Vector Kinematics Modelling and Simulation Analysis of Half-rotating Walking Mechanism

Yongming Wang1,2,*, Tengfei Ma1,2 and Xuetan Xu1,2

1School of Mechanical Engineering, Anhui University of Technology, Ma’anshan 243032, P.R. China
2Anhui Province Key Laboratory of Special Heavy Load Robot

*Corresponding author email: wangym@ahut.edu.cn

Abstract. In order to solve the stability problem existing in the robot mobile system based on the two-stage half-rotating mechanism, a swing balanced configuration is proposed. Combining the vector method and the D-H method, a forward kinematics modelling and analysis method for the half-rotating walking mechanism is proposed. First of all, using vector kinematics, according to the characteristics of the walking mechanism, combined with the D-H method to establish the relevant coordinate system and the transformation matrix between each coordinate system, derived the position vector, angular velocity vector, linear velocity vector and other relevant kinematic characteristic expressions of the key points of the mechanism, and calculated in MATLAB software to get its theoretical analytical curve. At the same time, the kinematics model of the half-rotating walking mechanism is established in the ADAMS software, and the ADAMS simulation results are compared with the theoretical analysis results. The two are basically consistent, which verifies the correctness of the forward kinematics modelling method of this half-turn walking mechanism.

Keywords: Half rotating walking mechanism; Balancing configuration; Kinematics modelling; Vector method; ADAMS simulation.

1. Introduction
Reasonable leg mechanism is the basis of design of walking robot. Typical walking leg mechanisms have bionic joint or connecting rod type, such as BigDog robot[1], WildCat robot[2], Cockroach-like robot[3], Cheetah-cub-S robot[4]. Its legs are all bionic joints, which are used to simulate animal limb movements and have good performance of terrain adaptability and stability. In addition, Wafa Batayne of Jordan University of Science and Technology proposed a Stephenson III robot consisting of a single-degree-of-freedom four-bar mechanism and a two-bar group[5], Doyoung Chang of Korea National Seoul University proposed a 2-DOF crank-slider-leg mechanism with linear spring[6], and the TITAN-XIII robot in Japan adopted 3-DOF[7].

In view of the stability walking problem of the half-rotating walking mechanism, the paper proposes the swing balancing configuration, which can dynamically adjust the center of mass of the mechanism to achieve stable walking of the walking mechanism. Firstly, the half-rotating walking mechanism is modeled by the vector kinematics, and its kinematics characteristics are derived. Then, combined with ADAMS simulation to verify the correctness of the forward kinematics modelling method, it provides a theoretical basis for the stability control and model prototype design of the subsequent half-rotating walking mechanism.
2. Swing Balancing Configuration
The swing balancing configuration of the half-rotating walking mechanism is shown in Figure 1. In this configuration, the balancing device is the balance pendulum which is installed in the center of the center axis. When the half-rotating walking mechanism is moving, the half-rotating leg moves relative to the center axis (frame), and the balance pendulum is driven by the motor to swing around the center axis at a certain angle, so as to adjust the position of the center of mass of the whole walking mechanism, and then maintain the dynamic stability during the walking process.

![Figure 1. Swing balance configuration of half-rotating walking mechanism.](image1)

3. Forward Kinematic Modelling

3.1. Coordinate System Description
In order to facilitate the subsequent theoretical analysis, firstly, it is necessary to establish the coordinate system of the half-rotating walking mechanism. In the paper, the right-hand coordinate system is used, as shown in Figure 2. W is the global coordinate system, the origin is located at a fixed point on the ground, the x axis points to the front of the initial position of the walking mechanism, and the y axis vertically points upward. R is the center coordinate system of the mechanism, \( \bar{R} \) is the follow-up coordinate system of the center of the mechanism, its origin coincides with the origin of the coordinate system R, and the azimuth angle is the same as the orientation of the global coordinate system W. Before establishing the kinematics model, for the convenience of analysis, it is assumed that all the rods and contact ground of the half-rotating walking mechanism are rigid bodies, and the stride rod directly contacts the ground without bouncing, ignored the influence of foot.

According to the coordinate system in Figure 2, the D-H parameters of the kinematics of the half-rotating walking mechanism can be obtained, as shown in Table 1. Table 1 only lists the D-H parameters in the right side of the half-rotating walking mechanism.

| Coordinate system | Joint angle \( \theta / \text{rad} \) | Joint offset \( d / \text{mm} \) | Length of connecting rod \( a / \text{mm} \) | Angle of torque \( \alpha / \text{rad} \) |
|-------------------|------------------|-----------------|------------------|------------------|
| \( R \rightarrow A_1 \) | \( -\pi / 2 \) | \( d_{RA} \) | \( a_{RA} \) | 0 |
| \( A_1 \rightarrow B_1 \) | \( -\theta \) | \( d_{AB} \) | \( a_{AB} \) | 0 |
| \( B_1 \rightarrow C_{11} \) | \( \pi + \theta / 2 \) | \( d_{BC} \) | \( a_{BC} \) | 0 |
| \( C_{11} \rightarrow D_{11} \) | \( \theta / 4 \) | \( d_{CD} \) | \( a_{CD} \) | 0 |

Due to the half-rotating walking mechanism is bilaterally symmetric, for the transformation of the left coordinate system, there only need to change the value of the joint offset \( d \) along the z axis to negative.

![Figure 2. Coordinate system and position vector relationship of walking mechanism.](image2)
3.2. Position Vector Analysis

Based on the D-H parameters of the coordinate system, the position and pose relationship between the coordinate systems can be deduced by using the principle of homogeneous matrix transformation after defining the coordinate system of the mechanism. Taking the right half-rotating leg as an example, from the initial moment, the following coordinate system \( R \) coincides with the central coordinate system \( R \) of the mechanism. As shown in Figure 2, the order of landing at the end points of the stride rod is \( D_{11}, D_{12}, D_{13}, D_{14}, D_{11}, \) and so on. When the end point \( D_{11} \) of the stride rod contacts the ground, the position vector relationship between the origins of each coordinate system can be expressed as follows:

\[
OD_{11} = OR + RA_1 + A_1B_1 + B_1C_1 + C_1D_{11}
\]

(1)

In this paper, Euler angle, which rotates the moving pedestal around the local coordinate system in \( z-y-x \) order, is selected to describe the rotation of the moving pedestal. According to Euler’s kinematics, the homogeneous transformation matrix \( T_{RD_{11}} \) from the origin of the central coordinate system \( R \) to the end point of the stride rod (taking the end of \( D_{11} \) as an example) is:

\[
T_{RD_{11}} = 
\begin{bmatrix}
    s_{123} & c_{123} & 0 & a_{AB} s_1 + a_{BC} s_{12} + a_{CD} s_{123} \\
    -c_{123} & s_{123} & 0 & -a_{AB} c_1 - a_{BC} c_{12} - a_{CD} c_{123} \\
    0 & 0 & 1 & d_{RA} + d_{AB} + d_{BC} + d_{CD}
\end{bmatrix}
\]

(2)

Where, \( \theta_1 = -\dot{\theta} \), \( \theta_2 = \frac{\pi}{2} + \frac{\theta}{2} \), \( \theta_3 = -\frac{\pi}{4} + \frac{\theta}{4} \), \( s_1 = \sin \theta_1 \), \( s_{12} = \sin(\theta_1 + \theta_2) \), \( s_{123} = \sin(\theta_1 + \theta_2 + \theta_3) \), the meaning is similar.

According to formula (2), the coordinates of the point \( D \) at the end of the stride rod:

\[
\begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix}
= 
\begin{bmatrix}
a_{AB} s_1 + a_{BC} s_{12} + a_{CD} s_{123} \\
-a_{AB} c_1 - a_{BC} c_{12} - a_{CD} c_{123} \\
d_{RA} + d_{AB} + d_{BC} + d_{CD}
\end{bmatrix}
\]

(3)

The coordinate \( \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix}^T_R \) of any point on the half-rotating walking mechanism in the central coordinate system \( R \) of the mechanism is converted to the coordinate system \( \overline{D} \) of the contact point.

\[
\begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix}
= 
Q \begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix}^T_R
\]

(4)

Where, \( Q \) represents the attitude transformation matrix from the follow-up coordinate system \( \overline{R} \) to Mechanism Central coordinate system \( R \), and \( Q = \begin{bmatrix} c\gamma \beta & -s\gamma \alpha + c\gamma \beta s\alpha & s\gamma \alpha + c\gamma \beta k\alpha \\
c\gamma \beta & c\gamma \alpha + s\gamma \beta s\alpha & -c\gamma \alpha + c\gamma \beta k\alpha \\
-s\beta & c\beta \alpha & c\beta k\alpha
\end{bmatrix} \).

3.3. Angular Velocity Vector Analysis

The motion process of the half-rotating walking mechanism is that the end points of the stride rod contact the ground alternately in order to realize the movement. In a step cycle, set the foot to keep contact with the ground without slipping, but the stride rod rotates relative to the ground with the contact point as the fulcrum. Assuming that the angular velocity vector of the ground relative to the stride rod is \( \omega_D \), the absolute rotational angular velocity of the ground can be expressed as the following. Because the ground is fixed, that is \( \omega_G = 0 \),

\[
\omega = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \omega_D = 0
\]

(5)
Where, $\omega_0$ is the angular velocity vector of the center $R$ of the mechanism, $\omega_1$ represents the angular velocity vector of the first rotating arm relative to the center axis of the mechanism, $\omega_2$ represents the angular velocity vector of the secondary arm relative to the first rotating arm, $\omega_3$ represents the angular velocity vector of the stride rod relative to the secondary rotating arm.

### 3.4. Linear Velocity Vector Analysis

The kinematic expression from the mechanism central coordinate system $R$ to the end point $D$ of the stride rod in contact with the ground is as follows:

$$v_D = v_0 + \sum_{i=0}^{3} \omega_i \times r_{i(i+1)}$$  \hspace{1cm} (6)

Where, $v_0$ is the linear velocity vector of the center $R$ of the mechanism.

For the convenience of calculation, the vector projection method is used to project all the vectors into the central coordinate system $R$ of the mechanism for calculation, and then the results are transformed into the results in the follow-up coordinate system $\tilde{R}$ by using the attitude transformation matrix $Q$. In the central axis coordinate system $R$ of the mechanism and the follow-up coordinate system $\tilde{R}$, the linear velocity of the center $R$ of the mechanism must be zero.

$$v_0 = 0$$  \hspace{1cm} (7)

Due to the half-rotating walking mechanism is symmetrical structure, and all its joints are rotated, so only the pitch angle $\varphi$ may change during the linear motion on the flat ground. So the attitude transformation matrix $Q$ in formula (9) can be simplified to

$$Q = \begin{bmatrix}
c_{\varphi_z} & -s_{\varphi_z} & 0 \\
s_{\varphi_z} & c_{\varphi_z} & 0 \\
0 & 0 & 1
\end{bmatrix}$$  \hspace{1cm} (8)

Then, according to formula (6), the following formula can be obtained.

$$v_D = \begin{bmatrix}
-\varphi_2 y_{04} + \varphi_2 z_{04} - \dot{\theta}(y_{12} + \frac{1}{2} y_{23} + \frac{1}{4} y_{34}) \\
\varphi_2 x_{04} - \varphi_2 z_{04} + \dot{\theta}(y_{12} + \frac{1}{2} y_{23} + \frac{1}{4} y_{34}) \\
-\varphi_2 x_{04} + \varphi_2 y_{04}
\end{bmatrix}$$  \hspace{1cm} (9)

The above formula is the velocity of the point $D$ at the end of the stride rod relative to the follow-up coordinate system $\tilde{R}$. According to the principle of relativity of motion, the velocity expression of the center $R$ is as follows:

$$v_R = v_{\tilde{R}} = -v_D$$  \hspace{1cm} (10)

### 4. Kinematics Simulation and Analysis

#### 4.1. Model Simplification

Taking the half-rotating walking mechanism as an example, the linear motion on the horizontal ground is simulated. In this state of motion, compared with the initial moment, the yaw angle and roll angle of the mechanism are not changed, only the pitch angle of the mechanism changes with the walking motion. Assuming the yaw angle, roll angle and pitch angle are zero at the initial moment, that is,
\[ \alpha = 0, \quad \beta = 0, \quad \gamma'_0 = 0, \quad \dot{\alpha} = 0, \quad \dot{\beta} = 0 \]  (11)

The position of center R of mechanism in the global coordinate system \( W \) is

\[
\begin{bmatrix}
    x_R \\
    y_R \\
    z_R
\end{bmatrix}
= \begin{bmatrix}
    c\gamma_0 - s\beta_0 \\
    s\gamma_0 + c\beta_0 \\
    z_0
\end{bmatrix}
\]  (12)

According to the linear velocity vector analysis in section 3.4, the velocity of the mechanism center R in the contact coordinate system \( D \) (that is, the velocity in the global coordinate system \( W \)) can be obtained, as shown in formula (13).

\[
\mathbf{v}_R = \begin{bmatrix}
    v_{xR} \\
    v_{yR} \\
    v_{zR}
\end{bmatrix}
= \begin{bmatrix}
    c\gamma [\dot{\phi}_0 y_4 - \dot{\theta}(y_{12} + \frac{1}{2}y_{23} + \frac{1}{4}y_{34})] + s\gamma [\dot{\phi}_0 x_4 - \dot{\theta}(x_{12} + \frac{1}{2}x_{23} + \frac{1}{4}x_{34})] \\
    s\gamma [\dot{\phi}_0 y_4 - \dot{\theta}(y_{12} + \frac{1}{2}y_{23} + \frac{1}{4}y_{34})] - c\gamma [\dot{\phi}_0 x_4 - \dot{\theta}(x_{12} + \frac{1}{2}x_{23} + \frac{1}{4}x_{34})] \\
    0
\end{bmatrix}
\]  (13)

4.2. Simulation Verification

In the simulation and calculation, the relative parameters of half-rotating walking mechanism are taken as shown in Table 2, and it is assumed that the half-rotating walking mechanism moves forward along the X-axis, the angular velocity of the first rotation arm is taken as \( \dot{\theta} = -\pi/3 \) rad/s. Taking the central axis \( R \) of half-rotating walking mechanism as an example, the values in Table 2 are respectively substituted into the analytical expressions (12) and formulas (13), and the theoretical analytical curves of the displacement and velocity can be obtained respectively.

**Table 2.** Parameters of half-rotating walking mechanism.

| Parameter                      | Value / mm |
|--------------------------------|------------|
| Length of the first rotating arm \( a_{AB} \) | 70         |
| Half length of the secondary rotating arm \( a_{BC} \) | 105        |
| Half length of the stride rod \( a_{CD} \) | 130        |
| Half length of the central axis \( d_{RA} \) | 200        |
| Lateral deviation \( d_{AB}, d_{BC}, d_{CD} \) | 20         |

![Figure 3. The screenshots of ADAMS simulation process.](image)

![Figure 4. Displacement and velocity curves of mechanism central axis R.](image)
In ADAMS simulation, constraints are set on the spindle of the mechanism so that it can only be moved relative to the ground without rotation. The corresponding driving speed is set in each rotating pair, in which the rotational speed of the first rotating arm is set as \( \frac{\pi}{3} \text{ rad/s} \), and the feet are set up three-dimensional contact with the ground. Figure 3 is the screenshots of ADAMS simulation process. After the ADAMS simulation, the displacement and velocity curves of the center axis \( R \) of the mechanism are obtained as shown in Figure 4. From Figure 4, it can be seen that the theoretical calculated values of the swing balancing half-rotating walking mechanism basically coincide with the simulation values, which verifies the accuracy of the forward kinematics model. The reason for the deviation is that the feet soles are added to the mechanism for ADAMS simulation analysis, while the analytical calculation in MATLAB assumes that the stride rod contacts directly with the ground, and there will be impact when landing.

5. Conclusion
Based on the motion principle of the two-stage half-rotating mechanism, the swing balanced configuration is proposed. Combined with vector method and D-H method, the forward kinematics model of half-rotating walking mechanism is derived. The kinematics of half-rotating walking mechanism is simulated in ADAMS software, and the simulation results are compared with MATLAB analysis results of kinematics model. The two results were basically consistent, which verifies the correctness of forward kinematics modeling method of half-rotating walking mechanism. This modeling method has the characteristics of rapid, simple and clear physical significance, which lays the foundation for the follow-up research on stable walking. In addition, this modeling and analysis method can be extended to other types of multi-body systems.

6. Conflict of Interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments
This study was funded by Natural Science Foundation of Anhui Province of China (No. 1908085ME131).

Reference
[1] BigDog: The First Advanced Rough-Terrain Robot [G/OL]. (2018-11-15) https://www.bostondynamics.com/bigdog
[2] WildCat: The World’s Fastest Quadruped Robot [G/OL]. (2018-11-15) https://www.bostondynamics.com/wildcat
[3] LI Fei, LIU Weiting and FU Xin 2012 Jumping like an insect: Design and dynamic optimization of a jumping mini robot based on bio-mimetic inspiration Mechatronics vol 22(2) pp 167-176
[4] Weinmeister K, Eckert P and Witte H 2016 Cheetah-cub-S: Steering of a quadruped robot using trunk motion// IEEE International Symposium on Safety, Security, and Rescue Robotics IEEE pp 1-6
[5] Wafa batayneh, Omar Al-Araidah and Salaheddin Malkawi 2013 Biomimetic Design of a Single DOF Stephenson III Leg Mechanism Mechanical Engineering Research vol 3(2) pp 43-50, DOI:10.5539/mer.v3n2p43.
[6] Doyoung Chang and Jeongryul Kim 2013 Design of a slider-crank leg mechanism for mobile hopping robotic platforms. Journal of Mechanical Science and Technology vol 27 (1) pp 207-214
[7] Kitano S, Hirose S and Horigome A 2016 TITAN-XIII: sprawling-type quadruped robot with ability of fast and energy-efficient walking. ROBOMECH Journal vol 3(1) pp 1-16