QED radiative corrections for elastic $e(\mu)p$ scattering in hadronic variables

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(April 26, 2022)

A numerical analysis of QED radiative corrections for elastic $e(\mu)p$ scattering in hadronic variables at energies of the current experiment at JLab is performed. The explicit formulae from the review of Akhundov et al. resulting from the integration over the phase space of leptonic variables plus photon are used to obtain the values of the cross sections and the radiative correction factor for unpolarized lepton-proton scattering. Our numerical results agree with the corresponding results arising from the formulae of Afanasev et al.

PACS numbers: Valid PACS appear here

The new detector generation allows to measure both scattered electron and hadronic final state in deep inelastic $ep$ scattering at HERA

$$e + p \rightarrow e + X,$$

and elastic $ep$ scattering at the Thomas Jefferson National Accelerator Facility (JLab)

$$e + p \rightarrow e + p.$$

The possibilities of these detectors have opened a new page in the field of the Radiative Corrections for electron-proton collisions. Physical analysis of the $ep$ collisions is based now not only on the familiar leptonic variables, when only scattered electrons are detected, but also on kinematical variables from the hadron measurement, or some combinations of both, such as mixed variables. This has to be met with a new treatment of the radiative corrections for elastic and deep inelastic scattering in different variables. Different choices of variables have no influence on the cross sections in the Born approximation, but make huge differences in the predictions for the radiative corrections to these processes, because the kinematics of the bremsstrahlung contributions to the processes with non-observed photon(s):

$$e + p \rightarrow e + X + n\gamma,$$

and

$$e + p \rightarrow e + p + n\gamma,$$

becomes quite different.

FIG. 1. Feynman diagrams contributing to the Born and Radiative corrections cross sections.

The first comprehensive contribution in this direction has been carried out by Akhundov et al. In that original review-paper, the model-independent QED corrections in leptonic, hadronic, mixed and Jaquet-Blondel variables were treated on a common base. Analytical and semi-analytical formulae were derived for deep inelastic scattering equation. The QED radiative corrections for the process have been recently calculated in the hadronic variables by considering unpolarized and polarized parts of the cross sections.

The main aim of this report is to present explicitly very compact analytical formulae for QED radiative corrections to elastic $e(\mu)p$ scattering (see Fig.1) in the hadronic variables resulting from the general approach of Akhundov et al. For this purpose we calculate the QED corrections for the energies of JLab using a new parametrization of the form factors, and compare our numerical results with corresponding ones of ref.

For the lepton-proton reactions and , with the competing processes , the hadronic variables are defined as follows:

$$Q_h^2 = (p_2 - p_1)^2, \quad y_h = \frac{p_1(p_2 - p_1)}{p_1k_1}, \quad x_h = \frac{Q_h^2}{y_hS}.$$
\[
\begin{align*}
s &= -(k_1 + p_1)^2 = S + m^2 + M^2, \\
m \text{ and } M \text{ are the masses of the incident lepton, and} \\
\text{the proton respectively.} \\
\text{For the elastic scattering } x_h = 1, \text{ and} \\
Q^2_h &= S y_h. \quad (6)
\end{align*}
\]

The Born cross section of the process \( \Box \) in the hadronic variables takes the form:

\[
\frac{d\sigma^B}{dQ^2_h} = \frac{2\pi\alpha^2}{\lambda_S} \sum_{i=1}^{3} A_i(Q^2_h) \frac{1}{Q^2_h} S^B_i(y_h), \quad (7)
\]

where \( \lambda_S = S^2 - 4m^2M^2 \). The leptonic functions \( S^B_i \) appearing in equation \( \Box \) are:

\[
\begin{align*}
S^B_1(y_h) &= Q^2_h - 2m^2, \\
S^B_2(y_h) &= 2(1 - y_h)S^2 - M^2Q^2_h, \\
S^B_3(y_h) &= 2Q^2_h(2 - y_h)S. \quad (8 - 10)
\end{align*}
\]

The hadronic functions \( A_i(Q^2_h) \) in equation \( \Box \) describe the electroweak interactions of leptons via the exchange of a photon or \( Z \) boson with unpolarized protons and are the generalized elastic form factors of the nucleon. These form factors were defined by formulae (20)-(22) of Akhundov et al. \( \Box \).

Using the results of ref. \( \Box \) (formulae (7.35), (1.10) and (1.11)) we obtain, for the QED corrected cross section of the elastic scattering process \( \Box \), the following expression:

\[
\frac{d\sigma}{dQ^2_h} = \frac{d\sigma^B}{dQ^2_h} \exp \left[ \frac{\alpha}{\pi} \delta^{\text{inf}}(Q^2_h) \right] + \frac{2\alpha^3}{S^2} \sum_{i=1}^{3} A_i(Q^2_h) \frac{1}{Q^2_h} S_i(y_h). \quad (11)
\]

The exponentiated dilogarithmic term \( \delta^{\text{inf}}(Q^2_h) \) takes into account the vertex corrections (Fig.1b) and the multiple soft photon emission:

\[
\delta^{\text{inf}}(Q^2_h) = (L_h - 1) \ln(1 - y_h),
\]

where \( L_h = \ln(Q^2_h/m^2) \). The second term in equation \( \Box \) is the hard bremsstrahlung correction of order \( \mathcal{O}(\alpha) \) resulting from the threefold analytical integration over the phase space of the process \( \Box \) with emission of the hard photon from the lepton (Fig.1d,1e):

\[
\begin{align*}
S_1(y_h) &= Q^2_h \left[ \frac{1}{16} \ln^2 y_h - \frac{1}{2} \ln^2(1 - y_h) - \ln y_h \ln(1 - y_h) \\
&\quad - \frac{3}{2} \text{Li}_2(y_h) + \frac{1}{2} \text{Li}_2(1) - \frac{1}{2} \ln y_h L_h \\
&\quad + \frac{1}{4} \left( 1 + \frac{2}{y_h} \right) L_{h1} + \left( 1 - \frac{1}{y_h} \right) \ln y_h \\
&\quad - \left( 1 + \frac{1}{4y_h} \right) \right], \quad (12)
\end{align*}
\]

\[
\begin{align*}
S_2(y_h) &= S^2 \left[ -\frac{1}{2} y_h \ln^2 y_h - (1 - y_h) \ln^2(1 - y_h) \\
&\quad - 2(1 - y_h) \ln y_h \ln(1 - y_h) - y_h \text{Li}_2(1) \\
&\quad - (2 - 3y_h) \text{Li}_2(y_h) + y_h \ln y_h L_h \\
&\quad + \frac{y_h}{2} (1 - y_h) L_{h1} - \frac{y_h}{2} (2 - y_h) \ln y_h \right], \quad (13)
\end{align*}
\]

\[
\begin{align*}
S_3(y_h) &= S Q^2_h \left[ -(2 - y_h) \left[ \frac{1}{2} \ln^2 y_h + \ln^2(1 - y_h) \\
&\quad + 2 \ln y_h \ln(1 - y_h) + 3 \text{Li}_2(y_h) - \text{Li}_2(1) \\
&\quad + \ln y_h + \ln y_h L_h \right] + \frac{3}{2} y_h L_{h1} + y_h \\
&\quad - \frac{7}{2} + 2 \left( 1 - 2y_h \right) \ln y_h \right], \quad (14)
\end{align*}
\]

where

\[
L_{h1} = \ln \left( \frac{Q^2_h}{m^2} \frac{y_h}{1 - y_h} \right). \quad (15)
\]

The formulae \( \Box \) are ultra-relativistic, i.e. \( S >> M^2 \). The running of the QED coupling \( \alpha \) may be taken into account in the standard way, as presented in refs. \( \Box \).

At low energies the dominant contribution to \( \Box \) comes from diagrams with \( \gamma \) exchange, and all numerical calculations can be performed using only the leptonic functions \( S^B_1(y_h), S_{1,2}(y_h) \) and the form factors \( A_{1,2}(Q^2_h) \):

\[
\begin{align*}
A_1(Q^2_h) &= Q^2_h G^2_{M}(Q^2_h), \quad (16) \\
A_2(Q^2_h) &= \frac{G^2_{E}(Q^2_h) + \tau G^2_{M}(Q^2_h)}{1 + \tau}, \quad (17)
\end{align*}
\]

where \( G^2_{E}(Q^2_h) \) and \( G^2_{M}(Q^2_h) \) are the standard electromagnetic form factors and \( \tau = Q^2_h/4M^2 \).
To compare our results with others, we use the radiative correction factor $\delta(Q^2_h)$ defined by:

$$\delta(Q^2_h) = \frac{d\sigma^{\text{theor}}/dQ^2_h}{d\sigma^{\text{B}}/dQ^2_h} - 1, \quad (18)$$

where $d\sigma^{\text{theor}}/dQ^2_h$ is the theoretical approximation to the measured cross section $d\sigma^{\text{meas}}/dQ^2_h$.

The cross section equation (11) and the contribution from vacuum polarization (Fig. 1c) have been included in $d\sigma^{\text{theor}}/dQ^2_h$. The running of the coupling $\alpha$ has been found to give a contribution of about 3% to the radiative correction factor $\delta$.

Figures 3 and 4 show $\delta(Q^2_h)$ for $S=5, 8, 30 \text{ GeV}^2$ from our calculations (solid line) and from the calculations using formulae of Afanasev et al.\(^1\) (dashed line) that contain the proton mass. As can be seen from the Figures 3 and 4, we have a very good agreement in the order and the shape of radiative corrections.

For precise comparison between the two independent calculations we have neglected the mass of proton in the formulae of Afanasev et al. (dotted lines on Figures 3 and 4) and found a good agreement.

**Acknowledgments**

Two of us A.A. and H.A. would like to thank the research center at KSU for support under project Phys/1420/27, and one of us H.H. would like to thank the NCMP at KACST.

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\(^1\) The formulae of Born cross section in the first paper \cite{Afanasev2001} contain an extra factor of 2.
FIG. 3. Radiative correction factor $\delta (Q^2)$ for $ep$-scattering at (a) $S=5 \text{ GeV}^2$, (b) $S=8 \text{ GeV}^2$ and (c) $S=30 \text{ GeV}^2$. Solid lines represent our calculations and dashed and dotted lines are as calculated from the formulae of ref. \[8\].

FIG. 4. Radiative correction factor $\delta (Q^2)$ for $\mu p$-scattering at (a) $S=5 \text{ GeV}^2$, (b) $S=8 \text{ GeV}^2$ and (c) $S=30 \text{ GeV}^2$. Solid lines are the present calculations and dashed and dotted lines are as calculated from the formulae of ref. \[8\].