Bondi-Metzner-Sachs symmetry, holography on null-surfaces and area proportionality of “light-slice” entropy

Dedicated to Detlev Buchholz on the occasion of his 65th birthday

submitted to Foundations of Physics

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Abstract

It is shown that certain kinds of behavior, which hitherto were expected to be characteristic for classical gravity and quantum field theory in curved spacetime, as the infinite dimensional Bondi-Metzner-Sachs symmetry, holography on event horizons and an area proportionality of entropy, have in fact an unnoticed presence in Minkowski QFT.

This casts new light on the fundamental question whether the volume proportionality of heat bath entropy and the (logarithmically corrected) dimensionless area law obeyed by localization-induced thermal behavior are different geometric parametrizations which share a common primordial algebraic origin. Strong arguments are presented that these two different thermal manifestations can be directly related, this is in fact the main aim of this paper.

It will be demonstrated that QFT beyond the Lagrangian quantization setting receives crucial new impulses from holography onto horizons.

The present paper is part of a project aimed at elucidating the enormous physical range of ”modular localization”. The latter does not only extend from standard Hamiltonian heat bath thermal states to thermal aspects of causal- or event- horizons addressed in this paper. It also includes the recent understanding of the crossing property of formfactors whose intriguing similarity with thermal properties was, although sometimes noticed, only understood in the modular setting [arXiv:0905.4006, the main part of the project].
1 Bondi-Metzner-Sachs symmetry, holography on null-surfaces and area proportionality of "light-slice" entropy

It has been known for a long time that the restriction of the vacuum state (or any other finite energy state) to localized quantum matter results in a thermal KMS state associated with a Hamiltonian which is uniquely associated with these data [1]. This knowledge has been mainly confined to a few theoreticians with a foundational knowledge of QFT (local quantum physics (LQP)). In the case where the localization behind a causal horizon is not a Gedanken-construction (as e.g. in the Unruh Gedankenexperiment), but is objectively fixed in form of an event horizon at a specific "place" in the spacetime metric, it has attracted general attention. The best known example is the case of a Hartle-Hawking state on global quantum matter in an extended Kruskal-Schwarzschild world to the outside of a Schwarzschild black hole with the Hamiltonian describing the timelike Killing movement, only in that case the thermal aspect in form of the Hawking radiation became part of popular knowledge.

It is perhaps less known that this localization-caused thermal behavior is also accompanied by a localization entropy [2] which behaves differently from the standard heat bath entropy. In the case of (inside or outside) black hole localization this entropy has nothing to do with the still illusive Quantum Gravity (QG) but belongs to the same thermal aspects as those discovered by Hawking.

The Localization-caused thermalization is methodologically related to the conceptual setting of "holographic projection" in which a bulk algebra is simplified by "projecting" it onto its causal horizon [3]. Spacetime localization of quantum matter with a sharp boundary causes infinitely large contributions from the vacuum fluctuation at the causal boundary (horizon); allowing an appropriately canonically defined "fuzzy" boundary in form of a light slice of thickness $\Delta R$ leads to a logarithmically corrected area law for entropy and energy. The physically relevant area is a dimensionless and consists of the (dimension-full) geometric area of the horizon divided by the square of the slice size $(\Delta R)^2$ i.e. $a = \frac{\text{Area}}{(\Delta R)^2}$. It is modified for $\Delta R \to 0$ by a factor $\ln a$. This law is the same for all quantum matter (apart from whatever substance may be related to quantum gravity), however as in the heat bath case a numerical factor in front does depend on the kind of quantum matter.

The logarithmic correction which comes from the lightlike direction of the horizon-slab may appear as an innocuous modification of the area law, however it is essential for our understanding of the unity (the common root) between heat

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1 The quoted article is the most recent in a series of previous publications [3] in which the modular localization formalism developed with the purpose to obtain a conceptually sustainable basis of holography. In order to avoid any confusion it should be mentioned that the rigorous quantum field theoretic holography in this paper is a genuine projection (reduction of degrees of freedom). The reader should be aware that there is also a more metaphoric use in discussions about quantum gravity where holography is thought to lead to an isomorphic storage of the information in the bulk matter onto a "screen".
bath and localization-caused thermal manifestation. Although this localization entropy is not the primary topic of this paper, for reasons of completeness we briefly present its derivation and add some new comments (section 1.3).

The occurrence of infinite vacuum polarization at sharp boundaries and their control by "softening" the boundary goes back to the dawn of QFT, but the thermal characterization of the restriction of the vacuum state to the operator algebra of a causally complete subregion is a combined result of black hole physics and, in a more abstract conceptual setting, the application of modular operator theory to the QFT subalgebra of operators localized in a wedge region. In this way the modular theory exposed the inexorable but often overlooked link between quantum field theoretic localization and thermal manifestations.

The first who realized a possible physically relevant connection between the observations of QFT in the presence of event horizons and modular situations in QFT was Sewell. The thermal aspects of modular theory should not came as a surprise since physicists who discovered certain aspects of this theory independent of mathematicians obtained their results while studying the conceptual problems of open systems in quantum statistical mechanics. The more recent quantum field theoretical use of modular theory for modular localization is an adaptation of the Tomita-Takesaki modular theory of operator algebras to causal localization of states and operator algebras.

It is important to view the black hole physics within curved spacetime QFT and the thermal consequences of localization within flat space QFT from a unified standpoint since on the one hand such a viewpoint takes away from black hole physics that connotation of thermal aspects mysteriously popping out of nowhere and on the other hand it reminds us that there are important aspects in standard QFT which, as a result of our Lagrangian prejudices, we have not been aware of. The fact that only the event horizons in black hole physics are objective placed horizons, whereas the causal horizons of quantum matter in Minkowski spacetime are somewhat subjective (observer-dependent) Gedanken-horizons, is no counter-argument since Gedanken-experiments as that of Unruh often lead to corrections in our way of thinking; in addition the constants which appear in the leading entropy behavior for the sheet size $\Delta R \to 0$ are interesting characteristics of the entire model, similar to Virasoro's constant in chiral theories and not just of individual observables within the model.

The modern development of modular localization of states and its use for classifying and constructing QFT models will play an important role in this paper.

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2The reader is reminded that Heisenberg became aware of the presence of vacuum polarization when he discovered that it is impossible to define a finite "partial charge" of a region $Q(V)$ without "softening" the boundary.

3The theory bears the name Tomita-Takesaki modular theory. Although Tomita discovered it, the theory would not have arrived at how we know it without essential contributions by Takesaki.

4The divergence of the localization entropy for $\Delta R \to 0$ is the only divergence with an intrinsic significance i.e. which cannot be subsumed under operator-valued distributions (as fields and their correlations) and renormalization.
Although Minkowski space QFT does not know anything about the gravitational interaction strength, and therefore by itself cannot produce a Bekenstein-like entropy formula (in which the dimensionless ratio is achieved with the help of the gravitational constant instead of $\Delta R$), it nevertheless does lead to drastic change from the volume proportionality of heat bath entropy to the (logarithmically modified) area behavior of localization entropy. The derivation of the formula for localization entropy resulting from vacuum polarization near horizons in which light sheets play a prominent role, is the subject of the fourth section. Its existence has apparently been overlooked as a result of a prejudice claiming that QFT cannot lead to an area law because this allegedly requires a thinning out of degrees of freedom which only a future quantum gravity (QG) could possibly achieve.

This more speculative role of holographic projections within a future theory of quantum gravity, where it is expected to encode the full information (holographic isomorphism) in the causally related bulk into a projected "null-screen" [10], has been the point of departure of many recent publications [3]. In conjunction with Bekenstein’s black hole entropy proposal it led to formulas for gravitational entropy bounds [12]. We have nothing to contribute to this problem which is part of the still elusive QG; our results concern localization-caused thermal behavior of QFT in Minkowski- and curved-spacetime and therefore aims at the entropic counterpart of the Hawking temperature.

Likewise the holography used in the present work is a rigorous property within the general setting of QFT [4] and there are two different ways of implementing it: one which starts with pointlike bulk fields and produces pointlike generators of the holographic projected quantum matter through intermediate semi-local steps, and the other starting from wedge-localized algebras with the local substructure on its causal horizon being constructed by algebraic intersections [3]. Both methods coalesce on observables which are local in the sense of the lightfront and only arise from integer dimensional bulk variables.

Whereas the algebraic method has been presented in previous work, the more recent pointlike field method is the more appropriate for the present purposes.

The intimate interrelation of modular localization, thermal aspects and gravitational localization (localization of quantum matter in front or behind event horizons) begs the question whether other observations which have been attributed to gravitation are also supported (possibly with a different interpretation) by local quantum physics. Since QFT is a very well researched subject

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5Ever since ’t Hooft’s seminal work in which the concept of holography appeared for the first time [10], there has been an abundance of speculative quantum gravity inspired papers about holography on null-surfaces. But in none of these papers a formulation of holography consistent with the rather restrictive setting of QFT was attempted which answers the question of how the holographic projection of a concrete bulk theory really looks like as a QFT. One of the reasons may be that the important mathematical instrument, namely modular localization, remained largely unknown. In this work, as in a series of before cited previous contributions, I try to redress this situation.

6In order to keep things simple, we restrict the bulk regions to wedges or double cones in Minkowski spacetime and leave the extension to curved spacetime for future work.
from the viewpoint of Lagrangian quantization, one suspects that the chances of making such observations increase if, as in the previous case of thermal manifestations, one moves beyond perturbation theory into the more general setting of local quantum physics.

In this note it will be shown that the Bondi-Metzner-Sachs (BMS) group [13], which originated in classical General Relativity within the setting of asymptotically flat models, is really the symmetry of the vacuum state restricted to local quantum matter holographically projected onto a lightlike horizon. Whether this lightlike surface is the horizon of a compact (double cone) or semi-compact region (Rindler wedge) in the bulk, or the lightlike boundary of the entire cosmos in the sense of the asymptotic flatness assumption of BMS, does not affect the mathematics but only the physical interpretation.

This section continues in the form of three subsections. The first is concerned with the definition and the basic properties of lightfront holography. The appearance of infinite dimensional symmetry groups, including the BMS group as a consequence of the symmetry enhancement of the holographic lightfront projection is the theme of the second subsection. Both algebras are local in their own right but somewhat nonlocal (semilocal in a well-defined sense) relative to each other. The issue of localization entropy is the subject of the third section. Here the knowledge of the lightfront algebra is not sufficient because it encodes only the lightlike vacuum polarizations within the lightfront and not those extending into ambient spacetime. This requires to consider a light-slice of thickness $\Delta R$ and to compute the leading power of the sheet entropy in the limit $\Delta R \rightarrow 0$. The similarity to the thermodynamical limit turns out to be more than an analogy. A precise definition of the light-slice entropy requires the notion of modular localization and the related split property.

Whereas the first and third subsection are extensions of already published results [3][2], the derivation of the BMS symmetry from null-space holography is new.

1.1 Holography on null-surfaces and the absence of transverse vacuum polarizations

As emphasized in the introduction, the only kind of holography which features in this paper is the one which permits an rigorous formulation in QFT. The most prominent case is the lightfront (LF) holography which is essentially the old lightfront quantization but now with a more careful formulation of what in the old days has been always neglected namely the relation with the original bulk description[3]. From the point of symmetries, the restriction of the global bulk to the LF leaves a 7-parametric subgroup of the 10-parametric Poincaré group of 4-dimensional Minkowski spacetime: 5 parameters account for a lightlike translation, a lightlike dilation (the wedge-preserving boost transformation projected onto the LF) and the 3-parametric transverse Euclidean group, whereas the re-

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[3] The lightfront quantization was mainly a computational device; how computed results connect with precise observables in the bulk theory was usually left open.
remaining two parameters are less obvious since they are the 2 "translations" of the Wigner little group (3 dimensional Euclidean subgroup of the 6 parametric Lorentz group) which leaves the lightray invariant \[3\].

If one uses characteristic data on the LF and propagates them into the bulk, then the symmetry off the LF (i.e. the LF changing Poincaré transformations) is certainly not encoded in the LF data; the first indication that the bulk data can only be fully reconstructed from those which are intrinsic to LF with extrinsic additional information.

The relation between LF characteristic data and bulk data is somewhat non-local, compactly localized data on LF do not correspond to compactly localized data in the bulk. A more informative way is to use the term semilocal since there is a global causal correspondence between a wedge $W$ and its (upper) horizon $H(W) \subset LF(W)$ where the associated lightfront results from the linear extension of $H(W)$. In classical (massive or massless) wave propagation the symplectic subspace of $H(W)$ (characteristic) data on $H(W)$ corresponds to the symplectic subspace of $W$— localized data. Obviously there are many proper subregions (i.e. regions whose causal completion is not the global spacetime) in the bulk whose horizon is on LF consist of all wedges $W \subset \mathcal{W}$ (the set of all wedges) with $H(W) \subset LF$; they are distinguished by the position of the $edge(W) \subset LF$ of their wedge.

A refinement of localization can be obtained by the formation of relative causal complements. For example by translating a wedge by $a_+$ along its upper lightlike direction into itself $W \rightarrow W_{a+}$ and forming the relative causal complement

$$W'_{a_+} \cap W \equiv H(0, a_+)$$

where our notation indicates that the resulting region is a transverse unbounded subspace on $H(W)$ of lightlike extension $(0, a_+)$. Since all $W$’s result from a fixed one by Poincaré transformations, this relative causal complement construction can be generalized to the other LF preserving transformations. It turns out that the relative causal complements define a local structure on LF which contains arbitrary small regions. However this does not work in the other direction: from the local data on LF one can only reconstruct the data for the LF compatible $W$’s and as we saw their relative commutants do not lead to new proper bulk regions. This means that we are unable to construct the bulk substructure e.g. that of double cones $D \subset W$ inside $W$. The group theoretic reason is that the holographically projected lightfront world is (as the name projection suggests) is not isomorphic to the Minkowski world since the holographical symmetry group is a proper subgroup. If on the other hand the symmetry groups are shared as in the famous AdS$_5$-CFT$_4$ correspondence \[9\], the projection may change into an

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\[8\] As a standard wedge one usually takes the t-z wedge $W = \{ |t| < z; \ (x, y) \in \mathbb{R}^2 \}$, all other wedges are Poincaré transforms. There is one Lorentz boost group $\Lambda_W$ which leaves $W$ invariant and there is one reflection $j_W$ which maps $W$ into its causal opposite $W'$.

\[9\] This is special case of a correspondence between certain regions of two manifolds which, although different by one dimension, share their maximal spacetime symmetry group. The correspondence extends to the operator algebras associated with those region, but its physical. Its physical usefulness is however doubtful \[11\].
isomorphism. However even in this case there is no invertible relation between generating pointlike quantum fields. In the present context holography will always refer to a horizon and all horizons are null-surfaces.

After having explained the kinematical prerequisites of lightfront holography one can now fill it with a dynamical content. In the classical setting of massive or massless linear wave theories when the full space is a symplectic space and the space of waves localized in the causal complement is identical to the symplectic complement of the sub space associated with the original localization. In the case of local quantum physics the local observables form operator algebras which act in a Hilbert space. The vacuum representation consists of all operators in a Hilbert space \( B(H) \), the commutants of \( \mathcal{O} \)-localized subalgebras \( \mathcal{A}(\mathcal{O})' \subseteq B(H) \) replace the classical symplectic complements and the Haag duality relation \( \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O})' \) extends the classical analogy. The nontriviality of the lightfront holography corresponds to the nontriviality of intersections within the family of wedge algebras whose horizon lies on the same LF. is nontrivial if the intersections are not the trivial algebra \( \{ \lambda 1 \} \).

The value of the holographic projection is a significant simplification of the dynamical problem at the expense of the loss of some information about the bulk. The quantum origin of this simplification is very interesting since it consists in the absence of transverse vacuum polarization; in other words the *vacuum tensor factorizes in transverse direction* (see below) so that the vacuum polarization is limited to the lightray direction and hence there is *no lightlike tensor factorization* of the vacuum. This kind of conceptual preparation helps to avoid making interpretational mistakes in attempting to define holographic projections directly in terms of pointlike fields. For our main purpose, namely the derivation of the BMS group from the symmetry enhancement of the holographic projection, we only need the holographic projection in the absence of interactions.

The crucial property which permits a direct holographic projection for a free field is the *mass shell representation* of a free scalar field

\[
A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{ipx} a^*(p) \frac{d^3p}{2p_0} + h.c.)
\]  

Using this representation one can directly pass to the lightfront by using lightfront adapted coordinates \( x_\pm = x^0 \pm x^3, \mathbf{x} \), in which the lightfront limit \( x_- = 0 \) can be taken without causing a divergence in the \( p \)-integration. Using a \( p \)-parametrization in terms of the wedge-related hyperbolic angle \( \theta : p_\pm = p_0 + \rho^3 \simeq e^{\mp \theta}, \mathbf{p} \) the \( x_- = 0 \) restriction of \( A(x) \) to LF reads

\[
A_{LF}(x_+, \mathbf{x}) \simeq \int \left( e^{i(p_-(\theta)x_+ + i\mathbf{p}\cdot\mathbf{x})} a^*(p_-, \mathbf{p}) dp_0 \frac{dp_3}{2p_3} + h.c. \right)
\]  

\[
\langle A_{LF}(x_+, \mathbf{x}), A_{LF}(x'_+, \mathbf{x'}) \rangle \simeq \delta(\mathbf{x} - \mathbf{x'}) \frac{1}{2\pi} \int e^{-ip_-(x_+ - x'_+)} \frac{dp_0}{2p_0} \frac{dp_3}{2p_3}
\]

\[
[A_{LF}(x_+, \mathbf{x}), A_{LF}(x'_+, \mathbf{x'})] \simeq \delta(\mathbf{x} - \mathbf{x'}) \frac{1}{2\pi} \int (e^{-ip_-(x_+ - x'_+)} - e^{-ip_-(x'_+ - x_+)} \frac{dp_0}{2p_0} \frac{dp_3}{2p_3})
\]

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Different from the bulk field, these formula for the two-point function on LF contain a logarithmic infrared divergence if interpreted pointwise. To see that this is harmless one recalls that fields are singular objects (operator-valued distributions) which only lead to observables after smearing with test functions. As a result of the mass shell restriction the equivalence class of test function which have the same restriction \( f|_{H_m} = P_m f \) to the mass hyperboloid of mass \( m \) is mapped to a unique test function \( f_{LF} \) on the lightfront \( [15] \) \( [3] \)

\[
A(f) = A(\{f\}) = A_{LF}(f_{LF})
\] (4)

However the image on LF does not contain all Schwartz test functions but only such functions whose total \( x^- \) integral vanishes i.e. which do not ”see” the logarithmic infrared divergence at \( p^- = 0 \) of the pointlike formula \( [3] \). This problem and its cure via testfunction restriction is well-known from the zero mass scalar field in \( d=1+1 \) and can be traced back to the fact that this field in lightlike direction is really a semiinfinite integral over a chiral current, which makes it string-localized\( [10] \). The simplifying feature of \( A_{LF}(x_+, x^-) \) is, as expected according to the previous considerations, the absence of transverse vacuum fluctuations as evidenced by the appearance of quantum mechanical \( \delta(x - x') \) delta function. In fact the LF generator behaves as a transverse extended chiral theory\( [11] \) of a chiral field which is the potential associated to a chiral current field. The vacuum polarization phenomenon which is closely related to the energy positivity in conjunction with a finite propagation speed has moved exclusively into the lightlike direction whereas in the transverse direction the vacuum factorizes. Holography on null-surfaces is the only construction in QFT which leads to a partial tensor factorization as a result of directional absence of vacuum polarization.

Although there is no local relation between the data on LF and those in the bulk, there is a remnant of ”semi”-locality between the data on that part of LF which coincides with the horizon the wedge \( H(W) \) and the bulk algebra on \( W \); instead of the equality of the associated classical subspaces one now has an equality of operator subalgebras of the algebra of all operators \( B(H) \). The meaning of the statement that the field \( A_{LF}(x, x_+) \) is a generator of the bulk algebra \( A(W) \) becomes now clear since the equality between individual operators of smeared fields \( A(f)|_{supp f \in W} = A_{LF}(f_{LF}) \) can be lifted to the equality of the

\( ^{10} \)the \( \partial_+ \) derivative of \( A_{LF} \) which is a pointlike field and can be smeared with unrestricted Schwartz test function serves as a better generator of the LF algebra. Derivatives of local fields generate (after Haag dualization) the same localized algebras \( A(\mathcal{O}) \) as the field.

\( ^{11} \)The derivative \( \partial_+ A_{LF} \) defines a transverse extended (direct integral) abelian chiral current operator.
where the last line denotes the (weakly closed) operator algebra generated by the Weyl elements\textsuperscript{13}, the corresponding LF algebras \( \mathcal{A}(H(W)) \) are defined correspondingly with \( \text{suppf}_{LW} \subset H(W) \subset LF \) leading to analytic functions on a strip in the appropriate momentum space rapidity which obey the mentioned vanishing of their \( x_+ \) Fourier transform at \( p_- = 0 \).

We are now getting to a conceptual subtlety of lightfront holography. In writing such relations between bulk and horizon algebras as above, the algebras are meant without any knowledge of their local substructure e.g. knowing \( \mathcal{A}(W) \) only means that the relative position (inclusion) \( \subset B(H) \) is known but not that that knows the local substructure of \( \mathcal{A}(W) \) i.e. the position of sharper localized algebras inside \( \mathcal{A}(W) \). Knowing a generating pointlike field for the bulk \textsuperscript{2} one of course knows the position of all operator algebras for arbitrary causally closed bulk regions, but in writing algebraic relations \textsuperscript{5} we have to forget about the local substructure. In particular the knowledge of \( \mathcal{A}(W) \) should not be confused with knowing the pointlike generators in the region \( W \).

A localized algebra is a holistic object in whose definition the localized test functions of individual elements which entered its construction are lost; sharper localized subalgebras can only be regained by either appropriately intersecting wedge algebras or going back to the generating bulk fields and repeating similar constructions for sharper localized test function. In particular \( \mathcal{A}_{LF} \) contains only lightlike vacuum fluctuations. The physical questions concerning localization entropy are however not related with vacuum polarization on the horizon but rather near the horizon and for their study the knowledge of \( \mathcal{A}(W) \subset B(H) \) is insufficient since it says nothing about the local substructure. In the third subsection we will address the localization entropy in a thin light sheet; this requires different techniques that those of this and the next section.

The knowledge of \( \mathcal{A}(W) \) but without its substructure is a state of affairs which one can only describe in terms of the position of an operator subalgebras within \( B(H) \), but it is not possible to encode this partial knowledge in terms of fields. Therefore it is not surprising that in a constructive approach where it is essential to dissect the difficult problem of constructing interacting fields (or, what is the same, the complete theory) into simpler parts and solve one problem at a time, operator algebras provide an important tool.

In connection with the \( AdS_{n+1}-CFT_n \) correspondence it was mentioned that in cases of relations between theories in different spacetime dimensions an en-

\textsuperscript{12}It is very important not to misunderstand the notation \( \mathcal{A}(W) \subset B(H) \); it does not imply any knowledge about the substructure about the local net inside \( W \). The local substructures of \( \mathcal{A}(W) \) and \( \mathcal{A}(H(W)) \) are of course very different (geometric subregions in \( H(W) \) have no geometric counterpart in \( W \)), only the position of these global algebras inside \( B(H) \) is identical.

\textsuperscript{13}In case of free Fermions the CCR (Weyl) algebra is replaced by the CAR algebra.
coding into pointlike fields has to be replaced by one between algebras of certain (usually noncompact wedge-like) regions. In order to arrive at arbitrarily small double cone algebras (the closest approximation to pointlike fields) one has to study intersections of those algebras which one obtained from the correspondence or holography, but the latter does not extend to these smaller algebras.

This can also be seen in terms of the LF fields. The fields on $H(W) \subset LF$ are subject to a significantly different localization from that of the bulk $W$. In particular it is not possible to reconstruct the local substructure of $A(W)$ even though globally $A(W) = A(H(W))$. The field $A_{LF}(x, x)$ does not know anything about the physical mass, but with its value added one can compute $A(W)$ with its bulk substructure. The relevant propagation formula from the null-surfaces into the bulk is contained in a recent paper $[29]$.

$$A_m(x) = -2i \int_{LF} dy_+ d^2 y_\perp \Delta_m(x - y)|_{y_\perp=0} A(y_+, y_\perp)$$

(6)

where $\Delta_m$ is the massive commutator function and $A(y_+, y_\perp)$ is the field on the lightfront. If one wants to compute the $A_m(x)$ for $x \in W$ we only have to integrate over the horizon $y \in H(W) \subset LF$ and not over all of LF; this is automatically incorporated in (6) via the support properties of $\Delta_m$. Note that the holographic data, in distinction to Cauchy data, do not know anything about the mass which has to be added from the outside. There are of course other, philosophically more challenging ways to supply the missing information. Their common feature is to get back from the 7 parametric subgroup to the full 10-parametric Poincaré group. A particular interesting way is to do this via a GPS-like positioning method using several (in $d=1+3$ one needs maximally 3) holographic projections (in certain relative modular positions $[2]$); in this case the mass enters indirectly via the 10-parametric Poincaré group which on her part result from 7-parametric subgroup from the positioning. Hence the concreteness of pointlike holographically projected fields is somewhat of a fake since there is no direct pointwise relation between them and the bulk; what is really identical objects are certain noncompact algebras which they generate and which by intersections lead to the local bulk structure.

The null-surface propagation formula can also be used to demonstrate that this kind of inverse holography is capable to solve important problems which other methods, as e.g. functional analytic methods, failed to do. It is well-known that the modular group of $A(W)$ acts on the generating field $A_m(x)$ as the $W$-preserving Lorentzgroup, whereas on the generator $A_{LF}(x, x)$ of $A(H(W))$ it acts as the $x$ dilation. Clearly the action on the holographic projection is much simpler. It is now easy to see that the holographic inversion formula does map the dilation group into the $W$-preserving Lorentz group. Since in (6) the scale transformation on $A(y_+, y_\perp)$ acts on $y_+$ and trivially on the fixed point $y_- = 0$, we can shift the scale transformation to the second argument of the commutator function. Using the Poincaré invariance of the commutator function and renaming integration variables one can shift the scale transformation $y_+ \to \lambda^{-1} y_+, y_- \to \lambda y_-$ to the unintegrated bulk variable whereupon fit takes the form of a $W$-preserving Lorentz group. Note that whereas for the Cauchy propagation
every spatial region independent of its size has its nontrivial causal shadow region, this is not the case for the inverse holography, in that case the region must be of the form \( H(W) \subset LF \) and for a given \( LF \) there are of course many \( H(W)'s \).

A similar formula to (6) serves in order to extend inverse holography for massive free fields to other horizons as those of (noncompact) spacelike cones \( H(C) \) or (compact) double cones \( H(D) \). In that case it is well known that the modular groups associated to the bulk algebras \( A(C), A(D) \), act in a "fuzzy" i.e. nongeometric way. Let us now see how the "fuzzyness" develops in the case of a double cone. For simplicity we stay in \( d=1+1 \) and chose a double cone symmetric around the origin as in [8]. Then the lower mantle of the cone with apex \((-1,0)\) is a Horizon whose causal shadow covers the double cone. Every signal which entered the double cone must have entered through the mantle.

In this case the propagation from the two pieces of the mantle leads to the sum

\[
A_m(x_+, x_-) = -2i \int_{-1}^{+1} dy \Delta_m(x-y) \bigg|_{y_-=0} A(y_+) +
\]

\[
+ -2i \int_{LF} dy \Delta_m(x-y) \bigg|_{y_+=0} A(y_-)
\]

Now the modular group on the horizon acts fractional namely the "dilation" which leaves the fixed points \( y_\pm = -1, +1 \) invariant (instead of 0, \( \infty \) as in the first case). The modular group on both parts of the horizon is

\[
x_\pm(s) = \frac{(1 + x_\pm) - e^{-s}(1 - x_\pm)}{(1 + x_\pm) + e^{-s}(1 - x_\pm)}
\]

Different from the previous case, one cannot transfer this fractional change from the \( y \) to the \( x \). There is no local fractional transformation on the bulk, rather the action is fuzzy but stays inside the double cone as it should. It is however not purely algebraic since it was obtained by combining the geometric group on the Horizon with the causal propagation whose "reverberation" (non-Huygens) aspect causes the fuzzyness. In a forthcoming joint article with Jens Mund it will be shown that the modular unitaries do not depend on the interaction. The reader will find a detailed mathematically precise presentation According to our results the interaction does not change anything, it is always this semi-geometric modular group. A more general discussion, including a calculation of the pseudodifferential generators of these modular actions, will be contained in a forthcoming paper by Brunetti and Moretti.

The most important message one can learn from this simple setting of holography is that contrary to popular belief one cannot encode the information about

\[\text{14If not stated otherwise the knowledge of a local algebra only means its positioning in the algebra of all operators in the Hilbert space } A(O) \subset B(H) \text{ and does not include its net-structure inside } O.\]

\[\text{15By construction the modular action restricted to the horizon is always geometric, which is closely related with the modular preservation of horizons.}\]

\[\text{16I thank Romeo Brunetti for informing me forthcoming results.}\]
the theory in the bulk into a holographic screen; in the above presentation we emphasized that the value of the mass must be added, but one could also say the holographically projected correlation functions do not allow to reconstruct the creation/annihilation operators \( a(p) \) or that the knowledge about the remaining 3 Poincaré transformation has to be supplied in addition to the system on the holographic screen. The difference between the old "lightcone quantization" and modern holography is that the "lightcone fields" were not considered as differently localized objects in the same theory, but rather as objects resulting from a different quantization. This created an enormous amount of confusion over many decades. Even though the free field holography is too simple to illustrate the advantage of the method in any convincing way, it does reveal its main philosophical basis: a radical change of the spacetime ordering device for a given kind of quantum matter\(^{17}\) in which part of the original information (e.g. mass spectra, particle properties) gets lost.

Physics is not defined in terms of abstract pure algebraic structure, but rather requires spacetime ordered quantum matter. Hence holography should be conceptually placed together with the change of spacetime ordering in passing between isometric submanifolds of different curved space time QFTs using the same abstract quantum matter (see the local covariance principle of QFT in CST \([16]\)). In both cases the work is still in its beginnings, but it is already clear that the spacetime ordering aspect in those new ways of looking at QFT is much more flexible than in the old "lightcone quantization" or in the old way of treating QFT in CST for each fixed CST separately.

There is another remarkable observation about a gain in spacetime symmetry. Up to now we only mentioned the symmetry loss from the Poincaré symmetry in the bulk to its 7-parametric LF subgroup, but there is also an enormous symmetry-gain which has no counterpart in the bulk theory. Transversely extended chiral theories share the infinite \( \text{Diff}(\mathbb{S}^1) \) invariance of chiral theories. In order to identify the LF projection with the well-known theory of an abelian current we take the \( x^+ \) derivative \( \partial_{x^+} A \) and obtain

\[
\langle \partial_{x^+} A_{\text{LF}}(x^+, x) \partial_{x^+} A_{\text{LF}}(x^+', x') \rangle \simeq \delta(x - x') \frac{1}{(x^+ - x^+' - i\varepsilon)^3} \tag{9}
\]

where the second factor describes the correlation of the chiral current whose commutator is a \( \delta' \) function describes the of fluctuating chiral current. Keeping the transverse coordinates fixed and only transforming the compactified (always possible for chiral theories) lightlike coordinates, the maximal symmetry is \( \text{Diff}(\mathbb{S}^1) \). This is a well-known consequence of properties of chiral energy-momentum tensors. Actually the symmetry is even larger because there are also transverse and mixed transverse-lightlike symmetry transformations. We will return to this problem in the next section.

The success of the pointlike formulated lightfront holography in the absence

\(^{17}\)In the free \( d \geq 4 \) case there is (according to the commutation structure) only CCR and CAR quantum matter.

\(^{18}\)The vacuum preserving symmetries consist only of the Moebius subgroup.
of interactions begs the question whether there exists a pointlike formulation in the presence of interactions. The main problem in this case is to find an analog of the mass shell representation (2). Such representations for interacting fields already appeared in the 60s; shortly after the formulation of LSZ scattering theory Glaser, Lehmann and Zimmermann introduced such representations which became known as “GLZ representations” [17]. They express the interacting Heisenberg field as a power series in incoming (outgoing) free fields. In case there is only one type of particles one has:

\[ A(x) = \sum \frac{1}{n!} \int_{V_n^+} \cdots \int_{V_n^+} a(p_1, \ldots, p_n) e^{i \sum p_i x} : A_{in}(p_1) \cdots A_{in}(p_n) : \frac{d^3 p_1}{2p_1 \cdot} \frac{d^3 p_n}{2p_n \cdot} \]  

\[ (10) \]

\[ A_{in}(p) = a^*_{in}(p) \text{ on } V_m^+ \text{ and } a_{in}(p) \text{ on } V_m^- \]

\[ a(p_1, \ldots, p_n)_{p_i \in V_m^+} = \langle \Omega | A(0) | p_1, \ldots, p_n \rangle \text{ vac.pol.component} \]

where the integration extends over the forward and backward mass shell \( V_m^\pm \subset V_m \) and the product is Wick ordered. Under the assumption of asymptotic completeness the incoming fields form a complete set and the GLZ formula (without a statement about its convergence status) is nothing more than a formal way of encoding all matrix elements of the field \( A(x) \) between l ket and k bra incoming states with \( n=l+k \). The coefficient functions \( a(p_1, \ldots, p_n) \) are mass shell restriction of retarded functions.

Even though there is no control about the convergence \(^{19}\), but at least superficially the formal lightfront restriction for each term in (10) does not seem to cause short distance divergences \(^{20}\). It is also clear that (as in the case of free fields) it is not possible to define a lightfront restriction in terms of vacuum expectations (Wightman functions). But within a mass shell representation as \(^{11}\) the limit can be formally taken term by term by setting \( x^- = 0 \) with the result

\[ A_{LF}(x_{LF}) = \sum \frac{1}{n!} \int_{H_m^{(2)}} \cdots \int_{H_m^{(2)}} a(p_{LF}^1, \ldots, p_{LF}^n) e^{i \sum p_{LF}^i x_{LF}} x_{LF} \times \]

\[ (11) \]

\[ \times : A_{in}(p_{LF}^1) \cdots A_{in}(p_{LF}^n) : dp_{LF} \cdots dp_{LF} \]

\[ x_{LF} = (x^+, x), \quad p_{LF} = (p^-, p), \quad dp_{LF} = \frac{dp_-}{2p_-} dp \]

where, just as in the GLZ representation before, the mass shell representation of the LF projection involves integrations over the positive and negative mass hyperboloid \( H_m^{(2)} \) (creation/annihilation part). Note that the coefficient functions \( a \) and the \( (\pm \text{ frequency}) \) creation/annihilation parts of \( A_{in}(p) \) in (11) remain

\(^{19}\)In contrast to the perturbative expansion which is known to diverge (even in the Borel sense), the convergence status of GLZ had not been settled. In \( d=1+1 \) factorizing models there are some indications in favor of convergence.

\(^{20}\)This is strictly speaking only true for \( d=1+1 \) theories which have no transverse degrees of freedom. For higher space-time dimension there are problems from composite fields (see below).
the same as in (10), only the parametrization of the mass shell is now specified in terms of $p^{LF}$. By construction the covariance of this field is given in terms of the mentioned 7-parametric subgroup, any other transformation would lead out of LF and therefore destroy the representation (11).

Superficially the LF projection seems to work but there is a hitch [29]. The holographic projection can only be defined on some but not on all composite fields from a class of local fields. The easiest way to see this problem is to apply the above projection formula to the lowest composite $A^2(x)$: (notation as before)

$$A^2(x)_{LF} = \frac{1}{(2\pi)^3} \int e^{i(p_1^{LF} + p_2^{LF}) \cdot x_{LF}} A(p_1^{LF}) A(p_2^{LF}) : dp_1^{LF} dp_2^{LF}$$  \hspace{1cm} (12)

The absence of any transverse damping (the constant $a(p_1^{LF}, p_2^{LF})$) brings about the appearance of the square of a meaningless transverse delta function $\delta(x)^2$ in the two-point function and the associated commutator. For the construction of the holographically projected algebras one does not need the existence of a LF projection for each individual composite field. But there is no theorem which secures the existence of even one field whose projection could serve as a LF generator.

The by far safest and most systematic, but unfortunately computationally least accessible approach to null-surface holography and its inversion is the algebraic method in the AQFT setting of nets of local algebras. This construction was indicated at the beginning of this section and more details can be found in [3][2]. As mentioned on a previous occasion the inversion of a holographic projection is highly nonunique. Knowing the local substructure on LF one can reconstruct all $\mathcal{A}(W)'s$ (without their local substructure!) with $H(W) \subset LF$, which falls short of the full local net. An attractive method to supply the missing LF external data for holographic inversion is the relative modular positioning which in the LF context amounts to knowing something about the relation between the algebras of several LFs ("GPS" in AQFT) [2].

The field generators whose holographic projection caused the problem of the ill-defined transverse part do not play any direct role. At the end of the day one may of course ask for generating fields of the holographic projection. There should be problem to construct these objects since a transverse extended chiral algebra is very similar to a chiral algebra and for the latter one knows how to extract generating fields from the net of operator algebras.

In algebraic terms, the absence of transverse vacuum fluctuations means that the global lightfront algebra tensor factorizes under “transverse subdivisions”:

$$\mathcal{A}_{LF}(\mathbb{R}^2 \times \mathbb{R}_+) \cong \mathcal{A}_{LF}(R \times \mathbb{R}_+) \otimes \mathcal{A}_{LF}(R' \times \mathbb{R}_+)$$  \hspace{1cm} (13)

$$\Omega = \Omega_R \otimes \Omega_{R'}$$  \hspace{1cm} (14)

where $R \subset \mathbb{R}^2$ and $R' = \mathbb{R}^2 \setminus R$, and this factorization is inherited by subalgebras associated with intervals $I \subset \mathbb{R}_+$ in the lightlike direction (see below). Although a detailed derivation of the localization structure on the horizon of the wedge requires a substantial use of theorems about modular inclusions and intersections
(for which we refer to [20][21]), the tensor factorization of the horizon algebra relies only on the following structural theorem in operator algebras:

**Theorem 1** (Takesaki [9]) Let \( (B, \Omega) \) be a von Neumann algebra with a cyclic and separating vector \( \Omega \) and \( \Delta_B \) its modular group. Let \( A \subset B \) be an inclusion of two von Neumann algebras such that the modular group \( Ad \Delta_B \) leaves \( A \) invariant. Then the modular objects of \( (B, \Omega) \) restrict to those of \( (A, \Omega) \) where \( e_A \) is the projection \( e_A H = \overline{\Omega A} \Omega \) as well as to those of \( (C e_A, \Omega) \) with \( C = A' \cap B \) the relative commutant of \( A \) in \( B \) and \( e C H = \overline{\Omega C} \Omega \). Furthermore the algebra \( A \vee C \) is unitarily equivalent to the tensor product \( A \otimes C \) in the tensor product Hilbert space.

In the application to lightfront holography we choose \( B = A(W) \equiv A_{LF}(\mathbb{R}^2 \times \mathbb{R}_+) \). Its modular group is the Lorentz boost \( \Lambda_W(−2\pi t) \) which in the holographic projection becomes a dilation. The dilation invariance of the algebra \( A = A_{LF}(A \times \mathbb{R}_+) \) is geometrically obvious and hence the prerequisite of the theorem concerning the modular group is met. The relative commutant is \( C = A_{LF}(R' \times \mathbb{R}_+) \). The lightlike nature of the subalgebra is absolutely crucial for the tensor factorization.

A particularly interesting situation is obtained for \( d=1+1 \) since in that case the holographic projection has no transverse part and hence every bulk field has a well defined chiral holographic projection and the conditions for an inverse holography appear especially favorable. In \( d=1+1 \) there exists a infinite family of so-called "factorizing models" [19] (in the 70s and 80s also referred to a "integrable QFTs"). They are distinguished by the presence of very simple generators of the wedge algebra \( A(W) \) which instead of the \( (\)commutation of creation/annihilation operators satisfy the so called Zamolodchikov-Faddeev (Z-F) commutation relations [26] which in the simplest case are of the form

\[
\tilde{Z}(\theta)\tilde{Z}^*(\theta') = S_2(\theta - \theta')\tilde{Z}^*(\theta')\tilde{Z}(\theta) + \delta(\theta - \theta') \tag{15}
\]

\[
\tilde{Z}(\theta)\tilde{Z}(\theta') = S_2(\theta' - \theta)\tilde{Z}(\theta')\tilde{Z}(\theta)
\]

\[
Z(x) = \frac{1}{\sqrt{2\pi}} \int (e^{ip(x)\theta} + h.c.)d\theta
\]

They deviate from the standard creation/annihilation operators by the appearance of the \( S_2 \) function which has the consequence that the \( Z(x) \) are not point-local, however they turn out to be still "wedge-local" [22][23]. They share with the standard creation/annihilation operators that they create vacuum polarization free states from the vacuum.

In that case the expansion of the interacting field in terms of the Z-F operators

\[
A(x) = \sum_n \frac{1}{n!} \int_{V_i} \cdots \int_{V_i} a(p_1, \ldots p_n) e^{i \sum p_k x} : \tilde{Z}(p_1) \ldots \tilde{Z}(p_n) : \frac{dp_1}{2p_{10}} \ldots \frac{dp_n}{2p_{n0}} \tag{16}
\]
The coefficient functions of this expansion have an important additional structural attribute: the crossing property [18]

\[ a(p_1, \ldots, p_n) = \langle 0 | A(0) | p_1, \ldots, p_n \rangle_{\text{in}} \]

\[ \langle 0 | A(0) | p_1, \ldots, p_n \rangle_{\text{in}} = \langle -p_{k+1}, \ldots, -p_n | A(0) | p_1, \ldots, p_n \rangle_{\text{c.o}} \]  

(17)

The various formfactors with different particle distributions between incoming and outgoing particles are related to one analytic master function which may be identified with the vacuum polarization component as in (17) where the c.o stands for the omission of contractions between in and out momenta. The factorizing models form an infinite family of nontrivial properly renormalizable theories whose existence can be, for the first time in the history of QFT, mathematically established [24]; this is also the first setting of interacting theories for which the constructive power of LF holography can be convincingly demonstrated.

But again, as in the case of the GLZ series, one has not been able to control the convergence of the formfactor series. The proof of their existence uses ideas from AQFT which avoid formfactor series as (16). There are however encouraging arguments that the holographically projected series can be summed. In the case of the massive Ising QFT, which is a well-known factorizing model, one can extract from its formfactor series another series representation for the two-point function of its holographic projection on the \( x_+ \)-horizon \( (x_- = 0) \). Of particular interest is an infinite series which represents the anomalous dimension of the order parameter which is known to be \( 1/16 \). The mere fact that the holographic projection reproduces this value is not surprising, but that this value results from summing a nontrivial infinite series is astonishing [22]. In other models for which one has series representations one also expects convergence of the series for anomalous dimensions and more generally for the series representing the holographic projection of (16).

The holographic method also sheds light on the conceptually mysterious but computationally successful Zamolodchikov proposal to consider factorizing models as resulting from suitably perturbing conformal models. In the Zamolodchikov setting the conformal model is viewed as the universality zero mass limit of a factorizing model. The concept of holography suggests to substitute the universality class limit by lightray holography in order to gain conceptual clarity. Where a zero mass universality class limit of a massive theory comes with a different Hilbert space, the holographic lightray projection of a massive 2-dimensional model lives in the same Hilbert space as the massive model and moreover their localization concepts are algebraically related. We expect that the usefulness of factorizing models as a theoretical laboratory for investigating in particular the idea of lightlike holography will continue for a long time to come.

\[ \text{[21] The short distance dimension of the order parameter in the massive Ising model is } 1/8 \text{ but only half of this value is seen in the holographic projection.} \]

\[ \text{[22] The calculation has been done in the critical limit which, although conceptually very different from the holographic projection, leads to the same anomalous dimensions. In contrast to the latter the critical limit yields a different theory in its own reconstructed Hilbert spaces.} \]
Factorizing models in d=1+1 also show an interesting structural phenomenon of holography with respect to the spin and statistics issue. Observable chiral fields can only come from Boson fields in the bulk with integer dimensions which are typically conserved currents or energy-stress tensors. Bosonic fields with anomalous dimensions (as most fields in factorizing models) pass to plektonic (nontrivial braid group representation) fields as a result of interlinking of spin, statistics and dimension in chiral theories. One expects of course that those plektonic chiral fields are precisely those which one obtains from the representation theory of the local observables.

For the remainder of this section some historical remarks are in order. The physical motivation for holography is not different from that of its predecessor the lightcone quantization, the main difference is the conceptual and mathematical precision coming from local quantum physics. Lightcone quantization was introduced during the 60s as a simplifying tool for exploring QFT in the nonperturbative regime especially for high energies. But it did not quite achieve what it was introduced for, partly because of old misunderstandings about the conceptual nature of QFT. As mentioned this already started with its name "lightcone quantization" which suggests a description in terms of a different theory whereas in reality it only should consist in a different spacetime view of the given (already quantized) quantum matter which focusses on certain physical aspects of interest at the expense of blanking out others. That holography was born on the umbilical cord of lightcone quantization as an attempt to overcome its conceptual misunderstandings becomes evident if one looks at the first papers in particular 't Hooft’s paper [10] where the presently used terminology appears for the first time.

There are no miracles in solving complicated physical problems as e.g. constructing models of interacting QFTs. The only available strategy is to chop the complicated problem into several simpler pieces. If there would be an isomorphism between the QFT of the bulk and that on its horizon, then holography could not be a constructive tool of QFT. Contrary to what one reads sometimes in papers, this is fortunately not the case: knowing only properties which are intrinsic to the LF (inverse holography is not intrinsic to LF!) it is not possible to reconstruct the quantum physics of the bulk. This is sometimes overlooked because the addition of e.g. the bulk mass to the massless LF generators is not perceived as non intrinsic to LF. If symmetries from LF into the bulk do not exist (e.g. a generic curved spacetime), one has to recover the bulk information, as mentioned before, from a "GPS system" of several holographic screens.

The theorists dream for such complex theories as QFT with infinite degrees of freedom is to decompose the original problem into a collection of simpler problems. Indeed, the QFT of extended chiral fields which appears after the holographic projection is much simpler than the bulk theory. But every simplification in QFT has its prize; in the present case there is no unique holographic inverse without invoking additional informations from outside the LF. In terms

\[\text{The idea behind GPS system of holographic screens is similar to the reconstruction of a full bulk QFT by the "modular positioning of a a rather small number of monads" [2].}\]
of degrees of freedom, the lightfront holography amounts to a thinning out of degrees of freedom. By adding the knowledge of how some Poincaré generators act on the holographic projection or (which amounts to the same), about the relative positioning of QFT on different "screens", one recovers the larger cardinality of degrees of freedom which is necessary to have a physically viable bulk theory. This would be very different for correspondences between bulk and time-like ("branes") subalgebras as the AdS-CFT correspondence \cite{27,18} with the CFT brane at infinity. In that case there is no adjustment of cardinality of degrees of freedom as in the holographic projection onto causal or event horizons; the cardinality stay the same which renders on side unphysical \cite{11}.

Although holography as an extension of the old lightcone quantization, it is foremost an instrument to make QFT more amenable to calculations and improve its conceptual understanding, there has been a lot of interest in making it also useful for problems of black-hole and cosmological horizons. It is interesting to look at these problems from a particle physics viewpoint. In the next subsection it will be shown that the Bondi-Metzner-Sachs symmetry is a consequence of the symmetry gain in the holographic projection.

1.2 The quantum origin of the Bondi-Metzner-Sachs symmetry

The holographic projection onto the lightfront inherits a 7-parametric subgroup from the 10 parametric Poincaré group of the bulk. As a result of the loss of transverse vacuum polarization and the fact that holography onto the horizon leads to a transverse extended chiral QFT there are infinitely many new symmetries. In addition to the Diff(S^1) invariance in lightlike direction, the compactification of the transverse plane (which is compatible with the quantum mechanical delta function) extends the infinity preserving E(2) symmetry to the SL(2,C) fractional action on the \(z, \bar{z}\) Riemann sphere with \(z=x+iy\). Both transformations together generate a very large symmetry group which contains in particular \(z, \bar{z}\) dependent \text{Diff}(S^1).

Even if one restricts to transformations which leave the vacuum invariant (proper symmetries) there are still infinite parameters since the parameters of the Moebius group can be functions of \(z, \bar{z}\) in such a way that a SL(2,C) transformation leads to a change of the Moebius parameters which is consistent with the composition law of the Moebius group. The full symmetry group of \(z, \bar{z}\) dependent lighlike diffeomorphisms is gigantic. As we will see the BMS group \cite{13} result from this vacuum preserving subgroup generated by \(z, \bar{z}\) dependent Moebius-transformations by imposing in addition the preservation of lightlike infinity which only leaves \(z, \bar{z}\) dependent transformation of the dilation-translation kind \(a + bx\).

In the following we want to argue that this is not an accidental consequence of our special choice of taking the horizon of a wedge as our null-surface, but that it holds as well for the holographic projection on the upper horizon of a double cone which is part of the mantle of a lower light cone shifted upward so that its lower end is \(t=0\) plane. The asymptotic situation envisaged by Penrose
results by moving the upper apex to timelike infinity in which the mantle
defines what one means by lightlike infinity in a Penrose sense.

In the case of conformal models one can try to compute generators for double-
cone holography by applying the appropriate conformal transformation to con-
vert the wedge into a double cone. The conformal map from the \( x_0 - x_3 \) wedge
\( W \) to the radius=1 double cone \( O_1 \) placed symmetrically around the origin is
\[
O_1 = \rho(W + \frac{1}{2}e_3) - e_3, \quad \rho(x) = -\frac{x}{x^2}
\]
\( W = \{(x_0, x_\perp, x_3) \mid x_3 > |x_0|, x_\perp \in \mathbb{R}^2\} \) (18)

with \( e_3 \) being the unit vector in the 3-direction. Restricted to the (upper)
horizon \( \partial W \) one obtains in terms of coordinates
\[
\partial O_1 \ni (\tau, \vec{e}(1 - \tau)), \quad \tau = \frac{t}{t + x_\perp^2 + \frac{1}{4}}, \quad \vec{e} = \frac{1}{x_\perp^2 + \frac{1}{4}}(x_\perp, \frac{1}{4} - x_\perp^2)
\]
where \( \partial W = \{(t, x_\perp, t) \mid t > 0, \, x_\perp \in \mathbb{R}^2\} \) (19)

If we use the unitary conformal transformation \( \mathcal{A}(W) \to \mathcal{A}(O_1) \) not only on
global generators for \( \partial \mathcal{A}(W) = \mathcal{A}(W) \) but also for their pointlike generating
fields \( A_{LF} \), we obtain the desired compact transverse proportionality factor
\( \sim \delta(\vec{e} - \vec{e}') \) replacing \( \delta(x_\perp - x_\perp') \) from the fact that the t-independent relation
between \( \vec{e} \) and \( x_\perp \) is that of a stereographic projection of \( S^2 \) to \( \mathbb{R}^2 \). The presence
of this factor corroborates the absence of vacuum polarization in the above
algebraic argument. The lightlike factor has the expected qualitative behavior
in terms of the variable \( \tau \) and the \( W \)-modular group \( t \to e^{\lambda t} \) passes to the \( O_1 \)
modular automorphism
\[
\tau \to \frac{-e^{-\lambda}(\tau + 1) + 1}{e^{-\lambda}(\tau + 1) + 1}
\]
(20)
The transverse additive group passes via inverse stereographic transformation
to the transverse rotational group.

The generators for \( \mathcal{A}(\partial O_1) \) are obtained by conformal transforming the light-
front generators (3). In order to notice that the full transverse symmetry is
6-parametric, we should realize that already before the transformation in the
lightfront setting the transverse quantum mechanics with the fluctuationless
vacuum state had a higher symmetry than just the 3-paramteric Euclidean
symmetry of a plane. For this purpose it is helpful to perform the stereographic
projection to the Riemann sphere. The latter has the 6-parametric \( SL(2,\mathbb{C}) \)
group as its symmetry group and this brings immediately to ones mind that
this is related to the fractional action of the (covering of the) Lorentz group on
the space of unit vectors (or lightlike directions). This action creates a conformal
factor which, as a result of the additional conformal factors arising from
the conformal covariant lightlike variable, can easily be compensated.

So no matter whether we study the holographic projection onto \( \partial W \) or
\( \partial O_1 \) we find the same symmetry acting on the transverse×lightlike coordinates
\( (z, \bar{z}) \times \mathbb{R}^3 \). The total dynamical symmetry groups is the infinite-parametric

\[28\]Here we pass to the cosmologically more costumary notation \( u \) instead of \( x_\perp \).
group of all $z, \bar{z}$ dependent diffeomorphisms on the line extended by $z, \bar{z}$ automorphism of the Riemann sphere whereas the proper (vacuum preserving) symmetry group is the $z, \bar{z}$ extended Moebius group of the circle. Here we are interested in the ax+b subgroup of the Moebius group which acts on the uncompactified lightray.

The $z, \bar{z}$ dependence leads to the Bondi-Metzner-Sachs group\footnote{There have been several attempts to relate the classical BMS group with quantum physics\cite{34}.}

$$u \to F_\Lambda(z, \bar{z})(u + b(z, \bar{z}))$$

$$(z, \bar{z}) \to U(\Lambda)(z, \bar{z}), \ U(\Lambda) \in SL(2, C)$$  \hspace{1cm} (21)

The group composition law $F_\Lambda(\Lambda(z, \bar{z}))F_\Lambda(z, \bar{z}) = F_{\Lambda \Lambda}(z, \bar{z})$ requires the multiplicative factor to be of the form

$$F_\Lambda(z, \bar{z}) = \frac{1 + |z|^2}{|az + b|^2 + |cz + d|^2}$$  \hspace{1cm} (22)

whereas the functions $b(z, \bar{z})$ are from a function space which is the closure of $C^\infty(z, \bar{z})$ functions on the Riemann sphere in some topology. The somewhat unexpected property is that the action of $SL(2, C)$ on the function space contains (in its linear part) the a copy of the semidirect product action of the Lorentz group on the translations i.e. the infinite dimensional BMS group contains the Poincaré group. For more informations especially on the position of the Poincaré inside the BMS group we refer to a comprehensive paper by Dappiaggi\cite{30}.

It is well known that historically this infinite dimensional group arose in general relativity as an asymptotic symmetry in asymptotically flat solutions of the Einstein Hilbert equations\cite{28}. In the present quantum context this group describes the vacuum preserving and infinity fixing part of a larger symmetry group which comes with the holographic projection onto horizons independent of the contest in which the "null-screen" arises. At first sight this is surprising because whereas one can envisage a nontrivial action of the Poincaré group on the asymptotic Penrose screen, it is not clear what this means in the case of the horizon of a finite double cone. What does the action on a causal horizon which is not left invariant by Poincaré transformation mean?

For this problem we use some properties of a so-called split inclusion of a double cone in a slightly bigger double cone $\mathcal{A}(\mathcal{D}(R)) \subset \mathcal{A}(\mathcal{D}(R + \Delta R))$ which will be the subject of a more detailed study in the last section. The important consequence of a split inclusion for the present problem concerning the action of a copy of the Poincaré group on the horizon of a double cone is that there exists a canonical factorization of the Hilbert space $H = H_1 \otimes H_2$ such that $\mathcal{A}(\mathcal{D}(R))$ acts on the factor $H_1$ and the causal disjoint of the bigger algebra $\mathcal{A}(\mathcal{D}(R + \Delta R))'$ (which is spatially separated from the smaller by a security distance $\Delta R$) acts on $H_2$. In this case it was shown that there exists in a canonically distinguished representation of the Poincaré group which locally (and for small group elements
so that the image stays inside $\mathcal{D}(R)$) acts on operators in $\mathcal{D}(R)$ as the original representation. This Poincare group acts in $H_1$ i.e. on the algebra $B(H_1)$ $\supset A(\mathcal{D}(R))$ where the localization of $B(H_1)$ in $\mathcal{D}(R + \Delta R)$ is sharp in $\mathcal{D}(R)$ and "fuzzy" in the surrounding sheet of size $\Delta R$. With respect to this only partially physical representation of the Poincare group the boundary horizon $\partial \mathcal{D}(R + \Delta R)$ behaves like a Penrose screen and the universal appearance of the BMS symmetry in holographic projections on null-surfaces looses some of its mystery. In other words the split inclusion of two double cones as above only describes the physical world inside the smaller double cone. Beyond its boundaries the split Poincare group acts mathematically correct as if the ring region and its boundary defined by the mantle of the larger double cone would be the infinite remainder of the world and its Penrose infinity. But this is an artificiality of the split construction since the action of the split Poincare group only coalesces with that of its split version inside the smaller double cone; the outside region up to the $R + \Delta R$ boundary is a fake part of the split universe on which, different from the region inside $R$, the full Poincaré group acts nongeometrically \[8\]. So at least the element of surprise of encountering a BMS-Penrose situation already on a null-surface in finite spacetime has been removed; the BMS group is a subgroup of the very big symmetry group which emerges after holographically projecting matter onto null horizons\[26\]. The physical relation with the classical BMS work is limited to the infinitely far Penrose horizon.

The universal validity of the BMS symmetry on horizons raises the question whether holography on finite lightlike surfaces as $\mathcal{D}(R)$, which are null-surfaces but not lightfronts, continues to hold for massive i.e. not conformal theories. It is well-known that a free massive QFT cannot have a geometric acting modular group, the action inside the double cone must be "fuzzy". In the previous section we argued that this results from the geometric modular action on the horizon because the massive propagation from a horizon into the bulk is "reverberating".

This problem has a classical analog: the propagation of characteristic data on a null-surface into the bulk. The relevant formula (6) which can be found in a recent paper \[29\] works for the classical and for the interaction free quantum case.

### 1.3 Split property and entropic area law

As well known, the Unruh effect \[14\] can be viewed as a thermal manifestation of the causal localization which underlies QFT but is absent in QM. The Hamiltonian is not the usual one associated with time translation in a Minkowski inertial system, but rather (up to a multiple which depends on the acceleration of the observer) the wedge-preserving Lorentz boost generator $K$ in a Rindler world. The vacuum restricted to the causally closed wedge region is a KMS state at the modular temperature $2\pi$ associated to $K$.

26Since holography is a process in which only the localization of quantum matter is changed whereas the Hilbert space is maintained, the infinite parametric holographic groups act also on the bulk, but this action is extremely fuzzy (non geometric) and therefore not of physical interest.
It is believed that this is not just a mathematical discovery with no other purpose then to highlight some unusual conceptual and structural aspects of QFT \[1\], but rather represents an in principal observational (but in practice not directly accessible) effect, which an appropriately uniformly accelerated thermal radiation counter will actually register. Its spacetime basis is the fact that the uniform acceleration of a particle counter causally confines the latter to a wedge-shaped spacetime region and at least subjectively converts the original global relativistic "world time" into the "Rindler time".

Whereas the observer’s time on each of the uniformly acceleration orbits inside a Rindler world is a concept of classical general relativity, the thermal aspect comes from the modular localization properties of QFT. The Unruh effect associated to the Rindler time on the different uniform acceleration orbits different temperatures in a way which maintains the well known (imaginary) time-temperature relation in passing from the abstract modular time/temperature to that of the Unruh Gedankenexperiment.

Trading an inertial system with a uniformly accelerated one entails changing a positive energy Hamiltonian (with the vacuum being the bottom state) with a boost Hamiltonian whose energy spectrum is two-sided. In this way the vacuum becomes a thermal state in which the zero energy only refers to a mean value in the middle. So what appears a small change in the spacetime situation has a conceptually large repercussion on the local quantum level. In the end it leads to a change in the association between the Hamiltonian and the measurement hardware as well as a change in the perception of the global vacuum state.

Although these radical conceptual changes lead to numerical modifications which remain way below observational accessibility, they are unavoidable consequences of QFT, a theory which has remained the most successful and comprehensive description of our material nature. Therefore they require the utmost conceptual and philosophical attention. Historically the thermal manifestation of localization has been first observed in curved spacetime QFT in the presence of event horizons, which in contrast to the fleeting causal horizons in flat spacetime, have an observer-independent status defined in terms of intrinsic properties of spacetime.

The Unruh situation is a special case of a more general setting of "modular localization" \[22\][31][32][33] which describes the position of the dense subspace in terms of domains of unbounded operators $S$. These domains are determined by in terms of the unitary representation of the Poincaré group but for knowing the operator itself one needs to know dynamical aspects of a QFT which leads to those localized states. The full $S$-operator is defined in the algebraic setting of QFT as

$$S_{\mathcal{O}}A\Omega = A^*\Omega, \ A \in \mathcal{A}(\mathcal{O})$$ (23)

where $\mathcal{A}(\mathcal{O})$ is the operator algebra localized in $\mathcal{O}$ and the state is (in most applications of QFT) the vacuum state. Although there is no operator which

\[27\]Without loss of generality we assume that the localization regions are causally closed i.e. $\mathcal{O} = \mathcal{O}''$ where one upper dash denotes the causal disjoint.
maps all bounded operators of the algebra of all operators into the adjoint, for certain kind of operator algebras which includes the spacetime localized algebras of QFT there does exist an unique unbounded such operator. The existence of an uniquely defined "Tomita" operator $S$ in QFT is guarantied for a large class of states including the vacuum $\Omega$. The necessary and sufficient condition is that the operator algebra $A(O)$ acts on the state $\Omega$ in a cyclic and separating way (or shorter that $(A(O), \Omega)$ is in "standard position"). Unless specified otherwise $\Omega$ in the sequel denotes the vacuum.

It turns out that this unbounded closed antilinear and involutive operator encodes the causal completion $O'$ in its domain, the change of localization region is precisely mirrored in the change of $domS_O$ in Hilbert space. $S$ has a polar decomposition

$$ S = J \Delta^{\frac{1}{2}} $$

where the modular group $Ad\Delta^{it}$ is an object of a "kinematical" rather than dynamical nature whereas the anti-unitary $J$ is "dynamic" since it depends on the scattering matrix (see below).

Another remarkable property following from its definition (involutive on its domain $S^2 \subset 1$) is its domain "transparent" in the sense of $ranS = domS = dom\Delta^{\frac{1}{2}}$. Hence the dynamics consists in a re-shuffling of vectors inside $domS$. It turns out that the global vacuum $\Omega$, defined as the lowest state in a theory with positivity of the (global) energy, after restriction to the subalgebra $A(O)$ becomes a thermal KMS state with respect the modular Hamiltonian $\Delta^{it} = e^{-itK}$

What is less known is that in this wedge situation the modular reflection $J_W$ has the following dynamic content

$$ J_W = J_0 S_{scat} $$

where $J_0$ is the TCP-related modular reflection of a free field (say the incoming free field in scattering theory) and $S_{scat}$ is the S-matrix. Note that $S_{scat}$ is defined in terms of large time scattering limits; its appearance as a relative (between interacting and free) modular invariant is surprising and has powerful consequences of which some will be mentioned later). Although the domain of the Tomita $S$-operator unlike for wedge-localized algebras allows no direct characterization in terms of the Poincaré group for subwedge regions, these domains can be build up from intersections of $S_{\text{edge}}$ domains. The dynamic content of subwedge reflections $J_O$ is however not known.

The general modular situation is more abstract than its illustration in the context of the Unruh Gedankenexperiment since the generic modular Hamiltonian is not associated with any spacetime diffeomorphism; it describes a "fuzzy"

---

28 In the case of $O = W$ the $\Delta^{it}$ is (up to a scaling factor) the $W$-preserving Lorentz boost. In a system of particles obeying the mass gap hypothesis one conventionally regards the particle spectrum "kinematical" (given) and considers as dynamical only those properties which depend on the interaction between those particles.

29 This observation of the author was obtained by rewriting the TCP covariance of the S-matrix in an asymptotically complete QFT and can be found in [23] (and earlier references therein).
movement which only respects the causal boundaries but is somewhat nonlocal inside. In this case the existence of such a Hamiltonian is nevertheless of structural value since it allows to give a mathematically precise quantum physical description of the locally restricted vacuum as a KMS state associated with the intrinsically determined modular Hamiltonian. As argued in the previous section there are strong indications that the holographic projections onto horizons will convert the fuzzy acting modular Hamiltonians associated with causally closed bulk subregions into geometric acting "surface Hamiltonians" [22] which is represented by a the generator of a dilation. In this way the possible loss of certain bulk symmetries is more than compensated for by the gain of infinitely many new symmetries after the projection.

None of the above properties holds in QM where the only localization is the probabilistic Born localization for which the space of $O-$localized wave functions at a fixed time is described by a projector $P_O$ which results from the spectral resolution of the position operator. In that case the vacuum simply factorizes, so that Born localization does not lead to a new entangled state; in particular any kind of entanglement from inside/outside localization factorization can never be of a thermal kind unless the global state was already thermal from the beginning. It can be shown that this factorization continues to hold for the ground states of nonrelativistic finite density zero temperature matter.

Wigner tried to adapt the Born localization to the relativistic realm and realized to his dismay$^{30}$ that this probability aspect is inconsistent with covariance $^{35}$ and reference dependent. The covariant modular localization, which underlies the formalism of relativistic QFT, deals with dense subspaces which cannot be described by projectors. Nevertheless the Born-Newton-Wigner localization plays a crucial role in scattering theory a fact which results from the fortunate circumstance that the correlation between asymptotically time-like separated Born-Newton-Wigner (BNW) localized events is covariant which leads to the consistency between covariance and the probability concept on the level of the S-matrix and the scattering cross-section.

The fundamental difference between BNW and modular localization is reflected in a radically different nature of local algebras $P_OB(H)P_O = B(P_OH)$ in QM and $A(O)$ in QFT. Localization in QM always ends up with the algebra of all bounded operators of a smaller Hilbert space, more precisely on a factor space$^{31}$ $B(H) = B(P_OH) \otimes B((1 - P_O)H)$ which corresponds to the spatial decomposition $H = P_OH \otimes (1 - P_O)H$.

Whereas the total algebra in QFT is still of the form $B(H)$, localized operator algebras in QFT are of hyperfinite type $\text{III}_1$ factor algebra in the classification of Connes, which constitutes a refinement of the original classification by Murray and von Neumann. For the sake of brevity (and also to avoid a shock and awe effect with the reader) we will call this algebra a monad, implying with

$^{30}$As a result of what he considered as a serious flaw, Wigner maintained a critical distance towards QFT in the later part of his life.

$^{31}$In order to facilitate the comparison with QFT we take the Fock space formulation of QM.
this notation that all localized $\mathcal{A}(\mathcal{O})$ in QFT\textsuperscript{32} are isomorphic copies of the monad which in turn is not isomorphic to the quantum mechanical algebras from bipartite splits done with Born localization. Again the monad $\mathcal{A}(\mathcal{O})$ commutes with causal disjoint (which happens to be equal to its commutant) $\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})'$ and both algebras span $\mathcal{B}(\mathcal{H}) = \mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O}')$ but this generation of the full algebra from its commuting parts cannot be brought into the form of a tensor product. In fact the whole conceptual framework of QM breaks down: the reduction of a pure state on $\mathcal{B}(\mathcal{H})$ gives an impure state which is not described in terms of a density matrix, and with the absence of the tensor factorization for the bipartite partition of $\mathcal{B}(\mathcal{H})$ the rug is pulled out from under the setting of entanglement.

The standard method ignores these structural properties and proceeds with a formal "as if" calculation based on a quantum mechanical tensor factorization. The unavoidable ultraviolet divergencies are interpreted as a shortcoming if not inconsistency of QFT which needs a high energy modification coming from a future quantum theory of gravity \textsuperscript{36}.

The view taken in the sequel of this paper is more prosaic namely that the divergence is not a shortcoming of QFT but rather the manifestation of the radically different nature of local monad algebras which do not tensor factorize under causal bipartite subdivision so that the prerequisite of the entanglement-setting for quantum information theory is violated. The split property permits to approximate a causally localized monad by a sequence of quantum mechanical algebras whose localization region is slightly larger by a distance $\Delta R$ which surrounds the localization region (a lightlike "sheet" of thickness $\Delta R$ instead of a horizon) and serves as a kind of attenuation distance for the otherwise infinite vacuum fluctuations at the sharp boundary horizon. This split approximation permits a quantitative description of the leading behavior in $\Delta R \to 0$. Of particular interest is the limiting behavior of the entropy, which, as will be argued below is a logarithmically corrected area law in which the sheet distance $\Delta R$ enters in such a way as to lead to a dimensionless result.

Although the split property leads to a very different kind of factorization as that resulting from spatial bipartite factorization\textsuperscript{33}, it comes with a saving grace which allows to recover at least some aspects of the quantum mechanical formalism; in particular a tensor factorization for localized algebras which, as in the case under consideration are separated by a finite spacelike distance. The trick is to approximate the desired region by a sequence of regions with "fuzzy" boundaries and to realize that this process only leads to vacuum polarization \textit{within} the fuzzy boundary. The observables in this localization-caused thermal behavior depend on the thickness $\Delta R$ of the boundary and similar to the entropy of heat bath systems, which can only be explicitly computed in the thermody-

\textsuperscript{32}The time slice property which holds in all physically relevant theories states that the algebra of a region is identical to that of the causally closed region. Therefore there is no loss of generality from assuming that all regions are causally complete.

\textsuperscript{33}In QM the ground state factorizes whereas the "split vacuum" (or finite energy particle states), as a result of vacuum polarization, is always a Gibbs-like thermal state with an infinite number of particles.
namic limit, one expects that the localization entropy at best be computed for \( \Delta R \to 0 \). We will now illustrate this split setting in the special context which will be of interest in the subsequent derivation of localization entropy.

For the following computation of localization entropy we will use the notation of the previous section. Hence let \( \mathcal{O} = \mathcal{D}(R) \) be the double cone which results from the causal completion of a ball of radius \( R \) around the origin and consider a slightly bigger concentric ball with associated double cone \( \mathcal{D}(R + \Delta R) \). Then the inclusion \( \{ \mathcal{A}(\mathcal{D}(R)) \subset \mathcal{A}(\mathcal{D}(R + \Delta R)), \Omega \} \) is called standard if \( \{ \mathcal{A}(\mathcal{D}(R)), \Omega \} \) and \( \{ \mathcal{A}(\mathcal{D}(R + \Delta R)), \Omega \} \) and \( \{ \mathcal{A}(\mathcal{D}(R + \Delta R)) \cap \mathcal{A}(\mathcal{D}(R))', \Omega \} \) are standard. For standard inclusions one defines the split property as the existence of an intermediate quantum mechanical type I algebra \( \mathcal{N} \) i.e.

\[
\mathcal{A}(\mathcal{D}(R)) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{D}(R + \Delta R))
\]

\[
\mathcal{A}(\text{ring}) \equiv \mathcal{A}(\mathcal{D}(R))' \cap \mathcal{A}(\mathcal{D}(R + \Delta R)),
\]

\[
\mathcal{N} = \mathcal{A}(\mathcal{D}(R)) \vee J_{\text{ring}} \mathcal{A}(\mathcal{D}(R)) J_{\text{ring}}
\]

If the standard inclusion is split, there are infinitely many intermediate type I algebras and among those there is a "canonical" one (for which we maintain the same name\(^{34}\)) which is uniquely determined by the modular data and given by the formula in the third line \[37\]. The tensor factorization \( B(H) = \mathcal{N} \otimes \mathcal{N}', \quad H = P_{\mathcal{N}} H \otimes (1 - P_{\mathcal{N}}) H = H_1 \otimes H_2 \) where \( P_{\mathcal{N}} \) is the projection onto the subspace \( \mathcal{N} \Omega \) together with the inclusions gives the desired tensor split

\[
\mathcal{A}(\mathcal{D}(R)) \vee \mathcal{A}(\mathcal{D}(R + \Delta R))' \simeq \mathcal{A}(\mathcal{D}(R)) \otimes \mathcal{A}(\mathcal{D}(R + \Delta R))'
\]

i.e. the statement that the operator algebra generated by the smaller and the commutant of the larger is isomorphic to their tensor product. But in contrast to the quantum mechanical factorization, the vacuum state does not factor but rather is highly entangled and leads upon reduction to the factor algebra \( \mathcal{N} \) to a thermal Gibbs state associated with Hamiltonian determined by the modular data of the split situation \[37\].

There is a useful analogy between the "funnel" limit \( \Delta R \to 0 \) in the thermal setting of local algebras of QFT and the thermodynamic limit \( V \to \infty \) in the setting ofQM. At this point it is important to be reminded of the fact that although the Born localized algebras of ground state QM are always type I and this continues to be the case for box-quantized Gibbs systems, the thermodynamic limits of Gibbs systems are hyperfinite type III\(_1\) algebras on which the Gibbs state changed into a more singular KMS state which cannot be described in terms of a density matrix. In fact the radical change of the type I algebra to a type III\(_1\) monad algebra in the thermodynamic limit is directly related to the volume divergence.

In both cases one approaches a monad in a KMS state by a sequence of Gibbs states on quantum mechanical type I algebras. The main difference is

\(^{34}\)The formula for the canonical \( \mathcal{N} \) by itself does not by itself secure the type I factor property; it only distinguishes a canonical split, if the inclusion is split to begin with.
one in physical interpretation; in one case one approximates a KMS state on
a global monad which is interpreted as a global algebra by an increasing se-
quence of type I algebras, whereas in the other case the approximating type I
sequence is shrinking for $\Delta R \to 0$ towards the monad which is interpreted as
$A(D(R))$. This analogy suggests to expect that the divergences in both cases
become identical after an appropriate reparametrization which takes care of the
different geometric aspects. In the rest of this section we will collect supporting
arguments for the following statement:

**Statement** Global heat bath systems in the thermodynamic limit and local
split systems in the limit of vanishing split distance have the same divergent
entropy factors after replacing the dimensionless volume by the logarithmically
modified dimensionless area

$$V_{n-1} (kT)^{n-1} |_{T=T_{\text{mod}}} \simeq \left( \frac{R}{\Delta R} \right)^{n-2} \ln \left( \frac{R}{\Delta R} \right)$$

(28)

where $V_{n-1}$ is the standard dimensionful volume factor which is made dimen-
sionless with the Boltzmann factor and on the right hand side appears the already
mentioned dimensionless area factor. The modifies area behavior should be seen
as a "lightlike volume factor" with the transverse directions giving the area factor
and the lightlike direction (of the light-like sheet of thickness $\Delta R$) contributing
the logarithmically parametrized missing length factor.

Here the left hand side is the dimensionless (as a result of the $kT$ fac-
tors) "volume" divergence which appears in the thermodynamic limit of an
n-dimensional QFT. The right hand side represents the divergence factors in the
in the split limit $\Delta R \to 0$. This relation for $n=4$ implies in particular that the
localization entropy follows an logarithmically corrected area behavior.

One may view this conjecture as resulting from an "inverse Unruh eff ect" i.e.
the idea that a thermodynamic limit state on a global operator algebra can be viewed as a localized subsystem of a larger system for which a global pure state
(e.g. a global vacuum) has been restricted to the subsystem. 

The necessity to fix the heat bath temperature at a particular value is no surprise in view of the ex-
istence of the Hawking temperature. For $n=2$ and conformal invariant theories
one can show that the two systems are isomorphic with the unitary corresponding
to a conformal transformation [3]. For higher dimensions the presence of the logarithmic factor expresses the contribution from the lightlike direction in
a light sheet. In any case the idea that heat bath thermality belongs to QFT
and the entropical area law is exclusively part of the totally different setting,
namely a still unknown QFT, seems to me untenable. Localization entropy is
without doubt a notion in QFT and the only role of curvature is to convert
observer-dependent "fleeting" causal horizons into intrinsically positioned event
horizons.

The support for the statement comes from three different directions.

1. The singularity of the entropy for $\Delta R \to 0$ and that of a dimensionless
"partial charge" in a similar geometric situation are consequences of the
same mechanism of vacuum polarization.
2. Both situations, that of a thermodynamic open system and that of a modular localized operator algebra are mathematically identical, namely hyperfinite type III\textsubscript{1} operator algebras in a KMS state, only the physical parametrization in terms of approximating quantum mechanical type I algebras is different.

3. For \( n=2 \) the relation \([25]\) expresses the inverse Unruh effect in which the heat bath length factor passes to the logarithmic short distance behavior from localization on a lightlike line. This is a consequence of the existence of a conformal transformation connecting the two systems.

As an historical interlude it is interesting to mention that Heisenberg’s discovery of vacuum polarization led to a behavior which is very similar the one in the conjecture. He found that if one integrates the zero component of the conserved current of a charged free field over a finite spatial region of radius \( R \), the so defined ”partial charge” diverges, which brings us to the first point. In the modern QFT setting it is possible to control the strength of such a divergence in terms specially prepared test functions. A finite partial charge with a vacuum polarization cloud within a ring of thickness \( \Delta R \) is is defined in terms of the following dimensionless operator

\[
Q(f_R, \Delta R, g_T) = \int j_0(x, t)f_R, \Delta R(x)g_T(t)dxdt
\]

\[
\|Q(f_R, \Delta R, g_T)\| = F(R, \Delta R)
\]

where the spatial smearing is in terms of a test function which is equal to one inside a sphere of radius \( R \) and zero outside \( R + \Delta R \) with a smooth transition in between and \( g_T \) is a finite support \([-T, T]\) interpolation of the delta function. As a result of charge conservation such expressions converge for \( R \to \infty \) to the global charge either weakly \([11]\) or (of one relates \( T \) with \( R \) appropriately) even strongly on a dense set of states \([38]\).

For an estimate of the vacuum polarization one would like to study the limit of \( \Delta R \to 0 \) for fixed \( R \) of \( F(R, \Delta R) \) \([29]\). As expected, the computation for the chiral conformal case shows a logarithmically divergent partial charge whereas for additional spatial dimension a factor of the dimensionless area \( \frac{\text{area}}{\Delta R^{\frac{1}{2}}} \) appears (note that the dimension of area increases with dimensions). The two-point function calculation of the smeared zero component of the current produces the naively expected result, apart from a logarithmic factor. Note that the smearing in timelike (or lightlike) direction in addition to the spatial \( f_R, \Delta R \) smearing is essential in order to obtain a finite operator, without it there would be no logarithmic divergent contribution. The rotational symmetry as well as the dimensionless nature of a partial charge would suggest a behavior as the right side of \([28]\). This time the calculation can be explicitly done in terms of two-point functions of a concrete operator.

The localization entropy on the other hand is not a computable property of a specific dimensionless testfunction-smeared operator-valued distribution as the
partial charge, but rather a manifestation of the vacuum polarization cloud residing in the lightlike sheet around the horizon $\partial A(D)$ of algebra $A(D)$. What links both cases is the universality of vacuum polarization caused by localization independent of whether the localization occurs through testfunction smearing leading to dimensionless operators or through localization of an entire algebra which takes one to the dimensionless entropy. Such analogies are however no replacement for computations, especially in cases in which the concepts entering the computation are more interesting than the result itself.

The assumption of conformal invariance simplifies the computation and confirms the area behavior which one could directly have obtained by a dimensional argument. The dimensionless logarithm (which is the only vacuum polarization contribution in two-dimensional theories) escapes such arguments and therefore the chiral case requires an explicit calculation which will be done directly for the inverse chiral Unruh effect in the later part of this section.

An analog behavior of $F$ to (28) may be taken as an indication that the vacuum polarization leads to a universal divergent behavior if the attenuation length of the vacuum polarization cloud is made to shrink $\Delta R \to 0$, independent of whether the vacuum polarization is caused by the algebraic split property for localized algebras or whether it is coming from individual operators whose formal limit describes (dimensionless) partial charges.

A mathematical proof that the localization property for a finite split localization entropy which in the $\Delta R \to 0$ limit behaves as claimed (28) amounts to a control of the density matrix $\rho_{\text{split}} \sim e^{-K_{\text{split}}}$ and the entropy is the von Neumann entropy of this Gibbs state (there is a corresponding density matrix on the commutant $\mathcal{N}'$ which leads to the same entropy). In the present state of QFT technology this is a hopeless task. As will be shown in the sequel there are special circumstances related to the knowledge about chiral theories which allow to do this for $n=2$.

The important aspect of the above split inclusion of two double cones is that the vacuum looks only different from what it was before the split in a ringlike region whose associated algebra is the relative commutant of the smaller within that localized in the bigger double cone. $\mathcal{N}$ is the canonically associated type I algebra in terms of which there is tensor factorization as in (27). The relative commutant in the second line is of special interest since geometrically it describes the finite shell region (or rather its causal completion) in which we expect the vacuum polarization to be localized. The restriction of the vacuum to $\mathcal{N}$ is a density matrix state $\rho_{\text{split}}$ since the algebra is quantum mechanical; In principle one could compute the entropy exactly on the basis of the modular data for $\mathcal{N}$ but in practice this is (as in the analog case of the thermodynamic limit) in the present state of knowledge about modular theory only possible in the funnel limit $\Delta R \to 0$.

The split tensor factorization leads to a notion of entanglement which is still distinctively different from the information theoretical entanglement which

\[35\text{Since the vacuum polarization in both cases of the fluctuation of the partial charge and of the entropy of the localized bulk matter is caused by the same localization mechanism, the identical behavior for } \Delta R \to 0 \text{ should be of no surprise.} \]
one encounters in QM [2]. A bipartite tensor factorization associated with the quantum mechanical Born localization creates (upon restriction to one tensor factor) a density matrix which is not of the thermal kind unless the global state was thermal to begin with. In other words quantum mechanical spatial bipartite partitions create information theoretic entanglement but generally do not lead to thermal KMS properties for the restricted states. Thermal manifestations of localization and hence of bipartite splitting is only possible in QFT; physically because one needs localization caused vacuum polarization and mathematically since the (sharply) localized algebras are monads whose properties are radically different from quantum mechanical type I factor algebras (monads have no density matrix states) even though they can be approximated by type I algebras.

In no way does modular localization and splitting create a real temperature which is associated with the physical Hamiltonian and the physical time. But there are zillions of other "Hamiltonians" within the same QFT model, i.e. Hermitian operators with respect to which the vacuum is not a state at the lower end of a one-sided spectrum, but for which the spectrum is two-sided. Modular theory selects a particular Hamiltonian with two-sided spectrum and what lends physical importance to this otherwise mathematical construction is the fact that the selection is inexorably coupled to localization which is the most important (and most subtle) property of particle physics. The modular groups associated with such Hamiltonians are automorphisms of the localized algebras which map of the bulk matter in a geometrically fuzzy way maintaining however the causal boundaries (horizons); that they represent a diffeomorphism is the exception and happens for massive theories in Minkowski spacetime only in the case of the Rindler wedge leading to the Unruh Gankenexperiment.

Even though effects related to modular localization theory will probably never be directly observational accessible (since they are orders of magnitudes smaller than quantum mechanical entanglement problems historically related to Schroedinger’s cat the violation of Bell’s inequality,....), their importance cannot be overestimated if it comes to the problem of nonperturbative classification and constructions of interacting models of QFT. A more detailed discussion of this point will be given in the next section.

A much more solid situation, in which horizons are defined by the system and not by the observer, results from event horizons in curved space time. The best known and historically first example of such a horizon is the Schwarzschild solution. In fact the thermal aspects of localization have been first observed by Hawking [7] while performing calculations on free scalar fields in the Schwarzschild metric.

In the present work we will avoid curved spacetime because it is our intention to convince the reader that many conjectured properties which arose in curved spacetime QFT and for which the presence of gravity was thought to be essential are in fact preempted by more abstract mathematical and conceptual properties in flat spacetime QFT. The basic difference between the flat and the curved spacetime situation is that the modular Hamiltonian associated with a thermal description of localized quantum matter bears no relation to the physical Hamiltonian associated with a time translation in an the inertial system, whereas in
case of event horizons as in the Schwarzschild spacetime the modular Hamiltonian of \((\mathcal{A}(\mathcal{O}_{\text{out}}), \Omega_{\text{H.H.}})\) is identical to that related to the timelike Killing symmetry; here \(\mathcal{O}_{\text{out}}\) denotes the part outside the black hole and \(\Omega_{\text{H.H.}}\) is the Hartle-Hawking state on the Kruskal extension of the Schwarzschild spacetime \(^{[39]}\).

We now return to the issue of localization entropy in QFT in the context of the before mentioned standard split inclusion of double cone.

Let us start with the case of a two-dimensional conformal QFT in which case the double cone is a two-dimensional spacetime region consisting of the forward and backward causal shadow of a spatial line segment of length \(R\) at \(t = 0\) sitting inside region obtained by extending the baseline on both sides by \(\Delta R\). As a result of the assumed conformal invariance of the theory, the canonical split algebra inherits the covariances, and hence the entropy of the canonical split algebra can only be a function of the cross ratio of the 4 points characterizing the split inclusion

\[
S = -\text{tr} \rho \ln \rho = f \left( \frac{(d-a)(c-b)}{(b-a)(d-c)} \right) \tag{30}
\]

with \(a < b < c < d = -R - \Delta R < -R < R < R + \Delta R\)

where for conceptual clarity we wrote the formula for the conformal invariant ratio in case of generic position of 4 points. For chiral theories the dependence of the entropy on the cross ratio of 4 points on the lightray expresses the fact that the entropy is a conformal invariant. In comparison with higher dimensions one does not need this generality, the symmetric case written in the last line \(^{[30]}\) is all we need. Our main interest is to determine the leading behavior of \(f\) in the limit \(\Delta R \to 0\) which is the analog of the thermodynamic limit \(V \to \infty\) for heat bath thermal systems.

The asymptotic estimate for \(\Delta R \to 0\) can be carried out with an algebraic version of the replica trick\(^{[36]}\) which uses the cyclic orbifold construction in \(^{[41]}\). First we write the entropy in the form

\[
S = -\frac{d}{dn} \text{tr} \rho^n |_{n=1}, \rho \in \mathcal{M}_{\text{can}} \subset \mathcal{A}(R + \Delta R) \tag{31}
\]

Then one uses again the split property, this time to map the \(n\)-fold tensor product of \(\mathcal{A}(L + \Delta L)\) into the algebra of the line (conveniently done in the compact \(S^1\)) with the help of the \(n^{th}\) root function \(\sqrt[n]{\cdot}\). The part which is invariant under the cyclic permutation of the \(n\) tensor factors defines the algebraic version \(^{[41]}\) of the replica trick. The transformation properties under Möbius group are

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\(^{[36]}\) The replica trick is well-known in mathematical work on solid state problems, including the calculation of entropy \(^{[40]}\).
now given in terms of the following subgroup of \( \text{DiffS}^1 \) written formally as

\[
\sqrt{\frac{\alpha z^n + \beta}{\beta z^n + \alpha}}, \quad L'_{\pm n} = \frac{1}{n} L_{\pm n}, \quad L'_0 = L_0 + \frac{n^2 - 1}{24n} c
\]  

(32)

\[
\dim_{\text{min}} = \frac{n^2 - 1}{24n} c
\]

where the first line is the natural embedding of the n-fold covering of Moeb in \( \text{DiffS}^1 \) and the corresponding formula for the generators in terms of the Virasoro generators. As a consequence the minimal \( L'_0 \) value (spin, anomalous dimension) is the one in the second line. With this additional information coming from representation theory we are able to determine at least the singular behavior of \( f \) for coalescing points \( b \to a, \ d \to c \):

\[
S_{\text{sing}} = -\lim_{n \to 1} \frac{d}{dn} \left[ \frac{(d-a)(c-b)}{(b-a)(d-c)} \right]^{\frac{n^2 - 1}{24n}} = \frac{c}{12} \ln \frac{(d-a)(c-b)}{(b-a)(d-c)}
\]  

(33)

Since the function is only defined at integer \( n \), one needs to invoke Carlson’s theorem.

The resulting entropy formula in the singular limit reads

\[
S_{\text{sing}} = \frac{c}{12} \ln \frac{(d-a)(c-b)}{(b-a)(d-c)} = \frac{c}{12} n \frac{R(R + \Delta R)}{(\Lambda R)^2}
\]  

(34)

where \( c \) in typical cases is the Virasoro constant (which appears also in the chiral holographic lightray projection).

This result was previously \[3\] obtained through establishing the validity of the "inverse Unruh effect" for chiral theories. This is a theorem stating that for a conformal QFT on a line, the KMS state obtained by restricting the vacuum to the algebra of an interval is unitarily equivalent to a global heat bath temperature state at a certain (geometry-dependent) value of the temperature \[42\] \[43\]. The transformation turns out to be a conformal transformation \[23\] which carries the \( L \) (one-dimensional volume) divergence into a \( \ln \varepsilon^{-1} \) factor with \( \varepsilon \sim e^{-L} \) and hence describes the same leading logarithmic divergence as the more detailed argument using the replica trick.

The existence of the inverse Unruh effect in chiral theories is an explicit demonstration that the above logarithmic divergence is nothing but the conventional volume (here length) factor conformal transformed into an exponential transformation of the affine length into the scaling group parametrization. This corresponds to a direct transformation of the large distance thermodynamic limit to a "funnel limit" in which a sequence of quantum mechanical type I algebras from splitting converge towards the monad algebra of the smaller region for shrinking split distance \[37\].

Although the inverse Unruh effect is apparently restricted to chiral theories, the intriguing analogy of the heat bath entropy with the localization entropy

37The funnel approximation can also be made from the inside.
continues to exert itself. Below it will be argued that the localization entropy in the n-dimensional case diverges for $\Delta R \to 0$, with $\Delta R$ the splitting distance, as

$$E^{\Delta R \to 0} = C(n) \frac{R^{n-2}}{(\Delta R)^{n-2}} c dt \frac{R^2}{(\Delta R)^2}$$

(35)

where we left out all constants except the constant $c$ which has the interpretation of a parameter corresponding to the holographically projected matter. It is the only recollection on the bulk quantum matter of this otherwise universal asymptotic behavior.

Compared to the chiral models, which can be controlled quite elegantly with the replica method, the question of higher dimensional localization entropy looks more involved. Neither the replica method is applicable nor a higher dimensional inverse conformal Unruh effect seems to be available. For conformal theories one uses dimensional arguments. The rotation symmetry together with the dimensionless of the entropy requires to multiply the two dimensional entropy with the dimensionless factor which generalizes the dimensionless area factor $(\frac{R}{\Delta R})^2$ of 4-dim QFTs.

But what about the remaining quantum matter parts? The inverse Unruh theorem for $n=2$ [3] shows that this is the case for chiral theories after we equate the heat bath temperature $kT_{h.b.}$ with the modular temperature $T_{mod} = 2\pi$ of the localized system. It should be possible to check this for higher dimensional conformal free fields, this would remove the last veil of mystery between localization and heat bath thermal aspects in QFT. The validation of such a universality would amount to a significant step in the understanding of local quantum physics i.e. in the QFT beyond the Lagrangian setting. It would support the idea that the approximation of a monad by tensor factor algebras is, after a ”kinematical” adjustment for the different spacetime situation[38] as in (28). The clarification about the presence/absence of this universality would be important for ideas on the interrelation of thermodynamics, geometry and gravitation and support ideas proposed by Jacobson [44]. Without such a clarification there is not much chance for making headway for finding the interface between QFT in CST and QG.

The important aspect of the split inclusion of two double cones is that the vacuum looks only different from what it was before the split in a ring-like region whose associated algebra is the relative commutant of the smaller within the bigger double cone. The relative commutant in the second line is of special interest since geometrically it describes the finite shell region (or rather its causal completion) in which we expect the vacuum polarization to be localized in that ring. The restriction of the vacuum to $\mathcal{N}$ is a density matrix state $\rho_{split}$ on $\mathcal{N}$ the subalgebra $\mathcal{A}(\mathcal{D}(R)) \subset \mathcal{N}$ is indistinguishable from the vacuum expectation values so that only if tested with operators in the ring region the ”split vacuum”

[38] In the heat bath case the monad is indexed by the entire Minkowski spacetime but the thermal representation is unitarily inequivalent to the vacuum representation. In the other case the spacetime indexing is a causally complete subregion and the state is the reduced vacuum.
differs from the original vacuum. The split entropy is the von Neumann entropy of the mixed state $\rho_{\text{split}}$; as in the inside/outside tensor-factorization in QM the density matrix of the opposite factor $\mathcal{N}'$ leads to the same entropy.

In principle one could compute the entropy exactly on the basis of the modular data for $\mathcal{N}'$ but practically this is (as in the analog case of the thermodynamic limit) only possible in the funnel limit $\Delta R \to 0$. The logarithmic factor is the camouflaged third length factor which hides the complete analogy of the area law with the thermodynamic volume factor. For the derivation of the formula for the canonical $\mathcal{N}'$ see [37].

The resulting formula (35) has a clear derivation in the conformal case because besides the length $R$ which determines the hypersurface "area" $R^{n-2}$ the only other dimension carrying parameter is $\Delta R$ so that the entropy is given by (34) with a kinematical proportionality factor $C(n)$ which depends on the spacetime dimension but unlike $c$ is independent on the quantum matter. It is believed that massive matter does not change the leading behavior for $\Delta R \to 0$.

The derivation of (35) based on the split inclusion is preceded by arguments using functional integral representations and momentum space cutoffs. They naturally inherit all the conceptual problems of the use of functional integrals in QFT [39]. On top of this they mask the fact that the spatial bipartite division does not lead to a factorization by the necessity of cutting off momentum space integrals in order to avoid infinities. Momentum space cutoffs in QFT have severe problems of their own. First there is no argument that a Euclidean cutoff functional integral is still associated with a quantum theory and second one cannot be sure whether the divergence only indicates an avoidable inappropriate argument which leads to metaphoric intermediate steps to a intrinsically consistent and correct result (example renormalization theory) or whether behind the divergence there is a universal structural limiting behavior of a physical quantity, as is the case for the split entropy law (35). The localization entropy only depends on geometric (localization) data, it is independent numerical factors which relate the modular Hamiltonian with that of an observer (e.g. the Unruh acceleration).

Perhaps the oldest entropy calculation for a bipartite situation in QFT is that in [36] which was done in the aftermath of Bekenstein's conjecture. Apart from the logarithmic factor the result (35) confirms this formal calculation [36].

The computation starts from the (as we know now) incorrect assumption of a splitless tensor factorization (as it is possible in QM but not in QFT [2]) and pays the prize in form of a momentum space divergence which, as customary in many textbook QFT, is dumped into a momentum space cutoff. The area dependence

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39 Whereas functional integrals have a solid mathematical status for standard problems of QM, they are limited to (Euclidean) free field actions and to fields whose short distance properties are not worse than those of free fields (superrenormalizable interactions). They are in particular not valid for fields with anomalous short distance dimension which includes all factorising (integrable) QFTs.
(without the logarithmic factor) is then inferred by dimensional reasoning from the cutoff dependence. The nature of the localized vacuum polarization near the horizon remains somewhat hidden and the interpretation of the entropy in terms of a momentum space cutoff goes into the wrong physical direction. One would be hard pressed to conclude from such computations that the vacuum polarizations which cause this phenomenon are localized near the horizon and that the momentum space cutoff is related to the size of the polarization sheet.

But whereas a momentum space cutoff limits the validity of the theory and modifies it in an uncontrollable way, the split method shows that although the localization entropy of sharply localized quantum matter is really infinite, there is no reason to modify the model by a momentum space cutoff\(^\text{40}\) permits an intrinsic approximation without modifying the model. The splitting procedure is a manipulation on the given (unsplit) system which, just as the holographic projection keeps the Hilbert space. Both methods are consistent with the local covariance principle (a fact which is worth mentioning because certain methods for estimating energy densities of cosmological reference states are not).

2 Concluding remarks, outlook

In this note we analyzed the quantum aspects of two problems which had their origin in classical general relativity, namely the BMS symmetry in asymptotically flat curved spacetime theories, and the Bekenstein area law for event horizons in classical geometric field theories of the Einstein-Hilbert type.

In the first case we found that the symmetry gain in holographic projections leads to an infinite symmetry group whose vacuum and infinity preserving part is the BMS group. This infinite parametric group contains the Poincaré group, but only if the nullsurface is that of conformal infinity in the sense of Penrose is the Poincaré group the physical one. The difficulty is to understand the interpretation of the Poincaré subgroup of BMS in case that the null-surface is not the Penrose infinite lightlike boundary. This somewhat paradoxical situation was interpreted by observing that the split situation creates a fictitious continuation of the smaller algebra into the ring-like slice region in which the larger boundary appears as an infinitely remote lightlike Penrose ”screen”. We emphasized that the gain in infinite dimensional symmetry through holographic projection is inexorably related to the thinning out of degrees of freedom and the impossibility to reconstruct the bulk from only the intrinsic properties of the projection.

QFT leads to a logarithmically corrected area law in which the logarithmic factor is the only singular factor in case of chiral theories on a light-line. The chiral inverse Unruh effect explains this factor as resulting from a conformal transformation applied to the length factor (i.e. the one-dimensional volume factor). This observation brings the volume proportional heat bath entropy and the logarithmically modified dimensionless area law into a much closer relation

\(^{40}\)The idea in favor of a finite black hole entropy cannot be based on arguments in \(^{36}\) since localization entropy can be consistently defined without momentum space cutoffs.
than hitherto imagined. The area aspect is the only one shared with the Bekenstein formula; whereas QFT produces a one-parametric family of $\Delta R$-dependent entropies, the Bekenstein formula achieves the dimensionlessness of entropy with the help of dimensionful constants from classical gravity. has no such parameter. Whereas the Hawking radiation is fully explained in terms of QFT in a Schwarzschild CST, the Bekenstein formula can only be reconciled with QFT if instead of ordinary quantum matter it refers to an unknown non-localizable gravitational form of quantum matter.

In both cases the principles of local quantum physics led to unexpected properties which were previously overlooked and which, at least in the case of the area proportionality of entropy near a horizon, add new aspects to the ideas around the Bekenstein entropy and its possible connection with the still illusive quantum gravity. It is important to have a good understanding of horizons and entropy caused by vacuum polarization near horizons first within the setting of QFT before embarking on the more ambitious program of quantum gravity.

Fundamental to all problems addressed in this article is the principle of causal localization, whose intrinsic mathematical formalization is the theory of modular localization and the mathematical (Tomita-Takesaki) modular theory of operator algebras. This is the best way to take care of the holistic aspects of local quantum physics.

Finally it may be helpful to remind the reader that the harbinger of the QM-QFT contrast with respect to localization [2] and the entailing dichotomy between information-theoretical and thermal entanglement is the influential observation made in 1964 by Haag and Swieca [45][11] that there exists a fundamental difference between the phase space densities in QM and QFT; thus destroying once and for all the once popular idea that QFT may be viewed as some kind of relativistic QM. Whereas, as everybody learns in a course on QM, a finite cell in phase space only accommodates a finite number of degrees of freedom, the phase space occupation in QFT is infinite, albeit a quite ”tame infinity”, namely the phase space densities form compact sets, later refined to nuclear sets [46]. This increase of degrees of freedom is a characteristic property of QFT, and although the aim of Haag and Swieca to understand the asymptotic completeness property of scattering theory in terms of phasespace degrees of freedom cardinality remains open up to date, the enormous fertility of this idea in the structural understanding of QFT is outside of any doubt.

Models with transnuclear phase space behavior may still mathematically exist as local theories, but they have a series of pathological properties which make particle physicists dismiss them as unphysical. In particular their thermal behavior has either a limiting temperature (Hagedorn temperature) or thermal states do not exist. Parallel to the thermal changes there is a change in the causal shadow picture in that the causal dependency region contains many more degrees of freedom than those which got there by propagation.

It is easy to provide illustrations: the AdS-CFT correspondence leads from a standard AdS QFT to an overpopulated CFT. For example a free field on $\text{AdS}_5$ leads to a generalized conformal free field on the compactified $M_4$ with a Kallen-Lehmann spectral function which fulfills a power law in the invariant
mass \cite{27}. It is comforting to know that this never happens in the holographic projection of localized bulk matter on its horizon. Writing the LF fields in terms of their intrinsic degrees of freedom on which only a 7-parametric subgroup of the Poincare group acts, there are lesser degrees of freedom on LF than in the bulk. This is a blessing because it makes holography a powerful constructive tool of QFT.

Although from a mathematical viewpoint there is no big difference between causal and event horizons, there are severe additional conceptual problems in passing from QFT to QFT in CST. One such problem is the question of what reference state to take, since there is no distinguished replacement for the vacuum state in generic CST. Since the holographic projection leads to a much simpler transverse extended chiral theory, it may be more natural to discuss the problem first in the holographic projection with its much simpler state structure and than to extend this state to the bulk. Such a procedure was proposed and illustrated in \cite{47}.

It would be desirable and add credibility to the ongoing discussions about the still elusive QG, if they would take place with the full knowledge of the holistic structure of QFT which is in sharp contrast to (relativistic) QM \cite{41}. Holography on horizons, the absence of tensor factorization under splitless causal bipartite dissection, the possibility to characterize a full QFT including the action of the Poincaré symmetry from the relative positioning of a finite number of monads and, last not least, the local covariance principle \cite{42}, all these properties illustrate the holistic aspects of QFT which have no counterpart in QM. As stressed by Hollands and Wald \cite{49} these aspects become important in applications of QFT to cosmology e.g. in estimates of energy densities in cosmic reference states which replace the Minkowski vacuum; simply adding up energy modes as in QM violates this holistic property of QFT.

References

[1] J. J. Bisognano and E. H. Wichmann, J. Math. Phys. 17, (1976) 30

[2] B. Schroer, Localization and the interface between quantum mechanics, quantum field theory and quantum gravity I (the two antagonistic localizations and their asymptotic compatibility) to appear in SHPMP, arXiv:0912.2874 Localization and the interface between quantum mechanics, quantum field theory and quantum gravity II (The search of the interface between QFT and QG), to appear in SHPMP, arXiv:0912.2886

[3] B. Schroer, Class. Quant. Grav. 24, (2007) 4239, and preceding articles by the same author

\textsuperscript{41}A relativistic QM or direct "particle interaction theory" (DPI) is a relativistic QM (no vacuum polarization) which fulfills all properties which one can formulate in terms of particles without using interpolating fields (invariant S-matrix, cluster properties) \cite{2}.

\textsuperscript{42}The implementation of this principle requires to study all theories on different but isometric manifolds simultaneously in order to construct a QFT on a particular QFT \cite{48}.
[4] G. L. Sewell, Ann. Phys. **141**, (1982) 201
[5] S. J. Summers and R. Verch, Lett. Mat. Phys. **37**, (1996) 145
[6] D. Guido, R. Longo, J. E. Roberts and R. Verch, Rev. Math. Phys. **13**, (2001) 125
[7] S. W. Hawking, Commun. Math. Phys. **43**, (1975) 19
[8] R. Haag, *Local Quantum Physics*, Springer Verlag 1996
[9] M. Takesaki, *Tomita’s theory of modular Hilbert algebras and its applications*, Lecture Notes in Mathematics Vol. **128**, Springer-Verlag, Berlin, Heidelberg and New York
[10] G. ’t Hooft, *Dimensional reduction in quantum gravity*, arXiv:gr-qc/9310026
[11] B. Schroer, *Jorge A. Swieca’s contributions to quantum field theory in the 60s and 70s and their relevance in present research*, to be published in EPJH - *Historical Perspectives on Contemporary Physics*, arXiv:0712.0371
[12] R. Bousso, Rev. of Mod. Phys. **74**, (2002) 825
[13] R. Sachs, Phys. Rev. **128**, (1962) 2851
[14] W. G. Unruh, Phys. Rev. **D14**, (1976) 870
[15] W. Driessler, Acta Phys. Austr. **46**, (1977) 63
[16] R. Brunetti, K. Fredenhagen and R. Verch, Commun. Math. Phys. **237**, (2003) 31
[17] V. Glaser, H. Lehmann and W. Zimmermann, Nuovo Cimento **6**, (1957) 1122
[18] B. Schroer, *A critical look at 50 years particle theory from the perspective of the crossing property*, arXiv:0905.4006
[19] H. Babujian, M. Karowski, *The "Bootstrap Program" for Integrable Quantum Field Theories in 1+1 Dim*, arXiv:hep-th/0110261
[20] B. Schroer, J. Phys. A **35** (2002) 9165, B. Schroer, Int. J. Math. Phys. A **18**, (2003) 1671
[21] B. Schroer, Int.J.Mod.Phys. **A19S2** (2004) 348
[22] B. Schroer, Nucl. Phys. **B499**, (1997) 547
[23] B. Schroer, Ann. Phys. **275**, (1999) 190
[24] G. Lechner, Commun.Math.Phys. **277**, (2008) 821
[25] H. Babujian and M. Karowski, Int. J. Mod. Phys. **A19S2**, (2004) 34
[26] A. B. Zamolodchikov and Al. Zamolodchikov, Ann. Phys. **120**, (1979) 253
[27] M. Duetsch and K.-H. Rehren, Henri Poincare 4, (2003) 613, arXiv:math-ph/0209035
[28] R. M. Wald, General Relativity, the University of Chicago Press 1984
[29] M. Bischoff, D. Meise, K.-H. Rehren and I. Wagner, *Conformal Quantum Field Theory in various Dimensions*, in preparation
[30] C. Dappiaggi, *Free field theory at null infinity and white noise calculus: a BMS invariant dynamical system*, arXiv:math-ph/0607055
[31] R. Brunetti, D. Guido and R. Longo, Rev. Math. Phys. **14**, (2002) 759
[32] L. Fassarella and B. Schroer, J. Phys. A **35**, (2002) 9123
[33] J. Mund, B. Schroer and J. Yngvason, Commun. Math. Phys. **268**, (2006) 621
[34] C. Dappiaggi, Phys. Lett. **B615**, (2005) 291
[35] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, (1949) 400
[36] L. Bombelli, R. K. Kaul, J. Lee and R. Sorkin, Phys. Rev. D **34**, (1986) 373
[37] S. Doplicher and R. Longo, Invent. Math. **75**, (1984) 493
[38] M. Requardt, Commun. Math. Phys. **50**, (1976) 256
[39] J. Hartle and S.W. Hawking, Phys. Rev. **D28** (1983) 2960
[40] J. Cardy, *Entanglement Entropy in Extended Quantum Systems*, arXiv:0708.2978
[41] R. Longo and F. Xu, Commun.Math.Phys. 251 (2004) 321
[42] H.-J. Borchers and J. Yngvason, J.Math.Phys. 40, (1999) 601
[43] B. Schroer and H.-W. Wiesbrock, Rev.Math.Phys. 12 (2000) 461, arXiv:hep-th/9901031
[44] T. Jacobson, Phys.Rev.Lett. **75** (1995) 1260
[45] R. Haag and J. A. Swieca, Commun. Math. Phys. **1**, (1965) 308
[46] D. Buchholz and E. H. Wichmann, Commun. math. Phys. **106**, (1986) 321
[47] C. Dappiaggi, V. Moretti and N. Pinamonte, Rev.Math.Phys.**18**, (2006) 349
[48] R. Brunetti, K Fredenhagen and R. Verch, Commun.Math.Phys. 237, (2003) 31

[49] S. Hollands and R. E. Wald, Gen.Rel.Grav. 36, (2004) 2595