Efficiently Finding Adversarial Examples with DNN Preprocessing

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Abstract. Deep Neural Networks (DNNs) are everywhere, frequently performing a fairly complex task that used to be unimaginable for machines to carry out. In doing so, they do a lot of decision making which, depending on the application, may be disastrous if gone wrong. This necessitates a formal argument that the underlying neural networks satisfy certain desirable properties. Robustness is one such key property for DNNs, particularly if they are being deployed in safety- or business-critical applications. Informally speaking, a DNN is not robust if very small changes to its input may affect the output in a considerable way (e.g. changes the classification for that input). The task of finding an adversarial example is to demonstrate this lack of robustness, whenever applicable. While this is doable with the help of constrained optimization techniques, scalability becomes a challenge due to large-sized networks. This paper proposes the use of information gathered by preprocessing the DNN to heavily simplify the optimization problem. Our experiments substantiate that this is effective, and does significantly better than the state-of-the-art.

Keywords: Adversarial Examples · Constrained Optimization · DNN Analysis

1 Introduction

It would not be an inexcusable exaggeration, if one at all, to say that Artificial Intelligence and Machine Learning touch every facet of our lives today. While this makes us more capable, it also necessitates that we be responsible in the use of these techniques and their artifacts, especially when we deploy them in safety- or business-critical applications. Consider a Deep Neural Network (DNN) guiding a self-driving car, signalling it to stop, slow down, or move at a traffic signal. We would like such a DNN to be trustworthy and robust. For instance, the DNN must classify a stop signal correctly even on a rainy day when the brightness
is low. In fact, we would like to reason about these formally, so that we can either produce an example which the DNN misclassifies merely due to a small, insignificant change, or guarantee that none exists. Such examples are called \textit{adversarial} examples, and they are useful not just for improving the network (through adversarial training) but also in deciding when the network should relinquish control to a more dependable entity.

The problem of finding adversarial examples, to demonstrate lack of robustness, has gained a lot of attention in the last several years. A number of techniques have been developed for this task, both \textit{complete} (e.g. [12,10]) and \textit{incomplete} (e.g. [7,21,25]), trading off scalability for precision and vice-versa. While we postpone the discussion of strengths and limitations of these to the related work (Sect. 4), the main challenge in this is to find the right balance of efficiency and completeness, particularly for large networks. This paper puts forth an approach that balances the two, by starting with an extremely light-weight incomplete method which can be refined, layer-by-layer, into a complete method.

In a recent work on abstraction-refinement of DNNs, Yizhak et al. [6] proposed preprocessing of DNNs to gather useful behavioral information of each neuron. In particular, they looked at how an increase or decrease in the value of an intermediate-layer (or, internal) neuron may affect (increase/decrease) the output values, and used this information to merge \textit{similarly-behaving} neurons into one. The key insight that we derive from their work is that this increment-decrement marking is capable of telling us how we may bring about a misclassification (if one exists) from \textit{any} layer. From the output layer, we know that we can get it by \textit{decreasing} the current winning class, and \textit{increasing} the runner-ups as much as possible. In the penultimate layer, because this layer is also marked \textit{w.r.t.} the output-marking, we know it is possible to bring about a misclassification by increasing the increment neurons and decreasing the decrement ones as much as possible. And, similarly, all the way to the input layer – we know that if we give the increment (decrement) input neurons as high (low) a value as we can, then we will get a misclassification at the output layer (and thus an adversarial example) if one exists.

This insight allows us to transform the problem of finding adversarial examples into an optimization problem. But there are a few challenges that arise in making this practicable. Firstly, many neurons may show a \textit{mixed} behavior, and an increment/decrement marking may not be possible in such cases. Secondly, between two increment neurons in the same layer, it is not clear which one should be prioritized (for increasing) over the other. The same is true for two decrement neurons. And even between an increment and a decrement neuron, it is not clear whether increasing the former is more important than lowering the latter, or the other way around. Essentially, a good objective function for the optimization task is unknown. Lastly, if we are unable to find an adversarial example at the first layer (or, for that matter, any layer except the last), can we guarantee that an adversarial example does not exist. This paper proposes the idea of using preprocessing information for finding adversarial examples efficiently, and also discusses how the aforesaid challenges may be addressed in this process.
In what follows, we cover the necessary background concept and present an illustrative example in Sect. 2. Due to lack of space, a formal presentation of our algorithm has been pushed to Appendix A. Sect. 3 shows the promise of our approach with the results of our initial experiments on ACAS Xu benchmarks, using a prototype implementation. We end with a discussion of the related and potential future work.

2 Background

A deep neural network (DNN) is described by an underlying weighted graph \( D = (N, E, W) \), where \( N \) is a set of nodes and \( E \) the set of edges. The set of nodes \( N \) is partitioned into successive layers, \( N_0, N_1, N_2, \ldots, N_k, N_y \), where layer \( N_0 \) is the input layer, \( N_1, N_2, \ldots, N_k \) are hidden layers and \( N_y, y = k + 1 \) is the output layer. The nodes of the graph are called neurons and are ordered within a layer with the \( j \)th neuron in the \( i \)th layer denoted as \( n_{ij} \). Each neuron in \( N_i \) is connected to neurons in \( N_{i+1} \) by directed edges from the edge set \( E \subseteq N \times N \), which have a real valued weight \( W : E \rightarrow \mathbb{R} \). When an input is applied to the neurons of \( N_0 \), every neuron \( n_{ij} \), for \( 0 < i \leq k+1 \) computes a weighted sum over values of neurons in the previous layer that are connected to it and adds a real valued bias to it. Moreover, every neuron \( n_{ij} \) in each hidden layer \( N_i, 1 \leq i \leq k \), applies an activation function to the weighted sum. We assume that all activation functions are Rectified Linear Units (ReLU), written as \( \text{ReLU}(x) = \max(0, x) \).

The DNN classifies an input vector \( x \) applied to neurons of the input layer \( N_0 \) based on a poll of the values of each output neuron \( n_{yi} \in N_y \). Let us call \( n_{yi} \) as the winning neuron for input \( x \) if it gets the highest value among all output neurons. In this case, we call \( i \) as the class of \( x \). Given a DNN \( D \) and an input \( x \) with winning neuron \( n_{yi} \), an adversarial input \( x' \) is one that is within the allowed perturbation range \( x' = x \pm \delta \) such that its winning neuron is not \( n_{yi} \).

2.1 Illustrative Example

Consider the DNN of Fig. 1 with ReLU activation function and zero bias for each neuron. The perturbation for each input neuron is \( \delta = 0.5 \). On applying the input vector \( x = (0.6, -1.9, -0.7, -1) \) the network produces the output \( (4.6, -0.25) \) with \( n_{21} \) as the winning neuron. Any input \( x' \) in the perturbation range \( x \pm 0.5 \) is adversarial if the winning neuron for it is \( n_{22} \). For \( n_{22} \) to become the winning neuron, its value needs to increase, while the value of \( n_{21} \) needs to decrease. To keep track of neurons
whose values need to either increase or decrease to effect a change in the winning neuron, we preprocess the DNN by adopting the neuron labelling scheme proposed by Elboher et. al. [9] with labels chosen from the set \{inc, dec\}. The neuron \( n_{23} \) is labelled inc (colored green), while \( n_{21} \) is labelled dec (colored red). The labelling scheme of [9], is then applied one layer at a time from the output layer back to the input layer. The resultant labels are shown in Figure 11 as node colors. However, in a departure from [9] our modified technique does not split neurons in the input layer if it can be labelled both inc and dec, simultaneously. This is because, no neurons need to be tracked for their increment/decrement behaviour beyond the input layer. These neurons are colored gray and are called mixed neurons. The inputs neurons \( n_{03} \) and \( n_{04} \) get labelled inc and dec, suggesting that their values need to be increased and decreased, respectively from their current values given by \( \mathbf{x} \), to effect a change in the winning neuron. Since, any change in the value of these neurons must be within the limits \( \mathbf{x} \pm 0.5 \), the value of \( n_{03} \) is set to \( -0.2 = -0.7 + 0.5 \) and the value of \( n_{04} \) is set to \( -1.5 = -1 - 0.5 \). But, values for the mixed neurons \( n_{01} \) and \( n_{02} \) cannot be similarly determined since they do not have a label and hence the direction of value change is unclear. To find values for these neurons, we pose an optimization query \( Q = \text{maximize}((n_{12} + n_{14}) - (n_{11} + n_{13})) \) to an SMT solver along with an encoding \( \pi_N \) of the DNN sub-graph structure for each neuron at layer \( N_1 \) and a set of value bound constraints \( \Delta \) for the mixed neurons \( n_{01} \) and \( n_{02} \). For example, the sub-graph encoding at neurons \( n_{11} \) and \( n_{12} \) are \( n_{11} = (n_{01} * 1 + 0) + (n_{02} * (-2.5) + 0) \) and \( n_{12} = (n_{01} * 2 + 0) + (n_{02} * (-1) + 0) \) respectively. The value bound constraints \( \Delta \) are encoded as \( 0.6 - 0.5 \leq n_{01} \leq (0.6 + 0.5) \) and \( (-1.9 - 0.5) \leq n_{02} \leq (-1.9 + 0.5) \). Thus, the choice of assignments for \( n_{01} \) and \( n_{02} \) are restricted to values that maximize the objective function of the optimization query, which in turn maximizes (minimizes) the values assigned to neurons labelled inc (dec). The satisfying assignments for \( n_{01} \) and \( n_{02} \) give a new set of values \( \mathbf{x}' \) for the input neurons. We note that restricting the encoding to \( N_0 \) and \( N_1 \) limits the size of the query which is a key strength of the technique and vastly improves the performance of the solver as we shall see later in Sect 3. For the example in Fig. 11 solving the query \( \varphi = Q \land \pi_N \land \Delta \) with the Z3 [15] solver, results in an input assignment \( \mathbf{x}' = (0.1, -1.4, -0.2, -1.5) \) with the corresponding network output \((1.1, 1)\), which does not change the winning class. At this point, we iterate by first strengthening the query \( \varphi \) with additional constraints, denoted \( \alpha \), for each neuron in \( N_1 \). For the example these constraints are \( n_{12} \geq 3.1, n_{14} \geq -0.3, n_{11} \leq 2.1, \) and \( n_{13} \leq -2.6 \), and require that inc labelled neurons \( n_{12} \) and \( n_{14} \) and dec labelled neuron \( n_{11} \) and \( n_{13} \) get values that are respectively \( \geq \) and \( \leq \) than the values they get because of the assignment \( \mathbf{x}' \). However, these constraints are added as soft constraints for each neuron to prevent over-constriction of the search space. To preclude a potential consequence that the solver returns the same input assignment \( \mathbf{x}' \) again, we add a constraint that blocks \( \mathbf{x}' \). With these additional constraints Z3 returns another input assignment \( \mathbf{x}'' = (0.85, -1.4, -0.2, -1.5) \), with the corresponding output \((1.1, 1.75)\), which changes the winning neuron and implying that \( \mathbf{x}'' \) is an ad-
versarial example. A formal presentation of our algorithm has been pushed to Appendix A due to lack of space.

3 Experiments

We have implemented a prototype tool and compared our results, on ACAS Xu (Airborne Collision Avoidance System) benchmarks \[11\], with that of \(\alpha, \beta\)-CROWN \[1\] and Marabou \[13\]. \(\alpha, \beta\)-CROWN is the winner of the 2nd International Verification of Neural Networks Competition (VNN-COMP 2021) \[2\], and Marabou is a popular SMT-based tool. The ACAS Xu benchmarks contain 45 DNNs, each having 5 input neurons, 6 hidden layers with 50 neurons each, and 5 output neurons. We check the robustness of these networks against 10 different properties as explained in \[12\] to find adversarial examples within allowed perturbation ranges for the inputs.

Table 1. Comparison of our tool with \(\alpha, \beta\)-CROWN and Marabou

| Properties | \#instances | \(\alpha, \beta\)-CROWN (with Gurobi) | Marabou (with Gurobi) | Our tool (with z3 solver) |
|------------|-------------|-------------------------------------|-----------------------|--------------------------|
|            |             | \#violated runtime(s)                | \#violated runtime(s)  | \#violated runtime(s)    |
| Prop 1     | 45          | 0 63.56                             | 0 648.94              | 0 83.78                  |
| Prop 2     | 45          | 38 425.82                           | 35 1489.19            | 34 29.69                 |
| Prop 3     | 45          | 3 93.26                             | 3 306.31              | 3 83.42                  |
| Prop 4     | 45          | 3 11.74                             | 3 95.99               | 3 83.07                  |
| Prop 5 to 10 | 6          | 0 578.19                            | 0 526.74              | 0 11.76                  |
| Total      | 186         | 44 1172.57                          | 41 3066.57            | 40 291.72                |

\#instance: No. of benchmarks instances, \#violated: No. of violations/adversarial examples found, runtime(s): Total tool execution time in seconds

Implementation Our prototype tool is implemented in Python, and we have used Z3\[15\] solver’s python API, Z3Py (v4.8.15), for constraint solving. The tool takes 2 inputs, a feed-forward DNN (using ReLU activation function), and a standard property file as used in VNN-COMP 2021 that describes the network’s output behavior like robustness w.r.t inputs or unchanged classification. Based of this input property, we identify the increment/decrement output neurons, and perform the neuron marking. We have implemented the iterative algorithm as explained in section 2.1 with the DNN sub-graph encoding restricted to the input layer and the first layer. The number of iterations, chosen heuristically, was fixed at 80. If the tool finds an adversarial example, it returns the input and prints the corresponding network output. Otherwise, it returns “unknown”.

Results Table 1 contains the results of our experiments. Properties 1 to 4 are applied on all 45 DNNs, whereas property 5 to 10 are applied on 6 separate ACAS Xu DNNs. We conducted these experiments on a machine with 16 GB memory. We have submitted all the artifacts as supporting documents along with the paper.
RAM, 3.60 GHz Intel processor, running Ubuntu 20.04, with a 116 seconds timeout for each benchmark instance running on single core.

Our tool finds violations (adversarial inputs) in 40 among a total of 186 benchmark instances. While comparing our tool with Marabou and $\alpha, \beta$-CROWN, we have used their respective versions submitted to VNN-COMP 2021 [2]. The number of violations found by our tool is comparable to those found by Marabou, which finds 41/186, and $\alpha, \beta$-CROWN which can find 44/186 violations. In terms of the total running time for all the 186 benchmarks, our tool is $\approx 75\%$ faster compared to $\alpha, \beta$-CROWN, and $\approx 90\%$ faster than Marabou. A comparison of the respective running times for all three tools on the 40 benchmarks for which we are able to find an adversarial violation reveals that our tool finds them in just 9.2s and performs significantly better in terms of total execution time compared to Marabou, which takes 799.7s. We also manage to improve upon the 13.9s taken by $\alpha, \beta$-CROWN for the same benchmark instances. The improvement in running time comes from the fact that our approach restricts the size of the problem/optimization instance to the first two layers of the DNN.

4 Related Work

In the past few years, there has been considerable work on proving adversarial robustness of deep neural networks. The vulnerability of deep neural networks to adversarial examples was first discovered in [22]. Following this there have been numerous subsequent results that study adversarial robustness using dynamic analysis techniques such as heuristic search [4,17,23,8,14,9]. Another line of work poses adversarial robustness as a deep neural network verification problem and borrows tools and techniques from classical program verification for its solution. Among these, [10,3,12,5] use constraint solving with various enhancements to enable an exhaustive search over the input perturbation interval for adversarial examples. Although, these techniques are complete they fail to scale as the network size increases. Abstraction based techniques work with an over-approximation of the network either by removing neurons [6] or by abstracting the state space computed by the network [7,20,24,27,21,19,16,26,18]. Although, these methods scale better than constraint solving based exact techniques they are incomplete and suffer from the problem of false adversarial examples. Our work proposes to bridge the gap between these classes of techniques by borrowing ideas from each viz. using constraint solving for optimization aided by behavioural abstraction of the DNN to efficiently generate adversarial inputs.

5 Conclusion and Future Work

The ability to generate adversarial examples is crucial for robustness and trustworthiness of DNNs, especially when they are used in safety-critical application domains such as autonomous vehicles and precision medicine. This paper presents an idea to find adversarial example efficiently. Our initial experiments demonstrate that the approach has promise. As an immediate future work, it
would be worthwhile to complement the behavioral marking of neurons with other quantitative measures e.g. significance or importance of a neuron, that can help obtain a good objective function for optimization. Moreover, since the benefit of this approach comes from the reduction in problem size, by shifting the problem to a layer closer to the input, it is important to lay down necessary and sufficient conditions for moving the optimization problem to a subsequent layer. It would also be interesting to apply and tune this idea to work on a large class of benchmarks including images, videos, and audio files.

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Algorithm 1 Adversarial Generation

1: \((N_E, N_P) \leftarrow \text{IncDecAdv}(D)\)
2: for each \(n' \in N_P\) do
3: if label of \(n'\) is inc then
4: set the value of \(n'\) to \(x(n') + \delta\)
5: else if label of \(n'\) is dec then
6: set the value of \(n'\) to \(x(n') - \delta\)
7: end if
8: end for
9: \(\phi \leftarrow \text{Encode}(D, Q, \pi_{N_1}, \Delta)\)
10: \(it \leftarrow 1\)
11: while \(it \leq C\) do
12: \(x' = \text{Solve}(\phi)\)
13: if \(x'\) is adversarial then
14: return \(x'\)
15: end if
16: \(\phi \leftarrow \text{AddSoftConstraint}(\phi, \alpha)\)
17: \(\phi \leftarrow \text{BlockExample}(\phi, x')\)
18: \(x = x'\)
19: \(it \leftarrow it + 1\)
20: end while
21: return unknown

For a DNN \(D\), let \(\text{IncDecAdv}(D)\) be the modified labelling scheme described in Sect. 2.1. A call to \(\text{IncDecAdv}(D)\) (Line 1) returns the set of mixed input neurons \(N_E\) and the set of labelled input neurons \(N_P\). If an input neuron \(n' \in N_P\) has label \(\text{inc} (\text{dec})\), then its value is set to \(x(n') + \delta (x(n') - \delta)\) where \(x(n')\) is the input value to neuron \(n'\). A call to \(\text{Encode}(D, Q, \pi_{N_1}, \Delta)\) (Line 9) generates the query \(\phi\), where \(Q\) is the optimization query, \(\pi_{N_1}\) is the sub-graph structure encoding for nodes in \(N_1\) and \(\Delta\) are the value bound constraints for the mixed neurons. The procedure then iterates for a pre-defined iteration count \(C\). The query \(\phi\) is solved by the solver to give a new input assignment \(x'\) (Line 12). If \(x'\) is adversarial, then the search exits successfully, else \(\phi\) is strengthened with soft constraints \(\alpha\) and the blocking constraint on \(x'\) via calls to \(\text{AddSoftConstraint}(\phi, \alpha)\) (Line 16) and \(\text{BlockExample}(\phi, x')\) (Line 17) before the solver is called again. We note that the maximization function that is used in the optimization query \(Q\) is only but one choice for an objective function. Algorithm 1 can work seamlessly with any other objective function that can better constrain \(\phi\) to improve the search for an adversarial example. The iterations continue till an adversarial input is generated or a fixed number of iterations is exhausted within a preset time-out duration for running the algorithm.

A.1 A note on completeness of the algorithm

We would like to record that the technique described by Algorithm 1 can be modified into a complete procedure for generating adversarial inputs by - encoding the sub-graph of the DNN between the input layer \(N_0\) and the output layer \(N_y\), which in effect is the entire graph of the DNN and generating the query \(Q\) over neurons in \(N_y\). Even the soft constraints can be written over neurons from \(N_y\). This query \(\phi_{N_y}\) when solved gives an input assignment that changes the winning neuron and hence is adversarial. The downside of this approach is that the algorithm does not scale and in essence is no better than existing techniques that take the full DNN into consideration. However, a more balanced approach generates queries \(\phi_{N_i}\), for \(0 < i \leq y\) at each layer starting from \(N_1\) to \(N_n\) and solves it for a fixed number of iterations. This also gives us a way to tune the
generalised version of the algorithm for a trade-off between scalability, which solves queries $\varphi_{0i}$ over layers closer to the input layer and completeness, which solves the query $\varphi_{0y}$ at the output layer.