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Creep Strain Analysis and an Improved Creep Model of Granite Based on the Ratio of Deviatoric Stress-Peak Strength under Different Confining Pressures

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Abstract: Creep refers to the deformation of rock with time under long-term applied stress, which occur in most underground engineering. The creep behavior of granite in Shuang jiangkou underground powerhouse in Western Sichuan Province, China, was studied by creep tests. Based on test results, a new parameter DPR, the ratio of deviatoric stress to peak strength, is proposed. DPR is found to be a key parameter to describe creep parameters such as instantaneous elastic modulus, creep elastic modulus, and viscosity coefficient of rock under different confining pressures. Creep tests show that instantaneous elastic modulus increases with the increase of DPR. Creep elastic modulus increases when DPR changes from 0.54 to 0.7004, but decreases when DPR is from 0.7004 to 0.88, indicating fractures in rock closes firstly and then new fractures are generated. The viscosity coefficient of the rock increases first and then decreases with the increase of DPR, and when DPR = 0.7171, viscosity coefficient is maximum, indicating the time for rock to reach stability is the longest in creep tests. By introducing DPR and confining pressure into creep model, which interconnect creep parameters in a unified expression, an improved generalized Kelvin creep model is proposed which can accurately describe the primary and the secondary creep behavior of granite under given deviatoric stresses and confining pressures.

Keywords: Deviatoric stress to peak strength ratio; Instantaneous strain; Creep strain analysis; Improved generalized Kelvin model

1. INTRODUCTION

The time-dependent (creep) behavior of rock refers to the continuous deformation under constant applied stress [1,2]. The deformation does not occur instantaneously, but is a time-dependent delayed deformation [3,4]. The creep behavior is obvious in soft rocks [5], but hard rocks under high stress can also exhibit obvious creep behavior. Excessive deformation caused by creep can affect the design function of rock infrastructure and increase the repair cost [6]. Therefore, creep characteristics is a significant mechanical basis to long-term stability of rock engineering [6], and the effect of creep on the stability of geotechnical engineering should be fully considered. This requires the establishment of a model that can predict the creep process of rock under different stress states and determine the creep converge time of rock.

A number of studies have been reported on creep characteristics of rock by analyzing creep strain or deformation. Wang R. et al. [6] and Goodman R. [8] indicated that the creep strain of a rock mass under different applied stresses can be divided into three stages: primary creep (attenuated creep), secondary creep (steady creep), tertiary creep (accelerated creep), as shown in Fig. 1. However, if the strain rate of rock in the second creep stage becomes zero, then the rock will not enter the tertiary creep stage and its strain will not increase. Zhang Y et al. [9] indicated that only when the deviatoric stress is increased to or exceeds a certain value, the rock will enter the tertiary creep stage and its strain rate increases rapidly. Lajtai et al. [10] and Li L. et al. [11] considered the certain value to be the long-term strength of the rock. Therefore, whether the creep strain rate of rock decreases to zero depends on whether the deviatoric stress is greater than the long-term strength or not. Zhang Q. et al. [12], Wu D. et al. [13] and Cui X. et al. [14] also indicated that when the deviatoric stress is less than the long-term strength, the steady-state creep strain rate should be zero, and the rock will not enter tertiary creep. Zhao Y et al. [15] have shown that the creep strain εc of rock consists of visco-elastic strain (reversible deformation) εve, and visco-plastic strain (irreversible deformation) εvp, and the visco-plastic strain εvp is the main reason of time-dependent delayed failure in rock engineering, as shown in Fig. 2. If the strain rate in the secondary creep stage is zero, only visco-elastic strain εve exists in creep process and the rock finally reaches a stable state. Therefore, it is of great significance to study the reversible deformation in the creep process for the long-term stability of rock engineering.

Many creep constitutive models have been developed based on different assumptions. Visco-elastic strain (reversible deformation) is one of assumptions considered in traditional models. Fahimifar A et al. [16] proposed a
new formulation to determine the wall displacement and convergence of tunnel based on the visco-elastic body, and compared the formulation with numerical analysis. It found that the results of numerical analysis were consistent with the results of the proposed solution. Zhao Y. et al. [15] analyzed the visco-elastic-plastic strain characteristics of rock based on the creep test results, and proposed a creep model which can describe the loading and unloading creep behavior precisely and the full stages of creep. Zhao J. et al. [17] conducted a true triaxial compression test, and proposed a creep model which can describe the creep behavior of hard rock under different 3D stress states. The model is composed of a parallel combination of Hooke component and damper component to describe visco-elastic behavior, and a nonlinear visco-plastic body to demonstrate irreversible creep behavior.

Most creep model can describe the relationship between the strain and time of rock, and the parameters in these creep models are identified by creep tests and data fitting. Zhang Y. [9] and Zhang H. et al. [18] utilized Burgers model to fit the strain-time relationship of rocks, and summed up the relationship between axial stresses and parameters of Burgers model. Cong L. et al. [19] proposed an improved Burgers model, and indicated the relationship between deviatoric stresses with initial elastic modulus and strain rate. Mansouri H. et al. [20] found that the axial stress has a linear relationship with the initial elastic strain, the initial elastic strain rate and the creep strain.

However, the proposed creep models need different parameters for different stress state, but these parameters are not related in a unified expression. In addition, confining pressures are not explicitly considered in the constitutive equations of these creep models, so the rock strain cannot be directly expressed by the creep models under different confining pressures and axial stresses.

In fact, confining pressure is an important factor affecting rock characteristics in creep tests. In this paper, a new parameter DPR, the ratio of deviatoric stress to peak strength, is proposed. The instantaneous strain and visco-elastic strain of rocks in the creep test were analyzed by DPR, and an improved generalized Kelvin creep model is proposed which can accurately describe the relationship between rock strain and time under different confining pressures and deviatoric stresses.

![Fig. 1. A schematic presentation of rheological behavior of rock](image1)

![Fig. 2. Representative strain-time curve with the different types of strains](image2)

2. MATERIAL AND METHODS

2.1. Rock samples and test equipment

Rock samples for tests were granite, taken from the middle guide cave of the main and auxiliary powerhouse at the Shuangjiangkou hydropower station in western Sichuan Province, China. These sample are gray-white, mainly composed of micro-plagioclase, quartz, feldspar, and muscovite, etc. As shown in Fig. 3, those rock samples were processed into Ø50mm×100mm cylindrical standard specimens in accordance with the experimental specification.
recommended by the International Society for Rock Mechanics (ISRM).

The equipment for tests is the MTS815 rock triaxial test machine at the College of Water Resources and Hydropower, Sichuan University, as shown in Fig. 4. The major technical parameters of MTS815 are listed in Table 1. The load in the test is applied by a force sensor, and the deformation of those samples is measured by extensometers. The precision of all measured parameters is 0.5%. The measurement information is collected by computers automatically to eliminate the manual error.

![Fig. 3. Granite sample](image1)

![Fig. 4. Test equipment](image2)

Table 1. The major technical parameters of MTS 815

| Parameter                              | Value       |
|----------------------------------------|-------------|
| Maximum axial force (kN)               | 4600        |
| Maximum confining pressure (MPa)       | 140         |
| Operating temperature (°C)             | 20-200      |
| Extensometer resolution                |             |
| Axial                                  | ±4mm        |
| Circumferential                        | -2.5~8mm    |

2.2. Experimental methods

The basic parameters of the granite samples were obtained by conventional triaxial compression test, listed in Table 2. According to the result of conventional triaxial compression tests, creep tests under different confining pressures were carried out. The creep test of four samples was carried out by Chen's loading method, i.e., multi-step loading method[21]. The creep test program for each granite sample was as follows:

1. Confining pressure was applied to each granite sample to a predetermined value at 0.1MPa/s, and remained stable.
2. The axial stress was applied to each granite sample to a predetermined value at 30kN/min. 55%~60% of the peak strength in the conventional triaxial compression test is taken as the first stress level of the creep test under the same confining pressure, maintaining the axial stress until the strain rate was stable.
3. The stress was increased by 10% of the peak strength as next stress level.
4. If the strain rate at the fourth stress level remains stable, the test was finished.

Basic parameters of granite samples and each stress level scheme of triaxial creep tests are listed in Table 3.

Table 2. The basic parameters of the granite samples

| Confining pressure σ3 (MPa) | Peak strength σp (MPa) | Poisson’s ratio | Elastic modulus (GPa) | Friction coefficient (°) | Cohesion (MPa) |
|-----------------------------|------------------------|-----------------|-----------------------|--------------------------|----------------|
| 1                           | 115.73                 | 0.0828          | 47.95                 |                          |                |
| 3                           | 142.36                 | 0.1215          | 48.37                 |                          |                |
| 5                           | 157.22                 | 0.1697          | 53.09                 | 50.16                    | 21.75          |
| 10                          | 197.97                 | 0.1886          | 57.87                 |                          |                |

Table 3. Basic parameters and triaxial creep tests scheme of granite samples

| Sample number | Height (mm) | Diameter (mm) | Density (g/cm³) | Confining pressure σ3 (MPa) | Axial stress σ1 (MPa) |
|---------------|-------------|---------------|-----------------|-----------------------------|-----------------------|
|               |             |               |                 | 1st level 2nd level 3rd level 4th level |                     |
| CR-1          | 99.92       | 51.31         | 2.68            | 1 68.75 80.36 91.49 102.63  |
| CR-2          | 101.76      | 51.34         | 2.66            | 2 79.35 92.92 104.38 117.04  |
| CR-3          | 100.38      | 50.34         | 2.68            | 3 92.47 107.05 121.63 136.20  |
| CR-4          | 100.33      | 50.36         | 2.66            | 5 119.50 137.58 155.67 173.75  |

3. ANALYSIS OF CREEP TESTS

In this study, a new dimensionless parameter DPR, the ratio of deviatoric stress to the peak strength, is used to
analyze creep tests. DPR can be expressed as:

\[
DPR = \frac{\sigma_1 - \sigma_3}{\sigma_p}
\]

(1)

where \(\sigma_1, \sigma_3\) is the deviatoric stress; \(\sigma_p\) is the peak strength.

\(\sigma_p\) can be obtained from conventional triaxial compression tests, as shown in Table 2; \(\sigma_p\) is related to confining pressures and can be expressed as:

\[
\sigma_p = k_1 \sigma_3 + b_1
\]

(2)

where \(k_1\) and \(b_1\) are parameters; in this creep test, \(k_1 = 8.8387\) and \(b_1 = 111.34\).

It is found that the new parameter DPR is a key variable to determine the creep behavior of granite instantaneous strain and creep strain under different confining pressures. Based on DPR, the traditional creep model is improved. Compared to previous creep model which need different parameters for different stress states, the improved model can conveniently describe the deformation of rock with time in a unified expression under given deviatoric stresses and confining pressures.

3.1. Strain of creep tests

According to the test program, four creep tests under different confining pressures were carried out. Fig. 5 shows the strain curves for granite samples with time under different confining pressures by using the Boltzmann superposition principle. In this study, if the creep strain rate \((h^{-1})\) is less than \(3.0 \times 10^{-5}\), the creep rate is considered to be zero. In Fig. 5, the creep strain rate is almost close to zero with time in the secondary creep of all creep tests under different confining pressures. It is indicated that the final deformation of each level in all creep tests is stable and the strain is no longer increased.

\[\begin{align*}
(\sigma_1, \sigma_3) & = 101.63 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 90.49 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 79.36 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 67.75 \text{ MPa}
\end{align*}\]

(a) \(\sigma_3 = 1 \text{ MPa}\)

\[\begin{align*}
(\sigma_1, \sigma_3) & = 114.04 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 101.48 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 88.92 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 76.35 \text{ MPa}
\end{align*}\]

(b) \(\sigma_3 = 3 \text{ MPa}\)

\[\begin{align*}
(\sigma_1, \sigma_3) & = 131.2 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 116.63 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 102.05 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 87.47 \text{ MPa}
\end{align*}\]

(c) \(\sigma_3 = 5 \text{ MPa}\)

\[\begin{align*}
(\sigma_1, \sigma_3) & = 163.75 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 145.67 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 127.58 \text{ MPa} \\
(\sigma_1, \sigma_3) & = 109.50 \text{ MPa}
\end{align*}\]

(d) \(\sigma_3 = 10 \text{ MPa}\)

Fig. 5. Curves for granite creep tests under different confining pressures and deviatoric stresses

3.2. Instantaneous strain and Instantaneous elastic modulus

The total strain \(\varepsilon\) in creep tests can be divided into the instantaneous strain \(\varepsilon_m\) and the creep strain \(\varepsilon_c\), which can be
written as\(^{(15)}\):

\[
\varepsilon = \varepsilon_m + \varepsilon_v
\]  \hspace{1cm} (3)

The instantaneous strain \( \varepsilon_m \) refers to the total strain of each rock when the loading stress reaches the predetermined value in the test, that is, the total strain when \( t = 0 \) in Fig. 5, and the creep strain \( \varepsilon_v \) refers to the strain increment of rock with time after axial stress. The value of \( \varepsilon, \varepsilon_m, \varepsilon_v \) and converge time \( t_c \) are listed in Table 4.

| Confining pressure (MPa) | DPR | Total strain \( \varepsilon \) (10\(^{-3}\)) | Instantaneous strain \( \varepsilon_m \) (10\(^{-3}\)) | Creep strain \( \varepsilon_v \) (10\(^{-3}\)) | Converge time \( t_c \) (h) |
|--------------------------|-----|----------------------------------|------------------|------------------|------------------|
| 1                        | 0.59 | 4.4609                           | 4.3016           | 0.1593           | 4.78             |
|                          | 0.69 | 5.0247                           | 4.8538           | 0.1709           | 9.75             |
|                          | 0.78 | 5.5444                           | 5.3735           | 0.1709           | 8.20             |
|                          | 0.87 | 6.2089                           | 5.8847           | 0.3242           | 6.63             |
| 3                        | 0.54 | 4.7880                           | 4.6697           | 0.1183           | 3.66             |
|                          | 0.62 | 5.3213                           | 5.2306           | 0.0907           | 6.02             |
|                          | 0.71 | 5.8729                           | 5.7790           | 0.0939           | 6.51             |
|                          | 0.80 | 6.4610                           | 6.3145           | 0.1465           | 6.02             |
| 5                        | 0.56 | 4.9905                           | 4.8839           | 0.1066           | 3.51             |
|                          | 0.65 | 5.6030                           | 5.5103           | 0.0927           | 7.72             |
|                          | 0.74 | 6.2495                           | 6.1449           | 0.1046           | 9.20             |
|                          | 0.83 | 6.9486                           | 6.7804           | 0.1682           | 7.21             |
| 10                       | 0.55 | 5.6188                           | 5.4970           | 0.1218           | 5.21             |
|                          | 0.64 | 6.3537                           | 6.2662           | 0.0875           | 5.95             |
|                          | 0.74 | 7.1092                           | 7.0236           | 0.0856           | 6.50             |
|                          | 0.83 | 7.8949                           | 7.7643           | 0.1306           | 5.31             |

The instantaneous elastic modulus \( E_M \) can be calculated by:

\[
E_M = \frac{\sigma_1 - \sigma_3}{\varepsilon_m}
\]  \hspace{1cm} (4)

Substituting Eq. (1) into Eq. (4), \( E_M \) can be expressed as:

\[
E_M = \frac{\sigma_p \times DPR}{\varepsilon_m}
\]  \hspace{1cm} (5)

\( E_M \) under different confining pressures and DPR are shown in Fig. 6. The instantaneous elastic modulus of rock increases with the increase of axial stresses, and shows a linear relationship which can be expressed as:

\[
E_M = E_0 + (E_p - E_0) \times DPR
\]  \hspace{1cm} (6)

where \( E_0 \) is the instantaneous elastic modulus at a certain confining pressure but without deviatoric stress; \( E_p \) is the instantaneous elastic modulus at peak strength \( \sigma_p \).

![Fig. 6. Relationship between \( E_M \) and DPR](image)

Table 5. The value of \( E_p \) and \( E_0 \) under different confining pressures

| Confining pressures \( \sigma_3 \) | \( E_p \) (GPa) | \( E_0 \) (GPa) |
|-----------------------------------|----------------|----------------|
| 1                                 | 17.91          | 12.62          |

Table 4. The value of \( \varepsilon, \varepsilon_m, \varepsilon_v \) and converge time \( t_c \) under different confining pressures
When $DPR = 1$ and 0, $E_p$ and $E_0$ can be obtained by the fitting expressions in Fig. 6, and the value of $E_p$ and $E_0$ under different confining pressures are shown in Table 5. The confining pressure has a linear relationship with $E_p$ and $E_0$. When $DPR = 1$, 100% of the peak strength and confining pressures are applied to the rock, then $E_p$ can be expressed as:

$$E_p = 17.87 + 0.41\sigma_3$$

(7)

Similarly, when $DPR = 0$, only the confining pressure is applied to the rock, then $E_0$ can be obtained by the confining pressure:

$$E_0 = 11.76 + 0.59\sigma_3$$

(8)

When $\sigma_3 = 0$, the instantaneous elastic modulus of the rock under the peak strength is 17.87 GPa at $DPR = 1$; the instantaneous elastic modulus of the rock without applied stress is 11.76 GPa at $DPR = 0$.

Therefore, Substituting Eq. (6) into Eq. (5), the instantaneous strain can be expressed as follows:

$$E = \frac{\sigma_1 - \sigma_3}{E_M} \times \frac{\sigma_p \times DPR}{E_p \times DPR + E_0(1 - DPR)}$$

(9)

### 3.3. Creep strain and creep elastic modulus

Creep strain is one of the most concerned parameters in the creep test. Under the applied stress state, the creep strain increases with time, and the creep strain rate also change with time. Hence, it is significant and necessary to study the creep strain in creep test.

In this creep test, the final creep strain rate of the secondary creep of each level is 0, indicating that the creep strain is visco-elastic strain $\dot{\varepsilon}_{ve}$, as shown in Table 4. The visco-elastic strain $\dot{\varepsilon}_{ve}$ in creep increases with time and finally reaches maximum.

The Kelvin model can well describe the visco-elastic strain in the creep with time, as shown in Fig. 7. The Kelvin model can be expressed as follows:

$$\dot{\varepsilon}_{ve}(t) = \frac{\sigma_1 - \sigma_3}{E_K} (1 - e^{-\frac{t}{\tau}})$$

(10)
where $\varepsilon_v(t)$ is the visco-elastic strain with time; $E_K$ and $\eta_K$ are Kelvin’s creep elastic modulus and viscosity coefficient, respectively.

It can be seen from Eq. (10) that when $t = 0$, $\varepsilon_v(t) = 0$, and when $t \to \infty$, $\varepsilon_v(t) = (\sigma_1 - \sigma_3)/E_K$. Therefore, $E_K$ can be defined by the final creep strain. The final creep strain can be represented by the deformation of the elastic element in Fig. 7. When $t \to \infty$, by Eq. (1) and (10), $E_K$ can be obtained as:

$$E_K = \frac{\sigma_1 - \sigma_3}{\varepsilon_{ve}} = \frac{\sigma_p \times DPR}{E - \varepsilon_{em}}$$

(11)

The relationship between $E_K$ and $DPR$ of rock under different confining pressures is shown in Fig. 8. The relationship between $E_K$ and $DPR$ is quadratic when $DPR = 0.54-0.88$ in this creep test. $E_K$ increases first with $DPR$ and then decreases. Based on the fitting curves of the relationship between $E_K$ and $DPR$, it can be calculated that when $DPR = 0.7134, 0.6827, 0.7137$ under $\sigma_3 = 1, 3, 5, 10$ MPa, $E_K$ reaches maximum, respectively 510.75, 1081.45, 1146.10 and 1670.52 MPa. The average $DPR$ when $E_K$ reaches maximum is 0.7004. The maximum $E_K$ at a given confining pressure can be expressed as:

$$E_{K,max} = 117.29\sigma_3 + 545.06$$

(12)

which $E_{K,max}$ is the maximum $E_K$ at a given confining pressure.

When $DPR < 0.7004$, with the increase of deviatoric stress, the internal fracture of the rock gradually close and the rock stiffness increase. When $DPR = 0.7004$, $E_K$ reaches the maximum, indicating that the closing process of internal fracture of the rock is finished and the stiffness of the rock reaches the maximum. When $DPR > 0.7004$, new fracture begins to form in the rock due to the increase of deviatoric stress and the rock stiffness decrease. Therefore, with the increase of $DPR$, the creep damage of rock can be divided into the fracture closure stage and fracture propagation stage. $E_K$ can be expressed by the closure degree of the internal fracture in the rock after the rock creep is stable, which can be expressed as:

$$E_K = D_{cr}(\sigma) \times E_{K,max}$$

(13)

where $D_{cr}(\sigma)$ is the closure degree of the internal fracture in the rock.

Fig. 9 shows the relationship between $D_{cr}(\sigma)$ and $DPR$ under different confining pressures. Under different confining pressures, $D_{cr}(\sigma)$ corresponding to the same $DPR$ is similar. Therefore, the relationship between $D_{cr}(\sigma)$ and $DPR$ under different confining pressures can be fitted. When $DPR = 0.7004$, $D_{cr}(\sigma) = 1$, which indicates that when the deviatoric stress is 70.04% of the peak strength, the internal fracture of rock can be bestly closed.

Fig. 9. Relationship between $D_{cr}(\sigma)$ and $DPR$ under different confining pressures

Table 4 shows the creep strain and converge time $t_c$ in each level of creep test. The value of $\eta_K$ can be calculated.
by substituting the value of creep strain and converge time obtained from the creep test into Eq. (10) and (11), expressing as:

\[ \eta_K = \frac{E_K \cdot t_c}{\ln(1 - \frac{E_K}{\sigma - \sigma_3})} \]  

(14)

As shown in Fig. 10, under different confining pressures, \( \eta_K \) at the same DPR are approximately the same. The quadratic function can well describe the relationship between \( \eta_K \) and DPR, expressed as:

\[ \eta_K = l \times DPR^2 + m \times DPR + n \]  

(15)

where \( l, m, n \) are parameters. In this study, the values of \( l, m, n \) are shown in Fig. 10.

When DPR = 0.7171, \( \eta_K \) reaches the maximum, that is, \( \eta_{K,\text{max}} = 927.83 \). Combined with \( E_K \) analysis, it is found that \( E_K \) and \( \eta_K \) have the same trend of change, and reach the maximum when DPR = 0.70-0.72, indicating that the larger \( E_K \) is, the smaller the creep strain rate of rock is, leading to longer time for rocks to reach stable state.

**4. IMPROVES KELVIN CREEP MODEL CONSIDERING DPR**

The time-dependent behavior of granite can be simulated by conventional creep models. The generalized Kelvin model can well describe primary creep and secondary creep with final strain rate of 0, as shown in Fig. 11. The generalized Kelvin model is composed of a Hooke body and a Kelvin model. The equation for the generalized Kelvin model can be expressed as follow:

\[ \varepsilon(t) = \frac{\sigma_1 - \sigma_3}{E_M} + \frac{\sigma_1 - \sigma_3}{E_K} (1 - e^{-\eta_K t}) \]  

(16)

Based on the analysis in Section 3.2-3.3, an improved model, which can accurately describe the time-dependent strain of rock, is obtained by substituting Eqs. (1), (6), (13) and (15) into Eq. (16):

\[ \varepsilon(t) = \frac{\sigma_p \times DPR}{E_p \times DPR + E_0 (1 - DPR)} + \frac{\sigma_p \times DPR}{D_0 (\sigma_p \times E_{K,\text{max}})} (1 - e^{-\eta_K t}) \]  

(17)

As shown in Fig. 12, comparing the rock strain curve obtained by Eq. (17) with the creep test, it is found that the improved model for the generalized Kelvin model can effectively describe the time-dependent behavior of the granite as well as the primary and secondary creep behaviors with final strain rate of 0 under a given deviatoric stress and confining pressure. Fig. 13 shows the relationship of creep strain with time and DPR under different confining pressures. When DPR is about 0.7, the creep strain will reach a minimum under different confining pressures.
(a) $\sigma_3 = 1$ MPa

(b) $\sigma_3 = 3$ MPa

(c) $\sigma_3 = 5$ MPa

(d) $\sigma_3 = 10$ MPa

Fig. 12. Creep test results and calculated curves for different confining pressures and deviatoric stresses
5. DISCUSSION

To develop a time-dependent creep model which can accurately describe the creep deformation of granite, improvements were made by introducing DPR and confining pressure into the generalized Kelvin model based on the analysis of instantaneous strain and visco-elastic strain.

The tests show that instantaneous elastic modulus increases with the increase of DPR under different confining pressures. Based on the analysis of the visco-elastic creep strain, it is found that the creep elastic modulus related to the final creep strain $E_K$ reaches the maximum when $DPR = 0.7004$. The viscosity coefficient $\eta_K$ related to the creep converge time of rock reaches maximum when $DPR = 0.7171$. Through data analysis of different rock tests in many previous studies[9,18,19,23,24], it is found that under different confining pressures, when $E_K$ reaches the maximum, DPR may varies from 0.45-0.76; and when $\eta_K$ reaches the maximum, DPR is between 0.42-0.80. In this paper, the maximum of $E_K$ and $\eta_K$ reach maximum when DPR = 0.70-0.72, showing the same trend of change with other researchers’ work. But for the diversity of rock types, the DPR corresponding to maximum $E_K$ and $\eta_K$ may change in a rather large range.

The generalized Kelvin model considering DPR can effectively predict the strain-time curve, but parameters in the model are different under different applied stresses. The relationship between creep parameters and DPR is set up, so the improved generalized Kelvin model can describe the strain-time relationship under different confining pressures and deviatoric stresses.

6. CONCLUSIONS

Through creep tests on rock samples from the Shuangjiangkou hydropower station, the creep strain–time curves under different stresses were investigated, and the influence of the stress on the creep deformation of the rock samples was obtained. In this study, a new parameter DPR is proposed, that is, the ratio of deviatoric stress to the peak strength.

The creep tests with DPR = 0.54-0.88 is analyzed, and the following conclusions are drawn:

(1) Under different confining pressures, with the increase of DPR, the instantaneous elasticity modulus of rock increases linearly.

(2) The creep converge strain of rock can be reflected by Kelvin’s elastic modulus $E_K$ which increases firstly and then decreases with the increase of DPR. When $DPR = 0.7004$, $E_K$ reaches the maximum. With the increase of DPR, the creep damage of rock can be divided into the fracture closure stage and fracture propagation stage.

(3) The Kelvin viscosity coefficient $\eta_K$ can reflect the relationship between creep strain rate and converge time. $\eta_K$ increases first and then decreases with the increase of DPR, and the value of $\eta_K$ is approximately equal under different confining pressures. When $DPR = 0.7171$, $\eta_K$ reaches the maximum, that is, the time for rock to reach creep stability is the longest.

(4) DPR and confining pressure determine rock creep behavior and parameters. By introducing DPR and confining pressure into creep model, an improved generalized Kelvin creep model is proposed which can effectively describe the primary creep behavior of granite and the secondary creep behavior under given deviatoric stresses and confining pressures.

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Declarations

Conflicts of interests: the authors have no conflicts of interest to declare that are relevant to the content of this article

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Code availability: Not applicable

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Figures

Figure 1

A schematic presentation of rheological behavior of rock
Figure 2

Representative strain-time curve with the different types of strains

Figure 3

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Figure 2

Representative strain-time curve with the different types of strains
Granite sample

Figure 4

Test equipment
Figure 5

Curves for granite creep tests under different confining pressures and deviatoric stresses
Figure 6

Relationship between EM and DPR

Figure 7
Figure 8

Relationship between $E_K$ and DPR under different confining pressures
Figure 9

Relationship between $D_{cr}(\sigma)$ and DPR under different confining pressures

The relationship is given by the equation:

$$D_{cr}(\sigma) = -14.29DPR^2 + 20.11DPR - 6.09$$

with $R^2 = 0.83$.
Figure 10
Relationship between $\eta_K$ and DPR under different confining pressures

$$\eta_K = -25432DPR^2 + 36473DPR - 12149$$

$R^2 = 0.77$

Figure 11
Illustrations of the generalized Kelvin model
Figure 12

Creep test results and calculated curves for different confining pressures and deviatoric stresses
Figure 13

Relationship of creep strain with time and DPR under different confining pressures