Proto-Neutron Star Winds, Magnetar Birth, and Gamma-Ray Bursts

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Abstract. We begin by reviewing the theory of thermal, neutrino-driven proto-neutron star (PNS) winds. Including the effects of magnetic fields and rotation, we then derive the mass and energy loss from magnetically-driven PNS winds for both relativistic and non-relativistic outflows, including important multi-dimensional considerations. With these simple analytic scalings we argue that proto-magnetars born with \( \sim \) millisecond rotation periods produce relativistic winds just a few seconds after core collapse with luminosities, timescales, mass-loading, and internal shock efficiencies favorable for producing long-duration gamma-ray bursts.

Keywords: neutron stars, stellar winds, supernovae, gamma ray bursts, magnetic fields

PACS: 97.60.Bw, 97.60.Gb; 97.10.Me

1. NEUTRINO-DRIVEN PNS WINDS

After a successful core-collapse supernova (SN), a hot proto-neutron star (PNS) cools and deleptonizes, releasing the majority of its gravitational binding energy (\( \sim 3 \times 10^{53} \) ergs) in neutrinos. With initial core temperature \( T > 10 \) MeV, a PNS is born optically-thick to neutrinos of all flavors because the relevant neutrino-matter cross sections scale as \( \sigma_{\nu m} \propto \epsilon_{\nu}^2 \propto T^2 \), where \( \epsilon_{\nu} \) is a typical neutrino energy. Indeed, because neutrinos are trapped, a PNS’s neutrino luminosity \( L_{\nu} \) remains substantial and quasi-thermal for a time after bounce \( \tau_{KH} \sim 10 - 100 \) s, as roughly verified by the 19 neutrinos detected from SN1987A 20 years ago [1],[2]. Although this Kelvin-Helmholtz (KH) cooling epoch is short compared to the time required for the shock, once successful and moving outward at \( \sim 10^4 \) km/s, to traverse the progenitor stellar mantle, \( \tau_{KH} \) is still significantly longer than the time over which the initial explosion must be successful. While the specific shock launching mechanism is presently unknown, it must occur in a time \( t < 1 \) s \( \ll \tau_{KH} \) after bounce for the PNS to avoid accreting too much matter.

Thus, even after the SN shock has cleared a cavity of relatively low density material around the PNS, \( L_{\nu} \) remains substantial. Detailed PNS cooling calculations [3] show that the electron neutrino(antineutrino) luminosity \( L_{\nu_e(\bar{\nu}_e)} \) is \( \sim 10^{52} \) erg/s at \( t \sim 1 \) s and declines as \( \propto t^{-1} \) until \( t \approx \tau_{KH} \), after which \( L_{\nu_e(\bar{\nu}_e)} \) decreases exponentially as the PNS becomes optically thin. This persistent neutrino flux \( F_{\nu_e(\bar{\nu}_e)} \) continues to heat the PNS atmosphere, primarily through electron neutrino(antineutrino)
absorption on nuclei ($\nu_e + n \rightarrow p + e^-$ and $\bar{\nu}_e + p \rightarrow n + e^+$). Because the inverse, pair capture rates dominate the cooling, which declines rapidly with temperature ($\dot{q}^+ \propto T^6$) and hence with spherical radius $r$, a region of significant net positive heating ($\dot{q} \equiv \dot{q}^+ - \dot{q}^- > 0$) develops above the neutrinosphere radius $R_{\nu}$. This heating drives mass-loss from the PNS in the form of a thermally-driven wind [4]. To estimate the dependence of the resultant mass-loss rate ($\dot{M}_{th}$) on the PNS properties explicitly, consider that in steady state the change in gravitational potential required for a unit mass element to escape the PNS ($GM/R_{\nu}$) must be provided by the total heating it receives accelerating outwards from the PNS surface:

$$\frac{GM}{R_{\nu}} \approx \int_{R_{\nu}}^{\infty} \frac{\dot{q}^+ dr}{v_r},$$

where $M$ is the PNS mass, $v_r$ is the outward wind velocity, and $\dot{q}$ is per unit mass. Because $\dot{q}$ is quickly dominated by heating from neutrino absorption, which scales as $\dot{q}^+ \propto F_{\nu_e} \sigma_{\nu e} \propto L_{\nu e} \epsilon_{e}^2 / 4 \pi r^2$, we see that equation (1) implies that

$$\frac{GM}{R_{\nu}} \propto \frac{L_{\nu e} \epsilon_{e}^2}{M_{th}} \int_{R_{\nu}}^{\infty} \rho dr \approx \frac{L_{\nu e} \epsilon_{e}^2}{M_{th}} \rho_{\nu} H_{\nu},$$

where we have used $\dot{M}_{th} = 4 \pi \rho r^2 v_r$ for a spherical wind, $\rho$ is the mass density, $H$ is the PNS’s density scale height, $\epsilon_{e}$ crudely defines a mean electron neutrino or antineutrino energy, and a subscript “$\nu$” denotes evaluation near $R_{\nu}$. Neglecting rotational support and assuming that the thermal pressure $P$ is dominated by photons and relativistic pairs (which also becomes an excellent approximation as the density plummets abruptly above the PNS surface), we have that $H_{\nu} \sim P_{\nu}/\rho_{\nu} g_{\nu} \propto T_{\nu}^6 R_{\nu}^2 / M \rho_{\nu}$, where $g_{\nu} \propto M/R_{\nu}^2$ is the PNS surface gravity and $T_{\nu} \propto (L_{\nu e} \epsilon_{e}^2 / R_{\nu}^2)^{1/6}$ is the PNS surface temperature. $T_{\nu}$ is set by the balance between heating and cooling at the PNS surface ($T_{\nu}^6 \dot{q}^- = \dot{q}^+ \propto L_{\nu e} \epsilon_{e}^2 / R_{\nu}^2$). Inserting these results into equation (2) and including the correct normalization from the relevant weak cross sections, one finds the expression for $\dot{M}_{th}$ first obtained by ref [4]:

$$\dot{M}_{th} \approx 10^{-4} L_{52}^{5/3} \epsilon_{10}^{10/3} M_{1.4}^{-2} R_{10}^{5/3} M_{\odot} / s,$$

where $L_{52} \equiv L_{\nu e} \times 10^{52}$ erg/s, $\epsilon_{10} \equiv 10 \epsilon_{e}$ MeV, $R_{10} \equiv 10 R_{10} km$, and $M \equiv 1.4 M_{1.4} M_{\odot}$.

Endowed with an enormous gravitational binding energy and a means, through this neutrino-driven outflow, for communicating a fraction of this energy to the outgoing shock, a newly-born PNS seems capable of affecting the properties of the SN that we observe. However, a purely thermal, neutrino-driven PNS wind is only accelerated to an asymptotic speed of order the surface sound speed: $v_{th}^{\infty} \sim c_{s,\nu} \approx \sqrt{2k T_{\nu}/m_p} \approx 0.1 L_{52}^{1/12} \epsilon_{10}^{1/6} R_{10}^{1/6} c$. Thus, the efficiency $\eta$ relating wind power $\dot{E}_{th} \approx \dot{M}_{th} (v_{th}^{\infty})^2 / 2$ to total neutrino luminosity ($L_{\nu} \sim 6 L_{\nu e}$) is quite low:

$$\eta \equiv \frac{\dot{E}_{th}}{L_{\nu}} \sim 10^{-5} L_{52}^{5/6} \epsilon_{10}^{11/3} R_{10}^{4/3} M_{1.4}^{-2}.$$
In particular, although neutrino energy deposited in a similar manner may be responsible for initiating the SN explosion itself at early times (i.e., the neutrino SN mechanism [5]), $\eta$ drops rapidly as the PNS cools. Quasi-spherical winds of this type are therefore not expected to affect the SN’s nucleosynthesis or morphology (although the wind itself is considered a promising r-process source [4]).

2. MAGNETICALLY-DRIVEN PNS WINDS

Some PNSs may possess a more readily extractable form of energy in rotation. A PNS born with a period $P = P_{\text{ms}}$ ms is endowed with a rotational energy $E_{\text{rot}} \simeq 2 \times 10^{52} P_{\text{ms}}^{-2} R_{10}^2 M_{1.4}$ ergs, which, for $P < 4$ ms, exceeds the energy of a typical SN shock ($\sim 10^{51}$ ergs). Given a mass loss rate $\dot{M}$ and torquing lever arm $\omega \tau$, a wind extracts angular momentum $J$ from the PNS at a rate $\dot{J} \simeq \Omega \omega \tau \dot{M}$, where $\Omega = 2\pi/P$ is the PNS rotation rate. With the PNS’s radius $R_{\nu}$ as a lever arm and the modest thermally-driven mass-loss rate given by equation (3), the timescale for removal of the PNS’s rotational energy, $\tau_{J} \equiv J/\dot{J} \sim M R_{\nu}^2/\dot{M} \omega \tau \sim M/\dot{M}_{\text{th}}$, is much longer than $\tau_{\text{KH}}$. However, if the PNS is rapidly rotating and possesses a dynamically-important poloidal magnetic field $B_p$ (through either flux-freezing or generated via dynamo action [6]), then both $\dot{M}$ and $\omega \tau$ can be substantially increased; this reduces $\tau_{J}$, allowing efficient extraction of $E_{\text{rot}}$.

For magnetized winds $\omega \tau$ is the Alfvén radius $\omega A$, defined as the cylindrical radius where $\rho v_r^2/2$ first exceeds $B^2/8\pi$ [7]. The magnetosphere of a PNS is most likely dominated by its dipole component, with a total (positive-definite) surface magnetic flux given by $\Phi_B = 2\pi B_{\nu} R_{\nu}^2$, where $B_{\nu}$ is the polar surface field. To estimate $\omega A$ for magnetized PNS outflows recognize that mass and angular momentum are primarily extracted from a PNS along open magnetic flux. For an axisymmetric dipole rotator this represents only a fraction $\approx 2(\pi \theta_{\text{LCFL}}^2)/4\pi \simeq R_{\nu}/2\omega Y$ of $\Phi_B$, where $\theta_{\text{LCFL}} \approx \sqrt{R_{\nu}/\omega Y}$ is the latitude (measured from the pole) at the PNS surface of the last closed field line (LCFL), $\omega Y$ is the radius where the LCFL intersects the equator (the “Y point”), and we have assumed that $\omega Y \gg R_{\nu}$ ($\theta_{\text{LCFL}} \ll 1$). Plasma necessarily threads a PNS’s closed magnetosphere and cannot be forced to corotate superluminally; thus $\omega Y$ cannot exceed the light cylinder radius $\omega L \equiv c/\Omega = 48 P_{\text{ms}}$ km, making it useful to write the PNS magnetosphere’s total open magnetic flux as $\Phi_{B,\text{open}} \approx \pi B_{\nu} R_{\nu}^2 (R_{\nu}/\omega Y)(\omega Y/\omega L)^{-1}$. Now, the overall latitudinal structure of a PNS magnetosphere (i.e., the allocation of open and closed magnetic flux, and the value of $\omega Y/\omega L$) is primarily dominated by the dipolar closed zone. However, recent numerical simulations [8] show that where the field is open it behaves as a “split monopole”. In this case the poloidal field scales as $B_p \sim \Phi_{B,\text{open}}/r^2 \approx 0.2 B_{\nu} P_{\text{ms}}^{-1} R_{10} (\omega Y/\omega L)^{-1} (R_{\nu}/r)^2$, rather than the dipole scaling $\propto (R_{\nu}/r)^3$. The constant of proportionality is chosen to assure that $B_p(R_{\nu}) \rightarrow B_{\nu}$ in the limit of vanishing closed zone ($\omega L, \omega Y \rightarrow R_{\nu}$) and is in agreement with numerical results (see eq. [28] of ref [8]).
2.1. Non-Relativistic Winds and Asymmetric Supernovae

Non-relativistic (NR) magnetically-driven winds reach an equipartition between kinetic and magnetic energy outside $\omega_A$ such that the kinetic energy flux at $\omega_A$ ($\dot{M}v_r(\omega_A)^2/2$) carries a sizeable fraction of the rotational energy loss extracted by the wind's surface torque $\dot{E}_{\text{rot}} = \dot{J} = \dot{M}\Omega^2\omega_A^2$: thus, we have that $v_r(\omega_A) \sim \Omega/\omega_A$. Combining this with the modified monopole scaling for $B_\nu$ motivated above and mass conservation $\dot{M}_{\Omega} \equiv \rho v_r^2$ ($\dot{M}_{\Omega}$ is the mass flux per solid angle) we find that:

$$\omega_A/R_\nu \simeq B_{15}^{4/3} P_{\text{ms}}^{-2/3} \dot{M}_{\Omega,-4} L_{10}^{4/3} (\omega_Y/\omega_L)^{-1},$$

where $\dot{M}_{\Omega} \equiv \dot{M}_{\Omega,-4} \times 10^{-4} M_{\odot}/s^{-1}$, $B_\nu \equiv B_{15} \times 10^{15}$ G, and we have concentrated on the open magnetic flux that emerges nearest the closed zone (polar latitude $\phi \approx \Omega\sin\nu$ on the open magnetic flux that emerges nearest the closed zone (polar latitude $\phi \approx \Omega\sin\nu$).

From equation (5) we see that winds from rapidly rotating PNSs with surface magnetic fields typical of Galactic "magnetars" ($B_\nu \sim 10^{14} \text{ - } 10^{15}$ G) possess enhanced lever arms for extracting rotational energy [9]. Furthermore, their total outflow power $\dot{E}_{\text{mag}}^{\text{NR}} \approx \dot{E}_{\text{rot}} \simeq 2\pi\theta_{\text{LCFL}}^2 \dot{M}_{\Omega} \omega_A^2 \approx 10^{49} B_{15}^{4/3} P_{\text{ms}}^{-1/3} \dot{M}_{\Omega,-4} L_{10}^{17/3} (\omega_Y/\omega_L)^{-3}$ ergs/s dominates thermal acceleration ($\dot{E}_{\text{mag}}^{\text{NR}} > \dot{E}_{\text{th}}$) for $B_{15} > 0.4 P_{\text{ms}}^{13/4} L_{52}^{23/24} \epsilon_{10}^{23/12} R_{10}^{-11/3} M_{1.41}^{1/3} (\omega_Y/\omega_L)^{9/4}$. This condition becomes easier to satisfy as the PNS cools, allowing magnetized winds to dominate later stages of the KH epoch for PNSs with even relatively modest $B_\nu$ and $\Omega$. NR magnetically-driven winds, in addition to being more powerful than spherical, thermally-driven outflows, are efficiently hoop-stress collimated along the PNS rotation axis [8]. The power they deposit along the poles may produce asymmetry in SN ejecta distinct from the shock-launching process itself.

Strong magnetic fields and rapid rotation can also increase the outflow's power through enhanced mass-loss because $\dot{E}_{\text{mag}}^{\text{NR}} \propto \dot{M}_{\Omega} L_{10}^4$. When the PNS's hydrostatic atmosphere is forced to co-rotate to the outflow's sonic radius $\omega_s = (GM\sin(\theta_{\text{LCFL}})/\Omega^2)^{1/3}$ then $\dot{M}_{\Omega}$ is enhanced by a factor $\phi_{\text{cf}} \sim \exp[(v_{\phi,\nu}/c_{s,\nu})^2]$ over $\dot{M}_{\text{th}}/4\pi$ due to centrifugal ("cf") slinging [9], where $v_{\phi,\nu} \approx R_\nu \omega_s \sin[\theta_{\text{LCFL}}]$ and $R_\nu \Omega / \sqrt{\omega_Y}$ is the PNS rotation speed at the base of the open flux. Using our estimate for $c_{s,\nu}$ from §1, we see that enhanced mass loss becomes important for $P_{\text{ms}} < P_{\text{cf,ms}} \equiv L_{52}^{-1/18} \epsilon_{10}^{-1/9} P_{10}^{10/9} (\omega_Y/\omega_L)^{-1/3}$ (i.e., only for PNSs with considerable rotational energy $E_{\text{rot}} > 10^{52}$ ergs).

Fully enhanced mass loss ($\dot{M}_{\Omega} = \dot{M}_{\text{th}} \phi_{\text{cf}} / 4\pi$) requires $\omega_A > \omega_s$, which in turn requires that $B_{15} > B_{\text{cf,15}} \equiv P_{\text{ms}}^{7/4} R_{10}^{-13/4} \dot{M}_{\Omega,-4}^{1/2} (\omega_Y/\omega_L)^{5/4} \approx 0.3 P_{\text{ms}}^{7/4} L_{52}^{5/6} \epsilon_{10}^{5/6} M_{1.4}^{-1/2} R_{10}^{29/12} \exp[0.5 (P/P_{\text{cf}})^{-3} (\omega_Y/\omega_L)^{5/4}$, where we have taken $\dot{M}_{\text{th}}$ from §1. For cases with $B_{15} < B_{\text{cf}}$ but $P < P_{\text{cf}}$, $\dot{M}_{\Omega}$ lies somewhere between $\dot{M}_{\text{th}}/4\pi$ and $\phi_{\text{cf}} \dot{M}_{\text{th}}/4\pi$ (see [10] for numerical results). Millisecond proto-magnetars generally attain $\phi_{\text{cf}}$, except perhaps at early times when the PNS is quite hot.
2.2. Relativistic Winds and Gamma-Ray Bursts

As the PNS cools, eventually $\omega_A \rightarrow \omega_L$ and the PNS outflow becomes relativistic (REL). This transition occurs after $\tau_{KH}$ for most PNSs (they become pulsars), but rapidly rotating proto-magnetar winds become relativistic during the KH epoch itself. Similar to normal pulsars, PNSs of this type lose energy at the force-free, “vacuum dipole” rate: $\dot{E}_{\text{REL}} \approx 6 \times 10^{49} B_{15}^2 P_{\text{ms}}^{-4} R_{10}^{-4} (\omega_Y/\omega_L)^{-2} \text{ergs/s}$ (again modulo corrections for excess open magnetic flux $\dot{E}_{\text{REL}} \propto \Phi_{\text{open}}^2 \propto (\omega_Y/\omega_L)^{-2}$ [8]), which gives a familiar spin-down timescale $\tau_J = \dot{E}_{\text{rot}} / \dot{E}_{\text{REL}} \approx 300 B_{15}^{-2} 15^{P_{\text{ms}}^{-4} R_{10}^{-4} (\omega_Y/\omega_L)^{-2}}$ s. On the other hand, the mass loading on a PNS’s open magnetic flux is set by neutrino heating, a process totally different from the way that matter is extracted from a normal pulsar’s surface. In fact, a proto-magnetar outflow’s energy-to-mass ratio $\sigma$ is given by

$$\sigma \approx \frac{\dot{E}_{\text{mag}}^{\text{REL}}}{2\pi \theta_{\text{LCFL}}^2 M_{\text{rhe}}} \approx 3 B_{15}^2 P_{\text{ms}}^{-3} L_{52}^{-5/3} \epsilon_{10}^{-10/3} P_{10}^{10/3} M_{1.4}^2 \exp \left[- \left( \frac{P}{P_{\text{cf}}} \right)^{-3} \right] \left( \frac{\omega_Y}{\omega_L} \right)^{-1} \quad (6)$$

From equation (6) we see that because a PNS’s mass-loss rate drops so precipitously as it cools, $\sigma \propto L_{\nu_{e}}^{-5/3} \epsilon_{\nu_{e}}^{-10/3}$ rises rapidly with time, easily reaching $\sim 10 - 1000$ during the KH epoch for typical magnetar parameters [9],[10]. Detailed evolution calculations indicate that $E_{\text{rot}}$ is extracted roughly uniformly in $\log(\sigma)$ [10].

To conclude with a concrete example, consider a proto-magnetar with $B_{\nu} = 10^{16}$ G and $P_{\text{ms}} = 3$ at $t = 10$ seconds after core collapse. From the cooling calculations of ref [3] we have $L_{52}(10 \text{ s}) \approx 0.1$ and $\epsilon_{10}(10 \text{ s}) \approx 1$ (see Figs. [14] and [18]) and so, under the conservative estimate that $\omega_Y = \omega_L$, equation (6) gives $\sigma \approx 500$. Because $\sigma$ represents the potential Lorentz factor of the outflow (assuming efficient conversion of magnetic to kinetic energy), we observe that millisecond proto-magnetar birth provides the right mass-loading to explain gamma-ray bursts (GRBs). Further, the power at $t = 10$ s is still $\dot{E}_{\text{mag}}^{\text{REL}} \approx 10^{50}$ erg/s with a spin-down time $\tau_3 \approx 30$ s, both reasonable values to explain typical luminosities and durations, respectively, of long-duration GRBs. Lastly, because $\sigma$ rises so rapidly with time as the PNS cools, in the context of GRB internal shock models a cooling proto-magnetar outflow’s kinetic-to-$\gamma$-ray efficiency can be quite high; our calculations indicate that values of $10 - 50\%$ are plausible. We conclude that magnetar birth accompanied by rapid rotation (but requiring less angular momentum than collapsar models) represents a viable long-duration GRB central engine.

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