A comparative Analysis of PV Cell Mathematical Model

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Abstract: Several methods are currently used to calculate the values of voltage, current, and power of a solar cell. A new method is suggested to numerically find these values using the popular methods Newton Raphson method (NRM) and the three-step method (TSM) at different values of load resistance R. Equation based on the equivalent circuit of a solar cell, so all calculations is performed in a MATLAB at room temperature. The obtained results of this new method were presented and compared with NRM. Additionally, a single diode model of a solar cell was demonstrated.

Keywords: Three-Step method; newton-Raphson method; iterations; physical parameters; maximum voltage.

1. Introduction
The fame and growth of numerical analysis are currently testimony to other evidence that applications are still the main source of predicting mathematical innovation. When new mathematical ideas develop, it is customary for new applications to be paved. The computer is a very illustrative one, so it responds to the urgent need for a quick calculation. The emergence of such calculators has made it possible to meet the requirements of modern applications, by developing new numerical methods, in many cases. This is the origin of modern numerical analysis. It is the numerical interface of the broad field of applied analysis [1-10]. The main idea is to reproduction electrical energy by means of photovoltaic cells, which form solar cells [11-20]. Semiconductor materials are the main part to fabricate these cells such as silicon. Many kinds of solar cells are fabricated based on the material used and fabrication technique such as organic and inorganic solar cells [21-30].

A new method for determining the physical parameters of a single diode model of a solar cell is presented and discussed. The proposed method is used to determine the voltage, current, power, and absolute error of the solar cell device. The Three-Step method proposed was compared to the other Newton–Raphson method. Values obtained from the proposed method were found to be more accurate than other methods studied in all cases. It is organized as follows: section 2 characterizing the analytical model of a single-diode design of the solar cell; Section 3 establishing the root-finding Newton Raphson Method (NRM); section 4 three step method (TSM); section 5 results and discussion; section 6 conclusions of the obtained results.

2. Characteristics of Single-Diode Solar Cells Equation
The simple equivalent electric circuit of a solar cell is shown in Figure 1.
Using Kirchhoff’s current law for the current $I$, the equation of this equivalent circuit is given by

$$I = I_{ph} - I_D$$  \hspace{1cm} (1)

$$I_D = I_0 \left( e^{\frac{-V_{pv}}{nV_T}} - 1 \right)$$  \hspace{1cm} (2)

$$I = I_{ph} - I_0 \left( e^{\frac{-V_{pv}}{nV_T}} - 1 \right)$$  \hspace{1cm} (3)

where:

- $I_{ph}$ is the photocurrent ($A$);
- $I_0$ is reverse saturation current of the diode ($A$);
- $I$ and $V_{pv}$ are the delivered current and voltage, respectively ($V$);
- $V_T = \frac{kT}{q} = 0.0259$ $V$ is thermic voltage = 27.5 $\pm$ 26 mV at $T = 25$ oC Air-Mass = 1.5;
- $m$ is the recombination factor closeness to an ideal diode ($1 < m < 2$),
- $k$ is Boltzmann constant = $1.38 \times 10^{-23} J/K$; $T$ is $p-n$ junction temperature ($K$);
- $q$ is the electron charge = $1.6 \times 10^{-19}$ $C$.

$$I_{ph} = I_{source}$$  \hspace{1cm} (4)

$$I_D = I_s \left( e^{\frac{V}{nV_T}} - 1 \right)$$  \hspace{1cm} (5)

Merge Eq. 4 in Eq. 5 we get

$$\left( I_{source} \right) - 10^{-12} \left( e^{\frac{-V}{1.2 + 0.026}} - 1 \right) = \frac{V}{R_L}$$  \hspace{1cm} (6)

where $I_s$ reverse saturation current = $10^{-12} A$. In parallel, $V_D = V_{pv} = V$

According to Eq. 6 one can calculate $V$ of the cell numerically based on the first derivative of this equation.

### 3. Newton Raphson Method

The following algorithm suggestion for solving Eq. 5 by using NRM (see Figure 1)

INPUT initial approximate solution $x_0 = 1$, tolerance $\epsilon$, the maximum number of iterations $N$

OUTPUT approximate solution $x_{n+1}$

Step 1: Set $x = 0$

Step 2: while $i \leq x_0$

Step 3: Calculate

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0, 1, 2, ...$$

Step 4: if $|x_i - x_{i-1}| < \epsilon$; then OUTPUT $x_{n+1}$ and stop.

Step 5: Set $n = n + 1$; $i = i + 1$ and go to Step 2.

Step 6: OUTPUT
4. Three Step Method (TSM)

Six-order convergences with three steps are investigated. Let \( f(x) = 0 \) is a nonlinear equation, suppose \( x_0 \) as an initial value, so the iteration results \( x_{n+1} \) can be calculated using the following scheme

\[
y_n = x_n - \frac{f(x_n)}{f'(x_n)}
\]

\[
z_n = x_n - \frac{f(x_n) + f'(x_n) - f(x_n)\times f'(x_n)}{f'(x_n)\times f'(x_n) - f(x_n)\times f'(x_n)}
\]

\[
x_{n+1} = z_n - \frac{f(x_n)\times f'(x_n) - f'(x_n)\times f(x_n)}{f(x_n)\times f'(x_n) - f'(x_n)\times f(x_n)}
\]

Eq. 7 has a six-order convergence called a three-step method (TSM); the proposed method.

5. Results and Discussion

Consider the Eq. 6 is modeled in the form single-diode solar cell has obtained the following approximate solutions and the TSM are applied with the first initial value (first value from NRM); while Newton-Raphson methods (NRM) with initial value \( x_0 = 1 \). In Table 1 the Newton Raphson method (NRM) and three step method (TSM) of the solution results (voltage \( V_{pv} \); current \( I_{pv} \) and power \( P_{pv} \) of the solar cell) and absolute error \( \varepsilon \) are given and listed in the last columns of this table when the load resistance \( R = 1 \).

Table 1. The \( V_{pv}, I_{pv}, P_{pv} \) and \( \varepsilon \) using NRM and TSM

| Iterations | \( V_{pv}\)-NRM | \( V_{pv}\)-TSM | \( I_{pv}\)-NRM | \( I_{pv}\)-TSM | \( P_{pv}\)-NRM | \( P_{pv}\)-TSM |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1          | 0.971416861    | 0.955060555   | 0.077576865    | 0.03263742     |
| 2          | 0.934845183    | 0.922427705   | 0.012422048    | 4.5709E-06     |
| 3          | 0.922076823    | 0.92242135    | 0.000283689    | 1.1102E-16     |
| 4          | 0.922423135    | 0.922423135   | 3.36712E-10    | 0.000000000    |
| 5          | 0.922423135    | 0.922423135   | 0.000000000    | 0.000000000    |
| 6          | 0.922423135    | 0.922423135   | 0.000000000    | 0.000000000    |

Figure 2 presents the obtained solutions of the study result.
Figure 2. The obtained solutions of the study result at the load resistance R=1.

In Table 2 the Newton Raphson method (NRM) and three step method (TSM) of the solution results (voltage $V_{pv}$, current $I_{pv}$ and power $P_{pv}$ of the solar cell) and absolute error $\varepsilon$ are given and listed in the last columns of this table when the load resistance $R = 2$

Table 2. The obtained values using NRM and TSM

| Iterations | $V_{pv}$-NRM | $V_{pv}$-TSM | $\varepsilon$-NRM | $\varepsilon$-TSM |
|------------|--------------|--------------|------------------|------------------|
| 1          | 0.971030472  | 0.923160241  | 0.082964618      | 0.037188279      |
| 2          | 0.932557944  | 0.917052987  | 0.015522562      | 1.7605E-05       |
| 3          | 0.917605401  | 0.917035382  | 0.000570019      | 1.55431E-15      |
| 4          | 0.917035387  | 0.917035382  | 5.03445E-09      | 0.000000000      |
| 5          | 0.917035382  | 0.000000000  | 0.000000000      |
| 6          | 0.917035382  | 0.000000000  |
| 7          | 0.917035382  | 0.000000000  |

$\varepsilon$-NRM | $\varepsilon$-TSM
|---------------|------------------|
| 0.485515236   | 0.471450089      |
| 0.466278972   | 0.434832159      |
| 0.458802701   | 0.420999836      |
| 0.458517694   | 0.420476951      |
| 0.458517691   | 0.420476946      |
| 0.458517691   | 0.420476946      |

Figure 3 Presents the obtained solutions of the study result.
Figure 3. The obtained solutions of the study result at the load resistance $R = 2$.

In Table 3 the Newton Raphson method (NRM) and three-step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$ and power $P_{pv}$ of the solar cell) and absolute error $\varepsilon$ are given and listed in the last columns of this table when the load resistance $R = 3$.

Table 3. The obtained values using NRM and TSM

| Iterations | $V_{pv}$-NRM | $V_{pv}$-TSM | $\varepsilon$-NRM | $\varepsilon$-TSM |
|------------|---------------|--------------|------------------|------------------|
| 1          | 1             | 0.953380693  | 0.08956626       | 0.042977319      |
| 2          | 0.970643792   | 0.919321269  | 0.060240418      | 0.008917895      |
| 3          | 0.930170085   | 0.91047401   | 0.019766711      | 7.06356E-05      |
| 4          | 0.911587131   | 0.910403374  | 0.001183757      | 4.4134E-13       |
| 5          | 0.910403456   | 0.910403374  | 8.19924E-08      | 0.00000000       |
| 6          | 0.910403374   | 1.11022E-16  |                  |                  |
| 7          | 0.910403374   | 0.000000000  |                  |                  |

Figure 4 Presents the obtained solutions of the study result.
Figure 4. The obtained solutions of the study result at the load resistance $R = 3$.

In Table 4 the Newton Raphson method (NRM) and three-step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$ and power $P_{pv}$ of the solar cell) and absolute error $\varepsilon$ are given and listed in the last columns of this table when the load resistance $R = 4$.

Table 4. The obtained values using NRM and TSM

| Iterations | $V_{pv}$-NRM | $V_{pv}$-TSM | $\varepsilon$-NRM | $\varepsilon$-TSM |
|------------|--------------|--------------|------------------|------------------|
| 1          | 0.952531681  | 0.098259398  | 0.050791079      |                  |
| 2          | 0.970256822  | 0.915170632  | 0.06851622       | 0.01343003       |
| 3          | 0.927675607  | 0.902042574  | 0.025935005      | 0.000301972      |
| 4          | 0.904333309  | 0.901740602  | 0.002592707      | 1.54978E-10      |
| 5          | 0.901742128  | 0.901740602  | 1.52598E-06      | 0.000000000      |
| 6          | 0.901740602  | 0.000000000  |                  |                  |
| 7          | 0.901740602  | 0.000000000  |                  |                  |

| $I_{pv}$-NRM | $P_{pv}$-NRM | $I_{pv}$-TSM | $P_{pv}$-TSM |
|--------------|--------------|--------------|--------------|
| 0.25         | 0.25         | 0.23813292   | 0.226829151  |
| 0.242564205  | 0.235349575  | 0.228792658  | 0.209384321  |
| 0.231918902  | 0.215145508  | 0.225510644  | 0.203420201  |
| 0.226083327  | 0.204454683  | 0.225435151  | 0.203284028  |
| 0.225435532  | 0.203284716  | 0.22543515   | 0.203284028  |
| 0.22543515   | 0.203284028  | 0.22543515   | 0.203284028  |
| 0.22543515   | 0.203284028  | 0.22543515   | 0.203284028  |

Figure 5 Presents the obtained solutions of the study result.
Figure 5. The obtained solutions of the study result at the load resistance $R = 4$.

In Table 5 the Newton Raphson method (NRM) and three step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$ and power $P_{pv}$ of the solar cell) and absolute error $\varepsilon$ are given and listed in the last columns of this table when the load resistance $R = 5$.

Table 5. The obtained values using NRM and TSM

| Iterations | $V_{pv}$-NRM   | $V_{pv}$-TSM   | $\varepsilon$-NRM | $\varepsilon$-TSM |
|------------|----------------|----------------|------------------|------------------|
| 1          | 0.9516766856   | 0.110907285    | 0.062583942      |                  |
| 2          | 0.96986956     | 0.910676781    | 0.080776845      | 0.021584067      |
| 3          | 0.925068092    | 0.890543397    | 0.035975378      | 0.001450682      |
| 4          | 0.895367304    | 0.889092803    | 0.006274589      | 8.81011E-08      |
| 5          | 0.889128976    | 0.889092715    | 3.62612E-05      | 0.000000000      |
| 6          | 0.889092715    | 0.889092715    | 1.36446E-13      |                  |
| 7          | 0.889092715    | 0.889092715    | 0.000000000      |                  |

| $I_{pv}$-NRM | $P_{pv}$-NRM | $I_{pv}$-TSM | $P_{pv}$-TSM |
|--------------|--------------|--------------|--------------|
| 0.2          | 0.2          | 0.190335331  | 0.181137692  |
| 0.193973912  | 0.188129393  | 0.182135356  | 0.16586644   |
| 0.185013618  | 0.171150195  | 0.178108679  | 0.158613508  |
| 0.179073461  | 0.160336522  | 0.177818561  | 0.158097202  |
| 0.177825795  | 0.158110067  | 0.177818543  | 0.158097171  |
| 0.177818543  | 0.158097171  | 0.177818543  | 0.158097171  |

Figure 6 Presents the obtained solutions of the study result.
Figure 6. The obtained solutions of the study result at the load resistance $R = 5$.

The obtained solution plot in the (no. of iteration)-$\varepsilon$-plane and the initial-output values proves that the proposed method (TSM) has six iterations indicated a fast behaviour. Parallel to this feature, it is noticed that the proposed method (TSM) has a behaviour of the solution in the initial value $x_0$, has the smallest error tolerance compared with (NRM) with initial value $x_0 = 1$.

A result of tables 1 to 5 is showing that the suggested method (TSM) has low absolute errors after relatively view iterations are computed and this in turn is demonstrating their efficiency.

6. Conclusion

This paper presents a new method to calculate the electrical parameter of the solar cell using two different methods; NRM and TSM. Values obtained from the proposed method (TSM) were found to be better compared to (NRM). Additionally, values for single diode solar cells were determined with fast convergence, more capable to determine these parameters, and establishing towards the final values.

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