$A_N$ in proton-proton collisions and the role of twist-3 fragmentation

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Abstract. We review and give an update on the current status of what causes transverse single-spin asymmetries (TSSAs) in semi-inclusive processes where a single hadron is detected in the final state, especially those involving proton-proton ($pp$) collisions. In particular, we provide a new analysis within collinear factorization of TSSAs in high transverse momentum charged and neutral pion production in $pp$ collisions at the Relativistic Heavy Ion Collider (RHIC). This study incorporates the so-called twist-3 fragmentation term and shows that one can describe RHIC data through this mechanism. Moreover, by fixing other non-perturbative inputs through extractions of transverse momentum dependent functions in $e^+e^-$ and semi-inclusive deep-inelastic scattering (SIDIS), we provide for the first time a consistency between certain spin/azimuthal asymmetries in all three reactions (i.e., $pp$, $e^+e^-$, and SIDIS).

1 Introduction

Transverse single-spin asymmetries (TSSAs) have received much attention from both the experimental and theoretical side starting in the mid-1970s. Initially, large effects were seen in transversely polarized $\Lambda$ production in proton-beryllium collisions [1]. These results were thought to contradict perturbative Quantum Chromodynamics (pQCD) because for such asymmetries (denoted by $A_N$) one should have $A_N \sim a_s m_q/P_{h\Lambda}$, with $a_s = g^2/4\pi$ ($g$ is the strong coupling constant), $m_q$ the mass of the quark, and $P_{h\Lambda}$ the transverse momentum of the outgoing hadron/particle [2]. Later it was shown how twist-3 quark-gluon-quark correlations in the nucleon could cause significant asymmetries [3], with benchmark calculations first performed in [4, 5] using collinear factorization. Experimental measurements of TSSAs in single-inclusive hadron production in proton-(anti)proton collisions also continued to show a sizable $A_N$ [6–10]. This led to much (still ongoing) theoretical work on these and similar observables — see, e.g., [4, 5, 11–17]. In Sec. 2 we summarize the theoretical formalism used to describe TSSAs, namely collinear twist-3 factorization. We also review attempts to explain this effect and why it has remained a puzzle for close to 40 years. We next show in Sec. 3 how the fragmentation mechanism could play a crucial role in TSSAs for single-inclusive pion production from proton ($pp$) collisions. This is the main content of the manuscript and is based on the work in [18]. In particular, we demonstrate that one can obtain a very good fit of all high transverse momentum RHIC data in $p^1p \rightarrow \pi X$ by including this fragmentation term while also fixing certain non-perturbative inputs from transverse momentum dependent (TMD) functions extracted in $e^+e^- \rightarrow h_1h_2X$ and semi-inclusive deep-inelastic scattering (SIDIS). Thus we provide for the first time in pQCD a simultaneous description of certain spin/azimuthal asymmetries in all three reactions (i.e., $pp$, $e^+e^-$, and SIDIS). In Sec. 4 we summarize our results and provide an outlook on the much fruitful work that lies ahead in order to fully understand TSSAs.

2 Collinear twist-3 formalism

We consider a process of the type $A(P, \vec{S}_\perp) + B(P') \rightarrow C(P_b) + X$, where the 4-momenta and polarizations of the incoming protons $A, B$ and outgoing hadron $C$ are specified. In collinear twist-3 QCD factorization one has

$$d\sigma(\vec{S}_\perp) = H \otimes f_a(A) \otimes f_b(B) \otimes D_{C(2)} + H' \otimes f_a(A) \otimes f_b(B) \otimes D_{C(2)} + H'' \otimes f_a(A) \otimes f_b(B) \otimes D_{C(3)},$$

(1)

where $f_a(A)$ denotes the twist-$t$ distribution function for parton $a$ in proton $A$, and similarly for the other distribution function $f_b(B)$ and fragmentation function $D_{C(2)}$ for hadron $C$ in parton $c$. The hard factors are given by $H, H'$, and $H''$, which are different for each term, and the symbol $\otimes$ represents convolutions in the appropriate momentum.
fractions. In Eq. (1) a sum over partonic channels and parton flavors in each channel is understood.

The first term in (1), where collinear twist-3 correlators appear for the transversely polarized proton, has been studied extensively [5, 12–14, 17, 19]. It contains three pieces that involve different matrix elements: i) quark-gluon-quark (qqg) soft-gluon pole (SGP) [5, 12, 14, 19]; ii) $qqg$ soft-fermion pole (SFP) [13]; and iii) tri-gluon ($ggg$) SGP [17]. The second term in (1), which involves collinear twist-3 functions for the unpolarized proton, was shown to be small [20]. For the third term in (1), in which collinear twist-3 functions enter for the outgoing (unpolarized) hadron, the complete analytical result was obtained only recently [21].

For quite some time it was assumed that the first term in (1) dominates $A_N$ in $p^1p \to hX$ for the production of light hadrons (see, e.g., Refs. [5, 12, 14]), where the $ggg$ SGP matrix element, called the Qiu-Sterman (QS) function $f_{1T}^{qg}$, is generally considered the most important piece that involves di-parton correlations for $\pi^-$. However, before we proceed to the phenomenology, let us first recall the important details of the analytical result and interpret scales) in such reactions can only be considered a physical observable in the $ZT$ region and for large $\chi$. The variables $x_f$, $\eta$ are further related by $x_f = 2P_h \sqrt{s}$, where $P_h$ is the longitudinal momentum of the hadron, as well as pseudo-rapidity $\eta = -\ln \tan(\theta/2)$, where $\theta$ is the scattering angle. The variables $x_f$, $\eta$ are further related by $x_f = 2P_h \sqrt{s}$, where $P_h$ is the transverse momentum of the hadron.

There are several non-perturbative functions that enter into Eq. (3). They are the transversity distribution $h_1$, the unpolarized parton density $f_1$, and the three (twist-3) fragmentation functions (FFs) $H$, $\hat{H}$, and $\hat{H}_{FU}$, with the last one being the imaginary part of a 3-parton correlator. In Ref. [21] one can also find the definition of those functions and the results for the hard scattering coefficients $S'$ for each channel $i$. (An alternative notation of the relevant FFs can also be found in Ref. [29], where twist-3 fragmentation effects in SIDIS were computed.)

Similar to the relation between $T_F$ and the Sivers function $f_1^{qg}$ in Eq. (2), the function $\hat{H}$ can be written in terms of the TMD Collins function $H_1^{qg}$ [30] according to [21, 31]

$$\hat{H}^{qg}(z) = z^2 \int d^2 k_\perp \frac{k_\perp^2}{2m_h^2} H_1^{qg}(z, z_1 k_\perp^2).$$ (4)

Exploiting the universality of the Collins function [32], one can simultaneously extract (see [33] and references therein) $H_1^{qg}$ and $h_1$ from data [34, 35] on the Collins TSSA $A_{Col}^{qg}$ in SIDIS [36] and data [37, 38] on the cos(2$\phi$) modulation $A_{cos(2\phi)}^{qg}$ in $e^+e^- \to h_1h_2X$ [39]. Such information for $H_1^{qg}$ and $h_1$, as well as that for the $f_{1T}^{qg}$ [40, 41], will be useful when describing $A_N$. The FFs $\hat{H}$ and $\hat{H}_{FU}$ are not independent, but rather satisfy [21]

$$H^{qg}(z) = -2z \hat{H}^{qg}(z) + 2z \int_0^\infty \frac{dz_1}{z_1} \frac{1}{z_1 - \frac{1}{z}} \hat{H}^{qg}_{FU}(z, z_1),$$ (5)

implying that in the collinear twist-3 framework one has two independent FFs. It is important to realize that this is different from the so-called TMD approach for $A_N$, where only $H_1^{qg}$ enters the fragmentation piece [42]. We note that the Sivers effect in the TMD formalism has also been applied to $A_N$ in $p^1p \to hX$ [41]. However, given that for single-inclusive processes there is only one large scale, using TMD factorization (which requires two different scales) in such reactions can only be considered a phenomenological model. In that sense, the collinear twist-3 formalism is the more rigorous theoretical framework.

3 Phenomenological fit of pion data

We analyze $A_N$ data for $p^1p \to \pi X$ in the forward region of the polarized proton, which has been studied by
To compute the fragmentation contribution we take $f_{\text{FF}}$, our "variables" because their allowed range is $[0, \beta]$ consistency. The calculation, so we adhere with this choice as a matter of DSS unpolarized FFs [44]. The GRV98 functions were the GRV98 unpolarized parton distributions [43] and the elements in order to safely apply pQCD. Throughout we use with the parameters $\alpha$, $\Phi$, $B$, $H$, $\delta$, $H_1$ (which fixes $\bar{H}$ through (4)) from [33]. For favored fragmentation into $\pi^+$ we make for $\bar{H}_{FU}^0$ the ansatz

$$
\frac{\bar{H}_{FU}^{\pi^+}(z, z_1)}{D^{\pi^+}(z_1)} = \frac{N_{\text{eff}}}{2 \lambda_{\text{eff}} N_{\text{full}}} \left(1 - \frac{\bar{H}_{FU}^{\pi^+}(z, z_1)}{\beta_{\text{eff}}} \right) \times (1 - \frac{\bar{H}_{FU}^{\pi^+}(z, z_1)}{\beta_{\text{eff}}} \beta_{\text{eff}}), \quad (6)
$$

with the parameters $N_{\text{eff}}$, $\alpha_{\text{eff}}$, $\alpha_{\text{eff}}$, $\beta_{\text{eff}}$, $\beta_{\text{eff}}$, and the unpolarized FF $D$. This parameterization follows the standard procedure of modifying the small and large “x” behavior of twist-2 unpolarized functions when trying to fit an unknown function. Note that $z$ and $z/z_1$ are chosen as our “variables” because their allowed range is $[0, 1]$ [47] and that our ansatz satisfies the constraint $\bar{H}_{FU}(z, z) = 0$ [47, 48]. With the use of DSS FFs [44], the factor $I_{\text{eff}}$ reads $I_{\text{eff}} = I_{\text{eff}} - I_0$ where $I_0 (i = u + \bar{u}, d, \bar{d})$ is defined as

$$
I_i = \frac{N_i (K_{1, i, \text{eff}} + \gamma_i K_{2, i, \text{eff}})}{B[i + \alpha_i, \beta_i + 1, 1] + \gamma_i[B[i + \alpha_i, \beta_i + 1, 1] + 1],
$$

with $K_{1, i, \text{eff}} = B[a_i, \beta_i, \gamma_i, \beta_i + 1, 1]$, and $K_{2, i, \text{eff}} = B[a_i + 1, \beta_i, \gamma_i, \beta_i + 1, 1]$, and $B[a, b, c]$, the Euler β-function. The parameters $N_i, \alpha_i, \beta_i, \gamma_i, \beta_i$ come from D FFs at the initial scale and are given in Table III of [44]. Note that $D^{\pi^+}$ in Ref. [44] differs from $D^{\pi^+}$ in (6) is similarly defined as $I_{\text{eff}} = J_{\text{eff}} - J_{\text{off}}$ where $J_i (i = u + \bar{u}, d, \bar{d})$ follows from $I_i$ through $a_i = (a_i + 4, \beta_i + 1, b_i + 1, 1)$. The factor $1/(2I_{\text{eff}} J_{\text{eff}})$ in (6) is convenient and implies $\int_0^1 2 \frac{d z}{I_{\text{eff}}(z)} = N_{\text{eff}}$. At the initial scale, $H_0$ represents the entire second term on the r.h.s. of (5). For the disfavored FFs $H_{FU}^{\pi^+}(z, z_1)$ we make an ansatz in full analogy to (6), introducing the additional parameters $N_{\text{dis}}, \alpha_{\text{dis}}, \alpha_{\text{dis}}, \beta_{\text{dis}}, \beta_{\text{dis}}, (J_{\text{dis}}$ and $J_{\text{dis}}$ are calculated using $D^{\pi^+} = D^{\pi^+}$ from [44]). The $\pi^-$ FFs are then fitted through charge conjugation, and the $\pi^0$ FFs are given by the average of the FFs for $\pi^+$ and $\pi^-$. The FFs $H_{FU}^{\pi}$ are computed by means of (5). All parton correlation functions are evaluated at the scale $\sqrt{s}$ with leading order evolution of the collinear functions.

Using the MINUIT package we fit the fragmentation contribution to data for $A_N^{\pi^+}$ [8] and $A_N^{\pi^-}$ [9]. In order to limit the number of free parameters, we only keep 7 of them free in $H_{FU}^{\pi^+}$. Since the large-x behavior of $h_1$ is mostly unconstrained by current SIDIS data, we also allow the β-parameters $\beta_{\pi^+} = \beta_{\pi^-} = 0$ of the transversity to vary within the error range given in [33]. All integrations are done using the Gauss-Legendre method with 250 steps. For the SV1 input the result of our 8-parameter fit is shown in Tab. 1. For the SV2 input the values of the fit parameters are similar, with an equally successful fit ($\chi^2/\text{d.o.f.} = 1.10$). The very good description of the RHIC $A_N$ data is explicitly evident in Fig. 1. We emphasize this is a non-trivial outcome if one keeps in mind the constraint in (5) and the need to simultaneously fit data for $A_N^{\pi^+}$ and $A_N^{\pi^-}$. Results for the FFs $H_{FU}^{\pi^+}$ and $H_{FU}^{\pi^+} = \int_0^1 \frac{d z}{I_{\text{eff}}(z)} (1 - \frac{\bar{H}_{FU}^{\pi^+}(z, z_1)}{\beta_{\text{eff}}} \beta_{\text{eff}})$.

**Figure 1.** Fit results for $A_N^{\pi^+}$ (data from [8]) and $A_N^{\pi^-}$ (data from [9]) for the SV1 input. The dashed line (dotted line in the case of $\pi^-$) means $H_{FU}^{\pi^+}$ switched off.

**Figure 2.** Results for the FFs $H_{FU}^{\pi^+}$ and $H_{FU}^{\pi^+}$ (defined in the text) for the SV1 input. Also shown is $H_{FU}^{\pi^+}$ without the contribution from $H_{FU}^{\pi^+}$ (dashed line).

The STAR [8], BRAHMS [9], and PHENIX [10] collaborations at RHIC. The data at $\sqrt{s} = 200 \text{GeV}$ typically has $P_{\text{ch}} > 1 \text{GeV}$, and we therefore focus on those measurements in order to safely apply pQCD. Throughout we use the GRV98 unpolarized parton distributions [43] and the DSS unpolarized FFs [44]. The GRV98 functions were also used in Refs. [33, 40, 41] for extracting the Sivers function and the transversity, which we take as input in our calculation, so we adhere with this choice as a matter of consistency. The $gqg$ SGP contribution to (1) is computed by fixing $T_G$ through Eq. (2) with two different inputs for the Sivers function — SV1: $f_I^T$ from Ref. [40], obtained from SIDIS data on $A_{1\text{SIDIS}}$ [45, 46]; and SV2: $f_I^T$ from Ref. [41], “constructed” such that, in the TMD approach, the contribution of the Sivers effect to $A_N$ is maximized while maintaining a good fit of $A_{1\text{SIDIS}}$. The input SV1 has a flavor-independent large-x behavior, while SV2 in that region has a flavor dependence and also falls off slower. To compute the fragmentation contribution we take $h_1$ and $H_1^+(z)$ which fixes $\bar{H}$ through (4)) from [33]. For favored fragmentation into $\pi^+$ we make for $\bar{H}_{FU}^0$ the ansatz

$$
\frac{\bar{H}_{FU}^0(z, z_1)}{D^0(z_1)} = \frac{N_{\text{eff}}}{2 \lambda_{\text{eff}} N_{\text{full}}} \left(1 - \frac{\bar{H}_{FU}^0(z, z_1)}{\beta_{\text{eff}}} \beta_{\text{eff}} \right) \times (1 - \frac{\bar{H}_{FU}^0(z, z_1)}{\beta_{\text{eff}}} \beta_{\text{eff}}).
$$

The FFs $H_{FU}^{\pi^+}$ are computed by means of (5). All parton correlation functions are evaluated at the scale $P_{\text{ch}}$ with leading order evolution of the collinear functions.
are displayed in Fig. 2. Similar to the Collins function \( H_F \), in either case the favored and disfavored FFs have opposite signs. Such reversed signs are actually “preferred” by the Schäfer-Teryaev (ST) sum rule 
\[
\sum_q \sum_{\beta} \int_{z_{min}}^{1} dz \, M_{q} H^{q/q}(z) = 0 \quad [49].
\]
Note that the ST sum rule, in combination with (5), implies a constraint on a certain linear combination of \( H^{q/q} \) and (an integral of) \( P_{FU}^{q/q} \). In view of that, one benefits from favored and disfavored FFs having opposite signs like in Fig. 2. Also depicted in Fig. 2 is \( H^{q/q} \) when \( H_{FU}^{q/q} \) is switched off. One sees \( H_{FU}^{q/q} \) causes a reasonable increase from this scenario. As shown in Fig. 1, when the 3-parton FF is turned off, one has difficulty describing the data for \( A_N \). According to Fig. 3, the \( H \) term (including its derivative) contributes only very little to \( A_N \). Also the \((ggq)\) SGP pole term is small, except for the SV2 input at large \( x_F \), where its contribution is opposite to the data. Note that with a Sivers function similar to SV2, there would definitely be serious issues with trying to match the \( A_N \) data without the 3-parton FF. Clearly \( A_N \) is governed by the \( H \) term. (Recall from (5) that this function involves both \( H^{q/q} \) and \( H_{FU}^{q/q} \).) This result can mainly be traced back to the hard scattering coefficients: e.g., for the dominant \( gg \to gg \) channel one has \( S_H \propto 1/\beta^2 \), but \( S_H \propto 1/\beta \) [21] in the forward region where \( \beta \) is small. Note also that \( S_{H_{FU}} \sim 1/\beta^2 \) for that channel, but it is suppressed by a color factor of \( 1/(N_c^2 - 1) \). Next, Fig. 4 shows the breakdown of \( A_N^{\pi} \) into favored and disfavored fragmentation contributions. One can see that

\[ A_N^{\pi} \] (or \( A_N^{\pi} \)) is dominated by favored (disfavored) fragmentation. Finally, Fig. 5 shows the \( P_{h\perp} \)-dependence of \( A_N \) for \( \sqrt{s} = 500 \text{ GeV} \). Preliminary data from STAR, extending to almost \( P_{h\perp} = 10 \text{ GeV} \), shows that \( A_N \) is rather flat [50]. Oftentimes it is stated that the collinear twist-3 calculation cannot reproduce this flat \( P_{h\perp} \) dependence of \( A_N \) due to the naive expectation that \( A_N \sim 1/P_{h\perp} \) for a subleading twist effect. However, as was first argued in [5] and later shown in [14], this does not have to be the case. Our calculation indeed does lead to a flat \( P_{h\perp} \) dependence, and also the magnitude of \( A_N \) is in line with the data. Note that the data of Ref. [50] were not included in our fit and that only statistical errors are shown in Fig. 5 [50].

**4 Summary and outlook**

For many years it was unclear what mechanism causes large TSSAs in hadron production from proton-proton collisions. Collinear twist-3 QCD factorization can be considered the most natural and rigorous approach to describe this observable, yet the sign-mismatch issue [25] threatened the validity of this formalism. Here we have shown for the first time that the fragmentation contribution in twist-3 factorization actually can describe high-energy RHIC data for \( A_N \) very well. By using a Sivers function fully consistent with SIDIS, we have demonstrated that this mechanism could also resolve the sign-mismatch crisis. We used the TMD Sivers, Collins, and transversity functions, which were extracted through spin/azimuthal asymmetries in SIDIS and \( e^+e^- \to h_1h_2X \), to fix certain non-perturbative inputs in our calculation. Together with the collinear 3-parton FF, these functions allowed for a very good fit of \( p^1p \to \pi X \) data. Thus we have shown that at present a simultaneous description of all three observables is possible (i.e., \( pp \), \( e^+e^- \), and SIDIS). We leave an analysis of \( A_N \) for kaons and etas and incorporation of SFPs for future work.

Ultimately in order to truly determine what mechanism underlies TSSAs, one must obtain information from

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*Table 1. Fit parameters for SV1 input.*

| Parameter | Value |
|-----------|-------|
| \( N_{\text{fav}} \) | -0.0338 |
| \( \alpha_{\text{fav}} = \alpha_{\text{fav}}' \) | -0.198 |
| \( \beta_{\text{fav}} \) | 0.0 |
| \( N_{\text{dis}} \) | 0.216 |
| \( \alpha_{\text{dis}} = \alpha_{\text{dis}}' \) | 3.99 |
| \( \beta_{\text{dis}} \) | 3.34 |
| \( \beta_{\text{dis}}' \) | 1.10 |

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*Figure 3. Individual contributions to \( A_N^{\pi} \) (data from [8]) for SV1 and SV2 inputs.*

*Figure 4. Individual contributions to \( A_N^{\pi} \) from favored and disfavored fragmentation (data from [9]) for SV1 input.*

*Figure 5. \( A_N \) as function of \( P_{h\perp} \) for SV1 input at \( \sqrt{s} = 500 \text{ GeV} \) (data from [50]).*
other reactions in order to independently determine the relevant collinear twist-3 functions and/or verify that previously extracted functions are consistent with other measurements. In this context, one already has data on $A_N$ in $p^3 p \rightarrow jetX$ available from the AN/DY Collaboration [51]. Experiments to determine $A_N$ for Drell-Yan and direct photon production would also be beneficial. Even measurements of the Sivers and Collins asymmetries at large $p_T$, would be helpful and could be performed at Jefferson Lab (JLab) 12, COMPASS, or a future Electron-Ion Collider. In addition, data on TSSAs for single-inclusive hadron production from lepton-nucleon collisions is currently available from JLab [52] and HERMES [53]. This reaction was also recently analyzed in [54] using the collinear twist-3 approach and in [55] within the TMD framework. The main question then becomes if one can find a formalism that can consistently describe TSSAs in all of these processes. Much work is left to be done on both the theoretical and experimental side in order to answer this.

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References

[1] G. Bunce et al., Phys. Rev. Lett. 36, 1113 (1976).
[2] G. L. Kane, J. Pumplin and W. Repko, Phys. Rev. Lett. 41, 1689 (1978).
[3] A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982) [Yad. Fiz. 36, 242 (1982)]; Phys. Lett. B 150, 383 (1985).
[4] J.-W. Qiu and G. F.Sterman, Phys. Rev. Lett. 67, 2264 (1991); Nucl. Phys. B 378, 52 (1992).
[5] J.-W. Qiu and G. F. Sterman, Phys. Rev. D 59, 014004 (1999).
[6] D. L. Adams et al. [ES81 and E704 Collaborations], Phys. Lett. B 261, 201 (1991); D. L. Adams et al. [E704 Collaboration], Phys. Lett. B 264, 492 (1991).
[7] K. Krueger et al., Phys. Lett. B 459, 412 (1999).
[8] J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 92, 171801 (2004) [hep-ex/0310058]; B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 101, 222001 (2008) [arXiv:0801.2990 [hep-ex]]; L. Adamczyk et al. [STAR Collaboration], Phys. Rev. D 86, 051101 (2012) [arXiv:1205.6826 [nucl-ex]].
[9] J. H. Lee et al. [BRAHMS Collaboration], AIP Conf. Proc. 915, 533 (2007); I. Arsene et al. [BRAHMS Collaboration], Phys. Rev. Lett. 101, 042001 (2008) [arXiv:0801.1078 [nucl-ex]].
[36] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders and M. Schlegel, JHEP 0702, 093 (2007) [hep-ph/0611265].

[37] R. Seidl et al. [Belle Collaboration], Phys. Rev. D 78, 032011 (2008) [Erratum-ibid. D 86, 039905 (2012)].

[38] J. P. Lees et al. [BaBar Collaboration], arXiv:1309.5278 [hep-ex].

[39] D. Boer, R. Jakob and P. J. Mulders, Nucl. Phys. B 504, 345 (1997); D. Pitonyak, M. Schlegel and A. Metz, Phys. Rev. D 89, 054032 (2014).

[40] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin and C. Turk, Eur. Phys. J. A 39, 89 (2009).

[41] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D 88, 054023 (2013).

[42] M. Anselmino, M. Boglione, U. D’Alesio, E. Leader, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D 86, 074032 (2012).

[43] M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C 5, 461 (1998).

[44] D. de Florian, R. Sassot and M. Stratmann, Phys. Rev. D 75, 114010 (2007).

[45] M. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 673, 127 (2009).

[46] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. 103, 152002 (2009).

[47] S. Meissner and A. Metz, Phys. Rev. Lett. 102, 172003 (2009).

[48] L. P. Gamberg, A. Mukherjee and P. J. Mulders, Phys. Rev. D 77, 114026 (2008); Phys. Rev. D 83, 071503 (2011).

[49] A. Schäfer and O. V. Teryaev, Phys. Rev. D 61, 077903 (2000); S. Meissner, A. Metz and D. Pitonyak, Phys. Lett. B 690, 296 (2010).

[50] S. Heppelmann [STAR Collaboration], PoS DIS 2013, 240 (2013).

[51] L. C. Bland et al. [AnDY Collaboration], arXiv:1304.1454 [hep-ex].

[52] K. Allada et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. C 89, 042201 (2014) [arXiv:1311.1866 [nucl-ex]].

[53] A. Airapetian et al. [HERMES Collaboration], Phys. Lett. B 728, 183 (2014) [arXiv:1310.5070 [hep-ex]].

[54] L. Gamberg, Z. -B. Kang, A. Metz, D. Pitonyak and A. Prokudin, arXiv:1407.5078 [hep-ph].

[55] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D 89, 114026 (2014) [arXiv:1404.6465 [hep-ph]].