Heavy quark production as sensitive test for an improved description of high energy hadron collisions

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QCD dynamics at small quark and gluon momentum fractions or large total energy, which plays a major role for HERA, the Tevatron, RHIC and LHC physics, is still poorly understood. For one of the simplest processes, namely $b\bar{b}$ production, next-to-leading-order perturbation theory fails. We show that the combination of two recently developed theoretical concepts, the $k_t$-factorization and the next-to-leading-logarithmic-approximation BFKL vertex, gives perfect agreement with data. One can therefore hope that these concepts provide a valuable foundation for the description of other high energy processes.

Existing QCD calculations describe many high energy observables which involve partonic transverse momentum rather poorly. This is also true for the theoretically especially clean case of $b\bar{b}$ production, which was investigated experimentally at Fermilab [1]. Since central quark-antiquark production at $\sqrt{s} = 1.8$ TeV is sensitive to very small gluon momentum fraction $x \approx 10^{-2} - 10^{-4}$, one probes the gluon content of the nucleon at small $x$, which is a central issue of current research. We consider this process and combine as essential new ingredients the $k_t$-factorization scheme with the next-to-leading-logarithmic-approximation BFKL production vertex derived in [2]. The $k_t$-factorization approach for the description of high energy processes [3-4] differs strongly from the conventional NLO collinear approximation (e.g. [5]) because it takes the non-vanishing transverse momenta of the scattering partons into account. The usual gluon densities are replaced by unintegrated gluon distributions which depend on the transverse momentum $k_t$. These together with the $k_t$-factorization form a basis for a general calculation scheme for high energy (i.e. small $x$). The standard collinear approximation has the advantage of being closely related to the operator product expansion. It is, however, only justified for the processes dominated by $x = O(1)$. In application to processes governed by small $x$ the $k_t$-factorization approach has the advantage that its approximations correspond to the dominant kinematics. Essential small $x$ contributions are included in the Born approximation which in the collinear approach are accounted for in higher orders only. This is well known from the case of structure functions where the DGLAP evolution is appropriate for $x = O(1)$ and the BFKL evolution for small $x$.

While the $k_t$-factorization formalism is very attractive theoretically, its phenomenological usefulness has been mostly tested in the case of the structure function $F_2(1,1)$. The NLLA BFKL vertices are just the ones needed to treat semi-hard central production at collider energies in this approach.

In our calculation we use one particular element of the NLLA BFKL formalism [6], namely the effective vertex for quark-antiquark production. Thus our calculation can be seen as a first phenomenological application of this vertex which decides whether the NLLA BFKL formalism can be hoped to converge.

One special aspect of the reaction we investigate is the possible loss of gauge invariance when a $q\bar{q}$ production vertex is incorporated into an amplitude with off-shell gluons. In the BFKL approach, however, gauge invariance is ensured automatically by the use of the just mentioned NLL effective vertex which is valid in quasi multi Regge kinematics (QMRK), i.e. when the $q$ and $\bar{q}$ have similar rapidities and form a cluster (in contrast to LLA, where the particles are produced with a large rapidity gap).

![FIG. 1. The basic diagram](image)

We begin with the following definition for the light cone coordinates and the momenta of the scattering hadrons in the c.m. frame

$$k^+ = k^0 + k^3, \quad k^- = k^0 - k^3, \quad k_\perp = (0, k^1, k^2, 0) = (0, \mathbf{k}, 0).$$

$$P_1^+ = P_2^+ = \sqrt{s}, \quad P_{1\perp} = P_{2\perp} = 0, \quad P_{1\perp} = P_{2\perp} = 0.$$

The Mandelstam variable $s$ is as usual the c.m. energy squared. As defined in fig. [2] $q_1$ and $q_2$ are the momenta of the gluons and the on-shell quark and antiquark have momentum $k_1$ respectively $k_2$. In the high energy (large $s$) regime we have

$$k_1^+ + k_2^+ = q_1^+ - q_2^+ \approx q_1^+,$$
\[ k_1 + k_2 = q_\perp - q_\perp' \approx -q_\perp', \]
\[ q_\perp^2 \approx q_\perp^2, q_\perp'^2 \approx q_\perp'^2. \]

The longitudinal momentum fractions of the gluons are
\[ x_1 = q_\perp^+ / P_1^+, x_2 = -q_\perp'^+ / P_2^+. \]

The cross section for heavy quark pair production in the $k_1$-factorization approach is then given by
\[
\sigma_{P_1 P_2 \rightarrow q\bar{q}X} = \frac{1}{16(2\pi)^4} \int \frac{d^3k_1 d^3k_2}{k_1^+ k_2^+} q_\perp q_\perp'^2 \frac{1}{(q_\perp^2)^2} \delta^2(q_\perp - q_\perp' - k_1 - k_2) F(x_1, q_\perp^2) \left( \frac{\psi^{(2c_1)}(x_1, x_2, q_\perp^2)}{(N^2 - 1)^2} \right) F(x_2, q_\perp'^2). \tag{1} \]

The factor $(N^2 - 1)^2$ reflects the projection on color singlet, where $N$ is the number of colors. The hard amplitude \( \psi^{(2c_1)}(x_1, x_2, q_\perp^2, k_1, k_2) \) is calculable in perturbation theory, whereas the unintegrated gluon distribution \( F(x, q_\perp^2) \) has to be measured or modelled. We choose the argument \( \mu^2 \) of the strong coupling constant \( \alpha_S(\mu^2) \) in the hard amplitude \( \psi^{(2c_1)} \) to be equal to \( q_\perp^2 = q_\perp'^2 \) respectively.

\[ \Gamma^{+ - \beta}(q_1, q_2) = 2(q_1 + q_2)^\beta - 2q_1^+ n^{-\beta} - 2q_2^+ n^{+\beta} - 2t_1 n^{-\beta} q_1 - 2t_2 n^{+\beta} q_2. \tag{4} \]

The factor \((N^2 - 1)^2\) presents in an obvious way in order to take the masses of the produced quarks into account. The resulting vertex \( \Psi^{(2c_1)} \) is given by a sum of two terms
\[
\Psi^{(2c_1)} = -q_\perp^2 \left( t^{c_1} t^{c_2} b(k_1, k_2) - t^{c_2 t^{c_1} b^T(k_2, k_1)} \right), \tag{2} \]
where \( t^{c} \) are the colour group generators in the fundamental representation. The connection between \( \psi^{(2c_1)} \) in eq. (1) and \( \Psi^{(2c_1)} \) in eq. (2) is given by
\[
\psi^{(2c_1)} = \pi(k_1) \psi^{(2c_1)}(k_2), \]
with the on-shell quark and antiquark spinors \( u(k) \) and \( v(k) \). The expression for \( b(k_1, k_2) \) is a sum of two terms
\[
b(k_1, k_2) = \gamma^+ \left( \frac{1}{(q_1 - k_1)^2} - m^2 \right) \gamma^- + \gamma^+ \Gamma^{+ - \beta}(q_2, q_1) \left( \frac{1}{(k_1 + k_2)^2} \right), \tag{3} \]

The first term on the r.h.s. of eq. (2) describes the production of a \( q\bar{q} \) pair by means of usual vertices (see fig. 2), the second term involves the light-cone projection of the effective vertex \( \Gamma^{+ - \beta}(q_2, q_1) \), which describes the transition of two \( t \)-channel gluons (reggeons) with momenta \( q_1 \) and \( q_2 \) to a gluon with momentum \( k_1 + k_2 \)

\[ \Gamma^{+ - \beta}(q_2, q_1) = 2(q_1 + q_2)^\beta - 2q_1^+ n^{-\beta} - 2q_2^+ n^{+\beta} - 2t_1 n^{-\beta} q_1 - 2t_2 n^{+\beta} q_2 \]

with \( t_{1/2} = q_1^2/2 \). This effective vertex differs from the usual triple-gluon vertex by the appearance of the last two terms. They are related to Feynman diagrams in which the \( q\bar{q} \) pair is not produced by the \( t \)-channel gluons but in other ways. These two last terms in eq. (4) are also required by gauge invariance, \( \Gamma^{+ - \beta}(q_2, q_1)(q_1 - q_2) = 0 \). Another consequence of gauge invariance is the vanishing of the matrix element of the effective vertex \( \Psi^{(2c_1)} \) between on-mass-shell quark and antiquark states in the limit of small \( q_1 \) or \( q_2 \)

\[ \pi(k_1) \Psi^{(2c_1)} v(k_2) \rightarrow 0 \text{ for } q_1 \rightarrow 0 \text{ or } q_2 \rightarrow 0. \]

The function \( b^T(k_2, k_1) \) is very similar to (3)
\[ b^T(k_2, k_1) = \gamma^+ \frac{q_1^+ - k_2 - m}{(q_1 - k_2)^2 - m^2} - \gamma^- \frac{\Gamma^{+ - \beta}(q_2, q_1)}{(k_1 + k_2)^2}. \]

The unintegrated gluon distribution is related to the standard gluon distribution by
\[ xg(x, q^2) = \int_0^\infty \frac{dk^2}{k^2} \Theta(q^2 - k^2) F(x, k). \]

Taking the derivative of this expression makes it obvious that \( F(x, k) \) includes the evolution of \( xg(x, q^2) \), which is given by the BFKL and/or DGLAP equation. Since the unintegrated gluon distribution is not known at small \( k \), we write this equation as
\[ xg(x, q^2) = xg(x, q_0^2) + \int_0^{q_0^2} \frac{dk^2}{k^2} \Theta(q^2 - k^2) F(x, k). \tag{5} \]

This formula has been repeatedly used and introduces the a priori unknown initial scale \( q_0 \) and the initial gluon distribution \( xg(x, q_0^2) \). Following (4), one may neglect the hard cross section dependence on \( q \) in the soft region \( |q| < q_0 \), so that
\[
\frac{1}{q_\perp^2} \int (N^2 - 1)^2 \frac{1}{q_\perp'^2} = S(q_\perp, q_\perp') \rightarrow \\
S(q_\perp, q_\perp') \Theta(q_\perp^2 - q_\perp'^2) \Theta(q_\perp'^2 - q_\perp^2) + S(q_\perp, q_\perp) \Theta(q_\perp^2 - q_\perp'^2) - S(q_\perp, q_\perp) \Theta(q_\perp^2 - q_\perp'^2) + S(q_\perp, q_\perp) \Theta(q_\perp'^2 - q_\perp^2) - S(q_\perp, q_\perp) \Theta(q_\perp'^2 - q_\perp^2). \tag{6} \]

Note that the very existence of the finite limit \( q_\perp \rightarrow 0 \) follows from the decrease of the production amplitude.
due to gauge invariance. Substituting this formula in (5) using eq. (6) one may easily perform the integration over $q_\perp$. As a result, $S(0,0)$ produces the standard expression of collinear factorization (ref. [10]), while $S(q_1,0)$, $S(0,q_2)$ correspond to the asymmetric configurations, where one of the gluons is described by the unintegrated distribution and the other by the integrated one. Here it is important to notice that when we insert (5), (6) in (1) the coupling constant $\alpha_s$ in the term proportional to $xg(x,q_0^2)$ is taken to be $\alpha_s(q_0^2)$.

In all our numerical calculations we used for the unintegrated gluon distribution $F(x,k)$ the code by Kwiecinski, Martin and Sta`s (see for example fig. 11 in [15]), because they use a combination of DGLAP and BFKL equations which governs simultaneously the evolution in $Q^2$ and $x$. They obtain an excellent description of $F_2(x,Q^2)$ in a very large $x$-$Q^2$ window. According to our knowledge this is the only unintegrated gluon distribution which has given such a satisfactory result, which justifies our choice. As in the case of the usual gluon distribution function one has to choose an initial scale and an initial distribution function which in the case of (4) are given by

$$q_0^2 = 1 \text{ GeV}^2, \quad xg(x,q_0^2) = 1.57(1-x)^{2.5}.$$  

We use these values, which are fixed by the fit to $F_2(x,Q^2)$, in our calculation.

We consider the production of $b\bar{b}$-pairs. For the computation we use eqs. (4) and (5) with the unintegrated gluon distribution from (4) and the corresponding values (6). The rapidities and the transverse masses of the produced quark and antiquark are defined as

$$y_{1/2} = \frac{1}{2} \ln(\frac{k_{1/2}^+}{k_{1/2}^-}), \quad m_{1/2} = \sqrt{m^2 - k_{1/2}^2}. $$

The Bjorken-variables of the gluons can then be written as

$$x_1 = \frac{1}{\sqrt{s}}(m_{1\perp}e^{y_1} + m_{2\perp}e^{y_2}),$$

$$x_2 = \frac{1}{\sqrt{s}}(m_{1\perp}e^{-y_1} + m_{2\perp}e^{-y_2}).$$

In fig. 3 we show our results for inclusive $b$ production, together with experimental results measured by the D0 Collaboration [16] (see Table II) in $\sqrt{s} = 1.8$ TeV $\overline{p}p$ collisions. We obtain this cross section by integrating out all antibottom variables in eq. (1).

The variable $k_{1\perp\min}$ is the lower integration cut on the transverse momentum of the produced $b$ quark. To get an indication of the theoretical uncertainties apart from higher order contributions which are not available at the moment we proceed in a similar way as the authors of ref. [10] and present our calculations for three different choices of $\Lambda_{\text{QCD}}$ and the bottom quark mass

- high : $\Lambda_{\text{QCD}} = 180$ MeV, $m_b = 4.5$ GeV,
- central : $\Lambda_{\text{QCD}} = 150$ MeV, $m_b = 4.7$ GeV,
- low : $\Lambda_{\text{QCD}} = 100$ MeV, $m_b = 4.9$ GeV,

Our result is in very good quantitative agreement with data over the whole range of $k_{1\perp\min}$. The corresponding central QCD NLO calculation has a similar shape, but is about a factor of 2–3 smaller than our central result (see for example fig. 11 in [15]).

In fig. 4 we consider the semi differential $b\bar{b}$ cross section at $k_{1\perp\min} = 6.5$ GeV, compared to CDF data and the NLO QCD result with MRSD0 structure functions and $m_b = 4.75$ GeV from [1].

We now turn to $b\bar{b}$ correlations in $\sqrt{s} = 1.8$ TeV $\overline{p}p$ collisions, which have been measured by the CDF collaboration at Fermilab [1]. The correlations of the quark and antiquark give an insight into the dynamics of the production mechanism and are important in order to study...
the limits of the collinear \((k_1 \perp = -k_2 \perp)\) LO QCD approximation. We present a comparison between our results and the experimental data in fig. 4 and fig. 5. The data points and uncertainties were taken from [1,17]. We find good agreement with experiment for both \(k_{1 \text{min}} = 6.5\ GeV\) (fig. 4) and \(k_{1 \text{min}} = 8.75\ GeV\) (fig. 5). In this case QCD NLO calculations underestimate the measured cross section roughly by factor of 3 (compare with fig.6 in [1]).

An interesting parameter concerning the correlation is the opening angle \(\phi\) between the momentum vectors of the produced quarks in the plane transverse to the beam axis. Our predictions for the corresponding differential cross sections at Fermilab and LHC energies are shown in fig. 6. As expected we find a peak at \(\phi = 180^\circ\) which shows the dominance of the collinear part.

Additionally we present our predictions for rapidity distributions of the \(b\bar{b}\) for the rapidity of the \(b\) being 0 and \(\sqrt{s} = 1.8\ TeV\) respectively \(\sqrt{s} = 16\ TeV\) in fig. 6. Our cross section for \(\sqrt{s} = 1.8\ TeV\) at \(y_2 = 0\) is about a factor of 3 larger than the corresponding QCD NLO result from [18].

Let us conclude. We have studied quark-antiquark hadroproduction within the \(k_t\)-factorization approach using an unintegrated gluon distribution and a specific effective BFKL vertex for \(q\bar{q}\) production. We found very good agreement with experiment for both single \(b\) production and \(b\bar{b}\) correlations at \(\sqrt{s} = 1.8\ TeV\). Our approach leads to nontrivial \(b\bar{b}\) correlations already at LO perturbation theory, whereas traditional collinear factor-

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