Thermodynamics of topological black holes in Hořava–Lifshitz gravity

Hernando Quevedo, Alberto Sánchez, and Safia Taj

1 Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, AP 70543, México, DF 04510, Mexico
2 Dipartimento di Fisica and ICRA, Università di Roma La Sapienza, I-00185 Roma, Italy
3 Departamento de Posgrado, CIIDET, AP 752, Querétaro, QRO 76000, Mexico
4 Center for Advanced Mathematics and Physics, National University of Sciences and Technology, H-12, Islamabad, Pakistan

E-mail: quevedo@nucleares.unam.mx, asanchez@nucleares.unam.mx, safiataaj@gmail.com

Abstract. We investigate the thermodynamic properties of the most general static, spherically symmetric, topological black holes of the Hořava-Lifshitz gravity. The mass of these black holes turns out to depend on the entropy and the cosmological constant that we consider as an additional thermodynamic variable. We study different thermodynamic ensembles and the corresponding heat capacities in order to show that second-order phase transitions occur at certain specific values of the horizon radius. Moreover, we study the geometric properties of the equilibrium space by using the formalism of geometrothermodynamics. We show that the phase transitions, which are characterized by divergencies of the heat capacities, are described in geometrothermodynamics by curvature singularities in the equilibrium space.

1. Introduction

Recently, a field theoretical model that can be interpreted as a complete theory of gravity in the ultraviolet (UV) limit was recently proposed by Hořava [1, 2]. The model is renormalizable [3] and non-relativistic in the UV regime. Moreover, in the infrared (IR) limit it can be reduced to Einstein’s gravity theory with a cosmological constant. In this model, space and time have different scalings at the UV fixed point, i.e., $x^i \to lx^i$, $t \to tz$ where $z$ is the scaling exponent. In particular, it is renormalizable if $z = 3$. It is usually named Hořava-Lifshitz (HL) theory in the literature [4, 5] to emphasize the anisotropy of space and time.

It was found that the Schwarzschild–anti de Sitter black hole solution is not recovered in the IR limit [6]. This difficulty was solved by introducing an additional parameter which modifies the IR behavior [7, 8, 9]. This theoretical model of quantum gravity has been extensively investigated in the context of black hole physics and classical cosmology (see, for instance, [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] and the references cited therein).

The HL gravity breaks general 4D covariance and splits it into 3D covariance plus reparametrization invariance of time. It is therefore convenient to formulate it in the $(3+1)$–ADM formalism, where an arbitrary metric can be written in the form

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

(1)
where $N^2$ is the lapse function and $N^i$ represents the shift. Then, the HL action is written as [20],

$$
\mathcal{I}_{HL} = \int \mathcal{L}_{HL} \, dt \, d^3x,
$$

(2)

where

$$
\mathcal{L}_{HL} = \sqrt{g} N \left[ \frac{2}{\kappa} \left( K_{ij} K^{ij} - \lambda K^2 \right) + \frac{\kappa^2 \mu^2 (\Lambda R - 3\Lambda^2)}{8(1 - 3\lambda)} + \frac{\kappa^2 \mu^2 (1 - 4\Lambda)}{32(1 - 3\lambda)} R^2 - \frac{\kappa^2}{2\omega^4} \left( C_{ij} - \frac{\mu\omega^2}{2} R_{ij} \right) \right].
$$

(3)

Here $R_{ij}$ and $R$ are the 3D Ricci tensor and curvature scalar, respectively. Moreover, the extrinsic curvature $K_{ij}$ and the Cotton tensor $C_{ij}$ are given by the expressions

$$
K_{ij} = \frac{1}{2N} \left( \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right), \quad C_{ij} = \epsilon^{ikl} \nabla_k \left( R^l - \frac{1}{4} R \delta^l_1 \right),
$$

(4)

where a dot represents differentiation with respect to the time coordinate. Finally, $\kappa^2$, $\lambda$, $\mu$, $\omega$ and $\Lambda$ are constants parameters.

The vacuum of this theory turns out to be the anti–de Sitter spacetime; however, it is possible to consider an additional term $(\mu^4 R)$ in the original action to obtain a Minkowski vacuum in the IR limit. This generalization is known as the deformed HL model.

A comparison of the HL action with the Einstein-Hilbert action leads to the conclusion that the speed of light, Newton’s constant and the cosmological constant $\tilde{\Lambda}$ are given by

$$
c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda}{1 - 3\lambda}}, \quad G = \frac{\kappa^2 c}{32\pi}, \quad \tilde{\Lambda} = \frac{3}{2} \Lambda.
$$

(5)

Consider now the spherically symmetric line element

$$
ds^2 = -\tilde{N}^2(r)f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega_k^2,
$$

(6)

where $d\Omega_k^2$ is the line element of the 2–dimensional Einstein space with constant curvature $2k$. Substituting the metric (6) into the action (2), we obtain [21],

$$
\mathcal{I}_{HL} = \frac{\kappa^2 \mu^2 \Omega_k}{8(1 - 3\lambda)} \int \tilde{N} \left[ \frac{(\lambda - 1)}{2} F'^2 - \frac{2\lambda}{r} FF' + \frac{(2\lambda - 1)}{r^2} F^2 \right] dt \, dr,
$$

(7)

where a prime denotes the derivative with respect to $r$, and $F$ is defined as,

$$
F(r) = k - \Lambda r^2 - f(r).
$$

(8)

The variation of (7) leads to the following set of equations

$$
\left( \frac{2\lambda}{r} F - (\lambda - 1) F' \right) \ddot{\tilde{N}} + (\lambda - 1) \left( \frac{2}{r^2} F' - F'' \right) \dot{\tilde{N}} = 0,
$$

(9)

$$
(\lambda - 1)r^2 F'^2 - 4\Lambda r FF' + 2(2\lambda - 1) F^2 = 0,
$$

(10)
whose solution is
\[ F(r) = \alpha r^s, \quad \tilde{N} = \gamma r^{1-2s}, \]  
(11)
where \( \alpha \) and \( \gamma \) are integration constants and \( s \) is given by
\[ s = \frac{2\lambda - \sqrt{2(3\lambda - 1)}}{\lambda - 1}. \]  
(12)

This solution was obtained recently by Cai, Cao and Ohta (CCO) [22]. In general, the value of \( s \) with a positive sign in front of the square root is also a solution of the above equations. However, in this case the asymptotic properties of the solution are not compatible with the properties of a black hole spacetime. In the allowed interval \( \lambda > 1/3 \), i.e. for \( s \in (-1, 2) \), the above solution is asymptotically anti–de Sitter and describes the gravitational field of a static black hole.

The main goal of the present work is to analyze the thermodynamic properties and the geometry of the equilibrium space of the CCO topological black holes. In particular, we find the different heat capacities that are derived by using different ensembles. It follows that these black holes posses a phase transition structure that is characterized by divergencies of the heat capacities, indicating that the phase transitions are of second order. This paper is organized as follows. In Sec. 2, we derive the most significant thermodynamic variables of the CCO black holes and study their behavior. The heat capacities are calculated explicitly and their divergencies are discussed. In Sec. 3, we briefly review the formalism of geometrothermodynamics that is then used to study the geometric properties of the equilibrium space. We show that the thermodynamic properties of the CCO black holes can be described correctly in terms of the geometric properties of the equilibrium space. Finally, in Sec. 4 we present the conclusions of our work. Throughout this work we use units with \( c = G = \hbar = k_B = 1 \).

2. Thermodynamics

The most popular method to derive the thermodynamic variables consists in computing the total energy of the system in terms of physical variables, say \( E^a \) with \( a = 1, \ldots, n \). In the case of black holes, the total energy coincides with the total mass \( M \) so that \( M = M(E^a) \). Moreover, usually one of the variables \( E^a \) can be associated with the area of the outermost horizon. Then, if we invoke the fundamental assumption of black hole thermodynamics, according to which the area of the horizon is proportional to the entropy, it turns out that the total mass depends on the entropy, i.e., \( M = M(S, E^i) \) with \( i = 2, \ldots, n \). It then follows that \( M = M(S, E^i) \) represents the fundamental thermodynamic equation from which the remaining variables can be derived. In fact, if we consider \( M \) as a thermodynamic potential, and assume the validity of the first law of thermodynamics
\[ dM = TdS + I_i dE^i, \quad I_i = \frac{\partial M}{\partial E^i}, \]  
(13)
the thermodynamic variables \( I_i \) dual to \( E_i \) can easily be computed. This strategy has been widely used in the context of black hole thermodynamics.

To determine the total mass of the CCO black hole, it is convenient to use the Hamiltonian approach that yields [21] the mass
\[ M = M_0 l^2 - 2s \left[ k + \frac{r^2}{l} \right]^2, \quad M_0 = \frac{c^3 \gamma \Omega_k}{16\pi G} \left( \frac{1 + s}{2 - s} \right). \]  
(14)

In the interval of values allowed for the parameter \( s \) the total mass turns out to be positive definite. Furthermore, the entropy can be shown to be given as
\[ S = \frac{4\pi M_0 l^2}{\gamma} \left( \frac{r^2}{l^2} + k \ln \frac{r^2}{l^2} \right). \]  
(15)
Here $\tilde{l}^2 = -1/\Lambda$ is the curvature radius and $r_+$ represents the radius of the outer horizon that is a function of $M$ and $l$ determined by the algebraic equation

$$r_+^2 - \frac{\mathcal{A}}{M^2 l^2} r_+^2 - \frac{\mathcal{A}k}{M^2 l} = 0, \quad \mathcal{A} = \frac{\kappa \mu \gamma^2 \Omega_k^2}{2 \pi^4 [3\lambda - 1]^2}. \quad (16)$$

According to the above equations, the total mass depends implicitly on $S$ and explicitly on $l$. Although only $S$ is clearly a thermodynamic variable, we assume here that the curvature radius $l$ can also be considered as a thermodynamic parameter, and we will investigate the consequences that follow from this assumption. Then, the first law of thermodynamics for this case reads

$$dM = T dS - L dl \quad (17)$$

where $L$ is the thermodynamic variable dual to the curvature radius $l$. It is then straightforward to derive the expressions for the Hawking temperature $T$ and the dual variable $L$ as

$$T = \frac{\gamma}{4\pi r_+^{2s}} \left[ (2 - s) \frac{r_+^2}{l^2} - ks \right], \quad (18)$$

$$L = 2M_0 l r_+^{2s} \left( \frac{r_+^4}{l^4} - k^2 \right). \quad (19)$$

The behavior of the temperature is illustrated in Fig.1 for different values of the parameters. Notice that for all the values of $k$ there exist unphysical regions with $T < 0$ that must be excluded from the analysis.

2.1. Phase transitions

According to Davies [23, 24, 25], second order phase transitions take place at those points where the heat capacity diverges. In this context, it is important to define the “heat” for the black hole. To this end, we simply use the analogy with ordinary thermodynamics and rewrite the first law of thermodynamics as

$$dM = dQ - L dl \quad (20)$$

where the “heat” is assumed to be determined as $dQ = T dS$. Then, keeping the value of $l$ constant we can define

$$C_l = \left( \frac{\partial Q}{\partial T} \right)_l = \left( \frac{\partial M}{\partial T} \right)_l = \left( \frac{\partial M}{\partial r_+} \right)_l \left( \frac{\partial T}{\partial r_+} \right)_l^{-1}. \quad (21)$$

In the present case, using the expressions (14) and (18), we obtain

$$C_l = \frac{4\pi M_0 l^2}{\gamma} \left[ \frac{\left( k + \frac{r_+^2}{l^2} \right) \left[ (2 - s) \frac{r_+^2}{l^2} - ks \right]}{(s - 1)(s - 2) \frac{r_+^2}{l^2} + ks^2} \right]. \quad (22)$$

It follows that the singularities of the heat capacity are located at

$$\frac{r_+^2}{l^2} = \frac{ks^2}{(s - 1)(2 - s)}. \quad (23)$$

We see that phase transitions can occur only for $k = 1$ and $s \in (1, 2)$, and for $k = -1$ and $s \in (-1, 1)$. For all the remaining values of $k$ and $s$, the corresponding black hole cannot
Figure 1. Behavior of the temperature for $k = -1$, $k = 0$ and $k = +1$, respectively, in terms of the horizon radius $r_+$ and the parameter $s$. Here we set $l = 1$ and $\gamma = 1$. Only the regions with $T > 0$ are physically allowed.

undergo a phase transition. Notice, however, that the phase transition condition (23) must be considered together with the inequality

\[
\frac{r_+^2}{l^2} > \frac{ks}{2 - s}
\]  

that follows from the condition $T > 0$ from Eq.(18).

An additional heat capacity can be obtained by applying the partial Legendre transform

\[
M_1 = M + IL
\]  

that corresponds to a different thermodynamic ensemble. Then, the first law of thermodynamics reads

\[
dM_1 = dQ + ldL .
\]
A straightforward computation yields

\[ M_1 = M_0 l^2 r^{-2s} \left( \frac{r^4}{l^4} + 2 \frac{r^2}{l^2} - k^2 \right). \tag{27} \]

It is then clear that for this ensemble we can define an additional heat capacity that takes into account the change of “heat” in terms of the temperature for a fixed \( L \), i.e.,

\[ C_L = \left( \frac{\partial Q}{\partial T} \right)_L = \left( \frac{\partial M_1}{\partial T} \right)_L = \frac{4\pi M_0 l^2}{\gamma} \frac{\left( 2s - 3 \right) \frac{r^4}{l^4} + 2 \frac{r^2}{l^2} - 3k^2}{(s-1)(s-2) \frac{r^4}{l^4} + ks^2}. \tag{28} \]

We see that the divergencies of \( C_L \) coincide with the roots of the equation (23); this means that both \( C_L \) and \( C_I \) show the same phase transition structure for the CCO black hole.

### 3. Geometrothermodynamics

Geometrothermodynamics (GTD) is a theory that has been formulated recently [26] in order to introduce in a consistent manner the Legendre invariance in the geometric description of the space of thermodynamic equilibrium states. In all the cases analyzed so far, GTD has delivered consistent results and allows us to describe geometrically the thermodynamic interaction and the phase transitions by means of Legendre invariant metrics. This theory has been applied to different thermodynamic systems the ideal gas, the van der Waals gas and a diverse black holes in different theories [27, 28, 29, 30, 31, 32]. In this Section we will show that GTD correctly describes the thermodynamic properties of the CCO black hole.

The main ingredient of GTD is a \((2n+1)\)-dimensional manifold \( \mathcal{T} \) where \( n \) is the number of thermodynamic degrees of freedom of the system. We introduce in \( \mathcal{T} \) the set of coordinates \( Z^A = (\Phi, E^a, I^a) \), where \( \Phi \) is an arbitrary thermodynamic potential, \( E^a \), \( a = 1, 2, ..., n \), are the extensive variables, and \( I^a \) the intensive variables. It is also possible to introduce in a canonical manner the fundamental one–form \( \Theta = d\Phi - \delta_{ab} I^a dE^b \), \( \delta_{ab} = \text{diag}(+1, ..., +1) \), that satisfies the condition \( \Theta \wedge (d\Theta)^n \neq 0 \), and is invariant with respect to Legendre transformations \((\Phi, E^a, I^a) \rightarrow (\tilde{\Phi}, \tilde{E}^a, \tilde{I}^a) \) with \( \tilde{\Phi} = \Phi - \delta_{ab} E^a \tilde{I}^b \), \( E^a = -\tilde{I}^a \), and \( I^a = \tilde{E}^a \). Moreover, we assume that on \( \mathcal{T} \) there exists a metric \( G \) which is also invariant with respect to Legendre transformations. The triad \((\mathcal{T}, \Theta, G)\) defines a Riemannian contact manifold that is called the thermodynamic phase space (phase manifold). The space of thermodynamic equilibrium states (equilibrium manifold) is an \( n \)-dimensional Riemannian submanifold \( \mathcal{E} \subset \mathcal{T} \) induced by a smooth map \( \varphi : \mathcal{E} \rightarrow \mathcal{T} \), i.e. \( \varphi : (E^a) \mapsto (\Phi, E^a, I^a) \), with \( \Phi = \Phi(E^a) \) and \( I^a = I^a(E^a) \), such that \( \varphi^*(\Theta) = \varphi^*(d\Phi - \delta_{ab} I^a dE^b) = 0 \) holds, where \( \varphi^* \) is the pullback of \( \varphi \). The manifold \( \mathcal{E} \) is naturally equipped with the Riemannian metric \( g = \varphi^*(G) \). The purpose of GTD is to demonstrate that the geometric properties of \( \mathcal{E} \) are related to the thermodynamic properties of a system with fundamental thermodynamic equation \( \Phi = \Phi(E^a) \).

The nondegenerate Legendre invariant metric [33]

\[ G = \Theta^2 + \frac{1}{2} \left( \delta_{ab} - \eta_{ab} E^a E^b \right) \left( \eta_{cd} dE^c dI^d \right), \tag{29} \]

where \( \eta_{ab} = \text{diag}(-1, 1, \ldots, 1) \), has been used extensively to describe second order phase transitions, especially in the context of black hole thermodynamics. The metric (29) induces on \( \mathcal{E} \), by means of \( g = \varphi^*(G) \), the thermodynamic metric

\[ g = \frac{1}{2} \left[ E^a \left( \frac{\partial \Phi}{\partial E^a} - \eta_{ab} \delta^{bc} \frac{\partial \Phi}{\partial E^c} \right) \right] \left( \eta_{ab} \delta^{bc} \frac{\partial^2 \Phi}{\partial E^c \partial E^d} dE^a dE^d \right). \tag{30} \]
Notice that the metric $g$ can be computed explicitly once the fundamental equation $\Phi = \Phi(E^a)$ is given whose existence is assumed to be guaranteed by the definition of the smooth map $\varphi$. Moreover, the second law of thermodynamics is equivalent to the condition

$$\pm \frac{\partial^2 \Phi}{\partial E^a \partial E^b} \geq 0,$$

where the sign $\pm$ depends on the choice of the thermodynamic potential. For instance, the positive sign must be selected, if $\Phi$ is chosen as the entropy. In the energy representation, $\Phi$ corresponds to the internal energy and the sign must be negative.

For the CCO topological black hole let us take, for instance, $\Phi = M$. The coordinates of the 5-dimensional phase manifold can be chosen as $Z^A = (M, S, l, T, L)$, where $T$ is the temperature dual to $S$ and $L$ is the dual of the curvature radius $l$. The fundamental one-form is then $\Theta = dM - T dS + L dl$ and the Legendre invariant metric (29) is written as

$$G = \Theta^2 + ST (-dSdT + dldL).$$

The smooth map $\varphi : \mathcal{E} \rightarrow \mathcal{T}$ or in coordinates

$$\varphi : (S, l) \mapsto \{ M(S, l), S, l, T(S, l), L(S, l) \}$$

determines the equilibrium manifold $\mathcal{E}$ with metric

$$g = \varphi^*(G) = S M_S \left( -M_S S dS^2 + M_0 dl^2 \right),$$

on which the first law of thermodynamics $dM = TdS - Ldl$ and the equilibrium conditions

$$T = \frac{\partial M}{\partial S} \equiv M_S, \quad -L = \frac{\partial M}{\partial l} \equiv M_l$$

hold.

In this particular case, the fundamental equation $M = M(S, l)$ cannot be written explicitly and so we use $r_+$ instead of $S$ as a coordinate. Then,

$$g = SM_S \left[ -M_S S r_+^2 dr_+^2 - 2M_S S r_+ S l dldr_+ + (M_0 - M_S S^2) dl^2 \right].$$

Using the expressions for the mass and the entropy, we obtain

$$g = \frac{\epsilon^3 \Omega^2 k^2 r_+^{-4s}}{16 \pi^2 G} \left( \frac{s + 1}{2 - s} \right) \left( s - 2 \right) \frac{r_+^2}{l^2} + ks \right] S(r_+, l) \left\{ \left( s - 1 \right) \left( s - 2 \right) \frac{r_+^2}{l^2} + ks^2 \right\}
\times \left[ \left( 1 + \frac{kl}{r_+^2} \right) dr_+^2 + 2 \frac{kl}{r_+} \left( \ln \frac{r_+^2}{l^2} - 1 \right) dldr_+ + F(r_+, l) dl^2 \right],$$

where

$$F(r_+, l) = \frac{2k^2 \left( \ln \frac{r_+^2}{l^2} - 1 \right)^2 \left[ s - 1 \right] \left( s - 2 \right) \frac{r_+^2}{l^2} + ks^2 \right] - \left( k^2 + 3 \frac{r_+^2}{l^2} \right) \left( k + \frac{r_+^2}{l^2} \right).$$
The curvature scalar corresponding to the metric (36) can be computed explicitly as

\[ R = \frac{N}{D}, \quad D = \left( k^2 + 3 \frac{r_+^4}{l^4} \right)^2 \left( k + \frac{r_+^2}{l^2} + k \ln \frac{r_+^2}{l^2} \right)^3 \left( \frac{r_+^2}{l^2} + k \ln \frac{r_+^2}{l^2} \right) \times \left[ (2 - s) \frac{r_+^2}{l^2} - ks \right]^4 \left[ (s - 1)(s - 2) \frac{r_+^2}{l^2} + ks^2 \right]^{\frac{2}{3}} \]

(39)

where \( N \) is a function of \( r_+ \) and \( l \) that is finite at those points where the denominator vanishes. Notice that there are several curvature singularities. The first set of singularities is situated at the roots of \( k + r_+^2/l^2 = 0 \). As follows from Eq.(14), these singularities correspond to the limit \( M \to 0 \). A second singularity is located at the roots of the equation \( r_+^2 + k \ln r_+^2 = 0 \); from Eq.(15) it follows that it can be interpreted as the limit in which the entropy vanishes. Furthermore, according to Eq.(18), the singularity situated at \( r_+^2/l^2 = ks/(2 - s) \) corresponds the limit \( T \to 0 \). Finally, if \( (s - 1)(s - 2)r_+^2/l^2 + ks^2 = 0 \) a singularity occurs that, according to Eq.(22), coincides with the limit \( C \to \infty \), i.e., with the points where second order phase transitions take place. The singularities at which the mass, the entropy or the temperature vanish indicate the limit of applicability of the thermodynamics of black holes and of GTD. We conclude that the curvature obtained within the formalism of GTD describes correctly and in an invariant manner the thermodynamic behavior of topological black holes in HL gravity.

4. Conclusions
In this work, we studied the thermodynamic properties of the Cai-Cao-Ohta (CCO) topological black holes in the Hořava-Lifshitz model of quantum gravity. We assumed the validity of the first law of thermodynamics of black holes to show the consistency of the thermodynamic variables. In particular, we derived two different heat capacities \( C_l \) and \( C_L \) that turned out to predict the existence of a non-trivial phase transition structure. In fact, it was shown that both heat capacities diverge at certain values of the horizon radius, indicating that second-order phase transitions occur at the singular points.

We applied the formalism of geometrothermodynamics (GTD) to describe the thermodynamics of the CCO black holes in terms of geometric concepts. In the thermodynamic phase manifold we introduce a particular Riemannian metric whose main property is its invariance with respect to Legendre transformations, i.e., its geometric characteristics are independent of the choice of thermodynamic potential. This is a property that holds in ordinary thermodynamics and that we assume as valid in GTD too. The Legendre invariant metric induces in a canonical manner a thermodynamic metric in the equilibrium manifold that is defined as a submanifold of the thermodynamic phase manifold.

We used the expressions for the main thermodynamic variables of the CCO black holes in order to compute the explicit form of the thermodynamic metric of the equilibrium manifold. The corresponding thermodynamic curvature turned out to be nonzero in general, indicating the presence of thermodynamic interaction. Moreover, it was shown that the phase transitions, which are characterized by divergencies of the heat capacities, are described in GTD by curvature singularities in the equilibrium manifold.

It was found that the geometrothermodynamic equilibrium manifold of the CCO black holes present additional curvature singularities which correspond to the vanishing of the mass, entropy and Hawking temperature. We interpret in general the vanishing of these thermodynamic variables as an indication of the limit of applicability of black hole thermodynamics. So we conclude that the formalism of GTD breaks down, with curvature singularities, exactly at those points where black hole thermodynamics fails.
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