Exploring Final State Hadron Structure 
and SU(3) Flavor Symmetry Breaking Effects 
in $D \to PP$ and $D \to PV$ Decays 

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Abstract 

The nonleptonic two body decays $D \to PP$ and $D \to PV$ are investigated based on the diagrammatic decomposition in a generalized factorization formalism. It is shown that to fit the experimental data, the SU(3) flavor symmetry breaking effects of the coefficients $a_i$s should be considered in $D \to PP$ decay modes. In $D \to PV$ decays, the final state hadron structure due to the pseudoscalar and vector mesons has more important effects on the coefficients $a_i$s than the SU(3) symmetry breaking effects. It is found that the nonfactorizable contributions as well as that of the exchange and annihilation diagrams are significant in these decays.
I. INTRODUCTION

Charmed meson nonleptonic two body decays have been an interesting subject of research for a long time as it can provide useful information on flavor mixing, $CP$ violation and strong interactions. The theoretical settlement of this transition type generally appeals to the factorization hypothesis. Empirically, nonfactorizable corrections which result from spectator interactions, final state interactions and resonance effects should be considered. The nonfactorizable corrections are believed to be significant, and they are relatively hard to be calculated because the charmed quark is not heavy enough to apply the QCD factorization approach or PQCD approach in a reliable manner. Fortunately, a great number of precise experimental data on charmed meson nonleptonic two body decays have been accumulated in recent years. Many new results are expected soon from the dedicated experiments conducted at BES, CLEO, E791, FOCUS, SELEX and the two $B$ factories BaBar and Belle. Phenomenological models based on all kinds of symmetries are of quite importance to guide the theoretical studies and explore new physics. But in some cases, the symmetry breaking effects can be significantly enhanced.

In the quark diagrammatic scenario, all two-body nonleptonic weak decays of charmed mesons can be expressed in terms of six distinct quark-graph contributions: (1) a color-favored tree amplitude $T$, (2) a color-suppressed tree amplitude $C$, (3) a $W$-exchange amplitude $E$, (4) a $W$-annihilation amplitude $A$, (5) a horizontal $W$-loop amplitude $P$ and (6) a vertical $W$-loop amplitude $D$. The $P$ and $D$ diagrams play little role in practice because the CKM matrix elements have the relation $V_{cs}^*V_{us} \approx -V_{cd}^*V_{ud}$ which will result in cancellations among these diagrams.

Based on SU(3) flavor symmetry, the $T$, $C$, $E$ and $A$ amplitudes were fitted from the measured $D$ meson decay modes. These amplitudes help one to understand the generality of charmed meson decays. But since SU(3) flavor symmetry breaking effects appear to be important, these fitted data can not describe the specific properties in certain decay modes. In , we investigated in detail both the Cabibbo-allowed and singly Cabibbo-suppressed $D \rightarrow PV$ decays based on the diagrammatic decomposition in the factorization formalism and found that the SU(3) symmetry breaking effects in the $D \rightarrow PV$ decays are significant. Two sets of solutions were found in the formalism of factorization. The case (I) solution can provide satisfactory explanation in a natural manner on the process $D^+ \rightarrow \overline{K}^0 K^{*+}$ which is thought to be a puzzle. But the solution is hard to be explained from the theoretical point of view because this solution requires such an unexpected large correction from nonfactorizable contributions that the strong phase of $a_{T_p}$ has around $150^\circ$ deviation from that of Wilson coefficients $c_1$. The case (II) solution shows relatively small correction from nonfactorizable contributions and hence seems more reasonable from theoretical point of view. But the solution predicts a relatively small branching ratio of the process $D^+ \rightarrow \overline{K}^0 K^{*+}$ in comparison with the experimental result. With such a treatment via solving fifteen equations for extracting out the same numbers of parameters, it is hard to consider the experimental uncertainties in . To investigate what impacts the experimental uncertainties will bring to the extracted parameters, it is useful to make a systematic analysis with taking into account the experimental uncertainties.

In this paper, we will perform a $\chi^2$ fitting procedure on charmed mesons decaying to a pseudoscalar and a vector meson ($D \rightarrow PV$) and also decaying into two light pseudoscalar mesons ($D \rightarrow PP$) by using the quark-graph description based on a generalized
factorization formalism which reflects SU(3) flavor symmetry breaking effects. Firstly by dividing these diagrams into factors including SU(3) flavor symmetry breaking effects and introducing parameters describing the overall properties, we arrive at two sets of solutions for the parameters from fitting experimental data. In the viewpoint of diagrammatic decomposition, the generalized QCD parameters $a_i (i = 1, 2)$ will be classified into two sets of parameters $a_i^p$ and $a_i^v$. The difference between $a_i^p$ and $a_i^v$ arises from the final state hadron structure of the pseudoscalar and vector mesons in $D \rightarrow PV$. In $D \rightarrow PP$ decays, we will show that, to fit the experimental data, one should classify the parameters $a_i$ into $a_i^d$ and $a_i^s$, which means that the SU(3) flavor symmetry breaking effects are important and need to be considered in the $a_i$s. Thus we can arrive at a conclusion that the coefficients $a_1$ and $a_2$ depend on either the final state hadron structure or SU(3) flavor symmetry breaking effects. For $D \rightarrow PP$ decay modes, the SU(3) flavor symmetry breaking effects play an important role in the coefficients $a_1$ and $a_2$, while for $D \rightarrow PV$ decays, the final state hadron structure becomes more important for the contributions to the coefficients than the SU(3) symmetry breaking effect does. The contributions of SU(3) flavor symmetry breaking effects to $a_1$ and $a_2$ can be neglected in $D \rightarrow PV$ decay modes. Using the fitted parameters as inputs, we are led to predictions for the branching ratios of other decay modes which are expected to be measured in the future. In studying the breaking of the SU(3) symmetry relations, we are able to quantify the SU(3) breaking effects. The breaking amount of the SU(3) symmetry relations in some channels can be significant so that it becomes unreliable to use the SU(3) relations to make predictions for some decay modes.

The paper is organized as follows. In section II we list the flavor decomposition of the corresponding mesons and present the quark-diagram description for the relevant decay modes. In section III the parameterized formalism based on factorization is introduced to investigate the processes. We then perform a fit procedure in section IV to extract the parameters and present predictions for thirty three $D \rightarrow PP$ decay modes and sixty two $D \rightarrow PV$ decay modes. The SU(3) flavor symmetry breaking effects are discussed in section V. A short summary and remark is given in the last section.

II. NOTATION AND QUARK-DIAGRAM FORMALISM

We adopt the following quark contents and phase conventions which have been widely used [10, 11, 12, 17].

- **Charmed mesons**: $D^0 = -c\bar{u}$, $D^+ = c\bar{d}$, $D^+_s = c\bar{s}$;
- **Pseudoscalar mesons** $P$: $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (u\bar{u} - d\bar{d} + s\bar{s})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$;
- **Vector mesons** $V$: $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\bar{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

In the above notations, $u$, $d$ and $s$ quarks transform as a triplet of flavor SU(3) group, and $-\bar{u}$, $\bar{d}$ and $\bar{s}$ as an antitriplet, so that mesons form isospin multiplets without extra
signs. In general, the $\eta\eta'$ mixing are defined as

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = 
\begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} 
\begin{pmatrix}
\eta_8 \\
\eta_0
\end{pmatrix}
$$

(1)

with $\eta_0 = (u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3}$ and $\eta_8 = (-u\overline{u} - d\overline{d} + 2s\overline{s})/\sqrt{6}$. For convenience, we have taken the mixing parameter as $\phi = 19.5^\circ = \sin^{-1}(1/3)$ which is close to the value $\phi = 15.4^\circ$ extracted from experiment\[18].

The partial width $\Gamma$ for $D \rightarrow PP$ and $D \rightarrow PV$ decays is expressed in terms of an invariant amplitude $A$. One has

$$
\Gamma(D \rightarrow PP) = \frac{p}{8\pi M_D^2} |A|^2
$$

(2)

for $D \rightarrow PP$ and

$$
\Gamma(D \rightarrow PV) = \frac{p^3}{8\pi M_D^2} |A|^2
$$

(3)

for $D \rightarrow PV$, where

$$
p = \sqrt{(M_D^2 - (m_1 + m_2)^2)(M_D^2 - (m_1 - m_2)^2)}
$$

$$
2M_D
$$

denotes the center-of-mass 3-momentum of each final particle.

In $D \rightarrow PP$ decays, to describe the flavor SU(3) breaking effects, a subscript $s$ or $d$ is attributed on $T$ and $C$ diagrams to distinguish the initial $c$ quark transits to $s$ quark or $d$ quark. The subscript $s$ or $d$ is attached to the diagrams $E$ and $A$ dominated by the weak process $c\overline{q}_1 \rightarrow q_2\overline{q}_3$ when the antiquark $\overline{q}_3$ is $\overline{s}$ or $\overline{d}$. In $D \rightarrow PV$ decays, a subscript $P$ or $V$ is assigned to $T$ and $C$, which are induced by $c \rightarrow q_3\overline{q}_3$ with the spectator quark containing in pseudoscalar or vector final meson. The subscript $P$ or $V$ is labelled to $E$ and $A$ graphs which are dominated by the weak process $c\overline{q}_1 \rightarrow q_2\overline{q}_3$ when the final antiquark $\overline{q}_3$ stays in the pseudoscalar or vector meson. $S$ is added before $E$ or $A$ to distinguish the exchange or annihilation graph involving in final singlet state contributions which result from disconnected graphs.

The total contributions of the $SE$, $SA$ graphs involving in $\pi^0$ and $\rho^0$ mesons are equal to zero because their contributions resulting from $u\overline{u}$ and $-d\overline{d}$ offset each other due to the isospin SU(2) symmetry. In the numerical analysis, we will assume that the contributions of the $SE_P$ and $SE_V$ graphs involving in $\omega$ and $\phi$ mesons are negligibly small since they seem not to contradict with the Okubo-Zweig-Iizuka rule. But the amplitude $SA_V$ seems to play an important role in the $D_s^+ \rightarrow \rho^+\eta$ and $D_s^+ \rightarrow \rho^+\eta'$ processes.\[10]. In the ideal mixing case, the process $D_s^+ \rightarrow \pi^+\omega$ has the amplitude representation as $\frac{1}{\sqrt{2}}(A_V + A_P + 2SA_P)$. Since $\omega$ has a similar quark structure in comparison with $\eta$ and $\eta'$, we assume that $SA_P$ has an important contribution in $D_s^+ \rightarrow \pi^+\omega$. In the present paper, we shall not consider the processes which receive contributions from $SA_V$ and $SA_P$ diagrams resulting from the final state particles $\eta$, $\eta'$ or $\omega$. The sign flips in the presentation of some relevant Cabibbo-favored modes, as well as that of some doubly Cabibbo-suppressed modes, come from the quark contents of final light mesons. In the singly Cabibbo-suppressed modes, the sign flips may come either from the quark contents of the final light mesons or from the CKM matrix element $V_{cd}^*V_{ud}$ since $V_{cs}^*V_{us} \approx -V_{cd}^*V_{ud}$ and we choose $V_{cs}^*V_{us}$ in the calculations. In Table IV\[14] and Table V\[11] a prime and double prime are added to the diagrams of singly Cabibbo-suppressed modes and doubly Cabibbo-suppressed modes respectively to distinguish them from the Cabibbo-favored ones.
### III. Flavor SU(3) Symmetry Breaking Description in Generalized Factorization Formalism

To investigate the SU(3) flavor symmetry breaking effects, we take the formalism of a generalized factorization approach [2, 20].

For $D \to PP$ decays, amplitudes can be written in the form as

$$T_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V_{q_3q_4}^* a_{T_{s,d}} f_{B_{s,d}}(m_{D_1}^2 - m_{P_1}^2) F_{0_{D_1} \to P_2}(m_{P_1}^2),$$

$$C_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V_{q_3q_4}^* a_{C_{s,d}} f_{B_{s,d}}(m_{D_1}^2 - m_{P_1}^2) F_{0_{D_1} \to P_2}(m_{P_1}^2),$$

$$E_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V_{q_2q_4}^* a_{E_{s,d}} f_{D_1},$$

$$A_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V_{q_1q_4}^* a_{A_{s,d}} f_{D_1}.$$  \hspace{1cm} (4)

For $D \to PV$ decays, amplitudes can be written in the form as

$$T_P = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V_{q_3q_4}^* a_{T_P} 2f_{P_{D_1}} A_{0_{D_1} \to V}(m_{P_1}^2),$$

$$C_P = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V_{q_3q_4}^* a_{C_P} 2f_{P_{D_1}} A_{0_{D_1} \to V}(m_{P_1}^2),$$

$$E_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V_{q_2q_4}^* a_{E_{V,P}} 2f_{D_1} m_{D_1},$$

$$A_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V_{q_1q_4}^* a_{A_{V,P}} 2f_{D_1} m_{D_1}.$$  \hspace{1cm} (5)

$D_i$ denotes $D^\pm$, $D_0$ or $D_s$. $F_0$, $F_1$ and $A_0$ are formfactors defined in the following formalism

$$\langle P(p)|\bar{q}\gamma^\mu c|D(p_D)\rangle = (p_D + p)_\mu - \frac{m_D^2 - m_p^2}{q^2} q^\mu \left[ F_1(q^2) + \frac{m_D^2 - m_P^2}{q^2} q^\mu F_0(q^2) \right],$$

$$\langle V(p)|\bar{q}\gamma^\mu (1 - \gamma^5)c|D(p_D)\rangle = -i (m_D + m_V) A_1(q^2) (\epsilon^\mu - \frac{\epsilon^\nu q^\mu}{q^2} q^\nu)$$

$$+ i \frac{A_2(q^2)}{m_D + m_V} (\epsilon^\nu (p_D + p)^\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu) - i \frac{2m_V}{q^2} (\epsilon^\nu q^\mu A_0(q^2) q^\nu) - \frac{2V(q^2)}{m_D + m_V} \epsilon^{\alpha\beta\gamma\delta} \gamma_\alpha p_D p_\beta p_\gamma,$$  \hspace{1cm} (14)

with $q = p_D - p$. $f_P$ and $f_V$ are decay constants defined as

$$\langle P(p)|\bar{q}\gamma^\mu \gamma_5 q_2|0\rangle = -i f_P p^\mu,$$

$$\langle V(p)|\bar{q}\gamma^\mu q_2|0\rangle = f_V m_V \epsilon^\mu.$$  \hspace{1cm} (15)
In naive factorization hypothesis, one has the following equalities

\begin{align}
    a_{T_s} = a_{T_d} = a_{T_v} = a_{T_p} = a_1(\mu), \\
    a_{C_s} = a_{C_d} = a_{C_v} = a_{C_p} = a_2(\mu),
\end{align}

with

\begin{align}
    a_1(\mu) = c_1(\mu) + \frac{1}{N_c}c_2(\mu), \\
    a_2(\mu) = c_2(\mu) + \frac{1}{N_c}c_1(\mu),
\end{align}

denoting the relations between quantities \(a_{1,2}\) and Wilson coefficients \(c_{1,2}\). \(N_c\) is the number of colors. \(\mu\) is the renormalization scale at which \(c_1\) and \(c_2\) are evaluated. So \(a_1\) and \(a_2\) are common real quantities of a certain process in quark level. To be more explicit, for decay modes induced by \(c \to s\) transition, \(a_1\) and \(a_2\) are invariant among all modes in naive factorization hypothesis.

However, naive factorization approach meets difficulties in describing all charmed meson decays, particularly for the decay modes which involve in the color-suppressed diagrams due to the smallness of \(|a_2|\). Furthermore, the coefficients \(a_1\) and \(a_2\) in Eqs. (18) and (19) depend on the renormalization scale and \(\gamma_5\) scheme at the next to leading order expansion. It is necessary to consider the nonfactorizable corrections which involve in hard spectator interactions, final state interactions and resonance effects etc. In a general case, one can express \(a_1\) and \(a_2\) in the form

\begin{align}
    a_1(\mu) = c_1(\mu) + \left(\frac{1}{N_c} + \chi_1(\mu)\right)c_2(\mu), \\
    a_2(\mu) = c_2(\mu) + \left(\frac{1}{N_c} + \chi_2(\mu)\right)c_1(\mu),
\end{align}

with \(\chi_1(\mu)\) and \(\chi_2(\mu)\) terms denoting the nonfactorizable effects. With nonfactorization corrections the equalities (18) and (19) are not yet satisfied because each \(a_i\) should contain terms from different corrections. The nonfactorization corrections can also bring phase differences among these coefficients, and then \(a_i\)s \((i = T_{s,d,V,P}, C_{s,d,V,P})\) become complexes. Currently, explicit calculations of total nonfactorizable corrections are not yet possible. In \(D \to PV\) decays, we shall take all \(a_i\)s as independent complex parameters and assume that the corrections do not depend on individual decay process at certain scale. In other words, we do not consider SU(3) flavor symmetry violation contributions to \(a_i\)s and it is supposed that mass factors, decay constants and formfactors have taken on the whole SU(3) symmetry breaking effects. While in \(D \to PP\) decays, the mass factors, the form factors and decay constants fail to account for the large SU(3) flavor symmetry breaking effects in \(D \to \pi\pi\), \(D \to \pi K\) and \(D \to K\bar{K}\). Nonfactorizable contributions may cause large SU(3) symmetry breaking effects \(14\). Two sets of coefficients \(a_i^s\) and \(a_i^d\) are introduced to describe the SU(3) flavor symmetry breaking effects induced by nonfactorizable contributions. In both \(D \to PP\) and \(D \to PV\) decays, the SU(3) symmetry breaking effects are not considered in the strong phases in our present analysis.

The exchange and annihilation diagrams have the following expressions in naive factorization approach:

\begin{equation}
    E_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_1 q_3} V_{q_2 q_4}^{*} a_{E_{s,d}}^{n_f} f_{D_i}(m_{P_1}^2 - m_{P_2}^2) P_1 P_2 (m_{D_i}^2),
\end{equation}
\[ A_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_2 q_3} V_{q_1 q_3}^{*} a_{A, s,d}^{n f} f_{D_i} (m_{P_1}^2 - m_{P_2}^2) F_0^{P_1 P_2} (m_{D_i}^2), \] (25)

\[ E_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_2 q_3} V_{q_1 q_3}^{*} a_{E, V,P}^{n f} 2 f_{D_i} m_{D_i} A_0^{PV} (m_{D_i}^2), \] (26)

\[ A_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_2 q_3} V_{q_1 q_3}^{*} a_{A, V,P}^{n f} 2 f_{D_i} m_{D_i} A_0^{PV} (m_{D_i}^2). \] (27)

The formfactors \( F_0^{P_1 P_2} (m_{D_i}^2) \) and \( A_0^{PV} (m_{D_i}^2) \) involving in the above formula are not manifest to relate directly to experimental measurements. The factorizable contributions of the exchange and annihilation diagrams are believed to be small. The main contributions of these diagrams may result from the nonfactorizable forms. Through intermediate states, these diagrams relate to the tree diagram \( T \) and color-suppressed diagram \( C \). Their contributions may be important and can not be ignored. In our present considerations, we use \( a_{E, A_i} (i = s, d, V, P) \) defined in eqs. (6), (7), (12) and (13) as global parameters to describe mainly the nonfactorizable contributions. By these definitions, the parameters \( a_{E, A_i} \) will have two dimensions of energy in \( D \to PP \) and will be dimensionless in \( D \to PV \).

**IV. NUMERICAL ANALYSIS AND RESULTS**

The explicit evaluation of the relevant formfactors in the factorization formula (4), (5) and (8)-(11) is a hard task because of the nonperturbative long distance effects of QCD. Various methods, such as QCD sum rules \[23, 24\], lattice simulations \[25, 26\] and phenomenological quark model \[27, 28\], have been developed to estimate the long distance effects to rather high certainties. The formfactors of \( D \) mesons decaying to light mesons have been widely discussed in \[29, 30, 31, 32, 33, 34\]. In our present considerations, we shall use the results of form factors obtained by Bauer, Stech and Wirbel \[2, 29\] based on the quark model. They have been found to be rather successful in describing a number of processes concerning \( D \) mesons. The values of the relevant formfactors evaluated at \( q^2 = 0 \) are listed in Table I. For the dependence on \( q^2 \), the formfactors are assumed to behave as a monopole dominance

\[ D \to P : \quad F_0(q^2) = \frac{F_0(0)}{1 - q^2/m_{F^*}^2}, \] (28)

\[ F_1(q^2) = \frac{F_1(0)}{1 - q^2/m_{F^*}^2}, \] (29)

\[ D \to V : \quad A_0(q^2) = \frac{A_0(0)}{1 - q^2/m_{F^*}^2}, \] (30)

where \( m_F, m_{F^*} \) and \( m_{F^{**}} \) are the pole masses given in Table I.

It is noted that the formfactors are more appropriate to be viewed as the relative scaling factors that characterize one source of SU(3) flavor symmetry breaking effects in hadronic matrix elements since we take the \( a_i \)s as free parameters that need to be extracted from experimental inputs in the present method. The relative ratio between the formfactors is what we really care about.

The input values for the light pseudoscalar and vector decay constants are presented in Table II \[35, 36\]. These values generally coincide with experiments. The decay constants...
\[ f_{\eta}^u, f_{\eta}^s, f_{\eta'}^u \text{ and } f_{\eta'}^s \text{ involving in factorization formula should be defined as follow:}\]

\[ \langle 0 | \pi^\mu \gamma_\gamma u | \eta^{(i)}(p) \rangle = if_{\eta}^u p^\mu, \]
\[ \langle 0 | \pi^\mu \gamma_\gamma s | \eta^{(i)}(p) \rangle = if_{\eta}^s p^\mu. \]

Then the quantities \( f_{\eta}^u, f_{\eta}^s, f_{\eta'}^u \) and \( f_{\eta'}^s \) take the formalism

\[ f_{\eta}^u = \frac{f_8}{\sqrt{6}} \cos \phi + \frac{f_0}{\sqrt{3}} \sin \phi, \]
\[ f_{\eta}^s = \frac{2f_8}{\sqrt{6}} \cos \phi - \frac{f_0}{\sqrt{3}} \sin \phi, \]
\[ f_{\eta'}^u = - \frac{f_8}{\sqrt{6}} \sin \phi + \frac{f_0}{\sqrt{3}} \cos \phi, \]
\[ f_{\eta'}^s = \frac{2f_8}{\sqrt{6}} \sin \phi + \frac{f_0}{\sqrt{3}} \cos \phi. \]

Making use of these definitions, the following factorization formalisms are adopted in the \( D \to \eta(\eta')V \) transition calculation

\[ 2C_V(D \to \eta V) = \frac{G_F}{\sqrt{2}} V_{q1q2} V_{cpq}^* a_{C_V} 2(f_{\eta}^u + f_{\eta}^s)m_D A_{D_i \to V}(m_{\eta}^2), \]
\[ C_V(D \to \eta' V) = \frac{G_F}{\sqrt{2}} V_{q1q2} V_{cpq}^* a_{C_V} 2(f_{\eta'}^s - f_{\eta'}^u)m_D A_{D_i \to V}(m_{\eta'}^2). \]

The other parameters used in the numerical calculation are the masses of relevant mesons, lifetimes of charmed mesons and relevant CKM matrix elements. We adopt the relevant values given in \[37\].

For convenience, we may express the complex parameters \( a_i \) as

\[ a_i = |a_i| e^{i\delta a_i}. \]

The \( \delta a_i \)s characterize the strong phases. One can always choose \( \delta a_{PV} = 0 \) in \( D \to PP \) and \( \delta a_{PV} = 0 \) in \( D \to PV \) so that all the other strong phases are relative to \( \delta a_{PV} \) and \( \delta a_{PV} \). There are 15 independent parameters to be extracted from experiments in both \( D \to PP \) and \( D \to PV \).

To conduct a fit procedure, we construct a \( \chi^2 \) function which has the following form

\[ \chi^2 = \sum_j \frac{(f_j(a_i) - \langle f_j \rangle)^2}{\sigma_j^2} \]

where \( \langle f_j \rangle \) and \( \sigma_j \) are the central values and corresponding errors of the experimentally measured observables. \( f_j(a_i) \) are the theoretical expressions for the observables. They are the functions of parameters \( a_i \)s. The set of \( a_i \)s which minimizes the \( \chi^2 \) function will be regarded as the best estimated values.

There are 17 experimental data points for 13 parameters (\( a_4^s \) does not appear in these experimental data) and 22 data points for 15 parameters, as shown in Table I and Table II respectively. We list in Table III the parameters with 1\( \sigma \) errors obtained in our present analysis. FIT \( \alpha \) and FIT A are obtained without any constraint to the parameters. A large \( |a_2^s/a_2^u| \approx 2.0 \) ratio predicted by FIT \( \alpha \) is an indication of inscrutably large flavor
SU(3) breaking effects. Constraining the ratio to the smallest extent, we get FIT β with the ratio $|a_2^f/a_2^f| \approx 1.1$. By "the smallest extent", we mean that, if we continue to suppress the ratio down, the predicted branching ratios of some decay modes in Table IV will be inconsistent with the experimental data. FIT A predicts an unusually large ratio $|a_2^f/a_2^f| \approx 1.1$ which indicates that the nonfactorizable contributions to $a_2^f$ are of great importance. By constraining the value of $|a_2^f|$ to be as small as possible, we obtain FIT B with the ratio 0.9. The next leading order Wilson coefficients $c_1(m_c) = 1.174$ and $c_2(m_c) = -0.356$ in the naive dimensional regularization (NDR) scheme or $c_1(m_c) = 1.216$ and $c_2(m_c) = -0.424$ in the 't Hooft-Veltman (HV) scheme are given in Ref. [38] when $\Lambda_{\overline{MS}} = 0.215$GeV. The present relatively large values of $|a_2^f|$, $|a_2^f|$, $|a_2^f|$ and $|a_2^f|$ can not be explained from formula [21]. They imply that nonfactorizable contributions are of significance in both $D \rightarrow PP$ and $D \rightarrow PV$ decays. To fit the experimental result of $Br(D^0 \rightarrow K^0\overline{K}^0) = (0.071 \pm 0.019)\%$, $a_E^f$ should differ much from $a_2^f$. In $D \rightarrow PV$ decays, because we have considered the errors of experimental data in the $\chi^2$ fit and used more experimental results as constraints, the present resulting parameters appear more reasonable than that of case (I) solution presented in [15], as the strong phases of the parameters $a_1^{PV}$ and $a_2^{PV}$ are not in contradiction to that predicted from QCD.

We present the resultant predictions for a variety of charmed meson decay processes in Table IV for $D \rightarrow PP$ decays and in Table V for $D \rightarrow PV$ decays. Note that there are no enough experimental data to extract the parameter $a_4^f$ in $D \rightarrow PP$ decays. To make predictions for some relevant decay modes which receive contribution from $A_4$ diagram, we take the assumption $a_4^f = a_4^f$. For a comparison, we also list the results obtained in Ref. [40]. The predictions for a number of singly and doubly Cabibbo-suppressed modes can be used to test our present analysis in the near future.

Note that in the assumption of $SA_P = 0$, we have the branching ratio 14% for the process $D^+_s \rightarrow \pi^+\omega$, which is much larger than the experimental result $(0.28 \pm 0.11)\%$. To accommodate the experimental data, significant contribution from $SA_P$ diagram, i.e. $SA_P \sim -A_p$, should be introduced [13].

V. SU(3) FLAVOR symmetry breaking

As pointed out in Ref. [13, 14], SU(3) breaking effects in charmed meson decays appear to be important. The violation may come from the finite strange quark mass, the final state interactions and resonances. In the SU(3) flavor symmetry limit, there are a number of relations among different decay modes. Based on the above extracted values for the parameters, we can discuss how large are the SU(3) breaking effects in $D \rightarrow PP$ and $D \rightarrow PV$ decays.

We present these relations in Table VI for $D \rightarrow PP$ and Table VII for $D \rightarrow PV$. The left hand side(LHS) values of the relations whose deviation from unit represents the breaking amounts of SU(3) flavor symmetry relations are listed in the second columns.

It is noted that though these relations deviating from unit reflects the SU(3) flavor symmetry breaking effects, the ones composed of three decay modes and those composed of two decay modes have different sources of breaking terms. To be clear, we take the expressions

$$\frac{|\lambda A(D^+ \rightarrow \pi^+K^-\overline{K}^0) + \sqrt{2} A(D^+ \rightarrow \pi^+\rho^0)|}{|\lambda\sqrt{2} A(D^+_s \rightarrow \pi^+\rho^0)|}$$

and

$$\frac{|A(D^0 \rightarrow K^+\overline{K}^-)|}{|A(D^0 \rightarrow \pi^+\rho^-)|}$$

as examples.
It is obvious that SU(3) flavor symmetry analysis is not applicable to such processes. Besides the mass factors, the form factors and decay constants, one should also consider the contributions of \( A_i \) factors when studying the SU(3) symmetry breaking effects in \( D \to PP \) decay modes. The situations are more complicated than that in \( D \to PV \) decay modes. General speaking, the SU(3) flavor symmetry breaking effects are more important in \( D \to PP \) decays. The first two relations in Table VII and Table VIII still conserve because all the decay modes in them form an isospin triangle respectively.
We have performed a $\chi^2$ fitting analysis on the $D \rightarrow PP$ and $D \rightarrow PV$ decays in the formalism of the factorization hypotheses. To fit the experimental data, it is vital to consider the SU(3) flavor symmetry breaking effects of the coefficients $a_i$s in $D \rightarrow PP$ decay modes. In $D \rightarrow PV$ decays, the final state hadron structure of the pseudoscalar and vector mesons has more important impact on the coefficients $a_i$s than the SU(3) symmetry breaking effects. The nonfactorizable contributions, as well as that of the exchange and annihilation diagrams, are found to be important in these decays. In the formalism of the relations obtained in the SU(3) symmetry limit, the total SU(3) symmetry breaking amount of certain processes in $D \rightarrow PP$ can reach 120% when the three symmetry breaking effects due to $a_i$ factors, mass factors and due to form factors and decay constants become to be coherently added. The total breaking amount of some processes in $D \rightarrow PV$ can add up to 50%. The breaking amount of the SU(3) symmetry relations in some channels is so significant that it becomes unreliable to use the SU(3) relations to make predictions for some decay modes. More precise measurement on the process $D^+ \rightarrow \bar{K}^0 K^{*+}$ is important for understanding the SU(3) symmetry breaking effects and nonfactorizable contributions. As an independent check, it is useful to measure the process $D_s^+ \rightarrow K^0 \rho^+$. The processes $D^0 \rightarrow \pi^+ \rho^-$, $D^0 \rightarrow \pi^- \rho^+$, $D^0 \rightarrow \pi^0 \rho^0$, $D^+ \rightarrow \pi^+ \omega$, $D^+ \rightarrow \pi^0 \rho^+$, $D^+ \rightarrow K^0 \rho^+$, $D^+ \rightarrow \pi^0 K^{*+}$, $D_s^+ \rightarrow K^+ \omega$ and $D_s^+ \rightarrow \pi^0 K^{*+}$ are predicted to be at the experimental sensitivity. It is expected to explore the final hadron structure and SU(3) flavor symmetry breaking effects in $D \rightarrow PP$ and $D \rightarrow PV$ decays in BES, CLEO-c, BaBar and Belle.

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TABLE I: Relevant formfactors at zero momentum transfer for $D \to P$ and $D \to V$ transitions and pole masses in BSW model.

| Decay          | $D \to \pi$ | $D \to \rho(\omega)$ | $D \to K$ | $D \to K^*$ | $D_s \to K$ | $D_s \to K^*$ | $D_s \to \phi$ | $D \to \eta/\eta'$ | $D_s \to \eta/\eta'$ |
|----------------|-------------|------------------------|-----------|-------------|-------------|-------------|----------------|----------------|----------------|
| $F_1$          | 0.692       | 0.762                  | 0.643     | 0.681/0.655 | 0.723/0.704 |
| $A_0$          | 0.669       | 0.733                  | 0.634     | 0.700       |
| $m_F$(GeV)     | 1.87        | 1.97                   | 1.87      | 1.97        |
| $m_{F^*}$(GeV) | 2.01        | 2.11                   | 2.01      | 2.01        | 2.11        |
| $m_{F^{**}}$(GeV) | 2.47      | 2.60                   | 2.47      | 2.47        | 2.60        |

TABLE II: Values of decay constants in MeV.

|          | $f_\pi$ | $f_K$ | $f_8$ | $f_0$ | $f_D$ | $f_D^*$ | $f_{K^*}$ | $f_\omega$ | $f_\phi$ | $f_{D^*}$ | $f_{D^*}$ |
|----------|---------|-------|-------|-------|-------|---------|-----------|-----------|---------|-----------|-----------|
|          | 134     | 158   | 168   | 157   | 200   | 234     | 210       | 214       | 195     | 233       | 230       | 275       |
TABLE III: Parameters $a_i$s fitted from experimental data at 1σ errors. The first entry is for amplitude and the second entry for the strong phase. $a_1^{s,d,V,P}$ and $a_2^{s,d,V,P}$ denote $a_{T_{s,d,V,P}}$ and $a_{C_{s,d,V,P}}$ respectively.

| $\chi^2$/d.o.f. | $D \rightarrow PP$ | $D \rightarrow PV$ |
|----------------|-------------------|-------------------|
| $\chi^2$/d.o.f. | FIT $\alpha$ | FIT $\beta$ | FIT A | FIT B |
| 4.06/4 | 1.08 ± 0.04 | 1.10 ± 0.03 | 1.13 ± 0.08 | 1.10 ± 0.07 |
| 8.16/4 | $a_1^{\alpha}$ | $a_1^{\beta}$ | $a_1^{A}$ | $a_1^{B}$ |
| 8.22/7 | 1.04 ± 0.09 | 1.09 ± 0.09 | 1.29 ± 0.04 | 1.29 ± 0.04 |
| 10.30/7 | (8.73 ± 7.96)$^o$ | (11.98 ± 7.85)$^o$ | (10.04 ± 16.62)$^o$ | (−1.36 ± 13.52)$^o$ |
| $a_2^s$ | $a_2^{\alpha}$ | $a_2^{\beta}$ | $a_2^{A}$ | $a_2^{B}$ |
| (−26.76 ± 1.60)$^o$ | (−26.25 ± 1.55)$^o$ | (−11.09 ± 20.01)$^o$ | (−10.74 ± 10.31)$^o$ |
| $a_2^d$ | $a_2^{\alpha}$ | $a_2^{\beta}$ | $a_2^{A}$ | $a_2^{B}$ |
| (−53.40 ± 28.65)$^o$ | (−35.12 ± 14.50)$^o$ | (−21.75 ± 1.38)$^o$ | (−22.15 ± 2.23)$^o$ |
| $a_3^s(\text{GeV}^2)$ | $a_3^{\alpha}(\text{GeV}^2)$ | $a_3^{\beta}(\text{GeV}^2)$ | $a_3^{A}(\text{GeV}^2)$ | $a_3^{B}(\text{GeV}^2)$ |
| (−49.43 ± 20.63)$^o$ | (−58.75 ± 19.48)$^o$ | (−166.87 ± 50.96)$^o$ | (−115.50 ± 12.78)$^o$ |
| $a_3^d(\text{GeV}^2)$ | $a_3^{\alpha}(\text{GeV}^2)$ | $a_3^{\beta}(\text{GeV}^2)$ | $a_3^{A}(\text{GeV}^2)$ | $a_3^{B}(\text{GeV}^2)$ |
| (−120.94 ± 2.92)$^o$ | (−122.02 ± 2.82)$^o$ | (82.82 ± 4.01)$^o$ | (78.55 ± 5.73)$^o$ |
| $a_4^s(\text{GeV}^2)$ | $a_4^{\alpha}(\text{GeV}^2)$ | $a_4^{\beta}(\text{GeV}^2)$ | $a_4^{A}(\text{GeV}^2)$ | $a_4^{B}(\text{GeV}^2)$ |
| $a_4^d(\text{GeV}^2)$ | $a_4^{\alpha}(\text{GeV}^2)$ | $a_4^{\beta}(\text{GeV}^2)$ | $a_4^{A}(\text{GeV}^2)$ | $a_4^{B}(\text{GeV}^2)$ |
| (90.04 ± 13.75)$^o$ | (89.94 ± 14.27)$^o$ | (−76.29 ± 41.25)$^o$ | (−77.93 ± 31.52)$^o$ |
TABLE IV: Predicted branching ratios for charmed mesons decaying to two pseudoscalar mesons. Single prime and double primes are added to the representations to denote the singly Cabibbo-suppressed processes and doubly Cabibbo-suppressed processes. \( C_{s1} \) and \( C_{s2} \), as well as \( C_{d1} \) and \( C_{d2} \), result from the exchange of the final mesons.

| Decay Modes | Representation | Experimental \( \mathcal{B} \times 10^{-2} \) | Present \( \mathcal{B} \times 10^{-2} \) | LP [40] \( \mathcal{B} \times 10^{-2} \) |
|-------------|----------------|---------------------------------|-----------------------------|-------------------|
| \( K^{-} \) \( \pi^{+} \) | \( T_{s} + E_{d} \) | 3.80 ± 0.09 | 3.79 | 3.80 | 3.847 |
| \( \bar{K}^{0} \) \( \pi^{0} \) | \( 1/\sqrt{2}(C_{s} - E_{d}) \) | 2.28 ± 0.22 | 2.27 | 2.24 | 1.310 |
| \( \bar{K}^{0} \eta \) | \( 1/\sqrt{3}C_{s} \) | 0.76 ± 0.11 | 0.80 | 0.81 | — |
| \( \bar{K}^{0} \eta' \) | \( 1/\sqrt{6}(C_{s} + 3E_{d}) \) | 1.87 ± 0.28 | 1.85 | 1.88 | — |
| \( \pi^{+} \pi^{-} \) | \( -(T'_{d} + E'_{d}) \) | 0.143 ± 0.007 | 0.144 | 0.144 | 0.151 |
| \( \pi^{0} \pi^{0} \) | \( 1/\sqrt{2}(C'_{d} - E'_{d}) \) | 0.084 ± 0.022 | 0.078 | 0.097 | 0.115 |
| \( K^{+}K^{-} \) | \( T'_{s} + E'_{s} \) | 0.412 ± 0.014 | 0.413 | 0.413 | 0.424 |
| \( K^{0}K^{0} \) | \( E'_{s} - E'_{d} \) | 0.071 ± 0.019 | 0.069 | 0.062 | 0.130 |
| \( K^{+} \sqrt{A_{0}^{0}} \) | \( - (T''_{d} + E''_{d}) \) | 0.0148 ± 0.0021 | 0.0150 | 0.0151 | 0.033 |
| \( \eta \pi^{0} \) | \( 1/\sqrt{6}(C'_{s} + C'_{d1} - C'_{d2} - 2E'_{d} - SE'_{d}) \) | — | 0.069 | 0.068 | — |
| \( \eta' \pi^{0} \) | \( 1/\sqrt{2}(2C'_{s} - C'_{d1} + C'_{d2} + 2E'_{d} + 4SE'_{d}) \) | — | 0.088 | 0.091 | — |
| \( \eta \eta \) | \( 1/\sqrt{3}(2C'_{s} + 2C'_{d} - 2E'_{s} + 2E'_{d} + 4SE) \) | — | 0.011 | 0.016 | — |
| \( \eta \eta' \) | \( 1/\sqrt{18}(3C'_{s1} - C'_{s2} - C'_{d1} - C'_{d2} - 4SE'_{d} - 2E'_{r} - 7SE) \) | — | 0.026 | 0.030 | — |
| \( K^{0} \pi^{0} \) | \( -1/\sqrt{2}(C''_{d} - E''_{s}) \) | — | 0.002 | 0.005 | 0.008 |
| \( K^{0} \eta \) | \( -1/\sqrt{6}(C''_{d} - E''_{s} + SE''_{d}) \) | — | 0.001 | 0.002 | — |
| \( K^{0} \eta' \) | \( -1/\sqrt{6}(C''_{d} + 3E''_{s} + 4SE''_{d}) \) | — | 0.0 | 0 | — |
| \( \bar{K}^{0} \pi^{0} \) | \( T_{s} + C_{s} \) | 2.77 ± 0.18 | 2.76 | 2.76 | 2.939 |
| \( \pi^{+} \pi^{0} \) | \( -1/\sqrt{2}(T'_{d} + C'_{d}) \) | 0.25 ± 0.07 | 0.25 | 0.19 | 0.185 |
| \( \eta \pi^{+} \) | \( 1/\sqrt{6}(T'_{d} + C'_{s} + 2A'_{d} + 2A'_{s} + 2A'_{s}) \) | 0.30 ± 0.06 | 0.34 | 0.37 | — |
| \( \eta' \pi^{+} \) | \( -1/\sqrt{6}(T'_{d} - 2C'_{s} + C'_{d} + 2A'_{d} + 4SA'_{d}) \) | 0.50 ± 0.10 | 0.45 | 0.42 | — |
| \( K^{+}K^{0} \) | \( T'_{s} - A'_{d} \) | 0.58 ± 0.06 | 0.62 | 0.62 | 0.764 |
| \( K^{0} \pi^{0} \) | \( -(C''_{d} + A''_{s}) \) | — | 0.012 | 0.026 | 0.053 |
| \( K^{+} \eta \) | \( -1/\sqrt{2}(T''_{d} - A''_{s}) \) | — | 0.021 | 0.023 | 0.055 |
| \( K^{+} \eta' \) | \( 1/\sqrt{3}(T''_{d} + SA'_{s}) \) | — | 0.011 | 0.012 | — |
| \( K^{+} K^{0} \) | \( -1/\sqrt{6}(T''_{d} + 3A''_{s} + 4SA'_{s}) \) | — | 0.005 | 0.006 | — |
| \( \bar{K}^{0} \eta^{0} \) | \( C_{s} + A_{d} \) | 3.6 ± 1.1 | 3.06 | 3.13 | 4.623 |
| \( \pi^{+} \eta \) | \( 1/\sqrt{3}(T_{s} - 2A_{d} - 2A_{s}) \) | 1.7 ± 0.5 | 1.05 | 1.09 | 1.131 |
| \( \pi^{+} \eta' \) | \( 2/\sqrt{6}(T_{s} + A_{d} + 2SA) \) | 3.9 ± 1.0 | 4.19 | 4.43 | — |
| \( \pi^{+} K^{0} \) | \( -(T'_{d} - A'_{s}) \) | < 0.8 | 0.24 | 0.26 | 0.373 |
| \( \pi^{0} K^{0} \) | \( -1/\sqrt{2}(C'_{d} + A'_{s}) \) | — | 0.047 | 0.090 | 0.146 |
| \( \eta K^{0} \) | \( 1/\sqrt{3}(T'_{s} + C'_{s} + C'_{d} - SA') \) | — | 0.055 | 0.040 | 0.300 |
| \( \eta' K^{0} \) | \( 1/\sqrt{6}(2T'_{s} + 2C'_{s} + C'_{d} + 3A'_{d} + 4SA') \) | — | 0.090 | 0.102 | — |
| \( K^{+} K^{0} \) | \( -(T''_{d} + C''_{d}) \) | — | 0.014 | 0.010 | 0.012 |
TABLE V: Predicted branching ratios for charmed mesons decaying to one pseudoscalar and one vector meson. Single prime and double primes are added to the representations to denote the singly Cabibbo-suppressed processes and doubly Cabibbo-suppressed processes.

| Decay Modes | Representation | Experimental $\mathcal{B}(\times 10^{-2})$ | Present $\mathcal{B}(\times 10^{-2})$ | LP[40] $\mathcal{B}(\times 10^{-2})$ |
|-------------|----------------|-------------------------------------------|--------------------------------------|-----------------------------------|
| $K^{*-}\pi^+$ | $T_V + E_P$ | 6.0 ± 0.5 | 5.93 | 5.97 | 4.656 |
| $K^-\rho^+$ | $T_P + E_V$ | 10.2 ± 0.8 | 9.99 | 9.90 | 11.201 |
| $K^{*0}\pi^0$ | $\frac{1}{\sqrt{2}}(C_P - E_P)$ | 2.8 ± 0.4 | 2.72 | 2.81 | 3.208 |
| $K^0\rho^0$ | $\frac{1}{\sqrt{2}}(C_V - E_V)$ | 1.47 ± 0.29 | 1.49 | 1.25 | 0.759 |
| $K^0\eta$ | $\frac{1}{\sqrt{3}}(C_P + E_P - E_V + SE_V)$ | 1.8 ± 0.4 | 1.50 | 1.94 |  |  
| $K^0\omega$ | $-\frac{1}{\sqrt{2}}(C_V + E_V)$ | 2.2 ± 0.4 | 2.11 | 1.80 | 1.855 |
| $K^0\phi$ | $-E_P - SE_P$ | 0.94 ± 0.11 | 0.95 | 0.90 |  |  
| $K^+K^-\pi^0$ | $T'_V + E'_P$ | 0.20 ± 0.11 | 0.25 | 0.25 | 0.290 |
| $K^+K^0\pi^0$ | $E'_V - E'_P$ | <0.17 | 0.08 | 0.16 | 0.052 |
| $K^0\pi^0\pi^0$ | $E'_P - E'_V$ | <0.09 | 0.08 | 0.16 | 0.062 |
| $\pi^0\phi$ | $\frac{1}{\sqrt{2}}(C'_P + SE'_P)$ | <0.14 | 0.12 | 0.12 | 0.105 |
| $K^0\eta'$ | $-\frac{1}{\sqrt{6}}(C_P + E_P + 2E_V + 4SE_V)$ | <0.10 | 0.004 | 0.003 |  |  
| $\eta\phi$ | $\frac{1}{\sqrt{3}}(C'_P - 2SE'_P + SE'_V)$ | <2.8 | 0.035 | 0.034 |  |  
| $\pi^+\rho^-$ | $-(T'_V + E'_P)$ | — | 0.34 | 0.35 | 0.485 |
| $\pi^-\rho^+$ | $-(T'_P + E'_V)$ | — | 0.62 | 0.61 | 0.706 |
| $\pi^0\rho^0$ | $\frac{1}{2}(C'_P + C'_V - E_P - E_V)$ | — | 0.19 | 0.16 | 0.216 |
| $\pi^0\omega$ | $\frac{1}{2}(C'_V - C'_P + E'_P + E'_V + 2SE'_P)$ | — | 0.020 | 0.003 | 0.013 |
| $\eta\omega$ | $-\frac{1}{\sqrt{6}}(C'_P + 2C'_V + SE'_V + 4SE'_P)$ | — | 0.13 | 0.10 |  |  
| $\eta'\omega$ | $\frac{1}{\sqrt{3}}(C'_P - C'_V - 4SE'_V - 2SE'_P)$ | — | 0.0007 | 0.0003 |  |  
| $\eta\rho^0$ | $\frac{1}{\sqrt{6}}(2C'_P - C'_V + SE'_V)$ | — | 0.0039 | 0.0015 |  |  
| $\eta'\rho^0$ | $\frac{1}{\sqrt{3}}(C'_P + C'_V + 4SE'_V)$ | — | 0.012 | 0.009 | 0.039 |
| $K^{*+}\pi^-$ | $-(T''_P + E''_P)$ | — | 0.029 | 0.029 | 0.025 |
| $K^+\rho^-$ | $-(T''_V + E''_P)$ | — | 0.016 | 0.016 | 0.004 |
| $K^{*0}\pi^0$ | $-\frac{1}{\sqrt{2}}(C''_P - E''_P)$ | — | 0.0052 | 0.0064 | 0.008 |
| $K^{0}\rho^0$ | $-\frac{1}{\sqrt{2}}(C''_V - E''_P)$ | — | 0.0069 | 0.0059 |  |  
| $K^{*+}\eta$ | $-\frac{1}{\sqrt{6}}(C''_P - E''_P + E''_V + SE''_V)$ | — | 0.0030 | 0.0041 |  |  
| $K^{*0}\eta'$ | $\frac{1}{\sqrt{6}}(C''_P + 2E''_P + E''_V + 4SE''_V)$ | — | 0.0 | 0.0 |  |  
| $K^0\omega$ | $\frac{1}{\sqrt{2}}(C''_V + E''_P)$ | — | 0.0076 | 0.0056 | 0.002 |
| $K^0\phi$ | $E''_V + SE''_P$ | — | 0.0 | 0.006 |  |  

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TABLE V: (continued).

| Decay Modes | Representation | Experimental $\mathcal{B}$($\times 10^{-2}$) | Present $\mathcal{B}$($\times 10^{-2}$) | LP [40] $\mathcal{B}$($\times 10^{-2}$) |
|--------------|----------------|---------------------------------------------|----------------------------------------|----------------------------------------|
|              |                | FIT A                                      | FIT B                                  |                                        |
| $K^{0\pi} \pi^+$ | $T_V + C_P$ | $1.92 \pm 0.19$ | $1.96$ | $1.96$ | $1.996$ |
| $\pi^+\phi$ | $C'_p - S_A'p$ | $0.61 \pm 0.06$ | $0.64$ | $0.62$ | $0.619$ |
| $\pi^0\rho^0$ | $T_p + C_V$ | $6.6 \pm 2.5$ | $7.56$ | $8.43$ | $12.198$ |
| $\pi^+\rho^0$ | $-\frac{1}{\sqrt{2}}(T_V' + C'_p - A'_p + A'_V)$ | $0.104 \pm 0.018$ | $0.088$ | $0.088$ | $0.104$ |
| $K^+K^{0}$ | $T_V - A'_V$ | $0.42 \pm 0.05$ | $0.44$ | $0.44$ | $0.436$ |
| $K^0K^{*+}$ | $T'_p - A'_p$ | $3.1 \pm 1.4$ | $1.43$ | $1.25$ | $1.515$ |
| $K^+\rho^0$ | $-\frac{1}{\sqrt{2}}(C''_V - A''_p)$ | $0.025 \pm 0.012$ | $0.030$ | $0.025$ | $0.029$ |
| $K^{\ast 0}\pi^+$ | $-(C''_p + A''_V)$ | $0.036 \pm 0.016$ | $0.024$ | $0.022$ | $0.027$ |
| $K^+\phi$ | $-(A''_V + S_A''_p)$ | $< 0.013$ | $0.0066$ | $0.0067$ | $< 0.013$ |
| $\pi^+\omega$ | $\frac{1}{\sqrt{2}}(T_V' + C'_p + A'_V + A'_p + 2S_A'p)$ | $-0.066$ | $-0.06$ | $-0.06$ | $-0.06$ |
| $\eta\pi^+$ | $\frac{1}{\sqrt{3}}(T_p + 2C'_V + A'_V + A'_p + S_A'V)$ | $-0.025$ | $0.022$ | $0.042$ | $0.042$ |
| $\eta'\rho^+$ | $-\frac{1}{\sqrt{6}}(T_p' - C'_V + A'_V + A'_p + 4S_A'V)$ | $-0.037$ | $0.036$ | $0.057$ | $0.057$ |
| $\pi^0\rho^+$ | $-\frac{1}{\sqrt{2}}(T_p' + C'_V + A'_p - A'_V)$ | $-0.025$ | $0.022$ | $0.042$ | $0.042$ |
| $K^0\rho^+$ | $-(C''_p + A''_p)$ | $-0.037$ | $0.036$ | $0.057$ | $0.057$ |
| $\pi^0K^{*+}$ | $-\frac{1}{\sqrt{2}}(C''_p - A''_V)$ | $-0.012$ | $0.011$ | $0.011$ | $0.011$ |
| $K^+\omega$ | $-\frac{1}{\sqrt{2}}(C''_V + A''_p)$ | $-0.012$ | $0.011$ | $0.011$ | $0.011$ |
| $K^{*+}\eta$ | $\frac{1}{\sqrt{3}}(T''_p - A''_V + A''_p + S_A''_V)$ | $-0.0014$ | $0.00016$ | $0.0015$ | $0.015$ |
| $K^{*+}\eta'$ | $-\frac{1}{\sqrt{6}}(T''_p + 2A''_p + A''_V + 4S_A''_V)$ | $-0.0014$ | $0.00016$ | $0.0015$ | $0.015$ |
| $K^{0\pi} K^+$ | $C_P + A_V$ | $3.3 \pm 0.9$ | $3.34$ | $3.42$ | $4.812$ |
| $K^0 K^{*+}$ | $C_V + A_P$ | $4.3 \pm 1.4$ | $4.98$ | $4.66$ | $2.467$ |
| $\pi^+\rho^0$ | $\frac{1}{\sqrt{2}}(A_V - A_P)$ | $0.06^\dagger<(0.07)$ | $0.06$ | $0.06$ | $0.06$ |
| $\pi^+\phi$ | $T_V + S_AP$ | $3.6 \pm 0.9$ | $3.08$ | $2.93$ | $4.552$ |
| $\pi^+K^{*0}$ | $-(T'_V - A'_V)$ | $0.65 \pm 0.28$ | $0.33$ | $0.35$ | $0.445$ |
| $K^+\rho^0$ | $-\frac{1}{\sqrt{2}}(C'_p + A'_p)$ | $< 0.29$ | $0.12$ | $0.12$ | $0.198$ |
| $K^+\phi$ | $T'_V + C'_p + A'_V + S_A'p$ | $< 0.05$ | $0.032$ | $0.033$ | $0.008$ |
| $K^0\rho^+$ | $-(T'_p - A'_p)$ | $-0.013$ | $0.013$ | $0.013$ | $0.013$ |
| $\pi^0K^{*+}$ | $-\frac{1}{\sqrt{2}}(C'_V + A'_V)$ | $-0.013$ | $0.013$ | $0.013$ | $0.013$ |
| $\eta K^+$ | $\frac{1}{\sqrt{3}}(T'_p + 2C'_V + A'_p - A'_V - S_A'V)$ | $-0.038$ | $0.047$ | $0.146$ | $0.146$ |
| $\eta'K^{*+}$ | $\frac{1}{\sqrt{3}}(2T'_p + C'_V + 2A'_V + A'_V + 4S_A'V)$ | $-0.068$ | $0.059$ | $0.0015$ | $0.0015$ |
| $K^{*0}K^+$ | $-(T'_V + C''_p)$ | $-0.0015$ | $0.0015$ | $0.006$ | $0.006$ |
| $K^{*+}K^0$ | $-(T''_p + C''_p)$ | $-0.0076$ | $0.0085$ | $0.018$ | $0.018$ |

$\dagger$ The central value of the E791 experiment [39].
TABLE VI: SU(3) flavor symmetry relations of $D \to PP$ decay modes and breaking of the relations. $\lambda = |V_{cs}V_{us}/V_{cs}V_{ud}| \approx 0.226$. $\kappa = |V_{cs}V_{us}/V_{cd}V_{us}| \approx 4.446.$

| SU(3) Symmetry Relations | LHS of Relations |
|--------------------------|------------------|
| $|\alpha_{D^0\to\pi^0\pi^0}| = 1$ | FIT $\alpha$   | FIT $\beta$ |
| $\sqrt{2}A(D^0\to\pi^0\pi^0)/\sqrt{2}A(D^+\to\pi^0\pi^0)$ | 1.00            | 1.00          |
| $\sqrt{2}A(D^0\to\pi^0\pi^0)/\sqrt{2}A(D^+\to\pi^0\pi^0)$ | 1.00            | 1.00          |
| $|\lambda A(D^+\to\pi^+\pi^-) + \kappa A(D^0\to\pi^0\pi^0)\rangle = 1$ | 0.49            | 0.79          |
| $\sqrt{2}\kappa A(D^+\to\pi^+\pi^-) + \sqrt{2}\kappa A(D^0\to\pi^0\pi^0)$ | 1.56            | 1.11          |
| $|\lambda A(D^+\to\pi^+\pi^-) + \kappa A(D^0\to\pi^0\pi^0)\rangle = 1$ | 2.21            | 1.82          |
| $\sqrt{2}\kappa A(D^+\to\pi^+\pi^-) + \sqrt{2}\kappa A(D^0\to\pi^0\pi^0)$ | 1.27            | 1.24          |
| $|\lambda A(D^0\to\pi^0\pi^-)\rangle = 1$ | 1.26            | 1.12          |
| $\kappa A(D^0\to\pi^-\pi^0) + \kappa A(D^0\to\pi^0\pi^0)$ | 1.78            | 1.10          |
| $|\lambda A(D^0\to\pi^0\pi^-)\rangle = 1$ | 0.89            | 0.86          |
| $\sqrt{2}\kappa A(D^0\to\pi^-\pi^0) + \sqrt{2}\kappa A(D^0\to\pi^0\pi^0)$ | 1.24            | 1.24          |
| $|\lambda A(D^0\to\pi^-\pi^0)\rangle = 1$ | 1.34            | 0.98          |
| $\sqrt{2}\kappa A(D^0\to\pi^-\pi^0) + \sqrt{2}\kappa A(D^0\to\pi^0\pi^0)$ | 1.08            | 1.14          |
| $|\lambda A(D^0\to\pi^-\pi^0)\rangle = 1$ | 0.55            | 0.67          |
TABLE VII: SU(3) flavor symmetry relations of $D \rightarrow PV$ decays and breaking of the relations. 
$\lambda = |V_{cs}^{*}V_{us}/V_{cs}V_{ud}| \approx 0.226$. $\kappa = |V_{cs}^{*}V_{us}/V_{cd}V_{us}| \approx 4.446.$

| SU(3) Symmetry Relations | LHS of Relations | FIT A | FIT B |
|--------------------------|-----------------|-------|-------|
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.00  | 1.00  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.00  | 1.00  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.99  | 0.99  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.00  | 1.00  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.10  | 1.10  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.88  | 0.88  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.60  | 0.59  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.10  | 1.10  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.03  | 1.03  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.48  | 1.10  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.95  | 0.97  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.58  | 0.57  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.17  | 1.16  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.96  | 0.97  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.05  | 1.19  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.98  | 0.97  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.42  | 1.31  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.92  | 0.95  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.11  | 1.07  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.91  | 0.89  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 0.93  | 0.94  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.05  | 1.05  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.14  | 1.14  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.08  | 1.08  |
| $|A(D^0 \rightarrow \pi^+ K^-)| + \sqrt{2}|A(D^0 \rightarrow 4\pi K^-)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.09  | 1.09  |
| $|A(D^0 \rightarrow K^- \phi)| = 1$ | $|A(D^0 \rightarrow \pi^+ K^-)|$ | 1.08  | 1.08  |

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