Power failure evolution model considering voltage stability

Jialin WANG\textsuperscript{1}, Qun YU\textsuperscript{1} and Qi WANG\textsuperscript{1}

1 College of Electrical Engineer and Automation, Shandong University of Science and Technology, Qingdao266590, Shandong Province, China
Email: wjl_94@163.com

Abstract. Voltage instability is often associated with power system blackout. In order to study the evolution mechanism of power failure and voltage instability in power failure, this paper proposes a new kind of power failure evolution model that considers voltage stability. The theory of static voltage stability analysis is introduced into the modelling. According to the simulation results of power failure in an IEEE39 system, the model proposed in this paper verifies the self-organized criticality of power network. It was also found that voltage instability lead to the increase of the frequency and scale of power failure. Furthermore, the frequency of load nodes in voltage instability accidents is calculated and the set of weak nodes is obtained.

1. Introduction

With the continuous development of the social economy, the interconnection of power grids is becoming a growing trend in power system development. As the scale of the power grid expands, system operation grows more complex, and the safe and reliable operation of power systems is affected. In recent decades, several power system blackouts have occurred around the world, resulting in huge economic losses. Generally, the evolution process of power failure includes two stages; the initial fault accumulation, rapid expansion of the fault area and the final stage of power failure. In the initial stage of power failure, some components exit the operation state due to short circuit failure, causing large-scale power flow transfer. In the stage of rapid expansion in the fault area, the power flow transfer causes an imbalance between the supply and demand of reactive power in the system, which may lead to voltage instability. In the final stage of power failure, the generator set adjacent to the unstable node may be out of step due to power imbalance. Voltage instability is one of the biggest factors that affects the development of power failure.

In recent years, in order to study the mechanism and reduce the risks involved in power failure, scholars have carried out a large number of studies and proposed a variety of power failure models. From the research methods, these models can be divided into two categories: those based on power flow analysis and those based on power grid topology analysis. The first type of model describes component characteristics through a probability model and mainly studies the overall characteristics of a power system, including OPA model \cite{1}, Hidden Failure model \cite{2}, Cascade model \cite{3}, SOC-power Failure model \cite{4}, and so on. The second model applies a complex network theory to analyse the topological characteristics of the power network, including a small-world network model \cite{5} and scale-free network model \cite{6}. Neither of the two models above consider the problem of voltage instability in power failure.

In order to describe the process of voltage instability, cellular automata theory is cited in the modelling of power failure. A cellular automata model is a discrete local dynamics model in time, space and state, and is mainly used in complex system simulation. Unlike general dynamics models, cellular automata
are not determined by physical equations or functions, but by a series of rules. In cellular automata, all cells follow the same transformation rules. And the evolution of cellular automata is realized by using transformation rules to update the cellular state at every moment. At present, cellular automata (CA) has been applied to many fields such as traffic flow simulation, forest fire spread simulation, and urban expansion simulation. In the literature [7], cellular automata theory was first applied to power failure modelling. In another study [8], fuzzy theory is combined with cellular automata theory to obtain the fuzzy cellular automata model. In the above model, the lines in the grid are taken as cells, while in this paper, the nodes in the power grid are taken as cells. Voltage is the basic parameter of the node. If the node is taken as a cell, it is convenient to introduce the theory of static voltage stability into the modelling of power failure. The theory of static voltage stability is based on power flow calculation. The critical point of voltage instability is analysed through a variety of indicators, and the distance between the running point and the critical point is given. In this paper, the cell state is defined by voltage offset and voltage stability, so as to establish a power failure model based on cellular automata theory while considering voltage stability factors. This model is used to study the overall characteristics of the power grid. On the one hand, the self-organized criticality of the power grid is proved; on the other, the influence of voltage instability factors on the occurrence scale of power failure is studied. Then, in order to analyse the weak nodes in the network, the frequency of voltage instability of the nodes in the simulation is counted.

2. The components of cellular automata

2.1. The definition of cellular state

Nodes and lines are the basic components of a power grid. In this paper, any node cell in the cellular automaton can be expressed as follows:

\[ C_i = i (i = 1, 2, \ldots, n) \]  

where, \( n \) is the number of nodes in the grid, and \( i \) is the cell number. For each cell \( C_i \), there is only one state at any given time. The set of cellular states is as follows:

\[ S_i \in \{0, 1, 2\} \]  

where, \( S_i \) is the state of cell numbered \( i \). When \( S_i \) is equal to 0, the cell is in a state of normal. When \( S_i \) is equal to 1, the cell is in a state of emergency. When \( S_i \) is equal to 2, the cell is in a state of failure. In the process of cellular automata evolution, the transformation of cellular state is always unidirectional; that is, from normal state to emergency state and from emergency state to the instability state. The state of the cell depends on three factors: voltage stability, load rate, and degree.

2.1.1. Cellular state definition based on voltage stability.

1) Definition of the cellular normal state

According to the definition of voltage stability in the IEEE and CIGRE report [9], voltage stability means that the voltage of the system can be maintained or restored to the allowable range after a small or large disturbance. The maximum deviation of cellular voltage relative to nominal voltage is defined as \( \Delta V \). When the system is disturbed and the cell voltage offset to the nominal voltage is not greater than \( \Delta V \), the cell is in the normal state.

\[ S_i = 0, \left( V_{\text{min}} \leq V \leq V_{\text{max}}, V_{\text{min}} = V_{\text{nom}} - \Delta V, V_{\text{max}} = V_{\text{nom}} + \Delta V \right) \]  

where, \( V_{\text{nomi}} \) is the nominal voltage. \( V_{\text{min}} \) is the lower limit of normal range. \( V_{\text{max}} \) is the upper limit of normal range. \( V \) is the voltage of the node at a certain moment. According to the power quality requirements, the absolute value of the positive and negative deviation of the voltage should not exceed 10% of the nominal voltage. Therefore, in this paper, the maximum voltage offset is set as 5%. The upper and lower limits of the normal range can be defined as follows:
2) Definition of the cellular emergency and failure state

According to TAYLOR’s definition of voltage instability [10], the result of voltage instability is overvoltage or low voltage. In this section, we define the emergency and failure state of the cell from the perspective of overvoltage and low voltage, respectively.

In the process of power failure, faults in electrical components or load shedding may cause overvoltage problems in some areas of the system, which may cause the insulation of the equipment to break down. Therefore, the over voltage problem is an important reason for the expansion of the scope of power failure. According to the analysis of overvoltage in power failure, the emergency and failure states of cell are defined as follows:

\[ S_i = \begin{cases} 2, & (V_{cr2} < V) \\ 1, & (V_{min} < V < V_{cr2}) \end{cases} \]  

(5)

where, \( V_{cr2} \) is the maximum permissible value of voltage. When the cell voltage is greater than the upper limit of the normal range and less than the maximum allowable voltage, the cell is in the emergency state. When the cell voltage is greater than the maximum allowable voltage, the cell is in the failure state.

According to the regulation of overvoltage limit of an AC electrical device, the maximum allowable voltage of electrical equipment is different with different voltage grades. For example, for 110kV and 220kV systems, the power frequency overvoltage should not be greater than 1.3p.u. In this paper, the maximum permissible value of voltage is given as follows:

\[ V_{cr2} = 1.3 \text{p.u.} \]  

(6)

In power failure, voltage instability can also cause low voltage in some areas. Therefore, the cell emergency and instability states are defined from the perspective of node voltage decrease as follows:

\[ S_i = \begin{cases} 2, & (V_{cr1} > V) \\ 1, & (V_{min} > V > V_{cr1}) \end{cases} \]  

(7)

where, \( V_{cr1} \) is the minimum permissible value of voltage. When the cellular voltage is less than the lower limit of the normal range and greater than the minimum allowable voltage, the cell is in the emergency state. When the cell voltage is less than the minimum allowable voltage, the cell is in the failure state.

According to the theory of static voltage stability, the minimum permissible value of voltage is taken as the value at the nose point of the PV curve. The PV curve is a typical static voltage stability analysis method that can directly analyse the voltage stability of the cell. The operating state of cells in the power grid at each moment corresponds to a point on the PV curve, which is usually called the operating point. When the operating point of the cell is in the upper half of the PV curve, the voltage of operating point is higher than the minimum permissible value, and the cell is in a normal or emergency state. When the operating point is in the lower half of the PV curve, the voltage of the operating point is lower than the minimum permissible value, and the cell is in a state of instability. When the operating point of the cell is at the inflection point of the PV curve, the cell voltage is equal to the critical voltage, and the cell is in the critical instability state.

Generally, due to the different nodes corresponding to PV curves, the minimum permissible voltage value for different nodes varies. In this paper, the minimum permissible value of voltage is given as follows:

\[ V_{cr1}(i) = V_{cpf}(i), (i = 1, 2, \ldots, n) \]  

(8)

where, the \( V_{cr}(i) \) is the minimum permissible voltage of cell \( i \). \( V_{cpf}(i) \) is the voltage at the nose of the PV curve of cell \( i \), which can be obtained by continuous power flow(CPF).
According to the above definition of cell state, the relationship between the cell state and the voltage limit is summarized and expressed on a PV curve, as shown in Figure 1.

\[ V_{	ext{min}} < V < V_{	ext{max}} \]

\[ V = V_{	ext{cr}} \]

\[ V = V_{	ext{c1}} \]

\[ V = V_{	ext{c2}} \]

Figure 1. The relationship diagram between cell states and cell voltage limits.

2.1.2. Cellular state definition based on load rate.
Considering the thermal stability of the grid elements, the load carrying capacity of each node in the grid is limited. The node may fail if the load current passing through the node is greater than its permitted carrying capacity. The load rate of the node is defined as follows:

\[ L = F / F_{\text{max}} \]

where, \( F \) is the active power flow through the node. \( F_{\text{max}} \) is the limit of the active power transmission capacity of node.

According to the load rate of the cell, the normal, emergency and failure states of the cell are defined as follows:

\[ S_c = \begin{cases} 0, & (L_{\text{c min}} < L < L_{\text{c nor}}) \\ 1, & (L_{\text{c nor}} < L < L_{\text{c max}}) \\ 2, & (L_{\text{c max}} < L) \end{cases} \]

where, \( L_{\text{c min}} \) is the minimum load rate of the node. \( L_{\text{c nor}} \) is the normal load rate of node. \( L_{\text{c max}} \) is the maximum load rate of the node. The cell in different states has different values of failure probability.

The cell fault probability model based on load rate is shown in figure 2. In this figure, \( P_{\text{max}} \) and \( P_{\text{min}} \) are the maximum and minimum shutdown probabilities, respectively.

2.1.3. Cellular state definition based on the degree.
In graph theory, the degree of a node is defined as the number of edges associated with that node. In the power grid, the degree of the electrical node is defined as the number of outgoing lines associated with the electrical node. For example, in figure 3, node 2 has two outgoing lines, so the degree of node 2 is two.
Cell states are defined based on the degree as follows:

\[ S_c = \begin{cases} 
0, (D = N) \\
1, (0 < D < N) \\
2, (D = 0) 
\end{cases} \quad (11) \]

where \( D \) is the degree of the node, and \( N \) is the degree of the node when the power grid is in the normal state. In power failure, when the outgoing line associated with the node fails and disconnects, the degree of the node decreases. When all the lines directly connected to the node are disconnected, the degree of the node is 0 and the cell is in the failure state. For example, in figure 1, when the line between node 2 and node 3 is disconnected, the node degree of node 3 is 0 and the node is out of service.

The degree of the node depends on the state of the outgoing line, and the state of the line depends on its load rate. The load rate of the line is defined as follows:

\[ L_{li} = \frac{F_{li}}{F_{li, max}} \quad (12) \]

where, \( F_{li} \) is active power flow over line \( i \). \( F_{li, max} \) is the maximum permissible value of active power flow over line \( i \). The failure probability of the line is related to load rate range. The corresponding relationship between line load rate and line failure probability is shown in the figure 4.

where, \( L_{li, min} \) is the minimum load rate of the line, \( L_{li, nor} \) is the normal load rate of the line, \( L_{li, max} \) is the maximum load rate of line, \( P_{max} \) is the maximum outage probability of the line, and \( P_{min} \) is the minimum outage probability of the line.

2.2. Definition of the cellular neighbour
The basic unit of a cellular automata is the cell. The state of the next moment of a cell is determined by the state of the cell itself and the state of its neighbour at the previous moment. In this paper, a node is defined as a cell. For two nodes directly connected by a line, any cell is the neighbour of another cell. In the three nodes system, node 1 and node 2 are neighbour cells to each other, so are node 2 and node 3.

2.3. Evolution rules of cellular automata
The power system is a complex system, and complex coupling relations exits between the components. When a component fails, it often affects the surrounding components as well. In cellular automata, cellular evolution rules need to be defined to describe the interaction between grid components in
power failure. Specifically, according to the state of the cell itself and its neighbours at the current moment, the state of the cell at the next moment can be determined by the evolution rule. This paper defines the evolution rules of cellular automata from the perspective of voltage stability.

In the process of power failure, the induction motor and on-load tap changer (OLTC) are considered to be one of the important factors that results in voltage collapse. Due to dynamic components such as OLTC, the instability range is gradually expended. In this paper, the evolution rules of cellular automata are defined according to the characteristics of bus voltage variation in power failure.

Suppose cell \( i \) and cell \( j \) are neighbours to each other, when cell \( i \) is in a failure state due to voltage decrease, the minimum permissible voltage of cell \( j \) will be affected as follows:

\[
V_{cr1,j}(t + 1) = (1 + \delta)V_{cr1,j}(t)
\]  

where, \( V_{cr1,j}(t + 1) \) is the minimum permissible voltage of cell \( j \) at the next moment. \( \delta \) is a constant factor.

When cell \( i \) is in a state of instability due to overvoltage, the maximum permissible voltage of cell \( j \) will be affected as follows:

\[
V_{cr2,j}(t + 1) = (1 - \delta)V_{cr2,j}(t)
\]

where, \( V_{cr2,j}(t + 1) \) is the maximum permissible voltage of cell \( j \) at the next moment. \( \delta \) is a constant factor.

3. Simulation and results

3.1. Analysis of power failure characteristics considering voltage instability

In order to verify whether the proposed grid cellular automaton model considering voltage instability is suitable for power failure simulation, the model was programmed and implemented in MATLAB. The simulation progress is as follows: 1) Read the initial grid data. 2) Add disturbance to the power grid and calculate the AC power flow. 3) Calculate the state of each component of the power grid based on the power flow results. 4) Judge whether the topology of the power grid changes. If the grid topology changes, the power flow will be recalculated. If the topology does not change, the power flow will not be calculated. 5) Judge whether there are cells in the failure state. If there are cells in failure state, cellular automata will evolve according to the evolution rules. If no cells are in the failure state, the disturbance will continue to be added to the power system 6) Loss loads are counted.

![Figure 5. Power failure evolution model considering voltage instability.](image-url)
In this paper, the model is used to simulate 200 power failure accidents in an IEEE39 system (Fig. 9). Loss load is calculated and the loss load sequence of power failure is obtained in figure 6. In order to study the influence of voltage instability on power failure, the loss load sequence considers voltage instability is compared with the those without considering voltage instability. Then, according to the loss load sequence, the number of power failure accidents under different loss load scales is counted. The logarithm of the scale of all power failure accidents and the corresponding number of power failure accidents are calculated, and the results are expressed in the scale-frequency double logarithm graph, that is, figure 7.

Figure 6. The loss-load sequence of power failure.

Figure 7. Scale-frequentness log-log graph.

The least square method is used to make linear regression for the fitting point at the tail, and the fitting curve regression equation and correlation coefficient of the scale of power failure taking into account the voltage instability factor are obtained:

\[ \lg N_1 = 11.12 - 3.11 \lg r_1 \]  \hspace{1cm} (15)

\[ R_1 = -0.954 \]  \hspace{1cm} (16)

The fitting curve regression equation and correlation coefficient of the scale of power failure, which ignores the voltage instability factor are obtained:

\[ \lg N_2 = 6.63 - 1.54 \lg r_2 \]  \hspace{1cm} (17)

\[ R_2 = -0.968 \]  \hspace{1cm} (18)

If significance level \( \alpha \) is equal to 0.01, the threshold value \( R_{0.01} \) is 0.7078. Based on the above critical value, the regression equation is verified as follows:

\[ |R_1| > R_{0.01}; |R_2| > R_{0.01} \]  \hspace{1cm} (19)

Therefore, the two regression equations above are valid and the simulation results are credible. It can be seen from the analysis of figure 6 that, on the one hand, the self-organized criticality of the power grid is verified, and on the other hand, after considering the voltage instability factor, the occurrence frequency and scale of power failure accidents increase.

3.2. Statistical analysis of weak points

The model is used to simulate 16,000 power failure accidents and extract the failure node set related to voltage instability in each accident. In this paper, voltage instability of PV node is not considered. In IEEE39 system, nodes 1 to 29 are PQ nodes and the statistical results of failure frequency related to voltage instability are shown in figure 8.
According to the statistical results of failure frequency, No.8, No.27, and No. 29 have higher occurrence frequency. It can be seen from the analysis that in the IEEE39 node system, node 8 and node 29 are both located at the terminal region of the system, and their voltages are susceptible to load fluctuations.

4. Conclusion
Based on the original model, a power failure model that considers voltage stability is established in this paper. The proposed model is used to simulate power failure. The self-organized criticality of the power grid is verified and the driving effect of voltage instability in power failure is verified. The failure frequency of the node is counted. By analysing the statistical data, it was found that the failure frequency of node 8 and node 29 is higher than other nodes. Node 8 and Node 29, which is in the terminal region of the system, is more prone to voltage instability.

In this paper, the constant power load model is adopted. In the next stage of work, the induction motor model can be used to analyse the influence of load dynamic characteristics on power failure.

5. References
[1] Dobson I, Carreras B A, et al. An initial model for complex dynamics in electric power system blackouts [DB/OL]. 2014-02-01.
[2] Phadeke A G, Thorp J S. Expose hidden failures to prevent cascading outages [J]. IEEE Computer Application in Power, 1996, 9(3): 20-23.
[3] Nedic D P, Dobson I, Kirschena D S, et al. Criticality in a cascading failure blackout model [J]. Electrical Power and Energy Systems, 2006, 28(9).
[4] Yu Qun, Cao Na, Zhao Xiaonan. An improved model for self-Organized criticality in electric power system blackouts [C]. International Conference on Electric Utility Deregulation & Restructuring & Power Technologies, 2010.
[5] Watts D J, Strogatz S H. Collective dynamics of ‘small-world’ networks [J]. Nature, 1998, 393(6684): 440-442.
[6] Barabási A L, Albert R. Emergence of scaling in random networks [J]. Science, 1999, 286(5439): 509-512.
[7] Yu qun, Cao Na, Guo Jianbo. Simulation model of self-organized criticality characteristics of power system based on Cellular Automata [J]. Automation of Electric Power Systems, 2011, 35(21): 1-5.
[8] Yu qun, Min Zhang, Cao Na. Fault evolution model of power grid based on Fuzzy Cellular Automata [J]. Journal of Computer Applications, 2015, 35 (9): 2682-2686.
[9] IEEE/CIGRE joint task force on stability terms and definitions. Definition and classification of power system stability [J]. IEEE Trans on Power System, 2004, 19(2): 1387-1401.
[10] CARSON W.TAYLOR. Power System Voltage Stability[M]. Beijing: China Electric Power Press, 2002.

Acknowledgments
This work was supported by the 2018 National Grid Corporation of Science and Technology Project “Interconnected power grid blackout early warning technology and system development based on multi-sand-pile theory”.