INFLATION VERSUS THE
COSMOLOGICAL MODULI PROBLEM

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Abstract

We show that in generic supergravity theories the mass of the moduli during inflation is larger (or at least of the same order of magnitude) than the Hubble constant. This fact does not depends on the details of the inflation and on the value of the Hubble parameter during it. The reason is that inflationary universe is dominated by large $F$-term (or $D$-term) density which is higher than the SUSY breaking scale in the present minimum and stabilizes the flat directions of the supersymmetric vacua. Therefore, in general even standard inflationary scenarios (with large $H$) may solve the cosmological moduli problem.

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Supergravity and superstring theories often include scalar fields with the weak scale \((M_W)\) masses and Planck scale \((M_P)\) suppressed interactions. Such fields have no supersymmetric mass term and/or renormalizable couplings in the superpotential and in the supersymmetric limit they parameterize the flat directions in the SUSY vacuum. Their masses are induced by the same mechanism that breaks supersymmetry and stabilizes flat directions. Since the message about the SUSY breaking is carried by Planck scale suppressed interactions, the resulting masses are of the order

\[
m^2 \sim \frac{|F_i|^2}{M_P^2}
\]  

(1)

where \(F_i\) are the expectation values of the F-terms that break supersymmetry. Since the same mechanism is responsible for the transfer of SUSY breaking to the visible (quark and lepton) sector the value of \(F_i\) has to be of order \(M_W M_P\) resulting in the weak scale mass. Independently from their origin, such fields having only Planck scale couplings and weak scale (non-supersymmetric) mass we will call collectively as moduli fields and will denote them by \(Z\). It is well known that in the cosmological context moduli exhibit serious problems. The cosmological evolution of the moduli have been analyzed in great detail before [1]. The key point of the problem is that in the early universe the expectation values of the moduli are expected to be far (at distance \(Z_0 \sim M_P\)) from the minimum (due to thermal or quantum fluctuations) and so one has a Bose condensate, which later starts oscillate about the minimum and contributes to much energy density to the mass of the universe. After late decay of this cold bosons the temperature of the
universe is too low for the standard nucleosynthesis.

It is assumed usually [1-4], that it is very difficult to eliminate moduli Bose condensate (reduce $Z_0$) by inflation [5], unless during inflation the Hubble constant ($H$) is of order $M_W$ (mass of the moduli at the minimum). (Alternative would be enormous number of e-foldings). Therefore in [3] the inflation with weak scale $H$ was proposed as a possible (and perhaps the only promising [4]) solution to the moduli problem. However, such late inflation in general has problems in explaining the large density fluctuations.

In the present note we show that for eliminating the initial moduli condensate by inflation such a small $H$ is not necessary, because moduli acquire large ($\sim H$) mass during inflation. By definition the moduli has a mass $\sim M_W$ “today” (in the true vacuum with zero cosmological constant), but there was no reason for this mass to be the same during inflation. In fact the opposite is true: in the generic supergravity theories the mass of the moduli during inflation is larger (or $\sim$) than the Hubble parameter. This has to do with the fact that any inflation brakes supersymmetry [6], since it provides a positive (and large) cosmological constant and thus nonzero F-term density (which automatically gives large mass to moduli). This mass is related to the value of the Hubble constant and as a matter of fact is larger (or at least of the same order of magnitude). This result is not surprising, since there is no generic reason why the flat directions of SUSY vacua may survive in the state with large vacuum energy and it is expectable that corresponding zero modes (moduli) will get masses of order the vacuum energy density ($\sim |F|_i|^2$) suppressed by the scale of messenger interaction ($M^2_P$).

To be more precise consider generic supergravity scalar potential:
\begin{equation}
V = \exp\left(\frac{K}{M^2}\right)[K_j^{i-1}F_iF^* j - 3\frac{|W|^2}{M^2}]
\end{equation}

where \(K(S_i, S_i^*)\) is a Kahler potential, \(W(S_i)\) is superpotential and \(S_i\) are chiral superfields (their scalar components we denote by the same symbols). \(F_i\)-terms are given by \(F_i = W_i + \frac{W}{M^2}K_i\) where upper (lower) index denotes derivative with respect to \(S_i\) \((S_i^*)\) respectively and \(M^2 = \frac{M^2}{8\pi}\) is a reduced Planck mass. (We neglect possible \(D\)-terms and assume that they vanish during inflation).

Let us start with a simplest case in which moduli is decoupled from the visible sector superfields both in Kahler potential and superpotential. For definiteness let us denote moduli field by \(Z\) and assume simplest possible dependence:

\begin{equation}
K = |Z|^2 + K'(S_i, S_i^*)
\end{equation}

Where \(K'\) and \(W\) are arbitrary functions independent of \(Z\). Now it is easy to see that whatever the quantities \(K', W\) are, in any state with zero or positive cosmological constant the only minimum (and even extremum) in \(Z\) is at \(Z = 0\) and it’s mass is given by:

\begin{equation}
m_Z^2 = \frac{e^{K'/M^2}}{M^2} [K_j^{i-1}F_iF^* j - 2\frac{|W|^2}{M^2}]
\end{equation}

In particular, in the state with vanishing vacuum energy and SUSY breaking scale \(F_i \sim M_W M\) this relation implies \(m_Z \sim M_W\).

Now let us consider how inflation affects this situation. Here we do not want to advocate any particular inflationary scenario and/or discuss whether such can be implemented in the supergravity framework. Our aim is to
point out some model-independent consequences of the inflation if it happens in some way in above system. The basic idea of inflationary scenario [5] is that for some time universe has to stay in the state with the positive (and large) cosmological constant \( V > 0 \) in order to undergo a period of exponential expansion. This state may or may not be a local minimum of the potential, but essential condition is that fields roll slowly enough and the Hubble constant is given by

\[
H^2 = \frac{V}{3M^2} \tag{5}
\]

From above we have learned two important things about the moduli behavior during (any) inflation:

1) Classical expectation value of \( Z \) is trapped in the minimum at \( Z = 0 \);
2) As it can be easily seen from (2),(4) and (5), the mass of the moduli is larger than the Hubble constant:

\[
m_Z^2 = 3H^2 + \frac{|W|^2}{M^4} e^{\frac{k'}{M^2}} \tag{6}
\]

Thus, there is no need to assume that \( H \sim M_W \) since (6) is valid for any \( H \). For the case \( H < m_Z \) reduction of the initial expectation value \( Z_0 \) of moduli goes through the factor \( e^{\frac{k'}{M^2}N} \) where \( N \) is the number of e-foldings since the beginning of inflation. Therefore, we expect that any inflation which can be implemented in the supergravity framework may dilute the coherent condensate and solve the moduli problem.

Now let us consider the situation in which moduli has some Planck scale suppressed couplings with the visible sector superfields in the superpotential:
\[ W' = \frac{1}{M^{n-2}} Z S_1 S_2 ... S_n + ... \]  

(7)

Clearly, whatever the origin of the \( S_i \) fields is, the following condition should be satisfied:

(*) In the phenomenological minimum with zero cosmological constant (present vacuum), in each \( n \)-linear invariant only \( n - 4 \) fields are allowed to have large (Planck scale) VEVs.

Otherwise \( Z \) will acquire large effective coupling and will not be “moduli” any more. In the case of GUT Higgs fields this requirement can be restricted by operators up to \( n = 11 \) or so, since the highers will be suppressed by large powers of \( M_{GUT}/M \).

Assuming that the minimum in \( Z \) during inflation is still at \( Z = 0 \), we are lead to the following correction to the right-hand side of (6)

\[ \delta m_z^2 = e^{\frac{K'}{M^2}} |W'_{zz}|^2 + K'_{ij} F_{iz} F^{*jz} - \frac{|W'_{z}|^2}{M^2} \]  

(8)

where lower (upper) index \( z \) denotes derivative with respect to \( z(z^*) \).

There are two model dependent possibilities:

a) If condition (*) is satisfied during inflation, then it is clear from (8) that positive correction to (6) is at best of order \( M^4/W^4 \) and the negative one is \( \sim M^6/W^4 \) and therefore relation (6) between \( m_z \) and \( H \) is valid with great accuracy.

b) In some models the condition (*) can be violated during inflation. This may be the case if some \( S \)-fields in (7) are coupled to the inflaton and get large (\( \sim M \)) displacement of the VEVs. In such a case the corrections of
the both sign in (8) can be of order $M^2$, but it would be very unnatural to
expect that they cancel each other and (6) in such a way that resulting mass
of the moduli is of order $M_{W}$. It is even more unnatural that such accidental
“fine tuning” may hold along the full inflationary trajectory. Therefore, we
conclude that in general moduli mass is of order (and in many cases larger)
the Hubble constant and therefore even standard inflation (with large $H$) can
be sufficient to solve the problem. Of course, in particular cases there can be
additional model-dependent factors (e.g. reheating temperature) which may
need some treatment and can not be discussed here.

At the end let us ask what may happen if the minimum in $Z$ is displaced
(by $Z_0 \sim M$) from the origin during inflation? Again, this is very model
dependent situation, but due to above arguments we expect that in this
“inflationary” minimum the moduli mass again will be $\sim H$. However, now
this may not be enough to solve the problem, since finally this displacement
has to disappear. If this happens while inflation is still going on and $m_z$
is large, problem may be avoided. However, if displacement occurs at the
very end, the problem will be recreated, since $Z$ will start in the same initial
condition as in the usual case without inflation.

References

1. G.Coughlan, W.Fischler, E.Kolb, S.Raby and G.Ross, Phys.Lett., B131
   (1983) 59; J.Ellis, D.V.Nanopoulos and M.Quiros, Phys.Lett., B174
   (1986) 176; G.German and G.G.Ross, Phys.Lett., B172 (1986) 305;
O.Bertolami, Phys.Lett B209 (1988) 277; R. de Carlos, J.A.Casas,
F. Quevedo and E. Roulet, *Phys. Lett.* **B318** (1993) 447.

2. T. Banks, D. Kaplan and A. Nelson *Phys. Rev., D49* (1994) 779.

3. L. Randall and S. Thomas, MIT preprint, LMU-TPW-94-17, hep-ph 9407248.

4. T. Banks, M. Berkooz and P. J. Steinhardt, preprint RU-94-92.

5. A. H. Guth, *Phys. Rev* **D23** (1981) 347; For the review see A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Switzerland, 1990); E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990);

6. G. Dvali, Q. Shafi and R. Schaefer, *Phys. Rev. Lett.* **73** (1994) 1886.