Generation of arbitrary cylindrical vector beams on the higher order Poincaré sphere

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We propose and experimentally demonstrate a novel interferometric approach to generate arbitrary cylindrical vector beams on the higher order Poincaré sphere. Our scheme is implemented by collinear superposition of two orthogonal circular polarizations with opposite topological charges. By modifying the amplitude and phase factors of the two beams, respectively, any desired vector beams on the higher order Poincaré sphere with high tunability can be acquired. Our research provides a convenient way to evolve the polarization states in any path on the high order Poincaré sphere.

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Light beam with spatially inhomogeneous state of polarization, also referred to as vector beam, has been investigated for many years due to its unique properties [1]. Comparing with the conventional homogeneous polarization represented by fundamental Poincaré sphere, the cylindrical vector beams can be represented by higher order Poincaré sphere (HOPS) [2–4]. Particular interests and investigations focused on the vector beams with radial and azimuthal polarizations, which can be represented as two points on the equator of the first-order Poincaré sphere. Such beams can be generated by twisted nematic liquid crystal [5–7], inserting phase elements in the laser resonator [8], computer-generated sub-wavelength dielectric gratings [9, 10], a conical Brewster prism [11], spatially variable retardation plates [12], and a binary phase mask [13]. The vector beams with special polarization symmetry can give rise to uniquely high-numerical-aperture focusing properties that may find important applications in nanoscale optical imaging and manipulation [14–19].

In this Letter, a novel interferometric method is proposed and experimentally demonstrated to generate arbitrary cylindrical vector beams on the HOPS. Homogeneous polarization on the fundamental Poincaré sphere can be seen as the superposition of two orthogonal circular polarizations corresponding to the two poles of the Poincaré sphere. Similarly, vector beams on the HOPS can be regarded as the linear superposition of two orthogonal circular polarizations with opposite topological charges. For homogenous polarization, two quarter-wave plates (QWPs) and one half-wave plate (HWP) with adjustable optical axis angles can transform it to any point on the fundamental Poincaré sphere [2]. For the HOPS, we use a modified Mach-Zender interferometer with which the amplitude and phase factors in each arm can be modified, respectively.

In the parameter space of the HOPS, the state of polarization \(\psi_\ell\) can be represented by [3]

\[
\psi_\ell(\upsilon, \phi) = \cos \left( \frac{\upsilon}{2} \right) e^{-i\phi/2} L_\ell + \sin \left( \frac{\upsilon}{2} \right) e^{i\phi/2} R_\ell.
\]

Here, \(\phi\) is the azimuthal angle and \(\upsilon\) the polar angle in the spherical coordinate, respectively. \(L_\ell\) and \(R_\ell\) are orthogonal circular polarization vortexes with \(L_\ell = (\hat{x} + i\hat{y}) e^{-i\sigma \ell} / \sqrt{2}\) and \(R_\ell = (\hat{x} - i\hat{y}) e^{i\sigma \ell} / \sqrt{2}\), possessing spin angular momentum \(\sigma \hbar\) (\(\sigma = \pm 1\)) per photon where \(\hbar\) is the Planck constant. The factor \(e^{i\phi/2}\) is the vortex phase associated with the orbital angular momentum \(\ell \hbar\) per photons where \(\ell\) is an integer number (\(\ell = \pm 1, \pm 2, \pm 3, \ldots\)) [20].

For arbitrary points on the HOPS, the state of polarization \(\psi_\ell(\upsilon, \phi)\) can be described as the linear combination of two orthogonal circular polarizations with opposite topological charges. Equation (1) indicates that \(\cos(\upsilon/2)\) and \(\sin(\upsilon/2)\) are amplitude factors, while \(\exp(-i\phi/2)\) and \(\exp(i\phi/2)\) are phase factors, respectively. By separately modifying the amplitude and phase factors of the two orthogonal components, any desired vector beam on the HOPS can be achieved. Generally, the equatorial points on the HOPS represent linear polarized...
where \( \alpha \) can convert the LG caded beam splitters (BSs) and an odd number of reflections modified Mach-Zender interferometer comprising of two cascaded beam splitters (BSs) and an odd number of reflections can convert the LG\(_0^1\) mode into LG\(_0^{-1}\) mode. In addition, we add PP1 and PP2 (two cascaded GLPs of which the second one is fixed) in each arm of the interferometer to justify the intensity factors. HH (a pair of HWPs within which only the second one can be rotated) is added to modify the phase factors.

\( \beta \) is the rotation angle of the second HWP. With appropriate rotation angles \( \alpha_1, \alpha_2, \) and \( \beta \), we can realize arbitrary cylindrical vector beams represented by the corresponding points \( (\nu, \phi) \) on the HOPS. This is the main point of the theory.

To examine the polarization of the generated vector beams, we will measure the Stokes parameters by a QP and a CCD camera (see Fig. 2), \( S_1, S_2, \) and \( S_3 \), which are given by

\[
S_1 = \frac{I_{45^\circ}^0 - I_{90^\circ}^0}{I_{45^\circ}^0 + I_{90^\circ}^0}, \quad S_2 = \frac{I_{45^\circ}^{135^\circ} + I_{135^\circ}^{45^\circ}}{I_{45^\circ}^{135^\circ} - I_{135^\circ}^{45^\circ}}, \quad S_3 = \frac{I_{135^\circ}^0 - I_{135^\circ}^{45^\circ}}{I_{135^\circ}^0 + I_{135^\circ}^{45^\circ}},
\]

where \( I_i^j \) stands for the intensity of the light recorded by the CCD, and \( i \) and \( j \) are the optical axis directions of the QWP and GLP, with respect to the \( x \) axis, respectively [22]. Note that the intensity profiles of \( S_3 \) depends on the polar angle \( \nu \).

The linear polarized vector beams represented by the equatorial points of the HOPS have \( S_3 = 0 \) at each transverse point. And \( S_3 = \pm 1 \) correspond to circular polarizations. But for the \( S_1 \) and \( S_2 \), the intensity patterns are both dependent on the angles \( \nu \) and \( \phi \).

We first generate the vector beams on the equator and the south pole (A and B) on the first-order Poincaré sphere as shown in Fig. 1. For the vector beam on the point A (\( \nu = \pi/2 \) and \( \phi = 0 \)), from Eqs. (2)-(4), we get \( \alpha_1 = 90^\circ, \alpha_2 = 0^\circ, \) and \( \beta = 0^\circ \). In order to achieve the vector beam on the points B, we must ensure that \( \alpha_1 = 90^\circ, \alpha_2 = 0^\circ, \) and \( \beta \) be an arbitrary value. The Stokes parameters of the generated beams are measured to verify the theoretical prediction as shown in Fig. 3, which indicates the high quality of the generated vector beams. Because the point A represents a linearly polarized vector beam, so \( S_3 = 0 \). While B is a circular polarized vortex beam, hence
mental results agree well with the theoretical calculations. In order to obtain the vector beams with more complex polarization distribution [24–27].

In conclusion, we have developed a modified Mach-Zender interferometer to generate arbitrary cylindrical vector beams on the HOPS. By suitably modifying the magnitude and phase factors of the interfering beams, we can achieve any desired vector beam on the HOPS. The theoretical prediction is verified by measuring the far-field Stokes parameters. Our scheme makes it possible to generate versatile cylindrical vector beams with high tunability and thereby providing a convenient way to evolve the polarization state in any path on the high order Poincaré sphere.

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Fig. 4. The Stokes parameters of the generated vector beams corresponding to $C$ and $D$ on the first-order Poincaré sphere (Fig. 1). The first column shows the intensity distribution and the next three columns are the Stokes parameters $S_1$, $S_2$, and $S_3$, respectively. First and third rows: the theoretical results for $C$ and $D$, respectively. Second and fourth rows: the corresponding experimental results.

$S_1 = S_2 = 0$ and $S_3 = -1$ with its intensity profile equals to the intensity of the beam.

We now expect to obtain vector beams on the points $C$ with $\nu = \pi/4$ and $\phi = 0$, as well as $D$ with $\nu = 3\pi/4$ and $\phi = \pi/2$ as shown in Fig. 1. This two kinds of vector beams can be transformed from the beam on the point $A$. According to Eqs. (2) and (4), we can generate the beam on the point $C$ by changing the angle $\alpha_2$ to 49.9°. Similarly, the vector beam on the point $D$ can be realized by rotating the PP1 with $\alpha_1 = 40.1°$ and HH with $\beta = 22.5°$, respectively. All the above transformations to realize the vector beams on points $C$ and $D$ are schematically illustrated in Fig. 1. Then we measure the Stokes parameters of the generated beams (Fig 4). Remarkably, the profile of $S_1$ and $S_2$ consists of four intensity lobes and the profile of $S_3$ exhibits a doughnut shape with a dark center. The experimental results agree well with the theoretical calculations. In order to obtain arbitrary cylindrical vector beams on the first-order Poincaré sphere, we can adjust the optical axis angles of PP1, PP2, and HH.

All the above discussions are limited to the case that the topological charge is $\ell = +1$. Actually, we can also obtain the cylindrical vector beams and realize its evolution on the HOPS by a metasurface [23], but it has less tunable to achieve a cylindrical vector beams with other topological charges ($\ell \neq 1$) than using the SLM. Here, this can be conveniently achieved by modulating the phase picture displayed on the SLM, and all the theoretical and experimental schemes are applicable in this manipulation. To generate the vector beams on the other HOPS with the opposite topological charge $-\ell$, one simple approach is to invert the phase picture on the SLM, and another way is to insert a HWP in each arm of the interferometer.

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