A New Equilibrium State for Singly Synchronous Binary Asteroids

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Abstract

The evolution of rotation states of small asteroids is governed by the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect, nonetheless some asteroids can stop their YORP evolution by attaining a stable equilibrium. The same is true for binary asteroids subjected to the binary YORP (BYORP) effect. Here we discuss a new type of equilibrium that combines these two, which is possible in a singly synchronous binary system. This equilibrium occurs when the normal YORP, the tangential YORP, and the BYORP compensate each other, and tidal torques distribute the angular momentum between the components of the system and dissipate energy. If unperturbed, such a system would remain singly synchronous in perpetuity with constant spin and orbit rates, as the tidal torques dissipate the incoming energy from impinging sunlight at the same rate. The probability of the existence of this kind of equilibrium in a binary system is found to be on the order of a few percent.

Key words: minor planets, asteroids: general

1. Introduction

The evolution of asteroids is known to be governed by the non-gravitational effects of sunlight. In one scenario an asteroid is sped up by the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect to its fission limit, then it settles into a binary, eventually loses its secondary component, and starts a new YORP cycle. Still, several equilibria exist along this evolutionary path, which can catch asteroids and thus stop their dynamic evolution. The diversity of equilibria is produced by the diversity of different manifestations of YORP (Vokrouhlický et al. 2015), the most important of which are the normal YORP (NYORP; Rubincam 2000), the binary YORP (BYORP; Ćuk & Burns 2005), and the tangential YORP (TYORP; Golubov & Krugly 2012).

First, a single asteroid can be in equilibrium if its NYORP and TYORP compensate each other (Golubov & Krugly 2012). Second, a doubly synchronous binary can be stuck in an equilibrium between NYORP and BYORP (Golubov & Scheeres 2016). Third, a semi-equilibrium state exists between BYORP and tides acting on the secondary (Jacobson & Scheeres 2011), although it does not correspond to a stable state of the primary.

Here we study a more complex and more physically rich equilibrium state, involving all four effects: NYORP, TYORP, BYORP, and tides (see the left panel of Figure 1). The secondary asteroid resides close to its BYORP-tides equilibrium (Jacobson & Scheeres 2011). The primary remains close to its TYORP-NYORP equilibrium (Golubov & Krugly 2012), still getting a positive torque from their difference, which compensates the tidal torque created by the secondary. Therefore, we combine the Jacobson & Scheeres (2011) equilibrium for the secondary with the Golubov & Krugly (2012) equilibrium for the primary to produce a global equilibrium.

In Section 2 we analytically study the simplest model for the effect. In Section 3 we consider observational data on asteroid shapes and on the observed properties of binary systems to estimate how realistic the considered equilibrium is. In Section 4 we briefly discuss the implications of this new type of equilibrium.

2. Theory

The simplest model of the phenomenon applies to a primary asteroid with radius $R_1$ and a secondary asteroid with radius $R_2$, their density $\rho$ is assumed to be the same, and their mass ratio $q = R_1^3/R_2^3$ is assumed to be small. The secondary rotates around the primary in a circular orbit of radius $r = R_1a$, which lies in the plane of the system’s heliocentric orbit. Rotation axes of both asteroids are perpendicular to this plane. The system is singly synchronous; i.e., the secondary’s rotation rate $\omega_2$ coincides with its orbital rotation rate, while the primary’s rotation rate $\omega_1$ is different.

The tidal torque created by the primary on the secondary is given by the expression (Jacobson & Scheeres 2011)

$$T_{\text{tides}} = \frac{2\pi q^2 K_1 \rho R_1^3 \omega_1^2}{Q a^6} \text{sgn}(\omega_1 - \omega_2).$$

(1)

In this equation $Q$ is the quality factor, $K_1$ is the tidal Love number, and $\omega = (4\pi G\rho/3)^{1/2}$ is the critical angular velocity of the primary.

The NYORP and BYORP torques are given by (Golubov & Scheeres 2016)

$$T_{\text{NYORP}} = \frac{C \Phi R_1^3}{c},$$

(2)

$$T_{\text{BYORP}} = \frac{B \Phi R_1^3}{c} q^{7/2} a.$$  

(3)

Here $C$ is the NYORP coefficient, $B$ is the BYORP coefficient, $\Phi$ is the solar constant at the asteroid’s heliocentric radius, and $c$ is the speed of light.

The approximate expression for TYORP is (Golubov 2017)

$$T_{\text{TYORP}} = \frac{D \Phi R_1^3}{c} e^{(\log\theta - \log\theta_0)^2/c^2},$$

(4)

where the constants are $\nu = 1.518$, $\log\theta_0 = 0.580$, and the TYORP coefficient is $D = 9\mu_0$, where $\mu = 0.00644$ and $n_0$ characterizes the surface density of boulders.
25143 Itokawa $n_0$ is estimated to be $0.028 \pm 0.018$ (Ševeček et al. 2015; Golubov 2017), thus $D = (1.6 \pm 1) \cdot 10^{-3}$. The thermal parameter is determined as

$$\theta = \frac{(C_r \rho \kappa \omega_1)^{1/2}}{(1 - \alpha) \Phi^{3/4} (\sigma T)^{1/4}}. \quad (5)$$

The physical meaning of the constants used in this equation, as well as their values used for calculations, are given in Table 1.

The dynamics of the primary and of the secondary are described by the following equations:

$$I_1 \frac{d \omega_1}{dt} = T_{\text{TYORP}} + T_{\text{NYORP}} - T_{\text{tides}}, \quad (6)$$

$$M_2 R_1^2 \frac{d (\omega_2 a^2)}{dt} = T_{\text{BYORP}} + T_{\text{tides}}, \quad (7)$$

with $M_2$ being the mass of the secondary and $I_1$ the moment of inertia of the primary. The torques on the right-hand sides should be substituted from Equations (1)–(4), and the orbit’s radius can be excluded from the equations via Kepler’s third law $a = (\omega_d/\omega_2)^{2/3}$. Thus we arrive at a closed system for two variables, $\omega_1$ and $\omega_2$.

The right panel of Figure 1 graphically illustrates the possibility of equilibrium in the system described by Equations (6)–(7). The right-hand side of the right panel shows $\omega_2$ as given by Equation (7) in the case $B < 0$ and $\omega_1 > \omega_2$, which we assume henceforth. We see that in this (and only in this) case a stable equilibrium for $\omega_2$ emerges. This equilibrium $\omega_2$ uniquely determines the tides, thus we treat tides as a constant in the right-hand side of Equation (6). If the total torque of NYORP and tides on the primary is slightly negative, then the configuration shown in the left-hand side of the right panel of Figure 1 can emerge, which has a stable equilibrium. Generally, stability in $\omega_1$ and $\omega_2$ separately does not guarantee stability during their simultaneous evolution. But if $q \ll 1$, then relaxation in $\omega_2$ occurs much faster than in $\omega_1$, and the system rapidly collapses to the equilibrium value of $\omega_2$, and then slowly settles to the equilibrium value of $\omega_1$. An example of such a stable equilibrium is shown in Figure 2, which is constructed by numerically solving Equations (6)–(7).

To find the equilibrium in the system, we must equate to zero the right-hand sides of Equations (6) and (7),

$$T_{\text{TYORP}} + T_{\text{NYORP}} = T_{\text{tides}} = -T_{\text{BYORP}}. \quad (8)$$

By substituting Equations (1)–(4) into Equation (8) and solving the resulting set of equations, we get expressions for the equilibrium dimensionless distance between the asteroids, $a^*$, and the equilibrium thermal parameter, $\theta^*$:

$$a^* = \left( -\frac{2 \pi c K_i R_i^2 \omega_d^2 q^{4/3}}{B \Phi Q} \right)^{1/7}, \quad (9)$$

$$\theta^* = \theta_0 \exp \left( \phi \log \frac{D}{-B q^{7/3} a^* - C} \right). \quad (10)$$

Then we find the equilibrium value $\omega_1^*$ from Equation (5) and $\omega_2^*$ from Kepler’s law

$$\omega_1^* = \frac{(1 - \alpha) \Phi^{3/4} (\sigma T)^{1/4} \theta^*}{C_r \rho_1 \kappa}. \quad (11)$$

| Table 1 | Constant Standard Values |
|---------|--------------------------|
| Notation | Value | Meaning |
| $q$ | 0.01 | mass ratio |
| $R_1$ | 1000 m | primary’s radius |
| $R_2$ | $R_1 q_{2/3}$ | secondary’s radius |
| $\omega_1$ | | primary’s rotation rate |
| $\omega_2$ | | secondary’s rotation rate |
| $\omega_d$ | $(4\pi G \rho/3)^{1/2}$ | critical rotation rate |
| $a$ | $(\omega_1/\omega_2)^{3/2}$ | distance between the components in terms of $R_1$ |
| $\Phi$ | 30,000 | ratio of the quality factor to the tidal Love number |
| $\kappa$ | $2 \frac{\alpha}{\omega_d}$ | TYORP constant |
| $\rho_1$ | | density of the asteroids |
| $\rho_s$ | 2.5 $\frac{m}{cm^3}$ | density of stones |
| $C_s$ | 2.8 $\frac{cal}{cm^2}$ | heat capacity of stones |
| $\epsilon$ | 0.26 $\frac{W}{m K}$ | heat conductivity of stones |
| $\alpha$ | 0.1 | Stefan–Boltzmann constant |
| $\sigma$ | 0.9 $\frac{W}{m^2 K^4}$ | thermal emissivity |

| Explanation of the equilibrium. Left: sketch of a system capable of YORP-BYORP equilibrium. Right: illustration of evolutionary equations for $\omega_1$ and $\omega_2$. Contributions of different torques are shown separately. Stable equilibria are marked by green circles, unstable equilibrium by a red circle. Small green arrows show the direction of evolution in the vicinity of stable equilibria. |
down to the equilibrium.

Starting from different initial conditions, the system settles

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(See Golubov & Scheeres 2016 for the definition of $a_{\text{max}}$). Still, these conditions are not very restrictive: $1 < a$ necessarily follows from Equation (14), while $a_{\text{max}}$ is never reached in our simulations, as discussed in the following section.

3. Applications

To estimate the importance of this kind of equilibrium for

We then use the same sets of asteroid shape models to

4. Discussion

We have found a new kind of equilibrium of a singly synchronous binary asteroid system. In this equilibrium, the

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secondary asteroid resides in an equilibrium between BYORP and tides (similar to Jacobson & Scheeres 2011), while the primary has an equilibrium between TYORP, NYORP, and tides (reminiscent to Golubov & Krugly 2012, but with tides added). In this system, radiation torques input angular momentum to the primary and take away the equivalent angular momentum from the secondary. Tides serve as a link for transporting angular momentum from the primary to the secondary. Tidal friction permanently consumes energy, but the energy of the system is perpetually supplanted by the mechanical work performed by radiation torques.

The probability of the existence of this equilibrium is found to be on the order of a few percent, and is thus comparable with the previously estimated probabilities of TYORP-NYORP equilibria of single asteroids (Golubov & Krugly 2012) and NYORP-BYORP equilibria of doubly synchronous asteroids (Golubov & Scheeres 2016). This mechanism can extensively exclude asteroids from their YORP-cycles and lock them in stable equilibria in the form of singly synchronous binaries. Such effects will strongly bias the observed statistics of binary asteroid spin rates.

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**Figure 3.** Probability of equilibria. All values of parameters are taken from Table 1, if not stated otherwise. Top left: distribution of the equilibrium rotation states over $\omega_1$ and $\omega_2$. Simulations for the DAMIT and radar shapes with the standard values of the relevant parameters (Table 1) are overplotted with the observed distribution in binaries. Top right: the same, but with $Q/K_1 = 10,000$ and $\kappa = 0.03$ W m$^{-2}$ K$^{-1}$. Bottom left: the same, but with $Q/K_1 = 1,000,000$ and all values of $C$ and $B$ divided by 100. Bottom right: probability of equilibrium as a function of the mass ratio $q$. Different curves correspond to different values of relevant parameters. The first two lines are calculated for the radar and DAMIT shapes with the standard values of the relevant parameters from Table 1. The third line ("decreased YORP") is computed for the YORP coefficients $C$ and $B$ that are twice smaller than in the radar database. For the remaining six lines the radar database is used, and all parameters are standard (Table 1), except for one parameter that is altered.
Erratum: “A New Equilibrium State for Singly Synchronous Binary Asteroids” (2018, ApJL, 857, L5)

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In the published article, there was an error in Equation (11): the power index of $\theta^*$ is missing. The correct equation reads as

$$\omega_1^* = \frac{(1 - \alpha \phi)^{3/2} (\sigma r)^{1/2} (\theta^*)^2}{C_1 \rho_s \mu}.$$  \hspace{1cm} (11)

This error affects Figure 3. The corrected version of Figure 3 is presented here. To get a better similarity to the observational data, different values of parameters are now used in the upper right and lower left panels of Figure 3, as given in the legend. This error does not affect the main conclusions of the published article.

Figure 3. Probability of equilibria. All values of parameters are taken from Table 1, if not stated otherwise. Top left: distribution of the equilibrium rotation states over $\omega_1$ and $\omega_2$. Simulations for the DAMIT and radar shapes with the standard values of the relevant parameters (Table 1) are overplotted with the observed distribution in binaries. Top right: the same, but with $\kappa = 0.1 \text{ W m}^{-2} \text{ K}^{-1}$. Bottom left: the same, but with $Q/K_1 = 100,000$ and all values of $C$ and $B$ divided by 10. Bottom right: probability of equilibrium as a function of the mass ratio $q$. Different curves correspond to different values of relevant parameters. The first two lines are calculated for the radar and DAMIT shapes with the standard values of the relevant parameters from Table 1. The third line (“decreased YORP”) is computed for the YORP coefficients $C$ and $B$ twice smaller than in the radar database. For the remaining six lines the radar database is used, and all parameters are standard (Table 1), except of one parameter that is altered.
