When does the choice of the refractive index of a linear, homogeneous, isotropic, active, dielectric medium matter?

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Abstract

Two choices are possible for the refractive index of a linear, homogeneous, isotropic, active, dielectric material. Either of the choices is adequate for obtaining frequency-domain solutions for (i) scattering by slabs, spheres, and other objects of bounded extent; (ii) guided-wave propagation in homogeneously filled, cross-sectionally uniform, straight waveguide sections with perfectly conducting walls; and (iii) image formation due to flat lenses. The correct choice does matter for the half-space problem, but that problem is not realistic.

1 Introduction

A century and a half after the unification of light with electricity and magnetism, certain aspects of electromagnetic wave propagation in a linear, homogeneous, isotropic, dielectric material continuum remain unsettled. One of those aspects is as follows. If \( \epsilon_r(\omega) = \epsilon'_r(\omega) + i\epsilon''_r(\omega) \) denotes the relative permittivity — where \( \epsilon'_r \in \mathbb{R}, \epsilon''_r \in \mathbb{R} \), and an \( \exp(-i\omega t) \) time-dependence is implicit — then the refractive index \( n(\omega) \) satisfies the dispersion relation \( n^2(\omega) = \epsilon_r(\omega) \). Is the refractive index equal to \( n^+ \) or \( n^- = -n^+ \), where \( \Im(n^+) > 0 \)?

Suppose that a material is passive at a specific frequency \( \Omega \), i.e., \( \epsilon''_r(\Omega) > 0 \). The general consensus is that the refractive index is equal to \( n^+(\Omega) \). If the material is active at the frequency \( \Omega \) (i.e., \( \epsilon''_r(\Omega) < 0 \)), however, disagreements on the correct choice of the refractive index have surfaced. We are exclusively concerned with linear, homogeneous, isotropic, active, dielectric materials in this communication, subject to the additional stipulations that the constitutive properties are spatially local and unalterable by the passage of an electromagnetic signal.

Many reasons have been provided for the refractive index of such a material to be \( n^-(\Omega) \) [1]–[7] and \( n^+(\Omega) \) [8]–[11]. A direct time-domain solution of the Maxwell equations has recently upheld the possibility of \( n^+(\Omega) \) as the refractive index [12]. Even more importantly, the adoption of the time-domain procedure implies that the refractive index cannot be determined at a single frequency by frequency-domain analyses, if the material is active at that frequency. By virtue of this argument, it is possible for two different active materials to have the same permittivity but opposite refractive indexes at a certain frequency [13].

Thus, in order to choose the refractive index of an active material at a specific frequency, the solution of a time-domain problem is needed with an incident signal of sufficient bandwidth to cover
the frequency of interest (though it appears that a frequency–domain algorithm may sometimes suffice to determine the variation of \( n(\omega) \) vs. \( \omega \) over a sufficiently wide \( \omega \)–range [13–15]). The time–domain problem should be such that the correct choice of the refractive index in the corresponding frequency–domain problem must be consequential — e.g., the reflection of a signal by a half–space filled with a material. This naturally leads to the question: when is knowledge of the correct choice of the refractive index essential for the solution of a frequency–domain problem?

In the following section, several commonly tackled frequency–domain boundary–value problems are discussed, in order to answer the foregoing question. The selected problems include scattering by objects of bounded extent; guided–wave propagation in homogeneously filled straight waveguide sections with perfectly conducting walls and uniform cross–section; and lensing by slabs.

2 Frequency–domain Boundary–value Problems

2.1 Planewave response of a slab

Let us begin by considering the electromagnetic response of the region \( 0 < z < L \) filled with an active material. The regions \( z < 0 \) and \( z > L \) are vacuous.

Because the dyadic Green function for free space can be expanded in terms of an angular spectrum of plane waves, we can consider the response of the slab to a plane wave without significant loss of generality. An incident plane wave can be represented by

\[
E_{\text{inc}}(r, \omega) = \left[ a_s \hat{y} + a_p \left( -\frac{\omega_0}{k_0} \hat{x} + \frac{\kappa}{k_0} \hat{z} \right) \right] \exp \left[ i (\kappa x + \omega_0 z) \right], \quad z \leq 0, \tag{1}
\]

where \( a_s \) and \( a_p \) are the amplitudes of the \( s \)– and \( p \)–polarized components, respectively; \( k_0 \) is the free–space wavenumber; and \( \omega_0 = +\sqrt{k_0^2 - \kappa^2} \) with \( \kappa \in \mathbb{R} \). The reflected and the transmitted plane waves are represented by

\[
E_{\text{ref}}(r, \omega) = \left[ r_s \hat{y} + r_p \left( \frac{\omega_0}{k_0} \hat{x} + \frac{\kappa}{k_0} \hat{z} \right) \right] \exp \left[ i (\kappa x - \omega_0 z) \right], \quad z \leq 0, \tag{2}
\]

and

\[
E_{\text{tr}}(r, \omega) = \left[ t_s \hat{y} + t_p \left( -\frac{\omega_0}{k_0} \hat{x} + \frac{\kappa}{k_0} \hat{z} \right) \right] \exp \left\{ i [\kappa x + \omega_0(z - L)] \right\}, \quad z \geq L, \tag{3}
\]

respectively, while the electric field phasor inside the slab region is

\[
E_{\text{int}}(r, \omega) = \left[ c_s \hat{y} + c_p \left( -\frac{\omega}{k_0 n} \hat{x} + \frac{\kappa}{k_0 n} \hat{z} \right) \right] \exp \left[ i (\kappa x + \omega z) \right]
+ \left[ d_s \hat{y} + d_p \left( \frac{\alpha}{k_0 n} \hat{x} + \frac{\kappa}{k_0 n} \hat{z} \right) \right] \exp \left[ i (\kappa x - \omega z) \right], \quad 0 \leq z \leq L. \tag{4}
\]

where \( \alpha = +\sqrt{k_0^2 n^2 - \kappa^2} \). Standard techniques yield the amplitudes

\[
\begin{align*}
  r_p &= -a_p (e^{2i\alpha L} - 1)(\alpha^2 - n^4 \alpha_0^2)/\Delta_p, \quad t_p = -4n^2 a_p e^{2i\alpha L} \alpha_0 / \Delta_p, \\
  c_p &= -2a_p n \alpha_0 (\alpha + n^2 \alpha_0) / \Delta_p, \quad d_p = -2a_p n \alpha_0 (\alpha - n^2 \alpha_0) / \Delta_p, \\
  r_s &= -a_s (e^{2i\alpha L} - 1)(\alpha^2 - \alpha_0^2) / \Delta_s, \quad t_s = -4a_s e^{2i\alpha L} \alpha_0 / \Delta_s, \\
  c_s &= -2a_s \alpha_0 (\alpha + \alpha_0) / \Delta_s, \quad d_s = 2a_s \alpha_0 (\alpha - \alpha_0) / \Delta_s
\end{align*}
\tag{5}
\]
where
\[ \Delta_p = \left( e^{2i\alpha_L} - 1 \right) \left( \alpha^2 + n^2 \alpha_0^2 \right) - 2n^2 \left( e^{2i\alpha_L} + 1 \right) \alpha \alpha_0 \]
\[ \Delta_s = \left( e^{2i\alpha_L} - 1 \right) \left( \alpha^2 + \alpha_0^2 \right) - 2 \left( e^{2i\alpha_L} + 1 \right) \alpha \alpha_0 \] \tag{6}

The reflected, transmitted, and the internal electric field phasors are not affected by the change \( n \rightarrow -n \); neither are the corresponding magnetic field phasors. Therefore, the fields everywhere are unaffected whether \( n = n_+ \) or \( n = n_- \) is chosen. The same conclusion is obtained if the plane \( z = L \) is assumed to be perfectly conducting.

### 2.2 Planewave response of a sphere

Let us next consider the spherical region \( r < a \) occupied by an active material, whereas the region \( r > a \) is vacuous. Without loss of generality, we take the sphere to be illuminated by a linearly polarized plane wave traveling along the \( +z \) axis. As is commonplace, the incident plane wave is represented in terms of vector spherical harmonics, \( \mathbf{M}^{(j)}_{\sigma \mu \nu}(w) \) and \( \mathbf{N}^{(j)}_{\sigma \mu \nu}(w) \) \cite{16}, as

\[ \mathbf{E}_{\text{inc}}(r, \omega) = A \sum_{\nu=1}^{\infty} i^\nu \frac{2\nu + 1}{\nu(\nu + 1)} \left[ \mathbf{M}^{(1)}_{\nu \mu \nu}(k_0 r) - i \mathbf{N}^{(1)}_{\nu \mu \nu}(k_0 r) \right], \] \tag{7}

where \( A \) is the amplitude. The scattered and the internal electric field phasors are also expressible in terms of vector spherical harmonics; thus \cite{16},

\[ \mathbf{E}_{\text{sc}}(r, \omega) = A \sum_{\nu=1}^{\infty} i^\nu \frac{2\nu + 1}{\nu(\nu + 1)} \left[ i a_{\nu} \mathbf{N}^{(3)}_{\nu \mu \nu}(k_0 r) - b_{\nu} \mathbf{M}^{(3)}_{\nu \mu \nu}(k_0 r) \right], \quad r \geq a \] \tag{8}

and

\[ \mathbf{E}_{\text{int}}(r, \omega) = -A \sum_{\nu=1}^{\infty} i^\nu \frac{2\nu + 1}{\nu(\nu + 1)} \left[ i d_{\nu} \mathbf{N}^{(1)}_{\nu \mu \nu}(n k_0 r) - c_{\nu} \mathbf{M}^{(1)}_{\nu \mu \nu}(n k_0 r) \right], \quad r \leq a, \] \tag{9}

where

\[ a_{\nu} = \frac{n^2 j_{\nu}(n k_0 a) \psi^{(1)}_{\nu}(k a) - j_{\nu}(n k_0 a) \psi^{(1)}_{\nu}(n k_0 a)}{n^2 j_{\nu}(n k_0 a) \psi^{(3)}_{\nu}(k a) - h^{(1)}_{\nu}(k a) \psi^{(1)}_{\nu}(n k_0 a)}; \] \tag{10}

\[ b_{\nu} = \frac{j_{\nu}(n k_0 a) \psi^{(1)}_{\nu}(k a) - j_{\nu}(k a) \psi^{(1)}_{\nu}(n k_0 a)}{j_{\nu}(n k_0 a) \psi^{(3)}_{\nu}(k a) - h^{(1)}_{\nu}(k a) \psi^{(1)}_{\nu}(n k_0 a)}; \] \tag{11}

\[ c_{\nu} = \frac{i \left[ j_{\nu}(k a) \right]}{i n / (k a)}; \] \tag{12}

\[ d_{\nu} = \frac{n^2 j_{\nu}(n k_0 a) \psi^{(3)}_{\nu}(k a) - h^{(1)}_{\nu}(k a) \psi^{(1)}_{\nu}(n k_0 a)}{n^2 j_{\nu}(n k_0 a) \psi^{(3)}_{\nu}(k a) - h^{(1)}_{\nu}(k a) \psi^{(1)}_{\nu}(n k_0 a)}; \] \tag{13}

\( j_{\nu}(\xi) \) and \( h^{(1)}_{\nu}(\xi) \) are the spherical Bessel function and the spherical Hankel function of the first kind, respectively; and

\[ \psi^{(1)}_{\nu}(\xi) = \frac{d}{d\xi} \left[ \xi j_{\nu}(\xi) \right], \quad \psi^{(3)}_{\nu}(\xi) = \frac{d}{d\xi} \left[ \xi h^{(1)}_{\nu}(\xi) \right]. \] \tag{14}

Expressions for the corresponding magnetic field phasors can be derived using the Faraday equation.
Let the change $n \rightarrow -n$ be effected in Eqs. (7)–(13). Because $j_\nu(-\xi) = (-1)^\nu j_\nu(\xi)$ and $\psi^{(1)}_\nu(-\xi) = (-1)^\nu \psi^{(1)}_\nu(\xi)$, it follows that both $a_\nu$ and $b_\nu$ are not affected, and neither are the scattered electric and magnetic field phasors. Although $c_\nu \rightarrow (1)^\nu c_\nu$ and $d_\nu \rightarrow (1)^{\nu+1} d_\nu$, the internal electric and magnetic field phasors are not affected because $M^{(1)}_{\nu m}(w) = (1)^\nu M^{(1)}_{\nu m}(w)$ and $N^{(1)}_{\nu m}(-w) = (1)^{\nu+1} N^{(1)}_{\nu m}(w)$. Therefore, again, the fields everywhere do not depend on the choice of the refractive index of the active material. We have numerically verified that the same conclusion holds for a dielectric sphere with a perfectly conducting, concentric, spherical core.

### 2.3 More general scattering problems

The conclusion garnered in Sec. 2.2 is valid in a far more general situation. Let all space be divided into three mutually disjoint regions $V_0$, $V_J$, and $V_\ell$ as follows. Whereas both $V_J$ and $V_\ell$ are bounded in extent, the vacuous region $V_0$ extends to infinity in all directions. The region $V_\ell$ is filled with an active material, whereas the sources of the electromagnetic field reside wholly in $V_J$.

The solution of the frequency–domain Maxwell curl equations everywhere can be stated as [17]

$$
\mathbf{E}(r, \omega) = i \omega \mu_0 \int_{V_J} G(k_0 \mathbf{r}, k_0 \mathbf{r}') \cdot \mathbf{J}_{so}(r', \omega) \, d^3r' + k_0^2 \int_{V_\ell} \left[ n^2(\omega) - 1 \right] G(k_0 \mathbf{r}, k_0 \mathbf{r}') \cdot \mathbf{E}(r', \omega) \, d^3r', \quad r \in V_J \cup V_\ell \cup V_0,
$$

(15)

where $\mu_0$ is the free–space permeability, $G(k_0 \mathbf{r}, k_0 \mathbf{r}')$ is the dyadic Green function for free space, and $\mathbf{J}_{so}(r, \omega)$ is the source electric current density phasor. An expression for the magnetic field phasor can be derived from Eq. (15) and the Faraday equation. Once again, the electric and the magnetic field phasors everywhere are not affected by the change $n \rightarrow -n$. The same conclusion is obtained if the active material is nonhomogeneous, for which case $n^2(\omega)$ must be replaced by $n^2(r', \omega)$ in Eq. (15).

### 2.4 Homogeneously filled waveguides of uniform cross–section

A commonplace problem in electromagnetism is the propagation of waves in straight waveguides of uniform cross–section and perfectly conducting walls. Let us begin with single–conductor waveguides. An example is the rectangular waveguide whose walls are formed by the planes $x = 0$, $x = a$, $y = 0$, and $y = b$. Let the propagation direction be along the $+z$ axis. The electric field phasor in this waveguide may be represented as the modal sum [18]

$$
\mathbf{E}(r, \omega) = \sum_{p \in \mathbb{Z}} \sum_{q \in \mathbb{Z}} A_{pq}^{(TE)} \exp(\gamma_{pq}z) \frac{i \omega \mu_0}{\Delta_{pq}} \left\{ - \frac{q \pi}{b} \cos \left( \frac{q \pi}{b} y \right) \sin \left( \frac{q \pi}{b} y \right) \hat{x} \\
+ \frac{p \pi}{a} \sin \left( \frac{p \pi}{a} x \right) \cos \left( \frac{q \pi}{b} y \right) \hat{y} \right\} \\
+ \sum_{p \in \mathbb{Z}} \sum_{q \in \mathbb{Z}} A_{pq}^{(TM)} \exp(\gamma_{pq}z) \left\{ \frac{\gamma_{pq}}{\Delta_{pq}} \left[ \frac{p \pi}{a} \cos \left( \frac{p \pi}{a} x \right) \sin \left( \frac{q \pi}{b} y \right) \hat{x} \\
+ \frac{q \pi}{b} \sin \left( \frac{p \pi}{a} x \right) \cos \left( \frac{q \pi}{b} y \right) \hat{y} \right] + \sin \left( \frac{p \pi}{a} x \right) \sin \left( \frac{q \pi}{b} y \right) \hat{z} \right\},
$$

(16)
wherein the combination \( p = q = 0 \) is not allowed; \( A_{pq}^{(TE)} \) and \( B_{pq}^{(TM)} \) are modal coefficients for transverse–electric and transverse–magnetic modes, respectively; and \( \gamma_{pq}^2 = \Delta_{pq} - k_0^2 n^2 \) with \( \Delta_{pq} = (\pi n/a)^2 + (gq/a)^2 \). A similar expression for magnetic field phasor can be derived using the Faraday equation. The value of the propagation constant \( \gamma_{pq} \) of the \((pq)^{th}\) mode is chosen to ensure that the integral of the modal time–averaged Poynting vector over any \( xy \) plane is coparallel with \( \hat{z} \). Neither the electric nor the magnetic field phasor is affected by the change \( n \rightarrow -n \). The same conclusion holds true for parallel–plate waveguides \((b \rightarrow \infty)\) and circular waveguides [18].

More generally, in a cylindrical coordinate system \((u, v, z)\), let \( \mathbf{E}(r, \omega) = [\mathbf{e}_t(u, v, \omega) + e_z(u, v, \omega)\hat{z}] \exp(\gamma z) \) and \( \mathbf{H}(r, \omega) = [\mathbf{h}_t(u, v, \omega) + h_z(u, v, \omega)\hat{z}] \exp(\gamma z) \), where \( \hat{z} \cdot \mathbf{e}_t = 0 \) and \( \hat{z} \cdot \mathbf{h}_t = 0 \). Then, the source–free Maxwell curl equations yield the transverse components of the field phasors in terms of the longitudinal components as [18]

\[
\begin{align*}
\mathbf{e}_t(u, v, \omega) &= \frac{\gamma}{\gamma^2 + k_0^2 n^2} \left[ (\text{grad}_t e_z - \frac{i \omega \mu_0}{\gamma} \hat{z} \times (\text{grad}_t h_z)) \right], \\
\mathbf{h}_t(u, v, \omega) &= \frac{\gamma}{\gamma^2 + k_0^2 n^2} \left[ (\text{grad}_t h_z + \frac{i \omega \epsilon_0 n^2}{\gamma} \hat{z} \times (\text{grad}_t e_z)) \right],
\end{align*}
\]

where \( \text{grad}_t \equiv \text{grad} - \hat{z} \{\partial/\partial z\} \). The longitudinal field components satisfy the Helmholtz equation as follows:

\[
\left( \nabla^2 - \frac{\partial^2}{\partial z^2} + k_0^2 n^2 + \gamma^2 \right) \begin{cases} e_z(u, v, \omega) \\ h_z(u, v, \omega) \end{cases} = \begin{cases} 0 \\ 0 \end{cases}.
\]

For guided–wave propagation, \( \gamma^2 \) emerges from a dispersion equation derived from enforcing the usual boundary conditions on the perfectly conducting walls. Clearly, to obtain modal solutions for \( e_z \) and \( h_z \), the refractive index is not needed by itself, but its square is needed. Therefore, the fields are unaffected whether \( n = n_+ \) or \( n = n_- \) is chosen.

In a two–conductor waveguide, in addition to transverse–electric and transverse–magnetic modes, a transverse–electromagnetic mode can also propagate along the \( +z \) axis. This latter type of mode is like a plane wave, and its description does require knowledge of the correct refractive index. The number of transverse–electromagnetic modes increases with the number of conductors [18]. As waveguide sections of finite length are used in practice, modal propagation along the \( -z \) and \( +z \) directions must be considered simultaneously [19]. The modal coefficients of the counterpropagating modes in a waveguide section would adjust according to the boundary conditions enforced at both ends of the section, just as in Sec. 2.1 for a slab; thus, the correct choice of the refractive index need not be critical.

### 2.5 Flat lenses

A major motivation for current research on isotropic dielectric–magnetic materials that display negative phase velocity (i.e., \( \text{Re}(n) < 0 \)) is their potential to form flat lenses. Ideally, a flat lens is a slab made of nondissipative and nondispersive material with \( \epsilon_r = -1 \) and relative permeability \( \mu_r = -1 \) (i.e., anti–vacuum). When this lens is sandwiched between vacuum half–spaces, an image of a source placed on one side of the lens is formed on the other side of lens and another image is formed within the lens itself. Both propagating and evanescent modes are brought to a focus at the image points in this ideal scenario [20]. A rigorous frequency–domain analysis reveals that the creation of images is independent of the choice of the refractive index of the lens material; furthermore, the focusing properties are seriously compromised if \( \epsilon_r = \mu_r \neq -1 \) [21]. The imaging capabilities are further compromised by dissipation and/or dispersion, and the extent to which the slab demonstrates paraxial
focusing or channeling of the illuminating field is independent of whether the slab’s refractive index is chosen to lie in the upper or lower half–space in the complex plane [21]. From these results, it follows that the nonmagnetic, active dielectric material of our interest here is unsuitable for flat lenses.

3 Discussion

There remains the problem of reflection of a plane wave from a half–space filled with an active material [13, 15]. As has been shown by reference to the time–domain solution [12], only one of the two choices of the refractive index yields the correct frequency–domain reflectance; that is also the conclusion from an analysis with the Laplace transform [15]. However, a time–domain solution can never access an entire half–space; thus, the removal of the ambiguity through a comparative study may not be considered by some researchers to be totally compelling. Such a comparative study may not be possible for many active materials anyway, because the fields will continue to grow, invalidate the assumption of linearity, and eventually counter the assumption of temporal stationarity of the constitutive properties. Furthermore, whereas when solving a frequency–domain problem, a half–space can be considered to be an adequate simplification of a sufficiently thick slab when the material is dissipative, the process of taking the limit $L \to \infty$ in Sec. 2.1 — for an active material [10, 11] — has unphysical consequences in general [15]. This is because the back face of an active slab can receive a sufficiently strong field which it can reflect back to reach the front face, but a half–space has no back face.

To conclude, we have shown that, for the most common frequency–domain problems handled by electromagnetism researchers, knowledge of the correct choice of the refractive index of a linear, homogeneous, isotropic, active, dielectric material is inessential. Either of the two choices would serve adequately. The major exception we encountered is the half–space problem, but that problem is significantly unrealistic.

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