A model that underlies the Standard model

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We assign the chiral fermion fields of the Standard model to triplets of flavor (family, generation, horizontal) \( SU(3)_f \) symmetry, for anomaly freedom add one triplet of sterile right-handed neutrino fields, and gauge that symmetry. First we demonstrate that the resulting quantum flavor \( SU(3)_f \) dynamics completely spontaneously self-breaks: Both the Majorana masses of sterile neutrinos and the masses of all eight flavor gluons come out proportional to the \( SU(3)_f \) scale \( \Lambda \). Mixing of sterile neutrinos yields new CP-violating phases needed for understanding the baryon asymmetry of the Universe. Second, the \( SU(3)_f \) dynamics with weak hypercharge radiative corrections spontaneously generates the lepton and quark masses exponentially suppressed with respect to \( \Lambda \). Three active neutrinos come out as Majorana particles extremely light by seesaw. The Goldstone theorem implies: (i) The electroweak bosons \( W \) and \( Z \) acquire masses. (ii) There are three axions, decent candidates for dark matter. Invisibility of the Weinberg-Wilczek axion \( a \) with mass \( m_a \sim \Lambda^2/\Lambda \) restricts the scale \( \Lambda \) from \( \Lambda \sim 10^{10}\text{GeV} \) upwards. Third, the composite ‘would-be’ Nambu-Goldstone (NG) bosons of all spontaneously broken gauge symmetries have their genuine composite massive partners: (i) One \( 0^+ \) flavorless Higgs-like particle \( h \) accompanying three electroweak ‘would-be’ NG bosons. (ii) Two \( 0^+ \) flavored Higgs-like particles \( h_3 \) and \( h_8 \) accompanying six flavored electroweak ‘would-be’ NG bosons. (iii) Three superheavy spin-zero sterile-neutrino-composites \( \chi_i \) accompanying eight flavored sterile-neutrino-composite ‘would-be’ NG bosons. We identify \( \chi_i \) with inflatons.

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I. INTRODUCTION AND SUMMARY

Standard model (SM) as a quantum field theory is firmly based on two general principles: The gauge principle, and the principle of spontaneous symmetry breaking. Principles are, however, more general than their particular realizations [1]. The SM realization of the gauge principle, defining the gauge particles of the underlying symmetries and fixing the form of their interactions with matter fields and with themselves is in full accord with data. The electroweak Higgs realization of the principle of spontaneous symmetry breaking giving particles softly their masses is less certain: Glorious CERN LHC discovery [2] of the spinless \( 0^+ 125 \text{GeV} \) boson with properties similar to the SM Higgs certainly does support the Higgs realization, technically all the way up to the Planck scale. It is, however, far from complete: First, it does not provide enough CP violation needed for the observed baryon asymmetry of the Universe. Second, it leaves neutrinos massless. Third, it does not offer any candidates for particles of dark matter. Fourth, it does not describe inflation of the early Universe. Fifth, if the CERN Higgs were indeed the Higgs boson of the Standard model the masses of quarks and charged leptons would stay theoretically arbitrary for ever.

This ‘environmental’ interpretation of the SM fermion mass spectrum is in sharp contrast with our understanding of the energy spectra of other quantum systems: Oscillators, nuclei, atoms and molecules have their spectra calculable. In the same vein, the spectrum of hadron masses is calculable in QCD in the chiral limit solely in terms of the QCD scale. That the laws of QCD at low momenta are known at present only to computers is another issue.

Here we suggest to replace the essentially classical Higgs sector of the SM with its ‘twenty-some’ parameters [1] by a new genuinely quantum non-Abelian dynamics. It is defined by gauging the flavor (family, generation, horizontal) \( SU(3)_f \) triplet index of three chiral SM lepton \((l^f_L, e^f_R)\) and quark \((q^f_L, u^f_R, d^f_R)\) families of the \( SU(2)_L \times U(1)_Y \) gauge invariant SM. This amounts to introduction of the octet of gauge flavor gluons \( C^a_u \), and for anomaly freedom to addition of one triplet of sterile right-handed neutrino fields \( \nu^f_R \). The resulting anomaly free, asymptotically free gauge \( SU(3)_f \) quantum flavor dynamics is characterized by one parameter. It is either the dimensionless gauge coupling constant \( h \) or, due to the dimensional transmutation, the theoretically arbitrary scale \( \Lambda \). Its Lagrangian is

\[
\mathcal{L}_f = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} + \bar{q}_L i \gamma_\mu D_L \bar{q}_R + \bar{u}_R i \gamma_\mu D_R u_L + \bar{d}_R i \gamma_\mu D_R d_L \\
+ \bar{l}_L i \gamma_\mu D_L e_R + \bar{\nu}_R i \gamma_\mu D_R \nu_R
\]

Treated in isolation \( \mathcal{L}_f \) should be appended by the \( SU(3)_f \) invariant hard Dirac fermion mass term

\[
\mathcal{L}_{\text{mass}} = -\sum_f (\bar{f}_R m_f f_L + h.c.)
\]

common to all three fermions \( f = u, d, e, \nu \) of a given electric charge. Such terms are, however, strictly prohibited by the gauge electroweak chiral \( SU(2)_L \times U(1)_Y \) symmetry tacitly always present in the game.

The form of \( \mathcal{L}_f \) is identical with the form of the QCD Lagrangian in the chiral limit. This is highly suspicious.
'In QCD we trust’, and it is a firm experimental fact that the flavor symmetry is not confining but badly broken.

Closer inspection reveals that the presence of the kinetic term \( \bar{\nu}_R i \gamma \cdot D \nu_R \) of the electrically neutral right-handed neutrinos makes the cardinal difference from QCD: The flavor gluon interaction likes to generate at the strong coupling dynamically the Majorana mass term

\[
L_{\text{Majorana}} = -\frac{1}{4} (\bar{\nu}_R M R (\nu_R) C + \text{h.c.})
\]

where \( M_{LR} \) are three different Majorana masses of order \( \Lambda \). There is no way how \( L_{\text{Majorana}} \) can be \( SU(3)_f \) invariant, so it is strictly prohibited at the Lagrangian level as a hard mass term. The Goldstone theorem applies, and the resulting composite ‘would-be’ NG bosons give rise to different masses of all eight flavor gluons \( C \) proportional to \( M_{LR} \). Hence the gauge \( SU(3)_f \) symmetry of \( \bar{L}_L \) gets dynamically completely self-broken. Because \( (\nu_R)C \) is a left-handed field transforming as an antitriplet of \( SU(3)_f \), the new dynamics is not QCD-like, but it is effectively chiral.

We will argue that the flavor gluon dynamics generates also the masses of the electroweakly interacting fermions and, due to the Goldstone theorem again the electroweak gauge symmetry is spontaneously broken down to \( U(1)_{em} \). The underlying ‘would-be’ NG bosons give rise to masses of the \( W \) and \( Z \) bosons.

Since it is not known at present how to put a chiral gauge theory on the lattice [2], the present attempt at computing the fermion mass spectrum should only be considered as a heuristic prototype computation. We believe nevertheless that it has all necessary attributes expected by common sense.

For analyzing the consequences of the fermion mass calculation we will borrow the strategy successfully applied in QCD in the \( SU(2)_L \times SU(2)_R \) chiral limit: The detailed properties of the QCD ground state responsible for the confinement of its colored constituents are not known. If, however, we assume that the essentially unknown vacuum breaks the global chiral symmetry spontaneously, the composite massless colorless NG pions, described in the effective chiral perturbation theory [4], are obliged to exist by the existence theorem of Goldstone.

To the best of our knowledge there is no existence theorem for other composite colorless (massive) hadrons. Because hadrons are phenomenologically so important many models of their formation were invented over the years. Grasping different expected properties of the confining vacuum they correspondingly differ: from the constituent quark model to the bag models of different types.

We will argue analogously: It is not known how the nonconfining vacuum of strongly coupled flavor dynamics acts in the infrared. If, however, we assume that it generates appropriate chiralities changing fermion proper self energies dynamically, the existence theorem of Goldstone implies definite firm predictions. There must be the whole spectrum of the true, ‘would-be’ and pseudo NG bosons of underlying global anomaly-free, local anomaly free, and global anomalous Abelian chiral symmetries with properties fixed by symmetry. The basic assumption thus implies:

(1) There is one true composite NG boson of a spontaneously broken global anomaly-free Abelian symmetry of the model. It is rather remarkable that the prediction of a massless spinless particle is not in flagrant conflict with data. The point is that it does not imply a new long-range force [3]. Moreover, it is always possible, if the experimental data demand, to gauge that symmetry. In such a case the ‘would-be’ NG boson disappears: We would cope with a new \( Z' \) characterized by a new gauge coupling \( g'' \) with mass proportional to the masses of all fermions to which \( Z' \) couples.

(2) There must be the massive gauge bosons of gauge chiral symmetries \( SU(2)_L \times U(1)_Y \) and \( SU(3)_f \). The ‘would-be’ NG bosons disappear from the physical spectrum and become the longitudinal polarization states of massive \( W, Z \) and of the flavor gluons \( C \) [3]. In the case of the electroweak \( SU(2)_L \times U(1)_Y \) there is a relation between Dirac masses of the electroweakly interacting fermions and the masses of \( W \) and \( Z \). In the case of gauge flavor dynamics the phenomenological viability requires that the flavor changing flavor gluons get very heavy masses. This comes out very naturally: The flavor gluon masses are the inevitable consequence of huge masses of sterile right-handed Majorana neutrinos.

(3) There must be three composite pseudo NG bosons of spontaneously broken global Abelian chiral anomalous symmetries. They acquire masses by instantons of three non-Abelian gauge forces present in the model. One of them we identify with the Weinberg-Wilczek axion [7], most welcome particle postulated originally to solve the strong CP problem. Some like it also as a hot candidate for cold dark matter [8]. Other particle is an ultra-light electroweak axion also postulated previously by Anselm and Uraltsev [9]. The third one is a new axion of quantum flavor dynamics. We fix its mass wishfully in the keV range, and offer it as a candidate for the explanation of several astrophysical puzzles [10].

Besides the collective excitations guaranteed by the Goldstone theorem it is natural to expect other massive composites of a strongly coupled dynamics. After the discovery of the Higgs boson this possibility became necessity. To the best of our knowledge there is no existence theorem for such particles i.e., the spectrum of composite non-NG excitations is model-dependent. Guided by the canonical Higgs model we look for the composite scalar fields with the SM Higgs field quantum numbers which can condense. We emphasize that the Standard model with its canonical Higgs sector is phenomenologically so successful that the existence of such a composite operator should be the necessary condition for the viability of the model.

The composite Higgs \( h \) is a massive partner of three
composite 'would-be' NG bosons which follow from the spontaneous electroweak symmetry breakdown by dynamically generated lepton and quark masses. They are identified in the composite scalar field by other means (by the Ward-Takahashi (WT) identity). Its Yukawa interaction extracted from the 'partnership' is identical with the SM one. Its interactions with the electroweak gauge bosons $W, Z, A$ are all the effective ones and uniquely come out from the ultraviolet (UV)-finite fermion loops.

Following the same reasoning we find analogous operator also in the sterile neutrino sector. It is a complex composite flavor sextet of $SU(3)_f$. Besides the eight composite 'would-be' NG bosons which follow from the spontaneous breakdown of $SU(3)_f$ by dynamically generated Majorana masses of sterile right-handed neutrinos it contains in this case three Higgs-like massive composite excitations $\chi_i(x)$. Twelveth component is the composite pseudo NG boson discussed in Sect.V.

To follow the same reasoning consistently we are enforced to consider also the composite multicomponent Higgs field associated with spontaneous breakdown of $SU(3)_f$ by masses of the electroweakly interacting leptons and quarks. It turns out that it is a composite octet of $SU(3)_f$. Because the Higgs octet condensate breaks in general the $SU(3)$ down to unbroken $U(1)\times U(1)$, the two composite Higgs-like particles $h_3$ and $h_8$ should remain in the physical spectrum of the model. Because the lepton and quark masses are tiny in comparison with huge Majorana masses of sterile neutrinos, the contributions of the underlying 'would-be' NG bosons to the masses of flavor gluons are neglected.

In contrast to the canonical SM Higgs boson the effective interactions of the composite Higgs-like particles are calculable. This conclusion is supported by the work [11] arguing in favor of equivalence of the weakly coupled model with the elementary scalar Higgs field and the strongly coupled renown BHL model [12]. It will become obvious in the following that our model comes close to the BHL one in a very crude low-momentum approximation.

Finally, it is of course mandatory to demonstrate that our basic assumption of the dynamical fermion mass generation is warranted. We support this assumption by finding explicitly, in separable approximation, the matrix chirality-changing lepton and quark proper self energies $\Sigma(p^2)$ as the nonperturbative strong-coupling solution of the Schwinger-Dyson (SD) equation. Separable Ansatz for the kernel represents a particular model of the momentum-dependent coupling $h_{ab}^2(q^2)$ of the gauge $SU(3)_f$ in the infrared, and enables to derive the exponentially sensitive formula for lepton and quark masses

$$m_i = \Lambda \exp(-1/4\alpha_i)$$

in terms of a handful of the effective low-momentum constants $\alpha_i$. It is gratifying that at the same time the tical approximation naturally yields the huge Majorana masses of all three sterile right-handed neutrinos.

The idea of gauging the flavor or family or generation or horizontal symmetry is so natural that it could hardly be new [13]. It ties together a number of troublesome global non-Abelian symmetries which necessarily emerge once the standard Higgs sector with its general Yukawa couplings is switched off. At the same time it is obvious that: (i) such a symmetry is badly broken, and (ii) its breakdown cannot be explicit. In all papers on this subject known to us this gauge flavor symmetry is spontaneously broken by an extended Higgs sector of elementary condensing scalar fields. We argue that no elementary scalars are necessary i.e., that the gauge flavor $SU(3)_f$ dynamics, strongly coupled in the infrared, due to its sterile neutrino sector, completely self-breaks. It should be noted that the idea of the dynamical breakdown of $SU(3)_f \times SU(2)_L \times U(1)_Y$ symmetry is mentioned by Yanagida in [13].

The paper is structured as follows. In Sect.II we demonstrate that the sole sector of sterile right-handed neutrinos yields the complete self-breaking of the gauge chiral $SU(3)_f$: The flavor gluon exchanges at low momenta between the right-handed sterile neutrino fields and their charge-conjugate left-handed counter parts generate three different Majorana masses $M_R$. They follow from the chiral symmetry breaking parts $\Sigma(p^2)$ found in a separable approximation as the low-momentum solutions of the Schwinger-Dyson equation for the full matrix neutrino propagator. All flavor gluons then necessarily acquire masses by absorbing the composite 'would-be' NG bosons as their longitudinal polarization states. It is gratifying that the formalism is convincingly supported by the description of spontaneous Majorana mass generation and of the consequent complete breakdown of gauge $SU(3)$ using the elementary scalar field. All what is needed is one Higgs field in the complex symmetric sextet representation of $SU(3)$ [14].

Sect.III is devoted to the aspects of electroweak symmetry breaking in the present model: First, we solve the Schwinger-Dyson equations for the Dirac chirality-changing proper self energies $\Sigma(p^2)$ of the SM fermions in separable approximation. Then we demonstrate that they result in a wide and wild spectrum of lepton and quark masses. Second, we compute the $W$ and $Z$ boson masses in terms of the fermion self-energies $\Sigma(p^2)$.

In Sect.IV we reveal the composite Higgs boson as a massive partner of the 'would-be' composite electroweak NG bosons, and indicate the derivation of its effective interactions with fermions, and with the electroweak gauge bosons. Similar argumentation leads to the expectation of the existence of new Higgs-like bosons $h_3$ and $h_8$.

In Sect.V we briefly analyze the basic properties of three composite axions, the pseudo NG bosons created by global anomalous Abelian currents. Axions acquire their masses by the non-perturbative effects of three different
non-Abelian gauge interactions present in the model.

In Sect.VI we collect several specific phenomenological consequences of the model, and Sect.VII contains our conclusions and an outlook.

II. FERTILE STERILE NEUTRINOS

In perturbation theory the Lagrangian of the sterile neutrino sector

\[ \mathcal{L}_{\nu_R} = \bar{\nu}_R \partial^\mu \nu_R + h \bar{\nu}_R \gamma_\mu \nu_R \chi_3 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]  

(2)
describes the triplet of massless right-handed neutrinos \( \nu_R \) interacting with the octet of massless spin-1 flavor gluons \( C_a \) in accordance with exact \( SU(3)_f \) gauge invariance (anomaly freedom is ignored for the moment). The hard Majorana mass term \( \bar{\nu}_R M_R \nu_R \) is not the \( SU(3)_f \) singlet \((3 \times 3 = 3_a + \bar{6}_a)\) and is therefore strictly prohibited by the \( SU(3)_f \) symmetry. (The subscripts \( a, s \) abbreviate the antisymmetric and symmetric representations, respectively.) The Lagrangian (2) obviously obeys also the global \( U(1)_a \) `sterility' symmetry.

We bring reasonable nonperturbative arguments that the dynamics defined by (2) in its strong coupling low-momentum regime is not confining, but it yields the complete spontaneous self-breaking: First, (2) describes three Majorana neutrinos with different masses \( M_R \) of order \( \Lambda \). Second, these dynamically generated different masses break the \( U(3)_f \) symmetry spontaneously and completely. (i) Since the chiral \( SU(3)_f \) is the gauge symmetry, its spontaneous breakdown generates eight composite ‘would-be’ NG bosons. They give rise to masses \( M \) of all eight flavor gluons proportional to \( M_R \). (ii) The composite NG boson of the global Abelian chiral symmetry remains in the spectrum. We will deal with it in detail in Sect.V, taking into account the axial anomaly. Third, as a remnant of the just described dynamical Higgs mechanism there should be three massive composite Higgs-like particles with calculable effective interactions with flavor gluons, with massive Majorana neutrinos and with themselves. Fourth, because below \( \Lambda \) the dynamics is strongly coupled and nonconfining, it is natural to expect that it generates other massive composite excitations. In particular, the flavored ones should contribute to the flavor gluon polarization tensor. This last point is mentioned here for its importance but its elaboration is beyond the scope of this paper. We use this expectation for justification of the Ansatz for the momentum-dependent sliding coupling \( \bar{h}_{ab}^2(q^2) \) in (3).

The impressionistic picture painted above is not unexpected: The massless fields can excite massive particles. This comes about by finding nonperturbatively the nontrivial poles of their full propagators. For fermions for which the masslessness is protected by the chiral symmetry this amounts to finding the chiral symmetry breaking self energies in their full propagators [13]. For gauge bosons for which the masslessness is protected by the gauge symmetry this amounts, according to Schwinger [16], to finding the residue at the massless pole of the gauge field polarization tensor. Moreover, it is a common knowledge that there exists the convincing (say phenomenological) realization of this picture, using the standard Higgs mechanism. Close relation between the strong-coupling microscopic and the weak-coupling phenomenological descriptions allows us to predict the existence of three massive composite Higgs-like particles which in the microscopic dynamics is difficult to identify.

We employ below the non-perturbative self-consistency reasoning underlying the concept of spontaneous symmetry breaking pioneered by Yoichiro Nambu: First we assume that three different \( SU(3)_f \) symmetry breaking Majorana masses of \( \nu_R \) are dynamically generated. This implies the existence of eight ‘would-be’ NG composite excitations which in turn give rise to different masses of all eight flavor gluons \( C \). Massive flavor gluons then imply the symmetry-breaking kernel in the Schwinger-Dyson equation for the fermion masses. Finally, using this form of the SD equation we find explicitly the assumed Majorana masses.

This genuinely quantum complete dynamical breakdown of the strong-coupling gauge \( SU(3)_f \) symmetry is considerably more complicated than the standard, essentially classical, Higgs realization: Using the elementary Higgs field \( \Phi \) in the complex symmetric sextet representation of \( SU(3)_f \) we construct the manifestly gauge \( SU(3)_f \) invariant Lagrangian using all three algebraically independent invariants which can be constructed from \( \Phi \). With appropriate choice of parameters in the Lagrangian the classical Hamiltonian has the minimum corresponding to the vacuum expectation value of \( \Phi \) with three different diagonal nonzero positive entries. It is then a manual work to demonstrate that such a condensate generates three different Majorana neutrino masses and eight different masses of flavor gluons. As a bonus there are necessarily three Higgs-like \( 0^+ \) scalars with different masses and prescribed interactions.

1. Dynamical Majorana neutrino mass generation

To see the possibility of the dynamical Majorana neutrino mass generation we rewrite the neutrino-flavor gluon interaction in (2) identically as

\[ \mathcal{L}_{\text{int}} = \frac{1}{2} h \bar{\nu}_R \gamma_\mu \nu_R - \frac{1}{4} \nu_R^C (\nu_R)^C \]  

(3)

where we have introduced the charge conjugate neutrino field \( (\nu_R)^C = C(\nu_R)^T \). It is important to realize that it is a left-handed field, \( (\nu_R)^C = (\nu^\ell)_L \), transforming as the
antitriplet of $SU(3)_f$: $T_a(L) = -\frac{1}{2}\lambda_a^T$. These ingredients are simply necessary: Any fermion mass term is a bridge between the left- and the right-handed fermion fields. In the present case these are not independent fields, but are related by charge conjugation. Moreover, because the left-handed neutrino field transforms as an antitriplet and the right-handed one as a triplet, the $SU(3)_f$ dynamics is not vector-like. As a result the Majorana mass matrix (or, more generally the Majorana chiral symmetry breaking self-energy), if dynamically generated, must be a general $3 \times 3$ matrix symmetric by Pauli principle.

We consider in this paper the full matrix fermion propagator $S(p)$ (with unimportant fermion wave function renormalization set to one) in the form devised by Petr Beneš [17]

\[ S^{-1}(p) = p - \hat{\Sigma}(p^2) \]  

where $\hat{\Sigma} = \Sigma_{PL} + \Sigma^+ P_R$ and $P_{LR} = \frac{1}{2}(1 \mp \gamma_5)$.

For sterile neutrinos $S^{-1}(p)$ corresponds to the effective bilinear Lagrangian [18]

\[ \mathcal{L}_{\text{eff}}^{(2)} = \frac{1}{2} \bar{\nu}_R \gamma^\mu \nu_R + \frac{1}{2} \bar{\nu}_R \gamma^\mu (\nu_R)^c - \frac{1}{2} \left[ (\bar{\nu}_R \Sigma (\nu_R)^c + \text{h.c.}) \right] \]

\[ = \frac{1}{2} \bar{n} \not{p} n - \frac{1}{2} \bar{n} \hat{\Sigma} n \]  

We emphasize that the same equation is valid also for the Dirac fermions. In the latter case the left- and the right-handed fermions are independent fields, and both transform as triplets. In both cases the fermion mass spectrum is given by the poles of $S(p)$ i.e., by solving the equation

\[ \det[p^2 - \Sigma(p^2)\Sigma^+(p^2)] = 0 \]  

The sliding coupling $\bar{h}_{ab}^2(q^2)$ in (6) defined in terms of the flavor gluon polarization tensor contains important informations about the assumed low-momentum properties of the model. In particular, it corresponds to the phase in which all flavor gluons are massive. Despite this, it remains unknown. The point is that the spectrum of the expected composites carrying flavor, which by definition below $\Lambda$ contribute to $\bar{h}_{ab}^2(q^2)$, is entirely unknown. Finding the fermion spectrum is therefore a formidable task.

In order to proceed we approximate the problem of finding the fermion mass spectrum as follows:

1. In the perturbative weak coupling high-momentum region from $\Lambda$ to $\infty$ which in technical sense guarantees the UV finiteness of $\Sigma(p^2)$ [19] we set the known perturbative i.e. small, $\bar{h}_{ab}^2(q^2)$

\[ \bar{h}_{ab}^2(q^2) \approx \frac{\delta_{ab}}{(11 - \frac{n_f}{2}) \ln(q^2/\Lambda^2)} \]

equal to zero. Here $n_f = 16$ is the number of chiral fermion triplets. The resulting model is thus not asymptotically, but strictly free above the scale $\Lambda$.

We project from (6) the equation for $\Sigma$, fix without loss of generality in the resulting SD equation the external euclidean momentum as $p = (p, \theta)$, integrate over angles and get

\[ \Sigma(p) = \int_0^\Lambda k^3 dk K_{ab}(p, k) T_a(R) \Sigma(k)[k^2 + \Sigma^+ \Sigma]^{-1} T_b(L) \]

where the unknown kernel

\[ K_{ab}(p, k) = \frac{3}{4\pi^3} \int_0^\pi \frac{\bar{h}_{ab}^2(p^2 + k^2 - 2pk \cos \theta)}{p^2 + k^2 - 2pk \cos \theta} \sin^2 \theta d\theta \]  

is separately symmetric in momenta and in the flavor octet indices.

2. Our key approximation is the separable approximation for the kernel $K_{ab}(p, k)$. In the following we
analyze explicitly the Ansatz
\[ K_{ab}(p, k) = \frac{3}{4\pi^2} \frac{g_{ab}}{pk} \]  
(10)

The Ansatz substitutes our ignorance of knowing the low-momentum \( h_{a}(q^2) \) and the low-momentum form of the flavor gluon propagators (to be found subsequently). Ultimately we should deal with a system of Schwinger-Dyson equations for several Green functions, an entirely hopeless task.

Here \( g_{ab} \) is a real symmetric matrix of the effective low-momentum dimensionless coupling constants. They reflect the complete breakdown of \( SU(3)_f \) and are ultimately calculable. We think they are analogous to the effective low-momentum dimensionless coupling constants of the chiral perturbation theory of the confining QCD.

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Separable approximation has several advantages (proved eventually a posteriori).

1. The nonlinearity of the integral equation is preserved. We expect that the non-analyticity of \( \Sigma \) upon the perturbation theory of the confining QCD.

The difficult part is that the numerical matrix \( \Gamma \) has to fulfil the homogeneous nonlinear algebraic self-consistency condition (gap equation)

\[ \Gamma_{ab} = g_{ab} \frac{3}{16\pi^2} \int_0^1 dx \langle T(R)\Gamma T(L) \rangle \left( x + (T(R)\Gamma T(L))^+ (T(R)\Gamma T(L)) \right)^{-1} \]

3a. For neutrinos \( \Sigma \) describes both the masses of Majorana neutrinos, and their mixing (including the new CP-violating phases): The general complex symmetric 3 \( \times \) 3 matrix \( \sigma \) can be put into a positive-definite real diagonal matrix \( \gamma \) by a constant unitary transformation

\[ \sigma = U^+ \gamma U^* \]

(13)

The gap equation becomes

\[ \gamma = UT_a(R)U^+ g_{ab}I(\gamma)U^* T_b(L)U^T \]

(14)

where

\[ I(\gamma) = \frac{3}{16\pi^2} \gamma \int_0^1 \frac{dx}{x + \gamma^2} = \frac{3}{16\pi^2} \gamma \ln \frac{1 + \gamma^2}{\gamma^2} \]

(15)

The diagonal entries of the equation (14) determine the sterile neutrino masses, the nondiagonal entries provide relations for the mixing angles and the new CP-violating phases. These phases are most welcome as a source of an extra CP violation needed for understanding of the baryon asymmetry of the Universe.

3b. For Dirac fermions, as in the Standard model, the generally complex 3 \( \times \) 3 matrix \( \sigma \) can be put into a positive-definite real diagonal matrix \( \gamma \) by a constant bi-unitary transformation:

\[ \sigma = U^+ \gamma V \]

(16)

The gap equation becomes

\[ \gamma = UT_a(R)U^+ g_{ab}I(\gamma)VT_b(L)V^+ \]

(17)

The diagonal entries of the equation (17) determine the fermion masses, the nondiagonal entries provide relations for the CKM mixing angles and the SM CP-violating phase.

It is natural to demonstrate the existence of the solutions of the gap equations simultaneously for both the sterile neutrino Majorana masses and for the Dirac masses of the electroweakly interacting leptons and quarks. This is done in the Appendix.

With neutrino mixing neglected and with rather simplifying assumptions on the effective couplings \( g_{ab} \) we obtain in the Appendix the desirable result

\[ M_{RR} \sim \Lambda \]

(18)

In Sect.V we set the scale \( \Lambda \) from the invisibility of the Weinberg-Wilczek axion \( \alpha \). Within a rather wide window we consider for definiteness \( \Lambda \sim 10^{19}\text{GeV} \).

2. Self-consistent generation of flavor gluon masses

It is self-evident that three different Majorana masses (self-energies \( \Sigma \)) break the \( U(3) \) symmetry of (2) completely. Because the appearance of different Majorana masses is spontaneous, there must be nine NG bosons. Eight of them are the ’would-be’, the ninth is the ’pseudo’, discussed in Sect.V.

Following [18] we reveal the eight ’would-be’ NG bosons as the massless poles in the Abelian approximation of the Ward-Takahashi identity for the Green’s function \( iT^a_0 = \langle 0|T [C^a_0(x)n(y)\bar{n}(z)]|0 \rangle_{1\Pi \Pi} \) (more precisely for its one-particle-irreducible part). It is related with the WT identity for the Green’s function \( \gamma^a_{0} = \langle 0|T [j^a_{\mu}(x)n(y)\bar{n}(z)]|0 \rangle_{1\Pi \Pi} \) associated with global symmetry generated by the current of Majorana neutrinos \( n \):

\[ j^a_{\mu} = \bar{\nu}_R \gamma^\mu_1 \lambda_a \nu_R = \frac{1}{2} \bar{\nu} \gamma^\mu \frac{1}{2} \lambda_a \nu \]

Here

\[ \frac{1}{2} \lambda_a = \frac{1}{2} \lambda_a P_R + \frac{1}{2} (\lambda^\mu_a P_L \]

(19)
The result is
\[ q_{\mu} \Gamma_\mu(p',p) = \hat{\Sigma}(p') \frac{1}{2} \Lambda_a - \frac{1}{2} \Lambda_a \hat{\Sigma}(p) \] (20)

The pole term is
\[ \Gamma_{\mu,\text{pole}}(p+q,p) = \frac{q_{\mu}}{q^2} \left[ \hat{\Sigma}(p+q) \Lambda_a - \Lambda_a \hat{\Sigma}(p) \right] \] (21)

where
\[ \frac{1}{2} \Lambda_a = \gamma_0 \Lambda_a \gamma_0 \]

The term in square brackets describes (with appropriate normalization) the effective vertices \( P_a \) between eight neutrino-antineutrino composite 'would-be' NG bosons \( \pi_a \) and the corresponding neutrino pair:
\[ P_a(p',p) \sim [\hat{\Sigma}(p') \Lambda_a - \Lambda_a \hat{\Sigma}(p)] \] (22)

Not surprisingly we recover the identical structure by computing the divergence of the current (19) using the Dirac equation with dynamically generated mass following from the bilinear Lagrangian (16):
\[ \frac{1}{2} \hat{n}(p') \gamma_{\mu} \Lambda_a n(p) = \frac{1}{2} \hat{n}(p') [\hat{\Sigma}(p') \Lambda_a - \Lambda_a \hat{\Sigma}(p)] n(p) \]

General strategy of computing the non-Abelian gauge boson mass matrix is described in detail in [18]. It amounts to computing the matrix effective loop-generated tadpoles between the 'would-be' NG bosons and the flavor gluons. They imply the massless pole in the effective tree-level longitudinal part of the flavor gluon polarization tensor. Its residue is the flavor gluon mass matrix. The apparently non-urgent explicit computation of the flavor gluon mass matrix along these lines is in progress. For an estimate of the value of the flavor gluon masses \( M_a \) it is quite sufficient at the moment to neglect the matrix structure, and to use the original Pagels-Stokar formula [21]
\[ F^2 = 8N \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma^2(p^2) - \frac{1}{2} \Sigma^2(p^2)}{(p^2 + \Sigma^2)^2} \] (23)

relating the 'would-be' NG boson decay constant \( F \) with the corresponding gauge boson mass \( M \) [22]: \( M^2 \sim h^2 F^2 \). Here \( N = 3 \) is a loop factor. With the explicit form of \( \Sigma(p^2) = (\Lambda^2 / p) \gamma \equiv M_f^2 / p \) at hand the integral is easily computed, and we have \( F^2 \approx \frac{15}{16\pi} M_f^2 \). Hence,
\[ M_{aC} \sim M_f \] (24)

Because the sterile neutrino masses are huge there is no problem with the flavor changing electric charge conserving processes transmitted by flavor gluons.

Finally, it is easy to show (see [22]) that for \( p' \to p \) the neutrino-'would-be' NG couplings have the matrix form
\[ P_a(p,p) \sim [\Sigma(p), \frac{1}{2} \lambda_a] \gamma_5, \quad a = 1, 3, 4, 6, 8, \] (25)

and
\[ P_a(p,p) \sim [\Sigma(p), \frac{1}{2} \lambda_a], \quad a = 2, 5, 7 \] (26)

We will use these formulas in the following for comparison with the canonical Higgs mechanism applied to the Lagrangian [2].

3. The Higgs sextet for the sterile neutrino sector

In the language of the many-body theory we have so far modified dynamically the dispersion laws of quasi-particles corresponding to the Lagrangian [2]. In general, the quasiparticles are the harmonic oscillator-like excitations created by the quantum fields (monomials) present in a Lagrangian. Explicitly, we have generated the masses of neutrinos and of the flavor gluons by the strong flavor gluon gauge interaction.

Dynamical generation of flavor gluon masses demands the existence of specific collective excitations, the composite 'would-be' NG bosons. They are guaranteed by the Goldstone theorem and visualized by the Ward-Takahashi identities. In general, the collective excitations (bound states) are the excitations created by certain polynomials of the original quantum fields. Their very formation in relativistic quantum field theory requires a strong force.

On the other hand, how to generate spontaneously the masses of sterile neutrino fields and the masses of the \( SU(3)_f \) flavor gluon fields in the Lagrangian [2] is notoriously known: Simply one has to add to it an appropriate weakly interacting Higgs sector.

In fact, the desired Higgs multiplet is known [14]: In the following we consider the condensing elementary scalar Higgs field \( \Phi(x) \) in the complex symmetric sextet representation of \( SU(3) \). Its condensate generates different Majorana masses to all sterile neutrinos, and at the same time it generates different masses to all eight flavor gluons. The general Higgs mechanism for the \( U(3) \) symmetry with the Higgs sextet was discussed in entirely different context of colored superconductors in great detail in [14]. For comparison with the previous Section we briefly summarize the main steps.

Under the \( SU(3) \) the \( \Phi \) transforms as a complex symmetric matrix, \( \Phi \to U \Phi U^T \). The general Higgs Lagrangian invariant under the \( SU(3) \times U(1) \) symmetry has the standard form
\[ \mathcal{L}_H = (D_\mu \Phi)^+ D_\mu \Phi - V(\Phi) + \mathcal{L}_Y \] (27)

where
\[ D_\mu \Phi = \partial_\mu \Phi - i h C_{\mu}^{\alpha} (\frac{1}{2} \lambda_\alpha \Phi + \Phi \frac{1}{2} \lambda_\alpha^T) \] (28)

and
\[ \mathcal{L}_Y = g_{\nu} \bar{\nu}_R \Phi(\nu_L)^c + h.c. \] (29)
The potential $V(\Phi)$ is a function of three independent invariants $\det(\Phi^+\Phi)$, $\text{tr}(\Phi^+\Phi)$, and $\text{tr}(\Phi^2)$. It determines the constant (due to the translational invariance of the vacuum) vacuum expectation value of the Higgs field $\Phi$:

$$\phi = \langle \Phi \rangle_0$$

Employing the liberty of writing $\phi$ as $\phi = UvU^T$ where $U$ is a convenient unitary matrix, we can cast $\phi$ into a real, diagonal matrix with non-negative entries, ordered by their values:

$$v = \text{diag}(v_1, v_2, v_3)$$

with $v_1 > v_2 > v_3 > 0$. This form of the v.e.v. follows from the particular form of $V(\Phi)$. Because we argue in terms of an effective field theory we ignore the fact that this particular form of $V$ violates the renormalizability (contains the polynomials in $\Phi$ of the order higher than four).

It is self-evident that the vacuum expectation value breaks the $SU(3) \times U(1)$ symmetry spontaneously and completely: The twelve real scalar fields of the complex sextet $\Phi$ decompose as follows: (1) There is one true NG mode $\theta(x)$ of the spontaneously broken global $U(1)$ symmetry. (2) There are eight ($a = 1, ..., 8$) 'would-be' NG bosons $\theta_a(x)$ which become the longitudinal components of massive flavor gluons, and disappear from the physical spectrum. (3) There are three ($i = 1, 2, 3$) massive radial modes, the Higgs bosons $\chi_i(x)$. Explicitly, and for small fields we have

$$\Phi = e^{i\frac{1}{2}x \lambda_a \theta_a} \frac{1}{\sqrt{2}} e^{i\theta} \text{diag} \left( v + \chi \right)e^{i\frac{1}{2}x \lambda_a \theta_a}$$

$$\approx [\text{diag} \left( v + \chi \right) + i\theta \pm i\theta \{\text{diag} \left( v + \chi \right), \frac{1}{2} \lambda_a \} \pm]$$

where $+$ means the anticommutator and $a = 1, 3, 4, 6, 8$ while $-$ means the commutator and $a = 2, 5, 7$.

For most of practical purposes it is convenient to work in the unitary gauge which eliminates the 'would-be- NG bosons. We set

$$\Phi(x) = \frac{1}{\sqrt{2}} e^{i\theta} \text{diag}(v_1 + \chi_1, v_2 + \chi_2, v_3 + \chi_3)$$

and ignore the small field $\theta(x)$ in considerations which follow. Due to the axial anomaly its fate is in fact non-trivial, and will be discussed in Sect.V in some detail.

1. Substitution of (32) into the kinetic term in (27) results in two terms: (i) The quadratic polynomial in the fields $C$ defines the mass matrix of flavor gluons,

$$M^2 = h^2 \times$$

$$\begin{bmatrix}
(v_1 + v_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (v_1 - v_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(v_1^2 + v_2^2) & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}}(v_1^2 - v_2^2) & 0 & 0 \\
0 & 0 & 0 & (v_1 + v_3)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}}(v_2^2 - v_3^2) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}}(v_3^2 - v_1^2) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

All masses are nonzero and unequal, with the $(3, 8)$ mixing to be done. For completeness we mention: All Majorana masses come out huge. The masses of flavor gluons which change flavor must also be huge. This is guaranteed. Masses of the flavor diagonal flavor gluons $C_3$ and $C_8$ can be, upon diagonalization, much lighter. (ii) The higher polynomials define the tree-level interactions of the Higgs fields $\chi_i$ with flavor gluons $C$.

2. Substitution of (32) into the potential $V$ results in the mass terms of the Majorana neutrinos

$$L_M = \bar{n} \frac{g_n}{\sqrt{2}} \nu n$$

and in the Yukawa interaction of the massive Majorana neutrinos with the massive Higgs fields $\chi_i$:

$$L_Y = \frac{g_n}{\sqrt{2}} \bar{n} \chi n$$

4. A lesson

The analysis presented above suggests a duality between the weak coupling canonical Higgs mechanism of spontaneous generation of fermion and gauge boson
masses and underlying strong coupling dynamical generation of fermion and gauge boson masses without elementary scalar fields. Such a conjecture is supported also by the paper [11]. It argues in favor of an equivalence of the top quark condensate model (strong coupling) and the SM with the elementary Higgs field (weak coupling). Without much imagination but with a lot of reservations our strong-coupling model with massive flavor gluons can be roughly approximated by the fourfermion interactions. Similar conjecture is formulated also in a recent paper [23].

Comparison of the Yukawa couplings [24, 25] of the composite ’would-be’ NG bosons with the Yukawa couplings of the elementary ’would-be’ NG bosons shown in [31] reveals that they are identical. The former ’would-be’ NG bosons are the bound states present in the original Lagrangian [4], but convincingly predicted on the basis of symmetry considerations by the WT identity. The latter ones are fully pre-prepared, together with their massive partners in the Lagrangian [24] in the elementary Higgs field Φ. The conjecture is that there are three composite massive χi also in quantum flavor dynamics, although we are not aware of an existence theorem for them. Their effective Yukawa interactions with the massive Majorana neutrinos should be the same as of the elementary ones. Their effective interactions with the massive flavor gluons should, however, be derivable by computing the corresponding UV finite neutrino loops. This apparently non-urgent computation is also in progress.

Can such superheavy scalar particles be useful? Apparently yes: We find amusing that one such a superheavy right-handed-neutrino-composite scalar bound by a NQL four-fermion interaction was suggested [23] as a candidate for the inflaton [25]. We point out that there are good reasons in the literature for considering more inflatons [26]. If the reasons for the existence of namely three of them are irresistible they could also be good for answering the question why there are three families [27].

We will refer to the duality discussed above further in the following: First, to fix the properties of the composite Higgs-like bosons in Sect.IV. Second, to argue against formation of the Majorana mass term of the left-handed neutrinos in Sect.III.2.

III. DYNAMICAL ELECTROWEAK SYMMETRY BREAKING

In accord with our strategy the first step is the generation of fermion masses (chirality-changing fermion self-energies) of the electroweakly interacting fermions by the strong-coupling quantum flavor dynamics.

In the second step we demonstrate that the composite electroweak ’would-be’ NG bosons resulting from the step one give rise to masses m_D^2, m_Z of the W and Z bosons. These masses are expressed in terms of the fermion masses by sum rules.

1. Dynamical generation of charged-lepton and quark masses

The chirality-changing fermion self-energy is a matrix bridge between the left-handed and the right-handed fermion field multiplets with given electric charge. In the Majorana neutrino case the left- and the right-handed neutrino fields are related by charge conjugation, and the resulting Σ (3 × 3 = 3a + 6a) is a symmetric matrix by Pauli principle. In the Dirac case the left- and the right-handed fermion fields are the independent fields both transforming as triplets (T_a(L) = T_a(R) = \( \frac{1}{2\lambda_a} \)), and the resulting Σ is a general complex 3 × 3 = 1 + 8 matrix. Its form is found by solving the SD equation as in the case of the Majorana case.

Clearly, with SU(2)_L × U(1)_Y electroweak gauge interactions switched off there is nothing in the model which would distinguish between Σ matrices of different fermion species f = u, d, e, ν. Consequently the flavor-dependent mass matrices of these different fermion species must come out equal:

\[
\begin{align*}
\Sigma_u &= \Sigma_b = \Sigma_s = \Sigma_v, \\
\Sigma_c &= \Sigma_s = \Sigma_v, \\
\Sigma_u &= \Sigma_d = \Sigma_v.
\end{align*}
\]

Hence for f = u, d, e, ν we get the fermion masses m_f(f) independent of f.

In the Appendix the corresponding SD equation is solved (with fermion mixing neglected), and the resulting mass formula is

\[
m_f(f) = \Lambda \exp \left(-1/4\alpha_{ii}^f\right)
\]

where

\[
\begin{align*}
\alpha_{11} &= \frac{3}{64\pi^2} (g_{33} + \frac{2}{\sqrt{2}}g_{38} + \frac{1}{4}g_{88}) \\
\alpha_{22} &= \frac{3}{64\pi^2} (g_{33} - \frac{2}{\sqrt{2}}g_{38} + \frac{1}{4}g_{88}) \\
\alpha_{33} &= \frac{3}{64\pi^2} g_{88}
\end{align*}
\]

In reality, the Abelian gauge field B of the gauge electroweak interactions, although interacting identically with different fermion families, does distinguish between different fermion species. Different types of chiral fermions differ by having different weak hypercharges \( Y \), given uniquely by different electric charges. For convenience they are reminded here:

\[
\begin{align*}
Y(l_L) &= -1, & Y(e_R) &= -2, & Y(\nu_R) &= 0 \\
Y(q_L) &= \frac{1}{3}, & Y(u_R) &= \frac{4}{3}, & Y(d_R) &= -\frac{2}{3}
\end{align*}
\]

Radiative corrections due to the non-Abelian electroweak gauge fields \( A_i \), interacting universally with all left-handed fermion fields give rise to a universal contribution and need not be considered.
Frankly, we do not know at the moment how to implement the $B$ radiative corrections into the separable Ansatz. Clarification of this important point requires further work. It is nevertheless justified to consider formally the effective couplings $\alpha_{ii}$ in the fermion mass formula above fermion-type dependent, and write it in a generic form

$$m_i(f) = \Lambda \exp \left( -1/4\alpha_{ii}(f) \right)$$

(36)

Here $f = u, d, e, \nu$ and $\alpha_{ii}(f)$ are, ultimately, the effective couplings $\alpha_{ii}$ with the calculable weak hypercharge radiative corrections included. A hope is that the exponential dependence of the fermion masses upon the effective couplings will describe the observed differences between fermion masses of fermions with different electric charges.

2. Neutrino mass spectrum

Computation of the neutrino mass spectrum in the present model is more subtle. It requires the knowledge of the neutrino mass matrix in its general form of a complex symmetric $6 \times 6$ matrix

$$\Sigma_\nu = \begin{pmatrix} \Sigma_L & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix}$$

(37)

Here $\Sigma_L$ is the Majorana self-energy of three active left-handed neutrinos to be discussed, $\Sigma_D \equiv \Sigma$ is the Dirac self-energy of the neutrinos computed in Sect.III.1 (without electroweak corrections), and $\Sigma_R$ is the Majorana self-energy of the triplet of the right-handed neutrinos computed in Sect.II.1.

What can we say about $\Sigma_L$? First of all, three $\nu_L$ belong to three electroweak doublets $l_L^f = (\nu_L, e_L)$. In accord with our reasoning of Sect.II.4 the spontaneous generation of $\Sigma_L \neq 0$ would imply considering the condensing complex triplet Higgs field having the quantum numbers of the elementary scalar Higgs field of the triplet Majoron model \cite{28}. The $SU(2)_L$ triplet is required by Pauli principle. To postulate the existence of the field

$$\phi_a = (\phi^{(0)}, \phi^{(+)}, \phi^{(++)})$$

does not cost anything provided it is elementary. Whether such a field has the right to exist as a bound state of two $l_L$ is a complicated issue of the underlying strong-coupling quantum flavor dynamics, definitely outside the scope of the present paper. Here we merely wishfully assume that the formation of a doubly charged composite Higgs-like field would be energetically very costly because of the Coulomb repulsion, and the complex composite triplet will not be formed. Consequently, there would be no condensate $\Sigma_L$ and we conclude that the neutrino mass eigenstates are determined by the neutrino mass matrix of the famous seesaw \cite{29} form

$$\Sigma_\nu = \begin{pmatrix} 0 & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix}$$

(38)

This implies that, upon diagonalization, there are three Majorana neutrinos with huge masses $\sim M_R$, and three active light Majorana neutrinos with masses

$$m_\nu \sim m_D^2/M_R$$

(39)

Clearly, for any dynamics pretending to compute the fermion masses the prediction of the neutrino mass spectrum is a crystalline challenge. Today, masses of the electrically charged leptons and quarks can ’only’ be postdicted. Referring to the simple analysis presented in the Appendix it seems that the masses $M_R$ cannot be made arbitrarily large: If we want to keep all $g_{ab}$ of the similar order of magnitude, we are restricted by the fact that those $g$ giving rise to the Dirac fermion masses are essentially fixed. Consequently, the masses of three active Majorana neutrinos cannot be apparently arbitrarily small. Other possibility, also mentioned in the Appendix, is that the Majorana and Dirac fermion masses are generated at different scales.

3. Dynamical generation of intermediate gauge boson masses

Fermion masses of the electroweakly interacting fermions or, more generally, their chirality-changing fermion proper self-energies $\Sigma(p^2)$, generated nonperturbatively by the strong gauge flavor dynamics, break spontaneously the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry down to unbroken electromagnetic $U(1)_{em}$. Consequently, the $W, Z$ gauge bosons must acquire masses proportional to $\Sigma$.

It is very important that the gauge electroweak $SU(2)_L \times U(1)_Y$ tie together in a unique way many otherwise independent global $SU(2)_L$ and $U(1)_{em}$ symmetries created by the chiral currents of different fermion species. With electroweak gauge interactions switched off there would be plenty of phenomenologically unacceptable real NG bosons. With electroweak interactions switched on there are just three composite multi-component ’would-be’ NG bosons which give rise to the masses of $W$ and $Z$ bosons.

We proceed as in the case of the dynamical generation of flavor gluon masses described in Sect.II.2. There we acted as if the fermion-gauge boson coupling $h$ were small. Here the couplings $g, g'$ are indeed small.

Consider the vertex parts $\Gamma^\alpha_W(p + q, p)$ and $\Gamma^\alpha_Z(p + q, p)$ following from the electroweak WT identities \cite{30} for one, heaviest, quark doublet $f = (t, b)$. In our simplified
world without mixing and without perturbative contributions from the electroweak interactions the self-energies are equal, \( \Sigma_f(p^2) = \Sigma_b(p^2) = \Sigma_\tau(p^2) = \Sigma_{v_\mu}(p^2) = \Sigma_f(p^2) \).

\[
\Gamma^\alpha_W(p + q, p) = \frac{g}{2\sqrt{2}} \left\{ \gamma^\alpha (1 - \gamma_5) - \frac{q^\alpha}{q^2} [(1 - \gamma_5) \Sigma_f(p + q) - (1 + \gamma_5) \Sigma_f(p)] \right\},
\]

\[
\Gamma^\alpha_Z(p + q, p) = \frac{g}{2 \cos \theta_W} \left\{ t_3 \gamma^\alpha (1 - \gamma_5) - 2Q \gamma^\alpha \sin^2 \theta_W - \frac{q^\alpha}{q^2} t_3 [\Sigma_f(p + q) + \Sigma_f(p)] \gamma_5 \right\}.
\]

Further steps are standard (see, e.g., [31]). We extract from the pole terms of the WT identities the effective fermion-'would-be' NG boson vertices \( P \)

\[
P^f_\pm = \frac{1}{4F} [(1 \mp \gamma_5) \Sigma_f(p + q) - (1 \pm \gamma_5) \Sigma_f(p)]
\]

\[
P^f_0 = \frac{1}{4F} \gamma_5 t_3 (\Sigma_f(p + q) + \Sigma_f(p)),
\]

and the effective f-component of the 'would-be' NG bilinear couplings with the gauge fields \( J^\mu_f \):

\[
J^\mu_{fW}(q) = \text{Tr} \int \frac{d^4k}{(2\pi)^4} P_S(k + q) \frac{g}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5) S_f(k)
\]

\[
J^\mu_{fZ}(q) = \text{Tr} \int \frac{d^4k}{(2\pi)^4} P_0 S_f(k + q) \frac{g}{2\cos \theta_W} \left\{ [t_3 \gamma^\mu (1 - \gamma_5) - 2Q \gamma^\mu \sin^2 \theta_W] S_f(k) \right\}
\]

Here \( F \) is the normalization factor to be computed explicitly in terms of all \( \Sigma_f \) in the model.

Without fermion mixing the Yukawa \( u, d \) quark-'would-be' NG boson vertices have for \( q^\mu = 0 \) and \( \Sigma(p^2 = m^2) = m \) the form

\[
P^f_+ = \frac{m_f}{2F} \gamma_5
\]

\[
P^f_0 = \frac{m_f}{2F} t_3 \gamma_5
\]

The effective vertices \( P \) will be employed in the next Section for the identification of the form of the operator of the composite Higgs field. The quantities \( J^\mu_f \) guarantee the massless pole in the longitudinal part of the gauge boson polarization tensors i.e., their masses. Here we compute them again using the Pagels-Stokar formula as it was done in technicolor [22]:

\[
F^f_2 = 8N \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma^2_f(p^2) - \frac{1}{2} p^2 (\Sigma^2_f(p^2))'}{(p^2 + \Sigma^2_f)^2}
\]

As before, \( F^2_f = \frac{5}{16N} N m_f^2 \). Here \( N = 3 + 1 = 4 \) where 3 stands for three colors of the \( t, b \) colored quarks, and 1 stands for \( \tau, \nu_\tau \) colorless leptons. Because the composite 'would-be' NG bosons made of all electroweak doublets and their corresponding singlets incoherently contribute, we have

\[
F^2 = \sum_f F^2_f
\]

Unlike the flavor gluon masses the numerical values of the electroweak gauge bosons \( W, Z \) are the very important parameters of the world at present energies. Considering only the heaviest fermions with the common mass \( m \), we have

\[
m^2_W = \frac{\alpha g^2}{4\pi} \sum_f m^2_f
\]

(43)

\[
m^2_Z = \frac{\alpha g^2}{4\pi} \sum_f m^2_f
\]

(44)

At the present exploratory stage not taking into account either the electroweak corrections or the fermion mixing the mass \( m \) is not known. Expecting its value of order of the electroweak scale \( v \) we conclude that the result is satisfactory. It also should be remembered that the relation between intermediate vector boson masses and the masses of electroweakly interacting fermions is sensitive to the explicit form of \( \Sigma(p^2) \).

**IV. THE CERN HIGGS AND ITS TWO RELATIVES**

So far we have shown how the strong gauge flavor dynamics generates the calculable masses of leptons and quarks and demonstrated how, as a consequence of the existence theorem, it generates the composite 'would-be' NG bosons giving masses to the intermediate electroweak bosons \( W, Z \).

Does the strong quantum flavor dynamics produce also composite Higgs particles ? As far as we can see, there is no existence theorem for such states. We will follow the logic of Sect.II and search for the fermion-antifermion composite operator which transforms according to a representation of the gauge group, and as the necessary consequence it contains the composite 'would-be' NG bosons found previously by the analysis of the WT identities. The remaining partner(s) will be identified with the composite Higgs(es).
We start with the electroweak symmetry. As a necessary condition the composite scalar operator constructed from the covariantly transforming fermion fields should contain three components of the 'would-be' NG bosons $\pi$, which in the elementary scalar Higgs field are prepared in the decomposition

$$\Phi(x) = \exp\left(\frac{i}{\sqrt{2}}(x) \tau^a\right) \left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{array} \right) \equiv \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right)$$

For small fields we have

$$\phi^+ = \frac{1}{\sqrt{2}}(\pi_2 + i\pi_1)$$
$$\phi^0 = \frac{1}{\sqrt{2}}(v + h - i\pi_3)$$

Here $h$ is the physical Higgs field and $v = 246$ GeV is the electroweak condensate.

1. The Yukawa couplings of the Higgs boson

In the Standard model the quark masses $m_u, m_d$ are generated from the $SU(2)_L \times U(1)_Y$ invariant Yukawa interaction

$$L_Y = y_u q_L u_R \Phi + y_d q_L d_R \Phi + h.c.$$  \hspace{1cm} (45)

where the charge conjugate Higgs field is $\Phi \equiv i\tau^2 \Phi^*$. In our approximation $m_u = m_d = y_u v/\sqrt{2} = y_d v/\sqrt{2} = m$ we get

$$L_Y = m(\bar{u}u + \bar{d}d) + m v \bar{u}u + \bar{d}d)$$

$$+ \left\{ \frac{m}{2v} \bar{u} \gamma_5 d(\pi_2 + i\pi_1) + h.c. \right\} + m v i[\bar{u} \gamma_5 u - \bar{d} \gamma_5 d] \pi_3$$

Standard interpretation of $L_Y$ is the following: In the unitary gauge, in which all $\pi_i$ vanish, $L_Y$ turns into the ordinary fermion mass term plus parity-conserving Yukawa interaction of the real $0^+$ field $h$ with couplings proportional to the fermion masses.

Comparison of $L_Y$ with our results obtained so far implies the following: First, we also generate spontaneously the fermion masses. While the Higgs mechanism is a tree-level effect, ours is genuinely quantal. Second, the terms in $L_Y$ containing the elementary 'would-be' NG bosons should be compared with the composite ones, signalled in the formulas [10] and [11] by their massless poles. Clearly, the correspondence is complete, provided

$$F = v$$  \hspace{1cm} (46)

Third, we believe that this coincidence justifies a conjecture that also in our model there is a composite multi-component $h$ as a massive partner of the NG bosons. It is seen as the real neutral component of the composite operator

$$\Phi = \sum (\Phi(\nu) + \Phi(\tau) + \Phi(d) + \Phi(u))$$

where in the quark sector

$$\Phi(d) = \frac{1}{F^2} \tilde{d}_{RL} L$$
$$\Phi(u) = \frac{1}{F^2} \tilde{u}_{RL} L$$

The leptonic composite doublets are constructed analogously.

Consequently, the Yukawa vertices of the composite $h$, which eventually enter the fermion loops, have the form

$$L_Y = \frac{h}{F} \sum [(\bar{u} L \Sigma u_R + \bar{u} R \Sigma u_L) + (\bar{d} L \Sigma d_R + \bar{d} R \Sigma d_L)]$$

Presence of fermion proper self-energies $\Sigma(p^2)$ provides necessary softening of the corresponding integrals at high momenta. The sum is over all upper ($u$) and lower ($d$) fermions (both quarks and leptons) in the electroweak doublets.

2. Gauge couplings of the Higgs boson

Phenomenologically the most important question to be answered is how the composite Higgs $h$ interacts with the electroweak gauge bosons $W, Z$ and $A$. Without any computations it is obvious that the resulting interactions must generically differ from the Standard model ones: In the present model all electroweak gauge fields interact directly only with the chiral lepton and quark fields. Consequently, all electroweak gauge fields interact with the composite $h$ merely via the UV finite fermion loops.

In contrast, the SM gauge interactions of the $W, Z$ fields with $h$ are the tree-level ones coming from the covariant derivative of the complex doublet Higgs field $\Phi$:

$$L_{h,W,Z} = \frac{1}{8}(2v h + h^2)[2g^2 W^\mu W^\nu + (g^2 + g^2) Z^\mu Z^\nu]$$
$$= + g m_W W^\mu W^\nu h + \frac{1}{2 \cos^2 \theta_W} g m_Z Z^\mu Z^\nu h$$
$$+ \frac{1}{4} g^2 W^\mu W^\nu h^2 + \frac{g^2}{8 \cos^2 \theta_W} Z^\mu Z^\nu h^2$$

The photon interacts in SM with the Higgs field via the fermion and the $W$ loops. A nasty remark might be that in this respect the unification of weak and electromagnetic interactions in the Standard model is rather strange. The detailed derivation of the effective interactions deserves separate work now in progress.

3. The Higgs boson mass

Finally, there is a question of the Higgs boson mass. In the Standard model the tree-level answer is exceedingly simple:

$$m_h^2 = 2\lambda v^2$$  \hspace{1cm} (47)
where $\lambda$ is the perturbative quartic Higgs field self-coupling.

We do not know how to compute reliably the mass of the non-NG-type collective excitation $h$ in our strong-coupling model. Referring to the similarity of our approach with the top quark condensate model (BHL)\(^{12}\) we weaken their strong coupling result for the Higgs boson mass $m_h = 2m_{\text{top}}$ into an estimate $m_h \sim O(F)$.

4. Two Higgs-boson relatives

Clearly, the dynamically generated masses of the electroweakly interacting leptons and quarks break spontaneously not only the electroweak symmetry but also the flavor symmetry $SU(3)_f$. More precisely, if the fermion self-energy $\Sigma$ (fermion mass) is written as

$$\Sigma = \Sigma_0 \lambda_0 + \Sigma_3 \frac{1}{2} \lambda_3 + \Sigma_8 \frac{1}{2} \lambda_8$$

$$\equiv \Sigma_0 \lambda_0 + \Sigma' \quad (\lambda_0 = \frac{1}{\sqrt{3}})$$

then only $\Sigma' = \Sigma_3 \frac{1}{2} \lambda_3 + \Sigma_8 \frac{1}{2} \lambda_8$ is $SU(3)_f$ symmetry-breaking, as mentioned in the Introduction. The corresponding composite ‘would-be’ NG bosons are visualized as massless poles by virtue of the $SU(3)_f$ WT identity for the vertex valid for arbitrary flavor triplet fermion $f = u, d, e, \nu$

$$\Gamma^\alpha_{ac}(p + q, p) = h \{ \lambda_a \quad - \quad \frac{q^\alpha}{q^2} \Sigma(p + q) \}$$

As before we can extract from the pole part of $\Gamma$ two quantities $|m'\lambda_a\lambda_a|$

First, the effective bilinear flavor gluon ‘would-be’ NG couplings (vectorial tadpoles) giving rise to small contributions of order $m'$ to the huge masses of flavor gluons. Detailed analysis of mixing of these composite ‘would-be’ NG bosons with the most important sterile neutrino-antineutrino composite ones requires extra work.

Second, the Yukawa couplings of fermions with the composite ‘would-be’ NG bosons $\theta_a$ are proportional to

$$P_a \sim -|m'\lambda_a\lambda_a|$$

(we set $\Sigma(p^2 = m^2 = m)$).

Notice that in the commutator the term $m_0$ proportional to the unit matrix, commuting with all eight $\lambda$ matrices, is present. Consequently, there are six composite ‘would-be’ NG bosons corresponding to the generators $(1, 2, 4, 5, 6, 7)$ (coupled with fermions $f$ in a uniquely prescribed way $m' = m_3 \lambda_3 + m_8 \lambda_8$).

Our task is now to find such an ordinary Higgs $SU(3)_f$ multiplet of the elementary spinless fields $\phi$ the condensate of which spontaneously generates $m'$ and at the same time contains six ‘pre-prepared’ elementary ‘would-be’ NG bosons coupled to fermions as in $|m'\lambda_a\lambda_a|$. Comparison with the dynamical picture described above then should yield the prediction of the composite Higgs-like particles and the form of their Yukawa couplings.

It is known $^{32,33}$ that the octet $\phi_a$, $a = 1, ..., 8$ breaks spontaneously the gauge $SU(3)$ symmetry by its vacuum condensates $\langle \phi_3 \rangle, \langle \phi_8 \rangle$ down to the unbroken $U(1) \times U(1)$ subgroup. To establish the correspondence with $^{18}$ we proceed heuristically as follows $^{33}$.

Consider eight small fields in the polar decomposition

$$\Phi(x) = \exp[i\theta_a \lambda_a](m' + s'(x))\exp[-i\theta_b \lambda_b] \sim m' + s'(x) - i[m', \lambda_a \lambda_a]$$

Simple inspection of the commutator reveals that the six fields

$$-m_3 \theta_2, +m_3 \theta_1, -\frac{1}{2}(m_3 + \sqrt{3}m_8)\theta_5, \frac{1}{2}(m_3 + \sqrt{3}m_8)\theta_4,$$

are the ‘would-be’ NG bosons which disappear from the physical spectrum and contribute to the masses of six flavor-changing flavor gluons.

From the correspondence between the formulas (48) and (49) we conclude: First, both the strong flavor dynamics and the elementary scalar octet generate the fermion mass term $m'$. While the dynamical fermion mass generation is a nonperturbative quantum loop effect, the use of the elementary scalar octet amounts merely to a tree-level vacuum condensation. Second, both approaches reveal in the intermediate state the existence of identically coupled ‘would-be’ NG bosons. Third, and most important, we predict from this correspondence the existence of two composite scalars $h_3(x)$ and $h_8(x)$ with peculiar Yukawa interactions with the electroweakly interacting fermions $f$ of the form

$$L'Y = \frac{m_3}{N}\bar{f}(x)\lambda_3 f(x)h_3(x) + \frac{m_8}{N}\bar{f}(x)\lambda_8 f(x)h_8(x)$$

In loops the hard fermion masses should be replaced by the corresponding momentum dependent self-energies $\Sigma(p)$. Here $N$ is a normalization factor analogous to the scale $F$ of the previous section i.e., of the order of the electroweak scale.

Detailed elaboration of the form of the effective interactions of these scalars with the electroweak gauge particles needs further work now in progress. The presence of the flavor matrices $\lambda_3, 8$ in the Yukawa couplings inevitably implies that the fermion loop can only be nonzero (and finite) with the fermion mass insertion proportional to $\Sigma_{3,8}(p)$.

V. FATE OF GLOBAL ABELIAN CHIRAL SYMMETRIES

There are six Abelian symmetries generated by the charges of six chiral fermion currents $J^i_\mu, i =
Taking into account the quantum effects of axial anomalies with four gauge forces in the game we have \( \partial_\mu j^\mu_{qL} = -A_Y - 9AW - 6AG - 6AF \)
\( \partial_\mu j^\mu_{uR} = 8AY + 3AG + 3AF \)
\( \partial_\mu j^\mu_{dR} = 2AY + 3AG + 3AF \)
\( \partial_\mu j^\mu_{l_L} = -3AY - 3AW - 2AF \)
\( \partial_\mu j^\mu_{e} = 6AY + AF \)
\( \partial_\mu j^\mu_{νR} = 3AF \)

Here \( A_X = \frac{g^2}{\Lambda^2} F_X \bar{F}_X \), and \( X \) abbreviates the gauge forces \( U(1) \gamma, SU(2)_L, SU(3)_c \) and \( SU(3)_f \), respectively.

### 1. Anomaly-free currents

There are two linear combinations of the currents \( j^\mu_i \) which are anomaly-free. They can be parameterized by two real parameters \( e, f \):
\[ j^\mu_{1f} = \frac{1}{3}(j^\mu_{qL} + j^\mu_{uR} + j^\mu_{dR} + j^\mu_{l_L} + j^\mu_{e} + j^\mu_{νR}) \]
\[ j^\mu_{2f} = \frac{1}{3}(j^\mu_{qL} - j^\mu_{uR} + j^\mu_{dR} - j^\mu_{l_L} + j^\mu_{e} - j^\mu_{νR}) \]

It creates the true fermion-antifermion massless composite NG boson. Because of its \( ν_R \) component its couplings with fermions are tiny. To prevent any conflict with data we better gauge the corresponding Abelian symmetry. We can, because it is anomaly-free. The NG boson becomes ‘would-be’ and the new \( Z’ \) very heavy, with mass mass \( m_{Z'} \sim g''M_R \) where \( g'' \) is a new gauge coupling constant.

### 2. The axions

One of the four anomalous linear combinations can be chosen as the \textit{vectorial} baryon current
\[ j^\mu_B = \frac{1}{3}(j^\mu_{qL} + j^\mu_{uR} + j^\mu_{dR}) \]
which does not create any pseudo NG boson.

We are left with three anomalous Abelian chiral currents which create three pseudo NG bosons. Their masses are assumed to be due to the nonperturbative effects of three non-Abelian forces present in the game. It is natural to identify two of them with those already known in the literature: First is the famous Weinberg-Wilczek axion \( a \), massive due to the instanton of the confining QCD. Second is the Anselm-Uraltsev ‘arion’ \( b \) with the mass expectedly associated with nonperturbative effects of the electroweak \( SU(2)_L \) gauge fields. The third one is the new axion \( c \) with mass expectedly associated with nonperturbative effects of the quark flavor. \( SU(3)_f \) gauge fields. We are not aware of any generally accepted formula for \( m_b \) and \( m_c \). Referring to \( [33] \) we merely expect that they contain the suppression factor due to the screened instantons.

Because the lack of data there is much freedom in fixing the coefficients in the linear combinations
\[ j^\mu_{a} = \sum a_i j^\mu_i, \quad j^\mu_{b} = \sum b_i j^\mu_i, \quad j^\mu_{c} = \sum c_i j^\mu_i \]

For definiteness we can simulate the data by demanding: 1. The axion \( a \) is invisible, hence \( aR ≠ 0 \). 2. The strongest gauge interaction that gives rise to its mass is QCD. Hence, \( \partial_\mu j^\mu_a \) must not contain \( A_f \). 3. The axion \( b \) becomes massive due to the electroweak interactions. Hence, \( \partial_\mu j^\mu_b \) must not contain \( A_f, A_c \). 4. The axion \( c \) does not interact with gluons. Hence, \( \partial_\mu j^\mu_c \) must not contain \( A_f, A_c \). 5. The requirement \( h_{eR} = 0 \) implies that the interactions of \( c \) with fermions are not suppressed by \( M_R \).

Explicit evaluation of the axion properties requires separate work. For example, the axions as described above are not the mass eigenstates, and in the present scheme there should be the axion mixing and, possibly the axion oscillations. Because the enormous hierarchy of scales the tiny mixing should be negligible.

The form of the pseudo NG currents fixes the effective interactions of the pseudo NG bosons. 1. The WT identities associated with the pseudo NG currents determine the effective Yukawa couplings of the pseudo NG bosons with fermions. In separable approximation they are pseudoscalar. 2. The divergences of the pseudo NG currents fix the effective interactions of the pseudo NG bosons \( a, b, c \) with the respective gauge fields. There is no way how they could interact with the gauge particles by the renormalizable interactions.

The effective interactions of the pseudo NG bosons with fermions and non-Abelian gauge boson give rise to their masses. For the axion \( a \) massive due to the instanton of the confining QCD we use the generally accepted estimate
\[ m_a \sim \Lambda_{QCD}/\Lambda ∼ 10^{-2} \text{eV} \]

For an estimate of masses of the bosons \( b \) and \( c \) associated with the dynamically massive non-Abelian gauge sectors \( SU(2)_L \) and \( SU(3)_f \), respectively, we are not aware of any generally accepted formula. Here we simply wishfully assume that the masses \( m_b \) and \( m_c \) of the axions \( b \) and \( c \) are such that they explain some astrophysical puzzles currently discussed.
the Standard model: First, it yields, at least in our approximation, the lepton and quark masses. Second, as a necessary consequence of spontaneous fermion mass generation the composite ‘would-be’ NG bosons inevitably give rise to masses of the electroweak gauge bosons $W$ and $Z$ proportional to the fermion masses. Third, a natural consequence of the existence of three electroweak composite ‘would-be’ NG bosons is that they have their genuine massive partner, the composite Higgs-like boson $h$. What are the main specific consequences of this very rigid model?

1. The Higgs-like $0^+$ boson $h$ is a fermion-antifermion composite. Although the Yukawa interactions of $h$ with leptons and quarks are the same as in the Standard model, its interactions with the electroweak gauge bosons $W, Z, A$ being all loop-generated, are different. Experimental data on these interactions provide the crucial test of the present model.

2. The model predicts two massive flavored Higgs-like fermion-antifermion $0^+$ scalars $h_3$ and $h_8$. These are the massive partners of the composite ‘would-be’ NG bosons inevitably following spontaneous breakdown of flavor $SU(3)_f$ by dynamically generated masses of the electroweakly interacting leptons and quarks. Both the tree-level Yukawa couplings of $h_3$ and $h_8$ with fermions and their loop-generated effective interactions with the electroweak gauge bosons are uniquely fixed.

3. For purely theoretical reason of anomaly freedom we were enforced to add to the list of the observed SM chiral lepton and quark fields one triplet of sterile right-handed neutrino fields. This by itself is most welcome: First, the interactions of sterile neutrinos with flavor gluons cause the complete dynamical self-breaking of $SU(3)_f$. Second, we believe that the $SU(3)_f$ with sterile right-handed neutrinos provides the origin of the seesaw mechanism: There is a hopefully good reason why the left-handed neutrino Majorana masses are not dynamically generated. As a consequence the huge Majorana masses of the right-handed neutrinos are responsible for the observed lightness of active neutrinos. The observed three active neutrinos are the extremely light Majorana particles.

4. Mixing of superheavy sterile Majorana neutrinos implies new complex phases and therefore a new source of CP violation in the model. An extra source of CP violation seems indispensable for understanding the baryon-antibaryon asymmetry of the Universe [20].

5. Global anomalous Abelian chiral symmetries of the microscopic Lagrangian, spontaneously broken by the dynamically generated fermion masses result in three pseudo-NG bosons. (i) The axions are the well motivated candidates for dark matter [8]. (ii) The axions naturally solve several astrophysical puzzles [10]. (iii) The Weinberg-Wilczek axion naturally solves the problem of strong CP violation [6].

6. The $SU(3)_f$ deals ultimately with one parameter, the the scale $\Lambda$. The electroweak sector adds to the list of parameters the unquantized weak hypercharges. These are, however, completely fixed in the SM by the lepton and quark electric charges, and the number of free parameters therefore does not increase. There should be plenty of relations between masses, mixing matrices and gauge couplings which can be tested. At the present fragmentary understanding of the strong-coupling infrared flavor dynamics the mass relations belong more to the realm of dreams rather than to reality. But sound dreaming is healthy [37], and we mention merely the sum rules for the gauge boson masses $m_W, m_Z$ in terms of the masses of electroweakly interacting fermions.

7. The model contains three superheavy spin-zero scalars $\chi_i$ composed of sterile neutrinos with fixed interactions. It is amazing that such particles can be phenomenologically useful in cosmology [24].

8. There should be new bound states with masses of order $\Lambda$. Although the scale $\Lambda$ of $SU(3)_f$ is very high, there is a possibility to look for traces of the new gauge flavor dynamics directly at the extremely high energies in the air showers.

VII. CONCLUSIONS AND OUTLOOK

Spontaneous generation of lepton and quark masses in the Standard model does not provide any understanding of their values: Fermion masses come out as the Higgs field condensate $v = 246$ GeV multiplied by independently renormalized i.e., theoretically arbitrary, vastly different, Yukawa couplings. This is the phenomenological description of fermion masses by construction. If the recently discovered spinless 125 GeV boson were indeed the Higgs boson of the Standard model such a sad state of affairs would stay for ever.

We have suggested in this paper to replace the essentially classical Higgs sector of the SM by a new non-Abelian genuinely quantum dynamics defined by properly gauging the flavor (family, generation, horizontal) $SU(3)_f$ index. We have argued that the $SU(3)_f$ gauge quantum flavor dynamics in its strong coupling regime, due to its sterile neutrino sector, is not confining but it self-consistently completely self-breaks. If true this implies that both its quasi-particle oscillator-type excitations as well as its bound-state collective excitations have masses which are the calculable multiples of $\Lambda$. This is the ultimate reason for our suggestion. The computations of particle masses presented here are, however, still rather illustrative. Although the Ansatz for the kernel of the SD equation is very crude, it nicely illustrates the important point: There are no large and small numbers in the microscopic Lagrangian. They come out only in solutions of the field equations. This happens as a robust, natural non-perturbative phenomenon, and should not be called fine tuning. Derivation of the effective couplings $g_{ab}$ is a dream.
In old days there was nothing wrong with the Higgs sector of the Standard model from the theory point of view: What could be better than a renormalizable weak coupling theory? Latter objection of 'unnaturalness' was always considered by some as unwarranted. With the triumphant discovery of the Higgs boson this theory is now very successful also phenomenologically, and has to be taken truly seriously, even though as an incomplete effective field theory.

Such a view does not preclude attempts at revealing an underlying microscopic dynamics. The experimentally confirmed properties of the canonical Higgs mechanism must be, first of all, reproduced by it. Its phenomenological parameters, e.g. its Yukawa couplings should, however, be the calculable numbers.

For some (including the author) the guiding idea in attempts to find the microscopic dynamics underlying the Higgs one always was the microscopic theory of superconductivity of Bardeen, Cooper and Schrieffer (BCS), known to underlie the phenomenological theory of Ginzburg and Landau (GL): First of all, there is nothing wrong with the GL theory. It is so beautiful and so deep that it lead to the prediction of two phenomena awarded by Nobel prizes: The Josephson effect, and type-II superconductivity. We cannot be sure so deep that it lead to the prediction of two phenomena: The Josephson effect [42], and type-II superconductivity [43]. We are inclined to argue that the Weinberg's famous dimension-five coupling should be an integral part of the SM effective Lagrangian today. It implies the firm prediction: The three active neutrinos are the massive Majorana particles.

The microscopic BCS superconductivity of course reproduces all good features of the phenomenological GL theory which, under certain assumptions, can be derived from it [44]. This is the necessary condition. The validity of BCS is, however, truly tested where GL has nothing to say: By measuring the dependence of the electronic specific heat on temperature below the superconducting critical temperature \( T_c \). In accordance with data this dependence is exponential due to the gap in the quasi-electron dispersion law. What is the analogous decisive test of the dynamics of the electroweak symmetry breaking? In the canonical Higgs model the Higgs condensate exists for ever. In quantum flavor dynamics the fermion masses are generated only below \( \Lambda \). Safe but impractical way to test the origin of the massiveness of fermions and of intermediate bosons is to go to very high energies.

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APPENDIX: DYNAMICAL FERMION MASS GENERATION

In the Appendix we present solutions of the gap equations (17) and (14) with fermion mixing neglected (\( U = V = 1 \) for the Dirac fermions and \( U = 1 \) for Majorana neutrinos), respectively:

\[
\gamma = \frac{1}{4} \lambda_a g_{ab} I(\gamma) \lambda_b
\]

\[
\gamma = -\frac{1}{4} \lambda_a g_{ab} I(\gamma) \lambda_b^T
\]

Here \( \lambda_a \) are the Gell-Mann matrices and \( \gamma \) is a real diagonal matrix with positive entries which determines the fermion masses as follows. Because \( \Sigma(p^2) = \frac{\Lambda^2}{p} \gamma \), the fermion mass, defined as a pole of the full fermion propagator is

\[
m = \Sigma(p^2 = m^2) = \Lambda \gamma^{1/2}
\]

With \( g_{11}, g_{22}, g_{33}, g_{44}, g_{55}, g_{66}, g_{77}, g_{88} \) different from zero the right hand sides of equations (50) and (51) are the diagonal matrices. The equations themselves can be rewritten as

\[
\gamma^D/M_i = \sum_{k=1}^{3} \alpha^{D/M}_{ik} \gamma^D/M_k \ln \frac{1 + \left( \frac{\gamma^D/M_i}{\gamma^D/M_k} \right)^2}{\left( \gamma^D/M_k \right)^2}
\]

where

\[
\alpha^{D/M} = \frac{3}{64\pi^2} \begin{pmatrix} g_{33} + \frac{2}{3} g_{38} + \frac{1}{3} g_{88} & g_{22} \pm g_{11} & g_{55} \pm g_{44} \\ g_{22} \pm g_{11} & g_{55} \pm g_{44} & g_{77} \pm g_{66} \\ g_{55} \pm g_{44} & g_{77} \pm g_{66} & g_{88} \end{pmatrix}
\]

and the upper and lower signs correspond to the Dirac fermion masses and the Majorana neutrino masses, respectively.

The goal is an immodest one: Demonstrate convincingly that there is a reasonable set of the effective low-momentum couplings \( g_{ab} \), which gives rise to the huge
masses of Majorana neutrinos \((M_{LR} \sim O(\Lambda))\) and at the same time to the hierarchical spectrum of many orders of magnitude lower masses \(m_i(f) \ll \Lambda\) of the electroweakly interacting fermions.

Simplifying as much as we can we consider only

\[ g_{33}, g_{38}, g_{88}; g_{11} = -g_{22}, g_{44} = -g_{55}, g_{66} = -g_{77} \]
different from zero.

(A) The matrix gap equation for the Dirac masses \(m_i\) becomes diagonal and decoupled, and it is easily solved. Provided the combinations

\[
\alpha_{11} = \frac{3}{64\pi^2} (g_{33} + \frac{2}{\sqrt{3}} g_{88} + \frac{1}{3} g_{88}) \\
\alpha_{22} = \frac{3}{64\pi^2} (g_{33} - \frac{2}{\sqrt{3}} g_{88} + \frac{1}{3} g_{88}) \\
\alpha_{33} = \frac{3}{64\pi^2} g_{88}
\]

This set of equations has a solution for any set of \(\gamma_i^M > 0\). To be explicit, let us put for an illustration \((\gamma_1^M, \gamma_2^M, \gamma_3^M) = (0.1, 0.2, 0.3)\) and \((\gamma_1^D, \gamma_2^D, \gamma_3^D) = (10^{-20}, 10^{-22}, 10^{-26})\) (this corresponds approximately to the hierarchy for charged leptons provided \(\Lambda = 10^{10}\) GeV). Then

\[
g = \begin{pmatrix}
8.08101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-8.08101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.7425 & 0 & 0 & 0 & 0 & 0.0899893 & 0 \\
0 & 0 & 0 & -21.8124 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 21.8124 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -34.029 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 34.029 & 0 & 0 \\
0 & 0 & 0.0899893 & 0 & 0 & 0 & 0 & 1.31887 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(B) Finding the solution for the Majorana masses is less straightforward. First, for \(g_{11} = g_{44} = g_{66} = 0\), the gap equations for the Majorana masses have no solution because of the minus sign in front of the \(\alpha_{ii}\). Consequently, \((g_{11}, g_{44}, g_{66}) \neq 0\). Second, in the case of sterile Majorana neutrinos we are not aware of the necessity of the hierarchical mass spectrum. With the constants \(\alpha_{ii}\) fixed by the numerical values of the Dirac masses the equations (52) for \(\gamma_i^M\) can be viewed as a system of three inhomogeneous linear equations for the unknown \((g_{11}, g_{44}, g_{66})\):

It is important that the precise size and hierarchy of \(\gamma_i^D\) does not play any important role for the numerical values of \(\gamma_i^M\).

We are far from making any strong conclusions from the solutions of the SD equation found here. They are nevertheless suggestive in the following respect: Often and naturally the observed hierarchy of fermion mass scales is attributed to 'tumbling' in asymptotically free gauge theories \([46]\). When the gauge coupling in the most attractive channel, growing towards infrared exceeds the critical value, the fermion-antifermion condensate (fermion mass \(m < M\)) is dynamically generated. The gauge coupling starts growing again towards smaller momenta until it reaches the critical value in the second most attractive channel, and another fermion-antifermion condensate (fermion mass \(m < M\)) is generated. It is not excluded that, as suggested by our simple analysis, the Majorana and Dirac masses are in fact dynamically generated at different scales.

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