Theory of Semi– and Nonleptonic Decays of Heavy Mesons

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Abstract:
I review some of the recent developments in the theoretical description of weak inclusive decays of heavy mesons. The topics cover the value of $|V_{cb}|$ as extracted from semileptonic inclusive decays and a short discussion of the theoretical errors. I also present the results of a recent calculation of next–to–leading order corrections to nonleptonic inclusive B decays which allows an improved prediction of the semileptonic branching ratio of B mesons.

1 Outline of Theoretical Foundations

During the recent two years, the theoretical description of inclusive decays of heavy hadrons has experienced considerable progress. For quark masses $m_Q \gg \Lambda_{\text{QCD}}$, the well–known short–distance expansion technique yields an expansion in inverse powers of the heavy quark mass, the so–called heavy quark expansion (HQE) \[1\]. The starting point for the HQE of, e.g., the decay rate of a B meson into a final state $X$ is its representation as imaginary part of the relevant forward–scattering amplitude:

$$\Gamma(B \rightarrow X) = \frac{1}{m_B} \text{Im} i \int d^4x \langle B|T\mathcal{L}(x)\mathcal{L}(0)|B\rangle.$$ \hspace{1cm} (1)

Here $\mathcal{L}_W$ is the effective weak Lagrangian mediating the decay $B \rightarrow X$. As shown in \[1\], for a very heavy b quark mass $m_b$, a short distance expansion of Eq. (1) yields:

$$\Gamma(B \rightarrow X) = \frac{C^2_{\mathcal{F}} m_b^5 |\text{CKM}|^2}{192 \pi^3} \left\{ C_0^{(X)}(\alpha_s(\mu)) \frac{\langle B|\bar{b}b|B\rangle}{2m_B} + C_2^{(X)}(\alpha_s(\mu)) \frac{1}{m_b^2} \frac{\langle B|\bar{b}g_s \sigma_{\mu\nu} F^{\mu\nu} b|B\rangle}{2m_B} + O\left(\frac{1}{m_b^3}\right) \right\}. \hspace{1cm} (2)$$

Here CKM denotes the appropriate CKM matrix elements, the $C_i^{(X)}$ are short–distance Wilson–coefficients, which depend on the parton model process underlying the decay $B \rightarrow X$. $b$ is the b quark field in full QCD, $m_B$ is the mass of the B meson and $F^{\mu\nu}$ the gluonic field–strength tensor.

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Without going into too much details, let me mention just a few general features of the above expansion. First we remark that HQE strongly resembles deep inelastic scattering with the difference that the value of the expansion parameter $m_b$ is fixed and cannot be controlled by the experimenter, so that higher twist effects in $1/m_b$ are important. On the other hand, the corresponding hadronic matrix elements can be expressed as moments of a universal distribution function [2, 3] and are thus measurable, at least in principle, cf. [4].

The first term in the above series, $\langle B|\bar b b|B\rangle$, just reproduces the free quark decay process; non-perturbative corrections to that picture are suppressed by terms of $\mathcal{O}(1/m_b^2)$ or higher. There are no terms of order $1/m_b$, since all possible gauge-invariant operators of suitable dimension either vanish or can be reduced to $\bar b b$ by the equations of motion.

The Wilson coefficients $C_j^{(X)}(\alpha_s(\mu))$ encode the short distance behaviour of Eq. (1) and are calculable within perturbation theory; they depend on the part on content of $X$, the renormalization scale $\mu$ and the renormalization scheme. In particular, $C_0^{(X)}$ contains the radiative corrections to the free quark decay and has been studied for various processes [4, 5-4].

Let me also mention some words of caution. In order to separate clearly the expansion in $1/m_Q$ from the one in $\alpha_s$, the HQE has to be done in terms of the pole mass. This mass definition, however, is unphysical and contradicts the confinement property of QCD. It has been shown, that this contradiction reflects itself in an intrinsic ambiguity of the definition of the pole mass which is said to be caused by a “renormalon” [10]. Although it was shown in [11] that the renormalon cancels in the semileptonic rate, at least up to order $1/m_b^2$, there is still a number of questions to be answered as far as nonleptonic and exclusive decays are concerned.

2 Applications I: $|V_{cb}|$ from Semileptonic Inclusive Decays

One immediate application of the HQE is the extraction of $|V_{cb}|$ from semileptonic inclusive decays. The decay rate $\Gamma(B \to X_c e\nu)$, of the generic form of Eq. (2), contains to order $1/m_b^2$ five unknown parameters: $V_{cb}$, $m_b$, $m_c$ and two hadronic matrix elements:

$$2m_B \lambda_1 = \langle B|\bar b_v(iD)^2b_v|B\rangle,$$

$$6m_B \lambda_2 = \langle B|\bar b_v \frac{\sigma_{\mu\nu}F^{\mu\nu}}{2}b_v|B\rangle,$$

where $b_v$ is defined as $b_v = e^{im_{B\ast}v}\bar{b}v$ and $v_{\mu}$ is the four-velocity of the B meson.

Whereas $\lambda_2$ is directly related to the observable spectrum of beautiful mesons,

$$\lambda_2 \approx \frac{1}{4} (m_{B\ast}^2 - m_B^2) = 0.12 \text{ GeV}^2,$$

the quantity $\lambda_1$ is difficult to measure, cf. [4]. Physically, $-\lambda_1/(2m_b)$ is just the average kinetic energy of the $b$ quark inside the meson. At present, only a QCD sum rule estimate is
available, according to which $\lambda_1 \simeq -0.6$ GeV$^2$ \cite{12}. This result has been met with caution (see, e.g. \cite{13}), since it corresponds in fact to a surprisingly large momentum of the $b$ quark inside the meson of order $\left(700-800\right)$ MeV. However, in a recent series of papers, cf. \cite{4, 3}, an upper bound on $\lambda_1$ was derived, to wit $\lambda_1 \leq -0.4$ GeV$^2$, which is in nice agreement with the QCD sum rule prediction. For a further discussion of the present status of $\lambda_1$, I refer to \cite{14}.

The next step is to fix $m_b$ and $m_c$. Here one makes use of the fact that in the framework of HQE the difference between $m_b$ and $m_c$ is given by

$$m_b - m_c = m_B - m_D + \frac{\lambda_1 + 3\lambda_2}{2} \left( \frac{1}{m_b} - \frac{1}{m_c} \right) + O\left( \frac{1}{m_Q^2} \right).$$

Thus either $m_b$ or $m_c$ remain to be fixed. Shifman et al. \cite{15} took $m_b = (4.8 \pm 0.1)$ GeV from spectroscopy. With $\lambda_1 = -0.5$ GeV$^2$ and for a $B$ lifetime $\tau_B = 1.49$ ps, they get $|V_{cb}| = 0.0415$. In \cite{14, 17, 18, 19} $m_c$ was determined from the experimental value of $B(D \to X e\nu)$ via Eq. (2) with $m_b$ replaced by $m_c$. Although this procedure has the advantage that both $m_b$ and $m_c$ are obtained by the same method, the validity of the HQE for the charm quark is not beyond controversy. The $b$ quark mass obtained is typically larger than 5 GeV and thus considerably larger as $m_b$ from spectroscopy. On the other hand, the $b$ quark mass used by Shifman et al. seems to underestimate the intrinsic renormalon ambiguity mentioned in the last section. But even using the same method, \cite{16} and \cite{17} obtain different results: $|V_{cb}| = 0.046 \pm 0.008$ and $\approx 0.042$, respectively (both numbers rescaled using $\tau_B = 1.49$ ps). Ref. \cite{19} takes into account the scheme–dependence of the free quark decay contribution and using running $\overline{MS}$ masses instead of pole masses obtains $|V_{cb}| = 0.036 \pm 0.005$. Since the decay rates only differ in terms of $O(\alpha_s^2)$, these results show that at present the theoretical error due to scale– and scheme–dependence is of paramount importance in exploiting inclusive decays of heavy mesons and for $|V_{cb}|$ amounts to nearly 20%.

### 3 Applications II: the Semileptonic Branching Ratio of B Mesons

The semileptonic branching ratio of B mesons is defined by

$$B(B \to X e\nu) = \frac{\Gamma(B \to X e\nu)}{\Gamma_{tot}}$$

with

$$\Gamma_{tot} = \sum_{\ell=e, \mu, \tau} \Gamma(B \to X\ell\nu\ell) + \Gamma(B \to X_c) + \Gamma(B \to X_{ce}).$$

The explicit formulas for the decay rates can be found in \cite{1}. The radiative corrections to $O(\alpha_s)$ to the semileptonic decay $b \to c e\nu$ were calculated in \cite{3, 4, 8}, the corrections
Figure 1: $B(B \to Xe\nu)$ as function of the pole mass $m_b$ for $\mu = m_b$. Solid line: $B$ calculated using pole masses, dashed line: $B$ calculated using running MS quark masses.

Table 1: $B(B \to Xe\nu)$ in different models depending on $\alpha_s(m_Z)$. Input parameters: $m_b = 4.8$ GeV, $m_c = 1.3$ GeV (pole masses) corresponding to $\lambda_1 = -0.6$ GeV$^2$. Renormalization scale: $\mu = m_b$. In the phase–space factor of $\Gamma(b \to ccs)$ $m_s = 0.2$ GeV is used. Only in [9, 24] full radiative corrections to the nonleptonic B decay modes were taken into account.

| $\alpha_s(m_Z)$ | Parton Model [22] | HQE [23] | HQE [9, 24] |
|-----------------|-------------------|----------|-------------|
|                 | pole masses       | pole masses | pole masses | MS masses |
| 0.110           | 0.132             | 0.130     | 0.121       | 0.111     |
| 0.117           | 0.128             | 0.126     | 0.116       | 0.103     |
| 0.124           | 0.124             | 0.121     | 0.111       | 0.097     |

There exists a number of theoretical analyses of $B(B \to Xe\nu)$ using different methods and obtaining different results. In [22], e.g., the branching ratio was investigated in a purely perturbative framework using the full radiative corrections for $b \to ce\nu$ [5, 8], but with $m_c = 0$ in the corrections to $b \to cud$ and $b \to ccs$ [1]. In Ref. [23], the same analysis was repeated taking into account non–leading terms in the HQE. Finally, in [1, 24] also the full radiative corrections to $b \to cud$ were calculated and those parts of the corrections to $b \to ccs$ available from [7] were taken into account. Also the effect of scheme–dependence was estimated by changing the definition of the quark mass. For the same set of input parameters, the results are given in Table 1. The table shows that the introduction of nonperturbative correction terms by Bigi et al. reduces $B(B \to Xe\nu)$ by 0.3% with respect to the free quark decay model, and that the account for the c quark mass in the radiative corrections yields an additional −0.9%, which again shows the importance of
perturbative corrections to the HQE. The scheme–dependence, however, is still tremendous and amounts to an uncertainty in \(B(B \to Xe\nu)\) of more than 1%. Whereas Refs. [22, 23] concluded that \(B(B \to Xe\nu) > 12.5\%\), Refs. [9, 24] find

\[
B(B \to Xe\nu) = (11.0 \pm 1.8 \pm 1.0)\% \tag{8}
\]

where the first error combines uncertainties in \(\alpha_s(m_Z)\), \(m_b\) (4.5 GeV < \(m_b\) < 5.1 GeV), the hadronic corrections, the renormalization scale and the uncertainty in the radiative corrections to \(b \to ccs\). The second error is a “guestimate” of the theoretical error due to scheme–dependence. The combined effect of complete radiative corrections, new results on \(\alpha_s(m_Z)\) [24] and the consideration of different definitions of the quark mass thus lowers the theoretical branching ratio, which now agrees with the experimental one within the errors and seriously restricts any possible “new physics” in nonleptonic B decays. A more detailed analysis is in preparation [24].

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