Dark Matter Accumulation near the Earth for the Long Range Forces case

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Abstract
The accumulation of dark matter near the Earth is considered. We analyze the case of long range interaction forces. Additional density of the dark matter at the Earth’s surface is calculated. We show that this density exceeds the mean density of the dark matter in our galaxy by more then $10^5$ times for some values of dark matter particle mass. Accumulation of WIMP’s near the Earth by the same mechanism is also analyzed.

1 Introduction
The dark matter (DM) in our Galaxy interacts with the Earth. The mean velocity of DM particles in the Galaxy is about 220 km/s. When such a particle is elastically scattered on the nuclei of the Earth it loses its energy and could be gravitationally absorbed by the Earth. As the escape velocity of the earth is equal to 11.2 km/s the DM particle should be slowed down to lower velocity to be captured. We are considering the following form of the potential of DM and ordinary particles

$$V(r) = G \frac{m_x m}{r} (1 + \alpha \frac{e^{-\lambda r}}{r})$$

Here $m_x$ - is the mass of the DM particle, $m$ - mass of the ordinary particle or body, $\alpha$ - parameter which is characterizing the relative strength of the gravitational force and the long range forth (LRF), $\lambda$ - is the range of the LRF interaction. In the previous parer [1] the possibility of accumulation of DM with LRF in the Sun system was assumed and in this paper we also fix the parameter $\alpha$ to be equal to $10^{26}$. The DM particle mass $m_x$ and parameter $\lambda$ will vary.

After the DM particle was gravitationally absorbed it can stay at the stationary orbit around the Earth until it is captured by the Earth. Staying inside the Earth the DM particle will be thermalized and get velocity

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distribution determined by the temperature of the Earth in the core. This temperature could be as high as 6000 K and the DM particle has noticeable probability to fly out of the Earth and in some cases even to escape the Earth if its velocity is higher than 11.2 km/s.

In this paper we will calculate capture rate of the DM particles by the Earth, and also the time life of them in the Earth $t_{Earth}$ and inside the Earth gravitation $t_{life}$. Using these quantities we can calculate the additional density of DM particles at the Earth surface. As it follows in some cases this additional density will considerably exceed the DM mean density in our Galaxy.

2 Earth model

The main properties of the Earth needed to describe the propagation of the DM in it are radial dependencies of the Earth density and the temperature, as well as the Earth elemental composition. Radial dependence on the Earth density and temperature we used are presented in Figure 1. We considered the Earth composition in the mantle and in the core to be different. Elemental composition of the Earth’s mantle and core is presented in Table 1 [2]. The depth of the mantle is equal to 2900 km. In our calculation we assume that the mantle consists of the single element with average atomic mass equal to 23.5 and the core consists of iron only. We also divide the Earth volume into concentric layers with constant density and temperature obtained by averaging the real density and temperature distribution over the layer volume. The number of layers we considered was equal to 20.

![Figure 1: Radial distribution of the Earth density and temperature](image-url)
3 DM propagation in the Earth

We consider separately the influence of the gravitation and LRF on DM particle propagation in the Earth. To simplify simulation of DM propagation in the gravitational potential of the Earth we approximate potential to be constant in the layers into which we divide the Earth. As in region with constant potential the gravitational force is zero, the DM particle propagates freely inside the layers. The trajectory of particle breaks only at the border of layers. The particle can be refracted or reflected from the border surface. The law of refraction has the following form

\[
\begin{align*}
\frac{v_1^2}{2} + \phi_1 &= \frac{v_2^2}{2} + \phi_2 \\
v_1 \sin \theta_1 &= v_2 \sin \theta_2
\end{align*}
\] (2)

where \(\phi_{1,2}\) are the gravitational potentials in regions 1 and 2, \(v_{1,2}\) - velocity of particle in these regions, \(\theta_{1,2}\) are the angles between the particle trajectory and the normal to the surface at the point where trajectory intersects the surface. In the case when equation system (2) has no solution the particle reflects mirrorly from the surface.

Scattering on Yukawa potential of (1) can be treated in Born approximation for interaction length \(\lambda < 3 \times 10^{-2} \text{ cm}\). Scattering amplitude in the c.m. system has the following form

\[
f(q) = \frac{2\mu}{\hbar^2} a \frac{\lambda^2}{(\lambda q)^2 + 1},
\] (3)

where \(\mu = \frac{m_x m}{m_x + m}\) is a reduced mass of scattering particles, \(q\) is wave vector transfer, and parameter \(a = -\alpha G m_x m\). Total cross section of scattering for different \(\lambda\) and \(m_x\) for \(m = 1 \text{ GeV}\) is presented in Figure 2.

As is seen from the figure total cross section strongly depends on the interaction length and DM mass. For

Table 1: The composition of the Earth’s core and mantle

| Element          | Atomic number | Mass fraction |        |        |
|------------------|---------------|---------------|--------|--------|
|                  |               | Core          | Mantle |
| Oxygen, O        | 16            | 0.0           | 0.440  |
| Silicon, Si      | 28            | 0.06          | 0.210  |
| Magnesium, Mg    | 24            | 0.0           | 0.228  |
| Iron, Fe         | 56            | 0.855         | 0.0626 |
| Calcium, Ca      | 40            | 0.0           | 0.0253 |
| Phosphor, P      | 30            | 0.002         | 0.0009 |
| Sodium, Na       | 23            | 0.0           | 0.0027 |
| Sulfur, S        | 32            | 0.019         | 0.00025|
| Nickel, Ni       | 59            | 0.052         | 0.00196|
| Aluminum, Al     | 27            | 0.0           | 0.0235 |
| Chromium, Cr     | 52            | 0.009         | 0.0026 |
\( \lambda > 10^{-6} \text{ cm} \) interaction length becomes rather short and the simulation of DM propagation becomes very time consuming. Scattering angle in the c.m. system can be simulated according to the following formula

\[
\cos \theta = \frac{1}{b} \left[ b + 1 - \frac{1 + 2b}{1 + 2b\xi} \right],
\]

where \( \xi \) is a uniform distributed in the interval \([0,1]\) random number. Parameter \( b = 2\mu^2 v^2 \lambda^2 / \hbar^2 \) determines the asymmetry of scattering. The velocity distribution of Earth particles is taken to be Maxwellian with the temperature of the layer where scattering occurs.

### 4 Capture rate of DM particles with the Earth

We take the velocity distribution of the Galaxy DM particles to be Maxwellian \([4]\),

\[
f_0(v)dv = n_x \frac{4}{\pi^{1/2} v_0^3} e^{-v^2/v_0^2} dv
\]

where the parameter \( v_0 = 220 \text{ km/s} \). We have taken the energy density of DM \( \rho_x = n_x m_x = 0.3 \text{ Gev/cm}^3 \).

As the Sun with the Earth is moving in the Galaxy coordinate system with the velocity \( v_S = 220 \text{ km/s} \), the velocity distribution of the DM particles, as we see it from the Earth has the following form:

\[
f(v) = n_x \frac{1}{\pi^{1/2} v_0 v_S} e^{-(v-v_S)^2/v_0^2} - e^{-(v+v_S)^2/v_0^2}.
\]

Velocity of DM particle increases when it approaches the Earth surface from the infinity according to the following equation: \( v'^2 = v^2 + \frac{2GM_E}{R_E} \). We consider the flux of DM particles to be isotropical neglecting its asymmetry due to the motion of the Sun.

Figure 2: Total scattering cross section of DM particle for different \( \lambda \)

![Figure 2](image-url)
After the DM particle falls on the Earth its propagation is considered according to the previous paragraph. During the elastic scattering on the earth nuclei the DM particle loses its energy. We consider the DM particle to be captured by the Earth if it is reflected several times from the surfaces of layers into which the Earth volume was divided. Knowing the initial flux of DM and calculating in this manner the probability of capturing we have calculated the capture rate of DM by the Earth.

4.1 Model testing

To test our model we have tried to calculate the test case of capture of weak interaction massive particles (WIMP) by the Earth. We suppose that these particles have fixed scattering cross section $\sigma_0$ with the nuclei of the Earth, and that this scattering is elastic and isotropic. We took this cross section to be equal to $\sigma = 10^{-34}$ cm$^2$. In paper [3] the capture rate of WIMP’s by the Earth was theoretically calculated. In this test calculation we neglect the motion of the Solar system relative to the Galaxy coordinate system when the analytical formulas for the capture rate are the most transparent. When the Sun motion is taken into account the caption rate changes only by a factor not higher than 2.5. Also analytical formula doesn’t take into account non zero temperature in the Earth. As was shown in [3] the influence of non zero Earth temperature on the capture rate is not crucial. Every element of the Earth gives its own contribution to the caption rate. Consider element of mass $m$ with number density $n$. Let its mass fraction in the Earth be $f$. It is convenient to introduce the following notations:

$$\mu = \frac{m_x}{m}, \quad \mu_\pm = \frac{\mu \pm 1}{2},$$

$$A = \frac{u^2}{v_0^2} \frac{\mu}{\mu_\pm},$$

where $u$ - is escape velocity of DM particle from the considered point inside the Earth. Let also $\phi$ be the gravitational potential in the Earth relative to this potential at the Earth surface. The value of $\phi$ is about 1.6 in the Earth center. We denote with $v_{\text{esc}}$ the escape velocity of particle at the Earth surface. This value is about 11.2 km/s. Then the caption rate of the DM by the considered element can be calculated as [3]:

$$C = \left[\frac{8}{3\pi}\sqrt{\sigma_0 \bar{v}}\right] \left[\frac{M_B}{m}\right] \left[\frac{v_{\text{esc}}^2}{v_0^2} \langle \frac{\phi}{v_0} \rangle \langle 1 - \frac{1 - e^{-A^2}}{A^2} \rangle\right],$$

where $\bar{v} = \sqrt{\frac{2}{m} \langle \phi \rangle}$, $M_B$ is the mass of the Earth and angle brackets indicate averaging over the mass of the Earth. The total capture rate for all elements can be calculated as $\sum f C_f$ where $C_f$ is the capture rate calculated according to equation (8). DM mass dependence on the capture rate for individual element has a maximum at $m_x = m$. Theoretically the calculated capture rate of WIMP’s according to this formulas for our model of the Earth is presented in Figure [3]. Here the result of our Monte Carlo simulation is also presented in comparison. We see that mass dependence on the theoretically calculated capture rate has two maxima according to two average elements we have considered: one with mass $m = 23.5$ GeV in the mantle of the Earth and the second at $m = 56$ GeV in the core. Monte Carlo results qualitatively agree with a theoretical prediction. Its mass dependence also has two maxima but the first maximum is about 20% smaller than the theoretical prediction.
and the width of the theoretical peaks is about two times smaller than in our model. Probably this discrepancy between curve forms can be a consequence of neglecting the temperature of the Earth in the theoretical formulae.

Figure 3: Capture rate of WIMP’s of different mass by the Earth

4.2 Capture rate of long range DM by the Earth

We have calculated the mass dependence on the capture rate for a long range interaction of DM with ordinary matter for different values of \( \lambda \). Results are presented in Figure 4. For a long range interaction DM the mass dependence of capture rate is smooth without maxima. This is due to the fact that scattering cross section for such matter is high and DM particle has a lot of interaction in the Earth with small transfer of energy during each interaction. Also the absolute value of capture rate for \( \lambda > 10^{-10} \) cm is about two orders of magnitude larger than for WIMP’s with cross section \( \sigma = 10^{-34} \) cm\(^2\). Capture rate increases with parameter \( \lambda \) and is even higher for larger \( \lambda \) than shown in Figure 4.

5 DM evaporation

Captured DM thermolizes in the Earth so that its position-velocity distribution follows Maxwell-Boltzmann law:

\[
    f_{th}(v, r) = \frac{n_0}{V_1} \frac{4}{\pi^{1/2}} \left( \frac{m_x}{2k_B T_x} \right)^{3/2} v^2 e^{-m_x v^2/2k_B T_x} e^{-m_x \Phi(r)/k_B T_x},
\]

where \( n_0 \) - is the number of captured particles, \( T_x \) - temperature of DM distribution, \( \Phi(r) \) is the Earth gravitational potential, and
\[ V_1 = \int_0^{R_E} 4\pi r^2 e^{-m_x \Phi(r)/k_B T_x} \, dr \]  

(10)

is the effective volume of the Earth and \( R_E \) is the Earth radius. Particles captured by the gravitational potential of the Earth can fly out of the Earth, move along the elliptic orbit around the Earth and then again return into the Earth. Sometimes if the velocity of the particle is larger than the escape velocity of the Earth the particle can fly out of the Earth. The latter process determines the total timelife of the captured DM particle. The rate at which DM particle fly out of the Earth is determined by the mass of DM particle, difference of gravitational potential at the Earth surface compared with the center of the Earth and by the cross section with which DM particle interacted in the Earth. In Figure 5 time life of DM particle in the Earth is presented for particle of mass equal to 10 GeV and for interaction of different length \( \lambda \).

Time life of DM particle in the Earth depends on the mass of DM particle \( m_x \) according to the Boltzmann distribution:

\[ t_{Earth}^{-1} \sim e^{-m_x \Delta \Phi/k_B T_x}, \]  

(11)

where \( \Delta \Phi \) is the difference of the gravitational potential between the center and the surface of the Earth. Calculated time life of DM particle in the Earth for different \( m_x \) is compared with this distribution in Figure 6. Calculated time life in Figure 6 for small \( m_x \) diverges from simple law (4) because the particles flying out of the Earth have nonzero kinetic energies.

Total time life of the captured DM particle in the Earth gravitation is presented in Figure 7 for particle with mass \( m_x = 10 \) GeV for different \( \lambda \). It has a peak for interaction length \( \lambda \) equal to \( 10^{-9} \). Total time life in the Earth gravitation is determined by the time life in the Earth \( t_{Earth} \) which is divided by the probability

Figure 4: Capture rate of DM by the Earth for different \( \lambda \)
of escaping the Earth. The latter probability depends on the form of the velocity spectrum of the particles which leave the Earth. Computer simulation shows that this spectrum for $\lambda > 10^{-9}$ can be described by simple Gaussian with nearly constant dispersion. On the other hand, the mean velocity of DM particle flying out of the Earth slightly rises with $\lambda$ so that probability of escaping the Earth gravitation increases and the total time life in Fig. 4 falls for large $\lambda$.

Dependence of total time life of DM particle on $m_x$ follows the similar law (11) but with $\Delta \Phi = \Phi(\infty) - \Phi(0)$, where $\Phi(0)$ is the gravitational potential in the center of the Earth. Calculated total life time is compared with law (11) in Figure 8. As is seen from the comparison total time life exactly follows Boltzmann law. Fitted temperature of the captured DM in the Earth is equal to 4900 K which is about 75% of the maximal temperature in the Earth core. Total time life of heavy DM particles in the gravitational potential of the Earth exceeds the age of the Earth. This time determines the time of DM accumulation by the Earth. The accumulation time is equal to the total time life of the DM particle if it doesn’t exceed the age of the Earth, or simply equals the age of the Earth if total time life is higher than the Earth age.

![Figure 5: Time life in the Earth for different $\lambda$](image)

6 DM density on the Earth surface

Captured by the Earth gravitation DM particles constantly fly out of the Earth, stay some time at elliptic orbit and then return to the Earth. As the age of the Earth is rather large (about 4.5 billion years) and the rate with which LRF DM is captured by the Earth is also large the amount of DM accumulated in the Earth may be up to $10^{37}$ particles. Accumulated by the Earth DM particles can produce additional DM density at the Earth
Figure 6: Time life in the Earth for different $m_x$

Figure 7: Total time life of DM particle before escaping for different $\lambda$
surface. This density can be calculated as

$$\rho_z = m_x \frac{C t_{\text{accum}}}{(t_{\text{Earth}} + t_{\text{orb}}) S_E \bar{v}_{\text{orb}}}$$

where $S_E$ is the area of the Earth surface, $t_{\text{orb}}$ is the average time the DM particles stay at the orbit outside the Earth which is usually not larger than 0.1 day, and $\bar{v}$ is the mean DM particle velocity when it leaves the Earth surface. In Eq. (12) $C$ is the capture rate of DM particles by the Earth, and $t_{\text{accum}}$ is the accumulation time of DM by the Earth, which is equal to total time life of DM particle captured by the Earth gravitation if it doesn’t exceed the Earth age. The results of the calculations of additional DM density for different interaction length $\lambda$ and DM particle mass $m_x$ are presented in Figure 9. As is seen from this figure for masses of DM particles smaller than 15 GeV the additional DM density could approach rather high values higher than $10^6$ times exceeding the mean DM density in our Galaxy.

7 Discussion

Additional DM density at the Earth surface presented in Figure 9 has a form of a mountain ridge. Mass dependence on $\rho_z$ for different $\lambda$ is presented in Figure 10. It has a sharp peak with exponential growth before peak and exponential falling after the peak. Such a behavior is determined by two times in Eq. (12) - $t_{\text{accum}}$ and $t_{\text{Earth}}$. These both times rise exponentially with $m_x$ before $t_{\text{accum}}$ reaches saturation determined by the age of the Earth. After these times ratio of these two times changes from exponential growth to exponential falling. Capture rate $C$ has a smooth behavior with $m_x$ and doesn’t significantly influence the $m_x$ behavior (see Fig. 4). The height of the peak in Fig. 9 only slightly decreases with $\lambda$ so that it is reasonable to suppose that

![Figure 8: Total time life of DM particle before escaping for different masses of DM particles](image-url)
Figure 9: Additional DM density on the Earth surface
for $\lambda$ up to $10^{-5}$ cm there will be a significant accumulation of DM at the Earth surface due to the considered mechanism. It is to be mentioned that the width of the peaks is not large (about several GeV at half width) that is why this mechanism of DM accumulation is selective for definite masses of DM if the range of LRF is fixed. Also the accumulated density of DM near the Earth is determined by the value of the constant $\alpha$ in Eq.(1). For smaller values of $\alpha$ the density of accumulated DM at the Earth surface will decrease.

Figure 10: Additional DM density on the Earth surface

Additional DM density at the Earth surface for the case of LRF can be compared with the same density but for the case of WIMP’s. Both in [3] and in a WIMP’s is considered the case of heavy Dirac neutrino. The total scattering cross section on the Earth element is assumed to be [3]

$$\sigma = \frac{\mu}{\mu_m} Q^2 \frac{m m_x}{(\text{GeV})^2} \times 5.2 \times 10^{-40} \text{ cm}^2,$$

(13)

where

$$Q = N - (1 - 4 \sin^2 \theta_W) Z \approx N - (0.124) Z$$

(14)

and $N, Z$ is the neutron and proton numbers of the Earth element. The calculated Capture rate using this cross section is presented in Fig. [11]. It has two peaks that appear when the WIMP’s mass coincides with that of the Earth element in our model of the Earth. This capture rate agrees well with that calculated theoretically in [3]. Additional density of WIMP’s at the Earth surface (due to WIMP’s evaporation) has also been calculated. It
is presented in Fig. 11. Additional density of WIMP’s achieves its maximal value for the WIMP’s mass about 8.5 GeV when the time life of WIMP’s reaches the age of the Earth. It is equal to 150 GeV/cm$^3$ which is about three orders of magnitude smaller than maximal additional density of DM with LRF. This discrepancy is due to the fact that the time life in the Earth is much larger for WIMP’s than for long range interaction DM. This is because the scattering cross section of WIMP’s is small so they need large time before they get in the collision at the energy sufficient to escape the Earth. As is seen from Fig. 2 this time increases for DM with LRF for small $\lambda$ where the cross section falls as is seen from Fig. 2.

![Graphs](image.png)

Figure 11: Additional WIMP’s density at the Earth surface (left panel) and capture rate of WIMP’s by the Earth (right panel)

In this paper we have considered only the case of DM capture and accumulation by the Earth. Besides this direct process there can also be a two step process. At the first step DM is captured by the Sun and then due to perturbations additional density of DM near the Earth is produced [1, 2, 3, 4]. Besides that due to this process DM is accumulated near the Earth, the velocity spectrum of DM particles is much softer in such a way that the probability of its capturing by the Earth is higher in this case than for the case of Galaxy DM. At the second step DM accumulated at the Earth orbit is captured by the Earth. Unfortunately this mechanism of accumulation is much more complicated than the one considered in this paper and there are also some uncertainties in time life of DM in the Sun system at the Earth orbit [7]. On the other hand, this mechanism is rather interesting because heavy DM particles can be accumulated by it.
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