Dark Matter Constraints on Low Mass and Weakly Coupled B-L Gauge Boson

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ABSTRACT: We investigate constraints on the new gauge boson ($Z_{BL}$) mass and gauge coupling ($g_{BL}$) in the $U(1)_{B-L}$ extension of the standard model (SM) with an extra SM singlet Dirac fermion ($\zeta$) which plays the role of dark matter (DM). The DM particle $\zeta$ is a non-singlet under the $B-L$ symmetry with an arbitrarily chosen charge $Q$ to guarantee its stability. We focus on the small mass and small $g_{BL}$ regions of the model, and find new constraints for both the cases where the DM relic abundance arises from thermal freeze-out and freeze-in mechanisms. We point out that in the thermal freeze-out case, the dark matter coupling is given by $g_{\zeta} \equiv g_{BL}Q \simeq 0.016 \sqrt{m_\zeta[GeV]}$ to reproduce the observed DM relic density and $g_{BL} \geq 2.7 \times 10^{-8} \sqrt{m_\zeta[GeV]}$ for the DM particle to be in thermal equilibrium prior to freeze-out. In this case, the direct detection constraints put a lower bound on the $Z_{BL}$ mass ($M_{Z_{BL}}$) of about 45 MeV. We find that $Z_{BL}$ mass range up to about 2 GeV with $g_{BL}$ in this region can be probed in the FASER experiment via displaced vertex searches and combined with future improved constraints from the direct detection limits, the $B-L$ gauge extension can be tested in this parameter region. For the freeze-in scenario, there are two possibilities to reproduce the observed DM relic density: $g_{\zeta}^2 g_{BL}^2 + \frac{18}{37} g_{\zeta}^4 \simeq 9.2 \times 10^{-24}$ for $g_{BL} \geq 2.7 \times 10^{-8} \sqrt{m_\zeta[GeV]}$, and $g_{\zeta} g_{BL} \simeq 3.0 \times 10^{-12}$ for $g_{BL} < 2.7 \times 10^{-8} \sqrt{m_\zeta[GeV]}$, independently of the DM mass. Even for the freeze-in case, the direct detection constraints set a lower bound on, for example, $M_{Z_{BL}} \gtrsim 70$ MeV for the DM mass of 30 GeV. We also discuss the bounds from SN1987A observations on the $g_{BL}$ analogous to the case of dark photon for $M_{Z_{BL}} \leq 100$ MeV.
1 Introduction

Extensions of the standard model (SM) were proposed many years ago with $U(1)_{B-L}$ as a possible new symmetry of electroweak interactions in the context of left-right symmetric models [1]. Its connections to the neutrino mass was demonstrated in Ref. [2] and it has recently attracted a great deal of attention. Theoretical constraints of anomaly cancellation allow two classes of such extensions: (i) one motivated by left-right symmetric and SO(10) models, where the $B-L$ generator contributes to the electric charge [1–3] and (ii) another where it does not contribute to electric charge [4–10] and is not embeddable into the left-right or SO(10) models. Both classes of models require the addition of three right handed neutrinos to satisfy the anomaly constraints and lead to the seesaw mechanism for neutrino masses [11–15]. There is however a fundamental difference between the two classes of models as regards the magnitude of their gauge couplings: in the first class of models where the $B-L$ contributes to electric charge [1–3], there is a relation between the electric charge of the positron and the $B-L$ gauge coupling of the form:

$$\frac{1}{e^2} = \frac{1}{g^2_L} + \frac{1}{g^2_R} + \frac{1}{g^2_{BL}} \quad (1.1)$$
As a result, there is a lower bound on the value of $g_{BL}$:

$$\frac{1}{g_{BL}} \leq \frac{\cos^2 \theta_W}{e^2} \quad \text{or} \quad g_{BL} \geq 0.34.$$  \hfill (1.2)

This lower bound gets strengthened to 0.416, when it is assumed that all $U(1)$ couplings in the $SU(2)_L \times U(1)_R \times U(1)_{B-L}$ model are demanded to be perturbative till the Grand Unified Theory scale \cite{16}.

In the second class of models on the other hand, there is no lower bound on $g_{BL}$ from theoretical considerations, so that it can be arbitrarily small, although there is an upper bound given by perturbativity requirements. In this paper, we focus on this second class of models in the small $g_{BL}$ and small $B-L$ gauge boson mass ($M_{Z_{BL}}$) regions to see what kind of phenomenological constraints exist, once we add a Dirac dark matter fermion $\zeta$ to the theory with an arbitrary $B-L$ charge, $Q$. Clearly, it is possible to choose a $B-L$ charge for $\zeta$ so that it is naturally stable. This class of models are completely realistic as far as the their fermion sector is concerned and have four parameters: $g_{BL}, g_{\zeta} \equiv g_{BL}Q$ and the two mass parameters, $m_\zeta$ and $M_{Z_{BL}}$ (see Refs. \cite{17, 18} for the case with the two mass parameters in the muti-TeV range). We ignore mixings between the $B-L$ gauge boson and the SM gauge bosons for simplicity so that $Z_{BL}$ is purely the $B-L$ boson. The dark matter in this case is the Dirac fermion $\zeta$ as just mentioned and anomaly cancellation is maintained.

We discuss constraints that $g_{BL}$ and $g_{\zeta}$ must satisfy from the requirements that there be a viable dark matter in the model. We consider the following two broad categories of gauge coupling parameter ranges of the theory: (i) one where the dark matter relic density arises via thermal freeze out of $\zeta$; (ii) the second case where the couplings $g_{BL,\zeta}$ are so weak that the dark matter was never in thermal equilibrium in the early universe with SM particles and the $\zeta$ had vanishing density in the early universe. The DM relic abundance was built up in this case via the freeze-in mechanism \cite{19–22}. In the first case, we find that the relic density constraint requires that $g_{\zeta} \simeq 0.016 \sqrt{m_\zeta[\text{GeV}]}$ and the condition for thermal equilibrium of $Z_{BL}$ in the early universe requires that $g_{BL} \gtrsim 2.7 \times 10^{-8} \sqrt{m_\zeta[\text{GeV}]}$. For the freeze-in case, we find that the product $g_{BL} g_{\zeta} \approx 10^{-12}$ almost independently of $m_\zeta \gg M_{Z_{BL}}$ to satisfy the constraint on the DM relic density $\Omega_{DM}$ (the freeze-in mechanism for a Majorana fermion DM and $g_{\zeta} = g_{BL}$ was investigated in Ref. \cite{23} and their results are consistent with ours). It is interesting that the spin-independent direct detection cross section also depends on the product $(g_{BL} g_{\zeta})^2 \mu_{\zeta N}^2 / M_{Z_{BL}}^4$ (where $\mu_{\zeta N}$ is the reduced mass of the DM-nucleon system) and therefore the $\Omega_{DM}$ constraint also puts lower limits on the $Z_{BL}$ mass. We explain the origin of these constraints and elaborate on the details in the body of the paper.

We also comment on two more cases: case (iiiA) where the $g_{\zeta}$ is large enough that both $Z_{BL}$ and $\zeta$ were in equilibrium with each other but not with the SM particles and case (iiiB) where both $g_{\zeta,BL}$ are so small that all three sectors were thermally sequestered from each other. These cases do not fall into either the freeze-in or freeze-out scenarios and are therefore listed separately.
There are also constraints on this model from Fermi-LAT observations that assume 100% branching ratio to either $b\bar{b}$ or $\tau^+\tau^-$ [24] which are compatible with the thermal freeze-out constraints only for $m_\zeta \geq \text{few GeV}$. The assumption of 100% branching ratio is however not the case for our model and we have more like 20% for the branching ratio. As a result, our bounds are weaker and we estimate it to be in the 2 GeV range using the Fig. 9 of the Fermi-LAT paper [24].

We note here that the $B - L$ models with [25] and without the dark matter [26] have been discussed previously. In Ref. [26], constraints are obtained for pure $B - L$ model with Dirac neutrinos. On the other hand, in our model, we have Majorana neutrinos and unlike the Refs. [25, 26], we consider the case where $B - L$ gauge interactions couple both matter and the dark matter, the latter with an arbitrary $B - L$ charge and obtain the constraints on the model. Constraints on dark photon portal models with an MeV dark matter have been considered in the literature in Ref. [27]. This has similarity to some of our considerations. In particular, we use the CMB bounds on dark matter using Fig. 3 of this reference. In the first two cases, the CMB constraints of Ref. [27] on the resulting cross section imply that $m_\zeta \geq 1$ GeV. In this paper, we will focus on the region of $\zeta$ mass with $m_\zeta \geq 1$ GeV.

The paper is organized as follows: in Sec. 2, we outline the details of the model; in Sec. 3, we discuss the case of thermal freeze-out of the dark matter and the constraints on the relevant model parameters from it. We note how the FASER experiment [28] combined with improvements on the spin-independent DM direct detection cross section in future can provide a test of the model; in Sec. 4, we switch to the parameter range of the model where the relic density arises out of the freeze-in mechanism and the constraints implied by it on the model. We also comment on constraints from the SN1987A and Big Bang Nucleosynthesis (BBN); in Sec. 5, we comment on the case where the “dark sector” with $\zeta$ and $Z_{BL}$ are decoupled from the SM thermal plasma, but they are produced from the inflaton decay after inflation. We conclude in Sec. 6 with a discussion of implications of our results and some additional comments.

2 The $B - L$ model with Dirac fermion dark matter

Our model is based on the $U(1)_{B-L}$ extension of the SM with gauge quantum numbers under $U(1)_{B-L}$ defined by their baryon or lepton number as is obvious. The gauge group of the model is $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where $Y$ is the SM hypercharge. We need three right handed neutrinos (RHNs) with $B - L = -1$ to cancel the $B - L$ anomaly. The RHN’s being SM singlets do not contribute to SM anomalies. The electric charge formula in this case is same as in the SM. We now add to this model a vector-like SM singlet fermion $\zeta$ with $B - L$ charge equal to $Q$. This being vector-like does not affect the anomaly cancellation of the model. The $B - L$ group is assumed not to contribute to electric charge formula as stated in the introduction. As a result, its couplings are theoretically not restricted. We assume that there is a Higgs boson with $B - L = +2$ which gives a Majorana mass to the RHNs and therefore helps to implement the seesaw mechanism for neutrino masses since the SM Higgs doublet already provides the Dirac mass to the neutrinos so
that seesaw mechanism works. The dark matter fermion in this case is a Dirac fermion.
The interaction Lagrangian in our model describing the interaction of the $B - L$ gauge boson (called $Z_{BL}$ here) is:

$$\mathcal{L}_{Z_{BL}} = (Z_{BL})_\mu \left[ g_{BL} \sum_f (B - L)_f \bar{f} \gamma^\mu f + g_\zeta \bar{\zeta} \gamma^\mu \zeta \right]. \quad (2.1)$$

This Lagrangian is enough to derive our conclusions. We start with letting the values of $g_{BL}$, $g_\zeta \equiv Qg_{BL}$, $M_{Z_{BL}}$ and $m_\zeta$ as free parameters and explore the smaller mass range of $M_{Z_{BL}}$ and as a benchmark point takes $m_\zeta$ in the range of $1 - 100$ GeV with $M_{Z_{BL}} < m_\zeta$. Clearly our results apply to a range of dark matter masses.

Note that for $g_{BL}^2/M_{Z_{BL}}^2 \lesssim 10^{-6}$ GeV$^{-2}$, the neutral current and other low energy constraints are automatically satisfied. This constraint implies that $g_{BL} \lesssim 10^{-3} M_{Z_{BL}}$ [GeV] is allowed by low energy observations and we seek other constraints in this domain when a dark matter is included in the theory. There are also ATLAS upper bounds on $g_{BL}$ as a function of $M_{Z_{BL}}$ but this bound for low mass $Z_{BL}$ is in the range of $g_{BL} \leq 2 \times 10^{-3}$ or so [29] and it becomes weaker as we go to higher masses.

3 Case (i): Thermal dark matter constraints

3.1 Dark matter relic density

We first consider the case where the parameter range of the model is such that $\zeta$ is a thermal dark matter. We will find these parameter ranges and their possible implications. We are thinking of the case where both $g_{BL}$ and $g_\zeta$ have such values that $Z_{BL}$, $\zeta$ and SM particles were all in thermal equilibrium in the early universe.

For the Dirac DM particle $\zeta$ to be a thermal freeze-out dark matter, it must be in thermal equilibrium with the the SM particles as well as the $Z_{BL}$ in the very early universe and as the temperature of the universe drops below the $m_\zeta$, the Boltzmann suppression makes the $\zeta$ particle density low and the particle goes out of equilibrium. Thermal freeze-out occurs and the DM then freely expands and forms the dark matter of the universe and its current abundance is determined by the values of $g_{BL}$, $g_\zeta$ and $m_\zeta$. In this case there are two constraints on the coupling parameters of the model which determine the freeze-out as well as the DM relic density. We now discuss how those constraints arise and what they are.

Typically in a thermal freeze-out situation, the fact that at one point the $\zeta$ particle was in equilibrium means that:

$$n_\zeta(T) \langle \sigma v \rangle \rightarrow \zeta \zeta \geq H \geq \sqrt{\frac{\pi^2}{90 g_\ast}} \frac{T^2}{M_P}, \quad (3.1)$$

where $n_\zeta(T) \simeq T^3$ is the DM number density for $T \gtrsim m_\zeta$, $g_\ast$ is the effective total degrees of freedom for SM particles in thermal equilibrium (we set $g_\ast = 106.75$ in the following analysis), and $M_P = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. Since we are interested in low mass $Z_{BL}$ boson, we obtain $\langle \sigma v \rangle \simeq \frac{g_{BL}^2 g_\zeta^2}{4 \pi T^2}$ for the $f \bar{f} \rightarrow \zeta \zeta$ process mediated by the
Z_{BL} boson. Requiring the thermal equilibrium condition to be satisfied at $T \simeq m_\zeta$, we obtain the following constraint on the gauge coupling parameters:

$$g_{BL}^2 \zeta^2 \geq 43 \frac{m_\zeta}{M_P}.$$  \hfill (3.2)

As we will see in the next subsection, the thermal equilibrium condition is not consistent with the direct DM detection constraints which are very severe for low $M_{Z_{BL}}$.

There is, however, another possibility for $\zeta$ to be in equilibrium with the SM particles i.e. via a two step process: first $Z_{BL}$ comes to equilibrium with SM fermions via the process $f \bar{f} \rightarrow Z_{BL} \gamma$ and then $\zeta$ goes into equilibrium with $Z_{BL}$ and hence with the SM fermions via the process $Z_{BL}Z_{BL} \rightarrow \zeta \zeta$. The thermal equilibrium condition for the first process is

$$n_{Z_{BL}}(T) \langle \sigma v \rangle_{f \bar{f} \rightarrow Z_{BL} \gamma} \geq H = \sqrt{\frac{\pi^2}{90}} g_* \frac{T^2}{M_P},$$  \hfill (3.3)

where $n_{Z_{BL}}(T) = \frac{2\xi_3}{\pi^2} T^3$ is the number density of $Z_{BL}$, and $\langle \sigma v \rangle_{f \bar{f} \rightarrow Z_{BL} \gamma} \simeq \frac{g_{BL}^4 \alpha_e}{T^2}$ with the fine-structure constant of $\alpha_e = 1/128$. We require that this condition is satisfied at $T = m_\zeta$ (at latest) and obtain

$$g_{BL} \geq 2.7 \times 10^{-8} \sqrt{m_\zeta}[\text{GeV}].$$  \hfill (3.4)

The second process depends only on $g_\zeta$ and the equilibrium condition gives a lower bound on $g_\zeta \geq 9.2 \times 10^{-5} \left( m_\zeta[\text{GeV}] \right)^{1/4}$ by using $\langle \sigma v \rangle_{Z_{BL}Z_{BL} \rightarrow \zeta \zeta} \simeq \frac{g_\zeta^4}{16 \pi T^2}$ in Eq. (3.1). Clearly if we want to get the DM relic density right, we need a larger $g_\zeta$ and therefore in our acceptable range for the DM relic density, $\zeta$ is in thermal equilibrium with $Z_{BL}$.

We next discuss the DM relic density constraints on the model. To evaluate the DM relic density, we solve the Boltzmann equation given by

$$\frac{dY}{dx} = -\frac{\langle \sigma v \rangle s(m_\zeta)}{x^2 H(m_\zeta)} \left( Y^2 - Y_{EQ}^2 \right),$$  \hfill (3.5)

where $x = m_\zeta/T$ is a “temperature” normalized by the DM mass $m_\zeta$, $\langle \sigma v \rangle$ is a thermally averaged DM annihilation cross section $\langle \sigma \rangle$ times relative velocity $\langle v \rangle$, $H(m_\zeta)$ is the Hubble parameter at $T = m_\zeta$, $s(m_\zeta)$ is the entropy density of the thermal plasma at $T = m_\zeta$, $Y$ is the yield of the DM particle which is defined as a ratio of the DM number density to the entropy density, and $Y_{EQ}$ is the yield of the DM in thermal equilibrium. Explicit forms for the quantities in the Boltzmann equation are as follows:

$$H(m_\zeta) = \sqrt{\frac{\pi^2}{90}} g_* \frac{m_\zeta^2}{M_P},$$

$$s(m_\zeta) = \frac{2\pi^2}{45} g_* m_\zeta^3,$$

$$Y_{EQ}(x) = \frac{g_{DM} x^2 m_\zeta^3}{2\pi^2 s(m_\zeta)} K_2(x).$$  \hfill (3.6)
where $K_2(x)$ is the modified Bessel function of the second kind, and $g_{DM} = 4$ is the number of degrees of freedom for the Dirac fermion DM particle $\zeta$. The thermal average of the DM annihilation cross section is given by the following integral expression:

$$\langle \sigma v \rangle = \frac{g_{DM}^2}{64\pi^4} \left(\frac{m_\zeta}{x}\right) \frac{1}{n_{EQ}^2} \int_{4m_\zeta^2}^{\infty} ds \left(\frac{1}{s} \right) K_1 \left(\frac{2\sqrt{s}}{m_\zeta}\right), \quad (3.7)$$

where $n_{EQ} = s(m_\zeta)Y_{EQ}/x^3$ is the DM number density, and $K_1$ is the modified Bessel function of the first kind. The DM annihilation occurs via the process $\zeta \bar{\zeta} \rightarrow Z_{BL}Z_{BL}$ for $m_\zeta > M_{Z_{BL}}$. In the non-relativistic limit, the annihilation cross section times relative velocity for this process is given by

$$\sigma(\zeta \bar{\zeta} \rightarrow Z_{BL}Z_{BL}) v \simeq \frac{g_{\zeta}^4}{16\pi m_\zeta^2} \left(1 - \frac{M_{Z_{BL}}^2}{m_\zeta^2}\right)^{3/2} \left(1 - \frac{M_{Z_{BL}}^2}{2m_\zeta^2}\right)^{-2} \simeq \frac{g_{\zeta}^4}{16\pi m_\zeta^2} \quad (3.8)$$

for $m_\zeta^2 \gg M_{Z_{BL}}^2$. By solving the Boltzmann equation of Eq. (3.5) with the initial condition $Y(x) = Y_{EQ}(x)$ for $x \ll 1$, we evaluate the DM yield at present, $Y(x \rightarrow \infty)$. With a good accuracy, the DM relic density is given as the asymptotic solution of the Boltzmann equation to be

$$\Omega_{DM} h^2 \simeq \frac{2.13 \times 10^8 x_f}{\sqrt{g_{DM}} M_P \langle \sigma v \rangle}, \quad (3.9)$$

where $M_P$ and $\langle \sigma v \rangle$ are evaluated in the unit of GeV, the freeze-out temperature of the DM particle is approximately evaluated as $x_f = m_\zeta/T_f \simeq \ln(x) - 0.5\ln(\ln(x))$ with $x \simeq 0.19\sqrt{g_{DM}/g_*}M_P m_\zeta \langle \sigma v \rangle$. Since the annihilation process occurs through s-wave, we can approximate $\langle \sigma v \rangle$ as $\sigma v$ in the non-relativistic limit.

In order to reproduce the observed DM relic density at present, $\Omega_{DM} h^2 = 0.12$ [33], we obtain a relation between the DM mass and the DM coupling with $Z_{BL}$ for the case of $M_{Z_{BL}}^2 \ll m_\zeta^2$, which is shown in Fig. 1. The observed DM relic density is reproduced along the line, which we find well approximated by

$$g_{\zeta} \simeq 0.016 \sqrt{m_\zeta[GeV]}. \quad (3.10)$$

As we expected, the thermal equilibrium condition for the process $Z_{BL}Z_{BL} \rightarrow \zeta \bar{\zeta}$ we have found before is always satisfied for $m_\zeta > 1$ GeV. In Fig. 2, we show the relation between the $Z_{BL}$ mass and the DM coupling with $Z_{BL}$ for fixed DM masses of 2 GeV (black), 6 GeV (red), 30 GeV (blue), and 100 GeV (green). The observed DM relic density is reproduced along each line. We can see that the coupling is almost constant for a fixed DM mass for $M_{Z_{BL}}^2 \ll m_\zeta^2$ and is well-approximated by Eq. (3.10). The coupling is sharply rising when the $Z_{BL}$ mass becomes very close to the DM mass because of the phase space/kinematic effect.

### 3.2 Direct detection constraints

Let us now turn to the direct detection constraints. In Fig. 3, we show the current upper bound on the spin-independent cross section for the elastic scattering of the DM particle
Figure 1. The relation between the DM mass and the DM coupling with $Z_{BL}$ for the case of $M_{Z_{BL}}^2 \ll m_{\zeta}^2$. The observed DM relic density is reproduced along the line. $g_\zeta \simeq 0.016 \times \sqrt{m_{\zeta}[\text{GeV}]}$ is a good approximation formula.

Figure 2. The relation between the $Z_{BL}$ mass and the DM coupling with $Z_{BL}$ for fixed DM masses of 2 GeV (black), 6 GeV (red), 30 GeV (blue), and 100 GeV (green). The observed DM relic density is reproduced along each line. For $M_{Z_{BL}}^2 \ll m_{\zeta}^2$, the coupling is almost constant for a fixed DM mass. The coupling is sharply rising when the $Z_{BL}$ mass becomes very close to the DM mass because of the phase space/kinematic effect.

with a nucleon for the DM mass of $m_{DM} \geq 2$ GeV. For the DM mass $m_{DM} \geq 6$ GeV, the most stringent upper bound is obtained by XENON1T experiment [34] while for $2$ GeV $\leq m_{DM} \leq 6$ GeV, the upper bound is obtained by a combination of DarkSide-50 [35], LUX [36] and PandaX-II [37]. As is well known the constraints are most severe for a DM mass around 30 GeV and become weaker on either side of this mass.

In our model, the elastic scattering of the DM particle with a nucleon $\zeta N \rightarrow \zeta N$ occurs
via the exchange of $Z_{BL}$ boson. The cross section for the process is given by [30]

$$
\sigma_{SI} = \frac{1}{\pi} g_{Z_{BL}}^2 g_{\zeta}^2 \frac{\mu_{\zeta N}^2}{M_{Z_{BL}}^4}, \tag{3.11}
$$

where $\mu_{\zeta N} = m_{\zeta} m_N/(m_{\zeta} + m_N)$ is the reduced mass for the DM-nucleon system with $m_N = 0.983$ GeV being the nucleon mass. For a given $m_{\zeta}$, say one GeV, which satisfies all the above constraints, we see that as $M_{Z_{BL}}$ goes down, the cross section rises and since $g_{BL}$ has a lower bound from Eq. (3.4) and $g_{\zeta}$ values are already fixed, this implies a lower bound on $M_{Z_{BL}}$ depending on the $\zeta$ mass. These constraints are shown in Fig. 4. For example, for $m_{\zeta} = 2$ GeV, we find the minimum $Z_{BL}$ mass to be $\simeq 45$ MeV.

**Figure 3.** The current experimental upper bound on the spin-independent cross section as a function of the DM mass.

**Figure 4.** Minimum $Z_{BL}$ mass to satisfy the conditions from the DM relic density, the spin-independent cross section and the thermal equilibrium between $Z_{BL}$ and the SM particles, as a function of the DM mass. For example, for $m_{\zeta} = 2$ GeV, we find the minimum $Z_{BL}$ mass to be $\simeq 45$ MeV, which is read off from the intersection of the black solid curve and the black dotted horizontal line in Fig. 5.
Figure 5. Solid curves show the upper bounds of $g_{BL}$ from the direct detection experiments for various values of dark matter mass, $m_\zeta = 2$ GeV (black), 6 GeV (red), 30 GeV (blue) and 100 GeV (green), with corresponding lower bounds (horizontal dotted lines) from the thermal equilibrium condition for $Z_{BL}$, along with the current excluded region (gray shaded), FASER and FASER 2 search reach (yellow shaded regions) and SHiP search reach [39] (dashed orange curve).

3.3 FASER tests of the freeze-out case

We now discuss a way to test the freeze-out scenario in the laboratory. One way is to search for a displaced vertex by the planned FASER detector [38] at the LHC which will probe the low $M_{Z_{BL}}$ ($\lesssim 1 - 2$ GeV) and low $g_{BL}$ region of the theory. This is a detector which will be installed in a tunnel near the ATLAS detector about 480 meters away to look for displaced vertices with charged particles from long-lived charge-neutral particles produced at the primary LHC vertex. In the very low $g_{BL}$ range, our model falls into this category since due to low $g_{BL}$ and low mass $M_{Z_{BL}}$, the high boost, the distance travelled by $Z_{BL}$ before decaying is given by $c\tau \sim \frac{12\pi E_{Z_{BL}}}{g_{BL}^2 M_{Z_{BL}}^2}$ and experiments such as FASER searching for displaced vertices can give useful constraints. In Fig. 5, we show the region of couplings $g_{BL}$ and mass $M_{Z_{BL}}$ which can be probed in this experiment. For $m_\zeta = 2$ GeV (black solid curve), for example, the region below the inclined line on the right is the allowed range from direct DM detection searches. The horizontal (black) dotted line below gives the lower limit on $g_{BL}$ from the thermal equilibrium condition of Eq. (3.4). As $m_\zeta$ increases, the horizontal line which depends like $\sqrt{m_\zeta}$ goes up whereas the inclined line goes down since the upper bound on $\sigma_{SI}$ becomes stronger till about $m_\zeta \simeq 30$ GeV (see Fig. 3). Above 30 GeV, the direct detection cross section becomes weaker and the inclined line goes higher. In Fig. 5, we give four benchmark values for $\zeta$ mass i.e. 2 GeV, 6 GeV, 30 GeV and 100 GeV to illustrate these points.

Note that as the limits on spin-independent DM scattering cross sections become sharper, the inclined lines will move to the right and reduce the intersecting area of the FASER search. Therefore, if the direct detection searches push the inclined line so much
that it goes outside the FASER searchable region and any evidence for a weakly interacting particle is found at FASER, this will rule out the $B-L$ model in our paper in the relevant parameter region and provide a test of the model. On the other hand, if there is a $B$-$L$ boson with mass coupling in the range being discussed, both the direct detection searches and FASER searches will give a positive signal.

4 Case (ii): Freeze-in dark matter constraints

4.1 Dark matter relic density

Let us now turn to the case where the gauge couplings $g_{BL}$ and $g_\zeta$ have much smaller values in which case the dark matter particle was never in equilibrium with the thermal plasma of the SM particles and had zero initial abundance. There are then two possible cases:

(a) the $Z_{BL}$ was in thermal equilibrium with SM particles. This corresponds to the case where $g_{BL} \geq 2.7 \times 10^{-8} \sqrt{m_\zeta [\text{GeV}]}$, and

(b) the $Z_{BL}$ was not in thermal equilibrium with SM particles i.e. $g_{BL} < 2.7 \times 10^{-8} \sqrt{m_\zeta [\text{GeV}]}$.

The most conservative conditions for the reaction $\zeta \bar{\zeta} \leftrightarrow f \bar{f}$ being out of equilibrium till the BBN epoch are:

$$g_{BL} g_\zeta \leq 10^{-10} \quad (4.1)$$

If DM mass is in the low GeV range, these constraints are even weaker. Similarly, for the process $\zeta \bar{\zeta} \leftrightarrow Z_{BL} Z_{BL}$, the corresponding condition is

$$g_\zeta \leq 9.2 \times 10^{-5} \ (m_\zeta [\text{GeV}])^{1/4}, \quad (4.2)$$
for case (a).

To evaluate the DM relic abundance, we numerically solve the Boltzmann equation of Eq. (3.5). Note that even for the freeze-in case the Boltzmann equation is of the same form as in the thermal dark matter case. This is because the term proportional to $Y_{EQ}^2$ in the right-hand side of Eq. (3.5) corresponds to the DM particle productions from the SM thermal plasma. The difference from the thermal dark matter case is that we set the boundary condition for the freeze-in case to be $Y(x_{RH}) = 0$, where $x_{RH} = m_\zeta/T_{RH} \ll 1$ with reheating temperature ($T_{RH}$) after inflation. The relic abundance of the DM in the present universe is given by

$$\Omega_{DM} h^2 = \frac{m_\zeta s_0 Y(\infty)}{\rho_c/h^2},$$

(4.3)

where $s_0 = 2890$ cm$^{-3}$ is the entropy density of the present Universe, and $\rho_c/h^2 = 1.05 \times 10^{-5}$ GeV/cm$^3$ is the critical density.

In evaluating the thermal average of the DM annihilation cross section in Eq. (3.7), we consider two processes for the DM particle creation, $f \bar{f} \rightarrow \zeta \bar{\zeta}$ mediated by $Z_{BL}$ and $Z_{BL}Z_{BL} \rightarrow \zeta \zeta$. Note that the second process is active only for case (a). The corresponding cross sections are given by those of the DM annihilation processes,

$$\sigma(\zeta \bar{\zeta} \rightarrow f \bar{f}) v \simeq \frac{37}{72\pi s} g_\zeta^2 g_{BL}^2,$$

$$\sigma(\zeta \bar{\zeta} \rightarrow Z_{BL}Z_{BL}) v \simeq \frac{g_\zeta^4}{4\pi s},$$

(4.4)

where we have assumed $m_6^2 \ll m_\zeta^2 < m_7^2$ and $m_{Z_{BL}}^2 \ll m_7^2$. Using these cross section formulas in Eq. (3.7), we numerically solve the Boltzmann equation. In Fig. 6, we show $Y(x)$ for $m_\zeta = 30$ GeV and $g_\zeta g_{BL} \simeq 3.0 \times 10^{-12}$ with $g_{BL} \gg g_\zeta$. As we can see, $Y(x)$ is growing from $Y(x_{RH} \ll 1) = 0$ and becomes constant at $x \simeq 1$. This behavior can be qualitatively understood as follows: For $x \lesssim 1$, we find $\langle \sigma v \rangle \propto x^2/m_\zeta^2$. Thus, Eq. (3.5) can be easily solved with $Y \ll Y_{EQ} \simeq$ constant and $Y(x_{RH} \ll 1) = 0$, and we find a solution to be $Y(x) \propto (x - x_{RH})/m_\zeta \simeq x/m_\zeta$. The DM particle creation from the thermal plasma stops at $T \sim m_\zeta$ because of the kinematics and hence $Y(\infty) \sim Y(x \simeq 1) \propto 1/m_\zeta$. Using Eq. (4.3), we find that the resultant DM relic density is independent of the DM mass.

By numerically solving the Boltzmann equation, we find that independently of $m_\zeta$, the observed DM relic density of $\Omega_{DM} h^2 = 0.12$ is reproduced in case (a) by

$$g_\zeta^2 g_{BL}^2 + \frac{18}{37} g_\zeta^4 \simeq 9.2 \times 10^{-24} \quad \text{for} \quad g_{BL} \geq 2.7 \times 10^{-8} \sqrt{m_\zeta[GeV]}.$$

(4.5)

In case (b), on the other hand, there is no $Z_{BL}$ initially, the condition is given by only the first term in the above equation i.e.

$$g_\zeta^2 g_{BL}^2 \simeq 9.2 \times 10^{-24} \quad \text{for} \quad g_{BL} < 2.7 \times 10^{-8} \sqrt{m_\zeta[GeV]}.$$

(4.6)

The first equation implies that $g_\zeta \sim 10^{-6}$ or lower whereas the second case corresponds to $g_\zeta \sim 10^{-4}$ or higher. This is displayed in Fig. 7.
Figure 7. The plot of $g_\zeta$ vs $g_{BL}$ (left panel) and $g_\zeta g_{BL}$ vs $g_{BL}$ (right panel) for $m_\zeta = 30$ GeV. The observed DM relic density is reproduced along the solid lines. Note that the discontinuity at the point where $Z_{BL}$ goes out of thermal equilibrium with the SM particles as a function of dark matter mass.

### 4.2 Direct detection constraints in the freeze-in case

One might think that because the couplings $g_\zeta$ and $g_{BL}$ are so small, there might be no useful constraints on the model in the freeze-in case. However, it turns out that this is strictly not true because the direct DM detection constraints are so stringent. We see that if in a region of parameter space, $g_{BL}^2 g_\zeta^2 \gtrsim 9.2 \times 10^{-24}$, then the spin-independent cross section is found to be $\sigma_{SI} \simeq 1.0 \times 10^{-7}$ [pb] $\times \left(\frac{10 \text{ MeV}}{M_{Z_{BL}}}\right)^4$ from Eq. (3.11). In Fig. 3, the most stringent constraint on $\sigma_{SI}$ is at $m_\zeta = 30$ GeV where we have $\sigma_{SI} = 4.2 \times 10^{-11}$ pb and at $m_\zeta = 2$ GeV, the limit is $9.9 \times 10^{-6}$ pb. These translate to lower bounds on $M_{Z_{BL}}$ in the several tens of MeV region. In Fig. 8 we show the lower bound on $M_{Z_{BL}} \gtrsim 70$ MeV for $m_\zeta = 30$ GeV, along the solid lines in Fig. 7. We can see that the direct DM detection constraints are more severe than the current collider bounds (gray shaded) for $2 \times 10^{-6} \lesssim g_{BL} \lesssim 7 \times 10^{-5}$.

### 4.3 Supernova constraints on 1 – 100 MeV $Z_{BL}$ for case (ii)

In the mass range of $Z_{BL}$ less than about 300 MeV, the $Z_{BL}$ boson can be produced from $e^+e^-$ and $\nu\bar{\nu}$ collisions in the supernova. There are then two kinds of constraints on $g_{BL}$ that can be derived from the SN1987A observations [40, 41]. The first kind of constraint arises when the $Z_{BL}$ escapes the SN and conflicts with the observed energy emitted in neutrinos i.e. $\sim 5 \times 10^{53}$ erg/sec. The second kind of bound arises, if $Z_{BL}$ mass is larger than an MeV, it can decay in principle to an $e^+e^-$ pair inside the supernova and leads to earlier X-ray and light signals than the three hour time lapse which was observed [42, 43]. We discuss these bounds now.

As far as the energy loss bound is concerned, we need to calculate the production rate of a light $Z_{BL}$ in the supernova and the typical energy loss $Q_{SN}$ given broadly speaking by the formula considering the electrons as the source

$$Q_{SN} \simeq n_{e^-} V \langle E \rangle \langle \sigma v \rangle \leq 5 \times 10^{53} \text{ erg/sec}. \quad (4.7)$$

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Figure 8. The lower bound on $M_{Z_{BL}}$ from the direct DM detection constraints for $m_\zeta = 30$ GeV (red solid line), along the solid lines in Fig. 7.

Taking $\langle \sigma v \rangle \sim g_{BL}^2$ and choosing $T \sim 50$ MeV, we find $g_{BL} \lesssim 10^{-9}$. This is assuming that the $Z_{BL}$ gets out of the supernova. However, if the mean free path of the $Z_{BL}$ is less than 10 km, the $Z_{BL}$ never gets out and there is no energy loss due to this extra physics. This happens when $g_{BL} \gtrsim 10^{-5}$. Thus, SN1987A energy loss constraint rules out the range $10^{-9} \lesssim g_{BL} \lesssim 10^{-5}$ for $M_{Z_{BL}} \gtrsim 100$ MeV.

Coming to the second case, in some range of the coupling $g_{BL}$, the $Z_{BL}$ will decay to $e^+e^-$ inside the star and will be in conflict with observations. To derive this bound, we observe that we can translate the bound obtained in Ref. [42] for dark photon mixed with regular photon by identifying $ee = g_{BL}$ and translating the bound in Fig. 6 of Ref. [42] to our case. Roughly, this implies that for $M_{Z'} \lesssim 30$ MeV, the region $g_{BL} \gtrsim 10^{-12}$ is ruled out by SN1987A observations. This is a very stringent constraint for low mass $Z_{BL}$.

4.4 BBN constraints on low mass $Z_{BL}$

In our model we assume that the RHNs required for anomaly cancellation acquire heavy Majorana mass ($M_{N_R} \geq 100$ GeV or more) so that the only new degree of freedom we have to consider at the epoch of BBN are the three modes of the vector boson $Z_{BL}$ (two transverse and one longitudinal). We assume $M_{Z_{BL}}$ to be in the one GeV or lower range. For the higher mass range, as long as $Z_{BL}$ is in thermal equilibrium, the density at decoupling is already suppressed enough so that there are no BBN constraints.

The physics of our considerations in the lower mass range are as follows: if the gauge coupling is large enough that the $Z_{BL}$ is in thermal equilibrium till $T = 1$ MeV, then how much it contributes to the quantity $\Delta N_{\text{eff}}$ depends on its mass. If its mass is larger than 10 MeV, its abundance at $T = 1$ MeV will be Boltzmann suppressed and its contributions to energy density will be within the current $\Delta N_{\text{eff}}$ limits. If it is smaller than 10 MeV,
it will contribute more and can be incompatible if the abundance of the $Z_{BL}$ is thermal and not exponentially suppressed. These constraints will put an upper limit on the value of $g_{BL}$. Furthermore, if the $g_{BL}$ is such that the $Z_{BL}$ production processes are out of equilibrium, it will decay above $T = 1$ MeV and the resulting SM fermions will thermalize and will not affect BBN. So, we only have to consider the case when $Z_{BL}$ mass is below 10 MeV to find the limit on the coupling. However as discussed above this range of masses are already in conflict with the direct detection bounds. Therefore, we conclude that in the range of $Z_{BL}$ masses that we are interested in, BBN does not impose any constraint unless the coupling $g_{BL}$ is really very small so that it does not decay above the BBN temperature. This requires that $g_{BL} \lesssim 10^{-11}(1 \text{MeV}/M_{Z_{BL}})$.

5 Case (iii): Small $g_{BL}$ and secluded dark sector with $\zeta$ and $Z_{BL}$

In this section we briefly comment on two more logical possibilities which arise when $g_{BL} < 2.7 \times 10^{-8} \sqrt{m_\zeta[\text{GeV}]}$ so that the SM particles are decoupled from the $\zeta$ and $Z_{BL}$ sectors. There are two possibilities here: case (iiiA) where $g_\zeta$ is large enough so that the DM particle can be in equilibrium with $Z_{BL}$ but not with the SM sector due to small $g_{BL}$, and case (iiiB) where $g_\zeta$ is small so that all three sectors are sequestered. Here we comment briefly on how the relic density can arise in both of the cases.

In either of cases (iiiA) and (iiiB), the decay of the inflaton will play a crucial role in building up the DM relic density. Assuming the inflaton $\phi$ being a gauge singlet scalar under the SM and $B - L$ gauge groups, we can consider couplings of the inflaton with particles in our model such as $\phi H \bar{H}$, $\phi \bar{\zeta} \zeta$ and $\phi Z_{BL}^{\mu\nu} Z_{BL\mu\nu}$, where $H$ is the SM Higgs doublet, and $Z_{BL}^{\mu\nu}$ is the field strength of $Z_{BL}$. After the end of inflation, the inflaton decays to particles through these couplings to reheat the universe and then the Big Bang Hubble era begins. For case (ii) in Sec. 4, we implicitly assumed that the branching ratio of the inflaton decay into the DM particles is negligibly small so that we employed the initial condition $Y(x_{RH}) = 0$ in solving the Boltzmann equation. Here we are considering the case where the inflaton branching ratio into the “dark sector” with $\zeta$ and $Z_{BL}$ is not negligible.

For case (iiiA), the early universe after reheating consists of two separate plasmas: one is the thermal plasma of the SM particles and the other is the plasma of the hidden sector, where $\zeta$ and $Z_{BL}$ are in thermal equilibrium. Although the temperatures of the SM sector and the dark sector are not the same, very roughly speaking, the evaluation of the DM relic density is similar to case (i) discussed in Sec. 3. For case (iiiB) on the other hand, all three sectors are sequestered and we may adjust the inflaton branching ratio into a pair of DM particles to reproduce the observed DM relic density.

6 Summary and conclusions

In summary, we have considered an extension of the standard model with a gauge $U(1)_{B-L}$ symmetry with a Dirac fermion dark matter. The $B - L$ symmetry is broken by a $B - L = 2$ Higgs field so that $Z_{BL}$ picks up a mass and it leads to the seesaw mechanism for neutrino masses. Ignoring the mixings of $Z_{BL}$ with SM gauge bosons, we show that in the weakly
coupled $B - L$ coupling range, there are constraints on the gauge couplings $g_{BL}$ and $g_{\zeta}$ as well as the masses of the dark matter and the $B - L$ gauge boson $Z_{BL}$ mass from different observations such as Fermi-LAT, CMB, $\Omega_{DM}h^2$ and direct dark matter detection experiments for the case when the dark matter is a thermal freeze-out type. We also point out that for even weaker gauge couplings where only way the dark matter relic density arises via the freeze-in mechanism, there are constraints on the couplings from the observed dark matter relic density and the direct detection experiments as well as from the supernova 1987A observations. We have pointed out that our model can be tested in the FASER experiment being planned at the LHC.

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Note added in proof

After this work was put in the arXiv, the paper arXiv:1908.09834 [44] with a similar study was brought to our attention.

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