Relativistic NN scattering equations without partial-wave decomposition

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Relativistic quasi-potential equations describing NN scattering are compared. Within the spectator formalism a cancellation is seen to occur between retardation and negative-energy effects.

1. INTRODUCTION

Modern experimental facilities such as TJNAF and Julich are probing the baryonic structure and interactions at intermediate energies (\(\sim\) GeV), through electromagnetic and strong reactions involving high momentum transfer \([1]\). To describe the NN interaction in the intermediate energy range (\(\sim\) 300 MeV-1 GeV), particle production mechanisms have to be included. Relativistic effects became also important. In this scenario an accurate relativistic treatment of NN scattering, including boosts and retardation effects, is needed. These dynamical aspects are included automatically when one solves de 4-dimensional Bethe-Salpeter integral equation \([2]\):

\[
T(p', p; P) = V(p', p; P) + i \int \frac{d^4k}{(2\pi)^4} V(p', k; P) g(k; P) T(k, p; P).
\]

(1)

We tested different 3-dimensional reductions of this equation, currently known as Quasi-Potential (QP) equations. These equations corresponds to different choices of the relative interaction energy. The QP equations considered are: the Gross or spectator equation \([3]\) (one particle on-mass-shell in all intermediate states), the Blankenbecler-Sugar equation (BbS) \([4]\) (both the two particles equally off-mass-shell in all intermediate states, which means no retardation effects), and the Equal-Time equation \([5]\) (BbS with effects from crossed-box diagrams included partially). The corresponding propagators for the intermediate states are respectively:

\[
g_{BbS}(k; P) = i2\pi \frac{M}{E_k E_k^2 - \frac{W^2}{4} - i\varepsilon},
\]

(2)

\[
g_{ET}(k; P) = i2\pi \frac{M}{E_k E_k^2 - \frac{W^2}{4} - i\varepsilon} \times \left(2 - \frac{W^2}{4E_k^2}\right).
\]

(3)
Figure 1. 300 MeV scattering amplitude. The bullets are from Ref. [6]. The solid line corresponds to a calculation with more mesh points.

\[ g_{\text{Gross}}(k; P) = i2\pi \frac{M M \delta(k_0 + W/2 - E_k)}{E_k W} \frac{E_k - W/2 - i\varepsilon}{E_k - W/2 - i\varepsilon} \]  

(4)

where \( m \) is the nucleon mass, \( E_k = \sqrt{m^2 + k^2} \) and \( W = \sqrt{P^2} \).

2. PARTIAL WAVE-DECOMPOSITION

Usually the 3-dimensional equations are solved by implementing a partial-wave decomposition in the angular variables. However, from the calculational point of view, the partial-wave decomposition of the NN scattering amplitude becomes less adequate and practical for high energies [6], where an increasing number of partial-waves is required. Fortunately, nowadays computational resources allow the evaluation of the scattering amplitude in terms of the on-shell momentum and scattering angle, without performing a partial-wave expansion. This one is traded for the resolution of an integral equation in two dimensions, instead of only one, after factoring out the integral in the azimuthal variable \( \phi \). Without using a decomposition in partial-waves, Ch. Elster et al. [6] solved already the Non Relativistic (NR) Lippman-Schwinger scattering equation for scalar particles interacting through the Malfliet-Tjon potential. Subsequently, they used the two-nucleon half-off-shell amplitudes in a three-nucleon system calculation [7]. We have reproduced their NR two-nucleon results. For that purpose we used non-relativistic relations between laboratory and the CM frames. Moreover, we compared them with results obtained from different relativistic scattering equations. We also compared the NR results with the ones obtained from de three different relativistic equations mentioned above. The results are presented in Figs. 1 and 2.
3. RESULTS

Our results quantify the increasing importance of relativistic effects with increasing energy. We have considered two energies cases: one (300 MeV) in the low energy and another (800 MeV) in the high energy region. The relativistic effects turn to be more significant for the BbS and spectator/Gross equations, as we can see in Figs. 1 and 2, particularly in the real part of the scattering amplitude: indeed, the Equal-Time amplitudes are the ones which are closer to the NR Lippman-Schwinger amplitudes. Deviations from the NR results increase with the energy.

As for retardation effects, we modified the Malfliet-Tjon such that the scalar exchanges would consider the transferred energy. By construction these effects can be present only in the spectator/Gross QP equation. With retardation effects included, the spectator amplitude (thick dashed line in Figs. 1 and 2) becomes closer to the NR limit than the Equal-Time amplitude.

To understand the previous results we analysed the relativistic effects included in the spectator/Gross equation, one by one. We decomposed the nucleon propagator in that equation into its positive and negative energy component components

\[ g_{\text{Gross}}(k; W) = i2\pi \frac{1}{2} \left( \frac{M}{E_k} \right)^2 \left[ \frac{1}{2E_k - W - i\varepsilon} + \frac{1}{W} \right] \delta(k_0 + W/2 - E_k). \tag{5} \]

The results of the decomposition are presented in Fig. 3. The negative energy effects (Gross +-) are essential and increase with energy. On the other hand, retardation contributions (Gross + and retardation) have the opposite effect. Most importantly, one concludes also that there is an almost perfect cancellation between retardation and negative energy state effects, with the net result that the spectator amplitude calculated with retardation and the NR amplitude are almost coincident. This agreement is not perfect and deteriorates with energy.
Figure 3. 300 MeV and 800 MeV scattering amplitude. Retardation and negative energy state effects.

Ref. [8] has extended in that direction the NR work of Ref. [6], by considering a realistic NN interaction. We have in progress a covariant two-nucleon calculation which deals with the complete nucleon structure arising from its nature as a Dirac particle.

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