Research Article

Robust Finite-Time $H_\infty$ Control for Nonlinear Markovian Jump Systems with Time Delay under Partially Known Transition Probabilities

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This paper is concerned with the problem of robust finite-time $H_\infty$ control for a class of nonlinear Markovian jump systems with time delay under partially known transition probabilities. Firstly, for the nominal nonlinear Markovian jump systems, sufficient conditions are proposed to ensure finite-time boundedness, $H_\infty$ finite-time boundedness, and finite-time $H_\infty$ state feedback stabilization, respectively. Then, a robust finite-time $H_\infty$ state feedback controller is designed, which, for all admissible uncertainties, guarantees the $H_\infty$ finite-time boundedness of the corresponding closed-loop system. All the conditions are presented in terms of strict linear matrix inequalities. Finally a numerical example is provided to demonstrate the effectiveness of all the results.

1. Introduction

Markovian jump systems, a class of hybrid dynamical systems, which consists of an indexed family of continuous or discrete-time subsystems and a set of Markovian chain that orchestrates the switching between them at stochastic time instants, have received extensive attention over the past few decades [1, 2]. Many real world processes, such as economic systems [3], manufacturing systems [4], electric power systems [5], and communication systems [6], may be modeled as Markovian jump systems when any malfunction of sensors or actuators cause a jump behavior in process performance. Recently, nonlinear Markovian jump systems have been extensively applied and developed in various disciplines of science and engineering, and a great number of excellent works have been developed [7–9].

Generally speaking, the behavior of nonlinear Markovian jump systems is determined by the transition probabilities in the jumping process. Usually, it is assumed that the information on transition probabilities was completely known. However, transition probabilities may be partially known for some real systems. For example, the networked control systems can be modeled by nonlinear Markovian jump systems with partially known transition probabilities when the packet dropouts or channel delays occur [10]. In addition, there are few results about the known bounds of transition probability rates or the fixed connection weighting matrices [11, 12]. Therefore, it is reasonable to study Markovian jump systems with partially known transition probabilities, especially, when it is difficult to measure the bounds of transition probability rates. It stimulates the research interests of the author.

Uncertainties and time delay frequently occur in various engineering systems, which usually is a source of instability and often causes undesirable performance and even makes the system out of control [14, 15]. Therefore, time delay systems with robustness have received an increasing attention among the control community [16–18]. On the other hand, one may be interested in not only system stability but also a bound of system trajectories over a fixed short time [19]. For instance, for the problem of robot arm control [7], when the robot works under different environmental conditions with changing payloads, it requests that the angle position of the arm should not exceed some threshold in a prescribed time interval. Meanwhile, the scholars attach more importance to the $H_\infty$ control problem, which is to find a stable controller such that the disturbance attenuation level $\gamma$ is below a prescribed level. There are a great number of useful and interesting results about $H_\infty$ control problem for linear and nonlinear Markovian jump systems in the literature [20–25]. To the best of our knowledge, the synthesis issue of
robust finite-time $H_\infty$ control for nonlinear Markovian jump systems with time delay under partially known transition probabilities has not been fully investigated until now, which motivates us to carry out the present study.

In this paper, we investigate the problem of robust finite-time $H_\infty$ control for nonlinear Markovian jump systems with time delay under partially known transition probabilities. The main contributions lie in the fact that some tractable sufficient conditions are provided to ensure $H_\infty$ finite-time boundedness or finite-time $H_\infty$ state feedback stabilization.

A robust finite-time $H_\infty$ state feedback controller is designed, which guarantees the $H_\infty$ finite-time boundedness of the closed-loop system. Seeking computational convenience, all the conditions are cast in the format of linear matrix inequalities. Finally, a numerical example is provided to demonstrate the effectiveness of the main results.

**Notations.** Throughout this paper, the notations used are fairly standard. For real symmetric matrices $A$ and $B$, the notation $A \succeq B$ (resp., $A > B$) means that the matrix $A-B$ is positive semi-definite (resp., positive definite). $A^T$ represents the transpose matrix of $A$, and $A^{-1}$ represents the inverse matrix of $A$. $\lambda_{\text{max}}(B)$ ($\lambda_{\text{min}}(B)$) is the maximum (resp., minimum) eigenvalue of a matrix $B$. $\text{diag}[A; B]$ represents the block diagonal matrix of $A$ and $B$. $I$ is the unit matrix with appropriate dimensions, and the term of symmetry is stated by the asterisk $*$ in a matrix. $\mathbb{R}^n$ stands for the $n$-dimensional Euclidean space, $\mathbb{R}^n_{\text{loc}}$ is the set of all $n \times m$ real matrices, and $\mathcal{M} = \{1, 2, \ldots, N\}$ means a set of positive numbers. $\| \cdot \|$ denotes the Euclidean norm of vectors. $E[\cdot]$ represents the mathematical expectation of the stochastic process or vector. $L^2_{\text{loc}}[0, +\infty)$ is the space of $n$-dimensional square integrable function vector over $[0, +\infty)$.

### 2. Problem Formulation and Preliminaries

Give a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\Omega$ is the sample space, $\mathcal{F}$ is the algebra of events, and $\mathbb{P}$ is the probability measure defined on $\mathcal{F}$. The random process $\{r(t), t \geq 0\}$ is a Markovian stochastic process taking values in a finite set $\mathcal{M} = \{1, 2, \ldots, N\}$ with the transition probability rate matrix $\Pi = \{\pi_{ij}\}, i, j \in \mathcal{M}$, and the transition probability from mode $i$ at time $t$ to mode $j$ at time $t + \Delta t$ is expressed as

$$P\{r(t + \Delta t) = j \mid r(t) = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

with the transition probability rates $\pi_{ij} \geq 0$, for $i, j \in \mathcal{M}$, $i \neq j$, and $\sum_{j=1,j\neq i}^{N} \pi_{ij} = -\pi_{ii}$, where $\Delta t > 0$, and $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$.

Consider the following nonlinear Markovian jump system with time delay in the probability space $(\Omega, \mathcal{F}, \mathbb{P})$:

$$\dot{x}(t) = (A(r_t) + \Delta A(r_t))x(t) + (A_d(r_t) + \Delta A_d(r_t))x(t - \tau)$$
$$+ (B(r_t) + \Delta B(r_t))u(t) + G(r_t)w(t)$$
$$+ f(r_t, x(t), x(t - \tau)),$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{L}^2_{\text{loc}}[0, +\infty)$ is an arbitrary external disturbance, $\tau(t) \in \mathbb{R}^n$ is the control output, $\varphi(t)$ represents a vector-valued initial function, and $r \in \mathbb{R}^+ \cup \{\text{constant}\}$ is the constant delay. $f(\cdot, \cdot, 
\cdot) : \mathcal{M} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is an unknown nonlinear function, $A(r_t), A_d(r_t), B(r_t), G(r_t), C(r_t), C_d(r_t), D(r_t),$ and $E(r_t)$ are known mode-dependent constant matrices with appropriate dimensions, $\Delta A(r_t), \Delta A_d(r_t),$ and $\Delta B(r_t)$ are unknown matrices, denoting the uncertainties in the system, and the uncertainties are time-varying but norm bounded uncertainties satisfying

$$\Delta A(r_t) = M_1(r_t) F(t, r_t) N_1(r_t),$$
$$\Delta B(r_t) = M_2(r_t) F(t, r_t) N_2(r_t),$$
$$\Delta A_d(r_t) = M_3(r_t) F(t, r_t) N_3(r_t),$$

where $M_1(r_t), N_1(r_t), M_2(r_t), N_2(r_t), M_3(r_t),$ and $N_3(r_t)$ are known mode-dependent matrices with appropriate dimensions and $F(t, r_t)$ is the time-varying unknown matrix function.

Consider the following state feedback controller:

$$u(t) = K(r_t) x(t) + K_d(r_t) x(t - \tau),$$

where $K(r_t)$ and $K_d(r_t)$ are the state feedback gains to be designed. Then the closed-loop system is as follows:

$$\dot{x}(t) = (A(r_t) + \Delta A(r_t) + B(r_t) K(r_t)) x(t)$$
$$+ \Delta B(r_t) K(r_t) x(t)$$
$$+ A_d(r_t) + \Delta A_d(r_t) + B(r_t) K_d(r_t)$$
$$+ \Delta B(r_t) K_d(r_t) x(t - \tau)$$
$$+ G(r_t) w(t) + f(r_t, x(t), x(t - \tau)), (6)$$

$$z(t) = (C(r_t) + D(r_t) K(r_t)) x(t)$$
$$+ C_d(r_t) + D(r_t) K_d(r_t) x(t - \tau)$$
$$+ E(r_t) w(t),$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0].$$

For notational simplicity, when $r(t) = i, i \in \mathcal{M}, A(r_t), A_d(r_t), B(r_t), G(r_t), K(r_t), K_d(r_t), C(r_t), C_d(r_t), D(r_t), E(r_t), \Delta A(r_t), \Delta B(r_t), M_1(r_t), N_1(r_t), M_2(r_t), N_2(r_t), M_3(r_t), N_3(r_t),$ and $f(r(t), x(t), x(t - \tau))$ are, respectively, denoted as $A_i, A_{d_i}$, $B_i, G_i, K_i, K_{d_i}, C_i, C_{d_i}, D_i, E_i, \Delta A_i, \Delta B_i, M_{1i}, N_{1i}, M_{2i}, N_{2i}, M_{3i}, N_{3i},$ and $f_i(x(t), x(t - \tau))$. 

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In addition, the transition probability rates are considered to be partially known; that is, some elements in matrix $\Pi = \{\pi_{ij}\}$ are unknown. For instance, for system (2) with four subsystems, the transition probability rate matrix $\Pi$ may be as

$$
\Pi = \begin{bmatrix}
\pi_{11} & \pi_{12} & ? & ? \\
? & ? & \pi_{23} & \pi_{24} \\
\pi_{31} & ? & \pi_{35} & ? \\
? & ? & ? & ? \\
\end{bmatrix},
$$

where "?" represents the unknown transition probability rate. \(\forall i \in \mathcal{M},\) we denote \(\mathcal{M} = L_k^1 \cup L_{uk}^1,\) and

$$
L_k^i \equiv \{j: \pi_{ij} \text{ is known, for } j \in \mathcal{M}\},
$$

$$
L_{uk}^i \equiv \{j: \pi_{ij} \text{ is unknown, for } j \in \mathcal{M}\}.
$$

Moreover, if $L_k^i \neq \emptyset$, it is further described as

$$
L_k^i = \{k_1^i, k_2^i, \ldots, k_m^i\}, \quad 1 \leq m \leq \mathcal{M},
$$

where $k_m^i \in \mathcal{M}$ represents the $m$th known transition probability rate of the set $L_k^i$ in the $i$th row of the transition probability rate matrix $\Pi$.

**Assumption 2.** The external disturbance $w(t)$ is varying and satisfies the constraint condition:

$$
\int_0^T w^T(s)w(s) \, ds \leq d, \quad d \geq 0. \tag{10}
$$

**Assumption 3.** \(\forall i \in \mathcal{M}, f_i(0,0) = 0,\) and $f_i(x(t), x(t - \tau))$ satisfies the following inequality

$$
\left\| f_i(x(t), x(t - \tau)) \right\|^2 \leq \begin{bmatrix} x(t) \\ x(t - \tau) \end{bmatrix}^T \begin{bmatrix} F_{11} & F_{12} \\ * & F_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau) \end{bmatrix}, \tag{11}
$$

where

$$
F_i := \begin{bmatrix} F_{11} & F_{12} \\ * & F_{22} \end{bmatrix} \geq 0. \tag{12}
$$

**Definition 4 (finite-time stability).** For a given time constant $T > 0$, system (2) ($u(t) = 0$, $w(t) = 0$) is said to be finite-time stable with respect to $(c_1, c_2, T, H_i)$, if the condition (13) holds, where $0 < c_1 < c_2, H_i > 0$.

$$
\mathbb{E}\{ x_0^T H_i x_0 \} \leq c_1 \implies \mathbb{E}\{ x(t)^T H_i x(t) \} \leq c_2, \quad \forall t \in [0, T], \tag{13}
$$

where $0 < c_1 < c_2, H_i > 0$.

**Definition 5 (finite-time boundedness).** For a given time constant $T > 0$, system (2) ($u(t) = 0$) is said to be finite-time bounded with respect to $(c_1, c_2, T, H_i, d)$, if the condition (13) holds, where $0 < c_1 < c_2, H_i > 0$.

**Definition 6 ($H_{\infty}$ finite-time boundedness).** For a given time constant $T > 0$, system (2) ($u(t) = 0$) is said to be $H_{\infty}$ finite-time bounded with respect to $(c_1, c_2, T, H_i, d)$, if there exists a positive constant $\gamma$, such that the following two conditions are true:

1. (1) system (2) is finite-time bounded with respect to $(c_1, c_2, T, H_i)$;
2. (2) under zero initial condition $x(t_0) = 0, t_0 = 0$, for any external disturbance $w(t) \neq 0$ satisfying condition (10), the control output $z(t)$ of system (2) satisfies

$$
\mathbb{E}\{ \int_0^T z^T(t)z(t) \, dt \} \leq \gamma^2 \int_0^T w^T(t)w(t) \, dt. \tag{14}
$$

**Definition 7 (finite-time $H_{\infty}$ state feedback stabilization).** The system (2) is said to be finite-time $H_{\infty}$ state feedback stabilizable with respect to $(c_1, c_2, T, H_i)$, if there exist a positive constant $\gamma$ and a state feedback controller in the form of (5), such that the closed-loop system (6) is $H_{\infty}$ finite-time bounded.

**Definition 8** (see [26]). In the Euclidean space $[\mathbb{R}^n \times \mathcal{M} \times \mathbb{R}^n]$, introduce the stochastic Lyapunov function for system (2) as $V(x(t), i)$, and the weak infinitesimal operator satisfies

$$
\mathcal{L} V(x(t), i) = \lim_{\Delta_t \to 0} \frac{1}{\Delta_t} [\mathbb{E}\{ V(x(t + \Delta_t) r(t + \Delta_t)) \} - V(x(t), i)]
$$

$$
= \frac{\partial}{\partial t} V(x(t), i) + \frac{\partial}{\partial x} V(x(t), i)x(t) + \sum_{j=1}^N \pi_{ij} V(x(t), j). \tag{15}
$$

**Remark 9.** It easily follows from (12) that $F_{11}' \geq 0, F_{22}' \geq 0$. So $F_{11}'$ and $F_{22}'$ can be decomposed as

$$
F_{11}' = \left( F_{11} \right)^{1/2} F_{11}' \left( F_{11} \right)^{1/2}, \quad F_{22}' = \left( F_{22} \right)^{1/2} F_{22}' \left( F_{22} \right)^{1/2}. \tag{16}
$$

**Remark 10.** It is noticed that finite-time stability can be regarded as a particular case of finite-time boundedness by setting $w(t) = 0$. That is, finite-time boundedness implies finite-time stability, but the converse is not true.

**Lemma 11** (see [27]). Let $T, M, F,$ and $N$ be real matrices of appropriate dimensions with $F^T F \leq 1$; then for a positive scalar $\varepsilon > 0$, there holds:

$$
T + M F N + N^T F^T M^T \leq T + \varepsilon M M^T + \varepsilon^{-1} N^T N. \tag{17}
$$
3. Main Results

3.1. Finite-Time Boundedness Analysis. In this subsection, we will consider the problem of finite-time boundedness for the nominal system of nonlinear Markovian jump system (2) with $F(t, r_i) = 0$ for all $t \geq 0$; that is,

$$
\dot{x}(t) = A(r_i) x(t) + A_d(r_i) x(t - \tau) + B(r_i) u(t) + G(r_i) w(t) + f(r_i, x(t), x(t - \tau)),
$$

$$
z(t) = C(r_i) x(t) + C_d(r_i) x(t - \tau) + D(r_i) u(t) + E(r_i) w(t),
$$

$$
x(t) = \varphi(t), \quad t \in [\tau, 0].
$$

Under the controller (5), the closed-loop system is

$$
\dot{x}(t) = (A(r_i) + B(r_i) K(r_i)) x(t) + (A_d(r_i) + B(r_i) K_d(r_i)) x(t - \tau) + G(r_i) w(t) + f(r_i, x(t), x(t - \tau)),
$$

$$
z(t) = (C(r_i) + D(r_i) K(r_i)) x(t) + (C_d(r_i) + D(r_i) K_d(r_i)) x(t - \tau) + E(r_i) w(t),
$$

$$
x(t) = \varphi(t), \quad t \in [-\tau, 0].
$$

Theorem 12. Given $T > 0$, if there exist positive constants $\alpha$ and $\epsilon_{fi}$, symmetric positive definite matrices $P_i \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{p \times q}$ and $S \in \mathbb{R}^{p \times p}$, and symmetric matrices $W_i \in \mathbb{R}^{n \times n}$, such that for all $i \in \mathcal{M}$

$$
\begin{bmatrix}
\Lambda_i & P_i A_{di} & \epsilon_{fi} F_{i1}^T P_i G_i \\
* & -Q + \epsilon_{fi} F_{i2}^T & 0 \\
* & * & -\alpha S
\end{bmatrix} < 0,
$$

$$
P_j - W_i \leq 0, \quad j \in L_{i}, \quad j \neq i,
$$

$$
P_j - W_i \geq 0, \quad j \in L_{i}, \quad j = i,
$$

$$
\epsilon_{fi} \left[ \lambda_{\text{max}}(P_i) + \tau \lambda_{\text{max}}(Q_i) \right] + d \lambda_{\text{max}}(S) \left( 1 - e^{-\alpha T} \right)
$$

$$
< e^{-\alpha T} \epsilon_{c_2}.
$$

then system (18) ($u = 0$) under partially known transition probabilities is finite-time bounded with respect to $(c_1, c_2, T, H_i, d)$, where

$$
\begin{align*}
\Lambda_{1i} &= A_{1i}^T P_i + P_i A_i + Q \\
&+ \sum_{j \in L_i} \pi_{ij} (P_j - W_i) + \epsilon_{fi} F_{i1}^T P_i + \epsilon_{fi} F_{i2}^T - \alpha P_i, \\
\bar{P}_i &= H_{i}^{-1/2} P_i H_{i}^{-1/2}, \quad \bar{Q}_i = H_{i}^{-1/2} Q H_{i}^{-1/2}.
\end{align*}
$$

Proof. For system (18) ($u = 0$), choose a Lyapunov function candidate

$$
V(x(t), i) = V_1(x(t), i) + \int_{-\tau}^{t} x^T(\xi) Q x(\xi) d\xi,
$$

where $P_i > 0$. Then by Definition 8, we get

$$
\mathcal{L}V_1(x(t), i) = \frac{\partial V_1}{\partial x}(x(t), i) \dot{x}(t)
$$

$$
\leq \epsilon_{fi} x^T(t) P_i f_i + \epsilon_{fi} x^T(t) P_i G_i w(t)
$$

$$
+ x^T(t) P_i A_{di} x(t) + x^T(t) P_i \alpha S x(t) + x^T(t) P_i \alpha S x(t)
$$

$$
+ \epsilon_{fi} x^T(t) P_i f_i x(t).
$$

Based on Lemma 11, there exist scalars $\epsilon_{fi}$ such that

$$
x^T(t) P_i f_i + \epsilon_{fi} x^T(t) P_i \alpha S x(t)
$$

$$
\leq \epsilon_{fi} \left[ x^T(t) P_i f_i x(t) + x^T(t) P_i \alpha S x(t) + x^T(t) P_i \alpha S x(t) \right]
$$

$$
+ x^T(t) f_{i1}^T x(t) + x^T(t) f_{i2}^T x(t) + x^T(t) f_{i3}^T x(t)
$$

$$
+ \epsilon_{fi} x^T(t) P_i f_i x(t).
$$

Substituting (27) into (26) yields

$$
\mathcal{L}V_1(x(t), i)
$$

$$
\leq x^T(t) \left[ A_{1i}^T P_i + P_i A_i + \epsilon_{fi} F_{i1}^T P_i + \sum_{j=1}^{N} \pi_{ij} P_j + \epsilon_{fi} f_i^T \right] x(t)
$$

$$
+ x^T(t) P_i A_{di} x(t) + x^T(t) P_i G_i w(t)
$$

$$
+ x^T(t) P_i f_i x(t) + \epsilon_{fi} x^T(t) P_i f_i x(t)
$$

$$
+ \epsilon_{fi} x^T(t) P_i f_i x(t).
$$

It is easy to obtain that

$$
\mathcal{L}V_2(x(t), i) = x^T(t) Q x(t) - x^T(t - \tau) Q x(t - \tau).
$$
From (28) and (29), the following holds: 
\[ \mathcal{L}V(x(t),i) \]
\[ = \mathcal{L}V_1(x(t),i) + \mathcal{L}V_2(x(t),i) \]
\[ \leq x^T(t) \left[ A^T_i P_i + P_i A_i + \varepsilon_{f_1}^i P_i P_i \right. \]
\[ + \sum_{j=1}^{N} \pi_{ij} (P_j - W_i) + \varepsilon_{f_2}^i P_i + \varepsilon_{f_1} F_{11} + Q \] \( x(t) \)
\[ + x^T(t) P_i G_i w(t) + x^T(t) \left[ P_i A_{di} + \varepsilon_{f_1} F_{12} \right] x(t - \tau) \]
\[ + w^T(t) G_i^T P_i x(t) + x^T(t - \tau) \left[ A_{di}^T P_i + \varepsilon_{f_2} F_{21} \right] x(t) \]
\[ + x^T(t - \tau) \left[ \varepsilon_{f_2} F_{22} - Q \right] x(t - \tau) \] \( t \).

Due to the fact that \( \sum_{j=1}^{N} \pi_{ij} W_i = 0 \) for arbitrary symmetric matrices \( W_i \), (30) can be written as
\[ \mathcal{L}V(x(t),i) \]
\[ \leq x^T(t) \left[ A^T_i P_i + P_i A_i + \varepsilon_{f_1}^i P_i P_i \right. \]
\[ + \sum_{j=1}^{N} \pi_{ij} (P_j - W_i) + \varepsilon_{f_1} F_{11} + Q \] \( x(t) \)
\[ + x^T(t) P_i G_i w(t) + x^T(t) \left[ P_i A_{di} + \varepsilon_{f_1} F_{12} \right] x(t - \tau) \]
\[ + w^T(t) G_i^T P_i x(t) + x^T(t - \tau) \left[ A_{di}^T P_i + \varepsilon_{f_2} F_{21} \right] x(t) \]
\[ + x^T(t - \tau) \left[ \varepsilon_{f_2} F_{22} - Q \right] x(t - \tau) \] \( t \).

Noticing that \( \pi_{ij} \geq 0 \) for all \( i \neq j \) and \( \pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij} < 0 \) for all \( i \in \mathcal{M} \), if \( i \in L_k^i \) (the elements of the diagonal are known), by inequalities (20) and (21), the following inequalities hold:
\[ \mathcal{L}V(x(t),i) < ax^T(t) P_i x(t) + aw^T(t) w(t) \]
\[ < ax^T(t) P_i x(t) + a \int_{t-\tau}^{t} x^T(\xi) Q x(\xi) d\xi \]
\[ + aw^T(t) W(t) \]
\[ = aV(x(t),i) + aw^T(t) W(t). \]

If \( i \in L_{ik}^i \) (the elements of the diagonal are unknown), according to the inequalities (20)–(22), inequality (32) holds. Multiplying (32) by \( e^{-at} \) yields
\[ \mathcal{L} \left( e^{-at} V(x(t),i) \right) < \alpha e^{-at} w^T(t) W(s) ds, \]

which shows
\[ V(x(t),i) < e^{-at} V(x(t),i) + a \int_{0}^{t} e^{-as} W(s) ds, \]

and combining (36) and (37), it follows that
\[ V(x(t),i) \geq \lambda_{\min}(\tilde{P}_i)x^T(t) H_i x(t) \]

and combining (36) and (37), it follows that
\[ \mathbb{E} \left\{ x^T(t) H_i x(t) \right\} \]
\[ < e^{-at} \left[ c_1 \left( \lambda_{\max}(\tilde{P}_i) + \tau \lambda_{\max}(\tilde{Q}_i) \right) + d \lambda_{\max}(S) \right] \left( 1 - e^{-at} \right) \]
\[ < c_2. \]

Condition (38) implies that, for \( t \in [0,T] \), \( \mathbb{E} \{ x^T(t) H_i x(t) \} < c_2 \).

The proof is complete.
Corollary 13. Given $T > 0$, if there exist positive constants $\alpha$, $\epsilon_f$, and $\gamma$, symmetric positive definite matrices $P_i \in \mathbb{R}^{n \times n}$, and $Q \in \mathbb{R}^{p \times q}$, and symmetric matrices $W_i \in \mathbb{R}^{n \times n}$, such that for all $i \in \mathcal{M}$

$$
\begin{align*}
\Lambda_{ii} &= A_i^T P_i + P_i A_i + Q \\
&\quad + \sum_{j \in \mathcal{L}_k} \pi_{ij} \left( P_j - W_j \right) + \epsilon_f^2 P_i + \epsilon_f^2 F_{11} - \alpha P_i,
\end{align*}
$$

(48)

Proof. From (44), the following inequality holds:

$$
\begin{align*}
\Lambda_{ii} &= A_i^T P_i + P_i A_i + Q \\
&\quad + \sum_{j \in \mathcal{L}_k} \pi_{ij} \left( P_j - W_j \right) + \epsilon_f^2 P_i + \epsilon_f^2 F_{11} - \alpha P_i
\end{align*}
$$

(49)

This together with (49) implies (39). Then based on (39)–(42), system (18) is finite-time bounded.

Then, let us prove that inequality (14) is satisfied for any external disturbance $w(t) \neq 0$ under zero initial condition. For system (18), choosing a Lyapunov function candidate (25), we have

$$
\dot{V}(x(t), i)
$$

(50)

for any symmetric matrices $W_i$. 

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Corollary 15. Given $T > 0$ and $w(t)$ satisfying (10), system (19) under partially known transition probabilities is finite-time $H_{\infty}$ state feedback stabilizable via a state feedback controller (5) with respect to $(c_1, c_2, T, H_i, d)$, if there exist positive scalars $\alpha, \gamma, \epsilon_f, \lambda_1$, and $\lambda_2$, symmetric positive definite matrices $X_i \in \mathbb{R}^{n \times n}$, symmetric matrices $W_i \in \mathbb{R}^{n \times n}$, and matrices $Y_i \in \mathbb{R}^{n \times m}$ and $K_{di} \in \mathbb{R}^{n \times m}$ such that for all $i \in \mathbb{M}$

\[
\begin{bmatrix}
\bar{A}_{11} + C_i^T C_i & P_i \bar{A}_{di} + \epsilon_f F_{11}^i + C_i^T \bar{C}_{di} & P_i G_i + C_i^T E_i \\
* & -Q + \epsilon_f F_{22}^i + C_i^T \bar{C}_{di} & C_i^T E_i \\
* & * & -\gamma^2 I + E_i^T E_i
\end{bmatrix} < 0,
\]

(54)

\[
P_j - W_j \leq 0, \quad j \in L_{uk}, \quad j \neq i,
\]

(55)

\[
P_j - W_j \geq 0, \quad j \in L_{uk}, \quad j = i,
\]

(56)

\[
c_1 \left[ \lambda_{\max} \left( \bar{P}_i \right) + \tau \lambda_{\max} \left( P_i \right) \right] + \frac{\gamma^2 d}{\alpha} \left( 1 - e^{-\alpha T} \right)
\]

\[
< \lambda_{\min} \left( \bar{P}_i \right) e^{-\alpha T} c_2,
\]

where

\[
\bar{A}_{11} = \bar{A}_i^T P_i + P_i \bar{A}_i + Q + \sum_{j \in L_{uk}} \pi_{ij} \left( P_j - W_j \right) + \epsilon_f \bar{P}_i \bar{P}_i + \epsilon_f F_{11}^i - \alpha P_i,
\]

\[
\bar{A}_i = A_i + B_i K_i,
\]

\[
C_i = C_i + D_i K_i,
\]

\[
\bar{P}_i = H_i^{-1/2} \bar{P}_i H_i^{-1/2}, \quad \bar{Q}_i = H_i^{-1/2} Q H_i^{-1/2}.
\]

(58)

Theorem 16. Given $T > 0$, system (18) under partially known transition probabilities is finite-time $H_{\infty}$ state feedback stabilizable via a state feedback controller with respect to $(c_1, c_2, T, H_i, d)$, if there exist positive scalars $\alpha, \gamma, \epsilon_f, \lambda_1$, and $\lambda_2$, symmetric positive definite matrices $X_i \in \mathbb{R}^{n \times n}$, symmetric matrices $W_i \in \mathbb{R}^{n \times n}$, and matrices $Y_i \in \mathbb{R}^{n \times m}$ and $K_{di} \in \mathbb{R}^{n \times m}$ such that for all $i \in \mathbb{M}$

\[
\begin{bmatrix}
\Pi_{11i} & \Pi_{12i} & G_i & \Pi_{14i} & I & \epsilon_f X_i F_{11}^i & \Pi_{16i} & S_i \left( \bar{x}_i \right)
\end{bmatrix}
\]

\[
* \quad \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 & 0 & 0
\]

(59)

Further, it implies that

\[
\mathbb{E} \int_0^T \lambda_i x_i < 0, \quad i \in L_{uk},
\]

(60)

\[
X_j - \mathbb{W}_j > 0, \quad j \in L_{uk}, \quad j = i,
\]

(62)

\[
-\epsilon^{-\alpha T} c_2 + c_1 \tau \lambda_2 + \frac{\gamma^2 d}{\alpha} \left( 1 - e^{-\alpha T} \right) \frac{\sqrt{c_1}}{\lambda_1} < 0,
\]

(63)

where

\[
\lambda_1 H_i^{-1} < X_i < H_i^{-1}, \quad 0 < Q < \lambda_2 H_i,
\]

(64)

It is clear that (54) is a nonlinear matrix inequality due to the existence of the nonlinear terms $K_i^T B_i^T P_i, P_i B_i K_i, K_{di}^T B_i^T P_i,$ and $P_i B_i K_{di}$. In order to solve the desired controller $K_i$, we give the following result.
Proof. It is clear that system (18) is finite-time $H_\infty$ state feedback stabilizable if the conditions (54)--(57) are satisfied. Notice that inequality (54) is equivalent to the following condition:

$$
\Sigma_i = \begin{bmatrix} 
\tilde{A}_{i1} & P_f A_{di} + \epsilon_f F^e_{12} & P_f G_i & \tilde{C}_i & P_i & \epsilon_f (F^e_{11})^{1/2} \\
* & -Q + \epsilon_f F^e_{22} & 0 & \tilde{C}_d^T & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 \\
* & * & * & -I & 0 & 0 \\
* & * & * & * & -\epsilon_f I & 0 \\
* & * & * & * & * & -\epsilon_f I 
\end{bmatrix} < 0.
$$

(65)

Pre- and postmultiplying inequality (66) by block diagonal matrix $\text{diag} \{P^{-1}_{i1}, I, I, I, I\}$, letting $X_i = P_{i1}^{-1} Y_i = K_i X_i$, and $W_i = P_{i1}^{-1} W_i P_{i1}^{-1}$, we have

$$
\Xi_i = \begin{bmatrix} 
\Xi_{i1} & \Pi_{12i} & G_i & \Pi_{i4i} & I & \epsilon_f X_i (F^e_{11})^{1/2} \\
* & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 \\
* & * & * & -I & 0 & 0 \\
* & * & * & * & -\epsilon_f I & 0 \\
* & * & * & * & * & -\epsilon_f I 
\end{bmatrix} < 0,
$$

(66)

where

$$
\Xi_{i1} = X_i A_i^T + A_i X_i + Y_i^T B_{i1}^T + B_i Y_i + Q_i + \sum_{j \in L_k} \pi_{ij} X_j X_j^{-1} X_i - \sum_{j \in L_k} \pi_{ij} W_i - \alpha X_i.
$$

(68)

Since $\pi_{ii} < 0$, $\forall i \in M$, inequality (67) is discussed in the following two cases.

**Case 1.** When $i \in L_k^i$, the left side of (67) becomes

$$
\Xi_{2i} = \begin{bmatrix} 
\Xi_{12i} & \Pi_{12i} & G_i & \Pi_{i4i} & I & \epsilon_f X_i (F^e_{11})^{1/2} \\
* & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 \\
* & * & * & -I & 0 & 0 \\
* & * & * & * & -\epsilon_f I & 0 \\
* & * & * & * & * & -\epsilon_f I 
\end{bmatrix} < 0,
$$

(69)

where

$$
\Xi_{2i} = X_i A_i^T + A_i X_i + Y_i^T B_{i1}^T + B_i Y_i + Q_i - \sum_{j \in L_k} \pi_{ij} W_i - \alpha X_i.
$$

(70)

Applying Schur complement lemma to (69), then (59) easily follows.

**Case 2.** When $i \in L_{nk}$, the inequality (69) turns into

$$
\Xi_3 = \begin{bmatrix} 
\Xi_{12i} & \Pi_{12i} & G_i & \Pi_{i4i} & I & \epsilon_f X_i (F^e_{11})^{1/2} \\
* & \Pi_{22i} & 0 & \Pi_{24i} & 0 & 0 \\
* & * & -\gamma^2 I & E_i^T & 0 & 0 \\
* & * & * & -I & 0 & 0 \\
* & * & * & * & -\epsilon_f I & 0 \\
* & * & * & * & * & -\epsilon_f I 
\end{bmatrix} < 0,
$$

(71)

where

$$
\Xi_3 = X_i A_i^T + A_i X_i + Y_i^T B_{i1}^T + B_i Y_i + Q_i - \sum_{j \in L_k} \pi_{ij} W_i.
$$

(72)

Similar to the proving process of the case one, we can prove that (60) is true. Pre- and postmultiplying inequalities (55) and (56) by $P^{-1}_i$, respectively, and letting $X_i = P^{-1}_i Y_i = K_i X_i$,
and $\mathcal{W}_i = P_i^{-1}W_i P_i^{-1}$, we have

$$X_j X_j^{-1} X_i - R_i < 0, \quad j \in L_{i,k}^i, \quad j \neq i,$$

$$X_j - R_j > 0, \quad j \in L_{i,k}^i, \quad j = i.$$  

(73)\(\)  \(\)  \(\)

Inequality (73) is equivalent to LMI (61). Denoting $\bar{X}_i = \bar{P}_i^{-1} = H_i^{1/2} X_i H_i^{1/2}$ and taking $\lambda_{\text{max}}(\bar{X}_i) = 1/\lambda_{\text{min}}(\bar{P}_i)$ into consideration, we conclude that condition (57) holds. Hence, the following conditions guarantee that

$$\lambda_1 < \lambda_{\text{min}}(\bar{X}_i), \quad \lambda_{\text{max}}(\bar{X}_i) < 1, \quad 0 < \lambda_{\text{min}}(Q),$$

$$\lambda_{\text{max}}(Q) < \lambda_2,$$

(75)\(\)

It should be easily observed that condition (76) implies LMI (63) and (75) is equivalent to (64). Therefore if LMIs (59)–(64) hold, the closed-loop system (19) is $H_{\infty}$ finite-time bounded, and then system (18) can be stabilized via the state feedback controller (5).

This completes the proof of Theorem 16. \(\Box\)

3.3. Robust Finite-Time $H_{\infty}$ Control. In this subsection, a robust finite-time $H_{\infty}$ state feedback controller is designed to guarantee the finite-time $H_{\infty}$ state feedback stabilization of system (2).

**Theorem 17.** Given $T > 0$, the problem of robust finite-time $H_{\infty}$ state feedback stabilizable for system (2) under partly known transition probabilities is solvable, if there exist positive scalars $\alpha, \gamma, \varepsilon_{f_1}, \varepsilon_{f_2}, \varepsilon_{f_3}, \varepsilon_{g_1}, \lambda_1$, and $\lambda_2$, symmetric positive definite matrices $X_i \in \mathbb{R}_{n \times n}^{m \times m}$, symmetric matrices $\mathcal{W}_i \in \mathbb{R}_{n \times m}^{n \times m}$, and matrices $Y_i \in \mathbb{R}_{n \times n}^{m \times m}$ and $K_{d i} \in \mathbb{R}_{n \times m}^{m \times n}$ such that for all $i \in \mathcal{M}$

$$\left[\begin{array}{cccccccc}
\bar{P}_{1_{i1}} & \bar{P}_{1_{i2}} & G_i & \bar{P}_{1_{i4}} & X_i N_i^T & Y_i^T & N_{i2i}^T & 0 & 0 & I & \varepsilon_{f_i} X_i (F_i)_{1/2} & S_{i1}(x)
\end{array}\right] < 0, \quad i \in L_{i,k}^i,$$  

(77)\(\)

$$\left[\begin{array}{cccccccc}
\bar{P}_{2_{i1}} & \bar{P}_{2_{i2}} & G_i & \bar{P}_{2_{i4}} & X_i N_i^T & Y_i^T & N_{i2i}^T & 0 & 0 & I & \varepsilon_{f_i} X_i (F_i)_{1/2} & S_{i2}(x)
\end{array}\right] < 0, \quad i \in L_{i,ak}^i,$$  

(78)\(\)

$$\left[\begin{array}{cccccccc}
-\mathcal{W}_i & X_i & \mathcal{W}_j & \mathcal{W}_j < 0, \quad j \in L_{i,ak}^i, \quad j \neq i,
\end{array}\right]$$  

(79)\(\)

$$X_j - \mathcal{W}_j > 0, \quad j \in L_{i,ak}^i, \quad j = i,$$  

(80)\(\)

$$\left[\begin{array}{cccccccc}
-e^{-\alpha T} c_2 + c_1 T \lambda_2 + \frac{\gamma^2d}{\alpha} (1 - e^{-\alpha T}) \sqrt{\lambda_1} & -e^{-\alpha T} c_2 + c_1 T \lambda_2 + \frac{\gamma^2d}{\alpha} (1 - e^{-\alpha T}) \sqrt{\lambda_1} & < 0,
\end{array}\right]$$  

(81)\(\)
\[ \lambda_1 H_i^{-1} < X_i < H_i^{-1}, \quad 0 < Q < \lambda_2 H_i, \]  
(82)

where

\[ \Pi_{11i}^1 = X_i A_i^T + A_i X_i + Y_i^T B_i^T + B_i Y_i + Q_i \]

\[ - \sum_{j \in L_i} \Pi_j^i \mathcal{W}_j + \varepsilon_i M_{ij} M_{ij}^T + \varepsilon_2 M_{2i} M_{2i}, \]

\[ \Pi_{i} = X_i A_i^T + A_i X_i + Y_i^T B_i^T + B_i Y_i + Q_i \]

\[ - \sum_{j \in L_i} \Pi_j^i \mathcal{W}_j + \varepsilon_i M_{ij} M_{ij}^T + \varepsilon_2 M_{2i} M_{2i}, \]

\[ + \varepsilon_3 M_{3i} M_{3i}^T + \varepsilon_4 M_{2i} M_{2i} - \alpha X_i, \]

\[ \Pi_{21i} = A_{di} + B_i K_{di} + \varepsilon_{fi} X_i F_{12}^i, \]

\[ \Pi_{22i} = -Q + \varepsilon_{fi} F_{12}^i, \]

\[ \Pi_{41i} = X_i C_i^T + Y_i^T D_i^T, \]

\[ \Pi_{42i} = C_i^T + K_{di} D_i^T, \]

\[ S_{1i}(x) = \left[ \sqrt{\pi_{k_i} x_i}, \ldots, \sqrt{\pi_{k_i} x_i}, \sqrt{\pi_{k_i} x_i}, \ldots, \sqrt{\pi_{k_i} x_i} \right], \]

\[ M_{1i}(x) = \text{diag} \left\{ x_{k_i}, \ldots, x_{k_i}, x_{k_i}, \ldots, x_{k_i} \right\}, \]

\[ S_{2j}(x) = \left[ \sqrt{\pi_{k_j} x_j}, \ldots, \sqrt{\pi_{k_j} x_j}, \ldots, \sqrt{\pi_{k_j} x_j} \right], \]

\[ M_{2j}(x) = \text{diag} \left\{ x_{k_j}, \ldots, x_{k_j}, \ldots, x_{k_j} \right\}, \]

(83)

with \( k_1^i, k_2^i, \ldots, k_m^i \) described in (9) and \( k_i^i = i \). Moreover, the finite-time \( H_{\infty} \) state feedback controller gains in (5) are given by \( K_i = Y_i X_i^{-1} \).

Proof. In (59) and (60), replacing \( A_i \), \( A_{di} \), and \( B_i \) with \( (A_i + \Delta A_i), (A_{di} + \Delta A_{di}) \), and \( (B_i + \Delta B_i) \), respectively, the following conditions are obtained:

\[ \Pi_{11i}^1 = X_i A_i^T + X_i \Delta A_i^T + A_i X_i + \Delta A_i X_i + Y_i^T B_i^T + Y_i^T \Delta B_i^T \]

\[ + B_i Y_i + \Delta B_i Y_i - \sum_{j \in L_i} \Pi_j^i \mathcal{W}_j + \pi_{ii} R_i + \pi_{ii} X_i, \]

\[ \Pi_{21i} = X_i A_i^T + X_i \Delta A_i^T + A_i X_i + \Delta A_i X_i + Y_i^T B_i^T \]

\[ + Y_i^T \Delta B_i^T + B_i Y_i + \Delta B_i Y_i - \sum_{j \in L_i} \Pi_j^i \mathcal{W}_j, \]

\[ \Pi_{12i} = A_{di} + \Delta A_{di} + B_i K_{di} + \Delta B_i K_{di} + \varepsilon_{fi} X_i F_{12}^i, \]

(84)

Based on Lemma II, there exist scalars \( \varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \) and \( \varepsilon_{4i} \) such that

\[ X_i \Delta A_i^T + \Delta A_i X_i = X_i N_{11i}^T F_{11i}^T (t) M_{11i}^T + M_{1i} F_i (t) N_{1i} X_i \]

\[ \leq \varepsilon_{1i} M_{1i} M_{1i}^T + \varepsilon_{1i}^{-1} X_i N_{11i} N_{11i} X_i, \]

\[ Y_i^T \Delta B_i^T + \Delta B_i Y_i = Y_i^T N_{21i}^T F_{21i}^T (t) M_{21i}^T + M_{2i} F_i (t) N_{2i} Y_i \]

\[ \leq \varepsilon_{2i} M_{2i} M_{2i}^T + \varepsilon_{2i}^{-1} Y_i N_{21i} N_{21i} Y_i. \]
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\[ \begin{bmatrix} 0 & \Delta B_i K_{di} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & M_2 F_1(t) N_{2i} K_{di} & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} 
\]

\[ = \begin{bmatrix} 0 & M_2 E_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leq \epsilon_i \]

Applying Schur complement lemma to (85), (77) can be obtained. Similar to the above proving process, we can prove that (78) holds. Therefore, if LMIs (77)–(82) hold, the closed-loop system (6) is robust $H_\infty$ finite-time bounded, and further system (18) can be stabilized via the state feedback controller (5).

The proof is complete.

**Remark 18.** It should be pointed out that the conditions in Theorems 16 and 17 are not strict linear matrix inequalities such as conditions (20), (39), (44), (54), (59), (60), (77), and (78), due to the product of unknown scalars and matrices. An efficient way to solve this problem is to choose the appropriate values of the unknown scalars and then solve a set of LMIs for the fixed values of these parameters. For example, if $\alpha, \epsilon_{ji}$ are fixed, then conditions (59) and (60) of Theorem 16 can be converted to LMIs conditions.

### 4. Numerical Examples

This section considers the following four-mode uncertain nonlinear Markovian jump systems with time delay as follows.

**Mode 1**

\[ A_1 = \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \]

\[ G_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \]

\[ D_1 = E_1 = 0.1, \quad M_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad N_{11} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \]

\[ M_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad N_{21} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \]

\[ M_{31} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad N_{31} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}. \]

\[(86)\]

**Mode 2**

\[ A_2 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \]

\[ G_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \]

\[ C_{d2} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \quad D_2 = E_2 = 0.2, \quad M_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \]

\[ N_{12} = \begin{bmatrix} 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad M_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad N_{22} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \]

\[ M_{32} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}, \quad N_{32} = \begin{bmatrix} 0.02 & 0.03 \\ 0 & 0.02 \end{bmatrix}. \]

\[(87)\]

**Mode 3**

\[ A_3 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} 0.1 & -0.3 \\ -0.2 & 0.3 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \]

\[ G_3 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 & 3 \end{bmatrix}. \]
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Table 1

| Case I | 1  | 2  | 3  | 4  | Case II | 1  | 2  | 3  | 4  |
|--------|----|----|----|----|---------|----|----|----|----|
| 1      | -1.2 | 0.3 | 0.5 | 0.4 | 1       | ?  | 0.3 | ?  | 0.4 |
| 2      | 0.2  | -1  | 0.3 | 0.5 | 2       | ?  | -1  | 0.3 | ?  |
| 3      | 0.8  | 0.1 | -1.3| 0.4 | 3       | 0.8 | -1  | 0.3 | ?  |
| 4      | 0.2  | 0.1 | 0.5 | -0.8| 4       | 0.2 | -1  | 0.3 | ?  |

| Case III | 1  | 2  | 3  | 4  | Case VI | 1  | 2  | 3  | 4  |
|-----------|----|----|----|----|---------|----|----|----|----|
| 1         | -1.2 | ?  | 0.5 | ?  | 1       | ?  | ?  | ?  | ?  |
| 2         | 0.2  | ?  | ?  | 0.5 | 2       | ?  | ?  | ?  | ?  |
| 3         | ?    | 0.1 | ?  | 0.4 | 3       | ?  | ?  | ?  | ?  |
| 4         | ?    | 0.1 | 0.5 | -0.8| 4       | ?  | ?  | ?  | ?  |

Table 2

| Case I | Controller gains | Case II | Controller gains | Case III | Controller gains | Case VI | Controller gains |
|--------|------------------|---------|------------------|---------|------------------|---------|------------------|
|        | $K_1 = \begin{bmatrix} -22.2335 & -19.0199 \end{bmatrix}$ | $K_1 = \begin{bmatrix} -22.5382 & -18.9685 \end{bmatrix}$ | $K_1 = \begin{bmatrix} -20.2412 & -18.1608 \end{bmatrix}$ | $K_1 = \begin{bmatrix} -21.8153 & -18.7884 \end{bmatrix}$ |
|        | $K_2 = \begin{bmatrix} -0.9097 & -0.9098 \end{bmatrix}$ | $K_2 = \begin{bmatrix} -0.9490 & -0.4701 \end{bmatrix}$ | $K_2 = \begin{bmatrix} -0.9283 & -0.5364 \end{bmatrix}$ | $K_2 = \begin{bmatrix} -0.8142 & -0.8142 \end{bmatrix}$ |
|        | $K_3 = \begin{bmatrix} -0.9498 & -0.2499 \end{bmatrix}$ | $K_3 = \begin{bmatrix} 0.6466 & -0.3225 \end{bmatrix}$ | $K_3 = \begin{bmatrix} 0.7272 & -0.2134 \end{bmatrix}$ | $K_3 = \begin{bmatrix} -0.9007 & -0.4391 \end{bmatrix}$ |
|        | $K_4 = \begin{bmatrix} -0.9008 & -0.4391 \end{bmatrix}$ | $K_4 = \begin{bmatrix} 0.6272 & -0.3112 \end{bmatrix}$ | $K_4 = \begin{bmatrix} 0.6272 & -0.3112 \end{bmatrix}$ | $K_4 = \begin{bmatrix} 0.6466 & -0.3225 \end{bmatrix}$ |

$C_{d3} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}$, $D_3 = E_3 = 0.3$, $C_{d4} = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}$, $D_4 = E_4 = 0.4$.

$A_4 = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$, $A_{d4} = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.1 \end{bmatrix}$, $B_4 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $H_1 = H_2 = H_3 = H_4 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $C_1 = 0.5$.

$G_4 = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}$, $C_4 = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $C_2 = 4$, $d = 4$, $T = 1.2$. (89)
Choose \( \tau = 1, \alpha = 0.5 \), the exogenous disturbance \( w(t) = [1/(5t + 1) 1/(t + 1)] \), and the nonlinearities

\[
\begin{align*}
 f_1(x(t), x(t - \tau)) &= \begin{bmatrix} 0.1 \sin(x(t)) \\ 0.1 \sin(x(t - \tau)) \end{bmatrix}, \\
 f_2(x(t), x(t - \tau)) &= \begin{bmatrix} 0.1 \sin(x(t - \tau)) \\ -0.15 \sin(x(t)) \end{bmatrix}, \\
 f_3(x(t), x(t - \tau)) &= \begin{bmatrix} 0.1 \sin(x(t)) \\ 0.1 \sin(x(t - \tau)) \end{bmatrix}, \\
 f_4(x(t), x(t - \tau)) &= \begin{bmatrix} 0.1 \sin(x(t - \tau)) \\ -0.15 \sin(x(t)) \end{bmatrix},
\end{align*}
\]

(90)

The four cases for the transition probability matrix considered in Table 1.

Solving the LMIs (77)–(82) in Theorem 17, the robust finite-time \( H_\infty \) state feedback controller gains of \( K_i \) are given by Table 2.

Figures 1, 2, and 3 are presented. For every figure, the four different transition probability matrices cases are included, which can be better to demonstrate the effectiveness of the design method. Figure 1 depicts the trajectories of system state \( x(t) \) and the corresponding switching signal. It can be seen that system (6) is robust finite-time stable, which implies that system (2) is robust finite-time \( H_\infty \) state feedback stabilizable via the designed state feedback controller (5). Figure 2 depicts the trajectories of system state \( x(t) \) with \( w(t) \neq 0 \) and the corresponding switching signal. It can be seen that system (6) is robust finite-time bounded. The trajectory of the output \( z(t) \) is described in Figure 3, which further shows the effectiveness of the designed controller (5).

5. Conclusions

In this paper, we have dealt with the problem of robust finite-time \( H_\infty \) control for a class of nonlinear Markovian jump systems with time delay under partially known transition probabilities. Based on the free-weighting matrices approach, all sufficient conditions have been firstly proposed to ensure...
Figure 2: The trajectory of $x(t)$ with $w(t) \neq 0$.

Figure 3: The trajectory of $z(t)$.
finite-time boundedness, $H_\infty$ finite-time boundedness, and finite-time $H_\infty$ state feedback stabilization for the given system. We have also designed a robust finite-time $H_\infty$ state feedback controller, which guarantees the $H_\infty$ finite-time boundedness of the closed-loop system. All the conditions have been presented in terms of strict linear matrix inequalities. Finally, a numerical example has been provided to demonstrate the effectiveness of all the results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] X. M. Yao, L. G. Wu, W. X. Zheng, and C. H. Wang, "Passivity analysis and passification of Markovian jump systems," Circuits, Systems, and Signal Processing, vol. 29, no. 4, pp. 709–725, 2010.

[2] T. Shi, H. Su, and J. Chu, "Robust $H_\infty$ control for uncertain discrete-time Markovian jump systems with actuator saturation," Journal of Control Theory and Applications, vol. 9, no. 4, pp. 465–471, 2011.

[3] W. H. Chen, J. X. Xu, and Z. H. Guan, "Guaranteed cost control for uncertain Markovian jump systems with mode-dependent time-delays," IEEE Transactions on Automatic Control, vol. 48, no. 12, pp. 2270–2277, 2003.

[4] L. J. Shen and U. Busker, "Solving the serial batching problem in job shop manufacturing systems," European Journal of Operational Research, vol. 221, no. 1, pp. 14–26, 2012.

[5] W. Assawinchaichote, S. K. Nguang, and P. Shi, "Robust $H_\infty$ fuzzy filter design for uncertain nonlinear singularly perturbed systems with Markovian jumps: an LMI approach," Information Sciences, vol. 177, no. 7, pp. 1699–1714, 2007.

[6] M. Atlans, "Command and control theory: a challenge to control science," IEEE Transactions on Automatic Control, vol. 32, no. 4, pp. 286–293, 1987.

[7] H. N. Wu and K. Y. Cai, "Mode-independent robust stabilization for uncertain Markovian jump nonlinear systems via fuzzy control," IEEE Transactions on Systems, Man, and Cybernetics B, vol. 36, no. 3, pp. 509–519, 2006.

[8] H. N. Wu and K. Y. Cai, "Robust fuzzy control for uncertain discrete-time nonlinear Markovian jump systems without mode observations," Information Sciences, vol. 177, no. 6, pp. 1509–1522, 2007.

[9] F. Liu and Y. Cai, "Passive analysis and synthesis of Markovian jump systems with norm bounded uncertainty and unknown delay," Dynamics of Continuous, Discrete & Impulsive Systems A, vol. 13, no. 1, pp. 157–166, 2006.

[10] L. X. Zhang and E. K. Boukas, "Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities," Automatica, vol. 45, no. 2, pp. 463–468, 2009.

[11] X. L. Luan, F. Liu, and P. Shi, "Finite-time filtering for non-linear stochastic systems with partially known transition jump rates," IET Control Theory & Applications, vol. 4, no. 5, pp. 735–745, 2010.

[12] Y. Yin, F. Liu, and P. Shi, "Finite-time gain-scheduled control on stochastic bioreactor systems with partially known transition jump rates," Circuits, Systems, and Signal Processing, vol. 30, no. 3, pp. 609–627, 2011.

[13] Y. Zhang, Y. He, M. Wu, and J. Zhang, "Stabilization for Markovian jump systems with partial information on transition probability based on free-connection weighting matrices," Automatica, vol. 47, no. 1, pp. 79–84, 2011.

[14] L. Weiss and E. F. Infante, "Finite time stability under perturbing forces and on product spaces," IEEE Transactions on Automatic Control, vol. 12, no. 2, pp. 54–59, 1967.

[15] F. Amato and M. Ariola, "Finite-time control of discrete-time linear systems," IEEE Transactions on Automatic Control, vol. 50, no. 5, pp. 724–729, 2005.

[16] G. D. Zong, R. H. Wang, W. X. Zheng, and L. I. Hou, "Finite-time stabilization for a class of switched time-delay systems under asynchronous switching," Applied Mathematics and Computation, vol. 219, no. 11, pp. 5757–5771, 2013.

[17] H. G. Li, Q. Zhou, B. Chen, and H. H. Liu, "Parameter-dependent robust stability for uncertain Markovian jump systems with time delay," Journal of the Franklin Institute, vol. 348, no. 4, pp. 738–748, 2011.

[18] K. Ramakrishnan and G. Ray, "Robust stability criterion for Markovian jump systems with nonlinear perturbations and mode-dependent time delays," International Journal of General Systems, vol. 41, no. 4, pp. 373–393, 2012.

[19] L. H. Hou, G. D. Zong, and Y. Q. Wu, "Observer-based finite-time exponential $L_\infty$ control for discrete-time switched delay systems with uncertainties," Transactions of the Institute of Measurement and Control, vol. 35, no. 3, pp. 310–320, 2013.

[20] Y. Zhang, S. Xu, and J. Zhang, "Delay-dependent robust $H_\infty$ control for uncertain fuzzy Markovian Jump systems," International Journal of Control Automation and Systems, vol. 7, no. 4, pp. 520–529, 2009.

[21] J. Gao, B. Huang, and Z. Wang, "LMI-based robust $H_\infty$ control for uncertain linear Markovian jump systems with time-delays," Automatica, vol. 37, no. 7, pp. 1141–1146, 2001.

[22] S. P. He and F. Liu, "Unbiased $H_\infty$ filtering for neutral Markov jump systems," Applied Mathematics and Computation, vol. 206, no. 1, pp. 175–185, 2008.

[23] S. P. He and F. Liu, "Robust finite-time $H_\infty$ control of stochastic jump systems," International Journal of Control Automation and Systems, vol. 8, no. 9, pp. 1336–1341, 2010.

[24] S. P. He and F. Liu, "Stochastic finite-time stabilization for uncertain jump systems via state feedback," Journal of Dynamic Systems, Measurement and Control, vol. 132, no. 3, Article ID 034504, 4 pages, 2010.

[25] P. Balasubramaniam, R. Krishnasamy, and R. Rakkiyappan, "Delay-dependent stability criterion for a class of non-linear singular Markovian jump systems with mode-dependent interval time-varying delays," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 9, pp. 3612–3627, 2012.
[26] X. Mao, “Stability of stochastic differential equations with Markovian switching,” *Stochastic Processes and Their Applications*, vol. 79, no. 1, pp. 45–67, 1999.

[27] Y. Y. Wang, L. H. Xie, and C. E. de Souza, “Robust control of a class of uncertain nonlinear systems,” *Systems & Control Letters*, vol. 19, no. 2, pp. 139–149, 1992.