Abstract

Motivated by Feynman’s 1983 paper on the simulation of physics by computers, we present a general approach to the description of quantum experiments which uses quantum bit registers to represent the spatio-temporal changes occurring in apparatus-systems during the course of such experiments. To illustrate our ideas, we discuss the Stern-Gerlach experiment, Wollaston prisms, beam splitters, Mach-Zender interferometers, von Neumann (PVM) tests, the more general POVM formalism, and a variety of modern quantum experiments, such as two-particle interferometry and the EPR scenario.

1 Introduction

It seems reasonable to state that at present, only about half of the laws of physics are understood; whilst we know very well how to predict the outcome probabilities of given quantum experiments, we have no idea as to why we find ourselves doing those experiments in the first place. In other words, we do not have a proper theory of the universe considered as a fully autonomous, self-referential quantum dynamical system.

In 1983, Feynman wrote a paper on the simulation of physics with computers [1]. Towards the end of the paper, he wrote:

“...we have an illusion that we can do any experiment that we want. We all, however, come from the same universe, have evolved with it, and don’t really have any “real” freedom. For we obey certain laws and have come from a certain past.”
A number of points arise in connection with Feynman’s paper, and with this quotation in particular, which have motivated the work presented in the present paper. First, Feynman’s paper explores the notion that the laws of physics, if not the universe as a whole, might be describable in terms of computation. Because of quantum mechanics, however, Feynman recognized that the computation involved could not be classical but has to involve what is now known as quantum computing. Secondly, and this comes as a surprise considering the pragmatic nature of Feynman’s lifetime contributions to quantum theory, Feynman seems to be advocating the study of endophysics as opposed to exophysics. Briefly, endophysics is physics described from within, whilst exophysics is physics described from the point of view of external observers looking into systems under observation. The latter approach to physics has been very successful ever since the time of Newton whilst the former remains a deep theoretical challenge.

Given the success of exophysics, it seems at first sight unreasonable to consider replacing it with an intractable alternative. However, there are no signs that quantum mechanics has any natural boundaries. On the contrary, there is increasing evidence for the applicability of quantum principles at scales much greater than the atomic. The hypothetical line between the classical and quantum worlds has been called the “Heisenberg cut”. It does not seem to exist. Sooner or later, we shall be forced to understand the dynamics of observers in a more fundamental way, not just the dynamics of the systems that those observers are looking at.

Given that we have been motivated to think of how observers and their measuring apparatus evolve dynamically, we are faced with the challenge of finding a dynamical description for them on a par with the quantum description we have for systems under observation, such as the Schrödinger equation. We are a long way from having anything like that, and it is possible that such a goal might never be achieved. However, ruling out such a possibility as a matter of principle seems a recipe for complacency, and besides, could be a serious mistake.

In this paper, therefore, we attempt to bring into a quantum framework a greater role for the physical apparatus involved in quantum experiments than is usual. It will be evident, after reading our quantum register description of experiments, what the limitations of our approach are. We present no theory as to why experiments are done, but a start is made to bring into the discussion some aspects involved in real quantum physics experiments which generally have not been modelled in conventional approaches. Quantum register physics has the potential to describe situations where not only might physical apparatus change in time, but also those situations where
many independent or coupled quantum experiments are being conducted simultaneously.

What is presented in this paper provides a consistent quantum computational framework for the description of simple and complex experiments, such as the Stern-Gerlach experiment, the Mach-Zender interferometer, experiments conventionally requiring a POVM description, and EPR-type experiments.

The plan of this paper is as follows. First we review the notions of quantum bits and quantum registers. Then we start to discuss more and more complex experiments from the point of view of quantum registers. Our aim here is to show how real physics experiments can be successfully modelled in a novel way which holds some promise of bringing physical apparatus into quantum discussions. Finally, we shall discuss some of the conceptual ramifications of quantum register physics.

## 2 Quantum bits

A classical bit $B$ is a system with two possible states: $|1\rangle \equiv \text{‘Yes’}= ‘\text{True’}= ‘\text{occupied’}$ and $|0\rangle \equiv \text{‘No’}= ‘\text{False’}= ‘\text{unoccupied’}$. We may represent these states by the two-dimensional real column vectors

$$|1\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |0\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(1)

and their duals $|1\rangle$, $|0\rangle$ by the row vectors

$$|1\rangle \equiv \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad |0\rangle \equiv \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

(2)

Then we have the orthonormality condition $\langle i|j \rangle = \delta_{ij}$ for any $i, j$ in the set $\{0, 1\}$.

Given that $|\psi\rangle$ is a classical bit state, but with no other information, we may write

$$|\psi\rangle = \alpha|1\rangle + \beta|0\rangle,$$

(3)

where $\alpha, \beta$ are in the set $\{0, 1\}$ and

$$\langle \psi|\psi \rangle = |\alpha|^2 + |\beta|^2 = 1.$$  

(4)

For a classical bit, there are only two possible sets of values for $(\alpha, \beta)$, viz., $(1, 0)$ or else $(0, 1)$.  

3
We turn a classical bit into a qubit (quantum bit) by regarding the classical bit states $|0\rangle$ and $|1\rangle$ as the basis vectors for a 2-dimensional Hilbert space $\mathcal{Q}$. These basis vectors will be referred to as the computational basis.

A general normalized qubit state is given by
\[ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \] (5)
where now $\alpha$, $\beta$ are allowed to be complex and satisfy the normalization condition
\[ (\psi|\psi) = |\alpha|^2 + |\beta|^2 = 1. \]

Here $|0 \rangle$ and $|1\rangle$ are the vectors dual to $|0\rangle$ and $|1\rangle$ respectively and form the computational basis for the dual qubit Hilbert space $\mathcal{Q}^*$.  

3 Qubit operators

For a given qubit $\mathcal{Q}$, we define the following operators:

i) the projection operators:
\[ P^0 \equiv |0\rangle(0|, \quad P^1 \equiv |1\rangle(1|) \] (6)

ii) the transition operators:
\[ A \equiv |0\rangle(1|, \quad A^+ \equiv |1\rangle(0|) \] (7)

iii) the identity operator:
\[ \sigma^0 \equiv P^1 + P^0 \] (8)

iv) the Pauli operators:
\[ \begin{align*}
\sigma^1 &\equiv A + A^+ \\
\sigma^2 &\equiv iA - iA^+ \\
\sigma^3 &\equiv P^1 - P^0.
\end{align*} \] flip operators (9)

All of these operators apart from $\sigma^2$ can be defined for classical bits. Also, all of these operators can be multiplied together and form a closed algebra, represented by Table 1. For example, the product $P^1A$ is found by the intersection of the row labelled by $P^1$ and the column labelled by $A$. From the table, we find $P^1A = 0$, the zero operator.

This table turns out to be invaluable in quantum register physics. It should be noted that it applies only to operators acting on the same qubit.
Table 1. The computational basis operator algebra for a single qubit.

From Table 1 we can read off the following fundamental property of the transition operators:

\[ AA = A^+ A^+ = 0, \]  

which at first sight suggests that these operators are related to fermionic or Grassmannian variables. However, qubits are neither spin-half fermions nor Grassmannian variables, but they can be used in the manner of Jordan and Wigner to construct fermionic quantum fields out of large collections (quantum registers) of qubits. The nilpotency property of the transition operators gives quantum register physics a very particular flavour, modelling the fact that a real physics apparatus is either void (is not being used) or else can “hold” only one state at a time.

### 4 Quantum registers

A rank-\(r\) quantum register \(R^r\) is the tensor product of \(r\) distinct, labelled qubits:

\[ R^r \equiv \mathcal{Q}_0 \otimes \mathcal{Q}_1 \otimes \ldots \otimes \mathcal{Q}_{r-1}. \]

It is a complex Hilbert space of dimension \(2^r\), containing separable and entangled states. This makes it ideal for discussing quantum physics.

In the following, the left-right ordering of tensor products is not significant, although labels are significant. For example, the rank-2 quantum registers \(\mathcal{Q}_0 \otimes \mathcal{Q}_1\) and \(\mathcal{Q}_1 \otimes \mathcal{Q}_0\) are equivalent.

A register computational basis \(B(R^r)\) is readily constructed by tensoring the computational bases for all of the qubits in the register in the following manner:

\[ B(R^r) = \left\{ |i_0\rangle_0 \otimes |i_1\rangle_1 \otimes \ldots \otimes |i_{r-1}\rangle_{r-1} : \begin{array}{c} i_j \in \{0,1\}, \\ 0 \leq j \leq r-1. \end{array} \right\}. \]
In our work, we shall always denote qubit and register states by Dirac bra-ket notation, modified by the replacement of angular brackets \( \langle \), \( \rangle \) with round brackets \( ( \), \( ) \) respectively. Elements of the register computational basis \( B(\mathcal{R}^r) \) can be represented in a number of equivalent ways, depending on context, as follows:

1. We can drop the tensor product symbol, as the individual qubit identifier labels suffice to carry the necessary information:

\[
|i_0\rangle_0 \otimes |i_1\rangle_1 \otimes \ldots \otimes |i_{r-1}\rangle_{r-1} = |i_0\rangle_0 |i_1\rangle_1 \ldots |i_{r-1}\rangle_{r-1}; \tag{13}
\]

2. We can write out register computational basis elements in terms of a sequence of ones and zeros:

\[
|i_0\rangle_0 \otimes |i_1\rangle_1 \otimes \ldots \otimes |i_{r-1}\rangle_{r-1} = |i_0i_1i_2\ldots i_{r-1}\rangle; \tag{14}
\]

3. We can interpret such a sequence as a binary number and use that instead of the sequence:

\[
|i_0\rangle_0 \otimes |i_1\rangle_1 \otimes \ldots \otimes |i_{r-1}\rangle_{r-1} = |i_02^0 + i_12^1 + \ldots + i_{r-1}2^{r-1}\rangle \tag{15}
\]

Then the register computational basis can be written in the form

\[
B(\mathcal{R}^r) = \{|a\rangle : 0 \leq a \leq 2^r - 1\}, \tag{16}
\]

with orthonormality condition

\[
(a|b) = \delta_{ab}, \quad 0 \leq a, b \leq 2^r - 1. \tag{17}
\]

For example, for a rank-3 quantum register, the element \(|1\rangle_0 \otimes |0\rangle_1 \otimes |1\rangle_2\) can be written in the following ways:

\[
|1\rangle_0 \otimes |0\rangle_1 \otimes |1\rangle_2 = |1\rangle_0 |0\rangle_1 |1\rangle_2 = |101\rangle = |1.2^0 + 0.2^1 + 1.2^2\rangle = |5\rangle. \tag{18}
\]

Occasionally, there will be possible ambiguity as to whether a number is written in binary or in decimal. For instance, in a rank-4 register, we have the state

\[
|1101\rangle = |1 + 2 + 8\rangle = |11_{10}\rangle \tag{19}
\]

In such cases, we shall always give the decimal representation the subscript 10, to denote “base ten”, and then we know that \(|11_{10}\rangle = |\text{“eleven”}\rangle\) and not \(|1.2^0 + 1.2^1\rangle = |3\rangle\). Whenever there is no possible ambiguity, we shall not need this subscript and so leave it out.
An arbitrary quantum register state $|\psi\rangle$ can always be expressed in terms of the register computational basis, i.e.,

$$|\psi\rangle = \sum_{a=0}^{2^r-1} \psi_a |a\rangle, \quad \psi_a \in \mathbb{C}. \quad (20)$$

Then the inner product between any two elements $|\psi\rangle$, $|\phi\rangle$ of the register is given by

$$(\psi|\phi) = \sum_{a=0}^{2^r-1} \psi_a^* \phi_a. \quad (21)$$

We are now ready to discuss real quantum physics experiments.

## 5 The Stern-Gerlach experiment

In 1922, Stern and Gerlach performed an experiment, passing electrons through a strong, inhomogeneous magnetic field \[4, 5\]. Their apparatus is represented in Figure 1.

![Figure 1: Idealized Stern-Gerlach experiment.](image)

In the conventional Hilbert space description of this experiment, an electron state evolves from an initial state $|\Psi_{in}\rangle$ to a final state $|\Psi_{out}\rangle$ which can be represented as the linear superposition of two possible outcome states, known as “spin up” and “spin down” respectively. Each of these states is associated with one of the outcome spots shown in Figure 1. These outcomes are represented by the orthonormalized kets $|\text{up}\rangle$ and $|\text{down}\rangle$ respectively:

$$|\Psi_{in}\rangle \rightarrow |\Psi_{out}\rangle = \alpha|\text{up}\rangle + \beta|\text{down}\rangle, \quad |\alpha|^2 + |\beta|^2 = 1. \quad (22)$$
The statistical results of the experiment are in agreement with the quantum theory Born probability rule

\[ P(\text{up} | \Psi_{in}) = |\langle \text{up} | \Psi_{out} \rangle|^2 = |\alpha|^2, \]
\[ P(\text{down} | \Psi_{in}) = |\langle \text{down} | \Psi_{out} \rangle|^2 = |\beta|^2. \]  

(23)

Our quantum register description of this and other experiments rests on several observations about what happens in such experiments.

First, in most real physics experiment, all parts of the apparatus exist before, during and after the experiment. Normally, many individuals runs or repetitions of such an experiment are performed, generally spaced over a significant interval of time and, during this time, the apparatus maintains a temporally enduring identity. We shall model this aspect of the physics in our quantum register description.

Second, for the Stern-Gerlach experiment illustrated in Figure 1, the two outcome possibilities for each emerging electron are detected at different spatial locations (of course, in any single run involving a single electron passing through the apparatus, the electron is detected at only one of these two places at the end of that run). This spatial separation is crucial to the success of the experiment, for without it, Stern and Gerlach would never have been able to observe anything unusual. This aspect of detection is also modelled in our quantum register physics.

Third, it is possible to have more than one Stern-Gerlach experiment being performed simultaneously in different parts of the world. The quantum register description permits us to describe such a scenario using a single large quantum register.

5.1 The quantum register description:

The essence of the quantum register description of the Stern-Gerlach and other experiments is to assign a qubit to each place where physicists could in principle detect new information. This is equivalent to using the space concept in a counterfactual way. For the Stern-Gerlach experiment, this means assigning at least three qubits as follows:

1) we assign a qubit \( Q_0 \) to the source of the electrons;
2) we assign a qubit \( Q_1 \) to the up state detector;
3) we assign a qubit \( Q_2 \) to the down state detector.

This assignment is represented in Figure 2:

Points to note are
1. Unlike the conventional description of the Stern-Gerlach experiment, states $|\text{up}\rangle$ and $|\text{down}\rangle$ are not regarded as orthogonal qubit states in the same qubit space;

2. We include the source in the description;

3. In principle we could imagine the physical space between source and detectors as filled with qubits, but these would be redundant here. We need only that number of qubits sufficient to model the essential physics of a given experiment. Later on in this paper we shall discuss quantum register cosmology and quantum register field theory, where there is a reason to consider space in terms of a very large (possibly infinite) rank quantum register.

Having set up a rank-three quantum register $\mathcal{R}^3 \equiv Q_0 \otimes Q_1 \otimes Q_2$ to model the Stern-Gerlach experiment, we now discuss a typical run of this experiment. This involves a time-dependent description of the state $|\Psi\rangle$ of the combined apparatus-system.

First, imagine the situation after all apparatus has been constructed but before the actual experiment has started. During such a time, the equipment is lying idle, i.e., unused. It exists, but no electron is being prepared and no detector is registering any result. Such a state of the apparatus-laboratory system will be called the void state (we shall not use the term vacuum in this context, as this will be reserved for other specific situations). We shall
represent the void state by the quantum register vector

\[ |\Psi_0 \rangle = |0 \rangle_0 |0 \rangle_1 |0 \rangle_2 = |000 \rangle = |0 \rangle_0 = |0 \rangle. \]  

(24)

Suppose now that the experiment has started. At some initial time \( t_{in} \), the experimentalists will be confident that the source has prepared an initial state, but nothing has yet been registered by either detector. We represent the lab-state (our terminology for the state of the apparatus and system) by the quantum register state

\[ |\Psi_{in} \rangle = |1 \rangle_0 |0 \rangle_1 |0 \rangle_2 = |100 \rangle = |1 \rangle = A_0^+ |0 \rangle, \]

(25)

where in this particular case

\[ A_0^+ \equiv A_0^+ \otimes \sigma_1^0 \otimes \sigma_2^0. \]

(26)

Note that we can be sure that there must be such an interval of time, because the detectors are spatially separated from the source, and therefore could only trigger a non-zero time after state preparation, according to the principles of special relativity. It does not matter that according to some quantum theorists, quantum states change instantaneously. This is irrelevant in quantum register physics. All signals registered in our qubits have to be consistent with relativity.

Finally, at a time \( t_{out} > t_{in} \), we may write down the lab-state immediately prior to detection:

\[ |\Psi_{out} \rangle = \alpha |010 \rangle + \beta |001 \rangle = \alpha |2 \rangle + \beta |4 \rangle = (\alpha A_1^+ + \beta A_2^+) |0 \rangle, \]

(27)

where

\[ A_1^+ \equiv \sigma_0^0 \otimes A_1^+ \otimes \sigma_2^0, \quad A_2^+ \equiv \sigma_0^0 \otimes \sigma_1^0 \otimes A_2^+. \]

(28)

Once we have determined \( |\Psi_{out} \rangle \), the Born probability rule adapted to the quantum register can be applied to give the outcome probabilities

\[ P (\text{up} | \Psi_{in} \rangle) \equiv |\langle 2 | \Psi_{out} \rangle|^2 = |\alpha|^2 \]
\[ P (\text{down} | \Psi_{in} \rangle) \equiv |\langle 4 | \Psi_{out} \rangle|^2 = |\beta|^2, \]
\[ P (\text{any other state} | \Psi_{in} \rangle) \equiv |\langle a | \Psi_{out} \rangle|^2 = 0, \quad a = 0, 1, 3, 5, 6, 7, \]

(29)

consistent with known physics. Of course, during any single run involving a single electron, only one detector gets triggered, so these probabilities have to be related to the frequencies of outcome built up over many runs of the basic experiment.
Normally, after each run is over and before the next one starts, the lab-state reverts to the void state $|0\rangle$. The specific mechanism for this is currently beyond known physics, as is the transition from the void state to the initial state at the start of a run.

Our quantum register approach permits a very interesting and physically meaningful situation to be contemplated. We could imagine starting the $(n + 1)^{th}$ run before the $n^{th}$ is complete. Such a situation would be modelled, for example, by the sequence

$$
|\Psi_n\rangle \equiv \alpha|010\rangle + \beta|001\rangle \rightarrow |\Psi_{n+1}\rangle \equiv \alpha|110\rangle + \beta|101\rangle.
$$

(30)

We shall discuss this and other exotic possibilities in §12.

More formally, we may consider changes in the lab-state to be described by unitary evolution over $\mathcal{H}^3$, viz.,

$$
|\Psi_{in}\rangle \rightarrow |\Psi_{out}\rangle \equiv U(t_{out}, t_{in})|\Psi_{in}\rangle,
$$

where $U(t_{out},t_{in})$ is unitary so as to preserve total probability. Exactly what this operator is or should be will not always be clear, because physics experiments will not in general deal with absolutely every possible state in a quantum register. The basic Stern-Gerlach experiment, for example, requires us only to consider four of the eight computational basis elements, viz, $|0\rangle, |1\rangle, |2\rangle$ and $|4\rangle$. For such basic systems, there is a degree of overkill in the quantum register description. This should not be regarded as a flaw; a similar situation occurs in most classical and quantum theories.

We can be sure of one or two things, however. First, if the laboratory is in a void state, then we do not expect that to change, unless we initiate a new run (which will not conserve probability anyway). Therefore, we may assume

$$
U(t_{out}, t_{in})|0\rangle = |0\rangle.
$$

(31)

Then we can represent the dynamics in terms of how the transition operators change, viz.,

$$
A_0^+ \rightarrow U(t_{out}, t_{in}) A_0^+ U^+(t_{out}, t_{in}) = \alpha A_1^+ + \beta A_2^+.
$$

(32)

More generally, we shall “modularise” our spatio-temporal description, meaning that individual transition operators will change at various times in their own ways. Typically, we shall leave out specific reference to the $U$ operators, writing for example

$$
A_0^+ \rightarrow \alpha A_1^+ + \beta A_2^+
$$

(33)
to describe a particular change in the operator $A^+_0$ at a particular place and time during a given run.

Two important points need to be noted. First, our quantum register description is not designed to give us the dynamical details of such transitions. For that we need to invoke standard quantum mechanics. The quantum register description is designed to show more consistently how sequences of such dynamical changes get distributed around in time and space, making overall calculations of complex processes easier to calculate.

Second, because real physics experiments are irreversible, we need to be cautious about what operators such as $U(t_{out}, t_{in})$ really mean. They will have the semi-group property

$$U(t_2, t_1) U(t_1, t_0) = U(t_2, t_0), \quad t_2 \geq t_1 \geq t_0$$

and satisfy

$$U(t_1, t_0) U^+(t_1, t_0) = I_R,$$

where $I_R$ is the register identity operator, but we may have no clear physical interpretation of what the operator $U(t_0, t_1)$ means, for $t_0 < t_1$.

## 6 The Wollaston prism

The two polarization degrees of freedom of photons make them analogous to electrons in certain situations. For instance, the passage of a monochromatic electromagnetic wave through a Wollaston prism splits the wave into two spatially distinct waves, identified with two distinct mutually orthogonal transverse polarization components, shown in Figure 3.

If $|\psi_0\rangle$ is a monochromatic photon state, we may write

$$|\psi_0\rangle = \psi_1 |x\rangle + \psi_2 |y\rangle,$$
where $|x\rangle$ and $|y\rangle$ represent the two mutually orthogonal transverse polarization vectors involved.

The quantum register description of a Wollaston prism, shown in Figure 4, turns out to be identical to that for the Stern-Gerlach experiment. In operator terms, we find

$$A_0^+ \rightarrow \psi_1 A_1^+ + \psi_2 A_2^+,$$

which is formally identical to (33).

Figure 4: Quantum register description of a Wollaston prism.

7 von Neumann tests

The Stern-Gerlach and Wollaston prism experiments are the most elementary and useful examples of the sort of quantum experiments discussed by von Neumann [6], where an ensemble of identically prepared initial states is passed through some test apparatus $A$ and a range of possible outcomes detected. The description of an idealized version of such an experiment leads to the so-called projection valued measure (PVM) description of quantum experiments. This is known to have its limitations, but remains an important concept.

The general PVM test is shown in Figure 5. For each run of an ensemble of runs, the initial state $|\Psi_{in}\rangle$, which will be assumed to be pure, is prepared by some apparatus $\Sigma_0$ at time $t_{in}$. Subsequently, the prepared state is passed through test apparatus $A$, and one out of $d$ possible outcomes detected at time $t_{out}$. In von Neumann’s approach, $|\Psi_{in}\rangle$ is assumed to be a normalized element of some $d$–dimensional Hilbert space $\mathcal{H}$. The test $A$ is represented by some non-degenerate Hermitian operator $\hat{A}$ acting over $\mathcal{H}$. Because of
non-degeneracy, the eigenstates $|a_1\rangle$, $|a_2\rangle$, \ldots, $|a_d\rangle$ can be normalized and form an orthonormal basis for $\mathcal{H}$, known as the preferred basis.

Because of completeness, we may write

$$|\Psi_{in}\rangle \rightarrow |\Psi_{out}\rangle = \hat{U} (t_{out}, t_{in}) |\Psi_{in}\rangle = \sum_{i=1}^{d} \Psi^i |a_i\rangle,$$  \hspace{1cm} (38)

where

$$\Psi^i = \langle a_i | \Psi_{out} \rangle = \langle a_i | \hat{U} (t_{out}, t_{in}) |\Psi_{in}\rangle.$$  \hspace{1cm} (39)

The Born probability interpretation then predicts the conditional outcome probabilities to be given by

$$P (a_i | \Psi_{in}) = |\langle a_i | \Psi_{out} \rangle|^2 = |\Psi^i|^2.$$  \hspace{1cm} (40)

The quantum register description of a PVM scenario follows the pattern outlined for the Stern-Gerlach and Wollaston prism experiments. We associate one qubit with every part of the apparatus wherever a state could be detected and new information acquired. This means one qubit for the preparation apparatus and one for each of the $d$ possible outcomes, shown in Figure 6. Therefore, we need a rank-$(1 + d)$ quantum register for such a test.

The quantum dynamics is given by the rule

$$\mathbb{A}_0^+ \rightarrow \sum_{i=1}^{d} \Psi^i \mathbb{A}_i^+,$$  \hspace{1cm} (41)
Figure 6: Qubit assignment for a general PVM, or von Neumann, test.

where the $\Psi^i$ are given by the conventional quantum calculation (39), so we find

$$ |\Psi_{in}\rangle \rightarrow |\Psi_{out}\rangle = \sum_{i=1}^{d} \Psi^i A^+_{i} |0\rangle = \sum_{i=1}^{d} \Psi^i |2^i\rangle. \quad (42) $$

The conditional probabilities for the $d$ possible outcomes of the experiment are then given by the quantum register Born rule

$$ P(a_i|\Psi_{in}) \equiv |\langle 2^i|\Psi_{out}\rangle|^2 = |\Psi^i|^2, \quad (43) $$
as before.

If all experiments were of this form, there would be little advantage in the quantum register description. This comes into its own when more than one von Neumann test are coupled together, a situation which occurs frequently in quantum optics experiments.

An important observation about the formalism developed thus far is that all physical states so far considered are linear combinations of only certain elements of the computational basis, viz, those of the form

$$ |2^k\rangle \equiv A^+_k |0\rangle. \quad (44) $$
States of the form $|2^k\rangle$ and linear combinations of such states, will be called rank-one states. We define rank-$p$ states to be linear combinations of elements of the computational basis given by

$$ A^+_i A^+_j \ldots A^+_p |0\rangle = |2^{i_1} + 2^{i_2} + \ldots + 2^{i_p}\rangle \quad (45) $$
where the $0 \leq i_j < i_{j+1} \leq r - 1$. For example,

$$ A^+_2 A^+_3 A^+_5 |0\rangle = |00110100000\ldots0\rangle = |2^2 + 2^3 + 2^5\rangle = |44\rangle \quad (46) $$
is a rank-3 state.
8 More general experiments

Eventually, it become apparent that the PVM formulation of quantum physics is too limited and it became superseded by the more general POVM (Positive Operator Valued Measure) approach. In this new approach, quantum experiments can have more or less outcomes than the dimension of the Hilbert space involved. For example, suppose we have an experiment with \( k \) possible outcomes, with \( k \) not necessarily equal to \( d \), the dimension of the Hilbert space \( \mathcal{H} \) used to model the states of the system. For each outcome \( |\phi_i^{\psi}\rangle \), \( i = 1, 2, \ldots, k \), there is an associated positive operator \( \hat{E}_i \), such that

\[
\sum_{i=1}^{k} \hat{E}_i = \hat{I}_\mathcal{H},
\]

where \( \hat{I}_\mathcal{H} \) is the identity operator over the Hilbert space. Given a normalized initial state \( |\Psi\rangle \in \mathcal{H} \), then the conditional probability \( P(\phi^i|\Psi) \) of outcome \( |\phi^i\rangle \) is given by

\[
P(\phi^i|\Psi) = \langle \Psi|\hat{E}_i^i|\Psi\rangle,
\]

with condition (47) ensuring probabilities sum to unity.

The above discussion involves pure states. In fact, the POVM approach is more general than this and can be extended to cover mixed states, which requires a density matrix approach involving the taking of traces. We shall not be interested in this paper in such situations. The generalization of our quantum register description to cover such cases is not anticipated to be particularly difficult and is left for future consideration.

We shall discuss now some situations where the POVM approach would normally be invoked and give an alternative quantum register description. We recall that an arbitrary POVM with a finite number of elements can always be converted into a von Neumann (maximal) test by the introduction of an auxiliary, independently prepared quantum system known as an ancilla, a result which utilizes Neumark’s theorem [7]. Essentially, the original Hilbert space \( \mathcal{H} \) is extended into one of higher dimension, \( \mathcal{H}' \), and von Neumann’s PVM formulation can be applied to \( \mathcal{H}' \) directly.

The disadvantage of this approach is that it masks the spatio-temporal structure of the measurements involved and suggests that the simple in–out structure of a single von Neumann test is all that is going on. In reality, complex experiments involve sequences of processes rather like what happens in a computer, which is why quantum computation is one possible way to approach physics [11].

Suppose for example that instead of irreversibly registering all information about the outcomes of a von Neumann test, we feed one or more of
its possible outcome channels into some new test. Quantum interference experiments, such as double-slit and Mach-Zender interferometer experiments, are of this form. As an example, consider the double Stern-Gerlach experiment shown in Figure 7. An electron is prepared by apparatus $\Sigma_0$ as in the original Stern-Gerlach experiment and passed through a Stern-Gerlach apparatus $SG_1$ which has quantization axis along vector $k$. Any spin down outcome $| -k \rangle$ is recorded, whereas any spin up outcome $| +k \rangle$ is not registered but channelled into a second Stern-Gerlach apparatus $SG_2$, which has quantization axis $a$, with each of its possible outcomes being detectable.

For any single run of the combined experiment, there are now three possible mutually exclusive outcomes, viz., $| +a \rangle$, $| -a \rangle$ and $| -k \rangle$, not two. For such an experiment, a PVM description involving a Hilbert space of dimension two is not adequate. Conventionally, either a POVM description with three positive operators over a two-dimensional Hilbert space is needed, or an ancilla has to be introduced if a PVM approach is desired, but this then requires an extension of the original Hilbert space.

The alternative we propose is a quantum register description, which describes such experiments quite readily. For the particular experiment shown in Figure 7, we require a rank-5 quantum register, as shown in Figure 8.

In this experiment, the outcome channel $| +k \rangle$ of test $SG_1$ serves as an *ideal measurement* [7] or preparation for $SG_2$; it is not absorbed by the detector but is used as an initial state for the subsequent test $SG_2$. This seems to be the only physically meaningful interpretation of the concept of “state reduction”. State reduction without subsequent testing is physically
meaningless. Therefore, rather than represent a final stage in a quantum process, state reduction should always be considered as a beginning.

Figure 8: A qubit assignment for the double Stern-Gerlach experiment shown in Figure 7.

9 Beam splitters

In optics, a beam splitter acts as a semi-transparent mirror, whereby part of an incident electromagnetic wave is reflected and part transmitted, as shown in Figure 9a:

In quantum optics, such a device is usually regarded as having two input ports and two output ports, as in Figure 9b, and is used in experiments involving quantum interference, such as the Mach-Zender experiment, discussed below.

We recover the single input channel picture when one of the two input channels acts as a vacuum or void port. More generally, for a lossless beam splitter, the input and output waves are consistent with unitary evolution, and can be written in the form [8]:

\[
\begin{bmatrix}
\psi_3 \\
\psi_4
\end{bmatrix} = e^{i\eta} \begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix} \begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix}
\]

(49)

where

\[
|a|^2 + |b|^2 = 1
\]

(50)
Figure 9: (a) The action of a beam splitter on a single incident electromagnetic wave, (b) a beam splitter with two in-ports is used in quantum interference experiments.

and $\eta$ is real. Then

$$|\psi_3|^2 + |\psi_4|^2 = |\psi_1|^2 + |\psi_2|^2. \quad (51)$$

A quantum register description of a beam splitter requires four qubits, as in Figure 10:

Figure 10: Qubit description of a beam splitter.

The correct temporal evolution is given by

$$A_1^+ \rightarrow e^{i\eta} \left\{ aA_3^+ - b^*A_4^+ \right\}$$
$$A_2^+ \rightarrow e^{i\eta} \left\{ bA_3^+ + a^*A_4^+ \right\}. \quad (52)$$
This analysis applies to rank-1 states as well as rank-2 states. Figure 10 on its own suggests that a rank-2 initial state is involved, such as

$$A_1^+ A_2^+ |0\rangle,$$

but the analysis applies equally to rank-1 initial states such as

$$\{A_1^+ + A_2^+\} |0\rangle,$$

and it is the latter which are involved in quantum interference usually.

10 The Mach-Zender interferometer

We are now in a position to consider more complex experiments via our quantum register formalism. First, we shall discuss the Mach-Zender interferometer, shown in Figure 11. A monochromatic beam of light $\Psi_0$ is incident on a beam splitter $BS_1$, with output channels $\Psi_1, \Psi_2$. The latter channel is passed through a device giving a phase-shift $\phi$. Beams $\Psi_2$ and $\Psi_3$ are then deflected by mirrors $M_1, M_2$ onto a second beam-splitter $BS_2$, identical to $BS_1$, and finally, its output channels lead on to photon detectors $D_1, D_2$.

![Figure 11: A Mach-Zender interferometer.](image)

Taking the individual modules of the apparatus in turn, a conventional wave-function description goes as follows:
i) Beam splitter $BS_1$:
\[
\begin{bmatrix}
\psi_0 \\
0
\end{bmatrix} \rightarrow \begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix} = e^{i\eta} \begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix} \begin{bmatrix}
\psi_0 \\
0
\end{bmatrix}
\] (55)

ii) Phase shift $\phi$:
\[
\begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix} \rightarrow \begin{bmatrix}
\psi_1 \\
\psi_3
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & e^{i\phi}
\end{bmatrix} \begin{bmatrix}
\psi_1 \\
\psi_2
\end{bmatrix}
\] (56)

iii) Mirrors $M_1, M_2$:
\[
\begin{bmatrix}
\psi_1 \\
\psi_3
\end{bmatrix} \rightarrow \begin{bmatrix}
\psi_4 \\
\psi_5
\end{bmatrix} = \begin{bmatrix}
0 & e^{i\mu} \\
e^{i\mu} & 0
\end{bmatrix} \begin{bmatrix}
\psi_1 \\
\psi_3
\end{bmatrix}
\] (57)
where we assume some phase shift $\mu$ due to reflection;

iv) Beam splitter $BS_2$:
\[
\begin{bmatrix}
\psi_4 \\
\psi_5
\end{bmatrix} \rightarrow \begin{bmatrix}
\psi_6 \\
\psi_7
\end{bmatrix} = e^{i\eta} \begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix} \begin{bmatrix}
\psi_4 \\
\psi_5
\end{bmatrix}
\] (58)

The result is that the waves incident on detectors $D_1$ and $D_2$ are given by
\[
\begin{aligned}
\psi_6 &= e^{i(2\eta + \mu)} \left[ ab - e^{i\phi}ab^* \right] \psi_0 \\
\psi_7 &= e^{i(2\eta + \mu)} \left[ |a|^2 + e^{i\phi} (b^*)^2 \right] \psi_0
\end{aligned}
\] (59)

The quantum register description follows the prescription used before, which is to place a qubit at every place where a quantum measurement/observation could in principle extract new information, as shown in Figure 12:

This suggests we need at least a rank-8 quantum register. However, if we ignored the effect of the mirrors, which simply give an unobservable change of phase in the overall amplitude, we could make do with two qubits less. The quantum register calculation goes as follows:

i) Beam splitter $BS_1$:
\[
A_0^+ \rightarrow e^{i\eta} a A_1^+ - e^{i\eta} b^* A_2^+,
\] (60)
Figure 12: Qubit assignment for a Mach-Zender interferometer.

ii) Phase shift $\phi$:
\[ A_2^+ \rightarrow e^{i\phi} A_3^+, \]  
(61)

iii) Mirrors $M_1, M_2$:
\[ A_1^+ \rightarrow e^{i\mu} A_5^+, \quad A_3^+ \rightarrow e^{i\mu} A_4^+, \]  
(62)

iv) Beam splitter $BS_2$:
\[ A_4^+ \rightarrow e^{i\eta} a A_6^+ - e^{i\eta} b^* A_7^+ \]
\[ A_5^+ \rightarrow e^{i\eta} b A_6^+ + e^{i\eta} a^* A_7^+ \]  
(63)

Hence the register dynamics gives
\[ A_0^+ \rightarrow e^{i(2\eta + \mu)} \left[ ab - e^{i\phi} ab^* \right] A_6^+ \]
\[ + e^{i(2\eta + \mu)} \left[ |a|^2 + e^{i\phi} (b^2) \right] A_7^+ , \]  
(64)

i.e.,
\[ |\psi_{in}\rangle \equiv A_0^+ |0\rangle \rightarrow |\psi_{out}\rangle = \begin{cases} 
   e^{i(2\eta + \mu)} \left[ ab - e^{i\phi} ab^* \right] A_6^+ \\
   + e^{i(2\eta + \mu)} \left[ |a|^2 + e^{i\phi} (b^2) \right] A_7^+ \end{cases} |0\rangle 
\]
\[ = e^{i(2\eta + \mu)} \left[ ab - e^{i\phi} ab^* \right] |2^6\rangle 
\]
\[ + e^{i(2\eta + \mu)} \left[ |a|^2 + e^{i\phi} (b^2) \right] |2^7\rangle . \]  
(65)
Figure 13: An interference experiment requiring a POVM description.

Note that the final state is still a rank-1 state. The amplitudes at the detectors are given by

\[
\begin{align*}
\text{at } D_1 & : \quad (2^6 |\psi_{\text{out}}\rangle) = e^{i(2\eta + \mu)} [ab - e^{i\phi} ab^*] \\
\text{at } D_2 & : \quad (2^7 |\psi_{\text{out}}\rangle) = e^{i(2\eta + \mu)} \left[|a|^2 + e^{i\phi} (b^2)\right]
\end{align*}
\]

in precise agreement with the conventional calculation shown earlier.

11 Quantum interference POVM example

We now consider a more complex experiment discussed recently, which requires a POVM description \[9, 10\]. In this experiment, a photon beam first passes through a Wollaston prism and its output channels pass through a beam splitter \(BS_1\) and a mirror \(M\) as shown in Figure 13. The beam reflected from the mirror has its polarization rotated by 90 degrees before passage through a second beam splitter \(BS_2\), where quantum interference takes place.

In this experiment, the initial state is given as a superposition of two non-orthogonal states,

\[
|\Psi_0\rangle = \alpha |u\rangle + \beta |v\rangle
\]

(67)
Figure 14: Qubit assignment for the experiment shown in Figure 13.

where $\langle u|v \rangle = \cos \theta$. The calculations in [9, 10] give the following set of POVM operators:

$$E_u = \frac{I_H - |v\rangle\langle v|}{1 + \cos \theta}, \quad E_v = \frac{I_H - |u\rangle\langle u|}{1 + \cos \theta}, \quad E_\gamma = I_H - E_u - E_v. \quad (68)$$

The outcome probabilities are found to be

$$P(u|\Psi_0) = \langle \Psi_0|E_u|\Psi_0 \rangle = |\alpha|^2(1 - \cos \theta)$$
$$P(v|\Psi_0) = \langle \Psi_0|E_v|\Psi_0 \rangle = |\beta|^2(1 - \cos \theta) \quad (69)$$
$$P(\gamma|\Psi_0) = \langle \Psi_0|E_\gamma|\Psi_0 \rangle = |\alpha + \beta|^2 \cos \theta.$$

The quantum register calculation goes as follows:

i) **Wollaston prism WP:**

$$A_0^+ \rightarrow (\alpha + \beta)\cos(\frac{1}{2}\theta)A_1^+ + (\alpha - \beta)\sin(\frac{1}{2}\theta)A_2^+, \quad (70)$$

ii) **Beam splitter BS$_1$:**

$$A_1^+ \rightarrow \sqrt{1 - \tan^2(\frac{1}{2}\theta)}A_3^+ + i\tan(\frac{1}{2}\theta)A_4^+, \quad (71)$$
iii) Mirror $M$ and $90^\circ$ Polarization Rotation $R$:

$$A_2^+ \rightarrow -A_3^+,$$  \hspace{1cm} (72)

iv) Beam splitter $BS_2$:

$$A_4^+ \rightarrow \frac{i}{\sqrt{2}} A_6^+ + \frac{1}{\sqrt{2}} A_7^+,$$

$$A_5^+ \rightarrow \frac{1}{\sqrt{2}} A_6^+ + \frac{i}{\sqrt{2}} A_7^+. \hspace{1cm} (73)$$

Hence finally,

$$A_0^+ \rightarrow (\alpha + \beta) \sqrt{\cos \theta} A_3^+ - \alpha \sqrt{1 - \cos \theta} A_6^+ + i \beta \sqrt{1 - \cos \theta} A_7^+,$$  \hspace{1cm} (74)

i.e.

$$|\Psi_{in} \rangle \equiv |1 \rangle \rightarrow |\Psi_{out} \rangle = (\alpha + \beta) \sqrt{\cos \theta} |2^3 \rangle - \alpha \sqrt{1 - \cos \theta} |2^6 \rangle + i \beta \sqrt{1 - \cos \theta} |2^7 \rangle. \hspace{1cm} (75)$$

This gives the conditional probabilities

$$P(\gamma | \Psi_0) = |(2^3 | \Psi_{out} \rangle|^2 = |\alpha + \beta|^2 \cos \theta$$

$$P(u | \Psi_0) = |2^6 | \Psi_{out} \rangle|^2 = |\alpha|^2 (1 - \cos \theta) \hspace{1cm} (76)$$

$$P(v | \Psi_0) = |2^7 | \Psi_{out} \rangle|^2 = |\beta|^2 (1 - \cos \theta)$$

exactly as in the conventional description.

It is here that the advantage of working with the quantum register description begins to show itself. The transition rule (74) not only has all the hallmarks of the PVM description (albeit in a Hilbert space of dimension $2^8$), but is conceptually more understandable than the set of POVM operators (68). In particular, all of the detector qubits $Q_3$, $Q_6$ and $Q_7$ are treated in the same way, whereas the status of the detector labelled “?” is considered different to the detectors labelled “u” and “v”. The quantum register description of each stage of the experiment makes it clear that the original formulation of the experiment in terms of non-orthogonal basis vectors is somewhat contrived and strictly speaking, not necessary.

12 Interpretation of higher rank states

There are certain situations in quantum register physics where states of rank higher than one are encountered. We discuss some of these next.
12.1 Independent experiments

Suppose two Stern-Gerlach experiments are performed separately, completely independently of each other in different parts of the world. In such a case, we can describe the two experiments by a single rank-6 quantum register with rank-2 states, as shown in Figure 15.

In this case, the initial state is given by

\[ |\Psi_{\text{in}}\rangle = A_0^+ A_3^+ |0\rangle = |100100\rangle = |2^0 + 2^3\rangle = |9\rangle. \]  

(77)

If each experiment is truly independent, then we can write

\[ A_0^+ \rightarrow \alpha A_1^+ + \beta A_2^+, \quad |\alpha|^2 + |\beta|^2 = 1, \]

\[ A_3^+ \rightarrow \gamma A_4^+ + \delta A_5^+, \quad |\gamma|^2 + |\delta|^2 = 1, \]

(78)

so

\[ |\Psi_{\text{in}}\rangle \rightarrow |\Psi_{\text{out}}\rangle = (\alpha A_1^+ + \beta A_2^+) (\gamma A_4^+ + \delta A_5^+) |0\rangle = |\psi_1\rangle \otimes |\phi_2\rangle, \]

26
where
\[
|\psi\rangle_1 \equiv \alpha|0\rangle_0|1\rangle_1|0\rangle_2 + \beta|0\rangle_0|0\rangle_1|1\rangle_2,
\]
\[
|\phi\rangle_2 \equiv \gamma|0\rangle_3|1\rangle_4|0\rangle_5 + \delta|0\rangle_3|0\rangle_4|1\rangle_5.
\] (79)

In other words, independent experiments are modelled in quantum register physics by separable states of rank higher than unity.

### 12.2 Change of rank experiments: (EPR)

Experiments of the type discussed by Einstein, Podolsky and Rosen\[11\] cause conceptual problems because they invoke quantum non-locality. However, we note that non-locality is already inherent in real experiments (recall the two distinct spots in the Stern-Gerlach experiment, shown in Figure 1). Therefore, what is conventionally regarded as non-locality is really a matter of scale or degree.

Suppose we prepared a spin-zero bound state of an electron and a positron, given in the conventional description by
\[
|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ |+\rangle \otimes |+\rangle - |\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle - |\rangle \otimes |-\rangle + |+\rangle \otimes |-\rangle \right\}.
\] (80)

Alice and Bob are two well-separated observers, each with their own particle species filters and Stern-Gerlach equipment. Alice can detect and test for electron spin only, whereas Bob can detect and test for positron spin only. Alice sets her quantization axis along \( k = (0, 0, 1) \), whereas Bob sets his along direction
\[
a = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)
\] (81)

Now whenever Alice finds an electron passes through her apparatus with spin \(|+\rangle\), Bob will find his positron passes through either of the \(|+a\rangle\) or \(|-a\rangle\) channels in a random way, with frequency given correctly by quantum mechanics.

The quantum register description requires five qubits, as shown in Figure 16:

The qubit assignment is
\begin{align*}
Q_0 & : \text{initial spinless bound state}, \\
Q_1 & : \text{electron spin } |+\rangle, \quad Q_2: \text{electron spin } |-\rangle \\
Q_3 & : \text{positron spin } |+a\rangle, \quad Q_4: \text{positron spin } |-a\rangle.
\end{align*}
Figure 16: Qubit assignment for an EPR experiment.

A conventional quantum mechanics calculation permits us to write

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ \sin\left(\frac{\theta}{2}\right)e^{-i\phi}|+\rangle_k + a|+\rangle_a + \cos\left(\frac{\theta}{2}\right)e^{-i\phi}|-\rangle_k - a|+\rangle_a \right\}$$

so we deduce

$$A_0^+ \rightarrow \frac{\sin\left(\frac{\theta}{2}\right)e^{-i\phi}}{\sqrt{2}} A_1^+ A_3^+ + \frac{\cos\left(\frac{\theta}{2}\right)e^{-i\phi}}{\sqrt{2}} A_1^+ A_4^+$$

$$- \frac{\cos\left(\frac{\theta}{2}\right)}{\sqrt{2}} A_2^+ A_3^+ + \frac{\sin\left(\frac{\theta}{2}\right)}{\sqrt{2}} A_2^+ A_4^+.$$ 

Hence the initial state

$$|\Psi_{in}\rangle = |1\rangle = A_0^+|0\rangle$$

is a rank-one state which changes into an entangled rank-two state.

12.3 Two-particle interferometry

In 1989, Horne, Shimony and Zeilinger discussed an experiment where an entangled two-photon state passes through the device shown in the quantum register representation, Figure 17. $M_1$, $M_2$, $M_3$ and $M_4$ are mirrors, $\phi_1$ and $\phi_2$ are variable phase shifts, and $BS_1$ and $BS_2$ are beam splitters. Qubits $Q_7$, $Q_8$, $Q_9$ and $Q_{10}$ are associated with photon detectors. The quantities of
interest are the two-particle coincidence count rates and their dependence on the phase-shift angles $\phi_1$, $\phi_2$, which can be varied at will throughout the experiment.

The conventional representation of the initial state is

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}} \{|k_A\rangle_1|k_C\rangle_2 + |k_D\rangle_1|k_B\rangle_2\},$$

(82)

where the wave vectors $k_A$, $k_B$, $k_C$ and $k_D$ are identified with qubits $Q_1$, $Q_2$, $Q_3$ and $Q_4$ respectively, and subscripts 1 and 2 refer to the two particles involved.

Details of the conventional calculation are not given here. The quantum register account goes as follows. The initial state is of rank one, regardless of the fact that it consists of an entangled two-photon state. There is only one source, which means the prepared state is of rank one. However, because two photons can be detected simultaneously at independent sites subsequent to state preparation, the initial state changes rank to a rank-two state. This is represented by the operator transition
\[ A_0^+ \rightarrow \frac{1}{\sqrt{2}} \left\{ A_3^+ A_3^+ + e^{i\theta} A_2^+ A_3^+ \right\} , \]  

(83)

where the angle \( \theta \) depends on the detailed placement of the various pieces of equipment, as discussed in [12]. We ignore any overall changes of phase due to the mirrors, as this will not affect probabilities. The effect of the phase shifts \( \phi_1, \phi_2 \) gives

\[ A_1^+ \rightarrow e^{i\phi_1} A_5^+ , \quad A_2^+ \rightarrow e^{i\phi_2} A_6^+ \]  

(84)

and finally, the beam splitters give the transitions

\[ A_6^+ \rightarrow \frac{1}{\sqrt{2}} \left\{ A_8^+ + iA_7^+ \right\} , \quad A_3^+ \rightarrow \frac{1}{\sqrt{2}} \left\{ A_7^+ + iA_8^+ \right\} \]

\[ A_4^+ \rightarrow \frac{1}{\sqrt{2}} \left\{ A_9^+ + iA_{10}^+ \right\} , \quad A_5^+ \rightarrow \frac{1}{\sqrt{2}} \left\{ A_{10}^+ + iA_9^+ \right\} . \]  

(85)

This is all that is required for the complete quantum register calculation. We find

\[ A_0^+ \rightarrow \frac{1}{2\sqrt{2}} \left\{ e^{i\phi_1} - e^{i(\theta + \phi_2)} \right\} A_7^+ A_{10}^+ + \frac{1}{2\sqrt{2}} \left\{ i e^{i\phi_1} + i e^{i(\theta + \phi_2)} \right\} A_7^+ A_0^+ \]

\[ + \frac{1}{2\sqrt{2}} \left\{ i e^{i\phi_1} + i e^{i(\theta + \phi_2)} \right\} A_8^+ A_{10}^+ + \frac{1}{2\sqrt{2}} \left\{ - e^{i\phi_1} + e^{i(\theta + \phi_2)} \right\} A_8^+ A_6^+ \]  

for the full experiment. The two-particle co-incidence probabilities are found to be

\[ P(7&9|\Psi_{in}) = \frac{1}{4} \left\{ 1 + \cos(\theta + \phi_2 - \phi_1) \right\} , \]

\[ P(7&10|\Psi_{in}) = \frac{1}{4} \left\{ 1 - \cos(\theta + \phi_2 - \phi_1) \right\} , \]

\[ P(8&9|\Psi_{in}) = \frac{1}{4} \left\{ 1 - \cos(\theta + \phi_2 - \phi_1) \right\} , \]  

(87)

\[ P(8&10|\Psi_{in}) = \frac{1}{4} \left\{ 1 + \cos(\theta + \phi_2 - \phi_1) \right\} , \]

in precise agreement with the calculation of Horne, Shimony and Zeilinger [12], assuming no losses in the system.

### 12.4 Other scenarios involving higher rank states

Obvious candidate experiments for future discussion are i) interference of photons from different sources, ii) teleportation and iii) experiments where a sequence of wave-pulses is set up moving towards target detectors. In
such cases, it is possible that the source apparatus gets destroyed before the
detectors register anything. This happens in astrophysics, where it is quite
normal for astronomers to receive light from sources which have long ceased
to exist.

Many interesting physical ideas remain to be explored. Of greatest inter-
est to us is the possibility of modelling physical space as a quantum register
of enormous, possible infinite rank. Work is in hand currently on this front.

Perhaps the ultimate development of quantum register physics would be
to provide an account of quantum cosmology, which we could call quantum
register cosmology \[13\]. Such a theory would be the ultimate vindication of
Feynman’s vision of physics simulated in terms of computation. We can only
speculate at this time as to the details of such a theory. In any programme
attempting to extending the quantum register description to the universe,
we would have to face great conceptual issues as well as technical problems.
It is not accepted universally that quantum mechanics can be applied to
the universe considered as a system, for instance. Certainly, such a concept
would require an endophysical account rather than the exophysical one we
have been forced to use thus far. But this is precisely what Feynman was
saying in the quote we gave at the start of this paper.

We believe that Feynman was not just advocating an endophysical vision
of physics. What he was referring to would, in our view, inevitably lead
to quantum register cosmology. In other words, a quantum computational
theory of everything.

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