Gauge Invariant Operators and Closed String Scattering in Open String Field Theory

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Abstract

Using the recent proposal for the observables in open string field theory, we explicitly compute the coupling of closed string tachyon and massless states with the open string states up to level two. Using these couplings, we then calculate the tree level S-matrix elements of two closed string tachyons or two massless states in the open string field theory. Up to some contact terms, the results reproduce exactly the corresponding amplitudes in the bosonic string theory.
1 Introduction

The open string tachyon condensation has attracted much interest recently. Regarding the recent development in string field theory, (for example see [1] and [2] and their references) it is believed that the open string field theory [3] might provide a direct approach to study the physics of string theory tachyon and could give striking evidence for the tachyon condensation conjecture regarding the decay of unstable D-branes or the annihilation of brane anti-brane system [4]. Therefore it would be very interesting to study and develop the structure of the open string field theory itself.

On the other hand the most difficult part of the Sen’s conjecture for open string tachyon is the way the closed string emerges in the tachyonic vacuum. So it would be a natural question to ask that how one can see the closed string states in the open string field theory. In fact it has been shown that the off-shell closed strings arise because certain one-loop open string diagrams can be cut in a manner that produces a closed string pole [5]. Therefore unitarity implies that they should also appear as asymptotic states. Of course one can not remedy this by adding an explicit closed-string field to the theory. This would just double the residue of the pole. They can not be also considered as a bound states, since they appear in the perturbation theory. Closed string in open string field theory has been studied in several papers including [6, 7, 8, 9].

In another attempt but related to the closed string states in the open string field theory, the gauge invariant operators in open string field theory have been considered in [10, 11]. These gauge invariant operators could also provide us the on-shell closed string in the open string field theory. In fact these operators are parameterized by on-shell closed string vertex operators and can arise from an open/closed transition vertex that emerged in one-loop open string theory. Actually this open/closed vertex was studied in [9] where it was shown that supplemented with open string vertex it would generate a cover of the moduli spaces of surfaces involving open and closed string punctures.

It has also been suggested in [10, 11] that the correlation function of these gauge invariant operators could be interpreted as the on-shell scattering amplitude of the closed strings from D-brane. This is the aim of this article to study this correspondence in more detail. We shall study the scattering amplitude of two closed string states off a D-brane in the framework of the open string field theory by making use of these gauge invariant operators.

The paper is organized as follows. In section 2, we shall review the open string field theory action as well as the gauge invariant operators introduced in [10, 11]. In section 3 we will evaluate the scattering amplitude of the closed strings in the framework of string field theory. In section 4 the same scattering amplitudes will be obtained in the bosonic string theory where we will show that up to some contact terms, the results are in agreement with the open string field theory results. The section 5 is devoted to the discussion and some comments.
2 Open String Field Theory

In this section we shall review the open string field theory and the structure of the gauge invariant operators which could provide observables of the open string field theory.

2.1 Cubic string field theory action

The cubic open string field theory action is given by

$$S(\Psi) = -\frac{1}{2\alpha'} \left( \int \Psi^* Q \Psi + \frac{2g_o}{3} \Psi^* \Psi^* \Psi \right) ,$$

(1)

which is invariant under the gauge transformation

$$\delta \Psi = Q \Lambda + g_o \Psi^* \Lambda - g_o \Lambda^* \Psi .$$

Here $g_o$ is the open string coupling, $Q$ is the BRST charge and the string field, $\Psi$, is a ghost number one state in the Hilbert space of the first-quantized string theory which can be expanded using the Fock space basis as

$$|\Psi \rangle = \int d^{p+1} k \left( \phi + A_\mu(k) \alpha^\mu_{-1} + \frac{i}{\sqrt{2}} B_\mu(k) \alpha^\mu_{-2} + \frac{1}{\sqrt{2}} B_{\mu\nu}(k) \alpha^\mu_{-1} \alpha^\nu_{-1} \right) + \beta_0 b_{-2} c_0 + \beta_1 b_{-1} c_{-1} + ik_\mu \alpha^\mu_{-1} b_1 c_0 + \cdots |k\rangle .$$

The gauge invariance of (1) can be fixed can by choosing Feynman-Siegel gauge $b_0 |\Psi \rangle = 0$. In this gauge the truncated field up to level two reads

$$|\Psi \rangle = \int d^{p+1} k \left( \phi(k) + A_\mu(k) \alpha^\mu_{-1} + \frac{i}{\sqrt{2}} B_\mu(k) \alpha^\mu_{-2} + \frac{1}{\sqrt{2}} B_{\mu\nu}(k) \alpha^\mu_{-1} \alpha^\nu_{-1} \right) + \beta_0 b_{-2} c_0 + \beta_1 b_{-1} c_{-1} + ik_\mu \alpha^\mu_{-1} b_1 c_0 + \cdots |k\rangle .$$

The corresponding string vertex is given by

$$\Psi(0) = \int d^{p+1} k \left[ \phi(k)c(0) + iA_\mu(k)c\partial X^\mu(0) + \frac{i}{\sqrt{2}} B_\mu(k)c\partial^2 X^\mu(0) - \frac{1}{2} \beta_0 \partial c(0) + \frac{1}{\sqrt{2}} B_{\mu\nu}(k)c\partial X^\mu \partial X^\nu(0) - \frac{1}{2} \beta_1 \partial^2 c(0) \right] e^{ik \cdot X(0)}.$$ (2)

In writing the above vertex, we have used the doubling trick. Hence, the worldsheet field $X^\mu(z)$ in above equation is only holomorphic part of $X^\mu(z, \bar{z})$.

To make sense out of the abstract form of the open string field theory action, one can use CFT method. In this method we usually use the conformal mapping

$${}^3$$Here, we use the convention fixed in [12] that uses the V and N matrices for projecting a space-time field to its component in the world-volume and transverse spaces, respectively. So in this convention $\mu, \nu = 0, 1, 2, ..., 25$, and $A_\mu \alpha^\mu_{-1} = A \cdot V \cdot \alpha_{-1} + A \cdot N \cdot \alpha_{-1}$. Our conventions also set $\alpha' = 2$.}

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and calculation of the correlation function of a CFT on a disk or upper-half plane \[13, 14\]. In the CFT language the \( n \)-string vertex is defined by

\[
\int \Psi \Psi \cdots \Psi = \left\langle f_1^{(n)} \circ \Psi(0) f_2^{(n)} \circ \Psi(0) \cdots f_n^{(n)} \circ \Psi(0) \right\rangle_{UHP},
\]

where \( f_k^{(n)} \circ \Psi(0) \) denotes the conformal transformation of the vertex operator \( \Psi(0) \) by the conformal map \( f_k^{(n)} \). Here \( \langle \cdots \rangle_{UHP} \) denotes correlation function on the upper-half plane and the conformal map \( f_k^{(n)} \) is defined as

\[
f_k^{(n)}(z_k) = g \left( e^{\frac{2\pi i}{n} (k-1)} \left( \frac{1 + iz_k}{1 - iz_k} \right)^n \right), \quad 1 \leq k \leq n,
\]

where \( g(\zeta) = -i \frac{\zeta - 1}{\zeta + 1} \). Therefore the open string action can be calculated as following in terms of correlation functions of the CFT on the UHP

\[
S = -\frac{1}{4} \left\langle f_2^{(2)} \circ \Psi(0) f_1^{(2)} \circ (Q \Psi(0)) + \frac{2g_0}{3} f_1^{(3)} \circ \Psi(0) f_2^{(3)} \circ \Psi(0) f_3^{(3)} \circ \Psi(0) \right\rangle_{UHP}.
\]

Form this expression the kinetic terms up to level two fields read

\[
S_{\text{quad}} = \int d^{p+1}x \left( -\frac{1}{2} \partial_{\mu} \phi \partial^\mu \phi + \frac{1}{2} \phi^2 - \frac{1}{2} \partial_{\mu} A_{\nu} \partial^\mu A^{\nu} - \frac{1}{2} \partial_{\mu} B_{\nu} \partial^\mu B^{\nu} - \frac{1}{4} B_{\mu} B^{\mu} \right.
\]
\[
- \frac{1}{4} \partial_{\lambda} B_{\mu \nu} \partial^\lambda B^{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} + \frac{1}{2} \partial_{\mu} \beta_1 \partial^\mu \beta_1 + \frac{1}{4} \beta_1^2 \right)
\]

which can be used to write the space-time propagators of the corresponding fields.

### 2.2 Gauge invariant operator

The gauge invariant operators in string field theory have been constructed in \[10, 11\]. The general form of these operators are given by

\[
\mathcal{O} = g_c \int V \Psi,
\]

where \( g_c \) is the closed string coupling and \( V \) is an on-shell closed string vertex operator with dimension \((0,0)\). In order to be gauge invariant, the closed string vertex operator has to be inserted at the midpoint of open string. From open string point of view, \( V \) is an operator which acts on a string field. Given any on-shell closed string vertex operator \( \mathcal{V} \), the gauge invariant operator \( \mathcal{O} \) can be obtained, using the CFT method, in terms of the open string field

\[
\mathcal{O} = g_c \int V \Psi = g_c \left\langle c \mathcal{V}(i) \bar{c} \mathcal{V}(-i) f_1^{(1)} \circ \Psi(0) \right\rangle_{UHP},
\]

where \( f_1^{(1)} = \frac{2s}{1-s} \) and \( \mathcal{V}(z) \bar{\mathcal{V}}(\bar{z}) \) is the matter part of the closed string vertex operator.

\(^4\) We assume that there is a normal order sign between fields at different points in the correlation functions.
This form of the gauge invariant operator can be understood from the closed/open vertex studied in [14], where it is shown that the extended open string field theory with the action

\[ S = -\frac{1}{4} \left( \int \Psi \ast Q \Psi + \frac{2g_o}{3} \Psi \ast \Psi \ast \Psi \right) + g_c \int V \Psi, \tag{5} \]

with \( V \) being an on-shell closed string vertex defined at the midpoint of the open string, would provide a theory which covers the full moduli space of the scattering amplitudes of open and closed string with a boundary. We note, however, that scattering amplitudes of open and closed string with a boundary are actually the closed string scattering off a D-brane. We should then be able to reproduce the closed string scattering amplitudes in the framework of the open string field theory. In the next section we are going to write down the explicit form of the gauge invariant operator as well as their correlation function among themselves to see to what extent we can reproduce the known results of the closed string scattering amplitudes from a D-brane in the bosonic string theory [15, 16].

3 Scattering amplitudes in string field theory

In this section we will consider the gauge invariant operators in the string field theory. Using CFT method we shall compute the explicit form of the operators in terms of space-time open string fields. According to the proposed action (5) the result can be thought as an space-time action representing the closed/open vertex. We shall also compute the correlation function of these operators among themselves. These correlators should be interpreted as the closed string scattering amplitude of the closed string vertex. According to the proposed action (5), one needs the transformation of the vertex (4) under conformal map \( f_1^{(1)} \). Using the following propagator

\[ \langle X^\mu(z)X^\nu(w) \rangle = -\eta^\mu\nu \ln|z-w|, \tag{6} \]

one finds that the different terms in (4) transform under a general conformal map \( f \) as

\[
\begin{align*}
f \circ (\omega^{2ikX}) &= f^{2k^2-1} \omega^{2ikX}, \\
f \circ (c\partial X^\mu e^{2ikX}) &= f^{2k^2} (\partial X^\mu - i k^\mu f''/f^2) \omega^{2ikX}, \\
f \circ (c\partial^2 X^\mu e^{2ikX}) &= f^{2k^2+1} [\partial^2 X^\mu + (f''/f^2)\partial X^\mu] - ik^\mu/6 (4f''/f^2 - 3(f''/f^2)^2) e^{2ikX}, \\
f \circ (c\partial X^\mu \partial X^\nu e^{2ikX}) &= f^{2k^2+1} [\partial X^\mu \partial X^\nu - 2i(f''/f^2) k^{(\mu} \partial X^{\nu)} - (f''/f^2)^2 k^\mu k^\nu - 1/12 (2f''/f^2 - 3(f''/f^2)^2) \eta^\mu\nu] e^{2ikX}, \\
f \circ (\partial^2 ce^{2ikX}) &= f^{2k^2+1} [\partial^2 c - (f''/f^2)\partial c - (f''/f^2)^2 - 2(f''/f^2)^2] c e^{2ikX}.
\end{align*}
\]

Note that in the above equations the world-sheet fields on the right hand side are functions of \( f(z) \), also \( k^2 = k \cdot k \). The function \( f_1^{(1)} \) and its derivatives at point \( z = 0 \)
that should be inserted in the above transformations are: \( f_1^{(1)}(0) = 0, f_1^{(1)}(0) = 2, f_1^{(m)}(0) = 0, f_1^{(m)}(0) = 12 \). The correlation functions over the ghost field that left over are \( \langle c(i) \bar{c}(-i) c(0) \rangle = 2i \) and \( \langle c(i) \bar{c}(-i) \partial^2 c(0) \rangle = 4i \).

Plugging the conformal transformation \( (7) \) into the equation \( (4) \) and using above correlations for ghost part, we get

\[
\mathcal{O} = 2ig_c \int d^{p+1}k \ 2^{2k^2} \left[ \frac{1}{2} \phi + \sqrt{2} B \cdot V \cdot k \right] \langle \mathcal{V}(i) \bar{\mathcal{V}}(-i) e^{2ikX} \rangle \\
+ iA_\mu \left[ \mathcal{V}(i) \bar{\partial}X^\mu e^{2ikX} \right] + i\sqrt{2}B_\mu \left[ \mathcal{V}(i) \bar{\partial}^2 X^\mu e^{2ikX} \right] \\
- \sqrt{2}B_{\mu \nu} \left[ \mathcal{V}(i) \bar{\partial}X^\mu \bar{\partial}X^\nu e^{2ikX} \right],
\]

(8)

where all correlations should be evaluated on the upper-half plane. Now we have all ingredients we need to compute the open string field theory observable \( (1) \). We will do this for both closed string tachyon and massless states.

### 3.1 Tachyon amplitude

The matter part of the vertex operator of closed string tachyon inserted at the midpoint of open string with momentum \( p^\mu \ (p \cdot p = 2) \) is given by

\[
\mathcal{V}(i) \bar{\mathcal{V}}(-i) = e^{ip \cdot X(i)} e^{ip \cdot D \cdot X(-i)},
\]

where \( 2V = \eta + D \), and we have used the doubling trick\[12\]. Plugging this operator into equation \( (8) \) and using the standard propagator \( (3) \), one can evaluate the correlators in \( (8) \). The result is

\[
\mathcal{O}(p^\mu) = \frac{ig_c}{8} (2\pi)^{p+1} e^{4ln2 p^\mu} \left( \phi + 4i p \cdot N \cdot A + 2\sqrt{2} p \cdot V \cdot B \right. \\
\left. - 8\sqrt{2} p \cdot N \cdot B \cdot N \cdot p + \frac{1}{\sqrt{2}} B_\mu^\mu - \beta_1 \right),
\]

here the space-time fields are functions of \(-pV\). Fourier-transforming to the position space, \( e.g., \phi(k) = \int \frac{d^{p+1}x}{(2\pi)^{p+1}} \phi(x) \ e^{-ik \cdot x} \), the operator \( \mathcal{O} \) becomes

\[
\mathcal{O}(p^\mu) = \frac{ig_c}{8} \int d^{p+1}x \ e^{ip \cdot x} \left( \tilde{\phi}(x) + 4i p \cdot N \cdot \tilde{A}(x) + 2\sqrt{2} p \cdot V \cdot \tilde{B}(x) \right. \\
\left. - 8\sqrt{2} p \cdot N \cdot \tilde{B}(x) \cdot N \cdot p + \frac{1}{\sqrt{2}} \tilde{B}_\mu^\mu(x) - \tilde{\beta}_1(x) \right),
\]

(9)

where the tilde sign over fields means, \( e.g., \tilde{\phi}(x) = e^{-4ln2 \partial^2} \phi(x) \). According to the proposed action \( (8) \) the expression \( (9) \) is space time action representing the coupling of the closed string tachyon with the open string fields.

Having a gauge invariant operator one would proceed to compute the correlation function of this gauge invariant operator. Since this operator is supposed to be state
corresponding to the on-shell closed string, therefore the correlation function of this operator should give the scattering amplitude of the closed string off a D-brane. Now we are going to compute explicitly this correlation function to see if we can reproduce the corresponding amplitudes in the bosonic string theory. We shall do this up to level two truncation in open string field.

Consider the following two point function

\[ \langle O(p_1)O(p_2) \rangle, \]

where \[ \langle \rangle \] denotes the correlation function in the string field theory. According to what we have said, this should be interpreted as the \( S \)-matrix elements of two closed string states. In order to evaluate the above correlation, one needs the propagator of the space time open string fields which can be obtained from the kinetic term of the string field theory action in (3), i.e.,

\[
\begin{align*}
\langle \phi(x)\phi(y) \rangle &= -i \int dp^{p+1} k e^{ik(x-y)} \frac{k^2}{k^2 + \frac{1}{2}}, \\
\langle \beta_1(x)\beta_1(y) \rangle &= i \int dp^{p+1} k e^{ik(x-y)} \frac{k^2}{k^2 + \frac{1}{2}}, \\
\langle B^{\mu\nu}(x)B^{\lambda\rho}(y) \rangle &= -i \int dp^{p+1} k \left( \eta^{\mu\lambda} \eta^{\nu\rho} + \eta^{\mu\rho} \eta^{\nu\lambda} \right) e^{ik(x-y)} \frac{k^2}{k^2 + \frac{1}{2}}. \\
\end{align*}
\]

(11)

Now inserting (9) into (10) and using above propagators, one finds

\[
\begin{align*}
\langle O(p_1)O(p_2) \rangle &= \frac{ig_2^2}{2} \frac{\delta^{p+1}(p_1 + p_2)}{2s - 1} \left\{ \frac{e^{8(s - \frac{1}{2})\ln 2}}{2s} - \frac{e^{8\sin 2p_1 \cdot N \cdot p_2}}{2s} \right. \\
&\quad \left. + \frac{e^{8(s + \frac{1}{2})\ln 2}}{8(s + \frac{1}{2}) - 1} \left[ \frac{1}{2} (p_1 \cdot N \cdot p_2)^2 - \frac{1}{8} \right] \right\},
\end{align*}
\]

where \( s = p_1 \cdot V \cdot p_1 \). In the above expression, those terms in each pole which are proportional to the denominator give contact terms in which we are not interested. Hence, the pole structure of the amplitude is

\[
\begin{align*}
\langle O(p_1)O(p_2) \rangle &= \frac{ig_2^2}{2} \frac{\delta^{p+1}(p_1 + p_2)}{2s - 1} \left\{ \frac{1}{2s - 1} - \frac{p_1 \cdot N \cdot p_2}{2s} \\
&\quad + \frac{1}{2} (p_1 \cdot N \cdot p_2)^2 - \frac{1}{8} \right\},
\end{align*}
\]

(12)

where dots represent some contact terms. We shall show that the above poles exactly reproduce the s-channel poles of the corresponding amplitude in the bosonic string theory.

### 3.2 Graviton amplitude

As an other example, let us to consider the massless states scattering off a D-brane in the framework of the string field theory. To do this we need to find observables
corresponding to the on-shell massless closed string states which could be dilaton, graviton or Kalb-Ramond field. In other words we need to compute the operator (8) for the corresponding vertex operators. The matter part of these vertex operators inserted at the midpoint are given by

\[ V(i)\hat{V}(-i) = (\varepsilon \cdot D)_{\mu \nu} \partial X^\mu(i)e^{ipX(i)}\partial X^\nu(-i)e^{ipDX(-i)} \]

with \( p_\mu p^\mu = 0 = p^\mu \varepsilon_{\mu \nu} = \varepsilon_{\mu \nu} p^\nu \). For graviton we have \( \varepsilon_{\mu \nu} = \varepsilon_{\nu \mu} \) and \( \varepsilon^\mu_\mu = 0 \) [12].

Plugging above closed string vertex operator into the equation (8) and using the world-sheet propagator (1), one finds

\[ O(\varepsilon, p) = \frac{igc}{8}(2\pi)^{p+1}e^{4\ln 2 p^{\nu}p_\nu} \left( \phi a + 4iA^\mu b^\mu + 2\sqrt{2}B^\mu c^\mu - 8\sqrt{2}B_{\mu \nu}d^{\mu \nu} - \beta_1 a \right) , \]

here the space-time fields are functions of \(-p \cdot V\). The kinematic factors \( a, b^\mu, c^\mu \), and \( d^{\mu \nu} \) are

\[ a = \text{Tr}(\varepsilon \cdot D) - p \cdot D \cdot \varepsilon \cdot D \cdot p , \]
\[ b^\mu = a p \cdot N^\mu + p \cdot D \cdot \varepsilon \cdot D^\mu - \varepsilon^\mu \cdot D \cdot p , \]
\[ c^\mu = a p \cdot V^\mu - 4p \cdot D \cdot \varepsilon \cdot D^\mu - 4\varepsilon^\mu \cdot D \cdot p , \]
\[ d^{\mu \nu} = a(p \cdot N^\mu p \cdot N^\nu - \frac{1}{16} \eta^{\mu \nu}) + 2(\varepsilon \cdot D)_{\{\mu \nu\}} + 2p \cdot D \cdot \varepsilon \cdot D(|p \cdot N^\nu - 2\varepsilon(\mu \cdot D \cdot p \cdot p \cdot N^\nu)|). \]

Fourier-transforming the open string fields to the position space, we get

\[ O(\varepsilon, p) = \frac{igc}{8} \int d^{p+1}x (\tilde{\phi}(x)a + 4i\tilde{A}_\mu(x)b^\mu + 2\sqrt{2}\tilde{B}_\mu(x)c^\mu - 8\sqrt{2}\tilde{B}_{\mu \nu}(x)d^{\mu \nu} - \tilde{\beta}_1(x)a) , \]

where the tilded fields are defined the same as those in (1). Inserting above operator in (11) and using the space-time propagators (11), one can evaluate the scattering amplitude of two massless states from D-brane, that is

\[ \langle O(\varepsilon_1, p_1)O(\varepsilon_2, p_2) \rangle = \frac{igc}{2}(2\pi)^{p+1}\delta^{p+1}(p_1 + p_2)\left\{ \frac{e^{8(s-\frac{1}{2})\ln 2}a_1a_2}{2s-1} - \frac{e^{8\ln 2}b_1b_2}{2s} \right. \]
\[ + \frac{e^{8(s+\frac{1}{2})\ln 2}c_1c_2 + \frac{1}{2}\text{Tr}(d_1 \cdot d_2) - \frac{1}{256}a_1a_2}{2s+1} \}
\[ = \frac{igc}{2}(2\pi)^{p+1}\delta^{p+1}(p_1 + p_2)\left\{ \frac{a_1a_2}{2s-1} - \frac{b_1b_2}{2s} \right. \]
\[ + \frac{c_1c_2 + \frac{1}{2}\text{Tr}(d_1 \cdot d_2) - \frac{1}{256}a_1a_2}{2s+1} + \ldots \right\} , \quad (13) \]

where dots represent some contact terms.

So far we have obtained the \( S \)-matrix elements of two closed string states in the framework of the open string field theory. Although we have only been able to find the \( s \)-channel poles using level truncated open string field, we will see that the \( t \)-channel can be also obtained by taking into account the infinite terms coming from the all level in the open string field. We will back to this point later in the conclusion. Now we are going to compare these results with the string theory computation.


## 4 Scattering amplitudes in string theory

In this section we study the scattering amplitude of two closed string states from D-brane in the bosonic string theory.

### 4.1 Tachyon amplitude

Scattering amplitude of two closed string tachyons from D-brane in the bosonic string theory is given by the following correlation function:

\[ A \sim \int d^2 z_1 d^2 z_2 \langle e^{ip_1 X(z_1)} e^{ip_1 DX(z_1)} e^{ip_2 X(z_2)} e^{ip_2 DX(z_2)} \rangle . \]

Using the propagator (6) it is straightforward calculations to evaluate the above correlation and show that the resulting integrand is \( SL(2, R) \) invariant. Fixing this symmetry by choosing \( z_1 = iy \) and \( z_2 = i \), one arrives at

\[ A = \frac{ig^2}{2} (2\pi)^{p+1} \delta^{p+1}(p_1 + p_2) B(-1 - t/2, -1 + 2s) , \quad (14) \]

where \( t = -(p_1 + p_2)^2 \) and \( s = p_1 \cdot V \cdot p_1 \).

Consider the following definition of the beta function:

\[ B(\alpha, \beta) = \sum_{n=0}^{\infty} \frac{1}{\alpha + n} \frac{(-1)^n}{n!} (\beta - 1) \cdots (\beta - n) . \quad (15) \]

From this expression we see that the beta function has simple poles at \( \alpha, \beta = 0, -1, -2, -3, \cdots \). Each term of the above expansion has one simple pole in the \( \alpha \)-channel, however, the poles in the \( \beta \)-channel appear when adding all infinite poles of the \( \alpha \)-channel in the above expansion.

By making use of this expression for the beta function the amplitude (14) reads

\[ A = \frac{ig^2}{2} (2\pi)^{p+1} \delta^{p+1}(p_1 + p_2) \left\{ \frac{1}{2s - 1} - \frac{(-2 - t/2)}{2s} \right. \\
\left. + \frac{1}{2s} (-2 - t/2)(-3 - t/2) \right\} , \]

Comparing it with the corresponding amplitude in the open string field theory (12), one finds that the tachonic pole is exactly the same. For the massless pole we write \(-2 - t/2 = p_1 \cdot N \cdot p_2 - s\). However, the term proportional to \( s \) in the massless pole gives contact term. So we get, up to some contact terms, exact agreement in two cases. Similarly, for the first massive pole one may rewrite it as

\[ \frac{1}{2} (-2 - \frac{t}{2})(-3 - \frac{t}{2}) = \frac{1}{2}(p_1 \cdot N \cdot p_2)^2 - (p_1 \cdot N \cdot p_2)(s + \frac{1}{2}) + \frac{1}{2}(s + \frac{1}{2})^2 - \frac{1}{8} . \]

Again terms that are proportional to \((2s + 1)\) give contact term. After dropping these terms, one finds exactly the massive pole as (12).
4.2 Graviton amplitude

The scattering amplitude of two massless closed string states from D-brane in the bosonic string theory is evaluated in [16]. The result is

\[ A = \frac{i g_s^2}{4} (2\pi)^{p+1} \delta^{p+1}(p_1 + p_2)\{e_1 B(-t/2, 1 + 2s) + e_2 B(-t/2, 2s) - e_3 B(1 - t/2, 1 + 2s) + e_4 B(1 - t/2, 1 + 2s) + e_5 B(-1 - t/2, 1 + 2s) + e_6 B(1 - t/2, -1 + 2s) + e_7 B(-1 - t/2, -1 + 2s) - e_8 B(-t/2, -1 + 2s) - e_9 B(2 - t/2, -1 + 2s) + e_{10} B(3 - t/2, -1 + 2s)\} \right\), \tag{16}

where the kinematic factors \(e_1, \ldots, e_{10}\) are\(^5\)

\[ e_1 = \frac{1}{2} p_2 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_1 + \frac{1}{2} p_2 \cdot \varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_1 + p_2 \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot p_2 + p_2 \cdot \varepsilon_2 \cdot D \cdot p_2 + (1 \leftrightarrow 2), \]
\[ e_2 = -\text{Tr}(\varepsilon_1 \cdot D) p_1 \cdot \varepsilon_2 \cdot p_1 + (1 \leftrightarrow 2), \]
\[ e_3 = -p_1 \cdot D \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 + p_2 \cdot \varepsilon_1 \cdot \varepsilon_2 \cdot D \cdot p_2 + (1 \leftrightarrow 2), \]
\[ e_4 = \text{Tr}(\varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D) - p_1 \cdot D \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 - p_2 \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_1, \]
\[ e_5 = \text{Tr}(\varepsilon_1 \cdot \varepsilon_2 \cdot T) - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_2 - p_2 \cdot \varepsilon_1 \cdot \varepsilon_2 \cdot p_1, \]
\[ e_6 = \frac{1}{2} \text{Tr}(\varepsilon_1 \cdot D) \text{Tr}(\varepsilon_2 \cdot D) - \text{Tr}(\varepsilon_1 \cdot D) p_2 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 + (p_2 \cdot \varepsilon_1 \cdot p_1) (p_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_1) + \frac{1}{2} (p_2 \cdot \varepsilon_1 \cdot D \cdot p_1) (p_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_1) + (1 \leftrightarrow 2), \]
\[ e_7 = (p_1 \cdot \varepsilon_2 \cdot p_1)(p_2 \cdot \varepsilon_1 \cdot p_2), \]
\[ e_8 = -(p_2 \cdot \varepsilon_1 \cdot p_2)(p_1 \cdot \varepsilon_2 \cdot D \cdot p_1) + p_1 \cdot D \cdot \varepsilon_2 \cdot p_1 + (1 \leftrightarrow 2), \]
\[ e_9 = -(p_2 \cdot D \cdot \varepsilon_1 \cdot D \cdot p_2)(p_1 \cdot \varepsilon_2 \cdot D \cdot p_1) + p_1 \cdot D \cdot \varepsilon_2 \cdot p_1 + (1 \leftrightarrow 2), \]
\[ e_{10} = (p_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_1)(p_2 \cdot D \cdot \varepsilon_1 \cdot D \cdot p_2). \]

Since in [16] the authors were interested in the low energy contact terms of the amplitude from which the low energy effective action can be found, the above amplitude was expanded in a limit in which \(s, t \to 0\). On the other hand, since in our point of view we are not looking for the low energy expansion, instead, we expand the amplitude in the s-channel using the expansion [15] for the beta functions, that is

\[ A = \frac{i g_s^2}{4} (2\pi)^{p+1} \delta^{p+1}(p_1 + p_2)\left\{ \frac{1}{2s - 1} (e_6 + e_7 - e_8 - e_9 + e_{10}) + \frac{1}{2s} [e_2 - e_3 + 2e_7 - e_8 + e_9 - 2e_{10} + \frac{t}{2} (e_6 + e_7 - e_8 - e_9 + e_{10})] \right\} \]

\(^5\)Note that our convention for matrix \(D,V,N\) is different from those in [16], i.e., \(D_{\text{here}} = -V_{\text{there}}, N_{\text{here}} = D_{\text{there}}, V_{\text{here}} = N_{\text{there}}.\)
\[
+ \frac{1}{2s+1} [e_1 + e_2 + e_4 + e_5 + 3e_7 - e_8 + e_{10} + \frac{t^2}{8}(e_6 + e_7 - e_8 - e_9 + e_{10}) \\
+ \frac{1}{2}(e_2 - e_3 + \frac{1}{2}e_6 + 5\frac{1}{2}e_7 - 3\frac{3}{2}e_8 + \frac{1}{2}e_9 - 3\frac{3}{2}e_{10})] + \cdots,
\]

where dots represent the \(s\)-channel poles for higher massive modes. Using the conservation of momentum on the world-volume of the D-brane, we recognize that the above \(s\)-channel poles, up to some contact terms, are exactly identical to the poles appear in the string field theory amplitude (13). This ends our illustration of the equality of the scattering amplitude of two closed string states from D-brane in the open string field theory and bosonic string theory.

5 Conclusion

In this paper we studied the closed string scattering amplitude off a D-brane in the framework of the open string field theory. Using the CFT method we have been able to show, explicitly, that the gauge invariant observable in open string field theory introduced in [10, 11] reproduces the correct pole structure of the scattering amplitude in the bosonic string theory. We have checked this in the level truncation method up to level two. We note, however, that although the \(s\)-channel poles can be directly obtained from the level truncation computation, to see the \(t\)-channel poles we need to evaluate the scattering amplitude to all level in the open string field. This can be seen from the form of the beta function (15). In fact each term in this expansion has a pole in \(s\), but to see a pole in \(t\) we need to take into account infinite terms in the summation. In principle one can compute the scattering amplitude to all level and then resume the expansion finding the exact expression for the \(S\)-matrix as (16) up to some contact terms. Therefore we conclude that the scattering amplitude of the closed string states in the open string field theory framework is the same as one in the bosonic string theory framework up to some contact terms. This contact terms might be related to a field redefinition.

As an other example of the scattering amplitude one can also compute the interaction of one closed string and two open string fields. In the open string field theory, this corresponds to \(\langle O \Psi \Psi \Psi \rangle\). The result should be the same as the corresponding amplitude in the bosonic string theory up to some contact terms. We left the detail computation of this case for the future work.

Probably more interesting problem is to find the gauge invariant operators in the open superstring field theory. Although the superstring field theory action is not as simple as the bosonic one, but since in the gauge invariant operator only the string field and on-shell closed string are involved one might suspect that the gauge invariant operator has the same form as the bosonic one. If this were that case one would proceed to compute the scattering amplitude in the superstring case.

Finally we note that there are two approaches in string field theory: (1) Open string field theory which we have used throughout this paper and (2) boundary
string field theory originally introduced in [17, 18] (for recent discussions see [19]). It is believed that these two string field theories are equivalent. More precisely there should be a field redefinition, though singular, which maps open string field theory to the boundary string field theory. Therefore an interesting question one might ask is what is the gauge invariant operators in the boundary string field theory? This gauge invariant operator would have the same role as one in the open string field theory, namely it should correspond to the on-shell closed string states [1].

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