Impurity effects on \(s+g\)-wave superconductivity in borocarbides \(Y(Lu)Ni_2B_2C\)

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Abstract: Recently a hybrid \(s+g\)-wave pairing is proposed to describe the experimental observation for a nodal structure of the superconducting gap in borocarbide \(YNi_2B_2C\) and possibly \(LuNi_2B_2C\). In this paper the impurity effects on the \(s+g\)-wave superconductivity are studied in both Born and unitarity limit. The quasiparticle density of states and thermodynamics are calculated. It is found that the nodal excitations in the clean system are immediately prohibited by impurity scattering and a finite energy gap increases quickly with the impurity scattering rate. This leads to an activated behavior in the temperature dependence of the specific heat. Qualitative agreement with the experimental results is shown. Comparison with \(d\)-wave and some anisotropic \(s\)-wave studied previously is also made.

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I. INTRODUCTION

The superconductivity in rare earth nickel borocarbides \(RNi_2B_2C\) (\(R=Y, Lu, Tm, Er, Ho, \) and \(Dy\)) is of great interest in recent years.1,2 Among them, the two nonmagnetic borocarbides \(YNi_2B_2C\) and \(LuNi_2B_2C\) have relatively high superconducting (SC) transition temperatures, which are 15.4K and 16.5K, respectively. Although they were initially understood by an isotropic \(s\)-wave pairing,3 recent various experimental results, including specific heat,4–7 thermal conductivity,8,9 Raman scattering,10,11 NMR relaxation rate,12 photoemission spectroscopy,13 scanning tunneling microscopy and spectroscopy,14 have shown that they may be another class of nodal superconductors. For example, both the \(\sqrt{H}\) dependence of the specific heat and \(H\) linear dependence of the thermal conductivity in the vortex state indicate the existence of nodal excitations.4–6 In particular, compelling evidences are presented by Izawa et al.8 based on the angular dependent thermal conductivity measurements in a magnetic field that the gap function of \(YNi_2B_2C\) is highly anisotropic with anisotropy ratio less than \(10^{-2}\), i.e., it has essentially nodes and moreover, the nodes are point-like which are located along \([1,0,0]\) and \([0,1,0]\) directions. Due to Doppler shift in the presence of supercurrent flow,15 the zero energy quasi-particle density of states (DOS) becomes finite and is sensitive to the relative orientation between the magnetic field and nodal direction. Thus the nodes can be unambiguously extracted from the angular modulation of the thermal conductivity. Based on the same physical mechanism the angular dependent specific heat in \(YNi_2B_2C\) was measured by Park et al.7 and the conclusion for the existence of the point nodes can be also drawn. In addition, the thermal conductivity in \(LuNi_2B_2C\) was measured by Boaknin et al. as a function of temperature and field strength.9 They claimed that the gap minimum is at least 10 times smaller than the gap maximum and possibly goes to zero at nodes. Thus \(YNi_2B_2C\) (and presumably \(LuNi_2B_2C\)) becomes the first superconductor in which the gap function with point nodes is clearly identified.

Theoretically, Maki et al. have proposed that the so called \(s+g\)-wave spin singlet gap function, i.e.,16,17\(\Delta_{k} = \Delta(1 - \sin^4 \theta \cos 4 \phi)/2 = \Delta_{f}(k)\)\)

can describe the experimental observation in \(Y(Lu)Ni_2B_2C\). Here \(\theta, \phi\) are the polar and azimuthal angles of \(k\), respectively. The second ‘\(g\)-wave’ component is given by a fourth degree \(A_{1g}\) basis function: \(k_x^4 + k_y^4 - 6k_x^2k_y^2\) in tetragonal crystal symmetry. In the gap function the amplitudes of \(s\)- and \(g\)-components are assumed to be equal. Thus 4 (and only 4) point nodes at \(\theta = \pi/2\) and \(\phi = 0, \pi/2, \pi, 3\pi/2\) are realized, which is exactly what has been observed experimentally.8

So far, a microscopic theory for the \(s+g\)-wave pairing is not available which might be complicated due to the complex Fermi surface of borocarbides18 and the possibility of strongly anisotropic Coulomb interactions. A phenomenological theory has been constructed by two of us,19 which shows that a stable coexistence of \(s\)- and \(g\)-wave with the fine tuning, i.e., the equal amplitudes of them are possible. On the other hand, the correctness of the \(s+g\) model could be checked by comparing the theoretical predictions with the available experimental data. Up to now, quite a few physical quantities based on the \(s+g\) model such as thermal conductivity,5 Raman spectra,20 sound attenuation21 etc. have been calculated and good agreement with the experimental results is obtained. This in turn stimulates us to continue studying more properties.

In this work the impurity effects on the \(s+g\) model will be investigated. Experimentally there have been extensive studies on Pt-substituted borocarbide \(YNi_{1-x}Pt_xNi_2B_2C\). The ultrahigh-resolution photoemission spectroscopy was performed by Yokoya et al.13 They found a small but significant difference for the spectra in the SC states between \(x = 0\) and \(x = 0.2\) compound,
which is contributed to the change in anisotropy of the SC gap: a strong anisotropic gap in \( x = 0 \) and an almost isotropic one in \( x = 0.2 \). That is, the gap anisotropy is wiped out by introducing impurities. Consistent result can be obtained from the electronic specific heat \( C \) measurements by Nohara et al.\textsuperscript{5} The temperature-dependence of \( C \) (under zero field) shows a power law behavior for \( x = 0 \) but a thermally activated behavior for \( x = 0.2 \). Very recently, the angular dependent thermal conductivity for \( Y(Ni_{1-x}Pt_x)2B2C \) with \( x = 0.05 \) in the SC state was measured.\textsuperscript{22} There is no angular modulation left. This shows that the gap anisotropy is nearly completely suppressed by only 5\% Pt substitution. It is the purpose of this work to see how soon a full energy gap will be opened by the impurities for the s+g-model through the calculation of the DOS. Furthermore, the thermodynamics like specific heat will be calculated. The crossover from a power law behavior in a pure system to an activated behavior in the presence of impurities will be exhibited.

II. FORMALISM

The effects of nonmagnetic impurities are treated by the self-consistent T-matrix approximation,\textsuperscript{23} For benefit of readers, we briefly summarize the formulation in the following. To begin with, the Matsubara Green’s function in Nambu representation is written as (quantities with a hat represent matrices)

\[
\hat{g}(k, i\omega_n) = (i\omega_n \sigma_0 - \xi_k \sigma_3 - \Delta_k \sigma_1 - \Sigma)^{-1},
\]

where \( \omega_n = (2n+1)\pi T \) (\( T \): temperature) is the Matsubara frequency, \( \xi_k \) the electron energy measured from the chemical potential, \( \Delta_k \) the gap function which has been assumed to be real. \( \sigma_0 \) and \( \sigma_{1,3} \) are unit and Pauli matrices, respectively. The self-energy \( \Sigma(i\omega_n) = \Gamma_u T(i\omega_n) \) with \( \Gamma_u = N_f/(\pi N_0) \). Here \( N_f \) is the total number of impurities and \( N_0 \) is the zero energy DOS per spin for the spectrum \( \xi_k \). The T-matrix \( T \) satisfies \( \hat{T}(i\omega_n) = \hat{K} + \hat{G}(i\omega_n)\hat{T}(i\omega_n) \), where \( \hat{G}(i\omega_n) = (1/\pi N_0) \sum_k \hat{g}(k, i\omega_n) \). \( \hat{K} \) is the T-matrix in the normal state: \( \hat{K} = -\sigma_3/c \) with \( c = \cot \delta_0 \). And \( \delta_0 \) is the s-wave phase shift controlled by the strength of a single impurity potential.\textsuperscript{24}

The matrix \( \hat{A} (A = \Sigma, T, G) \) is decomposed into \( \hat{A} = A_0 \sigma_0 + A_1 \sigma_1 + A_2 \sigma_3 \). Then one obtains \( G_3 = 0 \) if assuming particle-hole symmetry.\textsuperscript{23} Further \( T_3 \) and correspondingly \( \Sigma_3 \) vanish if we only concern Born \((c \gg 1)\) and unitarity \((c = 0)\) limit. The left \( \Sigma_0,1 \) are obtained as follows

\[
\Sigma_0 = \Gamma_u \frac{G_0}{c^2 - G_0^2 + G_1^2}, \quad \Sigma_1 = \Gamma_u \frac{-G_1}{c^2 - G_0^2 + G_1^2},
\]

where

\[
G_0 = \left\langle \sqrt{\frac{-i\omega_n}{\omega_n^2 + \Delta_k^2}} \right\rangle, \quad G_1 = \left\langle \sqrt{\frac{-\Delta_k}{\omega_n^2 + \Delta_k^2}} \right\rangle.
\]

Above \( \omega_n = \omega_n + i\Sigma_0, \Delta_k = \Delta_k + \Sigma_1 \), and \( \langle \cdots \rangle \) means angular average \( \int d\Omega/(4\pi) \). It deserves to point out that the off-diagonal self-energy \( \Sigma_1 \) is zero for d-wave but finite for the current s+g-wave. This difference will lead to different behavior of their respective DOS functions.

The gap function is connected to the matrix element \( g_{12} \) by \( \Delta_k = T \sum \sum_k V_{kk'} g_{12}(k', i\omega_n) \) with \( V_{kk'} \): the pair potential. It is often assumed \( V_{kk'} = -V f(k) f(k') \), which is valid for \( |\omega_n| < \omega_D \) (a cut-off). Then the following equation for the gap amplitude \( \Delta \) is obtained:

\[
\Delta = 2\pi T V \sum_{0<i\omega_n<\omega_D} \left( \frac{f(k)(\Delta_k + \Sigma_1)}{\sqrt{(\omega_n + i\Sigma_0)^2 + (\Delta_k + \Sigma_1)^2}} \right),
\]

where \( V = V N_0 \) is a dimensionless interaction constant.

By solving Eqs. (3)-(5) self-consistently, we can obtain the renormalized \( \Delta \) with impurity concentrations at all temperatures and further calculate the observable quantities like specific heat. In the following we adopt the impurity scattering rate parameter in the normal state \( \Gamma = \Gamma_u/(1 + c^2) \), which reduces to \( \Gamma_u/c^2 \) in Born limit \((c \gg 1)\) and \( \Gamma_u \) in unitarity limit \((c = 0)\).

III. DENSITY OF STATES

We first consider the DOS, which is given by (per spin)

\[
N(E)/N_0 = -\text{Im} G_0(i\omega_n)|_{i\omega_n=E+i0^+}.
\]

It is noticed that if one scales the input parameters \( \Gamma \) and \( E \) by \( \Delta \), the DOS can be obtained by only solving Eqs. (3) and (4) without explicit calculation of \( \Delta \) for the moment. The results for a few ratios \( \Gamma/\Delta \) are shown by Fig. 1 in both Born and unitarity limit. (Note that the scaled \( \Gamma/\Delta \) is still a monotonous function of \( \Gamma \).) From the figure a few features are clearly seen: i. There is no essential difference between the results in both limits, as seen by an example in Fig. 1b. Numerically we have \(-G_0^2 + G_1^2 \simeq 1\) in unitarity limit. Thus Eq. (3) can be rewritten into a unified form: \( \Sigma_0 = \Gamma G_0, \Sigma_1 = -\Gamma G_1 \) in both limits, leading to nearly the same DOS. ii. An energy gap opens up immediately when \( \Gamma \neq 0 \) and grows quickly with \( \Gamma \). As seen from the inset of Fig. 1a, a relatively big energy gap \( \omega_g \sim \Gamma \) is already present for small \( \Gamma/\Delta = 0.02 \). Actually \( \omega_g \) is well approximated by \( \omega_g = \Gamma/(1 + 2\Gamma/\Delta) \).

For intermediate \( \Gamma/\Delta = 1 \), the gap anisotropy is nearly suppressed and the DOS approaches the familiar shape of an isotropic s-wave gap. All the above results are quite different from those for d-wave.\textsuperscript{25–31} For example, a finite DOS is induced by impurities around zero energy for d-wave. Also, a resonance behavior at low energy exists in unitarity limit for d-wave, which does not appear in the s+g-state. We point out that the main result seen from the DOS based on the s+g model, i.e., the gap anisotropy is suppressed by impurity scattering, is consistent with the conclusion drawn from the photoemission data.\textsuperscript{13}
ally point or line nodes. 

\begin{align*}
\text{(a) Born} & \quad \Gamma/\Delta=0.00 \quad \Gamma/\Delta=0.02 \quad \Gamma/\Delta=0.20 \quad \Gamma/\Delta=1.00 \\
\text{(b) } \Gamma/\Delta=0.5 \\
\end{align*}

FIG. 1. The quasi-particle DOS for s+g-wave. (a) The results for various \( \Gamma \) in Born limit. The low energy region is enlarged in the inset. (b) Comparison between unitarity and Born limit for a fixed \( \Gamma/\Delta = 0.5 \). Only subtle differences exist in both limits.

At this stage, we would supplement a remark. Previously, in order to clarify the role of sign change in a gap function, some hypothetical anisotropic s-wave gap functions e.g. |cos2\( \phi \)|,\(^{26} \) which vanishes at the same nodes as d-wave but never change sign, have been studied in the presence of impurities. The obtained DOS for \( \Gamma \neq 0 \) has qualitatively similar properties to that shown here. This is understandable in the sense that the current s+g-wave (without sign change) is essentially an anisotropic s-wave. On the other hand, however, we would point out an important difference between the current s+g-wave gap and the one \( \sim |\cos2\phi| \): The former contributes only a few point nodes, while the latter gives line nodes, on a three-dimensional Fermi surface. This will lead to notable differences on some observable quantities, e.g., the angular dependent thermal conductivity, which has actually been used to identify whether the SC gap in YNi\(_2\)B\(_2\)C has really point or line nodes.\(^8 \) As far as the DOS is concerned, a quantitative difference is that the opened gap \( \omega_g \) increases with \( \Gamma \) for s+g-wave much faster than that for |cos2\( \phi \)| since point nodes are easily removed by impurity scattering. This is consistent with the experimental observation that YNi\(_2\)B\(_2\)C in the SC state is very sensitive to impurities as mentioned above.

**IV. TRANSITION TEMPERATURE, SC ORDER PARAMETER AND THERMODYNAMICS**

In this section we focus on the thorough solution of Eqs. (3)-(5). Considering the limit \( \Delta \to 0 \), we first extract the transition temperature \( T_c \) analytically. In both Born and unitarity limit, we obtain the following equation which is different from the standard Abrikosov-Gor'kov (AG) one\(^{32} \)

\[
-\ln \frac{T_c}{T_{c0}} = \frac{\langle f^2(k) \rangle - \langle f(k) \rangle^2}{\langle f^2(k) \rangle} \left( \psi \left( \frac{1}{2} + \frac{\Gamma}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right),
\]

where \( T_{c0} \) is the transition temperature at \( \Gamma = 0 \), and \( \psi(z) \) is the digamma function. The prefactor \( p = \frac{\langle f^2(k) \rangle - \langle f(k) \rangle^2}{\langle f^2(k) \rangle} \simeq 0.169 \) for the current s+g-wave. Note that for the usual nodal superconductors with only a single angular momentum pairing like d-wave or pure g-wave, the prefactor is unity, i.e., Eq. (7) becomes the standard AG equation.\(^{27,28} \) Therefore, for d-wave there exists a critical \( \Gamma \): \( \Gamma_c = \pi T_{c0}/(2e^\gamma) \approx 0.882T_{c0} \) at which \( T_c \) vanishes, whereas for s+g-wave \( \Gamma_c = \infty \). In the limit \( \Gamma \to 0 \), Eq. (7) gives

\[
T_c = T_{c0} - \frac{\pi}{4} p \Gamma.
\]

FIG. 2. \( T_c/T_{c0} \) (solid lines) and \( \Delta(\Gamma, T=0)/\Delta_{00} \) (dotted lines) vs. \( \Gamma/\Delta_{00} \) for s+g- and d-wave. Note that for each quantity the results in two limits are the same for s+g-wave. For d-wave the quantity \( \Delta(\Gamma, T=0)/\Delta_{00} \) is shown in unitarity limit.\(^{28} \)
The dependence of $T_c$ on $\Gamma$ is shown in Fig. 2, where $\Gamma$ is scaled by $\Delta_{00} = \Delta (\Gamma = 0, T = 0)$. The result for d-wave is also shown for comparison. From Eq. (8) we have the linear relation at $\Gamma \to 0$: $T_c / T_{c0} = 1 - q \Gamma / \Delta_{00}$ with $q = 0.366$ for s+g-wave and 1.68 for d-wave. Here we have used the BCS ratios $\Delta_{00} / T_{c0} \simeq 2.76$ and 2.14 for s+g- and d-wave, respectively.

We try to evaluate $\Gamma$ from the experimental data on $T_c$. For $Y(N_{1-x}Pt_x)2B_2C$, $T_c = 12.1K$ at $x = 0.2$, i.e., $T_c / T_{c0} \simeq 0.786$. Then we have $\Gamma / \Delta_{00} \simeq 0.94$. For $x = 0.05$, $T_c = 13.1K$, corresponding to $\Gamma / \Delta_{00} \simeq 0.55$.

The SC order parameter $\Delta(\Gamma, T)$ is solved numerically. In what follows we give the results only in Born limit. Those in unitarity limit are practically the same. In Fig. 3, $\Delta(\Gamma, T)$ is given as a function of $T$ for a few $\Gamma$. In addition, $\Delta(\Gamma, T = 0) / \Delta_{00}$ is shown in Fig. 2, which decreases slowly (slower than $T_c / T_{c0}$) with $\Gamma$.

![Graph of $\Delta(\Gamma, T)/\Delta_{00}$ vs. $T/T_{c0}$ for $\Gamma/\Delta_{00}$ and s+g-wave.](image1)

**FIG. 3.** $\Delta(\Gamma, T)/\Delta_{00}$ as a function of $T/T_{c0}$ at different $\Gamma/\Delta_{00}$ for s+g-wave.

We further calculate the thermodynamics. The entropy $S$ is given by

$$S = -\int_{-\infty}^{\infty} dE N(E) \{ f(E) \ln[f(E)] + [1 - f(E)] \ln[1 - f(E)] \}$$

(9)

with Fermi function $f(E) = 1/(1 + e^{\beta E})$. In Fig. 4 we have shown the entropy $S / S_n$ vs. $T / T_{c0}$ for a few $\Gamma$, where $S_n = \gamma T$ with $\gamma = 2k^2\pi^2 N_0$ is the entropy in the normal state. In the inset the same figure is replotted but $S$ is scaled by a constant $\gamma T_{c0}$. This is helpful to understand the specific heat shown below.

The specific heat is given by $C = T \frac{dS}{dT}$, which can be explicitly expressed as

$$\frac{C}{C_n} = \frac{3}{2\pi^2 T^3} \left[ \int_0^{\infty} dE \frac{N(E)}{N_0} E^2 \tanh^2 \left( \frac{E}{2T} \right) \right]$$

$$+ \frac{6}{\pi^2} \int_0^{\infty} dE \frac{\partial}{\partial T} \left[ \frac{N(E)}{N_0} \right] \left[ \ln(1 + e^{\beta E}) - \frac{\beta E}{1 + e^{-\beta E}} \right],$$

(10)

where $C_n = \gamma T$ is the specific heat in the normal state. In Eq. (10) the second term is contributed by the $T$-dependence of the DOS. It is important only at high temperatures because at low $T$, about $T < 0.2T_{c0}$ (see Fig. 3), $\Delta(\Gamma, T)$ is nearly $T$-independent and also the DOS. For an instructive look at how the DOS changes with $T$ in the whole range, we have shown in Fig. 5 an example for $\Gamma / \Delta_{00} = 0.2$.

![Graph of $S / S_n$ vs. $T / T_{c0}$ for $\Delta(\Gamma, T)/\Delta_{00}$ and s+g-wave.](image2)

**FIG. 4.** The entropy $S / S_n$ as a function of $T / T_{c0}$ for a few $\Gamma / \Delta_{00}$. The inset shows $S / (\gamma T_{c0})$ vs. $T / T_{c0}$.

![Graph of DOS vs. $E/\Delta_{00}$ for $\Gamma / \Delta_{00}$.](image3)

**FIG. 5.** The DOS with change of $T$ for $\Gamma / \Delta_{00} = 0.2$ (with corresponding $T_c / T_{c0} = 0.934$). It is seen that the DOS at $T/T_{c0} = 0.3$ is only slightly changed from that at $T = 0$. With increasing $T$, the bump at relatively high energy dissolves gradually, and the peak becomes narrow and simultaneously shifts towards zero. When $T \to T_c$, the DOS is close to an inverse square root function valid for an isotropic s-wave gap. Finally it will become the straight line “1” at $T = T_c$. 

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In the pure case, the DOS is linear at low energy. Present the analytical results for the specific heat at low energy. For the pure case into an activated behavior in the presence of impurities. This is due to the opening of a full gap by impurities as seen from the DOS. Thus the low energy quasi-particle excitations are exponentially small.

The above point becomes more transparent if we present the analytical results for the specific heat at low T where only the first integration in Eq. (10) contributes. In the pure case, the DOS is linear at low energy E \( \ll \Delta \): 
\[ \frac{N(E)}{N(0)} = \frac{E}{\Delta} \].
For about 0.1 \( \Delta / T > 5 \), i.e., \( T < 0.05 T_0 \), this linear formula can be used throughout the whole range in the integration. Then we have
\[ \frac{C}{C_n} = \frac{27}{4\pi} \zeta(3) \frac{T}{\Delta} \approx 0.93 \frac{T}{T_0} \],
(11)
i.e., a linear T-dependence (or \( T^2 \) dependence for \( C \)). When \( \Gamma > 0 \), the lower limit of the integration becomes \( \omega_g \). In view of \( \text{sech}^2(E/2T) \approx 4 \exp(-E/T) \) at low T (about \( \omega_g/T > 5 \)), an exponential factor \( \sim \exp(-\omega_g/T) \) is naturally expected after the integration irrespective of the concrete form of \( N(E) \). Explicitly, for \( T < \Gamma < \Delta_0 \) we have obtained \( \omega_g \approx \Gamma \)
\[ \frac{C}{C_n} = \frac{3\Gamma^{5/2}}{2\pi^{3/2}T^{3/2}\Delta} \left[ 1 - \frac{\Gamma}{\Delta \ln(1 + \frac{\Delta}{T})} \right]^{-1} \times \left( 1 + \frac{27T}{8\Gamma} \right)e^{-\Gamma / T} \].
(12)

Experimentally the specific heat was measured by Nohara et al.\(^5\) for Y(Ni\(_{1-x}\)Pt\(_x\))\(_2\)B\(_2\)C with \( x = 0 \) and 0.2. By fitting the experimental data they have exhibited a power law relation \( C \propto T^n \) with \( n \approx 3 \) for \( x = 0 \) compound and \( C \propto \exp(-\text{const.}/T) \) for \( x = 0.2 \). The essential features of the experimental data in both pure and impure systems have been reproduced by the theoretical results, although some differences exist for the accurate \( T \)-dependences between theory and experiment.

In addition, the specific heat jump at the transition temperature can be analytically derived. For a pure system, we have
\[ \frac{C - C_n}{C_n} \bigg|_{T=T_0} = 1.426 \frac{\langle f^2(k) \rangle^2}{\langle f^4(k) \rangle} \approx 0.885 \].
(13)
For \( \Gamma > 0 \), the formula is algebraically more complicated:
\[ \frac{C - C_n}{C_n} \bigg|_{T=T_c} = 12(1 + a)^2 \left[ 1 - \frac{a}{1 + a} w^{\psi(1)(1/2 + x)} \right]^2 \times \left\{ 7\zeta(3) - \frac{b}{2} w^{\psi(2)(1/2 + x)} + 3ax^{-1} \left[ \frac{\pi^2}{2} - \psi(1)(1/2 + x) \right] + ax \left[ \frac{\pi^2}{2} + \psi(1)(1/2 + x) - 2x^{-1} \left[ \psi(1/2) - \psi(1/2) \right] - \frac{1}{6} \psi(3)(1/2 + x) \right] \right\}^{-1} \],
(14)
where \( x = \Gamma / (2\pi T_c) \), \( a = \langle w^3 \rangle \), \( b = \langle w^4 \rangle \) with \( w = \sin^2 \theta \cos 4\phi \) and \( \psi(i)(z) \) (\( i = 1, 2, 3 \)) are the poly-gamma functions. From Eq. (14), we obtain \( \frac{C - C_n}{C_n} \bigg|_{T=T_c} = 0.9011(0.16) \) for \( \Gamma / \Delta_0 = 0.2(1) \), which are consistent with the numerical values in Fig. 6.

V. CONCLUSION

In the present paper we investigate the impurity effects on s+g-wave superconductivity in YNi\(_2\)B\(_2\)C and possibly LuNi\(_2\)B\(_2\)C. Unlike in the usual nodal superconductors, we have discovered that the results in Born and unitarity limit are practically the same. There is no resonant scattering associated with nonmagnetic impurities. Also unlike in the usual nodal superconductors, the quasi-particle energy gap opens up immediately by impurity scattering and grows quickly with the scattering rate. In other words the nodal excitations are prohibited immediately when impurities are introduced, which is clearly seen from the quasi-particle density of states. Indeed we could evaluate that the energy gap opens up even for Y(Ni\(_{1-x}\)Pt\(_x\))\(_2\)B\(_2\)C with \( x = 0.01 \). From the value for \( \Gamma \) at \( x = 0.05 \), we estimate \( \Gamma / \Delta_0 \) \( \sim 0.1 \) for the crystal with \( x = 0.01 \), which is substantial for a gap opening. Therefore the s+g-wave superconductivity is very sensitive to impurity scattering.

Also we have calculated the specific heat, which shows a change from a power law dependence of \( T \) in a pure system into an activated behavior in the presence of impurities. Compared with the experimental data by Nohara...
et al., the theoretical results have reproduced the main features. The accurate comparison needs improvements in both theoretical and experimental side.

Finally we point out that the sensitivity to impurities predicted by s+g-model is confirmed by the angular dependent thermal conductivity measurements on the single crystal Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C with $x = 0.05$, where the gap anisotropy has been cleared out. In a forthcoming paper we shall discuss other properties such as superfluid density etc. within the same formal treatment.

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