Power Allocation in Compressed Sensing of Non-uniformly Sparse Signals

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Abstract—This paper studies the problem of power allocation in compressed sensing when different components in the unknown sparse signal have different probability to be non-zero. Given the prior information of the non-uniform sparsity and the total power budget, we are interested in how to optimally allocate the power across the columns of a Gaussian random measurement matrix so that the mean squared reconstruction error is minimized. Based on the state evolution technique originated from the work by Donoho, Maleki, and Montanari, we revise the so called approximate message passing (AMP) algorithm for the reconstruction and quantify the MSE performance in the asymptotic regime. Then the closed form of the optimal power allocation is obtained. The results show that in the presence of measurement noise, uniform power allocation, which results in the commonly used Gaussian random matrix with i.i.d. entries, is not optimal for non-uniformly sparse signals. Empirical results are presented to demonstrate the performance gain.

I. INTRODUCTION

Compressed Sensing has been widely studied to reconstruct sparse signals from underdetermined observations [1]. The observation \( y \in \mathbb{R}^m \) is measured from the linear model

\[
y = Ax + w,
\]

where \( A \in \mathbb{R}^{m \times n} \) is the measurement matrix, \( x \in \mathbb{R}^n \) is the unknown sparse signal, and \( w \in \mathbb{R}^m \) is the white Gaussian noise with covariance \( \sigma^2 I \). In many applications, the sparse signals have inherent structures. For example, in gene expression and source positioning in sensor networks, the non-zero signals are usually clustered in certain areas. For natural images, most of the wavelet coefficients are distributed in a tree structure [2]. In video surveillance and real time MRI imaging, some of the non-zero signal components are unchanged compared to the previous time [3]. Relying on the sparse prior information, it is possible to adaptively design the measurement matrix to encourage a better signal reconstruction [4]. Existing methods include using Bayesian experimental methods to design measurement matrix based on an identified signal prior distribution [5]; adaptively selecting the precision and location of the measurements according to the extracted structured information from the previously collected data [6]; two-stage resource allocation in estimating sparse signals [7]; weighted \( \ell_1 \) minimization where the denser signal partition gets assigned a larger weight [8]. To name a few.

In this paper, we focus on how to optimize a Gaussian random measurement matrix benefiting from the sparse prior distribution. A non-uniformly sparse distribution is assumed where different signal components may have different probability to be nonzero. Given a total power budget, the question is how to allocate the power across the columns of a Gaussian random measurement matrix to minimize the reconstruction error.

Our technique is based on the approximate message passing (AMP) algorithm and the related analysis developed by Donoho et al. [9]. It was assumed that the entries of the measurement matrix were generated from i.i.d. Gaussian random variables. The key theoretical contribution was the so called state evolution, which exactly quantifies the phase transition curve previously derived from combinatorial arguments [1]. When noise is present, the reconstruction mean squared error (MSE) was derived for the worst case analysis [10], referred to as minimax MSE. More recently, the same technique had been applied to non-uniformly sparse signals in [11] and block separable signals in [12]. The phase-transition curve was characterized and the minimax MSE was quantified.

The main contribution of this paper is the asymptotically optimal power allocation to minimize the minimax MSE. More specifically, given a non-uniformly sparse signal, we revise the AMP algorithm for the Gaussian measurement matrix where different columns may have different \( \ell_2 \)-norm. We analyze the revised AMP algorithm and quantify the minimax MSE. Based on it, the optimal power allocation policy is derived by using Cauchy-Schwarz inequality. Simulations demonstrate that although the analysis focuses on the worst case, the results are ready to be applied to the sparse distributions that arise in real-world applications.

II. PROBLEM FORMULATION AND PRELIMINARIES

In standard compressed sensing (CS) settings, the entries of the measurement matrix \( A \) are generated from i.i.d. Gaussian random variables. However, this may not be optimal in terms of reconstruction distortion when the unknown signal \( x \) is non-uniformly sparse, i.e., the probabilities for different entries to be nonzero may be different. This claim can be demonstrated by the following example. Suppose that \( x = [x_{I_1}, x_{I_2}] \), where the entries in \( x_{I_1} \), \( x_{I_2} \in \mathbb{R}^{n/2} \) have different probabilities to be nonzero. In the extreme case, suppose that the entries in \( x_{I_1} \) have the same prior distribution where the probability to be nonzero is strictly positive, while the entries in \( x_{I_2} \) equal to zero with probability one. For fairness of comparison, fix the total power budget, that is, the squared \( \ell_2 \)-norm of each row of the measurement matrix is fixed to a constant. While the
standard CS yields equal power allocation across the columns, a more sensible way is to not spend any sensing power to $x_{k2}$, i.e., $A_k,j = 0$ for all $j \in \mathcal{I}_2$, but equally allocate power in the columns indexed by $\mathcal{T}_1$.

To be formal, assume the following block-sparsity for the signal $x$. Let $x = [x_{k1}; x_{k2}; \ldots; x_{k\ell}]$, where the prior distribution of $x_i$ is given by

$$p_i \in \mathcal{F}_i = \{p: p\{0\} = 1 - \epsilon_i\}, \quad i \in [n],$$

where $p_i$ has bounded second moment and $p_i = p_j$ if $i, j \in \mathcal{T}_k, k \in [s]$. For the purpose of power allocation, we assume that each column of $A$, denoted by $A_i$, $i \in [n]$, contains entries generated from i.i.d. Gaussian random variables with $\mathcal{N}(0, \sigma_i^2/m)$. Fix a total power budget $\Sigma_{i=1}^n \sigma_i^2 = n$. We aim at minimizing the reconstruction error subject to the total power budget, i.e.,

$$\min_{\sigma_i^2, \ldots, \sigma_n^2} \frac{1}{n} \mathbb{E}\left\{\|\hat{x} - x\|^2_2\right\}, \quad \text{s.t.} \quad \sum_{i=1}^n \sigma_i^2 = n, \quad (1)$$

where $\hat{x}$ is the compressed sensing reconstruction.

Our approach to tackle this problem relies on the AMP mechanism and analysis [9]. Consider a scalar sparse reconstruction problem $y = x + w$ where $x \sim p_x$ and $w \sim \mathcal{N}(0, \sigma_w^2)$. The AMP algorithm involves a soft thresholding function defined by

$$\hat{x} = \eta(y; \theta) \triangleq \begin{cases} x - \theta & \text{if } x > \theta, \\ x + \theta & \text{if } x < -\theta, \\ 0 & \text{otherwise}, \end{cases}$$

where $\theta > 0$ is the threshold. Define the reconstruction MSE

$$M(p_x, \sigma^2) = \inf_{\theta > 0} \mathbb{E}\{\|x - \hat{x}\|^2\}. \quad (2)$$

The optimal choice of $\theta$ depends on the prior sparse distribution $p_x$ and the value of $\sigma$. Henceforth, we assume that $\theta$ is optimally chosen. Given $\epsilon$, among all sparse distributions in the family of $\mathcal{F}_i$, the least favorable one is given by [13],

$$p_i^\# = \frac{\epsilon}{2} \delta_{-\infty} + (1 - \epsilon) \delta_0 + \frac{\epsilon}{2} \delta_{+\infty}, \quad (3)$$

where $\delta_c$ is the Delta function centered at $c$. Furthermore, if the least favorable prior distribution is assumed, the MSE exhibits the scale invariance property, i.e.,

$$M(p_i^\#, \sigma^2) = \sigma^2 M(p_i^\%, 1),$$

and the optimal threshold $\theta = \alpha \sigma$ where $\alpha$ only depends on $\epsilon$. To simplify the notations, we write $M(\epsilon) \triangleq M(p_i^\%, 1)$ for short and refer to it as minimax MSE. A closed form to compute $M(\epsilon)$ for an $\epsilon \in (0, 1)$ can be found in [13].

For the vector case appearing in the CS scenario, an AMP algorithm was derived in [13], [9], which reads

$$x^{t+1} = \eta(x^t + ATr^t; \Theta^t), \quad (4)$$

$$r^{t+1} = y - Ax^t + \frac{1}{m} \|x^t\|_0 r^{t-1}. \quad (5)$$

It has been proved [13] that as $n, m \rightarrow \infty$ simultaneously with a constant ratio $m/n \rightarrow \delta$, the asymptotic minimax MSE

$$\frac{1}{n} \mathbb{E}\left\{\|\hat{x} - x\|^2_2\right\}$$

can be exactly characterized as a function of $M(\epsilon)$ for a certain range of $\delta$.

III. REVISED AMP WITH A GIVEN POWER ALLOCATION

To analyze the effects of power allocation, the original AMP algorithm proposed in [9] needs to be tailored. Assume that a column of $A$, say $A_i$, contains entries generated from i.i.d. $\mathcal{N}(0, \sigma_i^2/m)$. We revise the AMP in [9] to the following form

$$x^{t+1} = \eta(x^t + \Theta^{-2} A^T r^t; \Theta^{-1} \theta^t), \quad (6)$$

$$r^{t+1} = y - Ax^t + \frac{1}{m} \|x^t\|_0 r^{t-1}, \quad (7)$$

where $\Theta^t \triangleq \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$, and refer to it as AMPP($\epsilon$). Compared to the standard AMP algorithm iterations in (4) and (5), the major difference is the terms $\Theta^{-2}$ and $\Theta^{-1}$ in (6). It is worth to note that the revised AMP is designed not particularly for the worst case on which the analysis in Section [X] focuses. The detailed derivation of the AMPP($\epsilon$) is given below.

A. Derivations

The derivation of the AMPP($\epsilon$) follows from the same idea behind the standard AMP [13]. Describe the statistical relationship between $x$ and $y$ by a bipartite graph, which includes variable nodes indexed by $i \in [n]$ for variables $x_i$ and factor nodes indexed by $a \in [m]$ corresponding to observations $y_a$. Denote the message passed from the factor node $a$ to the variable node $i$ by $r_{a \rightarrow i}$ and that from the variable node $i$ to the factor node $a$ by $x_{i \rightarrow a}$, where the superscript $t$ denotes the $t$th iteration. It can be verified that [13]

$$r_{a \rightarrow i}^{t+1} = y_a - \sum_{j \in [n]} A_{aj} x_{j \rightarrow a}^t, \quad (8)$$

$$x_{i \rightarrow a}^{t+1} = \frac{1}{\sigma_i^2} \bigg[ \sum_{b \in [m] \setminus a} A_{bi} r_{b \rightarrow i}^t \bigg], \quad (9)$$

where for notational convenience, $\eta(\cdot, \theta_i)$ is simplified to $\eta(\cdot)$ henceforth. The crux of the AMP is to approximate these messages so that the computational complexity can be significantly reduced.

In the approximation, only $O(1)$ and $O(n^{-1/2})$ terms are kept and all smaller terms are omitted. Here, it is assume that both $n$ and $m$ are large and $\delta \triangleq m/n$ is a constant strictly positive. Since $A_{ai} \sim \mathcal{N}(0, \sigma_i^2/m)$, it is clear $A_{ai}$ is of $O(n^{-1/2})$. Note that $r_{a \rightarrow i}^{t+1}$ is $y_a - \sum_{j \in [n]} A_{aj} x_{j \rightarrow a}^t + A_{ai} x_{i \rightarrow a}$ where the only last term of $(O(n^{-1/2}))$ depends on $i$. One can write $r_{a \rightarrow i}^{t+1} = r_{a \rightarrow i}^{t} + \delta r_{a \rightarrow i}^{t}$, where $r_{a \rightarrow i}^{t}$ is of $O(1)$ and both $\delta r_{a \rightarrow i}^{t}$ is of $O(n^{-1/2})$. By similar arguments, it holds that $x_{i \rightarrow a}^{t+1} = x_{i \rightarrow a}^t + \delta x_{i \rightarrow a}^{t}$, where again, $x_{i \rightarrow a}^t$ is of $O(1)$ and $\delta x_{i \rightarrow a}^{t}$ is of $O(n^{-1/2})$. Keeping only $O(1)$ and $O(n^{-1/2})$ terms, the equations (8) and (9) become

$$r_{a}^{t+1} + \delta r_{a \rightarrow i}^{t} = y_a - \sum_{j \in [n]} A_{aj} (x_{j}^t + \delta x_{j \rightarrow a}^t) + A_{ai} x_{i}^t, \quad (10)$$

$$x_{i}^{t+1} + \delta x_{i \rightarrow a}^{t} = \frac{1}{\sigma_i^2} \eta_i \left( \sum_{b \in [m]} A_{bi} (r_{b}^t + \delta r_{b \rightarrow i}^t) - A_{ai} r_{a}^t \right). \quad (11)$$
By Taylor expansion of \( \eta_t (\cdot) \), Equation (11) becomes
\[
x_i^t + \delta x_i^t - x_i^t \to \frac{1}{\sigma_i^t} \eta_t \left( \sum_{b \in [m]} A_{bi} (r_{b}^t + \delta r_{b-i}^t) + \frac{1}{\sigma_i^t} A_i r_{a}^t \right),
\]
from which it is clear that
\[
x_i^{t+1} = \frac{1}{\sigma_i^t} \eta_t \left( \sum_{b \in [m]} A_{bi} (r_{b}^t + \delta r_{b-i}^t) \right);
\]
\[
\delta x_i^{t+1} = \frac{1}{\sigma_i^t} A_i r_{a}^t \eta_t \left( \sum_{b \in [m]} A_{bi} (r_{b}^t + \delta r_{b-i}^t) \right).
\]
Substitute (13) into (15) and (16) into (12). Again omit the terms smaller than \( O(n^{-1/2}) \). We have
\[
x_i^{t+1} = \frac{1}{\sigma_i} \eta_t \left( \sigma_i^2 x_i^t + (A^T r_i^t) \right),
\]
\[
r_i^t = y_a - \sum_{j \in [n]} A_{aj} x_j^t + \sum_{j \in [n]} A_{aj} \sigma_j^2 \eta_{t-1} \left( \sigma_j^2 x_j-t-1 + (A^T r_j-t-1) \right) r_{a}^{t-1}.
\]
Note that for large \( n \), \( A_j^2 = \sigma_j^2 \sigma_j/m \). The last term on the right hand side of Equation (18) can be approximated as
\[
\sum_{j \in [n]} \frac{1}{\sigma_j^2} \eta_{t-1} \left( \sigma_j^2 x_j-t-1 + (A^T r_j-t-1) \right) r_{a}^{t-1} = \frac{1}{m} \| x_i^t \|_0 r_{a}^{t-1}.
\]
Combine Equation (17), (18), and (19). We obtain the AMP.P(\( \epsilon \)) iterations described by (6) and (7).

IV. RECONSTRUCTION MSE AND A HEURISTIC DERIVATION

We analyze the MSE performance of AMP.P(\( \epsilon \)). We focus on the minimax MSE as the analysis can be highly simplified by the scale invariance property described in (3) for the least favorable prior (2). Still the rigorous analysis is arduous. Instead, we follow the heuristic approach described in (14) which is much more straightforward to understand and helps in clarifying the key ideas. The main results can be summarized as follows. Consider the asymptotic region where \( n, m \to \infty \) simultaneously with a constant ratio \( m/n \to \delta \). Assume the block sparsity structure described at the beginning of Section II and the least favorable prior \( p^{\#}(\epsilon_i), i \in [n] \). Suppose that
\[
\lim_{m \to \infty} \frac{1}{m} \sum_{i=1}^n M^{\#}(\epsilon_i) < 1.
\]
Then the minimax MSE is given by
\[
\frac{1}{n} \mathbb{E} \left\{ \| x - x_i \|^2 \right\} = \frac{1}{n} \sum_{i=1}^n M^{\#}(\epsilon_i) / \sigma_i^2,
\]
where the symbol \( \frac{1}{\sigma_i} \) denotes the equality in the aforementioned asymptotic region.

Remark 1 (Relation with the Previous Result). Consider the uniform sparse signal \( x \) with \( \epsilon_i = \epsilon_j \) for all \( i, j \in [n] \). The minimax MSE in (20) becomes
\[
\frac{1}{n} \sum_{i=1}^n M^{\#}(\epsilon_i) / n = \frac{M^{\#}(\epsilon_i)}{1 - \sum_{i=1}^n M^{\#}(\epsilon_i) / \sigma_i^2},
\]
which consists with the result given in (10).

Remark 2 (Phase-Transition for the Noiseless Case). Set \( \sigma^2 = 0 \). Consider the asymptotic region where \( m, n, \) and \( n_i \), (the length of block \( x_L \), \( s \to \infty \) proportionally with \( m/n \to \delta \) and \( \sum_{i=1}^n \epsilon_i/m \to \rho \). The phase-transition curve that separates the sparse-undersampling (\( \rho - \delta \)) plane is given by
\[
\frac{1}{n} \sum_{i=1}^n M^{\#}(\epsilon_i) = \delta.
\]
That is, the reconstruction is exact if and only if
\[
\frac{1}{n} \sum_{i=1}^n M^{\#}(\epsilon_i) < \infty.
\]
This result is consistent with the one given in (11). Furthermore, note that the phase transition curve is independent of \( \sigma_i^2 \)’s. It can be concluded that power allocation will not affect the phase transition curve when there is no noise.

The heuristic derivation of (20) starts with the iterative algorithm that the term \( \frac{1}{m} \| x \|_0 r_i^t-1 \) in (7) is omitted, i.e.,
\[
x_i^{t+1} = \eta_t \left( x_i^t + \Theta^{-2} A_i^T r_i^t \right),
\]
\[
r_i^t = y - A x_i^t.
\]
At the same time, it also poses an artificial assumption that the matrix \( A \) at different iterations are independently generated. Note that in reality the matrix \( A \) is fixed for all the iterations. The heuristic derivation gives the correct analysis as adding the term (19) will make the residue noise from different iterations independent.

To proceed, the input of the thresholding function in (22) can be written as
\[
x_i + \Theta^{-2} A_i^T r_i = x_i + \Theta^{-2} A_i^T (y - A x_i) = x + e_i,
\]
where \( e_i \) is independent \( \Theta^{-2} A_i^T A_i - I \) \( (x - x_i) + \Theta^{-2} A_i^T w \). The explicit form of the matrix \( \Theta^{-2} A_i^T A_i - I \) in \( e_i \) is
\[
\begin{bmatrix}
\sigma_i^{-2} A_i^T A_i - 1 & \sigma_i^{-2} A_i^T A_2 & \cdots \\
\sigma_2^{-2} A_2^T A_1 & \sigma_2^{-2} A_2^T A_2 - 1 & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}.
\]
It can be verified that each diagonal entry \( \sigma_i^{-2} A_i^T A_i - 1 \) is approximately normal with zero mean and variance \( 2/m \).
each off-diagonal entry $\sigma_i^{-2} A_{ij}^T A_j$, $i \neq j$, has zero mean and variance $\sigma_i^{-2} \sigma_j^2/m$. By the fact that $w \sim N(0, \sigma^2 I)$, the following properties hold: 1) $\mathbb{E} \{ e_i^2 \} = 0$; 2) $\mathbb{E} \{ e_i^T e_j \} = 0$, $i \neq j$; 3) for large $n$,

$$\tilde{\sigma}^2_{t,i} = \mathbb{E} \left\{ |e_i|^2 \right\}$$

$$\tilde{\sigma}^2_{t,i} = \frac{1}{\sigma_i^2} \left( \sum_{j=1}^{n} \sigma_j^2 \mathbb{E} \left\{ |x_j - x_j'|^2 \right\} + \sigma^2 \right) .$$

This helps in quantifying the MSE at the $(t+1)^{th}$ iteration:

$$\tilde{\sigma}^2_{t+1,i} = \mathbb{E} \left\{ |e_{t+1,i}|^2 \right\}$$

$$\tilde{\sigma}^2_{t+1,i} = \frac{1}{\sigma_i^2} \left( \sum_{j=1}^{n} \sigma_j^2 \mathbb{E} \left\{ |x_j - \eta_t (x_j + e_j')|^2 \right\} + \sigma^2 \right) .$$

From the definition of $M^\# (\epsilon_j)$ in (3),

$$\mathbb{E} \left\{ |x_j - \eta_t (x_j + e_j')|^2 \right\} = M^\# (\epsilon_j) \tilde{\sigma}^2_{j}.$$  

(25)

As a result, when the steady state ($\tilde{\sigma}_{t,j} = \tilde{\sigma}_{t+1,j}$) is reached, it holds

$$\tilde{\sigma}_i^2 = \frac{1}{\sigma_i^2} \left( \frac{1}{m} \sum_{j=1}^{n} M^\# (\epsilon_j) \tilde{\sigma}_j^2 + \sigma^2 \right), \quad i \in [n] .$$  

(26)

The explicit form to compute $\tilde{\sigma}_i^2$ can be computed by observing that for all $i \in [n]$, $\tilde{\sigma}_i^2 = \sum_{j=1}^{n} M^\# (\epsilon_j) \tilde{\sigma}_j^2 + \sigma^2$ which is a constant independent of $i$. Hence,

$$\tilde{\sigma}_i^2 = \frac{1}{m} \tilde{\sigma}_i^2 \sum_{j=1}^{n} M^\# (\epsilon_j) + \sigma^2 .$$  

(27)

Or equivalently

$$\tilde{\sigma}_i^2 = \frac{\sigma_i^2}{\tilde{\sigma}_i^2} \left( 1 - \frac{1}{m} \sum_{i=1}^{n} M^\# (\epsilon_i) \right) , \quad i \in [n] .$$  

(28)

Combine (28) with the state evolution (25). We obtain the minimax MSE

$$\frac{1}{n} \mathbb{E} \left\{ \| \hat{x} - x \|^2 \right\} = \frac{1}{n} \sum_{i=1}^{n} M^\# (\epsilon_i) \tilde{\sigma}_i^2 ,$$

which gives (20).

V. OPTIMAL POWER ALLOCATION

Based on the derived minimax MSE, the optimal power allocation can be achieved. In particular, the power allocation problem can be formulated as a constrained optimization problem

$$\min_{\sigma_i, \ i \in [n]} \frac{1}{n} \sum_{i=1}^{n} M^\# (\epsilon_i) / \sigma_i^2 , \text{ s.t. } \sum_{i=1}^{n} \sigma_i^2 = n .$$

By the Cauchy-Schwarz inequality, one has

$$\sum_{i=1}^{n} M^\# (\epsilon_i) / \sigma_i^2 \geq \frac{1}{n} \left( \sum_{i=1}^{n} \sqrt{M^\# (\epsilon_i) / \sigma_i} \right)^2 = \frac{1}{n} \left( \sum_{i=1}^{n} \sqrt{M^\# (\epsilon_i)} \right)^2 ,$$  

(29)

where the equality holds if and only if $\sqrt{M^\# (\epsilon_i)} = c \sigma_i^2$ for some constant $c$. Recall the total power constraint $\sum \sigma_i^2 = n$. The constant $c$ can be characterized and the optimal power allocation is given by

$$\sigma_i^2 = \frac{\sqrt{M^\# (\epsilon_i)}}{n \sum_{i=1}^{n} \sqrt{M^\# (\epsilon_i)}} , \quad i \in [n] .$$  

(30)

VI. DISCUSSION

A. THEORETICAL RECONSTRUCTION ERROR

For theoretical demonstration of the effects of power allocation, we assume that the unknown sparse signal can be divided into two even-length blocks where the sparsity ratio $\epsilon^{(1)} / \epsilon^{(2)} = 100$. The blue solid line and the red dashed lines respectively present the minimax MSEs $\{0.1, 0.2, 0.5, 1, 2, 5, 10\}$ before and after the power allocation. The phase-transition curve for noiseless case is given by the black line. The upper right curved area is the inadmissible area under the sparsity ratio 100.
allocation. From the presented results, the average MSE after power allocation is always smaller than that before power allocation. The performance gain becomes larger when the sparsity ratio increases. Theoretical predictions drawn as dashed curves are very close to the curves obtained from simulations. In Fig. 3 we aim to demonstrate the linear relationship between the reconstruction MSE and the noise variance, predicted by (20). The settings are the same to those for Fig. 2 except that $\rho = 0.1$ and the sparsity ratios are chosen to be 5 and 100. From the simulations, the linear relationship is confirmed.

VII. CONCLUSION

In this paper we consider non-uniformly sparse signals. We first used an example to show in the presence of noise, Gaussian random measurement matrix with i.i.d entries may not be optimal in minimizing the mean squared reconstruction error. Then we considered how to allocate a given total power across the columns of the measurement matrix. Given a power allocation, we derived the AMP($\psi$) algorithm, and quantitatively analyzed the corresponding minimax MSE. Based on it, the optimal power allocation policy has been identified. Both theoretical and empirical results are presented with the clear consistency. Their consistency verified the performance gain.

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