Gradient field divergence-based small target detection in infrared images

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Received: 15 August 2021 / Accepted: 4 March 2022 / Published online: 29 June 2022
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Abstract
Infrared small target detection in complex cloud backgrounds has long been a research challenge. A novel robust target detection method based on the divergence of gradient field is proposed to enhance the target and suppress the complex background synchronously. The negative gradient field of the target intensity (NIG field) matches with characteristics of the positive source. The cloud cluster, on the other hand, lacks this feature. First, the NIG field is calculated based on the target’s property from the original image. The divergence values of NIG field are then calculated to produce a defined divergence map (D map), which highlights the target regions while suppressing the clutter regions. Meanwhile, a local vectors angle measure (LVAM) operator of the NIG field is designed to measure the angle distribution of 8-neighbour vectors and eliminate false target areas. Then, the defined local angle map (LA map) is obtained by measuring the local angle value of 8-neighbour vectors for each patch of NIG field. In addition, the divergence-local angle map (D-LA map) is obtained as the Hadamard product of the D map and LA map. Finally, we can easily obtain the target via a constant false alarm ratio based on the D-LA map. The performance evaluation results of real image sequences show that the proposed method is satisfactory for clutter suppression and target detection. Moreover, the results from comparative experiments show that the proposed method outperforms conventional methods in terms of detection accuracy and false alarm rate.

Keywords Small target detection · Divergence · NIG field · Infrared images

1 Introduction
Many infrared imaging systems use small target detection methods for searching and tracking targets (Eysa and Hamdulla 2019; Yueming et al. 2016; Maoxing et al. 2018; Jianxin et al. 2020), precision guidance, long-range early warning (He et al. 2016) and target monitoring (Yang et al. 2014). The targets typically occupy a small area in the image without
prior information. The infrared image cannot easily detect the target due to the low contrast between the small target and the background (Chen et al. 2013). The bright cloud clutter region, in particular, interferes with small target detection and increases the false alarm rate for a complex background that includes many clouds. As a result, detecting a small infrared target against a complex background is tricky.

A large number of detection methods for a small infrared target can be classified into two categories: track-before-detect (TBD) and detect-before-track (DBT), depending on the order of detection and tracking (Zhang et al. 2005). The spatial–temporal information of continuous images is used simultaneously to detect the target in the TBD method. TBD methods typically detect these targets based on the different characteristics of small targets and noise because the targets have their features of continuity and regularity (Zhang et al. 2017; Wan et al. 2017) under random noise. Existing TBD methods include the Hough transform-based method (Gao et al. 2012; Hu and Zhang 2010), the hypothesis test-based method (Li et al. 2009; Ying et al. 2016), the three-dimensional matched filtering-based method (Reed et al. 1983; Dragovic 2003; Xiong et al. 1997), the dynamic programming-based method (Huang et al. 2012; Li and Liu 2015) and the energy accumulation-based method (Zhang et al. 2005, 2007; Pan et al. 2014). TBD as mentioned above methods can detect small targets effectively, but their performance suffers noticeably when clutter interference becomes significant. In addition, the computational cost of TBD methods is high when dealing with a large number of images simultaneously. Therefore, the performance of TBD methods is limited.

Compared to TBD methods, DBT methods rely solely on the spatial information of the image to detect the target, resulting in a lower computational cost. Currently, DBT methods use three approaches for target detection. The first approach, which focuses on target features, directly estimates and removes the background component of the detected image before extracting the small targets (Zhao et al. 2014; Genin et al. 2012; Dabov et al. 2007). Recently, an infrared patch-image (IPI) model (Gao et al. 2013) was proposed to isolate the target from the background image based on the non-local self-correlation characteristic of the infrared background image. Wang et al. (Wang et al. 2017) proposed a stable multisubspace learning (SMSL) method to separate small targets from highly heterogeneous scenes to improve the IPI model’s background estimation performance. Another approach used by DBT methods is the detection operator. The operator can extract the small target based on the structure and brightness difference between the complicated background and the small target in a single frame (Bai and Zhou 2010; Zeng et al. 2006; Deshpande et al. 1999; Cao et al. 2008; Deng et al. 2021). These methods primarily design different filters to detect targets that take advantage of the target characteristics. However, the filters ignore that the clutter areas are very similar to targets, resulting in high false alarm rates. Recently, small target detection algorithms based on neural networks (Ju et al. 2021; Gao et al. 2019) have been proposed. However, due to the small size of the training data set and the lack of target features, the neural network method cannot fully exploit its advantages, resulting in a low probability of target detection.

The third approach utilized for detecting small targets emphasizes the contrast between the targets and the complex backgrounds. This method is based on the contrast mechanism of the human visual system (HVS). Two kinds of comparison mechanism-based methods have been considered. The first is an operator-based detection method. For instance, Shao et al. (Eysa and Hamdulla 2019; Shao et al. 2012) introduced the Laplacian of Gaussian (LoG) operator to detect small infrared targets to improve image contrast. Subsequently, Dong et al. (2014) utilized the Difference of Gaussians (DOG) operator to enhance the image contrast and capture a small infrared target. Aside from the difference of Gabor
(DoGb), an operator was introduced to the infrared small target detection field (Han et al. 2016). As a result, the IDoGb operator was found to possess the ability to improve contrast enhancement. The second one is the local contrast measure-based method. Chen et al. (2013) proposed an effective local contrast measure (LCM) method for the first time.

Given that LCM produced an excessive enhancement of pixel-sized noises with high brightness (PNHB), Han et al. (2014) proposed an improved local contrast measure (ILCM) to reduce the false alarm rate. Qin and Li (2016) found that ILCM cannot solve clutter with a high gray value in the background of the detected image and then presented a novel local contrast measure (NLCM) method. Given the poor adaptability of uniform window size, Deng et al. (2017) devised an adaptive window selection strategy and then constructed a contrast measure to calculate the local difference.

The contrast, as mentioned above, mechanism-based methods effectively detect small targets. However, in a complex background, the intensity of the clutter regions is enhanced simultaneously, resulting in relatively high false alarm rates. For this reason, Zhang et al. (2018) formulate the local intensity and gradient map using the local intensity and gradient properties, allowing the algorithm to detect the small target efficiently. Unfortunately, because this method does not adequately use the gradient, a dim target cannot be easily detected. This paper proposes a new methodology for detecting small targets in a cloudy background to address this issue. This methodology is based on the divergence of a negative gradient (NIG) field and better uses the gradient property. For defining the so-called divergence and the local angle (D-LA) map, the divergence is combined with a local angle map (LA map) that describes the gradient vector distribution. Infrared dim targets can be computed as a positive divergence value by using the D-LA map. As a result, the target features can be emphasized, whereas the clutter features can be suppressed. The proposed method applies a small target intensity gradient field in target detection. To demonstrate the potential of our method, we compared it to several state-of-the-art methods in the real-world database.

The paper is organized as follows. The divergence of the NIG field is explained in Sect. 2, and it also describes our method in detail. Section 3 presents the experimental results from the actual database. Finally, the conclusions are discussed in Sect. 4.

2 Analysis and methodology

This section introduces a new method for detecting small targets embedded in complicated cloud clutter backgrounds. The characteristics of the small targets are examined in Sect. 2.1 and described in the NIG field of the target intensity (NIG field) in Sect. 2.2. The small target of NIG field is then extracted using the divergence map (D map) in Sect. 2.3 and the LA map in Sect. 2.4. Target enhancement and background suppression are achieved simultaneously during the characteristic extraction process. Finally, in Sect. 2.5, we display the processing flowchart of our method in detail.

2.1 Analysis of small target characteristic

In practical, atmospheric conditions, imaging of long-distance targets is interfered with by various factors (e.g., atmospheric refraction and dispersion, mirror deformation, lens aberration and diffraction, and optical defocusing) (Eysa and Hamdulla 2019). In general, the small target in an infrared (IR) image is characterized by low contrast and low signal–noise
ratio (SNR). Moreover, the target’s size in IR image is only a few pixels with a high degree of regularity. To represent the IR small target (Eysa and Hamdulla 2019; Zhang et al. 2005; Shao et al. 2012), a parametric model of the point spread function (PSF) (Eysa and Hamdulla 2019) is specifically used. The model is as follows:

\[
s(m\_x, m\_y|m\_xc, m\_yc, \tau\_x, \tau\_y) = I \cdot \exp \left\{ -\frac{1}{2} \left[ \left( \frac{m\_x - m\_xc}{\tau\_x} \right)^2 + \left( \frac{m\_y - m\_yc}{\tau\_y} \right)^2 \right] \right\},
\]

where \( s \) denotes the small target, \( I \) represent the intensity of the peak, \( (m\_x, m\_y) \) represents the spatial coordinate of the target, \( (m\_xc, m\_yc) \) is the center position of target in the IR image, \( \tau\_x \) and \( \tau\_y \) represents the extent parameters in the horizontal and vertical directions, respectively. The actual small target image of an aircraft is displayed in Fig. 1.

In general, the small target image obtained by the IR detector consists of three components, namely the target, noise, and background. Accordingly, the observation of the IR image with the small target is expressed as

\[
g(m\_x, m\_y) = s(m\_x, m\_y) + b(m\_x, m\_y) + w(m\_x, m\_y),
\]

where \( g(m\_x, m\_y) \) denotes the intensity of the observation pixel in location \( (m\_x, m\_y) \), \( s(m\_x, m\_y) \) represents a small target, \( b(m\_x, m\_y) \) is a cluttered background, \( w(m\_x, m\_y) \) is the measurement noise in images. The clutter has a much more significant impact on target detection than the measurement noise. Therefore, the influence of measurement noise can be ignored. Small target detection could be summarized as the process of extracting small target signals from the clutter. This technology is based on finding the characteristic difference between the small target and the clutter.

### 2.2 The NIG field of infrared small target image

Following the analysis shown in Sect. 2.1, the small target characteristics are described in NIG field in this section. Before discussing the gradient, it is essential to understand the concept of the directional derivative. The directional derivative in mathematics is the rate of change of a scalar field function along a particular direction at a given point. Furthermore, the gradient of a scalar field function is a vector whose magnitude equals the value of the maximum directional derivative and whose direction represents the direction in which

![Fig. 1](image-url) An actual small target in infrared image a IR image of real scene, b small target with an enlarged view and contrast stretching, c Three-dimensional display of small target in (a)
the directional derivative of a scalar field function will be maximized. Consequently, the direction of the gradient at a certain point is the direction with the highest rate of increase of the scalar field function, and its magnitude is the slope of the function in that direction. In the two-dimensional Cartesian coordinate system, the gradient is expressed as:

\[ \nabla f = \frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j}, \]

where \( f \) is a scalar function. \( \vec{i}, \vec{j} \) are the basis vectors of the \( x \) and \( y \) coordinates, respectively.

Here, we use the scalar function \( f \) to represent an IR small target image. Then, a scalar field is represented by the intensity in the image. Thus, the intensity of the pixel \((x, y)\) is expressed as \( f(x, y) \). And the NIG field is investigated in this paper. At each pixel of the image, the NIG of \( f \) at that point will show the direction in which the intensity descends most quickly. The magnitude of the NIG will determine how fast the intensity descends in that direction. For a two-dimensional IR image, the NIG field is a two-dimensional vector field. In Fig. 2, a real scene infrared image is shown. Meanwhile, the local NIG fields of several key components in the image are given.

According to Fig. 2, it can be inferred that the NIG vectors in the target region roughly point outwards from the target center, which is consistent with the intensity distribution of the point spread function model. Nevertheless, the NIG vectors near the cloud edge region are generally consistent and orientated. On the contrary, the NIG vectors of the interior region of the cloud and sky background are irregular. In addition, their gradient magnitude is much smaller than that of the small target region and the cloud edge region. The property of the local NIG vectors of the four image regions is listed in Table 1. The most distinguishing feature of the small target is the gradient direction. Accordingly, the direction of local gradient vectors can be applied to detect small targets.

Fig. 2 An infrared small target image and the local negative gradient fields of four key components. \( a \) Is a real scene infrared image with a small target, cloud clutter and background. The red, blue, purple and yellow bounding boxes represent the small target region, the cloud edge region, the cloud’s interior region, and the sky background region, respectively. \( b \) Is the local negative gradient fields of the small target region. \( c \) Is the local negative gradient fields of the cloud edge region. \( d \) Is the local negative gradient fields of the interior region of the cloud. \( e \) Is the local negative gradient fields of the sky background region.
2.3 D map of the NIG field

In Sect. 2.2, the components of infrared images and their characteristic are analyzed in NIG field. In this section, we will extract the target characteristic of NIG field by the divergence. For a vector field, the divergence can express the extent to which there is some physical quantity exiting an infinitesimal region of space or entering it. If the divergence value is positive, it means that the exiting quantity is more than the entering quantity. If the divergence value is negative, the result is the opposite. If the divergence at a point is zero, it means that the exiting quantity is equal to the entering quantity or there is no physical quantity exiting and entering at this point. In other words, it is a local measure of the “outgoingness”. Simply, the divergence can show whether there is a source at a point and whether it is a positive source or a negative source. Supposing that \( \vec{F} \) is a two-dimensional differentiable vector function, namely, \( \vec{F} = U \cdot \vec{i} + V \cdot \vec{j} \), the divergence of \( \vec{F} \) is defined as follows: (Edwards 2014)

\[
div \vec{F} = \nabla \cdot \vec{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \cdot (U, V) = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}.
\]

When the intensity value of a local region is high, the NIG vectors expand in all directions and the vectors in the gradient field point outward from the center point. This is consistent with the NIG field characteristics of the small target region in infrared images. Thus, the region’s divergence would be a positive value. The gradient fields of the cloud edge region, cloud interior region and the sky background region, in comparison, lack the feature of pointing outwards from the center. As a result, such regions have no strong outgoingness, and the divergence is very small or close to zero.

In the case of an infrared small target image, the image’s negative intensity gradient defines a vector field known as the NIG field.

**Definition** The divergence values of all points in the NIG field form the D map of the infrared image.

An example of the D map is shown in Fig. 3.

Figure 3c shows a very bright small target region in the D map, indicating that the divergence value is high in that region. In contrast, the cloud edge region, cloud interior region and the sky background region are very dark, indicating that the divergence values
are minimal. Figure 3d shows the 3D display of the D map in (c). We can observe that the

target region has a much higher divergence value than the other regions. This point is also

supported by the analysis of Fig. 3b and d. In other words, the D map highlights the target

regions while simultaneously suppressing the clutter regions.

2.4 LA map of the NIG field

Although small targets are enhanced and the clutter is suppressed in the D map of the infra-
red image described in Sect. 2.3, some clutter regions have high divergence values. When
the local contrast of infrared image between the target and background is high, clutter
regions with high divergence values do not interfere with target detection. On the contrary,
when the local contrast is low, these regions will interfere with the target detection. To
overcome this difficulty, we extract the target characteristic of NIG field by the defined LA
map in this section.

The definition of divergence causes large divergence values of the clutter regions. The
outflow of physical quantities generally measures the divergence at a particular point. The
divergence value is positive as long as there is an outflow at a given point, regardless of
the distribution of the physical quantity at that point. For example, the gradient vectors of
the small target region are diffused from the center to the surrounding areas. In contrast,
the gradient vectors of the cloud edge regions are consistent in direction and outflow uniformly. The divergence values are positive regardless of the type of outflow of the vectors, which is inconsistent with the original intention of small target detection. To further suppress interference in some clutter regions, it is necessary to detect the distribution of local gradient vectors based on divergence. Thus, a local angle measure of the NIG field is proposed. A defined LA map is obtained by designing a local vectors angle measure (LVAM) operator to detect the distribution form of gradient vectors in the neighbourhood of a target. The radiation diffusion effect of the local gradient vectors from the target center to the surrounding areas is reflected in the map.

Definition The LA map of the infrared small target image is formed by measuring the whole NIG field with the LVAM operator.

The newly designed LVAM operator in this paper is shown in Fig. 4.

The small target studied in this paper takes up 3×3 pixels regions, which is no more than 9 pixels. The LVAM operator is an 8-neighbor measure template. Each square in Fig. 4 represents a pixel, and the 9 squares of the operator are numbered. The center square is numbered 0. The other squares are numbered counterclockwise from the upper right corner pixel, 1, 2, 3…, and 8, respectively. When measuring the angle of local vectors, the numbered 1, 2, 3…, and 8 square correspond to a pixel of 8-neighbors in the NIG field. A coordinate system is established in the No. 0 square of the operator, and eight angle intervals are defined in the other 8 squares. The angle interval defined in No. 1 to No. 8 square is shown in Eq. (5).

\[
\Psi = \{\Psi_k | \Psi_k = [(k - 1) \cdot \frac{\pi}{4}, \frac{\pi}{2} + (k - 1) \cdot \frac{\pi}{4}], k = 1, 2, \ldots, 8\}
\]

The pixels corresponding to the squares of the measured operator in NIG field are numbered when the operator measures the angle of local vectors in NIG field. The order of labels is shown in Fig. 5.

The pixel in the center is numbered 0. The remaining pixels are numbered counterclockwise from the upper right corner pixel, 1, 2, 3…, and 8, respectively. The same coordinate
system as the No. 0 square of the measured operator is established in the No. 0 pixel of the NIG field. The angle of the gradient vector in pixels 1 to 8 is represented as the Eq. (6).

\[ G = \{g_k, k = 1, 2, \ldots, 8\} \]

Each LVAM operator square computes the corresponding vector’s angle in the NIG field pixel. If the angle of the vector falls within the angle range of the measured operator, the value of the measured operator at this pixel is set to 1. If not, the value is set to 0. After analyzing all vector angles from No. 1 to No. 8, we obtained values at each pixel. Finally, the total 8-neighbour angle score is calculated by adding the eight values of the 8-neighbour pixel.

When the score is high, the local vectors distribution of the NIG field is closer to that of the point spread function model, and the probability of appearance of small targets is increased. On the contrary, small targets are unlikely to appear at lower scores. A threshold is set to evaluate the score to increase the robustness of the detection algorithm. If the score exceeds the threshold, the pixel’s value in the LA map is set to 1. If not, it is set to 0.

The whole LA map of the infrared image is obtained by measuring the entire NIG field with the LVAM operator. Algorithm 1 presents the measurement process of the measured operator. And the algorithm flow of the LA map of the infrared image is also obtained in Algorithm 1. Figure 6 shows the infrared image and its LA map.
Fig. 6 LA map of an infrared image. **a** Is an infrared image. The red dot represents the points in the LA map. **b** Is the corresponding LA map
The highlights in the LA map in Fig. 6b are the points satisfying the local vector’s radiation diffusion to the surrounding area. In conjunction with Fig. 6a, it is clear that the majority of the highlights are distributed in the inner region of the cloud.

In Sect. 2.3, we demonstrated that the divergence value of the cloud edge regions is relatively high, affecting the detection of small targets. Instead, the divergence value of cloud interior regions is small, so target detection is unaffected.

In this section, the score of pixels in cloud interior regions is high in the LA map. However, the score of pixels of the cloud edge regions is very low in the LA map. As a result, the LA map in this section and the D map in Sect. 2.3 are two different maps. Moreover, the angled score and the divergence value of the small target are high in both maps. Theoretically, the intersection of the two maps can prevent interference from both the cloud edge and the cloud interior at the same time.

### 2.5 Target detection based on D-LA map

In Sects. 2.3 and 2.4, we obtained the D map and LA map by processing the NIG field of the infrared image. The D map and the LA map are two different maps. The D map measures the divergent degree of local vectors around a certain pixel in the NIG field, while the LA map measures the divergent form of local vectors around a certain pixel in the NIG field. Both of them limit the divergence rule under the point spread function model, that is, the divergence rule of the small target. As a result, target detection and interference removal are achieved simultaneously. In this section, the overall detection process is presented.
2.5.1 D-LA map

As previously stated, the intersection of D and LA maps can accurately determine small targets. If the D map is represented as $D$ and the LA map is represented as $LA$, the intersection of the two maps is assumed to be a divergence-local angle (D-LA) map, which is represented as $DLA$, then

$$DLA = D \ast LA,$$  \hspace{1cm} (7)

where $\ast$ represents the Hadamard product.

According to Eq. (7), D-LA map, which is shown in Fig. 7, can be obtained by the Hadamard product of Figs. 3c and 6b. In the D-LA map, the clutter is effectively suppressed and the target signal is obviously enhanced.

2.5.2 CFAR judging

The D-LA map can reduce clutter while highlighting targets. However, in this paper, several interference pixels remain in the map, referred to as D-LA noise. The non-zero elements in the D-LA map are combined to form a set named $F$. The set $F$ comprises two components: the noise and the target. The constant false alarm ratio (CFAR) method is used for target detection in the D-LA map. It is assumed that the noise and the target in the D-LA map satisfy the binary probability hypothesis.

For elements in $F$, the expression of the binary probability hypothesis is shown in Eq. (8)

$$g \sim \begin{cases} 
N(\mu, \sigma^2) | H_0 \\
N(I + \mu, \sigma^2) | H_1 
\end{cases},$$  \hspace{1cm} (8)

where $g$ denotes the measurement. $H_0$ and $H_1$ are two hypotheses. $H_0$ is the hypothesis of the noise. Considering that the D-LA map makes the gray level of noise relatively uniform, we assume that the noise follows the normal distribution. $H_1$ is the hypothesis of the target, and we assume that the target has the same variance with the noise in the local area of the infrared image. $\mu$ is the mean of noise, $\sigma^2$ is noise variance, $I$ is the target and $g$ is the measurement.
If a desired permissive false alarm probability \( P_{FA} = \alpha \) is given, the decision of \( H_1 \) that maximizes detection probability \( P_D \) is expressed as below according to Neyman-Pearson theorem (Kay 1998)

\[
T(g) = \frac{p(g|I, H_1)}{p(g|H_0)} > \gamma \tag{9}
\]

where the threshold \( \gamma \) is found from \( P_{FA} = \int_{\{ g : T(g) > \gamma \}} p(g;H_0) dg = \alpha \).

If we substitute Eq. (8) into Eq. (9) and further simplify the formula, the next equation below can be acquired.

\[
T(g) = g > \frac{\sigma^2}{I} \cdot \ln \gamma + \frac{I}{2} + \mu = \gamma' \tag{10}
\]

Analysis of Eq. (10) shows that the false alarm probability \( P_{FA} \) is independent of the signal amplitude \( I \). Consequently, the detection threshold can be obtained when the signal is unknown.

On the basis of the above derivation, the desired permissive \( P_{FA} \) is represented by

\[
P_{FA} = \Pr \{ g > \gamma'|H_0 \} = Q\left( \frac{\gamma' - \mu}{\sigma} \right) \tag{11}
\]

where \( \gamma' \) is the detection threshold, and

\[
\gamma' = \sigma \cdot Q^{-1}(P_{FA}) + \mu \tag{12}
\]

Meanwhile, theoretical detection probability is acquired

\[
P_D = \Pr \{ g > \gamma'|H_1 \} = Q\left( \frac{Q^{-1}(P_{FA}) - \frac{I}{\sigma}}{\frac{I}{\sigma}} \right) \tag{13}
\]

where \( Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left( -\frac{t^2}{2} \right) dt \).

### 2.5.3 Algorithm flow

Figure 8 depicts a general description of the proposed method. Firstly, the NIG field is calculated by the NIG of the infrared image. Secondly, a D map can be obtained by calculating the divergence of every point in the NIG field. On the other hand, an LA map can be obtained by measuring the local angle value of the NIG field for each patch using

![Image](image-url)
Algorithm 1. Thirdly, the D-LA map is yielded as the Hadamard product of the D map and LA map. Finally, the detection result is acquired by CFAR based on the D-LA map.

The detection procedure is listed in Algorithm 2, where \( D_{ij}, LA_{ij}, \) and \( DLA_{ij} \) are the pixel intensity in D map, LA map, and D-LA map respectively; the width and height of the maps are represented by row and column; \( \gamma' \) is the adaptive threshold.

**Algorithm 2.** The target detection procedure based on D-LA map.

**Input:** Infrared image frame.

**Output:** The position of the target.

1: Achieve NIG field of the infrared image.
2: Compute the D map of the NIG field.
3: Compute the LA map of the NIG field according to Algorithm 1.
4: Compute the D-LA map.
5: for \( i = 1: \) row do
6: for \( j = 1: \) column do
   \[
   DLA_{ij} = D_{ij} \times LA_{ij}
   \]
7: end for
8: end for
9: Get the threshold by CFAR
   \[
   \gamma' = \sigma \cdot Q^{-1}(P_{fa}) + \mu
   \]
10: Achieve the detection result by the threshold \( \gamma' \).

It should be pointed out that solely the bright target is discussed here. To discover the dark target, only the positive intensity gradient (PIG) field of the infrared image is required. Then, the vector distribution feature of PIG field of the small dark target is consistent with the vector distribution feature of NIG field of the small bright target. The next steps are similar to the previous descriptions of this paper.

### 3 Experimental results

The assessment metrics, test data and baseline techniques used for comparison are presented in this section. After that, the main parameters of our approach are discussed. The validity and feasibility of our method are then demonstrated using a variety of infrared images with unique, complicated backgrounds.

#### 3.1 Assessment metrics, test data and baseline techniques

During detection of small targets in complicated backgrounds, it is essential to efficaciously suppress the background, and improve the target signal strength. Under the premise of the same detection probability, the key to reduce false alarm probability is less background
clutter and noise residue (Deng et al. 2017). Thus, the target can be detected more easily by eliminating more background clutters and noise.

For objective evaluation, a few metrics are brought which are regularly used to assess the overall performance of detection algorithms (Gao et al. 2018). The signal to clutter ratio gain ($G_{SCR}$), background suppression factor ($BSF$), detection probability $P_d$ and false alarm rate $F_a$ are adopted. $G_{SCR}$ and $BSF$ mirror the capability of target enhancement and background suppression. Therefore, the higher the magnitude of $G_{SCR}$ and $BSF$, the less difficult is it to discover a small target. The $G_{SCR}$ is defined as:

$$G_{SCR} = \frac{SCR_{out}}{SCR_{in}}$$  \hspace{1cm} (14)

where $SCR_{in}$ denote the signal to clutter ratio of the original IR image and $SCR_{out}$ denote the signal to clutter ratio of the resulting image. The $SCR$ is described by:

$$SCR = \frac{\mu_t - \mu_b}{\sigma_b}$$  \hspace{1cm} (15)

where $\mu_t$ is the average pixel intensity of the target region and $\mu_b$ is the average pixel intensity of the neighboring region around the target. $\sigma_b$ denotes the standard deviation of the neighboring region. The dimension of the neighboring area is $(a + 2d) \times (b + 2d)$. $a$ and $b$ point out the width and length of the target region, respectively. As shown in Fig. 9, $a = b = 3$ and $d = 15$ are set here.

Another assessment metrics is the background suppression factor ($BSF$), and its definition is,

$$BSF = \frac{C_{in}}{C_{out}}$$  \hspace{1cm} (16)

where $C_{in}$ represents the standard deviation of the original infrared image and $C_{out}$ represents the standard deviation of the infrared image processed by the detection algorithm.

In addition to the measurement of target enhancement and background suppression, the assessment metrics, namely the probability of detection $P_d$ and false alarm rate $F_a$, are also introduced. They are typically utilized to consider the overall detection performance of diverse methods. They are described as

![Fig. 9](image)

Fig. 9 The comparison between target region and background region
To verify the effectiveness and robustness of our method, we compare it with some well-known methods, including baseline methods such as Tophat (Zeng et al. 2006), Max-mean, and Max-median (Deshpande et al. 1999), and three state-of-the-art methods such as the IPI model (Gao et al. 2013), the SMSL method (Wang et al. 2017), the FAMSIS method (Chen et al. 2021) and the LIG method (Zhang et al. 2018).

The filter sizes used in Max-mean and Max-median methods are both $15 \times 15$ pixels. The parameters of IPI method, SMSL method, FAMSIS method and LIG method are the same as those in the original papers.

Six consecutive actual IR image sequences serve as the experimental images in this paper. The image’s size is $256 \times 318$ pixels. More statistics of the detector and experimental images are listed in Table 2. Every sequence consists of 300 frames. The six sequences are numbered in sequence from 1 to sequence 6. It need to be cited that different sequences have a different background. In addition, all sequences consist of a dim target and complicated cloudy clutter. The original images from the six sequences are shown in Fig. 10.

### 3.2 Selection of parameters

In this section, the effect of key parameters on our method is analyzed. Thereafter, the performance of the method under different parameters is compared.

There are two important parameters in the proposed method. One is the threshold of the score ($\text{TH}_s$) in LA map calculation step. The threshold can be used to separate the target from a complicated background. Meanwhile, it reflects the performance of background suppression algorithm. The other parameter is the constant false alarm rate ($\text{CF}_a$) set in the CFAR step. This parameter determines the real detection probability and false alarm probability.

To obtain optimal parameters, we change different values of the two parameters to derive the actual detection probability and false alarm probability. Here, sequence 1, sequence 2 and sequence 3 are selected as the test sequences.

Specifically, the threshold of the score ($\text{TH}_s$) and the constant false alarm rate ($\text{CF}_a$) are adjusted at the same time. The test image sequences are then processed using the proposed method. Then, the detection probability $P_d$ and false alarm probability $F_a$ are acquired and shown in Fig. 10 respectively. The threshold of the score ($\text{TH}_s$) varies from 1 to 8, and the false alarm rate ($\text{CF}_a$) varies from $10^{-1}$ to $10^{-10}$.

In practical application, the detection algorithm aims to achieve high detection probability and low false alarm probability simultaneously. For the various combinations of $\text{TH}_s$ and $\text{CF}_a$, which one will achieve the goal, and then the combination can be selected as the optimal parameters in this paper.

### Table 2 The parameters of infrared detector used in this paper

| Spectral range | Field of view | Focal length | Pixel size | Pixels | Frame frequency |
|----------------|---------------|--------------|------------|--------|-----------------|
| 3.7–4.8 μm     | $11^\circ \times 8^\circ$ | 46 mm        | $30 \times 30 \ \mu m$ | $256 \times 318$ | 50 Hz           |
Generally, for THs, when THs = 6, the method is considered to have achieved a higher detection probability and a lower false alarm probability than any other values of CFa. For CFa, when CFa = 10^{-8}, the method is considered to have achieved a higher detection probability and lower false alarm probability than any other values when THs = 6.

The above experimental results are regular with the theoretical analysis. For THs, the larger the value, the higher the missing alarm. On the contrary, the smaller the value, the higher the false alarm. The above conclusion can also be applied to CFa. Consequently, combining experimental results with theoretical analysis, we set THs = 6 and CFa = 10^{-8} in this paper.

3.3 Comparisons to baseline techniques

We compared our method’s background suppression, target enhancement, and target detection capabilities to baseline techniques. The difficulty of detecting targets is increased because the six image sequences have varying background complexities, target features, and noise levels. The R2014a MATLAB software is used to run these algorithms and data. The operating system is AMD A6-3420 M APU with 4 G RAM.

3.3.1 Target enhancement and background suppression performance

Figure 11 shows the processing results of the six image sequences using different methods. The results show that the proposed method produces a clearer background than the other six baseline techniques. Specifically, TopHat, MaxMedian, MaxMean and SMSL have relatively dense noise residuals. Although the IPI model, FAMSIS and LIG method
produce slightly better results, some sparse noise residuals exist. Even worse, some targets are missed entirely. In comparison, the proposed method demonstrates better results.

Tables 3 and 4 show the average GSCR and \( BSF \) average for the six methods on the six test image sequences to objectively assess the proposed method’s target enhancement and background suppression performance. We can find that the proposed method achieves the highest average \( G_{SCR} \) and average \( BSF \) in six sequences, confirming its superiority. In contrast, the IPI model, LIG method, FAMSIS method and SMSL have shown suboptimal performance in target enhancement and background suppression in different scenarios. The performance of the other four methods is relatively unobtrusive.

The high-quality small target detection method does more than just provide good background suppression. In the meantime, excellent target enhancement will be achieved.

![Fig. 11 Representative results of different methods for six real infrared image sequences. The image in the first column is a representative frame of six real infrared image sequences. The red rectangle represents the target and the yellow circle is a representative example of noise.](image)

| Methods   | Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 |
|-----------|------------|------------|------------|------------|------------|------------|
| Top-hat   | 5.961      | 1.949      | 11.534     | 0.954      | 1.779      | 1.567      |
| Max-mean  | 4.798      | 1.104      | 7.730      | 1.302      | 9.782      | 1.762      |
| Max-median| 8.894      | 2.239      | 14.977     | 2.123      | 11.815     | 3.105      |
| SMSL      | 7.547      | 1.267      | 10.129     | 2.553      | 18.886     | 3.247      |
| IPI       | 21.085     | 3.119      | 24.973     | 2.533      | 24.822     | 3.369      |
| LIG       | 11.260     | 3.371      | 27.771     | 1.388      | 6.474      | 3.305      |
| FAMSIS    | 4.638      | 2.609      | 18.798     | 2.987      | 8.436      | 2.855      |
| Our method| **21.852** | **4.084**  | **29.040** | **3.327**  | **29.279** | **3.946**  |
Consequently, our approach can detect small infrared targets that are submerged in cloudy and noisy background clutter.

### 3.3.2 Detection performance

Usually, if the pixel distance between centers of the detection result and the ground-truth is below a certain threshold [for example, 5 pixels (He et al. 2016), 4 pixels (Chen et al. 2013), etc.], then the detection is considered correct. Choosing a smaller threshold means that the distance error and workload in the practical application is less. Considering that the targets in all image sequences is in motion in each frame, we chose a threshold of 2 pixels in this paper.

Figure 12 shows the target movement trajectories of the ground-truth and the detected trajectories obtained by the proposed method on the six sequences. At the same time, the horizontal and vertical error distribution histograms corresponding to the six sequences are shown.

Here, we can make some conclusions from Fig. 12. For Sequences 1, as shown in Fig. 12 Seq.1(A), the ground-truth target trajectory approximates linearity, the detected trajectory of the proposed method does nearly match that of the target movement. Moreover, we can see from Seq.1(B) and Seq.1(C) that the most horizontal and vertical errors are zeros (only a few horizontal errors and vertical errors are less than 2 pixels). For Sequence 2 to Sequence 6, as shown in Fig. 12 Seq.2(A) to Seq.6(C), although the ground-truth trajectory is curvilinear, the trajectory, which is detected by the proposed method, does nearly match that of the target movement as well. Moreover, almost all horizontal and vertical errors are zeros (a few horizontal errors and vertical errors are less than 2 pixels). We can conclude from Fig. 12 that our approach achieves high detection accuracy and it is independent of the target’s trajectory.

The receiver operating characteristic (ROC) curve is a graph that shows the probability of detection and false alarm. The ROC curves are used to compare the detection performance of different methods. The curves are obtained by processing the six image sequences (Fig. 13) with our method’s six baseline methods. The comparative results in Fig. 13 show that our method has the highest detection probability and the lowest false alarm rate. Besides, the state-of-the-art methods, IPI model, the LIG, the FAMSIS and the SMSL outperform the four baseline methods. Therefore, our method has the best detection

| Methods      | Sequence 1 | Sequence 2 | Sequence 3 | Sequence 4 | Sequence 5 | Sequence 6 |
|--------------|------------|------------|------------|------------|------------|------------|
| Top-hat      | 2.79       | 3.05       | 2.73       | 2.98       | 2.72       | 3.32       |
| Max-mean     | 5.97       | 6.91       | 8.20       | 5.89       | 5.20       | 3.66       |
| Max-median   | 10.17      | 11.13      | 14.48      | 9.18       | 7.69       | 4.69       |
| SMSL         | 14.89      | 15.04      | 16.58      | 17.52      | 18.51      | 11.58      |
| IPI          | 16.74      | 20.58      | 31.65      | 23.52      | 35.22      | 23.08      |
| LIG          | 4097.80    | 4168.52    | 4231.24    | 3879.91    | 2409.67    | 2923.83    |
| FAMSIS       | 4.698      | 5.573      | 5.917      | 5.505      | 5.437      | 5.319      |
| Our method   | **5940.64**| **5501.39**| **6662.35**| **6388.56**| **4945.76**| **4537.660**|
Fig. 12 Seq.1(A) to Seq.6(A) are the ground-truth and detected trajectories of Sequence 1 to Sequence 6 obtained by our method. Seq.1(B) to Seq.6(B) and Seq.1(C) to Seq.6(C) are the histograms of detected errors of Sequence 1 to Sequence 6 obtained by our method: Seq.1(B) to Seq.6(B) are histograms of horizontal detected errors of Sequence 1 to Sequence 6. Seq.1(C) to Seq.6(C) are histograms of vertical detected errors of Sequence 1 to Sequence 6.
performance among the eight methods. The results shown in Fig. 13 indicate that our method effectively detects small targets in different complicated backgrounds.

The area under the ROC curve (AUC) is extensively used to quantitatively evaluate the overall performance of a method (Han et al. 2016, 2014). Its values range from 0 to 1. The larger the AUC value, the better the detection performance of the method. Table 5 shows the AUC values of the comparative methods for the above six image sequences. In addition, the average AUC values are listed in the last line of Table 5. We can see from the table that the AUC values of our method are larger than that of the comparative methods, which indicates that our method has superior detection performance to the comparative methods.

### 4 Conclusion

This paper presents an effective method for detecting small infrared targets embedded in cloudy background clutters based on D-LA map. The main idea of the proposed method is to utilize divergence operation and LVAM operator to construct D-LA map of an input image. D-LA map effectively enhances the targets while the background clutters and simultaneously suppresses noise. In this way, D-LA map can significantly improve SCR values compared to the original image while retaining minimal clutters and noise residuals. Therefore,

| Sequence | TopHat | MaxMedian | MaxMean | SMSL | IPI model | LIG | FAMSIS | Our method |
|----------|-------|-----------|---------|------|-----------|-----|--------|------------|
| Sequence 1 | 0.789 | 0.790 | 0.897 | 0.927 | 0.954 | 0.899 | 0.931 | 0.999 |
| Sequence 2 | 0.708 | 0.815 | 0.741 | 0.647 | 0.919 | 0.949 | 0.954 | 0.999 |
| Sequence 3 | 0.662 | 0.729 | 0.735 | 0.814 | 0.960 | 0.919 | 0.963 | 0.999 |
| Sequence 4 | 0.667 | 0.718 | 0.759 | 0.943 | 0.916 | 0.812 | 0.953 | 0.984 |
| Sequence 5 | 0.415 | 0.529 | 0.716 | 0.750 | 0.835 | 0.475 | 0.898 | 0.959 |
| Sequence 6 | 0.615 | 0.525 | 0.666 | 0.754 | 0.884 | 0.807 | 0.879 | 0.979 |
| Average | 0.643 | 0.684 | 0.752 | 0.806 | 0.911 | 0.810 | 0.929 | 0.987 |

Fig. 13 ROC curves: a–f ROC curves of Sequence 1–6
the map ensures that the proposed method has a low false alarm probability and high detection probability. Verification experiments are implemented on six different image sequences and over 1800 small target images against diverse cloudy backgrounds, demonstrating that the proposed method outperforms traditional baseline methods like TopHat, MaxMedian, MaxMean, SMSL, IPI and LIG. Simultaneously, the experiments show that the proposed method is more robust for specific target movements and complex and noisy backgrounds.

Although the empirical evidence supports the robustness and advantages of our method, further research incorporating different perspectives is required to improve it. For instance, we intend to test the applicability of our algorithm for the scale-changing targets and extend the algorithm to diverse, complicated backgrounds, such as sea-sky background, ground background, and sky-ground background.

Acknowledgements The authors would like to acknowledge the funding received from the National Natural Science Foundation of China (Grant Nos. 61903340 and 61773351), Postdoctoral Science Foundation of Henan Province (China) (Grant No. 001701002), Key Scientific Research Project of Colleges and Universities in Henan Province (Grant No. 19A413002) and Project of Young Talent Promotion of Henan Association for Science and Technology (Grant No. 2020HYTP028) to conduct this research investigation.

Funding National Natural Science Foundation of China (NSFC) (61903340); Key Specialized Research and Development Breakthrough in Henan Province (222102210158).

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