The Hilbert-Smith Conjecture

by

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Dedicated to the memory of Deane Montgomery

Abstract

The Hilbert-Smith Conjecture states that if $G$ is a locally compact group which acts effectively on a connected manifold as a topological transformation group, then $G$ is a Lie group. A rather straightforward proof of this conjecture is given. The motivation is work of Cernavskii (“Finite-to-one mappings of manifolds”, Trans. of Math. Sk. 65 (107), 1964.) His work is generalized to the orbit map of an effective action of a $p$-adic group on compact connected $n$-manifolds with the aid of some new ideas. There is no attempt to use Smith Theory even though there may be similarities.

It is well known that if a locally compact group acts effectively on a connected $n$-manifold $M$ and $G$ is not a Lie group, then there is a subgroup $H$ of $G$ isomorphic to a $p$-adic group $A_p$ which acts effectively on $M$. It can be shown that $A_p$ can not act effectively on an $n$-manifold and, hence, The Hilbert Smith Conjecture is true. The existence of a non empty fixed point set adds some complexity to the proof. In this paper, it is shown that $A_p$ can not act freely on a compact connected $n$-manifold. The basic ideas for the general case are more clearly seen in this case. The general proof will be given in another paper.

1. Introduction.

In 1900, Hilbert proposed twenty-three problems [8]. For an excellent discussion concerning these problems, see the Proceedings of Symposia In Pure Mathematics concerning “Mathematical Developments Arising From Hilbert Problems” [3]. The abstract by C.T.
Yang [22] gives a review of Hilbert’s Fifth Problem “How is Lie’s concept of continuous groups of transformations of manifolds approachable in our investigation without the assumption of differentiability?” Work of von Neumann [40] in 1933 showed that differentiability is not completely dispensable. This with results of Pontryagin [35] in 1939 suggested the specialized version of Hilbert’s problem: *If G is a topological group and a topological manifold, then is G topologically isomorphic to a Lie group?* This is generally regarded as Hilbert’s Fifth Problem. The first partial result was given by Brouwer [26] in 1909-1910 for locally euclidean groups of dimension \( \leq 2 \). The best known partial results were given for compact locally euclidean groups and for commutative locally euclidean groups by von Neumann [40] and Pontryagin [35], respectively.

In 1952, the work of Gleason [5] and Montgomery-Zippin [13] proved: *Every locally euclidean group is a Lie group.* This solved Hilbert’s Fifth Problem.

A more general version of Hilbert’s Fifth Problem is the following:

*If G is a locally compact group which acts effectively on a connected manifold as a topological transformation group, then is G a Lie group?* The Hilbert-Smith Conjecture states that the answer is yes.

Papers of Montgomery [34] in 1945 and Bochner-Montgomery [1] in 1946 established the partial result: *Let G be a locally compact group which acts effectively on a differentiable manifold M such that for any \( g \in G \), \( x \mapsto gx \) is a differentiable transformation of M. Then G is a Lie group and \((G, M)\) is a differentiable transformation group.* Another partial result was given by a theorem of Yamabe [43] and a Theorem of Newman [15] as follows: *If G is a compact group which acts effectively on a manifold and every element of G is of finite order, then G is a finite group.*

It has been shown [14] that an affirmative answer to the generalized version of Hilbert’s Fifth Problem is equivalent to a negative answer to the following: *Does there exist an effective action of a p-adic group on a manifold?*
It can be proved that the answer to this question is No. However, in this paper it is shown that there is no free action of a $p$-adic group on a connected compact $n$-manifold. The existence of a non empty fixed point set complicates the argument even though the techniques used are the same as in the free case. The proof is more easily understood in the free case. The general case is not as difficult to follow once the free case is understood. This will follow in another paper.

A brief review of some of the consequences of efforts to solve this problem is given below. There are examples in the literature of effective actions of an infinite compact 0-dimensional topological group $G$ (each $g \in G - \{\text{identity}\}$ moves some point) on locally connected continua. The classic example of Kolmogoroff [29] in 1937, is one where $G$ operates effectively but not strongly effectively [24] on a 1-dimensional locally connected continuum (Peano continuum) such that the orbit space is 2-dimensional. In 1957, R.D. Anderson [24] proved that any compact 0-dimensional topological group $G$ can act strongly effectively as a transformation group on the (Menger) universal 1-dimensional curve $M$ such that either (1) the orbit space is homeomorphic to $M$ or (2) the orbit space is homeomorphic to a regular curve.

In 1960, C.T. Yang [44] proved that if a $p$-adic group, $A_p$, acts effectively as a transformation group on $X$ (a locally compact Hausdorff space of homology dimension not greater than $n$), then the homology dimension of the orbit space $X/A_p$ is not greater than $n + 3$. If $X$ is an $n$-manifold, then the homology dimension of $X/A_p$ is $n + 2$. If $A_p$ acts strongly effectively (freely) on an $n$-manifold $X$, then the dimension of $X/A_p$ is either $n + 2$ or infinity. At about the same time (1961), Bredon, Raymond, and Williams [25] proved the same results using different methods. There are, of course, actions by $p$-adic groups on $p$-adic solenoids and actions by $p$-adic solenoids on certain spaces. See [25] for some of these results.

In 1961, Frank Raymond published the results of his study of the orbit space $M/A_p$ assuming an effective action by $A_p$ (as a transformation group) on an $n$-manifold $M$. 
Later (1967), Raymond [38] published work on two problems in the theory of generalized manifolds which are related to the (generalized) Hilbert Fifth Problem.

In 1963, Raymond and Williams [39] gave examples of compact metric spaces $X^n$ of dimension $n$ and an action by a $p$-adic group, $A_p$, on $X^n$ such that $\dim X^n/A_p = n + 2$. Work related to and used in [39] is the paper [41] by Williams. In [41], Williams answers a question of Anderson [24; p. 799] by giving a free action by a compact 0-dimensional group $G$ on a 1-dimensional Peano continuum $P$ with $\dim P/G = 2$.

In 1976, I described [32; 33] what I called $p$-adic polyhedra which admit periodic homeomorphisms of period $p$. Proper inverse systems $\{P_i, \phi_i\}$ of $p$-adic $n_i$-polyhedra have the property that the inverse limit $X = \lim\leftarrow P_i$ admits a free action by a $p$-adic group.

In 1980, one of my students, Alan J. Coppola [28] generalized results of C.T. Yang [44] which involve homologically analyzing $p$-adic actions. Coppola formalized these so that homological calculations could be done in a more algorithmic manner. He defined a $p$-adic transfer homomorphism and used it to produce all of the relevant Smith-Yang exact sequences which are used to homologically analyze $\mathbb{Z}_p$-actions on compact metric spaces. Coppola studied $p$-adic actions on homologically uncomplicated spaces. In particular, he proved that if $X$ is a compact metric $A_p$-space of homological dimension no greater than $n$ and $X$ is homologically locally connected, then the $(n + 3)$-homology of any closed subset $A \subset X/A_p$ vanishes.

In 1983, Robinson and I proved Newman’s Theorem for finite-to-one open and closed mappings on manifolds [10]. We formalized Newman’s Property (and variations) and studied this property for discrete open and closed mappings on generalized continua in 1984 [11].

In 1985, H-T Ku, M-C Ku, and Larry Mann investigated in [30] the connections between Newman’s Theorem involving the size of orbits of group actions on manifolds and
the Hilbert-Smith Conjecture. They establish Newman’s Theorem (Newman’s Property [11]) for actions of compact connected non-Lie groups such as the $p$-adic solenoid.

In 1997, D. Repovš and E.V. Ščepin [51] gave a proof of the Hilbert-Smith Conjecture for actions by Lipschitz maps. See also related work by Shchepin [52]. In the same year, Iozke Maleshick [53] proved the Hilbert-Smith conjecture for Hölder actions.

In 1999, Gaven J. Martin [54] announced a proof of The Hilbert-Smith Conjecture for quasiconformal actions on Riemannian manifolds and related spaces.

The crucial idea that works here is M.H.A. Newman’s idea used in his proof that for a given compact connected $n$-manifold $M$, there is an $\epsilon > 0$ such that if $h$ is any periodic homeomorphism of period $p$, a prime $> 1$, of $M$ onto itself, then there is some $x \in M$ such that the orbit of $x$, \( \{x, h(x), \ldots, h^{p-1}(x)\} \), has diameter $\geq \epsilon$. It is well known that the collection of orbits under the action of a transformation group $G$ on a compact Hausdorff space $X$ is a continuous decomposition of $X$.

The works [20; 21] of David Wilson and John Walsh [18] show that there exist continuous decompositions of $n$-manifolds $M^n$, $n \geq 3$, into Cantor sets. This paper shows that such decompositions cannot be equivalent to those induced by any action of a $p$-adic transformation group $A_p$ acting on $M^n$.

I owe a special debt of gratitude to Patricia Tulley McAuley who has been extremely helpful in reading drafts of numerous attempts to solve this problem and who has provided helpful insights with regard to Čech homology. Also, I am indebted to the work of Cernavskii [27].

**OUTLINE OF A PROOF**

It is well known that if a locally compact group $G$ acts effectively on a connected $n$-manifold $M$ and $G$ is not a Lie group, then there is a subgroup $H$ of $G$ isomorphic to a $p$-adic group $A_p$ which acts effectively on $M$. Thus, the Hilbert-Smith Conjecture can be established by proving that there is no effective action by a $p$-adic group $A_p$ on a
It is proved that if \( L(M,p) \), \( p \) is a prime greater than 1, is the class of all orbit mappings \( \phi : M \rightarrow M/A_p \) where \( A_p \) acts freely on a compact connected \( n \)-manifold \( M \), then \( M \) has Newman’s Property w.r.t. \( L(M,p) \). That is, there is \( \epsilon > 0 \) such that for any \( \phi \in L(M,p) \), there is some \( x \in M \) such that \( \text{diam} \ \phi^{-1}(x) \geq \epsilon \) (using the metric on \( M \)). This yields a contradiction to the well known fact that if \( L(M,p) \neq \emptyset \), then for given \( \epsilon > 0 \), there is \( \phi \in L(M,p) \) such that \( \text{diam} \ \phi^{-1}(x) < \epsilon \) for any \( x \in M \).

Lemma 2. (A consequence of a Theorem of Floyd [4].) Suppose that \( M \) is a compact connected \( n \)-manifold. There is a finite open covering \( W_1 \) of \( M \) such that (1) order \( W_1 = n + 1 \) and (2) there is a finite open refinement \( W_2 \) of \( W_1 \) which covers \( M \) such that if \( W \) is any finite open covering of \( M \) refining \( W_2 \), then \( \pi_{W_1} : \tilde{H}_n(M) \rightarrow H_n(W_1) \) maps \( \tilde{H}_n(M) \) isomorphically onto the image of the projection \( \pi_{W_1} : H_n(W) \rightarrow H_n(W_1) \).

[Here, if \( U \) is either a finite or locally finite open covering of \( M \), then \( H_n(U) \) is the \( n \)-th simplicial homology group of the nerve \( N(U) \) of \( U \). The coefficient group is always \( \mathbb{Z}_p \) and \( \tilde{H}_n(M) \) denotes the \( n \)-th Čech homology of \( M \).]

Now, choose \( U = W_1 \) and a finite open covering \( W \) of \( M \) which star refines \( W_2 \) where \( W_1 \) and \( W_2 \) satisfy Lemma 2.

Let \( \epsilon \) be the Lebesgue number of \( W_2 \). Choose \( \phi \in L(M,p) \) such that \( \text{diam} \ \phi^{-1}(x) < \epsilon \) for each \( x \in M \). Construct the special coverings \( \{V^m\} \) and the special refinements \( \{U^m\} \) as in Lemma 4 below where \( V^1 \) star refines \( W_2 \) such that order \( U^m = n + 1 \) and with projections \( \alpha_m, \beta_m, \) and \( \pi_m \) yielding the following commutative diagram:

\[
\begin{array}{ccccccc}
\leftarrow & H_n(V^m) & \xrightarrow{\nu_m} & H_n(U^m) & \xrightarrow{\nu_{m+1}} & H_n(V^{m+1}) & \xrightarrow{\nu_{m+1}} & H_n(U^{m+1}) & \leftarrow \\
\uparrow \nu_m & \beta_m & \downarrow \alpha_m & \uparrow \nu_{m+1} & \beta_{m+1} & \downarrow \alpha_{m+1} \\
\leftarrow & H_n(V_n^m) & \xrightarrow{\pi_m} & H_n(V_n^{m+1}) & \xrightarrow{\pi_{m+1}} & H_n(V_n^{m+2}) & \leftarrow \\
\end{array}
\]

Here \( \nu_m \) is the natural map of an \( n \)-cycle in \( H_n(V_n^m) \), the \( n \)-th simplicial homology group of the \( n \)-skeleton of the nerve \( N(V^m) \) of \( V^m \), into its homology class in \( H_n(V^m) \). The other
maps are those induced by the projections $\alpha_m$, $\beta_m$ and $\pi_m$. The upper sequence, of course, yields the Čech homology group $\check{H}_n(M)$ as its inverse limit. Furthermore, it can be easily shown, using the diagram, that $\check{H}_n(M)$ is isomorphic to the inverse limit $G = \varprojlim H_n(V^m_n)$, of the lower sequence. Specifically, $\gamma : \check{H}_n(M) \to G$ defined by $\gamma(\Delta) = \{\beta^*_m(\pi U_m(\Delta))\}$ is an isomorphism of $\check{H}_n(M)$ onto $G$. We shall use the isomorphism in what follows and for convenience we shall let $\gamma(\Delta) = \{z^n_m(\Delta)\}$, i.e. $z^n_m(\Delta) = \beta^*_m(\pi U_m(\Delta)) \in H_n(V^m_n)$. The group $\varprojlim H_n(V^m_n)$ is used because the operator $\sigma$ (introduced later) is applied to actual $n$-cycles rather than to elements of a homology class. This is the reason for the sequence $\{U^m\}$. Note that there is no attempt to use Smith Theory.

An operator $\sigma_m$ is defined on the $n$-chains of $N(V^{m+1})$ for each $m$. The operator $\sigma_m$ maps $n$-cycles to $n$-cycles and commutes with the projections $\pi^*_m : H_n(V^{m+1}_m) \to H_n(V^m_n)$ and, hence, induces an automorphism on $\pi_{V^{m+1}_m}(\check{H}_n(M)) \subset H_n(V^{m+1}_n)$. See Lemmas 5 and 6.

In the general case, it is shown in Lemma 10 that $\text{int} F_\phi = \emptyset$ where $F_\phi = \{x \mid \phi^{-1} \phi(x) = x\}$. Here, $F_\phi = \emptyset$.

Distinguished families of $n$-simplices in $N(V^m)$ are defined. Now, let $z_m = z^n_m(\Delta)$ where $\Delta$ is the generator of $G \cong \mathbb{Z}_p$. For each $n$-simplex $\delta^n$ in $z_m(\Delta)$, there is a unique distinguished family $S^m_j$ of $n$-simplices in $N(V^m)$ which contains $\delta^n$. If $C_j$ is the collection of all $n$-simplices in $z_m(\Delta)$ which are in $S^m_j$, then the sum of the coefficients of those members of $C_j$ (as they appear in $z_m(\Delta)$) is $0 \mod p$. Take the projection $\pi_{V^m U}$ from $V^m$ to $U = W_1$. Hence, $\pi_{V^m U}$ has the property that all members of a distinguished family $S^m_j$ of $n$-simplices in $N(V^m)$ project to the same simplex in $N(W_1)$. Thus, the projection of those members of $z_m(\Delta)$ which are in $S^m_j$ project to the same simplex $\delta_j$ in $N(W_1)$ and the coefficient of $\delta_j$ is $0 \mod p$. Thus, $\pi_{V^m U} : H_n(V^m_n) \to H_n(U)$ takes the nontrivial $n$-cycle $z_m(\Delta)$ to the $0-n$ cycle $\mod p$. This violates the conclusion of Lemma 2. Thus, $M$ has Newman’s Property w.r.t. the class $L(M, p)$. Hence, $\epsilon$ is a Newman’s number.
It is well known that if $A_p$ acts effectively (freely, in this case) on a compact connected $n$-manifold $M$, then given any $\epsilon > 0$, there is a free action of $A_p$ on $M$ such that $\text{diam} \phi^{-1}\phi(x) < \epsilon$ for each $x \in M$. That is, $M$ fails to have Newman’s property w.r.t. the class $L(M, p)$. It follows that $A_p$ can not act freely on a compact connected $n$-manifold $M$.

Details of the proof follow.

2. Some properties of the orbit mapping of an effective action by $A_p$ on a compact connected orientable $n$-manifold $M$.

Suppose that $\phi$ is the orbit mapping of a $p$-adic group $A_p$ acting as a transformation group on a connected compact $n$-manifold $M^n = M$ where $p$ is a prime larger than 1. By [12; 21], there is a sequence $A_p = H_0 \supset H_1 \supset H_2 \supset \ldots$ of open (and closed) subgroups of $A_p$ which closes down on the identity $e$ of $A_p$ such that when $j > i$, $H_i/H_j$ is a cyclic group of order $p^{j-i}$. Let $h_{ij} : A_p/H_j \to A_p/H_i$ and $h_i : A_p \to A_p/H_i$ be homomorphisms induced by the inclusion homomorphisms (quotient homomorphisms) on $A_p$ and $A_p/H_j$. Then $\{A_p/H_i; h_{ij}\}$ is an inverse system and $\{h_i\}$ gives an isomorphism of $A_p$ onto $\lim_{\leftarrow} A_p/H_i$. Now, let $a \in A_p - H_i$. For each natural number $i$, let $a_i$ be the coset $aH_i$ in $A_p/H_i$. Then $a_i$ is a periodic homeomorphism of $M/H_i$ onto $M/H_i$ with $a_i^q$ being the identity mapping where $q = p^i$ is the period of $a_i$. Consequently, $H_i$ acts as a transformation group on $M$ and $A_p/H_i$ acts as a cyclic transformation group on $M/H_i$.

As above, let $\{H_i\}$ be a sequence of open (and closed) subgroups of $A_p$ such that (a) $H_i \supset H_{i+1}$ for each $i$, (b) if $j \geq i$, then $H_i/H_j$ is a cyclic group of order $p^{j-i}$, and (c) $A_p/H_i$ is a cyclic group of order $p^i$. Since $A_p$ acts effectively on $M$ (a compact connected $n$-manifold), the cyclic group $A_p/H_i$ acts effectively on $M/H_i$ with orbit space $M/A_p$.

Let $\pi_{ij} : M/H_j \to M/H_i$ where $j > i$ and $\pi_i : M \to M/H_i$ be maps induced by the identity map of $M$. Also, $\pi_i = \pi_{ji} \circ \pi_j$. Thus, $\{M/H_i : \pi_{ij}\}$ is an inverse system and $\{\pi_i\}$ gives a homeomorphism of $M$ onto the inverse limit $M/H_i$ ($\lim M/H_i$). Notice that
$H_i/H_j$, a cyclic group of order $p^{j-i}$, acts on $M/H_j$ with orbit space $M/H_i$ [cf. 23; p. 211] where \( \pi_i = \pi_{ji} \circ \pi_j \) and \( \pi_{ji} : M/H_j \to M/H_i \) is the orbit map of the action of $H_i/H_j$ on $M/H_j$. Notice that $M/H_j$ is the orbit space of the action of $H_j$ on $M$ where $H_j$ is isomorphic to $A_p$. It follows that if $\epsilon > 0$, then there is a natural number $j$ such that $H_j \cong A_p$ acts (effectively) on $M$ such that if $\phi_j : M \to M/H_j$ is the orbit map of the action, then $\text{diam } \phi^{-1}\phi(x) < \epsilon$ for each $x \in M$. Observe that if $H$ is a non trivial open and closed subgroup of $A_p$, then for some $i$, $H = H_i$.

The following lemma is crucial to defining certain coverings of $M$ with distinguished families of open sets.

Lemma 1. Suppose that $\phi$ is the orbit mapping $\phi : M \to M/A_p$ where $A_p$ acts effectively (freely, in this paper) on a compact connected $n$-manifold $M$. For each $z \in M/A_p$ and $\epsilon > 0$, there is a connected set $U$ such that (1) $\text{diam } U < \epsilon$, (2) $z \in U$, and (3) $\phi^{-1}(U) = \{U_1, U_2, \ldots, U_p\}$ where $s$ is a natural number such that (a) $U_i$ is a component of $\phi^{-1}(U)$ for each $i$, $1 \leq i \leq p^s$, (b) $\bar{U}_i \cap \bar{U}_j = \emptyset$ for $i \neq j$, (c) $\phi(U_i) = U$ for each $i$, and (d) $U_1$ is homeomorphic to $U_j$ for each $j$, $1 < j \leq p^s$ (by maps compatible with the projections $\phi | U_j$). [The homeomorphism taking $U_1$ to $U_j$ is a power of a fixed element $g \in A_p - H_1$ and is used in Lemma 5.]

Proof. Since $z \in M/A_p$, $\phi^{-1}(z)$ is non degenerate. For $\epsilon > 0$, there is a connected open set $U$ such that (1) $\text{diam } U < \epsilon$, (2) $z \in U$, and (3) $\phi^{-1}(U)$ consists of a finite number (larger than one) of components $U_1, U_2, \ldots, U_m$ such that $\bar{U}_i \cap \bar{U}_j = \emptyset$ for $i \neq j$, and $\phi(U_i) = U$ for each $i$. This follows by Whyburn’s Theory of open mappings [19, pp. 78-80].

For each $U_j$, a component of $\phi^{-1}(U)$, there is an open and closed subgroup $G_j$ of $A_p$ which is the largest subgroup of $A_p$ which leaves $U_j$ invariant and the map induced by $\phi$ maps $U_j/G_j$ onto $U$. Since $G_j$ is a normal subgroup of $A_p$, $G_i = G_j$ for each $i$ and $j$. Furthermore, $A_p/G_j$ is a cyclic group of order $p^s$ where $s$ is a natural number. There are $p^s$ pairwise distinct components of $\phi^{-1}(U)$. (See [36: Lemma 2]). It follows that $G_1 = H_i$ for some $i$ where \( \{H_i\} \) is the sequence of open and closed subgroups of $A_p$. 

9
which closes down on the identity \( e \in A_p \) (mentioned above) and \( s = i \). Let \( a \in A_p - H_i \) such that \( aH_i \) generates the cyclic group \( A_p/H_i \). For each natural number \( i \), let \( a_i \) be the coset \( aH_i \) in \( A_p/H_i \). Thus, \( a_i \) is a periodic homeomorphism of \( M/H_i \) onto \( M/H_i \) with \( a_i^q = e \) where \( q = p^i \) is the period of \( a_i \).

Let \( f_i : M \rightarrow M/H_i \) be the orbit map of the action of \( H_i \) on \( M \) and \( g_i : M/H_i \rightarrow M/A_p \) be the orbit map of the action of the cyclic group \( A_p/H_i \) on \( M/H_i \). That is, \( \phi = g_if_i \). There are \( p^i \) cosets \( \{v_mH_i\}_{m=1}^{p^i} \) where \( v_1 = e \) (the identity) such that for each \( x \in U_1 \), \( \phi^{-1}(x) = \bigcup_{m=1}^{p^i} v_mH_i(x) \) where \( v_mH_i(x) = \{h(x) \mid h \in v_mH_i\} \), (2) \( v_mH_i(x) \in U_m/H_i \), (3) \( v_m \) is an orientation preserving homeomorphism of \( M \) onto \( M \), and (4) if \( A_p/H_i = \{a_i, a_i^1, a_i^2, \ldots, a_i^{p_i-1}\} \) (a cyclic group), then there are elements \( k_1, k_2, \ldots, k_{p_1} \) where \( k_1 = e \) such that (a) \( k_m(\pi_i(x)) = \pi_i(v_mH_i(x)) \) where \( \pi_i \) maps \( \phi^{-1}(U) \) onto \( \phi^{-1}(U)/H_i \) and (b) \( k_m \) maps \( U_1/H_i \) homeomorphically onto \( U_m/H_i \) with \( k_m = v_mH_i \in A_p/H_i \) which is a homeomorphism of \( M/H_i \) onto \( M/H_i \). Thus, \( v_m(x) \in U_m \). Let \( z \in U_m \). Hence, \( \pi_i(z) \in U_m/H_i \) and \( (v_mH_i)^{-1}(\pi_i(z)) = v_m^{-1}H_i(\pi_i(z)) \in U_1/H_i \). Consequently, \( v_m^{-1}H_i(\pi_i(z)) = \pi_i(v_m^{-1}(z)) \in U_1/H_i \) which implies that \( v_m^{-1}(z) \in U_1 \). Finally, \( v_m(v_m^{-1}(z)) = z \) and \( v_m \) maps \( U_1 \) homeomorphically onto \( U_m \).

Lemma 1 is proved.

3. Special coverings and distinguished families.

Let \( L(M,p) = \{\phi \mid \phi \) is the orbit mapping of a free action of a \( p \)-adic group \( A_p \) \( (p \) a prime with \( p > 1 \)) on a compact connected metric \( n \)-manifold without boundary, \( \phi : M \rightarrow M/A_p \} \). It would simplify the proof of lemmas which follow to know that \( M \) is triangulable. Without this knowledge, a theorem of E.E. Floyd is used.

**Notation.** Throughout this paper, \( \check{H}_n(X) \) will denote the \( n^{th} \) Čech homology group of \( X \) with coefficients in \( Z_p \), the integers mod \( p \), \( p \) a fixed prime larger than 1. Also, \( H_n(K) \) will denote the \( n^{th} \) simplicial homology of a finite simplicial complex \( K \), with coefficients
in $Z_p$. If $U$ is a finite open covering of a space $X$, then $N(U)$ denotes the nerve of $U$, $H_n(U)$ is the $n^{th}$ simplicial homology group of $N(U)$, and $\pi_U$ the usual projection homomorphism $\pi_U : \tilde{H}_n(X) \to H_n(U)$.

**Definition.** If $f$ is a mapping of $M$ onto $Y$, then an open covering $U$ of $M$ is said to be *saturated* (more precisely, saturated w.r.t. $f$) iff for each $u \in U$, $f^{-1}f(u) = u$. That is, $u$ is an open inverse set.

The next lemma follows.

**Lemma 2.** Suppose that $M$ is a compact connected metric $n$-manifold. There is a finite open covering $W_1$ of $M$ such that (1) order $W_1 = n + 1$ and (2) there is a finite open refinement $W_2$ of $W_1$ which covers $M$ such that if $W$ is any finite open covering of $M$ refining $W_2$, then $\pi_{W_1} : \tilde{H}_n(M) \to H_n(W_1)$ maps $\tilde{H}_n(M)$ isomorphically onto the image of the projection $\pi_{WW_1} : H_n(W) \to H_n(W_1)$.

**Proof.** Adapt Theorem (3.3) of [4] to the situation here and use (2.5) of [4].

If $M$ is triangulable, then there is a sufficiently fine triangulation $T$ such that if $U$ consists of the open stars of the vertices of $T$, then $\pi_U : \tilde{H}_n(M) \to H_n(U)$ is an isomorphism onto (where, of course, $\tilde{H}_n(M) \cong Z_p$).

**NOTE:** The reader can assume, for convenience, that $M$ is triangulable and that $W_1$ is the collection of open stars of a sufficiently fine triangulation $T$ such that the $n^{th}$ simplicial homology, $H_n(W_1)$, with coefficients in $Z_p$, of the nerve, $N(W_1)$, of $W_1$ is $Z_p$.

**Standing Hypothesis:** In the following, $M$ is a compact and connected metric $n$-manifold. Also, $L(M, p)$ is as defined above. The finite open coverings $W_1$ and $W_2$ which satisfy Lemma 2 will be used in certain lemmas and constructions which follow. Suppose also that $Y = M/A_p$ has a countable basis $Q = \{B_i\}_{i=1}^{\infty}$ such that (a) for each $i$, $B_i$ is connected and uniformly locally connected and (b) if $H$ is any subcollection of $Q$ and $\bigcap_{h \in H} h \neq \emptyset$, then $\bigcap_{h \in H} h$ is connected and uniformly locally connected (a consequence of a theorem due to Bing and Floyd [50]).
Lemma 3. Suppose that $\phi \in L(M,p)$. Then there exists a finite open covering $R$ of $Y = M/A_p$ such that (a) if $y \in Y$, then there is $r \in R$ such that $y \in r$, $r \in Q$ where $Q$ is the basis in The Standing Hypothesis, $\phi^{-1}(r) = r_1 \cup r_2 \cup \cdots \cup r_q$, $q = p^t$ for some natural number $t$, such that for each $i = 1,2,\ldots,q$, $r_i$ is a component of $\phi^{-1}(r)$, $r_i$ maps onto $r$ under $\phi$, $\overline{r_i} \cap \overline{r_j} = \emptyset$ for $i \neq j$, and $r_i$ is homeomorphic to $r_j$ for each $i$ and $j$ with a homeomorphism compatible with the projection $\phi$ (indeed, there is an element of $A_p$ which takes $r_i$ onto $r_j$), (b) $R$ is irreducible, (c) $V = \{ c \mid c$ is a component of $\phi^{-1}(r)$ for some $r \in R \}$ is such that $V$ star refines $W_2$, and (d) if $r_x \in R$, $r_y \in R$, $r_x \cap r_y \neq \emptyset$, $\phi^{-1}(r_x)$ consists of exactly $p^{m_x}$ components, $\phi^{-1}(r_y)$ consists of exactly $p^{m_y}$ components, and $m_x \geq m_y$, then each component of $\phi^{-1}(r_y)$ meets exactly $p^{m_x-m_y}$ components of $\phi^{-1}(r_x)$.

Proof. Since $Y$ is compact, use Lemma 1 to obtain a finite irreducible covering $R$ of $Y$ of sets $r$ satisfying the conditions of the lemma such that $R$ star refines $\{ \phi(u) \mid u \in W_2 \}$. Property (d) of the conclusion of Lemma 3 is satisfied by using the compactness of $Y$ and choosing $R$ such that each $r \in R$ has sufficiently small diameter and $r \in Q$ (the basis in The Standing Hypothesis). The lemma is established.

Lemma 3 is just the first step in establishing Lemma 4 below.

Lemma 4. There are sequences $\{V^m\}$ and $\{U^m\}$ of finite open coverings of $M$ cofinal in the collection of all open coverings of $M$ such that (1) $V^{m+1}$ star refines $U^m$, (2) $V^1$ star refines $W_2$ of Lemma 2, (3) $U^m$ star refines $V^m$, (4) order $U^m = n + 1$, (5) $V^m$ is generated by a finite open covering $R^m$ of $Y = M/A_p$, (6) $V^m$ and $R^m$ have the properties stated in Lemma 3 where $R^m$ replaces $R$ and $V^m$ replaces $V$, (7) $\{ \text{mesh } V^m \} \to 0$, and (8) there are projections $\pi_m : V^{m+1} \to V^m$ such that (a) $\pi_m = \beta_m \alpha_m$ where $\alpha_m : V^{m+1} \to U^m$ and $\beta_m : U^m \to V^m$, (b) $\pi_m$ takes each distinguished family $\{ f^{m}_{ij} \}_{j=1}^{t^m_{ij}}$ in $V^{m+1}$ (defined in a manner like those defined for $V^1$ and $V^2$ below) onto a distinguished family $\{ f^{m}_{ij} \}_{j=1}^{t^m_{ij}}$ in $V^m$, and (c) $\pi_m$ extends to a simplicial mapping (also, $\pi_m$) of $N(V^{m+1})$ into $N(V^m)$. (Also, $\alpha_m$ and $\beta_m$ denote the extensions of $\alpha_m$ and $\beta_m$.
to simplicial mappings \( \alpha_m : N(V^{m+1}) \to N(U^m) \) and \( \beta_m : N(U^m) \to N(V^m) \) where 
\( \pi_m = \beta_m \alpha_m \).

NOTE: As stated in The Standing Hypothesis, \( Y \) has a countable basis \( Q = \{ B_i \}_{i=1}^{\infty} \) such that (a) for each \( i \), \( B_i \) is connected and uniformly locally connected and (b) if \( H \) is any subcollection of \( Q \) and \( \bigcap_{h \in H} h \neq \emptyset \), then \( \bigcap_{h \in H} h \) is connected and uniformly locally connected (a consequence of a theorem due to Bing and Floyd [50]). Each \( r \in R^m \) can be chosen from \( Q \).

The proof of Lemma 4, although straightforward, is long and tedious. The existence of \( V = V^1 \) in Lemma 3 (which star refines \( W_2 \)) generated by \( R = R^1 \) is an initial step of a proof using mathematical induction. Additional first steps are described below. These should help make it clear how the induction is completed to obtain a proof of Lemma 4.

Call the collection \( \{r_1, r_2, \cdots, r_q\} \) consisting of all components of \( \phi^{-1}(r) \) a distinguished family in \( V \) (defined in Lemma 3) generated by \( r \in R \) where \( R \) satisfies Lemma 3. The finite open covering \( V \) can be partitioned into the subcollections \( \{r_1, r_2, \cdots, r_q\} \) consisting of the components of \( \phi^{-1}(r) \) for some \( r \in R \) which are defined to be distinguished families generated by \( r \in R \). Thus, \( V \) is generated by \( R = R^1 \).

Let \( V^1 = V \). Clearly, \( V^1 \) star refines \( W_2 \). Note that order \( V^1 \) may be larger than \( n + 1 \) since if \( \phi \) is the orbit mapping of an effective action by a \( p \)-adic transformation group, then \( \dim Y = n + 2 \) or \( \infty \) [22].

The covering \( V^1 = V \) is defined to be a special covering of \( M \) w.r.t. \( \phi \) generated by \( R \). Of course, \( \phi \) is fixed throughout this discussion as in the statement of Lemma 4. Observe that it follows from Lemma 1, that if \( \{f_{kj}^1\}_{j=1}^{q} \) and \( \{f_{mj}^1\}_{j=1}^{s} \) are two distinguished families (non degenerate) in \( V^1 \) such that for some \( i \) and \( t \), \( f_{ki}^1 \cap f_{mt}^1 \neq \emptyset \), then for each \( j \), the number of elements of \( \{f_{mj}^1\}_{j=1}^{s} \) which have a non empty intersection with \( f_{kj}^1 \) is a constant \( c_k \) and for each \( j \), the number of elements of \( \{f_{kj}^1\}_{j=1}^{q} \) which have a non empty intersection with \( f_{mj}^1 \) is a constant \( c_m \) where \( c_k = p^b \), \( b \geq 0 \), and \( c_m = p^d \), \( d \geq 0 \).

Construction Of \( U^1 \) Of Order \( n + 1 \) Which Refines \( V^1 \)
The reason that the sequence \( \{U^m\} \) is constructed is to prove (using the definitions of \( \alpha_m, \beta_m, \) and \( \pi_m = \beta_m\alpha_m \)) that the inverse limit of the \( n^{th} \) simplicial homology of the \( n \)-skeleta of the nerve of \( V^m \) is \( Z_p \) which permits the application of \( \sigma \) (defined below) to actual \( n \)-cycles. The operator \( \sigma \) can not be applied (as defined) to elements of a homology class.

The next step is to describe a special refinement \( U^1 \) of \( V^1 \) which has order \( n + 1 \) and other crucial properties. First, construct \( U^1_1 \). List the distinguished families of \( V^1 \) as \( F^1_1, F^1_2, \ldots, F^1_{n_1} \) where \( F^1_i = \{ f^1_{ij} \}_{j=1}^{t^1_i} \) where \( t^1_i = p^e_i \) which are non degenerate. Recall that \( f^1_{ij} \) is homeomorphic to \( f^1_{st} \) for each \( i, j, \) and \( t \) that makes sense. Since \( R^1 \) (which generates \( V^1 \)) is irreducible, it follows that if \( f^1_{ij} \in F^1_i \) and \( f^1_{st} \in F^1_s \) where \( F^1_i \) and \( F^1_s \) are distinguished families in \( V^1 \) with \( i \neq s \), then \( f^1_{ij} \) and \( f^1_{st} \) are independent, that is, \( f^1_{ij} \not\sim f^1_{st} \) and \( f^1_{st} \not\sim f^1_{ij} \). This ordering may be changed below.

For each \( i, 1 \leq i \leq n_1 \), choose a closed and connected subset \( K_i \) in \( \phi(f^1_{i1}) = r_i \in R^1 \) where \( F^1_i = \{ f^1_{ij} \}_{j=1}^{t^1_i} \) such that (1) \( K_{ij} = \phi^{-1}(K_i) \cap f^1_{ij} \), \( K_{ij} \) is homeomorphic to \( K_i \) for any \( s \) and \( j \) that makes sense, (2) \( C = \{ \text{int } K_{ij} \mid 1 \leq i \leq n_1 \text{ and } 1 \leq j \leq t^1_i \} \) covers \( M \) and \( K = \{ \text{int } K_i \mid 1 \leq i \leq n_1 \} \) covers \( M/A_\rho = Y \), and (3) \( K_{ij} \) is connected, \( 1 \leq j \leq t^1_i \). To see that this is possible, choose a closed subset \( A_i \) of \( r_i, 1 \leq i \leq n_1 \), such that \( A = \{ A_i \mid 1 \leq i \leq n_1 \} \) covers \( Y \) [47; 49]. Note that there exists a natural number \( k \) such that \( \{ A_i = r_i - N_{\frac{k}{m}}(\partial r_i) \mid 1 \leq i \leq n_1 \} \) covers \( Y \). To see this, suppose that for each \( k \), there is \( x_k \notin \bigcup_{i=1}^{n_1}(r_i - N_{\frac{k}{m}}(\partial r_i)) \). Since \( Y \) is compact, there is a subsequence \( \{ x_{n(k)} \} \) of \( \{ x_k \} \) which converges to \( x \in r_q \) for some \( q \). There is some \( m \) such that \( x \in r_q - N_{\frac{k}{m}}(\partial r_q) \) and \( x \in \text{interior } (r_q - N_{\frac{k}{m+1}}(\partial r_q)) \) which leads to a contradiction. Choose \( p_i \in r_i \) and \( m \) sufficiently large such that \( K_i \) is the component of \( r_i - N_{\frac{k}{m}}(\partial r_i) \) which contains \( p_i \). For each \( m \), let \( C_m(p_i) \) be the component of \( r_i - N_{\frac{k}{m}}(\partial r_i) \) which contains a fixed \( p_i \in r_i \) (with \( m \) large enough). It will be shown that \( \bigcup_{m=1}^{\infty} C_m(p_i) = r_i \). Suppose that there is \( q \in r_i \) such that \( q \notin \bigcup_{m=1}^{\infty} C_m(p_i) \). Since \( r_i \) is uniformly locally connected and locally compact (\( \bar{r_i} \) is a
Peano continuum), there is a simple arc $p_i q$ from $p_i$ to $q$ in $r_i$. Consequently, for some $m$, $r_i - N_{\frac{1}{m}}(\partial r_i) \supset p_i q$ which is in $C_m(p_i)$. This is contrary to the assumption above. Hence, $\bigcup_{m=1}^{\infty} C_m(p_i) = r_i$. For $p_i \in r_i$ fixed as above, choose $p_{ij} \in \phi^{-1}(p_i) \cap f_{ij}^1$ for $1 \leq j \leq t_i^1$. Let $C_m^i(p_{ij})$ be the component of $\phi^{-1}(C_m(p_i))$ which is in $f_{ij}^1$ and contains $p_j$. It will be shown that $\bigcup_{m=1}^{\infty} C_m^i(p_{ij}) = f_{ij}^1$. If this is false, then there is a $q_j \in f_{ij}^1 - \bigcup_{m=1}^{\infty} C_m^i(p_{ij})$. Since $f_{ij}^1$ is ulc and locally compact, there is a simple arc $p_{ij} q_j$ from $p_{ij}$ to $q_j$ in $f_{ij}^1$.

Now, $r_i \supset \phi(p_{ij} q_j)$. For $m$ large enough, $C_m(p_i) \supset \phi(p_{ij} q_j)$ and some component of $\phi^{-1}(p_{ij} q_j)$ contains $p_{ij} q_j$ and lies in $C_m^i(p_{ij})$. This is contrary to the assumption above.

Hence, $\bigcup_{m=1}^{\infty} C_m^i(p_{ij}) = f_{ij}^1$. Choose $m$ sufficiently large that condition (2) above is satisfied where $K_i = C_m(p_i)$ and (4) int $C_m(p_i) \supset A_i$. By [19], $\phi^{-1}(C_m(p_i))$ consists of a finite number of components each of which maps onto $K_i$ under $\phi$. Choose $m$ sufficiently large that the components of $\phi^{-1}(K_i)$ are $\{K_{ij}^1\}_{j=1}^{t_i}$ and conditions (1) - (4) are satisfied.

By a Theorem (Nagami and Roberts) and its Corollary [49; pp. 90-91], a metric space $X$ has dim $X \leq n$ if and only if $X$ has a sequence $\{G_i\}$ of open coverings of $X$ such that

1. $G_{i+1}$ refines $G_i$ for each $i$,
2. order $G_i \leq n + 1$ for each $i$, and
3. mesh $G_i < \frac{1}{t_i}$.

If $X$ is locally connected, then the elements of $G_i$ can be chosen to be connected for each $i$. If $X$ is an $n$-manifold, then the elements of $G_i$ can be chosen to be connected for each $i$. If $X$ is a triangulable $n$-manifold, then it is easy to see this by using barycentric subdivisions of a triangulation of $X$. Choose such a sequence $\{G_i\}$ of open coverings of $M$ such that $G_1$ star refines $K = \{\text{int } K_i \mid 1 \leq i \leq n_1\}$.

The next step is to show how to choose some $G_i$ from which $U_1^1$ is chosen and later modified to give $U^1$ with the desired properties.

Let $\epsilon$ be the Lebesgue number of the covering $K$. Choose $t$ such that (1) mesh $G_t < \frac{1}{t}$, (2) if $g \in G_t$, then diam $\phi(g) < \frac{\epsilon}{8}$, and (3) if $y \in Y$ and $G(y) = \{g \mid g \in G_t$ and $y \in \phi(g)\}$, then there is $s$, $1 \leq s \leq n_1$, such that int $K_s \supset \bigcup_{g \in G(y)} \phi(g)$. (Note that (3) follows from the choice of $\epsilon$, $t$, and the fact that $G_t$ star refines $K$.) Choose $U_1^1 \subset G_t$
such that \(U_1^1\) is an irreducible finite covering of \(M\). Since \(G_t\) is an open covering of \(M\) with connected open sets, mesh \(G_t < d\), and order \(G_t \leq n + 1\), \(U_1^1\) has the following properties:

1. order \(U_1^1 \leq n + 1\) and \(U_1^1\) star refines \(V^1\),
2. if \(u \in U_1^1\), then \(u\) is connected, and
3. if \(y \in Y\) and \(G(y) = \{g \mid g \in U_1^1\) and \(y \in \phi(g)\}\), then there is \(s\), \(1 \leq s \leq n_1\), such that \(\text{int } K_s \supseteq \bigcup_{g \in G(y)} \overline{\phi(g)}\).

**Construction Of \(U^1\) Which Refines \(U_1^1\) In A Special Way**

For each \(y \in Y\), let \(Q(y) = \{\phi(u) \mid u \in U_1^1\) and \(y \in \phi(u)\}\). There are at most a finite number of such sets distinct from each other. Order these sets as \(Q_1, Q_2, \ldots, Q_{m_1}\) such that for \(i \neq j\), \(Q_i \neq Q_j\) and \(\text{card } Q_i \geq \text{card } Q_j\). Let \(O_i = \cap Q_i - \bigcup_{j<i} Q_j\) where \(\cap Q_i = \{x \mid x \in \phi(u)\) for each \(\phi(u) \in Q_i, 1 \leq i \leq m_1\}\). For each \(i\), \(1 \leq i \leq m_1\), let \(B_{i\phi(u)} = (\partial O_i) \cap \partial \phi(u)\) where \(\phi(u) \in Q_i\) and \((\partial O_i) \cap \partial \phi(u) \neq \emptyset\). There are at most a finite number of such non empty sets distinct from each other. Let \(B_1, B_2, \ldots, B_{m_2}\) denote all those sets distinct from each other. Let \(B = \bigcup_{i=1}^{m_2} B_i\). For each \(y \in B\), let \(D(y) = \{B_t \mid y \in B_t\}\). There are at most a finite number of such non empty sets distinct from each other. Order these as \(D_1, D_2, \ldots, D_{m_3}\) such that if \(i < j\), then \(D_i \neq D_j\) and \(\text{card } D_i \geq \text{card } D_j\). For each \(i\), \(1 \leq i \leq m_2\), there is \(u \in U_1^1\) and a closed subset \(C_{iu}\) of \(\partial u\) such that \(\phi(C_{iu}) = B_i\).

It follows from the definition that \(\phi^{-1}(B)\) is closed and contains no open set. Hence, dimension \(\phi^{-1}(B) \leq n - 1\). For each \(y \in Y\), choose \(0 < \epsilon_y < (\frac{1}{8}) \min\{\rho(y, B_i) \mid y \notin B_i, 1 \leq i \leq m_2\} \cup \{\rho(\cap D_i, \cap D_j) \mid (\cap D_i) \cap (\cap D_j) = \emptyset\}\) and cover \(B\) with a finite irreducible collection \(E\) of \(\epsilon_y\)-neighborhoods for \(y \in B\). Choose \(t\) sufficiently large such that if \(g \in G_t\) and \(\phi(g) \cap B \neq \emptyset\), then there is \(e \in E\) such that \(e \supseteq \overline{\phi(g)}\). Let \(H'\) be a finite irreducible collection of open sets which refines \(G_t\), covers \(\phi^{-1}(B)\), refines \(U_1^1\), and order \(H' \leq n\) (see [48; pp. 133-34]). In addition, choose \(H'\) such that \(H'\) can be extended to a finite
irreducible cover $G$ of $M$ (i.e., $G \supset H'$) such that order $G \leq n + 1$, $G$ refines $U_1'$, and if $g \in G - H'$, then $\bar{g} \cap \phi^{-1}(B) = \emptyset$. Now, cover $\phi^{-1}(B)$ with a finite irreducible collection $H$ of open sets such that (a) $H$ refines $H'$, (b) order $H \leq n$, (c) order $(H \cup H') \leq n + 1$ (if $N[\delta^{n-1}]$ is the nucleous of an $(n-1)$-simplex in the nerve, $N(H')$, of $H'$, then one and only one element of $H$ contains $N[\delta^{n-1}] \cap \phi^{-1}(B)$), and (d) if $O = \{g \cap \phi^{-1}(O_i) \mid g \in G$ and $1 \leq i \leq m_1\}$ (recall that $\phi^{-1}(B) \cap \phi^{-1}(O_i) = \emptyset$), then $U_1' = H \cup O$ has the following properties: (1) $U_1'$ refines $U_1'$, (2) order $U_1 = n + 1$, (3) if $(\cap D_i) \cap (\cap D_j) = \emptyset$, $u \in U_1'$, $v \in U_1'$, $\phi(u) \cap (\cap D_i) \neq \emptyset$, and $\phi(v) \cap (\cap D_j) \neq \emptyset$, then $\bar{\phi(u)} \cap \bar{\phi(v)} = \emptyset$, (4) if $B_i \cap B_j = \emptyset$, $u \in U_1'$, $v \in U_1'$, $\phi(u) \cap B_i \neq \emptyset$, and $\phi(v) \cap B_j \neq \emptyset$, then $\bar{\phi(u)} \cap \bar{\phi(v)} = \emptyset$ (a consequence of (3)), (5) if $u \in U_1'$, $\phi(u) \cap (\cap D_i) \neq \emptyset$, and $\phi(v) \cap (\cap D_j) \neq \emptyset$, then $(\cap D_i) \cap (\cap D_j) \neq \emptyset$, (6) if $u \in U_1'$ and $\phi(u) \cap B = \emptyset$ where $B = \bigcup_{j=1}^{m_2} B_j$, then $O_i \supset \phi(u)$ for some $i$, $1 \leq i \leq n_1$ (it follows that $\phi(h) \supset \phi(u)$ for all $h \in U_1'$ such that $\phi(h) \cap \phi(u) \neq \emptyset$), and (7) if $u \in U_1'$ and $\phi(u) \cap (\cap D_1) \neq \emptyset$ for the smallest $t$, then $\phi(h) \supset \phi(u)$ for all $h \in U_1'$ such that there is no $B_x \in D_t$ with the property that $\partial \phi(h) \supset B_x$ and $\phi(h) \cap \phi(u) \neq \emptyset$. Observe that if $u \in U_1'$ and $\phi(u) \cap B = \emptyset$, then for some $i$, $O_i \supset \phi(u)$. It is not difficult to see that $U_1'$ has properties (1) - (7).

Construction of $V^2$ Which Refines $U_1'$

For $y \in B \subset Y$ choose $r_y \in Q$ (the basis for $Y$ described above) such that (1) $\bigcap_{u \in Q(y)} \phi(u) \supset \bar{r}_y$, (2) if $U(y) = \{u \mid u \in U_1' \text{ (not } U_1') \text{ and } y \in \phi(u)\}$, then $\bigcap_{u \in U(y)} \phi(u) \supset \bar{r}_y$, (3) $\text{diam } r_y < \left(\frac{1}{8}\right) \min\{\rho(y, \partial \phi(v)) \mid v \in U_1' \text{ and } y \notin \partial \phi(v)\}$, (4) $y \in r_y$, (5) $\phi^{-1}(r_y) = r_{y_1} \cup r_{y_2} \cup \cdots \cup r_{y_q}$, $q = p^{t_1}$ where $t_1 \geq 1$, $r_{y_i}$ maps onto $r_y$ under $\phi$, $\bar{r}_{y_i} \cap \bar{r}_{y_j} = \emptyset$ for $i \neq j$, and $r_{y_i}$ is homeomorphic to $r_{y_j}$ for each $i$ and $j$ with a homeomorphism compatible with the projection $\phi$ (indeed, there is an element of $A_p$ which takes $r_{y_i}$ onto $r_{y_j}$), and (6) for each $i$, there is $u \in U_1'$ such that $u \supset \bar{r}_{y_i}$. See the proof of Lemma 3 for the construction. Let $R^2_1$ denote a finite irreducible collection of such sets $r_y$ which covers $B$ such that $H \supset U(y)$. If $y \in Y - (R^2_1)^*$, then choose $r_y$ satisfying (1) - (6) above.
such that $\tilde{r}_y \cap B = \emptyset$. Let $R^2_2$ denote a finite irreducible collection of such $r_y$ described above which covers $Y - (R^1_1)^\ast$. Let $R^2 = R^2_1 \cup R^2_2 = \{r_{y_1}, r_{y_2}, \ldots, r_{y_{n_2}}\}$ which is a finite irreducible covering of $Y$. Now, $V^2 = \{c \mid c$ is a component of $\phi^{-1}(r_{y_i})$ for some $i$, $1 \leq i \leq n_2\}$. Also, $V^2$ can be chosen such that $V^2$ star refines $U^1$. The collection of components, $\{f^2_{ij}\}_{j=1}^{t^2_i}$, of $\phi^{-1}(r_{y_i})$ is defined to be a distinguished family in $V^2$.

Definitions Of $U^1$, $\alpha_1$, $\beta_1$, And $\pi_1 = \beta_1 \alpha_1$

Let $F^i_1 = \{f^i_{ij}\}_{j=1}^{t^i_1}$ be any distinguished family (nondegenerate) in $V^2$, $1 \leq i \leq n_2$. By definition, $r_{y_i}$ in $R^2_i$ generates $F^2_i$.

Case (1): $y_i \in B = \bigcup_{u \in U^1_i} \partial \phi(u)$ where $r_{y_i} \in R^2_i$. Let $c_i = \min \{t \mid y_i \in \cap D_i\}$. Let $s_i = \min \{s \mid \text{int } K_s \supset \overline{\phi(u)}$ for all $u \in U^1_i$ such that $u \in G(y_i)\}$. Choose $F^1_{s_i} = \{f^1_{s_i,j}\}_{j=1}^{t^i_1}$ for $F^2_i$. For each $j$, $1 \leq j \leq t^2_i$, choose $U_{ij} \in H$ such that $U_{ij} \supset f^2_{ij}$. There is a unique $z_{ij}$, $1 \leq z_{ij} \leq t^1_{s_i}$, such that $f^1_{s_i,z_{ij}} \supset U_{ij} \supset f^2_{ij}$. Let $\alpha_1(f^2_{ij}) = U_{ij}$, $\beta_1(U_{ij}) = f^1_{s_i,z_{ij}}$, and $\pi_1 = \beta_1 \alpha_1$.

Case (2): $y_i \not\in B$. Let $s_i = \min \{s \mid \text{int } K_s \supset \bigcup_{u \in G(y_i)} \overline{\phi(u)}\}$. Choose $F^1_{s_i} = \{f^1_{s_i,j}\}_{j=1}^{t^i_1}$ for $F^2_i$. For each $j$, $1 \leq j \leq t^2_i$, choose $U_{ij} \in O \subset U^1$ such that $U_{ij} \supset f^2_{ij}$. There is a unique $z_{ij}$, $1 \leq z_{ij} \leq t^1_{s_i}$, such that $f^1_{s_i,z_{ij}} \supset U_{ij} \supset f^2_{ij}$. Let $\alpha_1(f^2_{ij}) = U_{ij}$, $\beta_1(U_{ij}) = f^1_{s_i,z_{ij}}$, and $\pi_1 = \beta_1 \alpha_1$.

It will be shown that the mappings $\alpha_1$ and $\beta_1$ are well defined. If $\beta_1$ is not well defined, then there exists $F^2_i$ and $F^2_k$, two different distinguished families in $V^2$ such that (a) $s_i \neq s_k$, (b) $F^1_{s_i}$ is chosen for $F^2_i$, (c) $F^1_{s_k}$ is chosen for $F^2_k$, and (d) $U_{ij} = U_{kt} \supset f^2_{ij} \cup f^2_{kt}$ where $U_{ij}$ is chosen from $U^1$ such that $U_{ij} \supset f^2_{ij}$ and $U_{kt} = U_{ij}$ is chosen such that $U_{kt} \supset f^2_{kt}$ as described above.

Case (l): $U_{ij} = U_{kt} \in O \subset U^1$. In this case, $y_i \not\in B$ and $y_k \not\in B$. Indeed, $y_i \in O_x$ and $y_k \in O_x$ for some $x$, $1 \leq x \leq n_1$. In this case, it follows from Property (6) of the properties of $U^1$ that for each $u \in G(y_i)$, $y_k \in \phi(u)$, and for each $v \in G(y_k)$,
\( y_i \in \phi(v) \) since \( O_x \supset \phi(U_{ij}) = \phi(U_{kt}) \) and \( \cap Q_x \supset O_x \) as defined above. Consequently, \( G(y_i) = G(y_k) \) and \( s_i = s_k \) contrary to the assumption above.

Case (II): \( U_{ij} = U_{kt} \in H \subset U^1 \), \( y_i \in B \), and \( y_k \in B \). Recall that \( c_i = \min\{t \mid y_i \in \cap D_t\} \) and \( s_i = \min\{s \mid \text{int } K_s \supset \phi(u) \text{ for all } u \in U^1_1 \text{ such that } u \in G(y_i)\} \). Also, \( c_k = \min\{t \mid y_k \in \cap D_t\} \) and \( s_k = \min\{s \mid \text{int } K_s \supset \phi(u) \text{ for all } u \in U^1_1 \text{ such that } u \in G(y_k)\} \). Furthermore, \( U_{kt} \in H \) where \( \phi(U_{kt}) \cap (\cap D_{c_k}) \neq \emptyset \) and \( U_{kt} = U_{ij} \in H \) where \( \phi(U_{kt}) \cap (\cap D_{c_i}) \neq \emptyset \). Now, \( y_i \in \phi(U_{kt}) = \phi(U_{ij}) \) and \( y_k \in \phi(U_{ij}) \).

Clearly, \( y_i \in \phi(u) \) for each \( u \in G(y_i) \). Since \( y_i \in \phi(U_{ij}) \), it follows by construction of \( U^1 \) that \( \phi(h) \supset \phi(U_{ij}) \) for all \( h \in U^1_1 \) such that there is no \( B_x \in D_{c_i} \) such that \( \partial \phi(h) \supset B_x \) and \( \phi(h) \cap \phi(U_{ij}) \neq \emptyset \). If there is \( B_x \in D_{c_i} \) such that \( \partial \phi(u) \supset B_x \), then \( y_i \in B_x \) and \( y_i \notin \phi(u) \). Consequently, \( u \) is such an \( h \) and \( \phi(u) \supset \phi(U_{ij}) = \phi(U_{kt}) \) and \( y_k \in \phi(u) \). Now, \( y_k \in \phi(v) \) for each \( v \in G(y_k) \). It follows in a similar way that \( y_i \in \phi(v) \). Thus, \( G(y_i) = G(y_k) \) and \( s_i = s_k \) contrary to the assumption above.

It should be clear that \( \alpha_1 \) and \( \beta_1 \) are well defined. Since \( V^2 \) covers \( M \) and refines \( U^1 \) which is irreducible, \( \beta_1 \) is defined for each \( u \in U^1 \). Observe that \( \pi_1 \) maps distinguished families onto distinguished families.

The first steps in the proof of Lemma 4 are complete.

With \( V^i \) defined (as indicated for \( i = 1 \) and 2), define \( U^i \) in the manner that \( U^1 \) is defined for \( V^1 \). Use Lemma 3 to obtain a finite open covering \( R^{i+1} \) of \( Y \) satisfying the conditions of Lemma 3 where \( V^i \) replaces \( W_2 \) and \( R^{i+1} \) replaces \( R \) such that \( R^{i+1} \) generates a special covering \( V^{i+1} \) of \( M \) having the properties similar to those described above for \( V^2 \) w.r.t. \( U^1 \) and \( V^1 \) but w.r.t. \( U^i \) and \( V^i \). Similarly, define \( \alpha_i, \beta_i, \) and \( \pi_i \) in the manner that \( \alpha_1, \beta_1, \) and \( \pi_1 \) are described above. Extend \( \alpha_i, \beta_i, \) and \( \pi_i \) in the usual manner to the nerves \( N(V^{i+1}), N(U^i), \) and \( N(V^i) \), respectively.

It should be clear that the proof of Lemma 4 can be completed using mathematical induction and the methods employed above.

Orientation Of The Simplices In \( N(V^m) \)
Next, orient the simplices in \( N(V^m) \) for each special covering \( V^m \). Recall that for each special covering \( V^m \), there is associated a covering \( R^m \) of \( Y \) which generates \( V^m \).

Let \( R = R^m \) and \( V = V^m \). Suppose that \((v_0, v_1, v_2, \cdots, v_k) = \sigma^k\) is a \( k \)-simplex in \( N(R) \). For each \( i, 0 \leq i \leq k \), \( \phi^{-1}(v_i) = v_{i1} \cup v_{i2} \cup \cdots \cup v_{it_i} \) where \( \{v_{i1}, v_{i2}, \cdots, v_{it_i}\} \) is the distinguished family determined by \( v_i \). Consequently, \( \sigma^k \) determines a distinguished family of \( k \)-simplices in \( N(\phi^{-1}(R)) \) where \( \phi^{-1}(R) = \{c \mid c \text{ is a component of } \phi^{-1}(r) \text{ for } r \in R\} \).

If \( q \in N[\sigma^k] \), the nucleus or carrier of \( \sigma^k \), then \( \phi^{-1}(q) \cap v_{ij} \neq \emptyset \) for each \( i = 1, 2, \cdots, k \) and each \( j = 1, 2, \cdots, t_i \). The orientation of a \( k \)-simplex \( \delta^k = (v_{0j_0}, v_{1j_1}, \cdots, v_{kj_k}) \) is to be that of \( \sigma^k \) as indicated by the given order of the vertices \( (v_{0j_0}, v_{1j_1}, \cdots, v_{kj_k}) \) of \( \delta^k \). Since \( \phi^{-1}(v_i) \) has \( t_i \) components, there will be at least \( t_i \) \( k \)-simplices in \( N(\phi^{-1}(R)) \) which are mapped to \( \sigma^k \) by the simplicial mapping \( \phi^* : N(\phi^{-1}(R)) \to N(R) \) induced by \( \phi \). This collection of \( k \)-simplices is the distinguished family determined by \( \sigma^k \) (more precisely, determined by \( N[\sigma^k] \)). The distinguished family of \( k \) simplices in \( N(V^m) \) has cardinality \( p^c \) for some natural number \( c \). Note that \( N[\sigma^k] \) is connected and ulc.

**Distinguished Families Of \( n \)-Simplices In \( N(V^m) \)**

The distinguished families \( F^1_i = \{f^1_{ij} \}_{j=1}^{t_i^1} \) of members of the covering \( V^1 \) generate distinguished families of \( n \)-simplices. That is, for distinguished families \( F^1_{k_i} \), \( 0 \leq i \leq n \), in \( V^1 \) such that \( \bigcap_{i=1}^{n} (F^1_{k_i})^* \neq \emptyset \) where \( (F^1_{k_i})^* = \bigcup_{j=1}^{t_i^1} f^1_{k_i,j} \), the \( F^1_{k_i} \), \( 0 \leq i \leq n \), generate a distinguished family of \( n \)-simplices consisting of all \( n \)-simplices \( \{f^1_{k_0,j_0}, f^1_{k_1,j_1}, \cdots, f^1_{k_n,j_n}\} \) such that (1) \( f^1_{k_i,j_i} \in F^1_{k_i}, 0 \leq i \leq n \), and (2) \( \bigcap_{i=0}^{n} f^1_{k_i,j_i} \neq \emptyset \). The number of \( n \)-simplices in this family is \( \max\{t^1_{k_i} \mid 0 \leq i \leq n\} \). As pointed out above, a distinguished family of \( n \)-simplices is determined by an \( n \)-simplex \( \sigma^n \) in \( N(R) \) and such a family is a lifting of \( \sigma^n \) to \( N(V^m) \).

Consider a distinguished family \( S^1_k \) of \( n \)-simplices in \( N(V^1) \) as defined above such that a distinguished family \( S^2_q \) of \( n \)-simplices defined similarly in \( N(V^2) \) using distinguished families in \( V^2 \) is mapped onto \( S^1_k \) by \( \pi^*_1 : N(V^2) \to N(V^1) \). By construction, each \( n \)-
simplex in $S^1_1$ is the image of exactly $p^c$ $n$-simplices for fixed $c$, a non negative integer, where $\text{card } S^2_q = p^{c_q}$, $\text{card } S^1_k = p^{c_k}$, and $c = c_k - c_q$. Of course, as a chain in $N(V^1)$, this is a trivial $n$-chain using coefficients in $Z_p$.

For each natural number $m$, define distinguished families $S^m_i$ of $n$-simplices in $N(V^m)$ as described above. Each such family $S^m_i$ is the lifting of an $n$-simplex $\gamma^n_i$ in $N(R^m)$, that is, $\phi : M \to M/A_p$ induces a mapping $\phi^* : N(V^m) \to N(R^m)$. If $\gamma^n_i$ is an $n$-simplex in $N(R^m)$, then $(\phi^*)^{-1}(\gamma^n_i)$ is the union of a distinguished family of $n$-simplices $S^m_i$ in $N(V^m)$. The family $S^m_i$ is the lifting of $\gamma^n_i$ in $N(V^m)$. Each member of $S^m_i$ has the same orientation as $\gamma^n_i$.

Observe that if $S^m_i$ and $S^m_j$ are two non degenerate distinguished families of $n$-simplices such that $S^m_i = \{\delta^n_{ji} \}^{p_{ci}_{t=1}}$, $S^m_j = \{\delta^n_{ji} \}^{p_{cj}_{t=1}}$, and $\delta^n_{ji}$ shares and $(n-1)$-face with $\delta^n_{js}$ for some $j$ and $s$, then for each $t$, $1 \leq t \leq p^c_i$, there is some $s$, $1 \leq s \leq p^c_j$, such that $\delta^n_{ji}$ shares an $(n-1)$-face with $\delta^n_{js}$ and, conversely, for each $s$, $1 \leq s \leq p^c_j$, there is some $t$, $1 \leq t \leq p^c_i$, such that $\delta^n_{ji}$ and $\delta^n_{it}$ share an $(n-1)$-face. If $c_i < c_j$, then for each $t$, $1 \leq t \leq p^c_i$, $\delta^n_{it}$ shares an $(n-1)$-face with exactly $p^{c_j-c_i}$ members of $\delta^n_{ji}$.

The $n$-Skeleton, $N(V^m)_n$, Of $N(V^m)$, And Inverse Limit $H_n(V^m)_n \cong Z_p$.

Let $N(V^m)_n$ denote the $n$-skeleton of $N(V^m)$. Recall that $V^m$ denotes the special $m^{th}$ covering. The special projections $\pi^*_m : N(V^{m+1}) \to N(V^m)$, factors by $\alpha^*_m : N(V^{m+1}) \to N(U^m)$ and $\beta^*_m : N(U^m) \to N(V^m)$ where $\pi^*_m = \beta^*_m \alpha^*_m$.

Let $H_n(V^m)$ denote the $n^{th}$ simplicial homology of $N(V^m)_n$, the $n$-skeleton of $N(V^m)$. The coefficient group is always $Z_p$. The following two lemmas give facts concerning the homology which will be needed to finish the proof of the Theorem.

First, consider the following commutative diagram:

\[
\begin{array}{ccccccc}
\leftarrow & H_n(V^m) & \xleftarrow{\beta^*_m} & H_n(U^m) & \xleftarrow{\alpha^*_m} & H_n(V^{m+1}) & \xleftarrow{\beta^*_{m+1}} & H_n(U^{m+1}) & \leftarrow \\
& \nu_m \uparrow & \beta^*_m \nearrow & \kappa \alpha^*_m & \nu_{m+1} \uparrow & \beta^*_{m+1} \nearrow & \kappa \alpha^*_{m+1} & \\
\leftarrow & H_n(V^m)_n & \xleftarrow{\pi^*_m} & H_n(V^{m+1})_n & \xleftarrow{\pi^*_{m+1}} & H_n(V^{m+2})_n & \leftarrow
\end{array}
\]
Here $\nu_m$ is the natural map of a cycle in $H_n(V^m_n)$ into its homology class in $H_n(V^m)$. The other maps are those induced by the projections $\alpha_m$, $\beta_m$ and $\pi_m$. The upper sequence, of course, yields the Čech homology group $\check{H}_n(M)$ as its inverse limit. Furthermore, it can be easily shown, using the diagram, that $\check{H}_n(M)$ is isomorphic to the inverse limit $G = \varprojlim H_n(V^m_n)$, of the lower sequence. Specifically, $\gamma : \check{H}_n(M) \to G$ defined by $\gamma(\Delta) = \{\beta_m^*(\pi_{U_m}(\Delta))\}$ (where $\Delta$ is a generator of $\check{H}_n(M)$) is an isomorphism of $\check{H}_n(M)$ onto $G$. We shall use the isomorphism in what follows and for convenience we shall let $\gamma(\Delta) = \{z^n_m(\Delta)\}$, i.e. $z^n_m(\Delta) = \beta_m^*(\pi_{U_m}(\Delta)) \in H_n(V^m_n)$.

Let $\Delta$ be the generator of $\check{H}(M)$ where $\Delta = \{z^n_m(\Delta)\}$, a sequence of $n$-cycles such that $\pi_m^* : z^n_{m+1}(\Delta) \to z^n_m(\Delta)$ where $z^n_m(\Delta)$ is the $m^{th}$ coordinate of $\Delta$, i.e., $\pi_{V^m}(\Delta) = z^n_m(\Delta)$ an $n$-cycle in $N(V^m_n)$ where $N(V^m_n)$ is the $n$-skeleton of $N(V^m)$. If $s\Delta$, $s \in \mathbb{Z}_p$, is any $n$-cycle in $\check{H}(M)$, then the coordinate $n$-cycles $z^n_m(s\Delta)$ and $z^n_m(\Delta)$ contain exactly the same $n$-simplices in $N(V^m_n)$ (see Lemma 7 below).

It follows from Lemma 2 that there is no loss of generality in assuming that $\pi_{V^m} : \check{H}(M) \to H_n(V^m_n)$ has the property that $\pi_{V^m}(\check{H}(M)) \cong \mathbb{Z}_p$ for each natural number $m$.

A Cernavskii Operator $\sigma$

4. An Operator $\sigma$ is defined on $n$-Chains similar to the Cernavskii operator in [27].

Choose $g \in A_p - H_1$. Next, define an operator $\sigma$ [cf. 27, 10] on $n$-chains. If $\delta^n$ is an $n$-simplex in $N(V^{m+1})$, then $\delta^n$ is in a unique non degenerate distinguished family $S^{m+1}_i$ of $n$-simplices which has cardinality $p^{c_i}$. If $\pi_m^*(\delta^n)$ is an $n$-simplex, then each $n$-simplex in $S^{m+1}_i$ projects by $\pi_m^*$ to an $n$-simplex in $N(V^m)$. If $\pi_m^*(\delta^n)$ is a $k$-simplex, $k < n$, then each $n$-simplex in $S^{m+1}_i$ projects under $\pi_m^*$ to a $k$-simplex. Let $\sigma(\delta^n) = \sum_{s=0}^{p-1} g^s(\delta^n)$ where $g^0$ is the identity homeomorphism. Observe that if $\delta^n$ is an $n$-simplex in $N(V^{m+1})$ and $\pi_m^*(\delta^n)$ is an $n$-simplex in $N(V^m)$, then $\pi_m^*(\delta^n)$ is in a non degenerate distin-
guished family \( S^m_j \) of \( n \)-simplices in \( N(V^m) \). Also, \( \pi^*_m \) maps \( \{ g^s(\delta^n) \}_{s=0}^{p-1} \) one-to-one onto \( \{ \pi^*_m(g^s(\delta^n)) \}_{s=0}^{p-1} = \{ g^s(\pi^*_m(\delta^n)) \}_{s=0}^{p-1} \) and \( \pi^*_m \sigma_m \delta^n = \sigma_m \pi^*_m \delta^n \). It should be clear that if \( \delta^n \) is an \( n \)-simplex in \( N(V^{m+1}) \), then
\[
\pi^*_m \sigma_m \delta^n = \pi^*_m \sum_{s=0}^{p-1} g^s(\delta^n) = \sum_{s=0}^{p-1} \pi^*_m g^s(\delta^n) = \sum_{s=0}^{p-1} g^s(\pi^*_m(\delta^n)) = \sigma_m \pi^*_m \delta^n.
\]
Clearly, \( \sigma_m \) can be extended to any \( n \)-chain in \( N(V^m_n) \). If \( \delta^n \) is an \( n \)-simplex in \( N(V^m_n) \) and \( \pi^*_m \delta^n \) is a \( k \)-simplex, \( k < n \), then \( \pi^*_m \sigma_m (\delta^n) \) is a trivial \( n \)-chain.

Recall that the members of a distinguished family of \( k \)-simplices in \( N(V^m) \) have the same orientation (being the lifting of a \( k \)-simplex in \( N(R^m) \) where \( R^m \) is a certain cover of \( Y \)).

Lemma 5. [cf. 27] The special operator \( \sigma_m \) maps \( n \)-cycles to \( n \)-cycles and \( \sigma_m \) commutes with the special projections on \( n \)-chains. If \( z^m_{m+1}(\Delta) = z \) is a coordinate \( n \)-cycle in \( N(V^{m+1}_n) \), then either (a) \( \sigma_m z = 0 \) or (b) \( \sigma(z) = z \).

Proof. The homeomorphism \( g \) induces a simplicial homeomorphism \( \tilde{g} \) of \( N(V^m+1) \) onto itself since \( g \) maps "vertices" (elements of \( V^m+1 \)) one-to-one onto "vertices".

Let \( \Delta \) be a non zero \( n \)-cycle in \( \tilde{H}_n(M) = \text{lim}_{m} H_n(V^{m+1}_n) \cong Z_p \). Let \( z = z^m_{m+1}(\Delta) \) be the coordinate \( n \)-cycle of \( \Delta \) in \( H_n(V^{m+1}_n) \). Let \( \sigma = \sigma_m \). Consider \( \sigma(z) \) where \( z = \sum_{i=1}^{k} c_i \delta^n_i \). Thus, \( \sigma z = \sum_{i=1}^{k} c_i \sigma \delta^n_i \) where \( \sigma \delta^n_i = \sum_{s=0}^{p-1} g^s(\delta^n_i) \) and \( \{ g^s(\delta^n_i) \}_{s=0}^{p-1} \) is the distinguished subfamily of \( n \)-simplices in \( N(V^{m+1}) \) associated with \( \delta^n_i \). It follows by the construction that there is \( z^m_{m+2}(\Delta) \), a coordinate \( n \)-cycle in \( N(V^{m+2}_n) \) such that \( g^s(z^m_{m+2}(\Delta)) \) maps by \( \pi^*_m \) onto \( g^s(z) \). Hence, \( g^s(z) \) for \( 0 \leq s \leq p-1 \) contains the same \( n \)-simplices as \( z \) (see Lemma 7). Hence, \( \sigma z = \sum_{t=0}^{p-1} z_t \) where \( z_0 = z \) and \( z_t = \sum_{i=1}^{k} c_i g^t(\delta^n_i) \). Since \( g \) is a homeomorphism on \( M \), \( g \) induces an automorphism on \( \pi_{V^{m+1}}(G) \) in \( H_n(V^m_n) \) where \( G = \text{lim}_{m} H_n(V^m_n) \cong Z_p \), \( \pi_{V^{m+1}}(G) \cong Z_p \), and \( z_t \) is an \( n \)-cycle. It follows that \( \sigma z = \sum_{s=0}^{p-1} g^s(z) \) which is an \( n \)-cycle. If \( g \) induces the identity automorphism, then \( \sigma z = pz = 0 \mod p \) (the trivial \( n \)-cycle). If the induced automorphism is not the...
identity, then it will be shown that \( \sum_{t=1}^{p-1} z_t = 0 \mod p \) and that \( \sigma z = z \), that is, \( \sigma \) is
the identity automorphism. Now, \( \pi_{V_{n+1}^m}(G) \cong \mathbb{Z}_p = \{0, 1, 2, \ldots, p-1\} \). Since \( g \) induces an
automorphism on \( \pi_{V_{n+1}^m}(G) \), it induces an automorphism \( g_* \) on \( \mathbb{Z}_p \). Let \( g_*(1) = x \).
Hence, \( g_*^s(1) = x^s \). It is well known in number theory that \( p \) divides \( x^{p-1} - 1 \). Also,
\( x^{p-1} - 1 = (x-1)(1+x+x^2+\cdots+x^{p-2}) \). Consequently, \( p \) divides \( x(1+x+x^2+\cdots+x^{p-2}) =
x + x^2 + \cdots + x^{p-1} = \sum_{s=1}^{p-1} x^s = 0 \mod p \). It follows that \( \sigma(z) = \sum_{s=0}^{p-1} g^s(z) = z + \sum_{s=1}^{p-1} g^s(z) = z \mod p \). Thus, if \( g \) does not induce the identity automorphism, then \( \sigma \) is the identity automorphism.

If under the projection \( \pi_m^* : H_n(V_{n+1}^m) \to H_n(V_n) \), the image of an \( n \)-simplex \( \delta^n \)
is a \( k \)-simplex with \( k < n \), then the same is true for all members of the distinguished family to which \( \delta^n \) belongs. Thus, \( \pi_m^* \sigma_m \delta^n = \sigma_m \pi_m^* \delta^n = 0 \) where \( \sigma_m \pi_m^* \delta^n \) is defined above and is a trivial \( n \)-chain. If \( \pi_m^* (\delta^n) \) is an \( n \)-simplex, then by construction the distinguished subfamily of \( n \)-simplices with which \( \delta^n \) is associated is in one-to-one correspondence with the distinguished subfamily of \( n \)-simplices with which \( \pi_m^* (\delta^n) \) is associated. Thus, \( \pi_m^* \sigma_m \delta^n = \sigma_m \pi_m^* \delta^n \). Consequently, \( \sigma \) carries over to the \( n \)-cycles of \( M \) and to \( \tilde{H}(M) \). Lemma 5 is proved.

Lemma 6. If \( \Delta_1 \) and \( \Delta_2 \) are non zero elements of \( \tilde{H}_n(M) \), then for each \( m \), exactly the
same simplices appear in the chains \( z_m^n(\Delta_1) \) and \( z_m^n(\Delta_2) \) which are in the \( n \)-dimensional complex, \( N(V_n^m) \), the \( n \)-skeleton of \( N(V_n^m) \).

Proof. Suppose that this is not true. Without loss of generality, assume that the \( n \)-simplex \( \delta^n \) appears in \( z_m^n(\Delta_1) \) and not in \( z_m^n(\Delta_2) \) where \( z_m^n(\Delta_1) \) and \( z_m^n(\Delta_2) \) are the \( m \)th
coordinates of \( \Delta_1 \) and \( \Delta_2 \), respectively. Since \( N(V_n^m) \) is \( n \)-dimensional, these coordinates are \( n \)-cycles in \( N(V_n^m) \). Since \( \tilde{H}_n(M) = \mathbb{Z}_p \), assume that \( \Delta_1 \) generates \( \tilde{H}_n(M) \) and that \( \Delta_2 = s\Delta_1 \) for some natural number \( s \), \( 1 \leq s < p \). Consequently, \( z_m^n(\Delta_2) = s(z_m^n(\Delta_1)) \).
It follows that \( \delta^n \) appears in \( s(z_m^n(\Delta_1)) \) and hence in \( z_m^n(\Delta_2) \) – a contradiction.

Lemma 7. Suppose that \( \Delta \in \tilde{H}_n(M) \) with \( \Delta \neq 0 \) and \( z = z_{m+1}^n(\Delta) = \pi_{V_{n+1}^m}(\Delta) \), the
coordinate $n$-cycle of $\Delta$ in $H_n(V_{n+1}^{m+1})$. Let $z = \sum_{i=1}^{p} c_i \delta_i^n$. Then the collection $C_j = \{\delta_{j_1}^n, \delta_{j_2}^n, \ldots, \delta_{j_t}^n\}$ of all $n$-simplices in $\{\delta_1^n, \delta_2^n, \ldots, \delta_q^n\}$ which are in a fixed distinguished family $S_j^{m+1}$ of $n$-simplices in $N(V_{n+1}^{m+1})$ have the properties (1) if $x = \sum_{i=1}^{t} c_j, \delta_{j_i}^n$, then either (a) $\sigma_m x = 0$ when $g$ induces the identity automorphism or (b) $\sigma_m x = x$ when $g$ does not induce the identity automorphism and (2) $\sum_{i=1}^{t} c_{j_i} = 0 \mod p$.

Proof. By Lemma 5, either (a) $\sigma_m z = 0$ or (b) $\sigma_m (z) = z$.

Case (a): Since $\sigma_m (z) = 0$, it follows that $\sigma_m x = 0$ since for each $i$, $1 \leq i \leq t$, $\sigma_m \delta_{j_i}^n = \sum_{s=0}^{p-1} g^s(\delta_{j_i}^n)$ where for each $s$, $0 \leq s \leq p - 1$, $g^s(\delta_{j_i}^n)$ is an $n$-simplex in $C_q \subset S_j^{m+1}$. Choose notation such that (1) $S_j^{m+1} = \{\delta_{j_1}^n \}, (p^k = p^c_j$ in earlier notation), (2) $g \delta_{j_i} = \delta_{j_{i+1}}$ for $1 \leq j < p^k$ and $g \delta_{p^k} = \delta_1$ ($g$ permutes the $\delta_j$ in a cyclic order), and (3) $x = \sum_{i=1}^{p} c_i \delta_i$ where $c_i = 0$ iff $\delta_i \notin C_q$ and $c_i = c_{j_i}$ iff $\delta_i = \delta_{j_i}^n \in C_q$. Let $\sigma = \sigma_m$. Since $\sigma (z) = 0$, it follows that $\sigma (x) = 0$. Note that $0 = \sigma (x) = c_1 \sum_{i=1}^{p} \delta_i + c_2 \sum_{i=1}^{p} \delta_{i+1} + \cdots + c_j \sum_{i=1}^{p} \delta_{i+j-1} + \cdots + c_{p^k-p} \sum_{i=1}^{p} \delta_{i+p^k-p-1} + \cdots + c_{p^k} (\delta_{p^k} + \delta_1 + \delta_2 + \cdots + \delta_{p-1})$. Rearrange as

$$\sigma (x) = (c_1 + c_{p^k} + c_{p^k-1} + \cdots + c_{p^k-p+2}) \delta_1 + (c_2 + c_1 + c_{p^k} + \cdots + c_{p^k-p+1}) \delta_2 + \cdots + (c_{p} + c_{p-1} + c_{p-2} + \cdots + c_1) \delta_p + (c_{p+1} + c_1 + c_{p-1} + \cdots + c_2) \delta_{p+1} + \cdots + (c_{p^k} + c_{p^k-1} + \cdots + c_{p^k-p+1}) \delta_{p^k}.$$  

Since $\sigma (x) = 0$, it follows that the coefficient of $\delta_i$ is $0 \mod p$ for $1 \leq i \leq p^k$. A careful consideration of pairs of successive coefficients of $\delta_i$ and $\delta_{i+1}$ will give the following result. If $1 \leq i \leq p^k$, $1 \leq j \leq p^k$, and $i \equiv j \mod p$, then $c_i \equiv c_j \mod p$. Thus
\[
\sum_{i=1}^{p^k} c_i = \sum_{i=1}^{p} c_i + \sum_{i=p+1}^{3p} c_i + \sum_{i=(p^{k-1})p}^{(p^{k-1})p} c_i \text{ with } p^{k-1} \text{ summations each of length } p. \text{ Now, } c_i \equiv c_j \mod p \text{ if } i \equiv j \mod p \text{ gives that each of the } p^{k-1} \text{ summations is congruent to } 0 \mod p. \text{ Thus, } \sum_{i=1}^{p^k} c_i = p^{k-1} \left( \sum_{i=1}^{p} c_i \right) \equiv 0 \mod p \text{ if } k > 1. \text{ If } k = 1, \text{ then }
\]
\[
x = \sum_{i=1}^{p} c_i \delta_i, \\
\sigma(x) = c_1(\delta_1 + \delta_2 + \cdots + \delta_p) \\
+ c_2(\delta_2 + \delta_3 + \cdots + \delta_{p+1}) \\
\vdots \\
+ c_p(\delta_p + \delta_1 + \delta_2 + \cdots + \delta_{p-1}), \text{ rearrange as}
\]
\[
(c_1 + c_2 + \cdots + c_p)\delta_1 + \\
(c_1 + c_2 + \cdots + c_p)\delta_2 + \\
\vdots \\
(c_1 + c_2 + \cdots + c_p)\delta_p,
\]
and \(\sum_{i=1}^{p} c_i \equiv 0 \mod p.\)

It is instructive to consider a simple example. Let \(p = 3\) and \(x = \sum_{i=1}^{9} c_i \delta_i. \) Thus,
\[
\sigma x = c_1(\delta_1 + \delta_2 + \delta_3) + c_2(\delta_2 + \delta_3 + \delta_4) + c_3(\delta_3 + \delta_4 + \delta_5) + c_4(\delta_4 + \delta_5 + \delta_6) + c_5(\delta_5 + \delta_6 + \delta_7) + c_6(\delta_6 + \delta_7 + \delta_8) + c_7(\delta_7 + \delta_8 + \delta_9) + c_8(c_8 + c_9 + c_1) + c_9(c_9 + \delta_1 + \delta_2) = (\text{by rearrangement}) = (c_1 + c_8 + c_9)\delta_1 + (c_1 + c_2 + c_9)\delta_2 + (c_1 + c_2 + c_3)\delta_3 + (c_2 + c_3 + c_4)\delta_4 + (c_3 + c_4 + c_5)\delta_5 + (c_4 + c_5 + c_6)\delta_6 + (c_5 + c_6 + c_7)\delta_7 + (c_6 + c_7 + c_8)\delta_8 + (c_7 + c_8 + c_9)\delta_9.
\]
For each \(i, \ 1 \leq i \leq 9, \) the coefficient of \(\delta_i \equiv 0 \mod 3. \) Observe that from the coefficients of \(\delta_1 \) and \(\delta_2, \) it follows that \(c_2 \equiv c_8 \mod 3. \) The coefficients of \(\delta_2 \) and \(\delta_3 \) yield that \(c_3 \equiv c_9 \mod 3. \) Continuing, \(c_1 \equiv c_4, \ c_2 \equiv c_5, \ c_3 \equiv c_6, \ c_4 \equiv c_7, \ c_5 \equiv c_8, \ c_6 \equiv c_9, \) and \(c_7 \equiv c_1 \) all \(\mod 3. \) Thus, \((c_1 + c_2 + c_3) + (c_4 + c_5 + c_6) + (c_7 + c_8 + c_9) = 0 \mod 3 \) since \((c_4 + c_5 + c_6) \equiv (c_1 + c_2 + c_3) \mod 3, \ (c_7 + c_8 + c_9) \equiv (c_4 + c_5 + c_6) \equiv (c_1 + c_2 + c_3) \mod 3. \)
3 and \( \sum_{i=1}^{9} c_i \equiv 3(c_1 + c_2 + c_3) \equiv 0 \mod 3. \)

Case (b): \( \sigma_m(x) = x. \) Choose notation as in Case (a). Write \( \sigma(x) \) as in Case (a), but in this case, \( x = \sigma(x) \) rather than \( 0 = \sigma(x) \). Consider first the example \( p = 3 \) and \( x = \sum_{i=1}^{9} c_i \delta_i \) where \( k = 2 \). Now, \( \sigma x = (c_1+c_8+c_9)\delta_1+(c_1+c_2+c_9)\delta_2+(c_1+c_2+c_3)\delta_3+(c_2+c_3+c_4)\delta_4+(c_3+c_4+c_5)\delta_5+(c_4+c_5+c_6)\delta_6+(c_5+c_6+c_7)\delta_7+(c_6+c_7+c_8)\delta_8+(c_7+c_8+c_9)\delta_9 = x = c_1\delta_1+c_2\delta_2+c_3\delta_3+c_4\delta_4+c_5\delta_5+c_6\delta_6+c_7\delta_7+c_8\delta_8+c_9\delta_9. \) This is an identity. Thus, the coefficient of \( \delta_i \) on one side is equal \( \mod p \) to the coefficient of \( \delta_i \) on the other side. Hence, \( c_1 + c_8 + c_9 \equiv 1 \mod p, \) \( c_1 + c_2 + c_9 \equiv 2 \mod p, \) \( c_1 + c_2 + c_3 \equiv 3 \mod p \), and so forth. Thus, \( \sum_{i=1}^{9} c_i \equiv (c_1+c_8+c_9)+(c_1+c_2+c_9)+(c_1+c_2+c_3)+(c_2+c_3+c_4)+(c_3+c_4+c_5)+(c_4+c_5+c_6)+(c_5+c_6+c_7)+(c_6+c_7+c_8)+(c_7+c_8+c_9) = 3(c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_9) \equiv 0 \mod p. \)

Consider the general case as in Case (a) but with \( x = \sigma(x) \) rather than \( 0 = \sigma(x) \). Hence,

\[
x = \sigma(x) = (c_1 + c_{p^k} + c_{p^k-1} + \cdots + c_{p^k-p+2})\delta_1 + (c_2 + c_1 + c_{p^k} + \cdots + c_{p^k-p+1})\delta_2 + \cdots + (c_p + c_{p-1} + c_{p-2} + \cdots + c_1)\delta_p + (c_{p+1} + c_p + c_{p-1} + \cdots + c_2)\delta_{p+1} + \cdots + (c_{p^k} + c_{p^k-1} + \cdots + c_{p^k-p+1})\delta_{p^k} = \sum_{i=1}^{p^k} c_i \delta_i.
\]

It follows from this identity that the coefficient of \( \delta_i \) on one side is equal \( \mod p \) to the coefficient of \( \delta_i \) on the other side. Consequently, \( \sum_{i=1}^{p^k} c_i = p \sum_{i=1}^{p} c_i \mod p = 0 \mod p \) as claimed where \( k > 1. \) For \( k = 1, \) \( x = \sum_{i=1}^{p} c_i \delta_i = \left( \sum_{i=1}^{p} c_i \right) \delta_1 + \left( \sum_{i=1}^{p} c_i \right) \delta_2 + \cdots + \)
\[
\left(\sum_{i=1}^{p} c_i\right) \delta_p, \ c_t \equiv \sum_{i=1}^{p} c_i \mod p \text{ for each } t, \ 1 \leq t \leq p, \text{ and } \sum_{i=1}^{p} c_i \equiv p \left(\sum_{i=1}^{p} c_i\right) \mod p = 0 \mod p. \text{ Lemma } 7 \text{ is proved.}
\]

5. A proof that a $p$-adic group $A_p$ can not act freely on a compact connected $n$-manifold where $\phi : M \to M/A_p$ is the orbit mapping.

**Remarks.** If the compact connected $n$-manifold $M$ has a non empty boundary, then two copies of $M$ can be sewed together by identifying the boundaries in such a way that the result is a compact connected $n$-manifold $M'$ without boundary. If $A_p$ acts effectively on $M$, then $A_p$ acts effectively on $M'$. There is no loss of generality in assuming that $M$ is a compact connected $n$-manifold without boundary.

**Definition.** An $n$-manifold $(M, d)$ is said to have Newman’s Property w.r.t. the class $L(M, p)$ (as stated above) iff there is $\epsilon > 0$ such that for any $\phi \in L(M; p)$, there is some $x \in M$ such that $\text{diam } \phi^{-1}\phi(x) \geq \epsilon$.

Generalizations can be made to metric spaces $(X, d)$ which are locally compact, connected, and $\text{lc}$ [4] which have domains $D$ such that $\bar{D}$ is compact, $\text{lc}$, and $H_n(X, X - D), Z_p) \cong Z_p$.

**Theorem.** If $L(M, p)$ is the class of all orbit mappings $\phi : M \to M/A_p$ where $A_p$ acts freely on a compact connected $n$-manifold $M$, then $M$ has Newman’s Property w.r.t. $L(M, p)$.

**Proof.** There is no loss of generality in assuming that $M$ has empty boundary.

By hypothesis, $\hat{H}_n(M) \cong Z_p$. Consider a finite open covering $U = W_1$ where $W_1$ and $W_2$ satisfy Lemma 2 and such that if $z(\Delta)$ is the $V$-coordinate of a non-zero $n$-cycle $\Delta \in \hat{H}_n(M)$ where $V$ refines $W_2$, then $\pi_{VU} z(\Delta) \neq 0$. Let $\epsilon$ be the Lebesque number of $W_2$. Suppose that there is $\phi \in L(M, p)$ such that $\text{diam } \phi^{-1}\phi(x) < \epsilon$ for each $x \in M$. Construct the special coverings $\{V^m\}$ and the special refinements $\{U^m\}$ as in Lemma 4 such that the star of each distinguished family of $V^1$ lies in some element of
$W_2$. Furthermore, the special projections $\pi_m$ can be constructed such that if \( \{ \delta_{sj}^{m} \}_{j=1}^{t} \) is a distinguished family of $n$-simplices in $N(V_m^n)$, then $\pi_{V^n}U$ takes $\delta_{sj}^n$, $1 \leq j \leq t_s$, to the same simplex $\delta_s$ in $N(U)$. Now, let $z_m = z_m^n(\Delta)$.

Let $z_m = \sum_{i=1}^{k} c_i \delta_i^n$. Hence, for each $j$, $1 \leq j \leq k$, $\delta_j^n$ is in a non degenerate distinguished family $S_j^m$ of $n$-simplices in $N(V^n)$. Let $C_j = \{ \delta_{j_1}^n, \delta_{j_2}^n, \cdots, \delta_{j_t}^n \}$ denote the collection of all $n$-simplices such that (1) $\delta_{j_i}^n$ appears in $z_m$ for $1 \leq i \leq t$ and (2) $S_j^m \supset C_j$.

By Lemma 7, $\sum_{i=1}^{t} c_{j_i} = 0$ mod $p$. Since the $n$-simplices in $C_j$ are sent by $\pi_{V^n}U$ to a single simplex $\delta_j$ in $N(U)$, it follows that the coefficient of $\delta_j$ is 0 mod $p$ and $z$ is sent by $\pi_{V^n}U$ to the zero $n$-cycle in $N(U)$. Thus, the projection of $z_m$ by $\pi_{V^n}U : H_n(V^n_m) \to H_n(U)$ takes the nontrivial $n$-cycle $z_m(\Delta)$ to the 0 $n$-cycle mod $p$. This violates the conclusion of Lemma 2. Thus, $M$ has Newman’s Property w.r.t. the class $L(M, p)$. Hence, $\epsilon$ is a Newman’s number and the Theorem is proved.

It is well known that if $A_p$ acts effectively on a compact connected $n$-manifold $M$, then given any $\epsilon > 0$, there is an effective action of $A_p$ on $M$ such that $\text{diam } \phi^{-1}\phi(x) < \epsilon$ for each $x \in M$. That is, $M$ fails to have Newman’s property w.r.t. $L(M, p)$. It follows that $A_p$ can not act effectively on a compact connected $n$-manifold $M$. 

29
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