Pumping of twin-trap Bose-Einstein condensates

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We consider extensions of the twin-trap Bose-Einstein condensate system of Javaneinen and Yoo [Phys. Rev. Lett., 76, 161–164 (1996)] to include pumping and output couplers. Such a system permits a continual outflow of two beams of atoms with a relative phase coherence maintained by the detection process. We study this system for two forms of thermal pumping, both with and without the influence of inter-atomic collisions. We also examine the effects of pumping on the phenomenon of collapses and revivals of the relative phase between the condensates.
Both before and since the recent demonstrations of Bose-Einstein condensation in dilute alkali gases, the concept of the phase of a Bose-Einstein condensate (BEC) has attracted a deal of theoretical study. Traditionally, the existence of a phase is taken as a signature of spontaneous symmetry breaking and strictly it is only the relative phase of two BEC’s that can be assigned a definite value. Many people have discussed the difficulties associated with the fact that in many cases, we may know the number of atoms present fairly precisely. The condensate is thus in a state that cannot possess a well-defined phase. Recently, several authors have modelled systems containing two BEC’s, demonstrating that a relative phase can arise naturally, even when both condensates are initially in number states. Typically, atoms are permitted to leak out from both traps and are detected by some apparatus below. As it is unknown from which trap any individual atom comes, the distribution of positions at which atoms are detected shows interference fringes. At the same time, the quantum state of the two traps evolves from a simple product of number states into an entangled state of varying number differences between the traps allowing a well-defined relative phase to appear between the two condensates. The value of the relative phase is randomly distributed from run to run so that for an ensemble of runs, the phase symmetry is restored.

The influence of inter-atomic collisions on such a system has also been considered. The notable results are a reduction in the visibility of the observed interference pattern, and in the conditional visibility of the entangled state, due to diffusion of the condensate phase. Moreover, stopping the detection process after a relative phase has been established leads to collapses and revivals in the conditional visibility, as the collisions cause the phases of different components of the entangled state to rotate at different rates. As the total state is a discrete sum of number difference states, the phases realign periodically and the visibility is restored.

An obvious consequence of detecting atoms is that the system can not attain a steady
In time, the trap occupancies fall so low that the entangled state is reduced in size until eventually the atoms in one or both traps are exhausted altogether. If one envisaged using the two-trap system as a real “device” rather than merely an analogy for a single condensate, this might be a serious problem. A natural use would be as a kind of two-beam “atom laser” in which additional atoms would be tapped off from each trap through an output coupler into separate beams (see Fig. 1), as in the case of two remarkable recent experiments at MIT [12,13]. The relative phase between the BEC’s in the traps built up by the measurement process would then be reflected in the same well-defined relative phase between the two output beams, that could be exploited for some other interference experiment. While such a scheme could operate in a pulsed fashion where the traps were repeatedly filled with atoms, measured until exhausted and then the process repeated (as was the case for the experiments just mentioned), it would also be useful to have a continuous output. This would clearly require a continuous pumping of new atoms into the traps. We note of course, that interference measurements on the output beams would themselves act to produce a relative phase between the traps. It may be however, that the desired rate of measurements on the output beams is too low to stabilize the phase over long periods, whereas the detection rate directly on the traps may be as large as necessary. Here we assume that the rate at which subsequent measurements are made on the output beams is so low that they have a negligibly small feedback on the entangled state in the traps. (Note that the output rate may be relatively high as long as most of the leaked atoms are not actually detected in an interference experiment.)

In this paper, we explore the effects of pumping a two-trap BEC system. We investigate what kind of steady states may be reached when pumping is included, and study the competing effects of the collisions and measurements in such a steady state. We also allow output couplers on each trap as discussed above. We explore two types of pumping from thermal atom baths coupled to each trap. In the first, we allow two-way pumping where atoms are exchanged with the baths in both directions. Such a scheme was considered for a single-trap atom laser based on evaporative cooling by Wiseman et al [14]. In the second model, atoms
may enter the traps from a thermal bath, but the reverse process is forbidden. This kind
of irreversible pumping scheme has been considered in an atom laser model by Holland et
al [15]. We can then consider our whole system as a means of transferring non-condensate
atoms in a thermal bath, into two coherent beams with the coherence established by the
detection process. In this sense, the system might be considered a primitive two-beam atom
laser. We emphasize though, that in this paper we include no line-narrowing element, a
central component of true optical lasers [16–18]. The linewidth of the output beams would
be at least that of the output couplers.

The paper is structured as follows. In section II we describe our model in detail while
in section III we discuss the nature of the entangled state more fully. Using Monte-Carlo
wave function simulations, we consider pumping from a thermal bath of atoms in section IV
examining the visibility and other parameters, when the pumping of the trap is either
two-way or inwards-only. Finally in section V, we turn to the phenomenon of collapses
and revivals of the condensate phase and present some interesting effects associated with
pumping.

II. ELEMENTS OF THE MODEL

We now set out the model in full detail. In all the work below we assume a system
of two traps containing condensates with occupation numbers \(n_1\) and \(n_2\) (see Fig. I). We
occasionally write \(n_i\) to indicate either trap. Atoms may leak from either trap and be
detected below at a mean rate \(\gamma n_i\) establishing a relative phase. (We assume that an atom
in either trap has the same probability of detection). The traps are pumped from two
thermal reservoirs containing \(N_1\) and \(N_2\) non-condensate atoms with rate coefficients \(\chi_1\) and
\(\chi_2\) respectively. At different points in the paper, we assume that atoms may move either in
both directions between the traps and reservoirs, or only into the traps, so that we define
separate rates \(\chi_1^{in}, \chi_2^{in}\) and \(\chi_1^{out}\) and \(\chi_2^{out}\). In the simplest systems, we would expect \(\chi_1^{in} = \chi_1^{out}\),
but the inclusion of some irreversible pumping process can prevent atoms escaping from the
trap into the baths giving $\chi_i^{\text{out}} = 0$. One example would be to couple the thermal bath to an excited trap level $|e\rangle$. An atom in $|e\rangle$ can decay to the BEC mode $|g\rangle$ by spontaneous emission. If the medium is optically thin, the emitted photon is lost and excitation out of the ground state is impossible. We see in section IV that this one-way pumping leads to quite different behavior to the two-way pumping. The physical validity of the two types of thermal pumping has been discussed at some length in Ref. [14].

We also allow a separate leak from each trap into an empty mode at rates $\nu_1$ and $\nu_2$ to act as “output” beams. While we here leave the mechanism unspecified, we note that several techniques for creating an output coupler have been demonstrated by the MIT group [12] in which rf signals or a bias in the trapping field are used to couple a portion of the condensate to untrapped spin states [19]. Finally, the atoms in each trap experience collisions amongst themselves at a rate $\kappa$.

The master equation for the system may thus be written

$$
\frac{d\rho}{dt} = \frac{i}{\hbar} [\rho, H] + \gamma \int_0^{2\pi} D [\psi (\phi)] \, d\phi \rho + \chi_1^{\text{out}} (N_1 + 1) \, D [a_1] \rho + \chi_1^{\text{in}} N_1 D [a_1^\dagger] \rho
$$
$$
+ \chi_2^{\text{out}} (N_2 + 1) \, D [a_2] \rho + \chi_2^{\text{in}} N_2 D [a_2^\dagger] \rho + \nu_1 D [a_1] \rho + \nu_2 D [a_2] \rho,
$$

where the Hamiltonian describing collisions amongst the atoms is

$$
H = \frac{\kappa}{2} \left[ (a_1^\dagger a_1)^2 + (a_2^\dagger a_2)^2 \right],
$$

and $a_i$ is the annihilation operator for an atom in trap $i$. Also, for an arbitrary operator $c$, the superoperator $D [c] \rho$ is defined by

$$
D [c] \rho = c \rho c^\dagger - \frac{1}{2} (c^\dagger \rho c + \rho c^\dagger c).
$$

The field operator $\psi (\phi) = a_1 + a_2 e^{-i\phi}$, where $\phi = 2\pi x$, describes the detection of an atom at position $x$. Most of our results are obtained from Monte-Carlo wave-function simulations of Eq. (1) in which all leaks and additions of atoms to the traps are represented as quantum jumps, and the non-unitary evolution of the wave function is given by the effective Hamiltonian
\[ H_{\text{eff}} = H - i \frac{\hbar}{2} \left[ \gamma \left( a_1^\dagger a_1 + a_2^\dagger a_2 \right) + \left[ \chi_1^\text{out} (N_1 + 1) + \chi_1^\text{in} N_1 + \nu_1 \right] a_1^\dagger a_1 \\
+ \chi_1^\text{in} N_1 + \left[ \chi_2^\text{out} (N_2 + 1) + \chi_2^\text{in} N_2 + \nu_2 \right] a_2^\dagger a_2 + \chi_2^\text{in} N_2 \right]. \]

When the chosen jump is a detection, the phase \( \phi \) of the next detection is chosen randomly according to the conditional probability distribution

\[ P (\phi) = \langle t | \psi^\dagger (\phi) \psi (\phi) | t \rangle , \]

where \( |t\rangle \) is the state of the system immediately preceding the detection. It has been shown elsewhere [4,7,8] that this may always be written in the form

\[ P (\phi) \propto 1 + \beta \cos (\phi + \theta) , \]

where the conditional visibility \( \beta \) and conditional phase \( \theta \) are determined by the previous history of the system.

In order that the average number of atoms in the traps become constant in time for the thermal pumping schemes, we require a relation between the various coefficients in the master equation (1). Assuming equal detection and pumping rates for traps 1 and 2, the pumping rates must satisfy

- for two-way pumping with \( \chi^{\leftrightarrow} = \chi^\text{in} = \chi^\text{out} \):

  \[ \chi^{\leftrightarrow} = \frac{\gamma \langle n_1 + n_2 \rangle + \sigma_1 \langle n_1 \rangle + \sigma_2 \langle n_2 \rangle}{N_1 + N_2 - \langle n_1 + n_2 \rangle} , \]

- for one-way pumping with \( \chi^{-} = \chi^\text{in} \), and \( \chi^{\text{out}} = 0 \):

  \[ \chi^{-} = \frac{\gamma \langle n_1 + n_2 \rangle + \sigma_1 \langle n_1 \rangle + \sigma_2 \langle n_2 \rangle}{N_1 \left( \langle n_1 \rangle + 1 \right) + N_2 \left( \langle n_2 \rangle + 1 \right)} . \]

Note that for \( N_1 \approx N_2 \gg \langle n_i \rangle \), the two-way pumping rate \( \chi^{\leftrightarrow} \) is larger than the one-way rate \( \chi^{-} \) by a factor of approximately \( \langle n_1 + n_2 \rangle / 2 \). In the one-way case, on average one atom is added for each atom detected or lost to an output coupler, while in the two-way case, on average all the trapped atoms are exchanged with the reservoirs for every loss by detection or output coupling. This difference has important consequences below.
III. QUANTUM STATE OF THE TRAPS

Our main interest in this paper is to find the equilibrium state of the model just described, under a variety of conditions and explore the different influences of detection, pumping and collisions. For example, we naturally expect increasing collisions to reduce the phase coherence and drive the state towards a narrower number distribution. In preparation, however, we should first discuss the nature of the entangled state and our methods for characterizing it more fully.

In earlier studies that consider only detection of the atoms [4–8], the initial state is normally taken as the product state $|\rangle_0 = |n_1, n_2\rangle$ with $n_1$ atoms in trap 1 and $n_2$ atoms in trap 2. The unnormalized state following a single detection with phase $\phi$ is found by applying the field operator $\psi(\phi_1)$:

$$|\rangle_1 \propto (a_1 + a_2 e^{-i\phi_1}) |n_1, n_2\rangle = \sqrt{n_1} |n_1 - 1, n_2\rangle + e^{-i\phi_1} \sqrt{n_2} |n_1, n_2 - 1\rangle.$$  \hspace{1cm} (10)

By extension, after $m$ detections the state has the form

$$|\rangle_m = \sum_{k=0}^{m} c_k (m) |n_1 - m + k, n_2 - k\rangle,$$  \hspace{1cm} (11)

where the $c_k$ are functions of the phases of all the detected atoms $\{\phi_1, \ldots, \phi_m\}$. If collisions are included, the state experiences unitary evolution under the Hamiltonian (2) in between detections and the coefficients $c_k (m)$ are also functions of time. Here, with the inclusion of pumping the situation is similar, but as one can continue to detect atoms indefinitely, the entangled state can become very large (note of course that the number of detections $m$ can now arbitrarily exceed the initial occupancy of the traps). It becomes more natural to drop the dependence on $m$ and write the state at time $t$ as

$$|t\rangle \approx \sum_{k=-p}^{p} c_k (t) |n_1(t) - k, n_2(t) + k\rangle.$$  \hspace{1cm} (12)

This is an approximate relation because we truncate the sum at some cut-off $p$. This is particularly important numerically as the exact state can become prohibitively large for
calculations. Such a truncation is possible because the probability of all detections occurring from a single trap is small (assuming the initial trap occupancies are not wildly different) and hence the coefficients at the extreme ends of the entangled state are negligible. In our simulations, we drop terms for which $|c_k| < 10^{-12}$. We characterise the state in terms of the mean number of atoms in each trap, and the width of the number difference distribution with the natural definition

$$\sigma_n = \left( \langle (n_1 - n_2)^2 \rangle - \langle n_1 - n_2 \rangle^2 \right)^{1/2}. \quad (13)$$

Frequently we also wish to describe the phase distribution for which we use the width

$$\sigma_\phi = \left( 1 - |\langle \exp(i\phi) \rangle|^2 \right)^{1/2}$$

$$= \left( 1 - \sum_k |c_k^* c_{k+1}|^2 \right)^{1/2}. \quad (14)$$

For a minimum uncertainty state, we have $\sigma_n \sigma_\phi = 1$. Aside from the evolution of the conditional visibility, our main interest below is in the behavior of these two measures of the state.

**IV. STEADY STATES**

We now turn to finding the steady states of our system. With thermal pumping schemes however, a genuine steady state is achieved only for an average over many trajectories. For a given set of parameters, each trajectory differs not only in the actual relative phase established between the two traps, but more importantly, in the instantaneous atom numbers as a function of time. As the trajectory simulation proceeds, the occupancy of each trap exhibits thermal fluctuations which lead to time variations in other properties of the system such as the conditional visibility. Only the time-averaged properties approach a true steady state. As discussed in section [I], we treat two cases: two-way pumping in which atoms may be exchanged between the bath and trap in both directions, and one-way pumping in which atoms can only move from the bath into the trap. We begin with representative plots of
the visibility as a function of time for a single trajectory with no collisions ($\kappa = 0$). The visibility for two-way pumping is shown in Fig. 2a. There are initially $n = 100$ atoms in each trap and the pumping rate is chosen to balance the detection rate. On average, $2n$ atoms are detected in a time $\gamma t = 1$. The visibility is extremely noisy with frequent fluctuations of order 1. Our simulations show the occupancies of the traps also display large fluctuations as would be expected for coupling to thermal baths. In particular, a zero in the visibility is always associated with a zero in one or other of the atom numbers. The visibility for a typical trajectory with one-way pumping is shown in Fig. 2b. In this case, there are again large fluctuations but on a much longer time scale. This difference has a simple origin mentioned earlier: for the one-way case, on average one atom is added to the system for each atom detected and so all the atoms are replaced once in time $\gamma t = 1$. In the two-way case, $n$ atoms are exchanged with the baths for each atom detected, and so in $\gamma t = 1$, all the atoms are replaced $n$ times over and we expect a correspondingly shorter time scale for the fluctuations. As a comparison, in Fig. 2c we show a trajectory for a “regular” pumping model in which atoms are dripped into the trap at a constant rate to replace those lost by detection. In this case, the collisional rate is $\kappa = 0.5\gamma$, but the visibility shows a much improved response than in the (collisionless) thermal pumping cases, indicating the severe influence of the thermal pumping.

In fact, the visibility is degraded by the number fluctuations in two distinct ways. When the occupancy of one of the traps falls due to a fluctuation to within a few times $\sigma_n$ of zero, the extreme terms in the entangled state are removed, the number distribution narrows and the visibility falls. In particular, if one of the traps is completely emptied (as occurs several times in Figs. 2a and b,) the state is then a pure number difference state and any relative phase is completely destroyed. The visibility can of course be restored once fluctuations increase the atom number again, but there is no relation between the new relative phase and the phase before the trap was emptied.

Even when both traps have $\langle n \rangle \gg \sigma_n$, the visibility is reduced according to the number difference between the traps. This is obvious—if one trap has significantly more atoms than
the other, then we can predict with better than 50 % accuracy from which trap the next atom will be detected, and the visibility must fall accordingly. We can calculate this effect simply as follows. In our system, the atom numbers experience thermal fluctuations in time due to the pumping. Suppose for a moment we have a different situation in which there is no pumping, and we perform a series of detection runs with a thermal distribution in the initial trap numbers and measure the visibility after a well-defined relative phase has been set up (but before the traps are significantly depleted). If we picture the condensates as coherent states with some relative phase:

\[ |\rangle_1 = \sqrt{n_1} \exp(i\phi_1), \quad |\rangle_2 = \sqrt{n_2} \exp(i\phi_2), \]

the expected visibility is just \( \beta = 2\sqrt{n_1n_2} / (n_1 + n_2) \) which is a familiar expression for optical fields. Defining the relative occupancy

\[ f = \frac{n_1 - n_2}{n_1 + n_2}, \]

we have

\[ \beta(f) = \sqrt{1 - f^2}. \]

More correctly, we should derive Eq. (17) directly from the entangled state description of the twin-trap system. In general, the fringe visibility is given by

\[ \beta = |g^{(1)}| \sqrt{1 - f^2}, \]

where \( |g^{(1)}| \) is the normalized correlation function [21]. While in general, \( |g^{(1)}| \) is not easily evaluated for the entangled states with which we are concerned, it can be shown for example that for the projected two-mode coherent state [22] \( |\alpha, \beta\rangle = \sum_{k=0}^{N} \alpha^k \beta^{N-k} / \sqrt{k!(N-k)!} |k, N-k\rangle \), which is the most natural expression of a state with relative phase with fixed total atom number \( N \), \( |g^{(1)}| \) tends to unity in the limit of large \( N \).

Figures 3a and b test Eq. (17) in the form of scatter plots of the points \((f(t), \beta(t))\) for two-way and one-way pumping respectively with the same parameters as Figs 3a and
b. The prediction (17) is indicated by the black squares in each plot. The correlation is clearly much stronger in the one-way case. This difference is entirely due to the difference in time scales discussed above. If we are to have a well-defined phase, the number must be partially uncertain. Indeed, we see later that in the presence of collisions the average state has moderate number-squeezing but with a variance of the same order as a coherent state. So for a reasonable visibility we should require an entangled state of order \( \sigma = O(2\sqrt{n}) \) terms. Such an entangled state is set up by the same number of detections and requires a time of order \( \tau_e = 1/\sqrt{n\gamma} \). Now for two-way pumping, the time scale for replacement of all the atoms once over is \( \tau_r = 1/n\gamma \ll \tau_e \). Hence, the exchange of atoms with the baths occurs faster than an entangled state of a particular phase can be constructed and we may expect a reduced visibility. There can be only a weak correlation between the instantaneous visibility and the instantaneous relative occupancy \( f \), and the visibility is generally lower than the optimum given by Eq. (17). With one-way pumping however, the time scale for replacement of all the atoms is larger by a factor \( n \). The visibility is able to keep up with the drift in number and is then limited only by Eq. (17).

We can also calculate the mean visibility over time \( \bar{\beta} \), for an arbitrary pair of mean atom numbers \( \langle n_1 \rangle \) and \( \langle n_2 \rangle \). Again, we think of an ensemble of runs with no pumping and a thermal distribution of initial states \( |n_1 \rangle \) and \( |n_2 \rangle \). The mean visibility over many runs is the weighted average of \( \beta(f) \) over the probability distribution [see Eq. (17)]

\[
P_f(f) = \int \delta \left( f - \frac{n_1 - n_2}{n_1 + n_2} \right) P_{\tilde{n}_1}(n_1) P_{\tilde{n}_2}(n_2) \, dn_1dn_2,
\]

where \( P_{\tilde{n}_i}(n_i) = -\log(\gamma_i) \gamma_i^{n_i} \) are the probability distributions (in the continuous limit) of atom number for thermal distributions with mean number \( \tilde{n}_i \) and \( \gamma_i = \tilde{n}_i / (\tilde{n}_i + 1) \). For the case where the mean numbers are the same, \( P(f) \) is uniform and \( \bar{\beta} = \pi/4 \) [23]. Otherwise we find

\[
\bar{\beta} = \frac{2\pi \log(\gamma_1) \log(\gamma_2)}{[\log(\gamma_1/\gamma_2)]^2} \left( \frac{1}{\sqrt{1 - \left( \frac{\log(\gamma_1/\gamma_2)}{\log(\gamma_1\gamma_2)} \right)^2}} - 1 \right).
\]
which for \( \bar{n}_1, \bar{n}_2 \gg 1 \), gives

\[
\tilde{\beta} \simeq \frac{\pi \sqrt{p}}{(1 + \sqrt{p})^2},
\]

\[
(21)
\]

with \( p = \bar{n}_1/\bar{n}_2 \). In the pumped twin-trap setting, we also have thermal distributions in the atom number which occur not from run to run, but over time in a single trajectory, so it is reasonable to hope that the above analysis may still apply.

In Fig. 4, we show the average visibility for a thermal distribution as a function of the mean atom number ratio \( p \) given by Eq. (21). The plotted points show the time-averaged visibility calculated from trajectory simulations with one-way pumping and \( \bar{n}_1 + \bar{n}_2 = 200 \). Error bars are shown at 1 standard deviation. As expected, the mean visibility falls with increasing disparity in the mean atom number.

A. One-way pumping and output couplers

We have seen that the one-way pumping process shows significantly higher visibility than the two-way pumping. From this point on, we restrict our attention to the one-way model and add the effect of an output coupler from each trap. In our results we find two distinct regimes according to the length of the simulations. Figure 5 shows the visibility as a function of time averaged over 200 trajectories for a simulation with \( \kappa = 0, \nu_i = 0 \) and an initial state \( |n_1, n_2\rangle = |100, 100\rangle \). The one-way pumping rate was chosen to maintain the mean population at \( n = 100 \) in each trap. There are clearly two regimes: for \( \gamma t \ll n \), the mean visibility shows a steady decline, while for \( \gamma t \gg n \), the visibility tends to a steady-state value of \( \pi/4 \) consistent with the calculation of the previous section. In the initial stage of a particular run, the populations of the traps become decorrelated due to the thermal nature of the pumping until they are completely uncorrelated and the time-averaged visibility is \( \pi/4 \). The time for this decorrelation varies from run to run, having a characteristic length of \( \gamma t \approx n \). Thus the average over many trajectories shows a gradual decline until all members of the ensemble are likely to be decorrelated. We are thus led to examine the behavior of the...
system in the two regimes $\gamma t \ll n$, when the trap populations are likely to be quite close, and $\gamma t \gg n$, when there is no correlation between the populations. We treat these two cases in turn. In all cases, we start our simulations with the initial state $|n_1, n_2\rangle = |100, 100\rangle$ and calculate quantities averaged over 200 trajectories.

- $\gamma t \ll n$: For the short-time regime, we arbitrarily choose $\gamma t = 4$ to show results. Figure 6a shows the visibility as a function of $\kappa$ again averaged over five trajectories. As expected, the visibility decreases with increasing collisions which increasingly disrupt the relative phase \cite{footnote}. We have also performed simulations with a range of output couplings from $\nu_{i} = 0$ to $\nu_{i} = \gamma$. This leads to a small decrease in $\beta$ (of less than 0.025 for the strongest coupling). This effect is simply a result of the fact that the pumping rate is increased to balance the additional loss of atoms and so the trap populations decorrelate faster. The nature of the average state of the system is indicated in Fig. 6b. Shown are the widths of the number distribution $\sigma_n$ (dotted line) and phase distribution $\sigma_\phi$ (dot-dashed) and $\rho$: the root mean square of the product of the two (solid). The filled circles denote the actual simulations performed. The number distribution clearly narrows strongly with increasing collisional rate while the phase distribution spreads as collisions degrade the relative phase. The simulations with non-zero output coupling (not shown in Fig. 6b) produced a reduction of less than 5% in the number width and no discernible change in the other parameters. Note that for zero collisions, the product of the widths (solid) is unity indicating a minimum uncertainty state. Further, for all values of $\kappa$, the number width $\sigma_n < \sqrt{n} = 10$, which is the width we would expect if the state was a projection of a coherent state onto a basis of fixed total atom number. The real state is thus quite strongly number squeezed. This is consistent with recent analytic work by Dunningham et al\cite{Dunningham2015,Dunningham2016} using a Bose-broken symmetry model. They show that in the limit of a large collisional rate, the true state of the condensate is the amplitude-squeezed state that minimizes number fluctuations.

- $\gamma t \gg n$: In the large time regime, the mean visibility has no dependence on the output
coupling rate—once the atom numbers are completely uncorrelated, the precise rate
at which atoms enter the trap is irrelevant. The number and phase widths shown
in Figure 7 show very similar trends to the short time case. Note that even with the
uncorrelated trap numbers, the state is still minimum uncertainty for $\kappa = 0$, indicating
that the pumping rate is low enough for the visibility to adjust to changes in number.
Again, other simulations showed that the only effect of output coupling was to reduce
the number width by a few percent.

V. APPLICATION TO COLLAPSES AND REVIVALS

In this final section, we consider the application of pumping processes to the interesting
phenomenon of collapses and revivals in the relative phase. Several authors have shown that
if a relative phase is prepared by detection and the entangled state subsequently evolves
purely under the influence of the interatomic collisions, the visibility of the phase experiences
recurrent collapses and revivals of period $\pi/\kappa$ due to the differential rate of phase rotation
in the entangled state $[20, 21, 22]$. A demonstration of collapses and revivals of the phase,
perhaps through light scattering experiments, would be a significant result in BEC physics.
It is interesting to consider how the collapses and revivals are affected by pumping and
leaking of atoms through the traps. Naively, we might expect that the oscillations would be
destroyed by the time all the atoms had been replaced a few times over. In fact, we have
found the collapses and revivals to be remarkably robust to pumping processes.

In Fig. 8a, we show the visibility for a single trajectory without pumping or output
coupling in which there are initially 200 detections from a total of 1000 atoms, followed by
a period in which the system evolves only under the influence of collisions with $\kappa = 0.25$.
The oscillations in the visibility are clear. Fig. 8b shows a trajectory with the same number
of detections and collision strength, but with a continual flushing of the trapped atoms by
pumping and output coupling. On average, all the atoms are replaced in a period $\gamma t = 1$ and
the atom numbers exhibit large fluctuations (Fig. 8c). Despite this, the collapse and revivals
persist for a considerable period and only disappear when the atom number in trap 2 (thick line in Fig. 8c) approaches zero at $\gamma t \approx 250$. If the trajectory is such that neither atom number approaches zero, the oscillations may continue much longer still. As the simulation progresses however, while the period of the revivals is unchanged, the peaks broaden—the collapse time increases. This is associated with the gradual reduction in the width of the number difference of the initial entangled state due to the repeated addition and removal of atoms indicated in Fig. 8d. Essentially, every addition or removal of an atom through an output coupler tends to drive the state to a narrower number distribution. In their treatment of collapses and revivals for a fixed number of atoms, Wong et al. [8] have shown that the visibility during a collapse should decay according to

$$V \propto \exp \left( -2\sigma^2 n \kappa^2 t^2 \right),$$  \hspace{1cm} (22)$$

where $\sigma$ is the width of a Gaussian approximation to the coefficients

$$\mathcal{A}(k) = |c_k c_{k-1}| \sqrt{(n-k+1)(n-m+k)},$$ \hspace{1cm} (23)$$

and there have been $m$ detections from an initial state of $n$ atoms in each trap. For a broad distribution, and $m \ll n$, to lowest order we have $\mathcal{A}(k) \propto |c_k^2|$, so that in our notation $\sigma \approx \sigma_n/2$. The black squares in Fig. 8d are estimates of $\sigma_n$ calculated from the collapse widths in Fig. 8b using Eq. (22). The agreement with the directly measured values for the width of the number distribution (solid line) confirms that the increase in the collapse time is due purely to the change in $\sigma_n$.

Figure 8b also shows a variation in the height of the visibility peaks. Note that the variation is not monotonic, an effect we have found to be generally true. A natural guess is that the peak heights are associated with the relative occupancy $f = (n_1 - n_2) / (n_1 + n_2)$, which we found led to a maximum visibility for systems where the detections are not stopped in section IV. We have tested this using scatter graphs of the peak visibility similar to those in Fig. 3. We find a moderate confirmation of the connection. In cases for which the minima of the visibility remain small, there is a strong correlation between the peak visibility and

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the quantity $f$. In other cases, such as that in Fig. 8b for $\gamma t > 150$, for which the minima are significantly greater than zero, the correlation is poor and we conclude that the pumping process has produced an additional degradation of the state beyond that implied simply by the mean number difference.

VI. CONCLUSION

In this paper, we have studied the steady-state behavior for two pumping scenarios to show how an ongoing measurement process can generate a phase coherence between atoms derived from thermal baths, even in the presence of phase diffusion due to atomic collisions within the traps. We find important qualitative differences between systems with two-way and one-way pumping, the phase coherence being substantially improved for the one-way case. Systems displaying collapses and revivals of the condensate phase should provide an opportunity for examining the time-dependent effects induced by pumping. We remark finally that a natural extension to our model would be the inclusion of extra trap levels that would allow for line narrowing and genuine laser action.

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FIG. 1. Geometry of pumped twin-trap system. The straight solid arrows indicate the detection at rate \( \gamma \); the dotted arrows, the exchange of atoms with the reservoirs; the curved arrows, output coupling of the trapped atoms.

FIG. 2. a)-b) Visibility \( \beta \) as a function of time for a thermally pumped system with mean occupancy \( n = 100 \) for each trap and \( \kappa = 0 \). a) Two-way pumping, b) one-way pumping. c) Visibility for a regularly pumped model with \( \kappa = 1.0 \).

FIG. 3. Scatter plot of \( \beta \) as a function of relative occupancy \( f \) for a) two-way, and b) one-way pumping. There are 10000 points shown. Black squares indicate the relation Eq. (17).

FIG. 4. Mean visibility \( \bar{\beta} \) as a function of the atom number ratio \( p \). Error bars indicate 1 standard deviation errors in time-averaged simulations.

FIG. 5. Visibility for one-way thermal pumping with no collisions and no output coupling. The mean atom number in each trap is 100.

FIG. 6. Averaged state parameters as a function of collision rate for one-way pumping in short-time regime. a) Visibility and b) \( \sigma_n \) (dashed), \( \sigma_\phi \) (dot-dash), and \( \rho \) (solid).

FIG. 7. Averaged state parameters as a function of collision rate for one-way pumping in long-time regime: \( \sigma_n \) (dashed), \( \sigma_\phi \) (dot-dash), and \( \rho \) (solid).
FIG. 8. a) Visibility for collapses and revivals of relative phase with no pumping or output. Initially 200 detections were made from 1000 atoms. b) Visibility, c) atom numbers and d) $\sigma_n$ for the same parameters with pumping and output rates such that all atoms are replaced on average once in a period $\gamma t = 1$. 


\[ \beta = \frac{\text{constant}}{G50} \]
