High resolution kinematics of galactic globular clusters. II.
On the significance of velocity dispersion measurements

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Abstract. Small number statistics may heavily affect the structure of the broadening function in integrated spectra of galactic globular cluster centers. As a consequence, it is a priori unknown how closely line broadening measurements gauge the intrinsic velocity dispersions at the cores of these stellar systems. We have tackled this general problem by means of Monte Carlo simulations. An examination of the mode and the frequency distribution of the measured values of the simulations indicates that the low value measured for the velocity dispersion of M30 (Zaggia et al. 1992b) is likely a reliable estimate of the velocity dispersion at the center of this cluster. The same methodology applied to the case of M15 suggests that the steep inward rise of the velocity dispersion found by Peterson, Seitzer and Cudworth (1989) is real, although less pronounced. Large-aperture observations are less sensitive to statistical fluctuations, but are unable to detect strong variations in the dispersion which occur within the aperture itself.

Key words: clusters: globular; NGC 7099; NGC 7078

1. Introduction

Since the pioneering work of Gunn and Griffin (1979), much effort has been devoted to investigate the internal dynamics of galactic globular clusters (GGCs), both theoretically and observationally. The strategy is to derive the fundamental parameters of a GGC, such as mass and mass-to–light ratio, by constraining models through the radial luminosity and velocity dispersion profiles. The latter are measured in two ways:

i) either from the radial velocities of individual cluster stars, or

ii) from the line broadening of the spectrum integrated over some central area.

The second approach is more economical than the first one, but it suffers from the bias that the integrated spectrum is dominated by few bright stars only, whose light overwhelms that of all the others within the spectrograph aperture. For this reason it seems important to investigate the statistical significance of the measured values of the line broadening and their correspondence to the desired estimate of the velocity dispersion (σ²)² (see eq. 2).

The case of M15 is emblematic. Peterson, Seitzer, and Cudworth (1989, hereafter PSC) have reported that the velocity dispersion at the very center of this core–collapsed (King and Djorgovski 1984) GGC is as large as $25 \pm 7$ km s⁻¹, a fact implying a steep inward rise of $(v²)²$. This figure results from their interpretation of the measurements in the framework of the small number statistics: in fact, their sampling of $(v²)²$ in the $3'' \times 3''$ central area, made using a circular aperture with $2R = 1''2$, produces values of $σ_{cc}$ ranging from 8 to 30 km s⁻¹. (Hereafter the subscript $cc$ attached to $σ$ indicates a measure of $σ$ obtained with the cross–correlation method; cf. Tonry and Davis 1979.) Using a much larger aperture ($6'' \times 6''$) Dubath, Meylan, and Mayor (1992, hereafter DMM) measure directly a lower value $σ_{cc} = 14.0 \pm 0.3$ km s⁻¹. This second result, based on a larger number of stars, is statistically more significant, but its poorer spatial resolution may likely wipe out a rapid increase of $(v²)²$ in the inner 1–2 arc seconds, if any. Then the question is: how significant is the peak value of $σ_{cc}$ taken by PSC as the central value of $(v²)²$ in M15?

In order to shed some light onto the general problem of the statistical significance of the velocity dispersion measurements from integrated spectra of GGCs, we have investigated the effects of the various parameters on simulated spectra of the central regions of selected clusters. In the next Section we present the methodology, and in Sect. 3 we discuss its application to the case of M30, whose central velocity dispersion has been measured by Zaggia et al. (1992b = Paper I). The case of M15 is discussed in Sect. 4. Previous account of this matter has been given by Zaggia et al. (1992a).
2. Methodology for the simulations

The width \( \sigma_w \) of the broadening function of an integrated spectrum represents the luminosity–weighted dispersion

\[
\sigma_w = \left[ \frac{\sum_{i=1}^{N} \ell_i \times (v_{r_i})^2}{\sum_{i=1}^{N} \ell_i} \right]^{1/2}
\]

(1)

of the radial velocities \( v_{r_i} \) of the stars with luminosity \( \ell_i \), falling inside the area viewed by the spectrograph slit. (Other effects such as seeing convolution of the spectrograph aperture, convolution of the broadening function by stellar and cluster rotation, and presence of close binary systems are ignored). Thus \( \sigma_w \) may differ from the unweighted radial velocity dispersion for the totality \( N \) of the stars within the aperture,

\[
\langle v_r^2 \rangle \equiv \left[ \frac{\sum_{i=1}^{N} (v_{r_i})^2}{N - 1} \right]^{1/2}
\]

(2)

which is the quantity required in dynamical calculations. In fact, the broadening function is dominated by the brightest stars. Since they occur in a small number, the dispersion of their radial velocities does not necessarily represent the average velocity dispersion within the sampled area.

We have tested the effect of the small number statistics and of the star luminosity–weighting in determining the value of \( \sigma \), by means of simulated spectra with a priori known \( \langle v_r^2 \rangle_\infty \); this is the center of the Gaussian distribution for \( \langle v_r^2 \rangle \) produced by poor statistics, i.e., the asymptotic value provided by eq. 2 with \( N \to \infty \). Our Monte Carlo approach accounts also for the effects of the measuring technique used in Paper I, which is based on a fit of the peak of the cross–correlation function. Each experiment consists of the following steps.

1. First we generate a random set of stellar luminosity values \( \ell_i \) with probability distribution given by a luminosity function (LF) appropriate to metal poor GGCs (Ferraro 1990; see Fig. 3). The set is closed when a pre–determined total luminosity \( \sum \ell_i = L_A \) is reached.

2. An integrated spectrum is then simulated by summing up, for each star of the above set, a same (standard) spectrum weighted by the star luminosity \( \ell_i \), and shifted in wavelength by an amount corresponding to a random radial velocity with Gaussian probability distribution of fixed variance \( \langle v_r^2 \rangle_\infty \).

3. Photon and read–out noise are added to the synthetic cumulative spectrum; we decided to tailor these effects to the observations of Paper I (global efficiency of the 3.6 m + CASPEC, and characteristics of the CCD used there).

4. Finally, the broadening \( \sigma_w \) of the synthetic spectrum is measured by the same cross–correlation technique adopted for the real spectra.

Input parameters for a set of Monte Carlo simulations are:

– the total luminosity \( L_A = \sum \ell_i \) within the slit aperture \( A \);
– the cut–off value \( M_{lim} \) for the LF: stars fainter that \( M_{lim} \) are ignored in building up the set at step 1. Thus \( M_{lim} \) is the fixed value which controls the number of stars actually concurring to the formation of the simulated spectrum;
– the intrinsic (unweighted) radial velocity dispersion \( \langle v_r^2 \rangle_\infty \).

For all the experiments we made use of a high signal–to–noise spectrum of HD 122563, the template star of Paper I, obtained with CASPEC at the ESO 3.6 m telescope (resolution of FWHM = 0.156 Å, equivalent to 9.0 km s\(^{-1}\) at \( \lambda = 5200 \) Å). We have verified that the results do not change by utilizing a lower signal–to–noise spectrum of a typical GGC star. We have also tested the use of LFs other than the one adopted here (Fig. 3), finding no appreciable difference even with LFs of metal rich GGCs (e.g. NGC 2808; Ferraro et al. 1990). Finally, simple considerations show that no major consequence is expected in the results by the fact that stars are assumed to have all the same mass and so the same velocity dispersion, provided that \( M_{lim} \) is faint enough. In fact the difference between the mass of a turn–off star and one at 3.5 mag below it, i.e. our typical cut–off, is approximately 0.2 M\(_{\odot}\). With the assumption of isothermal velocity distribution, this difference corresponds to an increase of \( \approx 20\% \) of the velocity dispersion with respect to that of the turn–off stars.

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Fig. 1. The integrated luminosity function used in the simulations (solid line) gives the number \( \Phi(M_V) \) of stars brighter than \( M_V \) (per magnitude unit). The corresponding luminosity relative to the total luminosity (in percent) is also shown (dashed line). \( M_{lim} \) values adopted for M15 and M30 are plotted as upright dotted lines.
Fig. 2. The shaded area in each panel is the frequency distribution of the measured line broadening $\sigma_{cc}$ from 1000 Monte Carlo experiments with the indicated value of the input velocity dispersion $\langle v_r^2 \rangle_\infty^{1/2}$ (arrow) and fixed $M_{lim} = +3.5$ mag. The solid line is the distribution of the luminosity–weighted dispersion $\sigma_w$ for the input set of stars (see eq. (1)). The upper left panel reports also the histogram of $\langle v_r^2 \rangle_\infty^{1/2}$ (dotted line) for that particular set of experiments.

3. The meaning of $\sigma_{cc}$: an application to M30

Here we show how small number statistics and gauging of the broadening function operate by reporting on Monte Carlo experiments tailored to our observations of M30 (Paper I). This is a GGC which, by analogy with M15, is suspected to possess a central spike in the velocity dispersion (Zaggia et al. 1991). In all simulations $L_A$ has been fixed to $-3.5$ mag to match the total light ($m_A = 11.0$ mag) within the centered aperture of $4''6 \times 6''$ used for the spectra of Paper I. Simulations have been made for three values of $M_{lim} = 1.5$, 3.5, and 7.5 mag, and four values of the input velocity dispersion $\langle v_r^2 \rangle_\infty^{1/2} = 6, 12, 18,$ and $24$ km s$^{-1}$.

The results for groups of 1000 simulations are shown in the panels of Fig. 2 for the four values of $\langle v_r^2 \rangle_\infty^{1/2}$ (indicated by an arrow in the figures), keeping $M_{lim}$ fixed at 3.5 mag. The shaded area marks the distribution of the output values $\sigma_{cc}$ of the cross-correlation method applied to the simulated spectra. We also plot the luminosity–weighted velocity dispersion $\sigma_w$ of the input sets of stars (solid line), as it results from eq. (1).

The effect of the small number statistics stands out clearly in the upper-left plot of Fig. 2, where we have also plotted the distribution of $\langle v_r^2 \rangle_\infty^{1/2}$ (dotted line); this unweighted equivalent of $\sigma_w$ is calculated for each set of stars with eq. (2). The histogram of $\langle v_r^2 \rangle_\infty^{1/2}$ (truncated in its upper part) has a smaller dispersion than those of $\sigma_w$ and $\sigma_{cc}$. The mean of the distribution of $\langle v_r^2 \rangle_\infty^{1/2}$ is equal to the input value $\langle v_r^2 \rangle_\infty^{1/2}$, and the scatter is due to the small number of stars: this implies that the standard deviation of $\langle v_r^2 \rangle_\infty^{1/2}$ is Poissonian. In fact, with a mean number of $\sim 74$ stars per set (brighter than $M_{lim}$) and for
$\langle v_r^2 \rangle^{\frac{1}{2}} = 6$ km s$^{-1}$, the Poissonian standard deviation is 0.49 km s$^{-1}$, well in agreement with the standard deviation of the distribution of $\langle v_r^2 \rangle^{\frac{1}{2}}$ derived from our simulations (0.51 km s$^{-1}$).

The luminosity weighting permits some bright stars or group of stars to dominate the velocity dispersion. So, the distribution of $\sigma_w$, calculated with eq. (1), comes out broader than that of $\langle v_r^2 \rangle^{\frac{1}{2}}$. For the experiment with $\langle v_r^2 \rangle^{\frac{1}{2}} = 6$ km s$^{-1}$, the distribution of $\sigma_w$ has a mean value of 5.5 km s$^{-1}$ and a standard deviation of 1.1 km s$^{-1}$. This behavior is confirmed by the other simulations, and could be regarded as the limit to our kinematical measurements: in fact, what we measure in a spectrum is $\sigma_w$, which is most probably $\leq \langle v_r^2 \rangle^{\frac{1}{2}}$. For instance, in the application to the case of M30 we find that the most probable value of $\sigma_w$ is lower, by $\sim 10\%$, than the input $\langle v_r^2 \rangle^{\frac{1}{2}}$, and that the associated uncertainty (at 1 sigma level) is 20% of $\langle v_r \rangle$. While the cut-off of the LF moves towards fainter magnitudes, the intrinsic error lowers. This is the reason why we have chosen $M_{lim} = 7.5$ mag; the integrated light of all stars brighter than this limit is 95% of the total light encircled by the spectrograph aperture.

By looking at the successive panels of Fig. 3 it is apparent that the distributions of $\sigma_{cc}$ and of $\sigma_w$ differ more and more as $\langle v_r^2 \rangle^{\frac{1}{2}}$ increases. The distribution of $\sigma_{cc}$ is also increasingly skewed towards the low values. This is due to the somewhat unsmooth form of the cross-correlation function. In fact, with a high value of $\langle v_r^2 \rangle^{\frac{1}{2}}$, the cross-correlation function become multimodal, with peaks produced by bright single stars or groups of stars with radial velocity far from the mean. All but one of these peaks are ignored by the fitting algorithm, which fits the highest peak only; consequently the measured dispersion is systematically lower than the input value. Paradoxically, this effect increases with the spectral resolution.

**Fig. 3.** Comparison between input ($\langle v_r^2 \rangle^{\frac{1}{2}}$) and output ($\sigma_{cc}$) values of the velocity dispersion from Monte Carlo simulations tailored to our (Paper I) observations of M30. The horizontal dotted line represents the measure of $\sigma_{cc}$ reported in Paper I.

The difference between the input and output values of the velocity dispersion in these simulations of M30 can be better appreciated in Fig. 4, where the mode of the distribution of $\sigma_{cc}$, together with its half width at half maximum, is plotted against the corresponding input value of $\langle v_r^2 \rangle^{\frac{1}{2}}$ for three values of $M_{lim}$. It is clear that, with the instrumental set up of Paper I, any result giving $\sigma_{cc} \gtrsim 10$ km s$^{-1}$ would underestimate, on average, the intrinsic velocity dispersion of M30. In Paper I we have measured $\sigma_{cc} = 6.0 \pm 0.6$ km s$^{-1}$, well in the range where $\sigma_{cc} \simeq \langle v_r^2 \rangle^{\frac{1}{2}}$. Note however that, the same figure for $\sigma_{cc}$ would be (marginally) in accord also with values of $\langle v_r^2 \rangle^{\frac{1}{2}}$ as high as $\simeq 25$ km s$^{-1}$ or more. We shall expand the discussion on this important point in the next Section.

It can be of some interest to study the effects of changing the size of the integration aperture. We have thus investigated the relation between $\sigma_{cc}$ and $\langle v_r^2 \rangle^{\frac{1}{2}}$ for an aperture of $1'' \times 1''$ (about the same size of that used by PSC for M15), which, at the center of M30, encircles
a total luminosity $L_A$ of -0.7 mag (or $m_A = 13.8$). We have explored the parameter space using the same values of $M_{lim}$ and $\langle v_r^2 \rangle^\frac{1}{2}$ as in the previous experiments, taking care to reject all the experiments in which the total light is contributed by one star only. As expected, the departure of the output from the input velocity dispersion increases dramatically. The measured $\sigma_{cc}$ seems to reach a maximum between $\langle v_r^2 \rangle^\frac{1}{2} = 10$ and 20 km s$^{-1}$, then it decreases. Consequently, under these conditions a measure of $\sigma_{cc} = 5 \div 8$ km s$^{-1}$ from a single spectrum would have no other meaning but that of a possible lower limit.

In Fig. 5, we report the results of the simulations for the PSC parameter set. Here $\langle v_r^2 \rangle^\frac{1}{2}$ varies from 5 to 40 km s$^{-1}$ in steps of 5 km s$^{-1}$, $M_{lim} = 6.8$ mag, and $L_A = -1.7$ mag ($m_A = 13.5$) as this is the (average) luminosity collected by the circular aperture of 1$\arcmin$2 used by PSC to sample the velocity dispersion in the core of M15. What has been just discussed for M30 is still valid: $\sigma_{cc} < \langle v_r^2 \rangle^\frac{1}{2}$ for $\langle v_r^2 \rangle^\frac{1}{2} > 10$ km s$^{-1}$.

Fig. 5. Same as Fig. 3, but for M15 as observed by PSC. The data refer to the set of simulations with $M_{lim} = +6.8$ mag. The horizontal dashed line is the mean of all the PSC values, while the dotted line is the mean after rejection of two measurements deviating more than 3 sigma.

Fig. 6. The dotted shaded area in each panel represents the frequency distribution of $\sigma_{cc}$ for 1000 Monte Carlo experiments, tailored to the PSC observations of M15, for the indicated value of the input velocity dispersion $\langle v_r^2 \rangle^\frac{1}{2}$ and fixed $M_{lim} = +6.8$ mag. The solid shaded area is the histogram of the measurements of M15 velocity dispersion by PSC.

For a better comparison with the PSC observations, in the various panels of Fig. 4 we plot the frequency distributions of $\sigma_{cc}$ values resulting from the 1000 simulations made for each input value of $\langle v_r^2 \rangle^\frac{1}{2}$. Superimposed there is the histogram of the single measures of $\sigma_{cc}$ by PSC (their Table 4). It is clear from the panels of Fig. 4 that:

- Simulations allow to exclude that the central velocity dispersion of M15 can be as low as 10 km s$^{-1}$.
- There are a chance to measure $\sigma_{cc} \geq 20$ only if $\langle v_r^2 \rangle^\frac{1}{2} > 15$.
- The peak of the values measured by PSC is nearly coincident with the mode of the distributions of $\sigma_{cc}$ only when $15 < \langle v_r^2 \rangle^\frac{1}{2} < 30$.

4. The central $\sigma_{cc}$ in M15

The already mentioned claim by PSC of a steep gradient of the velocity dispersion in M15, consequence of the high value of the central $\sigma_{cc} \geq 25$ km s$^{-1}$, seems contradicted by the measure of $\sigma_{cc} = 14.0$ km s$^{-1}$ reported by DMM. We shall note here that the most immediate sign of a statistical bias, i.e. the presence of a correlation of the velocity dispersion with the corresponding absolute value of the excess cluster systemic velocity, is not present in the PSC data (Fig. 3). To reconcile the two apparently conflicting observations by PSC and DMM, we tailored the parameters of our Monte Carlo simulations machinery to M15.

Fig. 3. The dotted shaded area in each panel represents the frequency distribution of $\sigma_{cc}$ for 1000 Monte Carlo experiments, tailored to the PSC observations of M15, for the indicated value of the input velocity dispersion $\langle v_r^2 \rangle^\frac{1}{2}$ and fixed $M_{lim} = +6.8$ mag. The solid shaded area is the histogram of the measurements of M15 velocity dispersion by PSC.
The PSC claim of $\langle v_r^2 \rangle^{1/2} \geq 25 \text{ km s}^{-1}$ at the center of M15 rests on the distribution of the results of their eight independent measurements of $\sigma_{cc}$ made around the cluster center. Six of them lie in the interval $8.4 \leq \sigma_{cc} \leq 11.8 \text{ km s}^{-1}$ (with a mean value of 10.2 km s$^{-1}$), but the other two are of $23.6 \pm 3.1$ and $30.0 \pm 4.3 \text{ km s}^{-1}$ respectively, too large for $\langle v_r^2 \rangle^{1/2} \leq 15 \text{ km s}^{-1}$ according to our simulations. A more quantitative evaluation of the comparison between observations and simulations has been made using the Kolmogorov–Smirnov (KS) test. The KS probability of the match of PSC data with the Monte Carlo distributions of Fig. 4 is maximum (70%) for $\langle v_r^2 \rangle^{1/2} = 20 \text{ km s}^{-1}$, but it is still large at both $\langle v_r^2 \rangle^{1/2} = 15$ and $25 \text{ km s}^{-1}$ (63% and 53% respectively).

In conclusion, our numerical experiments confirm that, within $R \sim 1''5$ from the center of M15, the velocity dispersion is likely $\sim 20 \text{ km s}^{-1}$ (most probable value), in fair agreement with the qualitative interpretation given by PSC to their observations: “We conclude that the low values for the central dispersion are those dominated by one or two stars, and that the higher values are more representative of the dispersion of the background light” (verbatim). Indeed, the multiple sampling of $\sigma_{cc}$ is the winning strategy to overcome resolution and statistical problems at once. Clearly, a single measure of $\sigma_{cc}$ through an aperture as small as the one adopted by PSC would be of no meaning. At the same time, we shall see that the aperture used by DMM is large enough to wipe out fine kinematical features.

DMM observed M15 with CASPEC through a centered aperture of $6'' \times 6''$ (as we did for M30). The data plotted in Fig. 8 have been computed with the same set of parameters as in Fig. 3 but $L_A = -4.5 \text{ mag}$ (or $m_A = 10.7$). We see that DMM measure of $\sigma_{cc} = 14.0 \text{ km s}^{-1}$ is compatible with a similar low value for $\langle v_r^2 \rangle^{1/2}$, but it is consistent with values as large as $\langle v_r^2 \rangle^{1/2} = 30 \text{ km s}^{-1}$. This is confirmed by the frequency distributions of the $\sigma_{cc}$ values for fixed $\langle v_r^2 \rangle^{1/2}$ (Fig. 4). When $\langle v_r^2 \rangle^{1/2} > 15 \text{ km s}^{-1}$, the measure of DMM (arrow) is near the peak of the distribution of $\sigma_{cc}$.

Fig. 7. Same as Fig. 3 but for a larger aperture of $6'' \times 6''$ used for M15 by DMM. Again the data refer to the $M_{lim} = +6.8$ simulation. The horizontal dotted line represents the $\sigma_{cc}$ of M15 measured by DMM.

Fig. 8. Same as Fig. 4, with simulation parameters tailored to the observations of M15 by DMM. Their value of $\sigma_{cc}$ is marked by an arrow.

5. Conclusions

From the results of the simulations of the spectra of the central regions of GGCs, we can draw the following conclusions:

1. For M30, our measure of $\sigma_{cc} = 6.0 \pm 0.6 \text{ km s}^{-1}$ (Paper I) is well in the range where the simulations shows that $\sigma_{cc} \simeq \langle v_r^2 \rangle^{1/2}$.

2. We confirm quantitatively the results of the discussion by PSC about the central velocity dispersion of M15.
3. The observations of M15 by DMM are consistent with an inward rise of the radial velocity dispersion, although possibly not as steep as claimed by PSC.

Finally, we find that a single integrated spectrum through a large enough centered aperture is capable of providing just a lower value of \(\langle v_r^2 \rangle^{1/2}\). The multiple independent sampling technique pioneered by Peterson et al. (1989) seems thus essential to investigate fine kinematical structures of the cores of GGCs.

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