The theoretical predictions for the study of the $a_0(980)$ and $f_0(980)$ mesons in the radiative decays.

N.N. Achasov and V.V. Gubin
S.L. Sobolev Institute for Mathematics
630090 Novosibirsk 90, Russia

March 1, 2022

Abstract

The potentials of the production of the $a_0$ and $f_0$ mesons in the radiative decays are considered.

The central problem of light hadron spectroscopy has been the problem of the scalar $f_0(980)$ and $a_0(980)$ mesons. It is well known fact that these states possess peculiar properties from the naive quark $(qq)$ model point of view, see, for example [1, 2, 3, 4]. To clarify the nature of these mesons a number of models has been suggested. It was shown that all their challenging properties could be understood [1, 2, 3, 4] in the framework of the four-quark $(q^2 q^2)$ MIT-bag model [5] with the symbolic quark structure $f_0(980) = ss(uu + dd) = \frac{1}{2}$ and $a_0(980) = ss(uu + dd) = \frac{1}{2}$. Along with the $q^2 q^2$ nature of $a_0(980)$ and $f_0(980)$ mesons the possibility of their being the $KK$ molecule is discussed [6]. During the last few years it was established [7, 8, 9] that the radiative decays of the meson $f_0$ and $a_0$ could be a good guideline in distinguishing the $f_0$ and $a_0$ meson models. The branching ratios are considerably different in the cases of naive quark, four-quark or molecular models. As has been shown [7, 8, 9], in the four-quark model the branching ratio is

$$BR( \gamma f_0 q^2 q^2 \gamma) : BR( \gamma a_0 q^2 q^2 \gamma) \sim 10^4;$$

(1)

and in the $KK$ molecule model it is

$$BR( \gamma f_0 KK \gamma) : BR( \gamma a_0 KK \gamma) \sim 10^5;$$

(2)
It is easy to note that in the case \( f_0 = ss \) and \( a_0 = (uu \quad dd) = \frac{P - Z}{2} \) (so called ss model [10]) the branching ratios \( BR ( \! \! f_0 ! ! ) \) and \( BR ( \! \! a_0 ! ! ) \) are different by factor often, which should be visible experimentally.

In the case when \( f_0 = ss \) the suppression by the OZI rule is absent and the evaluation gives [7, 8]

\[
BR ( \! \! f_0 (ss) ! ! ) = 5 \times 10^6 ;
\]

whereas for \( a_0 = (uu \quad dd) = \frac{P - Z}{2} \) the decay \( \! \! a_0 ! ! \) is suppressed by the OZI rule and is dominated by the real \( K^+ K^- \) intermediate state breaking the OZI rule [7, 8]

\[
BR ( \! \! a_0 (qq) ! ! ) = (5 \times 8) \times 10^6 .
\]

Imposing the appropriate photon energy cuts \( ! < 100 \text{ MeV} \), one can show [9] that the background reactions \( e^+ e^- \) with regard to the mixing of the \( f_0 \) and \( mesons \). We consider the one-loop mechanism of the \( meson \) production, where \( R = f_0 \), through the charged kaon loop, \( \! \! K^+ K^- \! \! R \), see [7, 8, 9]. The whole formalism in the frame of which we study this problem is discussed in [9]. The parameters of the \( f_0 \) and \( mesons \) we obtain from fitting the scattering data, see [9].

In the four-quark model and ss model we consider the following parameters to be free: the coupling constant of the \( f_0 \) meson to the \( KK \) channel \( g_{f_0 KK} \), the coupling constant of the \( meson \) to the \( channel \) \( g' \), the constant of the \( f_0 \) transition \( C_{f_0} \), the ratio \( R = g_{f_0 KK}^2 = g_{f_0}^2 \), the phase of the elastic background and the \( meson \) mass. The mass of the \( f_0 \) meson is restricted to the region \( 0.97 < m_{f_0} < 0.99 \text{ GeV} \). Treating the \( meson \) as an ordinary two-quark state, we get \( g_{KK} = \frac{g}{\sqrt{2}} = 2' 0.35g + , \) where \( l = 2 \) takes into account suppression of the strange quark production. So the constant \( g_{KK} \) (and \( g' \)) is not essential in our text.

As for the reaction \( e^+ e^- \) the similar analysis of the scattering cannot be performed directly. But, our analysis of the nal state interaction for the \( f_0 \) meson production show that the situation does not changed radically, in any case in the region \( ! < 100 \text{ MeV} \). Hence, one can hope that the nal state interaction in the \( e^+ e^- \) reaction will not strongly a ect the predictions in the region \( ! < 100 \text{ MeV} \). Based on the analysis of the scattering and using the relations between coupling constants we predict the quantities of the \( BR ( \! \! a_0 ! ! ) \) in the \( q^2 q^2 \) model, \( KK \) model and the \( qq \) model where \( f_0 = ss \) and \( a_0 = (uu \quad dd) = \frac{P - Z}{2} \).
The analysis shows that in the four quark model \( g_{f_{0}K}^2 \) = 4 \( 1 \text{ GeV}^2 \) a number of parameters describe well enough the scattering in the region \( 0 \text{ GeV} < m < 1.5 \text{ GeV} \), see [5]. We predict \( B R(\phi \phi) \) \( 10^4 \) and \( B R(\phi \phi) \) \( 10^5 \).

In the model of the \( KK \) molecule we get \( B R(\phi \phi) \) \( 10^4 \) and \( B R(\phi \phi) \) \( 10^5 \).

In the \( gq \) model the \( f_0(500) \) meson is considered as a point-like object, i.e. in the \( KK \) loop, \( f_0(500) \) and in the transitions caused by the \( f_0 \) mixing we consider both the real and the virtual intermediate states. This model is different from \( \varphi^2 \) model by the coupling constant which is \( g_{f_0K}^2 = 4 \times 0.5 \text{ GeV}^2 \). In this model we obtain \( B R(\phi \phi) \) \( 10 \) and taking into account the imaginary part of the decay amplitude only, which violates the OZI rule, we get \( B R(\phi \phi) \) \( 8 \times 10 \).

The experimental data from SND and CMD-2 detectors support the four quark nature of the \( f_0 \) and \( a_0 \) mesons, see Fig. 1 and Fig. 2. and also [9,10,11,12]. The obtained parameters for \( f_0 \) meson from SND detector are \( m_{f_0} = 971 \text{ MeV}, \varphi_{f_0K}^2 = 4.2 \times 0.88 \text{ GeV}^2, \text{ and } B R(\phi \phi) \) \( 10^4 \), see the dashed line on Fig. 1.

As for reaction \( e^+e^-\phi \phi \), the analysis shows that the study of this reaction is an interesting and rather complex problem.

The main problem is the large background process of Dalitz radiation. The \( f_0 \) state in this reaction could be studied only by observing the interference patterns in the total cross-section and in the photon spectrum [13,14]. As it was shown in [13], since the Fermi-Watson theorem for the final state interaction due to the soft photons in the reaction \( e^+e^-\phi \phi \) is not valid, the phase of the amplitude \( \phi \phi \) is determined by the s-wave phase of scattering. The analysis of the interference patterns in the reaction \( e^+e^-\phi \phi \) \( f_0 \) and the phase of the \( f_0 \) should be performed taking into account the phase of the elastic background of the \( e^+e^-\phi \phi \), the phase of the triangle diagram \( f_0 \phi \phi \) and the phase of the \( f_0 \) amplitude. The whole form is for the description of these reactions and the resulting pictures were stated in [5,13,14].

References

[1] N. N. Achasov, S. A. Devyanin and G. N. Shestakov, Usp. Fiz. Nauk. 142, 361 (1984).

[2] N. N. Achasov, Nucl. Phys. B (Proc. Suppl.) 21, 189 (1991).

[3] N. N. Achasov and G. N. Shestakov, Usp. Fiz. Nauk 161, No 6, 53 (1991).
Figure 1: The simultaneous t of the spectrum of the differential cross section $d (e^+ e^- (f_0 + )^0 0^0 0^0) = d!$ with mixing of the $f_0$ and mesons (solid line) and of the scattering data (first row). The branching ratio for this t is $BR( 0^0 0^0) = 2.8 \times 10^9$. The dashed line is the spectrum of the $f_0$ meson without mixing with the meson. The scattering for this t is in the second row.
Figure 2: The spectrum of the differential cross section $d \left( e^+ e^- a_0^+ \right) d\Omega$. Parameters of the model are $m_{a_0} = 986 \ 22.1 \text{ MeV}$, $Q_{a_0 K}^2 = 4 = 1.5 \ 0.5 \text{ GeV}^2$ and $BR \left( a_0^+ \right) = (0.83 \ 0.23)$.  

[4] N. N. Achasov, this proceedings.

[5] R. L. Jaffe, Phys. Rev. D 15, 267, 281 (1977).

[6] J. W. Einstein and N. Isgur, Phys. Rev. D 41, 2236 (1990).

[7] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B 315, 465 (1989).

[8] N. N. Achasov, V. V. Gubin, and V. I. Shevchenko, Phys. Rev. D 56 203 (1997).

[9] N. N. Achasov and V. V. Gubin, Phys. Rev. D 56, 4084 (1997).

[10] N. A. Tornqvist, Phys. Rev. Lett., 49, 624 (1982).

[11] M. N. Achasov Phys. Lett. B 438 (1998) 441, Phys. Lett. B 440 (1998) 442.

[12] E. P. Solodov, this proceedings.

[13] V. N. Ivanchenko, this proceedings.

[14] V. B. Golubev, this proceedings.

[15] N. N. Achasov, V. V. Gubin and E. P. Solodov, Phys. Rev. D 55 (1997) 2672.

[16] N. N. Achasov and V. V. Gubin, Phys. Rev. D 57 (1998) 1987.