Persistence in Random Bond Ising Models of a Socio-Econo Dynamics in High Dimensions

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Abstract

We study the persistence phenomenon in a socio-econo dynamics model using computer simulations at a finite temperature on hypercubic lattices in dimensions up to 5. The model includes a ‘social’ local field which contains the magnetization at time $t$. The nearest neighbour quenched interactions are drawn from a binary distribution which is a function of the bond concentration, $p$. The decay of the persistence probability in the model depends on both the spatial dimension and $p$. We find no evidence of ‘blocking’ in this model. We also discuss the implications of our results for applications in the social and economic fields.

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I. INTRODUCTION

The persistence problem is concerned with the fraction of space which persists in its initial \((t = 0)\) state up to some later time \(t\). The problem has been extensively studied over the past decade for pure spin systems at both zero [1-4] and non-zero [5] temperatures. Typically, in the non-equilibrium dynamics of spin systems at zero-temperature, the system is prepared initially in a random state and the fraction of spins, \(P(t)\), that persists in the same state as at \(t = 0\) up to some later time \(t\) is studied. For the pure ferromagnetic Ising model on a square lattice the persistence probability has been found to decay algebraically [1-4]

\[ P(t) \sim t^{-\theta}, \]

where \(\theta \sim 0.22\) is the non-trivial persistence exponent [1-3]. The actual value of \(\theta\) depends on both the spin [6] and spatial [3] dimensionalities; see Ray [7] for a recent review.

At non-zero temperatures [5], consideration of the global order parameter leads to a value of \(\theta_{\text{global}} \sim 0.5\) for the pure two-dimensional Ising model.

It has been only fairly recently established that systems containing disorder [8-10] exhibit different persistence behaviour to that of pure systems. A key finding [8-9,11] is the appearance of ‘blocking’ regardless of the amount of disorder present in the system.

As well as theoretical models, the persistence phenomenon has also been studied in a wide range of experimental systems and the value of \(\theta\) ranges from 0.19 to 1.02 [12-14], depending on the system. A considerable amount of the recent theoretical effort has gone into obtaining the numerical value of \(\theta\) for different models.

In this work we add to the knowledge and understanding regarding persistence by presenting the initial results for the persistence behaviour of a modified version of a recently proposed spin model which appears to reproduce the intermittent behaviour seen in real financial markets [15]. In the next section we discuss the model in detail. In the subsequent section we give an outline of the method used and the values of the various parameters employed. Section IV describes the results and the consequent implications for using the models in a financial or social context. Finally, in Section V there is brief conclusion.
II. THE MODIFIED BORNHOLDT MODEL

The simulations were performed on a modified version of a spin model of financial markets proposed recently by Bornholdt [15].

In the original Bornholdt model, \( N \) market traders, denoted by Ising spins \( S_i(t), i = 1 \ldots N \), are located on the sites of a hypercubic lattice. The action of the \( i \)th trader of buying or selling at time step \( t \) corresponds to the spin variable \( S_i(t) \) assuming the value \(+1\) or \(-1\), respectively. A local field, \( h_i(t) \), determines the dynamics of the spins. In particular,

\[
h_i(t) = \sum_{<ij>} J_{ij} S_j(t) - \alpha C_i(t) \sum_{j=1}^{N} S_j(t),
\]

where the first summation runs over the nearest neighbours of \( i \) only (\( J_{ij} = J \), for nearest neighbours and \( J_{ij} = 0 \), otherwise), \( \alpha > 0 \) couples to the magnetization, and \( C_i(t) \) is a second spin used to represent the strategy of agent \( i \).

Subsequently, Yamano [16] worked with a model where the local field is given by

\[
h_i(t) = \sum_{<ij>} J_{ij} S_j(t) - \alpha | \sum_{j=1}^{N} S_j(t) |.
\]

Although here the strategy spin is omitted, the coupling constant is retained. Furthermore, the nearest neighbour interactions are now selected randomly, \( J_{ij} = \pm J \). Each agent is updated according to the following heat bath dynamics:

\[
S_i(t+1) = \begin{cases} 
+1 & \text{with } q = [1 + \exp(-2\beta h_i(t))]^{-1}, \\
-1 & \text{with } 1 - q,
\end{cases}
\]

where \( q \) is the probability of updating and \( \beta \) is the inverse temperature. In this model the return is defined in terms of the logarithm of the absolute value of the magnetization, \( M(t) = \sum_{j=1}^{N} S_j(t)/N \), that is

\[
\text{Return}(t) = \ln | M(t) | - \ln | M(t - 1) |
\]

Simulations [16] in spatial dimensions ranging from \( d = 1 \) to \( d = 5 \) indicate that the modified version of the model reproduces the required intermittent behaviour in the returns for suitable values of the coupling constant and the temperature \( T \); these are listed in Table 1.
TABLE I: Values of the linear dimension $L$ of the lattices used in the simulations. The coupling parameter $\alpha = 4.0$ in all cases. Intermittent behaviour was observed in the returns when the temperature was set at $T_{int}$ as given above.

In this work we investigate the persistence behaviour of the model where the local field is given by equation (2) but the nearest neighbour interactions are selected from

$$P(J_{ij}) = (1 - p)\delta(J_{ij} + 1) + p\delta(J_{ij} - 1),$$

(5)

where $p$ is the concentration of ferromagnetic bonds. Hence, we are interested in determining the fraction of traders who have been at time $t$ either buying or selling continuously since $t = 0$. We will also suggest a possible interpretation within sociophysics of the model later on.

III. METHODOLOGY

As mentioned in the previous section, for each spatial dimension $d$ we first fine tune the temperature to reproduce intermittent behaviour in the returns. As can be seen from Table 1, the temperature $T_{int}(d)$ decreases with $d$. For a given dimension, all subsequent simulations are performed at that temperature. Averages over at least 100 samples for each run were performed and the error-bars in the following plots are smaller than the data points.

The value of each agent at $t = 0$ is noted and the dynamics updated according to equation (3).

At each time step, we count the number of agents that still persist in their initial ($t = 0$) state by evaluating

$$n_i(t) = (S_i(t)S_i(0) + 1)/2.$$

(6)
Initially, \( n_i(0) = 1 \) for all \( i \). It changes to zero when an agent changes from buying to selling or vice versa for the first time. Note that once \( n_i(t) = 0 \), it remains so for all subsequent calculations.

The total number, \( n(t) \), of agents who have not changed their action by time \( t \) is then given by

\[
n(t) = \sum_i n_i(t). \tag{7}
\]

A fundamental quantity of interest is \( P(t) \), the persistence probability. In this problem we can identify \( P(t) \) with the density of non-changing agents \([1]\).

\[
P(t) = n(t)/N, \tag{8}
\]

where \( N = L^d \) is the total number of agents present.

**IV. RESULTS**

We now discuss our results. In figure 1 we show a semi-log plot of the persistence probability against time \( t \) for a range of bond concentrations \( 0 < p \leq 0.5 \) for \( d = 1 \). It’s clear from the plot that the data can be fitted to

\[
P(t) \sim e^{-\gamma t}, \tag{9}
\]

where we estimate \( \gamma \sim 0.56 \) from the linear fit.

Figure 2 displays the results for \( d = 2 \). Although once again there is evidence for exponential decay, this time it would appear that the value of the parameter \( \gamma \) depends on the \( p \). For \( p = 0.1 \) we estimate \( \gamma \sim 0.35 \). The results for the three-dimensional case are shown in figure 3. Here we see clear evidence of the qualitative nature of the decay depending on the bond concentration. For \( p = 0.5 \) we have behaviour very similar to the two cases considered earlier, namely exponential decay. However, the decay is clearly non-exponential for \( p = 0.1 \).

The results in \( d = 4 \) are very similar to those for \( d = 3 \) and we will not present them here. Instead, in figure 4 we show a log-log plot of the persistence against time for \( d = 5 \). The decay of \( P(t) \) is seen to be heavily dependent on the concentration of ferromagnetic bonds. For low values of \( p(\leq 0.3) \), we have a power-law decay at long times as given by equation (1) with an estimated value of \( \theta \sim 0.5 \). For higher value of \( p \) the decay would appear not to be a power-law but also not exponential in it’s nature.
FIG. 1: Here we plot $\ln P(t)$ versus $t$ for $d = 1$ over the range $0.1 \leq p \leq 0.5$. The straight line, which is a guide to the eye, has a slope of $-0.56$.

FIG. 2: A semi-log plot of the data for $d = 2$. We see that here, in contrast to figure 1 for $d = 1$, the slopes are dependent on the bond concentrations. The linear fit shown is that for $p = 0.1$ and the slope is $-0.35$. 
FIG. 3: A plot of $\ln P(t)$ against $t$ for $d = 3$ for the same bond concentrations as earlier. The straight line, which is a guide to the eye, has a slope of $-0.36$ and indicates that the decay for $p = 0.5$ is very similar to that found in lower dimensions. The behaviour for $p = 0.1$ is clearly non-exponential.

V. CONCLUSION

To conclude, we have presented the results of extensive simulations for the persistence behaviour of agents in a model capturing some of the features found in real financial markets. Although the model contains bond disorder, we do not find any evidence of ‘blocking’. The persistence behaviour appears to depend on both the spatial dimensionality and the concentration of ferromagnetic bonds. Generally, whereas in low dimensions the decay is exponential, for higher dimensions and low values of $p$ we get power-law behaviour.

The initial model was developed in an economic context. Power law persistence in the case means the existence of traders who keep on buying or selling for long durations. Furthermore, the presence of ‘blocking’ would be highly unrealistic for modelling the dynamics because the traders would have access to a finite amount of capital.

One can also interpret the model in a social context. Here the value $S_i(t) = +1$ or $-1$ could represent an opinion. Here ‘blocking’ would be realistic and correspond to the
FIG. 4: Here we display the data for $d = 5$ and selected bond concentrations as a log-log plot. Clearly the behaviour depends crucially on the value of $p$. For low ($p \leq 0.3$) values the decay is power-law. The straight line shown has a slope of $-0.5$.

proportion of the population that is stubborn. Hence, any model exhibiting exponential decay in the persistence probability would probably be an unrealistic model.

Hence, we can use the behaviour of the persistence probability a criterion to decide whether we have a realistic economic or social model.

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