Electroweak Chiral Lagrangian for a Hypercharge-universal Topcolor Model

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Abstract

Electroweak chiral Lagrangian for a hypercharge-universal topcolor model is investigated. We find that the assignments of universal hypercharge improve the results obtained previously from K.Lane’s prototype natural TC2 model by allowing a larger $Z'$ mass resulting in a very small $T$ parameter and the $S$ parameter is still around the order of +1.

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Topcolor-assisted technicolor (TC2) is a class of new physics models which combines technicolor and topcolor together to realize the electroweak symmetry breaking dynamically. In these theories, a technicolor condensate provides the masses to the weak vector bosons and an extended technicolor (ETC) sector gives masses to the light quarks and leptons, and a bottom-quark-sized mass to the top. The majority of the top-quark mass is due to the formation of a top-quark condensate through the dynamics of an extended color gauge sector. The typical gauge group of the TC2 models is

$$SU(N)_{TC} \otimes SU(3)_1 \otimes SU(3)_2 \otimes SU(2)_L \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$$

(1)

, in which the extended color and hypercharge groups $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2}$ spontaneously break to their diagonal subgroup $SU(3)_C \otimes U(1)_{Y}$ at a few TeVs and the remaining electroweak groups $SU(2)_L \otimes U(1)_{Y}$ spontaneously break to their electromagnetic subgroup $U(1)_{em}$ at the electroweak scale due to a combination of a top-quark condensate and a technifermion condensate. In the original TC2 model [1, 2], the extended hypercharge sector $U(1)_{Y_1} \otimes U(1)_{Y_2}$ is usually arranged nonuniversal in flavor to ensure that the bottom-quark and other light quarks and leptons do not condensate. Recently a new type of TC2 model with a flavor-universal extended hypercharge sector is proposed in Ref.[3], the authors there have examined various experimental and theoretical constraints, finding that precision electroweak measurements yield the strongest bounds on the model and the goodness of fit to all available Z-pole and LEP2 data for hypercharge-universal topcolor is comparable to that of the standard model (SM). In contrast, TC2 models with a flavor nonuniversal hypercharge sector are markedly disfavored by the data. The similar result on the nonuniversal hypercharge TC2 models is also obtained from our works [4, 5], where we have computed the coefficients of the bosonic part of electroweak chiral Lagrangian (EWCL) up to the order $p^4$ and found an upper bound for the mass of flavor nonuniversal Z$'$ boson. For Hill’s schematic TC2 model [1], Z$'$ mass $M_{Z'}$ is a few TeVs and the S parameter can be either positive or negative depending on whether the $M_{Z'}$ is large or small [4]. While for K.Lane’s prototype natural TC2 model [2], $M_{Z'}$ must be smaller than 400GeV and the S parameter is around order of $+1$ [5]. Since Ref.[3] already shows explicitly the experiment fit of the TC2 model due to the changes from the nonuniversal to the universal assignments for hypercharge sector, it is worthwhile to apply our formulation developed in Ref.[4] to the flavor-universal hypercharge topcolor model proposed in Ref.[3] to examine
the improvements from an alternative point of view. Our formulation offers an upper bound on nonuniversal $Z'$ mass previously, while Ref.\[3\] gives a lower bound of universal $Z'$ mass of roughly $2\text{TeV}$. We expect that applying our formulation to flavor-universal hypercharge topcolor model produces an upper bound on universal $Z'$ mass which will compensate the lower bound for the mass of universal $Z'$ boson obtained from Ref.\[3\]. In fact, from EWCL point of view, except technicolor and $Z'$ contributions, there are many other different sources to influence EWCL coefficients. In Ref.\[5\], we have made efforts to investigate the effective four-fermion interactions induced by extended technicolor (ETC). We find that their effects are small and we further point out that the walking technicolor (WTC) effects are worth future investigation. Considering that the authors in Ref.\[3\] assume that WTC effects do not generate large precision electroweak corrections, up to present stage, we ignore WTC effects in this work.

In this paper, we are mainly interested in the effects from flavor-universal hypercharge sector, to reduce the computations and to be convenient for comparison with flavor-nonuniversal hypercharge model, we base our calculations on the K.Lane's prototype natural TC2 model \[2\] discussed in Ref.\[5\], but change its hypercharge assignments to that given in Ref.\[3\]. The gauge charges are shown as Table I.

TABLE I. Gauge charge assignments of techniquarks for hypercharge universal TC2 model discussed in present paper. These techniquarks are $SU(3)_1 \otimes SU(3)_2$ singlets.

| field | $T^L_L$ | $U^R_L$ | $D^b_R$ | $T^L_R$ | $U^R_R$ | $D^b_R$ | $T^b_L$ | $U^b_R$ | $D^b_R$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $SU(N)$ | N | N | N | N | N | N | N | N | N |
| $SU(2)_L$ | 2 | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 1 |
| $U(1)_{Y_1}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ |
| $U(1)_{Y_2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In later numerical computations, technicolor group representation will be taken to be $N = 3$. 


The action of the symmetry breaking sector is
\[
S_{\text{SBS}}[G_{\mu}^a, A_{1\mu}^A, A_{2\mu}^A, W_{\mu}^a, B_{1\mu}, B_{2\mu}, \bar{T}^i, T^i, \bar{T}^b, T^b] = \int d^4x (\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{techniquark}} + \mathcal{L}_{\text{breaking}} + \mathcal{L}_{\text{4T}}), \tag{2}
\]
with \(\mathcal{L}_{\text{techniquark}}, \mathcal{L}_{\text{breaking}}\) and \(\mathcal{L}_{\text{4T}}\) being the same as those in Ref.\cite{5} and the modified techniquark Lagrangian with flavor-universal hypercharge is
\[
\mathcal{L}_{\text{techniquark}} = \bar{T}^i (i\partial - g_{\text{TC}} t^a \sigma^a - g_2 \frac{\tau^a}{2} W_{\mu}^a \gamma^\mu - \frac{1}{2} q_1 B_1 \tau^3 P_R) T^i + T^i (i\partial - g_{\text{TC}} t^a \sigma^a - g_2 \frac{\tau^a}{2} W_{\mu}^a \gamma^\mu - \frac{1}{2} q_1 B_1 \tau^3 P_R) T^b. \tag{3}
\]
Rotating hypercharge gauge fields \(B_{1\mu}\) and \(B_{2\mu}\) as
\[
(B_{1\mu} \ B_{2\mu}) = \begin{pmatrix} Z_{\mu}^i & B_{\mu} \end{pmatrix} \begin{pmatrix} \cos \theta' - \sin \theta' \\ \sin \theta' \cos \theta' \end{pmatrix}, \quad g_1 = q_1 \sin \theta' = q_2 \cos \theta'. \tag{4}
\]
The techniquark Lagrangian \(\mathcal{L}_{\text{techniquark}}\) is then reduced to
\[
\mathcal{L}_{\text{techniquark}} = \bar{\psi} (i\partial - g_{\text{TC}} t^a \sigma^a + V + \bar{A} \gamma^5) \psi, \tag{5}
\]
where all three doublets techniquarks are arranged in one by six matrix \(\psi = (U^T, D^T, D^T, U^b, D^b)^T\) and
\[
V_\mu = (-\frac{1}{2} g_2 \frac{\tau^a}{2} W_{\mu}^a - \frac{1}{2} q_1 \frac{\tau^3}{2} B_{\mu}) \otimes I + Z_{V\mu}, \quad A_\mu = (-\frac{1}{2} g_2 \frac{\tau^a}{2} W_{\mu}^a - \frac{1}{2} q_1 \frac{\tau^3}{2} B_{\mu}) \otimes I + Z_{A\mu} \tag{6}
\]
with \(I = \text{diag}(1, 1, 1)\), \(Z_{V\mu} = \text{diag}(Z_{V_{\mu}^i}, Z_{V_{\mu}^b}, Z_{V_{\mu}^b})\), \(Z_{A\mu} = \text{diag}(Z_{A_{\mu}^i}, Z_{A_{\mu}^b}, Z_{A_{\mu}^b})\) and
\[
Z_{V_{\mu}^i} = Z_{V_{\mu}^b} = Z_{V_{\mu}^b} = Z_{A_{\mu}^i} = Z_{A_{\mu}^b} = Z_{A_{\mu}^b} = -\frac{1}{4} q_1 \cot \theta' Z_{\mu}^3. \tag{7}
\]
As done in Ref.\cite{5}, the EWCL for present model is
\[
\exp \left( i S_{\text{EW}}[W_{\mu}^a, B_{\mu}] \right) = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}G_{\mu}^a \mathcal{D}Z_{\mu}^i e^{i S_{\text{SBS}}[G_{\mu}^a, 0, 0, W_{\mu}^a, B_{1\mu}, B_{2\mu}, T^i, T^i, \bar{T}^b, T^b]} = \mathcal{N}[W_{\mu}^a, B_{\mu}] \int \mathcal{D}\mu(U) \exp \left( i S_{\text{eff}}[U, W_{\mu}^a, B_{\mu}] \right), \tag{8}
\]
where \(U(x)\) is a dimensionless unitary unimodular matrix field in EWCL, and \(\mathcal{D}\mu(U)\) denotes the normalized functional integration measure on \(U\). The normalization factor \(\mathcal{N}[W_{\mu}^a, B_{\mu}]\) is determined through the requirement that when the technicolor interactions are switched off, \(S_{\text{eff}}[U, W_{\mu}^a, B_{\mu}]\) must vanish.
The following computation procedure is exactly the same as those given in Ref. [3], in which we integrated out the technigluons, the techniquarks and the colorons. We abbreviate the detailed process and only write down the resulted action,

$$\int D\phi(D\psi D\bar{\psi} D\bar{\psi}^{i} e^{iS_{\text{SNS}}^{A_{\mu}=A_{\mu}=0}}) = \int D\phi(D\psi D\bar{\psi}^{i} e^{iS_{\text{SNS}}^{A_{\mu}=A_{\mu}=0}}),$$

with

$$S_{\text{SNS}}[U, W_{\mu}, B_{\mu}, Z_{\mu}'] = -i \text{Tr} \log(i\bar{\psi} + V + A \gamma^{5}) + \int d^{4}x \left[ -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} \right.
\left. + \frac{1}{2} M_{0}^{2} Z_{\mu}' Z_{\mu} + 3 \text{tr} \left( (F_{0}^{1D})^{2} a^{2} - \kappa_{1}^{'1D,0} (d_{\mu} a_{\mu})^{2} - \kappa_{2}^{'1D,0} (d_{\mu} a_{\mu} - d_{\mu} a_{\mu})^{2} \right)
\left. + \kappa_{3}^{1D,0} (a_{\mu} a_{\mu})^{2} + \kappa_{4}^{1D,0} (a_{\mu} a_{\mu})^{2} - \kappa_{13}^{1D,0} V_{\mu\nu} V^{\mu\nu} + i \kappa_{14}^{1D,0} a_{\mu} a_{\mu} V^{\mu\nu} \right) \right] + O(p^{6}),$$

where $M_{0}$ is the bare mass of $Z'$ boson from spontaneously breaking of $SU(3)_{1} \otimes SU(3)_{2} \otimes U(1)_{Y_{1}} \otimes U(1)_{Y_{2}} \Rightarrow SU(3)_{C} \otimes U(1)_{Y}$, as in Ref. [3], its relation with vacuum expectation value $\tilde{v}$ causing breaking is $M_{0}^{2} = \frac{25}{36} \frac{g_{1}^{2}}{\sin^{2}\theta \cos^{2}\theta}$. The coefficients $F_{0}^{1D}$, $\kappa_{i}^{'1D,0}$ for $i = 1, 2, 3, 4, 13, 14$ are strong interaction coefficients for one doublet technicolor model which depend on techniquark self energy and are already computed numerically in Ref. [4], [5]. Further

$$v_{\mu} \equiv -\frac{1}{2} \left( g_{2} \frac{\tau^{a}}{2} W_{\xi_{\mu}} + g_{1} \frac{\tau^{3}}{2} B_{\xi_{\mu}} - \frac{1}{4} g_{1} \cot \theta' Z_{\mu}' \tau^{3} \right),$$

$$a_{\mu} \equiv \frac{1}{2} \left( g_{2} \frac{\tau^{a}}{2} W_{\xi_{\mu}} - g_{1} \frac{\tau^{3}}{2} B_{\xi_{\mu}} - \frac{1}{4} g_{1} \cot \theta' Z_{\mu}' \tau^{3} \right),$$

in which $W_{\xi_{\mu}}$ and $B_{\xi_{\mu}}$ are rotated electroweak gauge fields given in Eq.(26) and (27) in Ref. [3] which absorb Goldstone field $U$ into the definition of gauge fields.

We can further decompose (10) into

$$S_{Z'}[U, W_{\mu}, B_{\mu}, Z_{\mu}'] = \tilde{S}_{Z'}[U, W_{\mu}, B_{\mu}, Z_{\mu}'] + S_{Z'}[U, W_{\mu}, B_{\mu}, 0],$$

where $\tilde{S}_{Z'}[U, W_{\mu}, B_{\mu}, Z_{\mu}']$ is the $Z'$ dependent part of $S_{\text{eff}}[U, W_{\mu}, B_{\mu}, Z_{\mu}]$. We find that the $Z'$ independent part $S_{Z'}[U, W_{\mu}, B_{\mu}, 0]$ is just the same as that given in Ref. [5] which is three times of the one-doublet technicolor model result given in Ref. [4]. Similar as Ref. [5] $\tilde{S}_{Z'}[U, W_{\mu}, B_{\mu}, Z_{\mu}']$ has the structure

$$\tilde{S}_{Z'}[U, W_{\mu}, B_{\mu}, Z_{\mu}'] = \int d^{4}x \left[ \frac{1}{2} Z_{R,\mu} D^{-1}_{Z} Z_{R,\nu} Z_{R,\nu}^{\mu} J_{Z,\mu} + Z_{R,\mu}^{\mu} J_{Z,\mu} + Z_{R,\mu}^{\mu} J_{Z,\mu}^{3} + g_{4} \frac{g_{4}}{4} \right],$$
where \( D_{z}^{-1, \mu} = g^{\mu} (\partial^{2} + M_{z}^{2}) - (1 + \lambda_{g}) \partial^{\mu} \partial^{\nu} + \Delta_{Z}^{\mu} (X) \) and to normalize \( Z' \) field correctly, we introduce normalized field \( Z'_{R, \mu} \) as \( Z'_{\mu} = \frac{1}{c_{Z'}} z'_{R, \mu} \). Due to the present universal assignment of hypercharge, parameters appeared in \( \tilde{S}_{z} [U, W_{\mu}, B_{\mu}, Z'_{\mu}] \) are different from those in Ref. [5],

\[
\begin{align*}
c_{Z'}^2 &= 1 + 3 \kappa_{2} g_{1}^{2} \cot^{2} \theta' + \frac{3}{2} \kappa_{2}^{2} g_{1}^{2} \cot^{2} \theta' + \frac{3}{2} \kappa_{1}^{1D, \Sigma \neq 0} g_{1}^{2} \cot^{2} \theta', \\
M_{Z'}^2 &= \frac{1}{c_{Z'}^2} \left( M_{0}^{2} + \frac{3 g_{1}^{2} \cot^{2} \theta'}{4} (F_{0}^{1D})^{2} \right), \\
\lambda_{Z} &= - \frac{3 g_{1}^{2} \cot^{2} \theta'}{4 c_{Z'}^{2}} \kappa_{1}^{1D, \Sigma \neq 0}, \\
\Delta_{Z}^{\mu} (X) &= \frac{g_{1}^{2} \cot^{2} \theta' + \frac{3}{4} \kappa_{2}^{1D, \Sigma \neq 0} g_{1}^{2} \cot^{2} \theta'}{16 c_{Z'}^{2}} \left[ \left( -12 \kappa_{3}^{1D, \Sigma \neq 0} - 6 \kappa_{1}^{1D, \Sigma \neq 0} - 3 \kappa_{1}^{1D, \Sigma \neq 0} - 3 \kappa_{1}^{1D, \Sigma \neq 0} \right) \text{tr} [X_{\mu} \tau^{3}] \text{tr} [X^{\nu} \tau^{3}] \\
&\quad + (24 \kappa_{1}^{1D, \Sigma \neq 0} - 12 \kappa_{1}^{1D, \Sigma \neq 0} - 6 \kappa_{1}^{1D, \Sigma \neq 0} + 6 \kappa_{1}^{1D, \Sigma \neq 0}) \text{tr} [X_{\mu} X^{\nu}] \\
&\quad + g^{\mu} \left( -3 \kappa_{3}^{1D, \Sigma \neq 0} + 3 \kappa_{1}^{1D, \Sigma \neq 0} - 3 \kappa_{1}^{1D, \Sigma \neq 0} - 6 \kappa_{1}^{1D, \Sigma \neq 0} \right) \text{tr} [X_{k} X^{k}] \\
&\quad + g^{\mu} \left( -3 \kappa_{1}^{1D, \Sigma \neq 0} + 3 \kappa_{1}^{1D, \Sigma \neq 0} + 3 \kappa_{1}^{1D, \Sigma \neq 0} \right) \text{tr} [X_{k} X^{k}] \right],
\end{align*}
\]

\[
\begin{align*}
J_{Z}^{\mu} &= J_{Z0}^{\mu} + \frac{g_{1}^{2} \gamma}{c_{Z'}^{2}} \partial^{\mu} B_{\nu} + \tilde{J}_{Z}^{\mu}, \\
J_{Z0}^{\mu} &= \frac{3 g_{1} \cot \theta'}{4 c_{Z'}^{2}} i (F_{0}^{1D})^{2} \text{tr} [X_{\mu} \tau^{3}], \\
\gamma &= -3 \kappa \cot \theta' - \frac{3}{2} (\kappa_{2}^{1D, \Sigma \neq 0} + \kappa_{1}^{1D, \Sigma \neq 0}) \cot \theta', \\
\tilde{J}_{Z}^{\mu} &= - \frac{g_{1} \cot \theta'}{4 c_{Z'}^{2}} \left[ \kappa_{1}^{1D, \Sigma \neq 0} \{3 i \text{tr} [U^{\dagger} (D^{\nu} D_{\nu} U)] - 3 i \text{tr} [U^{\dagger} (D^{\nu} D_{\nu} U) \tau^{3}] U^{\dagger} D^{\mu} U] \\
&\quad - 3 i \partial^{\mu} \text{tr} [U^{\dagger} (D^{\nu} D_{\nu} U^{\dagger} \tau^{3}) U^{\dagger} D^{\mu} U] \\
&\quad + (\frac{3 i}{4} - \frac{3 i}{4} + 3 \kappa_{1}^{1D, \Sigma \neq 0} + 3 \kappa_{1}^{1D, \Sigma \neq 0} - 3 \kappa_{1}^{1D, \Sigma \neq 0} + 3 \kappa_{1}^{1D, \Sigma \neq 0}) \text{tr} [X^{\nu} X_{\nu}] \right],
\end{align*}
\]

\[
\begin{align*}
g_{4Z} &= (\kappa_{3}^{1D, \Sigma \neq 0} + \kappa_{4}^{1D, \Sigma \neq 0}) \frac{3 \cot^{4} \theta'}{128}, \\
J_{3Z}^{\mu} &= \frac{3 i}{32 c_{Z'}^{2}} (\kappa_{3}^{1D, \Sigma \neq 0} + \kappa_{4}^{1D, \Sigma \neq 0}) \text{tr} [X_{\mu} \tau_{3}],
\end{align*}
\]

where

\[
\mathcal{K} = - \frac{1}{48 \pi^{2}} \left( \log \frac{\kappa^{2}}{\Lambda^{2}} + \gamma \right), \quad \Lambda, \kappa: \text{ultraviolet and infrared cutoffs}.
\]

With similar procedure of Ref. [5] to integrate out the \( Z' \) field, we find that \( S_{\text{eff}} [U, W_{\mu}, B_{\mu}, Z'_{\mu}] \)
where $L$ discussed in Ref.[4].

$\text{p}$ can read out coefficients up to order of $p^4$ as follows,

$$f^2 = 3(F_0^{1D})^2, \quad \beta_1 = \frac{3(F_0^{1D})^2 g_1^2 \cot^2 \theta'}{8M_0^2 + 6(F_0^{1D})^2 g_1^2 \cot^2 \theta'},$$ (26)

$$\alpha_1 = 3L_{10}^{1D} + \frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 2\beta_1 \tan \theta' \gamma - 6\beta_1 L_{10}^{1D},$$

$$\alpha_2 = -\frac{3}{2} L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 2\beta_1 \tan \theta' \gamma + 3\beta_1 L_9^{1D},$$

$$\alpha_3 = (-\frac{3}{2} + 3\beta_1)L_9^{1D},$$

$$\alpha_4 = 3L_2^{1D} + 6\beta_1 L_9^{1D} + \frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1,$$

$$\alpha_5 = \frac{3}{2} L_3^{1D} + 3L_1^{1D} - \frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 - 6\beta_1 L_9^{1D},$$

$$\alpha_6 = -\frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 24\beta_1^2 L_1^{1D} - 6\beta_1 (4L_1^{1D} + L_9^{1D}),$$ (27)

$$\alpha_7 = \frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 6\beta_1^2 (L_3^{1D} + 2L_1^{1D}) - 2\beta_1 (3L_3^{1D} + 6L_1^{1D} - 3L_9^{1D}),$$

$$\alpha_8 = -\frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 12\beta_1 L_{10}^{1D},$$

$$\alpha_9 = -\frac{3(F_0^{1D})^2}{2M_Z^2} \beta_1 + 6\beta_1 (L_{10}^{1D} - L_9^{1D}),$$

$$\alpha_{10} = -4\beta_1^2 (-18L_1^{1D} - 3L_3^{1D}) + 32\beta_1^4 \cot^4 \theta' g_{4Z} - \frac{3}{2} \beta_1^3 (96L_1^{1D} + 16L_3^{1D}),$$

$$\alpha_{11} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 0,$$

where $L_i^{1D}$ for $i = 1, 3, 9, 10$ are EWCL coefficients for one doublet technicolor model discussed in Ref.[4].

The features of these results which are the same as those in K.Lane’s model are:

1. The contributions to the $p^4$ order coefficients are divided into two parts: the three doublets technicolor model contribution and the $Z'$ contribution.

2. All corrections from the $Z'$ particle are at least proportional to $\beta_1$ which vanish if the mixing disappear by $\theta' = 0$.

3. Since $L_{10}^{1D} < 0$, combining with positive $\beta_1$, (27) then tells us $\alpha_8$ is negative. Then $U = -16\pi \alpha_8$ coefficient given in Ref.[6] is always positive in present model.
Since $\alpha_1$ and $\alpha_2$ depend on $\gamma$ which from \cite{21} further rely on an extra parameter $\mathcal{K}$. We can combine (26), (15) and (16) together to fix $\mathcal{K}$.

\[
\frac{3(F_0^{1D})^2 g_1^2 \cot^2 \theta'}{8\beta_1 M_{Z'}^2} = 1 + 3\mathcal{K} g_1^2 \cot^2 \theta' + \frac{3}{2} K_{12} g_1^2 \cot^2 \theta' + \frac{3}{2} K_{13} g_1^2 \cot^2 \theta'.
\] (28)

Once $\mathcal{K}$ is fixed, with the help of \cite{25}, we can determine the ratio of infrared cutoff $\kappa$ and ultraviolet cutoff $\Lambda$, in Fig.1, we draw the $\kappa/\Lambda$ as functions of $T$ and $M_{Z'}$, we find natural criteria $\Lambda > \kappa$ offers stringent constraints on the allowed region for $T$ and $M_{Z'}$ that present theory prefers small $T$ parameter. The upper bound for $Z'$ mass increases as the value of $T$ decrease, for example, upper bound is below 1TeV for $T$ being order of $10^{-3}$ and 2-3TeV for $T$ being order of $10^{-5}$. In Fig.2 we draw $Z'$ mass as a function of $T$ parameter and the gray region is the forbidden zone where $\kappa \geq \Lambda$. Not like K.Lane’s model discussed in Ref.[5] where we have the upper bound of $Z'$ mass 400GeV, now this upper bound is pushed higher as long as we have a very small $T$ parameter. Considering that Ref.[5] already gives lower bound of $M_{Z'} = 2.08$TeV, from Fig.2 we find it corresponds to $T < 7.09 \times 10^{-5}$.

With this constraints on $M_{Z'}$, in Fig.3 we further draw the $S$ parameter in terms of $T$ and $M_{Z'}$. From this graph, we find that the $S$ parameter in the region of $T < 7.09 \times 10^{-5}$ and $M_{Z'} > 2$TeV is still at order of +1 which implies present model is still not fully matching with the experiment data. Compared to previous result for K.Lane’s natural TC2 model with nonuniversal hypercharge assignments, we find that the value of the $S$ parameter does decrease due to the universal hypercharge. For example, $S \approx 1.1$ at $T = 10^{-2}$ for K.Lane’s model, while $S \approx 0$ at $T = 10^{-2}$ for present model, this is compatible with result obtained in Ref.[3], but for more smaller $T$ parameter, $S$ increases and finally for $M_{Z'}$ at 2-3TeV, $S$ is still at order of +1. Finally for completion of our discussion, we depict all nonzero coefficients $\alpha_i$. Fig.4 is the graph for $\alpha_1$ and $\alpha_2$, Fig.5 is for $\alpha_3$, $\alpha_4$ and $\alpha_7$, Fig.6 is for $\alpha_5$, $\alpha_6$, $\alpha_9$ and $\alpha_8$, Fig.7 is for $\alpha_{10}$. In all these diagrams, we find that the curves are not sensitive to $M_{Z'}$ when $M_{Z'} > 1-2$TeV, therefore we do not label the $M_{Z'}$ on the graph. For Fig.5, Fig.6 and Fig.7, the $T$ axis starts from $10^{-3}$ instead of $10^{-6}$, since below $T = 10^{-3}$, all curves approach to zero.

To summarize, we apply the formulation developed in Ref.[4] to a hypercharge-universal topcolor model, compute all the coefficients of the bosonic part of EWCL up to the order of $p^4$. We find that the universal hypercharge does improve the model from the original nonuniversal hypercharge assignments by allowing a larger $Z'$ mass resulting in a very small
FIG. 1: The ratio of infrared cutoff and ultraviolet cutoff $\kappa/\Lambda$ as functions of the $T$ parameter and $Z'$ mass in unit of TeV.

FIG. 2: Upper bound of $Z'$ mass in unit of TeV as a function of the $T$ parameter and $\kappa/\Lambda$.

$T$ parameter, but the $S$ parameter is still kept at order of +1.
FIG. 3: The S parameter as functions of $T$ and $M_{Z'}$

![Graph of S parameter vs T and M_{Z'}]

FIG. 4: $\alpha_1$ and $\alpha_2$ as functions of $T$

![Graph of alpha_1 and alpha_2 vs T]

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FIG. 5: $\alpha_3$, $\alpha_4$ and $\alpha_7$ as functions of $T$

FIG. 6: $\alpha_5$, $\alpha_6$, $\alpha_8$ and $\alpha_9$ as functions of $T$

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FIG. 7: $\alpha_{10}$ as a function of $T$