A number of spectacular experimental anomalies\(^{1,2}\) have recently been discovered in certain cuprates, notably La\(_{2−x}\)Ba\(_x\)CuO\(_4\) and La\(_{1.6−x}\)Nd\(_{0.4}\)Sr\(_x\)CuO\(_4\), which exhibit unidirectional spin and charge order (known as “stripe order”). We have recently proposed to interpret these observations as evidence for a novel “striped superconducting” state, in which the superconducting order parameter is modulated in space, such that its average is precisely zero. Here, we show that thermal melting of the striped superconducting state can lead to a number of unusual phases, of which the most novel is a charge 4e superconducting state, with a corresponding fractional flux quantum \(\hbar c/4e\). These are never-before observed states of matter, and ones, moreover, that cannot arise from the conventional Bardeen-Cooper-Schrieffer (BCS) mechanism. Thus, direct confirmation of their existence, even in a small subset of the cuprates, could have much broader implications for our understanding of high temperature superconductivity. We propose experiments to observe fractional flux quantization, which thereby could confirm the existence of these states.

It is widely accepted that the mechanism of superconductivity in the cuprate high temperature superconductors is different than in conventional superconductors, and that the “normal” state above \(T_c\) is anything but normal. In contrast, the low energy low temperature properties of the superconducting state appears to be largely accounted for by the conventional BCS theory, albeit with a d-wave order parameter and a much suppressed superfluid density. In short, the perception is that the superconducting (SC) phase is simple while the “normal” phase is not. However, recent experiments\(^{3,4}\) in the “stripe ordered” materials, in which a novel form of highly two-dimensional partial SC order was found, suggest that the SC phases may have unconventional aspects. We have interpreted these anomalies as evidence of a new type of order, “striped superconductivity” or unidirectional pair density wave (PDW) order. Subsequent experiments\(^{5,6}\) have provided additional indirect evidence in favor of this interpretation.

In the present paper, we study the thermal melting of a striped superconducting groundstate by the proliferation of topological defects leads to a complex phase diagram, shown in Fig. 1 with several interesting phases, of which the most novel is a charge 4e superconducting state. The existence of such a state can be directly established experimentally by observing magnetic flux quantization with period \(\hbar c/4e\), half that of the usual superconducting flux quantum, in a variety of experimental geometries, as shown in Fig. 2. Moreover, it is likely that the charge 4e SC, resulting from partial melting the striped SC, has a finite density of states for gapless quasiparticles, reflecting the fact that it does not arise from the Bose condensation of literal four electron bound states.

In related developments, Agterberg and Tsunetsugu\(^{7,8}\) and Radzihovsky and Vishwanath\(^{9,10}\) have discussed related phenomena in connection with Larkin-Ovchinnikov (LO) phases in, respectively, the heavy fermion material CeCoIn\(_5\) at high fields and in partially spin-polarized ultra-cold atomic gases. The states we discuss here differ from the LO states in that there is no explicit time-reversal symmetry breaking. Also the structure of the striped SC reflects the discrete point group symmetry of the underlying (cuprate) lattice.

A striped superconductor is a unidirectional PDW state in which a spin singlet superconducting order parameter, \(\Delta(r)\) oscillates in space with a wave vector \(Q\)(\(^{11}\))(shown schematically in Fig. 1): \[
\Delta(r) = \Delta_q(r) e^{iQ \cdot r} + \Delta_{-q}(r) e^{-iQ \cdot r}. \tag{1}
\]

Of course, there will always be higher harmonics of the order parameter at wave vectors \(nQ\), but in the ordered state, these are slaved to the fundamentals and hence are not independent dynamical degrees of freedom. Generally, therefore, we will not treat them explicitly. However, two subsidiary order parameters play a special role in the thermal melting: Charge density wave (CDW) order, \(\rho(r) = \rho_0 + \rho_K(r) e^{iK \cdot r} + \rho_{\parallel K}(r) e^{-iK \cdot r}\), appears as a second harmonic of the fundamental ordering, with wave vector \(K = 2Q\), where \(\rho_K(r) \propto \Delta^* Q(r) \Delta_Q(r)\). Similarly, charge 4e SC order, represented by the complex scalar field \(\Delta_{4e}(r) \propto \Delta_{-Q}(r) \Delta_Q(r)\), occurs parasitically along with the fundamental striped SC order.

Although the stripe ordered superconductor need not be associated with a non-zero magnetization, it is in many ways similar to a LO state\(^{11}\). As such it has a \(U(1) \times U(1)\) symmetry corresponding to the independent uniform shifts of the phases of the complex components of the order parameter:

\[
\Delta_{\pm q}(r) \rightarrow e^{i\phi_{\pm q}} \Delta_{\pm q}(r) = e^{i(\theta_{\pm q} + \phi)} \Delta_{\pm q}(r), \tag{2}
\]

\[
\rho_K(r) \rightarrow e^{i2\phi} \rho_K(r) \quad \text{with} \quad \phi = (\theta_Q - \theta_{-Q})/2,
\]
\[
\Delta_{4e}(r) \rightarrow e^{i2\theta} \Delta_{4e}(r) \quad \text{with} \quad \theta = (\theta_Q + \theta_{-Q})/2.
\]
Much as other stripe electronic liquid crystal states, the PDW also typically breaks the point group symmetry; to be concrete, we will consider the case of a tetragonal crystal, in which the choice of direction for the unidirectional order breaks the $C_{4v}$ point group symmetry down to $C_2$. In analogy with classical liquid crystals, we refer to this as nematic ordering, although since the symmetry breaking is in fact, discrete, it is an “Ising nematic.”

This is the minimal set of broken symmetries associated with the striped superconductor. For simplicity, we will consider only the interplay of charge stripe and SC stripe orders, and ignore the striped spin order, even though all three are intertwined where striped superconductivity is conjectured to occur in the cuprates.

We consider a system which has a striped superconducting groundstate, and consider the manner in which thermal fluctuations gradually restore all the broken symmetries as the temperature, $T$, increases. Deep in the PDW phase, other than in a vortex core, we can ignore fluctuations in the magnitude of the order parameter, and write $\Delta_\pm Q(r) = \Delta_{SC} \exp\{i(\theta(r) \pm \phi(r))\}$, $\rho_K(r) = \rho_K \exp\{i2\theta(r)\}$, and $\Delta_{\mu}(r) = \Delta_{\mu} \exp\{i2\theta(r)\}$. Because the phase of the CDW order is subject to pinning by quenched disorder, the PDW is considerably more fragile than a uniform SC state, and quenched randomness easily leads to an XY superconducting glass phase. (Unless stated otherwise, below we ignore the effects of disorder.) Moreover, the striped superconductor state admits novel topological excitations, which play a central role in the thermal melting of the groundstate.

Deep in the striped SC phase, treating the system as 2D, and taking $\tilde{Q}$ in the $x$ direction, the effective Hamiltonian for the low energy thermal fluctuations is

$$\mathcal{H}[\theta, \phi] = \frac{\rho_s}{2} |D_s\theta|^2 + \frac{\kappa}{2} (D_c \phi)^2,$$

(3)

$D_s = \alpha_s[-i\hbar \partial_x - (2e/c)A_s x + \alpha_s^{-1}-i\hbar \partial_y - (2e/c)A_y y]$, $D_c = \alpha_c[-i\hbar \partial_x - \alpha_c^{-1}-i\hbar \partial_y]$, $\rho_s$ and $\kappa$ are, respectively, the superfluid stiffness and the CDW elastic constant, and $\alpha_s$ and $\alpha_c$ are the corresponding (finite) anistropies.\cite{12} We have shown, explicitly, the coupling to an external gauge field, but henceforth, unless otherwise specified, we will take $A = 0$.\cite{13}

The correlation functions of the PDW, CDW and charge 4e SC order parameters in this phase are

$$\langle \Delta_\pm Q(r) \Delta_\pm Q(r') \rangle \propto K_s(r - r') K_c(r - r')$$

$$\langle \rho Q(r) \rho Q(r') \rangle \propto |K_s(r - r')|^4$$

$$\langle 4e_c(r) 4e_c(r') \rangle \propto |K_s(r - r')|^4$$

(4)

where

$$K_s(r) \sim \left[(\alpha_s \epsilon)^2 + (\alpha_s^{-1} \epsilon)^2\right]^{-\frac{1}{2}}$$

$$K_c(r) \sim \left[(\alpha_c \epsilon)^2 + (\alpha_c^{-1} \epsilon)^2\right]^{-\frac{1}{2}}$$

$$\eta_s = 2\pi \rho_s T / \kappa$$

$$\eta_c = 2\pi T / \kappa.$$

(5)

Since the order parameters $\Delta_\pm Q(r)$ must be separately single valued, their phase fields must be invariant under the transformations $\theta_\pm Q(r) \rightarrow \theta_\pm Q(r) + 2\pi m_\pm Q$, where $m_\pm Q$ are integers. Correspondingly, the fields $\theta(r)$ and $\phi(r)$ must obey the conditions $\theta(r) \rightarrow \theta(r) + \pi(m Q + m_- Q)$, $\phi(r) \rightarrow \phi(r) + \pi(m Q - m_- Q)$. The integers $m_\pm Q$ then classify the topological excitations supported by the PDW state: vortices with topological charge $q_v = (m Q + m_- Q)/2$ and dislocations with topological charge $q_d = (m Q - m_- Q)/2$. We have three types of topological excitations ($q_v, q_d$):

1. Full vortices with $q_v = \pm 1$ and $q_d = 0$.
2. Double dislocations, with $q_v = \pm 1$ and $q_d = 0$.
3. Half-vortices, $q_v = \pm 1/2$, bound to single dislocations, $q_d = \pm 1/2$.

Much as in the case of the well understood theory of the Kosterlitz-Thouless (KT) phase transition\cite{15,16,17} the thermodynamic behavior of this system is also represented by a generalized (neutral) Coulomb gas of vortices $\{q_v\}$ and dislocations $\{q_d\}$, both with logarithmic interactions, where the strength of the interactions between vortices (and anti-vortices), both fractional and integral, is controlled by the superfluid density $\rho_s$ and the interaction between dislocations (and anti-dislocations) is controlled by the CDW stiffness $\kappa$. Let us denote by $g(q_v, q_d)$ the fugacity of a topological excitation with topological charges $(q_v, q_d)$. In the dilute limit, in which the fugacities are small, the partition function of the generalized vector Coulomb gas can be represented by a sine-Gordon type effective field theory (for a review see Refs.\cite{18}), that in this case requires two fields $\theta$ and $\varphi$, the dual of the superconducting phase $\theta$ and the CDW phase $\varphi$. The effective Hamiltonian density of the 2D field theory, dual to the degrees of freedom of Eq.(3), is

$$\mathcal{H}_{\text{dual}}[\hat{\theta}, \hat{\varphi}] = \frac{T}{2\rho_s} (D_s \hat{\theta})^2 + \frac{T}{2\kappa} (D_c \hat{\varphi})^2$$

$$-g(1,0) \cos(2\pi \hat{\theta}) - g(0,1) \cos(2\pi \hat{\varphi})$$

$$-2g(1,0) \cos(\pi \hat{\theta}) \cos(\pi \hat{\varphi})$$

(6)

There are three distinct pathways for the thermal melting of the PDW state\cite{13} by condensation of its topological excitations, as shown in the schematic phase diagram in Fig.\[1\]. In constructing this figure, the location of the phase boundaries were estimated by computing the scaling dimensions of the cosine operators at the non-interacting fixed point, $\Delta_{q_v, q_d} = \frac{\pi}{2}(q_v^2 \rho_s + q_d^2 \kappa)$, and then identifying the lines, $\Delta_{q_v, q_d} = 2$, at which each of the cosine operators in Eq.(6) first becomes relevant. Alternatively, a mean-field phase diagram can be obtained by treating Eq.(3) in a self-consistent phonon approximation. More sophisticated RG treatments of a Coulomb gas system with precisely the same formal structure as the above have been discussed previously for a model of the thermal melting of a charge-spin stripe state\cite{20} and the context of spinor Bose condensates.\cite{21}

All approaches yield the same topology of the phase diagram as the one shown in Fig.\[1\], although there are some
unresolved differences concerning the shape of the various phase boundaries and the nature of the multicritical points, $M_1$ and $M_2$. Moreover, since the specific problems being addressed in the field $\tilde{\nu}_\theta$ and $\tilde{\nu}_\sigma$ were different than those being considered here, the interpretation and the physics of the phases was different.

a. **PDW Phase:** At low temperatures, all three cosine operators are irrelevant and the topological excitations are uncondensed. The system is in a PDW state with quasi-long range order (QLRO), as given in Eq. (1).

b. **Charge 4e SC Phase:** This phase is accessed when the double dislocations proliferate, i.e. when the operator $\cos(2\pi\tilde{\varphi})$ becomes relevant. Hence, the PDW-Charge 4e SC phase boundary is the line $\Delta_{0,1} = 4\pi\kappa/T = 2$, or $\Delta_{\varphi} = 2\pi\tilde{\varphi} / \rho_s$ (see Fig. 1). In the charge 4e SC phase, the field $\tilde{\varphi}$ is pinned at integer values $n_\varphi \in \mathbb{Z}$, and has massive (gapped) fluctuations. In this phase, the dislocations of the CDW are screened, and the dual the CDW phase field $\varphi$ has wild fluctuations, leading to exponentially decaying correlations of both the PDW and CDW order parameters. In contrast, the field $\tilde{\theta}$ remains gapless, and the correlation function of the charge 4e SC have power-law correlations, with the same exponent $\eta_{4e}$. The phase transition from the charge 4e SC to the disordered non-superconducting nematic higher temperature phase proceeds through the subsequent unbinding and proliferation of vortices with fractional (half) topological charge.

c. **Stripe (CDW) Phase:** This phase is accessed through the proliferation of SC vortices with integer topological charge when the operator $\cos(2\pi\tilde{\varphi})$ becomes marginal, $\Delta_{1,0} = \pi\rho_s / T = 2$. There is CDW QLRO, with correlation functions that decay with a power law with an exponent $\eta_{CDW}$, and no SC order of any type. In this phase the field $\tilde{\varphi}$ is pinned at the integer values $n_\varphi \in \mathbb{Z}$, and its fluctuations are massive. The phase transition from this phase to the normal (nematic) phase occurs by the unbinding of single dislocations whose fractional vortex charge is screened in this phase.

d. **Direct PDW-nematic normal state phase transition:** This transition proceeds through the liberation of half-vortex-integer dislocation composite excitations, i.e. when the topological excitations $(\pm \frac{1}{2}, \pm \frac{1}{2})$ condense. This phase boundary is the $M_1 M_2$ line shown in Fig. 1 determined by the condition $\Delta_{\pm \frac{1}{2}, \pm \frac{1}{2}} = 2$.

e. **Multicritical Points:** The schematic phase diagram of Fig. 1 has two multi-critical points, $M_1$ and $M_2$, whose existence follows from the topology of the phase diagram. From the structure of Eq. (4) it also follows that there exist two special critical points with an emergent higher symmetry where two operators become marginal simultaneously: $(0, 1)$ and $(\frac{1}{2}, \pm \frac{1}{2})$ at $M'_1$, and $(1, 0)$ and $(\pm \frac{1}{2}, \pm \frac{3}{2})$ at $M'_2$. At these points, the general $U(1) \times U(1)$ symmetry of the model is enlarged to an $SU(3)_1$ symmetry\( ^{26} \) and in their vicinity the correlation length exhibits KT behavior with a modified exponent $\frac{\eta_{CDW}}{2}$. The simple scaling arguments used to construct Fig. 1 and the RG analysis in\( ^{20} \) suggest that the high symmetry and multicritical points are one and the same, $M_j = M'_j$, while the self-consistent phonon calculation and the RG treatment\( ^{23} \) suggest that they are distinct, with $M'_1$ and $M'_2$ lying part way along the segment of the phase boundary between $M_1$ and $M_2$.

While the present discussion of thermal melting was confined to 2D, it can be readily extended to the case of a 3D layered material with sufficiently weak inter-layer coupling. Some aspects of this extension depend on details of the interlayer geometry\( ^{23} \). For instance, depending on whether or not the inter-layer coupling frustrates the stripe ordering, the low temperature state may or may not have a gentle, time-reversal symmetry breaking superconducting spiral superimposed on the basic single plane stripe order. But, in general, all of the phases indicated in Fig. 1 persist in 3D, with the QLRO replaced by true long-range order, and a crossover, very near the phase boundaries, to 3D criticality.

At $T = 0$ the power-law correlations of the PDW phase are replaced by true long range order and gapless collective (Goldstone) modes. A simple mean-field treatment\( ^{24} \) of the spectrum of Bogoliubov quasiparticles in this phase reveals that, in common with a CDW, it generically has an only partially gapped Fermi surface, and hence a finite density of zero energy states. In principle, it should also be possible for the PDW phase to undergo quantum melting, possibly also by a quantum analog of the vortex unbinding mechanism described above.\( ^{24} \) This is a more complex problem than the thermal melting we have discussed here. In particular, the quantum phase transitions are affected by the existence of gapless quasiparticles. Hence, the structure and quantum dynamics of the vortices (both integer and fractional) is
expected to be generally damped and anisotropic, which affects the physics of quantum melting.

There are many unusual physical consequences of the nature of the PDW phase and of its thermally melted daughter phases. We have previously discussed some remarkable bulk features of a fully ordered PDW phase, including the possibility of dynamical layer decoupling (in a 3D layered material), an anomalous sensitivity to quenched disorder, the existence (in principle testable by STM and scanning SQUID) of pinned half-vortices associated with each disorder induced dislocation, and a tendency to formation of a superconducting glass phase likely spontaneously breaks time reversal symmetry. Here we focus on “phase sensitive” measurements, which have not yet been explored, and offer the possibility of directly establishing the existence of either the fully ordered phase or its uniform, translationally invariant charge 4e SC descendant.

Clearly, a superconducting SQUID loop, made of a charge 4e SC, will exhibit all the familiar features of a SQUID, but with a distinct half-flux quantum, $\hbar c/4e$ replacing the usual superconducting flux quantum, $\hbar c/2e$. From a practical point of view, however, it is undoubtedly easier to fabricate a SQUID loop in which only a single link consists of a charge 4e SC, as shown schematically in Fig. 2a. Such a SQUID will exhibit the same half-flux quantum as if it were an entire loop of charge 4e SC. In addition to the practical advantage, such a geometry will detect PDW related phases under a wider range of circumstances. For instance, in the disordered phase near criticality, so long as the width of the putative charge 4e link is not large compared to the SC coherence length, it will simply act as a Josephson weak link in a charge 4e SC SQUID. Moreover, under many circumstances, even if the link consists of a PDW state, the SQUID will still exhibit $\hbar c/4e$ periodicity.

To see this, consider a Josephson junction between a striped SC and an ordinary SC, as shown schematically in Fig. 2b. The dependence of the free energy on the phase difference, $\Delta \theta$, between the two SC can be expanded as

$$F(\Delta \theta) = S \sum_{n=1}^{\infty} J_n \cos[n\Delta \theta],$$

where $S$ is the junction area, and $J_n$ involves tunneling processes of $2n$ electrons, so that generally $J_n$ decays exponentially with increasing $n$. For the geometry shown in Fig. 2b, which is representative of the putative striped SC phase in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ near $x = 1/8$, $J_1$ (and indeed, all $J_{2n+1}$) vanish identically, since a $\pi$ phase change of the striped SC is equivalent to a translation by half a period. As a result, the (resistively shunted) Josephson coupling between the two SC is dominated by the intrinsically smaller higher order coupling, $J_2 \propto \frac{\hbar c}{4e}$. Not only does that mean that a striped SC in the place of the charge 4e SC link in Fig. 2a, will act in the same way, it also means that all the standard characteristics of a single Josephson junction, including Josephson oscillations and Shapiro steps, will occur with twice the usual frequencies.

$J_2$ is proportional to $J^2$, where $J$ is the “bare” (microscopic) Josephson coupling. A quantitative estimate of $J_2$ is difficult, since it depends exponentially on microscopic parameters. However, assuming that the coherence length $\xi$ is smaller than the period of the striped superconductor, an estimate based on an effective $x-y$ model (similar to that of Ref. 5, but adapted for a c-axis junction between a striped and a uniform superconductor) gives that $J_2 \sim J^2/|J'|$, where $J'$ is the inter-stripe Josephson coupling in the striped superconductor. Thus the energy denominator is the smallest energy scale of the single-plane problem. Moreover, since the total coupling energy is proportional to the area of the junction, it should be possible to use a large enough junction such that $SJ_2$ is substantial (i.e., much larger than $k_B T$).

It should be possible to perform the proposed experiments using small crystals of $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ with $x = 1/8$, where there is already strong circumstantial evidence of the existence of a striped SC phase. The observation either of $\hbar c/4e$ flux quantization in a SQUID of the sort shown in Fig. 2a, or a charge 4e in the Josephson relation in a Josephson junction of the sort shown in Fig. 2b, would constitute dramatic and direct evidence of the existence of these exotic superconducting phases.

E. Berez, berez@stanford.edu, is the corresponding author.

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All three authors contributed equally to all parts of this work. The authors declare that they have no competing financial interests.

1. Li, Q., Hückler, M., Gu, G. D., Tsvelik, A. M. & Tranquada, J. M. Two-Dimensional Superconducting Fluctuations in Stripe-Ordered La$_1.875$Ba$_{0.125}$CuO$_4$. Phys. Rev. Lett. 99, 067001 (2007).

2. Fujita, K., Noda, T., Kojima, K. M., Eisaki, H. & Uchida, S. Effect of disorder outside the CuO$_2$ planes on $T_c$ of copper oxide superconductors. Phys. Rev. Lett. 95, 097006 (2005).

3. Berg, E. et al. Dynamical layer decoupling in a stripe-ordered high $T_c$ superconductor. Phys. Rev. Lett. 99, 127003 (2007).

4. Berg, E., Fradkin, E. & Kivelson, S. A. Theory of the Striped Superconductor. Phys. Rev. B 79, 064515 (2009).

5. Berg, E., Fradkin, E., Kivelson, S. A. & Tranquada, J. M. Striped superconductors: How the cuprates intertwine spin, charge and superconducting orders (2009). (unpublished), arXiv:0901.4826.

6. Tranquada, J. M. et al. Evidence for unusual superconducting correlations coexisting with stripe order in La$_1.875$Ba$_{0.125}$CuO$_4$. Phys. Rev. B 78, 174529 (2008).

7. A. A. Schafgans, A. D. LaForge, S. V. Dordevic, M. M. Qazilbash, S. Komiy, Y. Ando, and D. N. Basov. Magnetic-field-induced spin order quenches Josephson coupling in La$_{2-x}$Sr$_x$CuO$_4$ (2008). (unpublished).

8. Agterberg, D. F. & Tsunetsugu, H. Dislocations and vortices in pair-density-wave superconductors. Nature Phys. 4, 639 (2008).

9. Agterberg, D. F., Sigrist, M. & Tsunetsugu, H. Order parameter and vortices in the superconducting Q-phase of CeCoIn$_5$ (2009). (unpublished), arXiv:0902.0843.

10. Radzihovsky, L. & Vishwanath, A. Quantum liquid crystals in imbalanced Fermi gas: fluctuations and fractional vortices in Larkin-Ovchinnikov states (2008). (unpublished), arXiv:0812.3945.

11. Here we consider the case of incommensurate PDW order (or with high order commensuration).

12. Larkin, A. I. & Ovchinnikov, Y. N. Nonuniform state of superconductors. Zh. Eksp. Teor. Fiz. 47, 1136 (1964). (Sov. Phys. JETP. 20, 762 (1965)).

13. In the continuum the charge anisotropy $\alpha_c$ diverges.

14. This is justified, provided that the penetration depth is long enough.

15. Kosterlitz, J. M. & Thouless, D. J. Ordering, metastability and phase transitions in two-dimensional systems. J. Phys. C 6, 1181 (1973).

16. Nelson, D. R. & Halperin, B. I. Dislocation-mediated melting in two dimensions. Phys. Rev. B 19, 2457 (1979).

17. Young, A. P. Melting and the vector coulomb gas in two dimensions. Phys. Rev. B 19, 1855 (1979).

18. Niemhuis, B. Coulomb Gas Formulations of Two-dimensional Phase Transitions. In Domb, C. & Lebowitz, J. (eds.) Phase Transitions and Critical Phenomena, vol. 11, 1 (Academic Press, London, 1987).

19. The physics is more complex if the (intertwined) spin stripe order is also considered depending on whether the full SU(2) spin symmetry remains intact (in which case there is no long range spin order in 2D for $T > 0$), or if the magnetic anisotropy is XY-like or Ising-like.

20. Krüger, F. & Scheidt, S. Nonuniversal ordering of spin and charge in stripe phases. Phys. Rev. Lett. 89, 095701 (2002).

21. Podolsky, D., Chandrasekharan, S. & Vishwanath, A. Phase Transitions of $S = 1$ Spinor Condensates in an Optical Lattice (2007). (unpublished), arXiv:0707.0695v2.

22. Kondiev, J. & Henley, C. L. Kac-Moody symmetries of critical ground states. Nucl. Phys. B 464, 540 (1996).

23. Baruch, S. & Orgad, D. Spectral signatures of modulated d-wave superconducting phases. Phys. Rev. B. 77, 174502 (2008).

24. The quantum melting of a charge-spin stripe state (without SC order) was discussed qualitatively in Ref. 22.

25. An analogous effect has been seen in Josephson junctions between two crystals of a d-wave superconductor with relative crystalline orientation chosen so that $J_1 = 0$.

26. Nussinov, Z. & Zaanen, J. Stripe fractionalization I: the generation of Ising local symmetry. Journal de Physique IV (Proceedings) 12, 9 (2002).

27. Schneider, C. W. et al. Half-$\pi$/2e critical current Oscillations of SQUIDs. Europhys. Lett. 68, 86 (2004).