Comprehensive Approach for Identification of Nonlinear Stiffness Characteristics of Bearing Supports for the Oxidizer Turbopump of the Liquid Rocket Engine

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Abstract. This article deals with the refinement of the mathematical and computational models of the oxidizer turbopump rotor considering bearing gaps, axial preloading, compliance of the housing parts and the effect of rotation. The loading scheme consists of four substeps is proposed considering preliminary displacement of the outer cage, axial displacement as a result of the support deformation due to the axial preloading force, radial displacement due to the support deformation, as well as centrifugal forces of inertia caused by rotation of the rotor with an inner cage. Modelling of contacts interactions using ANSYS software is carried out according to the appropriate models of contact behaviour. The contact areas between the rolling elements, inner and outer cases are obtained. The contact angle is determined. Isosurfaces of axial and radial displacements for the bearing supports are built. Nonlinear stiffness of bearing supports is determined as the tangent of the angle of inclination for the curve “radial load – radial displacement”. The proposed approach, which used for designing turbopump units for liquid rocket engines, will allow refining the reliable mathematical and computational models of rotor dynamics for turbopump units and providing appropriate computer simulation of forced oscillations of the rotor systems for given permissible residual imbalances considering nonlinear stiffness characteristics of bearing supports.

Keywords: compliance of housing, initial clearance, axial preloading, radial load, contact angle, radial stiffness, axial stiffness.

1 Introduction

The creation of reliable models of rotary systems is an urgent problem that allows designing high-power rotary machines including turbopump units of liquid rocket engines. The process of modelling should be based on the existing experience in designing of reliable equipment the requirements for which are permanently increased.

Based on the experience of the Faculty of Technical Systems and Energy Efficient Technologies (Sumy State University, Ukraine) in mathematical modelling of rotary systems, and the Faculty of Mechanical Engineering (University of West Bohemia, Czech Republic) in providing numerical simulations, as well as the Faculty of Manufacturing Technologies with a seat in Prešov (Technical University of Košice, Slovakia) in carrying out experimental research, this paper is devoted to refine the computational model of rotor dynamics for turbopump units and further computer simulation of forced oscillations of the rotor systems for given permissible residual imbalances considering the nonlinear stiffness of bearing supports.

The proposed clarifications are related to effect of rotation on deformation of moving parts of bearings, compliance of the housing, as well as gaps and axial preloading of the bearing supports.

The clarification of the stiffness parameters of the bearing supports is determined as the tangent of the angle of inclination for the curve “radial load – radial displacement”. The proposed approach, which used for designing turbopump units for liquid rocket engines, will allow refining the reliable mathematical and computational models of rotor dynamics for turbopump units and providing appropriate computer simulation of forced oscillations of the rotor systems for given permissible residual imbalances considering nonlinear stiffness characteristics of bearing supports.
2 Literature Review

The problem of identification of the nonlinear bearing stiffness is highlighted in recent research works. Particularly, the paper [1] deals with the investigation of nonlinear reactions in rotors’ bearing supports of turbopump units for liquid rocket engines. However, this work does not consider the impact of initial bearing gap and axial preloading.

The problem of dynamics and diagnostics of vertical rotors with nonlinear supports stiffness is solved in the doctoral thesis [2].

Clarifications considering the impact of gap seals with floating rings are presented in monograph [3]. Additionally, it is shown that the presence of gaps in the bearing supports reduces the bearing stiffness. However, gap seals with floating ring, axial preloading and rotation of the rotor cause an increasing dependence of the bearing stiffness on the rotor speed. Consequently, the critical frequencies of the rotor increase.

A review of investigations of nonlinear dynamic on bearings with rolling element is presented in the research work [4].

The calculation of forced oscillations under the system of imbalances (direct synchronous precession) performed using the computer program [5] considers the dependences of the bearing stiffness on the rotor speed and impact of the gyroscopic moments of inertia of bushing parts.

Paper [6] is aimed at theoretical and experimental study of spindle ball bearing nonlinear stiffness.

General scientific and methodological approach for the identification of mathematical models of mechanical systems using artificial neural networks is delivered in paper [7].

Nonlinear dynamic response for the system “cylindrical roller bearing – rotor system” is presented in the paper [8]. The proposed mathematical model with 9 degrees of freedom allows considering combined localized defect at inner–outer races of bearings.

The need to consider clarified mathematical model of rotor dynamics for investigation of critical frequencies considering stiffness of bearings and seals is justified in the work [9] on the example of on examples of the centrifugal compressor’s rotor.

The influence of bearing stiffness on the nonlinear dynamics of a shaft-final drive system is presented in the research paper [10].

3 Research Methodology

The connection of shafts of the turbopump unit causes a weak dynamic interaction between them. In this case, the partial critical frequencies of separate rotors do not differ much from the corresponding frequencies, obtained as a result of calculations for the entire rotor. This fact is confirmed by the calculations presented in the work [2]. Therefore, the oxidizer turbopump and fuel pump rotor systems should also be considered as the separate dynamic systems. The design scheme of the oxidizer turbopump rotor is presented in Figure 1.

The ANSYS software with the modules “Static Structural” and “Transient Structural” is used for determining the bearing stiffness considering initial gaps, axial preloading, rotation effect and compliance of the housing. The basic design schemes for loading bearing supports are presented in Figure 2.

![Figure 1 – Design scheme of the oxidizer turbopump rotor](image-url)
The loading scheme consists of four substeps:

1. Preliminary displacement of the outer cage (for the bearing support 45-216) and the shaft (for the bearing support 45-276214) towards the application of axial pre-loading force. Axial displacement $x_0$ is determined due to the maximum radial gap $h_0 = 0.095 \text{ mm}$ [11].

2. Determination of the axial displacement $x$ as a result of the support deformation due to the axial preloading force $T = 4.5 \text{ kN}$ [11]. Investigation of forced oscillations of the rotor on ball bearings.

3. Numerical calculation of the radial displacement $y$ of the shaft axis as a result of the support deformation due to the radial force $R = 10 \text{ kN}$.

4. Considering centrifugal forces of inertia caused by rotation of the rotor with an inner cage of the bearing support.

Modelling of contacts by using ANSYS software is carried out according to Table 1.

| Mating surfaces | Contact model |
|-----------------|---------------|
| Shaft           | Inner cage    |
| Inner cage      | Rolling elements |
| Rolling elements| Outer cage    |
| Outer cage      | Housing       |

“bonded” “frictional”

The contact areas between the rolling elements and cases are shown in Figure 3. The loading schemes according to substeps 2–4 of bearing loading are presented on Figures 4–6.

Isosurfaces of axial and radial displacements for the bearing supports are presented on Figures 7–8.
Figure 3 – Contact areas between the rolling bodies, inner (a, c) and outer (b, d) cages for the bearing supports 45-216 (a, b) and 45-276214 (c, d).

Figure 4 – Deformations of the bearing supports 45-216 (a) and 45-276214 (b) as a result of axial preloading.

Figure 5 – Deformations of the bearing supports 45-216 (a) and 45-276214 (b) as a result of axial preloading and radial force.
4 Results

The results of numerical simulation are summarized in Table 2.

The stiffness $c$ of the bearing support is determined due to the following formula:

$$ c = \frac{\partial R}{\partial y}, \quad (1) $$

where $R$ – radial force; $y$ – radial displacement.

As a result of numerical simulation (the determination of radial displacements of the rotor axis under the discrete values of the radial force), the approximated curves “radial load – radial displacement” are defined (Figure 9).

Analytical expressions describing the dependence “radial load – radial displacement” determined as a result of approximation of the experimental data are summarized in Table 3.
Table 2 – The results of numerical simulation

| Bearing       | Displacement, μm | Rotor speed, 10^3 rad/s |
|---------------|------------------|-------------------------|
|               | h₀  | x₀  | x  | 0  | 1.1 | 2.0 | 2.2 |
| 45-216        | 95  | 620 | 80 | 51 | 50  | 48  | 47  |
| 45-276214     | 95  | 620 | 100| 40 | 40  | 39  | 39  |

The linear radial stiffness of the support is defined as the tangent of the initial angle of inclination of the curve “radial load – radial displacement” (Figures 9, dash line):

\[ c_0 = \frac{\partial R}{\partial y} \bigg|_{y_0}. \]  

(2)

Figure 9 – Dependence “radial force – radial displacement” for the bearing supports 45-216 (a) and 45-276214 (b)

Table 3 – Analytical dependences “radial load – radial displacement”

| Bearing support | 45-216 | 45-276214 |
|-----------------|--------|-----------|
| Radial displacement y, μm | 5 | 100 | 150 | 5 | 100 | 200 |
| Radial force R, kN | 0.9 | 1.95 | 30 | 1 | 22 | 45 |
| Approximated curve (R, N; y, m) | \( R(y) = 1.94 \times 10^8 \cdot y + 4.0 \times 10^{10} \cdot y^2 \) | \( R(y) = 2.12 \times 10^7 \cdot y + 6.7 \times 10^{10} \cdot y^2 \) |
| Nonlinear radial stiffness (c, N/m; y, m; R, N) | \( c(y) = 1.94 \times 10^8 + 8.0 \times 10^{10} \cdot y \) | \( c(y) = 2.12 \times 10^7 + 1.34 \times 10^{11} \cdot y \) |
| \( c(R) = 1.94 \times 10^8 \cdot (1 + 4.2 \times 10^{-6} \cdot R)^{0.5} \) | \( c(R) = 2.12 \times 10^7 \cdot (1 + 6.0 \times 10^{-6} \cdot R)^{0.5} \) |

The linear radial stiffness of the support is defined as the tangent of the initial angle of inclination of the curve “radial load – radial displacement” (Figures 9, dash line):

\[ c_0 = \frac{\partial R}{\partial y} \bigg|_{y_0}. \]  

(2)

The linear radial stiffness of the support is defined as the tangent of the initial angle of inclination of the curve “radial load – radial displacement” (Figures 9, dash line):

\[ c_0 = \frac{\partial R}{\partial y} \bigg|_{y_0}. \]  

(2)

Taking into account the expressions given in Table 2, the values of the bearing stiffness \( c_0 \) of the supports 45-216 and 45-276214 are equal \( 1.94 \times 10^8 \) N/m and \( 2.12 \times 10^8 \) N/m respectively. Exceeding the bearing stiffness of the support 45-276214 in comparison with the bearing support 45-216 is explained by the relatively large number of rolling bodies.

Similar values of the bearing stiffness, determined without the rotor speed for the supports 45-216 and 45-276214 are equal \( 1.88 \times 10^8 \) N/m and \( 2.10 \times 10^8 \) N/m respectively.

For further designing the mathematical models of free and forced oscillations of the rotor systems for turbopump unit considering the impact of the rotor speed on bearing stiffness, the following analytical dependence is proposed:

\[ c(\omega) = c_0 + \alpha \omega^2. \]  

(3)

In this case, the estimation of the coefficient \( \alpha \) is carried out by the linear regression formula [1]:

\[ \alpha = \frac{\sum_{i=1}^{k} (c_k - c_0) \omega_k^2}{\sum_{i=1}^{k} \omega_k^4}, \]  

(4)

where \( c_k \) – bearing stiffness, determined as a result of the numerical simulation for the rotor speed \( \omega_k \) (Table 2); \( k \) – number of the experimental point.

As a result, values of the coefficient \( \alpha \) are obtained (Table 4), as well as the approximated curves are built (Figure 10).

The obtained data allows determining the dependence of the bearing stiffness on the radial force (Figure 11).

Table 5 and Figures 12–13 contain the data presenting the axial stiffness of the bearing supports.
Table 4 – Parameters of the nonlinear bearing stiffness

| Bearing support | Parameter | $c_0$, N/m | $\alpha$, N·s²/m |
|-----------------|-----------|------------|------------------|
| 45-216          | 1.88      | 1.223      |
| 45-276214       | 2.10      | 0.408      |

Figure 10 – Dependence of the bearing stiffness on the rotor speed

Figure 11 – Dependence of the bearing stiffness on the radial force

Figure 12 – Dependence “radial load – radial displacement” for the bearing supports 45-216 (a) and 45-276214 (b)

Figure 13 – Dependence “axial load – axial displacement” for the bearing supports 45-216 (a) and 45-276214 (b)
Thus, the proposed comprehensive approach is approved on the example of the oxidizer turbopump’s rotor for the liquid rocket engine that will allow refining the reliable mathematical and computational models of rotor dynamics for turbopump units and providing appropriate computer simulation of forced oscillations of the rotor systems for given permissible residual imbalances considering nonlinear stiffness characteristics of bearing supports.

5 Conclusions

As a result of numerical simulation (the determination of radial displacements of the rotor axis under the discrete values of the radial force), the approximated curves “radial load – radial displacement” are defined. Analytical expressions describing the mentioned dependence are determined as a result of approximation of the experimental data.

The results of more precise calculations of rotor dynamics for the turbopump considering bearing gaps, axial preloading, rotor speed and compliance of the housing parts will allow clarifying the detuning from the resonance.

Further research will be aimed at obtaining spectrums of critical frequencies and related mode shapes for the rotor systems in abovementioned bearing supports, as well as at the detailed analysis of forced oscillations of the turbopump rotor for the nonlinear stiffness of the bearing supports considering radial gaps, initial clearance, axial preloading, maximum rotor speed and compliance of the housing for the system of residual imbalances, which led to the maximum values of centrifugal forces.

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Комплексний підхід до ідентифікації нелінійних жорсткісних характеристик підшипників опор турбонасоса оцінювача рідкінного ракетного двигуна

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Анотація. Стаття присвячена уточненню математичних та обчислювальних моделей ротора турбонасоса оцінювача з урахуванням зазорів у підшипниках, попереднього осьового навантаження, податливості корпусних елементів та впливу обертання валу. Запропонована схема навантаження складається з чотирьох шарів з урахуванням попереднього зміщення зовнішньої обойми, осьового зміщення у результаті переміщення опори внаслідок попереднього осьового навантаження, радіального переміщення внаслідок деформації підшипникової опори, а також відцентривих сил інерції, викликаних обертанням ротора разом із внутрішньою обойми. Моделювання контактної взаємодії з використанням програмного забезпечення ANSYS здійснюється відповідно до достовірних моделей. Встановлені зони контактів між тілами кочення і внутрішньою та зовнішньою обоймами, а також визначений кут контактів. Побудовано іговерхні осьових та радіальних переміщень підшипниковых опор. Нелінійна жорсткість опор визначається як тангенс кута дотичної до кривої, що описує залежність «радіальне навантаження – радіальне зміщення». Запропонований підхід, який використовується для проєктування турбонасосних агрегатів рідкінних ракетних двигунів, дозволяє уточнити достовірні математичні та обчислювальні моделі динаміки ротора і забезпечити якісне моделювання вимушених коливань роторних систем для заданої системи допустимих залишкових дисбалансів з урахуванням нелінійних характеристик жорсткості підшипниковых опор.

Ключові слова: податливість корпуса, початковий зазор, попереднє осьове навантаження, радіальне навантаження, кут контралу, радіальна жорсткість, осьова жорсткість.