A NEW METAHEURISTIC APPROACH FOR THE ART GALLERY PROBLEM

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ABSTRACT

In the problem "Localization and trilateration with the minimum number of landmarks", we faced the 3-Guard and classic Art Gallery Problems. The goal of the art gallery problem is to find the minimum number of guards within a simple polygon to observe and protect its entirety. It has many applications in robotics, telecommunications, etc. There are some approaches to handle the art gallery problem that is theoretically NP-hard. This paper offers an efficient method based on the Particle Filter algorithm which solves the most fundamental state of the problem in a nearly optimal manner. The experimental results on the random polygons generated by Bottino et al. [1] show that the new method is more accurate with fewer or equal guards. Furthermore, we discuss resampling and particle numbers to minimize the run time.

Keywords  Art Gallery Problem · Localization · Visibility Polygon · Robotics · Particle Filter

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The classical Art Gallery problem was posed by Victor Klee in 1973 [2] and has been studied for several decades as one of the most important issues in computational geometry. It has many applications in minimizing landmarks, trilateration [3], pose estimation [4], and robot localization [5]. There are many ways to protect an art gallery. An effective way is using security cameras. Equipping an art gallery with many cameras will be expensive and difficult to maintain. On the other hand, if the number of cameras is small, some parts of the gallery may not be monitored. The goal is to find the minimum number of cameras (guards) to protect an art gallery. Every point in an art gallery should be visible to at least one guard. The art gallery may be considered as an $n$-gon, designated $P$ for short, that can be with or without holes.

The question is that how many guards are sufficient to cover $P$ [2].

There are several types of art gallery problems. The two-guard [6] problem in a simple polygon $P$ requires two guards on the boundary of $P$ from the starting vertex $u$ to the ending vertex $v$. One of the guards goes around the boundary clockwise and the other counterclockwise, such that they are able to see each other. The line segment connecting the two guards are lies completely in $P$ all the time [6].

The three-guard problem in a simple polygon $P$ asks whether three guards can move from a vertex $u$ to another vertex $v$ such that the first and third guards are separately on two boundary chains of $P$ from $u$ to $v$ and the second guard is always kept to be visible from two other guards inside $P$ [7].

The location of the guards in the variations of the art gallery problem is different. Guards can be located on the vertices of the polygon (vertex-guard), only on the edge (edge-guard), or without any restrictions (point-guard).

It has been known that the art gallery problem lies within the $NP$–hard problems. $NP$–hard problems are not solvable in polynomial times. Lee and Lin [8] constructed a reduction from 3 – SAT and proved that the art gallery problem is $NP$–hard. Finding the exact solution to these algorithms is time-consuming. Therefore, approximation and metaheuristic algorithms are used to find approximate solutions to some hard optimization problems. These algorithms can find near-optimal solutions in lower times. In 2018, Mikkel et al. [9] proved that the problem is $\exists \mathbb{R}$–complete. The class $\exists \mathbb{R}$–complete consists of problems that can be reduced in polynomial times to the problem of deciding whether a system of polynomial equations with integer coefficients and any number of real variables has a solution [9].

The art gallery problem has been widely studied over the years. In 1975, Chvátal established a theorem known as Chvátal’s Art Gallery Problem [10]. It states that $M = \lceil \frac{n}{4} \rceil$ guards are always sufficient and sometimes necessary to protect an $n$–gon art gallery. In 1978, another simple proof based on the polygon triangulation and coloring of vertices was proposed by Fisk [11]. Based on Fisk’s proof, Avis and Toussaint [12] proposed an $O(n \log n)$ time divide-and-conquer algorithm for placing guards in a simple polygon.

The art gallery problem has also been studied for orthogonal polygons [13]. As an important subclass of polygons, orthogonal polygons have internal angles of either $90^\circ$ or $270^\circ$ [14]. In 1983, Kahn et al. [15] proved that $M = \lfloor \frac{n}{4} \rfloor$ guards are always sufficient and sometimes necessary to protect an orthogonal polygon (galley). Later in 1983,
O’Rourke [13] proved again that the maximum guards in orthogonal polygons are \( M = \lceil \frac{n}{4} \rceil \). Katz and Roisman [16] proved that the art gallery problem for orthogonal polygons is NP-hard.

Gosh [17] proposed an approximation algorithm for the vertex-guard problem. Antonio et al. [18] presented an approximation algorithm based on general metaheuristic genetic algorithms to solve the vertex-guard problem. Amit et al. [19] introduced a heuristics algorithm for solving the point-guard problem. In 2011, Bottino et al. [1] introduced optimal solutions for the point-guard problem and in 2013 Tozoni et al. [20] introduced an algorithm that successfully found the exact solution when tested on a very large collection of instances from publicly available benchmarks, but whose convergence could not be in general guaranteed.

The remaining sections of this paper are organized as follows: Some important definitions related to the article are studied in Section 2. Then, in Section 3 a new metaheuristic method for the art gallery problem will be discussed and the simulation results are studied in Section 4. The conclusion and future work are discussed in Section 5. All symbols and variables used throughout the paper are introduced in Table 1.

## 2 Basic Definitions

In this section, some theoretical basics related to the proposed algorithm are going to be introduced.

### 2.1 Visibility Polygon

The visibility polygon is one of the foremost critical issues in computational geometry. Given a simple polygon \( P \), two points are said to be visible to each other if the line segment that joins them does not cross any obstacles [21]. Polygon \( P \) will be covered by point \( q \), if it is entirely visible from \( q \). In 1980, the first algorithm for computing the visibility polygon was introduced by Avis and ElGindy [22]. After that, different versions of this problem introduced each having their own applications [23].

Consider \( P \) as an \( n \)-gon and \( q \) as a point in it. The goal is to compute the visibility polygon or the covered area. In order to compute the visibility polygon in concave polygons, let \( P = (v_0, v_1, ..., v_n) \) and \( E = (v_0v_1, ..., v_{n-1}v_n) \) are respectively as polygon’s vertices and edges. Asano et al. [24] introduced an algorithm (Algorithm 1) to compute the visibility polygon in concave polygons using the Sweep Line algorithm which the complexity of it is \( O(n \log n) \).

**Algorithm 1 Computing the visibility polygon of a point \( q \) in a concave polygon \( P \)** [24]

**Input:** Polygon \( P = (v_0, v_1, ..., v_n) \) and point \( q \) inside \( P \)

**Output:** Visibility Polygon \( V(P, q) \)

Sort \( (v_0, v_1, ..., v_n) \) counterclockwise

Save \( E \) in the queue

if \( P \) is convex then

\[ V(P, q) = P \]

else

Draw lines from \( q \) to \( (v_0, v_1, ..., v_n) \)

Save the lines in the new queue

if the lines are outside \( P \) then

Delete the lines

else

Save them in the new queue

end if

end if

Return \( V(P, q) \)

### 2.2 Particle Filter

The particle filter is a recursive algorithm that was introduced in 1996 by Del Moral [25] and has been widely used in diverse fields such as robotics [26]. The general idea in this problem is to seek an approximate solution of a complex model rather than an accurate solution of a simple model.

The particle filter uses a set of particles to find feasible solutions to a problem. Each particle provides a feasible solution to the problem. Using the uniform random distribution, the algorithm spreads particles in the inner space of a polygon. At first, the particles have the same weight. According to the information obtained, the particle weights are being
changed and will be updated. So, the low-weighted particles are destroyed and are being produced again around the high-weighted particles. This step is known as resampling.

In the following, using the theoretical foundations put forwarded, the proposed algorithm for the art gallery problem will be introduced.

3 A New Algorithm for the Art Gallery Problem using the Particle Filter

First, some particles are uniformly distributed in the polygon space. The probability that each particle is a solution to the problem is considered to be the same. For example, 50 particles are uniformly dispersed. So, the probability of each particle being a solution is \( \frac{1}{50} \) that shows the particle weight or importance. Although each particle is regarded as a set of points (guards) in the proposed particle filter algorithm, each particle is considered as a set of points (guards) in the proposed algorithm. Consider \( n \)-gon \( P \) as the input of the algorithm.

As mentioned before, \( M = \lceil \frac{n}{4} \rceil \) guards are needed to protect an \( n \)-gon in the worst case. The number of guards in each stage is represented by \( k \). The maximum value of \( k \) is \( M \). The binary search is done in each stage until reaches an optimal solution.

3.1 Binary Search

The binary search algorithm is used to find the optimal solution (optimal \( k \)) in an ordered array (this set is started from 1 to \( M \)). In the binary search, the search space is split in half (\( \frac{M}{2} \)). First, \( \frac{M}{2} \) guards are checked that can cover the polygon or not. If \( \frac{M}{2} \) guards are enough to cover the polygon, the first split (\( \{1, 2, ..., \frac{M}{2}\} \)) is divided into two minor sets again and the binary search will be continued. If \( \frac{M}{2} \) guards are not adequate, the second split (\( \{\frac{M}{2}+1, ..., M\} \)) is divided into two minor sets again and the binary search will be continued. This process is continued until the optimal number of guards are obtained or all the elements are checked.

Let \( X \) be a set of \( N \) particles:

\[
X = (X_1, X_2, ..., X_N)
\]

Then, the particles are monotonously distributed in the polygon and the particle weights are defined as follows:

\[
w_i = \frac{1}{N} \quad st \quad 1 \leq i \leq N
\]

Each particle is regarded as a set of \( M \) random points (guards) in the polygon such as an \( M \)-footed spider as follows:

\[
X_1 = ([x_1^1, y_1^1], [x_1^2, y_1^2], ..., [x_{M^1}, y_{M^1}])
\]
\[
X_2 = ([x_1^2, y_1^2], [x_2^2, y_2^2], ..., [x_{M^1}, y_{M^1}])
\]

...  

\[
X_N = ([x_1^N, y_1^N], [x_2^N, y_2^N], ..., [x_{M^N}, y_{M^N}])
\]

Using the Algorithm the visibility polygon of each particle is computed by the union of the visibility polygons of the set of its points.

\[
VP(X_1) = VP([x_1^1, y_1^1]) \cup VP([x_1^2, y_1^2]) \cup VP([x_{M^1}, y_{M^1}])
\]
\[
VP(X_2) = VP([x_1^2, y_1^2]) \cup VP([x_2^2, y_2^2]) \cup VP([x_{M^1}, y_{M^1}])
\]

...  

\[
VP(X_N) = VP([x_1^N, y_1^N]) \cup VP([x_2^N, y_2^N]) \cup VP([x_{M^N}, y_{M^N}])
\]

Then, the particle weights are being updated:

\[
w_p(X_1) = (|VP(X_1)|)P
\]
\[ vp(X_2) = (|VP(X_2)|) P \]
\[ \vdots \]
\[ vp(X_N) = (|VP(X_N)|) P. \]

For each particle \( X_i \), the weight \( vp(X_i) \) is obtained from the ratio of the area of the visibility polygon for each particle \(|VP(X_i)|\) to the area of the polygon \(|P|\). Then, the resampling stage is started and repeated until \( vp(max) = 1 \) (the best ratio for the visibility area). The counter of the resampling stages is \( j \). It is started from 1 and is updated by incrementing by 1 at each stage until it reaches the maximum defined number \( RN \) (the defined resampling number). The best particle \( X \) is obtained when \( vp(X_i) = 1 \) which is indicative a feasible solution to the problem. No other particles are needed to be checked. There is no need to complete the sampling at this stage.

If \( vp(max) \neq 1 \), then there will be two states:

- All the resampling stages \( RN \) are completed and the particle that covers the polygon is obtained and there is no need to check the other particles.
- All the resampling stages \( RN \) are completed, but the particle that covers the polygon is not obtained yet. Therefore, the particle that covered the polygon in the last stage is selected as the optimal solution (Best \( X \)).

**Algorithm 2** Minimum Guards for the Art Gallery Problem

**Input:** Simple polygon \( P \) with \( n \) vertices

**Output:** Location of the minimum number of guards to guard \( P \)

\( k_{min} = 1, k_{max} = \lfloor n/3 \rfloor \) and \( k = k_{max} + k_{min} \)

while 1 do
  Generate \( X = \{X_1, X_2, ..., X_N\} \).
  for \( j = 1; j < RN; j++ \) do
    \( vp_{max}(X) = 0 \)
    for \( i = 1; i < N; i++ \) do
      \( vp(X_i) = \frac{|VP(X_i)|}{|P|} \)
      if \( vp(X_i) > vp_{max}(X) \) then
        \( vp_{max}(X) = vp(X_i) \)
      end if
    end for
    if \( vp_{max}(X) == 1 \) then
      \( BestX = X_i; \)
      break(); /* No need to other particles */
    end if
  end for
  if \( vp_{max}(X) == 1 \) then
    \( k_{max} = k; \)
    \( kopt = k; \)
    break(); /* No need to other resampling */
  else
    \( k_{min} = k; \)
  end if
end while

Return(\( kopt \)); /* When any particle cannot cover \( P \) with \( k \) cameras*/
Return(\( BestX \)) /* The best result for previous \( k \) */

3.1.1 The Resampling Stage

In this stage, low-weighted particles will be eliminated and reproduced again around the high-weighted particles. For example, high-weighted particles which their \( vp(X_i) \) are more than a specific threshold such as \( \tau = 0.9 \) (the particles that cover more than 90% of the polygon) are sampled and the particles that cover the polygon lower than 90% will be omitted. Then, the omitted particles are reproduced around the high-weighted particles. Therefore, the number of particles \( N \) is constant and their locations are changed.
In the following, the visibility polygon for each particle is computed using the Algorithm 1 (the weights are updated). Another resampling stage is done again (The maximum number of the resampling stages is limited, for example 20). A particle may be found that \( vp = 1 \) (the particle can cover the polygon). Then, this particle will be saved as an optimal solution. Regard that this particle is not unique. This procedure will be performed for all particles using the binary search. It is possible to obtain no particle after completing all the resampling stages. In this situation, the particle that covered the polygon in the last stage is regarded as the optimal solution. The proposed approach is presented in Algorithm 2. This algorithm finds the minimum number of required guards to protect a polygon using the particle filter algorithm.

If \( vp < 0.9 \) or \( vp > 0.9 \) for all the particles, any particles can enter the resampling stage. Reproducing omitted particles around other particles is based on their weights. The omitted particles are reproduced more around a particle which has higher \( vp \).

The complexity of the particle filter algorithm is \( O(N \log N) \) for \( N \) particles and the visibility polygon for an \( n \)-gon can be computed in \( O(n \log n) \) times. In the worst case, the steps are repeated \( O(\log M) \) times using the binary search method in which \( M = \frac{n}{3} \). The algorithm runs \( RN \) times which is resampling numbers and it is at most \( O(n/3) \). So, the whole complexity of the proposed algorithm is \( O(n^3 \log^2 n) \).

4 The Experimental Results

The simulation results are presented here to evaluate the proposed algorithm.

Example 1: According to [19], a simple polygon with 18 vertices is considered. The polygon’s area \( (P) \) is 92243.5\( m \) and its coordinates are as follows:

\[
P = ([558, 497], [513, 148], [477, 413], [439, 413], [403, 150], [384, 410], [339, 409], [298, 152], [267, 409], [228, 409], [192, 151], [161, 412], [124, 412], [80, 151], [74, 413], [52, 413], [25, 147], [11, 497]).
\]

According to \( M = \left\lceil \frac{21}{3} \right\rceil = 7 \) and the binary search algorithm, the proposed procedure for \([1, 2, 3, ..., 6]\) is as follows. The algorithm is first performed on \( 3D \) particles. If a particle can be found that covers the polygon, the algorithm will be performed on the first part of the array \([1, 2, 3]\) using the binary search. Here, the particle is not found, so the second part of the array \([3, 4, 5, 6]\) is checked using the binary search. In fact, the algorithm is examined for \( 5D \) particles. If a \( 5D \) particle that covers the polygon is found, then the algorithm will be performed for \( 4D \) particles. Since the covering \( 5D \) particle is not found here, the algorithm will be checked for \( 6D \) particles. After implementing the algorithm on particle dimension using the binary search, it is concluded that the algorithm for \( 6D \) particles (6 guards) reaches the optimal solution. The guards’ locations for the \( 6D \) particle are shown in Figure 1. Their coordinates are shown in

\[
Best(x) = ([83, 402], [22, 276], [227, 414], [514, 231], [320, 360], [404, 487]).
\]

Figure 1: The simulation results of the proposed algorithm for the given polygon in [19] shows that 6 guards (small black diamonds) are needed to be selected in this polygon.

Example 2: According to Amit et al. [19], another simple polygon with 21 vertices is considered. The polygons’ area \( (P) \) is 91201.5\( m \) and its coordinates are as follows:

\[
P = ([475, 512], [146, 512], [284, 486], [146, 480], [147, 366], [118, 415], [99, 288], [128, 295], [146, 336], [151, 226], [256, 226], [151, 191], [405, 190], [294, 226], [438, 223], [437, 316], [475, 290], [472, 418], [437, 343], [428, 474], [314, 480]).
\]

According to \( M = \left\lceil \frac{21}{3} \right\rceil = 7 \) and the binary search algorithm, for \([1, 2, 3, ..., 7]\) the proposed procedure is as follows. The algorithm is first performed on the \( 4D \) particles. The \( 4D \) particle that covers the polygon is found, so the first part of the array \([1, 2, 3, 4]\) is examined using the binary search. The algorithm tries to find a \( 2D \) particle to cover the
polygon. After detecting this particle, the algorithm tries to find a 1D particle that covers the polygon. Here, the 2D particle to cover the polygon that covers the polygon is not found. Then, the algorithm is checked to detect a 3D particle to cover the polygon and this particle is not found either. After implementing the algorithm on particle dimension using the binary search, it is concluded that the algorithm for 4D particles (4 guards) reaches the optimal solution. The guards’ locations for the 4D particle are shown in Figure 2. Their coordinates are

$$Best(X) = ([116, 348], [208, 209], [435, 324], [292, 495]).$$

Figure 2: The simulation results of the proposed algorithm for the given polygon in [19] shows that 4 guards (small black diamonds) are needed to be located in this polygon.

Figure 3: Comparison of the average guard numbers of the proposed algorithm with those in [1].

Using results obtained for the above examples and comparing them with Amit et al. [19], it can be inferred that the results are approximately optimal.

A comparison between the average accuracy of the proposed algorithm and that of Bottino et al. [1] for random polygons is shown in Table 2. In this comparison, 20 random 30–gons, 40–gons, 50–gons, and 60–gons are studied. These polygons are the same as the ones in [1] and some of them are shown in Figure 4.

Table 2 shows that the average optimal number of guards for random polygons in the proposed algorithm is less than or equal to the average optimal number of guards in [1] shown graphically in Figure 3.

Table 2: Comparison of the results of the method and Bottino et al. [1]

| n   | Proposed method | Bottino et al.’s algorithm [1] |
|-----|-----------------|---------------------------------|
| 30  | 3.9             | 4.2                             |
| 40  | 4.6             | 5.6                             |
| 50  | 6.1             | 6.7                             |
| 60  | 8.1             | 8.6                             |

Figure 3: Comparison of the average guard numbers of the proposed algorithm with those in [1].

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| 50  | 6.1             | 6.7                             |
| 60  | 8.1             | 8.6                             |
Figure 4: Some of the studied polygons and the locations of the guards using the proposed method.

Figure 5: According to the simulation results of the proposed algorithm, 6 guards are needed to be selected in this 50-gon.

In the following, the proposed algorithm is examined on an arbitrary 50-gon (Figure 5) and apply it to the different numbers of particles and thresholds ($\tau$). The optimal solutions are obtained at different times and illustrated in Figure 6. As shown there, the X, Y, and Z axes indicate the number of particles, threshold ($\tau$), and time, respectively. The color spectrum on the right shows the time in seconds and its color changes from blue to red with the increasing time. The points in the range of 1 to 100 seconds are blue. As the time increases, the color changes from dark blue to light blue. The darkest red point shows the optimal solution at the latest time (476 s) that is obtained when the number of particles is 90 and $\tau = 0.95$. The darkest blue point indicates the optimal solution with the minimum running time (28 s) and is obtained when the number of particles is 15 and the threshold is set at $\tau = 0.98$. Also, the darkest red point shows the
worst solution and the longest time used (476 s) and is obtained when the number of particles is 90 and the $\tau$ is 0.95. The results show that $\lfloor N = n/3 \rfloor$ and $\tau = 0.98\%$ are acceptable settings for our algorithm. The achievements of this paper can be used in issues such as SLAM and pose estimation.

Figure 6: The optimal solutions at different times
5 Conclusion and Future Work

The art gallery problem is one of the most vital NP-hard problems in computational geometry applicable in robot localization and SLAM. Using the particle filter, a new heuristic method has been proposed in this paper to find the near optimal solution of the art gallery problem. In this method, each particle provides an optimal solution to find the number and location of the adequate guards that would guard the given polygon. The experimental results show that our method finds fewer (or equal) number of guards to achieve this goal. In future work, guarded guard problem, 2—Guards and 3—Guards problems may be considered. Also, one can extend our results to solve the problem of achieving maximum coverage for a given polygon using a fixed number of guards.

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