Optimizing Gradient-driven Criteria in Network Sparsity: Gradient is All You Need

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Abstract

Network sparsity receives popularity mostly due to its capability to reduce the network complexity. Extensive studies excavate gradient-driven sparsity. Typically, these methods are constructed upon premise of weight independence, which however, is contrary to the fact that weights are mutually influenced. Thus, their performance remains to be improved. In this paper, we propose to further optimize gradient-driven sparsity (OptG) by solving this independence paradox. Our motive comes from the recent advances on supermask training which shows that sparse subnetworks can be located in a randomly initialized network by simply updating mask values without modifying any weight. We prove that supermask training is to accumulate the weight gradients and can partly solve the independence paradox. Consequently, OptG integrates supermask training into gradient-driven sparsity, and a specialized mask optimizer is designed to solve the independence paradox. Experiments show that OptG can well surpass many existing state-of-the-art competitors. Our code is available at https://github.com/zyxxmu/OptG.

1. Introduction

The explosive advances of convolution neural networks (CNNs) are mainly driven by continuously growing model parameters, incurring deployment difficulty on resource-constrained devices. By directly removing parameters for a sparse model, network sparsity emerges as an important technique to reduce model complexity (LeCun et al., 1989; Mozer & Smolensky, 1989; Hoefler et al., 2021).

Broadly speaking, methods in the literature can be divided into after-training sparsity, before-training sparsity and during-training sparsity (Liu et al., 2021a). After-training sparsity aims to remove parameters in pre-trained models (Han et al., 2015), while before-training sparsity attempts to refrain from the time-consuming pre-training process by constructing sparse models at random initialization (Lee et al., 2019). Recent advances advocate during-training sparsity which consults the sparsity process throughout network training (Evci et al., 2020; Kusupati et al., 2020).

Particularly, the core of network sparsity lies in selecting to-be-removed parameters such that it can satisfy: 1) desired sparse rate; 2) acceptable performance compromise. To this end, the most straightforward solution is to remove these parameters causing least increase on the training loss \( L \).

Then, by leveraging the first-order Taylor expansion of the loss function \( L \) to approximate the influence of removing parameter \( w_i \), the key format for measuring weight importance can be expressed as \( \frac{\partial L}{\partial w_i} w_i \), leading to gradient-driven sparsity. Recent advances rewrite this format using higher-order Taylor expansion (Wang et al., 2020; Molchanov et al., 2017), which will be detailed in Sec. 3.1.

Despite the progress, existing gradient-driven methods are built upon premise of independence among weights. However, this assumption contradicts with the practical implementation, in which, parameters are collectively making effort to derive the network output. Usually, existing methods remove weights once-for-all (Lee et al., 2019; Wang et al., 2020). Consequently, the computed loss change used to remove weights deviates a lot from the actual loss change. Thus, it is necessary to overcome this independence paradox in order to pursue a better performance.

Beyond the gradient-driven sparsity, recent developments on supermask training (Zhou et al., 2019; Ramanujan et al., 2020) show that high-performing sparse subnetwork can be located upon a randomly initialized networks without modifying any weight. Instead, they choose to update mask values using the straight-through-estimator (STE) (Bengio et al., 2013). In Sec. 3.3, we innovatively prove that the essence of supermask training is to accumulate gradients of both preserved and removed weights in gradient-driven spar-
Also, we show that this manner can partially solve the independence paradox. Unfortunately, the fixed weights in supermask training fail to eliminate independence paradox, thus, the performance is still sub-optimal.

In this paper, we propose to optimize the gradient-driven sparsity by integrating the advantage of supermask training in overcoming the independence paradox. Our method, termed OptG, introduces a novel mask optimizer that continuously accumulates the mask gradients of each training iteration. Differently, we only update the mask using SGD in the beginning of each training epoch. In this way, the remaining parameters can be well tuned on the training set to minimize the error gap, thus the independence paradox can be well solved. Extensive experiments demonstrate that OptG dominates its counterparts especially in the extreme sparse rate. For instance, OptG removes 98% parameters of ResNet-50 while still achieves 67.20% top-1 accuracy on ImageNet, surpassing the recent strong baseline STR (He et al., 2018) that only reaches 62.84% by a large margin.

Our main contributions are summarized below:

- We prove the existence of independence paradox in gradient-driven sparsity, which causes error gap in loss change for measuring weight importance.
- We reveal the essence of supermask training is to accumulate weights gradients in gradient-driven sparsity, which partly solves the independence paradox.
- We propose OptG which further optimizes the gradient-driven sparsity using a novel mask optimizer that overcomes the problem of independence paradox.
- Extensive experiments demonstrate the advantage of the proposed OptG over many existing state-of-the-arts in sparsifying modern CNNs.

### 2. Related Work

This section covers the spectrum of studies on sparsifying CNNs that closely related to our work. A more comprehensive overview can be found in the recent survey (Hoefler et al., 2021).

#### 2.1. Sparsity Granularity

The granularity of network sparsity varies from coarse grain to fine grain. The former is indicated to removing the entire channels or filters towards a structured subnetwork (He et al., 2017; Lin et al., 2020; He et al., 2019). Though well suited to a practical speedup on regular hardware devices, significant performance degradation usually occurs at a high sparse rate (Ding et al., 2018; Luo et al., 2017; He et al., 2020; 2018). The latter removes individual neurons at any location of the network to pursue an unstructured subnetwork (Han et al., 2015; Mocanu et al., 2018). It has been proved to well retain the performance even under an extremely high sparse rate (Kusupati et al., 2020; Gale et al., 2019). Moreover, many recent efforts also show great promise of unstructured sparse networks in practical acceleration (Gale et al., 2020; Elsen et al., 2020). In particular, recent 2:4 sparse pattern has been well supported by Nvidia A100 GPUs to accomplish 2x speedups.

### 2.2. When to Sparsify

According to the time point the sparsity is applied, we empirically categorize existing works into three groups (Liu et al., 2021a).

- **After-training sparsity** was firstly adopted by the optimal brain damage (LeCun et al., 1989). Since then, the followers obey a three-step pipeline, including model pretraining, parameter removing and network fine-tuning (Han et al., 2015; Molchanov et al., 2017; Ding et al., 2019; Lemaire et al., 2019). Unfortunately, in the cases that pre-trained models are missing and hardware resources are limited, the aforementioned approaches become impractical due to the expensive fine-tuning process.

- **Before-training sparsity** attempts to conduct network sparsity on randomly initialized networks for efficient model deployment. Through removing weights using gradient-driven measurement (Lee et al., 2019; Wang et al., 2020) or heuristic design (Tanaka et al., 2020), a sparse subnetwork can be available in a one-shot manner. Nevertheless, the performance gap of this group still exists compared with the after-training sparsity (Frankle et al., 2020).

- **During-training sparsity** has been drawing increasing attention for its performance retaining (Mocanu et al., 2018; Mostafa & Wang, 2019; Lin et al., 2020). In each training iteration, the parameters will be removed or revived according to a predefined criterion. Consequently, the sparse subnetwork can be obtained in a single training process. For instance, RigL (Evci et al., 2020) re-allocates the removed weights according to their dense gradients, while Sparse Momentum (Dettmers & Zettlemoyer, 2019) considers mean momentum magnitude of each layer as a redistribution criterion. Besides, the performance of during-training sparsity can be further enhanced if the desired sparsity is gradually achieved in an incremental manner (Zhu & Gupta, 2017; Liu et al., 2021a; 2021b).

### 2.3. Layer-wise Sparsity Allocation

It has been a wide consensus in the community that layer-wise sparsity allocation, i.e., sparse rate of each layer, is a core in network sparsity (Liu et al., 2019; Gale et al., 2019; Lee et al., 2020). Majorities of existing methods implement layer-wise sparsity using a static or dynamic
work sparsity can be viewed as multiplying a binary mask without the necessity of modifying weight values. For instance, (Orseau et al., 2020; Pensia et al., 2020) further proved that the lottery ticket hypothesis (Frankle & Carbin, 2019) reveals that there exist randomly-initialized sparse networks that are mostly removed, which further disables the network training. Therefore, recent studies (Kusupati et al., 2020; Savarese et al., 2019) pursue the layer-wise sparsity in a trainable manner, which, however, requires a complex hyper-parameter tuning and often leads to an unstable sparse rate.

2.4. Lottery Ticket Hypothesis and the Supermask

The lottery ticket hypothesis (Frankle & Carbin, 2019) reveals that there exist randomly-initialized sparse networks that can be trained independently to match the performance of the dense model. Following this conjecture, recent empirical studies (Zhou et al., 2019; Ramanujan et al., 2020) have further confirmed the existence of supermask, which simply updates the mask values to obtain sparse subnetworks using the straight-through-estimator (STE) (Bengio et al., 2013) without the necessity of modifying weight values. For instance, (Ramanujan et al., 2020) showed that a randomly initialized Wide ResNet-50 sparsified by a supermask, can initialized Wide ResNet-50 sparsified by a supermask, can match the performance of ResNet-32 trained on ImageNet. (Orseau et al., 2020; Pensia et al., 2020) further proved that the existence of supermask relies on a logarithmic over-parameterization. Despite the progress, an in-depth analysis remains unexplored on why a subnet exists without modifying weight values.

3. Method

3.1. Background

Notations. Denoting the weights of a convolution neural network as \( w \in \mathbb{R}^N \) where \( N \) is the weight number, network sparsity can be viewed as multiplying a binary mask \( m \in \{0, 1\}^N \) on \( w \) as \((w \odot m)\) where \( \odot \) represents the element-wise product. Consequently, the state of the \( i \)-th mask \( m_i \) indicates whether \( w_i \) is removed (0) or not (1).

Let \( \mathcal{L}(\cdot) \) and \( \mathcal{D} \) be the training loss and training dataset. Essentially, given a sparse rate \( P \), network sparsity aims to obtain a sparse \( m \) subject to \( \frac{|m|_0}{N} \leq 1 - P \) while minimizing \( \mathcal{L}(\cdot) \) on \( \mathcal{D} \). To this end, various scenarios have been proposed to derive \( m \). In what follows, we discuss two cases that are mostly related to our method.

Gradient-driven sparsity. The studies of network sparsity using weight gradient date back to the last few decades (LeCun et al., 1989; Mozer & Smolensky, 1989). The basic idea of these methods is to leverage weight gradients to approximate change in the loss function \( \mathcal{L}(\cdot) \) when removing some parameters. The overall optimization can be formulated as:

\[
\min_w \mathcal{L}(w \odot m; \mathcal{D}) \text{ s.t. } \frac{|m|_0}{N} \leq 1 - P. \tag{1}
\]

Then, it is easy to know that the loss change after removing a single weight \( w_i \) is:

\[
\Delta \mathcal{L}(w_i; \mathcal{D}) = \mathcal{L}(m_i = 0; \mathcal{D}) - \mathcal{L}(m_i = 1; \mathcal{D}). \tag{2}
\]

It is intuitive that \( \Delta \mathcal{L}(w_i; \mathcal{D}) < 0 \) indicates a loss drop, which means the removal of \( w_i \) results in better performance. To obtain a sparse \( m \), one naive approach is to repeatedly compute the loss change for each weight in \( w \), and then set masks of these parameters with larger loss changes to 0s, and 1s otherwise. However, modern CNNs tend to have parameters in millions, making it expensive to perform this one-by-one loss calculation.

Fortunately, \( \Delta \mathcal{L}(w_i; \mathcal{D}) \) can be approximated via the Taylor series expansion. Considering the first-order case (Molchanov et al., 2017), \( \Delta \mathcal{L}(w_i; \mathcal{D}) \) can be reformulated as:

\[
\Delta \mathcal{L}(w_i; \mathcal{D}) = \mathcal{L}(m_i = 0; \mathcal{D}) - \mathcal{L}(m_i = 1; \mathcal{D}) \\
= \mathcal{L}(m_i = 1; \mathcal{D}) - \frac{\partial \mathcal{L}}{\partial (w_i \odot m_i)}(w_i \odot m_i) \\
+ R_1(m_i = 0) - \mathcal{L}(m_i = 1; \mathcal{D}) \\
= -\frac{\partial \mathcal{L}}{\partial (w_i \odot m_i)}(w_i \odot m_i) + R_1(m_i = 0). \tag{3}
\]

If we ignore the first-order remainder \( R_1(m_i = 0) \), then:

\[
\Delta \mathcal{L}(w_i; \mathcal{D}) \approx -\frac{\partial \mathcal{L}}{\partial (w_i \odot m_i)}(w_i \odot m_i). \tag{4}
\]

Eq. (4) can be an efficient alternative to approximating \( \mathcal{L}(w_i; \mathcal{D}) \), since for all weights, the term \( w_i \odot m_i \) can be available in a single forward propagation and the term \( -\frac{\partial \mathcal{L}}{\partial (w_i \odot m_i)} \) can be derived in a single backward propagation. Consequently, the format of Eq. (4) has served as a basic in modern gradient-driven network sparsity (Mozer & Smolensky, 1989). Many recent variants are further excavated based on this format. Taylor-FO (Molchanov et al., 2017) considers \((\frac{\partial \mathcal{L}}{\partial (w \odot m)} \odot w)^2\) as a saliency metric to sparsify a pre-trained model, which is similar to the prune-at-initialization SNIP (Lee et al., 2019) that uses \(|\frac{\partial \mathcal{L}}{\partial (w \odot m)}| \) instead. Grasp (Wang et al., 2020) leverages the second-order Taylor series and derives the pruning criterion of \(-H \frac{\partial \mathcal{L}}{\partial (w \odot m)} \) instead, where \( H \) denotes the Hessian matrix. Besides, another variant \(|\frac{\partial \mathcal{L}}{\partial (w \odot m_i)}| \) is used to indicate if some pruned weights should be revived during sparse training.

Though great effort has been made, these existing works are developed on premise of independence assumption that
weights are irrelevant to each other, which however, is on
the contrary in practice as elaborately in Sec. 3.2. As results,
their performance remains an open issue.

**Supermask-driven sparsity.** In gradient-driven sparsity,
the values of weight vector \( \mathbf{w} \) are updated in the backward
propagation and the mask \( \mathbf{m} \) is recomputed using the above
criterion in the forward propagation. Instead, many recent
developments reveal that a sparse subnet can be found in
an untrained network without the necessity of modifying
any weight (Zhou et al., 2019). Typically, these methods
can be complemented by updating the mask vector \( \mathbf{m} \).
The corresponding learning objective can be formulated as:

\[
\min_{\mathbf{m}} \mathcal{L}(\mathbf{w} \odot \mathbf{m} ; \mathcal{D}) \quad \text{s.t.} \quad \frac{\|\mathbf{m}\|_0}{N} \leq 1 - P. \quad (5)
\]

To stress, the objective of Eq. (5) differs from that of Eq. (1)
in that its optimized variable is \( \mathbf{m} \) instead of \( \mathbf{w} \) which instead
is regarded as a costant vector in Eq. (5). Existing studies (Zhou et al., 2019; Ramanujan et al., 2020) optimize
Eq. (5) by first relaxing the discrete \( \mathbf{m} \in \{0, 1\}^N \) to
a continuous version of \( \hat{\mathbf{m}} \in \mathbb{R}^N \). Then, in the forward
propagation, a binary function \( h(\cdot) \) is applied to discretize
\( \hat{\mathbf{m}} \in \mathbb{R}^N \) as:

\[
h(\hat{\mathbf{m}}_i) = \begin{cases} 
0, & \text{if } \hat{\mathbf{m}}_i \text{ in the top-}P \text{ smallest of } \hat{\mathbf{m}}, \\
1, & \text{otherwise}. 
\end{cases} \quad (6)
\]

In the backward propagation, due to the non-differentiable in
above equation, the straight-through estimator (STE) (Ben-
gio et al., 2013) is used as an alternative to approximate the
costant gradient as:

\[
\frac{\partial \mathcal{L}}{\partial \hat{\mathbf{m}}_i} = \frac{\partial \mathcal{L}}{\partial (h(\hat{\mathbf{m}}_i) \odot w_i)} \frac{\partial h(\hat{\mathbf{m}}_i) \odot w_i}{\partial h(\hat{\mathbf{m}}_i)} = \frac{\partial \mathcal{L}}{\partial (h(\hat{\mathbf{m}}_i) \odot w_i)} w_i. \quad (7)
\]

By updating \( \hat{\mathbf{m}}_i \), a high-performing sparse subnet can be fi-
nally located. Nevertheless, to date, no one has dived into an
exploration of why a subnet exists without modifying weight
values. In what follows, we give a detailed explanation and
show that gradient-driven sparsity and supermask-driven
sparsity are the same in essence.

**3.2. Independence Paradox**

The gradient-driven sparsity using Eq. (4) neglects the high-
order terms of Taylor series as well as the remainder. Luck-
ily, it has been experimentally proved that the first-order
gradient (Lee et al., 2019) shows performance on par with
the higher-order ones (Wang et al., 2020). Nevertheless, ex-
isting gradient-driven sparsity is built upon the assumption
that weights are irrelevant to each other, which is contrary
to the practical implementation (Molchanov et al., 2017;
Lee et al., 2019) where a large number of weights are usu-
ally removed simultaneously. Considering the case that two
weights \( w_i \) and \( w_j \) are removed, independent, the loss
cchange of removing \( w_i \) using Eq. (3) is:

\[
\Delta \mathcal{L}(w_i; \mathcal{D}) = \mathcal{L}(m_i = 0, m_j = 1, w; \mathcal{D}) - \mathcal{L}(m_i = 1, m_j = 1, w; \mathcal{D}). \quad (8)
\]

However, considering that \( w_j \) has been removed as well, the
actual loss change due to the removal of \( w_i \) should become:

\[
\Delta \mathcal{L}(w_i; \mathcal{D}) = \mathcal{L}(m_i = 0, m_j = 0, w^*; \mathcal{D}) - \mathcal{L}(m_i = 1, m_j = 0, w^*; \mathcal{D}), \quad (9)
\]

where \( w^* \) indicates the state of original \( w \) after the removal
of \( w_j \) and a follow-up fine-tuning on the dataset \( \mathcal{D} \). It is
easy to know that weight-independent Eq. (8) is actually
built upon premise of preserving \( w_j \). However, the prac-
tice removes \( w_i \) and \( w_j \) simultaneously, which indicates a
loss change of Eq. (9). Thus, there exists an independence
paradox in existing studies and the error gap in gradient-
driven sparsity is proportional to the total number of re-
moved weights each time.

Note that, some recent advances (Evci et al., 2020; Zhu &
Gupta, 2017) advocate incremental pruning that removes a
small portion of weights each time. For instance, RigL (Evci
et al., 2020) removes a small fraction of weights and acti-
vates new ones iteratively, while (Zhu & Gupta, 2017) pro-
posed to gradually increase the number of removed weights
until the desired sparse rate is satisfied. Though not explicit-
lly stated, these works indeed accomplish network sparsity
by reducing the removed parameters so as to relieve the
error gap. Nevertheless, error gap still exists and thus their
performance is sub-optimal. An in-depth exploration to
overcome this independence paradox remains to be done.

**3.3. Our Insights on Supermask-driven Sparsity**

In this subsection, we show that the success of supermask-
driven sparsity is to use the first-order
based gradient-driven sparsity in essence. Also, supermask-driven sparsity, to
some extent, mitigates the aforementioned independence
paradox. Specifically, we denote the mask \( \hat{\mathbf{m}}_i \) at the \( t \)-th
training iteration as \( \hat{\mathbf{m}}_i^t \). Combining the mask gradient in
Eq. (7), \( \hat{\mathbf{m}}_i^t \) can be derived via SGD as:

\[
\hat{\mathbf{m}}_i^{t+1} = \hat{\mathbf{m}}_i^t - \eta \frac{\partial \mathcal{L}}{\partial \hat{\mathbf{m}}_i} = \hat{\mathbf{m}}_i^{t-1} - \eta \frac{\partial \mathcal{L}}{\partial (h(\hat{\mathbf{m}}_i) \odot w_i)} w_i, \quad (10)
\]

where \( \eta \) indicates the learning rate. When \( h(\hat{\mathbf{m}}_i^t) = 1 \), we
have \( h(\hat{\mathbf{m}}_i^t) \odot w_i = w_i \), which leads Eq. (10) to the result
We show that this can be optimized by adding back the \( \Delta \) This is exactly the pursued mask gradient in Eq. (10). Thus, \( \hat{m} \) is important to the network and should be revived in the case of
driven sparsity. Thus, we can conclude that the manner of supermask-driven sparsity to obtain a sparse mask \( m \) also indicates the removed weight is vital to the network performance. Similarly, a small \( \Delta^+ \) also indicates the removed weight is important to the network performance. Overall, from Eq. (12), we can know that the updating in supermask-driven sparsity is indeed to accumulate gradients of both preserved and removed weights in gradient-driven sparsity. Thus, we can conclude that the manner of supermask-driven sparsity to obtain a sparse mask \( m \) is indeed similar to that of gradient-driven sparsity. This well explains why supermask-driven sparsity can perform well in existing studies.

Further, we show that the key in supermask training is that it partially solves the independence paradox. Similarly, given that \( w_i \) and \( w_j \) are removed at the \( (t-1) \)-th training iteration, we have \( h(\hat{m}_{i-1}^{t-1}) = 1 \). According to Eq. (12), the loss change for adding back \( w_i \) becomes:

\[
\Delta^+ \mathcal{L}(w_i; D) = \mathcal{L}(m_i = 1, m_j = 0; D) - \mathcal{L}(m_i = 0, m_j = 0; D).
\]

(13)

Obviously, Eq. (13) is closer to the actual loss change of Eq. (9) than independent-based Eq. (8). Thus, the error gap from independence paradox can be well compensated by reviving \( w_i \) if it is important to the network performance.

### 3.4. Optimizing Gradient-driven Sparsity

Herein, we propose to interleave the supermask training into during-training learning to further optimize the gradient-driven sparsity. The learning objective of our method, termed OptG, can be formulated as:

\[
\min_{m, \mathcal{L}} \mathcal{L}(w \odot m; D) \quad \text{s.t.} \quad \frac{|m|_0}{N} \leq 1 - P.
\]

(14)

The motive of our OptG is to conduct weight optimization and mask training simultaneously. Therefore, when \( w_j \) is pruned, the remaining weights are further trained on \( D \) and the update item of \( \hat{m} \) in the \( t \)-th training iteration is:

\[
\Delta \mathcal{L}(w_i; D) = \mathcal{L}(m_i = 1, m_j = 0, w^t; D) - \mathcal{L}(m_i = 0, m_j = 0, w^t; D),
\]

(15)

where \( w^t \) is the trained weights after the \( t \)-th training iteration. Nevertheless, it is clear that \( w^t \) barely reflects the weight tuning on the whole training set as \( w^t \) has been trained for only one iteration. Moreover, if the binary function \( h(\cdot) \), i.e., Eq. (6), is applied during each forward propagation of masks, the network topology may be changed frequently, which can lead to unstable training process.

To solve the above-mentioned problem, we introduce a novel supermask optimizer towards comprehensively solving the independence paradox. In particular, we apply \( h(\cdot) \) to revive and prune weights at the beginning of each training epoch. Then, we continuously accumulate the mask gradient during each training iteration via Eq. (10), but keep the binary mask fixed. Therefore, the preserved weights can be sufficiently retrained on the training set, enabling our mask updating process to finally reach Eq. (9). Note that our OptG sorts the weights in a global manner to automatically decide a layer-wise sparsity budget, thus avoiding the rule-of-thumb design (Evci et al., 2020) or complex hyper-parameter tuning for learning sparsity distributions (He et al., 2018).
We summarize the workflow of the supermask optimizer in Alg. 1.

**Algorithm 1 Optimizing the gradient-driven sparsity.**

**Require:** Network weights $w$ and masks $\hat{m}$, target sparsity $P$, total training epoch $\tau$.

\begin{algorithmic}
1: $w \leftarrow$ randomly initialization, $\hat{m} \leftarrow 0$;
2: for $k \leftarrow 1, 2, \ldots, \tau$ do
3: \hspace{1em} Get current sparse rate via Eq. (16);
4: \hspace{1em} Get $m$ from $\hat{m}$ via Eq. (6);
5: \hspace{1em} for each training step $t$ do
6: \hspace{2em} Forward propagation via $(w \odot m)$;
7: \hspace{2em} Compute the gradient of $\hat{m}$ via Eq. (7);
8: \hspace{2em} Update $\hat{w}$, $\hat{m}$ using SGD optimizer;
9: \hspace{1em} end for
10: end for
\end{algorithmic}

We choose to increase the sparse rate from 0 to the target sparsity rate $P$ on the basis of the sigmoid function as:

$$P_k = \frac{P}{1 + e^{-\alpha(k-0.5\tau)}}, \quad (16)$$

where $k$ and $\tau$ represent the current and total training epoch, $\alpha$ is a hyperparameter that controls the total epoch for achieving sparsity. Fig. 1 shows that the sparsity ascent rate at initialization can be relatively smooth. The intuition behind this gradual sparsity schedule is that the weights require sufficient training as reviving a random-initialized weight at initialization can be relatively smooth. The intuition behind this gradual sparsity schedule is that the weights require sufficient training as reviving a random-initialized weight is usually meaningless from the perspective of Eq. (11).

Alternatively, we propose a novel learning rate schedule for the mask training with respect to this sparsity schedule. When the sparse rate of the network is low, the learning rate of masks is also small as the calculation of gradient score will be seriously interfered by the independence paradox. Moreover, when the sparse rate reaches $P$, the learning rate of masks can follow weights to sufficiently optimize the gradient-driven criteria while guarantee training convergence.

Overall, our optimizer assign the mask learning rate at epoch $k$ as:

$$\eta_{\hat{m},k} = \frac{\eta_{w,k}}{1 + e^{-\alpha(k-0.5\tau)}}, \quad (17)$$

where $\eta_{w,k}$ is the learning rate of weights at the $k$-th epoch. We summarize the workflow of the supermask optimizer in Alg. 1.

4. Experiments

4.1. Settings

We conduct extensive experiments to evaluate the efficacy of our OptG in sparsifying VGGNet-19 (Simonyan & Zisserman, 2015), ResNet-50 (He et al., 2016) on small scale CIFAR-10/100 (Krizhevsky et al., 2009) datasets and ResNet-50 (He et al., 2016), MobileNet-V1 (Howard et al., 2017) on large scale ImageNet (Deng et al., 2009) dataset. Besides, we compare our OptG with several the state-of-the-arts including SNIP (Lee et al., 2019), GraSP (Wang et al., 2020), Deep-R (Bellec et al., 2018) SynFlow (Tanaka et al., 2020), SET (Mocanu et al., 2020), DSNFS (Dettmers & Zettlemoyer, 2019), RigL (Evci et al., 2020), STR (Kusupati et al., 2020) and GraNet (Liu et al., 2021a).

We implement OptG with PyTorch (Paszke et al., 2019). Particularly, we set $\alpha = 0.5$ in all experiments and leverage the SGD optimizer to update the weights and their masks with a gradually-increasing sparsity rate Eq. (16). On CIFAR-10 and CIFAR-100, we train the networks for 160 epochs with a weight decay of $1 \times 10^{-3}$. On ImageNet, the weight decay is set to $5 \times 10^{-4}$ for ResNet-50 and $4 \times 10^{-5}$ for MobileNet-V1. We train ResNet-50 for 100 epochs and MobileNet-V1 for 180 epochs, respectively. Besides, the initial learning rate is set to 0.1, which is then decayed by the cosine annealing scheduler during training. All experiments are run with NVIDIA Tesla V100 GPUs.

4.2. CIFAR-10/100

**VGGNet-19.** Tab. 1 shows the performance of different methods for sparsifying the classic VGGNet with 19 layers. Compared with the competitors, our OptG yields better accuracy under the same sparse rate both on CIFAR-10 and CIFAR-100.

| Dataset    | CIFAR-10     | CIFAR-100    |
|------------|--------------|--------------|
| Sparse Rate| 90% 95%      | 90% 95%      |
| VGGNet-19  | 93.85        | -            |
| Deep-R     | 90.81        | 89.59        |
| SET        | 92.46        | 91.73        |
| SNIP       | 93.63        | 93.43        |
| GraSP      | 93.30        | 93.04        |
| SynFlow    | 93.35        | 93.45        |
| STR        | 93.73        | 93.27        |
| RigL       | 93.47        | 93.35        |
| GMP        | 93.59        | 93.58        |
| GraNet     | 93.80        | 93.72        |
| OptG       | 93.84        | 93.72        | 73.80 73.24 |
| ResNet-50  | 94.75        | -            | 78.23 - |
| SNIP       | 92.65        | 90.86        | 73.14 69.25 |
| GraSP      | 92.47        | 91.32        | 73.28 70.29 |
| SynFlow    | 92.49        | 91.22        | 73.37 70.37 |
| STR        | 92.59        | 91.35        | 73.75 70.45 |
| RigL       | 94.45        | 93.86        | 76.50 76.03 |
| GMP        | 94.34        | 94.52        | 76.91 76.42 |
| GraNet     | 94.49        | 94.44        | 77.29 76.71 |
| OptG       | 94.55        | 94.56        | 77.41 77.02 |
and CIFAR-100 datasets. For instance, compared with SNIP (Lee et al., 2019) that suffers serious performance degradation of 4.26% when pruning 95% parameters on CIFAR-10 (89.59% for SNIP and 93.85% for the baseline), the proposed OptG only lose negligible accuracy of 0.01% (93.84% for OptG), despite they are both built on gradient information. On CIFAR-100, our OptG provides significantly better accuracy against other gradient-driven approaches including GrasP (Wang et al., 2020) and RigL (Evci et al., 2020), which demonstrates the superiority of optimizing the gradient-driven criteria in network sparsity.

**ResNet-50.** It can also be referred from Tab. 1 that our OptG outperforms the state-of-the-art on both CIFAR-10 and CIFAR-100 datasets. In term of 90% sparse rate on CIFAR-10, OptG improve the performance of other during-training methods including GMP (Tanaka et al., 2020) and STR (Kusupati et al., 2020) by 0.25% and 0.11% (93.84%, 92.49%, 92.59% for OptG, GMP, STR). On CIFAR-100, our method again outperforms the most recent GraNet by 0.12% and 0.31% at 90% and 95% sparsity, respectively. Overall, OptG demonstrates its great capability over other methods to accelerate classic CNNs on small scale datasets.

### 4.3. ImageNet

**ResNet-50.** The comparison of compressing ResNet-50 (He et al., 2016) between the proposed OptG and its counterparts is presented in Tab. 2. As can be seen, our OptG well surpasses its competitors across different sparse rates. For example, in comparison with the gradient-driven approach RigL at a sparse rate of 90%, OptG greatly reduces the FLOPs to 342M with a remarkable accuracy of 74.28%, while RigL only reaches 73.00% with much higher FLOPs of 960M. When the sparse rate reaches 98.00%, all existing studies suffer severe performance degradation. In contrast, OptG presents an amazing result of 67.20% top-1 accuracy, which well surpasses the recent advances of STR by 4.36% and DNW by 9.00%. Furthermore, at an extreme sparse rate of around 99.00%, the proposed OptG still retains a high accuracy of 62.10%, which surpasses the second best STR by a large margin. These comparison results well demonstrate the efficacy of our OptG in compressing the large-scale ResNet.

Besides, it is worth mentioning that our OptG well outperforms STR by large margins across various sparse rates,
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Table 3. Performance comparison of MobileNet-V1 on ImageNet.

| Method   | Sparsity | Params  | FLOPs  | Top-1 Acc. |
|----------|----------|---------|--------|------------|
| MobileNet-V1 | 0.00     | 4.12M   | 569M   | 71.95      |
| GMP      | 74.11    | 1.09M   | 163M   | 67.70      |
| STR      | 75.28    | 1.04M   | 101M   | 68.35      |
| STR      | 79.07    | 0.88M   | 81M    | 66.52      |
| OptG     | 80.00    | 0.82M   | 124M   | **70.27**  |

| GMP      | 89.03    | 0.46M   | 82M    | 61.80      |
| STR      | 85.80    | 0.60M   | 55M    | 64.83      |
| STR      | 89.01    | 0.46M   | 42M    | 62.10      |
| STR      | 89.62    | 0.44M   | 40M    | 61.51      |
| OptG     | 90.00    | 0.41M   | 80M    | **66.80**  |

Table 4. Performance of different methods for sparsifying ResNet-50 on ImageNet under the same gradual sparsity schedule.

| Method | Sparsity | Params  | FLOPs  | Top-1 Acc. |
|--------|----------|---------|--------|------------|
| SET    | 90.00    | 5.12M   | 409M   | 67.19      |
| RigL   | 90.00    | 5.12M   | 960M   | 72.02      |
| GMP    | 90.00    | 5.12M   | 409M   | 73.80      |
| OptG   | 90.00    | 5.12M   | 342M   | **74.28**  |

Table 5. Performance of our OptG for sparsifying ResNet-50 on ImageNet with different layer-wise sparsity budget.

| Method | Sparsity | Params  | FLOPs  | Top-1 Acc. |
|--------|----------|---------|--------|------------|
| Uniform| 90.00    | 5.12M   | 409M   | 74.12      |
| GS     | 90.00    | 5.12M   | 697M   | 73.89      |
| ERK    | 90.00    | 5.12M   | 960M   | **74.39**  |
| OptG   | 90.00    | 5.12M   | 342M   | 74.28      |

Despite that STR has to conduct complex hyper-parameter tuning to obtain the overall sparsity, while OptG is much more flexible via a simple one-shot global sorting. To dive into a deeper analysis, given an overall sparse rate of 90%, we display the layer-wise sparsity budget in Fig. 2. Compared with other methods, on one hand, our OptG automatically assigns non-uniform sparsity budget across layers; on the other hand, it can allocate more budgets for the shallow layers, which contain more FLOPs than the deeper layers. This well explains why the FLOPs reduction of our OptG can be better than others when similar parameters are removed. As shown in Fig. 3, OptG again leads a frontier curve over other approaches for the accuracy performance under a similar FLOPs reduction. Although STR shows comparable accuracy, its performance is built upon preserving more parameters as displayed in Tab. 2. For example, our OptG obtains top-1 accuracy of 67.20% with 126M FLOPs and 0.51M parameters. In contrast, STR obtains a slightly better top-1 accuracy of 67.78% at the cost of more FLOPs of 127M and parameters of 0.99M.

MobileNet-V1. MobileNet-V1 (Howard et al., 2017) is a lightweight network with depth-wise convolution. Thus, compared with ResNet-50, it is more different to sparsify MobileNet-V1 without performance compromise. Nevertheless, results on Tab. 3 shows that our OptG still offers reliable performance on such a challenging task. Specifically, OptG achieves a top-1 accuracy of 70.27% at a sparse rate of 89.00%, which is 3.75% higher than that of STR that suffers more parameter burden as well. Similar observation can be found when the sparse rate is around 90.00%. Our OptG reduces the parameters to 0.41M and FLOPs to 80M, meanwhile it still preserve an accuracy of 66.80%. As for STR at a sparse rate of only 80.00%, it obtains a lower accuracy of 66.52%. Thus, OptG well demonstrates its ability to sparsify lightweight networks.

4.4. Ablation Studies

The gradual sparsity schedule. Tab. 4 shows the accuracy of different sparsity techniques under the same gradual sparsity schedule as defined in Eq. (16). Therefore, the difference among OptG and other methods falls in how to revive and prune weights in the during-training sparsity. As can be observed, OptG still take the lead over all methods, thus well demonstrates our point of optimizing the gradient-driven criteria.

It is worth mentioning that applying our schedule to other methods even leads to worth performance comparing with their origin schedule. This is reasonable for that other methods generally revive weights to 0s, which, if carried out in the latter training process, can not ensure a sufficient training. Therefore, different motivations lead to a unique sparse schedule for our OptG.

Layer-wise sparsity budgets. We further investigate the effect of layer-wise sparsity budgets by only sorting the weights inter-layer while keeping the layer-wise sparsity fixed with respect to different pre-defined budgets including including Uniform, GS (Han et al., 2015), ERK (Evci et al., 2020). As can be seen, using the global magnitude pruning (GS) as well as uniform budget both lead to accuracy drop. The ERK budget can achieve better performance with our global-sorting mechanism, nevertheless, the FLOPs of ERK is much higher than the layer-wise budget automatically decided by OptG. Therefore, the efficacy of sparsity budget chosen for OptG is also well identified.

5. Conclusion

In this paper, we have proposed to optimize the gradient-driven criteria in network sparsity, termed OptG. In particular, we first point out the independence paradox in previous approaches and show an effective trail to solve this paradox based on revealing the empirical success of supermask training. Following this trail, we further propose to solve the independence paradox by interleaving the supermask training process into during-training sparsity with a novel su-
permask optimizer. Extensive experiments on various tasks demonstrate that our OptG can automatically obtain layerwise sparsity burden, while achieving state-of-the-art performance at all sparsity regimes. Our work re-emphasizes the great potential of gradient-driven pruning and we expect future advances for the gradient-driven criteria optimizer.

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