Development of Control Method For An Optimal Quantum Receiver

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Abstract. We propose a method for optimal displacement controlling of an optimal quantum receiver for registrations a binary coherent signal. An optimal receiver is able to distinguish between two phase-modulated states of a coherent signal. The optimal receiver controlling method can be used later in practice in various physical implementations of the optimal receiver.

1. Introduction

With the rapid development of fiber-optic technology and single-photon detectors, we have possibility to create optimal quantum receiver, which can detect weak coherent signals with an error below the standard quantum limit [1]. We have developed an controlling method of an optimal quantum receiver for detecting coherent signals with discrimination error close to the Helstrom bound [2, 3]. This method can be used for further development of control methods that depend on the specific technical implementations of the optimal quantum receiver [4, 5]. In addition, a fiber-optic quantum receiver based on the Kennedy scheme at a telecommunication wavelength of 1.55 μm was experimentally demonstrated [6, 7]. However, fiber-optic receiver requires excellent thermo-optical stabilization [8].

2. Optimal receiver control

We assumed that the coherent signal has two states \(|\alpha\rangle\) and \(|-\alpha\rangle\). Where \(\alpha^2\) is mean photon number of the coherent signal. In order to distinguish two signal states, it is necessary to displace signal by local oscillator (LO) with state \(\Delta\) on a 99/1 beam splitter (see Figure 1) and then to measure by a single-photon detector with photon number resolving [9]. We can change intensity and phase of LO. We transform the phase-modulated signal into amplitude-modulated by mixing coherent signal with LO. The initial states of the signal is transformed to \(|-\alpha + \Delta\rangle\) and \(|\alpha + \Delta\rangle\). Where \(\Delta\) is the optical displacement. After a beam splitter the coherent signal interferes with LO. We can distinguish two states with an error below the standard quantum limit by measuring the beam intensity using a single-photon detector and analyzing the data. If the displacement is \(\Delta = \alpha\), then one state is completely nulling and the optimal receiver is transformed into Kennedy receiver [6, 10]. However, such a coherent signal detection mode is not the most effective than optimal receiver. For quantum optimal receiver we need to displace useful signal by Local Oscillator with the state \(\Delta = |\alpha + \beta\rangle\) on the beam splitter. Where \(\beta\) is an additional displacement. At the output of the beam splitter, a phase-modulated signal with the states \(|\beta|2\alpha + \beta\rangle\) is obtained.
The statistics of the photo counts of a signal is Poisson and we must know its:

\[ P(n, 2\alpha + \beta) = |2\alpha + \beta|^{2n} e^{-|2\alpha + \beta|^2} / n! \]

and

\[ P(n, \beta) = |\beta|^{2n} e^{-|\beta|^2} / n! \]

The duration of the received signal, which is the same for all signals.

Next, you need to use the strategy of distinguishing between the two signals.

We use next rule: if we got \( i \) number of photo counts per signal from a detector and probability \( P(i, 2\alpha + \beta) \gg P(i, \beta) \), that is, we got a logical “1”, else it is logical “0”. Clearly, those Poisson distributions of the photocounts may overlap. Then we have error of distinguishing between the two signals. In the work [11], error can be defined as:

\[ \varepsilon = 1 - \frac{1}{2} \sum_{i} \max[P(n, 2\alpha + \beta), P(n, \beta)] \]  

(1)

We can influence on an error by changing the parameter \( \beta \). It seems that we need to null one of the states by \( \Delta = |\alpha| \) for maximal decreasing error. However, as it was shown in [11] that the optimal displacement signal can be more effective if we do not nulling one of the states. There is effective displacement of a signal \( \Delta = |\alpha + \beta_{opt}| \). If we set optimal displacement \( \beta_{opt} \) for optimal quantum receiver that it can distinguish two states of signal with minimal error close to Helstrom bound. In order to receiver operates with maximal efficiency it is necessary find out \( \beta_{opt} \), and to take into account the imperfection of the optical scheme. We have two parameters, which characterize these imperfections. The first parameter \( \gamma_c \) is constant and is characterized dark counts of a single-photon detector and light background. The second parameter \( \gamma_0 \) is integral parameter and characterizes non-ideality of optical system. In fact, intensity of interference minimum is not zero. We assumed that \( \gamma_0 = k \cdot m_1 \). Where \( k \) – non-ideality coefficient. Then two states of output light we can describe by system of equations:

\[
\begin{align*}
(2\alpha + \beta)^2 + \gamma_0 + \gamma_c &= m_1 \\
\beta^2 + \gamma_0 + \gamma_c &= m_0
\end{align*}
\]  

(2)

Where \( m_1 \) and \( m_0 \) – mean photon numbers for constructive and destructive interferences, which corresponds logical “1” and “0” respectively.

In order to set the optimal displacement on a receiver, it is necessary to measure parameters \( \alpha, \gamma_c, k \) and to define \( \beta_{opt} \) in the system of equations (2). 1) The noise of a detector and background \( \gamma_c \) are measured. For this necessary turn off the signal and LO. 2) To measure power in the received phase-modulated signal \( \langle -\alpha |\alpha\rangle \). To do this, measure mean photon number \( m_s = |\alpha|^2 \) per signal on the output from the beam splitter without LO using a single-photon detector. 3) Non-ideality coefficient
can be determined if power of the LO will be equal power of signal. By changing the phase of the LO, we achieve that extinction, i.e. the ratio $m_1/m_0$ is maximum. In this case, the quantum optimal receiver transforms to Kennedy receiver. Then $k = 1/C_{\text{max}} - \frac{\gamma_c}{4\alpha^2}$. If the detector noise and background are very small $\gamma_c \ll 4\alpha^2$, then the expression takes a simpler form: $k = 1/C_{\text{max}}$. 4) Optimal displacement $\beta_{\text{opt}}$ can be determined when distinguishing error of two states of coherent signal is minimal. We use the formula (1) to calculate the dependence $\epsilon(\beta)$. Optimal displacement $\beta_{\text{opt}}$ is defined as the value of $\beta$ under which an error will be minimal in the first local minimum. 5) To use a receiver in optimal mode, we need to set optimal displacement $\beta_{\text{opt}}$, which depends on extinction. (3). We can set extinction by changing power of the LO.

$$C = \frac{(2\alpha+\beta_{\text{opt}})^2+\gamma_c}{\beta_{\text{opt}}^2+4\alpha(\alpha+\beta_{\text{opt}})+\gamma_c}$$

(3)

In fact, extinction $C$ and optimal displacement $\beta_{\text{opt}}$ depend on value $\alpha$. (see Figure 2).

![Figure 2. Extinction vs optimal displacement $\beta_{\text{opt}}$ for different $\alpha$](image)

3. Conclusion
Thus, we propose a method for controlling the optimal quantum receiver to distinguish between two coherent states of the measured signal. We take into account imperfections of the optical scheme and noise of single-photon detector. We can control optimal displacement and measure signal with minimal error close to Helstrom bound. Further refinement of the method should be made taking into account the specific technical implementation of the receiver.

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