Using Conditional Phase Change Operations to Probe the Nature of Quantum Mechanical Systems

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Abstract

Recent developments in quantum computing suggest that it could be possible to make conditional changes to the state of a quantum mechanical system without resorting to classical observation. It is accomplished through collective response of atoms comprising a lattice for the system and involves relative phase adjustments only. We exploit this possibility and describe experimental designs that could help elucidate quantum mechanical properties of these systems and distinguish between interpretations.

1 Introduction

A long-standing question in quantum mechanics is the following: If a measurement shows a quantum system to be in a state \( s \), in what state was the system before the measurement was made? There are three widely recognized plausible answers to this question that characterize the main schools of thought:

1. The system was in the state \( s \), but we were unable to determine that due to insufficient information.

2. The system was not in any particular state, and it was the act of measurement that forced it to choose a state in a quasi-random manner, respecting the probability distribution.

3. The question is irrelevant since we have no way of verifying what state the system was in prior to making the measurement.

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Until now, it has been widely believed that this question cannot be resolved by experiment. However, recent developments in the area of quantum computing have shown that it is possible to perform certain transform operations on quantum systems without observing the state—without disturbing the wave function. In this article we describe a possible experiment using these operations that may finally enable us to elucidate this question. In addition to the philosophical issues, this experiment may actually help us design other similar ones that can be used to decipher the mysteries of quantum mechanics.

2 Technique employed

The operation that we propose to use is the so called “Conditional Phase Flip” (CPF), described below.

Given: A quantum system in some superposition of \( n \) states \( s_1, s_2, ..., s_n \), and a (possibly) unknown but fixed value \( e \), which equals some \( s_i \), \( 1 \leq i \leq n \).

Output: A quantum system that is in the same superposition as the input, with the exception that the phase of \( s_i \) is rotated by \( \pi \) (i.e., inverted, in some sense).

This operation is a specific instance of the more general technique of Conditional Phase Shift. The ability to accomplish CPFs has been demonstrated using amplitude resonance of atomic lattice center-of-mass motion probed phonically[3]. Better, in terms of timescales, would be to accomplish the feat fully optically. Progress in this direction has also been documented[4]. The remarkable feature of these types of manipulation is that they seem to be able to carry out a CPF without resorting to a classical observation. Our purpose here is not to review the technology of cooling and trapping ions for construction of quantum gates[5], but instead we assume the existence and reliability of such tools for use in experiments that allow fundamental studies of the nature of quantum mechanics.

Models employing this physical operation have been used to design quantum algorithms, most notably the quantum database searching algorithm of Grover [2]. Grover combines the CPF with an “inversion about mean” operation, which can be achieved using the Walsh-Hadamard transform to increase the probability for the system to be found in a desired state, starting with a uniform distribution. Inversion about the mean is best understood as follows. Consider a quantum system in a superposition of \( n \) states, \( s_1, s_2, ..., s_n \), with amplitudes \( a_1, a_2, ..., a_n \), respectively. Further, for any two states \( s_i \) and \( s_j \), \( 1 \leq i, j \leq n \), \( s_i \) and \( s_j \) are either in phase or have a phase difference of \( \pi \). The phase thus induces a partitioning of the states into two subsets. Without loss of consistency,
we may assign a phase of 0 to one set and a phase of $\pi$ to the other. Since $e^{i0} = 1$ and $e^{i\pi} = -1$, we can think of each $s_i$ as having a “signed” amplitude, $+a_i$ if the phase is 0 and $-a_i$ if the phase is $\pi$. These signed amplitudes can be represented by means of a histogram. Let $\mu$ denote the arithmetic mean of the signed amplitudes of all the $n$ states. An inversion about the mean operation causes the histogram to be reflected about the line $y = \mu$. To date, there does not appear to be any successful attempt implementing this algorithm in all its complexity, though other quantum algorithms (viz., Shor’s quantum factoring algorithm) have been tested using ion traps [3].

3 Designing the set up

In our proposed experiment, we use a simple quantum system consisting of 2 qubits. In the article by Cleve et. al. [1] a simple generalized framework for representing quantum algorithms is discussed. The two key components in this framework are the Half-Silvered Mirror (HSM) and the Conditional Phase Shifter (CPS). Suppose we wish to force our 2-qubit system into a desired state, say $|11\rangle$. Initially the system is in state $|00\rangle$, i.e., we have 2 qubits, each in the state $|0\rangle$.

Figure 1 shows how this set up will look for each of the two qubits. In the first stage we employ a HSM (one for each qubit) to obtain the superposition $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$. Note that each
HSM performs the transform

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle). \quad (1)$$

Since we are in the initial state $|00\rangle$, we have the required result. The second stage employs a CPS that shifts the phase of $|11\rangle$ (which is our desired state) by $\pi$, to yield the superposition $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$.

The last three stages employ an HSM, a CFS and an HSM, respectively. The CFS in stage 4 shifts the phase of $|00\rangle$. These three steps are considered as a group, since their combined effect is to cause an inversion about the mean [2]. The mean of the amplitudes of the four states at the end of stage 2 is $1/4$. This means that the signed amplitude values of each state are reflected about the line $y = \frac{1}{4}$. A point $(x, \frac{1}{2})$ is reflected as $(x, 0)$ and the point $(x, -\frac{1}{2})$ is reflected as $(x, 1)$. Therefore, the inversion about the mean causes the amplitudes of $|00\rangle$, $|01\rangle$ and $|10\rangle$ to drop to zero, leaving the system in the state $|11\rangle$. This process, as explained in [1], is achieved by 2 crossed paths for each qubit, one path representing a 0 and the other path representing a 1. The expected behavior of this system is as predicted by QED.

What we have described above is essentially a simple version of Grover’s quantum search algorithm, where we are “searching” for the pre-defined state $|11\rangle$. For our experiment, we modify the the set up as shown in figure 2. In the new set up, after stage 2, we simply combine the two paths representing the state of each qubit into one, i.e., a simple additive superposition, instead of using the interference caused by the HSM. The result of this is a system in the exact same superposition, i.e., $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$, as we had earlier, except that each qubit is now traveling on a single path. We then employ the last three stages as described previously.

4 Expected results

The last three stages will cause an inversion about mean. Consider the two possible scenarios:

1. If the system were in any one of the four possible states, then the final state of the system will be of the form $\frac{1}{2}(\pm|00\rangle \pm |01\rangle \pm |10\rangle \pm |11\rangle)$, depending on the initial state. For instance, if the system were in state $|00\rangle$, the mean is $\frac{1}{4}$ and inversion about the mean implies reflection about the line $y = \frac{1}{4}$. This would assign the state $|00\rangle$ an amplitude of $\frac{1}{2}$ and the other states an amplitude of $\frac{1}{2}$. (The three other states will always end up with a phase opposite to that of the initial state.)
In other words, the system at the end of the experiment, is equally likely to be in any one of the four states.

2. If, on the other hand, the system were in a superposition of the states (and that it is the act of measurement that forces the system into a state), the final state of the system has to be $|11\rangle$, as explained in the previous section.

5 Another possible approach

Another possible approach would be a setup described in Figure 3. In this case we employ only one qubit and do not have a separation stage. The first three stages are identical to what we had earlier, with the exception that there are no phase shifts employed on any of the paths. In the fourth stage, instead of separating the states, we allow the photon in the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ to fall on the HSM. If the Copenhagen interpretation were to be followed, the $|0\rangle$ state results in a $|0\rangle$ state for the lower path with amplitude $1/2$ and a $|1\rangle$ state for the upper path with amplitude $1/2$, whereas the $|1\rangle$ state results in a $|0\rangle$ for the upper path and a $-|1\rangle$ state for the lower path, again with amplitudes of $1/2$. This leaves the lower path in the superposition $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, and the upper path in the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. In the fifth stage we combine the paths once again in an additive manner as explained earlier. If the phase could be controlled, this would leave the system in the final state $|0\rangle$. 

Figure 2: Proposed Quantum System for our experiment
If, on the other hand, we assume that in stage three the system were either in state $|0\rangle$ or in state $|1\rangle$ (but not both simultaneously), this would imply that after stage four, the system is either in state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ or in state $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, respectively. In either case, a measurement would be equally likely to find the system in state $|0\rangle$ or in state $|1\rangle$. Such a measurement can be made after the fifth stage using a polaroid filter.

6 Discussion

The techniques described above enable us to affect quantum systems, which have different expected results, depending upon the nature of the systems' state vector prior to measurement. Measuring the final state enables us to say something about the state in which the system existed prior to observation. If such experiments are shown to be feasible in practice, it would no longer be appropriate to retreat to an “agnostic” position, since we would have ways of knowing the state of a system prior to its direct measurement. Furthermore, these types of experiments have another potential benefit in terms of tapping additional power within quantum computers. It may be possible to generalize these ideas to develop other experiments that would get us closer to understanding the nature of quantum mechanical systems.
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