Cosmological bounds on open FLRW solutions of massive gravity

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Abstract

In this work we have analysed some cosmological bounds concerning an open FLRW solution of massive gravity. The constraints with recent observational $H(z)$ data were found and the best fit values for the cosmological parameters are in agreement with the ΛCDM model, and also point to a nearly open spatial curvature, as expected from the model. The graviton mass dependence with the constant parameters $\alpha_3$ and $\alpha_4$, related to the additional lagrangian terms of the model, are also analysed, and we have obtained a strong dependence with such parameters, although the condition $m_g \simeq H_0^{-1}$ seems dominant for a long range of the parameters $\alpha_3$ and $\alpha_4$. The limit $\alpha_2 \to 0$ forbid one of the branches of accelerated solution, which indicates the necessity of the corresponding lagrangian term.
I. INTRODUCTION

Current observations of supernovae type Ia [1, 2], cosmic microwave background radiation [3, 4] and Hubble parameter data [5, 6] indicate an accelerated expansion of the universe, being the ΛCDM model the best model to fit the observational data. The Λ term corresponds to a cosmological constant energy density which is plagued with several fundamental issues [7], which has motivated the search for alternatives models of gravity that could explain the observations.

Massive gravity theories [8–17] are good and old candidates to explain the accelerated expansion of the universe, since that the graviton mass could perfectly induce and mimic a cosmological constant term. However, such kinds of theories were considered for long time as being unsuitable due to the appearing of Boulware-Deser (BD) ghosts [9]. Recently it was discovered a nonlinear massive gravity theory that was shown to be BD ghost free [10], called dRGT (de Rham-Gabadadze-Tolley) model (see [11] for a review), bringing back the cosmological interest for such theories [12]. Self-accelerating cosmologies with ghost-free massive gravitons have been studied thereafter [13–16], although there is no full concordance about the stability of the solutions concerning the flat, open and closed Friedmann-Lemaitre-Robertson-Walker (FLRW) isotropic and homogeneous solutions of the metric. While some authors defend the existence of isotropic FLRW solutions [13, 14], others indicate the appearance of some kinds of nonlinear ghosts instabilities, thus just anisotropic FLRW solutions are ghost-free [15, 16]. Finally, in [17], there is a good explanation on these questions, showing that zero-curvature solutions are in one to one correspondence with exact open FLRW solutions.

We assume the existence of stable solutions in open isotropic FLRW cosmologies and study some bounds on the free parameters present in the massive gravity theory compared to recent observational data. In Section II we present the general massive gravity theory and the cosmological equations in Section III. In Section IV we present the constraints from $H(z)$ data and in Section V some bounds on the graviton mass are presented. We conclude in Section VI.

II. MASSIVE GRAVITY THEORY

Our starting point for the cosmological analysis is the massive theory for gravity proposed in [10]. The nonlinear action besides a functional of the physical metric $g_{\mu\nu}(x)$ include four spurious scalar fields $\phi^a(x)$ with $a = 0, 1, 2, 3$ called the St"uckelberg fields. They are introduced in order to make the action manifestly invariant under diffeomorphism, see for example [18]. Let us start by observing that these scalar fields are related to the physical metric and enter into the action as follows:
where it is defined the fiducial metric $f_{\mu\nu}$ which is written in terms of the Stückelberg fields:

$$f_{\mu\nu} \equiv \tilde{f}_{ab}(\phi^c) \partial_{\mu} \phi^a \partial_{\nu} \phi^b.$$  

Usually $\tilde{f}_{ab}$ is called the reference metric and for the purpose of this paper, one can use $\tilde{f}_{ab} = \eta_{ab} = (-, +, +, +)$ once the scheme proposed by dRGT respects Poincaré symmetry. Hence the fiducial metric in (1) is nothing but the Minkowski metric in the coordinate system defined by the Stückelberg fields. So we have automatically defined the covariant tensor $H_{\mu\nu}$ which propagates on Minkowski space and the action is then a functional of the fiducial metric and the physical metric $g_{\mu\nu}$.

The covariant action for massive general relativity that we are going to work with, can be written as:

$$S = M_{Pl}^2 \int d^4x \, \sqrt{-g} \left[ \frac{R}{2} + m^2 \mathcal{U}(g, H) \right]$$  

where $\mathcal{U}$ is a potential without derivatives in the interaction terms between $H_{\mu\nu}$ and $g_{\mu\nu}$ that gives mass to the spin-2 mode described by the Einstein-Hilbert term. As observed in [10] a necessary condition for the theory (3) to be free of the Bouware-Deser ghost in the decoupling limit is that $\sqrt{-g} \, \mathcal{U}(g, H)$ be a total derivative. The most general covariant mass term which respects this condition is composed by\(^1\):

$$\int d^4x \, \sqrt{-g} \, \mathcal{U}(g, H) = \int d^4x \sqrt{-g} \, (\alpha_2 \mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$  

where $\alpha_i$ are constants and the three lagrangians in (4) are written as:

\begin{align*}
\mathcal{L}_2 &= \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2]) \\
\mathcal{L}_3 &= \frac{1}{6} ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]) \\
\mathcal{L}_4 &= \frac{1}{24} ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]),
\end{align*}

where one has defined the tensor $\mathcal{K}_{\mu}^{\nu} = \delta_{\mu}^{\nu} - \sqrt{\partial^\mu \phi^a \partial_\mu \phi^b} \eta_{ab}$. In general there are other polynomial terms in $\mathcal{K}$ and the procedure to generate it can be found in [10], however it has been shown that all terms after the quartic order vanishes (see an explanation for this in [19]). Notice also that, contrary to most papers in this subject, we have maintained the constant term $\alpha_2$ in order to better study its consequences on the evolution equations.

\(^1\) In [17] a most general formulation is presented, where the parameters $\alpha_i$ are assumed to be dependent on the Stückelberg fields.
III. COSMOLOGY OF MASSIVE GRAVITY

Let us begin by considering an open \((K < 0)\), homogeneous and isotropic FRW universe for the physical metric:

\[
g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2dt^2 + a(t)^2 \left[ dx_i dx^i + \frac{K(x_i dx^i)^2}{1 - Kx_i x^i} \right],
\]

(8)

where, \(\mu, \nu = 0, 1, 2, 3\) and \(i, j = 1, 2, 3\), with \(x^0 = t, x^1 = x, x^2 = y, x^3 = z\). Adopting the same ansatz for the St"uckelberg fields used in \([16]\), i.e:

\[
\phi^0 = f(t) \sqrt{1 - Kx_i x^i}; \quad \phi^i = \sqrt{-K} f(t) x^i,
\]

(9)

after plugging back the metric and \((11)\) in \((13)\), one obtains the following Lagrangian \([16]\) for \(a(T)\) and \(f(T)\), where overdot will denote the time derivative:

\[
\mathcal{L}_g = \frac{1}{8\pi G} \left[ 3KN(t)a(t) - \frac{3\dot{a}(t)^2a(t)}{N(t)} + m_g^2(\alpha_2L_2 + \alpha_3L_3 + \alpha_4L_4) \right],
\]

(10)

where

\[
L_2 = (3a(t)^2 - 3a(t)\sqrt{-K} f(t))(2N(t)a(t) - \dot{f}(t)a(t) - N(t)\sqrt{-K} f(t)),
\]

\[
L_3 = (a(t) - \sqrt{-K} f(t))^2(4N(t)a(t) - 3\dot{f}(t)a(t) - N\sqrt{-K} f(t)),
\]

\[
L_4 = (a(t) - \sqrt{-K} f(t))^3(N(t) - \dot{f}(t)).
\]

Taking the Euler-Lagrange equation of \(\mathcal{L}_g\) with respect to \(f\) leads to

\[
(\dot{a}(t) - \sqrt{-K} N(t))[\alpha_2(3 - 2C) + \alpha_3(3 - 4C + C^2) + \alpha_4(1 - 2C + C^2)] = 0,
\]

(12)

where \(C = f(t)\sqrt{-K}/a(t)\). The two interesting solutions are\(^2\):

\[
C_\pm = \frac{(\alpha_2 + 2\alpha_3 + \alpha_4 \pm \sqrt{\Delta})}{(\alpha_3 + \alpha_4)},
\]

(13)

where \(\Delta = \alpha_3(\alpha_2 + \alpha_3 - \alpha_4) + \alpha_3^2\). We take Euler-Lagrange equation of \((11)\) with respect to \(N\) and use \((13)\) to obtain the Friedmann equation:

\[
\frac{\dot{a}(t)^2}{N(t)^2a(t)^2} + \frac{K}{a(t)^2} = \frac{8\pi G}{3} \rho_m + \frac{\Lambda_\pm}{3},
\]

(14)

\(^2\) The case \(\dot{a}(t) = \sqrt{-K} N(t)\) just reproduces a constant scale factor when we take \(N \to 1\) in order to recover the FLRW metric.
where $\Lambda_\pm = m_g^2 \beta_\pm$ and

$$\beta_\pm = -\frac{1}{(\alpha_3 + \alpha_4)^2} \left[ 2\alpha_3^2 + 3\alpha_2^2(\alpha_3 - \alpha_4 \pm \sqrt{\Delta}) + 3\alpha_2(\alpha_3 - \alpha_4)(\alpha_3 \pm \sqrt{\Delta}) - (\alpha_3 \pm \sqrt{\Delta})^2(-2\alpha_3 \pm \sqrt{\Delta}) \right]$$

(15)

is a dimensionless parameter depending only on the constants $\alpha_2$, $\alpha_3$ and $\alpha_4$. In this equation one can recognize $\Lambda_\pm$ as being the energy density of massive gravity ($\rho_g$). Equally, it is expected to find the pressure term ($p_g$) in the second Friedmann equation.

Such equation can be obtained by combining Eq. (14) with the variation of Eq. (10) with respect to $a(t)$. Namely, we subtract Eq. (14) from the Euler-Lagrange equation of $a(t)$ in order to get

$$-\frac{2\dot{H}(t)}{N(t)} + \frac{2K}{a(t)^2} = \frac{8\pi G}{3} (\rho_m + p_m)$$

(16)

where $H = \frac{\dot{a}(t)}{N(t)a(t)}$. Notice that the r.h.s of the above equation contains only the contribution from the matter part, which indicates that the graviton mass contribution satisfies an equation of state of the form $p_g = -\rho_g$, exactly as a vacuum behaviour. This same result was observed in [16].

Another combination is possible in order to get a direct relation among $\ddot{a}(t)$, pressure and energy density of matter and graviton. At this time we eliminate $\frac{\ddot{a}(t)^2}{N(t)a(t)^2}$ of the expression from variation of Eq. (10) with respect to $a(t)$. Thus, it is possible to get

$$\frac{\ddot{a}(t)}{N(t)a(t)} = \frac{\Lambda_\pm}{3} - \frac{4\pi G}{3} (\rho_m + 3p_m).$$

(17)

With such equation it is much easier to analyse the universe acceleration. We conclude that an accelerated expansion occurs when $\Lambda_\pm > 4\pi G(\rho_m + 3p_m)$.

It is also easy to see that $\Lambda_\pm$ acts exactly like an effective cosmological constant in Eq. (14). In both Friedmann equations, (14), (16) and also (17), there should be set $N = 1$ in order to reproduce a cosmological scenario. In order to reproduce a positive cosmological constant (which leads to an accelerating universe), we must have $\Lambda_\pm > 0$, which implies $\beta_\pm > 0$. We have kept the $\alpha_2$ parameter in order to analyse its consequences on the evolution equations.

Taking the limit $\alpha_2 \to 0$ into (15) leads to an interesting relation between the parameters $\alpha_3$ and $\alpha_4$ in order to have a positive effective cosmological constant, which is a minimal condition to have an accelerated expansion. We must have $\beta_\pm > 0$, and the only possibility is $\alpha_3 < 0$, so that $\beta_- > 0$. It is interesting to notice that the case $\beta_+ > 0$ is forbidden. The conditions are represented in the following table:

| $\beta_+$ | $\beta_-$ | $\beta_+ > 0$ | $\beta_- > 0$ |
|-----------|-----------|---------------|---------------|
| $\alpha_4 > 0$ | $\alpha_3 < 0$ | $\alpha_3 < 0$ | $\alpha_3 < 0$ |
| $\alpha_4 < 0$ | $\alpha_3 > 0$ | $\alpha_3 > 0$ | $\alpha_3 > 0$ |

5
Having analysed the effect of the $\alpha_2$ parameter in the evolution equation, from now on we will assume $\alpha_2 = 1$ according to the original dRGT theory [10]. The Friedmann equation (14) can be rewritten in terms of the present critical energy density $\rho_c = 3H_0^2/8\pi G$,

$$H(t)^2 = H_0^2\left(\frac{\rho_m}{\rho_c}\right) + m^2g\beta_- - \frac{K}{a^2},$$

where $H(t) = \dot{a}/a$ is the Hubble parameter and $H_0 \simeq 70$ km/s/Mpc is its present day value. Writing $\rho_m = \rho_{m0}(a/a_0)^{-3}$, where $\rho_{m0}$ is the present day value for the matter energy density, and introducing the density parameters

$$\Omega_m \equiv \frac{\rho_{m0}}{\rho_c}, \quad \Omega_g \equiv \beta_\pm \frac{m^2g}{H_0^2}, \quad \Omega_K \equiv -\frac{K}{a_0^2H_0^2},$$

the Friedmann equation (18) can be expressed as

$$H(t)^2 = H_0^2\left[\Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_K\left(\frac{a_0}{a}\right)^2 + \Omega_g\right],$$

or in terms of the redshift parameter, defined by $1 + z \equiv a_0/a$,

$$H(z)^2 = H_0^2\left[\Omega_m(1+z)^3 + (1 - \Omega_m - \Omega_g)(1+z)^2 + \Omega_g\right],$$

where we have used the Friedmann constraint

$$1 = \Omega_m + \Omega_K + \Omega_g,$$

that follows from (18) and (19). Observational data can be used to constrain the values of such parameters and this will be done in the next section.

IV. CONSTRAINTS FROM OBSERVATIONAL $H(z)$ DATA

Observational $H(z)$ data provide one of the most straightforward and model independent tests of cosmological models, as $H(z)$ data estimation relies on astrophysical rather than cosmological assumptions. In this work, we use the data compilation of $H(z)$ from Sharov and Vorontsova [6], which is, currently, the most complete compilation, with 34 measurements.

From these data, we perform a $\chi^2$-statistics, generating the $\chi^2_H$ function of free parameters:

$$\chi^2_H = \sum_{i=1}^{34} \left[\frac{H_0 E(z_i, \Omega_m, \Omega_g) - H_i}{\sigma_{H_i}}\right]^2,$$

where $E(z) \equiv \frac{H(z)}{H_0}$ and $H(z)$ is obtained by Eq. (21).

As the function to be fitted, $H(z) = H_0 E(z)$, is linear on the Hubble constant, $H_0$, we may analytically project over $H_0$, yielding $\tilde{\chi}^2_H$:

$$\tilde{\chi}^2_H = C - \frac{B^2}{A},$$

where
FIG. 1: **Solid lines:** Statistical confidence contours of massive gravity from $H(z)$ data. The regions correspond to 68.3%, 95.4% and 99.7% c.l. **Dashed line:** Flatness limit, where $\Omega_m + \Omega_g = 1$. Points above this line are not considered on the statistical analysis. **Star point:** Best fit, corresponding to $(\Omega_m, \Omega_g) = (0.242, 0.703)$, which leads to $\Omega_K = 0.055$. More details on the text.

where $A \equiv \sum_{i=1}^{n} \frac{E_i^2}{\sigma_{H_i}^2}$, $B \equiv \sum_{i=1}^{n} \frac{E_i H_i}{\sigma_{H_i}^2}$, $C \equiv \sum_{i=1}^{n} \frac{H_i^2}{\sigma_{H_i}^2}$ and $E_i \equiv \frac{H(z_i)}{H_0}$.

The result of such analysis can be seen on Figure 1. As can be seen, the results from $H(z)$ data alone yield nice constraints on the plane $\Omega_m - \Omega_g$. The flatness limit, which corresponds to $\Omega_m + \Omega_g = 1$, can be seen as a straight line on this plane (dashed line on Fig. 1). Points on and above this line were not considered, as they correspond to non-open models. One may see that the best fit relies right below this line, indicating that $H(z)$ data alone favour a slightly open Universe.

Furthermore, we have considered the prior $\Omega_m \geq \Omega_b$, with the baryon density parameter, $\Omega_b$, estimated by Planck and WMAP: $\Omega_b = 0.049$ [4], a value which is in agreement with Big Bang Nucleosynthesis (BBN), as shown on Ref. [20]. As a result of this prior, the 3$\sigma$ c.l. contour alone is cut for low matter density parameter, as we may see on Fig. 1.

The minimum $\chi^2$ was $\chi^2_{\text{min}} = 16.727$, yielding a $\chi^2$ per degree of freedom $\chi^2 = 0.523$. The best fit parameters were $\Omega_m = 0.242^{+0.041+0.065+0.090}_{-0.085-0.15-0.19}$, $\Omega_g = 0.703^{+0.069+0.085+0.10}_{-0.34-0.62-0.96}$ for 68.3%, 95.4% and 99.7% c.l., respectively, in the joint analysis.

As expected, this result is in agreement with $\Lambda$CDM constraints, as this model mimics the concordance model. Moreover, the best fit values of $\Omega_m$ and $\Omega_g$ leads to $\Omega_K = 0.055$, which corresponds to a negative value of $K$, as expected for this model. Sharov and Vorontsova [6] have found, for $\Lambda$CDM: $\Omega_m = 0.276^{+0.009}_{-0.008}$, $\Omega_\Lambda = 0.769 \pm 0.029$, for 1$\sigma$ c.l., where they have combined $H(z)$ with SN Ia and BAO data. Given the uncertainties on massive gravity
parameters above, the results are in good agreement, even considering the open Universe restriction for massive gravity, while ΛCDM has no restriction on curvature.

V. Bounds on the Graviton Mass

Having obtained the best fit values for the parameters, we show in Fig. 2 the plot of Massive Gravity theory (red line) with ±1σ limit (red dotted line). The ΛCDM according to best fit data of Sharov and Vorontsova [6] are also represented (black line).

This model also gives an expression to the graviton mass depending on the α₃ and α₄ parameters through β⁺⁻:

\[ m_g^2 = \frac{\Omega_g}{\beta_{\perp}} H_0^2. \]  

(25)

If we fix some of the parameters, we can see how the mass depends on the others. In Figure 3 we show a typical mass dependence with α₄ when we set the α₃ as a constant and we choose to work with β⁻. Such behaviour is also observed for others positive and negative values of α₃. It is easy to see that the mass increases and diverges for some specific value of α₄, corresponding to the limit β⁻ → 0. In some cases the mass can also abruptly decrease to zero, as shown in the cases α₃ = 6, α₃ = 4 and α₃ = 2. This shows that the graviton
mass is strongly dependent on the $\alpha$’s parameters, although in the limit of very negative $\alpha_4$ values the graviton mass goes to $m_g \simeq H_0^{-1}$.

![Graph](image)

**FIG. 3:** Some typical mass dependence with $\alpha_4$ for some specific values of $\alpha_3$.

**VI. CONCLUSION**

In this work we have analysed an open FLRW solution of massive gravity which admits accelerated expansion of the universe, in full concordance to observations. The constraints with recent observational $H(z)$ data were found and the best fit values obtained for the cosmological parameters are $(\Omega_m, \Omega_g, \Omega_K) = (0.242, 0.703, 0.055)$, in accordance with the $\Lambda$CDM model, and also point to a nearly open curvature ($K < 0$), for which the model is valid. The graviton mass dependence with the constant parameters $\alpha_3$ and $\alpha_4$ of the model were also analysed, and we have verified a strong dependence with such parameters. We have also obtained that the condition $m_g \simeq H_0^{-1}$ seems dominant for a long range of the parameters $\alpha_3$ and $\alpha_4$, although cosmological observations cannot be used to determine such parameters. The study of the positivity of the cosmological constant like term, $\Lambda_\pm = m_g^2 \beta_\pm$, when the constant $\alpha_2 \to 0$ indicates a complete supression of solutions for the case $\beta_+ \pm$ and admits solutions only if $\alpha_3 < 0$ for the case $\beta_- > 0$. This indicates the necessity of a $L_2$ term in the construction of the model, at least for the $\beta_+$ branch, in order to get a positive cosmological constant like term.
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