Digital watermarking in the singular vector domain

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Abstract

Many current watermarking algorithms insert data in the spatial or transform domains like the discrete cosine, the discrete Fourier, and the discrete wavelet transforms. In this paper, we present a data-hiding algorithm that exploits the singular value decomposition (SVD) representation of the data. We compute the SVD of the host image and the watermark and embed the watermark in the singular vectors of the host image. The proposed method leads to an imperceptible scheme for digital images, both in grey scale and color and is quite robust against attacks like noise and JPEG compression.

Key words: Watermarking, singular value decomposition, steganography.

1 Introduction

In the past one decade, there has been a phenomenal increase in the use and circulation of information in digital multimedia formats for various purposes. Today, many paintings, photographs, newspapers, books, music etc., are available over the internet in one or the other multimedia format. The increasing necessity to protect the intellectual property rights of such digital content has led to considerable research in that direction. Digital watermarking is one such approach. Watermarking is the process of embedding data into a multimedia element such as an image, audio or video [Langelaar et al. (2000)]. This embedded data can later be extracted from, or detected in, the multimedia for several purposes including copyright protection [Craver et al. (1998), Zeng and Liu (1999)], trade marks, access control or even to simply pass a secret piece of information hidden in an innocuous digital image [Zeng (1998), Petitcolas et al. (1999)]. A watermarking algorithm consists of the watermark structure, an embedding algorithm and an extraction and detection algorithm.

In this paper, we will be concerned with invisible watermarking of the digital images. In such images, the
basic idea is to embed the watermark image in a given host image such that the resultant (watermarked) image carries the watermark either in visible or invisible mode. It is important that the watermarked image should suffer least corruption due to the watermarking procedure. The broad strategy, then, is to embed the watermark, preferably, in the least significant part of the transformed domain of the image. Techniques that employ this strategy are the discrete cosine transformation (DCT) [Bors and Pitas (1996), Dickinson (1997), Piva et al. (1997), Piva et al. (1998)] and the wavelet transform [Kundur and Hatzinakos (1997), Kundur and Hatzinakos (1998)] that also happen to be most popular [Katzenbeisser and Petitcolas (2000)]. The DCT allows an image to be broken up into different frequency bands, making it much easier to embed watermarking information into the middle frequency bands of an image. These bands are chosen such that they minimize the changes to the important parts of the host image and remain unaffected by various image transformations [Lange-laar et al. (2000)]. The wavelet transform approach embodies a similar philosophy. The transformed domain method, such as this, works well also because the embedded data is located in that part of the frequency that is least sensitive for the human visual system hence, the watermark is imperceptible [Lacy et al. (1998), Prandoni and Vetterli (1998), Ohbuchi et al. (2002)].

However, notice that transform methods such as the DCT or wavelets attempt to decompose the image in terms of a standard basis set. This need not necessarily be the optimal representation for a given image. Singular value decomposition offers a method by which the transformed domain consists of basis states that is optimal in some sense; i.e., a tailor-made basis for a given image. Many of the earlier works using SVD for watermarking have attempted to manipulate the singular values of the host and the watermark images one way or the other [Liu and Tan (2002), Shieh et al. (2005)]. Another approach is to perform an SVD on various blocks of the image and then add the scaled singular values of the watermark image to that of the host image [Gorodetski et al. (2001)]. For a survey of existing watermarking methods using SVD, we refer the reader to Ganic et al. (2003). In the scheme proposed by Chang et al. (2005), both the singular values and singular vectors are explored for embedding the watermark. A watermarking scheme using minimax eigenvalue decomposition has been proposed by Davidson and Allen (1998). In our algorithm, we assume a situation in which those who have opted to watermark their digital images have access to their original images (without the watermark) and the watermark image for verification purposes later. This does not restrict the applicability of our technique as we argue below. In the proposed method we work at the level of singular vectors and embed the watermark in the singular vectors of the host image. The singular vectors of an image of size \((m > n) m \times n\) has \(n^2\) parameters and contains detailed,
graded information about the image as opposed to just \( n \) singular values. Hence, in the domain of singular vectors, we have more latitude to embed the watermark. We argue that this also leads to a level of digital security in watermark embedding. We also subject our algorithm to tests against additive noise, cropping and JPEG compression.

In the next section, we recall the singular value decomposition and then describe our algorithm. Then, we present our numerical simulations including the robustness to several possible attacks.

2 SVD-Based Watermarking

Singular value decomposition is a popular technique in linear algebra and it has applications in matrix inversion, obtaining low dimensional representation for high dimensional data, for data compression and even data denoising etc., [Golub and Reinsch (1970), Andrews and Patterson (1976), Leon (1994)]. If \( Z \) is any \( m \times n \) matrix, it is possible to find a decomposition of the form

\[
Z = U D V^T, \quad (1)
\]

where \( U \) and \( V \) are orthogonal matrices of order \( m \times n \) and \( n \times n \) respectively. Proof of Eq.(1) can be found in many standard linear algebra literature [Strang (1993)]. The diagonal matrix \( D \) of order \( n \times n \) has elements \( d_{ii}, (i = 1, 2, ..n) \), which are positive definite and are called the singular values of \( D \). We do not consider here the conditions under which a decomposition of type in Eq. (1) will fail for image matrices and hence we assume that, for all practical purposes, the image matrix \( Z \) can be decomposed in the form given in Eq. (1). Thus the SVD can be applied directly to digital images represented as matrix arrays.

2.1 Algorithm for embedding watermarking in gray scale image

In this section we present our algorithm to embed a gray scale image into another gray scale image of the same size using SVD. Let the matrix \( Z \) represent the host image which needs to be watermarked. Let \( W \) represent the matrix of the image to be embedded. As a first step, we compute the SVD of both \( Z \) and \( W \).

\[
Z = U_z D_z V_z^T = A_z V_z^T \quad (2)
\]

For the watermark image

\[
W = U_w D_w V_w^T = A_w V_w^T \quad (3)
\]

where \( A_{z/w} = U_{z/w} D_{z/w} \) are also called the principal components in the language of principal component analysis.

Now, we add the scaled eigenvector \( V_w \) of watermark to that of the original image,

\[
V = V_z + \lambda V_w \quad (4)
\]
where $\lambda$ is the scaling factor. Typically, $0 \leq \lambda \leq 1$, so that the intensity of the watermark $W$ is less compared to the original image $Z$. Note that, within the framework of SVD, $V_w V_w^T = I$, where $I$ is the identity matrix. Similar relation holds good for $V_z$ too. As $\lambda \to 0$, the approximation that $V$ is a orthogonal matrix, i.e, $V V^T \approx I$ gets better. This property is important in the next step for constructing the watermarked image. We get the watermarked image as,

$$Z_c = A_z V^T$$  \hspace{1cm} (5)

Thus, equations (2-5) constitute the algorithm for watermarking using SVD in the eigenvector domain.

### 2.2 Algorithm for extracting watermarks

Given the watermarked image $Z_c$, we can extract, possibly a corrupted watermark, if we have access to the matrices $A_z, A_w, V_z$ and the value of $\lambda$. That is, we assume that whoever wants to extract the watermark should have access to the original image as well as the embedded watermark image. We emphasize that this is not a restrictive assumption. In most cases, involving copyrights and trademarks embedded in digital images, the person or organization that embedded the watermark in their proprietary digital image will have access to both the clean original image and the watermark image.

Extraction algorithm is a straightforward reversal of the embedding algorithm given by equations (2-5). Starting from Eq.(5), we multiply both sides of Eq. (5) by $A_z^{-1}$ and substitute for $V^T$ from Eq. (4). It is straightforward to obtain an expression for $V_w^T$ as

$$V_{w}^{T} = \frac{A^{-1}_{z}Z_c - V_{z}^{T}}{\lambda}. \hspace{1cm} (6)$$

Finally, using Eq. (3), the watermark image can be constructed as,

$$\tilde{W} = A_{w}V_{w}^{T}. \hspace{1cm} (7)$$

Eq. (7), along with Eq. (6) constitutes the watermark extraction algorithm.

In general, the matrix $A_z$ is not a square matrix and by Eq. (6) we are required to take its inverse. This computation is simple because from Eq. (2), we have, $A_z^{-1} = D_z^{-1} U_z^T$, where inverse of a diagonal matrix and a transpose need to be computed. At this point, we also stress that if $d_{ii} > 0$, then $A_z^{-1}$ exists, even if $A_z$ is not a square matrix.

Firstly, note that even though we simulate with square images, the SVD based method presented above can handle rectangular images as well, without any changes to the algorithm. One important extension of this algorithm is to address the case of color images. We consider the case of RGB coded color images. If a color image is specified in RGB format then it comprises of three component matrices superimposed together, one each for red, green and blue. It can be written as, $Z = Z_r + Z_g + Z_b$ and
Similarly for the watermark image \( \mathbf{W} \). We can directly apply this algorithm to each of the respective component images, \( \{ \mathbf{Z}_r, \mathbf{W}_r \} \), \( \{ \mathbf{Z}_g, \mathbf{W}_g \} \) and \( \{ \mathbf{Z}_b, \mathbf{W}_b \} \).

Secondly, the algorithm provides certain implicit security features as well. The extraction algorithm given by Eq. (6) requires the knowledge of \( \mathbf{A}_z, \mathbf{V}_z, \mathbf{A}_w \). Notice that by performing a SVD on the watermarked image \( \mathbf{Z}_c \), it might be possible to obtain approximate estimates of \( \mathbf{A}_z \) and \( \mathbf{V}_z \). However, unless one knows \( \mathbf{A}_w \) and \( \lambda \), it is not possible to extract the watermark. This means that if \( \mathbf{Z}_c \) is an \( m \times n \) array, then to hack the embedded watermark, one has to estimate \( mn + 1 \) independent parameters. This becomes further complicated due to the fact that the watermark is embedded uniformly in the entire host image as given in Eq. (4). This is in contrast to the methods [Chandra (2002)] where the information about the watermark is embedded only in \( n \) values of the original image.

### 3 Numerical simulations for gray scale image

In Fig. (1) we show the original Lena image, the watermark in Fig. (2), the watermarked image in Fig. (3) and the extracted watermark in Fig. (4). All the images are arrays of size 128 \( \times \) 128 and \( \lambda = 0.2 \). In Fig. (5), we show the absolute difference between the original image and the watermarked image, i.e., \( \Delta_z = |\mathbf{Z}_c - \mathbf{Z}| \). The embedded watermark is completely invisible and we only see a texture of the original image. Even for larger values of \( \lambda \), we obtain a similar result. This means that even at 50% strength of watermark image in the host image, this method works well. As \( \lambda \to 0 \), the approximation that \( \mathbf{V} \) is an orthogonal matrix, i.e., \( \mathbf{V} \mathbf{V}^T \approx \mathbf{I} \) gets better, is shown by the linear graph of diagonal elements in Fig. (6).

In order to obtain a global picture of how this scheme performs as the scaling factor \( \lambda \) is increased, we compute the root-mean-square error (RMSE) and signal-to-noise ratio (PSNR) for the image difference matrix \( \Delta_z \) for
Fig. 6. Linear graph of diagonal element of the resultant vector $(V V^T)$ for each value of $\lambda$. If $\delta_{x,y}$ represent the elements of $\Delta z$, then the RMSE is defined as,

$$
\varepsilon = \sqrt{\frac{1}{mn} \sum_x \sum_y \delta_{x,y}^2}.
$$

For the case of square image arrays considered in our simulations, $m = n$. We also define the Peak Signal-to-Noise Ratio (PSNR) as,

$$
p = 10 \log_{10}(\max_{x,y}/\varepsilon).
$$

In Fig. (7), we show the $p$ and $\varepsilon$ as a function of $\lambda$. In this figure, the scaling factor $\lambda$ increases, there is an approximately linear increase in error as is to be expected. Beyond about $\lambda = 1.2$, the watermarked image becomes highly noisy. RMSE can be interpreted as the average error per pixel and it is seen that the error is only a small fraction of the pixel values.

4 Robustness of the algorithm

An important property of the watermarking algorithms is that they should be robust against several kinds of attacks. In this section, we show the results for (i) robustness against additive noise (ii) cropping (iii) JPEG compression.

4.1 Additive Noise

The noisy image $Z'$ can be represented as,

$$
Z' = Z + G
$$

where $G$ is a random matrix of same order as $Z$ with entries drawn from a standard Gaussian distribution $N(0, \sigma)$, where $\sigma$ is the variance.

Fig. 7. Error between the original Lena image and the watermarked image (squares). Error between the watermark and extracted watermark (circles). (a) PSNR ($p$) and (b) RMSE ($\varepsilon$) as a function of $\lambda$.

Fig. 8. Noisy watermarked image with $\lambda = 0.2$ (left) and recovered watermark (right). The PSNR error between watermark and the extracted watermark is $p = 16.3$ for $\sigma = 0.04$. 
Fig. 9. PSNR between the watermark and extracted watermark for different variance ($\sigma$).

Fig. (8) shows the result of extracting watermark from the noisy Lena image. Our simulations indicate that the image difference between watermark image and the one extracted from noisy image correlate well for a large range of $\lambda$. Fig. (9) shows PSNR between the watermark and extracted watermark for different variance ($\sigma$). It clearly indicates that as the variance of the Gaussian noise is increased the corresponding PSNR value decreases thereby leading to a deterioration in the quality of the extracted image.

4.2 Cropping

Cropping is the process of removing certain parts of an image. Here, we subject the original Lena image to cropping and the cropped image is displayed in Fig. (10). The extracted watermark is also displayed alongside. We recall that in this method the information about the watermark is stored in every part of the original image. When the watermarked image is cropped and subjected to an extraction procedure, then the watermark from the uncropped part survives. By design, there is some loss of information. However, the surviving part of the extracted watermark is very similar to the original watermark.

Fig. 10. Cropped Lena image (left) and the recovered watermark (right).

4.3 JPEG Compression

JPEG is one of the image encoding schemes that is currently popular [Wallace (1991)]. It is a lossy compression technique that uses discrete cosine transform or the wavelet transform (2000 standard) as the work horse that ensures a compressed image [Christopoulos et al. (2000)]. We show that our watermarking scheme is robust against JPEG compression to a great extent. In figures (12-15), we show the watermark extracted from JPEG compressed Lena image with various quality factors. Briefly, JPEG quality factor is an indication of the distortion, such that 100% quality factor corresponds to least distortion. It is worth mentioning here that, in our extraction algorithm as the JPEG quality increases the corresponding PSNR values also increase leading to a better extracted image. This fact is depicted in Fig. (11). This, in turn, corresponds roughly to the amount of information retained after wavelet or
DCT decomposition. In Fig. (3), we apply our watermark embedding algorithm with $\lambda = 0.2$ and we see that even at 50% and 30% JPEG quality factors, the extracted watermark carries a reasonable resemblance of the original watermark image. The PSNR values of $\Delta_w = |W - \tilde{W}|$ are provided as caption and as expected they decrease with the decrease in quality factor. Hence, the SVD based algorithm represented by equations (2-5) is robust against JPEG compression.

5 Simulations for color image

In Fig.(18), we show the simulations for the color images of Lena and the baboon. Clearly, at $\lambda = 0.02$, the watermarked image does not show any sign of the underlying watermark and it is invisible. The watermarking and extraction algorithm presented in sections (2.1, 2.2) can be applied to color images as well. Since most of the robustness can be generalized in a straightforward way from those for the grey scale images, we do not show the results of those tests here. However, we stress that the results of the robustness test are similar to the ones for the grey scale images presented above.

6 Conclusions

We have presented an algorithm for digital watermarking using singular
value decomposition in the domain of the singular vectors of the image matrices. We have implemented our method for gray and color images and shown that the technique is robust against noise, cropping and JPEG compression. The advantage of our algorithm, as opposed to other SVD based techniques, lies in the fact that most of the previous methods rely on the singular values of the image matrix, thereby the information content of an $n \times n$ image was translated into just $n$ values. In our approach, we use the scaled singular vectors to encode the watermark. The technique also inherently contains a level of security against the hacking of the watermark.

The algorithm presented in the manuscript is not closed. In fact there can be certain modification to the algorithm such that various other aspects can be included. For example, one can decrease the computational time of embedding and extracting the watermarks by a segmentation of the image. Then the algorithm can be applied to these various segments individually. Apart from this the embedded image can be encrypted thereby enhancing the security of the image content. Here, not only image data, but also other data can be embedded in a cryptic form. Finally, it must be mentioned that in the present algorithm the scaling factor $\lambda$ is not wholly integrated into the algorithm, meaning if one does not possess the value $\lambda$ then, extraction of the image becomes difficult. To do away with this one can include the scaling factor inside the image matrix itself. Currently the above mentioned features are under investigation and we hope to report them soon.

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