Causality detection and turbulence in fusion plasmas

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Abstract
This work explores the potential of an information-theoretical causality detection method for unravelling the relation between fluctuating variables in complex non-linear systems. The method is tested on some simple though non-linear models, and guidelines for the choice of analysis parameters are established. Then, measurements from magnetically confined fusion plasmas are analysed. The selected data bear relevance to the all-important spontaneous confinement transitions often observed in fusion plasmas, fundamental for the design of an economically attractive fusion reactor. It is shown how the present method is capable of clarifying the interaction between fluctuating quantities such as the turbulence amplitude, turbulent flux and zonal flow amplitude, and uncovers several interactions that were missed by traditional methods.

Keywords: fusion, magnetic confinement, plasma, turbulence, causality, data analysis

(Some figures may appear in colour only in the online journal)

1. Introduction
The problem of determining the causal relationship between various interacting fields or variables is of fundamental importance in many branches of science. Knowledge of the causal connection between variables is helpful for the elaboration of a realistic physical model and/or to check its validity. If one can intervene in the system under study and modulate the value of one variable, the observation of the (delayed) reaction of other variables to this modulation sometimes allows establishing the causal relationship. However, for various reasons many systems do not permit such intervention, or they are too complex to allow a straightforward interpretation of the observations. Other techniques to uncover causal relations are based on finding precursor events, time delays between extreme events (conditional averaging), correlations, etc. One may also attempt to match the system evolution to the predicted evolution from an analytic or numerical model, or to quantify parameters related to system evolution (growth rates, damping rates, etc). Most of these methods, however, do not provide a direct quantification of the causal interaction between variables. Even worse, linear analysis techniques (correlations, conditional averages) may lead to confusing or even erroneous conclusions regarding causality (see the well-known adage ‘correlation does not imply causation’). In this situation, how must one then determine the causal relation between variables?

Causality is notoriously hard to define in general [1]. In this work, we do not use the term ‘causality’ in its philosophical, absolute sense (if Y occurs, then X will occur; or: if X occurs, then Y must have occurred). Rather, we turn to the concept of ‘quantifiable causality’ introduced by Wiener [2] (rephrased slightly): For two simultaneously measured signals X and Y, if we can predict X better by using the past information from Y than without it, then we call Y causal to X. This idea led to the formulation of an algorithm for the detection of the causal relation between two measured signals, denoted by Granger causality [3]. This algorithm, however, is based on a linear prediction of the evolution of a time series involving multivariate minimization, which is inadequate for the analysis of turbulence. Although non-linear generalizations are possible and have indeed been elaborated [1], here we turn to a non-parametric procedure for causality detection originating in the field of information theory: the ‘transfer entropy’ [4].

In this work, we are mainly concerned with the interaction between zonal flows and turbulence. This interaction underlies the spontaneous confinement transitions often observed in fusion plasmas, fundamental for the design of an economically...
attractive fusion reactor. In recent years, more or less detailed models for this interaction have become available [5]. Much effort has been invested in demonstrating the relevance of these models for describing the observations, applying advanced analysis tools such as the biocorrelation [6–9].

An important aspect of these studies is the elucidation of the causal relation between turbulence, fluctuating zonal flows and steady state sheared flows. This issue is often implicitly present in the relevant publications, although causality is usually treated with circumspection due to the difficulty of addressing it directly; usually, all that can be said is that a certain sequence of events is observed (Y happens before X), which is a necessary but insufficient condition for the existence of a causal relation. Some ‘traditional’ methods for elucidating the causal relation between zonal flows and turbulence are considered relevant and analyse experimental data from the plasma. For this purpose, we will analyse a few model systems which other, in a highly non-linear situation characterized by causality questions of the type ‘which variable influences the transfer entropy technique can provide an answer to the bispectrum [15, 16]. This energy transfer reveals the direction involved in quadric three-wave coupling, based on the relevant to the study of the interaction between zonal flows [5] and turbulence. The structure of this paper is as follows. In section 2, we outline the method and perform some tests on relevant models of systems with non-linearly interacting variables. In section 3, we present some analysis results for data obtained in fusion devices, relevant in the framework of zonal flows and turbulence. In section 4, we discuss the findings and draw some conclusions.

2. Method

Consider two processes X and Y yielding discretely sampled time series data x and y. The data are assumed to correspond to a stationary state; any slow drifts of measurement signals have been removed by subjecting the time series to a suitable trend removal, if necessary. Their mutual information is defined as [17]:

$$I(X; Y) = \sum_{i,j} p(x_i, y_j) \log_2 \frac{p(x_i, y_j)}{p(x_i)p(y_j)}$$

where $p$ is a (joint) probability distribution function (pdf). It quantifies the mutual reduction of uncertainty of one of the variables due to knowledge of the other one, expressed in amount of bits. The mutual information $I(X; Y) = 0$ if and only if X and Y are statistically independent, in which case $p(x_i, y_j) = p(x_i)p(y_j)$. Thus, the mutual information detects common information content between the processes X and Y but does not reveal the direction of information flow (if any).

For this purpose, the temporal structure of the data patterns must be taken into consideration.

We introduce the multi-indices $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_l)$, such that the indices $\{ \alpha_i, \beta_j \} \in \mathbb{N}$ are monotonically increasing, i.e., $0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k$ and similar for $\beta$. We will use the shorthand notation $x^{(k)}_n = (x_{n-\alpha_1}, \ldots, x_{n-\alpha_k})$ to indicate a set of k data values preceding or coinciding with the time associated with time index n, and likewise $y^{(l)}_n = (y_{n-\beta_1}, \ldots, y_{n-\beta_l})$. A measure of information transfer between the two time series X and Y is given by the transfer entropy [4]:

$$T_{Y \rightarrow X} = \sum p(x_{n+1}, x^{(k)}_n, y^{(l)}_n) \log_2 \frac{p(x_{n+1} | x^{(k)}_n, y^{(l)}_n)}{p(x_{n+1} | x^{(k)}_n)}.$$  \hspace{1cm} (2)

The sum runs over the arguments of the probability distributions (or the corresponding bins, see the next section). The reason for using multi-indices (a minor extension of [4]) is to allow the possibility of including various time scales of influence on the effect variable. The transfer entropy can be rewritten in the form of a conditional mutual information [1]. It measures the excess amount of bits needed to encode the information of the process X at time point n + 1 with respect to the assumption that this information is independent from Y. In other words, the transfer entropy is an implementation of Wiener’s ‘quantifiable causality’. If Y has no influence on the immediate future evolution of system X, one has $p(x_{n+1} | x^{(k)}_n, y^{(l)}_n) = p(x_{n+1} | x^{(k)}_n)$, so that $T_{Y \rightarrow X} = 0$. $T_{Y \rightarrow X}$ can be compared with $T_{X \rightarrow Y}$ to uncover a net information flow.

Using $p(x | y) = p(x, y) / p(y)$, equation (2) can be rewritten as

$$T_{Y \rightarrow X} = \sum p(x_{n+1}, x^{(k)}_n, y^{(l)}_n) \log_2 \frac{p(x_{n+1} | x^{(k)}_n, y^{(l)}_n)p(x^{(k)}_n)}{p(x^{(k)}_n, y^{(l)}_n)p(x_{n+1})}.$$  \hspace{1cm} (3)

Thus, computing $T_{Y \rightarrow X}$ requires estimating four multidimensional probability distributions.

2.1. Implementation

Here, the probability distributions appearing in equations (2) and (3) are calculated using a discrete binning of m bins in each coordinate direction. The main joint pdf $p(x_{n+1}, x^{(k)}_n, y^{(l)}_n)$ has $l + k + 1$ dimensions, so there are $m^{l+k+1}$ bins, and this number should be much smaller than the available length of the data.
2.2. A system of coupled Van der Pol oscillators

A system of $M$ coupled Van der Pol oscillators may exhibit chaos without external driving. Such a system is described by

$$
\frac{\partial x_i}{\partial t} = y_i,
$$

$$
\frac{\partial y_i}{\partial t} = \left( \epsilon_i - \left( x_i + \sum_j \kappa_{ij} x_j \right)^2 \right) y_i - \left( x_i + \sum_j \kappa_{ij} x_j \right). \tag{4}
$$

The parameter $\epsilon_i$ determines the limit cycle of oscillator $i$ (for $\kappa = 0$), while $\kappa_{ij}$ specifies the non-linear coupling between oscillator $i$ and oscillator $j$ [18].

We have run a simulation with $M = 2$, $\epsilon = (1, 1.1)$ and

$$
\kappa = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{5}
$$

meaning that there are two oscillators with slightly different limit cycles, while oscillator 2 affects oscillator 1 (but not vice versa). With this choice of parameters, the system is in a quasi-periodic state. Time was integrated from $t = 0$ to $t = 1000$, and 10000 equally spaced data points were saved for analysis. Only the time interval from $t = 100$ to $t = 1000$ was used for analysis in order to remove the initial transient phase. A section of data is shown in figure 1.

From a spectral analysis, the mean oscillation period of the signals was about 6.72, corresponding to about 67 samples per period. Thus, $\alpha = \beta$ should be chosen less than about 17 ($\approx 67/4$). The latter value (17) also roughly corresponds to the first minimum of the self-mutual information of $x_1$.

The selected data are analysed with $\alpha = \beta = 8$, $m = 3$. Net information flow from signal $i$ to signal $j$ is computed as $T_{ij} = T_{ij}^\text{net} = T_{ij} - T_{ji}$, where the indices 1, . . ., 4 correspond to the signals $x_1, y_1, x_2, y_2$, respectively. The following net transfer entropy matrix is obtained (only the part of the matrix above the diagonal is shown; the rest follows from antisymmetry):

$$
T_{\text{net}} = \begin{pmatrix} 0 & -0.036 & -0.29 & 0.15 \\ -0.036 & 0 & 0.0007 & -0.20 \\ -0.29 & 0.0007 & 0 & 0.21 \\ 0.15 & -0.20 & 0.21 & 0 \end{pmatrix}. \tag{6}
$$

This can be represented graphically by drawing 4 dots representing the 4 signals in a plane, see figure 2. The four dots are connected by arrows, such that the direction of the arrow indicates the direction of net information flow, while the width of the arrow is proportional to the value $T_{ij}^\text{net}$. There is a strong flow from $x_2$ to $x_1$ (corresponding to $T_{21}^\text{net}$). This non-trivial component corresponds to the fact that $\kappa$ is such that oscillator 2 affects oscillator 1 (but not vice versa). Another strong flow is from $y_2$ to $y_1$ ($T_{21}^\text{net}$), for the same reason. The flow from $x_2$ to $y_2$ ($T_{22}^\text{net}$) is trivial (see equation (4)). The remaining
The stability of the analysis method was tested by computing the transfer entropy for a varying length of the data arrays, $N$, with the analysis settings as in the previous paragraph. It is seen that $T$ converges to a stable value for $N \gtrsim 10^4$, a rather modest number. Note that in this case, the total number of bins of the main pdf is $m^{n_{x+y}} = 3^3 = 27$. The value of the transfer entropy $T$, expressed in bits, can be calibrated against the total bit range, $\log_2 m = 1.58$, implying that the coupling strength is quite significant.

To understand the evolution of the transfer entropy with the analysis parameters $\alpha$, $\beta$, we calculated $T_{13}$ (which specifies the net flow from signal 1, $x_1$, to signal 3, $x_3$) for a range of $\alpha = \beta$ values, see figure 4. The graph shows that the net information transfer from $x_1$ to $x_2$ is negative for $\alpha, \beta \leq 17$ (as it should, for we know that the information transfer should go from oscillator 2 to oscillator 1). For higher values of $\alpha, \beta$, the net flow changes sign. This occurs when crossing the quarter-period value (17) or minimum self-mutual information value (17), and is caused by the fact that this system is quasi-periodic. It seems important, therefore, to keep $\alpha, \beta$ well below the mentioned reference values.

### 2.3. A simplified predator–prey model

In the context of fusion plasmas, spontaneous confinement transitions are of prime interest. In recent years, models have been developed to describe such transitions, involving the non-linear interaction between various fields. In this section, we will use the model of [19] to generate signals for analysis using the transfer entropy technique. The model equations are:

\[
\frac{dE}{dr} = \left(\frac{1}{1 + V^2 + U^2} - E\right)E
\]

\[
\frac{dV'}{dr} = (aE^2 + cU^2 - b) V'
\]

\[
\frac{dU'}{dr} = \left(\frac{aE^2}{1 + V^2} - b\right) U' + dE^2 V'.
\]

Here, $E$ represents the turbulence amplitude, $U'$ represents the zonal flow shear, and $V'$ the sheared flow.

We performed a simulation run with $a = 0.204$, $b = 0.16$, $c = 0.714$ and $d = 0.5$ (see figure 5 of the cited paper), generating a set of 10000 time points (at sampling rate $\Delta t = 1$). A short section of the model output is shown in figure 5. The mean period of the quasi-periodic oscillations is 40.09, corresponding to about 40 samples. The first minimum of the self-mutual information occurs at 8 (for $U'$), 10 (for $V'$) or 14 (for $E$) samples. Thus, $\alpha$ and $\beta$ should be chosen well below 8.

The transfer entropy is computed for all nine possible combinations of signals ($1 = E, 2 = V', 3 = U'$). The settings chosen are $\alpha = \beta = 5, m = 5$. The following transfer entropy matrix is obtained:

\[
T = \begin{pmatrix}
0 & 0.4913 & 1.0791 \\
0.6320 & 0 & 0.8540 \\
0.7084 & 1.0809 & 0
\end{pmatrix}.
\]
Figure 5. Signals from a run of the model of equation (7) with parameters $a = 0.204$, $b = 0.16$, $c = 0.714$ and $d = 0.5$.

Figure 6. Information flow between the three signals of the predator–prey model: the arrows indicate the direction of flow and the width is proportional to the amount of information transfer.

Comparing the values of $T$ with $\log_2 m = 2.32$, one concludes that the interactions are quite strong.

Net information flow from signal $i$ to signal $j$ is computed as $T_{ij}^{\text{net}} = T_{ij} - T_{ji}$. This is again represented graphically by drawing three dots representing the three signals in a plane. The three dots are connected by arrows, such that the direction of the arrow indicates the direction of net information flow, while the width of the arrow is proportional to the value $T_{ij}^{\text{net}}$. See figure 6.

The resulting diagram makes eminent physical sense: $E$ drives $U'$ (turbulence drives zonal flow); $U'$ drives $V'$ (zonal flow drives sheared flow); $V'$ drives $E$ (sheared flow controls turbulence). This is precisely the order of interaction that one would expect in this type of model. Of course, in this simplified case this sequence can also be obtained by simple inspection of the data (figure 5), as the mutual time delays are in clear accord with this result. On the other hand, the present method is generally applicable and allows quantifying the result.

3. Experimental data analysis results

In this section, we will apply the transfer entropy technique to data obtained from various magnetic confinement devices. The data have been selected for their relevance to the study of the interaction between zonal flows and turbulence. Zonal flows are large scale electrostatic potential structures that form spontaneously in magnetically confined toroidal fusion plasmas, and have zero toroidal wavenumber, small or zero poloidal wavenumber and finite radial wavenumber [5]. The global nature of these structures makes them hard to identify, as most measurements (of potential or radial electric field) are local. In the following, we will not worry about the precise identification of zonal flows, but confide in earlier published analyses showing that the presented data pertain to zonal flows with high probability. Our main goal here is to analyse the interaction between these hypothetical zonal flows (identified via potential or radial electric field fluctuations) and turbulence (identified via the density fluctuation amplitude or radial particle transport flux).

3.1. TJ-K

Here, we analyse data from the TJ-K stellarator, a torsatron operated at low magnetic field ($B = 72$ mT) and low plasma beta. The discharge analysed here corresponds to a helium plasma, heated by microwaves, with a central density of $n = 2.3 \times 10^{17} \text{ m}^{-3}$ and an electron temperature of $T_e = 8$ eV and cold ions, as reported in more detail elsewhere [20]. In this experiment, turbulence was dominated by electrostatic drift wave turbulence, and the total particle transport and zonal potential were found to be linked in a predator–prey cycle. Among other diagnostics, the device disposes of a set of 64 Langmuir probes, distributed over a poloidal circumference of the device. The probes are configured to measure floating potential and ion saturation current in an alternating fashion, at a sampling rate of 1 MHz. From these signals, we compute the zonal potential $\Phi_1(t)$ as the mean poloidal value of the floating potential, and the global radial particle flux $\Gamma_{\text{tot}}(t)$ as the poloidal mean of the local radial particle flux, proportional to the fluctuating ion saturation current times the local poloidal electric field, as described in more detail in the cited reference. We quantify the ‘global turbulence level’ by computing the root mean square (RMS) deviation of the 32 poloidally distributed ion saturation current measurements ($I_{\text{sat}}$), thus obtaining a quantifier of the turbulence level with the same time resolution as $\Phi_1(t)$ and $\Gamma_{\text{tot}}(t)$. A short section of data is shown in figure 7:
clearly, these data are much less regular than the model data shown in the preceding section, making it considerably more difficult to understand the non-linear relationship between the signals.

We apply the analysis described above, setting $m = 3$ (coarse graining). Figure 8 shows the transfer entropy between the two signals $\Phi_1(t)$ and $\Gamma_{\text{tot}}(t)$ as a function of $\alpha = \beta$. The amplitude of the transfer entropy is rather small, namely below $5 \times 10^{-3}$, compared with the full bit range $\log_2 m = 1.59$, which indicates that the causal link between these variables is not very strong. Nevertheless, it is unexpected and interesting to observe that $T_{\Phi_1 \rightarrow \Gamma_{\text{tot}}}$ peaks at about 20 $\mu$s, while $T_{\Gamma_{\text{tot}} \rightarrow \Phi_1}$ peaks at about 60 $\mu$s. Thus, zonal potential $\Phi_1$ has a rather fast impact on the total particle flux $\Gamma_{\text{tot}}$, while the total particle flux $\Gamma_{\text{tot}}$ acts back on the zonal potential $\Phi_1$ on a much longer time scale. In terms of net information transfer, it flows from $\Phi_1$ to $\Gamma_{\text{tot}}$ for timescales less than about 40 $\mu$s, and in the opposite direction for longer timescales. The two distinct timescales for mutual interaction would immediately give rise to oscillatory behaviour, as indeed observed. This seems coherent with the usual predator–prey models for the interaction between zonal flow and turbulence.

In the standard zonal flow model, the zonal flow has an impact on the global turbulence level. Figure 9 shows the interaction diagram between all three signals at the two most significant values of $\alpha = \beta$. On a short time scale (20 $\mu$s), the zonal flow affects the transport, and the transport in turn affects the turbulence level (presumably, by modifying the driving gradients). On a longer time scale (60 $\mu$s), the transport affects the zonal flow, but the interaction with the turbulence level is insignificant.

The short time scale result is interesting, as it confirms the analysis of [20], where it was observed that the zonal flow does not affect the turbulence amplitude strongly, but rather it affects the transport (as noted in the cited paper, by modifying the phase relation between density and potential fluctuations). The modification of the transport then affects the turbulence amplitude. Although there is an arrow showing that the zonal potential also affects the turbulence level directly, its strength is much less than the indirect route via the turbulent transport. The long time scale result is presumably simply due to a restoration of ambipolarity: a modification of transport must eventually lead to a modification of potential.

3.2. TJ-II: Doppler reflectometry

TJ-II is a Heliac type stellarator with four field periods. The experiments discussed below have been carried out in pure neutral beam injection (NBI)-heated plasmas (line-averaged electron density $\langle n_e \rangle = (2–4) \times 10^{19}$ m$^{-3}$, central electron temperature $T_e = 300–400$ eV, $T_i \simeq 140$ eV). The input NBI power was about 500 kW. These discharges have been reported elsewhere in more detail [21–23].

In this section, we will analyse data from the Doppler reflectometry diagnostic taken as the plasma experiences spontaneous confinement transitions. In Doppler reflectometry, a finite tilt angle is purposely introduced between the incident probing beam and the normal to the reflecting cut-off layer, and the Bragg back-scattered signal is measured [24]. The amplitude of the recorded signal, $A$, is a measure of the intensity of the density fluctuations, $\tilde{n}$. Furthermore, as the plasma rotates in the reflecting plane (flux surface), the scattered signal experiences a Doppler shift. The size of this shift is directly proportional to the rotation velocity of the plasma turbulence perpendicular to the magnetic field lines, $v_\perp$, and therefore to the plasma background $E \times B$
Figure 9. Graphical representation of the net transfer entropy between the zonal potential $\Phi_z(t)$, the global particle transport $\Gamma_{\text{tot}}(t)$, and the global turbulence level $\text{RMS}(I_{\text{sat}})$ at TJ-K for two values of $\alpha = \beta$.

Figure 10. Transfer entropy between $\tilde{v}_\perp$ (the fluctuating perpendicular flow velocity) and $\tilde{n}$ (the turbulence amplitude) in the I-phase versus $\alpha = \beta$ (in sampling units, $0.1 \mu s$).

Figure 11. Mean transfer entropy between $\tilde{v}_\perp$ (the fluctuating perpendicular flow velocity) and $\tilde{n}$ (the turbulence amplitude) for ten discharges in a magnetic configuration with $\iota(a)/2\pi = 1.553$ (the L–I transition occurs at $\Delta t = 0$).

First, we consider discharges in a magnetic configuration with edge rotational transform $\iota(a)/2\pi = 1.553$. In this configuration, a transition from L-mode to an intermediate (I) phase is often observed (intermediate between the L and H modes). In the I-phase, predator–prey oscillations occur, and bicoherence is relatively strong as reported elsewhere [23]. Figure 10 shows an example of the transfer entropy for data in a 20 ms long time window in the I-phase versus $\alpha = \beta$ (with $m = 3$). The graph bears similarity to the corresponding graph for TJ-K, figure 8, in that a clear peak in the transfer entropy curves, while $T_{\tilde{v}_\perp \rightarrow \tilde{n}}$ dominates over $T_{\tilde{n} \rightarrow \tilde{v}}$ for small values of $\alpha, \beta$. The position of the peak of the transfer entropy appears to be related to the autocorrelation time of the turbulence ($\sim 50 \mu s$ for TJ-K, 1–10 $\mu s$ for TJ-II). Thus, it is not related to the very slow predator–prey cycles reported in earlier work [21], with a period of about a ms. in other words, the analysis based on the transfer entropy has uncovered a novel interaction.

Next, we consider discharges in a magnetic configuration with $\iota(a)/2\pi = 1.630$. In this configuration, a relatively rapid transition from L-mode to H-mode is often observed, without intermediate (I) phase [22]. The average transfer entropy was computed for a number of discharges using an analogous procedure as described above; however, setting $\Delta t = 0$ at the L–H transition time. Figure 12 shows the average evolution of the transfer entropy for four discharges.
in this magnetic configuration (around the L–H transition). The transfer entropy \( T_{\tilde{v} \rightarrow \tilde{n}} \) increases sharply by a factor of 2 at the L–H transition, indicating the regulation of turbulence (\( \tilde{n} \)) by the zonal flow (\( \tilde{v} \)). This regulatory phase lasts for about 15–20 ms, in accord with the duration of enhanced bicoherence reported elsewhere [23].

We draw attention to an interesting difference between the L–I and L–H transitions. With the L–H transition, the transition is followed by a rapid increase in \( T_{\tilde{v} \rightarrow \tilde{n}} \), while \( T_{\tilde{n} \rightarrow \tilde{v}} \) remains approximately constant. Thus, the zonal flow is simply regulating the turbulence (suppressing it). With the L–I transition, the transition also shows a rapid increase in \( T_{\tilde{v} \rightarrow \tilde{n}} \), but this is mirrored (although at a lower intensity level) by a similar increase in \( T_{\tilde{n} \rightarrow \tilde{v}} \). This is consistent with the fact that not only does the zonal flow regulate the turbulence, but the turbulence also acts back on the zonal flow, which could be related to the observed (predator–prey type) oscillations. Also, in the case of the L–I transition, the values achieved by the transfer entropy are about three times higher than with the L–H transition. In both cases, the amplitude of the transfer entropy is modest compared with the bit range, \( \log_2 m = 1.59 \), although an order of magnitude above the TJ-K case reported in the preceding section.

### 3.3. TJ-II: Langmuir probes

In discharge 18080, heated by electron cyclotron resonant heating (\( P_{\text{ECRH}} \simeq 400 \text{kW} \)), a triple Langmuir probe was inserted to normalized radius \( \rho = 0.92 \). By raising the electron density, a spontaneous confinement transition was provoked, and a subsequent back-transition was achieved by bringing the density down again [26]. It should be noted that this transition is not an L–H transition, but is related to a change of neoclassical root [27] (a local sign change of the mean radial electric field, \( E_r \)). The density evolution is shown in figure 13 (top), showing the double crossing of the critical line-averaged density value.

The Langmuir probe measured floating potentials and ion saturation currents on various pins at a sampling rate of 1 MHz. The probe configuration allowed the computation of the fluctuating radial and poloidal electric fields, \( E_r \) and \( E_\theta \), and the fluctuating radial particle flux \( \Gamma \). Figure 13 (bottom) shows the transfer entropy between some of these signals, computed for successive 2 ms time sections using \( m = 3 \), \( \alpha = \beta = 5 \).

Interestingly, the transfer entropy is largest for the combination \( E_r \rightarrow \Gamma \). This is significant, in view of the fact that this corresponds to the impact of a possible zonal flow (\( E_r \propto v_\theta \)) on the radial particle flux. This quantity is seen to build up gradually before the transition, and essentially disappear during the enhanced confinement state (1115 < \( t \) < 1165 ms). The build-up phase presumably corresponds to the gradual development and growth of a zonal flow, which, however, disappears when the line-averaged density is above its critical value, \( n_{\text{crit}} \simeq 6 \times 10^{19} \text{ m}^{-3} \). The transfer entropy is also large for the combination \( E_r \rightarrow E_\theta \). This is also significant, as sheared flow is produced by Reynolds stress according to standard zonal flow models, which can only be large if \( E_r \) and \( E_\theta \) are phase-correlated. Traditional analyses have indeed shown that this phase correlation occurs [28], but the present analysis adds the information that it is the zonal flow (\( E_r \) or poloidal velocity) that drives the poloidal electric field (or poloidal particle velocity), and not the other way around. After the back-transition, all quantities return approximately to their pre-transition values. It is noted that very similar results are obtained for a set of six similar discharges, showing that these results are robust.

A similar analysis was made for two discharges with initial subcritical density in which external biasing was applied between \( t = 1100 \) and \( t = 1150 \text{ ms} \). A biasing probe was inserted about 2 cm into the plasma and biased with respect to a poloidal limiter tangent to the last closed flux surface. The triple Langmuir probe was inserted to normalized radius \( \rho \simeq 0.81 \). Detailed information about these discharges can be found elsewhere [26]. When applying positive biasing, turbulence was suppressed, leading to an improvement of
confined such that the density rose to values exceeding the critical density for spontaneous transitions \((n_c > n_{crit})\); however, in contrast to the spontaneous confinement transition, here \(T_{e \rightarrow r}^{\text{net}}\) remains positive. Figure 14 shows the evolution of \(T_{E_r \rightarrow E_e}^{\text{net}}\) and \(T_{E_r \rightarrow \Gamma}^{\text{net}}\). It is clear that biasing has a strong effect on these quantities.

Comparing the spontaneous and biasing-induced confinement transitions, one observes that \(T_{E_r \rightarrow \Gamma}^{\text{net}}\) is large for \(n_c < n_{crit}\) with the spontaneous transition, while it is large for \(n_c > n_{crit}\) with the induced transition. The explanation for this apparent contradiction is related to the evolution of the mean electric field profile \(E_r(\rho)\), and will be addressed in section 4.

4. Discussion and conclusions

4.1. General considerations

The analysis of the causal relation between fluctuating variables is of prime interest when studying complex nonlinear systems, and fundamental to reach a full understanding of such systems and develop realistic models. In this work, we use the concept of ‘causality’ in the restricted sense referred to in section 1 (Wiener’s ‘quantifiable causality’).

The transfer entropy technique [4] allows detecting a causal relation between variables that does not require very lengthy time series (although stationary state is still a requirement) and that does not rely on the assumption of weak turbulence. In essence, the analysis is based on the observation of a (significant) number of repetitive event sequences occurring in a pair of time series, which, however, may occur in an irregular manner.

As is the case with all methods for causality detection, an important caveat is due. The method only detects the information transfer between measured variables. If the net information flow suggests a causal link between two such variables, this may either be due to a direct cause/effect relation (in the restricted sense referred to above), or due to the presence of a third, undetected variable that affects both (e.g. with different delays, thus generating an apparent causal relation). Thus, physical insight into the system is always needed to determine whether all relevant variables are being measured and to decide whether the net information flow actually corresponds to a causal link.

4.2. Tests on numerical models

When computing the transfer entropy for numerical data generated by multivariate non-linear models, it was found that the direction of interaction between system variables could be recovered. For example, in the system of two coupled Van der Pol oscillators, it was clearly established that oscillator 2 affected oscillator 1, but not vice versa, in accordance with the design of the system. A similar statement can be made for the simplified predator–prey model. Numerical convergence of the analysis was tested and guidelines for an efficient choice of analysis parameters (\(m, \alpha, \text{and} \beta\)) are provided.

4.3. Application to experimental data and interpretation of results

We have explored the application of this technique to some data from turbulent fusion plasmas. The selected measurement data are relevant to the understanding of the important confinement transition, in which turbulence spontaneously generates a more ordered plasma state with reduced radial transport.

The analysis of the global potential and flux at TJ-K revealed the existence of two time scales: 20 and 60 \(\mu s\). On the short time scale, the zonal flow potential was shown to affect transport, which in turn affected turbulence. On the longer time scale, the transport affected the zonal flow potential, which was hypothesized to be due to a restoration of ambipolarity. The short time scale result is in accordance with the previous analysis of [20], whereas the longer time scale result is novel and reveals the potential significance of the technique to uncover new relationships.

Doppler reflectometry data from TJ-II taken across L–I and L–H transitions in NBI heated plasmas showed how the fluctuating perpendicular flow velocity \(v_r\), again associated with zonal flows, affects the turbulence. Across the L–H transition, \(T_{v_r \rightarrow E_r}\) was found to increase sharply, while the reverse interaction \(T_{E_r \rightarrow v_r}\) remained fairly constant. However, the L–I transition was characterized by an increase in both these quantities \((T_{E_r \rightarrow v_r}\) being dominant). The I-phase is characterized by quasi-periodic predator–prey oscillations [29], which, however, occur on a rather slow time scale (of the order of a ms) compared with the interactions found here, occurring on a \(\mu s\) time scale. Of course, if the predator–prey oscillations affect the turbulence, as one assumes must be the case, then this effect must be detectable on this fast time scale. This seems to be what the transfer entropy succeeds in doing. Note that the interaction between the slow time scale of the predator–prey cycle and the fast time scale of the turbulence autocorrelation time was hinted at already in a previous analysis based on the bicoherence [23].

Langmuir probe data from TJ-II taken across a low-density confinement transition in an ECR-heated plasma show how the transfer entropy between the fluctuating radial electric field (associated with zonal flows) and the particle flux, \(T_{E_r \rightarrow \Gamma}\), gradually grows prior to the transition and essentially disappears once the transition has taken place. By contrast,
during bias-induced transitions in ECR-heated plasmas, $T_{E_\rightarrow \Gamma}$ increases sharply while biasing is applied.

The observed behaviour of the transfer entropy in ECR-heated plasmas can perhaps be understood as follows. In the case of the spontaneous transition in ECR-heated plasmas, $T_{E_\rightarrow \Gamma}$ increases gradually as the density is raised towards the critical value, which is interpreted as a gradual growth of the zonal flow amplitude. Simultaneously, $T_{E_\rightarrow \Gamma}$ increases, which is interpreted as the build-up of Reynolds Stress, expected to produce a (steady state) sheared flow. However, as the density is raised slowly, the plasma adjust the profiles in an attempt to maintain the ambipolarity condition. At a certain point, the electron root solution of the ambipolarity equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. 

Following the transition to the ion flux are associated with large changes in the susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. Immediately prior to this point, the flow susceptibility is large (i.e. small changes in the ambipolar equation disappears. 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