Towards a Neural Network Determination of Charged Pion Fragmentation Functions

Emanuele R. Nocera

Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, University of Oxford, OX1 3NP, Oxford, United Kingdom

(Dated: November 14, 2018)

I present a first determination of a set of collinear fragmentation functions of charged pions using the NNPDF methodology. The analysis is based on a wide set of single-inclusive electron-positron annihilation data, including recent measurements from $B$-factory experiments, and is performed up to next-to-next-to-leading order accuracy in perturbative quantum chromodynamics. I discuss the results of the fits, highlighting their quality in the description of the data, their stability upon the inclusion of higher-order corrections, and their comparison to other sets of fragmentation functions.

PACS numbers: 13.87.Fh, 12.38.Bx, 13.85.Ni
Keywords: Fragmentation Functions, Pions, Hadronization

In the framework of perturbative Quantum Chromodynamics (QCD), the hadronization of partons, i.e. the emergence of bound states from quark and gluon interactions in a hard-scattering process, is encoded into Fragmentation Functions (FFs) \[^1\]. Because these are non-perturbative quantities, as Parton Distribution Functions (PDFs), they have to be determined from the data, possibly in a global QCD analysis combining results from a variety of processes \[^2\]. These include hadron production in electron-positron Single-Inclusive Annihilation (SIA), in lepton-nucleon Semi-Inclusive Deep-Inelastic Scattering (SIDIS) and in proton-proton ($pp$) collisions. All these processes are analyzed in light of factorization theorems \[^3\], which allow one to compute the relevant hard-scattering matrix elements perturbatively, and to absorb the collinear singularities arising from the masslessness of partons into FFs. Perturbative QCD corrections lead FFs to depend on the factorization scale, in a way which obeys time-like evolution equations \[^4\].

In this contribution, I present some recent progress towards a first determination of FFs based on the NNPDF methodology. Within this methodology, FFs are represented as a Monte Carlo sample, from which central values and uncertainties can be computed respectively as a mean and a standard deviation; also, FFs are parametrized by means of a flexible function, which is provided by a neural network with a redundant number of parameters. In comparison to the approach used in all the determinations of FFs achieved so far, the NNPDF methodology aims at reducing and keeping under control potential biases and procedural uncertainties as much as possible.

The NNPDF methodology was originally developed for the analysis of inclusive Deep-Inelastic Scattering (DIS) structure functions \[^5\] and for a determination of the PDFs of the proton, first from DIS data only \[^6\], then in a fit to data from a global set of processes \[^7\]. The NNPDF methodology has proven to be robust since then, and it has been successfully extended for instance to a global determination of unpolarized PDFs including a bunch of LHC data \[^8\], of threshold-resummed PDFs \[^9\], of PDFs with intrinsic charm \[^10\], and of polarized PDFs \[^11\].

It looks then sensible to extend the NNPDF methodology to a global determination of FFs. In this contribution, I present a first step into such a program, consisting of a determination of the FFs of charged pions from SIA data only. A dedicated forthcoming publication \[^12\] will provide extensive details on the preliminary results for charged pion FFs presented here. It will also include a determination of the FFs for other light hadrons, also based on SIA data only, specifically for charged kaons and protons/antiprotons, which constitute the largest fraction in frequently measured yields of hadrons.

This determination of FFs is based on a comprehensive set of cross section data from electron-positron annihilation into charged pions. It includes measurements from the experiments performed at CERN (ALEPH \[^13\], DELPHI \[^14\] and OPAL \[^15\]), DESY (TASSO \[^16\] and \[^17\]), KEK (BELLE \[^19\] and TOPAZ \[^20\]), and SLAC (BABAR \[^21\], HRS \[^22\], TPC \[^23\] and SLD \[^24\]). In the case of the BABAR experiment, the prompt yield is used, while a factor \(1/c\), with \(c = 0.65 \[^25\]\), is applied to the BELLE data in order to correct for initial and final state radiation effects not included in the original experimental analysis. On top of the inclusive measurements, flavor-tagged SIA data from DELPHI \[^14\], TPC \[^20\] and SLD \[^24\] are also included. The quark flavor refers to the primary quark-antiquark pair created by the intermediate photon or $Z$ boson. Available measurements of the sum of light quarks ($u, d, s$), and of individual charm and bottom quarks ($c, b$) differential cross sections are considered.

The data set included in this analysis is summarized in Tab. 1, where the name of the experiments, their corresponding publication reference, the centre-of-mass system (c.m.s.) energy $\sqrt{s}$, the relative normalization uncertainty (r.n.u.) and the number of data points included in the fit are specified. The kinematic coverage of the data set is displayed in Fig. 1.

The bulk of the data set comes from CERN-LEP and SLAC-SLC SIA experiments, at the scale of the $Z$-boson mass, $\sqrt{s} = M_Z$, and from $B$-factory experiments,
TABLE I. The data set included in this analysis of FFs. The experiment, the publication reference, the c.m.s. energy $\sqrt{s}$, the relative normalization uncertainty (r.n.u.) and the number of data points after (before) kinematic cuts are displayed.

| Exp. | Ref. | $\sqrt{s}$ [GeV] | r.n.u. [%] | $N_{\text{data}}$ |
|------|------|------------------|------------|------------------|
| BELLE| 199  | 10.52            | 1.4        | 70 (78)          |
| BABAR (prompt) | 21   | 10.54            | 0.098      | 37 (45)          |
| TASSO12 | 20   | 12.00            | 2.0        | 2 (5)            |
| TASSO14 | 20   | 14.00            | 8.5        | 7 (11)           |
| TASSO22 | 17   | 22.00            | 6.3        | 7 (13)           |
| TASSO34 | 15   | 34.00            | 6.0        | 8 (16)           |
| TASSO44 | 15   | 44.00            | 6.0        | 5 (12)           |
| TPC (incl.) | 23   | 29.00            |            | 12 (25)          |
| TPC (uds tag) | 20   | 29.00            |            | 6 (15)           |
| TPC (c tag) | 20   | 29.00            |            | 6 (15)           |
| TPC (b tag) | 20   | 29.00            |            | 6 (15)           |
| HRS    |      | 29.00            |            | 2 (7)            |
| TOPAZ  | 20   | 58.00            |            | 4 (17)           |
| ALEPH  | 135  | 91.20            | 3.0 - 5.0  | 22 (39)          |
| DELPHI (incl.) | 14   | 91.20            |            | 16 (23)          |
| DELPHI (uds tag) | 14   | 91.20            |            | 16 (23)          |
| DELPHI (b tag) | 14   | 91.20            |            | 16 (23)          |
| OPAL   | 135  | 91.20            |            | 22 (51)          |
| SLD (incl.) | 24   | 91.20            | 1.0        | 29 (40)          |
| SLD (uds tag) | 24   | 91.20            | 1.0        | 29 (40)          |
| SLD (c tag) | 24   | 91.20            | 1.0        | 29 (40)          |
| SLD (b tag) | 24   | 91.20            | 1.0        | 29 (40)          |

380 (602)

FIG. 1. The kinematic coverage in the $(z, \sqrt{s})$ plane of SIA data collected in Tab. I. Data sets are from DESY (black), KEK (green), SLAC (blue) and CERN (red).

BELLE and BABAR, at a significantly lower c.m.s. energy, $\sqrt{s} \approx 10$ GeV. All these experiments provide very precise data, with relative uncertainties of few percent, which accounts for about two thirds of the total data set. The remaining data points settle at intermediate energy scales, and are typically affected by larger uncertainties. The coverage in the hadron momentum fraction $z$ is rather limited, roughly $z \in [0.01, 0.95]$. The experiments at the highest c.m.s. energy provide the data at the lowest values of $z$ (down to $z \approx 0.006$), while the experiments at the lowest c.m.s. energy provide the data at the highest values of $z$ (very close to $z = 1$).

In this analysis, only the data which falls in the interval $[z_{\text{min}}, z_{\text{max}}]$ is retained, with $z_{\text{min}} = 0.05$ for experiments at $\sqrt{s} = M_Z$, $z_{\text{min}} = 0.1$ for all the other experiments, and $z_{\text{max}} = 0.9$ for all the experiments. These cuts exclude kinematic regions where resummation effects may be relevant, and have been chosen based on previous analyses of FFs. The total number of points before cuts is shown in parenthesis in Tab. I. In principle, resummed sets of FFs could be achieved [27], since all-order resummation has been developed both at small [28] and at large $z$ [29]. However, they are beyond the aim of this analysis.

All the available information on statistical and systematic uncertainties, including their correlations, is taken into account to reconstruct the covariance matrix for each experiment. Normalization uncertainties, see Tab. I are assumed to be fully correlated and, because of their multiplicative nature, which can lead to a systematically biased result [30], are included via an iterative procedure (the $l_0$ method [31]). As usual in the framework of the NNPDF methodology, the covariance matrix is used to generate a Monte Carlo sampling of the probability distribution defined by the data. The statistical sample is obtained by generating $N_{\text{rep}} = 100$ pseudodata replicas, according to a multi-Gaussian distribution centered at the data points and with a covariance equal to that of the original data (see e.g. Ref. [3] for details).

In this analysis, the leading observable is the SIA differential cross section for the production of a charged pion $\pi^{\pm}$ in the final state. This is usually defined in terms of the fragmentation (structure) function $F_2^{\pi^{\pm}}$ as

$$\frac{d\sigma^{\pm}}{dz}(z, Q^2) = \frac{4\pi\alpha^2(Q^2)}{Q^2} F_2^{\pi^{\pm}}(z, Q^2),$$

where $z = E^{\pi^{\pm}} / E_b = 2E^{\pi^{\pm}} / \sqrt{s}$ is the energy of the observed pion, $E^{\pi^{\pm}}$, scaled to the energy of the beam, $E_b$, $Q^2 > 0$ is equal to the c.m.s. energy squared, $s$, and $\alpha$ is the electromagnetic coupling. At leading twist, the factorized expression of the inclusive $F_2^{\pi^{\pm}}$ is given, as a convolution between FFs and coefficient functions, by

$$F_2^{\pi^{\pm}} = \langle \hat{c}^2 \rangle \left[ D_S^{\pi^{\pm}} \odot C_{2,q}^S + n_f D_g^{\pi^{\pm}} \odot C_{2,q}^S + D_{NS}^{\pi^{\pm}} \odot C_{2,q}^S \right],$$

where $n_f$ is the number of active flavors, $\langle \hat{c}^2 \rangle = n_f^{-1} \sum_q \hat{c}_q$ with $\hat{c}_q$ the effective electroweak charges, see e.g. Ref. [32] for their definition), $D_S^{\pi^{\pm}} = \sum_q \langle \hat{c}_q^2 / (D_q + D_\bar{q}) \rangle$ is the singlet FF, $D_{NS}^{\pi^{\pm}} = \sum_q (\hat{c}_q^2 / (\hat{c}_q^2 - 1)) (D_q + D_\bar{q})$ is a nonisnglet combination of FFs, $D_g^{\pi^{\pm}}$ is the gluon FF, and $C_{2,q}^S$ and $C_{2,q}^N$ are the corresponding coefficient functions (the explicit dependence on the scales has been omitted for brevity). In the case of tagged data, the sums on $q$ implicit in Eq. (2) run only over tagged quarks.

From Eq. (2), it is apparent that the SIA data has some limitations. Specifically, it is not sensitive to favored and unfavored FFs separately, as it involves the sum $D_q + D_\bar{q}$ only; also, it provides only a mild separation between different light quark flavors via the variation of their weighting effective electroweak charges with the energy. Also, the leading contribution to the coefficient functions is of order $\alpha_s$ for $C_{2,q}^S$ and $C_{2,q}^N$, while it is of order $\alpha_s^2$ for...
Each FF in the basis is parametrized as $D_i$, and the gluon, $D_g$, FFs, three nonsinglet combinations of FFs are chosen as

$$D_{T_3}^{q \pm} = \frac{2}{3}(2D_{u+}^{q \pm} - D_{d+}^{q \pm} - D_{s+}^{q \pm}),$$
$$D_{T_3}^{u \pm} = 2D_{u+}^{q \pm} + 3D_{d+}^{q \pm} + 3D_{s+}^{q \pm} - 3D_{v+}^{q \pm},$$
$$D_{T_3}^{s \pm} = D_{u+}^{q \pm} + D_{d+}^{q \pm} + D_{s+}^{q \pm} + 4D_{v+}^{q \pm},$$

where $D_{q+} = D_q + D_{\bar{q}}$. The contribution of heavy quarks fragmenting into light hadrons in Eq. (2) is not well described if they are assumed to be radiatively generated in the DGLAP evolution. For this reason, the two additional nonsinglet combinations $D_{T_3}^{q \pm}$ and $D_{T_3}^{s \pm}$ are parametrized independently and fitted to the data. Each FF in the basis is parametrized as $D_i(z, Q^0) = NN_i(z) - NN_i(1)$, $i = g, \Sigma, T_3 + \frac{1}{3}T_8, T_{15}, T_{24}$, where $NN_i(z)$ are five independent neural networks (multi-layer feed-forward perceptrons) with 37 free parameters each. The subtraction of the term $NN_i(1)$ ensures that $D_i(z = 1, Q^0) = 0$.

The FFs are evolved from the initial parametrization scale $Q_0$ to the scale of the data by solving time-like DGLAP equations. We use the zero-mass variable-flavor-number (ZM-VFN) scheme, with up to $n_f = 5$ active flavors, in which heavy-quark mass effects in the partonic cross sections are not taken into account. We choose $Q_0 = 5$ GeV, above the charm and bottom masses, but below the lowest value of $\sqrt{s}$ for which the data is available. This way, we avoid to deal with cross sections near and across heavy-quark thresholds, which would instead be better described in a matched general-mass VFN scheme [33], especially in the presence of non-negligible heavy-quark components.

This analysis is performed at leading, next-to-leading and next-to-next-to-leading order (LO, NLO and NNLO) accuracy in perturbative QCD. The computation of the cross sections and the evolution of the FFs is carried out with the APFEL program [37], and has been extensively benchmarked in Ref. [33]. We use the value $\alpha_s(M_Z) = 0.118$ as a reference for the strong running coupling at the mass of the $Z$ boson, $M_Z = 91.1876$ GeV, and the values $m_c = 1.51$ GeV and $m_b = 4.92$ GeV for the charm and bottom masses. We also take into account running effects of the fine-structure constant $\alpha$ to LO, taking $\alpha(M_Z) = 1/127$ as a reference value.

The FFs are fitted to the data by means of a Covariance Matrix Adaptation-Evolution Strategy (CMA-ES) learning algorithm [39], which ensures an optimal exploration of the parameter space and an efficient $\chi^2$ minimization. In order to make sure that the fitting strategy provides a faithful representation of FFs and their uncertainties, it has been validated by means of closure tests.

As discussed in detail in Ref. [8], closure tests are meant to quantify the robustness of the training methodology by fitting pseudodata generated using a given set of input FFs and checking whether the result of the fit is compatible with the input set. The successful outcome of closure tests ensures that, in the region covered by the data included in the fit, procedural uncertainties (including those related to the parametrization) are negligible, and that the ensuing extraction of FFs provides a faithful representation of the experimental uncertainties.

In Tab. [1] I report the values of the $\chi^2$ per data point, for each experiment and for the whole data set included in the fits, corresponding to the LO, NLO, and NNLO analyses. A good global fit quality is achieved at all perturbative orders, with the global $\chi^2$ being close to one in all cases. The inclusion of higher-order corrections improves the global description of the data clearly when going from LO to NLO, while only mildly when going from NLO to NNLO. If single experiments are considered, the improvement in the description of the corresponding data, accompanied by the inclusion of higher-order corrections, is not always clear, as already pointed out in Ref. [27]. For example, the description of the BELLE measurements, the most abundant and precise sample in the data set, improves by a significant amount when the perturbative order of the analysis is increased. However, the $\chi^2$ to the BABAR data, which settles at approximately the same energy as the BELLE data, deteriorates simultaneously. Incidentally, the anomalously small value of the $\chi^2$ to the BELLE data is comparable to that obtained in a similar independent analysis [27]. This should be taken with care, as correlations between systematics are not provided in the experimental analysis and hence not included in the fit. Such a value would then have been very unlikely if correlations had been taken into account.

In Fig. [2] I systematically compare theoretical predictions obtained from this analysis at NNLO with the data set. Specifically, I display data/theory ratios at the corresponding c.m.s. energy of each experiment. In all plots, shaded areas indicate regions excluded by kinematic cuts; bands represent one-$\sigma$ uncertainties.

In general, predictions based on this analysis provide a fairly good description of the whole data set, indicating that (N)NLO QCD is able to bridge low- and high-energy data without significant tensions. However, the data/theory ratios for some experiments, especially TASSO and TPC, show significant point-by-point fluctuations, which originate from corresponding fluctuations in the experimental data points. For this reason, the fit is not able to capture them all, and the corresponding $\chi^2$ is poor. Note that this problem worsens with the inclusion of higher-order corrections, as theoretical predictions become more accurate.

Furthermore, some signs of tension appear among experiments at equal or very close c.m.s. energies. First, in the case of BELLE and BABAR data (both at $\sqrt{s}$ ~ 10.5), theory tends to overestimate BELLE data and to
TABLE II. The experiment-by-experiment and total $\chi^2$ per data point, $\chi^2_{PO}/N_{\text{dat}}$, corresponding to the best FF set at each perturbative order, PO=LO, NLO, NNLO.

| Exp. | $N_{\text{dat}}$ | $\chi^2_{PO}/N_{\text{dat}}$ | $\chi^2_{\text{SLD}/N_{\text{dat}}}$ | $\chi^2_{\text{NNLO}/N_{\text{dat}}}$ |
|------|------------------|-------------------------------|-------------------------------------|-------------------------------------|
| BELLE | 70 | 0.54 | 0.13 | 0.12 |
| BABAR (prompt) | 37 | 1.04 | 1.28 | 1.37 |
| TASSO12 | 2 | 0.71 | 0.88 | 0.84 |
| TASSO14 | 7 | 1.54 | 1.60 | 1.68 |
| TASSO22 | 7 | 1.28 | 1.65 | 1.62 |
| TASSO34 | 8 | 1.09 | 1.08 | 0.99 |
| TASSO44 | 5 | 1.96 | 2.00 | 1.85 |
| TPC (incl.) | 12 | 0.79 | 1.02 | 1.13 |
| TPC (uds tag) | 6 | 0.70 | 0.66 | 0.62 |
| TPC (c tag) | 6 | 0.74 | 0.75 | 0.76 |
| TPC (b tag) | 6 | 1.59 | 1.58 | 1.57 |
| HRS | 2 | 2.91 | 4.77 | 4.22 |
| TOPAZ | 4 | 1.03 | 0.94 | 0.81 |
| ALEPH | 22 | 0.78 | 0.64 | 0.68 |
| DELPHI (incl.) | 16 | 2.63 | 2.62 | 2.59 |
| DELPHI (uds tag) | 16 | 1.39 | 2.00 | 1.93 |
| DELPHI (b tag) | 16 | 1.13 | 1.00 | 1.14 |
| OPAL | 22 | 1.87 | 1.79 | 1.77 |
| SLD (incl.) | 29 | 0.71 | 0.71 | 0.70 |
| SLD (uds tag) | 29 | 0.81 | 0.78 | 0.80 |
| SLD (c tag) | 29 | 0.61 | 0.65 | 0.65 |
| SLD (b tag) | 29 | 0.45 | 0.60 | 0.46 |

380 0.995 0.963 0.958

underestimate BABAR data close to the kinematic cut at high $z$. This behavior was already outlined in a previous dedicated analysis [27]. Second, in the case of TPC and HRS data (both at $\sqrt{s} = 29$ GeV), theory largely overestimates HRS data, as reflected by the very poor $\chi^2$ reported in Tab. III for this experiment. Third, in the case of experiments at $\sqrt{s} = M_Z$, theory describes all the experiments beautifully, with a slight deterioration at large values of $z$. The data from the DELPHI experiment is an exception, as it starts to deviate above theory (and the other data at the same c.m.s. energy) at $z \gtrsim 0.2$. This explains the poor value of the corresponding $\chi^2$ in Tab. III.

The agreement between the data and theoretical predictions in the small-$z$ region excluded by kinematic cuts rapidly deteriorates for the data at c.m.s. energies below the mass of the $Z$ boson, while it remains remarkably good for data at $\sqrt{s} = M_Z$, at least down to $z \sim 0.3$. This suggests that NNLO QCD is able to catch some of the beyond-fixed-order effects that kinematic cuts are meant to keep under control (see also Ref. [27]). Therefore, the cuts used in this analysis might be unnecessarily restrictive at NNLO.

In Fig. 2 I show, clockwise starting from the top left panel, the singlet, $D_{\pi}^{X}$, the gluon, $D_{g}^{X}$, the total charm, $D_{c}^{X}$, and the total bottom, $D_{b}^{X}$, FFs at $Q = M_Z$. In each panel FFs at LO, NLO, and NNLO are shown, together with their ratio to the corresponding LO distribution. Bands represent one-$\sigma$ uncertainties.

These plots confirm previous conclusions on the perturbative stability of this analysis. In all cases, the difference between the LO and the NLO determination is sizable, with the respective distributions not being compatible within their mutual uncertainties over most of the considered range in $z$. Conversely, the difference between the NLO and the NNLO determination is significantly smaller, with the distributions being in much better agreement. As expected, the uncertainty bands of FFs are larger in the LO determination than in the NLO and NNLO determinations. Larger uncertainties are indeed necessary to accommodate the data at LO, and they reflect the additional theoretical uncertainty from missing higher-order corrections. This effect, in conjunction with the deterioration of the $\chi^2$ of the LO analysis with respect to the NLO and NNLO analyses, emphasizes the adequacy of the LO approximation.

Finally, in Fig. 3 I compare the FFs obtained in this analysis with their counterparts determined in the recent DSS14 [38] and JAM16 [39] analyses. Because both the last two determinations were performed at NLO only, the NLO fit from this analysis is displayed consistently.
I show, clockwise starting from the top left panel, the singlet, $D_S^\pm$, the gluon, $D_g^\pm$, the total charm, $D_c^\pm$, and the total bottom, $D_b^\pm$, FFs are shown. The upper inset of each panel displays the FFs, while the lower inset displays their ratio to the corresponding LO FF. Bands represent one-σ uncertainties.

Note that the data set included in the JAM16 fit is very close to that used in this analysis: both are based on SIA data only, though it also includes ARGUS untagged cross section data and OPAL fully separated flavor-tagged data (given in terms of probabilities for a quark flavour to produce a jet containing a charged pion). We do not include ARGUS data because we find it to be in tension with the rest of the data set, and we do not include OPAL data because QCD does not allow for a clean, unambiguous interpretation of it beyond LO accuracy. The data set included in the DSS14 fit, instead, benefits from a wealth of additional measurements of hadron production in SIDIS and $pp$ collisions, on top of a SIA subset of data very similar to that used in this analysis. In both the DSS14 and JAM16 analyses, recent data samples from $B$-factory experiments, which represent the most abundant and accurate yields in the data set, are included.

From Fig. 3 it is apparent that the qualitative features of the shapes of the various FFs are similar across all parametrizations, except for the gluon FF. In this case, the results from the three analyses are all different, and not compatible within their mutual uncertainties in all the considered $z$ range. Specifically, the gluon FF determined here is significantly less suppressed than its DSS14 and JAM16 counterparts at large values of $z$. Its slope is nevertheless very similar to that obtained in the DSS14 analysis, while it is quite different from that obtained in the JAM16 analysis. The reason for this discrepancy is unclear. Possible explanations of the inconsistency among the three FF sets include a potential bias due to a too rigid FF functional form (in both the DSS14 and JAM16 analyses FFs are parametrized in terms of simple polynomials), and the treatment of heavy quark FFs (in both the DSS14 and JAM16 analyses they are included discontinuously above heavy quark thresholds). The discrepancy against the DSS14 analysis may also be explained by the rather different data set used to fit FFs.

Deviations of both the DSS14 and JAM16 results from the NNFF1.0 result are also observed for the singlet and the total charm and bottom FFs, especially at very large values of $z$, where FFs become very small. In the first case, deviations become larger than the NNFF1.0 one-σ uncertainty for both the DSS14 and JAM16 analyses at $z \gtrsim 0.7$; in the second case the DSS14 result deviates from the NNFF1.0 result up to two $\sigma$ in the region

![FIG. 3. A comparison among LO, NLO and NNLO FFs from this analysis at $Q = M_Z$. Clockwise starting from the top left panel, the singlet, $D_S^\pm$, the gluon, $D_g^\pm$, the total charm, $D_c^\pm$, and the total bottom, $D_b^\pm$, FFs are shown. The upper inset of each panel displays the FFs, while the lower inset displays their ratio to the correspondign LO FF. Bands represent one-σ uncertainties.](image1)

![FIG. 4. A comparison among NLO FFs from this analysis (NNFF1.0) and the DSS14 and JAM16 analyses at $Q = M_Z$. Clockwise starting from the top left panel, the singlet, $D_S^\pm$, the gluon, $D_g^\pm$, the total charm, $D_c^\pm$, and the total bottom, $D_b^\pm$, FFs are shown. The upper inset of each panel displays the FFs, while the lower inset displays their ratio to the corresponding NNFF1.0 determination. Bands represent one-σ Monte Carlo uncertainties for NNFF1.0 and JAM16, while they correspond to Hessian 90% CLs for DSS14.](image2)
$0.2 \lesssim z \lesssim 0.4$, while the JAM16 result is perfectly compatible within NNFF1.0 one-$\sigma$ uncertainties in all the $z$ range; in the third case, both the DSS14 and JAM16 analyses agree with the NNFF1.0 analysis within one-$\sigma$ uncertainties in all the $z$ range.

Finally, the size of the uncertainties in the three determinations of FFs is similar, with the NNFF1.0 bands being in general only slightly larger than JAM16 and DSS14 bands.

The results discussed in this contribution represent the first step towards a wider program. In the future, the fitted data set will be enlarged by including hadron production multiplicities in SIDIS and cross sections in $pp$ collisions. This will allow for a separation between favored and unfavored FFs and for a clearer investigation of the flavor dependence of the FFs, aspects not directly accessible from SIA data. Further theoretical sophistications might include the assessment of heavy-quark effects, which may be significant, and especially affect the determination of the gluon FF $\text{d}g/\text{d}x$. 

ACKNOWLEDGMENTS

I would like to thank the members of the NNPDF collaboration, in particular V. Bertone, S. Carrazza, N. Hartland and J. Rojo, R. Sassot and N. Sato for discussions and thoughtful advice. I also thank R. Seidl and I. Garzia for their help with BELLE and BABAR data respectively. This work is supported by a STFC Rutherford Grant ST/M003787/1.

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