The effect of density stratification on the resonant absorption of MHD waves in coronal loops

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August 20, 2010

Abstract

The standing quasi-modes of the ideal MHD in a zero-\( \beta \) cylindrical magnetic flux tube that undergoes a longitudinal density stratification and radial density structuring is considered. The radial structuring is assumed to be a linearly varying density profile. Using the relevant connection formulae of the resonant absorption, the dispersion relation for the fast MHD body waves is derived and solved numerically to obtain both the frequencies and damping rates of the fundamental and first-overtone, \( k = 1, 2 \) modes of both the kink \((m = 1)\), and fluting \((m = 2)\) waves. Where \( k \) and \( m \) are the longitudinal and azimuthal mode numbers, respectively.

\textbf{Key words:} Sun: corona – Sun: magnetic fields – Sun: oscillations

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1 Introduction

Transverse oscillations of coronal loops were first identified by Aschwanden et al. (1999) and Nakariakov et al. (1999) using the observations of TRACE. Nakariakov et al. (1999) reported the detection of spatial oscillations in five coronal loops with periods ranging from 258 to 320 s. The decay time was $(14.5 \pm 2.7)$ minutes for an oscillation of $(3.9 \pm 0.13)$ millihertz. Also Wang & Solanki (2004) described a loop oscillation observed on 17 April 2002 by TRACE in 195\AA.

They interpreted the observed loop motion as a vertical oscillation, with a period of 3.9 minutes and a decay time of 11.9 minutes. All these observations indicate strong dissipation of the wave energy that may be the cause of coronal heating.

Since the discovery of the hot solar corona about 66 years ago, different theories of coronal heating have been put forward and debated. Ionson (1978) was first to suggest that the resonant absorption of MHD waves in coronal plasmas could be a primary mechanism in coronal heating. Since then, much analytical and numerical work has been done on the subject. Rae and Roberts (1982) investigated both eikonal and differential equation approaches for the propagation of MHD waves in inhomogeneous plasmas. Hollweg (1987a,b) considered a dissipative layer of planar geometry to study the resonant absorption of coronal loops. Poedts et al. (1989, 1990) developed a finite element code to elaborate on the resonant absorption of Alfvén waves in circular cylinders.

Davila (1987) and Steinolfson & Davila (1993) did much work on resonant absorption through resistivity. Goossens et al. (2002) used the TRACE data of Ofman & Aschwanden (2002) to infer the width of the inhomogeneous layer for 11 coronal loops. Ruderman & Roberts (2002) did similar analysis with the data of Nakariakov et al. (1999). Van Doorsselaere et al. (2004a) used the LEDA code to study the resistive absorption of the kink modes of cylindrical models. They concluded that, when the width of the nonuniform layer was increased, their numerical results differed by as much as 25% from those obtained with the analytical approximation. In the vicinity of singularity, field gradients are large. Recognizing this, Sakurai et al. (1991a,b) and Goossens et al. (1992, 1995) developed a method to analyze dissipative processes in such regimes and to neglect them elsewhere.

Safari et al. (2006) studied the resonant absorption of MHD waves in magnetized flux tubes with a radial density inhomogeneity. Using the approximation that resistive and viscous processes are operative in thin layers surrounding the singularities of the MHD equations, they obtained the full spectrum of the eigenfrequencies and damping rates of the quasi-modes of the ideal MHD of the tube. Both surface and body modes were analyzed.

Verwichte et al. (2004), using the observations of TRACE, detected the multimode oscillations for the first time. They found that two loops are oscillating in both the fundamental and the first-overtone standing fast kink modes. According to the theory of MHD waves, for uniform loops the ratio of the period of the fundamental to the period of the first overtone is exactly 2, but the ratios found by Verwichte et al. (2004) are 1.81 and 1.64. However, these values were corrected to 1.82 and 1.58, respectively, by Van Doorsselaere et al. (2007) and thus clearly differ from 2. This may be caused by different factors such as the effects of curvature (see e.g. Van Doorsselaere et al. 2004b), leakage (see De Pontieu et al. 2001), magnetic twist (see e.g. Erdélyi & Fedun 2006; Erdélyi & Carter 2006) and density stratification in the loops (see Andries et al. 2005b; Erdélyi & Verth 2007). However, now it clear that the only cause of this deviation is the longitudinal density stratification. Curvature does not affect sufficiently the frequencies. Leakage can cause the damping of oscillations but practically does not affect the frequencies. Recently Ruderman (2007) showed that the magnetic twist also does not affect the frequencies.
Andries et al. (2005b) considered a coronal loop model with a straight cylindrically magnetized flux tube. They elaborated the effect of longitudinal density stratification on the oscillation frequencies and the damping rates of fast surface waves by resonant absorption.

Andries et al. (2005a), with the help of the method introduced by Andries et al. (2005b), showed that in the presence of a longitudinal density stratification the ratio of the periods of the fundamental kink mode of a coronal loop and of its first harmonic is lower than 2. They used this ratio to estimate the density scaleheight in the solar atmosphere.

Arregui et al. (2005) studied the effects of both radial and longitudinal density stratifications on the frequency and damping rate of resonantly fast kink modes. Using a numerical code (POLLUX), they solved the linear resistive MHD equations for a cylindrical flux tube model with a wide range of values of several loop parameters.

Safari et al. (2007) investigated the oscillations of coronal loops in presence of vertical stratification for a zero-\(\beta\) plasma. They solved the radial equation in the thin tube approximation and the transverse equation by both perturbational and numerical techniques. They confirmed the result obtained by Andries et al. (2005a) that the ratio of periods of the fundamental and first-overtone modes differ from 2 in loops with longitudinal density stratification.

Karami & Asvar (2007, hereafter Paper I) studied both the oscillations and damping of standing fast MHD body waves in coronal loops in the presence of both longitudinal density stratification and radial density structuring. The radial structuring was assumed to have a step-like density profile. They derived both the frequencies and damping rates of the fundamental, first-overtone and second-overtone frequencies of both the kink and fluting modes. They obtained that the ratios of the frequencies of the first-overtone and its fundamental mode for both the kink and fluting modes are lower than 2 (for unstratified loops).

In the present work, our aim is to investigate the effects of longitudinal density stratification and radial density structuring on resonant absorption of standing quasi-modes of the ideal MHD in the cold coronal loops observed by Verwichte et al. (2004) deduced from the TRACE data. This paper is organized as follows. In Sections 2 and 3 we combine the two techniques of Paper I and of Thompson & Wright (1993) to derive the equations of motion, introduce the relevant connection formulae and obtain the dispersion relation. In Section 4 we give numerical results. Section 5 is devoted to conclusions.

## 2 Equations of motion

The linearized MHD equations for a zero-\(\beta\) plasma are

\[
\frac{\partial \delta \mathbf{v}}{\partial t} = \frac{1}{4\pi \rho} \left\{ (\nabla \times \delta \mathbf{B}) \times \mathbf{B} + (\nabla \times \mathbf{B}) \times \delta \mathbf{B} \right\} + \frac{\eta}{\rho} \nabla^2 \delta \mathbf{v},
\]

\[
\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta \mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi \sigma} \nabla^2 \delta \mathbf{B},
\]

where \(\delta \mathbf{v}\) and \(\delta \mathbf{B}\) are the Eulerian perturbations in the velocity and magnetic fields; \(\rho\), \(\sigma\), \(\eta\) and \(c\) are the mass density, the electrical conductivity, the viscosity and the speed of light, respectively. The simplifying assumptions are the same as Paper I.

From Andries et al. (2005b), in the absence of dissipations, the perturbed quantities \(\delta \mathbf{B}(r, z)\) and \(\delta \mathbf{v}(r, z)\) can be expressed in terms of a complete set of orthonormal functions \(\psi^{(k)}(z)\) as follows

\[
\delta \mathbf{B}(r, z) = \sum_{k=1}^{\infty} \delta \mathbf{B}^{(k)}(r) \psi^{(k)}(z),
\]
where the same relation is kept for \( \delta v(r, z) \). Note that \( \psi^{(k)}(z) \) itself satisfies the eigenvalue relation \( L_A \psi^{(k)}(z) = \lambda_k \psi^{(k)}(z) \), where \( L_A \) is the Alfvén operator,
\[
L_A = \rho \omega^2 + \frac{B^2}{4\pi} \frac{\partial^2}{\partial z^2} = \rho \left( \omega^2 + v_A^2 \frac{\partial^2}{\partial z^2} \right),
\]
with Alfvén velocity \( v_A = \frac{B}{\sqrt{4\pi \rho}} \) and straight constant background magnetic field \( \mathbf{B} = B\hat{z} \).

One may show that the resonance absorption occurs where the Alfvén operator has a vanishing eigenvalue.

Let us denote the radius of the tube by \( R \) and a radius at with the resonant absorption occurs by \( R_1 < R \). The thickness of the inhomogeneous layer, \( l = R - R_1 \), will be assumed to be small. From Safari et al. (2006) and Paper I, the density profile is assumed to be
\[
\rho(r, z) = \rho_0(r) \exp \left[ -\mu \sin \left( \frac{\pi z}{L} \right) \right], \quad \mu := \frac{L}{\pi H},
\]
\[
\rho_0(r) = \begin{cases} 
\rho_{\text{in}}, & (r < R_1), \\
\frac{\rho_{\text{in}} - \rho_{\text{ex}}}{R - R_1} (R - r) + \rho_{\text{ex}}, & (R_1 < r < R), \\
\rho_{\text{ex}}, & (r > R),
\end{cases}
\]
where \( \mu \) is defined as stratification parameter, \( H \) and \( L \) are the density scale height and length of the loop, respectively. Also \( \rho_{\text{in}} \) and \( \rho_{\text{ex}} \) are the interior and exterior constant densities of the tube, respectively.

From paper I, in the absence of dissipations, in the interior region, \( r < R_1 \), solutions of Eqs. (1)-(2) are
\[
\delta B_z^{(\text{in},k)}(r) = \begin{cases} 
I_m(|k_{\text{in},k}|r), & k_{\text{in},k}^2 < 0, \text{ surface waves}, \\
J_m(|k_{\text{in},k}|r), & k_{\text{in},k}^2 > 0, \text{ body waves},
\end{cases}
\]
where \( J_m \) and \( I_m \) are Bessel and modified Bessel functions of the first kind, respectively. In the exterior region, \( r > R \), the waves should be evanescent. The solutions are
\[
\delta B_z^{(\text{ex},k)}(r) = K_m(k_{\text{ex},k} r), \quad k_{\text{ex},k} = -\frac{\lambda_{\text{ex},k}}{B^2/4\pi} > 0,
\]
where \( K_m \) is the modified Bessel function of the second kind.

Here we only consider the body waves. Because under coronal conditions, \( v_{\text{Ain}} < v_{\text{Aex}} \), magnetic flux tube supports fast body oscillations and there are no longer any surface modes (see Fig. 4 in Edwin & Roberts 1983). From Eq. (7) in Paper I and Eqs. (3), (6), one can obtain \( \delta B_z^{(\text{in},k)}(r, z) \) and \( \delta v_r^{(\text{in},k)}(r, z) \) in the interior region \( (r < R_1) \) as
\[
\delta B_z^{(\text{in})}(r, z) = \sum_{k=1}^{+\infty} A^{(\text{in},k)} J_m(|k_{\text{in},k}|r) \psi^{(\text{in},k)}(z),
\]
\[
\delta v_r^{(\text{in})}(r, z) = -\frac{i\omega B}{4\pi} \sum_{k=1}^{+\infty} \frac{k_{\text{in},k}}{\lambda_{\text{in},k}} A^{(\text{in},k)} J'_m(|k_{\text{in},k}|r) \psi^{(\text{in},k)}(z),
\]
where a prime on \( J_m \) and hereafter on each function indicates a derivative with respect to their appropriate arguments. The results for the exterior region are the same as Eq. (8), except that \( J_m \) and index "in" are replaced by \( K_m \) and "ex" everywhere.
3 Connection formulae, dispersion relation and damping

From Andries et al. (2005b) and according to the connection formulae developed by Thompson & Wright (1993), the jump across the boundary for $\delta B_z$ and $\delta \nu_r$ are

$$[\delta B_z] = 0,$$

$$[\delta \nu_r] = -\sum_{k=1}^{+\infty} \frac{B\bar{\omega} m^2}{4 r_A^2} \left( \phi^{(in,k)} \bigg| \delta B_z^{(in,k)} \right) \phi^{(in,k)},$$

where

$$\phi^{(in,k)} = \sqrt{\frac{\pi}{L}} \sum_{j=1}^{+\infty} \phi_j^{(in,k)} \sin \left( \frac{j \pi}{L} z \right),$$

$$L_A \phi^{(in,k)} = 0 \implies \phi_j^{(in,k)} = \left\{ \begin{array}{ll}
\frac{k^2 S_{kj}}{\rho \mu (1 + S_{kk})(j^2 - k^2)} & k \neq j \\
\frac{\rho \omega x_{kj}}{R} & k = j
\end{array} \right.,$$

and

$$L_{A1} = \frac{\partial L_A}{\partial r} \bigg|_{r=r_A} = \bar{\omega}^2 (1 + S_{kk}) \frac{\partial \rho_0 (r)}{\partial r} \bigg|_{r=r_A},$$

$$S_{kj} = \frac{2}{L} \int_0^L \sin \left( \frac{k \pi}{L} z \right) \left[ -\mu \sin \left( \frac{\pi}{L} z \right) \sin \left( \frac{j \pi}{L} z \right) \right] dz,$$

$$\left( \phi^{(in,k)} \bigg| L_{A1} \bigg| \phi^{(in,k)} \right) = \frac{\rho \omega x_{kj}}{R - R_1 \bar{\omega}} (1 + S_{kk}).$$

Note that $R_1 < r_A < R$ is the radius at which the singularity occurs and, $\bar{\omega} = \omega + i \gamma$ where $\gamma$ is damping rate. Substituting the fields of Eq. (8) in jump conditions, gives

$$\begin{pmatrix}
\Pi_1^{(ex,1)} \\
\Xi_1^{(ex,1)} \\
\Pi_1^{(ex,2)} \\
\Xi_1^{(ex,2)} \\
\Pi_1^{(in,1)} \\
\Xi_1^{(in,1)} + D_1^{(in,1)} \\
\Pi_1^{(in,2)} \\
\Xi_1^{(in,2)} + D_1^{(in,2)} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{pmatrix} \begin{pmatrix}
\Pi_1^{(ex,1)} \\
\Xi_1^{(ex,1)} \\
\Pi_1^{(ex,2)} \\
\Xi_1^{(ex,2)} \\
\Pi_1^{(in,1)} \\
\Xi_1^{(in,1)} + D_1^{(in,1)} \\
\Pi_1^{(in,2)} \\
\Xi_1^{(in,2)} + D_1^{(in,2)} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{pmatrix} = 0,$$

where

$$\Pi_j^{(in,k)} = J_m (x_k) \psi_j^{(in,k)},$$

$$\Pi_j^{(ex,k)} = K_m (y_k) \psi_j^{(ex,k)},$$

$$\Xi_j^{(in,k)} = \frac{J_m (x_k)}{\pi_k} \psi_j^{(in,k)},$$

$$\Xi_j^{(ex,k)} = \frac{K'_m (y_k)}{y_k} \psi_j^{(ex,k)},$$

$$D_j^{(in,k)} = -\frac{i B^2 m^2}{4 R^3} \sum_{l=1}^{+\infty} \phi_l^{(in,k)} \Pi_l^{(in,k)} \phi_j^{(in,k)},$$

$$A_j^{(ex,k)} = \frac{\rho \omega x_{kj}}{R - R_1 \bar{\omega}} (1 + S_{kk}).$$
oscillation frequency to the damping rate of the fundamental and first-overtone, in Figs. 1 to 7. Figures 1-4 show the frequencies and damping rates as well as the ratio of the frequencies obtained by numerical solution of the dispersion relation, Eq. (16). The results are displayed for the ratio \( \frac{\omega}{\rho} \) in Fig. 5. The results obtained for the ratio \( \frac{\omega}{\rho} \) are in accordance with those obtained by Dymova & Ruderman (2006). Also the results for all modes are in agreement with those obtained in Paper I.

The effects of density stratification on both the frequencies \( \omega \) and damping rates \( \gamma \) are calculated by numerical solution of the dispersion relation, Eq. (16). The results are displayed in Figs. 1 to 7. Figures 1-4 show the frequencies and damping rates as well as the ratio of the oscillation frequency to the damping rate of the fundamental and first-overtone, \( k = 1, 2 \), kink \( (m = 1) \) and fluting \( (m = 2) \) modes versus the stratification parameter \( \mu = L/\pi H \) for a loop with constant length and varying scale height. Figures 1 to 4 reveal that: i) Both frequencies, \( \omega_1, \omega_2 \) and their corresponding damping rates, \( |\gamma_1|, |\gamma_2| \) increase when \( \mu \) increases. The results for \( \omega_1 \) and \( \gamma_1 \) are in agreement with those obtained by Andries et al. (2005b) and Arregui et al. (2005). Also the results for all \( \omega_1 \) and \( \gamma_1 \) are in agreement with that obtained in Paper I. ii) For \( m = 1 \) with increasing \( \mu \), the damping rates, for instance \( |\gamma_1| \), increase (\( \simeq 2-170 \) percent) compared with a non-stratified loop. iii) The ratio of the oscillation frequency to the damping rate, \( \omega/|\gamma| \), is independent of stratification. This result is in good concord with those obtained numerically by Andries et al. (2005b) and analytically by Dymova & Ruderman (2006).

Figure 5 same as Fig. 1 shows the result of frequency, damping rate and ratio \( \omega/|\gamma| \) for the fundamental kink modes but for a thick inhomogeneous layer with \( l/R = 0.1 \). Comparing the results obtained for the ratio \( \omega_1/|\gamma_1| \) in Fig. 1 and 5 shows that the values of \( \omega/|\gamma| \) for \( l/R = 0.02 \) in Fig. 1, five times greater then the case of \( l/R = 0.1 \) in Fig. 5. This result is in good agreement with that obtained by Dymova & Ruderman (2006) who showed that under the assumption of uniform stratification, i.e. \( \rho(z) = \text{const.} \), the ratio \( \omega/|\gamma| \) for kink modes in thin tube-thin inhomogeneous layer approximation is proportional to \( (l/R)^{-1} \) and is independent of stratification.

The ratio of the frequencies \( \omega_2/\omega_1 \) of the first-overtone and its fundamental mode for both the kink \( (m = 1) \) and fluting \( (m = 2) \) modes are plotted in Fig. 6. Figure shows that for both modes, the ratio of the frequencies decreases from 2 (for unstratified loop) and approaches below 1.4 with increasing density stratification. The result of \( \omega_2/\omega_1 \) for \( m = 1 \) is in accordance with that obtained by Andries et al. (2005a) and Safari et al. (2007). Also the results for both \( m = 1 \) and \( m = 2 \) are in agreement with those obtained in Paper I. For \( m = 1, \mu = 0.59 \) and 0.89, the ratios \( \omega_2/\omega_1 \) are 1.821 and 1.576, respectively. These are in good agreement with the frequency ratios observed by Van Doorsselaere et al. (2007), 1.82±0.08 and 1.58±0.06, respectively, deduced from the observations of TRACE. Fig. 6 does not show a noticeable difference between the kink and fluting modes, but our numerical values shows that for a given stratification parameter \( \mu \), the frequency ratio \( \omega_2/\omega_1 \) increase slightly when the azimuthal mode number \( m \) increases.

4 Numerical results

As typical parameters for a coronal loop, we adopt radius = 10^3 km, length = 10^5 km, \( \rho_{in} = 2 \times 10^{-14} \) gr cm^{-3}, \( \rho_{ex}/\rho_{in} = 0.1 \), \( B = 100 \) G. For these parameters one finds \( v_{Ain} = 2000 \) km s^{-1}, \( v_{Aex} = 6400 \) km s^{-1} and \( \omega_{Ain} := \frac{\nu_A}{\lambda} = 0.02 \) rad s^{-1}.

The effects of density stratification on both the frequencies \( \omega \) and damping rates \( \gamma \) are calculated by numerical solution of the dispersion relation, Eq. (16). The results are displayed in Figs. 1 to 7. Figures 1-4 show the frequencies and damping rates as well as the ratio of the oscillation frequency to the damping rate of the fundamental and first-overtone, \( k = 1, 2 \), kink \( (m = 1) \) and fluting \( (m = 2) \) modes versus the stratification parameter \( \mu = L/\pi H \) for a loop with constant length and varying scale height. Figures 1 to 4 reveal that: i) Both frequencies, \( \omega_1, \omega_2 \) and their corresponding damping rates, \( |\gamma_1|, |\gamma_2| \) increase when \( \mu \) increases. The results for \( \omega_1 \) and \( \gamma_1 \) are in agreement with those obtained by Andries et al. (2005b) and Arregui et al. (2005). Also the results for all \( \omega_1 \) and \( \gamma_1 \) are in accordance with that obtained in Paper I. ii) For \( m = 1 \) with increasing \( \mu \), the damping rates, for instance \( |\gamma_1| \), increase (\( \simeq 2-170 \) percent) compared with a non-stratified loop. iii) The ratio of the oscillation frequency to the damping rate, \( \omega/|\gamma| \), is independent of stratification. This result is in good concord with those obtained numerically by Andries et al. (2005b) and analytically by Dymova & Ruderman (2006).

Figure 5 same as Fig. 1 shows the result of frequency, damping rate and ratio \( \omega/|\gamma| \) for the fundamental kink modes but for a thick inhomogeneous layer with \( l/R = 0.1 \). Comparing the results obtained for the ratio \( \omega_1/|\gamma_1| \) in Fig. 1 and 5 shows that the values of \( \omega/|\gamma| \) for \( l/R = 0.02 \) in Fig. 1, five times greater then the case of \( l/R = 0.1 \) in Fig. 5. This result is in good agreement with that obtained by Dymova & Ruderman (2006) who showed that under the assumption of uniform stratification, i.e. \( \rho(z)/\rho_{in}(z)=\text{const.} \), the ratio \( \omega/|\gamma| \) for kink modes in thin tube-thin inhomogeneous layer approximation is proportional to \( (l/R)^{-1} \) and is independent of stratification.

The ratio of the frequencies \( \omega_2/\omega_1 \) of the first-overtone and its fundamental mode for both the kink \( (m = 1) \) and fluting \( (m = 2) \) modes are plotted in Fig. 6. Figure shows that for both modes, the ratio of the frequencies decreases from 2 (for unstratified loop) and approaches below 1.4 with increasing density stratification. The result of \( \omega_2/\omega_1 \) for \( m = 1 \) is in accordance with that obtained by Andries et al. (2005a) and Safari et al. (2007). Also the results for both \( m = 1 \) and \( m = 2 \) are in agreement with those obtained in Paper I. For \( m = 1, \mu = 0.59 \) and 0.89, the ratios \( \omega_2/\omega_1 \) are 1.821 and 1.576, respectively. These are in good agreement with the frequency ratios observed by Van Doorsselaere et al. (2007), 1.82±0.08 and 1.58±0.06, respectively, deduced from the observations of TRACE. Fig. 6 does not show a noticeable difference between the kink and fluting modes, but our numerical values shows that for a given stratification parameter \( \mu \), the frequency ratio \( \omega_2/\omega_1 \) increase slightly when the azimuthal mode number \( m \) increases.
Figure 7 shows the ratios of frequencies $\omega_{\text{thick}}^{1}/\omega_{\text{thin}}^{1}$ and damping rates $\gamma_{\text{thick}}^{1}/\gamma_{\text{thin}}^{1}$ of the fundamental, $k = 1$, kink modes ($m = 1$) versus the stratification parameter for two tube radii $R/L = 0.02$ (thick) and 0.01 (thin). Figure 7 clears that both the frequencies and damping rates are independent of the tube radius, $R$, in the limit of slender tubes (see e.g. Van Doorsselaere et al. 2004a).

5 Conclusions

Resonant absorption of standing fast MHD body waves in coronal loops in presence of both the longitudinal density stratification and radial density structuring is studied. The radial structuring is assumed to be a linearly varying density profile. To do this, a typical coronal loop is considered as a straight pressureless cylindrical flux tube embedded in a constant background magnetic field. Using the relevant connection formulae, the dispersion relation is obtained and solved numerically for obtaining both the frequencies and damping rates of the fundamental and first-overtone kink and fluting modes. Our numerical results show that:

i) Both frequencies and damping rates of the fundamental and first-overtone modes of both the kink ($m = 1$) and fluting ($m = 2$) waves increase when the stratification parameter increases.

ii) The ratio of the oscillation frequency to the damping rate, $\omega/|\gamma|$ for both the kink ($m = 1$) and fluting ($m = 2$) modes is independent of stratification.

iii) The ratio of the frequencies $\omega_2/\omega_1$ for both the kink ($m = 1$) and fluting ($m = 2$) modes are lower than 2 (for unstratified loops), respectively, in presence of the longitudinal density stratification. The result of $\omega_2/\omega_1$ for kink modes is in accord with the TRACE observations.

Note that in our work, calculating the frequencies $\omega_1, \omega_2$ and their corresponding damping rates $\gamma_1, \gamma_2$ as well as the ratio of frequencies $\omega_2/\omega_1$, for fluting modes ($m=2$) is a new result in comparison with Andries et al. (2005a,b) and the rest is a repetition of the results previously obtained by other authors.

Acknowledgements. The authors thank to Prof. Michael Ruderman for his illuminating remarks that has enabled them to improve the clarity of the paper. This work was supported by the Department of Physics, University of Kurdistan, Sanandaj, Iran; the Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Maragha, Iran.

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Figure 1: Frequency of the fundamental kink mode \((m = 1)\) and its damping rate as well as the ratio of the oscillation frequency to the damping rate as a function of the stratification parameter \(\mu = L/\pi H\), for a loop with constant length and varying scaleheight. The loop parameters are: \(L = 10^5\) km, \(R/L = 0.01\), \(l/R = 0.02\), \(\rho_e/\rho_i = 0.1\), \(\rho_i = 2 \times 10^{-14}\) gr cm\(^{-3}\), \(B = 100\) G. Both frequencies and damping rates are in units of the interior Alfvén frequency, \(\omega_{\text{A}_i} = 0.02\) rad s\(^{-1}\).
Figure 2: Same as Fig. 1, for the first-overtone kink modes.

Figure 3: Frequency of the fundamental fluting mode ($m = 2$) and its damping rate as well as ratio of the frequency and the damping rate as a function of the stratification parameter $\mu = L/\pi H$. Auxiliary parameters as in Fig. 1.
Fluting Modes ($m=2$)

Kink Modes ($m=1$)

Figure 4: Same as Fig. 3, for the first-overtone fluting modes.

Figure 5: Same as Fig. 1, for the fundamental kink mode ($m = 1$) with $l/R = 0.1$. 

Figure 6: Ratio of the frequencies $\omega_2/\omega_1$ of the first-overtone and its fundamental mode versus $\mu = L/\pi H$ for both kink $(m = 1)$ and fluting $(m = 2)$ modes. Auxiliary parameters as in Fig. 1.

Figure 7: Ratios of the frequencies $\omega_1^{\text{thick}}/\omega_1^{\text{thin}}$ and the damping rates $\gamma_1^{\text{thick}}/\gamma_1^{\text{thin}}$ of the fundamental kink $(m = 1)$ mode versus $\mu = L/\pi H$ for two tube radii $R/L = 0.02$ (thick) and 0.01 (thin). Other auxiliary parameters as in Fig. 1.