New damage-sensitive feature for structures with bolted joints

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Abstract. Compared with riveted and welded joints, bolted joints have advantages of easily dismantled, high load carrying and load-transferring capacity. However, bolted joints are also weaker components of assembled structures. Structural damage detection (SDD) on bolted joints is much required in the field of structural health monitoring (SHM). A new SDD method is proposed for damage identification of structures with bolted joints based on residual error of AR model in time series analysis. Firstly, a new data standardization process is defined to maintain the information of damage location. Then, a new structural damage feature sensitive to structural damage is developed based on the standard deviation of AR model residual errors. To verify the proposed method, a bolted joint structure is designed and fabricated in laboratory, connection damages of structures are simulated by loosening the bolted joints. The acceleration responses of structures with bolted joints under healthy and damage cases are acquired. Finally, the SDD is performed by traditional DSF and the new DSF. The illustrated results show that the proposed method is a hybrid tool for the bolted joint damage detection with the new damage-sensitive feature. In addition, some related issues will be discussed as well.

1. Introduction
The vibration-based SDD technique has become an effective way in SDD [1, 2]. Normally, SDD can be achieved by comparing the structural characteristics extracted from structural reference state and damage state respectively. Zhou et al [3] used the hierarchical clustering analysis and similarity measure to distinguish structural damage state from health state. Most of these methods can be divided into two categories: model based and feature based. The model based methods need to construct the structural dynamic model. Khatir et al [4] used the co-ordinate modal assurance criterion for SDD. Khatir et al [5] used the modal scale factor and natural frequencies for structural damage detection and localization. To the feature based methods, especially for those based on time series analysis, are found to be less complicated and more sensitive to local damage [6].

Most of time series analysis based methods need to construct a time series model that fit for vibration response data and achieves SDD via comparing the extracted features from structural reference (healthy) and damage states. Chen and Yu [7] used the ARMA and GARCH model to analyze the vibration response data. There are kinds of time series models for different type of vibration data. Generally, the AR (Auto-Regressive) model, MA (Moving Average) model and ARMA (Auto-Regressive with Moving Average) model are suitable for modeling stationary time series while ARIMA (Auto-Regressive Integrated Moving Average) model is suitable for non-
stationary time series. One of the critical factors to successfully apply time series analysis based methods in SDD is to choose features that are sensitive to structural damage and have good robustness. Chen et al [8] used linear model to analyze the vibration response data from a nonlinear damage structure and extracted the damage features. Nair et al [9] used the first three autoregressive (AR) components to define damage features. Carden and Brownjohn [10] used ARMA coefficients as damage features and adopted classifier to distinguish normal data and damage data. Other researchers used residual errors to construct damage features. Yao and Pakzad [11] defined the Ljung-Box statistic of AR model residual error series as the damage feature. Yu and Lin [12] defined ratio of standard deviation as damage sensitive features. Yu and Zhu [13] used skewness and kurtosis to define a feature for structural non-linear damage diagnosis. Unfortunately, most of time series analysis based methods aim at determining existence of damages but they do not have good effectiveness when considering damage localization or extents, sometimes they cannot effectively locate the damage.

In this study, bolted joints damage detection is achieved by locating the damage region. If the bolted joints are included in the identified damage region, the bolted joint damage can be successfully identified. The acceleration time histories of structures are recorded and fitted by the AR model. A new damage-sensitive feature (DSF) is developed based on the standard deviation of residual errors. In order to verify the validity of the proposed method, a series of experiments are designed and some measured data are analyzed by traditional DSF and new one, respectively. The results will be compared and discussed, and some reasonable conclusions are made finally.

2. Theoretical background

The bolted joint damage will affect dynamic characteristics of structures, resulting in change of statistical feature in vibration signals. As a result, damage detection can be achieved by comparing the developed DSF under reference (healthy) and test states. In this study, AR model is adopted to model the acceleration time histories recorded from accelerometers. It is necessary to note that the AR model is limited to analyze linear stationary time series processes, which first and second moments are time invariant.

2.1. Data standardization

A stationary acceleration time history from reference state is shown in Figure 1. Supposing data \( x_i \) denotes the value of measured data with \( N \) samples at all \( l \) measurement points. In order to eliminate the effects caused by different loads, data standardization is necessary. In general, the data standardization process is performed by zero mean and unit variance as follows:

\[
\hat{x}_i = \frac{x_i - \bar{x}}{\sigma_x}, \text{ where } \sigma_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2
\]

(1)

Where \( \bar{x} \), \( \hat{x}_i \) and \( \sigma_x \) are mean, standardized value and standard deviation of \( x_i \), respectively. However, to a certain state of structure, data variances from different measurement points are related to their positions. The standardization of unit variance ensures that the features extracted from normal and damage structures can be compared but making the variance from different measurement points are the same to be unit and thus, losing the information of damage location. As the bolted joint damage detection is achieved by locating the damage region in this study, it is necessary to maintain the information of damage location and ensure that the same channel features extracted from normal and damage structures can be compared. A new data standardization is defined as follows:

\[
\hat{x}'_i = \frac{x_i - \bar{x}}{\sigma'_x}
\]

(2)

\[
\bar{f} = \frac{1}{N} \sum_{i=1}^{N} f_i, \sigma'_f = \frac{1}{N-1} \sum_{i=1}^{N} (f_i - \bar{f})^2
\]

(3)

Where \( \hat{x}'_i \), \( \bar{f} \) and \( \sigma'_f \) are new standardized value of \( x_i \), mean and variance of force \( f_i \), respectively. It can be found that using variance of force time histories as normalized parameter, the features
extracted from different structural states can be compared, and then the feature relationship between two different measurement points can be maintained as well.

![Figure 1](image1.png)

**Figure 1.** Acceleration time history of structures in reference state

2.2. Time series analysis based on AR model

The basic idea of time series analysis methods is that the development of things always keeping its inertia, which means that the future values in time series are dependent to its predecessors. In mathematical language, there are certain correlation relations in time series data and such correlation can be describe statistically. AR model is one of time series models that attempt to describe this regular pattern of time series by using the linear combination of its predecessors. The basic steps of AR model based time series analysis method includes model significance test, coefficients estimation and order determination.

As AR model is used to analysis stationary time series whose auto-correlation function (ACF) decays should gradually, but the partial auto-correlation function (PACF) would cut off after a few lags [14], a model significance test can be done by observing the changing trends of both sample ACF and PACF. Both sample ACF and PACF results from reference state are shown in Figure 2. It can be found that the behaviors of measured data are consistent with the behaviors of AR model process. Thus, AR model is adopted in this study. The coefficients $\phi_1, \phi_2, \ldots, \phi_p$ are estimating by the Yule-Walker equations [14]. An AR model of order $p$ at $j$-th measured acceleration time histories, or AR $(p)$, can be written as:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \epsilon_t$$  \hspace{1cm} (4)

Where $x_t, x_{t-1}, \ldots, x_{t-p}$ denote the current and previous values in time series and $\epsilon_t$ is the residual error.

![Figure 2](image2.png)

**Figure 2.** (a). Sample ACF and PACF at channel 4, (b). Sample ACF and PACF at channel 7

2.3. Order determination

The order of AR model is an unknown parameter. The high-order model has more variables making it can change more flexibly. As a result, a high-order model has better fitting effects to data than low-order model. However, the huge amount number of variables means there are more unknown factors in model resulting in the estimation precision decline and increasing the complexity of model. In contrast, a low-order model may not effectively describe the regular pattern of data. The determination of optimum order must consider fitting degree and variable quantity together. There are several criterion used in order determination, such as Akaike’s information criterion (AIC), Bayesian
information criterion (BIC), final prediction error (FPE) and partial auto-correlation function (PACF). Finally, the BIC is selected to assess the optimum order. The BIC is a weighted function of fitting precision and variable quantity as follow:

$$BIC = n \ln \sigma^2_e + (\ln n) p$$

Where $n = \text{number of time series data}; \sigma^2_e = \text{variance of residual error series}; p = \text{number of adjustable variables}$. The first term indicates the fitting precision. If the model is too simple, the variance of residual errors increases. The second term is a penalty factor which increase as the variable quantity grows. The optimum model is obtained when the responding BIC function reaches minimum and the model order is determined by the optimum model.

2.4. Traditional damage-sensitive feature

For structural reference (health) state, the measured acceleration time histories can be modeled by AR model, the corresponding AR coefficients $\Phi^\text{ref}_k$ and residual errors series $\varepsilon^\text{ref}$ can be obtained as well. Similarly, for the structural test state, the acceleration time histories at the same structural position can be measured. And then, the residual error series $\varepsilon^\text{test}$ can be obtained by fitting the AR model data in reference state to measured data in test state. As we know, the constructed AR model can effectively predict the data of structural health state. When the data comes from structural damage state, its regular pattern is different with the reference data. As a result, its residual error series will be different with the reference one as well. Therefore, the damage detection is performed through comparing the standard deviation (STD) of residual error series measured from same channel [13]. This method does not need to compare the features extracted from different channels and therefore the data standardization is performed by zero mean and unit variance in Equation (1). The traditional damage-sensitive index is defined as follows:

$$\varepsilon^\text{ref}_t = x^\text{ref}_t - \phi^\text{ref}_1 x^\text{ref}_{t-1} - \phi^\text{ref}_2 x^\text{ref}_{t-2} - \cdots - \phi^\text{ref}_p x^\text{ref}_{t-p}$$

$$\varepsilon^\text{test}_t = x^\text{test}_t - \phi^\text{test}_1 x^\text{test}_{t-1} - \phi^\text{test}_2 x^\text{test}_{t-2} - \cdots - \phi^\text{test}_p x^\text{test}_{t-p}$$

$$\gamma^\text{id} = \frac{\sigma(\varepsilon^\text{test})}{\sigma(\varepsilon^\text{ref})}$$

The damage detection is judged by the value of STD ratio in Equation (9). When the test samples come from the structural health state, the STD ratio $\gamma^\text{id}$ is approximately equal to one, when the test sample come from the structural damage state, $\gamma^\text{id}$ will larger than one.

2.5. New damage-sensitive features

The new DSF is designed for SDD and bolted joint damage detection is achieved by locating the damage location. The STD of residual error series is still being used to describe the structural dynamic characteristics. The new DSF is constructed through residual errors relationship between measured data from two different measurement points installed on a structure, where the area within two consecutive measured points can be regarded as a region. Therefore, the whole structure is divided into several regions. The new DSF is developed through comparing regular patterns of STDs from two consecutive measured points.

Assuming that there are totally $l$ measured points on a structure and $P_k$ and $P_{k+1}$ are two consecutive measured points, the data standardization is performed by newly defined data standardization process in Equations (2) and (3), and the new DSF is developed as follows:

$$\varepsilon^\text{ref}_t = x^\text{ref}_t - \phi^\text{ref}_1 x^\text{ref}_{t-1} - \phi^\text{ref}_2 x^\text{ref}_{t-2} - \cdots - \phi^\text{ref}_p x^\text{ref}_{t-p}$$

$$\varepsilon^\text{test}_t = x^\text{test}_t - \phi^\text{test}_1 x^\text{test}_{t-1} - \phi^\text{test}_2 x^\text{test}_{t-2} - \cdots - \phi^\text{test}_p x^\text{test}_{t-p}$$

$$\text{DSF}(k) = \sum_{n=1}^{k-1} \frac{\sigma_n(\varepsilon^\text{test})}{\sigma_n(\varepsilon^\text{test}) + \sigma_{k+1}(\varepsilon^\text{test})} - \frac{\sigma_n(\varepsilon^\text{ref})}{\sigma_n(\varepsilon^\text{ref}) + \sigma_{k+1}(\varepsilon^\text{ref})} \quad (k = 1, 2 \cdots l-1)$$
DSF\((k)\) is used for detecting the region \(k\) between points \(k\) to \(k+1\). When the damage occurs in a certain region, especially for bolted joints damage, the connection strength will be reduced, which result in the changes in load transfer characteristics or connection characteristics in the region. As a result, the relationship of standard deviation between two consecutive measured points may change as well. In other regions, the connection strength maintains as structural health state and the DSF changes a little. It is reasonable to consider that the new DSF is a sensitive to damage location as the value of DSF\((k)\) is mostly determined by damage in region \(k\) and slightly affected by damage in other regions.

If the test sample comes from the structural health state, the connection strength in region \(k\) remains unchanged and there are no any damage occurs in other regions. Therefore, the DSF\((k)\) should close to zero. When the test samples come from structural damage state but there is no location in region \(k\), the DSF\((k)\) should be a small value. When the damages are located in region \(k\), the DSF\((k)\) would be significant nonzero. If the identified damage location regions include the bolted joints, the new DSF can effectively detect structural damages caused by the bolted joints. Therefore, the DSF is used for detecting bolted joints damages by locating the damage region.

3. Experimental verification

In order to verify the validity of the proposed DSF for bolted joint damage detection, a steel cantilever beam, which is connected with a steel free-free beam via a bolted joint, is adopted in experiments, as shown in Figure 3. The spans of the test cantilever beam and the free-free beam are 800 mm and 400 mm respectively. The cross section is a rectangular section with 50 mm width and 10 mm height. The contact length is 50 mm.

As shown in Figure 4, a vibrator (HEV-200) together with a power amplifier and a force sensor (PCB, 208C02) are hanged on the beam at 0.7 m from the fixed end. A random excitation is generated by LMS Test.Lab and sent to the vibrator. Seven acceleration sensors (PCB, ICP 333B30) are mounted on points 1 to 7 and their positions are marked in Figure 4 by P1 to P7. The force and acceleration time histories from the reference state and damage state are recorded. The sampling frequency is 2048 Hz and the sampling duration is 4s for each data block and thus 8192 consecutive data samples are recorded. There are two structural states considered in experiments, i.e., the reference (healthy) and damage states, respectively. In structural reference state, 20 N·m torques are loaded by torque spanner. The damage state is simulated through loosening the bolted joints where the residual torque is reduced to 2.5 N·m. Three sets of acceleration data are measured, two of them are from structural health states, in which the first set is used as reference data while the second one as test sample named as Sample 1. The third set of data comes from structural damage state named as Sample 2. All the measured data are recorded by LMS Test.Lab and stored into the personal computer.

![Figure 3. Experiment setup](image)
3.1. Order determination for AR model

The effects of AR model orders on BIC of all measured channels in structural health state and damage state are show in Figure 5. The low order model cannot describe the regular pattern of measured data, so the BIC value is large and monotonically decrease with the increase of order at beginning. When the BIC curves reach minimum, they begins to increase gradually. It is because that the fitting precision changes a little with the increase of model order but the model becomes more and more complex. As a result, when the BIC curve reach minimum, the corresponding model is the optimum model and the corresponding order is obtained as shown in Table 1.

3.2. Bolted joint damage detection

In order to perform the proposed method and assess its validity, the traditional DSF and newly defined DSF are used to detect bolted joint damage and the results will be compared and discussed.
To calculate the traditional DSF $\gamma^{std}$, the data standardization is performed using Equation (1) firstly. After obtaining the AR orders, the residual errors of test samples and reference state are calculated by reference state’s AR coefficients as shown in Equations (6) and (7). The values of traditional DSF $\gamma^{std}$ are calculated by Equation (8) and the results are shown in Figure 6 and Table 2. It can be found when the test sample comes from structural health state, the $\gamma^{std}$ values at all measurement points are closer to one. Despite of the small errors that maybe caused by measured noises or environmental influences, the regular pattern of measured data from Sample 1 is the same as structural reference state. As a result, it can be seen that Sample 1 comes from structural health state, which is in agreement with the real situation. When the test sample comes from structural damage state, $\gamma^{std}$ values at all measurement points obviously away from one, which indicates that the Sample 2 comes from structural damage state. As a result, the traditional DSF $\gamma^{std}$ is successful in SDD. However, it is unable to perform damage localization by combining the $\gamma^{std}$ values at different measurement points. It is not sure that the changes in structural dynamic characteristics are caused by bolted joint damage or cantilever beam damage or free-free beam damage.

![Figure 6. Traditional DSF values at seven measurement points under health and damage states](image)

**Figure 6.** Traditional DSF values at seven measurement points under health and damage states

| Structural states | Measuring point number and corresponding DSF values |
|-------------------|---------------------------------------------------|
|                   | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
| Sample 1(Torque=20 N·m) | 1.050   | 0.9995  | 1.0220  | 0.9370  | 0.9060  | 0.9180  | 0.9397  |
| Sample 2(Torque=2.5 N·m) | 1.7832  | 1.8266  | 1.4170  | 1.4694  | 1.2028  | 1.2268  | 1.3271  |

For the newly defined DSF, the whole structure is divided into six regions by seven measurement points. First, the data is standardized as equation (2) and equation (3). The AR coefficients of three data sets are calculated and the AR orders are shown in Table 1. The residual errors of test samples and reference state are calculated by its corresponding AR coefficients as shown in equation (9) and equation (10) and then the new DSF is calculated by equation (11). DSF(k) is used for k-th region damage detection. It should be mentioned that the bolted joint locates in the fourth region. The values for new DSF from two samples are shown in Figure 7 and Table 3. For Sample 1, it can be found that DSF values in all regions are closer to zero, which indicates that the relationship is similar between Sample 1 and reference state in each region. As a result, state 1 is identified to be structural health state.

For Sample 2, the DSF(3) is obviously away from zero and the others are small values compared with DSF(3). The DSF values in regions 1, 2, 3, 5 and 6 are pretty small. According to the definition of new DSF, it can be seen that the damage occurs in structure and it is located in region (3), which is corresponding to the actual damage situation. Therefore, bolted joint damage detection is achieved by successfully locating the damage region in the proposed method.
3.3. Discussion and analysis

Comparing the results due to traditional DSF with ones due to new DSF, it can be found that both of them can achieve damage detection but only new SDF can achieve damage localization and ensures that the proposed method can be used in bolted joint damage detection.

Firstly, it is owed to the data standardization. When the structure with seven measurement points is excited under the same excitation force, a set of data can be measured at seven points. Their different regular patterns are related to themselves positions, their variance will also be different as well. So when data standardization of unit variance adopted, the different normalized parameters have affected the relationships of different measurement points and thus lost the location information. Secondly, when the residual error series $\varepsilon_{\text{test}}$ are obtained by fitting the reference state AR model to the measured data in health state, the residual error series $\varepsilon_{\text{test}}$ can describe actual regular pattern of measured data. When $\varepsilon_{\text{test}}$ were obtained by fitting the reference state AR model to the measured data in damage state, the residual error series $\varepsilon_{\text{test}}$ cannot describe actual regular pattern of measured data.

For the traditional DSF, the information of damage location is lost in data standardization. Even if the data at all measurement points were standardized by the same normalized parameter, the relationship data at different measurement points is influenced by measured position and inappropriate AR model together. Therefore, damage localization cannot be achieved by the traditional DSF.

For the new DSF, the corresponding AR models are constructed for each sample and the $\varepsilon_{\text{test}}$ are obtained by fitting the corresponding AR model to measured data. Therefore, $\varepsilon_{\text{test}}$ can describe the actual test states of structures. In addition, the relationship data at different measurement points is determined by its position via using the variance of force time histories to standardize the data. The experiment also verified that if there are any damage occurred in the region $k$, the DSF($k$) would be obviously away from zero. It is noticed that when there are no damage occurs in region $k$, ($k=2$, 3, 5, 6), DSF($k$) from Sample 2 are larger than those from Sample 1. It is because the damage of bolted joints has affected the structural vibration characteristics. The whole structure is depart from the reference state and the DSF($k$) would be depart from the reference state as well, so the regular pattern at each measurement point would be changed, but the connection strength in undamaged region maintain the same. Fortunately, the new DSF has been verified to be sensitive to damage location. Therefore, the bolted joint damage detection can be achieved by the proposed method.

![Figure 7. New DFS values for six region under structural health and damage states](image)

### Table 3. Values of new DSF

| Structural states | Region number and corresponding DSF values |
|-------------------|------------------------------------------|
|                   | 1            | 2            | 3            | 4            | 5            | 6            |
| Sample 1(Torque=20 N·m) | 0.0063      | 0.0023      | 0.0001      | 0.0123      | 0.0025      | 0.0001      |
| Sample 2(Torque=2.5 N·m) | 0.0034      | 0.0055      | 0.1146      | 0.0066      | 0.0208      | 0.0169      |
4. Conclusions
In this study, a new SDD method is proposed for bolted joint damage detection based on time series analysis. The measured data is standardized by variance of force time histories and modeled by AR model. The standard deviation of residual errors is used to construct the new structural DSF. The bolted joint damage detection is achieved by identifying the damage location. A series of experiments on bolted joint structure is designed to verify the proposed method. The illustrated results show that:
(1) The proposed method can effectively identify the bolted joint damage. (2) Data standardization by variance of force can maintain the information of damage location, the relationship between measured data at different points is sensitive to damage location. (3) The newly defined DSF is sensitive to local damage, it can be used to identify structural damage location as well. (4) The methodology should be verified with more measured data from different types of structures.

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