Optimal control of transient processes in an oscillating system with an electrorheological shock-absorber

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Abstract. The problem of optimal control of the oscillation of a driver’s seat with ER shock absorber is discussed in application to vehicles based on the principle of maximum. Coordinates of a vector of the control parameter and vertical motions of a seat are obtained. Test experimental investigations on working regimes for the driver’s seat model depending on the controlling impact of the electrical signal (the intensity of the electric field) were performed.

1. Introduction

The creation of “smart” shock-absorbers and the algorithm of their control is a key issue of vibroprotective systems, especially in application to vehicles. Nowadays, an intensive development of both “smart” (electro and magnetorheological) fluids for shock-absorbers [1, 2], and methods, and devices for the control of cushioning systems of vehicles [3, 4] is carried out. For each vehicle its own algorithm of control is needed, which can provide an optimal vibroprotection. The type of electrorheological (ER) shock-absorber is determined by electrorheological properties of ER fluid at given parameters of an oscillating system.

To characterize the described single mass oscillating system a state space of vertical motions is used. The calculation of the system results in the determination of the point of extremum of functional at given physical limitations. With the aid of one of the methods of variational calculus a control parameter is determined, which minimizes the initial functional.

2. Statement of the problem

In this paper an optimal control of transition processes of driver’s seat oscillations with an electrorheological shock-absorber in a vehicle is investigated. The controllable vibroprotection is shown in figure 1. The oscillation damping of the driver’s seat is controlled by a control unit of viscoelastic characteristics of a shock-absorber at applying the electric impact, depended on vibro acceleration, determined by sensors. A control unit (6) creates the control signal for providing a necessary force on a shock-absorber piston, which is connected with a sprung mass. Kinematical actuation of the shock-absorber forms its vertical displacement. Oscillations of the driver’s seat are described by the equation of motion:
and a balance of forces [5] on shock-absorber’s piston is described from the equation:

\[ m\ddot{z} + F(z, \dot{z}) = S_p\Delta P(E), \quad (2) \]

where the pressure drop [6] is determined from the equation of the fluid flow:

\[ \left( V_0 / 2G \right) \Delta P + k_1 \Delta P + S_p \dot{z} = k_2 E_c \quad (3) \]

with initial conditions \( \dot{z}(t) = \dot{z}_0; \Delta P(t_0) = \Delta P_0 \) for \( z(t_0) = z_0 \), with \( m \) – mass of the driver’s seat together with the weight of a driver; \( F(z, \dot{z}) = az + b\dot{z} \) – resistance force, conditional upon ER shock-absorber and consisting of elastic \( az \) and viscous \( b\dot{z} \) components [7]; \( c \) – rigidity of the system without ER shock-absorber; \( S_p \) – working area of ER shock-absorber piston; \( V_0 \) – fluid volume of a working chamber of ER shock-absorber; \( G \) – complex modulus of ER fluid; \( k_1, k_2 \) – coefficients, dependent on geometrical parameters of ER shock-absorber; \( \Delta P, \Delta P \) – a drop and change of a drop in pressure over time; \( E_c \) – the intensity of electric field; \( z, \dot{z}, \ddot{z} \) – vibro motion, vibro velocity, vibro acceleration of the system.

Substituting (3) into differentiated equations (1) and (2), we get a generalized equation

\[ \ddot{z} + a_1 \dot{z} + a_2 z + a_3 \dot{z} = a_4 E_c(t), \quad (4) \]

where

\[ a_1 = \left( bV_0 + 2Gk_1 \right)/mV_0, \quad a_2 = 2G \left( S_p + bk_1 \right)/mV_0, \quad a_3 = 2Gk_1 a / mV_0, \quad a_4 = 2Gk_2 S_p / mV_0. \]

Let \( x_1 = z; \ x_2 = \dot{z}; \ x_3 = \ddot{z}; \ x_4 = a_3 E_c(t) \), where \( x_1, x_2, x_3, u_c \) – time functions. It is needed to determine the controllable parameter for the system with the initial condition \( X(t_0) = [0 \ 0 \ 1] \) and a limitation \( |u_c| \leq u_{\text{max}} \), where \( u_{\text{max}} \) – the maximum of the value of the controllable parameter \( u_c \), which according to [8] satisfies the condition:

\[ \int_0^T \ddot{z} dt \rightarrow \min, \ t \in [0, T]. \quad (5) \]

Equation (4) is transformed to the system:

\[
\begin{align*}
\dot{x}_1 &= x_2; \\
\dot{x}_2 &= x_3; \\
\dot{x}_3 &= -a_2 x_2 - a_3 x_3 - a_4 x_4 + u_c.
\end{align*}
\]

System (6) in the matrix form takes the form:
\[ \dot{X} = AX + Bu_c, \quad (7) \]

where

\[
X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix}, \quad B = [0 \ 0 \ 1]^T.
\]

Hamilton function [8] to determine a controllable parameter \( u_c \) from (6) is:

\[
H = \phi_1 x_1 + \phi_2 x_3 + \phi_3 \left( -a_1 x_1 - a_2 x_2 - a_3 x_3 + u_c \right), \quad (8)
\]

where \( \phi_1, \phi_2, \phi_3 \) – auxiliary functions, dependent on time, according to [8] are determined from the system:

\[
\begin{align*}
\dot{\phi}_1 &= -\frac{\partial H}{\partial x_1} = \phi_2 a_1; \\
\dot{\phi}_2 &= -\frac{\partial H}{\partial x_2} = -\phi_1 + a_2 \phi_3; \\
\dot{\phi}_3 &= -\frac{\partial H}{\partial x_3} = -\phi_2 + a_3 \phi_1.
\end{align*} \quad (9)
\]

Necessary condition (5) is the finding of such operating parameter \( u_c \), at which functional \( H \) accepts the maximal values during each moment of time \( t \). A sufficient condition of minimization (5) is the correct choice of entry conditions for system (9).

To determine the parameter \( u_c \), equations (5), (8), (9) are used, providing that function \( H \) reaches extremum at each moment of time. The controllable parameter \( u_c \) is:

\[
u_c = \text{sgn} \left[ -\phi_3(t, z(t)) \right], \quad (10)
\]

where \( u_{\text{max}} = 9.81 \text{ m/s}^2 \).

Therefore, the obtained expression (10) for the controllable parameter \( u_c \) allows us to determine in the oscillating system the acceleration of a sprung mass \( x_3 = \ddot{z} \) from (6), which satisfies the condition (5), i.e. creates an optimal regime of damping of oscillations of the driver’s seat.

3. Results and discussion

To calculate the value of the controllable parameter \( u_c \) the following values of parameters are used: \( m = 80 \text{ kg}, \ b = 400 \text{ N·s/m}, \ c = 2 \cdot 10^3 \text{ N/m}, \ S_\phi = 5.2 \cdot 10^{-3} \text{ m}^2, \ V_0 = 50 \cdot 10^{-6} \text{ m}^3, \ G = 6 \cdot 10^7 \text{ N/m}^2 \). As a result of calculations a transient process, described by the dependence of vibro motions on time (figure 2) has been obtained for vertical oscillations \( x_3(t) \) for \( u_c = 0 \) (\( E_c = 0 \text{ kV/mm} \)) and \( u_c \neq 0 \) (\( E_c = 2.5 \text{ kV/mm} \)). Vector of optimal coefficients of the equation (6) has the following parameters: \( B = (0.56; 0.24; 0.15) \).

To perform the calculation (1) we experimentally investigated a model of the driver’s seat with ER shock-absorber at the condition of stimulated harmonic oscillations with the amplitude of 5 mm and the frequency of 1.6 Hz (figure 3).
Figure 2. Dependence of vibro motion of the driver’s seat on time at $E_c = 0$ kV/mm (1), $E_c = 2.5$ kV/mm (2) and the controllable parameter $u_c$ (3) on time.

Figure 3. Dependence of vibro motion of the model of the driver’s seat on time at stimulated harmonic frequencies with the amplitude of 5 mm and frequency 1.6 Hz at $E_c = 0$ kV/mm (1) and $E_c = 2.5$ kV/mm (2).

From figure 3 it is obvious that time of establishment of the constant amplitude of oscillations of the sprung mass $m$ at impact of the controllable parameter $u_c$ ($E_c = 2.5$ kV/mm) is 2-3 times smaller than that at $E_c = 0$ kV/mm which equals to 4.5 s. Calculated time of establishment of the constant amplitude of oscillations at $E_c = 2.5$ kV/mm is 1.76 s. Time of establishment $t_e$ of the constant amplitude of oscillations at the controllable parameter $u_c$ from the moment of switching on of the signal $t_{on}$ from the control unit is 1.5 s, which coincides with the calculated result for 85%.

Conclusion

Suggested calculation allows us to evaluate the efficiency of damping over time of establishment of oscillation amplitudes and to determine the optimal values of the intensity of the electric field $E_c(t)$ for a maximum damping of oscillations. Considered above scheme of the oscillating system of the driver’s seat is a part of a general scheme of controlling oscillations of vehicles.

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