Weak measurement combined with quantum delayed-choice experiment and implementation in optomechanical system

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Abstract. Weak measurement [1,19] combined with quantum delayed-choice experiment that use quantum beam splitter instead of the beam splitter give rise to a surprising amplification effect, i.e., counterintuitive negative amplification effect. We show that this effect is caused by the wave and particle behaviours of the system to be and can’t be explained by a semiclassical wave theory, due to the entanglement of the system and the ancilla in quantum beam splitter. The amplification mechanism about wave-particle duality in quantum mechanics lead us to a scheme for implementation of weak measurement in optomechanical system.

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1 Introduction

Postselection weak measurement was first proposed by Aharonov, Albert and Vaidman [1], where a pointer is weakly coupled to the system to be measured. It can help us to understand some counter-intuitive quantum paradoxes [2] and it is also used to measure and amplify small physical quantities or effects which are not directly detected by conventional techniques in experiment, such as direct measurement of wave function [3] and beam deflection [4,5]. Recently, the basics and application of weak measurement have been reviewed in [6].

Although postselection weak measurement have many applications, its application in optomechanics was rarely reported. The optomechanical system usually refers to a high finesse optical cavity with a tiny mirror attached to micromechanical oscillator [7,8], where the light in the cavity can give a radiation force on the mirror. When there is only one photon in the cavity the displacement of the mirror caused by the photon is hard to be measured since it is much smaller than the spread of the mirror wave packet. Recently, in the optomechanical system combined with weak measurement, the weak measurement amplification scheme that the displacement of the mirror caused by single photon can be amplified is proposed [9], where the optomechanical system is embedded in the March-Zehnder interferometer [10]. The amplification effect in [9] is obtained by retaining the Kerr phase in the Ref. [11].

It is known to all that the Mach-Zehnder interferometer [10], including the input beam splitter and the output beam splitter, can show wave-particle duality. When a photon enters the input beam splitter, the photon behaved as a wave behaviour if the output beam splitter is present or behaved as a particle behaviour if the output beam splitter is absent. Based on the Mach-Zehnder interferometer, the the delayed-choice experiment [12,13] is proposed to demonstrate Bohr’s complementarity [14] and it show that wave-particle duality depend on the classical detecting devices, i.e., whether the output beam splitter is inserted or removed. However, recently Ionicioiu et al propose a scheme for quantum delayed-choice experiment [15] based on quantum beam splitter in stead of the beam splitter and in combined with weak measurement whether there are some new features. In the paper, we will first give a general discussion about weak measurements [1,19] combined with quantum delayed-choice experiment that use quantum beam splitter in stead of the beam splitter and in combined with weak measurement whether there are some new features. In the paper, we will first give a general discussion about weak measurements [1,19] combined with quantum delayed-choice experiment that use quantum beam splitter in stead of the beam splitter and in combined with weak measurement whether there are some new features.
ter is to control the choice of the measured system basis. When the quantum beam splitter possessing the entanglement of the system and the ancilla play a role of the postselection, we find that the wave and particle behaviours of the measured system give rise to a surprising amplification effect and this amplification effect can’t be explained by Maxwell’s equations [20], due to the entanglement of the system and the ancilla in quantum beam splitter. We first show that the wave-particle duality can be used as the generation mechanism for weak measurement amplification, that has not previously been revealed. The scheme in this paper provide a new method for studying and exploiting postselection and weak measurement [1,19] in three-qubit system. On the other hand, the perspective in this paper lead us to a scheme for implementation of weak measurement [1,19] in optomechanical system. The amplification mechanism achieved here is different from the one in [9] which is caused by the Kerr phase. Moreover, the displacement of the mirror’s position in optomechanical system can appear within a large evolution time zone, which is a counterintuitive negative amplification effect.

The rest of this paper is organized in the following way. In Sec. 2, we give a general discussion about weak measurements combined with quantum delayed-choice experiment. In Sec. 3, we present a scheme for implementation of weak measurement in optomechanical system, and In Sec. 4, we give the conclusion about the work.

2 Weak measurement amplification via quantum beam splitter

To set the scene, we assume a general schematic diagram illustrated in Fig. 1. In the following the scheme involved a simple three-qubit system. The pointer is a continuous system and initially prepared in the ground state of the system, where $|0\rangle_s$ is an eigenstate of $\hat{\sigma}_z$. Set $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_s + |1\rangle_s)$ and $|\rangle = -|\rangle$, where $|+\rangle$ and $|\rangle$ are eigenstates of $\hat{\sigma}_z$. The ancilla is an arbitrary state $\cos \alpha |0\rangle_{anc} + \sin \alpha |1\rangle_{anc}$. Hadamard gate usually plays a role of the beam splitters in quantum circuits [21]. The Hadamard transformation is

$$|0\rangle_s = \frac{1}{\sqrt{2}} (|0\rangle_s + |1\rangle_s),$$

$$|1\rangle_s = \frac{1}{\sqrt{2}} (|0\rangle_s - |1\rangle_s),$$

(2)

where $|0\rangle_s$ and $|1\rangle_s$ are two input modes entering the beam splitter, $|0\rangle_s$ and $|1\rangle_s$ are two output modes exiting the beam splitter. It is obvious that when Hadamard gate is present, the measured system show wave behaviours. However, if Hadamard gate is absent, the transformation between input and output modes is

$$|0\rangle_s = |0\rangle_s,$$

$$|1\rangle_s = |1\rangle_s,$$

(3)

showing particle behaviours. However in Ref. [15], the Controlled-Hadamard C(H) gate is equivalent to the quantum beam splitter, where the quantum beam splitter controlled by ancilla [12] can be put in superposition of being present and absent. In other word, the state after quantum beam splitter in the interferometer is given by

$$|\psi_{QBS}\rangle = \cos \alpha |\text{particle}\rangle |0\rangle_{anc} + \sin \alpha |\text{wave}\rangle |1\rangle_{anc},$$

(4)

Defining an annihilation operator $\hat{c} = \frac{1}{\sqrt{2} \hbar} \hat{q} + i \hat{p}$, where $\hat{q}$ is position operator (the canonical variable conjugates to $\hat{p}$ and $[\hat{q}, \hat{p}] = i \hbar$). According to the result of Ref. [19], the Hamiltonian above can be rewritten as

$$\hat{H}_I = -i \frac{\hbar^2}{2\sigma} \hat{\sigma}_z (\hat{c} - \hat{c}^\dagger),$$

(5)

where $\sigma$ is the zero-point fluctuation and $\sqrt{2} \sigma$ is the width of the Gaussian distribution. Then the time evolution of the system and the pointer is given by

$$e^{-i \int H dt} |+\rangle = \exp[-\eta \hat{\sigma}_z (\hat{c} - \hat{c}^\dagger)\rangle |+\rangle |0\rangle_m$$

$$= \frac{1}{\sqrt{2}} (|0\rangle_s D(\eta) |0\rangle_m$$

$$+ |1\rangle_s D(-\eta) |0\rangle_m),$$

(6)

where $D(\eta) = \exp[\eta \hat{c}^\dagger - \eta^* \hat{c}]$ with $\eta = \frac{\hbar \sigma}{2}$ is a displacement operator and $\eta \ll 1$. After the quantum beam splitter and using the transformation of Eq. (2) and Eq. (3),
then the state of the total system becomes
\[
\begin{align*}
\frac{\cos\alpha}{\sqrt{2}}&(\langle 0\rangle_s D(\eta)|0\rangle_m + |1\rangle_s D(-\eta)|0\rangle_m)|0\rangle_{anc} \\
+ \frac{\sin\alpha}{2}&((|+\rangle(D(\eta)|0\rangle_m + D(-\eta)|0\rangle_m) \\
+ |\rangle(D(\eta)|0\rangle_m - D(-\eta)|0\rangle_m)\rangle_{anc}.
\end{align*}
\]
When the state $|1\rangle$ is detected, in the language of weak measurement we simultaneously measure the system in the orthogonal postselection state of wave behaviours $|\rangle$ [11, 9] and in the state of the particle behaviour $|1\rangle_s$ before the ancilla, i.e., before choosing if quantum beam splitter is present or absent, then the state of the pointer and the ancilla becomes
\[
|\psi\rangle_m = \frac{\cos\alpha}{\sqrt{2}} D(\eta)|0\rangle_m |0\rangle_{anc}
+ \frac{\sin\alpha}{2} (D(\eta)|0\rangle_m - D(-\eta)|0\rangle_m)|1\rangle_{anc}.
\]
Results indicate that even though the system is measured, we can vary the system from wave to particle behaviour through adjusting the angle $\alpha$ in ancilla. It is interesting to consider the fact that when the ancilla is measured in a superposition state, such as $|\varphi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{anc} - |1\rangle_{anc})$, then the final state of the mirror is a superposition state,
\[
|\psi\rangle_m = \frac{\cos\alpha}{\sqrt{2}} D(-\eta)|0\rangle_m
- \frac{\sin\alpha}{2} (D(\eta)|0\rangle_m - D(-\eta)|0\rangle_m).
\]
We note that $\cos\alpha$ is caused by the particle behaviour of the system, while $\sin\alpha$ is due to wave behaviour of the system. From the derivation of Eq. (9), it will be shown that the key advantage of the ancilla measurement is to control the choice of the measured system basis. This indicate that the Controlled-Hadamard gate or quantum beam splitter play a role of the postselection. Moreover, the state of Eq. (9) can also be obtained when we simultaneously measure the system and the ancilla in the entangled states, i.e., the postselection state
\[
|\phi\rangle = |1\rangle_s|0\rangle_{anc} - |\rangle|1\rangle_{anc}.
\]
Therefore, the superposition of the pointer in Eq. (9) caused by the particle and wave behaviours of the system is no classical analog.

Based on the Eq. (9), we can then perform a small quantity expansion about $\eta$ till the second order, where $\eta \ll 1$, then
\[
|\psi\rangle_m \approx \frac{\cos\alpha}{\sqrt{2}} (|0\rangle_m + \eta|1\rangle_m) - \eta \sin\alpha |1\rangle_m.
\]
For $\alpha = \pi/2$, the beam splitter is present, i.e., only wave behaviour occur, and we find the state $|\psi\rangle_m$ proportional to $|1\rangle_m$, which correspond to the case that the post-selected state of the system is orthogonal to the initial state of the system $|1\rangle$, the displacement of pointer’s position is zero.

For $\pi/2 - \alpha \ll 1$, the particle and wave behaviour simultaneously occur, here we use the approximation $\cos\alpha \approx \pi/2 - \alpha$ and $\sin\alpha \approx 1$ to get the superposition state
\[
|\psi\rangle_m \approx (\pi/2 - \alpha/\sqrt{2})|0\rangle_m - \eta|1\rangle_m.
\]
For Eq. (12), it can be seen that $\langle \hat{q} \rangle$ is none-zero (amplification) since the superposition between the states $|0\rangle_m$ and $|1\rangle_m$ is caused by the particle behaviour of the system, while the coefficient $\eta$ is due to $\eta \sin\alpha$ caused by the wave behaviour of the system.

The average displacement of the pointer position $\hat{q}$ is
\[
\langle \hat{q} \rangle = \frac{\langle |\psi\rangle_m |\psi\rangle_m^*\rangle_m}{\langle |\psi\rangle_m |\psi\rangle_m^*\rangle_m} - \langle 0|\hat{q}|0\rangle.
\]
where the coefficient $\pi/2 - \alpha/\sqrt{2}$ is due to cosm generated by the particle behaviour of the system, while the coefficient $\eta$ is due to $\eta \sin\alpha$ caused by the wave behaviour of the system.

![Fig. 2.](image-url)
3 Implementation in optomechanical system

In the following we will consider an weak measurement model in optomechanical system combined with quantum delayed-choice experiment.

3.1 The optomechanical model

Based on a Mach-Zehnder interferometer, a schematic diagram of quantum delayed-choice experiment to be considered is shown in Fig. 2. The optomechanical cavity $A$ is embedded in one arm of the March-Zehnder interferometer and a stationary Fabry-Pérot cavity $B$ is placed in another arm. And the first beam splitter is symmetric, but the second one is quantum beam splitter [18] which is made up of two components. The first is a polarization-dependent beam splitter (PDBS) which shows 100% reflection for horizontal polarized photons and 50/50 reflection/transmission for vertical polarized photons. The second is erasure device consisting of polarizing beam splitters (PBS1 and PBS2). In Fig. 2 on the left side, the electro-optic phase modulator is to rotate the polarization state of the ancilla’s photon by an angle $\alpha$ or $\beta$.

The Hamiltonian of optomechanical system in the interferometer is expressed as followed:

$$H = \hbar \omega_0 (a^\dagger a + b^\dagger b) + \hbar \omega_m c^\dagger c - \hbar g a^\dagger a (c^\dagger + c), \quad (14)$$

where $\hbar$ is Planck’s constant, $\omega_0$ is frequency of the optical cavity $A$, $B$ and the corresponding annihilation operators are $a$ and $b$, $\omega_m$ is frequency of mechanical system and the corresponding annihilation operator is $c$, and the optomechanical coupling strength $g = \frac{h}{2m} \sigma$, where $L$ is the length of the cavity $A$ or $B$, $\sigma = (h/2m\omega_m)^{1/2}$ which is the zero-point fluctuation and $m$ is the mass of mechanical system. Here it is a weak measurement model where the mirror is used as the pointer to measure the number of photon in cavity $A$. Noted that because $a^\dagger a$ of the Eq. (14) corresponds to $\hat{\sigma}_z$ of the Hamiltonian in the standard scenario of weak measurement and $c + c^\dagger$ corresponds to $\hat{p}$.

In the optomechanical cavity $A$ (see Fig. 2), if the initial state of the mirror is prepared at the ground state $|0\rangle$ and when one photon interacts with the mirror, the mirror will become a coherent displacement state $|\bar{x}(t)\rangle = e^{i\phi(t)}|\varphi(t)\rangle_m$, where $e^{i\phi(t)}$ is the Kerr phase of one photon with $\phi(t) = k^2 (\omega_m t - \sin \omega_m t)$, $\varphi(t) = k(1 - e^{-i\omega_m t})$ and $k = g/\omega_m$. According to the expression of the position displacement $\langle \bar{x}(t)|\bar{x}(t)\rangle = (q/q)|0\rangle$, then the displacement of the mirror is not more than $4\sigma$ for any time $t$ and can not be bigger than $0.25$ in weak coupling condition [26]. Therefore, the displacement of the mirror caused by one photon can not be detected. However, if considering optomechanical cavity with weak measurement combined with quantum delayed-choice experiment, we can amplify the mirror’s displacement.

3.2 Amplification with postselected weak measurement in optomechanics

Suppose the one of the photons from the maximally polarization-entangled Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|1, H\rangle_{anc} |1, H\rangle_s + |1, V\rangle_{anc} |1, V\rangle_s)$ is input into the interferometer, where $H$ ($V$) represent horizontal (vertical) polarized photonic modes, then the state of the photons after the first beam splitter becomes

$$|\psi_1\rangle = \frac{1}{2} [ |1, H\rangle_{anc} (|1, H\rangle_A + |1, H\rangle_B) + |1, V\rangle_{anc} (|1, V\rangle_A + |1, V\rangle_B) ]. \quad (15)$$

After interacting weakly with the mirror prepared at ground state $|0\rangle_m$ that can be achieved using sideband-resolved cooling technique [24, 25], the state of the photons and the mirror is

$$|\psi_2\rangle = \frac{1}{2} [ |1, H\rangle_{anc} (|1, H\rangle_A |\xi(t)\rangle_m + |1, H\rangle_B |0\rangle_m) + |1, V\rangle_{anc} (|1, V\rangle_A |\xi(t)\rangle_m + |1, V\rangle_B |0\rangle_m)]. \quad (16)$$

After the photon passes through the polarization-dependent beam splitter (PDBS), the state of the overall system becomes

$$|\psi_{PDBS}\rangle = \frac{1}{2} [ |1, H\rangle_{anc} (|1, H\rangle_A |\xi(t)\rangle_m + |1, H\rangle_B |0\rangle_m) + \frac{1}{\sqrt{2}} |1, V\rangle_{anc} (|1, V\rangle_A |\xi(t)\rangle_m + |1, V\rangle_B |0\rangle_m)]. \quad (17)$$

In order to erase all polarization information about PDBS output, we make the state of Eq. (17) go through the polarizing beam splitter (PBS1 and PBS2) oriented at $45^\circ$ to $\{H, V\}$ basis. Then Eq. (17) becomes

$$|\psi\rangle = \frac{1}{\sqrt{2}} [ |1, H\rangle_{anc} |\text{particle}\rangle_m + |1, V\rangle_{anc} |\text{wave}\rangle_m ] \quad (18)$$

with

$$|\text{particle}\rangle_m = \frac{1}{2} (|1\rangle_{\alpha'} + |1\rangle_{\alpha''}) |\xi(t)\rangle_m + (|1\rangle_{\alpha'} + |1\rangle_{\alpha''}) |0\rangle_m \quad (19)$$

and

$$|\text{wave}\rangle_m = \frac{1}{\sqrt{2}} (|\alpha\rangle_{\alpha'} + |\alpha\rangle_{\alpha''}) (|\xi(t)\rangle_m + |0\rangle_m) + (|\alpha\rangle_{\alpha'} + |\alpha\rangle_{\alpha''}) (|\xi(t)\rangle_m + |0\rangle_m). \quad (20)$$

Using the electro-optic phase modulator (EOM) rotate the polarization state of ancilla by an angle $\alpha$, we can obtain

$$|\psi_{QBS}\rangle = \frac{1}{\sqrt{2}} (|1, H\rangle_{anc} (\cos \alpha |\text{particle}\rangle_m - \sin \alpha |\text{wave}\rangle_m) + |1, V\rangle_{anc} (\cos \alpha |\text{wave}\rangle_m + \sin \alpha |\text{particle}\rangle_m)) \quad (21)$$
Based on the polarizing beam splitter (PBS) and detecting the ancilla’s photon in $D_H$, then we find that

$$|\Psi_{QBS}\rangle = \frac{1}{\sqrt{2}} (\cos \alpha |\text{particle}\rangle_m - \sin \alpha |\text{wave}\rangle_m) \tag{22}$$

Here we only consider detector $D_{k'}$. When a photon is detected at the dark port $D_{k'}$, in the language of weak measurement we simultaneously measure the single-photon in the orthogonal postselection state of wave behaviour $\sqrt{2}(|1\rangle_A - |1\rangle_B) \tag{11,9}$ and in the state of the particle behaviour $|1\rangle_A$. Then the state of the mirror after postselection is

$$|\psi_m\rangle = \frac{\cos \alpha}{2\sqrt{2}} (\xi(t))_m - \frac{\sin \alpha}{4} (\xi(t))_m - |0\rangle_m. \tag{23}$$

We note that $\cos \alpha$ is due to the particle behaviour of the photon, while $\sin \alpha$ is due to wave behaviour of the photon.

The average displacement of the mirror’s position operator $\hat{q} = (\hat{c} + \hat{c}^\dagger)$ is given by

$$\langle q \rangle = \frac{Tr(|\psi_m\rangle\langle \psi_m|q) - Tr(|0\rangle_m\langle 0|_m q)}{Tr(|\psi_m\rangle\langle \psi_m|) - Tr(|0\rangle_m\langle 0|_m)}. \tag{24}$$

Substituting Eq. (23) into Eq. (24), then

$$\langle q(t) \rangle = \sigma [(2 - \sin^2 \alpha - \sqrt{2}\sin 2\alpha)(\varphi(t) + \varphi^*(t)) + (\sin 2\alpha/\sqrt{2} - \sin^2 \alpha)e^{-i\phi(t)}(e^{i\phi(t)}\varphi(t) + e^{-i\phi(t)}\varphi^*(t))]/2 - \sqrt{2}\sin 2\alpha + (\sin 2\alpha/\sqrt{2} + \sin^2 \alpha)(e^{i\phi(t)} + e^{-i\phi(t)})] \tag{25}$$

To see this clearly, in Fig. 3, we plot the average displacement $\langle q(t) \rangle/\sigma$ of the mirror versus the time $\omega_m t$ with $k = 0.005$ for different angles $\alpha$ [$\alpha = 0.996 \times \pi/2$ (red-solid), $0.9995 \times \pi/2$ (orange-dashed) and $\pi/2$ (blue-dotted-dashed)]. It can be seen clearly from Fig. 2 that when $\alpha = 0.996 \times \pi/2$ the amplification (red-solid) is a time function of period $2\pi$ and at time $\omega_m t = (2n + 1)\pi$ $(n = 0, 1, 2, \cdots)$ the amplifications can reach strong coupling limits (the level of the ground state fluctuation) $\langle q \rangle = -\sigma$ [26]. However, when $\alpha \rightarrow \pi/2$, such as $\alpha = 0.995 \times \pi/2$, the amplification around time $\omega_m t = 2n\pi$ $(n = 1, 2, \cdots)$ is very prominent and reaches the level of the ground state fluctuation $\langle q \rangle = \pm \sigma$. The result is caused by the Kerr phase which can’t be explained by the standard weak measurement [1,19] and has been discussed in detail in Ref. [9]. Note that in the optomechanical cavity A (see Fig. 2) the maximal displacement of the mirror caused by one photon is $4k\sigma$ and the displacement $-\sigma$ can be obtained using quantum beam splitter in weak measurement, hence the amplification factor can be $Q = 1/4k$ which is $-50$ when $k = 0.005$. Moreover, the negative amplification is a counterintuitive result since intuitively there cannot exist negative displacement relative to the direction of the photon propagation.

### 3.3 Small quantity expansion about time for amplification

To explain the amplification phenomenon shown in Fig. 3, we only give an analysis about the special case that the maximal amplification appearing at the time $T = (2n + 1)\pi$ $(n = 0, 1, 2, \cdots)$. In the same way as the Eq. (12), for Eq. (23) we can perform a small quantity expansion about time $T$ till the second order. Suppose that $|\omega_m t - T| \ll 1$, and $k \ll 1$, then

$$|\psi_m\rangle \approx \frac{\cos \alpha}{\sqrt{2}} (|0\rangle_\pi + 2|k\rangle_1\alpha) - k \sin \alpha |1\rangle_m. \tag{26}$$

When $\pi/2 - \alpha \ll 1$, here we use the approximation $\cos \alpha \approx \pi/2 - \alpha$ and $\sin \alpha \approx 1$ to get the superposition state

$$|\psi_m\rangle \approx (\pi/2 - \alpha)/\sqrt{2}|0\rangle_m - k \sin \alpha |1\rangle_m. \tag{27}$$

Noted that the coefficient $(\pi/2 - \alpha)/\sqrt{2}$ is due to $\cos \alpha$ by generated the particle behaviour of the photon, while the coefficient $k$ is due to $k \sin \alpha$ caused by the wave behaviour of the photon. Substituting Eq. (27) into Eq. (24), the average value of the displacement operator $q$ is given by

$$\langle q(t) \rangle |_{|\omega_m t - T| \ll 1} = -\sigma \sqrt{2}k(\pi/2 - \alpha)/[k^2 + (\pi/2 - \alpha)^2], \tag{28}$$

which can get its minimal value $-\sigma$ when $k = (\pi/2 - \alpha)/\sqrt{2}$. Hence the state of the mirror achieving the maximal negative amplification is $\sqrt{2}(|0\rangle_m - |1\rangle_m)$. The results show that the key to the amplification is the superposition of the vacuum state and one phonon state of the mirror, which is due to the superposition of wave and particle behaviours of the photon. Such result has not previously been revealed. For Eq. (27), the amplification mechanism is accord with standard weak measurement [1,19] and it is caused by proper near-orthogonal postselection but different from the amplification mechanism for Eq. (16) in Ref. [9] which is caused by the Kerr phase in this case of orthogonal postselection.

Next we would like to discuss the amplification of momentum variable $p$ of the mirror. Substituting Eq. (23) into Eq. (24) which uses $p = -i(c - c^\dagger)\frac{\hbar}{2\sigma}$ instead of $q$, then we have

$$\langle p(t) \rangle = -i\frac{\hbar}{2\sigma}[(2 - \sin^2 \alpha - \sqrt{2}\sin 2\alpha)(\varphi(t) - \varphi^*(t)) + (\sin 2\alpha/\sqrt{2} - \sin^2 \alpha)e^{-i\phi(t)}(e^{i\phi(t)}\varphi(t) - e^{-i\phi(t)}\varphi^*(t))] /2 - \sqrt{2}\sin 2\alpha + (\sin 2\alpha/\sqrt{2} - \sin^2 \alpha)(e^{i\phi(t)} + e^{-i\phi(t)})] \tag{29}$$

The average displacement $\langle p(t) \rangle/\sqrt{2}$ of the mirror is shown in Fig. 4 as a function of $\omega_m t$ with $k = 0.005$ for different angle $\alpha$ [$\alpha = 0.996 \times \pi/2$ (red-solid), $0.999 \times \pi/2$ (green-solid) and $\pi/2$ (blue)].
It can be seen clearly that if the decay rate of the cavity which is due to the superposition of wave and particle behaviours of the measured system gives rise to a surprising amplification effect. We find that the wave-particle duality of the measured system can be used as a generation mechanism for weak measurement amplification, that has not previously been revealed. This amplification effect can't be explained by Maxwell's equations [20], due to the entanglement of the system and the ancilla in quantum beam splitter. This amplification mechanism about the wave-particle duality of the measured system lead us to a scheme for implementation in optomechanical system. Moreover, the displacement of the mirror's position in optomechanical system can appear within a large evolution time zone, which is a counterintuitive negative amplification effect. The results provide us an alternative method to discuss weak measurement and also deepen our understanding of the weak measurement.

4 Conclusion

In conclusion, we use the quantum beam splitter in stead of the beam splitter in the delayed-choice experiment. When combined with the weak measurement and the quantum beam splitter play a role of the postselection, it will be shown that the wave and particle behaviours of the measured system give rise to a surprising amplification effect. We find that the wave-particle duality of the measured system can be used as generation mechanism for weak measurement amplification, that has not previously been revealed. This amplification effect can't be explained by Maxwell's equations [20], due to the entanglement of the system and the ancilla in quantum beam splitter. This amplification mechanism about the wave-particle duality of the measured system lead us to a scheme for implementation in optomechanical system. Moreover, the displacement of the mirror's position in optomechanical system can appear within a large evolution time zone, which is a counterintuitive negative amplification effect. The results provide us an alternative method to discuss weak measurement and also deepen our understanding of the weak measurement.

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