Can an “impulse response” really be defined for a photoreceiver?

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Abstract

In this paper we examine the validity of the concept of impulse response employed to characterize the time response and the signal-to-noise ratio of p-i-n and similar photodetecting devices. We analyze critically the way in which the formalism of analog linear systems has been extrapolated, by employing results from macroscopic electromagnetic theory such as the Shockley–Ramo theorem or any equivalent approach, to the extreme case of a single-photon detection. We argue that the concept of “response to an optical impulse” is ill-defined in the customary terms it is envisioned in the literature, this is, as an output current pulse having a certain predictable, calculated temporal shape, in response to the detection of an optical “Dirac delta” impulse, conceived in turn as the absorption of a single photon.

1 Introduction

It is well known that the ultimate sensitivity of any photodetector is determined by the quantum noise of the radiation itself. Specifically for a photodiode receiver, this means that the noise present at the output current of the diode should be an exact reproduction of the intrinsic noise of the impinging radiation, with no any other excess noise contributions. Obviously, the current-voltage amplification at the electronic stage of the receiver should also be noiseless, consistent with the “ideal receiver” assumption. In other words, once all additional (electronic) noise sources have been removed, the process of noiseless photodetection amounts to photon-counting through ideally equivalent temporal electron-counting.

On the other hand, the functional modelling of a photoreceiver systematically makes use of an essential concept taken from linear systems theory: the “impulse response of the receiver,” [1] which is required for the analysis of both signal and noise performance of any linear, invariant system. For an analog system, the impulse response is defined as the output time signal when the input is an instantaneous impulse of unit area, i.e. a Dirac delta, $\delta(t)$, which contains all frequencies homogeneously, from 0 to $\infty$. Such an impulse is unrealizable (and surely unphysical), but its mathematical usefulness makes it convenient to assume its existence, at least in the approximate form of a physical impulse having a duration much shorter than any characteristic time of the system. Thus, in the case of an electrical circuit, one can think of a delta-like impulse of voltage, for example.
In the case of incoherent optical reception, the input “signal” is the time-varying optical power $P(t)$, so the input impulse is to be described mathematically as $P(t) = \delta(t)$.

It should be kept in mind that all signals are inherently analog in this formalism. Actually, to a great extent, the Dirac delta works as an *ad hoc* artifact intended to allow hypothetical point-like objects (masses, charges...) to “live” in continuous spatial or temporal domains, which would otherwise be unconceivable; if space-time is thought continuous, at least differential intervals are needed to contain a non-null amount of any magnitude, since a discrete *point* is, in mathematical terms, a zero measure set, thus meaningless. Only if one accepts that a space or time point can accommodate a “Dirac delta” (of charge, say), can the problem be skipped.

The above considerations lead us to the following point. Consider the optical signal to be a narrow-band modulated optical flux $\bar{Q}(t) = P(t)/(h\nu)$ (photons/s), where the over-bar denotes statistical averaging and $\nu$ is the central optical frequency. This corresponds well to the archetypical case of a laser (or even LED) beam modulated in intensity by a low frequency (baseband, RF, microwave) signal varying like $P(t)$. Contrary to what is frequently implied in the literature, the “unit” impulse at the input of the detector is not “one photon”—in spite of the cardinal number. This confusion, detected in many textbook presentations, arises surely from the fact that the electromagnetic field, roughly speaking, happens to be quantized in amplitude, whereas the Dirac delta formalism was never intended to deal with “quantized analog” signals—incidentally, a concept which does not exist in linear systems theory.

As far as the signal part of the signal and noise calculations is concerned, the problem can be surmounted easily for two related reasons. First, the unit amplitude of the Dirac delta is purely conventional and without consequences in a linear system; obviously, if the impulse $A\delta(t)$ is employed at the system input, the system response to the unit impulse will merely be the actual output divided by $A$. In other words, it is the temporal condensation that matters, not the amplitude. Second, in view of the previous consideration, any sufficiently short optical pulse, yet simultaneously intense enough to clear up any concern on signal level quantization, will be a perfectly valid approximation to the unit input impulse.

Things change when the focus is put on the noise. Particularly in the case of the photonic signal noise, the inherent amplitude quantization cannot be disguised anymore and the solution described above is unfeasible. One thus has to confront a frequently overlooked issue which threatens the typical automatic extension of the linear system formalism to handle noise of quantum origin. In Sections 2 and 3 we review, very briefly, the standard theory of the signal noise as routinely applied to the linear system model of photoreceiver. The problems carried out by the accepted formalism are discussed in Section 4. Section 5 contains the conclusions.

## 2 Optical shot noise in the photoreceiver model

Quantum noise is almost synonymous of shot noise as far as a photoreceiver is concerned. Only two or three simple statistical concepts are needed to describe the photodetection process in the simple fashion in which it is usually modelled, and a straightforward correspondence can be apparently established between the mathematical route and the physical route. Thus, assuming a coherent light source, the random arrival times of the photons are governed by a Poisson distribution characterized by its average $\bar{N}$, related in
turn to the average rate of the photon flux through $\bar{N} = \bar{Q}T$, with $T$ the “photocounting” period. One could anticipate at this point that $T$ will be roughly equal to the inverse of the bandwidth, which is basically true.

Next, as an optical-electrical transducer, the photodetector transmutes the photon absorptions into charge carriers—the mathematical consequence being a mere multiplication of the actual instantaneous photon flux, $Q(t) = \sum_k \delta(t - t_k)$, by the electron charge $q$ to arrive at the same delta train function, but this time as an electrical current rather than a photon flux: $i_\delta(t) = q \sum_k \delta(t - t_k)$. Certainly, this impossible current is only a conceptual intermediate step toward the “real” current, which in general is described by the expression

$$i(t) = q \sum_k M_k h_k(t - t_k),$$

where $h_k(t)$ is the shape of the current pulse generated across the terminals of the photodiode by the $k$-th absorbed photon. The prefactor $M_k$ accounts for the possibility of the detector being an avalanche photodiode (APD) with average gain $\bar{M}$, while the subindex $k$ of $h_k$ reflects the fact that, even in a p-i-n photodiode, the shape of current pulse will vary depending on the specific location within the photodiode where the photon has been absorbed [2]. Expression (1) is most often oversimplified by ignoring the random character of $h_k$ and writing a fixed $h(t)$, sketched in Fig. 1, which is then identified with the “impulse response” of the linear system, its Fourier transform $H(\omega)$ being the photodetector transfer function.

![Figure 1](image-url)

Figure 1: When applied to a photodetector, the formalism of linear systems seeks to calculate the “impulse response” as the photocurrent pulse at the output of the device which corresponds to the detection of just one photon. In absence of internal gain, such elementary current pulse is predicted to have the form $i(t) = q h(t)$, with $q$ the electronic charge and $h(t)$ the pulse shape, satisfying $\int_{-\infty}^{\infty} h(t)dt = 1$.

Considering the specificities of an APD is unnecessary for the purpose of the present discussion, so we will take $M_k = 1$ and focus on a p-i-n photodiode. The shape of $h_k(t)$ is determined by the geometry and structure of the diode, mainly the width of the intrinsic layer. Fig. 1 sketches the form of the current pulse, which—always within the frame of the described approach—arises from the transit of one electron-hole pair photogenerated (typically) somewhere in the space charge region, toward the positive and negative, respectively, electrodes of the structure. These transit times determine the ultimate bandwidth of the photodetector.
3 Impulse response and sub-electron charge

The area of any elementary current pulse as described above is given by

\[ \int_{-\infty}^{\infty} i(t) dt = q \int_{-\infty}^{\infty} h_k(t) dt = q, \]  

(2)

which manifests the transfer of one electron charge during the duration of the pulse, or, expressed more accurately, the passage of a total charge \( q \) across an imaginary plane located at any point along the electrical circuit. Thus, at the end of the “flight time” of the electron and the hole, assuming they do not recombine before being collected at the electrodes, one can safely say that a total charge of one electron has moved, as a conduction current, along the whole circuit. However, expression (1) has a very discomforting feature. If \( h_k(t) \) is truly a current shape and the actual pulse duration spreads, say, from \( t = 0 \) to \( t = T_p \), one should be able to observe a fractional charge \( q_f \) given by

\[ q_f = q \int_{t_1}^{t_2} h_k(t) dt \]  

(3)
during any finite interval \([t_1, t_2]\), with \( 0 \leq t_1 < t_2 \leq T_p \). However striking this consequence of the formalism may look, seemingly it has never deserved a remark in any textbook or article, passing completely unnoticed in the literature to the author’s knowledge.

It is necessary to recall the origin of the theory leading to this somewhat stunning result (3). Essentially, this is the Ramo or Shockley–Ramo theorem (SRT) [3], [4], first applied to determine the shape of the anode current of a vacuum tube by computing the charge electrostatically induced on the plate during the “flight” of the electronic space charge across the inter-electrode space. The SRT has been used intensively and extended to deal with other scenarios, including solid state devices (see, for example, [5], [6]).

Ramo’s original proof basically appeals to the energy balance, provides a relatively simple result which facilitates otherwise more cumbersome calculations. However, for the purpose of the present discussion, we will use an argument based directly on the Maxwell equations since, with the toy model to be used here—also frequently employed in the literature—, both approaches have about the same simplicity while the latter provides some more physical insight.

Figure 2 illustrates, with a very simple scheme, how the computation of the “impulse current” is almost universally made. To focus on the essential concepts, we will consider a typical one-dimensional, homogeneous structure of dielectric constant \( \varepsilon \) (it could equally be vacuum) bounded by two conducting planes. This could represent, for example, the intrinsic zone of a p-i-n photodiode sandwiched between p⁺ and n⁻ zones. As assumed in the ideal linear model, the voltage \( V \) across the dielectric remains constant, regardless of the photogenerated space charge and current. The “unit impulse” will then be materialized by the instantaneous absorption of one photon anywhere between the electrodes, say at \( x = x_0 \), and the corresponding generation of a single charge or an electron-hole pair at that point. Assume, for the sake of maximum simplicity, that just one electron is photogenerated. This “one-dimensional” electron will be described as a discrete charge sheet on the plane located at \( x_0 \), with a surface density charge \( \sigma_e \) such that \( \sigma_e A = -q \), with \(-q\) the electronic charge and \( A \) the transversal area considered. It should be noted, at this point, that we do accept the one-dimensional modelization of the electronic charge,
merely for obvious reasons of mathematical convenience, and this has nothing to do with the conceptual difficulties that are the object of this article.

Figure 2: One-dimensional model illustrating the way in which the photocurrent pulse corresponding to the detection of one photon is obtained.

The electron generated at \( x_0 \) will then move toward the positive electrode at \( z = d \) under the influence of the bias field created by \( V_b \). As is customarily made, we keep it simple and assume that it moves at a constant saturation velocity \( v_s \). The argument now is that, during the whole flight time of the electron, the time-dependent electric field it generates will electrostatically induce a continuously-varying current flowing through the electrodes at \( x = 0 \) and \( x = w \), thus resulting in the circulation of a short current pulse along the circuit. This should be the “impulse response” of the photodetector.

To calculate the shape of the aforementioned current pulse, we make use of the law of the conservation of charge, which follows readily from the Maxwell equations and reads, for the free charge and current density,

\[
\nabla \cdot J_f = -\frac{\partial \rho_f}{\partial t}.
\]

Considering a certain volume \( V \) and applying the Gauss theorem, we obtain the integral relation

\[
\int_S J_f \cdot dS = -\frac{\partial Q_f}{\partial t},
\]

with \( Q_f = \int_V \rho_f dV \), the total free charge enclosed. We choose to use a rectangular Gaussian box limited by the planes \( x = w^- \) (very close to the right-hand plate surface) and \( x = w^+ \) (just inside the right-hand plate). The electric field vector is defined as \( E(x) = \hat{e} \). In the surface integral, \( dS = \pm \hat{e} dS \) on the right/left plane and \( -\hat{e} dS \) on the left plane, while \( J_f = \hat{e} \). Thus,

\[
\int_S J_f \cdot dS = -J_f(w^-, t)A + J_f(w^+, t)A.
\]

There is no free current at \( w^- \), so \( J(w^-, t) = 0 \). The total free charge inside the volume is given by \( Q_f = \sigma_w A \), with \( \sigma_w \) the surface charge density at the right-hand plate, which can be related to the normal field at the conductor surface by the equation

\[
E(w, t) = -\frac{\hat{e} \sigma_w(t)}{\varepsilon},
\]

in the quasi-static approximation, so one finally obtains

\[
J_f(w^+, t) = \varepsilon \frac{\partial E(w, t)}{\partial t}.
\]

The field \( E(w, t) \), determined by the moving electron sheet charge \( \sigma_e \), can be computed using the Gauss law; it is easy, in this elementary case, to arrive at the result

\[
E(w, t) = -\frac{V}{w} + \frac{x_0(t) \sigma_e}{w^+}.
\]

Noting that \( dx_0(t)/dt = v_s \), the result \( J_f(w^+, t) = \sigma_e v_s / w \) follows. The total plate current is given by

\[
I(t) = \sigma_e \frac{v_s}{w} A = -q \frac{v_s}{w} \quad \text{during} \quad 0 < t < (w - x_0)/v_s
\]
(assuming the photocarrier is generated at $t = 0$ and instantly accelerated to $v_s$). The total charge crossing the $x = w$ plane from left to right during the time interval $[0, (w - x_0)/v_s]$ will be

$$q_T = \int_0^{(w-x_0)/v_s} I(t) dt = -\frac{q}{w} (w - x_0),$$

which is smaller than the charge of single electron. Actually, it is the photogenerated hole we have disregarded which provides, with a similarly shaped current pulse, the remaining charge, so that the total value $-q$ is obtained.

To summarize, we see that the formalism predicts a current pulse of rectangular shape (assuming equal electron and hole velocities) containing a total charge equal to the electron charge. Further refinements in the model, such as field-dependent carrier drift velocities and others, would lead to more or less complicated calculations and different resulting pulse shapes, but the issue of the sub-electron charge persists as all models are developed along the same key conceptual lines.

4 Discussion

As it is well known, when Maxwell’s classical equations are applied to material media, it is with the understanding that some process of macroscopic averaging is necessarily carried out (a comprehensive study can be found in [7], for example). Of course, in a microscopic view, one can consider individual charges formally described by a volume charge density of the type $\rho(\mathbf{r}) = \sum_j q_j \delta[\mathbf{r} - \mathbf{r}_j(t)]$, and still employ the corresponding spatially-continuous electromagnetic fields consistently calculated. However, one cannot reasonably expect that both approaches can be freely mixed in the same equations. When the electrostatic normal field on a conducting surface is written as $E(\mathbf{r}) = \hat{n} \sigma(\mathbf{r})/\varepsilon$, it has been tacitly accepted that $\sigma$ is a mathematically continuous function of $\mathbf{r}$, which can vary over $\mathbf{r}$ as smoothly as $E$ (which is a true continuous magnitude) demands. Physically, this is obviously never the case, so an implicit approximation is always under the rug; in this example, the approximation is that—contrary to the single, discrete photogenerated carrier—a very thin layer of atoms or molecules right below the surface contains so many free electrons, that it can be macroscopically treated as a continuous surface charge $\sigma$. Of course, this view breaks down if one looks at the surface too closely, but this limitation simply sets a scale limit beyond which it is recognized that the details (usually not needed) will be missed.

The problem in our case is of a different nature. The continuous variation of the field amplitude at $x = w$, determined by the instantaneous position of the photoelectron position during its “flight”, demands a continuously-varying surface charge density at the right-hand plate, which, according to the charge conservation law, should mandatorily result in a continuously-varying current density coming out of the $x = w$ plane. But of course, all the steps in such a derivation take for granted that the magnitudes involved are continuous or can be thought of as continuous with sufficient accuracy. As remarked in the previous paragraph, this premise is justified when many microscopic entities can be averaged out into a “continuous” fluid, but the averaging process becomes senseless when the set of (indivisible) microscopic entities to be averaged happens to contain just one. In a way, proceeding in this manner amounts to using a sort of a circular argument.

On the other hand, it is undeniable that the displacement of the single photoelectron
has to affect in some way the charges in the conducting left and right plates. What is by no means obvious, is the automatic, seemingly thoughtless assumption that the effect should be formalized through the same standard approach which is used macroscopically. In contrast, if rather than one single photogenerated electron located at point \( r \), there were a small charge contribution \( \Delta Q(r,t) = \rho(r,t)\Delta V \) within a “differential” volume \( \Delta V \) around \( r \) (which would be actually comprised of, say, millions of electrons), the formalism could then be applied straightforwardly, since a temporal fraction (say, \( \alpha < 1 \)) of the output current pulse would still contain \( \alpha \Delta Q(r,t)/q \) electrons, hopefully a number large enough to clear off any possible concern on charge discreteness.

Surprising as it may seem, the type of concerns expressed here never seem to have drawn any interest in the specialized literature through the years. It is futile to try to cite examples here, whether a few or many, since all known references could certainly be listed. With the sole purpose of not leaving this point without any bibliographic support, we will mention, for example, \([8]–[10]\) (references chosen almost at random with no intention at all to single them out as being precedential or more “original” than others; the idea of the “single-photon” current-impulse—reluctantly assumed even in \([2]\)—has propagated through the photonics literature as a routine, to the point that it is impossible to trace back its diffusion). Significantly, the first mentions to a similar problem have only appeared in the relatively new field of mesoscopic devices, as we exemplify next.

A single-electron transistor (SET) is a device where the so-called Coulomb blockade takes place \([11]\). This process involves the tunneling of individual electrons across a thin insulating barrier between two conducting electrodes. Among other conditions, the theoretical model of the Coulomb blockade requires a continuous charge transfer from an external source to the electrode. In this case, the conceptual problem posed by the necessity of considering a continuous charge has not passed unnoticed, and consensus seems to have been reached in recognizing that it is a continuous spatial displacement of the electronic charges around the atomic nuclei of the metal, during the intervals between tunneling events, that may provide the necessary “continuous charge”. We reproduce next a few, more or less similar, typical statements which can be found in the literature in regards with this issue. It will not go unnoticed, in any case, that they all tend to be rather qualitative.

In \([12]\), it is remarked that “(...) the current is determined by the current transferred through the conductor. Surprisingly this transferred charge can have practically any value, in particular, a fraction of the charge of a single electron. Hence, it is not quantized. This, at first glance counterintuitive fact, is a consequence of the displacement of the electron cloud against the lattice of atoms. This shift can be changed continuously and thus the transferred charge is a continuous quantity.” Under the section title “Continuous charge transfer,” the following statement can be found in \([13]\): “(...) \( q \) is not necessarily a charge transferred through some imaginary cross-section of the current leads (...) \( q \) is rather defined by the equation \( U = q^2/2C \) for the electrostatic energy of the capacitor, so that \( q \) is the net surface charge of its electrodes.” Or, in \([14]\), “in the macroscopic metallic leads ending at the barrier the electrons are in extended states, i.e. they can move freely. Consequently the accumulated charges \( +Q \) and \( -Q \) effectively result from a shift in the average positions of the electrons on the two sides of the barrier.” As a final example, we quote the claim in \([15]\) that “charge flow in a metal or a semiconductor is a continuous process because conduction electrons are not localized at specific positions. They form a quantum fluid which can be shifted by an arbitrary small amount.” Even if
the connection between the non-localizability of the electrons (of statistical nature) and
the electron flow being a "continuous process" appears somewhat obscure, the authors
seem in any case to appeal again to the continuous spatial displacement of the charges
to justify the formalism.

5 Conclusions

In view of all the previous considerations, we must finally decide whether it is reasonable
or not to expect that, upon absorption of just one photon at a specific point, a p-i-n
photodetector (or any similar device) will really be able to provide a photocurrent pulse
having a continuous, repeatable shape \( h(t) \), which can be calculated using the formalism
summarized above. It is important to make precise what is meant by this: one should
be able to observe perfectly, after suitable amplification, this current pulse shape on the
screen of an oscilloscope, say. [Realistically, some additional noise is to be expected,
due to electronic components or multiplicative noise, as in a APD photodetector or a
photomultiplier tube, but this should certainly not affect the alleged tangibility of a—
perhaps small albeit macroscopic—amplified output current instantaneously following the
continuous functional form \( h(t) \)]. Only the occurrence of this, and not any other situation
whatsoever, would fulfill the precise definition of "impulse response" of the photoreceiver
as a linear system, thus justifying the standard formalism under discussion.

It is interesting to note that, in all the descriptive accounts of the photodetection
process in a p-i-n photodiode, the argument provided to calculate the quantum efficiency
\( \eta \) excludes the photogenerated carriers that recombine before reaching the correspond-
ing electrode from the current contribution. Indeed, the same reasoning is applied to
compute the efficiency of solar cells. There obviously underlies the idea than only the
discrete electrons or holes effectively collected at the electrodes will contribute to the
output photocurrent. Actually if, as discussed with regards to the Coulomb blockade in
the previous section, the problematic "sub-electron" charge is really attributed to small
spatial displacements of the free electrons around the atomic nuclei in the conductors,
when the polarizing electric field causing this displacement ceases to exist, i.e. when
a traveling photoelectron, for example, suddenly disappears by recombination before it
reaches the positive contact, the corresponding electronic charges should simply shift back
to their original positions, with no final consequences for the circuit current. Obviously,
one-photon detection has existed for a long time, but a survey of the technical litera-
ture will show that virtually any approach to the subject ends up considering basically
a process of photocounting, thus electron-counting; to the author's knowledge, no true
experimental work clearly undertaking the verification of the elusive \( h(t) \) can be found in
the literature.

To summarize, within the linear system description of a photoreceiver, the impulse
response of the system can be taken as the output voltage/current of the receiver (which
will follow, properly amplified, the temporal shape of the device photocurrent) when a
very short, ("delta") impulse of photon flux is applied, with the proviso that a suffi-
ciently high number of photons "instantaneously" fill up the absorbing volume of the
device. Enough electrons/holes will then be created and drifted toward the terminal elec-
trodes forming a quasi-continuous current to which the familiar, macroscopic model can
be applied safely. This formalism will be valid for the calculation of both the signal and
the signal-to-noise ratio with additive noise (i.e., electronic noise: noise in the amplifier
circuitry, etc.) Indeed, it will also be valid for the quantum signal noise itself, as long as the signal level is not so low that the corpuscular character of the moving photocarriers becomes relevant, in the sense discussed here. For the latter case, the mechanical extrapolation of the SRT or any similar formalism to deal with the problem does not seem to be a very careful decision, to say the least; we could express it stating that the equations “have been pushed too far” and nobody seems to care... The idea that (even if the single incoming photon were to always be absorbed at the same location) a true, deterministic analog linear-system “impulse response” can be conceived for such an extreme situation, appears unrealistic, lacks any convincing theoretical or experimental support, and should therefore be abandoned.

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