Abstract. The critical star formation rate density required to keep the intergalactic hydrogen ionised depends crucially on the average rate of recombinations in the intergalactic medium (IGM). This rate is proportional to the clumping factor \( C_{\text{IGM}} \equiv \langle \rho_b^2 \rangle_{\text{IGM}}/\bar{\rho}_b^2 \), where \( \rho_b \) and \( \bar{\rho}_b \) are the local and cosmic mean baryon density, respectively and the brackets \( \langle \rangle_{\text{IGM}} \) indicate spatial averaging over the recombining gas in the IGM. We perform a suite of cosmological smoothed particle hydrodynamics simulations to calculate the volume-weighted clumping factor of the IGM at redshifts \( z \geq 6 \). We investigate the effect of photoionisation heating by a uniform ultraviolet background and find that photoheating strongly reduces the clumping factor as the increased pressure support smooths out small-scale density fluctuations. Even our most conservative estimate for the clumping factor, \( C_{\text{IGM}} = 6 \), is five times smaller than the clumping factor that is usually employed to determine the capacity of star-forming galaxies to keep the \( z \approx 6 \) IGM ionised. Our results imply that the observed population of star-forming galaxies at \( z \approx 6 \) may be sufficient to keep the IGM ionised, provided that the IGM was reheated at \( z \approx 9 \) and that the fraction of ionising photons that escape star-forming regions to ionise the IGM is larger than 0.25.

1. Introduction

The absence of a Gunn-Peterson trough in many of the absorption spectra towards high-redshift quasars suggests that the reionisation of intergalactic hydrogen was completed at redshifts \( z \gtrsim 6 \) (e.g., Fan, Carilli, & Keating 2006). Current observational estimates of the ultraviolet (UV) luminosity density at redshifts \( z \lesssim 6 \) (for a comprehensive discussion see, e.g., Bouwens et al. 2007), on the other hand, may imply star formation rate (SFR) densities several times lower than the critical SFR density required to keep the intergalactic medium (IGM) ionised.

The critical SFR density,

\[
\dot{\rho}_* \approx 0.027 \, \text{M}_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3} \, f_{\text{esc}}^{-1} \left( \frac{C_{\text{IGM}}}{30} \right) \left( \frac{1 + z}{7} \right)^3 \left( \frac{\Omega_b h_{70}^2}{0.0465} \right)^2 ,
\]

originally derived by Madau, Haardt, & Rees (1999) and here rescaled to match the most recent WMAP estimate for the cosmic baryon density (Komatsu et al. 2008), is inversely proportional to the escape fraction \( f_{\text{esc}} \), i.e. the fraction of
ionising photons produced by star-forming galaxies that escape the interstellar medium (ISM) to ionise the IGM. It is proportional to the average recombination rate in the IGM, parametrised using the dimensionless clumping factor $C_{\text{IGM}} \equiv \langle \rho_b^2 \rangle_{\text{IGM}} / \bar{\rho}_b^2$, where $\rho_b$ is the baryon density, $\bar{\rho}_b$ is the mean baryon density of the Universe and the brackets $\langle \rangle_{\text{IGM}}$ indicate spatial averaging over the gas constituting the recombining IGM.

Most observational studies that compare the SFR density derived from estimates of the UV luminosity density at redshift $z \approx 6$ to the critical SFR density assume an escape fraction $f_{\text{esc}} \lesssim 0.5$ and a clumping factor $C_{\text{IGM}} = 30$. Consequently, the observed population of galaxies has been found to be incapable of keeping the intergalactic hydrogen ionised, forming massive stars at a rate which is up to an order of magnitude lower than required by Eq. 1. It has, however, been noted that the discrepancy between the observationally inferred and critical SFR densities at $z \approx 6$ could be resolved if the employed clumping factor were too high (e.g., Sawicki & Thompson 2006).

In this work we perform cosmological Smoothed Particle Hydrodynamics (SPH) simulations to compute the clumping factor of the IGM. We pay particular attention to the fact that the clumping factor depends on the definition of which gas comprises the IGM. By comparing the critical SFR density, updated using our estimate of the clumping factor, with the observationally inferred SFR density we argue that the observed population of UV galaxies may well be capable of keeping the $z \approx 6$ Universe ionised. We also investigate the effect of photoionisation heating by a uniform UV background on the evolution of the clumping factor. We demonstrate that photoheating significantly lowers the clumping factor. Our results are insensitive to the redshift at which the UV background is turned on.

This work is described in more detail in Pawlik, Schaye, & van Scherpenzeel (2009).

2. Simulations

We use a modified version of the N-body/TreePM/SPH code GADGET-2 (Springel 2005) to perform a suite of cosmological SPH simulations including radiative cooling. We assume a flat $\Lambda$CDM universe and employ the set of cosmological parameters $[\Omega_m, \Omega_b, \Omega_\Lambda, \sigma_8, n_s, h]$ given by $[0.258, 0.0441, 0.742, 0.796, 0.963, 0.719]$, consistent with the WMAP 5-year results (Komatsu et al. 2008).

The gas is of primordial composition, with a hydrogen mass fraction $X = 0.752$ and a helium mass fraction $Y = 1 - X$. Radiative cooling and heating are included assuming ionisation equilibrium, as described in Wiersma, Schaye, & Smith (2009). Molecular hydrogen and deuterium and their catalysts are kept photo-dissociated by a soft UV background and never contribute to the cooling rate (e.g., Haaiman, Rees, & Loeb 1997). Star formation is modelled by turning gas particles into star particles using the recipe of Schaye & Dalla Vecchia (2008).

All simulations use $256^3$ DM particles and $256^3$ gas particles in a box of comoving size $L = 6.25 \, h^{-1} \, \text{Mpc}$. We perform simulations including photoionisation by a uniform, evolving Haardt & Madau (2001) UV background in the
optically thin limit at redshifts below the reheating redshift $z_r$. These simulations are denoted $r[z_r]L6N256$. To study the effect of photoheating, we compare these simulations to a simulation that does not include photoionisation and which is denoted $L6N256$. We explore a range of reheating redshifts to investigate the sensitivity of our conclusions to changes in this parameter. We note that for the simulations that included the UV background the employed box size is sufficiently large and the employed resolution is sufficient high to obtain converged results (for details see Pawlik, Schaye, & van Scherpenzeel 2009).

3. The clumping factor

Our main motivation for computing the clumping factor of the IGM is to evaluate the critical SFR density required to keep the IGM ionised. The critical SFR density describes a balance between the number of ionising photons escaping into the IGM (parametrised by $f_{\text{esc}}$) and the number of ionising photons that are removed from the IGM due to photoionisations of recombining hydrogen ions (parametrised by $C_{\text{IGM}}$). It is important to realize that only recombinations leading to the removal of ionising photons which escaped the ISM of the star-forming regions contribute to this balance.

We employ a threshold density criterion to separate the gas in the ISM from the gas in the IGM (e.g., Miralda-Escudé, Haehnelt, & Rees 2000). Ionising photons are counted as escaped once they enter regions with gas densities $\rho_b < \rho_{\text{thr}}$. We treat the threshold density $\rho_{\text{thr}} \equiv \rho_b \Delta_{\text{thr}}$ as a parameter and compute the clumping factor $C(< \Delta_{\text{thr}}) \equiv \langle \rho_b^2 \rangle_{\rho_b < \rho_{\text{thr}}} / \rho_b^2$ as a function of the corresponding threshold overdensity $\Delta_{\text{thr}}$. This is done by performing a volume-weighted summation over all SPH particles with overdensities $\Delta_i < \Delta_{\text{thr}}$, i.e. we set $C(< \Delta_{\text{thr}}) = \sum \Delta_i < \Delta_{\text{thr}} h_i^3 \Delta_i^2 / \sum \Delta_i < \Delta_{\text{thr}} h_i^3$, where $h_i$ is the radius of the SPH smoothing kernel associated with SPH particle $i$.

By definition, $C(< \Delta_{\text{thr}})$ increases monotonically with the threshold overdensity $\Delta_{\text{thr}}$. We set an upper limit $\Delta^*$ to $\Delta_{\text{thr}}$, corresponding to the threshold density $n_H^* \equiv 10^{-1} \text{ cm}^{-3}$ for the onset of star formation that we employ in our simulations. This choice is conservative, since the threshold density marking the escape of ionising photons and hence the clumping factor of the IGM to be used in Eq. 1 is likely lower. We conservatively identify the clumping factor $C(< \Delta^*)$ with the clumping factor of the IGM, i.e. $C_{\text{IGM}} \equiv C(< \Delta^*)$.

The left-hand panel of Fig. 1 shows $C(< \Delta_{\text{thr}})$ for the simulations $r9L6N256$ and $L6N256$ at redshifts shortly before (at $z = 9.08$) and well after (at $z = 6$) the reheating redshift $z_r = 9$. The inclusion of photoheating strongly reduces the clumping factor for large threshold overdensities. This is because the associated increase in pressure smoothes small-scale density fluctuations. Note that because of the reduction of the clumping factor, photoheating provides a positive feedback on reionisation, which acts in addition to the well-known negative feedbacks (for a review see, e.g., Ciardi & Ferrara 2005). At $z = 6$ the clumping factor reaches a maximum value of $C(< \Delta^*) \approx 6$. This is five times smaller

$^1$If $z_r > 9$, we use the $z = 9$ Haardt & Madau (2001) UV background for all redshifts $9 < z \leq z_r$, and employ the evolving Haardt & Madau (2001) UV background for redshifts $z \leq 9$. 

5 Keeping the Universe ionised
Figure 1. Left-hand panel: Clumping factor $C(<\Delta_{\text{thr}})$ of gas with overdensities $\Delta < \Delta_{\text{thr}}$. The inclusion of photoheating in $r9L6N256$ leads to a clumping factor that is substantially smaller than that obtained from $L6N256$, for threshold overdensities $\Delta_{\text{thr}} > 10$. The maximum threshold overdensity we consider corresponds to the critical density $n^*_H \equiv 10^{-1} \text{cm}^{-3}$ (translating into an overdensity $\Delta^*$) for the onset of star-formation. Right-hand panel: Evolution of the clumping factor $C_{\text{IGM}} \equiv C(<\Delta^*)$ for different reheating redshifts $z_r$. At $z = 6$ we find $C_{\text{IGM}} \approx 6$, insensitive to the reheating redshift provided that $z_r > 9$. Based on Pawlik, Schaye, & van Scherpenzeel (2009).

than the value quoted in Gnedin & Ostriker (1997), which has been employed in many (observational) studies. The difference with our result is probably because Gnedin & Ostriker (1997) computed the clumping factor using a density threshold implicitly set by the maximum overdensity resolved in their simulation. The right-hand panel of Fig. 1 shows the evolution of the clumping factor $C_{\text{IGM}} \equiv C(<\Delta^*)$ for different reheating redshifts. It demonstrates that our estimate of the clumping factor of the IGM at $z = 6$, $C_{\text{IGM}} \approx 6$, is insensitive to variations in the redshift at which the UV background is turned on.

4. Discussion

We computed the clumping factor $C_{\text{IGM}} \equiv \langle \rho^2 \rangle_{\text{IGM}}/\bar{\rho}^2$, a measure for the average recombination rate in the intergalactic medium (IGM), using cosmological smoothed particle hydrodynamics simulations. The clumping factor of the IGM depends critically on the definition of which gas is considered to be part of the IGM. Following Miralda-Escudé, Haehnelt, & Rees (2000), we assumed that all gas with densities below a threshold density constitutes the IGM and computed the clumping factor as a function of this threshold density.

Even our most conservative estimate for the clumping factor, $C_{\text{IGM}} \approx 6$, is five times smaller than the clumping factor that is usually employed to determine the capacity of star-forming galaxies to keep the $z \approx 6$ IGM ionised. Setting $C_{\text{IGM}} = 6$ in Eq. [1], the critical SFR density is $\dot{\rho}_* = 0.005 \, f_{\text{esc}}^{-1} \, M_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3}$. This is smaller than recent observational estimates for the SFR density at $z \approx 6$, $\dot{\rho}_* \approx 0.02 \pm 0.004 \, M_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3}$ (integrated to the observed $z \approx 6$ faint-end limit $L > 0.04 \, L^*_z = 3$ and with new dust-correction; Bouwens et al. 2009), provided that $f_{\text{esc}} \geq 0.25$. Our study thus suggests that the observed population...
of star-forming galaxies may be capable of keeping the $z \approx 6$ IGM ionised (see also, e.g., Bolton & Haehnelt 2007). In fact, recent studies suggest that, if we assume clumping factors as low as suggested by our work, observed galaxies may be capable of keeping the Universe substantially ionised out to redshifts as high as $z \lesssim 8$ (e.g., Finkelstein et al. 2009; Yan et al. 2009; Bouwens et al. 2009; Labbe et al. 2009; Bunker et al. 2009; Kistler et al. 2009).

The clumping factor is an important ingredient of (semi-)analytic treatments of reionisation (e.g., Barkana & Loeb 2001). We have studied the impact of photoionisation heating on the clumping factor of the IGM assuming a uniform ionising UV background in the optically thin limit. In reality the photoheating process will, however, be more complex, with self-shielding being only one example of the physical effects that we ignored. The validity of the approximations inherent to our simplified treatment should be assessed using high-resolution radiation-hydrodynamical simulations of reionisation that include cooling by metals and molecules and also allow for feedback from star formation. For instance, kinetic supernova feedback has been shown to amplify the effects of photoheating in evaporating the gas out of low-mass halos (Pawlik & Schaye 2009), which will affect the clumping factor of the IGM (Pawlik, Schaye, & van Scherpenzeel 2009).

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