From Dirac neutrino masses to baryonic and dark matter asymmetries

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We consider an $SU(3)_c \times SU(2)_L \times U(1)_Y$ dark sector, parallel to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ ordinary sector. The hypercharges, baryon numbers and lepton numbers in the dark sector are opposite to those in the ordinary sector. We further introduce three types of messenger sectors: (i) two or more gauge-singlet Dirac fermions, (ii) two or more $[SU(2)_L \times SU(2)_L']$-bidoublet Higgs scalars, (iii) at least one gauge-singlet Dirac fermion and at least one $[SU(2)_L \times SU(2)_L']$-bidoublet Higgs scalar. The lepton number conserving decays of the heavy fermion singlet(s) and/or Higgs bidoublet(s) can simultaneously generate a lepton asymmetry in the $[SU(2)_L']$-doublet leptons and an opposite lepton asymmetry in the $[SU(2)_L']$-doublet leptons to account for the cosmological baryon asymmetry and dark matter relic density, respectively. The lightest dark nucleon should have a mass about 5 GeV. By integrating out the heavy fermion singlet(s) and/or Higgs bidoublet(s), we can obtain three light Dirac neutrinos composed of the ordinary and dark neutrinos. If a mirror discrete symmetry is further imposed, our models will not require more unknown parameters than the traditional type-I, type-II or type-I+II seesaw models.

I. INTRODUCTION

Various neutrino oscillation experiments have established the phenomenon of massive and mixing neutrinos. To naturally understand the smallness of neutrino masses, we can consider the seesaw [1] extension of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model (SM). In the conventional seesaw scenario [2-8], the neutrinos have a Majorana nature which, however, has not been experimentally verified. Alternatively, we can build some Dirac seesaw models [9-11] to give the light Dirac neutrinos. Usually, the Dirac seesaw needs more parameters than the Majorana seesaw. In the Majorana or Dirac seesaw context, we can obtain the cosmological baryon asymmetry through the leptogenesis [12-17] or neutrino-genesis [18-20] mechanism. On the other hand, the dark matter relic density also indicates the necessity of supplementing the SM. It is intriguing that the baryonic and dark matter contribute comparable energy densities to the present universe [21] although they have very different properties. This coincidence can be elegantly explained if the dark matter relic density is an asymmetry between the dark matter and antimatter and its origin is related to the baryon asymmetry. Such asymmetric dark matter can naturally appear in the mirror universe models [22-47]. There are also other ideas connecting the dark matter asymmetry to the baryon asymmetry [48-73].

In this paper we shall propose a common genesis of the Dirac neutrino masses, the baryon asymmetry and the dark matter relic density. Specifically, we consider an $SU(3)_c \times SU(2)_L \times U(1)_Y$ dark sector, parallel to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ ordinary sector. The hypercharges, baryon numbers and lepton numbers in the dark sector are opposite to those in the ordinary sector. Besides the ordinary and dark sectors, there are three types of messenger sectors: (i) two or more gauge-singlet Dirac fermions, (ii) two or more $[SU(2)_L \times SU(2)_L']$-bidoublet Higgs scalars, (iii) at least one gauge-singlet Dirac fermion and at least one $[SU(2)_L \times SU(2)_L']$-bidoublet Higgs scalar. Through the lepton number conserving decays of the heavy fermion singlet(s) and/or Higgs bidoublet(s), we can simultaneously realize a lepton asymmetry in the $[SU(2)_L']$-doublet leptons and an opposite lepton asymmetry in the $[SU(2)_L']$-doublet leptons to account for the baryon asymmetry and the dark matter relic density, respectively. The lightest dark nucleon with a determined mass about 5 GeV can serve as the dark matter particle. The dark photon will become massive although the ordinary photon keeps massless. The kinetic mixing between the $U(1)_Y$ and $U(1)_Y'$ gauge fields can result in a testable scattering of the dark nucleons off the ordinary nucleons. Furthermore, we can get a tiny mass term between the ordinary and dark left-handed neutrinos by integrating out the heavy fermion singlet(s) and/or Higgs bidoublet(s). So, the ordinary and dark neutrinos can naturally form three light Dirac neutrinos [27-28, 41-43]. Finally, we can impose a mirror discrete symmetry to reduce the parameters. In this case, our models will not contain additional parameters compared with the traditional Majorana seesaw models.
II. THE ORDINARY AND DARK SECTORS

We denote the ordinary quarks, leptons and scalars by

$$q_L(3, 2, +\frac{1}{3}) = \begin{bmatrix} u_L \\ d_L \end{bmatrix}, \quad d_R(3, 1, -\frac{2}{3}), \quad u_R(3, 1, +\frac{1}{3})$$

$$l_L(1, 2, -1) = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}, \quad e_R(1, 1, -2),$$

$$\phi(1, 2, -1) = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix}, \quad \delta(1, 1, -4),$$

where the first and second numbers in parentheses are the dimensions of the $SU(3)_c$ and $SU(2)_L$ representations, while the third ones are the $U(1)_Y$ hypercharges $Y$. Accordingly, we define the dark quarks, leptons and scalars as below,

$$q'_L(3, 2, -\frac{1}{3}) = \begin{bmatrix} d'_L \\ u'_L \end{bmatrix}, \quad d'_R(3, 1, +\frac{2}{3}), \quad u'_R(3, 1, -\frac{1}{3})$$

$$l'_L(1, 2, +1) = \begin{bmatrix} e'_L \\ \nu'_L \end{bmatrix}, \quad e'_R(1, 1, +2),$$

$$\phi'(1, 2, +1) = \begin{bmatrix} \phi^{\prime+} \\ \phi^{\prime0} \end{bmatrix}, \quad \delta'(1, 1, +4),$$

where the first and second numbers in parentheses are the dimensions of the $SU(3)'_c$ and $SU(2)'_L$ representations, while the third ones are the $U(1)'_Y$ hypercharges $Y'$. Like the hypercharges, the baryon or lepton numbers of the dark fermions are assumed opposite to those of the ordinary fermions.

We then write down the Lagrangian of the ordinary and dark sectors,

$$\mathcal{L}^{OD} = \mathcal{L}^{OD}_R + \mathcal{L}^{OD}_Y - V^{OD}.$$  

(3)

Here the index "OD" is the abbreviation of "ordinary-dark". The kinetic terms include

$$\mathcal{L}^{OD}_R = \left(D_{\mu}\phi\right)^\dagger(D^{\mu}\phi) + \left(D_{\mu}\delta\right)^\dagger(D^{\mu}\delta) + i\bar{q}_L \gamma_\mu q_L + i\bar{d}_R \gamma_\mu d_R + i\bar{u}_R \gamma_\mu u_R + i\bar{e}_L \gamma_\mu e_L + i\bar{\nu}_L \gamma_\mu \nu_L$$

$$+ i\bar{e}_R \gamma_\mu e_R + \left(D_{\mu}\phi'\right)^\dagger(D^{\mu}\phi') + \left(D_{\mu}\delta'\right)^\dagger(D^{\mu}\delta') + i\bar{q}'_L \gamma_\mu q'_L + i\bar{d}'_R \gamma_\mu d'_R + i\bar{u}'_R \gamma_\mu u'_R + i\bar{\nu}'_L \gamma_\mu \nu'_L$$

$$+ i\bar{e}'_R \gamma_\mu e'_R - \frac{1}{4}G^{\mu\nu}_a G_{a\mu\nu} - \frac{1}{4}W^{\mu\nu}_a W_{a\mu\nu} - \frac{1}{4}B^{\mu\nu}_a B_{a\mu\nu} - \frac{1}{4}B^{\prime\mu\nu}_a B'_{a\mu\nu} - \frac{1}{2}B_{\mu\nu} B^{\mu\nu},$$

(4)

where the covariant derivatives are given by

$$D_{\mu} = \partial_{\mu} - ig_1 \frac{Y}{2} B_{\mu} - ig_2 \frac{\tau^a}{2} W_{\mu}^a \delta - ig_3 \frac{\lambda}{2} G_{\mu}^a \delta,$$

$$-ig_1 \frac{Y'}{2} B'_{\mu} - ig_2 \frac{\tau^a}{2} W'_{\mu}^a \delta' - ig_3 \frac{\lambda'}{2} G_{\mu}^{a'} \delta'$$

with

$$\delta_2 = \begin{cases} 1 & \text{for } SU(2)_L \text{ doublets}, \\
0 & \text{for } SU(2)_L \text{ singlets};
\end{cases}$$

$$\delta_3 = \begin{cases} 1 & \text{for } SU(3)_c \text{ triplets}, \\
0 & \text{for } SU(3)_c \text{ singlets};
\end{cases}$$

$$\delta_2' = \begin{cases} 1 & \text{for } SU(2)'_L \text{ doublets}, \\
0 & \text{for } SU(2)'_L \text{ singlets};
\end{cases}$$

$$\delta_3' = \begin{cases} 1 & \text{for } SU(3)'_c \text{ triplets}, \\
0 & \text{for } SU(3)'_c \text{ singlets}.
\end{cases}$$

We also show the Yukawa interactions:

$$\mathcal{L}^{YD}_r = -(y_d)_{ij} q'_L \bar{d}_R - (y_u)_{ij} \bar{u}_R \times (y_e)_{ij} \bar{e}_R \times (y_{\nu})_{ij} \bar{\nu}_L \times (y_{\nu}')_{ij} \bar{\nu}'_L \times i \bar{q}'_L \gamma_\mu \nu'_L \times i \bar{e}'_R \gamma_\mu e'_R \times i \bar{\nu}'_L \gamma_\mu \nu'_L \times i \bar{e}'_R \gamma_\mu e'_R \times H.c.,$$

(5)

and the scalar potential:

$$V^{OD} = \mu_2^2 \phi^\dagger \phi + \lambda_2 (\phi^\dagger \phi)^2 + \mu_3^2 \delta^\dagger \delta + \lambda_3 (\delta^\dagger \delta)^2 + \frac{1}{2} \lambda_{23} \phi^\dagger \phi \delta + \phi^\dagger \phi \lambda_{23}^* \phi + \lambda_{23} \delta + \delta^\dagger \delta + \lambda_{23}^* \delta + \delta^\dagger \delta + \lambda_{23} \lambda_{23}^* \delta + \lambda_{23}^* \delta + \delta^\dagger \delta + \lambda_{23} \lambda_{23}^* \delta + \lambda_{23}^* \lambda_{23} \delta.$$  

(6)

The symmetry breaking pattern is expected to be

$$SU(3)_c \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_c \times U(1)_{em},$$

$$SU(3)'_c \times SU(2)'_L \times U(1)'_Y \longrightarrow SU(3)'_c \times U(1)'_{em},$$

$$\longrightarrow SU(3)'_c.$$  

(7)

For this purpose, the ordinary and dark scalars should develop their vacuum expectation values (VEVs) as below,

$$\langle \phi \rangle = \begin{bmatrix} \langle \phi^0 \rangle \\ 0 \end{bmatrix} \approx 174 \text{ GeV}, \quad \langle \delta \rangle = 0,$$

$$\langle \phi' \rangle = \begin{bmatrix} 0 \\ \langle \phi'^0 \rangle \end{bmatrix}, \quad \langle \delta' \rangle \neq 0.$$  

(8)

This means the ordinary photon will keep massless while the dark photon will become massive.

It is straightforward to read the fermion masses in the ordinary sector,

$$\mathcal{L} \supset -m_d q_L d_R - m_u \bar{u}_L u_R - m_e \bar{e}_L e_R + \text{H.c.}$$

with

$$m_d = y_d \langle \phi \rangle, \quad m_u = y_u \langle \phi \rangle, \quad m_e = y_e \langle \phi \rangle.$$  

(9)
and the fermion masses in the dark sector,
\[ \mathcal{L} \ni \bar{L}'_R d'_R - m'_\nu \bar{u}'_R u'_L - m'_{\nu'} c'_R c'_L - \delta m'_{\nu'} c'_R c'_L + \text{H.c.} \]
\[ \text{with } m'_{\nu'} = y_{\nu'} (\phi''), \quad m'_c = y_c (\phi''), \quad \delta m'_{\nu'} = y_{\nu'} (\phi'). \] (11)

The dark charged leptons should be the so-called quasi-Dirac fermions for \( \delta m_{\nu'} \ll m_{\nu'} \). In the ordinary sector, the quark masses \( m_u \) and \( m_d \) are much smaller than the hadronic scale \( \Lambda_{\text{QCD}} \), so that they can only have a negligible contribution to the nucleon masses,
\[ m_p \simeq m_n \simeq 1 \text{ GeV} = m_N. \] (12)

In the dark sector, the quark masses \( m_{u'} \) and \( m_{d'} \) may be sufficiently larger than the hadronic scale \( \Lambda_{\text{QCD}} \). In this case, the dark nucleon masses can approximately equal the sum of the dark quark masses,
\[ m'_N = 2m_{u'} + m_{d'}, \quad m'_\nu = 2m_{d'} + m_{u'}. \] (13)

As we will demonstrate in the following, our completed models also contain a messenger sector. We will refer to \( \mathcal{L}^M \) (with the index "M" being the abbreviation of "messenger") as the Lagrangian involving the messenger fields. Note that our models will not have any baryon or lepton number violating interactions except the \( SU(2)_L \) and \( SU(2)'_L \) sphaleron processes \( \text{[27, 28, 41, 43]} \).

\section{III. THE MODEL WITH GAUGE-SINGLET DIRAC FERMIONS}

In this sector, we will give the completed model with two or more gauge-singlet Dirac fermions. The Lagrangian involving the fermion singlets (FSs) should be
\[ \mathcal{L}^M = \mathcal{L}^{FS}_K + \mathcal{L}_Y^{FS} + \mathcal{L}_m^{FS}, \] (14)
with the kinetic terms:
\[ \mathcal{L}_K^{FS} = i\bar{N}_R \gamma_i \partial^i N_R + i\bar{N}'_R \gamma_i \partial^i N'_R, \] (15)

the Yukawa terms:
\[ \mathcal{L}_Y^{FS} = -(y_N)_{ij} \bar{N}_R \phi N_R - (y_{\nu'})_{ij} \bar{N}'_R \phi' N'_R + \text{H.c.}, \] (16)

and the mass terms:
\[ \mathcal{L}_m^{FS} = -(M_N)_{ij} \bar{N}'_R \phi N_R + \text{H.c.}. \] (17)

Here we have introduced two types of gauge-singlet right-handed fermions:
\[ \left\{ \begin{array}{ll} \bar{N}_R(1,1,0)(1,1,0) & \text{with a lepton number } + 1, \\ \bar{N}'_R(1,1,0)(1,1,0) & \text{with a lepton number } - 1, \end{array} \right. \] (18)

where the first and second parentheses being the quantum numbers under the ordinary and dark gauge groups, respectively. Note that other gauge-invariant Yukawa and mass terms have been forbidden as a result of the lepton number conservation. After choosing a proper base, the fermion singlets can have a diagonal and real mass matrix:
\[ M_N = \text{diag}\{M_{N_1}, M_{N_2}, \ldots\}. \] (19)

We then can define the Dirac fermions:
\[ N_i = N_{R_i} + N'_{R_i}. \] (20)

\section{A. Dirac neutrino masses}

After the \([SU(2)_L]\)-doublet Higgs scalar \( \phi \) and the \([SU(2)'_L]\)-doublet Higgs scalar \( \phi' \) develop their VEVs, the ordinary and dark left-handed neutrinos as well as the gauge-singlet right-handed fermions will have a mass matrix as below,
\[ \mathcal{L} \ni \left[ \bar{\nu}_L \bar{N}'_R \right] \begin{pmatrix} 0 & y_N (\phi) \\ -y_N (\phi') & M_N \end{pmatrix} \left[ \begin{pmatrix} \nu'_L \\ N_R \end{pmatrix} \right] + \text{H.c.} \] (21)

We can block diagonalize the above mass matrix to be \( \text{[27, 28, 41, 43]} \)
\[ \mathcal{L} \ni \left[ \bar{\nu}_L \bar{N}'_R \right] \begin{pmatrix} m_{\nu'} & 0 \\ 0 & M_N \end{pmatrix} \left[ \begin{pmatrix} \nu'_L \\ N_R \end{pmatrix} \right] + \text{H.c.} \]
\[ m_{\nu'} = -y_N \langle \phi (\phi') \rangle / M_N, \]
if the seesaw condition is satisfied, i.e.
\[ y_N \langle \phi \rangle, \quad y_{\nu'} \langle \phi' \rangle \ll M_N. \] (23)

This means the ordinary and dark left-handed neutrinos will form the light Dirac neutrinos \( \nu = \nu_L + \nu'_L \), while the gauge-singlet right-handed fermions will form the heavy Dirac fermions \( N = N_R + N'_R \). Note that the Dirac neutrino mass matrix \( \text{[22]} \) will have one nonzero eigenvalue if there is only one gauge-singlet Dirac fermion. We thus need two or more gauge-singlet Dirac fermions to explain the neutrino oscillation data. Analogous to the usual type-I seesaw formula of the Majorana neutrino masses, we shall refer to the formula \( \text{[22]} \) of the Dirac neutrino masses to be the type-I Dirac seesaw. The relevant diagram is shown in Fig. [1].
FIG. 2: The lepton number conserving decays of the heavy gauge-singlet Dirac fermions $N_i = N_{Ri} + N_{Li}^\dagger$ in the type-I Dirac seesaw scenario. We need at least two fermion singlets to generate a nonzero lepton asymmetry in the $[SU(2)_L]$-doublet leptons $l_L$ and an opposite lepton asymmetry in the $[SU(2)_L]$-doublet leptons $l'_L$. The CP conjugation is not shown for simplicity.

B. CP asymmetry

As shown in Fig. 2, the lepton number conserving decays of the gauge-singlet Dirac fermions $N_i$ can simultaneously generate a lepton asymmetry $\eta_{L_i}$ in the $[SU(2)_L]$-doublet leptons $l_L$ and an opposite lepton asymmetry $\eta_{L'_i}$ in the $[SU(2)_L']$-doublet leptons $l'_L$, i.e.

$$\eta_{L_i} = -\eta_{L'_i} \propto \varepsilon_{N_i}.$$  

(24)

Here the CP asymmetry $\varepsilon_{N_i}$ can be calculated by

$$\varepsilon_{N_i} = \frac{\Gamma_{N_i \rightarrow l_i \phi^*} - \Gamma_{N_i \rightarrow l'_i \phi'}}{\Gamma_{N_i}}$$

$$= \frac{\Gamma_{N_i' \rightarrow l'_i \phi'}}{\Gamma_{N_i}} - \frac{\Gamma_{N_i \rightarrow l'_i \phi'}}{\Gamma_{N_i}}$$

$$= \frac{1}{4\pi} \sum_{j \neq i} \text{Im} \left[ \frac{(y_N^i y_N^j y_N^{i'})_{ij}}{(y_N^i y_N^{i'})_{ii} + (y_N^i y_N^{i'})_{jj}} \right] M_{N_i} M_{N_j} - \frac{M_{N_i}^2 - M_{N_j}^2}{\Gamma_{N_i}}.$$  

(25)

with $\Gamma_{N_i}$ being the decay width:

$$\Gamma_{N_i} = \Gamma_{N_i \rightarrow l_i \phi^*} + \Gamma_{N_i \rightarrow l_i \phi'} = \Gamma_{N_i \rightarrow l'_i \phi'} + \Gamma_{N_i \rightarrow l'_i \phi^*}$$

$$= \frac{1}{16\pi} |(y_N^i y_N^{i'})_{ii} + (y_N^i y_N^{i'})_{jj}| M_{N_i}.$$  

(26)

We should keep in mind that at least two gauge-singlet Dirac fermions are necessary to induce a nonzero CP asymmetry.

If the gauge-singlet Dirac fermions have a hierarchical mass spectrum, i.e. $M_{N_j}^2 \ll M_{N_i}^2$, we can simplify the CP asymmetry (25) to be

$$\varepsilon_{N_i} \approx \frac{1}{4\pi} \text{Im} \left[ \frac{(y_N^i m_{N_i})_{ii}}{(y_N^i y_N^{i'})_{ii} + (y_N^i y_N^{i'})_{jj}} \phi \langle \phi' \rangle \right] M_{N_i}.$$  

(27)

Similar to the Davidson-Ibarra bound [11] in the type-I Majorana seesaw scenario, the above CP asymmetry should have an upper bound:

$$|\varepsilon_{N_i}| \leq \frac{1}{8\pi} \text{Im} \left[ \frac{(y_N^i m_{N_i})_{ii}}{(y_N^i y_N^{i'})_{ii} + (y_N^i y_N^{i'})_{jj}} \phi \langle \phi' \rangle \right] M_{N_i}.$$  

(28)

Here $m_{N_i}^{\text{max}}$ is the maximal eigenvalue of the neutrino mass matrix $m_{N_i}$. Alternatively, the CP asymmetry (25) can be resonantly enhanced [8] if the gauge-singlet Dirac fermions have a quasi-degenerate mass spectrum, i.e. $M_{N_i}^2 \approx M_{N_j}^2 \gg |M_{N_i}^2 - M_{N_j}^2|$.

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1 In Ref. [32], the authors proposed a novel scenario where the gauge-singlet fermions with heavy Majorana masses can mediate some lepton number violating processes involving the dark leptons. This mechanism cannot apply to the present lepton number conserving model because the cross sections obey the relations $\sigma(l_i \phi^* \rightarrow l_i \phi^*) + \sigma(l_i \phi^* \rightarrow l'_i \phi') = \sigma(l'_i \phi \rightarrow l'_i \phi') + \sigma(l'_i \phi \rightarrow l'_i \phi'^*)$, $\sigma(l'_i \phi^* \rightarrow l'_i \phi^*) + \sigma(l'_i \phi^* \rightarrow l'_i \phi'^*) = \sigma(l_i \phi^* \rightarrow l_i \phi^*) + \sigma(l_i \phi \rightarrow l_i \phi'),$ and $\sigma(l_i \phi^* \rightarrow l_i \phi^*) - \sigma(l_i \phi \rightarrow l_i \phi') = 0$, $\sigma(l'_i \phi^* \rightarrow l'_i \phi'^*) - \sigma(l'_i \phi \rightarrow l'_i \phi') = 0$ as a result of CPT invariance.
IV. THE MODEL WITH $[SU(2)_L \times SU(2)_R]$-BIDOU LEPTONS

In this sector, we will give the completed model with two or more $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars:

$$\Sigma_a(1, 2, -1)(1, 2, +1) = \begin{bmatrix} \sigma_+^a & \sigma_0^a \\ \sigma_0^a & -\sigma_-^a \end{bmatrix}. \quad (29)$$

Here the first and second parentheses stand for the quantum numbers under the ordinary and dark gauge groups, respectively. The Lagrangian involving the Higgs bidoublets (HBs) should be

$$\mathcal{L}^M \equiv \mathcal{L}^{HB} = \mathcal{L}^{HB} + \mathcal{L}^Y - V^{HB}, \quad (30)$$

where the kinetic terms are

$$\mathcal{L}^{HB} = \text{Tr}[(D^\mu \Sigma_a)(D^\mu \Sigma_a)] \quad \text{with}$$

$$D_\mu \Sigma_a = \partial_\mu \Sigma_a - ig_1 \eta \Sigma_a \tau^i \Sigma_a - ig_2 \frac{\tau^i}{2} W^i \Sigma_a$$

$$-i \frac{g'_1}{2} B_\mu \Sigma_a - ig_2 \Sigma_a \frac{\tau^T}{2} \Sigma_a, \quad (31)$$

the Yukawa couplings contain

$$\mathcal{L}^{HB}_Y = -f^a_i \Sigma_a \nu^c_L + \text{H.c.}, \quad (32)$$

and the scalar potential includes

$$V^{HB} = (M^2_a)^{ab} \text{Tr}(\Sigma_a \Sigma_b) + (\lambda_\Sigma)_{abcd} \text{Tr}(\Sigma_a \Sigma_b \Sigma_c \Sigma_d)$$

$$+ 2(\lambda_{\phi} \Sigma_a)_{ab} \phi \delta \phi + (\lambda_{\phi} \Sigma_a)_{ab} \phi' \delta \phi' \text{Tr}(\Sigma_a) \Sigma_b$$

$$+ 2(\lambda_{\phi} \Sigma_a)_{ab} \phi | \delta | \phi' | \delta | \phi' \text{Tr}(\Sigma_a) \Sigma_b$$

$$+ \rho_a \phi^\dagger \phi^a \phi^* + \text{H.c.}. \quad (33)$$

Without loss of generality, we can choose a base to take

$$M^2_{\Sigma} = \text{diag}(M^2_{\Sigma 1}, M^2_{\Sigma 2}, \ldots), \quad \rho_1 = \rho_1^*, \quad \rho_2 = \rho_2^*, \ldots \quad (34)$$

Note that the above Yukawa couplings and scalar potential will exactly conserve the lepton number because the Higgs bidoublets $\Sigma_a$ don’t carry any lepton numbers.

A. Dirac neutrino masses

After the $[SU(2)_L]-$doublet Higgs scalar $\phi$ and the $[SU(2)_R]-$doublet Higgs scalar $\phi'$ acquire their VEVs, the heavy $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars $\Sigma_a$ can pick up the seesaw-suppressed VEVs:

$$\langle \Sigma_a \rangle = \begin{bmatrix} 0 \\ \langle \phi_a \rangle \end{bmatrix} \quad \text{with} \quad \langle \phi_a \rangle \simeq -\frac{\rho_a \langle \phi \rangle \langle \phi' \rangle}{M^2_{\Sigma_a}} \ll \langle \phi \rangle, \langle \phi' \rangle. \quad (35)$$

We hence can naturally obtain the light Dirac neutrinos composed of the ordinary left-handed neutrinos $\nu_L$ and the dark left-handed neutrinos $\nu'_L$, i.e.

$$\mathcal{L} \ni -m_\nu \bar{\nu}_L \nu'^*_L + \text{H.c. with}$$

$$m_\nu = \sum_a f_a \langle \Sigma_a \rangle = \sum_a m_\nu^{II}a = m_\nu^{II}. \quad (36)$$

FIG. 3: The type-II Dirac seesaw for generating the masses between the ordinary left-handed neutrinos $\nu_L$ and the dark left-handed neutrinos $\nu'_L$.

The above mechanism for the Dirac neutrino masses is very similar to the usual type-II seesaw [2] for the Majorana neutrino masses. So, it may be named as the type-II Dirac seesaw. We show the relevant diagram in Fig. 3.

B. CP asymmetry

From Fig. 4 it is straightforward to see that the lepton number conserving decays of the $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars $\Sigma_a$ can generate a lepton asymmetry $\eta_{L_a}$ in the $[SU(2)_L]$-doublets $l_L, l'_L$, and an opposite lepton asymmetry $\eta_{L'_a}$ in the $[SU(2)_L']$-doublet leptons $l'_L$, i.e.

$$\eta_{L_a} = -\eta_{L'a} \propto \varepsilon_{\Sigma_a}, \quad (37)$$

where $\varepsilon_{\Sigma_a}$ is the CP asymmetry:

$$\varepsilon_{\Sigma_a} = \frac{\Gamma_{\Sigma_a \rightarrow l_L l'_L} - \Gamma_{\Sigma_a \rightarrow l'_L l_L}}{\Gamma_{\Sigma_a}}, \quad (38)$$

with $\Gamma_{\Sigma_a}$ being the decay width:

$$\Gamma_{\Sigma_a} = \frac{\Gamma_{\Sigma_a \rightarrow l_L l'_L} + \Gamma_{\Sigma_a \rightarrow \phi' \phi'}}{\Gamma_{\Sigma_a}}$$

$$= \frac{\Gamma_{\Sigma_a \rightarrow l_L l'_L} + \Gamma_{\Sigma_a \rightarrow \phi' \phi'}}{\Gamma_{\Sigma_a}}. \quad (39)$$

We can calculate the decay width at tree level,

$$\Gamma_{\Sigma_a} = \frac{1}{16\pi} \left| \text{Tr} \left( f^*_a f^a \right) + \frac{\rho_a^2}{M^2_{\Sigma_a}} M_{\Sigma_a} \right| \quad (40)$$

and the CP asymmetry at one-loop order,

$$\varepsilon_{\Sigma_a} = -\frac{1}{4\pi} \sum_{b \neq a} \text{Im} \left[ \text{Tr} \left( f^*_a f^a \right) \right] \frac{\rho_b \rho_a}{M^2_{\Sigma_b} - M^2_{\Sigma_a}}. \quad (41)$$
Note that at least two $[SU(2)_L \times SU(2)_L^\prime]$-bidoublet Higgs scalars should be introduced to generate a nonzero CP asymmetry.

Obviously, the above CP asymmetry can be resonantly enhanced if the $[SU(2)_L \times SU(2)_L^\prime]$-bidoublet Higgs scalars have a quasi-degenerate mass spectrum, i.e. $M^2_{\Sigma_a} \simeq M^2_{\Sigma_b} \gg |M^2_{\Sigma_a} - M^2_{\Sigma_b}|$. In the hierarchical case with $M^2_{\Sigma_a} \ll M^2_{\Sigma_b}$, the CP asymmetry (44) can be simplified by

$$|\varepsilon_{\Sigma_a}| \simeq \frac{1}{4\pi} \text{Im} \left\{ \frac{\sum_{a \neq b} m^H_{\nu_b} m^H_{\nu_b}^{\prime a}}{\text{Tr} \left( f_a f_a^\dagger \right) + \frac{\phi^2}{M^2_{\Sigma_b}}} \right\} \frac{M^2_{\Sigma_a}}{\langle \phi \rangle^2} \frac{\langle \phi' \rangle^2}{M^2_{\Sigma_b}}$$

and then have an upper bound:

$$|\varepsilon_{\Sigma_a}| \leq \frac{1}{8\pi} \left( \text{Im} \left\{ \frac{\sum_{a \neq b} m^H_{\nu_b} m^H_{\nu_b}^{\prime a}}{\sqrt{\text{Tr} \left( f_a f_a^\dagger \right) \frac{\phi^2}{M^2_{\Sigma_b}}} \langle \phi \rangle^2 \langle \phi' \rangle^2} \right\} M^2_{\Sigma_a} \right) \frac{M^2_{\Sigma_b}}{\langle \phi \rangle^2} \frac{\langle \phi' \rangle^2}{M^2_{\Sigma_b}}$$

$$< \frac{1}{8\pi} \frac{M^2_{\Sigma_a} m^H_{\nu_b}^{\max}}{\langle \phi \rangle^2 \langle \phi' \rangle^2} = \varepsilon_{\Sigma_a}^{\max}.$$  

Here $m^H_{\nu_b}^{\max}$ is the maximal eigenvalue of the mass matrix $\sum_{b \neq a} m^H_{\nu_b}$. Unless there is a large cancellation between the mass matrix $m^H_{\nu_b}$ and the mass matrix $\sum_{b \neq a} m^H_{\nu_b}$, the eigenvalue $m^H_{\nu_b}^{\max}$ will not be much bigger than the largest neutrino mass $m^\nu_{\nu_b}^{\max}$. So, we can roughly constrain

$$|\varepsilon_{\Sigma_a}| < \frac{1}{8\pi} \frac{M^2_{\Sigma_a} m^\nu_{\nu_b}^{\max}}{\langle \phi \rangle^2 \langle \phi' \rangle^2}.$$  

V. THE MODEL WITH GAUGE-SINGLET DIRAC FERMION(S) AND [SU(2)_L x SU(2)_L'-BIDOUBLET HIGGS SCALAR(S)]

In this sector, we will give the completed model with at least one gauge-singlet Dirac fermion and at least one $[SU(2)_L \times SU(2)_L^\prime]$-bidoublet Higgs scalar. By taking the notations in the previous sections, the Lagrangian involving the fermion singlet(s) and the Higgs bidoublet(s) can be described by

$$\mathcal{L}^M = \mathcal{L}^{FS} + \mathcal{L}^{HB}.$$  

A. Dirac neutrino masses

The ordinary and dark left-handed neutrinos as well as the gauge-singlet right-handed fermions should have the following mass matrix:

$$\mathcal{L} \supset - \left[ \bar{\nu}_L \tilde{N}_R^c \right] \left[ \sum_{a \neq b} f_a \langle \Sigma_a \rangle \ y_{N'} \langle \phi' \rangle M_N \right] \left[ \nu_L^c \right] \tilde{N}_R^c + \text{H.c.}.$$  

Obviously, the ordinary left-handed neutrinos $\nu_L$ and the dark left-handed neutrinos $\nu'_L$ will form three light Dirac neutrinos as their mass term is just a sum of the type-I Dirac seesaw (22) and the type-II Dirac seesaw (43), i.e.

$$m_\nu = m_\nu' + m_\nu^I.$$  

FIG. 4: The lepton number conserving decays of the heavy $[SU(2)_L \times SU(2)_L^\prime]$-bidoublet Higgs scalars $\Sigma_a$ in the type-II Dirac seesaw scenario. We need at least two Higgs bidoublets to generate a nonzero lepton asymmetry in the $[SU(2)_L]$-doublet leptons $l_L$ and an opposite lepton asymmetry in the $[SU(2)_L^\prime]$-doublet leptons $l'_L$. The CP conjugation is not shown for simplicity.
B. CP asymmetry

As shown in Figs. 5 and 6 the lepton number conserving decays of the heavy gauge-singlet Dirac fermions \( N_i \) and/or \([SU(2)_L \times SU(2)_R] \)-bidoublet Higgs scalars \( \Sigma_a \) can simultaneously generate a lepton asymmetry in the \( [SU(2)_L] \)-doublet leptons \( l_L \) and an opposite lepton asymmetry in the \( [SU(2)_R] \)-doublet leptons \( l'_R \). The relevant CP asymmetries should be

\[
\varepsilon_{N_i} = \frac{1}{4\pi} \left( \frac{y_N^\dagger y_N}{M_N} + \frac{y_N^\dagger y_N}{M_N} \right) \times \left\{ \sum_{j \neq i} \text{Im} \left[ (y_N^\dagger y_N)_{ij} (y_{N'}^\dagger y_{N'})_{ij} \right] \frac{M_N M_{N_j}}{M_{N_j}^2 - M_{N_i}^2} \right. \\
+ 2 \text{Im} \left[ (y_N^\dagger f_a y_{N'})_{ii} \right] \frac{\rho_a}{M_N^2} \times \left[ 1 - \frac{M_{N_j}^2}{M_{N_i}^2} \ln \left( 1 + \frac{M_{N_j}^2}{M_{N_i}^2} \right) \right] \right\} ,
\]

for \( M_{N_i} \ll M_{N_j}, M_{\Sigma_a} \),

\[
\varepsilon_{\Sigma_a} = -\frac{1}{4\pi} \left\{ \sum_{j \neq a} \text{Im} \left[ \text{Tr} \left( f_b f_a \right) \right] \frac{\rho_a}{M_{\Sigma_b}^2 - M_{\Sigma_a}^2} \times \left[ \sum_i \text{Im} \left[ (y_N^\dagger f_a y_{N'})_{ii} \right] \frac{\rho_a}{M_{\Sigma_a}^2 - M_{\Sigma_i}^2} \ln \left( 1 + \frac{M_{\Sigma_a}^2}{M_{\Sigma_i}^2} \right) \right] \right\} ,
\]

(49)

and

\[
\varepsilon_{\Sigma_a} \approx -\frac{1}{4\pi} \left\{ \sum_{j \neq a} \frac{\text{Tr} \left[ (f_b f_a)_{ii} \right] \rho_a}{M_{\Sigma_b}^2 - M_{\Sigma_a}^2} \times \left[ \sum_i \frac{\rho_a}{M_{\Sigma_a}^2 - M_{\Sigma_i}^2} \ln \left( 1 + \frac{M_{\Sigma_a}^2}{M_{\Sigma_i}^2} \right) \right] \right\} ,
\]

(50)

Clearly, the vertex correction should be the unique source for the nonzero CP asymmetry (18) if there is only one gauge-singlet Dirac fermion. As for the CP asymmetry (19), it will not be affected by the self-energy correction if we only introduce one \([SU(2)_L \times SU(2)_R]\)-bidoublet Higgs scalar.

Similar to those in the pure type-I and type-II Dirac seesaw models, the CP asymmetries (48) and (49) can be simplified in the hierarchical cases, i.e.

\[
\varepsilon_{N_i} \approx \frac{1}{4\pi} \text{Im} \left[ (y_N^\dagger m_N y_{N'}^\dagger)_{ii} \right] \frac{M_{N_i}}{\langle \phi \rangle \langle \phi' \rangle} \text{ for } M_{N_i} \ll M_{N_j}, M_{\Sigma_a} ,
\]

(50)

\[
\varepsilon_{\Sigma_a} \approx -\frac{1}{4\pi} \text{Im} \left[ \text{Tr} \left[ (m_b + \sum_{b \neq a} m_{b}^{1\bar{b}}) m_{b}^{1\bar{b}} \right] \right] \frac{M_{\Sigma_a}^2}{\langle \phi \rangle \langle \phi' \rangle^2} \text{ for } M_{\Sigma_a}^2 \ll M_{N_i}^2 , M_{\Sigma_a}^2 ,
\]

(51)

and have the upper bounds given in Eqs. (28) and (14).

VI. ORDINARY AND DARK BARYON ASYMMETRIES

The \( SU(2)_L \) sphaleron processes [76] will partially transfer the lepton asymmetry \( \eta_L \) to a baryon asymmetry \( \eta_B \) in the ordinary sector,

\[
\eta_B = -\frac{28}{79} \eta_L .
\]

On the other hand, the lepton asymmetry \( \eta'_L \) will result in a baryon asymmetry \( \eta'_B \) in the dark sector,

\[
\eta'_B = \frac{28}{79} \eta'_L .
\]
Through the $SU(2)_{L}^{\prime}$ sphaleron processes. So, the correlation between the baryon asymmetries in the ordinary and dark sectors should be

$$\eta_B = -\eta_B \quad \text{for} \quad \eta_L = -\eta_L^{'}.$$  \hfill (54)

If the dark matter relic density is dominated by the dark proton and/or neutron, we should have

$$\Omega h^2 : \Omega_{DM} h^2 = m_p \eta_B : m_p \eta_L^{'} = m_p : m_p^{'} \approx 1 : 5,$$  \hfill (55)

to fit the cosmological observations. In this case, the dark matter mass should be determined by $m_p^{'} \approx 5 \text{ GeV}$. \hfill (56)

After the gauge-singlet Dirac fermions and/or the $[SU(2)_L \times SU(2)_L^{'},]\text{-bidoublet Higgs scalars are thermally produced and then go out of equilibrium, their CP-violating decays can generate the required ordinary and dark baryon asymmetries.}$ For this purpose, we can choose the proper values of the masses and couplings of the decaying particles, like those in the usual leptogenesis within the Majorana seesaw models. \hfill [12]

A. Baryon asymmetries from the decays of gauge-singlet Dirac fermions

In order to show the decays of gauge-singlet Dirac fermions can generate the required baryon asymmetries, as an example, let us consider a type-I Dirac seesaw model with two hierarchical fermion singlets $N_{1,2}$. In this case, the neutrino mass matrix $[22]$ is clearly rank 2 so that one neutrino mass eigenstate should be massless and then the maximal neutrino mass eigenvalue should be about $m_{\nu}^{\text{max}} \approx 0.05 \text{ eV}$. \hfill (57)

Without loss of generality, we assume $N_1$ much lighter than $N_2$. The final baryon asymmetries then could
mainly come from the decays of the lighter $N_1$. If the Yukawa couplings $(y_N)_{11}$ and $(y_N')_{11}$ of the decaying $N_1$ give

$$K_{N_1} = \left. \frac{\Gamma_{N_1}}{2H(T)} \right|_{T=M_{N_1}} \ll 1,$$  

the out-of-equilibrium condition is well satisfied and there is no washout effect associated to the $(y_N)_{11}$ and $(y_N')_{11}$ couplings. As a result, the final baryon asymmetry can be well described by \[50\],

$$\eta_B = -\frac{28}{79} \eta_L \approx -\frac{28}{79} \times \frac{\varepsilon_{N_1}}{g_*}. \quad (59)$$

Here and thereafter

$$H(T) = \left( \frac{8\pi^2 g_*}{90} \right) \left( \frac{T}{M_{Pl}} \right)^2, \quad (60)$$

is the Hubble constant with $M_{Pl} \approx 1.22 \times 10^{19}$ GeV being the Planck mass and $g_* = 2 \times (106.75 + 2) = 217.5$ being the relativistic degrees of freedom. For a quantitative estimation, we take

$$\langle \phi' \rangle = 500 \langle \phi \rangle, \quad M_{N_1} = 0.01 M_{N_2} = 10^{13} \text{GeV},$$

$$(y_N)_{11} = (y_N')_{11} \sim 0.1(y_N)_{22} = 0.1(y_N')_{22} = \mathcal{O}(0.01), \quad (61)$$

so that the elements of the neutrino mass matrix \[22\] can arrive at the values of the order of $\mathcal{O}(0.1 \text{eV})$. The above parameter choice also leads to

$$K_{N_1} = \mathcal{O}(0.01 - 0.1), \quad \varepsilon_{N_1}^{max} \simeq 1.3 \times 10^{-5}. \quad (62)$$

Therefore, we can obtain the needed baryon asymmetry $\eta_B \sim 10^{-10}$ \[24\] as the CP asymmetry $\varepsilon_{N_1}$ is allowed to be of the order of $\mathcal{O}(10^{-7})$.

\textbf{B. Baryon asymmetries from the decays of $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars}

We now study the baryon asymmetries from the decays of $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars. As an example, let us consider the type-II Dirac seesaw model with two hierarchical Higgs doublets $\Sigma_{1,2}$. We take

$$\langle \phi' \rangle = 500 \langle \phi \rangle, \quad M_{\Sigma_1} = 0.01 M_{\Sigma_2} = 10^{14} \text{GeV},$$

$$\rho_1 = 0.01 \rho_2 = 2 \times 10^{12} \text{GeV}, \quad (63)$$

to give

$$\langle \Sigma_1 \rangle = 100 \langle \Sigma_2 \rangle \simeq -3 \text{eV}. \quad (64)$$

We then assume

$$f_2 = f_1 e^{i \delta} \quad (65)$$
to simplify the neutrino mass matrix as

$$m_\nu = f_1 \langle \Sigma_1 \rangle + f_2 \langle \Sigma_2 \rangle \simeq f_1 \langle \Sigma_1 \rangle.$$  

By inputting

$$\text{Tr}(f_1^\dagger f_1) \simeq \text{Tr}(m_\nu^2) = \sum_i m_i^2 = 1.1 \times 10^{-3}$$

for

$$\sum_i m_i^2 = 0.01 \text{eV}^2, \quad (67)$$

we can read the CP asymmetry

$$\varepsilon_{\Sigma_1} = -2.3 \times 10^{-7} \sin \delta. \quad (68)$$

The above parameter choice also satisfies the out-of-equilibrium condition in the weak washout region,

$$K = \left. \frac{\Gamma_{\Sigma_1}}{2H(T)} \right|_{T=M_{\Sigma_1}} \simeq 0.08. \quad (69)$$

Note for the heavy masses in Eq. \[62\], the gauge interactions of the Higgs bidoublets can be safely kept out of equilibrium at the leptogenesis epoch \[10, 14\]. The final baryon asymmetry thus can be approximately calculated by \[50\]. Here we have added the factor 4 because the decaying particles are the $[SU(2)_L \times SU(2)_R]$-bidoublet Higgs scalars.

\textbf{VII. OTHER CONSTRAINTS AND IMPLICATIONS}

The dark photon $A'$ can decay into the ordinary fermion pairs $f\bar{f}$ and the dark fermion pairs $f'\bar{f}'$ as long as the kinematics is allowed. For example, if the dark photon is much heavier than the ordinary down quark but lighter than the ordinary strange quark and the dark charged fermions, its decay width should be

$$\Gamma_{A'} = \Gamma_{A'\rightarrow \nu\nu} + \Gamma_{A'\rightarrow e\bar{e}} + \Gamma_{A'\rightarrow u\bar{u}} + \Gamma_{A'\rightarrow d\bar{d}}$$

$$= \frac{5 e^2 \alpha \cos^2 \theta_W}{9 \cos^2 \theta_W} m_{A'}. \quad (71)$$

Here and thereafter

$$\theta_W = \arctan \frac{g_1}{g_2}, \quad \theta_W' = \arctan \frac{g_1'}{g_2'}, \quad (72)$$

are the Weinberg angles and

$$\alpha = \frac{g_2^2 \sin^2 \theta_W}{4\pi} \simeq \frac{1}{137}, \quad \alpha' = \frac{g_2'^2 \sin^2 \theta_W'}{4\pi}, \quad (73)$$
are the fine structure constants in the ordinary and dark sectors. We then find
\[
\tau_{A'} \simeq \left( \frac{4 \times 10^{-11}}{\epsilon} \right)^2 \left( \frac{\cos \theta_W}{\cos \theta_W'} \right)^2 \left( \frac{100 \text{ MeV}}{m_{A'}} \right) \text{sec}.
\]
(74)
So, the dark photon \(A'\) with a mass \(m_{A'} = 100\text{ MeV}\) can have a lifetime shorter than 1 second if we take \(\epsilon \gg 4 \times 10^{-11}\). Currently, the measurement on the muon magnetic moment constrains \(\epsilon^2 \cos^2 \theta_W \cos^2 \theta_W' / (1 - \epsilon^2) < 2 \times 10^{-5}\) for \(m_{A'} = 100\text{ MeV}\).

From the dark Higgs scalar \(\delta'\), which is responsible for the \(U(1)_{em}\) symmetry breaking, we will have a dark Higgs boson \(h_{\delta'}\) with the mass about \(m_{h_{\delta'}} = 2\sqrt{\lambda/\delta'}\).

This dark Higgs boson can mostly decay into the dark photon \(A'\) with the decay width,
\[
\Gamma_{h_{\delta'} \rightarrow A'A'} = 128 \pi \alpha'^2 \left( \frac{\delta'}{m_{h_{\delta'}}} \right)^2 \left[ 1 + \left( \frac{m_{h_{\delta'}}^2 - 2m_{A'}^2}{8m_{A'}^2} \right) \right] \times \sqrt{1 - \frac{4m_{A'}^2}{m_{h_{\delta'}}^2}}.
\]
(75)
The lifetime \(\tau_{h_{\delta'}}\) can be much shorter than 1 second. For example, we take \(\lambda_{\delta} = 1\) and \(\langle \delta' \rangle = 117\text{ MeV}\) to give \(m_{h_{\delta'}} = 234\text{ MeV}\) and then
\[
\tau_{h_{\delta'}} = \frac{1}{\Gamma_{h_{\delta'}}} = 2.9 \times 10^{-22} \left( \frac{\alpha'}{\alpha} \right)^2 \text{sec}
\]
for \(m_{A'} = 100\text{ MeV}\).
(76)
The dark charged leptons are the quasi-Dirac fermions so that their lepton asymmetry cannot survive \([79]\). The lightest dark charged lepton (the dark electron \(e'\)) thus should have a thermally produced relic density, which is determined by its pair annihilation into the dark photon \(A'\),
\[
\langle \sigma_{e'e' \rightarrow A'A'} \rangle \simeq \frac{\pi \alpha'^2}{m_{e'}^2} \left( \frac{256\text{ MeV}}{m_{e'}} \right)^2 \text{pb} = 10^6 \text{pb} \left( \frac{\alpha'}{\alpha} \right)^2 \left( \frac{\epsilon}{1.5 \times 10^{-7}} \right)^2 \left( \frac{m_{e'}}{A} \right)^2 \left( \frac{3Z + (A - Z)}{A} \right)^2.
\]
(77)
It is easy to check the dark electron will have a frozen temperature far below its mass. This means the dark electron will only give a negligible contribution to the dark matter relic density since its number density at the frozen temperature is highly suppressed by a Boltzmann factor. Similarly, the dark down quark \(d'\) and the dark up quark \(u'\) with the masses of the order of GeV will also have the frozen temperatures far below their masses.

The Big-Bang Nucleosynthesis (BBN) stringently restricts the existence of the new relativistic degrees of freedom. The constraint on the new degrees of freedom is conventionally quoted as \(\Delta N_e\), the effective number of additional light neutrinos. The seven-year WMAP observation has measured \([20]\)
\[
\Delta N_e = 1.34^{+0.86}_{-0.88} (68\% \text{ CL}) .
\]
(78)
We now check the dark left-handed neutrinos \(\nu'_L\) which form the light Dirac neutrinos with the ordinary left-handed neutrinos \(\nu_L\). We can estimate the decoupling temperature of the dark neutrinos by \([81]\)
\[
\frac{g_2/\cos \theta_W}{g_2/\cos \theta_W} \left( \frac{\langle \phi \rangle}{\langle \langle \phi' \rangle \rangle} \right)^4 G_F^2 T^5 = H(T)
\]
\[
\Rightarrow T \simeq 8.6\text{ GeV} \left( \frac{g_2}{100} \right)^{\frac{1}{4}} \left( \frac{\langle \langle \phi' \rangle \rangle}{500} \langle \langle \phi \rangle \rangle \right)^{\frac{1}{4}} \left( \frac{g_2/\cos \theta_W}{g_2/\cos \theta_W} \right)^{\frac{1}{4}},
\]
(79)
with \(G_F = 1.16637 \times 10^{-5}\text{ GeV}^{-2}\) being the Fermi constant \([81]\). At the temperature \(T \sim 8\text{ GeV}\), the relativistic degrees of freedom from the ordinary sector should be about \(g_\ast \sim 80\) \([80]\) while those from the dark sector (the dark photon \(A'\), the dark Higgs scalar \(\delta'\), the dark electron \(e'\), the dark down quark \(d'\), the dark up quark \(u'\) and the dark gluons) could be about \(g_\ast \sim 44.5\). The temperature of the dark neutrinos at the BBN epoch \(T \sim 1\text{ MeV}\) thus should be \([80]\)
\[
\left( \frac{T}{T'} \right)^4 \sim 10.75 \left( \frac{80 + 44.5}{80} \right) \sim 0.038,
\]
(80)
and hence give a negligible contribution to the effective number of additional light neutrinos, i.e.
\[
\Delta N_e = \frac{4}{7} \left( \frac{T}{T'} \right)^4 \lesssim 0.02 .
\]
(81)
Through the exchange of the dark photon, the dark matter nucleon can scatter off the ordinary nucleon. For example, the dark matter proton has the scattering cross section as below,
\[
\sigma_{p'N \rightarrow p'N} \simeq \frac{\pi \epsilon^2 \alpha' \epsilon_W^2}{c_W^2} m_{p'}^2 \left( \frac{3Z + (A - Z)}{A} \right)^2,
\]
\[
\simeq 10^{-41} \text{cm}^2 \left( \frac{\epsilon}{1.5 \times 10^{-7}} \right)^2 \left( \frac{\alpha'}{\alpha} \right)^2 \left( \frac{c_W}{c_W'} \right)^2 \left( \frac{\mu_r}{0.833\text{ GeV}} \right)^2 \left( \frac{100\text{ MeV}}{m_{A'}} \right)^4 \left( \frac{3Z + (A - Z)}{A} \right)^2,
\]
(82)
which can be close to the XENON10 limit \([83]\). Here \(Z\) and \(A - Z\) are the numbers of proton and neutron within the target nucleus, while \(\mu_r = m_{p'} \nu_{p'N}/m_{p'} + m_{p'N}\) is the reduced mass. Alternatively, the dark neutron can serve as the dark matter particle if it is lighter than the dark proton. The detailed studies can be found in \([43]\).
VIII. MIRROR SYMMETRY

There are many parameters in our models. To reduce the parameters, we can impose a mirror \[ \{21, 17\} \] discrete symmetry under which the fields transform as

\[
\begin{align*}
G^\mu_\mu & \leftrightarrow G^a_\mu, \quad W^a_\mu \leftrightarrow W^a_\mu, \quad B'_\mu \leftrightarrow B_\mu, \quad \phi' \leftrightarrow \phi, \\
\delta' & \leftrightarrow \delta, \quad q'_L \leftrightarrow q_L, \quad d'_R \leftrightarrow d_R, \quad u'_R \leftrightarrow u_R, \quad l'_L \leftrightarrow l_L, \\
e'_R \leftrightarrow e_R, \quad N'_R \leftrightarrow N_R, \quad \Sigma_a \leftrightarrow \Sigma_T.
\end{align*}
\]

The above mirror symmetry, which is assumed to softly break in the scalar potential, i.e.

\[
\mu_\phi^2 \neq \mu_\phi'^2, \quad \mu_\phi'^2 \neq \mu_\phi^2,
\]

will simplify the parameters to be

\[
\begin{align*}
g'_3 &= g_3, \quad g'_4 = g_2, \quad g'_1 = g_1, \quad \lambda_{\phi'} = \lambda_\phi, \quad \lambda_{\phi'} = \lambda_3, \\
\lambda_{\phi} &= \lambda_{\phi'}, \quad \lambda_{\phi} = \lambda_{\phi'}, \quad \lambda_{\phi} = \lambda_{\phi} = \lambda_{\phi'}, \\
y_{u'} &= y_u, \quad y_{e'} = y_e, \quad y_{\delta'} = y_\delta, \quad y_{N'} = y_N, \quad f_a = f'_a, \\
M_N &= M^T_N.
\end{align*}
\]

The dark charged fermion masses then should be \[ \{31\} \]

\[
\begin{align*}
\langle \phi' \rangle &= \frac{m_{u'}}{m_u} \frac{m_{d'}}{m_d} \frac{m_{s'}}{m_s} \frac{m_{c'}}{m_c} \frac{m_{b'}}{m_b} \frac{m_{t'}}{m_t},
\end{align*}
\]

By fixing the VEVs:

\[
\langle \phi' \rangle = 500 \langle \phi \rangle,
\]

we can read the dark charged fermion masses \[ \{81\} \]

\[
\begin{align*}
m_{u'} &= 2.5 \text{ GeV for } m_d = 5 \text{ MeV}, \\
m_{u'} &= 1.25 \text{ GeV for } m_u = 2.5 \text{ MeV}, \\
m_{s'} &= 50 \text{ GeV for } m_s = 100 \text{ MeV}, \\
m_{c'} &= 645 \text{ TeV for } m_c = 129 \text{ GeV}, \\
m_{b'} &= 2.095 \text{ TeV for } m_b = 4.19 \text{ GeV}, \\
m_{t'} &= 86.45 \text{ TeV for } m_t = 172.9 \text{ GeV}, \\
m_{e'} &= 0.256 \text{ GeV for } m_e = 0.511 \text{ MeV}, \\
m_{\mu'} &= 52.85 \text{ GeV for } m_{\mu} = 105.7 \text{ MeV}, \\
m_{\tau'} &= 888.5 \text{ GeV for } m_\tau = 1.777 \text{ GeV}.
\end{align*}
\]

The beta functions of the QCD in the ordinary and dark sectors then will yield

\[
A_{\text{QCD'}} = \left( \frac{\langle \phi' \rangle}{\langle \phi \rangle} \right)^4 (m_u m_d m_s) \frac{\hat{\Lambda}_{\text{QCD}}}{\hat{\Lambda}_{\text{QCD}} = 1.13 \text{ GeV}}\]  

for \[ \hat{\Lambda}_{\text{QCD}} = 200 \text{ MeV}. \]

The dark proton and neutron masses thus can be given by

\[
m_{p'} = 5 \text{ GeV}, \quad m_{n'} = 6.25 \text{ GeV}.
\]

In this case, the dark proton will serve as the dark matter particle.

Under the mirror symmetry, the Dirac neutrino mass matrices \[ \{22, 30, 17\} \] should have a symmetric structure. Compared with the traditional type-I, type-II or type-I+II Majorana seesaw, our Dirac seesaw will not contain new parameters since the VEV in the dark sector has been determined by the dark matter mass.

IX. SUMMARY

In this paper we have proposed a unified picture of the Dirac neutrino masses, the baryon asymmetry and the dark matter relic density. Specifically, we consider an \( SU(3)_c \times SU(2)_L \times U(1)_Y \) dark sector parallel to the \( SU(3)_c \times SU(2)_L \times U(1)_Y \) ordinary sector and then introduce three types of messenger sectors composed of the heavy gauge-singlet Dirac fermion(s) and/or \([SU(2)_L \times SU(2)'_L]-\)boson(s) or Higgs bidoublet(s). Through the type-I, type-II or type-I+II Dirac seesaw mechanism, the heavy fermion singlet(s) and/or Higgs bidoublet(s) can highly suppress the masses between the ordinary and dark left-handed neutrinos. So, the ordinary and dark neutrinos can form the light Dirac neutrinos in a natural way. In such Dirac seesaw context, the lepton number conserving decays of the heavy fermion singlet(s) and/or Higgs bidoublet(s) can simultaneously generate a lepton asymmetry in the \([SU(2)_L]\)-doublet leptons and an opposite lepton asymmetry in the \([SU(2)'_L]\)-doublet leptons. Benefited from the \( SU(2)_L \) and \( SU(2)'_L \) sphaleron processes, we eventually can obtain a baryon asymmetry in the ordinary sector and an opposite baryon asymmetry in the dark sector. The lightest dark nucleon thus should have a determined mass about 5 GeV to serve as the dark matter particle. In the presence of the kinetic mixing between the \( U(1)_Y \) and \( U(1)'_Y \) gauge fields, the dark nucleons can be verified in the dark matter direct detection experiments. By imposing a mirror discrete symmetry, our models needn’t more parameters than the conventional Majorana seesaw models.

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Appendix A: The \( SU(2) \) and \( U(1) \) gauge bosons

It is easy to find the charged gauge bosons,

\[
W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu),
\]

\[
W'^\pm_\mu = \frac{1}{\sqrt{2}} (W'^1_\mu \mp i W'^2_\mu).
\]
and their masses

$$m_W^2 = \frac{1}{2} g_\gamma^2 \langle \phi \rangle^2 + \sum_a (\Sigma_a)^2,$$  \hspace{1cm} (A2a)

$$m_{W'}^2 = \frac{1}{2} g_\gamma' \langle \phi' \rangle^2 + \sum_a (\Sigma_a)^2.$$  \hspace{1cm} (A2b)

Furthermore, we can diagonalize the kinetic term of the $U(1)_Y$ and $U(1)_Y'$ gauge fields by making a non-unitary transformation \[\mathcal{A}\],

$$\bar{B}_\mu = B_\mu + \epsilon B'_\mu, \quad \bar{B}'_\mu = \sqrt{1 - \epsilon^2} B'_\mu,$$  \hspace{1cm} (A3)

and then define the orthogonal fields,

$$\begin{cases}
A_\mu &= W_\mu^3 s_W + \bar{B}_\mu c_W, \\
Z_\mu &= W_\mu^3 c_W - \bar{B}_\mu s_W;
\end{cases} \hspace{1cm} (A4a)$$

$$\begin{cases}
A'_\mu &= W_\mu^{3'} s_W' + \bar{B}'_\mu c_W' \\
Z'_\mu &= W_\mu^{3'} c_W' - \bar{B}'_\mu s_W';
\end{cases} \hspace{1cm} (A4b)$$

with the ordinary and dark Weinberg angles,

$$\begin{cases}
s_W = \sin \theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \\
c_W = \cos \theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}};
\end{cases} \hspace{1cm} (A5a)$$

$$\begin{cases}
s_W' = \sin \theta_W' = \frac{g_1'}{\sqrt{g_1'^2 + g_2'^2}}, \\
c_W' = \cos \theta_W' = \frac{g_2'}{\sqrt{g_1'^2 + g_2'^2}}.
\end{cases} \hspace{1cm} (A5b)$$

Among the orthogonal fields \[\mathcal{A}\], $A_\mu$, as the ordinary photon is exactly massless, while $Z_\mu$, $Z'_\mu$ and $A'_\mu$ have the mass terms as below,

$$\mathcal{L} \supset \frac{1}{2} m_Z^2 [Z_\mu + \xi_1 s_W (c_W A'_\mu - s_W Z'_\mu)]^2$$
$$+ \frac{1}{2} m_Z^2 [Z'_\mu + \xi_2 s_W' (c_W' A'_\mu - s_W' Z'_\mu)]^2$$
$$+ \frac{1}{2} m_{A'}^2 (A'_\mu - s_W' Z'_\mu)^2$$
$$+ \frac{1}{2} \xi_3 m_Z^2 [Z_\mu - (\xi_3 s_W s_W' + c(1 - \xi_3 s_W^2)) Z'_\mu]$$
$$+ (\xi_1 s_W - c \xi_2 s_W') c_W' A'_\mu)^2.$$  \hspace{1cm} (A6)

Here we have denoted

$$m_Z^2 = \frac{g_2^2}{2c_W^2} \langle \phi \rangle^2,$$

$$m_Z'^2 = \frac{g_2'^2}{2c_W'^2} \langle \phi' \rangle^2,$$

$$m_{A'}^2 = \frac{8 g_2 s_W^2 c_W^2}{1 - \epsilon^2} \langle \delta' \rangle^2,$$

$$\approx (100 \text{ MeV})^2 \left( \frac{\alpha'}{\alpha} \right) \left( \frac{\langle \delta' \rangle}{117 \text{ MeV}} \right)^2.$$  \hspace{1cm} (A7)

and

$$\xi_1 = \frac{\epsilon}{\sqrt{1 - \epsilon^2}} \approx \epsilon, \quad \xi_2 = 1 - \frac{1}{\sqrt{1 - \epsilon^2}} \approx -\frac{1}{2} \epsilon^2,$$

$$\xi_3 = \frac{\sum_a (\Sigma_a)^2}{\langle \phi \rangle^2},$$  \hspace{1cm} (A8)

We can further define

$$Z'_\mu = Z'_\mu c_W' + A'_\mu s_W,'$$  \hspace{1cm} (A9a)

$$Z_\mu = Z_\mu c_\beta + (-Z'_\mu s_\alpha + A'_\mu c_\alpha) s_\beta,$$  \hspace{1cm} (A9b)

$$A'_\mu = -Z_\mu s_\beta + (-Z'_\mu s_\alpha + A'_\mu c_\alpha) c_\beta,$$  \hspace{1cm} (A9c)

with the rotation angles,

$$\begin{cases}
c_\alpha = \cos \alpha = \frac{1 - \xi_3 s_W^2}{\sqrt{1 - \xi_3^2 s_W^2 + \xi_3 s_W' c_W' s_W^2}}, \\
s_\alpha = \sin \alpha = \frac{\xi_3 s_W c_W'}{\sqrt{1 - \xi_3^2 s_W^2 + \xi_3 s_W' c_W' s_W^2}},
\end{cases} \hspace{1cm} (A10a)$$

$$\begin{cases}
c_\beta = \cos \beta = \frac{1 + \xi_3 s_W c_W'}{\sqrt{1 + \xi_3^2 s_W^2 + \xi_3 s_W' c_W' s_W^2}}, \\
s_\beta = \sin \beta = \frac{\xi_3 s_W' c_W}{\sqrt{1 + \xi_3^2 s_W^2 + \xi_3 s_W' c_W' s_W^2}},
\end{cases} \hspace{1cm} (A10b)$$

to rewrite the mass terms \[\mathcal{A}\] by

$$\mathcal{L} \supset \frac{1}{2} m_Z^2 \left[ (1 - \xi_2 s_W^2) + \xi_3 s_W' c_W' s_W^2 \right] Z_\mu^2$$
$$+ \frac{1}{2} m_Z^2 \left[ 1 + \xi_2 s_W^2 (c_W' s_W + s_W' c_W s_W^2)^2 \right] Z_\mu \nonumber$$
$$+ \xi_1 s_W (c_W' s_W - s_W' c_W) Z'_\mu \nonumber$$
$$+ \frac{1}{2} \xi_3 m_Z^2 \left[ 1 + \xi_3^2 s_W (c_W' s_W + s_W' c_W s_W^2)^2 \right] Z_\mu$$
$$- c(1 - 2 \xi_2 s_W^2 + \xi_3^2 s_W^2) Z'_\mu \nonumber$$
$$+ \left( c_W' + \xi_3 s_W' c_W \right) (A'_\mu c_\beta + \hat{Z}_\mu s_\beta)$$
$$+ \left( s_W - \xi_3 s_W' c_W \right) Z'_\mu.$$  \hspace{1cm} (A11)

Clearly, the definition \[\mathcal{A}\] can give us a physical dark photon in the case that the dark electromagnetic symmetry is unbroken. For $\langle \phi' \rangle \gg \langle \phi \rangle \simeq 174 \text{ GeV} \gg \langle \delta' \rangle = \mathcal{O}(100 \text{ MeV})$ and $\epsilon \ll 1$, the orthogonal fields $Z_\mu$, $Z'_\mu$ and $A'_\mu$ can approximate to the mass eigenstates. The dark photon $A'_\mu$ can couple to both of the dark and ordinary fermions,

$$\mathcal{L} \supset \frac{\epsilon c_W c_W'}{4c_W} A'_\mu \left[ \bar{e}_\gamma' \mu (3 + \gamma_5) e + \bar{\nu}_\gamma' \mu (1 - \gamma_5) \nu ight]$$
$$+ \bar{d}_\gamma' \mu \left( \frac{1}{3} + \gamma_5 \right) d - \bar{u}_\gamma' \mu \left( \frac{5}{3} + \gamma_5 \right) u$$
$$+ c_\alpha A'_\mu \left( \frac{2}{3} \bar{\gamma}' \gamma' d - \frac{2}{3} \bar{u}_\gamma' \gamma' u + \bar{e}_\gamma' \gamma' e \right)$$

with $e = g_2 s_W$, $e' = g_2' s_W'$. \hspace{1cm} (A12)
Appendix B: The dark QCD scale

The running of the ordinary QCD gauge coupling \( \alpha_s(\mu) \) is given by

\[
\alpha_s(\mu) = \frac{2\pi}{11 - \frac{2}{3} N_f \ln \left[ \frac{\mu}{\Lambda(N_f)} \right]},
\]

where \( N_f \) counts the number of the ordinary quarks involved at a given scale \( \mu \). By matching \( \alpha_s(\mu) \) at the scale \( \mu = m_t \) with \( N_f = 6 \) and \( \mu = m_b \) with \( N_f = 5 \), at the scale \( \mu = m_c \) with \( N_f = 4 \) and \( N_f = 3 \), respectively, we can deduce

\[
\Lambda_{(5)} = m_t^2 \Lambda_{(6)}^{\frac{27}{6}}, \quad \Lambda_{(4)} = m_b^2 \Lambda_{(5)}^{\frac{27}{6}},
\]

\[
\Lambda_{(3)} = m_c^2 \Lambda_{(4)}^{\frac{27}{6}},
\]

and then

\[
\Lambda_{\text{QCD}} = \Lambda_{(3)} = (m_x m_b m_t)^{\frac{27}{6} \Lambda_{(6)}^{\frac{27}{6}}},
\]

Similarly, the dark QCD gauge coupling \( \alpha_s'(\mu) \) should behave as

\[
\alpha_s'(\mu) = \frac{2\pi}{11 - \frac{2}{3} N_{f'} \ln \left[ \frac{\mu}{\Lambda(N_{f'})} \right]},
\]

with \( N_{f'} \) being the number of the involved dark quarks. For \( m_{u'} < m_{d'} < m_{s'} < m_c < m_{t'} < m_{b'}, \) we can have

\[
\Lambda_{(5)}' = m_t^2 \Lambda_{(6)}'^{\frac{27}{6}}, \quad \Lambda_{(4)}' = m_b^2 \Lambda_{(5)}'^{\frac{27}{6}},
\]

\[
\Lambda_{(3)}' = m_c^2 \Lambda_{(4)}'^{\frac{27}{6}}, \quad \Lambda_{(2)}' = m_s^2 \Lambda_{(4)}'^{\frac{27}{6}},
\]

\[
\Lambda_{(1)}' = m_d^2 \Lambda_{(2)}'^{\frac{27}{6}}, \quad \Lambda_{(0)}' = m_{u'}^2 \Lambda_{(1)}'^{\frac{27}{6}},
\]

and then

\[
\Lambda_{\text{QCD}'} = \Lambda_{(0)}' = (m_{u'} m_d m_s m_c m_{b'} m_{t'})^{\frac{27}{6} \Lambda_{(6)}^{\frac{27}{6}}}
\]

for \( \Lambda_{\text{QCD}'} < m_{u'} \).

At the sufficiently high scales \( \mu \gg m_{t'}, m_c \) the renormalization group invariants \( \Lambda_{(6)}' \) and \( \Lambda_{(6)} \) are only determined by the corresponding strong gauge couplings \( \alpha_s \) and \( \alpha_s' \). In the presence of a mirror symmetry which enforces

\[
\alpha_s(\mu) = \alpha_s'(\mu) \Rightarrow \Lambda_{(6)}' = \Lambda_{(6)},
\]

and

\[
\langle \phi' \rangle = \frac{\langle \phi \rangle}{m_t} = \frac{m_u}{m_d} = \frac{m_x}{m_s} = \frac{m_c}{m_b} = \frac{m_{b'}}{m_{t'}},
\]

the dark hadronic scale \( \Lambda_{\text{QCD}'} \) should arrive at

\[
\Lambda_{\text{QCD}'} = \left( \frac{\langle \phi' \rangle}{\langle \phi \rangle} \right)^{\frac{27}{6}} (m_u m_d m_s m_c m_{b'} m_{t'})^{\frac{27}{6} \Lambda_{\text{QCD}}}.
\]

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