BLIND ESTIMATION OF MULTIPLE CARRIER FREQUENCY OFFSETS

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ABSTRACT

Multiple carrier-frequency offsets (CFO) arise in a distributed antenna system, where data are transmitted simultaneously from multiple antennas. In such systems the received signal contains multiple CFOs due to mismatch between the local oscillators of transmitters and receiver. This results in a time-varying rotation of the data constellation, which needs to be compensated for at the receiver before symbol recovery. This paper proposes a new approach for blind CFO estimation and symbol recovery. The received base-band signal is over-sampled, and its polyphase components are used to formulate a virtual Multiple-Input Multiple-Output (MIMO) problem. By applying blind MIMO system estimation techniques, the system response is estimated and used to subsequently transform the multiple CFOs estimation problem into many independent single CFO estimation problems. Furthermore, an initial estimate of the CFO is obtained from the phase of the MIMO system response. The Cramer-Rao Lower bound is also derived, and the large sample performance of the proposed estimator is compared to the bound.

1. INTRODUCTION

In any communication system, the received signal is corrupted by carrier-frequency offsets (CFOs) due to the Doppler shift and/or local oscillators drift. The CFO causes a frequency shift and a time-varying rotation of the data symbols, which need to be compensated for at the receiver before symbol recovery. This can be achieved via pilot symbols. However, in the case of mobile systems and rich scattering environments the effects of CFO become time varying and even small errors in the CFO estimate tend to cause large data recovery errors. This necessitates transmission of pilots rather often, a process that lowers data throughput. In this paper we deal with CFO estimation without the need for pilot symbols. In single user systems, or in multi-antenna systems in which the transmitters are physically connected to the same oscillator, there is only one CFO that needs to be estimated. This is typically done via a decision feedback Phase Lock Loop (PLL) at the receiver end. The PLL is a closed-loop feedback control system that uses knowledge of the transmitted constellation to adaptively track both the frequency and phase offset between the equalized signal and the known signal constellation. However, depending on the constellation used during transmission, the PLL can have an M-fold symmetric ambiguity, resulting in a limited CFO acquisition range, i.e., $|F_k| < 1/(8T_s)$ for 4QAM signals, where $T_s$ is the symbol period. Moreover, the PLL typically requires a long convergence time. Alternatively, several methods have been proposed [2], [4], [5], [7] [9] that blindly estimate the CFOs and recover the transmitted symbols using second-order cyclic statistics of the over-sampled received signal. Blind CFO estimation has also been studied in the context of orthogonal frequency-division multiplexing (OFDM) systems, where the CFO destroys the orthogonality between the carriers (see [3] and the references therein).

In a spatially distributed antenna system where data are transmitted simultaneously from multiple antennas, the received signal contains multiple CFOs, one for each transmit antenna. A PLL does not work in that case as there is no single frequency to lock to. The literature on estimation of multiple CFOs is rather sparse. Existing literature on this topic focuses on pilot based CFO estimation. In [6], the multiple CFOs were estimated by using pilots that were uncorrelated between the different users. In [13], multiple CFOs were estimated via Maximum Likelihood based on specially designed pilots. To account for multiple offsets, [8] proposed that multiple nodes transmit the same copy of the data with an artificial delay at each node. The resulting system was modeled as a convolutive single-input/single-output (SISO) system with time-varying system response caused by the multiple CFOs. A minimum mean-square error (MMSE) decision feedback equalizer was used to track and equalize the channel and to recover the input data. Training symbols were required in order to obtain a channel estimate, which was used to initialize the equalizer.

Here we propose an approach for blind identification of multiple CFOs and subsequent symbol recovery. The received base-band signal is over-sampled, and its polyphase components are viewed as the outputs of a virtual MIMO system. The time-varying contribution of the CFOs, together with the transmitted symbols form the multiple-input/multiple-output (MIMO) inputs, while the time-invariant contribution of the CFOs along with fading channels comprise the system response. By applying blind MIMO system estimation techniques, the system response is estimated and used to subsequently transform the multiple CFOs estimation problem into many independent single CFO estimation problems. Furthermore, an initial estimate of the CFO obtained from the phase of the MIMO system response can be used to initialize the PLL eliminating the symmetrical ambiguity problem. The resulting method has full acquisition range for normalized CFOs, i.e., $|F_k| < P/(2T_s)$, where $P$ is the over-sampling factor. To evaluate large sample performance we establish the Cramer-Rao Bound (CRB) and compare the obtained mean square error to it.

2. SYSTEM MODEL

We consider a distributed antenna system, where $K$ users transmit simultaneously to a base station. Narrow-band transmission is assumed here, where the channel between any user and the base station is frequency non-selective. In addition, quasi-static fading is assumed, i.e., the channel gains remain fixed during the packet
length. The continuous-time base-band received signal \( y(t) \) can be expressed as
\[
y(t) = \sum_{k=1}^{K} a_k x_k(t - \tau_k) e^{j2\pi f_k t} + w(t)
\]  
(1)
where \( a_k \) represents the effect of channel fading between the \( k \)-th user and the base station and also contains the corresponding phase offset; \( \tau_k \) is the delay associated with the path between the \( k \)-th user and the base station; \( f_k \) is the frequency offset of the \( k \)-th user and \( w(t) \) represents noise; \( x_k(t) \) denotes the transmitted signal of user \( k \); \( x_k(t) = \sum s_k(i) p(t - iT_k) \) where \( s_k(i) \) is the \( i \)-th symbol of user \( k \); \( T_k \) is the symbol period; and \( p(t) \) is a pulse function with support \([0, T_k]\).

Our objective is to obtain an estimate of \( s(i) = [s_1(i), \ldots, s_K(i)]^T \) of the form
\[
\hat{s}(i) = \hat{A} \hat{P}^T s(i)
\]  
(2)
where \( P \) is a column permutation matrix and \( \hat{A} \) is a constant diagonal matrix. These are considered to be trivial ambiguities, and are typical in any blind inference problem.

The received signal \( y(t) \) is sampled at rate \( 1/T = P/T_s \), where the over-sampling factor \( P \geq K \) is an integer. In order to guarantee that all the users’ pulses overlap at the sampling times, the over-sampling period should satisfy: \( T_s/P \geq \tau_k, k = 1, \ldots, K \), which means that the over-sampling factor \( P \) is upper bounded by \( T_s/\min(\tau_1, \ldots, \tau_K) \). Let \( t = iT_s + mT \), \( m = 1, \ldots, P - 1 \) denote the sampling times. The over-sampled signal can be expressed as
\[
y_m(i) = y(iT_s + mT) = \sum_{k=1}^{K} a_k e^{j2\pi f_k (i + m/P)T_s} x_k((i + m/P)T_s - \tau_k) + w((i + m/P)T_s)
\]  
(3)
where \( f_k = f_kT_s/P \), \( (|f_k| \leq 0.5) \) is the normalized frequency offset between the \( k \)-th user and the base station, and the element of the virtual MIMO channel matrix \( A \) is given as
\[
a_{m,k} = a_k e^{j2\pi m f_k/P} e^{j\pi f_k T_s} - \tau_k, \quad m = 1, \ldots, P
\]  
(4)
Defining \( y(i) \triangleq [y_1(i), \ldots, y_P(i)]^T \); \( A \triangleq \{a_{m,k}\} \), a tall matrix of dimension \( P \times K \); \( \hat{s}(i) \triangleq [s_1(i)e^{j2\pi f_1/P}, \ldots, s_K(i)e^{j2\pi f_K/P}]^T \); and \( w(i) \triangleq [w(i + \frac{1}{2} P), \ldots, w(i + \frac{P}{2})]^T \), eq. (3) can be written in matrix form as
\[
y(i) = A\hat{s}(i) + w(i)
\]  
(5)

3. BLIND CHANNEL ESTIMATION AND COMPENSATION OF THE CFOs

Let us make the following assumptions.

- **A1** For each \( m = 1, \ldots, P \), \( w_m(.) \) is a zero-mean Gaussian stationary random processes with variance \( \sigma_w^2 \), and is independent of the inputs.

- **A2** For each \( k \), \( s_k(.) \) are a zero mean, independent identically distributed (i.i.d.) stationary with nonzero kurtosis, i.e., \( \gamma_k^2 = \text{Cum}[s_k(i), s_k(i), s_k(i), s_k(i)] \neq 0 \). The \( s_k \)'s are mutually independent, we can further assume that every user has unit transmission power, then \( C_n = I \).

- **A3** The over-sampling factor \( P \) is less than \( K \).

Under assumption (A2), it is easy to verify that the rotated input signals \( \hat{s}_k(.) \) are also zero mean, i.i.d. wide sense stationary with nonzero kurtosis. Also, the \( \hat{s}_k(.) \)'s are mutually independent for different \( k \)'s. Assumption (A3) guarantees that the virtual MIMO channel matrix \( A \) in (3) has full rank with probability one. If the delays of users are randomly distributed in the interval \([0, T_s/P)\), then each row of the channel matrix can be viewed as drawn randomly from a continuous distribution, thus the channel matrix has full rank with probability one.

One can apply any blind source separation algorithm (e.g., [1]) to obtain
\[
\hat{A} \triangleq \hat{A}_P \hat{A}_A
\]  
(6)
Subsequently, using a least-squares equalizer we can get an estimate of the de-coupled signals \( \hat{s}(i) \), within permutation and diagonal scalar ambiguities as
\[
\hat{s}(i) = (\hat{A}^H \hat{A})^{-1} \hat{A}^H \hat{y}(i) = e^{j\hat{\arg}(\hat{A})} \hat{A}^{-1} \hat{P}^T \hat{s}(i)
\]  
(7)
Denoting by \( \theta_k \) the \( k \)-th diagonal element of \( \text{Arg}\{A\} \), the \( k \)-th separated input signal can be expressed as
\[
\hat{s}_k(i) = s_k(i)e^{j(\theta_k - 2\pi f_k/P)}
\]  
(8)
Based on (8), any single CFO blind estimation method could be applied to recover the input signal. Those methods can benefit by a CFO estimate provided by the channel matrix estimate as follows. The phase of the estimated channel matrix \( \hat{A} \) equals
\[
\Psi = \text{Arg}\hat{A} = \begin{pmatrix}
2\pi f_1 + \phi_1 & \cdots & 2\pi f_K + \phi_K \\
\vdots & \ddots & \vdots \\
2\pi f_1 P + \phi_1 & \cdots & 2\pi f_K P + \phi_K
\end{pmatrix} P
\]  
(9)
where \( \phi_k = \text{Arg}\{a_k\} + \theta_k \), which accounts for both the phase of \( a_k \) and the estimated phase ambiguity in (8). The least squares estimate of \( f_k \) is given by
\[
\hat{f}_k = \frac{1}{2\pi} \frac{P(\sum_{p=1}^{P} p^2 \text{Re}(\Psi_{p,k})) - (\sum_{p=1}^{P} p \sum_{p=1}^{P} \text{Re}(\Psi_{p,k}))}{P(\sum_{p=1}^{P} p)^2 - (\sum_{p=1}^{P} p^2)^2}
\]  
(10)
We can write \( \hat{f}_k = f_k + \epsilon_k \) where \( \epsilon_k \) represents the estimation error.

Noting that the de-coupled signals \( \hat{s}_j(i) \) in (8) are shuffled in the same manner as the estimated CFOs in (10), we can use the estimated CFOs to compensate for the effect of CFO in the decoupled signals (8) and get estimates of the input signals as
\[
\hat{s}_j(i) = e^{j\hat{\arg}(\hat{A})} \hat{P}^T s(i)
\]  
(11)
Due to the residual error in the estimated CFOs, we can only compensate for a majority of the effect of CFO in (8) and obtain
\[
\hat{s}_j(i) = s_k(i)e^{j(\theta_k - 2\pi f_k/P)}
\]  
(12)
Let us apply the PLL to the recovered signals \( \hat{s}_j(i) \) in (12), to further mitigate the effect of residual CFO \( \epsilon_k \). For 4QAM signals, as long as \( |P\epsilon_k| < 1/8 \), it can be effectively removed by the PLL. Thus, the CFO estimator (10) can prevent the symmetric ambiguity of the PLL, and can also greatly reduce the convergence time of PLL. From (12), we can see that the CFO estimator will achieve full acquisition range for the normalized CFO, i.e., \( |f_k| < 1/2 \), which means we can deal with all continuous CFOs in the range \( F_k < P/(2T_s) \).
4. Cramer-Rao Lower Bound

To evaluate the large sample performance of the proposed method, we establish the Cramer-Rao lower bound according to [12]. Via central limit theory arguments, the received signals \( y \) can be approximated as complex Gaussian signal, with zero mean, and covariance matrix given by

\[
C_y = A C_z A^H + \sigma_w^2 I = A A^H + \sigma_w^2 I \tag{13}
\]

The covariance matrix is valid under assumption A1) and A2). The Gaussian assumption of the received signal is reasonable since the received signal is a linear mixture of i.i.d. signals.

Let \( \alpha = [ f^T, \rho^T, \sigma_w^2]^T \)

\[
\frac{\partial}{\partial \alpha} = \begin{bmatrix} \frac{\partial C_y}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial C_y}{\partial \alpha_1} \end{bmatrix}, \quad l, n = 1, ..., 2K + 1
\]

(14)

Since we are only interested in the CFO parameter \( f \), following the derivation in [12], we can obtain that

\[
\frac{1}{T} \text{CRB}^{-1}(f) = G^H G - G^H \Delta (\Delta^H \Delta)^{-1} \Delta^H G = G^H \Pi_{\Delta}^H G
\]

where \( G \) and \( \Delta \) are defined as

\[
\frac{1}{T} \text{FIM} = (\frac{\partial e_y}{\partial \alpha}^T)(C_y^T \otimes C_y^{-1}) (\frac{\partial e_y}{\partial \alpha}^T) = \begin{bmatrix} G^H & \Delta^H \end{bmatrix} \begin{bmatrix} G & \Delta \end{bmatrix}
\]

(15)

where \( e_y = \text{vec}(C_y) \) is a \( P^2 \times 1 \) vector constructed from columns of \( C_y \), and \( G \) is of dimension \( P^2 \times K \), while \( \Delta \) is of dimension \( P^2 \times (K + 1) \).

To proceed, we just need to evaluate the derivatives of \( e_y \) with respect to \( \alpha \). First consider \( \partial e_y / \partial f \), it holds that

\[
\frac{\partial e_y}{\partial f} = \text{vec} \left( \frac{\partial C_y}{\partial f} \right) = \text{vec} \left( \begin{bmatrix} 0 & \cdots & 0 & A^H \cdot A \cdot [0 \cdots 0]^T \end{bmatrix} \right)
\]

(16)

with \( d_k = \left[ 2^{25} f_k \cdot \rho_k \right] \cdot [0 \cdots 0] \), where \( \sigma \) is the Hadamard matrix product.

Similarly, we can get \( \partial e_y / \partial \sigma \):

\[
\frac{\partial e_y}{\partial \sigma} = \text{vec} \left( \frac{\partial C_y}{\partial \sigma} \right) = \text{vec} \left( \begin{bmatrix} 0 & \cdots & 0 & A^H \cdot A \cdot [0 \cdots 0]^T \end{bmatrix} \right)
\]

(17)

\[
\text{with } e_k = \left[ e^{2 \pi j f_k \cdot \rho(T_k - \tau_k)}, \ldots, e^{2 \pi j f_k \cdot \rho(T_K - \tau_k)} \right]^T.
\]

Finally, we have that \( \partial e_y / \partial \sigma_w^2 = \text{vec}(C_y^{-1}) \). Now we have all the ingredients to evaluate CRB(\( f \)) from (16).

5. SIMULATION RESULTS

In this section, we verify the validity of the proposed method via simulations, under the following assumptions. The channel coefficients \( a_k, \quad k = 1, ..., K \) are zero-mean Gaussian random variables. The waveform \( p(.) \) used here is hamming window. The continuous CFOs are randomly picked in the range \( [-\pi / T, \pi / T] \). The delays, \( \tau_k, \quad k = 1, ..., K \) are uniformly distributed in the range \([0, T_s / P]\). The input signals used here are 4QAM signals. The estimation results are averaged over 300 independent channels, and 20 Monte-Carlo runs for each channel.

The blind source separation algorithm used is the JADE method, which was downloaded from http://www.tsi.enst.fr/~cardoso/guidesepesou.html.

We show the performance of both the pilots-based method and the proposed method at different data lengths and SNR set to 30dB. For the pilots method, each user transmitted a pilot signal of length 32, and the pilots were random sequences uncorrelated between different users. In Fig. 1 we show the Mean Squares Error (MSE) for the CFO estimator [10] for different values of the over-sampling factor \( P \). To make the comparison fair for different over-sampling factor \( P \), the MSE is calculated based on \( \frac{1}{T} \sum_{k=1}^{K} ((f_k - \hat{f}_k) P)^2 \) = \( \frac{1}{P} \sum_{k=1}^{K} ((f_k - \hat{f}_k) T_s)^2 \). We can see that by increasing \( P \) we can get more accurate estimates of the CFOs. In Fig. 2 we show the Bit Error Rate (BER) for different \( P \)’s. For both the blind and the training based methods, the BER is calculated based on the recovered signals after the PLL. As expected, the BER performance also improves by increasing \( P \). The proposed method appears to work well even for short data length.

Next we show the performance of both methods at various noise levels. We set the packet length \( N \) to 1024. In Fig. 3 we show the MSE of the blind CFO estimator [10] as well as the training based method. We can see that by increasing \( P \) we can get more accurate estimates of the CFOs. In Fig. 4 we show the BER performance after PLL for both blind and training based methods. We can see that the proposed blind method has almost the same performance to the training based method for SNR lower than 20dB, while the training based method can achieve better BER performance for high SNR.

The mean square error of the CFO estimator of [10] is plotted against the stochastic CRB derived in Section 4. In Fig. 5, we plotted the MSE of the CFOs, as well as the CRB, as a function of the packet length \( T \). We can see that the MSE curves are parallel to the CRB, and error floor presented in the plot. Hence there is no bias in the estimates and the gap is only due to excess variance in the estimates. One possible reason for the existence of the excess variance is that we assume that we know the exact channel structure in the derivation of the CRB, i.e., the waveform used in transmission, which reduces the number of the unknown parameters. In the simulations, however, we did not assume any extra knowledge of the channel structure.

We should note that the PLL is important for good symbol recovery. For example, without the PLL, even if the residual error \( P_{\text{e}_{\text{t}}} = F_k - F_0 \) is only 0.001, the constellation will be rotated to a wrong position after 0.25/0.001 = 250 samples for 4QAM signals. To make sure that the PLL does not have the symmetrical ambiguity, we need to guarantee that \( |P_{\text{e}_{\text{t}}}| = |F_k - F_0| T_s | < 1/8 \) for 4QAM transmission. Thus, on the average, the maximum tolerable MSE for the CFO is in the order of \( 10^{-2} \). From the simulations, we can see that the achieved compensation is sufficient for practical systems and commonly used modulation schemes.

6. CONCLUSION

In this paper we have proposed a novel blind approach for identification of a distributed multiuser antenna system with multiple CFOs. By over-sampling the received base-band signal, we have converted the multiple-input/single-output (MISO) problem into a MIMO one. Blind MIMO system estimation yields the system response, and
MIMO input recovery yields the decoupled transmitted signals, each one containing a CFO. By exploring the structure of the MIMO systems response we can obtain a coarse estimate of the CFOs, which can be combined with a decision feedback PLL to compensate for the CFOs in the decoupled transmitted signals. The proposed blind method has full acquisition range for normalized CFOs. We have provided a Cramer-Rao bound (CRB) for the proposed blind CFO estimators. The analytical results have been validated via simulations.

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Fig. 4. BER vs SNR for K=2, 4QAM, T=1024

Fig. 5. MSE of the CFOs for SNR=30 dB
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keywords - Multi-user Systems, Distributed Antenna Systems, Carrier Frequency Offset, Blind MIMO System Identification

1. INTRODUCTION

In any communication system, the received signal is corrupted by carrier-frequency offsets (CFOs) due to the Doppler shift and/or local oscillators drift. The CFO causes a frequency shift and a time-varying rotation of the data symbols, which need to be compensated for at the receiver before symbol recovery. This can be achieved via pilot symbols. However, in the case of mobile systems and rich scattering environments the effects of CFO become time varying and even small errors in the CFO estimate tend to cause large data recovery errors. This necessitates transmission of pilots rather often, a process that lowers data throughput. In this paper we deal with CFO estimation without the need for pilot symbols. In single user systems, or in multi-antenna systems in which the transmitters are physically connected to the same oscillator, there is only one CFO that needs to be estimated. This is typically done via a decision feedback Phase Lock Loop (PLL) at the receiver end. The PLL is a closed-loop feedback control system that uses knowledge of the transmitted constellation to adaptively track both the frequency and phase offset between the equalized signal and the known signal constellation. However, depending on the constellation used during transmission, the PLL can have an M-fold symmetric ambiguity, resulting in a limited CFO acquisition range, i.e., $|F_k| < 1/(8T_s)$ for 4QAM signals, where $T_s$ is the symbol period. Moreover, the PLL typically requires a long convergence time. Alternatively, several methods have been proposed [2], [4], [5], [7] [9] that blindly estimate the CFOs and recover the transmitted symbols using second-order cyclic statistics of the over-sampled received signal. Blind CFO estimation has also been studied in the context of orthogonal frequency-division multiplexing (OFDM) systems, where the CFO destroys the orthogonality between the carriers (see [3] and the references therein).

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the over-sampling factor $P$ is no less than $K$.

Under assumption (A2), it is easy to verify that the rotated input signals $\tilde{s}_k(\cdot)$ are also zero mean, i.i.d. wide sense stationary with nonzero kurtosis. Also, the $\tilde{s}_k(\cdot)$’s are mutually independent for different $k$’s. Assumption (A3) guarantees that the virtual MIMO channel matrix $A$ in (5) has full rank with probability one. If the delays of users are randomly distributed in the interval $[0, T_s/\tau]$, then each row of the channel matrix can be viewed as drawn randomly from a continuous distribution, thus the channel matrix has full rank with probability one.

One can apply any blind source separation algorithm (e.g., [1]) to obtain

$$\hat{A} \triangleq \hat{A}P^T$$

(6)

Subsequently, using a least-squares equalizer we can get an estimate of the de-coupled signals $\hat{s}(i)$, within permutation and diagonal scalar ambiguities as

$$\hat{s}(i) = (\hat{A}^H \hat{A})^{-1} \hat{A}^H y(i) = e^{jArg(-\Lambda)} A^{-1} P^T \tilde{s}(i)$$

(7)

Denoting by $\theta_k$ the $k$–th diagonal element of $Arg\{A\}$, the $k$–th separated input signal can be expressed as

$$\hat{s}_k(i) = s_k(i)e^{j(-\theta_k + 2\pi f_k P t_i)}$$

(8)

Based on (8), any single CFO blind estimation method could be applied to recover the input signal. Those methods can benefit by a CFO estimate provided by the channel matrix estimate as follows. The phase of the estimated channel matrix $\hat{A}$ equals

$$\Psi = Arg\hat{A} = \begin{pmatrix} 2\pi f_1 + \phi_1 & \ldots & 2\pi f_K + \phi_K \\ \vdots & \ddots & \vdots \\ 2\pi f_1 P + \phi_1 & \ldots & 2\pi f_K P + \phi_K \end{pmatrix} P$$

(9)

where $\phi_k = Arg\{a_k\} + \theta_k$, which accounts for both the phase of $a_k$ and the estimated phase ambiguity in (8). The least squares estimate of $f_k$ is given by

$$\hat{f}_k = \frac{1}{2\pi} \frac{P(\sum_{p=1}^{P} P^2 \Psi_{p,k}) - (\sum_{p=1}^{P} P^2)^2}{P(\sum_{p=1}^{P} P^2)}$$

(10)

We can write $\hat{f}_k = f_k + \epsilon_k$ where $\epsilon_k$ represents the estimation error.

Noting that the de-coupled signals $\tilde{s}_j(i)$ in (8) are shuffled in the same manner as the estimated CFOs in (10), we can use the estimated CFOs to compensate for the effect of CFO in the decoupled signals (8) and get estimates of the input signals as

$$\hat{s}(i) = e^{jArg(-\Lambda)} P^T s(i)$$

(11)

Due to the residual error in the estimated CFOs, we can only compensate for a majority of the effect of CFO in (8) and obtain

$$\hat{s}_k(i) = s_k(i)e^{j(-\theta_k - 2\pi f_k \epsilon_k P t_i)}$$

(12)

Let us apply the PLL to the recovered signals $\hat{s}_j(i)$ in (12), to further mitigate the effect of residual CFO $\epsilon_k$. For 4QAM signals, as long as $|P\epsilon_k| < 1/8$, it can be effectively removed by the PLL. Thus, the CFO estimator (10) can prevent the symmetric ambiguity of the PLL, and can also greatly reduce the convergence time of PLL. From (9), we can see that the CFO estimator will achieve full acquisition range for the normalized CFO, i.e., $|f_k| < 1/2$, which means we can deal with all continuous CFOs in the range $F_k < P/(2T_s)$.  

3. BLIND CHANNEL ESTIMATION AND COMPENSATION OF THE CFOS

Let us make the following assumptions.

- **A1** For each $m = 1...P$, $w_m(\cdot)$ is a zero-mean Gaussian stationary random processes with variance $\sigma^2_w$, and is independent of the inputs.
- **A2** For each $k$, $s_k(\cdot)$ are a zero mean, independent identically distributed (i.i.d.) stationary with nonzero kurtosis, i.e., $\gamma_{sk} = Cum[s_k(i), s_k(i), s_k(i), s_k(i)] \neq 0$. The $s_k$’s are mutually independent, we can further assume that each user has unit transmission power, then $C_s = I$.
4. CRAMER-RAO LOWER BOUND

To evaluate the large sample performance of the proposed method, we establish the Cramer-Rao lower bound according to [12]. Via central limit theory arguments, the received signals $y$ can be approximated as complex Gaussian signal, with zero mean, and covariance matrix given by

$$C_y = A C_1 A^H + \sigma_w^2 I = A A^H + \sigma_w^2 I$$ (13)

The covariance matrix is valid under assumption A1) and A2). The Gaussian assumption of the received signal is reasonable since the received signal is a linear mixture of i.i.d. signals.

Let

$$\alpha = [f^T, \rho^T, \sigma_w^2]^T$$ (14)

where $f^T = [f_1, ..., f_K]^T$ is the vector of unknown CFOs, and $\rho^T = [\tau_1, ..., \tau_K]^T$ is the vector of random delays. The parameter we are interested in are the CFOs $f$, while $\rho$ and $\sigma_w^2$ are the nuisance parameters.

Under the previous assumptions and the Gaussian approximation, the Fisher Information Matrix (FIM) for the parameter vector $\alpha$ is given by [12]

$$\text{FIM}_{l,n} = \text{Tr}(\frac{\partial C_y}{\partial \alpha_l} C_y^{-1} \frac{\partial C_y}{\partial \alpha_n} C_y^{-1}), \quad l, n = 1, ..., 2K + 1$$ (15)

Since we are only interested in the CFO parameter $f$, following the derivation in [12], we can obtain that

$$\frac{1}{T} \text{CRB}^{-1}(f) = G^H G - G^H \Delta (\Delta^H \Delta)^{-1} \Delta^H G = G^H \Pi_{\Delta}^2 G$$

where $G$ and $\Delta$ are defined as

$$\frac{1}{T} \text{FIM} = (\frac{\partial c_y}{\partial f^T})^H (C_y^T \otimes C_y^{-1}) (\frac{\partial c_y}{\partial f}) = \begin{bmatrix} G^H \\ G \end{bmatrix} \begin{bmatrix} G \\ \Delta \end{bmatrix}$$

where $c_y = \text{vec}(C_y)$ is a $P^2 \times 1$ vector constructed from columns of $C_y$, and $G$ is of dimension $P^2 \times K$, while $\Delta$ is of dimension $P^2 \times (K + 1)$.

To proceed, we just need to evaluate the derivatives of $c_y$ with respect to $\alpha$. First consider $\partial c_y / \partial f^T$, it holds that

$$\frac{\partial c_y}{\partial f_k} = \text{vec}( \frac{\partial C_y}{\partial f_k} ) = \text{vec}( [0 \cdots d_k \cdots 0] A^H + A_0 [0 \cdots d_k^H \cdots 0]^T )$$

with $d_k = [2^{2\pi f_k} a_k \otimes [1, ..., P]^T]$, where $\otimes$ is the Hadamard matrix product.

Similarly, we can get $\partial c_y / \partial \tau_k$:

$$\frac{\partial c_y}{\partial \tau_k} = \text{vec}( \frac{\partial C_y}{\partial \tau_k} ) = \text{vec}( [0 \cdots e_k \cdots 0] A^H + A_0 [0 \cdots e_k^H \cdots 0]^T )$$

with $e_k = [2^{2\pi f_k} \frac{\partial p(\tau_k - \tau_0)}{\tau_k - \tau_0}, ..., 2^{2\pi f_k} \frac{\partial p(T_s - \tau_0)}{T_s - \tau_0}]^T$.

Finally, we have that $\frac{\partial c_y}{\partial \sigma_w^2} = \text{vec}(C_y^{-1})$. Now we have all the ingredients to evaluate $\text{CRB}(f)$ from (16).

5. SIMULATION RESULTS

In this section, we verify the validity of the proposed method via simulations, under the following assumptions. The channel coefficients $a_k, \ k = 1, ..., K$ are zero-mean Gaussian random variables. The waveform $p(.)$ used here is hamming window. The continuous CFOs are randomly picked in the range $[-\frac{2\pi}{T_s}, \frac{2\pi}{T_s}]$. For the wide interval in $\frac{2\pi}{T_s}$, we assume that the CFOs are uniformly distributed in the range $[0, 2\pi / T_s]$.

6. CONCLUSION

In this paper we have proposed a novel blind approach for identification of a distributed multiuser antenna system with multiple CFOs. By over-sampling the received base-band signal, we have converted the mufitple-input/single-output (MISO) problem into a MIMO one. Blind MIMO system estimation yields the system response, and

The Cramer-Rao lower bound is derived for both pilots-based method and the proposed method at different data lengths and SNR set to 30dB. For the pilots method, each user transmitted a pilot signal of length 32, and the pilots were random sequences uncorrelated between different users. In Fig. 1 we show the Mean Squares Error (MSE) for the CFO estimator (10) for different values of the over-sampling factor $P$. To make the comparison fair for different over-sampling factor $P$, the MSE is calculated based on $\frac{1}{K} \sum_{k=1}^{K} [ \frac{1}{K} \sum_{k=1}^{K} ((\hat{f}_k - f_k) P)_T ]^2$. We can see that by increasing $P$ we can get more accurate estimates of the CFOs. In Fig. 2, we show the Bit Error Rate (BER) for different $P$’s. For both the blind and the training based methods, the BER is calculated based on the recovered signals after the PLL. As expected, the BER performance also improves by increasing $P$. The proposed method appears to work well even for small data length.

Next we show the performance of both methods at various noise levels. We set the packet length $N$ to 1024. In Fig. 3, we show the MSE of the blind CFO estimator (10) as well as the training based method. We can see that by increasing $P$ we can get more accurate estimates of the CFOs. In Fig. 4, we show the BER performance after PLL for both blind and training based methods. We can see that the proposed blind method has almost the same performance to the training based method for SNR lower than 20dB, while the training based method can achieve better BER performance for high SNR.

The mean square error of the CFO estimator of (10) is plotted against the stochastic CRB derived in Section 4. In Fig. 5, we plotted the MSE of the CFOs, as well as the CRB, as a function of the packet length $T$. We can see that the MSE curves are parallel to the CRB, and no error floor is present in the plot. Hence there is no bias in the estimates and the gap is only due to excess variance in the estimates.

One possible reason for the existence of the excess variance is that we assume that we know the exact channel structure in the derivation of the CRB, i.e., the waveform used in transmission, which reduces the number of the unknown parameters. In the simulations, however, we did not assume any extra knowledge of the channel structure.

We should note that the PLL is important for good symbol recovery. For example, without the PLL, even if the residual error $P_{\epsilon_k} = \hat{F}_k - F_k$ is only 0.001, the constellation will be rotated to a wrong position after 0.25/0.001 = 250 samples for 4QAM signals. To make sure that the PLL does not have the symmetrical ambiguity, we need to guarantee that $|P_{\epsilon_k}| = |(\hat{F}_k - F_k) T_s| < 1/8$ for 4QAM transmission. Thus, on the average, the maximum tolerable MSE for the CFO is in the order of $10^{-2}$. From the simulations, we can see that the achieved compensation is sufficient for practical systems and commonly used modulation schemes.

The blind source separation algorithm used is the JADE method, which was downloaded from http://www.tsi.enst.fr/caro-dos/guidespou.html.

The simulation results show the performance of both the pilots-based method and the proposed method at various noise levels. We set the packet length $N$ to 1024. In Fig. 3, we show the MSE of the blind CFO estimator (10) as well as the training based method. We can see that by increasing $P$ we can get more accurate estimates of the CFOs. In Fig. 4, we show the BER performance after PLL for both blind and training based methods. We can see that the proposed blind method has almost the same performance to the training based method for SNR lower than 20dB, while the training based method can achieve better BER performance for high SNR.

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MIMO input recovery yields the decoupled transmitted signals, each one containing a CFO. By exploring the structure of the MIMO systems response we can obtain a coarse estimate of the CFOs, which can be combined with a decision feedback PLL to compensate for the CFOs in the decoupled transmitted signals. The proposed blind method has full acquisition range for normalized CFOs. We have provided a Cramer-Rao bound (CRB) for the proposed blind CFO estimators. The analytical results have been validated via simulations.

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**Fig. 4.** BER vs SNR for K=2, 4QAM, T=1024

**Fig. 5.** MSE of the CFOs for SNR=30 dB
MSE of CFOs

SNR dB

- $P=4$, blind
- CRB, blind, $P=4$
- $P=4$, pilots
- CRB, training, $P=4$