An Image Fusion Algorithm Based on Lifting Scheme

BAI Di FAN Qibin SHU Qian

ABSTRACT Taking the advantage of the lifting scheme's characters that can build wavelet transforms for transforming from integer to integer and the quality of the reconstructing image is independent of the topology way adopted by the boundary, an image fusion algorithm based on lifting scheme is proposed. This paper discusses the fundamental theory of lifting scheme firstly and then after taking transform analysis according to a kind of images that need to be confused.

KEYWORDS lifting schemes; image processing; integer wavelet transform; image fusion

Introduction

In the daily life, images always need to be further processed in many fields, and the lossless compression is required in many cases. The floating-point numbers are produced when the image data has been transformed by traditional DCT and wavelet function, therefore this method cannot be used in the image lossless compression. Sweldens used to introduce a new way to build wavelets—lifting scheme [1], which allows the entire construction of wavelets in the spatial domain without using the Fourier transform. All kinds of the traditional CDF (Cohen-Daubechies-Feauveau) biorthogonal wavelets can be built by using lifting scheme and the transform speed is much faster [2]. The inverse transform is easy to be found and always provides perfect reconstruction of the input data. So the lifting scheme will be applied in the broad field and the wavelets are known as "the second generation of wavelets".

The image fusion is a tool that can be used to process multi-source images by making use of the advanced technology of the image process. The aim of the advanced technology of the image process is to find a method having the following advantages: quick, reliable, antinoise, being able to fuse images from different sensors, adapted to large scale, rotation and translation changes. In general, the image fusion is divided into three levels: data fusion, character fusion and decision-making fusion. Data fusion, which is also called pixel fusion, is the base of the high level fusion and one of the emphases of the present image fusion field as well. The main algorithm of this article is based on it.

1 Postulates of lifting scheme

1.1 Basic frame

The foundational course of the lifting scheme is shown in Fig. 1. It is obvious that the lifting scheme has three parts that are split or merge, prediction and update.

1) Split Supposing \( r(n) \) demote a signal, and split \( r(n) \) into its even and odd polyphase components \( r_e(n) \) and \( r_o(n) \) firstly, where \( r_e(n) = r(2n) \) and \( r_o(n) = r(2n-1) \).

2) Predicting In the interpolating formulation of lifting, predict the odd polyphase coefficients having the following advantages: quick, reliable, antinoise, being able to fuse images from different sensors, adapted to large scale, rotation and translation changes. In general, the image fusion is divided into three levels: data fusion, character fusion and decision-making fusion. Data fusion, which is also called pixel fusion, is the base of the high level fusion and one of the emphases of the present image fusion field as well. The main algorithm of this article is based on it.
Fig. 1  Typical lifting steps: split/merge, predict and update

\[ x_o(n) \text{ from the neighboring even coefficient } x_e(n), \text{ the predictor for each } x_o(n) \text{ is a linear combination of neighboring even coefficients:} \]

\[ P[x_e(n)] = \sum p_x[n+l] \quad (1) \]

A new representation of the \( x(n) \) is obtained by replacing \( x_e(n) \) with the prediction residual. This leads to the first lifting step:

\[ d(n) = x_o - P[x_e(n)] \quad (2) \]

If the underlying signal is locally smooth, the prediction residuals \( d(n) \) will be small. Furthermore, the new representation contains the same information as the original signal \( x(n) \). If the even polyphase \( x_e(n) \) and the prediction residuals \( d(n) \) are given, the odd polyphase coefficients \( x_o(n) \) can be recovered.

This prediction procedure is equivalent to applying a high-pass filter to \( x(n) \). The prediction filter is typically designed to exactly predict local polynomials up to degree \( N-1 \). In wavelet terminology, the underlying synthesis scaling function corresponding to this prediction filter can reproduce polynomials of degree up to \( N-1 \), and the dual (analysis) wavelets have \( N \) zero moments.

3) Update  The third lifting step transforms the even polyphase coefficients \( x_e(n) \) into a low-pass filtered and subsampled version of \( x(n) \). This coarse approximation was obtained by updating \( x_e(n) \) with a linear combination of the prediction residuals \( d(n) \).

\[ x_o(n) = x_e(n) + U[d(n)] \quad (3) \]

Where \( U[d(n)] \) is a linear combination of neighboring values:

\[ U[d(n)] = \sum u_d[n+l] \quad (4) \]

1.2 Lifting scheme's character

In the case of the wavelets of first generation, the lifting scheme will never come up with wavelets which somehow could not be found by the techniques developed by Cohen, Daubechies and Feauveau\cite{42}. Nevertheless, using lifting to construct or reconstruct these wavelets has the following advantages:

1. It allows a faster implementation of the wavelet transform.
2. The lifting scheme allows a fully in-place calculation of the wavelet transform. In other words, no auxiliary memory is needed and the original signal or image can be replaced by its wavelet transform.
3. With the lifting scheme, the inverse wavelet transform can be found immediately by undoing the operations of the forward transform. In practice, this comes down to simply reversing the order of the operations and changing each + into a, and vice versa.

1.3 Examples

1.3.1 Haar wavelets

In the case of Haar wavelets, \( h(z) = 1 + z^{-1} \), \( g(z) = -1/2 + (1/2) z^{-1} \), \( h(z) = 1/2 + (1/2) z^{-1} \), and \( g(z) = -1 + z^{-1} \). Using the Euclidean algorithm, the polyphase matrix can be written as

\[ P(z) = \begin{bmatrix} 1 & -1/2 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1/2 \\ 1 & 1/2 & 0 & 1 \end{bmatrix} \quad (5) \]

So the forward haar wavelet transform using lifting can be got, whose frame is shown in Fig. 2.
1.3.2 D9/7 wavelets
Consider the popular D9/7 wavelets filter pair. The analysis filter \( \tilde{h} \) has 9 coefficients while the synthesis filter has 7 coefficients. Both high-pass filters \( g \) and \( \tilde{g} \) have 4 vanishing moments. Choose the filter with 7 coefficients to be the synthesis filter because it gives rise to a smoother scaling function than the 9 coefficients filter. For example, run the factoring algorithm starting from the analysis filter and get the polyphase matrix:

\[
\begin{bmatrix}
1 & a(1 + z^{-1}) & 1 & 0 \\
0 & 1 & \beta(1 + z) & 1 \\
1 & \gamma(1 + z^{-1}) & 1 & 0 \\
0 & 1 & \delta(1 + z) & 1 \\
\end{bmatrix}
\]

The lifting frame is shown in Fig. 3. And \( a = -1.586 \), \( \beta = -0.053 \), \( \gamma = 0.882 \), \( \delta = 0.443 \), \( \rho = 1.149 \).

2 Image fusion algorithm based on lifting scheme

2.1 General image fusion frame base on lifting scheme
Enlighten by Burt and Adelson’s Pyramid Image Decomposition and Reconstruction Theory, Mallat had advanced Mallat Celerity Algorithm and the image can be decomposed into different levels sub-image by exerting it.

An image can be considered as a two-dimension number array. Firstly, we choose a lifting scheme of wavelet, such as Haar lifting scheme, as it is shown in Fig. 2. Then transform the image by sending its first row vector to the Haar filter. In the split step, the row vector has divided into two parts, one part is even place vector and another is odd place vector. In the predictive step, the high pass coefficients are computed by using a Haar filter: \( H[n] = x[2n+1] - x[2n] \). In the update step, the low pass coefficients are computed by the prediction of the odd samples: \( L[n] = x[2n] + H[n] \). And store the results in the same place. Secondly, after transforming the first row, put the \( L[n] \) in the left and \( H[n] \) to the right, such as \([L[1], L[2], \cdots, H[1], H[2], \cdots]\). Thirdly, transform the rest row vector from top to bottom and the column vector from left to right in the same process. Then we get a decomposed image. This process is shown in Fig. 4.

Fig. 4 is a sketch map. \( L \) denotes low frequency and \( H \) denotes high frequency. After the wavelet transformation, the image turns to four parts: \( LL \) (contains the most original image information) and \( LH, HL, HH \) (reserve edge and detailed information).

Take two pictures for example, the fusion process is shown in Fig. 5.

1) Decompose image A and B separately to lev-
el $J$ by applying lifting scheme. Then arrange the coefficient sets according to the row and the column in different levels.

2) Fuse $A$ and $B$ coefficient sets in respective

3) Reconstruct the image by applying inverse lifting scheme to coefficient sets.

![Image of fusion process](image)

**Fig. 5 Fusion process of images $A$ and $B$**

### 2.2 The choice of wavelet and decomposition level

The choice of wavelet is very important to the quality of the image fusion. Take some interesting and useful properties into account, for example, symmetric, orthogonal, continuous and compactly supported, etc.\[81\], the choice of lifting scheme is very natural because of its entirely inverse and lossless characters.

The number of the wavelet decomposition level in this paper is 5 or 6 according to the Reference \[6\].

### 2.3 The rule of image fusion

It is very clearly from Fig. 4 that the image fusion rules play a very important role in the process. In order to prove the availability of the method, two images are taken under a fusion experiment later. As we can see from Fig. 6, Fig. 6(a) and 6(b) are fuzzed in different places. After fusion transform, we can get more accurate images.

Decompose image $A$ and $B$ separately to level $J$ by applying lifting, then the low frequency coefficient sets $S_A(2^j; x, y)$, $S_B(2^j; x, y)$ and high frequency coefficient sets $W_A^k(2^j; x, y)$, $W_B^k(2^j; x, y)$ can be got, where $k = 1, 2, 3$ represent three directions and $j = 1, 2, \ldots, J$. And then arrange the coefficient sets as Mallat's tower distributing form. For the most useful edge information may not concentrate only on one pixel, wavelet coefficients by the rules below will be chosen.

1) In the case of high frequency weight, the absolute value tool at the same level to judge should be adopted. In high-pass frequency band of $2^j$, for example, confirm the fusion window measure firstly (3X3 or 5X5 generally, this paper chooses 3X3 window). Supposing

\[
R_A = \sum_{m,n=-1}^1 W_A^k(2^j; x+m, y+n)
\]

\[
R_B = \sum_{m,n=-1}^1 W_B^k(2^j; x+m, y+n)
\]

\[
W_k(2^j; x, y) = \begin{cases} 
W_A^k(2^j; x, y), & \text{when } R_A > R_B \\
W_B^k(2^j; x, y), & \text{other}
\end{cases}
\]

where $k = 1, 2, 3$, and $j = 1, 2, \ldots, J$.

2) When the scale is $2^j$, which is refer to low frequency weight $LL$, take the following formula to calculate for the reason that this part has great influence on the quality of image.

\[
S(2^j; x, y) = k_1 \cdot S_A(2^j; x, y) + k_2 \cdot S_B(2^j; x, y)
\]

where $k_1$ and $k_2$ are power coefficients, and $k_1 + k_2 = 1$. 

![Fig. 6 Two images are fuzzed in different places](image)
3 Experiments and result analysis

Considering that the software Visual C++ is based on Windows and its products are excel at GUI advantage that make the users easy to handle, so the experiment is based on Visual C++. The testing image's measure is 256 × 256. Three images are shown in Fig. 7 in which there are an image based on space fusion, a standard image and an image based on lifting scheme. The space fusion algorithm is as follows:

\[ f(x,y) = (f_1(x,y) + k_1 \cdot f_2(x,y)) \cdot k_2 - \| f_1(x,y) - k_1 \cdot f_2(x,y) \| \cdot k_3 \]  

Fig. 7 Two experimental results and standard image

where \( k_1 = 1.30, k_2 = 0.44, k_3 = 0.13 \), which are chosen from the results of the experiments.

As the result shows, the image after space fusion is fairly faint and its detail is out of focus, while the image base on lifting is very clear and integral. It is very close to the standard image. Objectively, there are many ways to judge the quality of fusion image. And three kinds of quantitative evaluation criteria for image fusion are proposed.

1) Signal-to-Noise of the peak value

\[
\text{PSNR} = 10 \cdot \log(255 \times 255 / \text{MSE}) \tag{11}
\]

\[
\text{MSE} = \left( \sum_{i=1}^{M} \sum_{j=1}^{N} (R(i,j) - F(i,j))^2 / (M \times N) \right) \tag{12}
\]

where \( M \) and \( N \) denote the number of rows and columns of image respectively; \( R(i,j) \) and \( F(i,j) \) are the standard image’s and fusion image’s pixel value in coordinate \((i, j)\) respectively. PSNR take Decibel as a unit commonly and the larger its value, the better the quality of the fusion image.

2) Standard deviation

\[
\delta = \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} [A(i,j) - A_f(i,j)]^2 / N^2} \tag{13}
\]

where \( A(i,j) \) is the gray value of pixel \((i,j)\) in fusion image; \( N^2 \) is the size of the image. The image is 256 × 256 in paper, and the smaller the \( \delta \), the better the quality is.

3) Average error

\[
\Delta = A_r(i,j) - A_f(i,j) \tag{14}
\]

where \( A_r(i,j) \) and \( A_f(i,j) \) are the same as definition in standard deviation. So

\[
\Delta = \frac{1}{N^2} \sum_{i=1}^{M} \sum_{j=1}^{N} | \Delta | = \frac{1}{N^2} \sum_{i=1}^{M} \sum_{j=1}^{N} | A_r(i,j) - A_f(i,j) | \tag{15}
\]

Table 1 The quantization measurement results by the two algorithms

| Fusion algorithm | Signal-to-Noise of the peak value | Standard deviation | Average error |
|------------------|----------------------------------|--------------------|---------------|
| Space fusion     | 53, 213                          | 2, 611             | 0.893         |
| Lifting scheme   | 62, 796                          | 2, 320             | 0.683         |

From Table 1, the method based on lifting is much better than the space fusion.

4 Conclusions

Lifting provides insight into the construction of the wavelet transform, and the adaptive incorporation and nonlinear operating included into the transform are allowed. The nonlinear and adaptive filters can be introduced into the transform. (Continued on Page 200)