Features of the twistor formulation of the massless superparticle on $AdS_5 \times S^5$ superbackground

D V Uvarov
NSC Kharkov Institute of Physics and Technology, 61108 Kharkov, Ukraine
E-mail: d.uvarov@hotmail.com

Abstract. We study supertwistor formulations of the $D = 10$ massless superparticle model on $AdS_5 \times S^5$ superbackground of IIB supergravity. Product structure of the background suggests using $\text{Spin}(1,4)$ variables to express momentum components tangent to $AdS_5$ and $\text{Spin}(5)$ variables to express momentum components tangent to $S^5$ that yields eight-supertwistor formulation of the superparticle’s Lagrangian. We find incidence relations connecting supertwistor components with the $AdS_5 \times S^5$ superspace coordinates and the set of the constraints that supertwistors satisfy. Solving the constraints for the $\text{Spin}(1,4)$ and $\text{Spin}(5)$ variables it is possible to reduce eight-supertwistor formulation to the four-supertwistor one. Respective supertwistors agree with those introduced previously in other models. The advantage of the four-supertwistor formulation is the presence only of the first-class constraints that facilitates analysis of the superparticle model.

1. Introduction

$AdS_5/CFT_4$ correspondence is commonly formulated and studied in (super)space-time but $D = 4 \quad N = 4$ superYang-Mills theory can also be formulated in supertwistor space with manifest and linearly realized superconformal symmetry at the level of supermultiplet, field equations and Lagrangian, free and including interactions, as well as the scattering amplitudes [1], [2], [3], [4], [5], [6], [7], [8]. Basic ingredients are Penrose-Ferber supertwistors [9], ambitwistors and in the case of the scattering amplitudes – momentum twistors [7]. Thus it is natural to wonder whether IIB superstring/supergravity theories admit a twistor formulation? In Refs. [10], [11] $SU(2)$-doublet of Penrose twistors was extended to the Poincare patch of $AdS_5$ and found the twistor formulation of the massive particle’s action. Recently it was extended to the family of $Sp(4, \mathbb{K}) \simeq \text{Spin}(2, 2 + \dim \mathbb{K}) \quad (\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H})$ twistor formulations of particle models in $AdS_D$ with $D = 3 + \dim \mathbb{K}$ [12], [13]. Another definition of $AdS_5$ twistors was given in [14], there were also performed calculations reproducing some simple gauge theory correlators in the framework of $AdS_5/CFT_4$ duality using twistor methods in the bulk. Generalization of these results to $AdS_5 \times S^5$ superspace is not obvious since it is not superconformally flat [15] unlike Poincare patch of $AdS_D$. Independently I. Bars in the framework of his 2T approach proposed ‘parent’ superparticle and tensionless string models [16], [17] extended gauge symmetries of which can be fixed in a variety of ways. Massless superparticle on $AdS_5 \times S^5$ in the superspace and supertwistor formulations arises from the ‘parent’ superparticle upon choosing different gauge conditions. In the supertwistor gauge dynamical variables of the model are two $SU(2)$-doublets of supertwistors $Z^A_a$ and $\Psi^A_q$. In the ‘parent’ superparticle model by I. Bars superspace coordinates and supertwistors appear to be unrelated to one another so the incidence relations
cannot be derived. This motivated us to follow in [18], [19] the traditional root of deriving superwistors formulation for massless superparticle on $AdS_5 \times S^5$ superbackground described in this note that starts with the first-order form of the superspace Lagrangian and relies on the superparticle’s momentum representation in terms of the product of the spinor variables.

2. Massless superparticle on $AdS_5 \times S^5$ superbackground: eight-supertwistor formulation

Classical action of the massless superparticle on $AdS_5 \times S^5$ superbackground can be presented in the first-order form

$$S_{AdS_5 \times S^5} = \int dt \mathcal{L}_{AdS_5 \times S^5}, \quad \mathcal{L}_{AdS_5 \times S^5} = p_{m'} E^{m'}_\tau + p_T E_T - \frac{g}{2}(p_{m'} p^{m'} + p_T p_T),$$

where $E^{m'}_\tau$ and $E_T$ are world-line pullbacks of the $D = 10$ supervielbein bosonic components tangent to $AdS_5$ and $S^5$. To pass to the supertwistor formulation we need to express null 10-momentum $(p_{m'}, p_T)$ in terms of the product of spinor variables. Momentum components tangent to $AdS_5$ can be presented as

$$p_{m'} = -\frac{1}{2} v^\alpha_{m'} \gamma_m^{\alpha \beta} v_\beta = -\frac{1}{2} (v^b_{m'} \gamma_m^{\alpha \beta} v_\beta - v_b^{m'} \gamma_m^{\alpha \beta} v_\beta),$$

where

$$v^\alpha_{m'} = (-v_b^{m'}, v^{ab})$$

is the $4 \times 4$ matrix of the $Spin(1,4)$ variables. Its index from the beginning of the Greek alphabet is acted by the left $Spin(1,4)_L$ transformations resulting in the Lorentz rotations of the momentum components and index from the middle part of the Greek alphabet, that in (3) was decomposed into the $SU(2)_R \times SU(2)_L$ indices labelled by small letters from the beginning of the Latin alphabet, is acted by the right local $Spin(1,4)_R$ transformations, whose $SU(2)_R \times SU(2)_L$ subgroup leaves the momentum intact and enters the set of the gauge symmetries of the superparticle’s action. In (2) the two sets of $\gamma$-matrices in $D = 1 + 4$ dimensions are invariant under left and right $Spin(1,4)$ transformations respectively and satisfy the defining relations

$$\gamma_{m'}^{\alpha \sigma} \gamma_{m'}^{\beta \sigma} + \gamma_{m'}^{\alpha \sigma} \gamma_{m'}^{\beta \sigma} = -2 \eta_{m'n'} \delta^{\alpha}_{\beta}, \quad \eta_{m'n'} = \text{diag}(-,+,+,-,+)$$

$$\gamma^{(k')}_{\mu} \gamma^{(l')}_{\nu} + \gamma^{(k')}_{\mu} \gamma^{(l')}_{\nu} = -2 \eta^{(k')(l')} \delta_{\mu \nu}, \quad (k') = (0,\hat{I}), (l') = (0,\hat{J}).$$

In the last equality in (2) there has been used the diagonal realization of $\gamma^{(0)\nu} \lambda$

$$\gamma^{(0)\nu} \lambda = \left( \begin{array}{cc} -\delta^b_{\alpha} & 0 \\ 0 & \delta^b_{\alpha} \end{array} \right).$$

Matrix (3) obeys the reality conditions

$$\gamma^{(0)\nu} \lambda (v_{\nu}^{\alpha})^{\dagger} \gamma^{0\alpha} \beta = v^{T \lambda}_{\beta}$$

ensuring that $p_{m'}$ is real. In terms of the $4 \times 2$ rectangular blocks $v_b^{\alpha}$ and $v^{ab}$ they acquire the form of the $SU(2)$-Majorana conditions

$$(v_b^{\alpha})^{\dagger} \gamma^{0\alpha} \beta = v^{T b}_{\beta}, \quad (v^{ab})^{\dagger} \gamma^{0\alpha} \beta = v^{T b}_{\beta}.$$
so that
\[ p_{m'} p^{m'} = -\frac{1}{4} (e_{\alpha}^{a} e_{\bar{\alpha}}^{a} + e_{\dot{\alpha}}^{\dot{a}} e_{\bar{\dot{\alpha}}}^{\dot{a}})^2. \] (9)

The transformation and reality properties of the spinor variables (3) are the same as those
of \( D = 1 + 4 \) spinor Lorentz harmonics parametrizing the \( \text{Spin}(1,4)/(SU(2) \times SU(2)) \) coset
[19]. Also Eq. (2) coincides with the general relation that connects the time-like component
(first-column) of the matrix of the vector Lorentz harmonics and the spinor Lorentz harmonics
[22], [23], [24] up to the overall normalization chosen here to produce canonical form of
the superparticle’s kinetic term in the supertwistor formulation. The difference is that the
constraints (8) are weaker than those imposed on the \( D = 1 + 4 \) spinor Lorentz harmonics [19].
This is necessary to correctly balance the degrees of freedom of the null 10-momentum and
spinor variables.

Further introduce 4 × 4 matrix of the \( \text{Spin}(5) \) variables
\[ \ell^A = (\ell^A_p, \ell^A_{\dot{p}}). \] (10)

It transforms under left \( \text{Spin}(5)_L \simeq USp(4)_L \) transformations acting on the capital index from
the beginning of the Latin alphabet and local \( \text{Spin}(5)_R \simeq USp(4)_R \) transformations that act on
the capital index from the middle part of the Latin alphabet. \( SU(2)_R \times SU(2)_R \subset Spin(5)_R \)
subgroup will enter the set of the superparticle’s gauge symmetries, accordingly the index \( N \)
have been split into \( SU(2)_R \times SU(2)_R \) indices labelled by small letters from the middle part of
the Latin alphabet. \( \text{Spin}(5) \) variables satisfy the reality conditions
\[ (\ell^A)^\dagger_N = \ell^{TN}_A; \quad (\ell^A_p)^\dagger = \ell^T_{Ap}, \quad (\ell^{\dot{A}}^A)^\dagger = \ell^T_{\dot{A}p} \] (11)

and are constrained by the relations
\[ \ell^A_p \ell^A_{\dot{p}} = 0. \] (12)

Let us note that the properties of the \( \text{Spin}(5) \) variables are very close to those of the
\( USp(4)/(SU(2) \times SU(2)) \) harmonics [25]. They have the same transformation properties
and satisfy the same reality conditions but the harmonics obey extra constraints that adjust the
number of their independent degrees of freedom to the dimension of the \( USp(4)/(SU(2) \times SU(2)) \)
coset. In analogy with (2) momentum components tangent to \( S^5 \) can be expressed in terms of the
\( \text{Spin}(5) \) variables as
\[ p^I = \frac{1}{2} \ell^T A \gamma^A \gamma^B \ell^B N^N L = \frac{1}{2} (\ell^T A \gamma^A \delta^B_q (\ell^A q^B) + \ell^T \bar{A} \gamma^A \delta^B \bar{q} (\ell^A \bar{q}^B)), \] (13)

where the two sets of \( D = 5 \) \( \gamma \)-matrices are invariant under \( \text{Spin}(5)_L \) and \( \text{Spin}(5)_R \)
transformations respectively and obey
\[ \gamma^{\mu A}_C \gamma^{\nu B}_C + \gamma^{\mu A}_C \gamma^{\nu B}_C = 2 \delta^{\mu \nu} \delta^A_B, \]
\[ \gamma (I')^N L \gamma (J')^L M + \gamma (J')^N L \gamma (I')^L M = 2 \delta (I')^N (J')^L \delta^N_M, \quad (I')^5 = (I')^5, \quad (J')^5 = (J')^5. \] (14)

In the last equality in (13) there has been chosen the diagonal realization of \( \gamma^{(5)N}_L \)
\[ \gamma^{(5)N}_L = \begin{pmatrix}
-\delta^p_{q} & 0 \\
0 & \delta^\dot{\theta}_{\dot{p}}
\end{pmatrix}. \] (15)

\(^1 D = 1 + 4 \) Lorentz harmonics parametrizing other cosets were considered in [20], [21].
Then the square of the momentum components tangent to $S^5$ equals

$$p_Ip_I' = \frac{1}{4}(\ell_{q}^A\ell_{A}' - \ell_{q}^A\ell_{A}')^2$$

(16)

and the null-momentum condition in 10 dimensions acquires the form

$$\left(v_\alpha^a v_\alpha^a + v_\dot{\alpha}^a v_\dot{\alpha}^a\right)^2 = (\ell_{q}^A\ell_{q}^A - \ell_{q}^A\ell_{q}^A)^2.$$ 

(17)

At this point there is a discrepancy between the number of the independent degrees of freedom since null 10-momentum $(p_{m'},p_{I'})$ has 9 independent components, whereas spinors $(v_\alpha^a, v_\dot{\alpha}^a)$ and $(\ell_{q}^A, \ell_{q}^A)$ have $6 + 6 - 1 = 11$ independent components. Two additional constraints for the spinor variables will be obtained below.

To pass to the eight-supertwistor formulation of the $D = 10$ massless superparticle on the $AdS_5 \times S^5$ superbackground it is convenient to introduce diagonal supermatrix

$$V^A_N = \begin{pmatrix} v_\alpha^0 & 0 \\ 0 & \ell_{N}^A \end{pmatrix},$$

(18)

whose upper-diagonal block constitutes the matrix of the Spin$(1,4)$ variables (3), while in the lower-diagonal block there is the matrix of the Spin$(5)$ variables (10). Reality conditions (6) and (11) in the supermatrix form read

$$H^L_N(V^A_N)^\dagger H^A_B = V^T_L B = \begin{pmatrix} \nu_T^\lambda & 0 \\ 0 & \ell^T_L B \end{pmatrix},$$

(19)

where

$$H^A_B = \begin{pmatrix} \gamma^{0\alpha} & 0 \\ 0 & \delta_B^A \end{pmatrix}, \quad H^L_N = \begin{pmatrix} \gamma^{(0)\lambda} & 0 \\ 0 & \delta_N^L \end{pmatrix}. $$

(20)

Next introduce the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ representative [26], [27]

$$G^A_B = \begin{pmatrix} G_{\alpha\beta} & G_{\dot{\alpha}B} \\ G_{\alpha\beta} & G_{\dot{\alpha}B} \end{pmatrix} \in PSU(2,2|4)/(SO(1,4) \times SO(5))$$

(21)

that satisfies defining relations

$$(G^A_B)^\dagger = \mathcal{H}^B_\epsilon G^{-1}_\epsilon D \mathcal{H}^D_A.$$ 

(22)

Now we can define the twistor supermatrix

$$Z^A_N = G^A_B V^B_N = (Z^A_\nu \Psi^A_N) = (-Z^A_\nu Z^{ab} \Psi^A_p \Psi^{Ap})$$

(23)

and its dual

$$\bar{Z}^L_B = \mathcal{H}^L_A(Z^A_N)^\dagger \mathcal{H}^A_B = V^T_L c G^{-1}_c = \begin{pmatrix} \bar{Z}^\lambda_B \\ \bar{Z}_{B}^\dot{\lambda} \\ \bar{\Psi}_B^q \\ \bar{\Psi}_{B\dot{q}} \end{pmatrix}.$$ 

(24)

Let us note that our choice of the $\gamma$-matrices in $D = 2 + 4$ and $D = 1 + 4$ dimensions is such that $\gamma^0$ equals 'metric tensor' connecting $\mathbf{4}$ and $\mathbf{4}$ representations of $SU(2,2)$. As explained in detail
in [18] and [19] $\gamma^{0\alpha\beta}$ and $\gamma^{(0)\lambda}$ correspond to its off-diagonal (twistor) and diagonal (oscillator) realizations.

Supertwistors $Z^A_\alpha$, $Z^{Ab}$ and their duals are conventional ones and were named $c$-type supertwistors in [18] because their $SU(2)_L$ components $Z^\alpha_\beta$ are even and $SU(4)_L$ components $\eta^A_\beta$, $\eta^{Ab}$ are odd. On the contrary supertwistors $\Psi^A$, $\Psi^{Ab}$ and their duals were named $a$-type supertwistors since their $SU(2)_L$ components $\xi^\alpha_\beta$, $\xi^{\alpha\beta}$ are odd but $SU(4)_L$ components $L^A_p$, $L^{Ap}$ are even. Appearance of such supertwistors with unconventional statistics of their components in the superparticle model on $AdS_5 \times S^5$ superbackground was discussed in [16] and [17].

The 1-form that defines kinetic term of the superparticle’s Lagrangian (1) can be expressed in terms of the introduced supertwistors

$$ p_m E^m(d) + p_I E^I(d) = \frac{1}{2} \left( \tilde{Z}^\mathcal{L}_A dZ^A_N - d\tilde{Z}^\mathcal{L}_A Z^A_N \right) \Gamma^N_\mathcal{L} $$

$$ = \frac{1}{2} \left( \tilde{Z}_\alpha^c dZ_{\alpha}^C - d\tilde{Z}_\alpha^c Z_{\alpha}^C \right) + \frac{1}{2} \left( \tilde{Z}_{\alpha\beta} dZ_{\alpha\beta}^{Ac} - d\tilde{Z}_{\alpha\beta} Z_{\alpha\beta}^{Ac} \right) $$

$$ + \frac{1}{2} \left( \tilde{\Psi}^q_A d\Psi^A_q - d\tilde{\Psi}^q_A \Psi^A_q \right) - \frac{1}{2} \left( \tilde{\Psi}_{Aq} d\Psi_{Aq} - d\tilde{\Psi}_{Aq} \Psi_{Aq} \right) $$

\[ \text{(25)} \]

with the diagonal supermatrix

$$ \Gamma^N_\mathcal{L} = \begin{pmatrix} \gamma^{(0)\nu} & 0 \\ 0 & -\gamma^{(5)N}_L \end{pmatrix} = \begin{pmatrix} -\delta^c_b & 0 & 0 \\ 0 & \delta^p_b & 0 \\ 0 & 0 & -\delta^q_p \end{pmatrix} $$

\[ \text{(26)} \]

In its upper-diagonal block there is $\gamma^{(0)\nu}$ matrix that, as discussed in [19], not just breaks $SU(2,2)_R$ symmetry but actually 'switches' it to the $SU(4)_R$ symmetry. Similarly $\gamma^{(5)N}_L$ ‘switches’ $SU(4)_R$ symmetry to the $SU(2,2)_R$ one. So the supertwistor 1-form (25) is invariant under ‘twisted’ $SU(2,2|4)_R$ symmetry with $SU(4)_R$ parameters in the upper-diagonal block and $SU(2,2)_R$ parameters in the lower-diagonal block. This hints at another definition of the dual twistor supermatrix

$$ \tilde{Z}^\mathcal{N}_A = \Gamma^N_\mathcal{L} \tilde{Z}^\mathcal{L}_A = \tilde{\mathcal{H}}^N_\mathcal{L} (Z^B_L)^\dagger \mathcal{H}^B_A, $$

\[ \text{(27)} \]

where

$$ \tilde{\mathcal{H}}^N_\mathcal{L} = \begin{pmatrix} \delta^\nu_\lambda & 0 \\ 0 & -\gamma^{(5)N}_L \end{pmatrix} $$

\[ \text{(28)} \]

resulting in the following expression for the 1-form (25)

$$ \frac{1}{2} \left( \tilde{Z}^\mathcal{N}_A dZ^A_N - d\tilde{Z}^\mathcal{N}_A Z^A_N \right). $$

\[ \text{(29)} \]

This ‘twisted’ $SU(2,2|4)_R$ symmetry will, however, be broken by the supertwistor constraints that we discuss below. So let us write kinetic term of the superparticle’s Lagrangian in the form with manifest $(SU(2)_R)^4$ invariance

$$ \mathcal{L}_{\text{kin}} = \frac{1}{2} (\tilde{Z}_{\alpha}^c \dot{Z}_{\alpha}^c - \dot{\tilde{Z}}_{\alpha}^c Z_{\alpha}^c) + \frac{1}{2} (\tilde{Z}_{\alpha\beta}^c \dot{Z}_{\alpha\beta}^{Ac} - \dot{\tilde{Z}}_{\alpha\beta}^{Ac} Z_{\alpha\beta}^{Ac}) $$

$$ + \frac{1}{2} (\dot{\tilde{\Psi}}^q_A \dot{\Psi}^A_q - \tilde{\Psi}^q_A \Psi^A_q) - \frac{1}{2} (\dot{\tilde{\Psi}}_{Aq} \dot{\Psi}_{Aq} - \tilde{\Psi}_{Aq} \Psi_{Aq}) $$

\[ \text{(30)} \]

As is common supertwistors are constrained dynamical variables. The constraints are contained in the $SU(2,2|4)_L$-invariant supermatrix

$$ \tilde{Z}^\mathcal{L}_A Z^A_N. $$

\[ \text{(31)} \]
It is possible to decompose it on the constituents irreducible under all four $SU(2)_c$ subgroups of $SU(2,2|4)_R$ and calculate their Dirac bracket (D.B.) relations using the basic D.B. relations that follow from (30)

\[
\{Z^A_a, Z^B_b\}_{D.B.} = i\delta^b_a \delta^A_B, \quad \{Z^{A\dot{a}}, Z^{B\dot{b}}\}_{D.B.} = i\delta^\dot{b}_\dot{a} \delta^{A\dot{a}}_B, \\
\{\Psi^a_q, \Psi^p_\dot{b}\}_{D.B.} = i\delta^q_p \delta^a_\dot{b}, \quad \{\Psi^{A\dot{a}}, \Psi^{B\dot{b}}_q\}_{D.B.} = -i\delta^\dot{b}_\dot{a} \delta^{A\dot{a}}_B.
\]

(32)

In such a way one identifies 15 bosonic

\[
L^{a\dot{b}} = \bar{Z}^a_A Z^b_B - \frac{1}{2} \delta^a_b \bar{Z}^c_A Z^c_A \approx 0, \quad M^{\dot{a}\dot{b}} = \bar{Z}^{\dot{a}_A} Z^{\dot{b}_B} - \frac{1}{2} \delta^{\dot{a}}_{\dot{b}} \bar{Z}^{\dot{c}_A} Z^{\dot{c}_A} \approx 0,
\]

(33)

\[
R^q_p = \bar{\Psi}^q_A \Psi^A_p - \frac{1}{2} \delta^q_p \bar{\Psi}^r_A \Psi^r_A \approx 0, \quad S^{\dot{q}\dot{p}} = \bar{\Psi}^{\dot{q}_A} \Psi^{\dot{p}_A} - \frac{1}{2} \delta^{\dot{q}}_{\dot{p}} \bar{\Psi}^{\dot{r}_A} \Psi^{\dot{r}_A} \approx 0,
\]

(34)

\[
C + \bar{E} = \bar{Z}^c_c \bar{Z}^c + \bar{\Psi}^r_r \Psi^r - \bar{\Psi}^r_r \Psi^r \approx 0,
\]

(35)

\[
T = \bar{Z}^c_c \bar{Z}^c + \bar{\Psi}^r_r \Psi^r - \bar{\Psi}^r_r \Psi^r \approx 0,
\]

(36)

\[
U = \bar{Z}^c_c \bar{Z}^c - \bar{\Psi}^r_r \Psi^r + \bar{\Psi}^r_r \Psi^r \approx 0
\]

(37)

and 16 fermionic first-class constraints

\[
\bar{Z}^a_A \Psi^A_q \approx 0, \quad \bar{\Psi}^A_q \Psi^A_q \approx 0, \quad \bar{Z}^{\dot{a}_A} \Psi^{\dot{A}\dot{a}}_q \approx 0, \quad \bar{\Psi}^{\dot{A}\dot{a}}_q \Psi^{\dot{A}\dot{a}}_q \approx 0.
\]

(38)

The constraint (35) upon substitution of the incidence relations for supertwistors (23) and (24) equals the square root of the constraint (17), while (36) and (37) are two extra sought for constraints needed to reduce the number of independent components of the $Spin(1,4)$ and $Spin(5)$ variables to that of the null momentum in 10 dimensions. Besides that there are 16 bosonic

\[
\bar{Z}^a_A \bar{Z}^b_B \approx 0, \quad \bar{Z}^{\dot{a}_A} \bar{Z}^{\dot{b}_A} \approx 0, \quad \bar{\Psi}^q_A \Psi^{\dot{A}\dot{a}}_q \approx 0, \quad \Psi^{\dot{A}\dot{a}}_q \Psi^a_q \approx 0
\]

(39)

and 16 fermionic second-class constraints

\[
\bar{Z}^{\dot{a}_A} \Psi^{\dot{A}\dot{a}}_q \approx 0, \quad \bar{\Psi}^{\dot{A}\dot{a}}_q \bar{Z}^{\dot{a}_A} \approx 0, \quad \bar{Z}^{\dot{a}_A} \Psi^{\dot{A}\dot{a}}_q \approx 0, \quad \bar{\Psi}^{\dot{A}\dot{a}}_q \bar{Z}^{\dot{a}_A} \approx 0.
\]

(40)

Thus the number of the physical degrees of freedom in the eight-supertwistor formulation of the superparticle precisely matches that in the superspace formulation. The first-class constraints can be added to the Lagrangian with appropriate Lagrange multipliers so that the complete Lagrangian in the eight-supertwistor formulation equals

\[
\mathcal{L}_{\text{8-twistor}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{constr}},
\]

(41)

where expression for the kinetic term is given in (30) and

\[
\mathcal{L}_{\text{constr}} = \Lambda^a_{\dot{a}} L^{a\dot{a}} + \Lambda^{\dot{a}}_{\dot{a}} M^{a\dot{a}} + \Lambda^p_q R^{a\dot{a}}_p + \Lambda^{\dot{p}}_q S^{a\dot{a}}_p + \Lambda^{C+E}(C+E) + \Lambda^T T + \Lambda^U U + i\Lambda^p_q \bar{\Psi}^{a\dot{a}}_q \bar{Z}^{a\dot{a}}_p + i\Lambda^{\dot{p}}_q \bar{\Psi}^{\dot{a}_A} \bar{Z}^{\dot{a}_A} \Psi^{\dot{a}_A} q + i\Lambda^{\dot{a}}_{\dot{a}} \bar{\Psi}^{\dot{a}_A} \bar{Z}^{\dot{a}_A} \Psi^{\dot{a}_A} q + i\Lambda^{\dot{a}}_{\dot{a}} \bar{\Psi}^{\dot{a}_A} \bar{Z}^{\dot{a}_A} \Psi^{\dot{a}_A} q.
\]

(42)

3. Massless superparticle on $AdS_5 \times S^5$ superbackground: four-supertwistor formulation

The fact that in the eight-supertwistor formulation of the massless superparticle model there are 32 second-class constraints essentially hampers further Hamiltonian analysis, requiring introduction of the D.B. for these constraints. Since the presence of the second-class constraints signals redundancy of the degrees of freedom of the model, it is possible to remove some of them
that will simplify the algebra of the remaining constraints. The form of the incidence relations (23) and (24) shows that reduction of the Spin(1, 4) and Spin(5) variables will lead to reduction of the supertwistors. In the case of the Spin(1, 4) variables (3) using the constraints (8) and (35)-(37) one can show that in (2) contributions of the first and the second summands equal so that one can exclude, e.g. spinor variables with dotted SU(2) indices

\[ p_{m'} = -\frac{1}{2} v^b_{\alpha} m'_{\beta} v^b_{\beta}, \]  \hspace{1cm} (43)

where emerging factor of two has been absorbed in the definition of remaining Spin(1, 4) variables. Reduction of the Spin(5) variables proceeds analogously. Resulting expression for the momentum components tangent to \( S^5 \) is

\[ p_{I'} = -\frac{1}{2} p^q_B I'^a A q^A. \]  \hspace{1cm} (44)

Expressions for the momentum components (43) and (44) were the starting point to derive the four-supertwistor formulation of the massless superparticle on \( AdS_5 \times S^5 \) superbackground in [18]. Resulting four-supertwistor representation of the superparticle’s Lagrangian is

\[ \mathcal{L}_{4-\text{stwistor}} = \frac{1}{2} \left( Z^a_A \dot{Z}^a_A - \dot{Z}^a_A Z^a_A \right) + \frac{1}{2} (\bar{\Psi}^q_A \dot{\Psi}^q_A - \dot{\Psi}^q_A \Psi^q_A) \]
\[ + \Lambda^q_a E^q_{\alpha} + \Lambda^q_a E^q_{\beta} + \Lambda (\bar{Z}^a_A Z^a_A + \bar{\Psi}^q_A \Psi^q_A) \]
\[ + i \Lambda^q_a \bar{\Psi}^q_A Z^a_A + i \bar{\Lambda}^q_a \bar{Z}^a_A \Psi^q_A. \]  \hspace{1cm} (45)

Lagrangian (45) coincides with that obtained in Ref. [17] by partial gauge fixing 2T superparticle model in 2+10 dimensions. In our approach it became possible also to find the incidence relations for the supertwistors

\[ Z^a_A = G^A_B \begin{pmatrix} v^\beta_a \\ 0 \end{pmatrix} = \begin{pmatrix} Z^\alpha_a \\ \eta^a_{x_A} \end{pmatrix}, \quad \bar{Z}^a_A = (Z^a_A)^{\dagger} H^B_A = (\bar{Z}^a_A \bar{\eta}^a_{x_A}) \]  \hspace{1cm} (46)

and

\[ \Psi^q_A = G^A_B \begin{pmatrix} 0 \\ \epsilon^q_{x_A} \end{pmatrix} = \begin{pmatrix} \xi^a_{x_A} \\ \bar{L}^A_{q} \end{pmatrix}, \quad \bar{\Psi}^q_A = (\Psi^q_A)^{\dagger} H^B_A = (\bar{\xi}^q_{x_A} \bar{L}^A_{A}). \]  \hspace{1cm} (47)

that satisfy seven bosonic

\[ L^a_b = \bar{Z}^a_A Z^b_A - \frac{1}{2} \delta^a_c \bar{Z}^c_A Z^a_A \approx 0, \quad R^q_p = \Psi^q_A \Psi^p_A - \frac{1}{2} \delta^q_p \Psi^r_A \Psi^r_A \approx 0, \quad \bar{Z}^a_A Z^a_A + \bar{\Psi}^q_A \Psi^q_A \approx 0 \]  \hspace{1cm} (48)

and eight fermionic constraints

\[ \bar{\Psi}^q_A Z^a_A \approx 0, \quad \bar{Z}^a_A \Psi^q_A \approx 0 \]  \hspace{1cm} (49)

generating the superalgebra of the \( SU(2|2)_R \) color supergroup, whose notion was introduced in [16]. These constraints are the generators of the gauge symmetries of the superparticle’s action based on the Lagrangian (45) discussed in [18].

The fact that the constraints and the Lagrangian (45) are quadratic in supertwistors clears the way to the Dirac quantization of the model. In our previous work [18] we considered the simplest case when the superparticle propagates only within the \( AdS_5 \) subspace of the \( AdS_5 \times S^5 \) superspace, i.e. its momentum components in directions tangent to \( S^5 \) vanish. In supertwistor formulation this means that \( \Psi^q_A \) and \( \bar{\Psi}^q_A \) vanish so that only the c-type supertwistors \( Z^a_A \) and \( \bar{Z}^a_A \) remain as dynamical variables. It was demonstrated in [18] that the states of quantized superparticle in this case coincide with the \( D = 5 \) \( N = 8 \) gauged supergravity multiplet [28] both in the supertwistor and superoscillator approaches. Next task is to take into account \( \Psi^q_A \) and \( \bar{\Psi}^q_A \) supertwistors that corresponds to the case when superparticle moves both within the \( AdS_5 \) and \( S^5 \) subspaces of the \( AdS_5 \times S^5 \) superspace.
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