Extended electrodynamics and SHP theory

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Abstract. This work shows incompleteness and inconsistency in classical electrodynamics (CED) and quantum electrodynamics (QED). Extended electrodynamics (EED) resolves these issues. Stueckelberg-Horwitz-Piron (SHP) theory is equivalent to EED with important implications.

1. Introduction

QED has been called “the ‘most correct’ scientific theory in history” [1], “the most accurate physical theory ever built” [2], and “the most successful physical theory” [3]. CED is the basis for QED and has its foundation in Feynman’s ‘proof’ [4] of Maxwell’s equations. Thus, CED and QED are considered complete and closed, needing no re-evaluation. However, the assumptions for Feynman’s proof are based on empirical results, rather than first-principles logic. Falsifiability [5] states that an empirical theory cannot be proved, but can be disproved by contrary test result(s). This work first examines CED and QED and finds incompleteness and inconsistencies, as follows.

The magnetic ($B$) and electric ($E$) fields can be written in terms of the scalar ($\Phi$) and vector ($A$) potentials as [6]:

$$B = \nabla \times A$$

and

$$E = -\nabla \Phi - \frac{\partial A}{\partial t}.$$  

$B$ and $E$ are invariant under the gauge transformation: $\Phi \rightarrow \Phi - \frac{\partial \eta}{\partial t}$ and $A \rightarrow A + \nabla \eta$, where $\eta$ is the gauge function [7] that has an $\infty$-infinitude of choices. This transformation gauges away the irrotational component of $A$ [8-9] (also called the curl-free, gradient-driven, or longitudinal component, $A^L$).

$A^L$ has been measured in the micro-, meso-, and macro-scale domains over the full frequency spectrum. First, the Aharonov-Bohm effect (ABE) [10-11] involves a quantum particle ($q$ = charge) that travels along a path ($P$) in a region with zero magnetic field ($B=0$) and $A^L \neq 0$. A quantum particle can traverse different paths simultaneously, causing a phase difference ($\Delta \phi = q \Phi_B / \hbar$) by placing a magnetic solenoid between the slits of a double-slit experiment. Here, $\hbar$ is Planck’s constant divided by 2π; $\Phi_B$ is the magnetic flux enclosed by the particle’s path. Tests by Tonomura et al. [12] and Osakabe et al. [13] validated this prediction. The phase shift has also been observed in micrometer-sized metal rings [14-16], showing that electrons keep their phase coherence despite diffusion in mesoscopic samples. Second, Varma et al. [17-21] measured $A^L$ over centimeters via a low-current electron beam that propagated linearly along a $B$-field. The current showed time-periodic variations with a linear change in $A^L$, contrary to the CED prediction. The Maxwell-Lodge effect (MLE) is a third observation of $A^L$. Oliver Lodge applied an alternating voltage to an iron-core, toroidal solenoid...
and measured a voltage in the secondary coil via $E = -\partial A/\partial t$, despite no magnetic field outside the primary coil. Blondel [22] and Rousseaux et al. [23] replicated Lodge’s experiment by a long solenoidal magnet that was encircled at its mid-plane by a secondary coil. An alternating current produced a magnetic field inside the solenoid, but no magnetic field outside the solenoid. $A^L$ was time-varying both inside and outside the solenoid and parallel to the current. Daibo et al. [24-26] measured the MLE, using a coiled coil: a very long, flexible solenoid whose return-current wire ran through the coil’s center to create a pure vector potential with $B = 0$. The coiled-coil was wound around a hollow, (super)conducting cylinder (primary, also called the vector-potential transformer--VPT) and was driven with alternating current. Several secondary coils were passed through the hollow interior of the VPT. Path-independent voltages were measured in the secondary coils. These tests show that $A^L$ exists, contrary to being gauged away by CED.

An irrotational vector potential has the form, $A^L = \nabla \alpha$. Non-zero $A^L$ gives $B = \nabla \times A = \nabla \times \nabla \alpha = 0$ [27] and $E = -\nabla \Phi - \partial A/\partial t = -\nabla (\Phi + \partial \alpha/\partial t) = \nabla \epsilon \nabla \alpha$. Substitution of these expressions into Ampère’s law yields $\nabla \times B + \epsilon \mu \nabla \nabla \alpha = \epsilon \mu \nabla \nabla \alpha = \epsilon \mu \nabla \alpha$, showing that a longitudinal vector potential implies a longitudinal current of the form, $J^L = \epsilon \nabla \partial \partial / (\Phi - \partial \alpha/\partial t) = \nabla \kappa$. Here, $\epsilon$ and $\mu$ are the permittivity and permeability of the propagation medium (not necessarily vacuum); $\alpha$, $\epsilon$, and $\kappa$ are appropriate space-time, scalar functions. The net result is [28]: $A^L = \nabla \alpha \Leftrightarrow J^L = \nabla \kappa \Leftrightarrow E = \nabla \epsilon \Leftrightarrow B = 0$. Thus, CED and QED do not properly describe $A^L$, $E^L$, and $J^L$, leading to an incomplete description of longitudinal effects.

The incompleteness of the previous paragraph has been validated by experiments. All living processes involve ion-concentration-gradient-driven currents ($J^L$) across cell membranes [29]. These currents are the basis for electro-physiological, medical diagnostics and treatments. For example, scalp brain-waves arise from irrotational currents [30]. Longitudinal currents (driven by $E^L$) also occur in atmospheric lightning [31] and microscopic arcs in tape peeling [32-33]. The 2010 test [33] measured an angular X-ray distribution that is bounded from below by ordinary Bremsstrahlung and bounded from above by polarizational Bremsstrahlung. Neither CED model predicts the 20% peak between emission angles of 80° to 100°.

The Aharonov-Bohm effect [10-11] also predicts a phase shift ($\Delta \varphi$) that arises from the scalar potential ($\Phi$). Namely, $\Delta \varphi = -q \Phi t/h$ enables direct measurement of $\Phi$ for $E = 0$. Here, $t$ is the time spent in the potential. Van Oudenaarden et al. [34] validated this prediction by quantum interference of electrons in metal rings with two tunnel junctions and a voltage difference between the two ring segments. These results demonstrate that $A$ and $\Phi$ are independent, physically measureable fields, rather than mathematical conveniences, as commonly used in CED.

Richard Feynman (1965 Nobel Prize in physics) made the following comment on page 128 of his book [35]: “The shell game that we play…is technically called ‘renormalization.’ But no matter how clever the word, it is what I would call a dippy process! Having to resort to hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. … I suspect that renormalization is not mathematically legitimate.” Moreover, Dyson proved that all renormalized power-series, perturbation expansions in QED have zero-radius of convergence [36], making the results meaningless. Haag’s theorem [37] states that two Hilbert-space solutions can be unitarily inequivalent, requiring that a “proper” result be chosen from an $\mathcal{N}_\infty$-infinitude of forms. Haag’s theorem is inapplicable [38] to the Stueckelberg Lagrangian [39], which is the basis for EED. These examples show the inconsistency and incompleteness of QED (a local field theory), arising from analogous issues in CED (also a local field theory). New physics is needed [40].

These theoretical and experimental results falsify both CED and QED. Section 2 describes EED that resolves the above incompleteness and inconsistencies. Section 3 shows that EED is equivalent to SHP theory. Section 4 presents our conclusions, including implications of the EED-SHP equivalence.
2. Extended Electrodynamics

A remarkably large literature exists on extended electrodynamics and related topics from 1932 to the present [28,39,41-86]. Woodside’s seminal work [56] showed that EED is provably unique, using retarded potential solutions in Minkowski 4-space. The resultant dynamical equations are:

\[
\begin{align*}
E &= -\nabla \Phi - \frac{\partial A}{\partial t}; \\
B &= \nabla \times A; \\
C &= \nabla \cdot A + \frac{1}{c^2} \frac{\partial \Phi}{\partial t}; \\
\nabla \times B - \frac{1}{c^2} \frac{\partial E}{\partial t} - \nabla C &= \mu J; \\
\nabla \cdot E + \frac{\partial C}{\partial t} &= \frac{\rho}{\varepsilon}.
\end{align*}
\]

Here, \(\rho\) is the electrical charge density; \(c, \varepsilon,\) and \(\mu\) are the speed of light, the permittivity, and permeability of the propagation medium, respectively (not necessarily vacuum). Eqs. (1)-(2) are equivalent to Faraday’s law and the no-magnetic-monopoles equation, respectively. Eq. (4) uniquely decomposes \(J\) into solenoidal (\(\nabla \times B\)) and irrotational (\(\nabla C\)) components, in accord with the Helmholtz theorem [6]. Eq. (6) is the Stueckelberg Lagrangian [39] without the Maxwell-Proca term. Eq. (6) implies [45] Eqs. (1)-(5) and vice versa [51-52].

The \(A\)-wave equation [7] is obtained [28,60] by replacing \(B, E,\) and \(C\) in Eq. (4) with Eqs. (1)-(3); noting that \(\nabla \times \nabla \times A = \nabla (\nabla \cdot A) - \nabla^2 A\) [27]; and using \(\partial \nabla C / \partial t - \nabla \partial C / \partial t = 0\). The usual \(B\)-wave equation [7] arises from the curl of Eq. (4); use of Faraday’s law; use of \(\nabla \times \nabla \times B = \nabla (\nabla \cdot B) - \nabla^2 B\) [27]; and noting \(\nabla \times C = 0\) [27]. The usual \(E\)-wave equation [87] comes from the curl of Faraday’s law; use of \(\nabla \times \nabla \times E = \nabla (\nabla \cdot E) - \nabla^2 E\) [27]; substitution for \(\nabla \times B\) from Eq. (4); substitution for \(\nabla \cdot E\) from Eq. (5); and use of \(\partial \nabla C / \partial t - \nabla \partial C / \partial t = 0\). The \(\Phi\)-wave equation [7] is obtained by substitution of \(E\) and \(C\) from Eqs. (1) and (3) into Eq. (5); and use of \(\partial \nabla \cdot A / \partial t - \nabla \partial A / \partial t = 0\). The noteworthy results are: (a) derivation of the wave equations for \(A\) and \(\Phi\) without a gauge condition; (b) \(A\) and \(\Phi\) as independent, physically measurable fields; (c) classical TEM waves, together with longitudinal components.

The CED form of Gauss’ law has a well-known, media-interface matching condition [6]:

\[
\varepsilon_2 E_{2n} - \varepsilon_1 E_{1o} = \rho_d.
\]

Substitution of the expression for \(E\) is terms of \(A\) and \(\Phi\) into Eq. (7) gives [28,60]:

\[
\varepsilon_2 \left( -\nabla \Phi - \frac{\partial A}{\partial t} \right)_{2n} - \varepsilon_1 \left( -\nabla \Phi - \frac{\partial A}{\partial t} \right)_{1n} = \rho_d.
\]

Here, the subscript, ‘\(n\)’, denotes the component normal to the media interface. The subscripts ‘1’ and ‘2’ identify the two media. The \(\Phi\)-wave equation also has a media-interface matching condition by taking a Gaussian pill box with the end faces parallel to the interface in regions 1 and 2. Noting that \(\nabla^2 \Phi = \nabla \cdot \nabla \Phi\), the divergence theorem in the limit of zero pill-box height yields:

\[
-\left( \varepsilon \nabla \Phi \right)_{2n} + \left( \varepsilon \nabla \Phi \right)_{1n} = \rho_d.
\]
The interface surface-charge density is $\rho_A$. Eqs. (8)-(9) are inconsistent under CED [28,60], and cannot be ascribed to writing the equations in terms of $A$ and $\Phi$, since $B$ and $E$ are gauge invariant [7]. EED resolves this inconsistency.

The CED form of Ampere’s law has a media-interface matching condition [6]:

$\mu_i B_{1t} - \mu_2 B_{2t} = \mu_i \left( \nabla \times A \right)_{1t} - \mu_2 \left( \nabla \times A \right)_{2t} = J_A$.

Here, ‘$t$’ is the tangential field component at the media interface. The $A$-wave equation also has a matching condition by taking a Gaussian pill box with the end faces parallel to the interface in regions 1 and 2. Noting that $\nabla^2 A = \nabla \cdot \nabla A$, the divergence theorem in the limit of zero pill-box height gives:

$$-\left[ \frac{(\mathbf{n} \cdot \nabla) A}{\mu} \right]_1 + \left[ \frac{(\mathbf{n} \cdot \nabla) A}{\mu} \right]_2 = J_A.$$  \hspace{1cm} (11)

$J_A$ is the surface-current parallel to the interface; $\mathbf{n}$ is the unit vector normal to the interface. As before, the disparity between Eqs. (10)-(11) is not due to writing the equations in terms of $A$ and $\Phi$, since $B$ and $E$ are gauge invariant [7]. EED resolves this inconsistency.

A wave equation for C [28,60] arises from the divergence of Eq. (4); use $\varepsilon \mu (\partial / \partial t)$ on Eq. (5); noting $\nabla \cdot (\nabla \times B) = 0$ [27]; and summing of the results to yield:

$$\nabla^2 C - \frac{\partial^2 C}{\partial c^2 t^2} = -\mu \left( \frac{\partial \rho}{\partial t} + \nabla \cdot J \right).$$  \hspace{1cm} (12)

Eq. (12) is local in space and time. However, all experiments are performed over a finite spatio-temporal domain ($\Delta x$ and $\Delta T$), i.e., a spatio-temporal average. A long-time average of Eq. (12) gives $\partial \rho / \partial t + \nabla \cdot J = 0$, in accord with long-standing experiments that validate classical charge balance [88]. For example, the lower bound on electron lifetime for charge balance has been carefully measured as $\geq 6.6 \times 10^{28}$ years [89] (decay into two $\gamma$-rays, each at an energy of $me c^2 / 2 = 0.256$ MeV; $me =$ electron mass). Nevertheless, long-time charge conservation is not inconsistent with charge non-conservation over short-time scales, $\Delta T \leq \Delta t$, per the Heisenberg uncertainty relation, $\Delta E \Delta t \geq \hbar / 2$. Here, $\Delta E$ is the charged-quantum-fluctuation energy. Eq. (12) can be interpreted as charge non-conservation (particle-antiparticle fluctuations [PAPF]) driving $C$, and vice versa, not unlike energy-fluctuations driving mass-fluctuations in quantum theory and vice versa [28,60]. Equivalently, the right-hand-side (RHS) of Eq. (12) can be interpreted as PAPF over some non-local region ($\Delta x$), consistent with the Heisenberg uncertainty relation, $\Delta p \Delta x \geq \hbar / 2$. Here, $\Delta p$ is the charged-quantum-fluctuation momentum over a non-local region, $\Delta x$. (While this interpretation is novel, it is consistent with PAPF according to the Heisenberg uncertainty principle.) Eq. (12) then predicts charge conservation on long time-scales (consistent with CED), and exchange of energy between $C$ and PAPF for $\Delta T \leq \Delta t$. Validation of this prediction requires tests, consistent with the Heisenberg uncertainty relation. For example, a test could use the electron $[\Delta E$ (electron)]$=mc^2=0.511$ MeV] corresponding to $\Delta t \approx 6 \times 10^{-22}$ seconds. Such a test is feasible, because subzeptosecond dynamics have been measured [90].

A check of the scalar field ($C$) is needed. Specifically, $C$ should be derivable as a non-zero field by substitution of the Green’s function solutions for $A$ and $\Phi$ into Eq. (3), as follows.

$$C = \nabla \cdot \frac{\mu_0}{4\pi} \int \frac{J(x',t')d^3x'}{|x-x'|} + \varepsilon_o \mu_0 \frac{1}{4\pi \varepsilon_o} \int \frac{\rho(x',t')d^3x'}{|x-x'|}.$$  \hspace{1cm} (13)

The observer’s coordinates are unprimed and independent of the source coordinates (primed):

$$t' = t - \frac{|x - x'|}{c}.$$  \hspace{1cm} (14)

Eq. (15a) arises from interchange of the order of the derivatives in Eq. (13) with respect to $x$ (observer frame), and integrals with respect to $x'$ (source frame). Eq. (15b) cancels $\varepsilon_o$ in the numerator and denominator of the second term in Eq. (15a), and groups the integrand inside the square brackets.
Here, the notation is $R = x - x'$, and $R = |R|$. Eq. (15c) applies the divergence operator $[27]$ to $(J/R)$ and the partial-time derivative to $\rho$, since $r$ is time independent. Eq. (15d) groups the two terms with a factor of $(1/R)$ inside the square brackets, and evaluates the gradient of $(1/R)$ $[27]$.

Eq. (15d) requires evaluation of the terms inside the square brackets via the chain rule:

$$\nabla \cdot \mathbf{J}(x',t') = \sum_{i=1}^{3} \frac{\partial J_i(x',t')}{\partial x_i'} = \sum_{i=1}^{3} \frac{\partial J_i(x',t')}{\partial t'} \frac{\partial t'}{\partial x_i'} = -\sum_{i=1}^{3} \frac{\partial J_i(x',t')}{\partial t'} \frac{2(x_i - x_i')}{2c|x - x'|} = -\frac{R}{cR} \frac{\partial \mathbf{J}(x',t')}{\partial t'};$$  

$$\frac{\partial \rho(x',t')}{\partial t} = \frac{\partial \rho(x',t')}{\partial t'} \frac{\partial t'}{\partial t} = \frac{\partial \rho(x',t')}{\partial t'} \frac{t - |x - x'|}{c} = \frac{\partial \rho(x',t')}{\partial t'} \times 1.$$  

Substitution of Eqs. (16a)-(16b) into Eq. (15) then yields Eq. (17a), which can recast as Eq. (17b):

$$C = \frac{\mu_0}{4\pi} \int d^3x' \left[ \frac{1}{R} \left( \mathbf{E} \cdot \frac{\partial \mathbf{J}(x',t')}{\partial t'} - \mathbf{E} \cdot \frac{\partial \mathbf{J}(x',t')}{\partial t'} - \frac{\partial \mathbf{J}(x',t')}{\partial t'} \frac{\partial \mathbf{J}(x',t')}{\partial t'} \right) \cdot \mathbf{R} \right] \frac{R}{R^2};$$

$$= \frac{\mu_0}{4\pi} \int d^3x' \left[ \frac{1}{R} \left( \mathbf{E} \cdot \frac{\partial \mathbf{J}(x',t')}{\partial t'} - \mathbf{E} \cdot \frac{\partial \mathbf{J}(x',t')}{\partial t'} - \frac{\partial \mathbf{J}(x',t')}{\partial t'} \frac{\partial \mathbf{J}(x',t')}{\partial t'} \right) \cdot \mathbf{R} \right] \frac{R}{R^2}.$$  

Consequently, the term usually set to zero as the Lorenz gauge ($C=0$) is in fact non-zero, presenting a major inconsistency for CED. EED resolves this inconsistency as discussed above.

EED predicts that a scalar-longitudinal wave (SLW, also called the longitudinal-scalar wave or the electroscalar wave) can be obtained from the irrotational form of Eq. (4) in spherical coordinates:

$$-\frac{1}{c^2} \frac{\partial \mathbf{E}^L}{\partial t} - \nabla C = \mu \mathbf{J}^L.$$  

The RHS of Eq. (15) is zero on classical time scales, resulting in a lowest-order solution of [7]:

$$C = \frac{C_o \exp[j(kr - \omega t)]}{r}.$$  

$C_o$ is the scalar field amplitude; $j = \sqrt{-1}$; the wave number $k$ is $(2\pi/\lambda)$ for a wavelength ($\lambda$); $\omega = 2\pi f$ for a frequency ($f$); and $r$ is the spherical radius. $C(r\to\infty)\to 0$ is the boundary condition for Eq. (19), which is trivially satisfied. The energy density of the $C$-field is $(C^2/2\mu)$ $[28,60]$, yielding a constant total energy, $4\pi r^2 (C^2/2\mu)$, through a spherical boundary around a source in arbitrary media, as required. The corresponding electric field solution is $[28,60]$:

$$\mathbf{E}^L = \frac{\mathbf{E}_s \exp[j(kr - \omega t)]}{r}.$$  

Here, $\mathbf{E}_s$ is the longitudinal (radial in spherical coordinates for a monopolar source) E-field amplitude. As before, $\mathbf{E}'(r\to\infty)\to 0$ is satisfied, and the $\mathbf{E}$-field energy density through a spherical boundary
around a source is constant, $4\pi r^2 \left( \varepsilon E^2/2 \right)$. Substitution of Eqs. (19)-(20) into Eq. (18) yields the SLW impedance ($Z$), using $J^t=\sigma \varepsilon E^t$ with $[91] \sigma=\varepsilon_o \varepsilon \omega$, and $\varepsilon=\varepsilon_o (\varepsilon'-j \varepsilon'')$ [28,60]:

$$Z = \frac{\mu_0}{C / \mu_o \mu'} \left[ \frac{1}{\varepsilon_o} \right] 1 - \frac{1}{1 - \tan(\delta^o)}.$$  

(21)

Here, $\varepsilon_o$ and $\mu_o$ are the free-space permittivity and permeability, respectively; $\varepsilon'$ and $\mu'$ are the relative permittivity and permeability respectively (not necessarily vacuum); $\tan(\delta^o)=\varepsilon''/\varepsilon'$. The time-averaged SLW power ($P_{OUT}$) from a monopolar antenna is isotropic and attenuates as $r^{-2}$ [28,60]:

$$P_{OUT} = \frac{I^2 \mu}{2(4\pi r)^2} \sqrt{\frac{\mu}{\varepsilon}}.$$  

(22)

The wave equations for $A$, $C$, $E$, and $\Phi$ are time-reversal invariant, so a SLW transmitter can act as a receiver [28,60] with $\mu=\mu_o (\mu'-j \mu'')$. EED [28] predicts that the SLW is unconstrained by the skin effect, because no magnetic field exists to generate dissipative eddy currents in electrical conductors. Preliminary experimental results [28] are consistent the EED predictions: $r^{-2}$ free-space attenuation and propagation through thousands of skin-depths of solid copper using standard electronic instrumentation. CED cannot explain these test results.

3. Correspondence between EED and the pre-Maxwell equations in SHP

In the Stueckelberg-Horwitz-Piron formalism (SHP) [92,93], the position of a classical (non-quantum) event in spacetime is a function of an external chronological parameter $\tau$

$$x(\tau) = (t(\tau), x(\tau))$$  

(23)

where $\tau$ plays a role similar to that of time $t$ in non-relativistic Newtonian mechanics. Just as classical Maxwell fields are functions of position $x$ and time $t$, SHP fields and potentials are functions of spacetime position $x = (t, x)$ and $\tau$. SHP electrodynamics is found by defining five potentials

$$a(x, \tau) = (a^0, a, a^5)$$  

(24)

that in order to exploit the classical gauge invariance of the free particle Lagrangian (addition of a total $\tau$-derivative), and produce the field strengths

$$e = -\frac{1}{c} \frac{\partial a}{\partial t} - \nabla a^0$$

$$\epsilon = \frac{1}{c^2} \frac{\partial a}{\partial \tau} + \nabla a^5$$

$$b = \nabla \times a$$

$$\epsilon^0 = \frac{1}{c} \frac{\partial a^0}{\partial t} + \frac{1}{c} \frac{\partial a^5}{\partial t}$$  

(25)

where $\sigma = \pm 1$ depending on larger symmetry considerations, and $c_5 < c$ is a speed, so that $c_5 \tau$ is analogous to $c t$. The 3-vector $e$ and scalar $e^0$ are new field strengths arising from the $\tau$-dependence of the 4-vector potential and existence of the fifth potential $a^5$. These field strengths are invariant under the $\tau$-dependent gauge transformations

$$a^0(x, \tau) \rightarrow a^0(x, \tau) - \frac{1}{c} \frac{\partial a^0}{\partial \tau} \Lambda(x, \tau)$$

$$a(x, \tau) \rightarrow a(x, \tau) + \nabla \Lambda(x, \tau)$$

$$a^5(x, \tau) \rightarrow a^5(x, \tau) + \sigma \frac{1}{c} \frac{\partial a^5}{\partial \tau} \Lambda(x, \tau)$$  

(26)

and it is convenient to choose the gauge
which generalizes the Lorenz condition.

The SHP field equations (pre-Maxwell equations) in 3-vector and scalar form are

\[
\begin{align*}
\nabla \cdot \mathbf{e} - \frac{1}{c^2} \frac{\partial \mathbf{e}}{\partial \tau} &= e^0 \\
\nabla \times \mathbf{b} - \frac{1}{c} \frac{\partial \mathbf{e}}{\partial \tau} - \frac{1}{c^2} \frac{\partial \mathbf{e}}{\partial \tau} &= \mathbf{e} \\
\n\nabla \cdot \mathbf{e} + \frac{\partial e^0}{\partial \tau} &= e^j \frac{\partial \mathbf{e}}{\partial \tau} - \mathbf{b} \\
\n\nabla \cdot \mathbf{b} &= 0 \\
\n\nabla \mathbf{e} + \frac{\partial e^0}{\partial \tau} &= \mathbf{e}^j \mathbf{b} \\
\n\n
\nabla e^0 + \frac{1}{c} \frac{\partial e^0}{\partial \tau} + \frac{1}{c^2} \frac{\partial e^0}{\partial \tau} &= 0
\end{align*}
\]

which can be put into correspondence with extended electrodynamics (EED) in the following way. We define

\[
C = \frac{1}{c} \frac{\partial \mathbf{a}^0}{\partial \tau} + \nabla \cdot \mathbf{a}^0 (x, \tau)
\]

as in EED so that

\[
C = -\frac{1}{c^2} \frac{\partial \mathbf{a}^s}{\partial \tau}
\]

follows from the Lorenz condition (27). We now take the 4-vector potential to be \(\tau\)-independent

\[
\begin{align*}
\mathbf{a}^0 (x, \tau) &= \mathbf{A}^0 (x) \\
\mathbf{a} (x, \tau) &= \mathbf{A} (x)
\end{align*}
\]

so that the pre-Maxwell \(\mathbf{e}\) and \(\mathbf{b}\) fields behave like Maxwell \(\mathbf{E}\) and \(\mathbf{B}\) fields

\[
\begin{align*}
\mathbf{e} (x, \tau) &= -\frac{1}{c} \frac{\partial \mathbf{a}^0 (x, \tau)}{\partial \tau} - \nabla \mathbf{a}^0 (x, \tau) = -\frac{1}{c} \frac{\partial \mathbf{A} (x)}{\partial \tau} - \nabla \mathbf{A}^0 (x) = \mathbf{E} (x) \\
\mathbf{b} (x, \tau) &= \nabla \times \mathbf{a} (x, \tau) = \nabla \times \mathbf{A} (x) = \mathbf{B} (x)
\end{align*}
\]

as is also the case in EED. With these conditions, the scalar field strength reduces to:

\[
\begin{align*}
e^0 &= \frac{\sigma}{c} \frac{1}{c} \frac{\partial \mathbf{A}^0}{\partial \tau} + \frac{1}{c^2} \frac{\partial \mathbf{a}^s}{\partial \tau} - \frac{1}{c} \frac{\partial \mathbf{A}^s}{\partial \tau} \\
\frac{\partial e^0}{\partial \tau} &= \frac{1}{c} \frac{\partial \mathbf{a}^s}{\partial \tau} + \frac{1}{c} \frac{\partial \mathbf{a}^s}{\partial \tau} - \frac{1}{c^2} \frac{\partial \mathbf{a}^s}{\partial \tau} = -\frac{1}{c} \frac{\partial \mathbf{C}}{\partial \tau}
\end{align*}
\]

and similarly

\[
\begin{align*}
\mathbf{e} &= \sigma \frac{1}{c^2} \frac{\partial \mathbf{a}^0}{\partial \tau} - \nabla \mathbf{a}^s - \nabla \mathbf{a}^s \\
\frac{1}{c^2} \frac{\partial \mathbf{a}^s}{\partial \tau} &= -\frac{1}{c^2} \frac{\partial \mathbf{a}^s}{\partial \tau} - \nabla \mathbf{a}^s - \nabla \mathbf{a}^s
\end{align*}
\]

(34)
The four homogeneous pre-Maxwell equations now take the form
\[ \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \rightarrow \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \]  
(35)
and
\[ \nabla \cdot \mathbf{b} = 0 \rightarrow \nabla \cdot \mathbf{B} = 0 \]
so that we may ignore equations (36) because they vanish identically given the field definitions. The first inhomogeneous pre-Maxwell equation becomes
\[ \nabla \cdot \mathbf{e} - \frac{1}{c} \frac{\partial \mathbf{e}}{\partial \tau} = \mathbf{e} \mathbf{f}^0 \rightarrow \nabla \cdot \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{C}}{\partial t} = \mathbf{e} \mathbf{f}^0 \]  
(37)
reproducing the Gauss law in EED. The second inhomogeneous pre-Maxwell equation becomes
\[ \nabla \times \mathbf{b} - \frac{1}{c} \frac{\partial \mathbf{e}}{\partial t} - \frac{1}{c} \frac{\partial \mathbf{e}}{\partial \tau} = \mathbf{e} \mathbf{j} \rightarrow \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \nabla \mathbf{C} = \mathbf{e} \mathbf{j} \]  
(38)
reproducing the corresponding expression in EED. The third inhomogeneous pre-Maxwell equation becomes a wave equation for the fifth potential
\[ \nabla \cdot \mathbf{e} + \frac{1}{c} \frac{\partial \mathbf{e}}{\partial t} = -\nabla^2 \mathbf{a}^5 + \frac{1}{c^2} \frac{\partial^2 \mathbf{a}^5}{\partial t^2} = \mathbf{e} \mathbf{j}^5 (x, \tau) \]  
(39)
which can be solved as a Liénard-Wiechert potential from the current \( j^5 \) as
\[ \mathbf{a}^5 (x, \tau) = \frac{1}{4\pi} \int \frac{d^3 \mathbf{x}^\prime}{|\mathbf{x} - \mathbf{x}^\prime|} \left[ t - \frac{|\mathbf{x} - \mathbf{x}^\prime|}{c}, \mathbf{x}^\prime, \tau \right] \]  
(40)
leading to a determination of \( \mathbf{C} \) as
\[ \mathbf{C} = -\frac{\partial \mathbf{a}^5 (x, \tau)}{\partial \tau}. \]  
(41)
Alternatively, one can specify the field \( \mathbf{C} \) and interpret the RHS of (39) as an effective current.

In summary, the EED field structure can be described as the most general configuration of SHP fields in which the \( \mathbf{E} \) and \( \mathbf{B} \) fields are \( \tau \)-independent and behave like Maxwell fields. This is, of course, a very important case in electromagnetism.

4. Conclusions
This work shows that CED and QED are inconsistent and incomplete. Specifically, EED and QED gauge away the longitudinal components [8.9] of \( \mathbf{A}, \mathbf{E}, \) and \( \mathbf{J} \); experiments have measured these longitudinal components at the quantum-, meso-, and macroscopic scales [12-26, 29-34]. Moreover, EED gives inconsistent media-interface-matching conditions, Eqs. (8)-(11). EED and QED assume that the scalar field \( (\mathbf{C}) \) in Eq. (3) is zero (the Lorenz gauge), when a rigorous derivation proves that \( \mathbf{C} \)
is a non-zero, dynamical field. CED cannot explain the scalar-longitudinal wave, which propagates through thousands of skin-depths of solid copper [28]. EED corrects these issues by including the longitudinal fields, while leaving the transverse components unchanged.

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