Kinematic analysis of a hyper-redundant robot with application in vertical farming

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Abstract. The paper presents the forward kinematic analysis for a new hyper redundant robot with applications in vertical farming inspections and harvesting tasks. The high dexterity and the ability to adapt to various unstructured environments makes this type of robot a good solution for this kind of applications. The robot consists of seven 3-RPS (Revolute-Prismatic-Spherical) parallel modules resulting in a 21 DOF structure. The forward kinematic analysis of one module is presented first and then for the entire structure in detail. The obtained equations are then implemented in MATLAB and validated using a virtual robot model.

1. Introduction
It is estimated that about 80% of the global population will live in urban areas by 2050 [1] (more than 7 billion people). This trend will create a real challenge in producing and delivering the necessary general goods (food in particular) in these areas [2]. One proposed solution to tackle this challenge is vertical farming [1]. The concepts focus in creating dedicated buildings, in urban areas, that produce food in a controlled environment with increased efficiency [3]. One key aspect of this process is the increase automation that is implemented in order to control the plant growth, health and nutrients needs. The automation process assists the management of plants growth during the entire process starting from seeding, growing and harvesting the plants.

The assist of robotics in implementing the necessary automation systems in vertical farming is more and more present, as can be seen in the literature [4]. One area that robotics aid is the development of dedicated robotized structures for harvesting and inspection for example [5, 6]. Several structures that implement hyper redundant robots were developed with dedicated application in this area [7, 8]. The advantages of these robots (ability to operate in crowded and difficult workspaces, high robustness and the capability to perform complex movements) allow development of complex application tasks specific to the vertical farming operations.

Beside the advantages of these robots, one challenge in implementing these structures is the more complex kinematics comparing with the traditional robots [9, 10]. In implementing the direct and invers kinematics a mapping between actuator space, each module operational space and end effector (robot) operational space must be performed (figure 2). This increases the complexity of the equations and at the same time the necessary hardware performance that could compute in real time these equations are higher [11]. The path planning of these robots is also more complex to implement. For a given pose to
the end effect the robot can have an infinite number of values for the actuators that allows the robot to reach that pose.

In this paper a new hyper redundant structure is proposed for use in the vertical farming inspection and harvesting tasks. The proposed structure is made out of 7 modules that give the robot a total of 21 Degree-of-Freedom (DOF). The forward kinematics of the structure is analysed, and the performance evaluated for the use in the vertical farming process for the proposed tasks.

2. Robot kinematic analysis

The developed robot has a modular structure and is made of 7 modules (figure 1a, b). Each module consists of a 3-RPS parallel structure (figure 1c) and has seven mobile elements, six kinematic joints of class 5 (three revolute joints and three prismatic joints) and three kinematic joints of class 3 (three spherical joints). The number of DOFs for one module is 3, as can be seen from mobility equation:

\[ M = 6n - 5C_5 - 4C_4 - 3C_3 - 2C_2 - C_1 = 6 \cdot 7 - 5 \cdot 6 - 3 \cdot 3 = 3. \]  

where:

- \( M \) = Degrees-of-Freedom;
- \( n \) = number of mobile elements;
- \( C_k \) = number of kinematic joints of class \( k \) \((k = 1 \pm 5)\)

The actuation of the robot is done by the linear DC actuator RA-MINI Plast that produce a linear force of 70[N] at a speed of 3.6 [mm/s]. By multiplying the individual DOF of each module with the number of modules \( n_m = 7 \) results a number of 21 DOF for the entire structure. In design process of the structure one imposed parameter was the ability for the end-effector to have amplitude in orientation of \( \pm 180[\text{deg}] \) resulting the minimum number of the modules that form the structure (in this case \( n_m = 7 \)).
The kinematic of the robot can be divided in two steps that focus on the mapping process between actuator space and module operational space and the mapping between module operational space to the robot operational space (figure 2). In the first phase, based on the length of the actuators, for each module the orientation and position of each upper platform is calculated. These data are then used to calculate the position and orientation of the end effector in task space in the second phase. The propose kinematic analysis could be also extended to robots with similar architecture but with different number of modules.

2.1 Forward kinematic analysis of one module
Figure 3 presents the parallel mechanism used as a module for the proposed robot. It has three DOFs, two in orientation and one in cartesian position [12, 13]. The module consists of 3-RPS (revolute joint-prismatic joint-spherical joint) kinematic chains connected in parallel between a lower base \{A\} and an upper base \{B\}. Revolute joints \(R\) are connected at lower base at 120 degrees resulting the equilateral triangle \(\Delta A_1A_2A_3\) while the spherical joints \(S\) are connected at upper base at 120 degrees resulting the equilateral triangle \(\Delta B_1B_2B_3\). The three prismatic joints \(P\) are used as the inputs to the module to control the three DOFs of the upper base (two rotations about \(X\) and \(Y\)-axes and one translation about \(Z\) axis).
Each revolute joint $A_i (i=1 \pm 3)$ can rotate about its axis of rotation $J_i (i=1 \pm 3)$ in the direction described by the unit vector $\mathbf{j}_i = [j_x, j_y, j_z]^T$. Revolute joint $A_i (i=1 \pm 3)$ is described with respect to coordinate system $A(X,Y,Z)$ by vector $\mathbf{a}_i = [a_x, a_y, a_z]^T$ while spherical joint $B_i (i=1 \pm 3)$ with respect to coordinate system $B(x,y,z)$ by vector $\mathbf{b}_i = [b_x, b_y, b_z]^T$. Based on the previous observations, the coordinates of $A_i$ and $J_i$ are given as follows:

$$
\begin{align*}
\mathbf{a}_i &= [r_x, 0, 0]^T, \\
\mathbf{a}_2 &= \left[-\frac{1}{2}r_x - \frac{\sqrt{3}}{2}r_y, 0\right]^T, \\
\mathbf{a}_3 &= \left[-\frac{1}{2}r_x, -\frac{\sqrt{3}}{2}r_y, 0\right]^T, \\
\mathbf{b}_i &= [r_p, 0, 0]^T, \\
\mathbf{b}_2 &= \left[-\frac{1}{2}r_p - \frac{\sqrt{3}}{2}r_r, 0\right]^T, \\
\mathbf{b}_3 &= \left[-\frac{1}{2}r_p, -\frac{\sqrt{3}}{2}r_r, 0\right]^T, \\
\mathbf{j}_i &= [0, 0, 0]^T, \\
\mathbf{j}_2 &= \left[\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right]^T, \\
\mathbf{j}_3 &= \left[-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right]^T.
\end{align*}
$$

Forward kinematics describes the mathematical relationship between prismatic joints $P$ and position and orientation of the upper base $\{B\}$ with respect to the lower base $\{A\}$ fixed coordinate frame. That means, if prismatic joint lengths $d_1, d_2, d_3$ are provided, one can determine position vector $\mathbf{p} = [p_x, p_y, p_z]^T$ and rotation angles $\theta_x, \theta_y, \theta_z$ defined with respect to three successive rotation about fixed $X, Y, Z$-axes.

Let $\phi_1, \phi_2, \phi_3$ be the angles between the extensible limbs and lower base $\{A\}$ (see Figure 3). If these angles are known, the position vector $\mathbf{q}_i (i=1 \pm 3)$ of spherical joints $B_i (i=1 \pm 3)$ with respect to the lower base coordinate system $A(X,Y,Z)$ can be written as follows:

$$
\begin{align*}
\mathbf{q}_1 &= \begin{bmatrix} r_y - d_1 \cos(\phi_1) \\ 0 \\ d_1 \sin(\phi_1) \end{bmatrix}, \\
\mathbf{q}_2 &= \begin{bmatrix} \frac{-1}{2}(r_y - d_2 \cos(\phi_2)) \\ \frac{\sqrt{3}}{2}(r_y - d_2 \cos(\phi_2)) \\ d_2 \sin(\phi_2) \end{bmatrix}, \\
\mathbf{q}_3 &= \begin{bmatrix} \frac{-1}{2}(r_y - d_3 \cos(\phi_3)) \\ \frac{\sqrt{3}}{2}(r_y - d_3 \cos(\phi_3)) \\ d_3 \sin(\phi_3) \end{bmatrix}.
\end{align*}
$$

Because spherical joints $S$ are placed in the vertexes of triangle $\triangle AB_1B_2$, the distance between two consecutive sides of the equilateral triangle is $|B_1B_2| = |B_2B_3| = |B_3B_1| = \sqrt{3}r_p$. Substituting (3) in the previous mathematical relationship we get a system of three loop-closure nonlinear equations (4) with three unknown variables $(\phi_1, \phi_2, \phi_3)$ and three known variables $(d_1, d_2, d_3)$ where $\phi_i (i=1 \pm 3) \in [0, \pi]$. 


\[
\begin{align*}
    f_1 &= d_i^2 + d_2^2 + 3r_p^2 - 3r_p^2 + d_1d_2 \cos(\phi_1) \cos(\phi_2) - 2d_1d_2 \sin(\phi_1) \sin(\phi_2) \\
    &- 3r_1d_i \cos(\phi_1) - 3r_2d_2 \cos(\phi_2) \\
    f_2 &= d_2^2 + d_3^2 + 3r_p^2 - 3r_p^2 + d_2d_3 \cos(\phi_2) \cos(\phi_3) - 2d_2d_3 \sin(\phi_2) \sin(\phi_3) \\
    &- 3r_1d_2 \cos(\phi_2) - 3r_2d_3 \cos(\phi_3) \\
    f_3 &= d_3^2 + d_1^2 + 3r_p^2 - 3r_p^2 + d_1d_3 \cos(\phi_3) \cos(\phi_1) - 2d_1d_3 \sin(\phi_3) \sin(\phi_1) \\
    &- 3r_1d_3 \cos(\phi_3) - 3r_2d_1 \cos(\phi_1)
\end{align*}
\]

There are various options to solve this system of equations such as numerical methods for nonlinear system of equations, computer algebra with software like Maple or MATLAB or algorithmic elimination methods such as Sylvester's Dialytic elimination method [12-14].

In this paper we will use the Newton-Kantorovich numerical method for nonlinear system of equations to find the numerical values of \( \phi_i (i = 1 \pm 3) \) when \( d_i (i = 1 \pm 3) \) are known. Once the angles are known, the components of position vector \( \mathbf{q}_i (i = 1 \pm 3) \) can be computed from (3). Next, the cartesian position of the upper base \{B\} can be determined by calculating the position vector \( \mathbf{p} \) with the following equation:

\[
\mathbf{p} = \frac{1}{3} (\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3).
\]  

(5)

Regarding the orientation of the upper platform one can note that the position vector \( \mathbf{q}_i (i = 1 \pm 3) \) of spherical joints \( B_i (i = 1 \pm 3) \) can be expressed also as:

\[
\mathbf{q}_i = \mathbf{p} + \hat{A}_B \mathbf{R}_B \mathbf{b}_i.
\]  

(6)

where the rotation matrix \( \hat{A}_B \mathbf{R}_B \) of upper base \{B\} with respect to lower base \{A\} can be written as follows:

\[
\hat{A}_B \mathbf{R}_B = \begin{pmatrix}
    u_x & v_x & w_x \\
    u_y & v_y & w_y \\
    u_z & v_z & w_z
\end{pmatrix}.
\]  

(7)

Direction cosines \( u_x, u_y, u_z, v_x, v_y, v_z \) are solved from (3) and (6) while \( w_x, w_y, w_z \) from the orthogonality property of the rotation matrix [14]. Rotation matrix can be defined also using the following representation [15, 16]:

\[
\hat{A}_B \mathbf{R}_{XYZ} (\theta_x, \theta_y, \theta_z) = \\
\begin{pmatrix}
    \cos\theta_x \cos\theta_y & \cos\theta_x \sin\theta_y & -\sin\theta_x \\
    \cos\theta_z \sin\theta_x \sin\theta_y - \sin\theta_z \cos\theta_x \cos\theta_y & \cos\theta_z \sin\theta_y + \sin\theta_z \cos\theta_x \sin\theta_y & \sin\theta_z \sin\theta_x \\
    -\sin\theta_z \sin\theta_x \sin\theta_y + \cos\theta_z \cos\theta_x \cos\theta_y & -\sin\theta_z \sin\theta_x \cos\theta_y - \cos\theta_z \cos\theta_x \sin\theta_y & \cos\theta_z \sin\theta_x
\end{pmatrix}.
\]  

(8)

where: \( s = \text{sine} \) and \( c = \text{cosine} \)
Finally, by comparing (7) and (8), the three rotation angles $\theta_x, \theta_y, \theta_z$ defined with respect to fixed $X, Y, Z$-axes can be computed using the following [15, 16]:

$$
\begin{align*}
\theta_x &= A \tan 2(v_z / \cos \theta_y, w_z / \cos \theta_y) \\
\theta_y &= \sin^{-1}(-u_z) \\
\theta_z &= A \tan 2(u_y / \cos \theta_x, u_z / \cos \theta_x)
\end{align*}
$$

(9)

where: $\theta_x, \theta_z$ will always be in the range $[-\pi \text{ to } +\pi]$ and $\theta_y$ will be between $[-\pi/2 \text{ and } +\pi/2]$.

2.2 Kinematic analysis of the robot

Equations obtained in Section 2.1 allows to obtain the relative orientation (angles $\theta_x, \theta_y, \theta_z$) and position (vector $p = [p_x, p_y, p_z]^T$) between two consecutive platforms. Based on these parameters the pose of the end effector in respect with the global cartesian system $(O_0X_0Y_0Z_0)$ can be calculated. For this, the transformation matrix $^0T_i$ is calculated using an iterative process, where the transformation matrix is obtained using (10) based on the transformation matrix associated with the previous platform $^i-1T_i$ that is multiplied with the current module transformation matrix $^i-1T_i$ (where $i=1..7$).

$$
^0T_i = ^i-1T_0 \cdot ^i-1T_i \quad \{i=1..7\}
$$

(10)

The general form of the transformation matrix of one module $^i-1T_i$ is presented in (11). To obtain this matrix, the orientation $^i-1R$ and position vector $^i-1P$ obtained from the parameters $\theta_x, \theta_y, \theta_z, p_x, p_y, p_z$ are used. The rotation matrix is obtained using three successive rotations about the axis Z, Y, X.

$$
^i-1T_i = \begin{bmatrix}
^i-1R & ^i-1P \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(11)

Using the $^0T_i$ transformation matrix the pose parameters for the end effector (gripper), could be extracted.

3. Numerical results

The validation of the obtained equations was performed using a virtual model of the robot that was imported from SolidWorks in MATLAB/Simulink. The robot model is developed using Simscape library and the CAD model from SolidWorks. The Simulink model for a module and the virtual representation of the robot are presented in figure 4.

The equations that defined the forward kinematic of the robot were implemented using an M-Function block. The inputs in the block are the values provided by the actuator sensors that define the strokes of the prismatic joints. The outputs are the calculated value for the end effector pose. The parameters for the structure are $r_b = 54$ [mm], $r_p = 47$ [mm] and the actuator length vary from 98.66 [mm] to 128.66 [mm].

Using this setup, several numerical simulations were performed. The aim was to create different work scenarios for the robot and to evaluate the responses given by the Simscape model and the values calculated using the kinematic equations.
Figure 4. Robot virtual model a) Simulink model of one module b) 3D representation of the robot in Mechanics Explorer

The obtained results are presented in figure 5. The signal measured from the end effector pose from virtual robot sensors (blue) are compared with the values calculated using the proposed algorithm (red positions, green angles). It can be observed that variation for the end effector position and orientation matches the calculated results. A set of sine waves signals were used as inputs for the actuators, the amplitude of the signals was 10, 15 and 30 [mm] and a frequency of 0.1, 0.2 and 0.4 [Hz].

Figure 5. Numerical results: a) end effector position b) end effector orientation

4. Conclusions
The paper presented the forward kinematic analysis for a new hyper-redundant robot that is used in applications specific to vertical farming. Due to the complex structure of the robot a hybrid method was used where both analytical and numerical methods were implemented in order to compute the end-effector pose. The obtained equations were implemented in MATLAB/Simulink and the numerical results were validated using a virtual model of the robot. The proposed method can also be used to for robot with similar architecture but with a different number of modules.
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