Abstract

Dynamical gauge fields are essential to capture the short and long-distance behavior of gauge theories (confinement, mass gap, chiral symmetry breaking, asymptotic freedom). We propose two possible strategies to use optical lattices to mimic simulations performed in lattice gauge theory. We discuss how the use of biparticle lattices, atomic mixtures, Feshbach molecules or induced five-body rotations can be used to generate local invariance and link composite operators with adjoint quantum numbers that could play a role similar to the link variables used in lattice gauge theory.

Lattice gauge theory provides non-perturbative formulations of Quantum Chromodynamics (QCD) and of strongly interacting theories presented to describe possible new physics beyond the standard model of Electroweak Interactions. The gluons or more generally the gauge fields are space-time indices (the form depends on the type of gauge fields). In the temporal gauge, the unitary matrices in the time direction are gauged transformed to the identity and the quantum Hamiltonian has the form

\[ H = \frac{\beta}{2} \sum_{\text{space links}} \mathbb{E}^{\mu}_{a} \mathbb{E}_{a}^{\mu} - \sum_{\text{space plaques}} \left( 1 - (1/N) \text{Re Tr} \ln \mathbb{U}(p) \right) \]

with \( \mathbb{U}(x,t) \) the color electric fields. They can be seen as the generators of the local gauge transformations. They obey local commutation relations similar to the Lie algebra and the \( \mathbb{U}(x,t) \) transform like the adjoint representation under commutation with \( \mathbb{E}^{\mu}(x) \).

The generic form of the gauge boson interactions with fermions (quark-fermion interaction in QCD) is

\[ \sum_{x,y} \bar{\psi}^{a}(x) \mathbb{U}_{a}^{\dagger}(x,y) \mathbb{U}_{b}(y,x) \psi^{b}(y) \]

As explained above, it is essential to have dynamical \( \mathbb{U}(x,t) \) in order to obtain the main physical features. This also appears to be the most challenging part of the program. We see two possible types of strategies:

- **Strategy I:** quantum gauge fields and fermions
  - Engine quantum link variables having a Hamiltonian with quark-fermion interactions as in Eq. (4). This possibility seems to require an underlying local gauge symmetry. Correlation functions of gauge invariant products of fermions could be measured by introducing local source parameters coupled linearly to the gauge invariant products of fermions and taking “functional variations” as in quantum field theory.

- **Strategy II:** gauge variables and quantum fermions
  - Alternatively, one could use numerical link variables of MC simulations and replace the fermion determinants and propagators calculations in a fixed configuration for the link, by measurements of fermion correlations on the optical lattice. This possibility requires the ability to manipulate locally the hopping parameters appearing in Eq. (5) and to have fast enough communication between the classical computer and the optical lattice.

This is a list of problems that need to be solved in order to implement the above strategies.

- **Relativistic fermions with general color**
  - Using three of the hyperfine levels F=1/2 and 3/2 of Li Fermi gas near a Feshbach resonance, one can create a quantum degenerate three-state Fermi gas with approximate SU(3) symmetry [1]. On a honeycomb lattice, a single flavor Dirac theory with global SU(3) symmetry could be obtained.

- **Dynamical link variables**
  - An idea would be to map a particle physicist who was a graduate student in the technicolor era to build the link variable \( U_{a}^{\mu} \) as a “condensate” of the site variables \( \bar{q} q \) at the ends of the link.

- **Challenges**
  - Local manipulation of hopping parameters
  - Global non-abelian Berry phases can be obtained from abelian transformations in degenerate quantum mechanical systems [4]. Such phases can be obtained from “dark states” in a tripod system [5]. Global SU(N) potentials can also be created using N internal states of atoms and laser assisted state sensitive tunneling [6]. I am not aware of attempts to make these constructions local. However, locally rotating deformations of optical lattice have been studied recently [7].

- **Local symmetry?**
  - The principle of local gauge symmetry has played a central role in the development of the standard model of all known non-gravitational interactions. I believe it is also central for the present project. Local symmetry emerges in trapped alkalis with hyperfine states and the gauge field is the superfluid velocity [8].

**Figure 2:** RG flows obtained by the two lattice matching methods for Heisenberg Model in the complex \( \beta \) (inverse temperature) plane (figure made by Yuzhi Liu).

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