Developing Feasible Search Approach For Tackling Large Vehicle Routing Problem With Time Window Considering Service Disruption

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Abstract. In logistic system, a Vehicle Routing Problem with time windows (VRPTW) is defined as a problem to decide the optimal set of logistic routes used by homogeneity vehicles to deliver customers’ demand within scheduled service time. The objective is to minimize the total travel cost (related to the travel times or distances) and operational cost (related to the number of vehicles used). In this paper we address a variant of the VRPTW in which the time to deliver and the time departing from a customer are restricted. The problem also include service disruption. We use integer programming model to describe the problem. A feasible neighbourhood approach is proposed to solve the model.

1. Introducing

Vehicle Routing Problem (VRP) is an optimization model that have been used for many distribution systems which contains selecting best route for a fleet of vehicles through a set of nodes. This is a hard combinatorial optimization problem which belongs to the determination of an optimal set of routes utilized in a fleet of vehicles to serve a set of customers, considering various operational constraints. The first example of VRP was presented by Dantzig and Ramser [1] in truck dispatching. Afterward there are many papers which have been handling in this area to come across new methodologies with the aim to find the better solutions. There are a lot of survey papers can be found in literature for VRP, such as, [2], [3], [4], [5], and [6]. A compact review of vehicle routing can be found in [7] and [8].

The vehicle routing problem with time windows (VRPTW) is a generalisation of VRP. It can be seen as a problem of combined vehicle routing and scheduling, which often occurs in many real-world applications. It is intended to to use a fleet of vehicles efficiently, which which must carry out a number of visits in order to meet customer demand and to specify which customers should be served by each vehicle and in which order, subject to vehicle capacity and service time restrictions [9]. The problem involves assigning vehicles to travel in a way that minimizes assignment costs and the corresponding routing costs.

The VRPTW was the subject of intensive research in heuristic and exact optimisation approaches. Early studies of VRPTW solution techniques can be found in [10], [11] and [12]. Exact solutions techniques were the main focus of [13] and [14]. See [15] and [16] for details on these exact methods. Due to the high level of complexity of the VRPTW and its broad applicability to real-life situations,
solutions that can produce high-quality solutions in a limited time, i.e. heuristics, are of prime importance.

Heuristic used for VRP without time windows [17] and [18]. In [18], a heuristic TS was used to create different solutions to the problem of classical vehicle routing. The routes acquired are then paired to create vehicle workdays by resolving a bin packing problem, an idea previously introduced in [17]. Recent work in [19] reported on insertion heuristics that can efficiently handle various types of constraints, including time windows and multiple vehicle utilizes. The home delivery problem, which is closer to real-world applications, was introduced in [20].

In [21] present the Period Vehicle Routing Problem with Service Choice (PVRP-SC), which enables the endogenous determination of service levels. They acquire PVRP-SC integer programming using precise and heuristic solution methods. Solutions to the discrete PVRP-SC are restricted by instance size because of the computational complexity of the problem. Continuous approximation models are better suited for large problem instances; however, the use of continuous approximation models was limited for periodic routing problems.

This paper concerns with VRPTW with the existence of service disruption (VRPTW-SD). In order to model the problem, we propose a mixed integer programming formula. After solving the problem's continuous model, a feasible neighborhood heuristic search is addressed to find a feasible integer solution.

2. Methodology

2.1. Mathematical Formulation of The VRPTW

Consider a vehicle of capacity $Q$ is to deliver goods from a depot $N_0 = \{0\}$ to a set of customer nodes $N_1 = \{1,...,n\}$ in a complete directed graph with arc set $A$. A cost $c_{ij}$ is incurred and a travel time $t_{ij}$ is imposed for every arc $(i, j) \in A$, with $i \in N_0 \cup N_1$. Each customer $i \in N_1$ has a demand $q_i$, a service or time $s_i$ and a time window $[a_i, b_i]$, where $a_i$ is the earliest time to start service and $b_i$ is the latest time. Therefore, the vehicle must wait for customer $i$ to arrive before time $a_i$. It is necessary to define the set of route $K$ to be travelled by vehicles involved.

**Objective function**

Minimise

$$\sum_{k \in K} \sum_{(i, j) \in A} c_{ij} x_{ij}^k$$

Subject to:

$$\sum_{j \in N_1} x_{ij}^k = y_i^k, \quad i \in N_0 \cup N_1, \quad k \in K,$$

$$\sum_{k \in K} y_i^k = 1, \quad i \in N_0 \cup N_1,$$

$$\sum_{i \in N_1} x_{ih}^k - \sum_{j \in N_1} x_{bj}^k = 0, \quad h \in N_0 \cup N_1, \quad k \in K,$$

$$\sum_{i \in N_1} x_{ih}^k = 1, \quad k \in K,$$

$$\sum_{i \in N_1} x_{i(i+1)}^k = 1, \quad k \in K,$$

$$\sum_{i \in N_0 \cup N_1} q_i y_i^k = Q, \quad k \in K,$$

$$t_i^k + s_i + t_{ij} - M(1 - x_{ij}^k) \leq t_j^k, \quad (i, j) \in N_0 \cup N_1, \quad k \in K,$$

$$a_i y_i^k \leq t_i^k \leq b_i y_i^k, \quad i \in N_1, \quad k \in K,$$
\[ t^k_{0} \geq \sigma^k, \]  
\[ t^k_{n+1} + \sigma^{k+1} \leq t^k_{0}, \quad k = 1, \ldots, k-1 \]  
\[ \sigma^k = \beta \sum_{i \in N_1} s_i y^k_i, \quad k \in K, \]  
\[ t^k_i \leq t^k_{0} + t_{\max}, \quad i \in N_1, \quad k \in K \]  
\[ x^k_i = \{0, 1\} \]  
\[ (i, j) \in A, \quad k \in K, \]  
\[ y^k_i = \{0, 1\}, \quad i \in N_1, \quad k \in K. \]  

Define

- \( x^k_{ij} \) is 1 if arc \((i, j) \in A\) is in route \(k\), 0 otherwise; note that \( x^k_{0,n+1} \) is 1 if route \(k\) is empty;
- \( y^k_i \) is 1 if customer \(i\) is in route \(k\), 0 otherwise;
- \( t^k_i \) is the time of beginning of service at customer \(i\) in route \(k\);
- \( t^k_{0} \) is the start time of route \(k\);
- \( t^k_{n+1} \) is the end time of route \(k\).

According to equation (2), each customer should be visited once. Equations (3), (4) and (5) are flow conservation constraints that define the vehicle's path. Equation (6) states that total road demand should not exceed the vehicle's capacity. Equations (7) to (11) ensure timetable feasibility, while Equation (12) describes vehicle set-up time as the sum of service times multiplied by \(\beta\) for all customers on the route. Equation (13) finally corresponds to the time limit for serving a customer.

### 2.2. Modeling VRPTW-SD

We describe the problem of VRP undergoes disruption as follows: in a given directed graph \(G(V, A)\), a vehicle delivers customer's demand along vertices \(i\) and \(j\) in which \((i, j) \in A\). However, during the travelling a disruption may occur such that the vehicle must travel with different route. Therefore, in this case we should have a set \(Z\) a subset of \(A\) in which there would be disruption route. Then the VRPTW-SD is to discover optimal set of route \(R = K \setminus Z\) such that no disruption occurs.

Objective function

\[ \text{Minimise} \quad \sum_{k \in R} \sum_{(i, j) \in A \setminus Z} c_{ij} x^k_{ij} \]  

Subject to:

\[ \sum_{j \in N_1} x^k_{ij} = y^k_i, \quad i \in N_0 \cup N_1 \setminus Z, \quad k \in R, \]  
\[ \sum_{i \in R} y^k_i = 1, \quad i \in N_0 \cup N_1 \setminus Z, \]  
\[ \sum_{i \in N_1 \setminus Z} x^k_{ih} - \sum_{j \in N_1 \setminus Z} x^k_{bj} = 0, \quad h \in N_0 \cup N_1 \setminus Z, \quad k \in R, \]  
\[ \sum_{i \in N_1 / Z} x^k_{0i} = 1, \quad k \in R, \]  
\[ \sum_{i \in N_1 / Z} x^k_{i(i+1)} = 1, \quad k \in R, \]  
\[ \sum_{i \in N_0 \cup N_1 \setminus Z} q_i x^k_i = Q, \quad k \in R, \]  
\[ t^k_i + s_i + t_{ij} - M (1 - x^k_{ij}) \leq t^k_j, \quad (i, j) \in N_0 \cup N_1 \setminus Z, \quad k \in R, \]
The explanation of Equations (16) – (30) would be just the same as in the previous Eqs (1) to (15), except that the route taken by the vehicles involved through the route set R.

2.3. Neighbourhood Search

It should be noted that although the problems are convex, there is generally no reduced gradient vector in integer programming that is normally used to detect an optimal condition. Therefore, we need to impose a certain condition for the local search procedure to ensure that we have the "best" suboptimal solution for integer searches.

In addition, in Scarf [22], a quantity test was proposed to replace the price test in order to optimize the problem of integer programming by searching the neighbors of a proposed feasible point to see if a nearby point is also feasible and improves the objective function.

Let $[\beta]_k$ be an integer point belongs to a finite set of the $N([\beta]_k)$ neighborhood. We define a $[\beta]_k$-neighborhood system, that is, if such an integer point meets the following two requirements

a. if $[\beta]_j \in N([\beta]_k)$ then $[\beta]_k \in [\beta]_j, j \neq k$.

b. $N([\beta]_k) = [\beta]_k + N(0)$

In relation to the above mentioned neighborhood system, the proposed integration strategy can be described as follows.

Because of an optimal vector, $x_k$, a non-integer component, $x_k$'s adjacent $[x_k]$ dan $[x_k] + 1$ points are considered. If one of these points meets the constraints and results in a minimum deterioration of the optimum objective value, we move to another component if we do not have an integrated solution.

Let $[x_k]$ be the feasible integer point that meets the above conditions. If $[x_k] + 1 \in N([x_k])$ implies that point $[x_k] + 1$ is either unfeasible or yields a lower value than the objective function obtained in relation to $[x_k]$, we could say. In this case, $[x_k]$ is said to be an "optimal" integer programming solution. In our case, a neighborhood search is obviously carried out through feasible points, so that the integer solution is at least at a distance from the optimal continuous solution.

3. The Algorithm

The procedure for finding an optimal continuous solution can be described as follows after solving the relaxed problem.

Let $x = [x] + f, 0 \leq f \leq 1$ be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable $x$ and $f$ is the fractional component.

Stage 1.

Step 1. Get row $i*$ the smallest integer infeasibility, such that $\delta_* = \min\{f_j, 1 - f_j\}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v^T_* = e_*^T B^{-1}$$
Step 3. Calculate $\sigma_j = v_j^T \alpha_j$

With $j$ corresponds to

$$\min_j \left\{ \frac{d_j}{\alpha_j} \right\}$$

Calculate the maximum movement of nonbasic $j$ at lower bound and upper bound. Otherwise go to next non-integer nonbasic or superbasic $j$ (if available). Eventually the column $j^*$ is to be increased form LB or decreased from UB. If none go to next $i^*$.

Step 4. Solve $B\alpha_*=\alpha_*$ for $\alpha_*$

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic $j^*$ from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2.

Pass1: adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2: adjust integer feasible superbasics. The objective of this phase is to conduct a highly lovalized neighbourhood search to verify local optimality.

4. Conclusions

This paper was aimed to present a solution for among the most important problems in Supply Chain Management and Distribution problems. The aim of this paper was to create a model of vehicle Routing problem with time windows considering service-choice and Fleet Scheduling Problems. This problem has an additional constraint, which is limited by the number of vehicles. The proposed algorithm uses the nearest neighbor heuristic algorithm to solve the model.

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