Identification of the optimal parameters of the torsional vibration damper of the internal combustion engine crankshaft for normal power settings

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Abstract. This article is devoted to determining the parameters of torsional vibration dampers of the internal combustion engine crankshaft in the entire high-speed mode of its operation. Currently, these parameters are set only for one main harmonic of the distributing moment. At the same time, it is assumed that the remaining harmonics give a slight increase in the amplitude of torsional vibrations. Changing the rotation rate of the crankshaft in the entire range of the operating mode leads to the appearance of other resonant modes, although not as significant as the main resonant mode. Therefore, for more stable quenching of torsional vibrations, a quenching device with variable characteristics is required. The proposed method for determining the parameters of the torsional vibration damper under changing speed and load modes of the internal combustion engine, which ensure efficient use, can be useful in the design of new highly accelerated internal combustion engines.

1. Introduction
During the operation of the internal combustion engine (ICE), torsional vibrations occur in crankshaft sections, which negatively affect the ICE performance, reduce its durability and resource, and create additional voltage during operation [1, 2]. To reduce the torsional vibrations amplitude and thus to reduce their negative impact on the crankshaft, they are equipped with torsional vibrations dampers or silencers. Most of the existing dampers are designed and work in such a way that they work effectively only at a certain crankshaft rotation frequency, or in a narrow range without changing these frequencies [3, 4]. The authors designed a damper, which has a wide range of effective working area due to the variable moment of inertia of the oscillating element. A method for calculating such a damper is proposed below.

2. Materials and methods
The initial data for determining the parameters of the damper are the results calculated for the crankshaft torsional vibrations, and namely:

1) parameters of the equivalent torsion circuit;
2) own torsional vibrations frequencies and corresponding resonant modes of engine operation;
3) harmonious analysis of the engine torsion moment on all resonant operation modes;
4) oscillating motor masses amplitudes and resulting elastic moments;
5) form of vibrations and required place of damper installation.

The kinematic parameters of the damper are determined by the standard method of calculating the crankshafts for torsional vibrations [5, 6].

The geometric parameters of the damper are determined according to the structural parameters. At the same time, it is necessary to determine the initial value of flywheel disk moment of inertia and the dependence of its change on crankshaft rotational frequencies.

The authors developed a method for calculating the kinematic and design parameters of a torsional crankshaft damper in ICE using a flywheel with a variable moment of inertia [7, 8].

3. Results and discussion
Determination of the flywheel housing vibrations amplitude in case of constant and variable moment of inertia.

To determine the required flywheel moment of inertia and the resonant frequency of the rotation, it is assumed that the damper damps 85-95\% of the energy and distributing moment [1].

Let us assume that:

\[ 0.9 \times W_{\text{dist}} = W_{\text{damp}}, \]  

where \( W_{\text{dist}} \) – work of the disturbing moment;
\( W_{\text{damp}} \) – damper work.

Work of the disturbing moment:

\[ W_{\text{dist}} \times \pi \times M_{K} \times A_{1}^{\ast} \times R_{S}, \]

where:
- \( M_{K} \) – amplitude of the k-th harmonic of the torsion moment for the resonant mode of operation;
- \( A_{1}^{\ast} \) – absolute amplitude of the first motor mass;
- \( R_{S} = \sqrt{\left(\sum^{N}_{i=1} a_{i}^{\ast} \sin(k_{M} \Delta \theta_{1-i})\right)^{2} + \left(\sum^{N}_{i=1} a_{i}^{\ast} \cos(k_{M} \Delta \theta_{1-i})\right)^{2}}, \)

\( a_{i}^{\ast} \) - relative amplitudes of motor masses;
\( \Delta \theta_{1-i} \) - phase shift between the motor masses.

Let us assume that the modulus of resistance, acting on the damper housing from the flywheel side a is proportional to the difference and speed of relative displacement, then we get:

\[ M_{D} = \zeta_{D} \times (\dot{\theta}_{K} - \dot{\theta}_{M}), \]

where:
- \( \zeta_{D} \) – damping coefficient;
- \( \dot{\theta}_{K} \) - oscillation speed of the damper housing (crankshaft mounting);
- \( \dot{\theta}_{M} \) - flywheel oscillation speed.

If we assume that, in a steady resonance mode, the housing and the flywheel will oscillate at the same frequency, but with a phase shift \( \varphi_{D} \), then there will be a dependence between the relative amplitudes of the housing \( a_{K} \) and the flywheel \( a_{M} \):

\[ a_{M} = a_{K} / \sqrt{1 + \frac{J_{M} \times \omega_{V}}{\zeta_{D}}} = a_{K} / \sqrt{1 + \tan^{2} \varphi_{D}} = a_{K} \times \cos \varphi_{D}, \]

Where:

\[ \tan \varphi_{D} = \frac{J_{M} + \omega_{V}}{\zeta_{D}}. \]

In the case of resonant mode, from the flywheel side, the moment acts on the damper housing:
\[ M_M = J_M \times \dot{\theta}_M = J_M \times \cos^2 \varphi_D \times \dot{\theta}_K + 0.5 \times J_M \times \omega_V \times \sin 2 \varphi_D \times \dot{\theta}_K \]  

(7)

where \( \omega_V \) – frequency of the disturbing force.

The obtained equation shows that installing the damper is equivalent to connecting an additional mass with moment of inertia \( J_M \times \cos^2 \varphi_D \), which decreases the natural frequency, and a damping moment, which reduces oscillations amplitude:

\[ M_{MD} = 0.5 \times J_M \times \omega_V \times \sin 2 \varphi_D \times \dot{\theta}_K = (0.5 \times J_M \times \omega_V^2 \times a_K \times \sin 2 \varphi_D) \times \cos(\omega_V \times t) \]  

(8)

The amplitude of the damping moment reaches its maximum at \( \sin 2 \varphi_D = 1 \), i.e. at \( \varphi_D = 45^\circ \) and \( \tan \varphi_D = 1 \).

At \( \varphi_D = 45^\circ \) from the equation (7), we can obtain the equivalent value of the flywheel moment of inertia, connected to the oscillation system of the crankshaft:

\[ J^E_M = 0.5 \times J_M \]  

(9)

Moment of inertia of the damper mass connected to the system:

\[ J_D = J_K + 0.5 \times J_M \]  

(10)

From the condition of maximal efficiency of the damper \( \tan \varphi_D = 1 \) it is possible to determine the coefficient of its deformation:

\[ \zeta_D = J_M \times \omega_V \]  

(11)

and damping moment amplitude:

\[ M_{MAX_M D} = 0.5 \times J_M \times \omega_V^2 \times a_K \]  

(12)

The work of the modulus of resistance in the damper for a cycle of oscillations is determined by the expression:

\[ W_{damp} = 0.5 \times \pi \times J_M \times \omega_V \times A^2_K \times \sin 2 \varphi_D \]  

(13)

where \( A^2_K \) - absolute amplitude of the damper housing at the point of its attachment to the crankshaft.

Given the conditions of equations 1 and 2 we obtain:

\[ 0.9 \times \pi \times M_K \times A^2_K \times R_S = 0.5 \times \pi \times J_M \times \omega_V \times A^2_K \times \sin 2 \varphi_D \]

Hence the required flywheel moment of inertia for different frequencies of rotation is determined from the ratio:

\[ J_M = \frac{0.9 \times \pi \times M_K \times A^2_K \times R_S}{0.5 \times \pi \times \omega_V \times A^2_K \times \sin 2 \varphi_D} = \frac{9 \times M_K \times R_S}{5 \times \omega_V \times A^2_K \times \sin 2 \varphi_D} \]  

(14)

When the damper is installed, the oscillation amplitude of the first motor mass is limited taking into account the oscillation energy dissipation in the damper:

\[ A^2_{1D} = \frac{\pi \times M_K \times R_S}{\tau \times k \times \pi \times \omega_{ICE} \times \xi \times \sum_{i=1}^{N} \alpha_i^2 + 0.5 \times \pi \times J_M \times \omega_V \times A^2_K \times \sin 2 \varphi_D} \]  

(15)

where \( \tau = 4 \) – cycle coefficient; 
\( k \) - order of the resonant harmonic; 
\( \omega_{ICE} \) - crankshaft resonant frequency of rotation; 
\( \xi \) – damping coefficient of the crankshaft.

If we consider only the main and strong harmonics:
Then the relation of the amplitudes without damper and after its installation is determined by the expression:

\[ \frac{A_1D}{A_1} = \frac{1}{1 + \rho \times \sin 2\varphi_D}, \]  

(17)

where

\[ \rho = \frac{0.5 \times \pi \times f_M \times \omega_V \times a_k^2}{\frac{\pi}{k} \times \omega_{ICE} \times \xi \times \sum_{i=1}^{N} a_i^2} \]  

(18)

With an optimal choice of damper design parameters:

\[ \frac{A_1D}{A_1} = 1 \]  

(19)

The power expended to drive the damper is determined by:

\[ N_D = 0.25 \times J_M \times \omega_V^3 \times a_k^2 \]  

(20)

The calculation of the parameters of the designed damper according to the proposed method was carried out for a four stroke six cylinder diesel ICE: \( N_e = 130 \text{ kW} \) (\( n = 2600 \text{ rpm} \)), \( V = 8 \text{ l} \), compression ratio \( \varepsilon = 17 \), cylinder diameter \( D = 120 \text{ mm} \), piston stroke \( S = 120 \text{ mm} \), operating order of cylinders \( 153624 \), angle between flashes \( \Theta = 60^\circ \).

The obtained values of the parameter in a seven-mass torsion system, crankshaft wheel for the engine under study are given in Table 1,

| Moment of inertia, kg·m\(^2\) | Reduced length, m | Section stiffness, N·m |
|-----------------------------|-------------------|------------------------|
| \( J_1 = 0.0238859 \)       | \( l_{1,2} = 0.27755 \) | \( C_{1,2} = 793724,818 \) |
| \( J_2 = 0.0238859 \)       | \( l_{2,3} = 0.27755 \) | \( C_{2,3} = 793724,818 \) |
| \( J_3 = 0.0238859 \)       | \( l_{3,4} = 0.27755 \) | \( C_{3,4} = 793724,818 \) |
| \( J_4 = 0.0238859 \)       | \( l_{4,5} = 0.27755 \) | \( C_{4,5} = 793724,818 \) |
| \( J_5 = 0.0238859 \)       | \( l_{5,6} = 0.17646 \) | \( C_{5,6} = 1248418,614 \) |
| \( J_6 = 1.4105220 \)      | \( l_{6,7} = 0.17646 \) | \( C_{6,7} = 1248418,614 \) |

The equations for the natural vibrations of a torsional system are as follows:

\[
\begin{align*}
0.0238859^*\ddot{\theta}_1 + 793724,818^* (\ddot{\theta}_1 - \ddot{\theta}_2) &= 0 \\
0.0238859^*\ddot{\theta}_2 + 793724,818^* (\ddot{\theta}_2 - \ddot{\theta}_3) - 793724,818^* (\ddot{\theta}_1 - \ddot{\theta}_2) &= 0 \\
0.0238859^*\ddot{\theta}_3 + 793724,818^* (\ddot{\theta}_3 - \ddot{\theta}_4) - 793724,818^* (\ddot{\theta}_2 - \ddot{\theta}_3) &= 0 \\
0.0238859^*\ddot{\theta}_4 + 793724,716^* (\ddot{\theta}_4 - \ddot{\theta}_5) - 793724,818^* (\ddot{\theta}_3 - \ddot{\theta}_4) &= 0 \\
0.0238859^*\ddot{\theta}_5 + 793724,818^* (\ddot{\theta}_5 - \ddot{\theta}_6) - 793724,818^* (\ddot{\theta}_4 - \ddot{\theta}_5) &= 0 \\
0.0238859^*\ddot{\theta}_6 + 1248418,614^* (\ddot{\theta}_6 - \ddot{\theta}_7) - 793724,818^* (\ddot{\theta}_5 - \ddot{\theta}_6) &= 0 \\
1.4105222^*\ddot{\theta}_7 - 1248418,614^* (\ddot{\theta}_6 - \ddot{\theta}_7) &= 0
\end{align*}
\]  

(21)
where $\theta_i$ – actual coordinate of the oscillation motion of the $i$-th motor mass.

The lowest frequency $a$ of the natural vibrations of the system, determined by the Holzer method, has the value $\omega_1 = 1587 \frac{1}{C}$. The calculated values of the relative oscillation amplitudes $a_i$ of $i$-th motor masses and the corresponding one-node oscillation form are shown in Fig. 1.

![Figure 1. Relative amplitudes of single node oscillations of motor masses](image)

**Figure 1.** Relative amplitudes of single node oscillations of motor masses

According to the corrected phase diagram (Fig. 2), the main harmonic is $k = 12$, and the strong one $k = 6$.

![Figure 2. Phase diagrams](image)

**Figure 2.** Phase diagrams

The resonant speeds of the crankshaft are determined from the ratio:

$$n_{pk} = \frac{30\omega_0i}{\pi k} \text{ rpm}$$  \hspace{1cm} (22)

where $\omega_0i$ - frequency of natural torsional vibrations corresponding to the $i$-th nodal vibration, $k = 1,2,3,4…$ - motor harmonic index for 4-stroke engines.

For the main ($k=12$) and strong ($k=6$) harmonics, the resonant speed is

$$n_{p12} = \frac{30\omega_01}{\pi k} = \frac{30+1587}{\pi*12} = 1263 \text{ rpm and } n_{p6} = \frac{30\omega_01}{\pi k} = \frac{30+1587}{\pi*6} = 2527 \text{ rpm.}$$

The calculated values of the vibration amplitudes of the motor masses $A_i$ and the resulting elastic moments $M_{Elast,i,i+1}$ are given in Table 2.
Table 2. Oscillating amplitudes of engine mass $A_i^*$ and elastic moments on the sections $M_{\text{Elast},i+1}$

| Mass/section No. | $A_i^*$, rad | $A_{i+1}^*$, rad | $M_{\text{Elast},i+1,k}$, N*m, $k=6$ | $M_{\text{Elast},i+1,k}$, H*m, $k=12$ |
|------------------|---------------|-------------------|---------------------------------|---------------------------------|
| 1                | 0.0025        | 0.008             | 158.74                          | 555.6                           |
| 2                | 0.0023        | 0.0073            | 317.48                          | 873.09                          |
| 3                | 0.0019        | 0.0062            | 357.17                          | 1269.96                         |
| 4                | 0.00145       | 0.0046            | 515.92                          | 1428.7                          |
| 5                | 0.0008        | 0.0028            | 396.86                          | 1508.07                         |
| 6                | 0.0003        | 0.0009            | 394.5                           | 1747.78                         |
| 7                | -0.000016     | -0.0005           |                                 |                                 |

The maximum values of the resulting rotational oscillations of additional stresses $\tau$, power losses $\Delta N_{\text{oscil}}$ and increase in the specific effective fuel consumption $\Delta g_{\text{oscil}}$ are given in Table 3.

Table 3. Additional stress of torsional oscillation $\tau$ and capacity losses $\Delta N_{\text{oscil}}$ as well as fuel efficiency degradation $\Delta g_{\text{oscil}}$

| $k$ | $\tau$, MPa | $\Delta N_{\text{oscil}}$, kW | $\Delta g_{\text{oscil}}$, |
|-----|-------------|-------------------------------|---------------------------|
| 6   | 7.04        | 0.141                         | 0.258                     |
| 12  | 23.86       | 1.588                         | 3.815                     |

When calculating, we assume that the expansion amplitude for these frequencies remains unchanged.

Let us evaluate the work of the proposed design. For this, we determine the phase shift of the flywheel oscillations relative to the housing $\theta_M$. From the condition of maximum damper efficiency, we take for the rotational speed $\omega_i = 1263$ rpm $\theta_M = 45^\circ$. Then the oscillatory amplitude of the flywheel $a_O = \cos 45 \cdot a_a = 0.7$.

The results of calculations of the first motor mass oscillation amplitude (flywheel housing) for case without damper $A_{i0}^*$ and with damper from constant $A_{1D(J_1)}^*$ and variable moments of inertia $A_{1D(J_2)}^*$ are presented in Table 4 and reflected in fig. 3.

Table 4. The results of calculations of the first motor mass oscillation amplitude (flywheel housing) for case without damper $A_{i0}^*$ and with damper from constant $A_{1D(J_1)}^*$ and variable moments of inertia $A_{1D(J_2)}^*$

| $A_{i0}^*$, rad | $A_{1D(J_1)}^*$, rad | $A_{1D(J_2)}^*$, rad |
|-----------------|----------------------|----------------------|
| 0.008           | 0.0019               | 0.0019               |
| 0.0025          | 0.0008               | 0.0006               |

where $A_{i0}^*$, $A_{1D(J_1)}^*$ - oscillation amplitudes of the first motor mass without damper at resonant frequencies and with damper at different moments of inertia, respectively.
As shown by the calculations, the proposed design allows to increase damper efficiency in a wide range of crankshaft rotation frequencies. In particular, the calculated oscillation amplitudes of the first motor mass at the frequency $n = 2527$ rpm were reduced by 25%, which, in turn, reduces the loss of effective power and reduces the specific effective fuel consumption of the ICE.

4. Conclusions
As a result of the development of the proposed calculation method the following was established:

1) The moment of resistance in the damper depends on several factors: sizes of flywheel, gaps between the housing and the flywheel, fluid viscosity, shear rate, i.e. oscillation amplitude and frequency.

2) For more accurate calculations, the change in the characteristics of the fluid is taken into account both from the temperature and from the shear rate.

3) When the damper is installed, the system frequencies and modes of vibrations change; that is why, the determination of its parameters is a multiparameter iterative optimization task.

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