Dark matter stability from Dirac neutrinos in scotogenic 3-3-1-1 theory

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We propose the simplest TeV-scale scotogenic extension of the original 3-3-1 theory, where dark matter stability is linked to the Dirac nature of neutrinos, which results from an unbroken $B-L$ gauge symmetry. The new gauge bosons get masses through the interplay of spontaneous symmetry breaking à la Higgs and the Stueckelberg mechanism.

I. INTRODUCTION

Despite its amazing phenomenological success, almost no one thinks of the standard model as the final theory, so many are its drawbacks. Amongst these, the issues of neutrino mass, dark matter, the number of families and the strong CP problem stand out as important items in the wish list of theorists. Here we propose a standard model extension where these appear closely interconnected. To do this we build up upon a minimal gauge extension of the original Singer-Valle-Schechter (SVS) 3-3-1 model [1]. This was the first electroweak extension of the standard model in which the existence of three families of quarks and leptons is closely related to anomaly cancellation. Indeed, in this SU(3)$_c$ ⊗ SU(3)$_L$ ⊗ U(1)$_X$ theory one assumes that leptons transform as SU(3)$_L$ anti-triplets, while two families of left-handed quarks transform as triplets and the last one is an anti-triplet. This choice comes from anomaly cancellation and once adopted, leads to the prediction of three families of quarks and leptons [1–4]. In order to make the construction as minimal as possible we also adopt the choice made in [5] of identifying the third component of the leptons as a “right-handed” neutrino, so that neutrinos are of Dirac nature and their masses are generated at tree-level. However this early formulation is not compatible with the current neutrino oscillation data [6], as it predicts one massless and two mass-degenerate neutrinos. Besides, an unaesthetic feature of this construction is that lepton number symmetry emerges in SVS as a combination of a gauge symmetry and a global one.

In what follows we explore a simple scheme with a viable neutrino spectrum and realizing the scotogenic dark matter paradigm [7] which postulates that neutrino masses arise through the radiative exchange of a “dark matter” sector. The idea of relating dark matter stability to the Diracness of neutrinos has been proposed in [8] employing residual discrete symmetries arising from the partial breaking of a global $B-L$ symmetry [8–14]. An alternative proposal to link Dirac neutrino masses and dark matter stability is through a fully conserved global $B-L$ symmetry. This idea has been pursued in the context of bound-state dark matter [15], in which the radiative generation of Dirac neutrino masses is mediated by the exchange of bound-state-dark-matter constituents.

In this paper we choose a different route, namely, a scenario where dark matter stability is interconnected to the Diracness of neutrinos in the framework of a dynamical theory with gauged lepton number. In order to
achieve this we build upon a minimally extended class of SVS theories developed within the framework of the SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X ⊗ U(1)_N gauge symmetry [16–20]. The extra Abelian U(1)_N group allows a consistent embedding of B − L as a fully dynamical gauge symmetry. We propose a simple 3-3-1-1 extension of the original SVS theory where neutrino masses arise “scotogenically” (i.e. through the exchange of “dark” particles) at the one-loop level. The unbroken B − L symmetry acts as the protecting symmetry responsible for both neutrino Dirac masses and stabilization of dark matter. Our present construction follows the path as the simple Stueckelberg [21] U(1) extension of the standard model proposed in Ref. [22]. However this is achieved within a richer framework that provides not only a dynamical realization of the proposal that dark matter stability and Diracness are closely inter-related, but also touches other standard model shortcomings such as the number of families and the strong CP problem. In particular the existence of three families of quarks and leptons is linked to anomaly cancellation. Our present model also provides an example of “predestined” dark matter [23], in the sense that the specific quantum numbers of the new fermion and scalar multiplets automatically ensure the existence of a stable dark matter candidate, without the ad hoc imposition of any additional symmetry.

The paper is organised as follows. In Sec. II, we define the proposed model in terms of its field content and symmetries. The scalar sector is studied in Sec. III. In Sec. IV, we derive the extended electroweak vector boson spectrum taking into account contributions coming from both the spontaneous symmetry and Stueckelberg mechanisms. The charged fermion spectrum is presented in Sec. V, while the scotogenic neutrino masses are calculated in Sec. VI. Finally, the conclusions are presented in Sec. VII.

II. THE MODEL

In the present model, not only the Abelian electromagnetic symmetry U(1)_Q but also the U(1)_{B−L} symmetry emerges as a conserved residual subgroup of the SU(3)_c ⊗ SU(3)_L ⊗ U(1)_X ⊗ U(1)_N gauge symmetry, or 3-3-1-1 for short. In 3-3-1-1 models, the electric charge operator can be generically written as

$$ Q = T^3 + \beta T^8 + X, \quad (1) $$

while the B − L generator is expressed as

$$ B − L = \beta' T^8 + N, \quad (2) $$

with $T^a$ ($a = 1, \ldots, 8$), $X$ and $N$ as the respective generators of SU(3)_L, U(1)_X and U(1)_N [24]. The choices of the constants $\beta$ and $\beta'$ define different versions of the model, and for the SVS model, we have $\beta = 1/\sqrt{3}$ and $\beta' = 4/\sqrt{3}$. This specific choice ensures the B − L assignment in the SVS model with its original field content is anomaly-free and can be promptly promoted to local. On the other hand, other $\beta'$ values would require new fermions to cancel the $B − L$ anomalies\(^1\), see e.g. Refs. [16]. Here we stick to the SVS choice given in Table I. This gives all the quantum number assignments for the fields contained in our model. In addition to the fields present in the original SVS model, we have included three two-component Majorana fermion singlets $S^a_R, a = 1, 2, 3$ and one scalar anti-triplet $\Phi_4$. Notice that the Majorana fermions are full gauge singlets and hence carry no anomaly. The global U(1)_{PQ} symmetry forbids the term $(\bar{\psi}_L^a)^c \Phi_1 \psi_L^a$ which appears in Ref. [5] and leads to tree-level Dirac neutrino masses. However, as it will be discussed in Sec. III, this global symmetry is softly broken in the scalar sector, by the trilinear $\Phi_1 \Phi_2 \Phi_3$ coupling. As we will see, this avoids the disastrous presence of a visible axion field. Notice also that, since $B − L$ remains unbroken, the matter parity subgroup, generated by the matter-parity $M_P = (-1)^{3(B−L)+2s}$, where $s$ is the field’s spin, is also fully preserved. Under $M_P$, all the fields in the original SVS model transform trivially, whereas the new fields transform as $(S_{aR}, \Phi_4) \overset{M_P}{\rightarrow} -(S_{aR}, \Phi_4).$ Therefore, the lightest among the $M_P$-odd fields is stable and, if electrically neutral, it can play the role of dark matter.

\(^1\) Explicit calculation of anomaly coefficients for generic $\beta$ and $\beta'$ can be found in Ref. [24].
Our model contains four triplet scalars, three of them are Higgs-like, even under matter-parity, while $\Phi_4$ is “dark” or $M_P$-odd. The resulting scalar potential is given by

$$V_\Phi = \sum_{i=1}^{4} \left[ \mu_i^2 \Phi_i^\dagger \Phi_i + \lambda_i (\Phi_i^\dagger \Phi_i)^2 \right] + \sum_{i<j} \left[ \lambda_{ij} (\Phi_i^\dagger \Phi_i) (\Phi_j^\dagger \Phi_j) + \lambda_{ij} (\Phi_i^\dagger \Phi_j) (\Phi_j^\dagger \Phi_i) \right]$$

$$+ \left( \frac{-\mu_0}{\sqrt{2}} \Phi_1^\dagger \Phi_2 + \frac{\lambda'}{2} \Phi_2^\dagger \Phi_4 \Phi_4^\dagger \Phi_4 + \text{h.c.} \right),$$

where the cubic term characterized by $\mu_0$ breaks the U(1)$_{PQ}$ symmetry softly.

The scalar multiplets are decomposed as

$$\Phi_1 = \left( \begin{array}{c} \phi_1^- \\ \phi_1^0 \\ \phi_1^+ \\ \phi_1^0 \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} \phi_2^- \\ \phi_2^0 \\ \phi_2^+ \\ \phi_2^0 \end{array} \right), \quad \Phi_3 = \left( \begin{array}{c} \phi_3^- \\ \phi_3^0 \\ \phi_3^+ \\ \phi_3^0 \end{array} \right), \quad \Phi_4 = \left( \begin{array}{c} \phi_4^- \\ \phi_4^0 \\ \phi_4^+ \\ \phi_4^0 \end{array} \right),$$

where $v_1/\sqrt{2}$, $v_2/\sqrt{2}$ and $w/\sqrt{2}$ represent vacuum expectation values (vevs), with $w^2 \gg v_1^2 + v_2^2 \equiv v_{EW}^2$. Notice that, with the assumed vev alignment, the $B - L$ symmetry remains conserved, and the minimization of the potential leads to the tadpole equations

$$v_1 \left( 2\mu_1^2 + 2\lambda_1 v_1^2 + \lambda_{12} v_2^2 + \lambda_{13} w^2 \right) - v_2 w \mu_\phi = 0,$$
$$v_2 \left( 2\mu_2^2 + 2\lambda_2 v_2^2 + \lambda_{12} v_1^2 + \lambda_{23} w^2 \right) - v_1 w \mu_\phi = 0,$$
$$w \left( 2\mu_3^2 + 2\lambda_3 w^2 + \lambda_{13} v_1^2 + \lambda_{23} v_2^2 \right) - v_1 v_2 \mu_\phi = 0,$$

TABLE I: Field content and symmetry transformations

III. SCALAR SECTOR
which can be simultaneously solved for $\mu_1^2$, $\mu_2^2$ and $\mu_3^2$. In the following subsections, we present the physical states of the scalar sector and their respective masses.

### A. CP-even scalars

After spontaneous symmetry breaking, the CP-even components of the fields that acquire a vev mix according to the following squared mass matrix, written in the basis $(s_1, s_2, s_3)$,

$$M^2_{s_1, s_2, s_3} = \begin{pmatrix} 2v_1^2\lambda_1 + \frac{v_2 w \mu_2}{2v_1} & -v_1 v_2 \lambda_12 + \frac{w \mu_2}{2} & v_1 \lambda_12 - \frac{v_2 \mu_2}{2} \\ -v_1 v_2 \lambda_12 + \frac{w \mu_2}{2} & 2v_2^2 \lambda_2 + \frac{v_1 \mu_2}{2} & -v_2 \lambda_12 + \frac{v_1 \mu_2}{2} \\ v_1 \lambda_12 - \frac{v_2 \mu_2}{2} & -v_2 \lambda_12 + \frac{v_1 \mu_2}{2} & 2w^2 \lambda_3 + \frac{v_1 v_2 \mu_2}{2w} \end{pmatrix},$$

Diagonalization yields three physical mass-eigenstate scalars. Assuming for simplicity the hierarchy $\mu_2 \gg v_1, v_2$, the lightest one can be identified with the standard model Higgs boson discovered at the LHC

$$h \approx \frac{v_2 s_2 - v_1 s_1}{\sqrt{v_1^2 + v_2^2}}. \quad (7)$$

Its squared mass is given as

$$m_h^2 \approx \left(2\lambda_1 - \frac{\lambda_1^2}{\lambda_3}\right) v_1^2 + v_2^2 \left(2\lambda_12 - \frac{\lambda_12 \lambda_3^2}{\lambda_3^3} - \frac{\mu_2^2}{\lambda_3^2 \lambda_3^2}\right) + \left(2\lambda_2 - \frac{\lambda_2 \lambda_3^2}{\lambda_3^3}\right) v_1^2 + \mu_2 \left(\frac{\lambda_12 v_1^2 + \lambda_2 v_3^2}{\lambda_3 v_1 w}\right), \quad (8)$$

where all parameters, other than $\mu_2$ and $w$, lie at the electroweak scale. The remaining scalars are heavy and can be approximately identified as

$$H_1 \approx \frac{v_2 s_1 + v_1 s_2}{\sqrt{v_1^2 + v_2^2}}, \quad m_{H_1}^2 \approx \frac{(v_1^2 + v_2^2) w \mu_2}{2v_1 v_2},$$

$$H_2 \approx s_3, \quad m_{H_2}^2 \approx 2w^2 \lambda_3. \quad (9)$$

In principle, $\mu_2$ can be even lower than the electroweak scale. In that case this sector would give rise to two light scalars and a heavy one. In what follows we assume an arbitrary $\mu_2$ scale and a VEV hierarchy $w \gg v_1, v_2$.

### B. CP-odd scalars

Similar to the CP-even scalars, the CP-odd components also mix through the squared mass matrix,

$$M^2_{a_1, a_2, a_3} = \frac{\mu_2}{2} \begin{pmatrix} \frac{v_2 w}{v_1} & -w & v_2 \\ -w & \frac{v_1 w}{v_2} & -v_1 \\ v_2 & -v_1 & \frac{v_1 v_2 w}{w} \end{pmatrix}, \quad (10)$$

in the basis $(a_1, a_2, a_3)$. Upon diagonalisation, we find two Nambu-Goldstone bosons that can be identified as

$$G_1 = \frac{v_1 a_1 + v_2 a_2}{\sqrt{v_1^2 + v_2^2}}, \quad G_2 = \frac{-v_1 a_1 + w a_3}{\sqrt{v_1^2 + w^2}}, \quad (11)$$

and one physical pseudoscalar

$$A'_1 = \frac{v_2 w a_1 - v_1 w a_2 + v_1 v_2 a_3}{\sqrt{v_1^2 v_2^2 + v_1^2 w^2 + v_2^2 w^2}}, \quad (12)$$

with squared mass

$$m_{A'_1}^2 = \frac{\mu_2 \left(v_1^2 v_2^2 + v_1^2 w^2 + v_2^2 w^2\right)}{2v_1 v_2 w}. \quad (13)$$
The importance of the $U(1)_{\text{PQ}}$ soft-breaking term characterized by $\mu_\phi$ can be better understood by looking at the equations above and Table I. In the limit $\mu_\phi \to 0$, $U(1)_{\text{PQ}}$ becomes a classical global symmetry of the model, whose spontaneous breaking by the vevs of $\Phi_{1,2}$, where $\langle \Phi_1 \rangle^2 + \langle \Phi_2 \rangle^2 = v_{EW}^2$, would imply the existence of a massless Nambu-Goldstone boson, namely, the pseudoscalar defined in Eq. (12). However, the Peccei-Quinn-like symmetry has an associated $[SU(3)_c]^2 U(1)_{\text{PQ}}$ anomaly. Therefore, instead of a massless field, we would have a pseudo-Goldstone boson, an axion field getting its mass via nonperturbative effects. The existence of such an electroweak scale axion, à la Weinberg-Wilczek [25, 26], is ruled out phenomenologically, as noted in the context of 3-3-1 models in Ref. [27]. Alternative 3-3-1 proposals including gauge singlet scalars with non-vanishing $U(1)_{\text{PQ}}$ breaking scale axion, an axion field getting its mass via nonperturbative effects. The existence of such an electroweak symmetry has an associated [28, 29]. This way one can make the axion invisible, and thus viable by introducing a large $U(1)_{\text{PQ}}$ breaking scale. Here we do not follow this path. Instead, we avoid the presence of the visible axion simply by softly breaking $U(1)_{\text{PQ}}$ via the trilinear $\Phi_1 \Phi_2 \Phi_3$ term, instead of adding more scalars. Apart from minimality, this also ensures that tree-level neutrino masses are absent.

C. Complex neutral scalars

The complex neutral scalars that do not acquire vevs can be grouped in pairs according to their $B-L$ charges, as follows. First notice that, since $B-L$ is conserved, only fields with the same $B-L$ charges can mix. Since the fields $\phi_2^0$ and $\phi_3^0$ carry opposite $B-L$ charges, we define a $B-L = 2$ basis as $(\phi_2^0, \phi_3^0)$. In this basis, we can write down the following squared mass matrix

$$M_{\phi_2^0, \phi_3^0}^2 = \frac{1}{2} \begin{pmatrix} w (w \lambda_{23} + \frac{v_1 \mu_\phi}{v_2}) & -v_1 \mu_\phi - v_2 w \lambda_{23} \\ -v_1 \mu_\phi - v_2 w \lambda_{23} & v_2 (v_2 \lambda_{23} + \frac{v_1 \mu_\phi}{w}) \end{pmatrix}.$$ (14)

Upon diagonalisation, we find a massless complex scalar, shown in the next section to be absorbed à la Goldstone by the gauge sector,

$$G_3 = \frac{v_2 \phi_2^0 + w \phi_3^0}{\sqrt{v_2^2 + w^2}},$$ (15)

and a heavy complex scalar field

$$\varphi = \frac{-w \phi_2^0 + v_2 \phi_3^0}{\sqrt{v_2^2 + w^2}}, \quad m_\varphi^2 = \frac{(v_2^2 + w^2) (v_2 \lambda_{23} v_2 w + v_1 \mu_\phi)}{2 v_2 w}.$$ (16)

Likewise, coming to the remaining fields, these can be grouped in a basis with $B-L = 1$ as $(\phi_4^0, \phi_4^{0*})$. The corresponding squared mass matrix is

$$M_{\phi_4^0, \phi_4^{0*}}^2 = \frac{1}{2} \begin{pmatrix} v_1^2 \lambda_{14} + v_2^2 \lambda_{24} + w^2 (\lambda_{34} + \lambda_{24}^*) & 2 \mu_2^2 + \frac{1}{2} \lambda' v_2 w \\ -\frac{1}{2} \lambda' v_2 w & v_1^2 \lambda_{14} + v_2^2 (\lambda_{24} + \lambda_{24}^*) + w^2 \lambda_{34} + 2 \mu_2^2 \end{pmatrix},$$ (17)

that can be diagonalized as

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \phi_4^0 \\ \phi_4^{0*} \end{pmatrix},$$ (18)

with

$$\tan 2\theta = \frac{v_2 w \lambda'}{v_2^2 \lambda_{24} - w^2 \lambda_{34}}.$$ (19)

The physical neutral fields $\eta_1$ and $\eta_2$ defined above have squared masses

$$m_{\eta_1, \eta_2}^2 = \frac{1}{4} \left[ 4 \mu_2^2 + 2 \lambda_{14} v_2^2 + v_2^2 \left( 2 \lambda_{24} + \lambda_{24}^* \right) + w^2 \left( 2 \lambda_{34} + \lambda_{24}^* \right) \mp \mathcal{F} \sqrt{\lambda' v_2 w^2 + \left( \lambda_{24} v_2^2 - \lambda_{34} w^2 \right)^2} \right],$$ (20)

where $\mathcal{F} = \text{sign}(\lambda_{24} v_2^2 - \lambda_{34} w^2)$. 
D. Charged scalars

Again, for the charged scalars too, mixing takes place amongst those with the same $B - L$ charges, and they can be separated into three groups.

The basis $(\phi_1^{\pm}, \phi_2^{\pm})$ puts together the charged fields with $B - L = 0$ which mix according to the squared mass matrix

$$M_{\phi_1^{\pm}, \phi_2^{\pm}}^2 = \frac{1}{2} \begin{pmatrix} v_2 \left( \frac{w_\mu}{v_1} + \tilde{\lambda}_{12} v_2 \right) & -\mu_\phi w - \tilde{\lambda}_{12} v_1 v_2 \\ -\mu_\phi w - \tilde{\lambda}_{12} v_1 v_2 & v_1 \left( \frac{w_\mu}{v_2} + \tilde{\lambda}_{12} v_1 \right) \end{pmatrix}. \quad (21)$$

Upon diagonalisation, we find a (complex) “Goldstone” boson

$$G_3^+ = \frac{v_1 \phi_1^+ + v_2 \phi_2^+}{\sqrt{v_1^2 + v_2^2}}, \quad (22)$$

and a massive electrically charged physical scalar

$$H_1^+ = \frac{-v_2 \phi_1^+ + v_1 \phi_2^+}{\sqrt{v_1^2 + v_2^2}}, \quad m_{H_1^+}^2 = \frac{(v_1^2 + v_2^2)(w_\mu + v_1 v_2 \tilde{\lambda}_{12})}{2v_1 v_2}. \quad (23)$$

The charged scalars with $B - L = \pm 2$ are characterized by the following squared mass matrix, written in the basis $(\tilde{\phi}_1^+, \phi_3^+)$,

$$M_{\tilde{\phi}_1^+, \phi_3^+}^2 = \frac{1}{2} \begin{pmatrix} w \left( \frac{w_\mu}{v_1} + \tilde{\lambda}_{13} v_1 \right) & v_1 w \tilde{\lambda}_{13} + v_2 \mu_\phi \\ v_1 w \tilde{\lambda}_{13} + v_2 \mu_\phi & v_1 \left( \frac{w_\mu}{v_2} + v_3 \tilde{\lambda}_{13} \right) \end{pmatrix}. \quad (24)$$

from which one can identify another pair of charged Goldstones

$$G_5^+ = \frac{-v_1 \tilde{\phi}_1^+ + w \phi_3^+}{\sqrt{v_1^2 + w^2}}, \quad (25)$$

and the heavy charged states,

$$H_2^+ = \frac{w \tilde{\phi}_1^+ + v_1 \phi_3^+}{\sqrt{v_1^2 + w^2}}, \quad m_{H_2^+}^2 = \frac{(v_1^2 + w^2)(v_2 \mu_\phi + v_1 w \tilde{\lambda}_{13})}{2v_1 w}. \quad (26)$$

Finally, the only charged scalar with $B - L = 1$, $\phi_4^+$, remains unmixed after spontaneous symmetry breaking, and gets the squared mass

$$m_{\phi_4^+}^2 = \frac{1}{2} \left[ v_1^2 (\lambda_{14} + \tilde{\lambda}_{14}) + v_2^2 \lambda_{24} + w^2 \lambda_{34} + 2\mu_4^2 \right]. \quad (27)$$

IV. GAUGE SECTOR

In this section, we study the vector boson spectrum of the extended electroweak sector which contains ten gauge fields. After spontaneous symmetry breaking, gauge boson masses are generated, as usual, through the terms $L \supset (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1)$, where the covariant derivative acting on the scalar anti-triplets is defined as

$$D_\mu \Phi_i = \left[ \partial_\mu + ig_L \frac{\lambda_a^a}{2} W_\mu^a - ig_X X B_\mu - ig_N NC_\mu \right] \Phi_i = \left( \partial_\mu + ig_L \frac{\lambda_a^a}{2} \mathcal{P}_\mu \right) \Phi_i \quad (28)$$

where $W_\mu^a$ are the gauge fields of $SU(3)_L$, $\lambda^a$ are the Gell-Mann matrices, $B_\mu$ is the gauge field of $U(1)_X$, and $C_\mu$ is the gauge field of $U(1)_N$ and

$$\mathcal{P}_\mu = \begin{pmatrix} W^3 + \frac{W^8}{\sqrt{3}} - 2(t_X X B + t_N NC) & \sqrt{2}W^+ & \sqrt{2}W^- \\ \sqrt{2}W^+ & -W^3 + \frac{W^8}{\sqrt{3}} - 2(t_X X B + t_N NC) & \sqrt{2}X^0 \\ \sqrt{2}W^- & \sqrt{2}X^0 & -2 \left( \frac{W^8}{\sqrt{3}} + t_X X B + t_N NC \right) \end{pmatrix}_\mu. \quad (29)$$
with
\[
W_{\mu}^\pm = \frac{W_{\mu}^1 \mp iW_{\mu}^2}{\sqrt{2}}, \quad W_{\mu}^\pi = \frac{W_{\mu}^4 \mp iW_{\mu}^5}{\sqrt{2}}, \quad X_{\mu}^{0(*)} = \frac{W_{\mu}^6 \mp iW_{\mu}^7}{\sqrt{2}}.
\]  
(30)

In addition, we assume another source for gauge boson masses through the Stueckelberg mechanism for the Abelian $U(1)_N$ symmetry [21]. The masses and states of the ten electroweak gauge bosons are discussed below.

### A. Neutral Gauge Bosons and Stueckelberg Mechanism

After spontaneous symmetry breaking, the two gauge bosons of the abelian symmetries, $B_\mu$ and $C_\mu$, and the two fields associated with the diagonal generators of $SU(3)_L$, $W_\mu^8$ and $W_\mu^0$, mix among themselves. Assuming the kinetic mixing between the gauge bosons $B_\mu$ and $C_\mu$ can be neglected, the relevant terms contributing to the neutral boson masses, written in the basis $B_\mu^T = (W_\mu^3, W_\mu^8, B_\mu, C_\mu)$, are
\[
\mathcal{L} = \frac{1}{2} B_\mu^T M_0^2 B_\mu + \frac{1}{2} (mC^\mu - \partial^\mu \sigma)^2 + \mathcal{L}_{gf}^{St}.
\]  
(31)

Here $M_0^2$ is the squared mass matrix coming from the Higgs mechanism, $m$ is the Stueckelberg mass of the $C^\mu$ gauge field, and $\sigma$ is the scalar Stueckelberg compensator that renders the second term in Eq. (31) invariant under the gauge transformations
\[
C^\mu \rightarrow C^\mu + \partial^\mu \Omega(x),
\]
\[
\sigma \rightarrow \sigma + m\Omega(x),
\]  
(32)

with an arbitrary spacetime function $\Omega(x)$. The gauge fixing term $\mathcal{L}_{gf}^{St}$ can be chosen as
\[
\mathcal{L}_{gf}^{St} = -\frac{1}{2\xi} \left\{ \partial^\mu C_\mu + \xi \left[ m\sigma - \frac{2}{3} g_N \left( \sqrt{v_1^2 + v_2^2} G_1 + 2\sqrt{v_1^2 + w^2} G_2 \right) \right] \right\}^2,
\]  
(33)

ensuring (up to a total derivative) that the gauge field $C^\mu$ decouples from the gradients $\partial^\mu \sigma$, $\partial^\mu G_1$ and $\partial^\mu G_2$. Notice that after gauge-fixing, the Lagrangian is still invariant under a restricted set of gauge functions $\Omega(x)$, subject to the same equation of motion as $\sigma$, i.e. $(\partial^2 + \xi m^2)\Omega = (\partial^2 + \xi m^2)\sigma = 0$. This dynamical restriction guarantees the propagation of three degrees of freedom for the massive vector field $C_\mu$. Moreover, $\mathcal{L}_{gf}^{St}$ introduces a mixing between the scalars $\sigma$, $G_1$ and $G_2$.

After implementing the Stueckelberg mechanism outlined above, the squared-mass matrix of the neutral gauge bosons becomes
\[
M^2 = \frac{g_\mu^2}{2} \begin{pmatrix}
\frac{1}{2} (v_1^2 + v_2^2) & \frac{v_1^2 - v_2^2}{2\sqrt{3}} & -\frac{1}{3} (2v_1^2 + v_2^2) t_X & -\frac{2}{3} (v_1^2 - v_2^2) t_N \\
\frac{v_1^2 - v_2^2}{2\sqrt{3}} & \frac{1}{6} (v_1^2 + v_2^2 + 4w^2) & \frac{2}{3} (v_1^2 - v_2^2) -2(2v_1^2 + v_2^2) t_N \\
-(\frac{1}{3} (2v_1^2 + v_2^2) t_X & \frac{1}{3} (v_1^2 + v_2^2 -2v_1^2) t_X & \frac{2}{3} (2v_1^2 - v_2^2 + 2w^2) t_X & \frac{1}{3} (2v_1^2 - v_2^2 + 2w^2) t_N t_X \\
(\frac{2}{3} (v_1^2 - v_2^2) t_N & -2(\frac{v_1^2 + v_2^2 + 4w^2) t_N} & \frac{2}{3} (v_1^2 - v_2^2 + 2w^2) t_N t_X & \frac{2}{3} m^2 + \frac{8}{3} (2v_1^2 + v_2^2 + 4w^2) t_N^2 \\
\end{pmatrix}
\]  
(34)

with $t_X = g_X/g_L$ and $t_N = g_N/g_L$. In order to diagonalize $M^2$, several changes of basis will be required. In this analysis we follow the procedure described in [30].

We first identify the photon field $A_\mu$. The transformation matrix to the basis $(A_\mu, Z_{1,\mu}, Z'_{1,\mu}, C_\mu)$ is given by
\[
\begin{pmatrix}
A_\mu \\
Z_{1,\mu} \\
Z'_{1,\mu} \\
C_\mu
\end{pmatrix} = U_1 \begin{pmatrix}
W_\mu^3 \\
W_\mu^8 \\
B_\mu \\
C_\mu
\end{pmatrix}, \quad U_1 = \begin{pmatrix}
\frac{\sqrt{3} \chi}{\sqrt{4\chi^2 + 3}} & \frac{t_X}{\sqrt{4\chi^2 + 3}} & 0 & 0 \\
\frac{t_X}{\sqrt{4\chi^2 + 3}} & \frac{\sqrt{3} \chi}{\sqrt{4\chi^2 + 3}} & 0 & 0 \\
0 & 0 & \frac{\sqrt{3} \chi}{\sqrt{4\chi^2 + 3}} & -\frac{3\chi}{\sqrt{4\chi^2 + 3}} \\
0 & 0 & -\frac{3\chi}{\sqrt{4\chi^2 + 3}} & \frac{\sqrt{3} \chi}{\sqrt{4\chi^2 + 3}} \\
\end{pmatrix},
\]  
(35)
such that
\[ M'^2 = U_1 M^2 U_1^T = \begin{pmatrix} 0 & 0 \\ 0 & M'^2 \end{pmatrix}, \] (36)

with
\[ M'^2 = \frac{g_L^2}{2} \begin{pmatrix} 
\frac{(v_1^2 + v_2^2)(4t_X^2 + 3)}{2(t_X^2 + 3)} & \frac{\sqrt{4t_X^2 + 3}[v_1^2(4t_X^2 + 3) + v_2^2(2t_X^2 - 3)]}{6(t_X^2 + 3)} & -\frac{2}{3}(v_1^2 - v_2^2)t_N \left( \frac{4t_X^2 + 3}{t_X^2 + 3} \right)^{1/2} \\
-\frac{2}{3}(v_1^2 - v_2^2)t_N \left( \frac{4t_X^2 + 3}{t_X^2 + 3} \right)^{1/2} & \frac{\sqrt{4t_X^2 + 3}[v_1^2(3+4v_1^2 + v_1^2 - 3 t_X^2) + 4v_2^2 + 4w^2 t_X^2] - 2\sqrt{2}(v_1^2 - v_2^2) t_N \left( \frac{4t_X^2 + 3}{t_X^2 + 3} \right)^{1/2}}{9t_X^2 + 3} & \frac{2m^2}{g_L^2} + \frac{8}{9} (v_1^2 + v_2^2 + 4w^2) t_N^2 
\end{pmatrix}. \] (37)

Therefore, the photon is identified as
\[ A_\mu = \frac{1}{\sqrt{4t_X^2 + 3}} \left( \sqrt{3} t_N W^3 + t_N W^8 + \sqrt{3} B_\mu \right). \] (38)

For the second diagonalization to the basis \((A_\mu, Z_\mu, Z_2', C'_\mu)\), we use a “seesaw approximation” \[31\]
\[ U_2 \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \varepsilon_1 \\ 0 & -\varepsilon_1 & 1 & 0 \\ 0 & -\varepsilon_2 & 0 & 1 \end{pmatrix}, \] (39)

where \(\varepsilon_1\) and \(\varepsilon_2\) are the two components of a small vector given by
\[
\varepsilon \equiv -(m_{Z_1}^2, m_{Z_1}^2, m_{Z_2}^2, m_{Z_2}^2) \left( \begin{array}{c}
m_{Z_1}^2 \\
m_{Z_1}^2 \\
m_{Z_1}^2 \\
m_{Z_1}^2 \\
\end{array} \right)^{-1},
\] (40)
\[
\varepsilon_1 = -\frac{\sqrt{4t_X^2 + 3} \left( 8g_L^2 t_N^2 \left( w^2 v_1^2 + v_1^2 v_2^2 + v_1^2 v_2^2 \right) + 9m^2 \left( 2v_1^2 + v_2^2 \right) \right) + 9m^2 (v_1^2 - v_2^2) \right)}{4t_X^2 \left( 4g_L^2 t_N^2 \left( w^2 v_1^2 + v_1^2 v_2^2 + v_1^2 v_2^2 \right) + m^2 \left( 4v_1^2 + v_2^2 + w^2 \right) \right) + 3m^2 \left( 2v_1^2 - v_2^2 + 2w^2 \right) + 3 \left( v_1^2 + v_2^2 + 4w^2 \right) \right]},
\]
\[
\varepsilon_2 = \frac{4g_L^2 t_N^2 \left( w^2 v_1^2 + v_1^2 v_2^2 + v_1^2 v_2^2 \right) + m^2 \left( 4v_1^2 + v_2^2 + w^2 \right) \right) + 3m^2 \left( 2v_1^2 - v_2^2 + 2w^2 \right) + 3 \left( v_1^2 + v_2^2 + 4w^2 \right) \right]},
\]
which are suppressed by the hierarchy \(v_1, v_2 < < w, m\).

Then, after the second diagonalization we have
\[ M'^{\prime 2} = U_2 M'^2 U_2^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{Z_2}^2 & 0 \\ 0 & 0 & m_{Z_2}^2 \end{pmatrix}, \] (41)

with
\[ m_{Z_2}^2 \approx m_{Z_2}^2 + 2(\varepsilon_1, \varepsilon_2) \left( \begin{array}{c}
m_{Z_1}^2 \\
m_{Z_1}^2 \\
m_{Z_1}^2 \\
m_{Z_1}^2 \\
\end{array} \right) \]
(42)
\[ \approx \frac{g_L^2 (v_1^2 + v_2^2)(4t_X^2 + 3)}{4(t_X^2 + 3)}, \]

which can be identified with the squared mass of the physical electroweak \(Z_\mu\) boson and
\[ M'^{\prime 2} \approx \begin{pmatrix} m_{Z_2}^2 & m_{Z_2}^2 \mu' \\
m_{Z_2}^2 & m_{Z_2}^2 \mu' \end{pmatrix}. \] (43)
Finally we can diagonalize $M''^2$ to the $(A_\mu, Z_\mu, Z'_\mu, Z''_\mu)$ basis through

$$U_3 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & c_\zeta & s_\zeta \\
0 & 0 & -s_\zeta & c_\zeta
\end{pmatrix}, \tag{44}$$

and the diagonal squared mass matrix for the physical gauge bosons becomes

$$M'''' = U_3 M''^2 U_3^T \tag{45}$$

where the mixing angle is given by

$$\tan 2\zeta \approx \frac{8w^2 g_L^4 t_N \sqrt{T_X^2 + 3}}{w^2 g_L^2 (16t_N^2 - t_X^2 - 3) + 9m^2}, \tag{46}$$

and the diagonal entries can be identified as the squared masses for the physical $Z'$ and $Z''$ bosons

$$m_{Z', Z''}^2 = \frac{1}{18} \left\{ w^2 g_L^2 (16t_N^2 + t_X^2 + 3) + 9m^2 \pm \mathcal{G} \sqrt{[w^2 g_L^2 (16t_N^2 - t_X^2 - 3) + 9m^2]^2 + 64w^4 g_L^4 t_N^2 (t_X^2 + 3)} \right\}, \tag{47}$$

with $\mathcal{G} = \text{sign} [w^2 g_L^2 (16t_N^2 - t_X^2 - 3) + 9m^2]$.

**B. Complex Neutral Gauge Bosons**

The complex gauge boson $X_\mu^0$, with $B - L = 2$, does not mix with the other neutral vector fields. After spontaneous symmetry breaking, $X_\mu^0$, whose associated would-be Goldstone boson is $G^3$ in Eq. (15), gets the following mass term

$$m_{X^0}^2 = \frac{g_L^2}{4} (v_1^2 + w^2). \tag{48}$$

**C. Charged Gauge Bosons**

The charged gauge bosons present in the model, $W^\pm_\mu$ and $W'^\pm_\mu$, become massive after electroweak symmetry breaking but do not mix due to their different $B - L$ charges.

The first mass eigenstate is identified with the charged standard model electroweak W-boson, whose would-be Goldstone bosons given by $G^\pm_4$, and has the squared mass

$$m_{W^\pm}^2 = \frac{g_L^2}{4} (v_1^2 + v_2^2). \tag{49}$$

Finally, the other charged gauge boson is heavy and eats up the complex would-be Goldstone boson $G^\pm_5$ in order to acquire the squared mass

$$m_{W'^\pm}^2 = \frac{g_L^2}{4} (v_1^2 + w^2). \tag{50}$$

To sum up we note that, despite the conservation of $B - L$, all of the gauge bosons acquire adequate masses through the interplay of the standard Higgs mechanism with the Stueckelberg mechanism, leaving only the photon massless, as in the Standard Model.
After spontaneous symmetry breaking, the above interactions lead to the following mass matrices for the fermions:

\[-\mathcal{L}_{\text{Yuk}} = y_{ab}^{e} e_{R}^{j} \Phi_{1}^{\dagger} \psi_{L}^{j} + y_{ab}^{S} S_{R}^{j} \Phi_{4}^{\dagger} \psi_{L}^{j} + \frac{M_{R}^{2}}{2} \Phi_{S}^{j} S_{R}^{j} + y_{ab}^{d} W_{R}^{j} \Phi_{2}^{\dagger} Q_{L}^{j} + y_{ab}^{d} W_{R}^{j} \Phi_{2}^{\dagger} Q_{L}^{j} + y_{ab}^{d} W_{R}^{j} \Phi_{2}^{\dagger} Q_{L}^{j} + \text{h.c.}.\]

(51)

After spontaneous symmetry breaking, the above interactions lead to the following mass matrices for the fermions:

- Charged leptons:
  \[M_{\nu}^{\ell} = y_{ab}^{\ell} \frac{\nu_{1}}{\sqrt{2}}.\]
  (52)

- Up-type quarks, basis \((u, c, t, U_{3})\):
  \[M_{u} = \frac{1}{\sqrt{2}} \begin{pmatrix}
  v_{1} y_{11}^{u} & v_{1} y_{12}^{u} & -v_{2} y_{13}^{u}
  
v_{1} y_{21}^{u} & v_{1} y_{22}^{u} & -v_{2} y_{23}^{u}
  
v_{1} y_{31}^{u} & v_{1} y_{32}^{u} & -v_{2} y_{33}^{u}
  
0 & 0 & 0
\end{pmatrix} w_{y_{13}}^{U}.\]
  (53)

- Down-type quarks, basis \((d, s, b, D_{1}, D_{2})\):
  \[M_{d} = \frac{1}{\sqrt{2}} \begin{pmatrix}
  v_{2} y_{11}^{d} & v_{2} y_{12}^{d} & v_{1} y_{13}^{d}
  
v_{2} y_{21}^{d} & v_{2} y_{22}^{d} & v_{1} y_{23}^{d}
  
v_{2} y_{31}^{d} & v_{2} y_{32}^{d} & v_{1} y_{33}^{d}
  
0 & 0 & 0
\end{pmatrix} w_{y_{13}}^{D}.\]
  (54)

Realistic quark masses can be easily obtained from the above mass matrices, as the standard model and exotic quarks remain unmixed by virtue of the unusual \(B - L\) charges of the exotic sector. This is reflected by the block-diagonal form of the above matrices, which also implies the unitarity of the Cabibbo-Kobayashi-Maskawa matrix describing quark mixing.

Notice that, from the above Yukawa interactions, neutrinos remain massless at the tree level.

VI. SCOTOGENIC NEUTRINO MASSES

As previously shown, in the present model the gauged \(B - L\) symmetry remains unbroken, and so does the matter-parity \(M_{P}\). Furthermore, the U(1)\(_{PQ}\) symmetry, only broken softly in the scalar sector, forbids the appearance of a tree-level neutrino-mass-giving Yukawa term. However, the Yukawa interactions in Eq. (51) allow the emergence of a calculable one-loop contribution to the neutrino masses via the diagram in Fig. 1.

Assuming that the Majorana mass of the fermion singlets \(S_{R}\) is already diagonal \(M^{S} = \text{diag}(M_{1}, M_{2}, M_{3})\), the neutrino mass matrix generated by the scotogenic loop in the basis \((\nu_{L}, (\nu_{R})^{c})\) reads

\[M_{\nu} = \begin{pmatrix}
0 & m_{\nu}
\end{pmatrix}.\]

(55)

with neutrino Dirac masses

\[
(m_{\nu})_{ab} = \sum_{k=1}^{3} \frac{M_{k} y_{k a} S_{k} S_{k} \sin 2 \theta}{32 \pi^{2}} \left[ \frac{m^{2}_{\eta_{1}}}{m^{2}_{\eta_{1}} - M_{k}^{2}} \ln \frac{m^{2}_{\eta_{1}}}{M_{k}^{2}} - \frac{m^{2}_{\eta_{2}}}{m^{2}_{\eta_{2}} - M_{k}^{2}} \ln \frac{m^{2}_{\eta_{2}}}{M_{k}^{2}} \right].
\]

(56)
Notice that from Eq. (19), if the relevant quartic couplings are of the same order, the angle $\theta$ is already suppressed, of $O(v_2/w)$. Besides, the internal fields in the loop are odd under $M_P$, while the standard model fields are even. Thus, the lightest $M_P$-odd field is automatically stable and, if it is electrically neutral, can be identified as a dark matter candidate. In our model, the stable dark matter candidate will be the lightest field among the complex scalars $\eta_i$ and Majorana fermions $S_{aR}$.

\section{Summary and Conclusions}

In this work we have proposed a simple scotogenic extension of the original Singer-Valle-Schechter 3-3-1 model in which neutrinos are Dirac fermions as a result of a conserved $B - L$ gauge symmetry. In such minimal SVS gauge extension neutrino masses arise through the radiative exchange of the simplest scotogenic “dark” sector, as indicated by the diagram in Fig. 1. Conservation of $B - L$ gauge symmetry in the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ theory ensures the stability of dark matter, linked to the Dirac nature of neutrinos. By combining the Higgs and the Stueckelberg mechanisms one ensures that all neutral gauge bosons acquire adequate non-zero masses. Our present construction bears similarities with that in Ref. [22], but within a richer framework which also touches other standard model shortcomings, such as the existence of three families. The latter follows just from anomaly cancellation. Stable dark matter is “predestined” [23], in the sense that the imposition of additional symmetries is not required. We have given a detailed study of the basic structure of the theory. For example, we noted that due to our quantum numbers we have block-diagonal quark mass matrices, Eqs. (53) and (54), implying the unitarity of the CKM matrix describing quark mixing. However, the new neutral gauge bosons can have flavor-changing interactions at the tree level, as in the SVS model. These arise from the underlying structure of the neutral current dictated by the anomaly cancellation. However, in addition to direct searches through dilepton studies at the LHC, heavy neutral gauge bosons induce mass differences in neutral meson systems. These can lead to observable phenomena if they lie within the few TeV scale. For example, for $v_1 \sim v_2 \sim 173$ GeV if one takes $m \to \infty$, $w \sim 10^4$ GeV as a benchmark, one finds that the $B - L$ Stueckelberg gauge boson decouples, leaving adequate masses for the other new intermediate gauge bosons, around 4 TeV, consistent with limits from flavor changing neutral current and dilepton resonance searches at the LHC run 2 at 13 TeV [32]. Likewise, one can check that the scalar masses expected, e.g. from Eqs. (9) and (13), are also phenomenologically viable. The same happens for finite values of the Stueckelberg gauge boson mass parameter: in this case one also obtains gauge boson mass values in agreement with current limits. We expect, however, that they can lie within the sensitivities expected, for example, at High Luminosity-LHC, LHCb as well as upcoming B factories.
Last, but not least, we stress that, in contrast to previous 3-3-1-1 models, here neutrinos get radiative scotogenic Dirac masses, rather than Majorana masses from the conventional seesaw mechanism. A discovery of neutrinoless double beta decay would therefore invalidate our present construction.

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