Four-body baryonic decays of $B \to p \bar{p} \pi^+ \pi^- (\pi^+ K^-)$ and $\Lambda \bar{p} \pi^+ \pi^- (K^+ K^-)$

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**Abstract**

We study the four-body baryonic $B \to B_1 B_2 M_1 M_2$ decays with $B_{1,2} (M_{1,2})$ being charmless baryons (mesons). In accordance with the recent LHCb observations, each decay is considered to proceed through the $B \to M_1 M_2$ transition together with the production of a baryon pair. We obtain that $B(B^- \to \Lambda \bar{p} \pi^+ \pi^-) = (3.7^{+1.2}_{-1.1}) \times 10^{-6}$ and $B(B^0 \to p \bar{p} \pi^+ \pi^-) = (3.0 \pm 0.9, 6.6 \pm 2.4) \times 10^{-6}$, in agreement with the data. We also predict $B(B^- \to \Lambda \bar{p} K^+ K^-) = (3.0^{+1.2}_{-0.9}) \times 10^{-6}$, which is accessible to the LHCb and BELLE experiments.

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**1. Introduction**

One of the main purposes of the $B$ factories and current LHCb is to study CP violation (CPV), which is important for us to understand the puzzle of the matter–antimatter asymmetry in the Universe. As the observables, the (in)direct CP-violating asymmetries (CPAs) require both weak and strong phases [1–3], whereas the $\bar{p}_1 \cdot (p_2 \times \bar{p}_1)$ in a four-body decay, do not necessarily need a strong phase [4,5]. For example, the LHCb Collaboration has provided the first evidence for CPV from the TPCs in $\Lambda_b \to \pi^0 \pi^+ \pi^-$ [6], and measured TPCs in $\Lambda_b \to p K^- \mu^+ \mu^-$ [7]. As the similar baryonic cases, the four-body baryonic $B$ decays can also provide TPCs.

For a long time, the $B^- \to \Lambda \bar{p} \pi^+ \pi^-$ decay was the only observed decay mode in $B \to B_1 B_2 M_1 M_2$ [8]. Until very recently, more four-body baryonic $B$ decays have been observed by the LHCb [9], which motivate us to give theoretical estimations on the corresponding decay branching ratios. The experimental measurements for the branching ratios of $B^0/B^- \to B_1 B_2 M_1 M_2$ at the level of $10^{-6}$ are given by [8,9]

\[
B(B^0 \to p \bar{p} \pi^+ \pi^-) = (3.0 \pm 0.2 \pm 0.2 \pm 0.1) \times 10^{-6},
\]

\[
B(B^0 \to p \bar{p} K^+ \pi^\pm) = (6.6 \pm 0.3 \pm 0.3 \pm 0.3) \times 10^{-6}.
\]

where the resonant $B(B^- \to \Lambda \bar{p} (\rho^0, f_2(1270) \to )\pi^+ \pi^-)$ have been excluded from the data [8]. In comparison with $B(B^0 \to p \bar{p} K^+ K^-) = (1.3 \pm 0.3) \times 10^{-7}$ and $B(B^0 \to p \bar{p} \pi^+ \pi^-) \sim 7.3 \times 10^{-7}$ (90% C.L.) [9], the decays with $B \sim 10^{-6}$ in Eq. (1) are recognized to have the same theoretical correspondence, where $B^0/B^- \to B_1 B_2 M_1 M_2$ proceed through the $B \to M_1 M_2$ transition along with the $B_1 B_2$ production, as depicted in Fig. 1. Note that the $B^0$ decays of $B^0 \to p \bar{p} K^+ \pi^\pm$ and $p \bar{p} K^+ K^-$ with $\bar{s}$ being replaced by $d$ in $B^0 \to p \bar{p} \pi^+ \pi^-$ and $p \bar{p} K^+ K^-$ have also been found with the branching ratios of order $10^{-6}$ [9], respectively.

In this report, we will calculate the four-body baryonic $B$ decays in accordance with the decaying processes in Fig. 1, with the extraction of the $B \to M_1 M_2$ transition form factors from the $B \to D^{(*)0} M_1 M_2$ and $B \to M_1 M_2 M_3$ decays and the adoption of the timelike baryonic form factors from the two-body and three-body baryonic $B$ decays. Our theoretical approach will be useful for the estimations of TPCs in $B \to B_1 B_2 M_1 M_2$ to be compared to future measurements by the LHCb.

**2. Formalism**

In terms of the quark-level effective Hamiltonian for the charmless $b \to q_1 \bar{q}_2 q_3$ transition, the amplitudes of the four-body baryonic $B$ decays by the generalized factorization approach are derived as [10].
\[ A_1(B^0_{(s)} \rightarrow p \bar{p} M_1 M_2) = \frac{G_F}{\sqrt{2}} \left[ \langle \bar{p} p | \alpha_q^0 (\bar{u} u)_V - \alpha^q (\bar{u} u)_A | 0 \rangle \right] \]

\[ + \langle p \bar{p} | \bar{G}^0 (\bar{d} d)_V - \bar{G}^0 (\bar{d} d)_A | 0 \rangle \]

\[ + (\alpha_q^0 - \alpha_{10}^0/2) \langle \bar{p} p | (\bar{q} q) V - A | 0 \rangle \langle M_1 M_2 | (\bar{q} b) V - A | B^0_{(s)} \rangle \]

\[ + \alpha_q^0 \langle \bar{p} p | (\bar{q} q) s + p | 0 \rangle \langle M_1 M_2 | (\bar{q} b) s + p | B^0_{(s)} \rangle \right), \]

where \( G_F \) is the Fermi constant, \( V_{ij} \) are the CKM matrix elements, and \( (\bar{q} q_1)_{V(A)} \) and \( (\bar{q} q_2)_{S(P)} \) stand for \( q_1 \gamma_V (\gamma_S) q_2 \) and \( \bar{q}_1 (\gamma_S) q_2 \), respectively. The parameters \( \alpha_{1,2}^0 \) and \( \beta_0^0 \) in Eq. (2) are given by

\[ \alpha_{1,2}^0 = \alpha_2^0 + \alpha_3^0 \pm \alpha_9^0, \beta_0^0 = \alpha_3^0 \pm \alpha_9^0 - \alpha_7^0/2, \]

\[ \alpha_{1,2}^0 = V_{ub} V_{ut}^* a_{1,2}, \alpha_j^0 = -V_{tb} V_{ts}^* a_j, \alpha_q^0 = V_{tb} V_{ts}^* 2a_6, \]

with \( q = (d, s) \) and \( j = (3, 4, 5, 9, 10) \), where \( a_{1,2} \equiv c_{1,2}^{eff} + c_{1,2}^{eff/NNc} \) for \( i = \text{odd (even)} \) with the effective color number \( N_c^{eff} \) and Wilson coefficients \( c_{1,2}^{eff/NNc} \) in Ref. [10]. From \( A_1(B^0_{(s)} \rightarrow p \bar{p} M_1 M_2) \) and \( A_2(B^- \rightarrow \Lambda \bar{p} M_1 M_2) \) in Eq. (2), the allowed decays are

\[ B^0 \rightarrow p \bar{p} \pi^+ \pi^-, B^0 \rightarrow p \bar{p} K^+ K^-, \]

\[ B^- \rightarrow \Lambda \bar{p} \pi^+ \pi^-, B^- \rightarrow \Lambda \bar{p} K^+ K^-. \]

Note that the \( B^0 \rightarrow p \bar{p} \pi^+ K^- \) and \( B^0 \rightarrow p \bar{p} K^+ K^- \) decays have the matrix elements of \( \langle p \bar{p} | s \bar{s} V_{AS} | 0 \rangle \) with the \( s \bar{s} \) quark currents, which eventually cause the terms of \( \alpha_{s,6,10}^0 \) to give nearly zero contributions due to the OZI suppression of the \( s \bar{s} \) quark

For the matrix elements in Eq. (2), the baryon-pair productions from the quark currents are given by [5,12]

\[ B_1 B_2[q_1 \gamma_\mu q_2 | 0] = \bar{u} u \gamma_\mu \left[ F_1 + \frac{F_2}{m_{B_1} + m_{B_2}} i \sigma_{\mu \nu} q_\nu \right] v. \]

(3)

\[ B_1 B_2[q_1 \gamma_\mu q_2 | 0] = \bar{u} u \gamma_\mu \left[ g_A \gamma_\mu + \frac{h_A}{m_{B_1} + m_{B_2}} q_\nu \gamma_\nu \right] \gamma_5 v. \]

(4)

where \( q = p_{B_1} + p_{B_2}, \) \( s = q^2, \) \( u(v) \) is the (anti-)baryon spinor, and \( (F_{1,2}, g_A, h_A, f_5, g_5) \) are the timelike baryonic form factors. On the other hand, the \( B \rightarrow M_1 M_2 \) transition matrix elements are parameterized as [13]

\[ \langle M_1 M_2 | q_1 \gamma_\mu (1 - \gamma_5) b | B \rangle = \bar{u} u \gamma_\mu \left[ F_1 + \frac{F_2}{m_{B_1} + m_{B_2}} i \sigma_{\mu \nu} q_\nu \right] v. \]

(5)

\[ \langle M_1 M_2 | q_1 \gamma_\mu (1 - \gamma_5) b | B \rangle = \bar{u} u \gamma_\mu \left[ g_A \gamma_\mu + \frac{h_A}{m_{B_1} + m_{B_2}} q_\nu \gamma_\nu \right] \gamma_5 v. \]

(6)

where \( p = p_{M_1} + p_{M_2} \) and \( (r, w) \) are the form factors. Subsequently, one can also get \( (M_1 M_2 | q_1 \gamma_\mu | B) \) from Eq. (6) based on equations of motion. In terms of the approach of QCD counting rules, the momentum dependences for the \( 0 \rightarrow B_1 B_2 \) and \( B \rightarrow M_1 M_2 \) transition form factors are given by [14-17]

\[ F_1 = \frac{c_{\parallel}}{t^2}, \]

(7)

\[ F_1 = \frac{c_{\perp}}{t^2}, \]

where \( \zeta_c = \zeta_c(t/\Lambda_0^2) \) with \( \gamma = 2.148 \) and \( \Lambda_0 = 0.3 \) GeV. We note that since \( F_2 \) is derived to be \( F_2 = F_1/(t \ln(t/\Lambda_0^2)) \) [18], which is much less than \( F_1 \), while the small value of \( F(B^0 \rightarrow p \bar{p}) = (1.5 \pm 0.5) \times 10^{-8} \) [19,20] causes a tiny \( C_{b,a} \) [21] in \( h_A \approx C_{b,a}/t^2 \), we may not consider the effects from \( F_2 \) and \( h_A \). In addition, by following Ref. [16], we have neglected the terms related to \( r \) and \( w \), in Eq. (6) due to the wrong parity [22].

The integration over the phase space of the four-body \( B(p_B) \rightarrow B_1(p_{B_1}) B_2(p_{B_2}) M_1(p_{M_1}) M_2(p_{M_2}) \) decay relies on the five kinematic variables, that is, \( s, p_t, t \) and the three angles of \( \theta_B, \theta_M \) and \( \phi \). In Fig. 2, the angle \( \theta_M \) is between \( p_{B_1} (p_{B_2}) \) and the \( B_1 B_2 \) (\( M_1 M_2 \)) rest frame and the line of flight of the \( B_1 B_2 \) (\( M_1 M_2 \)) system in the \( B \) meson rest frame, while the angle \( \phi \) is from the \( B_1 B_2 \) plane to the \( M_1 M_2 \) plane, defined by the momenta of the
\( \vec{B}_1 \vec{B}_2 \) and \( M_1 M_2 \) pairs in the \( B \) rest frame, respectively. The partial decay width reads \([23, 24]\)

\[
d\Gamma = \frac{|\mathcal{A}|^2}{4(4\pi)^6m_B^2} X c_B d m c \theta_B d c \theta_M d \phi , \tag{8}\]

where \( X, \alpha_B \) and \( \alpha_M \) are given by

\[
X = \left[ \frac{1}{4} (m_B^2 - s - t)^2 - st \right]^{1/2} ,
\]

\[
\alpha_B = \frac{1}{t} \lambda^{1/2} (t, m_B^2, m_{\vec{B}}^2) ,
\]

\[
\alpha_M = \frac{1}{s} \lambda^{1/2} (s, m_M^2, m_M^2) , \tag{9}\]

respectively, with \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca \), while the allowed ranges of the five variables are given by

\[
(m_{M_1} + m_{M_2})^2 \leq s \leq (m_B - \sqrt{t})^2 ,
\]

\[
(m_{B_1} + m_{\vec{B}_2})^2 \leq t \leq (m_B - m_{M_1} - m_{M_2})^2 ,
\]

\[
0 \leq \theta_B, \theta_M \leq \pi , \quad 0 \leq \phi \leq 2\pi . \tag{10}\]

3. Numerical results and discussions

For the numerical analysis, the CKM matrix elements in the Wolfenstein parameterization are presented as

\[
(V_{ub}, V_{tb}) = (A^\lambda (\rho - i\eta), 1) ,
\]

\[
(V_{ud}, V_{td}) = (1 - \lambda^{1/2}, A^\lambda , \Lambda_1^\lambda) ,
\]

\[
(V_{us}, V_{ts}) = (\lambda, -A^\lambda) , \tag{11}\]

with \((\lambda, \rho, \eta) = (0.225, 0.814, 0.120 \pm 0.022, 0.362 \pm 0.013) [20].\)

To estimate the non-factorizable effects in the generalized factorization approach \([10]\), \( N_{\gamma}^{eff} \) ranges from 2 to \( \infty \). In Table 1, we show the values of \( a_i \) for the \( b \to d \) and \( b \to s \) transitions with \( N_{\gamma}^{eff} = (2, 3, \infty) \), respectively.

According to the extractions of \((C_{\gamma}, C_{\omega})\) in Refs. \([16, 17]\), we fit the \( B \to \pi \pi \) transition form factors with the branching ratios of \( B^0 \to D^{(\ast)} \pi^+ \pi^- \), \( B^+ \to \pi^+ \pi^- \pi^- \) and \( B^+ \to K^+ \pi^- \pi^- \), and the \( B \to (K, \pi, K) \) ones with those of \( B^0 \to D^{(\ast)} K^- K^0 \), \( B^0 \to D^0 K^- K^+ \) and \( B^+ \to K^+ \pi^- K^- \). Note that the contributions from the resonant \( B \to D^{(\ast)} M_1 \) to \( M_1 M_2 \) and \( B \to K^{\ast} M_1 \) to \( M_1 M_2 \) decays with \( \rho^0, f_2(1270) \to \pi^+ \pi^- \) or \( \phi K^+ K^- \) have been excluded from the data. Unfortunately, the current observations of \( B(B^0 \to M_1 M_2 M_3) \) are not sufficient for us to extract the \( M_1 \to M_1 M_2 \) transition form factors. As a result, we obtain

\[
(C_{\gamma}, C_{\omega})_{(b \to \pi \pi)} = (3.6 \pm 0.3, 0.7 \pm 0.2) \text{ GeV}^3 ,
\]

\[
(C_{\gamma}, C_{\omega})_{(b \to K^{\ast} K \pi)} = (-38.9 \pm 3.3, 14.2 \pm 2.3) \text{ GeV}^3 . \tag{12}\]

The timelike baryonic form factors in Eq. (5) can be related with the \( SU(3) \) flavor and \( SU(2) \) spin symmetries, such that

\[
\begin{align*}
C_{\gamma} & = (3.6 \pm 0.3, 0.7 \pm 0.2) \text{ GeV}^3 , \\
C_{\omega} & = (-38.9 \pm 3.3, 14.2 \pm 2.3) \text{ GeV}^3 .
\end{align*}
\]

Fig. 2. Three angles of \( \theta_B, \theta_M, \) and \( \phi \) in the phase space for the four-body \( B \to B_1 B_2 M_1 M_2 \) decays.
it is found that the contribution is mainly from the penguin-level dominant $B^0 \to p\pi^+\pi^−\pi^−$ mode. Note that $a_{3.5} \simeq b_{3.5} = -V_{td}V_{ce}(a_d + a_s) + a_s$ are also sensitive to the non-factorizable effects. With $N^{fj}_{ij} = 3$, we obtain $B(B^0 \to p\pi^+\pi^-K^-) = (6.6 \pm 2.4) \times 10^{-6}$, which suggests that the decay is free from the non-factorizable effects. In Table 2 we have included the data to constrain the non-factorizable effects, which results in $dN^{fj}_{ij} = 0.06$. We note that the two spectra in Fig. 3 for $B^- \to \Lambda_b\bar{p}M_1M_2$ and $B^0 \to p\bar{p}M_1M_2$ present the threshold effects as the peaks around the threshold areas of $m_{\Lambda_b}\simeq m_{\Lambda} + m_p$ and $m_{\bar{p}p}\simeq m_p + m_p$, respectively, which are commonly observed in the three and four-body baryonic $B$ decays [9,26].

Finally, we remark that we cannot explain the data of $B(B^0 \to p\bar{p}K^\pm\pi^\mp, p\bar{p}K^+K^-) = (1.5 \pm 0.7, 4.6 \pm 0.6) \times 10^{-6}$ measured by the LHCb [9] due to the lack of the information for the transition form factors of $B^0 \to (K^+ + K^-)(K^- + K^-)$. This calls for the theoretical and experimental studies of the three-body mesonic $B^0$ decays that could proceed with the $B^0 \to M_1M_2$ transitions, such as the $B^0 \to D^{*+}\pi^-K^0, B^0 \to D^0\pi^-K^+K^-)$ and $B^0 \to \rho^-\pi^+K^-$ decays with one of the mesons to be a vector one, in order to extract both ($h, w$) in Eq. (6). On the other hand, the observed $B^0 \to D^0K^+\pi^-$ and $B^0 \to D^0K^+\pi^-$ decays [20] are also important as they relate to $w$.

4. Conclusions

In sum, we have studied the charmless four-body baryonic $B \to B_1B_2M_1M_2$ decays, where the primary decaying processes are regarded as the $B \to M_1M_2$ transitions along with the baryon-pair productions. According to the new extractions of the $B \to M_1M_2$ transition form factors from the three-body $B \to D^{(*)}M_1M_2$ and $B \to M_1M_2M_3$ decays, we have shown that $B(B^- \to \Lambda\bar{p}\pi^-\pi^-) = (3.7 \pm 1.3) \times 10^{-6}$ and $B(B^0 \to p\bar{p}\pi^+\pi^-, p\bar{p}\pi^+K^-) = (3.0 \pm 0.9, 6.6 \pm 2.4) \times 10^{-6}$, which agree with the data. We have also predicted $B(B^- \to \Lambda\bar{p}K^+K^-) = (3.0 \pm 0.9) \times 10^{-6}$ to be accessible to the LHCb and BELLE experiments. The study of $B \to B_1B_2M_1M_2$ benefits the future test of T violation, as the T-odd triple momentum product correlation of $p_1 \cdot (\bar{p}_2 \times \bar{p}_3)$ can be directly constructed.

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