Analysis of the theory of calculation of fiber-reinforced concrete with non-steel fibers

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Abstract. The fiber-reinforced concrete finds more applications in modern construction. This material has improved strength and deformation characteristics. The fiber-reinforced concrete is considered to be one of the most promising building materials in the world practice. The experience of using this material in developed countries, such as the USA, UK, Japan, Germany, Italy, France, Australia, has shown the good technical and economic indicators in building constructions. Recently, the base of standards for preparation, calculation and design of such concretes has been prepared and introduced in Russia. The method of determination of the class of the fiber-reinforced concrete by residual tensile strength, the calculation of the strength of standard sections, the diagrams of the deformation is discussed in the article based on the new Code of Practice. The obtained results of analysis of the theory of calculation suggest further studies of the properties and physical characteristics.

1. Introduction
The uniformly distributed reinforcement of the entire volume of concrete eliminates the disadvantages of unreinforced concrete. In particular, it is a low flexural strength and shrinkage cracking during hardening and using of structures. This type of reinforcement can significantly increase the service life of reinforced concrete structures.

The fiber-reinforced concrete is a type of reinforced concrete dispersed-reinforced with various high-strength modified fibers (steel and non-steel). At the beginning of the 20th century, Russian engineer V P Nekrasov one of the first in the world proposed dispersion-reinforced concrete and conducted researches its building performance [1]. As the micro armature, they used pieces of wire of small diameters. Foreign scientists also worked on this topic, for example, Gary Porter in 1910 stated that the mechanical characteristics of concrete increase approximately 8 times with the introduction of wire and nails [2]. However, the idea of dispersed reinforcement was gradually forgotten, and only in the 1960s, it was rapidly developing again, due to the urgent need to improve the physical and mechanical properties of concrete in tension and increase the fracture toughness.

Currently, the nomenclature of the used fibers is very extensive. According to the accepted classification they are divided:

• According to modulus of elasticity on high modulus (steel, carbon, glass, etc.) and low modulus (polypropylene, viscose, etc.);
• According to origin on natural (asbestos, basalt, wool, etc.) and artificial (viscose, polyamide, etc.);
• According to the main material on the metal (most often steel) and non-metal (synthetic, mineral).

The code of practice «Fiber reinforced concrete structures and precast products with non-steel fibers. Design rules» was published in 2017 [3]. The industry-specific code of practice «Design and basic principles for technology for production of fiber reinforced concrete structures» was the first published document on this theme [4].
The fiber-reinforced concrete like any other structural material, has strength and deformation characteristics. Compared with the concrete without dispersed reinforcement, the fiber-reinforced concrete is characterized by high strength under various types of impact: tension, shear, crack strength. And this material is characterized by crack toughness. The strength properties of the fiber-reinforced concrete have been studied for a long time, and they are well studied by Russian and foreign experts [5-12]. After analysis of the scientific literature on the subject of fiber-reinforced concrete [13-20] had been carried out, investigation of definition of mechanical characteristics of this material was determined.

2. Method of research

According to the code of practice [3] analysis of the theory of determining the residual tensile strength of the fiber-reinforced concrete was carried out.

The main strength characteristics of fiber-reinforced concrete:
- The standard value of the resistance of the fiber-reinforced concrete to axial compression $R_{fb,n}$;
- The standard value of the resistance of the fiber-reinforced concrete to axial tension $R_{fb,t,n}$;
- The standard value of the residual resistance of the fiber-reinforced concrete to axial tension $R_{fb,t2,n}$;
- The standard value of the residual resistance of the fiber-reinforced concrete to axial tension $R_{fb,t3,n}$.

The standard and calculation value of the resistance of the fiber-reinforced concrete to axial compression are accepted according to the code of practice [12] for a similar class of the ordinary concrete.

The standard value of the resistance of the fiber-reinforced concrete to axial tension and the residual resistance of the fiber-reinforced concrete to axial tension are determined from the results of tests of control samples for axial tension.

It is required to make a test beam with a cut for the action of a concentrated load to determine the residual tensile strength of the fiber-reinforced concrete (figure 1).

The cut width should be less than 5 mm, depth - (25 ± 1) mm, the distance between the tip of cut and the upper side of the test beam $h_{sp} = (125 ± 1)$ mm.

As a result of testing, a diagram «$F$–$a_F$» is drawn (figure 2). Where $a_F$ is the displacement of the external side of the cut.

Figure 1. The scheme of the test beam (width $b = 150$ mm, height $h = 150$ mm, length $L = 550$ mm).

Figure 2. The diagram of “load – displacement of the external side of the cut”.

For the sample, the strength values are determined taking into account the inelastic properties of the fiber-reinforced concrete according to the formulas:

\[ R_{F,0.5} = \frac{3F_{0.5,l}}{2bh_{up}^2} k_{F,0.5}, \]  
\[ R_{F,2.5} = \frac{3F_{2.5,l}}{2bh_{up}^2} k_{F,2.5}, \]  
where \( F_{0.5} \) is the load value corresponding to the displacement of the external side of the cut \( a_F=0.5 \) mm; \( F_{2.5} \) is the load value corresponding to the displacement of the external side of the cut \( a_F=2.5 \) mm; \( k_{F,0.5}=0.4 \) and \( k_{F,2.5}=0.34 \) are coefficients of accounting for inelastic deformations in the fiber-reinforced concrete of the stretched zone of the test beam.

The standard values of the residual resistance of the fiber-reinforced concrete to axial tension are determined according to the formulas:

\[ R_{f_{bt},2m} = R_{F,0.5,m}(1-1.64\nu_{F,0.5,m}), \]  
\[ R_{f_{bt},3m} = R_{F,2.5,m}(1-1.64\nu_{F,2.5,m}), \]  
where \( R_{F,0.5,m} \) and \( R_{F,2.5,m} \) are average values of the residual tensile strength of the fiber-reinforced concrete; \( \nu_{F,0.5,m} \) and \( \nu_{F,2.5,m} \) are coefficients of variation determined by the formulas:

\[ \nu_{F,0.5,m} = \frac{S_{F,0.5,m}}{R_{F,0.5,m}}, \]  
\[ \nu_{F,2.5,m} = \frac{S_{F,2.5,m}}{R_{F,2.5,m}}, \]  
where \( S_{F,0.5,m} \) and \( S_{F,2.5,m} \) are the quadratic means determined by the formulas:

\[ S_{F,0.5,m} = \sqrt{\frac{\sum_{i=1}^{n}(R_{F,0.5,i} - R_{F,0.5,m})^2}{(n-1)}}, \]  
\[ S_{F,2.5,m} = \sqrt{\frac{\sum_{i=1}^{n}(R_{F,2.5,i} - R_{F,2.5,m})^2}{(n-1)}}, \]  
where \( n \) is the number of the test beam.

According to the code of practice [3], if axial tensile tests are not carried out, the class of the fiber-reinforced concrete for axial tensile strength can be assigned according to the results of the bending tests.

In this case, the value of strength is determined by the formula:

\[ R_{f_{el}} = \frac{3F_{el,l}}{2bh_{up}^2} k_{F_{el}}, \]  
where \( F_{el} \) is maximum value of the load in the range of the displacement of the external side of the cut in the range \( 0 < a_F < 0.05 \) mm; \( k_{F_{el}}=0.6 \) is coefficient of accounting for inelastic deformations in the fiber-reinforced concrete of the stretched zone of the test beam.

In this case the standard value of the resistance of the fiber-reinforced concrete to axial tension is determined by the formula:

\[ R_{f_{bt},n} = R_{f_{el},n}(1-1.64\nu_{F_{el},m}), \]
where $R_{Fel,m}$ is the average tensile strength of the fiber-reinforced concrete; $\nu_{Fel,m}$ is the coefficient of variation determined by the formula:

$$\nu_{Fel,m} = \frac{S_{Fel,m}}{R_{Fel,m}},$$  \hspace{1cm} (11)$$

where $S_{Fel,m}$ is the quadratic mean determined by the formula:

$$S_{Fel,m} = \left( \frac{1}{n} \sum_{i=1}^{n} (R_{Fel,i} - R_{Fel,m})^2 \right)^{1/2} \frac{1}{(n-1)}.$$  \hspace{1cm} (12)

The connection between deflection and displacement are determined by the formula:

$$f = 0.85a_F + 0.04,$$  \hspace{1cm} (13)

where $a_F$ is a value:
- If the distance $y$ from the axis of the displacement transducer to the lower face of the test beam is zero, it is taken equal to the value measured during the test;
- Otherwise this value is calculated by the formula:

$$a_F = a_{Fy} \frac{h}{h+y}.$$  \hspace{1cm} (14)

3. Results and discussion

As a result of the testing and determining the standard value of the resistance of the fiber-reinforced concrete to axial tension, the class of the fiber-reinforced concrete can be chosen, taking into account the subclass "a", "b", "c", "d" and "e" according to the code of practice [3].

According to the code of practice [3], the minimum standard value of the resistance of the fiber-reinforced concrete to axial tension is 0.77 MPa (for the class of the fiber-reinforced concrete $B_{ft3}i$ with the subclass index $i=e$), and the maximum - 16 MPa (for the class of the fiber-reinforced concrete $B_{ft3}8i$ with the subclass index $i=a$).

The analysis of the formulas for calculating the standard value of the resistance of the fiber-reinforced concrete to axial tension has shown a large using of empirical coefficients, the physical meaning of these cannot be explained.

According to the code of practice [3], the working diagram of deformation of the fiber-reinforced concrete to axial compression is the curved diagram with a falling branch, or a simplified three-line diagram with parametric points of the compressed fiber-reinforced concrete as for the ordinary concrete (figure 3). The working diagram of deformation of the fiber-reinforced concrete to axial tension is the simplified three-line diagram (figure 4). The tensile stresses of the fiber-reinforced concrete $\sigma_{fbt}$ are determined depending on the relative deformations $\varepsilon_{fbt}$ according to the formulas of the code of practice [3].

![Figure 3. The diagram of the deformation of the ordinary concrete according to the code of practice [12].](image-url)
The stress diagram in the compressed zone of the fiber-reinforced concrete and the ordinary concrete is taken a triangular shape (as for an elastic solid), and for a tensile zone of fiber-reinforced concrete - a trapezoidal shape with stresses in the tensile section (figure 5).

The calculation of the bent elements of critical force for the fiber-reinforced concrete and the ordinary concrete is produced from the condition:

\[ M \leq M_{ult} \]  

where \( M \) is the bending moment of the external load; \( M_{ult} \) is the breaking bending moment that can be perceived by the section of the element, \( M_{ult} = R_{fb}W_{pl} \) - for the fiber-reinforced concrete; \( M_{ult} = R_{bt}W \) - for the ordinary concrete.

The section modulus is different for the fiber-reinforced concrete and the ordinary concrete.

For example, the elastoplastic section modulus of the fiber-reinforced concrete element of a rectangular cross-section for an extreme tensile fiber (without working reinforcement) is determined by the formula:

\[ W_{pl} = \frac{bh^2}{3.6} \]  

The elastoplastic section modulus of the ordinary concrete for an extreme tensile fiber is determined by the formula:

\[ W = \frac{bh^2}{6} \]  

The calculation of the strength of standard sections of the fiber-reinforced concrete and the concrete elements with working reinforcement is made depending on the ratio between the value of
the relative height of the compression side of the fiber-reinforced concrete and the concrete accordingly \( \xi = \frac{x}{h_0} \). The value of the boundary relative height of the compression side \( \xi_R \) is determined taking into account the relative deformation of the fiber-reinforced concrete \( \varepsilon_{fb} \) and the ordinary concrete \( \varepsilon_{b} \), the calculation value of the critical relative deformations of the reinforcement \( \varepsilon_s \).

\[
\xi_R = \frac{x_R}{h_0} = \frac{\omega}{1 + \frac{\varepsilon_s}{\varepsilon_{fb2}(\varepsilon_{b2})}}
\]

where \( \omega \) is a characteristic of the compression side of fiber-reinforced concrete (ordinary concrete).

According to the code of practice [5] the values of relative deformations of the compressed fiber-reinforced concrete and the concrete is allowed to take equal \( \varepsilon_{fb2} = \varepsilon_{b2} \).

When calculation of the flexural fiber-reinforced concrete and the concrete elements of rectangular cross-section with the working reinforcement (figure 6), the value \( M_{ult} \) and the height of the compression side are determined according to the standards (table 1):

**Table 1.** The calculation of the flexural fiber-reinforced concrete and the concrete elements.

| The fiber-reinforced concrete element | The concrete element |
|--------------------------------------|----------------------|
| \( M_{ult} = R_{fb} b x (h_0 - 0.5x) - \) | \( M_{ult} = R_b b x (h_0 - 0.5x) + \) |
| \(-R_{fb3} b (h - x) \times \frac{h - x}{2} - a + R_{s'} A' \) \( h_0 - a' \) | \( +R_{s'} A' \) \( h_0 - a' \) |
| \( x = R_s A_s - R_{s'} A'_s + R_{fb3} b h (R_{fb} + R_{fb3}) b \) | \( x = \frac{R_s A_s - R_{s'} A'_s}{R_b b} \) |

**Figure 6.** The design of stresses and stress diagram across the rectangular cross-section with the reinforcement in its strength calculation: a) for the fiber-reinforced concrete; b) for the ordinary concrete;

4. Conclusion

1. The conducted analysis of the theory of calculation of strength characteristics of the fiber-reinforced concrete according to the code of practice [3] has shown a large using of empirical coefficients to take into account inelastic deformations in the tensile side.
2. To determine the class of the fiber-reinforced concrete by residual tensile strength, it is an imperative to carry out the experiment of the test beams for the bending.

3. The minimum standard value of the resistance of the fiber-reinforced concrete to axial tension is 0.77 MPa (for the class of the fiber-reinforced concrete $B_{03i}$ with the subclass index $i = e$), and the maximum - 16 MPa (for the class of the fiber-reinforced concrete $B_{038i}$ with the subclass index $i = a$). For the ordinary concrete, according to the code of practice [6], the minimum standard value of the resistance to axial tension is 0.39 MPa (for the class of the concrete B3.5), and the maximum – 3.8 MPa (for the class of the concrete B100). The standard value of the resistance of the fiber-reinforced concrete to axial tension is analyzed to be about 4 times greater than that of the ordinary concrete.

4. The standard value of the resistance of the fiber-reinforced concrete to axial tension was happen to be about 4 times greater than that of the ordinary concrete.

5. The stress diagram in the compressed zone of the fiber-reinforced concrete and the ordinary concrete is taken a triangular shape (as for an elastic solid), and for a tensile zone of the fiber-reinforced concrete - a trapezoidal shape with stresses in the tensile section.

6. The obtained results of analysis of the theory of calculation of fiber-reinforced concrete with non-steel fibers suggest further studies of the properties and physical characteristics. Series of experiment is planned to conduct due to researching the theory data. The main purpose of experimental researches will be proof or refutation of the application of the theory of calculation of ordinary concrete structures to fiber-reinforced concrete.

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