Adiabatic fission barriers in superheavy nuclei

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Abstract

Using the microscopic-macroscopic model based on the deformed Woods-Saxon single-particle potential and the Yukawa-plus-exponential macroscopic energy we calculated static fission barriers $B_f$ for 1305 heavy and superheavy nuclei $98 \leq Z \leq 126$, including even-even, odd-even, even-odd and odd-odd systems. For odd and odd-odd nuclei, adiabatic potential energy surfaces were calculated by a minimization over configurations with one blocked neutron or/and proton on a level from the 10-th below to the 10-th above the Fermi level. The parameters of the model that have been fixed previously by a fit to masses of even-even heavy nuclei were kept unchanged. A search for saddle points has been performed by the "Imaginary Water Flow" method on a basic five-dimensional deformation grid, including triaxiality. Two auxiliary grids were used for checking the effects of the mass asymmetry and hexadecapole non-axiality. The ground states were found by energy minimization over configurations and deformations. We find that the non-axiality significantly changes first and second fission barrier in many nuclei. The effect of the mass-asymmetry, known to lower the second, very deformed barriers in actinides, in the heaviest nuclei appears at the less deformed saddles in more than 100 nuclei. It happens for those saddles in which the triaxiality does not play any role, what suggests a decoupling between effects of the mass-asymmetry and triaxiality. We studied also the influence of the pairing interaction strength on the staggering of $B_f$ for odd- and even-particle numbers. Finally, we provide a comparison of our results with other theoretical fission barrier evaluations and with available experimental estimates.

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I. INTRODUCTION

Although fission barrier heights $B_f$ are not directly measurable quantities, i.e. are not quantum observables, they are very useful in estimating nuclear fission rates. As the activation energy $E_a$ (per mole) in chemistry gives a rate $k$ of a chemical reaction at temperature $T$ via the Arrhenius law: $k = A e^{-E_a/RT}$ ($R$ - the gas constant; $A$ - the frequency factor) [1,2], the fission barrier gives the fission rate $\Gamma_f$ of an excited (as they usually are in nuclear reactions) nucleus via: $\Gamma_f \sim e^{-B_f/kT_{eff}}$, where $T_{eff}$ is an effective temperature derived from the excitation energy, and $k$ - the Boltzman constant. For example, knowing fission barriers of possible fusion products helps predicting a cross section for a production of a given evaporation residue in a heavy ion reaction: one can figure out whether neutron or alpha emission wins a competition with fission at each stage of the deexcitation of a compound nucleus. Moreover, one can try to understand the experimentally established, intriguing growth of the total cross sections around $Z=118$; for its correlation with $B_f$, see e.g. Fig. 6 and the related discussion in [3]. On the other hand, the prediction of the spontaneous or low energy (i.e. from a weakly excited state) fission rates, governed by the regime of the collective quantum tunneling, requires an additional knowledge of the barrier shape and mass parameters.

A non-observable status of the fission barrier, again in analogy to that of the activation energy in chemistry, is reflected in its possible dependence on a reaction type and/or the excitation energy (effective temperature) range. This leads to some uncertainty in calculations of fission barriers. In particular, it is not clear whether intrinsic configurations should be conserved along the level crossings, which increases $B_f$, or the adiabatic state should be followed. This is especially relevant for odd-A and odd-odd nuclei, in which sharp crossings of levels occupied by the odd particle exclude the strictly adiabatic scenario. It is known that if the projection of the single-particle angular momentum on the symmetry axis of a nucleus $\Omega$ is conserved, the diabatic effect on the fission barrier can be huge, see e.g. [4]. As there is no accepted formula for a barrier correction due to the non-adiabaticity, it is usually ignored, even in odd-N and/or odd-Z nuclei. A general idea is that at the excitation energies close to, and higher than the barrier, but still not inducing sizable dissipative corrections, the adiabatic barrier could be used for calculating fission rates.

Since calculations of potential energy surfaces (PES’s) for odd-A and odd-odd nuclei involve a repetition of calculations for many low-lying quasiparticle states which multiplies the effort (especially in odd-odd systems), systematic studies of their fission barriers are rather scarce. Up to now, they were provided mainly by the Los Alamos microscopic-macroscopic (MM) model and recently by some self-consistent models [5]. The current state of the-

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oretical predictions in fission of even - even nuclei (with \(Z \geq 100\)) has been discussed recently in \([6]\).

In the present paper we extend our MM model based on the deformed Woods-Saxon potential, which up to now was applied mainly to even-even nuclei \([2]\) to odd-\(A\) and odd-odd SH systems. We study a wide range of isotopes which, perhaps, may be of some use for astrophyysical purposes. The fission barriers are calculated using the adiabatic assumption, i.e. they are the smallest possible. Since the model has been quite reasonable, in particular in reproducing first \([2]\) and second \([8]\) fission barriers in actinides, as well as super- \([9]\) and hyper-deformed \([10, 11]\) minima, we prefer to keep its parameters unchanged. The shell and pairing correction for an odd nucleon system is done by blocking the lowest-lying quasiparticle states. The modification of the macroscopic energy by including the average pairing energy contribution which we introduced for nuclear masses in \([12]\) is irrelevant for fission barriers.

The other motivation of our study is to improve the predictions for the fission saddles. This requires simultaneously taking into account a large number of shape variables \([8, 11]\) and relying on an in principle exact method for finding saddles to escape errors inherent in the mostly used constrained minimization method, see \([13, 14]\). As usual, to make the involved computational effort manageable one has to make some compromises which will be discussed in detail. The need for a simultaneous consideration of many shape variables in PES’s calculations is common to all nuclear models, including self-consistent theories based on some effective interactions \([15]\). The results on fission saddles obtained up to now in the SH region clearly show the great importance of triaxial deformation, neglected in many published work. A recent study \([16]\) of barriers within both the MM Woods-Saxon and Skyrme SLy6 Hartree-Fock plus BCS models shows that triaxiality is even more crucial beyond \(Z = 126\).

A description of our method of calculations is given in section II. The results, details of the additional calculations, and comparisons with other calculated barriers are presented and discussed in section III. Finally, the conclusions are summarized in section IV.

## II. The Method

Multidimensional energy landscapes are calculated within the MM model based on the deformed Woods-Saxon potential \([17]\). The Strutinski shell and pairing correction \([18]\) is taken for the microscopic part. For the macroscopic part we used the Yukawa plus exponential model \([19]\) with parameters specified in \([20]\). Thus, all parameter values are kept exactly the same as in all recent applications of the model to heavy and superheavy nuclei.

The main point in fission barrier calculations is its reliability which, once the model for calculating energy of a nucleus as a function of deformation is fixed, hangs on two main ingredients: 1) the kind and dimension of the admitted deformation space and 2) a method applied to the search for saddles.

Mononuclear shapes can be parameterized via spherical harmonics \(Y_{lm}(\theta, \varphi)\) (for brevity we will just use the symbol \(Y_{lm}\)). by the following equation of the nuclear surface:

\[
R(\theta, \varphi) = c(\{\beta\})R_0\{1 + \sum_{\lambda=1}^{\infty} \sum_{\mu=1}^{+\lambda} \beta_{2\lambda, Y_{l, \mu}}\},
\]

where \(c(\{\beta\})\) is the volume-fixing factor and \(R_0\) is the radius of a spherical nucleus. This parameterization has its limitations; certainly, it is not suitable for too elongated shapes. However, for moderately deformed saddle points in superheavy nuclei it excellently reproduces all shapes generated by other parametrizations, e.g. by \([21]\), as we checked in numerous tests.

For nuclear ground states it is possible to confine analysis to axially-symmetric shapes, with the expansion truncated at \(\beta_{80}\):

\[
R(\theta, \varphi) = c(\{\beta\})R_0\{1 + \beta_{20}Y_{20} + \beta_{30}Y_{30} + \beta_{40}Y_{40} + \beta_{50}Y_{50} + \beta_{60}Y_{60} + \beta_{70}Y_{70} + \beta_{80}Y_{80}\}.
\]

Thus, a seven dimensional minimization is performed using the gradient method. For odd systems, the additional minimization over configurations is performed at every step of the gradient procedure. Considered configurations consist of the odd particle occupying one of the levels close to the Fermi level and the rest of the particles forming a paired BCS state on the remaining levels. Ten states above and ten states below the Fermi level have been blocked and energy minimized over these configurations.

The main problem in a search for saddle points is that, since they are neither minima nor maxima, one has to know energy on a multidimensional grid of deformations (the often used and much simpler method of minimization with imposed constraints may produce invalid results \([8, 13, 15]\). To find saddles on a grid we used the Imaginary Water Flow (IWF) technique. This conceptually simple and at the same time very efficient (from a numerical point of view) method was widely used and discussed before \([13, 15, 22, 25]\). The number of numerically tractable deformation parameters \(\{\beta_{lm}\}\) is practically limited. More than five-dimensional grids, keeping in mind a subsequent interpolation, are intractable in calculations for many (\(\sim 1000\)) nuclei. Including mass- and axially-symmetric deformations \((\beta_{20}, \beta_{40}, \beta_{60}, \beta_{80} - \text{see} \ [26, 24]\) together with both, mass-asymmetry \((\beta_{30}, \beta_{50}, \beta_{70})\) and triaxiality (at least \(\beta_{22}\)) would mean at least an eight-dimensional mesh and was impossible at present.

Based on our previous results showing that triaxial saddles are abundant in SH nuclei \([7]\), we consider that quadrupole triaxial shapes have to be necessarily included. We treated the effects of mass-asymmetry and

\[
\text{axially-symmetric deformations (\(\beta_{20}\)) and triaxiality (at least \(\beta_{22}\)) would mean at least an eight-dimensional mesh and was impossible at present.}
\]
nonaxial higher multipoles as corrections and analysed them at the second stage of calculations. A rationale for a lesser importance of mass-asymmetric saddles is that, while they constitute a second, more deformed ($\beta_{20} \approx 0.7 - 0.8$), prominent barrier peak in actinides, their heights are much reduced in SH nuclei where they become irrelevant. In the remaining, less deformed saddles the mass asymmetry occurs less frequently. As to the nonaxial multipoles of higher order, they are less important. The additional studies of the mass-asymmetry and higher nonaxial multipoles are described in the proper subsections of the Results section.

Thus, at the first stage, for all 1305 investigated nuclei the saddle points were searched in a five dimensional deformation space spanned by: $\beta_{20}, \beta_{22}, \beta_{40}, \beta_{60}, \beta_{80}$, using the IWF technique. The appropriate nuclear radius expansion has the form:

$$R(\vartheta, \varphi) = c(\{\beta\}) R_0 \{ 1 + \beta_{20} Y_{20} + \frac{\beta_{22}}{\sqrt{2}} [Y_{22} + Y_{2-2}] + \beta_{40} Y_{40} + \beta_{60} Y_{60} + \beta_{80} Y_{80} \}.$$  \hfill (3)

The five-dimensional calculations are performed on the following deformation mesh:

$$\begin{align*}
\beta_{20} &= 0.00 (0.05) 0.60 \\
\beta_{22} &= 0.00 (0.05) 0.45 \\
\beta_{40} &= -0.20 (0.02) 0.20 \\
\beta_{60} &= -0.10 (0.02) 0.10 \\
\beta_{80} &= -0.10 (0.02) 0.10
\end{align*}$$  \hfill (4)

This makes a grid of 29250 points which was subsequently interpolated to a fivefold denser grid of 50735286 points with the step 0.01 in each dimension. On the latter, the saddle point, or rather several saddle points - if there were a few of comparable heights within the 0.5 MeV energy window - were searched for by means of the IWF procedure. For odd or odd-odd nuclei, at each grid point we were looking for low-lying configurations by blocking particles on levels from the 10-th below to the 10-th above the Fermi level (in neutrons or/and protons).

The fact that searches for ground states and for saddles are separated - performed using different deformation spaces - allows saving some number of deformation parameters in Eq. (3). This is equivalent to assuming that the fission saddles have mostly prolate deformations large enough to make nonaxial deformations of multipolarity $\lambda \geq 3$ less important. One has to check this assumption afterwards and separately treat nuclei in which the inclusion of nonaxial deformations with $\lambda \geq 4$ is necessary.

Although, as mentioned before, in SH nuclei the second barriers at large deformations are usually smaller than the first one or do not exist at all, for $Z = 98-101$ the mesh (4) was extended to $\beta_{20} = 1.5$ and the second saddles were searched for by the IWF technique. It turned out that these more deformed barriers are indeed mostly smaller than the first ones and decrease with increasing $Z$. Only in Cf isotopes with $N = 134 - 160$ there were some second saddles (at $\beta_{20} \approx 0.9$) higher than the first one by at most 0.5 MeV. However, even those saddles were lowered by at least 1 MeV after including the mass-asymmetry. Therefore, we have reasons to believe that the range of $\beta_{20}$ in (4) is sufficient for knowing the height of the fission barrier in the whole studied region.

### III. RESULTS AND DISCUSSION

In the present paper we have systematically calculated fission-barrier heights $B_f$ as the energy difference between the saddle point and the ground state. The saddle point is defined as the minimum over possible fission paths of the maximal energy along the path. Let us emphasize that the calculations presented here have been performed without adding any zero-point vibration energy. We have included 1305 heavy and superheavy nuclei with proton numbers $98 \leq Z \leq 126$ and neutron numbers in the range $134 \leq N \leq 192$, with the smallest $N$ for a given $Z$ increasing by one with every step in $Z$. All obtained barriers have been collected in Table III. On all PES’s presented here, energy is normalized in such a way that its macroscopic part is set to zero at the spherical shape.

#### A. Potential Energy surfaces

Some idea about the positions of ground states, secondary minima and saddles may be gained from PES’s. Chosen examples are shown in figures for: $^{252}{\text{Lr}}$ - Fig. 2, $^{270}{\text{Db}}$ - Fig. 2, $^{276}{\text{Mt}}$ - Fig. 2, $^{280}{\text{Cn}}$ - Fig. 4, and $^{297}{\text{119}}$ - Fig. 5. Overall evolution of ground states with increasing $Z$ from prolate to spherical can be seen there. In some nuclei one can see multiple saddles of which the one defining the fission barrier should be properly chosen. Sometimes the saddles between competing minima can be important, therefore the determination of all saddles on the map is necessarily needed.

The energy landscapes Fig. 1-5 were obtained by minimizing energy on the 5D grid with respect to $\beta_{40}, \beta_{60}$ and $\beta_{80}$. One should be aware of two related circumstances: 1) As the grid Eq. (2) does not include nonaxial deformations $\lambda \geq 4$, the axial deformations $\lambda = 4, 6, 8$ with respect to the $x$-axis cannot be reproduced, so the landscapes are inexact around the oblate $\gamma = 60^\circ$ axis. 2) A reduction of a $n$-dimensional grid of energy values via the minimization over $n - 2$ deformations sometimes leads to an energy surface composed from disconnected
patches, corresponding to multiple minima in the auxiliary (those minimized over) dimensions. This can distort the picture of the barrier (actually, a reduction of multi-dimensional data to a two-dimensional map is a general problem).

With these reservations in mind, one can still explore some of the details shown in the maps. In particular, the prolate g.s. minimum with strongly nonaxial first saddle point at $\beta_{20} = 0.41$ and $\beta_{22} = 0.18$ is visible in $^{252}\text{Lr}$. One can notice that the axially symmetric saddle lies more than 2 MeV higher. A slightly less steep, prolate g.s. minimum and a gently emerging second minimum is visible in Fig. 2 for $^{270}\text{Db}$. The triaxial saddle at $\beta_{20} = 0.52$ and $\beta_{22} = 0.13$ has a smaller triaxiality $\gamma$ than the saddle in $^{252}\text{Lr}$. A decrease in barrier height due to triaxiality is $\approx 2$ MeV, Fig. 2.

In a heavier nucleus $^{276}\text{Mt}$, a prolate deformation of the g.s. is clearly smaller than in $^{252}\text{Lr}$, see Fig. 3. The second minimum, which was barely outlined in $^{270}\text{Db}$, is more pronounced here, giving the fission barrier a double-hump structure. The deformation $\beta_{20} \approx 0.5$ of the second saddle is much smaller than that of the second barriers in actinides. Thus, a two-peak structure of the barrier in SH nuclei may be viewed as a result of a division (split) of the first barrier, occurring with growing $Z$. The higher second axial saddle is lowered by triaxiality by $\approx 1.5$ MeV, but eventually is still higher than the first axial saddle.

For $^{280}\text{Ds}$ a topology of the PES is even more complicated. We see several minima: prolate - the g.s. and a superdeformed one, and a shallow oblate. The map shows also a few saddles. The axially deformed saddle point at $\beta_{20} = 0.3$ has a similar height as the nonaxial saddle at $\beta_{20} = 0.54$ and $\beta_{22} = 0.12$. It follows from the IWF calculation that the second fission barrier is nonaxial in this case. The axial second saddle is lowered by $\approx 1$ MeV owing to the nonaxiality.

The nucleus $Z = 119, N = 178$ is spherical in its g.s. - Fig. 4. There is a secondary oblate minimum (whose depth is underestimated in the map due to omission of nonaxial $\lambda = 4, 6$ deformations). There is a low triaxial

\begin{figure*}[h]
\centering
\begin{minipage}{0.49\textwidth}
\includegraphics[width=\textwidth]{figure1}
\caption{Energy surface, $E - E_{mac}(\text{sphere})$, for $Z = 103$ and $N = 149$.}
\end{minipage}\hspace{1cm}
\begin{minipage}{0.49\textwidth}
\includegraphics[width=\textwidth]{figure2}
\caption{The same as in (1) but for $Z = 105$ and $N = 165$.}
\end{minipage}
\begin{minipage}{0.49\textwidth}
\includegraphics[width=\textwidth]{figure3}
\caption{The same as in (1) but for $Z = 109$ and $N = 167$.}
\end{minipage}
\end{figure*}
B. Role of the mass asymmetry

To study the effect of the reflection (mass) - asymmetry on the fission barriers, a two-step procedure has been performed. At the first stage, we have checked the stability of all the saddles found on the basic 5D mesh (the first, the second, ..., axially symmetric or triaxial, of energy within 0.5 MeV of the highest saddle) against the mass-asymmetry. This was done by a 3D energy minimization with respect to $\beta_{30}$, $\beta_{50}$ and $\beta_{70}$ around each saddle. Since most of the saddles are non-axial, the most general version of our Woods-Saxon code had to be used. In this case, when both symmetries (axial and mass symmetry) are broken simultaneously, the nuclear shapes are defined by the following equation of the nuclear surface:

$$R(\theta, \varphi) = R_0 c(\{\beta\}) \left\{ 1 + \beta_{20}Y_{20} + \frac{\beta_{22}}{\sqrt{2}} [Y_{22} + Y_{2-2}] + \beta_{30}Y_{30} + \beta_{40}Y_{40} + \beta_{50}Y_{50} + \beta_{60}Y_{60} + \beta_{70}Y_{70} + \beta_{80}Y_{80} \right\}$$

It turned out that this minimization lowers energy of only those saddles in which: i) there is no triaxiality, ii) deformation $\beta_{20} \approx 0.3$. This supports an often expressed conventional ”wisdom”, that the mass-asymmetry and triaxiality effects on fission saddle are decoupled. This is why, at the second step of the procedure, we could carry out a full IWF analysis on a grid including only axially-symmetric deformations: $\beta_{20}, \beta_{30}, \beta_{40}, \beta_{50}, \beta_{60}, \beta_{70}, \beta_{80}$, with $\beta_{20}$ restricted to a quite short interval $0.25 - 0.40$:

$$\begin{align*}
\beta_{20} &= 0.25 \ (0.05) \ 0.40 \\
\beta_{30} &= 0.00 \ (0.05) \ 0.25 \\
\beta_{40} &= -0.15 \ (0.05) \ 0.20 \\
\beta_{50} &= 0.00 \ (0.05) \ 0.15 \\
\beta_{60} &= -0.10 \ (0.05) \ 0.10 \\
\beta_{70} &= 0.00 \ (0.05) \ 0.15 \\
\beta_{80} &= -0.10 \ (0.05) \ 0.10.
\end{align*}$$

This seven-dimensional grid, composed of 76800 deformations, was subject to the fivefold interpolation in all directions before it was used in the IWF procedure. This means that the IWF calculations have been performed on the grid containing 1 690 730 496 (!) points. We have made such 7-dimensional analysis for more than 100 nuclei, for which the effect of minimization was greater than 300 keV. Results for these nuclei are shown in Table I. The rest of 127 cases shown in Table I are the test nuclei, in which the effect of the minimization was smaller than 0.3 MeV. The results for these additional nuclei allow to appreciate whether the (in principle exact) IWF method could produce a greater effect than the (inexact) minimization method which is not always reliable.

Still another type of PES, typical of nuclei with the superdeformed oblate g.s., is presented in Fig 8 in the subsection C.
For example, for \( Z = 118 \) and \( N = 165 \), the discussed effect resulting from the minimization amounts to 0.44 MeV, which, just in this case, is quite similar to 0.46 MeV obtained from the IWF technique; however, in \( Z = 113 \) and \( N = 163 \) one obtains \( \approx 0.5 \) MeV difference between saddles obtained by both methods. In this particular nucleus, the \( \approx 0.77 \) MeV barrier lowering by the mass-asymmetry is the largest among all studied nuclei. It should be also noted that for the isotopes of \( Z = 113 \) the effect of the mass-asymmetry is particularly large, see the top panel in Fig. 6.

In the bottom panel of Fig. 6 we show the difference between the results of the both methods - the minimization - (MIN) and “Imaginary Water Flow” - (IWF). One can see that this difference increases with the neutron number. In particular, there is practically no effect derived from the mass-asymmetry in \(^{281}113\) when IWF is used. On the contrary, the approach based on minimization suggests still a quite substantial (spurious) effect (0.55 MeV). One might notice that our conclusion concerning decoupling of the variables describing the axial and reflection asymmetries is in a delicate contradiction with the studies [30].

C. Role of the triaxiality

The importance of including triaxiality in a calculation of fission barrier heights was indicated many times before [31–40]. In particular, it was shown that the effect of both quadrupole and a general hexadecapole nonaxiality, when accounted for within the nonexact method of constrained minimization (used generally in all selfconsistent studies), may reach 2.5 MeV for some superheavy even-even nuclei, see Fig. 5 in [8]. Here, we extend our previous discussion of its role to the odd and odd-odd nuclei and, at the same time, improve the treatment by employing the exact IWF method in potentially most interesting cases.

By using the original 5D mesh (4) we have obtained saddles with quadrupole nonaxiality for about 900 nuclei, what constitutes more than 70% of all fission barriers. We illustrate this conspicuous effect in Fig. 6 on the example of two isotopic chains, \( Z = 103 \) and 113.

We show the difference between axial and nonaxial barriers in these nuclei. One can see that for lighter Lawrencium isotopes the effect of nonaxiality is quite considerable. Starting with \( N = 164 \), it is weakening quickly and finally vanishes for \( N \geq 176 \). Somewhat different dependence of the effect on the neutron number occurs in \( Z = 113 \) isotopes. The maximum lowering of the barrier of more than 1.5 MeV occurs for \( N \approx 165 \), there is a second maximum at \( N = 179 \), and the effect becomes large again at \( N = 192 \). Inbetween, for \( N \approx 154 \) and \( N \approx 174 \), there is no effect at all. Thus, the effect of nonaxiality has to be studied carefully, indeed.

Another task is to consider the influence of the hexadecapole nonaxiality, namely: \( \beta_{12}, \beta_{14} \) in Eq. 1 on the fission barriers. The unconstrained inclusion of these shapes would lead to a 7D grid which is too much for now. To evaluate the effect without increasing the grid dimension we constrained \( \beta_{42} \) and \( \beta_{44} \) to be functions of the quadrupole nonaxial deformation \( \beta_{22} \), or actually \( \gamma \), and \( \beta_{40} \), in a well known manner [45]. Using the conventional notation:

\[
\beta = \sqrt{\beta_{20}^2 + \beta_{22}^2},
\]
\[
\gamma = \arctg(\beta_{22}/\beta_{20}),
\]

the following form of Eq. 1 was used:
| Z = 109 | Z = 114 | Z = 117 |
|---------|---------|---------|
| 157     | 0.39    | 155     | 0.28    | 157     | 0.24    | 157     | 0.28    |
| 158     | 0.22    | 156     | 0.14    | <0.30   | 158     | 0.28    | <0.30   |
| 159     | 0.54    | 157     | 0.72    | 0.83    | 159     | 0.24    | 0.34    |
| 160     | 0.31    | 158     | 0.46    | 0.46    | 160     | 0.12    | <0.30   |

| Z = 110 |         |         |
|---------|---------|---------|
| 157     | 0.41    | 160     | 0.45    | 0.66    | 161     | 0.26    | <0.30   |
| 158     | 0.19    | 161     | 0.53    | 0.79    | 166     | 0.23    | <0.30   |
| 159     | 0.52    | 162     | 0.42    | 0.64    | 167     | 0.19    | 0.50    |
| 160     | 0.50    | 163     | 0.58    | 0.65    | 168     | 0.07    | <0.30   |
| 161     | 0.43    | 164     | 0.40    | 0.63    | 169     | 0.05    | 0.37    |
| 162     | 0.35    | 165     | 0.42    | 0.65    |         |         |         |

| Z = 111 |         |         |
|---------|---------|---------|
| 166     | 0.38    | 163     | 0.53    | 0.68    | 164     | 0.23    | <0.30   |
| 158     | 0.36    | 168     | 0.06    | 0.41    | 165     | 0.46    | 0.44    |
| 159     | 0.61    | 157     | 0.28    | 0.64    | 167     | 0.20    | 0.63    |

| Z = 115 |         |         |
|---------|---------|---------|
| 160     | 0.85    | 157     | 0.28    | 0.64    | 167     | 0.20    | 0.63    |
| 161     | 0.89    | 158     | 0.25    | 0.50    | 168     | 0.15    | 0.39    |
| 162     | 0.80    | 159     | 0.34    | 0.49    |         |         |         |

| Z = 112 |         |         |
|---------|---------|---------|
| 163     | 0.56    | 160     | 0.39    | 0.38    | 165     | 0.46    | 0.57    |
| 164     | 0.58    | 161     | 0.56    | 0.58    | 166     | 0.33    | 0.37    |
| 166     | 0.48    | 162     | 0.42    | 0.39    | 167     | 0.34    | 0.49    |

| Z = 116 |         |         |
|---------|---------|---------|
| 163     | 0.46    | 168     | 0.54    | 0.64    |

| Z = 117 |         |         |
|---------|---------|---------|
| 157     | 0.57    | 164     | 0.49    | 0.45    | 169     | 0.31    | 0.57    |
| 158     | 0.32    | 165     | 0.47    | 0.60    | 170     | 0.24    | 0.38    |
| 159     | 0.58    | 166     | 0.53    | 0.54    | 171     | 0.23    | 0.32    |
| 160     | 0.60    | 167     | 0.42    | 0.80    |         |         |         |

| Z = 120 |         |         |
|---------|---------|---------|
| 161     | 0.51    | 168     | 0.20    | 0.55    | 165     | 0.39    | 0.38    |
| 162     | 0.53    | 169     | 0.13    | 0.31    | 166     | 0.17    | <0.30   |
| 163     | 0.56    | 170     | 0.07    | 0.30    | 167     | 0.20    | 0.49    |
| 164     | 0.44    | 168     | 0.15    | <0.30   |
| 165     | 0.33    | 155     | 0.40    | 0.41    | 169     | 0.10    | 0.46    |
| 166     | 0.34    | 156     | 0.19    | <0.30   |

| Z = 113 |         |         |
|---------|---------|---------|
| 155     | 0.14    | 159     | 0.35    | 0.44    | 167     | 0.38    | 0.52    |
| 156     | 0.24    | 160     | 0.28    | 0.49    | 168     | 0.31    | 0.34    |
| 157     | 0.80    | 161     | 0.40    | 0.44    | 169     | 0.36    | 0.60    |
| 158     | 0.50    | 162     | 0.33    | 0.37    | 170     | 0.30    | 0.43    |
| 159     | 0.56    | 163     | 0.48    | 0.54    |         |         |         |

| Z = 122 |         |         |
|---------|---------|---------|
| 160     | 0.61    | 164     | 0.40    | 0.38    | 164     | 0.00    | <0.30   |
| 161     | 0.72    | 165     | 0.46    | 0.50    | 165     | 0.21    | <0.30   |
| 162     | 0.57    | 166     | 0.33    | 0.40    | 166     | 0.12    | <0.30   |
| 163     | 0.76    | 167     | 0.30    | 0.38    | 167     | 0.19    | 0.31    |
| 164     | 0.49    | 168     | 0.11    | <0.30   | 168     | 0.11    | <0.30   |
| 165     | 0.54    | 169     | 0.09    | 0.32    | 169     | 0.10    | 0.45    |
| 166     | 0.40    | 0.86    |         |         |         |         |         |
| 167     | 0.19    | 0.78    |         |         | 166     | 0.06    | <0.30   |
| 168     | 0.10    | 0.55    |         |         | 167     | 0.10    | 0.32    |
the $\beta\gamma$-mixing in more reliable for barriers at small $\gamma$, while the original mesh Eq. (3) may be expected to be better for saddles closer to $\beta\gamma$-mixing at $\gamma = 0^\circ$ and around the $x$ axis at $\gamma = 60^\circ$, which allows to better approximate energy at oblate shapes. For this reason, while the original mesh Eq. (3) may be expected more reliable for barriers at small $\gamma$, the one of Eq. (8) is better for saddles closer to $\gamma = 60^\circ$, like those in nuclei with well- or super-deformed oblate ground states.

Our method of proceeding is analogous to that used in the study of the mass-asymmetry. The difference is that we do not have to perform the first step: a minimization with respect to $\beta_{42}$ and $\beta_{44}$ at the saddles found from the grid Eq. (3). Such calculations were already done in the previous studies of the effect of nonaxial deformations of higher multipolarity on the fission barrier in heaviest nuclei [46–49]. We know that the minimization gave the largest effect in the following four regions of nuclei, see Fig. 2 in [49]: (I) $Z \approx 122$, $N \approx 160$ - up to 1.5 MeV, and a $\sim 3$ times smaller effect for nuclei with larger $N$ and $Z > 120$, (II) $Z \approx 110$, $N \approx 146$ - up to 1 MeV, (III) $Z \approx 114$, $N \approx 184$ - up to 1 MeV, and (IV) $Z \approx 104$, $N \approx 170$ - up to 0.4 MeV.

By applying the IWF method on the mesh Eq. (8) we have found the saddles for a dozen of nuclei from the last three regions, for which the effect of minimization was the largest. It turned out that, compared to saddles found on the original grid Eq. (3), they were lowered by less than 150 keV in the region (II), by less than 100 keV in the region (III), and even increased by $\sim 100$ keV in the region (IV). On this basis we conclude that the lowering of the fission saddles found by the minimization in [3, 48] in these three regions is in a large measure a spurious effect which mostly vanishes when saddles are fitted by a proper method.

On the contrary, the substantial effect (up to $\sim 1$ MeV) of the nonaxial hexadecapole in the region (I), although smaller than found by the minimization, survives in the exact IWF treatment. This might be expected as these are very heavy $Z \geq 119$ nuclei with short barriers and oblate (also superdeformed) ground states, so $\beta_{42}$ and $\beta_{44}$ are necessary to reproduce energy in the vicinity of the oblate axis. Therefore, in the whole region of nuclei with $Z \geq 118$ we calculated triaxial barriers by the IWF method using the mesh Eq. (8) and then selected the proper fission barriers from two 5D calculations.

Three types of saddles in nuclei from the region (I) are shown for a very heavy and exotic nucleus $^{285}_{122}$ in Fig. 8. The landscape was created from the 5D mesh Eq. (8).

\begin{align*}
R(\vartheta, \varphi) &= c(\beta)R_0 \{ 1 + \beta \cos (\gamma)Y_{20} \\
&+ \frac{\beta \sin (\gamma)}{\sqrt{2}} [Y_{22} + Y_{-22}] \\
&+ \beta_{40} \frac{1}{6} (5 \cos^2 (\gamma) + 1)Y_{40} \\
&- \beta_{40} \frac{1}{6} \sqrt{\frac{15}{2}} \sin (2\gamma) [Y_{42} + Y_{-42}] \\
&+ \beta_{40} \frac{1}{6} \sqrt{\frac{35}{2}} \sin^2 (\gamma) [Y_{44} + Y_{-44}] \\
&+ \beta_{60} Y_{60} + \beta_{80} Y_{80} \}. \quad (8)
\end{align*}

On this 5D grid, the hexadecapole nonaxiality (but not the $\beta_{60}$ and $\beta_{80}$ terms) preserves the modulo-60° invariance in $\gamma$, so, in particular, the parameter $\beta_{40}$ describes a deformation which is axially symmetric around the $z$ axis at $\gamma = 0^\circ$ and around the $x$ axis at $\gamma = 60^\circ$, which allows to better approximate energy at oblate shapes. For this reason, while the original mesh Eq. (3) may be expected more reliable for barriers at small $\gamma$, the one of Eq. (8) is better for saddles closer to $\gamma = 60^\circ$, like those in nuclei with well- or super-deformed oblate ground states.

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&+ \beta_{60} Y_{60} + \beta_{80} Y_{80} \}. \quad (8)
\end{align*}
This nucleus has a global superdeformed oblate (SDO) minimum with the quadrupole deformation $\beta_{20} = -0.455$ (spheroid with the axis ratio $\approx 3:2$). It represents a neutron-deficient area of superheavy nuclei according to recent predictions [50]. These intriguing SDO minima were already confirmed, as the global ones, by various self-consistent models [51,52]. There is a saddle close to the oblate axis, separating the SDO g.s. from the wide minimum near the spherical shape - type a); the axially symmetric saddle is designated as b). One fission path may go through the saddles a) and b), the higher of which would define the barrier along this path. The second fission path goes through a triaxial saddle of type c) at $\beta_{20} \approx 0.4, \gamma \approx 35^\circ$. The fission barrier of $B_f = 3.6$ MeV corresponds to the saddle c) as found by using the grid Eq. (8). It turns out that saddles of type a) and c) are much lowered by including $\beta_{42}, \beta_{44}$, the first usually more than the second.

Table II summarizes the effect of nonaxial hexadecapole on the barriers in the region (I). It contains 75 nuclei in which the barrier lowering is greater than 300 keV. The most frequent saddle type in the region (I), on both grids, is c), but there are also more complicated cases in which the saddle type changes when $\beta_{42}$ and $\beta_{44}$ are included. The largest effect of 1.167 MeV occurs in the nucleus $Z = 125, N = 163$.

Let us remark that the difference between the results of the constrained minimization and the IWF method for the nonaxial hexadecapole is the main source of the discrepancy between the current fission barriers and those published in [7] for even-even nuclei.

D. Isotopic dependence

Calculated fission barriers given in Table III are illustrated along isotopic chains in Figures: 9 - 13. Generally, it can be seen that: i) in the whole region $Z=98 - 126$ the fission barrier heights are limited by: $B_f \leq 8.06$ MeV; ii) there are characteristic maxima of fission barriers at $Z \approx 100, N \approx 150$, near $Z = 108, N = 162$ (deformed magic shells) and $Z = 114, N = 178$ (not 184); high barriers occur also at the border of the studied region, for $Z = 98, N \approx 183$; iii) over intervals of $N$ where $B_f(N)$ increase or are on average constant, the fission barriers in a neighboring system $N_{even} + 1$ are higher than $B_f(N_{even})$; it may the opposite over intervals where $B_f(N)$ strongly decrease; the same behaviour can be seen when comparing barriers for isotones - see Fig. 20. This quite pronounced odd-even staggering in barriers is related to a decrease in the pairing gap due to blocking as it will be discussed in the next subsection.

In the isotopic dependence of the fission barriers for Cf, Es and Fm nuclei, shown in Fig. 21 there are two peaks of a similar size, at $N = 152$ and $N = 184$. The minima of $B_f(N)$ occur at $N \approx 170$. Odd-even staggering in $B_f$ for Es is stronger around $N = 152$, while for Cf it is stronger near $N = 184$.

FIG. 9: Isotopic dependence of fission barriers for $Z = 98, 99$ and $Z = 100$.

FIG. 10: The same as in Fig.9 but for $Z = 101, 102$ and $Z = 103$.

For Md, No and Lr isotopes (Fig. II), the second maximum around $N = 184$ is weakening. A maximum associated with the semi-magic deformed shell at $N = 162$ appears. As before, the minima of $B_f(N)$ are located at $N \approx 170$. For Rf, Db, Sg, Bh and Hs nuclei (Fig. III and IV), previously distinct maximum at $N = 152$ becomes more flat, and a kind of plateau forms between $N = 152$ and 162. For Mt isotopes this plateau changes into a local minimum in the isotopic dependence $B_f(N)$, located
TABLE II: The barrier lowering (in MeV) greater than 0.3 MeV in nuclei $Z \geq 118$, in particular in those with SDO ground states, from the IWF calculations on the 5D mesh including $\beta_{42}$ and $\beta_{44}$ according to Eq. (8). Also reported is the associated change in the saddle type (for a description of saddle types see text); no entry means that a c-type saddle results from both grids, Eq. (3) and (8).

| $N$ | $\Delta B_f$ | saddle | $N$ | $\Delta B_f$ | saddle | $N$ | $\Delta B_f$ | saddle |
|-----|-------------|--------|-----|-------------|--------|-----|-------------|--------|
| 155 | 0.597       |        | 158 | 0.779       |        | 161 | 1.083       |        |
| 156 | 0.482       |        | 159 | 0.959       |        | 162 | 0.958       |        |
| 157 | 0.472       |        | 160 | 0.807       |        | 163 | 1.167       |        |
| 158 | 0.566       |        | 161 | 0.731       |        | 164 | 0.936       |        |
| 159 | 0.585       |        | 162 | 0.690       |        | 165 | 0.439       |        |
| 160 | 0.508       |        | 163 | 0.469       | $a \rightarrow c$ | 166 | 0.806       | $b \rightarrow c$ |
| 161 | 0.315 $b \rightarrow c$ | 164 | 0.364 $b \rightarrow c$ | 167 | 0.806 |        |        |
| 162 | 0.471 $a \rightarrow c$ | 169 | 0.403 $b \rightarrow c$ | 168 | 0.800 |        |        |
| 170 | 0.343 $b \rightarrow c$ | 170 | 0.365 |        | 169 | 0.714 |        |        |
| 172 | 0.480 $b \rightarrow c$ |        |        |        | 170 | 0.551 |        |        |
| 173 | 0.501       | 159 | 0.831 |        |        |        |        |        |
| 174 | 0.400       | 160 | 0.821 | 162 | 0.955 |        |        |        |
| 156 | 0.613       | 162 | 0.924 | 164 | 1.034 |        |        |        |
| 157 | 0.731       | 163 | 0.496 $a \rightarrow c$ | 165 | 0.802 |        |        |        |
| 158 | 0.652       | 164 | 0.480 $a \rightarrow c$ | 166 | 0.912 |        |        |        |
| 159 | 0.778       | 168 | 0.357 $b \rightarrow c$ | 167 | 0.807 |        |        |        |
| 160 | 0.696       | 169 | 0.300 $b \rightarrow c$ | 168 | 0.845 |        |        |        |
| 161 | 0.658       |        |        | 169 | 0.911 |        |        |        |
| 162 | 0.581 $a \rightarrow c$ | 160 | 0.819 | 170 | 0.735 |        |        |        |
| 163 | 0.323 $a \rightarrow b$ | 161 | 0.868 | 171 | 0.534 |        |        |        |
| 157 | 0.747       | 163 | 0.741 |        |        |        |        |        |
| 158 | 0.774       | 164 | 0.739 $a \rightarrow c$ |        |        |        |        |        |
| 159 | 0.690       | 165 | 0.333 $b \rightarrow c$ |        |        |        |        |        |
| 160 | 0.830       | 166 | 0.334 $b \rightarrow c$ |        |        |        |        |        |
| 161 | 0.688       | 167 | 0.455 $b \rightarrow c$ |        |        |        |        |        |
| 162 | 0.633 $b \rightarrow c$ | 168 | 0.519 $b \rightarrow c$ |        |        |        |        |        |
| 169 | 0.459       |        |        |        |        |        |        |        |
| 170 | 0.328       |        |        |        |        |        |        |        |

around $N = 155$. The highest barriers in Bh, Hs and Mt isotopic sets occur at $N \approx 162$.

For Ds, Rg and Cn nuclei (Fig. 13), with the increasing proton number the $N = 184$ spherical shell starts to dominate. However, not much lower barriers are obtained near the deformed gap $N = 162$.

For nuclei: $Z = Nh, Fl, Mc$ (Fig. 14), one can see one region with high barriers, around $N = 180$. One can notice that the maxima in $B_f(N)$ are already shifted towards smaller $N < 184$. Slight residues of the formerly observed shells at $N = 152$ and $N = 162$ can be spotted.

For nuclei: $Z = Lv, Ts, Og$ (Fig. 15), the main maximum in $B_f$ progresses further towards smaller $N$, reaching finally $N \approx 175$. The minima in $B_f(N)$, observed before at $N = 172$, gradually disappear. For nuclei: $Z = 119, Z = 120, Z = 121$ (Fig. 14), the situation is similar to that described above. Barriers in nuclei $Z = 122, 123, 124$ (Fig. 17), compared to the previous set, are clearly lower. The maximum is even more shifted towards smaller $N$. For nuclei: $Z = 125, 126$ (Fig. 18) the fission barriers are still lower. Their maxima occur at $N = 171$ and 173.

All calculated fission barriers heights are collected together and shown as a map $B_f(Z, N)$ in Fig. 19. One can see three areas with clearly raised barriers: around $N \approx 152$, $N = 162$ and $N \approx 180$, and the region of low barriers around $N = 170$, as discussed above. The effect of the odd particle, i.e. an often (but not always) higher
FIG. 11: The same as in Fig. 9 but for \( Z = 104 \), \( Z = 105 \) and \( Z = 106 \).}

E. Role of the pairing interaction and the odd-even barrier staggering

It is known that the blocking procedure often causes an excessive reduction of the pairing gap in systems with an odd particle number. This effect is much more pronounced in the g.s. than in the fission saddle, as the pairing gap is never small in the latter. One device to avoid an excessive even-odd staggering in nuclear binding was to assume a stronger (typically by \( \sim 5\% \)) pairing interaction for odd-particle-number systems, see [41–44]. Here, instead of performing another grid calculation with modified pairing strengths, we tested the magnitude of their effect on fission barriers by increasing them by 5 and 10 percent for odd particle numbers (neutrons or protons) at previously found ground states and saddle points. The results of this test are presented in Fig. 20 for the \( N=169 \) isotones and in Fig. 21 for the \( Z=109 \) isotopic chain.

Both the isotopic and isotonic dependence show that
increasing the intensity of pairing leads to a reduction of the fission barrier by a variable amount. When the pairing strengths are increased by 5% for odd particle numbers, the fission barriers decrease in odd-even, even-odd and odd-odd systems by up to 0.5 MeV; the 10% increase in the pairing strengths can decrease the barriers at most by about 1 MeV. The same pairing change leads to the suppression, and then the inversion of the staggering effect.

The even-odd barrier staggering related to pairing is convoluted with the isotopic or isotonic dependence related to the mean-field. With the original pairing, when one separates a linear part of the latter by calculating:

\[ B_f(Z_{\text{odd}}, N) - \frac{1}{2}[B_f(Z_{\text{odd}} + 1, N) + B_f(Z_{\text{odd}} - 1, N)] \]

and an analogous quantity for odd neutron numbers, one obtains numbers between 1.053 and −0.947 MeV, with the average of \( \approx 0.22 \) MeV for protons and \( \approx 0.26 \) MeV for neutrons. As shown by black points in Fig. 20, 21.
the effect is indeed irregular and, when present, typically at the level of several hundred keV.

The 5% increase in pairing for odd particle numbers reduces the staggering in \( N = 169 \) isotones and nearly cancels it in \( Z = 109 \) isotopes (red points in Fig.20 and 21). The important point is that the 10% increase in pairing for odd number of particles \( \text{inverts} \) the staggering, at least locally: near \( Z = 120 \) in \( N = 169 \) isotones and near \( N = 153, \ N = 162 \) and \( N = 180 \) in Mt isotopes (green points in Fig.20 and 21).

Although the spontaneous fission rates of odd-particle number nuclei are smaller by 3-5 orders of magnitude than those of their even neighbors, the experimental fission barriers in actinides show only a moderate odd-even staggering, c.f. [52, 54]. Still, it is inconceivable that the fission barriers in odd-\( Z \) or odd-\( N \) systems should be on average smaller than in their even neighbors. This indicates that the 10% increase in pairing strengths in odd-\( N \) or odd-\( Z \) systems would be too large. A qualitative argument which follows is that even if the blocking method overestimates the pairing decrease, the fission barriers of odd-\( Z \) or/and odd-\( N \) nuclei should fall in a strip between the black and red points in Fig.20 and 21. Thus, the test of the pairing influence on barriers points that a possible overestimate of barriers in odd-\( A \) and odd-odd nuclei, induced by the blocking, should not be much larger than 0.5 MeV. One may add in this context that the barriers from the FRLDM model do not show any even-odd staggering due to the way the pairing was included there.

F. Comparison with other theoretical calculations and some empirical data

Let us discuss the results in Table III in relation to available empirical data and to the other theoretical estimates.

As an empirical check of our model, one can use the barriers in the actinide region. We have reported quite a spectacular agreement of the calculated first [7] and second [8] fission barriers in even-even actinides with the data [53, 54], with root mean square deviation 0.5 MeV and 0.7 MeV, respectively.

The heaviest nucleus in which the fission barrier height has been measured recently is \(^{254}\text{No}\). The value \( B_f = 6.0 \pm 0.5 \text{ MeV} \) at spin \( 15\hbar \), giving by extrapolation \( B_f = 6.6 \pm 0.9 \text{ MeV} \) at the spin \( 0\hbar \), has been deduced from the measured distribution of entry points in the excitation energy vs. angular momentum plane [55]. This result perfectly agrees with our evaluation: \( B_f = 6.88 \text{ MeV} \) (at spin \( 0\hbar \)) and with the MM model [64] which gives: \( B_f = 6.76 \text{ MeV} \). The selfconsistent calculations, mainly based on the Skyrme interaction, overestimate this barrier significantly [56–58] (9.6 and 8.6 or 12.5 MeV, respectively). There are experimental estimates of barriers in a few SH nuclei, based on observed ER production probabilities [61], which again well agree with our barriers, see [7]. Apart from those, fission barriers in the SH region are generally unknown.

As a supplementary insight, one can crosscheck barriers evaluated within various models. Quite recently we noted a dramatic divergence in calculated fission barriers [59]. Since, as it was discussed previously, the inclusion of traxiality is absolutely necessary in the SH region, we have chosen only models which take this into account. In fact, there is only one systematic calculation, including triaxiality and odd-particle-number nuclei - the Finite Range Liquid Drop Model [13, 60, 64].
FIG. 19: Calculated fission barrier heights $B_f$ for superheavy nuclei.

(FRLDM) developed by Los Alamos group. It can be noted though, that the inner fission barrier is fixed there in only three-dimensional deformation space, what is certainly not enough.

The first conclusion from the comparison between our results and those of FRLDM is that a conspicuous barrier staggering between odd- and even-particle number nuclei is obtained in the Woods-Saxon model. As mentioned before, this results from the blocking treatment of pairing. At present it is not certain how large this staggering should be.

One can include more models for comparison if one confines it to even - even nuclei. We take the covariant density functional model [63] with the nonlinear meson-nucleon coupling, represented by the NL3* parametrization of the relativistic mean-field (RMF) Lagrangian and the Hartree-Fock-Bogoliubov (HFB) approach with the SkM* Skyrme energy density functional [62].

As can be seen in Fig. 22, fission barriers in Hassium nuclei are quite similar in all models. The values of $B_f$ differ up to 2 MeV, but never more. Regarded as a function of $N$, they show a maximum close to the semi-magic number $N = 162$ while the second maximum is related with the $N = 184$ spherical gap. In the FRLDM this maximum is barely outlined and slightly shifted to the neutron deficient side. The minimum in barriers is obtained in both MM models at the similar place ($N = 170$), while the RMF gives the smallest barriers at $Z = 174$.

As one can see in Fig. 23 for Flerovium isotopes the barriers calculated here are in agreement with the experimental (empirical) estimates [61] and with the self-consistent calculations [62] based on the SKM* interaction. The FRLDM [64] overestimates these quasi-empirical barriers [61] significantly. Although only the lower limit for the barrier height has been estimated in [61], which would reproduce the known cross sections on the picobarn level, such a high barrier seems problematic, see discussion in [65, 66]. On the other hand, with extremely small barriers obtained within the RMF model one cannot explain experimentally known millisecond fission half-life in $^{284}$Fl. One should note, however, that a slight tuning of the RMF model [67] gives higher barriers,
FIG. 20: Effect of the pairing strength increase (while keeping fixed the g.s. and saddle deformations) in $N=169$ isotones: standard $G_{n}$ and $G_{p}$ - black points, $G_{n}$ and $G_{p}$ increased by 5% (10%) for odd-$Z$ and odd-$N$ nuclei - red (green) points.

FIG. 21: Effect of the pairing strength increase (while keeping fixed the g.s. and saddle deformations) in $Z=109$ isotopes: standard $G_{n}$ and $G_{p}$ - black points, $G_{n}$ and $G_{p}$ increased by 5% (10%) for odd-$Z$ and odd-$N$ nuclei - red (green) points.

FIG. 22: Fission barriers predicted by various models for Hassium isotopes: black - WS model, green FRLDM [64], blue SkM* [62], red RMF with NL3 parametrization [63]. Experimental data taken from [61]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

FIG. 23: The same as in Fig. 22 but for $Z=114$.

thus, closer to ours. This is true, especially in Cn and Fl isotopes, see details in Fig. 5 in [67] and discussion included there. For $Z=120$ our results, shown in Fig. 24 are very
FIG. 24: The same as in Fig 22 but for Z=120.

FIG. 25: The same as in Fig 22 but for Z=126.

IV. CONCLUSIONS

We have determined fission barriers for 1305 heavy and superheavy nuclei, including odd-A and odd-odd systems, within the macroscopic-microscopic method by following the adiabatic configuration in each nucleus. The applied Woods-Saxon model was widely used for heavy nuclei and well reproduces experimental fission barriers in actinides. For odd-Z or/and odd-N nuclei pairing was included within the blocking procedure. Triaxial and mass-asymmetric deformations were included and the IWF method used for finding the saddles which allowed to escape errors inherent in the constrained minimization approach. To find saddles, energy for each nucleus was calculated on a 5D deformation grid and then 5-fold interpolated in each dimension for the IWF search. Two additional energy grids: a second 5D and another 7D, were calculated in order to include nonaxial hexadecapole and mass-asymmetry effects on fission barriers. The following conclusions can be drawn from our investigation:

i) Global calculations confirm the existence of two physically important areas in the Z-N plane with prominent barriers: one located around the semi-magic quantum numbers $Z = 100 - 108$ and $N = 150 - 162$ (connected with deformed closed shells) and the second - of nearly spherical nuclei around $Z = 114$ and $N = 176 - 180$. The highest fission barrier among the studied nuclei occurs in very exotic Es$_{250}$.

ii) The well-known effect of the mass asymmetry on the second barrier in actinides is not very relevant for the heaviest nuclei since very deformed barriers at $\beta_{20} \approx 0.8$

In the case of $Z = 126$, shown in Fig. 25 both MM models give significantly smaller barriers than the model based on the SKM* force. For example, the barrier $B_f \approx 9$ MeV for $^{310}126$, calculated with this Skyrme interaction, is still impressively large. This might induce thoughts on the ways of synthesis of such superheavy systems, but one has to remember that the predicted half-lives with respect to $\alpha$ decay are below the present-day $10^{-5}$ s time-limit for the experimental identification. On the contrary, $B_f \approx 2$ MeV obtained in the MM approach does not induce any hopes; it only points to a quite striking disagreement between models.

close to those obtained within the RMF model. The results of [64] are systematically higher by $\approx 1$ MeV. This is in an evident contrast with the Skyrme SkM* prediction [62] of the highest barriers for $Z = 120$ [62] - related to the proton magic gap. Three models: FRLDM, RMF and ours converge at $N=182-184$ to $B_f \simeq 5$ MeV. The nucleus $^{302}120$ is particulary interesting, as two unsuccessful attempts to produce it have already taken place in GSI, providing a cross-section limit of 560 fb [68] or 90 fb in [69], and in Dubna [70], providing the limit of 400 fb. The cross-section estimates [71] do not support a possibility of an easy production of this SH isotope in the laboratory. It seems that with the barrier of the order of 10 MeV, as obtained in the frame of the self-consistent theory, producing superheavy $Z=120$ nuclei should not pose any difficulties.

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ii) The well-known effect of the mass asymmetry on the second barrier in actinides is not very relevant for the heaviest nuclei since very deformed barriers at $\beta_{20} \approx 0.8$
decrease with increasing $Z$ and fission barriers are fixed by the less deformed saddles. However, in some nuclei with $Z \geq 109$ the mass(reflection) asymmetry effect lowers the first saddles which are sometimes split into two humps. It seems that this concerns only axially-symmetric saddles. The largest barrier lowering (by 0.8 MeV) has been observed for $Z = 113$ and $N = 157$.

iii) It has been demonstrated that the inclusion of triaxial shapes significantly reduces the fission barriers by up to 2.5 MeV; about 70% of the found fission barriers correspond to triaxial saddles. Besides the quadrupole nonaxiality we checked also the effect of hexadecapole nonaxiality which significantly lowers the fission barrier in $Z \geq 119$ nuclei, especially neutron-deficient ones.

iv) Rather strong, irregular odd-even $Z$ or $N$ barrier staggering effect resulted from the blocking formalism used for pairing. The barrier of an odd nucleus or $N_{even} + 1$ is typically by several hundred keV higher than that of its even neighbor.

v) The existing theoretical evaluations of fission barriers differ significantly. Even the results of the two models based on the microscopic-macroscopic approach differ dramatically for some nuclei. Our calculations indicate, in contrast to the self-consistent mean-field studies, that fission barriers, still quite substantial for some $Z = 118$ nuclei, become lower than 5.5 MeV for $Z = 126$.

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TABLE III: Calculated fission barrier heights (in MeV).

| N  | A  | B_f | N  | A  | B_f | N  | A  | B_f | N  | A  | B_f |
|----|----|-----|----|----|-----|----|----|-----|----|----|-----|
| Z=98 |   |     | Z=99 |   |     | Z=100 |   |     | Z=101 |   |     |
| 134 | 232 | 2.28 | 135 | 233 | 2.74 | 136 | 234 | 2.83 | 137 | 235 | 3.45 |
| 138 | 236 | 3.62 | 139 | 237 | 4.64 | 140 | 238 | 4.78 | 141 | 239 | 5.86 |
| 142 | 240 | 5.90 | 143 | 241 | 6.71 | 144 | 242 | 6.61 | 145 | 243 | 7.35 |
| 146 | 244 | 6.88 | 147 | 245 | 7.41 | 148 | 246 | 6.86 | 149 | 247 | 7.08 |
| 150 | 248 | 6.79 | 151 | 249 | 7.36 | 152 | 250 | 6.67 | 153 | 251 | 6.26 |
| 154 | 252 | 5.98 | 155 | 253 | 5.62 | 156 | 254 | 5.19 | 157 | 255 | 5.00 |
| 158 | 256 | 4.73 | 159 | 257 | 4.99 | 160 | 258 | 4.48 | 161 | 259 | 5.06 |
| 162 | 260 | 4.60 | 163 | 261 | 4.41 | 164 | 262 | 4.10 | 165 | 263 | 3.97 |
| 166 | 264 | 3.71 | 167 | 265 | 3.71 | 168 | 266 | 3.62 | 169 | 267 | 4.38 |
| 170 | 268 | 3.85 | 171 | 269 | 4.81 | 172 | 270 | 4.48 | 173 | 271 | 5.13 |
| 174 | 272 | 5.13 | 175 | 273 | 6.00 | 176 | 274 | 5.58 | 177 | 275 | 6.63 |
| 178 | 276 | 6.62 | 179 | 277 | 6.17 | 180 | 278 | 6.72 | 181 | 279 | 7.25 |
| 182 | 280 | 6.91 | 183 | 281 | 7.65 | 184 | 282 | 7.14 | 185 | 283 | 5.73 |
| 186 | 284 | 5.43 | 187 | 285 | 4.59 | 188 | 286 | 4.00 | 189 | 287 | 4.13 |
| 190 | 288 | 5.52 | 191 | 289 | 3.53 | 192 | 290 | 3.08 |      |      |      |

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TABLE IV: Calculated fission barrier heights (in MeV).

| N  | A  | $B_f$ | N  | A  | $B_f$ | N  | A  | $B_f$ | N  | A  | $B_f$ |
|----|----|------|----|----|------|----|----|------|----|----|------|
| 139| 242| 3.13 | 140| 243| 3.14 | 141| 244| 4.01 | 142| 245| 3.92 |
| 143| 246| 4.55 | 144| 247| 4.66 | 145| 248| 5.29 | 146| 249| 5.19 |
| 147| 250| 6.10 | 148| 251| 5.79 | 149| 252| 6.54 | 150| 253| 6.28 |
| 151| 254| 7.26 | 152| 255| 6.81 | 153| 256| 6.62 | 154| 257| 6.45 |
| 155| 258| 6.49 | 156| 259| 6.30 | 157| 260| 6.33 | 158| 261| 6.10 |
| 159| 262| 6.15 | 160| 263| 5.89 | 161| 264| 5.36 | 162| 265| 5.83 |
| 163| 266| 5.19 | 164| 267| 4.83 | 165| 268| 4.17 | 166| 269| 3.97 |
| 167| 270| 3.82 | 168| 271| 3.41 | 169| 272| 3.52 | 170| 273| 3.19 |
| 171| 274| 3.68 | 172| 275| 3.30 | 173| 276| 4.07 | 174| 277| 3.67 |
| 175| 278| 3.65 | 176| 279| 3.30 | 177| 280| 4.07 | 178| 281| 4.46 |
| 179| 282| 4.68 | 180| 283| 5.01 | 181| 284| 5.42 | 182| 285| 5.62 |
| 183| 286| 5.91 | 184| 287| 5.31 | 185| 288| 4.38 | 186| 289| 4.17 |
| 187| 290| 2.74 | 188| 291| 2.76 | 189| 292| 2.44 | 190| 293| 2.20 |
| 191| 294| 2.21 | 192| 295| 1.70 | 193| 296| 1.45 | 194| 297| 1.12 |
TABLE V: Calculated fission barrier heights (in MeV).

| Z=108 | N  | A   | Bf  | Z=109 | N  | A   | Bf  | Z=110 | N  | A   | Bf  | Z=111 | N  | A   | Bf  | Z=112 | N  | A   | Bf  |
|-------|----|-----|-----|-------|----|-----|-----|-------|----|-----|-----|-------|----|-----|-----|-------|----|-----|-----|
| 144   | 252| 2.72|     | 145   | 253| 3.49|     | 146   | 254| 3.58|     | 147   | 255| 4.50|     | 148   | 256| 4.88|     |
| 149   | 257| 5.60|     | 150   | 258| 5.29|     | 151   | 259| 6.40|     | 152   | 260| 5.98|     | 153   | 261| 6.13|     |
| 154   | 262| 6.07|     | 155   | 263| 6.07|     | 156   | 264| 5.93|     | 157   | 265| 5.93|     | 158   | 266| 5.83|     |
| 159   | 267| 6.19|     | 160   | 268| 6.09|     | 161   | 269| 6.87|     | 162   | 270| 6.46|     | 163   | 271| 6.28|     |
| 164   | 272| 5.52|     | 165   | 273| 5.00|     | 166   | 274| 4.62|     | 167   | 275| 4.16|     | 168   | 276| 3.80|     |
| 169   | 277| 3.40|     | 170   | 278| 3.20|     | 171   | 279| 3.80|     | 172   | 280| 3.31|     | 173   | 281| 4.20|     |
| 174   | 282| 3.88|     | 175   | 283| 5.00|     | 176   | 284| 4.50|     | 177   | 285| 4.16|     | 178   | 286| 4.30|     |
| 179   | 287| 5.61|     | 180   | 288| 5.29|     | 181   | 289| 6.06|     | 182   | 290| 5.47|     | 183   | 291| 6.05|     |
| 184   | 292| 5.61|     | 185   | 293| 4.20|     | 186   | 294| 4.23|     | 187   | 295| 3.04|     | 188   | 296| 2.83|     |
| 189   | 297| 1.75|     | 190   | 298| 1.44|     | 191   | 299| 1.30|     | 192   | 300| 0.75|     |       |     |     |     |       |     |     |     |

Note: The table continues with similar entries for Z=109, Z=110, Z=111, and Z=112.
TABLE VI: Calculated fission barrier heights (in MeV).

| N  | A  | B_f | N  | A  | B_f | N  | A  | B_f | N  | A  | B_f |
|----|----|-----|----|----|-----|----|----|-----|----|----|-----|
| Z=113 |    |     | Z=114 |    |     | Z=115 |    |     | Z=116 |    |     | Z=117 |    |     |
| 149 | 262 | 2.02 | 150 | 263 | 2.02 | 151 | 264 | 2.95 | 152 | 265 | 2.44 | 153 | 266 | 2.33 | 154 | 267 | 2.25 | 155 | 268 | 2.44 |
| 156 | 269 | 2.34 | 157 | 270 | 2.55 | 158 | 271 | 2.66 | 159 | 272 | 3.65 | 160 | 273 | 3.45 | 161 | 274 | 4.58 | 162 | 275 | 4.41 |
| 163 | 276 | 4.88 | 164 | 277 | 4.44 | 165 | 278 | 4.58 | 166 | 279 | 4.23 | 167 | 280 | 4.67 | 168 | 281 | 4.34 | 169 | 282 | 4.89 |
| 170 | 283 | 4.46 | 171 | 284 | 5.19 | 172 | 285 | 4.98 | 173 | 286 | 5.74 | 174 | 287 | 5.54 | 175 | 288 | 6.43 | 176 | 289 | 6.28 |
| 177 | 290 | 6.95 | 178 | 291 | 6.61 | 179 | 292 | 7.26 | 180 | 293 | 6.82 | 181 | 294 | 6.93 | 182 | 295 | 6.71 | 183 | 296 | 7.13 |
| 184 | 297 | 6.63 | 185 | 298 | 5.36 | 186 | 299 | 5.43 | 187 | 300 | 4.11 | 188 | 301 | 4.14 | 189 | 302 | 2.67 | 190 | 303 | 2.70 |
| 191 | 304 | 1.30 | 192 | 305 | 1.28 | 193 | 306 | 0.91 | 194 | 307 | 1.23 | 195 | 308 | 0.75 | 196 | 309 | 1.18 | 197 | 310 |     |

Z = 113, 114, 115, 116, 117
TABLE VII: Calculated fission barrier heights (in MeV).

| Z=118 | Z=119 | Z=120 | Z=121 | Z=122 |
|-------|-------|-------|-------|-------|
| N A  | Bf   | N A  | Bf   | N A  | Bf   | N A  | Bf   |
|-------|------|------|------|------|------|------|------|
| 154   | 272  | 0.66 |      |      |      |      |      |
| 155   | 273  | 1.39 | 155  | 274  | 1.82 |      |      |
| 156   | 274  | 1.41 | 156  | 275  | 1.83 | 156  | 276  | 1.38 |
| 157   | 275  | 2.30 | 157  | 276  | 2.26 | 157  | 277  | 1.79 |
| 158   | 276  | 2.25 | 158  | 277  | 2.25 | 158  | 278  | 1.88 |
| 159   | 277  | 2.06 | 159  | 278  | 2.78 | 159  | 279  | 2.39 |
| 160   | 278  | 3.15 | 160  | 279  | 2.79 | 160  | 280  | 2.44 |
| 161   | 279  | 4.30 | 161  | 280  | 3.49 | 161  | 281  | 3.23 |
| 162   | 280  | 3.94 | 162  | 281  | 3.27 | 162  | 282  | 3.07 |
| 163   | 281  | 4.32 | 163  | 282  | 3.96 | 163  | 283  | 3.89 |
| 164   | 282  | 4.13 | 164  | 283  | 3.68 | 164  | 284  | 3.57 |
| 165   | 283  | 3.79 | 165  | 284  | 3.87 | 165  | 285  | 3.65 |
| 166   | 284  | 3.64 | 166  | 285  | 3.67 | 166  | 286  | 3.43 |
| 167   | 285  | 4.09 | 167  | 286  | 4.43 | 167  | 287  | 4.08 |
| 168   | 286  | 4.01 | 168  | 287  | 4.45 | 168  | 288  | 4.21 |
| 169   | 287  | 5.03 | 169  | 288  | 5.21 | 169  | 289  | 5.01 |
| 170   | 288  | 5.05 | 170  | 289  | 5.23 | 170  | 290  | 5.01 |
| 171   | 289  | 6.03 | 171  | 290  | 6.26 | 171  | 291  | 6.08 |
| 172   | 290  | 5.75 | 172  | 291  | 5.75 | 172  | 292  | 5.48 |
| 173   | 291  | 6.40 | 173  | 292  | 6.55 | 173  | 293  | 6.06 |
| 174   | 292  | 6.09 | 174  | 293  | 6.21 | 174  | 294  | 5.62 |
| 175   | 293  | 6.62 | 175  | 294  | 6.95 | 175  | 295  | 6.28 |
| 176   | 294  | 6.09 | 176  | 295  | 6.32 | 176  | 296  | 5.79 |
| 177   | 295  | 6.64 | 177  | 296  | 6.71 | 177  | 297  | 6.02 |
| 178   | 296  | 6.12 | 178  | 297  | 6.20 | 178  | 298  | 5.56 |
| 179   | 297  | 6.21 | 179  | 298  | 6.20 | 179  | 299  | 5.57 |
| 180   | 298  | 5.79 | 180  | 299  | 5.72 | 180  | 300  | 5.08 |
| 181   | 299  | 6.02 | 181  | 300  | 5.88 | 181  | 301  | 5.24 |
| 182   | 300  | 5.52 | 182  | 301  | 5.38 | 182  | 302  | 4.71 |
| 183   | 301  | 5.71 | 183  | 302  | 5.50 | 183  | 303  | 4.81 |
| 184   | 302  | 5.20 | 184  | 303  | 4.98 | 184  | 304  | 4.30 |
| 185   | 303  | 3.93 | 185  | 304  | 3.82 | 185  | 305  | 3.02 |
| 186   | 304  | 4.03 | 186  | 305  | 3.85 | 186  | 306  | 3.14 |
| 187   | 305  | 2.80 | 187  | 306  | 2.78 | 187  | 307  | 2.17 |
| 188   | 306  | 2.78 | 188  | 307  | 2.63 | 188  | 308  | 1.95 |
| 189   | 307  | 1.78 | 189  | 308  | 2.03 | 189  | 309  | 1.51 |
| 190   | 308  | 1.57 | 190  | 309  | 1.97 | 190  | 310  | 1.39 |
| 191   | 309  | 0.79 | 191  | 310  | 1.67 | 191  | 311  | 1.05 |
| 192   | 310  | 0.80 | 192  | 311  | 1.63 | 192  | 312  | 1.05 |

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TABLE VIII: Calculated fission barrier heights (in MeV).

| Z=123 | N   | A   | B_f | Z=124 | N   | A   | B_f | Z=125 | N   | A   | B_f | Z=126 | N   | A   | B_f |
|-------|-----|-----|-----|-------|-----|-----|-----|-------|-----|-----|-----|-------|-----|-----|-----|
| 159   | 282 | 2.14|     | 160   | 283 | 2.25|     | 161   | 284 | 3.01|     | 162   | 285 | 2.87|     |
| 163   | 286 | 4.03|     | 164   | 287 | 3.69|     | 165   | 288 | 4.75|     | 166   | 289 | 4.15|     |
| 167   | 290 | 4.43|     | 168   | 291 | 4.08|     | 169   | 292 | 5.10|     | 170   | 293 | 4.94|     |
| 171   | 294 | 5.96|     | 172   | 295 | 5.52|     | 173   | 296 | 5.52|     | 174   | 297 | 5.23|     |
| 175   | 298 | 5.16|     | 176   | 299 | 4.82|     | 177   | 300 | 4.82|     | 178   | 301 | 4.75|     |
| 179   | 302 | 4.83|     | 180   | 303 | 4.07|     | 181   | 304 | 3.90|     | 182   | 305 | 3.39|     |
| 183   | 306 | 3.54|     | 184   | 307 | 2.91|     | 185   | 308 | 2.20|     | 186   | 309 | 2.01|     |
| 187   | 310 | 1.89|     | 188   | 311 | 1.69|     | 189   | 312 | 1.47|     | 190   | 313 | 1.31|     |
| 191   | 314 | 0.90|     | 192   | 315 | 0.76|     |       |     |     |     |       |     |     |     |

TABLE VIII: Calculated fission barrier heights (in MeV).