Entropic Risk Constrained Soft-Robust Policy Optimization

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Abstract
Having a perfect model to compute the optimal policy is often infeasible in reinforcement learning. It is important in high-stakes domains to quantify and manage risk induced by model uncertainties. Entropic risk measure is an exponential utility-based convex risk measure that satisfies many reasonable properties. In this paper, we propose an entropic risk constrained policy gradient and actor-critic algorithms that are risk-averse to the model uncertainty. We demonstrate the usefulness of our algorithms on several problem domains.

1. Introduction
Reinforcement Learning (RL) aims to learn how to map situations to actions in order to maximize the rewards accrued over the long run (Sutton & Barto, 2018; Szepesvári, 2010). Markov Decision Processes (MDPs) provide a functional framework to model RL problems (Bertsekas & Tsitsiklis, 1996; Puterman, 2005). In general, transition dynamics and rewards of MDPs are computed from limited and noisy samples. Which often makes it difficult to build a good model of the world. This results in policies that can fail catastrophically when deployed (Petrik et al., 2016; Hanasusanto & Kuhn, 2013). To mitigate the risk of failure in high-stakes domains, such as autonomous driving or robotic manipulation, it is important to account for the uncertainty about models.

A common approach to computing policies that are reliable even with imprecise models is to use robust optimization (Iyengar, 2005; Wiesemann et al., 2013). This approach is simple and can be computationally effective (Ho et al., 2018), but unfortunately too conservative (Russel & Petrik, 2019). A class of methods that build on robust optimization but mitigate its conservativeness are described as epistemic risk aversion (Eriksson & Christos, 2019) or soft robustness (Ben-Tal et al., 2010; Derman et al., 2019). These methods also estimate the range of possible models consistent with the observed data and then optimize a policy with respect to a risk metric across different models. In one early example of this approach, the percentile criterion optimizes the value-at-risk (VaR) of the policy’s performance with respect to uncertain model (Delage & Mannor, 2010).

It is important to distinguish between soft-robustness with respect to epistemic uncertainty, which we address in this work, and standard risk-averse MDPs. Risk-averse MDPs optimize a risk-sensitive objective that penalizes the variability in returns caused by stochastic transitions, also referred to as the aleatoric uncertainty. Policy gradient and actor-critic algorithms to optimize risk-averse objective for MDPs have been developed recently (Tamar et al., 2013; Chow & Ghavamzadeh, 2014) for several common risk-measures like Value-at-risk (VaR) and conditional value-at-risk (CVaR) (Rockafellar & Uryasev, 2000). These methods do not consider model uncertainty and are very different from our work in several crucial aspects. The use of VaR and CVaR in sequential optimization is complicated because they are not dynamically-consistent and the optimal policy may need to be history-dependent (Ruszczynski, 2010).

In this paper, we propose entropic risk constrained policy gradient and actor-critic algorithms under epistemic uncertainty within a Bayesian framework. Our choice of relying on the entropic data and then optimize a policy with respect to a risk metric across different models. In one early example of this approach, the percentile criterion optimizes the value-at-risk (VaR) of the policy’s performance with respect to uncertain model (Delage & Mannor, 2010).

It is important to distinguish between soft-robustness with respect to epistemic uncertainty, which we address in this work, and standard risk-averse MDPs. Risk-averse MDPs optimize a risk-sensitive objective that penalizes the variability in returns caused by stochastic transitions, also referred to as the aleatoric uncertainty. Policy gradient and actor-critic algorithms to optimize risk-averse objective for MDPs have been developed recently (Tamar et al., 2013; Chow & Ghavamzadeh, 2014) for several common risk-measures like Value-at-risk (VaR) and conditional value-at-risk (CVaR) (Rockafellar & Uryasev, 2000). These methods do not consider model uncertainty and are very different from our work in several crucial aspects. The use of VaR and CVaR in sequential optimization is complicated because they are not dynamically-consistent and the optimal policy may need to be history-dependent (Ruszczynski, 2010).

In this paper, we propose entropic risk constrained policy gradient and actor-critic algorithms under epistemic uncertainty within a Bayesian framework. Our choice of relying on the entropic risk measure is motivated by the fact that, this risk measure is convex and time-consistent. Our contributions in this paper are as follows: 1) We derive gradient update rule for the entropic risk constrained optimization with model uncertainty where the sampling based gradients are estimated from a Bayesian posterior. 2) We propose a trajectory-based policy gradient algorithm and actor-critic algorithm with function approximation.

The remainder of the paper is organized as follows: Section 2 formally describes the MDP framework and entropic risk constrained objective. Section 3 derives the gradient update rules and presents the policy gradient and actor-critic algorithms. Section 4 presents empirical results on several problem domains. And we finally draw conclusions in Section 5.
We consider an MDP model \( \Upsilon \) with a finite number of states \( S = \{1, \ldots, S\} \) and finite number of actions \( A = \{1, \ldots, A\} \). Every action \( a \in A \) is available for the decision maker to take in every state \( s \in S \). After taking an action \( a \in A \) in state \( s \in S \), the decision maker receives a reward \( r_{s,a} \in \mathbb{R} \) and transitions to a next state \( s' \) according to the true and unknown transition probability \( p_{s,a}^\star \in \Delta S \). We parameterize a class of stationary randomized policies as \( \pi(\cdot|s; \theta) \) where \( s \in S \) and \( \theta \in \Theta \subseteq \mathbb{R}^k \) is a \( k \)-dimensional parameter vector. We use \( \pi \) and \( \theta \) interchangeably for the rest of the paper. The return \( \rho^\theta \) for a policy \( \theta \) and a sampled trajectory \( \xi \) is defined as: \( g^\theta(\xi) = \sum_{t=0}^{\infty} \gamma^t r_{s_t, \pi(s_t)} \) (Puterman, 2005), where \( \xi = [s_0, a_0, \ldots] \). The expected values of the random variables \( g^\theta(\xi) \) when \( \xi \) starts from a specific state \( s \) is defined as the value function of that state: \( v^\theta(s) = \mathbb{E}[g^\theta(\xi)] \). We can estimate the gradients of the return \( \rho^\theta \) w.r.t the parameters \( \theta \) from sampled trajectories \( \xi \). The objective is then to maximize the infinite horizon \( \gamma \)-discounted return \( \rho^\theta \) by adjusting the parameters \( \theta \) in the direction of the gradients (Sutton & Barto, 2018). Ideally, the optimal policy \( \pi^* \in \arg\max_{\pi \in \Pi} \rho^\pi(\Pi^*) \) could be computed with a known \( P^* \), where \( \Pi \) is the set of all stationary deterministic policies. This is possible when the true transition probabilities \( P^* \) are known and only estimated from samples.

### Entropic Risk Measure
Risky methods address the challenge of computing a policy that is not too conservative in the worst-case scenario when \( P^* \) is unknown. The idea is to compute a policy that maximizes the expected return and satisfies a constraint that the worst-case return is above some preset threshold. Entropic risk measure \( \rho : \mathbb{X} \rightarrow \mathbb{R} \) is a popular risk measure based on exponential utility function and for a risk-aversion parameter \( \alpha > 0 \), it takes the form:

\[
\rho_{\alpha}(X) = -\frac{1}{\alpha} \log \left( \mathbb{E}[e^{-\alpha X}] \right)
\]

The entropic risk measure defined in Equation (1) satisfies the properties of monotonicity, translation invariance and convexity, but does not satisfy the positive homogeneity property (Föllmer & Knispel, 2011). Similarly, we define the exponential utility based entropic Bellman operator as:

\[
T'[v](s) = \max_{a \in A} \left[ -e^{-R(s,a)} + \gamma \sum_{s' \in S} P(s'|s,a)v(s') \right]
\]

This entropic Bellman operator of (2) is a contraction and satisfies other standard properties.

### Soft-Robust Objective
We define the entropic-risk constrained soft-robust objective with the below optimization problem:

\[
\max_{\theta} \mathbb{E}_\Upsilon \left[ \mathbb{E}_\xi \left[ g^\theta(\xi) \right] \right] \\
\text{s.t.} \quad -\frac{1}{\alpha} \log \left( \mathbb{E}_\Upsilon \left[ e^{-\alpha \mathbb{E}_\xi [g^\theta(\xi)]} \right] \right) \geq \beta
\]

where \( \beta \in \mathbb{R} \) is the cost tolerance and \( \mathbb{E}_\Upsilon \) represents the expectation with respect to different models. We assume that there exists a policy \( \theta^* \) such that the optimization problem in (3) is feasible. The policy \( \theta^* \) computed for this entropic-risk constrained MDP is history independent, thanks to the time-consistency property of entropic risk measure.

We solve Equation (3) by applying Lagrange relaxation procedure (see e.g. Chapter 3 of (Bertsekas, 2003)), which turns it into an unconstrained optimization problem:

\[
\min_{\lambda \geq 0} \max_{\theta} \left( L(\theta, \lambda) = \sum_{m} P(m) \sum_{\xi} P_{\theta,m}(\xi) g(\xi) + \lambda \left( \sum_{m} P(m)e^{-\alpha \sum_{\xi} P_{\theta,m}(\xi) g(\xi)} - e^{-\alpha \beta} \right) \right)
\]

where \( \lambda \) is the Lagrange multiplier. The goal here is to find a saddle point \((\theta^*, \lambda^*)\) that satisfies \( L(\theta, \lambda^*) \geq L(\theta^*, \lambda) \geq L(\theta^*, \lambda^*), \forall \theta, \lambda \geq 0 \). This is achieved by descending in \( \theta \) and ascending in \( \lambda \) using the gradients.

### 3. Entropic Risk Constrained Policy Optimization
We compute the gradient estimates of (4) with respect to \( \theta \) and \( \lambda \) to optimize the objective.

\[
\nabla_\theta L(\theta, \lambda) = \sum_{m} P(m) \sum_{\xi: P_{\theta,m}(\xi) \neq 0} g(\xi) P_{\theta,m}(\xi) \left( 1 - \alpha e^{-\alpha \sum_{\xi: P_{\theta,m}(\xi) \neq 0} P_{\theta,m}(\xi) g(\xi)} \right) \sum_{k=0}^{T-1} \nabla_\theta \pi_{\theta}(a_k|s_k) / \pi_{\theta}(a_k|s_k)
\]

\[
\nabla_\lambda L(\theta, \lambda) = \sum_{m} P(m) e^{-\alpha \sum_{\xi: P_{\theta,m}(\xi) \neq 0} P_{\theta,m}(\xi) g(\xi)} - e^{-\alpha \beta}
\]

See Appendix A for the detailed derivation of the gradients. We use this gradient update rule to develop policy-gradient (PG) and actor-critic (AC) algorithms.

### Policy gradient algorithm
At each episode, several MDPs are sampled from the posterior distribution. The PG method then updates its parameters \( \theta \) and \( \lambda \) based on the expected gradients estimated from several trajectories drawn from the sampled MDPs. Algorithm 2 in the Appendix B presents the pseudo-code of the policy gradient algorithm.
We use value function approximation to estimate the critic.

Algorithm 1: Entropic Risk Constrained Soft-Robust Actor-Critic Algorithm

**Input:** A differentiable policy parameterization $\pi(\cdot, \theta)$, a differentiable state-value function parameterization $\hat{v}(s, w)$, confidence level $\alpha$, budget constraint $\beta$, model posterior $\mathcal{M}$ and initial state distribution $p_0$. Step size schedules $\zeta_3$, $\zeta_2$ and $\zeta_1$.

**Output:** Policy parameters $\theta$

1. Initialize actor parameters $\theta \leftarrow \theta_0$, $\lambda \leftarrow \lambda_0$ and critic parameter $w \leftarrow w_0$.
2. for $k \leftarrow 0, 1, 2, \ldots$ do
3. \hspace{1em} $\theta \leftarrow 0$, $\lambda \leftarrow 0$, $w \leftarrow 0$;
4. \hspace{1em} /* Computed expected gradient from M sampled MDPs */
5. \hspace{2em} for $m \leftarrow 0, 1, 2, \ldots, M$ do
6. \hspace{3em} Sample model from posterior: $\hat{\mathcal{M}} \sim \mathcal{M}$;
7. \hspace{3em} Sample initial state: $s_0 \sim p_0$;
8. \hspace{3em} $\hat{\theta} \leftarrow 0$, $\hat{\lambda} \leftarrow 0$, $\hat{w} \leftarrow 0$;
9. \hspace{3em} /* Simulate trajectories with current policy $\theta$ from sampled MDP */
10. \hspace{4em} for $t \leftarrow 0, 1, 2, \ldots, T$ do
11. \hspace{5em} Sample action: $a_t \sim \pi(\cdot|s_t, \theta)$;
12. \hspace{5em} Observe reward $R(s_t, a_t)$ and next state $s_{t+1} \sim \hat{\mathcal{M}}(\cdot|s_t, a_t)$;
13. \hspace{5em} $\delta = \rho_a(R(s_t, a_t)) + \hat{v}(s_t, w) - \hat{v}(s_{t+1}, w)$; // Compute TD error
14. \hspace{5em} $\hat{\theta} \leftarrow \hat{\theta} + \delta (1 - \alpha \lambda e^{-\alpha t}) \nabla_{\pi(\pi(\pi(s_t, a_t)))} v_{\pi(\pi(\pi(s_t, a_t)))}$;
15. \hspace{5em} $\hat{\lambda} \leftarrow \hat{\lambda} + e^{-\alpha t} - e^{-\alpha t}$;
16. \hspace{5em} $\hat{w} \leftarrow \hat{w} + \delta \nabla_{\pi(\pi(\pi(s_t, a_t)))} \hat{v}(s_t, w)$;
17. \hspace{5em} $\hat{\theta} \leftarrow \hat{\theta} + \hat{\theta} / T$, $\hat{\lambda} \leftarrow \hat{\lambda} + \hat{\lambda} / T$, $\hat{w} \leftarrow \hat{w} + \hat{w} / T$;
18. $\lambda$ update: $\lambda \leftarrow \lambda - \zeta_3(k) \hat{\lambda} / M$; // Actor update
19. $\theta$ update: $\theta \leftarrow \theta + \zeta_2(k) \hat{\theta} / M$; // Actor update
20. $w$ update: $w \leftarrow w + \zeta_1(k) \hat{w} / M$; // Critic update
21. return $\hat{\theta}$;

**Actor-Critic algorithm** The policy gradient algorithm proposed in Algorithm 2 has a very high variance. We address this issue by using bootstrapped function approximation. We propose a risk-averse incremental actor-critic algorithm that converges to a (local) saddle point of the entropic risk constrained objective function $L(\theta, \lambda)$ defined in Equation (4). The gradient update rule for the actor-critic algorithm follows directly from Equation (5) and Equation (6). We use value function approximation to estimate the critic and update the parameters incrementally with expected gradient estimates. Algorithm 1 presents the pseudo-code of the actor-critic algorithm.

**4. Empirical Evaluation**

In this section, we empirically evaluate policy gradient and actor critic methods on different problem domains. All the experiments are run with risk parameter $\alpha = 0.9$ and budget constraint $\beta = 1$ unless otherwise specified. We start with some samples $D$ drawn by arbitrary baseline policies $\pi_b$ from the underlying true distribution $P^*$. We then compute the Bayesian posterior from the prior $p$ and data $D$. We use linear combination of features in all the experiments to approximate the value functions for critic.

![Figure 1. Distribution of return for different assets.](image-url)

**Asset Management Problem** We first evaluate the policy gradient methods on a simple asset management problem (Tamar et al., 2015) with 3 assets, where the distribution of return for the first asset is standard normal. Asset 2 has a normal distribution with mean $\mu = 4$ and standard deviation $\sigma = 6$. Asset 3 has a pareto distribution with shape parame-
The outcome of an action is uncertain and that contributes to the uncertainty about model.

Figure 2 shows the probability of picking each asset as the algorithm progresses. The risk neutral method on the left prioritizes asset 2 with higher mean return, avoiding the fact that it has high variance and the worst case return can be very bad. On the other hand, the soft-robust method on the right first avoids the most risky asset by allocating probabilities to less risky assets 1 and 3. It then realizes that asset 1 is riskier compared to asset 3 and thus allocates all the probabilities to asset 3.

Inventory Management We now evaluate the actor-critic method on an instance of inventory management problem (Behzadian et al., 2019), described in Appendix C.1. The violin plot of Figure 3 (left) shows the return distributions computed by different actor-critic methods. The risk neutral method has very high variance and an arbitrarily bad worst-case return. As this domain involves both epistemic and aleatoric uncertainty, the performance of risk-averse and robust methods are competitive. The risk-averse method is able to reduce the variance due to the inherent stochasticity. Our soft-robust method has a slightly higher mean return and smaller variance compared to the risk-averse version.

Cart-Pole Next, we evaluate our methods on Cart-Pole (Brockman et al., 2016), a domain containing only epistemic uncertainty, details in Appendix C.2. Return distributions computed by different actor-critic methods are shown in the violin plot of Figure 3 (right). The risk-averse actor-critic method performs very poorly having a variance as high as the risk-neutral case. This is because the environment dynamics of this domain are deterministic. But our soft-robust method obtains an expected return near to the risk-neutral method, and also reduces the variance by more than a factor of 2. The worst-case return estimate is also way higher compared to other methods.

5. Summary and Conclusion

In this paper, we derived soft-robust gradient update rules for problems with model uncertainty. We proposed entropic risk constrained policy gradient and actor-critic algorithms with value function approximation. Our empirical results further establishes the usefulness of the proposed methods. Theoretical analysis of our algorithms remain to be done. Future work may also include the study of a novel class of algorithms that can be both risk-averse and soft-robust at the same time.
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A. Gradient Update Rule for Soft-Robust Objective

From Equation (4), we have:

$$L(\theta, \lambda) = \sum_m P(m) \sum_\xi P_\theta,m(\xi) g(\xi) + \lambda \left( \sum_m P(m) e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)} - e^{-\alpha \beta} \right)$$

First we compute the gradient of $L(\theta, \lambda)$ with respect to $\theta$.

$$\nabla_\theta L(\theta, \lambda) = \sum_m P(m) \sum_{\xi: P_\theta(\xi) \neq 0} \nabla_\theta P_\theta,m(\xi) g(\xi) + \lambda \sum_m P(m) \nabla_\theta e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)}$$

$$= \sum_m P(m) \left( \sum_{\xi: P_\theta(\xi) \neq 0} \nabla_\theta P_\theta,m(\xi) g(\xi) - \alpha \lambda e^{-\alpha \sum_\xi P_\theta(\xi) P_\theta,m(\xi) g(\xi)} \sum_{\xi: P_\theta(\xi) \neq 0} \nabla_\theta P_\theta,m(\xi) g(\xi) \right)$$

$$= \sum_m P(m) \sum_{\xi: P_\theta(\xi) \neq 0} \nabla_\theta P_\theta,m(\xi) g(\xi) \left( 1 - \alpha \lambda e^{-\alpha \sum_\xi P_\theta(\xi) P_\theta,m(\xi) g(\xi)} \right)$$

$$= \sum_m P(m) \sum_{\xi: P_\theta,m(\xi) \neq 0} g(\xi) P_\theta,m(\xi) \left( 1 - \alpha \lambda e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)} \right)$$

$$\nabla_\theta \log \left( \prod_{k=0}^{T-1} P_m(s_{k+1}|s_k, a_k) \pi_\theta(a_k|s_k) \mathbb{1}\{x_0 = x^0\} \right)$$

$$= \sum_m P(m) \sum_{\xi: P_\theta,m(\xi) \neq 0} g(\xi) P_\theta,m(\xi) \left( 1 - \alpha \lambda e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)} \right)$$

$$\nabla_\theta \left( \sum_{k=0}^{T-1} \log P_m(s_{k+1}|s_k, a_k) + \log \pi_\theta(a_k|s_k) + \log \mathbb{1}\{x_0 = x^0\} \right)$$

$$= \sum_m P(m) \sum_{\xi: P_\theta,m(\xi) \neq 0} g(\xi) P_\theta,m(\xi) \left( 1 - \alpha \lambda e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)} \right)$$

$$= \sum_m P(m) \sum_{\xi: P_\theta,m(\xi) \neq 0} g(\xi) P_\theta,m(\xi) \left( 1 - \alpha \lambda e^{-\alpha \sum_\xi P_\theta,m(\xi) g(\xi)} \right)$$

$$\sum_{k=0}^{T-1} \nabla_\theta \log \pi_\theta(a_k|s_k)$$

$$\sum_{k=0}^{T-1} \frac{\nabla_\theta \pi_\theta(a_k|s_k)}{\pi_\theta(a_k|s_k)}$$
Next we compute the gradient of $L(\theta, \lambda)$ with respect to $\lambda$:

$$\nabla_\lambda L(\theta, \lambda) = \sum_m P(m) e^{-\alpha \sum_{\xi \in \Xi} P_{\theta}(\xi) \lambda e^{-\alpha \sum_{\xi \in \Xi} P_{\theta}(\xi) g(\xi)}}$$

**B. Policy Gradient Algorithm**

**Algorithm 2:** Entropic Risk Constrained Soft-Robust Policy Gradient Algorithm

**Input:** A differentiable policy parameterization $\pi(\cdot, \theta)$, confidence level $\alpha$, budget constraint $\beta$, model posterior $M$ and initial state distribution $p_0$, step size schedules $\zeta_2$ and $\zeta_1$.

**Output:** Policy parameters $\theta$

1. Initialize policy parameter $\theta \leftarrow \theta_0$ and Lagrange parameter $\lambda \leftarrow \lambda_0$;
2. for $k \leftarrow 0, 1, 2, \ldots$ do
   3. $\hat{\theta} \leftarrow 0$, $\hat{\lambda} \leftarrow 0$; /* Estimate gradients from $M$ sampled MDPs */
   4. for $m \leftarrow 0, 1, 2, \ldots, M$ do
      5. Sample model from posterior: $\hat{M} \sim M$;
      6. Sample initial state: $s_0 \sim p_0$;
      7. Generate trajectories for current policy $\theta$: $\Xi_\theta \sim \hat{M}$;
      8. $\hat{\theta} \leftarrow \hat{\theta} + \sum_{\xi \in \Xi_\theta} P_{\theta}(\xi) g(\xi) \left(1 - \alpha \lambda e^{-\alpha \sum_{\xi \in \Xi_\theta} P_{\theta}(\xi) g(\xi)} \right) \sum_{l=0}^{T-1} \nabla_{\theta} \pi_{\theta}(a_l | s_l)$;
      9. $\hat{\lambda} \leftarrow \hat{\lambda} + \left( e^{-\alpha \sum_{\xi \in \Xi_\theta} P_{\theta}(\xi) g(\xi)} - e^{-\alpha \beta} \right)$;
   /* Update parameters with expected gradient estimates */
10. $\theta$ update: $\theta \leftarrow \theta + \zeta_2(k) \hat{\theta}/M$;
11. $\lambda$ update: $\lambda \leftarrow \lambda - \zeta_1(k) \hat{\lambda}/M$;
12. return $\theta$;

**C. Experiment Details**

**C.1. Inventory Management**

This is a full MDP setup with discrete state and action spaces. There is inherent stochasticity in transition dynamics between states and also the model parameters are not known precisely because of limited samples. So this domain involves both aleatoric and epistemic uncertainty. It starts from an empty inventory level and the inventory evolves based on a normally distributed demand $\sim \mathcal{N}(\mu = 8, \sigma = 3)$. The purchase cost and sale price are 2.49 and 3.99 respectively. Ordering products to restock the inventory helps to meet demands, but unsold products incur a holding cost of 0.03.

**C.2. Cart-pole**

Cart-pole is a standard RL benchmark problem where the evolution of state space is deterministic (no aleatoric uncertainty). But the model parameters are not known precisely and the domain involves epistemic uncertainty. We build a linear model of transition dynamics with data-sets generated from the true distribution. We then generate synthetic samples from the fitted model and use K-nearest neighbor strategy to aggregate nearby states with a resolution of 200.