Double slip effects of Magnetohydrodynamic (MHD) boundary layer flow over an exponentially stretching sheet with radiation, heat source and chemical reaction

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Abstract: The double slip effect on the magnetohydrodynamic boundary layer flow over an exponentially stretching sheet with suction/blowing, radiation, chemical reaction and heat source is presented in this analysis. By using the similarity transformation, the governing partial differential equations of momentum, energy and concentration are transformed into the non-linear ordinary equations. These equations are solved using Runge-Kutta-Fehlberg method with shooting technique in MAPLE software environment. The effects of the various parameter on the velocity, temperature and concentration profiles are graphically presented and discussed.

1. Introduction

Boundary layer flow over a stretching sheet becomes essential in chemical and manufacturing processes such as polymer extrusion, drawing of copper wires, continuous casting of metals, wire drawing and glass blowing [1]. Many researchers consider the exponentially stretching sheet in their studies like Bidin and Nazar [2], Sajid and Hayat [3] and Eid [4].

MHD flows find its applications in MHD power generators, petroleum industries, plasma studies and geothermal energy extractions [5]. MHD flow was considered by Parida et al [6] and Raju et al [7].

Many processes in engineering occur at high temperature and knowledge of radiative heat transfer becomes more important to produce products of desired qualities [8]. MHD flow with radiation effects was studied by Gorfe and Shankar [9] and Ishak [10].

Fluids that exhibit boundary slip have important technological applications like in the polishing of artificial heart valves and internet cavities [11]. Various aspects of MHD flow with slip conditions have been investigated by authors such as Jain and Choudhary [12] and Ibrahim and Shankar [13].

In chemical engineering process, chemical reaction between foreign mass and fluid take place in numerous industrial applications such as polymer production, manufacturing of ceramic, food processing, etc. [14]. In light of the above applications, the present study is to investigate the double slip effects on MHD boundary layer flow over an exponentially stretching sheet with radiation, heat source and chemical reaction.

2. Mathematical formulation

This study will consider a steady two dimensional laminar boundary layer flow of an incompressible viscous and electrically conducting fluid over an exponential stretching sheet. The x-axis is along the continuous stretching surface in direction of fluid motion while y-axis is perpendicular to the surface. The flow is generated by the stretching sheet such that the velocity of the boundary sheet is of an
exponential order of the flow directional coordinate $x$. The governing continuity, momentum, energy and concentration equations are as below following [4], [15] and [16]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u
\]

(2)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \left( \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{\rho C_p} \left( \frac{\partial q_r}{\partial y} \right) + \frac{Q_0}{\rho C_p} (T - T_\infty)
\]

(3)

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \left( \frac{\partial^2 C}{\partial y^2} \right) - k_1 (C - C_\infty)
\]

(4)

where $u$ and $v$ are component of velocity in $x$ and $y$ directions respectively, $\nu = \mu / \rho$ is the kinematic viscosity, $\rho$ is the fluid density, $\mu$ is the coefficient of fluid viscosity, $\sigma$ is the electrical conductivity, $B(x) = B_0 e^{x/L}$ is the variable magnetic field where $B_0$ is the constant and $L$ is the characteristic length, $T$ is the fluid temperature, $\kappa$ is the thermal conductivity, $C_\rho$ is the specific heat at constant pressure, $q_r$ is the radioactive heat flux, $Q_0$ is the heat generation coefficient, $C$ is the fluid concentration. $D_b$ is mass diffusivity, $k_1$ is the variable rate of chemical conversion where $k_1 = k_0 e^{x/L}$, $k_0$ is a constant, $T_\infty$ and $C_\infty$ are the ambient temperature and concentration respectively. Using the Rosseland approximation for radiation [4], the radiative heat flux is then simplified as:

\[
q_r = -4\sigma^* \frac{\partial T^4}{\partial y}
\]

(5)

where $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ is the absorption coefficient. The temperature difference within the flow is assumed to be sufficiently small such that $T^4$ can be expressed as a linear function of temperature. Thus, expanding $T^4$ in a Taylor series about a free stream temperature $T_\infty$ and neglecting higher-order terms we obtain $T^4 \approx 4T_\infty^3 - 3T_\infty^4$. Equations (3) becomes:

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \left( \frac{\partial^2 T}{\partial y^2} \right) - \frac{16\sigma^* T_\infty^2}{3\rho C_p k^*} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty)
\]

(6)

The appropriate boundary conditions for the problem are given by:

\[
u = -V(x), \quad \frac{\partial u}{\partial y} = N_D \frac{\partial u}{\partial y}, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = 0
\]

\[
u = -V(x), \quad \frac{\partial u}{\partial y} = N_D \frac{\partial u}{\partial y}, \quad \frac{\partial C}{\partial y} = 0 \quad \text{as} \quad y \to \infty
\]

(7)

(8)

Here $U = U_0 e^{x/L}$ is the stretching velocity where $U_0$ is the reference velocity, $N = N_0 e^{x/L}$ is the velocity slip factor changes with $x$ while $N_0$ is the initial value of velocity slip, $V(x) > 0$ is the velocity of suction while $V(x) < 0$ is the velocity of blowing, $V(x) = V_0 e^{x/L}$ is the velocity at the wall where $V_0$ is the initial strength of suction. $D = D_0 e^{x/L}$ is the thermal slip factor changes with $x$ while $D_0$ is the initial value of thermal slip factor, $T_\infty = T_\infty + T_0 e^{x/L}$ is the temperature at the sheet and $T_0$ is the reference temperature, $C_\infty$ is the wall concentration. Introducing the similarity variables as:

\[
\eta = \sqrt{\frac{(U_0 / 2\nu L)e^{x/L}}{y}}, \quad u = U_0 e^{x/L} f'(\eta), \quad v = -\sqrt{(\nu U_0 / 2L)e^{x/L}} (f(\eta) + \eta f'(\eta))
\]

(9)

\[
T = T_\infty + T_0 e^{x/L} \theta(\eta), \quad C = C_\infty + C_0 e^{x/L} \phi(\eta)
\]

(10)

Upon substitution of Eqs.(9) and (10) into the Eqs. (1) to (6), the governing equations reduce to:
\[ f'''(\eta) + f'(\eta)f''(\eta) - 2f'(\eta)f'(\eta) - M^2 f'(\eta) = 0 \] (11)

\[ (1 + (4/3)R)\varphi''(\eta) + Pr(\vartheta'(\eta)f'(\eta) - f'(\eta)\vartheta(\eta)) + Pr 2Q\vartheta(\eta) = 0 \] (12)

\[ \varphi''(\eta) - Sc(f'(\eta)\varphi'(\eta) - f'(\eta)\varphi(\eta)) - 2\varphi(\eta) = 0 \] (13)

and the boundary conditions take the following form:

\[ f(0) = S, \quad f'(0) = 1 + \lambda f'(0), \quad \vartheta(0) = 1 + \delta \vartheta'(0), \quad \varphi(0) = 1 \text{ at } \eta = 0 \] (14)

\[ f'(\infty) \to 0, \quad \vartheta'(\infty) \to 0, \quad \varphi'(\infty) \to 0 \text{ as } \eta \to \infty \] (15)

where \( M = \left(2\sigma h_0^2 L / \rho U_0\right)^{1/2} \) is the magnetic parameter, \( R = \left(4\sigma^* T_\infty^3 \right) / k \) is the radiation parameter, \( Pr = \left(\mu C_p\right) / \kappa \) is the Prandtl number, \( Q = \left(Ue^{x/L}Q_0\right) / U_0 \) is the heat source, \( Sc = \nu / D_\theta \) is the Schmidt number, \( \gamma = (Lk)/U_0 \) is the chemical reaction parameter, \( S = U_0 / \left(vU_0 / 2L\right)^{1/2} \) is the suction parameter, \( \lambda = N\nu \left(U_0 / 2vL\right)^{1/2} \) is the velocity slip parameter and \( \delta = D_l \left(U_0 / 2vL\right)^{1/2} \) is the thermal slip parameter. The physical quantities of the skin-friction coefficient, the reduced Nusselt number, and reduced Sherwood number were obtained as:

\[
\left(\frac{x}{L}\right)f''(0) = 2C_f, \quad \frac{1}{\sqrt{Re_x}}N_{ux} = -\left[1 + (4/3)R\right]\vartheta'(0), \quad \frac{1}{\sqrt{Re_x}}Sh_x = -\varphi'(0)
\] (16)

3. Results and discussion

Table 1 shows the result of the present study are in favorable agreement with studies done by Bidin and Nazar [2], Ishak [10] and Mukhopadhyay [15].

| Pr | R | M | Bidin and Nazar [2] | Ishak [10] | Mukhopadhyay [15] | Present study |
|----|---|---|---------------------|------------|---------------------|---------------|
| 1  | 0 | 0 | 0.9547              | 0.9548     | 0.9547              | 0.9548        |
| 2  | 1.4714 | 1.4715 | 1.4714             | 1.4714     | 1.4715              | 1.4715        |
| 3  | 1.8691 | 1.8691 | 1.8691             | 1.8691     | 1.8691              | 1.8691        |
| 5  | 2.5001 | 2.5000 | 2.5000             | 2.5001     | 2.5001              | 2.5001        |
| 10 | 3.6604 | 3.6603 | 3.6603             | 3.6604     | 3.6604              | 3.6604        |
| 1  | 0.8611 | 0.8610 | 0.8610             | 0.8615     |                     |               |

Figure 1 shows that the velocity profile decreases when magnetic parameter \( M \) increases. The existence of the Lorentz force produce more resistance to the fluid flow and reduce the rate of transport. Figure 2 and 3 indicates that as magnetic parameter \( M \) increases, the thermal and concentration boundary layer thickness also increases. Figure 4 depicts that an increase in the suction parameter \( (S > 0) \) decreases the velocity profile. The imposition of the wall suction \( (S > 0) \) resulted in the decrease of the boundary layer thickness and reduced the velocity field. Figure 5 and 6 illustrate that as suction parameter \( (S > 0) \) increases, the temperature and concentration profiles decrease but they increase due to blowing \( (S < 0) \). Figure 7 demonstrates the effect of the chemical reaction parameter \( \gamma \) on the concentration profile. As the chemical reaction parameter \( \gamma \) increases, the solute concentration decrease and its boundary layer. Figure 8 shows that the increases in the heat source \( Q \) will increase the heat transport rate, thus the increase in temperature and the accompanying thermal boundary layer thickness. Figure 9 represents the influence of radiation parameter \( R \) as the temperature profiles. It is observed that the temperature increases with the radiation parameter \( R \). The thermal boundary layer thickness increases with increasing radiation parameter \( R \). As radiation
parameter $R$ increases, the radiative heat transfer rate to the fluid increases, resulting in the increase of the fluid temperature. Figure 10 presents the slip parameter $\lambda$ effects on the velocity profile. It is observed that velocity is decreasing as the slip parameter $\lambda$ increases accompanied by the reduction in momentum boundary layer thickness. The momentum provided by the stretching sheet is transmitted partly to the fluid under the condition of velocity slip and causes the fluid velocity to decrease. Figure 11 displays the effects of the thermal slip parameter $\delta$ on temperature. As thermal slip parameter $\delta$ increases, the heat transferred to the fluid from the sheet is reduced, thus the decrease in the distribution of temperature and thickness of thermal boundary layer.

Figure 1. Velocity profiles $f'(\eta)$ for some values of $M$.

Figure 2. Temperature profiles $\theta(\eta)$ for some values of $M$.

Figure 3. Concentration profiles $\phi(\eta)$ for some values of $M$.

Figure 4. Velocity profiles $f''(\eta)$ for some values of $S$.

Figure 5. Temperature profiles $\theta(\eta)$ for some values of $S$.

Figure 6. Concentration profiles $\phi(\eta)$ for some values of $S$. 
Figure 7. Concentration profiles $\phi(\eta)$ for some values of $\gamma$.

Figure 8. Temperature profiles $\phi(\eta)$ for some values of $Q$.

Figure 9. Temperature profiles $\theta(\eta)$ for some values of $R$.

Figure 10. Velocity profiles $f(\eta)$ for some values of $\lambda$.

Figure 11. Temperature profiles $\theta(\eta)$ for some values of $\delta$.

4. Conclusion
A theoretical study of double slips effect on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing, radiation, heat source and chemical reaction has been carried out. The numerical solution shows that when the magnetic parameter $M$ increases, it will reduce the velocity field. Suction ($S > 0$) reduces the velocity profile, temperature profile and concentration profile but the opposite behaviour was noted for blowing ($S < 0$). The increases of chemical reaction parameter $\gamma$ will reduce the concentration boundary layer thickness. Temperature increases with the increasing heat source parameter $Q$. Thermal boundary layer thickness increases with radiation parameter $R$. Velocity and temperature decrease with increasing values of the velocity and thermal slip respectively.

Acknowledgement
The authors acknowledge the financial support of Universiti Teknologi Mara under the Lestari Fund 600-IRMI/DANA 5/3/LESTARI (0140/2016). The authors wish also to express their deepest thanks to the reviewer for the valuable comments and suggestions.
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