Identification of Rheological Model of the Layer Composite Rock Mass and Its Application

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Abstract. The layer composite rock mass is composed of soft rock and hard rock. The difference of their mechanical characteristics is obvious. It is difficult to describe comprehensively the complicated property with the existent constitutive model. Based on the rheological properties of soft rock and hard rock in the composite rock mass and the creep experiment curves, the rheological model can be used by connecting Burgers model in series with a plastic component. The rheological curves of the composite model under diverse stress level can be attained by multi-staged loading. The simulated result is proved good by comparing the theory curves and the experiment curves. The rheology principium of the complicated rock mass can be illuminated well by the model so that it could be popularly used in engineering practice.

1. Introduction
A lot of experimental analyze in site and in laboratory tell us that rheological characteristic is diverse if the nature and structure of rock is different. When the mechanical parameters are given, it is very important to set up appropriate rheological model to analyze. The complex characteristic of rock mass can be represented visually by the model and it will help to know elastic component, plastic component and viscous component, so the relevant formulas can describe creep, stress relaxation and steady deformation. The common models contain experiential models based on experiments or data in site and combined models by connecting spring, dashpot with sliding block in series or parallel. In recent years, many people studied rheological model of rock. Many models are presented and they are appropriate to soft rock, fractured hard rock, integrating faces and interlayer respectively [1-9]. There is not a followed principle as to how to choose rheological model for a certain rock mass engineering. For layer compose rock mass, not only the soft characteristic but also the elasto-brittle of rock should be thought. If just the elasto-plastic is be thought, the result is trustless [10].

2. The theory of model identification

2.1. The constitute of model
Selection of a suitable rheological model and determination of the corresponding parameters according to the rheological test data are the important parts of rheological study on rock. The rheological constitutive models of rock are roughly divided into three categories: Common models include empirical models obtained on the basis of test or measured data and composite rheological mechanics models with a Hookean elastic body (spring), Newtonian viscous body (dashpot) and St. Venant rigid plastic body
(slider) as basic elements, obtained from series-parallel connection in different ways.[11-13]. For layer compose rock mass, the rheological characteristic of soft rock is very obvious. The process of rheological deformation is different along with the change of stresses and it always includes many deformations, such as elastic, plastic, visco-elastic and visco-plastic etc[14]. The hard rock is always appearing the elasto-plastic when stresses are low, but the shear creep will increase when stresses rise. For simulating the curve of stress-strain-time, the Burgers model is chosen in this paper, and combine Mohr-Coulomb shear failure and tension failure compose law to simulate the constitutive law of layer compose rock mass (Fig.1).

![Figure 1. The suggested rheological model.](image)

In Fig.1, $E_M$, $E_K$, $\eta_M$, $\eta_K$ are Maxwell elastic modulus, Kelvin elastic modulus, Maxwell viscosity modulus and Kelvin viscosity modulus, $\eta_M$, $\eta_K$ and $\varepsilon_P$ are Maxwell strain, Kelvin strain and plastic strain.

### 2.2. The constitutive law

The deviatoric increment:

$$\Delta e_{ij} = \Delta e_{ij}^M + \Delta e_{ij}^K + \Delta e_{ij}^P$$  \hspace{1cm} (1)

Because stresses of each element are diverse along with time, time must be discretized and element stresses is invariable in each small time step $\Delta t$.

Maxwell component:

$$\Delta e_{ij}^M = \frac{\Delta S_{ij}}{2G^M} + \frac{\bar{S}_{ij}}{2\eta_M} \Delta t$$  \hspace{1cm} (2)

Kelvin component:

$$\bar{S}_{ij} \Delta t = 2\eta^K \Delta e_{ij}^K + 2G^K \bar{\varepsilon}_{ij}^K \Delta t$$  \hspace{1cm} (3)

Plasticity component:

$$\Delta e_{ij}^P = \Delta \lambda \frac{\partial g}{\partial \sigma_{ij}} - \frac{1}{3} \Delta \varepsilon_{vol} \delta_{ij}$$  \hspace{1cm} (4)

Where,

$$\Delta \varepsilon_{vol}^P = \Delta \lambda \left[ \frac{\partial g}{\partial \sigma_{11}} + \frac{\partial g}{\partial \sigma_{22}} + \frac{\partial g}{\partial \sigma_{33}} \right]$$  \hspace{1cm} (5)

$G$ is shear modulus, $\eta$ is viscosity modulus, $g$ is plasticity potential function. The plasticity component use Mohr-Coulomb shear failure and tension failure compose law. The plastic yield condition is $f=0$, in the principal axes,

Shear yielding
Tension yielding

\[ f^* = \sigma_1 - \sigma_3 N_\phi + 2c \sqrt{N_\phi} \]  \hspace{1cm} (6)

Based on the global creep curve of rock under diverse stresses, the rheological parameters can be obtained. Using two couples of stresses and creep velocities in the creep curve:

\[ \eta_M = \frac{\sigma_2 - \sigma_1}{\dot{\varepsilon}_2 - \dot{\varepsilon}_1} \]  \hspace{1cm} (8)

Using steady strain \( \varepsilon_c(\infty) \):

\[ E_K = \frac{\sigma_0}{\varepsilon_c(\infty)} \]  \hspace{1cm} (9)

Using any one point \([\varepsilon_c(t), t]\) in the creep curve,

\[ \eta_K = \frac{E_K t}{\ln\left(\frac{\sigma_0}{\sigma_0 - E_K \varepsilon_c(t)}\right)} \]  \hspace{1cm} (10)

### 3. Application

#### 3.1. Laboratory test

The rock rheological test is a main means of knowing the rheological properties of soft rock, and the results are the basis of establishing a rheological constitutive model and obtaining the corresponding rheological mechanics parameters. There are many methods of rock rheological test, and the uniaxial compression, direct shear, conventional triaxial creep test, etc., are the most common. To reveal the distribution and engineering geological characteristics of soft rock and weak interlayers in an underground workshop, an inclined exploration well was excavated in the main workshop, extended to the D_3x soft rock layer underneath the main workshop. A great deal of indoor and on-site rock mechanical testing researches have been conducted on the typical rock stratum, including indoor compression creep test of soft rock and weak interlayer that controls the stability of surrounding rock of the main workshop. Specimen selected include the carbonaceous marl and 031\# shear zone (thickness of about 1 m) in the bearing stratum P_{1q} rock stratum for crane beam on the rock wall at the main workshop, and the carbonaceous marl in the C_{2h} stratum of the Huanglong Formation and the D_{3x} Shale of Xiejingsi Formation.

All specimens were taken manually. It was difficult to take samples of soft rock and the rock samples were likely to be loose. Therefore, the undisturbed samples could only be obtained manually or by using special fixtures. To keep the rock samples obtained undisturbed, they were bundled, wax-sealed and transported back to the laboratory after the samples was broken away from the matrix. The specimens were processed separately based on their hardness or softness. The extremely soft rock and weak interlayers were very difficult to be made into core samples due to breaking away from the matrix, so non-standard samples were adopted to be changed carefully in square shape and the upper and lower ends had similar lengths on each side, with the sample height being about twice the length of the side. A core from a slightly harder rock sample was drilled to make it into a standard cylindrical specimen with a size of \( \varnothing 50 \text{ mm} \times 100 \text{ mm} \). All specimens were stored in a sealed container after being processed to keep their natural state.
The test was conducted on a RLW-2000 rock triaxial rheological tester. Gas-liquid loading system in the equipment could prevent it from effects of power outages. An energy storage device was used for pressure stabilization, allowing automatic pressurization in case of drop in pressure caused by increase in deformation, and this type of automatic pressure controller helped offer better pressure stabilization. The equipment worked in a laboratory with constant temperature and humidity. In order to prevent the moisture loss of the sample and keep the specimens in the natural water condition as far as possible during the test, all the specimens were wrapped in transparent plastic bags and placed on the test equipment together.

Before the creep test, uniaxial compressive strength and conventional shear test of rock were conducted. It served as the basis of estimating the applied load in order to obtain the instantaneous compressive strength and shear strength of rock. The specimens of conventional and creep tests were taken from the same rock sample. Creep test was conducted by incremental loading step by step. Before the tests, low normal stress preloading was applied to the specimens, and the general preloading duration was controlled at 20 hours. The main purpose of the creep test was to eliminate or reduce the deformation of the contact joints between the specimens and the bearing plates and make the deformation measured more realistic. The variation of deformation with time was observed with a dial gauge. The duration of each load depended on the deformation rate. When the general displacement increment was not greater than 0.001 mm/d, the next order load was applied. For the occurrence of a constant strain rate, the total observation duration was usually 14 days.

3.2 Test results
During the test, it was found that the creep characteristics of different rocks were approximately the same in the process of deformation:

1. At the moment of each phase of stress loading, the specimen produced instantaneous strain, and its measured value increased with the increase in stress level; the instantaneous strain accounted for the main part of the total deformation at most stress levels, and for extremely soft rock (ultimate breaking strength<5MPa), the creep deformation sometimes accounted for more than 1/3 of the total deformation.

2. There was no obvious initial rheological strength in rock rheology, i.e. the deformation of rock increased with time at a low stress level. The deformation duration curve of each specimen attenuated rheological phase (first phase rheology) at a low stress level and stable flow phase (second phase rheology) at a high stress level. It was metastable creep.

3. For some specimens which were unloaded at the same time during the loading, the specimen produced a rapid reversible deformation at the moment of unloading, indicating that the rock had an instantaneous elastic recovery. With the increase of time, there was lagged rebounding deformation, i.e. elastic aftereffect phenomenon. When the stress was unloaded to zero, the deformation could not be fully recovered, i.e. there was permanent residual deformation.

4. When the deformation increased rapidly at a high stress level, the specimen usually broke down quickly. The circumstance that the second phase directly enters the third phase (accelerating creep phase) did not occur generally. It took a very short time from the deformation of the specimen to the buckling. The cracks in the outer side of the rock mass were visible to naked eyes, the phenomena of rapid expansion, opening and falling of corner were observed, and most of the rock blocks were damaged along the primary cracks and bedding. The breaking of specimens was usually accompanied by obvious lateral bulging.

The global creep curve based on some underground plant surrounding rock can be obtained as is shown in Fig.2. The real line is the result that is simulated based on the method as above. It is obvious the presented model in this paper is appropriate to the rheological characteristic of layer compose rock mass.
3.3. Calculation of rheological parameters

According to the rheological load test curve, after entering the rheological deformation phase, the test point has obvious rheological deformation in the first phase, and the change of rheological displacement curve in the second phase is relatively smooth. Therefore, the rock mass in this test area is a typical viscoelastic body with elastic deformation and rheological deformation. According to the formula established above, the expression of the analytical solution of viscoelasticity under the experimental conditions was obtained by using the elastic solution and the Laplace transform and inverse transform, and the explicit solution of the viscoelastic analytical solution of BURGERS rheological model under load test was established. According to the rheological curve of the test, the rheological model identification and the inversion analysis of various elastic and viscoelastic parameters were conducted. The optimum rheological model of rock mass in the test area was given, and the elastic modulus, long-term elastic modulus, viscoelastic modulus and viscoelastic coefficient of the rock mass in different test areas were determined.

The rheological parameters are obtained by Eqs. (8), (9), (10) as been written in table 1.

| Sress (MPa) | Rock characteristic | $E_M$ (GPa) | $E_K$ (GPa) | $\eta_M$ (GPa·h) | $\eta_K$ (GPa·h) |
|------------|---------------------|------------|------------|----------------|----------------|
| 2.35       | soft                | 1.755      | 1.781      | 791.91         | 2.004          |
|            | hard                | 8.332      | 38.257     | 851.65         | 40.058         |
| 3.99       | soft                | 2.311      | 2.865      | 507.53         | 3.448          |
|            | hard                | 12.219     | 50.515     | 1292.2         | 65.194         |
| 5.64       | soft                | 2.677      | 2.771      | 645.97         | 6.484          |
|            | hard                | 16.067     | 78.225     | 1364.98        | 77.414         |
| 7.28       | soft                | 2.787      | 3.298      | 631.50         | 4.229          |
|            | hard                | 18.339     | 83.325     | 2259.97        | 108.75         |

4. Conclusion

(1) The rheological characteristic of rock is relevant to the physical mechanics characteristic of itself, and the deformation is subject to many factors. Because of the qualities of inhomogeneous, discontinuous and anisotropic, the rheological model of layer compose rock mass is diversiform and flexible.

(2) The calculating curve by simulation is parallel with the experiment curve of rock, so the rheological characteristic of layer compose rock mass can be represented well with the given model.
(3) The rheological parameters of rock are relevant to the loading condition. Each parameter will change along with the increment of load. This rheological model can be identified through loading and unloading curve under diverse stresses level and the rheological parameters can be obtained.

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