Pion fragmentation functions from $e^+e^-$ production

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Abstract. We present the new sets of fragmentation functions for charged pions at leading order. They are fitted to data on inclusive charged-hadron production in $e^+e^-$ annihilation taken by different collaborations, who discriminated between events with charm, bottom, and light-flavors fragmentation in their charged-hadron sample. They also agree with data on the production of neutral pions.

1. Introduction

In finding any physics signatures beyond the standard model and any new hadronic systems in high-energy hadron reactions it is necessary to have accurate Quantum Chromodynamics (QCD) predictions for cross sections or decay rates. As the new generation of high energy experiments with the detection of a final state hadron are taking place, further tests of QCD and the Standard Model require an accurate knowledge not only of the parton distribution functions (PDFs) and $\alpha_s(M_Z)$, but equally of the fragmentation functions (FFs) $D_h^i(x, Q^2)$. A fragmentation function indicates the probability of the transition of a parton $i$ at scale $Q$ into a hadron $h$ carrying away a fraction $x$ of the parton’s momentum or energy in the center-of-mass (c.m.) frame.

The fragmentation functions are related to a nonperturbative aspect of QCD, hence they cannot be precisely calculated by theoretical methods. The situation is similar to the determination of the PDFs, where high-energy experimental data are used for their determination instead of theoretical calculations. In the data sample without discrimination of the hadron species, they distinguished between four cases, namely fragmentation of $u$, $d$, and $s$ quarks; $c$ quarks only; $b$ quarks only; and all five quark flavors ($u$, $d$, $s$, $c$, and $b$).

Like $\alpha_s(M_Z)$, PDFs and FFs are important quantities because they are universal: according to the factorization theorem, once they are known at some suitably defined scale $Q = Q_0$, they can be calculated at any other scale $Q$ and used in any type of process [1]. The most reliable way to determine them at a given scale is by fitting to inclusive single hadron production data in which the fraction $x$ of available momentum or energy in the c.m. frame carried away by the hadron is measured.
2. Single inclusive cross section in $e^+e^-$ annihilation

The inclusive production of a hadron $h$ by $e^+e^-$ annihilation,

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow h + X,$$

is described first by a partonic subprocess $e^+e^- \rightarrow q\bar{q}$ to produce a quark-antiquark pair and a gluon at NLO and second by a hadronization process in which a parton fragments to produce a hadron. The latter is a non-perturbative process that is described by a non-perturbative fragmentation function (non-pff). The hadron-production cross section for inclusive production of a hadron $h$ is given by [2]

$$F^h(x,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dx}.$$  \hspace{1cm} (2)$$

To be able to compare our calculations with experimental data, we normalize the cross section to

$$\sigma_{tot} = N_c \sum_{i=1}^{N_f} e_{q_i}^2 \sigma_0,$$

where $\sigma_0 = (4\pi\alpha^2/3s)$ is the total cross section of $e^+e^- \rightarrow \mu^+\mu^-$ for massless leptons, $N_c = 3$, $e_{q_i}$ is the electroweak charge of quark $q_i$ in units of the positron charge, and $N_f$ is the number of active flavors. The variable $Q^2$ is the virtual photon or $Z^0$ momentum squared in $e^+e^- \rightarrow \gamma$ (or $Z^0$) and it is expressed by the center-of-mass energy $\sqrt{s}$ as $Q^2 = s$. The variable $x$ is the hadron energy $E_h$ scaled to the beam energy $\sqrt{s}/2$, and it is defined by the fraction:

$$x \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{\sqrt{Q^2}}.$$  \hspace{1cm} (3)$$

QCD factorization theory predicts that the fragmentation process is described by the summation of hadron productions from quarks, antiquarks, and gluons and this is similar to the structure functions in DIS [3, 4, 5]

$$F^h(x,Q^2) = \sum_i C_i(x,\alpha_s) \otimes D^h_i(x,Q^2).$$  \hspace{1cm} (5)$$

According to the notation $\otimes$ that indicates a convolution integral definition

$$f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f(y)g\left(\frac{x}{y}\right),$$

and also coefficient functions $C_i(x,\alpha_s)$ describing partonic subprocess [6], the hadron-production cross section is given by

$$F^h(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} D^h_i(z,M_f^2) \frac{1}{\sigma_{tot}} \frac{d\sigma_i}{dy}\left(\frac{x}{z},\mu^2, M_f^2\right).$$  \hspace{1cm} (6)$$
Figure 1. $\pi^+$ fragmentation functions at initial sale $Q = Q_0$ and $Q = M_Z$.

3. QCD analysis
The $x$ dependence of the pion fragmentation functions are parameterized at the initial $Q^2 (\equiv Q_0^2)$ in the same way as the PDF analysis [7, 8, 9, 10, 11, 12, 13]. Since they should vanish at $x=1$, we choose a simple polynomial form

$$D_i^\pi (x, Q_0^2) = N_i^\pi x^{\alpha_i^\pi} (1 - x)^{\beta_i^\pi},$$

where $N_i^\pi$, $\alpha_i^\pi$, and $\beta_i^\pi$ are parameters to be determined by a $\chi^2$ analysis of $e^+e^- \rightarrow \pi X$ data. Since we have built in the $c\bar{c}$ and $b\bar{b}$ thresholds, we have three different starting scales $Q_0$ [14, 15]

$$Q_0 = \begin{cases} 
1 \text{ GeV}, & \text{if } i = u, d, s, g \\
m_c = 1.43 \text{ GeV}, & \text{if } i = c \\
m_b = 4.3 \text{ GeV}, & \text{if } i = b 
\end{cases}.$$  

(9)

According to partonic structure of $\pi^+ (ud)$ we take the same favored fragmentation functions for $\pi^+$ from $u$ and $\bar{d}$ quarks. Also the pion productions from $\bar{u}$, $d$, $s$, and $\bar{s}$ are considered the same at the $Q_0^2 = 1$ GeV.
Figure 2. Comparison of our results with pion experimental data at $Q = M_Z$.

\[ D_{u}^{+}(x, Q_0^2) = D_{d}^{+}(x, Q_0^2) = N_u^{+} x^{\alpha_{u}^{+}} (1 - x)^{\beta_{u}^{+}}, \]
\[ D_{\bar{u}}^{+}(x, Q_0^2) = D_{\bar{d}}^{+}(x, Q_0^2) = D_{s}^{+}(x, Q_0^2) = N_{\bar{u}}^{+} x^{\alpha_{\bar{u}}^{+}} (1 - x)^{\beta_{\bar{u}}^{+}}. \]

Gluon and heavy quarks fragmentation functions are defined separately.

\[ D_{g}^{+}(x, Q_0^2) = N_{g}^{+} x^{\alpha_{g}^{+}} (1 - x)^{\beta_{g}^{+}}, \]
\[ D_{c}^{+}(x, m_c^2) = D_{\bar{c}}^{+}(x, m_c^2) = N_{c}^{+} x^{\alpha_{c}^{+}} (1 - x)^{\beta_{c}^{+}}, \]
\[ D_{b}^{+}(x, m_b^2) = D_{\bar{b}}^{+}(x, m_b^2) = N_{b}^{+} x^{\alpha_{b}^{+}} (1 - x)^{\beta_{b}^{+}}. \]

These 15-dimensional parameters are determined by the charged-hadron production data of $e^+ + e^- \rightarrow h^\pm + X$ in which the deviation of the theoretical prediction from the data becomes minimal. We use the data obtained by the collaborations SLD [16], ALEPH [17], OPAL [18], DELPHI [19, 20], TASSO [21, 22, 23], TPC [24], and TOPAZ [25].
4. Conclusions

We presented new sets of FF for charged pions, at LO. $Q^2$ evolution results of FFs are shown in Fig. 1. The initial functions of $\pi^+\pi^-$ are supplied at $Q^2=1$ GeV$^2$ for $g$ and light quarks FFs and for heavy quarks at $Q^2=m_{b}^2$ and $m_{c}^2$. They were fitted to data on inclusive charged-pion production in $e^+e^-$ annihilation. In Fig. 2, our LO hadron-production cross section results are compared with the data at $Q=M_Z$ without separation on initial partons by the SLD, ALEPH, OPAL and DELPHI collaborations.

Acknowledgments

The authors would like to thank the ICPP-Istanbul II organizing committee for providing this opportunity and support us to participate in the conference.

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