Highly Chiral Exceptional Point in Perturbed Coupled Resonators

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Abstract. Exceptional point (EP) is a non-Hermitian spectral degeneracy that has application in ultrasensitive sensors and laser mode selectivity. By employing strong chirality in an optical system, the direction of light propagation can be controlled and subwavelength particles can be detected. Here, we show that EP with high chirality can appear in the coupled resonators perturbed by a scatterer, in which both the distance and position of the scatterer can be tuned. We achieve strong chiral EP in two different distances between the resonators, with chirality around 0.99 in both cases.

1. Introduction
Exceptional point (EP) is a degeneracy in the space parameter at which two or more eigenvalues and their corresponding eigenfunctions coalesce, simultaneously. It has attracted many interests due to its exotic functionalities in such as unidirectional lasing and supersensitive sensors [1, 2, 3]. Highly directional emission can be achieved from a chaotic deformed microresonator [4], and from an elliptical resonator with a wavelength-size notch at the boundary [5]. Besides, chiral EP in a couple of asymmetry disk-like cavities have application in nanoparticle detection [6].

In this paper, we show that incorporating both gain/loss and scatterers in a coupled pair ring resonators EP with high chirality can be achieved. In this structure, with tuning the distances between the resonators, and the position of the scatterer, a strong chiral EP appears in two different distances between the resonators. It can be considered as a tunable device in which with tuning both distance between the resonators, and position of the scatterer the possible imperfections during the fabrication process can be compensated. Using tunable components such as scatterers and/or Kerr nonlinearity [7] can open a new way to achieve a regime extremely close to EP and chiral EP, enabling super-sensitivity/directionality.
2. Coupled Ring Resonators

In a ring resonator, the WGMs reveal two degenerate states, even and odd modes. Incorporating two scatterers along the surface of a microcavity, and tuning their position, we can achieve unidirectional propagation, so-called chiral EP, related to the either clockwise (CW) or counter clockwise (CCW) direction, due to the destructive optical interference for CCW or CW propagation, respectively.

In a coupled ring resonators, the degeneracy of WGMs would be lifted due to the coupling between the rings. The Hamiltonian of this system is described by:

\[ \hat{H} = \begin{pmatrix} \Omega_0 & \kappa \\ \kappa & \Omega_0 \end{pmatrix} \]  

where \( \kappa \) is the coupling rate between two rings and \( \Omega_0 \) is the resonant frequency of rings. Therefore, the eigenvalues of the coupled system are \( \Omega_0 \pm \kappa \). In an ideal case, the system would have four eigenvalues, two of them are degenerated by the value \( \Omega_0 - \kappa \), and the other two eigenvalues are degenerated by \( \Omega_0 + \kappa \). However, the numerical result of (unperturbed) coupled resonator shows four different eigenvalues, with close values two by two. This slight difference between the analytical and numerical values can be attributed to the difference between the coupling rate of even and odd modes in the rings. These four eigenvalues can be analogous to a pair of two-level system (TLS).

Analogous to the chiral EP in a single resonator, in which two scatterers can bring the system to the chiral EP, in the coupled ring resonators, two mechanisms: 1) the scatterer which unevenly affects the resonators, and 2) coupling between the resonators, can create the conditions of chiral EP in the resonators. Our result shows that this kind of destructive interference can occur close to the eigenvalues related to the (inter-)EP in gain/loss coupled resonators. To this end, first, we bring the unperturbed coupled ring resonator to the EP (or close to it), by tuning the distance (coupling) and dissipation between them, and then we incorporate a scatterer, and tune its position to create the chiral EP. The schematic of such a mechanism is shown in Fig. 1(a) (top).

To calculate the field distribution, we solve the wave equation \(- \nabla^2 \psi = n^2(x,y) \frac{\omega^2}{c^2} \psi\), numerically in Comsol, where \( \psi \) is associated to \( E_z \) for TM polarizaion, and \( n(x,y) \), \( \omega \) and \( c \) are the refractive index distribution, eigenfrequency and speed of light in vacuum, respectively. To calculate CW and
CCW components, we expand the wavefunction inside the cavity in cylinder harmonics as \( \psi(\rho, \phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(n \omega_c \rho) \exp(i m \phi) \), where \( J_m \) is the \( m \)th order of first kind bessel function, and the positive (negative) angular momentum, \( m \) is corresponding to CCW (CW) component. The chirality can then be defined as
\[
\alpha = 1 - \frac{\min\{\sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2\}}{\max\{\sum_{m=-\infty}^{-1} |\alpha_m|^2, \sum_{m=1}^{\infty} |\alpha_m|^2\}} \tag{2}
\]
[8]. We analyse a coupled resonators with the radii of rings \( r = 0.5 \mu m \), and the permittivity \( \varepsilon = 6 \). The dissipation is considered for the right resonator by adding an imaginary part to its refractive index equal to 0.012. The radius and gap of scatterer are tuned to 0.03 \( \mu m \), and its angle is 35° with respect to the left resonator (here, the gap means center to center distance minus the radii of particles). The maximum chirality is obtained for two different gaps between the resonators \( g = 0.115 \mu m \) (Fig. 1(a)-1(b)), and \( g = 0.075 \mu m \) (Fig. 1(c)-1(d)).

Figure 1(a) depicts |\( \alpha_m \)| components of both left and right resonators for the gap between them \( g = 0.115 \mu m \). From the |\( \alpha_m \)| components, it can be inferred that WGM in the left (right) resonator is mainly in CCW (CW) direction, and the field is mostly distributed in the right resonator. Its field distribution is shown in Fig. 1(b). The vanished zero nodes of the WGM structure is the sign of unidirectional traveling wave characteristic. The chirality of the left (right) resonator related to the modes 3 and 4, \( \alpha_{l,3}, \alpha_{l,4} (\alpha_{r,3}, \alpha_{r,4}) \) are estimated \( \alpha_{l,3} = 0.9863, \alpha_{l,4} = 0.9916, \alpha_{r,3} = 0.9903 \) and \( \alpha_{r,4} = 0.9910 \).

Figure 1(c) and 1(d) depicts |\( \alpha_m \)| components of resonators and their field distribution for the gap \( g = 0.075 \mu m \). It shows that field distribution is distributed equally between the resonator, and WGM structure has mainly CCW (CW) direction in the left (right) resonator. The chirality is calculated \( \alpha_{l,3} = 0.9890, \alpha_{l,4} = 0.9733, \alpha_{r,3} = 0.9879 \) and \( \alpha_{r,4} = 0.9781 \).

In order to observe the interference of the energy levels at chiral EP, Fig. 2 shows real and imaginary parts of four eigenvalues related to the perturbed coupled resonators versus the gap between them. The zoomed regions of the dotted circle shows that the real parts of third and fourth eigenvalues depart and interfere locally, while their corresponding imaginary parts are pretty close to each other. It is similar to the chiral EP in a single resonator with two scatterers, where one of the scatterer lifts the degeneracy, and the another one creates the degenerated states.

3. conclusion
We study chiral EP in a coupled ring resonators containing scatterer(s) as a perturbation. The chirality around 0.99 could be achieved for two different distances between the resonators. Due to the tunability of both the distance between the resonators and the position of the scatterer, the proposed structure enables...
achieving high chirality, and can compensate the possible imperfections during the fabrication process.

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