Optimization in work modeling of a mixer

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Abstract. The paper presents a list of indicators used to optimize the working process of rotary machines using the example of a mixer. The shortcomings of polynomial statistical models are indicated and the use of functions which nature of variation corresponds to the theoretical positions is recommended. The theoretical dependencies of the mass of the prepared portion of the mixture, the quality of the mixture, the duration of mixing, the productivity of the mixer, the drive power, the specific energy consumption are revealed, taking into account the uniformity of the mixture. These dependencies and the refinement coefficients that appear in them have the form of power and exponential functions. These functions do not have an extremum, therefore optimal parameters are impossible.

1. Introduction

Devices providing formation of various mixtures and composite substances are widely used in industry, construction, and agriculture. There are numerous studies justifying the conditions for a fast and efficient process of mixing components at low energy costs [1-4]. Rotation of working bodies acting on different material media are generally placed inside a container of the investigated machines [1, 2]. To describe working process of substances moving and mixing, specialist reveal many dependences as theoretical (based on the laws of mechanics and hydraulics [5]) and/or functional (statistical) ones based on the justification of quantitative (productivity/flow, duration of the mixing cycle), qualitative (uniformity / uniformity of the mixture, average particle size, temperature, etc.) and energy (drive power, specific energy consumption / energy consumption of the process) indicators of the process flow from the dimensional characteristics of the working bodies [1-5]. Agricultural mixers, according to the requirement of regulatory and normative documents in Russia and Belarus, have to follow controlled parameters like productivity, process duration, power and energy consumption, and unevenness, the coefficient of variation of the distribution of the control component in 15-20 samples of a given mass and concentration of the check component. By analogy with other devices [1-5], the operation of mixers was simulated, and indicators were monitored on the basis of statistical dependences [6].

According to the theory of experimental multifactor planning [6], processing the results of interpolation studies needs polynomial dependences which describe technical processes [6, 7]. The linear model is firstly tested when describing the data. If the obtained results are not adequately described by linear model, it is necessary to proceed to the regression equations of the second and subsequent orders. With an adequate description of the results revealed by the model, a researcher
should remove insignificant coefficients in the polynomial regression equation from the obtained order equation using the Student's test, thereby simplifying, if it is possible, the given equation.

Obtained functional statistical expressions do not reflect the physical essence of the phenomenon, but due to the refinement of the coefficient values, the regression equations tend to compress the reference surface of the model to experimental values with a minimum error. Extrapolation of the revealed dependence beyond the experimental limits is practically impossible [1].

It is known [8-10] that the power expended on the work process adequately describes the power functions, and exponential functions are applicable to describe the change in the quality of the mixture. These dependencies are of a functional nature, not reflecting the physical essence of the process studied. Therefore, to understand the physical essence of the phenomenon, one should use theoretical expressions.

The aim of our work is to establish the type of dependencies theoretically describing the working process of mixing components in a mixer and to reveal effective parameters of the mixer construction design.

2. Experimental Part
The quality index of the mixture, the cost of power, the energy consumption of mixing, and the productivity in the mixing cycle were studied in the research.

The research methodology provided for a number of actions. In order to achieve the aim we firstly established main indicators of the working process of the stirrer on the basis of theoretical studies analysis [9, 10]. Then we found the extremum (optimum) for the indicators of the stirrer working process. We analyzed the data to determine the effective performance of the stirrer process as a criterion for process optimization. Finally, we analyzed the possibility of description of technical and technological processes by polynomial expressions.

Based on the known information [9-11], we describe the working process of the agitator-mixer (as an example of rotary devices) on the basis of theoretical expressions with the introduction of additional correctional coefficients and we will try to optimize the parameters of the technological process.

The mass of the portion of the prepared mixture $M$ is a particular case of the power function, where the exponent is equal to "one" and is described, kg:

$$M = V_0 \cdot E \cdot \rho.$$  \hspace{1cm} (1)

where $V_0$ is the volume of the mixing tank, m$^3$; $E$ is the degree of filling the mixing tank by the ingredients, shares; $\rho$ is the density of the heap of the mixture, kg / m$^3$.

A number of authors propose to describe the mixing process by analogy with the diffusion process by an exponential function. Uniformity of the mixture (0.01%):

$$\theta = 1 - e^{-kT_c},$$  \hspace{1cm} (2)

where $k$ is the empirical coefficient of mixing intensity; $T_c$ is the mixing time of the material, s.

The uniformity of the mixture (0.01%) (as "relative uniformity" [12]) can be written with the help of the coefficient of variation $\nu$ (0.01%) of the content of the control component in the samples:

$$\theta = 1 - \nu, \quad \text{or} \quad \nu = e^{-kT_c},$$  \hspace{1cm} (3)

For different mixers, the empirical mixing intensity $k$ will be different [9, 10]. Therefore, the graphs of variance of the uniformity of the mixture $Q$ for such mixers will differ in intensity (Figure 1). The uniformity of the mixture before mixing is $Q_n$, required at the end of mixing – $Q_c$. The current values of the uniformity of the mixture for different mixers are $Q_1(T)$ and $Q_2(T)$.

To minimize energy costs and financial costs, it is desirable to shorten the mixing time ($[T_k - T_n]$ and $[t_e - t_n]$). Accordingly, if we know the values of $k$ of several mixers or their operating regimes, we can find the required mixing time for ingredients in a particular case.

The conditional time before the mixing is determined by:
The conditional time before the end of mixing is determined by:

$$T_s = -\frac{1}{k} \ln(1 - \theta_s),$$

(4)

The required mixing time of the components in the mixer in general form is determined as follows:

$$T_c = T_k - T_H = -\frac{1}{k} \ln \left(1 - \frac{\theta_k}{\theta_H}\right).$$

(5)

$$T_c = T_k - T_H = -\frac{1}{k} \ln \left(1 - \frac{\theta_k}{\theta_H}\right).$$

(6)

Figure 1. Change in the uniformity of the mixture Q (0.01%), depending on the mixing time T, s.

According to the technological requirements for the quality of the mixture, a number of uniformity values of the mixture are regulated as critical (permissible) values: 80%, 90%, 95%. Then: $\theta_k \geq \theta_{zoo}$, where $\theta_{zoo} = 0,80 \equiv v_{zoo} = 0,20$; $\theta_{zoo} = 0,90 \equiv v_{zoo} = 0,10$; $\theta_{zoo} = 0,95 \equiv v_{zoo} = 0,05$. In this case, the mixing time should be, s:

$$T_c \geq -\frac{\ln(v_{zoo})}{k} = \frac{1}{Sk_i},$$

(7)

where $Sk_i$ –is the total factor of the empirical coefficients of the mixing duration, taking into account the constructive, kinematic and technological features of the work, given, for example, in the work [10] in the form of power functions.

The form of this expression corresponds to a "hyperbola" which is the particular case of a power function with a negative value. The productivity of the mixing tact of the ingredients in the mixer is a special case of the power function [10], kg / s:

$$Q_c = \frac{M}{T_c} = \frac{V_a \cdot E \cdot \rho}{T_c}.$$  

(8)

In mechanics the power is determined by:

$$N = F \cdot v,$$

(9)

where $F$ – is the motion resistance force, H; $v$ –is the displacement velocity of the object in the environment, m/s.

In the resistance of materials, the voltage is determined by [6], Pa:
\[
\sigma = F / S , \text{ where the force } F = \sigma \cdot S , \quad (10)
\]

where \( \sigma \) – working stresses, PaN/m²; \( S \) – is the area of force application, m².

The area of application of force is determined by, m²:

\[
S = (l_s \cdot \sin(\alpha)) \cdot dR \cdot Z = (K_1 \cdot D \cdot \sin(\alpha) \cdot Z) \cdot dD , \quad (11)
\]

where \( l_s \) – is the blade width, m; \( \alpha \) – the angle of attack when installing the blade, rad.; \( dR \) – the length of the current radius section of the force application, m; \( Z \) – the number of blades, pcs; \( D \) – the diameter of the mixing tank, m; \( K_1 \) – the coefficient of proportionality of the width of the blade and the diameter of the tank; \( dD \) – the length of the section of the current radius of the force application, expressed in terms of the diameter, m.

The velocity of the end of the blades during rotation is described by, m/s:

\[
\vartheta = \omega \cdot R = \omega \cdot D / 2 ,
\]

where \( \omega \) – is the angular velocity of the rotor, rad/s; \( R \) – is the maximum radius of rotation of the point of application of force, m.

Considering the fact that the radius of the force application for the blades of mixer is not constant, the power equation for mixing the components in the mixer will be written as follows:

\[
N = \int_0^{D/2} \left[ \sigma \cdot (K_1 \cdot D \cdot \sin(\alpha)) \cdot dD \cdot Z \right] \cdot (\omega \cdot D / 2) \cdot dD = K_1 \cdot \sigma \cdot Z \cdot D \cdot \sin(\alpha) \cdot \omega / (2 \cdot 3 \cdot 0.5 \cdot 180) .
\]

Power at mixing is determined as follows, W:

\[
N = \pi \cdot K_1 \cdot \sigma \cdot Z \cdot \left(\frac{D}{2}\right)^2 \cdot \sin(\alpha) \cdot n / 180 ,
\]

where \( n \) – is the rate of rotation of mixer, min⁻¹.

The power expression is a particular case of a power function, where the exponent of some of the indicators exponents is equal to "one" (Figure 3). The value of the stress \( O \) is very difficult to determine theoretically, therefore it can be found experimentally as a correction factor in the power expression 14. For example, for a blade mixer where \( K_1 = D / 6 \), the exponent is determined by the power function:

\[
\sigma = 10,125,86 \cdot M \cdot D^{-4} \cdot n^{-0.185863} \cdot Z^{-0.606518} \cdot \sin(\alpha)^{0.653631} \cdot (l_s^{0.543368} + L^{0.62341})
\]

where \( L \) – is the length of the blade, m; \( l_s \) – is the width of the lining plate on the blade, m.

Specific energy consumption per tact of mixing is a power function of the indicators, J/kg:

\[
Y = \frac{N \cdot T_c}{M} = \frac{\pi \cdot K_1 \cdot Z \cdot \sin(\alpha) \cdot n \cdot (D / 2)^3 \cdot T_c}{180 \cdot V_o \cdot E \cdot \rho}
\]

It is also possible to define the optimization criterion as the specific energy consumption per tact, taking into account the uniformity of the mixture, J/kg:

\[
Y_k = Y / (1 - \nu) . \quad (17)
\]

The specific energy consumption of mixing, taking into account the uniformity of the mixture for the mixer at the current value of uniformity \( \Theta \) is determined by:

\[
Y = \frac{N \cdot T_c}{\Theta \cdot M} = \frac{\pi \cdot K_1 \cdot Z \cdot \sin(\alpha) \cdot n \cdot (D / 2)^3 \cdot T_c}{180 \cdot \Theta \cdot V_o \cdot E \cdot \rho}
\]

Analyzing the formulas (1...14), it is clear that they are based on a number of indicators: constructive \( (V_o, D, Z, K_1, \alpha) \), kinematic \( (n) \) and technological \( (E, T_c, \rho) \). Some indicators \( (\sigma, k) \)
depend both on the physico-mechanical properties of the material of the mixture and on the design-regime parameters, i.e. are an empirical power function of these arguments.

We will analyze the function of the energy intensity of mixing, taking into account the uniformity of the mixture (14) as the most complex one which comprised of complex of other specified functions of the mixing process. This function, as well as other functions, is a function of several variables which were specified earlier.

To confirm the optimal values of design-regime and technological parameters, these functions must be checked for externality.

If the differentiable function \( y = f(x_1, x_2, \ldots, x_n) \) has an extremum \( \mathbf{M}_o = (x_{10}, x_{20}, \ldots, x_{no}) \), then all its parts of the first-order derivative at this point are equal to zero:

\[
\frac{\partial y}{\partial x_1}\bigg|_{\mathbf{M}_o} = \frac{\partial y}{\partial x_2}\bigg|_{\mathbf{M}_o} = \cdots = \frac{\partial y}{\partial x_{n-1}}\bigg|_{\mathbf{M}_o} = \frac{\partial y}{\partial x_n}\bigg|_{\mathbf{M}_o} = 0, \tag{19}
\]

By the necessary condition of extremum of function of several variables, all its partial derivatives at the extremum point must be equal to zero.

3. Results and discussion

These functions (1...14) are the product of several functions of one variable. Therefore, all its partial derivatives (functions of one variable) are exponential and monotonically increasing or decreasing. Consequently, the functions (14) do not have an extremum at positive values of the arguments (\( V_o, D, Z, K_l, \alpha, n, E, T_c, \rho, \sigma \)).

The optimization of the process is possible if the mixer parameters are experimentally justified, when the stable steady pattern of motion is disturbed (the growth of a certain parameter of the blade / mixer / starts to interfere with the movement of the material, which makes the quality of the mixture worse, i.e. it changes the nature, proceeding and movement of material in the mixer tank). These cases are determined by the features of the functions of the coefficients \( S_{ki} \) and \( \sigma \).

For example, other constant values of the parameters, increasing the height of the blade of the vertical mixer with the minimum blade height does not set in motion the entire mass of material in the tank. As the height of the blade increases, all material begins to move and mix. Further growth of the blade height will lead to the formation of sectors that prevent typical free movement of the material, the material will accumulate inside with minimal mixing, which will increase the duration of mixing and the energy intensity of the process. In this case, we can see the possibility of an extremum with increasing blade height. However, such an experiment is counterproductive. Another algorithm of actions is more effective.

The blade height can be different with a nominally loaded engine of a constant installed power, but the lowering of the altitude will increase the speed of the mixer, which will increase turbulence, reduce processing time, increase productivity and, possibly, reduce energy intensity. We will get a set of combinations of parameters where it will be possible to find the highest device performance or the lowest energy costs and thereby rational values of constructive, kinematic and technological parameters by means of exhaustive search method with the loaded engine.

To describe the working processes and to find the optimum, it is widely recommended to use polynomial functions [6]. However, there are questions to the validity of the application of these expressions.

The resulting polynomial models of the process will seek to obtain an extremum, which the functions will seek to obtain forcibly. In this case, the optimization problem is to find the min / max values. At the same time, the nature of the experimental values may not have an extremum.

An example of the absence of extremum is a number of processes previously studied and described [9, 11, 12]. As the number of impacts increases, the quality of the mixture will improve. At the same time, there is no possibility of effective separation. The coefficient of variation of the content of the component in the sample will decrease. The values of the coefficient tend to zero, slowing to zero
without reaching it as shown in [11, 12]. When cleaning the surface, the mass of dirt on the surface to be cleaned will decrease with the increase in the number of treatment effects. New pollution to take at the same time nowhere. It turns out that the second, increasing branch of the parabola cannot grow in principle on the graph of the experiment as demonstrated previously [9]. As the column of material in the vertical container increases, the pressure on the bottom will increase. At the same time due to friction of the material on the walls of the tank, the pressure increase will not be linear. The nature of the change in pressure corresponds to the exponential function [9].

The inexpediency of the application of polynomial equations is well seen from [9], the data of which is in Figure 2. To process the results of the pressure data (using its analogue – the force of the column of material P on the bottom of the tank) at a low height of the column of material H (points on the graph near zero, up to 50 cm) different formulas can be used. In particular, linear dependence, a second-degree polynomial, a third-degree polynomial, or exponential function (Table 1).

![Figure 2](image)

**Figure 2.** Influence of the height column (H, cm) on the pressure (P, Pa) by the third-order polynomial (a), the second order (b), and the exponential function (c).

To control the values, an additional point at 270 cm was taken. After processing the data by Statistica 5.5, the expressions in Table 1 were obtained. The results are presented in Figure 2. Height of polynomial graphs (Figure 2a and 2b) initially increases at different rates, and then falls down to negative values. In reality, it is impossible for the values of the material column pressure to the bottom.

| Material    | Type of model                                                   | Index of correlation with experimental data, % |
|-------------|-----------------------------------------------------------------|-----------------------------------------------|
| Stockfeed   | Linear function: \( P = 10.9 + 0.071 \cdot H \)               | 72.96                                         |
|             | Second-degree polynomial: \( P = 1.33 + 0.485 \cdot H - 0.0015 \cdot H^2 \) | 98.40                                         |
|             | Third-degree polynomial: \( P = -1.89 + 0.73 \cdot H - 0.005 \cdot H^2 - 0.0001 \cdot H^3 \) | 99.86                                         |
|             | Exponential function: \( P = 35.78 \cdot (1 - e^{-0.02 \cdot H}) \) | 99.13                                         |

The character of the exponential function increases (Figure 2c). The increase in values gradually decreases. The character of the change in the exponential function is close to theoretical dependences on the basis of the laws of theoretical mechanics. Such regressive exponential function can predict
values in neighboring plots. In this case there occurs an error. Thus, problems of extrapolating functions to neighboring plots with the experimental zone are visible, despite the high statistical coefficients, e.g. correlation coefficient in Table 1. The highest correlation coefficient was for the third-degree polynomial. The exponential function also had a high value. The pressure of the material column height was also theoretically described by the exponential function. Therefore, the exponential function should be preferred.

Figure 3. Change of Y from the value of factor X for functional dependencies of different types (genus):
\[ y = e^{kx^2} \] - of an exponent;\[ y = k - x \] - a power function (positive power);\[ y = k - x^2 \] - a linear function (the power of the power function is equal to one);\[ y = k - x^c \] - a hyperbola (the power of the power function is equal to the negative value);\[ y = k - x^2 + n - x + c \] - a parabola.

Thus, possible restrictions on the use of polynomial expressions were established. An analysis of the nature of the change in the arguments of the functions (Figure 3) for various functions showed that for small interval of study of indicators (red square), experimental points can be described by equations both power and polynomials. With the expansion of the research interval, there are contradictions between functions. Therefore, an incorrectly established relationship (the form of which does not correspond to theoretical expressions) can lead to incorrect results of the mixer studies. In this case, the expressions used for the statistical description of the process should have the form close to the theoretical dependencies.

Conclusion
Functional expressions describing the working process of the mixture formation have been established. The impossibility of theoretical determination of the optimum of the stirrer parameters was revealed. Functional expressions describing the mixer's working process are power or exponential functions that in principle do not have an extremum.

A method for finding rational valid values of the mixer parameters on the basis of the engine's load, observing the standards for the quality of the mixture, and finding the highest productivity or lowest power consumption of mixing from a possible combination of the values of the mixer parameters was proposed.

The impossibility of finding the extremum of the expressions for the mixer's working process and some series of processes due to the use of polynomial expressions that experimentally statistically describe these processes was revealed. The expressions used for the statistical description of the process should have the form close to the theoretical dependences.

References
[1] Pezo M, Pezo L, Jovanović AP, Lončar B and Kojić P 2018 Discrete element model of particle transport and premixing action in modified screw conveyors. Powder Technol. 336 255
[2] Pezo L, Pezo M, Jovanović A, Banjac V, Đuragić 2018. The joint mixing action of the static pre-mixer and the rotating drum mixer – Discrete element method approach. *Adv. Powder. Technol.* **29**(7) 1734

[3] Leš K, Kowalski K and Opaliński I 2015. Optimisation of process parameters in high energy mixing as a method of cohesive powder flow ability improvement. *Inz. Chem. Procesow.* **36**(4) 449

[4] Li W, Guo H, Huang Q, Hou Y and Zou W 2018. Effect of stirring rate on microstructure and properties of microporous mullite ceramics. *J. Mater. Process. Techn.* **261** 159

[5] Tamrin K F, Sheikh N A and Rahmatullah B 2016. Numerical analysis of swirl intensity in turbulent swirling pipe flows. *Sci. Eng.* **78**(5-10) 133

[6] Bilodeau M and Brenner D 1999. *Theory of Multivariate Statistics* (New-York: Springer-Verlag) p 289

[7] Jiang W 2011. *Asymptotic Theory of General Multivariate GARCH*, PhD Thesis, University of Western Ontario.

[8] Chemezov D, Tyrina S, Bayakina A and Lukyanova T 2018. Multi-factor experiment to determining of vibrations of steel pipes induced by vortex of air flow. *ISJ Theoretical & Applied Science* **03**(59) 201 [in Russian]

[9] Konovalov VV 2003. *Workshop on the processing of the results of scientific research with the help of PC*. (Penza: RIO PGSHA) 176 [in Russian]

[10] Ivanets V N, Bakin A I and Ratnikov S A 2004. *Processes and devices of food production: a Training manual* (Kemerovo: Kemerovo Institute of Food Technology) [in Russian] p 180, available at: http://dump.vstu.ru/files/storage/Kafedry/MAPT/Biblioteka_uchebno_metodicheskoy_literatury/PAPP/Metodicheskaya_literatura/processsy_ivanec.pdf

[11] Fomina M, Konovalov V, Cupshev A and Teruskov V 2016. Modeling of capacity vertical paddle mixer on the basis of statistical expressions [in Russian] *Innovative Techniques and Technol.* **3/08** 50

[12] Konovalov V, Cupshev A and Fomina M 2016. Modeling of the quality change of the blade mixer mixture on the basis of technological parameters *Innovative Techniques and Technol.* [in Russian] **3/08** 56