Research on the influence of air resistance in spacecraft moment of inertia measurement

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Abstract. MOI values of spacecraft are critical values to influence the control accuracy and attitude in the orbit. Traditionally, MOI values are measured by air floating MOI measurement equipment, but the air resistance may influence the accuracy of MOI measurement. How to evaluate the scale of the influence and to reduce the influence will reduce measurement error. The thesis designs a kind of simulation boards to research the air resistance influence, then using the air influence factors to calculate the influence in quantity, and finally using the method to compensate the air resistance influence. The influence of air resistance can be compensated by calculating the coefficient of air resistance, and compensation will eliminate 96% of the error.

1. Introduction
Moment of Inertia (MOI) values of spacecraft are critical values to influence the control accuracy and attitude in the orbit [1]. MOI measurement equipment can measure MOI value of spacecraft, and the measurement method is to measure the rotary period of product [2]. So, the rotary period accuracy will influence the accuracy of MOI value directly. For large scale spacecraft, because the section of spacecraft is large, air resistance may have important influence in MOI measurement, which will affect the accuracy of MOI value.

In order to analysis the influence of air resistance in MOI accuracy in quantity, the thesis research difference of object’s measured MOI value and true value under different section, different position and different angle of same section to figure out the scale of air resistance in MOI accuracy and the method of compensation and effect.

2. Principle of MOI measurement
The relationship of sum moment of external force and MOI can be shown as (1)[3]:

\[ M = I \dot{\theta} \]  

(1)

For MOI measurement equipment, the sum moment of external force has 2 parts. One is equal to the spring coefficient multiply the rotary angle; the other is related to air resistance. So relationship can be express as (2).

\[ I \ddot{\theta} + F \dot{\theta} + K \theta = 0 \]  

(2)

In the formula, F is air resistance factor, K is the spring coefficient.

The root of equation can be express as (3).

\[ \theta = e^{-\frac{F}{2K}t} c_1 \cos \left( \frac{\left(4IK - F^2\right)^{1/2}}{2I} t + \phi \right) + c_2 \]  

(3)

Because \( F^2 \ll 4IK \left(1 - \frac{F^2}{4IK} \right)^{1/2} \). The rotary period of oscillation is shown as (4).
Thus,\
\[ T = 2\pi \left( \frac{I}{K} \right)^{1/2} \] (4)
\[ I = \frac{K \tau^2}{4\pi^2} \] (5)

3. Problem and solution

In the rotary period formula, \( \left(1 - \frac{F^2}{41K}\right)^{1/2} \) is assumed as 1, which will neglect the influence of air resistance, which will cause less accuracy of MOI measured value. Traditionally, the influence of air resistance is less than the required accuracy of product, so it can be neglected. But for large scale spacecraft, bigger section will produce large air resistance. In order to make the measurement value more accurate, the influence of air resistance should be analysis in quantity.

In order to research the influence of air resistance under different section, different position, and different angle, the thesis design two simulation boards, and design different position for the boards, at the same time design different angle for assembly the boards. The proE models of simulation boards are shown as figure 1 and figure 2.

![Figure 1](image1.png)

Figure 1. Simulation boards on far end.

![Figure 2](image2.png)

Figure 2. Simulation boards on middle position.

4. Test data

4.1. Test data of simulation boards on far end

The test data of simulation boards on far end is shown in table 1, and the figure of the test data is shown in figure 3.
Table 1. Test data of simulation boards on far end.

| T0 (s)   | Td (s)   | I (kgm²)   | ΔI          | ΔI (%)     |
|----------|----------|------------|-------------|------------|
| 0        | 1.195255563 | 1.668395223 | 222.473935  | 3.029      | 1.38       |
| 15       | 1.195244006 | 1.667793406 | 222.1487975 | 2.704      | 1.232      |
| 30       | 1.195242431 | 1.666847027 | 221.6312317 | 2.186      | 0.996      |
| 45       | 1.195230374 | 1.66555148  | 220.9270724 | 1.482      | 0.675      |
| 60       | 1.195217807 | 1.663653321 | 219.8943745 | 0.449      | 0.205      |
| 75       | 1.195215818 | 1.662970812 | 219.5223501 | 0.077      | 0.035      |
| 90       | 1.195185536 | 1.662807555 | 219.4450829 | 0          | 0          |

Figure 3. The figure of influence of simulation boards on far end.

4.2. Test data of simulation boards on middle position

The test data of simulation boards on far end is shown in table 2, and the figure of the test data is shown in figure 4.

Table 2. Test data of simulation boards on middle position.

| T0 (s)   | Td (s)   | I (kgm²)   | ΔI          | ΔI (%)     |
|----------|----------|------------|-------------|------------|
| 0        | 1.168331261 | 1.443549136 | 118.0319683 | 1.542261393 | 1.323946496 |
| 15       | 1.168323697 | 1.443650347 | 118.0828524 | 1.593145433 | 1.367627642 |
| 30       | 1.168318205 | 1.443009843 | 117.7813695 | 1.291662561 | 1.108821195 |
| 45       | 1.168307834 | 1.441809707 | 117.2168637 | 0.727156702 | 0.624223994 |
| 60       | 1.168303699 | 1.441107564 | 116.886076  | 0.396369045 | 0.340261002 |
| 75       | 1.168302042 | 1.440610236 | 116.6513887 | 0.161681787 | 0.138794912 |
| 90       | 1.168283741 | 1.440253595 | 116.4897069 | 0          | 0          |

Figure 4. The figure of influence of simulation boards on middle position.
4.3. Test data of simulation boards on near end

The test data of simulation boards on far end is shown in table 3, and the figure of the test data is shown in figure 5.

Table 3. Test data of simulation boards on near end.

| T0 (s)         | Td (s)         | I (kgm²)          | ΔI          | ΔI (%)       |
|---------------|---------------|------------------|------------|-------------|
| 0             | 1.148224934   | 1.255415465      | 42.30538419 | 0.624496944 | 1.49828131 |
| 15            | 1.148217598   | 1.25504734       | 42.15640421 | 0.475516958 | 1.140851333|
| 30            | 1.148215758   | 1.254707059      | 42.01686882 | 0.335981572 | 0.806080663|
| 45            | 1.148215376   | 1.254686533      | 42.00855519 | 0.327667938 | 0.786134749|
| 60            | 1.14821373    | 1.254036602      | 41.74145018 | 0.060562934 | 0.145301452|
| 75            | 1.148211157   | 1.253990467      | 41.72342136 | 0.042534111 | 0.102047039|
| 90            | 1.148209774   | 1.25388591       | 41.68088725 | 0 | 0 |

Figure 5. The figure of influence of simulation boards on near end.

4.4. Summary

According to test data, the following conclusions can be analysed: boards with bigger injection area will influence the accuracy more; boards with same section but bigger distance will influence the accuracy more; the total influence of air resistance will less than 2%.

5. Analysis of air resistance influence

5.1. Coefficient of air resistance

In formula (3), if the influence of air resistance can’t be neglected, which means \(1 - \frac{P^2}{4IK}\) doesn’t equal to 1, the rotary period can be expressed as formula (6).

\[
T = \frac{2\pi}{\omega} = 2\pi \left( \frac{K}{I} \right)^{-1/2} \left( 1 - \frac{P^2}{4IK} \right)^{-1/2}
\]  

(6)

F is proportional to coefficient of air resistance \(\lambda\), section area \(S\) and distance \(r\), which can be expressed as formula (7).

\[
F = \lambda \int S \cdot r
\]

(7)

According to formula (6) and formula (7), \(\lambda\) can be solved.

\[
\lambda = \left[ 1 - \left( \frac{2\pi}{(\frac{K}{I})^{1/2} \cdot 4IK} \right)^2 \right]^{1/2} \times (\int Sr)^{-1}
\]

(8)

When \(\lambda\) is calculated, the actual MOI of product can be calculated using rotary period \(T\), section area \(S\) and distance \(r\).
5.2. Calculate air resistance coefficient
Because the error of $\lambda$ is inverse proportional to the square of section area and square of distance, the bigger section area and distance will calculate more accuracy $\lambda$. So, choose the rotary period of simulation board at 0 angle of far end, the MOI value of simulation board at 90 angle of far end, and MOI equipment coefficient $K$ to solve the coefficient of air resistance $\lambda$.

| Angle       | Equipment coefficient $K$ | Rotary period $T$ (s) | MOI value (kgm$^2$) |
|-------------|---------------------------|-----------------------|---------------------|
| Simulation board at far end | 90 | - | 219.44508 |
| 0 | 6482.31 | 1.164 | - |

5.3. MOI error compensation
Using the calculated air resistance coefficient, the error compensation data of simulation board at far end is shown as table 5.

| Angle | Measured MOI | Error of measured MOI | Calculated value including air resistance | Error after compensation of air resistance |
|-------|--------------|-----------------------|------------------------------------------|------------------------------------------|
| 0     | 222.47393    | 3.02885               | 222.47394                                | -0.00001                                 |
| 15    | 222.1488     | 2.70371               | 222.26843                                | -0.11963                                 |
| 30    | 221.63123    | 2.18615               | 221.70891                                | -0.07768                                 |
| 45    | 220.92707    | 1.48199               | 220.94913                                | -0.02206                                 |
| 60    | 219.89437    | 0.44929               | 220.19454                                | -0.30016                                 |
| 75    | 219.52235    | 0.07727               | 219.6454                                 | -0.12305                                 |

The error compensation data of simulation board at middle position is shown as table 6.

| Angle | Measured MOI | Error of measured MOI | Calculated value including air resistance | Error after compensation of air resistance |
|-------|--------------|-----------------------|------------------------------------------|------------------------------------------|
| 0     | 118.03197    | 1.54226               | 118.03421                                | -0.00224                                 |
| 15    | 118.08285    | 1.59315               | 117.92947                                | 0.153382                                 |
| 30    | 117.78137    | 1.29166               | 117.64426                                | 0.137111                                 |
| 45    | 117.21686    | 0.72716               | 117.25687                                | -0.04001                                 |
| 60    | 116.88608    | 0.39637               | 116.87203                                | 0.014045                                 |
| 75    | 116.65139    | 0.16168               | 116.5919                                 | 0.059484                                 |

The error compensation data of simulation board at near end is shown as table 7.

| Angle | Measured MOI | Error of measured MOI | Calculated value including air resistance | Error after compensation of air resistance |
|-------|--------------|-----------------------|------------------------------------------|------------------------------------------|
| 0     | 42.305384    | 0.6245                | 42.147144                                | 0.15824                                  |
| 15    | 42.156404    | 0.47552               | 42.115585                                | 0.040819                                 |
| 30    | 42.016869    | 0.33598               | 42.029605                                | -0.01274                                 |
| 45    | 42.008555    | 0.32767               | 41.912719                                | 0.095836                                 |
| 60    | 41.74145     | 0.06056               | 41.796482                                | -0.05503                                 |
| 75    | 41.723421    | 0.04253               | 41.711798                                | 0.011623                                 |

5.4. Summary
According to the data, it can conclude that the error will reduce largely after compensation of air resistance, which means air resistance takes a large part of error between measured value and actual
value; the same air resistance coefficient at different position will have effective result, which means the air resistance analysis can be applied to other conditions.

6. Conclusion
In the MOI measurement, air resistance will influence the accuracy, and the scope of influence will less than 2%; the influence of air resistance can be compensated by calculating the coefficient of air resistance, and compensation will eliminate 96% of the error.

References
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