Motivated by the emerging possibilities to study threshold pion electroproduction at large momentum transfers at Jefferson Laboratory following the 12 GeV upgrade, we provide a short theory summary and an estimate of the nucleon axial form factor for large virtualities in the $Q^2 = 1 - 10$ GeV$^2$ range using next-to-leading order light-cone sum rules.

Keywords: QCD, nucleon form factors, axial current, light-cone sum rules

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In our opinion such measurements would be very interesting and the task of this note is to provide one with the corresponding QCD expectations. To this end we present a calculation of the nucleon axial form factor for photon virtualities in the $Q^2 = 1 - 10$ GeV$^2$ range using next-to-leading order (NLO) light-cone sum rules.

II. LIGHT-CONE SUM RULES

It is generally accepted that hadron form factors in the formal $Q^2 \to \infty$ limit are dominated by hard gluon exchanges between the valence quarks at small transverse separations. However, there is overwhelming evidence that the hard rescattering regime is not achieved for realistic momentum transfer accessible at modern accelerators and the so-called “soft” or Feynman-type contributions play the dominant role. Soft contributions can be estimated using the light-cone sum rule (LCSR) technique that is based on the light-cone operator product expansion of suitable correlation functions combined with dispersion relations and quark-hadron duality. This technique is attractive because it can be applied to all elastic and transition form factors and involves the same universal non-perturbative functions that enter the pQCD calculation; there is no double counting and (almost) no new parameters.

The LCSRs for the electromagnetic nucleon form factors have been derived in [10, 11] to the leading order (LO) and recently in [12] to the NLO in the QCD coupling. For the axial form factor only the LO results are available [11, 13]. It turns out, however, that the NLO LCSRs for the axial form factor do not require a new calculation and can be obtained using the expressions presented in [12] with minor modifications.

The starting point is the correlation function

$$T_{\mu\nu}(p, q) = i \int d^4x e^{ipx} \langle 0| T\{\eta(0)j_{\mu\nu}(x)\}|P(p)\rangle,$$  

(1)

where $|P(p)\rangle$ is the proton state with momentum $p_{\mu}, p^2 = m^2$, and $\eta(x)$ is a suitable local operator with proton quantum numbers (Ioffe current). The corresponding coupling is $\langle 0|\eta(0)\langle P(p)\rangle = \lambda_1 m u(p)$. For technical reasons it is more convenient to consider the neutral axial vector current

$$j_{\mu\nu} = \frac{1}{2} [\bar{u}\gamma_{\mu\nu}u - \bar{d}\gamma_{\mu\nu}d].$$  

(2)

Following [10] we consider the “plus” projection of the correlation function in the Lorentz and spinor indices which can be parametrized by two invariant functions:

$$\Lambda_5 T_{55} = p_+ \left[ m A_5(Q^2, p^2) + g_5 B_5(Q^2, p^2) \right] \gamma_5 u_+(p),$$  

(3)

where $p' = p + q$. The invariant functions can be calculated for large Euclidean momenta $Q^2, -p^2 \ll A_{QCD}^2$ using the light-cone OPE. The results can be written in the form of a dispersion integral

$$A_5^{QCD}(Q^2, p^2) = \frac{1}{\pi} \int_0^\infty \frac{ds}{s - p^2} \text{Im} A_5^{QCD}(Q^2, s) + \ldots$$  

(4)

where $\text{Im} A_5^{QCD}(Q^2, s)$ is given by the convolution of perturbatively calculable coefficient functions $C_5^T$ and the matrix elements of three-quark operators at light-like separations, $F(x, \mu_F)$, dubbed distribution amplitudes (DAs),

$$\text{Im} A_5^{QCD} = \sum_{\mathcal{F}} C_5^T(x, Q^2, s, \mu_F, \alpha_s(\mu_F)) \otimes F(x, \mu_F).$$  

(5)

The sum goes over all existing DAs of increasing twist, $x = \{x_1, x_2, x_3\}$ stands for the quark momentum fractions and $\mu_F$ is the factorization scale.

Leading-order (LO) expressions are available from [11], see Eq. (A.7). For consistency with our NLO calculation we expand all kinematic factors in the LO results in powers of $m^2/Q^2$ and neglect terms $O(m^4/Q^4)$. This truncation is also consistent with taking into account contributions of twist-three, -four, -five (and, partially, twist-six) in the OPE. The NLO expressions for $A_5$ can be obtained from the results in [12] (see Appendix E) with the following replacements:

- For the $d$-quark contribution, replace $e_d \to 1/2$.
- For the $u$-quark contribution, replace $e_u \to 1/2$ and interchange symmetric and antisymmetric parts of the DAs: $V_1 \leftrightarrow -A_1, V_2 \leftrightarrow A_2, V_3 \leftrightarrow A_3$.

The sum rules are constructed by matching the QCD representation [4] to the dispersion representation in terms of hadronic states

$$A_5^{QCD}(Q^2, p^2) = \frac{2\lambda_1 G_A(Q^2)}{m^2 - p^2}$$

$$+ \frac{1}{\pi} \int_0^\infty \frac{ds}{s - p^2} \text{Im} A_5^{QCD}(Q^2, s) + \ldots$$  

(6)

where it is assumed that contributions of nucleon resonances and scattering states are effectively taken into account by the QCD expression above a certain threshold $s_0 \simeq (1.5\text{ GeV})^2$ (interval of duality). Applying the Borel transformation $p^2 \to M^2$ to get rid of the subtraction constants and suppress higher-mass contributions, one obtains the LCSR [10, 11]

$$2\lambda_1 G_A(Q^2) = \frac{1}{\pi} \int_0^{s_0} ds e^{(m^2 - s)/M^2} \text{Im} A_5^{QCD}(Q^2, s)$$  

(7)

that we analyze in what follows [14].

III. RESULTS

The results are shown in Fig. 1 where on the left panel we plot $G_A(Q^2)$ in absolute normalization and on the right panel the ratio of $G_A(Q^2)$ to the dipole formula [4] with the axial mass $M_A = 1.069$ GeV corresponding to the average value [4] from the pion electroproduction measurements [1].

The LCSR for the axial form factor in Eq. (7) does not contain free parameters. The results are shown for two realistic models of the leading- and higher-twist nucleon DAs, ABO1 (solid curves) and ABO2 (dashed curves), defined in in Table I of Ref. [12]. These models have been obtained by combining...
the available lattice QCD constraints [15, 16] with the fit to the electromagnetic proton form factors, $F_1(Q^2)$ and $F_2(Q^2)$, see Fig. 3 in [12]. The NLO corrections that are the subject of this work are large and positive (up to 40%) at $Q^2 = 1 - 2$ GeV$^2$ but decrease (to below 15%) at larger momentum transfers and change sign at $Q^2 \sim 6$ GeV$^2$ for the both DA models. We show the results starting at $Q^2 > 1$ GeV$^2$. However, the experience with the LCSRs for $B$-decay and nucleon electromagnetic form factors indicates that momentum transfers in the 1-2 GeV$^2$ range are still too low for a fully quantitative treatment in this approach.

A compilation of the low-$Q^2$ measurements can be found in [1] (see also [3]). For the neutrino scattering, in order not to overload the plot we show the standard dipole parametrization with the axial mass $M_A = 1.026(21)$ GeV by the narrow shaded area, and, in addition, by a broader shaded area extending to $Q^2 = 4$ GeV$^2$, the one-sigma envelope from the recent analysis using a more general $z$-parametrization that also includes newer deuterium data [3]. For the same reason we do not show "old" electroproduction data except for [17] in the range $Q^2 = 0.45 - 0.88$ GeV$^2$. The three shown sets of data points correspond to the form factor extraction using the strict soft-pion limit (filled triangles) and two models for the hard pion corrections (open triangles). The recent CLAS data [4] are shown by filled squares. These results were obtained by employing the low-energy theorem in the chiral limit and extracting the $E_{0+}$ multipole from the fit to the total cross section $\gamma^*p \to \pi^+n$ at the energy $W = 1.11$ GeV, closest to the threshold. Our predictions for the large $Q^2$ region match the existing neutrino scattering data at smaller momentum transfers [3] very well, and are about 20-30% below the CLAS extraction from pion electroproduction in the soft pion limit [4]. Since the corrections to the soft pion limit are expected to be negative [8, 9, 17] and can well be in the 20% range, there is no contradiction. A more detailed analysis of such corrections within realistic models would be very welcome.

To summarize, we argue that studies of pion electroproduction at threshold $\gamma^*p \to \pi^+n$ at large photon virtualities accessible at the Jefferson Laboratory following the 12 GeV upgrade supplemented by the measurements of the neutron magnetic form factor in the same $Q^2$ range provide one with a viable method to determine the axial proton form factors with the theoretical accuracy that is currently limited to 20 - 30% but, very likely, can be improved in the future. These results can be confronted with QCD predictions based on LCSRs and, potentially, lattice QCD (e.g. [18, 19]) and Dyson-Schwinger equations [20] although the extension to the large-$Q^2$ region in both approaches can be challenging. A combination of lattice calculations with models can offer additional insights, e.g. [21].

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