MATCHING MODELS UNDER SCRUTINY: AN APPRAISAL OF THE SHIMER PUZZLE

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Abstract. Two papers have recently questioned the quantitative consistency of the search and matching model. Shimer has argued that a textbook matching model is unable to explain the cyclical variation of unemployment and vacancies in the US economy. Costain and Reiter have found the existence of a trade-off in the model’s performance: any attempt to change the calibrated values to improve the model’s ability to predict the business cycle jeopardizes its predictions of the impact of unemployment benefits on unemployment. In surveying the literature originating in these findings, I distinguish three different avenues that have been followed to correct the model: change in wage formation, change in the calibration and changes in the model specification. The last approach seems to achieve the best results both from a business cycle and from a microeconomic viewpoint.

Keywords. Business cycles; Labour markets; Search-matching equilibrium

1. Introduction

In an influential paper, Shimer (2005) evaluates the business cycle performance of the search and matching model, nowadays the standard workhorse adopted by macro and labour economists to study aggregate labour markets. The most important claims of his paper can be summarized as follows.

1. A textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the US economy in response to productivity shocks of plausible magnitude. The results of the job-finding rate are 12 times more volatile in the data than in the model, whereas the standard deviation of the vacancies in the data is 10 times larger than in the model.

2. Such discrepancies stem from the wage formation assumptions. In a standard matching model, a Nash bargaining solution is introduced in order to share the total surplus between the firm and the worker. When a positive productivity shock hits the economy, wages instantaneously increase, dampening vacancy creation. Thus, vacancies (and, as a consequence, the job-finding rate) are much more variable in the data than in the model.

3. The model also exhibits no propagation as it implies a contemporaneous correlation between the vacancy/unemployment ratio and the productivity of 1, while in the data it is about 0.4.
The aim of this paper is to present the literature that sprung from Shimer’s findings. As far as the first two claims are concerned, I distinguish three different avenues that have been taken by scholars.

In the first group, I list papers that agree with Shimer’s first and second claims: the model fails to replicate the US business cycle facts because wages are too responsive to changes in productivity. Hence, the most straightforward way to reconcile model and data is to modify the wage formation rules, for instance, by introducing some form of wage stickiness or imposing imperfect information in the firm–worker negotiation.

Although the sticky wage hypothesis has gained the initial attention of many economists, two caveats suggest that it may not be the right answer to the lack of amplification exhibited by the model. First, wage stickiness (or even a completely rigid wage) is not sufficient to fit the model with the data; a low calibrated profit share is also needed, so that the percentage increase in profits is large for a given percentage increase in productivity, and vacancy creation is boosted. Second, as Pissarides (2007) and Haefke et al. (2007) have recently pointed out, introducing sticky wages into matching models is difficult to justify on empirical grounds. What is key in amplifying fluctuations is the cyclical behaviour of the wages of the newly hired workers. To augment the unemployment and vacancy volatility, these wages must be nearly acyclical; however, the data show a near-proportional relation between them and labour productivity.

The second avenue has been pursued by Hagedorn and Manovskii (2007, 2008). They do not agree with Shimer’s first two conclusions. According to them, the model is unable to match the data because of an erroneous parametrization of two key variables: the instantaneous utility of being unemployed and workers’ bargaining power. With a higher calibrated value for the utility of unemployment and a lower bargaining power for workers, a standard matching model succeeds in replicating the observed business cycle fluctuations.

Even this second approach has been called into question on different grounds. Mortensen and Nagypál (2007b) claim that Hagedorn and Manovskii’s calibration postulates a too small and unrealistic difference between the instantaneous utility of employment and unemployment.

More importantly, Costain and Reiter (2007) checked the quantitative consistency of the model in response not only to productivity and separation rate shocks, but also to changes in the level of unemployment benefits. Their conclusion is that any attempt to calibrate a standard matching model in order to match the business cycle unemployment and vacancies data produces unrealistic results about the effects of an increase in unemployment benefits on the unemployment rate, and vice versa. As a possible answer to the amplification puzzle, Costain and Reiter propose the introduction of embodied technological progress.

This naturally leads to a third way in which Shimer’s findings have been challenged. What this vein of the literature (more or less) implicitly argues is that a standard matching framework contains some simplified assumptions that inevitably jeopardize the quantitative consistency of the model. Enriching the
basic set-up – considering for instance turnover costs, on-the-job search or market power – would bridge the gap between data and theory.

This last approach seems to be the most successful in improving the quantitative performance of the model, both at business cycle and at a policy analysis level. In particular, assuming cohort-specific productivity shocks as in Costain and Reiter (2007) and Reiter (2007) or introducing turnover costs as in Silva and Toledo (2007) improves the business cycle performance of the model and fits the microeconomic estimates, while keeping it analytically tractable.

A final remark concerns the lack of propagation exhibited by the model. This problem has received much less attention from the scholars than the amplification puzzle explained above. Fujita (2003), Fujita and Ramey (2007b) and Hagedorn and Manovskii (2007) have focused on this issue, caused by the excessive responsiveness of vacancies to shocks. They show that the introduction of planning lags or increasing marginal costs in vacancy creation ameliorates the dynamics of the model.

1.1 Related Surveys

The present paper is not the first survey that aims to present Shimer’s claims and the subsequent literature they motivated.

In a detailed review, Hornstein et al. (2005) clearly spell out the importance of a low profit share to improve the performance of the model and explain the crucial differences between Shimer’s and Hagedorn and Manovskii’s calibration. Since the publication of their paper, however, many other papers have tried to answer Shimer’s points, focusing in particular on the third approach mentioned above. The present survey tries to account for this more recent strand of research.

Costain and Reiter (2007), Mortensen and Nagypál (2007b) and Pissarides (2007), while taking part in the contest by proposing alternative ways to fit the model with the data, also provide helpful summaries of the state-of-the-art literature. Of course, the main objective of these papers is not to provide an overview, and some studies are not presented there.

Finally, Yashiv’s (2007a) survey covers a much wider subject, namely the recent advances in macroeconomic models with search frictions in the labour market. So, the so-called Shimer critique comprises only a fraction of the topics covered in his review.3

1.2 Structure of the Paper

This paper is organized as follows. Section 2 describes the empirical and theoretical framework. Section 3 presents the quantitative inconsistency in the model. Sections 4, 5 and 6, respectively, survey the three different approaches followed in reaction to Shimer’s claims. Section 7 deals with the propagation problem, and Section 8 concludes.
Table 1. Shimer’s Summary Statistics.

|                  | $u$   | $v$   | $v/u$ | $f$   | $s$   | $p$   |
|------------------|-------|-------|-------|-------|-------|-------|
| Standard deviation | 0.190 | 0.202 | 0.382 | 0.118 | 0.075 | 0.020 |
| Quarterly autocorrelation | 0.936 | 0.940 | 0.941 | 0.908 | 0.733 | 0.878 |

Correlation matrix

|     | $u$   | $v$   | $v/u$ | $f$   | $s$   | $p$   |
|-----|-------|-------|-------|-------|-------|-------|
| $u$ | 1     | -0.894| -0.971| -0.949| 0.709 | -0.408|
| $v$ | -1    | 0.975 | 0.897 | -0.684| 0.364 |
| $v/u$ | -    | -     | 1     | 0.948 | -0.715| 0.396 |
| $f$ | -     | -     | -     | 1     | -0.574| 0.396 |
| $s$ | -     | -     | -     | -     | 1     | -0.524|
| $p$ | -     | -     | -     | -     | -     | 1     |

Quarterly US data (1951–2003). All variables are reported as log-deviations from a Hodrick–Prescott trend with smoothing parameter $10^5$.

Source: Shimer (2005).

2. Empirical and Theoretical Framework

2.1 US Labour Market Facts

Table 1, taken from Shimer (2005), summarizes the statistics on the economic variables of interest. In bold are the numbers on which scholars have concentrated.

I first present Shimer’s data collection method and then, in Section 2.2, I briefly discuss the differences between his work and the previous literature. The variables are the following: $u$ unemployment, $v$ vacancies, $f$ the job-finding rate, $s$ the separation rate, and $p$ labour productivity. Data are quarterly and refer to the period from 1951 to 2003.

What emerges from a first inspection of the data is the relatively high volatility of the level of unemployment and vacancies. The standard deviation of unemployment $\sigma_u$ is 0.19, meaning that this variable can be as much as 38% above or below trend. Since unemployment is countercyclical and vacancies are procyclical, labour market tightness, defined as $\theta \equiv v/u$, is extremely procyclical. Moreover, both unemployment and vacancies are very persistent variables, with an autocorrelation of around 0.94.

Shimer assumes a constant returns to scale (CRS) matching technology, $m(v, u)$, increasing and concave in both arguments, representing the measure of new jobs created as a function of the level of unemployment $u$ and vacancies $v$. Because of the CRS assumption, the job-finding rate $f \equiv m(v, u)/u = m(\theta, 1)$ is an increasing and concave function of $\theta$ only. Shimer gets an average monthly hazard rate of around 0.45 and a standard deviation of 0.118. As Shimer points out, the high positive correlation between $f$ and $\theta$ is a strong argument in favour of a CRS matching technology.

Once $f_t$ and $\theta_t$ have been computed, Shimer looks at the matching function. With a Cobb–Douglas functional form, $m(v, u) = \mu u^\alpha v^{1-\alpha}$, two parameters need
to be estimated, $\alpha$ and $\mu$. Using his data on $f_t$ and $\theta_t$, he gets a value of $\alpha$ between 0.70 and 0.75, beyond the plausible range of 0.3–0.5 reported by Petrongolo and Pissarides (2001).

The separation rate is computed using another labour market flows equation and yields a value of 0.034. Such a variable displays a lower volatility compared to the job-finding rate and has a negative correlation with labour market tightness.

Labour productivity is computed as the ratio between real output and the number of workers in the non-farm business sector. Data show that labour productivity is positively correlated with tightness and, more crucially, its standard deviation is 10 times lower than that of vacancies and almost 20 times lower than the standard deviation of labour market tightness.\(^6\)

Some other papers I will survey later also consider the volatility of real wages with respect to productivity. In this respect, a clear distinction has to be made. Scholars who look at time-series statistics on the aggregate wage in the economy estimate an elasticity of the real wage with respect to productivity, denoted by $\eta_{wp}$, in a range between 0.3 and 0.7 and a ratio of the standard deviations $\sigma_w/\sigma_p$ in an interval between 0.4 and 0.9 (Gertler and Trigari, 2006; Rotemberg, 2006; Hagedorn and Manovskii, 2007, 2008).\(^7\)

Two more recent studies (Haefke et al., 2007; Pissarides, 2007), however, contend that for a correct comparison between the data and the search and matching model it is important to distinguish between the volatility of the wages of newly hired workers and the volatility of the wages in ongoing matches. Examining various panel regressions of individual workers (Pissarides), or using micro-data from the CPS (Haefke et al.), both papers reach the same conclusion: the wages of newly hired workers are as volatile as labour productivity (implying unit elasticity $\eta_{wp}$), whereas those in existing jobs are about half as cyclical (the elasticity is around 0.5 for Pissarides, 0.27 for Haefke et al.).

2.2 Job Creation versus Job Destruction Volatility

The strong procyclicality of the job-finding rate and the weak countercyclicality of the job separation rate with respect to output contradict the conclusion reached by Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1992) and Davis et al. (1996) that job destruction is the main source of unemployment fluctuations. The question can be put in these terms: Shimer, both in his (2005) paper and in a subsequent one in which he uses gross flow data (Shimer, 2007), argues that recessions are essentially periods where it is extremely difficult to find a job, whereas most of the previous literature identifies recessions as periods mainly characterized by high job-loss rates. What can explain such varying results?

According to Shimer (2007), the reason lies in the different facts that he and Davis and Haltiwanger measure. Shimer considers the dynamic behaviour of monthly unemployment levels to get $f_t$ and $\theta_t$, whereas Davis and Haltiwanger measure job creation and job destruction. The former is defined as the net employment gains at establishments that experience positive net gains in a certain period; the latter as the net job losses at establishments experiencing negative net
employment gains in a certain period. In the job separation estimates both the firing of existing employees and the decision not to hire new workers to replace leavers are computed. Firings represent an increase in the separation rate but the decision not to hire represents a decrease in the job-finding rate, i.e. therefore underestimated in Davis and Haltiwanger’s analysis.  

Even with respect to the previous literature on gross worker flow data, the divergence in the results may depend on the different statistics measured. Much of this literature (such as Blanchard and Diamond, 1990) refers to the levels, i.e. \( f_iu_t \) and \( s_i\epsilon_t \), whereas Shimer’s findings are about the rates, \( f_i \) and \( s_i \); \( u_t \) and \( f_i \) move in opposite directions over the cycle, so that \( f_iu_t \) remains fairly stable even with a strongly procyclical job-finding rate. Moreover, according to Shimer, the gross flow literature fails to account for time aggregation problems. For instance, models in which workers are not assumed to lose and find a job within the same period (a week or a month) may deliver a biased measurement of the job-finding rate.

Shimer’s conclusions have not put an end to the dispute. A more recent strand of research (e.g. Davis and Kennan, 2005; Elsby et al., 2007; Fujita and Ramey, 2007a; Yashiv, 2007b) has questioned his assertions, arguing that the volatility of the separation rate makes a decisive contribution to unemployment fluctuations.

Since the present survey primarily concentrates on the inconsistency between matching models and data, and not on the measurement of the flows in and out of unemployment, space limitations do not allow me to discuss these papers. It is undeniable, however, that understanding the contribution of the separation rate to the unemployment variability has significant theoretical implications. Indeed, a strongly countercyclical separation rate would make considering it exogenous much less justifiable. A definitive conclusion on the ins and the outs of unemployment – whether the ins win, as the ‘old’ literature concluded, or the ‘outs’, as Shimer maintains, or we have a tie, as Fujita and Ramey (2007a) and Elsby et al. (2007) argue – is urgently needed.

### 2.3 Policy Evaluation Estimates

The search and matching model can also be quantitatively assessed with respect to labour market micro-data. We can consider, for instance, estimates of the impact of unemployment benefits both on unemployment duration and on the level of unemployment.

Because labour supply is fixed in a standard matching model and the unemployment benefits affect unemployment only via their negative impact on vacancy creation, the studies conducted by Layard et al. (1991) and Layard and Nickell (1999) that evaluate the general equilibrium effects of some labour market policies in various countries appear to be the most appropriate references. They consider OECD data that go back to 1960 and get a semi-elasticity of unemployment with respect to unemployment benefits replacement ratio of around 1.3. Costain and Reiter (2007) also ran some cross-country regressions on Layard and Nickell’s data set and obtained a semi-elasticity close to 2.0. In surveying other
recent estimates, Baker et al. (2003) found values not substantially larger than those obtained by Layard and Nickell.

2.4 Theoretical Framework

The model built by Shimer is a standard matching framework with the addition of a stochastic economy-wide shock in two parameters, productivity and the separation rate. I present the case of productivity shocks, the environment with a stochastic separation rate being symmetric. All the endogenous variables that depend on the current value of productivity are henceforth denoted by the subscript \( p \).

The economy is composed of \( L \) risk-neutral workers. Time is continuous and the discount rate is denoted by \( r \). Unemployed workers search for a job, while every firm can post one and only one vacancy. All jobs are identical and every firm–worker pair produces the unique consumption good at the flow rate of \( p \). Autocorrelated shocks affect the value of \( p \). More precisely, the time sequence \( \{p_t\} \) is a jump process with an arrival rate \( \lambda \) and a conditional distribution of new values represented by the c.d.f. \( F_p : P \times P \rightarrow [0, 1] \), where \( P \) is the support of the process.

The flow of new matches is denoted by \( m(v, u) \) and the job-finding rate is given by \( f(\theta) \equiv m(v, u)/u \); the rate at which vacancies are filled is \( m(v, u)/v = f(\theta)/\theta \), which is positive, decreasing and a convex function of \( \theta \). At an exogenous rate a firm–worker pair is destroyed. The law of motion of unemployment is

\[
\dot{u}_p = s(L - u_p) - f(\theta_p)u_p
\]

In steady state, we have

\[
u_p = \frac{sL}{s + f(\theta_p)} \tag{1}\]

The downward-sloping relationship in \( v/u \) space obtained from equation (1) is called the Beveridge curve.

Once a worker finds a firm with a vacant job, a surplus of the match arises. This is given by the difference between the expected discounted value that the two parties will receive by forming a match and the expected discounted value they renounce by being employed. A zero profit condition implies that the expected discounted value of an unfilled vacancy is zero. So, the surplus is defined as \( S_p \equiv J_p + W_p - U_p \), in which \( J_p \) is the value of a filled vacancy, \( W_p \) is the value of being employed for a worker and \( U_p \) is the value of unemployment. Denoting by \( E_p \) the expectation operator conditional on the current state \( p \), the Bellman equations take the form

\[
r U_p = z + f(\theta_p)(W_p - U_p) + \lambda(E_p U_p - U_p) \tag{2}\]

\[
r W_p = w_p + s(U_p - W_p) + \lambda(E_p W_p - W_p) \tag{3}\]

\[
r J_p = p - w_p - sJ_p + \lambda(E_p J_p - J_p) \tag{4}\]
in which $z$ denotes the instantaneous utility enjoyed by the unemployed worker, while $w_p$ is the wage. Hence,

$$rS_p = p - z - f(\theta_p)(W_p - U_p) - sS_p + \lambda(E_pS_p' - S_p)$$  \hspace{1cm} (5)

To determine the wage allocation, Shimer considers a Nash bargaining solution: the wage $w_p$ is chosen to maximize the product $(W_p - U_p)^\beta(J_p)^{1-\beta}$. Parameter $\beta \in [0, 1]$ represents the workers’ bargaining power. The unique solution to this maximization problem is

$$(1 - \beta)(W_p - U_p) = \beta J_p$$  \hspace{1cm} (6)

Notice that $W_p - U_p = \beta S_p$ and $J_p = (1 - \beta)S_p$. The free-entry zero profit condition implies that the expected cost of filling a vacancy (given by the expected duration of finding a worker multiplied by the flow cost of keeping a vacancy open, denoted by $c$) must be equal to the value of a match to the employer:

$$\frac{c\theta_p}{f(\theta_p)} = J_p$$  \hspace{1cm} (7)

Using equations (2)–(4), (6) and (7), a standard wage equation can be derived even in this stochastic set-up:

$$w_p = \beta(p - rU_p) + rU_p = \beta(p + c\theta_p) + (1 - \beta)z$$  \hspace{1cm} (8)

Finally, the match surplus (5) can be rewritten using $W_p - U_p = \beta c\theta_p/[(1 - \beta)f(\theta_p)]$ as

$$\frac{c\theta_p}{f(\theta_p)} = \frac{(1 - \beta)(p - z) - \beta c\theta_p + (1 - \beta)\lambda E_pS_p'}{r + s + \lambda}$$  \hspace{1cm} (9)

Equation (9) is the equilibrium condition for labour market tightness, with the other endogenous variable being the expectation of future surplus $E_pS_p'$.9

2.4.1 Elasticities

In his paper, Shimer (2005) shows that the elasticity of tightness with respect to productivity when there are no shocks, obtained by differentiating equation (9) with $\lambda = 0$, is a useful approximation to the volatility of tightness in the dynamic stochastic set-up. Indeed, Mortensen and Nagypál (2007b) prove that the two outcomes coincide in the limit when the arrival rate $\lambda$ is close to 0 or the change in productivity is small (see Proposition 2 of their paper).10 This result is extremely useful, since it allows different set-ups to be compared without the need to compute numerical simulations.

The elasticity of tightness with respect to productivity when $\lambda = 0$ is

$$\eta_{\theta p} \equiv \frac{\partial \ln \theta}{\partial \ln p} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \frac{p}{p - z}$$  \hspace{1cm} (10)

where $\alpha = 1 - f'(\theta)\theta / f(\theta)$ is the elasticity of the expected duration of filling a vacancy with respect to tightness. Recalling that $f(\theta) = \mu \theta^{1-\alpha}$, the elasticity of the
job-finding rate with respect to productivity is given by $\eta_{f,p} = (1 - \alpha)\eta_{\theta p}$. Using equation (1), I also get the elasticity of unemployment with respect to productivity, $\eta_{u,p} = (1 - \alpha)(1 - u)\eta_{\theta p}$.

To deal with the labour policy implications of the model and the volatility of the wage, I compute two other related elasticities. Parameter $z$ is the sum of the level of unemployment benefits $b$ and the value of leisure. The semi-elasticity of $u$ with respect to $b$ is

$$
\zeta_{u,b} = \frac{\partial \ln u}{\partial b} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} = \eta_{u,p}
$$

Finally, the elasticity of the wage with respect to productivity is given by

$$
\eta_{wp} \equiv \frac{\partial \ln w}{\partial \ln p} = \left(\frac{\beta}{w/p}\right) \left[\frac{\alpha(r + s) + f(\theta)}{\alpha(r + s) + \beta f(\theta)}\right]
$$

3. Comparing the Model with the Data

The business cycle data to which scholars have paid most attention concern the volatility of unemployment, vacancies and the job-finding rate. The dynamic correlation between productivity and the labour market variables, as well as the autocorrelation of vacancies, has received less attention. Some papers have also focused on wage statistics. As regards the policy analysis estimates, the performance of the model is mainly evaluated by looking at the (semi)elasticity of unemployment with respect to unemployment benefits.

3.1 Business Cycle Viewpoint

Shimer’s calibrated values are $r = 0.012$, $p$ normalized to 1, $s = 0.10$, $f = 1.355$ (remember that he obtained a monthly separation rate of 0.034 and a monthly job-finding rate of 0.45) and $\alpha = 0.72$. Moreover, Shimer considers $z$ only as the level of unemployment benefits, ignoring the value of leisure. He set $z = 0.4$. Since mean labour income in the stochastic model is 0.993, this value of $z$ is at the upper end of the range of replacement ratios in the USA.

Substituting these values in equation (10) and imposing the Hosios (1990) condition $\beta = \alpha$, which ensures the efficiency of the decentralized equilibrium, gives $\eta_{\theta p} = 1.72$ and $\eta_{fp} = 0.481$. A comparison between the data and the model is presented in Table 2. Assuming, as Shimer does, that shocks on productivity are the only source of fluctuations in labour market tightness, we can compare the elasticities $\eta_{\theta p}$ and $\eta_{fp}$ with the corresponding ratios $\sigma_{\theta}/\sigma_{p}$ and $\sigma_{f}/\sigma_{p}$ found in the data. The difference is striking: $\sigma_{\theta}/\sigma_{p}$ is 11 times larger than $\eta_{\theta p}$, while $\sigma_{f}/\sigma_{p}$ is more than 12 times larger than $\eta_{fp}$. This is Shimer’s main point: a standard matching model can explain less than 10% of the observed fluctuations in the vacancy/unemployment ratio.
3.2 The Propagation Problem

The other deficiency of the model is the absence of propagation of the labour productivity shock. This can be seen by comparing the contemporaneous correlations between productivity and all the labour market variables of interest. In the model, these moments are equal to 1, and in the data they are close to 0.4 in absolute value. Other shortcomings concern the autocorrelation of vacancies and the time response of tightness to productivity. These problems – as well as the various solutions proposed in the literature – will be discussed in Section 7.

3.3 Conditional versus Unconditional Moments

Not all the papers analysing the amplification puzzle target the ratio of standard deviations as Shimer does. Mortensen and Nagypál (2007b) argue that an empirical correlation between productivity and tightness of 0.396 renders the assumption that productivity is the only explanation of fluctuations in tightness problematic. Rather, such a value suggests the coexistence of several driving forces behind the volatility of tightness and unemployment. Thus, instead of considering the ratios of standard deviations, Mortensen and Nagypál gauge the consistency of the model by comparing the empirical OLS regression coefficients $\rho_\theta \cdot \sigma_\theta / \sigma_p$ and $\rho_f \cdot \sigma_f / \sigma_p$ with the simulated ones. Data show that the latter coefficient is 7.56 and the former is 2.34, while the simulated counterparts are close to the corresponding ratios of standard deviations because of the unit correlations delivered by the model. The lack of amplification highlighted by Shimer (2005) is still present, but in less dramatic terms.

3.4 Separation Rate Shock

Shimer also considers the effects of a shock on the separation rate. The main result is that it delivers a positive counterfactual correlation between unemployment and vacancies. To understand why, consider equation (9): a higher separation rate lowers $\theta$, since it makes entry less profitable for firms. A decrease in labour market tightness is depicted in the $v-u$ space by a less steep ray starting from the origin; yet, the Beveridge curve (1) moves to the right, so it is not possible
to discern the behaviour of the vacancies (see Figure 1). With a Cobb–Douglas matching technology, the shift of the Beveridge curve may be large enough to make both unemployment and vacancies increase, explaining why a positive correlation between these two variables is observed in the stochastic simulation.

A framework with both productivity and separation rate shocks does not deliver significant improvements. Unemployment appears to be more cyclical, but both tightness and the job-finding rate are still less than 10% as volatile as expected.

3.5 Wage Share and Wage Volatility

Two distinctive features of Shimer’s calibrated model are the large value of the wage share and the high volatility of the wage. As regards the former, inserting the calibrated values into equation (8) gives a wage ratio $w/p = 0.973$. As Hornstein et al. (2005) clearly show, the wage share is large in Shimer’s calibrated model because both the job-finding rate $f(\theta)$ and the workers’ bargaining power $\beta$ are much greater than the job separation rate and the discount rate.$^{12}$

From equation (12), it is also clear that $\eta_{wp} \approx p/w$ if $\beta$ is close to 1. So, in Shimer’s calibrated model, the wage elasticity is around 1. This value is far beyond the range of estimates obtained by examining time-series data on aggregate wages, but is consistent with the results of Pissarides (2007) and Haefke et al. (2007) on the volatility of wages for new employees. So a natural question is which of the two statistics is more relevant to the search and matching model. Haefke et al. contend that it is the latter. In a matching framework, firms decide to post a vacancy on the basis of the expected present values of productivity, wages and search costs. Hence, the variable $\eta_{wp}$ represents the elasticity of the expected present value of wage payments with respect to the expected present value of productivity, not observable in the data. Haefke et al. show that, over a plausible range of parameters, the elasticity of the current period wage of newly hired workers with respect to productivity is an accurate proxy for $\eta_{wp}$.
3.6 Policy Evaluation Viewpoint

Now, I will compare the semi-elasticity of unemployment with respect to $b$ predicted by the model with its empirical counterpart. Substituting the calibrated values chosen by Shimer in equation (11), I get $\zeta_{u,b} = 0.45$, lower than the values reported in Section 2.3.13. But the crucial point here is different. Since $p$ is set equal to 1, the semi-elasticity of unemployment with respect to the unemployment benefits $\zeta_{u,b}$ is equal to the elasticity of unemployment with respect to productivity $\eta_{u,p}$. Data show $\sigma_u/\sigma_p = 9.5$. The trade-off is clear: either the parametrization is constructed to match the business cycle volatility of unemployment, or it is constructed to match the policy estimates cited above. No calibration can do both.

4. First Approach: Changes in the Wage Formation

Why is the matching model unable to replicate the observed fluctuations in unemployment and vacancies? Shimer argues that the main culprit is the Nash bargaining solution:

An alternative wage determination mechanism that generates more rigid wages in new jobs, measured in present value terms, will amplify the effects of productivity shocks on the $v-u$ ratio, helping to reconcile theory and evidence. (Shimer, 2005)

4.1 Exogenous Wage Rigidity

The most straightforward way to break the link between wages and productivity is to impose wage rigidity.

In this respect, an important point deserves to be stressed. Assuming wage rigidity only in the existing matches (meaning that firms and workers negotiate the wage only the first time they match and it does not change following subsequent shocks) has no impact on the vacancy/unemployment ratio. The reason is that this kind of rigidity does not affect the discounted expected profits of the firms, but simply the timing of the wage payments. Since in a matching model a firm decides to post a vacancy only on the basis of its future expected profits, the level of tightness takes the same value as in the flexible wage set-up.

As Shimer (2004) shows, the results change dramatically when rigid wages also apply to new matches. Under this hypothesis, firms and workers never bargain and take the wage as an exogenous variable. The new equilibrium equation when $w = \bar{w}$ and $\lambda = 0$ is

$$\frac{c\theta_p}{f(\theta_p)} = \frac{p - \bar{w}}{r + s}$$

With $\bar{w} = 0.967$ (chosen to match an average US unemployment rate of 5.7), the elasticity of tightness with respect to productivity is $\eta_{\theta,p} = p/[\alpha(p - \bar{w})] = 42.08$,
a value more than twice as large as the ratio of standard deviations found in the data.\textsuperscript{14}

However, the success of the rigid wage model can be questioned on various grounds.

1. The large values for the elasticities mainly depend on the value $\bar{w} = 0.967$. If $\bar{w} = 0.5$, then $\eta_{\theta p} = 2.7$, higher than in the flexible wage set-up but inconsistent with the data. Wage rigidity is insufficient to improve the business cycle consistency of the model. A small profit share $(p - w)/p$ is also needed, so that the percentage increase in profits is large for a given percentage increase in productivity.

2. As we have seen at the end of Section 2.1, imposing wage rigidity for new hires is not empirically grounded, Haefke \textit{et al.} (2007) and Pissarides (2007) have shown that the corresponding elasticity $\eta_{w, p}$ for this subset of wages is close to unity.

3. With perfect wage rigidity, the level of unemployment benefits does not affect tightness and employment. The semi-elasticity $\zeta_{ub}$ is equal to zero. So, the price of the business cycle consistency of the model is to make it useless for a policy evaluation analysis.\textsuperscript{15}

4.1.1 \textit{Sticky Wages}

‘Milder’ forms of wage rigidity may also be introduced. Farmer and Hollenhorst (2006) build a fully blown DSGE model, in which households take consumption-saving decisions and search effort is endogenous, and assume that only 19\% of the wage is bargained while the remaining fraction is unaffected by productivity shocks. Their framework fits the US data particularly well. The sticky wage economy matches the unemployment standard deviation and slightly overshoots on the vacancy standard deviation. As the authors stress, two parameters are decisive: the disutility of effort parameter and the degree of wage stickiness. The latter in particular is key to replicating both a positive correlation between output and vacancies and a negative correlation between output and unemployment.

4.1.2 \textit{Staggered Wage Contracts}

Staggered wage contracts constitute a middle way between perfectly rigid and flexible payments. Firms and workers bargain over the wage at an exogenous Poisson rate $1 - \varphi$. The expected duration of a contract is therefore equal to $1/(1 - \varphi)$. The aggregate wage in the economy is $w_t = (1 - \varphi)w^*_t + \varphi w_{t-1}$, with $w^*_t$ being the payment negotiated at time $t$. Gertler and Trigari (2006) and Bodart \textit{et al.} (2006) pursue this approach.

The data Gertler and Trigari want to replicate are $\sigma_\theta/\sigma_p = 12.10$, $\sigma_u/\sigma_p = 5.81$ and $\sigma_w/\sigma_p = 0.46$. Assuming three or four quarters as the average length of the contract, their model explains 81\% of unemployment volatility, 89\% of tightness volatility and 95\% of wage volatility. Yet, the success of both papers hinges on
the assumption that wage stickiness also applies to new matches (i.e. newly hired workers receive the same wage as the other employees16). This lays them open to the same criticism as that addressed to the rigid wage hypothesis by Pissarides and Haefke et al. Moreover, as emphasized by Mortensen and Nagypál (2007b), staggered contracts imply that labour market tightness increases more than in the flexible set-up after a positive shock on $p$ because only a fraction $1 - \varphi$ of the new employees bargain the wage at time $t$. But when the wages are finally renegotiated in accordance with the new productivity value, tightness decreases to a level below its initial response. This is at odds with the data.

4.2 Endogenous Wage Stickiness

4.2.1 Social Norm

Imposing exogenous sticky wages on the model implies that the agents are not fully rational, since they are not exploiting all the advantages of the negotiation. Hall (2005) overcomes this critique by imposing a wage norm that never lies outside the bargaining set. Adopting the same notation as in the previous sections, the workers’ reservation wage is equivalent to $rU_t$, whereas $p_t$ is the highest level of wage that an employer is willing to pay. Consider an idiosyncratic random shock, $\epsilon$, normally distributed with zero mean and standard deviation $\sigma$, that shifts the bargaining set so that it becomes $[rU_t + \epsilon, p_t + \epsilon]$. Then, the current wage takes the form

\[
\begin{align*}
w_t &= rU_t + \epsilon & \text{if } w_{t-1} < rU_t + \epsilon \\
&= p_t + \epsilon & \text{if } w_{t-1} > p_t + \epsilon \\
&= w_{t-1} & \text{otherwise}
\end{align*}
\]

The wage does not change if it remains in the bargaining set; otherwise, it takes the value of the nearest boundary. Hall imposes the norm $w_t = E(w_t(\epsilon))$. The average wage at time $t$ is a function of $w_{t-1}$, $rU_t$ and $p_t$. It is worth noticing that such a norm also has an impact on the wages of new matches, which become stickier.

Hall’s analysis differs from Shimer’s in that he considers a permanent price shock. Productivity jumps from 1 to $1 + \Delta$ and then remains at that level. His model succeeds in replicating the behaviour of key labour market variables in the US economy. A reduction in productivity by 1% produces the classical hump-shaped form for the dynamics of the unemployment rate: it starts at 5.6%, reaches the maximum value of 6.7% after 7 months and then starts to decline.

4.2.2 Long-term Wage Contracts

In Rudanko’s (2007) paper, wage rigidity is the consequence of long-term wage contracts proposed by risk-neutral firms to risk-neutral workers in order to smooth
income in response to labour productivity shocks. Rudanko considers three types of contract: full commitment, where both the firm and the employee commit to not parting even if such choice may not be the optimal one \textit{ex post}; one-sided limited commitment, where only firms commit to contracts; and two-sided limited commitment, in which either party can break the contract.

The first type of contract features perfect wage rigidity, the firm bearing all the risk caused by fluctuations in productivity. Under the second type of contract, should a positive shock on productivity mean that workers could get a higher utility elsewhere, wages would adjust up to the new value of the opportunity cost of employment to stop them leaving. Under two-sided limited commitment, the contract also foresees a wage decrease during troughs in order to keep firms from firing workers.

The similarity between the last type of contract and Hall’s model is evident. However, the simulation results show that none of these contracts amplifies the fluctuations in unemployment and vacancies. Moreover, the first two types of contract feature a volatility of the aggregate wage lower than the targeted ratio $\frac{\sigma_w}{\sigma_p} \approx 0.5$.

The reason for this twofold failure is that the wage contracts studied by Rudanko augment the rigidity of wages in ongoing matches – so that the volatility of the aggregate wage is lower than in the data – but do not dampen the cyclicality of the wages for new hires. Indeed, under each of the three types of contracts, the firm makes a wage offer at the moment of matching that depends on the current conditions in the economy. As Rudanko observes, in her model wage rigidity fails to fit both tightness and the wage fluctuations observed in the data.

### 4.3 Changing the Bargaining Threat Points

A productivity shock affects wages both directly and via an increase in the opportunity cost of employment, $rU$. The latter enters the wage equation (8) because in a standard Nash solution the threat points are constituted by the utility for firms and workers of being unmatched (the so-called outside options, $U$ and $V = 0$ for workers and firms, respectively). By changing the threat points, Hall and Milgrom (2008) avoid a higher opportunity cost of employment $rU$ being translated into higher wages.

Following the non-cooperative bargaining approach of Binmore \textit{et al.} (1986), Hall and Milgrom distinguish between outside options and disagreement pay-offs. A disagreement pay-off is what the party gets by prolonging the bargaining period – refusing the counterpart’s offer and making a counterproposal – and is independent of the outside conditions in the labour market (namely, market tightness). It is assumed to be a flow cost for the firm and a flow benefit for the worker. So, in Hall and Milgrom’s set-up the threat points are given by a weighted average of the outside option and the disagreement pay-off, the weights being the exogenous probability that the match is broken during bargaining and its complement, respectively.
Such probability is crucial in Hall and Milgrom’s analysis; the less likely a separation during bargaining, the weaker the impact of productivity on the wage via $U$. On the other hand, if the probability is equal to 1, the bargaining process coincides with the canonical Nash solution.

By setting the daily separation probability during the bargaining process at 0.0055, Hall and Milgrom succeed in matching the US data on vacancies and unemployment fluctuations. The elasticity of the wage with respect to productivity is 0.69, a result mid-way between the unit value suggested by Haefke et al. (2007) as a good target for the wage elasticity in matching models and the estimates obtained by examining aggregate wage statistics.

4.4 Asymmetric Information

As a possible solution to the poor quantitative performance of the matching model, Shimer (2005) also suggests the introduction of asymmetric information in the wage setting process. The results summarized in this section show that asymmetric information is not sufficient per se; to augment the volatility in the model, the amount of private information held by the employer must also increase during booms.

4.4.1 Acyclical Informational Rents

Brügemann and Moscarini (2007) explain the limited impact of asymmetric information on amplifying fluctuations by examining the properties of the wage equation. When the matching between a worker and a firm gives rise to quasi-rents, the wage can be divided into two parts: a fraction of the total rent generated by the match and the opportunity cost of employment, which is always procyclical. For instance, in the Nash solution, the former is given by $\beta(p - rU)$ and the latter by $rU$.

Brügemann and Moscarini first show that, in many models with asymmetric information in the bargaining process, workers’ rents are at most acyclical, but never countercyclical. Then, they prove that an acyclical worker’s rent is not sufficient to generate the observed business cycle fluctuations. In their baseline framework, the output of an employer–worker match is the sum of an aggregate productivity component and a match-specific one, the latter being the employer’s private information. Similarly, a match-specific amenity value of the job, known only by the worker, must be added to the wage to determine his or her utility in employment. Nesting three different wage determination schemes (a take-it-or-leave-it offer, sequential bargaining with one-sided asymmetric information, and a bilateral asymmetric information scheme) in such a set-up, they show that none of these arrangements is able to amplify unemployment and vacancy fluctuations.

Their conclusion is corroborated by the results of Guerrieri (2007). She constructs a competitive search model in which workers privately observe their type. Her numerical exercises show that asymmetric information does not help in replicating the vacancy and unemployment fluctuations observed in the data.
4.4.2 Procyclical Informational Rents

Brügemann and Moscarini analyse a symmetric set-up, in which both the employer and the employee hide some information from their counterpart and the outside economic conditions do not affect such informational rents. Two papers show that the performance of the model is improved if it is assumed that the gain that firms obtain by being more informed than workers increases during booms.

Kennan (2007) develops this idea by considering two aggregate states in the economy (1, the bad state, and 2, the good state) and two different idiosyncratic values for the productivity of a job (high or low). After matching with a worker, only the firm knows the idiosyncratic value of a job. So, if the firm makes the offer, it will get all the surplus and the worker will receive the opportunity cost of employment. On the other hand, if the worker makes the offer, he faces a potential trade-off between demanding a higher wage and being sure to reach an agreement. Kennan imposes two crucial assumptions: (1) workers always demand the low surplus; (2) when the aggregate state of the economy is good, there are more jobs with high idiosyncratic productivity.

Given these two assumptions, an average firm will also earn more profit during booms because the higher idiosyncratic value of the match does not translate into higher wages. Kennan shows that such a procyclical informational rent enjoyed by the firm magnifies the fluctuations in unemployment and vacancies.\(^{21}\)

A similar approach is pursued by Menzio (2005) in a wage-posting model with intra-firm bargaining. Firms have private information about their productivity type (composed of a permanent and a transitory part), and every period they make a wage offer both to their employees and to a fraction of unemployed workers contacted. Once the worker has observed all the offers received, he or she chooses a trading partner and the firm–worker negotiation begins. In the stable equilibrium of the extensive-form bargaining game, identical workers employed in the same firm cannot be paid differently; otherwise, a worker who had been discriminated against could ask for a renegotiation of payment conditions. Because of this non-discrimination constraint, the cost for a firm willing to adjust the wage bill in response to a positive productivity shock is proportional to the number of its employees. On the other hand, the benefit of raising the wage is given by a higher acceptance rate among contacted workers and a lower separation rate among the employees searching on-the-job. The shorter the expected duration of the shock, the smaller will be the marginal benefit of raising the wage. So, the wage is independent of the realization of productivity shocks if the shock’s persistence is below a certain threshold. This kind of wage rigidity, together with a low calibrated profit share, amplifies the volatility of unemployment and vacancy fluctuations (Menzio’s numerical results are \(\sigma_v/\sigma_p = 7.83\) and \(\sigma_v/\sigma_p = 6.45\)).\(^{22}\)

4.5 Final Remarks on the First Approach

It is undeniable that the wage flexibility implied by the Nash solution plays a role in weakening the volatility of unemployment and vacancies in response
to a productivity shock. However, Shimer (2005) and the subsequent literature summarized in this section have probably put too much emphasis on this aspect. Indeed, loosening or even breaking the link between wages and productivity in the model is not *per se* sufficient to replicate business cycle facts. A low profit share is also needed. In addition, the criticism raised by Pissarides (2007) and Haefke et al. (2007) concerns all the sticky wage models presented in this section except Hall and Milgrom (2008).

Models with asymmetric information deliver conflicting results. The business cycle consistency of the model is improved only by imposing procyclical information rents. Such an assumption however, is, difficult to test empirically.

5. Second Approach: Changes in the Calibration

The papers examined so far are based on the belief that Shimer is right both when he denounces the quantitative inconsistency of a standard matching model and when he identifies the Nash wage bargain as the main culprit. Hagedorn and Manovskii (2008) take a different route and focus on the calibration of the model.

Their model is substantially similar to the one presented in Section 2.4, the only difference being the introduction of capital into the production technology in order to measure the capital cost of vacancy creation. Some calibrated values are the same as in Shimer’s paper: $r = 0.012$, $f = 1.355$ and $s = 0.10$. Mean labour productivity is normalized to 1.

Hagedorn and Manovskii’s calibration method essentially differs in computing $\beta$ and $z$. To pin down these two variables and the matching function parameter, they use as targets $f = 1.35$, $\theta = 0.634$ (obtained by choosing a quarterly job-filling rate $f/\theta = 2.13$) and, more importantly, the elasticity $\eta_{w,p} = 0.449$. They obtain $z = 0.955$ and $\beta = 0.052$, in striking contrast to the values chosen by Shimer ($z = 0.4$ and $\beta = \alpha = 0.72$). Hagedorn and Manovskii claim that such a discrepancy can be explained by focusing on two aspects: (1) in the model, the profit share $(p − w)/p$ is small; and (2) wages are moderately procyclical in the data.

The first fact has already been emphasized in previous sections. A low profit share is necessary (but not sufficient) to amplify the business cycle fluctuations in unemployment and vacancies. As noted in Section 3.1, Shimer obtains a large value for $(p − w)/p$ because he sets $\beta = 0.72$, so $\beta f(\theta)$ is large relative to $s$ and $r$. But we can also have a high labour share by choosing a high value for $z$. By manipulating equations (5) and (8), we get

$$\frac{w}{p} = \frac{(r + s)[\beta + (1 - \beta)z/p] + \beta f(\theta)}{r + s + \beta f(\theta)}$$  \hspace{1cm} (13)

The labour share is close to 1 if the term inside the square brackets is close to 1. This can be obtained either with a large $\beta$ or with a high fraction $z/p$. But a high $\beta$ implies an elasticity $\eta_{w,p}$ close to unity (see equation (12)). Since Hagedorn and Manovskii target $\eta_{w,p} = 0.449$, they opt for a high fraction $z/p$ and a low $\beta$. 24

Hagedorn and Manovskii show that their results are not sensitive to the kind of matching technology assumed. Keeping the usual Cobb–Douglas matching
function, with $\alpha = 0.72$, gives the following elasticities: $\eta_{\theta p} = 26.8$ and $\eta_{fp} = 7.5$. Compared to the results presented in Table 2, Hagedorn and Manovskii’s calibrated model succeeds in amplifying the fluctuations in tightness and in unemployment. The reason stems from the high value of $z$ that raises the values of the elasticities $\eta_{\theta p}$ and $\eta_{fp}$.

5.1 Pros and Cons of the Second Approach

The parametrization performed by Hagedorn and Manovskii can be questioned on three different grounds.

First, with $z = 0.955$ the gap between the utility of being employed and the utility of being unemployed becomes extremely small (the difference $w - z$ is only 0.022). Is it realistic to think that employees work for a 2.2% surplus? Hagedorn and Manovskii list a series of reasons in favour of a high $z$. They claim, for instance, that since in the model the expected duration of the job search is low, it is plausible to imagine that, for people who are unemployed for 2.5 months on average, the utility gap is almost zero. Further, a standard real business cycle set-up with indivisibility of labour and without search frictions would imply no difference $w - z$, so their calibrated model can be viewed as a linear approximation to a richer framework in which workers take consumption-saving decisions and firms face a downward-sloping demand.

Second, targeting an elasticity $\eta_{wp}$ of 0.449 is in contrast with the conclusions of Haefke et al. (2007) that an elasticity of the wage for the new matches close to unity is the correct approximation in a canonical matching model with Nash bargaining.

Third, using Hagedorn and Manovskii’s calibration gives $\zeta_{ub} = 6.96$, three times larger than the value reported in Section 2.3. This confirms the existence of a business cycle/policy analysis trade-off. 25

6. Third Approach: Enriching the Standard Model

Papers following the third approach agree with Shimer’s diagnosis of a business cycle inconsistency in the standard matching model, but doubt that the Nash wage bargain is the main culprit for the quantitative failings of the model. The underlying motivation of these papers is that a textbook matching model is a useful tool for looking at the qualitative effects of a policy change or a shock, but is too stylized to be expected to be also consistent with data. Therefore, embedding other realistic features (such as on-the-job search, hiring and firing costs, imperfectly competitive product markets and firms’ heterogeneity) in the standard setting should make the model more suitable for quantitative scrutiny.

6.1 Turnover Costs

Silva and Toledo (2007) insert two different kinds of turnover costs into a standard matching model: training costs that firms spend on new entrants and separation costs suffered by employers when a job is destroyed.
Both the model and their calibration do not stray much from Shimer (2005). It is worth stressing that they set $\beta = 0.34$ and $z = 0.715$ in order to obtain $\zeta_{ub}$ equal to 2.0, in line with the policy evaluation estimates presented in Section 2.3, so avoiding the criticisms addressed to Hagedorn and Manovskii’s (2008) set-up.

Inserting Silva and Toledo’s calibrated values into their equations for the elasticities yields $\eta_{\theta p} = 6.62$, almost four times larger than in Shimer’s setting, $\eta_{fp} = 1.85$, $\eta_{u,p} = 1.747$ and $\eta_{w,p} = 1$. Recall that in the US data $\sigma_f/\sigma_p = 5.9$ and $\sigma_u/\sigma_p = 9.5$, while Haefke et al. (2007) suggest that a unit wage elasticity is a good proxy in a standard matching framework. Silva and Toledo’s paper improves upon the business cycle consistency of the model while keeping it consistent with the microeconometric estimates presented in Section 2.3.26

Why do turnover costs improve the performance of the model? The mechanism is analogous to that obtained by a low profit share. Turnover costs lower the value of a filled job, so a higher $p$ yields a large percentage increase in profits. More vacancies are posted in order to restore the free-entry equilibrium, enhancing the tightness elasticity.

A similar effect is present in Garibaldi’s (2006) paper. He considers two types of large (multiple-job) firms in his model: those that, having a high productivity value, react to adverse shocks by simply posting fewer vacancies, and those ‘at the margin’, with a very low profit share. When a negative productivity shock hits the economy, this second kind of firm either declares bankruptcy; firing all its employees, or freezes its hirings, not replacing the workers who leave. If the expected losses are greater in absolute value than the firing costs, the firm declares bankruptcy; otherwise it experiences a ‘hiring freeze’. For firms ‘at the margin’ a small change in productivity can make a great difference, and the number of vacancies they post in good times is much higher than the number of vacancies posted in bad times. The volatility of this variable greatly increases. Hiring freezes and bankruptcy allow Garibaldi to explain up to 35% of the tightness volatility displayed in the data.

6.2 On-the-job Search

6.2.1 Hiring Costs and On-the-job Search

Nagypál (2005) shows that a combination of hiring costs and on-the-job search may reconcile the model with the empirical evidence.

In her set-up, firms incur hiring costs – i.e. they pay a fixed amount of resources upon matching – and employees search for jobs with a higher idiosyncratic pay-off. Hence, some matches have a negative pay-off for firms, for an employee working in a job with a low idiosyncratic pay-off can leave before the employer’s initial investment has been recouped. Firms do not know the idiosyncratic value of the match, but they realize that unemployed searchers are more likely to accept matches with a low match quality than employed searchers. This is due to a positive selection effect that shifts workers into match qualities towards
the top of the distribution. So, with high hiring costs firms have a lower expected pay-off from contacting an unemployed searcher than an employed one. After a positive shock on $p$, the fraction of employed searchers in the total number of searchers increases, making firms even more willing to post vacancies.

Nagypál computes the elasticity of the job-finding rate in response to a positive shock on productivity and a negative one on the separation rate, in order to amplify the effects in terms of vacancy creation. If hiring costs are set at two or three times the quarterly profit flow $p - w$, this implies a $\sigma_f/\sigma_p$ of 3.086 and 7.168, respectively, close to the value of 5.9 found by Shimer in the data.

6.2.2 Wage Heterogeneity

An increase in the number of job seekers in response to a positive shock on productivity may enhance the volatility of vacancies and unemployment in the model, because it increases the job-filling rate, boosting vacancy creation. This is the mechanism at work in Krause and Lubik’s (2007) paper. They consider an economy with a high-paid sector and a low-paid one. The employees who work in the low-paid sector exert some effort in searching on-the-job. Following an economy-wide positive productivity shock, firms post more vacancies both in the high-paid and in the low-paid sectors. Increased tightness in the high-paid sector raises the search effort of the low-paid employees, implying a larger number of job seekers in terms of search units. The rate at which a high-paid job is filled goes up, so making firms even more willing to post vacancies there. The process ends because of a convexity assumption of the search cost, delivering a larger amplification in vacancy posting in response to a productivity shock. A complementarity between sectors also arises. If the search effort of low-paid employees goes up, congestion effects in the matching technology will make it more difficult for unemployed workers to find a high-paid job. Then, they will direct their search towards the low-paid sector. This in turn will boost vacancy creation in that sector. The model almost succeeds in replicating 90% of tightness fluctuations, but it predicts too little wage volatility ($\sigma_w/\sigma_p = 0.117$).

6.3 Embodied Technical Change

Shimer assumes that the productivity shock hits all the matches in the economy in the same way. Costain and Reiter (2007) and Reiter (2007) argue that a model with embodied technical change helps to solve the amplification puzzle. In their model, the productivity of a match is denoted by $Y$ and takes the form

$$Y = (1 - \xi_p) p + \xi_p P_m$$

$0 \leq \xi_p \leq 1$

in which $p$ is the current level of productivity and $P_m$ is the level of aggregate productivity at the time the match was formed. The higher the value of the parameter $\xi_p$, the more embodied is the technical change in the match. When $\xi_p = 0$, the model is identical to the standard discussed in Section 2.4.
Why should embodied technical change amplify fluctuations? The reason is twofold. First, if the observed productivity is an average of past vintages, then the productivity for new matches, which is the source of fluctuations in unemployment and vacancies, is underestimated. The calibrated model must therefore account for the higher variability of current productivity.

Second, embodied technical change makes employers’ surpluses more procyclical. The productivity of a match is only partially affected by current shocks. Anticipating this, firms will post many vacancies when the shock is positive. Fluctuations in vacancies and unemployment are bigger than in the disembodied productivity case. With $\xi_p = 0.302$ or $\xi_p = 0.576$, Reiter (2007) gets a ratio $\sigma_\theta/\sigma_p$ equal to 7.9 or 18.65, respectively, while the same set-up with $\xi_p = 0$ delivers a relative standard deviation of 4.15.

Costain and Reiter (2007) highlight two shortcomings of the model. First, the endogenous separation rate should be taken into account, because workers could find it optimal to quit their job when new matches with higher productivity are created in the economy. This kind of separation raises the number of unemployed people in upturns, destroying the Beveridge curve. The second shortcoming concerns the excessive wage volatility delivered by the model. 27

To overcome these problems, Reiter (2007) introduces two features: a long-term wage contract à la Rudanko (2007) and turnover costs à la Silva and Toledo (2007). The former reduces the wage volatility, while post-match training costs paid by the workers make it more costly for them to quit a job and search for another opportunity. Reiter’s results seem to go in the right direction.28

6.4 The Role of the Separation Rate

What is the role of the separation rate in amplifying fluctuations in unemployment and vacancies? Several scholars have addressed this question. To clarify their conclusions, it is important to distinguish between models that consider an exogenous separation rate and models in which it is endogenous.

In the former case, the idea consists of incorporating a countercyclical shock on the separation rate into the standard model with a productivity shock. Recall from Section 3.1 that Shimer (2005) has already performed such an exercise, with ambiguous results. Fluctuations in unemployment are amplified, because the level of this variable along the cycle is affected not only by procyclical job creation but also by countercyclical separation rates. The problems concern the behaviour of vacancies. An increase in the separation rate has two effects, opposite in sign. On the one hand, it raises the factor to which profits are discounted, stifling vacancy creation (job creation effect). On the other hand, it shifts the Beveridge curve to the right, so more vacancies must be posted for the equality of labour market flows to hold; this is obtained by an increase in the job-filling rate $f(\theta)/\theta$ that induces more firms to enter the market (Beveridge curve effect). As a result, a positive shock on the separation rate may deliver a counterfactual increase in vacancies. Figure 1
illustrates the two mechanisms at work. The fluctuations in tightness and the job-finding rate are insufficiently low, since they are bigger the more procyclical is the level of vacancies.

Such discouraging conclusions are in part reversed by Mortensen and Nagypál (2007b). They show that the counterfactual increase in vacancies during troughs can be ruled out if a wage bargaining à la Hall and Milgrom (2008) is introduced into the model. Loosening the link between wages and tightness amplifies the negative effect of the separation rate on vacancy creation, for the decrease in expected profits is not offset by a lower wage. So, the equality of labour market flows is reached at a level of vacancies lower than before. The job creation 0-ray shifts down more than under the canonical Nash solution.29

More recent papers have sought to answer Shimer’s puzzle by introducing endogenous separation rates. Despite its theoretical appeal, such a move does not seem to produce satisfactory results. Whether analysing the standard Mortensen and Pissarides (1994) model (Pissarides, 2007) or considering an extended version of it (Mortensen and Nagypál, 2007a), the conclusions are similar: endogenous separations have little impact on job creation and the volatility of tightness. The reason is well explained by Pissarides. When the separation rate is exogenous, all jobs are equally likely to be destroyed. Job creation is dampened because a positive shock on \( s \) reduces the expected profits of any match, regardless of its productivity value. When the separation rate is endogenous, only matches with a value close to the reservation productivity are destroyed. Job creation is scarcely affected because the expected returns of such jobs are close to zero and firms and workers are indifferent about continuing the match or separating.

Elsby and Michaels (2008) and Andrés et al. (2006) are able to match the data in a model with endogenous separations, but their models include several mechanisms that may potentially amplify vacancy fluctuations. In Elsby and Michaels’ paper, the assumption of decreasing marginal productivity and intra-firm bargaining is key. Firms hire more than one worker and, under Nash bargaining, the wage is a constant fraction of firms’ marginal revenues. The employers take into account the marginal decrease in the wage bill caused by hiring one additional worker, so, ceteris paribus, they will post more vacancies than in the standard linear case.30 The increase in job creation caused by a positive shock on productivity outweighs the Beveridge curve effect, and vacancies are strongly procyclical.

6.5 Price Rigidity

Andrés et al. (2006) construct a DSGE model in which several mechanisms affect vacancy behaviour: these include endogenous separation rates, capital, taxes, intertemporal substitution and price rigidity. The latter gives the most decisive contribution in terms of tightness amplification.

Their model features a two-tier productive scheme: wholesale firms operate in a competitive market using labour and capital, and retail firms adopt as their only input the good bought by wholesale firms and are monopolistically
competitive. The final consumption good is a composite of different varieties produced in the retail sector. Like the wage contracts in Gertler and Trigari (2006), the price decisions of the retail firms are staggered over time.

When a positive shock affects the productivity of the wholesale firms, the relative price of their good immediately goes down, due to the price rigidities in the retail sector. Yet, it soars in the following periods because of the downward adjustment of prices in the retail sector. Such a jump is less smooth than in the case of flexible prices, so that the value of matching for the wholesale firms varies more and vacancy creation is boosted.

6.6 Other Sources of Shocks

If changes in labour productivity are tiny in comparison with the variations in the labour market, this may be because labour productivity is not the (only) exogenous driving force in the economy. As a result, the failure of the model does not depend on the lack of an amplification mechanism, but on the misidentification of the shock hitting the economy.

Such a reading of the Shimer puzzle has attracted the attention of several scholars. The papers following this route present a rich framework that encompasses several departures from the standard search and matching model. While the results in terms of amplification and propagation mechanisms are often promising, it is difficult to disentangle all the effects at work in these set-ups.

6.6.1 A VAR Approach

Yashiv (2005, 2006) considers a reduced-form vector autoregression (VAR) of the actual data to specify the driving shocks. Assuming that three variables (the rate of productivity growth, the separation rate and the interest rate) follow a first-order VAR, his model captures the persistence, the volatility and some co-movements of the main labour market variable in the data. The high persistence of vacancies is obtained by imposing convex hiring costs that make vacancy creation more sluggish. However, convex costs also tend to reduce the volatility of the vacancies. To amplify fluctuations, according to Yashiv, the stochastic properties of the separation rate play a crucial role. He rightly argues that the separation rate is key in evaluating the expected discounted value of a match. Nevertheless, it is not clear why this is so essential in engendering the correct volatility in Yashiv’s VAR approach, while Shimer’s setting with contemporaneous shocks in both $p$ and $s$ does not deliver analogous outcomes.

6.6.2 Market Power

While in Shimer’s model a technological shock hits all jobs, Rotemberg (2006) considers a change in firms’ market power as the driving force in the economy. The key is that in Shimer’s set-up the marginal productivity of labour and, in turn, the wages increase; in Rotemberg’s model, firms react to fiercer competition by
producing more, and labour productivity goes down. Workers have to moderate their wage demand, while employment goes up.$^3$1

Rotemberg’s main objective is to match data on wage volatility. In his model $\sigma_w/\sigma_p = 0.56$, a value that fits the data on the aggregate wage targeted by the author, but that is inconsistent with the conclusions of Haefke et al. (2007) about a unit wage elasticity (and so, an even higher ratio of standard deviations) being the right target in a matching model with Nash bargaining. Further, the model overshoots on vacancies volatility, whereas it captures one-third of the employment fluctuations and half of unemployment volatility.

However, as Trigari (2008) documents in a detailed comment, the success of the paper depends less on the imperfect competition and diminishing returns of labour than on the high value assigned to the instantaneous utility in unemployment (Rotemberg sets $z/w = 0.9$), confirming once again the importance of a low profit share in amplifying fluctuations.

6.6.3 Establishment-level Shocks

Cooper et al. (2007) build a matching model with establishment-level profitability shocks and succeed in fitting the cyclical behaviour of unemployment, vacancies and wages. In their framework, firms can adjust both the intensive and the extensive margins (hours worked and level of employment) and face fixed and variable costs when hiring or firing workers. Firms’ production function is subject both to aggregate and idiosyncratic (establishment-level) shocks. The objective of their paper is not only to solve Shimer’s puzzle but also to match some establishment observations, such as the negative correlation between hours growth and employment growth. It is difficult to isolate all the effects that are at work in such a rich setting. The authors argue that their set-up is able to fit the aggregate data partly because of the distribution of the idiosyncratic shocks that are not smoothed out by aggregation, while another potentially interesting feature of the model – the ability of firms to fire workers as in Garibaldi (2006) – is unexplained.

6.6.4 Monetary Shocks

A recent strand of the literature has investigated the contribution of demand (monetary and non-monetary) shocks to the labour market. In this respect, these papers belong to the vein of New Keynesian models, in which several departures from a standard real business cycle framework contribute to the response of the labour market variables.

The paper that most convincingly stresses the importance of monetary shocks in understanding the Shimer puzzle is Barnichon (2007). Barnichon argues that the cyclical changes in labour productivity, considered by Shimer as the only determinant of unemployment and tightness fluctuations, are actually caused by...
exogenous shocks on monetary policy. Productivity and tightness co-move in response to the same shock, but there is no causal relationship between them. In his model a shock that increases the supply of money is able to explain about 50% of the tightness volatility.

Braun’s (2005) model presents many features that may potentially amplify the effects of monetary policy shocks: wage stickiness, price rigidity in the intermediate sectors, a high opportunity cost of employment and training costs. Having estimated an identified VAR on the US data, Braun finds that wage rigidity and a high opportunity cost of employment contribute more decisively to matching the empirical responses.

Braun et al. (2007) also adopt a VAR approach to investigate which shock is most crucial in driving labour market fluctuations. They identify three kinds of shocks: supply shocks that are required to have opposite effects on output and the price level; demand shocks that move output and prices in the same direction; and monetary shocks that also push down the interest rate. Their conclusion is that, although the response of hours worked, unemployment and vacancies is qualitatively the same regardless of the shock considered, demand shocks may be more important in driving labour market fluctuations.

7. The Propagation of Shocks

A standard matching model not only falls short of replicating labour market fluctuations but also exhibits no propagation of productivity shocks. Three facts are worth stressing. (1) Data show that the maximum correlation between vacancies and productivity is observed when productivity is lagged by one or two quarters, while in the model the peak is reached at zero lags. (2) In the data, labour market tightness follows productivity by one year and the contemporaneous correlation between these two variables is 0.40. Simulation results, on the contrary, predict a correlation $\rho_{\theta p}$ of 0.999. The correlations between productivity and the other labour market variables are also greatly overestimated. (3) The autocorrelation of vacancies is lower in the model than in the data.

The absence of propagation in a matching model depends on how vacancy behaviour is modelled. As Pissarides (2000, pp. 26–31) stresses, the vacancy rate – and consequently market tightness – is a ‘jump variable’, so it adjusts too rapidly in response to a productivity shock. Thus, any mechanism allowing for more sluggish behaviour of the vacancy rate should yield more realistic dynamics.

Fujita (2003), Fujita and Ramey (2007b) and Hagedorn and Manovskii (2007) address this issue. Hagedorn and Manovskii introduce time-to-build into job creation. Any vacancy created at a certain moment enters the market with some delay. Their model can match the contemporaneous correlation of tightness and productivity, but still fails to predict the autocorrelation of vacancies.

In Fujita and Ramey’s (2007b) model, a firm–worker pair can be destroyed for two reasons: obsolescence, meaning that the worker becomes unemployed and the position disappears, or ‘normal’ separation, meaning that both the worker and the
job position enter the matching pool in the following period. Moreover, the zero profit equilibrium equation is assumed to be \( V_t = Kn_t \). The value of a vacancy at time \( t \) is equal to a sunk cost that Fujita and Ramey assume increases with the number of positions created in the economy at \( t, n_t \).

These changes have two effects. First, marginal costs that increase with the number of positions induce firms to spread out vacancy creation. In turn, this translates into a more realistic value for the autocorrelation of vacancies, as in the Yashiv (2006) framework with convex hiring costs. Second, since the value of a vacancy is strictly positive at the equilibrium, firms are willing to keep a position open after a separation. Vacancies become a predetermined variable. Unlike Shimer (2005) and Hagedorn and Manovskii (2007), Fujita and Ramey first estimate a reduced-form VAR to study the dynamics of employment, tightness and productivity. Then they compare their simulation results with conditional empirical correlations. Their model delivers a more realistic propagation mechanism, both in terms of cross-correlations and in terms of impulse response.

8. Concluding Remarks

Shimer (2005) and Costain and Reiter (2007) have called the quantitative consistency of matching models into question. Considering the relatively short period that has elapsed since the publication of these papers, many scholars have reacted to their findings, and they have done so with competing approaches.

Shimer’s results concern the inability of the model to reproduce realistic fluctuations and propagation of shocks. As far as the propagation problem is concerned, Fujita and Ramey (2007b) and Hagedorn and Manovskii (2007) are able to improve the dynamics of the model by making the response of vacancies more sluggish.

As regards the amplification puzzle, scholars have followed three different routes, which are summarized in Table 3. Unfortunately, it is not possible to adopt a unique criterion in evaluating all the papers: some of them ignore policy analysis issues, others do not report statistics on the real wage, and some present their findings in terms of impulse response functions and not in terms of elasticities or relative standard deviations. Table 3 can therefore only give an idea of the results presented in the previous sections.

However, the general conclusion that may be drawn is that the third approach appears to be the most effective. A model with turnover costs improves the business cycle fluctuations and fits the policy analysis long-run estimates. Cohort-specific shocks on productivity match the data quite well and are empirically plausible.

Nevertheless, some questions remain open. To what extent is job separation important in explaining unemployment fluctuations? Can a standard matching model match business cycle facts in other countries, in Europe for instance? The macroeconomic performance of matching models is far from being an exhausted research area.
Table 3. The Amplification Puzzle.

| First approach | Second approach | Third approach |
|----------------|-----------------|---------------|
| Completely rigid wages | Shimer (2004) | Hagedorn and Manovskii (2007, 2008) |
| Fit the data if profit share is low. Wage elasticity empirically inconsistent (see Haefke et al., 2007; Pissarides, 2007) | Fit the business cycle data. However, the policy analysis consistency of the model is reduced and the wage elasticity they target is too low (see Haefke et al., 2007; Pissarides, 2007) | Fit the model both from a business cycle and from a policy analysis viewpoint |
| Sticky wages | Farmer and Hollenhorst (2006) |  |
| Amplify fluctuations. The calibrated value for the leisure disutility parameter is decisive |  |
| Staggered contracts | Gertler and Trigari (2006), Bodart et al. (2006) |  |
| Improve the model. Wage elasticity empirically inconsistent (see Haefke et al., 2007; Pissarides, 2007) |  |
| Social norm | Hall (2005) |  |
| Improve the model. Wages of new hires too sticky (see Haefke et al., 2007; Pissarides, 2007) |  |
| Long-term contracts | Rudanko (2007) |  |
| No amplification. Wages too rigid in two out of three contract types |  |
| Changing the bargaining threats | Hall and Milgrom (2008) |  |
| Fit the empirical conditional moments on \( u \) and \( v \). The separation during bargaining parameter is crucial |  |
| Acyclical informational rents | Brugemann and Moscarini (2007), Guerrieri (2008) |  |
| No amplification |  |
| Cyclical informational rents | Kennan (2007), Menzio (2005) |  |
| Amplify fluctuations. Kennan’s model hard to check empirically |  |
| Changes in the calibration |  |
|  |
|  |
|  |  |
|  |  |
|  |  |
Table 3. Continued.

| Mechanism                                      | Author(s)                                      | Description                                                                 |
|------------------------------------------------|-----------------------------------------------|-----------------------------------------------------------------------------|
| Hiring freeze                                  | Garibaldi (2006)                              | Amplify fluctuations                                                        |
| Hiring costs + on-the-job search               | Nagypál (2005)                                | Amplify fluctuations                                                        |
| On-the-job search + job quality                | Krause and Lubik (2006)                       | Amplify fluctuations                                                        |
| Embodied technical change                      | Costain and Reiter (2007), Reiter (2008)     | Fit the model. Excessive wage volatility is corrected by long-term wage contracts. The Beveridge curve is restored by introducing turnover costs |
| Shock on the separation rate + shock on $p$    | Mortensen and Nagypál (2007)                  | If the bargaining process is as in Hall and Milgrom (2008), both unemployment and vacancies fluctuations are fitted and the Beveridge curve is not destroyed |
| Endogenous separation rate                     | Mortensen and Nagypál (2008), Pissarides (2007) | The vacancy fluctuations are not amplified as in the case of a shock in the separation rate |
| Price rigidity                                 | Andrés et al. (2006)                          | Along with other amplification mechanisms, it succeeds in matching the data |
| VAR approach                                   | Yashiv (2005, 2006)                           | The separation rate is key in matching data on unemployment and vacancies volatility |
| Market power                                   | Rotemberg (2006)                              | No satisfactory results                                                      |
| Establishment-level shocks                     | Cooper et al. (2007)                          | Fit the data                                                                |
| Monetary policy                                | Barnichon (2007), Braun (2005), Braun et al. (2007) | Barnichon explains 50% of the Shimer puzzle as a misidentification of shocks |
|                                                 |                                               | Braun (2005) shows that wage rigidity and a low profit share are crucial even in the presence of a monetary shock |
|                                                 |                                               | Braun et al. (2007) argue that demand shocks are at least as important as supply shocks in driving labour market fluctuations |
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Notes

1. This model was pioneered by Diamond, Mortensen and Pissarides. See Mortensen and Pissarides (1999) and Pissarides (2000) for a detailed exposition.
2. This point was first spelt out by Hornstein et al. (2005) and by Mortensen and Nagypály (2007b).
3. The same space limitations do not allow me to review some recent dynamic stochastic general equilibrium (DSGE) matching models that bear some relationship to the issues raised in this survey, but whose main scope is not to react to Shimer’s findings. For instance, Blanchard and Gali (2005) and Walsh (2005) are more oriented towards monetary issues, while Veracierto (2002) focuses on the dynamic behaviour of labour force participation. Quadrini and Trigari (2007) show that accounting for the public sector may enhance the employment fluctuations, but they do not provide data on vacancies and the job-finding rate.
4. Statistics on unemployment are constructed by the Bureau of Labor Statistics from the Current Population Survey (CPS), while measures on job vacancies are proxied using the the Conference Board help-wanted index that computes the number of help-wanted advertisements in 51 major newspapers. The Job Openings and Labor Turnover Survey (JOLTS) would be the ideal source for an analysis of job vacancies, but it only covers data from December 2000. However, a comparison of the two measures from 2000 to 2003 shows that the Conference Board help-wanted index does not differ substantially from the JOLTS.
5. To measure the job-finding rate, Shimer prefers not to use gross worker flow data, for the data set is only available since 1976 and measurement and classification errors could bias his estimation. Instead, he constructs the data by using the monthly number of unemployed people and assuming that workers are homogeneous and neither enter nor exit the labour force. In a subsequent paper, he shows that relaxing these strong assumptions does not bias his computations (see Shimer, 2007).
6. Hagedorn and Manovskii (2007) have recently pointed out that the correlation between productivity and market tightness is strikingly different depending on whether CPS data or the Current Employment Statistics (CES) are used. The reason has still to be ascertained. The papers I will survey in the following sections have followed Shimer (2005) and considered the CES data.
7. Since the variables are expressed in logs, ordinary least squares (OLS) estimation tells us that $\eta_{wp} = \rho_{wp} \cdot \sigma_w / \sigma_p$.
8. Moreover, Davis and Haltiwanger only consider manufacturing establishments in which job separation is more volatile than in the rest of the economy.
9. Mortensen and Nagypály (2007b) prove that under some conditions an equilibrium solution defined as a vector of functions $(w_p, \theta_p, U_p, W_p, J_p, S_p)$ for any possible
value of productivity $p$ exists, is unique, and that all the functions are increasing in $p$. For the proof, I refer to them.

10. Shimer sets $\lambda = 4$, a relatively large value, but also imposes $\Delta$ equal to 0.0083, so the approximation can be accepted.

11. I compute the semi-elasticity of $u$ with respect to $b$ and not the elasticity simply to follow Costain and Reiter (2007) and to compare the results of the model more easily with the estimates presented in Section 2.3.

12. The wage share is $w/p = \beta + \beta c \theta/p + (1 - \beta)z/p$. Hornstein et al. (2005) show that if $r$ and $s$ are much smaller than $\beta f(\theta), c \theta/p \approx [(1 - \beta)/\beta](1 - z/p)$. So $w/p \approx 1$.

13. Actually, Layard and Nickell (1999) consider the semi-elasticity of unemployment with respect to the replacement ratio. This gives $b = \nu w$, with $\nu$ being the replacement ratio. However, in the present calibration, $w \approx p = 1$. So $\zeta_{u,b} \approx \zeta_{u,v}$.

14. Hornstein et al. (2005) show that, when wages are rigid, the comparative statics elasticity $\eta_{\theta p}$ overestimates the response of $\theta$ to a productivity shock. For simplicity, I still compute $\eta_{\theta p}$, considering it as an upper bound for volatility. Shimer (2004) sets $\alpha = 0.5$. In this case, $\eta_{\theta p} = 60.6$.

15. Another shortcoming of the rigid wage model is the lack of persistence in vacancies: their autocorrelation is 0.715, while in the data it is 0.930. I will discuss this issue in Section 7.

16. This is what Gertler and Trigari assume. Bodart et al. allow a fraction of new jobs to have a freely negotiated wage, but their sensitivity analysis shows that such a fraction must be close to 0 to yield realistic unemployment fluctuations.

17. Actually, the model gets close to the data only by setting the instantaneous utility of unemployment $z$ higher than 0.8. This confirms again the crucial role played by a low profit share.

18. Following Mortensen and Nagypál (2007b), they disentangle the volatility captured by fluctuations in productivity from that beyond the reach of a productivity explanation.

19. The rationale behind this point goes as follows. If the worker’s fraction of the total rent is rigid and large, this implies that the capital gain from finding a job is also high. Data show that $f(\theta)$ is extremely volatile and procyclical. But a high job-finding rate and a large capital gain in booms will enhance the opportunity cost of employment that, in turn, will generate strongly procyclical wages, so dampening the incentives for vacancy creation during booms. Brügemann and Moscarini call such a mechanism the feedback effect. On the other hand, if the rent accruing to the worker is low and acyclical, the same problem occurs as in the rigid wage model. A lower rent going to the employee means large profits for the firm, so that a percentage increase in productivity will enhance only them by a small percentage amount. Firms will not create many vacancies in booms. This second mechanism is called the congestion effect by the authors, because it depends on the free-entry condition that links the number of vacancies posted directly with firms’ profits. Both the congestion effect and the feedback effect limit the response of tightness to a productivity shock.

20. Workers are heterogeneous in terms of the sunk training costs they incur at the beginning of their employment.

21. Kennan compares the steady-state rate of unemployment both in the bad and in the good state of the economy (denoted as $u_1$ and $u_2$, respectively) with the corresponding values obtained in a model without asymmetric information. The
informational rent moves unemployment by about 40%, even though the difference in productivity levels is only 3%: \( u_1 = 5.6\% \) and \( u_2 = 5.5\% \) in the case of complete information, whereas \( u_1 = 7.5\% \) and \( u_2 = 5.2\% \) when productivity is observed privately by the employer.

22. Menzio also introduces concave vacancy costs. This assumption can by itself magnify the volatility of vacancies.

23. Apart from the values of \( \beta \) and \( z \), other departures from Shimer’s calibration are the following: (i) a different matching function \( m(u, v) = uv/(u^l + v^l)^{1/l} \), with \( l \) being the only parameter to be estimated; (ii) a quarterly job-filling rate \( f/\theta = 2.13 \), so that \( \theta = 0.634 \); (iii) the total flow cost of opening a vacancy defined as \( c = c^K p + c^W p^{\eta_w} \), with \( c^K = 0.474 \) being the capital cost and the labour cost \( c^W \) being equal to 4.5% of the quarterly wages of a new hire or 11% of labour productivity.

24. As stressed in Section 3.1, the correlation between tightness and productivity \( \rho_{\theta p} \) is 0.393 in the data and close to 1 in the model. So, the wage elasticity \( \eta_{w p} \) and the ratio of standard deviations \( \sigma_w/\sigma_p \) are virtually identical in the model, while in the data the former is about 60% lower than the latter. In a subsequent paper, Hagedorn and Manovskii (2007) address this question and show that adding an additional shock to non-market activity to the model allows both \( \eta_{w p} \) and \( \sigma_w/\sigma_p \) to be targeted. The calibration method they employ is the same as that shown in this section.

25. Hagedorn and Manovskii react to this point, raising some doubts about the endogeneity problems, that in cross-country regressions such as those performed by Costain and Reiter (2007) cannot be ruled out.

26. Actually, in their simulation part, Silva and Toledo get \( \sigma_{\theta}/\sigma_{p} = 20 \), much larger than \( \eta_{\theta p} \) and very close to the ratio \( \sigma_{\theta}/\sigma_{p} \) found in the data. Such a discrepancy probably depends on the choice of the grid step size \( \Delta = 0.053 \) and the arrival rate \( \lambda = 0.4 \) made by the authors. These values are not sufficiently close to 0 for the approximation to hold.

27. Wages are volatile because the worker’s outside option that depends on the current value of productivity, and not on the productivity of the match, is strongly procyclical.

28. With long-term contracts and \( \xi_p = 0.302 \), the elasticity of the wages for new entrants with respect to productivity is about 1, but the elasticity of the aggregate wage is too small (less than 0.1). Further, turnover costs reduce the number of endogenous separations.

29. A similar effect is obtained by Nagypál (2005) by introducing on-the-job search into the standard model. A higher separation rate encourages firms to post more vacancies because it raises the number of unemployed people, who are the only job seekers in the economy. In a framework with on-the-job search, an increase in \( s \) has a smaller impact on the total number of searching workers and the incentive for firms to post more vacancies as \( s \) goes up is weakened.

30. On intra-firm wage bargaining within a matching framework see Cahuc and Wasmer (2001).

31. Rotemberg also assumes concave vacancy costs that induce firms to post more vacancies in booms, so amplifying the volatility of such variable. Comparing Yashiv’s and Rotemberg’s models leads to the conclusion that concave vacancy costs tend to amplify the shocks but worsen the ability of the model to propagate them. Convex costs engender the opposite effects.
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