Electromagnetically induced transparency metamaterial with strong toroidal dipole response

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Abstract

This paper designs a planar electromagnetically induced transparency (EIT) metamaterial, which comprises an asymmetric ellipse split resonance ring (AESRR) and cut wire (CW). The proposed EIT metamaterial works in the wide range of incident angles and has polarization-sensitive at two transmission dips. The frequency of transparency peak is 10.67 GHz and maintains a high quality-factor (180.84). By calculating the multipole’s radiated power, it can be found that the toroidal dipole response is enhanced, while the electric dipole response is suppressed at the transparency peak. Interestingly, this paper firstly uses the radiated power of electric dipole to elucidate the polarization sensitivity in two minimal transmissions. Meanwhile, the coupling mechanism of the EIT metamaterial is analyzed by the two-oscillator model and equivalent circuit.

1. Introduction

In the electromagnetic multipoles family, classical electric and magnetic multipoles play essential and fundamental roles in electromagnetic investigation [1]. The third family member—toroidal dipole—was first presented by Zel’dovich [2] in 1957 and has been widely applied in the domain of nuclear and particle physics [3]. As is the case for electric and magnetic dipoles, the toroidal dipole is a lower-order and primary member of the toroidal multipoles family, but its response is hindered due to the stronger excitation of other electric and magnetic multipoles [4]. Detection of the toroidal dipole is quite challenging in the electromagnetic multipole field. Generally, a toroidal dipole’s surface current can be viewed as a circular head-to-tail arrangement of magnetic dipoles [5], and its applicability in metamaterials research has proved pretty diverting. A metamaterial [6, 7] is an artificial material with many unique characteristics not found in natural materials, such as a negative refractive index [8, 9], perfect absorption [10, 11], and the superlens phenomenon [12]. The strong toroidal dipole response in metamaterials regularly exhibits unattainable electromagnetic phenomena, such as resonant transparency [13, 14], unconventional optical activity [15], and optical switch [16].

Electromagnetically induced transparency (EIT) [17–19] is the classical resonant transparency in metamaterials. The transmission spectrum forms a sharp transparency peak at a specific frequency through energy level coupling [20]. The different energy levels can be viewed as bright and dark modes in metamaterials [21]. The bright mode represents a low-quality factor (Q-factor) that can directly couple with an electromagnetic wave; this mode is always excited by an electric dipole. When an electromagnetic wave enters the dark mode, which is not existing couple directly and possessing high Q-factor, the dark mode can be severed as a magnetic or toroidal dipole. The electric and toroidal dipole coupling permits EIT in metamaterials, which becomes a novelty research direction.

In this paper, a high quality-factor EIT metamaterial is designed, which shows a strong toroidal dipole response at the transparency peak. Due to its asymmetric structure, the proposed metamaterial exhibits polarization-sensitive at two transmission dips. Simulation, experimental, and calculation results all support
each other, and the EIT metamaterial reveals a large group index at the transparency peak. We envisage that eventually, the proposed EIT metamaterial is able to achieve a refractive index sensor.

2. Design and fabrication

The diagrams of asymmetric ellipse split resonance ring (AESRR) and cut wire (CW) are drawn in figure 1(a). The two structures constitute the proposed metal pattern of the EIT metamaterial, which is etched on a 1 mm thickness loss F4B substrate ($\varepsilon_r = 3$ and $\tan \theta = 0.001$) with 0.035 mm copper (the conductivity of copper, $\sigma = 5.96 \times 10^7$ S m$^{-1}$). The specific geometric parameters are plotted in figure 1(b): $L = 12$ mm, $R = 3.5$ mm, $D = 9$ mm, $t = 0.6$ mm and line width is 0.5 mm. The part of the fabricated experimental sample is displayed in figure 1(c). The dimensions of the whole sample are 240 mm × 240 mm, which is a 20 × 20 unit cell in the x and y directions. The commercial simulation software CST Microwave Studio was employed to acquire electromagnetic parameters (transmission and reflection coefficients), using the finite-integration time-domain (FITD) method. As the boundary conditions, the x- and y- axes are set as the unit cell, and the electromagnetic wave is incident in the z- direction. The setup represents a normally incident electromagnetic wave with x-polarization of the electric field.

3. Results and analysis

Figure 2 shows the simulation transmission results of the three different structures: the AESRR, CW, and AESRR combined with the CW. The AESRR’s transmission spectrum shows a dip at 10.38 GHz. When the structure only has the CW, the transmission curve is a line. There is no doubt that the AESRR plays the bright mode, and the CW serves as the dark mode. When the AESRR is combined with the CW, the transmission spectrum appears sharply with a transparency peak at 10.67 GHz and a transmission coefficient of 0.79.

As shown in figure 3(a), the simulated result of transmission spectra has a transparency peak and two transmission dips. The transparency peak locates at 10.67 GHz, where the transmission coefficient is 0.79. The frequencies of the two minimal transmissions are 10.31 and 10.88 GHz. The sharp transparency peak has a width of only 0.049 GHz at 3 dB; the calculated formula of Q-factor is a resonance frequency of transparency peak divided by the 3-dB width. Therefore, the Q-factor of the proposed EIT metamaterial could attain 180.84 in the microwave region. The experimental result was measured with a vector network analyzer (Agilent PNA E8362B) and a pair of horn antennas in free space. The simple schematic of the measurement of transmission spectra is presented in figure 3(b). The transparency peak locates at 11.13 GHz in the experiment result, for which the
The transmission coefficient is 0.82. The frequencies of the two transmission dips are 10.49 GHz and 11.28 GHz. The trend of the transmission spectrum of the experimental result is in good agreement with that of the simulation; however, the experimental data of transparency peak have a 0.4 GHz frequency shift compared with the

Figure 2. The transmission spectrum of the AESRR, CW, and AESRR combined with the CW.

Figure 3. (a) Simulation and experimental transmission spectra. (b) The schematic of the measurement.
simulation data, the cause of which the F4B's permittivity discrepancy between the experimental sample and the simulation model. Moreover, fabrication errors during the process of the printed circuit board (PCB) also lead to discrepancies between the two curves.

The radiated power of multipoles [22] is an essential tool in quantitative analysis of the EIT metamaterial coupling mechanism. The calculated formulas of the radiated power of multipoles are based on the conducting current density \( j \) and are given as:

\[
\text{electric dipole moment: } P = \frac{1}{i\omega} \int j d^3r \\
\text{magnetic dipole moment: } M = \frac{1}{2c} \int (r \times j) d^3r \\
\text{toroidal dipole moment: } T = \frac{1}{10c} \int [(r \cdot j) r - 2r^2] d^3r \\
\text{electric quadrupole moment: } Q_e = \frac{1}{12\omega} \int \left[ r_{\alpha} j_{\beta} + r_{\beta} j_{\alpha} - \frac{2}{3} (r \cdot j) \delta_{\alpha\beta} \right] d^3r
\]

Here \( c \) is the speed of light in vacuum, \( \omega \) is frequency, and \( r \) is the distance vector from the origin to point \((x, y, z)\) in a Cartesian coordinate system \((\alpha, \beta = x, y)\). Therefore, the decomposed far-field scatter power by these multipole moments can be calculated by using the following equations:

\[
I_P = \frac{2\omega^4}{3c^3} |P|^2 \\
I_M = \frac{2\omega^4}{3c^3} |M|^2 \\
I_T = \frac{2\omega^6}{3c^3} |T|^2 \\
I_Q = \frac{\omega^6}{3c^3} |Q_e|^2
\]

(5)–(8) represent the radiated power of an electric dipole, magnetic dipole, toroidal dipole and electric quadrupole, respectively. The corresponding results are displayed in figure 4. The radiated power of the magnetic dipole remains almost unchanged within the whole frequency range, and it is remarkably lower than other radiated power of multipoles. At the transparency peak, the radiated power of the electric quadrupole rises somewhat. However, the increment of the toroidal dipole is far larger than that of the electric quadrupole, which is about 83 times. With regard to the electric dipole, the radiated power decrease to a minimum. Evidently, the intensity of the toroidal dipole is much higher than that of the electric dipole at the transparency peak. At 10.31 GHz and 10.88 GHz, the radiated power of the electric dipole is high, higher than that of the other dipoles. Clearly, the EIT metamaterial coupling mechanism primarily arises from the interaction between the electric dipole and toroidal dipole, forming a sharp transparency peak.

Figure 4. Radiated power of four elementary multipoles.
The polarization angle ($\theta$) is a significant parameter in an EIT metamaterial, and due to its asymmetric structure, the proposed EIT metamaterial is polarization-sensitive in two minimal transmissions. Figure 5(a) shows simulation results with different polarization angles. As the polarization angle increases from $0^\circ$ to $45^\circ$ by a step of $15^\circ$, the transmission coefficient of the transparency peak increases and it rises slightly reaching 0.9 at $45^\circ$. However, the frequency of the transparency peak remains almost unchanged, near 10.67 GHz. With increasing polarization angle, the frequencies of transmission dip blue-shifts, and the transmission coefficient increases gradually that reveals the polarization-sensitive. The measured results are shown in figure 5(b). The frequency and coefficient of the transparency peak move little with polarization angle rotation. The most obvious variation in the transmission spectra is that of the transmission dips, and the transmission coefficient rises with increasing polarization angles. The frequencies of the experimental results, however, blue-shift in comparison with the simulation results. The trends of the transmission spectra in the experimental and simulation results in good agreement. The radiated power of the electric dipole reveals the polarization-sensitive mechanism of two transmission dips in figure 5(c). At different polarization angles, the radiated power of the electric dipole varies in the same way. Meanwhile, at the two transmission dips, the intensity of the electric dipole increase to a different degree, which leads to the transmission coefficient of transmission dips also increases. We infer that the electric dipole is the primary influence in two minimal transmissions, which explain the proposed EIT metamaterial’s property of polarization sensitivity.

Another crucial parameter is the incident angle ($\phi$). The simulated and measured results for the wide range of incident angles are plotted in figure 6. It reveals that the EIT is able to achieve wide angles. When incident angles increase from $0^\circ$ to $60^\circ$, the transmission spectrum almost coincides. The frequency and amplitude of transparency peak are slightly changed in figure 6(a). The experiment results are exhibited in figure 6(b). It reveals perfect superposition in different incident angles, and the trend of transmission spectra shows well coincide with simulated results. However, the frequency of transparency peak still has 0.4 GHz deviation between the simulated and measured.

Another analytical method used to evaluate EIT metamaterials is the two-oscillator model [23]. The two oscillators can be regarded as the proposed EIT metamaterial’s bright and dark modes, which should satisfy the following equations:

Figure 5. Polarization angle results: (a) Simulation transmission spectrum. (b) Experimental transmission spectrum. (c) Radiated power of electric dipole.
\[
\begin{align*}
\ddot{x}_1(t) + \gamma_1 \dot{x}_1(t) + \omega_1^2 x_1(t) + \Omega \dot{x}_2(t) + g E_0(t) & = 0 \quad (9) \\
\ddot{x}_2(t) + \gamma_2 \dot{x}_2(t) + (\omega_0 + \delta)^2 x_2(t) + \Omega \dot{x}_1(t) & = 0 \quad (10)
\end{align*}
\]

\( x_1, x_2 \) and \( \gamma_1, \gamma_2 \) are two-oscillator parameters, which represent a resonant amplitude and damping. The parameter \( g \) is the coupling strength between bright and dark modes when excited by an electric field. The external electric field \( E_0(t) = E_0 e^{i\omega t} \). \( \delta \) and \( \Omega \) represent the frequency difference between the transparency peak and resonance point of intrinsic oscillators, as well as the coupling strength of the two different oscillators. The displacement vector is depicted \( x_n(t) = x_n e^{i\omega t} (n = 1, 2) \) and the approximation \( \omega_1^2 - \omega^2 \approx -2\omega_1(\omega - \omega_1) \) is used. According to (9) and (10), the transmission coefficient can be written in (11) [24, 25]:

\[
T = 1 - \text{Im} \left( \frac{g(\omega - \omega_0 - \delta + i\gamma_2/2)}{(\omega - \omega_0 + i\gamma_1/2)(\omega - \omega_0 - \delta + i\gamma_2/2) - \Omega^2/4} \right)
\]  

(11)

After calculating, the parameters of the fitting equation are following: \( \gamma_1 = 2.090 \) GHz, \( \gamma_2 = 0.038 \) GHz, \( \Omega = 0.513 \) GHz, \( \delta = 0.141 \) GHz, \( g = 0.925 \) GHz and \( \omega_0 = 10.67 \) GHz. The analytically fitted curve based on the two-oscillator model is shown in figure 7. Compared with the simulation results, the calculation results show a perfect fit.

In order to better understand the mechanism of the EIT, we give the equivalent circuit of the metamaterial to analyze its transmission spectrum characteristic [26]. In the passive electric system, the four passive components can imitate EIT formation. The schematic diagram equivalent circuit is shown in figure 8(a), in which two series LC circuits are represented in different modes. The resonance frequency is \( f = \frac{1}{2\pi\sqrt{LC}} \). According to the frequencies of transmission dips, the value of inductors and one capacitor are calculated to the following: \( L_1 = 2.137 \) nH, \( C_1 = 1 \) pF, \( L_2 = 2.378 \) nH and \( C_2 = 1 \) pF. The calculated results of the transmission spectrum are shown in figure 8(b), which reflects the excellent fit with the simulation results.
The slow light effect [27] is the crucial feature of the EIT phenomenon, which is generally described by the group index \( n_g \). The calculation formula of \( n_g \) is as follows:

\[
n_g = n + \omega \frac{dn}{d\omega}
\]  \hspace{1cm} (12)

Here \( n \) is the effective refractive index of the metamaterial, and \( \omega \) is the angular frequency. \( n_g \) and \( \text{Im}(n) \) (imaginary part of the effective refractive index) are shown in figure 9. \( n_g \) increases to 1537 at 10.67 GHz, which is the most massive value in the range 9–12 GHz. It means that the speed at which electromagnetic waves travel through this EIT metamaterial is reduced to \( 1/1537 \) of the speed of light. Since \( \text{Im}(n) \) is close to zero at 10.67 GHz, the loss at transparency peak is minimal. \( \text{Im}(n) \) has large values at 10.31 GHz and 10.88 GHz, so the incident wave is strongly scattered and absorbed at the two transmission dips.

Due to the high coefficient of the transparency peak, the EIT metamaterial can be used to make the refractive index sensor. Figure 10 shows the transmission spectra with different surrounding medium refractive index. When the surrounding medium refractive index varies from 1 to 1.3, the transparency peak red-shifts and the transmission coefficient has slightly increased. Therefore, we can make the refractive index sensing through the EIT-like effect in our structure. Sensor sensitively (S) is defined as the frequency shift at the transparency peak divided by the refractive index unit (RIU), for which the value is 1.9 GHz/RIU in the proposed EIT metamaterial. The figure of merit (FOM) is another most critical indicator in sensor technology, and its formula is as follows:

\[
\text{FOM} = \frac{S}{\text{FWHM}}
\]  \hspace{1cm} (13)
where FWHM is the full width at half maximum bandwidth. In the proposed EIT metamaterial, the FWHM closes to 0.142 GHz, and the FOM reaches up to 13.38, which means it can be used as an efficient refractive index sensor.

4. Conclusions

In summary, we have imitatively, experimentally, and numerically demonstrated an EIT metamaterial with a high Q-factor. The polarization-sensitivity at two transmission dips and the wide range of incident angle implementation of EIT has been proved. The coupling mechanism of the transmission spectra is explained by the two-oscillator model and equivalent circuit. The strong toroidal dipole response at transparency peak is verified by the radiated power of multipoles. In the meantime, the character of polarization-sensitivity in the two minimal transmissions is explained by the radiated power of the electric dipole. Due to the sharp transparency peak, the proposed EIT metamaterial has excellent potential in the refractive index sensor.

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