Instantaneous Interaction between Charged Particles

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Abstract  The interaction between charged particles through quasi-static fields must occur instantaneously; otherwise a violation of the energy principle would occur. As a consequence, the instantaneous transmission of both energy and information over macroscopic distances is feasible by using the quasi-static fields which are predicted by Maxwell’s equations.

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• Transmission of information

I Introduction
Since the Special Theory of Relativity has become a constitutive part of physics, it is commonly believed that the transmission of energy or information between distant locations can only occur with the maximal speed of light. Maxwell’s equations are usually formulated as wave equations for the scalar and vector potentials suggesting (in Lorenz gauge) that the influence of a source propagates with finite speed to a field point where an observer may be placed. This conception is undoubtedly true as far as electromagnetic waves are concerned. There exists, however, a further method of energy transmission effected by quasi-static fields which are also predicted by Maxwell’s theory. In contrast to the radiation fields, they decay in the far field zone inversely proportional to the third power of the distance from the sources so that the range of transmission is much shorter than in case of travelling waves. As far as the magnetic quasi-static fields are concerned, they have a most important application for the transmission of energy in transformers.

Due to the linearity of Maxwell’s equations the fields may be split up into quasi-static, instantaneous contributions, and into wave parts: \( \vec{B} = \vec{B}_i + \vec{B}_w \), \( \vec{E} = \vec{E}_C + \)
\[ \vec{E}_i + \vec{E}_w. \] In case of the electric field the instantaneous part consists of the irrotational Coulomb field \( \vec{E}_C \) and an induced rotational field \( \vec{E}_i \). The wave fields obey inhomogeneous hyperbolic equations:

\[
\Delta \vec{E}_w - \frac{1}{c^2} \frac{\partial^2 \vec{E}_w}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \vec{E}_i}{\partial t^2}
\]

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\]

(1)

where we have used, e.g.: \( \text{rot rot } \vec{E}_w = \nabla (\text{div } \vec{E}_w) - \Delta \vec{E}_w = -\Delta \vec{E}_w \). The instantaneous fields appear as sources in (1) and can be calculated from a given charge and current distribution:

\[
\vec{E}_C (\vec{x}, t) = \iiint \rho (\vec{x}', t) \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} d^3x'
\]

\[
\vec{E}_i (\vec{x}, t) = -\frac{1}{4\pi c} \iiint \frac{\partial \vec{B}_i (\vec{x}', t)}{\partial t} \times \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} d^3x'
\]

\[
\vec{B}_i (\vec{x}, t) = \frac{1}{c} \iiint \left( \vec{j} (\vec{x}', t) + \frac{1}{4\pi} \frac{\partial \vec{E}_C (\vec{x}', t)}{\partial t} \right) \times \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} d^3x'
\]

(2)

The set of equations (1, 2) is entirely equivalent to Maxwell’s equations which may be checked by insertion into the first order system [1]. In the present formulation it becomes obvious that Maxwell’s theory predicts not only wave propagation at the velocity of light (1), but also instantaneous fields as described by the quasi-static integrals (2). A complete cancellation of the two kinds of fields is not possible, as was pointed out in [2] and is obvious from (1): If the sum \( \vec{E}_w + \vec{E}_i \), e.g., would vanish for a certain interval of time, one would have \( \Delta \vec{E}_w = 0 \) so that the wave field \( \vec{E}_w \) would vanish everywhere when the usual boundary condition \( \vec{E}_w (\infty) = 0 \) is imposed.

In this note we show that the transmission of energy by quasi-static fields must occur instantaneously as suggested by (2); otherwise the energy principle would be violated. In Sect. II we discuss the interaction of two charges, and in Sect. III the interaction of two current loops. In both cases we come to the conclusion that energy can be transmitted instantaneously.

II Interaction of two charges coupled by the Coulomb field

Let us consider two positive charges which are placed at a distance \( R \). In Fig. 1 charge B is rigidly attached to a heavy wall, whereas charge A can be moved by a mechanical force along a distance \( r \). When charge A is pushed against B, a force must be applied against the repulsive electric force, and a certain amount of potential energy is invested. Since charge A moves in a conservative potential which does not vary in time, the invested work is recovered when the charge returns to its initial position.

Next we consider Fig. 2 where charge B is attached to a flexible spring. Initially the spring is somewhat compressed due to the repulsive force between the charges. When charge A is pushed against B, the spring is compressed even more, and, because of its inertia, charge B will oscillate after charge A has returned to its initial position.
Obviously, energy has been transmitted from A to B through the quasi-static electric field. The oscillation energy must have been supplied by the mechanical force acting on charge A. This is easily explained: Charge A was moving in a conservative potential which was, however, not constant in time, since charge B was yielding to the increased force due to its elastic fixture. If one carries out the exact calculation assuming that the charges were coupled by the Coulomb force:

\[ F(t) = \frac{q_A q_B}{4\pi \varepsilon_0 R^2(t)} \]  

one finds, of course, that the energy appearing in the oscillation of charge B was exactly supplied by the mechanical force which moved charge A.

In this seemingly trivial example we have assumed in (3) that both charges are coupled instantaneously. Let us now assume that a time \( R/c \) elapses before charge B can “realize” that charge A is moving. If the cyclic motion of charge A is completed within a short time \( \tau < R/c \), charge B cannot react during the cycle so that charge A still moves in a constant conservative potential like in Fig. 1. The mechanical force does not produce any total work during the cycle, but after a delay time \( R/c \) charge B will start to oscillate. Its energy comes out of nothing as a consequence of our assumption that the action of charge A on B is delayed. As long as we believe in the conservation of energy we must conclude that the coupling is determined by (3), and the energy transfer occurred instantaneously.

One might argue that the acceleration of charge A produced a wave containing energy which was transported to B at finite velocity. It is, of course, true that the
acceleration of charges produces waves according to Maxwell’s theory as described by (1). This holds, however, in both cases of Fig. 1 and Fig. 2. It is entirely independent of charge B being rigidly or elastically attached to the wall. In both cases the mechanical force must supply a small amount of energy which is carried away by the wave and must be accounted for in the energy balance. The salient point is, however, that the energy balance completed at charge A during a cycle cannot depend on charge B oscillating in the future or not. There must be an instantaneous feedback from charge B to charge A in order to balance correctly the work at charge A with any oscillation energy produced at charge B. This necessary feedback is provided by the Coulomb force (3), or equation (2), in general.

Furthermore, it should be noted that the wave travels perpendicular to the acceleration of charge A and does not reach charge B at all, as it is placed in line with the acceleration vector. Thus, the production of waves cannot explain the missing energy source in case of Fig. 2, when delayed action is postulated.

III Interaction of two current loops
All the power produced by the electric companies and transmitted to the consumers passes several times through transformers. In this Section we show that the transmission from the primary to the secondary circuits must occur instantaneously as described by Maxwell’s equations.

In principle, a transformer consists of two current loops as sketched in Fig. 3. Applying Faraday’s law of induction and the laws of Ampère and Ohm one has the transformer equations [3]:

\[ U = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \]  \hspace{1cm} (4)
\[ 0 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} + R I_2 \]  \hspace{1cm} (5)

where \( L_{1,2} \) are the self-inductances of the loops and \( M \) is the coupling inductance. When there is no resistive load in the secondary circuit \((R = 0)\), it follows from (5): \( L_2 I_2 + M I_1 = 0 \) for alternating currents. \( I_1 \) and \( I_2 \) are out of phase by 180 degrees. From (4) follows then that there is a phase difference of 90 degrees between the applied voltage \( U \) and the current \( I_1 \). The power \( U I_1 \) injected into the primary circuit oscillates forth and back so that no energy is deposited into the (ideal) transformer on average.
If the secondary resistance is finite, a phase shift occurs which may be calculated by solving the differential equations (4) and (5). As a result the time integral \( \int U I_1 \, dt \) does not vanish anymore, but supplies the energy \( \int R I_2^2 \, dt \) which is dissipated in the secondary circuit. Energy is obviously transmitted over the distance between the loops. Since in (4, 5) Maxwell’s displacement current was neglected, the coupling of the loops was assumed to be instantaneous which holds then also for the energy transfer.

If we would assume that it takes some time for the magnetic field produced in loop 1 to travel to loop 2, induce a current there which in turn produces a magnetic field travelling back to loop 1, one would have a phase shift of more than 90 degrees between voltage and current in the primary circuit, even in the case of vanishing resistance \((R = 0)\). The integral \( \int U I_1 \, dt \) would not be zero on average, and energy would be lost in the ideal transformer which is supposed to contain only superconducting loops. As in the previous Section, we must conclude that the coupling in a transformer cannot be effected by an electromagnetic wave, but must be caused by quasi-static instantaneous fields, in order to avoid a clash with the energy principle. The quasi-static magnetic field can apparently be used like the quasi-static electric field to transmit information faster than light.

In industrial transformers the distance between the primary and the secondary circuit is chosen to be very small, but one can arrange the two loops of Fig. 3 at an appreciable distance in order to measure the time of transmission. This was in fact done by Kholmetskii and coworkers [4]. They found experimentally that a “bound” magnetic field as described by (2) is spreading at a velocity “highly exceeding the velocity of light”.

IV Conclusion
In two simple examples it was demonstrated that the coupling of electric charges and currents through quasi-static fields must occur instantaneously, in order to maintain the conservation of energy. In technical applications this kind of coupling is assumed anyway, but it is frequently thought that engineers just use a practical approximation, whereas the ‘correct’ interaction requires a description in terms of travelling wave fields. Our analysis shows that this is not the case. It proves that instantaneous transmission of information over macroscopic distances is possible in agreement with Maxwell’s theory and with very recent experiments. The argument brought forward in Sect. II may also be applied to the mechanical force acting between the current loops of Fig. 3. It could be extended to the gravitational force as well.
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