Extraction of proton trace anomaly energy from near-threshold $\phi$ and $J/\psi$ photo-productions

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Abstract The trace anomalous energy contribution to the proton mass is a very important topic in non-perturbative QCD and hadron physics. In experiments, it is under the hot discussions on how to measure the trace anomalous energy. The QCD interpretation of proton trace anomaly is still not clear. To connect the theory with the experiment, we extract the trace anomaly by analyzing the near-threshold photo-production data of $\phi$ and $J/\psi$ vector mesons. Based on the vector-meson-dominance model and QCD Van der Waals representation, we find that the percentage of trace anomaly in the proton mass ranges from 16 to 24%, which is of similar order of magnitude as the 23% given by Lattice QCD. We also provide the approximate magnitudes of the systematic uncertainties of the extracted results from the model assumptions as well as the data fitting procedures. We argue that the near-threshold $Y$ photo-production experiments are more beneficial for the measurement of trace anomaly in proton mass in the future.

1 Introduction

The majority of the visible mass of the universe resides in the two types of nucleons—protons and neutrons. Nucleons are made of massless gluons and almost massless quarks. The generation of nucleon mass is one of the puzzles in modern particle physics. The origin of the masses of fundamental particles (leptons, quarks, and massive gauge bosons) are delicately explained by the famous Higgs mechanism [1–3]. The proton mass comes mainly from the complicated workings of non-perturbative Quantum Chromodynamics (QCD) [4–6]. QCD theory originates from Yang–Mills theory [7] in the 1950s, which belongs to a non-abelian gauge theory. To calculate the proton mass in the first principle view, one encounters two main challenges: the color confinement for the constitutes (quarks and gluons) [8] and the divergent strong coupling constant at low energy scale.

The proton is a composite particle made of quarks and gluons, hence its mass is usually argued to be from some different sources. The concept of trace anomaly was first introduced from Refs. [9,10]. In Refs. [11,12], Ji first define the proton mass decomposition with QCD Hamiltonian operators. Four gauge-invariant Hamiltonian operators are introduced as,

$$H_{QCD} = H_q + H_g + H_m + H_a,$$

where

$$H_q = \int d^3x \left[ \bar{\psi} \left( -i \nabla \cdot \alpha \right) \psi \right],$$

$$H_g = \int d^3x \frac{1}{2} \left( E^2 + B^2 \right),$$

$$H_m = \int d^3x \left( 1 + \frac{1}{4} \gamma_m \right) \bar{m} \psi \psi,$$

$$H_a = \int d^3x \frac{1}{4} \beta(g) \left( E^2 - B^2 \right).$$

In Ref. [12], the author assumed that the hadron mass is calculated as the expectation value of the Hamiltonian at the hadron rest frame:

$$M_N = \left. \frac{\langle P | H_{QCD} | P \rangle}{\langle P | P \rangle} \right|_{\text{rest frame}},$$
which is decomposed into four terms characterized by the QCD trace anomaly parameter $b(\mu^2)$ and the momentum fraction $a(\mu^2)$ carried by all quarks [12]. The four terms of the proton mass partitions are written as [12],

$$
M_q = \frac{3}{4} \left( a - \frac{b}{1 + \gamma_m} \right) M_N, \quad M_s = \frac{3}{4} (1 - a) M_N, \\
M_m = \frac{4 + \gamma_m}{4 (1 + \gamma_m)} b M_N, \quad M_a = \frac{1}{4} (1 - b) M_N, 
$$

(4)

where the anomalous dimension of quark mass $\gamma_m$ [13] describes the renormalization information. The momentum fraction $a(\mu^2)$ of the quarks is easily computed with all the quark distributions determined in experiments as,

$$
a(\mu^2) = \sum_f \int_0^1 x \left[ q_f \left( x, \mu^2 \right) + \bar{q}_f \left( x, \mu^2 \right) \right] dx. \quad (5)
$$

The proton mass decomposition is from the analysis of the energy-momentum tensor (EMT) in QCD theory [12]. The first three terms in Eq. (4) can be easily understood with the classical field theory. However the last term is an extension of the classical description in the quantum field theory—the quantum anomaly. In the recent papers [14,15], the authors provide further discussion on the the source of quantum anomalous energy (QAE) to the proton mass and argue that it arises from the scale symmetry breaking due to the ultraviolet (UV) divergences in the quantum field theories. The trace anomaly part to the proton mass resembles the dynamical Higgs mechanism to the mass of the fundamental fermions. The author mentions that the quantum anomalous contribution is scale-independent in the chiral limit [14]. However the quark mass is not exactly zero. Therefore, the quantum anomalous energy should be scale-dependent due to the contribution of the quark mass [16,17].

Although the proton mass is mainly generated by the dynamical chiral symmetry breaking, a small portion does arise from the masses of the quarks. The quark mass contribution is usually characterized by a matrix element parameter $b$ [12], which is used to represent the Hamiltonian of quark mass term in Eqs. (2) and (4). The parameter $b$ is related to the quark masses themselves and the quark scalar charges of the proton [10,12],

$$
bM_N = \langle N | m_u \bar{u} u + m_d \bar{d} d | N \rangle + \langle N | m_s \bar{s} s | N \rangle = m_t \langle N | \bar{u} u + \bar{d} d | N \rangle + m_s \langle N | \bar{s} s | N \rangle = \sigma_{\pi N} + \sigma_{sN}. \quad (6)
$$

The scalar nucleon matrix element of up and down quarks $\sigma_{\pi N}$ is about 45 MeV, determined by the low energy $\pi - N$ scattering amplitude [18-20]. In Ref. [21], the authors present a new result using new experimental data and chiral effective field theory $\sigma_{\pi N} = 59(7)$ MeV. Phenomenological estimation using $\pi - N$ scattering data was performed in the Ref. [22]. Too little is known about the $\sigma_{sN}$, which describes the strange scalar charge $m_s \langle N | \bar{s} s | N \rangle$ in the nucleon [10,11]. It is believed to be large because the strange quark is heavier. However, in theory, the QCDSF Collaboration finds $\sigma_{sN} = 11 \pm 13$ MeV [23] and the $\chi$QCD Collaboration finds $\sigma_{sN} = 40 \pm 12$ MeV [24]. The strange part of nucleon matrix element $\sigma_{sN}$ is also suggested to be small around 16 MeV from an effective field theory [25]. The nucleon scalar charge is an important parameter for calculating the scattering between nucleon and the dark matter particles [26-31]. Hence studying the QCD trace anomaly parameter $b$ might be helpful in investigating the method of detecting the weakly-interacting dark matter particles.

At present, the direct experimental measurement of quantum effects inside protons is impossible to be realized. But the low-energy scattering between heavy quarkonium and nucleon can be used to probe the trace anomaly of the nucleon, since the two-gluon exchange is dominant for the process [16,17,32,33]. It is not difficult to compute the scattering amplitude using the local operator product expansion (OPE) with the gluon operator [34]. Recently, in Refs. [35,36] the authors had tried to extract the trace anomaly from the $\phi$ photo-production data near threshold, based on a holographic QCD framework. Based on the Lattice QCD, the result of proton anomaly is discussed in Ref. [37]. For more information on the importance of the proton trace anomaly, one can refer to the EiCUG Yellow Report [38]. In this work, we begin with the vector-meson-dominance model (VMD) suggested by Refs. [10,39]. Following the recent analysis based on this method [40], we try to extract the trace anomalous energy of the proton firstly from the diffractive production data of both $\phi$ and $J/\psi$ vector mesons near the thresholds [41,42]. The paper is organized as follows. We describe the forward meson-nucleon scattering amplitude in the VMD model in Sec. 2, relating the parameter $b$ to the photo-production data. We subsequently show our extraction of the trace anomaly part by fitting the experimental data in Sect. 3. We generally discuss and analyze the uncertainties introduced by the model and data fitting in Sect. 4. Finally we give the conclusions and discussions in Sect. 5.

2 Methods

With the VMD model [43], the forward cross section of the vector meson $X (\phi, J/\psi, \Upsilon, \text{etc.})$ photo-production on the nucleon target is formulated as [39],

$$
\frac{d\sigma_{\gamma N \rightarrow X N}}{dt} \bigg|_{t=0} = 3 \Gamma \left( X \rightarrow e^+ e^- \right) \left( \frac{k_{X N}}{k_{\gamma N}} \right)^2 \frac{d\sigma_{X N \rightarrow X N}}{dt} \bigg|_{t=0}, \quad (7)
$$
where \( \alpha = 1/137 \) denotes the fine structure constant, 
\( k_{ab}^2 = \left[ s - (m_a + m_b)^2 \right] \left[ s - (m_a - m_b)^2 \right] / 4s \) describes 
the center-of-mass momentum square of the corresponding 
two-body system, and \( \Gamma \) is the partial decay width of the \( X \) 
meson decaying into a \( e^+e^- \) pair. The \( \frac{d\sigma_{XN \rightarrow XN}}{dt} \bigg|_{t=0} \) term of 
elastic scattering in the forward limit is given by,

\[
d\sigma_{XN \rightarrow XN} \bigg|_{t=0} = \frac{1}{64\pi m_N^2 (\lambda^2 - m_X^2)} |F_{XN}|^2 , \tag{8}
\]

where \( \lambda = (p_N p_X / m_X) \) is the nucleon energy at the quarkonium 
rest frame [39]. \( F_{XN} \) denotes the invariant amplitude of 
\( X \rightarrow N \) elastic scattering. Using these definitions, the amplitude 
takes the form [10,39],

\[
F_{XN} \simeq r_0^3 d_2 \frac{8\pi^2 M_N m_X}{27} \left( M_N - \sum_{i=u,d,s} m_i q_i \right) N \right) \bigg|_{N}
\]

\[
= r_0^3 d_2 \frac{8\pi^2}{27} (1 - b) M_N^2 m_X , \tag{9}
\]

which is dominated by the QCD trace anomaly part. For 
low-energy scattering and in the chiral limit, the mass of a 
hadron state comes purely from the quantum fluctuations of 
the gluons. Away from the chiral limit, the factor \((1 - b)\) is 
used to characterize the QCD trace anomaly contribution to 
the proton mass. In Eq. (9), the “Bohr” radius \( r_0 \) of the meson \( X \) 
is given by [10],

\[
r_0 = \left( \frac{4}{3\alpha_s} \right) \frac{1}{m_q} , \tag{10}
\]

where \( m_q \) represents the mass of quark \((s\text{ for the } \phi \text{ meson and charm quark } c \text{ for the } J/\psi \text{ meson})\).
In this work, we choose the constituent quark mass (low-
energy scale) for the extraction, e.g. \( m_c = 1.67 \text{ GeV} \) and 
\( m_s = 0.486 \text{ GeV} \) [44]. We discuss the effect of quark mass 
selection on the results in the final section. In Eq. (9), the 
Wilson coefficient \( d_2 \) is found in Refs. [10,45,46] as,

\[
d_2^{1S} = \frac{32}{N_c} \frac{\sqrt{\pi}}{\Gamma(n + \frac{3}{2})} \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n + 5)} , \tag{11}
\]

where \( N_c \) is the number of colors. The strong coupling con-
stant \( \alpha_s \) depends on the renormalization scale \( \mu^2 \), and 
the scale is chosen to be the “Ryderberg” energy \( \epsilon_0 \) for the 
bound state of the quarkonium \( X \) [10,46].

The running strong coupling constant \( \alpha_s \) is an important 
parameter in QCD evolution equations. In this work, we 
use a renormalization-group-invariant process-independent 
effective strong coupling constrained by the calculation of 
LQCD [47]. The effective strong coupling shows a saturated 
plateau approaching the infrared region, which agrees well 
with the the Bjorken sum rule with meson PDFs at low \( \mu^2 < 1 \text{ GeV}^2 \) [47]. The saturated effective strong coupling is given 
by [47,48],

\[
\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{m^2 + \mu^2}{\Lambda^2_{QCD}} \right)} , \tag{12}
\]

where \( \beta_0 = (33 - 2n_f) / 3 \) refers to the one-loop \( \beta \) function 
coefficient, \( n_f \) is the number of flavors, \( m_a = 0.43 \text{ GeV} \) is 
the effective gluon mass owing to the dynamical breaking of 
scale invariance [47]. For the QCD cutoff, we chose \( \Lambda_{QCD} = 0.34 \text{ GeV} \) [49]. Based on this analysis, we use the saturated 
form of strong coupling in order to take into account of the 
non-perturbative effect.

Applying the theoretical and phenomenological framework 
discussed above, we extract the QCD trace anomaly \( M_a \) 
from the extrapolated value of the differential photo-
production cross section at Mandelstam variable \(-t = 0 \text{ GeV}^2\). 
We take the differential cross section data from the 
diffractive \( \phi \) photo-production near the threshold published 
by the LEPS, SAPHIR and CLAS Collaborations [41,50–52] 
and the diffractive \( J/\psi \) photo-production data near the 
threshold by GlueX Collaboration at Jefferson Laboratory (JLab) 
[42]. The experimental data are fitted with an exponential function 
\( \frac{d\sigma}{dt} = \frac{d\sigma}{dt}|_{t=0} \times e^{-kt} \), \tag{13}

where \( \frac{d\sigma}{dt}|_{t=0} \) represents the forward differential cross-
section and \( k \) denotes the slope parameter. Eq. (13) 
describes the \(-t\)-dependence of the cross section. The details of our 
study and the results are shown in the next section, focusing on 
the formulae mentioned in this section.

### 3 Extraction of trace anomaly

Based on Eq. (13), the forward differential cross-sections are 
obtained by fitting the experimental data of \( \phi \) and \( J/\psi \) 
production near the thresholds from Refs. [41,42,50,51]. 
According to Eqs. (7–12), the trace anomaly parts of proton 
mass are extracted by simple algebraic operations. Fig. 1 
shows the GlueX collaboration’s near-threshold differential cross 
section data for \( J/\psi \) photo-production on the proton. It 
is suggested that the \(-t\)-dependence of the differential cross section is well described with the exponential form [42].

Table 1 lists the key parameters and the experimental settings 
for the extraction of the QCD trace anomaly. In this 
work we assume that the energy scale of the strong interac-
tion \( (\mu) \) is equal to the “Ryderberg” energy \( \epsilon_0 \) of the quark-
antiquark pair [39,46], for the production near the threshold. 
For \( J/\psi \) production, the binding energy \( \epsilon_0^2 = \mu^2 = 0.41 \text{ GeV}^2 \) is 
taken in Ref. [10]. Thinking about pulling apart a \( c\bar{c} \) 
pair to generate a \( D\bar{D} \) pair, a naive estimate of the “Ryderberg” 
energy \( \epsilon_0 = m_D + m_{\bar{D}} - m_{J/\psi} \) [10,40]. Similarly, we choose
Table 1 Forward cross section $\frac{d\sigma}{dt}|_{t=0}$ and trace anomaly $M_d/M_N$ extracted by fitting the experimental data from GlueX collaboration (corresponding to Fig. 1)

| $E_\gamma$ (GeV) | $\frac{d\sigma}{dt}|_{t=0}$ (nb/GeV²) | $\chi^2/d.o.f.$ | $M_d/M_N$ (%) |
|-----------------|-------------------------------------|----------------|---------------|
| 10.72           | 3.79 ± 1.32                         | 0.14           | 19.2 ± 3.3    |

Only statistical uncertainties are considered. The systematic uncertainty analysis is described in Sect. 4.

Table 2 Forward cross section $\frac{d\sigma}{dt}|_{t=0}$ and trace anomaly $M_d/M_N$ extracted by fitting the experimental data from LEPS collaboration (corresponding to Fig. 2)

| $E_\gamma$ (GeV) | $\frac{d\sigma}{dt}|_{t=0}$ (μb/GeV²) | $\chi^2/d.o.f.$ | $M_d/M_N$ (%) |
|-----------------|-------------------------------------|----------------|---------------|
| 1.62            | 1.29 ± 0.68                         | 0.10           | 18.9 ± 4.5    |
| 1.72            | 0.91 ± 0.28                         | 0.60           | 16.5 ± 2.5    |
| 1.82            | 1.20 ± 0.33                         | 0.66           | 20.0 ± 2.8    |

Only statistical uncertainties are considered. The systematic uncertainty analysis is described in Sect. 4.

$\epsilon_0^2 = 0.14$ GeV² as the energy scale corresponding to the $\phi$ photo-production.

The $\phi$ production data from CLAS, SAPHIR and LEPS collaborations are presented in Figs. 2 and 4. We still use the exponential function fittings to get the forward differential cross sections, respectively. In the following paragraphs we present some of the details.

Figure 2 shows the LEPS collaboration’s near-threshold differential cross section data for $\phi$ photo-production on the proton target.

The results for some important physical quantities are listed in Table 2.

We present the trace anomaly extracted from cross-section data at three energy points near the threshold measured by the LEPS collaboration [41]. Due to the low statistics, the uncertainty is relatively large. We also analyze the measurements from the SAPHIR collaboration [50] several decades ago for the comparisons. The differential cross section data of SAPHIR are shown in Fig. 3. From the fittings of the exponential form of Eq. (13), the SAPHIR data give the proton trace anomaly parts to be 18.3 ± 1.8% and 16.9 ± 1.1% at $E_\gamma = 1.7$ GeV and $E_\gamma = 1.95$ GeV, respectively (summarized in Table 3).
Due to the high luminosity of the accelerator and the large acceptance of the CLAS spectrometer, the CLAS data is of high precision over a wide \(-t\) range \([51,52]\). In the following, we perform a similar analysis on the CLAS data on the proton. Figure 4 shows the CLAS collaboration’s near-threshold differential cross section data for \(\gamma p \rightarrow \phi p\) on the hydrogen target. The fits to the data based on Eq. (13) are shown in the figure. The differential cross section data are described reasonably well by the exponential function. However, in the large \(-t\) region, we see the cross section rising with \(-t\). This behavior in the large \(-t\) region may be due to the direct \(\phi\)-radiation contributions from \(u\)-channel and \(s\)-channel with \(\phi NN\) coupling or \(\phi NN^*\) coupling \([53–60]\).

The fitting qualities and extraction of trace anomaly are summarized in Table 4. To avoid the \(u\)-channel or \(s\)-channel contamination, we narrow the fit range to the small \(-t\) region. We carefully studied the fits restricted to different \(-t\) ranges, as the CLAS data covers a wide \(-t\) range and of high precision. To understand the effect of large \(-t\) data on the extraction of forward differential cross section and trace anomaly, we perform a series of fits excluding the large \(-t\) data requiring \(-t < 0.6\) GeV\(^2\), \(-t < 0.7\) GeV\(^2\), \(-t < 0.8\) GeV\(^2\), \(-t < 0.9\) GeV\(^2\) and \(-t < 1.0\) GeV\(^2\) respectively. The proton trace anomaly obtained from these fits are shown in Fig. 5. We find that below 1 GeV\(^2\), the extracted trace anomaly does not strongly depend on the choice of large \(-t\) cuts. This is probably because the \(u\)-channel or \(s\)-channel contribution only dominates in the large \(-t\) region where the error bars are comparatively large \([54–60]\). Therefore, the large \(-t\) data have little effect on the fits. The uncertainty arising from changing the fit range of \(-t\) is consistent with the statistical uncertainty in the data.

### 4 Uncertainty analysis

Now let us try to systematically estimate the uncertainty of the analysis in the previous section. From the method we used to extract the trace anomaly, the forward scattering cross sections are fitted from the data and the model calculating the \(X - N\) elastic scattering amplitude (9), which result in the statistical and systematical uncertainties. We start with the model and consider the uncertainty that may arise from the selection of the model’s arguments. It can be seen in Eq. (9) that the \(X - N\) elastic scattering amplitude depends on the previously mentioned “Bohr” radius \(r_0\). In Eq. (10), the expression tells us that the “Bohr” radius of vector meson depends on the quark mass \(m_q\) and the strong coupling constant \(\alpha_s\). First, we take the results in Tables 1 and 2 as the benchmarks \((m_s = 0.486\) GeV and \(m_c = 1.67\) GeV). Then we consider the quark masses with 5% fluctuations, i.e. \(m_{q\pm} = m_q (1 \pm 5\%)\). The trace anomaly \(\Delta_\alpha (1 - b)\) obtained for different quark masses are shown in the following tables.

Tables 5 and 6 show the mean deviation of the trace anomaly for different quark masses for each \(E_\gamma\) set. We easily get a relative deviation in the order of 15%, i.e. 5% of the quark mass fluctuations cause 15% deviation of the extracted trace anomaly. For the impact of the uncertainty...
of strong coupling $\alpha_s$, we perform the same analysis as that with the quark mass. We take the uncertainty of the coupling constant in Ref. [19] and fix the fluctuation of $\alpha_s$ at the level of 5%. Then we find that the trace anomaly also has 15% deviation with different strong couplings.

The extracted proton trace anomaly from the experiments also relies on the data fitting settings. We consider the statistical uncertainty in the experimental data in the previous discussions. As a supplement, we try to discuss some additional uncertainties that exist in terms of experiments. We consider the standard deviations associated with different fitting ranges for the Mandelstam’s variable $| -t | < | -t_{\text{max}} |$ at different energies $E_y$. In the previous sections we describe in detail about the VMD model [10,39,46]. Then we propose how to obtain the trace anomaly in the proton mass decomposition using $\phi$ and $J/\psi$ near-threshold photo-production experimental data [41,42,50,51]. Based on our extracted results, we give a value of the trace anomaly inside the proton, of which the percentage is from 16% to 24%. Previous Lattice QCD study gives the trace anomaly value about 23% [61]. This work is an attempt to extract the proton trace anomaly using more types of data, and we find that the trace anomaly maybe not depend on the type of the vector meson probe. The trace anomaly $M_0/M_N = 0.25(1 - b)$ is very sensitive to the parameter $r_0$, the “Bohr” radius of vector meson. By Eq. (10) we find that it eventually depends on the constituent quark mass $m_q$ and the running coupling constant $\alpha_s$. Thus we provide a complete uncertainty analysis in Sect. 4. It also indicates that the trace anomaly we extracted is model parameter dependent, based on the VMD model adopted in this analysis. The VMD model is successful in describing the vector meson photo-production process, at least as the first step of more extensive theoretical studies. In the VMD model, a real photon fluctuates into a virtual vector meson, which subsequently scatters off the proton target. This model assumption is based on the fact that the vector meson has the same quantum numbers of the photon. The VMD model was used for determining the $\phi - p$, $\omega - p$ and $J/\psi - p$ scattering lengths [62–65]. But the applicability of the VMD model in this case requires special attentions [65]. For a critical review of the VMD model, the papers by Boretsky et. al. [66,67] and the references therein give a very comprehensive discussions. In this work, the values of quark mass and running coupling are based on the assumptions in the non-perturbative energy region. The strong coupling $\alpha_s$ saturates around 3, which is constrained by the calculation of Lattice QCD [47]. For the “Bohr” radius, more experimental con-

| Table 5 | Trace anomaly contributions (in percentage) to the proton mass with different values of strange quark mass based on LEPS data [41] |
| $E_y$ (GeV) | $m_{s-}$ | $m_s$ | $m_{s+}$ | Mean deviation |
| 1.62 | 15.9 ± 4.2 | 18.5 ± 4.9 | 21.4 ± 5.6 | 2.75 |
| 1.72 | 14.1 ± 2.2 | 16.5 ± 2.5 | 19.1 ± 2.9 | 2.5 |
| 1.82 | 17.2 ± 2.4 | 20.0 ± 2.8 | 23.2 ± 3.2 | 3.0 |

| Table 6 | Trace anomaly contributions (in percentage) to the proton mass with different values of charm quark mass based on GlueX data [42] |
| $E_y$ (GeV) | $m_{c-}$ | $m_c$ | $m_{c+}$ | Mean deviation |
| 1.62 | 15.9 ± 4.2 | 18.5 ± 4.9 | 21.4 ± 5.6 | 2.75 |
| 1.72 | 14.1 ± 2.2 | 16.5 ± 2.5 | 19.1 ± 2.9 | 2.5 |
| 1.82 | 17.2 ± 2.4 | 20.0 ± 2.8 | 23.2 ± 3.2 | 3.0 |

Table 7 The standard deviations of trace anomaly contributions (in percentage) with different fitting ranges $| -t | < | -t_{\text{max}} |$ at different energies $E_y$ based on CLAS data [51,52]

| $E_y$ (GeV) | 1.65 | 1.75 | 1.85 | 1.95 |
| Std. Dev. | 0.37 | 0.70 | 1.13 | 1.03 |

The standard deviations of trace anomaly contributions (in percentage) with different energies $E_y$ from different Collaborations (CLAS: $E_y = 1.65, 1.75, 1.85, 1.95$ GeV; SAPHIR: $E_y = 1.70, 1.95$ GeV; LEPS: $E_y = 1.62, 1.72, 1.82$ GeV)

| Collaborations | CLAS | SAPHIR | LEPS |
| Std. Dev. | 5.84 | 2.57 | 9.22 |

5 Conclusions and discussions

In the previous sections we describe in detail about the VMD model [10,39,46]. Then we propose how to obtain the trace anomaly in the proton mass decomposition using $\phi$ and $J/\psi$ near-threshold photo-production experimental data [41,42,50,51]. Based on our extracted results, we give a value of the trace anomaly inside the proton, of which the percentage is from 16% to 24%. Previous Lattice QCD study gives the trace anomaly value about 23% [61]. This work is an attempt to extract the proton trace anomaly using more types of data, and we find that the trace anomaly maybe not depend on the type of the vector meson probe.

The trace anomaly $M_0/M_N = 0.25(1 - b)$ is very sensitive to the parameter $r_0$, the “Bohr” radius of vector meson. By Eq. (10) we find that it eventually depends on the constituent quark mass $m_q$ and the running coupling constant $\alpha_s$. Thus we provide a complete uncertainty analysis in Sect. 4. It also indicates that the trace anomaly we extracted is model parameter dependent, based on the VMD model adopted in this analysis. The VMD model is successful in describing the vector meson photo-production process, at least as the first step of more extensive theoretical studies. In the VMD model, a real photon fluctuates into a virtual vector meson, which subsequently scatters off the proton target. This model assumption is based on the fact that the vector meson has the same quantum numbers of the photon. The VMD model was used for determining the $\phi - p$, $\omega - p$ and $J/\psi - p$ scattering lengths [62–65]. But the applicability of the VMD model in this case requires special attentions [65]. For a critical review of the VMD model, the papers by Boretsky et. al. [66,67] and the references therein give a very comprehensive discussions. In this work, the values of quark mass and running coupling are based on the assumptions in the non-perturbative energy region. The strong coupling $\alpha_s$ saturates around 3, which is constrained by the calculation of Lattice QCD [47]. For the “Bohr” radius, more experimental con-
straints should be found in the future to achieve the goal of reducing the uncertainty. Higher-twist calculations should be investigated as well. From this work, we give the result for the proton anomaly energy, which is similar to the result of the lattice point QCD, and for the first time we provide the statistical and systematical uncertainty analyses. The model uncertainties generated by the current method in this work are significant and more statistics are needed on the experimental side in future.

Furthermore, the $\Upsilon(1S)$ [68] photo-production data generated by Electron-Ion Colliders in the United States and China [38, 69–72] will become even more important in the future. Since the “Rydberg” energy $\epsilon_0$ of $\Upsilon(1S)$ is higher than that of other quarkonia, Eq. (9) is more valid for the near threshold $\Upsilon(1S)$ production [39, 46]. The theoretical uncertainty from the strong coupling $\alpha_s$ is much smaller at a higher energy scale $\epsilon_0$. Therefore, the theoretical framework in this work is more suitable for the analysis of near-threshold $\Upsilon(1S)$ photo-production [40] in the future.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The analyzed data are taken from GlueX, LEPS, CLAS and SAPHIR which are mentioned in text. The extracted results are clearly presented in the tables and the main text.]

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