Dynamical Higgs Mechanism without Elementary Scalars:

A Lesson from Instantons

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Abstract

The generation of gauge–dependent fermion vacuum condensates in Yang–Mills theory would dynamically break the gauge symmetry and thus provide an alternative to the Higgs mechanism in unified theories. We explore a simple example: instanton-induced quark pair (diquark) condensation in QCD with 2 flavors and $N_c$ colors. We do find diquark condensates in the vacuum, but only for $N_c = 2$, where they are equivalent to the standard quark-antiquark condensate. At $N_c \geq 3$ diquark condensates exist as meta–stable saddle points of the instanton-induced effective QCD action and may strongly affect the properties of matter under extreme conditions. The scalar diquark excitation, which is a Goldstone boson at $N_c = 2$, becomes unbound for $N_c \geq 3$. 

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Instanton-induced quark interactions provide a well-known mechanism for spontaneous chiral symmetry breaking by quark-antiquark pair condensation in the QCD vacuum \[1\]. In this letter we pursue the question whether these interactions, which are attractive in certain quark-quark channels, might also induce diquark condensates. Since such condensates are not gauge invariant (except for \(N_c = 2\)), they would signal dynamical gauge symmetry breaking.

Diquark condensates, which are a direct analogue of Cooper pair condensates in superconductive materials, could furthermore serve as a prototype for dynamical gauge symmetry breaking in other theories and thus as an alternative to the standard Higgs mechanism. Even if they are absent in the ground state of a theory, however, they might still form in a state corresponding to a saddle point of the effective action. Such meta-stable states could, for example, play an important role in cosmology and in the evolution of the universe.

Our study is based on the effective quark interaction \[1,2\]

\[
\mathcal{L} = g \left\{ \left( 1 - \frac{1}{N_c} \right) \left[ (\bar{q}q)^2 + (\bar{q}\gamma_5 q)^2 - (\bar{q}\gamma_5 \tau q)^2 \right] - \frac{1}{N_c} \left[ (\bar{q}C \tilde{\lambda}_A i \tau_2 q^T) (q^T C \tilde{\lambda}_A i \tau_2 q) + (\bar{q}\gamma_5 C \tilde{\lambda}_A i \tau_2 q^T) (q^T \gamma_5 C \tilde{\lambda}_A i \tau_2 q) \right] \right\},
\]

which is induced by the fermionic zero-modes in the instanton background (in euclidean space-time). Here, \(\tau\) are the \(SU(2)\) flavor Pauli-matrices, the \(\tilde{\lambda}_A\) form the antisymmetric subset of the \(SU(N_c)\) color generators and \(C\) is the Dirac charge conjugation matrix.

In order to probe the condensation of quark-quark and quark-antiquark pairs in the vacuum, one can bosonize the interaction \[1\] by introducing auxiliary scalar, pseudoscalar, scalar diquark and anti-diquark fields and integrating the quarks out \[3\]. The resulting bosonic effective action can then be treated in leading loop approximation, i.e. by neglecting meson and diquark loops. Instead, we will use an equivalent \[4\], but technically more efficient approach. It is based on a somewhat more general, bilocal effective action functional \(\Gamma[\Delta, D, \bar{D}]\) \[4,5\], which determines in our case the ground state energy of the dynamics \(\|\) under the constraint that the vacuum expectation values of three time-ordered quark bilinears,
\[ \Delta(x) = <0|T q(x)\bar{q}(0)|0>, \]  
\[ D(x) = <0|T q(x)q(0)|0>, \]  
\[ \bar{D}(x) = <0|T \bar{q}(x)\bar{q}(0)|0>. \]  

are fixed. In terms of \( \Delta, D \) and \( \bar{D} \), the effective action can be expressed as \[4,5\]

\[
\Gamma[\Delta, D] = \frac{-1}{2} \text{Tr} \log \begin{pmatrix} \bar{D} - \Delta^T \\ \Delta \\ D \end{pmatrix} + \text{Tr} \left( \Delta^{-1} - 1 \right) + V[\Delta, D, \bar{D}],
\]

which consists of a kinetic term, containing the free quark propagator \( \Delta_0(p) = \frac{i(p - m_0 + i\epsilon)^{-1}}{} \)

with the current quark mass \( m_0 \), and the sum of all two-particle-irreducible vacuum diagrams \( V[\Delta, D, \bar{D}] \). (The symbol Tr denotes a trace over color, flavor and Dirac indices as well as over the space-time dependence.)

The physical propagator \( \Delta \) and \( D, \bar{D} \) are obtained as the stationary point of \( \Gamma \) with minimal energy. It is straightforward to show \[8\] that all stationary points of the effective action correspond to solutions of the coupled set of gap equations

\[
\frac{\delta V}{\delta \Delta} = \left( \Delta + D(\Delta^T)^{-1} \bar{D} \right)^{-1} - \Delta_0^{-1},
\]

\[
\frac{\delta V}{\delta D} = \frac{1}{2} \Delta^{-1} \bar{D} \left( \Delta + D(\Delta^T)^{-1} \bar{D} \right)^{-1},
\]

\[
\frac{\delta V}{\delta \bar{D}} = \frac{1}{2} \left( \Delta + D(\Delta^T)^{-1} \bar{D} \right)^{-1} D(\Delta^T)^{-1}.
\]

We now specify the potential \( V \) by adopting the Bogoliubov-Hartree approximation \[9\], which is exactly equivalent to the zero-meson and -diquark loop approximation in the

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\(^1\)Note that the solutions of the gap equations do not necessarily satisfy the condition \( \gamma_0 \bar{D} \gamma_0 = D^\dagger \) (see, e.g., eqs. (11 – 12) which one would naively expect from (9), (10) and \( \bar{q} = q^\dagger \gamma_0 \). In the effective action functional, \( \Gamma \), however, \( D \) and \( \bar{D} \) are independent quantities, and the constraint from the operator identity \( \bar{q} = q^\dagger \gamma_0 \) is not mandatory. In the path integral formulation the invariance of the measure under an infinitesimal transformation \( q \rightarrow q + i\delta \alpha q \) and \( \bar{q} \rightarrow \bar{q} - i\delta \alpha \bar{q} \) leads to a less stringent constraint, \( \text{Tr} D \frac{\delta \Gamma}{\delta D} = \text{Tr} \bar{D} \frac{\delta \Gamma}{\delta \bar{D}} \) (consistent with (11 – 12)).
bosonization language \[4\]. Restricted to the terms which can affect scalar quark pair and quark-antiquark condensation, \(V\) reads

\[
V[\Delta, D, \bar{D}] = i g V_4 \frac{N_c - 1}{N_c} [(\text{tr} \Delta(0)) (\text{tr} \Delta(0)) - (\text{tr} \bar{\tau} \Delta(0)) (\text{tr} \bar{\tau} \Delta(0))]
\]

\[
+ i \frac{g V_4}{2 N_c} (\text{tr} \tau_2 \bar{\lambda}_A \gamma_5 C D^\dagger(0))(\text{tr} \tau_2 \bar{\lambda}_A \gamma_5 C^{-1} D(0))
\]

\[
+ i \frac{g V_4}{2 N_c} (\text{tr} \tau_2 \bar{\lambda}_A \gamma_5 C \bar{D}(0))(\text{tr} \tau_2 \bar{\lambda}_A \gamma_5 C^{-1} \bar{D}^\dagger(0)),
\]

\(9\)

where \(V_4 \equiv \int d^4x\) is the spacetime volume and the trace (tr) is over color, flavor and Dirac indices only.

Under the assumption of a flavor-symmetric vacuum and with \(N_c = 2\) or 3, the most general ansatz\[^2\] for \(D\) and \(\bar{D}\) yields solutions to the gap equations of the form

\[
\Delta(p) = \frac{i}{\bar{p} - m + i\epsilon} \left(1 + (\bar{\alpha} \cdot \bar{\lambda}_A)^2 \frac{m_c^2}{p^2 - m^2 - m_c^2 + i\epsilon}\right),
\]

\(10\)

\[
D(p) = \tau_2 \bar{\alpha} \cdot \bar{\lambda}_A C \gamma_5 \frac{i m_c}{p^2 - m^2 - m_c^2 + i\epsilon},
\]

\(11\)

\[
\bar{D}(p) = \tau_2 \bar{\alpha} \cdot \bar{\lambda}_A \gamma_5 C^{-1} \frac{i m_c}{p^2 - m^2 - m_c^2 + i\epsilon},
\]

\(12\)

which are expressed in terms of the constituent quark mass \(m\), an analogous mass parameter, \(m_c\), related to the quark pair condensation strength, and the \(N_c(N_c - 1)/2\) dimensional unit vector \(\bar{\alpha}\) (i.e. \(\bar{\alpha} \cdot \bar{\alpha} = 1\)), which determines the orientation of a potential diquark condensate in the \(so(N_c)\) subalgebra of color space. Since only for \(N_c = 2\) and 3 the expression \((\bar{\alpha} \cdot \bar{\lambda}_A)^2\) is a projection operator\[^3\] with \((\bar{\alpha} \cdot \bar{\lambda}_A)^4 = (\bar{\alpha} \cdot \bar{\lambda}_A)^2\), and since this property is necessary

\[^2\]We preselect the standard chiral orientation of the quark-antiquark condensate (i.e. \(<\bar{q} \gamma_5 q> = 0\) and do not consider potentially interesting colored quark-antiquark condensates without simultaneous quark pair condensation in the present paper.

\[^3\] For all \(N_c\), \(A^4 = \frac{1}{N_c} \text{tr} (A^2) A^2 + (\bar{\beta} \cdot \bar{\lambda}_A) A\), where \(A \equiv (\bar{\alpha} \cdot \bar{\lambda}_A)\) and \(\beta_i = \alpha_j \alpha_k \alpha_l d_{ijm} d_{klm}\) in terms of the totally symmetric d-symbols of the \(su(N_c)\) Gell-Mann matrices. Therefore, only for \(N_c = 2\) with \(\beta_i = 0\) and for \(N_c = 3\) with \(\beta_i = \frac{1}{6} \alpha_i \text{tr} (A^2)\), does \(A^2\) become a projector for all \(\bar{\alpha}\) of unit length.
to satisfy the color structure of the gap equations, the above solutions do not generalize to \( N_c > 3 \) for general \( \vec{\alpha} \). Special choices of \( \vec{\alpha} \), however, as for example \( \vec{\alpha} = \{1,0,0,...\} \), lead to solutions for all \( N_c \).

Inserting these solutions into (3), we obtain their energy density \( \epsilon = i \Gamma/V_4 \):

\[
\epsilon[m^2, m_c^2] = -(N_c - 2) \epsilon_0[m^2] - 2 \epsilon_0[m^2 + m_c^2] \\
+ (N_c - 2) m(m - m_0) I[m^2] + 2(m(m - m_0) + m_c^2) I[m^2 + m_c^2] \\
- g \frac{N_c - 1}{N_c} \left( (N_c - 2) m I[m^2] + 2 m I[m^2 + m_c^2] \right)^2 \\
- g \frac{1}{N_c} \left( 2 m_c I[m^2 + m_c^2] \right)^2,
\]

where the integrals \( \epsilon_0[m^2] \) and \( I[m^2] \), properly regularized by the cutoff \( \lambda \), are given by

\[
\epsilon_0[m^2] = 4 i \int \frac{d^4 p}{(2\pi)^4} \log[p^2 - m^2 + i \epsilon] \\
= \frac{2}{\pi^2} \int_0^\lambda d p p^2 \sqrt{p^2 + m^2}, \quad \text{(14)}
\]

\[
I[m^2] = \frac{2}{\pi} \frac{\partial \epsilon_0}{\partial m^2}. \quad \text{(15)}
\]

The gap equations in terms of the variables \( m \) and \( m_c \) can now be recovered from the stationarity requirement for the energy density:

\[
m - m_0 = g \frac{N_c - 1}{N_c} \left[ 2(N_c - 2) m I[m^2] + 4 m I[m^2 + m_c^2] \right], \quad \text{(16)}
\]

\[
m_c = g \frac{1}{N_c} 4 m_c I[m^2 + m_c^2]. \quad \text{(17)}
\]

Following reference [1] the coupling constant \( g \) can be related to the gluon condensate \( \langle F_{\mu\nu}^2 \rangle = 32\pi^2 \bar{n} \) or the instanton density \( \bar{n} \) by adding to the vacuum energy the term

\[
\Delta \epsilon = 2 \bar{n} \log g
\]

and minimizing the resulting expression with respect to the coupling strength \( g \). This leads to a further equation,

\[
\frac{2\bar{n}}{g} = \frac{N_c - 1}{N_c} \left( (N_c - 2) m I[m^2] + 2 m I[m^2 + m_c^2] \right)^2 \\
+ \frac{1}{N_c} \left( 2 m_c I[m^2 + m_c^2] \right)^2.
\]

\[
\text{(19)}
\]
We fix the instanton density at the value $\bar{n} = N_c/3$ (229 MeV)$^4$ \cite{1}, which incorporates a straightforward extrapolation of the $N_c$ dependence for $N_c \neq 3$. The cutoff is set to $\lambda = 640$ MeV, which leads to the value $\langle \bar{q}q \rangle = -251$ MeV of the quark condensate for $N_c = 3$, consistent with phenomenology.

The explicit expressions (13) for the energy density and (16), (17) for the gap equations show a pronounced $N_c$ dependence. We will thus discuss the three distinct cases $N_c = 2$, $N_c = 3$ and $N_c > 3$ separately.

a) $N_c = 2$:

In the chiral limit ($m_0 = 0$), this is the unique case in which the energy density $\epsilon = \epsilon[m^2 + m_c^2]$ depends solely on the combination $m^2 + m_c^2$, and where the gap equations are consequently symmetric under exchange of $m$ and $m_c$. Both of these properties reflect the $SU(4)$ (Pauli-G"ursey \cite{9}) symmetry of the instanton-induced dynamics \cite{10} and of QCD for two colors \cite{11}, which mixes quark and antiquark states and leaves $m^2 + m_c^2$ invariant. If the coupling $g$ becomes strong enough to generate a quark-antiquark condensate, the latter can be transformed into an equivalent, degenerate quark-quark pair condensate by an $SU(4)$ transformation.

For the same reason, such a diquark condensate does not break any more symmetries than the usual $<\bar{q}q>$ condensate, and in particular neither baryon number nor color symmetry. Indeed, these diquark condensates have the quantum numbers of colorless $N_c = 2$ “baryons” and are thus gauge invariant. Furthermore, should one decide to quantize the theory in a vacuum sector containing condensed diquarks, then the corresponding $SU(4)$ transformation of the baryon charge generator would reveal that it also has zero baryon number.

With the coupling $g$ fixed from the instanton dynamics via eq. (19), the energy density $\epsilon$ has the form of a mexican hat, and thus the symmetry breaking solution with $m^2 + m_c^2 = (271$ MeV)$^2$ is energetically favored compared to the symmetry preserving one, $m = m_c = 0$. The energy density is plotted in Fig.1a.

The above discussion implies that the diquark vacuum condensate is not at odds with the Vafa-Witten theorem \cite{12}, which states that vector symmetries cannot be spontaneously
broken in vector-like gauge theories. Since this theorem has been established only for finite
current quark masses, however, we also considered the case $m_0 \neq 0$, which breaks the $SU(4)$
symmetry explicitly and thereby lifts the degeneracy of the vacuum in favor of a unique
ground state. For $m_0 = 11$ MeV, the resulting energy density is shown in Fig. 1b. Indeed,
the mexican hat potential is tilted such that the absolute minimum occurs at $m_c = 0$ and
$m = 346$ MeV, which is in accord with the Vafa-Witten theorem.

b) $N_c = 3$

In the (physical) case of three colors, the energy density is shown in Fig. 2a for $m_0 = 0$.
Its absolute minimum corresponds to a constituent quark mass of $m = 346$ MeV and a
vanishing quark pair parameter $m_c = 0$. This implies (see ref. [13])\[<\bar{q}q> = -(251 \text{ MeV})^3\]
for the quark condensate, a vanishing diquark condensate, and $f_\pi = 95$ MeV for the pion
decay constant.

Fig. 2a also shows a saddle point of $\epsilon$ at $m = 0$ MeV and $m_c = 449$ MeV, corresponding
to a pure diquark condensate state. Its energy density is $(282\text{MeV})^4$ larger than that of
the vacuum solution. The symmetry preserving local maximum with $m = m_c = 0$ lies
$(629\text{MeV})^4$ above the vacuum.

Such saddle points can acquire physical significance in the presence of large temperatures
or baryon densities, e.g. as a transition state for baryon number or color symmetry violating
processes. They could thus play a role in the evolution of the early universe, inside stars
and in ultrarelativistic heavy-ion collisions. These interesting possibilities deserve further
study.

Fig. 2b shows the energy density for a finite current quark mass of $m_0 = 11$ MeV. The
absolute minimum is shifted to $m = 353$ MeV with $m_c = 0$, whereas the meta-stable saddle
point moves to $m_c = 451$ MeV and $m = -4$ MeV. Thus the constituent mass of quarks
propagating in this state becomes slightly negative for finite $m_0$, which can be directly
understood from the $m_0$-dependent terms in the energy density [13]. The latter are repulsive
for negative constituent masses, which are prefered since the saddle point is a maximum in
the $m$ direction. Of course negative quark masses do not render the theory tachyonic and
occur, for example, in QCD with a finite \( \theta \) vacuum angle.

c) \( N_c > 3 \):

As mentioned above (below eq. (11)), our quantitative results for \( N_c > 3 \) are based on a somewhat more restrictive ansatz for the color structure of the quark and diquark propagators than that for \( N_c = 2,3 \). Besides the trivial solutions with \( m = 0 \) and \( m_c = 0 \), one still finds symmetry breaking solutions (for sufficiently large, attractive \( g \)) with \( m > 0 \) and \( m_c = 0 \) or with \( m = 0 \) but \( m_c > 0 \). However, it is easily established that the gap equations (16) and (17) do not support solutions with both nonzero \( m \) and \( m_c \).

Estimating the coupling as before from eq. (19), moreover, we find the same qualitative condensation pattern as for \( N_c = 3 \): The vacuum contains solely a quark-antiquark condensate, whereas a state with condensed quark pairs exists as a saddle point of the effective action. The energy difference between these two states increases with \( N_c \), as expected from eq. (1): for large \( N_c \) the attraction in the diquark channel is suppressed relative to that in the quark-antiquark channels, and at \( N_c \to \infty \) diquark degrees of freedom can be neglected altogether.

We checked that an artificial enhancement of the attraction in the diquark channel relative to that in the quark-antiquark channel\( ^4 \) generates a diquark condensate also in the ground state. Inspection of the \( m_0 \)-dependent terms in the energy density (13) reveals, furthermore, that the diquark vacuum condensate appears at \( m \geq 0 \). This is in contrast to the saddle point solutions discussed above.

A diquark-condensed vacuum phase such as the one generated above (or also colored \( \bar{q}q \) condensates) could be of interest as a prototype for dynamical gauge symmetry breaking by instantons. In view of possible applications to unified theories, one should keep in mind that the Vafa-Witten theorem does not exclude diquark vacuum condensates in the presence of diquarks.

\[^4\]Note that this is an ad-hoc modification of the instanton-generated interaction, which leaves the realm of \( N_c \)-QCD.
axial-vector gauge couplings, as they occur in the weak interaction sector.

We finally turn to the calculation of the mass of the scalar diquark excitation on top of the ordinary vacua (with $g$ fixed by eq. (19)). This mass can be readily obtained by solving the Bethe-Salpeter equation with the appropriate kernel given by $K_{BS} = \delta^2 V/(\delta D \delta \bar{D})$. The mass equation reads:

$$J[m_s^2] = \frac{N_c}{2g} = N_c(N_c - 1) I[m^2]$$

with

$$J[q^2] = 2 \left( 2 + \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n + 1)!!} \left( \frac{q^2}{2} \frac{\partial}{\partial m^2} \right)^{n+1} \partial \epsilon_0 \right) \frac{\partial \epsilon_0}{\partial m^2}$$

Identifying the quark condensate $<\bar{q}q>$ and the square of the pion decay constant $f_\pi^2$ with appropriate derivatives of the free vacuum energy $\epsilon_0$ as given below,

$$<\bar{q}q> = -N_c m \frac{\partial}{\partial m^2} \epsilon_0[m^2],$$

$$f_\pi^2 = -N_c m^2 \left( \frac{\partial}{\partial m^2} \right)^2 \epsilon_0[m^2],$$

we can write the scalar diquark mass as

$$m_s^2 = -2 <\bar{q}q> \frac{m}{f_\pi^2} (N_c(N_c - 1) - 2) + O[m_s^4]$$

We observe that for $N_c = 2$ we recover $m_s = 0$; i.e. the scalar diquark and the pion are the Goldstone bosons of the spontaneously broken $SU(4)$ symmetry. However, already for $N_c = 3$ the scalar diquark mass is pushed above the quark-quark threshold, up to about 1 GeV. The instanton sector of QCD does therefore not support the existence of a light scalar diquark for $N_c = 3$. We consider this result an important by-product of our study, since the possibility of a light diquark is widely discussed in the current literature [14].

To summarize, we studied instanton-induced quark interactions in $SU(N_c)$ QCD with self-consistently fixed couplings. In mean-field approximation (i.e. neglecting meson and diquark loops), these interactions do not generate quark-pair condensates in the vacuum state, except for the special case of two colors, where the additional $SU(4)$ symmetry renders
them equivalent to the usual quark-antiquark condensate. We also do not find light scalar
diquark states for $N_c \geq 3$. Since quark-pair condensates for $N_c > 2$ would break color and
further discrete and continuous symmetries, our results are in accord with phenomenolgy
for QCD with $N_c = 3$ and with general expectations for $N_c = 2$ and $N_c > 3$.

In other dynamical settings, however, as for example in grand unified theories, instanton-
induced interactions could induce vacuum condensates which break the gauge symmetry of
the interactions down to a subgroup. Such a mechanism could furnish a viable alternative
to the standard Higgs sector of unified theories, which is of dynamical origin and does not
require the presence of scalar fields. A prototype of such a situation can also be generated in
a modified version of our dynamical framework, where the coupling in the diquark channel
is increased beyond a critical value.

Even in the case of QCD with $N_c$ colors (i.e. with the effective coupling self-consitently
fixed by the instanton sector), however, quark pair condensates do exist in metastable states,
corresponding to saddle points of the energy density. It is tempting to speculate that such
states could have played a role in earlier phases of the evolution of the universe or that they
could be found inside of hot stars or in ultra-relativistic heavy-ion collisions.

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$$\text{Tr} \log \left( \begin{array}{cc} \bar{D} & -\Delta^T \\ \Delta & D \end{array} \right) = \text{Tr} \log \Delta + \text{Tr} \log \left( \Delta + D(\Delta^T)^{-1}\bar{D} \right)$$

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FIGURES

FIG. 1. The energy density $\epsilon$ at $N_c = 2$ for a) $m_0 = 0$ (i.e. in the chiral limit) and b) $m_0 = 11\text{ MeV}$.

FIG. 2. The energy density $\epsilon$ at $N_c = 3$ for a) $m_0 = 0$ (i.e. in the chiral limit) and b) $m_0 = 11\text{ MeV}$.
Energy density [MeV$^4$]
Energy density [MeV$^4$] vs $m$ [MeV] and $m_c$ [MeV].
Energy density [MeV$^4$]

-500 0 500

$m$ [MeV]

$m_c$ [MeV]

$(233)^4$

$(277)^4$

$N_c = 2$ & $m_0 = 11$ MeV
$N_c = 3 \& m_0 = 11 \text{ MeV}$