Strongly Coupled Supersymmetry as the Possible Origin of Flavor

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Abstract

The existence of the electroweak hierarchy gives one strong reason to suspect that nonperturbative physics lurks between the electroweak and the Planck scales. In this talk I speculate that these nonperturbative interactions might also be behind the fascinating but obscure flavor structure we see at low energies in the form of fermion masses and mixing angles. A “dual” Froggatt-Nielsen mechanism is shown to explain how hierarchies can arise from quark and lepton substructure.

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1 New strong interactions and flavor

A commonly held picture of physics beyond the standard model these days consists of the minimal supersymmetric extension, a desert with perturbative interactions up to the GUT scale, and just beyond, string theory. Occasional oddities populate the desert in various versions — perhaps an axion, an inflaton, or massive right-handed neutrinos. The machinery required for supersymmetry breaking is tastefully hidden behind a veil and called the hidden sector.

That machinery is expected to entail new strong interactions. The motivation is that the large hierarchy between the Planck and electroweak scale looks very much like the large hierarchy between the Planck and the QCD scales — after all, the QCD and electroweak scales only differ by two decades. The latter is due to nonperturbative physics, and results from the large value of $e^{1/\alpha_s}$, where $\alpha_s$ is the QCD coupling at the Planck scale. Similarly, it is plausible that the electroweak scale (or the SUSY breaking scale, in a supersymmetric theory) arises from some new nonperturbative physics. This ought to be an exciting conclusion, since one might expect nonperturbative physics to entail a rich phenomenology for experimentalists to explore. However, the success of precision electroweak predictions in the standard model, the
absence of observable flavor changing processes, the stability of the proton, and the apparent perturbative unification of coupling constants all serve to make new nonperturbative physics an unwelcome guest at the party. So it is usually relegated to the hidden sector where it won’t bother anyone.

In spite of the prevalent prejudice, it seems to my collaborators and me worth exploring whether new strong interactions at short distance scales might not also be useful in explaining the outstanding question in particle physics — namely the origin of flavor. There are few hints as to why we have three families, and why fermions have the peculiar mass ratios and mixing angles we observe. We have seen in the hadronic spectrum that complicated structure can arise from nonperturbative physics, and it seems natural to explore whether the spectrum of the patterns seen in the Yukawa interactions of the standard model might likewise be the result of strong dynamics. Our motivation is in part opportunistic — with the recent advance in understanding strongly coupled supersymmetric theory [2, 3], one can now reliably discuss theories with massless composite fermions, and so it is now at last possible to explore an old idea that the distinctions between families are due to internal substructure.

2 The flavor problem

Guessing at the dynamics underlying the quark and lepton masses and mixing angles is particularly difficult since one needs more information than is experimentally accessible. We would like to measure the 54 real parameters characterizing the three (up quark, down quark and charged lepton), $3 \times 3$, complex Yukawa matrices of the standard model; probably there is some natural basis for these matrices which exposes the underlying interactions that give rise to the observed flavor structure. However, out of those 54 parameters, all we can measure at low energy are thirteen: the three charged lepton and six quark masses, as well as the three angles and one complex phase in the CKM matrix (for simplicity I will ignore the possibility of neutrino masses). In the pure standard model, the other parameters are unphysical and can be eliminated by redefining the quark and lepton fields.

In an extension of the standard model which has flavor physics at short distances, many additional parameters of the Yukawa matrices are physical, and are encoded in higher dimension operators, such as dimension 6 four-fermion interactions (including flavor changing neutral currents, proton decay, etc.). When such operators are present in the theory, redefining the quark and lepton fields does not eliminate the “unphysical” parts of the Yukawa matrices, as they reappear in flavor changing (and possibly CP violating) higher dimensional operators. Unfortunately, such operators are experimentally inaccessible if the scale of the new physics is above $\sim 1000$ TeV.

3 Texture and broken flavor symmetry

A popular recourse is to guess at what possible form the Yukawa matrices take in the basis that is natural from the point of view of the short distance flavor physics, and then to try to construct the dynamics that would give rise to those Yukawa matrices. The game is “guess the texture”, where texture refers to the pattern and hierarchies of entries that appear in the Yukawa matrices. For example, in ref. [4], a model is described in which the Yukawa matrices
for the up and down quarks look like

\[
Y_{\text{up}} = \begin{pmatrix}
\epsilon_2^2 & 0 & \epsilon_2 \\
\epsilon_1 \epsilon_2 & \epsilon_2 & \epsilon_1 \\
\epsilon_2 & 0 & 1
\end{pmatrix} \quad Y_{\text{down}} = \begin{pmatrix}
\epsilon_2^2 & \epsilon_1 \epsilon_2 & \epsilon_1 \epsilon_2 \\
\epsilon_1 \epsilon_2 & \epsilon_1^2 & \epsilon_1^2 \\
\epsilon_2 & \epsilon_1 & \epsilon_1
\end{pmatrix},
\]

(3.1)

with the predictions

\[
|V_{cb}| \sim \frac{m_s}{m_b} \sim \frac{m_b}{m_t} \sim \epsilon_1 \quad |V_{ub}| \sim \frac{m_u}{m_c} \sim \frac{m_c}{m_t} \sim \epsilon_2 \quad |V_{us}| \sim \sqrt{\frac{m_s}{m_b}} \sim \frac{\epsilon_2}{\epsilon_1}
\]

(3.2)

which looks reasonable with \(\epsilon_1 \sim \frac{1}{25}, \epsilon_2 \sim \frac{1}{120}\).

In turn, the texture (3.2) can be explained as arising from broken approximate flavor symmetries. In the standard model, in the limit that the Yukawa matrices vanish, there is a \(U(3)^5\) chiral symmetry — a \(U(3)\) family symmetry for each of the five different fermion representations (\(q, \ell, u^c, d^c, e^c\)). With realistic Yukawa matrices, the only exact remnant of this symmetry is \(U(1)_B \times U(1)_L\), lepton and baryon number (ignoring the electroweak anomaly). However, a much larger subgroup of the \(U(3)^5\) remains as an approximate symmetry, since some of the fermion Yukawa interactions are so weak (e.g., the electron). For example, if we approximated the real world by setting all Yukawa couplings to zero except the top quark’s, the flavor symmetry would be \(U(3)^3 \times U(2)^2 \times U(1) \subset U(3)^5\).

What is confusing when trying to guess short distance physics is that some approximate flavor symmetries at low energy might also be approximate symmetries at high energies, others might be accidental symmetries that are badly broken at high energy. An example of the latter is baryon number in \(SU(5)\) — it is approximately conserved at low energy, even though interactions at the GUT scale violate it badly. If one assumes that some subgroup \(H \subset U(3)^5\) is an approximate flavor symmetry at the scale of flavor physics, and is broken by small parameters \(\epsilon\), then the Yukawa matrices are forced to have a certain texture built up out of powers of the \(\epsilon\) parameters, according to how the various fermions transform. In the above example, the texture (3.2) can be explained by an approximate \(U(1) \times U(1)\) flavor symmetry at high energy broken by the parameters \(\epsilon_{1,2}\), with appropriate charge assignments for the quarks [4]. (Note that in this example, the limit \(\epsilon_{1,2} \to 0\) makes the \(U(1)^2\) symmetry exact at high energy, while the accidental symmetry group of the renormalizable interactions at low energy is the much larger \(U(3)^2 \times U(2) \times U(1)\).)

The game becomes one of finding a fundamental approximate flavor symmetry \(G\), choosing the representations of both the fermions and the small parameters(s) \(\epsilon\) which break \(G\) explicitly, thus determining the Yukawa texture. This idea was first discussed in detail by Frogatt and Nielsen [5], who broke the flavor symmetries spontaneously, so that the \(\epsilon\) parameters were proportional to various scalar vacuum expectation values. One then must see if the model explains the observed masses and mixings, while being consistent with FCNC constraints. The drawback with this procedure is that there is quite a lot of freedom in choosing symmetries and charges, and little predictability.

## 4 Families from compositeness

The type of model mentioned above considers solutions to the flavor problem which are perturbative. In ref. [1], François Lepeintre, Martin Schmaltz and I propose how flavor structure could arise from new strong interactions. The basic idea is that quarks and leptons, and possibly the Higgs, might be composite at short distance scales, with Yukawa interactions being generated.
by interactions among the constituents. Families are distinguished by different constituents, and therefore have different strength couplings to the Higgs. Thanks to recent advances in understanding strongly coupled SUSY theories, it is possible to discuss such theories with some confidence.

A possible paradigm is found in nuclear physics. Consider, for example, the three isotopes of hydrogen: they each have the same chemistry yet have dramatically different masses — a fact simply understood once it is realized that the nucleus is composite, and that the three isotopes each contain a single proton but varying numbers of neutrons. Similarly, quarks and leptons could be bound states of both charged and neutral constituents, with different numbers or types of neutral constituents for the different generations. The nature of these composites will be determined by the underlying strong interactions, and the interactions of the “neutrons” will largely determine the flavor structure observed at low energies.

One consequence of this picture is that the number of families can be determined by the properties of the strong dynamics, much as the number of isotopes of hydrogen is determined by the strong interactions. Another consequence is that the approximate flavor symmetry at short distances will be a product of $U(1)$’s which essentially count constituents. Continuous nonabelian symmetries won’t appear because particles with different numbers of constituents will be dissimilar when regarded closely, even if they have similar standard model properties. Texture in the Yukawa interactions can easily arise since, with different numbers of constituents in the different families, the interactions giving rise to Yukawa couplings will have different dimensions — and thus different strengths — depending on which families participate.

An interesting example of a composite model that predicts three families is an $Sp(6)$ gauge theory with 6 fundamentals $Q$ and an antisymmetric tensor $A_{[6, 7]}$. The theory has an $SU(6) \times U(1)$ global symmetry, as well as an $R$ symmetry. The confined description involves the $Sp(6)$ neutral fields

\[
T_m = \text{Tr} A^m, \quad m = 2, 3 \\
M_n = QA^n Q, \quad n = 0, 1, 2
\]

(4.1)

where $Sp(6)$ indices are contracted with the appropriate metric, which can be taken to be $J = i \sigma_2 \times I_3$. The number of $M$-particle families equals 3 because of properties of $Sp(6)$ representations. The quantum numbers of the fields are:

| $Sp(6)$ | $SU(6)$ | $U(1)$ | $U(1)_R$ |
|--------|---------|--------|---------|
| $A$    | $\begin{array}{c}
\text{defining} \\
\text{representation}
\end{array}$ | $1$    | $-3$    | $0$     |
| $Q$    | $\begin{array}{c}
\text{antisymmetric} \\
\text{tensor}
\end{array}$ | $2$    | $\frac{1}{3}$ |
| $T_m$  | $\begin{array}{c}
\text{defining} \\
\text{representation}
\end{array}$ | $1$    | $-3m$   | $0$     |
| $M_n$  | $\begin{array}{c}
\text{antisymmetric} \\
\text{tensor}
\end{array}$ | $4 - 3n$ | $\frac{2}{3}$ |

(4.2)

I have indicated symmetry representations by Young tableaux in this table: $\square$ is the defining representation, while $\begin{array}{c}
\text{antisymmetric} \\
\text{tensor}
\end{array}$ denotes an antisymmetric tensor. This model has a number of desirable features. If weak gauge interactions are embedded in the $SU(6)$ symmetry of this model, then there is a replication of “families” of 3 fields $\begin{array}{c}
\text{defining} \\
\text{representation}
\end{array}$. Furthermore, in spite of having 3 families, the family symmetry of the model is not $U(3)$, but only $U(1)$. Family replication arises because the $A$ field only carries this global $U(1)$ charge, and so the SM gauge charges of a composite particle are independent of the number of $A$ fields it contains. Breaking this $U(1)$ flavor symmetry will
allow us to generate flavor in a manner analogous to the Froggatt-Nielsen mechanism. The model realizes the isotope paradigm of the introduction, with the \((QQ)\) and \(A\) fields playing the roles of the proton and neutron respectively.

### 5 Texture from compositeness

In order to create a realistic model based on the family replication example above, one needs more than a single \(Sp(6)\) group, as a family cannot fit in the antisymmetric tensor of \(SU(6)\). A possible extension is to use an \(Sp(6)^3\) gauge group, where one \(Sp(6)\) binds together the left-handed quarks, the second confines the right-handed quarks, and the third produces composite Higgs fields. The three groups could be made to talk to each other by having three massive fields \(V_1, V_2, V_3\), each of which transforms as a fundamental under two of the \(Sp(6)\) groups at a time. On can introduce \(VQQ', VVA\) and \(VVV\) type interactions in the superpotential. When the \(V\) fields are integrated out of the theory at their mass scale (assumed to be above the confining scale), they produce multifermion interactions; after confinement, these lead to Yukawa interactions among the composite fields (Fig. 1). If the \(VVA\) coupling is small, then every time an additional \(A\) field is included in the graph, there is a suppression factor. Since the number of \(A\) constituents distinguishes families, the theory provides a new mechanism for generating texture. Graphs of the sort pictured in Fig. 1 can give rise to Yukawa coupling matrices of the form

\[
Y \propto \begin{pmatrix} 19\epsilon^4 & 9\epsilon^3 & 3\epsilon^2 \\ 9\epsilon^3 & 5\epsilon^2 & 2\epsilon \\ 3\epsilon^2 & 2\epsilon & 1 \end{pmatrix}
\]

where \(\epsilon = g_{VVA}\Lambda/M\), with \(g_{VVA}\) being the \(VVA\) coupling, \(M\) being the mass of the \(V\) fields, and \(\Lambda\) being the confinement scale of the \(Sp(6)\) groups. The peculiar numbers appearing in Eq. (5.1) arise as combinatoric factors. The eigenvalues of the matrix \(Y\) are approximately \(\{1, \epsilon^2, \epsilon^4\}\), and could explain the hierarchy observed in up quark masses if \(\epsilon \sim 1/20\), for example. This mechanism is in a sense dual to the Froggatt-Nielsen proposal, as the \(A\) fields are confined, as opposed to getting vacuum expectation values.
This example shows quantitatively how texture can arise from compositeness. The three families are distinguished by the number of $A$ constituents, and the $U(1)$ symmetries carried by the $A$ fields would prohibit masses and mixings among families. However the $VVA$ and $VVV$ couplings violates these $U(1)$’s explicitly, and lead to masses and mixing. Particles with many $A$ constituents require many powers of $g_{VVA}$ in their masses, and are hence quite light. This mechanism for generating texture is more constrained than the Frogatt-Nielsen mechanism since the structure and of the light composite fermions are fixed by the strong dynamics and can’t be tweaked to one’s desire. In ref. [1] more complicated and realistic examples are given.

6 What should we look for?

I began this talk motivating the existence of new strong interactions by the existence of the electroweak hierarchy, and then proceeded to ignore SUSY (electroweak) symmetry breaking. Putting flavor and SUSY breaking together in a single strongly interacting theory is an interesting and challenging enterprise. It is hard to even figure out where to begin.

One possible starting point is to ask whether the quantum numbers of the standard model particles suggest any particular compositeness structure. For inspiration, consider the quark model and the way it explains the baryon spectrum. If one classifies the baryon octet and decuplet first under $SU(2)$ isospin, then $SU(3)$, then finally $SU(6)$, one finds the following representations (again I use Young tableaux, as they provide a particularly suggestive way to visualize group representations):

$$ N \oplus \Lambda \oplus \Sigma \oplus \Xi \oplus \Delta \oplus \Sigma^* \oplus \Xi^* \oplus \Omega = \begin{array}{c} \Box + 1 + \Box + \Box + \Box + \Box + \Box + \Box + 1 \\ \rightarrow \Box + \Box + \Box \\ \rightarrow \Box + \Box + \Box \end{array} \quad (SU(2)) \quad (SU(3)) \quad (SU(6)) $$

These representations do not provide striking evidence for compositeness at the level of $SU(2)$; under $SU(3)$ one sees a threefold structure, but the symmetry properties are obscure; however, classification under $SU(6)$ suggests immediately that the baryons are a bound state of three constituents totally symmetric under spin/flavor.

The analogous procedure for the standard model particles is to look at their charges when unified into larger and larger groups, then gaze at the relevant Young tableaux and see if any particular substructure suggests itself. A clue that this is the right procedure to follow is the apparent perturbative unification of the standard model coupling constants in a manner consistent with $SU(5)$, groups which contain $SU(5)$, such as $SO(10)$ and $E_6$, and $SU(3) \times SU(3) \times SU(3)$. $SU(5)$ representations for a family $[\Box + \Box]$ do not readily suggest substructure, unless one says that the $\Box$ is fundamental and the $\Box$ is a bound state of a pair of constituents $[\Box, \Box]$. Under $SO(10)$, a family is a spinor, and if composite, at least one of the constituents would likewise have to be a spinor. So $SO(10)$ doesn’t readily suggest compositeness, and neither does $E_6$ for similar reasons.

In an $SU(3) \times SU(3) \times SU(3)$ GUT, families do have structure that suggests compositeness:

$$ \text{one family} = (\Box, \Box, 1) \oplus (1, \Box, \Box) \oplus (\Box, 1, \Box) \quad (6.2) $$

which has a natural explanation if each particle in the standard model is a boundstate of two constituents, a fermion and a boson. This is the structure explored in ref. [1], but none of the realistic examples there are consistent with unification. It must be admitted that maintaining $SU(3) \times SU(2) \times U(1)$ coupling constant unification in a theory of composite quarks and leptons is not easy, although possible [3]).
7 Conclusions

Perhaps for a physicist today to try to understand the origin of quark and lepton masses with our limited experimental information about flavor is analogous to asking Mendel to try to deduce the structure of DNA from his experiments on breeding peas. Nevertheless, it is the outstanding question for particle theorists to answer, and attempts to do so may eventually lead to a solution, if not a sudden epiphany. The work described here uncovers a new mechanism that is capable of generating the observed spectrum we observe. One of the virtues of our work is that the models presented are renormalizable field theories, so that none of the required dynamics is hidden. By analyzing quantitatively how mass hierarchies can arise from internal structure for quarks and leptons, a new paradigm is offered which is hoped to contain a germ of truth.

Putting aside the usual apologies and lies that are the common refuge of theorists, I, like many of my experimental colleagues at this conference, must conclude that my collaborators and I have looked for something radically new — in this case a composite model that explains flavor, unification, and SUSY breaking — but that we have not found it yet at the 95% confidence level. Nevertheless, we have seen promising signs of something new on the horizon, and have high hopes for the next run.

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References

[1] D. B. Kaplan, F. Lepeintre, M. Schmaltz, *Flavor from Strongly Coupled Supersymmetry*, [hep-ph/9705411](http://arxiv.org/abs/hep-ph/9705411).

[2] N. Seiberg, *Phys. Rev.* D49, 6857 (1994), [hep-th/9402044](http://arxiv.org/abs/hep-th/9402044); *Nucl. Phys.* B435, 129 (1995), [hep-th/9411149](http://arxiv.org/abs/hep-th/9411149).

[3] for reviews see e.g. K. Intriligator and N. Seiberg, *Nucl. Phys. Proc. Suppl.* 45BC, 1 (1996), [hep-th/9509060](http://arxiv.org/abs/hep-th/9509060); M.E. Peskin, [hep-th/9702094](http://arxiv.org/abs/hep-th/9702094).

[4] M. Leurer, Y. Nir, N. Seiberg, *Nucl. Phys.* B398, 319 (1993), [hep-ph/9212278](http://arxiv.org/abs/hep-ph/9212278).

[5] C.D. Froggatt and H.B. Nielsen, *Nucl. Phys.* B147, 277 (1979).

[6] P. Cho and P. Kraus, *Phys. Rev.* D54, 7640 (1996), [hep-th/9607200](http://arxiv.org/abs/hep-th/9607200).

[7] C. Csáki, M. Schmaltz and W. Skiba, *Nucl. Phys.* B487, 128 (1997), [hep-th/9607210](http://arxiv.org/abs/hep-th/9607210).

[8] M. Strassler, *Phys. Lett.* 376B, 119 (1996), [hep-ph/9510342](http://arxiv.org/abs/hep-ph/9510342).

[9] A. Nelson and M. Strassler, [hep-ph/9607362](http://arxiv.org/abs/hep-ph/9607362).