Dynamics and quantum Zeno effect for a qubit in either a low- or high-frequency bath beyond the rotating-wave approximation

Xiufeng Cao*\(^{1,2}\), J. Q. You\(^{1,3}\), H. Zheng\(^{4}\) and Franco Nori\(^{1,5}\)

\(^ {1}\)Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan
\(^ {2}\)Department of Physics and Institute of Theoretical Physics and Astrophysics, Xiamen University, Xiamen, 361005, China
\(^ {3}\)Department of Physics and Surface Physics Laboratory (National Key Laboratory), Fudan University, Shanghai 200433, China
\(^ {4}\)Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China
\(^ {5}\)Physics Department, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA

(Dated: April 14, 2010)

We use a non-Markovian approach to study the decoherence dynamics of a qubit (with and without measurement) in either a low- or high-frequency bath modeling the qubit environment. This approach is based on a unitary transformation and does not require the rotating-wave approximation. We show that without measurement, for low-frequency noise, the bath shifts the qubit energy towards higher energies (blue shift), while the ordinary high-frequency cutoff Ohmic bath shifts the qubit energy towards lower energies (red shift). In order to preserve the coherence of the qubit, we also investigate the dynamic of qubit with measurement (quantum Zeno effect) in this two cases: low- and high-frequency baths. For very frequent projective measurements, the low-frequency bath gives rise to the quantum anti-Zeno effect on the qubit. The quantum Zeno effect only occurs in the high-frequency cutoff Ohmic bath, after considering counter-rotating terms. For a high-frequency environment, the decay rate should be faster (without measurements) or slower (with frequent measurements, in the Zeno regime), compared to the low-frequency bath case. The experimental implementation of our results here could distinguish the type of bath (either a low- or high-frequency one) and protect the coherence of the qubit by modulating the dominant frequency of its environment.

PACS numbers: 03.65.Yz, 03.67.-a, 85.25.-j

* Email: xfcao@xmu.edu.cn
I. INTRODUCTION

There is considerable interest in low-frequency noise (see, e.g., the review in Ref. [1] and references therein), because this type of noise limits the coherence of qubits based on superconducting devices such as flux or phase qubits [2, 3]. Also, the dephasing of flux qubits is due to low-frequency flux noise with intensity comparable to the one measured in dc SQUIDs (e.g., Refs. [1, 4, 5]). Under certain conditions, noise can enhance the coherence of superconducting flux qubit [6]. There are also several models (e.g., Refs. [7–9]) for the microscopic origin of low-frequency flux noise in Josephson circuits. Therefore, the study of low-frequency noise has becomes very important for superconductor qubits [1]. Moreover, the quantum Zeno effect has been proposed as a strategy to protect coherence [10], entanglement [11, 12], and to control thermodynamic evolutions [13]. The quantum Zeno effect and anti-Zeno effect have been widely discussed [14, 15]. So it is an interesting topic to investigate the quantum Zeno effect of a qubit coupled to a low-frequency bath.

The description of low-frequency noise [1] (such as 1/f noise) is complicated by the presence of long-time correlations in the fluctuating environment, which prohibit the use of the Markovian approximation. In addition, the rotating wave approximation (RWA) is also unavailable in an environment with multiple modes. In the case of a time-depend external field for a qubit coupled to a thermal bath, Kofman and Kurizki developed a theory [16] which considers the counter-rotating terms of the fast modulation field through the negative-frequency part \( G(\omega) (\omega < 0) \) in the bath-correlation function spectrum. The dynamics in two significant models (the spin-boson model with Ohmic bath, and a qubit coupled to a bath of two level fluctuators) have been calculated within a rigorous Born approximation and without the Markovian approximation [17, 18]. Refs. [17] and [18] describe the structure of the solutions in the complex plane with branch cuts and poles.

Here we present an analytical approach, based on a unitary transformation. We use neither the Markovian approximation nor the RWA in order to discuss the transient dynamics of a qubit coupled to its environment. This method has already been used [19] to study the decoherence of the Ohmic bath, sub-Ohmic bath, and structured bath. In this paper, we calculate the coherence dynamics of the qubit (with and without measurement) respectively in two kinds of baths. Besides producing an energy shift, the environment can change the decay rate of the qubit. To preserve the coherence, we also investigate the decay rate of the qubit subject to the quantum Zeno effect. The low-frequency noise uses a Lorentzian-type spectrum with the peak of the spectrum in the low-energy region and for the high-frequency noise we choose the ordinary Ohmic bath with Drude cutoff.

Our results show that for low-frequency noise, the qubit energy increases (blue shift) and an anti-Zeno effect takes place. For a high-frequency cutoff Ohmic bath, the qubit energy decreases (red shift) and the Zeno effect dominates. For a high-frequency environment, the decay rate should be faster (without measurements) or slower (with frequent measurements, in the Zeno regime), compared to the low-frequency bath case. The experimental implementation of our results here could distinguish the type of bath (either a low- or high-frequency one) and protect the coherence of the qubit by modulating the dominant frequency of its environment. The coherence dynamics without measurement also indicates that the coherence time of the low-frequency noise is longer than the high-frequency noise, which demonstrates the powerful temporal memory of the low-frequency bath and stem from non-Markov process of qubit-bath interaction within the bath memory time [20].
II. METHOD BEYOND THE RWA BASED ON A UNITARY TRANSFORMATION

We describe a qubit coupled to a boson bath, modeling the environment, by the Hamiltonian

\[ H = -\frac{1}{2}\Delta \sigma_z + \frac{1}{2} \sum_k g_k (a_k^+ + a_k) \sigma_z + \sum_k \omega_k a_k^+ a_k. \]  

(1)

The boson bath and the spin-$\frac{1}{2}$ (fluctuators) bath will have the same dissipative effect on the qubit at zero temperature, \( T = 0 \), if both baths have the same correlation function \[21\]. However, for finite temperatures, the spin bath has a smaller effect on the qubit because of the likely saturation of the populations in the spin bath. Here, for simplicity, we only consider the case of zero temperature. That is, our results are also applicable to a fluctuator-bath at zero temperature. Our approach is based on a unitary transformation and can be used for different types of environmental baths. Below we give detailed derivations for the low-frequency noise case.

The spectral density \( J(\omega) \) of the environment considered here is given by

\[ J(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k) = \frac{2\alpha \omega}{\omega^2 + \lambda^2}, \]  

(2)

where \( \lambda \) is an energy lower than the qubit two-energy spacing \( \Delta \) and \( \alpha \) describes the coupling strength between the qubit and the environment. Choosing \( \Delta \) as the energy unit, \( \alpha/\Delta^2 \) is a dimensionless coupling strength. When \( \omega \geq \lambda \), \( J(\omega) \sim 1/\omega \), corresponding to a \( 1/\omega \) noise.

We apply a canonical transformation to the Hamiltonian \( H \): \( H' = \exp[S] H \exp[-S] \). Further explanations on the validity of the transformation can be found in the Appendix. Note that \( \sum_k g_k^2 \xi_k^2 / (2\omega_k^2) > 0 \) in Eq. \( \text{(A.7)} \), so \( 0 < \exp[- \sum_k g_k^2 \xi_k^2 / (2\omega_k^2)] < 1 \). Then the solution of \( \eta \) will be in the region from 0 to 1. Actually, the existence and uniqueness of the solution of \( \eta \) in Eq. \( \text{(A.7)} \) can be used as a criterion for the validity of our method. The parameter \( \eta \) can be regarded as a renormalization factor of the energy spacing \( \Delta \) and is calculated as

\[ \eta = \exp \left\{ \frac{\pi \lambda \eta}{\alpha} \Delta + \left[ -\lambda^2 - \eta^2 \Delta^2 + (\lambda^2 - \eta^2 \Delta^2) \log \left| \frac{\lambda}{\eta \Delta} \right| \right] \right\}. \]  

(3)

Obviously, \( \eta \) is determined self-consistently by the above equation.

Thus, the effective transformed Hamiltonian can be derived as

\[ H' \approx -\frac{1}{2} \eta \Delta \sigma_z + \sum_k \omega_k a_k^+ a_k + \sum_k V_k (a_k^+ \sigma_- + a_k \sigma_+). \]  

(4)

where

\[ V_k = \eta \Delta \frac{g_k \xi_k}{\omega_k}. \]  

(5)

Comparing \( H' \) in Eq. \( \text{(4)} \) with the ordinary Hamiltonian in the rotating-wave approximation (RWA):

\[ H_{\text{RWA}} = -\frac{1}{2}\Delta \sigma_z + \sum_k \omega_k a_k^+ a_k + \sum_k \frac{g_k}{2} (a_k^+ \sigma_- + a_k \sigma_+), \]  

(6)

one can see that the unitary transformation plays the role of renormalizing two parameters in the Hamiltonian, i.e., the energy spacing \( \Delta \) is renormalized:

\[ \Delta \longrightarrow \eta \Delta; \]  

(7)
and the coupling strength $g_k/2$ between the qubit and the bath is renormalized:

\[
\frac{g_k}{2} \rightarrow \left(\frac{2\eta \Delta}{\omega_k + \eta \Delta}\right) \frac{g_k}{2}.
\]  

Below we study the decoherence dynamics of the qubit and the quantum Zeno effect using the transformed Hamiltonian $H'$.

### A. Non-measurement decoherence dynamics

We diagonalize the transformed Hamiltonian $H'$ in the ground state $|g\rangle = |\uparrow\rangle |0_k\rangle$ and lowest excited states, $|\downarrow\rangle |0_k\rangle$ and $|\uparrow\rangle |1_k\rangle$, as

\[
H' = -\frac{1}{2}\eta \Delta |g\rangle \langle g| + \sum_E E |E\rangle \langle E|,
\]

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of $\sigma_z$, i.e., $\sigma_z |\uparrow\rangle = |\uparrow\rangle$, $\sigma_z |\downarrow\rangle = -|\downarrow\rangle$ and $|n_k\rangle$ denotes the state with $n$ bath excitations for mode $k$. The state $|E\rangle$ is

\[
|E\rangle = x(E) |\downarrow\rangle |0_k\rangle + \sum_{k'} y_k(E) |\uparrow\rangle |1_k\rangle,
\]

with $x(E) = \left[1 + \sum_k \frac{V_k^2}{(E + \eta \Delta/2 - \omega_k)^2}\right]^{-1/2}$ and $y_k(E) = \frac{V_k}{E + \eta \Delta/2 - \omega_k} x(E)$.

Here we calculate the dynamical quantity $\langle \sigma_x(t) \rangle$, which is the analog of the population inversion $\langle \sigma_z(t) \rangle$ in the spin-boson model [22]. Since the coupling to the environment will be “always present” in essentially all physically relevant situations, a natural ground state is given by the dressed state of the two-level qubit and bath. Therefore, the ground state of $H$ is $\exp[-S] |\uparrow\rangle |0_k\rangle$, under counter-rotating terms, and the corresponding ground-state energy is $-\eta \Delta/2$, then the ground state of $H'$ becomes $|\uparrow\rangle |0_k\rangle$ with the identical ground-state energy $-\eta \Delta/2$. We now prepare the initial state, which is also a dressed state of the qubit and bath, from the ground state as

\[
|\psi(0)\rangle = \frac{(1 + \sigma_x)}{\sqrt{2}} \exp[-S] |\uparrow\rangle |0_k\rangle.
\]

Then the initial state in the _transformed Hamiltonian_ becomes $|\psi(0)\rangle' = (|\uparrow\rangle + |\downarrow\rangle) |0_k\rangle / \sqrt{2}$, which is the eigenstate of $\sigma_x$. Starting from this initial state, we obtain [23]

\[
\langle \sigma_x(t) \rangle = \Tr_B \langle \psi(t) | \sigma_x | \psi(t) \rangle
\]

\[
= \Tr_B \langle \psi(0) | \exp[iHt] \sigma_x \exp[-iHt] | \psi(0) \rangle
\]

\[
= \frac{1}{2} \sum_E x(E)^2 \exp\left[-i \left(E + \frac{\eta \Delta}{2}\right)t\right] + \frac{1}{2} \sum_E x(E)^2 \exp\left[i \left(E + \frac{\eta \Delta}{2}\right)t\right]
\]

\[
= \frac{1}{4\pi i} \int_{-\infty}^{\infty} dE' \exp\left[-iE't\right] \left(E' - \eta \Delta - \sum_k \frac{V_k^2}{E' + i0^+ - \omega_k}\right)^{-1}
\]

\[
+ \frac{1}{4\pi i} \int_{-\infty}^{\infty} dE' \exp\left[iE't\right] \left(E' - \eta \Delta - \sum_k \frac{V_k^2}{E' - i0^+ - \omega_k}\right)^{-1}
\]

\[
= \text{Re} \left[ \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\exp\left[-iE't\right]}{E' - \eta \Delta - \sum_k \frac{V_k^2}{E' + i0^+ - \omega_k}} dE' \right].
\]
Here we denote the real and imaginary parts of $\sum_k V_k^2/(\omega - \omega_k \pm i0^+)$ as $R(\omega)$ and $\mp \Gamma(\omega)$, respectively. It follows that

$$R(\omega) = \varphi \sum_k \frac{V_k^2}{\omega - \omega_k} = (\eta \Delta)^2 \varphi \int_0^\infty \frac{J(\omega')}{(\omega - \omega')(\omega' + \eta \Delta)^2} d\omega'$$

$$= 2\alpha \Delta^2 \eta^2 \left( \frac{\omega \log |\omega|}{(\eta \Delta + \omega)^2(\lambda^2 + \omega^2)} \right)$$

$$+ 2\alpha \Delta^2 \eta^2 \pi \lambda \frac{[\lambda^2 + \eta \Delta (-\eta \Delta + 2\omega)] - 2[\lambda^2 (2\eta \Delta - \omega) + \eta^2 \Delta^2 \omega \log |\lambda|]}{2(\lambda^2 + \eta^2 \Delta^2)^2(\lambda^2 + \omega^2)}$$

$$+ 2\alpha \Delta^2 \eta^2 \left( \frac{[\lambda^2 + \eta \Delta (-\eta \Delta + 2\omega)] - 2[\lambda^2 (2\eta \Delta - \omega) + \eta^2 \Delta^2 \omega \log |\lambda|]}{(\lambda^2 + \eta^2 \Delta^2)^2(\eta \Delta + \omega)^2} \right).$$

(13)

and

$$\Gamma(\omega) = \pi \sum_k V_k^2 \delta(\omega - \omega_k) = \pi (\eta \Delta)^2 \frac{J(\omega)}{(\omega + \eta \Delta)^2},$$

(14)

where $\varphi$ stands for the Cauchy principal value and $J(\omega)$ is the spectral density. Then, we have

$$\langle \sigma_x(t) \rangle = \frac{1}{\pi} \int_0^\infty \frac{\Gamma(\omega) \cos \omega t}{(\omega - \eta \Delta - R(\omega))^2 + \Gamma(\omega)^2} d\omega.$$

(15)

The integration in Eq. (15) can be calculated numerically or approximately using residual theory.

**B. Measurement dynamics: quantum Zeno effect**

It is known that the quantum Zeno effect can effectively slow down the quantum decay rate of a quantum system. We study this effect using an approach that goes beyond the Markovian approximation and RWA. Here we consider the low-frequency bath as in Sec. II.A and derive the effective decay rate. The Hamiltonian $H'$ is given in Eq. (11),

$$H' = -\frac{1}{2} \eta \Delta \sigma_z + \sum_k \omega_k a_k^+ a_k + \sum_k \eta \Delta \frac{g_k \xi_k}{\omega_k} (a_k^+ \sigma_- + a_k \sigma_+).$$

(16)

In this paper, the effective decay rate is defined in the same form as in Ref. [14].

Write the wave function in the transformed Hamiltonian as

$$|\Phi(t)\rangle' = \chi(t) |\downarrow\rangle |0_k\rangle + \sum_k \beta_k(t) |\uparrow\rangle |1_k\rangle,$$

(17)

with probability in the excited state at initial time $|\chi(0)|^2$. Substituting $|\Phi(t)\rangle'$ into the Schrödinger equation with $H'$, we have

$$i \frac{d\chi(t)}{dt} = \frac{\eta \Delta}{2} \chi(t) + \sum_k V_k \beta_k(t),$$

(18)

$$i \frac{d\beta_k(t)}{dt} = \left( \omega_k - \frac{\eta \Delta}{2} \right) \beta_k(t) + \sum_k V_k \chi(t).$$

(19)

When the transformations

$$\chi(t) = \overline{\chi}(t) \exp \left[ -\frac{i \eta \Delta t}{2} \right],$$

(20)
\[ \beta_k(t) = \tilde{\beta}_k(t) \exp \left[ -i \left( \omega_k - \frac{\eta \Delta}{2} \right) t \right] \]  

are applied, Eqs. (18) and (19) can be written as

\[ \frac{d\tilde{\chi}(t)}{dt} = -i \sum_k V_k \tilde{\beta}_k(t) \exp \left[ -i(\omega_k - \eta \Delta) t \right], \]

\[ \frac{d\tilde{\beta}_k(t)}{dt} = -i V_k \tilde{\chi}(t) \exp \left[ i(\omega_k - \eta \Delta) t \right]. \]

Integrating Eq. (23) and then substituting it into Eq. (22), we obtain

\[ \frac{d\tilde{\chi}(t)}{dt} = -\sum_k V_k^2 \int_0^t \tilde{\chi}(t') \exp \left[ -i(\omega_k - \eta \Delta)(t - t') \right] dt'. \]

This integro-differential equation Eq. (22) is exactly soluble by a Laplace transformation.

As for the present study of the quantum Zeno effect, i.e., using frequent measurements, it suffices to obtain the short-time behavior and the equation can be solved iteratively. With the initial excited-state probability amplitude \( \chi(0) \), in the first iteration, Eq. (24) is solved as

\[ \frac{\tilde{\chi}(t)}{\chi(0)} \approx 1 - \int_0^t (t - t') \sum_k V_k^2 \exp \left[ -i(\omega_k - \eta \Delta)t' \right] dt'. \]

We can approximately write \( \tilde{\chi}(t) \) using an exponential form:

\[
\frac{\tilde{\chi}(t)}{\chi(0)} = \exp \left[ -\int_0^t (t - t') \sum_k V_k^2 \exp \left[ -i(\omega_k - \eta \Delta)t' \right] dt' \right],
\]

\[ = \exp \left[ -t \sum_k V_k^2 \exp \left[ -i(\omega_k - \eta \Delta)t \right] \frac{1 + i(\omega_k - \eta \Delta)t}{(\omega_k - \eta \Delta)^2} \right] \]

\[ = \exp \left[ -t \sum_k V_k^2 \left( \frac{2 \sin \left( \frac{\omega_k - \eta \Delta}{2} \right)^2}{(\omega_k - \eta \Delta)^2} - i \frac{(\omega_k - \eta \Delta)t \sin \left[ (\omega_k - \eta \Delta)t \right]}{t(\omega_k - \eta \Delta)^2} \right] \right]. \]

The instantaneous ideal projections are assumed to be performed at intervals \( \tau \). If single measurement, the probability amplitude is \( \tilde{\chi}(t = \tau) \). For a sufficiently large frequency of measurements, the survival population in the excited state is

\[ \rho_{\text{ee}}(t = n\tau) = |\tilde{\chi}(t = n\tau)|^2 = |\chi(0)|^2 \exp[-\gamma(\tau)t], \]

where the subscript “ee” refers to the initial and final excited state. And \( \gamma(\tau) \), with projection intervals \( \tau \), is obtained as

\[ \gamma(\tau) = 2\pi \int_0^\infty d\omega \sum_k \left( \frac{g_k}{2} \right)^2 \left( \frac{\omega_k}{\omega} \right)^2 \left( \frac{2\xi_k}{\omega} \right)^2 \frac{2 \sin^2 \left( \frac{\eta \Delta - \omega \tau}{2} \right)}{\pi(\eta \Delta - \omega)^2 \tau}, \]

\[ = 2\pi \int_0^\infty d\omega J(\omega) \left[ 1 - \frac{\omega - \eta \Delta}{\omega + \eta \Delta} \right]^2 \frac{2 \sin^2 \left( \frac{\eta \Delta - \omega \tau}{2} \right)}{\pi(\eta \Delta - \omega)^2 \tau}. \]

We now prepare the qubit to the dressed excited state \( \exp[-S]|\uparrow\rangle|0_k\rangle \) \((\sigma_z |\downarrow\rangle = -|\downarrow\rangle)\) at the initial time \( t = 0 \), which can be achieved by acting the operator \( \sigma_x \) on the ground state \( \exp[-S]|\uparrow\rangle|0_k\rangle \),

\[ |\Phi(0)\rangle = \exp[-S]|\downarrow\rangle|0_k\rangle = \sigma_x \exp[-S]|\uparrow\rangle|0_k\rangle. \]
In this case, the initial state in the transformed Hamiltonian is $|\downarrow\rangle \langle 0_k|$ and $\chi(0) = 1$.

Note that in Eq. (31), the renormalization factor $\eta$ of the characteristic energy $\Delta$ appears in the decay rate $\gamma(\tau)$. This is different from the formulas for $\gamma(\tau)$ in Refs. 24 and 25. In the case of spontaneous emission, the coupling strength between the electromagnetic field and atom is the fine structure constant $1/137$, so it belongs to the weak-coupling case and $\eta$ then becomes extremely close to 1. Therefore, besides spontaneous emission, this result for the quantum Zeno effect can apply to other cases of strong coupling between the qubit and the bath.

Under the RWA, the Hamiltonian becomes $H_{\text{RWA}}$ as in Eq. (6). We prepare the initial excited state of $H$ through the operator $\sigma_x$ acting on the ground state under RWA $|\uparrow\rangle \langle 0_k|$, $\sigma_x |\uparrow\rangle \langle 0_k| = |\downarrow\rangle \langle 0_k|$. Then, following the derivation in Refs. 14, the decay rate is reduced to

$$\gamma_{\text{RWA}}(\tau) = 2\pi \int_0^\infty d\omega \sum_k \frac{(g_k^2/2) 2 \sin^2(\Delta - \omega/2) \pi(\Delta - \omega)^2 \tau}{\pi(\Delta - \omega)^2 \tau}.$$  

To compare with the high-frequency Ohmic bath, we choose the ordinary Ohmic bath with Drude cutoff:

$$J_{\text{oh}}(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k) = 2\alpha_{\text{oh}} \omega (\omega/\omega_c)^2 + 1.$$  

This is a realistic assumption for, e.g., an electromagnetic noise. In Eq. (35), $\alpha_{\text{oh}}$ is the coupling strength between the qubit and the Ohmic bath. The cutoff frequency $\omega_c$ in the spectral density $J_{\text{oh}}(\omega)$ is typically assumed to be the largest frequency in the problem. As for the low-frequency noise, we use $J(\omega) = 2\alpha \omega/\omega_c^2 + \lambda^2$, which is the same as in Eq. (2). The difference between these two baths is that $\lambda$ corresponds to an energy lower than the qubit energy. In the next section, we will show our numerical results for these two baths.

III. RESULTS AND DISCUSSIONS

To show the effects of either a low- or a high-frequency noise on the qubit states respectively, we study the dynamical quantities $\langle \sigma_x(t) \rangle$ and the quantum Zeno decay rate $\gamma(\tau)$. The energy shift and the quantum Zeno decay rate exhibit evidently different features for the low- and high-frequency noises. Thus, these two quantities (energy shift and decay rate) can be used as criteria to distinguish the type of noise. In Fig. 1, we show the spectral densities $J(\omega)$ and $J_{\text{oh}}(\omega)$ of the two baths in the cases of both weak dissipation $\alpha/\Delta^2 = 0.01$ ($\alpha_{\text{oh}} = 0.01$) and strong dissipation $\alpha/\Delta^2 = 0.1$ ($\alpha_{\text{oh}} = 0.1$). As examples, we choose $\lambda = 0.09\Delta$ and $\lambda = 0.3\Delta$ in Fig. 1(a) and 1(b), respectively. Here, each value of $\lambda$ corresponds to the position of the peak in the low-frequency spectral density. For an Ohmic bath, the cutoff frequency is fixed at $\omega_c = 10\Delta$. As expected, these very different low- and high-frequency spectral densities should give rise to different decoherence behaviors of the qubit. In Fig. 1, we also show the characteristic energy of the isolated qubit $\Delta$ (see the vertical dotted line in Fig. 1).

A. Non-measurement decoherence dynamics

Results from the numerical integration of Eq. (26) are shown in Fig. 2. They are qualitatively consistent with the results obtained using residual theory. This indicates that the branch cuts considered in Refs. 17 and 18 do not affect the oscillation frequency. The time evolution of $\langle \sigma_x(t) \rangle$ is given in Fig. 2(a) for the case of weak coupling between the
FIG. 1: (Color online) The spectral density \( J(\omega) \) of the low- and high-frequency baths. (a) The case of weak interaction between the bath and the qubit, where the parameters of the low-frequency Lorentzian-like spectrum are \( \alpha/\Delta^2 = 0.01 \) and \( \lambda = 0.09\Delta \) (red solid curve), while for the high-frequency Ohmic bath with Drude cutoff the parameters are \( \alpha^{\text{oh}} = 0.01 \) and \( \omega_c = 10\Delta \) (green dashed-dotted curve). (b) The case of strong interaction between the bath and the qubit, where the parameters of the low-frequency bath are \( \alpha/\Delta^2 = 0.1 \) and \( \lambda = 0.3\Delta \) (red solid curve) and the parameters of the high-frequency Ohmic bath are \( \alpha^{\text{oh}} = 0.1 \) and \( \omega_c = 10\Delta \) (green dashed-dotted curve). The characteristic energy of the isolated qubit is indicated by a vertical blue dotted line. Here, and in the following figures, the energies are shown in units of \( \Delta \).

qubit and the bath, where \( \alpha/\Delta^2 = 0.01 \) and \( \lambda = 0.09\Delta \) for the low-frequency noise and \( \alpha^{\text{oh}} = 0.01 \) and \( \omega_c = 10\Delta \) for the Ohmic bath. Figure 2(b) presents the time evolution of \( \langle \sigma_x(t) \rangle \) in the strong coupling case with the parameters \( \alpha/\Delta^2 = 0.1 \) and \( \lambda = 0.3\Delta \) for the low-frequency noise, as well as \( \alpha^{\text{oh}} = 0.1 \) and \( \omega_c = 10\Delta \) for the ohmic bath. As expected, the quantum oscillations of \( \langle \sigma_x(t) \rangle \) dampen faster in the strong-coupling case. We approximately evaluate the oscillation frequency or the effective energy of the qubit, \( \omega_0 - \eta\Delta - R(\omega_0) = 0 \), using the residue theorem. The decay rate can be obtained from \( \Gamma(\omega) \). We will now show the numerical values of \( \eta \) in the corresponding cases. In Fig. 2(a), the renormalized factor \( \eta = 0.98336 \), the oscillation frequency is \( \omega_0 = 1.0225\Delta \) and the decay rate is \( \Gamma(\omega_0) = 0.014654\Delta \) for the low-frequency noise; while \( \eta = 0.98447 \), \( \omega_0 = 0.97720\Delta \), and \( \Gamma(\omega_0) = 0.015318\Delta \) for the Ohmic bath. In Fig. 2(b), the renormalized factor \( \eta = 0.91444 \), the oscillation frequency is \( \omega_0 = 1.0868\Delta \) and the decay rate is \( \Gamma(\omega_0) = 0.11215\Delta \) for the low-frequency noise, while \( \eta = 0.84469 \), \( \omega_0 = 0.77221\Delta \), and \( \Gamma(\omega_0) = 0.13163\Delta \) for the Ohmic bath.

The energy spectral densities in Fig. 1 and the results in Fig. 2 indicate two opposite shifts of the characteristic
FIG. 2: (Color online) Time evolution of the coherence $\langle \sigma_x(t) \rangle$ versus the time $t$ multiplied by the qubit energy spacing $\Delta$.

(a) The case of weak interaction between the bath and the qubit, where the parameters of the low-frequency Lorentzian-type spectrum are $\alpha/\Delta^2 = 0.01$, $\lambda = 0.09\Delta$ (red solid curve); while for the high-frequency Ohmic bath with Drude cutoff the parameters are $\alpha_{\text{oh}} = 0.01$, $\omega_c = 10\Delta$ (green dashed-dotted curve). (b) The case of strong interaction between the bath and the qubit, where the parameters of the low-frequency bath are $\alpha/\Delta^2 = 0.1$, $\lambda = 0.3$ (red solid line), and for the high-frequency Ohmic bath are $\alpha_{\text{oh}} = 0.1$, $\omega_c = 10\Delta$ (green dashed-dotted line). These results show that the decay rate for the low-frequency bath is shorter than for the high-frequency Ohmic bath. This means that the coherence time of the qubit in the low-frequency bath is longer than in the high-frequency noise case, demonstrating the powerful temporal memory of the low-frequency bath. Also, our results reflect the structure of the solution with branch cuts \[17\]. The oscillation frequency for the low-frequency noise is $\omega_0 > \Delta$, in spite of the strength of the interaction. This can be referred to as a blue shift. However, in an Ohmic bath, the oscillation frequency is $\omega_0 < \Delta$, corresponding to a red shift. The shifting direction of the energy is independent of the interaction strength and only determined by the spectral properties. Thus, it can be used as a criterion for distinguishing the low- and high-frequency noises.

energy $\Delta$ for the two kinds of baths considered here. These opposite energy shifts are equivalent to energy repulsion. The energy shift is determined by the interaction term. In the transformed Hamiltonian, the interaction is $H'_1$ in Eq. \[A.5\]. Also, dipolar interactions, such as $H'_{JC} = g (a^+ \sigma_- + a \sigma_+) \text{ in the Jaynes-Cummings model, decrease the qubit’s ground-state energy and increase its excited-state energy. Thus, now we ask the following questions: how a qubit is affected by either a multimode bath or a single-mode cavity? How a qubit is influenced by these two kinds of multimode baths: low-frequency and high-frequency ones?}
For a low-frequency bath, the energy peak of the bath is located between the ground-state energy and the excited-state energy, that is to say the main part of the spectrum is in the region $\omega_k < \Delta$. Then (as seen in Fig. 1) the interaction of the bath with the two qubit states is "opposite", i.e. the ground-state energy becomes lower and the excited-state energy becomes higher. So the energy spacing for the case of a low-frequency bath exhibits a blue shift. This result is similar to the single-mode Jaynes-Cummings model. The energy difference of the two-state qubit is increased by the low-frequency bath.

For a high-frequency cutoff Ohmic bath, the energy peak of the bath is located above the excited-state energy. So the main part of the spectrum is in the region $\omega_k > \Delta$. The effect of the bath on the qubit mainly comes from the frequencies higher than the excited-state energy of the qubit. Then (as seen in Fig. 1) the bath repels both the excited-state and the ground-state energies to lower energies. But the effect of the high-frequency bath on the excited state is much larger than on the ground state. As a result, on the whole, the qubit energy difference in a high-frequency bath is red shifted. Thus, the effective energy difference of the qubit is reduced by the high-frequency bath.

For example, if the initial state of the qubit is an excited state, in the interaction picture, the main part of the coupling is

$$\sum_k g_k a_k^+ \exp(i \omega_k t) \sigma_- \exp(-i \Delta t) = \sum_k g_k a_k^+ \sigma_- \{ \cos[(\omega_k - \Delta)t] + i \sin[(\omega_k - \Delta)t] \},$$

where the real part contributes to the decay rate and the imaginary part results in the energy shift. For the low-frequency bath, the main part of the spectrum is in the region $\omega_k < \Delta$. Thus, the term for the energy shift is

$$\sin[(\omega_k - \Delta)t] < 0.$$  \hspace{1cm} (37)

However, for a high-frequency cutoff Ohmic bath, the main part of the spectrum is in the region $\omega_k > \Delta$, where the term for energy shift becomes

$$\sin[(\omega_k - \Delta)t] > 0.$$  \hspace{1cm} (38)

These results show that the energy shift for the two kinds of baths moves in opposite directions. Note that there is a minus in the interaction term in the expressions for the dynamical quantities such as Eq. (21) and Eq. (33). The above observations help us understand why the energy levels repel. The contribution by the real part of the interaction on the decay rate will be discussed below, when studying the quantum Zeno effect.

B. Measurement dynamics: quantum Zeno effect

The quantum Zeno effect can be a useful tool to preserve the state coherence of a quantum system, with the help of repeated projective measurements. Below we investigate the quantum Zeno effect in the qubit system and propose another criterion for distinguishing low- and high-frequency noises. In general, without using the RWA, the effective decay rate can be obtained as

$$\gamma(\tau) = 2\pi \int_0^\infty d\omega J(\omega) \left( 1 - \frac{\omega - \eta \Delta}{\omega + \eta \Delta} \right)^2 \frac{2 \sin^2(\frac{\eta \Delta - \omega}{2} \tau)}{\pi(\eta \Delta - \omega)^2 \tau}.\hspace{1cm} (39)$$
This expression includes three terms, i.e., the spectral density $J(\omega)$ of the bath, the projection time modulating function

$$F(\omega, \tau) = \frac{2 \sin^2(\frac{\Delta - \omega \tau}{2})}{\pi(\Delta - \omega)^2 \tau},$$

and the interaction contribution function of both the rotating and counter-rotating terms

$$f(\omega) = \left(1 - \frac{\omega - \eta \Delta}{\omega + \eta \Delta}\right)^2.$$  \hspace{1cm} (41)

The counter-rotating term contributing to $f(\omega)$ is $(\omega - \eta \Delta)/(\omega + \eta \Delta)$. If the RWA is applied, $f(\omega) = 1$. In Fig. 3(a), the decay rate $\gamma(\tau)/\gamma_0$ for the low-frequency bath is plotted in the weak-coupling case with $\alpha/\Delta^2 = 0.01$. Here $\gamma_0$ is the decay rate for $|e\rangle \rightarrow |g\rangle$ in the long-time limit with the RWA,

$$\gamma_0 = \gamma(\tau \rightarrow \infty) = 2\pi J(\Delta)/4.$$ \hspace{1cm} (42)

From the energy spectrum in Fig. 1(a), we can see that $\gamma_0$ is proportional to the spectrum density of the bath, with the magnitude corresponding to the crossing of the energy $\Delta$ of the isolated qubit and the bath spectrum. For comparison, we also plot $\gamma_{\text{RWA}}(\tau)/\gamma_0$ as a dashed-dotted curve, with

$$\gamma_{\text{RWA}}(\tau) = 2\pi \int_0^\infty d\omega J(\omega) \frac{2 \sin^2(\frac{\Delta - \omega \tau}{2})}{\pi(\Delta - \omega)^2 \tau}. \hspace{1cm} (43)$$

As we know, $\gamma(\tau)/\gamma_0 < 1$ means that repeated measurements slow down the decay rate $\gamma(\tau) < \gamma_0$, which is the quantum Zeno effect. On the contrary, $\gamma(\tau)/\gamma_0 > 1$ means an anti-Zeno effect. The curves in Fig. 3(b) show results for the Ohmic bath with $\alpha^\text{oh} = 0.01$. In Fig. 4, we show the decay rate in the strong-coupling case for the low-frequency bath with $\alpha/\Delta^2 = 0.1$ and for the Ohmic bath with $\alpha^\text{oh} = 0.1$. It can be seen that, for the low-frequency bath, the anti-Zeno effect appears as shown in Figs. 3 and 4. For the high-frequency cutoff Ohmic bath, the Zeno effect always dominates and no anti-Zeno effect occurs. Also we can see, from Figs. 3 and 4, that $\gamma(\tau)$ and $\gamma_{\text{RWA}}(\tau)$ approach $\gamma_0$ when the measurement interval $\tau \rightarrow \infty$. In particular, if $\tau \rightarrow 0$, $F(\omega, \tau) \rightarrow 0$. Thus, $\gamma(\tau) \rightarrow 0$. This implies that in a sufficiently short time interval of a projective measurement, the quantum Zeno effect occurs, regardless of the bath spectrum. When the interval $\tau$ increases, the projection interval modulation function $F(\omega, \tau)$ displays a number of oscillations. Then, the energy peak of the bath spectrum will act on the decay rate and it is possible to implement the anti-Zeno effect. However, this result still depends on the function $f(\omega)$ and the given spectral density $J(\omega)$ of the bath. Now, we emphasize again that the second term in the bracket of the function $f(\omega) = [1 - (\omega - \eta \Delta)/(\omega + \eta \Delta)]^2$ is due to the counter-rotating terms; when neglecting the counter-rotating terms, $f(\omega) = 1$.

For low-frequency noise, the noise mainly comes from the region $\omega < \Delta$, so $f(\omega) > 1$. Here, $f(\omega)$ as well as $F(\omega, \tau)$ magnify the effect of the energy peak of the bath spectrum. Thus, the counter-rotating terms accelerate the decay and the anti-Zeno effect occurs.

In the high-frequency cutoff Ohmic bath, the noise mainly comes from the region $\omega > \Delta$, which leads to $f(\omega) < 1$. Thus, it is mainly the counter-rotating term in $f(\omega)$ that reduces the effect of the energy peak of the bath on the decay. This slows down the decay and only the quantum Zeno effect can now take place. The projection-intervals-modulating function $F(\omega, \tau)$, together with the interaction-modulating function $f(\omega)$, causes the Zeno effect to dominate in the high-frequency cutoff Ohmic bath.
FIG. 3: (Color online) The effective decay $\gamma(\tau)/\gamma_0$, versus the time interval $\tau$ between consecutive measurements, for a weak coupling between the qubit and the bath. In the horizontal axis, the time-interval $\tau$ is multiplied by the qubit energy difference $\Delta$. The curves in (a) correspond to the case of a low-frequency bath with parameters $\alpha/\Delta^2 = 0.01$ and $\lambda = 0.09\Delta$ (red solid curve). (b) corresponds to the case of an Ohmic bath with parameters $\alpha_{\text{oh}} = 0.01$ and $\omega_c = 10\Delta$ (red solid curve). The green dashed-dotted curves are the results under the RWA when the same parameters are used. Note how different the RWA result is in (b), especially for any short measurement interval $\tau$.

IV. SUMMARY

In summary, we have studied a model of a qubit interacting with its environment, modeled either as a low- or as a high-frequency bath. For each type of bath, the quantum dynamics of the qubit without measurement and the quantum Zeno effect on it are shown for the cases of weak and strong couplings between the qubit and the environment. Our results show that, for a low-frequency bath, the qubit energy increases (blue shift) and the quantum anti-Zeno effect occurs. However, for a high-frequency cutoff Ohmic bath, the qubit energy decreases (red shift) and the quantum Zeno dominates. Moreover, for a high-frequency environment, the decay rate should be faster (without measurements) or slower (with frequent measurements, in the Zeno regime), compared to the low-frequency bath case. These very different behaviors of the quantum dynamics and the Zeno effect in different baths should be helpful to experimentally distinguish the type of noise affecting the qubit and protect the coherence of the qubit through modulating the dominant frequency of its environment.

Acknowledgements
FIG. 4: (Color online) The effective decay $\gamma(\tau)/\gamma_0$, versus the time interval $\tau$ between successive measurements, for a strong coupling between the qubit and the bath. The time-interval $\tau$ is multiplied by the qubit energy difference $\Delta$. The curves in (a) correspond to the case of a low-frequency bath with parameters $\alpha/\Delta^2 = 0.1$ and $\lambda = 0.3\Delta$ (red solid curve). (b) corresponds to the case of an Ohmic bath with parameters $\alpha_{oh} = 0.1$ and $\omega_c = 10\Delta$ (red solid curve). The green dashed-dotted curves are the results under RWA when the same parameters are used. Note how different the RWA result is in (b), especially for any short measurement interval $\tau$.

FN acknowledges partial support from the National Security Agency (NSA), Laboratory for Physical Sciences (LPS), Army Research Office (USARO), National Science Foundation (NSF) under Grant No. 0726909, and JSPS-RFBR under Contract No. 06-02-91200. X.-F. Cao acknowledges support from the National Natural Science Foundation of China under Grant No. 10904126 and Fujian Province Natural Science Foundation under Grant No. 2009J05014. J.-Q. You acknowledges partial support from the National Natural Science Foundation of China under Grant No. 10904126 and Fujian Province Natural Science Foundation under Grant No. 2009CB929300 and the ISTCP under Grant No. 2008DFA01930.
Appendix: Examining the validity of the unitary transformation

In this appendix, we show the main results of the canonical transformation to the qubit-bath Hamiltonian $H$ considered here, and prove that the contribution of $H'_2$ to physical quantities is of the order $O(g_k^4)$ and higher, so we ignore $H'_2$ in the calculations. We now apply a canonical transformation to the Hamiltonian $H$:

$$H' = \exp{[S]} H \exp{[-S]} ,$$

with

$$S = \sum_k \frac{g_k}{2\omega_k} \xi_k (a_k^+ - a_k) \sigma_z .$$

Here a $k$-dependent variable, $\xi_k = \omega_k / (\omega_k + \eta \Delta)$, is introduced in the transformation. It is clear that this canonical transformation is unitary, because $(\exp{[S]})^+ = \exp{[-S]}$. The transformed Hamiltonian $H'$ can now be decomposed in three parts:

$$H' = H'_0 + H'_1 + H'_2 ,$$

with

$$H'_0 = -\frac{1}{2} \eta \Delta \sigma_z + \sum_k \omega_k a_k^+ a_k - \sum_k \frac{g_k^2}{4\omega_k} \xi_k (2 - \xi_k) ,$$

$$H'_1 = \sum_k \eta \Delta \frac{g_k \xi_k}{\omega_k} (a_k^+ \sigma_+ + a_k \sigma_-) ,$$

$$H'_2 = -\frac{1}{2} \Delta \sigma_z \left\{ \cosh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right] - \eta \right\}$$

$$-\frac{1}{2} \Delta \sigma_z \left\{ \sinh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right] - \eta \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right\} ,$$

where

$$\eta = \exp \left[ -\sum_k \frac{g_k^2}{2\omega_k^2} \xi_k^2 \right] .$$

Note that no approximation was used during the transformation, so $H' = \exp{[S]} H \exp{[-S]}$ is exact. Because the constant term $\sum_k \frac{g_k^2}{4\omega_k} \xi_k (2 - \xi_k)$ in Eq. (A.4) has no effect on the dynamical evolution, we neglect it. Considering at low temperatures, the multiple-step process is so weak that all the higher-order terms can be neglected. In the following derivation, we will prove that the contribution of $H'_2$ to physical quantities is of the order $O(g_k^4)$ and higher, so we ignore $H'_2$ and obtain the effective transformed Hamiltonian $H' = H'_0 + H'_1$.

Now let us expand the first term of $H'_2$, $\cosh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right]$, as a series in $g_k \xi_k / \omega_k$. We define $\chi_k$ as $\chi_k = g_k \xi_k / \omega_k$, which is proportional to $g_k$:

$$\cosh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right] = \cosh \left[ \sum_k \chi_k (a_k^+ - a_k) \right]$$

$$= \frac{1}{2} \left\{ \exp \left[ \sum_k \chi_k (a_k^+ - a_k) \right] + \exp \left[ -\sum_k \chi_k (a_k^+ - a_k) \right] \right\} .$$
The first term in Eq. (A.9) can be written as

\[
\exp \left[ \sum_k \chi_k (a_k^+ - a_k) \right] = \exp \left[ \sum_k \chi_k a_k^+ \right] \exp \left[ \sum_k -\chi_k a_k \right] \exp \left[ -\frac{1}{2} \left[ \sum_k \chi_k a_k^+, \sum_k -\chi_k a_k \right] \right],
\tag{A.10}
\]

where \( \left[ \sum_k \chi_k a_k^+, \sum_k -\chi_k a_k \right] \) means the commutator of two operators. Using the commutation relation \([a_k^+, a_k] = -1\), it is simplified to

\[
\exp \left[ -\frac{1}{2} \left[ \sum_k \chi_k a_k^+, \sum_k -\chi_k a_k \right] \right] = \exp \left[ -\frac{1}{2} \sum_k \chi_k^2 \right] = \exp \left[ -\sum_k \frac{g_k^2}{2\omega_k^2} \xi_k^2 \right] = \eta,
\tag{A.11}
\]

which is the definition of \( \eta \).

Afterwards, we expand Eq. (A.10) as follows,

\[
\exp \left[ \sum_k \chi_k (a_k^+ - a_k) \right] = \eta \exp \left[ \sum_k \chi_k a_k^+ \right] \exp \left[ \sum_k -\chi_k a_k \right] \tag{A.12}
\]

\[
= \eta \left[ 1 + \sum_k \chi_k a_k^+ + \frac{(\sum_k \chi_k a_k^+)^2}{2} + \frac{(\sum_k \chi_k a_k^+)^3}{3!} + \ldots \right]
\tag{A.13}
\]

\[
\times \left[ 1 - \sum_k \chi_k a_k + \frac{(-\sum_k \chi_k a_k)^2}{2} - \frac{(\sum_k \chi_k a_k)^3}{3!} + \ldots \right]
\tag{A.14}
\]

\[
= \eta \left[ 1 + \sum_k \chi_k a_k^+ - \sum_k \chi_k a_k - \sum_k \chi_k a_k^+ \sum_k \chi_k a_k + \ldots \right]
\tag{A.15}
\]

Now we see the second-order terms in \( \chi_k \) in Eq. (A.15):

\[
\sum_k \chi_k a_k^+ \sum_k \chi_k a_k = \sum_k \chi_k^2 a_k^+ a_k (\text{diagonal}) + \sum_{k \neq k'} \chi_k a_k^+ \chi_{k'} a_{k'} (\text{off-diagonal}).
\tag{A.16}
\]

The off-diagonal terms are related to the multi-boson transition and their contributions to the physical quantities are fourth order in \( \chi_k \). Furthermore, the initial state \( |0_k\rangle \langle 1| \) is used in the calculation, so the direct effect of the second-order diagonal term on physical quantities is zero and its effect through the interaction will also be fourth order in \( \chi_k \). Thus we now ignore the terms higher than second order in \( \chi_k \) in the following calculation and obtain the expression of Eq. (A.10),

\[
\exp \left[ \sum_k \chi_k (a_k^+ - a_k) \right] \approx \eta \left[ 1 + \sum_k \chi_k a_k^+ - \sum_k \chi_k a_k \right].
\tag{A.17}
\]

Similarly, the second term in Eq. (A.9) becomes

\[
\exp \left[ -\sum_k \chi_k (a_k^+ - a_k) \right] = \eta \exp \left[ -\sum_k \chi_k a_k^+ \right] \exp \left[ \sum_k \chi_k a_k \right] \tag{A.18}
\]

\[
\approx \eta \left[ 1 - \sum_k \chi_k a_k^+ + \sum_k \chi_k a_k \right].
\tag{A.19}
\]

Therefore, Eq. (A.9) is reduced to

\[
cosh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right] = \cosh \left[ \sum_k \chi_k (a_k^+ - a_k) \right] \tag{A.20}
\]

\[
= \frac{1}{2} \left\{ \exp \left[ \sum_k \chi_k (a_k^+ - a_k) \right] + \exp \left[ -\sum_k \chi_k (a_k^+ - a_k) \right] \right\} \tag{A.21}
\]

\[
\approx \eta.
\tag{A.22}
\]
In the same way, the third term in $H'_2$ is simplified to

$$\sinh \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) \right] = \eta \left[ \sum_k \frac{g_k}{\omega_k} \xi_k (a_k^+ - a_k) + O(\chi_k^3) \right]. \tag{A.23}$$

In $H'_2$, we have subtracted the terms of zero and first-order in $g_k$, which are included in $H'_0 + H'_1$, and only left the terms equal to and higher than second-order in $g_k$, whose contribution to the physical quantities is of order $O(g_k^4)$ and higher. Thus, $H'_2$ can be omitted.

We expanded several series in the variable $\chi_k$, with the real variable $\eta \Delta \chi_k$ less than $g_k$. That is: $\eta \Delta \chi_k = \eta \Delta g_k \xi_k / \omega_k = \eta \Delta g_k / (\omega_k + \eta \Delta) < g_k$. In other words, through this transformation we find a variable smaller than $g_k$ for the series expansion. Therefore, our method can be extended to the case of strong interaction between the qubit and the environment.

[1] R. McDermott, IEEE Trans. Appl. Supercond. 19, 2 (2009).
[2] J. Q. You and F. Nori, Phys. Today 58, No. 11, 42 (2005).
[3] J. Clarke and F. K. Wilhelm, Nature (London) 453, 1031 (2008).
[4] F. Yoshihara, K. Harrabi, A. O. Niskanen, Y. Nakamura, and J. S. Tsai, Phys. Rev. Lett. 97, 167001 (2006).
[5] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, A. Shnirman, Phys. Rev. Lett. 98, 047004 (2007).
[6] A. N. Omelyanchouk, S. Savel’ev, A. M. Zagoskin, E. Il’ichev, and Franco Nori, Phys. Rev. B 80, 212503 (2009).
[7] R. H. Koch, D. P. DiVincenzo, and J. Clarke, Phys. Rev. Lett. 98, 267003 (2007).
[8] R. de Sousa, Phys. Rev. B 76, 245306 (2007).
[9] L. Faoro and L. B. Ioffe, Phys. Rev. Lett. 100, 227005 (2008).
[10] L. Zhou, S. Yang, Y.-X. Liu, C. P. Sun and F. Nori, Phys. Rev. A 80, 062109 (2009).
[11] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, F. Plastina, Phys. Rev. Lett. 100, 090503 (2008).
[12] X.-B. Wang, J. Q. You, and F. Nori, Phys. Rev. A 77, 062339 (2008).
[13] N. Erez, G. Gordon, M. Nest, and G. Kurizki, Nature 452, 724 (2008).
[14] A. G. Kofman and G. Kurizki, Nature (London) 405, 546 (2000).
[15] P. Facchi and S. Pascazio, Phys. Rev. A 62, 023804 (2000); Phys. Lett. A 241, 139 (1998); P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. 86, 2699 (2001).
[16] A. G. Kofman and G. Kurizki, Phys. Rev. Lett. 93, 130406 (2004).
[17] D. P. DiVincenzo and D. Loss, Phys. Rev. B 71, 035318 (2005).
[18] G. Burkard, Phys. Rev. B 79, 125317 (2009).
[19] X.-F. Cao and H. Zheng, Phys. Rev. A 77, 022320 (2008); P.-H. Huang and H. Zheng, J. Phys.: Condens. Matter 20, 395233 (2008); X.-F. Cao and H. Zheng, Phys. Rev. B 76, 115301 (2007); Z.-G. Lü and H. Zheng Phys. Rev. B 75, 054302 (2007).
[20] G. Gordon, G. Bensky, D. Gelbwaser-Klimovsky, D D B. Rao, N. Erez and G. Kurizki, New J. Phys. 11, 123025 (2009).
[21] U. Weiss, Quantum dissipative systems (3rd Edition) (World Scientific, Singapore, 2008).
[22] X.-F. Cao and H. Zheng, Phys. Rev. A 75, 062121 (2007).
[23] H. Zheng, Eur. Phys. J. B 38, 559 (2004).
[24] H. Zheng, S. Y. Zhu, and M. S. Zubairy, Phys. Rev. Lett. 101, 200404 (2008).
[25] Z.-H. Li, D.-W. Wang, H. Zheng, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A 80, 023801 (2009).