Causality and entropic arguments pointing to a null Big Bag hypersurface

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Abstract. I propose a causality argument in order to solve the homogeneity (horizon) problem and the entropy problem of cosmology. The solution is based on the replacement of the spacelike Big Bang boundary with a null boundary behind which stays a chronology violating region. This solution requires a tilting of the light cones near the null boundary and thus it is based more on the behavior of the light cones and hence on causality than on the behavior of the scale factor (expansion). The connection of this picture with Augustine of Hippo famous philosophical discussion on time and creation is mentioned.

1. Introduction
The usual mathematical approach to cosmology starts with the Einstein equations,

$$G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}, \quad T_{ab} = (\rho + p)u_a u_b + pg_{ab},$$

passes through the assumption of the cosmological principle, namely through the assumption that all the observers moving with the cosmological flow (i.e. covariant velocity $u$) should see an isotropic universe as we do, which implies $g = -dt^2 + a(t)^2[dr^2/(1 - kr^2) + r^2d\Omega^2]$, where $k = -1, 0, +1$. Finally, it arrives at the Friedmann equation $H^2 + k/a^2 = (\Lambda + 8\pi\rho)/3$, which together with the equation of state $p = p(\rho)$ should determine the dynamics of the large scale geometry of the Universe (the FLRW model). Most work of modern cosmology revolves around these two equations but as I shall argue in this work, observations point to a different model according to which the cosmological principle holds only at late times and cannot be extrapolated to the beginning of the Universe. Indeed, I will argue that the mathematical framework above removes completely the non-conformal dynamics, that is, the tilting of the light cones, which is fundamental in order to solve in a very simple way the homogeneity and entropy problems of cosmology.

2. The homogeneity (horizon, isotropy) problem
The first suggestion comes from the homogeneity problem. The cosmic microwave background radiation (CMB) is the remnant of the decoupling between light and matter which happened at about $z = 1000$ when the baryon density decreased so much that the mean free path of light became infinite and light could travel freely to reach us today. Observations show that it has a quite good black body spectrum at 2.7K with very high isotropy: variations are of the
Figure 1. The conformal diagram of an FLRW universe which displays the homogeneity problem and the inflation solution to it. The Big Bang is a spacelike hypersurface in the conformal completion.

order of $10^{-5}$ for angles as large as 80°. This spherical thermal photograph of the sky tells us that the intersection of our past light cone with the decoupling hypersurface was a region very homogeneous in temperature. If the cosmological principle holds from the time of decoupling to our days, this means that the whole Universe at decoupling time was thermally homogeneous.

This conclusion is puzzling because according to the FRLW universe the opposite portions of the night sky that we observe today had no past in common at decoupling time, so it is not clear why their temperature should have been related. It is useful to rewrite the metric in terms of the conformal time $\eta(t) = \int_0^t \frac{dt'}{a(t')}$, since it makes the coordinate speed of light constant (Fig. 1).

The theory of inflation solves the homogeneity problem by claiming that the scale factor $a$ grows enormously in the first instants of the Universe. The idea is that if a patch of space expands so much to include the whole surface we see today, then it should be natural to observe homogeneity. This argument works only if homogeneity is assumed at a different scale, actually at a much smaller scale, prior to inflation, namely if the initial patch is considered homogeneous.

This criticism has been moved to inflation by several authors, as rather that solving the problem of homogeneity, inflation seems to replace a type homogeneity assumption with another [1]. R. Penrose argues that inflation may well prove to be correct but not for the initial arguments moved in its favor [1].

Mathematically, inflation pushes up the decoupling time in the conformal diagram. In this way the past cones of two decoupling events seen from us today intersect and could influence each other. The idea is that they have roughly the same temperature because there has been time for thermalization before decoupling. This solution based on thermalization increases the entropy difference from the Big Bang to the decoupling time and in particular makes worse the problem of justifying why the entropy of the Big Bang state was so small - the cosmological entropy problem.

3. The cosmological entropy problem

As Penrose clarifies one has to distinguish between matter and gravity when talking about homogeneity and order [2]. The second law of thermodynamics which we know to hold would not hold if the Universe were already at a state of maximum entropy. We are lead to the idea that either the Universe we see was generated by a large entropy fluctuation around the equilibrium or, more reasonably, that the entropy was small already at the beginning of the Universe. According to Penrose’s argument in the coarse graining of the phase space the Universe was located in a very small cell at the Big Bang, a fact which made possible the increase of entropy (by the way, Penrose argues that the entropy has nothing to due with the size of the Universe). This idea can be quantified by comparing the observed entropy of the Universe with its maximum that would be reached after every baryon enters a black hole and using Bekenstein-Hawking formula $S_{BH} = \frac{k\hbar c^3}{4\pi g}A$ which relates the area of the black hole with its entropy. This calculation
shows that the cell at decoupling is about one part of $10^{10^{12}}$ than that at end of the Universe. This figure gives an idea of how peculiar was the the Big Bang. According to Penrose the Big Bang was special because the gravity degrees of freedom were in a special state (the CMB shows that on the contrary matter and radiation were in a hot disordered state). Looking for some mathematical expression of the gravitational entropy he observes that the Weyl tensor $C_{ab}{}^{dc} = R_{ab}{}^{dc} - 2 R_{[a}{}^{[c} g_{b]}{}^{c]} + \frac{1}{3} R g_{[a} g_{b]} d^c$, increases when the gravitational entropy increases, indeed an homogeneous matter configuration through the action of gravity will tend to clump and the Weyl tensor would increase. He proposes that the Universe at the Big Bang had to have a vanishing or very small Weyl tensor, in such a way that the gravitational degrees of freedoms were frozen. This is Penrose’s Weyl tensor hypothesis [2, 3]. (In order to solve this problem Penrose proposes a cyclic picture of the Universe. His proposal differs from mine.) Even if one could prove observationally that the Big Bang satisfied the Weyl tensor hypothesis, one would still be faced with the problem: if the Big Bang is made by causally unrelated events on the boundary how could it be special?

4. A simple causality solution: The Big Bang hypersurface is null

The solution I propose to these two problems, simple as it is, is that the Big Bang in the conformal completion diagram should not be spacelike but rather lightlike and generated by lightlike geodesics. The mechanism I propose works particularly well if the Big Bang hypersurface is also compact thus I will consider only this case. The idea is that every two points at the Big Bang share the same future, that is, future distinction [4] fails at the Big Bang. This is accomplished not through a particular behavior of the conformal factor $a(t)$ but rather of the light cones which tilt near the Big Bang hypersurface as they have to be tangent to it. Exact solutions of the Einstein equations that exhibit this kind of behavior exist, e.g. the Taub-NUT and the A-Taub-NUT metrics.

Thus, any two points at the last scattering hypersurface see the whole Big Bang and thus it is clear that they have roughly the same temperature. However, in this picture the null Big Bang hypersurface is generated by lightlike geodesics and is compact, thus it is made by closed or almost closed causal curves. In practice the events in the Big Bang hypersurface are in (almost) causal contact and hence the matter and radiation content is highly homogeneous already at the Big Bang. There is no need to assume thermalization from Big Bang to decoupling time in order to account for the homogeneity of the CMB radiation. In particular the problem of the specialness of the Big Bang is made less severe.

5. The role of rigidity

This picture for the beginning of the Universe solves naturally also the cosmological entropy problem. Indeed, if $\mathcal{B}$ is the Big Bang hypersurface then $\mathcal{B}$ is the boundary of the Universe that generates from it $\mathcal{B} = \partial I^+(\mathcal{B})$. Since $\mathcal{B}$ is an achronal boundary, the lightlike geodesics that run on it must be achronal and hence cannot have conjugate points. However, $\mathcal{B} = H^{-}(S)$ where $S$ is a Cauchy hypersurface on the future of $\mathcal{B}$ (for simplicity we are making some causality assumptions) and because of a theorem due to Hawking [4, Lemma 8.5.5], since $\mathcal{B}$ is compact it is future null complete. Further, by using the area theorem [4, p.297] [5] one obtains that $\theta = R_{ab} K^a K^b = 0$ on the geodesics of $\mathcal{B}$ where $K^a$ is the tangent vector.

Consider the equations for the expansion $\theta$ (measuring the divergence of the transverse section to the flow) and for the shear $\sigma_{mn}$ (measuring the deformation) of the geodesics running on such hypersurface [4, Sect. 4.2]

$$\frac{d\theta}{dv} = -R_{ab} K^a K^b - 2\sigma^2 - \frac{1}{2} \dot{\theta}^2, \quad \frac{d\sigma_{mn}}{dv} = -C_{m4n4} - \theta \sigma_{mn} - \sigma_{mp} \sigma_{pn} + \delta_{mn} \sigma^2, \quad \text{(sum over } p)$$
Last scattering hypersurface

Big Bang

Here?

\begin{align}
A(n) = B(n) & \quad (A) \\
B(n) & \quad (B)
\end{align}

Figure 2. The chronology violating region behind the Big Bang (case \(A\)) guarantees the Big Bang stability and hence that of our solution to the homogeneity and entropy problems.

where \(m, n = 1, 2\), and \(v\) is the affine parameter and a base adapted to the congruence has been chosen. From the first equation we obtain that the shear vanishes and from the second that \(C_m\hat{n}_4 = 0\) on \(B\). The proposal implies that several components of the Weyl tensor vanish at the Big Bang and hence that the geometry there is very special. Penrose’s Weyl tensor hypothesis is then at least supported if not proved.

6. The chronology violating set and the connection with Augustine’s cosmology

One can also ask if the causal structure (but not the metric, as one expects a singularity due to the singular behavior of the conformal factor at the Big Bang) of the spacetime can be continued before the Big Bang \(B\). We argue that if so then there must be a chronology violating region \(C\) before the Big Bang. Indeed, in this case the achronal hypersurface of the Big Bang would be stable under small perturbations of the metric as \(B\) would be the slice between the non-chronological region \(C\) and the chronological region \(M\setminus C\) of spacetime (Fig. 2).

I have proved a theorem [6] which shows that under weak conditions the chronological region \(M\setminus(B \cup C)\) is actually stably causal [4] and hence admits a time function \(t : M \rightarrow \mathbb{R}\), namely a function which increases over every causal curve. The physical content of this theorem recalls Augustine of Hippo’s conception of time and creation [7, 6] according to which God \((C)\) created the whole Universe \((M = I^+(C))\) and time \((t)\) began to exist at the act of creation \((B)\) and throughout the created world \((M\setminus(B \cup C))\), but still outside God which indeed, in the middle age philosophical thought, was usually considered atemporal [8, 6].

References

[1] Penrose R 2005 The road to reality: A complete guide to the laws of the Universe (New York: A. A. Knopf)
[2] Penrose R 1979 Singularities and time-asymmetry vol General relativity: An Einstein centenary survey (Cambridge: Cambridge University Press) pp 581–638
[3] Goode S W 1991 Class. Quantum Grav. 8 L1–L6
[4] Hawking S W and Ellis G F R 1973 The Large Scale Structure of Space-Time (Cambridge: Cambridge University Press)
[5] Chruściel P T, Delay E, Galloway G J and Howard R 2001 Ann. Henri Poincaré 2 109–178
[6] Minguzzi E Can God find a place in physics? St. Augustine’s philosophy meets general relativity preprint: 0909.3876
[7] St Augustine 1992 Confessions World’s Classics (Oxford: Oxford University Press) translated by H. Chadwick
[8] Craig W L and Smith Q 1993 Theism, Atheism and Big Bang Cosmology (Oxford: Oxford University Press)