Brownian heat engines use local temperature gradients in asymmetric potentials to move particles against an external force. Here we show that, by using a suitably chosen energy filter, electrons can be transferred reversibly between reservoirs that have different temperatures and electrochemical potentials. We apply this result to propose heat engines based on mesoscopic semiconductor ratchets, which can quasi-statically operate arbitrarily close to Carnot efficiency.
Ratchets combine asymmetry with non-equilibrium processes to generate directed particle motion [1]. When non-equilibrium is induced by contact with heat baths at different temperatures, a ratchet can act as a so-called Brownian heat engine, converting local spatial or temporal temperature variations into useful work [2, 3]. This mechanism is currently attracting considerable interest ([4-6] and references therein) and in the future may be applied to power artificial micro-machines such as chemical motors [5, 7].

Many proposed Brownian heat engines suffer from a maximum thermodynamic efficiency that is significantly lower than the ideal Carnot efficiency. The critical issue is that the kinetic energy of particles making contact with different heat baths is considered irreversible. A different approach has been proposed by Derényi and Astumian [5]. Their ratchet is described by an overdamped Brownian particle moving between two electron reservoirs with different temperatures and electrochemical potentials. The efficiency of such a ratchet is given by the Carnot efficiency:

\[
\varepsilon = \frac{T_L}{T_L + T_R}
\]

where \(T_L\) and \(T_R\) are the temperatures of the reservoirs and \(\mu_L\) and \(\mu_R\) are their electrochemical potentials. Here, the kinetic energy of particles is considered as a degree of freedom that is not exchanged between the reservoirs. This approach can achieve efficiencies that approach the Carnot limit.

We begin our analysis by considering the transport of single electrons between two reservoirs, denoted \(L\) and \(R\) (Fig. 1a). Each reservoir is described by an equilibrium Fermi-Dirac distribution with temperatures \(T_L\) and \(T_R\) and electrochemical potentials \(\mu_L\) and \(\mu_R\), respectively. A bias voltage \(V\) applied to reservoir \(R\) creates a difference in the electrochemical potentials, \(\mu_R - \mu_L = eV\), where \(e\) is the charge on the electron. Except for the flow of electrons between reservoirs, the reservoirs are assumed to be thermally isolated. The electron mean free path for inelastic processes is taken to be much larger than the distance between the reservoirs, but much smaller than the characteristic length scale of the system.

The heat associated with the addition of one electron to a reservoir is given by

\[
Q = U - \mu
\]

where \(U\), the increase in internal energy, is negated by the change in electrochemical potential.

We begin our analysis by considering the transport of single electrons between two reservoirs.
As an example, we consider an electron traveling from reservoir $L$ to reservoir $R$ at constant energy $\mu_L$ and $\mu_R$, respectively, shown in Fig. 1(a). The Fermi-Dirac distribution functions in $L$ and $R$ are given by

$$\frac{n_L}{n_R} = \frac{1}{1 + e^{(\mu_L - \mu_R)/kT}}.$$ 

where $n_L$ and $n_R$ are the Fermi-Dirac distribution functions in reservoirs $L$ and $R$, respectively. In particular, the entropy $S_L$ of the system is the sum of the entropy of $L$ and $R$.

It is instructive to understand how the entropy relates to the energy distributions in the two reservoirs. Figure 1 illustrates the meaning of this equation. Consider two reservoirs at the same temperature $T$, but with different electrochemical potentials, $(\mu_L - \mu_R) > 0$ (Fig. 1a). Independent of the electron energy, the transfer of an electron from $L$ to $R$ increases the system entropy by $S_L = eV/T > 0$. In the complementary case, when $T_R > T_L$ but $\mu_L = \mu_R = 0$ (see Fig. 1b),

$$S_L = \left[\frac{1}{T_R} - \frac{1}{T_L}\right].$$

This means it is thermodynamically advantageous ($S_L > 0$) if, on average, 'cold' electrons ($\mu < \mu_L$) move from $L$ to $R$ and 'warm' electrons ($\mu > \mu_L$) move from $R$ to $L$. If the probability for electron transmission across the junction is independent of $\mu$, no net electric current is generated, but heat flows from $R$ to $L$. The increase in total entropy in both examples, Figs. 1(a) and (b), shows that these processes are spontaneous and irreversible.

Now consider the general case where $T_R > T_L$ and $\mu_L > \mu_R$ (Fig. 1c). This is the situation encountered in Brownian heat engines, when particles driven by a temperature gradient do work against an external force. Significantly, Eq. (1) yields $S_L = 0$ for $T_R = T_L$ and $\mu_L = \mu_R$. This is the main result of the present work: two electron reservoirs with arbitrary temperatures and electrochemical potentials can exchange electrons at arbitrary energy costs, and the system entropy remains constant. This fact is illustrated in Fig. 1(d) for $T_R > T_L$ and $\mu_L > \mu_R$.

The overall increase in entropy due to the transfer of one electron from $L$ to $R$ is

$$S_L = eV/T.$$ 

Because the electron picks up kinetic energy in the electric field between reservoirs, the removal of heat leads to $S_L > 0$, and excesses of heat are transferred. The change of heat in associated with the electron is measured relative to the same voltage-independent zero energy. As an example, we consider an electron traveling from reservoir $L$ to reservoir $R$.
gradient from L to R. For S, where the probability for finding an electron is the same on both sides, the two driving forces cancel. One may say that at this particular energy the two reservoirs behave as if they were in thermal equilibrium with each other.

If the two reservoirs were connected via an ideal energy filter that was transparent for electrons at S and at no other energy, no time-averaged particle or heat current would occur spontaneously. The warm bath would not cool, and the voltage would drive no current.

We will now apply Eq. (2) to propose an electron ratchet that can be operated as a heat engine and we will demonstrate analytically how Carnot efficiency can be achieved using ideal energy filters. Specifically, we will consider a 'rocked' electron ratchet - that is, electrons in an asymmetric potential that is tilted periodically and symmetrically by an external force [1]. An adiabatically rocked electron ratchet is essentially a non-linear rectifier; the magnitude and spectral composition of the current depend, for asymmetric devices, on the voltage sign. Consequently, a symmetric AC 'rocking' voltage generates, on time-averaging, a net electric current.

Fig. 2(a,b) illustrates a hypothetical non-linear device connecting two equal two-dimensional (2D) reservoirs, L and R, rocked by a square-wave voltage of amplitude \( V_0 \). We assume that switching between the values \( \pm V_0 \) occurs on a time scale slower than any characteristic electronic times (so-called adiabatic rocking), but much faster than the rocking period. It is therefore sufficient to analyse the device for the two DC situations \( V = \pm V_0 \), while transient behaviour can be neglected. The probability for electrons to be transmitted across the device is taken as a single Lorentzian resonance, with amplitude 0.01 and a full width at half maximum of 2\( \mu \). The other resonances, \( \phi \pm \left( \frac{V_0}{2} \right) \eta \), are symmetrically arranged around \( \phi = 0 \).

As can be confirmed using Eq. (3) below, this ensures that the time-averaged electric current \( I_{\text{net}}(V_0) = 0.5 \left[ I(V_0) + I(-V_0) \right] \) is zero, thus avoiding the trivial condition where a finite \( I_{\text{net}} \) is accompanied by a heat current.

As a realization of a filter with this transmission function, one can consider coherent resonant tunneling via an asymmetric quantum dot, in which the resonant energy level shifts its value when a bias voltage is applied. This approach has been experimentally demonstrated [8].
voltage deforms the band structure, and which is connected by 1D quantum point
contacts to 2D electron gas (2DEG) reservoirs.

The steady-state electric current from $L$ to $R$ generated by a DC bias voltage $V$ applied to $R$ is given by a Landauer equation

$$I(V) = \frac{e}{h} \int \frac{dE}{\pi} \left( T_L(E) - T_R(E) \right)$$

where $T_L(E)$ and $T_R(E)$ are the transmission coefficients at energy $E$. The electrical power input into the device, averaged over a full cycle of rocking, is given by

$$W = \langle I(V) V \rangle$$

where $\langle \rangle$ denotes the average over a full cycle of rocking.

The efficiency of a heat engine, $\eta$, is given by the ratio of the work output to the heat removed from the warmer of the two reservoirs

$$\eta = \frac{W}{Q_{in}}$$

where $Q_{in}$ is the heat input to the device. Assuming that $T_R > T_L$, we can write

$$\eta = \frac{W - (Q_L + Q_R)}{W}$$

where $Q_L$ and $Q_R$ are the heat inputs to the left and right reservoirs, respectively.

The corresponding Carnot values are

$$\eta_C = \frac{T_L}{T_R - T_L}$$

and

$$\eta_{EC} = \frac{T_R - T_L}{T_R}$$

respectively.

The net efficiency of a refrigerator, cooling the colder reservoir $L$ using work $W$, is given by

$$\eta_{ref} = \frac{W}{Q_L}$$

where $Q_L$ is the heat removed from the warmer reservoir.

The corresponding Carnot values are

$$\eta_{ref, C} = \frac{T_L}{T_R - T_L}$$

and

$$\eta_{ref, EC} = \frac{T_R - T_L}{T_R}$$

respectively. Calculating the efficiencies, normalized to the Carnot values, for the device in Fig. 2(a,b) is shown in Fig. 3(a). The line along which the condition $\kappa_{res}^R(V_0) > \kappa_{res}^L(V_0)$ is fulfilled (and where, by symmetry, $\kappa_{res}^R(-V_0) < \kappa_{res}^L(-V_0)$) is visible as a ridge of high normalized efficiency values. To the left of this ridge, where $\kappa_{res}^R(V_0) < \kappa_{res}^L(V_0)$, the ratchet operates as a refrigerator: during each half cycle of rocking, the electron flow follows the electrochemical potential gradient, and heat flows against the thermal gradient (Peltier effect). Therefore, the normalized efficiency of a refrigerator, $\eta_{ref}/\eta_{ref, C}$, is plotted. For $\kappa_{res}^R(V_0) < \kappa_{res}^L(V_0)$, the heat flow in each half cycle follows the thermal gradient, while electrons flow against the potential gradient. The ratchet does work against the battery, using thermal energy provided by the warm reservoir to do work. The net efficiency of a refrigerator, $\eta_{ref}/\eta_{ref, C}$, is plotted. For $\kappa_{res}^R(V_0) < \kappa_{res}^L(V_0)$, the heat flow in each half cycle follows the thermal gradient, while electrons flow against the potential gradient. The ratchet does work against the battery, using thermal energy provided by the warm reservoir to do work.

Along the line $\kappa_{res}^R(V_0) = \kappa_{res}^L(V_0)$, the efficiency coefficients approach their corresponding Carnot values. To show this analytically, we note that after several
algebraic steps and using the symmetry of $tV$ one can simplify $qVRL_{\text{net}}/0u$ to $qVRL/vw$. Assuming that the width of the 'energy filter', $2y$ is much smaller than the energy scales $kT_R/L$ over which the Fermi-Dirac distributions vary, {\eqref{eq:5}} can be approximated

$$\mathcal{F}_e \approx \mathcal{E}_e \mathcal{E}_S \mathcal{S}_e \mathcal{R}_e$$

In the quasistatic limit $\gamma \approx \varepsilon$ (where electron flow is quenched) one obtains the efficiency parameters

$$\eta_e \approx \frac{\mathcal{F}_e}{\mathcal{E}_e}, \quad \eta_S \approx \frac{\mathcal{F}_S}{\mathcal{E}_S}$$

where we used the definition of $\mathcal{E}_S$ and the relation $[\mu_L - \mu_R (+V_0)] = eV_0$. For $\mathcal{E}_S(0,0,2\gamma,0,2\gamma)$ one then recovers the respective Carnot efficiencies. As for any reversible heat engine, the power output goes to zero as reversibility is approached. This is apparent in Fig. 3(c) where the power of the device exhibits a valley along the line where $\mathcal{E}_S$ approaches the exact Carnot efficiencies and heat pumping mix within the transmission range and contribute

$$\mathcal{R}_e \approx \mathcal{S}_e \mathcal{S}_R \mathcal{S}_L$$

In Fig. 3(b) we compare the exact calculations of $\mathcal{F}_e$ \mathcal{E}_e \mathcal{S}_e \mathcal{R}_e$ (bolder lines) and $\mathcal{F}_S$ \mathcal{E}_S \mathcal{S}_S \mathcal{R}_e$ (thinner lines) with $\mathcal{S}_e \mathcal{R}_e$ \mathcal{S}_S \mathcal{R}_e$ and $\mathcal{S}_S \mathcal{R}_e$ \mathcal{S}_S \mathcal{R}_e$, respectively. As $\mathcal{E}_S$ approaches $\mathcal{E}_S$ from above, the cooling power \mathcal{R}_e \mathcal{S}_e \mathcal{S}_L$ of the refrigerator turns into a heating power. This is also apparent in Fig. 3(d) where $\mathcal{R}_e$ and $\mathcal{S}_e$ are shown for

$$\mathcal{E}_e \approx \frac{(0\Lambda)^{2\gamma} \mathcal{E}_e}{(0\Lambda)^{2\gamma} \mathcal{E}_e}$$

Maximum power is always obtained when the energy filter is at a value away from $\mathcal{E}_S$. When $\gamma \approx 0.5 eV_0$, Joule heating exceeds the cooling power, as shown in Fig. 2(a, b).

The analysis presented here for the case of an adiabatically rocked ratchet is not possible in a rocked device as shown in Fig. 2(a, b).
Edwards et al. proposed the use of resonant tunneling for heat pumping purposes [12, 13]. Energy selectivity has been observed in semiconductor quantum dots, using gates to control position and width of the transmission resonances, which were confirmed to be Lorentzian with a width determined only by the lifetime of the energy states [14]. To realize one cold-hot-cold section of the device in Fig. 2(c), one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D section of the device in Fig. 2(c) one could deviate the temperature of a small 2D
Electron transfer between reservoirs: (a) in the presence of a difference in electrochemical potentials, (b) in the presence of a temperature difference, and (c) in the presence of both. (d) The Fermi-Dirac distributions in the energy range around $\epsilon_0 = 0.5(\mu_L + \mu_R)$ for $T_R = 2 \text{ K}$, $T_L = 0.5 \text{ K}$, and $V = 0.1 \text{ mV}$.

(a) A hypothetical non-linear rectifier. The energy band structure of the device indicates that of an asymmetric quantum dot forming a resonant tunneling structure, with an energy level position that depends on the voltage. Any higher resonant levels are assumed to be out of the reach of thermally excited electrons. The dashed lines are calculated for $T_R > T_L$ is assumed.

(b) A Brownian heat engine consisting of a periodic, static ratchet potential. Particle flow is against a potential gradient of $\epsilon$ per period. With the position of ideal energy filters (indicated as horizontal lines between warm and cold reservoirs) tuned according to Eq. (2), this ratchet works reversibly. The full line is for $\epsilon = 0.1 \text{ eV}$ (Eq. (5)). The dashed lines are calculated for $\epsilon = 10^{-2}, 10^{-3}, \text{ and } 10^{-4} \text{ meV}$. Bold lines (left hand half) are for a refrigeration thermoelectric heat engine, and thin (right hand half) are efficiencies.

(c) Efficiency (normalized to the Carnot value) of the model device in Fig. 2(a, b) for $T_R = 1 \text{ K}$, $T_L = 0.5 \text{ K}$, and $\epsilon_0 = 0.1 \text{ meV}$, as a function of the position of the resonant level, $\epsilon$ (see main text for further details). Bold (q) normalized efficiency for $\epsilon = 1.1 \text{ meV}$, $\epsilon = 0.1 \text{ meV}$, and $\epsilon = 0.0 \text{ meV}$. Dashed (r) efficiency for $\epsilon = 20 \text{ meV}$ (Eq. (2)).

(d) Efficiency (normalized to the Carnot value) of the model device in Fig. 2(a, b) for $T_R = 1 \text{ K}$, $T_L = 0.5 \text{ K}$, and $\epsilon_0 = 0.1 \text{ meV}$. Bold (s) efficiency for $\epsilon = 10^{-2}, 10^{-3}, \text{ and } 10^{-4} \text{ meV}$. Data are shown only where the efficiencies are defined.

(e) and (f) show the power output of the device in Fig. 2(a, b) for the same parameters as used for the efficiencies are defined. The power output of the device is shown only where the efficiencies are defined. For $\epsilon = 20 \text{ meV}$ (Eq. (5)). The dashed lines are calculated for $\epsilon = 10^{-2}, 10^{-3}, \text{ and } 10^{-4} \text{ meV}$.

(f) For $\epsilon = 10^{-2}, 10^{-3}, \text{ and } 10^{-4} \text{ meV}$, the full line is for $\epsilon = 10^{-2} \text{ meV}$ (Eq. (5)). The dashed lines are calculated for $\epsilon = 10^{-2}, 10^{-3}, \text{ and } 10^{-4} \text{ meV}$. Data are shown only where the efficiencies are defined.

To put the numerical values into context, a cooling power of $10^7 \text{ meV/s}$ corresponds to the heat leaked via phonons to a 2DEG with temperature of 0.2 K and area 20 µm$^2$ in a crystal externally cooled to 0.3 K.

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