A Conjecture about Hadrons

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Abstract

We conjecture that in the chiral limit of QCD the spectrum of hadrons is comprised of decoupled, reducible chiral multiplets. A simple rule is developed which identifies the chiral representations filled out by the ground-state hadrons. Our arguments are based on the algebraic structure of superconvergence relations derived by Weinberg from the high-energy behavior of pion-hadron scattering amplitudes.

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I. INTRODUCTION

Understanding the observed structure and decay patterns of the hadrons continues to be a Holy Grail for nuclear and particle physicists. A vast amount of information about hadrons presently exists, and the ongoing experimental program to measure hadronic properties continues to flourish with a wealth of high precision data streaming in from Jefferson Laboratory, and other facilities. During the past decade or so important progress has been made toward understanding hadrons from a fundamental perspective with tools such as the large-$N_c$ limit of QCD [1–3] and an increasingly powerful effort in lattice QCD. In this work we return to ideas about the relation between the high-energy behavior of scattering amplitudes and the low-energy properties of hadrons developed long ago in Ref. [4–11]. These ideas were cast in a modern framework by Weinberg [10–12], who has long advocated their central role in hadronic physics. By comparing models in this framework with data we are led to conjecture that in the chiral limit, the spectrum of hadrons is comprised of decoupled, reducible chiral multiplets, and that axial matrix elements between the states in each reducible chiral multiplet are determined by the mass spectrum. We develop a simple rule for constructing the ground-state chiral multiplets of the hadrons.

The observations and arguments we use are conceptually straightforward but have deep implications. One considers forward pion-hadron scattering in the chiral limit of QCD, where pions are derivatively coupled to hadrons involved in any given process via the axial-current operator [10]. The general analytic structure of the scattering amplitude enables the amplitude at very low energies, described by an appropriate effective field theory of hadrons, to be related to the behavior of the scattering amplitude at asymptotically high energies. By working in a collinear frame in which the pion-hadron interaction conserves the helicity of the hadron, and noting the absence of isospin-one ($I=1$) Regge-trajectories with $\alpha_1(0) \geq 1$ in the crossed $t$-channel $^1$, one can show that the matrix elements of the axial current between hadron states of helicity $\lambda$, $X^\alpha$, along with the isospin matrices, $T^\alpha$, form an $SU(2)_L \otimes SU(2)_R$ chiral algebra,

$$ [T^\alpha, X^\beta_\lambda] = i\epsilon^{\alpha\beta\gamma} X^\gamma_\lambda, \quad [X^\alpha_\lambda, X^\beta_\lambda] = i\epsilon^{\alpha\beta\gamma} T^\gamma, \quad (1) $$

for two-flavor QCD, and $SU(N_f)_L \otimes SU(N_f)_R$ for QCD with $N_f$ flavors [13]. In arriving at eq. (1) it has been assumed that the low-energy amplitude is saturated by single-particle pole diagrams. The absence of $I=2$ Regge-trajectories with $\alpha_2(0) \geq 0$ in the crossed $t$-channel allows one to derive algebraic relations involving the axial-current matrix elements and the hadronic mass matrix, $M$,

$$ [ X^\alpha_\lambda, [ X^\beta_\lambda, M^2 ] ] \propto \delta^{\alpha\beta}, \quad [ X^\alpha_\lambda, [ X^\beta_\lambda, M J_{x,y} ] ] \propto \delta^{\alpha\beta}, \quad (2) $$

$^1$The forward scattering amplitudes have the following asymptotic behavior with $\omega$:

$$ (M^{(-)}_{\beta h',\alpha h}(\omega))_{I=1} \rightarrow \omega^{\alpha_1(0)-1}, \quad (M^{(+)}_{\beta h',\alpha h}(\omega))_{I=0,2} \rightarrow \omega^{\alpha_0,2(0)}, $$

where $h, h'$ are the isospin indices of the initial and final state hadron while $\alpha, \beta$ are the isospin indices of the initial and final state pion.
where \( J_{x,y} \) are angular-momentum generators in the transverse direction (where we have chosen \( z \) to be the collinear direction). One can show [10] that the first commutator in eq. (2) implies that \( M^2 \) is a sum of two components: one transforms as a chiral singlet, \( M^2_1 \), and one as the isosinglet component of a \((2,2)\) representation, \( M^2_{22} \), of \( SU(2)_L \otimes SU(2)_R \). The second commutator in eq. (2) implies that the same is true for \( M J_{x,y} \). There is a further constraint on the structure of the mass matrix arising from the empirical fact that the cross section for inelastic diffractive scattering is significantly smaller than that for elastic scattering. If the inelastic scattering cross section were to vanish asymptotically then [10]

\[ [M^2_1, M^2_{22}] = 0 \]  

(3)

We will present results both with and without this constraint. From the constraints in eq. (1) and eq. (2) Weinberg [12] showed that

**W1**: any set of hadronic states that furnish a representation of the commutation relations in eq. (1) and eq. (2) in which, for each helicity, any given isospin appears at most once, must be degenerate.

**W2**: any set of *degenerate* hadronic states that furnish a representation of the commutation relations in eq. (1) and eq. (2) also furnish a representation of an \( SU(4) \otimes O(3) \) algebra.

It follows that by assuming only that the \( I = J \) tower of baryon states that naturally arises in the large-\( N_c \) limit of QCD saturates the commutation relations in eq. (1) and eq. (2), the states in the tower must be degenerate and the axial matrix elements must be those of the naive constituent quark model (NCQM) \(^2\). By construction they exhibit a spin-flavor \( SU(4) \otimes O(3) \) symmetry (which becomes a contracted \( SU(4) \otimes O(3) \) symmetry in the large-\( N_c \) limit [2]). The ground-state chiral multiplet has been constructed for the \( I = J \) tower and shown to give the (contracted) spin-flavour \( SU(4) \) results [17].

These are indeed beautiful results and allow one to understand how the NCQM can provide a rigorous mnemonic for describing properties of hadrons in the chiral limit with a single assumption about the hadronic spectrum. However, there are certain aspects of this construction that we wish to investigate further. First, if we want to understand hadrons in QCD with \( N_c = 3 \), we need to consider schemes which saturate the commutation relations in eq. (1) and eq. (2) with more than just the ground-state \( I = J \) tower. Put another way, if we want to understand \( 1/N_c \) corrections to the picture described above then we must consider more complicated saturation schemes that allow mixings between the large-\( N_c \) tower states (which for \( N_c = 3 \) contains only the nucleon and \( \Delta \)) and other states [12,17]. Second, there are non-zero mass splittings between the low-lying baryons that will survive in the chiral limit which must appear in any consistent description. Third, while the ratio of axial matrix elements of the NCQM agree reasonably well with available experimental data, their absolute values are too large by \( \sim 30\% \).

\(^2\)By NCQM we mean the barest form of the constituent quark model (CQM) where there are no spin-dependent interaction between quarks arising from either the phenomenological “gluon exchange” [14] or “pseudo-Goldstone boson exchange” [15,16] as in the chiral quark model, i.e. the \( \Delta \) and nucleon are degenerate.
II. THE LIGHT BARYONS

Consider the lowest-lying baryons with valence structure composed of up and down quarks only, such as the nucleon and the ∆-resonance. The only representations of $SU(2)_L \otimes SU(2)_R$ that contain only $I = \frac{1}{2}$ and $I = \frac{3}{2}$ states are $(1, 2), (2, 1), (1, 4), (4, 1), (2, 3)$ and $(3, 2)$. Given one’s bias from the NCQM, it is natural to consider chiral multiplets that are completely filled out by the nucleon and the ∆-resonance alone. Clearly there is only a small number of chiral-multiplets (for each helicity state) that these hadrons can belong to, without admixtures of other states. For the $\lambda = \frac{1}{2}$ helicity states they are $(1, 2) \oplus (1, 4), (2, 1) \oplus (1, 4), (1, 2) \oplus (4, 1), (2, 1) \oplus (4, 1), (2, 3)$ and $(3, 2)$, while for the $\lambda = \frac{3}{2}$ helicity states they are $(1, 4)$ and $(4, 1)$. Given the results of $W_1$ and $W_2$ it is clear that the only mass spectrum possible for all of these chiral embeddings, either reducible or irreducible, is one in which the nucleon and ∆ are degenerate, and have axial matrix elements that are those of the NCQM. Clearly the degenerate masses and the excessively-large axial-current matrix elements render these multiplet structures insufficient. However, from the NCQM point of view one uses this as a starting point and perturbatively includes contributions that appear somewhat natural, such as spin-dependent quark interactions and the quenching of the constituent quark axial coupling from $g_A^q = 1$ to some lesser value to reproduce the nucleon axial matrix elements. In deriving the algebraic constraints, the analysis has been performed in the chiral limit and single-particle contributions have been taken to saturate the commutators. Given the small value of the pion mass and the extensive studies in chiral perturbation theory in the nucleon-∆ sector [18,19], the quark masses will not bring the NCQM chiral multiplet structure into agreement with data. Furthermore, the saturation of the commutators with single-particle states is precisely the assumption that one makes in constructing the low-energy effective field theory description of the $\pi N \Delta$ system, and this construction is very successful. Thus, one is led to conclude that the NCQM chiral multiplet structure is incomplete and likely not a sensible basis for a perturbative expansion.

To determine the minimal realistic chiral multiplet structure, consider the isospin content of the QCD interpolating fields that have non-zero overlap with the nucleon or ∆:

$$\varepsilon_{abc} q^a q^b q^c \rightarrow \frac{1^+}{2} \oplus \frac{1^+}{2} \oplus \frac{3^+}{2} ,$$

where $a, b, c$ are color indices and all the Dirac and flavour indices have been suppressed. We have used the schema $I^P$ where $P$ is parity. This interpolating field contains an additional $I = \frac{1}{2}$ baryon beyond the nucleon and ∆.

We conjecture that a chiral multiplet which describes the low-lying hadrons is the minimal chiral representation which includes all of the isospin multiplets in the QCD interpolator for that hadron at least once, and for which there is no degeneracy within the multiplet, unless required by an additional symmetry like heavy-quark symmetry or flavor $SU(3)$ symmetry.

Consider the ground-state chiral multiplet for the $\lambda = \frac{1}{2}$ baryons. From eq. (4) we require a representation that contains at least two $I = \frac{1}{2}$ states and one $I = \frac{3}{2}$ state, all of like-parity. In order to avoid degeneracy within the multiplet while allowing pion transitions we require a representation with nonvanishing matrix elements of $M_{22}$. Hence the ground-state chiral multiplet for the $\lambda = \frac{1}{2}$ baryons is uniquely determined by the conjecture to
be \((2, 3) \oplus (1, 2)\) \(^3\). This multiplet was considered by Weinberg [10] (and also by Gilman and Harari [9]). As discussed in detail below, the additional baryon in the chiral multiplet is identified as the Roper resonance, \(N(1440)\) [20]. The actual Dirac and lorentz structure of the interpolating fields is not at all obvious to us. An early discussion of this problem can be found in work by Casher and Susskind [21] and a recent discussion can be found in Ref. [22].

The helicity states of the nucleons, \(N\), and excited nucleons, \(N'\), are in \(I = \frac{1}{2}\) representations of \(SU(2)_I\), described by a tensor with a single fundamental index. Likewise, the helicity states of the \(\Delta\)'s and the excited \(\Delta\)'s are in \(I = \frac{3}{2}\) representations of \(SU(2)_I\), described by a symmetric tensor with three fundamental indices. We will now construct the \((2, 3) \oplus (1, 2)\) representation which contains \(N, N'\) and \(\Delta\). At leading order (LO) in the chiral expansion the axial matrix elements are defined through the currents [18]

\[
J^{α,5}_{\hat{T}, LO} = g_A N_1^α T^α N_1 + g''_A \left( N_1^α T^α N_1^α + \text{h.c.} \right) + g''_A N_1^α T^α N_1^α \nonumber
\]

\[
- C_{\Delta N} \left( \sqrt{\frac{2}{3}} N_1^α T^α \Delta_↑ + \text{h.c.} \right) - C_{\Delta N'} \left( \sqrt{\frac{2}{3}} N_1^α T^α \Delta_↑ + \text{h.c.} \right) \nonumber
\]

\[
- \mathcal{H}_{\Delta N} \frac{1}{\sqrt{3}} T^α \Delta_↑, \nonumber
\]

\[
J^{α,5}_{\hat{T}, LO} = - \mathcal{H}_{\Delta N} \frac{1}{\sqrt{3}} T^α \Delta_↑ \tag{5}
\]

for the \(\lambda = \frac{1}{2}\) helicity states (\(↑\)) and \(\lambda = \frac{3}{2}\) helicity states (\(↑↑\)), respectively. The NCQM places the \(N\) and \(\Delta\) in the 20-dimensional representation of spin-flavor \(SU(4)\), and the \(N'\) and a \(\Delta'\) in the 20' representation. This leads to the familiar NCQM predictions: \(g_A = g''_A = \frac{5}{3}\), \(g''_A = 0\), \(C_{\Delta N} = -2\) and \(\mathcal{H}_{\Delta N} = -3\).

In order to construct the ground-state baryon chiral multiplet consistent with our conjecture, we introduce the fields \(S_a, T_{a,b,c}\) to include the \(\lambda = +\frac{1}{2}\) helicity states and the field \(D_{abc}\) to include the \(\lambda = +\frac{3}{2}\) helicity states. The field \(S_a\) transforms as \((1, 2)\) under \(SU(2)_L \otimes SU(2)_R\); that is, \(S \rightarrow LS\), while the field \(D_{abc}\) transforms as \((1, 4)\) under \(SU(2)_L \otimes SU(2)_R\); that is, \(D \rightarrow LLLD\). It is straightforward to embed an \(I = \frac{1}{2}\) and an \(I = \frac{3}{2}\) state into a single irreducible representation of \(SU(2)_L \otimes SU(2)_R\), the \((2, 3)\). The field \(T_{a,b,c}\) transforms as \(T \rightarrow RLLT\), and in terms of fields transforming as \(I = \frac{1}{2}\), \(S_T\), and \(I = \frac{3}{2}\), \(D_T, T\) can be written as

\[
T_{a,b,c} = \frac{1}{\sqrt{6}} \left( S_{T,b,} \epsilon_{ac} + S_{T,c,} \epsilon_{ab} \right) + D_{T,abc}. \tag{6}
\]

We also introduce a spurion field, \(v^a_6\), which transforms as \(v \rightarrow L v R^\dagger\), such that \(\langle v^a_6 \rangle = M_{22}^2 \delta^a_6\). The free-field dynamics of the helicity states are determined by the two-dimensional effective Lagrange densities constructed from the available tensors,

\[
\mathcal{L}_↑ = \partial_+ T^{a,b,c}_↑ \partial_- T_{a,b,c} + \partial_+ S^{a}_↑ \partial_- S_a - M_{1T}^2 T^{a,b,c}_↑ T_{a,b,c} - M_{1S}^2 S^{a}_↑ S_a
\]

\(^3\)Parity interchanges \(SU(2)_L\) and \(SU(2)_R\) representations. Therefore if we assign the \(\lambda = +\frac{1}{2}\) states to an \((2, 3) \oplus (1, 2)\) representation, parity requires that the \(\lambda = -\frac{1}{2}\) states are in the \((3, 2) \oplus (2, 1)\) representation [10].
\[ -\mathcal{A} \left( T^{a,bc} \lambda^d_{a} S_{b d c} + \text{h.c.} \right), \]
\[ \mathcal{L}_{\hat{\alpha}} = \partial_{+} D^{a,bc}_{\hat{\alpha}} \partial_{-} D_{a,bc} - M^{2}_{T I D} \ D^{a,bc}_{\hat{\alpha}} D_{a,bc}, \]

where \( \mathcal{A} \) is an undetermined parameter and \( x_{\pm} = z \pm t \) with \( z \) the collinear direction. Notice that the helicity components of the baryons act as scalar fields. The current operators that satisfy the constraints imposed by eq. (1) and eq. (2) take the form
\[
\hat{T}^{\alpha}_{\hat{\alpha}} = T^{a,bc}_{\hat{\alpha}} (T^{\alpha})^{d}_{a} T_{d,bc} + 2 T^{a,bc}_{\hat{\alpha}} (T^{\alpha})^{d}_{b} T_{a,dc} + S^{\alpha}_{a} (T^{\alpha})_{a}^{d} S_{d},
\]
\[
\hat{X}^{\alpha}_{\hat{\alpha}} = T^{a,bc}_{\hat{\alpha}} (T^{\alpha})^{d}_{a} T_{d,bc} - 2 T^{a,bc}_{\hat{\alpha}} (T^{\alpha})^{d}_{b} T_{a,dc} - S^{\alpha}_{a} (T^{\alpha})_{a}^{d} S_{d},
\]
\[
\hat{T}^{\alpha}_{\hat{\beta}} = 3 D^{abc}_{\hat{\alpha}} (T^{\alpha})^{d}_{a} D_{d,bc},
\]
\[
\hat{X}^{\alpha}_{\hat{\beta}} = -3 D^{abc}_{\hat{\alpha}} (T^{\alpha})^{d}_{a} D_{d,bc}. \]

The mass eigenstates are linear combinations of the chiral eigenstates with a mixing angle \( \psi \). Setting \( M^{2}_{IT} = M^{2}_{IT} \) one can easily check that the commutators of eq. (1) and eq. (2) are satisfied. Diagonalizing the mass matrix and matching to the chiral perturbation theory current in eq. (5) leads to
\[
g_{A} = 1 + \frac{2}{3} \cos^{2} \psi, \quad g'_{A} = \frac{2}{3} \sin \psi \cos \psi, \quad g''_{A} = 1 + \frac{2}{3} \sin^{2} \psi, \quad c_{\Delta N} = -2 \cos \psi, \quad c_{\Delta N'} = -2 \sin \psi, \quad \mathcal{H}_{\Delta \Delta} = -3, \quad M^{2}_{N} \cos^{2} \psi + M^{2}_{N} \sin^{2} \psi = M^{2}_{\Delta}, \]

where \( \psi \) is the mixing angle between the two \( I = \frac{1}{2} \) multiplets. \(^4\) If we further impose the inelastic diffraction constraint, we find that \( M^{2}_{IT} = M^{2}_{IS} \) and consequently \( \psi = \frac{\pi}{4} \), which corresponds to maximal mixing. \(^5\) This then gives
\[
g_{A} = \frac{4}{3}, \quad g'_{A} = \frac{1}{3}, \quad g''_{A} = \frac{4}{3}, \quad c_{\Delta N} = -\sqrt{2}, \quad c_{\Delta N'} = -\sqrt{2}, \quad \mathcal{H}_{\Delta \Delta} = -3, \quad M^{2}_{\Delta} - M^{2}_{N} = M^{2}_{N'} - M^{2}_{\Delta}. \]

These values are impressively close to those in nature and it is conceivable that the agreement may improve as the physical values are extrapolated to the chiral limit. Using the nucleon and \( \Delta \) masses as input one finds \( M^{2}_{N'} = 1467 \text{ MeV} \), consistent with the Roper resonance. Notice that both \( g_{A} \) and \( c_{\Delta N} \) are decreased from their NCQM in the direction of experiment (see Table I). The phenomenology of the axial couplings in this scenario is discussed in detail in Ref. [20]. Recent work on \( \pi N \rightarrow \pi \pi N \) scattering by Fettes [23] has determined a range for the higher-order contributions to the axial current in chiral perturbation theory, described by the constant \( \tilde{d}_{16} \). The range of values for \( \tilde{d}_{16} \) is consistent with a value of \( g_{A} = \frac{4}{3} \) in the chiral limit. Finally, we point out that it is likely that since the Roper-nucleon mass splitting is less than the chiral symmetry breaking scale, the non-vanishing quark-mass corrections to this chiral multiplet can be computed using chiral perturbation theory [24].

\(^4\)We believe the mass relation in Ref. [10] to be incorrect.

\(^5\)This choice of the mixing angle corresponds to a discrete symmetry of the free lagrange density which interchanges \( S \) and \( S_{T} \) [22]. In all cases we study in this paper, the constraint of no inelastic diffraction corresponds to a discrete symmetry of the collinear field theory.
TABLE I. Axial couplings for the light baryons, heavy baryons and heavy mesons. The third and second columns give the predictions of the chiral conjecture both with (*) and without the inelastic diffraction constraint of eq. (3). The fourth column gives the axial couplings of the NCQM. The experimental values have been determined via branching fractions that appear in the particle data group [25]. The extractions of $C_{\Delta N}$ and $H_{\Delta \Delta}$ from data were made in $SU(3)$ chiral perturbation theory [26]. For a discussion of the experimental value of $g$ and other allowed values, see the text.

### III. THE HEAVY BARYONS

For baryons containing one heavy quark one naturally considers the NCQM states as being partners, such as the $\Lambda_c^+$, the $\Sigma_c^{++},+0$ and the $\Sigma_c^{*++},+0$ in the charmed sector. The minimal chiral multiplet structure for these states gives a continuous spectrum of axial-current matrix elements when the algebraic constraints of eq. (1) and eq. (2) are imposed. However, also requiring heavy-quark symmetry (HQS) yields the axial coupling constants of the NCQM. An interpolating field for the heavy baryons takes the form

$$
\varepsilon_{abc} Q^a q^b q^c \rightarrow 0^+ \oplus 1^+ ,
$$

where $Q$ denotes a heavy quark and again we have used the schema $I^P$ where $P$ is parity. In order to describe the $\Lambda_Q$, the $\Sigma_Q$ and the $\Sigma_Q^*$ two copies of the heavy-baryon interpolator must be present, and thus a $\Lambda_Q'$ will be present in the ground-state chiral multiplet. The unique chiral multiplet consistent with our conjecture is $(2,2) \oplus (1,3) \oplus (1,1)$. One additional ingredient in the heavy-quark sector that is absent in the light-quark sector is heavy quark spin-symmetry (HQSS). In the experimentally-determined heavy-baryon spectrum, the $\Sigma_Q$ and $\Sigma_Q^*$ are identified as members of an irreducible representation of HQSS, and become degenerate in the heavy quark limit. HQSS greatly simplifies the form of the axial-current matrix elements and one finds [27]

$$
J_{Q^*;LO}^{\alpha,5} = g_2 \left( \frac{2}{3} \text{Tr} \left[ \Sigma_{Q^*} T^\alpha \Sigma_{Q^*} \right] + \frac{1}{3} \text{Tr} \left[ \Sigma_{Q} T^\alpha \Sigma_{Q} \right] + \frac{\sqrt{2}}{3} \left( \text{Tr} \left[ \Sigma_{Q}^* T^\alpha \Sigma_{Q} \right] + \text{h.c.} \right) \right) + g_3 \left( \frac{1}{\sqrt{3}} \Lambda Q^* \text{Tr} \left[ T^\alpha Q_{Q} \right] - \sqrt{2} \frac{\Lambda Q^* \text{Tr} \left[ T^\alpha Q_{Q}^* \right]}{3} + \text{h.c.} \right)
$$

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6The discussion also holds for strange baryons such as the $\Lambda$ or $\Sigma$. 

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The NCQM values for the axial couplings are \( g_2 = 2 \) and \( g_3 = -\sqrt{2} \) [27,28].

We will now construct the \((2, 2) \oplus (1, 3) \oplus (1, 1)\) representation that contains \( \Lambda_Q\), \( \Lambda_Q'\), \( \Sigma_Q\) and the \( \Sigma_Q^*\). We introduce the fields \( \lambda_1\), \( Z\) and \( Y\) to describe the \( \lambda = \frac{1}{2}\) helicity states and the field \( Q\) to describe the \( \lambda = \frac{3}{2}\) helicity state. \( \lambda_1\) transforms as \((1, 1)\) under the chiral group, \( Z_0^a\) transforms as \((1, 3)\) under the chiral group, \( Z \rightarrow L Z L^\dagger\), and \( Y_0^a\) transforms as \((2, 2)\) under the chiral group, \( Y \rightarrow L Y R^\dagger\), with \( Y_0\) the \( I = 0\) component, and \( Y_1\) the \( I = 1\) component. The field \( Q_0^a\) transforms as \((1, 3)\) under the chiral group, \( Q \rightarrow L Q L^\dagger\). The free-field dynamics of the helicity states are determined by the two-dimensional effective Lagrange densities constructed from \( \lambda_1\), \( Z\) and \( Y\),

\[
\mathcal{L}_+ = \partial_+ \lambda_1 \partial_- \lambda_1 + \text{Tr} \left[ \partial_+ Y^\dagger \partial_- Y \right] + \text{Tr} \left[ \partial_+ Z^\dagger \partial_- Z \right] - M_1^2 \text{Tr} \left[ Y^\dagger Y \right] - M_2^2 \text{Tr} \left[ Z^\dagger Z \right] - M_3^2 \lambda_1 \left( A_1 \text{Tr} \left[ Y^\dagger Z v \right] + A_2 \text{Tr} \left[ Y^\dagger v \right] \lambda_1 + \text{h.c.} \right),
\]

\[
\mathcal{L}_L = \partial_+ Q_\pm \partial_- Q_\mp - M_4^2 \text{Tr} \left[ Q^\dagger Q \right],
\]

where \( A_{1,2}\) are unknown constants. The \( \Lambda_Q^{(\lambda)}\) mass eigenstates are linear combinations of the \( \lambda_1\) and \( Y_0\) fields with a mixing angle \( \psi\), while the \( \Sigma_Q^{(\lambda)}\) mass eigenstates are linear combinations of the \( Z\) and \( Y_1\) fields, with mixing angle \( \phi\). The constraints imposed by eq. (1) and eq. (2) along with HQSS have a non-trivial solution for the axial coupling constants and masses

\[
g_2 = 2 , \quad g_3 = -\sqrt{2} \cos \phi , \quad g_3' = -\sqrt{2} \sin \phi , \quad M_1^2 \cos^2 \phi + M_2^2 \sin^2 \phi = M_4^2 \Sigma_Q^{(\lambda)}.
\]

Imposing the constraint in eq. (3) arising from the absence of inelastic diffractive scattering requires \( \psi = \frac{\pi}{4} \) which gives

\[
g_2 = 2 , \quad g_3 = g_3' = -1 , \quad M_4^2 = M_2^2 - M_2^2 = M_2^2 \Sigma_Q^{(\lambda)} ,
\]

or \( \psi = 0\), which decouples the \( \Lambda_Q'\) as a stand-alone \((1, 1)\) representation and recovers the NCQM values for the heavy-baryon multiplet axial transitions \((g_2 = 2, g_3 = -\sqrt{2}\) and \(g_3' = 0\)).

Information continues to be accumulated about the properties and decays of baryons containing heavy quarks (for a recent review of experimental data see Ref. [29,30], and references therein and also the particle data group [25]). Ideally, one would like to have detailed information on b-baryons so as to be as close to the HQ limit as nature will allow. However, there is much more information on charmed baryons, simply due to the number that have been produced in the laboratory. Experimentally, the measured width of the \( \Sigma_Q^{++}\) is \( \Gamma(\Sigma_Q^{++}) = 17.9^{+3.4}_{-3.3} \pm 4.0\) MeV, which fixes the axial coupling \( g_3\) to be \(|g_3| = 0.95 \pm 0.08 \pm 0.08\). This value is significantly smaller than the NCQM value of \(|g_3| = \sqrt{2}\), but consistent with \(|g_3| = 1\) that one obtains with \(\psi = \frac{\pi}{4}\). Thus our conjecture predicts
there to be a chiral partner to the $\Lambda_c$, $\Sigma_c$ and the $\Sigma_c^*$, the positive-parity $\Lambda_c'$ with a mass of $M_{\Lambda_c'} \sim 2688$ MeV that is the analogue of the Roper in the light baryon sector and is not yet observed. The axial coupling between the $\Lambda_c'$ and the $\Sigma_c$ and the $\Sigma_c^*$ is predicted to be $|g_3'| = 1$. The fact that the charmed spectrum is not as close to the heavy-quark limit as one would like means that the $\Lambda_c'$ mass may be somewhat heavier than we have estimated here.

IV. THE LIGHT MESONS

In the case of the light mesons, our conjecture has been proven to be true in the large-$N_c$ limit of QCD [13] using eq. (1), eq. (2) and eq. (3). The $\lambda = 0$ helicity states of the $\pi$, $\rho$, $f_0$ and $a_1$ form a chiral quartet that is decoupled from the other mesons. This provides a QCD-based interpretation of the work by Gilman and Harari [9] in which it was shown that the complete set of Adler-Weisberger sum-rules in the pion sector could be satisfied with just these four meson states. The one free mixing angle introduced by Gilman and Harari is predicted to be $\psi = \frac{\pi}{4}$ and gives a $\rho$ width that is consistent with experiment. We now demonstrate how these results are implied by our conjecture. In effect, we will see that our conjecture recovers precisely the full saturation scheme of Gilman and Harari [9]. The interpolating fields for the mesons are of the form $\bar{q}^a q_a$ and contain isospin representations $I = 0 \oplus 1$ only. The chiral multiplets which can give rise to these isospins and no others are $(1, 1)$, $(1, 3)$, $(3, 1)$, and $(2, 2)$. As the mesons have $\lambda = 0$ helicity states normality, $\eta \equiv P(-1)^I$, constrains the structure of the mass matrix and axial currents. Moreover, $G$-parity adds a new complication as there is an additional symmetry, $G\eta$, which commutes with the axial operator, $\hat{X}_a$. Hence one should consider sectors with $G\eta = +1$ and $G\eta = -1$ separately. Meson interpolators with space-time quantum numbers, $J^{PC}$, of pseudoscalar, vector, axialvector and scalar character decompose to

\[
(\bar{q}^a q_a)_P \to 0^- \oplus 1^- , \quad (\bar{q}^a q_a)_V \to 0^+ \oplus 1^+ ,
\]

\[
(\bar{q}^a q_a)_A \to 0^- \oplus 1^- , \quad (\bar{q}^a q_a)_S \to 0^+ \oplus 1^+ ,
\]

(16)

where we use the schema $I^n_{G\eta}$. Consider first the sector with $G\eta = +1$ which includes the pion. Since normality interchanges $(1, 3)$ and $(3, 1)$, the minimal reducible representation that allows for non-zero mass splittings is $(2, 2) \oplus (1, 3) \oplus (3, 1)$ which corresponds to $1^_, 1^+_1$ and $0^+_1$ from eq. (16). (This representation is denoted by $v \oplus t$ in Ref. [10].) Hence the pion belongs to a chiral representation composed of the $\lambda = 0$ helicity states of $\pi$, $f_0(600)$, $\rho(770)$ and $a_1(1260)$. We introduce the fields $h$, $k$ and $t$ which transform as $h \to LhL^\dagger$, $k \to RkR^\dagger$ and $t \to LtR^\dagger$ under chiral transformations, and whose free-field dynamics are described by the Lagrange density

\[
\mathcal{L}_0^+ = \text{Tr} \left[ \partial_+ t^\dagger \partial_+ t \right] + \text{Tr} \left[ \partial_+ h^\dagger \partial_+ h \right] + \text{Tr} \left[ \partial_+ k^\dagger \partial_+ k \right] - M_{II}^2 \text{Tr} \left[ t^\dagger t \right] - M_{II}^1 \text{Tr} \left[ (h + k)^\dagger (h + k) \right] - M_{I_1}^2 \text{Tr} \left[ (h - k)^\dagger (h - k) \right]
\]

\[
- \left( \mathcal{A} \text{ Tr} \left[ t^\dagger (hv + v^\dagger k) \right] + \text{h.c.} \right),
\]

(17)

where $\mathcal{A}$ is an unknown constant and the current operators are

\[
\hat{T}_0^a = \text{Tr} \left[ t^\dagger T^a t - t^\dagger t T^a \right] + 2 \text{Tr} \left[ h^\dagger T^a h + k^\dagger T^a k \right],
\]

\[
\hat{X}_0^a = \text{Tr} \left[ t^\dagger T^a t + t^\dagger t T^a \right] + 2 \text{Tr} \left[ h^\dagger T^a h - k^\dagger T^a k \right].
\]

(18)
One makes the following particle identifications

\[
\rho = \frac{1}{\sqrt{2}} \left[ h - k \right], \quad a_1 = \cos \psi t_1 + \sin \psi \frac{1}{\sqrt{2}} \left[ h + k \right], \\
\pi = -\sin \psi t_1 + \cos \psi \frac{1}{\sqrt{2}} \left[ h + k \right], \quad f_0 = t_0,
\]

which lead to

\[
\begin{align*}
\langle \rho^0 | \hat{X}_0^1 - i \hat{X}_0^0 | \pi^+ \rangle &= -\sqrt{2} \cos \psi, \\
\langle f_0 | \hat{X}_0^1 - i \hat{X}_0^0 | \pi^+ \rangle &= -\sqrt{2} \sin \psi, \\
\langle \rho^0 | \hat{X}_0^1 - i \hat{X}_0^2 | a_1^+ \rangle &= -\sqrt{2} \sin \psi, \\
\langle f_0 | \hat{X}_0^1 - i \hat{X}_0^2 | a_1^+ \rangle &= \sqrt{2} \cos \psi, \\
M_{a_1}^2 &= M_{\rho}^2 + M_{f_0}^2, \quad M_{\rho}^2 = \sin^2 \psi M_{a_1}^2.
\end{align*}
\]

The absence of inelastic diffractive scattering, as encapsulated in eq. (3), requires that \( \psi = \frac{\pi}{4} \) which, through the decay rate \( \Gamma(\rho \to \pi \pi) \sim 283 \cos^2 \psi \text{MeV} \), is in excellent agreement with experiment. This value of the mixing angle is consistent with the large and uncertain widths of \( f_0 \) and \( a_1 \). (It is interesting that the mixing angle takes a value halfway between the sigma model scenario \( \psi = \frac{\pi}{2} \) where \( \pi \) is paired with the scalar \( f_0 \) in a degenerate multiplet, and the vector-limit scenario \( \psi = 0 \) where \( \pi \) is paired with the vector \( \rho \) in a degenerate multiplet.) One also obtains the well-known mass relation \( \psi \)

\[
M_{\rho}^2 = M_{\sigma}^2 = M_{a_1}^2 - M_{\rho}^2.
\]

The phenomenology of this embedding is discussed in detail in Refs. [9,10,13,33,34], and expressed in terms of chiral perturbation theory parameters in Ref. [35].

Next we consider the sector with \( G_{\eta} = -1 \). The minimal representation with mass splittings is \( (2, 2) \oplus (1, 1) \oplus (1, 1) \) which corresponds to \( 0_-, 0_-, 0_+ \) and \( 1_+ \) from eq. (16). This chiral representation is composed of the \( \lambda = 0 \) helicity states of \( \eta, \omega(782), a_0(980) \) and \( f_1(1285) \). In order to construct this chiral multiplet, we introduce the fields \( \lambda_1, \lambda_2 \) which transform as \( (1, 1) \) and \( y_0^\alpha \) which transforms as a \( (2, 2) \) under the chiral group, \( y \to L y R^\dagger \) with \( I = 0 \) component \( y_0 \), and \( I = 1 \) component, \( y_1 \). The free-field dynamics of the helicity-zero states are determined by the two-dimensional effective Lagrange density constructed from \( \lambda_1, \lambda_2 \) and \( y \),

\[
L_0 = \partial_+ \lambda_1^\dagger \partial_- \lambda_1 + \partial_+ \lambda_2^\dagger \partial_- \lambda_2 + \text{Tr} \left[ \partial_+ y^\dagger \partial_- y \right] - M_{1y}^2 \text{Tr} \left[ y^\dagger y \right] - M_{1\lambda_1}^2 \lambda_1^\dagger \lambda_1 - M_{1\lambda_2}^2 \lambda_2^\dagger \lambda_2 - \left( A_1 \text{Tr} \left[ y^\dagger v \right] \lambda_1 + A_2 \text{Tr} \left[ y^\dagger v \right] \lambda_2 + \text{h.c.} \right),
\]

where \( A_{1,2} \) are unknown constants, and the axial-current operator is

\[
\hat{X}_0^\alpha = \text{Tr} \left[ y^\dagger T^\alpha y + y^\dagger y T^\alpha \right].
\]

Since \( \omega \) carries opposite normality to \( \eta \) and \( f_1 \), it is necessarily unmixed \( (A_1 = 0) \) and transforms as a chiral singlet. Hence we have the particle identifications

\[
\begin{align*}
\eta &= \cos \psi \lambda_2 + \sin \psi y_0, \\
f_1 &= -\sin \psi \lambda_2 + \cos \psi y_0, \\
a_0 &= y_1, \quad \omega = \lambda_1,
\end{align*}
\]

(24)
and we find the following non-trivial solution for the axial coupling constants and masses,

\[
\langle a_0^+ | \hat{X}_0^1 + i \hat{X}_0^2 | f_1 \rangle = \sqrt{2} \cos \psi, \quad \langle a_0^+ | \hat{X}_0^1 + i \hat{X}_0^2 | \eta \rangle = \sqrt{2} \sin \psi,
\]

\[
M_{f_1}^2 \cos^2 \psi + M_{\eta}^2 \sin^2 \psi = M_{a_0}^2.
\]

The masses in this multiplet are well-known experimentally and using the mass relation to fix the mixing angle gives \( \psi = 44.5^\circ \), in excellent agreement with the inelastic diffraction constraint imposed by eq. (3). The predicted value for the \( f_1 \) width, \( \Gamma(f_1 \to a_0 \pi) \approx 76 \cos^2 \psi \) MeV, is then quite respectable. The value for the \( a_0 \) width, \( \Gamma(a_0 \to \eta \pi) \approx 647 \sin^2 \psi \) MeV, would appear to over predict the (rather uncertain) experimental width. However, as in the case of the \( a_1 \) and \( f_0 \), one should not expect to do better in the scalar sector given the assumptions that have been made.

Finally, we consider the remaining degrees of freedom necessary to fill out the interpolators of eq. (16): the \( \lambda = 1 \) \((\uparrow)\) components of \( \rho, a_1, \omega \) and \( f_1 \). Evidently, these states are in two decoupled \((2, 2)\) representations. One immediately finds

\[
\langle \rho^+ | \hat{X}_1^1 + i \hat{X}_1^2 | \omega \rangle = \sqrt{2}, \quad \langle a_1^+ | \hat{X}_1^1 + i \hat{X}_1^2 | f_1 \rangle = \sqrt{2},
\]

\[
M_{\rho}^2 = M_{\omega}^2, \quad M_{a_1}^2 = M_{f_1}^2,
\]

and hence the masses of the \( G\eta = \pm 1 \) sectors are related by eq. (2). While the mass relations are in excellent agreement with data, the axial couplings cannot be confronted with data at present.

**V. THE HEAVY MESONS**

Heavy mesons fall into degenerate doublets labeled by \( j^P_\pm = (j_\ell \pm 1/2)^P \), where \( P \) \((P_\ell)\) is the parity of the heavy meson (light degrees of freedom) and \( j \) \((j_\ell)\) is the angular momentum of the heavy meson (light degrees of freedom). The ground-state mesons have \( j_\ell = 1/2 \) and \( P_\ell = (-) \) and are denoted \( P \) \((0^-)\) and \( P^* \) \((1^-)\), while the first excited heavy-meson doublet also have \( j_\ell = 1/2 \) and \( P_\ell = (+) \), and are denoted \( P^*_0 \) \((0^+)\) and \( P^*_1 \) \((1^+)\). Consider first the \( \lambda = 0 \) heavy mesons. The axial couplings between the \( \lambda = 0 \) states are defined as [36]

\[
\langle P | \hat{X}_0^a | P^* \rangle = g \, T^a, \quad \langle P | \hat{X}_0^a | P^* \rangle = \langle P_1^* | \hat{X}_0^a | P^* \rangle = h \, T^a, \quad \langle P_1^* | \hat{X}_0^a | P^* \rangle = g' \, T^a.
\]

Generally the heavy mesons are sums of any number of \((2, 1)\) and \((1, 2)\) representations. This case is particularly easy to analyze. Weinberg has shown that \(|g| \leq 1 \) [37], a bound which holds for all axial transitions among all heavy mesons [38]. If one imposes the constraint of no inelastic diffraction, then Weinberg has further shown that \(|g| = 0, 1 \) are the only possibilities. Heavy meson interpolators with space-time quantum numbers, \( J^P \), of pseudoscalar, vector, axialvector and scalar character decompose to

\[
(\overline{Q}^a q_a)_P \to \frac{1}{2}^-, \quad (\overline{Q}^a q_a)_V \to \frac{1}{2}^+, \quad (\overline{Q}^a q_a)_S \to \frac{1}{2}^+, \quad (\overline{Q}^a q_a)_A \to \frac{1}{2}^-.
\]

(28)
where we have used the schema $I^\alpha$. These quantum numbers correspond to $P$, $P^*$, $P_0^*$ and $P_1^*$, respectively. Now in order to construct states of definite normality we require at least the representation $(2, 1) \oplus (1, 2)$, and we might naively place the ground state heavy mesons in this representation. HQSS then requires that $M_{22}^2$ vanish between these states and the absence of the $P \rightarrow P$ (required by normality) matrix element yields $|g| = 1$, which is the NCQM value. Placing the excited doublet in an analogous multiplet then gives $|g'| = 1$ and $|h| = 0$ as well. However, there is a subtlety with these embeddings. As one moves away from the heavy-quark limit and turns on $M_{22}^2$, one finds that the $P$ and $P^*$ masses are of the form $M_1^2 \pm M_{22}^2$. However, the form of the leading spin-symmetry violating operator, $\sigma \cdot G$ with matrix elements parameterized by $\lambda$, requires that the spin-averaged combination $3 M_{2*}^2 + M_{3*}^2$ be independent of $M_{2*}^2 - M_{3*}^2$, and there is an analogous constraint for the excited doublet. Hence one expects that the ground- and first excited-state heavy mesons fill out a reducible $(2, 1) \oplus (1, 2) \oplus (2, 1) \oplus (1, 2)$ representation. It is straightforward to show that

$$g = \cos(\theta + \phi) = -g', \quad h = \sin(\theta + \phi),$$  

(29)

where $\theta$ ($\phi$) is the mixing angle between the states with $\eta = -1$ ($\eta = +1$). The constraint of no inelastic diffraction implies $\theta = \phi = \frac{\pi}{4}$ and one finds $|g| = 0$, $|h| = 1$, consistent with the theorem of Ref. [37].

Finally, we have the remaining degrees of freedom necessary to fill out the interpolators of eq. (16): the $\lambda = 1$ ($\dagger$) components of $P^*$ and $P_1'$. The axial matrix elements between these states are

$$\langle P^* | \hat{X}^\alpha_\dagger | P^* \rangle = g T^\alpha, \quad \langle P_1' | \hat{X}^\alpha_\dagger | P_1' \rangle = g' T^\alpha, \quad \langle P_1' | \hat{X}^\alpha_\dagger | P^* \rangle = h T^\alpha.$$

(30)

Consistency with eq.(2) and eq. (29) requires that the $\lambda = 1$ components of $P^*$ and $P_1'$ be in a reducible $(2, 1) \oplus (1, 2)$ with a mixing angle given by $(\theta + \phi)/2$ which takes the value $\frac{\pi}{4}$ when the inelastic diffraction constraint of eq. (3) is imposed.

Experimentally, the coupling $g = g_\pi$ is known to be smaller than unity through different determinations. The width of the $D^*$, combined with the branching fraction for $D^* \rightarrow D\pi$, gives $g = 0.59 \pm 0.01 \pm 0.07$ [39] at tree-level in the chiral expansion, which is consistent with recent lattice QCD calculations [40], and with a determination from $D^* \rightarrow D\pi$ and $D^* \rightarrow D\gamma$ at order $\sqrt{m_\pi}$ in $SU(3)$ chiral perturbation theory [41]. At order $m_\pi$ and $1/m_c$ in chiral perturbation theory [42] there are two solutions for $g$ derived from $D^* \rightarrow D\pi$ and $D^* \rightarrow D\gamma$. One solution, $g = 0.76 \pm 0.03 \pm 0.15$ [42], is significantly larger than our prediction and it is unlikely that the discrepancy can be accounted for through chiral corrections. However, the second solution, $g = 0.27 \pm 0.03 \pm 0.04$, is consistent with our conjecture and is conceivably non-zero because of quark mass effects alone. A small value of $g$ has also been found by Bardeen and Hill who developed a dynamical model that incorporates a chiral multiplet structure very similar to that found here [43].

VI. CONCLUSIONS

The observed high-energy behavior of pion-hadron scattering amplitudes, together with the assumption that pole graphs dominate the amplitude at low energies, is sufficient
provide algebraic constraints on the structure of hadronic axial-current matrix elements and mass matrices [9–13]. These constraints allow one to prove that the $\pi$, $\rho$, $f_0$ and $a_1$ form a decoupled chiral multiplet in the large-$N_c$ limit of QCD [13]. We have conjectured that the entire hadron spectrum is composed of decoupled reducible chiral representations and we have given a prescription for finding these multiplets. Our conjecture provides a map of the chiral structure of the QCD ground state. In summary, we find:

$$
qqq:\begin{cases}
\lambda = \frac{1}{2} & : (2, 3) \oplus (1, 2) \\
\lambda = \frac{3}{2} & : (1, 4)
\end{cases}
$$

$$
Qqq:\begin{cases}
\lambda = \frac{1}{2} & : (2, 2) \oplus (1, 3) \oplus (1, 1) \\
\lambda = \frac{3}{2} & : (1, 3)
\end{cases}
$$

$$
\overline{q}q:\begin{cases}
\lambda = 0 & : (2, 2) \oplus (1, 3) \oplus (3, 1) \ (G\eta = +1) \\
\lambda = 1 & : (2, 2) \oplus (2, 2)
\end{cases}
$$

$$
\overline{Q}q:\begin{cases}
\lambda = 0 & : (1, 2) \oplus (2, 1) \oplus (1, 2) \oplus (2, 1) \\
\lambda = 1 & : (1, 2) \oplus (2, 1)
\end{cases}
$$

for ground-state light baryons, heavy baryons, light mesons and heavy mesons, respectively. Evidently, the chiral multiplet structure that nature has chosen contains the minimal particle content necessary to saturate the interpolating fields for the hadrons, and to allow for non-zero mass splittings between members of the multiplet. The claim that chiral multiplets are small and decoupled might appear odd given that there are observed axial transitions — say in the light-baryon sector — from excited states to the ground-state multiplet. However, the decoupled chiral multiplets mix when the quark masses are turned on. Therefore, the smallness of the axial transitions from excited multiplets to the ground-state multiplet as compared to those within the ground-state multiplet is due to the smallness of the quark masses.

Our conjecture agrees with all experimental data available for the low-lying hadrons and leads to new and exciting predictions. We look forward to exploration of this conjecture both theoretically and experimentally.

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