Birefringence of high-energy $\gamma$-quanta in the single crystals

V.A.Maisheev *

Institute for High Energy Physics, 142284, Protvino, Russia

Abstract

Problems of the experimental observation of the birefringence of high energy $\gamma$-quanta propagating in single crystals are discussed.

1 Introduction

The birefringence of $\gamma$-quanta with energies $> 1$ GeV propagating in single crystals was predicted in [1]. The main process by which $\gamma$-quanta are absorbed in single crystals is the electron-positron pair production. The cross section of the process depends on the direction of linear polarization of the $\gamma$-quanta relative to the crystallographic planes. As a result of interaction with the electric field of the single crystal, a monochromatic, linearly polarized beam of $\gamma$-quanta comprises two electromagnetic waves with different refractive indices, so that linear polarization is transformed into circular polarization or vice versa. This polarization phenomenon would be observed for symmetric orientations of single crystals with respect to the direction of motion of $\gamma$-quanta.

The general case of the propagation of $\gamma$-quanta in single crystals was considered in [2, 3, 4]. In these papers it was shown that the propagating $\gamma$-beam is a superposition of the two elliptically polarized waves and unpolarized $\gamma$-beam obtain some degree of circular and linear polarization after passage through a single crystal. In case, describing in [1], the beam of $\gamma$-quanta is a superposition of the two linearly polarized waves and unpolarized beam obtain only some degree of linear polarization after propagation in single crystals.

It is important to note that no experiments have been performed to date to corroborate the transformation of $\gamma$-beam polarization in single crystals, despite the notable lapse of time since the publication of [1]. It is at least two essential purposes for experimental investigations of the birefringence in single crystals. There are:

1) The nature of phenomenon is a manifestation of the nonlinearity of Maxwell’s equations for the electromagnetic vacuum. Of course, a single crystal contain carriers of electric charge (electron, ions, etc), but their direct presence is significant only if the frequencies of the electromagnetic radiation passing through the single crystal are low, while at high frequencies the fields formed by these charges play the main role. Thus the observation of the birefringence in single crystals is indirect experimental proof of existence of the similar effect in electromagnetic vacuum (see [3] and the literature cited therein);

2) Some possibilities exist to utilize this phenomenon in experiments on modern accelerators (see [4, 6, 7] and the literature cited therein).

*E-mail maisheev@mx.ihep.su
2 Refractive indices of $\gamma$-quanta in single crystals.

Now we have found the refractive indices of $\gamma$-quanta propagating in a single crystal. Below we rewrite the components of a complex permittivity tensor $\varepsilon_{ij} = \varepsilon'_{ij} + \varepsilon''_{ij}, \ i, j = 1, 2$ (the process is determined by the transverse part of the tensor) from paper \[2\]. Let us consider a high-energy beam of $\gamma$-quanta moving at a small angle $\theta$ with a reciprocal lattice axis defined by vector $\mathbf{G}_1$. Then in the Cartesian system of coordinates such that one axis is oriented approximately parallel to the direction of motion of the lattice axis defined by vector $\mathbf{G}_1$ and other two axes lie in planes determined by the vectors $\mathbf{G}_1, \mathbf{G}_2$ and $\mathbf{G}_3$, the tensor $\varepsilon_{ij}$ is a sum over reciprocal lattice vectors $\mathbf{g} = n_2 \mathbf{G}_2 + n_3 \mathbf{G}_3$ ($n_1 = 0, \theta \ll 1$) and has the following components:

\[
\varepsilon'_{11} = \frac{S'}{2} \frac{BN\bar{\sigma}}{8\pi mc} \sum_{\mathbf{g}} \Phi(g) (g_2^2 - g_3^2) z_g^2 F'_1(z_g).
\]
\[
\varepsilon'_{22} = \frac{S'}{2} \frac{BN\bar{\sigma}}{8\pi mc} \sum_{\mathbf{g}} \Phi(g) (g_2^2 - g_3^2) z_g^2 F'_1(z_g).
\]
\[
\varepsilon'_{12} = \varepsilon'_{21} = \frac{BN\bar{\sigma}}{8\pi mc} \sum_{\mathbf{g}} \Phi(g) (2g_2g_3) z_g^2 F'_1(z_g).
\]
\[
S' = 2 + \frac{BN\bar{\sigma} \hbar}{\pi mc} \sum_{\mathbf{g}} \Phi(g) (g_2^2 + g_3^2) z_g^2 F'_2(z_g, 1).
\]
\[
z_g = \frac{2mc^2}{E_0\theta(g_2 \cos \alpha + g_3 \sin \alpha)} = \frac{1}{n_2W_V + n_3W_H}.
\]

The summation over $\mathbf{g}$ satisfies the condition

\[
z_g > 0. \tag{3}
\]

\[
\varepsilon''_{11} = \frac{S''}{2} \frac{BN\bar{\sigma}}{16 mc} \sum_{\mathbf{g}} \Phi(g)(g_2^2 - g_3^2) F''_1(z_g),
\]
\[
\varepsilon''_{22} = \frac{S''}{2} + \frac{BN\bar{\sigma}}{16 mc} \sum_{\mathbf{g}} \Phi(g)(g_2^2 - g_3^2) F''_1(z_g),
\]
\[
\varepsilon''_{12} = \varepsilon''_{21} = -\frac{BN\bar{\sigma}}{16 mc} \sum_{\mathbf{g}} \Phi(g)(2g_2g_3) F''_1(z_g),
\]
\[
S'' = \varepsilon_A + \frac{BN\bar{\sigma}}{2 mc} \sum_{\mathbf{g}} \Phi(g)(g_2^2 + g_3^2) z_g^2 F''_2(z_g, 1).
\]

The summation over $\mathbf{g}$ satisfies the condition

\[
0 < z_g \leq 1 \tag{5}
\]

The functions $F'_1, F'_2, F''_1, F''_2, F'_3$ are equal to:

\[
F'_1(z) = \begin{cases} 
[\sqrt{1 - z + \frac{z}{2}L_+}]^2 + [\sqrt{1 + z - \frac{z}{2}L_+}]^2 - \frac{z^2}{4}, & 0 < z \leq 1, \\
-\left[\frac{\arccot \sqrt{z - 1}}{2} + [\sqrt{1 + z - \frac{z}{2}L_+}]^2]\right. & z > 1.
\end{cases}
\]

\[1\] Note that these functions are also used for description of the birefringence in the laser electromagnetic wave \[3, 8\].
absorption of \( \gamma \)-light, \( N \) is the number of the material of the single crystal, \( \Delta \) is the volume of the elementary cell and \( A \) is the mean-square amplitude of thermal vibrations of the atoms. \( N \) is the number where \( S(g) \) is the structure factor, \( F(g) \) is the form factor of an atom in the single crystal \( \alpha \) where \( \lambda \) constant is measured in units of \( E \). In these equations \( E_{\gamma} \) is the energy of \( \gamma \)-quanta, \( m \) is the electron mass, \( c \) is the speed of light, \( \alpha \) is the angle between planes \( (G_1, G_2) \) and \( (G_1, K) \), where \( K \) is the momentum of \( \gamma \)-quanta. The value \( \Phi(g) \) is determined by following relation:

\[
\Phi(g) = |S(g)|^2(1 - F(g))^2\exp^{-Ag^2}/g^4,
\]

where \( S(g) \) is the structure factor, \( F(g) \) is the form factor of an atom in the single crystal and \( A \) is the mean-square amplitude of thermal vibrations of the atoms. \( N \) is the number of atoms per unit of volume.

\[
B = \frac{16\pi^2}{N_S\Delta}, \quad \bar{\sigma} = \alpha_e Z^2 r_e^2,
\]

where \( \alpha_e \) is the fine-structure constant, \( r_e \) is the classical electron radius, \( Z \) is the atomic number of the material of the single crystal, \( \Delta \) is the volume of the elementary cell and \( N_S \) is the number of atoms per this cell. The term \( \varepsilon_A \) in Eqs.(5) takes into account the absorption of \( \gamma \)-quanta on the thermal vibrations of the lattice and is equal to

\[
\varepsilon_A = \frac{\bar{\sigma} N c h}{E_{\gamma}} \left( \frac{2}{3} \psi_1^{am} + \frac{1}{9} \psi_2^{am} \right),
\]

where the values \( \psi_1^{am} \) and \( \psi_2^{am} \) are approximately constants and these quantities are determined in theory [3]. In Eqs.(1-4,12-14) the system of units was used in which the reciprocal lattice constant is measured in units of \( \lambda_e^{-1} \) (\( \lambda_e = h/mc \)) and the direct lattice constant is measured in units of \( \lambda_e \); this is adopted in the theory of coherent radiation and pair-production [4].

The choice of the basic vectors \( G_1, G_2, G_3 \) is not unique. It is convenient to choice these vectors along axes of symmetry of the crystallographic lattice. So, for instance, let choice the vector \( G_1 \) along the \(<110> \)-axis in a silicon single crystal. Then one can choice the vectors \( G_2 \) and \( G_3 \) along the \(<001> \) and \(<110> \) axes, correspondingly. In this case the lengths of these vectors are equal to

\[
G_2 = 2\pi/a, \quad G_3 = 2\sqrt{2}\pi/a,
\]
where the $a$ is the side of the sell. One can see from the expressions (1)-(4) that the components of the tensor $\varepsilon_{ij}$ are the functions of the two universal parameters $W_H$ and $W_V$ (if the term $\varepsilon_A$ is ignored). These parameters for silicon crystal and orientation determined by Eq.(15) are equal to

$$W_H = 6.183E_\gamma \theta \sin \alpha \quad W_V = 4.372E_\gamma \theta \cos \alpha$$

where $E_\gamma$ and $\theta$ are measured in GeV and radians, correspondingly.

Knowing the permittivity tensor $\varepsilon_{ij}$ one can find the refractive indices of $\gamma$-quanta \[3\]

$$\tilde{n}^2 = (\varepsilon_{11} + \varepsilon_{22})/2 \pm \sqrt{(\varepsilon_{11} - \varepsilon_{22})^2/4 + \varepsilon_{12}\varepsilon_{21}},$$

(17)

Thus two waves with different indices of refraction $\tilde{n}_1$ and $\tilde{n}_2$ can propagate in the single crystals. In general, these refractive indices are complex quantities. Besides, in general case these two waves are elliptically polarized. However in particular case when the coordinate system exists in which the tensors $\varepsilon'_{ij}$ and $\varepsilon''_{ij}$ are simultaneously diagonal (i.e., complex tensor $\varepsilon_{ij}$ is reduced to principal axes) the both waves are linearly polarized. It is obviously that the permittivity tensor is diagonal when the momentum of $\gamma$-quanta lies strictly in $(G_1, G_2)$ or $(G_1, G_3)$ planes (angle $\alpha = 0$ or $\pi/2$). Then the refractive indices are equal to:

$$\tilde{n}_1 = \sqrt{\varepsilon_{11}}, \quad \tilde{n}_2 = \sqrt{\varepsilon_{22}},$$

(18)

In this case the differences of the real and imaginary parts of refractive indices are equal to ($\alpha = 0$):

$$\text{Re}(\tilde{n}_1 - \tilde{n}_2) = \frac{BN\tilde{\sigma}}{8\pi \frac{\hbar}{mc}} \sum_g \Phi(g) (g_2^2 - g_3^2)z_g^2F'_1(z_g)\vartheta(z_g),$$

(19)

$$\text{Im}(\tilde{n}_1 - \tilde{n}_2) = -\frac{BN\tilde{\sigma}}{16 \frac{\hbar}{mc}} \sum_g \Phi(g)(g_2^2 - g_3^2)F''_1(z_g)\vartheta(1-z_g)\vartheta(z_g),$$

(20)

$$z_g = 1/(n_2W_V)$$

(21)

where $\vartheta$ is the Heaviside unit step function. The similar case was considered in paper \[4\].

However, the $\gamma$-beam obtaining for experiments has some nonzero phase volume and, strictly speaking, the number of $\gamma$-quanta, which have different angles $\theta$ but fixed angle $\alpha = 0$, is equal to zero. In other words, a real $\gamma$-beam have some distribution over the angle $\alpha$. Now we show in detail that this fact change noticeably the relations for calculation of the refractive indices.

The components of permittivity tensor are the sum over the reciprocal lattice vectors and summation over $g$ satisfies the conditions $z_g > 0$ or $0 < z_g < 1$ for the real and imaginary components, correspondingly. Let us consider the first condition (for real components). One can rewrite its in the following form: $n_2W_V + n_3W_H > 0$. When $\alpha = 0$ we have $W_H = 0$ and get $n_2 > 0$ and $n_3$ is an arbitrary integer number ($W_V \neq 0$). Now let the angle $\alpha$ is nonzero small angle. Then we get

$$n_2(G_2\theta \cos \alpha) + n_3(G_3\theta \sin \alpha) > 0$$

(22)

It easy to see that set of numbers $n_2 = 0$, $n_3 = (1, 2, 3...)$ sign $\alpha$ (sign is the function equal to $\pm 1$ according to sign of $\alpha$) satisfies to Eq.(22). The set of obtained numbers (in case $\alpha = 0$ ) is also satisfied Eq.(22). Note that is true for any small nonzero angle $\alpha$. For imaginary components the set of $n_2$, $n_3$-numbers is the same in both cases, if only the angle $\alpha$ is enough small.
Now we can calculate the components of the permittivity tensor in limits $\alpha \to \pm 0$. It is clear that permittivity tensor (in pointed limit) have a diagonal form. Finally we get the following quantity of the difference of real parts of the refractive indices:

$$\text{Re}(\tilde{n}_1 - \tilde{n}_2) = \frac{BN\bar{\sigma}}{8\pi mc} \left\{ \sum g \Phi(g) (g_2^2 - g_3^2) z^2 F_1'(z_g) \vartheta(z_g) + \frac{8}{15} \sum_{n_3=1}^{\infty} \Phi(n_3 G_3)(G_3 n_3)^2 \right\} \quad (23)$$

Note that the left and right limits are equal in value. The difference of the imaginary parts of refractive indices is described as before by Eq(20). The add term in Eq(23) is equal to the mean-square value of the interplanar electric field (within a multiplier) $\bar{\sigma}$.

One can pointed to the similar effect in the coherent bremsstrahlung in single crystals. Let the electron beam motion in single crystal is determined by the $W_H$ and $W_V$ parameters. Then the theory predicts that the intensity of radiation of the low energy photons is small enough, when $W_H = 0$ and $W_V$ is reasonably large. However the experiments show the significant exceeding of intensity of these photons relative to calculated values $\bar{\sigma}$, if the calculations is not take into account the angular divergence of the electron beam.

3 Influence of the $\gamma$-beam divergence on propagation

As it was shown in paper [2], in general case the $\gamma$-beam propagate in the single crystal as superposition of the two elliptically polarized waves. Birefringence is a special case of propagation of high-energy $\gamma$-quanta in single crystals, when the elliptical polarization of these waves (eigenfunctions of the problem) degenerate into linear one. The linear polarization point to the space symmetry of the problem as it was shown previously.

Now we consider the important problem for the experimental observation of birefringence. We want to get the answer on the following question: Is the refractive indices and polarization states of waves, when $\gamma$-beam move near the axis of symmetry in the single crystal (in other words, when $W_H \neq 0$, but $W_H \ll W_V$), essentially changed? With the aim of investigation of this problem we carry out calculations of the refractive indices and polarization states of waves at the small values of $W_H$. These calculations are based on papers [2, 3, 4] where the general case of $\gamma$-quanta propagation in the anisotropic medium was considered. Besides, we examine only the case when the beam of $\gamma$-quanta move under a small angle $\theta$ with respect to one of the "strong" crystallographic axis (in other words when $W_H, W_V \simeq 1$). The difference of real parts of refractive indices is more significant at these orientations in compare with the motion of beam near crystallographic planes [2].

Figures 1 and 2 show the results of calculations of the refractive indices as functions of $W_V$ at some values of $W_H$. One can see that the variations of the refractive indices difference are insignificant as a whole when the parameter $W_H$ is within 0 - 0.01. However the peaks of curve at $W_H = 0$ are spreading enough when the parameter $W_H$ rise to 0.01. The curve at $W_H = 0.1$ is differ from curve at $W_H = 0$ for all practically values of the $W_V$. In all calculations the Moliere form factor was employed [11].

Figure 3 illustrates the absolute value of circular polarization, which have the normal electromagnetic waves (eigenfunctions of the problem). The circular polarization $P_c$ is small when $W_H \sim 0.01$ and it rise to 0.5 with increasing of the parameter $W_H$ to 0.1. Nevertheless, the value of linear polarization $P_L = \sqrt{1 - P_c^2}$ of the normal waves is dominant at all considered here values of $W_H$ and $W_V$. Besides, the turn of the semi-axes of polarization ellipse in plane $(G_2, G_3)$ is take a place (see figure 4).
Thus once can say that the pure birefringence in the single crystal takes a place for γ-beam with a small angle divergence (the value of this divergence one can found from relation $\delta W_H \sim 0.01$). On the other hand, the variations of polarized state of the γ-quanta propagating in single crystals at different values of $W_H$ is of more direct interest to practical goals.

Figures 5,6 show the variation of the circular polarization of the 100 GeV γ-beam propagating in the silicon single crystal as a function of its thickness. The system of coordinate is chosen so that the Stokes parameter of γ-beam $\xi_3 = \pm 1$ when 100 % linear polarization lies in planes (110) and (001), correspondingly. We take for illustration the cases of partially polarized beam and unpolarized one at the point of entry in the single crystal. In the case of pure birefringence (see figure 5) the unpolarized γ-beam can obtain only some degree of the linear polarization on any thickness of a single crystal (i.e. $\xi_2(x) = 0$). In the case when the normal electromagnetic waves is elliptically polarized the propagating unpolarized beam of γ-quanta can obtain some degree of the linear and circular polarization (see figure 6). The transformation of linear polarization to circular one (under angle in $\pm 45^\circ$ with respect to above-mentioned coordinate system) one can see also on these figures. The analogous curves for parameter $W_H = -0.1$ are mirror-symmetric with respect to x-coordinates. The intensity of γ-beam is decreased in $\sim 10^9$ times on 100 cm of the silicon single crystal.

Note that our consideration of the bierfrigence base on the theory of coherent e±-pair production in single crystals [3, 4]. However this theory is violated at some orientations of single crystals (in regions of so called ”strong field”) [5]. For silicon crystallographic planes this violation is expected at very high energy of γ-quanta $\gg 1\,\text{TeV}$.

4 Conclusion

The pure birefringence of high energy γ-beam propagating near crystallographic axis (when the eigenfunctions of a problem is two linearly polarized electromagnetic wave) take a place for special (predominantly symmetric) orientations. In general case the propagating γ-beam is the superposition of two elliptically polarized waves, because of this some peculiarities in the propagation of γ-beam exist even for orientations near to the pointed symmetrical ones. For these orientations we can point on the following:
1. Some noticeable degree of circular polarization of eigenfunctions exists.
2. Some angle shift of the axes of the polarization ellipse takes a place also.
3. The quantities of refractive indices is changed sharply for close orientations.

Thus it needs to take into account these effects in experimental observation of birefringence of a beam of γ-quanta with some phase volume.

In addition we consider in detail the procedure of calculation of refractive indices for real γ-beams.

References

[1] N.Cabibbo, G. Da Prato, G. De Franceschi, and U. Mosco Phys. Rev. Lett, 1962. V.9 P.435.

[2] V.A. Maisheev, V.L. Mikhalev, A.M. Frolov, Zh. Eksp. Teor. Fiz., V.101, p.1376, (1992) [in Russian]; Sov.Phys. JETP v.74, p.740, (1992).
[3] V.A. Maisheev, preprint IHEP 98-79, Protvino, 1998; E-print Archive hep-ph/9710202.

[4] V.A. Maisheev, E-print Archive hep-ph/9712211.

[5] W. Dittrich and H. Gies, E-print Archive hep-ph/9806417.

[6] V.G. Baryshevskii and V.V. Tikhomirov, Usp. Fiz. Nauk v.154, p.529, (1989) Sov. Phys. Usp. v. 32, p. 1013 (1989).

[7] K. Piotrzkowski Nucl. Instr. and Meth. B, 1996, B119, p.253;

[8] V.A. Maisheev, Zh.Eksp. Teor. Fiz., V. 112, p.2016, (1997) [in Russian]; Sov.Phys.JETP V.85, p.1102, (1997); Preprint IHEP 97-25, Protvino, 1997.
   In English version is the translation error: in Eqs(21),(22),(26) instead the function \( \arctan(x) \) must be \( \text{arccot}(x) \). In Russian version the function \( \text{arccot}(x) \) denote by \( \text{arcctg}(x) \) (\( \text{arccot}(x) \equiv \text{arcctg}(x) \)).

[9] M.L. Ter-Mikaelian, High Energy Electromagnetic Processes in Condensed Media (Wiley, New York, 1972).

[10] H. Bilokon et al. Nucl. Instr. and Meth., 1983, V.204, p.299.

[11] V.N.Baier, V.M. Katkov, and V.S.Fadin, Radiation from Relativistic Electrons [in Russian], Atomizdat, Moscow, 1973.

[12] H. Uberall, Phys.Rev.,v.103, p. 1055, 1956; v.107, p. 223, 1957.

[13] R. Moore at al. Nucl. Instr. and Meth. B, 1996, B119, p.149.
Figure 1: The differences of the real parts of the refractive indices in silicon as functions of \( W_H \)-parameter. Curves 1 and 2 are calculated at \( W_H = 0 \) by the use of Eq.(19) and Eq.(23), correspondingly. Curves 3 and 4 are calculated according to Eq.(18) at \( W_H = 0.01 \) and 0.1.
Figure 2: The differences of the imaginary parts of the refractive indices in silicon as functions of $W_V$-parameter. Curves 1, 2, 3 are calculated at $W_H = 0, 0.01, 0.1$, correspondingly.

Figure 3: Absolute value of the circular polarization of the normal electromagnetic waves in silicon as the function of the $W_V$-parameter. The solid curve is calculated for $W_V = 0.1$ and the dotted line is for $W_V = 0.01$. 
Figure 4: Absolute value of the angle shift of the normal electromagnetic waves in silicon as the function of the $W_V$-parameter. The solid curve is calculated for $W_H = 0.1$ and the dotted line is for $W_H = 0.01$.

Figure 5: The variation of circular polarization of the 100 GeV $\gamma$-beam propagating in a silicon single crystal as function of its thickness (in centimeters). For all curves the $W_H$-parameter is equal to 0. The parameter $W_V = 1.1, 0.9, 1.1$ for solid, dotted and dot-and-dashed curves, correspondingly. The values near the curves are the initial quantity of $\xi_1$ Stokes parameter for $\gamma$-beam. The other initial Stokes parameters are equal to 0 for all curves.
Figure 6: The variation of circular polarization of the 100 GeV $\gamma$-beam propagating in a silicon single crystal as function of its thickness (in centimeters). For all curves the $W_H$-parameter is equal to 0.1. The parameter $W_V = 1.0, 0.9, 1.1$ for solid, dotted and dot-and-dashed curves, correspondingly. The values near the curves are the initial quantity of $\xi_1$ Stokes parameter for $\gamma$-beam. The other initial Stokes parameters are equal to 0 for all curves.