Impact of ribs on flow parameters and laminar heat transfer of water–aluminum oxide nanofluid with different nanoparticle volume fractions in a three-dimensional rectangular microchannel

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Abstract
This article aims to study the impact of ribs on flow parameters and laminar heat transfer of water–aluminum oxide nanofluid with different nanoparticle volume fractions in a three-dimensional rectangular microchannel. To this aim, compulsory convection heat transfer of water–aluminum oxide nanofluid in a rib-roughened microchannel has been numerically studied. The results of this simulation for rib-roughened three-dimensional microchannel have been evaluated in contrast to the smooth (unribbed) three-dimensional microchannel with identical geometrical and heat–fluid boundary conditions. Numerical simulation is performed for different nanoparticle volume fractions for Reynolds numbers of 10 and 100. Cold fluid entering the microchannel is heated in order to apply constant flux to external surface of the microchannel walls and then leaves it. Given the results, the fluid has a higher heat transfer with a hot wall in surfaces with ribs rather than in smooth ones. As Reynolds number, number of ribs, and nanoparticle volume fractions increase, more temperature increase happens in fluid in exit intersection of the microchannel. By investigating Nusselt number and friction factor, it is observed that increase in nanoparticle volume fractions causes nanofluid heat transfer properties to have a higher heat transfer and friction factor compared to the base fluid used in cooling due to an increase in viscosity.

Keywords
Numerical simulation, three-dimensional microchannel, nanofluid, friction factor, heat transfer

Introduction
One of the most fundamental issues in electronics industry is thermal management of electronic devices, particularly with regard to maximum working temperature limitations, and uniform heat distribution in the devices. These factors directly influence performance, cost, and reliability of electronic devices.¹ An investigation of recent articles shows an increasing demand for reducing weight and volume of electronic devices and

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increasing their power. Nowadays, the main constraint in producing electronic devices is lack of an efficient technique to decrease heat in these devices. In the last two decades, manufacturing micro devices with micron structures and compressing electronic devices have developed. Future microelectronic components have been designed for consumptions higher than 1000 (W/m²). These high heat fluxes cannot be easily transferred using current cooling methods. In recent years, efforts have been made to produce cooling devices such as compressed electronic components.1

Recently, several possible cooling solutions including two-phase flow, nucleate boiling, microchannel heat sinks, using nanofluids, and collision of gases with high speed have been investigated by different authors. Among the cooling methods, microchannel heat sink has proved to be a high-performance cooling method.2 Significant development of micro-manufacturing methods in the past decade has made microchannel reactors able to have different practical applications in several fields. Due to business benefits and practical applications of the current discussion, many studies have been carried out by researchers in this field.

Gunnasegaran et al.3 investigated the effect of geometric parameters and heat transfer of water fluid flow in a microchannel. Their results showed that the best uniform distribution of temperature and heat transfer coefficient can be obtained in channels with the smallest hydraulic diameters.

Chen and Ding4 investigated the heat transfer performance of microchannels with water–aluminum oxide nanofluid in different nanoparticle volume fractions concluding that thermal resistance and temperature distribution in microchannels significantly vary in response to the effect of inertia force.

Mohammed et al.5 also investigated numerical heat transfer in a rectangular heat exchanger microchannel. Their results showed that when there is a slight increase in pressure drop, nanofluids increase heat properties and performance of heat exchangers. In addition, these authors6 numerically evaluated the effects of using nanofluids on heat transfer and nanofluid flow parameters in a rectangular heat sink for Reynolds numbers of 100–1000 using water–aluminum oxide nanofluid with volume percentage of 1%–5%. They found that as volume percentage of nanoparticles increases, heat transfer coefficient and shear stress increase, and that using nanofluids leads to a slight pressure drop along the microchannel.

Manca et al.7 performed a numerical analysis on air displacement flow in square, rectangular, triangular, and trapezoid channels and concluded that in turbulent fluid flow regime, friction factor increases as Nusselt number increases.

Through their experimental research on friction effects in turbulent flow and heat transfer behaviors of the nanofluid composed of TiO2 and Al2O3 particles in a round tube, Pak and Cho8 concluded that Nusselt number increases by increasing nanoparticle concentration. Maiga et al.9 investigated benefits of using nanofluid and showed that using nanoparticles causes intense effects of shear stress on the wall.

Izadi et al.10 investigated compulsory displacement flow in extended laminar mode in between two concentric circles and concluded that volume fraction of nanoparticles has an important effect on thermal profiles but no effect on dimensionless velocity. Till date, many studies have been conducted to evaluate the fluid flow in macro and microchannels with surface roughness.11–13 Among these investigations, methods to enhance the heat transfer are suggested and studied.

This study was also carried out to study heat transfer and computational fluid dynamics of nanofluid in microchannel, for Reynolds numbers 10 and 100, and volume fraction of $\varphi = 0%–4%$ solid nanoparticles of Al2O3. The main difference between this study and other studies is as follows:

1. The use of non-circular surfaces to increase the surface heat transfer;
2. The use of nanofluids for working fluid;
3. The use of a tooth for better mixing of fluid flow.

**Statement of the problem**

The analysis was performed on a three-dimensional rectangular microchannel investigated in two individual modes of (a) and (b). In mode (a), two rectangular ribs have been made in the middle of the length of the microchannel in opposite upper and lower walls. Mode (b) is the same as mode (a) with the difference that there is only one rectangular rib in this mode whose height is half of the ribs’ heights in mode (a). To create the mode of developed hydrodynamic flow, the microchannel in entrance and exit areas has the entrance length with the dimensions shown in the figure. This length is insulated in the entrance and exit of the microchannel from four sides. Considering Figure 1, a uniform heat flux equal to $q'' = 25,000$ (W/m²) is applied to the central area of the microchannel from four sides.

Figure 1 shows the microchannel investigated in this study in terms of heat boundary conditions and entrance and exit length areas in two and three dimensions. The microchannel length is $l = 2.5$ mm and its height is $h = 25$ µm.

In Figure 2, the lengths are $l_1 = l_3 = 750$ µm and the microchannel width is $w = 25$ µm. The length is $l_2 = 1000$ µm. The investigated modes (a) and (b) are like Figure 1 in terms of boundary conditions in walls and general dimensions. The only distinction between
modes (a) and (b) and the smooth channel is the position of ribs in \( l_2 \) area in upper and lower walls of the microchannel as in Figure 3.

The temperature of the inlet fluid to the microchannel is \( T_c = 293 \text{ K} \). The flow in laminar mode is investigated for Reynolds numbers 10 and 100. Agent fluid is water and aluminum oxide powder (Al\(_2\)O\(_3\)) with a volume fraction of \( \varphi = 0 - 0.04 \).

In this study, three-dimensional flow is assumed to be incompressible, Newtonian, laminar, and single-phased. In the entrance areas of the microchannel, fluid is entered with uniform velocity, and the shape of nanoparticles is assumed to be uniform and spherical.

**Governing equations for laminar nanofluids**

Governing dimensionless equations include equations of continuity, momentum, and energy, which are solved for permanent and laminar mode in Cartesian coordinates\(^{14}\)

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} &= 0 \\
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} &= -\frac{\partial P}{\partial X} + \frac{1}{\rho_{nf} \nu_f \text{Re}} \left( \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial U}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \mu_{nf} \frac{\partial U}{\partial Z} \right) \right) \\
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} &= -\frac{\partial P}{\partial Y} + \frac{1}{\rho_{nf} \nu_f \text{Re}} \left( \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial V}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial V}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \mu_{nf} \frac{\partial V}{\partial Z} \right) \right)
\end{align*}
\]
Table 1. Thermophysical properties of the nanofluid and solid nanoparticle.\textsuperscript{17}

|          | Water | Al\textsubscript{2}O\textsubscript{3} | Nanofluid \(\varphi = 0.01\) | Nanofluid \(\varphi = 0.02\) | Nanofluid \(\varphi = 0.03\) | Nanofluid \(\varphi = 0.04\) |
|----------|-------|-------------|-----------------|-----------------|-----------------|-----------------|
| \(C_p\) (J/kg K) | 4179  | 765         | 4047            | 3922.4          | 3804.7          | 3693.2          |
| \(\rho\) (kg/m\textsuperscript{3}) | 997.1 | 3970        | 1026.8          | 1056.6          | 1086.3          | 1116            |
| \(k\) (W/m K) | 0.613 | 40          | 0.6408          | 0.6691          | 0.698           | 0.7276          |
| \(\mu\) (Pa s) | 8.91 \times 10\textsuperscript{-4} | --          | 9.13 \times 10\textsuperscript{-4} | 9.37 \times 10\textsuperscript{-4} | 9.61 \times 10\textsuperscript{-4} | 9.86 \times 10\textsuperscript{-4} |

\[
U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = - \frac{\partial P}{\partial Z} + \frac{1}{\rho_{nf} \nu_f} \frac{1}{Re} \left( \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial W}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial W}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( \mu_{nf} \frac{\partial W}{\partial Z} \right) \right)
\]

\[
\frac{\partial}{\partial X} \left( K_{nf} \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( K_{nf} \frac{\partial \theta}{\partial Y} \right) + \frac{\partial}{\partial Z} \left( K_{nf} \frac{\partial \theta}{\partial Z} \right)
\]

(4)

(5)

In the above equations, the following dimensionless parameters are used\textsuperscript{14}

\[
X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}, \quad Z = \frac{z}{D_h}, \quad D_h = 4A_p \quad \frac{p}{\mu_f}
\]

\[
\theta = \frac{T - T_c}{\Delta T}, \quad U = \frac{u}{u_c}, \quad V = \frac{v}{u_c}, \quad W = \frac{w}{u_c}, \quad Re = \frac{u_c D_h}{\nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad \Delta T = \frac{q'' D_h}{k_f}
\]

(6)

In the above equation, \(X, Y, Z\) are the coordinates of the axes along the \(x, y, z\), respectively. Also, parameters of \(U, V, W\) are the dimensionless velocities along the axes, \(x, y, z\), respectively. \(D_h, \theta, P, Pr, Re, \Delta T\) parameters are the hydraulic diameter, temperature and pressure dimensionless, Prandtl number, Reynolds number, and the temperature difference, respectively.

**Nanofluid properties**

Thermophysical properties of the nanofluid and aluminum powder nanoparticles are presented in Table 1. The following relationship is used for calculating nanofluid density\textsuperscript{15,16,17}

\[
\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_p
\]

In equation (7), parameters \(\rho_{nf}, \rho_f, \varphi, \rho_p\) refer to nanofluids and base fluid density, the volume fraction of solid nanoparticles and solid nanoparticle density, respectively.

Effective heat distribution coefficient of the nanofluid is calculated from the following formula\textsuperscript{18}

\[
\alpha_{nf} = \frac{k_{eff}}{(\rho C_p)_{nf}}
\]

In the above equation, parameters \(\alpha_{nf}, C_{p,nf}, k_{eff}\) are thermal diffusivity, specific heat capacity, and effective thermal conductivity of nanofluids, respectively.

Specific heat capacity of the nanofluid is calculated by the following formula\textsuperscript{19}

\[
(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_p
\]

Brinkman relationship\textsuperscript{20,21} is used to calculate effective dynamic viscosity of nanofluid

\[
\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{1.5}}
\]

(10)

In equation (10), expressions \(\mu_{nf}, \mu_f\) are dynamic viscosity of fluid and nanofluids, respectively. To calculate effective thermal conductive coefficient of nanofluid for suspensions having sphere-like particles, the relationship provided by Patel et al.\textsuperscript{22} is used

\[
k_{eff} = k_f \left[ 1 + \frac{k_n A_p}{k_f A_f} + c k_p Pe \frac{A_p}{k_f A_f} \right]
\]

(11)

where experimental constant is \(c = 36,000\), and terms \(k_f, k_n, Pe\) refer to the thermal conductivity of the base fluid and solid nanoparticles and Peclt number, respectively

\[
\frac{A_p}{A_f} = \frac{d_f}{d_p} \varphi
\]

(12)

\[
Pe = \frac{u_c d_p}{\alpha_f}
\]

(13)

where water molecule diameter equals \(d_f = 2 \text{ Å}\), aluminum nanoparticle molecule diameter equals \(d_p = 50 \text{ nm}\), and \(u_c\) value is Brownian motion velocity of nanoparticles and is calculated by the following formula

\[
u_c = \frac{2 \kappa_b T}{\pi \mu_f d_p^2}
\]

(14)

where \(\kappa_b = 1.3807 \times 10^{-23} \text{ J/K}\) value is Boltzmann constant.\textsuperscript{23}
To calculate the local Nusselt number along the microchannel walls, the following equation is used, where the values of \( T_w \) and \( T_m \) are average temperature of the wall and bulk temperature of the fluid, respectively, and \( q^0 \) is the heat flux applied to the surfaces of microchannels

\[
\text{Nu}_{\text{ave}} = \frac{q^0 D_h}{k_f (T_w - T_m)}
\]  

The following equation is used for calculate fanning friction factor

\[
f = \frac{2 \Delta P D_h}{L \rho u_{in}^2}
\]  

In equation (16), \( \Delta P \), \( L \), \( u_{in} \) refer to pressure drop, the length of channel and inlet velocity of the fluid in the microchannels, respectively. The degree of pumping power is calculated from the following equation

\[
P_p = A u_{in} \Delta P
\]  

In equation (17), \( A \) is the inlet cross-sectional area of microchannel. For general evaluation of heat–fluid performance of the rib-roughened microchannel, performance evaluation criterion (PEC) parameter is defined as heat efficiency as follows

\[
\text{PEC} = \left( \frac{\text{Nu}_{\text{ave}}}{\text{Nu}_{\text{ave,s}}} \right) \frac{f_s}{f}^{1/3}
\]  

In the definition of factor PEC, expressions \( \text{Nu}_{\text{ave}} \), \( \text{Nu}_{\text{ave,s}} \), \( f \), \( f_s \) refer to average Nusselt number in smooth and the toothed microchannel, respectively, and terms \( f \), \( f_s \) are the average friction coefficient in smooth and toothed microchannel, respectively. Poiseuille number is calculated from the following equation

\[
C_f = f \text{Re}
\]  

**Numerical method**

Equations governing laminar flow of the water–aluminum oxide nanofluid in three-dimensional and steady state mode are continuity, momentum, and energy equations. These equations are solved and discretized by finite-volume method. Semi-Implicit Method for Pressure Linked Equations—Consistent (SIMPLEC) algorithm is used to solve velocity–pressure coupled equations. The remainder of \( 10^{-6} \) is selected as the acceptable remainder to obtain exact answers and ignorable error from the current problem solving and using less computer memory in numerical simulation process maximum.

**Numerical procedure validation**

In the numerical solution of the current problem, the results from the numerical simulation were validated through the results of the reference with identical geometric and boundary conditions to ensure the accuracy of the results. With regard to Figure 4, the average Nusselt number in this study is in good agreement with the study performed in the reference. The average Nusselt number in smooth channel increases by increasing Reynolds number, which is due to better confounding of fluid layers as a result of an increase in velocity in higher Reynolds number, which in turn causes a decrease in heat gradient in-between the layers far from the hot surface of the microchannel. As nanoparticle volume fraction increases, the average Nusselt number increases as well due to the existence of solid nanoparticles in between fluid layers. Existence of nanoparticles improves heat conductivity properties of the fluid and reinforces heat transfer mechanisms in micron scale.

**Grid independence**

In the research ahead, to achieve optimal grid number, the effect number of grid will be investigated on the heat transfer and fluid flow parameters. The grid independence study of the present numerical simulation is shown in Table 2, which illustrates the results for Reynolds numbers of 100 and volume fractions of 4% in case (b) through average value of Nusselt number, friction factor variation, and convection heat transfer coefficient. The results indicate that for the grid number of 220,000, the average Nusselt number and friction
factor variation and convection heat transfer coefficient have a maximum error of less than 5%. Furthermore, for this number of grids, compared to 295,000 grids, the computation time is reduced and the numerical error is negligible.

Results and discussions

In this numerical study, laminar flow of the water–aluminum oxide nanofluid was investigated in a rectangular three-dimensional microchannel. This research studied the effect of ribs on dynamic behavior of the computational fluids of nanofluid in different modes of (a) and (b). The results embrace the average Nusselt number, average convection heat transfer coefficient, heat–fluid PEC, pumping power, friction factor, and Poiseuille number. These results were considered for two modes (a) and (b) and are presented as a function of nanoparticle volume fraction. Calculated values for each parameter in each mode of (a) and (b) are compared with similar parameters in the smooth microchannel with identical geometric and boundary (heat–fluid) conditions with pure water cooling fluid. The results of this study are calculated for Reynolds numbers of 10 and 100 and volume fractions of $\varphi = 0\%–4\%$ aluminum nanoparticle. Figure 5 shows the friction factor diagram in the smooth microchannel. It is observed that friction factor in the smooth channel decreases by increasing Reynolds number. With regard to the figure, friction factor of the flow with Reynolds number of 10 is about 10 times as large as that with Reynolds number of 100 due to more fluid contact with microchannel surfaces in lower velocities. By increasing nanoparticle volume fraction, friction factor increases due to an increase in shear stress in walls as a result of the existence of solid particles in cooling fluid. The main reason should be that the dynamic viscosity increased very quickly as the nanoparticle volume fraction increased, so the flowability of nanofluid turned worse, which led to an increase in the thickness of boundary layer. As a result, the friction factor increased.

Figure 6 compares friction factor in each mode of (a) and (b) to the smooth microchannel. It is observed that due to the existence of the ribs, friction factor has been significantly increased. Friction factor in mode (a) is larger due to the pressure drop resulting from an
increase in the number of ribs with higher heights compared to mode (b). In both modes, increasing nanoparticle volume fraction increases friction factor as well. In comparison to Reynolds numbers 10 and 100, in the case of (a), the coefficient of friction is increased, which causes a further drop in velocity in this case, and graphs in case (b) to have a higher level.

Figure 7 shows the value of average Nusselt numbers in each mode of (a) and (b) compared to the smooth microchannel. The average Nusselt number increases as Reynolds number and nanoparticle volume fraction increase. The average Nusselt number ratio in microchannel (a) has increased compared to mode (b) in each Reynolds number of 10 and 100, which is due to lower heat gradient in rib-roughened surfaces resulting from the existence of more ribs with higher heights. By putting a rib in the direction of fluid flow, a better mixing of the fluid layers happens, and better heat distribution between layers of the fluid is performed. With regard to Figure 7, the increase in heat transfer in comparison to smooth microchannels in both Reynolds numbers 10 and 100 is considerable.

Figure 8 shows the average convection heat transfer coefficient in each mode of (a) and (b) compared to the smooth microchannel. Like the average Nusselt number, the value of this coefficient increases by an increase in Reynolds number and nanoparticle volume fraction in both modes. The increasing trend of this coefficient is larger in higher Reynolds and with higher nanoparticle volume fractions. Increase in heat transfer coefficient at Reynolds number 100, compared to Reynolds number 10, is considerably more. In case (a), when Reynolds number is 100 and the 4% volume fraction of nanoparticles, it is possible to achieve a heat transfer equivalent to 4.5 times as large as that in smooth microchannel.

Figure 9 shows the Poiseuille number ratio for the modes of (a) and (b) compared to the smooth channel. Due to the existence of ribs, Poiseuille number has increased in the rib-roughened microchannel. Ribs cause more contact surface of fluid and walls and
create an obstruction in the microchannel, which has a large effect on pressure drop and causes an increase in Poiseuille number compared to the smooth channel. Increasing volume fraction of solid nanoparticles increases fluid viscosity, which in turn increases friction factor and Poiseuille number.

In Figure 10, pumping power ratio for each modes of (a) and (b) is compared to the smooth microchannel. It is observed that due to the existence of nanoparticles, the value of density and dynamic viscosity of the cooling fluid increases; and on the other hand, the existence of the ribs in the microchannel causes an obstruction, which increases the pumping power given the above-mentioned points. Increased flow rate in microchannels caused by an increase in the fluid inlet velocity requires a higher pumping power. For both cases (a) and (b), the pumping power diagram at Reynolds number 100, in comparison to the Reynolds number 10, is at a higher level.

In Figure 11, PEC of rib-roughened microchannels (a) and (b) evaluates the relation of heat transfer value to friction effects and compares both channels in relation to the smooth channel in identical conditions. By comparing the diagrams, it can be found that an increase in the number of ribs with higher heights causes higher efficiency of microchannel (a). This criterion increases by an increase in volume fraction for both modes of (a) and (b) which is due to higher effects of heat transfer compared to nanoparticle friction effects. In Reynolds number 10 for both cases, (a) and (b), diagram PEC, with an increase in volume fraction, the trend is the decreasing, which expresses a significant increase in the coefficient of friction against an increase in heat transfer. In Reynolds number 100, the behavior of diagram PEC is contrary to Reynolds number 10, and increase in heat transfer is significant. The diagram with increasing volume fraction is uptrend.

Figures 12 and 13 evaluate local Nusselt number contours in the walls of microchannels (a) and (b) in volume fractions of $\phi = 0\%-4\%$ for Reynolds number 100. Figure 12(a) relates to mode (a) in Reynolds number 100 and pure water fluid. Figure 12(b) relates to mode (a) in Reynolds number 100 and volume fractions of 4% of the solid nanoparticle. Figure 13(a) relates to mode (b) in Reynolds number 100 and pure water fluid. Figure 13(b) relates to mode (b) in Reynolds number 100 and volume fraction of 4% of the solid nanoparticle. It is observed that in mode (a), due to the existence of more ribs, in each nanoparticle volume fraction, the areas with higher local Nusselt number and those with lower heat distribution gradient increase compared to the mode (b). In addition to the number of ribs and higher height of ribs, increase in nanoparticle volume fraction in each Figures 12(b) and 13(b) compared to Figures 12(a) and 13(a) causes better distribution of heat transfer in the microchannel.

**Conclusion**

In this study, heat transfer of laminar flow of water-aluminum oxide nanofluid was investigated in rib-roughened three-dimensional microchannel, and the results were compared with those from the smooth microchannel in identical geometric and boundary
conditions. According to the results obtained in this study, it can be found that using ribs in microchannels causes an increase in heat transfer rate and a decrease in heat gradient in between layers of cooling fluid.

Existence of nanoparticles in the cooling fluid also has a significant effect on increasing heat transfer, which is reinforced with an increase in Reynolds number. However, on the other hand, existence of ribs in microchannel causes a pressure drop due to the obstruction compared to the smooth channel, which in turn causes an increase in friction factor and pumping power. Although using nanofluid causes an increase in heat transfer, existence of nanoparticles in the cooling fluid causes an increase in density and viscosity, which in turn increases shear rate in walls and friction factor and pumping power. Finally, efficiency of using ribs and nanofluid in the microchannel should be evaluated by PEC. Given the diagrams of Nusselt number ratio, a heat transfer of up to 4.5 times of the smooth channel can be obtained using nanofluid and ribs in the microchannel.

Figure 12. Local Nusselt number contours for the mode in Reynolds number 100 with volume fraction of (a) \( \varphi_a = 0 \) and (b) \( \varphi_b = 0.04 \).

Figure 13. Local Nusselt number contours for the mode in Reynolds number 100 with volume fraction of (a) \( \varphi_a = 0 \) and (b) \( \varphi_b = 0.04 \).

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References
1. Hung T and Yan W. Enhancement of thermal performance in double-layered microchannel heat sink with nanofluids. Int J Heat Mass Tran 2012; 55: 3225–3238.
2. Farsad E, Abbasi SP, Zabihi MS, et al. Numerical simulation of heat transfer in a microchannel heat sinks using nanofluids. Heat Mass Transfer 2011; 47: 479–490.
3. Gunnapasaran P, Mohammad HA, Shuaib NH, et al. The effect of geometrical parameters on heat transfer...
characteristics of microchannel heat sink with different shapes. *Int Commun Heat Mass* 2010; 37: 1078–1086.
4. Chen CH and Ding CY. Study on the thermal behavior and cooling performance of a nanofluid-cooled microchannel heat sink. *Int J Therm Sci* 2010; 50: 378–384.
5. Mohammed HA, Gunnasegaran P and Shuaib NH. Influence of nanofluids on parallel flow square microchannel heat exchanger performance. *Int Commun Heat Mass* 2011; 38: 194–201.
6. Mohammed HA, Gunnasegaran P and Shuaib NH. Heat transfer in rectangular microchannels heat sink using nanofluids. *Int Commun Heat Mass* 2010; 37: 1496–1503.
7. Manca O, Nardini S and Ricci D. Numerical investigation of air forced convection in channels with differently shaped transverse ribs. *Int J Numer Method H* 2010; 21: 618–639.
8. Pak BC and Cho YI. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. *Exp Heat Transfer* 1998; 11: 151–170.
9. Maiga SEB, Nguyen CT, Galanis N, et al. Heat transfer behaviours of nano-fluids in a uniformly heated tube. *Superlattice Microstruct* 2004; 35: 543–557.
10. Izadi M, Behzadmehr A and Jalali-Vahida D. Numerical study of developing laminar forced convection of a nanofluid in an annulus. *Int J Therm Sci* 2009; 48: 2119–2129.
11. Zhang CB, Chen YP, Deng Z, et al. Role of rough surface topography on gas slip flow in microchannels. *Phys Rev E* 86: 016319.
12. Chen YP, Zhang CB, Fu PP, et al. Characterization of surface roughness effects on laminar flow in microchannels by using fractal cantor structures. *J Heat Transf* 2012; 134: 051011.
13. Chen YP and Cheng P. Heat transfer and pressure drop in fractal tree-like microchannel nets. *Int J Heat Mass Tran* 2002; 45: 2643–2648.
14. Sheikhzadeh GA, Ebrahim Qomi M, Hajjialilog N, et al. Effect of Al2O3–water nanofluid on heat transfer and pressure drop in a three-dimensional microchannel. *Int J Nano Dimens* 2013; 3: 281–288.
15. Aminossadati SM and Ghasemi B. Natural convection cooling of a localized heat source at the bottom of a nanofluid-filled enclosure. *Eur J Mech B Fluid* 2009; 28: 630–640.
16. Malvandi A and Ganji DD. Magnetohydrodynamic mixed convective flow of Al2O3–water nanofluid inside a vertical microtube. *J Magn Magn Mater* 2014; 369: 132–141.
17. Mahdy A. Unsteady mixed convection boundary layer flow and heat transfer of nanofluids due to stretching sheet. *Nucl Eng Des* 2012; 249: 248–255.
18. Karimipour A, Esfe MH, Safaei MR, et al. Mixed convection of copper-water nanofluid in a shallow inclined lid driven cavity using lattice Boltzmann method. *Phys A* 2014; 402, 150–168.
19. Togun H, Safaei MR, Sadri R, et al. Heat transfer to turbulent and laminar Cu/water flow over a backward-facing step. *Appl Math Comput* 2014; 239, 153–170.
20. Brinkman HC. The viscosity of concentrated suspensions and solution. *J Chem Phys* 1952; 20: 571–581.
21. Karimipour A, Nezhad AH, D’Oraio A, et al. Simulation of copper–water nanofluid in a microchannel in slip flow regime using the lattice Boltzmann method. *Eur J Mech B Fluid* 2015; 49: 89–99.
22. Patel HE, Sundararajan T, Pradeep T, et al. A micro-convection model for thermal conductivity of nanofluids. *Praman* 2005; 65: 863–869.
23. Goodarzi M, Safaei MR, Ahmadi G, et al. An investigation of laminar and turbulent nanofluid mixed convection in a shallow rectangular enclosure using a two-phase mixture model. *Int J Therm Sci* 2014; 75: 204–220.
24. Xia G, Zhai Y and Cui Z. Numerical investigation of thermal enhancement in a micro heat sink with fan-shaped reentrant cavities and internal ribs. *Appl Therm Eng* 2013; 58: 52–60.
25. Zhai YL, Xia GD, Liu XF, et al. Heat transfer in the microchannels with fan-shaped reentrant cavities and different ribs based on field synergy principle and entropy generation analysis. *Int J Heat Mass Tran* 2014; 68: 224–233.
26. Liu H and Wang J. Numerical investigation on synthetical performances of fluid flow and heat transfer of semi-attached rib-channels. *Int J Heat Mass Tran* 2011; 54: 575–583.
27. Xie G, Chen Z, Sunden B, et al. Numerical analysis of flow and thermal performance of liquid-cooling microchannel heat sinks with bifurcation. *Numer Heat Tr A Appl* 2013; 64: 902–919.
28. Hung TC and Yan W-M. Optimization of a microchannel heat sink with varying channel heights and widths. *Numer Heat Tr A Appl* 2012; 62: 722–741.
29. Xie G, Li S, Sunden B, et al. A Numerical study of the thermal performance of microchannel heat sinks with multiple length bifurcation in laminar liquid flow. *Numer Heat Tr A Appl* 2013; 65: 107–126.
30. Ho CJ and Chen WC. An experimental study on thermal performance of Al2O3/water nanofluid in a minichannel heat sink. *Appl Therm Eng* 2013; 50: 516–522.
31. Li P, Zhang D and Xie Y. Heat transfer and flow analysis of Al2O3–water nanofluids in microchannel with dimple and protrusion. *Int J Heat Mass Tran* 2014; 73: 456–467.
32. Lewis FM and Princeton NJ. Friction factors for pipe flow. *Trans ASME* 1944, http://user.engineering.uio wa.edu/~me_160/lecture_notes/MoodyLFPaper1944.pdf
33. Papautsky I, Gale BK, Mohanty S, et al. Effects of rectangular microchannel aspect ratio on laminar friction constant. *Proc SPIE Microfluid Dev Syst II* 1999; 3877: 147–158.
34. Aminossadati SM, Raisi A and Ghasemi B. Effects of magnetic field on nanofluid forced convection in a partially heated microchannel. *Int J Nonlinear Mech* 2011; 46: 1373–1382.

**Appendix 1**

**Notation**

- $A$: area (m$^2$)
- $C_f$: Poiseuille number
- $C_p$: specific heat (J/kg K)
- $D$: solid particle molecule diameter (mm)
- $D_h$: hydraulic diameter (m)
\begin{align*}
f & \text{ friction factor} \\
h & \text{ microchannel height (nm)} \\
h & \text{ convection heat transfer coefficient (W/m}^2\text{ K)} \\
k & \text{ heat conductivity coefficient (W/m K)} \\
L & \text{ length (m)} \\
Nu & \text{ Nusselt number} \\
p & \text{ perimeter (m)} \\
P & \text{ pressure (Pa)} \\
\text{PEC} & \text{ performance evaluation criterion} \\
Pp & \text{ pumping power (W)} \\
Pr & \text{ Prandtl number} \\
q'' & \text{ heat flux (W/m}^2\text{)} \\
Re & \text{ Reynolds number} \\
T & \text{ temperature (K)} \\
u & \text{ velocity component along } x \\
v & \text{ velocity component along } y \\
w & \text{ velocity component along } z \\
\alpha & \text{ heat permeability (m}^2\text{/s)} \\
\beta & \text{ volumetric expansion coefficient (k}^{-1}\text{)} \\
\mu & \text{ viscosity (Pa s)} \\
\rho & \text{ density (kg/m}^3\text{)} \\
\phi & \text{ volume fraction} \\
\text{Super- and subscripts} \\
\text{ave} & \text{ average} \\
f & \text{ fluid} \\
f & \text{ nanofluid} \\
p & \text{ solid articles} \\
S & \text{ smooth}
\end{align*}