The interplay of hadronic amplitudes and Coulomb phase in LHC measurements at 13 TeV

A. K. Kohara a, E. Ferreira b, and M. Rangel b
a Centre de Physique Théorique, École Polytechnique, CNRS, Université Paris-Saclay, F-91128 Palaiseau, France
b Instituto de Física, Universidade Federal do Rio de Janeiro C.P. 68528, Rio de Janeiro 21945-970, RJ, Brazil

Detailed analysis of the measurements of differential cross sections in the forward region of pp elastic scattering at 13 TeV in LHC is performed. The structures of the real and imaginary parts of the scattering amplitude are investigated, both requiring exponential and linear factors. Representations of the data in different conditions are compared, investigating the role of the Coulomb-nuclear interference and satisfying predictions from dispersion relations. The additivity of nuclear and Coulomb eikonal phases to determine the interference phase in pp scattering is submitted to a comparison with assumption of phase equal to zero. The structures of the real part of the scattering amplitude under the two assumptions are examined, and direct comparison is performed with the information extracted from the data. The alternative criteria for the treatment of the interference phase lead to different values for the $\rho$ parameter between 0.1 and 0.13 while $\sigma$ differ by 0.18 mb, and it is shown that the available data cannot discriminate the two choices. As an alternative study, we also present the results for the amplitude parameters and $\chi^2$ values when constant values of 2 and 3 radians are subtracted from the analytical expression for the interference phase.

I. INTRODUCTION AND FORMALISM

Measurements of elastic pp scattering at 13 TeV have been presented with emphasis by the experimental group on the determination of the $\rho$ parameter. We here analyse the same data with a fundamental framework, investigating the structure of the real and imaginary parts of the complex elastic amplitude. We stress our view that in the analysis of elastic data it is essential to obtain clearly the identification of the real and imaginary parts of the complex amplitude

$$T(s,t) = T_R(s,t) + i T_I(s,t).$$

The quantitative, as much as possible model-free, description of the structure of the amplitudes provides essential connection between measurements and possible theoretical interpretation. The foundations of the strong and electromagnetic interactions (unitarity, causality, dispersion relations) deserve to be assumed as valid until clear deviation is imposed by the data. In the laboratory, measurements are made of production rates $dN/dt$ and $d\sigma/dt$, while the identification of the two parts in the sum

$$\frac{d\sigma}{dt} = \frac{d\sigma_R}{dt} + \frac{d\sigma_I}{dt} = (hc)^2[(T_I)^2 + (T_R)^2]$$

is not at all trivial, depending on analytical forms and phenomenological models to be submitted to detailed analysis. This assertion seems obvious, but it is important to avoid ambiguities and misunderstandings of essential points in the interpretation of the data.

The disentanglement of the two terms in the observed modulus $d\sigma/dt$ is the crucial task. At each energy, parameterizations must exhibit clearly the properties of magnitudes, signs, slopes and zeros of the real and imaginary parts. Complementary support, as dispersion relations and connections with analyses at other energies, give important clues and control. The intervention of the electromagnetic interactions must be treated coherently with a proposed analytical form for the nuclear part, and the role of the phase of the Coulomb-Nuclear Interference (CNI) must be investigated. The determination of scattering parameters require that analytical forms for the amplitudes be written explicitly and checked for reliability.

In the present study of forward data, each part of the amplitude is written with an exponential factor with a slope, multiplying a linear term in $t$, thus with three parameters. We consider that this is (almost) a model free construction, as these analytical forms are necessary and sufficient to describe the properties of the nuclear parts and yet are able to describe forward data with accuracy. The six parameters are studied using fits to data with appropriate statistical control. The partial contributions to the differential cross section are written in the forms

$$\frac{d\sigma_R}{dt} = \pi \frac{(hc)^2}{4\pi} \left[ \frac{\sigma(\rho - \mu t)}{4\pi (hc)^2} e^{B_{Rt} t/2} + F^C(t) \cos(\alpha \phi) \right]^2$$

and

$$\frac{d\sigma_I}{dt} = \pi \frac{(hc)^2}{4\pi} \left[ \frac{\sigma(1 - \mu t)}{4\pi (hc)^2} e^{B_{It} t/2} + F^C(t) \sin(\alpha \phi) \right]^2,$$
where \( t \equiv -|t| \), \( \alpha \) is the fine structure constant and \((\hbar c)^2 = 0.3894 \text{ mb GeV}^2\). \( F^C(t) \) and \( \alpha\phi(t) \) account for the proton form factor and phase of the Coulomb-nuclear interference.

In Eqs. (3.3) we have the amplitudes

\[
T_R(t) = T_R^N(t) + T_R^C(t) , \quad T_I(t) = T_I^N(t) + T_I^C(t) ,
\]

with separate terms

\[
T_R^N(t) = \frac{\sigma(\rho - \mu_R t)}{4\sqrt{\pi} \hbar c^2} e^{B_R t/2} , \quad T_R^C(t) = \sqrt{\pi} F^C \cos(\alpha\phi)
\]

and

\[
T_I^N(t) = \frac{\sigma(1 - \mu_I t)}{4\sqrt{\pi} \hbar c^2} e^{B_I t/2} , \quad T_I^C(t) = \sqrt{\pi} F^C \sin(\alpha\phi) .
\]

The normalization (optical theorem) and \( \rho \) parameter are

\[
\sigma(s) = 4\sqrt{\pi}(\hbar c)^2 T_I^N(s, t = 0) , \quad \rho = \frac{T_R^N(0)}{T_I^N(0)} .
\]

With positive \( \rho \) and negative \( \mu_R \) in pp at high energies \cite{2}, there is a zero in the real amplitude, namely Martin’s zero \cite{2}, located at

\[
t_R = \frac{\rho}{\mu_R} ,
\]

while, with negative \( \mu_I \) the imaginary part points to a zero with \( t \) outside the forward range, responsible for the dip in the differential cross section \cite{3}.

The derivatives of the nuclear amplitudes at \( t = 0 \)

\[
D_I = \frac{d}{dt} \log T_I^N(t) \bigg|_0 = \frac{1}{2} [B_I - 2\mu_I] = \frac{1}{2} B_I^\text{eff},
\]

\[
D_R = \frac{d}{dt} \log T_R^N(t) \bigg|_0 = \frac{1}{2} [B_R - 2\mu_R/\rho] = \frac{1}{2} B_R^\text{eff}
\]

are related through the dispersion relations for slopes \cite{3}.

The average slope calculated directly from \( d\sigma/dt \) is the quantity

\[
B = \frac{1}{2} \left[ \frac{d}{dt} (d\sigma/dt) \right]_0 = \frac{1}{1 + \rho^2} [B_I^\text{eff} + \rho^2 B_R^\text{eff}] .
\]

The parameters \( \mu_I \) and \( \mu_R \), with their roles of pointing towards zeros in the amplitudes, are very important for the accurate description of the forward elastic data \cite{2,3}.

We thus have the framework necessary for the data analysis, with clear identification of the amplitudes and of the role of the six free parameters \( \sigma, B_I, \mu_I, \rho, B_R, \mu_R \).

In the range of the data, the real part \( d\sigma_R/dt \), that contains the \( \rho \) parameter, is about 1/100 of the imaginary part \( d\sigma_I/dt \), and evidently the determination of parameters of the real part requires neat subtraction of the 99% magnitude due to the imaginary part. Although this separation is not explicitly identified in the fitting procedure exhibited in the experimental paper \cite{1}, it occurs inside the fitting algorithm. Thus the value of \( \rho \) informed as “best fit” results from a delicate mathematical separation, that must be clearly exhibited. There is a crucial role of the Coulomb interference phase in the extraction of parameters, that we investigate in detail. In particular, as we did in our previous analysis of LHC data at 7 and 8 TeV \cite{3}, to establish a reference, we compare results including the conventional interference phase \( \phi \), based on the assumption of direct addition of the nuclear and Coulomb eikonal phases, with results obtained with phase put equal to zero.

The assumption of additivity of eikonal phases in the superposition of interactions is believed to be successful in Glauber type calculations of hadronic collisions with nucleus, where addition is made of interactions of similar nature (superposition of strong interactions of the incident hadron with the nucleons of the nucleus). In the description of pp elastic scattering the interference occurs between nuclear and electromagnetic forces that act on very different ranges. There is obvious possibility that in this case the addition of eikonal phases is unrealistic, particularly at high energies. On the other hand, we can question about the relativistic derivation of the Coulomb phase in the pioneering works of Solov’ev \cite{6} and West-Yennie \cite{7}, where the authors calculate the Coulomb phase using the simplest Feynman diagrams. The results obtained provide phases with the same sign and similar magnitudes compared to the eikonal Coulomb phase. However it is important to stress that these derivations were in the pre-QCD era, without concept of quarks and gluons. Since the quarks couple with photons and gluons it is hard to separate strong and electromagnetic interactions, specially at high energies where the gluon density increases within the hadrons and nonlinear effects dominate the interactions. In purely hadronic terms, Feynman diagrams of higher order include proton excitations (\( N^* \)) and resonances (\( \Delta \)), and the pure additivity of separate electromagnetic and nuclear amplitudes is not satisfied. Thus there are complicated situations, and, in a phenomenological treatment \cite{3}, in the present letter we investigate the amplitudes entering in the calculation of \( d\sigma/dt \) with interference phase put equal to zero, and also examine the influence of constant (non \( t \)-dependent) displacements in the values of the phase.

\[\text{II. RESULTS}\]

The results of the analysis of the data in the \( |t| \) range 0.0008 - 0.1996 GeV\(^2\) are presented in Table I. The headings of the table indicate the quantities determined in fits, namely the six parameters \( \sigma, B_I, \mu_I, \rho, B_R, \mu_R \). A binned maximum likelihood fit is performed by MI-NUIT through the RooFit library available in the software toolkit ROOT 6.14/04 \cite{8}. The analysis accounts
for statistical and systematic uncertainties and for correlations. However, since the values of \( \chi^2 \) do not change much compared with the statistical uncertainties only, we show in the table only the statistical errors, that are needed for the determination of the parameters of the amplitudes.

As in the previous paper for 7 and 8 TeV [2], we identify in the 13 TeV data the real and imaginary amplitudes, and extract from the data the information on the parameters. The results for the real part are compared with predictions for \( \rho \) and for amplitude derivative \( D_R \) from dispersion relations [3]. Using the amplitudes written in Eqs. (6,7), we compare calculations including Coulomb interference phase and with phase put equal to zero. We also show the fitting parameters assuming \( \rho = 0.131 \) suggested by dispersion relations based on usual parametrization of cross section data [3] and putting zero phase. It is important to remark that in the inputs for the dispersion relations we found no need for inclusion of odd terms, since the total cross section data are not sensitive to these contributions. We observe important differences in the values obtained for the parameter \( \rho \), while no significant differences between the \( \chi^2 \) values are found, but it is interesting that \( \chi^2 \) values tend to be smaller for zero phase.

In order to isolate possible influences of mathematical nature, in the numerical work the Coulomb phase is treated with two different methods: using the implicit expression in Eq.(17) of the experimental paper [11] that is called KL phase [3], and using appropriate analytical expressions [2], called KF phase , that are a generalization of Cahn’s calculation [10] appropriate for amplitudes with independent exponential and linear factors. As shown in Table [1] the values of parameters are practically the same in the two cases, and this seems natural, since the implicit interference phases are nearly the same for very small \(|t|\), dominated by the term in \( \log(-t) \) present in all phases based in the sum of eikonals. Thus numerically the two methods of treating the calculation with interference phase (KL and KF) are here equivalent.

There is an important difference of 0.16-0.18 mb in the values of \( \sigma \) obtained with and without contribution of phase. This difference is only due to the pure Coulomb contribution \( T^C_I \) to the imaginary amplitude \( T_I \) in Eqs. (11,17). The effects are obvious: \( F^C \sin \alpha \phi \) is negative (while \( T^N_I \) is positive), and its presence forces higher \( \sigma \) to fit the data maintaining the same \( d\sigma/dt \) at the origin. With higher value for \( \sigma \) and \( d\sigma/dt \), the value of \( \rho \) in \( T_R \) must be smaller to fit the measured \( d\sigma/dt \) near the origin. Thus: positive phase in \( \sin \phi \) causes smaller \( \rho \), and determination of \( \rho \) goes influenced by the assumption of the presence of the Coulomb-nuclear interference phase.

We remark that the value of the derivative \( D_R \) of the real part, given by \( D_R \) in Eq. (10), is connected with \( D_I \) and satisfies the prediction by the Dispersion Relations for Slopes [3]. This indicates that the true interference phase may be smaller than the calculation based on the additivity of eikonals. Thus we observe that the much discussed determination of the value of the parameter \( \rho \) as 0.1 is consequence of the assumption of the interference phase determined by the additivity of nuclear and Coulomb eikonal phases. The question is not about details in calculation of the phase, but rather it is whether the assumption of sum of Coulomb and nuclear eikonal phases is justified in pp scattering at high energies. We show below that a test of this question requires data much more accurate than is presently available.

In Fig. (1) we show how \( \rho \) and phase affect the \(|t|\) structure of the real amplitude. We illustrate the superposition of the quantities \( T^N_R \) and \( -T^C_R \) that appear in the real part \( d\sigma/dt \) of the differential cross section. We remark that \( d\sigma/dt \) is zero in the points where \( T^N_R \) and \( -T^C_R \) cross each other, and this causes a marked dip in \( d\sigma/dt \). Depending on the value of \( \rho \) the curves may not cross, just approaching each other, with a marked reduction in \( d\sigma/dt \) in this region. The two situations are illustrated in the top and bottom parts plots of the figure. In the RHS plots we show \( d\sigma/dt \) for the two values of \( \rho \), without and with \( \phi \), illustrated in the LHS.

To exhibit how the separation of real and imaginary parts appear in the data, we introduce in Fig. (2) plots of the ratio \( T^2_R/T^2_I \) against \(|t|\), where

\[
\frac{T^2_R}{T^2_I} = \frac{|T^N_R(t) + \sqrt{\pi} F^C(t) \cos(\alpha \phi)|^2}{|T^N_I(t) + \sqrt{\pi} F^C(t) \sin(\alpha \phi)|^2}.
\]

The figure shows how the separation is nearly a linear function of \(|t|\) in the region where the ratio cancels \( T^2_R/T^2_I \) cancels. The ratio conveniently cancels most of the contributions to other terms plots of the figure. In the RHS plots we show \( d\sigma/dt \) with separation of the imaginary part. The ratio conveniently cancels most of the normalization uncertainty of \( d\sigma/dt \). The figure shows plots for the extracted quantity in the two investigated cases, and we observe the enormous difficulty of the measurements to reach a statistical precision sufficient to distinguish \( t \) dependences of the magnitude of the real part. It is thus difficult to confirm the determination of \( \rho \) at 0.1, that disagrees with the expectation from even signature dependences of the magnitude of the real part.

A. Displacements by subtracting constants in the phase

obtained from the \( d\sigma/dt \) data with separation of the imaginary part. The ratio conveniently cancels most of the normalization uncertainty of \( d\sigma/dt \). The figure shows plots for the extracted quantity in the two investigated cases, and we observe the enormous difficulty of the measurements to reach a statistical precision sufficient to distinguish \( t \) dependences of the magnitude of the real part. It is thus difficult to confirm the determination of \( \rho \) at 0.1, that disagrees with the expectation from even signature dependences of the magnitude of the real part.
TABLE I. Results of fittings of the 138 points of the Totem measurements at 13 TeV in the $-t$ range from 0.0008 to 0.1996 GeV$^2$. The Coulomb interference phase is calculated according to Kundrát-Lokajíček [9] and KF [2], and also phase is put equal to zero. The quantity $B_{I}^{\text{eff}}$ represents $B_I - 2\mu_I$ that is connected with the derivative $D_I$ of the imaginary amplitude at the origin according to Eq. (10) and is input for dispersion relation for slopes. Attention must be given to the apparently small changes in $\sigma$ (with and without phases), as they are direct cause of the changes in $\rho$, according to explanation given in the text.

| $\phi$ (mb) | $\sigma$ | $\rho$ (mb) | $B_I$ (GeV$^{-2}$) | $B_R$ (GeV$^{-2}$) | $\mu_R$ (GeV$^{-2}$) | $\mu_I$ (GeV$^{-2}$) | $t_R$ (GeV$^2$) | $B_{I}^{\text{eff}}$ (GeV$^{-2}$) | $\chi^2$/ndf |
|-------------|----------|-------------|-------------------|-------------------|---------------------|---------------------|-----------------|---------------------|----------------|
| KL          | 111.82±0.06 0.099±0.005 16.07±1.00 22.68±0.91 -3.78±0.37 -2.27±0.47 -0.026±0.004 20.71±1.20 126.48/132=0.958 |
| KF          | 111.84±0.06 0.097±0.005 16.13±1.33 22.72±1.19 -3.76±0.47 -2.26±0.62 -0.026±0.004 20.75±1.34 126.83/132=0.961 |
| 0           | 111.66±0.06 0.125±0.005 15.85±1.19 22.65±1.11 -3.84±0.44 -2.31±0.55 -0.033±0.004 20.47±1.22 123.43/132=0.935 |

TABLE II. Results of fittings of the 138 points of the Totem measurements at 13 TeV in the $-t$ range from 0.0008 to 0.1996 GeV$^2$. The Coulomb interference phase is calculated according to KF [2] subtracting constant values 2 and 3 radians. Attention must be given to the apparently small changes in $\sigma$, since they are direct cause of the changes in $\rho$.

| $\phi$ (mb) | $\sigma$ (fix) | $\rho$ (mb) | $B_I$ (GeV$^{-2}$) | $B_R$ (GeV$^{-2}$) | $\mu_R$ (GeV$^{-2}$) | $\mu_I$ (GeV$^{-2}$) | $t_R$ (GeV$^2$) | $B_{I}^{\text{eff}}$ (GeV$^{-2}$) | $\chi^2$/ndf |
|-------------|----------------|-------------|-------------------|-------------------|---------------------|---------------------|----------------|---------------------|----------------|
| KF-2        | 111.67±0.06 0.131 (fix) 15.78±1.53 22.79±1.43 -3.96±0.53 -2.35±0.70 -0.033±0.004 20.48±1.60 125.34/133=0.942 |
| KF-3        | 111.58±0.06 0.119±0.005 15.92±1.27 22.96±1.17 -3.73±0.47 -2.26±0.59 -0.032±0.004 20.47±1.63 126.61/132=0.959 |

Condition I - all six parameters free

Condition II - $\rho$ fixed by even signature dispersion relations; phase zero $\phi = 0$
FIG. 1. Study of the properties of the real amplitude $T^N_R(t)$ and its superposition with the Coulomb amplitude $T^C_R(t) = \sqrt{\pi}F_C(t) \cos \alpha \phi(t)$ in calculations with and without interference phase $\phi(t)$. Since $T^N_R$ is positive in the forward range, while $T^C_R$ is negative, the plots show the effects of the possible cancellations, or proximity, of the two quantities. The lines in the plots are calculated with the solutions given in Table I for the cases of phase zero ($\rho = 0.125$) and phase non-zero KF ($\rho = 0.100$). Since the quantity $\alpha \phi$ is very small, the lines for $T^N_R(t)$ are similar with and without phase. On the contrary, the line for $T^C_R(t)$ in the plots at the LHS, is lower when the phase is present, because $\rho$ is smaller. The effect of the influence of the phase is dramatic. With zero phase, there are two zeros in $T^N_R + T^C_R$, located at $|t_0| = 0.0068$ and $|t_1| = 0.0245$ GeV$^2$; these cancellations appear as dips in the quantity $d\sigma/dt$ as shown in the RHS at the top. On the other hand, with the presence of the phase, with smaller $\rho$, illustrated in the plots of the bottom, there is no cancellation in the sum $T^N_R + T^C_R$, but rather an approximation of the $T^N_R$ and $-T^C_R$ lines, causing a minimum in $d\sigma/dt$; this case is illustrated in the RHS of the lower part of the figure. If the data are available with high precision, the two situations can be distinguished. The reality with the present data is shown in the next figure.
FIG. 2. Study of the real part of differential cross sections of the 13 TeV data. The lines represent the squared ratios of the parametrized forms of $T_R$ and $T_I$ given in Eq. (12). The points are obtained as in Eq. (13): the data points of $d\sigma/dt$ minus the calculated imaginary background $T_I^2$, divided by the same.

FIG. 3. Plot of the $t$ dependence of the phase $\phi$ (KF), together with representations of solutions with changes in the expression of the phase subtracting constant (no $t$ dependence) values of 2 and 3 radians. In the range of data, the phase $\phi$ (KF) is dominated by a log(-$X$ $t$) term where $X$ depends on the parameters of the amplitudes and of the proton form factor. This term has a zero in the middle of the data range, with a maximum in $\cos \phi$. As explained in the text, the changes in $\rho$ are determined mainly by the changes in the imaginary part and in the total cross section, caused by differences in $\sin \phi$. 
III. REMARKS

With amplitudes given by Eqs. (3) and (4), we analyse the 13 TeV data to investigate the interplay between Coulomb interference phase and the parameters of the amplitudes, with particular attention to the $\rho$ parameter of the real part. We stress that $\rho$ and $\sigma$ are parameters fit to data using proper analytical forms of the amplitudes in the forward range, and are obtained through limits as $t \to 0$.

To investigate the role of the Coulomb interference phase, we calculate with the usual construction based on additivity of eikonal phases, and also obtain results with phase put equal to zero.

In Fig. 1 we exhibit the superposition of the hadronic and Coulombic contributions to the real amplitude $T_R = T_R^N + T_R^C$ of Eqs. (3,4) in the low $|t|$ region, comparing the calculations with and without phase of Table 1. Since $T_R^N$ is positive and $T_R^C$ is negative, they may cancel in two points (case of larger $\rho=0.125$) as in the top part of the figure, or they just pass close (case of smaller $\rho=0.100$) as in the bottom part of the figure. In the RHS of each case we show the effects of the two situations on the real part of cross section $d\sigma_R/dt$. In a very close scale there is a marked difference between the two cases. The results for the real part extracted from the data presented in Fig. 2 show that the determination of $\rho$ and the test of the phase influence require much more data than it is available. To identify the dips or minimum in the data for the real part it is necessary to improve the statistics and the regularity of the original $d\sigma/dt$ data.

To avoid doubts about the numerical procedures, as a test of the correct treatment of the influence of the phase, Table I shows that as far as $\chi^2$ and parameter values are concerned, the fittings with phase KL treated using Eq.(17) of the experimental paper [1] (where the phase is not represented by resolved analytical explicit form but is evaluated in each step of the fitting procedure), and the phase KF treated with the explicit calculation [2] are equivalent. The table shows that, as far as $\chi^2$ values are concerned, the fits with zero phase may be considered equivalent to those with included phase, but the values of the parameters are different. Even the calculation with fixed $\rho = 0.131$ suggested by dispersion relations and zero phase is statistically equivalent to those with free $\rho$. There is important difference in the first decimal digit in the values of $\sigma$, which is connected with the difference in the values of $\rho$.

It is remarkable that our previous analysis at 7 and 8 TeV [2] presents similar features.

The imaginary part makes a very high contribution of 99%, so that $d\sigma/dt$ must be measured with precision better than 0.1% in order to discriminate between models with and without phase. In particular, $\sigma$ must be obtained with error less than 0.1 mb, namely error in the second decimal digit in mb.

Values of $t$ where the interference with the Coulomb force has important local effects (the minimum and the dips shown in Fig. [1], namely $|t|$ from 0.005 to 0.05 GeV$^2$, appear as more important than a forward range below 0.001 GeV$^2$. This information may be useful for the experimental effort.

To clarify with more detail the connection between the structure of the hadronic amplitudes and the phase of the Coulomb-nuclear interference, in subsection II A we present in Table [1] and Fig. [3] results obtained with changes in the expressions for the phase subtracting constant values 2 and 3 radians.

The parametrization of the amplitudes in equations (15) and (16) of the experimental paper [1] assumes that real and imaginary parts run parallel in their $t$ dependences. However, the ratio $T_R(t)/T_I(t)$ is not constant, dispersion relations for slopes say that $B_R \neq B_I$, $T_R$ has a zero (Martin zero) interior to the range of the data. The parametrization of the experimental paper is not convenient also in the imaginary part, that behaves unrealistically for higher $|t|$, while it should point towards a zero around 0.5 GeV$^2$.

Importance of the real part at high energies and the theory for the interference phase

The significance of the real part of the elastic amplitude for possible signs of new physics was pointed out [11] in 2005, before LHC operation. The deviation from conservative dependence with the energy may signify violation of principles that support dispersion relations, or induce new ideas in the theory of the strong interactions. The possibility of a small value $\rho = 0.1$ at 13 TeV brought renewed interest in the dynamics generated by contribution of the odderon [12] that was proposed a long time ago. The Tel-Aviv group remarks [13] that contribution of odderon should be seen also at lower energies, and suggested that small $\rho$ can be investigated through screening (shadowing) and saturation effects in the gluon dynamics.

The standard parametrization of the total cross section, without odd term, in dispersion relations, provides a simple and efficient framework for the description of elastic scattering, which should not be abandoned before investigation of other possibilities for the phenomenology of elastic scattering. Recalling the epistemological principle of Occam’s razor (law of parsimony), it is not reasonable to look for a sophisticated dynamical model without investigating simpler mechanisms.

The present paper shows that the value of $\rho$ obtained from $d\sigma/dt$ may be consistent with the natural prediction 0.131 from dispersion relations, if the calculation of the Coulomb interference phase is modified, assuming or not the additivity of the eikonal phases of nuclear and electromagnetic forces.

The success of Glauber calculation of the superposition of hadronic interactions in hadron-nucleus collision is not necessarily a strong guideline for the interference of Coulomb and strong interactions. The caution is particularly obvious at high energies, with participation of sea quarks, gluons and active intervention of QCD vacuum.
in the nuclear part. We do not know how the electromagnetic field interferes with this nuclear dynamics. We then point out that at high energies the theory for the phase based on the additivity in the eikonal representation of the nuclear and electromagnetic interactions, must be tested against experiment.

The usual construction of the phase since Solovev and West-Yennie [6, 7, 10, 14] is based on the Born approximation, with diagram of one-photon exchange. The multi-photon exchanges are seen to be important even at low energies [15] and may modify strongly the calculation of the interference of electric and nuclear forces. On another hand, the proton form factor is now studied in terms of proton generalized structure functions [16] leading to the change in the traditional parametrization of the dipole form factor. The influence of the changes may apparently be not very large in the very forward range, but we must observe that in the study of the real part any small influence may become important.

The important questions raised by the value of $\rho$ (violation of fundamental principles, screening effects in the gluonic dynamics, existence of odderon) should stimulate more experiments of elastic pp scattering. Hopefully, richer data will be produced in 14 TeV runs at LHC that will occur after two years from now.

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