Gravitational corrections to Yukawa systems

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Abstract. We compute the gravitational corrections to the running of couplings in a scalar-fermion system, using the Wilsonian approach. Our discussion is relevant for symmetric as well as for broken scalar phases. We find that the Yukawa and quartic scalar couplings become irrelevant at the Gaussian fixed point.

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I. INTRODUCTION

The lack of renormalizability of Einstein’s theory does not preclude the possibility of calculating quantum corrections to low energy processes due to graviton loops [1]. This effective field theory approach has been applied to calculate corrections to the gravitational potential [2] and the running of Newton’s constant [3, 4]. Graviton loops also contribute to the beta functions of matter couplings. This has been studied in the case of a scalar field in [5]. More recently, there has been considerable interest in (and controversy about) the corrections to the beta function of gauge couplings [6]. Aside from the intrinsic theoretical interest, such effects could have obvious applications to grand unified theories, whose characteristic energy scale is not too distant from the Planck scale, where gravity becomes strong. In fact, it has been argued recently [7] that in the determination of the GUT scale, quantum gravitational effects could be more important than two loop effects.

With these motivations in mind, and in the same spirit, we will calculate here the gravitational effects on the beta functions of a simple Yukawa theory, consisting of one scalar and Nf fermion fields. We will do our calculations in flat Euclidean space, and therefore we will not calculate here the effect that the matter has on the running of the gravitational couplings (e.g., Newton’s constant), but at least in the limit where the matter couplings are negligible, this effect is easily calculable [8].

In addition to the above, there is also another reason for studying this problem. If we look for a fundamental, as opposed to effective, theory of quantum gravity, there is now the concrete possibility that a purely field theoretic solution can be obtained, provided that the renormalization group has a fixed point with a finite number of UV attractive (relevant) directions. A theory with these properties is said to be asymptotically safe and has the same good properties (finiteness, predictivity) as, for example, QCD. The failure of perturbation theory means that the Gaussian fixed point of gravity does not have the desired properties. Work done in the last ten years has provided rather convincing evidence for the existence of a suitable nontrivial fixed point in pure gravity; see [10] for reviews. It is then important to make sure that this fixed point persists also when interacting matter is brought in. In the case of scalar interactions, this was discussed in [11]. It was shown that there exists a “Gaussian matter fixed point”, where the gravitational couplings are nonzero and slightly shifted relative to pure gravity, but all scalar selfinteractions are asymptotically free or zero. Our results imply that such a fixed point exists also in the presence of a Yukawa coupling.

Finally we mention that asymptotic safety may play a role also in the standard model. Some evidence for a nontrivial fixed point in Yukawa systems has appeared recently [12]. If this was the case, then the calculations presented here are necessary to complete the picture by including also the gravitational interactions.

II. YUKAWA SYSTEM

In this section we set up the calculation. The flow of the renormalized couplings will be computed on a flat Euclidean background using an exact flow equation. An infrared cutoff, denoted $k$, is introduced via a cutoff term $\Delta S_k[\Phi]$. A theory with these properties is said to be asymptotically safe and has the same good properties (finiteness, predictivity) as, for example, QCD. The failure of perturbation theory means that the Gaussian fixed point of gravity does not have the desired properties. Work done in the last ten years has provided rather convincing evidence for the existence of a suitable nontrivial fixed point in pure gravity; see [10] for reviews. It is then important to make sure that this fixed point persists also when interacting matter is brought in. In the case of scalar interactions, this was discussed in [11]. It was shown that there exists a “Gaussian matter fixed point”, where the gravitational couplings are nonzero and slightly shifted relative to pure gravity, but all scalar selfinteractions are asymptotically free or zero. Our results imply that such a fixed point exists also in the presence of a Yukawa coupling.

Finally we mention that asymptotic safety may play a role also in the standard model. Some evidence for a nontrivial fixed point in Yukawa systems has appeared recently [12]. If this was the case, then the calculations presented here are necessary to complete the picture by including also the gravitational interactions.

In flat space the cutoff term has the general form $\Delta S_k[\Phi] = \frac{1}{4} \int d^4x \Phi R_k^2 (-\partial^2)\Phi$ and $R_k^2(z)$ is constructed so as to suppress the contributions to the functional integral from the infrared modes of the field $\Phi$. For a scalar $\phi$, we choose $R_k^2(z) = k^2 r(z/k^2)$, with $r(y) = (1-y)(1-y/2)$, leading to the substitution $-\partial^2 = z \rightarrow P_k(z) = z + k^2 r(z/k^2)$, a kind of cutoff-propagator. For a fermion $\psi$, $R_k^2(i\not\partial) = (\sqrt{P_k(-\partial^2)/(-\partial^2)} - 1)i\not\partial$.

The cutoff-corrected Legendre transform $\Gamma_k = W_k - \int d^4x J\phi - \Delta S_k[\phi]$ defines the effective average action $\Gamma_k$ satisfying the renormalization group equation [14, 15],

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} + R_k \right] \partial_t R_k,$$

where $t = \ln k$ and $\text{STr}$ denotes a functional trace, including a factor $-1$ for fermions. We will restrict our considerations to functionals $\Gamma_k$ of the following form

$$\Gamma_k[g_{\mu\nu}, \phi, \psi, \bar{\psi}] = \int d^4x (L_b + L_f + L_g + L_{G\phi} + L_{gh}) .$$

The theory contains a single scalar field with Lagrangian $L_b = \sqrt{g} \left[ \frac{1}{2} Z_\phi \nabla^\mu \phi \nabla_\mu \phi + V(\phi) \right]$. 

We choose the potential $V$ to be even in $\phi$. Then, there are $N_f$ Dirac fermions $\psi$ with $U(N_f)$-symmetric Lagrangian

\[
L_f = \sqrt{g} \left( \bar{\psi} \gamma^\mu \partial_\mu \psi - \bar{\psi} \gamma^\mu \gamma^5 \psi \right) + i H(\phi) \bar{\psi} \psi .
\]

The covariant derivative is $D_\mu \psi = \partial_\mu \psi + \frac{1}{2} \omega_{\mu c d} J^c_d \psi$, and $D_\mu \bar{\psi} = \partial_\mu \bar{\psi} - \frac{1}{2} \omega_{\mu c d} J^c_d \bar{\psi}$, where $\omega_{\mu c d}$ is the spin connection and $J^c_d$ are the $O(4)$ generators. We will choose the $O(4)$ gauge such that the vierbein is symmetric, so that all vierbein fluctuations can be written in terms of the metric fluctuations and there are no $O(4)$ ghosts. For the time being we keep the function $H(\phi)$ general. On the other hand we will set $Z_\phi = Z = 1$ and neglect anomalous dimensions.

For gravity we have the Einstein-Hilbert Lagrangian

\[
L_g = -Z \sqrt{g} R[g_{\mu \nu}]
\]

where $Z = 1/(16\pi G)$. Similarly to previous analyses, we shall expand around constant backgrounds, which we still denote $g_{\mu \nu} = \delta_{\mu \nu}, \phi, \psi$ and $\bar{\psi}$ with corresponding fluctuations $\delta g_{\mu \nu}, \bar{\psi}, \phi$. For diffeomorphisms we fix a covariant background gauge, with gauge fixing term

\[
L_{GF} = \frac{Z}{2\alpha} \delta_{\mu \nu} F_\mu F_\nu ; \quad F_\mu = \left( \delta_\mu^\beta \partial^\alpha - \frac{1 + \beta}{4} \partial^{\beta} \partial^\alpha \right) g_{\alpha \beta}
\]

and the ghost action term consequently given by

\[
L_{gh} = c_{\mu} \left( -\delta_{\mu \nu} \partial^2 + \frac{\beta - 1}{4} \partial^\mu \partial^\nu \right) c_{\nu} .
\]

We also employ the tensor decomposition

\[
\delta g_{\mu \nu} = \partial_\mu v_\nu + \partial_\nu v_\mu + (\partial_\mu \partial_\nu - \frac{1}{4} \partial_\rho \partial^\rho ) \sigma + \frac{1}{2} \delta_\mu \partial_\nu + \frac{1}{2} \delta_\nu \partial_\mu ,
\]

where $h_{\mu \nu} = \eta_{\mu \nu} h_{\mu \nu} = \partial_\mu v_\nu = 0$ and $h = \delta_g h_{\mu \nu}$, for tensor $(h_{\mu \nu})$, vector $(v_\mu)$ and scalar $(\sigma, h)$ fluctuations of the metric.

The second order expansion of the Lagrangian $\mathcal{L}$ in the fluctuations $h_{\mu \nu}, v_\mu, \sigma, h, \phi, \chi$ and $\bar{\chi}$ is given by:

\[
\mathcal{L}^{(2)} = \frac{h_{\mu \nu} \left( Z \partial^2 + V + iH \bar{\psi} \psi \right) h_{\mu \nu}}{4} - \frac{i}{16} h_{\mu \lambda} \bar{\partial}_\mu h_{\nu \nu} \bar{\psi} \gamma^{\mu \nu \rho} \psi
\]

\[+ \frac{3}{32} \bar{\partial}^2 \sigma \left( \frac{\alpha - 3}{\alpha} Z \partial^2 - 2V - 2iH \bar{\psi} \psi \right) \partial^2 \sigma + \frac{3 - \alpha}{16\alpha} \bar{\partial}^2 \sigma \partial^2 h - \frac{1}{32} \bar{\psi} \bar{\partial}^2 \sigma \left( h \left( \frac{2}{\alpha} Z \partial^2 - 2V - 2iH \bar{\psi} \psi \right) + \frac{1}{2} (V + iH \bar{\psi} \psi) \right)
\]

\[+ \frac{1}{2} h (\bar{\psi} \bar{\psi} \phi) \bar{\nu} - \frac{1}{2} \bar{\nu} (-\partial^2 + V'' + iH' \bar{\psi} \psi) \bar{\phi} - \frac{1}{2} \bar{\phi} \partial^2 \bar{\psi} \bar{\psi} + \frac{1}{2} \bar{H} \bar{h} (\bar{\psi} \bar{\psi} \bar{\psi} \bar{\chi} + \bar{\chi} \bar{\chi}) + \frac{1}{16} \bar{\partial}^2 \bar{\sigma} \partial^2 h - \frac{1}{16} \bar{\partial}^2 h \left( \bar{\psi} \bar{\psi} \bar{\chi} - \bar{\chi} \bar{\psi} \bar{\chi} \right)
\]

III. BETA FUNCTIONS.

Let us define the dimensionless field $\tilde{\phi} = \phi/k$, and the dimensionless functions $v(\tilde{\phi}) = V(k\tilde{\phi})/k^4$ and $h(\tilde{\phi}) = H(k\tilde{\phi})/k$. The running of $V$ and $H$ is obtained matching $\tilde{\phi} \sim \int d^4 x \left( \tilde{V} + iH \tilde{\psi} \psi \right)$. Then, $\dot{\tilde{\psi}} = -4v \tilde{\phi} \bar{v}' + k^{-4} \tilde{V}$, and $\tilde{h} = -h + \tilde{\phi} h' + k^{-1} \tilde{H}$. We present here the beta functionals for $v$ and $h$, in the gauge $\beta = 1$ and expanding to first order in the dimensionless Newton constant $\tilde{G} = G/k^2$ (the full expressions are nonpolynomial in $\tilde{G}$):

\[
\dot{v} = -4v + \tilde{\phi} v' - \frac{N_f}{8\pi^2 (1 + h^2)} + \frac{3 + 2v''}{32\pi^2 (1 + v'')^2} - \frac{\tilde{G}(3-\alpha)v''}{2\pi} \left( 2 + v'' \right) + \frac{\tilde{G}v(3+2\alpha)}{\pi} + O(\tilde{G}^2),
\]

\[
\dot{h} = -h + \tilde{\phi} h' - \frac{h''}{32\pi^2 (1 + v'')^2} + \frac{hh''}{16\pi^2 (1 + h^2)^2 (1 + v'')^2} + \frac{\tilde{G}(3-\alpha)v''}{\pi} \left( \frac{1}{2} h'' (3 + v'') - \frac{hh''}{2} \left( 4 + 3h^2 + (2 + h^2) v'' \right) \right) \frac{1}{(1 + h^2)^2} + O(\tilde{G}^2)
\]

Fixing the form of the potentials and expanding around an appropriate basis of operators one may find the

\[
\begin{align*}
\dot{v} &= -4v + \tilde{\phi} v' - \frac{N_f}{8\pi^2 (1 + h^2)} + \frac{3 + 2v''}{32\pi^2 (1 + v'')^2} - \frac{\tilde{G}(3-\alpha)v''}{2\pi} \left( 2 + v'' \right) + \frac{\tilde{G}v(3+2\alpha)}{\pi} + O(\tilde{G}^2), \\
\dot{h} &= -h + \tilde{\phi} h' - \frac{h''}{32\pi^2 (1 + v'')^2} + \frac{hh''}{16\pi^2 (1 + h^2)^2 (1 + v'')^2} + \frac{\tilde{G}(3-\alpha)v''}{\pi} \left( \frac{1}{2} h'' (3 + v'') - \frac{hh''}{2} \left( 4 + 3h^2 + (2 + h^2) v'' \right) \right) \frac{1}{(1 + h^2)^2} + O(\tilde{G}^2)
\end{align*}
\]
following local power-law potentials, expanding either around \( \langle \phi \rangle = 0 \) or \( \langle \phi \rangle = \sqrt{\kappa} \). Concerning \( h \), from now on we restrict ourselves to a simple Yukawa interaction \( h = y \phi \).

a. Expansion around \( \langle \phi \rangle = 0 \). For a quartic potential

\[
v(\phi) = \lambda_0 + \lambda_2 \phi^2 + \lambda_4 \phi^4,
\]

inserting in (4) we find, in the gauge \( \alpha = 0 \) and in the approximation \( \lambda_0 = 0 \),

\[
\lambda_0 = \frac{3 + 4 \lambda_2}{32 \pi^2 (1 + 2 \lambda_2)} - \frac{N_f}{8 \pi^2},
\]
\[
\lambda_2 = \frac{-2 \lambda_2 + N_f y^2}{8 \pi^2} = \frac{3 \lambda_4}{8 \pi^2 (1 + 2 \lambda_2)^2} + \frac{3 \tilde{G} \lambda_2}{\pi (1 + 2 \lambda_2)^2},
\]
\[
\lambda_4 = \frac{-9 \lambda_4^2}{2 \pi^2 (1 + 2 \lambda_2)^3} - \frac{N_f \lambda_4^4}{8 \pi^2}
+ \frac{3 \tilde{G} \lambda_4}{\pi (1 + 2 \lambda_2)^3} + O(\tilde{G}^2),
\]
\[
y' = \frac{y^2 (1 + 2 \lambda_2)}{8 \pi^2 (1 + 2 \lambda_2)^2} + \tilde{G} y \frac{27 + 12 \lambda_2 (1 + 2 \lambda_2)}{16 \pi (1 + 2 \lambda_2)^2}. \tag{6}
\]

In general, the beta functions would depend nonpolynomially on \( \lambda_0 \) and \( \tilde{G} \). In the approximation \( \lambda_0 = 0 \), \( G \) appears only polynomially: the highest power of \( \tilde{G} \) occurs in \( \lambda_4 \) and is 2. In all other terms \( \tilde{G} \) appears at most linearly.

When \( \alpha \neq 0 \) one has to add the following correction terms:

\[
\Delta y = \alpha \tilde{G} y \frac{29 + 180 \lambda_2 (1 + 2 \lambda_2)}{16 \pi (1 + 2 \lambda_2)^2},
\]
\[
\Delta \lambda_2 = 2 \alpha \tilde{G} \lambda_2 \frac{1 + 6 \lambda_2 (1 + 2 \lambda_2)}{(1 + 2 \lambda_2)^2},
\]
\[
\Delta \lambda_4 = 2 \alpha \tilde{G} \lambda_4 \frac{1 + 14 \lambda_2}{\pi (1 + 2 \lambda_2)^3}. \tag{7}
\]

b. Expansion around a VEV. Depending on the sign of \( \lambda_2 \), the potential \( \langle \phi \rangle \) can be used to describe both the symmetric and the broken phase of the theory. In the latter case it may be more convenient to expand \( v \) around the VEV \( \langle \phi \rangle = \sqrt{\kappa} \) \( (\kappa \geq 0) \), such that

\[
v' (\sqrt{\kappa}) = 0. \tag{8}
\]

If we restrict ourselves to fourth order polynomials, \( v \) has the form

\[
v(\hat{\phi}) = \theta_0 + \theta_4 (\hat{\phi}^2 - \kappa^2). \tag{9}
\]

The new couplings, in the broken phase where \( \lambda_2 < 0 \), are related to those in (5) by \( \theta_4 = \lambda_4, \kappa = -\lambda_2/2 \lambda_4, \theta_0 = \lambda_0 - \lambda_2^2/4 \lambda_4 \). The beta functions of these couplings can be derived from these relations and (5). Alternatively, one can obtain the running of \( \kappa \) by deriving (8), which yields

\[
\dot{\kappa} = -2 \sqrt{\kappa} \dot{v}' (\sqrt{\kappa}) / v'' (\sqrt{\kappa}). \tag{10}
\]

For the broken phase, using Eq. (4) and retaining terms up to first order in \( \tilde{G} \), we then obtain

\[
\dot{\theta}_0 = \frac{-4 \theta_0 + 3 + 16 \kappa \theta_4}{32 \pi^2 (1 + 8 \kappa \theta_4)} - \frac{N_f}{8 \pi^2 (1 + \kappa \theta_4^2)} + \frac{3 \dot{G} \theta_0}{\pi}, \tag{11}
\]
\[
\dot{\kappa} = \frac{-2 \kappa + 3}{16 \pi^2 (1 + 8 \kappa \theta_4)^2} - \frac{N_f \dot{y}^4}{16 \pi^2 (1 + \kappa \theta_4^2)^2}, \tag{12}
\]
\[
\dot{\lambda}_4 = \frac{-9 \lambda_4^2}{2 \pi^2 (1 + 8 \kappa \theta_4)^3} - \frac{N_f \lambda_4^4}{8 \pi^2 (1 + \kappa \theta_4^2)^3} + \frac{3 \dot{G} \lambda_4}{\pi (1 + \kappa \theta_4^2)^3} \tag{13}
\]
\[
\dot{y} = \frac{3 \dot{G} y}{16 \pi (1 + \kappa \theta_4^2)^2} \left[ 2 - 16 \kappa \theta_4 (3 + 8 \kappa \theta_4^2) \right] - 3 \kappa y^2 (1 + 8 \kappa \theta_4 (7 + 16 \kappa \theta_4) - \kappa y^4 (1 + 56 \kappa \theta_4^2)]
+ \frac{3 \dot{G} y}{16 \pi (1 + \kappa \theta_4^2)^2} \left[ 9 + 16 \kappa (1 + 4 \kappa \theta_4) \right] - 9 \kappa y^2 (1 + 8 \kappa \theta_4) (9 + 8 \kappa \theta_4^2) + 192 \kappa y^4 \kappa (3 + 16 \kappa \theta_4^2) + 256 \kappa y^6 \theta \kappa (1 + 4 \kappa \theta_4^2). \tag{14}
\]

We do not give here the \( O(\alpha) \) corrections to these formulae. We notice that unlike in the expansion around \( \langle \phi \rangle = 0 \), here \( \theta_0 \) appears only in its own beta function. Up to order \( \tilde{G} \), there is no approximation involved in setting \( \theta_0 = 0 \) in the beta functions of \( \kappa \), \( \theta_4 \) and \( y \), as is natural in an expansion around flat space.

IV. DISCUSSION.

The standard \( \overline{\text{MS}} \) result for the beta function of the Yukawa coupling is \( \dot{y} = \frac{5 y^3}{16 \pi^2} + \ldots \). On the other hand, neglecting \( \tilde{G} \) and \( \lambda_2 \) in (5) or neglecting \( \tilde{G} \) and \( \kappa \) in (11), we remain with \( \dot{y} = \frac{y^3}{8 \pi^2} + \ldots \). The difference is due to the fact that here we neglect the anomalous dimensions of \( \phi \) and \( \psi \). Since their contribution is not very small, our results are not quantitatively accurate, but they should still give a reasonable qualitative picture of the gravitational corrections. We also stress that even though here we analyze a toy model, our result for the leading one loop gravitational correction applies also to realistic theories. In particular when the Yukawa couplings form a matrix \( y_{ij} \), every beta function \( \dot{y}_{ij} \) will receive the same correction \( (27/16 \pi) \tilde{G} y_{ij} \). The inclusion of anomalous dimensions is currently under study. Switching off the gravitational corrections, our results are in agreement with those of [12], when the anomalous dimensions are neglected. Furthermore, the results for \( \lambda_i \) in (5) are also in agreement with those of [11]. We have given in Eqs. (4) and (11) also the beta functions of the vacuum energy \( \lambda_0 \) and \( \theta_0 \). One can see the leading contributions, proportional to \( (3 - 4 N_f) \), the difference between the number of bosonic and fermionic degrees of freedom.

Having used an expansion around flat space, gravity is off shell. This is the cause of the dependence of the results on the gauge parameter \( \alpha \) and \( \beta \), the dependence on which we have computed but not reported here for simplicity. We note that the sign of the leading corrections
does not change as long as $\alpha > 0$; we have also checked that it remains the same at least for $0 \leq \beta \leq 1.8$, which comprises the most popular gauge choices. Furthermore, there are arguments showing that if $\alpha = 0$ would correspond to a nonperturbative fixed point [17]. This suggests that the results obtained for $\alpha = 0$ are probably the most reliable.

The procedure also generically depends on the choice of cutoff scheme, and in particular on the cutoff function $r(y)$. The leading terms in the beta functions of $\lambda_4$ and $y$ turn out to be independent on this choice, but not the gravitational corrections, which are related to a dimensionful coupling. In the results presented above we only used the cutoff $r(y) = (1 - y)\theta(1 - y)$, so the scheme dependence is not manifest, but the numerical coefficients of the gravitational correction would change if we used another cutoff function. We have checked that the leading gravitational correction is proportional to a single integral involving $r(y)$, so that the ratio of the leading correction terms in 10 and 1 is independent of $r$. Furthermore, the sign of the gravitational correction would be the same for any choice of $r(y)$ that satisfies the boundary and monotonicity conditions to be a good cutoff.

The system 16 has a (Gaussian) fixed point when $\lambda_2 = \lambda_4 = y = 0$. Without gravity both $\lambda_4$ and $y$ are marginal, but the gravitational correction makes them irrelevant. In fact the critical exponents are $2 - (3 + 2\alpha)G/\pi$, $-(3 + 2\alpha)\tilde{G}/\pi$ and $-(27 + 29\alpha)\tilde{G}/16\pi$, corresponding to the eigenvectors $\lambda_2 = 3\lambda_4/16\pi^2$, $\lambda_4$ and $y$ respectively. (Note that the gravitational corrections depend on $\alpha$ but are always negative.) This is a remarkable result, because in the standard model these couplings are free parameters, to be determined by experiment, whereas here they are predicted to be zero at high energy. Any value they have at low energy is due to the nonlinearity of the RG flow. This result may change in the presence of other matter fields: it was shown in 11 that minimally coupled matter fields can change the sign of the critical exponent, making $\lambda_4$ relevant. Then its value at low energy would be a free parameter, while at high energy we would have asymptotic freedom.

All this holds both for positive and negative $\lambda_2$. However for negative $\lambda_2$ we may obtain an improved perturbation series 12 expanding both $v$ and $h$ around the VEV. Then, the beta functions are those given in 11.

Most of the comments made above holds also in this case. The main difference lies in the fact that, in the absence of gravitational corrections, the fixed point now has $\theta_4 = y = 0$ and $\kappa = 3/32\pi^2$. Remarkably, the beta function of $\kappa$ does not receive any gravitational correction, as was already noted in 9 for the potential 9 with $\theta_0 = 0$, even taking into account the scalar field anomalous dimension. This is a general property: for any scalar potential $v$, using 10 and 4,

$$\dot{\kappa} = -2\kappa + \frac{\sqrt{\kappa} v''}{16\pi^2 v'' (1 + v'')^2} - \frac{hN\sqrt{\kappa}v'}{2 (1 + h^2)^2 \pi^2 v''} \left|_{\dot{\phi} = \sqrt{\kappa}} \right..$$

We stress again that the beta function of $\kappa$ obtained from the relation $\dot{\kappa} = -\lambda_2(2\lambda_4 + 2\lambda_4^2)/\pi$ together with 9 has a $G$ dependence in it. Also note that the general beta functional of $h$ in 4 can be used to calculate the running of any term of the form $\phi^n\tilde{v}\psi$, in particular of an explicit fermionic mass.

The gravitational corrections are of order $\tilde{G} = k^2/M_{Planck}^2$, and therefore can be treated perturbatively at low energies. They may not be negligible at the GUT scale, though. Beyond the Planck scale the gravitational corrections seem to be large and unbounded. The theory may still be meaningful provided all couplings (in particular $\tilde{G}$) reach a fixed point. It is known that in the Einstein-Hilbert truncation gravity has a nontrivial fixed point, also in the presence of minimally coupled matter fields. Since the Yukawa system has a Gaussian fixed point, one can conclude that the theory of gravity coupled to scalars and fermions also has a fixed point, which we may call a “Gaussian matter” fixed point. However, it is clear that to study the properties of this fixed point, in particular the critical exponents, it is necessary to calculate also the beta function of $\tilde{G}$. There is also the possibility that the matter sector exhibits a nontrivial fixed point 12. Preliminary results indicate that, as long as $G_s \lesssim 1$, this fixed point would also exist in the presence of gravity. We plan to discuss these matters in more detail elsewhere.

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