Effect of bubble size on Lagrangian pressure statistics in homogeneous isotropic turbulence

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Abstract. The study of bubble’s behavior in turbulent flows is fundamental to the understanding of many engineering applications that are concerned with bubbly/two-phase flow. In turbomachinery, for example, tiny gas nuclei present in the liquid may grow to macroscopic size if the instantaneous pressure dips below the vapor pressure for a time long enough to incite cavitation events. In this paper, the Lagrangian pressure statistics of finite sized bubbles in homogeneous isotropic turbulence is investigated using highly-resolved direct numerical simulations of the Navier-Stokes equations at $Re_\lambda = 150$. A modified Maxey-Riley equation is used for Lagrangian tracking of bubbles in the turbulence field. The Lagrangian pressure statistics (probability density function, frequency of low-pressure events and their duration) are analyzed as functions of the bubble size. The overall picture that emerges is consistent with finite-sized bubbles being driven towards vortex cores, resulting in an average pressure further below the mean value and longer and more frequent low-pressure events as the considered size is increased.

1. Introduction
Turbulent flows with dispersed particles or bubbles play an important role in different natural and industrial applications. Examples include sediment, pollen and plankton transport and dispersion, atmospheric pollution, mixing in combustion engines, gas nuclei transport around turbines and propellers, mixing in chemical reactors, and many others [1, 2, 3]. Suspended particles in turbulent flows have been explored previously both numerically and experimentally (see for instance Balachandar & Eaton [4] for a comprehensive review). In this contribution we focus on so-called Lagrangian time-histories of quantities, i.e., time-histories recorded along the particles’ trajectories, considering light particles that model small bubbles. Several experimental and numerical works have been reported on the Lagrangian velocity and acceleration of light particles [5, 6, 7, 8, 9].

Qureshi et al. [3] discussed the transport of material properties in turbulence based on their experiments on neutrally buoyant particles in isotropic turbulence. Volk et al. [7] compared the effect of particle types (light, neutral, and heavy), while Cartwright et al. [2] reviewed the dynamics of particles in chaotic flow. It is now established that particles lighter than the fluid
accumulate near the high vorticity regions, whereas heavier particles tend to move away from the high vorticity regions gathering near the high strain regions [4, 2].

The statistics of the pressure field along the particles’ trajectories, on the other hand, have received little or no attention. Our work aims to fill this void because these statistics are directly relevant to cavitation modeling. In a recent work [10] initial results were obtained that hold in the limit of zero particle size, in which the particle moves with the fluid’s velocity. Here we study the effect of particle size on the pressure statistics, computing the trajectories by solving the Maxey-Riley equation [11] with coefficients appropriate for gas bubbles in liquids. The model is valid for low concentrations of bubbles smaller than the Kolmogorov length scale, and it is thus applicable to the microscopic gas nuclei suspended in a liquid that undergoes incipient cavitation. Other authors have modeled bubble motion with (variants of) the same equation, such as Reutsch & Meiburg [12] and Marchioli et al. [13] to study bubbles in vortex flows, Spelt & Biesheuvel [14] to study turbulent dispersion, Volk et al. [6] to study acceleration statistics, among others.

Constructing a statistical description of the pressure fluctuations undergone by bubbles of different sizes in homogeneous isotropic turbulence will allow us later on to use this description on top of a Computational Fluid Dynamics (CFD) code without explicitly tracking the bubbles in the flow field. As discussed by Arndt & George [15] and La Porta et al. [16], there are two main statistical descriptors of the fluctuating pressure that are relevant to cavitation: The (average) frequency with which the bubbles undergo low-pressure events (i.e., events in which the pressure at the bubble surface dips below some specified threshold), and the duration of such events. Therefore we compute the average frequency as function of the pressure threshold and the PDF of the events’ durations, and study how they are affected by the bubble size. The results show a significant increase of both frequency and duration of low-pressure events as the size of the bubbles is increased. This provides a quantitative estimate consistent with the preferential concentration near vortex cores already discussed.

### 1.1. Dataset and non-dimensionalization

The Lagrangian pressure statistics for finite sized bubbles were obtained by suitably tracking particles along an isotropic and homogeneous turbulent flow. Data from a Direct Numerical Simulation (DNS) available at Universidad Politécnica de Madrid by Javier Jiménez and coworkers were used. The database contains numerical solutions of the three-dimensional, periodic, incompressible Navier-Stokes equations with forcing in the low-wavenumber modes, ensuring statistically steady turbulent flow. From it, pressure, velocity and acceleration fields are retrieved at interpolation points as discussed later on.

The turbulent kinetic energy $\langle k \rangle$ and the viscous dissipation $\langle \epsilon \rangle$ averages are the defining turbulence variables of the flow. From them, one defines a velocity scale, 

$$u' = \sqrt{\frac{2 \langle k \rangle}{3}},$$

and a length scale, 

$$\lambda = u' \sqrt{\frac{15 \nu}{\langle \epsilon \rangle}},$$

where $\nu$ is the kinematic viscosity and $\lambda$ is the Taylor length microscale. In what follows, all fluid-dynamic variables are in non-dimensional form, i.e., without changing the notation the velocity $\mathbf{u}$ refers to $u/u'$, and similarly for the position vector ($\mathbf{x} \leftarrow \mathbf{x}/\lambda$), time ($t \leftarrow tu'/\lambda$), acceleration ($\mathbf{a} \leftarrow a\lambda/u'^2$), pressure ($p \leftarrow p/(\rho u'^2)$), where $\rho$ is density, etc.

Once in non-dimensional form, the turbulence is characterized by a single non-dimensional parameter, $Re_\lambda = u'\lambda/\nu$, which is the Taylor-scale Reynolds number. The dataset used here
corresponds to $Re_{\lambda} = 150$, with spectral resolution of $256^3$ modes. Notice that with the adopted non-dimensionalization the Kolmogorov scales are given by

$$\eta_K \text{ (length scale)} = 15^{-\frac{1}{4}}Re_{\lambda}^{-\frac{1}{2}} = 0.0415, \text{ and } \tau_K \text{ (time scale)} = 15^{-\frac{1}{2}} = 0.2582.$$ 

Therefore, the spatial resolution is given by $k_{\text{max}}\eta_K \approx 1.5$.

1.2. Bubble tracking equations

The finite-sized bubbles are initially placed in an equally spaced cubic grid in the domain, with initial velocity equal to the fluid velocity. The bubbles are modeled as small, spherical, and non-interacting particles that move according to

$$\frac{dx(t)}{dt} = v(t), \quad (1)$$

where $x(t)$ and $v(t)$ are the bubble position and velocity at time $t$, respectively, and $d/dt$ represents the time derivative along the particle trajectory.

Due to the density difference, the velocity $v$ of a finite-sized bubble differs from that of the fluid surrounding it. In the case of bubble diameter small compared to the Kolmogorov length scale $\eta_K$, a simplified equation for $v$ can be written as [11, 1]

$$\frac{dv}{dt} = \frac{1}{\tau}(u + w - v) + 3a, \quad (2)$$

where $u$ and $a$ are the fluid velocity and acceleration at the particle position $x$ and at time $t$, respectively, and $w$ is the non-dimensional bubble terminal velocity (i.e., the steady upwards velocity a bubble attains in a still liquid in the presence of gravity). The non-dimensional inertial time $\tau$ is given by Stokes’ law, i.e.,

$$\tau = \frac{Re_{\lambda}}{9} R^{*2}, \quad (3)$$

with $R^{*}$ the non-dimensional bubble radius (i.e., $R/\lambda$). Equation (2) includes the effects of fluid acceleration, Stokes drag, added mass and buoyancy (the Faxén correction and the Basset-Boussinesq history term are neglected). In this study, the impact of bubble radius $R^{*}$ (or inertial time $\tau$) on Lagrangian pressure statistics in a flow without gravity (i.e., $w = 0$) is evaluated. The bubble radii used and the corresponding inertial times are listed in Table 1. Note that in (2) and (3) the inertial time $\tau$ is assumed independent of the relative velocity $u - v$. This assumption holds if the Stokes drag law does not require any correction for particle Reynolds number and thus is valid if $\tau < 1/9$ or, equivalently, $R^{*} < 0.08$ [12]. The bubble sizes tested here satisfy this criterion.

The added-mass effect term was originally defined for the motion of small particles in an unsteady Stokes flow by Maxey and Riley [11] in a slightly different form (with a $d\mathbf{u}/dt$ term instead of $a$), generating a different equation for the particle velocity model:

$$\frac{dv}{dt} = \frac{1}{\tau}(u + w - v) + 2a + \frac{d\mathbf{u}}{dt}. \quad (4)$$

As mentioned by Maxey and Riley [11], for low-Reynolds number the difference between the two approaches is negligible. In turbulent flows, however, this difference can have a significant impact on the modeling of the particle trajectory. In this study, the impact of this alternative added-mass modeling approach on the the statistics of the Lagrangian pressure is also evaluated.
1.3. Numerical algorithm

The numerical integration of (2) (and similarly for (4)) is not completely straightforward because (a) the fields $u(x,t)$ and $a(x,t)$ are only available at discrete times $t^n$, with $n = 0, 1, \ldots$ (the time step is variable), and each evaluation at a point $x$ requires spatial interpolation, which is rather costly; and (b) the intrinsic time $\tau$, for $R*$ small enough, can be smaller than $\Delta t$. These considerations make the use of explicit multi-stage Runge-Kutta schemes not suitable for this problem. Instead, we propose a predictor-corrector implementation of the second-order implicit scheme in the form

\begin{align*}
x^{n+1} &= x^n + \frac{\Delta t}{2} (v^n + v^{n+1}), \\
u^{n+1} &= u(x^{n+1}, t^{n+1}), \\
a^{n+1} &= a(x^{n+1}, t^{n+1}), \\
v^{n+1} &= v^n + \frac{\Delta t}{2} \left[ \frac{u^n + u^{n+1} + 2w - v^n - v^{n+1}}{\tau} + 3(a^n + a^{n+1}) \right].
\end{align*}

In these equations, the variables at time $t^n$ are assumed already computed, and the operations in (6) and (7) represent spatial interpolation at time $t^{n+1}$, which is performed with three-dimensional polynomials of order three. The proposed implementation is second-order-accurate in time, semi-implicit in $v$ and unconditionally stable. It reads as follows:

**Predictor:**

\begin{align*}
x^* &= x^n + \Delta t v^n, \\
u^* &= u(x^*, t^{n+1}), \\
v^* &= v^n + \Delta t \left[ \frac{u^n + u^* + 2w - v^n - v^*}{\tau} + 3(a^n + a^*) \right].
\end{align*}

**Corrector:**

\begin{align*}
x^{n+1} &= x^n + \frac{\Delta t}{2} (v^n + v^{n+1}), \\
u^{n+1} &= u(x^{n+1}, t^{n+1}), \\
a^{n+1} &= a(x^{n+1}, t^{n+1}), \\
v^{n+1} &= v^n + \frac{\Delta t}{2} \left[ \frac{u^n + u^{n+1} + 2w - v^n}{\tau} + 3(a^n + a^{n+1}) \right].
\end{align*}
To reduce computational cost, Equations (9) to (15) are solved for every fourth DNS time step, which corresponds to $\Delta t = 4.3 \times 10^{-3}$. The adopted method is prone to some oscillations after rapid variations of $u$ if $\Delta t/\tau > 2$. However, for decreasing $R^*$ the bubble velocity approaches asymptotically the liquid velocity, thus velocity changes are limited to changes in liquid velocity in a time step, which are small for small time steps. $\Delta t$ is approximately equal to $\tau_k/60$, ensuring a very fine temporal resolution for the tracking of bubbles and ruling out abrupt variations of $u$. The maximum value of $\Delta t/\tau$ in this study is 2.57, corresponding to $R^* = 0.01$. For this value the oscillation is negligible and decays after 2 or 3 time steps. Simulations are carried from $t_0 = 0$ to $t = 21.5$, which corresponds to 5000 time-steps (or 20000 DNS time steps and $84\tau_k$). Pressure is recorded along the Lagrangian bubble trajectories for each bubble size. The simulation parameters are listed in Table 2.

### Table 2. Simulation parameters.

| Parameter          | Value       |
|--------------------|-------------|
| $Re_\lambda$       | 150         |
| Box edgelength     | 21.1        |
| Grid               | $256^3$     |
| Number of bubbles  | $64^3$      |
| Time step          | $4.3 \times 10^{-3}$ |
| Total simulation time | 21.5       |

### 1.4. Low-pressure events

A Lagrangian low-pressure event is defined for several negative pressure thresholds ($p_- = -2.0, -2.2, -2.4, ...$), and corresponds to events with starting time $t_{start}$ (when $p$ goes from above to below $p_-$) and ending time $t_{end}$ (when $p$ goes from below to above $p_-$). The duration of the event is $d = t_{end} - t_{start}$. More details are given by Bappy et al. [10], where Lagrangian pressure fluctuations for $R^* = 0$ were explored in two homogeneous and isotropic turbulence databases ($Re_\lambda = 150$ and 418). The average low-pressure event rate or frequency is defined as

$$\zeta(p_-) = \frac{n(p_-, T)}{MT}, \quad (16)$$

where $n(p_-, T)$ is the number of Lagrangian events with pressure threshold $p_-$ for $M$ bubbles evolving over a total time $T$ in the turbulence field. From the low-pressure event frequency $\zeta$ (number of events per bubble per unit time), the frequency curves for all defined pressure thresholds are calculated for different bubble sizes.

### 2. Results

The basic pressure statistics obtained along the trajectory of each bubble size are reported in Figure 1 (corresponding values are available in Table 3). The Lagrangian Taylor time scale is defined as

$$\tau_p = \left[ \frac{\text{Var}(p)}{\text{Var}(dp/dt)} \right]^{1/2}. \quad (17)$$

For Lagrangian tracers ($R^* = 0$), results are similar to the values reported by Bappy et al. [10] for the same $Re_\lambda$ (small differences are caused by the difference in simulation time step, which here is approximately a factor of ten smaller than in the previous study). As reported by Bappy...
et al. [10], the value of \( \text{Var}(p) \) is within the trend of values observed in previous studies in the range \( 21.6 \leq Re \lambda \leq 172 \) [17, 18, 19]. The other variables are not available from the literature.

The effect of bubble size on pressure statistics is quite noteworthy. The mean pressure goes from close to zero for tracers (the Eulerian mean pressure of the simulation) to increasingly negative values with increase in bubble size, a consequence of their tendency to cluster in vortex cores [1]. Larger bubbles experience more deviation of pressure from the mean value, as both \( \text{Var}(p) \) and \( \text{Var}(dp/dt) \) increase with \( R^* \). This is due to the fact that bubbles are attracted toward low pressure regions in space. These low pressure regions are concentrated at the cores of vortices. In an ideal vortex (for instance a Lamb-Oseen vortex) a tracer particle, moving at the speed of the liquid, rotates around the core. A bubble rotates around the core but also develops a velocity toward it, thus experiencing a decrease in pressure with time not present for tracer particles. In the context of HIT this results in both lower pressures and more changes in \( dp/dt \), contributing to both higher variance in \( p \) and \( dp/dt \). The effect is more dramatic for larger bubbles, which experience higher velocity toward the vortex cores. As shown in Figure 2, the pressure autocorrelation also increases with increasing bubble radius, indicating an increase in the integral time scale.

**Table 3.** Pressure statistics for different bubble sizes

| \( R^* \)  | \( \bar{p} \)  | \( \text{Var}(p) \) | \( \text{Var}(dp/dt) \) | Taylor timescale |
|----------|---------------|-----------------|-----------------|-----------------|
| 0.00     | +5.95 \times 10^{-4} | 0.7768          | 1.2199          | 0.7979          |
| 0.01     | -2.34 \times 10^{-2} | 0.8226          | 1.2331          | 0.8167          |
| 0.02     | -7.41 \times 10^{-2} | 0.9273          | 1.2792          | 0.8514          |
| 0.03     | -1.44 \times 10^{-1} | 1.0729          | 1.3607          | 0.8879          |
| 0.04     | -2.23 \times 10^{-1} | 1.2531          | 1.4859          | 0.9183          |
| 0.05     | -3.02 \times 10^{-1} | 1.4148          | 1.6404          | 0.9287          |
Figure 2. Pressure autocorrelation for different bubble sizes

Figure 3. Probability density function of pressure for different bubbles sizes

2.1. Lagrangian pressure PDFs

The probability density function (PDF) of pressure for different bubbles sizes are shown in Figure 3. The standard shape for pressure PDF in homogeneous isotropic turbulence, with an exponential tail in the low pressure side and an approximately Gaussian shape from near zero to positive pressure is observed [17, 18]. The negative pressure tails move upwards as the bubble size increases, indicating higher probability of being in low pressure regions of the flow, consistent with the mean pressure trend. On the positive-pressure side the PDFs are close to Gaussian and much less dependent on $R^*$. 
Figure 4. Frequency distribution of events of any duration (a) as a function of pressure threshold for different sizes (b) as a function of bubble size for different pressure thresholds.

Figure 5. PDF of duration of low pressure events for different bubble sizes for (a) $p_- = -2$ and (b) $p_- = -8$.

2.2. Frequency and duration of low-pressure events

As a direct consequence of the decrease in pressure experienced by larger bubbles, the frequency of low-pressure events also increases with bubble size for all values of pressure threshold and bubble sizes evaluated (see Figure 4). Between $R^* = 0$ and $0.05$ the increase factor goes from the order of two for $p_- = -2$ to as high as fifty for $p_- = -12$.

The higher rate of low-pressure events for larger bubbles takes place together with longer...
duration of these events. The distribution of the duration of events for the thresholds \( p = -2 \) and \(-8\) are shown in Figure 5. For \( p = -2\), the distribution peaks at a duration of approximately \( 0.17 = \frac{2}{3} \tau_K \) \[10\]. The height of the peak decreases with increasing bubble size, whereas the probability of a longer duration increases (Figure 5a). For \( p = -8\) the peak of the distribution is more sensitive to \( R^*\), moving towards a duration around \( 0.2582 = \tau_K \) as \( R^*\) increases (Figure 5b). Combined with the increase in frequency of low-pressure events, these results show that larger bubbles have a higher probability of experiencing extreme low-pressure events of longer duration. Size effects are thus crucial for accurate modeling of turbulence-induced cavitation inception.

2.3. Modeling of the added-mass effect

We also analyze how the results change if the bubble-tracking equation (2) is replaced by its variant (4), which was originally proposed by Maxey and Riley \[11\]. Figure 6 shows comparisons of the PDF of pressure, of the frequency of low-pressure events and of the PDF of events duration. As expected, no difference is observed for \( R^* = 0\), but notice that there is already a non-negligible difference for \( R^* = 0.02\) in all three plots. A decrease is observed in the probability of finding very low pressures as well as the frequency of low pressure events (Figures 6a and 6b, respectively). Minor changes in the PDF of event duration (Figure 6c) are also observed. This difference indicates that the original formulation (which replaces \( a\) by \( du/dt\) in one term, see equation 4) leads to different bubble dynamics and in particular predicts fewer and shorter (in average) low-pressure events.

3. Discussion & Conclusion

Lagrangian pressure statistics for different bubble sizes in homogeneous isotropic turbulence of \( Re_\lambda = 150\) are presented in this study. A modified Maxey-Riley equation was used to transport the bubbles in the turbulent field. The bubble size was limited to the acceptable range in which the equation is valid. For larger bubbles the tracking equation should also include a particle Reynolds number correction for the Stokes drag, the Faxen correction and Basset memory terms, which have been neglected in this work. In addition, larger bubbles can change shape and affect the turbulence in the flow in a two-way coupling fashion, effects not considered in this study.

The pressure statistics of finite-sized bubbles vary significantly with respect to Lagrangian tracers (\( R^* = 0\)). The pressure PDF shows a higher probability of low-pressure fluctuations with increasing bubble size, in agreement with the knowledge that larger bubbles are more attracted to the low pressure regions of the flow. As a consequence, we observe a higher frequency of low pressure events as the bubble radius is increased, for all pressure thresholds tested here. In addition, the PDF of the duration shows an increase in the probability of longer events with increased bubble size.

The results presented here represent another effort towards the improvement of cavitation inception modeling. They indicate that, even for idealized conditions (homogeneous and isotropic turbulence without gravity), the effect of bubble size on the pressure history experienced by suspended bubbles is significant. The validity of these results is of course limited by the hypotheses of the study. The fluid’s velocity field is obtained from single-phase DNS, neglecting the effect of the bubbles on the flow. The volume concentration of bubbles must thus be very small, as is the case in cavitation inception conditions. Because of the database used, applicability is also restricted to flows that, at least approximately, are locally homogeneous and isotropic. In addition, the bubble radius must be smaller than the Kolmogorov scale of the flow, since the bubbles are tracked as mathematical points. Despite these limitations, to our knowledge these are the first quantitative estimates of the rate of low-pressure events experienced by bubbles in turbulent flows. They can be used to enrich current deterministic models of cavitation inception by incorporating the stochastic effect of pressure fluctuations.
Figure 6. Effect of added-mass model on the (a) pressure PDF, (b) frequency of low-pressure events and (c) PDF of event duration, for different bubble sizes. Dashed lines correspond to trajectories obtained by solving Equation (2) and solid lines correspond to those obtained by solving Equation (4).
It remains to analyze the effect of gravity on particle transport (i.e., $w \neq 0$), which is the subject of ongoing research. Gravity effects can be significant for conditions where the hydrostatic pressure gradient overcomes the dynamic pressure gradient.

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