The history of the cosmological constant problem *

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Abstract

The interesting early history of the cosmological term is reviewed, beginning with its introduction by Einstein in 1917 and ending with two papers of Zel’dovich, shortly before the advent of spontaneously broken gauge theories. Beside classical aspects, I shall also mention some unpublished early remarks by Pauli on possible contributions of vacuum energies in quantum field theory.

1 Introduction

One of the contributions in the famous volume *Albert Einstein: Philosopher–Scientist* [1] is an article by George E. Lemaître entitled “The Cosmological Constant.” In the introduction he says: “The history of science provides many instances of discoveries which have been made for reasons which are no longer considered satisfactory. It may be that the discovery of the cosmological constant is such a case.” When the book appeared in 1949 – at the occasion of Einstein’s seventieth birthday – Lemaître could not be fully aware of how right he was, how profound the cosmological constant problem really is, especially since he was not a quantum physicist.

During this week we shall hear why we are indeed confronted with a deep mystery and the current evidence for the unexpected finding that the recent (z < 1) Universe is dominated by an exotic homogeneous energy density

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with negative pressure will be summarized by various speakers. The simplest candidate for this exotic energy density is a cosmological term in Einstein’s field equations, a possibility that has been considered during all the history of relativistic cosmology. The organizers of this interesting meeting asked me to review the main aspects of the history of the Λ-term, from its introduction in 1917 up to the point when it became widely clear that we are facing a deep mystery.

2 Einstein’s original motivation of the Λ-term

The cosmological term was introduced by Einstein when he applied general relativity for the first time to cosmology. In his paper of 1917 [2] he found the first cosmological solution of a consistent theory of gravity. In spite of its drawbacks this bold step can be regarded as the beginning of modern cosmology. It is still interesting to read this paper about which Einstein says: “I shall conduct the reader over the road that I have myself travelled, rather a rough and winding road, because otherwise I cannot hope that he will take much interest in the result at the end of the journey.” In a letter to P. Ehrenfest on 4 February 1917 Einstein wrote about his attempt: “I have again perpetrated something relating to the theory of gravitation that might endanger me of being committed to a madhouse. (Ich habe wieder etwas verbrochen in der Gravitationstheorie, was mich ein wenig in Gefahr bringt, in ein Tollhaus interniert zu werden.)”[3].

In his attempt Einstein assumed – and this was completely novel – that space is globally closed, because he then believed that this was the only way to satisfy Mach’s principle, in the sense that the metric field should be determined uniquely by the energy-momentum tensor. In these years and for quite some time Mach’s ideas on the origin of inertia played an important role in Einstein’s thinking. This may even be the primary reason that he turned so soon after the completion of general relativity to cosmology. Einstein was, in particular, convinced that isolated masses cannot impose a structure on space at infinity. It is along these lines that he postulated a universe that is spatially finite and closed, a universe in which no boundary conditions are needed. Einstein was actually thinking about the problem regarding the choice of boundary conditions at infinity already in spring 1916. In a letter to Michele Besso from 14 May 1916 he also mentions the possibility of the world being finite. A few month later he expanded on this in letters to Willem de Sitter.

In addition, Einstein assumed that the Universe was static. This was not unreasonable at the time, because the relative velocities of the stars as
observed were small. (Recall that astronomers only learned later that spiral nebulae are independent star systems outside the Milky Way. This was definitely established when in 1924 Hubble found that there were Cepheid variables in Andromeda and also in other galaxies. Five years later he announced the recession of galaxies.)

These two assumptions were, however, not compatible with Einstein’s original field equations. For this reason, Einstein added the famous $\Lambda$-term, which is compatible with the principles of general relativity, in particular with the energy-momentum law $\nabla_{\nu} T^{\mu\nu} = 0$ for matter. The modified field equations in standard notation (see, e.g., [15]) and signature $(+−−−)$ are

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}.$$ (1)

The cosmological term is, in four dimensions, the only possible complication of the field equations if no higher than second order derivatives of the metric are allowed (Lovelock theorem). This remarkable uniqueness is one of the most attractive features of general relativity. (In higher dimensions additional terms satisfying this requirement are allowed.)

For the static Einstein universe the field equations (1) imply the two relations

$$8\pi G \rho = \frac{1}{a^2} = \Lambda,$$ (2)

where $\rho$ is the mass density of the dust filled universe (zero pressure) and $a$ is the radius of curvature. (We remark, in passing, that the Einstein universe is the only static dust solution; one does not have to assume isotropy or homogeneity. Its instability was demonstrated by Lemaître in 1927.) Einstein was very pleased by this direct connection between the mass density and geometry, because he thought that this was in accord with Mach’s philosophy. (His enthusiasm for what he called Mach’s principle later decreased. In a letter to F. Pirani he wrote in 1954: “As a matter of fact, one should no longer speak of Mach’s principle at all. (Von dem Machschen Prinzip sollte man eigentlich überhaupt nicht mehr sprechen”.) [4]

Einstein concludes with the following sentences:

“In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It has to be emphasized, however, that a positive curvature of space is given by our results, even if the supplementary term is not introduced. That term is necessary only for the purpose of making possible a quasi-static distribution of matter, as required by the fact of the small velocities of the stars.”
3 From static to expanding world models

In the same year, 1917, de Sitter discovered a completely different static cosmological model which also incorporated the cosmological constant, but was anti-Machian, because it contained no matter. The model had one very interesting property: For light sources moving along static world lines there is a gravitational redshift, which became known as the de Sitter effect. This was thought to have some bearing on the redshift results obtained by Slipher. Because the fundamental (static) world lines in this model are not geodesic, a freely-falling particle released by any static observer will be seen by him to accelerate away, generating also local velocity (Doppler) redshifts corresponding to peculiar velocities. In the second edition of his book published in 1924, Eddington writes about this:

“de Sitter’s theory gives a double explanation for this motion of recession; first there is a general tendency to scatter (...); second there is a general displacement of spectral lines to the red in distant objects owing to the slowing down of atomic vibrations (...), which would erroneously be interpreted as a motion of recession.”

I do not want to enter into all the confusion over the de Sitter universe. This has been described in detail elsewhere (see, e.g., [7]). An important discussion of the redshift of galaxies in de Sitter’s model by H. Weyl in 1923 should, however, be mentioned. Weyl introduced an expanding version of the de Sitter model. For small distances his result reduced to what later became known as the Hubble law.

Until about 1930 almost everybody knew that the Universe was static, in spite of the two fundamental papers by Friedmann in 1922 and 1924 and Lemaître’s independent work in 1927. These path breaking papers were in fact largely ignored. The history of this early period has – as is often the case – been distorted by some widely read documents. Einstein too accepted the idea of an expanding Universe only much later. After the first paper of Friedmann, he published a brief note claiming an error in Friedmann’s work; when it was pointed out to him that it was his error, Einstein published a retraction of his comment, with a sentence that luckily was deleted before publication: “[Friedmann’s paper] while mathematically correct is of no physical significance”. In comments to Lemaître during the Solvay meeting in 1927, Einstein again rejected the expanding universe solutions as physically unacceptable. According to Lemaître, Einstein was telling him: “Vos calculs sont corrects, mais votre physique est abominable”. On the other hand,

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1I recall that the de Sitter model has many different interpretations, depending on the class of fundamental observers that is singled out.
I found in the archive of the ETH many years ago a postcard of Einstein to Weyl from 1923 with the following interesting sentence: “If there is no quasi-static world, then away with the cosmological term”. This shows once more that history is not as simple as it is often presented.

It also is not well-known that Hubble interpreted his famous results on the redshift of the radiation emitted by distant ‘nebulae’ in the framework of the de Sitter model. He wrote:

“The outstanding feature however is that the velocity-distance relation may represent the de Sitter effect and hence that numerical data may be introduced into the discussion of the general curvature of space. In the de Sitter cosmology, displacements of the spectra arise from two sources, an apparent slowing down of atomic vibrations and a tendency to scatter. The latter involves a separation and hence introduces the element of time. The relative importance of the two effects should determine the form of the relation between distances and observed velocities.”

However, Lemaître’s successful explanation of Hubble’s discovery finally changed the viewpoint of the majority of workers in the field. At this point Einstein rejected the cosmological term as superfluous and no longer justified [1]. He published his new view in the Sitzungsberichte der Preussischen Akademie der Wissenschaften. The correct citation is:

Einstein. A. (1931). Sitzungsber. Preuss. Akad. Wiss. 235-37.

Many authors have quoted this paper but never read it. As a result, the quotations gradually changed in an interesting, quite systematic fashion. Some steps are shown in the following sequence:

- A. Einstein. 1931. Sitzsber. Preuss. Akad. Wiss. ...
- A. Einstein. Sitzber. Preuss. Akad. Wiss. ... (1931)
- A. Einstein (1931). Sber. preuss. Akad. Wiss. ...
- Einstein. A .. 1931. Sb. Preuss. Akad. Wiss. ...
- A. Einstein. S.-B. Preuss. Akad. Wis. ...1931
- A. Einstein. S.B. Preuss. Akad. Wiss. (1931) ...
- Einstein, A., and Preuss, S.B. (1931). Akad. Wiss. 235
Presumably, one day some historian of science will try to find out what happened with the young physicist S.B. Preuss, who apparently wrote just one important paper and then disappeared from the scene.

At the end of the paper Einstein adds some remarks about the age problem which was quite severe without the Λ-term, since Hubble’s value of the Hubble parameter was almost ten times too large. Einstein is, however, not very worried and suggests two ways out. First he says that the matter distribution is in reality inhomogeneous and that the approximate treatment may be illusionary. Then he adds that in astronomy one should be cautious with large extrapolations in time.

Einstein repeated his new standpoint much later [12], and this was also adopted by many other influential workers, e.g., by Pauli [13]. Whether Einstein really considered the introduction of the Λ-term as “the biggest blunder of his life” appears doubtful to me. In his published work and letters I never found such a strong statement. Einstein discarded the cosmological term just for simplicity reasons. For a minority of cosmologists (O.Heckmann, for example [14]), this was not sufficient reason. Paraphrasing Rabi, one might ask: ‘who ordered it away’?

After the Λ-force was rejected by its inventor, other cosmologists, like Eddington, retained it. One major reason was that it solved the problem of the age of the Universe when the Hubble time scale was thought to be only 2 billion years (corresponding to the value $H_0 \sim 500 \text{ km s}^{-1}\text{Mpc}^{-1}$ of the Hubble constant). This was even shorter than the age of the Earth. In addition, Eddington and others overestimated the age of stars and stellar systems.

For this reason, the Λ-term was employed again and a model was revived which Lemaître had singled out from the many solutions of the Friedmann-Lemaître equations\footnote{I recall that Friedmann included the Λ-term in his basic equations. I find it remarkable that for the negatively curved solutions he pointed out that these may be open or compact (but not simply connected).}. This so-called Lemaître hesitation universe is closed and has a repulsive Λ-force ($\Lambda > 0$), which is slightly greater than the value chosen by Einstein. It begins with a big bang and has the following two stages of expansion. In the first the Λ-force is not important, the expansion is decelerated due to gravity and slowly approaches the radius of the Einstein universe. At about the same time, the repulsion becomes stronger than gravity and a second stage of expansion begins which eventually inflates into a whimper. In this way a positive Λ was employed to reconcile the expansion of the Universe with the age of stars.
The repulsive effect of a positive cosmological constant can be seen from the following consequence of Einstein’s field equations for the time-dependent scale factor $a(t)$:

$$\ddot{a} = -\frac{4\pi G}{3} (\rho + 3p) a + \frac{\Lambda}{3} a,$$  \hspace{1cm} (3)

where $p$ is the pressure of all forms of matter.

Historically, the Newtonian analog of the cosmological term was regarded by Einstein, Weyl, Pauli, and others as a Yukawa term. This is not correct, as I now show.

For a better understanding of the action of the $\Lambda$-term it may be helpful to consider a general static spacetime with the metric (in adapted coordinates)

$$ds^2 = \varphi^2 dt^2 + g_{ik} dx^i dx^k,$$  \hspace{1cm} (4)

where $\varphi$ and $g_{ik}$ depend only on the spatial coordinate $x^i$. The component $R_{00}$ of the Ricci tensor is given by $R_{00} = \Delta \varphi/\varphi$, where $\Delta$ is the three-dimensional Laplace operator for the spatial metric $-g_{ik}$ in (4) (see, e.g., [13]). Let us write Eq. (1) in the form

$$G_{\mu\nu} = \kappa (T_{\mu\nu} + T^\Lambda_{\mu\nu}) \hspace{1cm} (\kappa = 8\pi G),$$  \hspace{1cm} (5)

with

$$T^\Lambda_{\mu\nu} = \frac{\Lambda}{8\pi G} g_{\mu\nu}.$$  \hspace{1cm} (6)

This has the form of the energy-momentum tensor of an ideal fluid, with energy density $\rho_\Lambda = \Lambda/8\pi G$ and pressure $p_\Lambda = -\rho_\Lambda$. For an ideal fluid at rest Einstein’s field equation implies

$$\frac{1}{\varphi} \Delta \varphi = 4\pi G \left[ (\rho + 3p) + (\rho_\Lambda + 3p_\Lambda) - 2\rho_\Lambda \right].$$  \hspace{1cm} (7)

Since the energy density and the pressure appear in the combination $\rho + 3p$, we understand that a positive $\rho_\Lambda$ leads to a repulsion (as in (3)). In the Newtonian limit we have $\varphi \approx 1 + \phi$ (\phi: Newtonian potential) and $p \ll \rho$, hence we obtain the modified Poisson equation

$$\Delta \phi = 4\pi G (\rho - 2\rho_\Lambda).$$  \hspace{1cm} (8)

This is the correct Newtonian limit.

As a result of revised values of the Hubble parameter and the development of the modern theory of stellar evolution in the 1950s, the controversy over ages was resolved and the $\Lambda$-term became again unnecessary. (Some tension
remained for values of the Hubble parameter at the higher end of recent determinations.)

However, in 1967 it was revived again in order to explain why quasars appeared to have redshifts that concentrated near the value $z = 2$. The idea was that quasars were born in the hesitation era \[16\]. Then quasars at greatly different distances can have almost the same redshift, because the universe was almost static during that period. Other arguments in favor of this interpretation were based on the following peculiarity. When the redshifts of emission lines in quasar spectra exceed 1.95, then redshifts of absorption lines in the same spectra were, as a rule, equal to 1.95. This was then quite understandable, because quasar light would most likely have crossed intervening galaxies during the epoch of suspended expansion, which would result in almost identical redshifts of the absorption lines. However, with more observational data evidence for the $\Lambda$-term dispersed for the third time.

4 Quantum aspects of the $\Lambda$-problem

Let me conclude this historical review with a few remarks on the quantum aspect of the $\Lambda$-problem. Since quantum physicists had so many other problems, it is not astonishing that in the early years they did not worry about this subject. An exception was Pauli, who wondered in the early 1920s whether the zero-point energy of the radiation field could be gravitationally effective.

As background I recall that Planck had introduced the zero-point energy with somewhat strange arguments in 1911. The physical role of the zero-point energy was much discussed in the days of the old Bohr-Sommerfeld quantum theory. From Charly Enz and Armin Thellung – Pauli’s last two assistants – I have learned that Pauli had discussed this issue extensively with O. Stern in Hamburg. Stern had calculated, but never published, the vapor pressure difference between the isotopes 20 and 22 of Neon (using Debye theory). He came to the conclusion that without zero-point energy this difference would be large enough for easy separation of the isotopes, which is not the case in reality. These considerations penetrated into Pauli’s lectures on statistical mechanics \[17\] (which I attended). The theme was taken up in an article by Enz and Thellung \[18\]. This was originally written as a birthday gift for Pauli, but because of Pauli’s early death, appeared in a memorial volume of Helv.Phys.Acta.

From Pauli’s discussions with Enz and Thellung we know that Pauli estimated the influence of the zero-point energy of the radiation field – cut off at the classical electron radius – on the radius of the universe, and came
to the conclusion that it “could not even reach to the moon”.

When, as a student, I heard about this, I checked Pauli’s unpublished remark by doing the following little calculation:

In units with $\hbar = c = 1$ the vacuum energy density of the radiation field is

$$< \rho >_{\text{vac}} = \frac{8\pi}{(2\pi)^3} \int_0^{\omega_{\text{max}}} \frac{\omega}{2} \omega^2 d\omega = \frac{1}{8\pi^2} \omega^4_{\text{max}},$$

with

$$\omega_{\text{max}} = \frac{2\pi}{\lambda_{\text{max}}} = \frac{2\pi m_e}{\alpha}.$$

The corresponding radius of the Einstein universe in Eq.(2) would then be ($M_{\text{pl}} \equiv \frac{1}{\sqrt{G}}$)

$$a = \frac{\alpha^2}{(2\pi)^\frac{3}{2}} \frac{M_{\text{pl}}}{m_e} \frac{1}{m_e} \sim 31\text{km}.$$

This is indeed less than the distance to the moon. (It would be more consistent to use the curvature radius of the static de Sitter solution; the result is the same, up to the factor $\sqrt{3/2}$.)

For decades nobody else seems to have worried about contributions of quantum fluctuations to the cosmological constant, also physicists learned after Dirac’s hole theory that the vacuum state in quantum field theory is not an empty medium, but has interesting physical properties. As an important example I mention the papers by Heisenberg and Euler \cite{20} in which they calculated the modifications of Maxwell’s equations due to the polarization of the vacuum. Shortly afterwards, Weisskopf \cite{21} not only simplified their calculations but also gave a thorough discussion of the physics involved in charge renormalization. Weisskopf related the modification of Maxwell’s Lagrangian to the change of the energy of the Dirac sea as a function of slowly varying external electromagnetic fields. Avoiding the old fashioned Dirac sea, this effective Lagrangian is due to the interaction of a classical electromagnetic field with the vacuum fluctuations of the electron positron field. After a charge renormalization this change is finite and gives rise to electric and magnetic polarization vectors of the vacuum. In particular, the refraction index for light propagating perpendicular to a static homogeneous magnetic field depends on the polarization direction. This is the vacuum analog of the well-known Cotton-Mouton effect in optics. As a result, an initially linearly polarized light beam becomes elliptic. (In spite of great efforts it has not yet been possible to observe this effect.)

\footnote{A trace of this is in Pauli’s Handbuch article \cite{19} on wave mechanics in the section where he discusses the meaning of the zero-point energy of the quantized radiation field.}
In the thirties, people like Weisskopf were, however, not interested in gravity. As far as I know, the first who came back to possible contributions of the vacuum energy density to the cosmological constant was Zel’dovich. He discussed this issue in two papers [22] during the third renaissance period of the \( \Lambda \)-term, but before the advent of spontaneously broken gauge theories. The following remark by him is particularly interesting. Even if one assumes completely ad hoc that the zero-point contributions to the vacuum energy density are exactly cancelled by a bare term, there still remain higher-order effects. In particular, gravitational interactions between the particles in the vacuum fluctuations are expected on dimensional grounds to lead to a gravitational self-energy density of order \( G\mu^6 \), where \( \mu \) is some cut-off scale. Even for \( \mu \) as low as 1 GeV (for no good reason) this is about 9 orders of magnitude larger than the observational bound.

This illustrates that there is something profound that we do not understand at all, certainly not in quantum field theory (so far also not in string theory). We are unable to calculate the vacuum energy density in quantum field theories, like the Standard Model of particle physics. But we can attempt to make what appear to be reasonable order-of-magnitude estimates for the various contributions. All expectations are in gigantic conflict with the facts. Trying to arrange the cosmological constant to be zero is unnatural in a technical sense. It is like enforcing a particle to be massless, by fine-tuning the parameters of the theory when there is no symmetry principle which implies a vanishing mass. The vacuum energy density is unprotected from large quantum corrections. This problem is particularly severe in field theories with spontaneous symmetry breaking. In such models there are usually several possible vacuum states with different energy densities. Furthermore, the energy density is determined by what is called the effective potential, and this is dynamically determined. Nobody can see any reason why the vacuum of the Standard Model we ended up as the Universe cooled, has – for particle physics standards – an almost vanishing energy density. Most probably, we will only have a satisfactory answer once we shall have a theory which successfully combines the concepts and laws of general relativity about gravity and spacetime structure with those of quantum theory.

For more on this, see e.g. [23] (and references therein), as well as other contributions to this meeting.

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