Recovering hidden signals of statistical anisotropy from a masked or partial CMB sky

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Based on arXiv:1506.00550
Isotropy violation: Eg. Dipole anisotropy

Figure: Left: CMB anisotropies as observed by NASA’s COBE satellite, in Mollweide projection. Right: High-\(l\) effects of exaggerated Doppler boost on CMB (PLANCK2013-XXVII).
Isotropy violation: Eg. Doppler Boost

**Figure:** Doppler boost signal recovered in PLANCK2013-XXVII, using high-$l$ correlation from the multipole range $l = [500, 2000]$. 
Dipole modulation of CMB temperature anisotropies:

\[ \Delta T(\hat{n}) = \Delta T_{\text{iso}}(\hat{n})(1 + A\hat{\lambda} \cdot \hat{n}). \]

**Figure:** Y. Akrami et al., 2014, ApJ, 784, L42
CMB Foreground : spatial extent

Figure: Commander estimated CMB and three foreground components from PLANCK 2015 data release.
Figure: Variation of some of the foreground component levels in the frequency range in which WMAP and PLANCK made their observations.
Figure: Contours of foreground level in WMAP data. **Masking is necessary** to prevent leakage of bias due to foreground contamination in estimating quantities of interest.
BipoSH

\[ T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi). \]  

(1)

Two-point correlation function:

\[ C(\hat{n}, \hat{n}') = \langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle. \]  

(2)

If CMB is isotropic then \( C(\hat{n}, \hat{n}') \rightarrow C(\Theta = \hat{n} \cdot \hat{n}') \), otherwise

\[ C(\hat{n}_1, \hat{n}_2) = \langle \Delta T(\hat{n}_1)\Delta T(\hat{n}_2) \rangle \]  

(3)

\[ A_{l_1 l_2}^{LM} = \int d\Omega_1 d\Omega_2 C(\hat{n}_1, \hat{n}_2)\{ Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2) \}^*_{LM} \]  

(4)

Bipolar Spherical Harmonic (BipoSH) coefficients:

\[ A_{l_1 l_2}^{LM} = \sum_{m_1 m_2} a_{l_1 m_1} a_{l_2 m_2} C_{l_1 m_1 l_2 m_2}^{LM}. \]  

(5)
Effect of velocity boost on CMB

Doppler boost = dipole + aberration × freq. dep. modulation

\[ \Delta T(\hat{n}) = T_0 \vec{\beta} \cdot \hat{n} + \Delta T' \left( \hat{n} - \nabla (\vec{\beta} \cdot \hat{n}) \right) \left( 1 + b_\nu \vec{\beta} \cdot \hat{n} \right), \quad (6) \]

where
\[ \vec{\beta} = \frac{\vec{v}}{c}, \]
\[ \Delta T = \text{boosted temperature anisotropies}, \]
\[ \Delta T' = \text{unboosted temperature anisotropies}, \]
\[ b_\nu = \frac{\nu}{\nu_0} \coth \left( \frac{\nu}{2\nu_0} \right) - 1 \text{ and}, \]
\[ \nu_0 = \frac{k_B T_{cmb}}{h}. \]

The BipoSH coefficients of Doppler boosted CMB anisotropies upto \( O(\beta) \) are given by

\[ A_{l_1 l_1}^{LM} = (A_{l_1 l_2}^{LM})^{iso. \ space \ cmb} + \beta_{LM} H_{l_1 l_2}^L. \quad (7) \]
Here $H_{l_1l_2}^L$ is a function describing Doppler boost effects in BipoSH space.

For $L > 0$, one can define a \textit{minimum variance estimator} as

$$\beta_{LM} = \sum_{l_1l_2} w_{l_1l_2}^L \frac{A_{l_1l_2}^{LM}}{H_{l_1l_1}^L}, \quad (8)$$

with weights (that add up to ‘one’) and reconstruction noise given by

$$w_{l_1l_2}^L = \frac{(H_{l_1l_2}^L)^2}{C_{l_1} C_{l_2}} \left[ \sum_{l_1l_2} \frac{(H_{l_1l_2}^L)^2}{C_{l_1} C_{l_2}} \right]^{-1}, \quad (9)$$

$$N_L = \left[ \sum_{l_1l_2} \frac{(H_{l_1l_2}^L)^2}{2 C_{l_1} C_{l_2}} \right]^{-1}, \quad (10)$$
**Figure:** Doppler signal recovery from full sky estimator from simulated full sky Doppler boosted CMB maps generated using CoNIGS [S. Mukherjee S. & T. Souradeep (2014)] with nominal PLANCK noise level for 217 GHz.
**Figure:** Common analysis mask to exclude regions with significant foreground contamination used in Planck 2015 data.
Masked sky BipoSH’s in terms of full sky BipoSH:

\[ \tilde{A}_{LM}^{l_1l_2} = \sum_{l_3l_4} \frac{\prod_{l_3} \prod_{l_4}}{\sqrt{4\pi}} \sum_{l_5l_6} \frac{\prod_{l_5} \prod_{l_6}}{\sqrt{4\pi}} C_{l_30l_50}^{l_10} C_{l_40l_60}^{l_20} \]

\[ \times \sum_{L'M'JK} \left\{ \begin{array}{ccc} L & l_1 & l_2 \\ L' & l_3 & l_4 \\ J & l_5 & l_6 \end{array} \right\} \]

\[ \times \prod_{L'} \prod_{J} A_{l_3l_4}^{L'M'} W_{l_5l_6}^{JK} C_{L'M'JK}^{LM} , \quad (11) \]

This is a generalization of the pseudo-C\_l MASTER algorithm of Hivon et al. (2002) relating masked and full sky C\_l:

\[ \tilde{C}_l = \sum_{l''} M_{l''l'} C_{l''} , \quad (12) \]

since \[ A_{l''l'}^{00} \propto C_l \delta_{l''l'} . \]
BipoSH due to Doppler boosted CMB anisotropies from a masked sky:

\[ \tilde{A}_{LM}^{l_1 l_2} = \left( \tilde{A}_{LM}^{l_1 l_2} \right)^{iso.cmb} + \sum_{L'M'} \beta_{L'M'} K_{LM}^{L'M'} K_{LM l_1 l_2} \] \quad (13)

Further simplifications:

- The modified shape function (MSF), \( K_{LM}^{L'M'} \), is diagonal i.e., \( M = M' \)
- There is no significant leakage from intrinsic Doppler signal \( L' = 1 \) to \( L \neq 1 \)

Under these approximations: \( L = L' (=1) \) and \( M = M' \) we have

\[ \tilde{A}_{LM}^{l_1 l_2} = \left( \tilde{A}_{LM}^{l_1 l_2} \right)^{iso.cmb} + \beta_{LM} K_{LM}^{LM} K_{LM l_1 l_2} \] \quad (14)
With these approximations we can define an estimator for masked sky as

\[ \hat{\beta}_{LM} = \sum_{l_1 l_2} \hat{w}_{l_1 l_2}^L \frac{\hat{A}_{l_1 l_2}^{LM}}{K_{LM}^{LM} l_1 l_2}, \] (15)

where

\[ \hat{w}_{l_1 l_2}^L = \frac{1}{\sum_M \frac{\hat{\sigma}_{l_1 l_2}^{LM}}{(K_{LM}^{LM} l_1 l_2)^2} \left[ \sum_{l_1' l_2'} \frac{1}{\hat{\sigma}_{l_1' l_2'}^{LM}} (K_{LM}^{LM} l_1' l_2') \right]^{-1}}, \] (16)

\[ \hat{A}_{l_1 l_2}^{LM} = \tilde{A}_{l_1 l_2}^{LM} - \langle \tilde{A}_{l_1 l_2}^{LM} \rangle_{iso. cmb}, \] (17)

\[ \hat{\sigma}_{l_1 l_2}^{LM} = \langle |\tilde{A}_{l_1 l_2}^{LM}|^2 \rangle_{iso. cmb} - |\langle \tilde{A}_{l_1 l_2}^{LM} \rangle_{iso. cmb}|^2. \] (18)

▶ One more simplification: BipoSH covariance is diagonal

\[ \langle A_{l_1 l_2}^{LM} A_{l_1' l_2'}^{LM*} \rangle \rightarrow \left( \delta_{l_1 l_2}^{LM} \right)^2 \delta_{l_1' l_1} \delta_{l_2' l_2}^{LM}. \]
Justification - 1:

**Figure:** MSF of the apodized common analysis mask used in Planck 2015 data.
BipoSH Doppler rec. - Masked sky (contd.)

Justification - 2:

Figure: Validating the consistency of the relation
\[ \tilde{A}_{l1}^{LM} - (\tilde{A}_{l1}^{LM})^{iso.cmb} = \beta_{LM} K_{LM}^{l1}, \]
between MSF in diagonal approximation and mask bias corrected anisotropic BipoSH coefficients.
BipoSH Doppler rec. - Masked sky (contd.)

Justification - 3 :

Figure: Validating the diagonal approximation to the covariance of bias corrected BipoSH coefficients: \( \langle A_{l_1, l_2}^{LM} A_{l_1', l_2'}^{LM*} \rangle \rightarrow (\hat{\sigma}_{l_1 l_2}^{LM})^2 \delta_{l_1' l_1} \delta_{l_2' l_2} \).

Normalized BipoSH covariance matrix is defined as,

\[
R_{ll'} = \frac{C_{ll'}}{\sqrt{|C_{ll'}| |C_{l'l'|}}}
\]

(19)

\[
C_{ll'} = \langle A_{l+1}^{1M} A_{l'+1}^{1M*} \rangle
\]

(20)
A novel approach to reconstructing signals of isotropy violation from a masked CMB sky

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Statistical isotropy (SI) is one of the fundamental assumptions made in cosmological model building. This assumption is now being rigorously tested using the almost full sky measurements of the CMB anisotropies. A major hurdle in any such analysis is to handle the large biases induced due to the process of masking. We have developed a new method of analysis, using the bipolar spherical harmonic basis functions, in which we semi-analytically evaluate the modifications to SI violation induced by the mask. The method developed here is generic and can be potentially used to search for any arbitrary form of SI violation. We specifically demonstrate the working of this method by recovering the Doppler boost signal from a set of simulated, masked CMB skies.

PACS numbers: 98.70.Vc, 98.80.Es

**Figure:** arXiv:1506.00550
Results

Figure: Doppler signal estimated from masked simulated Doppler boosted CMB skies, from different multipole bin windows.
Application to data : PLANCK 2015 I&S

Planck 2015 results. XVI. Isotropy and statistics of the CMB

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Figure: Planck 2015 results. XVI. Isotropy and statistics of the CMB - arXiv:1506.07135
Application to PLANCK 2015 data (contd.)

(a) Low−l dipole modulation

(b) Doppler boost
Application to PLANCK 2015 data (contd.)

(c) Low-$l$ dipole modulation

(d) Doppler boost

Table 24. Amplitude ($A$) and direction of the dipole modulation in Galactic coordinates as estimated for the multipole range $\ell \in [2, 64]$ using a BipoSH analysis. The measured values of the dipole amplitude and direction are consistent for all maps.

| Method    | $A$       | Direction $(l, b)$ [$^\circ$] |
|-----------|-----------|-------------------------------|
| Commander | 0.067 ± 0.023 | (230, −18) ± 31              |
| NILC      | 0.069 ± 0.022 | (228, −17) ± 30              |
| SEVEM     | 0.067 ± 0.023 | (230, −17) ± 31              |
| SMICA     | 0.069 ± 0.022 | (228, −18) ± 30              |

Table 25. The Doppler boost amplitude ($|\beta|$) and direction in Galactic coordinates derived over the multipole range $\ell \in [640, 1024]$ as evaluated from a BipoSH analysis. The errors are estimated from an identical analysis of a set of 1000 Doppler boosted simulations for each frequency.

| Map       | $|\beta| \times 10^{-3}$ | Direction $(l, b)$ [$^\circ$] |
|-----------|--------------------------|-------------------------------|
| SEVEM-100 | 1.24 ± 0.66              | (277, 40) ± 50               |
| SEVEM-143 | 1.35 ± 0.56              | (264, 39) ± 39               |
| SEVEM-217 | 1.28 ± 0.45              | (257, 42) ± 32               |
Acknowledgements

▶ We acknowledge the use of NASA’s WMAP satellite data made available at LAMBDA site, and ESA’s PLANCK satellite data available at PLA web page.

▶ We acknowledge the use of HEALPix package created for representing/manipulating data on a sphere for CMB analysis.

▶ Some of the images are taken from NASA’s WMAP, and ESA’s PLANCK mission pages.
Thank You

For your patience