Seed the Self-Modulation in a Long Proton Bunch by Charge Cancellation with a Short Electron Bunch

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Abstract

In plasma wakefield accelerators (e.g. AWAKE) the proton bunch self-modulation is seeded by the ionization front of a high-power laser pulse ionizing a vapour and by the resulting steep edge of the driving bunch profile inside the created plasma.

In this paper, we present calculations in 2D linear theory for a concept of a different self-modulation seeding mechanism based on electron injection. The whole proton bunch propagates through a preformed plasma and the effective beam current is modulated by the external injection of a short electron bunch at the centre of the proton beam. The resulting sharp edge in the effective beam current in the trailing part of the proton bunch is driving large wakefields that can lead to a growth of the seeded self-modulation (SSM). Furthermore, we discuss the feasibility for applications in AWAKE Run 2.

Keywords: AWAKE, Seeding of Proton Bunch Self-Modulation, Plasma Wakefield Acceleration, Electron Injection

1. Introduction

The proton bunch self-modulation in plasma wakefield accelerators (e.g., the Advanced Wakefield Experiment (AWAKE) located at CERN) is usually seeded by using a high-power laser pulse co-propagating in the centre of the proton bunch (see Fig. 1a). The laser ionizes a vapour and creates a plasma, resulting in a sharp relativistic ionization front separating vapour and plasma. Due to this ionization front, the proton bunch self-modulation is seeded and consequently growing within a distance of a few meters before reaching its saturation level. The long proton bunch is split into a train of micro-bunches with a period on the order of the plasma wavelength. This modulation is caused by the self-modulation instability (SMI), a transverse beam-plasma instability. A witness electron bunch can thus deterministically be injected into one of the buckets between the micro-bunches and accelerated.

2. A Simplified Model in Linear Theory

For a more quantitative understanding of the approach towards seeding, we introduce the following simplified model: The proton and electron bunch longitudinal density distribution is described by a $\cos$-profile, whereas their radial profiles are Gaussian. We use 2D-linear plasma wakefield theory to describe the seeding effect by calculating the initial wakefields. The total (proton and electron) longitudinal charge distribution $n_{bl}$ (see Fig. 2) is given by

$$n_{bl}(\xi) = \begin{cases} A_0 \cdot \sin \left( \frac{\pi (\xi + L)}{2L - U} \right) & \xi \in [-L, -U] \\ A_{\text{max}} \cdot \cos \left( \frac{\pi \xi}{U} \right) & \xi \in [-U, U] \\ A_0 \cdot \cos \left( \frac{\pi \xi}{2L - U} \right) & \xi \in [U, L] \\ 0 & \text{elsewhere} \end{cases}$$

where $A_0$ is the effective beam current to be accelerated and $n_{bl}$ is the longitudinal charge distribution of the proton bunch.

$$\tau = 120 \text{fs}, E = 450 \text{mJ}.$$
where \( \xi = ct - z \) is the spatial coordinate in the co-moving bunch frame, \( 2L \) the longitudinal size of the proton bunch and \( 2U \) the longitudinal size of the electron bunch. Outside \([-L, L]\) the bunch charge density is assumed to be zero. The amplitude of the proton charge density is \( A_0 \) and the amplitude of the electron charge density is \( A_0 - A_{num} \). The absolute number of charges in one bunch for our model is given by \( N = 8A_0L\rho_0\sigma_{r,p}^2 \cdot 0.683 \). For comparison in case of a Gaussian longitudinal bunch profile \( N \) is given by \( N = (2\pi)^{3/2}\rho_0\sigma_{r,p}^2\). According to (8) the parallel and perpendicular wakefields \( W_\parallel(\xi, r) \) and \( W_\perp(\xi, r) \) are given by

\[
W_\parallel(\xi, r) = \frac{4}{\pi} \int_{-\infty}^{\infty} n_\parallel(r') I_0(k_{pe}(\xi - r')) \, dr'
\]

and

\[
W_\perp(\xi, r) = \frac{4}{\pi} \int_{-\infty}^{\infty} n_\parallel(r') I_0(k_{pe}(\xi - r')) \, dr'
\]

where \( e \) is the elementary charge, \( \epsilon_0 \) the vacuum permittivity and \( k_{pe} = \omega_{pe}/v_b \approx \omega_{pe}/c \) the plasma wave number in case of a relativistic particle bunch with bunch velocity \( v_b \approx c \). The plasma electron angular frequency \( \omega_{pe} \) is

\[
\omega_{pe} = \frac{n_\parallel e^2}{\epsilon_0 m_e},
\]

where \( m_e \) is the electron mass and \( n_\parallel \) the plasma charge density. The transverse component \( R(r) \) in Eqn. 2 and 3 is given by

\[
R(r) = k_{pe}^2 K_0(k_{pe}r) \int_{0}^{\infty} r' n_{\perp L}(r') I_0(k_{pe}r') \, dr'
\]

and

\[
R(r) = k_{pe}^2 K_0(k_{pe}r) \int_{0}^{\infty} r' n_{\perp L}(r') K_0(k_{pe}r') \, dr',
\]

with \( I_0 \) and \( K_0 \) the modified Bessel functions of the first and second kind, respectively. Assuming the radial profile and the radial size of the electron and proton bunch are the same, \( R(r) \) and \( \frac{dR}{dr} \) are just multiplying factors. Considering different radii is trivial.

Piecewise integration of the separate bunch parts of Eqs. 2 and 3 along \( \xi \) (see Fig. 2) gives the contribution of each single part to the resulting wakefields.

The wakefields driven by the leading part of the proton bunch \((-L < \xi < -U)\) are described by

\[
E_{1+}(\xi) = \begin{cases} 
0 & \text{if } -L < \xi < -U \\
\frac{\partial}{\partial \xi} \left[ (C_+ + C_-) \cos \left( \frac{\xi^2 (L - U) + \cos k_{pe}(\xi + U)}{2L} \right) \right] & \text{if } -U < \xi < 0 \\
\frac{\partial}{\partial \xi} \left[ (C_+ + C_-) \sin \left( \frac{\xi^2 (L - U) + \sin k_{pe}(\xi + U)}{2L} \right) \right] & \text{if } 0 < \xi < L \\
0 & \text{if } L < \xi < U 
\end{cases}
\]

and

\[
E_{1-}(\xi) = \begin{cases} 
0 & \text{if } -L < \xi < -U \\
\frac{\partial}{\partial \xi} \left[ (C_+ - C_-) \sin \left( \frac{\xi^2 (L - U) + \sin k_{pe}(\xi + U)}{2L} \right) \right] & \text{if } -U < \xi < 0 \\
\frac{\partial}{\partial \xi} \left[ (C_+ - C_-) \cos \left( \frac{\xi^2 (L - U) + \cos k_{pe}(\xi + U)}{2L} \right) \right] & \text{if } 0 < \xi < L \\
0 & \text{if } L < \xi < U 
\end{cases}
\]

where the first term is valid for \( \xi < -L \), the second term for \(-L \leq \xi \leq -U\) and the third term for \( \xi > -U \). The parallel
and perpendicular wakefields in the part where the electrons are injected and therefore cause a drop in the effective charge density are given by

\[
E_{2,\parallel}(\xi) = \begin{cases} 
0, & \xi < \xi_1 \\
\frac{\omega_p}{\pi} \left[ (D_+ + D_-) \sin \left( \frac{\omega_p}{L} \xi \right) - (D_+ - D_-) \sin(k_{\parallel} \xi + U) \right] \\
\frac{\omega_p}{\pi} \left( (D_+ - D_-) \sin(k_{\parallel} \xi - U) - (D_+ - D_-) \sin(k_{\parallel} \xi + U) \right) \\
- \frac{\omega_p}{\pi} \left[ \sin(k_{\parallel} \xi - U) - \sin(k_{\parallel} \xi + U) \right] 
\end{cases}
\]

and

\[
E_{2,\perp}(\xi) = \begin{cases} 
0, & \xi < \xi_1 \\
\frac{\omega_p}{\pi} \left[ (D_+ + D_-) \cos \left( \frac{\omega_p}{L} \xi \right) - (D_+ - D_-) \cos(k_{\parallel} \xi + U) \right] \\
\frac{\omega_p}{\pi} \left( (D_+ - D_-) \cos(k_{\parallel} \xi - U) + (D_+ - D_-) \cos(k_{\parallel} \xi + U) \right) \\
- \frac{\omega_p}{\pi} \left[ \cos(k_{\parallel} \xi - U) - \cos(k_{\parallel} \xi + U) \right] 
\end{cases}
\]

Here, the first term is valid for \( \xi < 0 \), the second for \(-U \leq \xi \leq L \) and the third for \( \xi > U \).

The wakefields caused by bunch section between \( U \) and \( L \) are described by

\[
E_{3,\parallel}(\xi) = \begin{cases} 
0, & \xi < \xi_1 \\
\frac{\omega_p}{\pi} \left[ (C_+ + C_-) \sin \left( \frac{\omega_p}{L} \xi \right) + (C_+ - C_-) \sin(k_{\parallel} \xi - L) \right] \\
\frac{\omega_p}{\pi} \left( (C_+ + C_-) \cos(k_{\parallel} \xi - L) + (C_+ - C_-) \sin(k_{\parallel} \xi - L) \right) \\
- \frac{\omega_p}{\pi} \left[ \cos(k_{\parallel} \xi - L) - \cos(k_{\parallel} \xi - L) \right] 
\end{cases}
\]

and

\[
E_{3,\perp}(\xi) = \begin{cases} 
0, & \xi < \xi_1 \\
\frac{\omega_p}{\pi} \left[ (C_+ - C_-) \cos \left( \frac{\omega_p}{L} \xi \right) + (C_+ + C_-) \cos(k_{\parallel} \xi - L) \right] \\
\frac{\omega_p}{\pi} \left( (C_+ + C_-) \sin(k_{\parallel} \xi - L) + (C_+ - C_-) \cos(k_{\parallel} \xi - L) \right) \\
- \frac{\omega_p}{\pi} \left[ \sin(k_{\parallel} \xi - L) - \sin(k_{\parallel} \xi - L) \right] 
\end{cases}
\]

For this case, the first line is valid for \( \xi < U \), the second for the interval \( 0 \leq \xi \leq L \) and the third line for \( \xi > L \).

The constants in Eqns. 6 to 11 are given by

\[
A_1 = \frac{\Delta s - A_0}{2}, \quad A_2 = \frac{\Delta s + A_0}{2}, \quad C_+ = \frac{\pi}{\sqrt{2}} \frac{L}{v_p} \quad \text{and} \quad D_+ = \frac{1}{2} \frac{L}{v_p}.
\]

Making use of the superposition theorem of waves in linear theory, the final wakefield distribution can be determined by adding the different contributions of the bunch segments in every region. Hence, the resulting wakefields are described by

\[
E(\xi) = \begin{cases} 
0, & \xi < L \\
E_1(\xi)|_{\xi \in [-L; -U]} + E_2(\xi)|_{\xi \in [-L; -U]} + E_3(\xi)|_{\xi \in [-L; -U]} \\
E_1(\xi)|_{\xi \in [U; L]} + E_2(\xi)|_{\xi \in [U; L]} + E_3(\xi)|_{\xi \in [U; L]} \\
E_1(\xi)|_{\xi > L} + E_2(\xi)|_{\xi > L} + E_3(\xi)|_{\xi > L} 
\end{cases}
\]

Fig. 3 shows the longitudinal and perpendicular wakefields (Eq. 12) for a ratio \( L/U = 1/10 \) and \( U/\lambda_{pe} = 1.6 \). The front of the proton bunch (\( \xi < -U \)) propagates in a preformed plasma where its length is very long when compared to the plasma wavelength \( \lambda_{pe} \) and its profile is smooth. It is therefore not effective at driving wakefields, but still drives a low amplitude fields that could lead to development of the SMI process over long plasma distances. The adiabatic response of the plasma generates a globally focussing force by charge neutralisation that, upon propagation, can lead to an increase in bunch density and thus also of the wakefield response. This effect could also lead to SMI growth (as opposed to SSM) over long plasma distances. From the position of the electron bunch (\( \xi > U \)) and all along the second half of the proton bunch, much larger wakefields are driven, which subsequently provide the strong seed for the SM process. For bunch parameters similar to AWAKE (\( L = 6 \, \text{cm}, \, U = 2 \, \text{mm}, \, \frac{L}{U} = 1/10 \), \( U/\lambda_{pe} = 1.6 \), peak currents \( |Q_{p,\text{peak}}| = |Q_{e,\text{peak}}| \) (see Fig. 3) and \( k_{pe} = 5000 \, \text{m}^{-1}, \, n_p = 7 \times 10^{16} \, \text{cm}^{-3}, \, \sigma_r = 200 \, \mu\text{m} \) the wakefields in the trailing part of the bunch could reach values up to the GV/m scale.

It can be easily seen from Eqns. 6-11 that the amplitudes of the wakefields in the trailing part of the bunch are scaling linearly with the electron bunch charge \( Q_e \), i.e. the depth of the gap in the effective charge density \( A_0 - A_{min} \). Furthermore, the wakefields are strongly depending on the width of the gap in the effective charge density \( 2U \), corresponding to the longitudinal size of the injected electron bunch \( \sigma_{r,z} \).

The amplitude of the wakefields can be maximized for a given configuration of \( A_{min} \) and \( L \) and follow the distribution shown in Fig. 4. For an electron bunch length \( U \) on the order of \( \lambda_{pe} \), the peak wakefields are the highest (see Fig. 4).

3. Discussion of the Method

The seeding method described above could be an interesting approach for a seeding concept for the planned extension.
of the AWAKE experiment after 2021, called AWAKE Run 2. Current design studies for the plasma accelerator consist of a split plasma, a short one (~ 4 m) for the seeding of the SSM and a second, longer one (~ 10 m) driven by the modulated proton bunch for the acceleration of the electrons (see Fig. 5). The injection point for the witness electron bunch is foreseen to be between the two plasmas. The short plasma source is similar to the one used in AWAKE Run 1, whereas the long plasma cell could be either a Helicon or a discharge source. By seeding the SMI with an electron bunch, as described here, the ionisation front is not necessary for seeding and thus the first plasma could also be preformed in one of these two types of sources. Hence, there would be no longer need for a maintenance-intensive high-power laser system nor for the complicated Rubidium handling and storage procedures required by a Rubidium based vapour source.

The major uncertainty concerning the feasibility of this approach is the effect of the first part of the proton bunch. Even though, according to the simulations of Kumar et al. and according to the wakefields described in Eq. [2] there is no strong seeding of the SMI expected by the leading part of the bunch, the weak amplitude wakefields or those from noise may grow. Subsequently, wakefields in the front of the bunch would interfere with the seeded wakefields in the trailing part of the bunch. Hence, a phase-stable seeding of the SSM would no longer be possible.

First preliminary experimental results from the AWAKE-experiment show some evidence for a growth of wakefields along the bunch on a scale of a few meters as well as micro-bunching even without seeding, i.e. propagation in a preformed plasma.

In case the unseeded SM-growth in the leading part of the proton bunch is too high for a seeding with an electron bunch in the centre of the beam, another option would be to shift the electron bunch to an earlier position with respect to the proton bunch or even ahead of it (see Fig. 1c). With this, the full proton bunch charge could be used for driving wakefields, while removing the risk of growth of SMI ahead of the seeding point.

4. Conclusions

We presented an alternative concept for the seeding of the SSM in plasma-wakefield accelerators. Instead of seeding by an ionisation front created by a high-power laser pulse, the seeding is archived by the injection of a short electron bunch in the centre of a long proton bunch propagating through a preformed plasma. Calculations in 2D linear theory show that the resulting steep rising edge in the effective charge density of the proton bunch drives large amplitude wakefields that seed the SSM. Although the approach is very promising and could simplify the design for AWAKE Run 2, first experimental results from the latest AWAKE Runs give evidence that wakefields do grow from noise in the front of the proton bunch, when the bunch propagates in a preformed plasma and no seeding is provided.

Appendix A. References

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