Infrared small target detection based on robust principal component analysis joint directional derivative penalty

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Abstract. Many robust principal component analysis based methods have been proposed for infrared small target detection recently. However, due to the lack of local prior information, these methods show poor performance under complex backgrounds. In this paper, a novel infrared patch-image model joint the directional derivative saliency feature is proposed. First, Laplace norm minimization is used instead of nuclear norm minimization to approximate low rank background can automatically capture the inherent rank information, which has higher matching with the real rank. Then, considering that the $l_1$-norm constrains target patch-image inexactly, we construct a saliency map by using the directional derivative feature of the target and incorporate it into the $l_1$-norm to enhance target and suppress background clutter simultaneously. Extensive experiments show the superiority of the proposed method to the other state-of-the-art methods in robustness and efficiency.

1. Introduction

Small target detection plays a vital role in infrared search and track (IRST) systems. It has been widely applied to precision-guided weapons, medical imaging, and early-warning system. Among the existing methods, these methods based on robust principal component analysis (RPCA) achieve great performance, such as IPI model[1]. However, under the scene with strong background edges and pixel-sized noise, there is a high false alarm rate in IPI detection result. The first reason is that nuclear norm is not accurate enough to estimate the background. We apply Laplace norm to approach low rank matrix. The second reason is that the strong background edge and pixel-sized noise meet the same sparseness as the small target[2]. To further separate the small target from background residuals and pixel-sized noise, the saliency feature of the small target can be taken into account. Therefore, a novel directional derivative based saliency map is constructed and is combined with the $l_1$-norm as a sparsity penalty term in this letter. With the novel sparsity penalty term, we propose a robust infrared small target detection method based on directional derivative weighted IPI (DDWPI).

2. Introduction of IPI model

Generally speaking, an infrared image with small target consists of three parts: target, background and noise, which can be described by an additive model as follows:

$$f(x,y) = f_b(x,y) + f_T(x,y) + f_N(x,y)$$

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where \( f_1(x, y), f_t(x, y), f_b(x, y), f_n(x, y) \) represent the original image, target image, background image and noise image respectively. Gao et al.[1] transformed the original image into patch image \( I \), which can be separated into target patch image \( T \) and background patch image \( B \):

\[
I = B + T + N
\]  

Then, with low-rank hypothesis of the background and sparse hypothesis of the target, the small target detection task is transformed into low-rank and sparse matrix decomposition problem:

\[
\min_{B,T} \|B\|_\ast + \lambda \|T\|_1 \quad \text{s.t.} \quad I = B + T
\]  

where \( \| \cdot \|_\ast \) is the nuclear norm, (i.e. the sum of singular values), \( \| \cdot \|_1 \) is the \( l_1 \)-norm (i.e. \( \| A \|_1 = \sum |A_{ij}| \)), and \( \lambda \) is a positive weighting parameter.

3. Methodology

3.1. Laplacian low rank matrix approximation

When facing the complex infrared scene, the performance of IPI is very weak. Firstly, in the IPI model, the nuclear norm used for low rank approximation adds all singular values, which indicates that the larger singular value is more severely punished than the smaller one. It leads to the excessive shrinkage of the rank component[3]. Secondly, the convergence speed of the nuclear norm based method is generally poor due to a large number of iterations. Thirdly, IPI model needs some methods to define the rank information in advance, which is impractical in the real detection process[4]. To approach the real rank more accurately, Chen et al.[5] proposed Laplace norm to approach low rank matrix, which is defined as:

\[
\text{rank}(L) \approx \|L\| = \sum_{i=1}^{\min\{r,c\}} \left(1 - e^{-\alpha(L)/\gamma}\right)
\]  

where \( \gamma > 0 \).

It can be seen from figure 1 that when the singular value is greater than 1, the nuclear norm, log determinant function and Schatten-p norm deviate from 1 obviously, which indicates that they make the rank component shrink excessively. In contrast, the Laplacian norm has a higher matching with the real rank, which indicates that the Laplacian norm is closer to the rank function than other functions. Also, Laplace norm does not need to obtain prior information, and can automatically capture the inherent rank information. Therefore, this chapter uses Laplace norm to constrain the background block group.

![Figure 1](image_url)  

**Figure 1** The effect of different rank functions approximating real rank.

3.2. Saliency mapping based on directional derivative

With a long detection distance in IRST, the small target can be regarded as a point source. Based on the prior information, a small target can be modelled by using a 2D Gaussian function[6]:

\[
f(x,y) = \gamma \exp \left[ -\frac{1}{2} \left( \frac{x}{\sigma_x} \right)^2 + \left( \frac{y}{\sigma_y} \right)^2 \right]
\]
where $\gamma$ is target amplitude, $\sigma_x, \sigma_y$ are horizontal and vertical extent parameters, respectively. As shown in figure 2(a), in both horizontal and vertical derivatives of the target, positive and negative peaks appear in turn in the specified direction because of the rotational symmetry of 2D Gaussian function. Furthermore, figure 2(b) illustrates that there is directional consistency in the corresponding directional derivatives of background edge. Thus, the difference between the first-order directional derivative distribution of background and small target is significant. According to the positive and negative peaks in the directional derivative of the target, a method of regional division is adopted to construct a saliency map[7], as shown in figure 3. With a given direction, the energy intensity of each sub-area is defined by:

$$D_i = \frac{1}{N_i} \sum_j d_{ij}$$

(6)

where $N_i$ is the pixel number in sub-area $\Psi_j (i=1,2)$, and $d_{ij}$ is the directional derivative of each pixel. When the direction angle is $\alpha$, the energy intensity of the local region is defined by:

$$D^\alpha = \begin{cases} \sum_{i=1}^{2} |D_i|, & \text{if } D_1 > 0, D_2 < 0 \\ 0, & \text{otherwise} \end{cases}$$

(7)

where $\alpha = 0', 90'$. Finally, by fusing the energy intensity of each direction, the directional derivative based saliency map can be accumulated as:

$$D_{ij} = \prod_{\alpha} D^\alpha$$

(8)

where $i,j$ is the current pixel position.

Combining the directional derivative based saliency map and $l_1$-norm, we propose directional derivative weighted $l_1$-norm, which is defined by:

$$\| W_D \odot T \| = \sum W_D(i,j)T_{i,j}$$

(9)

$$W_D(i,j) = \frac{1}{D^\alpha_{ij} + \epsilon_T}$$

(10)

where $\odot$ is Hadamard product, $D^\alpha_{ij}$ is denoted as patch-map constructed by saliency map $D$, $W_D(i,j)$ is one element of the weight matrix $W_D$ at position $(i,j)$, and $\epsilon_T$ is a positive parameter to avoid division zero. Thus, DDWPI model can be formulated as follows:

$$\min_{B,T} \| B \|_1 + \lambda \| W_D \odot T \| \quad \text{s.t. } I = B + T$$

(11)
3.3. Optimization procedure of DDWIPI model

In this paper, the optimization algorithm based on alternating direction method of multipliers (ADMM)[8] is used to solve (11), then the augmented Lagrangian function of (11) is as follows:

\[
L(B, Y, \mu) = \|B\|_F + \lambda \|W \odot T\|_F + \langle Y, B + T - I \rangle + \frac{\mu}{2} \|B + T - I\|_F^2
\]  

(12)

where \(Y\) is the Lagrange multiplier, \(\langle \cdot, \cdot \rangle\) is the inner product of two matrices, \(\mu\) is a positive penalty parameter, and \(\|\cdot\|_F\) is the Frobenius norm. The minimization problem of (12) can be decomposed into several subproblems by ADMM. When fixing other variables, \(B\), \(T\), and \(Y\) can be updated respectively. Update \(B\):

\[
B^{k+1} = \arg \min_B \|B\|_F + \frac{\mu}{2} \|B + T^k - I + \frac{Y^k}{\mu}\|_F^2
\]  

(13)

\[
B^{k+1} = U D_{\mu}(\Sigma) V^T
\]  

(14)

where \(Z = I - T^k + \left(Y^k/\mu\right)\), singular value decomposition of \(Z\) is \(Z = U \Sigma V^T\), \(D_{\mu}(\Sigma) = \text{diag}\left(\max\left(\sum_{\sigma_i, \mu_i} - (\nabla \Phi(\sigma_i)/\mu)\right), 0\right)\), \(\Phi(\sigma) = 1-e^{-\sigma/\lambda}\). Update \(T\):

\[
T^{k+1} = \arg \min_T \|W \odot T\|_F + \frac{\mu}{2} \|B^{k+1} + T - I + \frac{Y^k}{\mu}\|_F^2
\]  

(15)

\[
T^{k+1} = S_{\mu/\lambda}(I - B^{k+1} - \frac{Y^k}{\mu})
\]  

(16)

where \(S_{\mu/\lambda}(\cdot)\) is the soft threshold operator. Update \(Y\):

\[
Y^{k+1} = Y^k + \mu \left(I - B^{k+1} - T^{k+1}\right)
\]  

(17)

Figure 4 shows the overall framework of DDWIPI model detecting small and weak targets.

4. Experimental analysis

In the proposed DDWIPI method, the patch size is set as 50×50, and the sliding step is set as 10. The sparse penalty \(\lambda = \sqrt{\max(m, n)}\), and \(\mu = \min(\rho \mu_{\max}, \mu_{\max})\), where \(\rho = 1.6, \mu_{\max} = 10^6\). We compare the proposed method with FKRW[9], PSTNN[10], and IPI by using three infrared image sequences, which come from datasets collected by Hui et al[11]. Each image sequence contains 50 images with the size 256×256. Figure 5 presents the detection results of the four methods. It can be observed in the
result of FKRW that the sparse clutter and the target are enhanced simultaneously. In the result of PSTNN and IPI, the target is completely preserved but sporadic noise still exists. Based on detection results in figure 5, the proposed method detects the target effectively with less clutter and noise residues in different scenes.

To further demonstrate the robustness of the proposed method, we adopt the receiver operating characteristic (ROC). The probability of detection is defined as the ratio of the detected target’s number to the real targets’ number, and the false alarm rate is defined as the ratio of the false alarms’ number to the images’ number. As shown in figure 6, compared with the other three methods, the proposed method achieves the best performance. To further quantitatively verify the performance of the proposed method, the signal-to-clutter ratio gain (SCRg) and the background suppression factor (BSF) are adopted in our experiment, which are defined by:

\[
SCR = \frac{\mu_t - \mu_b}{\sigma_b}, \quad SCR_g = \frac{SCR_{out}}{SCR_{in}}, \quad BSF = \frac{\sigma_{in}}{\sigma_{out}}
\]

(18)

where \(\mu_t\) is the average grey value of the target area, \(\mu_b\) and \(\sigma_b\) denotes the average grey value and the standard deviation of the neighbouring area around the target. \(\sigma_{in}\) and \(\sigma_{out}\) are the standard deviation of the neighbouring area in the original image and the result image. The experimental data for the four methods in figure 5 is given in Table 1. Based on the comparison result in Table 1, the background suppression ability of our method is better than other methods.
Table 1: SCRG and BSF of four methods

| Methods | Metrics | Sequence 1 | Sequence 2 | Sequence 3 |
|---------|---------|------------|------------|------------|
| FKRW    | SCRG    | Inf        | Inf        | 1.89       |
|         | BSF     | Inf        | Inf        | 500.46     |
| IPI     | SCRG    | 2.67       | 7.63       | 4.62       |
|         | BSF     | 6122.94    | 3311.09    | 845.98     |
| PSTNN   | SCRG    | Inf        | Inf        | Inf        |
|         | BSF     | Inf        | Inf        | Inf        |
| DDWIPI  | SCRG    | Inf        | Inf        | Inf        |
|         | BSF     | Inf        | Inf        | Inf        |

Inf means the target neighbourhood is suppressed completely.

5. Conclusion
In this letter, we proposed a DDWIPI model, which combined directional derivative penalty with $l_1$-norm to efficiently suppress background residual and enhance the sparsity in the target image. Experimental results on different infrared sequences illustrated that the proposed method outperforms other baseline methods in background suppression and target detection.

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