Response of round-hole tubes submitted to pure bending creep and pure bending relaxation

Kuo-Long Lee¹, Bo-You Liu² and Wen-Fung Pan²

Abstract
This paper presents experimental study on the response of 6061-T6 aluminum alloy round-hole tubes with five different hole diameters of 2, 4, 6, 8, and 10 mm and four different diameter-to-thickness ratios of 30, 40, 50, and 60 submitted to pure bending creep and pure bending relaxation. Pure bending creep or relaxation is defined as bending the tube to the required moment or curvature and maintaining that moment or curvature for a period of time. The experimental results of pure bending creep show that the curvature increases with time. In addition, larger holding moment, diameter-to-thickness ratio, or hole diameter results in larger creep curvature. As the curvature continues to increase, the round-hole tube eventually breaks. The experimental results of pure bending relaxation show that the relaxation moment decreases sharply with time and tends to a stable value. In addition, larger holding curvature, diameter-to-thickness ratio, or hole diameter results in larger drop of the relaxation moment. Due to fixed curvature, the round-hole tube does not break. Finally, formulas proposed by the research team of Pan et al. were respectively improved to simulate the creep curvature-time relationship for pure bending creep in the initial and the secondary stages and the relaxation moment-time for pure bending relaxation. After comparing with the experimental results, it is found that theoretical analysis can reproduce the experimental results reasonably.

Keywords
6061-T6 aluminum alloy round-hole tubes, hole diameters, diameter-to-thickness ratios, pure bending creep, pure bending relaxation, moment, curvature, time

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Introduction
Round-hole tube (abbreviation: RHT) refers to a tube with a round hole, as shown in Figure 1. RHT is usually used for connecting components of bicycles, motorcycles, or automobiles. When RHT submits to a bending load, the bending rigidity of RHT will progressively reduce as the amount of bending increases, which is called a phenomenon of deterioration. RHT will encounter fracture when the curvature reaches a certain critical value.

The study of smooth tube under bending has achieved plentiful results. In 1987, Kyriakides et al. set up a mechanical device that can perform cyclic bending tests on tubes of various materials (1020 steel, 1018 steel, 304 stainless steel, 6061-T6 aluminum alloy, and NiTi). They have carried out many experimental and
theoretical studies on bending with or without internal or external pressure.\textsuperscript{1–9} Several other researchers have also published related results. Elchalakani et al.\textsuperscript{10} determined the slenderness limits of cold-formed CHS for fully ductile section by conducting cyclic bending tests with variable amplitudes. Mathon and Liman\textsuperscript{11} experimentally studied the pure bending collapse of cylindrical thin shells. Elchalakani and Zhao\textsuperscript{12} investigated the cyclic bending behavior of cold-formed steel tubes with concrete filled. Yazdani and Nayebi\textsuperscript{13} examined the damage of pipes that undergo periodic bending with a stable internal pressure. Shariati et al.\textsuperscript{14} experimentally discussed the cyclic bending behavior of SS316L cantilevered cylindrical shells. Elchalakani et al.\textsuperscript{15} used the strain detected in bending test to find a new ductility slender limit for the CFT structural plastic designs. Li and Wang\textsuperscript{16} conducted cyclic bending tests with variable amplitudes. Mathon and Liman\textsuperscript{11} experimentally studied the pure bending collapse of cylindrical thin shells. Elchalakani and Zhao\textsuperscript{12} investigated the cyclic bending behavior of cold-formed steel tubes with concrete filled. Yazdani and Nayebi\textsuperscript{13} examined the damage of pipes that undergo periodic bending with a stable internal pressure. Shariati et al.\textsuperscript{14} experimentally discussed the cyclic bending behavior of SS316L cantilevered cylindrical shells. Elchalakani et al.\textsuperscript{15} used the strain detected in bending test to find a new ductility slender limit for the CFT structural plastic designs. Li and Wang\textsuperscript{16} constructed a new measurement equipment to measure the ovalization (change of the outer diameter divides by the outer diameter, $\Delta D_o/D_o$) and curvature ($\kappa$) of tubes submitted to bending. They confirmed the function of the equipment by conducting tubes under cyclic bending. Lee et al.\textsuperscript{17} tested the stability of 304 stainless steel tubes with four diameter-to-thickness ratios ($D_o/t$ ratios) submitted to cyclic bending. They found that the log $k$-log $N_b$ (number of cycles required to initiate buckling) relationships demonstrate four parallel lines. Lee and Pan\textsuperscript{20} examined the pure bending creep behavior of 304 stainless steel tubes with four $D_o/t$ ratios. By modifying the Bailey-Norton law for uniaxial creep, the creep curvature-time relationships of the first two stages were simulated. Pan and Lee\textsuperscript{21} explored the mean curvature effect on the behavior of tubes undertook cyclic bending. Different curvature ratios of $-1.0$, $-0.5$, and $0$ were considered in their experimental investigation. Chang and Pan\textsuperscript{22} experimentally explored cyclic bending instability of tubes with four $D_o/t$ ratios. They introduced a new formula for estimating tube's buckling life.

In 1998, Pan et al. started a series of experimental and theoretical investigations on behavior of tubes under monotonic or cyclic bending with various loading paths. Pan et al.\textsuperscript{18} invented and constructed a new measurement equipment to measure the ovalization (change of the outer diameter divides by the outer diameter, $\Delta D_o/D_o$) and curvature ($\kappa$) of tubes submitted to bending. They confirmed the function of the equipment by conducting tubes under cyclic bending. Lee et al.\textsuperscript{19} tested the stability of 304 stainless steel tubes with four diameter-to-thickness ratios ($D_o/t$ ratios) submitted to cyclic bending. They found that the log $k$-log $N_b$ (number of cycles required to initiate buckling) relationships demonstrate four parallel lines. Lee and Pan\textsuperscript{20} examined the pure bending creep behavior of 304 stainless steel tubes with four $D_o/t$ ratios. By modifying the Bailey-Norton law for uniaxial creep, the creep curvature-time relationships of the first two stages were simulated. Pan and Lee\textsuperscript{21} explored the mean curvature effect on the behavior of tubes undertook cyclic bending. Different curvature ratios of $-1.0$, $-0.5$, and $0$ were considered in their experimental investigation. Chang and Pan\textsuperscript{22} experimentally explored cyclic bending instability of tubes with four $D_o/t$ ratios. They introduced a new formula for estimating tube's buckling life.

In 2010, Pan et al. started to experimentally and analytically investigate the behavior of tubes with a circumferential notch submitted to bending. Lee et al.\textsuperscript{23} examined the changes of $\Delta D_o/D_o$ for circumferential sharp-notched tubes submitted to cyclic bending. The $\Delta D_o/D_o-N$ (number of cycles) curve was identified into three different growing stages. Later, Lee\textsuperscript{24} investigated cyclic bending response of circumferentially sharp grooved tubes with different depths. A theoretical formula for estimating $N_b$ was proposed. Thereafter, Lee et al.\textsuperscript{25} examined the stability of circumferential sharp-notched 304 stainless steel tubes submitted to cyclic bending at different curvature rates (viscoplastic response). They employed three curvature rates of 0.35, 0.035, and $0.0035 \text{ m}^{-1} / \text{s}$ to demonstrate the viscoplastic response. Lee et al.\textsuperscript{26} inspected the pure bending creep behavior and pure bending relaxation behavior of tubes with a circumferential sharp notch. The formula proposed by Lee and Pan\textsuperscript{20} was improved for describing the creep curvature-time and relaxation moment-time relationships. Chung et al.\textsuperscript{27} investigated the cyclic bending collapse of circumferential sharp-notched 6061-T6 aluminum alloy tubes. The $\kappa$-$N_b$ relationship exhibited considerable differences from that discovered by Lee\textsuperscript{24} in circumferential sharp-notched 304 stainless steel tubes subjected to cyclic bending.

In 2016, Lee et al.\textsuperscript{28} explored the mechanical behavior of local sharp-cut 6061-T6 aluminum alloy tubes subjected to cyclic bending. In their study, the ANSYS was employed to simulate the $\Delta D_o/D_o-\kappa$ and $M$ (moment)-$\kappa$ relationships. After that, Lee et al.\textsuperscript{29} researched the failure of local sharp-grooved 304 stainless steel tubes submitted to cyclic bending. The $M$-$\kappa$ relationships were very similar to that obtained by Lee et al.\textsuperscript{25}; but, the contours of the $\Delta D_o/D_o-\kappa$ relationships were completely different. Lee et al.\textsuperscript{30} inspected the deterioration and failure of local sharp-dented 6061-T6 aluminum alloys tubes submitted to cyclic bending.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Schematic diagram of an RHT.}
\end{figure}
The dent was created by contacting the mold on the tube’s surface. In addition, a theoretical formula was put forward to express the $k-N_f$ (number of cycles required to initiate failure) relationship.

In this study, the response of 6061-T6 aluminum alloy round-hole tubes (abbreviation: Al-RHTs) with different hole diameters of 2, 4, 6, 8, and 10 mm, and different diameter-to-thickness ratios of 30, 40, 50, and 60 submitted to pure bending creep and pure bending relaxation are examined. In the experimental part, a bending machine was employed to perform relative experiments on Al-RHTs. The magnitudes of $M$ and $k$ were obtained by the measuring devices on the machine. Additionally, time ($t$) was also recorded. In the theoretical part, the formulas proposed by Lee and Pan\textsuperscript{20} and Lee et al.\textsuperscript{26} were respectively improved to simulate the creep curvature ($k_c$-$t$) relationships for pure bending creep in the initial and secondary stages and the relaxation moment ($M_r$-$t$) relationships for pure bending relaxation.

**Experiment**

**Experimental equipment**

The experiments executed by a tube-bending machine. Figure 2(a) and (b) respectively show a schematic diagram and a picture of the machine. It was designed to perform a pure bending test (four-point bending test), which can apply monotonic and cyclic bending. The machine consists of two rotating sprockets, placed symmetrically on two support beams. Two sprockets support two rollers, which apply point loads at each end of the tube. The chains run around these sprockets and are connected to two hydraulic cylinders and load cells to form a closed loop. Once the top or bottom cylinder is contracted, the sprocket will rotate to achieve pure bending of the tube. The load transferred to the tube is the moment formed by the concentrated load of the two rollers. For a detailed description of the tube-bending machine, please refer to the following papers (Lee et al.,\textsuperscript{19} Lee and Pan,\textsuperscript{20} and Pan and Lee\textsuperscript{21}).
Figure 3(a) and (b) respectively depict a schematic diagram and a picture of an apparatus to measure the $\Delta D_o/D_o$ and $\kappa$ of the tube invented by Pan et al. When the tube is bent, the two side-inclinometers in the device are able to detect the tube’s angle changes. The $\kappa$ can be easily calculated according to angle changes. In addition, the central part of the equipment can be used to monitor the change in the diameter of the tube cross section using a magnetic detector. Thus, the $\Delta D_o/D_o$ can be measured. For a detailed explanation of the apparatus, please refer to the Pan et al. study.

**Materials and specimens**

The chemical composition by weight percentage of the 6061-T6 aluminum alloy is 0.916% Mg, 0.733% Si, 0.293% Cu, 0.268% Ti, 0.256% Fe, 0.132% Mn, 0.0983% Zn, 0.0682% Cr, and the remaining part Al. The mechanical properties are ultimate tensile stress ($\sigma_u$) = 320 MPa, yield stress ($\sigma_y$) = 283 MPa, and percent elongation (%$\varepsilon_f$) = 13%.

Smooth 6061-T6 aluminum alloy tubes with $D_o = 36.0$ mm and $t = 3.0$ mm were machined on the outer surface to acquire the desired $D_o/t$ ratios of 30, 40, 50, and 60, as shown in Figure 4. However, the inner radius (29.0 mm) of all tested Al-RHTs was unchanged. Next, the tube with every $D_o/t$ ratio was drilled again to obtain the Al-RHT with the required hole diameter $d$. In this study, five $d$ values were considered, 2, 4, 6, 8, and 10 mm.

Because the hole is local, the hole direction $\phi$ (Figure 5) may also affect the RHT’s behavior. Since the degradation and failure of the RHT undergoing bending is the most
severe when the bending moment direction (z direction) is perpendicular to the $\Phi$. Therefore, this study only considers $\phi = 0^\circ$ (y direction).

**Test procedures**

The experiment includes pure bending creep (bend the tube to the required $M$ and hold that $M$ for a period of time) and pure bending relaxation (bend the tube to the required $k$ and hold that $k$ for a period of time). For each Al-RHT with certain $D_o/t$ and $d$, different $M$ and $k$ were respectively held for pure bending creep and pure bending relaxation. The $M$ and $k$ were measured and controlled by the related equipment. Furthermore, the $t$ was also recorded.

**Experimental results, simulated results, and discussion**

**Experimental response of Al-RHTs under pure bending creep**

Figure 6 shows the experimental $M$-$k$ curves for Al-RHTs with different $D_o/t$ and different $d$ subjected to pure bending. For each $D_o/t$ ratio, five $d$ values, 2, 4, 6, 8, and 10 mm, were tested. It can be seen that the linear $M$-$k$ relationship is observed when the deformation is in the elastic range. Once the deformation is in the plastic range, the $M$-$k$ relationship becomes nonlinear. It can also be observed that the RHT with a larger $D_o/t$ ratio has a smaller $t$. Therefore, the $M$-$k$ curve shows a lower $M$ value. In addition, for each $D_o/t$ ratio, the $M$-$k$ curves are similar for different $d$. However, $d = 2$ mm has the largest breaking curvature and $d = 10$ mm has shortest breaking curvature. The breaking points are denoted as “*,” “×” “△,” “○” and “■” for $d = 2$, 4, 6, 8 and 10 mm, respectively. Subsequently, the held moments of Al-RHTs with different $D_o/t$ and different $d$ for pure bending creep and the held curvatures of Al-RHTs with different $D_o/t$ and different $d$ for pure bending relaxation were according to the $M$-$k$ curves in Figure 6.

Figure 7 demonstrates the experimental $k$–$\ddot{t}$ curves of Al-RHTs with $D_o/t = 50$ and $d = 2$ mm for the overall bending process (monotonic bending and pure bending creep). The creep starting and creep rupture points are respectively marked by “•” and “×.” It can be found that the curvature of the Al-RHT increases rapidly when the creep begins. Larger held moment results in larger creep curvature. As the curvature continues to increase, the Al-RHT eventually ruptures. Note that the time of rupture for held moment of 120 N·m is 4258 s.

Next, We focus on the pure bending creep part from the creep starting point “•” to the creep rupture point “×” in Figure 7. Figure 8 depicts the $k_c$–$\ddot{t}$ relationship when the held moment is 150 N·m. It can be seen that the $k_c$–$\ddot{t}$ curve can be divided into three stages, which are called in this paper to be the initial stage, secondary stage, and tertiary stage. Once pure bending creep begins, the amount of $k$ increases, which is called the initial stage. When $k_c$ gradually decreases, pure bending creep enters the secondary stage, which is called the secondary stage. The $k_c$ increases steadily with $\ddot{t}$ and $k_c$ is approximate a constant in this stage. The third stage is called the tertiary stage. The $k_c$ increases sharply and the Al-RHT ruptures in this stage. In addition, most of pure bending creep time is spent in the initial and secondary stages.
Theoretical simulation for Al-RHTs under pure bending creep

In 2002, Lee and Pan\textsuperscript{20} tested smooth 304 stainless steel tubes with different $D_o/t$ ratios submitted to pure bending creep. They proposed a theoretical formulation to describe the $k_c–\tilde{C}t$ relationships in the initial and secondary stages as

$$k_c = CM^{n_1}t^{n_1},$$ \hspace{1cm} (1)

in which $M$ is the held moment, $C$, $m_1$, and $n_1$ are material parameters. In their study, $m_1$ and $n_1$ were respectively determined to be 5.70 and 0.53 and $C$ was a function of the $D_o/t$ ratio. In 2014, Lee et al.\textsuperscript{26} studied the $k_c–\tilde{C}t$ relationships for circumferential sharp-notched 304 stainless steel tubes submitted to pure bending creep. Due to the same material and similar trend of the $k_c–\tilde{C}t$ relationships, equation (1) was also used as the theoretical formulation. In their study, $m_1$ and $n_1$ were also used the same values, and $C$ was proposed to be a function of circumferential sharp notch depth.

In this study, the $k_c–\tilde{C}t$ relationships for Al-RHTs with different $D_o/t$ ratios and different $d$ under pure bending creep are similar to that of smooth 304 stainless steel tubes with different $D_o/t$ ratios submitted to pure bending creep tested by Lee and Pan.\textsuperscript{20} Therefore, equation (1) is used in this investigation. However, the material parameter $C$ is proposed to relate to $D_o/t$ ratios and $d$. As for material parameters $m_1$ and $n_1$, they can be determined from smooth 6061-T6 aluminum alloy tubes subjected to pure bending creep. Figure 9 demonstrates the experimental $k_c–\tilde{C}t$ in the initial and secondary stages for smooth 6061-T6 aluminum alloy tubes subjected to pure bending creep. Note that the smooth 6061-T6 aluminum alloy tubes are with $D_o = 36.0$ mm and $t = 3.0$ mm. By curve fitting with equation (1), the values of $m_1$ and $n_1$ are respectively obtained to be 1.086 and 0.046.

Table 1. Amounts of $C$ for different $D_o/t$ ratios and different $d/t$

| $D_o/t$ | 30  | 40  | 50  | 60  |
|---------|-----|-----|-----|-----|
| $d/t$   |     |     |     |     |
| 0.667   | 0.00187 | 0.00287 | 0.00395 | 0.00567 |
| 1.333   | 0.00193 | 0.00314 | 0.00449 | 0.00596 |
| 2.000   | 0.00205 | 0.00339 | 0.00463 | 0.00669 |
| 2.667   | 0.00221 | 0.00366 | 0.00535 | 0.00724 |
| 3.333   | 0.00243 | 0.00395 | 0.00549 | 0.00845 |

Figure 8. Experimental $k_c–\tilde{C}t$ curve of Al-RHTs with $D_o/t = 50$ and $d = 2$ mm for held moment of 150 N m under pure bending creep.

Figure 9. Experimental and simulated $k_c–\tilde{C}t$ in the initial and secondary stages for smooth 6061-T6 aluminum alloy tubes subjected to pure bending creep.
Due to increasing of the curvature, the Al-RHT breaks eventually. Using equation (1) with $m_1 = 1.086$ and $n_1 = 0.0458$, the amounts of $C$ for different $D_o/t$ ratios and different $d/t$ are determined by curve fitting from the experimental data in Figures 10 to 13 which are shown in Table 1.

When we consider the $\ln C$ and $d/t$ relationship, four straight lines for $D_o/t = 30, 40, 50,$ and $60$ are observed (see Figure 14). Thus, the parameter $C$ can be proposed as

$$\ln C = c_1(d/t) + c_2$$

where $c_1$ and $c_2$ are material parameters which are related to $D_o/t$ ratio. The amounts of $c_1$ and $c_2$ can be determined in Figure 14.

From Figure 15(a) and (b), linear relationships of $\ln c_1$-$\ln D_o/t$ and $\ln c_2$-$\ln D_o/t$ are respectively observed. Therefore, parameters $c_1$ and $c_2$ are respectively proposed to be

$$\ln c_1 = \alpha_1 \ln D_o/t + \alpha_2$$

and

$$\ln c_2 = \alpha_3 \ln D_o/t + \alpha_4$$
where $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ are material parameters which are respectively determined from Figure 15(a) and (b) to be $0.679$, $-1.172$, $-0.263$, and $2.749$. The solid lines in Figure 10(a) to (e) respectively indicate the simulated $\kappa_c^{-t}$ curves of Al-RHTs with $D_o/t = 30$ and $d = 2$, 4, 6, 8, and 10 mm under pure bending creep in the initial and secondary stages, the solid lines in Figure 11(a) to (e) respectively indicate the simulated $\kappa_c^{-t}$ curves of Al-RHTs with $D_o/t = 40$ and $d = 2$, 4, 6, 8, and 10 mm under pure bending creep in the initial and secondary stages, the solid lines in Figure 12(a) to (e) respectively indicate the simulated $\kappa_c^{-t}$ curves of Al-RHTs with $D_o/t = 50$ and $d = 2$, 4, 6, 8, and 10 mm under pure bending creep in the initial and secondary stages, and the solid lines in Figure 13(a) to (e) respectively indicate the simulated $\kappa_c^{-t}$ curves of Al-RHTs with $D_o/t = 60$ and $d = 2$, 4, 6, 8, and 10 mm under pure bending creep in the initial and secondary stages. It can be seen that the simulation and experimental data have a good correlation.

Experimental response of Al-RHTs under pure bending relaxation

Figure 16 demonstrates a typical $M^{-t}$ curves of Al-RHTs with $D_o/t = 40$ and $d = 10$ mm under monotonic pure bending followed by pure bending relaxation. The relaxation starting points are marked by “•” It can be
found that the moment of the tube drops sharply when the relaxation begins and approaches to a stable value. In addition, a larger held curvature results in a larger drop of the relaxation moment. Due to fixed curvature, the Al-RHT does not break.

**Theoretical simulation for Al-RHTs under pure bending relaxation**

In 2014, Lee et al. tested circumferential sharp-notched 304 stainless steel tubes submitted to pure bending relaxation. They proposed a theoretical formula to describe the relationship between relaxation moment \( M_r \) and \( t \) to be

\[
M_r = R \kappa m_2 t^{n_2},
\]

where \( \kappa \) is the held curvature, \( R, m_2, \) and \( n_2 \) are material parameters. In their study, \( m_2 \) and \( n_2 \) were respectively determined to be 0.15 and 0.07 and the parameter \( R \) was proposed as a function of notch depths for circumferential sharp-notched 304 stainless steel tubes.

In this study, the \( M_r - t \) relationships for Al-RHTs with different \( D_o/t \) ratios and different \( d \) under pure bending relaxation are similar to that of circumferential sharp-notched 304 stainless steel tubes subjected to pure bending relaxation tested by Lee et al. Therefore, equation (5) is used in this investigation. However, the material parameter \( R \) is proposed to relate to the \( D_o/t \) and \( d \). As for material parameters \( m_2 \)
and \( n_2 \), they can be determined from smooth 6061-T6 aluminum alloy tubes subjected to pure bending relaxation. Figure 17 depicts experimental \( M-t \) curves (dotted lines) for smooth 6061-T6 aluminum alloy tubes under monotonic pure bending followed by pure bending relaxation. Note that the relaxation starting points are marked by “•”. In addition, the smooth 6061-T6 aluminum alloy tubes are with \( D_o = 36.0 \) mm and \( t = 3.0 \) mm. By curve fitting of equation (5) (solid lines) with the experimental data (dotted lines) for pure bending relaxation, the values of \( m_2 \) and \( n_2 \) are respectively obtained to be 0.405 and -0.012.

Figure 18(a) to (e) respectively indicate the experimental \( M-t \) curves (dotted lines) of Al-RHTs with \( D_o/t = 60 \) and \( d = (a) 2, (b) 4, (c) 6, (d) 8, and (e) 10 \) mm under pure bending relaxation in the initial and secondary stages.

Figure 13. Experimental and simulated \( k_\tau-t \) curves of Al-RHTs with \( D_o/t = 60 \) and \( d = (a) 2, (b) 4, (c) 6, (d) 8, and (e) 10 \) mm under pure bending in the initial and secondary stages.

Figure 14. Relationship between \( \ln C \) and \( d/t \).
$t = 30$ and $d = 2, 4, 6, 8, \text{ and } 10 \text{ mm under monotonic bending followed by pure bending relaxation, Figure 19(a) to (e) respectively indicate the experimental } M - t \text{ curves (dotted lines) of Al-RHTs with } D_o/t = 40 \text{ and } d = 2, 4, 6, 8, \text{ and } 10 \text{ mm under monotonic bending followed by pure bending relaxation.}

When we consider the $R^{1/3} - d/t$ relationship, four straight lines for $D_o/t = 30, 40, 50, \text{ and } 60$ are observed (see Figure 22). Therefore, parameter $R$ is proposed to be

$$R^{1/3} = r_1(d/t) + r_2$$

where $r_1$ and $r_2$ are material parameters which are related to $D_o/t$ ratio.

From Figure 23(a) and (b), linear relationships of $\ln r_1 - \ln D_o/t$ and $\ln r_2 - \ln D_o/t$ are observed. Therefore, parameters $r_1$ and $r_2$ are respectively proposed to be

$$\ln r_1 = \beta_1 \ln D_o/t + \beta_2$$
\[ \ln r_2 = \beta_3 \ln \frac{D_0}{t} + \beta_4 \]  

(8)

where \( \beta_1, \beta_2, \beta_3, \) and \( \beta_4 \) are material parameters which are respectively determined from Figure 23(a) and (b) to be \(-0.750, -0.459, -0.199, \) and \(2.759.\) Figure 18(a) to (e) respectively indicate the simulated \( M_r - t \) curves (solid lines) of Al-RHTs with \( \frac{D_0}{t} = 30 \) and \( d = 2, 4, 6, 8, \) and \(10 \) mm under pure bending relaxation, and Figure 21(a) to (e) respectively indicate the simulated \( M_r - t \) curves (solid lines) of Al-RHTs with \( \frac{D_0}{t} = 60 \) and \( d = 2, 4, 6, 8, \) and \(10 \) mm under pure bending relaxation. Again, it can be seen that the simulation and experimental data have a good correlation.

**Conclusions**

This paper presents the experimental and theoretical research on the response of Al-RHTs with different \( \frac{D_0}{t} \) ratios and different \( d \) submitted to pure bending creep and pure bending relaxation. Based on experimental and theoretical results, this paper draws the following important conclusions:
(1) It is seen from the $M$-$\kappa$ curves for monotonic bending that once the deformation is in the plastic range, the $M$-$\kappa$ relationship becomes nonlinear. It can also be observed that the larger the $D_o/t$ ratio, the smaller the tube wall thickness. Therefore, a lower $M$ value is found. In addition, for each $D_o/t$ ratio, the $M$-$\kappa$ curves are similar for different $d$. However, a smaller $d$ leads to a larger breaking curvature.

(2) It is found from pure bending creep that once the pure bending creep begins, the curvature increases quickly. In addition, the $k_c-C^2_2t$ relationship can divide into three stages. Most of the time for pure bending creep is spent in the initial and secondary stages. For a certain $D_o/t$ ratio under a constant moment for pure bending creep, a smaller $d$ leads to a faster increasing and larger $k_c$. As the curvature continues to increase, Al-RHT eventually breaks.

(3) It is observed from pure bending relaxation that once the relaxation begins, the moment drops sharply and gradually becomes a stable value. In addition, the $M_r-C^2_2t$ relationship is similar to the reverse $k_c-C^2_2t$ relationship. For a certain $D_o/t$ ratio under a constant curvature for pure bending relaxation, a smaller $d$ leads to a larger drop of the $M_r$. Due to the fixed curvature, the Al-RHT does not break.
The theoretical formula proposed by Lee and Pan was employed for simulating the $k_c-t$ relationships for Al-RHTs with different $D_o/t$ ratios and different $d$ submitted to pure bending creep in the initial and secondary stages. In addition, a new formulation of $C$ in equations (2)–(4) was proposed. Moreover, the theoretical formula proposed by Lee et al. was employed for simulating the $M_r-t$ relationships for Al-RHTs with different $D_o/t$ ratios and different $d$ submitted to pure bending relaxation. In addition, a new formulation of $R$ in equations (6)–(8) was proposed. By comparing with experimental data, the theoretical formulas can accurately simulate the experimental results, as shown in Figures 10 to 13 and 18 to 21.

Figure 20. Experimental $M-t$ (dotted lines) and simulated $M_r-t$ (solid lines) curves of Al-RHTs with $D_o/t = 50$ and $d =$ (a) 2, (b) 4, (c) 6, (d) 8, and (e) 10 mm under monotonic pure bending followed by pure bending relaxation.
Figure 21. Experimental $M$–$t$ (dotted lines) and simulated $M_r$–$t$ (solid lines) curves of Al-RHTs with $D_o/t = 60$ and $d =$ (a) 2, (b) 4, (c) 6, (d) 8, and (e) 10 mm under monotonic pure bending followed by pure bending relaxation.

Table 2. Amounts of $R$ for different $D_o/t$ rations and different $d/t$.

| $D_o/t$ | 30   | 40   | 50   | 60   |
|---------|------|------|------|------|
| 0.667   | 494.15 | 407.98 | 350.69 | 323.07 |
| 1.333   | 477.12 | 389.93 | 328.53 | 302.60 |
| 2.000   | 460.62 | 371.72 | 311.48 | 279.78 |
| 2.667   | 438.96 | 352.46 | 290.37 | 264.28 |
| 3.333   | 418.73 | 333.14 | 277.52 | 246.54 |

Figure 22. Relationship between $R^{1/3}$ and $d/t$. 
Declaration of conflicting interests

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References

1. Kyriakides S and Shaw PK. Inelastic buckling of tubes under cyclic bending. J Press Vessel Technol 1987; 109: 169–178.
2. Corona E and Kyriakides S. An experimental investigation of the degradation and buckling of circular tubes under cyclic bending and external pressure. Thin-Walled Struct 1991; 12: 229–263.
3. Vaze SP and Corona E. Degradation and collapse of square tubes under cyclic bending. Thin-Walled Struct 1998; 31: 325–341.
4. Corona E, Lee LH and Kyriakides S. Yield anisotropy effects on buckling of circular tubes under bending. Int J Solids Struct 2006; 43: 7099–7118.
5. Limam A, Lee LH, Corona E, et al. Inelastic wrinkling and collapse of tubes under combined bending and internal pressure. Int J Mech Sci 2010; 52: 637–647.
6. Limam A, Lee LH and Kyriakides S. On the collapse of dented tubes under combined bending and internal pressure. Int J Solids Struct 2012; 55: 1–12.
7. Bechle NJ and Kyriakides S. Localization in NiTi tubes under bending. Int J Solids Struct 2014; 51: 967–980.
8. Jiang D, Kyriakides S, Bechle NJ, et al. Bending of pseudoelastic NiTi tubes. Int J Solids Struct 2017; 124: 192–214.
9. Kazinakis K, Kyriakides S, Jiang D, et al. Buckling and collapse of pseudoelastic NiTi tubes under bending. Int J Solids Struct 2021; 221: 2–17.
10. Elchalakani M, Zhao XL and Grzebieta R. Variable amplitude cyclic pure bending tests to determine fully ductile section slenderness limits for cold-formed CHS. Eng Struct 2006; 28: 1223–1235.
11. Mathon C and Limam A. Experimental collapse of thin cylindrical shells submitted to internal pressure and pure bending. Thin-Walled Struct 2006; 44: 39–50.
12. Elchalakani M and Zhao XL. Concrete-filled cold-formed circular steel tubes subjected to variable amplitude cyclic pure bending. Eng Struct 2008; 30: 287–299.
13. Yazdani H and Nayebi A. Continuum damage mechanics analysis of thin-walled tubes under cyclic bending and internal constant pressure. Int J Appl Mech 2013; 5: 1350038 [20 pages].
14. Shariati M, Kolasiangani K, Norouzi G, et al. Experimental study of SS316L cantilevered cylindrical shells under cyclic bending load. Thin-Walled Struct 2014; 82: 124–131.
15. Elchalakani M, Karrech A, Hassanine MF, et al. Plastic and yield slenderness limits for circular concrete filled tubes subjected to static pure bending. Thin-Walled Struct 2016; 109: 50–64.
16. Li P and Wang L. Nonlinear stability behavior of cable-stiffened single-layer latticed shells under earthquakes. Int J Struct Stab Dyn 2018; 18: 1850117 [24 pages].
17. Chegeni B, Jayasuriya S and Das S. Effect of corrosion on thin-walled pipes under combined internal pressure and bending. Thin-Walled Struct 2019; 143: 106218 [8 pages].
18. Pan WF, Wang TR and Hsu CM. A curvature-ovalization measurement apparatus for circular tubes under cyclic bending. Exp Mech 1998; 38: 99–102.
19. Lee KL, Pan WF and Kuo JN. The influence of the diameter-to-thickness ratio on the stability of circular
tubes under cyclic bending. *Int J Solids Struct* 2001; 38: 2401–2413.

20. Lee KL and Pan WF. Pure bending creep of SUS 304 stainless steel tubes. *Steel Compos Struct* 2002; 2: 461–474.

21. Chang KH, Pan WF and Lee KL. Mean moment effect on circular thin-walled tubes under cyclic bending. *Struct Eng Mech* 2008; 28: 495–514.

22. Chang KH and Pan WF. Buckling life estimation of circular tubes under cyclic bending. *Int J Solids Struct* 2008; 28: 495–514.

23. Chang KH and Pan WF. Buckling life estimation of circular tubes under cyclic bending. *Int J Solids Struct* 2009; 46: 254–270.

24. Lee KL, Hung CY and Pan WF. Variation of ovalization for sharp-notched circular tubes under cyclic bending. *J Mech* 2010; 26: 403–411.

25. Lee KL, Hsu CM and Pan WF. Viscoelastic collapse of sharp-notched circular tubes under cyclic bending. *Acta Mech Solida Sin* 2013; 26: 629–641.

26. Lee KL, Hsu CM and Pan WF. Response of sharp-notched circular tubes under bending creep and relaxation. *Mech Eng J* 2014; 1: 1–14.

27. Chung CC, Lee KL and Pan WF. Collapse of sharp-notched 6061-T6 aluminum alloy tubes under cyclic bending. *Int J Struct Stab Dyn* 2016; 16: 1550035. [24 pages].

28. Lee KL, Chang KH and Pan WF. Failure life estimation of sharp-notched circular tubes with different notch depths under cyclic bending. *Struct Eng Mech* 2016; 60: 387–404.

29. Lee KL, Chang KH and Pan WF. Effects of notch depth and direction on stability of local sharp-notched circular tubes subjected to cyclic bending. *Int J Struct Stab Dyn* 2018; 18: 1850099 [23 pages].

30. Lee KL, Chung CC and Pan WF. Cyclic bending deterioration and failure of locally-dented circular tubes. *J Chi Soc Mech Eng* 2018; 39: 53–63.