Semileptonic $B$–decay as a test of CKM unitarity

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Abstract

I point out that $B \to X_q \ell^+ \ell^-$ decays ($q = s, d$) are sensitive probes of possible violation of CKM unitarity. I compute the decay rates and asymmetries in a minimal extension of the Standard Model containing an additional isosinglet charge (-1/3) quark, which leads to a deviation from CKM unitarity. It is shown that even for small mixing ratios $|z_{qb}/(V_{tq}^* V_{tb})| \sim O(10^{-2})$, the contribution of the tree-level Z–FCNC appearing in the model should change the rates and asymmetries significantly. Especially the CP asymmetry, $A_{CP}(B \to X_s \ell^+ \ell^-)$, can be enhanced to be few percents, while in the standard model the size is less than $O(10^{-3})$. On the other hand, $A_{CP}(B \to X_d \ell^+ \ell^-)$ is not altered so much. Constraints for the mixing ratios are extracted from the experiments of $B \to X_s \gamma$ for $q = s$ and $B^0_d - \bar{B}^0_d$ mixing for $q = d$ under a natural assumption that the couplings of the tree-level $Z f \bar{f}$ are almost unity, i.e. $z_{\alpha \alpha} \sim 1$.

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Beyond the experimentally measured $B \to X_s \gamma$, the study of another flavor-changing neutral current (FCNC) processes, $B \to X_q \ell^+ \ell^-$ with $q = s$ or $d$, may also provide important test of the Standard Model (SM) as well as open a window for physics beyond it.

In the present letter, I point out that these decays are sensitive and has crucial dependence on the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. For definiteness, I adopt a typical model that violates the unitarity, that is the SM containing an additional isosinglet charge (-1/3) quark [1]. Then the decay rates and asymmetries of the channels will be analysed in the framework of the model. Phenomenologically, the unitarity violation in the new CKM matrix is not affecting the branching fraction $B \to X_s \gamma$ to any level of significance [4]. However, I will show that $B \to X_q \ell^+ \ell^-$ even for $q = s$ can change significantly. Furthermore, there are also theoretically appealing motivations behind its consideration, i.e. $E_6$ GUT, some superstring-inspired model and a solution to the strong CP problem [2].

Now I briefly describe the model. An extra down-type (charge -1/3) quark, whose left and right handed components are both SU(2) singlets, is introduced. Then the particle content for the quark sector is,

\[
\begin{pmatrix}
  u_i \\
  d_i
\end{pmatrix}_L , d_L , u_R , d_R
\]

with generation indices $\alpha = 1, 2, 3, 4$ and $i = 1, 2, 3$. Throughout this paper I use the following notations for chirality, $L/R \equiv (1 \mp \gamma_5)/2$. Consequently, in the present model the gauge interactions in terms of the mass eigenstates become,

\[
\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} V_{i\alpha} \bar{u}_i \gamma^\mu L d_\alpha W^{\mu+} + \text{h.c.},
\]

\[
\mathcal{L}_Z = \frac{g}{2 \cos \theta_W} \bar{d}_\alpha \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W \delta_{\alpha\beta} - z_{\alpha\beta} \right) L + \frac{2}{3} \sin^2 \theta_W \delta_{\alpha\beta} R \right] d_\beta Z_\mu,
\]

where

\[
z_{\alpha\beta} \equiv \sum_{i=1}^3 U^d_{i\alpha} U^{d*}_{i\beta} = \delta_{\alpha\beta} - U^d_{\alpha4} U^{d*}_{\beta4},
\]

and the new CKM matrix is a $3 \times 4$ one, that is

\[
V_{i\alpha} = \sum_{j=1}^3 U^{u*}_{ij} U^d_{j\alpha}.
\]
Here $U^{u,d}$ are unitary matrices that relate the weak and mass eigenstates as below,

$$
\left( \begin{array}{c}
  u_i \\
  d_\alpha
\end{array} \right)_{\text{mass}} = \left( \begin{array}{cc}
  U^{u}_{ij} & 0 \\
  0 & U^{d}_{\alpha\beta}
\end{array} \right) \left( \begin{array}{c}
  u_j \\
  d_\beta
\end{array} \right)_{\text{weak}}. \quad (5)
$$

Indeed, using the unitarity of $U^{u,d}$, the CKM unitarity violation in the present interest can be expressed as follows,

$$
V^*_u V_{ub} + V^*_c V_{cb} + V^*_t V_{tb} = z_{qb}.
$$

From Eq. (2), it is clear that tree-level FCNC is appearing in the model. Of course other tree-level FCNC also exist in the neutral Higgs sector, but it should be suppressed by a factor $1/M_W$. So I ignore the contribution here. Remark that the photon interaction is not altered at all.

In most of models, including the SM and the present model, $b \to q \ell^+ \ell^-$ decays can be expressed by three effective Wilson coefficients and are governed by the following effective Hamiltonian [3]:

$$
H_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tq} V_{tb} \left\{ C_7^{\text{eff}} \left[ \tilde{q} \gamma_\mu L b \right] \left[ \tilde{\ell} \gamma^\mu \ell \right] + C_{10}^{\text{eff}} \left[ \tilde{q} \gamma_\mu L b \right] \left[ \tilde{\ell} \gamma^\mu \gamma_5 \ell \right] - 2 C_7^{\text{eff}} \left[ \tilde{q} i \sigma_{\mu\nu} \tilde{\ell}^{\nu} (R + \hat{m}_q L) b \right] \left[ \tilde{\ell} \gamma^\mu \ell \right] \right\}. \quad (7)
$$

where $q^\mu$ denotes four-momentum of the dilepton and $s = q^2$. Notations with hat on the top means it is normalized with the $b-$quark mass. In the tree-level approximation, new interactions in Eq. (2) contributes to $O_9$ and $O_{10}$. Therefore, involving the continuum and resonances parts into calculation gives

$$
C_7^{\text{eff}} = C_7 + \eta_{\text{QCD}} C_7^{\text{new}}, \quad (8)
$$

$$
C_9^{\text{eff}} = \left( C_9 + C_9^{\text{new}} \right) \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s}) \right] + C_9^{\text{con}}(\hat{s}) + C_9^{\text{res}}(\hat{s}), \quad (9)
$$

$$
C_{10}^{\text{eff}} = C_{10} + C_{10}^{\text{new}}, \quad (10)
$$

where

$$
C_9^{\text{con}}(\hat{s}) = \left[ \left( 1 + \frac{V_{uq}^* V_{ub}}{V_{tq} V_{tb}} - \frac{z_{qb}}{V_{tq}^* V_{tb}} \right) g(\hat{m}_c, \hat{s}) - \frac{V_{uq}^* V_{ub}}{V_{tq} V_{tb}} g(\hat{m}_u, \hat{s}) \right] \times (3 C_1 + 3 C_2 + 3 C_3 + C_4 + 3 C_5 + C_6)
$$
\[-\frac{1}{2} g(1, \hat{s}) \left(4C_3 + 4C_4 + 3C_5 + C_6 \right) \]

\[-\frac{1}{2} g(0, \hat{s}) \left(C_3 + 3C_4 \right) + \frac{2}{9} \left(3C_3 + C_4 + 3C_5 + C_6 \right), \quad (11)\]

\[C_9^{\text{res}}(\hat{s}) = -\frac{16\pi^2}{9} \left(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6 \right) \]

\[\times \left[ \left(1 + \frac{V_{ub}^* V_{tb}}{V_{tb}^* V_{tb}} - \frac{z_{qb}}{V_{tb}^* V_{tb}} \right) \sum_{V=q,\bar{q}} F_V(\hat{s}) - \frac{V_{ub}^* V_{ub}}{V_{tb}^* V_{tb}} \sum_{V=p,\omega} F_V(\hat{s}) \right] \quad \text{(12)}\]

\[C_7^{\text{new}} = -\frac{1}{3} \frac{z_{qb}}{V_{tb}^* V_{tb}}, \quad (13)\]

\[C_9^{\text{new}} = \frac{\pi}{\alpha} \frac{z_{gb}}{V_{tb}^* V_{tb}} \left(4\sin^2 \theta_W - 1 \right), \quad \text{(14)}\]

\[C_{10}^{\text{new}} = \frac{\pi}{\alpha} \frac{z_{gb}}{V_{tb}^* V_{tb}}. \quad \text{(15)}\]

Here \(C_i (i = 1, \cdots, 10)\) are the Wilson coefficients for each operator \(O_i\) calculated in the SM including the QCD corrections \[6\], \(\omega(\hat{s})\) represents the \(O(\alpha_s)\) correction from the one gluon exchange in the matrix element of \(O_9\), \(g(\hat{m}_u, \hat{s})\) describes the continuum part of \(u_i \bar{u}_i\) pair contributions \((u_i = u, c)\) and lastly \(F_V(\hat{s})\) denotes the resonances due to vector mesons including its momentum dependences. Remark that I keep \(C_7^{\text{new}}\), although it is occured in the one-loop level, to get a constraint for the mixing ratio \(z_{gb}/V_{tb}^* V_{tb}\) from \(B \to X_s \gamma\) decay which all contributions in the magnetic moment operator are coming from the one-loop level. However in the discussion of \(B \to X_q \ell^+ \ell^-\) one can ignore it. The result of \(C_7^{\text{new}}\) is under a natural assumption that \(z_{\alpha\alpha} \sim 1\) and an approximation that \((m_{d_i}/m_Z)^2 \sim 0\) \[4\]. Note that the dependence on the extra down-type quark mass \((m_{d_4})\) is supressed due to the assumption. I also assume the same QCD correction factor for both SM and new diagrams for all processes discussed in the present letter. This should be a good approximation since QCD corrections above the scale of \(m_Z\) are negligible. For efficiency, the reader should refer \[7\] and references therein for explicit expressions of each auxiliary functions above which are not given here. I use the following values for further analysis \[3\]

\[C_1 = -0.2404, \ C_2 = 1.1032, \ C_3 = 0.0107, \ C_4 = -0.0249, \ C_5 = 0.0072, \]

\[C_6 = -0.0302, \ C_7 = -0.3109, \ C_8 = -0.1478, \ C_9 = 4.1990, \ C_{10} = -4.5399, \]

and \(\eta_{QCD}^{\beta\gamma} = 0.6745\) by putting \(\Lambda_{QCD}^{(5)} = 0.214\) (GeV) and the renormalization scale
\[ \mu = 5 \text{ (GeV)}. \]

The additional terms proportional to the mixing ratios \( z_{qb}/V_{tb}^*V_{tb} \) in Eqs. (14) and (15) are coming from the tree \( Z \)–exchange diagram respectively, while in Eqs. (11) and (12) are due to the unitarity violation relation in Eq. (3) in the calculation of \( b \rightarrow q u_i \bar{u}_i \) processes. One may expect that these terms will contribute to the CP violation in the channel together with the usual contribution in \( C_9^{\text{eff}} \), because generally \( z_{qb} \) has different phases with \( V_{tb}^*V_{tb} \), i.e.

\[
\frac{z_{qb}}{V_{tb}^*V_{tb}} = \left| \frac{z_{qb}}{V_{tb}^*V_{tb}} \right| e^{i\theta_q},
\]

where \( \theta_q = \arg \left( \frac{z_{qb}}{V_{tb}^*V_{tb}} \right) \). However, as pointed out later \( C_{10}^{\text{new}} \) contribute nothing to the CP asymmetry, while \( C_9^{\text{new}} \) gives small change. From Eqs. (14) and (15), one can predict easily that \( C_i^{\text{new}} \) \( (i : 9, 10) \) should be large even for small mixing ratios, i.e. \( |z_{qb}/V_{tb}^*V_{tb}| \sim O(10^{-2}) \). Especially, a large enhancement is expected in \( C_{10}^{\text{eff}} \), because of no suppression due to Weinberg angle. High dependences of \( C_{10}^{\text{eff}} \) are expected in the forward-backward (FB) and lepton-polarization (LP) asymmetries.

Before going on analysing the decay rates and asymmetries, I consider experimentally well-known \( B_d^0 - \bar{B}_d^0 \) mixing and \( B \rightarrow X_s \gamma \) to obtain some constraints for the mixing ratios. First, from the \( B_d^0 - \bar{B}_d^0 \) mixing, a constraint for \( z_{db}/V_{td}^*V_{tb} \) can be obtained from the measurement of \( x_d \). In general \( B_q^0 - \bar{B}_q^0 \) mixings, \( x_q \) is given as [3]

\[
x_q = C_{B_q\bar{B}_q} |F_{\Delta B=2}| \left| V_{tq}^*V_{tb} \right|^2 \left( 1 + \frac{4\pi \sin^2 \theta_W}{\alpha} |F_{\Delta B=2}| \right) \left| \frac{z_{qb}}{V_{tb}^*V_{tb}} \right|^2 e^{2i\theta_q},
\]

where \( C_{B_q\bar{B}_q} = G_F^2/(6\pi^2) \tau_{B_q} \eta_{QB}^{B\bar{B}} m_{B_q} m_{W^2} \left( f_{B_q}^2 B_q \right). \) Numerically, using QCD correction factor \( \eta_{QB}^{B\bar{B}} = 0.55 \), \( m_W = 80.33 \text{ (GeV)} \) and \( \sqrt{f_{B_d}^2 B_{B_d}} = 173 \pm 40 \text{ (MeV)} \), one obtains \( C_{B_d\bar{B}_d} = 13834.6^{+7137.1}_{-5637.9} \). Here, for \( m_{B_d} \) and \( \tau_{B_d} \), I use \( m_{B^0} = 5279.2 \pm 1.8 \text{ (MeV)} \) and \( m_{B^0} = 1.28 \pm 0.06 \text{ (ps)} \) and from the box diagram calculation in the SM, \( |F_{\Delta B=2}| = 0.543 \) for \( m_t = 175 \text{ (GeV)} \). On the other hand, recent experiment gives \( x_d = 0.73 \pm 0.05 \) [2]. Secondly, from \( B \rightarrow X_s \gamma \) which experimentally has been measured to be \( B(B \rightarrow X_s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \) [8], one can extract a constraint for \( z_{sb}/V_{ts}^*V_{tb} \). Generally in terms of the semi-leptonic \( B \) decay, the
branching ratio for $B \rightarrow X_q \gamma$ is expressed as

$$\mathcal{B}(B \rightarrow X_q \gamma) = C_{bq\gamma} \left| V_{tq}^* V_{tb} \right|^2 \left| C_7^{\text{eff}} \right|^2,$$

where $C_{bq\gamma} = \mathcal{B}(B \rightarrow X_c \ell \bar{\nu}) (6 \alpha) / (\pi f(m_c) \kappa(m_c) |V_{cb}|^2)$. The value is $C_{bq\gamma} = 1.910^{+0.399}_{-0.315}$ by using $|V_{cb}| = 0.041 \pm 0.003$, $\mathcal{B}(B \rightarrow X_c \ell \bar{\nu}) = (10.4 \pm 0.4)\%$, while $f(m_c) = 0.542$ and $\kappa(m_c) = 0.885$ for $m_c = 0.29$ [7].

Lastly, substituting the experiment results of $x_d$ and $B \rightarrow X_s \gamma$ into Eqs. (17) and (18), one obtains the bounds for the mixing ratios as depicted in Fig. 1. In the left figure, the solid and dashed curves denote the central, upper and lower bounds for $|z_{db}|$ as a function of $|V_{td}^* V_{tb}|$, while in the right one the bounds are given for $q = s$ with varying $\theta_s$. Since my emphasis is on a case that a relatively small CKM unitarity violation may change the distributions in the decays, further I put both mixing ratios to be small, that is $|z_{qb}/V_{td}^* V_{tb}| \sim 0.01$ which correspond to $|V_{td}^* V_{tb}| \sim 0.0095$ and $|V_{ts}^* V_{tb}| \sim 0.036$. Because of the smallness, its contributions in Eqs. (8), (11) and (12) can be ignored, while in Eqs. (14) and (15) must be kept as it stands. Particularly in the case of $q = s$ the ratio $V_{ts}^* V_{ub}/V_{td}^* V_{tb}$ is negligible, while for $q = d$ it should be kept and roughly I put the ratio and the phase to be same with the SM [4] for simplicity.

Now I turn to analyse the dilepton invariant mass distribution of the decay rates and asymmetries in the channels. Involving the lepton and quark masses, the differential branching ratio (BR) is

$$\frac{d\mathcal{B}}{ds} = \int_{-1}^{1} dz \frac{d^2\mathcal{B}}{d\hat{s}dz}$$

$$= \frac{4}{3} C_{bq\ell\ell} \sqrt{1 - \frac{4 \hat{m}_q^2}{\hat{s}}} \hat{u}(\hat{s}) \left\{ 6 \left| C_9^{\text{eff}} \right|^2 - \left| C_{10}^{\text{eff}} \right|^2 \right\} \hat{m}_q^2 \left[ 1 - \hat{s} + \hat{m}_q^2 \right]$$

$$+ \left[ \left| C_9^{\text{eff}} \right|^2 + \left| C_{10}^{\text{eff}} \right|^2 \right] \left[ (1 - \hat{m}_q^2)^2 + \hat{s} (1 + \hat{m}_q^2) - 2 \hat{s}^2 + \hat{u}(\hat{s}) \frac{2 \hat{m}_q^2}{\hat{s}} \right]$$

$$+ 4 \left| C_7^{\text{eff}} \right|^2 \frac{1 + 2 \hat{m}_q^2/\hat{s}}{\hat{s}}$$

$$\times \left[ 2 (1 + \hat{m}_q^2)(1 - \hat{m}_q^2)^2 - (1 + 14 \hat{m}_q^2 + \hat{m}_q^4) \hat{s} - (1 + \hat{m}_q^2) \hat{s}^2 \right]$$

$$+ 12 \text{Re} \left( C_9^{\text{eff}} \right)^* C_7^{\text{eff}} \left[ 1 + \frac{2 \hat{m}_q^2}{\hat{s}} \right] \left[ (1 - \hat{m}_q^2)^2 - (1 + \hat{m}_q^2) \hat{s} \right] \right\}.$$

(19)
where \(\hat{u}(\hat{s}) = \frac{1}{\sqrt{\hat{s} - (1 + \hat{m}_q)^2}}\), \(\hat{s} = \cos \theta\) is the angle of \(\ell^+\) measured with respect to the \(b\)–quark direction in the dilepton CM system and \(C_{bq\ell\ell} = B(B \to X_c \ell^\nu \ell) \left(3 \alpha^2 |V_{tq}^* V_{tb}|^2 \right) \left(16 \pi^2 |V_{cb}|^2 f(\hat{m}_c) \kappa(\hat{m}_c) \right)\).

The normalized forward-backward (FB) asymmetry is defined \([10]\) and calculated as follows

\[
\hat{A}_{FB} = \frac{\int_0^1 dz \frac{d^3B}{d\hat{s}dz} - \int_0^\hat{s} dz \frac{d^3B}{d\hat{s}dz}}{\int_0^1 dz \frac{d^3B}{d\hat{s}dz} + \int_0^\hat{s} dz \frac{d^3B}{d\hat{s}dz}}
\]

\[
= -\frac{4 C_{bq\ell\ell}}{dB(\hat{s})/d\hat{s}} \sqrt{1 - \frac{4 \hat{m}_t^2}{\hat{s}}} \hat{u}(\hat{s})^2 C_{10} \left[ \text{Re} \left( C_9^{\text{eff}} \right) * \hat{s} + 2 C_7^{\text{eff}} \left(1 + \hat{m}_q^2\right) \right]
\]

(20)

Doing same treatment as \([9]\) in the amplitude level, the normalized CP asymmetry can be written simply as

\[
\hat{A}_{CP} = \frac{d\mathcal{B}/d\hat{s} - d\mathcal{B}/d\hat{s}}{d\mathcal{B}/d\hat{s} + d\mathcal{B}/d\hat{s}} = -\frac{2 d\mathcal{A}_{CP}/d\hat{s}}{d\mathcal{B}/d\hat{s} + 2 d\mathcal{A}_{CP}/d\hat{s}}
\]

(21)

where \(\mathcal{B}\) and \(\mathcal{B}\) denote the BR of the \(\bar{b} \to q \ell^+ \ell^-\) and its complex conjugate \(b \to \bar{q} \ell^+ \ell^-\) respectively. Different with the SM, in the present model there is a possible new source of CP violation in \(C_{10}^{\text{eff}}\). Redefine the Wilson coefficients \(C_i^{\text{eff}} (i : 9, 10)\) as \(C_i^{\text{eff}} = \tilde{C}_i + (V_{uq}^* V_{ub}/V_{tq}^* V_{tb}) C_i^{\text{CP}} + (z_{qb}/V_{tq}^* V_{tb}) \tilde{C}_i^{\text{CP}}\), the result is

\[
\frac{d\mathcal{A}_{CP}}{d\hat{s}} = \frac{4}{3} C_{bq\ell\ell} \sqrt{1 - \frac{4 \hat{m}_t^2}{\hat{s}}} \hat{u}(\hat{s}) \left\{6 [C_9 - C_{10}] \hat{m}_t^2 \left[1 - \hat{s} + \hat{m}_q^2\right] + [C_9 + C_{10}] \left[(1 - \hat{m}_q^2)^2 + \hat{s} (1 + \hat{m}_q^2) - 2 \hat{s}^2 + \hat{u}(\hat{s})^2 \frac{2 \hat{m}_t^2}{\hat{s}}\right] + 6 \hat{C}_9 C_7^{\text{eff}} \left[1 + \frac{2 \hat{m}_t^2}{\hat{s}}\right] \left[(1 - \hat{m}_q^2)^2 - (1 + \hat{m}_q^2) \hat{s}\right]\right\}
\]

(22)

where

\[
C_i = \text{Im} \left( \frac{V_{uq}^* V_{ub}}{V_{tq}^* V_{tb}} \right) \text{Im} \left( \tilde{C}_i^{\text{CP}} \right) + \text{Im} \left( \frac{z_{qb}}{V_{tq}^* V_{tb}} \right) \text{Im} \left( C_i^{\text{CP}} \right)
\]

\[
+ \text{Im} \left( \frac{V_{uq}^* V_{ub}}{V_{tq}^* V_{tb}} \right) \text{Im} \left( C_i^{\text{CP}} \right)
\]

(23)

\[
\tilde{C}_i = \text{Im} \left( \frac{V_{uq}^* V_{ub}}{V_{tq}^* V_{tb}} \right) \text{Im} \left( C_i^{\text{CP}} \right) + \text{Im} \left( \frac{z_{qb}}{V_{tq}^* V_{tb}} \right) \text{Im} \left( \tilde{C}_i^{\text{CP}} \right)
\]

(24)
As mentioned before, it is clear that $C_{10}^{\text{eff}}$ will not contribute to the CP asymmetry in the model. Because one needs both complex CKM factor and complex Wilson coefficients to produce CP asymmetry, that is $\text{Im} \left( \bar{C}_i \right)$, $\text{Im} \left( C_i^{CP} \right)$, $\text{Im} \left( \bar{C}_i^{CP} \right)$ and the imaginary of its combination must not be zero.

For the longitudinal polarization in the $\ell^{-}$ rest frame, the normalized lepton-polarization (LP) asymmetry is given as [11]

$$
\mathcal{A}_{\text{LP}} = \frac{\text{d}B(n_{\ell^-})/d\hat{s} - \text{d}B(-n_{\ell^-})/d\hat{s}}{\text{d}B(n_{\ell^-})/d\hat{s} + \text{d}B(-n_{\ell^-})/d\hat{s}} = \frac{8 C_{7}^{\text{eff}}}{3 \text{d}B(\hat{s})/d\hat{s}} \left( 1 - \frac{4 \hat{m}_q^2}{\hat{s}} \right) \hat{u}(\hat{s}) C_{10}^{\text{eff}} \left[ (1 - \hat{m}_q^2)^2 - \hat{s} (1 + \hat{m}_q^2) \right] + \text{Re} \left( C_{9}^{\text{eff}} \right)^* \left[ (1 - \hat{m}_q^2)^2 + \hat{s} (1 + m_q^2) - 2 \hat{s}^2 \right].
$$

(25)

with $n_{\ell^-} = p_{\ell^-}/|p_{\ell^-}|$.

As the results, the distributions of differential BR and asymmetries on dilepton invariant mass are plotted in Figs. 2~4. New contribution in the model changes the differential BR significantly for both $q = d, s$. It also contributes to FB and LP asymmetries without no significant differences for both $q = d, s$ since the over- whole CKM factor is eliminated by definition. Furthermore, in the differential BR the distribution is dominated by $C_{9}^{\text{eff}}$, while in the FB and LP asymmetries are dominated by $C_{10}^{\text{eff}}$. On the other hand, the new source of CP violation does not contribute to the CP asymmetry as large as expected. However, for $q = s$ the new term in $C_{9}^{\text{eff}}$ enhance the CP asymmetry for about one order. Note that in the SM, $\mathcal{A}_{\text{CP}}(B \to X_s e^+ e^-) \sim O(10^{-3})$.

In conclusion, the measurements of the decay rates and asymmetries in $B \to X_q \ell^+ \ell^-$ decays together with the measurement of CP asymmetry in flavor changing charge-curent $B$ decays will provide a crucial test of CKM unitarity as well as leading to the discovery of unitarity violation.

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References
[1] G. C. Branco and L. Lavoura, *Nucl. Phys.* B278 (1986) 738;  
L. Lavoura and J. P. Silva, *Phys. Rev.* D47 (1993) 2046;  
V. Barger, M. S. Berger and R. J. N. Philips, *Phys. Rev.* D52 (1995) 1663.

[2] J. L. Hewett and T. G. Rizzo, *Phys. Rep.* 183 (1989) 193;  
L. Bento, G. C. Branco and P. A. Parada, *Phys. Lett.* B267 (1991) 95.

[3] G. C. Branco, T. Morozumi, P. A. Parada and M. N. Rebelo, *Phys. Rev.* D48 (1993) 1167.

[4] G. Bhattacharyya, G. C. Branco and D. Choudhury, *Phys. Lett.* B336 (1994) 487 [Err. *Phys. Lett.* B340 (1994) 266];  
L. T. Handoko and T. Morozumi, *Mod. Phys. Lett.* A10 (1995) 309.

[5] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* B319 (1989) 271;  
R. Grigjanis, P. J. O’Donnel, M. Sutherland and H. Navelet, *Phys. Lett.* B223 (1989) 239.

[6] M. Ježabek and J. H. Kühn, *Nucl. Phys.* B320 (1989) 20;  
M. Misiak, *Nucl. Phys.* B393 (1993) 23 [Err. B439 (1995) 461];  
A. J. Buras and M. Münz, *Phys. Rev.* D52 (1995) 186.

[7] L. T. Handoko, submitted to *Phys. Rev.* D, [hep-ph/9707222] (1997).

[8] M. S. Alam et.al. (CLEO collaboration), *Phys. Rev. Lett.* 74 (1995) 2885.

[9] F. Krüger and L. M. Sehgal, *Phys. Rev.* D55 (1997) 2799.

[10] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett.* B373 (1991) 505.

[11] J. L. Hewett, *Phys. Rev.* D53 (1996) 4964;  
F. Krüger and L. M. Sehgal, *Phys. Lett.* B380 (1996) 199.

[12] Particle Data Group, *Phys. Rev.* D54 (1996) 1.
Figure 1: Allowed values for $|z_{qb}|$ (left) and $|z_{sb}|$ (right) according to its counterpart $|V_{td}^* V_{tb}|$ and $|V_{ts}^* V_{tb}|$.

Figure 2: Differential BR for $e^+e^-$ in the SM (thin solid curve) and in the present model with $\theta_q = 0^o$ (dashed curve) and $\theta_q = 180^o$ (solid thick curve).
Figure 3: FB (left) asymmetry for $e^+e^-$ and LP asymmetry for $\mu^+\mu^-$ in the SM (thin solid curve) and in the present model with $\theta_q = 0^\circ$ (dashed curve) and $\theta_q = 180^\circ$ (solid thick curve).

Figure 4: CP asymmetry for $e^+e^-$ in the SM (thin solid curve) and in the present model with $\theta_q = 90^\circ$ (solid thick curve) and $\theta_q = -90^\circ$ (dashed curve).