The Effect of Viscosity Ratio and Peclet Number on Miscible Viscous Fingering in a Hele-Shaw Cell: A Combined Numerical and Experimental Study

Daniel Keable¹ · Alistair Jones¹ · Samuel Krevor¹ · Ann Muggeridge¹ · Samuel J. Jackson²

Received: 11 November 2021 / Accepted: 30 March 2022 / Published online: 25 May 2022
© The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract
The results from a series of well characterised, unstable, miscible displacement experiments in a Hele-Shaw cell with a quarter five-spot source-sink geometry are presented, with comparisons to detailed numerical simulation. We perform repeated experiments at adverse viscosity ratios from 1 to 20 and Peclet numbers from $10^4$ to $10^6$ capturing the transition from 2D to 3D radial fingering and experimental uncertainty. The open-access dataset provides time-lapse images of the fingering patterns, transient effluent profiles, and meta-information for use in model validation. We find the complexity of the fingering pattern increases with viscosity ratio and Peclet number, and the onset of fingering is delayed compared to linear displacements, likely due to Taylor dispersion stabilisation. The transition from 2D to 3D fingering occurs at a critical Peclet number that is consistent with recent experiments in the literature. 2D numerical simulations with hydrodynamic dispersion and different mesh orientations provide good predictions of breakthrough times and sweep efficiency obtained at intermediate Peclet numbers across the range of viscosity ratios tested, generally within the experimental uncertainty. Specific finger wavelengths, tip shapes, and growth are hard to replicate; model predictions using velocity-dependent longitudinal dispersion or simple molecular diffusion bound the fingering evolution seen in the experiments, but neither fully capture both fine-scale and macroscopic measures. In both cases, simulations predict sharper fingers than the experiment. A weaker dispersion stabilisation seems necessary to capture the experimental fingering at high viscosity ratio, which may also require anisotropic components. 3D models with varying dispersion formulations should be explored in future developments to capture the full range of effects at high viscosity ratio and Peclet number.

Keywords Viscous instability · Miscible · Quarter five-spot · Hele-Shaw · Fingering
1 Introduction

Viscous fingering is an instability that occurs when a less viscous fluid displaces a more viscous fluid in a porous medium or a Hele-Shaw cell. Small perturbations at the fluid-fluid interface (whether a sharp, immiscible interface or a smooth miscible interface) grow, forming distinct fingers with characteristic wavelength and finger size. A similar form of fingering also results from gravitationally driven instabilities, occurring when a denser fluid sits above a lighter fluid (known as the Rayleigh–Taylor instability). The gravitationally driven instability was first described by Rayleigh (1882) and further discussed by Taylor (1950) while the viscous driven instability was first investigated by Saffman and Taylor (1958) and is now often referred to as the Saffman–Taylor instability. Both types of instabilities have been studied extensively as they can influence the efficiency of carbon dioxide sequestration in the subsurface (e.g. Kopp et al. 2009; Ennis-King et al. 2005), chromatographic experiments (e.g. Rousseaux et al. 2007) and enhanced oil recovery (e.g. Gerritsen and Durlofsky 2005). This paper focuses on viscous-driven instabilities.

The fingering pattern structure for miscible, viscous driven instabilities is controlled by the viscosity ratio of the displaced to invading fluid ($M = \mu_2/\mu_1$), the transverse Peclet number ($P_{eT}$) and the anisotropy between longitudinal and transverse diffusion/dispersion ($D_L/D_T$) (Tan and Homsy 1986; Zimmerman and Homsy 1991, 1992; Hamid and Mugggeridge 2020) where the transverse Peclet number for linear displacements is defined as:

$$P_{eT} = \frac{vL}{D_T}$$  

where $v$ is average fluid velocity, $L$ is a characteristic length and $D_T$ is the transverse diffusivity. For linear displacements (line drives) at a given rate, larger viscosity ratios result in an increased number of fingers with smaller characteristic wavelength, while higher transverse dispersions result in fewer fingers. A higher longitudinal dispersion, for a given flow rate, will delay (or if very large, prevent) the onset of fingering by dispersing the interface between displacing and displaced fluids and making the displacement more stable. The ratio of longitudinal to transverse diffusivity controls the number of fingers and their growth rate (Tan and Homsy 1986; Zimmerman and Homsy 1991, 1992; Hamid and Mugggeridge 2020).

Miscible viscous fingering in radial flows is of practical relevance, as this may occur around injection wells in CO$_2$ storage schemes and during miscible gas injection for enhanced oil recovery. The fingering behaviors are more complex than in linear flows because the flow velocity decreases with distance from the injection point. This means that the impact of diffusion and dispersion on the fingering also changes with time and distance from the injector. This is further complicated because the dispersion may be velocity dependent at higher rates. In porous media, the dispersion typically varies approximately linearly with velocity (Perkins and Johnston 1963) whilst in Hele-Shaw cells the longitudinal dispersion varies with the square of velocity (Taylor 1953) and the transverse dispersion is only dependent on molecular diffusion (Tan and Homsy 1987; Chen and Meiburg 1998; Petitjeans et al. 1999). In the absence of such velocity dependence, the Peclet number will decrease with distance from the injection point, so the number of fingers and their growth rate will also decrease (Tan and Homsy 1987). For higher velocities in porous media, the impact of dispersion will tend to decrease with time until the flow is only affected by molecular diffusion (Riaz et al. 2004). Some authors assert that the dependence of dispersion on velocity squared is not important (Petitjeans et al. 1999; Riaz et al. 2004).
Numerical simulation is usually used to describe the impact of viscous fingering on systems of practical interest, however very high mesh resolutions are required to ensure that the solutions are dominated by physical diffusion and dispersion rather than numerical diffusion. This can make such simulations very computationally expensive, in particular, if a fixed mesh is used. A further limitation is that most finite volume methods are affected by mesh orientation error when modeling miscible displacements, as observed in radial flows (Brand et al. 1991). Some researchers have explored the use of dynamic adaptive meshing to focus resolution in the fingered zone and thus reduce the computational effort required (e.g. Edwards and Christie 1993; Lee and Wheeler 2017; Kampitsis et al. 2020). The mesh orientation effort is also much reduced in finite element or control-volume/finite element approaches using unstructured triangular meshes (e.g. Lee and Wheeler 2017; Kampitsis et al. 2020) or by using a nine-point method in finite volume methods such as described by Yanosik and McCracken (1979). At larger scales, many researchers use upscaled, homogenized models (e.g. Koval 1963; Todd and Longstaff 1972) to estimate the average impact on macroscopic flow, however, these models usually need to be calibrated by comparison with detailed simulation. Some insights can be obtained for early time behaviors from analytical solutions, usually in the form of linear stability analysis (e.g. Tan and Homsy 1986, 1987; Zimmerman and Homsy 1992; Riaz et al. 2004) but these approaches are not suitable for predicting the late time, non-linear behaviors seen in most practical systems, especially when there is geological heterogeneity.

Physical experiments provide important insights into the physics of viscous fingering (as well as being essential in validating the predictions of detailed numerical simulations) however, there are few experiments for radial flow reported in the literature. Not all of those that are reported provide sufficient information for validation purposes or they only investigate a small range of the possible physical behaviors. Ideally, there should be information regarding the values of all input parameters needed for simulation (system dimensions, fluid properties, porosity, permeability) as well as the experimental results in terms of volumes or rates of fluids produced over time and images of the displacement at different time intervals, including early times.

Tables 1 and 2 provides a summary of the experimental investigations of unstable miscible viscous fingering in radial flow for Hele-Shaw cells and porous media, respectively. We include references that provide sufficient information to characterise the pack or cell properties, the viscosity ratio(s) used and the Peclet number and/or provide some results in terms of fingering patterns or other measures of fingering dynamics. Most of these investigations used Hele-Shaw cells (Mahaffey et al. 1966; Paterson 1985; Stoneberger and Claridge 1988; Petitjeans et al. 1999; Bischofberger et al. 2014; Djabbarov et al. 2016; Videbæk and Nagel 2019). Zhang et al. (1997) used glass bead packs whilst (Habermann 1960) used a system in which sand was glued to the glass plates. Lacey et al. (1961) performed experiments in glass bead packs but did not provide data on their porosity and permeability. Nonetheless they are included in our review as they provide data on recovery efficiencies. The table also provides an estimate of the Peclet number, $P_e$ for radial flow (as defined by Petitjeans et al. 1999):

$$P_e = \frac{Q}{\frac{C\pi D_m}{h}}$$

where $Q$ is the flow rate (m$^3$/s), $D_m$ is the molecular diffusivity (m$^2$/s), $h$ is the thickness of the cell (m) and $C = 1/2$ for flow in a quarter five-spot well pattern and $C = 2$ when the source is in the center of a circle. We used the same value of diffusivity ($3 \times 10^{-10}$ m$^2$/s),
Table 1 Comparison of Hele-Shaw experiments investigating unstable miscible displacements in radial flow from the literature. The average velocity is calculated from the time taken for a stable displacement to travel from injector to producer for the specified flow rate and cell dimensions. Open B.C.s have the entire cell circumference open to the atmosphere.

| Hele-Shaw Pattern | This experiment Outlet B.C. | Mahaffey et al. (1966) Point sink in opposite corner | Paterson (1985) Point sinks in corners | Stoneberger and Claridge (1988) Point sink in opposite corner | Petitjeans et al. (1999) Point sink | Bischofberger et al. (2014) Circle | Djabbarov et al. (2016) Point sink in opposite corner | Videbæk (2019) Circle |
|------------------|-----------------------------|--------------------------------------------------|-------------------------------------|-----------------------------------------------------|--------------------------------|---------------------------------|--------------------------------|---------------------|
| System side length or diameter [cm] | 40 | 30.5 | 60 | 28.6 | 68 | 28 | 50 | 14, 28 |
| Plate spacing [cm] | 0.025 | 0.00381, 0.0119 | 0.15, 0.3 | 0.01 | 0.061 | 0.0076, 0.1143 | 0.0375 | 0.0076, 0.0419 |
| Permeability [D] | 5280 | 122.57, 1200 | 190000, 760000 | 844 | 31400 | 488, 110000 | 11900 | 488, 14800 |
| Flow rate [ml/min] | 0.1–10 | 0.049 | 7.38, 73.80 | 8.30 | 0.0156–23.5 | 0.4–40 | 1 | 0.001–10 |
| Average velocity [cm/s] | 0.0024–0.24 | <0.02 (small gap) <0.006 (large gap) | 0.0290–0.580 | 19.6 | 0.00603–9.09 | 0.0443–0.29 | 0.0126 | 0.00022–0.040 |
| Viscosity ratio, $M$ | 1, 2, 5, 10, 20 | 1, 3.3, 12.5, 39.4 | Water and glycerin | 0.1, 2, 1.3, 5.3, 50 | 3.32, 4.48, 12.2, 110, 153 | 1–1350 | 1, 2, 5, 10, 20, 30, 100 | 5.2, 31, 909 |
| $P_e = \frac{Q}{C \pi D_m h}$ | 14150 – 1415000 | 1100–3400 | 6500–131000 | 881000 | 70–102000 | 3100–4650000 | 28300 | 6–350000 |

Note: $P_e$ is the dimensionless parameter defined as $P_e = \frac{Q}{C \pi D_m h}$.
Table 2  Comparison of porous media experiments investigating unstable miscible displacements in radial flow from the literature. Velocity calculation as Table 1. Note, this experiment was performed in a Hele-Shaw cell but is shown here for comparison

| Porous media | This experiment | Habermann (1960) | Lacey et al. (1961) | Zhang et al. (1997) |
|--------------|----------------|------------------|---------------------|---------------------|
| Pattern      | Confined quarter five spot | Confined quarter five spot | Confined and unconfined quarter five spot | Confined quarter five spot |
| System side length [cm] | 40 | 11.4, 38.1 | 22.5, 45, 90 | 40 |
| Plate spacing [cm] | 0.025 | 0.3175, 0.635 | 0.3, 0.8, 1.25, 7.62 | 1 |
| Permeability [D] | 5280 | 4.5, 12, 20 | - | 5 |
| Porosity (%) | 100 | 24.2, 31 | - | 38 |
| Approx diameters of beads [µm] | - | ≈53–600 | - | ≈100 |
| Flow rate [ml/min] | 0.1–10 | 0.07–30 | - | 0.83–3.33 |
| Average velocity [cm/s] | 0.0024–0.24 | 0.00022–0.242 | 0.0006–0.0210 | 0.0011–0.0042 |
| Viscosity ratio, $M$ | 1, 2, 5, 10, 20 | 0.037, 0.222, 1, 1.4, 2.4, 2.95, 4.6, 12.9, 17.3, 23.4, 27.0, 38.2, 71.5, 130.7 | 1, 10, 42 | 4, 12, 25 |
| $P_e = \frac{Q}{C \pi D_m h}$ | 14150 – 1415000 | 390–330000 | - | 3000–12000 |
for water/glycerol (D’Errico et al. 2004) in all cases, although this value will be slightly different for other fluid pairs or if there is velocity-dependent dispersion. The value of $P_e$ is very large in all experiments indicating the dominance of viscous effects over diffusion, although it ranges over several orders of magnitude.

All of the experiments investigated the flow patterns in homogeneous systems, with the exception of those described by Djabbarov et al. (2016). In addition, in the experiments identified in the literature, only Zhang et al. (1997) and Djabbarov et al. (2016) compared the predictions of numerical simulations with their experimentally observed fingering patterns. In both papers the simulations did not provide a good prediction of the fingering patterns although they were able to provide reasonable estimates of effluent profiles.

In this paper, we describe a series of well-characterised, unstable, miscible displacement experiments in a glass Hele-Shaw cell with a uniform plate separation. The impact of viscosity ratio and Peclet number on the fingering patterns are investigated. Both the fingering patterns and the effluent profiles are compared against the predictions from numerical simulations for varying viscosity ratio for an intermediate Peclet number case. The resulting dataset represents an ideal validation case for numerical methods.

2 Methodology

A series of miscible displacement experiments were performed in a Hele-Shaw cell (1898) and then simulated with a finite-difference simulator that has previously been validated against a variety of linear, miscible displacement experiments (Christie and Bond 1985, 1987; Christie et al. 1990; Davies et al. 1991).

2.1 Physical Experiments

The physical experiments were performed in a horizontally orientated, square Hele-Shaw cell, 0.4 m on each side, see the supporting information Figures S1 and S2 for a photo of the setup and schematic of the Hele-Shaw cell, respectively. The 15 mm glass plates were separated by a 0.25 mm thick Mylar insert which ran the circumference of the cell. They were mounted in an aluminum frame that kept them centralised and allowed the plates to be clamped together to ensure the cell was sealed around the edges. Fluid was injected through a 15 mm port in the top glass plate using an ISCO pump at constant rate (nominal accuracy of 0.001 ml/min, i.e. possible fluctuations much below the minimum rate of 1 ml/min rate). Fluid was produced from a 15 mm port on the opposite corner at constant atmospheric pressure and collected in a sealed electronic scale unit. The displacement represented that seen in a quarter five-spot injection-production pattern, as is typical in subsurface operations. The permeability, $k$, of the low Reynolds number system can be calculated using the depth-averaged Stokes flow through the parallel plates of spacing $h$ (Greenkorn et al. 1964) as $k = h^2/12$. For our system, $k = 5.208 \times 10^{-9} \text{ m}^2 = 5280 \text{ D}$. The uniformity of the gap and thus the homogeneity of the cell was confirmed by examining the interface observed during a stable (matched viscosity) displacement run in two

---

1 Estimated for a 20% volume fraction of glycerol using data in D’Errico et al. (2004). This is only approximate for all experiments except for those described in this paper, but it enables a rough comparison of Peclet numbers.
The fluids selected for the experiments were glycerol solution (displaced phase) and deionised water (DI, displacing fluid) since they can be used to achieve a range of viscosity ratios (by diluting the glycerol with water), have similar wetting properties on all materials used in the cell and are fully miscible. They have been used by several researchers for similar experiments including (Habermann 1960; Paterson 1985; Zhang et al. 1997; Petitjeans et al. 1999). Analytical expressions are available for estimating both the binary mixture density (Volk and Kähler 2018) and viscosity (Cheng 2008) for different mixture proportions, making it straightforward to prepare the desired mobility ratios, (see Table 3). The glycerol solution was dyed with Lissamine Green B at a concentration of 0.6 g per 100 ml. This provided enough color contrast between the fluids for imaging whilst being easy to clean from the cell and injection pump after finishing each experiment. The different concentrations used for the displaced fluid and associated viscosity ratios are given in Table 3. These were determined using the relationship found in Cheng (2008) at 20 °C under atmospheric conditions, and verified by direct measurements in several experiments, using a Brookfield cone and plate rheometer at various shear rates. The measured viscosity
ratios were found to be within ±10% of those calculated, with differences largely due to temperature variations; these uncertainties are reflected in the horizontal error bars on the $M$ plots below.

Images of the fluid displacement were recorded using a camera mounted above the cell and time-lapse software. These images were post-processed to estimate breakthrough times for comparison with the measurements of produced fluids, as well as metrics such as interfacial lengths. They were first cropped and processed to account for small misalignments with the camera objective, using a projective geometric transformation in Matlab. The image brightness was scaled using pre and post-experiment images to account for uneven illumination across the imaging period.

Images were segmented using the FIJI image processing software (Schindelin et al. 2012). We utilised a flat-field correction (FFC) to correct for uneven illumination or dirt/dust on the lenses using the pre-injection reference image. The image segmentation steps are then: conversion from RGB to 16-bit greyscale, FFC, brightness and contrast correction (in some cases), application of a sharpening mask followed by auto-thresholding based on the RenyiEntropy algorithm (Kapur et al. 1985). The auto-threshold limits were manually checked, with resulting metrics largely independent of threshold choice (e.g. variations of ±5% generated <2% change in areal sweep calculations).

### 2.2 Numerical Modelling

The experiments were modeled using a finite-difference numerical method based on that first described by Christie and Bond (1987). This was originally developed to model unstable, first contact, miscible displacements and has been subsequently developed to include various higher-order methods, velocity-dependent dispersion where the dispersion depends on a specified power of the velocity and inactive cells, amongst other features. It has been previously validated against a range of physical experiments investigating linear displacements in porous media (see for example Christie and Bond 1987; Christie et al. 1990; Davies et al. 1991), but has not previously been fully validated for radial flows or flows in Hele-Shaw cells.

For first contact miscible fluids we solve the following mass conservation equation:

\[
\frac{\partial c}{\partial t} + \nabla \cdot \mathbf{v} = \phi \nabla \cdot (D \nabla c) \tag{3}
\]

where $c$ is the mass fraction of the injected fluid, $\phi$ is the porosity (=1 in a Hele-Shaw cell), $D$ is the dispersion tensor, $\nabla$ is the vector differential operator and $\mathbf{v}$ is the Darcy velocity given by:

\[
\mathbf{v} = -\frac{k}{\mu_m} \nabla P \tag{4}
\]

where $k$ is the permeability of the porous medium, $\mu_m(c)$ is the mixture viscosity and $P$ is the fluid pressure. The exponential mixing rule (as suggested by Cheng 2008) was used to describe the mixture viscosity:

\[
\mu_m = \mu_1 e^{-c \ln M} \tag{5}
\]

where $M = \mu_2/\mu_1$, subscript 1 refers to the injected fluid, and subscript 2 to the displaced fluid. The dispersion tensor, $D$ in Eq. (3), is given by:
\[ D = \begin{bmatrix} D_L & 0 \\ 0 & D_T \end{bmatrix} \]  \hspace{1cm} (6)

where \( D_T \) and \( D_L \) are the transverse and longitudinal dispersion coefficient, respectively. In this work, we set \( D_T \) equal to the molecular diffusivity in the bulk fluid, \( D_m \). The tensor is orientated with longitudinal in the direction of flow and transverse perpendicular to this, at all times. \( D_L \) is given by:

\[
\frac{D_L}{D_m} = 1 + \frac{2}{105} \frac{h^2 |v|^2}{D_m^2}
\]  \hspace{1cm} (7)

where \(|v|\) is the local speed. Equations (3) and (4) are constrained by the equation for the conservation of total volume (as the system is assumed to be incompressible):

\[ \nabla \cdot v = 0 \]  \hspace{1cm} (8)

Combining Eq. (8) with (4), an elliptic pressure equation is formed:

\[ \nabla \cdot \left( \frac{k}{\mu_m} \nabla P \right) = 0 \]  \hspace{1cm} (9)

To solve the system of equations numerically we employ an implicit pressure, explicit saturation (IMPES) technique with finite-differences, solving the elliptic pressure Eq. (9) implicitly, with a first-order backward Euler scheme, and spatially with a second-order accurate central scheme. The solution technique uses a modified incomplete Cholesky (MIC(0)) preconditioner with Cholesky conjugate gradient method. The hyperbolic concentration transport Eq. (3) is then solved explicitly using a flux-corrected transport algorithm (FCT) by Zalesak (1979). It uses operator splitting to include diffusion and dispersion which are solved implicitly.

A dimensionless Cartesian mesh of size \( 400^2 \) (effective resolution 1 mm) was used for all 2D simulations with the mesh orientated parallel, or diagonal, to the line joining injection and production wells. This mesh resolution was chosen following a mesh refinement study to ensure that physical diffusion dominated numerical diffusion, see the supporting information Figures S3 and S4. Fingers were triggered by specifying a log-normal permeability distribution with a standard deviation equivalent to \( \sigma = 0.03 \) mm and a correlation length of 2 mm across the entire mesh. \( \sigma \) was estimated by considering the manufacturing imperfections in the glass plates, and any bending due to the injection pressures; 0.03 mm is the maximum estimated value. The correlation length was chosen from the same manufacturing consideration, but also the minimum length-scale of fingers observed in experiments, which was of order 3–5 mm. A correlation length of 2 mm can trigger fingers of initial size 2 mm, using a mesh at least 2× as resolved (to mitigate orientation effects).

Five different realisations (with the same statistics) were simulated for the \( M = 20 \) displacement in order to estimate the influence of different permeability distributions on the simulated behavior. Using a random permeability field for each simulation mimics the natural variability in finger initiation observed in the experiment, see Fig. 2 repeated experiments. Previous work by Davies et al. (1991) obtained good results when the simulations were initialised with the experimental concentration distributions at early time, however, here we choose to initialise with a random permeability field to capture the inherent variability in the experiment, and the field process.
A constant rate injector and producer were specified in opposite corners of the square of active cells to act as a source and sink of fluid. These had the same radius as the inlet and outlet ports and locations with respect to the model boundaries as used in the experiments.

**Fig. 2** Evolution of miscible fingering patterns over time, $t$ (expressed as fractions of breakthrough time, $t_b$, left to right) seen in experiments with different viscosity ratios ($M$, increasing top to bottom), at $P_e = 1.4 \times 10^5$. For $M = 10, 20$ we show repeat experiments with source and sink swapped (images rotated) under the top images ($M = 20$ has two repeats in the opposite direction). Note the bottom two $M = 20$ repeat experiments have some shadow artifacts on the bottom of the cell but are shown for qualitative comparisons. Mass fraction of injected fluid $c = 1$ is white, $c = 0$ is light blue—this applies to all experimental images.

A constant rate injector and producer were specified in opposite corners of the square of active cells to act as a source and sink of fluid. These had the same radius as the inlet and outlet ports and locations with respect to the model boundaries as used in the experiments.
3 Results

3.1 Varying Viscosity Ratio, $M$

Figure 2 shows the evolution of the fingering pattern in the experiment for different viscosity ratios where time is expressed in fractions of the breakthrough time, $t_b$, for that particular viscosity ratio ($P_e = 1.4 \times 10^5$ in all cases). The mass fraction $c$ is shown by the color with $c = 1$ as white and $c = 0$ as light blue. Quantitative concentration information between these limits is not possible due to the polychromatic lighting. We show the breakthrough time in pore-volumes injected for the varying viscosity ratio cases quantitatively in Fig. 3, along with other literature results. Experimental results are also shown for all cases in Table 4 in Appendix A. The volume-based breakthrough time is determined from the point at which the recovered vs. injected volume gradient $dV_r/dV_i$ inflects below unity. The image-based breakthrough times are calculated as the time at which the displacing plume (in white) passes into the sink outlet. There is an uncertainty of $\pm 1$ frame with this method, with an error bar within the symbols in Fig. 3.

In line with previous experimental results, we find that breakthrough time is reduced as the viscosity ratio increases. Linear stability analyses (Tan and Homsy 1987; Riaz et al. 2004) suggest that in the early-time regime, the number and the length of the fingers increases with viscosity ratio, as is observed here, which could explain the earlier breakthrough in the non-linear regime. There is increasing variability in finger velocities at higher $M$, with more shielding of small finger growth (Nicolaides et al. 2015). This leads to a more dominant single finger, along the central source-sink streamline (see Fig. 2 bottom.
right), which largely controls breakthrough and becomes invariant to $M$ as $M$ becomes very large.

The experimental results in Figs. 2 and 3 show the variability in repeated experiments, along with sources of experimental uncertainty. This is largely lacking in previous experimental literature but provides realistic bounds for assessing model conformance. Experiments were repeated in opposite flow directions in the cell, swapping the source and sink (for $M = 5, 10, 20$) as well repeated in the same direction (for $M = 20$) to highlight typical variability. Figure 2 shows that similar fingering structures, with characteristic wavelength, are created in repeat runs, but the specific finger growth and tip-splitting occurrence are distinct in each case. This again highlights the cell homogeneity—there are no inclusions or regions that specifically trigger instabilities. Instead, fingering variations between repeat runs are likely due to small instabilities in the initial fluid configuration as it leaves the injection port and the interaction thereafter with the cell varying thickness. The fingering variation is carried through to the variability in the macroscopic measures in Fig. 3. Small variations in the cell thickness create variations in spatial permeability; this is how instability is triggered in the simulations and represents a realistic finger formation mechanism. The stochastic nature is captured in the simulations by varying the statistical realization of the permeability field, mimicking the experimental uncertainty (see Fig. 6 below). Variations in permeability of this relative magnitude are typically of subsurface rocks (e.g. sandstones and carbonates), and would therefore be encountered in many field scenarios.

The volume-based experimental results in Fig. 3 have uncertainty bars which stem from the difference in recovered volume of displaced fluid (estimated from mass-produced and calculated mixture density), and the injected volume of displacing fluid (estimated from constant pump volume rate) at the breakthrough point—they should be equal if there were no uncertainty. Generally the image-based breakthrough is slightly faster than the volume-based method, largely due to the precision of the volume based method which has
a detection like ‘threshold’ for when the recovered volume has clearly diverged from the injection volume. The correspondence of the image and volume based methods (when considering the uncertainty) gives confidence in the experimental precision, and the accuracy of the results when compared to other literature values across a range of conditions. The image-based method is more reliable than the volume-based methods when the fingering is truly 2D (i.e not at high $P_e$ in further results), however it does not give information after breakthrough. Generally the repeat breakthrough times vary by $\approx 0.02 PV$ through the image-based methods, which represent variation of 5–8% of the breakthrough time over the range considered here. This variability is smaller than the volume-based uncertainty in most cases.

In Fig. 4 we show the dimensionless interface length against time for the different viscosity ratio experiments. The interface length was calculated directly from the segmented images using the perimeter of the displacing phase (this is generally smooth so there are

Fig. 5 Comparison of fingering patterns predicted by simulation with experimental observations for $M = 2 - 20$ ($P_e = 1.4 \times 10^5$) with a parallel mesh at different pore-volume injected times. Simulation concentration legend is shown at the bottom of the figure
The simulation results from Chen and Meiburg (1998) for $M = 12$, $Pe = 255, 510$ and 765 (converted to the $Pe$ definition here, their $Pe$ values are 400, 800 and 1200, respectively) are shown for comparison. In all cases, the interface length increases with time to a maximum at breakthrough. After breakthrough the interface length decreases again. This is mainly due to the decay of side fingers as the displacing fluid channels preferentially along the finger(s) that have broken through. We also show the theoretical scaling of a stable growing circle subject to mass conservation ($l^* \sim \sqrt{t^*}$), which the $M = 1$ case follows for its displacement until breakthrough. The Chen and Meiburg (1998) results are qualitatively similar but the interface length increases more rapidly than in our experiments. This is despite their much lower Peclet number which would suggest their flows are more influenced by diffusion and should thus have fewer fingers growing more slowly.

In our experiment results, we see a significant delay in the onset of fingering in all cases (see Fig. 2). This has been observed and discussed previously in linear displacements (see Hamid and Muggeridge 2020 and references therein). It was also seen for radial flows in the experiments reported by Perkins and Johnston (1963); Paterson (1985); Videbæk and Nagel (2019) although only discussed by Perkins and Johnston (1963). The phenomenon is also discussed briefly in the analytical study by Sharma et al. (2019). This delay occurs because initially, the step interface between displaced and displacing fluid spreads more rapidly by diffusion/dispersion than the fingers can grow. This also reduces the initial number of fingers that form and their subsequent growth rate (see for example Perkins and Johnston 1963). We hypothesize that this initially stable zone is larger in radial flows subject to velocity-dependent dispersion because of increased velocities near the injector. This may be exacerbated in Hele-Shaw cells compared with porous media because the dispersion is dependent on the square of velocity, see Eq. (7). This may also explain why the interface length obtained from our experiments (Fig. 4) grows more slowly than the Chen and Meiburg (1998) simulations. Their simulations neglected hydrodynamic dispersion and were performed using much lower Peclet numbers. We estimate that velocity...
dependent dispersion dominates over molecular diffusion in our experiments for the duration of each displacement.

We now discuss the numerical simulation predictions of the experiments, which are shown in Figs. 5, 6, 7, for both parallel and diagonal meshes. For the parallel mesh, shown first in Fig. 5, the fingering patterns predicted by the simulations are qualitatively similar although (a) there are more short, fine fingers seen in the low viscosity ratio simulations and (b) there is a greater tendency for one finger to dominate the flow in the higher viscosity ratio displacements. Generally, the simulation predicts a slightly later breakthrough time (Fig. 6) for all viscosity ratios, although the results are within 10% of the experimental results in most cases. There are mesh orientation impacts on both the finger structure (Fig. 7), and the corresponding macroscopic measures. When the flow is aligned with the mesh in the parallel case, fingers initiate preferentially along the inlet-outlet flow path, as opposed to the diagonal mesh case where they initiate preferentially along the sides of the main flow path. The breakthrough time of the parallel cases reduces more quickly with $M$ than the experimental results, and is more in line with the experiments at higher $M$—at higher mobility ratios a single larger finger dominates at later times, as is seen in the experiments.

Figure 6 shows the macroscopic predictions using both diagonal and parallel meshes, as well as predictions from different realisations of the underlying permeability field. Here we also show simulation uncertainty by using a threshold in concentration which the outlet must reach for break through to have occurred, $c = 0.03 \pm 0.02$. This highlights a ‘detection’ limit, in a similar manner to the experimental results. It also accounts for numerical
errors that can occur when considering breakthrough times as $c > 0$. These uncertainties can be seen to be less than those inherent in the mesh orientation, and permeability realization, and are largely within the plot symbols. Generally, we see that the breakthrough times for diagonal meshes are later than for the parallel mesh, as expected, with the parallel mesh case much closer to the experimental results. Changing the permeability realization can alter the breakthrough time by $\pm 0.03 PV$, similar to the uncertainty in repeat experimental runs. Earlier simulations by Djabbarov et al. (2016) used a nine-point centred numerical approximation; these provided predictions with less mesh orientation error, however, their simulations did not model physical diffusion or velocity dependent dispersion. Here, we can see that the simulation uncertainties largely overlap with the experimental uncertainty, and provide good overall predictions of the macroscopic measures.

Fig. 8  

a Influence of Peclet number on the fingering pattern for $M = 20$, $t = 0.25 PV$. First column are experimental results, middle and right columns are simulation results using a parallel mesh—the middle column features dispersion and molecular diffusion, whilst the right column only has molecular diffusion. Note, the highest $P_e$ experiment was not simulated as the experimental fingering pattern was 3D rather than 2D. 

b High peclet number experimental fingering patterns at $P_e = 1.4 \times 10^6$. In the middle ($t = 0.25 PV$) and right plots ($t = 0.20 PV$), the image has been inverted and contrast enhanced to highlight the small wavelength streaks discussed in the main text.
3.2 Varying Peclet Number, $P_e$

The fingering patterns observed in the $M = 20$ experiments at 0.25 PVI for different transverse Peclet numbers (calculated using $D_T = D_m$) are shown in Fig. 8, left column with simulation results in the middle and right columns. In these displacements, the longitudinal dispersion (dependent on $v^2$) controls the time at which fingers start to grow, whilst the transverse diffusion (which only depend on molecular diffusivity) and $M$ control the number of fingers and their growth rate. This means that at low $P_e$ transverse diffusion is relatively high and longitudinal dispersion is relatively low so the fingers grow sooner in the displacement, and are able to develop into defined finger structures. As we transition to higher $P_e$ the longitudinal dispersion becomes more important, meaning fingers start growing later from a smeared, more stable interface.

The fingering pattern in Fig. 8 changes as $P_e$ is increased from $P_e = 1.4 \times 10^4$ to $P_e = 7 \times 10^5$ and $1.4 \times 10^6$, with numerous fine fingers forming along the interface between the water and the glycerol mixture. For the highest $P_e$ case, we show enhanced experimental images in Fig. 8b, middle and right for $t = 0.25$ and 0.2 PVI, respectively. This highlights the small, fine fingers along the interface which appear across the gap-width of the cell. The fingering pattern for $P_e = 7 \times 10^5$ is a combination of one larger scale finger, similar in width to those seen in the lower Peclet number experiments, and very fine fingers, as seen for $P_e = 1.4 \times 10^6$. We infer that this shows a transition from 2-dimensional fingering (where fingers fill the gap between the glass plates) to 3-dimensional fingering where the finger widths are smaller than the plate separation. This transition from 2D to 3D fingering in a Hele Shaw cell has been investigated in more detail by Videbæk and Nagel (2019). They observed a sudden transition from one type of fingering to the other at a critical Peclet number, where they defined the Peclet number as:

$$P_{e, VN} = \frac{Vh}{D}$$

(10)

where $V = \frac{Q}{C \pi rh}$ and $C = 1/2$ or 2 depending upon whether a quarter five spot or radial injection is considered. However, they do not specify the value of interface radius, $r$, where this critical value was calculated. From our results with $r = 10$cm, the critical value of $P_{e, VN}$ can be estimated as $\approx 1800$ compared with the value of $\approx 1000$ reported by Videbæk and Nagel (2019). The critical onset is the same order of magnitude in both cases, however, uncertainty in the radius definition precludes further quantitative comparison. In the earlier work by Petitjeans et al. (1999), they found 3D effects for high viscosity ratio cases, even at relatively low $P_e$ numbers. They used the Atwood number, $At = (\mu_2 - \mu_1)/(\mu_1 + \mu_2)$ to describe the flow regime where a significant portion of the displaced fluid is left trailing on the walls of the cell, at $At > 0.5$. In this regime, there exists an effective interfacial tension between the fluids, forming different fluid-fluid curvatures in the plane of the cell, similar to the immiscible case. The fingers can be impacted by gravity, depending on the density contrast, and rotational flow altering the shape and propagation through the cell. For the experiments here, $At > 0.95$ for $M = 20$, and the density of the displacing and displaced fluids are $\rho_1 = 1$ g/cm and $\rho_2 = 1.15$ g/cm$^3$, respectively. This suggests 3D effects could be occurring, and the invading finger could buoyantly lift in the plane of cell.

Simulation predictions at varying $P_e$ are shown in Fig. 8a middle and right columns, with macroscopic predictions shown in Fig. 9. We show two model results, one using the full dispersion tensor $D$ in Eqs. (6) and (7), the other using purely molecular diffusion, e.g. $D_L = D_T = D_m$. In general, we see that the two models with dispersion and pure
diffusion, respectively, somewhat bound the experimental finger evolution. The model with pure molecular diffusion predicts a more unstable finger regime at the (high) Peclet numbers considered here, with the results largely invariant to changes in $Pe$. All $Pe$ regimes have similar wavelength and number of fingers. This is further reflected in the breakthrough time and recovery, which is essentially constant for the molecular diffusion model. This shows the influence (or lack there-of) of velocity dependent longitudinal dispersion on the time at which fingers grow.

The model with dispersion predicts a stabilisation of the interface as $Pe$ is increased. This is similar to the numerical results of Petitjeans et al. (1999), but at much higher $Pe$ here. This enhanced dispersion increases the breakthrough time as $Pe$ is increased, which is in-line with the experiment results. However, in the experiment, it is clear that 3D fingering is also occurring, with fingers forming in the plane of the cell-thickness, which cause a greater (areal) sweep of the cell and corresponding increased breakthrough time and recovery. In the simulation, we do not have these 3D effects, but the general influence of dispersion is to increase the breakthrough time, whilst recovery is largely unaffected. It is clear from the experiments that there is greater instability and more fingers than the dispersive model, but the resulting macroscopic measures are similar to the simulations.

In both simulation models, the fingering is sharper than the experiments, and finer fingers are not well captured through their full evolution. The simulations use a fine mesh resolution of 1mm, however some of the experimental fingers approach this size and may not be well resolved. There may also be 3D effects, even in the lower $Pe$ cases, as noted in Petitjeans et al. (1999) as the viscosity ratio is still high. The Atwood number, $At > 0.95$ for the $M = 20$ cases here, meaning a significant fraction of the cell thickness may be left uninvaded, changing the apparent ‘sharpness’ of the fingers themselves. With this behaviour, the flow may diverge from Poiseuille behaviour, meaning the velocity dependence in the dispersion model may not be quadratic. We also note that we assume transverse dispersion is equal to molecular diffusion here; this too could be dependent on velocity and
have impacts on the interactions between fingers, potentially enabling more intricate finger structures as seen in the experiments here (Ghesmat and Azaiez 2007).

In this section, we have shown various model predictions of the miscible fingering in a quarter five-spot Hele-Shaw configuration. The models show that we can predict macroscopic behaviour relatively well, and trends in $P_e$ and $M$ dependence. However, finger growth mechanisms are still mismatched with the experiment. The experimental dataset herein provides a useful starting point to explore extensions to existing theory to cover the full spectrum of $M$ and $P_e$ effects observed in experiments, with time-lapse images also allowing the onset of fingering (and subsequent evolution) to be rigorously explored.

4 Conclusion

A series of well characterised experiments investigating miscible viscous fingering in radial flow using a Hele Shaw cell have been performed and their results compared with the predictions of high-resolution finite difference simulations using various mesh and model choices, across a range of Peclet numbers and viscosity ratio. We performed repeat experiments in opposite flow directions, and in the same flow direction, to quantify the underlying uncertainty, which is paramount for assessing model conformance.

The dynamics of the fingering patterns seen in the Hele-Shaw experiments were consistent with previous experiments. For modest Peclet numbers, the number of fingers and their growth rate increases with the viscosity ratio between the injected and displaced fluids. In addition, we see a transition from 2D to 3D fingers as Peclet number increases. This transition occurs at the same (order of magnitude) critical Peclet number as seen by Videbaek and Nagel (2019).

Our simulations generally provided good predictions of the macroscopic behaviour in terms of breakthrough time and recovery as a function of viscosity ratio, for intermediate Peclet numbers. Mesh orientation effects were observed, with parallel and diagonal orthogonal meshes producing results which effectively bounded the behaviour seen in the experiments. The difference in mesh behaviour may stem from the initiation of the fingers from the underlying permeability structure—the orientation of the mesh impacts the primary direction with which fingers first initiate. The experiments show that although macroscopic behaviour is repeatable between experiments, the specific fine-scale fingers that form are not, although they have overall globally similar wavelength and number. The difference appears to stem from the inlet of the fluid, and the initial interaction of the stable interface with slightly different parts of the ‘imperfect’ cell.

We used two models to investigate the Peclet number behaviour; one with longitudinal dispersion dependent on the squared interface velocity, and the other using pure molecular diffusion. We found the dispersive model generally predicted the macroscopic breakthrough and recovery behavior across the $P_e$ range well, although the specific fine finger growth did not match the experiments. The simulations had a more dispersive interface in the plane of the cell than the experiments. The molecular diffusion model on the other hand did predict the formation of smaller fingers, but showed little $P_e$ dependency. Both models predicted sharper fingers than were found in the experiments. In the experiments, at $M = 20$ for the highest Peclet numbers, 3D effects in the plane of the cell thickness appeared considerable (and potentially for lower $P_e$ cases, although direct measurement with the current setup was precluded), altering the finger shape and evolution.
Further work should model the full 3D problem with various dispersion models with modern, parallelised code to fully explore the range of $M$ and $P_e$ behaviour. When 3D effects occur, the classical Hele-Shaw model breaks down; modifications are needed to make to incorporate the in-plane effects in the lubrication approximation (Videbæk 2020). This work, alongside experiments, should explore the impacts of varying the in-plane thickness of the cell, which could impact the 3D effects in a regime not directly predicted from the obvious changed in $P_e$ and permeability. Furthermore, future research should also explore the benefits of using alternative discretisation schemes such as the nine-point method with Cartesian meshes or control-volume-finite element approaches with unstructured meshes (e.g. as used by Kampitsis et al. 2020) for immiscible viscous fingering) to reduce the influence of the grid orientation error.

All the results from these experiments are available in open-access form, see the Data access section below. They represent a valuable dataset for the future development of models and numerical methods to simulate viscous instabilities. The fine-scale results here can also be used to calibrate upscaling approaches e.g. the Todd-Longstaff mixing model for field-scale displacements (Cusini et al. 2018).

Appendix A.

See Table 4

| Mobility ratio, $M$ | Flow rate, $[\text{ml/min}]$ | Peclet number, $P_e$ | Break through time [PV], Image | Break through time [PV], Volumetric | Recovery at $1PVI$ [PV], Volumetric | Error [PV], Volumetric |
|-------------------|-----------------------------|-------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 2                 | 1.0                         | $1.4 \times 10^5$ | 0.448                       | 0.484                       | 0.706                       | 0.076                       |
| 5                 | 1.0                         | $1.4 \times 10^5$ | –                           | 0.396                       | 0.610                       | 0.052                       |
| 5                 | 1.0                         | $1.4 \times 10^5$ | 0.341                       | 0.374                       | 0.699                       | 0.003                       |
| 10                | 1.0                         | $1.4 \times 10^5$ | 0.308                       | 0.352                       | 0.606                       | 0.021                       |
| 10                | 1.0                         | $1.4 \times 10^5$ | 0.286                       | 0.330                       | 0.552                       | 0.067                       |
| 20                | 1.0                         | $1.4 \times 10^5$ | 0.286                       | 0.286                       | 0.549                       | 0.026                       |
| 20                | 1.0                         | $1.4 \times 10^5$ | 0.264                       | 0.286                       | 0.543                       | 0.051                       |
| 20                | 1.0                         | $1.4 \times 10^5$ | 0.246                       | 0.286                       | 0.597                       | 0.039                       |
| 20                | 0.1                         | $1.4 \times 10^4$ | 0.255                       | 0.264                       | 0.419                       | 0.082                       |
| 20                | 0.5                         | $7.0 \times 10^4$ | 0.264                       | 0.242                       | 0.607                       | 0.039                       |
| 20                | 5.0                         | $7.0 \times 10^5$ | 0.268                       | 0.308                       | 0.553                       | 0.069                       |
| 20                | 10.0                        | $1.4 \times 10^6$ | 0.272                       | 0.396                       | 0.776                       | 0.117                       |
Supplementary information The online version contains supplementary material available at (https://doi.org/10.1007/s11242-022-01778-4).

Acknowledgements Dr. Wawrzyniec Kostorz is thanked for adding several new features to the MISTRESS code that enabled this study to be performed. We thank Vincenzo Cunsolo and Henry Arthur for their help in the laboratory experiments.

Funding The majority of this work was undertaken as part of the MSc Petroleum Engineering degree at Imperial College London by Daniel Keable in 2018.

Data availability Experimental and simulation data associated with this work are hosted on Zenodo at https://doi.org/10.5281/zenodo.5567913. The matlab codes for processing the images, analysing results and generating figures, as well as example simulation datafiles are available on Github at https://github.com/sci-sjj/MiscibleViscousFingering. The Mistress simulation code is currently under-development; up to date code is available from the authors on request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work in this paper.

References

Bischofberger, I., Ramachandran, R., Nagel, S.R.: Fingering versus stability in the limit of zero interfacial tension. Nature Commun. (2014). https://doi.org/10.1038/ncomms6265
Brand, C., Heinemann, J., Aziz, K.: The Grid Orientation Effect in Reservoir Simulation. Society of Petroleum Engineers, In SPE Symposium on Reservoir Simulation (1991)
Chen, C.Y., Meiburg, E.: Miscible porous media displacements in the quarter five-spot configuration. Part 1. The homogeneous case. J. Fluid Mech. 371, 233–268 (1998). https://doi.org/10.1017/S0022112098002195
Cheng, N.S.: Formula for the viscosity of a glycerol-water mixture. Ind. Eng. Chem. Res. 47(9), 3285–3288 (2008). https://doi.org/10.1021/ie071349z
Christie, M., Bond, D.: Multidimensional Flux-Corrected Transport for Reservoir Simulation. Society of Petroleum Engineers, In SPE Reservoir Simulation Symposium (1985)
Christie, M., Bond, D.: Detailed simulation of unstable processes in miscible flooding. SPE Reserv. Eng. 2(04), 514–522 (1987). https://doi.org/10.2118/14896-pa
Christie, M.A., Jones, A.D.W., Muggeridge, A.H.: Comparison between laboratory experiments and detailed simulations of unstable miscible displacement influenced by gravity, North Sea Oil and Gas Reservoirs–II, 245–250. Springer, Netherlands. (1990). https://doi.org/10.1007/978-94-009-0791-1_20
Cusini, M., Gielisse, R., Groot, H., van Kruisjadijk, C., Hajibeygi, H.: Incomplete mixing in porous media: Todd-longstaff upscaling approach versus a dynamic local grid refinement method. Comput. Geosci. 23(2), 373–397 (2018). https://doi.org/10.1007/s10596-018-9802-0
Davies, G., Muggeridge, A., and Jones, A.: Miscible displacements in a heterogeneous rock: Detailed measurements and accurate predictive simulation. In SPE Annual Technical Conference and Exhibition, Dallas, Texas,., SPE (1991)
D’Errico, G., Ortona, O., Capuano, F., Vitagliano, V.: Diffusion coefficients for the binary system glycerol + water at 25 °C a velocity correlation study. J. Chem. Eng. Data 49(6), 1665–1670 (2004). https://doi.org/10.1021/je049917u
Djabbarov, S., Jones, A.D.W., Krevor, S., Muggeridge, A.H.: Experimental and Numerical Studies of First Contact Miscible Injection in a Quarter Five Spot Pattern. In SPE Europec featured at 78th EAGE Conference and Exhibition. Society of Petroleum Engineers (2016)
Edwards, M., Christie, M.: Dynamically adaptive godunov schemes with renormalization in reservoir simulation. Society of Petroleum Engineers, In SPE Symposium on Reservoir Simulation (1993)
Ennis-King, J., Preston, I., Paterson, L.: Onset of convection in anisotropic porous media subject to a rapid change in boundary conditions. Phys. Fluids (2005). https://doi.org/10.1063/1.2033911
Gerritsen, M.G., Durlofsky, L.J.: Modeling fluid flow in oil reservoirs. Annu. Rev. Fluid Mech. 37(1), 211–238 (2005). https://doi.org/10.1146/annurev.fluid.37.061903.175748

Ghesmat, K., Azaiez, J.: Transp. Porous Media. Viscous fingering instability in porous media: effect of anisotropic velocity-dependent dispersion tensor. 73(3), 297–318 (2007). https://doi.org/10.1007/s11242-007-9171-y

Greenkorn, R., Haring, R., Jahns, H.O., Shallenberger, L.: Flow in Heterogeneous Hele-Shaw Models. Soc. Petrol. Eng. J. 4(04), 307–316 (1964). https://doi.org/10.2118/8999-PA

Habermann, B.: The efficiency of miscible displacement as a function of mobility ratio (1960)

Hamid, S.A.A., Muggeridge, A.H.: Fingering regimes in unstable miscible displacements. Phys. Fluids 32(1), 016601 (2020). https://doi.org/10.1063/1.5128338

Habermann, H.S.: The flow of water. Nature 58(1489), 34–36 (1898). https://doi.org/10.1038/058034a0

Kampitsis, A.E., Adam, A., Salinas, P., Pain, C.C., Muggeridge, A.H., Jackson, M.D.: Dynamic adaptive mesh optimisation for immiscible viscous fingering. Comput. Geosci. 24(3), 1221–1237 (2020). https://doi.org/10.1007/s10596-020-09938-5

Kapur, J.N., Sahoo, P.K., Wong, A.K.: A new method for gray-level picture thresholding using the entropy of the histogram. Comp. Vision Gr. Image Proc. 29(3), 273–285 (1985)

Kopp, A., Class, H., Helmig, R.: Investigations on CO2 storage capacity in saline aquifers. Int. J. Greenh. Gas Control 3(3), 263–276 (2009). https://doi.org/10.1016/j.ijggc.2008.10.002

Koval, E.: A method for predicting the performance of unstable miscible displacement in heterogeneous media. Soc. Petrol. Eng. J. 3(02), 145–154 (1963). https://doi.org/10.2118/450-pa

Koval, E.: A method for predicting the performance of unstable miscible displacement in heterogeneous media. Soc. Petrol. Eng. J. 3(02), 145–154 (1963). https://doi.org/10.2118/450-pa

Lacey, J., Faris, J., Brinkman, F.: Effect of bank size on oil recovery in the high-pressure gas-driven lpg-bank process. J. Petrol. Technol. 13(08), 806–816 (1961). https://doi.org/10.2118/1619-G-PA

Lee, S., Wheeler, M.F.: Adaptive enriched galerkin methods for miscible displacement problems with entropy residual stabilization. J. Comput. Phys. 331, 19–37 (2017). https://doi.org/10.1016/j.jcp.2016.10.072

Mahaffey, J., Rutherford, W., Matthews, C.: Sweep efficiency by miscible displacement in a five-spot. Soc. Petrol. Eng. J. 6(01), 73–80 (1966). https://doi.org/10.2118/1233-PA

Nicolaides, C., Jha, B., Cueto-Felgueroso, L., Juanes, R.: Impact of viscous fingering and permeability heterogeneity on fluid mixing in porous media. Water Resour. Res. 51(4), 2634–2647 (2015). https://doi.org/10.1002/2014wr015811

Paterson, L.: Fingering with miscible fluids in a hele shaw cell. Phys. Fluids 28(1), 26–30 (1985). https://doi.org/10.1063/1.865195

Perkins, T., Johnston, O.: A review of diffusion and dispersion in porous media. Soc. Petrol. Eng. J. 3(01), 70–84 (1963). https://doi.org/10.2118/480-pa

Petitjeans, P., Chen, C.Y., Meiburg, E., Maxworthy, T.: Miscible quarter five-spot displacements in a Hele-Shaw cell and the role of flow-induced dispersion. Phys. Fluids 11(7), 1705 (1999). https://doi.org/10.1063/1.870037

Paterson, L.: Fingering with miscible fluids in a hele shaw cell. Phys. Fluids 28(1), 26–30 (1985). https://doi.org/10.1063/1.865195

Perkins, T., Johnston, O.: A review of diffusion and dispersion in porous media. Soc. Petrol. Eng. J. 3(01), 70–84 (1963). https://doi.org/10.2118/480-pa

Petitjeans, P., Chen, C.Y., Meiburg, E., Maxworthy, T.: Miscible quarter five-spot displacements in a Hele-Shaw cell and the role of flow-induced dispersion. Phys. Fluids 11(7), 1705 (1999). https://doi.org/10.1063/1.870037

Rayleigh, J.W.S.: Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density. Proceedings of the London Mathematical Society s1-14(1): 170–177. (1882) https://doi.org/10.1112/plms/s1-14.1.170

Riaz, A., Pankiewitz, C., Meiburg, E.: Linear stability of radial displacements in porous media: influence of velocity-induced dispersion and concentration-dependent diffusion. Phys. Fluids 16(10), 3592–3598 (2004). https://doi.org/10.1063/1.1775431

Rousseaux, G., Wit, A.D., Martin, M.: Viscous fingering in packed chromatographic columns: linear stability analysis. J. Chromatogr. A 1149(2), 254–273 (2007). https://doi.org/10.1016/j.chroma.2007.03.056

Saffman, P.G., Taylor, G.: The penetration of a fluid into a porous medium or hele-shaw cell containing a more viscous liquid. Proc. R. Soc. A Math. Phys. Eng. Sci. 245(1242), 312–329 (1958). https://doi.org/10.1098/rspa.1958.0085

Schindelin, J., Arganda-Carreras, I., Frise, E., Kaynig, V., Longair, M., Pietzsch, T., Preibisch, S., Rueden, C., Saalfeld, S., Schmid, B., Tinevez, J.Y., White, D.J., Hartenstein, V., Eliceiri, K., Tomancak, P., Cardona, A.: Fiji: an open-source platform for biological-image analysis. Nat. Methods 9(7), 676–682 (2012). https://doi.org/10.1038/nmeth.2019

Sharma, V., Nand, S., Pramanik, S., Chen, C.Y., Mishra, M.: Control of radial miscible viscous fingering. J. Fluid Mech. (2019). https://doi.org/10.1017/jfm.2019.932

Stoneberger, M., Claridge, E.: Graded-viscosity-bank design with pseudoplastic fluids. SPE Reserv. Eng. 3(04), 1221–1232 (1988). https://doi.org/10.2118/14230-pa

Tan, C.T., Homsy, G.M.: Stability of miscible displacements in porous media: rectilinear flow. Phys. Fluids 29(11), 3549 (1986). https://doi.org/10.1063/1.865832
Tan, C.T., Homsy, G.M.: Stability of miscible displacements in porous media: radial source flow. Phys. Fluids 30(5), 1239 (1987). https://doi.org/10.1063/1.866289
Taylor, G.: The instability of liquid surfaces when accelerated in a direction perpendicular to their planes. I. Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 201(1065): 192–196. (1950) https://doi.org/10.1098/rspa.1950.0052
Taylor, G.I.: Dispersion of soluble matter in solvent flowing slowly through a tube. Proc. R. Soc. Lond. Series A Math. Phys. Sci. 219(1137), 186–203 (1953). https://doi.org/10.1098/rspa.1953.0139
Todd, M., Longstaff, W.: The development, testing, and application of a numerical simulator for predicting miscible flood performance. J. Petrol. Technol. 24(07), 874–882 (1972). https://doi.org/10.2118/3484-pa
Videbæk, T.E.: Delayed onset and the transition to late time growth in viscous fingering. Phys. Rev. Fluids (2020). https://doi.org/10.1103/physrevfluids.5.123901
Videbæk, T.E., Nagel, S.R.: Diffusion-driven transition between two regimes of viscous fingering. Phys. Rev. Fluids (2019). https://doi.org/10.1103/physrevfluids.4.033902
Volk, A., Kähler, C.J.: Density model for aqueous glycerol solutions. Exp. Fluids 59(5), 75 (2018). https://doi.org/10.1007/s00348-018-2527-y
Yanosik, J., McCracken, T.: A nine-point, finite-difference reservoir simulator for realistic prediction of adverse mobility ratio displacements. Soc. Petrol. Eng. J. 19(04), 253–262 (1979). https://doi.org/10.2118/5734-pa
Zalesak, S.T.: Fully multidimensional flux-corrected transport algorithms for fluids. J. Comput. Phys. 31(3), 335–362 (1979). https://doi.org/10.1016/0021-9991(79)90051-2
Zhang, H., Sorbie, K., Tsibuklis, N.: Viscous fingering in five-spot experimental porous media: new experimental results and numerical simulation. Chem. Eng. Sci. 52(1), 37–54 (1997). https://doi.org/10.1016/S0009-2509(96)00382-X
Zimmerman, W.B., Homsy, G.M.: Nonlinear viscous fingering in miscible displacement with anisotropic dispersion. Phys. Fluids A 3(8), 1859–1872 (1991). https://doi.org/10.1063/1.857916
Zimmerman, W.B., Homsy, G.M.: Viscous fingering in miscible displacements: unification of effects of viscosity contrast, anisotropic dispersion, and velocity dependence of dispersion on nonlinear finger propagation. Citation Phys. Fluids A Fluid Dyn. 4, 2348 (1992). https://doi.org/10.1063/1.858476

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.