CURRENT ALGEBRA BASED EFFECTIVE CHIRAL THEORY OF MESONS AND A NEW EW THEORY

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A current algebra based effective chiral theory of pseudoscalar, vector, axial-vector mesons is reviewed. A new mechanism generating the masses and gauge fixing terms of gauge boson is revealed from this effective theory. A EW theory without Higgs is proposed. The masses and gauge fixing terms of W and Z are dynamically generated. Three heavy scalar fields are dynamically generated too. They are ghosts.

Keywords: Effective theory; Standard model

Current algebra is a very successful theory of pseudoscalar, vector, and axial-vector mesons at low energies. Based on current algebra the chiral theory of pseudoscalar, vector, and axial-vector mesons is constructed[1]
\[ L = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi}(x)M\psi(x) \]
\[ + \frac{1}{2}m_0^2(\rho_i^{\mu}\rho_{\mu i} + \omega^{\mu}\omega_{\mu} + a_i^{\mu}a_{\mu i} + f^{\mu}f_{\mu}) \]
where \( a_{\mu} = \tau_ia_{\mu}^i + f_{\mu}, \ v_{\mu} = \tau_i\rho_{\mu i} + \omega_{\mu}, \ u = exp{i\gamma_5(\tau_i\pi_i + \eta)} \). The theory has been extended to three flavors. In this theory vector and axial-vector mesons are defined by corresponding quark vector and axial-vector currents, pseudoscalar mesons are introduced by nonlinear \( \sigma \) model. Therefore, this theory is the filed theory realization of current algebra. Integrating out quark fields the effective Lagrangian of mesons is derived. As a matter of fact, the meson Lagrangian is the effect of one quark loop diagrams(in Euclidean space).

This effective theory has dynamical chiral symmetry breaking \(<\bar{\psi}\psi> \sim m^3\), and explicit chiral symmetry in the limit \( m_q \rightarrow 0 \). A cut-off \( \Lambda \sim 1.8GeV \) is introduced. In chiral limit there are two parameters: \( f_\pi \) and a universal coupling constant g which is determined to be 0.39 by fitting \( \rho \rightarrow ee^+ \). They are defined by loop integrals. Both \( f_\pi^2 \) and \( g^2 \) are proportional to \( N_C \). The order of physical meson fields in \( N_C \) expansion are defined to be \( O(\sqrt{N_C}) \). Therefore \( N_C \) expansion is carried out in this theory. Tree diagrams are at the leading order and loop diagrams of mesons are at higher order. The mass difference of \( \omega - \rho \) and \( a_1 - f_1 \) and \( \phi \rightarrow \rho\pi \) are at next leading order of \( N_C \) expansion. \( N_C \) expansion governs the calculations of this theory.
Because of the cancellation of the two diagrams obtained from
\[ -im\bar{\psi}\gamma_5\tau^i\psi\pi^i \quad \frac{1}{2} m\bar{\psi}\psi\pi^2 \]
the Goldstone theorem is revealed,
\[ m_{\pi}^2 = -\frac{2}{f_{\pi}^2} (m_u + m_d) < 0|\bar{\psi}\psi|0 > . \]
The KSFR sum rule leads to
\[ m_{\rho}^2 = 2 \frac{f_{\rho}^2}{g_{\rho}^2} = 6m^2. \]
A modified Weinberg’s second sum rule is obtained
\[ (1 - \frac{1}{2\pi^2 g^2})m_{a_1}^2 = 2m_{\rho}^2. \]
The pion mass is from the explicit chiral symmetry breaking. The \( m_{\rho} \) originates in dynamical chiral symmetry breaking. The \( m_{a_1} \) is obtained from a new mechanism of chiral symmetry breaking. This new mechanism is a new discovery from this current algebra based effective chiral field theory.

The strong decay widths are calculated \( \Gamma_{\rho} = 150MeV, \quad \Gamma_{K^*} = 45MeV, \quad \Gamma_{\phi} = 4.2MeV, \quad \Gamma_a = 326MeV,... \) EM and weak decays are calculated too. Theory agrees with data. Weinberg’s first sum rule is satisfied analytically. In the amplitude of \( a_1 \rightarrow \rho\pi \) there are two terms. The theoretical ratio of d-wave to s-wave agrees with data well. There is strong cancellation between the two terms. In the limit of soft pion the relation found by current algebra is satisfied. The puzzle of current algebra for long time is solved. The decay widths of \( K_1(1400) \) and \( f_1(1420) \) are much narrower than \( a_1 \). The theory provides the explanation and numerical results agree with data.

The scattering lengths and slopes \( \pi - \pi \) scattering are derived and they are the same as obtained by Weinberg. The amplitudes of \( I = 1, 2 \) agree with data. The s-wave\((I = 1) \) agrees with data at lower energies and the contribution of \( \sigma \) meson must be taken into account. \( \pi - K \) scattering has been studied too. Theory agrees with data very well. Besides the \( \rho \) pole an intrinsic form form factor is found in \( \pi \) form factor. This new form factor redeems the shortcomings of the \( \rho \) pole form factor.

\[
f_{\pi}(q^2) = \frac{1 + \frac{q^2 - m_{\rho}^2}{2m_{\rho}f_{\rho}^2}[(1 - \frac{2c}{g})^2 - 4\pi^2c^2]}{q^2 - m_{\rho}^2 + i\sqrt{q^2}\Gamma_{\rho}(q^2)}
\]
where \( c = \frac{f_{\pi}^2}{2gm_{\pi}^2} \). This form factor agrees with data in both time-like and space-like regions.

The kaon form factors are studied too. Theory agrees with data very well.

PCAC is satisfied. Many \( \tau \rightarrow mesons + \nu \) are calculated. Theory agrees with data.
The Lagrangians of normal parity and Wess-Zumino-Witten anomaly are derived from the same Lagrangian. The two parameters of WZW anomaly are determined. The anomalous $L$ related to $\omega$ is derived as

$$L = \frac{N_c}{(4\pi)^2} \frac{2}{3} \varepsilon^{\mu\nu\alpha\beta} \omega_\mu \text{Tr} \partial_\nu U U^\dagger \partial_\alpha U U^\dagger + \frac{2N_c}{(4\pi)^2} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \text{Tr} \{i [\partial_\nu U U^\dagger (\rho_\alpha + a_\alpha) - \partial_\nu U (\rho_\alpha - a_\alpha)] - 2(\rho_\alpha + a_\alpha) U (\rho_\beta - a_\beta) U^\dagger - 2\rho_\alpha a_\beta \}.$$ 

The low energy theorem of $\gamma \rightarrow 3\pi$ which is less than data by 30% is corrected[2] $A_{\gamma 3\pi}(0,0,0) = \frac{2e}{\pi^2 f_\pi^2} (1 + \frac{6c^2}{g^2}) = 12.2 GeV^3$, it agrees with data. The cross section of $\pi^+ (A,Z) \rightarrow \pi\pi + (A,Z)$ is calculated $\sigma = 1.34 nb$. It agrees with data.

$ee^+ \rightarrow \pi^0\pi^+\pi^-$ is dominant by the anomaly $\gamma\pi\pi\pi$. The contribution of the WZW anomaly(leading order) is far too small. The next leading order has to be taken into account. This effective chiral theory can go beyond the WZW anomaly and the next leading terms are found. Theory agrees with data very well. There are many other anomalous processes in which next leading terms must be taken into account.

Comparison with other chiral theories of mesons
1) The Chiral Perturbation Theory is the low energy limit of this effective theory, all the 10 coefficients are predicted 2) Hidden gauge theory. All the 13 parameters of the hidden gauge theory are predicted. Three new terms are found[3]. 3) Nambu-Jano-Lasinio model is a model of four quark interactions. This effective theory is not. There are highly nonlinear quark interactions. 4) In 1966[4] we first used the effective $L$ of four quark interactions to study the physics of mesons and baryons. We have obtained a mass relations between meson and baryons

$$\frac{m_K^2 - m_\pi^2}{m_\rho^2 - m_\pi^2} = \frac{3}{4} \frac{M_{\Sigma^*} - M_{\Delta}}{M_{\Delta} - M_N}.$$ 

Based on the studies in this paper the strong interactions $e_Y \bar{\psi} Y\gamma_\mu\psi B_\mu$, $e_A \bar{\psi} Y\gamma_\mu\gamma_5\psi C_\mu$, are proposed.

New symmetry breaking and a possible new EW theory
In chiral limit, the original $L$ has global chiral symmetry, $m_a = m_\rho$. After the quark loops are taken into account the mass formula of $a_1$ is obtained, which agrees with data. The global chiral symmetry is broken. Additional mass term and $(\partial_\mu a_\mu)^2$ (gauge fixing term) are dynamically generated by the vacuum polarization of $a_1$-fields. The gauge fixing term is related to the new factor in $a_1$ mass formula. The same happens to the vacuum polarization of charged vector currents. Therefore, a new mechanism upon which masses of gauge bosons and the gauge fixing terms can be dynamically generated is revealed from this effective field theory. It is known that scalar fields are required to cancel $\frac{k_i k_i}{m^2}$ after the intermediate bosons gain masses. The Higgs mechanism satisfies these two requirements. However, they are added by hand and they cause other problems. So far, Higgs has not been found.
In Ref.[5] the new mechanism found in the effective field theory has been used to construct a new Electro-Weak theory without Higgs. One neutral and charged scalar fields are dynamically generated. EW theory without Higgs can be expressed as

\[ \mathcal{L} = - \frac{1}{4} A_{\mu
u} A^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q}_L \left\{ i \gamma^\mu \partial^\mu + \frac{g}{2} \gamma^\mu A^\mu + g \frac{Y}{2} \gamma^\mu B \right\} q_L + \bar{q}_R \left\{ i \gamma^\mu \partial^\mu + \frac{g'}{2} \gamma^\mu B \right\} q_R - m_q \bar{q} q \]

There are neutral axial-vector and charged vector and axial vector currents of fermions. One-loop vacuum polarization of Z-fields is expressed as

\[ \Pi_{\mu\nu}^Z = \frac{1}{2} F_{Z1}(z)(p_\mu p_\nu - p^2 g_{\mu\nu}) + F_{Z2}(z)p_\mu p_\nu + \frac{1}{2} \Delta m_2^2 g_{\mu\nu}, \]

where \( F_2 \) is finite and \( \Delta m_2^2 \) is a constant. In the limit of zero fermion mass, these two terms disappear[5]. The vacuum polarization of W-fields has similar expression. The masses of W and Z and gauge fixing terms are dynamically generated by corresponding vacuum polarization diagrams of W and Z. The gauge fixing terms lead to dynamical generation of scalar fields

\[ Z'_\mu = Z'_\mu + \frac{1}{m_{\phi Z}} \partial_\mu \phi_Z, \quad \partial_\mu Z'_\mu = 0, \quad W'_\mu = W'_\mu + \frac{1}{m_{\phi W}} \partial_\mu \phi_W, \quad \partial_\mu W'_\mu = 0. \]

Their masses are determined to be \( m_{\phi Z} = 3.78 \times 10^{14} \text{GeV} \) and \( m_{\phi W} = 9.31 \times 10^{13} \text{GeV} \). They are ghosts. The masses of W and Z and propagators of W and Z fields are determined to be

\[ m_w^2 = \frac{1}{2} g^2 m_t^2, \quad m_Z^2 = \frac{1}{2} (g^2 + g'^2) m_t^2, \quad G_F = \frac{1}{2 \sqrt{2} m_t^2}. \]

where \( m_t \) is the top quark mass. They agree with data very well.

\[ \Delta_{Z\mu\nu} = \frac{1}{p^2 - m_Z^2} \left\{ -g_{\mu\nu} + \left( 1 + \frac{1}{2 \xi_Z} \right) \frac{p_\mu p_\nu}{p^2 - m_{\phi Z}^2} \right\}, \]

\[ \Delta_{W\mu\nu} = \frac{1}{p^2 - m_W^2} \left\{ -g_{\mu\nu} + \left( 1 + \frac{1}{2 \xi_W} \right) \frac{p_\mu p_\nu}{p^2 - m_{\phi W}^2} \right\}, \]

\( \xi_Z = -1.18 \times 10^{-25} \) and \( \xi_W = -3.73 \times 10^{-25} \). The dynamically generated scalars behave like Higgs. In this new EW theory the Faddeev-Popov procedures are not required.

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