Galactic halo magnetic fields and UHECR deflections

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ABSTRACT
We consider the synchrotron emission from electrons out in the Galactic halo region where the Fermi bubble structures reside. Utilising a simple analytical expression for the non-thermal electron distribution and a toy magnetic field model, we simulate polarised synchrotron emission maps at a frequency of 30 GHz. Comparing these maps with observational data, we obtain constraints on the parameters of our toy Galactic halo magnetic field model. Utilising this parameter value range for the toy magnetic field model, we determine the corresponding range of arrival directions and suppression factors of ultra high energy cosmic rays (UHECRs) from potential local source locations. We find high levels (down to 5%) of flux suppression and large deflection angles (> 80°) for source locations whose line-of-sight pass through the Galactic halo bubble region. We conclude that the magnetic field out in this Galactic halo region can strongly dominate the level of deflection UHECR experience whilst propagating from local sources to Earth.

Key words: galaxies: magnetic fields, astroparticle physics, radiation mechanisms: non-thermal

1 INTRODUCTION
The origin and structure of the Galactic magnetic field remains a long standing unresolved problem in astrophysics. What has become apparent, however, is the vital role it plays, especially in terms of cosmic ray propagation within the Galaxy. The incompleteness of the observational data, required to probe the Galactic magnetic field structure on many different length scales, limits significantly our ability to describe cosmic ray propagation through the Galaxy. This is especially true when it comes to the modelling of cosmic ray propagation out in the Galactic halo region where our knowledge of the magnetic field is particularly weak.

A variety of methods allow observational probes of Galactic magnetic fields, such as starlight polarisation and infrared emission from dust grains, and Zeeman splitting of spectral radio lines in the dust clouds (Beck 2007). Galactic magnetic fields are also probed by Pulsar dispersion with Faraday rotation, which is sensitive to the magnetic-field component parallel to the line of sight, $B_\parallel$, and synchrotron radiation which probes the component perpendicular to the line of sight, $B_\perp$.

A major drawback in using the Pulsar dispersion measure along with the Faraday rotation measure method for probing Galactic magnetic fields is that it relies heavily on the lines of sight along which Pulsars are found, which places a strong focus on the regions close to the Galactic plane. Therefore, this method is of most use for probing the magnetic field in the Galactic disc region, and is not so useful for probing the magnetic field out in the Galactic halo. Synchrotron radiation on the other hand is produced by the gyration of non-thermal electrons around magnetic field lines. Since it is produced anywhere where sufficient non-thermal electrons and a magnetic field are present, this emission can act as a natural probe of magnetic fields also in the Galactic halo.

The observations made using FERMI-LAT (Dobler et al. (2010); Su et al. (2010); Su & Finkbeiner (2012)) in the gamma ray regime unveiled giant bipolar gamma ray bubbles extending up to $\approx 3$ kpc radially and $\approx 8$ kpc in the $z$-direction, having a total energy of $\approx 10^{54-55}$ ergs. Recently, observations made using eROSITA (Predehl et al. (2020)) in the X-ray regime further suggest the existence of even larger bubbles going up to $\approx 7$ kpc radially and $\approx 14$ kpc in the azimuthal direction, having an estimated total thermal energy of $\approx 10^{56}$ ergs. These recent observations strongly motivate further investigations into the magnetic field present out in the Galactic halo region. Henceforth, for the sake of simplicity we will address the two bubbles together as the Galactic halo bubbles.

With the help of the aforementioned techniques, we can estimate the strength and direction of the magnetic field in different parts of the Galaxy. For the Galactic halo the S-PASS observations made at 2.3 GHz (Carretti et al. (2013)) seem to suggest that the field strength in the halo bubbles can be anywhere between 6 – 10 $\mu$G depending on the proton-electron ratio value adopted in the minimum energy calculation. S-PASS observations, however, are subject to depolarisation of polarised synchrotron radiation via Faraday rotation due to its relatively low observation frequencies. Additionally, this data set is not sensitive to a portion of the Fermi bubble region of the sky due to the ground-based location of the

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instrument allowing only observations in the southern terrestrial hemisphere. For this reason, data from Planck and WMAP are more helpful when probing magnetic fields in the Galactic halo due to their all-sky coverage and observation bandwidths which are not sensitive to Faraday rotation effects.

Knowledge of the non-thermal electron distribution is critical in the modelling of Galactic magnetic fields, which are both required inputs for the determination of synthetic synchrotron maps. We have direct information on the distribution of cosmic ray electrons at Earth from the observations made, for example, by AMS (Aguilar et al. (2002); Aguilar et al. (2014)), CALET (Adriani et al. (2017)) and DAMPE (Ambrosi et al. (2017)). However, we do not have direct knowledge of the electron distribution in the Galaxy. Currently there are a few ways to model the spatial distribution of these relativistic electrons; for example, either on theoretical grounds using the GALPROP diffusive transport code (Waelkens, A. et al. (2009); Strong et al. (2011)) or on more phenomenological grounds as done in the WMAP data analysis (Page et al. (2007)).

Several efforts have been made to model the magnetic fields in the Galaxy, for example by Sun et al. (2008); Jaffe et al. (2010, 2011); Jansson & Farrar (2012). It should be noted that our understanding of the magnetic field in the disc region of the Galaxy is much better than that of the halo region due to the larger amount of observational data present at varying frequencies. However, widely used models like JF12 (Jansson & Farrar (2012)) have also made some efforts towards the modelling of the Galactic halo magnetic field. One drawback of JF12 is that it masks out the Fermi bubble regions in its evaluation of the model agreement with the data, whereas S-PASS observations (Carretti et al. 2013) tell us that magnetic field strength in this region is not negligible. Therefore, it is important to consider modelling the Galactic halo including the Fermi bubbles.

An understanding of the propagation of cosmic rays is vital for resolving their sources. However, this understanding is limited by our current knowledge about the intervening magnetic fields. Extragalactic cosmic rays (ultra high energy cosmic rays (UHECRs) with energies higher than 10^{18} eV) are constituted by charged protons or nuclei, and their original directions are, therefore, scrambled by the magnetic fields in the path between the source and Earth. Different models of the Galactic magnetic field give vastly different predictions for the deflection of UHECRs (see e.g. Sun et al. (2008); Sun & Reich (2010); Pshirkov et al. (2011); Jansson & Farrar (2012); Farrar (2014)). Recently, significant anisotropies in the UHECR sky have been discovered (see Abbasi et al. (2014); Aab et al. (2018); Caccianiga (2019); Kim et al. (2021)). Due to the deflections in the Galactic magnetic fields, the interpretation of these results in terms of the localisation of the UHECR sources is extremely hard and hence, knowledge of Galactic magnetic fields is extremely important.

The structure of this paper is the following. In section 2 we provide a description of the electron distribution and the toy magnetic field model adopted in our study. In section 2.4 synthetic polarised synchrotron maps are produced adopting this model, which are then compared against the Planck data. A grid scan of the model against the data is then made in order to obtain constrained model parameters. In section 3 we determine the arrival directions of ultra high energy cosmic rays with \( E = 40 \text{ PeV} \) from our toy model and discuss how the uncertainties in the parameters can propagate errors in estimating the cosmic ray deflections. Lastly, in section 4 we summarise our conclusions.

2 GALACTIC HALO MAGNETIC FIELD MODEL

2.1 Toy Model for the Galactic Halo Bubbles

In this paper we follow the philosophy of West et al. (2020), adopting a simple toy model as means of a preliminary attempt to provide a model for the Galactic halo bubbles.

For our toy model, we adopt an axisymmetric toroidal structured field along with a Kolmogorov turbulent field, with \( B_{\text{tur}} \) as the mean-field strength and a power-law spectrum of index 5/3. The expression for the toroidal field is:

\[
B_{\text{tor}} = B_{\text{str}} \delta(z/R_{\text{mag}})(-\frac{z}{|z|})e^{-\frac{|z|}{L_{\text{mag}}}}e^{-\frac{|r|}{R_{\text{mag}}}}.
\]

(1)

The structured field has 3 free parameters: \( B_{\text{str}} \) as the strength of the magnetic field and \( R_{\text{mag}} \) and \( Z_{\text{mag}} \) describing the radial and azimuthal cut off distances, respectively. The value of \( z_{\text{min}} = 100 \text{ pc} \), which dictates the cut in the Galactic plane, is fixed. The model spans radially up to 20 kpc with the observer being centered at Earth, (-8.5,0,0) kpc. The direction of the toroidal field is orientated in opposite directions above and below the Galactic plane. A visualisation of our magnetic field in \( xz \) and \( xy \) cross-sections is shown in Fig. 1.

We use CRPropa 3 (Batista et al. (2016)) for generating turbulent fields with a power law spectrum, with the magnitude of this component being \( B_{\text{tur}} \). The minimum and maximum values of wavelength to generate these fields are \( L_{\text{min}} = 200 \text{ pc} \) and \( L_{\text{max}} = 400 \text{ pc} \). For computational reasons we stick to this restricted dynamic range of \( L_{\text{min}} \) and \( L_{\text{max}} \). The turbulent field has effectively only 1 free parameter which is the magnitude of the turbulent field strength, \( B_{\text{tur}} \), with the coherence length of the field being kept fixed at 150 pc. This value of \( L_{\text{coh}} \), although in the range of values considered (Ohno & Shibata (1993); Chepurnov et al. (2010); Iacobelli et al. (2013); Giacinti et al. (2018)), nevertheless may be too large. Regardless, for the sake of simplicity, we fix the coherence length at this length scale. In Appendix B we show a power-spectrum plot for the turbulent magnetic field realisation adopted.

Since we focus only on the Galactic regions of the sky which probe the Galactic halo, we do not include any disc magnetic field component in this model. For the purposes of comparison, we use the JF12 model as a comparative reference since it is a widely known Galactic magnetic field model. However, it should be noted that the JF12 model was motivated by observations which masked out a large part of the Galactic bubble region that we focus on, and adopts magnetic fields strengths and spatial extensions both weaker and smaller than those suggested by the S-PASS observations (Carretti et al. 2013) in these regions.

2.2 Electron Distribution

In order to calculate synthetic synchrotron maps, both a non-thermal electron distribution and magnetic field model are required. For the non-thermal electron distribution, the
JF12 model considered both the WMAP analytical expression (Eq. 2) and a simulated electron distribution from GALPROP. They used the latter for their model in their paper. The two models are quite different. The WMAP model (Page et al. (2007)) is an analytical expression whereas the GALPROP distribution (Waelkens, A. et al. (2009)) is more theoretical in motivation, being obtained from a solution to the diffusive transport equation assuming a specific spatial distribution for the sources. As our current knowledge of the non-thermal electron distribution in the Galaxy, especially in the Galactic halo region, is very limited, we choose to adopt the simple WMAP analytical model in order to avoid adding further layers of complexity. The WMAP electron density distribution model we adopt has the form:

$$\frac{dn_e}{d\log E_e} = C_{\text{norm}} \left( \frac{E_e}{E_{10\text{GeV}}} \right)^{-p+1} e^{-r/R_{\text{el}}} \text{sech}^2 \left( \frac{z}{Z_{\text{el}}} \right), \quad (2)$$

where $dn_e/d\log E_e$ is the differential electron density in logarithmic energy bins, in units of cm$^{-3}$, and $p = 3$ is the spectral index of the electron spectrum. The parameter $C_{\text{norm}}$ describes the electron density for electrons with an energy of 10 GeV, and $R_{\text{el}}$ & $Z_{\text{el}}$ describe the radial and azimuthal spatial cut-offs. For reference, in Fig. 2 we show a spatial distribution of 10 GeV electrons both in linear and logarithmic space.

It should be noted that in our description of the halo, it is assumed that both the magnetic field and electron distribution possess an exponential cut-off in their spatial extent beyond a cut-off distance scale, whereas in reality they may have a power-law decay beyond this distance (Waelkens, A. et al. (2009); Samui et al. (2018); Bell & Matthews (2022)). However, since we are primarily interested in regions dominating the total synchrotron emission, the actual distribution of the particles and field beyond the scale height distance are not our focus. Provided that the synchrotron emissivity decays faster than $l^{-1}$ along the line of sight at distances beyond the cut off distance, the contribution to the synchrotron emission from further distances can be safely neglected.

### 2.3 Synchrotron Emission

#### 2.3.1 Intensity & polarisation

Synchrotron radiation or magneto-bremsstrahlung radiation is the radiation produced due to charged particles that gyrate at relativistic speeds around a static magnetic field. Synchrotron radiation is sensitive to $B_{\perp}$, the magnetic field component perpendicular to the line of sight. The radiation produced via synchrotron is often linearly polarised. The polarised emissivity (emission per unit volume) spectral distribution can be visualised as an ellipse where the major axis is the perpendicular component ($J_\perp$) and the minor axis is the parallel ($J_\parallel$) component (see Appendix A for further discussion). The two polarisation emission components, $J_\perp$ and $J_\parallel$, describe the emission spectrum for a given peak photon energy $E_{\gamma}^{\text{peak}}$. Expressions for these two components averaged over an isotropic distribution of pitch angles of electrons are provided below in Eqs. 3 and 4.

$$J_\perp = \frac{1}{\pi} \int \frac{dE_e^{\max}}{\log E_e^{\min}} dE_e \frac{dn_e}{d\log E_e} \left[ F \left( \frac{E_\gamma}{E_{\gamma}^{\text{peak}}} \right) + G \left( \frac{E_\gamma}{E_{\gamma}^{\text{peak}}} \right) \right]$$

and

$$J_\parallel = \frac{1}{\pi} \int \frac{dE_e^{\max}}{\log E_e^{\min}} dE_e \frac{dn_e}{d\log E_e} \left[ F \left( \frac{E_\gamma}{E_{\gamma}^{\text{peak}}} \right) - G \left( \frac{E_\gamma}{E_{\gamma}^{\text{peak}}} \right) \right]$$

Figure 1. Cross-section of the toy model for the Galactic magnetic field (for best fit parameter values see table 1) in the Galactic halo region in the $xy$ plane at $z = 1$ kpc and $xz$ plane at $y = 1$ kpc (with the Galactic plane in the $xy$ plane at $z = 0$) showing their drop in two dimensions. We omit the disc region in the left plot since its not a part of our model.
where

$$t^{-1} = \frac{4\alpha}{g} \frac{B}{B_{\text{crit}}} \frac{m_e c^2}{h}, \quad E_{\text{peak}} = \Gamma^2 \frac{B}{B_{\text{crit}}} m_e c^2,$$

and

$$F(x) = x \int_x^\infty K_{\alpha/3}\left(x'\right) dx', \quad G(x) = x K_{2/3}. $$

These expressions are provided in terms of the critical magnetic field strength, $B_{\text{crit}} = \frac{m_e c^2}{e\alpha} = 4.414 \times 10^{13}$ G, where $m_e c^2 = 0.511$ MeV is the rest-mass energy of the electron, $h = 4.136 \times 10^{-15}$ eV s is Planck’s constant, $\Gamma$ is the Lorentz factor and $\alpha \approx \frac{1}{137}$ is the electromagnetic fine structure constant.

For clarity, several of the conventions we adopted are noted here. The parallel component of polarisation ($J_\parallel$) is orientated in the same direction as $B_\perp$, and the perpendicular component of polarisation ($J_\perp$) is perpendicular to $B_\perp$. The Stokes parameters at each point along the line of sight can be written in terms of the intrinsic polarisation angle $\Psi_{\text{in}}$, which is the angle between the line-of-sight perpendicular component of the magnetic field $B_\perp$ and Galactic south at each step. The conventions adopted here match those used by the Planck Collaboration (Planck Collaboration et al. 2015) based on the HEALPix$^3$ software by Gorski et al. (2005). For each step along the line of sight, both $J_\perp$ and $J_\parallel$ are subsequently used to obtain the $Q$ and $U$ Stokes parameters. We obtain the values the intrinsic Stokes parameters $Q_{\text{in}}$ and $U_{\text{in}}$ by integrating over $Q$ and $U$ along the line of sight:

$$Q_{\text{in}} = \frac{1}{4\pi} \int_0^L dl \left( J_\perp - J_\parallel \right) \cos(2\Psi_{\text{in}}),$$
$$U_{\text{in}} = \frac{1}{4\pi} \int_0^L dl \left( J_\perp - J_\parallel \right) \sin(2\Psi_{\text{in}}).$$

The polarised flux ($I_{\text{pol}}$) can then be expressed in terms of $Q_{\text{in}}$ and $U_{\text{in}}$ as

$$I_{\text{pol}} = \sqrt{(Q_{\text{in}})^2 + (U_{\text{in}})^2} = J_\perp - J_\parallel.$$

Similarly, $I_{\text{tot}}$ is computed by summing the contributions of $J_\parallel$ and $J_\perp$ for each point along the line of sight,

$$I_{\text{tot}} = \frac{1}{4\pi} \int_0^L dl (J_\parallel + J_\perp).$$

$J_\parallel$ and $J_\perp$ are the resultant magnitudes of emissions in perpendicular and parallel directions and can be given by:

$$J_\parallel = (I_{\text{tot}} + I_{\text{pol}})/2,$$
$$J_\perp = (I_{\text{tot}} - I_{\text{pol}})/2.$$

The intrinsic polarisation angle $\Psi_{\text{in}}$ is the resulting angle of polarisation:

$$\tan(2\Psi_{\text{in}}) = \frac{U_{\text{in}}}{Q_{\text{in}}}.$$ 

In Appendix A an example case for these calculations is provided for further understanding.

**2.3.2 Simulation setup for the polarised synchrotron emission**

Utilising the setup described in Section 2, we generate a synthetic polarised synchrotron emission map for each parameter.

\[1\] https://healpix.jpl.nasa.gov/
set of our toy model. The toy model comprises of 5 free parameters, (see Table 1). The radial cut off of the magnetic field and electron distribution is kept identical ($R_{\text{Mag}} = R_{\text{el}}$) and the same applies to the azimuthal cut-off ($Z_{\text{Mag}} = Z_{\text{el}}$). The reason for this constraint is that the synchrotron radiation level depends on both the non-thermal electron density and the magnetic field strength. Thus, even if the spatial extend of the magnetic field differs from the electron distribution, one can only probe the magnetic field in the region where both the magnetic field and non-thermal electrons are present. For the spatial parameter scan, the parameter values scanned over for $R_{\text{el}}$ and $Z_{\text{el}}$ are 2 kpc to 19 kpc, with a scanned step size of 1 kpc. Likewise, the range over which both $B_{\text{el}}$ and $B_{\text{tur}}$ are scanned is 2 $\mu$G to 19 $\mu$G, with a step size of 1 $\mu$G. We calculate the polarised emission for one particular value of $\log_{10}(C_{\text{norm}}[\text{cm}^{-3}]) = -12.43$, and subsequently re-scale these results to obtain the polarised emission maps for different values of $\log_{10}(C_{\text{norm}}[\text{cm}^{-3}])$, ranging this scan from $-9.5$ to $-15.5$ with a step size of 0.3 (ie. in 20 steps).

In our study, we mask out three regions of the sky from our skymaps. The first is in the Galactic disc region between $b = (-15^\circ, 15^\circ)$. For the second region, based on observations from Su et al. (2010) and Predehl et al. (2020), we block out longitudes $\geq \pm 90^\circ$ from the Galactic center (i.e. all directions pointing away from the Galactic center direction), so as to ensure that our analysis only covers the region occupied by the Galactic Halo (Fermi and eRosita) bubbles. Lastly, we block out the region associated with the North Polar Spur (NPS). Our motivation here is that there are indications that the higher latitudes of the NPS are originating locally rather than from the Galactic center, based on starlight polarisation observations (Panopoulou et al. 2021). In order to remain as impartial as possible for the designation of this region, we adopt a cut for it selected in Wolleben (2007). In Fig. 3 observational and synthetic skymaps are shown with these three regions removed.

To obtain the best-fit parameters for our model and their constraints, we ran a grid search over the 5 free parameters, sampling in total $2 \times 10^6$ parameter sets. For each model parameter configuration, a synthetic skymap was generated using Healpix (Gorski et al. 2005), adopting a resolution with NSide = 32. Since the interests of our study are focused on large scale structures, both the synthetic skymaps and observational data were smoothed out, using a Gaussian kernel, on a size scale of $15^\circ$, to wash out smaller scale features. We then compare the simulated polarised emission with the Planck data at 30 GHz by evaluating the $\chi^2$ value of the model fit to the data.

### 2.3.3 Observational data

For our synchrotron emission study, we use the publicly available data from the Planck satellite mission\(^2\). Specifically, we use the polarised radio data at 30 GHz from Planck where the peak frequency is at 28.4 GHz, with a band width of 9.8 GHz. At this frequency a considerable level of polarised synchrotron emission is observed, with only a small level of Faraday rotation occurring at these high frequencies. However, we also note that in this 30 GHz band, the Planck data cannot be used to probe synchrotron intensity directly, since at this frequency the unpolarised sky receives considerable contributions from both thermal bremsstrahlung and anomalous microwave emission, as well as synchrotron radiation (Planck Collaboration et al. 2015, 2016a,b,c).

### 2.4 Constraints on Magnetic Field Model

We obtain 1σ constraints on each of our model parameters (see Table 1). For the structured magnetic field strength, $B_{\text{str}}$, we obtain the best-fit value of 3 $\mu$G with the upper extreme being 11 $\mu$G and the lower extreme 1 $\mu$G. Similarly, for the turbulent magnetic fields, $B_{\text{tur}}$, the mean value is 6 $\mu$G with lower and upper extreme values of 4 $\mu$G and 12 $\mu$G, respectively. For the spatial extent of the field, we obtain a best-fit value of 7 kpc for the azimuthal extent, $Z_{\text{Mag}}/Z_{\text{el}}$, with uncertainties of $\pm 1$ kpc. These values are in agreement with the observations made by FERMI (Su et al. 2010), S-PASS (Carretti et al. 2013) and eROSITA (Predehl et al. 2020). The dominance of turbulent to structured fields are consistent with the findings from studies of other local galaxies (??). We do not, however, obtain an upper extreme value for $R_{\text{Mag}}/R_{\text{el}}$, only a lower extreme value of 4 kpc with the best-fit value being 5 kpc. Again, this best-fit value is in agreement with the radial extent as obtained by the observations.

In Fig. 3 the smoothened skymap obtained from the best-fit values of the parameters and the smoothened polarised Planck data is shown along with the residuals. The best-fit values used for the parameters are provided in Table 1. The polarisation fraction obtained by our best-fit toy-model, given in Fig. 3, was calculated taking the ratio of the polarised to the total intensity. The polarisation fraction for the best-fit model is comparable to the values as seen in the observation data of Page et al. (2007) and Carretti et al. (2013).

### 3 COSMIC RAY DEFLECTIONS DUE TO THE MAGNETIC FIELD MODEL

Charged particles propagating through magnetic fields precess around the field lines by virtue of the Lorentz force

$$\frac{d\boldsymbol{p}}{dt} = \frac{1}{r_L} \boldsymbol{p} \times \hat{B},$$

where $\boldsymbol{p}$ is the particle’s velocity vector, $\hat{B}$ is the magnetic field unit direction vector, and $r_L$ is the particle’s Larmor radius. Thus

$$r_L = \frac{p}{eB} = \frac{v}{c},$$

where $R = E/eZ$ is the particle’s rigidity and $Z$ is the nucleus’s proton number.

UHECRs experience deflection effects when propagating through both extragalactic and Galactic magnetic fields. The extragalactic magnetic field is considered to be weak, with $B < nG$ for $\lambda_{\text{coh}} = 1$ Mpc (Blasi et al. 1999; Kronberg et al. 2007). For UHECRs with rigidity $R > 10^{19}$ V in weak (sub nG) extragalactic magnetic fields, $r_L > 10$ Mpc, giving rise to a deflection of $\theta \approx \lambda_{\text{coh}}/r_L < 6^\circ$ each coherence length. Thus the angular deflection expected from UHECR propagating from local (< 4 Mpc) sources a few coherence lengths away is $\lesssim 10^\circ$. In comparison, within the Galactic magnetic field structure, field strengths of order 5 $\mu$G are experienced. An UHECR with rigidity 10 EV in a 5 $\mu$G field, has a Larmor radius of $r_L \approx 2$ kpc. Thus, UHECR in this rigidity range

\(^2\)http://pla.esac.esa.int/pla/
from a nearby source will be pick up their largest angular deflections from their source positions upon passing through the large scale Galactic magnetic field region.

We use the publicly available cosmic ray propagation code CRPropa 3 (Batista et al. 2016) for studying the effects of toy model magnetic fields on the arrival directions of cosmic rays. Within this software we use the Boris pusher scheme in order to ensure a particle’s trajectory evolution satisfies the Lorentz force equation. It is important to note that CRPropa conserves the total energy of each particle during the propagation. We propagate $10^7$ cosmic rays starting at Earth isotropically through the toy model using the backtracking scheme out to a distance of 20 kpc from the Galactic center. We use nitrogen as the choice of our cosmic ray particles at 40 EeV, with rigidity $R \approx 6 \times 10^{18}$ V.

### 3.1 Effects of the Magnetic Field Model on UHECR Arrival Directions

In figure 4 we show:

(i) **Left column:** the magnification maps in log(particles/str), obtained by backtracking an isotropic distribution of cosmic rays from Earth. These magnification maps are made for the best-fit values (top), a set of minimum values (middle) and a set of maximum values (bottom) for the magnetic field model parameter values. To create these histograms we binned the cosmic ray distribution into angular bins (with respect to the Earth) on the escape surface, with 180 bins for both latitudes and longitudes. The histogram values in the maps are normalised to the histogram values obtained for simulations without any magnetic fields present (giving rise to uniform sky brightness). In each map the blue regions denote the areas of the sky where cosmic rays are suppressed and the red regions are the ones where the cosmic rays are enhanced.

(ii) **Right column:** skymaps for arrival directions of cosmic rays from UHECRs candidate sources Cen A and NGC 253. Similar to the above case of the magnification maps we backtrack cosmic rays starting from Earth until they reached an escape radius of 20 kpc from the Galactic centre.

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**Table 1.** Table of best-fit parameters with uncertainties

| Parameter | Best-fit value | Description |
|-----------|----------------|-------------|
| $B_{str}$ | $3^{+8}_{-1} \mu$G | Structured field strength |
| $B_{tur}$ | $6^{+12}_{-2} \mu$G | Turbulent field strength |
| $R_{Mag} = R_{el}$ | $5^{+1}_{-1}$ kpc | Radial cut off |
| $Z_{Mag} = Z_{el}$ | $7^{+1}_{-1}$ kpc | Azimuthal cut off |
| $\log_{10}(C_{norm}[cm^{-3}])$ | $-12.43^{+0.32}_{-1.31}$ | Electron normalisation at 10 GeV |
Figure 4. **Left:** magnification maps of the extragalactic sky obtained by backtracking an isotropic distribution of cosmic rays (with $R \approx 6 \times 10^{18}$ V) from Earth. These maps are normalised relative to results obtained without magnetic fields with the same total number of events. **Right:** the binned arrival directions of the cosmic rays (with $R \approx 6 \times 10^{18}$ V) from two candidate sources: Cen A and NGC 253. In the legend we denote for both sources, the ratio (‘Magn.’) of the number of backtracked cosmic rays within 5° from the source location for the magnetic field model configuration, to the equivalent number obtained in the absence of magnetic fields. The number of particles in each bin are again normalised by the peak value of a binned histogram obtained without magnetic fields, as represented by the grey colour bars. The mean direction in each plot is denoted by a $\phi$. **Top row:** results obtained for best-fit magnetic field parameters, **middle row:** lower extreme magnetic field parameters & **bottom row:** upper extreme magnetic field parameters ($R_{\text{Mag}} = 20$ kpc adopted).

The cosmic ray arrival directions from a region of 5° from the source are then binned. Like the magnification maps, we normalise these maps to the peak value of the histogram densities in the source region for the case of no magnetic fields. This gives a normalised value of the number of hits (‘Magn.’) obtained. Analogous to the magnification maps the **top**, **middle** and **bottom** plots denote the ‘Best-fit’, ‘Minimum’ and ‘Maximum’ cases, respectively.

The deflections of UHECR in the Galactic magnetic field are sensitive to both the structured and turbulent field com-
ponents of the field in different ways. One of the first effects worth noting is the suppression effect for cosmic rays from certain regions of the sky. For the 'Maximum' toy model magnetic field model giving rise to the largest suppression factor in the magnification map in comparison to the 'Best-fit' and 'Minimum' cases (see Table 1). These suppression and enhancement of UHECRs from different regions of the sky are also seen in the skymaps provided for two potential UHECR sources, namely Cen A (lon = -50.49°, lat = 19.42°) and NGC 253 (lon = 97.36°, lat = -87.96°). Because of their positions in the sky, both of these sources lie in the suppression region of the magnification maps for both the 'Best-fit' and 'Maximum' cases. In particular, we note that for NGC 253, the 'Maximum' toy model case leads to a suppression level of \( \approx 5\% \) of the level that would arrive for the no magnetic field case, and \( \approx 40\% \) for Cen A.

A second effect introduced by the turbulent magnetic fields is the spreading effect (mean deflection angle) of cosmic rays around their originating source direction. In order to quantify this effect, a list is provided below of the mean deflection angle, \( \langle \sigma_{\text{source}} \rangle \), between the mean direction and the arrival directions of the cosmic rays for the two candidate sources considered:

- **Best-fit** - \( \sigma_{\text{NGC 253}} = 38^\circ \), \( \sigma_{\text{Cen A}} = 38^\circ \)
- **Minimum** - \( \sigma_{\text{NGC 253}} = 27^\circ \), \( \sigma_{\text{Cen A}} = 30^\circ \)
- **Maximum** - \( \sigma_{\text{NGC 253}} = 83^\circ \), \( \sigma_{\text{Cen A}} = 69^\circ \)

In comparison to the magnitude of these spreading angles, the mean deflection angle for Cen A and NGC 253 from the JF12 toroidal halo field (JF12 halo) (Jansson & Farrar 2012) are \( \sigma_{\text{NGC 253}} = 9^\circ \), and \( \sigma_{\text{Cen A}} = 22^\circ \). The spread of the cosmic rays obtained for our toy model are therefore potentially considerably larger (up to 5-10 times bigger) than those obtained for the JF12 toroidal halo. The primary driver of this difference is that our toy model possesses a larger level of turbulent magnetic fields than structured fields. It is also worth noting that the mean deflection angle of Cen A (\( \sigma_{\text{Cen A}} = 38^\circ \)) for the 'Best-fit' case of our toy model is comparable with the Pierre Auger Observatory (PAO) observations (Abreu et al. 2021).

Additional to this spreading effect, the presence of a structured field component in the magnetic field model leads to the coherent deflection of the mean direction of the ensemble of cosmic rays away from the source direction. Following the propagation of cosmic rays from the two candidate source NGC 253 and Cen A, the mean source position (lon, lat) for the three cases are as follows:

- **Best-fit** - NGC 253: \((2^\circ, -72^\circ)\) & Cen A: \((-45^\circ, 10^\circ)\)
- **Minimum** - NGC 253: \((9^\circ, -78^\circ)\) & Cen A: \((-39^\circ, 16^\circ)\)
- **Maximum** - NGC 253: \((41^\circ, -19^\circ)\) & Cen A: \((-53^\circ, 1^\circ)\)

In comparison, the mean shift positions from the JF12 halo model for the two sources are NGC 253: \((1^\circ, -33^\circ)\) & Cen A: \((-47^\circ, -1^\circ)\).

Additionally, for the case of cosmic rays from NGC 253, an interesting difference between our toy model and the JF12 halo model is worth noting. For the JF12 halo model (see Appendix C), the mean position of cosmic rays from NGC 253 is situated at roughly a latitude of \(-33^\circ\) (also seen in van Vliet et al. 2021). In contrast to this, in our 'Best-fit' toy model case this value is at approximately \(-72^\circ\), which would be in better agreement with the PAO observations (Aab et al. 2018) if this southern Galactic hemisphere hotspot does indeed originate from NGC 253. This difference in the position of mean direction is again an effect of our toy model having weaker structured fields in comparison to turbulent fields (see Table 1), since structured fields dictate the extent of the mean deflection angle from the source position.

The suppression, spreading and coherent deflection effects place challenges on the association of cosmic rays to their originating sources. It can be seen that in the 'Maximum' toy model magnetic field would make associating cosmic rays to their source extremely challenging at the energies considered, whereas the 'Best-fit' or 'Minimum' cases make this possible. This is due to the fact that the structured fields are responsible for the overall direction of the particle deflections, whereas turbulent fields are responsible for spreading out the directions of the particle deflections around this overall deflected direction. For cases in which the turbulent magnetic field component dominates, and this field strength component is large, the source directions can be completely washed-out. This washing-out of the source association is evident for the 'Maximum' case in Fig. 4, cosmic rays from sources like NGC 253 are largely deflected from the source position by the magnetic field structure with a mean deflection angle of \( \sigma_{\text{NGC 253}} = 83^\circ \). Likewise, in the case of Centaurus A the final positions are spread out over a large region of the sky with a mean deflection angle of \( \sigma_{\text{Cen A}} = 69^\circ \), making association with the source position challenging. From both the magnification and arrival direction maps in Fig. 4, it is evident that the best-fit and lower extreme ('Minimum') parameters allow some degree of association of the deflected UHECRs with their original source position. However, in the upper extreme ('Maximum') parameters, such a connection between the point of origin of cosmic rays and their final positions is heavily erased.

### 4 CONCLUSIONS

Utilising our toy model for the Galactic halo magnetic field, and making comparisons of the synchrotron emission predicted by it to the Planck 30 GHz data, we explore the region of model parameters capable of providing a good description of the data. Significant evidence is found for the presence of an extended magnetic field component out in the Galactic halo region. Our results are compatible with the Galactic halo magnetic field extending to 7 kpc in height above the Galactic disc. The total magnetic field content in the halo region from our model fits is \( \approx 10^{55} \) ergs (with a comparable total energy in 1 GeV cosmic ray protons of \( \approx 4 \times 10^{55} \) ergs). We note that these values are comparable with observational inference made in Predel et al. (2020) which indicated the presence of some \( 10^{56} \) ergs of thermal particles in the Galactic halo. In comparison to these energy contents, the total magnetic field energy content in the halo field component of the JF12 model is \( 4 \times 10^{54} \) ergs and \( 3 \times 10^{54} \) ergs for the toroidal halo and X-field respectively (Taylor & Hillas 2019).

Using the maximum and minimum constraints on the magnetic field model parameter values, the range of deflection that UHECRs experience in passing through such Galactic halo magnetic field structure was subsequently investigated. A significant range in predictions of both: a) the magnification of different regions of the extragalactic sky, and b) the
deflection of cosmic rays arriving from different local extragalactic sources was found. The predictions from our magnetic field model are also in agreement with PAO observations (Aab et al. 2018; Abreu et al. 2021), for both the hotspot around Cen A and the potential hotspot from NGC 253.

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APPENDIX A: POLARISED SYNCHROTRON EMISSION

As discussed in section (2.3) the line of sight components for Stokes parameters are given by:

\[ Q_{\text{in}} = (J_{\parallel}^1 - J_{\parallel}^4) \cos(2\Psi_{\text{in}}), \]  
\[ U_{\text{in}} = (J_{\parallel}^1 - J_{\parallel}^4) \sin(2\Psi_{\text{in}}). \]  

We can take two test cases (I & II) given in the left and right panel of Fig. A1, respectively. We consider that there are two steps along a line of sight for which \( J_{\parallel}^{1,2} = 0.85 \) and \( J_{\parallel}^{1,2} = 0.15 \).

In case I the angles \( \Psi_{\text{in}}^{1,2} = 90^\circ \) and \( 0^\circ \). The resultant value of \( I_{\text{tot}} \) is 0 by virtue of Eq. 7 and we only have a contribution to \( I_{\text{rot}} \). This implies that for case I the resultant emission is seen only in total intensity, since the values of \( J_{\parallel}^{1,2} = J_{\parallel}^{1,2} \).

In case II we apply similar calculations to case I, however, now the angles are \( \Psi_{\text{in}}^{1,2} = 90^\circ \) and \( 45^\circ \). This in turn results in contributions to both polarised emission \( I_{\text{pol}} \) and total intensity \( I_{\text{tot}} \). Thus the values of \( J_{\parallel}^{1,2} \neq J_{\parallel}^{1,2} \). In the right panel of Fig. A1 we only show the polarised intensity for simplicity, however, there will be both total intensity and polarised intensity present.

APPENDIX B: TURBULENT MAGNETIC FIELD

As discussed in Section 2.1 we generate the turbulent fields for our model using CRPropa 3 (Batista et al. 2016). The minimum and maximum wavelength we use to generate these fields are \( L_{\text{min}} = 200 \) pc and \( L_{\text{max}} = 400 \) pc and \( L_{\text{coh}} \approx 150 \) kpc. One of the major reasons why we do not have more decades covered for the wavelength is because of the time it takes to generate these fields using CRPropa. We investigated power spectra for different realisations of the turbulent field. In Fig. B1 we plot power spectra in \( x, y \) and \( z \) directions, after averaging over the other two directions. We chose a step size of 1 pc and integrate up to \( \approx 9 \times 10^7 \) pc. We chose
Figure A1. Diagram depicting visually the resultant ellipse (blue), obtained from the summation of two ellipses of equal magnitude (pink), but different orientations. The resultant ellipse dictates the resultant total and polarised intensities.

This particular realisation since it followed closely a power-law spectrum of index $5/3$, with a similar amount of power in each direction (i.e. was reasonably isotropic).

APPENDIX C: ARRIVAL DIRECTIONS FOR THE JF12 TOROIDAL HALO FIELD

We calculate the arrival directions of cosmic rays for two candidate sources, Cen A and NGC 253, for the JF12 toroidal halo model (Jansson & Farrar 2012), shown in Fig. C1. We normalise these binned arrival directions by the peak value of the histogram obtained from the same setup without magnetic field present. It can be seen from Fig. C1 that the JF12 toroidal halo displaces the binned arrival directions to much higher latitudes in the case of NGC 253. This is because the structured field strength in the JF12 toroidal halo is stronger than the turbulent field and hence the mean deflection from the source position is larger.
**Figure B1.** Power spectra of turbulent magnetic fields, evaluated along three orthogonal directions, namely the $x$, $y$ and $z$ directions.

**Figure C1.** Arrival direction map of cosmic rays (with $R \approx 6 \times 10^{18}$ V) deflected from the JF12 halo for two potential UHECR sources. It can be seen that the mean direction of the deflection for the JF12 toroidal halo is at a higher latitude than for the toy model, shown in Fig. 4.