Decaying Sterile Neutrinos and the Short Baseline Oscillation Anomalies

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The MiniBooNE experiment has observed a significant excess of electron neutrinos in a muon neutrino beam, in conflict with standard neutrino oscillations. We discuss the possibility that this excess is explained by a sterile neutrino with a mass ~ 1 keV that decays quickly back into active neutrinos plus a new light boson. This scenario satisfies terrestrial and cosmological constraints because it has neutrino self-interactions built-in. Accommodating also the LSND, reactor, and gallium anomalies is possible, but requires an extension of the model to avoid cosmological limits.

I. INTRODUCTION

Many major discoveries in neutrino physics have started out as oddball anomalies that gradually evolved into incontrovertible evidence. In this work, we entertain the possibility that history is repeating itself in the context of the MiniBooNE anomaly. From 2002 to 2019, the MiniBooNE experiment has been searching for electron neutrinos ($\nu_e$) appearing in a muon neutrino ($\nu_\mu$) beam [1–3], and has found a corresponding signal at 4.8$\sigma$ statistical significance. For some time, the simplest explanation for this signal appeared to be the existence of a fourth neutrino species $\nu_s$, called “sterile neutrino” because it would not couple to any of the Standard Model interactions, but would communicate with the Standard Model only via neutrino mixing. If $\nu_s$ has small but non-zero mixing with both $\nu_e$ and $\nu_\mu$, and if the corresponding mostly sterile neutrino mass eigenstate $\nu_4$ is somewhat heavier ($\sim$ 1 eV) than the Standard Model neutrinos, the MiniBooNE signal could be explained. This explanation would also be consistent with a similar 3.8$\sigma$ anomaly from the earlier LSND experiment [4], and with several reported hints for anomalous disappearance of electron neutrinos in reactor experiments [5, 6] and in experiments using intense radioactive sources [7, 8]. However, the sterile neutrino parameter space consistent with MiniBooNE and these other anomalies is in severe tension with the non-observation of anomalous $\nu_\mu$ disappearance [9–19], unless several additional new physics effects are invoked concomitantly [20, 21].

In this work, we propose a different explanation for the MiniBooNE anomaly, and possibly also for the LSND, reactor, and gallium anomalies. In particular, we consider a sterile neutrino that rapidly decays back into Standard Model (“active”) neutrinos $\nu_a$ [22–24]. The MiniBooNE excess is then interpreted as coming from these decay products. We will see that this scenario requires only very small mixing between $\nu_s$ and $\nu_\mu$, thus avoiding the strong $\nu_\mu$ disappearance constraints. It also requires somewhat larger mixing between $\nu_s$ and $\nu_e$, in line with the hints from reactor and radioactive source experiments. Finally, we will argue that decaying sterile neutrinos may avoid cosmological constraints because the model automatically endows sterile neutrinos with self-interactions (“secret interactions” [25, 26]).

II. DECAYING STERILE NEUTRINO FORMALISM

We extend the Standard Model by a sterile neutrino $\nu_s$ (a Dirac fermion) and a singlet scalar $\phi$. The relevant interaction and mass terms in the Lagrangian of the model are

$$\mathcal{L} = -g \bar{\nu}_s \nu_s \phi - \sum_{a=\alpha, \mu, \tau, s} m_{\alpha\beta} \bar{\nu}_a \nu_\beta .$$

(1)

The neutrino flavor eigenstates $\nu_\alpha$ are linear combinations of the mass eigenstates $\nu_j$ ($j = 1, 2, 3, 4$) according to the relation $\nu_\alpha = U_{\alpha j} \nu_j$, where $U$ is the unitary $4 \times 4$ leptonic mixing matrix. The first term in eq. (1) can thus be rewritten as

$$-g \bar{\nu}_F \nu_F \phi - g |U_{s4}|^2 \bar{\nu}_4 \nu_4 \phi - (g U_{s4}^* \bar{\nu}_4 \nu_F \phi + h.c.),$$

(2)

with

$$\nu_F \equiv \sum_{i=1}^3 U_{si} \nu_i .$$

(3)

We assume initially that the fourth, mostly sterile, mass eigenstate $\nu_4 \simeq \nu_s$ has a mass $m_4$ between $\mathcal{O}(eV)$ and...
$O(100 \text{ keV})$, and that the mass of $\phi$ is of the same order, but smaller. The last term in eq. (2) will then induce $\nu_4 \rightarrow \nu_F + \phi$ decays, while the first term is responsible for $\phi \rightarrow \nu_F + \bar{\nu}_F$ decays. When these decays occur in a neutrino beam, they will produce lower-energy neutrinos at the expense of higher-energy ones, and they may also alter the flavor structure of the beam. In particular, they can produce excess low-energy $\nu_e$ in a $\nu_\mu$ beam, as suggested by the MiniBooNE anomaly.

The phenomenology of the model depends mainly on five new parameters. Besides $m_4$ and $m_\phi$, these are the coupling $g$ and the mixings $|U_{e4}|^2$, $|U_{\mu 4}|^2$ between $\nu_4$ and $\nu_e$, $\nu_\mu$. We will assume the mixing with $\nu_\tau$ to be zero and neglect the complex phases, as these parameters do not play an important role in explaining the MiniBooNE excess. For practical purposes, it is convenient to quote $m_4 \Gamma_4$ instead of $g$, as $m_4 \Gamma_4$ appears directly in the laboratory frame decay length $E/(m_4 \Gamma_4)$. Also, it is convenient to use the ratio $m_\phi/m_4$ instead of just $m_\phi$ because the ratio measures more directly the kinematic suppression in $\nu_4$ decays.

The evolution in energy $E$ and time $t$ of a neutrino beam in our model can be described by a neutrino density matrix $\hat{\rho}_\nu(E, t)$ (a $4 \times 4$ matrix in flavor space), the corresponding antineutrino density matrix $\hat{\rho}_{\bar{\nu}}(E, t)$, and the scalar density function $\rho_\phi(E, t)$. The evolution equations are [27, 28],

\[
\frac{d\hat{\rho}_\nu(E, t)}{dt} = -i[\hat{H}, \hat{\rho}_\nu] - \frac{1}{2} \left\{ \frac{m_\phi}{E} \hat{\Gamma}, \rho \right\} + \mathcal{R}_\nu[\hat{\rho}_\nu, \rho_\phi, E, t] \tag{4}
\]

\[
\frac{d\hat{\rho}_{\bar{\nu}}(E, t)}{dt} = -\frac{m_\phi}{E} \hat{\Gamma}_\phi \rho_\phi + \mathcal{R}_{\bar{\nu}}[\hat{\rho}_{\bar{\nu}}, E, t] \tag{5}
\]

where $\hat{H} = \frac{1}{2E} \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2)$ is the standard neutrino oscillation Hamiltonian, written here in the mass basis, and $\hat{\Gamma} = \Gamma_4 \hat{\Pi}_4$ is the decay term, which contains the projection operator $\hat{\Pi}_4 = |\nu_4\rangle \langle \nu_4|$ onto the fourth, mostly sterile, mass eigenstate as well as the decay width $\Gamma_4$ of $\nu_4$ in its rest frame. Similarly, $\Gamma_\phi$ is the rest frame decay width of $\phi$. The functional $\mathcal{R}_\nu[\hat{\rho}_\nu, \rho_\phi, E, t]$ describes the appearance of the daughter neutrinos from $\nu_4$ and $\phi$ decay. Neglecting the masses of $\nu_1$, $\nu_2$, and $\nu_3$, it is given by

\[
\mathcal{R}_\nu[\hat{\rho}_\nu, \rho_\phi, E, t] = \hat{\Pi}_F \int_E^{\infty} dE_4 \sum_{k} \hat{\rho}_{\nu,44}(E_4, t) \frac{d\Gamma_{\text{lab}}(\nu_4 \rightarrow \nu_k \phi)}{dE_k} + \hat{\Pi}_F \int_E^{\infty} dE_\phi \rho_\phi(E_\phi, t) \frac{d\Gamma_{\text{lab}}(\phi \rightarrow \nu_k \bar{\nu}_j)}{dE_k}, \tag{6}
\]

where $d\Gamma_{\text{lab}}(X \rightarrow Y)/dE_k$ are the differential decay widths for the various decays $X \rightarrow Y$ in the lab frame, and $x_{4i} \equiv m_\phi/m_4$. The projection operator

\[
\hat{\Pi}_F = \frac{|\nu_F\rangle \langle \nu_F|}{|\nu_F\rangle \langle \nu_F|} = \sum_{i,j=1}^{3} \frac{U_{ei}^* U_{ej}}{|U_{ei}|^2} |\nu_i\rangle \langle \nu_j| \tag{7}
\]

isolates the specific combination of mass eigenstates that appears in $\nu_4$ and $\phi$ decays, and the integrals run over all parent energies $E_4$, $E_\phi$ that lead to daughter neutrinos of energy $E$. Analogously, $\mathcal{R}_{\bar{\nu}}[\hat{\rho}_{\bar{\nu}}, E, t]$ describes the appearance of scalars from $\nu_4$ decay:

\[
\mathcal{R}_{\bar{\nu}}[\hat{\rho}_{\bar{\nu}}, E, t] = \int_E^{E/x_{44}^2} dE_4 \sum_{k} \hat{\rho}_{\bar{\nu},44}(E_4, t) \frac{d\Gamma_{\text{lab}}(\nu_4 \rightarrow \nu_k \phi)}{dE_\phi} + \hat{\rho}_{\bar{\nu},44}(E_4, t) \frac{d\Gamma_{\text{lab}}(\bar{\nu}_4 \rightarrow \bar{\nu}_k \phi)}{dE_\phi}. \tag{8}
\]

With the appearance terms $\mathcal{R}_\nu[\hat{\rho}_\nu, \rho_\phi, E, t]$ and $\mathcal{R}_{\bar{\nu}}[\hat{\rho}_{\bar{\nu}}, E, t]$ defined, the equations of motion (4) and (5) can be solved analytically if we neglect matter effects. Neglecting furthermore the small mass splittings between the three light neutrino mass eigenstates, the electron neutrino flux $\phi_e(L, E)$ appearing in a muon neutrino beam of energy $E$ after a distance $L$ due to oscillations and decay is given by

\[
\phi_e(L, E) = \phi_\mu(0, E) |U_{e4}|^2 |U_{\mu 4}|^2 \left[ 1 + e^{-\frac{m_4^2 L}{2E}} - 2e^{-\frac{m_4^2 L}{2E}} \cos \left( \frac{\Delta m_{31}^2 L}{2E} \right) \right] + |U_{\mu 4}|^2 \frac{|\langle \nu_4 | \nu_F \rangle|^2}{|\langle \nu_4 | \nu_F \rangle|^2} I. \tag{9}
\]

Here, $|\nu_F\rangle$ is the superposition of mass eigenstates into which the $\nu_4$ decay (defined in eq. (3)), and the decay integral
\[ I = \int_{E/(1-x^2_{\phi}L)}^{\infty} dE_4 (1 - e^{-m_{\phi}^4 E_4}) \phi_{\mu}(0, E_4) \sum_j \frac{m_{\phi}^4 \Gamma_4}{E_4} \frac{d\Gamma_{\text{lab}}(\nu_4 \to \nu_j \phi)}{dE} + \int_E^{\infty} dE_\phi \int_{E_{\phi}/x^2_{\phi}}^{\infty} dE_4 \frac{m_{\phi}^4 \Gamma_4}{E_4} \frac{m_{\phi}^4 \Gamma_4 L}{E_\phi} \left[ (1 - e^{-m_{\phi}^4 E_\phi}) - (1 - e^{-m_{\phi}^4 E_4}) \right] \frac{m_{\phi}^4 \Gamma_4 L}{E_\phi} \right] \times \frac{1}{m_{\phi}^4 \Gamma_4} \sum_j \left[ \phi_{\mu}(0, E_4) \frac{d\Gamma_{\text{lab}}(\nu_4 \to \nu_j \phi)}{dE} + \phi_{\mu}(0, E_4) \frac{d\Gamma_{\text{lab}}(\bar{\nu}_4 \to \nu_j \phi)}{dE} \right] \sum_{i,j} \frac{m_{\phi}^4 \Gamma_4}{E_\phi} \frac{d\Gamma_{\text{lab}}(\phi \to \nu_i \bar{\nu}_j)}{dE}. \]  

(10)

In the above equations, \( \phi_{\mu}(0, E) \) and \( \tilde{\phi}_{\mu}(0, E) \) are the initial \( \nu_\mu \) and \( \bar{\nu}_\mu \) fluxes, respectively. A completely analogous equation describes \( \nu_e \) appearance.

The physical interpretation of eq. (9) is straightforward: the first term on the right-hand side describes \( \nu_\mu \to \nu_e \) oscillations, altered by the removal of neutrinos at energy \( E \) due to \( \nu_4 \) decay. In fact, this contribution matches the result of ref. [29], on invisible \( \nu_4 \) decay. The second term gives the contribution from neutrinos generated in \( \nu_4 \) and \( \phi \) decays. The factor \( |U_{\mu 4}|^2 \) arises because \( \nu_4 \) is the only mass eigenstate that decays. It describes the amount of \( \nu_4 \) in the \( \nu_4 \) beam. The factor \( |\langle \nu_e | \nu_F \rangle|^2 \) is the probability of the decay product to be detected as an electron neutrino, and the integral \( I \) controls the energy distribution of the decay products.

Analytic expressions for the decay widths appearing in eqs. (4) to (10) are given in appendix B.

III. FIT TO MINIBOONE DATA

To compare the predictions of the decaying sterile neutrino scenario to MiniBooNE data, we evolve the un-oscillated beam following the formulas given above. We then follow the fitting procedure recommended by the MiniBooNE collaboration (see the data releases accompanying refs. [1, 3]), but go beyond it by accounting for the impact of \( \nu_\mu \) and \( \nu_e \) disappearance on the signal and background normalization (see Appendix for details).

Illustrative results are shown in fig. 1, where we have chosen parameter values that give an optimal fit to MiniBooNE data while being consistent with null results from other oscillation experiments, as well as no-oscillation constraints. At \( m_4 \Gamma_4 = 2.1 \text{eV}^2 \), most \( \nu_4 \) will have decayed before reaching the detector. The value \( m_\phi / m_4 = 0.82 \) implies mild phase space suppression in \( \nu_4 \) decays, which tends to shift the \( \nu_e \) spectrum to lower energies, in excellent agreement with the data. Compared to models with massless \( \phi \) [22, 23], our scenario also has the advantage that it allows \( \phi \to \nu_F \bar{\nu}_F \) decays, further boosting the \( \nu_e \) flux at low energies. It is therefore favored compared to the \( m_\phi = 0 \) case at more than 99% confidence level. The fit in our model is better than in oscillation-only scenarios (blue dotted histogram in fig. 1) [19], which by themselves already offer an excellent fit as long as only MiniBooNE data are considered (MiniBooNE quotes a \( \chi^2 \) per degree of freedom of 9.9/6.7 [3]). Our model, however, is also consistent with all constraints. Notably, it reproduces the angular distribution of the neutrino interaction products in MiniBooNE because it predicts an actual flux of electron neutrinos instead of attempting to mimic the signal with other particles [30–35]. In particular, the angle between the parent \( \nu_4 \) and the daughter \( \nu_e \) is suppressed by a large Lorentz boost \( \gamma \sim \mathcal{O}(1000) \) [36]. This boost is sufficient to ensure that the daughter neutrinos enter the MiniBooNE detector, which is a \( \sim 6 \text{m sphere located} \sim 500 \text{m from the primary target, under essentially the same angle as the parent neutrino would have done.} \)

IV. CONSTRAINTS

We now discuss the various constraints that an explanation of the MiniBooNE anomaly in terms of decaying sterile neutrinos has to respect. The most relevant constraints are also summarized in figs. 2 and 3.

(1) Oscillation null results. Putting MiniBooNE into context with other \( \nu_e \) appearance searches, we show in fig. 3 two slices through the 5-dimensional parameter space of the decaying sterile neutrino model along the plane spanned by \( |U_{e4}|^2 \) and \( |U_{\mu 4}|^2 \). To produce this figure, we have used fitting codes from refs. [9, 12, 19] (based partly on refs. [37–39]). We see that most of the parameter region preferred by MiniBooNE is well compatible with the KARMEN short-baseline oscillation search [40] and with the OPERA long-baseline experiment [41]. We have checked that the limits from ICARUS [42–44] and E776 [45] are significantly weaker.

All constraints on \( |U_{e4}|^2 \) \((|U_{\mu 4}|^2)\) from \( \nu_e \) \((\nu_\mu)\) disappearance experiments are avoided [17, 19, 46]. This is mostly because in pure oscillation scenarios the number of excess events in MiniBooNE and LSND is proportional to \( |U_{e4}|^2 |U_{\mu 4}|^2 \), while in our scenario it is proportional only to \( |U_{\mu 4}|^2 \) as long as \( |U_{e4}|^2 \gg |U_{\mu 4}|^2 \). Therefore, it agrees well even with the tightest constraints [47, 48].

We can already see from fig. 3 that MiniBooNE is also
Majorana particles, the non-observation so far of neutrinos looking for anomalous features in beta decay 
black dashed lines in fig. 3). Direct searches for sterile neutrinos in the early Universe. We will see be-
ferred by the reactor anomaly, but only in a parameter 
ophysical particles in the early Universe
consistent with all null results (blue dotted histogram, parameter values $\Delta m^2_{41} = 0.13 \text{eV}^2$, $|U_{e4}|^2 = 0.024$, $|U_{\mu 4}|^2 = 0.63$).

(4) $N_{\text{eff}}$, a measure for the energy density of relativistic particles in the early Universe (green region in fig. 2). The measured value of $N_{\text{eff}}$ is very close to the SM value of $\sim 3$ both at the BBN and recombination epochs [54, 55]. Naively, one might expect that this observation precludes the existence of a fourth neutrino species with $m_4 \lesssim \text{MeV}$. In our model, however, the $N_{\text{eff}}$ constraint is avoided by the “secret interactions” mechanism [25, 26]: any small abundance of $\nu_s$ generates a temperature-dependent potential $V_{\text{eff}} \propto g^2 T$, reducing the $\nu_s - \nu_\alpha$ mixing by a factor $\sqrt{\Delta m^2 / (E V_{\text{eff}})}$. Hence, the production of $\nu_s$ is suppressed until the temperature drops low enough. For the parameter range that the short-baseline anomalies are pointing to, this can easily be postponed to late times ($T \ll \text{MeV}$), after neutrino-electron decoupling. Consequently, when $\nu_s$ are eventually produced, they are produced at the expense of active neutrinos, so $N_{\text{eff}}$ does not change any more and constraints are automatically satisfied. More quantitatively, $N_{\text{eff}}$ constraints are avoided when

$$ (m_4 \Gamma_4)^{\text{eff}} \gtrsim 2 \times 10^{-14} \text{eV}^2 \left(\frac{m_4}{\text{eV}}\right)^4, $$

where we have defined

$$ (m_4 \Gamma_4)^{\text{eff}} \equiv \frac{m_4 \Gamma_4}{|U_{e4}|^2 |U_{\mu 4}|^2 \left(1 - \frac{m_4^2}{m_4^2}\right)^2}. $$
This constraint can be easily satisfied in the mass range allowed by beta decay limits.

\(5\) \(\sum m_\nu\), the sum of neutrino masses. Massive neutrinos affect the CMB as well as structure formation, and this has for instance allowed the Planck collaboration to set a limit \(\sum m_\nu \lesssim 0.12\) eV [55]. In our model, this constraint is easily satisfied because in the interesting parameter range with \(m_4 \gg 1\) eV and \(m_4 \Gamma_4 \gtrsim 1\) eV\(^2\), any \(\nu_4\) that are produced in the early Universe will have decayed via \(\nu_4 \rightarrow \nu_{1,2,3} + (\phi \rightarrow \nu_{1,2,3}\nu_{1,2,3})\) long before recombination and the onset of structure formation.

\(6\) Neutrino Free-Streaming (blue region in fig. 2 and gray dotted lines in fig. 3). Via the mixing with \(\nu_4\), also the light neutrino mass eigenstates \(\nu_{1,2,3}\) feel \(\phi\)-mediated interactions and are therefore not fully free-streaming. This may put the model in tension with CMB observations, which require that neutrinos should free-stream from about redshift 10\(^5\) onwards [56–60].\(^3\) This requirement bounds the squared coupling among the lightest neutrino mass eigenstate and the scalar \(\phi\), i.e., \((g|U_{e4}|)^2\). (Heavier mass eigenstates are not relevant as they decay quickly.) Here we are taking \(\nu_4\) to be the lightest mass eigenstate, as favoured by current data. Quantitatively,

\[
(m_4 \Gamma_4)^{\text{eff}} \lesssim 4 \times 10^{-10}\,\text{eV}^2 \left(\frac{m_4}{\text{eV}}\right)^4 \left(\frac{0.1}{|U_{e4}|}\right)^4 x_{\phi}^2, \quad (13)
\]

with \(m_4 \lesssim 200\) eV required for \(g^2 \gtrsim 10^{-6}\) [60]. Note that in fig. 3, this constraint is present even for very small mixings. This is because, at fixed \(m_4 \Gamma_4\), small mixings need to be compensated by a large coupling \(g\), strengthening the free streaming constraint. The value of \(|U_{e4}|^2\) is fixed in terms of \(|U_{e1}|^2\) and \(|U_{e3}|^2\) by unitarity, assuming the active neutrino mixing angles to be fixed at their values from Ref. [66].

However, the constraint could be substantially weakened in extensions of our model, see for instance refs. [67–70]. A minimalist example is the production of extra species of light particles at the expense of the neutrino sector after neutrino decoupling. These would compensate for the lack of free-streaming in active neutrinos.

\(7\) SN 1987A. The fact that neutrinos from supernova 1987A could be observed at Earth without being absorbed through scattering on the cosmic neutrino background constrains neutrino self-interactions [71]. We have checked that, due to mixing suppression, these constraints are avoided in our scenario. Note that supernova cooling, which is sensitive to non-interacting sterile neutrinos, does not constrain our model as \(\nu_4\) and \(\phi\) quickly decay to lighter neutrinos that remain trapped in the supernova core.

\(8\) Decays of SM neutrinos. We have checked that decays of the form \(\nu_{2,3} \rightarrow \tilde{\nu}_1 + 2\nu_1\), mediated by an off-shell \(\phi\), are always sufficiently rare to be consistent with solar neutrino constraints [29, 72]. Note, however, that we predict the cosmic neutrino background today to consist exclusively of \(\nu_1\) or \(\nu_3\), for normal and inverted neutrino mass ordering, respectively.

\(9\) Perturbativity (red region in fig. 2). Requiring that the \(\nu_e-\phi\) coupling constant \(g\) in eqs. (1) and (2) is \(< \sqrt{4\pi}\) imposes the bound

\[
(m_4 \Gamma_4)^{\text{eff}} \lesssim 0.25\,\text{eV}^2 \left(\frac{m_4}{\text{eV}}\right)^2. \quad (14)
\]

Similarly to the free-streaming bound, this constraint applies even for very small mixing when \(m_4 \Gamma_4\) is fixed. This bound restricts \(m_4\) in our model to be \(\gtrsim\) 100 eV for \(m_4 \Gamma_4\) values large enough to explain the MiniBooNE anomaly.

In summary, the sterile neutrino mass range to explain the MiniBooNE anomaly is between 100 eV and 2.5 keV.

V. THE LSND AND REACTOR ANOMALIES

As shown in fig. 3, decaying sterile neutrinos can simultaneously fit the MiniBooNE and LSND anomalies, but only if cosmological neutrino free-streaming constraints can be avoided (see discussion under point (6) above for possible scenarios). Quantitatively, a parameter goodness-of-fit test [73] reveals that LSND is incompatible with the rest of the data at the 4.7\(\sigma\) level if free-streaming constraints hold. If the free-streaming problem is solved by other means, this reduces to 2.1\(\sigma\), implying consistency. The best fit to all data including LSND, but excluding free-streaming is found at \(m_4 = 97\) eV, \(|U_{e4}|^2 = 0.018\), \(|U_{e1}|^2 = 0.0015\), \(m_4 \Gamma_4 = 0.87\) eV\(^2\), \(m_\phi/m_4 = 0.89\).

Interestingly, at this value of \(|U_{e4}|^2\), the model can also explain the flux deficit observed in reactor and gallium experiments [5–8, 19, 74]. We test our model against reactor data by comparing to Daya Bay’s generic flux-weighted cross section [75]. To estimate the viable parameter space we perform a chi-square test using the covariance matrix given in the same reference. In addition we introduce a 2.4\% systematic flux normalization error corresponding to the theoretical uncertainty, in accordance with fig. 28 of ref. [75]. The \(|U_{e4}|^2\) region preferred by reactor experiments is included in fig. 3, and a comparison of the reactor neutrino spectrum to our model prediction is shown in fig. 4.

VI. DETAILED INVESTIGATION OF THE PARAMETER SPACE

To supplement fig. 3 and give the reader a broader overview of the preferred parameter regions of decaying sterile neutrinos, we show in figs. 5 and 6 additional slices through the 5-dimensional parameter space.

The color coding in the figure is the same as in fig. 3: the yellow, banana-shaped regions are preferred

\(^3\) It is noteworthy, though, that some cosmological fits have actually found a preference for neutrino self-interactions [56, 61–65] that could be accommodated in our model.
FIG. 3. Allowed values of the squared mixing matrix elements $|U_{e4}|^2$ and $|U_{44}|^2$ (measuring the mixing of $\nu_e$ with $\nu_4$ and $\nu_\mu$, respectively) in the decaying sterile neutrino scenario. We show two representative slices through the 5-dimensional 99% confidence regions. Our fits include MiniBooNE, OPERA, ICARUS, E776, and KARMEN data, as well as constraints from nuclear beta decay spectra and from the requirement of neutrino free-streaming in the early Universe. For the null results from oscillation experiments, the region to the right of the curves is excluded. For the free-streaming constraint, the region to the left of the gray dotted contour is excluded. We also show, as a black rule at the bottom of the plot, the $|U_{e4}|^2$ range preferred by the reactor neutrino anomaly. Constraints on $\nu_\mu$ disappearance are significantly weaker here than in the 3 + 1 scenario without decay, and are hence not shown. We also do not show a fit including both LSND and cosmology as the goodness of fit would be very poor. Note that the global combinations are sensitive to five degrees of freedom, namely $m_4$, $|U_{e4}|^2$, $U_{e4}^2$, $m_4 \Gamma_4$, and $m_\phi/m_4$; oscillation experiments are sensitive only to the last four of these; beta decay spectra depend on two degrees of freedom ($m_4$ and $|U_{e4}|^2$); reactor experiments depend only on $|U_{e4}|^2$; and the free-streaming constraint depends only on the parameter combination $m_4/|U_{e4}|$.

by MiniBooNE, the large dark red ones by LSND; the orange regions at low $|U_{e4}|^2$ correspond to a global fit to MiniBooNE, OPERA, ICARUS, E776, KARMEN, nuclear beta decay spectra, and cosmological free-streaming constraints; bright red regions show instead a global fit to MiniBooNE, LSND, OPERA, ICARUS, E776, KARMEN, and nuclear beta spectra, but excluding the free-streaming constraint. Solid lines indicate constraints from OPERA (blue), ICARUS (purple), KARMEN (cyan), E776 (green), nuclear beta decay spectra (black dashed), and free-streaming in the early Universe (black dotted). The region to the right of the lines is excluded.

We observe that, at smaller values of $m_4 \Gamma_4$, the allowed parameter regions from short-baseline oscillations (MiniBooNE, LSND, KARMEN) shift towards larger values of $|U_{e4}|^2$ and $|U_{44}|^2$. In this case, only a small fraction of neutrinos decays before reaching the detector, making the phenomenology more similar to that of 3 + 1 models without decay. Strong constraints from beta decay spectra and from cosmology imply that a good global fit cannot be achieved at $m_4 \Gamma_4 \ll 1 \text{ eV}^2$.

Regarding the dependence of the fit on $m_4$, we note that smaller values of $m_4$ are favored by beta decay spectra, but disfavored by cosmology, in agreement with fig. 2. Exclusion limits from oscillation experiments do not depend on $m_4$ for $m_4 \gg \text{eV}$.

Comparing fig. 5 with $m_\phi/m_4 = 0.5$ and fig. 5 with $m_\phi/m_4 = 0.9$, we see that it becomes in general more difficult to fit all experiments at smaller $m_\phi/m_4$. The reason is that, at small $m_\phi/m_4$, the active neutrinos produced in $\nu_4$ and $\phi$ decays have a harder spectrum. This in particular makes it more difficult to explain the MiniBooNE low-energy excess. In fact, for even smaller values of $m_\phi/m_4$, and in particular for nearly massless $\phi$ (as considered in refs. [22, 23]), the MiniBooNE-preferred region would disappear completely from the plots.

VII. CONCLUSIONS

In summary, we have shown that scenarios in which the SM is extended by a sterile neutrino that has a decay mode to active neutrinos can well explain the MiniBooNE anomaly without violating any constraints. An explanation of the LSND and reactor/gallium anomalies is possible if the model is extended to avoid constraints on neutrino free-streaming in the early Universe. The preferred mass of the sterile neutrino is of order few hundred eV.
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Appendix A: Impact of Oscillations on the Background Prediction in MiniBooNE

In this appendix, we briefly discuss our fit to MiniBooNE data, and in what ways it differs from the collaborations’ fit as described in the supplemental material to ref. [1], and using the data released with ref. [3]. In particular, we consider the following three effects, which are relevant in a fit to a 3+1 scenario, but are not encountered in a 2-flavor fit.

1. Normalization of the $\nu_\mu \rightarrow \nu_e$ oscillation signal. To predict the number of expected $\nu_e$ events from $\nu_\mu \rightarrow \nu_e$ oscillations for a given set of oscillation parameters, the initial $\nu_\mu$ flux must be known. It is obtained in situ using MiniBooNE’s own sample of $\nu_\mu$ events. Note, however, that in a 3+1 model, the measured $\nu_\mu$ flux will be reduced by an amount $\sim |U_{\mu 4}|^2$ due to $\nu_\mu \rightarrow \nu_e$ oscillations. (This effect is unimportant in a 2-flavor model, where the deficit is only of order $\sin^2 2\theta_{\mu e}$, where $\theta_{\mu e}$ is the effective 2-flavor mixing angle.) We account for this effect by first computing the expected $\nu_e$ signal based on the unoscillated MiniBooNE flux, and then dividing it by the $\nu_\mu$ survival probability in each bin.

The impact of this change in normalization is illustrated in the top panels of fig. 7. The colored region in panel (a) of this figure shows our reproduction of the official MiniBooNE fit, which is shown as black contours. In panel (b), we have included the change in normalization for the signal.

2. Oscillations of the $\nu_e$ backgrounds. Part of the MiniBooNE background is constituted by the intrinsic $\nu_e$ contamination in the beam. In a 2-flavor fit, this contribution to the total event rate is only modified by a factor of order $\sin^2 2\theta_{\mu e}$, but in the full 3+1 framework, it is reduced by a factor of order $|U_{e4}|^2$ instead. The impact of this modification to the background sample is shown in fig. 7 (c).

3. Oscillations of the $\nu_\mu$ sample. The fit described in the supplemental material to ref. [1] which we are following includes also MiniBooNE’s sample of $\nu_\mu$ events. This is necessary to properly account for systematic uncertainties which are correlated between the two samples. But of course, in a 3+1 scenario, the $\nu_\mu$ sample suffers from $\nu_\mu$ disappearance into $\nu_e$, proportional to $|U_{\mu 4}|^2$. (Once again, in a 2-flavor model, only a much smaller fraction $\propto \sin^2 2\theta_{\mu e}$ will disappear, which is usually negligible.) The impact of including $\nu_\mu$ disappearance is shown in panel (d) of fig. 7.

We see that including the effect of 3+1 oscillations on the normalization in the control regions and on the background prediction reduces the significance of the MiniBooNE anomaly, though it remains above $3\sigma$. These effects are thus unable to fully explain the MiniBooNE anomaly, but they could well be part of an “Altarelli cocktail” of several effects conspiring to lead to the large observed excess [78].

FIG. 4. Comparison of the reactor anti-neutrino spectrum predicted in the decaying sterile neutrino scenario discussed in this work (blue) to the standard Huber–Mueller prediction (orange-dashed) [76, 77] and to Daya Bay data (black data points with error bars) [75]. For model parameters motivated by the MiniBooNE and LSND anomalies, a flux deficit consistent with the reactor anomaly can be accommodated. (See text for details, and for a discussion of how possible cosmological constraints can be avoided.)

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1. Normalization of the $\nu_\mu \rightarrow \nu_e$ oscillation signal. To predict the number of expected $\nu_e$ events from $\nu_\mu \rightarrow \nu_e$ oscillations for a given set of oscillation parameters, the initial $\nu_\mu$ flux must be known. It is obtained in situ using MiniBooNE’s own sample of $\nu_\mu$ events. Note, however, that in a 3+1 model, the measured $\nu_\mu$ flux will be reduced by an amount $\sim |U_{\mu 4}|^2$ due to $\nu_\mu \rightarrow \nu_e$ oscillations. (This effect is unimportant in a 2-flavor model, where the deficit is only of order $\sin^2 2\theta_{\mu e}$, where $\theta_{\mu e}$ is the effective 2-flavor mixing angle.) We account for this effect by first computing the expected $\nu_e$ signal based on the unoscillated MiniBooNE flux, and then dividing it by the $\nu_\mu$ survival probability in each bin.

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FIG. 5. Slices through the 5-dimensional parameter space of decaying sterile neutrinos at $m_\phi/m_4 = 0.5$ fixed. The color code is the same as in fig. 3.

Let us finally mention one caveat with the above corrections to the MiniBooNE fit. Namely, we can only apply the corrections at the level of reconstructed events as the mapping between true and reconstructed neutrino energies is not publicly available for muon neutrinos. This means we have to assume that the reconstructed neutrino energy is a faithful representation of the true neutrino energy. While this is true for quasi-elastic scatter-
ing events which constitute the majority of events, it is not the case for other event categories. For instance, a neutrino–nucleon interaction may create an extra pion, and if this pion is reabsorbed as it propagates out of the nucleus, the event will be misinterpreted as a quasi-elastic interaction, and the kinematic reconstruction of the neutrino energy based on the observed charged lepton energy and direction will fail.
FIG. 7. Impact of oscillations in the background and control regions on the MiniBooNE fit in a simple 3+1 model (oscillations only, no decay). All panels show $\Delta m^2_{41}$ vs. the effective 2-flavor mixing angle $\sin^2 2\theta_{\mu e}$, which in a 3+1 scenario is given by $|U_{e4}|^2|U_{\mu4}|^2$. Panel (a) shows our reproduction (colored regions) of the official MiniBooNE fit (black contours), based on the instructions given in the supplemental material to ref. [1] and using the data released with ref. [3]. In panel (b), we include in addition the impact of $\nu_\mu \rightarrow \nu_s$ disappearance on the normalization of the signal in each bin. The colored contours in panel (c) include on top of this the effect of $\nu_s$ disappearance on the intrinsic $\nu_e$ contamination in the beam. Panel (d) finally shows the additional impact of $\nu_\mu$ disappearance on the sample of $\nu_\mu$ events that is included in the fit along with the $\nu_e$ sample. In all panels, we show projections of the three-dimensional parameter space spanned by $\Delta m^2_{41}$, $|U_{e4}|^2$, and $|U_{\mu4}|^2$ onto the $\Delta m^2_{41}$–$\sin^2 2\theta_{\mu e}$ plane, imposing the constraint $|U_{e4}|^2 < 0.2$ due to bounds from reactor neutrino experiments.

Appendix B: Decay Widths and Transition Probability

Based on the interaction terms from eq. (2), we can compute the differential decay rates of the heavy neutrino $\nu_4$ and of the scalar $\phi$. In the massless light neutrino...
is the total rest frame decay width of $\nu_4$, $x_{\phi 4} \equiv m_\phi/m_4$ is the ratio of scalar and neutrino masses, and $E_j$, $E_\phi$ are the daughter neutrino and scalar energies, respectively. In the $\nu_4$ rest frame, $E_j$ is restricted to the interval $[0, m_4(1 - x_{\phi 4}^2)]$.

The lab frame decay rate of the scalar is

$$\sum_{i,j} \frac{1}{E_\phi} \frac{d\Gamma_{\phi \to \nu_i \bar{\nu}_j}}{dE_i} = \frac{1}{E_\phi}, \quad (B4)$$

with the total rest frame decay width of $\phi$

$$\Gamma_\phi = \frac{g^2}{8\pi} m_\phi \sum_{i,j=1}^{3} |U_{s_i}^* U_{s_j}|^2. \quad (B5)$$

The kinematic constraint on the daughter neutrino energies is $E_i, E_j \in [0, m_\phi]$.

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