Large-$N$ limit of the two-dimensinal Yang-Mills theory on surfaces with boundaries

M. Alimohammadi$^1$ * and M. Khorrami$^2$ †

$^1$ Department of Physics, University of Tehran,
North Karegar Ave., Tehran, Iran.

$^2$ Department of Physics, Alzahra University, Tehran 19938-91167, Iran.

Abstract

The large-$N$ limit of the two-dimensional $U(N)$ Yang-Mills theory on an arbitrary orientable compact surface with boundaries is studied. It is shown that if the holonomies of the gauge field on boundaries are near the identity, then the critical behavior of the system is the same as that of an orientable surface without boundaries with the same genus but with a modified area. The difference between this effective area and the real area of the surface is obtained and shown to be a function of the boundary conditions (holonomies) only. A similar result is shown to hold for the group $SU(N)$ and other simple groups.

1 Introduction

In recent years, the two-dimensional Yang-Mills theory ($YM_2$) has been studied by many authors [1–7]. It is an important integrable model which can shed light on some basic features of QCD$_4$. Also, there exists an equivalence between $YM_2$ and the string theory. It was shown that the coefficients of the $1/N$ expansion of the partition function of $SU(N)$ $YM_2$ are determined by a sum over maps from a two-dimensional surface onto the two-dimensional target space.

The partition function and the expectation values of the Wilson loops of $YM_2$ have been calculated in lattice- [1,8] and continuum-formulations [4,9–11]. All of these quantities are described as summations over the irreducible representations of the corresponding gauge group. In general, it is not possible to perform these summations explicitly. For large gauge groups, however, these summations may be dominated by some specific representations, and it can be possible to perform the summations explicitly. There are other physical reasons as well (for example the relation between large-$N$ $YM_2$ and the string theory), that the study of the $YM_2$ for large groups is important.

*alimohmd@ut.ac.ir
†mamwad@mailaps.org
In [12], the large-$N$ limit of the $U(N)$ YM$_2$ on a sphere was studied. There it was shown that the above mentioned summation is replaced by a (functional) integration over the continuous parameters of the Young tableau corresponding to the representation. Then the saddle-point approximation singles out a so-called classical representation, which dominates the integration. In this way, it was shown that the free energy of the $U(N)$ YM$_2$ on a sphere with the surface area $A < A_c = \pi^2$ has a logarithmic behavior [12]. In [13], the free energy was calculated for areas $A > \pi^2$, from which it was shown that the YM$_2$ on a sphere has a third-order phase transition at the critical area $A_c = \pi^2$, like the well known Gross-Witten-Wadia phase transition for the lattice two dimensional multicolour gauge theory [14,15]. The phase structure of the large-$N$ YM$_2$, generalized YM$_2$, and nonlocal YM$_2$ on a sphere were further discussed in [16–19].

It can be easily seen that for surfaces with no boundaries, only the genus-zero surface (i.e. the sphere) has a nontrivial saddle-point approximation, and all other surfaces have trivial large-$N$ behavior [12].

For surfaces with boundaries, the situation is much more involved. In these cases, for each boundary, the character of the holonomy of the gauge field corresponding to that boundary appears in the expression of the partition function. These characters complicate the saddle-point equation. In [20], the large-$N$ properties of YM$_2$ on cylinder and on zero-area vertex manifold (a sphere with three holes) have been studied. If we denote the $j$'th boundary by $C_j$, then each boundary condition is specified by the conjugacy class of the holonomy matrix $U_j = \text{Pexp} \int_{C_j} dx^\mu A_\mu(x)$. So, the boundary condition corresponding to $C_j$ is fixed by the eigenvalues of $U_j$. These eigenvalues are unimodular, so that each of them is specified by a real number $\theta$ in $[-\pi, \pi]$. In the large-$N$ limit, the eigenvalues of these matrices become continuous and one can denote the set of these eigenvalues (corresponding to the $j$'th boundary) by an eigenvalue density $\sigma_j(\theta), \theta \in [-\pi, \pi]$.

In the case of cylinder where only two boundaries $U_1$ and $U_2$ exist, it has been shown that the free energy of YM$_2$ satisfies a Hamilton-Jacobi equation with a Hamiltonian describing a fluid of a negative pressure. The time coordinate of the system is the area of the cylinder between one end and a loop ($0 \leq t \leq A$), and its position coordinate is $\theta$, with two boundary conditions $\sigma(t)|_{t=0} = \sigma_1(\theta)$ and $\sigma(t)|_{t=A} = \sigma_2(\theta)$. In this way it was shown that the classical Young tableau density $\rho_c$ at large-$N$, satisfies $\pi \rho_c [-\pi \sigma_0(\theta)] = \theta$ where $\sigma_0(\theta)$ is $\sigma(\theta,t)$ at a time (area) $t$ at which the fluid is at rest. The presence of a phase transition is then related to the existence of a gap in the eigenvalue density $\sigma(\theta)$ [20]. Note that by this method, only the existence of the phase transition is proved and no explicit expression can be obtained for critical area. Only for a disc, $\sigma_2(\theta) = \delta(\theta)$, an explicit expression for critical area $A_c$ has been given, using the Itzykson-Zuber integral at large-$N$ limit [21]. Similar calculation were performed for the large-$N$ generalized YM$_2$ and nonlocal YM$_2$, in [22] and [23] respectively.

For a vertex manifold, and other spheres with more than two holes, the above Hamilton-Jacobi description does not work since the time coordinate can
not be defined in a vertex (3-way) point. Therefore, in [20] only a selection rule has been obtained for a zero area vertex.

In this paper we want to study the large-$N$ behaviour of YM$_2$ on an orientable genus-$g$ surface with $n$ boundaries ($\Sigma_{g,n}$). As explained above, to investigate the partition function and hence the possible phase transition of these systems is difficult. So we restrict ourselves to the cases in which the boundary holonomies $U_j$s are very close to identity. In this case, one can expand the character $\chi_R(U)$, where $R$ denotes the irreducible representations of $U(N)$, around $U = I$. It will be shown that the critical behavior of YM$_2$ on $\Sigma_{g,n}$ with area $A(\Sigma_{g,n})$ is the same as a genus-$g$ surface with no boundary ($\Sigma_{g,0}$), but with the area $A(\Sigma_{g,0}) = A(\Sigma_{g,n}) + V(U_1,\ldots,U_n)$. Therefore, it is seen that in the large-$N$ limit, the phase structure of YM$_2$ on $\Sigma_{g>0,n}$ is trivial, while YM$_2$ on $\Sigma_{0,n}$ exhibits a third-order phase transition, as long as all boundary holonomies are close to identity.

The plan of the paper is as following. In section 2, the expansion of $\chi_R(U)$ with $U \in U(N)$, is obtained in terms of the eigenvalues $s_1,\ldots,s_N$ of $U$, up to second order in $(s_j - 1)$’s. In section 3, the large-$N$ limit of the partition function of $U(N)$ YM$_2$ on $\Sigma_{g,n}$, and its critical behavior is obtained. In the special case ($g = 0, n = 1$), where the surface is a disc, it is shown that our result coincides with the only known one, obtained by Gross and Matytsin [20]. Finally in section 4, a similar result for simple gauge groups is obtained by a different method.

2 The $U(N)$ characters

The partition function of a YM$_2$ on a two-dimensional surface $\Sigma_{g,n}$ is [4]

$$Z_{g,n}(U_1,\ldots,U_n; A) = \sum_R d_R^{2-2g-n} \chi_R(U_1) \cdots \chi_R(U_n) e^{-\frac{1}{N} C_{2R}}. \quad (1)$$

$C_{2R}$ is the second Casimir of the group in the representation $R$, $A$ is the surface area, and the factor $N^{-1}$ (as the coupling for the gauge group $U(N)$ or $SU(N)$) has been inserted to give the system a nontrivial saddle-point expansion. As it can be seen from (1), corresponding to each boundary a factor $\chi_R(U_i)/d_R$ appears in the expression for the partition function. Let us expand this factor around $U \approx I$ for the group $U(N)$.

The representation $R$ of the gauge group $U(N)$ is characterized by $N$ integers $l_1$ to $l_N$, satisfying

$$+\infty > l_1 > l_2 > \cdots > l_N > -\infty. \quad (2)$$

The group element $U$ has $N$ eigenvalues $s_1 = e^{i\theta_1}$ to $s_N = e^{i\theta_N}$. The character $\chi_R(U)$ is then

$$\chi_R(U) = \frac{\det \{ e^{il_j s_k} \}}{\text{van}(s_1,\ldots,s_N)}, \quad (3)$$
where \( \text{van}(s_1, \ldots, s_N) \) is the van der Monde determinant

\[
\text{van}(s_1, \ldots, s_N) := \left| \begin{array}{cccc}
    s_1^{N-1} & s_1^{N-2} & \cdots & 1 \\
    s_2^{N-1} & s_2^{N-2} & \cdots & 1 \\
    \vdots & \vdots & \ddots & \vdots \\
    s_N^{N-1} & s_N^{N-2} & \cdots & 1
  \end{array} \right|,
\]

\[= \prod_{i<j}(s_i - s_j). \quad (4)\]

From (3) it is seen that if \( s_0 \) is a phase and \( U = s_0 U' \), then

\[\chi_R(U) = s_0^{l_1 + \cdots + l_N - [N(N-1)/2]} \chi_R(U'). \quad (5)\]

Any member of \( U(N) \) can be decomposed as a product of a phase and a member of \( SU(N) \). The above result then shows the relation between the character of any element of \( U(N) \) and the character of the corresponding element of \( SU(N) \). (One can take \( U' \) to be in \( SU(N) \), which means that the product of its eigenvalues is equal to one.)

Denoting the numerator of (3) by \( F(s, l) \):

\[F(s, l) := \det \left\{ e^{il_j \theta} \right\},\]

\[= \left| \begin{array}{cccc}
    s_1^{l_1} & s_1^{l_2} & \cdots & s_1^{l_N} \\
    s_2^{l_1} & s_2^{l_2} & \cdots & s_2^{l_N} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_N^{l_1} & s_N^{l_2} & \cdots & s_N^{l_N}
  \end{array} \right|,\]

\[= \prod_{i<j}(s_i - s_j). \quad (6)\]

it is seen that it has roots at \( l_i = l_j \) and \( s_i = s_j \), so it is proportional to \( \text{van}(s_1, \ldots, s_N) \) and \( \text{van}(l_1, \ldots, l_N) \). Expanding the remaining part around \( s = (1, \ldots, 1) =: e \), it is found that

\[F(s, l) = \text{van}(s_1, \ldots, s_N) \text{van}(l_1, \ldots, l_N) \left\{ E + B \sum_i (s_i - 1) + C \sum_i (s_i - 1)^2 + D \sum_{i<j} (s_i - 1)(s_j - 1) + \cdots \right\}. \quad (7)\]

Defining

\[\xi_i := \ln s_i, \quad (8)\]

and

\[\mathcal{O} = \left( \frac{\partial}{\partial \xi_1} \right)^{N-1} \left( \frac{\partial}{\partial \xi_2} \right)^{N-2} \cdots \left( \frac{\partial}{\partial \xi_{N-1}} \right), \quad (9)\]

acting by \( \mathcal{O} \) on (7), and putting \( s = e \), one arrives at

\[\text{van}(l_1, \ldots, l_N) = E \text{van}(N, N-1, \ldots, 1) \text{van}(l_1, \ldots, l_N), \quad (10)\]
from which one arrives at

$$E = \frac{1}{\text{van}(N, N-1, \ldots, 1)}. \quad (11)$$

Defining

$$O_1 := \left( \frac{\partial}{\partial \xi_1} \right) O, \quad (12)$$

applying $O_1$ on (7), and putting $s = e$, one arrives at

$$\left( \sum_i l_i \right) \text{van}(l_1, \ldots, l_N) = \text{van}(l_1, \ldots, l_N) \left( q_N + \frac{NB}{A} \right), \quad (13)$$

where

$$\frac{\partial^n}{\partial s^n} = s^n \left( \frac{\partial}{\partial s} \right)^n + q_n s^{n-1} \left( \frac{\partial}{\partial s} \right)^{n-1} + p_n s^{n-2} \left( \frac{\partial}{\partial s} \right)^{n-2} + \cdots \quad (14)$$

has been used in which

$$q_n = \frac{n(n-1)}{2},$$

$$p_n = \frac{n(n-1)(n-2)(3n-5)}{24}. \quad (15)$$

From (13), one has

$$\frac{B}{E} = \frac{1}{N} \sum_i l_i - \frac{N-1}{2}. \quad (16)$$

Defining

$$O_2 := \left( \frac{\partial}{\partial \xi_1} \right)^2 O, \quad (17)$$

applying $O_2$ on (7), and putting $s = e$, one arrives at

$$\left( \sum_i l_i^2 + \sum_{i<j} l_i l_j \right) \text{van}(l_1, \ldots, l_N) = \text{van}(l_1, \ldots, l_N) \left[ p_{N+1} + q_{N+1} \frac{NB}{E} + \frac{N(N+1)C}{E} \right], \quad (18)$$

from which

$$\frac{C}{E} = \frac{1}{N(N+1)} \left( \sum_i l_i^2 + \sum_{i<j} l_i l_j \right) - \frac{1}{2} \left( \sum_i l_i \right) + \frac{(3N+2)(N-1)}{24}. \quad (19)$$

Finally, defining

$$O_3 := \frac{\partial^2}{\partial \xi_1 \partial \xi_2} O, \quad (20)$$
applying O₃ on (7), and putting \( s = e \), one arrives at
\[
\left( \sum_{i < j} l_il_j \right) \text{van}(l_1, \ldots, l_N) = \text{van}(l_1, \ldots, l_N) \left[ q_N q_{N-1} - q_N + q_{N-1} \frac{NB}{E} \right.
\]
\[
\left. + \frac{N(N-1)(D-C)}{E} \right],
\]
which results in
\[
\frac{D}{E} = \frac{C}{E} + \frac{1}{N(N-1)} \left( \sum_i l_il_j \right) - \frac{N-2}{2N} \left( \sum_i l_i \right) + \frac{(N-2)(3N-5)}{24}. \tag{22}
\]
So, one has
\[
\ln \left[ \chi_R(U) \right] = a \sum_i (s_i - 1) + b \sum_i (s_i - 1)^2 + c \left[ \sum_i (s_i - 1) \right]^2 + \cdots, \tag{23}
\]
where
\[
a := \frac{B}{E} = \frac{1}{N} \sum_i l_i - \frac{N-1}{2},
\]
\[
b := \frac{C}{E} - \frac{D}{2E} = \frac{1}{2(N^2-1)} \left( \sum_i l_i^2 \right) - \frac{1}{2N(N^2-1)} \left( \sum_i l_i \right)^2
\]
\[
- \frac{1}{2N} \left( \sum_i l_i \right) + \frac{5N-6}{24},
\]
\[
c := \frac{1}{2} \left[ \frac{D}{E} - \left( \frac{B}{E} \right)^2 \right] = - \frac{1}{2N(N^2-1)} \left( \sum_i l_i^2 \right) + \frac{1}{2N^2(N^2-1)} \left( \sum_i l_i \right)^2
\]
\[
+ \frac{1}{24}. \tag{24}
\]

3 The large-\(N\) limit of the \(U(N)\) partition function

In the large-\(N\) limit, one introduces the continuous variables [12]
\[
\phi(x) := -\frac{l(x)}{N},
\]
\[
0 \leq x := \frac{i}{N} \leq 1,
\]
which represent the irreducible representation. (Note that \( l_i \) in this paper is the same as \( n_i - i + N \) in [12, 16]). In the large-\(N\) limit,
\[
\sum_i f(l_i) \to N \int_0^1 dx \, f[-N\phi(x)]. \tag{26}
\]
So, in the large-$N$ limit,

$$a = -N \left[ \int_0^1 dx \phi(x) + \frac{1}{2} \right],$$

$$b = \frac{N}{2} \left\{ \int_0^1 dx \phi^2(x) - \left[ \int_0^1 dx \phi(x) \right]^2 + \int_0^1 dx \phi(x) + \frac{5}{12} \right\},$$

$$c = \frac{1}{2} \left\{ -\int_0^1 dx \phi^2(x) + \left[ \int_0^1 dx \phi(x) \right]^2 + \frac{1}{12} \right\}, \quad (27)$$

In the large-$N$ limit, the discrete eigenvalues $s_j = e^{i\theta_j}$ are also represented by the eigenvalue density function $\sigma(\theta)$ with $\theta \in [-\pi, \pi]$, and one has [20]

$$\sum_i f(\theta_i) \to N \int_{-\pi}^{\pi} d\theta \sigma(\theta) f(\theta). \quad (28)$$

So, if $U \approx I$ (which means $s \approx e$) using

$$s_j - 1 = i\theta_j - \theta_j^2/2 + \cdots, \quad (29)$$

one arrives at

$$\sum_j (s_j - 1) \to N \left[ i \int d\theta \sigma(\theta) \theta - \frac{1}{2} \int d\theta \sigma(\theta) \theta^2 \right],$$

$$\sum_j (s_j - 1)^2 \to -N \int d\theta \sigma(\theta) \theta^2,$$

$$\left[ \sum_j (s_j - 1) \right]^2 \to -N^2 \left[ \int d\theta \sigma(\theta) \theta \right]^2. \quad (30)$$

Inserting (27) and (30) in (23), one finds

$$\ln \left[ \frac{\chi_R(U)}{d_R} \right] = -N^2 \left[ \int dx \phi(x) + \frac{1}{2} \right] \left[ i \int d\theta \sigma(\theta) \theta \right]$$

$$- \frac{N^2}{2} \left\{ \int dx \phi^2(x) - \left[ \int dx \phi(x) \right]^2 - \frac{1}{12} \right\}$$

$$\times \left\{ \int d\theta \sigma(\theta) \theta^2 - \left[ \int d\theta \sigma(\theta) \theta \right]^2 \right\}. \quad (31)$$

Defining

$$Q(U) := \int d\theta \sigma(\theta) \theta,$$

$$V(U) := \int d\theta \sigma(\theta) \theta^2 - \left[ \int d\theta \sigma(\theta) \theta \right]^2, \quad (32)$$

7
it is seen that under the translation \( \theta \to \theta + \alpha \), \( Q \to Q + \alpha \), while \( V \) remains invariant. The translation \( \theta \to \theta + \alpha \) does not change the SU(\( N \)) factor of \( U \), but it does change the U(1) part of \( U \). So, the logarithm of the character is the sum of two terms, one coming from the SU(\( N \)) part, the other from the U(1) part, as was expected from (5).

For the remaining part of the partition function one has
\[
d_R^n e^{-\frac{A}{N^2} C^2 R} = e^{S_0}, \quad \eta := 2 - 2g,
\] (33)
and (following [12])
\[
S_0[\phi] = \frac{N^2 A}{2} \int_0^1 dx \left[ \phi(x) + \frac{1}{2} \right]^2 + \frac{N^2 \eta}{2} \int dx \, dy \, \log |\phi(x) - \phi(y)| + C,
\] (34)
where \( C \) is a constant. The large-\( N \) limit of the partition function then becomes the following functional integral
\[
Z_{g,n}(U_1, \cdots, U_n; A) = \int \mathcal{D}\phi \, e^{S[\phi]},
\] (35)
where
\[
S[\phi] = S_0[\phi] + S'[\phi],
\] (36)
in which
\[
S'[\phi] = -iN^2 Q \left[ \int dx \, \phi(x) + \frac{1}{2} \right]
- \frac{N^2}{2} V \left\{ \int dx \, \phi^2(x) - \left[ \int dx \, \phi(x) \right]^2 - \frac{1}{12} \right\},
\] (37)
where
\[
Q = \sum_j Q(U_j),
\]
\[
V = \sum_j V(U_j).
\] (38)
Defining
\[
q := \int_0^1 dx \left[ \phi(x) + \frac{1}{2} \right],
\]
\[
\psi(x) := \phi(x) + \frac{1}{2} - q,
\] (39)
one arrives at
\[
Z_{g,n}(U_1, \cdots, U_n; A) = Z_1 Z_2,
\] (40)
where

\[ Z_1 := N \int dq \exp \left[ -N^2 \left( \frac{A q^2}{2} + iQ q \right) \right], \]

\[ = \exp \left( -\frac{N^2 Q^2}{2A} \right), \tag{41} \]

and

\[ Z_2 := e^{N^2 V/24} \int D\psi \exp \left\{ -\frac{N^2}{2} \left[ (A + V) \int dx \psi^2(x) \right. \right. \]

\[ \left. \left. - \eta \int dx \, dy \, \log |\psi(x) - \psi(y)| \right\} \right\}. \tag{42} \]

\( \mathcal{N} \) is a normalization constant. It is seen that \( Z_2 \) is in fact equal to the partition function on a genus-\( g \) surface without boundaries, with the surface area equal to \( A + V \):

\[ Z_2 = Z_{g,0}(A + V). \tag{43} \]

So,

\[ \log[Z_{g,n}(U_1, \cdots, U_n; A)] = \frac{N^2}{2} \left( \frac{V}{12} - \frac{Q^2}{A} \right) + \log[Z_{g,0}(A + V)]. \tag{44} \]

As the first term is a smooth function of \( A \), the phase transition comes from the second term, which is known to be trivial for \( g > 0 \) [12]. So there is a phase transition only for \( \Sigma_{0,n} \), and the transition occurs at

\[ A_c = \pi^2 - V. \tag{45} \]

The boundary conditions do not change the structure of the phase transition: it is still a third order phase transition. They do, however, change the critical area.

As an example, let us consider a sphere with one hole, that is a disc. The critical area of a disc with a boundary condition such that the eigenvalue density \( \sigma(\theta) \) is even, has been found in [20]:

\[ A_c = \pi \left[ \int \frac{d\theta \, \sigma(\theta)}{\pi - \theta} \right]^{-1}. \tag{46} \]

For \( U \approx I \), \( \sigma(\theta) \) is nonnegligible only at \( \theta \) near zero. Expanding the denominator of (46) up to second order in \( \theta \), one arrives at

\[ A_c = \pi^2 \left[ \int d\theta \, \sigma(\theta) + \frac{1}{\pi^2} \int d\theta \, \sigma(\theta) \theta + \frac{1}{\pi^2} \int d\theta \, \sigma(\theta) \theta^2 + \cdots \right]^{-1}, \]

\[ = \pi^2 \left[ 1 + \frac{1}{\pi^2} \int d\theta \, \sigma(\theta) \theta^2 + \cdots \right]^{-1}, \]

\[ = \pi^2 \left[ 1 - \int d\theta \, \sigma(\theta) \theta^2 + \cdots \right]. \tag{47} \]
This is consistent with our general result, since in this case the second term of $V$ in (32) vanishes.

4 The partition function for other groups

The character expression introduced in eq. (3) is for the group $U(N)$. It can be used, however, for $SU(N)$ as well. If $U \in SU(N)$, then the product of the eigenvalues of $U$ is equal to one, and it is easily seen that in this case, translating all $l_j$’s by a fixed integer does not change the character. In fact, for $SU(N)$ one can use the same $l_1$ to $l_{N-1}$ to characterize the representation, or use the same results obtained for $U(N)$ but with $Q = 0$. So, one arrives at

$$Z_{g,n}(U_1, \ldots, U_n; A) = Z_{g,0}(A + V).$$

(48)

For gauge groups other than $U(N)$ and $SU(N)$, another approach is followed. Suppose that the gauge group is simple. A group element is characterized by $D$ parameters $x^\alpha$ like

$$U = \exp(x^\alpha T_\alpha),$$

(49)

where $T_\alpha$’s are the generators of the group. If $U \approx I$, then

$$U = 1 + x^\alpha T_\alpha + \frac{1}{2} x^\alpha x^\beta T_\alpha T_\beta + \cdots,$$

(50)

from which,

$$\chi_R(U) = d_R + \frac{1}{2} x^\alpha x^\beta \chi_R(T_\alpha T_\beta) + \cdots,$$

(51)

in which use has been made of the fact that the representations of the generators of simple groups are traceless. For a simple group,

$$\chi_R(T_\alpha T_\beta) = d_R \frac{C_{2R}}{d_G} \Omega_{\alpha\beta},$$

(52)

where $\Omega$ is the Killing form of the group, and $d_G$ is the dimension of the group. So, up to second order in $x^\alpha$’s,

$$\frac{\chi_R}{d_R} = \exp \left( \frac{C_{2R}}{2d_G} \Omega_{\alpha\beta} x^\alpha x^\beta \right).$$

(53)

So from (32), one arrives at (48) with

$$V = -\frac{N}{d_G} \Omega_{\alpha\beta} \sum_j x^\alpha_j x^\beta_j,$$

(54)

where the summation is over the boundaries.
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