Hidden Supersymmetries of Deformed Supersymmetric Mechanics

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Abstract. We consider quantum models corresponding to supersymmetrizations of the two-dimensional harmonic oscillator based on worldline $d=1$ realizations of the supergroup $SU(N/2|1)$, where the number of supersymmetries $N$ is arbitrary even number. Constructed models possess the hidden supersymmetry $SU(N/2|2)$. Degeneracies of energy levels are spanned by representations of the hidden supersymmetry group.

1. Introduction
In [1, 2, 3] there were studied a new class of $\mathcal{N}=4$, $d=1$ supersymmetric quantum mechanics (SQM) models known as “Weak Supersymmetry” models, where the appropriate superalgebra involves, besides Hamiltonian, extra bosonic generators not commuting with supercharges. In [4, 5] we reproduced these models, associated with the multiplets $(1, 4, 3)$ and $(2, 4, 2)$, from a superfield approach based on the $d=1$ worldline supersymmetry $SU(2|1)$. We identified the weak supersymmetry superalgebra with the superalgebra $su(2|1)$, where we considered it as a deformation of the standard $\mathcal{N}=4$, $d=1$ Poincaré superalgebra by a mass parameter $m$.

The simplest model of weak supersymmetry considered in [1] corresponds to $\mathcal{N}=4$ supersymmetric extension of the one-dimensional harmonic oscillator. One of the distinct features of this model is the non-trivial degeneracy of energy levels (see Figure 1), that was explained in [4] in the framework of the $SU(2|1)$ representation theory [6]. The first excited level is given by unequal numbers of fermionic and bosonic states forming the fundamental representation of $SU(2|1)$. All other higher excited levels show up the standard 4-fold degeneracy containing equal numbers of fermionic and bosonic states and corresponding to the simplest typical $SU(2|1)$ representation.

In [4], we also considered the chiral multiplet $(2, 4, 2)$ and constructed its general superfield action. One of the simplest models corresponds to $SU(2|1)$ supersymmetric extension of the two-dimensional harmonic oscillator and possesses the hidden supersymmetry $SU(2|2)$. Degeneracies at each energy level, except the ground state, are spanned by $SU(2|2)$ representations characterized by equal numbers of fermionic and bosonic states [7].

In this paper, we consider $SU(\mathcal{N}/2|1)$ supersymmetric extensions of the two-dimensional harmonic oscillator, where the number of supersymmetries $\mathcal{N}$ is even and not restricted from

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1 The numbers $(k, \mathcal{N}, \mathcal{N} - k)$ display the field content of $\mathcal{N}$, $d=1$ multiplets. For example, the multiplet $(1, 4, 3)$ contains 1 physical bosonic field, 4 physical fermionic fields and 3 auxiliary bosonic fields.
We show that the presence of hidden supersymmetry is a common feature of such models.

2. Two-dimensional harmonic oscillator
The quantum Hamiltonian of the two-dimensional oscillator reads

\[ H_{\text{bos.}} = \frac{1}{2} \sum_{a=1,2} \left( -\partial_a^2 + m^2 x_a^2 \right) \Rightarrow H_{\text{bos.}} = -\partial_z \partial_{\bar{z}} + m^2 z \bar{z}, \tag{1} \]

where

\[ z = \frac{1}{\sqrt{2}} (x_1 + ix_2), \quad \bar{z} = \frac{1}{\sqrt{2}} (x_1 - ix_2) \tag{2} \]

The Hamiltonian can be rewritten as

\[ H_{\text{bos.}} = a^+ a^- + b^+ b^- + m, \tag{3} \]

where creation and annihilation operators are given by

\[ a^+ = \frac{i}{\sqrt{2}} (\partial_z - m\bar{z}), \quad a^- = \frac{i}{\sqrt{2}} (\partial_z + m\bar{z}), \quad [a^-, a^+] = m, \]
\[ b^+ = \frac{i}{\sqrt{2}} (\partial_{\bar{z}} - mz), \quad b^- = \frac{i}{\sqrt{2}} (\partial_{\bar{z}} + mz), \quad [b^-, b^+] = m. \tag{4} \]

Wave functions are written via the creation operator \( a^+ \) and \( b^+ \) acting on the ground state \( |0\rangle = \exp(-mz\bar{z}) \) as

\[ |n_1, n_2\rangle = (a^+)^{n_1} (b^+)^{n_2} |0\rangle, \]
\[ |n_1 + 1, n_2\rangle = a^+ |n_1, n_2\rangle = (a^+)^{n_1+1} (b^+)^{n_2} |0\rangle, \]
\[ |n_1, n_2 + 1\rangle = b^+ |n_1, n_2\rangle = (a^+)^{n_1} (b^+)^{n_2+1} |0\rangle. \tag{5} \]
The operators $a^-$ and $b^-$ annihilate the ground state and reduce the number of creation operators:

$$a^- |0\rangle = 0, \quad a^- |n_1, n_2\rangle = n_1 m |n_1 - 1, n_2\rangle,$$

$$b^- |0\rangle = 0, \quad b^- |n_1, n_2\rangle = n_2 m |n_1, n_2 - 1\rangle.$$  \hspace{1cm} (6)

The spectrum of the Hamiltonian $H_{\text{bos}}$ is then

$$H_{\text{bos}} |n_1, n_2\rangle = (n_1 + n_2 + 1) m |n_1, n_2\rangle.$$  \hspace{1cm} (7)

2.1. Hidden symmetry

One can introduce SU(2) symmetry generators commuting with the Hamiltonian:

$$J_+ = \frac{1}{m} b^a^-, \quad J_- = \frac{1}{m} a^+ b^- , \quad J_3 = \frac{1}{2m} (b^+ b^- - a^+ a^-).$$  \hspace{1cm} (8)

They commute as

$$[J_+, J_-] = 2J_3, \quad [J_3, J_\pm] = \pm J_\pm .$$  \hspace{1cm} (9)

It is none other than the well known hidden SU(2) symmetry of the two-dimensional harmonic oscillator. In general, $N$-dimensional harmonic oscillator possesses SU($N$) symmetry (see e.g. [9]).

Since the generators (8) commute with the Hamiltonian, we have a degeneracy of energy levels corresponding to SU(2) representations. As shown in Figure 2, the energy level $n$ is given by $n + 1$ states with the wave function defined as a sum of (5):

$$|n\rangle = \sum_{n_1=0}^{n} C_{n_1} |n_1, n - n_1\rangle , \quad H |n\rangle = (n + 1) m |n\rangle , \quad C_{n_1} = \text{const}.$$  \hspace{1cm} (10)

Indeed, the action of $2J_3$ on these states takes the integer eigenvalues from $-n$ to $n$.

\begin{center}
\begin{tikzpicture}
\draw[->] (0,0) -- (0,6);
\foreach \y in {2,4,6} {\draw (0,\y) -- (0.5,\y) node[above] {$\y m$};}
\draw (0,6) -- (5,6);
\foreach \y in {0,2,4} {\draw (0,\y) -- (4,\y) node[below] {$\y$};}
\end{tikzpicture}
\end{center}

**Figure 2.** The degeneracy of energy levels of the two-dimensional oscillator. Each energy level is given by $n + 1$ states.
3. Supersymmetric two-dimensional harmonic oscillator

Supersymmetric Weyl-ordered Hamiltonian reads

\[ H = a^+ a^- + b^+ b^- + m \psi^K \bar{\psi}_K + m \left( 1 - \frac{N}{4} \right), \]  

where the fermionic fields \( \psi^I \) and \( \bar{\psi}_J \) anticommute as

\[ \{ \psi^I, \bar{\psi}_J \} = \delta^I_J. \]  

The capital indices \( I, J, K, L \) refer to the SU\((N/2)\) fundamental and anti-fundamental representations. The supersymmetric Hamiltonian (11) commute with supercharges defined as

\[ Q^I = \sqrt{2} \psi^I a^-, \quad \bar{Q}_J = \sqrt{2} \bar{\psi}_J a^+. \]  

They close on the centrally-extended superalgebra \( \hat{su}(N/2|1) \) given by the following non-vanishing (anti)commutators:

\[ \{ Q^I, Q_J \} = 2m L^I_J - 2m \delta^I_J R + \delta^I_J H, \quad [L^I_J, L^K_L] = \delta^K_I L^I_J - \delta^I_J L^K_L, \]
\[ [L^I_J, Q^K] = \delta^K_I Q^J - \frac{2}{N} \delta^I_J Q^K, \quad [L^I_J, Q_L] = \frac{2}{N} \delta^I_J Q_L - \delta^I_J Q_J, \]
\[ [R, Q^I] = \left( 1 - \frac{2}{N} \right) Q^I, \quad [R, Q_J] = - \left( 1 - \frac{2}{N} \right) Q_J. \]  

The SU\((N/2) \times U(1)\) subgroup generators \( L^I_J \) and \( R \) are written as

\[ L^I_J = \psi^I \bar{\psi}_J - \frac{\delta^I_J}{4} \psi^K \bar{\psi}_K, \]
\[ R = \frac{1}{2m} (b^+ b^- - a^+ a^-) + \left( \frac{1}{2} - \frac{2}{N} \right) \psi^K \bar{\psi}_K - \frac{N}{4} \left( \frac{1}{2} - \frac{2}{N} \right). \]  

The mass-dimensional generator \( H \), associated with Hamiltonian, is a central charge generator. In the limit \( m = 0 \), the SU\((N/2) \times U(1)\) generators become automorphism generators of the standard \( \mathcal{N}, d=1 \) Poincaré superalgebra.

3.1. Hidden supersymmetry

One can check the hidden SU\((2)\) symmetry generators (8) commutes with the supersymmetric Hamiltonian. However their commutators with (13) give new supercharges

\[ [J^-, Q^I] = - S^I, \quad [J^+, Q_J] = S_J. \]  

The new supercharges are written as

\[ S^I = \sqrt{2} \psi^I b^-, \quad \bar{S}_J = \sqrt{2} \bar{\psi}_J b^+. \]  

Thus, the superalgebra (20) is extended by the new supercharges and SU\((2)\) generators.

Splitting the U\((1)\) generator \( R \) as

\[ R = J_3 + \left( 1 - \frac{4}{N} \right) F \quad \Rightarrow \quad F = \frac{1}{2} \psi^K \bar{\psi}_K - \frac{N}{8}, \quad J_3 = \frac{1}{2m} (b^+ b^- - a^+ a^-), \]  

...
As instructive example, let us consider in details 3.2. N=4 supersymmetric extension of SU(2) we obtain the centrally-extended superalgebra \( \hat{su}(N/2|2) \) given by

\[
\{Q^i, \bar{Q}_j\} = 2m L^i_j - 2m \delta^i_j J_3 + 2m \left( \frac{4}{N} - 1 \right) \delta^i_j F + \delta^i_j H, \quad \{Q^i, S_j\} = 2m \delta^i_j J_+ ,
\]

\[
\{S^i, \bar{S}_j\} = 2m L^i_j + 2m \delta^i_j J_3 + 2m \left( \frac{4}{N} - 1 \right) \delta^i_j F + \delta^i_j H, \quad \{S^i, \bar{Q}_j\} = 2m \delta^i_j J_-, \]

\[
[L^i_j, \bar{Q}_k] = \frac{2}{N} \delta^i_j \bar{Q} - \delta^i_k \bar{Q}_j, \quad [L^i_j, Q^K] = \delta^i_j Q - \frac{2}{N} \delta^i_j Q^K ,
\]

\[
[L^i_j, S^K] = \frac{2}{N} \delta^i_j S - \delta^i_k S_j, \quad [L^i_j, S^k] = \delta^i_j S^k - \frac{2}{N} \delta^i_j S^K,
\]

\[
[F, Q^i] = \frac{1}{2} Q^i, \quad [F, \bar{Q}_j] = -\frac{1}{2} \bar{Q}_j, \quad [F, S^i] = \frac{1}{2} S^i, \quad [F, \bar{S}_j] = -\frac{1}{2} \bar{S}_j, \]

\[
[J_3, Q^i] = \frac{1}{2} Q^i, \quad [J_3, \bar{Q}_j] = -\frac{1}{2} \bar{Q}_j, \quad [J_+, Q^i] = S_j, \quad [J_+, S^i] = -Q^i,
\]

\[
[J_3, S^i] = -\frac{1}{2} S^i, \quad [J_3, \bar{S}_j] = \frac{1}{2} \bar{S}_j, \quad [J_-, S^i] = Q_j, \quad [J_-, Q^i] = -S^i,
\]

\[
[L^i_j, L^k_L] = \delta^i_j L^k - \delta^i_L L^k_j, \quad [L^i_j, J_+] = 2J_3, \quad [J_3, J_\pm] = \pm J_\pm. \tag{19}
\]

3.2. N=4 supersymmetric extension

As instructive example, let us consider in details \( \mathcal{N} = 4 \) supersymmetric extension. Our studies of SU(2|1) supersymmetric mechanics \(^3\) were based on the deformation of the standard \( \mathcal{N} = 4 \), \( d = 1 \) Poincaré superalgebra to the centrally-extended superalgebra \( \hat{su}(2|1) \) given by

\[
\{Q^i, \bar{Q}_j\} = 2m I^i_j - 2m \delta^i_j R + \delta^i_j H, \quad \{I^i_j, I^k_L\} = \delta^i_j I^k - \delta^k_j I^i ,
\]

\[
[I^i_j, Q_l] = \frac{1}{2} \delta^i_j Q_l - \delta^i_l Q_j, \quad \{I^i_j, Q^K\} = \delta^i_j Q^i - \frac{1}{2} \delta^i_j Q^K ,
\]

\[
[R, Q^i] = \frac{1}{2} Q^i, \quad \{R, Q_l\} = -\frac{1}{2} \bar{Q}_l. \tag{20}
\]

The indices \( i, j \) (\( i = 1, 2 \)) are SU(2) indices. The generators \( I^i_j \) and \( R \) correspond to the SU(2) \( \times U(1) \) subgroup.

The quantum SU(2|1) generators are written as

\[
Q^i = \sqrt{2} \psi^i a^-, \quad \bar{Q}_j = \sqrt{2} \bar{\psi}_j a^+ ,
\]

\[
H = a^+ a^- + b^+ b^- + m \psi^k \bar{\psi}_k ,
\]

\[
R = \frac{1}{2m} (b^+ b^- - a^+ a^-) \quad \Rightarrow \quad R \equiv J_3 ,
\]

\[
I^i_j = \psi^i \bar{\psi}_j - \frac{\delta^i_j}{2} \psi^k \bar{\psi}_k . \tag{21}
\]

It is straightforward to check that they satisfy the superalgebra relation \(^2\).

As was shown in \(^3\) the model possesses hidden supersymmetry SU(2|2). The corresponding

\(^3\) The superalgebra relations here differ from those in \(^3\) such that \( H \rightarrow 2H \).
centrally-extended superalgebra $\tilde{su}(2|2)$ is given by

\[ \{Q^i, \bar{Q}_j\} = \delta^i_j H + 2m (I^i_j - \delta^i_j J^3) , \quad \{Q^i, \bar{S}_j\} = 2m \delta^i_j J^+ , \]

\[ \{S^i, \bar{S}_j\} = \delta^i_j H + 2m (I^i_j + \delta^i_j J^3) , \quad \{S^i, \bar{Q}_j\} = 2m \delta^i_j J^- , \]

\[ [I^i_j, \bar{Q}_k] = \frac{1}{2} \delta^i_j Q_k - \delta^i_k Q_j , \quad [I^i_j, Q^k] = \delta^k_j Q^i - \frac{1}{2} \delta^k_j Q^k , \]

\[ [I^i_j, S^k] = \delta^k_j S^i - \frac{1}{2} \delta^k_j S^k , \]

\[ [J^3, Q^i] = \frac{1}{2} Q^i , \quad [J^3, \bar{Q}_j] = -\frac{1}{2} Q_j , \quad [J^+, \bar{Q}_j] = \bar{S}_j , \quad [J^+, S^i] = -Q^i , \]

\[ [J^3, S^i] = -\frac{1}{2} S^i , \quad [J^3, \bar{S}_j] = \frac{1}{2} \bar{S}_j , \quad [J^-, \bar{S}_j] = \bar{Q}_j , \quad [J^-, S^i] = -S^i , \]

\[ [I^i_j, I^k_l] = \delta^k_l I^i_j - \delta^i_j I^k_l , \quad [J^+, J^-] = 2 J^3 , \quad [J^3, J^\pm] = \pm J^\pm . \]

### 3.2.1. Dual superfield description

Corresponding Lagrangian of the two-dimensional $SU(2|1)$ supersymmetric harmonic oscillator reads

\[ \mathcal{L}_{\text{on-shell}} = \dot{z} \ddot{z} + i \left( \bar{\psi}^i \dot{\psi}_i - \dot{\bar{\psi}}^i \bar{\psi}_i \right) - m^2 z \ddot{z} - m \bar{\psi}^k \dot{\bar{\psi}}_k . \]

This on-shell Lagrangian can be constructed from superfield approach in two ways, which is exceptional for $\mathcal{N} = 4$ supersymmetric extension.

First way is given in the framework of $SU(2|1)$ superfield approach elaborated in [4]. Simple superfield Lagrangian for the chiral multiplet $(2,4,2)$ reads

\[ \mathcal{L}_{\text{deformed}} = \int d^2 \theta \ d^2 \bar{\theta} \ (1 + 2m \bar{\psi}^k \partial_k) \Psi \bar{\Psi} . \]

The second superfield construction is based on the chiral multiplet of non-deformed (standard) SQM, where the parameter $m$ introduced via superpotential term:

\[ \mathcal{L}_{\text{non-deformed}} = \int d^2 \theta \ d^2 \bar{\theta} \ (\Phi \bar{\Phi} + \frac{m}{2} \left[ \int d^2 \theta \ (\Phi^2 + \int d^2 \bar{\theta} \ (\bar{\Phi}^2) \right] . \]

After elimination of auxiliary fields in both actions, we set equivalence of these Lagrangians. The superalgebra $(20)$ and the standard $\mathcal{N} = 4$, $d = 1$ superalgebra are realized on the same on-shell set $(z, \bar{z}, \psi^i, \bar{\psi}_i)$. The superalgebra $(22)$ is recovered as the closure of these two superalgebras.

Define the new supercharges

\[ \Pi^i = \frac{1}{\sqrt{2}} (Q^i + \bar{S}^i) , \quad \bar{\Pi}_j = \frac{1}{\sqrt{2}} (\bar{Q}_j - \bar{S}_j) . \]

One can check that they form the standard $\mathcal{N} = 4$ superalgebra,

\[ \{\Pi^i, \bar{\Pi}_j\} = \delta^i_j H . \]

Thus, it is a subalgebra of $(22)$, while the generators $(8)$ become external automorphism generators:

\[ [J^3, \bar{\Pi}_j] = -\frac{1}{2} \bar{\Pi}_j , \quad [J^3, \Pi^i] = \frac{1}{2} \Pi^i , \]

\[ [J^+, \bar{\Pi}_j] = \bar{\Pi}_j , \quad [J^-, \Pi^i] = \Pi^i . \]
3.2.2. Wave functions. We construct supersymmetric wave functions in terms of (10) satisfying the same spectrum

\[ H |n\rangle = n m |n\rangle. \]  

(29)

We impose the standard physical condition on the bosonic states \(|n\rangle\) as

\[ \bar{\psi}_j |n\rangle = 0. \]  

(30)

Then, the fermionic states \(\psi_k |n\rangle\) have shifted spectrum \((n + 1) m\):

\[ H \psi_k |n\rangle = (n + 1) m \psi_k |n\rangle. \]  

(31)

Additional bosonic states defined as \(\psi_k \psi^k |n\rangle\) also have shifted spectrum \((n + 2) m\):

\[ H \psi_k \psi^k |n\rangle = (n + 2) m \psi_k \psi^k |n\rangle. \]  

(32)

\[ \begin{array}{c}
\begin{array}{c}
H
\end{array}
\end{array} \]

\[ \begin{array}{c}
4m
\end{array} \]

\[ \begin{array}{c}
3m
\end{array} \]

\[ \begin{array}{c}
2m
\end{array} \]

\[ \begin{array}{c}
m
\end{array} \]

\[ \begin{array}{c}
0
\end{array} \]

Figure 3. The degeneracy of energy levels of the two-dimensional SU(2|1) supersymmetric oscillator. There are \(4n\) states at the excited energy level \(n\). This picture can be interpreted as a multiplication of the pictures drawn in Figures 1 and 2. One can easily see that each tower divided by dashed line is a tower of the same type drawn in Figure 1.

As shown in Figure 3, each excited energy level is given by equal numbers of fermionic and bosonic states. States on the excited level \(n\) form a short SU(2|2) representation of the dimension \(4n\). Note that the first excited level corresponds to the fundamental representation of SU(2|2).

4. Conclusions
We considered superextensions of the two-dimensional harmonic oscillator with the relevant supersymmetry group SU(\(N/2\|1\)). We performed quantization and constructed the Hilbert space of wave functions in terms of wave functions of bosonic harmonic oscillator. We showed that the hidden on-shell symmetry SU(2) of the two-dimensional harmonic oscillator reveals the hidden supersymmetry SU(\(N/2\|2\)) defining degeneracies of energy levels. The case of \(N = 4\) superextension is exceptional, since it has dual superfield description.
Another feature of the two-dimensional harmonic oscillator is the presence of conformal \( \text{SO}(2, 1) \) invariance of the trigonometric type \([10], [11]\), where superconformal Hamiltonian must be an even function of the deformation parameter \( m \) \([12]\). Thus, the Hamiltonian is redefined as

\[
H = H_{\text{conf}} + 2mF, \quad H_{\text{conf}} = -\partial_{\bar{z}} \partial_z + m^2 z \bar{z} = a^+ a^- + b^+ b^- + m. \tag{33}
\]

Then the closure of the relevant superconformal group \( \text{SU}(N/2) \) and the hidden symmetry \( \text{SU}(2) \) yields some extended conformal supersymmetry corresponding to the superalgebra \( \sim \text{osp}(N|4) \), where the extra supercharges are written as

\[
\begin{align*}
Q_I &= \sqrt{2} \psi_I a^+, \\
\bar{Q}_J &= \sqrt{2} \bar{\psi}_J a^-, \\
S_I &= \sqrt{2} \psi_I b^+, \\
\bar{S}_J &= \sqrt{2} \bar{\psi}_J b^-.
\end{align*} \tag{34}
\]

It can be interpreted as a spectrum-generating supersymmetry.

It would be interesting to study superextensions of three- or four-dimensional harmonic oscillators for hidden supersymmetries. For example, the multiplet \((4, 4, 0)\) can describe \( N = 4 \) superextension of the four-dimensional harmonic oscillator \([13]\).

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