Cooperative Inverse Reinforcement Learning

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Abstract

For an autonomous system to be helpful to humans and to pose no unwarranted risks, it needs to align its values with those of the humans in its environment in such a way that its actions contribute to the maximization of value for the humans. We propose a formal definition of the value alignment problem as cooperative inverse reinforcement learning (CIRL). A CIRL problem is a cooperative, partial-information game with two agents, human and robot; both are rewarded according to the human’s reward function, but the robot does not initially know what this is. In contrast to classical IRL, where the human is assumed to act optimally in isolation, optimal CIRL solutions produce behaviors such as active teaching, active learning, and communicative actions that are more effective in achieving value alignment. We show that computing optimal joint policies in CIRL games can be reduced to solving a POMDP, prove that optimality in isolation is suboptimal in CIRL, and derive an approximate CIRL algorithm.

1 Introduction

“If we use, to achieve our purposes, a mechanical agency with whose operation we cannot interfere effectively . . . we had better be quite sure that the purpose put into the machine is the purpose which we really desire.” So wrote Norbert Wiener (1960) in one of the earliest explanations of the problems that arise when a powerful autonomous system operates with an incorrect objective. This value alignment problem is far from trivial. Humans are prone to mis-stating their objectives, which can lead to unexpected implementations. In the myth of King Midas, the main character learns that wishing for ‘everything he touches to turn to gold’ leads to disaster. In a reinforcement learning context, Russell & Norvig (2010) describe a seemingly reasonable, but incorrect, reward function for a vacuum robot: if we reward the action of cleaning up dirt, the optimal policy causes the robot to repeatedly dump and clean up the same dirt.

A solution to the value alignment problem has long-term implications for the future of AI and its relationship to humanity (Bostrom, 2014) and short-term utility for the design of usable AI systems. Giving robots the right objectives and enabling them to make the right trade-offs is crucial for self-driving cars, personal assistants, and human–robot interaction more broadly.

The field of inverse reinforcement learning or IRL (Russell, 1998; Ng & Russell, 2000; Abbeel & Ng, 2004) is certainly relevant to the value alignment problem. An IRL algorithm infers the reward function of an agent from observations of the agent’s behavior, which is assumed to be optimal (or approximately so). One might imagine that IRL provides a simple solution to the value alignment problem: the robot observes human behavior, learns the human reward function, and behaves according to that function. This simple idea has two flaws. The first flaw is obvious: we don’t want the robot to adopt the human reward function as its own. For example, human behavior (especially in the morning) often conveys a desire for coffee, and the robot can learn this with IRL, but we don’t want the robot to want coffee! This flaw is easily fixed: we need to formulate the value

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alignment problem so that the robot always has the fixed objective of optimizing reward for the human, and becomes better able to do so as it learns what the human reward function is.

The second flaw is less obvious, and less easy to fix. IRL assumes that observed behavior is optimal in the sense that it accomplishes a given task efficiently. This precludes a variety of useful teaching behaviors. For example, efficiently making a cup of coffee, while the robot is a passive observer, is an inefficient way to teach a robot to get coffee. Instead, the human should perhaps explain the steps in coffee preparation and show the robot where the backup coffee supplies are kept and what do if the coffee pot is left on the heating plate too long, while the robot might ask what the button with the puffy steam symbol is for and try its hand at coffee making with guidance from the human, even if the first results are undrinkable. None of these things fit in with the standard IRL framework.

Cooperative inverse reinforcement learning. We propose, therefore, that value alignment should be formulated as a cooperative and interactive reward maximization process. More precisely, we define a cooperative inverse reinforcement learning (CIRL) game as a two-player game of partial information, in which the “human”, $H$, knows the reward function (represented by a generalized parameter $\theta$), while the “robot”, $R$, does not; the robot’s payoff is exactly the human’s actual reward. Optimal solutions to this game maximize human reward; we show that solutions may involve active instruction by the human and active learning by the robot.

Reduction to POMDP and Sufficient Statistics. As one might expect, the structure of CIRL games is such that they admit more efficient solution algorithms than are possible for general partial-information games. Let $(\pi^H, \pi^R)$ be a pair of policies for human and robot, each depending, in general, on the complete history of observations and actions. A policy pair yields an expected sum of rewards for each player. CIRL games are cooperative, so there is a well-defined optimal policy pair that maximizes value.\(^2\) In Section 3 we reduce the problem of computing an optimal policy pair to the solution of a (single-agent) POMDP. This shows that the robot’s posterior over $\theta$ is a sufficient statistic, in the sense that there are optimal policy pairs in which the robot’s behavior depends only on this statistic. Moreover, the complexity of solving the POMDP is exponentially lower than the NEXP-hard bound that (Bernstein et al., 2000) obtained by reducing a CIRL game to a general Dec-POMDP.

Apprenticeship Learning and Suboptimality of IRL-Like Solutions. In Section 3.3 we model apprenticeship learning (Abbeel & Ng, 2004) as a two-phase CIRL game. In the first phase, the learning phase, both $H$ and $R$ can take actions and this lets $R$ learn about $\theta$. In the second phase, the deployment phase, $R$ uses what it learned to maximize reward (without supervision from $H$). We show that classic IRL falls out as the best-response policy for $R$ under the assumption that the human’s policy is “demonstration by expert” (DBE), i.e., acting optimally in isolation as if no robot exists. But we show also that this DBE/IRL policy pair is not, in general, optimal: even if the robot expects expert behavior, demonstrating expert behavior is not the best way to teach that algorithm.

We give an algorithm that approximately computes $H$’s best response when $R$ is running IRL under the assumption that rewards are linear in $\theta$ and state features. Section 4 compares this best-response policy with the DBE policy in an example game and provides empirical confirmation that the best-response policy, which turns out to “teach” $R$ about the value landscape of the problem, is better than DBE. Thus, designers of apprenticeship learning systems should expect that users will violate the assumption of expert demonstrations in order to better communicate information about the objective.

2 Related Work

Our proposed model shares aspects with a variety of existing models. We divide the related work into three categories: inverse reinforcement learning, optimal teaching, and principal–agent models.

Inverse Reinforcement Learning. Ng & Russell (2000) define inverse reinforcement learning (IRL) as follows: “Given measurements of an [actor]’s behavior over time. . . . Determine the reward function being optimized.” The key assumption IRL makes is that the observed behavior is optimal in the sense that the observed trajectory maximizes the sum of rewards. We call this the demonstration-by-expert (DBE) assumption. One of our contributions is to prove that this may be suboptimal behavior in a CIRL game, as $H$ may choose to accept less reward on a particular action in order to convey more information to $R$. In CIRL the DBE assumption prescribes a fixed policy.

\(^2\)A coordination problem of the type described in Boutilier (1999) arises if there are multiple optimal policy pairs; we defer this issue to future work.
Figure 1: The difference between demonstration-by-expert and instructive demonstration in the mobile robot navigation problem from Section 4. Left: The ground truth reward function. Lighter grid cells indicate areas of higher reward. Middle: The demonstration trajectory generated by the expert policy, superimposed on the maximum a-posteriori reward function the robot infers. The robot successfully learns where the maximum reward is, but little else. Right: An instructive demonstration generated by the algorithm in Section 3.4 superimposed on the maximum a-posteriori reward function that the robot infers. This demonstration highlights both points of high reward and so the robot learns a better estimate of the reward.

for $H$. As a result, many IRL algorithms can be derived as state estimation for a best response to different $\pi^H$, where the state includes the unobserved reward parameterization $\theta$.

Ng & Russell (2000), Abbeel & Ng (2004), and Ratliff et al. (2006) compute constraints that characterize the set of reward functions so that the observed behavior maximizes reward. In general, there will be many reward functions consistent with this constraint. They use a max-margin heuristic to select a single reward function from this set as their estimate. In CIRL, the constraints they compute characterize $R$’s belief about $\theta$ under the DBE assumption.

Ramachandran & Amir (2007) and Ziebart et al. (2008) consider the case where $\pi^H$ is “noisily expert,” i.e., $\pi^H$ is a Boltzmann distribution where actions or trajectories are selected in proportion to the exponent of their value. Ramachandran & Amir (2007) adopt a Bayesian approach and place an explicit prior on rewards. Ziebart et al. (2008) places a prior on reward functions indirectly by assuming a uniform prior over trajectories. In our model, these assumptions are variations of DBE and both implement state estimation for a best response to the appropriate fixed $H$.

Natarajan et al. (2010) introduce an extension to IRL where $R$ observes multiple actors that cooperate to maximize a common reward function. This is a different type of cooperation than we consider, as the reward function is common knowledge and $R$ is a passive observer. Waugh et al. (2011) and Kuleshov & Schrijvers (2015) consider the problem of inferring payoffs from observed behavior in a general (i.e., non-cooperative) game given observed behavior. It would be interesting to consider an analogous extension to CIRL, akin to mechanism design, in which $R$ tries to maximize collective utility for a group of $H$s that may have competing objectives.

Fern et al. (2014) consider a hidden-goal MDP, a special case of a POMDP where the goal is an unobserved part of the state. This can be considered a special case of CIRL, where $\theta$ encodes a particular goal state. The frameworks share the idea that $R$ helps $H$. The key difference between the models lies in the treatment of the human (the agent in their terminology). Fern et al. (2014) model the human as part of the environment. In contrast, we treat $H$ as an actor in a decision problem that both actors collectively solve. This is crucial to modeling the human’s incentive to teach.

Optimal Teaching. Because CIRL incentivizes the human to teach, as opposed to maximizing reward in isolation, our work is related to optimal teaching: finding examples that optimally train a learner (Balbach & Zeugmann, 2009; Goldman et al., 1993; Goldman & Kearns, 1995). The key difference is that efficient learning is the objective of optimal teaching, while it emerges as a property of optimal equilibrium behavior in CIRL.

Cakmak & Lopes (2012) consider an application of optimal teaching where the goal is to teach the learner the reward function for an MDP. The teacher gets to pick initial states from which an expert executes the reward-maximizing trajectory. The learner uses IRL to infer the reward function, and the teacher picks initial states to minimize the learner’s uncertainty. In CIRL, this approach can be characterized as an approximate algorithm for $H$ that greedily minimizes the entropy of $R$’s belief.

Beyond teaching, several models focus on taking actions that convey some underlying state, not necessarily a reward function. Examples include finding a motion that best communicates an agent’s intention (Dragan & Srinivasa, 2013), or finding a natural language utterance that best communicates
We write the reward for a state–parameter pair as $R_A$, while hoping for $B$. In economics, this is known as the principal–agent problem: the principal (e.g., the employer) specifies incentives so that an agent (e.g., the employee) maximizes the principal’s profit (Jensen & Meckling, 1976).

Principal–agent models study the problem of generating appropriate incentives in a non-cooperative setting with asymmetric information. In this setting, misalignment arises because the agents that economists model are people and intrinsically have their own desires. In AI, misalignment arises entirely from the information asymmetry between the principal and the agent; if we could characterize the correct reward function, we could program it into an artificial agent. Gibbons (1998) provides a useful survey of principal–agent models and their applications.

3 Cooperative Inverse Reinforcement Learning

This section formulates CIRL as a two-player Markov game with identical payoffs, reduces the problem of computing an optimal policy pair for a CIRL game to solving a POMDP, and characterizes apprenticeship learning as a subclass of CIRL games.

3.1 CIRL Formulation

Definition 1. A cooperative inverse reinforcement learning (CIRL) game $M$ is a two-player Markov game with identical payoffs between a human or principal, $H$, and a robot or agent, $R$. The game is described by a tuple, $M = (S, \{A^H, A^R\}, T(\cdot|\cdot,\cdot), \{\Theta, R(\cdot,\cdot,\cdot)\}, P_0(\cdot,\cdot), \gamma)$, with the following definitions:

$S$ a set of world states: $s \in S$.
$A^H$ a set of actions for $H$: $a^H \in A^H$.
$A^R$ a set of actions for $R$: $a^R \in A^R$.
$T(\cdot|\cdot,\cdot)$ a conditional distribution on the next world state, given previous state and action for both agents: $T(s'|s,a^H,a^R)$.
$\Theta$ a set of possible static reward parameters, only observed by $H$: $\theta \in \Theta$.
$R(\cdot,\cdot,\cdot)$ a parameterized reward function that maps world states, joint actions, and reward parameters to real numbers. $R : S \times A^H \times A^R \times \Theta \rightarrow \mathbb{R}$.
$P_0(\cdot,\cdot)$ a distribution over the initial state, represented as tuples: $P_0(s_0,\theta)$.
$\gamma$ a discount factor: $\gamma \in [0,1]$.

We write the reward for a state–parameter pair as $R(s,a^H,a^R;\theta)$ to distinguish the static reward parameters $\theta$ from the changing world state $s$. The game proceeds as follows. First, the initial state, a tuple $(s,\theta)$, is sampled from $P_0$. $H$ observes $\theta$, but $R$ does not. This observation model captures the notion that only the human knows the reward function, while both actors know a prior distribution over possible reward functions. At each timestep $t$, $H$ and $R$ observe the current state $s_t$ and select their actions $a_t^H, a_t^R$. Both actors receive reward $r_t = R(s_t,a_t^H,a_t^R;\theta)$ and observe each other’s action selection. A state for the next timestep is sampled from the transition distribution, $s_{t+1} \sim P_T(s'|s_t,a_t^H,a_t^R)$, and the process repeats.

Behavior in a CIRL game is defined by a pair of policies, $(\pi^H, \pi^R)$, that determine action selection for $H$ and $R$ respectively. In general, these policies can be arbitrary functions of their observation histories; $\pi^H : [A^H \times A^R \times S]^* \times \Theta \rightarrow A^H$, $\pi^R : [A^H \times A^R \times S]^* \rightarrow A^R$. The optimal joint policy is the policy that maximizes value. The value of a state is the expected sum of discounted rewards under the initial distribution of reward parameters and world states.

Remark 1. A key property of CIRL is that the human and the robot get rewards determined by the same reward function. This incentivizes the human to teach and the robot to learn without explicitly encoding these as objectives of the actors.
3.2 Structural Results for Computing Optimal Policy Pairs

The analogue in CIRL to computing an optimal policy for an MDP is the problem of computing an optimal policy pair. This is a pair of policies that maximizes the expected sum of discounted rewards. This is not the same as ‘solving’ a CIRL game, as a real world implementation of a CIRL agent must account for coordination problems and strategic uncertainty (Boutilier, 1999). The optimal policy pair represents the best H and R can do if they can coordinate perfectly before H observes θ. Computing an optimal joint policy for a cooperative game is the solution to a decentralized-partially observed Markov decision process (Dec-POMDP). Unfortunately, Dec-POMDPs are NEXP-complete (Bernstein et al., 2000) so general Dec-POMDP algorithms have a computational complexity that is doubly exponential. Fortunately, CIRL games have special structure that reduces this complexity.

Nayyar et al. (2013) shows that a Dec-POMDP can be reduced to a coordination-POMDP. The actor in this POMDP is a coordinator that observes all common observations and specifies a policy for each actor. These policies map each actor’s private information to an action. The structure of a CIRL game implies that the private information is limited to H’s initial observation of θ. This allows the reduction to a coordination-POMDP to preserve the size of the (hidden) state space, making the problem easier.

**Theorem 1.** Let M be an arbitrary CIRL game with state space S and reward space Θ. There exists a (single-actor) POMDP M_C with (hidden) state space S_C such that |S_C| = |S| · |Θ| and, for any policy pair in M, there is a policy in M_C that achieves the same sum of discounted rewards.

Theorem proofs can be found in the supplementary material. An immediate consequence of this result is that R’s belief about θ is a sufficient statistic for optimal behavior.

**Corollary 1.** Let M be a CIRL game. There exists an optimal policy pair (π^H*, π^R*) that only depends on the current state and R’s belief.

**Remark 2.** In a general Dec-POMDP, the hidden state for the coordinator-POMDP includes each actor’s history of observations. In CIRL, θ is the only private information so we get an exponential decrease in the complexity of the reduced problem. This allows one to apply general POMDP algorithms to compute optimal joint policies in CIRL.

It is important to note that the reduced problem may still be very challenging. POMDPs are difficult in their own right and the reduced problem still has a much larger action space. That being said, this reduction is still useful in that it characterizes optimal joint policy computation for CIRL as significantly easier than Dec-POMDPs. Furthermore, this theorem can be used to justify approximate methods (e.g., iterated best response) that only depend on R’s belief state.

3.3 Apprenticeship Learning as a Subclass of CIRL Games

A common paradigm for robot learning from humans is apprenticeship learning. In this paradigm, a human gives demonstrations to a robot of a sample task and the robot is asked to imitate it in a subsequent task. In what follows, we formulate apprenticeship learning as turn-based CIRL with a learning phase and a deployment phase. We characterize IRL as the best response (i.e., the policy that maximizes reward given a fixed policy for the other player) to a demonstration-by-expert policy for H. We also show that this policy is, in general, not part of an optimal joint policy and so IRL is generally a suboptimal approach to apprenticeship learning.

**Definition 2.** (ACIRL) An apprenticeship cooperative inverse reinforcement learning (ACIRL) game is a turn-based CIRL game with two phases: a learning phase where the human and the robot take turns acting, and a deployment phase, where the robot acts independently.

**Example.** Consider an example apprenticeship task where R needs to help H make office supplies. H and R can make paperclips and staples and the unobserved θ describe H’s preference for paperclips vs staples. We model the problem as an ACIRL game in which the learning and deployment phase each consist of an individual action. The world state in this problem is a tuple (p_a, q_a, t) where p_a and q_a respectively represent the number of paperclips and staples H owns. t is the round number. An action is a tuple (p_a, q_a) that produces p_a paperclips and q_a staples. The human can make 2 items total: \( A^H = \{(0, 2), (1, 1), (2, 0)\} \). The robot has different capabilities. It can make 50 units of each item or it can choose to make 90 of a single item: \( A^R = \{(0, 90), (50, 50), (90, 0)\} \). We let \( \Theta = [0, 1] \) and define \( R \) so that \( \theta \) indicates the relative preference between paperclips and staples: \( R(p_a, q_a; \theta) = \theta p_a + (1 - \theta) q_a \). R’s action is ignored when \( t = 0 \) and H’s is ignored when \( t = 1 \). At \( t = 2 \), the game is over, so the game transitions to a sink state, (0, 0, 2).
Deployment phase — maximize mean reward estimate. It is simplest to analyze the deployment phase first. $R$ is the only actor in this phase so it get no more observations of its reward. We have shown that $R$’s belief about $\theta$ is a sufficient statistic for the optimal policy. This belief about $\theta$ induces a distribution over MDPs. A straightforward extension of a result due to Ramachandran & Amir (2007) shows that $R$’s optimal deployment policy maximizes reward for the mean reward function.

**Theorem 2.** Let $M$ be an ACIRL game. In the deployment phase, the optimal policy for $R$ maximizes reward in the MDP induced by the mean $\theta$ from $R$’s belief.

In our example, suppose that $\pi^H$ selects $(0, 2)$ if $\theta \in [0, \frac{1}{3})$, $(1, 1)$ if $\theta \in [\frac{1}{3}, \frac{2}{3}]$ and $(2, 0)$ otherwise. $R$ begins with a uniform prior on $\theta$ so observing, e.g., $\pi^H = (0, 2)$ leads to a posterior distribution that is uniform on $[0, \frac{1}{3})$. Theorem 2 shows that the optimal action maximizes reward for the mean $\theta$ so an optimal $R$ behaves as though $\theta = \frac{1}{6}$ during the deployment phase.

**Learning phase — expert demonstrations are not optimal.** A wide variety of apprenticeship learning approaches assume that demonstrations are given by an expert. We say that $H$ satisfies the demonstration-by-expert (DBE) assumption in ACIRL if she greedily maximizes immediate reward on her turn. This is an ‘expert’ demonstration because it demonstrates a reward maximizing action but does not account for that action’s impact on $R$’s belief. We let $\pi^E$ represent the DBE policy.  

Theorem 2 enables us to characterize the best response for $R$ when $\pi^H = \pi^E$: use IRL to compute the posterior over $\theta$ during the learning phase and then act to maximize reward under the mean $\theta$ in the deployment phase. We can also analyze the DBE assumption itself. In particular, we show that $\pi^E$ is not $H$’s best response when $\pi^H$ is a best response to $\pi^E$.

**Theorem 3.** There exist ACIRL games where the best-response for $H$ to $\pi^R$ violates the expert demonstrator assumption. In other words, if $br(\pi)$ is the best response to $\pi$, then $br(br(\pi^E)) \neq \pi^E$.

The supplementary material proves this theorem by computing the optimal equilibrium for our example. In that equilibrium, $H$ selects $(1, 1)$ if $\theta \in [0.51, 0.54]$. In contrast, $\pi^E$ only chooses $(1, 1)$ if $\theta = 0.5$. The change arises because there are situations (e.g., $\theta = 0.49$) where the immediate loss of reward to $H$ is worth the improvement in $R$’s estimate of $\theta$.

**Remark 3.** We should expect experienced users of apprenticeship learning systems to present demonstrations optimized for fast learning rather than demonstrations that maximize reward.

Crucially, the demonstrator is incentivized to deviate from $R$’s assumptions. This has implications for the design and analysis of apprenticeship systems in robotics. Inaccurate assumptions about user behavior are notorious for exposing bugs in software systems (see, e.g., Leveson & Turner (1993)).

### 3.4 Generating Instructive Demonstrations

Now, we consider the problem of computing $H$’s best response when $R$ uses IRL as a state estimator. For our toy example, we computed solutions exhaustively, for realistic problems we need a more efficient approach. Section 3.2 shows that this can be reduced to a POMDP where the state is a tuple of world state, reward parameters, and $R$’s belief. While this is easier than solving a general Dec-POMDP, it is a computational challenge. If we restrict our attention to the case of linear reward functions we can develop an efficient algorithm to compute an approximate best response. Specifically, we consider the case where the reward for a state $(s, \theta)$ is defined as a linear combination of state features for some feature function $\phi : R(s, a^H, a^R; \theta) = \phi(s)^\top \theta$. Standard results from the IRL literature show that policies with the same expected feature counts have the same value (Abbeel & Ng, 2004). Combined with Theorem 2, this implies that the optimal $\pi^H$ under the DBE assumption computes a policy that matches the observed feature counts from the learning phase. This suggests a simple approximation scheme. To compute a demonstration trajectory $\tau^H$, first compute the feature counts $R$ would observe in expectation from the true $\theta$ and then select actions that maximize similarity to these target features. If $\phi_\theta$ are the expected feature counts induced by $\theta$ then this scheme amounts to the following decision rule:

$$\tau^H \leftarrow \arg\max_{\tau} \phi(\tau)^\top \theta - \eta ||\phi_\theta - \phi(\tau)||^2.$$  

(1)

This rule selects a trajectory that trades off between the sum of rewards $\phi(\tau)^\top \theta$ and the feature dissimilarity $||\phi_\theta - \phi(\tau)||^2$. Note that this is generally distinct from the action selected by the demonstration-by-expert policy. The goal is to match the expected sum of features under a distribution of trajectories with the sum of features from a single trajectory. The correct measure of feature
4.1 Cooperative Learning for Mobile Robot Navigation

Our experimental domain is a 2D navigation problem on a discrete grid. In the learning phase of the game, $H$ teleoperates a trajectory while $R$ observes. In the deployment phase, $R$ is placed in a random state and given control of the robot. We use a finite horizon $H$, and let the first $\frac{1}{2}H$ timesteps be the learning phase. There are $N_\phi$ state features defined as radial basis functions where the centers are common knowledge. Rewards are linear in these features and $\theta$. The initial world state is in the middle of the map. We use a uniform distribution on $[-1, 1]^{N_\phi}$ for the prior on $\theta$. Actions move in one of the four cardinal directions $\{N, S, E, W\}$ and there is an additional no-op $\emptyset$ that each actor executes deterministically on the other agent’s turn.

Figure 1 shows an example comparison between demonstration-by-expert and the approximate best response policy in Section 3.4. The leftmost image is the ground truth reward function. Next to it are demonstration trajectories produced by these two policies. Each path is superimposed on the middle of the map. We use a uniform distribution on $[-1, 1]^{N_\phi}$ for the prior on $\theta$. Actions move in one of the four cardinal directions $\{N, S, E, W\}$ and there is an additional no-op $\emptyset$ that each actor executes deterministically on the other agent’s turn.

4.2 Demonstration-by-Expert vs Best Responder

**Hypothesis.** When $R$ plays an IRL algorithm that matches features, $H$ prefers the best response policy from Section 3.4 to $\pi^E$: the best response policy will significantly outperform the DBE policy.

**Manipulated Variables.** Our experiment consists of 2 factors: $H$-policy and num-features. We make the assumption that $R$ uses an IRL algorithm to compute its estimate of $\theta$ during learning and maximizes reward under this estimate during deployment. We use Maximum-Entropy IRL (Ziebart et al., 2008) to implement $R$’s policy. $H$-policy varies $H$’s strategy $\pi^H$ and has two levels: demonstration-by-expert ($\pi^E$) and best-responder (br). In the $\pi^E$ level $H$ maximizes reward during the demonstration. In the br level $H$ uses the approximate algorithm from Section 3.4 to compute an approximate best response to $\pi^R$. The trade-off between reward and communication $\eta$ is set by cross-validation before the game begins. The num-features factor varies the dimensionality of $\phi$ across two levels: 3 features and 10 features. We do this to test whether and how the difference between experts and best-responders is affected by dimensionality. We use a factorial design that leads to 4 distinct conditions. We test each condition against a random sample of $N = 500$ different reward parameters. We use a within-subjects design with respect to the $H$-policy factor so the same reward parameters are tested for $\pi^E$ and br.
Depending on the problem of computing an optimal policy pair to solving a POMDP. This is a useful theoretical tool and can be used to design new algorithms, but it is clear that optimal policy pairs are only part of the story. In particular, when it performs a centralized computation, the reduction assumes that we can effectively program both actors to follow a set coordination policy. This is clearly infeasible in reality, although it may nonetheless be helpful in training humans to be better teachers. An important avenue for future research will be to consider the coordination problem: the process by which two independent actors arrive at policies that are mutual best responses. Returning to Wiener’s warning, we believe that the best solution is not to put a specific purpose into the machine at all, but instead to design machines that provably converge to the right purpose as they go along.

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6 Appendix: Supplementary Material & Proofs

This appendix contains the supplementary material and proofs from the NeurIPS version of the paper. It is somewhat redundant with the original text to make it more self-contained.

6.1 CIRL Formulation

This section formulates CIRL as a two-player Markov game with identical payoffs, reduces the problem of computing an optimal equilibrium for a CIRL game to solving a POMDP, and characterizes apprenticeship learning as a subclass of CIRL games.

Definition 1. A cooperative inverse reinforcement learning (CIRL) game \( M \) is a two-player Markov game with identical payoffs between a human or principal, \( H \), and a robot or agent, \( R \). The game is described by a tuple, \( M = (S, \{A^H, A^R\}, T(\cdot|\cdot, \cdot, \cdot), \{\Theta, R(\cdot, \cdot; \cdot, \cdot)\}, P_0(\cdot, \cdot), \gamma) \), with the following definitions:

- \( S \) a set of world states: \( s \in S \).
- \( A^H \) a set of actions for \( H \): \( a^H \in A^H \).
- \( A^R \) a set of actions for \( R \): \( a^R \in A^R \).
- \( T(\cdot|\cdot, \cdot, \cdot) \) a conditional distribution on the next world state, given previous state and action for both agents: \( T(s'|s, a^H, a^R) \).
- \( \Theta \) a set of possible static reward parameters, only observed by \( H \): \( \theta \in \Theta \).
- \( R(\cdot, \cdot; \cdot, \cdot) \) a parameterized reward function that maps world states, joint actions, and reward parameters to real numbers: \( R : S \times A^H \times A^R \times \Theta \rightarrow \mathbb{R} \).
- \( P_0(\cdot, \cdot) \) a distribution over the initial state, represented as tuples: \( P_0(s_0, \theta) \).
- \( \gamma \) a discount factor: \( \gamma \in [0, 1) \).

We write the reward for a state–parameter pair as \( R(s, a^H, a^R; \theta) \) to distinguish the static reward parameters \( \theta \) from the changing world state \( s \).

The game proceeds as follows. First, the initial state, a tuple \( (s, \theta) \), is sampled from \( P_0 \). \( H \) observes \( \theta \). This parameter represents the human’s internal reward function. This observation models that only the human knows the reward function, while both actors know a prior distribution over possible reward functions. At each timestep \( t \), \( H \) and \( R \) observe the current state \( s_t \) and select their actions \( a^H_t, a^R_t \). Both actors receive reward \( r_t = R(s_t, a^H_t, a^R_t; \theta) \) and observe each other’s action selection. A state for the next timestep is sampled from the transition distribution, \( s_{t+1} \sim P_T(s'|s_t, a^H_t, a^R_t) \), and the process repeats.

Behavior in a CIRL game is defined by a pair of policies, \( (\pi^H, \pi^R) \), that determine action selection for \( H \) and \( R \) respectively. In general, these policies can be arbitrary functions of their observation histories; \( \pi^H : [A^H \times \Theta]^{\infty} \rightarrow A^H \), \( \pi^R : [A^R \times \Theta]^{\infty} \rightarrow A^R \). The optimal joint policy is the policy that maximizes value. The value of a state is the expected sum of discounted rewards under the initial distribution of reward parameters and world states.

Remark 1. A key property of CIRL is that the human and the robot get rewards determined by the same reward function. This incentivizes the human to teach and the robot to learn without explicitly encoding these as objectives of the actors.

6.2 Structural Results for Optimal Equilibrium Computation

The analogue in CIRL to computing an optimal policy for an MDP is the problem of computing an optimal policy pair. This is a pair of policies that maximizes the expected sum of discounted rewards. This is not the same as ‘solving’ a CIRL game, as a real world implementation of a CIRL agent must account for coordination problems and strategic uncertainty (Boutilier, 1999). The optimal policy pair represents the best \( H \) and \( R \) can do if they can coordinate perfectly before \( H \) observes \( \theta \).

Computing an optimal joint policy for a cooperative game is the solution to a decentralized partially observed Markov decision process (Dec-POMDP). Unfortunately, Dec-POMDPs are NEXP-complete (Bernstein et al., 2000) so general Dec-POMDP algorithms have a computational complexity that is doubly exponential. Fortunately, CIRL games have special structure that makes optimal equilibrium computation more efficient.

Nayyar et al. (2013) shows that a Dec-POMDP can be reduced to a coordination-POMDP. The actor in this POMDP is a coordinator that observes all common observations and specifies a policy for each actor. These policies map each actor’s private information to an action. The structure of a CIRL game
implies that the private information is limited to H’s initial observation of θ. This allows the reduction to a coordination-POMDP to preserve the size of the (hidden) state space, making the problem easier.

**Definition 2.** Let M be a CIRL game between H and R. The corresponding coordination POMDP MC is a POMDP where the single actor is a coordinator C. States are tuples of world state and reward parameters: S = S × Θ. The initial state distribution places the same distribution on S × Θ as P0. C’s actions are tuples (αH, aR) that specify an action for R and a decision rule for H that maps its private information (θ) to an action δH : Θ → A, C observes H’s action and the world state. Transitions are defined analogously to those in M.

**Theorem 1.** Let M be an arbitrary CIRL game with state space S and reward space Θ. There exists a (single-actor) POMDP MC with (hidden) state space SC such that |SC| = |S| · |Θ| and, for any policy pair in M, there is a policy in MC that achieves the same sum of discounted rewards.

**Proof.** We take MC to be the coordination POMDP associated with M. The second component of C’s action is an action for R. R has no private observations, so for any policy πR R could choose to follow. C can match it by simulating πR and outputting the corresponding action. Similarly, C only observes common observations, so R can implement any coordinator strategy by simulating C and directly executing the appropriate action.

By a similar argument, H can also simulate any given πC to compute her decision rule δH, and then execute the corresponding action. To see that there is a πC that can reproduce the behavior of any πH, let h be the action-observation history for H. C can choose the following decision rule

$$\delta^H(\theta) = \pi^H(\theta; h)$$

to produce the same behavior. □

**Corollary 1.** Let M be a CIRL game. There exist optimal policies (πH∗, πR∗) that only depend on the current state and R’s belief.

$$\pi^H_\ast: S \times \Delta_\Theta \rightarrow A^H, \quad \pi^R_\ast: S \times \Delta_\Theta \rightarrow A^R.$$

**Proof.** Smallwood & Sondik (1973) showed that an optimal policy in a POMDP only depends on the belief state. R’s belief uniquely determines the belief for C. From this, an appeal to Theorem 1 shows the result. □

### 6.3 Apprenticeship CIRL

**Example.** Consider an example apprenticeship task where R needs to help H make office supplies. H and R can make paperclips and staples and the unobserved θ describe H’s preference for paperclips vs staples. We model the problem as an ACIRL in which the learning and deployment phase each consist of an individual action.

The world state in this problem is a tuple (p, q, t) where p and q respectively represent the number of paperclips and staples H owns. t is the round number. An action is a tuple (p, q) that produces p paperclips and q staples. The human can make 2 items in total: $A^H = \{(0, 2), (1, 1), (2, 0)\}$. The robot has different capabilities. It can make 50 units of each item or it can choose to make 90 of a single item: $A^R = \{(0, 90), (50, 50), (90, 0)\}$.

We let Θ = [0, 1] and define R so that θ indicates the relative preference between paperclips and staples: $R(s, (p, q); \theta) = \theta p + (1 - \theta) q$. R’s action is ignored when t = 0 and H’s is ignored when t = 1. At t = 2, the game is over, so we transition to a sink state, (0, 0, 2). Initially, there are no paperclips or staples, and we use a uniform prior on θ.

H only acts in the initial state, so πH can be entirely described by a single decision rule $\delta^H: [0, 1] \rightarrow A^H$. R only observes one action from H and so the reachable beliefs are in one-to-one correspondence with H’s actions. This lets us characterize R’s policy as πR : $A^H \rightarrow A^R$.

**Theorem 2.** Let M be an ACIRL game. In the deployment phase, the optimal policy for R maximizes reward in the MDP induced by the mean θ.

**Proof.** If R never observes another action from H, then there are no common observations, so the coordination POMDP has no observations. The unobserved component of the state is static, so this distribution does not change over time. This reduces the problem to solving an MDP under a fixed distribution over reward functions so Theorem 3 from Ramachandran & Amir (2007) shows the result. □
The DBE assumption in our example assumes that $H$ maximize reward in the first round. Let $\theta = 0.49$. $H$ maximizes reward and chooses to make 0 paperclips and 2 staples. $R$ observes this and updates its belief (using $\delta^E$ to define the observation distribution). In this case, we get $b^R = \text{Unif}([0, 0.5])$. Given this belief, $R$ maximizes expected reward and chooses to make 0 paperclips and 90 staples. Thus, the expert decision rule $\delta^E$ and its best response $br(\delta^E)$ are defined by

\[
\delta^E(\theta) = \begin{cases} 
(0, 2) & \theta < 0.5 \\
(1, 1) & \theta = 0.5 \\
(2, 0) & \theta > 0.5 
\end{cases} 
\]

(2)

\[
br(\delta^E)(a^H) = \begin{cases} 
(0, 90) & a^H = (0, 2) \\
(50, 50) & a^H = (1, 1) \\
(90, 0) & a^H = (2, 0) 
\end{cases} 
\]

(3)

Note that when $\theta = 0.49$ $H$ would prefer $R$ to choose (50, 50). $H$ is willing to forgo immediate reward during the demonstration to communicate this to $R$: the best response chooses (1, 1) when $\theta = 0.49$. This leads to the following result.

**Theorem 3.** There exist ACIRL games where the best-response for $H$ to $\pi^R$ violates the expert demonstrator assumption. In other words, if $br(\pi)$ is the best response to $\pi$, then $br(br(\pi^E)) \neq \pi^E$.

**Proof.** Our office supply example gives a counter-example that shows the theorem. When $H$ accounts for $R$’s actions under $br(\delta^E)$, $H$ is faced with a choice between 0 paperclips and 92 staples, 51 of each, or 92 paperclips and 0 staples. It is straightforward to show that the optimal decision rule is given by

\[
\delta^H(\theta) = \begin{cases} 
(0, 2) & \theta < \frac{41}{92} \\
(1, 1) & \frac{41}{92} \leq \theta \leq \frac{51}{92} \\
(2, 0) & \theta > \frac{51}{92} 
\end{cases} 
\]

This is distinct from Equation 2, so we conclude the result. $\square$
References

Abbeel, P and Ng, A. Apprenticeship learning via inverse reinforcement learning. In ICML, 2004.

Balbach, F and Zeugmann, T. Recent developments in algorithmic teaching. In Language and Automata Theory and Applications. Springer, 2009.

Bernstein, D, Zilberstein, S, and Immerman, N. The complexity of decentralized control of Markov decision processes. In UAI, 2000.

Bostrom, N. Superintelligence: Paths, dangers, strategies. Oxford, 2014.

Boutilier, Craig. Sequential optimality and coordination in multiagent systems. In IJCAI, volume 99, pp. 478–485, 1999.

Cakmak, M and Lopes, M. Algorithmic and human teaching of sequential decision tasks. In AAAI, 2012.

Dragan, A and Srinivasa, S. Generating legible motion. In Robotics: Science and Systems, 2013.

Fern, A, Natarajan, S, Judah, K, and Tadepalli, P. A decision-theoretic model of assistance. JAIR, 50 (1):71–104, 2014.

Gibbons, R. Incentives in organizations. Technical report, National Bureau of Economic Research, 1998.

Goldman, S and Kearns, M. On the complexity of teaching. Journal of Computer and System Sciences, 50(1):20–31, 1995.

Goldman, S, Rivest, R, and Schapire, R. Learning binary relations and total orders. SIAM Journal on Computing, 22(5):1006–1034, 1993.

Golland, D, Liang, P, and Klein, D. A game-theoretic approach to generating spatial descriptions. In EMNLP, pp. 410–419, 2010.

Jensen, M and Meckling, W. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of Financial Economics, 3(4):305–360, 1976.

Kerr, S. On the folly of rewarding A, while hoping for B. Academy of Management Journal, 18(4): 769–783, 1975.

Kuleshov, V and Schrijvers, O. Inverse game theory. Web and Internet Economics, 2015.

Leveson, N and Turner, C. An investigation of the Therac-25 accidents. IEEE Computer, 26(7): 18–41, 1993.

Natarajan, S, Kunapuli, G, Judah, K, Tadepalli, P, and Kersting, Kand Shavlik, J. Multi-agent inverse reinforcement learning. In Int’l Conference on Machine Learning and Applications, 2010.

Nayyar, A, Mahajan, A, and Teneketzis, D. Decentralized stochastic control with partial history sharing: A common information approach. IEEE Transactions on Automatic Control, 58(7): 1644–1658, 2013.

Ng, A and Russell, S. Algorithms for inverse reinforcement learning. In ICML, 2000.

Ramachandran, D and Amir, E. Bayesian inverse reinforcement learning. In IJCAI, 2007.

Ratliff, N, Bagnell, J, and Zinkevich, M. Maximum margin planning. In ICML, 2006.

Russell, S. and Norvig, P. Artificial Intelligence. Pearson, 2010.

Russell, Stuart J. Learning agents for uncertain environments (extended abstract). In COLT, 1998.

Smallwood, R and Sondik, E. The optimal control of partially observable Markov processes over a finite horizon. Operations Research, 21(5):1071–1088, 1973.
Waugh, K, Ziebart, B, and Bagnell, J. Computational rationalization: The inverse equilibrium problem. In *ICML*, 2011.

Wiener, N. Some moral and technical consequences of automation. *Science*, 131, 1960.

Ziebart, B, Maas, A, Bagnell, J, and Dey, A. Maximum entropy inverse reinforcement learning. In *AAAI*, 2008.