RESEARCH ARTICLE

A novel multi-item joint replenishment problem considering multiple type discounts

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Abstract

In business replenishment, discount offers of multi-item may either provide different discount schedules with a single discount type, or provide schedules with multiple discount types. The paper investigates the joint effects of multiple discount schemes on the decisions of multi-item joint replenishment. In this paper, a joint replenishment problem (JRP) model, considering three discount (all-unit discount, incremental discount, total volume discount) offers simultaneously, is constructed to determine the basic cycle time and joint replenishment frequencies of multi-item. To solve the proposed problem, a heuristic algorithm is proposed to find the optimal solutions and the corresponding total cost of the JRP model. Numerical experiment is performed to test the algorithm and the computational results of JRP under different discount combinations show different significance in the replenishment cost reduction.

1 Introduction

In the multi-item inventory environment, a joint replenishment policy can generally be defined as the coordination of multiple items that may be ordered jointly from a single supplier [1–3]. Traditionally, two types of ordering cost, the major ordering cost related to ordering times and the minor ordering cost related to each item, in a two-layer supplying system, within which a buyer placing an order to a supplier for a number of different items, are assumed [4]. It is believed that a well planned joint replenishment policy can bring great savings for both buyers and suppliers [2, 5–8]. Henceforth, the joint replenishment problem (JRP) has received extensive attention from both practitioners and researchers.

Practically, a buyer is more willing to accept a price break after purchasing a large amount of the supplier’s product, while the motivations for a supplier offering quantity discounts is either to pursue the price discriminate or to reduce the operating cost [9] and control operating risks [10]. For example, several discounts, e.g. percentage-based discount, dollar value discount, and free shipping or free gift on different products are adopted by some B2C e-business to attract more consumers’ buying. In the early stage of the JRP research, the benefits obtained by performing joint replenishment policy are solely assumed as the savings in ordering cost.
through group replenishing different items [1, 4, 11]. However, a performed joint replenishment policy with conventional JRP assumptions increases the inventory level and the system cost of the buyer for controlling inventory [2]. While in another aspect, in order to promote the buyer to purchase more items, the supplier usually provides the buyer discount offers to balance the buyers’ inventory level and the inventory carrying cost.

Furthermore, in light of different items show different cost features in manufacturing, supplying and storing, some more flexible discount offers are preferred by the supplier according to the specific supplied items. In reality, comparing to all items that being offered with one discount type, it is very common for suppliers to make a comprehensive decision according to the orders on hand and provide a mixed discount type offer, the reason lies in that multiple discounts can help the supplier make a more flexible selling strategy. Hence, before constructing JRP model with multiple discounts, the differences of discounts should be specified.

Thanks to the positive benefits of discounts, various discounting schemes are offered by the suppliers in practice and discussed by researchers. For example, all-unit quantity scheme is a widely utilized scheme as it directly links the ordering prices and quantities of the items and is easy to perform in practice [12]. Incremental quantity discount is another commonly applied discount scheme that the supplier would benefit more as only those ordered unit exceeds certain amount can be offered a lower price. While the total business volume discount scheme is very convenient to apply in the multi-item situation [13, 14]. The motivations for the buyers and suppliers to perform different discount schemes may differ, but it has been testified that the buyer and the supplier would be coordinated if the transfer price or cost is set optimally based on the discounting schemes [15].

The presence of different discount schemes often complicates the item purchasing decisions [16] and sometimes looms the information risks [17, 18]. Thus, in most studies, for ease of processing the discount settings, environments of a single item with multiple discount schemes [13] or multi-item with a single discount scheme [2, 19] are the most welcomed and prevalent research assumptions. However, the research considering both joint replenishment of multi-item and multiple quantity discount scheme offers is rare. Therefore, this study aims to contribute to a supplement research to JRP with multiple discount considerations. The main contributions addressed in this research are elaborated as follows.

(1) A new JRP model considering three discount types, all-unit quantity, incremental quantity and total business volume, simultaneously is constructed. In practice, a supplier can provide more flexible discount offers in light of particular types of different items. Within this background, the new model is constructed to investigate the joint effects of different discount combinations to the total cost of JRP.

(2) An iterative heuristic algorithm is presented to solve the proposed model. In light of the NP-hard nature of JRP, we design an iterative heuristic algorithm to deal with three quantity discounts sequentially based on two designed solving procedures.

(3) A numerical case is presented to test the effectiveness of the algorithm in solving JRP under different discount combinations. In the numerical experiments, JRP with no discount, JRP with quantity discount, JRP with incremental discount, JRP with total business volume discount, and JRP with three discounts, are compared and analyzed, respectively.

The rest of this paper is organized as follows. In Section 2, literature on the evolvement of JRP with discount considerations are reviewed. Section 3 presents the assumptions, notations, and the formulation process of the model, the corresponding solving procedures are also given. In Section 4, a JRP case is presented as a numerical example to test effects of JRP with different discounts. Section 5, conclusions and focus for future research are provided.
2 Literature review
Numerous researchers have made contributions in researching JRP since JRP was presented in 1970s. Currently, JRP has already become one of the most important research branch that deal with multi-items. In this part, we limit our focus on the researches of multi-items replenishment with can provide us a clear understanding of the JRP models and the solving methodologies in discount environment.

2.1 Item replenishment with discount considerations
Item replenishment with discount considerations is a common practice in commercial purchasing activities, however, it is always a great challenge in making a decision on replenishing multi-item with different discount combinations. In general, item replenishment involves numerous processes and activities, such as demand prediction, supplier selection, price negotiation, and so on [20–22]. The offered discounted prices for the buyer making the replenishment decision becomes even more complicated [16]. Thus, the vast majorities of researchers construct mathematical models to study item replenishment with discount considerations to investigate the connections of the ordering quantities and the ordering cost. Basically, based on the types of items with discount offers, the researches can be classified as the single item replenishment problem and the multi-items replenishment problem.

The single item replenishment problem with discount consideration often reduces to the problem of multi-supplier selection. Within this circumstance, Xia and Wu [16] once noted, no one supplier can fulfill the whole order so that the order is divided from one supplier to multiple suppliers. Thus, multiple sources of items and their extensions are generally considered in many researches, but each supplier is generally assumed to supply a single type of item. For example, Yang et al. [23] focused on obtaining the satisfied replenishment policy to minimize the transportation time and inventory cost in a multi-supplier multi-retailer supply chain, where the transportation cost are discounted according to the ordering quantities of different items. Zhang and Chen [21] constructed a mixed integer programming model to allocate the discounted ordering quantities of a single type of item to multiple suppliers, the objective of the model is to minimize the total cost, including the selecting cost, the procurement cost, the holding cost and the shortage cost. On deciding the purchasing prices of single items, Lee et al. [24] assumed that both all-unit quantity discounts and incremental discounts were provided by parts of suppliers, respectively.

The multi-item replenishment models with discount consideration are usually constructed under the assumption that a supplier fulfills the whole order. Haksever and Moussourakis [25] presented a mixed integer programming model to determine the best-found order quantities of multi-item with incremental quantity discount offered by multiple suppliers. Zhang [26] examined a multi-item newsboy problem and formulated a mixed integer model to investigate the impact of quantity discount and budget constraint to the optimal ordering quantity. Considering the multi-suppliers with the all-unit quantity discount, Shi and Zhang [27] formulated a model to determine the best selling prices and ordering quantities of multi-items simultaneously. Manerba and Mansini [28] made a further extension to the single supplier selection problem and assumed the orders can be fulfilled among different suppliers with the total quantity discount (TQD). Based on the work of these forerunners, our research would contribute the literature on investigating the multi-item jointly replenishment problem with multiple discounts.

A general summary of pertinent papers is provided in Table 1.
From Table 1, we observe that (1) a large share of papers are focused on supplier selection problem and supplying assignment problem, only a small number of research papers consider
the multi-item joint replenishment problem. (2) three typical discount schemes, all-unit quantity discount, incremental discount and total volume discount, are the most favorite discount structures in the model constructions, but the papers considering multiple discounts are rare. (3) the mixed integer programming (MIP) models are constructed in most papers, but their solution algorithms are different. Therefore, in light of above researches in item replenishment modeling without mixed discount type considerations, our research would provide supplement literature on joint replenishment problem with multiple discounts.

### 2.2 JRP with discount schemes

Since Shu [11] presented JRP, JRP has drawn worldwide researchers’ attention. Khouja and Goyal [1] reviewed several extension of JRP, including JRP under stochastic [32] and JRP under dynamical demand [33]. Other extensions, such as all-unit quantity discount [19], JRP under continuous unit cost decrease JRP with supplying capacity constraints [34], JRP with delivery [35], JRP with imperfect items [2] and so on, are developed. Of all JRP extensions, one extension of JRP, JRP with multiple quantity discount schemes, has not been fully considered, though multiple discount combinations are practiced by the practitioners. In general, two strategies, the direct grouping strategy (DGS) and the indirect grouping strategy (IGS) are raised for grouping items [1]. However, before DGS is performed, a predetermined number of groups should be provided under the minimized total cost [36]. Under IGS, the replenishment cycle

| Article | Discount | Item | Buyer | Supplier | Model and Solution Algorithm |
|---------|----------|------|-------|----------|------------------------------|
| [16]    | All unit discounts | Multi-item | – | Multi-supplier | Multi-objective programming, optimization tool box of Matlab |
| [23]    | All unit discount | Single item | Multi-retailer | Multi-supplier | MIP, Genetic Algorithm |
| [21]    | All unit discount | Single item | – | Multi-supplier | MIP, Bender’s decomposition heuristic |
| [24]    | All-unit and incremental discounts | Single item | – | Multi-supplier | MIP, Genetic Algorithm |
| [25]    | Incremental quantity discount | Multi-item | A warehouse | – | MIP, multiple software packages |
| [26]    | All unit discount | Multi-item | A newsboy | – | MIP, lagrangian relaxation |
| [27]    | All unit discount | Multi-item | A retailer | Multi-supplier | MIP, lagrangian relaxation |
| [28]    | Total quantity discount | Multi-item | – | Multi-supplier | MIP, a branch-and-cut approach |
| [29]    | Total quantity discount | Multi-item | A buyer | Multi-supplier | MIP, a heuristic algorithm |
| [30]    | All-unit discount | Multi-lane | A distributor | Multi-carrier | MIP, a tabu search algorithm |
| [2]     | All unit discount | Multi-item | A buyer | A supplier | MIP, heuristic algorithms |
| [13]    | All unit discounts, incremental and total volume discounts | Single item | A buyer | Multi-supplier | MIP, a scatter search algorithm |
| [31]    | All unit and incremental discounts | Single item | A centralized buyer | Multi-vendor | Integer lot-sizing model and heuristic algorithms |

* MIP is the abbreviation of Mixed Integer Programming.

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of each item is an integer multiplier of the basic cycle time. The problem is simplified as to
determine the basic cycle time and the replenishment frequencies of all items simultaneously.
Thus, IGS is adopted in the following analysis.

In traditional JRP, the ordering quantities are assumed deterministic [37], in which the
superiorities of joint replenishment are reflected in but not limited to acquire the savings of
ordering cost by group purchasing multi-items. By introducing the all-unit quantity discount
to JRP, Cha and Moon [19] constructed a JRP model with quantity discounts and an efficient
heuristic algorithm was developed to solve the proposed model. Moon et al. [38] transformed
the single supplier JRP with all-unit quantity discount to a multi-supplier and each item is
assumed to be purchased from one supplier. Paul et al. [2] formulated a JRP model considering
the imperfect items and all-unit quantity discount. However, there are no researches consider-
ing the mixed quantity discount scheme in JRP.

When talking about the discount structures, Munson and Rosenblatt [9] pointed that the
form of discount may be either all-units or incremental. Three common discount schemes, the
all-unit quantity discount, the incremental quantity discount and the total volume discount
are commonly applied in model constructions. According to the definition from Lee et al.
[24], under the all-unit quantity discount, if the ordering quantity belongs to a specified quan-
tity level predetermined by the supplier, the discounted price is applied to all-units starting
from the first unit, see Fig 1a. The incremental quantity discount shows the only difference in
that the discounted price of incremental quantity discount is applied to the units inside two
continuous quantity breaks, see Fig 1b. While the total business volume discount (TBD)
scheme or TQD presented in Ebrahim et al. [13], Manerba and Mansini [28] and Xia and Wu
[16] to illustrate the fluctuation of total business values over the total ordering quantities of all
items, which means that a TBD represents item aggregation where the price breakpoints are
based on the total dollar volume of business over all items ordered from the supplier [9].
Therefore, TBQ can be considered as the variation of all-unit or incremental discount. A gra-
phical illustration of the two (all-unit and incremental) discounts is presented in Fig 1.

3 Description of the proposed model and the solving algorithm

3.1 Problem description, assumptions and notations

In the proposed model, a two-layer supply chain with a supplier (e.g. an item manufacture)
and a buyer (e.g. a distribution center or a retailer) is considered. At the supplying side, besides
the items supplied with no discount, the supplier also offers three discounts, all-unit quantity
discount, incremental discount, and total business volume discount, to the buyer according to
the stored items. Moreover, each kind of item can only have one discount type. At the buying side, four types of cost, the major ordering cost, the minor ordering cost, the inventory holding cost, and the item purchasing cost, are considered during the replenishment process. The aim is to find the optimal combination of the basic cycle time and the ordering frequencies of all items with the context of multiple discounts.

The assumptions of the general JRP are inherited from the assumptions of the economic ordering quantity (EOQ) problem. For example, the demand is assumed to be deterministic and conforms to a uniform distribution, no shortages are allowed, no quantity discount, the holding cost is linear [1], and so on. Based on these assumptions, the assumptions considered throughout this paper are given below:

The demand of each item is deterministic and constant.

No shortages are allowed.

The items are replenished when the inventory level drops to zero.

The inventory holding cost is known and constant.

The order is delivered instantly without the lead-time consumption.

Three discount offered by the supplier.

The discount structures are offered by the supplier and known by buyer.

Each type of items is offered one and only one possible discount scheme

Accordingly, the vectorial sets, indices, and decision variables are given as follows:

\(i\): the index of items, and set \(I = \{i | i = 1, 2, \ldots, n\}\),

\(j\): the index of discount intervals, and set \(J = \{j | j = 1, 2, \ldots, J_i\}\),

\(n_0\): the number of items that are offered no discount (ND) by the supplier, and set \(\mathbb{N}_0\) means the items with ND in \(\mathbb{N}_0\),

\(n_1\): the number of items that are offered all-unit quantity discount (AD) by the supplier, and set \(\mathbb{N}_1\) means the items with AD in \(\mathbb{N}_1\),

\(n_2\): the number of items that are offered incremental discount (ID) by the supplier, and set \(\mathbb{N}_2\) means the items with ID in \(\mathbb{N}_2\),

\(n_3\): the number of items that are offered total business volume discount (BD) by the supplier, and set \(\mathbb{N}_3\) means the items with BD in \(\mathbb{N}_3\),

\(c_i\): unit cost/price of item \(i\) that the buyer pays to the supplier with ND,

\(\alpha_{ij}\): discounted unit price of item \(i\) in the \(j\)-th interval under the AD scheme,

\(\beta_{ij}\): discounted unit price of item \(i\) in the \(j\)-th interval under the ID scheme,

\(\gamma_{ij}\): discounted rate of item \(i\) in the \(j\)-th interval under the BD scheme,

\(x_{ij}\): binary variable: if and only if the order quantity of item \(i\) falls on the interval of \(j\), \(x_{ij} = 1\), otherwise \(x_{ij} = 0\),

\(\mu_{ij}\): threshold (breakpoint) of each discount interval, and to item \(i\), \(0 = \mu_{i,0} < \mu_{i,1} < \cdots < \mu_{i,J_i} \leq \infty\),

\(TC\): total annual cost of all items,
\( S \): major ordering cost of each order, \\
\( s_i \): minor ordering cost of each item, \\
\( D_i \): demand rate of item \( i \), \\
\( h_i \): annual holding cost of item \( i \), \\
\( T \): basic cycle time (decision variable), and \\
\( k_i \): integer multiplier of item \( i \) (decision variable), \( k_i \in K \).

### 3.2 Model formulation

#### 3.2.1 The general JRP model

Under the indirect grouping strategy (IGS) [1], \( T_i \) for each item \( i \) is an integer multiple \( k_i \) of \( T \). Thus, the replenishment cycle of item \( i \) is:

\[
T_i = k_i T
\]

and the order quantity \( Q_i \) of item \( i \) is:

\[
Q_i = T_i D_i = D_i k_i T
\]

The annual total holding cost per unit time is:

\[
C_h = \sum_{i=1}^{n} Q_i h_i / 2 = \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i
\]

And the annual total ordering cost per unit time is:

\[
C_o = S / T + \sum_{i=1}^{n} \left( s_i / (k_i T) \right) = \frac{1}{T} \left( S + \sum_{i=1}^{n} (s_i / k_i) \right)
\]

Accordingly, the annual total cost per unit time is:

\[
TC_0(T, K) = C_h + C_o = \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i + \frac{1}{T} \left( S + \sum_{i=1}^{n} (s_i / k_i) \right)
\]

where \( k_i \in K, i = 1, 2, \ldots, n \), and \( K \) is a set of integer multipliers. Here we call the annual total cost per unit time as the total cost \( TC \), and the objection is to find the minimized \( TC \) of JRP. For a fixed \( K = (k_1, \ldots, k_n) \in \mathbb{N}^n \) the optimal value of \( T^* \) is given by Eq (6) below:

\[
T^* = \sqrt{2 \left( S + \sum_{i=1}^{n} (s_i / k_i) \right) / \sum_{i=1}^{n} k_i D_i h_i}
\]

Thus, the optimal \( TC \) is obtained after \( T \) and \( k_i \)'s have been fixed. The \( k_i \) is obtained by referring to the optimal condition presented by Goyal [4], such that

\[
k_i (k_i - 1) \leq \frac{2s_i}{D_i h_i T^2} \leq k_i (k_i + 1)
\]

In general, the purchasing cost of items is not included in the total cost of joint replenishment process. In practice, however, most of the practitioners prefer to perform the joint replenishment strategy not only for the sake of acquiring benefits in ordering cost decreasing, but also eager to save more cost through ordering different items in large batches with different...
of discount offers. Therefore, the total joint cost of JRP with no item discount is presented as

$$TC(T, K) = C_u + C_o + C_p = \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i + \frac{1}{T} \left( S + \sum_{i=1}^{n} (s_i/k_i) \right) + \frac{1}{T} \left( \sum_{i=1}^{n} c_i Q_i \right)$$

(8)

where $C_p$ is the total purchasing cost (we can also call it the occupational cost or inventory carrying cost per unit time) of items in each order for the buyer, $n = n_0 + n_1 + n_2 + n_3$, and $Q_i$ can be substituted by $D_i k_i T$.

3.2.2 JRP with multiple discounts. The total purchasing cost of the buyer depends on the cost structure offered by the supplier. In the following the structures of three mentioned discounts are presented. Here below the cost function of each discount structure is given as

(1) All-unit quantity discount

In the all-unit quantity discount scheme, the supplier offers price discount according to the possible order quantities of different items. The price is stepped down as the ordering quantity of an item increases progressively in different intervals, and the ordering quantity intervals are divided according to the maximum and the minimum ordering data in the supplier’s supplying history. Thus, the total purchasing cost per unit time with all-unit discount is formulated as:

$$C_{AD} = \frac{1}{T} \sum_{i=1}^{n} \sum_{j \in J} x_{ij} Q_{ij}$$

(9)

where $\sum_{j=1}^{l+1} x_{ij} Q_{ij} = Q$, and $\sum_{j=1}^{l+1} x_{ij} = 1$, which means that for item $i$ for $j \in J$, $Q_{ij} = Q$, if and only if $\mu_{ij-1} \leq Q < \mu_{ij}$. It is also assumed that the unit price is stepped down as $\alpha_1 > \alpha_2 > \cdots > \alpha_f$ for item $i$. Fig 1a gives a simple illustration of the all-unit discount. Therefore, if $j$ is fixed, the $C_{AD}$ can be simplified as $C_{AD} = \sum_{i=1}^{n} \frac{1}{T} Q \sum_{j \in J} x_{ij} Q_{ij} = \sum_{i=1}^{n} \frac{1}{T} \sum_{j \in J} x_{ij} Q_{ij} D_j$.

(2) Incremental discount

For the incremental discount scheme, the slightly difference comparing to the all-unit quantity discount lies in that the incremental discount applies only when quantity exceeds the price break quantity. The cost function $C_{ID}$ under incremental discount scheme is given as:

$$C_{ID} = \frac{1}{T} \sum_{i=1}^{n} \sum_{j \in J} \left( \beta_{ij}(Q_{ij} - x_{ij} \mu_{ij-1}) + x_{ij} \sum_{k=1}^{l} \beta_{ij}(\mu_{ij} - \mu_{ij-1}) \right)$$

(10)

where $\sum_{j \in J} x_{ij} = 1$, and if and only if $\mu_{ij-1} \leq Q_{ij} < \mu_{ij}, x_{ij}$ equals to 1 and the others equal to 0 for $j \in J$. It is also assumed that the unit price in this scheme is stepped down as $\beta_{11} > \beta_{12} > \cdots > \beta_{1f}$ for item $i$. Fig 1b gives a simple illustration of the incremental discount.

(3) Total business volume discount

In the total business volume discount scheme, supplier offers discount rate according to the total business value of the ordered items, but not to the ordering quantities, and the discount rate breaks are a function of total business volume discount. The structure of total business volume discount has been testified similarly to that in all-unit discount scheme by [13] for single item purchasing. Following the model construction principal for total business volume discount in [14] and [16], the total purchasing cost function $C_{BD}$ per unit time with total business volume discount is modeled as:

$$C_{BD} = \frac{1}{T} \sum_{i=1}^{n} \sum_{j \in J} (1 - \gamma_{ij}) x_{ij} c_i Q_i$$

(11)
where $\sum_{j=1}^{n} x_{ij} = 1$ and if and only if $\mu_{ij-1} \leq \rho_i Q_j < \mu_{ij} x_{ij} = 1$, otherwise, $x_{ij} = 0$ for $j \in J$. In this case, there is a need to calculate the total cost of the order firstly before the total business volume discount scheme takes effect. Then, by examining which discount interval the total cost lies in, the price (discount rate) offer is decided. It is also assumed that the unit discount rate is stepped down as $\gamma_{i1} > \gamma_{i2} > \cdots > \gamma_{ij}$. Similarly, if $j$ is fixed, the $C_{BD}$ can also be simplified as $C_{BD} = \sum_{i=1}^{n} \frac{1}{T} C_i Q_j \sum_{j=1}^{m} (1 - \gamma_{ij}) x_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{m} (1 - \gamma_{ij}) x_{ij} c_i D_i$.

After three cost functions have been formulated, the total joint cost of $C_{TP}$ can be decomposed as

$$TC(T, K) = C_A + C_B + C_P$$

where $C_A$ is the total item purchasing cost, including the total cost of items purchased with no discount, the all-unit quantity discount, the incremental discount and the total business volume discount, and $C_P$ is modeled as

$$C_P = C_{P(n-n_0)} + C_{AD(n-n_0)} + C_{ID(n-n_0)} + C_{BD(n-n_0)}$$

$$= 1 \left( \sum_{i=1}^{n} c_i Q_i + \sum_{j=1}^{n} \sum_{j=1}^{m} x_{ij} Q_j x_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} (\beta_{ij} Q_j - x_{ij} \mu_{ij-1}) \right)$$

$$+ x_{ij} \frac{1}{1-k_i} \beta_{ij} (\mu_{ij} - \mu_{ij-1}) + \sum_{i=1}^{n} \sum_{j=1}^{m} (1 - \gamma_{ij}) x_{ij} c_i Q_j$$

where $\sum_{j=1}^{n} x_{ij} = 1$.

### 3.2.3 Solutions for JRP with multiple discounts

In order to obtain the optimal combination of $T$ and $K$ that minimizes $TC'$, two remarks below are presented to illustrate the solving process of the proposed model. JRP has been testified as the NP-hard problem [33], the most effective and efficient methodologies for JRPs are the heuristic algorithms. Henceforth, a simple heuristic algorithm is presented in the following contents.

For a given set of $k_i$s, taking the derivative of $TC'(T, K)$ with respect to $T$ and let it equal to 0, we have

$$\frac{\partial TC(T, K)}{\partial T} = \frac{1}{T} \sum_{i=1}^{n} k_i D_i h_i - \frac{1}{T^2} \left( S + \sum_{i=1}^{n} (s_i / k_i) \right) + \frac{\partial C_P}{\partial T}$$

while $\frac{\partial C_p}{\partial T}$ can be decomposed as $\frac{\partial C_p}{\partial T} = \frac{\partial C_{P(n-n_0)}}{\partial T} + \frac{\partial C_{AD(n-n_0)}}{\partial T} + \frac{\partial C_{ID(n-n_0)}}{\partial T} + \frac{\partial C_{BD(n-n_0)}}{\partial T} = 0$, $Q_{ij} = Q_j$ for a fixed $j$, taking the derivative of $C_{ID}$ with respect to $T$ considering $x_{ij} = 1$, we can obtain

$$\frac{\partial C_{ID}}{\partial T} = \frac{\partial}{\partial T} \left( \sum_{i=1}^{n} \sum_{j=1}^{m} \beta_{ij} (Q_j - x_{ij} \mu_{ij-1}) + x_{ij} \sum_{j=1}^{m} (\mu_{ij} - \mu_{ij-1}) \right)$$

$$= \frac{1}{T} \left( \sum_{i=1}^{n} \sum_{j=1}^{m} \beta_{ij} Q_j \right)$$

$$+ \frac{\partial}{\partial T} \left( \frac{1}{T} \sum_{i=1}^{n} \sum_{j=1}^{m} \beta_{ij} (\mu_{ij} - \mu_{ij-1}) \right)$$

$$= \frac{\sum_{i=1}^{n} \left( \beta_{ij} \mu_{ij-1} - \sum_{j=1}^{m} \beta_{ij} (\mu_{ij} - \mu_{ij-1}) \right)}{T^2}$$
Table 2. Discount data of the two items.

| Item | Discount intervals | Price  |
|------|---------------------|--------|
| 2    | $0 \leq Q_i < 500$  | 3.25$  |
|      | $500 \leq Q_i < 1,000$ | 3.20$  |
|      | $1,000 \leq Q_i < 2,000$ | 3.15$  |
| 5    | $Q_i \geq 2,000$    | 3.10$  |
|      | $0 \leq Q_i < 300$  | 3.25$  |
|      | $Q_i \geq 300$      | 3.20$  |

Hence, if we define $\frac{\Delta}{p_T} = \frac{\partial G_{ij}}{\partial T}$, we have $\frac{\partial G_{ij}}{\partial T} = \frac{\Delta}{p_T}$. $\Delta$ can also be expressed as

$$\Delta = \sum_{i=1}^{n} (\beta_i \mu_{ij-1} - \sum_{j=1}^{i-1} \beta_j (\mu_{i,g} - \mu_{i,g-1}))$$

at the premise that the best purchasing interval of item $i$ is ascertained and $x_{ij} = 1$. Through decomposition, $\Delta$ can be rewritten as

$$\Delta = \sum_{i=1}^{n} \sum_{j=2}^{i} (\beta_i - \beta_{i,j-1})\mu_{i,g-1} + \beta_{i,0}\mu_{i,0})$$

(16)

Since $\mu_{i,0} = 0$ and $\beta_{i,j-1} > \beta_{i,0}$, we have $\Delta < 0$ and $\frac{\partial G_{ij}}{\partial T} < 0$. Consequently, if $S + \sum_{i=1}^{n} (s_i/k_i) + \Delta \geq 0$, by solving Eq (14), the optimal $T$ (denoted by $\bar{T}$) can be expressed as

$$\bar{T} = \sqrt{2 \left( S + \sum_{i=1}^{n} (s_i/k_i) + \Delta \right) / \sum_{i=1}^{n} k_i D_i h_i}$$

(17)

where $S + \sum_{i=1}^{n} (s_i/k_i) + \Delta \geq 0$, from which we can also obtain that $- (S + \sum_{i=1}^{n} (s_i/k_i)) \leq \Delta < 0$. The next problem is to find feasible $\Delta$s. Hence, taking a two-item case for example, the data of the case are provided in Table 2. For each item, the values of $\Delta_{ij} = \sum_{j=2}^{i} (\beta_{i,j-1} - \beta_{i,j-1})\mu_{i,j-1} + \beta_{i,1}\mu_{i,0})$ for all the intervals are given, then the summation of two items is $\Delta$. If there is more than one $\Delta < 0$, we choose the smallest feasible one ($\Delta = -90$ in the box in

Table 3. Computational results for $\Delta$.

| Item 5 | Item 2 | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ |
|--------|--------|--------|--------|--------|--------|
| $\Delta$ | 0      | -25    | -75    | -175   |        |
| $\Delta_{ij}$ | $\Delta = 0$ | $\Delta = -25$ | $\Delta = -75$ | $\Delta = -175$ |        |
| $S + \sum_{i=1}^{n} (s_i/k_i)$ | 0      | -15    | -15    | -15    | -90    |
| $\min\{ (S + \sum_{i=1}^{n} (s_i/k_i) + \Delta) \geq 0 \}$ | 141    | 141    | 141 - 90 | 51 > 0 |        |

$j'$ denotes $j$-th interval, $S = 50$, $s_1 = 46$, $s_2 = 45$, $K = [1, 1, 1, 1, 1, 1]$. 

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Therefore, to solve the proposed model, a heuristic algorithm is developed to deal with these complicated [16, 28], not to say multiple discount schemes are considered simultaneously. Even to the models with only one type of discount scheme, the solving algorithms heuristics are always problem pertinent, and the trivia in solving the discounted model is spread-sheet technique [41], and Silver’s heuristic [37] and its extensions [42–44]. Since the numerous scholars development algorithms so solve JRP [39], such as

3.3 An iterative heuristic algorithm

Numerous scholars development algorithms so solve JRP [39], such as Power of Two [40], spread-sheet technique [41], and Silver’s heuristic [37] and its extensions [42–44]. Since the heuristics are always problem pertinent, and the trivia in solving the discounted model is apparent, even to the models with only one type of discount scheme, the solving algorithms are complicated [16, 28], not to say multiple discount schemes are considered simultaneously. Therefore, to solve the proposed model, a heuristic algorithm is developed to deal with these multiple discounts. To the simple JRP, an iterative method was presented by Goyal [4] to find the optimal T and kset, based on which the proposed algorithm is constructed. However, comparing to T, the optimal T̂ is also interfered by Δ, so the most intricate part goes to find a best Δ. The simple case in Table 2 only provides us a rough sketch for computing Δ. As the number of items increases to 3 or more, it is not so easy to obtain Δ. Therefore, the following procedure provides a quick solution to find Δ, see Algorithm 1.

Algorithm 1 The procedure for obtaining T̂

1: Compute \( \nabla = S + \sum_{i,j} \left( s_i/k_j \right) \) and set a very large positive number \( M \).
2: for \( i = 1 \) to \( n_s \) do
3:   for \( j = 1 \) to \( J_i \) do
4:     Compute and output \( \Delta_{i,j} \).
5:   end for
6: end for
7: Formulate vector \( V_j \) as \( V_j = [\Delta_{i,1}, \Delta_{i,2}, \ldots, \Delta_{i,J_i}] \) and define set DM to contain all \( V_j \), \( DM(i) = V_j \).
8: Open spaces \( P \) for positioning the candidate \( \Delta_{i,j} \) in \( V_j \), and \( val \) for containing the value of the candidate \( \Delta_{i,j} \).
9: for run = 1 to Max.run do
10:   for \( i = 1 \) to \( n_s \) do
11:     \[ \text{val}(i), P(i) = \min(DM(i)(1 \text{ to } J_i)) \].
12: end for
13: Compute Delta, and Delta = sum(val).
temp = $\nabla + \Delta$.

if temp $\leq 0$ then
    // Select the minimum element in val, then position it in DM.
    [result, index] = min(val).
    result = result + M.
    DM[index](P(index)) = result.
else
    intermediateV(run) = temp;
    current best $\Delta$: CurBesDel = $\Delta$.
end if

end for

best_intermediate_v = min(intermediateV), output the best found $\Delta$ as BesFouDel = CurBesDel, and corresponding position of discount interval of each item.

Compute the current best $T$ based on Eq (17).

In Algorithm 1, a nabla symbol $\nabla$ is applied to denote '$S + \sum_{i=1}^{n} (s_i/k_i)$' and a very large number $M$ is given for eliminating illegal numbers in line 1. Lines 2-6 is to calculate and output $\Delta_{ip}$, the result of which is then sent to a vector $V_i$ in line 7, and a cell array $DM$ is generated to contain all $V_i$s. In lines 8-12, vectors $val$ and $P$ are defined as two arrays to contain the minimum value and the corresponding position of $\Delta_{ij}$, respectively. Lines 9-24 are presented to illustrate the procedure for obtaining all feasible values of $\nabla + \Delta$, which is contained in an intermediate vector $intermediateV$. The minimized element in $intermediateV$ is output in line 25 and which is then to applied to compute $T$ in line 26.

Based on the obtained $T$, Algorithm 2 is provided to update $k_i$s.

Algorithm 2 The procedure for updating $k_i$

1: Set ‘$n$’ as the total number of items, and predefine vectors $MinK$ and $MaxK$ to contain the smallest and largest $k_i$ of item $i$, and initially, $MinK = MaxK = \text{ones}(1, n)$.
2: for $i = 1$ to $n$ do
3:     if item $i \notin N^*_i$ then
4:         Compute $k_i$ based on Eq (7).
5:     else
6:         // Compute the lower and upper bounds of $k_i$ according to $\left\lfloor \frac{\mu_i}{\Delta} \right\rfloor \leq k_i < \left\lceil \frac{\mu_i}{\Delta} \right\rceil$ and the thresholds of $j$-th interval of item $i$, and
7:         $MinK(i) = \left\lfloor \frac{\mu_i}{\Delta} \right\rfloor$ and $MaxK(i) = \left\lceil \frac{\mu_i}{\Delta} \right\rceil$.
8:         for $k = MinK(i)$ to $MaxK(i) - 1$ do
9:             Compute and output the minimum TC.
10:        end for
11:     end if
12: end for
13: Output the current best $k_i$ according to the minimum TC.
14: Output the current best $K$.

Algorithm 2 offers two main ways for computing the $k_i$s, for items purchased under ND, AD and BD, the $k_i$s are obtained by Eq (7), the procedure of which is provided in lines 3-4, for items purchased under ID, a new $k_i$ is obtained through the interval thresholds and updated by finding a smaller total cost, the procedure of which is provided in lines 5-11. Then, the current best $K$ is output.

The synthetical procedure for solving the proposed problem is presented in Algorithm 3, the pseudocodes of which are given as follows. The first three lines of Algorithm 3 are
preparation procedures for initializing the input parameters, such as \( K, TC \), and the maximum accumulative computation times as \( X \). The counter is initialized as ‘counter = 0’ in line 3. The loop in lines 5-15 depicts the main procedure for updating \( T \) (by calling Algorithm 1) and \( K \) (by calling Algorithm 2) when and only when the new total cost is smaller than current best total cost. Lines 16-20 present the loop for running the counter and when the maximum accumulative computation times reached to \( X \), Algorithm 3 is terminated.

Algorithm 3 The synthetical procedure for the proposed model

1: Initialize \( K \) as a unit vector, assign \( TC \) a very large number and set it as ‘CurBesTC’.
2: Set accumulative computation times as \( X \) and ‘count’ as the counter.
3: \( \text{count} = 0 \).
4: \( \text{for} \ G = 1 \ \text{to} \ \text{Max}_G \ \text{do} \)
5: Call Algorithm 1 for computing a new\( T \) and output the position of \( j \)-th interval of item \( i \).
6: Call Algorithm 2 for updating \( k_i \), output the local best found \( K \) as new\( K \) and the corresponding minimum \( TC \) as the new new\( TC \).
7: \( \text{if} \ \text{newTC} < \text{CurBesTC} \text{ then} \)
8: \( T = \text{newT} \). // \( T \) is updated
9: \( K = \text{newK} \). // \( K \) is updated
10: \( \text{CurBesTC} = \text{newTC} \). // \( TC \) is updated
11: \( \text{count} = 0 \). // recount
12: \( \text{else} \)
13: Keep \( T(G), K(G) \) and CurBesTC unchanged.
14: \( \text{count} \text{++} \).
15: \( \text{end if} \)
16: \( \text{if} \ \text{count} = X \text{ and CurBesTC is unchanged} \text{ then} \)
17: Break. // Break out of the loop
18: \( \text{else} \)
19: Continue. // Continue the loop
20: \( \text{end if} \)
21: \( \text{end for} \)

The flow chart of the synthetical heuristic algorithm is given as Fig 2. In Fig 2, the mini-figure in the middle is to illustrate the 6 searching steps of Algorithm 3. The left mini-figure is to call Algorithm 1 to calculate the current-best \( T \); see line 5 of Algorithm 3. The right mini-figure is to call Algorithm 2 to update \( k_i \) based on the returned current-best \( T \). After certain steps of iteration, the final result is output as our best-found result.

4 Numerical experiment

In this section, a \( JRP \) case with 6 items is presented to demonstrate the constructed model and the heuristic algorithm. In the case, a supplier supplies multi-item to a single B2C company. In order to promote the sales of these items, the supplier offers different promoting discount schemes. The basic data for the case is presented in Table 4 and the data on quantity discounts are presented in Table 5.

Specifically, from Table 5 we can observe that \( n_0 = 1, n_1 = 2, n_2 = 2, n_3 = 1 \), the proposed algorithm is applied to solve the case, and if all items are purchased without any discount considerations, the solving algorithm is reduced to solve \( JRP \) with ND. To make clearly comparisons, we assume all the items are purchased with all-unit quantity discount (AD) considering the same purchasing cost structure as that in Table 5 and quantity structure as that in Table 4. The comparison results of \( JRP \) with (all items are presumed to be purchased under) AD, \( JRP \) with (all items are presumed to be purchased under) AD,
ID, JRP with (all items are presumed to be purchased under) BD and JRP with multiple dis-
counts (MD) using the proposed heuristic are presented in Tables 6 and 7.

The results in Tables 6 and 7 tell that,

(1) Taking the basic cycle time and replenishment frequencies for discussion. The basic
cycle time $T$ under different discount schemes shows different features, $T_s$ and $K_s$ of JRP with
ND, with AD and with BD are the same, that is because all these $T_s$ and $K_s$ are obtained by
Eq (7), but $T_s$ of JRP with ID and MD are shortened as the value of $T$ is interfered $\nabla$ and $\Delta$, the

![Flow chart of the algorithm.](https://doi.org/10.1371/journal.pone.0194738.g002)

**Table 4. The data for the JRP case.**

| Item $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|----------|---|---|---|---|---|---|
| $D_i$    | 10,000 | 5,000 | 3,000 | 1,000 | 600 | 200 |
| $h_i$    | 1 | 1 | 1 | 1 | 1 | 1 |
| $s_i$    | 45 | 46 | 47 | 44 | 45 | 47 |
| $S$      | 100 | 100 | 100 | 100 | 100 | 100 |
| $c_i$    | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |

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replenishment frequencies of JRP with ID and JRP with MD are interfered by the obtained T, correspondingly.

(2) Taking the total cost for discussion, the results of TC reveal the roles and magnitudes of different schemes on TC. From the perspective of the supplier, the best discount offer for him/her is to adopt the incremental discount scheme, as it can bring him/her more benefits. When standing at the side of the buyer, the best offer is definitely the JRP with AD or with BD, as he/she can more cost decreasing than JRP with ND and with ID. However, the suppliers and buyers who want to build a long term stable supply chain, the MD scheme may be the most promising scheme form them. MD scheme plays mediate intermediate role comparing to JRP with AD (with BD) and JRP with ID, also TC under MD is smaller than that under ND in above case.

(3) On how different discount schemes impact the order quantity per order, Table 7 gives some hints. To the items ordered under ND, AD, BD, the role of introduction of discount mainly reflects in decreasing the total cost. To the items ordered under ID and MD, respectively, the role of introduction the discount reflects both in decreasing the total cost of JRP and order quantities of relevant items. Also, our model testified the assumption of [28] that the all-

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Table 5. Discount schedule.

| Item i | Discount types | Schedule | Price |
|--------|----------------|----------|-------|
| 1      | AD             | $0 \leq Q_1 < 500$ | $0.10$ |
|        |                | $500 \leq Q_1 < 1,000$ | $0.09$ |
|        |                | $1,000 \leq Q_1 < 2,000$ | $0.08$ |
|        |                | $Q_1 \geq 2,000$ | $0.07$ |
| 2      | ID             | $0 \leq Q_2 < 500$ | $0.10$ |
|        |                | $500 \leq Q_2 < 1,000$ | $0.09$ |
|        |                | $1,000 \leq Q_2 < 2,000$ | $0.08$ |
|        |                | $Q_2 \geq 2,000$ | $0.07$ |
| 3      | AD             | $0 \leq Q_3 < 500$ | $0.10$ |
|        |                | $500 \leq Q_3 < 1,000$ | $0.09$ |
|        |                | $Q_3 \geq 1,000$ | $0.08$ |
| 4      | ND             | — | $0.10$ |
| 5      | ID             | $0 \leq Q_5 < 300$ | $0.10$ |
|        |                | $Q_5 \geq 300$ | $0.09$ |
| 6      | BD             | $0 \leq C_{BD} < 10$ | $0.00$ |
|        |                | $10 \leq C_{BD}$ | $10$ |

Table 6. Comparisons of JRP with different discounts.

|        | K          | T     | TC       |
|--------|------------|-------|----------|
| JRP with ND | 1,1,1,1,1,4 | 0.1822 | 8,337.86 |
| JRP with AD  | 1,1,1,1,1,4 | 0.1822 | 8,049.86 |
| JRP with ID  | 1,1,1,1,2,2 | 0.1523 | 8,555.01 |
| JRP with BD  | 1,1,1,1,1,1 | 0.1822 | 8,049.96 |
| JRP with MD  | 1,1,1,1,2,4 | 0.1628 | 8,204.21 |

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Table 7. The schedule information under different discount schemes and Q’.

| Item | ND Schedule | Q’ | AD and ID Schedule | Price | Q’ | BD Schedule | Price | Q’ | Types | MD Schedule | Price | Q’ |
|------|-------------|----|-------------------|-------|----|-------------|-------|----|--------|-------------|-------|----|
| 1    | 0 ≤ Q₁ < 10000 | 1822 | 0 ≤ Q₁ ≤ 500 | 0.10% | 500 ≤ Q₁ ≤ 1000 | 0.09% | 1000 ≤ Q₁ ≤ 2000 | 0.08% | 1822 | AD | 0 ≤ Q₁ ≤ 500 | 0.10% | 1822 |
|      |             |     |                  |       | 500 ≤ Q₁ ≤ 1000 | 10% | 1000 ≤ Q₁ ≤ 2000 | 20% | 1822 |               |       |     |
|      |             |     |                  |       | Q₁ ≥ 2000 | 0.07% | Q₁ ≥ 2000 | 30% | Q₁ ≥ 2000 |               |       |     |
| 2    | 0 ≤ Q₂ < 5000 | 911 | 0 ≤ Q₂ ≤ 500 | 0.10% | 500 ≤ Q₂ ≤ 1000 | 0.09% | 1000 ≤ Q₂ ≤ 2000 | 0.08% | 911 | ID | 0 ≤ Q₂ ≤ 500 | 0.10% | 911 |
|      |             |     |                  |       | 500 ≤ Q₂ ≤ 1000 | 10% | 1000 ≤ Q₂ ≤ 2000 | 20% | 911 |               |       |     |
|      |             |     |                  |       | Q₂ ≥ 2000 | 0.07% | Q₂ ≥ 2000 | 30% | Q₂ ≥ 2000 |               |       |     |
| 3    | 0 ≤ Q₃ < 3000 | 547 | 0 ≤ Q₃ ≤ 500 | 0.10% | 1000 ≤ Q₃ ≤ 2000 | 0.09% | 2000 ≤ Q₃ ≤ 3000 | 0.08% | 547 | AD | 0 ≤ Q₃ ≤ 500 | 0.10% | 547 |
|      |             |     |                  |       | Q₃ ≥ 1000 | 0.08% | Q₃ ≥ 1000 | 20% | 547 |               |       |     |
| 4    | 0 ≤ Q₄ < 1000 | 182 | 0 ≤ Q₄ ≤ 1000 | 0.10% | 2000 ≤ Q₄ ≤ 3000 | 0.09% | 3000 ≤ Q₄ ≤ 4000 | 0.08% | 182 | ND | 0 ≤ Q₄ ≤ 1000 | 0.10% | 182 |
| 5    | 0 ≤ Q₅ < 600 | 109 | 0 ≤ Q₅ ≤ 300 | 0.10% | 3000 ≤ Q₅ ≤ 4000 | 0.09% | 4000 ≤ Q₅ ≤ 5000 | 0.08% | 109 | ID | 0 ≤ Q₅ ≤ 300 | 0.10% | 109 |
|      |             |     |                  |       | Q₅ ≥ 300 | 0.09% | Q₅ ≥ 300 | 10% | Q₅ ≥ 300 |               |       |     |
| 6    | 0 ≤ Q₆ < 200 | 146 | 0 ≤ Q₆ ≤ 100 | 0.00% | 4000 ≤ Q₆ ≤ 5000 | 0.09% | 5000 ≤ Q₆ ≤ 6000 | 0.08% | 146 | BD | 0 ≤ Q₆ ≤ 100 | 0.00% | 146 |
|      |             |     |                  |       | Q₆ ≥ 100 | 10.0% | Q₆ ≥ 100 | 10% | Q₆ ≥ 100 |               |       |     |

Q’: Q’ under AD, Q’’: Q’’ under ID.

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5 Conclusions

In this paper, we provide a new focus on JRP with multiple discount schemes. By referring to the work of predecessors on supplier selection and multi-item replenishment considering different discount types, a new JRP model is constructed considering three discount types simultaneously to investigate the joint effects of discount schemes on the decisions of replenishment cycle time and frequencies of each item. In light of the NP-hard nature of JRP, a heuristic algorithm is presented to solve the proposed model. Through numerical experiments on different JRPs with different discount type combinations, we verify that both the supplier and the buyer would be benefited by formulating a multiple discount contract.

This research aims to give a new extension of JRP and a simple heuristic for solving the new model, but the performance of the proposed heuristic is not fully verified comparing to the existed evolutionary algorithms. Thus, in our following research, we would spare our energy in finding some more efficient and effective algorithms to solve the proposed model, and the other is to extend the current problem to JRP with delivery consideration.

Author Contributions

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