Mathematical and Mental Rotation Skill in Internal Representation of Elementary Students

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Abstract. In this article, we review how the mental rotation of students in their internal representation and what is mathematic skill they used. Researchers found three geometry problems in elementary schools in Kediri Indonesia. Students have difficulty to make a cube net from the cube and they have opinion that to make a cube they need parallelogram and square. It is a big problem in geometry to transform 2 dimensional objects to 3 dimensional objects and vice versa. Based on the geometry problem of student in Kediri Indonesia, the researcher gives them a cube to modify as a cube net. Internal process is activities in their mind that they used for make a transformation an object into another object. The form of an object is different in this case 3 dimensional object into 2 dimensional object. They used mental rotation in transformation process. This article describes process mental rotation and mathematic skill student that they used to transform 3 dimensional object to 2 dimensional object. The result shown that encoding of features, imagined angular stimuli rotations and fast stimulus matching are they used in mental rotation processes and they used symbolic number sense, ordinal magnitude sense and line sub division are they used in mathematic skill.

Keywords: spatial cognition, internal process, mental rotation, mathematic skill, representation.

1. Introduction
Spatial thinking is important thing in our daily activities. Spatial thinking contributes to performance in science, technology, and mathematics to controlling verbal, visual and kinesthetic of personal abilities. It has shown that spatial thinking has a big support to develop cognitive and build concept to improve mathematic learning. One of spatial thinking components is internal representation. Internal representation is a mental activities carried out of someone to interpret the idea. Internal representation process includes the transformation process in which is mental rotation. The mental rotation skill is the ability to mentally rotate a two or three dimensional figure quickly and accurately [1]. The great attention because it appeared to attest that mental rotation taken a role in spatial thinking.

1.1 Spatial Thinking
Spatial thinking is amalgam concept, representation, transformation and reasoning. The same idea NRC told that spatial thinking can be defined as constructive combination of cognitive skills comprised of
knowing concepts of space, using tools of representation, and applying processes of reasoning [2]. People have used spatial thinking to answer a problem solving, find answers, express and communicate solutions. Spatial thinking has always been a fundamental cognitive skill and space is a key for organizing the concept.

1.2 Internal Representation
Representation in most general sense seems to mean “something” that substitutes something else. The representation is not always used to refer to ‘something’ but also to some activities or operations, and the representation may be either the act of representation or the product of representation. This distinction is a unity that cannot be separated from each other’s, product of representation and process of representation. Process of representation is mental process, that happened only in human mind and we called internal representation. Human describes their ideas become words, images or kinesthetic, there are products of representing and we called external representation. This paper is using two kinds of representation, that are internal representation and external representation. There are two kinds of representation i.e. internal representation and external representation [3]. Internal representations are defined as individual cognitive configuration inferred from human behavior describing some aspect of the process of mathematical thinking and problem solving and external representations can be described as structured physical situations that can be seen as embodying mathematical ideas[4]. Internal representation also called mental representation [5] [6] [7]. There are abstraction and distortion process in internal representation. An abstraction is remove the real object form, there is omitting the conditions because an abstraction involves the omission of a truth [8]. In the abstraction process, we do not describe the system in it is entirety, so there is not telling the whole truth. An abstraction is included “freedom” distorted, irrelevant information is not to use. Omission necessarily results in distortion [9]. Thus, an abstraction can result the distortion, an abstraction and a distortion are very different in each case. The result of abstraction is described in verbal or visual namely external representation. This paper is limited to abstractions and distortions that they are a part of internal representations that can be observed and only refers to information that can be explored through the interview guidelines and the task of solving spatial problems.

1.3 Mental Rotation
Mental image or mental picture is a representation of an object that individuals see in their mind. Mental images can be either static or dynamic. Mental rotation is the most intimately known among others which has been defined by Shepard and Metzler (1971) as “the ability to imagine how an object would look if rotated away from the plane or depth in which it is actually presented”. Several studies have explored mental rotation because of its importance in mathematics, especially 2D and 3D geometry [10], [11]. The researchers defined ‘mental rotation’ as a cognitive process whereby one imagines some rotating object. In their pilot study, participants were presented with pairs of 3-dimensional objects (in a 2-dimensional representation) positioned in different orientations or 'cut into' cubes.

2. Methods
Exploratif descriptive through qualitative approach was used in this study.

2.1 Subject
The subjects of this study were the 5th grade of visual learning style of both boy and girl students. The 5th-grade students were chosen because they have learned space matter in school and still in concrete operation phase’s, also it is important to study this subject in the earliest of cognitive development. By choosing a visual subject, students are expected to optimize spatial thinking that is closely related to visualization. The study result shows that the gender influence students’ performance and concept in mathematics and that boy’s do better in space [11-13] than the girls. Thus, besides exploring the spatial thinking of students, the author also distinguishes between boys’ and girls’ work to provide comprehensive information.
2.2 Technique
Subjects did geometry test for three days with one problem in each day. The subjects’ work in solving geometry test were explored and analyzed through an unstructured interview where the interviewer follows the interviewees’ narration and generates questions spontaneously based on his or her reflections on that narration. A deep interview conducted to the subject regarding what they thought, done, written and spoken while doing the test. The audio and video format were used to record subjects along the research process from solving geometry test until interview section.

2.3 Instrument
Two types of instruments were used in this study: main instrument and tracer instrument. The main instrument contained statements about the spatial concepts with which the students could explore the cube nets from the given cube object. The tracer instrument was used to classify student learning style. For validity test data were used in this study: 1) credibility test (internal validity), 2) test of dependency (reliability), 3) confirmation test (objectivity) and 4) transferability test.

3. Result and Discussion
Subject used sketch to describe their way to make a cube become a cube net. They used mental rotation, they mark relevant features of the object are focused on side of cube and a square of cube net. Subjects tend to use a fast flipping transformation equivalent to matching features for simple, 2D stimuli. Each of mental rotation performance has a potential role to play in mental rotation and various mathematical skill of development. They used symbolic number for marking equivalent features, they used ordinal magnitude sense and line sub division for make a cube net. Another set of findings that they used gesture, rich manipulatives have been found to facilitate learning mathematical concepts. The images illustrated below.

![Figure 1. Exercise student A](image1.jpg)

![Figure 2. Exercise student B](image2.jpg)
They have similar characteristics to make decision, i.e. imagine the cube nets in their mind, they imagine how the shape rotate, the used manipulation to reach the final form and they sketch it into the paper. Students imagined the shape, rotated 2D shapes (made of paper) and then describe what they would have to do make them look the same (mental rotations) into the paper. They verified their mental rotations with put the mark in the congruent shapes. There are abstractions in manipulation process, they take the similar characteristics and make a perception. They represented their perception by the sketch. They draw the sketch without ruler, but result of member check shown that congruent shapes are shapes with the similar mark. It means students have ability get congruent shape. Students imagine flipped the shape become cube, one side is congruent with reverse side. The students gave different mark in every couple of side. Students improve their spatial thinking through build the cubes from the cube nets and vice versa. They imagine how 2D shape transform to 3D and vice versa and then draw it in the paper. Students more capable than anticipated at 3D mental rotation, students make a result from naturally language i.e “flip” and “turn” without previous and prior instruction, students attaching meaning 3D figures. This is an important finding for several reasons. The result of this research showed that it is possible to accelerate the growth of student’s mental rotation skill through a variety task and activities. Although there was many research have provided evidence for the malleability of spatial thinking, these results are largely derived from carefully controlled of research with extremely precise of experimental manipulations. The current research offers a different approach to teaching spatial thinking become authentic contexts. Given the spatial thinking is intimately linked to success in overall mathematics, where improvements in spatial reasoning may also be seen in overall mathematics.

4. Conclusions

In this review, we evaluated how mental rotation of students in the internal representation and what is mathematic skill they used. The findings that can emerge from this approach are important for increasing our basic understanding mental rotation of internal representation and mathematical thinking are connected. The skill of mental rotation can improve by the time and students. Students were enthused to explore geometry in the new ways which increased their understanding of “what is possible”, “how to make something different by their own idea”. Our focus on spatial thinking has expanded our theoretical and practical conceptions of what might be included in geometry and curriculum for students. The geometry and spatial thinking task implemented to know mental rotation of student’s school in the city. Schools are particularly dynamic in nature i.e transforming of objects, rotating, both in the mind’s eyes and in our physical 3D word, and moved well beyond what the teachers of the study had previously considered as geometry (in theoretically and their classroom program). Future research effort are needed to determine the extent to which improving spatial learning generalizes to gains in mathematics performance.
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