Magnetic Moment Generation from non-minimal couplings in a scenario with Lorentz-Symmetry Violation

H. Belich\textsuperscript{a,e,f}, L.P. Colatto\textsuperscript{b,e}, T. Costa-Soares\textsuperscript{d,e}, J.A. Helayël-Neto\textsuperscript{c,e}, M.T.D. Orlando\textsuperscript{a,e}

\textsuperscript{a}Universidade Federal do Espírito Santo (UFES), Departamento de Física e Química, Av. Fernando Ferrari S/N, Vitória, ES, CEP 29060-900, Brasil
\textsuperscript{b}CEFET-Campos, Rua Dr. Siqueira 273, Campos dos Goytacazes, RJ, CEP 28030-130, Brasil
\textsuperscript{c}CBPF - Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, Rio de Janeiro, RJ, CEP 22290-180, Brasil
\textsuperscript{d}Universidade Federal de Juiz de Fora (UFJF), Colégio Técnico Universitário, Av. Bernardo Mascarenhas 1283, Juiz de Fora, MG, CEP 36080-001, Brasil
\textsuperscript{e}Grupo de Física Teórica José Leite Lopes, C.P. 91933, CEP 25685-970, Petrópolis, RJ, Brasil and
\textsuperscript{f}International Center for Condensed Matter Physics, Universidade de Brasília, Caixa postal 04667, 79109000, Brasília, DF, Brasil

This paper deals with situations that illustrate how the violation of Lorentz symmetry in the gauge sector may contribute to magnetic moment generation of massive neutral particles with spin-\(\frac{1}{2}\) and spin-1. The procedure we adopt here is based on Relativistic Quantum Mechanics. We work out the non-relativistic regime that follows from the wave equation corresponding to a certain particle coupled to an external electromagnetic field and a background that accounts for the Lorentz symmetry violation, and we read thereby the magnetic dipole moment operator for the particle under consideration. We keep track of the parameters that govern the non-minimal electromagnetic coupling and the breaking of Lorentz symmetry in the expressions we get for the magnetic moments in the different cases we contemplate. Our claim is that the tiny magnetic dipole moment of truly elementary neutral particles might signal Lorentz symmetry violation.

PACS numbers: 11.30.Cp, 12.60.Cn, 13.40.Em.

I. INTRODUCTION

Lorentz-violating theories have been extensively studied and used as an effective probe to test the limits of Lorentz covariance. Nowadays, these theories are encompassed in the framework of the Extended Standard Model (SME),\textsuperscript{1} as a possible extension of the minimal Standard Model of the fundamental interactions. Such kind of idea has driven much attention mainly after some authors argued the possibility of Lorentz and CPT spontaneous breaking in the context of string theory\textsuperscript{2}. The SME is the suitable framework to investigate properties of Lorentz violation on physical systems involving photons\textsuperscript{3, 4}, radiative corrections\textsuperscript{5}, fermions\textsuperscript{6}, neutrinos\textsuperscript{7}, topological defects\textsuperscript{8}, topological phases\textsuperscript{9}, cosmic rays\textsuperscript{10}, supersymmetry\textsuperscript{11}, particle decays\textsuperscript{12}, and other relevant aspects\textsuperscript{13, 14}. The SME has also been used as a framework to propose Lorentz symmetry violation\textsuperscript{15} and CPT\textsuperscript{16} probing experiments, which have amounted to the imposition of stringent bounds on the Lorentz-symmetry violating (LV) coefficients.

To take into account how this violation is implemented, in the fermion sector of the SME, for example, there are two CPT-odd terms,\textsuperscript{17} \(e_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi\), \(b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi\), where \(e_\mu, b_\mu\) are the LV backgrounds. The modified Dirac theory has already been examined in literature\textsuperscript{18}, and its non-relativistic limit, with special attention to the hydrogen spectrum\textsuperscript{19}, is realized. A similar study has also been developed for the case of a non-minimal coupling with the background, with new outcomes\textsuperscript{20}. Atomic and optical physics are other areas in which Lorentz symmetry violation has been intensively studied. Indeed, there are several works examining Lorentz violation in electromagnetic cavities and optical systems\textsuperscript{21}, which contributed to establish upper bounds on the LV coefficients.

\textsuperscript{1}Electronic address: belich@cce.ufes.br, lcolatto@cefetcampos.br, tcsoares@cbpf.br, helayel@cbpf.br, orlando@cce.ufes.br
Works by Belinfante, Case, Fronsdal and Schwinger in the fifties [22], and Yang and Lee in the early sixties [23], came to the general result that charged truly-elementary particles, coupled to the electromagnetic field, exhibit a gyromagnetic ratio given by the inverse of its corresponding spin. For spin-half particles, described by the Dirac field, this is a well-celebrated result. However, for higher spins, this general result is not correct. Indeed, theoretical evidences, based on the high-energy behavior of amplitudes and unitarity bounds [24] and on the dynamics of higher-spin particles propagating in electromagnetic backgrounds as dictated by string theories, indicate that, for charged genuinely elementary particles, the gyromagnetic ratio is always 2, no matter what the spin of the particle is. For charged spin-1 vector bosons, like the W-particles of the electroweak interactions, the value 2 is reconciled by means of the Yang-Mills interactions that yield a non-minimal (but renormalisable) coupling between the electromagnetic field-strength and the potentials associated to the charged bosons [25].

Charged spin-1 matter fields with self-interactions and topological terms have been studied to provide a possible microscopic description for the origin of Lorentz symmetry violation [26]. In our contribution, we reassess this issue in an environment dominated by a background vector that parametrises a tiny violation of Lorentz symmetry. We come to the conclusion that, also independently of the spin of the particle, the Lorentz-symmetry violating background vector may yield the same contribution to the magnetic moment and the Aharonov-Casher phase of the particle, even if it is electrically neutral; whenever a particular non-minimal coupling of the particle to the electromagnetic field and the Lorentz-breaking vector is considered. Everything goes as if Lorentz-symmetry violating background endows each elementary particle, even those spinless and electrically neutral, with a universal contribution to its magnetic moment and, consequently, to its Aharonov-Casher phase. In this context, we come back to the interesting question that concerns the magnetic properties of neutrinos [27]. Incidentally, from Neutrino Physics, more specifically, from the observation of non-zero neutrino masses, there emerges a striking evidence in favour of a Physics Beyond the Standard Model [28]. More recently, theoretical bounds for the neutrino mass and magnetic moment have been calculated that could be tested in the new experiments [30]. In our considerations, we propose that magnetic moment contributions to neutral and Majorana fermions can be obtained already at the tree-level approximation by means of non-minimal couplings. Besides the cases of the charged and neutral massive spin-1 particles and the Majorana fermions themselves, another contribution we shall present in this work refers to the way the \((k_F)_{\mu\nu\lambda\kappa}\) parameter [1] may contribute to the magnetic dipole moment of neutral spin-1 bosons. This result shall be used to give us a possible experimental bound on the magnetic moment of a neutral massive spin-1 particle.

One of our motivations to consider the magnetic moment contributions from Lorentz-symmetry breaking for massive neutral particles is partly based on the fact that this issue is always discussed in connection with (loop) radiative corrections in field-theoretic models. Our focus is to set up a discussion, at the level of Quantum Mechanics which, in field theory, would correspond to the generation of magnetic moment contributions (for neutral particles) at the tree-level. We understand that, in a scenario where the Lorentz covariance is violated, the symmetry breaking parameters may be responsible for the appearance of a magnetic moment for neutral particles at the tree-level. So, we adopt this scenario to discuss magnetic properties of neutral particles at the quantum-mechanical level. We stress that this is one of the main goals of our work. Indeed, the discussion on magnetic dipole moments for neutral particles is a question of relevance in connection with results coming from some Physics Beyond the Standard Model. Finally, we would like to point out that the paper of ref. [31] reports an interesting calculation of the photon magnetic moment in connection with (external) strong magnetic fields.

The organization of our paper is given as follows: in Section II, we briefly report on the attainment of the value 2 for the gyromagnetic ratio for massive charged spin-1 bosons non-minimally coupled to an electromagnetic field. In Section III, we introduce the Lorentz symmetry violation term and we discuss the magnetic moment of a neutral massive spin-1 boson non-minimally coupled to the background associated to the breaking of Lorentz symmetry and an external electromagnetic field. A \((k_F)\) contribution to the magnetic moment of spin-1 bosons is also reported in this Section. The discussion involving Majorana fermions is carried out in Section IV. Finally, we cast our Concluding Remarks in Section V.
II. MASSIVE AND CHARGED SPIN-1 FIELD.

We start off from the Lagrangian that describes a massive charged vector matter field, \( W_\mu \), minimally coupled to an external electromagnetic field as below:

\[
\mathcal{L} = -\frac{1}{2} (W^{\mu\nu})^* W_{\mu\nu} + m^2 (W^\mu)^* W_\mu,
\]

where \( W^{\mu\nu} = D^\mu W_\nu - D^\nu W_\mu \), \( D_\mu = \partial_\mu + ieA_\mu \). The minimal electromagnetic coupling yields a wrong (\( g = 1 \)) gyromagnetic ratio: this may be found by considering the Pauli-type equation for the charged vector boson in the presence of a magnetic field, namely,

\[
\frac{1}{2m} \left( \vec{p} - e\vec{A} \right)^2 W_i - \frac{e}{2m} \vec{B} \cdot \vec{S}_{ij} W_j = E_{nr} W_i,
\]

where \( E_{nr} \) means the non-relativistic energy, and \( \vec{S}_{ij} \) is the spin matrix:

\[
\vec{\mu} = \frac{e}{2m} \vec{S},
\]

\((S_k)_{ij} = -i\varepsilon_{kij}\). To by-pass the conflict of the gyromagnetic ratio, we have to introduce a renormalisable non-minimal electromagnetic coupling [25]. The motivation for this new interaction term comes from the Electroweak Theory: its \( SU(2) \times U(1) \) gauge symmetry dictates the coupling between \( A_\mu \) and the charged gauge bosons as given below, after the spontaneous breaking of \( SU(2) \) takes place:

\[
ie F_{\mu\nu} W^\mu W^\nu.
\]

This indeed cancels high-energy divergences and corrects the gyromagnetic factor to the right value \( g = 2 \), as it should be. So, taking into account this new interaction, one may consider the vector field equation that follows:

\[
D_\mu W^{\mu\nu} + m^2 W_\nu + ieW_\mu F^{\mu\nu} = 0.
\]

It directly implies the subsidiary condition \( D_\mu W^\mu = 0 \). In the non-relativistic limit, this condition yields:

\[
W^0 \approx \frac{\vec{p} - e\vec{A}}{m} \cdot \vec{W} - \frac{e}{m} \vec{B} \cdot \vec{W},
\]

which shows that the time component of the \( W \)-field is of the order \( \left( \frac{\vec{v}}{c} \right) \) of its space components. It is worthy to remark that, by considering the time component of the field equation above, one exactly arrives at the same relation that follows from the subsidiary condition: we then go straight to consider the space components of the field equation for \( W^\mu \). By properly carrying out the non-relativistic approximation, after some algebraic steps, we show that the gyromagnetic ratio comes out with its correct value equal to 2:

\[
\frac{1}{2m} \left( \vec{p} - e\vec{A} \right)^2 W_i - \frac{e}{2m} \vec{B} \cdot \vec{S}_{ij} W_j = EW_i.
\]

In the course of these calculations, it becomes clear that the net effect of the non-minimal coupling, inherited from the non-Abelian \( SU(2) \) symmetry of the Electroweak Theory, is to add up the piece which was missing to yield the right value for \( g \). We now turn into the discussion of the gyromagnetic ratio in situations where there occurs violation of Lorentz symmetry. This is motivated by the fact that one may use magnetic moment measurements of higher spin particles to get new bounds on the Lorentz symmetry violation parameter.

III. A LORENTZ-SYMMETRY VIOLATING BACKGROUND PARAMETRISED BY A 4-VECTOR, \( v_\mu \)

Assuming that the background vector couples to the electromagnetic field, the covariant derivative operator can be modified to introduce a non-minimal coupling according to the expression below:

\[
D_\mu = \partial_\mu + ieA_\mu + igu^\alpha \tilde{F}_{\mu\alpha},
\]
where $\tilde{F}_{\mu\alpha}$ stands for the dual of the electromagnetic field strength. It is worthy to mention that Lorentz-symmetry violation does not conflict with gauge invariance. Gauge symmetry is not violated by the action term $\varepsilon_{\mu\nu\kappa\lambda} v^\mu A^\nu F^{\kappa\lambda}$ introduced by Carrol, Field and Jackiw. So, if we are to consider the dynamics of charged particles under the action of the electromagnetic field, a gauge covariant derivative must be adopted. In our proposal, we go a step further: we extended the usual covariant derivative by adding up a term that implements a (non-minimal) coupling of the particle to the external $A^\mu$-field and, contemporarily, to the background vector, $v^\mu$. The term $v^\mu \tilde{F}_{\mu\alpha}$ is clearly gauge invariant, so it does not harm the status of $D_\mu$ as a covariant derivative. This means that a local phase transformation, $e^{i\alpha}$, performed on the charged matter fields acts upon $D_\mu$ according to the usual gauge transformation of a genuine covariant derivative: $D'_\mu = e^{i\alpha} D_\mu e^{-i\alpha}$.

In $(1+2)D$, due to the fact that the Levi-Civita tensor is a rank-$3$ tensor, the dual of the field strength is a vector; so, we can define a covariant derivative as below:

$$D_\mu = \partial_\mu + ieA_\mu + ig\tilde{F}_\mu.$$  \hfill (9)

A direct consequence of the non-minimal coupling introduced in $D_\mu$ is that scalar particles display a non-trivial magnetic moment. Another contribution of this covariant $(1+2)D$ derivative is the generation of electrically charged vortices in the Abelian Higgs Model.

Based on the result referred to above, and considering that the $v^\mu-$ background may, in some special case, lead to an effective $(1+2)D$ model $(v^\mu = (v^0, v^1, v^2, 0))$, we then introduce the term $igv^\mu \tilde{F}_{\mu\nu}$ as the $4-$dimensional counterpart (whenever there is Lorentz-symmetry breaking) of the non-minimal term studied in $29$. As a consequence, we may investigate electrically charged vortices in the $4D$ Abelian Higgs model and the anomalous magnetic moment generation of spin-$\frac{1}{2}$ particles also in 4-dimensional space-times.

The effect of the non-minimal interaction term above on a charged vector field, as considered in the previous Section (but, now, with the covariant derivative modified as above), is to endow the particle associated to the $W$-field with a universal magnetic moment given by

$$\bar{\mu} = \frac{1}{2} g v^\mu,$$  \hfill (10)

as it is the case for the scalar and the spin-$\frac{1}{2}$, according to the results reported in the work of reference $9$. This is a very peculiar outcome. Everything happens as if the presence of the background modifies the structure of the particle and endows it with the universal magnetic moment given above. According to the previous studies carried out by Colladay and Kostelecký, in the works of reference $1$, different particle species may have different independent Lorentz-breaking parameters. Our non-minimal coupling present in $D_\mu$ is taken the same for all charged particle species, for it accompanies the minimal coupling term in the covariant derivative defined above. In the same way the minimal coupling is universal, our term $gv^\mu \tilde{F}_{\mu\nu}$ follows the same pattern. What is highlighted here is that its net effect, no matter which spin the particle possesses, is to yield the same value for $\bar{\mu}$, as given above, in eq. $10$.

This is also the situation in the case of neutral vector particles, once the non-minimal coupling above is switched on. Indeed, to explicitly see this result, we take the simpler case where a non-charged ($e = 0$) massive spin-1 particle is non-minimally coupled to the background and to the electromagnetic field as described in the wave equation given below:

$$
\left( \partial_\mu + igv^\kappa \tilde{F}_{\mu\kappa} \right) Z^{\mu\nu} + m^2 Z^{\nu} = 0,
\right)
\hfill (11)

where $Z_\mu$ is the wave function of the spin-1 particle. The subsidiary condition in this case takes the form:

$$
\partial_\mu Z^\mu + igv^\kappa \tilde{F}_{\mu\kappa} Z^\mu = 0,
\right)
\hfill (12)

where we can notice that, differently from the Proca case, it sets up non-trivial relations among the $Z$-field components. (Incidentally, by introducing an external electric field, we can get how the Aharonov-Casher (AC) phase looks like.) In the non-relativistic limit, the subsidiary condition yields:

$$
Z^0 \approx \frac{\bar{p}}{m} \cdot \bar{Z} - \frac{1}{m} (gv \times E) \cdot \bar{Z},
\hfill (13)
$$
where we point out the presence of a sort of AC term. By replacing the expression above for $Z^0$ in the space components of the $Z^\mu$-equations, and by properly keeping the terms that survive the non-relativistic approximation, we get:

$$
\frac{1}{2m} \left[ \vec{p} + \frac{g}{2} \left( \vec{v} \times \vec{E} \right) \right]^2 Z_i = E_{nr} Z_i.
$$

(14)

This result suggests that the quantity $\frac{1}{2} g \vec{v}$ could be interpreted as the magnetic moment acquired by the neutral particle due to the presence of the background vector, $\vec{v}$. We can observe that the immediate consequence is the appearance of a universal AC phase for different spins, by virtue of the breaking of Lorentz symmetry under the particle point of view. Moreover, in the presence of an external electric field, $\vec{E}$, the wave function of every particle, charged or neutral, with or without spin, acquires a non-trivial phase given by $\frac{g}{2} \left( \vec{v} \times \vec{E} \right)$. To show that the quantity $\frac{1}{2} g \vec{v}$ is actually the magnetic moment, we consider our neutral vector particle under the action of an external magnetic field, $\vec{B}$. To do that, we take Eq. (11) and we switch on a constant magnetic field given by

$$
F_{ij} = -\varepsilon_{ijk} B_k.
$$

(15)

From the subsidiary condition, we get that

$$
Z^0 \approx \frac{\vec{p}}{m} \cdot \vec{Z} - \frac{g v^0}{m} \vec{B} \cdot \vec{Z},
$$

(16)

and, by means of this relation and the space components of Eq. (11) taken in the non-relativistic limit, the resulting wave equation for the space components, $Z_i$, reads as below:

$$
\frac{\vec{p}^2}{2m} Z_i + \frac{1}{2} g \vec{v} \cdot \vec{B} Z_i = E_{nr} Z_i,
$$

(17)

where it is easy to recognise the gyromagnetic ratio and to see that it gets the same expression as in the scalar and spin-$\frac{1}{2}$ cases, as we had already mentioned.

Before closing this Section and getting to the discussion on the Majorana fermions, we believe it is worthwhile to mention another result valid for the case of the neutral spin-1 bosons, namely, the contribution of the $k_F$—parameter [1] to the magnetic dipole moment of this sort of particle, previously described by $Z^\mu$(present section).

The $k_F$—violating term modifies the $Z^\mu$—field equations as given below:

$$
D_\mu Z^{\mu \nu} - \frac{1}{2} k_F^{\nu \lambda \rho} D_\kappa Z_{\lambda \rho} + m^2 Z^\nu = 0,
$$

(18)

where

$$
D_\mu = \partial_\mu + ig v^\alpha F_{\mu \alpha}.
$$

(19)

Following along the same steps as we have shown previously (Section II), we place the spin-1 particle in an external magnetostatic field and consider the non-relativistic regime of the corresponding field equation to read off its corresponding magnetic moment contribution. We calculate the subsidiary condition out of the equation above and, by considering the space components of these field equations, where we insert the expression for $Z^0$ coming from the subsidiary condition, we get, after some algebraic manipulations and the use of the conditions for the non-relativistic regime, that the $k_F$—parameter induces the correction given by

$$
\vec{B} \cdot \vec{\mu}_{ij} Z_j,
$$

(20)

where the n-th component of $\vec{\mu}_{ij}$ is given by

$$
(\mu_n)_{ij} = \frac{1}{2} g v^0 (k_F)_{nij0}.
$$

(21)

With this result, in our Concluding Remarks, we shall be able to present a bound on the $Z^0$—s magnetic moment. For that, we propose a discussion on the magnetic moment of Majorana-type neutrinos in the sequel, from which we will be able to get information on the product of parameters $g v^0$. As for $k_F$, we shall be adopting a result presented in the work of ref. [22], so that an estimation of the magnetic moment given in the expression above can be obtained.
IV. THE CASE OF MAJORANA FERMIONS

Neutrino magnetic dipole moments in the Standard Model are calculated as radiative corrections and the tiny values obtained from loop calculations may be used as good precision tests. In our case, we adopt the same procedure followed to study the case of (massive) neutral vector bosons: we assume a tiny deviation from the situation of Lorentz symmetry and we non-minimally couple the (neutral) Majorana fermions to an external electromagnetic field and the background vector that parametrizes the breaking of the relativistic symmetry. To implement this scenario, we set up the Dirac equation below for a Majorana spinor:

\[ i\gamma^\mu (\partial_\mu + igv^\nu F_{\mu\nu}\gamma_5)\Psi - m\Psi = 0. \] (22)

The introduction of the chirality matrix in the non-minimal electromagnetic coupling is dictated by the Majorana character of the fermion we consider. We adopt to work with the Majorana fermion by writing its wave function, \( \Psi \), in the Majorana representation for the \( \gamma \)-matrices:

\[ \gamma^0 = \begin{pmatrix} \sigma_y & 0 \\ 0 & -\sigma_y \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_x & 0 \\ 0 & -i\sigma_x \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma_z & 0 \\ 0 & -i\sigma_z \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & i1 \\ i1 & 0 \end{pmatrix}, \]

\[ \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & -i1 \\ i1 & 0 \end{pmatrix}. \] (23)

The charge conjugation matrix is \( C = -\gamma^0 \); thus, in this representation, a Majorana spinor \( \Psi^c = C\Psi^\dagger = \Psi \) exhibits 4 real components. We should remark that parity may be preserved despite the appearance of the chirality matrix in the action term \( \bar{\Psi}\gamma^\mu\gamma_5\Psi v^\nu F_{\mu\nu} \); the particular property of \( v^\mu \) under parity (vector or a pseudo-vector) may be suitably chosen so as to make this term parity-preserving.

As first step, we must probe the neutral particle by subjecting it to an external magnetic field, \( \vec{B}_{0\text{t}} = \vec{B} \). This shall reveal the (eventual) contributions of the Lorentz-symmetry violating parameters, \( v_0 \) and \( \vec{v} \), to the magnetic moment of the neutral fermion. We start of from the coupled Dirac’s Eq. (22),

\[ \left( \gamma^0 E - \gamma^\mu\vec{p} - m - g\vec{v} \cdot \vec{B}\gamma^0\gamma_5 + g\vec{v}_0 \vec{B} \cdot \gamma\gamma_5 \right) \Psi = 0 \] (24)

The coupled Dirac’s equation above splits up into 2 equations for the 2-component spinors, \( \xi \) and \( \chi \). They read as follows:

\[ M\xi + N\chi = 0, \]
\[ P\xi - Q\chi = 0 \] (25)

where

\[ M \equiv m - E\sigma_y + ip_x\sigma_x + ip_y\sigma_z + g\nu^0B_z; \quad N \equiv ip_z - igv^\nu \cdot \vec{B}\sigma_y - g\nu^0B_x\sigma_x - g\nu^0B_y\sigma_z, \]
\[ P \equiv -ip_z + igv^\nu \cdot \vec{B}\sigma_y + g\nu^0B_x\sigma_x + g\nu^0B_y\sigma_z; \quad Q \equiv m - E\sigma_y + ip_x\sigma_x + ip_y\sigma_z + g\nu^0B_z; \] (26)

\[ \chi = Q^{-1}P\xi; \]
\[ (M + NQ^{-1}P)\chi = 0. \] (27)

Using the quaternionic unities \[33\], we cast the operators \( M, N, P \) an \( Q \) in the form below:

\[ M = (m + g\nu^0B_z) + p_xI + iEJ + p_yK; \]
\[ N = ip_z + ig\nu^0B_xI - g\vec{v} \cdot \vec{B}J + ig\nu^0B_yK, \]
\[ P = -ip_z - ig\nu^0B_xI + g\vec{v} \cdot \vec{B}J - ig\nu^0B_yK; \]
\[ Q = (m + g\nu^0B_z) + p_xI - iEJ + p_yK. \] (28)
The $(M + NQ^{-1}P)$-operator, that yields a Pauli-type equation, is worked out, but, by analyzing its explicit form, the magnetic moment cannot be properly identified. This shows us that the Majorana representation is not suitable for the sake of taking the non-relativistic approximation. We better go over into the (usual) Dirac’s representation and we also propose a more general situation, where we try to compare the competitive effect of two non-minimal couplings that may be contemporarily present in the Dirac’s equation for a Majorana fermion; both the couplings are collected in the expression below:

\[ i\gamma^\mu (\partial_\mu + igv^\nu \tilde{F}_{\mu\nu}\gamma_5)\Psi + \tilde{g}F_{\mu\nu}\Sigma^\mu\nu\gamma_5\Psi - m\Psi = 0. \] (29)

So, from this complete equation, we follow the necessary steps to work out the non-relativistic approximation and to arrive at a Pauli-type equation for the $\xi$, and to arrive at a Pauli-type equation for the $\xi$-component. By properly treating the terms that dominate in the non-relativistic limit and taking care of the relations imposed by the Clifford algebra, we find out the non-relativistic equation for $\xi$, which turns out to be:

\[ i\hbar \frac{\partial}{\partial t} \xi = \left\{ \frac{1}{2m} \left( \vec{\nu} + 2 \tilde{g} \vec{\sigma} \times \vec{B} \right)^2 + g v^0 \vec{\sigma} \cdot \vec{B} - g \vec{E} \cdot (\vec{\nu} \times \vec{\sigma}) - \tilde{g} \vec{\sigma} \cdot \vec{E} \right\} \xi. \] (30)

The expression above opens up a number of interesting remarks on the (non-minimal) electromagnetic effects of the spin of a neutral self-conjugated fermion. We identify the interaction term that leads to the magnetic dipole moment as being given by $g v^0 \vec{\sigma} \cdot \vec{B}$; this then shows that, instead of the space component, $v^0$, it is now the time component, $v^0$, the responsible for the magnetic moment generation, and the Pauli-type coupling (the one given by $\tilde{g}$) does not contribute to the magnetic interaction as in the ordinary case. Instead, it induces a coupling to the electric field and a new type of phase ($\Phi$) in the fermion wave function, given by

\[ \Phi = \frac{2 \tilde{g}}{g v^0} \int d\vec{l} \cdot (\vec{\mu} \times \vec{B}). \] (31)

V. CONCLUDING REMARKS

The main effort in our work has been to show how, in an environment where Lorentz symmetry is violated, truly elementary neutral particles may show up magnetic properties only due to their spin, once non-minimal couplings to the electromagnetic field are allowed. The background vector responsible for the Lorentz-symmetry violation couples to both the electromagnetic field and the particle itself and then the electromagnetic properties of the spin are revealed through Aharonov-Casher and Pauli-type couplings of the magnetic dipole moment of the particle. For charged spin-0, spin-$\frac{1}{2}$, spin-1 particles and neutral vector bosons, we have seen that there appears a universal magnetic dipole moment for each particle, $\vec{\mu} = \frac{1}{2}g \vec{v}$, as a result of the presence of the background vector. Nevertheless, other contributions for the magnetic moment may be derived which depend on the type of the particle, as discussed in the treatment of the Majorana fermion, for which the non-minimal coupling with the presence of the chirality matrix produced a new sort of phase which involves the magnetic field. This means that, for neutral particles like the neutrinos, the tiny magnetic dipole moments they have (less than $10^{-10}$ Bohr magnetons), which in the framework of the Electroweak Theory are understood and computed as an effect of the radiative corrections, may also be attributed to possible effects of an eventual violation of Lorentz symmetry. What we conclude is that purely electromagnetic effects of the spin may emerge if neutral particles interact with an external electromagnetic field via a background that realizes the tiny breaking of Lorentz symmetry. In the present paper, this has been done for a background vector; however, from our results, we can safely state that the same conclusions hold through if the Lorentz-symmetry breaking background is of a tensor nature.

With the result on the magnetic moment for Majorana-type fermions presented in the previous Section, and the experimental bounds on the neutrino magnetic moment [34], we can set a bound on the product $g v^0$, namely, $g v^0 < 0.9 \times 10^{-10} \mu_B$. By considering the results of the work of ref. [35], and a recent paper by Klinkhamer and Shereck [32], it is reasonable to take the bound $|k_F|_{\eta_i g_\eta} < 2 \times 10^{-7}$, so that we can get an estimation on the magnetic moment for $Z^0$: $\mu(Z^0) < 1.65 \times 10^{-14} \mu_N$, where $\mu_N$ stands for the nuclear magneton. In possess of
the results presented in this work, we are now concentrating our efforts to systematically get bounds on Lorentz-
symmetry breaking parameters from our investigation of their influence on the calculation of gyromagnetic ratios
and magnetic moments for different particle species, with special interest on the sector of neutral fundamental
fermions and vector bosons. These results shall be soon reported in a forthcoming paper.

Acknowledgments

J. Moraes and R. Turcati are kindly acknowledged for long discussion. HB, TCS, JAHN and MDTO acknowledge CNPq for the invaluable financial support.

[1] D. Colladay and V. A. Kostelecký, Phys. Rev. D 55, 6760 (1997); D. Colladay and V. A. Kostelecký, Phys. Rev. D 58, 116002 (1998).
[2] V. A. Kostelecký and S. Samuel, Phys. Rev. Lett. 63, 224 (1989); Phys. Rev. Lett. 66, 1811 (1991); Phys. Rev. D 39, 683 (1989); Phys. Rev. D 40, 1886 (1989), V. A. Kostelecky and R. Potting, Nucl. Phys. B 359, 545 (1991); Phys. Lett. B 381, 89 (1996); V. A. Kostelecky and R. Potting, Phys. Rev. D 51, 3923 (1995)
[3] S.M. Carroll, G.B. Field and R. Jackiw, Phys. Rev. D 41, 1231 (1990); C. Adam and F. R. Klinkhamer, Nucl. Phys. B 607, 247 (2001); Nucl. Phys. B 657, 214 (2003); Phys. Lett. B 513, 245 (2001); C. Kaufhold and F. R. Klinkhamer, Nucl. Phys. B 734, 1 (2006); R. Montemayor and L.F. Urrutia, Phys. Rev. D 72, 045018 (2005); X. Xue and J. Wu, Eur. Phys. J. C 48, 257 (2006); A.A. Andrianov and R. Soldati, Phys. Rev. D 51, 5961 (1995); Phys. Lett. B 435, 449 (1998); A.A. Andrianov, R. Soldati and L. Sorbo, Phys. Rev. D 59, 025002 (1999); Q. G. Bailey and V. Alan Kostelecky, Phys. Rev. D 70, 076006 (2004); M. Frank and I. Turan, Phys. Rev. D 74, 033016 (2006); B. Altschul, Phys. Rev. D 71, 085005 (2005); R. Lehnert and R. Potting, Phys. Rev. Lett. 93, 110402 (2004); R. Lehnert and R. Potting, Phys. Rev. D 70, 125010 (2004); B. Altschul, Phys. Rev. Lett. 98, 041603 (2007); Phys.Rev. D 72, 085003 (2005); Phys.Rev. D 73, 036005 (2006); Phys. Rev. D 73, 045004 (2006).
[4] H. Belich M. M. Ferreira Jr, J.A. Helayel-Neto, M. T. D. Orlando, Phys. Rev. D 67, 125011 (2003); -ibid, Phys. Rev. D 69, 109903 (E) (2004); H. Belich, M. M. Ferreira Jr, J.A. Helayel-Neto, M. T. D. Orlando, Phys. Rev. D 68, 025005 (2003); A. P. Baêta Scarpelli, H. Belich, J. L. Boldo, J.A. Helayel-Neto, Phys. Rev. D 67, 085021 (2003); H. Belich et al., Nucl. Phys. B Suppl. 127, 105 (2004); H. Belich, M. M. Ferreira Jr and J. A. Helayel-Neto, Eur. Phys. J. C 38, 511 (2005); H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayêl-Neto, Eur. Phys. J. C 42, 127 (2005), Cantcheff M. B Eur. Phys. J. C 46, 247 - 254, (2006), H. Belich, et al. REV. BRAS. ENS. FIS. 29, 57 (2007).
[5] R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. 82, 3572 (1999); J. M. Chung and B. K. Chung Phys. Rev. D 63, 105015 (2001); J.M. Chung, Phys.Rev. D 60, 127901 (1999); G. Bonneau, Nucl.Phys. B 595, 398 (2001); M. Perez-Victoria, Phys. Rev. Lett. 83, 2518 (1999); M. Perez-Victoria, JHEP 0104, 032 (2001); O.A. Battistel and G. Dallabona, Nucl. Phys. B 610, 316 (2001); O.A. Battistel and G. Dallabona, J. Phys. G 28, L23 (2002); J. Phys. G 27, L53 (2002); A. P. Baêta Scarpelli, M. Sampaio, M.C. Nemes, and B. Hiller, Phys. Rev. D 64, 046013 (2001); F.A. Brito, T. Mariz, J.R. Nascimento, E. Passos, R.F. Ribeiro, JHEP 0510 (2005) 019.
[6] B. Altschul, Phys. Rev. D 70, 056005 (2004); G. M. Shore, Nucl. Phys. B 717, 86 (2005); D. Colladay and V. A. Kostelecky, Phys. Lett. B 511, 209 (2001); M. M. Ferreira Jr, Phys. Rev. D 70, 045013 (2004); M. M. Ferreira Jr, Phys. Rev. D 71, 045003 (2005); M. M. Ferreira Jr and M. S. Tavares, Int. J. Mod. Phys. A 22, 1685 (2007); J. Alfaro, A.A. Andrianov, M. Cambiaso, P. Giacconi, R. Soldati, Phys. Lett. B 639 (2006) 586-590.
[7] V. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, Phys. Rev. Lett. 85, 5055 (2000); V. A. Kostelecky and M. Mewes, Phys. Rev. D 69, 016005 (2004); Phys. Rev. D 70, 031902 (R) (2004); Phys. Rev. D 70, 076002 (2004); T. Katori, A. Kostelecky, R. Tayloe, Phys.Rev. D 74, 105009 (2006); R. Brustein, D. Eichler, S. Foffa, Phys.Rev. D 65, 105006 (2002); Y. Grossman, C. Kilic, J. Thaler, and D.G. E. Walker, Phys. Rev. D 72, 125001 (2005); L. B. Auerbach et al., Phys. Rev. D 72, 076004 (2005); D. Hooper, D. Morgan, and E. Winstanley, Phys. Rev. D 72, 065009 (2005).
[8] M. Lubo, Phys. Rev. D 71, 047701 (2005); M.N. Barreto, D. Bazeia, and R. Meneses, Phys. Rev. D 73, 065015 (2006).
[9] H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayêl-Neto, M.T. D. Orlando, Phys. Lett. B 639, 678 (2006); H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayêl-Neto, Eur. Phys. J. C 41, 421 (2005).
References

[10] O. Gagnon and G. D. Moore, Phys. Rev. D 70, 065002 (2004); J.W. Moffat, Int. J. Mod. Phys. D 12 1279 (2003); F. W. Stecker and S.T. Scully, Astropart. Phys. 23, 203 (2005); F. W. Stecker and S.L. Glashow, Astropart. Phys. 16, 97 (2001).

[11] P. A. Bolokhov, S. G. Nibbelink, M.Pospelov, Phys. Rev. D 72, 015013 (2005); H. Belich et al., Phys. Rev. D 68, 065030 (2003); S. G. Nibbelink and M. Pospelov, Phys. Rev. Lett. 94, 081601 (2005), H. Belich et al., Phys. Lett. A 370, 126 - 130 (2007).

[12] E. O. Iltan, Eur. Phys. J. C 40, 269 (2005); Mod. Phys. Lett. A19, 327 (2004); JHEP 0306 (2003) 016.

[13] V.A. Kostelecky and R. Lehnert, Phys. Rev. D 63, 065008 (2001); R. Lehnert, Phys. Rev. D 68, 085003 (2003).

[14] H. Belich, M. A. De Andrade, M. A. Santos, Mod.Phys.Lett. A 20, 2305 (2005)H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayèl-Neto and M.T.D. Orlando, Int. J. Mod. Phys. A 21, 2415 (2006), A. P. Baeta Scarpelli and J. A. Helayel-Neto, Phys. Rev. D 73, 105020 (2006); N.M. Barraz, Jr., J.M. Fonseca, W.A. Moura-Melo, and J.A. Helayel-Neto, Phys. Rev. D76, 027701 (2007).

[15] V.A. Kostelecky and C. D. Lane, J. Math. Phys. 40, 6245 (1999); R. Lehnert, M.M. Ferreira Jr and F. M. O. Moucherek, Int. J. Mod. Phys. A 21, 6211 (2006); Nucl. Phys. A 790, 635 (2007); S. Chen, B. Wang, and R. Su, Class. Quant.Grav. 23, 7581,(2006);O. G. Kharlanov and V. Ch. Zhukovsky, J. Math. Phys. 48, 092302 (2007), M. Frank, I. Turan, I. Yurdusen, hep-th 07094276.

[16] H. Belich, T. Costa-Soares, M.M. Ferreira Jr., J. A. Helayèl-Neto, and F. M. O. Moucherek, Phys. Rev. D 74, 065009 (2006).

[17] H. Muller, C. Braxmaier, S. Herrmann, and A. Peters, Phys. Rev. D 67, 056006 (2003); H. Müller, A. Saenz, A. Peters, and C. Lämmerzahl, Phys. Rev. D 68, 116006 (2003); H. Müller, Phys. Rev. D 71, 045004 (2005); N. Russell, Phys. Scripta 72, C38 (2005).

[18] D. F. Phillips, M. A. Humphrey, E. M. Mattison, R. E. Stoner, R. F C. Vessot, R. L. Walsworth, Phys.Rev. D63, 111101 (2001); D. Bear, R.E. Stoner, R.L. Walsworth, V. Alan Kostelecky, Charles D. Lane, Phys.Rev.Lett. 85, 5038 (2000); Erratum-ibid. 89 (2002) 209902; M.A. Humphrey, D.F. Phillips, R.L. Walsworth, Phys. Rev. A 68, 063807 (2003).

[19] F. J. Belinfante, Phys. Rev. 92, 997 (1953); K. M. Kase Phys. Rev. 94, 1442 (Z6) (1954); C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958); J. Schwinger, Particles, Sources, and Fields (Addison-Wesley, Reading, MA, 1970).

[20] T. D. Lee and C. N. Yang, Phys. Rev. 128, 2, 885 - 898 (1962).

[21] S. Weinberg, Lectures on Elementary Particles and Quantum Field Theory, Proceedings of the Summer Institute, Brandeis University, 1970, edited by S. Deser (MIT Press, Cambridge, MA, 1970), Vol. I.

[22] S. Ferrara, M. Porrati, V. Telegdi, Phys.Rev. D46 3529-3537,1992.

[23] P. Colato, A.L.A. Penna, W.C. Santos, Eur.Phys.Jour. C36 79-87, 2004 .

[24] J. Schechter, J.W.F. Valle Phys.Rev. D24:1883-1889,1981, Erratum-ibid. D25:283,1982.

[25] M. Trodden and S. M. Carroll, [arXiv:astro-ph/0401547].

[26] S. K. Paul, and A. Khare, Phys. Lett. B174, 420 (1986).

[27] N.F. Bell, M. Gorchtein, M.J. Ramsay-Musolf, P. Vogel, P. Wang, Phys.Lett. B642:377-383,2006.;N.F. Bell, Int. J. Mod. Phys A22 (2007):4891; N.F. Bell, V. Cirigliano, M.J. Ramsay-Musolf, P. Vogel and M.B.Wise, Phys. Rev. Lett. 95 (2005):151802.

[28] S. Villalba-Chávez, H. Pérez-Rojas, hep.th/0609008; S. Villalba-Chávez, H. Pérez-Rojas, hep.th/0604059.

[29] F. R. Klinkhamer and M. Shereck arXiv:0809.321.

[30] A. J. Hanson, Visualizing Quaternions, Elsevier-Morgan Kaufman Publishers, San Francisco, 2006.

[31] W. -M. Yao et al., Review of Particle Physics, Journal of Physics G 33 (2006) 1.

[32] S. Reinhardt et al., Nature Phys. 3, 861 (2007).