INTRODUCTION
The following study assesses which of two alternative investments give the investor greater benefits. The alternative investments under consideration are gold and crude oil, and these investments are assessed both separately and together. In addition to these alternative investments, investments in companies included in the WIG20 index are also examined, as the assessment is made from the perspective of a Polish investor. In the study, in addition to the Nobel prize-winning portfolio theory by Markowitz [1952], measures such as rate of return, risk, investment efficiency (measured by the Sharpe ratio [Sharpe 1966]), and maximum cumulative loss were also used [Bacon 2004]. The use of these measures made it possible to assess whether the application of portfolio theory allowed an investor investing on the Warsaw Stock Exchange to achieve an additional benefit. The two main investment objectives considered in the following study were: reducing investment risk and increasing investment efficiency. The purpose of the following study is, therefore, to assess whether an investment portfolio consisting of shares of enterprises included in the WIG20 index as well as gold and/or crude oil positively affects the four measures of the investment portfolio.
results obtained using portfolio theory are compared with a naive strategy to assess whether a portfolio strategy improves the investor’s situation. A naive strategy is understood as a strategy for which the shares of all investments in the portfolio are equal.

According to the basic assumption of portfolio theory, adding another investment to the investment portfolio should lead to a reduction of risk if the rates of return of the considered investments are not marked by a positive correlation coefficient. Investments in gold and crude oil were selected for research in the context of the Polish stock exchange, as these investments are considered to be safe havens. This means that if the stock market falls, the return on these alternative investments stays positive, which allows better parameters of the investment portfolio to be achieved in a difficult period.

**LITERATURE REVIEW**

Adding investment in gold to the investment portfolio is an aspect of the alternative investment market. This topic was studied by Baur and Lucey [2010]. The authors found that gold can be treated as a hedge investment in the capital market. In addition, gold acts as a safe haven in the event of shocks in financial markets. The study analysed stock and bond markets in the USA, the UK, and Germany. The authors emphasized, however, that gold is a safe haven for the stock market, but has never been such for the bond market.

Another study by Baur and McDermott [2010] found that gold acts both as a hedge and a safe haven for major European stock markets and the USA but not for Australia, Canada, Japan, or large emerging markets such as the BRIC countries. The authors based their research on 14 capital markets (13 countries and 1 world index). A period of 30 years from 1979 to 2009 was analysed using the daily rate of return. The authors emphasized that gold has the potential to act as a stabilizing force for the global financial system by reducing losses when most needed.

The study of Anand and Madhogaria [2012] checked whether gold secures the investor’s position on the capital market. The authors analysed the six capital markets of the USA, the Great Britain, Germany, Japan, India, and China. Developed and developing countries were considered separately, and the study was conducted for data from January 2002 to December 2011. The authors found a weak correlation between gold and major capital market indices.

Research devoted only to the American capital market and the role of gold can be found in Hood and Malik [2013]. On the basis of daily data from November 1995 to November 2010, the authors concluded that gold serves as a hedge and a weak safe haven for the US stock market. They also stated that in periods of extremely low or high volatility, gold does not have a negative correlation with the US stock market.

It was also checked whether gold is an investment that protects the investor against changes in exchange rates. A study was conducted by Joy [2011], in which 16 exchange rates against the US dollar were analysed based on weekly data from 10 January 1986 to 29 August 2008. It was found that during the past 23 years, gold has behaved as a hedge against the USD, but gold has been a poor safe haven.

Investment in gold and crude oil was tested in Ciner et al. [2013]. The authors tested the alternative investments in the context of the capital markets of the USA and the Great Britain using data from January 1990 to June 2010. It was found that gold as an investment can be seen as a hedge against sudden movements in capital markets, but this was not the case for crude oil.

Selmi et al. [2018] compared investment in gold and investment in bitcoin to determine which better protects the investor in the context of rapid movements on the crude oil market. The authors stated that both bitcoin and gold serve the roles of a hedge, a safe haven, and a diversifier for crude oil price movements. What is more, they concluded that by controlling for new and relevant US and global uncertainty indicators, both bitcoin and gold, but not crude oil, are assets where investors may park their cash during times of political and economic turmoil. Furthermore, the relationship between bitcoin and crude oil returns was found to be stronger than the one between gold and crude oil.

The role of gold and crude oil in the context of currency markets as well as capital markets was assessed by Śmiech and Papież [2017], using weekly data from 2 January 1995 to 28 December 2015. The authors
concluded that gold can act as a hedge for stocks in normal market conditions in all tested subperiods. Additionally, gold was generally positively correlated with bonds, while crude oil was negatively correlated with them.

As the literature review shows, the role of gold and crude oil has been evaluated in numerous studies. Despite this, no research on the participation of gold or crude oil in an optimal investment portfolio in the context of the Polish capital market has been found. The following study should, therefore, fill this gap.

**DATA AND RESEARCH METHODOLOGY**

The data used for the analysis was obtained from three websites, infostrefa.com – a source of stock prices for Polish companies, lbma.org – a source of gold prices, and opec.org – a source of crude oil prices.

The data were obtained for the period from 30.12.2013 to 31.07.2019. Data for Polish companies were quoted on PLN, and for gold and crude oil in USD. For each day, the logarithmic rates of return were calculated (the research was made only on the basis of percentage changes without taking into consideration exchange rate) so finally, the oldest investment prices for each time series were lost due to the following formula [Jajuga and Jajuga 2006]:

\[
i = \ln P_t - \ln P_{t-1},
\]

where:
- \(i\) – daily logarithmic rate of return,
- \(P_t\) – investment price on day \(t\),
- \(P_{t-1}\) – investment price on day \(t-1\).

The portfolio rate of return, the risk of the portfolio, and the Sharpe ratio were calculated from the following formulas, respectively [Dębski 2007]:

\[
i_p = \sum_{k=1}^{n} w_k i_k
\]

\[
\sigma_p = \sqrt{\sum_{k=1}^{n} w_k^2 \sigma_k^2 + 2 \sum_{k=1}^{n} w_k w_m \sigma_k \sigma_m p_{km}}
\]

\[
s_p = \frac{i_p - i_f}{\sigma_p}
\]

For the calculations, two additional assumptions were made:

\[
\sum_{k=1}^{n} w_k = 1 \quad w_k \geq 0
\]

where:
- \(i_p\) – portfolio rate of return;
- \(i_k\) – rate of return from investment \(k\);
- \(w_k\) – weight of investment \(k\) in the portfolio;
- \(\sigma_p\) – portfolio risk (standard deviation of the rate of return);
- \(\sigma_k\) – risk of investment \(k\) (standard deviation of the rate of return);
- \(p_{km}\) – coefficient of correlation of return rates on investment \(k\) and investment \(m\);
- \(S_p\) – effectiveness measure (Sharpe ratio);
- \(i_f\) – risk-free rate of return;
- \(n\) – number of investments in portfolio.

In turn, the maximum cumulative loss (MDD) was defined as the maximum loss an investor can suffer in the investment by buying at the highest point and selling at the lowest point [Bacon 2004].

In this paper, portfolios with three (without any alternative investments), four (with gold or with crude oil), and five (with gold and crude oil) elements were examined. Subscript “mr” was added to all portfolios with minimum risk, “me” was added to portfolios with maximum efficiency, and “n” was added to naive diversification.

The study was conducted by dividing the study period into two windows. The first is the estimation window, on the basis of which the portfolio weights were determined according to portfolio theory. The second window is the evaluation window, for which the portfolio weights were tested and for which the final results of all measures tested were obtained. The evaluation window was unchanged and always covered the period from 2 January 2018 to 31 July 2019. In this period, 394 daily rates of return were recorded. In order to obtain comprehensive conclusions, five cases were considered for the estimation window.

The first estimation window covered all data from 2 January 2014 to 29 December 2017, which constituted 1,001 observations, the second window was the time interval from 2 January 2015 to 29 December 2017.
2017 (752 observations), the third was from 4 January 2016 to 29 December 2017 (501 observations), and the last estimation window was data from the period 3 July 2017 to 29 December 2017 (126 observations). In order to better present the results for each estimation window length, the estimation windows have been respectively given the following abbreviations: ALL; 151617; 1617; 17; 17half.

This distinction also made it possible to check whether the length of the estimation window has an impact on the diversity of the results obtained. A total of 22,800 investment portfolios with minimal risk and the same number of portfolios with maximum efficiency were constructed. This number was chosen due to the fact that there are 1,140 possible three-element combinations for companies included in the WIG20 index. In addition to the three-element portfolios for the analysed companies, for each estimation window, 1,140 portfolios consisting of three capital investments and investments in gold, 1,140 portfolios consisting of three capital investments and investments in crude oil, and 1,140 portfolios of three capital investments and investments in gold and crude oil at the same time were also created. Hence, for each of the five estimation windows, 4,560 portfolios with minimal risk and 4,560 portfolios with maximum efficiency were created. In addition, 4,560 portfolios with naive diversification were created to compare the results. These were the same as above: 1,140 portfolios for three-element portfolios consisting of investments in company shares, 1,140 portfolios consisting of capital investments and gold, 1,140 portfolios consisting of capital investments and crude oil, and 1,140 portfolios of capital investments and gold and crude oil at the same time. Altogether, nearly 55,000 investment portfolios were constructed. All calculations were carried out in the R program [Kopczewska et al. 2009].

RESEARCH RESULTS AND DISCUSSION

As stated at the beginning of this study, whether the addition of a given investment will positively affect the characteristics of the investment portfolio largely depends on the value of the linear correlation coefficient. Therefore, Table 1 presents the values of these coefficients for all the studied investments, as before, broken down into the longest estimation window and the evaluation window.

Table 1. Value of selected correlation coefficients

| Investment | Longest estimation window | Evaluation window |
|------------|--------------------------|-------------------|
|            | gold  | crude oil | gold  | crude oil |
| ALIOR      | –0.05 | 0.12      | 0.03  | 0.11      |
| CCC        | –0.05 | 0.13      | 0.06  | 0.08      |
| CDPROJEKT  | –0.07 | 0.10      | –0.01 | 0.09      |
| CYFRPLSAT  | –0.07 | 0.12      | –0.01 | 0.01      |
| ENERGA     | –0.02 | 0.08      | 0.06  | 0.09      |
| EUROCASH   | –0.01 | 0.06      | 0.07  | 0.05      |
| JSW        | 0.02  | 0.14      | 0.07  | 0.11      |
| KGHM       | 0.04  | 0.26      | 0.14  | 0.13      |
| LOTOS      | –0.03 | 0.10      | 0.04  | 0.13      |
| LPP        | –0.04 | 0.16      | 0.11  | 0.09      |
| MBANK      | –0.04 | 0.15      | 0.07  | 0.04      |
| ORANGEPL   | –0.01 | 0.13      | 0.13  | 0.03      |
| PEKAO      | –0.07 | 0.14      | 0.06  | 0.04      |
| PGE        | –0.04 | 0.11      | 0.05  | 0.12      |
| PGNIG      | –0.05 | 0.21      | 0.06  | 0.20      |
| PKNORLEN   | –0.05 | 0.02      | 0.03  | 0.04      |
| PKOBP      | –0.08 | 0.12      | –0.02 | 0.08      |
| PZU        | –0.03 | 0.03      | 0.06  | –0.04     |
| SANPL      | –0.02 | 0.13      | 0.07  | 0.09      |
| TAURONPE   | 0.01  | 0.10      | 0.07  | 0.13      |
| Gold       | 1.00  | 0.01      | 1.00  | 0.05      |
| Crude oil  | 0.01  | 1.00      | 0.05  | 1.00      |

Source: Authors’ own calculations.

Based on the data in Table 1, it can be concluded that there is no correlation between rates of return on investment in gold and other investments, as evidenced by the low and often negative values of the calculated coefficients. The same is true for crude oil investments. At the same time, it seems that gold is a better choice for diversifying the investment portfolio, as for most of the researched investments it had a lower correlation coefficient than crude oil. For gold, the average value of correlation coefficients in relation to other analysed investments was –0.03 in the longest estimation window and 0.06 in the evaluation window.
window. For crude oil, these measures were 0.12 and 0.08, respectively. This means that both investments can be considered as diversification investments [Potrykus 2015].

Table 2 presents the average results for portfolios created using a naive strategy. These results represent the rate of return, standard deviation, the Sharpe ratio, and maximum cumulative loss for the assessment window, assuming that the share of all investments in the portfolio was identical.

As shown in Table 2, if the investor decided to create a portfolio of only three investments in WIG20 index shares, then a naive strategy in the evaluation window would lead to a loss equal to 0.046% per day at a risk of 1.5%, efficiency at the level of –0.034, and a cumulative maximum loss of 32%. Expanding the portfolio with an investment in crude oil had a positive impact on all the measures analysed. The rate of return increased, risk decreased, efficiency increased, and the maximum cumulative loss decreased. Even more favourable results were obtained if investment in crude oil was replaced with investment in gold. For four-element portfolios, investment in gold was a better source of diversification than investment in crude oil assuming a naive strategy. However, even better results were obtained by five-element portfolios, for which the risk decreased below 1% and the cumulative loss was on average 21.3%. Despite achieving the best values for these portfolios, the rate of return and efficiency remained on average negative, which is due to the fact that most of the investments in the evaluation window had an average negative rate of return. The majority of the 20 companies – 16 – reported mean average negative returns in the evaluation window. Positive mean rates of return were recorded for gold and crude oil as well as for companies such as ORANGEPL, CYFRPLSAT, LOTOS, and CDPROJEKT.

Table 3 presents the average results of the examined portfolio characteristics for each estimation window, broken down into the strategies analysed. In virtually all cases, the application of portfolio theory contributed to obtaining better characteristics of the investment portfolio in the evaluation window than for a naive strategy. The exceptions here are portfolios created only from WIG20 shares, which on average achieved a higher risk than a naive strategy for all the estimated window lengths. Rate of return and efficiency for portfolios created only from WIG20 shares were also higher only when the estimation window was the shortest. In addition, even better results for investment risk were found for the naive strategy than for crude oil portfolios when the estimation window length was 151,617. In other cases, the use of portfolio theory achieved better results than the naive strategy. It can also be seen that the best characteristics were in the portfolios that contained gold and crude oil in addition to the shares of the analysed companies. On average, these portfolios had the highest rates of return, the lowest risk, the highest efficiency, and the lowest maximum cumulative loss.

Importantly, the inclusion of gold and crude oil in the portfolios allowed for a positive return, despite the fact that most companies had average negative returns in the evaluation window. Results were similar for the strategy that included only gold in the portfolio. It can also be seen that the shorter the estimation window, the lower the risk and maximum cumulative loss of designated portfolios. The combination of shares in companies on the WIG20 index and investment in crude oil has better portfolio characteristics than portfolios constructed only on the basis of shares or naive portfolios; however, this improvement is not as significant as in the case of investments in gold. Table 4 presents

| Strategy                              | ip_n (%) | σp_n (%) | Sp_n | DDp_n (%) |
|---------------------------------------|----------|----------|------|-----------|
| Naive without gold and crude oil      | -0.046   | 1.50     | -0.034 | -32.0     |
| Naive only with crude oil             | -0.033   | 1.22     | -0.031 | -25.9     |
| Naive only with gold                  | -0.028   | 1.15     | -0.029 | -25.4     |
| Naive with gold and crude oil         | -0.022   | 0.99     | -0.027 | -21.3     |

Source: Author’s own calculations.
Table 3. Values of tested measures for portfolios with minimal risk

| Measure | Strategy | Estimation window |
|---------|----------|-------------------|
|         |          | ALL  | 151 167 | 1 617 | 17  | 17half |
| ip_mr (%) | without gold and crude oil | -0.040 | -0.038 | -0.035 | -0.045 | -0.047 |
|          | only with crude oil | -0.025 | -0.026 | -0.023 | -0.020 | -0.021 |
|          | only with gold | 0.006 | 0.007 | 0.007 | 0.006 | 0.008 |
|          | with gold and crude oil | 0.007 | 0.008 | 0.008 | 0.006 | 0.008 |
| σp_mr (%) | without gold and crude oil | 1.54 | 1.55 | 1.51 | 1.50 | 1.50 |
|          | only with crude oil | 1.20 | 1.23 | 1.21 | 1.08 | 1.09 |
|          | only with gold | 0.66 | 0.66 | 0.66 | 0.63 | 0.61 |
|          | with gold and crude oil | 0.61 | 0.61 | 0.61 | 0.59 | 0.58 |
| Sp_mr | without gold and crude oil | -0.029 | -0.028 | -0.026 | -0.033 | -0.034 |
|          | only with crude oil | -0.026 | -0.025 | -0.023 | -0.023 | -0.024 |
|          | only with gold | 0.002 | 0.003 | 0.002 | 0.001 | 0.005 |
|          | with gold and crude oil | 0.003 | 0.004 | 0.004 | 0.001 | 0.005 |
| DDp_mr (%) | without gold and crude oil | -31.75 | -31.65 | -30.39 | -31.47 | -31.33 |
|          | only with crude oil | -24.71 | -25.14 | -24.27 | -24.65 | -24.83 |
|          | only with gold | -15.58 | -15.46 | -15.49 | -15.54 | -15.06 |
|          | with gold and crude oil | -13.71 | -13.80 | -13.63 | -13.24 | -12.92 |

Source: Author’s own calculations.

Table 4. Values of tested measures for maximum efficiency portfolios

| Measure | Strategy | Estimation window |
|---------|----------|-------------------|
|         |          | ALL  | 151 167 | 1 617 | 17  | 17half |
| ip_me (%) | without gold and crude oil | -0.016 | -0.018 | -0.022 | -0.039 | -0.053 |
|          | only with crude oil | -0.016 | -0.006 | -0.003 | -0.017 | -0.010 |
|          | only with gold | 0.010 | 0.002 | 0.009 | 0.001 | -0.011 |
|          | with gold and crude oil | 0.010 | 0.002 | 0.011 | 0.002 | -0.002 |
| σp_me (%) | without gold and crude oil | 2.12 | 2.10 | 1.95 | 1.76 | 1.92 |
|          | only with crude oil | 2.12 | 1.84 | 1.38 | 1.22 | 1.24 |
|          | only with gold | 1.44 | 1.43 | 0.95 | 0.80 | 0.86 |
|          | with gold and crude oil | 1.44 | 1.39 | 0.82 | 0.74 | 0.79 |
| Sp_me | without gold and crude oil | -0.010 | -0.012 | -0.016 | -0.026 | -0.031 |
|          | only with crude oil | -0.010 | -0.007 | -0.009 | -0.019 | -0.013 |
|          | only with gold | 0.006 | 0.002 | 0.001 | -0.007 | -0.014 |
|          | with gold and crude oil | 0.006 | -0.002 | 0.004 | -0.006 | -0.009 |
| DDp_me (%) | without gold and crude oil | -38.91 | -38.77 | -37.16 | -35.73 | -38.90 |
|          | only with crude oil | -38.91 | -37.45 | -30.46 | -26.41 | -33.90 |
|          | only with gold | -27.64 | -28.37 | -20.28 | -17.78 | -19.79 |
|          | with gold and crude oil | -27.64 | -28.36 | -17.19 | -15.58 | -26.16 |

Source: Author’s own calculations.
the values of the studied characteristics for portfolios with maximum efficiency.

Higher performance variation was observed for portfolios with maximum efficiency than for portfolios with minimal risk. The use of portfolio theory obtained higher efficiency than a naive strategy regardless of the length of the estimation window for all the portfolios studied. This confirms the legitimacy of using portfolio theory when maximizing portfolio efficiency. A similar situation was visible for the rate of return, with one exception: for the estimation window 17, the naive strategy turned out to be more profitable than portfolios consisting only of WIG20 index shares. Other values were observed for risk measures in maximum efficiency portfolios. Here, generally, the shorter the estimation period, the lower the risk. This rule does not apply to the estimation window 17 because for the shortest estimation window, an increase in risk measure was again observed. Once more, the best results were obtained for portfolios that contained gold or gold and crude oil at the same time. This is a similar conclusion to the situation for portfolios with minimal risk. Table 5 presents the mean values of the alternative investments in designated investment portfolios, broken down into portfolios with minimal risk and maximum efficiency.

What is surprising is the fact that alternative investments constitute a significant part of optimal investment portfolios. The share of gold in portfolios with minimal risk is over 65% in portfolios containing both gold and crude oil, and over 72% in portfolios containing gold but not crude oil. On the other hand, the average share of crude oil amounted to nearly 38% in portfolios containing crude oil and less than 12% in portfolios containing both crude oil and gold. For portfolios with maximum efficiency, there was an average significant decrease in the share of investments in gold by over 20 p.p. and an increase in the share of investments in crude oil by an average of over 8 p.p. It can, therefore, be concluded that investment in gold is a source of investment risk reduction for Poland, while investment in crude oil in addition to gold is an additional value for investors seeking to maximize efficiency.

**SUMMARY**

Based on the research, it can be concluded that gold is a better source of investment portfolio diversification than crude oil. However, both investments are a desirable component of the investment portfolio, especially for investors seeking to maximize the effectiveness of their investment. On the other hand, gold also has assets that, in combination with the Polish market of large companies, offer benefits in reducing investment risk.

In addition, it has been demonstrated that the use of portfolio theory contributes to better results than a naive strategy. Such conclusions have been obtained on the basis of research on the rate of return, risk measured by the standard deviation of the rate of return, portfolio efficiency, and maximum cumulative loss. In portfolios with minimal risk that included gold and crude oil, compared to the naive strategy, the average portfolio interest rate increased by 0.029% per day, risk measured by standard deviation decreased by 0.39 p.p. on average, increased in efficiency by 0.034% and decreased in maximum cumulative loss by 7.85 p.p. on average. In portfolios with maximum efficiency, the use of portfolio theory compared to the naive strategy on average

| Strategy                  | Mean weight of investment | Portfolio with (%) |
|---------------------------|---------------------------|--------------------|
|                           |                           | (mr)               |
| Only with crude oil       | crude oil                 | 37.57              |
| Only with gold            | gold                      | 72.51              |
| With gold and crude oil   | gold                      | 65.36              |
|                           | crude oil                 | 11.73              |

Source: Author’s own calculations.
increased the return on investment by 0.024% and increased efficiency by 0.0256 on average.

Research shows that an investor using portfolio theory almost always gets better portfolio characteristics regardless of the length of the estimation window. However, in this study, the best results in terms of risk were obtained when the length of the estimation window did not exceed one year, while efficiency was usually higher for estimation windows longer than one year.

An important element is also the fact that the examined alternative investments constitute a significant share in designated investment portfolios. For gold, this was 56% on average in all designated portfolios, and for crude oil this was close to 26%. The share of gold increased in portfolios with minimal risk and the share of crude oil increased while maximizing efficiency.

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