Role of Dual Higgs Mechanism in Chiral Phase Transition at Finite Temperature

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Abstract

The chiral phase transition at finite temperature is studied by using the Schwinger-Dyson equation in the dual Ginzburg-Landau theory, in which the dual Higgs mechanism plays an essential role on both the color confinement and the spontaneous chiral-symmetry breaking. At zero temperature, quark condensate is strongly correlated with the string tension, which is generated by QCD-monopole condensation, as \( \langle \bar{q}q \rangle^{1/3} \propto \sqrt{\sigma} \). In order to solve the finite-temperature Schwinger-Dyson equation numerically, we provide a new ansatz for the quark self-energy in the imaginary-time formalism. The recovery of the chiral symmetry is found at high temperature; \( T_C \sim 100\text{MeV} \) with realistic parameters. We find also strong correlation between the critical temperature \( T_C \) of the chiral symmetry restoration and the strength of the string tension.
In SU($N_c$) gauge theory, the appearance of magnetic monopoles was pointed out with the idea of the abelian gauge fixing based on the topological argument, $\pi_2(SU(N_c)/U(1)^{N_c-1}) = Z_{\infty}^{N_c-1}$, by 't Hooft. He conjectured that the dual Meissner effect would be realized if QCD-monopoles were condensed. The basic idea for color confinement is that the color-electric flux between quarks is squeezed like a string or a tube, because the color-electric fields are excluded in the QCD-vacuum. This is similar to the Meissner effect in the superconductor, where the ordinary magnetic fields are excluded. The linear-confining potential is produced, since the squeezed flux has a uniform energy per unit length. The recent lattice QCD simulations support this idea that QCD-monopoles play a crucial role on color confinement through their condensation.

The effective theory of non-perturbative QCD may be derived from QCD by using the abelian gauge fixing, which brings the non-abelian gauge theory into the abelian gauge theory with QCD-monopoles. In order to describe the above mentioned dual Meissner effect, the dual Ginzburg-Landau (DGL) theory is constructed following Nambu’s demonstration, which uses the relativistic version of the Ginzburg-Landau theory on the Abrikosov vortex solution in the superconductor. QCD-monopole condensation causes strong and long range correlations between a quark and antiquark pair, which produce the linear-confining potential through the dual Higgs mechanism. We assume in addition the abelian dominance, in which the non-abelian part does not contribute to non-perturbative phenomena at low energy and is neglected. We then studied chiral symmetry breaking in the DGL theory. It was found that QCD-monopole condensation plays an essential role on the dynamical mass generation of the quark, which means spontaneous chiral-symmetry breaking, by using the Schwinger-Dyson (SD) equation for the dynamical quark. We mention here that the lattice QCD study supports this idea of providing spontaneous chiral-symmetry breaking due to QCD-monopole condensation.

The high temperature and/or high density region would be realized in the laboratories as the intermediate states of the relativistic heavy-ion collisions, the study of which will be
made at RHIC in BNL and at LHC in CERN in the near future. The possible signals of a new phase, in which the liberation of colors, namely quarks and gluons, takes place, are expected to be seen in these experiments. Anticipating these experiments, the quark-gluon plasma (QGP) have been studied by many theorists. It is necessary for both confinement-deconfinement phase transition and chiral phase transition to understand the QCD phase transition. However, we do not have yet an effective theory to study the QGP physics based on the systematic treatment of these phase transitions, except for the lattice QCD simulations, which are not accessible for all the necessary physical quantities at the present.

We study the QGP physics on the basis of the DGL theory, which describes both color confinement and chiral symmetry breaking at zero temperature \[13\]. In particular, we concentrate on the manifestation of chiral symmetry at finite temperature with confinement properties, which are based on the dual Meissner effect in the QCD vacuum. In this respect, we would like to mention the successful use of the DGL theory on the confinement-deconfinement phase transition at finite temperature \[13\]. We shall formulate the finite-temperature SD equation using the imaginary-time formalism.

The DGL theory is an infrared effective theory of non-perturbative QCD based on the dual Higgs mechanism in the abelian gauge \[10\],

\[
\mathcal{L}_{\text{DGL}} = \text{tr} \hat{K}_{\text{gauge}}(A_\mu, B_\mu) + \bar{q}(i\gamma^\mu - eA^\mu - m_q)q + \text{tr}[\hat{D}_\mu, \chi] + \text{tr}[\hat{D}^\mu, \chi] - \lambda \text{tr}(\chi^\dagger \chi - v^2)^2 ,
\]

where \(\hat{D}_\mu \equiv \hat{\partial}_\mu + ig B_\mu\) is the dual covariant derivative. The dual gauge coupling \(g\) obeys the Dirac condition \(eg = 4\pi\) with \(e\) being the gauge coupling. The diagonal gluon \(A_\mu\) and the dual gauge field \(B_\mu\) are defined on the Cartan subalgebra \(T_3, T_8\) on \(\text{SU}(3)\):

\[A^\mu \equiv A_3^\mu T_3 + A_8^\mu T_8 , \quad B^\mu \equiv B_3^\mu T_3 + B_8^\mu T_8 .\]

The QCD-monopole field \(\chi\) is defined on the \(\text{SU}(3)\) root vectors \(E_\alpha\):

\[\chi \equiv \sqrt{2} \sum_{\alpha=1}^3 \chi_\alpha E_\alpha .\]

\(\hat{K}_{\text{gauge}}\) is the kinetic term of the gauge fields \((A_\mu, B_\mu)\) in the Zwanziger form \[17\],

\[
\hat{K}_{\text{gauge}} \equiv -\frac{1}{n^2} [n \cdot (\partial \wedge A)]^2 - \frac{1}{n^2} [n \cdot (\partial \wedge B)]^2 - \frac{2}{n^2} [n \cdot (\partial \wedge A)]_\nu [n \cdot (\partial \wedge B)]^\nu ,
\]

where the duality of the gauge theory is manifest. The DGL lagrangian has the \([\text{U}(1)_e]^2 \times [\text{U}(1)_m]^2\) gauge symmetry.
It is straightforward to calculate the static confining potential using the static quark-antiquark pair sources, which are totally color-singlet, located at a distance \( r = |b - a| \).

\[
\mathbf{j}_\mu = Q g_{\mu 0} \{ \delta^3(\mathbf{x} - \mathbf{b}) - \delta^3(\mathbf{x} - \mathbf{a}) \} \; ,
\]

where \( Q = (Q_3, Q_8) \) is the color charge of the static quark. The linear potential; \( \sigma r \) in the long distance is derived from the quark color-electric current-current correlation; \(-\frac{1}{2} j_\mu D^{\mu \nu} j_\nu \) with the static source (3). Here, \( D^{\mu \nu} \) denotes the diagonal gluon propagator. We then get the string tension as the simple expression [15],

\[
\sigma \simeq \frac{Q^2 m_B^2}{8 \pi} \ln \left( \frac{1 + \sqrt{1 - 3g^2 \lambda}}{1 - \sqrt{1 - 3g^2 \lambda}} \right) = \frac{2\pi v^2 \kappa}{\sqrt{\kappa^2 - 2}} \ln \left( \frac{\kappa + \sqrt{\kappa^2 - 2}}{\kappa - \sqrt{\kappa^2 - 2}} \right)
\]

(4)

with \( Q^2 = \frac{N_c - 1}{2N_c} e^2 = \frac{e^2}{3} \) for \( N_c = 3 \) and \( \kappa = \sqrt{\frac{2\pi}{3g^2}} \) corresponding to the Ginzburg-Landau parameter. The mass of \( B_\mu, m_B = \sqrt{3g} v \), is proportional to the QCD-monopole condensate \( v \) [5,6]. For the type-II limit \( (\kappa \gg \sqrt{2}) \), one finds \( \sigma \simeq 2\pi v^2 \ln(2\kappa^2) \), corresponding to the formula for the energy per unit length of the Abrikosov vortex in the type-II superconductor [5,8]. We may set parameters as \( e = 5.5, \lambda = 25 \) and \( v = 0.126 \text{GeV} \), which reproduce the string tension \( (\sqrt{\sigma} \simeq 0.44 \text{GeV}) \) and the cylindrical radius of the hadron flux tube \( (R \sim m_B^{-1} \simeq 0.4 \text{fm}) \) [3].

We get the ordinary SD equation by using the non-perturbative gluon propagator, obtained through QCD-monopole condensation in the DGL theory, as the full gluon propagator of QCD in order to include the non-perturbative effect in the infrared region. Hereafter, we concentrate on the case of the chiral limit for the simplicity of the argument. The SD equation for the dynamical quark propagator \( S_q(p) \) in the rainbow approximation is written as

\[
S_q^{-1}(p) = i\mathbf{p} + \int \frac{d^4k}{(2\pi)^4} Q^2 \gamma_\mu S_q(k) \gamma_\nu D_{\mu \nu}(p - k) \; ,
\]

(5)
in the Euclidean metric.

In the QCD-monopole condensed vacuum, the non-perturbative gluon propagator is derived by integrating out \( B_\mu \) [3,8,10],

\[4\]
\[ D_{\mu\nu}(k) = -\frac{1}{k^2} \left( \delta_{\mu\nu} + (\alpha_e - 1) \frac{k_\mu k_\nu}{k^2} \right) - \frac{1}{k^2} \frac{m_B^2}{m_B^2 + m_B^2} \epsilon_{\lambda\mu\alpha\beta} \epsilon_{\lambda\nu\gamma\delta} n_\alpha n_\beta k_\gamma k_\delta, \]

where \( \alpha_e \) is the gauge fixing parameter on the residual abelian gauge symmetry \([U(1) e]^2\). It is noted that this modified gluon propagator is asymptotically reduced to the original gluon propagator of QCD in the ultraviolet region; \( k^2 \gg m_B^2 \). We have introduced the infrared cutoff \( a \) in this propagator \( (6) \) corresponding to the dynamical quark-antiquark pair creation and/or the color confinement in the size of hadrons \([10]\).

We take the angular average on the direction of the Dirac string \( n_\mu \) in the SD equation,

\[
\left< \frac{1}{(n \cdot k)^2 + a^2} \right>_{\text{average}} \equiv \frac{1}{2\pi^2} \int d\Omega_n \frac{1}{(n \cdot k)^2 + a^2} = \frac{2}{a} \frac{1}{a + \sqrt{k^2 + a^2}}. \]

Here, the dynamical quark is considered to move in various directions inside hadrons, and hence the constituent quark mass would be regarded as the quark self-energy in the angle-averaged case \([10]\). Taking a simple form for the quark propagator as \( S_{q}^{-1}(p) = i\not{p} - M(p^2) \), the SD equation for the quark self-energy \( M(p^2) \) is obtained by taking the trace of Eq.(5) in the Landau gauge \( (\alpha_e = 0) \),

\[
M(p^2) = \int \frac{d^4k}{(2\pi)^4} Q^2 \frac{M(k^2)}{k^2 + M^2(k^2)} \times \left[ \frac{2}{k^2 + m_B^2} + \frac{1}{k^2} + \frac{4}{a} \frac{1}{a + \sqrt{k^2 + a^2}} \left( \frac{m_B^2 - a^2}{k^2 + m_B^2 + a^2} \right) \right] \]

with \( \tilde{k}_\mu \equiv p_\mu - k_\mu \). It is noted that the r.h.s. of Eq.(8) is always non-negative.

We solve Eq.(8) numerically using the Higashijima-Miransky approximation \([16]\), \( Q^2 = 4\pi C_F \cdot \alpha_s^{\text{eff}}(\max\{p^2, k^2\}) \) with \( C_F = \frac{N_c^2 - 1}{2N_c} \) for SU\((N_c)\) in order that the SD equation is reduced to the usual one of the QCD-like theory \([14,17]\) in the ultraviolet region. As a consequence, the quark self-energy \( M(p^2) \) has the asymptotic form, which is obtained by a study of the operator product expansion and the renomalization group \([10]\), in the ultraviolet limit. The running coupling with the hybrid type behavior is obtained as

\[
\alpha_s^{\text{eff}}(p^2) = \frac{12\pi}{(11N_c - 2N_f) \ln[(p_c^2 + p_c^2)/\Lambda_{\text{QCD}}^2]} : \ p_c^2 = \Lambda_{\text{QCD}}^2 \exp\left[\frac{48\pi^2(N_c + 1)}{(11N_c - 2N_f) \cdot e^2} \right], \]

where \( p_c \) approximately divides the momentum scale into the infrared region and the ultraviolet region with the QCD scale parameter \( \Lambda_{\text{QCD}} \) fixed at 200MeV. Here, \( p_c \) is fixed by
$\alpha_{s}^{\text{eff}}(p^2 = 0) = \alpha_{s}/(N_c + 1); \alpha_{s} = e^2/4\pi$. The magnitude of the quark condensate plays the role of an order parameter of spontaneous chiral-symmetry breaking. We can calculate the quark condensate with the thus obtained quark self-energy $M(p^2)$ from the SD equation as

$$\langle \bar{q}q \rangle^\Lambda = -\frac{N_c}{4\pi^2} \int_0^{\Lambda^2} dk^2 \frac{k^2 M(k^2)}{k^2 + M^2(k^2)},$$

where the ultraviolet cutoff $\Lambda$ is introduced to regularize the ultraviolet divergence of the integral. If the ultraviolet cutoff is taken in the deeply asymptotic region, $\Lambda \gg m_B$, one can get the quark condensate in the renormalization-group invariant form for $N_c = N_f = 3$,

$$\langle \bar{q}q \rangle_{\text{RGI}} = \langle \bar{q}q \rangle^\Lambda \cdot \{\ln(\Lambda^2/\Lambda_{\text{QCD}}^2)\}^{-4/9}.$$

The ultraviolet cutoff is taken to be large enough as $\Lambda/\Lambda_{\text{QCD}} = 10^3$ hereafter. One finds that QCD-monopole condensation provides a large contribution to spontaneous chiral-symmetry breaking [6,8–10], because the mass of the dual gauge field, $m_B$, is proportional to the QCD-monopole condensate [5,6] and the quark condensate increases with $m_B$ [6,8–10]. In other words, the strength of spontaneous chiral-symmetry breaking seems originated from QCD-monopole condensation by the sufficient strength of the string tension as shown in Fig. 1. The parameter set is taken as $e = 5.5, m_B = 0.5\text{GeV}$ and $a = 85\text{MeV}$, where these parameters except for $a$ are set by fitting the confinement phenomena at zero temperature. In this case, the quark condensate and the pion decay constant at zero temperature are well-reproduced as $\langle \bar{q}q \rangle_{\text{RGI}} \simeq -(247\text{MeV})^3$ and $f_\pi \simeq 88\text{MeV}$, respectively [3,10].

Now we study the chiral symmetry restoration at finite temperature using the imaginary-time formalism for the SD equation [13]. The finite-temperature SD equation is obtained by making the following replacement in Eq.(8) [14]:

$$p_4 \rightarrow \omega_n = (2n + 1)\pi T,$$

$$\int \frac{d^3k}{(2\pi)^3} \rightarrow T \sum_{n=-\infty}^{\infty} \int \frac{dk}{(2\pi)^3},$$

$$M(p^2) \rightarrow M_T(\omega_n; \mathbf{p}).$$

The resulting equation is very hard to solve even numerically since the quark self-energy depends not only on the three dimensional momentum $\mathbf{p}$, but also on the Matsubara fre-
quencies $\omega_n$ [20]. We propose the covariant-like ansatz [18], in which a replacement is made for the quark self-energy at $T \neq 0$ instead of (11c) as

$$M(p^2) \rightarrow M_T(p^2 + \omega_n^2) \equiv M_T(\hat{p}^2)$$  \hspace{1cm} (12)

with $\hat{p}^2 \equiv p^2 + \omega_n^2$ and $\omega_n = (2n + 1)\pi T$ [18]. It is noted that this ansatz guarantees that the finite-temperature SD equation in the limit $T \rightarrow 0$ is exactly reduced to the SD equation (8) at $T = 0$. This fact is also confirmed by numerical calculations as shown in Fig. (2). The final form of the SD equation at $T \neq 0$ is derived as

$$M_T(\hat{p}^2) = \frac{T}{8\pi^2} \sum_{m=\infty}^{\infty} \int_{\omega_m^2}^\infty dk^2 \int_{-1}^1 d\zeta Q^2 \sqrt{k^2 - \omega_m^2} \frac{M_T(k^2)}{k^2 + M_T^2(k^2)}$$

$$\times \left[ \frac{2}{k_{nm}^2 + m_B^2} \right. + \frac{1}{k_{nm}^2 + m_B^2} + \frac{4}{a} \frac{1}{\sqrt{k_{nm}^2 + a^2}} \left( \frac{m_B^2 - a^2}{k_{nm}^2 + m_B^2} + \frac{a^2}{k_{nm}^2 + m_B^2} \right) \right] ,$$  \hspace{1cm} (13)

where $\tilde{k}_{nm}^2 = \hat{k}^2 + \hat{p}^2 - 2\zeta \sqrt{(\hat{p}^2 - \omega_n^2)(\hat{k}^2 - \omega_m^2)} - 2\omega_m\omega_n$. It is noted that the large number of the Matsubara frequencies have negligible contribution at high temperature in Eq.(13) since $\tilde{k}_{nm}^2 \sim (\omega_m - \omega_n)^2$ at $T \gg |p|, |k|$. In other words, it is important for the small number of the Matsubara frequencies to generate the non-trivial solution of the quark self-energy $M_T(\hat{p}^2)$ in r.h.s. of Eq.(13). We then solve the SD equation by setting $\omega_n = 0$ in the r.h.s. of Eq.(13). In addition, it is necessary to truncate the infinite sum of $m$ in order to solve it numerically. We check the numerical convergence by this truncation for the $m$ sum, which is quite fast at high temperatures. In the effective running coupling eq.(9), there is only a slight modification as $Q^2 = 4\pi C_F \cdot \alpha_s^{\text{eff}}(\max\{\hat{p}^2, \hat{k}^2\})$ due to the covariant-like ansatz. We show in Fig. 2, the quark self-energy $M_T(\hat{p}^2)$ as a function of $\hat{p}^2$ at finite temperature. No nontrivial solution is found in the high temperature region, $T > 110\text{MeV}$. In other words, the chiral symmetry is restored at high temperature.

The quark condensate is easily calculated using the quark self-energy $M_T(\hat{k}^2)$ as

$$\langle \bar{q}q \rangle_T^\Lambda = -4N_c \cdot T \sum_{n=-n_{\text{max}}}^{n_{\text{max}}} \int_{\omega_n^2}^\Lambda d\hat{k}^2 \sqrt{\hat{k}^2 - \omega_n^2} \frac{M_T(\hat{k}^2)}{\hat{k}^2 + M_T^2(\hat{k}^2)}$$

$$\simeq -\frac{2N_c}{\pi^2} \cdot T \sum_{n=0}^{n_{\text{max}}} \int_{\omega_n^2}^\Lambda d\hat{k}^2 \sqrt{\hat{k}^2 - \omega_n^2} \frac{M_T(\hat{k}^2)}{\hat{k}^2 + M_T^2(\hat{k}^2)}$$  \hspace{1cm} (14)
with \( n_{\text{max}} \equiv \left[ \frac{\Lambda}{2\pi T} - \frac{1}{2} \right] \). The quark condensate, \( \langle \bar{q}q \rangle^\Lambda \), shown in Fig.3, decreases gradually with temperature in the low temperature region \( (\lesssim 0.5T_c) \) and vanishes suddenly near the critical temperature \( T_c \). We mention that the behavior of the quark condensate in the low temperature region is similar to that of the calculations on the chiral perturbation theory, namely \( \langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_{T=0} = 1 - T^2/8f_{\pi}^2 + O(T^4) \) \[21\]. In addition, these results are slightly different from the lattice QCD simulation \[22\], which shows that the temperature dependence of the chiral condensate is small until near the critical temperature. In these calculations, we do not take any temperature dependence in the parameters used for simplicity, although we should take it into account as more information on high temperature QCD becomes available.

Finally, we examine the correlation between the critical temperature \( T_c \) of the chiral symmetry restoration and the confinement quantity such as the string tension \( \sigma \). As shown in Fig. 4, there exists a strong correlation between them. From the physical point of view, higher \( T_c \) would be necessary to restore the chiral symmetry in the system where the linear-confining potential is much stronger, because there exists the close relation \[3,10–12\] between color confinement and dynamical chiral-symmetry breaking through QCD-monopole condensation as shown in Fig. 1. The critical temperature is estimated as \( 100\text{MeV} \lesssim T_c \lesssim 110\text{MeV} \), when one takes the standard value of the string tension \( \sqrt{\sigma} \simeq 0.44\text{GeV} \). We remark here that the critical temperature would increase if we take smaller cutoff parameter \( a \) at finite temperature, which is expected as due to the increase of the size of hadrons because of smaller confining force \[13\]. Hence, it would be very interesting to couple the calculations of QCD-monopole condensation and chiral symmetry breaking at finite temperature.

In summary, we have studied spontaneous chiral-symmetry breaking and its restoration at finite temperature using the SD equation in the DGL theory, which is known to provide both quark confinement and dynamical chiral-symmetry breaking. We have solved the finite-temperature SD equation numerically with the covariant-like ansatz. The chiral symmetry is restored at \( T_c \sim 100\text{MeV} \). We have found the strong correlation between the critical
temperature $T_c$ and the string tension $\sigma$.

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FIGURE CAPTIONS

FIG.1. The correlation between the quark condensate and the mass of the dual gauge field and also the string tension. The other parameters are fixed as \( e = 5.5 \) and \( a = 85 \text{MeV} \). The unit is taken by \( \Lambda_{\text{QCD}} \simeq 200 \text{MeV} \).

FIG.2. The quark self-energy \( M_q(\hat{p}^2) \) as a function of \( \hat{p}^2 \) at \( T = 0 \), 60 and 100MeV with \( e = 5.5 \), \( m_B = 0.5 \text{GeV} \) and \( a = 85 \text{MeV} \).

FIG.3. The ratio of the quark condensate at finite temperature to that of zero temperature as a function of temperature normalized to the critical temperature \( T_C \). The same parameters are used as in Fig.2. \( T_C \) is found at \( T_C \simeq 110 \text{MeV} \) in this case.

FIG.4. The critical temperature \( T_C \) as a function of the square root of the string tension \( \sigma \). \( T_C \) rises almost linearly with \( \sqrt{\sigma} \).
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