A method to correct third and fourth order moments in turbulent flows

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Abstract. It is well known that spatial averaging, resulting from the finite size of a hot-wire probe, significantly affects the accuracy of such measurements in turbulent flows close to the wall. Here, a theoretical model which describes the effect of the spatial filtering of hot-wire probes on the third and fourth order moments of the streamwise velocity is presented. The model, which is based on the three (four) point velocity correlation function for the third (fourth) order moment, shows that the filtering can be related to a characteristic length scale which is an equivalent of the Taylor transverse micro-scale for the second order moment. The capacity of the model to accurately describe the attenuation is validated against direct numerical simulation (DNS) data of a zero pressure-gradient turbulent boundary layer. The DNS data allow the filtering effect to be appraised for different wire lengths and for the different moments. A procedure, based on the developed model, to correct the measured moments in turbulent flows is finally presented. The method is applied by combining the response of two single hot-wire sensors with different wire lengths. The technique has also been validated against spatially averaged DNS data showing a good capacity to reconstruct the actual profiles over the entire height of the boundary layer except, for the third order moment, in the region where the latter is close to zero.

1. Introduction

While, to date, there are no measurement techniques capable to reach the versatility of hot-wire probes and their frequency response, the issue of their spatial resolution is still a matter of debate regarding the effective accuracy achievable with such a measurement tool. Naturally, the trend in the past years has been driven by the idea that the spatial resolution could only be improved by decreasing the hot-wire length, which has been confirmed by recent experiments (Bailey et al., 2010). On the other hand, new theoretical approaches and empirical fits have quantified such an effect in particular for the variance of the measured streamwise velocity component (Hutchins et al., 2009) and proposed correction schemes (Monkewitz et al., 2010; Chin et al., 2010; Smits et al., 2011). Despite the fact that this latter issue is still far from being completely analyzed, there is even more uncertainty about the measured higher order moments where almost no experimental work has been done in order to quantify such effects as a function of the wire length (Örlü & Alfredsson, 2010).
Following the approach used by Dryden et al. (1937) and Frenkiel (1949), Segalini et al. (2011a) developed an analytical model which describes the effect of spatial filtering of hot-wire probes on the mean and the variance of the streamwise velocity in turbulent wall-bounded flows. The model, which is based on the two point velocity correlation function, showed that the filtering could mainly be related to the transverse Taylor micro-scale. In their work, Segalini et al. analyzed also the effects of the second order terms in the model equation and used the same relation to assess the effect of a probe misalignment.

Under the same assumptions, Segalini et al. (2011b) developed a simplified model to correct the filtering effect by assuming that this process is only related to the transverse Taylor microscale. Therefore the model was used to formulate a procedure to evaluate both turbulence intensity and the transverse Taylor microscale in turbulent flows. The method was applied by combining the response of two single hot-wire sensors with different wire lengths and was firstly employed to high Reynolds number data from turbulent boundary layer experiments.

The aim of the present work is to show that by using a similar procedure it is possible to extend the theoretical arguments to the measured higher order velocity moments. The drawbacks of this approach is the expected increased complexity of the integrals involved which take into account the filtering process.

2. The proposed model

2.1. The third order moment

In the following, for the sake of simplicity, we assume a linear filtering of the hot-wire probe, namely:

\[ u_m = \frac{1}{L} \int_0^L u(s) \, ds, \]  

where \( s \) is a scalar coordinate along the wire direction, the subscript \( m \) denotes the measured quantity and \( L \) is the wire length. From now on, in order to simplify the notation, the time dependence of the instantaneous quantities inside the average operator \( \langle \cdot \rangle \) will be omitted.

Under the same assumptions previously discussed by Segalini et al. (2011b), the measured third order moment can be expressed as:

\[ \langle u'^3_m \rangle = \frac{1}{L^3} \int_{[0,L]^3} \langle u'(s) u'(q) u'(r) \rangle \, dsdqdr. \]  

By introducing now the three point coefficient \( \chi_3(s, q, r) \) such that

\[ \chi_3(s, q, r) \langle u'^3(s) \rangle = \langle u'(s) u'(q) u'(r) \rangle, \]  

and assuming a statistically homogeneous flow in the spanwise direction, equation (2) can be re-written as:

\[ F_3 = \frac{\langle u'^3_m \rangle}{\langle u'^3 \rangle} = \frac{1}{L^3} \int_{[0,L]^3} \chi_3(s, q, r) \, dsdqdr. \]  

It is worth pointing out that, since \( \langle u'^3 \rangle \) is a continuous statistic that assumes positive or negative values at different points, a zero crossing is expected somewhere. Since the triple-point product at the right hand side of (3) is not necessarily zero in this position, this means that the triple-point correlation coefficient may diverge towards \( \pm \infty \), breaking all the hypotheses that are assumed in the following. Therefore we will restrict our attention to regions far from zero crossings of \( \langle u'^3 \rangle \).

Since the flow is statistically homogeneous \( \chi_3 \) is only a function of the relative distances between the points \( s, q \) and \( r \). By defining now \( r_1 = q - s \) and \( r_2 = r - q \), it is possible
to introduce the change of variables \((s, q, r) \rightarrow (s, s + r_1, s + r_1 + r_2)\) in order to state that \(\chi_3 (s, q, r) = \chi_3 (r_1, r_2)\) to which the following symmetry conditions apply:

\[
\begin{cases}
\chi_3 (0, 0) = 1 \\
\chi_3 (A, B) = \chi_3 (B, A) \\
\chi_3 (A, B) = \chi_3 (-A, -B) \\
\chi_3 (A, B) = \chi_3 (B + A, -B)
\end{cases}
\forall A, B \in \mathbb{R}.
\tag{5}
\]

The simplest polynomial expression which satisfies the constraints (5) is:

\[
\chi_3 (r_1, r_2) = 1 + \gamma \left( r_1^2 + r_2^2 + r_1 r_2 \right) + O \left( r_1^3, r_2^3 \right),
\tag{6}
\]

where

\[
\gamma = \frac{1}{2} \frac{\partial^2 \chi_3}{\partial r_1^2} (0, 0) = \frac{\text{sign} (\gamma)}{\lambda_3^2}.
\tag{7}
\]

The above defined \(\lambda_3 > 0\) is the equivalent of the transverse Taylor microscale for the two-point correlation coefficient. However, differently from the latter, \(|\chi_3|\) is not limited to 1 and can reach values up to infinity when \(\langle u'^3 \rangle \rightarrow 0\). Consequently \(\gamma\) can be positive or negative. The characteristic scale \(\lambda_3\) can be related to velocity statistics by means of the following expression:

\[
\lambda_3^2 = \frac{2 \left| \langle u'^3 \rangle \right|}{\left| \langle u' (\partial u'/\partial z)^2 \rangle \right|} = 2 \frac{\langle u'^3 \rangle}{\langle u'^2 (\partial u'/\partial z)^2 \rangle},
\tag{8}
\]

since \(\frac{\partial}{\partial z} \langle u'^2 (\partial u'/\partial z)^2 \rangle = 0\) by spanwise homogeneity.

The application of the above mentioned change of variables to equation (4) leads, after the application of Fubini’s theorem, to:

\[
F_3 = \frac{2}{L^3} \int_0^L \left( L \int_{-L}^{-r_2} \chi_3 dr_1 - r_2 \int_{-r_2}^{L-r_2} \chi_3 dr_1 - \int_0^{L-r_2} r_1 \chi_3 dr_1 + \int_{-L}^{-r_2} r_1 \chi_3 dr_1 \right) dr_2,
\tag{9}
\]

where the integration in \(s\) has already been performed. A further simplification can be done to expression (9) by applying another change of variables, viz. \((r_1, r_2) \rightarrow (\hat{r}_1 \lambda_3, \hat{r}_2 \lambda_3)\) and by defining the quantity \(\beta = L/\lambda_3\):

\[
F_3 = \frac{2}{\beta^3} \int_0^\beta \left( \beta \int_{-\beta}^{-\hat{r}_2} \chi_3 d\hat{r}_1 - \hat{r}_2 \int_{-\hat{r}_2}^{\beta-\hat{r}_2} \chi_3 d\hat{r}_1 - \int_0^{\beta-\hat{r}_2} \hat{r}_1 \chi_3 d\hat{r}_1 + \int_{-\beta}^{-\hat{r}_2} \hat{r}_1 \chi_3 d\hat{r}_1 \right) d\hat{r}_2.
\tag{10}
\]

To proceed further, we need an ansatz of the function \(\chi_3 (r_1, r_2)\). Of course it is expected that this function changes with the Reynolds number and with the distance from the wall but we will instead assume \(\chi_3\) to be self-similar and only related to a single characteristic length scale. The simplest ansatz is the polynomial expression (6), truncated at the second order terms, which leads to the simple formula:

\[
F_3 \approx 1 + \text{sign} (\gamma) \frac{\beta^2}{4},
\tag{11}
\]

and has the same drawbacks as the one proposed for the variance by Frenkel (1949) and Segalini et al. (2011b), namely it is valid only for very low \(\beta\). This requirement is critic, especially in the neighborhood of zero crossings of \(\langle u'^3 \rangle\) where \(\chi_3 \rightarrow \pm \infty\), \(\lambda_3 \rightarrow 0\) and \(\beta \rightarrow +\infty\) for any wire length \(L\). Away from these regions, expression (11) is valid but only for very low \(L\) values, due to the quadratic behavior of equation (6).
An exponential expression can alternatively be proposed to reduce the decay rate of the polynomial $\chi_3$ ansatz, assuming that $\gamma < 0$ (which is usually the case away from the zero crossing points). The simplest exponential expression which satisfies the symmetry conditions is:

$$\chi_3 = \exp \left( -\frac{r_1^2 + r_2^2 + r_1 r_2}{\lambda_3^2} \right) = \exp \left[ - \left( \hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_1 \hat{r}_2 \right) \right].$$  \hspace{1cm} (12)

It is important to stress that this correction will work properly only if $\gamma < 0$, namely with a decay of the triple-point coefficient close to the origin. The substitution of equation (12) into (10) leads to a new expression for the attenuation (and attenuation only, since $\gamma$ is assumed to be negative) of the third order moment, viz.:

$$F_3 \approx \frac{2}{\beta^3} \int \left[ (\beta - \hat{r}_2) I_1 (\hat{r}_2; \beta - \hat{r}_2) - \beta I_1 (\hat{r}_2; -\beta) + \hat{r}_2 I_1 (\hat{r}_2; -\hat{r}_2) - I_2 (\hat{r}_2; \beta - \hat{r}_2) + I_2 (\hat{r}_2; 0) + I_2 (\hat{r}_2; -\hat{r}_2) - I_2 (\hat{r}_2; -\beta) \right] d\hat{r}_2,$$  \hspace{1cm} (13)

which does not allow to be expressed in analytical form because the integration in $\hat{r}_2$ must be performed numerically. The functional expressions $I_1(\hat{r}_2; \alpha)$ and $I_2(\hat{r}_2; \alpha)$ are:

$$I_1(\hat{r}_2; \alpha) = \frac{\sqrt{\pi}}{2} \exp \left( -\frac{3}{4} \hat{r}_2^2 \right) \left[ \text{erf} \left( \frac{\alpha + \hat{r}_2}{2} \right) - \text{erf} \left( \frac{\hat{r}_2}{2} \right) \right],$$

$$I_2(\hat{r}_2; \alpha) = -\frac{1}{2} \exp \left( -\frac{3}{4} \hat{r}_2^2 \right) \left[ \exp \left[ - \left( \alpha + \frac{\hat{r}_2}{2} \right)^2 \right] - \exp \left[ - \left( \frac{\hat{r}_2}{2} \right)^2 \right] + \frac{\sqrt{\pi}}{2} \hat{r}_2 \left[ \text{erf} \left( \alpha + \frac{\hat{r}_2}{2} \right) - \text{erf} \left( \frac{\hat{r}_2}{2} \right) \right] \right].$$

Following the spirit of Segalini et al. (2011b), a third ansatz is also proposed here because the exponential ansatz (c.f. equation (12)) decays too rapidly for high $\beta$. This task can be accomplished by defining a new spatial variable:

$$\tilde{r} = \sqrt{r_1^2 + r_2^2 + r_1 r_2},$$

and a simple exponential series as a new ansatz:

$$\chi_3 = \sum_{j=1}^{4} A_j \exp \left( -j \frac{\tilde{r}}{\lambda_3} \right) \text{ with } A_1 = 1 \quad A_2 = 2 \quad A_3 = -3 \quad A_4 = 1.$$  \hspace{1cm} (17)

Segalini et al. (2011b) found that this expression worked better than the others in the variance attenuation estimation and therefore it is reported here as well. Despite the simplicity of the ansatz, the integration of equation (10) is not possible even at the first integration level and therefore a 2D numerical integration procedure must be pursued to calculate the shape of the attenuation function $F_3$ with this latter assumption.

It is noteworthy that for small wire lengths the measured skewness factor goes as:

$$S_{\text{meas}} = \frac{\langle u_m^3 \rangle}{\langle u_m^2 \rangle^{3/2}} \approx S \left[ 1 + \frac{L^2}{4} \left( \frac{1}{\lambda_3^2} + \frac{\text{sign}(\gamma)}{\lambda_3^2} \right) \right],$$

where $S$ is the real skewness. It follows that, when $\gamma < 0$, the wire length terms will subtract each other, with a consequent reduction of spatial resolution effects on the skewness ratio. The opposite happens for $\gamma > 0$ where both effects add to each other leading to a larger error.
2.2. The fourth order moment
Under the same assumptions of linear filtering previously discussed, the measured fourth order moment is:

\[
\langle u_{m}^{4} \rangle = \frac{1}{L^{4}} \int_{[0,L]}^{4} \langle u'(s) u'(q) u'(r) u'(m) \rangle ds dq dr dm.
\] (19)

By introducing now the four points coefficient \(\chi_{4}(s,q,r,m)\) such that

\[
\chi_{4}(s,q,r,m) \langle u_{m}^{4} \rangle = \langle u'(s) u'(q) u'(r) u'(m) \rangle,
\] (20)

equation (19) becomes:

\[
F_{4} = \frac{\langle u_{m}^{4} \rangle}{\langle u_{m}^{2} \rangle^{2}} = \frac{1}{L^{4}} \int_{[0,L]}^{4} \chi_{4}(s,q,r,m) ds dq dr dm.
\] (21)

Differently from the previous section, the fourth order moment is a positive statistic, therefore no zero crossings are possible, and the present approach should work better than that in section 2.1. Also it can be demonstrated that \(|\chi_{4}| \leq 1\) through the use of the Cauchy-Schwarz inequality.

Since the flow is statistically homogeneous \(\chi_{4}\) is expected to be a function of the relative distances between the points \(s, q, r\) and \(m\) only. By defining \(r_{1} = q - s\), \(r_{2} = r - q\) and \(r_{3} = m - r\), it is possible to introduce the change of variables \((s,q,r,m) \rightarrow (s + r_{1}, s + r_{1} + r_{2}, s + r_{1} + r_{2} + r_{3})\) so that \(\chi_{4}(s,q,r,m) = \chi_{4}(r_{1},r_{2},r_{3})\) and the following symmetry conditions apply to \(\chi_{4}\):

\[
\begin{align*}
\chi_{4}(0,0,0,0) &= 1 \\
\chi_{4}(A,B,C) &= \chi_{4}(C,B,A) \\
\chi_{4}(A,B,-C) &= \chi_{4}(-A,-B,-C) \\
\chi_{4}(A,B,C) &= \chi_{4}(A+B,-B,-C) \\
\chi_{4}(A,B,-C) &= \chi_{4}(A,B+C,-C)
\end{align*}
\] (22)

The simplest polynomial expression which satisfies the constraints in (22) is:

\[
\chi_{4}(r_{1},r_{2},r_{3}) = 1 + \gamma_{4} \left[ r_{1}^{2} + r_{2}^{2} + \frac{4}{3} r_{2}(r_{1} + r_{2} + r_{3}) + \frac{2}{3} r_{1} r_{3} \right] + \mathcal{O}(r_{1}^{2}, r_{2}^{2}, r_{3}^{2})
\] (23)

where

\[
\gamma_{4} = \frac{1}{2} \frac{\partial^{2} \chi_{4}}{\partial r_{1}^{2}}(0,0) = -\frac{1}{\lambda_{4}^{2}}.
\] (24)

The characteristic scale \(\lambda_{4}\) can be related to velocity statistics by means of the following expression:

\[
\lambda_{4}^{2} = 2 \left[ \langle u'^{4} \rangle \frac{\partial^{2} \chi_{4}}{\partial u'^{2} \partial u'^{2}} \right].
\] (25)

The application of the above mentioned change of variables to equation (21) leads to:

\[
F_{4} = \frac{2}{L^{4}} \int_{0}^{L} \left\{ \int_{-L}^{L} \left[ \int_{-L}^{0} (L + r_{1} + r_{2}) \chi_{4} dr_{1} + \int_{0}^{-r_{2}} (L + r_{2}) \chi_{4} dr_{1} + \right. \right.
\]

\[
+ \int_{-r_{2}}^{L} (L - r_{1}) \chi_{4} dr_{1} \left. \right] dr_{2} + \int_{0}^{0} \left[ \int_{-L}^{L} (L - r_{1}) \chi_{4} dr_{1} + \int_{-r_{2}}^{L} (L + r_{1} + r_{2}) \chi_{4} dr_{1} + \right. \right.
\]

\[
+ \int_{-r_{2}}^{L} (L - r_{3}) \chi_{4} dr_{1} + \int_{-r_{2}}^{L} (L - r_{1} - r_{2} - r_{3}) \chi_{4} dr_{1} \left. \right] dr_{2} + \int_{0}^{L} \left[ \int_{-L}^{L} (L - r_{1}) \chi_{4} dr_{1} + \int_{-r_{2}}^{L} (L + r_{1} + r_{2}) \chi_{4} dr_{1} + \right. \right.
\]

\[
+ \int_{-L}^{L} (L - r_{1} - r_{2} - r_{3}) \chi_{4} dr_{1} \left. \right] dr_{2} \right\} dr_{3},
\] (26)
where the integration in \( s \) has already been performed. A further simplification can be done to (26) by applying the equivalent change of variables applied in section 2.1, namely

\[(r_1, r_2, r_3) \rightarrow (\hat{r}_1 \lambda_4, \hat{r}_2 \lambda_4, \hat{r}_3 \lambda_4)\]

and by defining the quantity \( \eta = L/\lambda_4 \):

\[
F_4 = \frac{2}{\pi^4} \int_0^\eta \left\{ \int_{\eta-r}^{\hat{r}_1} \left[ \int_{\eta-\hat{r}_2}^{\eta-r} (\eta + \hat{r}_1 + \hat{r}_2) \chi_4 d\hat{r}_1 + \int_0^{\hat{r}_3} (\eta + \hat{r}_2) \chi_4 d\hat{r}_1 + \int_{\eta-\hat{r}_2}^{\eta-r} (\eta + \hat{r}_1 + \hat{r}_2) \chi_4 d\hat{r}_1 + \int_{\eta-\hat{r}_2}^{\eta-r} (\eta - \hat{r}_3) \chi_4 d\hat{r}_1 + \int_0^{\hat{r}_3} (\eta - \hat{r}_2 - \hat{r}_3) \chi_4 d\hat{r}_1 \right] d\hat{r}_2 \right\} d\hat{r}_3.
\]

To proceed further, we need an ansatz of the function \( \chi_4 (r_1, r_2, r_3) \). The use of the polynomial expression (23), truncated at the second order terms, leads to the simple formula:

\[
F_4 \approx 1 - \frac{\eta^2}{3},
\]

which has the same drawbacks of the one proposed in section 2.1, namely it is valid only for very low \( \eta \) or, equivalently, very low \( L \) due to the quadratic behavior of equation (28).

Two different ansatz can alternatively be used to improve the behavior for large \( L \), namely the quadratic exponential one:

\[
\chi_4 (r_1, r_2, r_3) = \exp \left[ - \left( \hat{r}_1 + \hat{r}_3 + \frac{4}{3} \hat{r}_2 (\hat{r}_1 + \hat{r}_2 + \hat{r}_3) + \frac{2}{3} \hat{r}_1 \hat{r}_3 \right) \right],
\]

and the simple exponential one:

\[
\chi_4 (r_1, r_2, r_3) = \sum_{j=1}^{4} A_j \exp \left( -j \frac{\hat{r}}{\lambda_4} \right) \quad \text{with} \quad \hat{r} = \sqrt{\hat{r}_1^2 + \hat{r}_3^2 + \frac{4}{3} \hat{r}_2 (\hat{r}_1 + \hat{r}_2 + \hat{r}_3) + \frac{2}{3} \hat{r}_1 \hat{r}_3},
\]

and the \( A_j \) coefficients are the same as the ones stated in equation (17). Unfortunately equation (27), with both expressions (29) and (30), cannot be solved analytically but must be integrated with 2D or even 3D numerical schemes, hence no closed-form expression is reported here.

In analogy with section 2.1, a correction for the flatness factor is proposed under the hypothesis of small \( L \) values, viz.:

\[
K_m = \frac{\langle u_{m_4}^4 \rangle}{\langle u_{m_2}^2 \rangle^2} \approx K \left[ 1 + \frac{L^2}{3} \left( \frac{1}{\lambda_4^2} - \frac{1}{\lambda_9^2} \right) \right],
\]

which demonstrates that the measured flatness factor \( K_m \) is less affected by the spatial resolution problem as compared to \( \langle u_{m_2}^2 \rangle \).

3. Model validation and filtering effects

The model presented in section 2 is tested using fully-resolved direct numerical simulation (DNS) data. We employ the data provided by a recent large-scale simulation of a spatially evolving zero-pressure-gradient (ZPG) turbulent boundary layer (TBL) by Schlatter & Örlü (2010) which covers the range \( Re_\theta \approx 200 \) up to \( Re_\theta = 4300 \), with \( \theta \) being the momentum loss thickness. Given the long domain used for the simulation, it is assured that the turbulence has reached a fully
developed state independent of initial and boundary conditions. For the current purposes, we focus on a Reynolds number of $Re_\theta = 4000$ corresponding to about $Re_\tau \approx 1300$, where $Re_\tau$ denotes the ratio between the boundary layer thickness and the viscous length scale. At each of the 768 spanwise ($\Delta z^+ = 5.4$) and 50 wall-normal positions 55,000 equidistantly spaced ($\Delta t^+ \approx 0.25$) time samples for the streamwise velocity have been recorded and are used in the present analysis.

In order to mimic the finite length of a hot-wire sensor by means of the DNS data, the raw time series are filtered along the spanwise direction with a physical-space top-hat filter, according to equation (1). The filter operation is conveniently implemented in Fourier space as a direct multiplication with the corresponding transfer function.

Figure 1 shows the estimated attenuation functions according to the model developed above. For sake of completeness the second order moment has also been added. The shaded area shows the attenuation obtained by filtering the DNS data across the boundary layer. The grey intensity gives the inner scaled wall normal position. As expected the attenuation factors decrease monotonically with the increase of the wire length. As it appears from figures a) and c) the exponential form of the attenuation function seems to work better, at least for the second and the fourth order moment. In these cases, both polynomial and Gaussian attenuation functions decay too rapidly. Note that the developed models correctly estimate the actual filtering at low values of $L/\lambda$ and in the region of the boundary layer close to the wall (light grey area). A different behavior is shown in figure b). Here a polynomial function seems to be more consistent with the filtered DNS data. Note that data in the region $10 < y^+ < 20$ shown in have been excluded in the estimation since in this region, due to the zero-crossing of the third order moment, the equations employed are ill-conditioned.

4. The correction procedure

Following a similar procedure to the one described in Segalini et al. (2011b) the models developed in section 2 can be used to estimate the real third and fourth order moments $\langle u'^3 \rangle$, $\langle u'^4 \rangle$ and the corresponding length scales ($\lambda_3$, $\lambda_4$) from the measurements provided by two hot-wire probes.

![Figure 1. Attenuation function, $F_2$, for different ($L/\lambda_g$) (see Segalini et al., 2011b) b) $F_3$ vs. ($L/\lambda_3$) c) $F_4$ vs. ($L/\lambda_4$) with the polynomial (diamonds), Gaussian (square) and exponential (circle) behavior. Shaded area corresponds to the respective attenuation function based on the spanwise filtered DNS data across the boundary layer, where the inner-scaled wall normal position, $y^+$, has been given through the colorbar. Note, that data in the region $10 < y^+ < 20$ has been excluded in b) due to the zero-crossing in the third order moment.](image)
with different wire lengths. Since a nonlinear algebraic system is obtained the solution must be performed numerically. Additional measurements obtained with different wire lengths can be included. In this case a least squared fitting procedure can be employed similarly to what has been done in Segalini et al. (2011b).

Figure 2 shows the three statistical moments of the streamwise velocity component for the original DNS data together with the signals filtered for various sensor lengths $L^+ = 22 - 87$. These lengths have been chosen since they correspond, in the simulated flow field, to probes which have a physical dimension close to real ones. While the small values are generally thought to give limited spatial resolution effects in the experimental practice, the largest one is considered as a measurement condition that should significantly affect the quality of the measured velocity variance and of the higher order moments. As expected, the second and fourth order moments decrease with increasing filter/sensor length. This effect is of course dominant near the wall and becomes almost negligible in the outer edge of the boundary layer. Moreover the attenuation seems to increase for higher order moments. The third order moment, on the other hand, does not show this monotonic behavior. As an example, for large wire lengths the third order moments does not show any minimum value at $y^+ = 30$, but a dramatic change in the distribution is observed.

The correction method can be finally used to reconstruct the unattenuated data using pairing of different wire lengths. The collapse of the reconstructed signals with the original data seems good for all the moments and at all distances from the wall. Only the third order moment in the region where it is very close to zero seems to deviate significantly from the real value. In the fourth order moment case the corrected signal is slightly underestimated in the region close to the near-wall peak, but remarkably accurate elsewhere.

Figure 3 shows that similar conclusions can be drawn when the same correction is applied to the skewness and flatness factors inside the boundary layer. As demonstrated in sections 2.1 and 2.2, the skewness and flatness ratio are less affected by spatial resolution than their corresponding statistical moments. This is especially true for the flatness ratio where $\lambda_4 \approx \lambda_g$ so that the spatial resolution effect decreases up to 25% of the original value. For the skewness ratio it works similarly except for the region close to the zero crossing of the third order moment. These observations were made already by Örlü & Alfredsson (2010) through the analysis of experimental data acquired with different wire-lengths.

5. Conclusions

An analytical model has been developed to describe the filtering effect of a finite length hot-wire sensor on the third and fourth order statistical moment of the stream-wise velocity component in a turbulent flow. By introducing a general expansion of the third and fourth order correlation function, it is possible to formulate a correction procedure which can be performed by using two (or more) single hot-wire sensors of different sensor lengths $L$. The analytical models describing the sensor attenuation have been validated against DNS data from a zero pressure-gradient turbulent boundary layer showing a good performance at low values of $L$ and in the region close to the wall. The exponential form of the expansion seems to be the best in reproducing the actual probe response for the variance and the flatness. Conversely, for the third order moment, it is the polynomial expansion which is more consistent with the actual probe response. However, in this case, due to the zero-crossing of the third order moment, the equations become ill-conditioned. Consequently, the proposed correction method seems to work quite well except in those regions where the third order moment is close to zero.
Figure 2. a) Inner-scaled variance profiles from DNS at $Re_\theta \approx 4000$: black solid line. Profiles with $L^+$ values of 22 (magenta), 33 (blue), 49 (cyan), 65 (green), and 87 (red), are simulated by spanwise filtering of the DNS. Circles show the estimates obtained by application of the proposed correction scheme. b) Profile of the inner-scaled third order moment. c) Profiles of the inner-scaled forth order moment. Data and color code in b) and c) correspond to those shown in a). Shaded area in b) denotes the region excluded in figure 1b), due to the zero-crossing of the third-order moment.

Figure 3. Skewness and flatness factor profiles for the same data as shown in Figure 2.

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