Error analysis in solving the linear equation system of three variables using Polya’s problem-solving steps

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Abstract: Recently, students still often have difficulties and share common mistakes in solving the linear equation system of three variables. This qualitative descriptive research describes the errors and causes of students’ errors in solving the linear equation system of three variables using Polya’s problem-solving steps. The subjects were 33 students of Grade 10 Science at one of the Senior High School in Indralaya. The data collection techniques used observation, student worksheets, and interviews. The results showed there were several students’ errors and the causes of students’ errors in solving the problems. These errors are errors in identifying the known and asked information, errors in making mathematical models of problems, errors in calculations, errors in arranging the linear equation system of three variables, and errors in giving valid reasons for their answers. The causes of these errors are the limited time for understanding the problem, careless in doing calculations, forgetting the formulas, and lacking understanding of variables concept.

1. Introduction
Mathematics is one of the compulsory subjects in Senior High Schools. One of the topics taught in Grade Tenth is a linear equation system of three variables. Based on the 2013 curriculum, this topic is an absolute prerequisite topic for studying matrices and linear programs [1]. So, this topic is essential for students to understand so that they can learn the next topic well, without any difficulties and obstacles in using the prerequisite topic. Also, problems related to this topic are often used in daily life. But in reality, there are still many students who have difficulties and share common errors in that topic.

Some of the students’ difficulties in this topic are students did not understand the information in the problem, so they were unable to write the known and asked information, and they often made errors in calculations [2]. In other studies, most students’ difficulty is students did not understand how to make mathematical models [3]. Other students’ difficulties are students did not understand the problem well, students could not communicate the information in the problem into the correct variables, and students were still confused about choosing terms that would be the variables [4]. Several students’ errors in this topic are errors in misconceptions, procedures, and calculations [5]. The causes of these students’ errors are lacking understanding of the concepts, inconsistency in answering the questions, and low ability in generalizing questions [6].

Based on these students’ difficulties and errors in the linear equation system of three variables topic, it can be concluded that this topic is classified as a difficult topic. These students’ difficulties caused students’ answers errors, which have impacts on students’ learning outcomes [7]. Therefore,
alternative solutions are needed to overcome and minimize these students’ errors. Moreover, if there are many types of students’ errors, then it is necessary to know in advance what problems students did so that the most appropriate solution can be given. So, it is necessary to analyze the students’ errors in the topic of a linear equation system of three variables. Error analysis is an investigation of students’ answers errors [8]. These errors need to be analyzed to know the causes and find a solution, so it can be used as an evaluation for further learning [9,10].

As we know, the linear equation system of three variables is related to daily problems. And in fact, these problems are often met and experienced by students. Therefore, the researcher gave a linear equation system of three variables problems to students with steps using Polya’s problem-solving steps. Problem-solving is a fundamental ability that must be possessed and mastered by every student [11]. Problem-solving is a process to overcome the difficulties to achieve goals [13]. One of the famous and often used problem-solving is problem-solving expressed by Polya [12]. There are four steps of Polya’s problem-solving, they understand the problem, devising a plan, carrying out the plan, and looking back. Here is table 1 shows the descriptions of Polya’s problem-solving steps [13].

Table 1. Descriptions of Polya’s problem-solving steps.

| Description of Polya’s problem-solving steps | Description |
|---------------------------------------------|-------------|
| Understanding the problem                   | Students need to specify and identify what information is known and asked from the problem and restate the information in their language. |
| Devising a plan                             | Students create a mathematical model, select an appropriate strategy that will be used, make estimates, and reduce things that are not related to the problem. |
| Carrying out the plan                       | Students implement the plans and strategies that have been chosen and arranged to solve the problem. |
| Looking back                                | Students look back at the solutions and results obtained from the problem-solving steps so that there are no errors in students’ answers. |

Besides that, the contexts of the problem are coins and beach tourism, which are familiar to students certainly. Students are also required to solve problems sequentially because the steps contain regularity patterns. Based on the descriptions above, the researcher is interested in researching the analysis of the students’ errors in solving the linear equation system of three variables problem using Polya’s problem-solving steps. This research aims to describe the students’ errors and the causes of students’ errors in solving the problems in each step of Polya’s problem-solving.

2. Method
The type of research is qualitative descriptive. The subjects were 33 students of Grade 10 Science in one of the Senior High Schools in Indralaya. The subjects were selected through purposive sampling, while the data were collected through observation, student worksheets, and interviews. Two observers have observed students during learning by using validated observation sheets. And the worksheets given to students consist of two problems that have been developed and validated through the development studies [14]. The data obtained from the student worksheets were students’ errors in solving the linear equation system of three variables problem. These errors are described based on Polya’s problem-solving steps [13]. This research used unstructured interviews. The interviews are still related to the aspects of the data collected. Interviews were conducted to know more about the causes of students’ errors in solving the problems on student worksheets deeply. Students selected for interviews were students who have different abilities (heterogeneous). The students who had different answer errors, and the students who had the most errors were selected. The student was also asked about their willingness to be interviewed. Then, the data obtained were analyzed and concluded.
3. Result and Discussion

Based on observations during classroom learning, several students asked the essence of the problem on the student worksheets. Students also often asked what they should do in each step; most students were still confused about interpreting the words in some of the steps. Based on the student worksheets, there were several students’ errors in each problem-solving step. The students’ errors would be described based on Polya’s problem-solving step below.

3.1. Understanding the problem

Based on students’ answers in the first step, students did not write the known and asked information about the problem correctly. Some of them copied and rewrote all the information, and could not choose essential information that was used to solve the problems. The following is one of the students’ incorrect answers in understanding the problem.

From the answer, the student did not write the known information about the problem correctly. The information written by the students was incomplete and inaccurate because it did not explain the real information. This error is also similar to the results of previous studies in that students could not select essential information from the problems [7,15]. Based on the interview, the student did not understand the information about the problem. They said that the text was too long and it took a long time to understand the problem. Students also did not read the instructions carefully, so the answers written by
students did not match the existing instructions. Besides, those who did not write the asked information reasoned that they wanted to solve the problem quickly. However, students who were careless and wanted to solve it quickly did not mean that they could not solve the problem, since, in the next step, they could answer the problem well [15].

3.2. Devising a plan
Some of the students’ errors in this step are errors in modifying sentences into variables and making mathematical models. The following is the students’ incorrect answer in modifying the sentences into variables.

Translation:
- Use variables or mathematical symbols to represent the amount of time that Boy, Mondy, and Reva had spent visiting 24 beaches.
  - The amount of time that has been spent by Boy to visit 24 beaches = \( \frac{24}{x} \) days
  - The amount of time that has been spent by Mondy to visit 24 beaches = \( \frac{24}{x} \) days
  - The amount of time that has been spent by Reva to visit 24 beaches = \( \frac{24}{x} \) days

Figure 2. Error in devising a plan.

Based on the student’s answer above, the student cannot modify the sentence into a variable. After being asked about the written answers, the student could not give an appropriate reason. Then, the researcher asked about the concept of the variable, and it turned out that the student had not yet understood the concept of the variable. Most of the students remembered about variables are only the symbols of x, y, and z; they did not understand how to use these variables. The researcher also asked the concept of variables to students who answered correctly, but the student answered hesitantly. They only knew that the variable was related to x, y, and z. So, these students also did not understand the real concept of variables. Yet, because students often found questions about variables, then they remembered and could write the requested variables. Based on all students’ answers in this step, the variables used by the students are only about the symbols x, y, z. Only a few students used other symbols such as symbols a, b, and c. While students who made errors in making mathematical models because they did not understand the problem and did not read the problem until the end. So, the mathematical model or equation made was not precise and did not match the existing problem. This cause of the error is also the same as the previous study. Students’ errors in making mathematical models were caused by students who were still making errors in understanding the information about the problem [16].

3.3. Carrying out the plan
In this step, most of the students made errors in calculations. Some of them could not choose and use the appropriate formulas for answering the problem, and also could not arrange the system of linear
The following is one of the students’ incorrect answers in performing a fractional operation.

Translation:
The number of beaches that Boy has visited in one day:
24 beaches = x days
24 beaches = x \cdot 1 days
\frac{24}{x} beaches = \frac{x}{x} days = \frac{24}{x} beaches

Figure 3. Error in carrying out the plan.

From that students’ answer, students made errors in the fractional operation. After being interviewed about the answer, it turned out that the student did not understand the concept of dividing fractions containing variables. The student connected the division of x to x was like the division of one by one, so that the number itself is obtained. And based on interviews with students who did not write systems of equations, this was because students did not understand the system means, so the answers were left blank. Students who could not choose and use the appropriate formulas for calculations were because they forgot the formulas. Students tended to memorize formulas, so they forgot about them quickly. The same thing was also found in earlier research that there are students who already understood the problem-solving instructions. Yet, they made an error in using the appropriate concept to solve the problem [6]. And other errors did not only occur due to the lack of students’ concepts of mathematical arithmetic operations understanding but also due to the students’ lacking incautious in calculating. So, there were several students’ correct answers in the previous step, but wrong in getting the final answer. Another cause of this error was because students did not solve the problem according to the prepared plan; this cause is similar to the results of previous related studies [15].

3.4. Looking back
In the last step of Polya’s problem-solving, many students did not check the result of the answer, and they were not precise in giving valid reasons for their written answers. The following is one of the students’ incorrect answers in giving valid reasons for the answer.

Translation:
- Recheck whether the equation system you get is a linear equation system of three variables or not. Give your reasons.
  Yes, because to change an equation’s fraction, the equation must be in the form of variables such as a, b, and c or x, y, and z.

Figure 4. Error in looking back.
In the students’ answers, they did not give valid reasons for the answers obtained, even though the answers obtained were correct. When the student was interviewed with the same question, the student continued to answer what he wrote on the student worksheet. So, from the students’ answers, we know that students did not understand the concept of the linear equation system of three variables. The same thing was shown by a previous study that students who answered problems, but the answer was not appropriate and inaccurate because these students did not understand the whole concept of the topic [17]. The reason given by the student was an explanation of the problem-solving steps he had done, and it did not lead to the concept of a linear equation system of three variables. In this step, many students were also confused about the meaning of the solution instructions, so they did not recheck the solution that has been made and left the answer column blank.

4. Conclusion
Based on the results and discussion above, it could be concluded that there were several students’ errors and the causes of students’ errors in each step of Polya’s problem-solving. (1) These students’ errors are errors in identifying the known and asked information, errors in creating mathematical models of problems, errors in performing mathematical arithmetic operations, errors in arranging the linear equation system of three variables, and errors in giving valid reasons for their answers. (2) The causes of these students’ errors are the limited time for understanding the problem; students did not understand the problem, the concept of variable and fractional operations, students, forgot the appropriate formulas, incautious in doing calculations, students did not follow the procedures according to the plan, students did not understand the problem instructions and the whole concept of a linear equation system of three variables. Therefore, teachers are expected to prepare better learning for the linear equation system of three variables topic to minimize the students’ errors and the causes of students’ errors.

Based on the experience, the limitation of this study is the problem description, and the solution instructions were too long, so students need a long time to understand and solve the problem. The information provided by students in the interview was also not in-depth, because the interview was conducted after the last class so that the student answered hurriedly. So, it is suggested for further researches to present interesting problems among students with a not too long description. Also, the interview should be conducted at an exclusive time so that students focus on answering interview questions, and then the information provided is clear.

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