Dynamics of Black-Hole Nuclei

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Abstract. Space-based observations are beginning to yield detailed information about the stellar kinematics at the very centers of galaxies, within the sphere of gravitational influence of the black hole. The structure and dynamics of these regions is probably determined in part by the infall and coalescence of black holes during galaxy mergers. A goal of N-body simulations is to reproduce the kinematics near the black holes as well as the relations that exist between the nuclear and global properties of galaxies. However, the problem is computationally difficult due to the wide range of length and time scales, and no single N-body code can efficiently follow the evolution from kiloparsec to sub-parsec scales. We review existing N-body work on this problem and present the first, fully self-consistent merger simulations of galaxies containing dense stellar cusps and black holes.

1. Introduction

Supermassive black holes (BHs) are generic components of galaxies, and they appear to be linked in fundamental ways to the dynamics of the stellar component, both on large and small scales. An astonishingly strong correlation exists between BH mass and stellar velocity dispersion, $M_\bullet \sim \sigma^\alpha$, $\alpha \approx 5$ (Ferrarese & Merritt 2000; Gebhardt et al. 2000b). The $M_\bullet - \sigma$ relation as defined by the best-determined BH masses is so tight that it surpasses in predictive accuracy what can be achieved from detailed dynamical modelling of high-quality stellar-kinematical data in most galaxies. On small scales, the approximately power-law variation of stellar density with radius (Ferrarese et al. 1994) would be difficult to understand in the absence of BHs. Low luminosity ellipticals and bulges have steep nuclear density profiles, $\rho \sim r^{-\gamma}$, $\gamma \approx 2$; such steep cusps form naturally in stellar systems where the BHs grow on time scales long compared to crossing times (Peebles 1972; Quinlan, Hernquist & Sigurdsson 1995). Brighter galaxies typically have weaker cusps, $\rho \sim r^{-1}$ (Merritt & Fridman 1995; Gebhardt et al. 1996). While no universally accepted model has yet been proposed for the origin of the weak cusps, we note here one feature that suggests a link to BHs. Two galaxies, NGC 3379 and M87, have weak cusps with well-determined structural parameters and also have BHs with accurately determined masses. Table 1 gives for each galaxy the “break” radius $r_b$ at which the central power law turns over to a steeper outer profile; and also the stellar mass $M_*$ contained within $r_b$. In both galaxies, $M_*(r_b)$ is identical within the uncertainties with $M_\bullet$, the mass of the BH; this is in spite of a factor of \(~30\) difference in $M_\bullet$ and \(~6\) in $r_b$. 

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The rough equality between $M_\bullet$ and $M_\star(\epsilon)$, and the exclusive association of weak cusps with bright galaxies, is consistent with the suggestion that weak cusps are relics of galaxy mergers (Begelman et al. 1980; Ebisuzaki et al. 1991). In this model, the two BHs from the merging galaxies fall to the center and heat the stars, reducing the stellar density and forming a tight binary. The “damage” done by the binary, and hence the extent of the shallow cusp, would be roughly proportional to the total BH mass.

Table 1.

| Galaxy   | $\gamma$ | $r_\epsilon$ (pc) | $M_\bullet$ ($M_\odot$) | $M_\star(r_\epsilon)$ ($M_\odot$) |
|----------|----------|-------------------|------------------------|----------------------------------|
| NGC 3379 | 1.1(1)   | 51(1)             | $1.35 \times 10^8$(2)  | $3.5 \times 10^8$                |
| M87      | 1.26(3)  | 315(3)            | $3.6 \times 10^9$(4)   | $4.8 \times 10^9$                |

1Gebhardt et al. 1996
2Gebhardt et al. 2000a
3van der Marel 1994
4Macchetto et al. 1997

New instruments and observational techniques, both ground-based (e.g. SAURON; Miller 2000) and space-based (HST; e.g. Joseph et al. 2000), are giving us increasingly detailed glimpses of the stellar kinematics very near the centers of galaxies. These data are useful both for constraining BH masses, but also as fossil signatures of the stellar dynamical processes that shaped nuclei. Interpreting these new data in the context of models for galaxy formation will be a challenging computational task. Stellar densities in a steep stellar cusp are $\geq 10^7 M_\odot$ pc$^{-3}$ implying dynamical times of $\lesssim 10^4$ yr, and even less in the vicinity of the BH; thus an N-body code that simulates both the large- and small-scale evolution of merging galaxies must be able to handle a range of $\sim 10^5$ in time scales as well as a sufficiently large number of particles to resolve the stellar cusps. Until recently, N-body codes satisfying these requirements were almost non-existent, but happily that situation has now changed. Tree codes that efficiently implement variable time steps on parallel architectures now exist (e.g. GADGET, Springel et al. 2000); such codes are ideal for following the early stages of galaxy mergers before the BHs form a bound pair. In the later stages, when the BH binary begins to interact “collisionally” with individual stars, a code like NBODY6++ (Spurzem & Baumgardt 1999) is more appropriate: this code implements Aarseth’s efficient hierarchical algorithm using block time steps on parallel computers and can treat particle-particle interactions on arbitrarily small scales.

Merger simulations will need to reproduce a number of observed regularities in the properties of the stellar cusps. One is the tight correlation between $M_\bullet$ and $\sigma$ mentioned above. Another observational result is the dependence of cusp slope $\gamma$ on galaxy luminosity (Gebhardt et al. 1996): faint ellipticals, $|M_v| \gtrsim 20$, have $\gamma \sim 2$ while $\gamma$ for bright ellipticals varies from $\sim 2$ to $\sim 0$. There are at least three interesting questions here: (1) What determines the characteristic luminosity separating galaxies with strong and weak cusps? (2) Assuming that...
all galaxies begin life with strong cusps, how do the weak cusps form? (3) How are the weak cusps in bright galaxies maintained in spite of mergers with dense, low-luminosity galaxies? Another empirical result is the rough proportionality between break radius \( r_b \) and galaxy luminosity (Faber et al. 1997). A direct link between cusp slope and BH mass has also been suggested (van der Marel 1999).

In addition to these well-established parameter relations, the new, high-resolution kinematical data will provide detailed stellar velocity fields in galactic nuclei which can be compared to \( N \)-body simulations.

In this article, we summarize the existing \( N \)-body work on interactions between galaxies with high central densities and BHs. We then present preliminary results from new simulations which, for the first time, follow the evolution of BH nuclei through a realistic merger to the formation and decay of a hard BH binary.

2. \( N \)-Body Studies of Mergers with Stellar Cusps and Black Holes

In the merger of two galaxies each containing a supermassive nuclear BH, the dynamical evolution can usefully be divided into two stages, before and after the formation of a hard BH binary. Quinlan (1996) defines a “hard” BH binary as one in which the semimajor axis \( a \) satisfies

\[
a < a_h = \frac{Gm_2}{4\sigma^2} = 2.7 \text{pc} \left( \frac{m_2}{10^8 M_\odot} \right) \left( \frac{200 \text{ km s}^{-1}}{\sigma} \right)^2
\]

with \( m_2 \) the mass of the smaller of the two BHs and \( \sigma \) the stellar velocity dispersion. Before the binary becomes hard, each BH acts as an independent point mass and its orbit decays in roughly the way described by Chandrasekhar’s dynamical friction formula (except that the relevant mass is that of the BH plus any stars from the original cusp that remain bound to it). The evolution in this regime is essentially collisionless, i.e., independent of the masses of the background stars. After the formation of the hard binary, the two BHs move about the galactic center like a single particle of mass \( M_{12} = m_1 + m_2 \) and their interaction with passing stars is dominated by three-body encounters. The latter almost always result in the transfer of energy from the BH binary to the stars; as a consequence, stars are ejected from the nucleus and the binary shrinks (Hills 1992; Quinlan 1996).

Simulating the merger of two galaxies, each containing a dense stellar cusp and a central point mass, is computationally demanding, and essentially no such simulations have appeared in the literature to date. Barnes (1999) described mergers between identical spherical galaxies with Dehnen’s (1993) density law, having central power-law cusps with indices \( \gamma = (1, 1.5, 2) \), but no BHs. He used a tree code with fixed time step. Barnes found that the \( \gamma = 1 \) and \( \gamma = 1.5 \) cusps became slightly shallower following the merger but that the \( \gamma = 2 \) cusps were nearly unchanged. Holley-Bockelmann & Richstone (1999) obtained similar results in simulations of unequal-mass mergers, also without BHs; they fixed the potential corresponding to the larger galaxy and assumed that the smaller galaxy remained spherical as its orbit decayed. They found that the secondary
remained intact as long as its initial density was higher than that of the primary, as expected in mergers between real galaxies. Neither of these studies was able to produce weak cusps in galaxies that did not contain them initially.

Merger simulations including nuclear BHs have been carried out but only from rather unrealistic initial conditions, typically consisting of galaxies with large, constant-density cores (Ebisuzaki et al. 1991; Makino et al. 1993; Governato et al. 1994; Makino & Ebisuzaki 1996). The BHs were found to generate still larger cores in the merger products due to heating of the stars; the core tended to grow in such a way as to maintain rough proportionality between core radius and half-mass radius, and between core mass and (total) BH mass. This result is consistent with observed parameter relations (Faber et al. 1997) if core radius is identified with break radius $r_b$. The central density profiles in some of these simulations showed weak, power-law cusps. Nakano & Makino (1999a,b) argued that this was due simply to the removal of low-energy stars by heating from one or both of the BHs. Holley-Bockelmann & Richstone (2000) extended their earlier, BH-free simulations to the case of a fixed primary containing a central point mass and an evolving secondary with no BH. They found that the secondary was usually disrupted by tidal forces from the primary’s BH; however this result might be modified if a BH were added to the secondary as well.

A collisional $N$-body code is required to follow the evolution once the BH binary becomes hard. This statement is at first sight surprising, since the rate of evolution of a hard binary in a background of low-mass perturbers depends only on their total density, not their individual masses (Hills 1992; Quinlan 1996). However the rate of binary decay depends also on how widely the binary wanders about the center of the potential. A fixed binary rapidly ejects all of the stars that pass near it; continued evolution requires that the binary wander to regions of higher density. It is able to do this because the low stellar density implies a small gravitational restoring force, and because the binary receives kicks from ejected stars. The resultant random walk depends on the size of the kicks, hence on the mass ratio $m_*/M_*$ and the number of particles $N$. The decay rates observed in the simulations with largest $N$, $N \approx 3 \times 10^5$ (Makino 1997; Quinlan & Hernquist 1997; Hensendorf 1999), imply a binary lifetime (until gravitational radiation coalescence) of order a Hubble time (Merritt 2000); if one extrapolates the decay rate to still larger $N$ using the $N$-dependence estimated by Makino (1997), $\sim N^{-1/2}$, one concludes that BH binaries in real galaxies might be expected to often survive long enough for a third BH to find its way to the center (Valtonen 1996). However Quinlan & Hernquist (1997) saw evidence that the decay rate may saturate for $N \gtrsim 10^5$. Another possibility is that gas dynamical processes accelerate the decay (Gould & Rix 2000).

Although the timescale for evolution of massive BH binaries to complete coalescence is still unclear, most of the damage that the binary inflicts on the stellar cusp takes place shortly after it becomes hard. The mass ejected is

$$M_{ej} \approx M_{12} \ln(a_h/a)$$

(Quinlan 1996), hence the binary ejects of order its own mass after shrinking by a factor of only two. This is probably sufficient to convert a strong cusp into a weak cusp (Table 1). Quinlan & Hernquist (1997) presented density profiles that look qualitatively similar to those in bright galaxies, with a clear break
Figure 1. Upper panel: Separation between the BHs in a set of merger simulations. Time is measured from the start of the simulation in units such that the crossing time in a single galaxy is $\sim 2.2$. The BHs form a hard binary at $t = t_h \approx 11.1$. Squares are from a $1.31 \times 10^5$-particle integration with the tree code GADGET; solid line is from the NBODY6++ integration with $N = 65,536$. The binary separation saturates at roughly the softening length $h$ in the tree code simulation, while the direct-summation code is able to follow the decay to arbitrarily small scales. Lower panel: Stellar density as a function of time at the center of the $N = 65,536$, NBODY6++ integration. Density is defined as number of particles within a sphere of radius 0.05 centered on the origin. The density drops sharply at $t \approx t_h$ when the hard binary forms, then decays more slowly thereafter.
Figure 2. Transformation of two density cusps into one. Contours show the surface density in the NBODY6++ merger simulation with 65,536 particles. Contours are separated by 0.13 in the logarithm; dark circles are the BHs. Projection is onto the initial orbital plane. Frames are separated by unequal time intervals; the first frame is at $t = 10.6$, the last at $t = 11.5$, and the BH binary becomes hard approximately at frame 8, $t = 11.0$. The two cusps remain nearly intact until after the BHs form a hard binary, at which point the density drops suddenly (see also Fig. 1b).
radius; some of these profiles even exhibit central minima. The decaying binary also has a strong influence on the nuclear kinematics, since ejected stars are predominantly on eccentric orbits; the remaining stars exhibit a tangentially-biassed velocity ellipsoid. Detecting this velocity polarization in real galaxies would be difficult due to the low surface brightnesses of weak-cusp galaxies.

Taken together, these simulations probably reproduce much of the interesting dynamics that takes place during the merger of two galaxies containing nuclear BHs. Among the interesting questions yet to be answered are: (1) Can a shallow power-law cusp be produced from the merger of two galaxies with steep cusps and nuclear BHs? (2) Can a shallow cusp survive the accretion of a small dense galaxy with a nuclear BH? (3) What is the precise $N$-dependence of the wandering amplitude and decay rate of a BH binary in a galactic nucleus? (4) Is the structure of the resultant stellar core also $N$-dependent?

In addressing these questions, one would ideally like to simulate mergers in a continuous way from the largest, kiloparsec scales down to the sub-parsec scales that characterize the BH binary. We describe in the next section a first attempt to carry out this ambitious program.
Figure 4. Wandering of the BH binary in the 32,768-particle, NBODY6++ merger simulation. Each line traces the location of one BH, from time $t = 10.6$, shortly before the binary becomes hard, to $t = 20.0$. The upper left panel is projected onto the initial orbital plane of the two galaxies. Wandering begins suddenly when the binary becomes hard, and its amplitude increases with time.
Figure 5. Stellar velocity fields at the end of the $N = 65,536$ merger simulation, as viewed from the initial orbital plane. Left panels show contours of the mean velocity and the major-axis rotation curve. Right panels show the velocity dispersion.

3. A Merger Simulation

We report the simulation of a merger between two spherical, equal-mass galaxies with steep central density cusps surrounding nuclear BHs. We first integrated the entire merger using the tree code GADGET (Springel et al. 2000), a parallel algorithm with continuously variable time steps. GADGET has a fixed spatial resolution determined by a softening length $h$, which was chosen to be slightly smaller than $a_h$, the expected separation of the hard BH binary. To follow the evolution of the binary at late times when $a < h$, we continued the integration using NBODY6++, a parallel, multiple-time-step version of Aarseth’s direct-summation code. The particle coordinates as computed with GADGET were extracted at a time before the formation of the hard binary and used to generate initial conditions for NBODY6++. The early evolution as computed by NBODY6++ was compared to that of GADGET over the same time interval to ensure that the tree code had accurately integrated the equations of motion prior to extraction of the coordinates.

The initial galaxies were generated from Dehnen’s (1993) law with central density dependence $\rho \propto r^{-\gamma}$, $\gamma = 2$. Initial velocities were assigned based on an isotropic distribution function that accounts for the presence of the BH
Each galaxy was given $N = 1.31 \times 10^5$ equal-mass particles; the two particles representing the BHs were given masses 1% that of their host galaxies. Two, randomly-selected subsets of particles were extracted for the later integration with \texttt{NBODY6++}, with $N = 32,768$ and $N = 65,536$. The initial separation between the galaxies was four times the half-mass radius of each, where $r_{1/2} = 1.0$, and the initial orbital velocity was 1/2 that of a circular orbit. The internal orbital period at the half-mass radius of either galaxy was 8.8 in $N$-body units; the \texttt{GADGET} integration was carried out for $\sim 2$ of these periods and required approximately 10 days using 8 processors on the Rutgers Sun E10000 supercomputer. At the time of this writing, the \texttt{NBODY6++} integration with $N = 32768$ had advanced until a time of $\sim 20$ and the integration with $N = 65,536$ until a time of $\sim 12$.

The separation between the two BH particles as a function of time is shown in Fig. 1a. As computed by \texttt{GADGET}, the binary separation stalls at $a \approx h$, the softening length, soon after the BHs have fallen to the center and formed a hard binary. With \texttt{NBODY6++}, the binary continues to slowly decay as it exchanges energy with the stars. However most of the damage done by the binary to the cusp takes place very shortly after the binary forms. The central density (Fig. 1b) drops by a large factor in just a few orbital periods of the binary. This behavior is consistent with a simple model (Merritt 2000) which predicts a mass ejected by the binary of

$$M_{ej} = M_{12} \ln \left( \frac{4(t - t_h)}{t_0} \right)$$

with $t_h$ the time of formation of the hard binary and $t_0 \approx 1.76GM_{12}/\sigma_3^2 \approx 0.02$ in $N$-body units. The transformation of two density cusps into one is shown in Fig. 2; here again, the very sudden drop in density at $t \approx t_h$ is apparent.

Fig. 3 shows the change in the stellar density profile during the time of its most rapid evolution, $t \gtrsim t_h$. The cusp evolves quickly from its initial $r^{-2}$ dependence to a shallower power law, $\rho \sim r^{-1}$, and a break appears at a radius $\sim 0.03 \sim 2.4 \times a_h$. Thereafter the profile continues to evolve toward shallower slopes but very slowly, as the binary continues to eject stars. This figure confirms for the first time that the merger of two galaxies with steep cusps and BHs can produce a weak, power-law cusp. Bright ellipticals and bulges sometimes have even shallower central profiles (Gebhardt et al. 1996). Increasing the masses of the BHs would probably produce weaker cusps (e.g. Quinlan & Hernquist 1997), although the BHs in our simulation are already uncomfortably large. The continued, slow evolution in the central density as the binary ejects stars (Fig. 1b) might also produce a shallower profile; repeated mergers would probably have a similar effect. Future work should be directed toward understanding which of these explanations accounts for the observed trend of cusp slope with galaxy luminosity.

The sharply lowered density at $t \gtrsim t_h$ causes another change in the behavior of the binary, as shown in Fig. 4. The binary begins suddenly to wander over a region much larger than $a_h$. Wandering was inferred by Quinlan & Hernquist (1997) and Makino (1997) in their $N$-body studies but little is known about its properties. The wandering is enabled by the reduced force gradients in the low-density core; it is driven by the ejection of single stars that interact strongly.
with the binary and hence its amplitude depends on the mass ratio $m_\ast/M_{12}$. Fig. 4 suggests that the wandering amplitude is constant or even increasing with time. The $N$-dependence of the wandering amplitude is another important topic for future study.

The kinematical structure of the merged galaxy at late times is shown in Fig. 5. The rotation curve reflects primarily the initial orbital angular momentum of the galaxies and not the presence of the BHs; it peaks at a radius of $\sim$ a few times $r_b$, similar to what is seen in merger simulations without BHs (e.g. Bendo & Barnes 2000) and in real, high-luminosity ellipticals (e.g. Statler et al. 1999). It is very different from the rotation curves seen in fainter galaxies (e.g. M32, Joseph et al. 2000) which remain flat or rising into the sphere of influence of the BH. The velocity dispersions do continue to rise toward the center, roughly as $r^{-1/2}$, as they did in the initial galaxy models.

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