Gauge invariant structures and Confinement

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June 1995

Abstract

By looking at cooled configurations on the lattice, we study the presence of peaks in the action density, or its electric and magnetic components, in the SU(2) gauge vacuum. The peaks are seen to be of instanton-like nature and their number variation takes care of the drop in the string tension observed when cooling. Possible explanations of this finding are analysed.

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1 Introduction

Our purpose in this paper is to present results aiming at the understanding of the Confinement mechanism in the gauge theory vacuum. We will study SU(2) Yang-Mills theory for simplicity reasons, but it is generally believed that there is a common origin for the Confinement problem for all gauge groups and in the presence of quarks. Quantitative differences are nonetheless expected, about which we have nothing to say at present. In seeking for a simple understanding of the Confinement problem, many workers in the field have pursued the relationship of the non-abelian theory with an abelian theory with charges and monopoles, which is known as the abelian projection [1]. Confinement is then seen as dual superconductivity [2, 3] arising from the condensation of magnetic charge. Despite the appealing aspects of this approach, there are still some aspects which should be clarified. For example, the class of acceptable gauge fixing projections has to be understood, and the relationship between different gauges clarified (see for example [4]). Here, we will pursue a different avenue, not necessarily incompatible with the previous one. We will look for gauge invariant structures which might reveal to be associated with Confinement. Their existence is a common feature of all models of Confinement based on classical Yang-Mills configurations. In our preferred scenario, the gauge-invariant structures would, after the abelian projection, give rise to monopole currents.

Now we should proceed to analyse the possible methodology. Monte Carlo generated lattice configurations are seen to reproduce the statistical properties of the Yang-Mills vacuum and give rise to a non-zero string tension. However, it is very hard to see any structure directly in these configurations, since the short wavelength noise is much bigger than any signal. A traditional way out of this problem is to make use of the technique known as cooling [5, 6]. This local minimization mechanism is intended to relax the noise at a much faster rate than the long wave-length signal. This strategy was already clear in Polikarpov and Veselov work [7]. Since then, other authors have pursued similar roads [8, 9, 10, 11, 12].

A fundamental question is to understand the effect of cooling on the
resulting configurations. One should make sure that what one sees in the end is not an artifact of the cooling procedure, and if it is not, that there is no distortion introduced by this procedure. Fortunately, cooling is not a unique technique, and there are several versions of it, which allow to test the universality of the results. In particular, we have found convenient to consider a 1-parameter family of cooling procedures, which we will refer to as overimproved cooling [13]. This is nothing else but the naive cooling method, but associated with a one parameter ( $\epsilon$ ) family of lattice actions formed by $1 \times 1$ and $2 \times 2$ plaquettes. The advantage, is that in going from the Wilson action ( $\epsilon = 1$ ) to $\epsilon = -1$, one has reversed the sign of the $O(a^2)$ corrections to the continuum action. The point is quite relevant, since order $a^2$ terms break the degeneracy of the zero-modes and induce distortion of the resulting cooled configuration. In particular, the well-known instability of lattice instantons under cooling can be seen to be a result of the wrong sign (positive) of $\epsilon$. Choosing negative values of $\epsilon$ is hence recommended. Nonetheless, the larger the absolute value of $\epsilon$, the faster the motion along zero-mode directions of the original configuration, with the corresponding modification of the original distribution (instanton size distribution for example). For our lattice sizes and values of $a$, $\epsilon = -0.3$ is a good compromise. A fairly complete analysis of the effect of cooling on the lattice configurations for different values of $\epsilon$ has been presented elsewhere [14, 15].

A mechanism which results from cooling, but is fairly independent of the value of $\epsilon$, is the annihilation of instanton-antiinstanton pairs. Although preserving the value of the topological charge, this mechanism induces a reduction of the number of instantons with cooling. The typical number of cooling steps necessary to produce an annihilation, depends very much on distance and width of the 2 intervening instantons, but it is not untypical to have destructions within a few tens of cooling steps. We do not know at present if it is possible to devise a different technique which is free of this problem. Hence, we should understand at least how this mechanism affects our results.
2 Analysing the gauge invariant content of the vacuum

Our analysis is based on SU(2) Yang-Mills configurations obtained by Monte Carlo simulations with Wilson action and $\beta = 2.325$ in lattices of size $8^3 \times 64$. We imposed twisted boundary conditions in all spacelike planes, corresponding to twist vector $\vec{m} = (1, 1, 1)$. In total we have used 56 configurations separated by several thousand heat bath sweeps on the average. Our estimates show that the configurations are fairly statistically independent from each other. The choice of the point and parameters of the simulation was done based on our previous analysis \cite{16, 17}. In these previous works, we studied the electric flux ground state energies by measuring the temporal correlation between Polyakov loops of different winding numbers in the (short) spatial directions. Our results showed a fairly good scaling behaviour for these quantities even for $N_s = 4$: The energies depended on the lattice spatial length $N_s$ and the plaquette coupling constant $\beta$, only through the physical length $l_s = N_s a(\beta)$. Furthermore, we had a fairly extensive coverage of the dependence of the energies with $l_s$, showing that, at values of $l_s$ slightly exceeding $1 fm$, the results obtained on these intermediate lattices were consistent with both the Confinement prediction and the results of much larger lattices. More specifically, what we had is that the electric flux ground state energies $E_{\vec{e}}$ behaved as

$$E_{\vec{e}} = |\vec{e}| \sigma l_s$$

(1)

where $\sigma$ is the string tension. At $N_s = 8$ and $\beta = 2.325$, the values of $\sigma$ obtained for the three different fluxes $\vec{e} = (1, 0, 0)$, $\vec{e} = (1, 1, 0)$ and $\vec{e} = (1, 1, 1)$ are $4.75(17) fm^{-2}$, $4.46(44) fm^{-2}$ and $5.01(30) fm^{-2}$ respectively. We see that the value is consistent for the three determinations and also consistent with the large volume result, which is taken for normalization, equal to $5 fm^{-2}$. The $l_s$ dependence in this range of volumes is also seen to be consistent with Formula (1). From these results we conclude, that at this value of $\beta$ and this size, we are in fact measuring the string tension with fairly small finite size corrections (10 %).

We have performed up to 50 cooling steps on our configurations with our
over-improved ansatz and $\epsilon = -0.3$. One can measure the electric flux energies $E_\sigma$ on the cooled configurations, and from them extract a value of $\sigma$ by means of Eq. (1). Again, the three values obtained from the different fluxes are consistent, but these values are now smaller than the ones obtained before cooling and with the full statistics. Furthermore, the value of $\sigma$ decreases with the number of cooling steps. Our results were presented in Ref. [14], where we pointed out that at 50 cooling steps, the value of the string tension had only dropped around 40-50% of its uncooled value. Given the degree of self-duallity observed in these cooled configurations, we argued that a big fraction, if not all, of the physical string tension is due to classical (smooth and almost self-dual) configurations. We knew not, however, what could be the origin of the observed decrease with cooling steps.

Our results, and in particular the afore-mentioned decrease, is apparently in contradiction with some statements made previously by other authors on similar issues. For example, in Ref. [7] the authors claimed a much smaller fraction of the string tension to be attributed to instantonic configurations. Nonetheless, there are differences between their work and ours in several respects: These authors cool for a longer period and with the naive cooling method, which given the tendency to decrease, might well explain the differences. Another work going in the opposite direction is that of Ref. [18]. The authors claim that the string tension becomes stable under cooling, but with very rapidly decreasing errors. The apparent disagreement with our results, could be understood if we take into account that again the authors use a different cooling method but this time they cool for a shorter interval than us. Actually, in their work they acknowledge an eventual drop of the string tension beyond some number of cooling steps. To understand what could be the origin of the difference, we tried using their cooling procedure instead of ours. What we observed, is that the cooled trajectory seems to be the same with both cooling methods, but that every ordinary cooling step corresponded to several of their steps. Indeed, our results show a plateau for the first 3 to 5 steps, resulting in a longer one for the method of Ref. [18]. Presumably, hence, there is no inconsistency between our results and those of other authors. Finally, we should mention the remark made by Teper [19].
that, since cooling is a local operation, all masses including the string tension have to remain invariant under cooling. This result, although correct, still permits the appearance of a mass parameter at intermediate length scales, which is the one we are referring to. We claim that this new string tension exists and has a physical meaning. Indeed, since cooling rapidly brings lattice configurations to smooth ones, its following effect can be understood in purely classical terms. The afore-mentioned instanton-antiinstanton annihilation mechanism for example, can only take place, in a finite number of cooling steps, among pairs that are separated less than a certain distance. Only beyond this scale can one recover the asymptotic invariant string tension. In summary, the results of our paper, when taken in this context, are both meaningful and consistent with Teper’s argument.

In this work we have looked at the structure and content of cooled configurations. We have concentrated in local maxima of the action density and of its electric and magnetic components. We also triggered on similar maxima for the absolute value of the topological charge density. Our definition of the lattice densities is as follows. We first construct the colour electric and magnetic field at each lattice point, by averaging over all plaquettes emerging from the point in question in the appropriate plane, in a clover-like fashion. We also used a naively improved version, which uses both $1 \times 1$ and $2 \times 2$ plaquettes (clover=2). Given the fields, the densities are obtained by scalar products in colour space and ordinary space according to the continuum formulas. Next, our algorithmic definition of a maximum, is given by the points where the value of the quantity in question is larger than in all its first neighbours. We call such a maximum: a peak. For each peak, its position and the value of the quantity under consideration at this point and at its nearest neighbours is stored. It is possible to calculate its position with better than a lattice spacing precision, by fitting a paraboloid with the afore-mentioned data and extracting its maximum. This gives excellent results on latticized classical configurations usually.

Our first result concerns the global statistics on the number of observed peaks. First, we mention that there is no difference observed between the number of electric and magnetic peaks (There is a tendency to have a higher
number of peaks in the topological charge density, due to noise caused by it being a signed quantity). Not only the number of peaks agrees, but also the location and the size is the same in more than 80 percent of the cases. We are hence dealing with self-dual structures associated with the peaks. We expect that instantons are present and cause some of the observed peaks. In the continuum, the action density associated with an instanton is given by:

\[ S(x) = \frac{48\rho^4}{(x^2 + \rho^2)^4} \]  

(2)

Since \( \rho \) is the only scale present, both the height and the width are functions of it. This suggests considering the following two functions of the peak structure:

\[ \rho_1 = \left( \frac{48}{S(0)} \right)^{\frac{1}{4}} \]

\[ \rho_2 = \sqrt{\frac{-32S(0)}{\Delta S(0)}} \]  

(3)

Notice that, while the first quantity is only sensitive to the height of the peak and not to the width, the second is invariant under a scale transformation. They are, hence, quite independent from each other in general. However, for an instanton both quantities are equal to the size parameter \( \rho \). For the set of our sample peaks we have computed \( \rho_1 \) and \( \rho_2 \) by using the information of the peak and its nearest neighbours in an obvious way. In Fig. 1 a contour plot is given which shows the combined distribution of both variables for the 50 cooling steps data. No essential difference is obtained by looking at 25 cools, electric or magnetic and clover 1 or 2. The message of Fig. 1 is that our peaks are locally similar to instanton peaks. This behaviour is not a general consequence of self-duallity and the existence of a peak. The range of observed values of \( \rho_1 \) is determined by our ultraviolet and infrared cut-offs. We can produce instanton-like configurations with values of \( \rho \) close to the extremes observed in the contour plot. We cannot claim, nevertheless, that our peaks are BPST instantons, since there are other configurations showing a similar structure at the center of the peak but differing away from it. For
example, that is the case for the $Q = \frac{1}{2}$ instanton observed with twisted boundary conditions [20, 21].

The observed number of peaks decreases with the number of applied cooling steps. Is there any correlation between this decrease and the one of the string tension? On very general grounds one can relate both quantities to a scale. The number of peaks per unit volume gives a certain density which has dimensions of length to the power -4. The string tension has dimension of length to the power -2. Hence, we may form the scale invariant combination

$$K = \frac{\sigma}{\sqrt{N_{\text{peaks}}/\text{Volume}}}$$

(4)

Being dimensionless, it can be computed both in lattice or in physical units. In Fig. 2 we show the value of $K$ as a function of the number of cooling steps. We see that after 5 coolings, the value of $K$ shows a striking plateau. Although errors are shown, one must take into account that data at different number of cooling steps are strongly correlated. Hence, the plateau is much better that what errors might allow. In the same plot the data of the string tension (divided by 2 in $fm^{-2}$ units) is shown, to let the reader appreciate how the computation of the quotient does correct the decreasing behaviour.

The quality of the data leaves no doubt that the drop in the number of peaks and in the string tension have the same origin.

The extraction of the string tension from our lattice data is done by measuring correlations of straight-line Polyakov loops $C(t)$ at time separation $t$. As usual, the corresponding mass is obtained by considering first the effective mass defined as

$$M(t) = -\ln \left( \frac{C(t+1)}{C(t)} \right).$$

(5)

The mass parameter (which divided by $l_s$ gives the string tension) should be seen as a plateau in the value of $M(t)$ for sufficiently large $t$. Our data values for $M(t)$ do grow with $t$ for small values, but show a levelling up for distances $t$ of 5 or 6. Unfortunately, $t$ cannot exceed values of 6 or 7 since the errors grow too much. Combining this data with the definition of $K$, we might define

$$K(t) = \frac{M(t)}{(l_s\sqrt{N_{\text{peaks}}/\text{Volume})}}.$$

The plateau behaviour is obtained with
similar quality for all values of $t$, except for 5 coolings for which we have a better behaviour for large $t$. From the results obtained from 20 to 50 cooling steps our plateau values obtained are: $K(2) = 0.939 \pm 0.024 \pm 0.027$, $K(3) = 1.463 \pm 0.048 \pm 0.037$, $K(4) = 1.881 \pm 0.097 \pm 0.036$, $K(5) = 2.184 \pm 0.205 \pm 0.027$, $K(6) = 2.331 \pm 0.427 \pm 0.028$, $K(7) = 2.236 \pm 0.813 \pm 0.048$. The first error is statistical and is basically equal for all cooling steps. The second is the maximum difference observed in absolute value between the average and the value for all cooling steps from 20 to 50. It is a measure of the quality of the plateau. We stress that while the number of peaks drops by a factor of 2 from 20 to 50 cooling steps, $K$ varies by less than 2% in this interval.

3 Results and Conclusions

The results shown in the previous section demonstrate the existence of a clear correlation between the value of the string tension and the occurrence of peaks in the action density, or its electric and magnetic components. Furthermore, we have shown some evidence that the observed peaks are self-dual and of instanton-like character. Can we understand how this can occur? As mentioned previously, the quantity which shows a plateau is a dimensionless combination. Hence, we can explain such a behaviour if we consider a theory with a single length scale which could enter both the density of ”objects” and the string tension. Furthermore, in order to explain the drop in the string tension, it should happen that cooling produces a growth of this scale. We only know of one model that predicts such a behaviour, although there could be others as well. In this model, proposed by some of us \[16, 17\], Confinement is a property of a certain class of multi-instanton configurations, which are argued to dominate the path integral (with their cloud of perturbative fluctuations). The set of configurations is conjectured to be describable as a 4 dimensional liquid of $Q = \frac{1}{2}$ instantons. The main parameter of this fluid is its density, which is fixed dynamically to a value that was estimated to be in the order of $(0.7 fm)^{-4}$. It was essentially this value which set the scale to the string tension, which was estimated to be roughly of the size known
phenomenologically. In what follows we will show that our data are in other respects in agreement with the predictions of this model.

Obviously one of the characteristic ingredients of the model of Ref. [16, 17] is that the objects which make up the gas are self-dual and have fractional topological charge. Unfortunately, as mentioned previously, it is not possible to distinguish $Q = \frac{1}{2}$ instantons from $Q = 1$ instantons by looking in the neighbourhood of the maximum. By now we have been unable to find a simple algorithmic procedure to distinguish both types of configurations. We might however proceed in an statistical fashion and compute the mean action divided by $4\pi^2$ per peak configuration by configuration. This quantity also turns out to be fairly insensitive to cooling, varying from 1.072 to 1.132 as we move from 20 to 50 cooling steps. A more informative plot is that of Fig. 3, where we have plotted the ratio $\frac{S}{4\pi^2}$ versus the number of peaks $N_{\text{peaks}}$ for every configuration and for all cooling steps from 20 to 50 (in steps of 5). The two displayed lines correspond to the predictions for a gas of instantons (slope 2) and of $Q = \frac{1}{2}$ instantons (slope 1). Clearly all points cluster around the second line.

That the mean action per peak is equal to $4\pi^2$ doesn’t necessarily mean that all our peaks are $Q = \frac{1}{2}$ instantons. For example if some peaks carry no action and others have the typical instanton action $8\pi^2$, one can still end up getting the same mean action. To convince ourselves that this is not what is happening, we have been monitoring individual configurations as we cool the system. A good advantage of our elongated lattice shapes ($64 \times 8^3$), is the nice aspect of the energy profile, defined as

$$\mathcal{E}(t) = \int d^3 \vec{x} S(x).$$

(6)

As observed previously [20], this quantity behaves very approximately as if it was additive. If we have a collection of $Q = \frac{1}{2}$ instantons centered at different times, we might simply add the known profile of each object to have a prediction for the total profile. In some cases we need to add an ordinary $Q = 1$ instanton profile with a given width $\rho$ to have a good description. Fig 4. shows the comparison of the lattice data with the prediction for one of our configurations at 240 cooling steps. Essentially identical results are
gotten for 130 cooling steps. The discontinuous curve is obtained by adding the energy profiles of a set of $Q = \frac{1}{2}$ instantons centered at the time positions where we have observed peaks in the action density (no free parameters).

To describe the data better, 3 of the peaks are set to $Q = 1$ instantons with values of $\rho$ which agree with the $\rho_1$ or $\rho_2$ parameters of the peak. If we leave the positions and widths as free parameters, we can obtain a line which goes basically through all our data points, which is also shown in Fig 4. The new locations of the instantons are in most cases within one lattice spacing of the peak position. The displacement is larger in those places where there are other nearby peaks, as expected. This case shows that indeed, at least for this configuration and cooling value, the majority of our peaks are associated with $Q = \frac{1}{2}$ instantons. The other configurations that we have looked at show essentially the same behaviour.

Now suppose that indeed the ratio $K$ is the same all the way down to zero coolings. Of course it is very hard to count the number of peaks for zero cooling steps, since most of the peaks are purely the result of high frequency noise. This might explain also why at 5 cooling steps there is a drop in the measured value of $K$: together with the instantonic peaks there are also some noise peaks. Notice nonetheless that at 5 cooling steps one can extract values of the string tension which are fairly in agreement with the uncooled ones and that the departure of $K$ from the plateau value there is not too big. Hence, we might be seeing indeed a property of the uncooled vacuum. Since the string tension value is known ($\sigma = 5\, fm^{-2}$), we can use the measured value of $K$ to predict the density. A fairly safe estimate for $K$, coming from $t = 6$ is $K = 2.33(43)$, which gives for the density $(0.68(7)\, fm)^{-4}$. Notice that the resulting value is very close to the estimated density for our model.

Finally, let us give our explanation for the decrease of the number of peaks or of the string tension. Indeed, if those peaks were instantons we should observe a decrease in their number due to the previously mentioned instanton-antiinstanton annihilation mechanism. An analogous annihilation takes place for opposite charge $Q = \frac{1}{2}$ instantons. The density decrease should then drive the observed one for the string tension. Actually, our monitoring of individual configurations is quite consistent with this mechanism.
being the main cause of peak destruction.

In summary, we have shown that the decrease of the string tension with cooling goes parallel with the decrease in the number of peaks in the action density. This shows a clear relation between the occurrence of gauge invariant structures and Confinement. We have offered an explanation in which the decrease in peak number is due to the annihilation of instanton-antiinstanton pairs. One can explain the observed dependence of the string tension with the instanton density, within the model of Ref. [16, 17]. This is a $Q = \frac{1}{2}$ instanton liquid model with density close to $(0.7 \text{fm})^{-4}$. We have seen that the observed total action per peak $(4\pi^2)$ is not far from the prediction of this model. Although we lack a determination of the number of $Q = \frac{1}{2}$ instantons present in our configurations, we have analysed individual configurations and verified that indeed they have a large number of the latter objects. It is possible that other models also explain the data. In any case, the association of gauge invariant structures with Confinement need not be necessarily in contradiction with the dual superconductor description. Indeed, we have recently observed that the $Q = \frac{1}{2}$ instantons give rise, when maximally abelian projected, to monopole (both elementary and extended) currents [22] going exactly through their centers. A monopole-instanton correlation study is in progress.

Acknowledgements

This work was supported by the CICYT grants AEN93-0693 and the EC network CHRX-CT93-0132. Useful conversations with M. Polikarpov are acknowledged.

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Figure Captions

Figure 1: Contour plot which shows the combined distribution of variables $\rho_1$ (vertical axis) and $\rho_2$ (horizontal axis) defined in Eq. (3) for all our peaks obtained at 50 cooling steps.

Figure 2: The value of $K$ (circles) defined in Eq. (4) is plotted as a function of the number of cooling steps $N_c$. The horizontal line $K = 2.17$ comes from a fit to the data from 15 to 50 cooling steps. For comparison, in the same plot the value of $\frac{\sigma^2}{2}$ (triangles) in $fm^{-2}$ is displayed. In both cases the value of the string tension $\sigma$ is extracted from correlations of Polyakov loops at distances 5 and 4.

Figure 3: A density plot showing for all our configurations and all cooling steps from 20 to 50 (in steps of 5) the total action of the configuration in units $4\pi^2 \left( \frac{S}{4\pi^2} \right)$ versus the number of peaks $N_{\text{peaks}}$ of this configuration. The two lines correspond to the predictions for a gas of $Q = 1$ instantons (slope 2) and of $Q = \frac{1}{2}$ instantons (slope 1).

Figure 4: We show the energy profile (Eq. (6)) for one of our configurations at 240 cooling steps. The discontinuous line is obtained by adding the energy profiles of a set of $Q = \frac{1}{2}$ and $Q = 1$ instantons centered at the time positions where we have observed peaks in the action density (no free parameters). Three out of the eleven peaks are $Q = 1$ instantons with values of $\rho$ which agree with the $\rho_1$ and $\rho_2$ parameters of the peak. By allowing the centers of the instantons to vary slightly one gets the continuous line.
Fig. 4