Skewness Premium with Lévy Processes

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Abstract

We study the skewness premium (SK) introduced by Bates (1991) in a general context using Lévy Processes. Under a symmetry condition Fajardo and Mordecki (2006) obtain that SK is given by the Bate’s $x\%$ rule. In this paper we study SK under the absence of that symmetry condition. More exactly, we derive sufficient conditions for SK to be positive, in terms of the characteristic triplet of the Lévy Process under the risk neutral measure.

Keywords: Skewness Premium; Lévy Processes.

JEL Classification: C52; G10

1 Introduction

The stylized facts of option prices have been studied by many authors in the literature. An important fact from option prices is that relative prices of out-of-the-money calls and puts can be used as a measure of symmetry or skewness of the risk neutral distribution. Bates (1991), called this diagnosis “skewness premium”, henceforth SK. He analyzed the behavior of SK using

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three classes of stochastic processes: Constant Elasticity of Variance (CEV), Stochastic Volatility and Jump-diffusion. He found conditions on the parameters for the SK be positive or negative.

But, as many models in the literature have shown, the behavior of the assets underlying options is very complex, the structure of jumps observed is more complex than Poisson jumps. They have higher intensity, see for example Aït-Sahalia (2004). For that reason diffusion models cannot consider the discontinuous sudden movements observed on asset prices. In that sense, the use of more general process as Lévy processes have shown to provide a better fit with real data, as was reported in Carr and Wu (2004) and Eberlein, Keller, and Prause (1998). On the other hand, the mathematical tools behind these processes are very well established and known.

When the underlying follows a Geometric Lévy Process, Fajardo and Mordecki (2006) obtained a relationship between calls and puts, that they called Put-Call duality and obtain as a particular case the Put-call symmetry, and obtain that SK is given by the Bate’s $x\%$ rule. The Put-Call duality has important applications, in particular the Put-Call symmetry, as Bowie and Carr (1994) and Carr, Ellis, and Gupta (1998) show using symmetry we can construct static hedges for exotic options..

In this paper we study the SK under absence of symmetry and obtain sufficient conditions for the excess of SK be positive or negative. The main idea behind the proofs is to exploit the monotonicity property of option prices with respect to some parameter of the Lévy measure. This monotonicity is not an easy task, monotonicity with respect to the intensity parameter of the jump have been recently address by Ekström and Tysk (2007), while the monotonicity with respect to the symmetry parameter have not been totally addressed in previous works. A particular answer is given for the case of GH distributions in Bergenthum and Rüschendorf (2007).

The paper is organized as follows: in Section 2 we introduce the Lévy processes and we present the duality results. In Section 3 we discuss market symmetry and present our main results. In Section 4 we study the skewness premium. Section 5 discuss monotonicity with respect to the symmetry parameter and Section 6 concludes.


2 Lévy processes and Duality

Consider a real valued stochastic process \( X = \{X_t\}_{t \geq 0} \), defined on a stochastic basis \( \mathcal{B} = (\Omega, \mathcal{F}, \mathcal{F}_t = (\mathcal{F}_s)_{s \leq t}, Q) \), being càdlàg, adapted, satisfying \( X_0 = 0 \), and such that for \( 0 \leq s < t \) the random variable \( X_t - X_s \) is independent of the \( \sigma \)-field \( \mathcal{F}_s \), with a distribution that only depends on the difference \( t - s \). Assume also that the stochastic basis \( \mathcal{B} \) satisfies the usual conditions (see Jacod and Shiryaev (1987)). The process \( X \) is a Lévy process, and is also called a process with stationary independent increments (PIIS). For general reference on Lévy processes see Jacod and Shiryaev (1987), Skorokhod (1991), Bertoin (1996), Sato (1999). For Lévy process in Finance see Boyarchenko and Levendorski (2002), Schoutens (2003) and Cont and Tankov (2004).

In order to characterize the law of \( X \) under \( Q \), consider, for \( q \in \mathbb{R} \) the Lévy-Khinchine formula, that states

\[
E e^{i q X_t} = \exp \left\{ t \left[ i a q - \frac{1}{2} \sigma^2 q^2 + \int_{\mathbb{R}} (e^{i y} - 1 - i q h(y)) \Pi(dy) \right] \right\}, (1)
\]

with

\[
h(y) = y 1_{\{y < 1\}}
\]

a fixed truncation function, \( a \) and \( \sigma \geq 0 \) real constants, and \( \Pi \) a positive measure on \( \mathbb{R} \setminus \{0\} \) such that \( \int (1 \wedge y^2) \Pi(dy) < +\infty \), called the Lévy measure. The triplet \((a, \sigma^2, \Pi)\) is the characteristic triplet of the process, and completely determines its law.

Consider the set

\[
\mathbb{C}_0 = \left\{ z = p + i q \in \mathbb{C} : \int_{\{|y| > 1\}} e^{py} \Pi(dy) < \infty \right\}. (2)
\]

The set \( \mathbb{C}_0 \) is a vertical strip in the complex plane, contains the line \( z = i q \ (q \in \mathbb{R}) \), and consists of all complex numbers \( z = p + i q \) such that \( E e^{p X_t} < \infty \) for some \( t > 0 \). Furthermore, if \( z \in \mathbb{C}_0 \), we can define the characteristic exponent of the process \( X \), by

\[
\psi(z) = az + \frac{1}{2} \sigma^2 z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zh(y)) \Pi(dy) \quad (3)
\]
this function \( \psi \) is also called the *cumulant* of \( X \), having 
\[
E|e^{zX_t}| < \infty \quad \text{for all} \quad t \geq 0, \quad \text{and} \quad E e^{zX_t} = e^{t\psi(z)}.
\] 
The finiteness of this expectations follows from Theorem 21.3 in Sato (1999). Formula (3) reduces to formula (1) when \( \operatorname{Re}(z) = 0 \).

### 2.1 Lévy market

By a *Lévy market* we mean a model of a financial market with two assets: a deterministic savings account \( B = \{B_t\}_{t \geq 0} \), with 
\[
B_t = e^{rt}, \quad r \geq 0,
\]
where we take \( B_0 = 1 \) for simplicity, and a stock \( S = \{S_t\}_{t \geq 0} \), with random evolution modelled by
\[
S_t = S_0 e^{X_t}, \quad S_0 = e^x > 0,
\]
where \( X = \{X_t\}_{t \geq 0} \) is a Lévy process.

In this model we assume that the stock pays dividends, with constant rate \( \delta \geq 0 \), and that the given probability measure \( Q \) is the chosen equivalent martingale measure. In other words, prices are computed as expectations with respect to \( Q \), and the discounted and reinvested process \( \{e^{-(r-\delta)t}S_t\} \) is a \( Q \)-martingale.

In terms of the characteristic exponent of the process this means that
\[
\psi(1) = r - \delta, \tag{5}
\]
based on the fact, that 
\[
E e^{-(r-\delta)t+X_t} = e^{-t(r-\delta+\psi(1))} = 1,
\]
and condition (5) can also be formulated in terms of the characteristic triplet of the process \( X \) as
\[
a = r - \delta - \sigma^2/2 - \int_{\mathbb{R}} \left( e^y - 1 - h(y) \right) \Pi(dy). \tag{6}
\]
In the case, when 
\[
X_t = \sigma W_t + at \quad (t \geq 0), \tag{7}
\]
where \( W = \{W_t\}_{t \geq 0} \) is a Wiener process, we obtain the Black–Scholes–Merton (1973) model (see Black and Scholes (1973), Merton (1973)).
In the market model considered we introduce some derivative assets. More precisely, we consider call and put options, of both European and American types. Denote by $\mathcal{M}_T$ the class of stopping times up to a fixed constant time $T$, i.e:

$$\mathcal{M}_T = \{ \tau : 0 \leq \tau \leq T, \tau \text{ stopping time w.r.t } F \}.$$ 

Then, for each stopping time $\tau \in \mathcal{M}_T$ we introduce

$$c(S_0, K, r, \delta, \tau, \psi) = E e^{-r\tau}(S_\tau - K)^+, \quad (8)$$

$$p(S_0, K, r, \delta, \tau, \psi) = E e^{-r\tau}(K - S_\tau)^+. \quad (9)$$

In our analysis (8) and (9) are auxiliary quantities, anyhow, they are interesting by themselves as random maturity options, as considered, for instance, in Schroder (1999) and Detemple (2001). If $\tau = T$, formulas (8) and (9) give the price of the European call and put options respectively.

2.2 Put Call duality and dual markets

**Lemma 2.1** (Duality). Consider a Lévy market with driving process $X$ with characteristic exponent $\psi(z)$, defined in (3), on the set $\mathbb{C}_0$ in (2). Then, for the expectations introduced in (8) and (9) we have

$$c(S_0, K, r, \delta, \tau, \psi) = p(K, S_0, \delta, r, \tau, \tilde{\psi}), \quad (10)$$

where

$$\tilde{\psi}(z) = \tilde{a}z + \frac{1}{2}\tilde{\sigma}^2z^2 + \int_{\mathbb{R}} (e^{zy} - 1 - zh(y))\tilde{\Pi}(dy) \quad (11)$$

is the characteristic exponent (of a certain Lévy process) that satisfies

$$\tilde{\psi}(z) = \psi(1 - z) - \psi(1), \quad \text{for } 1 - z \in \mathbb{C}_0,$$

and in consequence,

$$\begin{cases}
\tilde{a} = \delta - r - \sigma^2/2 - \int_{\mathbb{R}} (e^y - 1 - h(y))\tilde{\Pi}(dy), \\
\tilde{\sigma} = \sigma, \\
\tilde{\Pi}(dy) = e^{-y}\Pi(-dy).
\end{cases} \quad (12)$$

**Proof.** See Fajardo and Mordecki (2006).
The above Duality Lemma motivates us to introduce the following market model. Given a Lévy market with driving process characterized by \( \psi \) in (3), consider a market model with two assets, a deterministic savings account \( \tilde{B} = \{ \tilde{B}_t \}_{t \geq 0} \), given by \( \tilde{B}_t = e^{\delta t}, \quad \delta \geq 0 \), and a stock \( \tilde{S} = \{ \tilde{S}_t \}_{t \geq 0} \), modelled by \( \tilde{S}_t = Ke^{\tilde{X}_t}, \quad \tilde{S}_0 = K > 0 \), where \( \tilde{X} = \{ \tilde{X}_t \}_{t \geq 0} \) is a Lévy process with characteristic exponent under \( \tilde{Q} \) given by \( \tilde{\psi} \) in (11). The process \( \tilde{S}_t \) represents the price of \( KS_0 \) dollars measured in units of stock \( S \). This market is the auxiliary market in Detemple (2001), and we call it dual market; accordingly, we call Put–Call duality the relation (10). It must be noticed that Peskir and Shiryaev (2001) propose the same denomination for a different relation. Finally observe, that in the dual market (i.e. with respect to \( \tilde{Q} \)), the process \( \{ e^{-(\delta-r)t}\tilde{S}_t \} \) is a martingale. As a consequence, we obtain the Put–Call symmetry in the Black–Scholes–Merton model: In this case \( \Pi = 0 \), we have no jumps, and the characteristic exponents are

\[
\psi(z) = (r - \delta - \sigma^2/2)z + \sigma^2 z^2/2,
\]

\[
\tilde{\psi}(z) = (\delta - r - \sigma^2/2)z + \sigma^2 z^2/2.
\]

and relation (10) is the result known as put–call symmetry. In the presence of jumps like the jump-diffusion model of Merton (1976), if the jump returns of \( S \) under \( Q \) and \( \tilde{S} \) under \( \tilde{Q} \) have the same distribution, the Duality Lemma, implies that by exchanging the roles of \( \delta \) by \( r \) and \( K \) by \( S_0 \) in (10) and (12), we can obtain an American call price formula from the American put price formula. Motivated by this analysis we introduce the definition of symmetric markets in the following section.

### 3 Market Symmetry

It is interesting to note that in a market with no jumps (i.e. in the Black–Scholes model), the distribution of the discounted and reinvested stock both in the given risk neutral and in the dual Lévy market, taking equal initial
values, coincide. It is then natural to define a Lévy market to be symmetric when this relation hold, i.e. when

\[ L(e^{-(r-\delta)t+X_t} \mid \mathbb{Q}) = L(e^{-(\delta-r)t-X_t} \mid \tilde{\mathbb{Q}}), \]  

meaning equality in law. Otherwise we call the Lévy market asymmetric. In view of (12), and due to the fact that the characteristic triplet determines the law of a Lévy processes, we obtain that a necessary and sufficient condition for (13) to hold is

\[ \Pi(dy) = e^{-y} \Pi(-dy). \]  

This ensures \( \tilde{\Pi} = \Pi \), and from this follows \( a - (r - \delta) = \tilde{a} - (\delta - r) \), giving (13), as we always have \( \tilde{\sigma} = \sigma \). As pointed out by Fajardo and Mordecki (2006) condition (14) answers a question raised Carr and Chesney (1996). With this condition in mind we can obtain the following result.

**Corollary 3.1** (Bates’ \( x \% \) Rule). Take \( r = \delta \) and assume (14) holds, we have

\[ c(F_0, K_c, r, \tau, \psi) = (1 + x) p(F_0, K_p, r, \tau, \psi), \]  

where \( K_c = (1 + x)F_0 \) and \( K_p = F_0/(1 + x) \), with \( x > 0 \).

**Proof.** Follows directly from Lemma 2.1. Since \( r = \delta \) and \( \psi = \tilde{\psi} \).

From here calls and puts at-the-money (\( x = 0 \)) should have the same price. As we mention this \( x\% \)−rule, in the context of Merton’s model was obtained by Bates (1997). That is, if the call and put options have strike prices \( x\% \) out-of-the-money relative to the forward price, then the call should be priced \( x\% \) higher than the put.

### 3.1 Empirical Evidence of Symmetry

In Fajardo and Mordecki (2006) several concrete models proposed in the literature are reviewed. More exactly, Lévy markets with jump measure of the form

\[ \Pi(dy) = e^{\beta y} \Pi_0(dy), \]  

with

\[ \beta = \frac{a}{\sigma^2}, \]  

are considered. Here, \( \Pi_0(dy) \) represents the initial measure, \( \beta \) is a constant, and \( a \) is a parameter that depends on the specific model. This form of measure is known as the exponential affine jump-diffusion model.
where \( \Pi_0(dy) \) is a symmetric measure, i.e. \( \Pi_0(dy) = \Pi_0(-dy) \), everything with respect to the risk neutral measure \( \mathbb{Q} \).

As a consequence of (14), Fajardo and Mordecki (2006) found that the market is symmetric if and only if \( \beta = -1/2 \). Then, as we have seen when the market is symmetric, the skewness premium is obtained using the \( x\%- \)rule.

Although from the theoretical point of view the assumption (16) is a real restriction, most models in practice share this property, and furthermore, they have a jump measure that has a Radon-Nikodym density. In this case, we have

\[
\Pi(dy) = e^{\beta y} p(y) dy,
\]

where \( p(y) = p(-y) \), i.e. the function \( p(y) \) is even. More precisely, all parametric models that we found in the literature, in what concerns Lévy markets, including diffusions with jumps, can be reparametrized in the form (17): The Generalized Hyperbolic model proposed by Eberlein and Prause (2000), The Meixner model proposed by Schoutens (2001) and The CGMY model proposed by Carr, Geman, Madan, and Yor (2002). Recently, ? shows that under some conditions the Time Changed Brownian motion with drift is also included in this class. Then, they show that the resulting processes will satisfy the above symmetry if and only if the drift equal -1/2.

Using the risk neutral market measure and the Esscher transform measure as EMM, Fajardo and Mordecki (2006) obtain evidence that empirical risk-neutral markets are not symmetric. Then, the question naturally arises: How to obtain a Put-call symmetry, under absence of symmetry? In what follows we try to answer this question.

Henceforth take \( r = \delta \). We need the following assumption

**Assumption 1.** Option prices are monotonic with respect to the symmetry parameter \( \beta \).

Our main result is stated as follows.

**Theorem 3.1.** Consider Lévy measures given by (16). Under Assumption 1, if \( \beta \geq -1/2 \) then

\[
C(F_0, K_c, r, \tau, \psi) \geq (1 + x) P(F_0, K_p, r, \tau, \psi),
\]

(18)
where \( K_c = (1 + x)F_0 \) and \( K_p = F_0/(1 + x) \), with \( x > 0 \).

**Proof.** We have that

\[
\beta \gtrless -1/2 \iff \beta \gtrless \tilde{\beta} := -\beta - 1.
\]

Then, \( \Pi(dy) = e^{\beta y}\Pi_0(dy) \) has \( \beta \gtrless \tilde{\beta} \) of \( \tilde{\Pi} = e^{-(1+\beta)y}\Pi_0(dy) \). By monotonicity

\[
C(F_0, K_c, r, \tau, a, \sigma, \Pi) \gtrless C(F_0, K_c, r, \tau, a, \sigma, \tilde{\Pi}) = (1 + x)P(F_0, K_c, r, \tau, a, \sigma, \Pi),
\]

were the last equality is obtained from duality and the fact that \( \tilde{\Pi} = \Pi \).

The same can be obtained if put prices were decreasing on \( \beta \), we have: \( \beta \gtrless -1/2 \) implies

\[
(1 + x)P(F_0, K_c, r, \tau, a, \sigma, \Pi) \lesssim (1 + x)P(F_0, K_c, r, \tau, a, \sigma, \tilde{\Pi}) = C(F_0, K_c, r, \tau, a, \sigma, \Pi), \ \forall x > 0,
\]

\[\square\]

**Remark 3.1.** In the particular case of the GH distributions Assumption 1, can be guaranteed by Th. 4.2 in Bergenthum and Rüschendorf (2007).

### 3.2 Diffusions with jumps

Consider the jump—diffusion model proposed by Merton (1976). The driving Lévy process in this model has Lévy measure given by

\[
\Pi(dy) = \lambda \frac{1}{\delta\sqrt{2\pi}}e^{-(y-\mu)^2/(2\delta^2)}dy,
\]

and is direct to verify that condition (14) holds if and only if \( 2\mu + \delta^2 = 0 \). This result was obtained by Bates (1997) for future options, that result is obtained as a particular case.

Note that in that model \( \beta = \frac{\mu}{\delta^2} \), so we obtain that sufficient conditions can be replaced by \( \mu + \delta^2/2 \gtrsim 0 \), as also Bates (1997) found.
4 Skewness Premium

In order to study the sign of SK, let’s analyze the following data on S&P500 American options in 08/31/2006 that matures in 09/15/2006 with future price $F = 1303.82$. To verify if the Bates’ rule holds we need to interpolate some non-observed option prices. To this end we use a cubic spline, as we can see in Fig. 1.

The $x\%$ Skewness Premium is defined as the percentage deviation of $x\%$ OTM call prices from $x\%$ OTM put prices. The interpolating calls and put prices for the non-observed strikes are presented in Tables 1 and 2 at the end. We can see in both tables that this rule does not hold. Moreover, for OTM options usually $x_{obs} < x$, what implies $\frac{c}{p} - 1 < x$ and for ITM options, $x_{obs} > x$, implying $\frac{c}{p} - 1 > x$.

Then we want to know for what distributional parameter values we can capture the observed vies in these option price ratios. To this end we use the following definition introduced by Bates (1991).

$$SK(x) = \frac{c(S, T; X_c)}{p(S, T; X_p)} - 1, \text{ for European Options,}$$

$$SK(x) = \frac{C(S, T; X_c)}{P(S, T; X_p)} - 1, \text{ for American Options,}$$

Figure 1: Observed Call and Put prices on S&P500 in 08/31/2006
where $X_p = \frac{F}{1+x} < F < F(1 + x), \ x > 0$.

The SK was addressed for the following stochastic processes: Constant Elasticity of Variance (CEV), include arithmetic and geometric Brownian motion. Stochastic Volatility processes, the benchmark model being those for which volatility evolves independently of the asset price. And the Jump-diffusion processes, the benchmark model is the Merton’s (1976) model. For that classes Bates (1996) obtained the following result.

**Proposition 1** (Bates (1996)). For European options in general and for American options on futures, the SK has the following properties for the above distributions.

1. $SK(x) \leq x$ for CEV processes with $\rho \leq 1$.
2. $SK(x) \leq x$ for jump-diffusions with log-normal jumps depending on whether $2\mu + \delta^2 \leq 0$.
3. $SK(x) \leq x$ for Stochastic Volatility processes depending on whether $\rho_S \sigma \leq 0$.

Now in equation (19) consider

$$X_p = F(1 - x) < F < F(1 + x), \ x > 0.$$  

Then,

4. $SK(x) < 0$ for CEV processes only if $\rho < 0$.
5. $SK(x) \geq 0$ for CEV processes only if $\rho \geq 0$.

When $x$ is small, the two SK measures will be approx. equal. For in-the-money options ($x < 0$), the propositions are reversed. Calls $x\%$ in-the-money should cost $0\% - x\%$ less than puts $x\%$ in-the-money.

**Proof.** See Bates (1991).

Now using Th. 3.1 we can extend Bates’ result to Lévy processes.

**Proposition 2.** Consider Lévy measures given by (16). Under Assumption 1, we have

$$\beta \geq -1/2 \Rightarrow SK(x) \geq x, \ x > 0.$$  

And from here we obtain the sign of the excess of skewness premium for a huge class of Lévy processes.


5 Monotonicity and Symmetry Parameter

As we have seen in the last section we need the monotonicity of option prices with respect to the symmetry parameter to obtain our main result. The literature had study extensively the monotonicity properties of option prices. The main idea is to exploit the convexity preserving property\(^1\), to obtain the monotonicity of option prices with respect to certain parameter of the model. See Bergman, Grundy, and Wiener (1996), El Karoui, Jeanblanc-Pique, and Shreve (1998) and Ekström and Tysk (2007).

On the other hand, this question is very related to the ordering of option prices by changing the equivalent martingale measure. That is, imposing conditions on the predictable characteristic of the underlying process, an ordering in option prices with respect to the equivalent martingale measures is established, see Bellamy and Jeanblanc-Picque (2000), Henderson and Hobson (2003), Henderson (2005), Jakubenas (2002), Gushchin and Mordecki (2002) and Bergenthum and Rüschendorf (2006).

But we are interested in the possible mispecification in the models when using a fixed equivalent martingale measure. That is, if we change the parameter \(\beta\) on the Lévy measure described by (16) what happen with the option price. To answer partially that question, we can apply Lemma 5.1 in Gushchin and Mordecki (2002) for a certain group of Lévy measures (16), that is if the Lévy measure satisfy the assumptions of that lemma, we obtain and order in option prices. Then, Assumption 1 will be satisfied.

6 Conclusions

Under a given risk neutral probability measure. We use a measure of symmetry, introduced by Fajardo and Mordecki (2006), to address the Skewness premium under absence of symmetry. First, we analyze the sign of the Skewness premium using data from S&P500 and we obtain evidence that Bates’ \(x\)% rule does not hold. In that case we derive sufficient conditions for excess

\(^1\)We say that a model is convexity preserving, if for any convex contract function, the corresponding price is convex as a function of the price of the underlying asset at all times prior to maturity. Many models do not satisfy this property as for example general stochastic volatility models.
of SK to be positive or negative. In particular on the symmetry parameter. In this way we obtain simply diagnostic to observe what Lévy model deals with both the behavior of the underlying and with the sign of SK.

Interesting issues to study in future works are the empirical evidence of Assumption 1 and under what conditions, on the symmetry parameter, the monotonicity of option prices with respect to symmetry parameter holds. In that sense the results obtained by Bergenthum and Rüschendorf (2007) for the GH distributions can bring some insights.

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Table 1: Options prices Interpolating Put prices

| $K_c$ | $K_p = F^2/K_c$ | $x = K_c/F - 1$ | $x_{obs} = c_{obs}/p_{int} - 1$ | $x - x_{obs}$ |
|-------|----------------|-----------------|-------------------------------|--------------|
| 1230  | 1382.07        | -0.05662        | 0.050681                      | -0.1073      |
| 1235  | 1376.475       | -0.05278        | 0.13642                       | -0.1892      |
| 1240  | 1370.925       | -0.04895        | 0.115006                      | -0.16395     |
| 1245  | 1365.419       | -0.04511        | 0.197696                      | -0.24281     |
| 1250  | 1359.957       | -0.04128        | 0.277944                      | -0.31922     |
| 1255  | 1354.539       | -0.03744        | 0.280729                      | -0.31817     |
| 1260  | 1349.164       | -0.03361        | 0.536286                      | -0.5699      |
| 1265  | 1343.831       | -0.02977        | 0.574983                      | -0.60476     |
| 1270  | 1338.541       | -0.02594        | 0.606719                      | -0.63266     |
| 1275  | 1333.291       | -0.0221         | 0.675372                      | -0.69748     |
| 1280  | 1328.083       | -0.01827        | 0.691325                      | -0.70959     |
| 1285  | 1322.916       | -0.01443        | 0.966306                      | -0.98074     |
| 1290  | 1317.788       | -0.0106         | 0.904839                      | -0.91544     |
| 1295  | 1312.7         | -0.00676        | 0.794059                      | -0.80082     |
| 1300  | 1307.651       | -0.00293        | 0.78018                       | -0.78311     |
| 1305  | 1302.641       | 0.000905        | 0.614561                      | -0.61366     |
| 1310  | 1297.669       | 0.00474         | 0.532798                      | -0.52806     |
| 1315  | 1292.735       | 0.008575        | 0.427299                      | -0.41872     |
| 1320  | 1287.838       | 0.01241         | 0.108911                      | -0.0965      |
| 1325  | 1282.979       | 0.016245        | -0.11658                      | 0.132826     |
| 1330  | 1278.155       | 0.020079        | -0.45097                      | 0.471053     |
| 1335  | 1273.368       | 0.023914        | -0.50378                      | 0.527697     |
| 1340  | 1268.617       | 0.027749        | -0.61306                      | 0.640807     |
| 1345  | 1263.901       | 0.031584        | -0.73872                      | 0.770305     |
| 1350  | 1259.22        | 0.035419        | -0.81448                      | 0.849896     |
| 1355  | 1254.573       | 0.039254        | -0.80297                      | 0.842224     |
| 1360  | 1249.961       | 0.043089        | -0.82437                      | 0.867454     |
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$K_p$ & $K_c = F^2 / K_p$ & $x = F / K_p - 1$ & $x_{obs} = c_{int} / p_{obs} - 1$ \\
\hline
1250 & 1359.957 & 0.043056 & -0.88837 & 0.931421 \\
1255 & 1354.539 & 0.0389 & -0.86897 & 0.907873 \\
1260 & 1349.164 & 0.034778 & -0.85655 & 0.891331 \\
1265 & 1343.831 & 0.030688 & -0.78107 & 0.81176 \\
1270 & 1338.541 & 0.02663 & -0.70531 & 0.731941 \\
1275 & 1333.291 & 0.022604 & -0.63926 & 0.661869 \\
1280 & 1328.083 & 0.018609 & -0.51726 & 0.535865 \\
1285 & 1322.916 & 0.014646 & -0.31216 & 0.326801 \\
1290 & 1317.788 & 0.010713 & -0.20329 & 0.214005 \\
1295 & 1312.7 & 0.006811 & -0.03659 & 0.043397 \\
1300 & 1307.651 & 0.002938 & 0.090739 & -0.13175 \\
1305 & 1302.641 & -0.0009 & 0.130843 & -0.13175 \\
1310 & 1297.669 & -0.00472 & 0.252541 & -0.25726 \\
1315 & 1292.735 & -0.0085 & 0.261905 & -0.27041 \\
1320 & 1287.838 & -0.01226 & 0.242817 & -0.25507 \\
1325 & 1282.979 & -0.01598 & 0.346419 & -0.3624 \\
1330 & 1278.155 & -0.01968 & 0.183207 & -0.20289 \\
1335 & 1273.368 & -0.02336 & 0.237999 & -0.26135 \\
1340 & 1268.617 & -0.027 & 0.145858 & -0.17286 \\
1345 & 1263.901 & -0.03062 & 0.152637 & -0.18325 \\
1350 & 1259.22 & -0.03421 & 0.101211 & -0.13542 \\
1355 & 1254.573 & -0.03777 & -0.03964 & 0.001869 \\
1360 & 1249.961 & -0.04131 & 0.028337 & -0.06965 \\
1365 & 1245.382 & -0.04482 & -0.0101 & -0.03472 \\
1375 & 1236.325 & -0.05177 & -0.0451 & -0.00667 \\
\hline
\end{tabular}
\caption{Options prices Interpolating Call prices}
\end{table}