Some Recent Developments in Sphalerons

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ABSTRACT
We review briefly the sphaleron and list some of its properties. We summarize
some of the results in models which have an extended scalar sector. We also
present our work on models dealing with physics beyond the standard model. We
focus on the energy of the sphaleron which is important in determining the rate
of baryon number violation at the electroweak scale.

I. Introduction
In the standard model baryon number and lepton number are not conserved
due to the existence of the anomaly, and non-trivial vacua topology of the the-
ory, as pointed out by 't Hooft[1]. Manton and Klinkhamer[2] showed that in
the SU(2) gauge-Higgs theory there exist non-contractible loops in the configura-
tion space which is composed of stationary, finite energy solutions of the classical
equations of motion. The highest-energy configuration on a minimal energy path
is a saddle point and it can be interpreted as the minimal energy barrier sepa-
ating the neighbouring vacua of different Chern-Simon numbers. This is called
a sphaleron[2]. The minimal height of the barrier is the energy of the sphaleron
which will be denoted as $E_{sp}$.

This interpretation of the sphaleron led to its application to electroweak baryo-
genesis[3]. In the high temperature environment of the early universe thermal
excitation becomes important, and the transition from one vacuum to another
can be effected by passing over the sphaleron barrier. This observation has two
important implications for baryogenesis: one is that the existence of processes of
rapid baryon number violation will invalidate the GUT baryogenesis with B-L in-
variance. Another is the possibility of producing and maintaining excess baryon
number at the electroweak energy scale, i.e., electroweak baryogenesis (EWB).

We will focus on some of the properties of the sphaleron. The references listed
here will be quite incomplete and we apologize for all the omissions.
II. Some Important Facts about Sphalerons

(II.a) Energy of the sphaleron[2][4]: The gauge and Higgs profile functions of the sphaleron in the SU(2)-Higgs theory are spherically symmetric and the energy of the sphaleron is given by 
\[ E_{sp} = \frac{2M_W}{\alpha_W} B(\frac{\lambda}{g_2}) = (5\text{TeV}) B(\frac{\lambda}{g_2}), \]
where \(2.52 \leq B(\frac{\lambda}{g_2}) \leq 2.70\), \(\lambda\) is the Higgs quartic coupling constant, and \(g_2\) the SU(2) coupling constant.

(II.b) Topological charge of the sphaleron[2]: The sphaleron has a half-odd integer topological charge. This allows the interpretation that it lies on the top of the potential barrier between the vacua with Chern-Simon numbers \(n\) and \(n + 1\).

(II.c) Sphaleron of finite Weinberg angle[2][5]: Upon including the U(1), the sphaleron is no longer spherically symmetric. However, the energy of the sphaleron differs very little from the SU(2)-Higgs theory, slightly lower by about 1% at the physical Weinberg angle.

(II.d) Deformed sphaleron[6]: Multiple solutions due to bifurcation arise when the Higgs mass becomes large. The first bifurcation takes place with the appearance of new solutions for \(M_H \geq 12m_W\). More solutions appear when the Higgs mass increases further. The first bisphaleron which lies below the sphaleron has the lowest energy, which is about 8% lower asymptotically.

(II.e) Effect of fermions[7]: The effect of the back action of the fermion on the sphaleron energy is generally small for light fermions less than 300 GeV. For fermions as heavy as 1 TeV, the fermion effect on the energy of sphaleron is still less than 10%.

(II.f) Sphaleron at finite temperature[8]: The sphaleron energy can be approximated by 
\[ E_{sp}(\lambda, T) \simeq E_{sp}(\lambda, 0) \frac{\langle \Phi(T) \rangle}{\langle \Phi(0) \rangle}, \]
where \(\langle \Phi(T) \rangle\) is the Higgs vacuum expectation value at the temperature \(T\).

(II.g) Strong sphaleron[9]: The analysis of the Standard EW model above the symmetry restoration temperature leads to a similar study of axial baryon number violation in QCD at finite temperature. There exists a sphaleron-like object in QCD, which has an important implication in EWB.

(II.h) Sphalerons in extended models: The study of sphalerons in extended models, one-doublet with a singlet[10] and two-doublet Higgs models[11][8], is motivated by the difficulties of the one-Higgs doublet SM with EWB: one is that the CP violation is too small to produce the observed density ratio of baryons to photons. Another is that preserving the excess baryons produced during the
EW phase transition requires $M_H < 45\text{GeV}$, which is below the current LEP experimental lower bound of $67\text{GeV}$. In general, the energy of the sphaleron is remarkably stable against the variation of models of the Higgs sector.

**III. Beyond the standard model**

The approach we used to describe physics beyond the SM is to add high dimension effective operators to the SM. To simplify the matter we take the limit of vanishing Weinberg angle and ignore the fermion.

(III.a) Dimension 6 operators\cite{13}\cite{14}: In the absence of fermion fields, the lowest dimension is 6. There are the following operators:

$$\frac{1}{4\Lambda^2}(|\Phi\Phi|^2); \frac{1}{4\Lambda^2}(D_\mu \Phi)(D_\mu \Phi)^\dagger; \frac{1}{2\Lambda^2}|\Phi|^2; \frac{1}{2\Lambda^2}(\Phi^\dagger \Phi)W_{\mu\nu} W^{\alpha\mu\nu}. $$

To compute the energy of the sphaleron we add the above operators individually to the SM Lagrangian, and recalculate the energy. The ansatz of the SM profile functions is still applicable. The contribution to the sphaleron energy depends on the value of $\Lambda$, which we took to be $1\text{ TeV}$. All contributions are small, to within a few percent of that of the SM sphaleron, except for the first operator for small values of $\lambda$.

The first operator is anomalous for small values of $\lambda$, where the sphaleron energy becomes very large and negative, whether one takes the negative or positive sign of the operator. This change of behavior can be understood from the fact that the sphaleron, being classical, can probe only a limited region of the scalar potential, i.e., $|\Phi| \leq \frac{v^2}{2}$. When the potential in this region is not affect significantly, such is the case of not too small $\lambda$, the sphaleron energy will not suffer much change. For sufficiently small $\lambda$, which corresponds to $M_H \leq 20\text{GeV}$, the sphaleron energy can be modified drastically. But this case is unphysical, below the experimental lower bound of the Higgs mass.

To conclude, the inclusion of a *reasonably behaved* dimension 6 operators will not modify the sphaleron energy significantly for a reasonable cutoff like $1\text{ TeV}$. Some of the effects of including dimension 6 operators in the SM scalar potential is to raise the upper limit on the Higgs mass from $45\text{GeV}$ to about $100\text{GeV}$\cite{13}.

(III.b) Dimension 8 operators\cite{15}: There are many dimension 8 operators involving the gauge and Higgs fields. We found an interesting one in the form,

$$\frac{1}{\Lambda^4}\{(D_\mu \Phi)^\dagger (D_\mu \Phi)\}^2. $$

For large $\lambda$ the sphaleron energy is proportional to $E_\text{sp} \sim \left(\frac{\Lambda}{g^2}\right)^4$, which blows up when the Higgs mass approaches $\infty$. We calculated the
energy numerically. The result shows that $E_{sp}^{BSM}$ is very close to $E_{sp}$ for $\frac{\lambda}{g^2} \leq 10^3$. $E_{sp}^{BSM}$ starts to depart from $E_{sp}$ significantly for $\frac{\lambda}{g^2} = 10^7$ and increases more rapidly when $\lambda$ increases further, as shown in Fig. I.

(III.c) Sphaleron in the non-linear $\sigma$ model[16]: The above result has an interesting consequence: the sphaleron energy in the non-linear $\sigma$ model is infinity. This result may have some bearing on EWB in a dynamically broken theory, where the Higgs sector is realized by a non-linear $\sigma$ model. As the sphaleron has a large energy in the broken phase and will satisfy the Shaposhnikov[12] criterion, the baryon asymmetry produced in the symmetric phase and in the bubble wall will not be washed out in the broken phase.

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