Chirp amplification of entropy growth in an open chirped-driven anharmonic oscillator

Shihui Zhang¹, Zhanyuan Yan and Yuanyuan Ren

Department of Mathematics and Physics, North China Electric Power University, Baoding, P. R. China

¹Email: physxzhang@foxmail.com

Abstract. The dependence of decoherence on chirp is investigated in an open one-dimensional chirped-driven anharmonic oscillator. The latter is perturbed by an Ohmic environment and the decoherence of the system is measured by the linear entropy. The entropy growth is found to be significantly dependent on the value of the chirp. Especially, the curve of the entropy growth versus the chirp value has a remarkable peak, near which the decoherence of the is greatly amplified. Further analyses shows that the enhancement of the entropy growth by the chirp is closely related to the influences of the chirped drive on the energy levels participating in the quantum evolution. This can be confirmed by the correspondence between the chirp dependence of the entropy growth, the quantum spectrum and the classical squared amplitude.

1. Introduction

Coherence is of great importance in quantum-enhanced technologies [1]. However, a quantum system will inevitably suffer decoherence, due to the perturbation of the environment. Decoherence is viewed as a major obstacle to the development of quantum technologies. Thus, it has received a mount of attentions from the end of the 20th century and is still of great interest [2-6]. Early studies have shown that decoherence is closely related to chaos [2]. Besides, it is also found that macroscopic quantum resonators [3] and universal decoherence exists in systems with gravitational potential [5, 6].

During the past decade, mechanical systems are emerging as good candidates for studying quantum effects and quantum-enhanced technologies [7-11]. In fields related to mechanical systems, driven anharmonic systems are of fundamental interests [8-14]. For a driven anharmonic system, it is chirped when the frequency of the external drive is time-dependent. Chirp is commonly used in classical sonar or radar systems. In optics, laser pulses exhibit chirp due to the material dispersion. As the driving frequency varies linearly with time, the chirp is linear. Linear chirp plays key roles in quantum ladder systems [15], optimal control of quantum systems [16] and quantum auto-resonance [17]. However, the effect of chirp on decoherence in driven anharmonic systems has not been reported before to our knowledge. This is the main incentive behind this paper.

The investigations are performed with a chirped-driven anharmonic oscillator, which has a quadric potential and is a typical driven nonlinear system. The system is perturbed by an Ohmic environment in high temperature and its quantum evolution starts with coherent state. Due to the perturbation from the environment, decoherence, which is measured by the linear entropy, increases with time during the quantum evolution. It is found that the growth of the linear entropy has significant dependence on the chirp value. Furthermore, the chirp-dependence curve of the entropy growth has a pronounced peak, near which the decoherence is significantly amplified. The chirp dependence of the entropy growth
can be attributed to the influence of the chirped drive on the energy levels involved in the quantum evolution. This can be confirmed by the correspondence between the chirp-dependence of the entropy curve and that of the quantum spectrum. Besides, correspondence is found between the chirp-dependence curve of the entropy growth and that of the classical mean squared amplitude. This is consistent with the argument that the chirp-dependence of the entropy curve can be attributed to the influence of the chirp on energy levels, since the classical mean squared amplitude can reveal the energy levels involved in the underlying classical dynamics.

2. Chirped-driven anharmonic oscillator and the linear entropy

When a quantum system is driven by external chirped drive, its Hamiltonian generally reads

\[ H = H_0 + \dot{\phi} \cos[\phi(t)]. \]  

(1)

In equation (1), \( H_0 \) stands for the Hamiltonian of the isolated system with

\[ H_0 = \frac{p^2}{2m} + \frac{1}{2} ma_0^2(aq^2 + bq^4), \]

(2)

where \( m \) is the mass, \( \omega_0 \), \( a \) and \( b \) are the parameters of potential. It is a nonlinear oscillator with a quartic potential, which may serve as a prototype of nano-mechanical or opto-mechanical devices [11-14]. The last term in equation (1), \( \dot{\phi} \cos[\phi(t)] \), denotes the chirped drive. Its phase

\[ \phi(t) = \omega_1 t + \gamma t^2/2, \]

(3)

where \( \omega_1 \) and \( \gamma \) are the central frequency and the chirp rate respectively. The chirp rate satisfies, \( \gamma = (\omega_2 - \omega_1)/T \), where \( \omega_1 \) is the starting frequency, \( \omega_2 \) is the final frequency and \( T \) is the width of the chirp pulse [18].

Here, the wave-function of the system at the time \( t \) is denoted by \(|\Psi(t)\rangle\). Its corresponding Wigner function reads

\[ W(q, p; t) = (2\pi \hbar)^{-1} \int dq' dp' W(q', p; t)\langle q' - q/2|\Psi(t)\rangle\langle q + q'/2|\Psi(t)\rangle. \]

(4)

When perturbed by the environment, the quantum state of the system will suffer decoherence. The environment considered here is a general Ohmic one [19]. Generally, the coupling of the quantum system to the environment is weak. In this case, the time evolution of the Wigner function follows a master equation [2, 19]. In high-temperature limit, it can be described as [19-21]

\[ \partial_t W = \{H, W\}_PB + \sum_{n=1}^{\infty} \frac{(-1)^n(\hbar/2)^{3n}}{(2n+1)!} \partial_q^{2n+1}W \partial_p^{2n+1}W + D\partial_p^2W, \]

(5)

where \( \{\}_{PB} \) is the Poisson bracket, the last term on the right-hand side arises from the coupling of the system to the environment and \( D \) is the diffusion coefficient. In natural units, \( D = 2\gamma T \), where \( \gamma \) is the relaxation rate and \( T \) is the temperature of the environment [19].

The decoherence of the system, which is induced by the environment, can be evaluated by the linear entropy. In the Wigner representation, the linear entropy can be expressed as [2, 22]

\[ S(t) = 1 - 2\pi \hbar \int dq \int dp W^2(q, p; t). \]

(6)

It can be rewritten as \( S(t) = 1 - Tr\rho^2(t) \), where \( \rho = |\Psi(t)\rangle\langle\Psi(t)| \) is the reduced density of the system. Its growth indicates the decay of the off-diagonal elements of the reduced density matrix of the system and thus corresponds to the loss of the quantum coherence [23]. Specifically, the decoherence increases as \( S \) increases from 0 to 1 \((0 \leq S \leq 1)\).

3. Dependence of the entropy growth on the chirp value

Following equation (1) to (6), the decoherence of the system (1) and its relations to the chirp are investigated. The quantum state of the system is initially prepared in coherent state, i.e.

\[ |\Psi(0)\rangle = e^{-|\alpha|^2/2} \sum_n^{\infty} (\alpha^n / \sqrt{n!}) |n\rangle, \]

(7)
where the orthonormal vectors $\{|n\rangle\}$ satisfy $a'\{a|n\rangle = n|n\rangle$ and $\alpha$ are phase-space coordinates with $\alpha = (m\alpha_0q_0 + ip_0)/\sqrt{2ma_0h}$. Its Wigner function is
\[
W(q, p; 0) = (\pi \hbar)^{-1} \exp[-\sigma_q^2(q - q_0)^2 - \sigma_p^2(p - p_0)^2].
\]
where $\sigma_q = \sqrt{\hbar / ma_0}$ and $\sigma_p = \sqrt{ma_0 \hbar}$. Coherent states play important roles and are key ingredients in quantum physics [24-27]. With the above coherent state as the initial state, we numerically investigate the decoherence during the quantum evolution of the chirped anharmonic oscillator by the spectral method [21]. The parameter values are $a = 2$, $b = 1/20$, $\delta = 2$, $D = 1/500$ and $q_0 = p_0 = 1$ (natural units are used here with $m = \hbar = \omega_0 = 1$).

The linear entropy of the system is initially equal to zero. During the quantum evolution, it grows with the time, due to the perturbation of the environment. Moreover, it is found that the growth rates of the linear entropy for different values of chirp are markedly different. This can be observed in figure 1(a). Figure 1(a) presents the time growth of the linear entropy for four typical values of chirp. From bottom to top, they correspond to $\gamma = 0$ (black solid line), $\gamma = 0.5$ (blue dotted line), $\gamma = 1$ (red dashed line) and $\gamma = 3$ (purple dash-dotted line). As can be seen figure 1, the increase of the linear entropy for $\gamma = 0.5$ is faster than those chirp value has great influence on the growth of the entropy.

Figure 1. (a) Linear entropy $S$ versus the time $t$. The values of the chirp $\gamma$ in (a) are 0 (blue dotted line), 0.5 (black solid line), 1 (red dashed line) and 3 (purple dash-dotted line). (b) Entropy growth versus the chirp value $\gamma$. $S_\tau$ in (b) indicates the growth rate of the entropy. The increment of the chirp value $\gamma$ in (b) is 0.1.

More detailed numerical investigations are made with the chirp increasing from 0 to 3. The results for the growth rate of the linear entropy are presented in figure 1(b). The growth rate of the linear entropy $S$ in figure 1(b) is characterized by $S_\tau$, i.e., the value of $S$ at the time $\tau$ ($\tau = 500$). The latter is proportional to the average growth rate $S_\tau$ reveals the growth rate of the linear entropy. As can be seen from figure 1(b), the value of $S_\tau$ varies significantly with the chirp value $\gamma$ and reach its maximum when $\gamma = 0.5$. Especially, its chirp-dependence curve has a pronounced peak, near which the value of $S_\tau$ is very close to its maximum. By contrast, the values of $S_\tau$ are relatively very small, as the value of the chirp is far from the peak. This means that the decoherence of the system can be greatly influenced by the chirp and is significantly amplified near the peak.

As is shown above, the increase of the line entropy $S$ arises from the enhancement of the decay of the off-diagonal elements of the reduced density of the system. The latter is induced by the interference between energy levels during the quantum evolution. For a driven anharmonic oscillator like (1), the presence of the chirp force can induce transitions between energy levels. Therefore, it can affect the interference between different energy levels and further influence the growth of the entropy. This can be seen from the following analytical investigations.
4. Entropy growth during the quantum evolution

The wave-function $\psi(q,t)$ of the system at the time $t$ can be represented in terms of the eigenfunctions of $H_0$, i.e.

$$\psi(q,t) = \sum_n c_n(t) \varphi_n(q),$$

where $\{\varphi_n(q)\}$ satisfy $H_0 \varphi_n(q) = E_n \varphi_n(q)$. Its corresponding Wigner function becomes

$$W(q,p;t) = \sum_{n,m} C_{nm}(t) w_{nm}(q,p),$$

with $C_{nm}(t) = c_n(t) c_m^*(t)$ and

$$w_{nm}(q,p) = \frac{2\pi \hbar}{(2\pi \hbar)^2} \int dp dq' \varphi_n(q-y/2) \varphi_m^*(q+y/2).$$

Without the chirp force and the perturbation of the environment, the evolution of the quantum state of the isolated system follows the Schrödinger equation

$$i\hbar \partial_t \psi(q,t)/\partial t = H_0 \psi(q,t).$$

Its corresponding quantum Liouville equation is

$$\partial W(q,p;t)/\partial t = \hbar_0 W(q,p;t),$$

where $\hbar_0 = (a^2 + 2b^2^3)(\partial/\partial \psi) - (\hbar^2/2)q (\partial^3/\partial \psi^3).$

The solutions of equation (12) and (13) for small $dt$ in the Heisenberg picture are respectively

$$\psi(q,t+dt) = \exp(-iH_0 dt/\hbar) \psi(q,t)$$

and

$$W(q,p;t+dt) = \exp(h_0 dt) W(q,p;t).$$

Substituting equation (9) into equation (14), it is easy to see that

$$c_n(t+dt) = e^{-iE_n dt/\hbar} c_n(t).$$

Accordingly, we have $C_{nm}(t+dt) = e^{-i(E_n-E_m) dt/\hbar} C_{nm}(t)$. That is, in terms of the energy-eigenfunctions of $H_0$, equation (15) can be expressed as

$$\exp(h_0 dt) W(q,p;t) = \sum_{n,m} \exp(-i\omega_{nm} dt) C_{nm}(t) w_{nm}(q,p),$$

where $\omega_{nm} = (E_n - E_m)/\hbar$. It corresponds to the evolution dependent on $H_0$.

In the presence of the chirped drive and the perturbation of the environment, the time evolution of the Wigner function follows equation (5). In this case, the evolution of the Wigner function in a small time interval $dt$ in the Heisenberg picture is

$$W(q,p;t+dt) = \exp[[h_0 + F(t) \frac{\partial}{\partial p} + D \frac{\partial^2}{\partial p^2}] dt] W(q,p;t),$$

where $F(t) = \delta \cos[\phi_0(t)]$. Since $dt$ is very small, equation (18) can be approximately expressed as

$$W(q,p;t+dt); \quad \exp[dt(F(t) \frac{\partial}{\partial p} + D \frac{\partial^2}{\partial p^2})] \exp(h_0 dt) W(q,p;t) + O(dt^2).$$

In equation (19), the evolution of the Wigner function is divided into two parts. One is only related to $H_0$, while the other is determined by the external influences (the chirped drive and the environment).

The two exponential functions on the right-hand side of equation (19) can be expanded into a Taylor series about $dt$. In this case, equation (19) can be rewritten as

$$\sum_{n,m} C_{nm}(t+dt) w_{nm}(q,p) \sum_{n,m} C_{nm}(t) w_{nm}(q,p) - i dt \sum_{n,m} C_{nm} \omega_{nm} w_{nm}(q,p)$$

$$+ \sum_{n,m} C_{nm}(t) [dt F(t) \frac{\partial}{\partial p} + dt D \frac{\partial^2}{\partial p^2}] W_{nm}(q,p) + O(dt^2)$$

(20)
by means of equation (10) and (17). According to equation (11) and the methods related to the Fourier Transform [28], we have
\[
\frac{\partial^n}{\partial p^n} w_{nm}(q, p) = (2\pi\hbar)^{-1} \int_{\mathbb{R}^2} dy e^{i q y} \varphi_n(q - y/2) \varphi_n^*(q + y/2) = (\frac{i}{\hbar})^n Y_{nm}^n(q, p),
\]
where
\[
Y_{nm}^n(q, p) = (2\pi\hbar)^{-1} \int_{\mathbb{R}^2} dy e^{i q y} \varphi_n(q - y/2) \varphi_n^*(q + y/2).
\]
By means of equation (21), equation (20) can be written as
\[
\sum_{n,k} C_{nk}(t + dt) w_{nm}(q, p) + \sum_{n,k} C_{nk}(t) w_{nn}(q, p) - i\hbar \sum_{n,k} C_{nk}(t) \omega_{nk} w_{nm}(q, p)
+ i\hbar dt F(t) \sum_{n,k} C_{nk}(t) Y_{nm}^n(q, p) - \hbar^2 dt^2 \sum_{n,k} C_{nk}(t) Y_{nm}^{n+2}(q, p).
\]
Since \[dq \varphi_n^*(q) \varphi_n(q) = \delta_{nk},\] we can easily obtain the orthogonality of \( w_{nm}(q, p) \) with equation (11), i.e.
\[
\int dq \int dp w_{nk}^*(q, p) w_{nm}(q, p) = (2\pi\hbar)^{-1} \delta_{nk} \delta_{nk'},
\]
where \( \delta_{nk} = \delta(n-k) \) and \( \delta_{nk'} = \delta(n'-k') \).

Multiplying equation (23) by \( w_{nk}(q, p) \) and integrating over \( q \) and \( p \), we can get the following equation by means of equation (24) and (25), i.e.
\[
C_{nk}(t + dt) C_{nk}(t) - i\hbar dt C_{nk}(t) \omega_{nk} + i\hbar^2 dt F(t) Z_{nk}(t) - \hbar^2 dt^2 Z_{nk}(t),
\]
where \( \omega_{nk} = (E_n - E_k)/\hbar \) and
\[
Z_{nk}(t) = \sum_{n,k} C_{nk}(t) \int_{\mathbb{R}^2} dz (z - z')^n \varphi_n(z) \varphi_n^*(z') \varphi_k(z) \varphi_k^*(z').
\]
Equation (26) can be rewritten as
\[
\frac{dC_{nk}(t)}{dt} = -i\{\omega_{nk} C_{nk} - \hbar^{-1} \cos[\phi_n(t)] Z_{nk}^n\} - D\hbar Z_{nk}^2,
\]
with \( dC_{nk} = C_{nk}(t + dt) - C_{nk}(t) \). Equation (28) describes the evolution of the components of the Wigner function. In the right-hand side of equation (28), the first item is for the free evolution and the second item is brought forth by the chirp force. The last item in equation (28), induced by the coupling to the environment, is responsible for the deocherenence.

Since the Wigner function is real, \( W'(q, p, t) = W(q, p, t) \). Thus, the entropy can be written as

Figure 2. Quantum spectra with (a) \( \gamma = 0 \), (b) \( \gamma = 0.5 \), (c) \( \gamma = 1 \) and (d) \( \gamma = 3 \). They are obtained with the same parameter values as in figure 1. The four spectra correspond to the four entropy curves in figure 1(a) one-by-one.
\[ S(t) = 1 - 2\pi h \sum_{k,k'} \sum_{n,n'} C_{k,n} C_{k',n'} \int dq dp w_{k,n}(q,p) w_{k',n'}. \]  

With equation (24), equation (29) becomes \[ S(t) = 1 - \sum_{k,k'} C_{k} C_{k'} \] and then

\[ \frac{dS}{dt} = -\sum_{k,k'} (C_{k} C_{k'} \frac{dC_{k'}}{dt} + C_{k'} C_{k} \frac{dC_{k}}{dt}). \]  

As is shown above by equation (28), without the presence of the chirp source and the diffusion, the time evolution of the component of the Wigner function \( C_{k} \) is only related to the energy levels \( \phi_k \) and \( \phi_{k'} \). However, the presence of the driving source induces energy level transitions and makes the evolution of \( C_{k'} \) dependent on the energy levels in additional to \( \phi_k \) and \( \phi_{k'} \). This is because \( Z_{k,k'}^i \) results in the connections between \( C_{k'} \) and the energy levels in additional to \( \phi_k \) and \( \phi_{k'} \), as given by equation (27). That is, the energy levels involved in the quantum evolution are influenced by the chirped driving source and thus can be influenced by the chirp value. On the other hand, it can be seen from equation (28) that the decoherence resulted by the last item depends on \( Z_{k,k'}^2 \). Similar to \( Z_{k,k'}^1 \), \( Z_{k,k'}^2 \) is closely related to the energy level transitions. Therefore, the decoherence can also affected by the chirped force, according to equation (30). This can be confirmed by the energy spectral analysis.

5. Influences of the chirp on entropy growth and quantum spectrum

According to the spectral method [21], the energy levels involved in the quantum evolution can be estimated by the Fourier spectrum of the correlation function \( \langle \psi(0) | \psi(t) \rangle \). The latter here can be estimated by \( R(t) = \int W(q,p;0)W(q,p;t) dq dp \). In figure 2, we illustrate four typical Fourier spectra of \( R(t) \) (hereinafter referred to as quantum spectrum) for \( \gamma = 0, \gamma = 0.5, \gamma = 1 \) and \( \gamma = 3 \). The values of the other parameters are the same as in figure 1. The four quantum spectrum in figure 2 correspond to the four entropy curves in figure 1(a). One can see from figure 2 that the width and density of the energy spectrum depend on the value of chirp. Furthermore, the denser and wider the energy spectrum is, the faster the entropy growth is, as can be found by comparing figure 1 and 2. This is consistent with the above conclusion that the value of the chirp influence the entropy growth via influencing the energy levels involved in the quantum evolution.

Indeed, the value of the correlation function reveals the ability of the wave-packet revival during the quantum evolution [29]. The more the energy levels involved in the quantum evolution are, the more difficult the revival of the wave-packet is. Thus, the average of the correlation function over time decreases with the increase of the energy levels involved during the quantum evolution. In this case, the average value of \( R(t) \) , i.e., \( R_A = \int R(t) dt \) , can reveal the energy levels involved during the quantum evolution. The smaller \( R_A \) is, the more the energy levels participate in the quantum evolution are. That is, the value of \( R_A \) decreases with the increase of the energy levels involved in the quantum evolution. The latter can be influenced by the value of the chirp, as mentioned above. Thus, the dependence of \( R_A \) on the chirp value should be consistent to some extent to the dependence of the entropy growth on the chirp value. This can be seen from figure 3(a). Figure 3(a) presents the chirp-dependence of \( R_A \). Comparing figure 1(b) and 3(a), good correspondence can be found between the chirp-dependence curve of \( R_A \) and that of \( S_{\gamma} \). This further supports the argument presented above.

As is shown above, the chirped driving source influences the decoherence via influencing the energy levels involved in the quantum evolution. Moreover, in the further investigations, a good correspondence is found between chirp-dependence curve of the entropy growth and that of the classical mean square amplitude. The latter reads

\[ A_{\gamma} = \tau^{-1} \int_0^\tau q^2(t) dt, \]  

(31)
where \( q(t) \) is obtained by classical Hamilton's equation with \((q_0, p_0)\) as the initial condition. It is also an average over the time scale.

The dependence of \( A_{CI} \) on the chirp value \( \gamma \) is presented in figure 3(b). They are obtained with the same parameter values as in figure 1. Clear correspondences can be seen between the chirp-dependence curves for the entropy growth and those for the classical mean square amplitude. These suggest the connections between the chirp dependence of the decoherence and that of the mean square amplitude of the underlying classical dynamics.

According to the power spectrum method [30], the energy levels involved in the classical evolution can be estimated by the power spectrum of \( q(t) \). \( q(t) \) is the position value of the underlying of the classical dynamics. According to Parserval’s theorem [31], the energy level transitions revealed by the power spectra of \( q(t) \) can be estimated by the classical mean square amplitude. This is because

\[
\tau^{-1} \int_0^\tau q^2(t) dt = \int P(\eta) d\eta,
\]

for a large \( \tau \). In equation (32), \( P(\eta) \) is the power spectrum of \( q(t) \) and \( \eta \) is the frequency with respect to \( q \). The integral of \( P(\eta) \) with respect to \( \eta \) can be used as an indicator for the energy levels involved in the classical evolution. It indicates the energy-level transitions participating in the quantum evolution. Accordingly, the variation of \( A_{CI} \) with the chirp value \( \gamma \) can reveal the chirp dependence of the energy level transitions involved in the quantum evolution, due to the classical-quantum correspondence. Thus, the correspondence between the curves of \( A_{CI}(\gamma) \) and \( S_\tau(\gamma) \) further confirms the influence of the chirped driven force on the entropy growth be means of energy levels involved in the quantum evolution.

6. Conclusions and discussions

In short, the influence of chirp on of the decoherence is investigated in a chirped nonlinear system. It is shown that the growth of the linear entropy is obviously dependent on the chirp value, which is closely related to the influences of the chirp force on the energy levels participating in the quantum evolution. This can be confirmed by the correspondence between the chirp-dependence curve of the entropy growth, the autocorrelation function and the classical mean squared amplitude. However, the relationship between the chirp and the decoherence has not been completely and fully clarified. This needs further investigation by future studies. Besides, the environment considered here is Ohmic in high temperature limit. The influence of the chirp on the decoherence in low temperature should also be explored in future researches. Nevertheless, the conclusions drawn here will hold true to some extent in the low temperature, due to the influences of the chirp on the entropy growth via influencing the energy levels. Thus, the investigations presented here may shed some lights on the decoherence in the chirped driven nonlinear systems.

**Figure 3.** (a) Average of correlation function \( R_A \) versus the value of chirp \( \gamma \) (the values of \( R_A \) are plotted in reverse scale for comparison purpose). (b) Classical mean squared amplitude \( A_{C_1} \) versus the chirp value \( \gamma \). The increment of \( \gamma \) and the parameter values are the same as in figure 1(b).
Chirp is widely used in fields related to information systems, such as optical fiber communication, radar systems and sonar systems [32]. In recent years, quantum radar is emerging as a promising application of quantum-enhanced technologies. Therefore, the chirp-amplification of the decoherence shown here may be of value to the further applications of chirp especially in quantum systems.

Acknowledgements
This work is support by the Fundamental Research Funds for the Central Universities in China (No. 2015MS78 and 2017MS165).

References
[1] Silverman M P 2008 Quantum superposition: counterintuitive consequences of coherence, entanglement, and interference Springer, Berlin
[2] Zurek W H 2003 Rev. Mod. Phys. 75 715
[3] Hu B L 2014 Journal of Physics: Conference Series 504 012021
[4] Mavadia S, et al. 2017 Nat. Commun. 8 14106
[5] Pikovski I, Zych M, Costa F, et al. 2015 Nat. Phys. 11 668
[6] Pfister C, et al 2016 Nat. Commun. 7 13022
[7] Vacanti G, et al 2013 Phys. Rev. A 88 013851
[8] Rips S, Kiffner M, Wilson-Rae I and Hartmann M J 2012 New J. Phys. 14 023042
[9] Aspelmeyer M, Kippenberg T J and Marquardt F 2014 Rev. Mod. Phys. 86 1391
[10] Peano V and Thorwart M 2004 Phys. Rev. B 70 235401
[11] Imboden M, Williams O A and Mohanty P 2013 Appl. Phys. Lett. 102 103502.
[12] Almog R, Zaitsev S, Shtempluck O and Buks E 2007 Phys. Rev. Lett. 98 078103
[13] Rips S, Wilson-Rae I and Hartmann M J 2014 Phys. Rev. A 89 013854
[14] Balling P, Maas D J and Noordam L D 1994 Phys. Rev. A 50 4276
[15] Murch K W, et al 2011 Nat. Phys. 7 105
[16] Amstrup B, et al 1993 Phys. Rev. A 48 3830
[17] Habib S, Shizume K and Zurek W H 1998 Phys. Rev. Lett. 80 4361
[18] Ravinder R P 2001 Mathematical Methods of Quantum Optics, Springer, Berlin
[19] Feit M D, Fleck J A and Steiger A 1982 J Comput. Phys. 47 412
[20] Grosshans F and Grangier P 2002 Phys. Rev. Lett. 88 057902
[21] Cerf N J, Leuchs G and Polzik E S 2007 Quantum information with continuous variables of atoms and light, Imperial College Press, London
[22] Lobo M, et al. 2008 Science 322 563
[23] Rao S S 2007 Vibration of continuous systems, Wiley, Hoboken
[24] Robinett R W 2004 Phys. Rep 392 1
[25] Dumont R S and Brumer P 1988 J Chem Phys. 88 1481
[26] Haykin S and Moher M 2006 An Introduction to Digital and Analog Communications, Wiley, Hoboken
[27] Rihaczek A W 1969 Principles of High-Resolution Radar, McGraw-Hill, New York