Anisotropy of spin-polarized transport in a ferromagnet/d-wave superconductor bilayer: Role of the small exchange field

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The current–voltage characteristic of a ferromagnet/d-wave superconductor (FD) bilayer is calculated as a function of the orientation of the superconductor crystallographic axes relative to the FD boundary by solving the spin-dependent Bogoliubov–de Gennes equations. We found that, regardless of the superconductor orientation, the conductance \( G(V) \) always reaches a maximum when the bias voltage matches the exchange field in the ferromagnet, \( eV = h \), providing that the exchange field is larger than the order parameter \( \Delta_1 \). For small exchange fields, conductance \( G(V) \), and zero-bias conductance \( G(0) \) in particular, is more sensitive to the rotation of the superconductor crystallographic axis. Zero-bias conductance (ZBC) is isotropic in two limiting cases: when the exchange field in the ferromagnet is strong enough \( (h \gg \Delta) \), and in the opposite case of normal metal \( (h = 0) \) at \( T = 0 \). In between these two extreme cases the low-temperature ZBC is anisotropic, and the anisotropy is most pronounced for \( h \approx \Delta \). The maximum of ZBC always occurs when the direction of the gap node is perpendicular to the FD boundary.

Subject Index 1, 60, 64

1. Introduction

Theoretical and experimental studies of spin-polarized transport in hybrid structures realized with a ferromagnet (F) and d-wave superconductor (D) are interesting from the viewpoint of both spin-dependent spectroscopy and the possible device applications of these materials. For more than 20 years it has been widely accepted that most high-Tc superconductors have anisotropic pairing symmetry, with an order parameter that depends on the position on the Fermi surface \( \Delta(k_F) \) [1,2]. The order parameter may change sign, and thus there are directions on the Fermi surface where the order parameter is zero. The anisotropy of the order parameter, and the sign change, have a profound impact on various superconducting properties and various macroscopic quantities in the mixed state of a superconductor. For example, magnetization, specific heat, and heat conductivity depend on the angles between the applied magnetic field and the crystallographic axes of the superconductor [3–5]. Therefore, based on the information on the positions of the minimum and maximum of magnetization and specific heat as well, one can determine the position of gap nodes on the Fermi surface and thus indirectly obtain information about the order parameter symmetry [6,7].

One can expect that the pairing potential anisotropy will result in a direction-dependent current–voltage characteristic \( I(V) \) and a corresponding conductance \( G(V) = dI/dV \), as well as zero-bias conductance \( \Gamma = G(0) \) in FD bilayer junctions; i.e., all these characteristics should depend on...
Fig. 1. Schematic picture of the ferromagnet/$d$-wave superconductor junction. The anisotropic pair potential of $d_{x^2-y^2}$ wave symmetry is also shown.

the angle $\theta$ of orientation of the superconducting electrode with respect to the F/D interface (see Fig. 1).

In a number of papers concerning FD bilayers only cases where the superconductor (100) and (110) axes are oriented normal to the interface are studied [8–15]. The development of ramp-edge tunnel junctions [16–18] has allowed us to fabricate the junction at any arbitrary angle $\theta$. This was the motivation for the recent study of orientation-dependent Andreev transport in DFD trilayers [19]. Namely, the intuitive picture, in which the electron–hole coherence is suppressed by an increasing exchange field, and as a consequence the subgap current should be reduced, does not hold in cases in which the exchange field is smaller than or of the order of the superconducting gap. The current non-monotonically changes with exchange field, and for small exchange fields the current could even be enhanced [19,20]. All of this has motivated us to clarify how the anisotropy of the $d$-wave symmetry within the $ab$ plane manifests itself through the transport properties of the FD bilayer, especially in the case of a weak ferromagnet.

Early investigations of transport properties of junctions composed of normal metal and $d$-wave superconductor, with an isolator in between (NID), focused on the determination of spectral conductance $\sigma(\epsilon)$ (conductance for quasiparticles with energy $\epsilon$) [21,22]. It is shown that the height of the zero-energy peak in $\sigma(\epsilon)$ significantly depends both on the orientation of the superconducting electrodes and on the isolation barrier strength. The isolation barrier between the normal metal and $d$-wave superconductor is responsible for the large anisotropy of the zero-energy peak that is experimentally observed [18]. For a clean bilayer contact, i.e., in the absence of the isolation barrier, the spectral conductance $\sigma(\epsilon)$ does not depend on the orientation of the $d$-wave superconductor [21].

When normal metal in a bilayer is replaced with ferromagnet, then the transport properties also depend on the value of the exchange field in the ferromagnet [15,23–28]. Based on the results shown by Hirai et al. [28], it seems that the values of ZBC (conductance at zero bias, which is spectral conductance integrated over all quasiparticle energies) in an FD bilayer without an isolation barrier are identical for the considered orientations of the superconductor ($\theta = 0$ and $\theta = \pi/4$) regardless of the value of the exchange field. The region of small exchange fields was not the focus of their paper, but we have found that the orientation dependence of ZBC is most pronounced exactly in this region. It is shown that in between these two extreme cases—normal metal/$d$-wave superconductor and strong ferromagnet/$d$-wave superconductor, where ZBC is isotropic—there is a region of small exchange fields $h$, of the order of $\Delta$, where ZBC of the FD bilayer depends on the orientation of the $d$-wave superconductor. This becomes important in the context of advances in the fabrication of ferromagnetic materials, with exchange fields smaller than or of the order of the superconducting gap in the last few years [29–31].
In this paper we investigate the orientation dependence of transport properties in the simplest case of a voltage-biased FD bilayer in the ballistic limit without an isolation barrier in between, which is readily available nowadays. In such contacts, within the quasiclassical approximation [32], electrons and holes exhibit only Andreev reflection from the FD boundary. Note that the addition of an insulating barrier (FID contacts) will provide the possibility for electrons (holes) to be normally reflected from the boundary, beside Andreev reflection, and therefore experience a change in sign of the $d$-wave order parameter along their classical trajectory. As a result, a zero-energy state (ZES) is formed, which affects the transport properties of the FD contacts. In the FD contacts that are considered here, ZES is absent. We solve the Bogoliubov–de Gennes (BdG) equations [33] within the Blonder–Tinkham–Klapwijk approach generalized to take into account the anisotropic form of the superconducting order parameter [15,23–26]. We calculate conductance, and ZBC as well, as the superconducting order parameter [15,23–26]. We calculate conductance, and ZBC as well, as a function of several parameters such as the angle $\theta$, the exchange field $h$ in the ferromagnet, and temperature $T$.

The paper is organized as follows. In Sect. 2 we briefly formulate the model used for calculation of the current–voltage characteristic. We also present some analytical expressions in limiting cases. In Sect. 3 we present our numerical results. Finally, Sect. 4 is devoted to the conclusion.

2. Formulation

A voltage-biased FD junction with semi-infinite electrodes and a transparent interface in the clean limit is assumed (Fig. 1). The ferromagnet is assumed to be at the potential $V$, while the superconductor is at the potential zero. The Stoner model with an exchange energy shift $2h$ between two sub-bands is used in F. An anisotropic Bardeen–Cooper–Schrieffer (BCS) model of superconductivity is used in D. The Fermi wave vectors $k_F$ and the effective masses $m$ are assumed to be equal in both F and D. The temperature-dependent $d$-wave pair potential is a function of the angle $\varphi$ between the quasiparticle wave vector and the interface normal, given by $\Delta(r) = \Theta(z) \Delta(T) \cos(2\varphi \mp 2\theta) \exp(i\chi) = \Theta(z) \Delta_{\pm}$, where $\Theta(z)$ is the Heaviside step function, $\theta$ is the angle between the $a$-axis of the crystal and the interface normal, $\chi$ is the order parameter phase, and $\Delta(T) = \Delta(0) \tanh[1.76(T_c/T - 1)]$ [34], where $T_c$ is the critical temperature of D. Note that $\Delta_{\pm}$ ($\Delta_{\mp}$) stands for the pair potential for electron-like (hole-like) quasiparticles. The spatial dependence of the pair potential is assumed to be the step function, because there is no remarkable difference between the conductance in the non-self-consistent calculation and that in the self-consistent calculation [28].

We adopt the Bogoliubov–de Gennes (BdG) equation [33] to study the FD bilayer. Electron-like (ELQ) and hole-like (HLQ) quasiparticles with energy $\varepsilon$ (measured relative to the Fermi energy $E_F$) and spin projection $\sigma = \uparrow, \downarrow$ are described by wave functions $u_\sigma(r)$ and $v_\sigma(r)$, where $r$ is the spatial coordinate. In these junctions, the quasiparticle states are generally expressed by wave functions of four components, respectively, for electron-like quasiparticles and hole-like quasiparticles with spin-up and -down. The four-component BdG equations, in the case when spin-flip scattering is absent, may be decoupled into two sets of two-component equations, one for the spin-up electron-like and spin-down hole-like quasiparticle wave function $(u_\uparrow, v_\downarrow)$, and the other for the spin-down electron-like and spin-up hole-like quasiparticle wave function $(u_\downarrow, v_\uparrow)$ [15,26,35]. The BdG equations are given by

$$
\begin{bmatrix}
H_0(r) - \eta_\sigma h(r) & \Delta(r) \\
\Delta^*(r) & -H_0(r) - \eta_\sigma h(r)
\end{bmatrix}
\begin{bmatrix}
u_\sigma(r) \\
v_\sigma(r)
\end{bmatrix}
= \varepsilon \begin{bmatrix}
u_\sigma(r) \\
v_\sigma(r)
\end{bmatrix},
$$

(1)
The wave vectors of the electron and hole in $F$ are given by
\[ k_{\sigma} = k_{\sigma}^e \mathbf{e}_x + k_{\sigma}^h \mathbf{e}_y, \]
where $H_0(r) = -(h^2/2m)\nabla^2 - E_F + eV\Theta(-z)$, $h(r) = h\Theta(-z)$, and $\eta_\sigma = 1$ for $\sigma = \uparrow$ and $\eta_\sigma = -1$ for $\sigma = \downarrow$; $\sigma$ stands for the spin opposite to $\sigma$.

There are four solutions of the scattering problem for Eq. (1), which correspond to the four types of quasiparticle injection processes: an ELQ or HLQ injected from the ferromagnet or superconducting electrode [36]. For example, considering a beam of spin-$\sigma$ ELQ incident on the interface at $z = 0$ from F to D, there are four possible trajectories: normal reflection ($h_\sigma$), Andreev reflection ($a_\sigma$) as a hole with spin $\bar{\sigma}$, transmission to D as a spin-$\sigma$ ELQ ($c_\sigma$), and transmission to D as a spin-$\bar{\sigma}$ HLQ ($d_\sigma$). The junction is translatory invariant in directions perpendicular to the $z$-axis; consequently the parallel component of the wave vector, $k_z = k_x e^x + k_y e_y$, is conserved and the wave function can be written as
\[ \begin{bmatrix} u_\sigma(r) \\ v_\sigma(r) \end{bmatrix} = e^{ik_z r} \begin{bmatrix} u_\sigma(z) \\ v_\sigma(z) \end{bmatrix}. \] (2)

In this way, owing to translational invariance in the direction parallel to the interface, the BdG equations are reduced to effective 1D equations. Thus, the solutions of the BdG equations for $\sigma$-spin electron injections from F (the first of the four solutions of the scattering problem) are given by
\[ \Psi^F_\sigma(z < 0) = \begin{bmatrix} u_\sigma(z < 0) \\ v_\sigma(z < 0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i\theta_0 z} + b_\sigma \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{-i\theta_0 z} + a_\sigma \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{i\theta_0 z}, \] (3)
\[ \Psi^D_\sigma(z > 0) = \begin{bmatrix} u_\sigma(z > 0) \\ v_\sigma(z > 0) \end{bmatrix} = c_\sigma \begin{bmatrix} U^+_0 e^{i\chi/2} \\ V^+_0 e^{-i\chi/2} \end{bmatrix} e^{ik^+_z z} + d_\sigma \begin{bmatrix} V^-_0 e^{i\chi/2} \\ U^-_0 e^{-i\chi/2} \end{bmatrix} e^{-ik^-_z z}. \] (4)

The wave vectors of the electron and hole in F are given by
\[ q^e_\sigma \simeq k_F \left\{ 1 + \frac{1}{2E_F} (\epsilon + \rho_\sigma h - eV) \right\}, \quad q^h_\sigma \simeq k_F \left\{ 1 - \frac{1}{2E_F} (\epsilon + \rho_\sigma h + eV) \right\}, \] (5)
while the wave vectors of ELQ and HLQ in D are given by
\[ k^+_e = k_F \left\{ 1 + \frac{1}{E_F} \left( \epsilon^2 - \Delta^2 \right)^{1/2} \right\}^{1/2}, \quad k^-_h = k_F \left\{ 1 - \frac{1}{E_F} \left( \epsilon^2 - \Delta^2 \right)^{1/2} \right\}^{1/2}. \] (6)

Here $k_F$ and $E_F$ are the $z$-component of the Fermi wave vector and Fermi energy, respectively. In the following we apply the quasiclassical approximation [32], and the wave vectors $q^e_\sigma, q^h_\sigma, k^+_e$, and $k^-_h$ are approximately equal to $k_F$. Note that
\[ U^+_0 = \frac{1}{\sqrt{2}} \left\{ 1 + \sqrt{\epsilon^2 - \Delta^2 \pm \epsilon} \right\}^{1/2}, \quad V^+_0 = \frac{1}{\sqrt{2}} \left\{ 1 - \sqrt{\epsilon^2 - \Delta^2 \pm \epsilon} \right\}^{1/2}. \] (7)

Similarly, one can solve the BdG equations for three other cases: injection of a $\sigma$-spin hole from F, and injection of a $\sigma$-spin hole (electron) from D. All coefficients for the four scattering processes can be determined by matching the boundary conditions at $z = 0$: $\Psi^F_\sigma(0) = \Psi^D_\sigma(0)$ and $d\Psi^F_\sigma/dz|_{z=0} = d\Psi^D_\sigma/dz|_{z=0}$.

The current density in the FD bilayer can be obtained from the solutions associated with the four ($i = 1, \ldots, 4$) processes listed above [33]:
\[ \mathbf{j} = \sum_{i=1}^{4} \sum_{\sigma=\uparrow,\downarrow} j^i_\sigma, \] (8)
where

\[ j_\sigma^i = \frac{e}{2m} \sum_i (f_{\sigma}^i u_{\sigma}^i \mathbf{P} u_{\sigma}^i + (1 - f_{\sigma}^i) v_{\sigma}^i \mathbf{P} v_{\sigma}^i) + \text{c.c.} \]

The sum goes over all possible states; the gauge-invariant gradient is \( \mathbf{P} = -i\hbar \nabla - e\mathbf{A}/c \). In the calculation we have neglected the magnetic field \( \mathbf{A} \) created by the current flowing through the contact. Note that \( f_{\sigma}^i (i = 1, \ldots, 4) \) are Fermi distribution functions: \( f_{\sigma}^1 = f(\varepsilon + \rho_\sigma h - eV) \), \( f_{\sigma}^2 = f(\varepsilon + \rho_\sigma h + eV) \), \( f_{\sigma}^3 = f(\varepsilon) \). To calculate the current density in the FD bilayer one has to do a summation over all possible states in \( \mathbf{k} \)-space. After switching from the sum to the integral the \( z \)-component of the current density is given by

\[ j = e^2 v_F N(0) V + \int_0^{\pi/2} (j_-(\varphi) - j_+(\varphi)) \sin 2\varphi d\varphi, \]  

where

\[ j_+(\varphi) = \frac{e v_F N(0)}{4} \int \frac{1}{|\Delta_+|} \left[ 1 - \left[ \frac{v_0^+}{U_0^+} \right]^2 \right] (f(\varepsilon + h - eV) + f(\varepsilon - h - eV) - 2f(\varepsilon)) d\varepsilon, \]

\[ j_-(\varphi) = \frac{e v_F N(0)}{4} \int \frac{1}{|\Delta_-|} \left[ 1 - \left[ \frac{v_0^-}{U_0^-} \right]^2 \right] (f(\varepsilon + h + eV) + f(\varepsilon - h + eV) - 2f(\varepsilon)) d\varepsilon. \]

Note that the factor \( \sin 2\varphi \) enters Eq. (9) since the factor \( \cos \varphi \) is coming from the \( z \)-component of the Fermi wave vector \( (k_F = k_F \cos \varphi) \), while the factor \( \sin \varphi \) is coming from the elementary volume \( d\mathbf{k} \) in \( \mathbf{k} \)-space. Further, \( N(0) = m k_F / 2 \pi^2 h^2 \) is the density of states, and \( v_F \) is the Fermi velocity. Note that, at \( T = T_c \), when the order parameter drops to zero, \( \Delta_+ = \Delta_- = 0 \) and \( V_0^\pm = 0 \), we recover the standard result for current density between two normal metals (N):

\[ j_{NN} = \frac{1}{2} e^2 v_F N(0) V. \]  

Also, for an isotropic \( s \)-wave (S) superconductor \( (\Delta_+ = \Delta_- = \Delta) \) in contact with normal metal \( (h = 0) \) at zero temperature, we have \( j_+ = j_- = 0 \) and

\[ j_{NS} = 2j_{NN} = e^2 v_F N(0) V. \]

The corresponding conductance \( G(V) \) can be calculated from the current density (Eq. (9)) as

\[ G(V) = \frac{dI}{dV} = L_x L_y \frac{dj}{dV} = L_x L_y \left[ e^2 v_F N(0) - \int_0^{\pi/2} (G_+(\varphi) + G_-(\varphi)) \sin 2\varphi d\varphi \right], \]  

where \( L_x L_y \) is the cross-sectional area of the FD contact and

\[ G_+(\varphi) = \frac{e^2 v_F N(0)}{4 k_B T} \int \frac{1}{|\Delta_+|} \left[ 1 - \left[ \frac{v_0^-}{U_0^-} \right]^2 \right] \left\{ \frac{1}{\cosh^2 \left[ \frac{\varepsilon + h - eV}{k_B T} \right]} + \frac{1}{\cosh^2 \left[ \frac{\varepsilon - h - eV}{k_B T} \right]} \right\} d\varepsilon, \]

\[ G_-(\varphi) = \frac{e^2 v_F N(0)}{4 k_B T} \int \frac{1}{|\Delta_-|} \left[ 1 - \left[ \frac{v_0^-}{U_0^-} \right]^2 \right] \left\{ \frac{1}{\cosh^2 \left[ \frac{\varepsilon + h + eV}{k_B T} \right]} + \frac{1}{\cosh^2 \left[ \frac{\varepsilon - h + eV}{k_B T} \right]} \right\} d\varepsilon. \]
We are particularly interested in zero-bias conductance

$$\Gamma = G(0) = \frac{dI}{dV}|_{V=0},$$

(16)

because the effect of anisotropy is the most pronounced in this case.

Before proceeding to numerical calculation of the current density and conductance, according to Eqs. (9) and (13), respectively, it would be useful to get some analytical expressions, which are readily available in the limit of zero temperature and zero exchange field.

2.1. Limiting case: normal metal/\(d\)-wave superconductor at low temperatures

Note that in the case of a normal metal/\(d\)-wave superconductor at low temperatures, the \(T \to 0\) expression for current density can be further simplified, since we have that

$$j_{-}(\phi) \approx 0,$$

(17)

and

$$\frac{j_{+}(\phi)}{|\Delta_{+}|} = \frac{e^{\frac{\sqrt{\Delta_{2}^{2} - \Delta_{+}^{2}}}{2}}}{|\Delta_{+}|} \left\{ a^{3} \left[ \left(1 - \frac{1}{a^{2}}\right)^{3/2} - 1 \right] + a - \frac{2}{3} \right\}, \quad a = \frac{eV}{|\Delta_{+}|}.\tag{18}$$

Further, in a limit of high voltages \(eV \gg \Delta(0)\) (and \(eV \ll E_{F}\)) the current density could be approximated as a sum of ohmic current density (linear in \(V\)) excess current density (\(V\) independent):

$$j = \frac{1}{2} e^{2} v_{F} N(0) V + \frac{1}{3} e v_{F} N(0) \Delta(0) \{\cos 2\theta + 2\theta \sin 2\theta\}, \quad 0 \leq \theta \leq \pi/4.\tag{19}$$

Therefore in this limit the corresponding conductance,

$$G(V) = \frac{1}{2} e^{2} v_{F} N(0) L_{x} L_{y},\tag{20}$$

is independent of the superconducting angle \(\theta\).

In the opposite limit of low voltages, \(eV \ll \Delta(0)\), only quasiparticles that fly along directions close to the gap nodes \(\phi \approx \pi/4 + \theta\), where \(\Delta_{+} \approx 0\), contribute to the current \(j_{+}\). After expanding \(j_{+}(\phi) \sin 2\phi\) around \(\phi = \pi/4 + \theta\) it is straightforward to show that

$$j = e^{2} v_{F} N(0) \left\{ V - \frac{e^{2} \Delta(0)}{\Delta(0)} \left(1 - \frac{\pi}{4}\right) \cos 2\theta\right\}, \quad 0 \leq \theta \leq \pi/4.\tag{21}$$

So in the limit of small voltages conductance is weakly anisotropic:

$$G(V) = e^{2} v_{F} N(0) \left\{1 - \frac{2eV}{\Delta(0)} \left(1 - \frac{\pi}{4}\right) \cos 2\theta\right\} L_{x} L_{y},\tag{22}$$

with the anisotropy being of the order of the small parameter \(eV/\Delta(0)\).

Evidently ZBC at low temperatures in a normal metal/\(d\)-wave superconductor bilayer is isotropic:

$$\Gamma = G(0) = e^{2} v_{F} N(0) L_{x} L_{y},\tag{23}$$

i.e., does not depend on the orientation of the \(d\)-wave superconductor with respect to the contact boundary, in the same way as in Ref. [21].
Fig. 2. Conductance $G(V)$ for $T/T_c = 0.05$, $\theta = 0$, and three values of exchange field $\tilde{h} = h/\Delta(0) = 1, 2, \text{and } 3$.

3. Results

The conductance $G(V)$ (Eq. (13)) can be calculated analytically in only a few limiting cases at $T = 0$. In order to investigate quasiparticle transport through a voltage-biased ferromagnet/$d$-wave superconductor bilayer for various values of temperature $T$, the exchange field $h$ in the ferromagnetic layer, and the angle $\theta$ of orientation of the $d$-wave electrode, one has to calculate the conductance $G(V)$ numerically. In the following, we introduce reduced units: $\tilde{h} = h/\Delta(0)$, $\tilde{V} = eV/\Delta(0)$, $t = T/T_c$, $\tilde{G} = G/e^2 v_F N(0)L_x L_y$, and $\tilde{\Gamma} = \Gamma/e^2 v_F N(0)L_x L_y$.

The effect of the exchange field on the conductance curve at low temperature and for angle $\theta = 0$ is shown in Fig. 2. The three curves correspond to three values of exchange field $\tilde{h} = 1, 2, \text{and } 3$. Obviously conductance $G(V)$ peaks at $eV = h$, which could be used to determine the value of the exchange field by measuring the conductance $G(V)$ at low temperature when the full gap of the $d$-wave superconducting electrode is facing the F/D boundary. This is similar to the method proposed by Ozaeta et al. [20] for an S–F–N hybrid structure with an $s$-wave superconductor (S) and normal metal (N) electrode. The height of the peak $\tilde{G}_{\text{max}}$ (for $T = 0$) and for $\tilde{h} > 0.5$ is given by

$$\tilde{G}_{\text{max}} = \frac{3}{4} + \frac{1}{4} \int_0^{\pi/2} \frac{2\tilde{h} - \sqrt{4\tilde{h}^2 - \Omega_+^2}}{2\tilde{h} + \sqrt{4\tilde{h}^2 - \Omega_+^2}} \sin 2\varphi d\varphi,$$

where $\Omega_+ = \tanh (1.76\sqrt{T_c/T} - 1) \cos(2\varphi - 2\theta)$. $\tilde{G}_{\text{max}}$ weakly depends on $\tilde{h}$ and to a good approximation $\tilde{G}_{\text{max}} \approx 3/4$, which can also be seen in Fig. 2.

The influence of the orientation of the superconducting electrode with respect to the F/D interface on $G(V)$ at low temperatures is shown in Fig. 3 for two values of exchange field $\tilde{h} = 0.8$ (Fig. 3(a)) and $\tilde{h} = 3$ (Fig. 3(b)), and for two angles $\theta = 0$ and $\theta = \pi/4$. For higher exchange fields, a change in the superconducting electrode orientation will just change the shape of the peak at $eV = h$ (Fig. 3(b)). For small exchange fields $\tilde{h} \leq 1$, the difference in conductance for two orientations is more visible and the difference is maximal for zero bias, as can be seen in Fig. 3(a). This is in contrast to the case of normal metal in contact with a $d$-wave superconductor (see Eq. (23)) because when the normal metal is replaced with ferromagnet then ELQ and HLQ with different spin orientations make different contributions to the total current. This imbalance paves the way for ZBC anisotropy to appear even at low temperatures. Since the anisotropy of spin-dependent transport is most pronounced at ZBC, we will study this particular case of conductance in more detail.
Fig. 3. Conductance $G(V)$ for $T/T_c = 0.05$ and two values of angle $\theta = 0$ (black dashed line) and $\pi/4$ (red solid line). (a) $\tilde{h} = h/\Delta(0) = 0.8$. (b) $\tilde{h} = h/\Delta(0) = 3$.

Fig. 4. The dependence of zero-bias conductance $\Gamma$ on angle $\theta$ (of orientation of the $d$-wave electrode with respect to the F/D interface) for $T/T_c = 0.05$ and $\tilde{h} = h/\Delta(0) = 0.8$.

The angular dependence of ZBC is shown in Fig. 4. The ZBC has the same periodicity $\pi/2$ as $|\Delta_{\pm}|$ but with a maximum at $\theta = \pi/4$ and a minimum at $\theta = 0, \pi/2$. We find that this is a universal characteristic in FD bilayer in the sense that for all ranges of temperatures and exchange fields the maximum of ZBC always appears when the superconducting gap node is perpendicular to the interface with metal. The physical origin of this effect is in the following. Namely, low-voltage conductance at low temperatures in ND, and in FD junctions as well, is mainly realized by Cooper pairs, except for electrons (holes) with momenta close to the positions of gap nodes at the Fermi surface. The number of these electrons (holes) in ND junctions is proportional to $eV$ at low voltages, so that low-voltage conductance due to Cooper pairs, $e^2v_FN(0)$, is reduced by linearity in the $eV$ term, as can be seen from Eq. (22). When normal metal is replaced with weak ferromagnet $h \ll \Delta(0)$, then all arguments also hold in the case in which $eV$ is replaced by $h \pm eV$. ZBC due to Cooper pairs, $e^2v_FN(0)$, is reduced by linearity in the $h$ term (see Fig. 6(b) below). To explain why this correction
term to ZBC is minimal for $\theta = \pi/4$ one should analyze the expression for current density in FD bilayer (Eq. (9)). In the correction term for the current density there is a weighting factor $\sin 2\varphi$. When $d_{x^2-y^2}$ superconductor is oriented with its (100) axis normal to the boundary with ferromagnet (i.e. $\theta = 0$), then the direction $\varphi = \pi/4$ corresponds to the gap node, and the correction, which is due to electrons (holes) close to the gap node, is maximal. For the (110) axis normal to the boundary with ferromagnet (i.e., $\theta = \pi/4$), the direction $\varphi = \pi/4$ corresponds to the gap maximum, and the correction due to electrons (holes) close to the gap node is minimized, i.e., current density and conductivity is maximized.

It can also be seen that the difference between the value of maximum and minimum is most pronounced when $h$ is small ($h < \Delta_1$). This characteristic may be used for determination of the gap node position by measuring the angular dependence of ZBC in FD bilayer especially in ferromagnet with a small exchange field.

On the other hand, it is known that the low-energy transport in these systems is governed by Andreev reflection (AR), which causes switching from a particle-like to a hole-like quasiparticle state in a superconductor. Since the incoming electron and Andreev-reflected hole in ferromagnet occupy an energy band with opposite spins, the mechanism of charge transport at the interface between the superconductor and ferromagnet is modified in comparison with the ND case, which leads to the fact that ZBC is sensitive to the degree of spin polarization in the ferromagnet [28]. Therefore, the AR is broken due to the exchange potential and the magnitude of $\Gamma$ diminishes with increasing exchange field in the ferromagnet for each angle $\theta$; see Fig. 5(a).

To measure the influence of exchange field or temperature on ZBC anisotropy let us introduce the relative anisotropy:

$$A = \left| \frac{\Gamma_{\text{max}}}{\Gamma_{\text{min}}} - 1 \right|. \quad (25)$$

In Fig. 5(b) the relative anisotropy of ZBC as a function of exchange field $A(\tilde{h})$ is shown. As one can see, the anisotropy of ZBC for $h = 0$ (ND junction) is approximately zero, but for $h \approx \Delta(0)$ it may be increased up to 30% at low temperatures. Further increase of the exchange field suppresses anisotropy, which diminishes for $h \gg \Delta$. ZBC in FD bilayers as a function of exchange field $h$, for two different orientations, is shown by Hirai et al. [28]. Both curves for the two different orientations look identical (see Fig. 2 in Ref. [28]), which means that there is no ZBC anisotropy. Actually, they are using different units for the exchange field, so the ZBC dependence on $h$ shown in Fig. 2 of Ref. [28] mainly covers the region of large exchange fields, where there is no ZBC anisotropy, like in our calculation for large $h$. But the region of very small exchange fields was not a focus of their investigation. Nowadays, weak ferromagnets are readily available [29–31], and several reliable detection methods to measure small exchange field have been proposed [19,20,37,38].

Next we show the temperature dependence of $\Gamma$ for various $h$ and $\theta$ (Fig. 6(a)). The exchange potential in ferromagnets affects the temperature dependence of $\Gamma$. For $h = 0$ and for small $h$ ($h \approx \Delta(0)$), $\Gamma$ decreases with increasing temperature $T$ for every angle $\theta$. This is because the current at low voltages is mainly carried by two electron processes through AR and the amplitude of the Andreev reflection is suppressed for $T \rightarrow T_c$. For larger values of $h$ (but still $h \ll E_F$) AR is mainly suppressed by the large exchange potential and the current is mainly carried by a single electron process, so the ZBC is practically constant with temperature and tends to a value of $\tilde{\Gamma} = 0.5$. Note that, comparing Figs. 5(a) and 6(a), one can notice that for large exchange fields and
small temperatures (Fig. 5(a)), as well as for zero exchange field and $T \rightarrow T_c$ (Fig. 6(a)), we have $\tilde{\Gamma} \approx 0.5$, but the underlying physics is different. The order parameter $\Delta(T)$ decreases with increasing temperature and disappears at $T \rightarrow T_c$, in which case we have contact of two normal metals with zero-bias conductance $\tilde{\Gamma} = 0.5$. On the other hand ZBC for large exchange fields $h \gg \Delta(0)$ (but still $h \ll E_F$) also approaches a value of $\tilde{\Gamma} = 0.5$ because large exchange fields suppress one spin orientation of the current, while the other component survives.

In Fig. 6(b) we focus on the temperature dependence of ZBC relative anisotropy $A(t)$. We note that in the case of normal metal ($h = 0$) we have $A(T = 0) = A(T = T_c) = 0$. Since ZBC anisotropy is a
macroscopic manifestation of the anisotropy of the order parameter, it is quite natural that, when the order parameter disappears for \( T \to T_c \), the ZBC anisotropy also disappears. In the opposite limit when \( T \to 0 \), the contribution to the current density that goes beyond linearity in \( V \) (Ohm’s law) comes from the region in the near vicinity of the gap node, which is of the order of \( eV/\Delta(0) \), and the correction to the current is of the order of \( e^2V^2/\Delta^2(0) \) (see Eq. (21)). This means that there is no correction of ZBC beyond the standard result \( \Gamma = 1 \). The effect of exchange field on the temperature dependence of ZBC anisotropy is also examined. We find that the most prominent ZBC anisotropy could be achieved at low temperature and for a value of exchange field \( h \lesssim \Delta \).

4. Conclusion

In this paper we investigate the mutual influence of the exchange field of ferromagnetic metal and temperature on the anisotropy of transport properties (especially on ZBC) in a ballistic voltage-biased FD bilayer junction. For exchange fields greater than \( \Delta \), the conductance \( G(V) \) at low temperatures has a well defined maximum for \( eV = h \) and for all orientations of the superconducting electrode. Measuring the position of the maximum, one may determine the value of exchange field in the ferromagnet, regardless of the superconductor orientation. But in weak ferromagnets with \( h \lesssim \Delta(0) \), the position and height of \( G_{\text{max}} \) strongly depends on the superconducting electrode orientation. It appears that the difference in \( G(V) \) for two different superconductor orientations at low temperatures is most pronounced at zero bias, i.e., we found that ZBC exhibits the strongest anisotropy. Actually, for \( h = 0 \) (the normal metal case), the ZBC anisotropy disappears for low \( (T \to 0) \) and high \( (T \to T_c) \) temperatures, while in the case of a strong ferromagnet \( \Delta(0) \ll h \ll E_F \), ZBC is isotropic for all temperatures. In the FD bilayer the maximum of ZBC always occurs when the \( d \)-wave superconductor is oriented with its gap node facing the boundary with the ferromagnetic contact. This result is universal in the sense that it holds for all temperatures and all values of exchange field \( h \). In this way, measurement of the angular dependence of ZBC could provide information about the position of gap nodes in a superconductor, assuring that it is in contact with a weak ferromagnet. Again, one should note that ZBC anisotropy is detectable only when the exchange field in the ferromagnet layer is of the order of the order parameter \( (h \approx \Delta) \).

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