Influence of density inversion and sinusoidal heating on dual diffusive convection in a water saturated square porous box

K Janagi 1, S Sivasankaran 2, M Bhuvaneswari 3 and M Eswaramurthi 4

1Department of Mathematics, Sri Shannughha College of Engineering and Technology, Pullipalayam, Sankari, Tamilnadu, India
2Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
3Department of Mathematics, Kongunadu Polytechnic College, D Gudalur, Dindugal, Tamilnadu, India
4Department of Mathematics, Velalar College of Engineering and Technology, Erode, Tamilnadu, India

Abstract. The motive of the present numerical study is to analyze the density maximum effect of double diffusive buoyancy-driven convection in a square porous box. The horizontal walls are treated as adiabatic and the vertical walls are maintained with different temperatures. The right wall is heated sinusoidally while the left wall kept with constant temperature. The concentration on the left and right walls of the box are $C_L$ and $C_h$, respectively, with $C_h > C_L$. The present study uses finite volume method to solve the governing equations. The effects of buoyancy ratio, density inversion parameter, Darcy number and porosity are studied by using the Brinkmann-Forchheimer extended Darcy model. It is observed that the bicellular structure is formed due to density inversion. When $\frac{mT}{m} = 1$, the average mass and thermal energy transfer rates are high for every values of buoyancy ratio. When $\frac{mT}{m} = 0.4$, the average mass and thermal energy transfer rates are low for all values of $N$ due to density maximum of water.

Keywords: Double diffusion, porous medium, sinusoidal heating, density inversion.

1. Introduction

The investigation on double diffusive buoyancy convection plays a major role in enormous engineering applications for the last few decades. Various authors studied the effect on double diffusive convection in a fluid saturated porous medium with different phenomena. Initially, the authors has been used the Darcy’s law to do their research in porous medium. After the analytical study on steady nonlinear dual diffusive convection in a porous medium based upon the Brinkmann-Forchheimer model by Kaloni and Guo [1], the researchers understood and used this model as one of the best model to study the convection in porous medium. On the whole, increase in temperature leads to decrease in density of the fluid. But the reversed phenomenon is occurred in pure water. i.e., at a pressure of one atmosphere, around 4°C, the maximum density of 999.972kgm⁻³ is attained for pure water. This phenomenon, named density inversion effect of cold water, is studied numerically by several authors. Janagi et al. [2] numerically examined the density inversion effect and the effect of sinusoidal heating of cold water in a square box with porous materials. The result exposed that the density inversion parameter leaves strong effect in the variation of thermal energy transfer rate.

Tong and Koster [3] and Nansteel et al. [4] numerically simulated the buoyancy convection of water in a rectangular box with density inversion. It is reported that the density inversion shows significant effect on thermal energy transfer. Sivasankaran and Ho [5] numerically investigated the free convection of water adjacent to its density maximum in a box with various properties depends on temperature. It is identified that the distribution of heat was highly subjective with the effect on density maximum of water. An experimental study is made by Bukreev et al. [6] to find the effect of water density on free convection which depends nonmonotonically. It is found that the height of the convective plume is limited because
of density inversion of water. Free Marangoni convection of water in a box with density inversion effect is studied numerically by Sivasankaran and Ho [7]. When the buoyancy force is weakened, it is found that the convective thermal energy transfer is improved by thermo-capillary force. The consequence of water density inversion on double-diffusive convection is numerically studied by Sivasankaran et al. [8]. The results clearly indicates that the maximum density acts a vital role on mass and thermal energy transfer.

Several authors numerically deliberated the natural convection in various shaped boxes with sinusoidal heating on the walls. Sivasankaran and Pan [9] made numerical experiment on mixed convection in a square porous lid-driven box to study the influence of non-uniform temperature on vertical walls. It is found that thermal energy transfer rate increases on increasing amplitude ratio. Sivakumar and Sivasankaran [10] performed a numerical study on mixed convection in lid-driven box which is inclined in position and with non-uniform heating on both sidewalls. It is identified that both walls with non-uniform heating case produced more convection thermal energy transfer than the case of wall with uniform heating. Bilgen and Yedder [11] numerically analyzed the natural convection in a rectangular box having a sidewall with sinusoidal temperature. It is observed that the thermal energy transfer is higher when the heated section is in the lower half of the enclosure than that in which the heated section is at the upper half of the enclosure. Mansour et al. [12] numerically examined the effect of sinusoidal variations of boundary conditions on unsteady double diffusive natural convection in a square porous box. Some more papers are dealing the sinusoidal heating or nonuniform heating in enclosed spaces [13-22].

However, the survey of literature exhibits that there are numerous investigations on dual diffusive convection with opposing flows and the studies with density maximum effect, no such work carried out on dual diffusive natural convection with density maximum and sinusoidal heating effect. The present study deals with sinusoidal heating and density maximum effect together to study the dual diffusive convection in water saturated square box with porous material.

2. Mathematical formulation

An isotropic water saturated square box of size L containing porous materials L is considered for the present numerical study as displayed in figure 1. The sinusoidal temperature (θ_h) is considered on the left wall with concentration C_l and θ_v, C_h are the temperature and concentrations on the right wall, respectively. The top and bottom walls are adiabatic and impermeable to heat and mass transfer. The thermal and solutal buoyancy are the two active forces in the present study. The laminar incompressible flow is considered. The action of gravitational force towards the downward direction is considered. The velocity components (u, v) are considered in the x and y directions, respectively. In the present study the porous medium is modeled by Brinkman-Forchheimer extended Darcy model. Also it is presumed to be homogeneous and thermodynamic equilibrium with the fluid. The constant thermal properties of the fluid are considered except the density. The specific heat ratio(σ) is assumed to be one. The density variation
of water follows the relation \( \rho = \rho_m \left[ 1 - \beta_T (\rho - \rho_m) \right] \), where \( \beta_T \) represents the coefficient of thermal expansion and \( \beta_c \) indicates the coefficient of solutal expansion. Subscript ‘m’ symbolizes the density inversion state. The Boussinesq approximation is valid. Insigificant viscous dissipation is considered.

The equations governed by the system are as follows:

\[
\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} = \alpha \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{F_c}{\sqrt{K}} u \sqrt{u^2 + v^2} + \beta_T (\theta - \theta_m)^b + g \beta_c (c - c_m)
\]

\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)
\]

where \( F_c = \frac{1.75}{\sqrt{150 \rho c v^2}} \).

The initial and boundary conditions for the problem can be written as follows:

\[
t = 0: \quad u = v = 0, \quad \theta = \theta_0; \quad \rho = \rho_m; \quad 0 \leq (x, y) \leq L
\]

\[
t > 0: \quad u = v = 0, \quad \frac{\partial \theta}{\partial y} = 0; \quad \frac{\partial c}{\partial x} = 0; \quad y = 0 \& L
\]

\[
u = 0, \quad \theta = \theta(y) = \sin \pi y; \quad c = c_m; \quad x = 0
\]

\[
u = 0, \quad \theta = \theta_0; \quad c = c_m; \quad x = L
\]

The dimensionless equations in the vorticity and stream function formulation are obtained with the treatment of the dimensionless variables \( X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{v}, V = \frac{vL}{v}, T = \frac{\theta - \theta_m}{\theta_h - \theta_m}, C = \frac{c - c_m}{c_h - c_m}, \zeta = \frac{\alpha}{\nu L^2} \), and \( \Psi = \frac{\psi}{v} \), and the resulting equations are as follows:

\[
\frac{1}{\epsilon^2} \left( \frac{\partial \zeta}{\partial \tau} + \frac{1}{\epsilon} \left( \frac{U \partial \zeta}{\partial X} + V \partial \zeta}{\partial Y} \right) \right) = \epsilon \nabla^2 \zeta - \frac{1}{Da} \zeta - \frac{F_c |U|}{\sqrt{Da}} \zeta
\]

\[
- \frac{F_c \left( U \frac{\partial |U|}{\partial Y} - V \frac{\partial |V|}{\partial X} \right)}{\sqrt{Da}} + Gr_l \left[ \frac{\partial}{\partial X} (T - T_m) \right] + N \frac{\partial C}{\partial X} + \frac{\partial^3 \Psi}{\partial X^2} + \frac{\partial^3 \Psi}{\partial Y^2} = -\zeta
\]

\[
\frac{\partial T}{\partial \tau} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \nabla^2 T
\]

\[
\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \nabla^2 C
\]
The dimensionless parameters in the current problem are the Darcy number $Da = \frac{K}{L}$, thermal Grashof number $Gr_i = \frac{g\beta_i\theta_i}{\nu^2} L^3$, the solutal Grashof number $Gr_c = \frac{g\beta_c(\kappa_c-c_c)}{\nu^2} L^3$, the buoyancy ratio $N = \frac{Gr_c}{Gr_i}$, the Prandtl number $Pr = \frac{\nu}{\alpha} = 11.6$, the Schmidt number $Sc = \frac{\nu}{D}$ and $Tm$ density inversion parameter.

The initial and boundary conditions in dimensionless form are

$$
\tau = 0: \quad U = V = \Psi = T = C = 0; \quad \xi = \theta = 0; \quad 0 \leq X \leq 1, \quad 0 \leq Y \leq 1
$$

$$
\tau > 0: \quad U = V = \Psi = T = C = 0; \quad \xi = \theta = 0; \quad Y = 0 & 1
$$

$$
\tau > 0: \quad U = V = \Psi = T = C = 0; \quad X = 0
$$

$$
\tau > 0: \quad U = V = \Psi = T = C = 0; \quad X = 1
$$

The thermal energy transfer is accounted for the hot wall by the use of the local Nusselt number $Nu = \left. \frac{dT}{dY} \right|_{Y=0}$ and the average thermal energy transfer rate is given by a Nusselt number $Nu = \int_0^1 Nu dX$. The rate of mass transfer is measured using the local Sherwood number $Sh = \left. \frac{dC}{dY} \right|_{Y=1}$ and the average Sherwood number is given by $Sh = \int_0^1 Sh dX$ which is calculated by using Trapezoidal method. The solution of dimensionless equations governed the system, is obtained by using finite volume method. The criteria of convergence used for the field variables $\phi(=\xi,\Psi,T,C)$ is

$$
\frac{\text{Max}_{i,j} |\phi_{i+1,j}(i,j)-\phi_{i,j}(i,j)|}{\text{Max}_{i,j} \phi_{i+1,j}(i,j)} \leq 10^{-6}, \quad \text{where} \quad \phi_{i,j}(i,j) \quad \text{and} \quad \phi_{i,j+1}(i,j)
$$

are the previous and present iteration values of the parameter respectively. The solution methodology and code validation is clearly clarified in Janagi et al. [2].

3. Results and discussion

A numerical study on dual diffusive convective flow of cold water near its density inversion in a square box with porous materials with sinusoidal heating on the left wall is analyzed. The controlling parameters for the present study are, buoyancy ratio ($N$), Darcy number ($Da$), porosity ($\epsilon$) and the density inversion parameter ($Tm$). The range of controlling parameters considered for numerical calculations are Buoyancy ratio $-5 \leq N \leq 5$, Darcy number $10^{-3} \leq Da \leq 10^{-4}$, the porosity $0.4 \leq \epsilon \leq 0.8$ and the density inversion parameter $0 \leq Tm \leq 1$.

Figure 2 represents the streamlines and distribution of heat across the box for different values of $Tm$ and keeping $Da = 10^{-3}$, $\epsilon = 0.6$, $N=1$, $Ra = 10^6$. When $Tm = 0$, the fluid particles move alongside the hot wall, travel consistently in the direction of the right wall and moves downwards along it. This flow pattern forms a cell which occupies the whole box. Sinusoidal heating leads to the formation of a secondary cell near the top-left corner of the box. With the value of $Tm$ as 0.4, the box is seen with the development of a bi-cellular structure inside it, the secondary cell becomes much more strengthen. That is, the convective thermal energy transfer is reduced and the thermal energy transfer is due to conduction in the middle of the box due to the location of density inversion plane near the center of the box. The secondary cell enlarged in its size constantly and occupies the whole box. Now the primary cell becomes a small eddy near bottom-left corner of the box as $Tm$ increases. It is clearly understood that the convective mode of thermal energy transfer is maintained inside the box for all values of $Tm$ except $Tm = 0.4$. When $Tm = 0.4$, the conductive thermal energy transfer takes place due to density maximum effect. Also the
thermal boundary layer exists on the left hot wall at different position on it. i.e., for $T_m=0$, thermal boundary layer exists at the lowermost part of the heated wall whereas it exists at the upper part of the hot wall for $T_m=1$ due to sinusoidal heating. There is no boundary layer along hot wall for $T_m=0.4$.

**Figure 2.** Streamlines and isotherms for various $T_m$ with $Da=10^{-3}$, $\varepsilon=0.6$, $N=1$ and $Ra=10^6$.

Figure 3 shows the mass transfer across the box for distinct values of $T_m$ with $Da=10^{-3}$, $\varepsilon=0.6$, $N=1$, $Ra=10^6$. Solutal boundary layer exists along the bottom left side wall and top right side wall for all values of $T_m$ except at $T_m=0.4$. Due to density maximum effect the rate of mass transfer is moderate at $T_m=0.4$. The isoconcentration lines for $T_m=1$ is the reverse image of those for $T_m=0$. In general smooth mass transfer is enhanced throughout the box.
Figure 3. Isoconcentration lines for different $T_m$ with $Da=10^{-3}$, $\varepsilon=0.6$, $N=1$ and $Ra=10^6$.

Figure 4. Local Nusselt number (a) and Sherwood number (b).

The variations of local thermal energy transfer rate on the left wall is shown in figure 4a for various values of $T_m$ and keeping $N=1$, $Da=10^{-3}$, $\varepsilon=0.6$, $Ra=10^6$. When density inversion plane is located either on the right side wall ($T_m=0$) or on left wall ($T_m=1$), the local thermal energy transfer attains its maximum level. When density inversion plane is moved towards the left wall, the local thermal energy transfer reduces slowly, and gradually increases, when it comes near to the left wall. When $T_m<0.5$, the local thermal energy transfer is high near the bottom of the wall (i.e., at $Y=0.35$) and when $T_m>0.5$, the local thermal energy transfer is high near the upper part of the wall (i.e., at $Y=0.7$). These variations in local thermal energy transfer exist due to sinusoidal heating. It is identified from the shape of the local Nusselt number curve that the sinusoidal heating affect local thermal energy transfer directly. Fig. 4b exhibits the impact of $T_m$ on the local mass transfer rate for various density inversion parameter with $N=1$, $Da=10^{-3}$, $\varepsilon=0.6$ and $Ra=10^6$. When $T_m=0$, 0.2 the diffusion starts gradually from the lowest part of the box near cold wall and the local mass transfer rate attains its maximum in the upper portion of the wall. When $T_m<0.3$, the local mass transfer is low at the bottom of the wall and is exponentially increasing from the bottom to the upper part of the right side wall. When $T_m>0.3$, we get a reverse phenomenon, that
is, the mass transfer is high at the bottom of the wall and slowly decreases and reaches its minimum level at the top of the right wall. This non-linear behavior of mass transfer is due to the existence of density inversion.

Figure 5. Average Nusselt number (a) average Sherwood number (b) on variation of Darcy number

Figure 6. Averaged Nusselt number (a) & averaged Sherwood number (b) vs. $T_m$

Figure 5a represents the impact of Darcy number on the mean value of $Nu$ for different values of density inversion parameter with $N=1$, $\varepsilon=0.6$ and $Ra=10^6$. It is observed that a contradict result is obtained. In general, increase in Darcy number increases the average thermal energy transfer rate. But it is evidently seen that the obtained average thermal energy transfer rate for $Da=10^{-1}$ is lower than the obtained average thermal energy transfer rate for $Da=10^{-2}$. This contradict result occurs due to sinusoidal temperature on the left side wall. Figure 5b reports the consequence of the Darcy numbers on average Sherwood number for various values of density inversion parameter with $N=1$, $\varepsilon=0.6$ and $Ra=10^6$. It is found that the increase in Darcy number increases the average Sherwood number. When $T_m=0$ to $T_m=0.4$, the value of average Sherwood number decreases and gradually increases as increasing the value of $T_m$. This is because of density maximum effect. Since Darcy number is directly proportional to permeability of the porous medium, for lower value of Darcy number, the average rate of mass transfer is very low due to low circulation of flow.

Figure 6 clearly shows the impact of porosity on average $Nu$ and average Sherwood number for different values of density inversion parameter with $N=1$, $Da=10^{-3}$, $Ra=10^6$. The average mass and thermal energy transfer rate rises accordingly as rise in porosity. This is because of strong movement of fluid particles for higher value of porosity. When $T_m=0.4$, the circulation of fluid particles divide the box into two major cells and the mode of thermal energy transfer becomes conductive which leads to low thermal energy transfer rate (see figure 6a). Also the low circulation of flow leads to the lower mass transfer rate (see figure 6b).
Figure 7. Average Nusselt number (a) & average Sherwood number (b) vs. density inversion parameter

Figure 7a exhibits the performance of density inversion parameter on average Nu for various values of buoyancy ratio with $\varepsilon = 0.6$, $Da = 10^{-3}$ and $Ra = 10^6$. When $T_m = 1$, the average thermal energy transfer rate is high for every values of buoyancy ratio $(-5 \leq N \leq 5)$. When $T_m = 0.4$, the average thermal energy transfer rate is very low for every values of N. This is because of density maximum effect of water.

Figure 7b shows the result of $T_m$ on average Sherwood number and buoyancy ratio with $\varepsilon = 0.6$, $Da = 10^{-3}$ and $Ra = 10^6$. It is evidently seen that the average mass transfer is high for $T_m = 1$ for the value of N ranging from -5 to 5. When $T_m = 0.4$, the average mass transfer rate is very low for all values of N. This is due to the inversion of water density.

4. Conclusion
Free convective mass and thermal energy transfer of cold water in a square porous box with sinusoidal heating on vertical wall are examined numerically in the presence of density inversion. In this study finite volume approach is used to solve the governing equations. The following conclusions are arrived from the numerical results.

1. The location of density maximum plane acts an important role in fluid motion. The streamlines form a single major cell at $T_m = 0$ and 1, i.e., density maximum plane located near the side walls. As $T_m$ increases the secondary cells are formed and the strong bicellular structure is formed at $T_m = 0.4$ where the density maximum plane is located at the center of the box.
2. The average thermal energy transfer rate for $Da = 10^{-1}$ is lower than the average thermal energy transfer rate for $Da = 10^{-2}$. This is a contradict result to the fact that, increase in the value of Da increases the thermal energy transfer rate.
3. The mean value of Sh increases as the Da increases.
4. The average mass and thermal energy transfer rates increase as increase in porosity.
5. When $T_m = 1$, the average mass and thermal energy transfer rates are high for every values of buoyancy ratio. When $T_m = 0.4$, the average mass and thermal energy transfer rates are very low for all values of N because of density inversion effect of water.

References
[1] Kaloni PN and Guo J 1996 J. Math. Anal. Appl. 204 pp 138-155
[2] Janagi K Sivasankaran S Bhuvaneswari and M Eswaramurthi M 2017 Int. J. Numer. Methods Heat Fluid Flow 27(4) pp 1000-1014
[3] Tong W and Koster JN 1993 Int. J. Heat and Fluid Flow 14(4) pp 366-375
[4] Nansteel MW Medjani K and Lin DS 1987 Phys. Fluids. 30(2) pp 312-317
[5] Sivasankaran S and Ho C J 2008, Numer. Heat Trans. A 53 pp 507-523
[6] Bukreev VI Gavrilov NV and Chebotnikov AV 2011 J. Appl. Mech. Tech. Phy. 52(1) pp 24-30
[7] Sivasankaran S and Ho C J 2010 Numer. Heat Trans. A 58 pp 457-474
[8] Sivasankaran S Kandaswamy P and Ng CO 2008 Trans. Por. Media 71(2) pp 133–145
[9] Sivasankaran S and Pan K L 2012 Numer. Heat Trans. A 61 pp 101-121
[10] Sivakumar V and Sivasankaran S 2014 J. Appl. Mech. Tech. Phy. 55(4) pp 634-649
[11] Bilgen E and Yedder RB 2007 *Int. J. Heat Mass Trans.* 50 pp 139-150
[12] Mansour MA, Abd-Elaziz MM, Abdalla R and Elsayed S 2012 *Int. J. Numer. Methods Heat Fluid Flow* 22(1) pp 129-146
[13] Cheong HT, Sivasankaran S and Bhuvaneswari M 2017 *Int. J. Numer. Meth. Heat Fluid Flow* 27(2) pp 287-309
[14] Sivasankaran S and Bhuvaneswari M 2013 *Numer. Heat Trans. A* 63(1) pp 14-30
[15] Sivasankaran S, Ananthan SS and Abdul Hakeem AK 2016 *Scientia Iranica Trans. B. Mech Engg* 23(3) pp 1027-1036
[16] Bhuvaneswari M, Sivasankaran S and Kim YJ 2011 *Numer. Heat Trans. A* 59 pp 167-184
[17] Sivasankaran S, Cheong HT Bhuvaneswari M and Ganesan P 2016 *Numer. Heat Trans. A* 69(6) pp 630-642
[18] Cheong HT, Siri Z and Sivasankaran S 2013 *Int. Comm. Heat Mass Trans. 45* pp 75-85
[19] Sivasankaran S, Ananthan SS, Bhuvaneswari M and Abdul Hakeem AK 2017 *Sadhana, 42(11)* pp 1929–1941
[20] Sivakumar V and Sivasankaran S 2014 *J. Appl. Mech. Tech. Phy.* 55(4) pp 634 – 649
[21] Sivasankaran S and Pan KL 2014 *Numer. Heat Trans. A* 65 pp 247–268
[22] Sivasankaran S and Pan KL 2012 *Numer. Heat Trans. A* 61(2) pp 101-121