Chiral Effects of Quenched $\eta'$ Loops

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Preliminary results of a study of quenched chiral logarithms at $\beta = 5.7$ are presented. Four independent determinations of the quenched chiral log parameter $\delta$ are obtained. Two of these are from estimates of the $\eta'$ mass, one from the residue of the hairpin diagram and the other from the topological susceptibility combined with the Witten-Veneziano formula. The other two determinations of $\delta$ are from measurement of virtual $\eta'$ loop effects in $m_\pi^2$ vs. quark mass and in the chiral behavior of the pseudoscalar decay constant. All of our results are consistent with $\delta = 0.080(15)$. The expected absence of quenched chiral logs in the axial-vector decay constant is also observed.

1. Quenched Chiral Logs

One of the most distinctive physical effects of light quark loops in QCD is the screening of topological charge, which is responsible for the large mass of the $\eta'$ meson. The absence of screening in the quenched approximation gives rise to singular chiral behavior (quenched chiral logs)\footnote{Talk presented by H. Thacker} arising from soft $\eta'$ loops. In full QCD $\eta'$ loops are finite in the chiral limit, while in quenched QCD they produce logarithmic singularities which exponentiate to give anomalous power behavior in the limit $m_\pi^2 \to 0$. The first evidence of quenched chiral logs in the pion mass was reported last year\footnote{\textsuperscript{2,3}}. Here, and in a forthcoming paper\footnote{\textsuperscript{4}}, we present the results of a more detailed study of this effect.

In the context of the effective chiral Lagrangian for QCD, the $U(3) \times U(3)$ chiral field $U = \exp \left( i \sum_{i=0}^{3} \phi_i \lambda_i / f_\pi \right)$ exhibits singular chiral behavior in the quenched theory arising from logarithmically divergent $\eta' = \phi_0$ loops:

$$U \to \exp \left( - (\phi_0^2 / 2 f_\pi^2) \right) \tilde{U} = \left( \frac{\Lambda^2}{m_\pi^2} \right)^\delta \tilde{U}$$

where $\tilde{U}$ is finite in the chiral limit, and $\Lambda$ is an upper cutoff on the $\eta'$ loop integral. (We consider, for simplicity, the case of three equal mass light quarks.) The anomalous exponent $\delta$ is determined by the $\eta'$ hairpin mass insertion $m_0^2$,

$$\delta = \frac{m_0^2}{48 \pi^2 f_\pi^2}$$

(Here $f_\pi$ is normalized to a phenomenological value of 95 MeV.) From the chiral behavior of the field $U$, we can infer the singularities expected in the matrix elements of various quark bilinears. Matrix elements of operators which flip chirality, such as the pseudoscalar charge $\bar{\psi} \gamma^5 \psi \propto U - U^\dagger$ should exhibit a chirally singular factor $(m_\pi^2)^{-\delta}$. By contrast, operators which preserve chirality, such as the axial vector current $\bar{\psi} \gamma^5 \gamma^\mu \psi \propto iU^{-1} \partial^\mu U + h.c.$ are expected to be finite in the chiral limit. From PCAC, it follows that

$$m_\pi^2 \propto m_q$$

In the data presented here, we verify all of these predictions by studying the pion mass and the vacuum-to-one-particle matrix elements

$$\langle 0 | \bar{\psi} \gamma^5 \psi | \pi(p) \rangle = f_P$$

$$\langle 0 | \bar{\psi} \gamma^5 \gamma^\mu \gamma^a \psi | \pi(p) \rangle = f_A P^a$$

The modified quenched approximation (MQA)\textsuperscript{5} provides a practical method for resolving the
problem of exceptional configurations, and allows an accurate investigation of the chiral behavior of quenched QCD with Wilson-Dirac fermions. The pole-shifting prescription for constructing improved quark propagators is designed to remove the displacement of real poles in the quark propagator, which is a lattice artifact, while retaining the contribution of these poles to continuum physics. We have used the MQA procedure to study the masses and matrix elements of flavor singlet and octet pseudoscalar mesons in the chiral limit of quenched QCD. A complete discussion of these results will be presented in a forthcoming publication. Here we present results from 300 gauge configurations at \( \beta = 5.7 \) on a \( 12^3 \times 24 \) lattice, with clover-improved quarks (\( C_{sw} = 1.57 \)) at nine quark mass values covering a range of pion masses from .2387(53) to .5998(17).

In addition to observing \( \eta' \) loop effects, we also present two direct estimates of the chiral log parameter \( \delta \), one from a calculation of the \( \eta' \) hairpin diagram, and the other from a calculation of the topological susceptibility combined with the Witten-Veneziano relation. The results for \( \delta \) agree well with each other and with the exponents extracted from \( f_P/f_A \) and \( m_\pi^2 \), thus giving four independent and consistent determinations of \( \delta \). All four results fall within one standard deviation of \( \delta = .080(15) \). This is about a factor of two smaller than the result \( \delta = .17 \) expected from the phenomenological values of \( m_0 \approx 850 \text{ MeV} \) and \( f_\pi = 95 \text{ MeV} \). The agreement between the four determinations of \( \delta \) indicates that relations imposed by chiral symmetry and the Witten-Veneziano formula are approximately valid, even at \( \beta = 5.7 \). Possible reasons for the overall suppression of the exponent \( \delta \) compared to phenomenological expectations will be discussed in Ref. [4].

2. The \( \eta' \) hairpin mass insertion

Following Ref. [4] we calculate \( \gamma^2 \) quark loops using quark propagators with a source given by unit color-spin vectors on all sites. The statistical errors are dramatically improved by the MQA procedure, allowing a detailed study of the time-dependence even at the lightest masses. We determine the value of \( m_\pi^2 \) with a one-parameter fit to the overall magnitude of the hairpin correlator, assuming a pure double-Goldstone pole form. In the chiral limit, we find \( m_\pi a = .601(30) \), or \( m_\pi = 709(35) \text{ MeV} \) using \( a^{-1} = 1.18 \text{ GeV} \). (For unimproved \( C_{sw} = 0 \) fermions, we get a much smaller value of 464(24) MeV.) Using Eq. (2), we obtain the values for the chiral log parameter \( \delta \). Extrapolating to the chiral limit, this gives \( \delta = .068(8) \) if we use the unrenormalized lattice value for \( f_\pi \). If a tadpole improved renormalization factor is included, this becomes \( \delta = .095(8) \).

3. Topological susceptibility

The fermionic method for calculating the topological susceptibility\(^6\) can be implemented with the same allsource propagators used for the hairpin calculation. For each configuration, we compute the integrated pseudoscalar charge \( Q_5 \). The winding number \( \nu = -im_\pi Q_5 \) is then obtained and the ensemble average \( \langle \nu^2 \rangle \) is calculated. In the chiral limit, this gives \( \chi_t = (188 \text{ MeV})^4 \).

(Here we have used \( a^{-1} = 1.18 \text{ GeV} \).) Using the Witten-Veneziano formula (with unrenormalized \( f_\pi \)) to obtain \( m_0 \), the chiral log parameter \( \delta = .065(8) \) is found.

Figure 1. Chiral log effect in \( m_\pi^2 \) vs. \( m_q \). Solid line is a perturbative (quadratic) fit to the five largest masses (four are not shown). Dotted line is fit of all masses to Eq. (3).
4. Quenched chiral logs in the pion mass

The pion masses are obtained for nine kappa values over a range of hopping parameters from .1400 to .1428, with $C_{sw} = 1.57$, corresponding to quark masses from roughly the strange quark mass down to about four times the up and down quark average (i.e. a pion mass of ≈ 270 MeV). A perturbative (linear+quadratic) fit of $m_\pi^2$ as a function of $m_q$ works well for the five heaviest masses, up to $\kappa = .1420$. The value of $m_\pi^2$ for the lighter quark masses deviates significantly below this perturbative fit (see Fig. 1) showing clear evidence of a quenched chiral log effect. Fitting to the formula (3) gives an anomalous exponent $\delta = .079(8)$.

5. Pseudoscalar and axial-vector matrix elements

By a combined fit of smeared-local pseudoscalar and axial-vector propagators, we obtain values for the decay constants $f_P$ and $f_A$ defined in (4)-(5). The behavior of these two constants as a function of pion mass squared is shown in Figs. 2 and 3. We see that there is a very significant chiral log enhancement at light masses for the pseudoscalar constant, but the axial-vector shows no chiral log effect, just as theoretical arguments predicted. From the phenomenology of $O(p^4)$ terms in the chiral Lagrangian, it can be argued that the perturbative slopes of $f_P$ and $f_A$ in full QCD should be approximately equal. Indeed, if a chiral log factor is removed from $f_P$, the remaining slopes seen in our lattice data are equal within errors. To obtain an estimate of $\delta$, we fit the ratio $f_P/f_A$ to a pure chiral log factor:

$$\frac{f_P}{f_A} = \text{const} \times (m_\pi^2)^{-\delta}$$

This fit gives $\delta = .080(7)$.

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