Variation Characteristics and Complexity Measurement of Urban Precipitation Based on Multi-scale Permutation Entropy

Dong Liu*, Tingqi Yan, Sicheng Liu and Ge Yan
School of Water Conservancy & Civil Engineering, Northeast Agricultural University, Harbin, Heilongjiang 150030, China

*Corresponding author email: liudong@neau.edu.cn

Abstract. In order to better judge the complexity of precipitation, this paper took some major cities of Heilongjiang Province as the research object, and used the Mann-Kendall test and multi-scale permutation entropy theory to study the complexity of precipitation. The results show that there are many sudden change points of precipitation in cities in Heilongjiang Province, which have a strong complexity, which can provide a basis for the study of precipitation complexity. Among urban precipitation, the complexity of precipitation in Hegang City is the strongest, and the complexity of precipitation in Daxinganling is the weakest. Among them, water resources and living area have a significant impact on the precipitation complexity. This study can provide a new method for studying hydrological changes and complexity intensity in the future.

Keywords: Rainfall change; Mann-Kendall test; Multi-scale permutation entropy; Complexity measure.

1. Introduction
In recent years, global warming has intensified and climate change has intensified significantly[1]. Precipitation, as one of the important signs of climate warming, has gradually become an indispensable part of the hydrological research field[2]. Changes in precipitation constantly affect human production and life, and human activities also affect the formation of precipitation. The increase in urban population, the construction industry, and the rapid development of industry and agriculture, urbanization has intensified[3], resulting in an increase in urban temperature. It also causes air convection inside and outside the city, exacerbating the complexity of precipitation. Therefore, it is imperative to study the changes and complexity of urban precipitation.

In order to explore the law of precipitation change, Mann-Kendall test is selected to detect the precipitation change preliminarily[4], the method has strong interference ability, simple calculation method, and can be calculated under the condition of irregular sequence, which is suitable for discovering the changing law of precipitation. Precipitation has many influences due to its uncertainty and factors[5], so it is not suitable to find its changing mechanism, and it often shows the characteristics of complexity change. In the 1980s, complexity science began to rise, which triggered a strong discussion in all walks of life in the world and was increasingly applied to all levels of Natural Science[6]. In the early 1990s, Professor Qian proposed to explore complexity methodology[7]. In 1999, Professor Cheng proposed that complex science is a science which mainly research complexity and system complexity[8]. Nowadays, complexity science has penetrated into various fields of research, such as fractal theory[9], chaos theory[10], are widely used in complexity analysis. However, the above
methods have the disadvantages of low accuracy and long calculation time. The multi-scale permutation entropy algorithm is selected to study the complexity\cite{11}, which improves the accuracy while making up for the defects such as the influence of noise and the inability to distinguish the characteristics fault features. At the same time, precipitation, as a common natural weather phenomenon, is scale-free\cite{12}, long-term precipitation data can be regarded as scale-free network (SF). The typical SF is more fault-tolerant when it loses mild data in a uniform way, and is significantly higher than the nonuniform way\cite{13}. The coarse-grained algorithm is used in the calculation of multi-scale permutation entropy, when the scale increases, the data distribution tends to be uniform. When the scale increases, the data distribution tends to be uniform, which makes SF lose the data as much as possible when it is subjected by nonuniform attack, and improves the fault tolerance of the algorithm, so that the robustness of the algorithm is enhanced. Therefore, Mann-Kendall test and multi-scale permutation entropy algorithm are used to analyze the characteristics and complexity of precipitation change in Heilongjiang Province, providing reference for the research of Hydrometeorology and the protection and utilization of water resources in Heilongjiang Province.

2. Materials and Methodology

2.1. Study Area

The Heilongjiang Province is located in the northeast of China, mainly including 13 cities (the specific location is shown in Figure 1). The climatic characteristics of Heilongjiang Province mainly show two climates from north to south: temperate and cold temperate monsoon climate. The climate is characterized by clear four seasons, large temperature differences throughout the year, and uneven distribution of precipitation, which leads to the phenomenon of drought. As the country's major forestry provinces and agricultural provinces, the development of agriculture and forestry will rely too much on precipitation and the use of water resources, making the complexity of precipitation in Heilongjiang Province even more significant. Therefore, taking Heilongjiang Province as the research area and analyzing the complexity of precipitation in cities in Heilongjiang Province can be of great significance to the urban construction, people's production and life, and economic development of Heilongjiang Province.

![Figure 1. Urban Spatial Distribution of Heilongjiang Province](image)
2.2. Data Sources
This research data is from monthly rainfall data of 13 major cities (Harbin, Qiqihar, Jixi, Hegang, Shuangyashan, Daqing, Jiamusi, Mudanjiang, Suihua, Qitaihe, Daxinganling, Yichun, Heihe) provided by Heilongjiang Meteorological Bureau from 1967 to 2016 (within 50 years). The precipitation data conforms to the characteristics of the SF network, and its complexity can be analyzed.

2.3. Mann-Kendall Test
This study takes 13 major cities in Heilongjiang as an example, and uses the Mann-Kendall test to analyze precipitation trends based on known rainfall data from 1967 to 2016. This method has no specific requirements for the distribution of data, and when there is a certain amount of data missing, it will ensure that the results are correct[14].

In the Mann-Kendall test, select the random time sequence $H_0(x_1, x_2, ..., x_n)$, where $n$ is the number of samples, and $x$ is the independent and random data; assuming $H_i$ is a two-sided test, that is, $H_i$ has a trend. The statistic $S$ is calculated as follows:

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} Sgn(x_j - x_k)$$

$$Sgn(\phi) = \begin{cases} +1 & (\phi) > 0 \\ 0 & (\phi) = 0 \\ -1 & (\phi) < 0 \end{cases}$$

$S$ follows normal distribution making the mean value of 0. The variance is calculated as follows:

$$Var(S) = n(n-1)(2n+5)/18$$

When $n > 10$, Standardize Eq.1 to:

$$Z = \begin{cases} \frac{S - 1}{\sqrt{Var(S)}} & S > 0 \\ 0 & S = 0 \\ \frac{S + 1}{\sqrt{Var(S)}} & S < 0 \end{cases}$$

Thus, in the bilateral trend test, when $|Z| \geq Z_{1-\alpha/2}$, it is proved that there is an obvious change trend in the time series, in which $\alpha$ is the significance level. When $Z < 0$, the overall data shows a downward trend, and vice versa. Among them, when $|Z| \geq 1.28$, it means that it has passed the significance test.

When further mutation tests are performed, the test statistics are different from the above $Z$, by constructing an order column:

$$S_k = \sum_{i=1}^{k} \sum_{j=1}^{i-1} \alpha_{ij} \quad (k=2,3,4...,n)$$

$$\alpha_{ij} = \begin{cases} 1 & (x_j > x_i) \\ 0 & (x_j \leq x_i) \quad (1 \leq j \leq i) \end{cases}$$

Define statistical variables:
\[ UF_k = \frac{S_k - E(S_k)}{\sqrt{Var(S_k)}} \quad (k=1,2,...,n) \] (7)

\[ E(S_k) = k(K + 1)/4 \] (8)

\[ Var(S_k) = k(k - 1)(2k + 5)/72 \] (9)

\( UF_k \) is the standard normal distribution, if |\( UF_k \)| > \( U_{a/2} \), it indicates that the sequence has obvious trend changes. Reverse time sequence \( x \) and calculate it according to the above formula. Let the equation satisfy the next condition:

\[
\begin{align*}
UB_k &= -UF_k \\
k &= n + 1 - k \\
(k=1,2,...,n)
\end{align*}
\] (10)

By analyzing the statistical sequences \( UF_k \) and \( UB_k \) and drawing the curve, the mutation time of sequence \( x \) can be judged, and the mutation time domain can be clarified. If \( UF_k > 0 \), it indicates that the sequence is upward; \( UF_k < 0 \), it indicates that it is downward; When the critical line is exceeded, the trend changes significantly. If the intersection point of the two curves is between \( |\alpha| \), then this point is the beginning of the mutation.

2.4. Multi-scale Permutation Entropy (MSPE)

MSPE is based on permutation entropy algorithm and multi-scale coarse-grained improvement to perform complexity analysis\(^{[15]}\). Assume that a time sequence of length \( L \) is \( X_L = \{x_1, x_2, ..., x_L\} \), and the scale factor \( s \) (\( s = 1, 2, ... \)). Coarse graining process is as follows:

\[
y^s_j(i) = \frac{1}{s} \sum_{i = (j-1)s+1}^{j} x(i), 1 \leq j \leq L / s
\] (11)

\( y^s_j \) is a multi-scale time sequence. \( L/S \) is the length of the time sequence after coarse granulation. When \( s = 1 \), the time sequence is the original sequence, and the calculated entropy is the calculated permutation entropy of the time sequence.

(1) The time reconstruction \( y^s_{j} \) is obtained (\( m \) is the embedding dimension and \( \tau \) is the delay time):

\[
Y^s_t = \{y^s_{t+\tau}, y^s_{t+2\tau}, ..., y^s_{t+(m-1)\tau}\}
\] (12)

(2) Sort the time series \( y^s \), in ascending order:

\[
y^s_{t+(j_1-1)\tau} \leq y^s_{t+(j_2-1)\tau} \leq \cdots \leq y^s_{t+(j_m-1)\tau}
\] (13)

When present \( x_{t+\tau r} = x_{t+\tau r} \), sort by \( r \), if \( r_j < r_k \), so \( x_{t+\tau r_j} < x_{t+\tau r_k} \). The sequence has \( m! \) Permutations possible.

(3) Count the number of occurrences \( N_i \) of each permutation type, and calculate the probability of its corresponding permutation:

\[
P^s_i = \frac{N_i}{L / s - m + 1}
\] (14)

(4) The multi-scale permutation entropy of this sequence is defined as:

\[
H^s_p = -\sum_{i=1}^{m!} P^s_i \ln P^s_i
\] (15)
At $P^s = 1/m!$, $H^s_f$ reached the maximum value $\ln (m!)$, normalize the permutation entropy of the time series at scale $s$ to obtain the normalized multi-scale permutation entropy:

$$h^s_f = H^s_f / \ln(m!)$$  \hspace{1cm} (16)

According to the above definitions, the permutation entropy calculation is greatly affected by the embedding dimension $m$ and the delay time $\tau$: too large or too small will affect the calculation time; too large $\tau$ will make the reconstruction vector between adjacent delay coordinates. Correlation is too small, $\tau$ is too small, it is easy to cause information loss, so generally $\tau<2$; the larger the embedded dimension $m$, the more detailed the calculated information will be, but too large $m$ will make the calculated data length too long \cite{16}. Generally, $2$ to $3$ are selected. According to the above rules, $m = 3$ and $\tau = 1$ are selected in this paper. In general, when $s > 10$, the sequence dynamics changes will appear. Under the condition that the calculation process is not influenced by the length of the coarse-grained sequence, the maximum value of the scale factor is usually It is about $12$, so we choose $s = 1 \sim 12$ in this paper.

3. Result

3.1. Mann-Kendall Mutation Test to Analyze Precipitation Trends

According to the Mann-Kendall test based on the average precipitation data of major cities from 1967 to 2016, the following results were obtained, the precipitation of the 13 major cities showed a decreasing trend with varying degrees of time ($Z < 0$), and the precipitation data for most cities passed the significance test of confidence, as shown in Table 1.

|             | Harbin | Mudanjiang | Qiqihar | Daqing | Jixi  | Hegang | Yichun |
|-------------|--------|------------|---------|--------|-------|--------|--------|
| $Z$         | -1.0361| -2.0106    | -1.4287 | -0.8399| -1.8348| -1.2920| -1.7945|
|             | Shuangyashan | Jiamusi | Suihua | Qitaihe | Daxinganling | Heihe |
| $Z$         | -1.2664| -2.3938    | -1.2179 | -1.1091| -0.9635| -1.0034|

After obtaining the precipitation trends of 13 major cities in Heilongjiang Province, further catastrophe tests were conducted: Because the false set letter level is 0.05, the critical value can be obtained by querying the normal distribution table $U_{0.05/2} = 1.96$, when $|U_{F_k}| < U_{a2}$, the trend is insignificant, otherwise the significance trend is not obvious. Therefore, the significance trend of precipitation in the above 13 major cities is judged, as shown in Figure 3-1. When the two curves fluctuate smoothly, it is found that in 13 cities, Except for the three cities of Yichun, Jixi and Jiamusi, the absolute values of the $U_{F_k}$ statistics of the other cities are all less than 1.96, indicating that among the 13 cities in Heilongjiang Province, the precipitation trend is not significant compared to most cities; The length is too long, making the number of intersection points of the two curves of $U_{F_k}$ and $U_{B_k}$ more, which cannot be identified one by one. It also proves that there are more sudden change points of precipitation in 13 cities in Heilongjiang Province, and the trend cannot be judged. The change of precipitation in cities in the past 50 years is more complicated. In summary, the trend and complexity of precipitation in Heilongjiang Province are preliminarily determined.
A) Harbin
B) Mudanjiang
C) Qiqihar
D) Daqing
E) Jixi
F) Hegang
G) Shuangyashan
H) Jiamusi
3.2 Complexity Analysis of Heilongjiang Precipitation

Coarse graining analysis was performed on the measured data of precipitation at 13 selected stations in Heilongjiang Province from 1967 to 2016 to calculate the entropy values at various scales, as shown in Table 2:

Figure 2. Results of Mann-Kendall Mutation Test in Heilongjiang Province.
Table 2. Multi-scale Permutation Entropy of Urban Precipitation in Heilongjiang Province (\(r = 1 \sim 12\)).

| Main City     | \(\tau=1\) | \(\tau=2\) | \(\tau=3\) | \(\tau=4\) | \(\tau=5\) | \(\tau=6\) | \(\tau=7\) | \(\tau=8\) | \(\tau=9\) | \(\tau=10\) | \(\tau=11\) | \(\tau=12\) | Average value |
|---------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|--------------|
| Harbin        | 1.664      | 1.517      | 1.508      | 1.455      | 1.715      | 1.526      | 1.721      | 1.771      | 1.597      | 1.740      | 1.737      | 1.790      | 1.645        |
| Mudanjiang    | 1.756      | 1.59       | 1.557      | 1.394      | 1.713      | 1.660      | 1.713      | 1.624      | 1.715      | 1.685      | 1.78        | 1.739      | 1.660        |
| Qiqihar       | 1.715      | 1.518      | 1.528      | 1.308      | 1.714      | 1.469      | 1.706      | 1.760      | 1.688      | 1.763      | 1.767      | 1.748      | 1.640        |
| Daqing        | 1.714      | 1.546      | 1.513      | 1.361      | 1.713      | 1.601      | 1.704      | 1.787      | 1.683      | 1.785      | 1.769      | 1.787      | 1.664        |
| Jixi          | 1.741      | 1.586      | 1.591      | 1.538      | 1.710      | 1.54       | 1.756      | 1.743      | 1.645      | 1.734      | 1.776      | 1.671      |              |
| Hegang        | 1.754      | 1.627      | 1.599      | 1.452      | 1.712      | 1.695      | 1.717      | 1.624      | 1.745      | 1.771      | 1.738      | 1.760      | 1.683        |
| Shuangyashan  | 1.696      | 1.558      | 1.570      | 1.455      | 1.712      | 1.611      | 1.739      | 1.785      | 1.693      | 1.760      | 1.784      | 1.776      | 1.678        |
| Jiamusi       | 1.768      | 1.583      | 1.591      | 1.41       | 1.715      | 1.663      | 1.709      | 1.737      | 1.763      | 1.753      | 1.767      | 1.733      | 1.682        |
| Suihua        | 1.73       | 1.589      | 1.55       | 1.439      | 1.715      | 1.44       | 1.713      | 1.640      | 1.767      | 1.744      | 1.696      | 1.779      | 1.651        |
| Daxinganling  | 1.706      | 1.529      | 1.411      | 1.245      | 1.715      | 1.384      | 1.674      | 1.732      | 1.726      | 1.723      | 1.776      | 1.768      | 1.616        |
| Qitaihe       | 1.739      | 1.556      | 1.581      | 1.411      | 1.712      | 1.642      | 1.721      | 1.684      | 1.689      | 1.681      | 1.755      | 1.733      | 1.659        |
| Heihe         | 1.697      | 1.548      | 1.500      | 1.221      | 1.714      | 1.556      | 1.716      | 1.741      | 1.640      | 1.776      | 1.78        | 1.772      | 1.638        |
| Yichun        | 1.720      | 1.527      | 1.48       | 1.308      | 1.714      | 1.497      | 1.719      | 1.776      | 1.601      | 1.760      | 1.771      | 1.757      | 1.636        |

After calculating MSPE of each site, each site is divided into four levels according to the average entropy value, for comparison of the complexity of each site: where the entropy value is 1.610 ~ 1.630 for Level I, 1.630 ~ 1.650 is Level II, 1.650 ~ 1.670 is Level III, and 1.670 ~ 1.690 is Level IV, as shown in Table 3:

Table 3. City complexity Grade of Heilongjiang Province.

| Main City     | Average Entropy | Complexity level | Main City     | Average Entropy | Complexity level |
|---------------|-----------------|------------------|---------------|-----------------|------------------|
| Harbin        | 1.645           | II               | Jiamusi       | 1.682           | IV               |
| Mudanjiang    | 1.660           | III              | Suihua        | 1.651           | III              |
| Qiqihar       | 1.640           | II               | Daxinganling  | 1.616           | I                |
| Daqing        | 1.664           | III              | Qitaihe       | 1.659           | III              |
| Jixi          | 1.671           | IV               | Heihe         | 1.638           | II               |
| Hegang        | 1.683           | IV               | Yichun        | 1.636           | II               |
| Shuangyashan  | 1.678           | IV               |               |                 |                  |
As can be seen from Table 3, the complexity levels of precipitation in the 13 cities are divided into: Jixi, Hegang, Shuangyashan, and Jiamusi are IV-Level; Mudanjiang, Daqing, Suihua, and Qitaihe is III-Level; Harbin, Qiqihar, Heihe, and Yichun is II-Level; the complexity of precipitation in Daxinganling is I-Level. Among them, the rainfall complexity levels of Jixi, Hegang, Shuangyashan and Jiamusi are all classified as IV, and the complexity is high, indicating that these four stations are affected by the most influencing factors, and precipitation changes are difficult to predict; Daxinganling is classified as a level of complexity I, with the weakest intensity of complexity, indicating that it has the least impact factor and is the easiest to predict among the 13 cities. The entropy values calculated from the above calculations are arranged in complexity measures, from high to low: Hegang> Jiamusi> Shuangyashan> Jixi> Daqing> Mudanjiang> Qitaihe> Suihua> Harbin> Qiqihar> Heihe> Yichun> Daxinganling, description Of the 13 major cities in Heilongjiang Province, precipitation in the Daxinganling is the easiest to predict, and precipitation in Hegang is the most difficult to predict.

4. Discussion
In order to more accurately explore the impact element of the complexity of precipitation, this paper selects several influencing factors such as green area, population density, living area, arable land area, total water resources, gross agricultural output, and industrial output, and each city improves the entropy value.

Table 4. Correlation between precipitation complexity and impact factors.

| Impact factor | Green Area | Population Density | Living Area | Cultivated Area |
|---------------|------------|-------------------|-------------|----------------|
| Correlation Coefficient | -0.220    | 0.254             | -0.452*     | -0.184         |

| Impact factor | Total Water Resources | Gross Agricultural Output | Industrial output |
|---------------|-----------------------|---------------------------|-------------------|
| Correlation Coefficient | -0.660** | -0.135 | -0.143 |

Note: * indicates significant correlation at the 10% level, ** indicates significant correlation at the 1% level
It was found in the calculations that the population size and entropy value of each city were significantly significant ($p < 0.01$) and showed a negative correlation; the area of living area and the entropy value were significant ($p < 0.1$) and showed a negative correlation. Sex. This shows that the population complexity of Heilongjiang Province has a significant relationship with the scale of expansion and room area. Because of the interchange of human activities, these two factors have a deep potential impact on the complexity of Heilongjiang Province. Thanks to others. Although there is no correlation between the factors in the calculation process, it is not significant. Therefore, there is only a small degree of influence on the amount of degradation in Heilongjiang Province. The decomposition of possible influencing factors cannot be explained here one by one, and will be explored in more depth in future research.

5. Conclusions
In this paper, Mann-Kendall test and multi-scale permutation entropy method are used to analyze the complexity of precipitation in 13 major cities in Heilongjiang Province:

(1) According to the Mann-Kendall test, the precipitation tendency of 13 major cities is obtained: Among the 13 cities in Heilongjiang Province, the precipitation trend is not significant compared with most cities; because the intersection points of the two curves of $U_{F_k}$ and $U_{B_k}$ The numbers are all large, which proves that there are many mutation points and cannot be specifically identified, and the change trend cannot be clearly judged, which indicates that the change of precipitation in 13 major cities in Heilongjiang Province within 50 years is more complicated, and it can better perform complexity analysis.

(2) After performing multi-scale permutation entropy calculation, the 13 cities' complexity level of precipitation is divided into: Jixi City, Hegang City, Shuangyashan City, and Jiamusi City, the complexity level of precipitation is IV; Mudanjiang, Daqing, and Suihua, Qitaib, the complexity level of precipitation is III; Harbin, Qiqihar, Heihe, Yichun, the complexity level of precipitation is II; Daxinganling, the complexity level of precipitation is level I. The order of the complexity of the 13 major cities of Heilongjiang Province is: Hegang> Jiamusi> Shuangyashan> Jixi> Daqing> Mudanjiang> Qitaib> Suihua> Harbin> Qiqihar> Heihe> Yichun> Daxinganling, of which the Daxinganling is the easiest. It is predicted that the precipitation in Hegang is the most difficult to predict. This study is of great significance for future precipitation prediction and hydrometeorological research.

(3) By calculating the correlation between the influencing factors of the precipitation complexity of each city and the average entropy value of each city, it is found that the total amount of water resources and the entropy value are significantly more significant ($p < 0.01$) and show a negative correlation; The living area and entropy value are significant ($p < 0.1$), and have a negative correlation. In summary, it shows that there is a significant relationship between the complexity of precipitation in Heilongjiang Province and the changes in total water resources and living area, and that these two factors have a deep influence on the precipitation complexity of Heilongjiang Province. It proved that the complexity of precipitation can be affected by different factors and not be disorderly. This result provides a deeper scientific basis for practical issues such as protection and utilization of water resources and precipitation prediction.

Acknowledgement
This study is supported by the National Natural Science Foundation of China (No.51579044, No.41071053), National Science Fund for Distinguished Young Scholars (No.51825901), National Key R&D Program of China (No.2017YFC0406002), Natural Science Foundation of Heilongjiang Province (No.E2017007).

References
[1] Canqiang Zhang, Biao Zhang, Wenhua Li, et al. Response of streamflow to climate change and human activity in Xitiaoxi river basin in China [J]. Hydrological Processes, 2014, 28(1):1-8.
[2] Huaihui Chen, C. V. Chandrasekar. Ground validation of satellite measurements of precipitation using upgraded dual polarization WSR-88D radar network[C]// AGU Fall Meeting Abstracts, 2013.
[3] Tan Jianguo, Wen Gu. Advances in Urbanization Precipitation Effects Research[J]. Advances in Meteorological Science and Technology, 2015, 5(06):17-22.
[4] Le Wang, Dedi Liu, Tianyuan Li, et al. Trend Analysis of Precipitation in Beijiang River Basin Based on Multivariate Mann-Kendall Test[J]. Journal of China Hydrology, 2015.
[5] Maoyun Liu. Analysis of factors affecting precipitation [J]. Scientific Consulting: Decision Management, 2007(06): 17.
[6] Xuefeng Song. Research Status and Prospect of Complexity Science, J. Complex Systems and Complexity Science. 2005 (01): 10-17.
[7] Xinrong Huang. Research on Qian Xuesen's Complexity Thoughts--Also on the Characteristics of Chinese Complexity Research[J]. Journal of Systems Science, 2004, 012(004):12-18.
[8] Haijun Huang. International Symposium on Complexity Science Held in Shanghai[J]. Progress in Natural Science, 2002(11):48.
[9] Zhengli Mao. Quantification method of terrain complexity based on fractal theory[J]. Geographic Information World, 2015, 000(001):43-46.
[10] Junlong Yu, Shufeng Yan. Application of Non-Linear Dynamics in Clinical Medicine -- Chaos Theory, Fractal and Complexity (Translation)[J]. Guangzhou Medical, 2004, 35(6):5-6.
[11] Aziz Wajid, Arif Muhammad, Multiscale Permutation Entropy of Physiological Time Series, International Multitopic Conference [D]. 2005.
[12] Zuofang Zheng, Xiaoyan Cao, Hongxing Cao, et al. Long-range variation characteristics of temperature and precipitation in Beijing [C]// Proceedings of the 2005 Annual Meeting of the Chinese Meteorological Society. 2005.
[13] Yilun Shang. Subgraph Robustness of Complex Networks Under Attacks[J]. IEEE Transactions on Systems Man & Cybernetics Systems, 2017:1-12.
[14] Tao Liu, Zhuangxiao Ma, Nan Du, Application of Multiscale Permutation Entropy in Scroll Compressor Fault Diagnosis [J]. Journal of Lanzhou University of Technology, 2018, 44 (1).
[15] Kehui Sun, Shaobo He, Linzi Yin, et al. Application of Fuzzy Entropy Algorithm to Chaotic Sequence Complexity Analysis [J]. Acta Phys. Sin, 2012, 61 (13): 71-77.