Research on Vibration of Magnetic Suspension Rotor System Caused By Magnetic Bearing Model Error —Closed-Loop Parameter Identification Method

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Abstract. Aiming at the inaccuracy of the magnetic model parameters in the magnetic levitation flywheel’s unbalanced vibration control algorithm, this paper proposes an online closed-loop identification method for the magnetic parameters, and feeds the identification results back to the unbalanced vibration control algorithm to obtain higher vibration control precision.

1. Introduction

Compared with mechanical ball bearings, the active magnetic bearing has the characteristics of contactless friction and active support of the bearing force [1]. The magnetic suspension rotor system allows the rotor to rotate at a high speed, and the magnetic bearing force can be adjusted to achieve active suppression of the disturbance caused by the rotor mass imbalance, which significantly reduces the vibration level of the magnetic bearing-rotor system. It has a good application prospect in the field of space attitude control using inertia momentum wheel and control torque gyro, which are sensitive to vibration level [2] [3].

The same-frequency vibration caused by mass imbalance is the main vibration component of the magnetic suspension rotor system [4], and it is also a research hotspot of vibration control [5]. There are two main methods for achieving co-frequency vibration suppression: one class identifies the position of the inertia main shaft, controls the rotation of the rotor around the inertia main axis [6], and the other class uses the same frequency disturbance power or torque as the feedback amount to achieve the imbalance by adjusting the current. Disturbance compensation [7]. The accuracy of the two types of co-frequency vibration suppression methods depends on the accuracy of the magnetic bearing model parameters. The reduction of the compensation accuracy caused by the parameter error is one of the main problems that restrict the vibration level of the magnetic bearing rotor system. The magnetic bearing model parameters are time variables related to the operating conditions of the magnetic bearing rotor system. Therefore, in order to improve the compensation accuracy of the unbalanced vibration, it is necessary to study the online identification method of the magnetic bearing model parameters, and obtain the model parameters in real time with high precision.
2. Research status at home and abroad

The linear control model near the equilibrium position is generally used in the design of the feedback control algorithm for the magnetic bearing rotor system. The displacement stiffness and current stiffness are the core parameters of the linearization model. At present, the methods for obtaining the current stiffness displacement stiffness can be roughly divided into two categories, namely analytical method and experimental method. The analytical method [8] [9] is to obtain the expression of the magnetic force of the rotor with respect to the rotor displacement and the control current. The magnetic force is linearized as a function of displacement and current near the working point. The primary coefficient of the displacement and current in the resulting expression is the displacement stiffness and Current stiffness. Since the magnetic bearing system is a complex mechanical electrical system, the structure and electrical parameters affecting the magnetic force are many. The commonly obtained magnetic analytical formula is based on many structural simplifications and assumptions, so the theoretical calculation results in poor displacement stiffness and current stiffness accuracy. For the experimental acquisition method of current stiffness and displacement stiffness, many scholars at home and abroad have done a lot of research. Among them, Kim S J [10] proposed an online LMS algorithm using three kinds of measurement information of the rotor as the excitation identification magnetic parameter, but the algorithm is only applicable to a magnetic bearing system equipped with a force sensor, and when the phase difference between the input and output is small, the convergence speed of the algorithm is significantly slower. Hou Eryong and Liu Kun proposed a magnetic bearing parameter identification algorithm based on online dynamic balance, but the algorithm needs to install a dynamic balance instrument, and the experimental operation is complicated.

In this paper, an online magnetic bearing parameter identification method based on active magnetic bearing unbalance vibration suppression algorithm is proposed. This method obtains the amplitude frequency of the magnetic bearing system by performing FFT transformation on the collected control current and displacement signals during the operation of the magnetic suspension rotor system. Characteristic.

3. Influence of Magnetic Bearing Model Error on Unbalanced Vibration Suppression

Due to the non-uniformity of the magnetic bearing rotor material, machining error and other factors, the magnetic suspension rotor inevitably has mass imbalance, so the rotor’s inertia main axis and the geometrical main axis are deviated, as shown in Fig. 1.

![Fig. 1 Rotor imbalance diagram](image-url)
The magnetic bearing rotor system generally uses a contactless displacement sensor to detect the geometrical axis position of the rotor, and controls the rotor to rotate around the geometric axis. Due to the deviation between the rotor geometry axis and the inertia axis, the magnetic bearing rotor system generates the same frequency disturbance power/torque under the combined action of the rotor unbalance force and the magnetic bearing control force. In order to simplify the analysis, after linearizing the magnetic bearing force at the equilibrium position, the magnetic bearing force equation with the same frequency unbalance disturbance is:

\[
\begin{bmatrix}
F_{Ax}'

F_{Ay}'

F_{Bx}'

F_{By}'
\end{bmatrix} = \begin{bmatrix}
k_x & 0 & 0 & 0 & k_x (x_x + \Theta_{Ax}) \\
0 & k_y & 0 & 0 & k_y (x_y + \Theta_{Ay}) \\
0 & 0 & k_x & 0 & k_x (y_x + \Theta_{Bx}) \\
0 & 0 & 0 & k_y & k_y (y_y + \Theta_{By}) 
\end{bmatrix} \mathbf{1}
\]

(1)

Where: \(k_x\) and \(k_y\) are the current stiffness and displacement stiffness of the equilibrium position of the magnetic bearing rotor system, respectively. \(\Theta_{Ax}\), \(\Theta_{Ay}\), \(\Theta_{Bx}\) and \(\Theta_{By}\) respectively represent the equivalent displacement perturbations introduced at the ends of the magnetic bearings A and B due to the existence of rotor mass imbalance.

On the basis of the identification of the same-frequency unbalance of the rotor by the adaptive notch filter, the same-frequency disturbance component in the control current is filtered out, and the feedforward compensation is performed on the unbalanced displacement stiffness component, that is, in the control current of each magnetic bearing. Add compensation for displacement stiffness \(i_{Ax}'\), \(i_{Ay}'\), \(i_{Bx}'\) and \(i_{By}'\), it is possible to realize the rotation of the rotor about the inertia axis, which is derived:

\[
\begin{bmatrix}
i_{Ax}'
i_{Ay}'
i_{Bx}'
i_{By}'
\end{bmatrix} = \begin{bmatrix}
k_x & 0 & 0 & 0 & k_x (x_x + \Theta_{Ax}) \\
0 & k_y & 0 & 0 & k_y (x_y + \Theta_{Ay}) \\
0 & 0 & k_x & 0 & k_x (y_x + \Theta_{Bx}) \\
0 & 0 & 0 & k_y & k_y (y_y + \Theta_{By}) 
\end{bmatrix} \mathbf{1}
\]

(2)

The unbalanced vibration control method with feedforward compensation of displacement stiffness is shown in the figure:

Fig.2 Displacement stiffness feedforward compensation
It can be seen that current stiffness and displacement stiffness are important parameters in unbalanced vibration control. The inaccuracy of current stiffness and displacement stiffness will affect the effect of vibration suppression.

4. Magnetic bearing parameter identification algorithm
In the unbalanced vibration control system, considering the low-pass filter time constant \(t_{fs}\) of the sensor and the AD sampling amplification factor \(k_{ad}\), the transfer function of the magnetic bearing-rotor system portion is:

\[
G(s) = \frac{k_{ad} \cdot k_i}{m_s^2 - k_i} \cdot \frac{1}{t_{fs}^2 \cdot s^2 + 2t_{fs} \cdot s + 1}
\]  

(3)

Let \(s = j\omega\), the frequency response function will be:

\[
G(j\omega) = \frac{k_{ad} \cdot k_i}{-m_s^2 - k_i} \cdot \frac{1}{1 - t_{fs}^2 \omega^2 + 2t_{fs} \cdot \omega \cdot j}
\]  

(4)

Thus the amplitude-frequency characteristics and phase-frequency characteristics of the system can be obtained:

\[
\left|G(j\omega)\right| = \frac{k_{ad} \cdot k_i}{-m_s^2 - k_i} \cdot \frac{1}{\sqrt{1 - t_{fs}^2 \omega^2}^2 + 4t_{fs}^2 \omega^2}
\]

\[
\angle G(j\omega) = -\arctan \left[ \frac{2t_{fs} \omega}{1 - t_{fs}^2 \omega^2} \right]
\]

(5)

Equation (4) shows that the amplitude-frequency characteristic of the magnetic suspension rotor system is a function of the parameter \(k_{ad}\) and \(k_i\). If the value of the amplitude-frequency characteristic \(\left|G(j\omega_1)\right|\) and \(\left|G(j\omega_2)\right|\) at two frequency points is known, the current stiffness and displacement stiffness values can be obtained from (4):

\[
k_i = \frac{-m \left|G(j\omega_1)\right| \left|G(j\omega_2)\right| \eta_1 \left(\omega_2^2 - \omega_1^2\right)}{k_{ad} \left|G(j\omega_1)\right| \eta_1 - \left|G(j\omega_2)\right| \eta_2}
\]  

(6)

In the formula:

\[
\eta_i = \sqrt{1 - t_{fs}^2 \omega_i^2}^2 + 4t_{fs}^2 \omega_i^2
\]  

(7)

Due to the influence of processing and installation errors and material unevenness, the detection surface of the displacement sensor will have a certain roundness error, remanence and non-uniformity, and it will be affected by electrical non-ideal characteristics. The sensor also has certain nonlinear characteristics. The displacement sensor output signal contains the multi-harmonic component of the same speed and multiplier of the speed, which named Sensor Runout.
\[
q_\nu = \begin{bmatrix}
q_{i,\text{max}} \\
q_{i,\text{min}} \\
q_{i,\text{nom}}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{n} \sigma_{i,m} \cos(i\omega t + \xi_{i,m}) \\
\sum_{i=1}^{n} \sigma_{i,b} \cos(i\omega t + \eta_{i,b}) \\
\sum_{i=1}^{n} \sigma_{i,m} \sin(i\omega t + \xi_{i,m}) \\
\sum_{i=1}^{n} \sigma_{i,b} \sin(i\omega t + \eta_{i,b})
\end{bmatrix}
\] (8)

\[i\] is the harmonic order, \(i = 1, 2, \ldots, n\); \(\sigma_{a,i}\) and \(\sigma_{b,i}\) are the amplitude of the second harmonic of the output signals of the sensors, \(\xi_{i,m}\) and \(\eta_{i,b}\) are the corresponding initial phase.

In addition, due to the non-ideal characteristics such as the magnetic center of the radial magnetic bearing and the center of the sensor detection surface, the roundness error of the radial magnetic bearing rotor, and the non-uniform magnetic field, the Magnet Runout, the radial direction of the magnetic suspension rotor in the magnetic bearing coordinate system The displacement will have multiple harmonic components of the same frequency and frequency multiplication as the rotational speed, resulting in multi-harmonic displacement stiffness forces and moments.

![Fig. 3 Radial rotor force surface roundness error](image)

It can be seen from the system vibration waterfall diagram (Fig. 4) that in the magnetic suspension rotor system, the amplitude of the same frequency \(\Omega\), the second frequency \(2\Omega\) and the triple frequency \(3\Omega\) of the rotational speed co-frequency signal and the sensor harmonic caused by the unbalance amount are large. Let \(w_1\) and \(w_2\) in equation 5 be \(\Omega\) and \(2\Omega\) respectively:

![Fig.4 Magnetic suspension rotor system vibration waterfall diagram](image)

According to (4), proposed the following identification system (Fig.5).
Taking into account the inter-spectral interference of unrelated frequency components, the author uses a tracking filter to obtain the rotor displacement signal of the sensor output and the same-frequency and multi-frequency components of the control current signal output from the power amplifier. In the figure, $N_r(s)$ is the frequency selector, $\varepsilon$ is the notch parameter, the negative feedback loop composed of the frequency selector and the notch parameter is the tracking filter $N(s)$.

The input of the tracking filter is $r(t)$, Output is $c(t)$, Then and satisfy the following expression:

$$c(t) = \left[ \sin(\Omega t) \cos(\Omega t) \right] \int \sin(\Omega t) r(t) dt$$  
$$ \int \cos(\Omega t) r(t) dt$$  \hspace{1cm} (9)

The transfer function $N(s)$ is:

$$N(s) = \frac{s}{s^2 + \Omega^2}$$  \hspace{1cm} (10)

When $\varepsilon \neq 0$, let $s = j\omega$, then the Amplitude-frequency characteristic of subsystem represented by $N(s)$ is:
\[
\begin{align*}
N_f(j\omega) & = 1 \quad \omega \in (\Omega - \Delta \omega, \Omega + \Delta \omega) \\
N_f(j\omega) & \approx 0 \quad \omega \in (-\infty, \Omega - \Delta \omega) \cup (\Omega + \Delta \omega, +\infty)
\end{align*}
\] (11)

Where: \(\Delta \omega\) means the bandwidth of \(N(s)\), the size of bandwidth is determined by the notch parameter \(\epsilon\). \(N(s)\) providing a gain of 1 at the frequency of \(\Omega\) and almost 0 at the other frequency points, thus achieving the purpose of frequency selection. Replace \(\Omega\) with \(2\Omega\), the multiplier component in the current signal and the displacement signal can be obtained. The amplitude of the two frequency components is then calculated using the FFT transform as in (7), where \(N\) is the number of points sampled by the selected frequency signal.

\[
X(k) = \sum_{k=0}^{N-1} x(n)W_{nk}^k, \quad k = 0, \ldots, N-1
\]

\[
W_{nk} = e^{-j\frac{2\pi nk}{N}}
\] (12)

Assume that the magnitude of the rotor displacement signal at \(\Omega\) and \(2\Omega\) are \(A_\Omega\) and \(A_{2\Omega}\), The amplitude of the current signal are \(I_\Omega\) and \(I_{2\Omega}\), Substitute these data to equation 5, get the analytical expression of the magnetic parameter

\[
k_x = \frac{m}{I_{2\Omega}} \left( \frac{A_{2\Omega} \eta_2}{I_{2\Omega}} \Omega^2 - 4 \frac{A_{2\Omega} \eta_1}{I_{2\Omega}} \Omega^2 \right)
\]

\[
k_\Omega = -\frac{m}{I_{2\Omega}} \frac{A_{2\Omega} \eta_2}{I_{2\Omega}} \left( 4 \Omega^2 - \Omega^2 \right)
\]

\[
k_{ad} = \frac{A_{2\Omega}}{I_{2\Omega}} \left( \frac{A_{2\Omega} \eta_2}{I_{2\Omega}} - \frac{A_{2\Omega} \eta_2}{I_{2\Omega}} \right)
\] (13)

5. Simulation analysis
In order to determine the current stiffness and displacement stiffness measurement error as the influence of the structural parameters of the displacement stiffness feedforward compensation in the active vibration control of the magnetic levitation rotor on the performance of the control system, in the Matlab simulation, the theoretical design values and experimental test values of the bearing stiffness are used respectively. The rotor is modeled and the theoretical design values of the magnetic parameters are used in the feedforward channel (taking the X channel as an example, The theoretical value of current stiffness parameter \(k_x\) is 132.26, the experimental test value is 184.74. The theoretical value of displacement stiffness \(k_x\) is 352000, and the experimental test value is 351400.) The obtained unbalanced vibration force output are shown in Fig 7:
Fig. 7 Influence of magnetic parameter accuracy on unbalanced vibration force

As the simulation results showed, when there is an error in the current stiffness displacement stiffness and the true value used by the current feed channel, the effect of vibration suppression will be obviously weakened.

In MATLAB simulation, two sinusoidal signals are used to simulate the harmonics of the sensor as the input signal for parameter identification, so that the influence of the identification signal on the rotor unbalance vibration force amplitude does not exceed 50%. Set the sampling frequency to 5KHz, buffer 1024 data points for FFT transformation, perform FFT transformation to the input signal for identify without the notch and after the notch trap, obtain the spectrum diagram as

Fig. 8 FFT transform spectrogram
It can be seen from the spectrum graph that when added the notch, the accuracy of the FFT transformation is reduced due to the interference of other frequencies in the system are removed, and the amplitude at the second frequency of the rotation speed is hardly visible, and after the trap is added, The spectrogram of the FFT transform is four symmetrical and clear vertical lines. This shows that other frequency components in the system have a great influence on the calculation of the amplitude of the identification point, and the trap can eliminate the interference.

When $\varepsilon =10$, the true value of the current stiffness in the radial x-axis direction and the identification value curve are obtained as shown in the Fig.9, and the values of the feedback coefficients of the notch filter are set to 1, 2, 5 and 10 respectively, and the simulation analysis is performed. The current stiffness and displacement stiffness values in the radial x direction are compared with the true values as shown in Table 1. When $\varepsilon$ is 1, the deviation rate between the experimental value of current stiffness and the design value is about 0.8%, and the deviation between the experimental value of displacement stiffness and the design value is about 3.9%. The deviation is related to the size of the feedback coefficient. The simulation shows that the larger the feedback coefficient is, the smaller the deviation between the identification value and the design value is.

| $\varepsilon$ | 1    | 2    | 5    | 10   |
|---------------|------|------|------|------|
| $k_{ix}/(V \cdot m^{-1})$ | 131.1752 | 132.1776 | 132.2592 | 132.2604 |
| $k_{ix}/(V \cdot m^{-1})$ | 338310   | 351160   | 351990   | 352000   |

6. Conclusion
The magnetic bearing magnetic parameter identification method based on the harmonic signal in the rotor system only needs the displacement sensor and current sensor of the magnetic bearing control system, and does not need to install external instruments such as a dynamometer. The method is simple and easy to implement, and has wide application range. The feedback coefficient of the notch in the algorithm affects the current stiffness displacement stiffness identification result. The larger the feedback coefficient, the more accurate the identification result.

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