Computational studies of the EDL effect in 3-D developing flow in microchannel

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Abstract. Electric double layer (EDL) effect has been proven to have significant effect in microchannel. However most of models were based on 2-D flow, the first analysis on EDL effect on 3-D flow field was reported by Tan and Ng. In this paper, one of 3-D effects, which are the aspect ratio on the microchannel with EDL, will be shown and discussed. Parameters such as friction coefficient, $fR$ and Nusselt Number, $Nu$ were used to determine the performance microchannel. Nernst and Plank model was used here to model the EDL effect. It is found that in decreasing of aspect ratio, the values of $fRe$ increase. It is also concluded that the decrease in aspect ratio is resulted with an increase in the $Nu$ values.

1. Introduction
Microchannel, with its high area to volume ratio and low liquid mass, has great potential in area heat transfer and bioengineering. Electric double layer (EDL) effect has been proven to have significant effect in microchannel. However the most of model was based on 2-D flow, the first analysis on EDL effect on 3-D flow field was found in Tan and Ng [1]. In this study, the predictions base on assumption of Boltzmann's distribution of charge. Ng and Tan [2] further presented the effects of the charge density on the 3-D flow field including developing region. Nusselt numbers for each case such as with or without EDL effect are also included; so that the actual effectiveness of the micro-channel can be understood. When comparing to 2-D model, a 3-D model is far more challenging to handle and more CPU time is needed. However, for complex shapes and in cases where aspect ratio, AR is not closed to zero, 3-D model is unavoidable. In this paper, the main focus is on the EDL effect at different aspect ratio (AR)s, which demonstrate the important of 3-D model.

2. Mathematical model
A simple rectangular channel as shown in Figure 1. is used as the current physical model. The coordinate system is set such that the X, Y Z directions depict the width W, length L, and height H of the rectangular respectively. The main stream is along Z-direction. The origin is place at the mid-width of the rectangular and this permits computation to be done on half of the model, as the physical model is symmetric, only one half of the rectangular microchannel is considered for the computational domain (as shown in Figure 2.).
2.1. Surface electrostatic potential

In microscale flow, the effect of the EDL near the solid/liquid interface on liquid flow through a rectangular microchannel is significant and cannot be ignored. According to the theory of electrostatics, the relationship of the electrostatics potential, $\psi$ and charge density, $\rho_e$ is given by the Poisson’s equation [3]:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\kappa^2 \frac{\rho_e}{2n_0 z_1 e}$$

where $\psi$ is nondimensional electrical potential, $n_0$ is ion concentration in the fluid, $z_1$ is valence of the charge carrier, $e$ is charge of electron ($1.6 \times 10^{-19}$C) and $\kappa$ is Debye-Huckel parameter ($1/\kappa$ is referred to as the characteristic thickness of EDL). $x$, $y$ and $z$ is nondimensional coordinate in X, Y, Z direction.

The charge density, $\rho_e$ is defined as

$$\rho_e = (n^+ - n^-)z_1 e$$

$n^+$ and $n^-$ are the nondimensional ion concentration and are modeled by two Poisson equations named after Nernst-Plank [4]

$$\frac{\partial n^+}{\partial t} + \frac{\partial (n^+ u)}{\partial x} + \frac{\partial (n^+ v)}{\partial y} + \frac{\partial (n^+ w)}{\partial z} = \frac{1}{Sc Re} \left( \frac{\partial^2 n^+}{\partial x^2} + \frac{\partial^2 n^+}{\partial y^2} + \frac{\partial^2 n^+}{\partial z^2} \right)$$

$$\pm \frac{1}{Sc Re} \left[ \frac{\partial}{\partial x} (n^+ \partial \psi) + \frac{\partial}{\partial y} (n^+ \partial \psi) + \frac{\partial}{\partial z} (n^+ \partial \psi) \right]$$

where $u$, $v$, $w$ are nondimensional velocity value at X, Y, Z direction. $Sc$ and $Re$ is Schmidt number and Reynolds number respectively.
2.2. Navier-Stoke equations with EDL effect

An additional body force in Z-direction originating due to the presence of the EDL is considered so as to modify the conventional Navier-Stokes equation. Hence, the Z-direction momentum equation of 3-D steady state developing laminar flow for micro-channel is given as:

\[
\frac{u}{\partial x} + \frac{v}{\partial y} + \frac{w}{\partial z} = \frac{1}{Re} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial p}{\partial z} - G_1 E_z \bar{\rho}_e
\]

(4)

\(\bar{\rho}_e\) is nondimensional charge density. \(G_1\), \(E_z\) are defined as follows

\[
\bar{G}_1 = \frac{2 \nu \xi_o e \bar{\xi}_o}{\rho W_{in}^2}
\]

(5)

\[
E_Z = \left( \frac{D_h}{W_c} \right) \left( \frac{D_h}{L_c} \right) \bar{G}_2 \int \rho_e w \left( \frac{\partial n^+}{\partial z} - eD_f \frac{\partial n^-}{\partial z} \right) dA
\]

where \(\bar{\xi}_o\) is nondimensional electrical potential at wall, \(W_{in}\) is the inlet velocity in Z direction, \(\rho\) is the density of fluid and \(D_h\) is the hydraulic diameter.

3. Results and discussions

In this study, in this section, a total of 5 aspect ratio values, 0.30, 0.50, 0.67, 0.75 and 1.00 were chosen to study the effect of aspect ratio for microchannels. Figure 3 shows the fRe value distribution along z-direction (flow direction) for different AR values. The trend of the fRe value is similar. However, the values of fRe increase with decreasing of aspect ratio.

![fRe along Z direction for different aspect ratio, AR](image)

Figure 3. Friction Coefficient, fRe distribution along z-direction for different aspect ratios, AR
Figure 4. Friction coefficient, $f_{Re}$, at fully developed region for different aspect ratios, $AR$

Figure 4. summarizes the $f_{Re}$ value at fully developed region for flow field with and without the EDL effect. The EDL body force is an opposing force; therefore it is found in Figure 4, that the $f_{Re}$ values for flow field with EDL are generally higher than that without EDL effect. Similar to the $f_{Re}$ values, the distribution along $z$-direction at different aspect ratios have similar trend from the viewpoint of thermal aspect. In inlet region, the gradient in decreasing in $Nu$ value is very steep (as shown in Figure 5.). The $Nu$ value of each $AR$ value at gradient fully developed region is achieved when reaches about 8.

Figure 6. summarizes the $Nu$ value at the fully developed region for different aspect ratios. It is found that the smaller aspect ratio would result in higher $Nu$ values. With the higher opposing force, extra heat is produced and hence reduces the efficient of heat transfer. Therefore, the $Nu$ values with EDL effect are lower than that without EDL effect.

Figure 5. Nusselt Number, $Nu$ distribution along $z$-direction for different aspect ratios, $AR$
4. Conclusions
In all, the discrepancy of $fRe$ and $Nu$ values for different ARs are significant. Therefore a 3-D analysis is concluded to be important and necessary for investigating the EDL effect in microchannel. Once again, with and without EDL effect predict same trend of result but different in magnitudes.

5. References
[1] Tan S.T., and, Ng, E. Y-K., 2003, Numerical Studies of Developing Flow in Microchannel, *International Journal of Computational Engineering Science*, Special Issue on MEMs, Vol. 4, No:2, Imperial College Press, pp. 389-392.
[2] Ng, E. Y. K. and Tan, S.T., 2004, Computation of 3D Developing Pressure-driven Liquid Flow in Microchannel with EDL Effect, *Numerical Heat Transfer*, Part A: Applications, Vol. 45 1013-1027.
[3] Mala, G. M., Li, D., and Dale, J. D., 1997, Heat transfer and fluid flow in microchannels. *Int. J. Heat Mass Transfer* Vol 40 3079-3088.
[4] Yang, C. and Li, D., 1997, Electrokinetic effects on pressure-driven liquid flows in rectangular microchannels. J. Colloid Interface Science. Vol 194 95-107.