Spectra of heavy-light mesons at finite temperature

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Abstract

Spectra of heavy-light meson are studied using potential model and thermofield dynamics prescription. The mass spectra of different heavy-light mesons are calculated at different temperatures and compared with those at $T = 0$. It is found that the binding mass of heavy-light meson decreases as temperature increases.

1 Introduction

Recent experiments on heavy-light mesons at BABAR [1], CLEO [2] and BELLE [3] collaborations stimulated extensive theoretical studies. Within QCD, relativistic potential models, chiral perturbation theory and non-relativistic potential models that are analogs of the hydrogen-like atoms. All numerical work is based on lattice QCD gives probably the best results [4]-[10].

An effective method to explore hadron properties at finite temperatures is the lattice QCD approach that leads to results of higher precision for spectra and transition rates. However it requires powerful computer resources. It is therefore interesting to explore methods that are simpler and yield interesting results for the hadron spectra at finite temperature. Recently such a method has been used to calculate quarkonium spectra at finite temperature in potential models [11]. In this paper we extend this method to heavy-light mesons with its dynamics described by the Dirac equation. The finite temperature effects are included by using thermofield dynamics (TFD), a real time finite temperature field theory. We apply the prescription to the Dirac Hamiltonian with Coulomb and a linear potential. The physical content of TFD will be described below. It is important to note that TFD has been used to study the Casimir effect and quantum chaos in Yang-Mills-Higgs system at finite temperature [12].

With the results emerging from RHIC (Relativistic Heavy Ion Collider) and the anticipated results from LHC (Large Hadron Collider) properties at finite temperature...
will be needed. In the process of converting baryonic system to quark-gluon plasma and subsequently the hadronization of this plasma would have mesons and baryons at finite temperature. This would require how these hadrons behave at high temperatures. The question of reactions, change of coupling constants and decay rates finite temperatures have been considered already [13]. The present calculation provides additional information as to how the mass changes with temperature.

The plan of the paper is to consider briefly the description of the heavy-light mesons using the Dirac equation at zero temperature. In section 3, we give a short review of TFD in order to make clear the finite temperature formulation in general, stating clearly the underlying details of the theory. In section 4, TFD is applied to calculate the spectra of heavy-light mesons at finite temperature. Some concluding remarks are given in the final section.

2 Dirac equation for heavy-light meson

Starting point for our study of finite-temperature mass spectra of heavy-light meson is the Dirac equation written as [14, 15]

\[ i \frac{\partial \Psi}{\partial t} = H \Psi \] (1)

where \( H = \alpha \vec{p} + \beta (m_q + \lambda r) - \frac{Z}{r} + V_0 \). We note that the confining part of the potential is included into this equation as a Lorentz scalar (not as fourth component of the 4-vector). Introducing the following time scaling \( \tau = r^{-1} t \), we can rewrite this equation as

\[ i \frac{\partial \Psi}{\partial \tau} = r H \Psi. \] (2)

Using standard substitution \( \Psi(r, \theta, \tau) = e^{-iE\tau} \Psi(r, \theta) \) and separating angular variables we get the time-independent radial equation:

\[ E \Psi = r H_r \Psi, \] (3)

where

\[ H_r = \begin{pmatrix} m_q + \lambda r + V_0 - \frac{Z}{r} & -\frac{d}{dr} + \frac{\epsilon}{r} \\ \frac{d}{dr} + \frac{\epsilon}{r} & -m_q - \lambda r - V_0 - \frac{Z}{r} \end{pmatrix} \] (4)

being the radial Dirac Hamiltonian. For our purpose we need to represent this Hamiltonian in terms of annihilation and creation operators which are defined as

\[ a = \frac{1}{\sqrt{2}} \frac{d}{dr} + \frac{1}{\sqrt{2}} r, \quad a^\dagger = -\frac{1}{\sqrt{2}} \frac{d}{dr} + \frac{1}{\sqrt{2}} r. \] (5)

It is easy to see that these operators obey the following commutation relation

\[ [a, a^\dagger] = 1. \]

The radial Dirac Hamiltonian (4) in terms of these operators can be written as

\[ H_r = r^{-1} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \] (6)
where
\[ A = \frac{m_q + V_0}{\sqrt{2}}(a + a^\dagger) + \frac{\lambda}{2}(a + a^\dagger)^2 - Z, \]
\[ B = -\frac{1}{2}(a + a^\dagger)(a - a^\dagger) + \kappa, \]
\[ C = \frac{1}{2}(a + a^\dagger)(a - a^\dagger) + \kappa, \]
and
\[ D = -\frac{m_q + V_0}{\sqrt{2}}(a + a^\dagger) - \frac{\lambda}{2}(a + a^\dagger)^2 - Z. \]

The eigenvalues of the Hamiltonian written in terms of annihilation and creation operators can be calculated by diagonalizing it in a harmonic oscillator basis. The wave function is expanded in terms of nonrelativistic harmonic oscillator wave functions as
\[ |\Psi\rangle = \sum_n \begin{pmatrix} a_n \\ b_n \end{pmatrix} |n\rangle. \quad (7) \]

Then the energy eigenvalues of the heavy-light meson are to be calculated by diagonalization of the matrix
\[ E = \langle \Psi' | rH_r | \Psi \rangle. \]

This method will be combined with the thermofield dynamics to calculate spectra of heavy-light meson at finite temperature. The next section presents briefly basic thermofield dynamics.

3 Thermofield Dynamics

Thermofield dynamics is a real-time operator formalism of quantum field theory at finite temperature. It has been recognized that for a theory at finite temperature, the Hilbert space has to be doubled [16, 17]. The usual Hilbert space may be defined by creation, \( a^\dagger(k) \) and annihilation operators, \( a(k) \). Then the second Hilbert space is defined by tilde operators, \( \tilde{a}^\dagger(k) \) for creation and \( \tilde{a}(k) \) for annihilation operators. These operators, for a boson system, satisfy
\[ [a(k), a^\dagger(k')] = \delta_{k,k'}, \]
\[ [\tilde{a}(k), \tilde{a}^\dagger(k')] = \delta_{k,k'}, \]
\[ [a(k), \tilde{a}^\dagger(k')] = 0 \]
and all other commutators are zero. For fermion systems anti-commutator are to be used.

At finite temperature a Bogolyubov transformation is used to mix these two sets of operators to obtain the finite temperature creation and annihilation operators. Any operator \( A(x) \) is then written at finite temperature as
\[ A(x, \theta) = U(\theta) A(x) U^{-1}(\theta) \]
where \( U(\theta) \) is the Bogolyubov transformation, i.e.
\[ U(\theta) = e^{-iG(\theta(\beta))} \]
where \( G = i\theta(\beta)(a^\dagger a^\dagger - a\tilde{a}) \) with \( \beta = \frac{1}{k_B T} \). Here \( T \) is the temperature and \( k_B \) is the Boltzmann constant.

This implies that

\[
\begin{align*}
\tilde{a}(k, \theta) &= U(\theta)\tilde{a}(k)U^{-1}(\theta) = u(\theta)\tilde{a}(k) - \vartheta(\theta)a^\dagger(k) \\
\tilde{a}^\dagger(k, \theta) &= U(\theta)\tilde{a}^\dagger(k)U^{-1}(\theta) = u(\theta)\tilde{a}^\dagger(k) - \vartheta(\theta)a(k)
\end{align*}
\]

where \( u(\theta) = \cosh(\theta) \), \( \vartheta(\theta) = \sinh(\theta) \) such that \( u^2(\theta) - \vartheta^2(\theta) = 1 \). The vacuum state at finite temperature is defined similarly as

\[
\left| 0(\theta) \right> = U(\theta) \left| 0, \tilde{0} \right>.
\]

Then the temperature dependent creation and annihilation operators acting on the vacuum state, \( \left| 0(\theta) \right> \) give

\[
\begin{align*}
a(k, \theta) \left| 0(\theta) \right> &= \tilde{a}(k, \theta) \left| 0(\theta) \right> = 0.
\end{align*}
\]

Thus there is a procedure to work with the two set of operators

\[
\begin{align*}
(A_i, A_j) &= (\tilde{A}_i, \tilde{A}_j) \\
(cA_i + A_j) &= c^*\tilde{A}_i + \tilde{A}_j \\
A_i^\dagger &= (\tilde{A}_i)^\dagger \\
(\tilde{A}_i)^\dagger &= A_i
\end{align*}
\]

and \( [\tilde{A}_i, A_j] = 0 \). It is to be noted that \( \vartheta^2(\theta) = \sinh^2 \theta = [e^\beta - 1]^{-1} \) where \( \beta = \frac{\omega}{k_B T} \).

From a group theoretical approach, it has been established that for any kinematical symmetry there are two sets of operators, physical observable \( \tilde{O} \) and generators of symmetry, \( \tilde{O} = O - \hat{O} \). This establishes a connection to an algebraic approach based on kinematic symmetry and the doubling does not happen due to an arbitrary ansatz [18].

Such a procedure has been used to calculate scattering cross section and decay rates at finite temperature. The use of the Bogolyubov transformations that mixes the operators from the two Hilbert spaces is a reminder of superconductivity where this implied a condensation of quanta in the vacuum state.

Thus TFD is a powerful tool for exploring quantum dynamics of a system at finite temperature provided its Hamiltonian can be represented in terms of annihilation and creation operators. It has found many applications in condensed matter physics [19, 20, 21], especially in superconductivity theory and related topics. Recently TFD has been applied to explore quantum chaos in the Yang-Mills-Higgs system [22] and for the calculation of the spectra of a strongly interacting bound system [11]. In this work we apply the TFD prescription to the Jaynes-Cummings model. It should be noted that TFD has been applied earlier to the Jaynes-Cummings model [23] where the thermal noise effects in quantum optics are studied. In the next section we apply the TFD prescription for the calculation of the energy spectra of heavy-light mesons.
4 Spectra of heavy-light meson at finite temperature

Applying TFD prescription to the Hamiltonian of the heavy-light meson we can write it in temperature-dependent form. Then finite-temperature energy eigenvalues of heavy-light meson can be calculated by diagonalization of the following temperature-dependent matrix:

\[ E(\beta) = \langle \Psi_\beta | rH_r | \Psi_\beta \rangle = \begin{pmatrix} a_{n',n}(\beta) & b_{n',n}(\beta) \\ c_{n',n}(\beta) & d_{n',n}(\beta) \end{pmatrix}, \]

where

\[ \Psi_\beta = \sum_n \begin{pmatrix} a_n \\ b_n \end{pmatrix} | n(\beta) \rangle, \]

\[ a_{n',n}(\beta) = \frac{m_q + V_0}{\sqrt{2}} A_1(\beta) + \frac{\lambda}{2} A_2(\beta) - Z \delta_{n',n}, \]

\[ b_{n',n}(\beta) = -\frac{1}{2} A_3(\beta) + (\kappa + \frac{1}{2}) \delta_{n',n}, \]

\[ c_{n',n}(\beta) = \frac{1}{2} A_3(\beta) + (\kappa - \frac{1}{2}) \delta_{n',n}, \]

\[ d_{n',n}(\beta) = -\frac{m_q + V_0}{\sqrt{2}} A_1(\beta) - \frac{\lambda}{2} A_2(\beta) - Z \delta_{n',n}, \]

and

\[ A_1(\beta) = \left( \sqrt{n} \delta_{n',n-1} + \sqrt{n+1} \delta_{n',n+1} \right) (\cosh \theta + \sinh \theta), \]

\[ A_2(\beta) = \left( \sqrt{n(n-1)} \delta_{n',n-2} + \sqrt{(n+1)(n+2)} \delta_{n',n+2} + (1+2n)\delta_{n',n} \right) (\cosh^2 \theta + \sinh^2 \theta) + 2 \cosh \theta \sinh \theta (n+1) \delta_{n',n+1} + n \delta_{n',n-1}, \]

\[ A_3(\beta) = \left( \sqrt{n(n-1)} \delta_{n',n-2} - \sqrt{(n+1)(n+2)} \delta_{n',n+2} \right) (\cosh^2 \theta + \sinh^2 \theta). \]

Using this method we can calculate finite-temperature energy and mass spectra of heavy-light mesons.

5 Results

In table 1 mass spectra of \(c\bar{s}\)-meson is presented for different values of principle and orbital quantum numbers at different temperatures. The parameters are chosen as follows: \(m_q = 0.5 \text{ GeV}, m_c = 1.486 \text{ GeV}, \lambda = 0.8 \text{ GeV}^2, V_0 = -0.6 \text{ GeV}, \alpha_s = 0.32\). It is clear from this table that increasing the temperature leads to decreasing of the mass. Such effect has been found earlier in the case of quarkonium spectra at finite temperature [11, 24]. Table 2 represents the spectrum of \(b\bar{s}\)-system for the following set of parameters: \(m_q = 0.5 \text{ GeV}, m_b = 4.88 \text{ GeV}, \lambda = 0.8 \text{ GeV}^2, V_0 = -0.6 \text{ GeV}, \alpha_s = 0.22\). Table 3 represents the spectrum of \(b\bar{c}\)-system for the following parameters: \(m_q = 1.486 \text{ GeV}, m_b = 4.88 \text{ GeV}, \lambda = 0.2 \text{ GeV}^2, V_0 = -0.3 \text{ GeV}, \alpha_s = 0.22\).

Again, decreasing of the mass by increasing the temperature is clear from these tables. This effect is shown in Fig. 1 which is the pictorial presentation of some states for \(c\bar{s}\)-system at finite temperature for the values of temperature 0, 0.1, 0.15, 0.25 (in GeV). Results for \((b\bar{s})\) and \((b\bar{c})\) have a similar behaviour of changing with temperature.
6 Conclusions

In this work we have studied spectra of heavy-light mesons at finite temperature. By combining quark potential model and real-time finite-temperature field theory, the thermofield dynamics Dirac Hamiltonian for heavy-light meson is presented in the temperature-dependent form. Diagonalizing this Hamiltonian in the non-relativistic harmonic oscillator basis finite-temperature energy and mass spectra of heavy-light mesons with different heavy and light quark contents are obtained. It is found that the masses of heavy-light mesons at finite-temperature are less than those at $T = 0$.

It is important to remark that TFD has proved to be a convenient approach for such a study that calculates the effect of temperature on the spectra of heavy-light quark mesons [18]. Furthermore the results will provide a useful input to studies of quark-gluon plasma at RHIC and LHC.

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Table 1: The mass spectra (in GeV) of $(c\bar{s})$-meson at finite temperature (in GeV). The parameters are chosen as follows: $m_s = 0.5$ GeV, $m_c = 1.486$ GeV, $\lambda = 0.8$ GeV$^2$, $V_0 = -0.6$ GeV, $\alpha_s = 0.32$.

| $n(l)/T$ | 0    | 0.1  | 0.15 | 0.25  |
|----------|------|------|------|-------|
| 1 (l=0)  | 1.7575 | 1.7199 | 1.6809 | 1.6623 |
| 2        | 1.9462 | 1.8730 | 1.8336 | 1.8157 |
| 3        | 2.1005 | 2.0794 | 2.0323 | 2.0151 |
| 4        | 2.3061 | 2.2585 | 2.2108 | 2.1968 |
| 5        | 2.4911 | 2.4693 | 2.4139 | 2.3996 |
| 6        | 2.7056 | 2.6681 | 2.6124 | 2.6018 |
| 7        | 2.9113 | 2.8835 | 2.8196 | 2.8095 |
| 1 (l=1)  | 2.1138 | 2.0231 | 1.9846 | 1.9645 |
| 2        | 2.2549 | 2.1606 | 2.1207 | 2.1001 |
| 3        | 2.3950 | 2.3295 | 2.2823 | 2.2626 |
| 4        | 2.5635 | 2.4896 | 2.4429 | 2.4251 |
| 5        | 2.7252 | 2.6737 | 2.6196 | 2.6035 |
| 6        | 2.9122 | 2.8554 | 2.8011 | 2.7869 |
| 7        | 3.0982 | 3.0517 | 2.9900 | 2.9786 |
| 1 (l=2)  | 2.5330 | 2.4093 | 2.3732 | 2.3531 |
| 2        | 2.6414 | 2.5287 | 2.4886 | 2.4663 |
| 3        | 2.7646 | 2.6673 | 2.6213 | 2.6003 |
| 4        | 2.9053 | 2.8103 | 2.7629 | 2.7416 |
| 5        | 3.0498 | 2.9685 | 2.9160 | 2.8976 |
| 6        | 3.2101 | 3.1318 | 3.0777 | 3.0597 |
| 7        | 3.3757 | 3.3059 | 3.2470 | 3.2332 |
Table 2: The mass spectra (in GeV) of \((b\bar{s})\)-meson at finite temperature (in GeV). The parameters are chosen as follows: \(m_s = 0.5\) GeV, \(m_b = 4.88\) GeV, \(\lambda = 0.8\) GeV\(^2\), \(V_0 = -0.6\) GeV, \(\alpha_s = 0.22\).

| \(l\) | \(n(T)/T\) | 0   | 0.1 | 0.15 | 0.25 |
|------|-------------|-----|-----|------|------|
| 1    | (l=0)       |     |     |      |      |
|      | 1           | 5.1481 | 4.9984 | 4.9756 | 4.9317 |
|      | 2           | 5.3432 | 5.1995 | 5.1741 | 5.1230 |
|      | 3           | 5.4986 | 5.3576 | 5.3361 | 5.2764 |
|      | 4           | 5.7082 | 5.5695 | 5.5445 | 5.4757 |
|      | 5           | 5.8936 | 5.7548 | 5.7343 | 5.6574 |
|      | 6           | 6.1098 | 5.9715 | 5.9476 | 5.8603 |
|      | 7           | 6.3158 | 6.1767 | 6.1585 | 6.0625 |
| 1    | (l=1)       |     |     |      |      |
|      | 1           | 5.5057 | 5.3519 | 5.3269 | 5.2898 |
|      | 2           | 5.6503 | 5.4992 | 5.4712 | 5.4251 |
|      | 3           | 5.7907 | 5.6414 | 5.6165 | 5.5607 |
|      | 4           | 5.9627 | 5.8159 | 5.7877 | 5.7233 |
|      | 5           | 6.1253 | 5.9817 | 5.9591 | 5.8858 |
|      | 6           | 6.3147 | 6.1718 | 6.1461 | 6.0641 |
|      | 7           | 6.5013 | 6.3601 | 6.3400 | 6.2476 |
| 1    | (l=2)       |     |     |      |      |
|      | 1           | 5.9259 | 5.7708 | 5.7454 | 5.7147 |
|      | 2           | 6.0364 | 5.8820 | 5.8535 | 5.8138 |
|      | 3           | 6.1600 | 6.0062 | 5.9780 | 5.9270 |
|      | 4           | 6.3028 | 6.1501 | 6.1200 | 6.0609 |
|      | 5           | 6.4480 | 6.2986 | 6.2718 | 6.2023 |
|      | 6           | 6.6106 | 6.4623 | 6.4348 | 6.3583 |
|      | 7           | 6.7771 | 6.6320 | 6.6082 | 6.5203 |
Table 3: The mass spectra (in GeV) of ($b\bar{c}$)-meson at finite temperature (in GeV). The parameters are chosen as follows: $m_{\bar{c}} = 1.486$ GeV, $m_b = 4.88$ GeV, $\lambda = 0.2$ GeV$^2$, $V_0 = -0.3$ GeV, $\alpha_s = 0.22$.

| $n(l)/T$ | 0   | 0.1 | 0.15 | 0.25 |
|----------|-----|-----|------|------|
| 1 (l=0)  | 6.4132 | 6.0423 | 5.9721 | 5.7302 |
| 2        | 7.0008 | 6.7803 | 6.4376 | 6.3066 |
| 4        | 7.4043 | 7.1883 | 6.7436 | 6.4567 |
| 5        | 7.8131 | 7.5976 | 7.1403 | 6.7371 |
| 6        | 7.9541 | 7.7572 | 7.6095 | 7.1157 |
| 7        | 8.3235 | 8.1021 | 7.9986 | 7.5723 |
| 1 (l=1)  | 7.1444 | 7.0931 | 6.4040 | 5.9279 |
| 2        | 7.3652 | 7.2417 | 7.1465 | 6.4397 |
| 3        | 7.7834 | 7.3741 | 7.3373 | 7.1281 |
| 4        | 7.9654 | 7.6891 | 7.6270 | 7.3249 |
| 5        | 8.2946 | 7.9966 | 7.6666 | 7.6077 |
| 6        | 8.6206 | 8.3817 | 7.9835 | 7.8986 |
| 7        | 9.0263 | 8.7682 | 8.3063 | 7.9818 |
| 1 (l=2)  | 7.3533 | 7.3150 | 7.1480 | 7.0523 |
| 2        | 8.2777 | 8.2737 | 8.0734 | 8.0376 |
| 3        | 9.0333 | 8.7053 | 8.6196 | 8.5776 |
| 4        | 9.2232 | 8.8580 | 8.7972 | 8.7778 |
| 5        | 9.4450 | 9.1152 | 9.0458 | 9.0084 |
| 6        | 9.6959 | 9.3808 | 9.3020 | 9.2559 |
| 7        | 10.0100 | 9.7292 | 9.6397 | 9.5752 |
Figure 1: The mass spectra of $(c\bar{s})$-meson at finite temperature.
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