the influence of such a relativistic velocity on breakup calculations, we repeat both CDCC and DEA calculations using a relativistic velocity $v_{rel}$. The results are displayed in Fig. 2 with dotted green lines (CDCC with $\lambda_{max} = 3$) and full orange line (DEA).

The modification induced by the relativistic velocity is identical in both reaction models, confirming their equivalence. We observe a small increase of the breakup cross section (see Fig. 2(a)). That increase amounts to 4% at the maximum of the energy distribution, which is in good agreement with the analysis of Ref. [16]. However, its major effect is to spread the angular distribution towards larger angles (see Fig. 2(b)). Therefore, these effects would be more visible in that observable. The small increase observed in the energy distribution may affect the ANC extracted from CDCC analyses of breakup experiments.

5. Conclusion and perspectives
In this work, we have compared the CDCC [9, 10] and DEA [8, 11] reaction models for the breakup of $^{15}$C on Pb at 68.4 MeV. The results show an excellent agreement between the two models, indicating their equivalence at these intermediate energies. Only small discrepancies have been observed in the angular distribution at large angles. Some partial waves seem also to be differently populated in the two reaction models. Albeit negligible in the energy and angular distributions considered in this study, these differences might affect other breakup observables.

In order to check this possibility, we plan to extend our comparison to parallel-momentum distributions. If the equivalence of CDCC and DEA is confirmed, it would mean that, at intermediate energies, the heavy CDCC calculations could be advantageously replaced by the DEA, which is more economical, computationally speaking.

We have also analysed the influence of special relativity upon breakup calculations at intermediate energy. For that part of the study, we have considered a relativistic formula to derive the $P-T$ relative velocity. The influence of that relativistic velocity is first to slightly increase the breakup cross section, and second to spread the angular distribution to larger angles. The first effect may influence the extraction of physical values, like ANC, from breakup data [13]. The second, being more visible experimentally, may be a way to assess the validity of this simulation of special relativity in breakup calculations. However, the use of relativistic velocity is not the only one effect that relativity may have on breakup calculations. To better evaluate relativistic effects, breakup should be simulated with a Klein-Gordon-like equation as proposed in Ref. [15].

Acknowledgments
This work is supported by the National Science Foundation grant PHY-0800026.

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