Discovery reach of $CP$ violation and non-standard interactions in low energy neutrino factory

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In low energy neutrino factory ($E_\mu < 10$ GeV) using MIND detector, we have studied the optimization of $CP$ violation discovery reach in the leptonic sector for different baselines and different parent muon energy considering only Standard Model interactions of neutrinos with matter. Considering such optimized experimental set-up of baseline and energy we have addressed the question of how $CP$ violation discovery reach could get affected by the presence of non-standard interactions of neutrinos with matter during the propagation of neutrinos. For off diagonal NSI elements there could be complex phases $\phi_{ij}$ which could also lead to $CP$ violation. In presence of these complex phases we have shown the contours showing the discovery reach of $\delta$ and $\phi_{ij}$. We have also shown the discovery reach of NSIs in the same experimental set-up which is optimized for discovery of $CP$ violation in the leptonic sector.

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I. INTRODUCTION

The present experiments on neutrino oscillations confirm that there is mixing between different flavours of neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$). For mixing of three active neutrinos there could be $CP$ violating phase in the mixing matrix. This could be probed in future neutrino oscillation experiments. The probability of neutrino oscillations depends on various parameters of the neutrino mixing matrix-the PMNS matrix [1]. The current experiments tells us about two of the angles $\theta_{23}$ and $\theta_{12}$ [2] with some accuracy. The reactor neutrino experiments like Daya Bay[3] and Reno[4] provided compelling evidences for a non-zero $\theta_{13}$, with 5.2$\sigma$ and 4.9$\sigma$ results respectively. These recent reactor neutrino results indicate $\theta_{13}$ very close to 8.8$^\circ$. The $CP$ violating phase $\delta$ is totally unknown. Although the mass squared difference of the different neutrinos ($\Delta m_{ij}^2 = m_i^2 - m_j^2$) are known to us but the sign of $\Delta m_{31}^2$ (which is related to mass hierarchy) is still unknown.

In this work we shall study the discovery reach of $CP$ violating phase $\delta$ in neutrino factory for low parent muon energy around 1-10 GeV for different baselines and have explored for which baselines and low parent muon energy this discovery could be optimized considering Standard Model (SM) interaction of neutrinos with matter. Low energy neutrino factory was first discussed in references [5, 6]. Recently it has been discussed in the International Design Study for neutrino factory that MIND detector (Toroidal magnetized iron neutrino detector) with low muon energy around 10 GeV has somewhat similar performance level as compared to experimental set-up with higher parent muon energy and longer baselines provided that $\sin^2 2\theta_{13} > 0.01$ [7]. If the value of $\theta_{13}$ is large then a low energy neutrino factory provides the ideal scenario [6] for the extraction of the unknown oscillation parameters as well as for resolving the discrete degeneracies which corresponds to oscillation probability $P_{\nu_\mu \nu_\mu}(\theta_{23}) = P_{\nu_\mu \nu_\mu}(\pi/2 - \theta_{23})$ is symmetric under $\theta_{23} \rightarrow \pi/2 - \theta_{23}$ [5]. As recent reactor neutrino experiments indicates large value of $\theta_{13}$ it is important to study the discovery potential of different so far unknown oscillation parameters in low energy neutrino factory.

There is another advantage in choosing low muon energy. There could be non-standard interactions of neutrinos with matter and that could affect the $CP$ violation discovery. As in general there is depletion in the effect of NSIs for shorter baselines on the discovery reach of $CP$ violation in the leptonic sector due to $\delta$ so to get the $CP$ violation discovery lesser affected under such scenario the shorter baselines would be preferred. For shorter baselines relatively lower muon energy is more favourable for the discovery of unknown oscillation parameters. In this work, we shall study what could be the NSI effect on $CP$ violation discovery reach in low energy neutrino factory for the experimental set-up which is better optimized for $CP$ violation discovery considering only SM interactions of neutrinos with matter.

There could be various kind of non-standard interactions of neutrinos with matter. In this work we have considered non-standard interactions of neutrinos with matter fermions ($u$, $d$ and $e$) during propagation of neutrinos only. This could affect oscillations of different flavors of neutrinos as sub-leading effect. We have discussed it in further detail in the next section. There could be other different kind of interactions beyond Standard model leading to non-unitarity of 3 $\times$ 3 PMNS neutrino mixing matrix. Considering non-standard interactions of neutrinos at the source and the detector in neutrino oscillation experiments also lead to such possibility. However, such NSIs' at the source and detector have highly stringent constraints [8] and as such the effect on neutrino oscillation is expected to be lesser affected than that due to NSI in matter during propagation of neutrinos. We have not considered NSIs' at the source and detector in this analysis. There are some studies on the performance of low energy neutrino factories [7] in the context of standard [5, 6, 9–15]
and non-standard interactions (NSI) [16, 17] mainly for small $\theta_{13}$. For large $\theta_{13}$ sensitivity of experiments like MINOS, NOvA and LBNE to NSI has been studied in [18]. Considering large $\theta_{13}$ as indicated by Daya Bay, RENO and other experiments we have analysed the discovery reach of CP violation and NSIs’.

The paper is organized as follows: In section II we discuss the non-standard interactions of neutrinos with matter. In section III, we have discussed $\nu_e \rightarrow \nu_\mu$ oscillation probability and how the $\delta$ dependent and independent part varies with the variation of matter density for baseline $L$ for standard and non-standard interactions as $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ channels are the most important channels for discovery of CP violation. In section IV we discuss about the MIND detector, the experimental set-ups and the assumptions in doing the numerical simulations using GLoBES. In section V, we have presented our results on CP violation discovery reach and also NSI discovery reach. The effect of complex NSI phases in CP violation discovery reach also has been discussed. The analysis in presence of NSIs’ have been done for a few chosen baselines which are optimized for CP violation discovery reach. In section VI, we conclude with remarks on the interplay of CP violating Dirac phase $\delta$, NSIs’ as well as the NSI phases for off-diagonal NSI elements.

II. NON-STANDARD INTERACTIONS

We consider the non-standard interactions of neutrinos which could be outcome of effective theory at low energy after integrating out the heavy mediator fields at the energy scale of neutrino oscillation experiments. Apart from Standard Model (SM) Lagrangian density we consider the following non-standard fermion-neutrino interaction in matter defined by the Lagrangian:

$$\mathcal{L}_M^{\text{NSI}} = -2\sqrt{2}G_F \varepsilon^{fP}_{\alpha\beta} \bar{f} \gamma_\mu Pf \bar{\nu}_\beta \gamma_\mu L \nu_\alpha$$

(1)

where $P \in (L, R)$, $L = \left(1 - \gamma_5 \right) / 2$, $R = \left(1 + \gamma_5 \right) / 2$, $f = e, u, d$ and $\varepsilon^{fP}_{\alpha\beta}$ are termed as non-standard interactions (NSIs) parameters signifying the deviation from SM interactions. These are non-renormalizable as well as not gauge invariant and are dimension-6 operators after heavy fields are integrated out [8]. Although at low energy NSIs’ look like this but at high energy scale where actually such interactions originate there they have different form. These NSI parameters can be reduced to the effective parameters and can be written as:

$$\varepsilon^{fP}_{\alpha\beta} = \sum_{f, P} \varepsilon^{fP}_{\alpha\beta} \frac{n_f}{n_e}$$

(2)

where $n_f$ and $n_e$ are the fermion and the electron number density respectively in matter. As these NSIs modify the interactions with matter from the Standard Model interactions the effective mass matrix for the neutrinos are changed and as such there will be change in the oscillation probability of different flavor of neutrinos. Although NSIs could be present at the source of neutrinos, during the propagation of neutrinos and also during detection of neutrinos [19] but as those effects are expected to be smaller at the source and detector due to their stringent constraints [8, 20], we consider the NSI effect during the propagation of neutrinos only.

Model dependent [8, 20–34] and independent [35, 36] bounds are obtained for these matter NSI parameters and are shown in the following table. In obtaining model dependent bounds on matter NSI the experiments
with neutrinos and charged leptons - LSND, CHARM, CHARM-II, NuTeV and also LEP-II have been considered. Bounds coming from loop effect have been used for model dependent bounds. However, model independent bounds are less stringent and could be larger than the model dependent bounds by several orders and have been obtained first by Biggio et al [8, 20] and discussed in [8]. Considering recent results from

| NSI      | Model dependent bound on NSI [Reference [8]] | Model independent bound on NSI [Reference [20]] |
|----------|-----------------------------------------------|-----------------------------------------------|
| $\varepsilon_{ee}$ | $\gg -0.9; < 0.75$ | $< 4.2$ |
| $|\varepsilon_{e\mu}|$ | $\ll 3.8 \times 10^{-4}$ | $< 0.33$ |
| $|\varepsilon_{e\tau}|$ | $\ll 0.25$ | $< 3.0$ |
| $\varepsilon_{\mu\mu}$ | $\gg -0.05; < 0.08$ | $< 0.068$ |
| $|\varepsilon_{\mu\tau}|$ | $\ll 0.25$ | $< 0.33$ |
| $\varepsilon_{\tau\tau}$ | $\ll 0.4$ | $< 21$ |

TABLE I: Strength of Non standard interaction terms used for our Analysis

experiments in IceCube-79 and DeepCore more stringent bound on $\varepsilon_{\mu\mu}$, $|\varepsilon_{\mu\tau}|$ and $\varepsilon_{\tau\tau}$ have been obtained in [35]. However, the analysis has been done considering two flavor only. In section IV, we shall consider both model dependent and independent allowed range of values of different NSIs as shown in the table above for earth like matter using numerical simulations while showing discovery reach for CP violation and NSIs'.

III. $\nu_e \rightarrow \nu_\mu$ OSCILLATION PROBABILITIES WITH NSI

The flavor eigenstates $\nu_\alpha$ is related to mass eigenstates of neutrinos $\nu_i$ as

$$|\nu_\alpha > = \sum_i U_{\alpha i} |\nu_i >$$

in vacuum where $U$ is PMNS matrix [1] consisting four parameters- three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and one $CP$ violating phase $\delta$. In the flavor basis the total Hamiltonian consisting both standard ($H_{SM}$) and non-standard interactions ($H_{NSI}$) of neutrinos interacting with matter during propagation can be written as:

$$H = H_{SM} + H_{NSI}$$

where

$$H_{SM} = \frac{\Delta m_{31}^2}{2E} \left[ U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right],$$

$$H_{NSI} = A \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix}$$
In equations (5) and (6)

\[ A = \frac{2E\sqrt{2}G_Fn_e}{\Delta m_{31}^2}; \quad \alpha = \frac{\Delta m_{31}^2}{\Delta m_{31}^2}; \quad \Delta m_{ij}^2 = m_i^2 - m_j^2 \]

(7)

where \( m_i \) is the mass of the \( i \)-th neutrino, \( A \) corresponds to the interaction of neutrinos with matter in SM and \( G_F \) is the Fermi constant. \( \varepsilon_{ee}, \varepsilon_{e\mu}, \varepsilon_{e\tau}, \varepsilon_{\mu\mu}, \varepsilon_{\mu\tau} \) and \( \varepsilon_{\tau\tau} \) correspond to the non-standard interactions (NSIs) of neutrinos with matter. In equation (6), \((^*)\) denotes complex conjugation. The NSIs - \( \varepsilon_{ee}, \varepsilon_{e\tau} \) and \( \varepsilon_{\mu\tau} \) could be complex. Later on, in the expressions of probability of oscillation we have expressed these NSIs as \( \varepsilon_{ij} = |\varepsilon_{ij}|e^{i\phi_{ij}} \). In our numerical analysis we have considered the NSIs - \( \varepsilon_{ee}, \varepsilon_{e\tau} \) and \( \varepsilon_{\mu\tau} \) as both real as well as complex.

For baselines of length 730 Km, 1290 Km and 1500 Km in the low energy range of 1-10 Gev (which has been considered in this work) the oscillation probability \( P_{\nu_e \rightarrow \nu_\mu} \) is presented below. Following the perturbation method adopted in references [37, 38] the oscillation probability \( P_{\nu_e \rightarrow \nu_\mu} \) upto order \( \alpha^2 \) (considering \( \sin \theta_{13} \sim \sqrt{\alpha} \) as follows from recent reactor experiments) and small NSI of the order of \( \alpha \) and the matter effect parameter \( A \) in the leading order of perturbation and NSI parameters \( \varepsilon_{\alpha\beta} \) of the order of \( \alpha \) one obtains [39]

\[ P_{\nu_e \rightarrow \nu_\mu} = P_{\nu_e \rightarrow \nu_\mu}^{SM} + P_{\nu_e \rightarrow \nu_\mu}^{NSI} \]

(8)

where

\[
P_{\nu_e \rightarrow \nu_\mu}^{SM} = 4\sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right) \frac{s_{13}^2 s_{23}^2}{s_{13}^2 \cos \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)} \left( (A - 1)^2 - (1 + A)^2 s_{13}^2 \right) \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)
\]

\[
+ A(A - 1) \frac{\Delta m_{31}^2 L}{E} \frac{s_{13}^2 \cos \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)}{1 + (A - 1)^2} \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right) - 8A \sin^2 \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)
\]

\[
+ \frac{\alpha s_{13}^2 s_{23}^2 (A - 1)^3}{(A - 1)^2} \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{2E} \right) \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)
\]

\[
+ \frac{\alpha s_{13} s_{23}^2}{A(A - 1)} \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right) \sin \left( \frac{(A - 1)\Delta m_{31}^2 L}{4E} \right)
\]

(9)
where

$$a_1 = A \varepsilon_{ee}$$

$$|a_2| e^{i \phi_{a_2}} = A \left( e^{i \phi_{ee}} |e_{ee}| c_{23} - e^{i \phi_{e\tau}} |e_{e\tau}| s_{23} \right)$$

$$|a_3| e^{i \phi_{a_3}} = A \left( e^{i \phi_{ee}} |e_{ee}| c_{23} + e^{i \phi_{e\tau}} |e_{e\tau}| s_{23} \right)$$

$$|a_4| e^{i \phi_{a_4}} = A \left( |e_{e\tau}| e^{i \phi_{e\tau}} - 2 |e_{e\tau}| s_{23} + (|e_{e\mu}| - |e_{e\tau}|) c_{23} s_{23} \right)$$

$$a_5 = A \left( e_{e\tau} c_{23}^2 + e_{e\mu} s_{23}^2 + |e_{e\tau}| |e_{e\mu}| s_{23} \right)$$

and

$$\phi_{a_2} = \tan^{-1} \left( \frac{|e_{ee}| c_{23} \sin \phi_{ee} - |e_{e\tau}| s_{23} \sin \phi_{e\tau}}{|e_{ee}| c_{23} \cos \phi_{ee} - |e_{e\tau}| \cos \phi_{e\tau} s_{23}} \right)$$

$$\phi_{a_3} = \tan^{-1} \left( \frac{|e_{ee}| s_{23} \sin \phi_{ee} + |e_{e\tau}| c_{23} \sin \phi_{e\tau}}{|e_{e\tau}| c_{23} \sin \phi_{e\tau} + |e_{e\mu}| \cos \phi_{e\mu} s_{23}} \right)$$

$$\phi_{a_4} = \tan^{-1} \left( \frac{|e_{e\tau}| \sin \phi_{e\tau}}{|e_{e\tau}| c_{23} \cos \phi_{e\tau} + (|e_{e\mu}| - |e_{e\tau}|) c_{23} s_{23}} \right)$$

where \( s_{ij} = \sin \theta_{ij}, \ c_{ij} = \cos \theta_{ij}, \ s_{2\times ij} = \sin 2\theta_{ij}, \ c_{2\times ij} = \cos 2\theta_{ij} \).

For CP violation there is difference of probability in the neutrino oscillation and probability of antineutrino oscillation. The oscillation probabilities for antineutrinos can be obtained from the oscillation probabilities
given for neutrinos above by using the following relation:

$$P_{\bar{\alpha}\bar{\beta}} = P_{\alpha\beta}(\delta_{CP} \rightarrow -\delta_{CP}, A \rightarrow -A).$$

(13)

In addition, while considering non-standard interactions we also have to replace $\varepsilon_{\alpha\beta}$ with their complex conjugates, in order to deduce the oscillation probability for the antineutrino.

To estimate the order of magnitude of $\delta$ dependent and $\delta$ independent but matter dependent (i.e., $A$ dependent) part in the above two oscillation probability, following reactor experiments we shall consider $\sin \theta_{13} \sim \sqrt{\alpha}$. For only SM interactions, (i.e $\varepsilon_{\alpha\beta} \rightarrow 0$) in above expressions of oscillation probabilities one finds that the $\delta$ dependence occurs at order of $\alpha^{3/2}$ for both neutrino oscillation and antineutrino oscillation probabilities.

However, when NSIs are also taken into account one can see that $\delta$ dependence in oscillation probability could occur at the order of $\alpha^{3/2}$ also through $a_2$ and $a_3$ (which are NSI dependent) containing terms in (10) for NSIs of the order of $\sqrt{\alpha}$. We have checked that for slightly higher NSIs of the order of $\sqrt{\alpha}$ using perturbation method the same $\delta$ dependent terms appear with $a_2$ and $a_3$ in the oscillation probability for long baseline as given in (10) and this slightly higher NSI makes these terms at the order of $\alpha$ which could compete with the $\delta$ independent but matter dependent part (which could mimic $CP$ violation) for long baseline as that is also at the order of $\alpha$. So presence of slightly higher NSIs of order $\sqrt{\alpha}$ present in $a_2$ and $a_3$ improves the discovery reach of $CP$ violation for longer baseline. As $a_2$ and $a_3$ contains NSIs like $\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$ and also these are coupled with $\delta$ dependent term in the oscillation probability, these NSIs’ could have significant effect in changing the discovery reach of $CP$ violation. Interestingly, sometimes these NSIs’ could improve the prospect of discovering $CP$ violation due to $\delta$ as discussed in section V provided that we know those NSI values.

IV. NUMERICAL SIMULATION

In this work we have analyzed $CP$ fractions over various baselines over the range (100-4500 kms) with muon energies in the range (1-10 GeV) for SM interactions as shown in figure 1. Here $CP$ fraction is the fraction of the total allowed range (0 to $2\pi$) for the $CP$ violating phase over which $CP$ violation can be discovered. Based on high $CP$ fraction discovery potential as found in this figure we have chosen 10 GeV muon energy and a few baselines which are : 730 Km (FNAL-Soudan), 1290 Km (FNAL-Homestake) and 1500 Km (FNAL-Henderson). Next we have asked the question that had there been Non-Standard interactions what could have been their effect on the $CP$ violation discovery reach for such experimental set-ups. We have considered $5 \times 10^{21}$ number of stored muons and anti-muons decays per year with running time of 10 years for each type of decays. The numerical simulation has been done by using GLoBES [40, 41]. Different oscillation channels which have been considered as signals and backgrounds [42] in the analysis are shown in table II.

We consider the true values [43] of the neutrino oscillation parameters as $|\Delta m^2_{31}| = 2.5 \times 10^{-3}$ eV$^2$, $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV$^2$, $\sin^2 2\theta_{13} = 0.094$, $\sin^2 \theta_{12} = 0.31$ and $\theta_{23} = 38.3^\circ$. Also in calculating the priors we consider an error of 5% on $\sin^2 \theta_{12}$, 5% on $\sin^2 2\theta_{13}$, 8% on $\theta_{23}$, 3% on $|\Delta m^2_{31}|$ and 3% on $\Delta m^2_{21}$. Also we consider an error of 2% on matter density . In our analysis we have taken the uncertainty in the hierarchy of neutrino masses .
TABLE II: Different oscillation channels considered as signals and backgrounds in the analysis.

| Channel Name            | $\mu^+$   | $\mu^-$   |
|-------------------------|-----------|-----------|
| Signal                  |           |           |
| Golden Channel          | $\nu_e \rightarrow \nu_\mu$ | $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ |
| Silver Channel          | $\nu_e \rightarrow \nu_\tau$ | $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ |
| Background              |           |           |
| $\nu_e$ disappearance channel | $\nu_\mu \rightarrow \nu_e$ | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ |
| $\nu_\mu$ disappearance channel | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | $\nu_\mu \rightarrow \nu_\mu$ |
| Platinum Channel        | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | $\nu_\mu \rightarrow \nu_\tau$ |
| Dominant Channel        | $\nu_\mu \rightarrow \nu_\tau$ | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ |

In this work we have used a large magnetised iron neutrino detector (MIND) \cite{42} with a toroidal magnetic field having a mass of 100 KTon. MIND can also be described as an iron-scintillator calorimeter. This detector has the capability of excellent reconstruction and charge detection efficiency. Furthermore, it has the capacity to identify the $\nu_e \rightarrow \nu_\tau$ silver channel oscillation as signal. This reinforces the golden channel signal. In this work we have considered muons in a storage ring consisting of both $\mu^+$ and $\mu^-$ which decay with energies of 10 GeV. We consider $5 \times 10^{21}$ stored muons. The golden channel ($\nu_e \rightarrow \nu_\mu$ oscillation channel) where the charged current interactions of the $\nu_\mu$ produce muons of the opposite charge to those stored in the storage ring (generally known as wrong-sign muons), is the most promising channel to explore CP violation at a neutrino factory. The detector that we are considering in this work - MIND is optimized to exploit the golden channel oscillation as this detector has the capacity to easily identify signal i.e. a muon with a sign opposite to that in the muon storage ring. We have taken the migration matrices for the true and reconstructed neutrino energies as given in reference \cite{42}. The signal and background efficiencies are taken into account in those matrices. We have considered systematic errors to be 1%. In this work we have considered a running time of 10 years for both $\mu^+$ and $\mu^-$. 

V. RESULTS

In this section in figure 1 we first address the question of optimization for different baselines and different parent muon energy for the discovery reach of CP violation when only SM interactions of neutrinos with matter during propagation is present. It is found that at 5$\sigma$ confidence level the CP fraction of about $(0.9 \gtrsim F_3 \gtrsim 0.8 )$ is possible for baselines ranging from 200 to 700 Km and 450 to 2500 Km for energies lesser than 5 GeV and for energies 5-10 GeV respectively.

Based on the CP violation discovery optimization analysis we have chosen some baselines of length 730 Km, 1290 Km and 1500 kms and are also found to be optimized for CP violation discovery reach as in figure 1 with MIND detector. Although lower energy around 4 Gev could be possible for shorter baselines which has also the potential of very good CP violation discovery reach but if we want to get in the same experimental set-ups good NSI discovery reach also then we should consider relatively higher possible muon energy for which good CP fraction discovery reach is also possible. Keeping this in mind we have considered parent muon energy of 10 GeV \cite{42} although our main concern is to study the CP violation discovery reach, Considering a few optimized baselines and energy 10 GeV we have studied the effect of NSIs' on the CP violation discovery reach for these few optimized experimental set-ups. While taking into account NSI effect, for off-diagonal NSIs' we have also taken into account the effect of NSI phases also over $\delta$ CP violation. We
FIG. 1: Fraction ($F_δ$) of $δ_{CP}$ discovery with only SM interactions for different baselines $L$ and different muon energies ($E_μ$) at 5$σ$.

have also addressed the question of NSI discovery reach in the same experimental set-ups optimized for $CP$ violation in absence of $δ_{CP}$.

In figure (2), we have studied $δ_{CP}$ fraction in the presence of real NSIs’ (NSI phases have been chosen to be zero) for different baselines of length 730 Km, 1290 Km and 1500 Km. Here we have considered the model independent bounds on NSIs’ as shown in table I. For lower values of NSIs’ there is essentially negligible effect on discovery reach of $CP$ violation which is seen in the figure as horizontal straight line. This part of the figure corresponds to the $δ_{CP}$ fractions with SM interactions only which can be found in figure 1. For $ε_{ee} ≥ 0.6$, $ε_{eμ} ≥ 0.03$, $ε_{eτ} ≥ 0.1$, $ε_{μτ} ≥ 0.2$, $ε_{ττ} ≥ 0.8$ there is noticeable effect of NSIs’ on $δ_{CP}$ fractions. Particularly, for $ε_{eμ}$ and $ε_{eτ}$ the effect on $δ_{CP}$ fraction is more with respect to other NSIs’ at their relatively smaller values. This feature can be understood from the expression of oscillation probability ($ν_τ → ν_µ$) in equation (10) where we see that particularly two NSIs’ $ε_{eτ}$ and $ε_{eμ}$ have more effect in the oscillation probability in comparison to other NSIs’ being at lower order in $α$. The $δ_{CP}$ fractions in presence of these two NSIs’ could be even more than the SM value. However, there is no noticeable effect due to NSI-$ε_{μμ}$.

Next in figures 3 and 4 considering model dependent and independent NSI bounds respectively we have considered the case where the $CP$ violation might come from $δ_{CP}$ as well as from NSI phase $φ_{αβ}$. In figure 3 we have chosen uppermost value of NSI with model dependent bound and in 4 we have chosen uppermost value of NSI with model independent bound. In these plots unshaded regions correspond to the discovery of total $CP$ violation. For $δ = 0, π, 2π$ and the NSI phases also having those values obviously one can not get $CP$ violation discovery. Corresponding to $δ$ values very near to $δ = π$ with NSI phases having one of those $CP$ conserving values, sometimes the region for which $CP$ violation can not be discovered, is too small to be seen in the figures. In figure 3 for $φ_{eμ}$ and $φ_{μτ}$ slightly away from 0, π and 2π it is found that total $CP$ violation discovery reach could be possible. For $φ_{eτ}$ $CP$ violation discovery reach is much better over almost entire region. However, in the next figure 4 with the increase in NSI values one can see that total...
$CP$ violation discovery reach further improves. Since the sensitivity of $CP$ violation from the NSI phases is coupled with the magnitude of the modulus of respective NSIs' so if that value is relatively lesser then the discovery region of total $CP$ violation decreases and vice versa. One important point is to be noted here that for some NSI phases one may not be able to see $CP$ violation for any value of Dirac phase $\delta$. As the pattern of such no observation does not change much going to higher or lower baselines, it seems combination of short and long baselines may not help much to solve this problem.

In figure 5 we have addressed the question of what could be the $CP$ fraction for discovery of $CP$ violation if Dirac phase $\delta$ is absent in PMNS mixing matrix and $CP$ violation comes from purely NSI phases. We observe that for longer baselines the $CP$ fraction is more in comparison to the shorter baselines. With the increase of $|\epsilon_{\alpha\beta}|$ there is increase in discovery of $CP$ fraction in general. However, for $|\epsilon_{\epsilon\tau}| \gtrsim 0.3$ there is no further increase in $CP$ fraction.

In figure 6 we have explored the discovery of $\delta_{CP}$ fraction in the presence of two off-diagonal NSIs’ in the $H_{NSI}$ matrix where we have taken $|\epsilon_{\alpha\beta}| \in (0.001, 0.01)$ except for $|\epsilon_{\mu\tau}|$ for which only 0.001 value has been considered due to the model dependent stringent upper bound on it. Here we see that only for the left
hand side panel the \(\delta_{CP}\) fraction varies from 83% to 87% and different regions are shaded differently based on this variation in the fraction as stated in the figure caption. For other NSI combinations with various combination of NSI values as mentioned above it is always found that the entire region correspond to almost same \(CP\) fractions of about 87.5% like the one combination of NSIs’ shown on the right hand side panel.

In figures 7 and 8 we have addressed the question of what could be the NSI discovery reach at maximal \(CP\) violation due to purely Dirac phase \(\delta = \frac{3\pi}{2}\) and purely NSI phase \(\phi_{ij} = \frac{3\pi}{2}\) respectively. Considering parent muon energy to be 10 GeV we have studied the discovery reach of NSIs’ for different baselines with length ranging from about 100 Km to 4500 Km. The shaded regions in both the figures correspond to the discovery reach for NSIs’ at different confidence levels as shown in figures. One can see that in general for
FIG. 4: $\delta$ versus phase ($\phi_{ij}$) considering the value of NSIs($\varepsilon_{ij}$) at the upper limit of model independent bounds.

FIG. 5: Discovery reach of $CP$ violation due to the NSI phases $\phi_{ij}$ over entire allowed range when $\delta_{CP} = 0$ at 5$\sigma$ confidence level.
FIG. 6: Contours for $\delta_{CP}$-fraction in the plane of two NSI phases $\phi_{ij}$ for two different pair of modulus of NSI values as shown in the figure at 5$\sigma$ confidence level. The black, yellow, grey regions (colors online available) correspond to $> 83\%$ and $\leq 84\%$; $> 83\%$ and $\leq 86\%$; $> 83\%$ and $< 88\%$ fractions respectively for the left hand side panel. For right hand side panel the whole region correspond to $> 87\%$ and $< 88\%$ fractions.

FIG. 7: Contours showing discovery limit of real NSIs($\varepsilon_{ij}$) at 3$\sigma$ and 5$\sigma$ confidence levels.

longer baselines better discovery reach is possible as compared to shorter baselines.
VI. CONCLUSION

Considering only SM interactions of neutrinos with matter we have studied the optimization of CP violation discovery reach in different baselines of length ranging from 100 to 4500 Km with different low parent muon energy upto 10 GeV with MIND detector for neutrino oscillation experiments in neutrino factory. Our analysis shows that for baselines of length 450 Km - 2500 Km with parent muon energy 5-10 GeV and lengths 200 - 700 Km with energy lesser than 5 GeV it is possible to have CP fraction \( F_\delta \) in the range of 0.8 to 0.9. On the basis of optimization analysis we have chosen a few baselines between accelerator facilities and underground laboratories which are of length 730 Km, 1290 Km and 1500 Km with parent muon energy 10 GeV to study the NSI effect on the CP violation discovery reach. For real NSI \( |\varepsilon_{\alpha\beta}| \lesssim O(\alpha \approx 0.027) \) there is no noticeable effect in the CP violation discovery reach. However, above that for different values different NSIs' start showing the effect on CP violation discovery reach.

The CP violation discovery reach in neutrino factory with MIND detector has the potential to have CP violation discovery reach at around \( F_\delta \sim 85\% \) for SM interactions only. But if we consider the NSI effects then for relatively shorter baseline like 730 Km length the NSI effect could change this CP violation discovery reach. Considering their upper model independent bound for the NSI values above \( \alpha \) value the CP fraction \( F_\delta \) for \( |\varepsilon_{ee}|, |\varepsilon_{e\mu}|, |\varepsilon_{e\tau}|, |\varepsilon_{\mu\mu}|, |\varepsilon_{\mu\tau}|, |\varepsilon_{\tau\tau}| \) could decrease to 0.6, increase to 0.9, decrease to 0.65, decrease to 0.75, does not change noticeably, could decrease to zero respectively with NSI phases zero. Although NSIs' are real here but one can see from the expression of oscillation probabilities that the contribution of \( \delta \) dependent terms to oscillation probability change due to non-zero NSIs'. Thus real NSIs' change the CP violation discovery reach.

For off-diagonal NSIs' with phases there is new source of CP violation and if we explore the total CP violation then it turns out that for \( \varepsilon_{e\mu}, \varepsilon_{\mu\tau} \) for NSI phases slightly away from 0, \( \pi \) and 2\( \pi \) there could be total CP violation discovery for NSIs' with model dependent stringent bounds. For \( \varepsilon_{e\tau} \) there is better CP violation discovery reach for such NSI phase. For higher values of NSIs' satisfying model independent bounds there is better prospect to find total CP violation. However, it is found that there could be some values of NSI phases for which CP violation may not be found for any value of \( \delta \). Particularly for NSIs' \( \varepsilon_{e\mu}, \varepsilon_{\mu\tau} \) (when these are nearer to their present model dependent upper bound for the NSI phases nearer to 0, \( \pi \) and 2\( \pi \) values this problem is severe. Even this may not be solved by considering the combination of short
and long baselines. Obviously, in the $|\varepsilon_{ij}| \to 0$ limit this problem will disappear. So getting more stringent constraint on $|\varepsilon_{ij}|$ from various experiments will give better idea on this problem.

If we assume that the $CP$ violation in the leptonic sector is coming purely from NSIs’ then it is found that in general the $CP$ violation discovery reach increases with the increase in the length of the baseline. In our experimental set-up with low muon energy at around 10 GeV the pure NSI $CP$ violation could be observable for modulus of NSI at least above 0.001. For two NSIs’ $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ with their modulus being 0.01 the $\delta_{CP}$ fraction could vary from 83% to 88%. However, for other off-diagonal NSIs’ with one of their modulus being 0.001 and that of the other NSI being 0.01 the $\delta_{CP}$ fraction is around 87% to 88%. For further lower values of NSIs’ this fraction is at around the value with only SM interaction. If there is maximal $CP$ violation coming purely from Dirac phase $\delta$ then the discovery reach for $|\varepsilon_{\alpha\beta}|$ improves for longer baselines. However, if there is maximal $CP$ violation coming purely from NSI phase $\phi_{ij}$ then around 2000 Km there is better NSI discovery reach. For some NSIs’ the discovery reach could go to the lower value of NSI upt to around $3 \times 10^{-3}$.

Although with SM interactions there is wide range of length of baseline as well as parent muon energy for which the $CP$ violation discovery reach remains almost same ($\delta_{CP}$ fraction around 0.8 to 0.9) with MIND detector, however, in presence of NSI this discovery reach could change significantly. In fact, sometimes its’ presence could improve the prospect of $CP$ violation discovery also provided that we know its’ value. With MIND detector in comparison to other detectors the prospect of $CP$ violation discovery is much better. But the presence of real or the complex NSI could make the discovery of non-zero $CP$ violating Dirac phase $\delta$ difficult.

It seems in general with shorter baselines the NSI effect on $CP$ violation discovery reach will be lesser. However, for smaller baselines below 300 Km with lower parent muon energy although NSI effect will be lesser but $\delta_{CP}$ fraction also starts getting reduced. However, if there is NSI of neutrinos with matter then considering relatively shorter baselines with low parent muon energy might be better to have to some extent lesser NSI effect on the discovery of $CP$ violation in the leptonic sector.

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