Time evolution of quantum correlations in presence of state dependent bath

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Abstract
The emerging quantum technologies heavily rely on the understanding of dynamics in open quantum systems. In the Born approximation, the initial system-bath correlations are often neglected which can be violated in the strong coupling regimes and quantum state preparation. In order to understand the influence of initial system-bath correlations, we study the extent to which these initial correlations and the distance of separation between the qubits influence the dynamics of quantum entanglement and coherence. It is shown that at low temperatures, the initial correlations have no role to play while at high temperatures, these correlations strongly influence the dynamics. Furthermore, we have shown that the distance of separation between the qubits in presence of a collective bath helps to maintain entanglement and coherence at long times.

1. Introduction
Quantum correlations (entanglement) [1] form a key concept for the foundational understanding of quantum mechanics and has been realized as a precious resource for various tasks that are impossible in a classical domain like quantum teleportation, cryptography, and various other processings [2–5]. The deeper insights into the dynamics of quantum correlations is of great importance since the real quantum system always interacts with its bath which leads to decoherence of quantum superposition and entanglement degradation [6–10]. A complete understanding of the dynamics of entanglement [8–10] relies on an available measure that can reflect the time variation of the system of interest. Here, we utilize Concurrence [11, 12] as the entanglement measure to understand the underlying dynamics in presence of initial system-bath correlations. In addition to the quantum entanglement, coherence has been proposed as an alternative resource for quantum information processing [13–17]. It has attracted much attention over the past decade both in theory and experiment [18–23]. For example, the relation between coherence and other quantum correlations like quantum discord has been explored in [24]. Also, it has been shown that the coherence exists in photosynthetic complexes [25–27], therefore can play an important role in explaining the high efficiency of these complexes, which in turn have technological benefits. There exist several quantifiers for coherence [14, 24] in a given quantum system like $l_1$-norm, relative entropy, etc. In this work, we utilize the $l_1$- norm which in simple terms represents the sum of the absolute values of off-diagonal elements of a given density matrix. Due to its important role in quantum mechanics [28, 29], quantum information [30, 31], and quantum biology [32–35], the behaviour of coherence during the evolution of a system is necessary to be investigated.

The dynamics of a given quantum system coupled to a bath is categorized either as Markovian or non-Markovian [36–38]. In Markovian dynamics, the system loses information to the environment irreversibly. In contrast, many quantum systems show non-Markovian behaviour in which there exists a flow of information from the bath to the system, signifying the presence of quantum memory effects [39–47]. These memory effects play an important role in the physical realization of certain protocols like recent proposals for the generation of entangled states [48, 49], for schemes of dissipative quantum computation [50, 51], for the design of quantum memories [52] and the enhancement of the efficiency in quantum metrology [53]. Furthermore, the presence of
non-Markovian effects and system-bath correlations invalidates the initial state in which the system and the bath are independent [54, 55]. Especially in the experimentally relevant case where the qubit system is excited out of equilibrium and the subsequent dynamics is probed, proper treatment of the initial state is crucial.

There has been a tremendous amount of work towards understanding the non-Markovian behaviour in single and many qubits coupled with a bosonic bath [56–62]. In most of these cases, the Born approximation is used which means that the joint initial state of the system and bath are assumed to be uncorrelated. However, this assumption is often violated when there exists strong interaction between the system and the bath [46, 63–71]. In particular, quantum state preparation can lead to strong system-bath correlations thus affecting the subsequent dynamics. In this regard, various works have critically examined the initial system-bath correlations in the spin–Boson model [72], superconducting qubits [73], quantum dots [74] etc. For example, Morozov et al [66, 72], have considered an exactly solvable dephasing model of a single qubit interacting with a bosonic bath in presence of initial system–bath correlations, while Smirne et al [75] have considered the Jaynes–Cummings model and Majeed et al [76] have studied the entanglement dynamics in presence of spin bath. These models, however, consider either single qubit or many coupled with individual baths. Furthermore, the distance of separation between the qubits in presence of initial correlations is not taken into account in these works. Our main objective in this work is to study the effect of distance-dependent interaction between qubits and the collective bath in presence of initial system–bath correlations. Such kind of settings can be obtained using cold-atom impurities immersed in a Bose–Einstein condensate (BEC) [77]. The positioning of immersing cold atoms in the BEC would yield a distance-dependent interaction with non-trivial bath spectral density. Thus the present work would be mainly important for quantum information processing using cold atoms.

This paper is organised in the following way. In section II, we introduce the model of qubits interacting with a collective bath in presence of initial system-bath correlations and calculate the time evolution of two qubits density matrix. In section III, we discuss the dynamics of entanglement given by concurrence and coherence. Finally, we conclude in section IV.

2. Model calculations

In this section, we introduce our model of qubits interacting with a bosonic bath. In spin representation, the total Hamiltonian of the model is written as ($\hbar = 1$).

$$H = H_S + H_B + H_{int}$$

$$= \frac{\omega_0}{2} \sum_i \sigma_i^z + \sum_k \omega_k b_k^\dagger b_k + \sum_{ik} \sigma_i^z (g_k e^{-i\kappa} b_k + h.c.)$$

where $\omega_0$ is the energy splitting of the qubit. $b_k, b_k^\dagger$ are annihilation and creation operators corresponding to the $k$-th bath mode with energy $\omega_k$. $\sigma_i^z$ are Pauli matrix and position vector of $i$th–qubit respectively. Here, h.c. means Hermitian conjugate. We assume the distance of separation between two qubits equal to $L = \tilde{r}_i - \tilde{r}_j$, $g_k$ is the system-bath coupling. This kind of model can be realized in an ultracold setting by immersing an ultra cold gas trapped in an optical lattice in Bose–Einstein condensate [77, 78]. The low lying excitations of BEC i.e. Bogoliubov phonons will act as the bosonic bath. We assume linear dispersion for bath modes (phonon or photon type). In the BEC setting, it will correspond to phonon like excitations in the long-wavelength domain instead of particle like excitations at large momentum.

In this work, we will assume a particular type of initial state of the bath and system as a case study of the dynamics of quantum correlations. Before $t = 0$, we initially start in the state for the combined system and bath that are thermal states of the whole system

$$\rho = \frac{e^{-\beta H}}{Z}$$

where $Z$ is the partition function and $\beta = \frac{1}{T}$. This should be compared to usual situation where the thermal state is considered only with respect to the bath states i.e. $\rho_B = \frac{e^{-\beta H_B}}{Z_B}$. This type of state could arise in a situation where the measurement apparatus is prepared in the vicinity of the system prior to the state preparation. Since the system is rather small, the time taken to reach thermal equilibrium can be rather short, and the state (1) is attained quickly before the state preparation is performed.

Now we would like to prepare the state of the system in a manner such that a projective measurement is made on the system. For the system the projection operators are

$$P_S = \{|\psi\rangle\langle\psi|, |\psi_0\rangle\langle\psi_0|\}$$

where $|\psi\rangle$ is the initial state of the system and $|\psi_0\rangle$ is the state that is orthogonal to this state. Now we post-select on the state $|\psi\rangle$ such that the system is ensured to have the state.
\[ \rho_s(0) = \ket{\psi}\bra{\psi}. \]  

and the bath states then take a form

\[ \rho_b^\psi(0) = \frac{\bra{\psi}\exp(-\beta H)\ket{\psi}}{Tr_b(\bra{\psi}\exp(-\beta H)\ket{\psi})} \]  

where there is a dependence on the state of the system because the original state (1) were thermal states in the space of the system and bath. The initial state of the whole system is therefore written as

\[ \rho(0) = \rho_s(0) \otimes \rho_b^\psi(0) = \ket{\psi}\bra{\psi} \otimes \frac{\bra{\psi}\exp(-\beta H)\ket{\psi}}{Tr_b(\bra{\psi}\exp(-\beta H)\ket{\psi})}. \]  

The primary difference to uncorrelated case \( \rho(0) = \rho_s(0) \otimes \rho_b(0) = \ket{\psi}\bra{\psi} \otimes \frac{e^{-\beta H}}{Z} \) is that the bath state according to this preparation [equation (5)] depends in a non-trivial way on the system state. We can understand such type of density matrices in the following way. A system is considered to be in the equilibrium with bath at \( t < 0 \) and at time \( t = 0 \), we make a measurement on the system only. Using the principles of quantum measurement, the state after a projective measurement will be of the form \( \sum_i \hat{\Pi}_i \frac{e^{-\beta H}}{Z} \hat{\Pi}_i^\dagger \), where \( \hat{\Pi}_i \) is the projection operator onto a given state of the system, and \( \frac{e^{-\beta H}}{Z} \) is the equilibrium state of system and bath. In case of selective measurement, the system is prepared in the state \( \ket{\psi} \), and this sum collapses to a single term as given in equation (5). Thus if an open quantum system interacts with a bath, then the process of preparation of the state of the bath will effect the state of bath as well. In this regard, we say the state in equation (5) contains initial correlations which are due to pre-measurement equilibrium state. These density matrices were used to study dynamics of some spin-boson models [79] and the environment-induced decoherence in quantum Brownian motion [80]. The usual initially correlated states considered by Ming-Liang and Zhang [54, 55] are pure states and valid for zero temperature. Furthermore, in these pure states the number of excitations is fixed and the overall state of the bath is a Gibb’s state. However, the correlated state considered in equation (5) is a non-Gibb’s state for bath and is in general a mixed state.

We assume the initial state of the system to be a general two qubit state \( \ket{\psi} = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \) with \( |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1 \). For this state, the corresponding bath density matrix can be written as (for detailed calculations see appendix A):

\[ \rho_b^\psi(0) = \frac{1}{Z} [d|\beta|^2 e^{-\beta H_{ab}} + |b|^2 e^{-\beta H_{bb}} + |c|^2 e^{-\beta H_{cc}} + |d|^2 e^{-\beta H_{dd}}]. \]  

Next, our main interest is to calculate the reduced density matrix of the system by tracing out degrees of freedom of the bath:

\[ \rho_s(t) = Tr_b[U(t) \rho(0) U(t)\dagger] \]  

where \( U(t) = T e^{-i \int_0^t dt' H_I(t')} \) is the time evolution operator and \( H_I(t) \) is the interaction Hamiltonian in interaction picture. We write \( U(t) = e^{-i f(t)} e^{i \sum \lambda(t)} \) where \( f(t) = 2 \sum_k \sin \omega_k(t) \) is a function of time only and \( \lambda(t) = \sum_k (\alpha_k(t) b_k + \alpha_k^* b_k^\dagger) \) with \( \alpha_k(t) = \frac{1 - e^{-i \omega_k t}}{\omega_k} \). Therefore, we can write

\[ \rho_s(t) = Tr_b[U(t) \rho(0) U(t)\dagger] \]  

\[ = Tr_b[e^{i \sum \lambda(t)} \rho_b^\psi(0) e^{-i \sum \lambda(t)}] \]  

after a cumbersome calculations (see appendix B), we get

\[ \rho_s(t) = \left(\begin{array}{cccc}
|a|^2 & ab^* \varphi(t) & ac^* \zeta(t) & ad^* \kappa(t) \\
ab a^* \varphi(t) & |b|^2 & bd^* \zeta(t) & cd^* \kappa(t) \\
ac a^* \zeta(t) & bd^* \kappa(t) & |c|^2 & de^* \varphi(t) \\
ad^* \kappa(t) & db^* \zeta(t) & de^* \varphi(t) & |d|^2
\end{array}\right) \]  

where the different functions are written explicitly in appendix. In the next section we examine the entanglement and coherence in various approximations.
3. Dynamics of entanglement and coherence

3.1. Concurrence

In this section, we analyze quantum entanglement measured by concurrence in a two qubit system considered in this work. For a density matrix \( \rho \), the concurrence is defined as

\[
\mathcal{C} = \max \{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, \}
\]

where \( \lambda_i, i = 1, 2, 3, 4 \) are the eigenvalues of the matrix \( \rho \rho^\dagger \), taken in descending order and \( \rho = (\sigma_1^z \otimes \sigma_1^z) \rho^B (\sigma_1^z \otimes \sigma_1^z) \) is the time reversal density matrix. \( \rho^B \) is the conjugation obtained in standard basis.

For an unentangled state \( \mathcal{C} = 0 \) while for maximally entangled state \( \mathcal{C} = 1 \). In order to simplify our calculations, we assume \( a = \sqrt{p}, d = \sqrt{1 - p} \) and \( b = 0 = c \) so that our initial state is the class of states \( \psi = \sqrt{p}[00] + \sqrt{1 - p}[11] \), with \( 0 \leq p \leq 1 \). The same analysis holds for other types of states as well. For this state, time evolved concurrence can be written as

\[
\mathcal{C}(t) = 2 \max \{ 0, \mathcal{C}(t) \},
\]

with

\[
\mathcal{C}(t) = 2 \sqrt{p(1 - p)} \sqrt{\cos^2 \Phi + \sin^2 \Phi \tanh^2 (\beta \omega_0) e^{-\gamma(t)}}
\]

where

\[
\Phi(t) = 8 \sum_k |g_k|^2 \sin \omega_k t [1 + \cos (\vec{k}, \vec{l})] |g_k|^2 \sin \omega_k t [1 + \cos (\vec{k}, \vec{l})]
\]

\[
\gamma(t) = 4 \sum_k |g_k|^2 [1 + \cos (\vec{k}, \vec{l})] \frac{1 - \cos \omega_k t}{\omega_k} \coth \frac{\beta \omega_k}{2}
\]

We rewrite the concurrence \( \mathcal{C}(t) = e^{-\gamma(t)} \mathcal{C}(0) \), with \( \mathcal{C}(0) = 2 \sqrt{p(1 - p)} \) as the initial entanglement of the system. Thus unentangled states \( p = 0 \) and \( p = 1 \) remain always unentangled. This is due to the dephasing nature of the interaction. Thus, all values of \( 0 < p < 1 \) show the same type of behaviour. Therefore, without loss any generality we take \( p = 1/2 \). Also, \( \Gamma(t) = \gamma_0(t) + \gamma(t) \) with

\[
\gamma_0(t) = -\frac{1}{2} \log [\cos^2 \Phi + \sin^2 \Phi \tanh^2 (\beta \omega_0)]
\]

as the decoherence function due to initial correlations and \( \gamma(t) \) is the standard decoherence function.

3.2. Coherence

Quantum coherence directly stems from the superposition principle that enables it to show quantum interference phenomena. There have been several proposals to quantify coherence in a given quantum system [13, 24]. However, we take \( l^1 \)-norm quantification of coherence in the present work. It is an intuitive measure related to the magnitude of off-diagonal terms of the density matrix. From the resource theory point of view, any coherence measure \( \mathcal{N}^3 \) must satisfy the following axioms [15]:

1. Let \( \mathcal{I} \) be the set of incoherent states, which are diagonal in a preferred basis, then \( f \) for a density matrix \( \rho \in \mathcal{I} \).
2. It should be monotonic under incoherent selective measurements, i.e. \( \mathcal{N}(\rho) \geq \sum_i p_i \mathcal{N}(\rho_i) \). Here \( \rho_i = K_i \rho K_i^\dagger \) and \( p_i = Tr(K_i \rho K_i^\dagger) \), where \( K_i \) are Kraus operators \( \sum_i K_i^\dagger K_i = 1 \) and \( K_i K_j^\dagger \subset \mathcal{I} \).
3. It should be convex, i.e. \( \mathcal{N} (\sum_i p_i \rho_i) \leq \sum_i p_i \mathcal{N}(\rho_i) \) for any set of states \( \rho_i \) with probability distributions \( p_i \).

Based on these axioms, relative entropy and \( l^1 \)-norm were shown to be valid measures. \( l^1 \)-norm \( \mathcal{N}_1 \) is defined as

\[
\mathcal{N}_1 = \min_{\delta \in \mathcal{I}} \| \rho - \delta \|_1
\]

After the optimization, we get the following expression for the coherence in the standard basis

\[
\mathcal{N}_1 = \sum_{i,j} | \langle i| \rho | j \rangle |.
\]

This simply represents the sum of the absolute values of off-diagonal elements of a density matrix under consideration and therefore captures the notion of interference in a quantum state. For the states under

\(^3\)To avoid confusion with concurrence \( \mathcal{C} \), we use \( \mathcal{N}^3 \) for coherence.
consideration we have

\[ \mathcal{N}_g = 2\sqrt{\cos^2 \Phi + \sin^2 \Phi \tanh^2 (\beta \omega_k)} e^{-\gamma(t)} = 2\mathcal{C}(t). \]

(17)

This is true for other types of states \((\sqrt{\rho} |01\rangle + \sqrt{1-\rho} |10\rangle)\) as well. Thus, quantum entanglement given by concurrence is equal to half of the coherence given by \(\mathcal{N}_g\). Therefore, it is sufficient to analyze the concurrence \(\mathcal{C}(t)\). Furthermore, this provides an explicit example where entanglement can be measured by coherence \([16, 20]\).

### 3.3. Decay of concurrence

Next, we analyze the decoherence functions \(\Phi(t)\) and \(\gamma(t)\) using various approximations. Before evaluating the sum in equations (15), we realize that \(\gamma(t)\) becomes zero for certain modes of the bath. For \(k, L = (2n + 1)\pi, n = 0, 1, \ldots\), we have \(\cos \theta = -1\), which makes \(\gamma(t) = 0\) and \(\Phi(t) = 0\). It implies there exists certain bath modes for \(\theta = \cos^{-1}\left(\frac{(2n + 1)\pi}{kL}\right)\) that do not lead to the decay of entanglement (or coherence), \(\theta\) is the angle between \(k\) and \(L\). In other words, all bath modes do not couple to the qubits which can lead to decoherence. This can be intuitively understood from the fact that these bath modes do not resolve the coherence of the bath modes and the temperature is measured with \(\Theta\), \(0 < \Theta < 1\), reflects the fact that the modes do not resolve the coherence.

Therefore, we write

\[ \gamma(t) = 16 \int \frac{d^3k}{(2\pi)^3} |g_{kk}|^2 \cos^2 (k, L) \frac{\sin \omega_k t}{\omega_k^2} \coth \frac{\beta \omega_k}{2} = \frac{4g_0}{\Omega^2 \pi^3 v^3} \int_0^\infty d\omega \omega e^{-\frac{\omega^2}{8}} \left(1 + \frac{\sin \omega s}{\omega s}\right) \sin \frac{\omega_k t}{2} \coth \frac{\beta \omega}{2} \]  

(19)

and

\[ \Phi(t) = 8 \int \frac{d^3k}{(2\pi)^3} |g_{kk}|^2 \cos^2 (k, L) \frac{\sin \omega_k t}{\omega_k^2} = \frac{4g_0}{\pi v^2 \Omega^2} \int_0^\infty d\omega \omega e^{-\frac{\omega^2}{8}} \left(1 + \frac{\sin \omega s}{\omega s}\right) \sin \omega t \]  

(20)

where \(s = \frac{L}{v}\) is the time scale provided by the interactions mediated by the bath modes between qubits separated by the distance \(L\) and \(v\) is the velocity of bath modes. In a typical experimental setup \([82]\) for cold atoms, we can vary distance between two qubits from \(L = 100 \text{ nm}\) to 10 \(\mu\text{m}\) with speed of sound \(v = 350 \text{ ms}^{-1}\), which yields \(s = 0.1 \text{ ns}\) to 10 ns. Also, we have different energy scales arising in our model. The highest energy scale is given by the cut-off frequency \(\Omega\) which provides the relaxation time scale for the bath \(\tau_\Omega \sim \frac{1}{\Omega}\); the energy scale \(\omega_0\) provides a natural time scale for the relaxation of qubits \(\tau_s \sim \frac{1}{\omega_0}\). Now we can parametrize the above equations (21) and (22) in the following way:

\[ \omega \rightarrow \frac{\omega}{\Omega} \quad t \rightarrow \Omega t, \quad s \rightarrow \Omega s \]  

and the temperature is measured with respect to \(\Omega\): \(\beta \rightarrow \beta \Omega\). For notational convenience, we take \(\Omega = 1\) without loss of generality.

In order to understand the time evolution of concurrence, we first observe the behaviour of \(\gamma(t)\), \(\gamma(t)\) and \(\Phi(t)\) for different temperature regimes and time scales \(s\). For this, we plot \(\gamma(t)\) in figure 1(a) for low temperatures \(\beta \Omega \gg 1\), while in figure 1(b) we have \(\gamma(t)\) plotted for temperatures \(\beta \Omega \ll 1\) which correspond to high temperature cases. From these plots, we observe that in the long time limit \(\gamma(t)\) saturates at a certain value. We see that for short distances \(L\) or small \(s\), \(\gamma(t)\) varies appreciably at high and low temperatures in comparison to large \(s\) values i.e. large \(L\), thus \(L\) plays an important role to protect system from decoherence. Furthermore, the non-monotonic variation of \(\gamma(t)\) leads to the identification of three regions of dynamics, which are due to three time scales arising in the system dynamics. These time scales are set by the cutoff energy \(\Omega\) of the bath modes and the temperature \(\beta\) (thermal fluctuations). In the first regime, where \(t < \Omega^{-1}\) or \(\Omega t < 1\), we have \(\gamma(t) \propto t^2\) which can be obtained from the Taylor series expansion of equation (21). This results in an \(e^{-\tau^2}\)-type behaviour of entanglement decay, thus showing a non-Markovian behaviour \([83]\). In the second
regime $\Omega^{-1} \leq t < \beta$ i.e. $1 \leq \Omega t < \beta \Omega$, the quantum fluctuations and thermal fluctuations start to compete before thermal fluctuations take over. In such a regime, we have $\gamma(t) \propto t$ and thus Markovian ($e^{-t}$) behaviour sets in the dynamics. Thirdly, the long time limit, there is a saturation region that implies existence of finite value of quantum correlations. Qualitatively, this crossover from non-Markovian to Markovian then finally to saturation region can be understood from the nature of bath spectral function $f(\omega)$ and the derivatives of $\gamma(t)$, as pointed by Ming-Liang et al [84]. The dynamics for a certain interval of time can be non-Markovian if $\frac{d\gamma(t)}{dt} < 0$ for a given set of initial states. Therefore, by choosing a suitable bath spectral density we can tune dynamics from non-Markovian to Markovian. Due to complicated nature of coupling $g_a$, we do not have a simple analytical formula for $f(\omega) = \sum_\omega g_k^2 \delta(\omega - \omega_k)$. However, modelling phenomenologically, we can write $f(\omega) \propto \omega e^{-\omega^2/\Omega^2}$. It can be shown [79] that for a qubit dephasing, $s \geq 1$, the bath feeds information back to the system. A detailed analysis for this purely quantum mechanical phenomenon associated with the interplay between quantum and thermal fluctuations will be treated separately in terms of a given non-Markovian measure [85].

In figure 1 (c), we have decay function due to initial correlations $\gamma_0(t)$, plotted for different values of $s$ and $\beta \Omega \ll 1$. We see that for small $s$, the $\gamma_0(t)$ has pronounced non-Markovian decay in comparison to large $s$. Since concurrence has an $e^{-\gamma_0(t)}$ factor, which therefore implies initial system-bath correlations contribute to concurrence $C(t)$ for small $L$, while for large $L$, thermal fluctuations influence the decay. Also, in figure 1 (d), we plot $\Phi(t)$, for different values of $s$. We see that with the overall behaviour of $\Phi(t)$ is same for all values of $s$, in other words, distance of separation of qubits does not influence the dephasing. In the long time limit, $\Phi(t)$ goes to zero which implies that influence of initial correlations on the entanglement remains over long periods of time.

Now, we plot time evolution of concurrence for different values of $s$ and temperature. At low temperatures $\beta \Omega \gg 1$ (i.e. in the vicinity of $T \to 0$) we see from the equation (13), the initial correlations $\gamma_0(t)$ do not contribute to entanglement dynamics. This can be understood as follows: at low temperatures, we have joint state of the system and bath $\rho = e^{-\beta H}/Z \rightarrow |GS\rangle \langle GS|$, where $|GS\rangle$ is some ground state of the Hamiltonian system (S) plus bath (B) $H_S + H_B$. For the model under consideration, $|GS\rangle = |1\rangle_S \otimes |0\rangle_B$. Thus

$$
\rho_S^0 = \frac{\langle \psi | \exp(-\beta H) |\psi \rangle}{Tr_B(|\psi \rangle \langle \psi| \exp(-\beta H))} \rightarrow |0\rangle_S \langle 0|_S.
$$

Therefore, $\rho_S^0$ has no non-trivial dependence on the system parameters and thus we have an initially uncorrelated state. This is purely due to thermal nature of the initial state. However, for other temperatures $\beta \Omega \sim 1$ and high temperatures $\beta \Omega \ll 1$, we have $\gamma_0(t)$ influencing the

Figure 1. Variation of decoherence functions $\gamma_1(t)$, $\gamma_0(t)$ and dephasing function $\Phi(t)$ for different values of $s$ in (a) and (b) we have $\gamma(t)$ versus $t$ in units of $\Omega$ for $\beta \Omega \gg 1$ (low temperature) and $\beta \Omega \ll 1$ (high temperature) cases respectively. In (c) we have $\gamma_0(t)$ versus $t$ for $\beta \Omega \ll 1$, (d) $\Phi(t)$ does not have any temperature dependence and varies non-monotonically with qubit separation. In the long time limit, in all cases, $\gamma(t)$ saturates at some particular value, which suggest suppression of decoherence due to long wavelength modes of the bath.
concurrence. This can be seen from the plot for $t_0$ in figure 1(c). We see from this plot for small qubit separations, the correlations are least influenced by thermal fluctuations of the bath than the qubits with larger separation. The overall contribution to the concurrence from initial system-bath correlations and damping factors $\gamma(t)$ is plotted in figures 2(a) and (b). In figure 2(a), we have concurrence plotted with respect $t$ at temperature $\beta\Omega \sim 1$. From this plot, we see that concurrence decays in a non-Markovian way with saturation to a non-zero value of entanglement in the long time limit. Furthermore, concurrence variation at the high temperature case ($\beta\Omega \ll 1$) (figure 2(b)) is mostly similar in behaviour to low temperature case with slightly lower value for long time entanglement.

We see from these plots, that the qubit separation plays an important role in long time entanglement. The parameter can be used to protect the entanglement decay in an optimal way. We observe from figure 2(a) and 2(b) that initial correlations superimposed with thermal fluctuations enhance the long time entanglement for large $s$ values in comparison to small $s$ or shorter qubit distances $L$. Qualitatively this can be understood in the following way. Since we are assuming a common bath coupled to two qubits which are separated by certain distance. Therefore, there will be less energetic modes to scatter the qubits that would cause decoherence. In other words, at large $L$, there will be only few modes that couple to the qubits and lead to low decoherence rate while for small qubit separations, the number of modes increase (as we are integrating over solid angle in equation (21)) that cause fast decoherence. However, the initial correlations suppress this rate so that we have finite concurrence in the long time limit. This can be seen from the comparison of time evolution of concurrence with no initial correlations and with that of exponential (Markovian) decay for (a) $\beta\Omega \sim 1$ (b) $\beta\Omega \ll 1$. We see that the initial system-bath correlations help to maintain coherence for longer times.

![Figure 2](image)

**Figure 2.** Time variation of entanglement $C(t)$. We see that for different separation lengths between qubits, entanglement saturates over long time scales with different values for (a) $\beta\Omega \sim 1$ and (b) $\beta\Omega \ll 1$.

![Figure 3](image)

**Figure 3.** A comparison is made for the time evolution of concurrence with and without initial correlations and with that of exponential (Markovian) decay for (a) $\beta\Omega \sim 1$ (b) $\beta\Omega \ll 1$. We see that the initial system-bath correlations help to maintain coherence for longer times.

concurrence. This can be seen from the plot for $\gamma_0(t)$ in figure 1(c). We see from this plot for small qubit separations, the correlations are least influenced by thermal fluctuations of the bath than the qubits with larger separation. The overall contribution to the concurrence from initial system-bath correlations and damping factors $\gamma(t)$ is plotted in figures 2(a) and (b). In figure 2(a), we have concurrence plotted with respect $t$ at temperature $\beta\Omega \sim 1$. From this plot, we see that concurrence decays in a non-Markovian way with saturation to a non-zero value of entanglement in the long time limit. Furthermore, concurrence variation at the high temperature case ($\beta\Omega \ll 1$) (figure 2(b)) is mostly similar in behaviour to low temperature case with slightly lower value for long time entanglement.

We see from these plots, that the qubit separation plays an important role in long time entanglement. The parameter can be used to protect the entanglement decay in an optimal way. We observe from figure 2(a) and 2(b) that initial correlations superimposed with thermal fluctuations enhance the long time entanglement for large $s$ values in comparison to small $s$ or shorter qubit distances $L$. Qualitatively this can be understood in the following way. Since we are assuming a common bath coupled to two qubits which are separated by certain distance. Therefore, there will be less energetic modes to scatter the qubits that would cause decoherence. In other words, at large $L$, there will be only few modes that couple to the qubits and lead to low decoherence rate while for small qubit separations, the number of modes increase (as we are integrating over solid angle in equation (21)) that cause fast decoherence. However, the initial correlations suppress this rate so that we have finite concurrence in the long time limit. This can be seen from the comparison of time evolution of concurrence with no initial correlations and with that of an exponential decay in figure 3(a) and (b). We see from these plots, that initial correlations help to maintain coherence for longer times. Therefore, in the long time limit, the soft modes (long wavelength modes) are suppressed and thus slow down the decoherence.

4. Conclusions

In conclusion, we have studied quantum entanglement between two qubits coupled via distance-dependent interaction with a common bath. The initial correlated state is obtained via a projective measurement of the system while assuming a joint thermal equilibrium state of the system and bath. Such a procedure is important
for quantum state preparation where the bath state can depend on the system parameters. We investigated the influence of such initial system-bath correlations, as well as the distance between the qubits, on the dynamics of concurrence and coherence in a vast class of two-qubit states. It is shown that concurrence is half of the coherence for all such kinds of states, thus enabling to measure entanglement in terms of coherence.

Next, we have shown that at sufficiently low temperatures ($\beta \omega \gg 1$), initial system-bath correlations play no role in dynamics as the system and bath become uncorrelated which is due to pre-measurement thermal state. While at other temperatures ($\beta \omega \sim 1$, $\beta \omega \ll 1$), these correlations substantially modify the entanglement decay. Furthermore, we have shown the distance between the qubits forms an important parameter to control decoherence effects both at temperature $\beta \omega \sim 1$ (transition temperature) and $\beta \omega \ll 1$ (high temperature), i.e., we can tune $L$ in such a way that there exist only few modes which cause decoherence and some modes do not interact with the qubits. In order to characterize these modes and their influence in case of interacting qubits with the transition from non-Markovian to Markovian then saturation region will be treated separately [85].

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**Data availability statement**

No new data were created or analysed in this study.

**Appendix A. Bath density matrix $\rho_B^{(0)}(0)$**

In this appendix, we derive the state dependent bath density matrix $\rho_B^{(0)}(0)$. Since we assume a two qubit state $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ and bath density matrix is

$$
\rho_B^0 = \frac{\langle \psi \exp(-\beta H) \rangle}{Tr_B(\psi \exp(-\beta H))}.
$$

(A1)

Using $\sigma^z(0) = |0\rangle$, $\sigma^x|1\rangle = -|1\rangle$, we have

$$
e^{-\beta H}|00\rangle = e^{-\beta \omega_a}|00\rangle, \ e^{-\beta H}|01\rangle = e^{-\beta \omega_b}|01\rangle
$$

(A2)

$$
e^{-\beta H}|10\rangle = e^{-\beta \omega_c}|10\rangle, \ e^{-\beta H}|11\rangle = e^{-\beta \omega_d}|11\rangle
$$

(A3)

where

$$H_B^k = H_B + (B_{ik} \pm B_{2k})H_B^k = H_B - (B_{ik} \pm B_{2k})
$$

(A4)

and $B_{ik} = g_{ik}b_i + g_{ki}^*b_i^\dagger$. Therefore, we write

$$
\langle \psi | e^{-\beta H} | \psi \rangle = |a|^2e^{-\beta \omega_a} + |b|^2e^{-\beta \omega_b} + |c|^2e^{-\beta \omega_c} + |d|^2e^{-\beta \omega_d}.
$$

(A5)

Next, the partition function $Z$, after a straightforward calculation, we write

$$Z = Tr_B(\psi | e^{-\beta H}) = |(a|^2 e^{-\beta \omega_a} + |d|^2 e^{-\beta \omega_d})^\frac{\Sigma_{k=1}^2 \omega_k^2}{\omega_k^4} + |(b|^2 + |c|^2)^\frac{\Sigma_{k=1}^2 \omega_k^2}{\omega_k^4} |Z_B \equiv Z' \cdot Z_B
$$

(A6)

where $Z_B = Tr_B(e^{-\beta H_B})$ and $Z'$ is the rest of the expression. Therefore, using .5 and .6 in equation .1, we get the bath density matrix depending on the system parameters.

**Appendix B. Time evolved density matrix $\rho(t)$**

In this appendix, we give explicit derivation of the time evolved density matrix given in equation (11). Let $\rho(0)$ be the joint density matrix for the system coupled with a bath at $t = 0$. At time $t$, this density matrix evolves in the interaction picture as

$$
\rho(t) = U_t(t) \rho(0) U_t(t)^\dagger.
$$

(A7)

Here $U(t)$ is the time evolution operator defined by $U(t) = T e^{-\int_0^t (dt/H_t(t))}$ and $H_t(t)$ is the Hamiltonian in the interaction picture. Our main interest is to calculate the reduced density matrix of the system by tracing out degrees of freedom of the bath:

$$
\rho_s(t) = Tr_B[U(t) \rho(0) U(t)^\dagger].
$$

(A8)

We write $U(t) = e^{i(t) \omega_{k}^\dagger \hat{A}_k(t)}$ where $f(t)$ is a function of time only and $\hat{A}_k(t) = \sum_\alpha (\alpha_k(t) b_k - \alpha_k^*(t) b_k^\dagger)$ with $\alpha_k(t) = g_k \left( 1 - e^{-\omega_k t} \right)$. Therefore, we can write

$$
\rho_s(t) = Tr_B[U(t) \rho(0) U(t)^\dagger]
$$

(A9)
We see that diagonal terms do not change, therefore we look for off-diagonal terms.

(i) $|00\rangle \langle 01|$ matrix element ($ab^\beta$):

$$\phi_{00} |00\rangle \langle 01| = \frac{1}{Z} \mathcal{T}_B \{ e^{\Sigma_{\alpha}\hat{\chi}_{\alpha}^{(t)}} |00\rangle \langle 01| \} = \frac{1}{Z} \mathcal{T}_B \{ e^{\Sigma_{\alpha}\hat{\chi}_{\alpha}^{(t)} |00\rangle \langle 01|} \}$$

Next, using these unitary transformations, we write

$$O_k \exp \{-\beta H_0 \} O_k^{-1} = e^{-\beta H_0} e^{\frac{\Sigma_{\alpha} \hat{g}_k + \hat{g}_k^\beta}{\omega_k}} O_k \exp \{-\beta H_0 \} O_k^{-1} = e^{-\beta H_0} e^{\frac{\Sigma_{\alpha} \hat{g}_k + \hat{g}_k^\beta}{\omega_k}}$$

Next, using these unitary transformations, we arrive at the following expression for the matrix element $|00\rangle \langle 01|$ with coefficient $a^\beta \phi(t)$ where

$$\phi(t) = \frac{e^{-\gamma(t)}}{e^t} \left\{ a^\beta \right\} e^{-\gamma(t)} \left\{ a^\beta \right\} \mathcal{T}_B \{ e^{\Sigma_{\alpha}\hat{\chi}_{\alpha}^{(t)}} |00\rangle \langle 01| \}$$

and after further simplification, we arrive at the final expression
\[ \varphi(t) = \left[ |a|^2 e^{-\beta \omega_k} e^{i \Phi_k(t, t)} + |d|^2 e^{\beta \omega_k} e^{-i \chi(t, t) - \Phi_k(t, t)} \right] e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} + |b|^2 e^{-\beta \omega_k} e^{-i \chi(t, t) - \Phi_k(t, t)} e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \exp \left( -4 \Sigma_k \frac{\lvert g_k \rvert^2}{\omega_k} (1 - \cos \omega_k t \coth \frac{\beta \omega_k}{2}) \right) \] (A20)

where

\[ \Phi_k(r, t) = 4 \sum_k \frac{|g_k|^2}{\omega_k^2} \sin \omega_k t (1 \pm \cos k.(\vec{n} - \vec{r})) \] (A21)

\[ \chi(r, t) = 4 \sum_k \frac{|g_k|^2}{\omega_k^2} \sin \omega_k t (1 - \cos \omega_k t) \] (A22)

In the similar way, we obtain all other matrix elements which are given by:

- **(ii) \( \langle 00 \rvert 10 \rangle \) matrix element (ac*):**

\[ \zeta(t) = \langle 00 \rvert \frac{1}{Z} T_{H} \left[ e^{\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 10 \rvert \otimes \left[ |a|^2 e^{-\beta \omega_k} e^{-\beta H_{\text{in}}} + |b|^2 e^{-\beta H_{\text{in}}} \right] \]

\[ + |c|^2 e^{-\beta H_{\text{in}}} + |d|^2 e^{\beta \omega_k} e^{-\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 10 \rvert \right] \]

\[ = \left[ |a|^2 e^{-\beta \omega_k} e^{i \Phi_k(t, t)} + |d|^2 e^{\beta \omega_k} e^{-i \chi(t, t) - \Phi_k(t, t)} \right] e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \exp \left( -4 \Sigma_k \frac{\lvert g_k \rvert^2}{\omega_k} (1 - \cos \omega_k t \coth \frac{\beta \omega_k}{2}) \right) \] (A23)

- **(iii) \( \langle 00 \rvert 11 \rangle \) matrix element (ad*):**

\[ \kappa(t) = \langle 00 \rvert \frac{1}{Z} T_{H} \left[ e^{\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 11 \rvert \otimes \left[ |a|^2 e^{-\beta \omega_k} e^{-\beta H_{\text{in}}} + |b|^2 e^{-\beta H_{\text{in}}} \right] \]

\[ + |c|^2 e^{-\beta H_{\text{in}}} + |d|^2 e^{\beta \omega_k} e^{-\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 11 \rvert \right] \]

\[ = \left[ |a|^2 e^{-\beta \omega_k} e^{2 i \Phi_k(t, t)} + |d|^2 e^{\beta \omega_k} e^{2i \chi(t, t) - 2 \Phi_k(t, t)} \right] e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} + |b|^2 e^{i \chi(t, t)} \]

\[ + |c|^2 e^{i \chi(t, t)} e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \exp \left( -4 \Sigma_k \frac{\lvert g_k \rvert^2}{\omega_k} (1 + \cos k.(\vec{n} - \vec{r})) (1 - \cos \omega_k t) \coth \frac{\beta \omega_k}{2} \right) \] (A24)

- **(iv) \( \langle 01 \rvert 10 \rangle \) matrix element (bc*):**

\[ \tilde{\kappa}(t) = \langle 01 \rvert \frac{1}{Z} T_{H} \left[ e^{\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 10 \rvert \otimes \left[ |a|^2 e^{-\beta \omega_k} e^{-\beta H_{\text{in}}} + |b|^2 e^{-\beta H_{\text{in}}} \right] \]

\[ + |c|^2 e^{-\beta H_{\text{in}}} + |d|^2 e^{\beta \omega_k} e^{-\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 10 \rvert \right] \]

\[ = \left[ |a|^2 e^{-\beta \omega_k} e^{i \chi(t, t) + \Phi_k(t, t)} + |d|^2 e^{\beta \omega_k} e^{-i \Phi_k(t, t)} \right] e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \]

\[ + |b|^2 e^{i \Phi_k(t, t)} + |c|^2 e^{i \chi(t, t) - \Phi_k(t, t)} e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \exp \left( -4 \Sigma_k \frac{\lvert g_k \rvert^2}{\omega_k} (1 - \cos \omega_k t) \coth \frac{\beta \omega_k}{2} \right) \] (A25)

- **(v) \( \langle 01 \rvert 11 \rangle \) matrix element (bd*):**

\[ \tilde{\zeta}(t) = \langle 01 \rvert \frac{1}{Z} T_{H} \left[ e^{\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 11 \rvert \otimes \left[ |a|^2 e^{-\beta \omega_k} e^{-\beta H_{\text{in}}} + |b|^2 e^{-\beta H_{\text{in}}} \right] \]

\[ + |c|^2 e^{-\beta H_{\text{in}}} + |d|^2 e^{\beta \omega_k} e^{-\Sigma \sigma_l \hat{J}_l(t)} \right] \langle 11 \rvert \right] \]

\[ = \left[ |a|^2 e^{-\beta \omega_k} e^{i \chi(t, t) + \Phi_k(t, t)} + |d|^2 e^{\beta \omega_k} e^{-i \Phi_k(t, t)} \right] e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \]

\[ + |b|^2 e^{i \Phi_k(t, t)} + |c|^2 e^{i \chi(t, t) - \Phi_k(t, t)} e^{\beta \omega_k \frac{\lvert g_k \rvert^2}{\omega_k}} \exp \left( -4 \Sigma_k \frac{\lvert g_k \rvert^2}{\omega_k} (1 - \cos \omega_k t) \coth \frac{\beta \omega_k}{2} \right) \] (A26)
(vi) $|10\rangle\langle11|$ matrix element ($bc^*$):

$$\tilde{\varphi}(t) = \langle10|\sum_{\alpha} T_{\alpha} \{ |\alpha\rangle^2 e^{-\beta_\alpha t} e^{-\beta H_\alpha} + |\beta\rangle^2 e^{-\beta H_\beta} + |c|^2 e^{-\beta H_1} + |d\rangle^2 e^{-\beta H_2} |11\rangle \}
+ \langle11| \{ |\alpha\rangle^2 e^{-\beta_\alpha t} e^{-\beta H_\alpha} + |\beta\rangle^2 e^{-\beta H_\beta} + |c|^2 e^{-\beta H_1} + |d\rangle^2 e^{-\beta H_2} |11\rangle \}

\begin{align*}
&= \left[ \left| a \right|^2 e^{-\beta_\alpha t} e^{i(\chi_{\alpha} t) - \Phi_{\alpha}(t)} + \left| d \right|^2 e^{-\beta_\beta t} e^{i(\chi_{\beta} t) + \Phi_{\beta}(t)} \right] e^{i0} \frac{\left| a \right|^2 + \left| d \right|^2}{2}
&+ \left[ \left| b \right|^2 e^{-i(\chi_{\beta} t) + \Phi_{\beta}(t)} + \left| c \right|^2 e^{i(\chi_{\alpha} t) - \Phi_{\alpha}(t)} \right] e^{i0} \frac{\left| b \right|^2 + \left| c \right|^2}{2}
&\times \exp \left[ -4 \Sigma_2 \frac{\left| b \right|^2}{\omega_2^2} (1 - \cos \omega_2 t) \coth \frac{\beta_\beta}{2} \right]
\end{align*}
(A27)

Using all these equations, we get the time evolved density matrix used in the main text 11:

$$\rho(t) = \begin{pmatrix}
|a|^2 & ab^* \varphi(t) & ac^* \zeta(t) & ad^* \kappa(t) \\
ba^* c^* & \left| b \right|^2 & bc^* \xi(t) & bd^* \eta(t) \\
ca^* \zeta^*(t) & cb^* \xi^*(t) & \left| c \right|^2 & cd^* \varphi(t) \\
d\kappa^*(t) & db^* \eta^*(t) & dc^* \varphi^*(t) & \left| d \right|^2
\end{pmatrix}
(A28)

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