Flatness measurement of large flat with two-station laser trackers

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ABSTRACT
We present a novel method to accurately measure the flatness of a large flat. The method can be seen as a task-specific error correction of the laser tracker. Laser trackers are positioned at two specific measuring stations and measure the coordinates of the same sample points. The angular errors of the primary laser tracker are compensated with constraint information provided by an additional laser tracker. Using this method, we measure the flatness of a 4.5 m flat. The flatness measurement uncertainty is 1.4 \( \mu \text{m} \) in rms (root mean square).

KEYWORDS
Flatness measurement; large-scale metrology; error compensation; coordinate measuring machine; laser tracker

SUBJECT CLASSIFICATION CODES
120.3940; 120.6650

1. Introduction
The flatness measurement of components for precision optomechanical systems has become increasingly challenging for the following reasons. The components, in addition to being larger, are tested in the workshop, and required accuracy is more stringent. For example, the flatness of optical flat is 10–20 \( \mu \text{m} \) (PV, Peak-to-Valley) during the fine grinding process. Another example is the flatness of contact surface of telescope’s turning table needs figuring to 20 \( \mu \text{m} \) (PV).

A variety of methods exists to measure the flatness of a large flat. These methods can be classified into two categories: comparison methods and coordinate methods. Comparison methods measure the differences between sample points, while coordinate methods directly measure the coordinates of sample points. Both methods have drawbacks. The comparison methods require custom-made measuring tools and are not available for discontinuous surfaces, while the coordinate methods are sensitive to coordinate measurement error\[^{[1,2]}\].

The laser tracker is widely used in industry and it can be applied to the requirement of large-scale metrology, mainly due to flexibility, mobility, and convenience. However, the three-dimensional accuracy of measuring the position of the sample point is typically limited by the angular errors.

Researchers have proposed various ways to reduce the impact of the angular error in the surface form measurement. The first way to reduce the surface form measurement uncertainty is by minimizing its sensitivity to angles, this would require placing laser tracker far away toward the surface, so the laser beam can be pointed approximately perpendicular to the surface. The dissatisfaction with this method is that a huge test tower is needed to keep an optimum relative position\[^{[3]}\]. Another way to reduce the measurement uncertainty is by using a set of laser trackers (at least four), based on multilateration principle coordinates are calculated from the measured interferometric distances, but it is costly and inefficient\[^{[4]}\].

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Color versions of one or more of the figures in this article can be found online at www.tandfonline.com/uopt.
Therefore, we present a new method to accurately measure the flatness of a large flat using two laser trackers. An additional laser tracker is used to compensate for angular error of the primary laser tracker, therefore, high accuracy is achieved.

2. Methodology

A laser tracker is essentially a portable coordinate measuring machine (CMM), it measures spherical coordinates instead of Cartesian coordinates. The tracker utilizes a laser interferometer (IFM) and two encoders to track and measure the location of a sphere-mounted retro-reflector (SMR) as it moves through space\(^5,6\). A laser tracker has excellent accuracy for the line-of-sight distance, but the angular measurements are less accurate. (Table 1)\(^7-9\).

The single-station laser tracker flatness measurement is shown in Figure 1. A laser tracker is positioned above the flat, the vertical coordinate \(z\) and projective length \(x\) are calculated by using the distance \(\rho\) and angle \(\beta\):

\[
\begin{align*}
z &= \rho \cos \beta \\
x &= \rho \sin \beta
\end{align*}
\]

(1)

The measurement uncertainties, \(u_z\), \(u_x\) are:

\[
\begin{align*}
u_z &= \sqrt{(u_\rho \cos \beta)^2 + (\rho u_\beta \sin \beta)^2} \\
u_x &= \sqrt{(u_\rho \sin \beta)^2 + (\rho u_\beta \cos \beta)^2}
\end{align*}
\]

(2)

where \(u_\rho\) and \(u_\beta\) denote the measurement uncertainty of \(\rho\) and \(\beta\).
According to Equation (2), when $\beta \rightarrow 0$,

$$
\begin{align*}
  u_z &\approx u_{\rho} \\
  u_x &\approx \rho u_{\beta}
\end{align*}
$$

(3)

when $\beta \rightarrow \pi/2$,

$$
\begin{align*}
  u_z &\approx \rho u_{\beta} \\
  u_x &\approx u_{\rho}
\end{align*}
$$

(4)

Thus, the relation between error $\Delta z$ and $\Delta x$ is (see Figure 1):

$$
\Delta z_{\beta} = \Delta x_{\beta} \times \tan \beta
$$

(5)

This means that if $\Delta x_{\beta}$ can be determined, $z$ coordinate error can be compensated.

Figure 2 shows the two-station laser trackers flatness measurements. The primary tracker is located at a certain height above the flat, while the additional tracker is placed on the surface. Both of the trackers are positioned at the flat center and measure same sample points. The error of measured projective length $x$ under primary and additional station are $\sqrt{(\Delta \rho \sin \beta_p)^2 + (\rho \cos \beta_p \Delta \beta_p)^2}$ and $\Delta \rho_R$, respectively. Typically, the former is 5–10 times larger than the latter. Taking the projective length $x$ measured by additional tracker as a “true value”, then we can obtain $\Delta x_{\beta}$ of the primary tracker.

The measurement procedure is summarized here:
2.1. Set up two measuring stations

The position and orientation of two trackers do not need strictly alignment. However, the closer the additional tracker is placed to the surface, the more accurate $\Delta x_\beta$ can be determined. The primary tracker is placed above the surface, the compensation is more effective when $\beta$ is less than $\pi/4$.

2.2. Measure the flatness

The two trackers should measure exactly the same sample points. The laser beam should not be interrupted because the “locked-on” process will bring in up to 10 $\mu$m distance measurement error.

2.3. Compensate angle error of the primary tracker

The measured coordinates of the primary and additional laser trackers are $(x_{pti}, y_{pti}, z_{pti})$ and $(x_{rti}, y_{rti}, z_{rti})$, the corresponding projective coordinates (on the fitted plane) are $(x_{ppi}, y_{ppi})$ and $(x_{rpi}, y_{rpi})$.

Since the two groups of coordinates are from the same sample points, we can calculate coordinate transformation coefficients and transform two sets of coordinates in a unified plane coordinate system by using the least square optimization:

$$
x_{pti} = x_{ppi} \times a + y_{ppi} \times b + c
$$

$$
y_{pti} = y_{ppi} \times a - x_{ppi} \times b + d
$$

$$
\sum_{i=1}^{n} \Delta l_i = \sum_{i=1}^{n} \left( \sqrt{x_{pti}^2 + y_{pti}^2} - \sqrt{x_{rpti}^2 + y_{rpti}^2} \right) \rightarrow \text{min}
$$

where $a$, $b$ are rotation coefficients, $c$, $d$ are translation coefficients, and $x_{pti}, y_{pti}$ are transformed coordinates of $x_{ppi}, y_{ppi}$. $\Delta l_i$ is the projective radial length error of the primary station tracker. Therefore, the compensated angle $\beta'_p$ can be calculated as: (see Figure 3):

$$
\beta'_p \approx \arcsin \left[ (\rho_p \sin \beta_p - \Delta l_i) / \rho_p \right]
$$

Figure 3. The relation between $\Delta l_i$ and $\beta'_p$. 
The compensated spherical coordinates are then transformed to Cartesian coordinates, which are used to calculate best-fit plane and flatness error.

3. Experiment and measurement uncertainty analysis

The preliminary experiment was carried out on a 4.5 m annular flat and only one tracker (API radian) measured the repeatable sample points twice by changing stations. The primary station was on a 1.6 m tripod and the additional station was mounted on a 0.35 m magnetic chuck above the surface center. Thirty auxiliary tools were glued onto the flat that offers precisely repeatable positioning of the SMR (see Figure 4). The samplings of the two stations were in the same sequence in 15 minutes.

ASME B89 standard prescribes a number of tests of laser tracker performance including an extensive set of length measurements and two-face repeatability tests\[10\]. However, the tests do not attempt to quantify instrument accuracy and directly use the manufacturer’s specification to evaluate measurement uncertainty is not accurate, thus the measurement uncertainty analysis is as follows:

- Distance measurement uncertainty.

The distance measurement uncertainty is determined by separately evaluating uncertainty from “random” sources ($u_{\mu 1}$) and “systematic” sources ($u_{\mu 2}$)\[11\].

The tracker was located at the flat center and the radial displacement of 30 sample points was only 20 mm that could be measured with IFM with sub-micron resolution. The distance

![Figure 4. Experiment of measuring a 4.5 m annular flat flatness.](image)

![Figure 5. Distance dynamic repeatability.](image)
measurement uncertainty $u_{p1}$, which is caused by radial run-out of the axis, sphericity of SMR, air turbulence, and refractive index error can be evaluated by testing the tracker’s dynamic repeatability (put an SMR into a fixed nest repeatedly and observe the distance variations).

Figure 5 shows the distance variations seen at a 2.4 m range. The distance measurement uncertainty $u_{p1}$ is 0.58 $\mu$m.

Distance measurement uncertainty $u_{p2}$ mainly comes from geometrical misalignment of the tracker. According to the tracker’s model, the corrected distance $\rho_c$ is:

$$\rho_c = \rho_m + x_2 \cdot \sin(\beta_m) + x_8$$

where $x_2$ is transit offset, $x_8$ is bird bath error, $\rho_m$ and $\beta_m$ are measured distance and vertical angle. The change of $\beta_m$ is within 2 degrees in the experiment, so residual of $\rho_c$ can be evaluated by using the tracker to measure two fixed target nests at two specific positions. As shown in Figure 6, the tracker measures incremental length $L_1$ without systematic error at position 1; the tracker measures length $L_2$ with doubled systematic error at position 2. The distance measurement uncertainty $u_{p2}$ is calculated as:

$$u_{p2} = \sqrt{\frac{\sum_{i=1}^{N} [(L_{2i} - L_{1i})/2]^2}{N}}$$

The result of six measurements gives: the measurement uncertainty $u_{p2}$ is 3 $\mu$m.

- Angle measurement uncertainty.

The measurement is insensitive to tracker’s horizontal angle error in the experiment. According to the tracker’s model, the corrected vertical angle $\beta_c$ is:

$$\beta_c = \beta_m - \frac{x_{1m}}{\rho_m} + \frac{x_2 \cdot \cos(\beta_m)}{\rho_m} + x_3 + x_{7n} \cdot \cos \beta_m - x_{7z} \cdot \sin \beta_m + x_{10a} \cdot \sin 2\beta_m + x_{10b} \cdot \cos 2\beta_m$$

where $x_{1m}$ is beam offset, $x_3$ is vertical offset index, $x_{7n}$, $x_{7z}$ are encoder eccentricity, $x_{10a}$, $x_{10b}$ are errors in the encoder scale. The systematic vertical angle error slightly changes during the measurement and thus can be neglected.

The vertical angle measurement uncertainty $u_\beta$ arising from “random” sources was evaluated by using a high precision rotary table. As shown in Figure 7, the tracker was mounted on a rotary table center and an SMR was fixed at a 2.4 m range. The tracker kept tracking SMR and recording the coordinates dynamically when the rotary table was rotating. If the tracker has no vertical angle error, we can fit an ideal plane to the measured coordinates, which also means the error of vertical angle $\beta$ can be calculated from flatness error:

$$\Delta \beta_i = \frac{e_i}{R}$$

where $\Delta \beta_i$ is vertical angle error, $e_i$ is flatness error, $R$ is distance. The result of the experiment gives: the measurement uncertainty $u_\beta$ is 0.38 arcsecond.
Distance measurement uncertainty arises from environmental influences. The IFM can measure displacement with sub-micron resolution. However, the accuracy is limited by the environment. Environmental influences include atmospheric compensation, material expansion, and optical thermal drift. The distance measurement uncertainty $u_{q3}$ arises from environmental influences was evaluated by using a tracker monitoring a fixed SMR at a 2.4 m range. The distance variations measured by the tracker during 30 minutes are shown in Figure 8.

The measured distance changed regularly in a short period of time. It can be partly compensated by re-measuring the first few points at the end of the measurement sequence and do a linear compensation. The measurement experiment took 15 minutes, thus the distance measurement uncertainty $u_{q3}$ is 2.3 μm.

Monte-Carlo simulation.

The flatness is measured indirectly by using the compensation method, so there is no mathematical model to follow. To understand how individual uncertainty components contribute to final uncertainty, we used a Monte-Carlo simulation (MCS) method[14]. The supplement 1 of GUM (guide to the expression of uncertainty in measurement) provides basic guidelines for using the MCS for the propagation of distributions in metrology[15,16]. For each input, the MCS procedure generates a numeric value drawn at random from its respective probability density functions. Numeric values derived in this manner are produced for all inputs to the known functional relationship, which is then used to produce a single numeric value as output. This process is repeated sufficiently large number of times so as to produce a set of simulated results as output. The statistic value of these output results is then the measurement uncertainty.

According to the experiment setting, the sphere coordinates of 30 points, which are on an ideal plane are generated under a coordinate system of the primary and additional station, respectively $((\rho_{gp}, \alpha_{gp}, \beta_{gp}), (\rho_{gr}, \alpha_{gr}, \beta_{gr}))$. Then, each of these coordinate is added with random sampling variables, $u_{\rho1}$, $u_{\rho2}$, $u_{\rho3}$, $u_{\beta}$ as described in the following equation:

$$
\begin{align*}
\rho_{pi} & = (\rho_{gpi}) + (u_{\rho1pi} + u_{\rho2pi} + u_{\rho3pi}) \\
\alpha_{pi} & = (\alpha_{gpi}) + (u_{\alpha1pi} + u_{\alpha2pi} + u_{\alpha3pi}) \\
\beta_{pi} & = (\beta_{gpi}) + (u_{\beta1pi} + u_{\beta2pi} + u_{\beta3pi})
\end{align*}
$$

Figure 7. Evaluating vertical angular measurement uncertainty of a laser tracker with using a high-precision rotary table.
Figure 8. Distance variations introduced by environment during 30 minutes.

Figure 9. Comparison of two-station and one station flatness measurement results.

Figure 10. Flatness measured by the level method.

Figure 11. Results of two-station laser trackers measurement and level measurement.
Two groups of coordinates are then processed by compensation algorithms and output a simulated result. Since the generated points are from an ideal plane, the simulated flatness can be considered from the measurement. Hundred times MCS results indicate that the flatness measurement uncertainty in PV is 5.7 μm and the flatness measurement uncertainty in rms is 1.4 μm.

In the experiment, the flatness measured with the primary tracker is 25.9 μm in PV and 7.2 μm in rms. The flatness measured with the additional tracker is 31.9 μm in PV and 9.3 μm in rms. The flatness measured with two-station trackers method is 23.1 μm in PV and 6.9 μm in rms (Figure 9). Although all three results have similar trends, the maximum point-to-point deviation is up to 10 μm. Furthermore, the two-station result is clearly more continuous.

The flatness is also measured with an electronic level for comparison (Figure 10). The level method measures the slopes between neighboring points and calculates the flatness, the measurement uncertainty is 1 μm (rms)\(^{[17]}\), but it is not applicable to discontinue surfaces and it is less efficient.

Figure 11 compares the results of two-station laser tracker measurement and level measurement. An experimental deviation of 1.4 μm (rms, see Figure 12) is within “root-sum-square” of two method measurement uncertainties, demonstrating that the experimental flatness measurement uncertainty of two-station laser trackers is consistent with the theoretical uncertainty analysis.

\[
Deviation_{\text{rms}} < \sqrt{u_{\text{rms(Two-station)}}^2 + u_{\text{rms(Level)}}^2}
\]

4. Conclusion

We presented a method to use two-station laser trackers to accurately measure the large flat flatness. The tracker’s angular error is eliminated by error compensation. Experimental results have confirmed the feasibility of the proposed method.

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