Damage Constitutive Model of Loaded Orthotropic Rock under Freeze-thaw Cycles

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Abstract. In this paper the orthotropic damage constitutive model of loaded rock in cold environment is established according to the orthotropic characteristics of the one in freeze-thaw cycles, based on the strength of rock microelements subject to Weibull random distribution, and the macroscopic statistical damage model of rock. This model could reflect the environment of natural rock occurrence more truthfully, and revealed the law of rock damage evolution under freeze-thaw environment. The research results would have important theoretical and practical significance for the evaluation of safety and stability of engineering construction in cold area.

1. Introduction
With the rapid development of water conservancy project, transportation and mining, the study of mechanical properties and failure mechanism of rock materials has been paid great attention by academic circles and engineering circles. In large-scale basic engineering construction and resource exploitation in cold area, the influence of severe climate and environmental factors is often encountered, especially the freeze-thaw damage caused by seasonal turnover and day and night cycle poses a great threat to rock body engineering [1, 2]. Therefore, the study of rock physical and mechanical properties under freeze-thaw cycles has become one of the most concerned problems in engineering field.

In the past few decades, many scholars have carried out some research on rock freeze-thaw damage. Matsuoka [3] carried out a large number of laboratory tests to study the process of freeze-thaw failure of the three kinds of major rock in semi-immersion environment; Mu [4] analyzed the mechanism of rock freezing-thawing failure by means of microscopic scanning by electron microscope. Park [5] studied the relationship between thermal and physical parameters and temperature of typical Korean granite and sandstone through experiments. Huseyin [6] tested the physical and mechanical properties of andesite, such as hard wave velocity and compressive strength, after being subjected to thermal shock cycle and freeze-thaw cycles, and obtained the deterioration law of andesite. Xu [7], Zhang [8] studied the mechanical properties and damage deterioration mechanism of rocks under freeze-thaw conditions. Zhang [9, 10] established the damage model of loaded rock under freeze-thaw cycles, fully considered the non-uniformity of the microstructure of rock. Jia [11] analyzed the freeze-thaw damage of sandstone under different freezing conditions and its dominant mechanism. Jihwan [12] studied the physical and mechanical properties of frozen and melted basalt, diorite and tuff, and analyzed the changes of rock microstructure during freeze-thaw cycles by SEM technique and CT scan. Nicholson [13] studied the relationship between the deterioration grade and lithology of freeze-thaw damage in 10 kinds of rocks under cyclic freeze-thaw conditions. Zhou Coping [14] carried out an experimental study on the physical characteristics of weathered granite by freeze-thaw cycles.
Most of the widely accepted rock and soil failure criteria in rock mechanics, such as the Mohr-Coulomb criterion and the Drucker-Prager criterion, as well as the rock damage constitutive model proposed by most scholars, are based on isotropic assumptions and are only applicable to homogeneous isotropic bodies. The actual geological body has extremely complex mechanical properties. Rocks in the diagenesis process and the crustal movement after diagenesis, can form layers, joints, faults and other structures, and the different sizes and combinations of mineral particles in the rock have resulted in the directional arrangement of structural structures in the rock, these directional arrangements make the rock present anisotropy, such as the ubiquitous shale, sandstone, granite in nature, marble and so on have obvious anisotropy. Li [15] confirmed that the anisotropy of rock mechanical properties exists objectively, and obtains a difference of 24.4% between the maximum and minimum values of elastic modulus in different directions; there is a difference of 35.9% between the maximum and minimum values of Poisson's ratio, and the difference between the maximum and minimum values of uniaxial compressive value is about 23% through experimental research. Chao [16] revealed that the intensity of sandstone was the lowest when it was 30° intersection the angle of the principal stress axis and the weak surface is obtained. Deng [17] studied the anisotropic mechanical properties of layered sandstone, and the results showed that the anisotropic characteristics of layered sandstone were obvious under the condition of uniaxial and triaxial compression, the laminar angle increased from 0° to 90°, and the elastic modulus increased gradually, while the deformation modulus, compressive strength, adhesion force and friction angle decreased first and then increased, and showed U-type distribution.

In geotechnical engineering, the anisotropy of rock is usually divided into transversely isotropic and orthogonal anisotropy, and red sandstone belongs to orthotropic rock. Taking red sandstone as the object, this paper studied the orthotropic damage constitutive model of fractured rock in freeze-thaw environment, and analyzed the damage expansion characteristics of rock, and the research results would have important theoretical and practical significance for the evaluation of safety and stability of engineering construction in cold area.

2. Isotropic Loaded Rock Damage Constitutive Model under Freeze-thaw Cycles

From the point of view of damage, the microscopic cracks and pores randomly distributed inside the rock can be regarded as a kind of damage field, which is affected by freeze-thaw cycles and load, and the microscopic defects continue to expand, showing the deterioration of the macroscopic mechanical properties of the rock, and when the change of the microscopic structure reaches a certain degree, the material is destroyed. Under the action of load, rock grains produce slip and staggered, and the behavior of material progressive damage caused by the continuous accumulation of micro-defects can be expressed by damage parameter D. Based on the inhomogeneity of rock microstructure [18], the distribution of mechanical properties of the internal elements of the material is probabilistic, and the statistical distribution characteristics of rock strength are expressed by Weibull function distribution, and according to stress-strain relation[19], the damage evolution equation of the loaded rock can be

\[ D = \int_0^\varepsilon \varphi(x) \, dx = 1 - e^{-\left(\frac{x - \varepsilon}{m}\right)^m} \]  \hspace{1cm} (1)

where \( \varphi(\varepsilon) \) is the damage rate of primitive bodies of loaded rocks, \( \varepsilon \) is the strain, \( \varepsilon_i \) is the maximum strain, \( m = 1/\ln \left(\frac{E_0\varepsilon_i}{\sigma_i}\right) \) is the rock uniformity coefficient, \( \sigma_i \) is the maximum stress.

In the literature [9], the concepts of freeze-thaw damage, load damage and total damage are put forward, and freeze-thaw cycles and load have different mechanical mechanisms to promote the initiation and expansion of cracks in rocks, thus showing different damage characteristics. After the rock undergoes the freeze-thaw cycles, it is equivalent to the first stage loading, then the loaded rock under the freeze-thaw can be equivalent to two-stage loading. According to the principle of strain equivalence after popularization in the literature [20], the reference damage state of rock is treated as
the first damage state, and the state after freeze-thaw damage is used as the second damage state, and the damage variable is defined as

\[ D_n = \frac{A_0 - A_n}{A_0} \]  

(2)

Where \( A_0 \) and \( A_n \) are the effective bearing area of reference damage state and freeze-thaw damage state respectively, \( D_n \) is the freeze-thaw damage variable, the subscript \( n \) is a freeze-thaw cycles coefficient. It is assumed that the rock material is isotropic, that is, the damage variable is isotropic.

According to the concept of macroscopic image damage mechanics, the response of rock macroscopic physical properties can represent the degree of deterioration within the material. The elastic modulus of the material is more convenient to be analyzed and measured during the freeze-thaw cycles, and the rock freeze-thaw damage variable can be expressed by Eq. (2) as

\[ D_n = 1 - \frac{E_n}{E_0} \]  

(3)

Where, \( E_0 \) and \( E_n \) are isotropic elastic modulus of initial damage state and freeze-thaw damage state respectively.

The total damage variable of loaded rock under freeze-thaw cycles is

\[ D_n = D + D_n - DD_n \]  

(4)

Where \( D_n \) is the total damage variable of loaded rocks under freeze-thaw cycles.

Thus, the total damage evolution equation is obtained

\[ D_m = 1 - \frac{E_n}{E_0} e^{-\frac{1}{m(n)}} \]  

(5)

The damage constitutive relationship of the isotropic loaded rocks under different freeze-thaw cycles is

\[ \sigma = E_0 \left(1 - D_m\right) \varepsilon = E_0 e^{-\frac{1}{m(n)}} \varepsilon \]  

(6)

3. Establishment of Damage Constitutive Model of Anisotropic Loaded Rock under freeze-thaw Cycles

3.1. Generalized Hooker’s Law of Orthotropic Body

For isotropic materials, the normal stress only produces normal strain, the shear stress component only produces the corresponding shear strain component. Unlike isotropic materials, the normal stress of anisotropic materials not only produces normal strain, but also produces shear strain. Similarly, shear stress not only produces shear strain, but also produces normal strain; In addition to producing the shear strain component corresponding to it, the shear stress component also produces other shear strain components. The coupling effect is determined by the physical properties of anisotropic materials.

In general, the constitutive equations of elastic anisotropic material can be written as the following matrix expressions.
Where \( [C]_{6\times6} \) is the 6-order matrix composed of 36 material constants.

If there are three elastic symmetrical surfaces perpendicular to each other at each point of the uniform elastomer, the elastic properties are the same in the symmetrical direction on both sides of each symmetrical surface, but the elastic properties are not the same in these three directions, and objects with this elastic properties are called orthotropic bodies. In general, sedimentary rocks and metamorphic rocks, due to the sedimentary conditions during deposition and the optimal orientation of mineral crystal parts during metamorphism, make the physical and mechanical properties of such rocks show orthogonal anisotropy, which belongs to orthotropic bodies.

Stress components and strain components or their increments are represented in the material principal axis coordinate system of orthotropic materials. The stress component is not coupled with the strain component, and its elastic stress-strain relationship is determined by generalized Hooker’s law.

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

(7)

Where, \( \varepsilon_x, \varepsilon_y, \cdots, \gamma_{yz} \) and \( \sigma_x, \sigma_y, \cdots \) are the strain and the stress components in the principal direction of the material. \( E_x, E_y, \cdots, G_{yz} \) are the elastic modulus in the principal direction of the material and the corresponding shear modulus respectively.

According to the symmetry of the elastic coefficient matrix are:

\[
E_x\mu_{yz} = E_y\mu_{zx}, E_x\mu_{xy} = E_y\mu_{xy}, E_x\mu_{xz} = E_y\mu_{zx}
\]

(9)

The stress-strain relationship that can be expressed by flexibility

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{E_x} & \frac{-\mu_{yz}}{E_y} & \frac{-\mu_{zx}}{E_z} & 0 & 0 & 0 \\
\frac{-\mu_{yz}}{E_y} & \frac{1}{E_y} & \frac{-\mu_{zy}}{E_z} & 0 & 0 & 0 \\
\frac{-\mu_{zx}}{E_z} & \frac{-\mu_{zy}}{E_z} & \frac{1}{E_z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{zy}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zx}}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

(8)

Where

\[
\Delta = \frac{1}{E_x E_y E_z} \begin{bmatrix}
1 & -\mu_{yx} & -\mu_{zx} \\
-\mu_{xy} & 1 & -\mu_{zy} \\
-\mu_{xz} & -\mu_{yz} & 1
\end{bmatrix}
\]
The stress and strain are converted from the elastic principal coordinate system to any coordinate system, and there are

\[
\{\sigma^\prime\} = [D^\prime][\varepsilon^\prime]
\]

(11)

where \([L] = [D][L]^T\)

\[
\begin{bmatrix}
I_1^2 & I_2^2 & I_3^2 & 2I_1I_2 & 2I_1I_3 & 2I_2I_3 \\
 m_1^2 & m_2^2 & m_3^2 & 2m_1m_2 & 2m_1m_3 & 2m_2m_3 \\
 n_1^2 & n_2^2 & n_3^2 & 2n_1n_2 & 2n_1n_3 & 2n_2n_3 \\
 l_1n_1 & l_2n_1 & l_3n_1 & l_1m_1 + l_2n_1 & l_1m_2 + l_3n_1 & l_1m_3 + l_2n_1 \\
m_1n_1 & m_2n_2 & m_3n_3 & m_1n_2 + m_2n_1 & m_1n_3 + m_2n_2 & m_2n_3 + m_1n_1 \\
m_1l_1 & m_2l_2 & m_3l_3 & n_1l_1 + n_2l_1 & n_1l_2 + n_2l_2 & n_1l_3 + n_2l_3 \\
\end{bmatrix}
\]

Where, \(l_i\), \(m_i\), \(n_i\) are the cosine of the angle between the elastic principal axes and any coordinate system \(i = 1, 2, 3\).

3.2. Establishment of the Constitutive Model of the Damage of Anisotropic Loaded Rock under Freeze-thaw Cycles

Due to the complexity along the inelastic principal axis, only three independent damage variables are considered for freeze-thaw damage variables along the elastic principal direction, assuming that the rock damage occurs in the direction of any principal axis, the other 2 principal directions do not consider damage. The main variables of orthotropic damage are set, which are the damage variables along the material principal direction \(x, y, z\), which are defined as

\[
D_{s,i} = \frac{A_{s,i} - A_{s,i}}{A_{0,i}} (i = 1, 2, 3)
\]

(12)

Where \(A_{0,1}, A_{0,2}\) and \(A_{0,3}\) are the cross-sectional area in the material principal direction \(x, y, z\), respectively. \(A_{s,i}\) is the apparent cross-sectional area corresponding to the damage.

According to the concept of macroscopic phenomenological damage mechanics, the response of rock macroscopic physical properties can represent the degree of deterioration within the material. The elastic modulus of the material is more convenient to be analyzed and measured during the freeze-thaw cycles, and the rock freeze-thaw damage variable can be expressed by Eq (12) as

\[
D_{s,i} = 1 - \frac{E_{s,i}}{E_{0,i}} (i = 1, 2, 3, 4, 5, 6)
\]

(13)

where \(E_{0,1}, E_{0,2}, E_{0,3}\) are the elastic modulus along elastic principal direction \(x, y, z\) in the initial damage state respectively, \(E_{s,4}, E_{s,5}, E_{s,6}\) are the shear modulus in the plane \(yz, xz, xy\) in the initial damage state, respectively, \(E_{s,1}, E_{s,2}, E_{s,3}\) are the elastic modulus along elastic principal direction \(x, y, z\) respectively in the freeze-thaw damage state, \(E_{s,4}, E_{s,5}, E_{s,6}\) are the shear modulus in the plane \(yz, xz, xy\) respectively in the freeze-thaw damage state.

Assuming that the damage along the direction of a certain elastic principal does not affect the damage of the other two principal directions, the total damage variables of loaded rock under freeze-thaw cycles are deduced according to the generalized strain equivalence hypothesis in the literature [20].

\[
D_{m,i} = D + D_{s,i} - DD_{s,i} (i = 1, 2, 3, 4, 5, 6)
\]

(14)
Where $D_{nj} (i=1,2,3)$ is the total damage variable of loaded rock along the $x$, $y$, $z$ direction under freeze-thaw cycles. $D_{nj} (i=4,5,6)$ is the total damage variable of rock in the plane $yz$, $xz$, $xy$.

Therefore, the damage constitutive relationship of the loaded orthotropic rock under different freeze-thaw cycles is obtained by substituting Eq. (6) into Eq. (12).

\[
\begin{align*}
\sigma_x &= 1 - \mu_{yx} - \mu_{zx} \\
\sigma_y &= 1 - \mu_{xy} - \mu_{zy} \\
\sigma_z &= 1 - \mu_{xz} - \mu_{yz} \\
\tau_{yz} &= E_{n4} - E_{n5} - E_{n6} \\
\tau_{xz} &= E_{n4} - E_{n5} - E_{n6} \\
\tau_{xy} &= E_{n4} - E_{n5} - E_{n6}
\end{align*}
\]

(15)

Where, $\Delta = \frac{1}{E_{n1}E_{n2}E_{n3}}$

For the stress-strain relationship in the direction of the inelastic principal axes has a more complex form, the specific expression can be derived from the Eq (11), which is abbreviated here. It can be seen that in the direction of the inelastic principal, even the orthotropic rock shows the general anisotropic properties, and there is a coupling effect between the shearing strain and the normal stress, as well as the shearing stress and the linear strain.

4. Examples

The paper cited the test results in the literature [21]. The red sandstone was processed along the sedimentary direction into a cylindrical specimen with a diameter of 50 mm and a height of 100 mm, and the friction angle was measured as 36°, and the conventional triaxial compression mechanical properties of the red sandstone were tested in the non-freeze-thaw state and after freeze-thaw state. The displacement is applied along the sedimentary direction, which is carried out in the confining pressure 2, 4 and 6 MPa respectively, and the mechanical parameters of red sandstone under different freeze-thaw cycles and confining pressure are obtained, as shown in Table 1.

| Freeze-thaw cycles | Ultimate stress /MPa | Ultimate strain | Elastic modulus /GPa | Poisson’s ratio |
|-------------------|----------------------|----------------|---------------------|----------------|
|                   | 2 MPa | 4 MPa | 6 MPa | 2 MPa | 4 MPa | 6 MPa | 2 MPa | 4 MPa | 6 MPa | 2 MPa | 4 MPa | 6 MPa |
| 0                 | 14.6 | 19.7 | 24.9 | 0.011 | 0.013 | 0.016 | 1.39 | 1.63 | 1.65 | 0.258 | 0.255 | 0.254 |
| 5                 | 13.1 | 19.1 | 24.3 | 0.011 | 0.013 | 0.017 | 1.30 | 1.45 | 1.45 | 0.259 | 0.257 | 0.255 |
| 10                | 12.7 | 18.9 | 23.5 | 0.011 | 0.015 | 0.019 | 1.16 | 1.29 | 1.33 | 0.262 | 0.26  | 0.259 |

*2 indicates confining pressure 2Mpa.

As the experiment only measured the parameters of the sedimentary direction, the determination of the parameters of the other two confining pressure directions is based on the results of the
determination of the longitudinal and transverse wave velocity of sandstone samples in the literature [22], combined with the test results of the anisotropic parameters of the red sandstone in the document, and other material parameters can be determined by analogy.

The theoretical curves of the constitutive model were calculated and compared with the experimental curves, and since only the stress-strain data along the sedimentary direction were recorded in the experiment, only the calculation of the same direction was given here, which was in good agreement with the experimental results, as shown in Figure 1.

![Figure 1. Verification of damage constitutive model for rock](image)

5. Conclusions
In this paper, based on the rock engineering in cold area, considering the orthogonal anisotropy of fractured loaded rock under freeze-thaw cycles the orthotropic damage constitutive model of fractured rock in freeze-thaw environment was studied on the basis of the macroscopic statistical damage model of rock by using the characteristics of the stochastic distribution of rock micro-element intensity subject to Weibull function. The main conclusions were as follows:

(1) The damage evolution equation and damage extended constitutive relationship of orthotropic rock were established, based on the formation process of natural rocks, the non-uniformity of rock microscopic structure was fully considered, and the damage evolution and its influence on the macroscopic mechanical behavior of materials were described from the fact that the microscopic strength of rock is randomly distributed. It was revealed that the damage of rock in all directions affected each other.

(2) Compared with the constitutive model of freeze-thaw damage based on rock isotropic, the freeze-thaw damage model was established in this paper, which considers rock anisotropy, and can reflect the environment of natural rock occurrence more realistically. The theoretical results were more consistent with the experimental results. Because of the large number of material parameters of anisotropic rock to be measured, the experimental scale was relatively large. The isotropic material parameters could be obtained only by one-way compression experiment, and the accuracy of the constitutive model of freeze-thaw damage was also able to meet the engineering requirements.

6. References
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