General solutions of state machine replication have to ensure that all replicas apply the same commands in the same order, even in the presence of failures. Such strict ordering incurs high synchronization costs caused by distributed consensus or by the use of a leader.

This paper presents a protocol for linearizable state machine replication of conflict-free replicated data types (CRDTs) that neither requires consensus nor a leader. By leveraging the properties of state-based CRDTs—in particular the monotonic growth of a join semilattice—synchronization overhead is greatly reduced. In addition, updates just need a single round trip and modify the state ‘in-place’ without the need for a log. Furthermore, the message size overhead for coordination consists of a single counter per message. While reads in the presence of concurrent updates are not wait-free without a coordinator, we show that more than 97% of reads can be handled in one or two round trips under highly concurrent accesses.

Our protocol achieves high throughput without auxiliary processes like command log management or leader election. It is well suited for all practical scenarios that need linearizable access on CRDT data on a fine-granular scale.

1 INTRODUCTION

The implementation of a replicated state machine (RSM) is a well-established approach for designing fault-tolerant services. In its common form, clients submit update commands that modify the state of the replicated object, or read commands returning (part of) its state back to the client. To guarantee linearizable access to an RSM, all replicas must apply the same commands in the same order. This is commonly achieved by using a consensus protocol like Paxos [17, 18], Raft [25], or variations thereof [14, 20, 24]. However, the use of consensus often incurs significant synchronization overhead. In particular, most approaches require the use of a central coordinator (leader) to achieve acceptable performance and require to maintain a command log, which must be regularly truncated to prevent unbounded memory consumption. This often makes the correct implementation of RSMs a challenging task [8].

A wealth of previous work exists that aims to reduce the cost associated with fault-tolerant replication. Some approaches reduce synchronization by leveraging the commutativity of some submitted commands by solving generalized consensus [19]. Other approaches avoid the cost associated with consensus by using a weaker consistency model such as strong eventual consistency (SEC). SEC was formalized by Shapiro et al. [31] with the introduction of conflict-free replicated data types (CRDTs). CRDTs are data structures whose mathematical properties ensure the convergence of all replicas as long as all updates are propagated to them in arbitrary order. They do not require protocol-level conflict resolution mechanisms, as conflicting updates can be resolved computationally. This allows the conflict-free execution of both reads and updates in relaxed consistency models like SEC. Data structures that can be implemented as a CRDT include counters, sets, and certain types of graphs [31]. Due to their low synchronization costs, numerous practical systems have employed CRDTs to this date, such as Redis [9], Riak [6], SoundCloud [5], and Akka [16]. However, their usage is restricted to cases where relaxed consistency models suffice. For example, this prevents their use to implement atomic counters, which are a ubiquitous primitive in distributed computing.

This paper introduces a protocol to implement a special class of replicated state machines that allows linearizable access on CRDTs. These RSMs support update operations that modify the state and read
operations that return a value but do not modify the state. Operations that both modify the state and return a value are not supported.

By leveraging the mathematical properties of CRDTs, our protocol achieves high throughput even in the absence of a leader, thereby eliminating the needs for implementing leader election mechanisms and allowing for continuous availability as long as a majority of replicas is reachable. In addition, our protocol does not replicate a log of commands, which is commonly the case for consensus protocols. Instead, we can replicate the state directly and update it ‘in-place’. This results in a protocol with memory overhead of a single counter per replica, which avoids the complexity associated with command log state and memory management.

Our approach relies on solving generalized lattice agreement (GLA). Similar to CRDTs, values proposed in GLA belong to a join semilattice—a partially ordered set that defines a join (least upper bound) for all element pairs. In contrast, for generalized consensus it is not required that such a join always exists. This difference makes generalized lattice agreement an easier problem to solve. In fact, previous work has shown that wait-free [12] solutions to this problem exist [10], which is proven to be impossible for consensus [11] in an asynchronous system in the presence of process failures. However, the protocol described by Faliero et al. [10] requires sending an ever-increasing set of proposed values in its messages. In contrast, the message size of our approach is bounded by the state of the CRDT but is not wait-free in all cases.

The main contributions of this paper are as follows:

- We present a protocol that provides linearizable state machine replication of state-based CRDTs by solving generalized lattice agreement. The protocol is light-weight as it does not rely on auxiliary processes for leader election or state management of any kind.
- The protocol processes updates in a single round-trip. Reads are not wait-free in the presence of concurrent updates. However, as we show in an experimental evaluation, more than 97% of reads can be processed in one or two round-trips.
- We compare the performance of our protocol with open-source implementations of Paxos and Raft, two well-known approaches for implementing linearizable RSMs.

2 PRELIMINARIES

In this section, we briefly discuss the assumed system model and give an introduction to CRDTs.

2.1 System Model

We consider a distributed system of \( N \) independent and asynchronous processes \( \Pi = \{p_1, p_2, \ldots, p_N\} \), which communicate by message passing. Processes can fail under the crash-recovery model, i.e., each process can crash and recover indefinitely often without losing its internal state upon recovery. Byzantine failures are not handled and out of the scope of this paper.

We assume over \( \Pi \) a fixed quorum system \( QS \) [36], i.e., a set of sets of processes with mutual overlap:

\[
\forall Q \in QS : Q \subseteq \Pi, \quad \forall Q_1, Q_2 \in QS : Q_1 \cap Q_2 \neq \emptyset
\]

Elements in \( QS \) are called quorums. A necessary condition for progress is that at least a quorum of processes does not crash and is able to pairwise exchange messages for a sufficiently long time. Message transfer is unreliable. Messages can arrive out of order, can be delayed arbitrarily, or can be lost.

2.2 State-Based Conflict-Free Replicated Data Types

Eventual consistency promises better performance and availability in large scale systems in which the coordination required for linearizable approaches is not feasible [35]. Updates are applied at some replica and at a later time propagated across the system. Eventually, all replicas receive all updates, possibly in different orders. However, concurrent updates may create conflicts. Resolving them often requires roll-backs and consensus decisions.

The use of conflict-free replicated data types (CRDTs) [31], introduced as part of the strong eventual consistency model, eliminates the need for roll-backs or consensus by leveraging mathematical properties
We consider a state-based CRDT (Algorithm 1).

**Algorithm 1** State-based G-counter with n replicas as (non-linearizable) CRDT.

1. \( S := \mathbb{N}^n, \sqcup := \text{compare}, \sqcap := \text{merge}, \)
2. \( Q := \{\text{query}\}, \ U := \{\text{update}\} \)
3. \( \text{compare} (x \in S, y \in S) \rightarrow \text{boolean} \)
4. return \( \wedge_{i=1}^n x[i] \leq y[i] \)
5. \( \text{merge} (x \in S, y \in S) \rightarrow S \)
6. \( z[i] \leftarrow \max(x[i], y[i]) \); return \( z \)

result to the client, whereas queries do not modify the CRDT’s state but return its value as result. As a

Next, we discuss how to leverage the properties of state-based CRDTs to provide fast, linearizable access.

3 LINEARIZABLE AND LOGLESS RSM FOR STATE-BASED CRDTs

Next, we discuss how to leverage the properties of state-based CRDTs to provide fast, linearizable access.

3.1 Problem Statement

We consider a state-based CRDT \((S, Q, U)\) replicated on \(N\) processes. Each process starts with an initial state \(s_0 \in S\). Clients can submit \(u \in U\) or query \(q \in Q\) commands to any process and each process may receive an arbitrary number of commands. Updates modify the state of the CRDT without returning a result to the client, whereas queries do not modify the CRDT’s state but return its value as result. As a

preventing the emergence of conflicts. **Operation-based** CRDTs require the commutativity of all its update operations, whereas **state-based** CRDTs rely on monotonicity in a join semilattice [30]. Both types have advantages and disadvantages. In general, operation-based CRDTs have lower bandwidth needs but require reliable, i.e., exactly once, and causally ordered delivery of updates [30]. As our system model assumes unreliable communication, we only focus on **state-based CRDTs** in this paper. However, both types of CRDTs can emulate each other [30].

State-based CRDTs are based on the concept of join semilattices:

**Definition 1 (Join Semilattice).** A join semilattice \( S \) is a set \( S \) equipped with a partial order \( \sqsubseteq \) and a least upper bound (LUB) \( \sqcup \) for all pairs of elements \( x, y \in S \).

The LUB of two elements \( x, y \in S \) is the smallest element in \( S \) that is equal or larger than both \( x \) and \( y \).

**Definition 2 (Least Upper Bound).** \( m = x \sqcup y \) is a LUB of \( \{x, y\} \) under partial order \( \sqsubseteq \) iff:

\[
\forall m' \in S, \ x \sqsubseteq m' \land y \sqsubseteq m' \implies x \sqsubseteq m \land y \sqsubseteq m \land m \sqsubseteq m' \\]

From this definition it follows that \( \sqcup \) is idempotent \((x \sqcup x = x)\), commutative \((x \sqcup y = y \sqcup x)\), and associative \(( (x \sqcup y) \sqcup z = x \sqcup (y \sqcup z) ) \).

**Definition 3 (State-Based CRDT).** A state-based CRDT consists of a tuple \((S, Q, U)\), where \( S \) is a join semilattice defining the possible payload states, \( Q \) is a set of query functions, and \( U \) is a set of monotonically non-decreasing update functions, i.e., \( \forall u \in U, \ s \in S : s \sqsubseteq u(s) \).

Two payload states \( s_1, s_2 \in S \) are **equivalent** \((s_1 \equiv s_2)\) if all queries return the same result for both, i.e., \( s_1 \sqsubseteq s_2 \land s_2 \sqsubseteq s_1 \implies s_1 \equiv s_2 \). They are **comparable** if they can be ordered, i.e., \( s_1 \sqsubseteq s_2 \lor s_2 \sqsubseteq s_1 \).

**Example.** One of the most simple state-based CRDTs is a monotonically increasing counter, called G-counter (grow-only counter). Its state-based definition is shown in Algorithm 1. The payload state of such a counter, replicated on \(n\) processes, consists of an array of length \(n\). All replicas, which are assumed to be distinguishable by an ID, manage their own local copy of the counter’s state. Locally incrementing the counter increments the array element corresponding to the ID of the respective replica. The **merge** and **compare** functions implement \( \sqcup \) and \( \sqcap \).

In a system that provides **SEC**, a replica that receives an increment command from a client increments its counter by calling **update**. It periodically propagates its counter state \(g\) to the other replicas. Any replica that receives such a counter state updates its own counter state using the **merge** function. As all replicas only increment their own slot, no updates are lost and eventually all replicas converge to the same state.

3 LINEARIZABLE AND LOGLESS RSM FOR STATE-BASED CRDTs

Next, we discuss how to leverage the properties of state-based CRDTs to provide fast, linearizable access.

3.1 Problem Statement

We consider a state-based CRDT \((S, Q, U)\) replicated on \(N\) processes. Each process starts with an initial state \(s_0 \in S\). Clients can submit update \(u \in U\) or query \(q \in Q\) commands to any process and each process may receive an arbitrary number of commands. Updates modify the state of the CRDT without returning a result to the client, whereas queries do not modify the CRDT’s state but return its value as result. As a
prerequisite to achieve linearity for a query \( q \) that is submitted to a process \( p \), \( p \) must first learn a state \( s \in S \) by exchanging messages with the other processes before returning \( q(s) \) to the client. We say that \( s \) is the state learned by query \( q \) at process \( p \).

All learned values must satisfy the following conditions:

**Validity** Any learned state is equivalent to some set of submitted update functions applied on \( s_0 \).

**Stability** For any two states \( s_1, s_2 \in S \) learned by subsequent queries \( q_1 \) and \( q_2 \in Q \) at any two processes:

\[
 s_1 \sqsubseteq s_2.
\]

**Consistency** Any two states \( s_1 \) and \( s_2 \) learned at any two processes are comparable.

These conditions are based on Generalized Lattice Agreement (GLA) [10], which is similar to our problem. However, our protocol processes update and query commands, whereas processes in GLA receive and return values of a semilattice. Note, that our Stability semantics are slightly different. In Sect. 3.4 we discuss the changes needed to provide GLA’s semantics, which are more closely related to generalized consensus.

The conditions stated above define the behavior of submitted query commands. However, the behavior of updates must also be considered providing linearize access [10]. We say that a state \( s \) includes update \( u \), if \( s = u(s') \) or \( s = u'(s') \) where \( s' \) includes \( u \), or \( s = s_1 \sqcup s_2 \) with \( s_1 \) and/or \( s_2 \) being a state that includes \( u \).

**Update Stability** If the execution of update \( u_1 \) completes before update \( u_2 \) is submitted, then every learned state that includes \( u_1 \) also includes \( u_2 \).

**Update Visibility** If the execution of update \( u \) completes before query \( q \) is submitted, then the state learned of the query \( q \) includes \( u \).

### 3.2 The Protocol

The success path of the protocol is depicted in Algorithm 2. We consider two roles that processes can assume: proposer and acceptor. Roughly speaking, proposers process incoming requests from clients and acceptors act as the replicated storage of the CRDT. For simplicity, we assume that all processes implement both the acceptor and proposer role.

**Conventions.** To keep the presented code brief, we follow several conventions. First, we assume that messages are tuples with a tag and an arbitrary number of elements and are denoted as \( \langle TAG, e_0, \ldots, e_n \rangle \). Processes wait until they have received enough messages with a specific tag before executing its corresponding action. If an action requires messages from a set of processes, we aggregate the received messages element-wise into multisets. For example, two messages \( \langle TAG, a_0, b_0 \rangle \), \( \langle TAG, a_1, b_1 \rangle \) would be aggregated into the message \( \langle TAG, \tilde{A} = \{a_0, a_1\}, \tilde{B} = \{b_0, b_1\} \rangle \) on the receiver side. At any time, each process executes at most one action (serial processes).

The second concept we use are rounds. Ranges are pairs of a round number and a round ID. Round \( r \) is denoted as \( r = (number, ID) \), with \( r_{nr} \) and \( r_{id} \) providing access to its number and ID, respectively. Round numbers are used to order concurrent requests, and round IDs guarantee that the round of each request is unique. A common way to generate unique round IDs is that each process appends its process ID to a local counter, which is incremented for each new round. Rounds can be totally ordered by first comparing their round numbers and then their IDs.

We furthermore assume that proposers implement a mechanism to keep track of ongoing requests and can differentiate to which request an incoming message belongs to. In practice, this can be achieved by generating a request ID for each request and including it in each message it produces.

**Internal State.** Each acceptor holds as its internal state the current payload state \( s \) of the CRDT and the highest round \( r \) it has observed so far. In the beginning, each acceptor’s state is initialized with some initial payload state \( s_0 \) and some round with round number 0 and an ID that cannot be generated by proposers.

Proposers only have to temporarily store information of ongoing requests and still unprocessed messages (as they have to wait for replies from a quorum). No further state is required.

**Update Commands.** Update commands are processed in a single round trip. They do not require any synchronization. If a proposer receives an update command, which includes update function \( f_u \in U \), it
Algorithm 2 Linearizable state machine replication of state-based CRDTs

| Proposer: | Acceptor: |
|-----------|-----------|
| **Update Commands** | **Update Commands** |
| 1. on receive ⟨UPDATE, f_u⟩ from client c: | 25. on initialise: |
| 2. store c | 26. r ← (0, ⊥) |
| 3. s ← apply_update(f_u) | 27. s ← s₀ |
| 4. send ⟨MERGE, s⟩ to all remote acceptors | | |
| 5. on receive ⟨MERGED⟩ from some quorum: | function apply_update(f_u): |
| 6. send ⟨UPDATE_DONE⟩ to c | 29. s ← f_u(s) |
| | 30. r_id ← write |
| | 31. return s |
| | | |
| | 32. on receive ⟨MERGE, s’⟩ from proposer p: |
| | 33. s ← s ⊔ s’ |
| | 34. r_id ← write |
| | 35. send ⟨MERGED⟩ to p |
| | | |
| | 36. on receive ⟨PREPARE, r’, s’⟩ from proposer p: |
| | 37. s ← s ⊔ s’ |
| | 38. if r_nr = ⊥ then |
| | 39. r’ ← (r_nr + 1, r_id) |
| | 40. if r_nr > r_nr then |
| | 41. r ← r’ |
| | 42. send ⟨ACK, r, s⟩ |
| | | |
| | 43. on receive ⟨VOTE, r’, s’⟩ from proposer p: |
| | 44. s ← s ⊔ s’ |
| | 45. if r’ = r then |
| | 46. r ← r’ |
| | 47. send ⟨VOTED, s’⟩ |
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acceptors reply with NACK messages (not shown for brevity) so that the proposer can retry its request. An incremental prepare is always accepted, as the local round number of the acceptor increases (line 39).

The prepare is successful if a quorum has replied with ACK messages (line 11). Depending on the replies, $p$ can either (a) immediately learn a state, (b) propose a state to learn, or (c) retry the prepare phase.

(a) If all acceptors of the quorum replied with the same payload state, then this state can be considered to be learned by $p$. Thus, the second phase can be skipped, $p$ can apply $f_Q$ on the learned state, and send the result to the client. We refer to such state as learned by consistent quorum (lines 13–15). The second phase can be skipped here as $p$ is already certain of a payload state that is established in a quorum.

(b) If a quorum of acceptors replied with the same round, the first phase was successful. In the second phase, the proposer can propose a payload state to learn, which is the LUB of all received acceptors’ payloads. This state is sent with the first phase’s round in VOTE messages to all acceptors (lines 16–17).

(c) If neither payload states nor rounds are consistent, the first phase has failed. In this case, the proposer used an incremental prepare. It can then retry with a fixed prepare by choosing a round number that is larger than all seen round numbers (lines 19–21).

Each acceptor that received a $\langle$VOTE, $s'$, $r'$ $\rangle$ message has to decide whether the proposal is valid. This is the case when the acceptor has received $p$’s PREPARE message and its state was not modified by a concurrent update or query in the meantime (line 45). If the proposal is valid, then the acceptor replies with a VOTED message. Otherwise, it denies the proposal by optionally sending a NACK so that $p$ can retry (not shown). If $p$ receives a quorum of VOTED messages, then its proposed state is learned. We refer to this as a state learned by vote. Then, $p$ can apply the received query and include the result in a message to the client.

**Retrying Requests.** On concurrent query requests, acceptors may deny them by sending NACKs to the respective proposer. It helps to include the current payload state of the acceptor in this message. Any proposer that received a NACK before receiving a quorum of ACK or VOTED messages must retry its request. It can compute the LUB of all received payloads as the state to include in its next PREPARE messages. By always retrying with an incremental prepare, eventual liveness (see Sect. 3.5) can be guaranteed. However, retrying with a fixed prepare also does not violate any correctness condition of Sect. 3.1.

### 3.3 Proof of Safety

In the following, we proof the conditions outlined in Sect. 3.1. The query protocol begins by either an incremental or fixed prepare. The following invariants hold for them, according to the code in Algorithm 2:

- **I1** If a proposer learns some state, then it has received ACK’s from a quorum (line 14, line 23 via line 17).
- **I2** Any learned state is the LUB of all payload states received in ACK messages from a quorum (line 12).
- **I3** If a proposer sends a VOTE message, then it has received the same round in ACK messages from a quorum (lines 16–17).
- **I4** If a proposer has received an ACK message from an acceptor (line 11), then this acceptor has increased its stored round number due to the proposer’s PREPARE message (line 41).

**Theorem 3.1 (Validity).** Any learned state is equivalent to some set of submitted update functions applied on $s_0$.

**Proof.** All acceptors start with payload $s_0$ (line 27). Payload modifications only happen by either application of a received update function or by LUB computation. Computing the LUB of two payload states merges the update sets included in them (see [31]). Thus, no other state than that of a set of submitted updates or $s_0$ can be learned.

**Lemma 3.2.** The payload state of each acceptor increases monotonically.

**Proof.** Both LUB and update functions are monotonically increasing.

**Corollary 3.3.** If messages $\langle$ACK, $r$, $s$ $\rangle$ and $\langle$ACK, $r'$, $s'$ $\rangle$ are send by the same acceptor in this order, then $s \subseteq s'$. This follows without proof directly from Lemma 3.2.
LEMMA 3.4. If state $s$ is learned by any proposer, then there exists a quorum $Q$ with $s \subseteq a.s$, $\forall a \in Q$, where $a.s$ designates the local state variable $s$ of a particular acceptor process $a$.

Proof. State $s$ can be learned (i) by consistent quorum from messages of a quorum $Q_{cons}$ (line 14) or (ii) by vote from messages of a quorum $Q_{vote}$ (line 23).

(i) Trivial, as all acceptors in $Q_{cons}$ have included a state $s' \equiv s$ in their ACK message (lines 37 and 42).

(ii) $p$ sent $s$ in VOTE messages. At least all acceptors in $Q_{vote}$ must have received the message and have merged their payload state with $s$ by LUB computation (line 44) before replying with VOTED (line 47).

THEOREM 3.5 (Stability). For any two states $s_1$ and $s_2$ learned by subsequent queries $q_1$ and $q_2$ at any two processes: $s_1 \subseteq s_2$.

Proof. From Lemma 3.4 it follows that once a proposer $p$ has received the QUERY message of $q_2$, there exists a quorum $Q$ such that $s_1 \subseteq a.s$, $\forall a \in Q$. To learn a state, $p$ eventually receives ACK messages from quorum $Q'$. As $Q \cap Q' \neq \emptyset$, there exists some $a' \in Q'$ with $s_1 \subseteq a'.s$. The state learned by $p$ is the LUB of all received states included in the ACK messages. Thus, $s_1 \subseteq a'.s \subseteq s_2$.

LEMMA 3.6. Two learned states $s_1$ and $s_2$ are comparable if at least one state is learned by consistent quorum.

Proof. (By contradiction) Let $s_1$ and $s_2$ be learned due to queries handled at proposer $p_1$ and $p_2$, respectively. $p_1$ and $p_2$ have received ACKs from quorums $Q_1$ and $Q_2$, respectively. Assume $s_1$ is learned by consistent quorum and $s_2$ is not comparable to $s_2$. In this case, the following conditions must hold:

C1 $\forall a \in Q_1 \cap Q_2$: $a$ must send an ACK to $p_2$ with state $s : (s \subseteq s_1) \land \neg(s \equiv s_1)$, otherwise $s_1 \subseteq s_2$. This implies that $a$ receives $p_2$'s PREPARE message before $p_1$'s.

C2 $\forall a \in Q_1$: $a$ must receive a PREPARE message from $p_1$ before receiving VOTE from $p_2$ (otherwise $s_2 \subseteq s_1$). $s_2$ cannot be learned by consistent quorum, as this would imply $s_2 \equiv s \subseteq s_1$ (C1 and Corollary 3.3). Thus, to learn $s_2$, $p_2$ must receive VOTED messages from a quorum with at least one acceptor $a$ in $Q_1$. For that, $p_2$ sends a (VOTE, $r$, $s_2$) message to $a$. It follows from C1 and C2 that $a$ has received $p_1$'s PREPARE message in between $p_2$'s PREPARE and VOTE message. Due to invariant I4, $a$ has modified its round and $r \neq a.r$. Therefore, $a$ does not reply with a VOTED message and $s_2$ cannot be learned.

LEMMA 3.7. Two learned states $s_1$ and $s_2$ are comparable if both are learned by vote.

Proof. Let $s_1$ and $s_2$ be learned due to query requests handled at proposer $p_1$ and $p_2$, respectively. $p_1$ has received ACKs from quorum $Q_1$ and $p_2$ from quorum $Q_2$. As $Q_1 \cap Q_2 \neq \emptyset$, there is at least one acceptor $a$ that has sent ACKs to both $s_1$ and $s_2$. Assume $a$ sends an ACK to $p_1$ first. Therefore, $p_1$ sends (VOTE, $r_1$, $s_1$) and $p_2$ sends (VOTE, $r_2$, $s_2$) with $r_1 < r_2$. Let $Q_v$ be the quorum of acceptors that replied to $p_1$ with VOTED messages. All acceptors $a \in Q_v \cap Q_2$ must receive $p_1$'s VOTE before $p_2$'s PREPARE message, as otherwise either $p_2$ receives inconsistent rounds or $a$ does not reply to $p_1$. Therefore, $a$ includes state $s$ with $s_1 \subseteq s$ in its ACK message to $p_2$. As $p_2$ computes the LUB of all states received in ACK messages, $s_1 \subseteq s \subseteq s_2$.

THEOREM 3.8 (Consistency). Any two states $s_1$ and $s_2$ learned at any two processes are comparable.

Proof. Follows from Lemma 3.6 and Lemma 3.7.

THEOREM 3.9 (Update Stability). If the execution of update $u_1$ completes before update $u_2$ is submitted, then every learned state that includes $u_2$ also includes $u_1$.

Proof. (By contradiction) As $u_1$ and $u_2$ are subsequent requests, there is a quorum $Q_u$ that has received MERGE messages with a payload including $u_1$ before any acceptor includes $u_2$. Thus, there cannot be a quorum at any time that includes $u_2$ but not $u_1$.

Assume a proposer $p$ learns state $s$ that includes $u_2$, but not $u_1$. So, there must be a quorum $Q_{ack}$ that has replied to $p$ an ACK message before receiving the MERGE message and at least one acceptor replied with a payload including $u_2$. It follows that $s$ is not learned by consistent quorum. It also follows that all acceptors
in $Q_u$ received the $\text{MERGE}$ before $p$ received all replies from $Q_{\text{ack}}$. To propose a state in $\text{VOTE}$ messages, $p$ must have received the same round $r$ from all acceptors in $Q_{\text{ack}}$. However, $\exists Q : a.r = r, \forall a \in Q$, as $\forall a \in Q_u$ updated their round. Therefore, $p$‘s proposal cannot succeed and $s$ is not learned by vote.  

**Theorem 3.10 (Update Visibility).** If the execution of update $u$ completes before query $q$ is submitted, then the learned state of the query $q$ includes $u$.

**Proof.** Once the execution of update $u$ completes, there exists a quorum of acceptors including $u$. Thus, any subsequent proposer that processes a query, receives at least one $\text{ACK}$ message that includes $u$.  

### 3.4 GLA-Stability

Both generalized lattice agreement and generalized consensus define a slightly different Stability condition as the one stated in Sect. 3.1:

**GLA-Stability** The states learned at the same process increase monotonically.

The protocol as described in section Sect. 3.2 does not satisfy GLA-Stability, as a proposer that processes two concurrent queries may learn a higher state first if messages from acceptors arrive out of order. However, this can be easily solved by letting proposers remember the largest payload state it has ever learned:

Each proposer stores its largest learned state $s_{\text{learned}}$. Every time a proposer learns a new state $s$, it compares $s$ with $s_{\text{learned}}$. The larger state is used as the learned state for the respective query request and $s_{\text{learned}}$ is updated accordingly. As the protocol satisfies Consistency, $s$ is always comparable to $s_{\text{learned}}$. In any case, the returned state is at least as large as $s$. Therefore, Update Stability and Update Visibility are still satisfied.

### 3.5 Liveness

Falerio et al. [10] show that wait-free protocols for solving GLA exist. To achieve this is costly, however, as their approach requires exchanging a growing set of accepted input commands.

In contrast, the protocol for executing queries as presented in Sect. 3.2 is not wait-free. Concurrent proposers can block each other indefinitely without ever learning a state. This problem is reminiscent to Paxos-style protocols for solving consensus in asynchronous systems. In fact, wait-freedom is proven to be impossible [11] for consensus in the presence of process failures. The common solution to this problem is to assume the existence of a leader process. The other proposers forward their commands to the leader, which acts the sole proposer that is allowed to propose commands. However, this design makes the leader process the single point of failure. If the leader crashes, the system becomes unavailable until a new leader is elected. However, leader election also requires consensus and is thereby not live. Furthermore, the system’s performance is limited to the throughput of the leader. While a leader deployment is possible for our approach, we show in our evaluation in Sect. 4 that our protocol is able to terminate within one or two round-trips for a high proportion of requests even during highly concurrent access and without elected leader. Furthermore, our approach satisfies a weaker liveness condition:

**Eventual Liveness(p)** If a finite number of updates are submitted and proposer $p$ receives a query, then $p$ will eventually learn some state.

The requirement holds under the assumption that $p$ and any quorum does not crash and is able to exchange messages with each other that eventually arrive at their destination. Under these conditions, eventual liveness is fulfilled by our protocol if an incremental prepare is used for retrying failed queries:

As there are a finite number of updates, there is a point in time in which the $apply\_update$ function is called for the last time, i.e., no new updates are included in any acceptor. Any proposer that is executing a query after this point will execute incremental prepares (possibly interleaved with fixed prepares) until it learned a state. Each time an incremental prepare is executed, the proposer will either learn a state by consistent quorum or receive at least one reply with a different payload. Does the request fail, the proposer retries with the LUB of all received payloads from the previous iteration. In each unsuccessful iteration, the
updates of at least one additional acceptor are included in the LUB. As there is a finite number of acceptors, eventually all acceptors include all updates and the proposer learns a state by consistent quorum.

3.6 Optimizations

The base protocol described in Sect. 3.2 can be optimized in several ways.

**Sending fewer payload states.** To reduce network traffic, the number of payloads that are send by the protocol can be reduced. First, proposers do not have to send $s_0$ in their initial PREPARE messages, as LUB computation with $s_0$ will never increase an acceptor’s payload state. This saves bandwidth if $s_0$ is large and prevents unnecessary LUB computations. Second, acceptors do not need to include any payload state in VOTED messages, as this is the state they received from the proposer. Instead, each proposer can simply remember the payload state it proposes and use it once it receives a quorum of VOTED messages.

**Batching.** Batching is a common strategy to reduce synchronization overhead and bandwidth needs in workloads with high concurrent access by sacrificing some latency. Implementing batching on a per-proposer basis is simple. Each proposer manages a separate update and query batch in which it buffers all commands it has received since the previous batch. To process a batch, the proposer executed the update or query protocol as normal except for applying all buffered commands in arbitrary order. Note, that it is not necessary to send the buffered commands over the network, as all commands are applied locally. This means that the achievable throughput is limited by processing speed of the respective proposer, as the number of messages and the required bandwidth is independent from the batch size.

4 EVALUATION

We implemented our protocol as part of the distributed key-value store Scalaris [29], which is written in Erlang. The implementation’s correctness was tested using a protocol scheduler that enforces random interleavings of incoming messages. For comparison, we use an open-source Erlang implementation of Multi-Paxos [18, 34] and Raft [25, 27]. We configured both approaches to write their respective command logs on a RAM disk to minimize their performance impact. The protocol proposed by Faliero et al. [10] exchanges an ever growing set of accepted input commands between its participants. This set needs to be truncated for this approach to be practical. Unfortunately, such a truncation mechanism is not described. As we found that designing one is a non-trivial task, we deemed that it is out of scope for our evaluation. Thus, the protocol is not included in the evaluation despite its theoretical importance.

All benchmarks were performed on a cluster equipped with two Intel Xeon E5-2670 v3 2.4 GHz per node running Ubuntu 16.04.6 LTS. The nodes are fully connected with 10 Gbit/s. For all measurements, we implemented a replicated counter that is replicated on three nodes using the respective approaches. In our approach, to which we will refer to as CRDT Paxos, we implemented a G-Counter as described in Sect. 2.2. In our implementation, we also applied the optimizations outlined in Sect. 3.6. For Multi-Paxos and Raft, we used a simple replicated integer as the counter. All experiments were executed using Erlang 19.3. Up to three separate nodes were used to generate load using the benchmarking tool Basho Bench [33]. All measurements ran over a duration of 10 minutes with request data aggregation in 1 s intervals. For Figure 1 and Figure 2, we show the median with 99 % confidence intervals (CI). The CI is always within three percent of the reported medians.

4.1 Failure-free Operation

In this experiment, we measured the throughput of the approaches under different loads and increasing number of clients (see Figure 1), which were distributed evenly across three load generators. Each client independently submits requests to one of the three replicas and then waits for a reply before submitting the next request. CRDT Paxos and Multi-Paxos perform better for read heavy workloads as they distinguish between read and update requests. For CRDT Paxos, a decrease in update increases the probability of observing a consistent quorum, which also increases the ability to process requests in a single round trip.

1https://github.com/scalaris-team/scalaris/tree/master/src/crdt (git commit hash used for benchmarks: 8effc6e)
whereas the Multi-Paxos implementation employs leader read leases. The Raft implementation appends both updates and consistent reads to its command log, which results in its consistent performance for all load types. Overall, CRDT Paxos achieves a higher throughput for mixed workloads with a low percentage of updates and less than 1500 clients. This is partly due to its better load distribution across all replicas compared to the leader based designs. For more clients, its performance degrades because of the interference between updates and reads. Note, that the 95th percentile read latency of our approach is slightly higher compared to the other approaches as a small percentage of reads must be retried due to updates conflicts (see Figure 2 and Figure 3). Since updates can always be answered within a single round trip, update latencies are consistently low as long as the nodes and network are not saturated.

The issues of read-updates conflicts can be resolved by applying a simple batching scheme (see Sect. 3.6). Even though no leader is used, conflicts are greatly reduced, when using 5 ms batches, and more than 97% of reads can be processed within two round trips. Similarly, read- or update-only workloads are conflict-free, increasing the performance of CRDT Paxos by one order of magnitude compared to mixed update-heavy workloads.

4.2 Node Failure

One drawback to leader-based approaches is their brief unavailability during leader failure and the added complexity of implementing a leader election algorithm. As our approach does not require a leader, continuous availability can be achieved as long as a quorum of replicas is reachable. Figure 4 shows the impact of a node failure on the 95th percentile latency for 64 clients and 10% updates. Latencies increase slightly for the base protocol without batching as all the remaining replicas must be consistent to reach a consistent quorum. This increases the likelihood of updates interference.

5 RELATED WORK

As previously mentioned, a wealth of consensus protocols were invented with the advent of the state machine approach [28], most notably Paxos [17, 18], Raft [25] and variations of them [14, 20, 24]. To partially alleviate the high synchronization costs incurred by consensus, numerous protocol were designed to exploit commutative operations [19, 24, 32]. In contrast to these generalized consensus protocols, which allow any pair of commands to commute with each other or not, our approach solves generalized lattice agreement [10] by requiring that all update commands commute with each other. This restriction simplifies the problem so that a high number of concurrent clients can be supported without the need for leader or central coordinator. In contrast, solving (generalized) consensus often relies on efficient leader election [1, 21, 25] or multi-leader
approaches [7, 22] to alleviate the leader performance bottleneck and impact on the system’s availability during a leader failure.

Starting with the original formalization of CRDTs [31], numerous works discuss the design and composition of these data structures [3, 4, 23, 26, 30]. Normal usage of state-based CRDTs require the transmission of the complete state while dispersing updates to remote replicas. This becomes costly when CRDTs grow larger. A solution to this problem is discussed by Almeida et al. [2] by only transmitting state-deltas instead of the complete data structure. In addition, certain CRDT designs suffer from state inflation, e.g., due to accumulation of tombstone values. Garbage collection mechanisms are discussed in [30]. Further research is needed to show how these ideas can be incorporated into our protocol.

Several protocols that solve generalized lattice agreement exist. Falerio et al. [10] discusses a protocol in which a value is always learned in $O(N)$ messages delays, where $N$ is the number of proposers. Recent work published on arXiv [37] improves this upper bound to $O(\log f)$, with $f$ being the maximum number of crash failures and also addresses the problem of truncating the internally managed command sets. Imbs et al. [15] solves lattice agreement by introducing a Set-Constrained Delivery (SCD) broadcast primitive, which is build on top of FIFO broadcast. SCD broadcasting a message requires $O(N^2)$ messages.

6 CONCLUSION

In this paper, we presented a protocol that provides linearizable state machine replication for state-based CRDTs. The protocol guarantees that updates always terminate in a single round trip. Even though wait-freedom is not provided for read commands in the presence of concurrent updates, our experimental evaluation showed that high throughput can be sustained even under highly concurrent access and without the typical leader-based deployment commonly used for consensus-related problems. In addition, our protocol is lightweight and requires no growing log as it has the memory and message size overhead of a single counter in addition to the replicated data. Thereby, no auxiliary processes for leader election or state management are required for a practical deployment of our approach. This contrasts our design to the original solution of generalized lattice agreement [10], which is wait-free but requires additional effort to truncate the managed state or message sizes.

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