Electroweak Penguin Effects In Some $B^0_s$ Two-Body Decays

Dong-sheng Du$^{a,b}$ and Mao-zhi Yang$^b$

a CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, China

b Institute of High Energy Physics, Academia Sinica, P.O.Box 918(4), Beijing, 100039, P. R. China*

Abstract

Using the next-to-leading order low energy effective Hamiltonian for $\Delta B = 1$ transitions, the effects of electroweak penguin operators in some two-body decay modes of $B^0_s$ meson are estimated in the Standard Model (SM). We find that in $B^0_s \rightarrow \pi^+ K^-$ and $B^0_s \rightarrow K^+ K^-$ decay modes, the electroweak penguin effects are small, while in $B^0_s \rightarrow \pi^0 K^0$, $\phi K^0$, $\phi \phi$, the electroweak penguin operators enhance or reduce the pure QCD penguin and tree level contributions by $20\% \sim 80\%$ in decay width. We also present the results of CP asymmetries in these $B^0_s$ decay modes.

*mailing address.
Recently Buras et al. generalized the low energy effective Hamiltonian for $|\Delta B| = 1$ transitions to next-to-leading order QCD corrections[1~4]. Now, we are in a position to investigate not only the pure QCD penguin and tree level contributions to B meson decays, but also the effects caused by electroweak penguin diagrams beyond the leading logarithmic approximation. Naively, people believe that electroweak penguin contributions to B meson decays are suppressed by a factor of $\alpha_{em}/\alpha_s \approx O(10^{-2})$ relative to QCD penguin contributions [5,6]. So electroweak corrections comparing with QCD penguin contributions in B system may be negligible. However this is not always true. In some B meson decay modes, electroweak penguin contributions can play a significant role[7,8]. In this paper, we investigate some $B_s^0$ two-body decay channels. They are $B_s^0 \to \pi^+ K^-$, $K^+ K^-$, $\pi^0 K^0$, $\phi \phi$, and $\phi \bar{K}^0$, which are induced by QCD and electroweak penguin diagrams. We calculate the partial decay widths, branching ratios, and the CP asymmetries of these decay modes. We compare the decay widths including full electroweak penguin contributions with that including only tree and QCD penguin diagrams. We find that in $B_s^0 \to \pi^+ K^-$, and $K^+ K^-$, electroweak correction is small, it just modify the pure QCD corrections by $1\% \sim 8\%$. While in $B_s^0 \to \pi^0 \bar{K}^0$, $\phi \bar{K}^0$, and $\phi \phi$, the electroweak penguin contributions are quite large relative to QCD penguin corrections. They enhance or reduce the results obtained by only taking into account tree and QCD penguin operators by $20\% \sim 80\%$. The decay mode $B_s^0 \to \phi \phi$ has been calculated in [8]. We include it here just for completeness and comparison. Our results for this channel agree with that in [8].

In the standard model, the number of colors $N_c$ is 3, while some people argue that the number favored by experimental data is 2 [9]. So in this paper we take not only $N_c = 3$, but also $N_c = 2$. We also present the results with $N_c = \infty$ as in [10].

The next-to-leading order low energy effective Hamiltonian relevant to charmless B decays
can be taken as the following form [7]:

\[
H_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c} v_q \left\{ Q_1^u C_1(\mu) + Q_2^u C_2(\mu) \right\} \sum_{k=3}^{10} Q_k C_\mu(\mu) \right] ,
\]

(1)

where \( C_k(\mu) \) (k=1, ..., 10) are Wilson coefficients which are calculated in renormalization group improved perturbation theory and include leading and next-to-leading order QCD corrections. \( v_q \) is the product of Cabibo-Kobayashi-Maskawa (CKM) matrix elements and defined as

\[
v_q = \begin{cases} 
V_{qd} V_{q\bar{b}}^* & \text{for } \bar{b} \to d \text{ transitions} \\
V_{qs} V_{q\bar{s}}^* & \text{for } \bar{b} \to s \text{ transitions} 
\end{cases}
\]

In our numerical calculations the CKM matrix elements are taken as \( \lambda = 0.22, A = 0.8, \eta = 0.34, \rho = -0.12 \) in Wolfenstein parametrization, which are the preferred values obtained from the fit to the experimental data[11]. The ten operators are taken as the following form [2,3]

\[
\begin{align*}
Q_1^u &= (\bar{b}_a u_\beta)_{V-A}(\bar{u}_\beta q_\alpha)_{V-A}, & Q_2^u &= (\bar{b} u)_{V-A}(\bar{u} q)_{V-A}, \\
Q_3 &= (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A}, & Q_4 &= (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}, \\
Q_5 &= (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A}, & Q_6 &= (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}, \\
Q_7 &= \frac{3}{2} (\bar{b} q)_{V-A} \sum_{q'} e_q (\bar{q}' q')_{V+A}, & Q_8 &= \frac{3}{2} (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}, \\
Q_9 &= \frac{3}{2} (\bar{b} q)_{V-A} \sum_{q'} e_q (\bar{q}' q')_{V-A}, & Q_{10} &= \frac{3}{2} (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}
\end{align*}
\]

(2)

where \( Q_1^u \) and \( Q_2^u \) are current-current operators, \( q=u, c \), for \( q=c \) case, the two operators \( Q_1^c \) and \( Q_2^c \) are obtained through making substitution \( u \to c \) in \( Q_1^u \) and \( Q_2^u \). \( Q_3 \sim Q_6 \) are QCD penguin operators, the sum \( \sum_{q'} \) is runing over all the quark flavors being active at \( \mu = m_b \) scale, \( q'=\{u, d, s, c, b\} \). \( Q_7 \sim Q_{10} \) are electroweak penguin operators, \( e_{q'} \) are the electric charges of the relevant quarks in unit e which is the charge of the proton. The subscripts \( \alpha, \beta \) are \( SU(3)_c \) color indices. \( (V \pm A) \) refers to \( \gamma_\mu (1 \pm \gamma_5) \).
The Welson coefficient functions $C_i(\mu)$ are renormalization scheme (RS) dependent beyond the leading order approximation. Thus we should cancel this renormalization scheme dependence. So define renormalization scheme independent Wilson Coefficients[12]

$$\bar{C}(\mu) = (\hat{1} + \hat{r}_s^T \alpha_s(\mu)/4\pi + \hat{r}_e^T \alpha_{em}(\mu)/4\pi) \cdot C(\mu),$$

(3)

and treat the matrix elements to one-loop level[7],

$$< Q^T(\mu) >= < Q^T >_0 \cdot \left[ \hat{1} + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s^T(\mu) + \frac{\alpha_{em}(\mu)}{4\pi} \hat{m}_e^T(\mu) \right],$$

(4)

Combine eq.(3) and (4) we obtain

$$< Q^T(\mu) \cdot C(\mu) > = < Q^T >_0 \cdot \left[ \hat{1} + \frac{\alpha_s(\mu)}{4\pi} (\hat{m}_s(\mu) - \hat{r}_s)^T + \frac{\alpha_{em}(\mu)}{4\pi} (\hat{m}_e(\mu) - \hat{r}_e)^T \right] \cdot \bar{C}(\mu)

\equiv < Q^T >_0 \cdot C'(\mu)$$

(5)

The corresponding elements of the matrices $\hat{r}_s$, $\hat{r}_e$, $\hat{m}_s(\mu)$ and $\hat{m}_e(\mu)$ are given by [1, 4, 13]. Substituting these matrix into eq.(5), we can obtain $C'_i(\mu)$ as

$$C'_1 = \bar{C}_1, \quad C'_2 = \bar{C}_2,$$

$$C'_3 = \bar{C}_3 - P_s/N_c, \quad C'_4 = \bar{C}_4 + P_s,$$

$$C'_5 = \bar{C}_5 - P_s/N_c, \quad C'_6 = \bar{C}_6 + P_s,$$

$$C'_7 = \bar{C}_7 + P_e, \quad C'_8 = \bar{C}_8,$$

$$C'_9 = \bar{C}_9 + P_e, \quad C'_{10} = \bar{C}_{10},$$

(6)

where $P_{s,e}$ are given by

$$P_s = \frac{\alpha_s}{8\pi} \bar{C}_2(\mu) \left[ \frac{10}{9} - G(m_q, q, \mu) \right],$$

$$P_e = \frac{\alpha_{em}}{3\pi} \left( \bar{C}_1(\mu) + \frac{\bar{C}_2(\mu)}{N_c} \right) \left[ \frac{10}{9} - G(m_q, q, \mu) \right],$$

$$G(m_q, q, \mu) = - 4 \int_0^1 x(1-x) dx ln \frac{m_q^2 - x(1-x)q^2}{\mu^2},$$

here $q=u,c$, for numerical calculation, we take $m_u = 0.005GeV$, $m_c = 1.35GeV$, and use $q^2 = m_b^2/2$, which represents the average “physical” value.
We take $m_t = 174\text{GeV}, m_b = 5.0\text{GeV}, \alpha_s(M_z) = 0.118, \alpha_{em}(M_z) = \frac{1}{128}$, and take the numerical values of the renormalization scheme independent Wilson coefficients $\bar{C}_i(\mu)$ as \[8\]

$$
\begin{align*}
\bar{C}_1 &= -0.313, \quad \bar{C}_2 = 1.150, \quad \bar{C}_3 = 0.017, \\
\bar{C}_4 &= -0.037, \quad \bar{C}_5 = 0.010, \quad \bar{C}_6 = -0.046, \\
\bar{C}_7 &= -0.001 \cdot \alpha_{em}, \quad \bar{C}_8 = 0.049 \cdot \alpha_{em}, \quad \bar{C}_9 = -1.321 \cdot \alpha_{em}, \\
\bar{C}_{10} &= 0.267 \cdot \alpha_{em},
\end{align*}
$$

Using vacuum-saturation approximation the decay amplitude $< XY|H_{eff}(\Delta B = 1)|B^0_0 >$ can be factorized into a product of two current matrix elements $< X|J^\mu|0 >$ and $< Y|J'_\mu|B^0_s >$. The hadronic matrix elements are calculated in BSW method\[14\]. We define $M_{XYq_1q_2q_3}$ as in Ref.\[10\].

$$
M_{XYq_1q_2q_3}^{q_1q_2q_3} = \frac{G_F}{\sqrt{2}} < X|(\bar{q}_1 q_2)V-A|0> < Y|(\bar{b} q_3)V-A|B>, 
$$

Now we can express the amplitudes $< XY|H_{eff}(\Delta B = 1)|B^0_0 >$ in terms of $C'_i$ and
\[ M_{q_1q_2q_3}^{XY} \]

\[
< \pi^+ K^- | H_{\text{eff}} | B_s^0 > = \left[ v_u a_1 + \sum_{q=u,c} v_q \left\{ a_3 \frac{M_2^2}{(m_u + m_d)(m_b - m_u)} (2a_5 + 3e_u a_7) + a_0 \cdot \frac{3}{2} e_u \right\} \right] \cdot M_{\pi^+ K^-}^{\text{eff}},
\]

\[
< K^+ K^- | H_{\text{eff}} | B_s^0 > = \left[ v_u a_1 + \sum_{q=u,c} v_q \left\{ a_3 \frac{M_2^2}{(m_u + m_d)(m_b - m_u)} (2a_5 + 3e_u a_7) + a_0 \cdot \frac{3}{2} e_u \right\} \right] \cdot M_{K^+ K^-}^{\text{eff}},
\]

\[
< \pi^0 K^0 | H_{\text{eff}} | B_s^0 > = \left[ v_u a_2 + \sum_{q=u,c} v_q \left\{ a_4 - a_6 - a_8 \cdot \frac{3}{2} e_u + a_0 \cdot \frac{3}{2} e_u \right\} \right] M_{\pi^0 K^0}^{\text{eff}}
\]

\[
+ \left[ \sum_{q=u,c} \left\{ \left( \frac{1}{N_c} + 1 \right)(C_3' + C_4') - a_0 - a_8 \cdot \frac{3}{2} e_d + \frac{M_2^2}{m_d (m_b - m_d)} (a_5 + 2e_d \cdot a_7) \right\} + \left( \frac{1}{N_c} + 1 \right) \cdot \frac{3}{2} e_d (C_9' + C_{10}') \right] \right] M_{\pi^0 K^0}^{\text{eff}}
\]

\[
< \phi \phi | H_{\text{eff}} | B_s^0 > = \left[ \sum_{q=u,c} v_q \left\{ (1 + \frac{1}{N_c})(C_3' + C_4') + a_6 + \frac{3}{2} e_s a_8 \right\} \right] + \left( 1 + \frac{1}{N_c} \right)(C_9' + C_{10}') \cdot e_s \right\} \right] M_{\phi \phi}^{\text{eff}}
\]

\[
< \phi \bar{K}^0 | H_{\text{eff}} | B_s^0 > = \left[ \sum_{q=u,c} v_q \left\{ a_3 + \frac{M_2^2}{(m_s + m_d)(m_s + m_b)} (-2a_5 - 3e_s a_7) + \frac{3}{2} e_s a_9 \right\} \right] M_{\phi \bar{K}^0}^{\text{eff}}
\]

\[
+ \left[ \sum_{q=u,c} \left\{ a_3 + 3e_s (a_8 + a_{16}) \right\} \right] M_{\phi \bar{K}^0}^{\text{eff}}
\]

\[
(9)
\]

where the masses of d quark and s quark are taken as \( m_d = 0.01 \text{GeV}, m_s = 0.175 \text{GeV} \) in numerical calculation, and \( a_k \) is defined as

\[
a_{2i-1} \equiv \frac{C_{2i-1}'}{N_c} + C_{2i}', \quad a_{2i} \equiv C_{2i-1}' + \frac{C_{2i}'}{N_c}, \quad (i = 1, 2, 3, 4, 5)
\]

The authors of Ref.[8] have calculated \( < \phi \phi | H_{\text{eff}} | B_s^0 > \). Our results in eq.(9) are consistent with theirs after making a charge conjugation. We follow the definition of relevant decay constants and form factors in Ref.[14], and use eq.(9~ 13) in Ref.[10] to calculate
\[
\sum_{q=u,c} |M_{q_1 q_2 q_3}^{XY}|^2. \] In the rest frame of \(B_s^0\), the two body decay width is calculated by

\[
\Gamma(B_s^0 \to XY) = \frac{1}{8\pi} |\langle XY | H_{eff} | B_s^0 \rangle|^2 \left| \frac{p}{M_B} \right|, \tag{10}
\]

where \(p = \frac{[(M_B^2-(M_X+M_Y)^2)(M_B^2-(M_X-M_Y)^2)]^{1/2}}{2M_B}\) is the magnitude of momentum of the particles X and Y. \(M_B, M_X\) and \(M_Y\) are the mass of \(B_s^0\), X and Y, respectively. For calculating the branching ratio, the total decay width of \(B_s^0\) is taken as \(\Gamma_{tot} = 4.91 \times 10^{-13}\) GeV\([15]\).

We give our numerical results in Table 1-5, where “QCD+EW” means the decay width or branching ratio with full QCD penguin and EW penguin corrections, “QCD” with only QCD penguin corrections. \(\frac{\Gamma_{QCD+EW} - \Gamma_{QCD}}{\Gamma_{QCD}}\) represents the enhancement percentage of EW penguin contributions to the decay width. \(R\left(\frac{QCD+EW}{tree}\right), R\left(\frac{EW}{tree}\right),\) and \(R\left(\frac{QCD}{tree}\right)\) represent the relevant ratios of amplitudes of tree and penguins.

From Table 1 and 2, we can see that for \(B_s^0 \to \pi^+ K^-\) and \(K^+ K^-\), the EW penguin effects are small, they only reduce or enhance the decay width by 0.6% \(\sim\) 8%, and the results are not sensitive to \(N_c\). For \(B_s^0 \to \pi^+ K^-\), the tree level contribution is dominant, while for \(B_s^0 \to K^+ K^-\), the QCD penguin contribution dominant. From Table 3 - 5, we find that for \(B_s^0 \to \pi^0 K^0, \phi K^0\) and \(\phi\phi\), the EW penguin correction is significant. The percentage of correction is almost as high as \(-80.3\%\) for \(B_s^0 \to \phi K^0\) decay modes when taking \(N_c = 3.\)

In Table 6 and 7, we present the branching ratios and CP asymmetry parameters of the five \(B_s^0\) decay modes discussed above. For \(f\) being non-CP-eigenstate cases, the CP asymmetry is defined as the following

\[
\mathcal{A}_{cp} = \frac{\Gamma(B_s^0 \to f) - \Gamma(\bar{B}_s^0 \to \bar{f})}{\Gamma(B_s^0 \to f) + \Gamma(\bar{B}_s^0 \to \bar{f})}, \tag{11}
\]

while for \(f\) being CP-eigenstate case, we calculate the CP asymmetry through\([16]\)

\[
\mathcal{A}_{cp} = \frac{\int_0^\infty [\Gamma\left(B_{phys}^0(t) \to f\right) - \Gamma\left(\bar{B}_{phys}^0(t) \to \bar{f}\right)]dt}{\int_0^\infty [\Gamma\left(B_{phys}^0(t) \to f\right) + \Gamma\left(\bar{B}_{phys}^0(t) \to \bar{f}\right)]dt} = \frac{1 - |\xi|^2 - 2 Im \xi \left(\frac{\Delta m}{\Gamma}\right)\left(1 + |\xi|^2\right)\left[1 + \left(\frac{\Delta m}{\Gamma}\right)^2\right]}{\left(1 + |\xi|^2\right)^2\left[1 + \left(\frac{\Delta m}{\Gamma}\right)^2\right]^2}. \tag{12}
\]
where $\Delta m/\Gamma \simeq 20$ which is the preferred value in the Standard Model[11].

We find that only for $B^0_s \rightarrow \pi^0\bar{K}^0$, and $\phi\bar{K}^0$ two cases, CP violation parameter is sensitive to the number of colors $N_c$, the other four decay modes are not.

This work is supported in part by the China National Natural Science Foundation and the Grant of State Commission of Science and Technology of China.
References

[1] A.J.Buras, M.Jamin, M.E.Lautenbacher, P.H.Weisz, Nucl.Phys.B370(1992)69; Nucl.Phys.B375(1992)501.

[2] A.J.Buras, M.Jamin, M.E.Lautenbacher, P.H.Weisz, Nucl.Phys.B400(1993)37.

[3] A.J.Buras, M.Jamin, M.E.Lautenbacher, Nucl.Phys.B400(1993)75.

[4] A.J.Buras, M.Jamin, M.E.Lautenbacher, Nucl.Phys.B408(1993)209.

[5] F.J.Gilman, J.S.Hagelin, Phys.Lett.B126(1983)111.

[6] A.J.Buras, J.M.Gerard, Phys.Lett.B203(1988)272.

[7] Robert Fleischer, Phys.Lett.B321(1994)259;

Robert Fleischer, Phys.Lett.B332(1994)419;

Robert Fleischer, Z.Phys.C62(1994)81;

[8] N.G.Deshpande and X.G.He, Phys.Rev.lett.74(1994)26;

N.G.Deshpande and X.G.He, Phys.Lett.B336(1994)471;

N.G.Deshpande and X.G.He, Phys.Lett.B345(1995)547.

[9] A.Deandrea, N.Di Bartolomeo, R.Gatto and G.Nardulli, Phys.Lett.B318 (1993)549.

[10] D.S.Du and Z.Z.Xing, Phys.Rev.D48(1993)4155.

[11] A.Ali and D.London, Preprint CERN-TH.7408/94

[12] A.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys.B370(1992)69.

[13] R.Fleischer, Z.Phys.C58(1993)483

[14] M.Wirbel, B.Stech and M.Bauer, Z.Phys.C29(1985)637; Z.Phys.C34(1987) 103.
[15] Particle Data Group, M.Aguilar-Benitez, et.al., Review of Particle Properties, Phys.Rev.D50,1173(1994).

[16] Michael Gronau, Phys.Rev.Lett.63(1989)1451.
Table 1 Numerical results of $B_s^0 \to \pi^+ K^-$

| $N_c$ | $\Gamma$ (10^{-18} GeV) | $\Gamma_{QCD+EW}/\Gamma_{QCD}$ | $R(QCD+EW_{tree})$ | $R(QCD_{tree})$ | $R(EW_{tree})$ |
|-------|-------------------------|-------------------------------|-------------------|----------------|----------------|
| 2     | 1.83                    | 1.86                          | -1.23%            | 0.197          | 0.187          |
| 3     | 2.016                   | 2.025                         | -0.42%            | 0.197          | 0.193          |
| $\infty$ | 2.45                | 2.42                          | 0.69%             | 0.194          | 0.200          |

Table 2 Numerical results of $B_s^0 \to K^+ K^-$

| $N_c$ | $\Gamma$ (10^{-18} GeV) | $\Gamma_{QCD+EW}/\Gamma_{QCD}$ | $R(QCD+EW_{tree})$ | $R(QCD_{tree})$ | $R(EW_{tree})$ |
|-------|-------------------------|-------------------------------|-------------------|----------------|----------------|
| 2     | 3.19                    | 2.93                          | 8.61%             | 3.61           | 3.43           |
| 3     | 3.58                    | 3.48                          | 2.93%             | 3.62           | 3.56           |
| $\infty$ | 4.22                | 4.43                          | -4.82%            | 3.56           | 3.68           |

Table 3 Numerical results of $B_s^0 \to \pi^0 \bar{K}^0$

| $N_c$ | $\Gamma$ (10^{-19} GeV) | $\Gamma_{QCD+EW}/\Gamma_{QCD}$ | $R(QCD+EW_{tree})$ | $R(QCD_{tree})$ | $R(EW_{tree})$ |
|-------|-------------------------|-------------------------------|-------------------|----------------|----------------|
| 2     | 1.59                    | 1.97                          | -19.0%            | 0.426          | 0.617          |
| 3     | 0.45                    | 0.69                          | -34.7%            | 1.831          | 2.516          |
| $\infty$ | 0.59                | 0.58                          | 1.72%             | 0.501          | 0.647          |

Table 4 Numerical results of $B_s^0 \to \phi \bar{K}^0$

| $N_c$ | $\Gamma$ (10^{-20} GeV) | $\Gamma_{QCD+EW}/\Gamma_{QCD}$ | $R(QCD+EW_{tree})$ | $R(QCD_{tree})$ | $R(EW_{tree})$ |
|-------|-------------------------|-------------------------------|-------------------|----------------|----------------|
| 2     | 1.78                    | 3.46                          | -48.6%            |                |                |
| 3     | 0.12                    | 0.62                          | -80.3%            |                |                |
| $\infty$ | 3.50                | 2.22                          | 58.0%             |                |                |
Table 5 Numerical results of $B_s^0 \rightarrow \phi\phi$

| $N_c$ | $\Gamma$ (10$^{-18}$GeV) | $\Gamma_{QCD+EW}/\Gamma_{QCD}$ |
|-------|-----------------|-----------------|
| 2     | 2.46            | -24.8%          |
| 3     | 1.56            | -26.1%          |
| $\infty$ | 0.38            | -26.9%          |

Table 6 Branching ratios and CP asymmetry parameters with only tree and QCD penguin contributions

| decay mode | $B_s^0 \rightarrow \pi^+K^-$ | $B_s^0 \rightarrow \phi\phi$ |
|------------|-----------------------------|-----------------------------|
| $N_c = 2$  | $3.78 \times 10^{-6}$       | $6.67 \times 10^{-6}$       |
| $N_c = 3$  | $4.12 \times 10^{-6}$       | $4.29 \times 10^{-6}$       |
| $N_c = \infty$ | $4.95 \times 10^{-6}$ | $1.06 \times 10^{-6}$       |
| $A_{cp}$ | $-7.0\%$                  | $-0.010\%$                  |
| $N_c = 2$  | $4.12 \times 10^{-6}$       | $7.66 \times 10^{-7}$       |
| $N_c = 3$  | $4.95 \times 10^{-6}$       | $7.66 \times 10^{-7}$       |
| $N_c = \infty$ | $-2.31\%$                 | $-0.016\%$                 |

Table 7 Branching ratios and CP asymmetry parameters with full tree, QCD and EW penguin contributions

| decay mode | $B_s^0 \rightarrow \pi^+K^-$ | $B_s^0 \rightarrow \phi\phi$ |
|------------|-----------------------------|-----------------------------|
| $N_c = 2$  | $3.73 \times 10^{-6}$       | $5.02 \times 10^{-6}$       |
| $N_c = 3$  | $4.11 \times 10^{-6}$       | $3.19 \times 10^{-6}$       |
| $N_c = \infty$ | $4.98 \times 10^{-6}$ | $7.66 \times 10^{-7}$       |
| $A_{cp}$ | $-7.28\%$                  | $-0.011\%$                  |
| $N_c = 2$  | $4.11 \times 10^{-6}$       | $7.66 \times 10^{-7}$       |
| $N_c = 3$  | $4.98 \times 10^{-6}$       | $7.66 \times 10^{-7}$       |
| $N_c = \infty$ | $-2.22\%$                 | $-0.016\%$                 |

12