Testing the Chiral Magnetic Effect with Central $U + U$ collisions

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A quark interaction with topologically nontrivial gluonic fields, instantons and sphalerons, violates $\mathcal{P}$ and $\mathcal{CP}$ symmetry. In the strong magnetic field of a noncentral nuclear collision such interactions lead to the charge separation along the magnetic field, the so-called chiral magnetic effect (CME). Recent results from the STAR collaboration on charge dependent correlations are consistent with theoretical expectations for CME but may have contributions from other effects, which prevents definitive interpretation of the data. Here I propose to use central body-body $U + U$ collisions to disentangle correlations due to CME from possible background correlations due to elliptic flow. Further more quantitative studies can be performed with collision of isobaric beams.

PACS numbers: 25.75.Ld

I. INTRODUCTION

Quantum chromodynamics (QCD), the theory of strong interactions, is a non-Abelian gauge theory that possesses multiple vacua characterized by Chern-Simons numbers. QCD links chiral symmetry breaking and the origin of hadron masses to the existence of topologically nontrivial classical gluonic fields, instantons and sphalerons, describing the transitions between the vacuum states with different Chern-Simons numbers. Quark interactions with such fields change the quark chirality and are $\mathcal{P}$ and $\mathcal{CP}$ odd. For a review, see Refs. [1, 2]. Though theorists have little doubt in the existence of such fields, they have never been observed directly, e.g. at the level of quarks in the deep inelastic scattering. It was suggested in Ref. [3] to look for metastable $\mathcal{P}$ and $\mathcal{CP}$ odd domains, space-time regions occupied by a classical field with a nonzero topological charge, in ultrarelativistic heavy ion collision. Some earlier discussion of how one could observe this local strong parity violation can be found in Refs. [3–5].

The situation with experimental search for the local strong parity violation drastically changed once it was noticed [6, 7] that in noncentral nuclear collisions it would lead to the asymmetry in the emission of positively and negatively charged particle perpendicular to the reaction plane. Such charge separation is a consequence of the difference in the number of quarks with positive and negative helicities positioned in the strong magnetic field ($\sim 10^{15}$ T) of a noncentral nuclear collision, the so-called chiral magnetic effect (CME) [6, 8]. The same phenomenon can also be understood as an effect of the induced electric field that is parallel to the static external magnetic field, chiral magnetic induction, which occurs in the presence of topologically nontrivial gluonic fields [9]. It has been also argued that the charge separation could have origin in nonzero vorticity of the system created in noncentral collisions [7]. Chiral magnetic effect has been observed in the lattice QCD calculations [10, 12]. Newer developments in this field has been recently discussed at the RIKEN BNL workshop [13].

An experimental observation of CME would be a direct proof for the existence of topologically nontrivial vacuum structure and would provide an opportunity for a direct experimental study of the relevant physics. The difficulty in experimental observation of CME comes from the fact that the direction of the charge separation varies in accord with the sign of the topological charge of the domain. Then the observation of the effect is possible only by correlation techniques. According to Refs. [6, 8] the charge separation could lead to asymmetry in particle production $(N_- - N_+)/N_+ \sim (N_- + N_+) \sim Q/N_+ $, where $Q = 0, \pm 1, \pm 2, ...$ is the topological charge and $N_+$ is the positive pion multiplicity in one unit of rapidity – the typical scale of such correlations. It results in correlations of the order of $10^{-4}$, which is accessible in current high statistics heavy ion experiments. An observable directly sensitive to the charge separation effect has been proposed in Ref. [14]. It is discussed in more detail below.

Recent STAR results [15, 16] on charge separation relative to the reaction plane consistent with the expectation for CME can be considered as evidence of the local strong parity violation. The ambiguity in the interpretation of experimental results comes from possible contribution of (the reaction plane dependent) correlations not related to CME. As the detailed quantitative predictions for CME do not yet exist, it is difficult to disentangle different contributions. A key ingredient to CME is the strong magnetic field, while the background effects originate in the elliptic flow. In noncentral collisions of spherical nuclei such as gold, both, magnetic field, and the elliptic flow are strongly (cor)related to each other. In order to disentangle the two effects one has to find a possibility to significantly change the relative strengths of the magnetic field and the elliptic flow. The discussion of such a possibility provided by $U + U$ collisions is the subject of this Letter. Necessary quantitative estimates are obtained using Glauber Monte-Carlo simulations. I estimate the magnetic field following the approach of Ref. [17]. Estimates of elliptic flow are based on the assumption that it scales with initial (participant) eccentricity of the nuclei overlap region.

In noncentral nuclear collisions particle distribution in azimuthal angle is not uniform. The deviation from a flat distribution is called anisotropic flow and often is
described by the Fourier decomposition [18] (for a review, see Ref. [20]):

\[
\frac{dN_n}{d\phi} \propto 1 + 2v_{1,\alpha}\cos(\Delta\phi) + 2v_{2,\alpha}\cos(2\Delta\phi) + \ldots
\]

\[
+ \ 2a_{1,\alpha}\sin(\Delta\phi) + 2a_{2,\alpha}\sin(2\Delta\phi) + \ldots,
\]

where \(\Delta\phi = (\phi - \Psi_{RP})\) is the particle azimuth relative to the reaction plane, \(v_1\) and \(v_2\) account for directed and elliptic flow. Subscript \(\alpha\) denotes the particle type. Because of the “up-down” symmetry of the collisions \(a_n\) coefficients are usually omitted. CME violates such a symmetry. Although the “direction” of the violation fluctuates event to event and on average is zero, in events with a particular sign of the topological charge, the average is not zero. As a result, it leads to a nonzero contribution to correlations. One expects that the first harmonic would account for the most of the effect. To measure \(\langle a_{1,\alpha}a_{1,\beta}\rangle\), it was proposed [14] to use the correlator:

\[
\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle =
\]

\[
\langle \cos(\Delta\phi_\alpha \cos(\Delta\phi_\beta) - \sin(\Delta\phi_\alpha \sin(\Delta\phi_\beta) - \sin(\Delta\phi_\alpha \sin(\Delta\phi_\beta)) \right)
\]

\[
\approx \langle a_{1,\alpha}a_{1,\beta} \rangle + [B_{in} - B_{out}].
\]

This correlator represents the difference between correlations “projected” onto the reaction plane and the correlations projected onto an axis perpendicular to the reaction plane (a more detailed discussion of that can be found in Refs. [14, 16]). The key advantage of using such a difference is that it removes the correlations among particles \(\alpha\) and \(\beta\) that are not related to the reaction plane orientation. The remaining background in Eq. 3, \(B_{in} - B_{out}\), is due to processes in which particles \(\alpha\) and \(\beta\) are products of a cluster (e.g. resonance, jet, di-jets) decay, and the cluster itself exhibits elliptic flow or decays (fragments) differently when emitted in-plane compared to out-of-plane. The corresponding contribution to the correlator can be estimated as [14, 16]:

\[
\frac{N_{\text{clust}} N_{\text{pairs, in}}}{N_{\text{pairs, out}}} \langle \cos(\phi_\alpha + \phi_\beta - 2\phi_{\text{clust}}) \rangle_{\text{clust}} v_2,\text{clust},
\]

where \(\langle \ldots \rangle_{\text{clust}}\) indicates that the average is performed only over pairs consisting of two daughters from the same cluster. This kind of background can not be easily eliminated or suppressed and constitute the main uncertainty in the interpretation of the STAR results. To address its contribution one has to rely on model calculations [16]. A better approach would be to perform experiments where the relative contribution of CME and background can be varied. Note that the background is proportional to the elliptic flow present in the event. \(v_{2,\text{clust}}\) in Eq. 4.

It is not clear how one could suppress elliptic flow and at the same time preserve strong magnetic field needed for CME. But the opposite, collisions with strong elliptic flow and no (or almost no) magnetic field, seems to be possible. This can be achieved in central body-body \(U + U\) collisions. Uranium nuclei are not spherical and have roughly ellipsoidal shape. Central collision, when most of the nucleons interact, can have different geometry, ranging from the so-called tip-tip collisions to body-body collisions [21], see Fig. 1. Unlike tip-tip collisions, body-body ones would exhibit strong elliptic flow. Neither would lead to a strong magnetic field; consequently, a very weak signal due to CME is expected, while background would be much stronger in body-body configuration compared to tip-tip configuration.

Collisions of uranium nuclei were first proposed for RHIC by P. Braun-Munzinger [22] with the goal to achieve higher energy density compared to Au+Au collisions. The idea was later elaborated in Refs. [23, 24], in particular pointing out an important possibility to study elliptic flow at such high energy densities.

At RHIC one can select central collisions by requiring low signal in the zero degree calorimeters that detect spectator neutrons. Below I discuss how one might “control” the geometry of the collision, and, consequently, the relative strengths of the signal due to CME and background, by selecting events based on multiplicity, signal in the zero degree calorimeters, and the magnitude of the flow vectors.

The magnetic field strength at a position \(\mathbf{r}\) and time \(t\) is defined by the Lienard-Wiechert potentials

\[
eB(t, \mathbf{r}) = e_{\text{EM}} \sum_n e_n \left(1 - \frac{v_n^2}{(R_n - R_n v_n)^3}R_n \times \mathbf{v}_n \right) \times \mathbf{r},
\]

where \(e_{\text{EM}} \approx 1/137\) is the fine-structure constant, and \(e_n\) is the electric charge of the \(n\)th particle in units of the electron charge. \(R_n = \mathbf{r} - \mathbf{r}_n\), where \(\mathbf{r}_n\) is the radius vector of particle, \(\mathbf{v}_n\) is particle velocity. The quantities \(v_n\) and \(r_n\) are taken at retarded time \(t' = t - |\mathbf{r} - \mathbf{r}_n(t')|\). Summation runs over all charged spectators. Spectator
contribution to the magnetic field is dominant at early times \[8, 17\]; we also use this approximation in our estimates. Because of the Lorentz contraction, in collisions of ultrarelativistic nuclei, the longitudinal size of the nucleus is negligible compared to the transverse size, and the time dependence of the magnetic field is totally determined by the gamma-factor (energy) of colliding nuclei. We are interested only in a relative change in the strength of the magnetic field in collisions at different centralities and different configurations. For this, it is sufficient to calculate the magnetic field only at \(t = 0\) (the time the two nuclei collide). At \(\sqrt{s_{NN}} = 200\) GeV, the collision energy used in our estimates, the magnetic field at \(t = 0\) is about factor of 2 smaller compared to the maximum value reached approximately at \(t \approx 0.05\) fm/c \[17\].

Elliptic flow is determined by the geometry of the overlapping zone. We assume \(v_2 = \kappa \varepsilon_p\), where \(\varepsilon_p\) is the so-called participant eccentricity. We consider only events within \(< 5\%\) of the most central collisions, for which \(\kappa \approx \text{const.}\) For definitions of eccentricity and details of experimental measurements of \(v_2/\varepsilon\), see review \[20\]. We take \(\kappa = 0.2\) based on experimental measurements. Initial eccentricity, magnetic field, and charged particle multiplicity at midrapidity are calculated using Glauber Monte-Carlo with all parameters taken the same as used in Ref. \[25\].

The effect of nonsphericity of uranium nuclei is clearly visible in Fig. 2 which shows the distribution of events in elliptic flow \(v_2\) in event samples with number of spectators, \(N_{sp} < 20\). The average elliptic flow is almost a factor of 2 larger in \(U + U\) collisions compared to \(Au + Au\), which would mean a strong increase in the background correlations compared to that due to CME. The requirement of the same number of spectators assures that the magnetic field is not very different in the two systems; it is slightly lower in \(U + U\) collisions [see Fig. 3b]. The condition \(N_{sp} < 20\) selects about 1.5% of the most central events in \(U + U\) collisions and about 2.3% in \(Au + Au\) collisions.

Within the event sample selected on the basis of number of spectators, it would be instructive to study the dependence of the signal on the magnitude of the elliptic flow. As a measure of the latter we use the magnitude of the flow vector \(q = Q_2/\sqrt{M}\), where

\[
Q_{2,x} = \sum_{i=1}^{M} \cos(2\phi_i), \quad Q_{2,y} = \sum_{i=1}^{M} \sin(2\phi_i),
\]

and the sum runs over all particles in a given momentum window. We calculate the flow vector based on charged particle multiplicity in 2 units of rapidity. As shown in Fig. 3, the elliptic flow is strongly correlated with \(q\), and at the same time the magnetic field has almost no \(q\)-dependence. It means that the correlator, Eq. 3 used to measure the signal would stay constant if the signal is mostly determined by CME, and increase strongly with \(q\) if it is due to the background effects. For such a test \(U + U\) collisions provide a significantly better opportu-
dependence is different for two systems, with U magnetic field on charged multiplicity. The elliptic flow Figure 4 presents the dependence of the elliptic flow and on the dependence of the signal on charged multiplicity. 

In collisions exhibiting a characteristic kink (cusp) at multiplicity $\sim 1000$ [25], reflecting the fact that high(er) multiplicity events have predominantly tip-tip orientation; the latter also leads to a decrease in elliptic flow. Being mostly determined by correlation of the multiplicity with the number if participants, the magnetic field has similar dependence on multiplicity for both collision systems. The difference in the dependencies of the magnetic field and elliptic flow on charged multiplicity can be used as a test for the nature of correlations contributing to the signal.

The charge separation dependence on the strength of the magnetic field can be further studied with collision of isobaric nuclei, such as $^{44}\text{Ru}$ and $^{40}\text{Zr}$. These nuclei have the same mass number, but differ by the charge. The multiparticle production in the midrapidity region would be affected very little in collision of such nuclei, and one would expect very similar elliptic flow. At the same time the magnetic field would be proportional to the nuclei charge and can vary by more than 10%, which can results in 20% variation in the signal. Such variations should be readily measurable. The collisions of $^{44}\text{Ru}$ and $^{40}\text{Zr}$ isotopes have been successfully used at GSI [26] in a study of baryon stopping. Collisions of isobaric nuclei at RHIC will be also extremely valuable for understanding the initial conditions, and in particular the initial velocity fields, the origin of directed flow, etc.

In summary, the estimates presented in this Letter show that a detailed analysis of central $\text{Au+Au}$ and $U+U$ collisions should be able to disentangle CME and background correlations contributing to the signal observed by STAR.

Discussions with J. Dunlop and P. Filip are gratefully acknowledged. This work was supported in part by the US Department of Energy, Grant No. DE-FG02-92ER40713.

![FIG. 4: (Color online) Elliptic flow and the magnetic field (arbitrary units) in $\text{Au+Au}$ and $U+U$ collisions as function of multiplicity. The arrows indicate the multiplicities corresponding to the top 2% of the collision cross section.](image-url)
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