Is the $a_0(1450)$ a candidate for the lowest $q\bar{q} \ 3P_0$ state?

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Abstract

For the $a_0(1450)$, considered as the $q\bar{q} \ 1 \ 3P_0$ state, “experimental” tensor splitting, $c_{exp} = (-150 \pm 40)$ MeV, appears to be in contradiction with conventional theory of fine structure. There is no such discrepancy if the $a_0(980)$ belongs to the $1 \ 3P_J \ q\bar{q}$ multiplet. The hadronic shift of the $a_0(980)$ is shown to be strongly dependent on the value of the strong coupling in spin-dependent interaction.

1 Two possible candidates for the ground $3P_0$ state

At present a few low-lying $P$-wave light mesons, both in isovector and isoscalar channels, were experimentally observed [1, 2], however, theoretical identiﬁcation of these states faces with serious diﬃculties, ﬁrst of all, because the $3P_0$ member of the lowest $3P_J$ multiplet is still not identiﬁed in unambiguous way. Here we shall discuss only more simple isovector multiplet.

As it is well known the $a_0(980)$ couples strongly to $\eta\pi$ and $K\bar{K}$ channel and therefore this state was suggested to be interpreted as a multiquark meson [3] or $K\bar{K}$ molecule [4]. Then under such assumptions the $1 \ 3P_0 \ q\bar{q}$ state is to be identiﬁed with the $a_0(1450)$ meson [5]. There exist also many arguments in favour to consider the $a_0(980)$ as the ground $q\bar{q} \ 3P_0$ state, which however has large $K\bar{K}$ component in its wave function [6].

We shall discuss here both possibilities performing the ﬁne structure (FS) analysis of the lowest $q\bar{q} \ 3P_J$ multiplet. It is evident that a identiﬁcation of the members of this multiplet should be in accord with values and sign of FS parameters. We shall show here that in two cases depending on whether the $a_0(980)$ or the $a_0(1450)$ is $q\bar{q} \ 1 \ 3P_0$ state, the drastic diﬀerence in value and even sign of tensor splitting takes place. In particular, if the $a_0(1450)$ is identiﬁed as the $q\bar{q} \ 1 \ 3P_0$ state then large magnitude and negative sign of tensor splitting appears to be in contradiction with conventional QCD theory of the FS. Also in this case the center of gravity of the $1 \ 3P_J$ multiplet $M_{cog}$ has large value and large shift with respect to the mass of the $b_1(1235)$ meson (see the discussion in Ref. [7]). In other case if the $a_0(980)$ is the $1 \ 3P_0$ state, FS splittings and $M_{cog}$ appear to be in agreement with theoretical picture and the value of the hadronic shift is strongly correlated with the strong coupling in spin-dependent interaction.
interaction. For large $\alpha_s(1 - \text{loop}) = 0.53$ the hadronic shift appears to be equal zero and the masses of the $a_0, a_1, a_2$ mesons just coincide with their experimental values. For the $\alpha_s(2 - \text{loop}) = 0.43$ the hadronic shift about 100 MeV is obtained.

2 Fine-structure splittings

For any $n^3P_J$ multiplet the FS parameters: spin-orbit (SO) splitting $a(nP)$ and tensor splitting $c(nP)$ can be expressed through the masses $M_J(J = 0, 1, 2)$ of the members of the multiplet in the standard way [8]:

$$a = \frac{5}{12}M_2 - \frac{1}{4}M_1 - \frac{1}{6}M_0, \quad (1)$$

$$c = \frac{5}{6}M_1 - \frac{5}{18}M_2 - \frac{5}{9}M_0. \quad (2)$$

where the experimental values of $M_1$ and $M_2$ for the $a_1$ and $a_2$ mesons are well known [1],

$$M_1 = (1230 \pm 40) \text{ MeV}, \quad M_2 = (1318 \pm 0.6) \text{ MeV}, \quad (3)$$

while the $1^3P_0$ state can be identified either with the $a_0(1450)$ with the mass $M_{0A}$ [1],

Case A: $M_{0A} = (1452 \pm 8) \text{ MeV} \quad (4)$

or with the $a_0(980)$ having the mass $M_{0B}$ [1]:

Case B: $M_{0B} = (984.8 \pm 1.4) \text{ MeV} \quad (5)$

Then from Eqs. (4)-(6) in case A the following “experimental” values of SO and tensor splittings can be extracted,

$$a_A(\text{exp}) = (-0.3 \pm 11.6) \text{ MeV} \quad (6)$$

$$c_A(\text{exp}) = (-148 \pm 38) \text{ MeV}, \quad (7)$$

i.e SO splitting is compatible with zero or even small negative value while tensor splitting (7) is always negative and has large magnitude:

$$-186 \text{ MeV} \leq c_A(\text{exp}) \leq -110 \text{ MeV} \quad (8)$$

Note that large experimental error in $c_A(\text{exp})$ (7) comes from the error in the $a_1(1260)$ mass [3].

In case B, if the $a_0(980)$ is the ground $q\bar{q} \quad 3^P_0$ state, the situation is complicated by strong coupling of the $q\bar{q}$ channel with other hadronic channels, $\eta\pi$ and $K\bar{K}$, and for our analysis it is convenient to introduce the mass $\tilde{M}_0$ of the $1^3P_0$ state in one-channel approximation. Then

$$M_{0B}(\text{exp}) = 985 \text{ MeV} = \tilde{M}_0 - \Delta_{\text{had}}, \quad (9)$$

where the hadronic shift $\Delta_{\text{had}}$ is an unknown value while the mass $\tilde{M}_0$ can be calculated in different theoretical approaches, e.g. in the paper [9] of Godfrey, Isgur $\tilde{M}_0(GJ) =$
1090 MeV which corresponds to $\Delta_{had}(GJ) = 105$ MeV. In QCD string approach our calculations give close number for the choice of $\alpha_s(FS) = 0.42$.

It is of interest to note that if the $a_0(980)$ is the $q\bar{q}$ $1\,^3P_0$ state then both SO and tensor splittings are positive and the situation looks like in charmonium. For the $\chi_c(1P)$ mesons $a_{exp}(c\bar{c})$ and $c_{exp}(c\bar{c})$ are both positive with tensor splitting being by $13\%$ larger than SO one [10]:

$$a_{exp}(c\bar{c}) = (34.6 \pm 0.2) \text{ MeV}$$

$$c_{exp}(c\bar{c}) = (39.1 \pm 0.6) \text{ MeV}$$

(10)

3 Theory of the $P$-wave fine structure

Spin–dependent interaction in light $n\bar{n}$ mesons for the $L$-wave multiplets with $L \neq 0$ can be considered as a perturbation since FS splittings are experimentally small with compare to the meson masses. It is also assumed that the short-range one-gluon-exchange (OGE) gives the dominant contribution to perturbative potentials, in particular, to SO potential $V_{SO}^P(r)$ and tensor potential $V_T^P(r)$ [11]. As a result the matrix elements (m.e.) $a_P = \langle V_{SO}^P \rangle$ and $c_P = \langle V_T^P \rangle$ in light mesons are defined as the first order terms in $\alpha_s$:

$$a_P(nP) = a_P^{(1)} = \frac{2\alpha_s}{m_n^2} \langle r^{-3} \rangle_{nP},$$

$$c_P(nP) = c_P^{(1)} = \frac{2}{3} a_P^{(1)} = \frac{4}{3} \frac{\alpha_s}{m_n^2} \langle r^{-3} \rangle_{nP},$$

(11)

These expressions contain the constituent mass $m_n$ of a light quark, which was supposed to be fixed and just the same for all states with different quantum numbers $nL$; $m_n \approx 300 \div 350$ MeV is usually taken [11].

More detailed analysis of spin-dependent potentials for light mesons, both perturbative and nonperturbative (NP), was done in Ref. [12] where Feynman-Schwinger-Fock (FSF) representation of light meson Green’s function was used. Keeping only bilocal correlators of the fields, the spin–dependent potentials were expressed through these correlators (see Appendix) from which perturbative SO and tensor splittings are given by the expressions:

$$a_P^{(1)}(nL) = \frac{2\alpha_s}{\mu^2(nL)} \langle r^{-3} \rangle_{nL}, \quad c_P^{(1)}(nL) = \frac{2}{3} a_P^{(1)}(nL),$$

(12)

and NP contributions are

$$a_{NP}(nL) = -\frac{\sigma}{2\mu^2(nL)r} \langle r^{-1} \rangle_{nL}$$

$$c_{NP}(nL)$$

is compatible with zero

It is important to stress that in FSF representation the expansion in inverse quark masses as in heavy quarkonia is not used, and the constituent mass $\mu(nL)$ in
Eqs.(12),(13) is defined by the average over the kinetic energy term of the string Hamiltonian [13, 14]. This “constituent”, or dynamical, mass \( \mu(nL) \) for light quark with current mass \( m = 0 \) appears to be

\[
\mu(nL) = \langle \sqrt{p^2} \rangle_{nL} = \frac{1}{2} \sigma \langle r \rangle_{nL} \quad \text{for linear } \sigma r \text{ potential}
\]

and can be expressed only through string tension \( \sigma \) and the universal number.

In contrast to the constant mass \( m_n \) in Eqs.(11), the mass \( \mu(nL) \) depends on the quantum numbers \( n, L \) of a given state and it is increasing with growing \( n \) and \( L \).

In light mesons which have large radii linear static potential \( \sigma r \) dominates for all states with exception of the 1S and 1P states where Coulomb term turns out to be important [14].

The choice of the string tension \( \sigma \) and coupling \( \alpha_{st} \) in static potential can be fixed from the slope and the intercept of leading Regge \( L \)–trajectory which is defined for spin-averaged masses \( M_{cog}(L) \) for the multiplets \( 1L \). As shown in Ref. [14] the experimental values of \( M_{cog}(L) \) put strong restrictions on \( \sigma \) and also strong coupling \( \alpha_{st} \): \( \sigma = (0.185 \pm 0.005) \text{ GeV}^2, \alpha_{st} \lesssim 0.40 \). Here we present our calculations for linear \( \sigma r \) potential with \( \sigma = 0.18 \text{ GeV}^2 \) and also for linear plus Coulomb potential with

\[
\alpha_{st} = 0.42, \quad \sigma = 0.18 \text{ GeV}^2
\]

and take into account that the experimental Regge \( L \)-trajectory [14]: \( M_{cog}^2(L) = (1.60 \pm 0.04) \text{ GeV}^2 \) gives

\[
M_{cog}(1P) = 1260 \pm 10 \text{ MeV}
\]

Linear potential with \( \sigma = 0.18 \text{ GeV}^2 \) fits very well the orbital excitations with \( L \geq 2 \). From Ref. [14] the m.e. can be taken,

\[
\mu_0(1P) = 448 \text{ MeV}, \quad \langle r^{-1} \rangle_{1P} = 0.236 \text{ GeV}, \quad \langle r^{-3} \rangle_{1P} = 0.0264 \text{ GeV}^3.
\]

and the constituent mass for the 1S state,

\[
\mu(1S) = 335 \text{ MeV},
\]

coincides with the conventional value \( m_n \approx 300 \div 350 \text{ MeV}, \) usually used in potential models. For the 1P state calculated m.e. for Cornell potential are

\[
\mu(1P) = 486 \text{ MeV}, \quad \langle r^{-1} \rangle_{1P} = 0.260 \text{ MeV}, \quad \langle r^{-3} \rangle = 0.0394 \text{ GeV}^3
\]

so here \( \langle r^{-3} \rangle_{1P} \) turns out to be by 50% larger than for linear potential.

NP contributions to SO and tensor potentials can be correctly defined in bilocal approximation (see Appendix) when the Thomas term dominates in NP SO splitting, 

\[
a_{NP}(nP) = -\frac{\sigma}{2\mu^2(nP)} \langle r^{-1} \rangle_{nP},
\]

while NP tensor splitting \( c_{NP}(1P) \) for light mesons (as well as for heavy mesons) is compatible with zero:

\[
0 \leq c_{NP}(1P) < 5 \text{ MeV}
\]
and therefore can be neglected in our later analysis. This result follows from the fact, established in lattice QCD, that vacuum correlator $D_1$, which defines $c_{NP}$, is small \[15,16\].

Thus NP contribution is present only in SO splitting so that total SO splitting,

$$a = a_P + a_{NP} = \frac{2\pi_s}{\mu^2(1P)}\langle r^{-3}\rangle_{1P} - \frac{\sigma}{2\mu^2(1P)}\langle r^{-1}\rangle_{1P}$$ (22)

Due to the negative Thomas precession term a cancellation of perturbative and NP terms in $a_{(total)}$ takes place and in principle $a_{(total)}$ could be small or even negative number.

In contrast to that, NP tensor splitting is small and positive, so that total tensor splitting appears to be always positive,

$$c = c_P = \frac{4}{3}\pi_s\langle r^{-3}\rangle_{1P}$$ (23)

Note that in the $\chi_c$ mesons the correction of order $\pi_s^2$ to $c_P$ was obtained to be also positive and small \[10\].

### 4 Remarks about strong coupling $\alpha_s$

In heavy quarkonia strong coupling $\alpha_s(\mu_{ren})$ at the renormalization scale $\mu_{ren}$ can explicitly be extracted from experimental values of FS splittings due to rather simple, renorm-invariant relation between $\alpha_s(\mu_{ren})$ and the combination $\eta = \frac{3}{2}c - a$ \[10,15\]. In light mesons one-loop perturbative corrections ($\alpha_s^2$ order) are still not calculated and OGE contributions (12) with a fitting $\pi_s$ are assumed to be dominant.

However, at present we know some useful features of $\alpha_s$.

1. The strong coupling freezes at large distances and therefore $\pi_s$ in OGE terms have to be less, or equal, the critical value $\alpha_{cr}$ \[17,19\].

2. The critical value $\alpha_{cr}$ was calculated in background field theory \[17\] and obtained $\alpha_{cr}(r)$ have appeared to be in good agreement with lattice measurement of static potential in quenched approximation \[18\]. For QCD constant $\Lambda_{QCD}^{(0)} = (385 \pm 30)$ MeV, defined in lattice calculations \[19\], in Ref.\[17\] calculated $\alpha_{cr}$ is

$$\alpha_{cr}(1 - \text{loop}) = 0.59; \quad \alpha_{cr}(2 - \text{loop}) = 0.43^{+0.05}_{-0.04} \ (n_f = 0)$$ (24)

3. The characteristic size of FS interaction in the $P$-waves, $R_{FS}$, can be defined as

$$R_{FS} = \left[\frac{3}{\sqrt{\langle r^{-3}\rangle_{1P}}}\right]^{-1} \cong 0.60 \text{ fm}$$ (25)

From the study of $\alpha_s(r)$ in coordinate space \[17\] it was observed that at distances $r \sim 0.6$ fm strong coupling $\alpha_s(r)$ is already close to the critical value (24) being only by about 10% smaller. Therefore one can expect that $\pi_s$ in OGE splittings (22),(23) has to be equal

$$\pi_s(R_{FS}) = 0.41 \pm 0.02$$ (26)
4. The size $R_{FS}$ (25) remarkably coincides with the radius of the 1P $c\pi$ and also of the 2P $b\pi$ state which both are about 0.60 fm. For them the extracted from experiment strong coupling is

$$\alpha_{exp}(c\pi, 1P) = 0.38 \pm 0.03 (exp),$$

(27)

i.e. this number is very close to that in Eq.(26). To check a sensitivity of FS splitting to the choice of $\pi_s$ here in our calculations we shall take

$$\pi_s(2 - loop) = 43, \quad \pi_s(1 - loop) = 0.53 \cong 0.9\alpha_{cr}$$

(28)

5  **The $a_0(1450)$ is the $q\bar{q}$ $1^3P_0$ state**

We start with $NP$ contribution to SO splitting $a_{NP}$ and for $\sigma = 0.18$ GeV$^2$ ($\mu(1P) = 448$ MeV) $a_{NP} = -106$ MeV

(29)

It can be shown that $a_{NP}$ is weakly dependent on the choice of the parameters of static interaction varying in the range 99 MeV – 106 MeV.

From Eq.(6) $a_A(exp)$ is compatible with zero and from the Eq.(22) this condition can be reached only if $a_P = |a_{NP}|$ from which

$$a_P = 106 \text{ MeV}, \text{ and therefore } \pi_s(fit) = 0.40.$$  

(30)

Note that this $\pi_s$ is in agreement with expected values (26). Correspondingly, from (23) in theory tensor splitting $c_P = \frac{2}{3}a_P$ is positive,

$$c = 71 \text{ MeV}$$

(31)

Thus positive sign of $c(1P)$ appears to be in contradiction with the “experimental” number [4]: $c_A(exp) = (-148 \pm 38) \text{ MeV}$.  

The second discrepancy is that spin-averaged mass in considered case A,

$$M_{cog} = 1303 \text{ MeV},$$

(32)

is larger than the experimental number (16) following from linear Regge $L$-trajectory for spin-averaged masses [1].

6  **The $a_0(980)$ is the $q\bar{q}$ $1^3P_0$ state**

Here the mass $M_0$ ($^3P_0$) will be calculated in one-channel approximation,

$$\tilde{M}_0 = M_{cog}(1^3P_J) - 2a - c$$

(33)

with $M_{cog}(1^3P_J) = (1260 \pm 10)$ MeV from experimental Regge trajectory. We give below FS splittings $a(1P)$ and $c(1P)$ for two values of strong coupling (24).

For $\alpha_s = 0.43$ when radiative corrections of $\alpha_s^2$ order were supposed to be small, $\langle r^{-3}\rangle_{1P} = 0.0394$ GeV$^3$, one obtains ($a_p = 143$ MeV, $a_{NP} = -99$ MeV)

$$a = 44 \text{ MeV}, \quad c = 96 \text{ MeV}$$

(34)
and from the definition (33)
\[ \tilde{M}_0(\alpha_s = 0.43) = (1076 \pm 10) \text{ MeV}, \] (35)
so that corresponding hadronic shift of the \( a_0 \) meson, according to the definition (9), is
\[ \Delta_{\text{had}}(\alpha_s = 0.43) = (91 \pm 10) \text{ MeV}, \] (36)
If radiative corrections are not suppressed then it would be more consistent to take in OGE terms the strong coupling in one-loop approximation when from Ref.[17]
\[ \tilde{\alpha}_s(1 - \text{loop}) \cong 0.9\alpha_{\text{cr}}(1 - \text{loop}) = 0.53 \] (37)
Then FS splittings appear to be in good agreement with the experimental numbers
\[ a = 78 \text{ MeV}, \quad c = 118 \text{ MeV}, \] (38)
which correspond to the hadronic shift equal zero and,
\[ \tilde{M}_0 = (986 \pm 10) \text{ MeV}, \] (39)
coincides with the mass of the \( a_0(985) \). Thus the hadronic shift turns out to be very sensitive to the choice of strong coupling and:
\[ \Delta_{\text{had}}(\alpha_s = 0.53) \cong (3 \pm 10) \text{ MeV} \] (40)
It is essential that in Ref.[9], as well as in eqs.(37), large value of one-loop coupling was used. It assumes that \( \alpha_s^2 \) corrections to FS splitting are not small being about \( (30 \div 20)\% \). To understand which choice of \( \alpha_s \) is preferable one needs to study many other \( P \)-wave multiplets. In any case one can conclude that predicted value of hadronic shift is correlated with the choice of strong coupling in SO and tensor potentials.
For FS splittings (38) the masses of the \( a_1 \) and \( a_2 \) mesons are equal 1242 and 1325 MeV, i.e. very close to their experimental values.

7 Hyperfine shift as a tool to identify the members of the \( ^3P_J \) multiplet

Hyperfine (HF) splitting of the \( P \)-wave multiplet which comes from spin-spin interaction, \( \Delta_{HF}(1P) \), is defined as
\[ \Delta_{HF}(nP) = M_{\text{cog}}(n^3P_J) - M(n^1P_1) \] (41)
where \( M_{\text{cog}} \) is not sensitive to taken value \( \tilde{M}_0 \) since it enters \( M_{\text{cog}} \) with small weight equal 1/9. In (38) \( M(1^1P_1) \) is the mass of the \( b_1(1235) \).
Then if the \( a_0(1450) \) belongs to the lowest \( q\overline{q}^3P_J \) multiplet, \( M_{\text{cog}} = 1303 \text{ MeV} \) and
\[ \Delta_{HF}^{(1P)} = (73 \pm 3(\text{exp})) \text{ MeV}, \quad \text{case A} \] (42)
Identifying the $a_0(980)$ as the $q\bar{q}$ ground $^3P_0$ state, one obtains $M_{cog} = 1252$ MeV and

$$\Delta_{HF}^{(1P)} = 28 \pm 6(\text{th}) \pm 3(\text{exp}) \text{ MeV}, \quad \text{case B}$$

(43)
i.e. essentially smaller. One can compare these numbers with theoretical predictions from Ref.[7] where perturbative contribution to HF shift was shown to be small and negative while NP term is positive and not small,

$$\Delta_{HF}(th) = (30 \pm 10) \text{ MeV.}$$

(44)

In $\Delta_{HF}(th)$ (44) theoretical error comes from uncertainty in our knowledge of gluonic correlation length $T_g$ in lattice calculations: $T_g = 0.2$ fm in quenched QCD and $T_g = 0.3$ fm in full QCD [16]. From comparison (44) and (43) one can see that theoretical number (44) appears to be in good agreement with experimental one (43) if the $a_0(980)$ is the $q\bar{q}$ $^1{}^3P_0$ and this statement does not depend on a value of hadronic shift $\Delta_{had}$.

On the contrary if the $a_0(1450)$ is taken as the $q\bar{q}$ $^1{}^3P_0$ state, then “experimental” value (42) is two times larger than $\Delta_{HF}(theory)$.

8 Conclusions

Experimental data on FS splittings in light mesons appear to be a useful tool to identify the members of the $^3P_J$ multiplet. Our study of the FS has shown that

The $a_0(1450)$ cannot be a candidate for the ground $^3P_0$ state since under such identification
i) “experimental” value of tensor splitting turns out to be negative (with large magnitude), $c_{exp} = (-150 \pm 40)$ MeV, in contradiction with the conventional theory.
ii) Also for such interpretation spin-averaged mass $M_{cog} = 1303$ MeV would be too large and lie above linear Regge trajectory for spin-averaged masses.
iv) Hyperfine shift of the $b_1(1235)$ with respect to $M_{cog}$ would be two times larger than predicted number in Ref.[1].

There is no such discrepancies if the $a_0(980)$ is the $q\bar{q}$ $^1{}^3P_0$ state. In this case the mass $\tilde{M}_0(^3P_0)$ can be calculated in one-channel approximation and from the difference $\tilde{M}_0(^3P_0) - M_0(a_0(980)) = \Delta_{had}$ hadronic shift is found to be very sensitive to the chosen value of strong coupling $\alpha_s$.

If radiative corrections of $\alpha_s^2$ order are small, as in charmonium, and $\alpha_s \approx 0.40$ is used, then $\Delta_{had}$ is large, $\Delta_{had} = (100 \pm 10)$ MeV. If for $\alpha_s$ the value $\alpha_s (1 - \text{loop}) = 0.53$ is taken, the hadronic shift appears to be equal zero.

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Appendix.

Spin-dependent potentials in light mesons

In Refs. [12,21] all NP spin-dependent potentials in light mesons: $\hat{V}_{LS} = L_S V_{LS}$; tensor potential $\hat{V}_T = S_{12} V_T$, and HF potential $\hat{V}_{HF} = S_1 S_2 V_{HF}$ were obtained being expressed through bilocal vacuum correlation functions (v.c.f.) $D(x)$ and $D_1(x)$

$$V_{LS}^{NP}(r) = -\frac{1}{\mu_0 r} \int_0^\infty d\nu \int_0^r d\lambda \left( 1 - \frac{4\lambda}{r} \right) D(\sqrt{\lambda^2 + \nu^2}) + \frac{3}{2\mu_0^2} \int_0^\infty d\nu D_1^{NP}(\sqrt{r^2 + \nu^2}) \quad (A.1)$$

$$V_T^{NP}(r) = -\frac{2r^2}{3\mu_0^2} \int_0^\infty d\nu \frac{\partial}{\partial r^2} D_1^{NP}(\sqrt{r^2 + \nu^2}) \quad (A.2)$$

$$V_{HF}^{NP}(r) = \frac{2}{\mu_0^2} \int_0^\infty d\nu \left[ D(\sqrt{r^2 + \nu^2}) + D_1^{NP}(\sqrt{r^2 + \nu^2}) + \frac{2r^2}{3} \frac{D_1^{NP}(\sqrt{r^2 + \nu^2})}{\partial r^2} \right] \quad (A.3)$$

Here v.c.f. $D$ and $D_1$ are defined though the gauge-invariant bilocal vacuum correlators:

$$\frac{g^2}{N_c} \langle F_{\mu\nu}(x)\phi(x,y)F_{\lambda\sigma}(y)\phi(y,x) \rangle =$$

$$(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda})D(x-y) + \frac{1}{2} \partial_{\mu} \left[ h_{\lambda\delta_{\nu\sigma}} - h_{\sigma\delta_{\nu\lambda}} + \text{permutation} \right] D_1(x-y) \quad (A.4)$$

where $h_{\mu} = x_{\mu} - y_{\mu}$ and the factor

$$\phi(x,y) = P \exp \int_y^x A_{\mu}(z) dz_{\mu} \quad (A.5)$$

provides the gauge invariance of the correlators (A.4). In Ref.[22] it was shown that in bilocal approximation there is no perturbative contribution to v.c.f. $D(x)$ while the correlator $D_1$ contains both perturbative and NP contributions:

$$D_1 = D_1^{NP} + D_1^{\text{pert}} \quad (A.6)$$

with

$$D_1^{\text{pert}}(x) = \frac{16}{3\pi} \alpha_s \frac{x^4}{s^4} \quad (A.7)$$

To derive the expressions (A.1)-(A.3) the meson Green’s function in FSF representation (which is gauge – invariant) was studied and spin terms enter $G_M(x,y)$ through the exponential factors,

$$\exp(g \int_0^{\tilde{s}} d\tau \sigma_{\mu\nu}^{(1)} F_{\mu\nu}) \exp(-g \int_0^{\tilde{s}} d\tilde{\tau} \sigma_{\mu\nu}^{(2)} F_{\mu\nu}) \quad (A.8)$$
Here \( \sigma_{\mu\nu} = \frac{1}{4} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \), \( F_{\mu\nu} \) is the field strength, and \( s(\bar{s}) \) is the proper time of the quark (antiquark). The proper time \( \tau(\bar{\tau}) \) plays the role of ordering parameters along the quark (antiquark) trajectory \( z(\tau) \) (\( \bar{z}_\mu(\bar{\tau}) \)).

To obtain Hamiltonian and potentials from the meson Green’s function it is necessary to go over from the proper time to the actual time \( t \) of the quark thus defining the new quantity \( \mu(\tau) \):

\[
2\mu(t) = \frac{dt}{d\tau}, \quad 2\bar{\mu}(\bar{t}) = \frac{dt}{d\bar{\tau}} \tag{A.9}
\]

Then in (A.8) the integrals can be rewritten as

\[
J_q = \int_0^s d\tau \, \sigma_{\mu\nu} F_{\mu\nu} = \int_0^T \frac{dt}{2\mu(t)} \sigma_{\mu\nu}^{(1)} F_{\mu\nu}(z(t)) \tag{A.10}
\]

and correspondingly the integral

\[
\bar{J}_q = \int_0^T \frac{dt}{2\bar{\mu}(t)} \sigma_{\mu\nu}^{(2)} F_{\mu\nu} \tag{A.11}
\]

is defined for the antiquark. In bilocal approximation after averaging the exponents (A.8) will contain the bilocal correlators, or the cumulants. To obtain spin-dependent potentials the important approximation is that the spin factors (A.8) are considered as a perturbation and therefore \( \mu(t) \) and \( \bar{\mu}(t) \) in (A.9),(A.10) can be changed by corresponding values \( \mu_0 \) and \( \bar{\mu}_0 \) (\( \mu_0 = \bar{\mu}_0 \)) calculated for the unperturbed Hamiltonian \( H_R \) which is defined for a meson with spinless quark and antiquark. Notice that in FSF representation to derive spin-dependent potentials (in light mesons) the expansion in inverse powers of quark mass was not used.

The derivation of the meson relativistic Hamiltonian \( H_R \) and the definition of the constituent mass,

\[
\mu_0 = \langle \sqrt{p^2 + m^2} \rangle_{nL} \tag{A.12}
\]

is discussed in details in Refs. [12-13].

The v.c.f. \( D \) and \( D_1 \) were calculated in lattice QCD [16] where it was shown that \( D_1^{NP} \) is small with compare to \( D(x) \) and even compatible with zero in full QCD. Therefore in the potentials (A.1)-(A.3) the terms containing \( D_1^{NP} \) can be omitted, in particular, NP contribution to tensor splitting

\[
c_{NP} = \langle V_T \rangle \text{ is compatible with zero} \tag{A.13}
\]

The perturbative contribution to SO and tensor potentials which are defined by v.c.f. (A.7) just corresponds to OGE terms (12),(13).

Lattice measurements has also shown that v.c.f. \( D \) can be parametrized with a good accuracy as the exponent at the distances \( x \gtrsim 0.2 \text{ fm} \), i.e.

\[
D(x) = d \exp \left( -\frac{x}{T_g} \right) \tag{A.14}
\]
Then from (A.1) and (A.9)

\[
V_{LS}^{NP}(r) = -\frac{\sigma}{\mu_0^2 r \pi} \int_0^{r/T_g} t K_1(t) + \frac{4\sigma}{\pi \mu_0^2} \left[ \frac{2T_g}{r^2} - \frac{1}{T_g} K_2\left(\frac{r}{T_g}\right) \right]
\]  

(A.15)

where the string tension,

\[
\sigma = 2 \int_0^\infty d\nu \int_0^\infty d\lambda D(\sqrt{\lambda^2 + \nu^2}),
\]  

(A.16)

for the exponential form of \(D(x)\) is

\[
\sigma = \pi dT_g^2.
\]  

(A.17)

From the expression (A.15) it can easily be shown that

\[
V_{LS}^{NP}(r \gg T_g) \to -\frac{\sigma}{2\mu^2 r},
\]  

(A.18)

i.e. coincides with the Thomas precession term if \(T_g\) is supposed to be small. We shall not take into account here the positive correction to the Thomas potential coming from second term in (A.15) since there exist two other contributions: from the interference of perturbative and NP terms [23] and from Coulomb term with correct strong coupling in background fields \(\alpha_B(r)\) which imitates linear \(\sigma^* r\) potential at \(r \lesssim 0.3\) fm [17],[18].
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