Update on very light \( CP \)-odd scalar in the Two-Higgs-Doublet Model

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In a previous work we have shown that a general two-Higgs-doublet model (THDM) with a very light \( CP \)-odd scalar can be compatible with electroweak precision data, such as the \( \rho \) parameter, \( \text{BR}(b \to s \gamma) \), \( R_h \), \( A_h \), \( \text{BR}(\Upsilon \to A \gamma) \), \( \text{BR}(\eta \to A \gamma) \), and \( (g - 2) \) of muon. Prompted by the recent significant change in the theoretical status of the latter observable, we comment on the consequences for this model and update the allowed parameter region. It is found that the presence of a very light scalar with a mass of 0.2 GeV is still compatible with the new theoretical prediction of the muon anomalous magnetic moment.

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I. INTRODUCTION

The possibility of a Higgs boson decaying into a pair of light \( CP \)-odd scalars was considered in Ref. \textsuperscript{[1]}. Although it is very unlikely that this particle can be accommodated in the minimal supersymmetric standard model (MSSM), in the light of the restrictions imposed by the current low-energy data on the parameters of this model, a very light \( CP \)-odd scalar \( A \) can still arise in some other extensions of the standard model (SM), such as the minimal composite Higgs model \textsuperscript{[2]}, or the next-to-minimal supersymmetric model \textsuperscript{[3]}. Therefore, the existence of a very light \( CP \)-odd scalar not only proves new physics but also casts the most commonly studied MSSM in doubt. Furthermore, studying the couplings of the light \( CP \)-odd scalar to the SM fermions may help discriminating models of electroweak symmetry breaking – either a weakly interacting model (e.g., the next-to-minimal supersymmetric model) or a strongly interacting model (e.g., the minimal composite Higgs model). Apart from the above implications arising from the existence of a very light \( CP \)-odd scalar, our main interest in studying this particle stems from the fact that its phenomenology is indeed rather exciting: an interesting aspect of a light \( A \) is that if its mass \( M_A \) is less than twice that of the muon \( m_\mu \), i.e. less than about 0.2 GeV, it can only decay into a pair of electrons \( (A \to e^+ e^-) \) or photons \( (A \to \gamma \gamma) \). Hence, the decay branching ratio \( \text{BR}(A \to \gamma \gamma) \) can be sizable. Consequently, \( A \) can behave like a fermiophobic \( CP \)-odd scalar and predominantly decay into a photon pair, which would register in detectors of high energy collider experiments as a single photon signature when the momentum of \( A \) is much larger than its mass \( \sqrt{2} M_A \).

In a previous work \textsuperscript{[4]} we performed an extensive analysis within the framework of the two Higgs doublet model (THDM) and found that a very light \( CP \)-odd scalar can still be compatible with precision data, such as the \( \rho \) parameter, \( \text{BR}(b \to s \gamma) \), \( R_h \), \( A_h \), \( \text{BR}(\Upsilon \to A \gamma) \), and the muon anomalous magnetic moment \( \mu_\mu \) \textsuperscript{[5]}. We considered different values for \( \sin^2(\beta - \alpha) \) and found the constraints imposed on the remaining parameters of the model, which we summarize in Table \textsuperscript{[6]} where \( M_H \) (\( M_h \)) stands for the heavy (light) \( CP \)-even scalar and \( M_{H^\pm} \) for the charged scalar. As for the soft breaking term \( \mu_{12} \), it is not involved in any of the above observables, so they cannot be used to constrain it. Since \( \mu_{12} \) has no relevance for the present discussion (the purpose of this work is to update the bounds derived from the changes in the status of the theoretical value of the muon anomaly), we refer the reader to Ref. \textsuperscript{[4]}, where LEP-2 data were used to set bounds on this parameter. In obtaining the bounds shown in Table \textsuperscript{[6]} we have used the lower value of 110 GeV for \( M_h \). We recall that the LEP-2 direct search bound requires \( M_h > 114.1 \) GeV at the 95\% C.L. \textsuperscript{[6]}. However, in the presence of new physics such a bound can be substantially relaxed. As explained in Ref. \textsuperscript{[4]}, the reason why the LEP-2 bound \( (M_h > 114.1 \text{ GeV}) \) does not apply in our model is because this bound is based on the SM specific value of \( \text{BR}(h \to b\bar{b}) \) \textsuperscript{[4]}. In the THDM, the new \( h \to AA \) decay mode can significantly reduce the \( h \to b\bar{b} \) branching ratio. This was clearly illustrated in the Fig. 9 of Ref. \textsuperscript{[4]} for some allowed parameter space of the model. In any case, the new decay channel \( (h \to AA) \) registers as a di-photon signature \( (h \to \gamma \gamma) \) for...
which LEP-2 has already set a lower bound. By taking both the $AA$ and $b\bar{b}$ decay modes into consideration, a lower bound for $M_h > 103$ GeV can be established in our light $A$ scenario [4].

At this point we would like to emphasize that, given the recent measurements of $a_\mu$ at Brookhaven National Laboratory (BNL) [7], the bounds on new physics effects imposed by the muon $(g - 2)$ data depend largely on the theoretical value predicted by the SM for the nonperturbative hadronic contribution to $a_\mu$. In our analysis [4], we followed a conservative approach and considered various predictions for the hadronic correction $a^{\text{had}}_\mu$ [8, 9, 10, 11], which in fact has been the source of debate recently [12, 13, 14]. For instance, the bounds shown in Table I were obtained from the calculation presented in Ref. [8], which was the one allowing the largest parameter space.

| Constraint | Type-I THDM | Type-II THDM |
|------------|-------------|-------------|
| $(g - 2)_\mu$ | $\tan \beta > 0.4$ | $\tan \beta < 2.6$ |
| $[\tan \beta > 1] b \to s\gamma$ | $M_{H^+} > 100$ GeV | $M_{H^+} > 200$ GeV |
| $[0.5 < \tan \beta < 1] b \to s\gamma$ | $M_{H^+} > 200 - 350$ GeV | |
| $0.6 < [\tan \beta < 1] R_h$ | $M_{H^+} > 200 - 600$ GeV | $M_{H^+} > 200 - 600$ GeV |
| $[\sin^2(\beta - \alpha) = 1] \Delta \rho$ | $M_H \sim M_{H^+}$ | $M_H \sim M_{H^+}$ |
| $[\sin^2(\beta - \alpha) = 0.8] \Delta \rho$ | $M_H \sim 1.2 M_{H^+}$ | $M_H \sim 1.2 M_{H^+}$ |
| $[\sin^2(\beta - \alpha) = 0.5] \Delta \rho$ | $M_H \sim 1.7 M_{H^+}$ | $M_H \sim 1.7 M_{H^+}$ |

After the completion of our work, it was evident that the latest precision measurement of $a_\mu$ at BNL [7] along with some theoretical predictions for $a^{\text{had}}_\mu$ disfavored the presence of a light $A$ in the THDM. As is well known, the BNL data opened the prospect for new physics as the experimental value of $a_\mu$ appeared to be more than 2.6 $\sigma$ above the theory prediction based on some calculations of the hadronic vacuum polarization. At the one-loop order, a light $CP$-odd scalar can give a significant negative contribution to $a_\mu$, making it harder for this type of model to be consistent with experiment. However, the two-loop calculation can yield a large correction to the one-loop result as pointed out in [15]. Although this fact seems to contradict perturbation theory, the unusual situation in which a two-loop diagram can give a contribution of similar size or even larger than that from the one-loop diagrams within a perturbative calculation was noted first by Bjorken and Weinberg when evaluating the Higgs scalar contribution to the $\mu \to e\gamma$ decay [16]. It is straightforward to see that this situation also occurs in the calculation of the Higgs scalar contribution to $a_\mu$. The reason is that the coupling of the Higgs scalar to the muon enters twice in the one-loop diagram, whereas at the two-loop level there appears a diagram in which this coupling enters just once, together with a line where the Higgs scalar couples to a heavy fermion pair (see Fig. 1). This gives rise to an enhancement factor, due to the couplings, that compensates the suppression factor $g^2/(16\pi^2)$, due to an additional loop. It turns out that the diagram of Fig. 1(c) gives by far the most dominant contribution at the two-loop level. Therefore, we do not expect large uncertainties arising from unknown higher order terms. In our previous analysis, even when we considered the two-loop calculation for the $CP$-odd scalar contribution to $a_\mu$, together with the hadronic correction quoted in Ref. [8], i.e. that by Davier and Höcker [10], we found that there was no allowed parameter space (in the type-II THDM) in the $\tan \beta$ vs. $M_A$ plane when $M_A$ was below 3 GeV. Nevertheless, there were other SM calculations yielding $a_\mu$ close enough to the experimental value as to allow a very light $A$.

![FIG. 1: Contribution from the THDM to the anomalous magnetic moment of muon: (a) neutral Higgs bosons, (b) charged Higgs boson, and (c) the leading two-loop contribution from the $CP$-odd scalar.](image-url)
TABLE II: Contributions to the anomalous magnetic moment of muon in the SM [19], prior to the discovery of a wrong sign in the pion pole correction to \(a_\mu\)\(^\text{theory}\)(1.b.l.), which significantly changed the \(a_\mu\)\(^\text{theory}\) prediction. All values are given in units of \(10^{-11}\).

| Contribution | SM prediction |
|--------------|---------------|
| \(a_\mu\)\(^\text{had}\) | 116584705.7 (1.8) |
| \(a_\mu\)\(^\text{weak}\) | 151 (4) |
| \(a_\mu\)\(^\text{had}(1.b.l.)\) | -79.2 (15.4) \(^a\) |
| \(a_\mu\)\(^\text{had}(\text{h.o.})\) | -101 (6) |

\(^a\)This value has been found to be wrong in Ref. [17].

Since the publication of [8], there has been a lot of controversy regarding the theoretical value of the muon anomalous magnetic moment. It is evident that before claiming the presence of any new physics effect, an extensive reexamination of every contribution to \(a_\mu\) is necessary [12, 13, 14]. Along these lines, a reevaluation [17] of the hadronic light by light correction found a sign error in earlier calculations [18] of this contribution, which has resulted in a significant change of the \(a_\mu\) prediction. Once the corrected value is taken into account, the discrepancy between experiment and theory reduces down to the level of 1.6 \(\sigma\). In the light of this result, we believe it is worth revisiting our work and reexamining our previous bounds.

II. ALLOWED PARAMETER RANGE FOR \(M_A\) AND \(\tan \beta\)

The SM prediction of \(a_\mu\) is composed of the following three parts [19]:

\[a_\mu^{\text{theory}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}},\]

where the electroweak corrections have been computed to a very good accuracy: they have a combined error of the order of \(5 \times 10^{-11}\), which is already about one order of magnitude smaller than the ultimate goal of the E821 experiment [8]. In contrast, the hadronic contribution \(a_\mu^{\text{had}}\) contains the bulk of the theoretical error (\(\sim 70 \times 10^{-11}\)) and can be decomposed into three parts [19], namely the hadronic vacuum polarization contribution \(a_\mu^{\text{had}(\text{h.v.p.})}\), the hadronic light-by-light correction \(a_\mu^{\text{had}(1.b.l.)}\), and other hadronic higher order terms \(a_\mu^{\text{had}(\text{h.o.})}\):

\[a_\mu^{\text{had}} = a_\mu^{\text{had}(\text{h.v.p.})} + a_\mu^{\text{had}(1.b.l.)} + a_\mu^{\text{had}(\text{h.o.})}.\]

In our previous analysis we used the values shown in Table [12] for each contribution to \(a_\mu^{\text{theory}}\) together with the \(a_\mu^{\text{had}(\text{h.v.p.})}\) predictions to be discussed below. In the months following the publication of our work, a new situation arose: the sign of the pion pole contribution to the hadronic light by light correction was found to be wrong [17]. Very interestingly, this contribution alone represents about 70\% of the full \(a_\mu^{\text{had}(1.b.l.)}\). It turns out that after correcting this mistake, the \(a_\mu^{\text{had}(1.b.l.)}\) value gets significantly changed and even its sign gets flipped. As a result, the discrepancy between the experiment and theory reduces down to the level of 1.6 \(\sigma\). Subsequent publications have confirmed this finding [20, 21, 22]. In Table [12] we list the most recent evaluations of \(a_\mu^{\text{had}(1.b.l.)}\). In addition, there is one more calculation that is based on chiral perturbation theory [23]:

\[a_\mu^{\text{had}(1.b.l.)} = \left(55^{+50}_{-60} + 31\hat{C}\right) \times 10^{-11},\]

where \(\hat{C}\) is an unknown low-energy constant that parametrizes some subdominant terms. We will not consider this result here but only mention it as an example of a calculation that is still open to debate. For the purpose of this work we will take an average of the top three results shown in Table [12] and study the consequences on the allowed parameter space of the THDM.

After introducing the corrected value of \(a_\mu^{\text{had}(1.b.l.)}\), the sum of all the contributions to \(a_\mu^{\text{theory}}\) except \(a_\mu^{\text{had}(\text{h.v.p.})}\) is:

\[a_\mu^{\text{theory}} - a_\mu^{\text{had}(\text{h.v.p.})} = 116584845.3 (17.1) \times 10^{-11},\]
TABLE III: The most recent evaluations of the hadronic light by light contribution to $a_{\mu}^{\text{had}}(\text{h.b.l.})$. These corrected values contrast with the wrong one shown in Table I.

| Author | $a_{\mu}^{\text{had}}(\text{h.b.l.)} \times 10^{11}$ |
|--------|----------------------------------|
| KN     | 83 (12)                          |
| HK     | 89 (15)                          |
| BPP    | 83 (32)                          |
| BCM    | 56\textsuperscript{a}           |

\textsuperscript{a}This value accounts only for the pion pole contribution.

TABLE IV: Some of the most recent calculations of $a_{\mu}^{\text{had}}(\text{h.v.p.})$ together with the respective theory prediction $a_{\mu}^{\text{theory}}$ and the discrepancy $\Delta a_{\mu}$ between experiment and theory. The last column represents the bounds on any new physics contribution $a_{\mu}^{\text{NP}}$ at the 95\% C.L. All of the values are given in units of $10^{-11}$.

| Author | $a_{\mu}^{\text{had}}(\text{h.v.p.})$ | $a_{\mu}^{\text{theory}}$ | $\Delta a_{\mu}$ | Allowed range for $a_{\mu}^{\text{NP}}$ |
|--------|----------------------------------|-----------------|-----------------|----------------------------------|
| ADH    | 7011(94)                         | 116591883.6 (95.34) | 166.7 (179.53)  | [-185 – 519]                     |
| DH     | 6921(62)                         | 116591769.3 (64.31) | 253.7 (164.92)  | [-70 – 577]                      |
| J      | 6974(105)                        | 116591833.3 (112.3) | 189.7 (188.8)   | [-180 – 560]                     |
| N      | 7031(77)                         | 116591876.3 (78.88) | 146.7 (171.25)  | [-189 – 482]                     |
| TY     | 6952(64)                         | 116591797.3 (66.25) | 225.7 (165.81)  | [-99 – 551]                      |

where the errors have been composed quadratically. \textsuperscript{1} As for the $a_{\mu}(\text{h.v.p.})$ term, its evaluation has also been the source of renewed interest lately. \textsuperscript{2} As in our previous work, here we will use a conservative approach and consider some representative evaluations of $a_{\mu}(\text{h.v.p.})$. In the second column of Table \textsuperscript{I} we show some of the most recent results, which were compiled in \textsuperscript{24}, whereas in the third column we show the full theory prediction, which is obtained after adding up each value in the second column to Eq. (3).

As for the experimental value $a_{\mu}^{\text{exp}}$, the data obtained during the 1999 running period combined with previous measurements give \textsuperscript{7}

$$a_{\mu}^{\text{exp}} = 116592023 (152) \times 10^{-11}. \quad (4)$$

One thus can obtain the discrepancy between experiment and theory $\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{theory}}$ for each different evaluation of $a_{\mu}(\text{h.v.p.})$, as shown in the fourth column of Table \textsuperscript{I}. Finally, if we assume that the discrepancy between theory and experiment is to be ascribed to new physics effects, we can obtain the bounds shown in the last column of the same Table for the new physics contribution to the anomalous magnetic moment at the 95\% C.L., which is denoted by $a_{\mu}^{\text{NP}}$. Those bounds on $a_{\mu}^{\text{NP}}$ should be compared to those used in our previous analysis, cf. Eq. (2) in Ref. \textsuperscript{4}.

Given the new bounds on $a_{\mu}^{\text{NP}}$, we update the constraint imposed by it on the tan $\beta - M_A$ plane within the THDM. The analytical expressions for the contribution of either a CP-even or a CP-odd scalar (Fig. \textsuperscript{1}) can be found in Appendix A of Ref. \textsuperscript{4}. We will use the two-loop calculation for the contribution from the CP-odd scalar \textsuperscript{13}. In order to satisfy the bounds shown in Table \textsuperscript{I}, we are assuming that the remaining four Higgs scalars are much heavier than the CP-odd scalar $A$, so their contribution to $a_{\mu}$ turns out to be negligibly small as compared to that coming from the latter. Also, we are considering that $\sin^2(\beta - \alpha) = 1$. The reason why we make this choice is because in our scenario with a very light CP-odd scalar the most convenient way to meet the constraint imposed by the $\rho$ parameter is to have $M_H$ and $M_{H^+}$ nearly degenerate and $\sin^2(\beta - \alpha)$ close to 1 \textsuperscript{11}. For comparison purposes, we will analyze the bounds arisen from the theoretical predictions based on the DH \textsuperscript{10}, J \textsuperscript{11}, N \textsuperscript{22} and TY \textsuperscript{24} calculations of $a_{\mu}^{\text{had}}(\text{h.v.p.})$, which are the most representative and recent ones. We would like to note that, as observed through Fig. \textsuperscript{2} to Fig. \textsuperscript{3}, the bounds from the J \textsuperscript{11} and N \textsuperscript{22} calculations are almost indistinguishable.

In Figs. \textsuperscript{4} \textsuperscript{6} we show the allowed regions in the tan $\beta - M_A$ plane for both types of THDMs. In Fig. \textsuperscript{5} which shows the low $M_A$ regime, it can be seen clearly that even if one considers the DH calculation of $a_{\mu}^{\text{had}}$, which is the one with the smallest error, there is still the possibility of having a CP-odd scalar with a mass of the order of 0.2 GeV in the type-II THDM as long as tan $\beta < 1.43$, whereas for a type-I THDM tan $\beta$ has to be greater than 0.87. This is a very significant change with respect to the results obtained when using the old (uncorrected) value of $a_{\mu}^{\text{theory}}$. In that case,
the DH calculation did not allow for a light \( CP \)-odd scalar in either type of THDM, though other calculations did allow such a possibility.

![Graph](image)

**FIG. 2:** The regions (above the curves for type-I and below the curves for type-II THDM) in the \( \tan \beta \) versus \( M_A \) plane allowed by the \( a_\mu \) data at the 95% CL. Four different curves are displayed depending whether the SM prediction is obtained from the DH, J, N or TY calculation of \( a_\mu^{\text{had}} \) (h.v.p.). The two-loop contribution from the light \( A \) has been used.

![Graph](image)

**FIG. 3:** The region (above the curves) in the \( \tan \beta \) versus \( M_A \) plane of a type-I THDM allowed by the \( a_\mu \) data at the 95% CL. The allowed regions based on the DH, J, N and TY calculations are above the curves. The two-loop contribution from the light \( A \) has been used.

As stated above, so far our results have been derived from the two-loop contribution from the \( CP \)-odd scalar to \( a_\mu^{\text{NP}} \). It is also interesting to repeat the above analysis using only the one-loop calculation for \( a_\mu^{\text{NP}} \). Its result is depicted in Figs. 4 and 5. The old (uncorrected) theory prediction based on the DH calculation required any new physics contribution to \( a_\mu \) to be positive. However, the one-loop contribution from a light \( CP \)-odd scalar is always negative. Therefore, the old SM theory prediction for \( a_\mu \) combined with the THDM one-loop correction strongly disfavored the existence of a very light \( CP \)-odd scalar. This is to be contrasted with the conclusion drawn from the corrected value of \( a_\mu^{\text{theory}} \). In that case, there is indeed an allowed region of \( \tan \beta \) when \( M_A \sim 0.2 \) GeV, though this region is smaller than the one allowed by the two-loop calculation of \( a_\mu^{\text{NP}} \) (cf. Figs. 3 and 5, and Figs. 4 and 6). As shown in Fig. 4, there is an interesting feature in the \( \tan \beta \) versus \( M_A \) plane of a type-II THDM when \( M_A \) is around 2.6 GeV. It is because for \( M_A \sim 2.6 \) GeV, the two-loop contribution from a light \( CP \)-odd scalar becomes as large as the respective one-loop contribution but with an opposite sign, so the total effect cancels.

**A. Bounds on \( \tan \beta \) from meson decays**

For completeness we now turn to analyze the bounds obtained on THDMs with a very light \( CP \)-odd scalar from meson decays. A very light Higgs scalar (\( CP \)-odd or \( CP \)-even) can be a decay product of some hadrons, like the \( \eta \) and \( T \) mesons. For the latter, a measured upper bound to the \( X + \gamma \) decay channel has been set \({27}\) that can be
FIG. 4: The regions (below the curves) in the tan$\beta$ versus $M_A$ plane of a type-II THDM allowed by the $a_\mu$ data at the 95\% CL. The allowed regions based on the DH, J, N and TY calculations are below the curves. The two-loop contribution from the light $A$ has been used.

FIG. 5: Same as Fig. 4, but only the one-loop contribution from the light $A$ is considered.

FIG. 6: Same as Fig. 5, but only the one-loop contribution from the light $A$ is considered.

used to constrain the $A\bar{b}b$ coupling. Denote the Yukawa coupling of $A\bar{b}b$ to be $k_d m_b/v$, with $k_d = \tan \beta$ (cot $\beta$) in the type-II (type-I) model. Then, the data of the meson decay $\Upsilon \rightarrow \gamma + X$ requires $k_d < 1$. (We refer the reader to Refs. 3-4 for a detailed discussion.)

As shown in Ref. 28, there is another decay process that can strongly constrain tan $\beta$, namely $\eta \rightarrow \pi S$, where $S$ is a very light $CP$-even scalar. Those results can be translated into the case of a $CP$-odd scalar. In particular, the experimental upper limit
\[ \text{BR}(\eta \rightarrow \pi^0 e^+ e^-) \leq 5 \times 10^{-5} \]  

(5)
can be used to obtain the following constraint on a THDM \( CP \)-odd scalar with mass \( M_A \) lying in the range \( 2m_e \leq M_A \leq 2m_{\mu} \):

\[ (k_d - k_u)^2 \lambda \left( \frac{m_{\eta}^2}{m_{\eta}^2}, \frac{m_{\eta}^2}{m_{\eta}^2} \right) \leq 1.5 \]  

(6)

where \( k_u \) is cot\( \beta \) for either type-I or type-II THDM and \( k_d \) has been defined above. The function \( \lambda \) is given by \( \lambda^2(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \). From here we can conclude that cot\( \beta \geq 0.65 \) for type-I THDM and 0.55 \( \leq \) tan\( \beta \) \( \leq 1.8 \) for type-II THDM. We thus can confirm that the hadron decay data together with the muon \((g - 2)\) measurement require tan\( \beta \) to be of order 1 if there exists a very light pseudoscalar with a mass smaller than \( 2m_{\mu} \).

III. OVERALL DESCRIPTION OF THE GENERAL THDM WITH A LIGHT \( A \)

Once the allowed parameter range for tan\( \beta \) and \( M_A \) has been updated, there remains five other parameters to consider: the \( CP \)-even neutral Higgs mixing angle \( \alpha \), the soft breaking term \( \mu_{12} \) and the three other Higgs masses: \( M_h, M_H \) and \( M_{H^+} \). Since we already know that tan\( \beta \) has to be of order 1 we can address the status of the charged Higgs mass \( M_{H^+} \) independently of the other parameters. It turns out that both the \( b \rightarrow s\gamma \) and the \( R_b \) data require \( H^+ \) to be considerably heavy \([4, 29]\):

\[ M_{H^+} \gtrsim 350 \text{ GeV} \].

(7)

Such a high lower bound for the \( H^+ \) mass affects the allowed values of the mixing angle \( \alpha \). In Ref. \([4]\) we show that the \( \rho \) parameter requires \( M_H \) and \( M_{H^+} \) to be very correlated depending on the value of \( \sin^2(\beta - \alpha) \). In fact, if a very light \( CP \)-odd scalar is to be allowed, the easiest way to satisfy the bound imposed by \( \rho \approx 1 \) is to have \( M_H \) and \( M_{H^+} \) degenerate and \( \sin^2(\beta - \alpha) = 1 \). With this choice, \( M_h \) is not restricted since it does not contribute to the \( \rho \) parameter. As we consider values of \( \sin^2(\beta - \alpha) \) smaller than 1, it turns out that \( \rho \) is very sensitive to the masses of \( H \) and \( H^+ \). For instance, if \( \sin^2(\beta - \alpha) = 0.5 \), \( M_H \) must be at least of the order of 500 GeV \([6]\). Generally speaking, our conclusion on the bounds on a very light \( CP \)-odd scalar in the THDM does not change significantly for \( 0.5 < \sin^2(\beta - \alpha) < 1 \) as long as the other Higgs bosons in the model are heavy enough. For a very small value of \( \sin^2(\beta - \alpha) \), much less than 0.5, the \( \rho \)-parameter data would have required the mass of \( H \) to be at the TeV order.

IV. CONCLUSION

In conclusion, with the recent correction to the SM prediction of \( a_\mu \), the current muon \((g - 2)\) data, together with other precision data (cf. Table I), still allows a light (\( M_A \sim 0.2 \text{ GeV} \)) \( CP \)-odd scalar boson in the THDM. Due to this new development in the SM theory calculation of muon’s \((g - 2)\), the allowed range of tan\( \beta \) in the Type-I or Type-II THDM is modified, and our result is summarized in Table \([4]\). It is interesting to note that the phenomenology at high energy colliders predicted by the THDM with a light \( CP \)-odd Higgs boson is dramatically different from that predicted by the usual THDM in which the mass of the \( CP \)-odd scalar is at the weak scale. A detailed discussion on this point can be found in Ref. \([4]\). In particular, various potential discovery modes were studied in there: it was found that the Fermilab Tevatron, the CERN large hadron collider (LHC) and the planned \( e^+e^- \) linear collider (LC) have a great potential to either detect or exclude a very light \( A \) in the THDM.

Finally, we note that while a light \( CP \)-odd scalar in THDM is still compatible with all the precision data, it has been shown recently in Ref. \([30]\) that a light \( CP \)-odd scalar in the MSSM will violate the constraint derived from the \( Zb\bar{b} \) coupling. This is because in the MSSM, the masses of the five Higgs bosons are related by the mass relations required by supersymmetry. Hence, with a light \( CP \)-odd scalar, the mass of the other Higgs bosons cannot be arbitrary large, and it is difficult to yield the decoupling limit when calculating low energy observables.

Note added:
TABLE V: Constraints on $\tan \beta$ from the muon $(g - 2)$ data for Type-I and Type-II THDM, with $M_A = 0.2$ GeV, based on various SM theory predictions of $a_{\mu}^{\text{had}}(h.v.p)$. The two-loop contribution for the $CP$-odd scalar has been used.

| Theory prediction | Type-I THDM | Type-II THDM |
|-------------------|-------------|--------------|
| DH [10]           | $\tan \beta > 0.87$ | $\tan \beta < 1.43$ |
| J [11]            | $\tan \beta > 0.54$ | $\tan \beta < 2.19$ |
| N [24]            | $\tan \beta > 0.53$ | $\tan \beta < 2.24$ |
| TY [24]           | $\tan \beta > 0.73$ | $\tan \beta < 1.67$ |

During the review process of this manuscript, the muon $(g - 2)$ collaboration announced a new result based on data collected in the year 2000 [31], in which the experimental uncertainty has been reduced to one half that of the previous measurement while the central value of $a_{\mu}^{\exp}$ remains about the same. [The new data yields $a_{\mu}^{\exp} = 11659204(7)(5) \times 10^{-10}$ (0.7 ppm).] According to the latest experimental data, we have updated Figs. 2 to 6 in this paper to Figs. 7 to 11. The new data suggests that a very light $CP$-odd scalar is not allowed in the type-I or type-II THDM based on the SM calculation done by DH [10] and TY [24]. However, based on the N [25] and J [11] calculations, a very light $CP$-odd scalar is still possible though the allowed parameter space of the THDM has been tightly constrained.

FIG. 7: Same as Fig. 3, but with the latest experimental data from the muon $(g - 2)$ collaboration [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

FIG. 8: Same as Fig. 4, but with the latest experimental data from the muon $(g - 2)$ collaboration [31]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.
FIG. 9: Same as Fig. 4, but with the latest experimental data from the muon \((g - 2)\) collaboration [3]. The region allowed by the DH [10] and TY [24] calculations is bounded by the respective lines.

FIG. 10: Same as Fig. 5, but with the latest experimental data from the muon \((g - 2)\) collaboration [3]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.

FIG. 11: Same as Fig. 6, but with the latest experimental data from the muon \((g - 2)\) collaboration considered [3]. There is no allowed region in this range of parameters according to the DH [10] and TY [24] calculations.
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[1] B. A. Dobrescu, G. Landsberg and K.T. Matchev, Phys. Rev. D 63, 075003 (2001).
[2] B. A. Dobrescu, Phys. Rev. D 63, 015004 (2001).
[3] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, The Higgs Hunter’s Guide, Addison-Wesley, 1996. Errata: arXiv: hep-ph/9302272.
[4] F. Larios, G. Tavares-Velasco and C.-P. Yuan, Phys. Rev. D 64, 055004 (2001).
[5] ALEPH, DELPHI, L3 and OPAL (The LEP working group for Higgs searches), talk presented at 2001 International Symposium on Lepton and Photon Interactions at High Energies, Rome, 2001, arXiv: hep-ex/0107024. U. Schwickerath, arXiv: hep-ph/0205129.
[6] M. Aciarri et al., Phys. Lett. B 489, 115 (2000); R. Barate et al., Phys. Lett. B 487, 241 (2000); P. Abreu et al., ibid. 507, 89 (2001).
[7] H.N. Brown, et. al., Phys. Rev. Lett. 86, 2227 (2001).
[8] J. A. Casas, C. López and F.J. Ynduráín, Phys. Rev. D 32, 736 (1985).
[9] K. Adel and F. J. Ynduráin, Rev. Acad. Ciencias (Esp.) 92, 113 (1998) [arXiv: hep-ph/9509308].
[10] M. Davier and A. Höcker, Phys. Lett. B 435, 427 (1998); Phys. Lett. B 419, 419 (1998); for a recent summary of these results see A. Höcker, arXiv: hep-ph/0111245.
[11] F. Jegerlehner, DESY 01-028, arXiv: hep-ph/0104304.
[12] F. J. Ynduráin, arXiv: hep-ph/0102312.
[13] W. J. Marciano and B. L. Roberts, arXiv: hep-ph/0105057.
[14] K. Melnikov, Int. J. Mod. Phys. A 16, 4591 (2001).
[15] D. Chang, W.-F. Chang, C.-H. Chou, and W.-Y. Keung, Phys. Rev. D 63, 091301 (2001).
[16] J. D. Bjorken and S. Weinberg, Phys. Rev. Lett. 38 622 (1977).
[17] M. Knecht and A. Nyffeler, Phys. Rev. D 65, 073034 (2002); M. Knecht, A. Nyffeler, M. Perrottet and E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002).
[18] J. Bijnens, E. Pallante, and J. Prades, Nucl. Phys. B474, 379 (1996); M. Hayakawa, T. Kinoshita, and A. I. Sanda, Phys. Rev. D 54, 3137 (1996); M. Hayakawa and T. Kinoshita, ibid. 57, 465 (1998).
[19] V. Hughes and T. Kinoshita, Rev. Mod. Phys. 71, S133 (1999).
[20] M. Hayakawa and T. Kinoshita, arXiv: hep-ph/0112102.
[21] J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B626, 410 (2002).
[22] I. Blokland, A. Czarnecki and K. Melnikov, Phys. Rev. Lett. 88, 071803 (2002).
[23] M. Ramsey-Musolf and M. Wise, arXiv: hep-ph/0201297.
[24] J. F. de Troconiz and F. J. Yndurain, Phys. Rev. D 65, 093001 (2002).
[25] S. Narison, Phys. Lett. B 513, 53 (2001); Erratum ibid. B 526, 414 (2002); for an update on this calculation see arXiv: hep-ph/0203053.
[26] R. Alemany, M. Davier and A. Höcker, Eur. Phys. J. C 2, 123 (1998).
[27] Particle Data Group, K. Hagiwara et. al., Phys. Rev. D66, 010001 (2002).
[28] A. Pich and J. Prades, Phys. Lett. B245, 117 (1990); A. Pich, J. Prades and P. Yepes, Nucl. Phys. B388, 31 (1992).
[29] P. Gambino and M. Misiak, Nucl. Phys. B611, 338 (2001).
[30] A. G. Akeroyd, S. Baek, G. C. Cho and K. Hagiwara, arXiv: hep-ph/0205094.
[31] G. W. Bennett et al., arXiv: hep-ex/0208001.