Spectral efficiency analysis of millimeter-wave heterogeneous cellular networks based on PCP

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Abstract. To solve the interference problem in densely populated areas, this paper proposes a new scheme that taking user equipment (UE) as the center of millimeter-wave heterogeneous networks based on Poisson cluster process (PCP) model. Under the network model, the UE is divided into cluster-center UE or cluster-edge UE by using the ratio of the first and second closest distances of the UE from pico base stations. The random geometry and PCP methods are used to study the downlink spectral efficiency and the correct derivation results are verified. At last, the influence of relevant parameters about system performance is analyzed.

1. Introduction
With the development of wireless communication technologies, there is an increasing demand for performance such as high data rates. Hence, considering the combination of millimeter-wave (mm-Wave) and heterogeneous networks (HetNets), the capacity of cellular networks can be increased[1].

The work [2] mentions that mm-Wave communication has abundant spectrum resources and it is expected to provide high data rates. Despite the huge potential in terms of spectrum, the near-field path loss in the mm-Wave band is severe. High directional antennas need be deployed to improve performance. And the side lobes will support simultaneous communications with very little interference[3]. Network densification is another promising solution to improve the network capacity, where large number of small base stations (BSs) are deployed giving rise to HetNets[4]. Moreover, the overlaps of all small, pico and femto cells with the existing macro cells will improve spectrum reuse efficiency. As a result, mm-Wave heterogeneous cellular networks have wide range research and development perspective.

Since the BSs are generally deployed in dense user equipment (UE) areas, the inevitable UE-BS coupling problem has followed[5]. Therefore, according to the potential connection between hotspot formation and UE-BS coupling, the UE distribution is modeled and analyzed by the Poisson cluster process (PCP)[6]. In the work [7], there is an analysis scheme based on distance ratio. It applies to the independent Poisson point process (PPP) heterogeneous cellular networks.

In summary, this paper proposes a scheme based on PCP model. The multi-tier BSs which is centered on UE are deployed on the two-tier mm-Wave HetNets composed of pico base stations (PBSs) and femto base stations (FBSs). Then, this paper classifies UE clusters by using the ratio of the first and second closest distances between the UE and the PBSs. Combining propagation model, the spectral efficiency of the downlink is derived.
2. System model and channel assumption

2.1 System model
This paper considers two-tier mm-Wave HetNets containing PBSs and FBSs. Their locations are modelled by the Thomas cluster process (TCP) expressed as \( \Phi^p_{TCP} (\lambda_c, M_p, C_p) \) and \( \Phi^f_{TCP} (\lambda_c, M_f, C_f) \), where \( \lambda_c \) represents the density of the parent PPP \( \Phi_c \), \( M_H \) represents the number of members in the cluster, the number of simultaneously active members in the cluster is assumed to be a Poisson distribution variable with an average value of \( C_H \), \( H \in \{P, F\} \). The subprocesses \( \Phi^p_{TCP} (.) \) and \( \Phi^f_{TCP} (.) \) are distributed around the same stable parent PPP \( \Phi_c \). The heterogeneity between PBS and FBS determines the value of \( M_H \) is different from \( C_H \). The subprocess \( \Phi^p_{TCP} (.) \) and \( \Phi^f_{TCP} (.) \) have densities of \( \lambda_c C_p \) and \( \lambda_c C_f \), respectively. The PBS and FBS sets centered on \( x \) are denoted as \( N^p_x \) and \( N^f_x \), \( x \in \Phi_c \). Correspondingly, \( S^p_x \) and \( S^f_x \) are used to indicate the active PBS and FBS sets. It is assumed that the randomly selected UE is located in a representative cluster centered on \( x_0 \in \Phi_c \) with 0 as the origin.

2.2 Obstruction model
It is supposed that the whole network works on mm-Wave. A distinguishing feature of mm-Wave is the provision of an antenna array at the BS and mobile receiver to overcome near field losses due to high frequency. In order to facilitate the analysis of the problem, this paper approximates the actual antenna pattern into a sectorized model. In this model, when the angle of deviation from the visual axis is within the main lobe area, the antenna gain is expressed as \( M \). When the angle of deviation from the visual axis is within the side lobes, the antenna gain is expressed as \( m \).

Another significant feature of mm-Wave is the poor penetration capability, which results in line-of-sight (LOS) and non-line-of-sight (NLOS) link. Since the path loss factors of the LOS link are different from the NLOS link, the parameters of the channel fading are distinct. This paper proposes an approximate calculation method that the LOS range can be considered as a sphere, where the sphere radius is \( \mu \). It is defined as the average distance of the UE to the nearby blockage. When the BS is located within the \( \mu \) area, the link between the BS and the UE is called LOS link. Otherwise, the link between the BS and the UE is called NLOS link. By using this model, the network path loss theorem with distance is given:

\[
L(r) = U(\mu - r)C_\mu r^{-\alpha_k} + U(r - \mu)C_\mu r^{-\alpha_k} \tag{1}
\]

Where \( U(.) \) represents the unit step function, \( C_\mu \) represents the intercept, \( \alpha_k \) represents the path loss exponent, \( k = L \) and \( N \) represent the LOS and NLOS link, respectively.

2.3 Cluster classification method
In the PBS and FBS clusters, the UE meets more severe intra-cluster interference, which greatly degrades performance. In order to overcome this problem, this paper proposes an effective interference management scheme, which classifies UE clusters according to the ratio of the nearest distances.

Assume that \( N^p_0 \) refers to the PBS sets of representative clusters which centered on \( x_0 \in \Phi_c \). In particular, a cluster is divided into two disjoint sub-regions, named cluster-center region and cluster-edge region. The cluster-center area refers to an area where the distance between the UE and the dominant interference PBS. Otherwise, the UE is defined as the cluster-edge UE. If the target UE is satisfied \( r^e / r^d > \xi (\xi \in [0,1]) \) in the representative cluster \( \Phi^p_{TCP} \), the target UE is classified as a cluster-edge user equipment (CEUE). If
not, it is classified as a cluster-center user equipment (CCUE), where \( r_{op}^v \) and \( r_{op}^d \) are the first and second closest distance between the target UE and the PBS.

### 3. DL spectrum efficiency analysis

This section gives the signal to interference plus noise ratio (SINR) received by the target UE firstly:

\[
\text{SINR}^B_{\text{CYUE}} \left( r_{ob} || x_0 \right) = \frac{P_B M_B M_L \left( r_{ob}^v \right)}{\sigma_i^2 + I^B_{\text{CYUE}}} \cdot \left( r_{ob}^v \alpha_{TB} < r_{in}^v \right) \tag{2}
\]

Where \( B(b) \in \{ P(p), F(f) \} \), \( Y(y) \in \{ C(c), E(e) \} \), \( T(t) \in \{ P(p), F(f) \} \setminus B(b) \). \( \sigma_i^2 \) represents the additive Gaussian noise power of the UE. \( \text{SINR}^B_{\text{CYUE}} \left( r_{ob} || x_0 \right) \) indicates the received SINRs when the CYUE is connected to the BSs with the closest distance \( r_{ob}^v \). \( P_B \) refers to the transmission power of BSs, \( I^B_{\text{CYUE}} \) is the total interference received by the CYUE. According to the system model considered in this paper, received SINRs will have four forms.

Assuming that the target UE located at the origin \( 0 \) is connected with the BS located in \( y_d \), the interference can be divided into two types, namely intra-cluster interference \( I^{B-\text{Intra}}_{\text{CYUE}} \) and inter-cluster interference \( I^{B-\text{Inter}}_{\text{CYUE}} \). The total interference \( I^B_{\text{CYUE}} \) is calculated as:

\[
I^B_{\text{CYUE}} = I^{B-\text{Intra}}_{\text{CYUE}} + I^{B-\text{Inter}}_{\text{CYUE}} \tag{3}
\]

where \( I^{B-\text{Intra}}_{\text{CYUE}} = \sum_{y_d \in y_d} P_I G_I L(\| x_0 + y_d \|) \). It signifies the interference from the PBS and the FBS in the cluster when the CYUE is connected with the BS. In this equation, \( G_I^v \) and \( G_I^d \) represent the transmit antenna gain at the BS and the receive antenna gain at the UE, respectively. The mm-Wave path loss model \( L(.) \) is given by (1). \( I^{B-\text{Inter}}_{\text{CYUE}} = \sum_{x \in y_d' \setminus y_d} P_I G_I^v L(\| x + y \|) \) represents interference from inter-cluster PBS and FBS.

Therefore, when CYUE is connected with PBS by the access distance \( r_{ob}^v \), the Laplace transform of the interference \( I^P_{\text{CYUE}} \) is calculated as:

\[
\mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{op}^v, \eta \right) = \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{op}^v \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{op}^v, \eta \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, \eta \right) \tag{4}
\]

Respectively, the Laplace transform of \( I^{P-\text{Intra}}_{\text{CYUE}}, I^{P-\text{Inter}}_{\text{CYUE}}, \) and \( I^{P-\text{Inter}}_{\text{CYUE}} \) are:

\[
\mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{op}^v \right) = \sum_{i} P_i \exp \left( - (\pi r_{op}^v)^2 \right) \times \left( C_L \int_{\min (\mu, \eta \mu)}^\infty w \cdot f_{\eta w} \left( \mu \right) dw + C_N \int_{\max (\mu, \eta \mu)}^\infty w \cdot f_{\eta w} \left( \mu \right) dw \right) \tag{5}
\]

\[
\mathcal{L}^{P-\text{Inter}}_{\text{CYUE}} \left( z, \eta \right) = \exp \left( - 2 \pi C_c \sum_{i} P_i \left( z \cdot \bar{e}_r \right) \left( \bar{a}_r \right) \right) \times \left( C_L \int_{0}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw + C_N \int_{\infty}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw \right) \tag{6}
\]

\[
\mathcal{L}^{P-\text{Inter}}_{\text{CYUE}} \left( z, r_{op}^v, \eta \right) = \sum_{i} P_i \exp \left( (1 - \eta) \bar{e}_r \left( \bar{a}_r \right) \right) \times \left( C_L \int_{0}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw + C_N \int_{\infty}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw \right) \tag{7}
\]

\[
\mathcal{L}^{P-\text{Inter}}_{\text{CYUE}} \left( z, \eta \right) = \exp \left( - 2 \pi C_c \sum_{i} P_i \left( (1 - \eta) \bar{e}_r \left( \bar{a}_r \right) \right) \right) \times \left( C_L \int_{0}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw + C_N \int_{\infty}^{\infty} w \cdot f_{\eta w} \left( \mu \right) dw \right) \tag{8}
\]

When CYUE is connected with an FBS by the access distance \( r_{of}^v \), the Laplace transform of the received interference is:

\[
\mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{of}^v, \eta \right) = \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{of}^v \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{of}^v, \eta \right) \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, \eta \right) \tag{9}
\]

The Laplace transform of \( \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{of}^v \right), \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, r_{of}^v, \eta \right), \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z \right) \) and \( \mathcal{L}^{P-\text{Intra}}_{\text{CYUE}} \left( z, \eta \right) \) are:
The general form of downlink (DL) spectral efficiency is derived by (2) as follows:

\[ S_{\text{DL}}(z) = \int_{0}^{\infty} \exp \left( -\frac{z \sigma_{c}^{2}}{2} \right) \left( \int_{0}^{\tau_{c}} \left( 1 - \exp \left( -z P_{c} M_{L} M_{L}(r) \right) \right) L_{\text{DL}}(z, r, \eta) F_{\text{DL}}(r \alpha_{L}) f_{\text{DL}}(r) dr \right) dz \]  

(14)

where \( L_{\text{DL}}(z, r, \eta) \) is Laplace transform of interference \( I_{\text{DL}}(z, r, \eta) \). \( F_{\text{DL}}(\cdot) \) is complementary cumulative distribution function (CDF) of \( r_{\text{DL}} \). \( f_{\text{DL}}(\cdot) \) is probabilities density function (PDF) of \( r_{\text{DL}} \).

**4. Simulation results and analysis**

This paper constructs two-tier heterogeneous cellular networks consisting of PBSs and FBSs. All simulations and analysis use the parameter values in Table 1, unless otherwise stated.

| Table 1. Simulation parameter values. |
|--------------------------------------|
| Parameter | Values |
| \( \lambda_{c} \) | 100/Km² |
| \( \mu \) | 100m |
| \( P_{c} \) | -10~45dBm |
| \( P_{F} \) | 0~35dBm |
| \( C_{P} \) | 1~20 |
| \( C_{F} \) | \{1, 5, 10\} |

Figures show the relationship between activity factors \( c_{P} \) and \( c_{F} \) with DL spectral efficiency of PBS and FBS. Shown in the figures that the DL spectral efficiency of PBS and FBS decreasing when the activity factor \( c_{P} \) increasing, because of increased interference from active PBS and FBS. Besides, it was found that when the activity factor \( c_{F} \) is small, the activity factor \( c_{P} \) impact the DL spectral efficiency of PBS and FBS obviously. On contrary, the influence is ignorable.

![Figure 1. Spectrum efficiency related to PBS.](image)
Figure 1 demonstrates that when the target UE is connected with the PBS, the DL spectral efficiency of the CCUE is better than CEUE.

![Figure 1. Spectrum efficiency related to PBS.](image)

Different from figure 1, figure 2 reveals that the DL spectrum efficiency of the CEUE is optimal when the target UE is connected with the FBS.

![Figure 2. Spectrum efficiency related to FBS.](image)

5. **Conclusion**

This paper classifies user equipment for limiting severe interference and increasing throughput across the network under the UE communication hotspot scene. According to the propagation model of the mm-Wave signal from the line of sight ball, the Laplace transform for the interference of CYUE in all possible connection is calculated. Furthermore, the DL spectrum efficiency is derived on the parent PPP and the influence of relevant parameters is analysed. The next step is to study the coverage probability of mm-Wave cellular networks based on the Poisson cluster model of three-dimensional space.

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