Nuclear Structure Studies of Some Neutron Rich Nuclei Produced in $^{252}$Cf Spontaneous Fission

A.V. Ramayya, J. H. Hamilton, J. K. Hwang
Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235

E-mail: a.v.ramayya@vanderbilt.edu

Abstract. High spin states of neutron-rich nuclei such as $^{133,134}$Te, $^{93}$Sr, $^{105}$Nb have been studied by measuring $\gamma - \gamma - \gamma$ coincidences (cube), $\gamma - \gamma - \gamma - \gamma$ coincidences (hypercube) and angular correlations from the spontaneous fission of $^{252}$Cf with the LBNL Gammasphere detector array. Four types of particle-hole bands built on the known 334.3 keV isomer in $^{133}$Te were identified. The level structure of $^{92}$Sr is interpreted, in part, as arising from the weak coupling of the 1d5/2 neutron hole to the yrast states of the $^{94}$Sr core. The $g$-factor of the 4+ state in $^{134}$Te was measured, for the first time, by using a new technique developed for measuring angular correlations with Gammasphere. A new level scheme of $^{105}$Nb was established. Three new collective bands were identified with a total of 14 new levels and 36 new $\gamma$ transitions. In $^{117-122}$Cd, a shift to more slightly deformed structures was found where the excited levels do not fit the long held picture of one, two and three phonon bands.

This paper is dedicated to Prof. Aldo Covello for his achievements, for the past two decades, in nuclear structure calculations.

1. Experimental technique
In the fission of $^{252}$Cf about 200 neutron-rich nuclei are produced. Since each nucleus emits 10-50 $\gamma$-rays mostly in the energy region of 10 to 2000 keV, the $\gamma$ spectrum becomes very complex. The only available technique to study these nuclei is through the study of prompt triple and higher fold $\gamma - \gamma$ coincidences with large detector arrays such as Gammasphere and Eurogam.

The data for the present high spin studies of $^{133,134}$Te, $^{93}$Sr, $^{105}$Nb were obtained using the Gammasphere detector array with 101 detectors at Lawrence Berkeley National Laboratory. A $^{252}$Cf spontaneous fission source with an $\alpha$ activity of 62 $\mu$Ci was sandwiched between two 10 mg/cm$^2$ iron foils, which were used to stop the fission fragments and eliminate the need for a Doppler correction. A total of $5.7 \times 10^{11}$ triple- and higher-fold $\gamma$-ray coincidence events were measured. A $\gamma - \gamma - \gamma$ coincidence matrix (cube) and a $\gamma - \gamma - \gamma - \gamma$ coincidence matrix (hypercube) were constructed. More details about this experiment and data analysis procedure can be found in [1,2,3]. A newly developed technique for measuring angular correlations with the Gammasphere detector array by sorting our high statistics data into 17 angle bins was used here to measure the $g$ factors and assign spins and parities to excited states in neutron-rich nuclei produced in the spontaneous fission of $^{252}$Cf. More details of this technique can be found in [4]. In case of Cd isotopes, the low energy excited states were obtained with our $^{252}$Cf data and $\beta$-decay work.
2. Particle-hole states in $^{133}$Te [1]

The nucleus $^{133}$Te is expected to have excited states originating from two protons in the $\pi g_{7/2}$ orbitals above the $Z=50$ shell gap and one neutron hole in the $\nu h_{11/2}$ orbital below the $N=82$ shell gap. The level structure of $^{133}$Te is very interesting and extends our understanding of the proton particle-neutron hole states in this region. From the $\beta^{-}$ decay of $^{133}$Sb [5] the $(11/2^{-})$ isomer at 334.3 keV and 14 other states between the $(7/2^{+}, 5/2^{-})$ and $(5/2^{+})$ levels at 1096 and 2755 keV, respectively, were known. Below the $(11/2^{-})$ isomer, the $d_{3/2}$ and $s_{1/2}$ neutron-hole states were observed. In [1], new high spin levels in $^{133}$Te from spontaneous fission work of $^{252}$Cf were identified including neutron particle-hole states. Shell-model calculations within the 50–82 shells are in good agreement with the experimental data for the excitation energy of levels up to 4.3 MeV.

Four types of particle-hole bands were observed in $^{133}$Te. The spins, parities, and dominant configurations of these bands were assigned by the theoretical predictions of the shell-model calculations. In these calculations, $^{132}$Sn is assumed to be a closed core and the valence protons and neutrons occupy the five orbits of the 50 – 82 shells. As a two-body interaction between the valence nucleons, a realistic effective interaction derived from the Bonn-A free nucleon-nucleon ($NN$) potential was employed [6]. This effective interaction already produced quite satisfactory results for the light $N=82$ isotones as well as for $^{132}$Sb [7]. As regards the single-particle proton and single-hole neutron energies, they were taken from the experimental spectra of $^{133}$Sb and $^{131}$Sn, respectively [7]. The latter, however, have been slightly modified to accurately reproduce the $1/2^{+}$ and $11/2^{-}$ neutron-hole states in $^{133}$Te.

The configurations assigned to the bands are confirmed by comparing the $^{133}$Te band head energies to those in $^{134}$Te and $^{135}$I. In figure 1, the band head energies of the four bands in $^{134}$Te and $^{135}$I are shown for energy comparison. One proton excited states from the $1g_{7/2}$ orbital to the $2d_{3/2}$ orbital are identified at 2.3, 2.4, and 2.0 MeV in $^{133}$Te, $^{134}$Te, and $^{135}$I, respectively. The one proton excited states from the $g_{7/2}$ orbital to $h_{11/2}$ orbital are identified at 3.9, 4.0, and 3.7 MeV for $^{133}$Te, $^{134}$Te, and $^{135}$I, respectively. The states corresponding to the $v_{f_{7/2}}(h_{11/2})^{-1}$ particle-hole first excited state at 4041 keV of the doubly magic core nucleus, $^{132}$Sn, are reported at 4.6 MeV in $^{134}$Te and at 4.2 MeV in $^{135}$I. The appearance of the neutron particle-hole excited states around 4–5 MeV has been suggested by shell-model calculations employing empirical $NN$ interactions. On this basis, the band beginning at 5.2 MeV in $^{133}$Te is interpreted as the particle-hole excitation from the core. It is impressive that the particle-hole excitation energy of 5.2 MeV in $^{133}$Te is the highest ever observed. The spins of this band were assigned on the basis that $E1$ transitions dominate between the negative parity bands and the positive parity band, such as those in the cases of $^{134}$Te and $^{135}$I.

![Figure 1. Band head energies of the four bands in $^{133}$Te, $^{134}$Te and $^{135}$I [1].](image-url)
3. High spin states in $^{93}$Sr [2]

As a first attempt to interpret the level scheme of $^{93}$Sr, we have tried to identify the observed states as arising from the weak coupling of the 1$d_{5/2}$ neutron hole to the states of the $^{94}$Sr core. While some features of the experimental spectrum may be understood in this simple way, a detailed description can only be obtained in terms of the shell model. In principle, one may think of a large-scale calculation with $^{100}$Sn as a closed core, where one has to consider 12 proton holes in the 50 closed shell and five neutrons beyond the 50 shell. A much simpler approach, which greatly reduces the dimensions of the model space, is based on the assumption that $Z=38$ is, to a large extent, a good magic number. This assumption has indeed been made in most of the existing calculations for the Sr and Zr isotopes. Motivated by the data made available by this experiment, a shell-model calculation was performed for this nucleus as well as for the three adjacent isotopes, $^{90,92,94}$Sr, assuming $^{88}$Sr as a closed core and letting the 5, 2, 4, and 6 valence neutrons, respectively, occupy the five levels of the 50-82 shell. In these calculations a realistic effective interaction derived from the CD-Bonn NN potential [8] was used. As a result, no adjustable parameter appears in the calculations.

Twenty one new $\gamma$ transitions were identified in $^{93}$Sr. It turns out that the calculated results provide only a partial interpretation of the experimental spectra of the above nuclei. In particular, for $^{93}$Sr the calculated level energies are not in a good agreement with the experimental ones as shown in Fig. 2. This is likely to be a consequence of our neglecting the proton degrees of freedom. However, only large-scale calculations may shed light on this point.

4. g-factor of the 4$^+$ state in $^{134}$Te [4]

Since the fission fragments were stopped in iron foils, they were therefore subjected to the magnetic hyperfine fields ($B_{\text{hf}}$) in the iron lattice caused by their implantation in substitutional sites. For a nuclear state with lifetime $\tau$, the spin vector of the nucleus will rotate about $B_{\text{hf}}$ over the lifetime of the state, with the frequency of the rotation proportional to the strength of the field and the $g$-factor of the state. For this experiment, the magnetic domains in the iron foils are randomly oriented; the foils were not cooled and there were no applied external fields. The net result of the rotation of the implanted nuclei about the randomly oriented fields is an attenuation of the expected angular correlation. The attenuation factors $G_{2,4}$ are related to the Larmor precession frequency $\omega_L$ and the lifetime. Experimentally, the attenuation factors $G_{2,4}$ are defined by fitting the measured angular correlation to the parameters $A_2$ and $A_4$. By comparing experimental values with the theoretical values, one can calculate attenuation coefficients. The attenuation of the angular correlation coefficients are used to calculate the g-factors of the 4$^+$ state in $^{134}$Te.
The $g$-factor of the $4^+$ state in $^{134}\text{Te}$ was measured here for the first time. The unattenuated coefficients for the $4^+ \rightarrow 2^+ \rightarrow 0^+$ cascade are $A_2 = 0.101(3)$, $A_4 = 0.006(4)$. The angular correlation between two gamma rays in a cascade is unchanged by an unobserved intermediate stretched $E2$ transition in the cascade, provided the intermediate transition does not cause attenuation through precession in the $2^+$ state. The $2^+$ state is so short-lived as to preclude measurable precession. Therefore, the $4^+ \rightarrow 2^+$ transition can be skipped in the $6^+ \rightarrow 4^+, 2^+ \rightarrow 0^+$ angular correlation without affecting the $A_2$ and $A_4$ values. The $6^+ \rightarrow 4^+, 2^+ \rightarrow 0^+$ correlation is shown in Fig. 3. This correlation shows attenuation from the theoretical values of $A_2 = 0.102, A_4 = 0.009$, with the attenuation factor $G_2 = 0.28(5)$. From the two different magnetic hyperfine fields of $34(16)$ T and $19(6)$ T [9] previously reported for the $4^+$ state, the $g$-factors of $0.70^{+0.55}_{-0.38}$ and $1.31^{+0.31}_{-0.14}$ were obtained, that consistent with the previously calculated value of $0.83$ [10].

**Figure 3.** The correlation for the $6^+ \rightarrow 4^+, 2^+ \rightarrow 0^+$ cascade in $^{134}\text{Te}$. This correlation gives an attenuation coefficient of $G_2 = 0.28(5)$ [4].

### 5. Multi-phonon $\gamma$-vibrational bands in $^{105}\text{Nb}$ [3]

A new level scheme with three new collective bands was established in $^{105}\text{Nb}$. The $^{105}\text{Nb}$ with $Z = 41$ and $N = 64$ is located in the $A = 100$ neutron-rich region. Searching for multi-phonon $\gamma$-vibrational bands in this nucleus is important for systematically understanding the structural characteristics in this region. In [3], we reinvestigated the level structure of $^{105}\text{Nb}$. Many new levels and transitions were observed, and the $1\gamma$ and $2\gamma$ bands proposed. Total Routhian surface and triaxial projected shell model (TPSM) calculations [11] were carried out. These calculations support our $1\gamma$ and $2\gamma$ assignments. TPSM calculations are performed for $^{105}\text{Nb}$ with the fixed axial deformation $\varepsilon = 0.350$ and the triaxial deformation of 0.130. The calculations indicate that the yrast band in $^{105}\text{Nb}$ originates from the $\pi 5/2^+[422]$ Nilsson orbital with $K^\pi = 5/2^+$. The obtained energy levels compared with the experimental data are presented in Fig. 4. It can be seen that the calculations are in excellent agreement with the available data. Not only the band-head energies for the yrast and excited bands, but also the intervals between the levels (i.e., the moments of inertia), are well described by the calculation. This strongly supports the interpretation for these bands as $1\gamma$ and $2\gamma$ collective bands, respectively, built on the yrast $K^\pi = 5/2^+$ structure. Another band based on the $1686.5$ keV level may be a three-phonon $\gamma$-vibrational band or a three quasiparticle band.

**Figure 4.** Comparison of the calculated and experimental level energies [3].
6. Low energy collective states in $^{116-126}$Cd [12]

It has been observed that there are significant deviations from the expected U(5) dynamical symmetry for $^{110,112,114,116}$Cd nuclei. However, a significant mixing with intruder states in this region is expected. In [11], we investigated states in the heavier $^{120,124,126}$Cd populated via beta decay. The decay patterns of $^{120,124}$Cd are similar to lighter Cd isotopes even though the intruder states are much higher in energy. In particular, $^{120}$Cd and $^{124}$Cd show a similar pattern as lighter Cd isotopes, with no good candidates for $2^+$ and $0^+$ 3-phonon states. The situation for $^{126}$Cd is unclear since 3-phonon states are not observed. Whether this is due to fact that these states are not populated in beta decay or they are simply absent cannot be determined. We have shown that the deviations from U(5) dynamical symmetry for the heavy even-mass Cd isotopes continue to higher masses even though the intruder states are further away in energy. While these states do not appear to be multiphonon states, their exact nature remains unclear. Future studies using other experimental methods (g-factors, etc.) will be needed to better understand the nature of these states. Overall, the multiphonon vibrational approach fails to explain the low-energy structure of these Cd isotopes. An alternative way to describe the low-lying states in Cd isotopes is to place them as independent band structures rather than as N-phonon states. In all of these cases, there is a ground state band, a quasi-gamma band, and an intruder band as shown in Fig. 5 for $^{116}$Cd. This interpretation is more consistent with the experimental data especially for the $0^+_4$ and $2^+_4$ states, than the three-phonon picture, as the decay pattern from these levels is what would be expected from the decay of independent bands. Also, in $^{116-122}$Cd a shift to more slightly deformed structures was found where the excited levels do not fit the long held picture of one, two and three phonon bands. The low-lying states in Cd may be understood as independent band structures rather than as N-phonon states.

7. Summary

In summary, level schemes of neutron rich nuclei near the line of $\beta$-stability can be studied using spontaneous fission sources such as $^{252}$Cf and $^{248}$Cm. The $\gamma$-spectroscopy techniques can be successfully applied with large detector arrays such as Gammasphere and Eurogam. Since the conference is in honor of Prof. Aldo Covello’s contributions to nuclear structure calculations, we have chosen to present the results for a few nuclei wherein he and his group performed several calculations in collaboration with Vanderbilt’s group.

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