Modeling Barkhausen Noise in magnetic glasses with dipole-dipole interactions

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Abstract – Long-ranged dipole-dipole interactions in magnetic glasses give rise to magnetic domains having labyrinthine patterns on the scale of about 1 micron. Barkhausen Noise then results from the movement of domain boundaries which is modeled by the motion of elastic membranes with random pinning. Here we propose that on the nanoscale new sources of Barkhausen Noise can arise. We propose an atomistic model of magnetic glasses in which we measure the Barkhausen Noise which results from the creation of new domains and the movement of domain boundaries on the nanoscale. The statistics of the Barkhausen Noise found in our simulations is in striking disagreement with the expectations in the literature. In fact we find exponential statistics without any power law, stressing the fact that Barkhausen Noise can belong to very different universality classes. In the present model the essence of the phenomenon is the fact that the spin response Green’s function is decaying too rapidly for having sufficiently large magnetic jumps. A theory is offered in excellent agreement with the measured data without any free parameter.

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Introduction. – The statistics of the so-called “Serrated Noise” is a subject of wide-ranging interest from earthquakes with stress fluctuations on a global scale to Barkhausen Noise in small magnetic samples with magnetization jumps that are barely measurable. Typically one finds in such problems a wide range of sharp variations in some measurable quantity, and the question is how to model the statistics of these variations. In this letter we return to Barkhausen Noise which is one of the most studied examples of serrated responses since its discovery in 1919 [1]. The phenomenon manifests itself as a series of jumps in the magnetization of a ferromagnetic sample when subjected to a varying external magnetic field [2–7]. The phenomenon has practical importance for magnetic recordings [8] and for non-invasive material characterization [9]. When the magnetic field is ramped up and then down the magnetization describes a hysteresis loop which is however punctuated by sharp jumps in the measured value. Here we report simulations in an atomistic model of a magnetic glass and observe a serrated Barkhausen Noise on scales much smaller than those found in experiment, see, for example, fig. 1.

Some of the more careful experimental realizations of Barkhausen Noise involve magnetic systems with labyrinthine magnetic domains in which the serrated response can be linked to the movement of the domain boundaries [10–12]. In these cases a theory was proposed using a model of an elastic membrane that is pinned by random impurities and is moving under the action of a force. In their excellent review of both experiments and theory Durin and Zapperi [12] warn the reader that even in these well-chosen experiments the interpretation of the results is far from obvious, not the least because the statistics of Barkhausen Noise is not invariant along the magnetization hysteresis loop. In less well-characterized experiments Barkhausen Noise appears to be a very complex physical phenomenon with many different appearances. Its character may depend on the type of ferromagnetic specimen under study, the character of the disorder in the material, the external field driving rate, thermal effects, strength of the demagnetization fields, and other experimental details.

For these reasons it is worthwhile to construct microscopic theoretical models in which the measurement can be done with arbitrary accuracy and in which the interpretation can be fully justified by comparing careful...
simulations with the appropriate theory. Indeed, in recent papers we initiated the microscopic study of Barkhausen Noise in magnetic glasses based on a model Hamiltonian that couples the mechanical properties of an amorphous solid to its magnetic degrees of freedom. In this letter we announce a model that contains long-ranged dipole-dipole interactions such that the magnetic domains appear labyrinthine (see fig. 2) in accordance with the expectation that Barkhausen Noise will be associated with the movement of domain boundaries. Nevertheless we will report here results that are quite surprising, in strong contradiction with many of the expectations in the field. In doing so we do not mean to put doubt on previous results, but rather to highlight the richness of possible statistics of serrated noises in general and of Barkhausen Noise in particular. Different choices of the microscopic Hamiltonian can result in different universality classes of the observed statistics. Thus the example presented below is used to enrich rather than contradict the existing literature.

The model. – Our model Hamiltonian represents a binary glass with magnetic degrees of freedom [13–15]:

\[ U(\{r_i\}, \{S_i\}) = U_{\text{mech}}(\{r_i\}) + U_{\text{mag}}(\{r_i\}, \{S_i\}), \]

(1)

where \(\{r_i\}_{i=1}^N\) are the 2-dimensional positions of \(N\) particles in an area \(L_x \times L_y\) and \(S_i\) are spin variables. The mechanical part \(U_{\text{mech}}\) represents a standard binary mixture of 50% particles A and 50% particles B, with Lennard-Jones potentials having a minimum at positions \(\sigma_{AA} = 1.17557\), \(\sigma_{AB} = 1.0\) and \(\sigma_{BB} = 0.618034\) [16]. These parameters are known to provide good glass formation without crystallization. The energy parameters are selected as \(\epsilon_{AA} = \epsilon_{BB} = 0.5\) and \(\epsilon_{AB} = 1.0\), in units for which the Boltzmann constant equals unity. All the potentials are truncated at distance \(2.5\sigma\) with two continuous derivatives. \(N_A\) “A” particles carry spins \(S_i\); the \(N_B\) “B” particles are not magnetic. Of course \(N_A + N_B = N\). In the present model the spins \(S_i\) are classical Heisenberg spins in 3 dimensions; these can point anywhere on the unit sphere.

The magnetic contribution to the potential energy is chosen to allow the creation of labyrinthine magnetic domains

\[
U_{\text{mag}}(\{r_i\}, \{S_i\}) = -\sum_{<ij>} J(r_{ij})S_i \cdot S_j - B \cdot \sum_i S_i - \sum_i K_i \cos^2(\phi_i - \psi_i(\{r_i\})) - K_2 \sum_i S_{iz}^2 - \sum_{\langle ij \rangle} 3(\mu_i \cdot r_{ij})(\mu_j \cdot r_{ij}) - (\mu_i \cdot \mu_j) \rho_{ij}^2.
\]

(2)

Here \(r_{ij} \equiv |r_i - r_j|\) and the sums are only over the A particles that carry spins. The exchange parameter \(J(r_{ij})\) is a deterministic function of a changing inter-particle position (either due to affine motions induced by an external strain or an external magnetic field or due to non-affine particle

Fig. 1: (Color online) A typical hysteresis loop in our atomistic simulations showing the sharp changes in magnetization when a magnetic field in the \(z\)-direction is ramped first up until saturation \((m = 1)\) and then down until saturation with \(m = -1\). Our interest in this letter is in the statistics of the sharp changes \(\Delta m\) seen in this figure.

Fig. 2: (Color online) A typical labyrinthine structure of the magnetic domains in the present model with \(N = 2100\) and dipole-dipole interactions. Here \(J_0 = 6\), \(K^z = 0.25\), \(K_0 = 5\), \(\mu_B = 0.1\). Upper panel: zero magnetic field. The “small” particles in black are the non-magnetic particles. The other two colors represent 50% spins “up” and 50% spins “down”. Middle and lower panels: the effect of an increasing magnetic field on the labyrinthine pattern shown in the upper panel. Middle panel: \(B = 0.3\). Lower panel: \(B = 0.4\). Note the coarsening of the domains of “up” spins (in magenta) which occur via movements of domain boundaries.
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displacements, and see below). We choose for concreteness the monotonically decreasing form \( J(x) = J_0 f(x) \) where \( f(x) \equiv \exp(-x^2/0.3) + H_0 + H_2 x^2 + H_4 x^4 \) with \( H_0 = -5.51 \times 10^{-8} \), \( H_2 = 1.68 \times 10^{-8} \), \( H_4 = -1.29 \times 10^{-9} \) [17,18]. This choice cuts off \( J(x) \) at \( x = 2.5 \) with two smooth derivatives. Finally, in our case \( J_0 = 6 \). The second term is the interaction with the external magnetic field. The next term represents the effect of an in-plane \((x-y)\) local anisotropy, where the local axis of anisotropy \( \psi_i \) is determined by the local structure. The angle \( \phi_i \) is determined by the projection of the spin \( S_i \) on the \( x-y \) plane and is measured with respect to the \( x \)-axis. To find \( \psi_i \) we define the matrix \( T_{ij} \):

\[
T_{ij}^\alpha \equiv \sum_j J(r_{ij}) \bar{r}_{ij}^\alpha \bar{r}_{ij}^\beta / \sum_j J(r_{ij}), \quad \bar{r}_{ij} \equiv |r_i - r_j|, \quad (3)
\]

where we sum over the particles that are within the range of \( J(r_{ij}) \). The matrix \( T_{ij} \) has two eigenvalues in 2 dimensions that we denote as \( \kappa_{i,1} \) and \( \kappa_{i,2} \). The eigenvector that belongs to the larger eigenvalue \( \kappa_{i,1} \) is denoted by \( \bar{n}_i \). The easy axis of anisotropy is given by \( \psi_i = \sin^{-1}(|\bar{n}_i|) \). Finally, the coefficient \( K_i \) is defined as

\[
K_i \equiv \tilde{C} \left( \sum_j J(r_{ij}) \right)^2 (\kappa_{i,1} - \kappa_{i,2})^2, \quad \tilde{C} = K_0 / J_0 \sigma_{AB}. \quad (4)
\]

The parameter \( K_0 \) determines the relative strength of this random local anisotropy term with respect to other terms in the Hamiltonian\(^1\). The next term in the Hamiltonian represents the perpendicular (out-of-plane) anisotropy in the \( z \)-direction. The last term is the dipole-dipole weak but long-ranged interaction; here \( \mu_i \) is defined as \( \mu_i S_i \) where \( \mu_B \) is taken as \( \mu_B^2 = 0.1 \). We have chosen \( B \) in the range \([-0.65, 0.65]\). At the two extreme values all the spins are aligned along the direction of \( B \).

Barkhausen Noise. – The model has an obvious enormous parameter space with very many interesting effects that are beyond the scope of this letter. Here we explore parameters that result in a labyrinthine pattern of “up” and “down” spins. In fig. 2 upper panel we show a snapshot of the magnetic domains of the present model when the external magnetic field is zero. We reiterate that the parameters were chosen such that the competition between the dipole-dipole interaction and the perpendicular anisotropy result in all the spins pointing either “up” or “down” in the \( z \)-direction. Having this structure with \( B = 0 \) we next switch on a magnetic field in the \( z \)-direction which we ramp up in small steps (quasi-statically), applying conjugate gradient energy minimization after each such step to bring the system back to mechanical and magnetic equilibrium. The effect of the increasing magnetic field is exemplified by the middle and lower panels of fig. 2; we observe the creation of new domains and the coarsening of the existing domains of “up” spins at the expense of “down” spins. The coarsening occurs by a movement of the domain boundary.

The creation of new domains and the movement of the domain boundary occurs in jerks (sometimes referred to as “avalanches”) such that a number of spins \( s \) flip from “down” to “up” when \( B \) is increasing, and later, after saturation, when all the spins are pointing “up” the opposite changes occur when the magnetic field is decreased to the point of being negative. The flip of \( s \) spins is equivalent to a change in magnetization \( \Delta m \equiv s / N_A \). A typical hysteresis loop exhibiting the sharp changes in the magnetization \( M \) is shown in fig. 1. The magnetization curve has smooth sections punctuated by discontinuities whose size and distribution will be the focus of this letter.

Statistics of the Barkhausen Noise. – There exists a large body of literature that expects the statistics of Barkhausen Noise, as well of many other serrated responses, to be modeled by a power law multiplied by a cutoff function, i.e.

\[
P(\Delta m) = \Delta m^{-\alpha} f(\Delta m), \quad (5)
\]

where \( f(x) \) is falling off rapidly for large values of \( x \). Accordingly, in fig. 3 we present the measured data from a system of 2100 particles for \( P(\Delta m) \). The data was collected in logarithmic bins and plotted accordingly as \( \log_{10} P(\Delta m) \) as a function of \( \log_{10} \Delta m \). The reader can convince herself that the plot appears to agree with the expected form of eq. (5) with \( \alpha \approx -1 \). In the rest of this letter we will show that this is in fact incorrect, and that in the present example there is no power law whatsoever, notwithstanding the apparent scaling presented in fig. 3.

To understand what is the actual statistics in the present model we need to think what is happening when the magnetic field is ramped up or down. Indeed, the magnetization is changed due to flips of some number of spins from “down” to “up” when the magnetic field is

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\(^1\)The reader should note that the anisotropy term could be chosen in the form \( \bar{n} \cdot S \), rather than being restricted to the \( x-y \) plane. This will result in a richer model that we plan to study in the future.
increased or from “up” to “down” in the opposite case. Our model here is not close to any apparent criticality, so we should expect that there exist an average number of spins \( \langle s \rangle \) that flip in a typical avalanche, and that this average number does not increase like the number of particles \( N \) when the latter is increased. In fact, we have measured this average as a function of the system size, cf. fig. 4, where it becomes clear that \( \langle s \rangle \) tends to a system-size–independent value when \( N \to \infty \). Having a size-independent average of the number of spins involved in an avalanche furnishes a strong constraint on the statistics \( P(s) \), together with the normalization condition

\[
\sum_{s = s_{\text{min}}}^{s_{\text{max}}} P(s) = 1.
\]  

The average number of flipping spins is fixed by

\[
\sum_{s = s_{\text{min}}}^{s_{\text{max}}} s P(s) = \langle s \rangle.
\]

In the light of the system-size–independent constraint on the statistics, we employ the principle of maximum entropy [19] to find the actual distribution \( P(s) \). We should maximize the information entropy

\[
S = - \sum_{s = s_{\text{min}}}^{s_{\text{max}}} P(s) \ln P(s),
\]

subject to the constraints defined by eq. (6) and eq. (7). The standard method of Lagrange multipliers [19] is employed with the final result

\[
P(s) = \frac{e^{-(s - s_{\text{min}})/\langle s \rangle}}{\langle s \rangle}.
\]

Note that if the reasoning leading to eq. (9) is accepted, we have no free parameter when comparing this prediction to our data, since in the present case \( s_{\text{min}} = 1 \) and \( \langle s \rangle \) is known for every system size \( N \).

The fundamental reason why the dipole-dipole long-range term does not bring us any closer to a mean-field theory of the Barkhausen Noise is that in 2 dimensions it falls off too rapidly. In principle it could happen in this model that the slow decay of the stress fluctuations could influence also the rate of decay of magnetic correlations. To demonstrate quantitatively that this does not happen we employ the Green’s function for this system and its relation to the Hessian matrix [20]. In the present case \( \mathcal{H} \) takes on the form [13]:

\[
\mathcal{H} = \left( \begin{array}{cc}
\frac{\partial^2 U}{\partial \mathbf{r}_i \partial \mathbf{r}_j} & \frac{\partial^2 U}{\partial \mathbf{r}_i \partial \mathbf{S}_j} \\
\frac{\partial^2 U}{\partial \mathbf{S}_i \partial \mathbf{r}_j} & \frac{\partial^2 U}{\partial \mathbf{S}_i \partial \mathbf{S}_j}
\end{array} \right).
\]
Since the Hessian is a real symmetric matrix it is diagonalizable, and as long as the system is stable all the eigenvalues are real positive. Denote the eigenfunctions and eigenvalues of \( \mathcal{H} \) as \( \Psi_i(x) \) and \( \lambda_i \), where \( x \) is taken collectively to denote \( \{r_i\}, \{S_i\} \).

\[
\mathcal{H}(\Psi_i(x) = \lambda_i \Psi_i(x).
\] (11)

According to the Fredholm theory, the Green’s function for this system is then given by

\[
G(x, x') \equiv \sum_i \frac{\Psi_i(x) \Psi_i(x')}{\lambda_i}.
\] (12)

In other words, the Green’s function is determined by the inverse Hessian \( \mathcal{H}^{-1} \). We can therefore check now, using the inverse Hessian, what is the range of interaction of the positional vs. the spin degrees of freedom. While we expect the Green’s function projection on the positional degrees of freedom to decay slowly due to elasticity, we do not know \textit{a priori} what is the decay rate of the projection onto the spin degrees of freedom. A direct calculation shows that as a function of \( r_{ij} \) the positional entries of the inverse Hessian decay slowly, as a power law with an asymptotic \( 1/r^2 \) law. In contrast the spin entries decay more rapidly, like \( 1/r^3 \), cf. fig. 6. This power law is integrable in 2 dimensions, explaining the non-existence of a divergence of \( \langle s \rangle \) when \( N \to \infty \). On the other hand in 3 dimensions \( 1/r^3 \) is not integrable and one may expect that for thin films there would be a crossover from 2-dimensional to 3-dimensional statistics of the Barkhausen Noise. Such effects are under present study.

Summary and conclusions. – The present letter indicates a couple of important conclusions for the large community that is interested in serrated responses. Firstly, Barkhausen Noise and other similar phenomena can appear with statistics that vary enormously depending on the underlying microscopic dynamics, on the dimensionality of the samples and on the scale of magnetic jumps under study. This conclusion was also drawn on the basis of experimental measurements, see for example [21–23]. These two papers found very different power laws for the Barkhausen noise although the domain structure was similar. The likely answer is that there is no power law and that avalanches are bound by a characteristic size, as shown in the present paper. The authors of ref. [23] reached a similar conclusion based on a phase field model \( e.g. \) avalanches are bound by intrinsic length, not related to the system size, and power law fits can give widely fluctuating outcomes. To reiterate again that apparent power laws can be misleading we present in fig. 7 the data shown before in fig. 3 and the lower panel of fig. 5. Here we present both the apparent scaling law and the exponential form. The exponential form comes from a theoretical argument without any fitting parameter. Note that in the present example we considered the cleanest possible case with temperature \( T = 0 \) and without any mechanical strains or stresses, and yet the expected behavior eq. (5) did not materialize itself. Secondly, we conclude that the richness of behaviors that begins to unfold itself with different microscopic models underlines the usefulness of such models — their simulation is straightforward, the quality of the data is excellent and in general it is relatively easy to understand what is the nature of the serrated response under study. We thus plan to continue along the lines presented here and study further universality classes of serrated responses with the same care and precision.

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