Computational Simulation of Semiconductor Laser with Optical Injection

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Abstract. In this paper we study the dynamics of a semiconductor laser with optical injection. The time behaviour of solutions of a system of three coupled nonlinear rate equations, describing the electric field amplitude and the carrier concentration and the phase difference within the resonator, is discussed both qualitatively and numerically. We then concentrate on the periodic orbits that emanate from Hopf bifurcations. Depending on the injection strength and the phase difference two types of oscillations can be found, such as relaxation and periodic oscillations.

1. Introduction
The nonlinear processes play important roles in the development of new communication systems, computing elements. Understanding of laser instabilities is necessary for developing control techniques. From a dynamical systems point of view semiconductor laser systems are very attractive because they show an intriguing variety of complicated dynamics. Lasers including semiconductor, solid-state with the additional external perturbation produce temporal and spatial instability. An optical injection is an example of such perturbation. External perturbations may destabilize or induce the sustain intensity oscillation.

The first unstable fluctuations instead of continuous wave emission were observed in a continuously pumped maser in 1958[1]. The connection between laser and instabilities was published in 1975 by Haken [2]. It was shown that a set of nonlinear differential equations from Maxwell- Bloch equations for a laser model resembles the Lorentz equations that are the basic model for deterministic chaos. Instabilities in lasers have become broad subject in quantum optics. In the present paper we consider the laser rate equations with an optical injection. One of the first studies on the effects of the laser with optical injection was carried out in 1992 [3]. Since then, several papers have discussed the instabilities associated with optical injection [4-8]. The recent books chapters [9-11] give an excellent overview of the dynamical instabilities in lasers.

The objective of this work is to investigate the effects of main parameters, such as detuning and the injection level, on dynamical behavior of laser system using numerical and nonlinear dynamics methods.

This paper is organized as follows. In Section 2 and 3 we consider the model of semiconductor laser with optical injection based on the rate equations and its linear stability analysis. Section 4 discusses...
the numerical results and compares them with obtained analytical results. The time series and phase portraits are plotted for the solutions of the given system. Section 5 concludes the paper.

2. Model

The emission in semiconductor lasers results from electron – hole recombination between energy bands. The dynamics of the semiconductor laser model with optical injection can be studied by three-dimensional set of ordinary differential equations. These equations describe the electric field and the number of electron-hole pairs in the laser active medium. The semiconductor laser is injected with light from an external laser source (master laser). The laser that is influenced by the master laser is called the ‘slave’ laser.

The rate equations that describe the dynamics of the slave semiconductor laser in response to the injected signal from a master laser can be written as [9]:

\[
\begin{align*}
\frac{dR}{dt} &= ZR + \eta \cos \psi \\
\frac{d\psi}{dt} &= \Omega - \alpha Z - \eta \sin \psi / R \\
T \frac{dZ}{dt} &= P - Z - (1 + 2Z)R^2
\end{align*}
\]

, where \(R\) - the electric field amplitude, \(Z\)- carrier concentration over threshold, \(\alpha\) - the linewidth enhancement factor, \(\Omega\) - detuning frequency (the frequency difference between the master and the slave lasers), \(T\) - is the ratio of the photon lifetime to the carrier lifetime, \(P\) - pumping current, \(\eta\) - the injection strength, \(\psi\) - the phase difference.

Thus, equations (1), (2) and (3) simulate the evolution of the amplitude of the electric field, phase difference, carrier concentration respectively.

Solutions of nonlinear differential equations depend on their parameters. There may be critical parameter values, at which the character of the solution changes completely. For this reason, it is extremely important to know the qualitative behavior of the solutions before starting a numerical calculation.

Let us determine the stability of the steady states solution of the system.

The steady state solutions for \(R_0\), \(Z_0\) and \(\psi_0\) have the following implicit form:

\[
\begin{align*}
R_0^2 &= (P - Z_0) / (1 + 2Z_0) \\
Z_0^3(1 + \alpha^2) + Z_0^2(-P - \alpha^2 P - 2\alpha \Omega) + Z_0(2\alpha^2 + 2\alpha \Omega P + \Omega^2) + \eta^2 - P_0^2 &= 0 \\
\eta \sqrt{1 + \alpha^2 \sin(\psi_0 - \arctan(\alpha))} &= \Omega R_0
\end{align*}
\]

Steady state solution of the cubic equation (5) \(Z_0\) can be calculated, and then we can obtain the corresponding solutions for \(R_0\) and \(\psi_0\).

To analyze equilibrium points, let’s linearize the system (1), (2), (3) near equilibrium points.

The Jacobian matrix of the system (1), (2), (3) near equilibrium point is given by

\[
J = \begin{bmatrix}
Z_0 & R_0(\alpha Z_0 - \Omega) & R_0 \\
\Omega - \alpha Z_0 & Z_0 & -\alpha \\
-2\gamma(1 + 2Z_0)R_0 & 0 & -\gamma(1 + 2R_0^2)
\end{bmatrix}
\]

Here \(\gamma = 1/T\). The characteristic polynomial of matrix (7) has the following form:
\[ p(\lambda) = |I - \lambda I| = \begin{vmatrix} Z_0 - \lambda & R_1(\alpha Z_0 - \Omega) & R_2 \\ \Omega - \alpha Z_0 & Z_0 - \lambda & -\alpha \\ -2(1 + 2Z_0)R_0 & 0 & -\gamma(1 + 2R_0) - \lambda \end{vmatrix} \] (8)

From (8):
\[ -p(\lambda) = \lambda^3 + C_1\lambda^2 + C_2\lambda + C_3, \]
where the coefficients \( C_1, C_2, C_3 \) are equal to the following expressions:
\[
C_1 = \gamma \frac{1 + 2P}{1 + 2Z_0},
C_2 = Z_0^2(\Omega - \alpha Z_0)^2 + 2\gamma(P - Z_0) - 2\gamma Z_0 \frac{1 + 2P}{1 + 2Z_0},
C_3 = \gamma Z_0^2 \frac{1 + 2P}{1 + 2Z_0} + 2\gamma(\alpha - \alpha Z_0)(\Omega - \alpha Z_0) - 2\gamma Z_0(P - Z_0) + \gamma \frac{1 + 2P}{1 + 2Z_0}(\Omega - \alpha Z_0)^2.
\]

The eigenvalues of the polynomial (9) can be calculated from the following characteristic equation to identify the type and stability of the equilibrium points:
\[ -p(\lambda) = \lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0 \] (10)

According to the theorem of Hurwitz Eq.(5) has only roots with negative real part, which implies asymptotic stability of \( Z_0 \), if and only if \( C_1 > 0, C_2 > 0, C_1C_2 - C_3 = 0 \). The condition \( C_1C_2 - C_3 = 0 \) implies that a complex pair of eigenvalues is crossing over the imaginary axis, so that the Hopf bifurcation is occurred [9].

3. Discussion and results
Let's now examine how the change in the main control parameter, such as injection strength and detuning frequency affects the system dynamics behaviour. Using the approximate numerical methods, in particular the Runge-Kutta fourth order method, we present the results of numerical solutions of the rate equations (1), (2), (3) and the corresponding phase portraits.

Consider the values of the system parameters \( \Omega = -0.1, \eta = 0.01 \). In this case, the cubic equation (5) has one real root or one equilibrium point of the original differential equations. The solution is unstable, since the real parts of the eigenvalues of a complex-conjugate pair of the characteristic polynomial (10) are positive. The phase space trajectory and time responses confirm these analytical results (see Fig. 1) as well.

Raise the value of the injection \( \eta = 0.03 \). In this case the cubic equation has three real roots. According to the eigenvalues of a polynomial, two equilibrium points are stable whereas the third point is unstable. The saddle-node bifurcation is occurred. The system performs the relaxation oscillations.

Consider the value of the injection force \( \eta = 0.04 \). For a given value of the parameter the pair of complex eigenvalues crosses the imaginary axis. There is a qualitative change in the phase portrait. The system goes to the periodical mode. The curve is a limit cycle in the phase space of \( R - \psi - Z \). The Hopf bifurcation is occurred.

Let us raise the value of the injection to the value \( \eta = 0.9 \). The detuning parameter is left unchanged. The saddle node bifurcation is occurred for given values of the parameters. The stability of the equilibrium point turns back.

Consider the positive values of the detuning \( \Omega \), in particular \( \Omega = 0.1 \) and \( \eta = 0.01 \). In this case, the system has one unstable solution. The real parts of the eigenvalues of a complex-conjugate pair of the characteristic polynomial are positive. The time series and phase portraits for positive detuning value are plotted in Fig. 2.

Let's see what happens if we raise the value of \( \eta = 0.04 \). According to their eigenvalues all three solutions are unstable again. The reason is that in this case, the leading coefficient of characteristic
The equation for positive values of detuning is always negative. \( \Delta_C = -1.9984 < 0 \). According to the criterion of Routh–Hurwitz, this condition is not satisfied with the condition of stability of equilibrium point. Thus, the transitions between relaxation and periodic oscillations (see Fig. 1) can occur only for negative detuning values.

4. Conclusions

We have studied the model of laser system with optical injection using numerical analysis. The results from numerical calculations show different types of laser dynamical behaviors. The transitions from relaxation oscillations to a state of periodic oscillations are observed. The behavior of system is sensitive to variations in the main parameters. A slight raise in the parameter \( \eta \) leads to a qualitative change in the behavior system dynamics. A fourth order Runge-Kutta method is used to plot the time integration and phase portraits of the system. It was observed that for certain parameters, raising the value of \( \eta \) can bring stability back. As a result of the simulation was also point out that the saddle node and Hopf bifurcations are occurred only for negative detuning values.

**Figure 1.** The time series and phase spaces of laser rate equations \((\alpha = 4, P=1)\) for negative detuning.

**Figure 2.** The time series and phase spaces of laser rate equations \((\alpha = 4, P=1)\) for positive detuning.
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