Transverse Energy per Charged Particle and Freeze-Out Criteria in Heavy-Ion Collisions

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Abstract

In relativistic nucleus-nucleus collisions the transverse energy per charged particle, \( E_{T}/N_{\text{ch}} \), increases rapidly with beam energy and remains approximately constant at about 800 MeV for beam energies from SPS to RHIC. It is shown that the hadron resonance gas model describes the energy dependence, as well as the lack of centrality dependence, qualitatively. The values of \( E_{T}/N_{\text{ch}} \) are related to the chemical freeze-out criterium \( E/N \approx 1 \text{ GeV} \) valid for primordial hadrons.

1 Introduction

In this paper we investigate the transverse energy per charged particle, \( E_{T}/N_{\text{ch}} \), for beam energies ranging from about 1 AGeV up to 200 AGeV. In this energy range, \( E_{T}/N_{\text{ch}} \) at first increases rapidly from SIS [1] to AGS [2,3], then saturates to a value of about 800 MeV at SPS [4–6] energies and remains constant up to the highest available RHIC energies [7–9]. The present analysis of \( E_{T}/N_{\text{ch}} \) uses the hadron resonance gas model (thermal model) which describes the final state in relativistic heavy-ion collisions as composed of hadrons, including heavy hadronic resonances as being in thermal and chemical equilibrium. It has been known for many years [10] that the chemical freeze-out can be described by the condition \( E/N \approx 1 \text{ GeV} \), where \( E \) and \( N \) are, respectively the total energy and particle number of the primordial hadronic resonances before they decay into stable hadrons. This quantity cannot be determined directly from experiment unless the final state multiplicity is low and hadronic resonances can be identified, which is not the case in relativistic heavy-ion collisions. Our analysis therefore starts by relating the number of charged particles seen in the detector to the number of primordial hadronic resonances and the transverse energy to the energy \( E \) of primordial hadrons. In this paper all thermal model calculations were performed using the THERMUS package [12].
The transverse energy, $E_T$, is defined as the energy deposited transverse to the beam direction in a given interval of pseudo-rapidity $\eta$. The transverse energy has two components, the hadronic one, $E_{\text{had}}^T$, and the electromagnetic one, $E_{\text{em}}^T$, coming from the electromagnetic particles (photons, electrons and positrons). Electromagnetic calorimeters are used to measure $E_{\text{em}}^T$ whereas hadronic calorimeters or the Time Projection Chamber (for particle identification and momentum information) are used to measure $E_{\text{had}}^T$. The energy of a particle is defined as being the kinetic energy for nucleons, for anti-nucleons as the total energy plus the rest mass and for all other particles as the total energy [7,8,14].

In the experiments the transverse energy and the charged particle multiplicity are measured in a similar way so that most of the systematic uncertainties cancel out in the ratio. Experiments have reported a constant value of the ratio $E_T/N_{\text{ch}} \sim 0.8$ GeV from SPS to RHIC [7,9], with the ratio being almost independent of centrality of the collision for all measurements at different energies. In all cases the value of $E_T/N_{\text{ch}}$ has been taken for the most central collisions, at the end of this paper we consider the centrality dependence of $E_T/N_{\text{ch}}$. When this ratio is observed for the full range of center of mass energies, it shows two regions [9]. In the first region from lowest $\sqrt{s_{NN}}$ to SPS energy, there is a steep increase of the $E_T/N_{\text{ch}}$ ratio with $\sqrt{s_{NN}}$. In this regime, the increase of $\sqrt{s_{NN}}$ causes an increase in the $\langle m_T \rangle$ of the produced particles. In the second region, SPS to higher energies, the $E_T/N_{\text{ch}}$ ratio is very weakly dependent on $\sqrt{s_{NN}}$.

The energy pumped into the system by the increase of $\sqrt{s_{NN}}$ is converted mainly into particle production. This observation is quite remarkable and requires the help of models for a better understanding of the underlying physics.

To estimate $E_T/N_{\text{ch}}$ in the thermal model we relate the number of charged particles, $N_{\text{ch}}$, to the number, $N$, of primordial hadrons. To estimate the charged particle multiplicity at different center of mass energies from the thermal model, we proceed as follows. First we study the variation of the ratio of the total particle multiplicity in the final state, $N_{\text{decays}}$, and that in the primordial i.e. $N_{\text{decays}}/N$ with $\sqrt{s_{NN}}$. This ratio starts from one, since there are only very few resonances produced at low beam energy and becomes almost independent of energy after SPS energy. The value of $N_{\text{decays}}/N$ in the region where it is independent of $\sqrt{s_{NN}}$ is around 1.7. The excitation function of $N_{\text{decays}}/N$ is shown in Fig. 1(a). Secondly, we have studied the variation of the ratio of charge particle multiplicity and the particle multiplicity in the final state ($N_{\text{ch}}/N_{\text{decays}}$) with $\sqrt{s_{NN}}$. This is shown in Fig. 1(b). The $N_{\text{ch}}/N_{\text{decays}}$ ratio starts around 0.4 at lower $\sqrt{s_{NN}}$ and shows an energy independence at SPS and higher energies. At lower SIS energy, the baryon dominance at mid-rapidity makes $N_{\text{ch}}/N_{\text{decays}} \sim N_{\text{proton}}/N_{(\text{proton}+\text{neutron})}$ which has a value of 0.45 for Au-Au collisions.

As the next step we connect the transverse energy $E_T$ to the the energy of the primordial hadrons $E$. We start by relating the two quantities for a static fireball. In this case one has

$$\langle E \sin \theta \rangle = V \int \frac{d^3p}{(2\pi)^3} E \sin \theta \ f(E)$$

(1)
where \( f(E) \) is the statistical distribution factor, e.g. for a Boltzmann distribution it is given by \( f(E) = \exp\left(-\frac{E}{T}\right) \). It is straightforward to re-write this expression as

\[
\langle E \sin \theta \rangle = V \frac{\pi}{4} \int \frac{d^3p}{(2\pi)^3} E f(E)
\]

\[
= \frac{\pi}{4} \langle E \rangle
\]  

(2)

Thus, for a static fireball, the transverse energy is related to the total energy by a simple factor of \( \pi/4 \). In the hadronic resonance gas model there is a sum over all hadrons; furthermore, taking into account the experimental configuration which leads to adding the mass of the nucleon for anti-nucleons and subtracting the same for nucleons one has

\[
\langle E_T \rangle \equiv V \sum_{i=Nucleons} \int \frac{d^3p}{(2\pi)^3} (E_i - m_N) \sin \theta \ f(E_i)
\]

\[
+ V \sum_{i=Anti-nucleons} \int \frac{d^3p}{(2\pi)^3} (E_i + m_N) \sin \theta \ f(E_i)
\]

\[
+ V \sum_{i=All \ Others} \int \frac{d^3p}{(2\pi)^3} E_i \sin \theta \ f(E_i),
\]

\[
= \frac{\pi}{4} \left( \langle E \rangle - m_N \langle N_B - N_{\bar{B}} \rangle \right).
\]  

(3)
The above equation relates the transverse energy measured from the data and that estimated from the thermal model. In the limit of large beam energies one has

\[
\lim_{\sqrt{s_{NN}} \to \infty} \frac{\langle E_T \rangle}{N_{ch}} = \frac{\langle E_T \rangle}{0.6 N_{\text{decay}}} = \frac{\pi}{4} \frac{E}{0.6 + 1.7N} \approx 0.83,
\]

which is close to the value measured at RHIC. The measured \( E_T \) will be affected by the radial flow and by the difference between chemical freeze-out and kinetic freeze-out temperatures; these effects will lead to corrections which tend to largely cancel each other. A detailed comparison of this, in the framework of a single freeze-out temperature model and limited to RHIC energies, has been made in Ref. [13].

In Fig. 2 we plot lines of constant \( E_T/N_{ch} \) in the \((T, \mu_B)\)-diagram. For low values of \( E_T/N_{ch} \), these lines are almost independent of \( \mu_B \), this is mainly a consequence of subtracting \( m_N \) in the definition of \( E_T \), thus taking away much of the influence of nucleons. Only towards larger values of \( \mu_B \) there is a notable dependence on this variable. To compare with the chemical freeze-out condition, we show also the chemical freeze-out curve in the same plane (Fig. 2). At higher energies, when \( \mu_B \) nearly goes to zero, the transverse energy production is mainly due to the meson content in the matter. The intersection points of lines of constant \( E_T/N_{ch} \) and the freeze-out line give the values of \( E_T/N_{ch} \) at the chemical freeze-out. Hence at freeze-out, given the values of \( E_T/N_{ch} \) from the experimental measurements we can determine \( T \) and \( \mu_B \) of the system. In Fig. 3, we plot the ratio \( E_T/N_{ch} \) as a function of the temperature \( T \) and as a function of \( \mu_B \). It can be seen that the relation between the \( E_T/N_{ch} \) and \( T \) is linear to a good approximation, similarly for the relation with \( \mu_B \).

For the most central collisions, the variation of \( E_T/N_{ch} \) with center of mass energy is shown in Fig. 4. The data have been taken from Ref. [1–9], and are compared with the corresponding calculation from the thermal model with chemical freeze-out. We have checked explicitly that other freeze-out criteria discussed in the literature give almost identical results for the behavior of \( E_T/N_{ch} \) as a function of \( \sqrt{s_{NN}} \); this is the case for the fixed baryon plus anti-baryon density condition [15] and also for fixed normalised entropy density condition, \( s/T^3 = 7 \) [16–18]. It has been observed from SPS to RHIC [7,9], that the ratio \( E_T/N_{ch} \), is almost independent of the centrality of the collisions which is represented by the number of participant nucleons. To understand the variation of \( E_T/N_{ch} \) with collision centrality, we have estimated \( E_T/N_{ch} \) for 130 GeV Au+Au collisions at RHIC, for different centrality classes [19]. This is compared with the corresponding data in Fig. 5. The effect of flow is not taken into account in the model calculations for the centrality behavior. The model agrees well with the experimental data for the centrality behavior. Again, we have checked explicitly that other freeze-out criteria lead to similar results [15–18].
Fig. 2. Lines of constant $E_T/N_{ch}$ from thermal model without flow are shown in the $(T, \mu_B)$ plane. The chemical freeze-out condition of $E/N = 1.08$ GeV is also shown.

Fig. 3. The variation of $E_T/N_{ch}$ with $T$ (left) and the variation of $E_T/N_{ch}$ with $\mu_B$ (right).

3 Summary

In conclusion, we have discussed the connection between $E_T/N_{ch}$ and the ratio of primordial energy to primordial particle multiplicity, $E/N$, from the thermal model. This model, when
combined with chemical freeze-out criteria explains the data over all available measurements for the $\sqrt{s_{NN}}$ behavior of $E_T/N_{ch}$. It has to be noted that variables like $E_T/N_{ch}$, the chemical freeze-out temperature $T_{ch}$, $N_{decays}/N_{primordial}$ and $N_{ch}/N_{decays}$ discussed in this paper, show saturation starting at SPS and continuing to higher center of mass energies. This observation along with the centrality independence of $E_T/N_{ch}$ is not inconsistent with the simultaneity of chemical and kinetic freeze-out at higher energies [20].

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Fig. 5. The variation of $E_T/N_{ch}$ with $N_{part}$ for 130 GeV Au+Au collisions at RHIC with the corresponding thermal model estimate.

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