Scaling laws for planetary sediment transport from grain scale-resolved numerical simulations

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Aeolian sediment transport shapes erodible surfaces and affects the dust cycles and climates of planetary bodies. For the approximately unidirectional near-surface winds often temporarily prevailing in planetary atmospheres, sediment transport approaches an equilibrium state when given enough fetch to adapt. Here, we use grain scale-resolved numerical sediment transport simulations to derive scaling laws for equilibrium transport. The simulations cover seven orders of magnitude in the particle-fluid-density ratio $s$, ranging from water to extremely rarefied air on Pluto. They suggest that the threshold fluid shear velocity needed to sustain aeolian transport exhibits a parabolic dependency on the grain size with a pronounced threshold minimum. We show that this behavior originates from the equations of motion describing quasicontinuous saltation driven by Stokes drag in a hydrodynamically smooth turbulent boundary layer. In particular, the threshold minimum, rescaled using units of particle density, gravity, and dynamic fluid viscosity, scales as $s^{1/3}$, contrary to the $s^{1/2}$-scaling measured in nonequilibrium wind tunnel experiments. Furthermore, the simulations suggest that the aeolian transport rate and grain impact energy flux, which drives dust emission into a planetary body’s atmosphere, obey scaling laws resembling the classical law by Ungar and Haff (\textit{Sedimentology} 34, 289-299, 1987), but with nonconstant scaling coefficients proportional to $s^{1/3}$ and $s^{1/4}$, respectively. The simulation-derived semiempirical master curves for the cessation threshold and the threshold-dependent transport rate and impact flux, supported by their agreement with terrestrial measurements in air and water, provide a simple means to make predictions of aeolian processes across a large range of planetary conditions.

Key words:

1. Introduction

Aeolian (wind-driven) transport of nonsuspended grains, including sand, ice, and snow, is a ubiquitous phenomenon that leads to a rich variety of multiscale bedforms on Earth and other planetary bodies (Bourke \textit{et al.} 2010; Kok \textit{et al.} 2012; Diniega \textit{et al.} 2017).

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As suggested by the presence of wind streaks and dunes, it may even occur in the very rarefied atmospheres of Neptune’s moon Triton (Sagan & Chyba 1990), Pluto (Telfer et al. 2018), and the comet 67P/Churyumov-Gerasimenko (Thomas et al. 2015; Jia et al. 2017). Aeolian transport also affects planetary bodies’ climates via the emission of dust into their atmospheres, predominantly caused by the bombardment of the regolith by moving sand grains (Kok et al. 2012)

Driven by fluid drag and gravity, most transported sand-sized and larger grains regularly interact with the bed surface as flow turbulence is too weak to suspend them. For denser fluids, like water and most liquids, this near-surface grain motion occurs in the form of rolling, sliding, and small hops (bedload), while for lighter fluids, like most gases, grains move in more energetic hops (saltation). At equilibrium, the deposition of transported grains on the bed is exactly balanced by the entrainment of bed grains into the transport layer. The rate at which equilibrium aeolian transport takes place and the threshold wind speed below which it ceases constitute the two arguably most important statistical transport properties in the context of bedform formation and evolution in natural environments (Kok 2010a; Durán Vinent et al. 2019). In particular, in natural environments, topography inhomogeneities, strong turbulent fluctuations, and a variety of wind-unrelated mechanisms to generate airborne grains, along with very long natural sediment fetches, can plausibly initiate transport and lead to equilibrium transport above the cessation threshold (see discussion in Section 3.3.3.4 of Pähtz et al. 2020). This may even be true in environments where the aeolian transport initiation threshold for an idealized flat sediment bed is much larger than the cessation threshold, like potentially on Mars (Kok 2010a), Pluto (Telfer et al. 2018), and Saturn’s moon Titan (Comola et al. 2022), as well as in Antarctica. In fact, although Antarctica’s surface is covered by very cohesive (Pomeroy & Gray 1990) old snow and ice (cohesion increases the initiation threshold probably much more than the cessation threshold, Comola et al. 2019, 2022; Pähtz et al. 2021), aeolian snow and ice transport occurs there even at relatively low wind speeds that are likely much below the initiation threshold (Leonard et al. 2011).

Since the highly random, collective motion of bed and transported grains eludes a rigorous analytical description, physical models of equilibrium aeolian transport have relied on drastic simplifications of reality, such as representing the grain motion by a family of deterministic saltation trajectories (Ungar & Haff 1987; Claudin & Andreotti 2006; Kok 2010b; Berzi et al. 2016; Lämmel & Kroy 2017). This issue is aggravated by the strong effect different simplification choices can have on model predictions. For example, the two most widely used physically-based expressions for the equilibrium aeolian transport rate by Ungar & Haff (1987) and Durán et al. (2011), which are based on two distinct, mutually exclusive physical assumptions about the bed boundary conditions that ensure equilibrium conditions, make fundamentally different predictions for extraterrestrial environments (see discussion in the introduction of Pähtz & Tholen 2021). Naturally, one would like to test such expressions with experimental data across environmental scales, especially across a large range of air surface pressures. However, existing cessation threshold and transport rate measurements carried out in air pressure-adjustable wind tunnels suffer from serious limitations. Existing wind tunnels that allow for an increase of the air pressure above the ambient level are usually very small and therefore produce only very thin turbulent boundary layers (∼2 cm, Greeley et al. 1984; Greeley & Marshall 1985), while existing low-pressure wind tunnel experiments were carried out using relatively short working sections (2–4 m, Greeley et al. 1982; White 1982; Andreotti et al. 2021). Both limitations make it seem implausible that equilibrium transport conditions were established in these studies (most of them did not address this point). One of the problems is that the length sediment transport needs to reach equilibrium conditions increases with decreasing air pressure (Claudin & Andreotti 2006; Pähtz et al.
2013). For example, Andreotti et al. (2021) reported that the entire 3.25 m long sand bed of their low-pressure wind tunnel, which was actually designed to make predictions for the air pressure conditions at the Martian surface (≈ 600 Pa, Sullivan et al. 2005), was undergoing erosion (i.e., equilibrium was nowhere reached) already for air pressures below of 30000 Pa.

Here, to overcome the lack of reliable data, we use an existing discrete element method (DEM)-based numerical model (Durán et al. 2012) of equilibrium aeolian and subaqueous transport of cohesionless nonsuspended sediments (section 2). It resolves both grain-grain and fluid-grain interactions and the resulting change in the average flow field, and successfully reproduces terrestrial measurements (Pähtz & Durán 2018a, 2020). We perform simulations for environmental conditions that exceed previously simulated conditions by almost four orders of magnitude in the particle-fluid-density ratio \( \frac{1}{s} \), ranging from transport driven by water (\( s \approx 65 \) ) to transport driven by air on Pluto (\( s \approx 10^7 \) ). We find simple scaling laws for the cessation threshold, equilibrium transport rate, and grain impact energy flux, valid across the entire simulated range of planetary conditions (section 3).

2. Numerical model
We use the numerical model of Durán et al. (2012), modified as described by Pähtz & Durán (2017), which couples a discrete element method for the grain motion under gravity, buoyancy, and fluid drag with a continuum Reynolds-averaged description of hydrodynamics. Spherical grains (\( 10^4 - 10^5 \) ) with mild polydispersity are confined in a quasi-two-dimensional domain of length \( \approx 10^3 d \) (where \( d \) is the median grain diameter), with periodic boundary conditions in the flow direction, and interact via normal repulsion (restitution coefficient \( e = 0.9 \) ) and tangential friction (contact friction coefficient \( \mu_c = 0.5 \) ). The bottom-most grain layer is glued on a bottom wall, while the top of the simulation domain is reflective but so high that it is never reached by transported grains. The Reynolds-averaged Navier-Stokes equations are combined with a semiempirical mixing length closure that ensures a smooth hydrodynamic transition from high to low particle concentration at the bed surface and quantitatively reproduces the mean turbulent flow velocity profile in the absence of transport. Simulations with this numerical model are insensitive to \( e \) and therefore insensitive to viscous damping. The simulations reproduce measurements of the rate and cessation threshold of terrestrial aeolian transport, and viscous and turbulent subaqueous transport (figures 1 and 3 of Pähtz & Durán (2018a) and figure 4 of Pähtz & Durán (2020)), height profiles of relevant equilibrium transport properties (figure 2 of Pähtz & Durán (2018a) and figure 6 of Durán et al. (2014a)) and aeolian ripple formation (Durán et al. 2014b).

2.1. Average of simulated quantities
We define two types of averages for a particle property \( A_p \). Based on the spatial homogeneity of the simulation, the mass-weighted average of \( A_p \) over all particles within an infinitesimal vertical layer \((z, z + dz)\) and all time steps (after reaching the steady state) is (Pähtz et al. 2015)

\[
\langle A \rangle(z) = \frac{\sum_{z_p \in (z, z + dz)} m_p A_p}{\sum_{z_p \in (z, z + dz)} m_p},
\]

where \( m_p \) and \( z_p \) is the particle mass and elevation, respectively. We also define the average of a vertical profile \( A(z) \) over the transport layer as (Pähtz & Durán 2018a)

\[
\bar{A} = \int_0^\infty \rho \langle A \rangle dz / \int_0^\infty \rho dz,
\]
where \( \rho \) is the local particle concentration. The bed surface elevation \( z = 0 \) is defined as the elevation at which \( p_g d \langle v_x \rangle / dz \) is maximal (Pähtz & Durán 2018b), where \( \langle v_x \rangle \) is the average grain velocity in the streamwise direction and \( p_g(z) = -\int_z^\infty \rho(a_z)dz' \) the normal-bed granular pressure, with \( a \) the acceleration of grains by noncontact forces.

### 2.2. Calculation of transport rate, cessation threshold, and impact flux

We calculate the sediment transport rate \( Q \) as (Pähtz & Durán 2018b)

\[
Q = \int_{-\infty}^\infty \rho \langle v_x \rangle dz. \tag{2.3}
\]

When \( Q \) vanishes, also the grain-borne shear stress at the bed surface \( \tau_g(0) \) vanishes, with

\[
\tau_g(z) = \int_z^\infty \rho(a_z)dz' \tag{2.4}
\]

the grain-borne shear stress profile. We therefore extrapolate the cessation threshold value \( \tau_t \) of the fluid shear stress \( \tau \) at which \( Q \) vanishes using the approximate relation (Pähtz & Durán 2018b)

\[
\tau_g(0) = \tau - \tau_t, \tag{2.4}
\]

where we treat \( \tau_t \) as a fit parameter. Furthermore, the grain impact energy flux \( q_{Ez} \) is defined as the negative vertical energy flux \( -q_{Ez} \) at the bed surface: \( q_{E1} \equiv -q_{Ez}(0) \). The vertical energy flux \( q_{Ez} \) consists of two additive contributions: a kinetic contribution \( \frac{1}{2} \rho \langle v_z v^2 \rangle \) from the motion of grains and a contact contribution associated with grain-grain contacts (Pähtz et al. 2015). Denoting the contact force applied by grain \( q \) on grain \( p \) as \( F_{pq} \) (defining \( F_{pp} \equiv 0 \)), \( q_{Ez} \) is calculated as (Pähtz et al. 2015)

\[
q_{Ez}(z) = \frac{1}{2} \rho \langle v_z v^2 \rangle + \frac{1}{2\Delta} \sum_{z_p \in (z,z+dz)} \sum_q F_{pq} \cdot v_p(z_p - z_q) l_{dz}^{z_pq}, \tag{2.5}
\]

where \( \Delta \) is the simulated bed area and \( l_{dz}^{z_pq} \) the length fraction of the line connecting \( z_p \) and \( z_q \) that is contained in the interval \((z,z+dz)\).

### 2.3. Dimensionless control parameters and rescaling of physical quantities

The average properties of equilibrium sediment transport are mainly determined by a few grain and environmental parameters: the grain and fluid density (\( \rho_p \) and \( \rho_f \), respectively), median grain diameter (\( d \)), kinematic fluid viscosity (\( \nu \)), fluid shear velocity (\( u_* = \sqrt{\nu / \rho_f} \)), and gravitational constant (\( g \)), or its buoyancy-reduced value \( \tilde{g} \equiv (1 - \rho_f / \rho_p) g \) (for air, \( \tilde{g} \approx g \)). Physical quantities with a superscript ‘+’ are rescaled using units of \( \rho_p, \tilde{g}, \) and \( \nu \). For example,

\[
\begin{align*}
    d^* & = \tilde{g} d / (\tilde{g} \nu)^{2/3}, \tag{2.6} \\
    u_*^* & = u_* / (\tilde{g} \nu)^{1/3}, \tag{2.7} \\
    Q^* & = Q / (\rho_p \nu), \tag{2.8} \\
    q_{E1}^* & = q_{E1} / (\rho_p \tilde{g} \nu). \tag{2.9}
\end{align*}
\]

As we will see, this rescaling is well suited to describe the relevant physical processes underlying the cessation threshold scaling. A given environmental condition is fully determined
Sediment transport is always nonsuspended. Near the cessation threshold (subscript numerical model neglects turbulent fluctuations around the mean turbulent flow, simulated Sincethemixinglength-basedReynolds-averageddescriptionofhydrodynamicsusedinthe
we distinguish through the criterion (Pähtz & Durán 2018
nonsuspended transport occurs as either bedload or saltation (see introduction), which
conditions near its cessation threshold value
Numerical simulations are carried out for various combinations of the particle-fluid-density
Table 1: Simulated particle-fluid-density ratios s and Galileo numbers Ga. *The
condition $s = 2.5 \times 10^5$, $Ga = 1$ corresponds to a typical transport environment on Mars
($d \approx 100 \ \mu m$) and $s = 10^7$, $Ga = 0.2$ to a hypothetical transport environment on Pluto
($d \approx 200 \ \mu m$). Simulations with significantly larger respective values of $Ga$ are unstable
for these large-s conditions. We have been unable to fix this issue and do not know
whether it has numerical or physical causes.
by the values of three dimensionless numbers (Pähtz & Durán 2020):
$$s \equiv \frac{\rho_p}{\rho_f} = \frac{1}{\rho_f^+}, \quad (2.10)$$
$$Ga \equiv \sqrt{s \tilde{g} d^3/\nu} = \sqrt{sd^{+3/2}}, \quad (2.11)$$
$$\Theta \equiv \frac{\bar{u}_z^2}{(s \tilde{g} d)} = \frac{u_z^2}{(sd^+)}. \quad (2.12)$$
Numerical simulations are carried out for various combinations of the particle-fluid-density
ratio $s$ and Galileo number $Ga$ (table 1), and for Shields numbers $\Theta$ ranging from weak
conditions near its cessation threshold value $\Theta_t$ to intense conditions far above $\Theta_t$.

2.4. Sediment transport regimes for near-threshold conditions
Since the mixing length-based Reynolds-averaged description of hydrodynamics used in the
numerical model neglects turbulent fluctuations around the mean turbulent flow, simulated sediment transport is always nonsuspended. Near the cessation threshold (subscript $t$),
nonsuspended transport occurs as either bedload or saltation (see introduction), which
we distinguish through the criterion (Pähtz & Durán 2018a)
$$\text{Transport regime} = \begin{cases} 
\text{bedload} & \text{if } \frac{\bar{v}_z^2}{\tilde{g}} < d \\
\text{saltation} & \text{if } \frac{\bar{v}_z^2}{\tilde{g}} \geq d .
\end{cases} \quad (2.13)$$
The quantity $\bar{v}_z^2/\tilde{g}$ describes the contribution of hopping grains to the characteristic transport
height of all transported grains $\bar{z}$, including those that role and slide. In particular, for saltation
near the cessation threshold, $\bar{v}_z^2/\tilde{g} \approx \bar{z}_t$, whereas $\bar{v}_z^2/\tilde{g}$ is significantly smaller than $\bar{z}_t$ for
bedload transport (figure 1). Henceforth, $\bar{v}_z^2/\tilde{g}$ and $\bar{z}$ are termed hop height and transport
layer thickness, respectively, for simplicity.

3. Results

3.1. Cessation threshold

3.1.1. Scaling of the cessation threshold for saltation

Of the physical parameters affecting the shear velocity at the cessation threshold $u_{st}$, the air pressure $P$ varies most strongly with the planetary environment. Furthermore, for a given planetary environment, the grain size $d$ is the most strongly varying relevant physical parameter. To isolate the effect of $P$ on $u_{st}$, we normalize $u_{st}$ in terms of relevant parameters that do neither depend on $P$ nor on $d$, $U_{st} \equiv u_{st} / (\mu g / \rho_p)^{1/3}$ (using that the dynamic viscosity $\mu = \rho_f \nu$ does not depend on $P$), and compare it with the density ratio $s$, which incorporates the effect of $P$ isolated from that of $d$.

For saltation, the numerical simulations of planetary transport reveal a lower bound for $U_{st}$ scaling as $s^{1/3}$ (figure 2, circles). This is distinct from the classical scaling of the saltation initiation threshold with $s^{1/2}$ (Greeley et al. 1976, 1980; Iversen & White 1982; Greeley et al. 1984; Burr et al. 2015, 2020; Swann et al. 2020) (figure 2, gray crosses), which follows from a balance between flow-induced and resisting forces or torques acting in bed surface grains (Pähtz et al. 2020). Roughly the same $s^{1/2}$-scaling was also found for the nonequilibrium cessation threshold measurements by Andreotti et al. (2021) carried out in a low-pressure wind tunnel (figure 2, black crosses). This hints at a fundamental qualitative difference between these nonequilibrium cessation thresholds and the equilibrium ones obtained from the numerical transport simulations. In particular, note that Andreotti et al. (2021) did not find a significant quantitative difference between their measured initiation and cessation thresholds, which is in conflict with a number of previous wind tunnel studies reporting hysteresis between the two for terrestrial conditions (Bagnold 1937; Chepil 1945; Ho 2012; Carneiro et al. 2015), with substantial differences especially for coarse sand grains (Ho 2012, table 5.1).
Figure 2: Cessation threshold shear velocity normalized using air pressure- and grain size-independent natural units $U_{st} \equiv u_{st}/(\mu g/\rho_p)^{1/3}$ versus density ratio $s$. Symbols that appear in the legend correspond to initiation (Greeley et al. 1976, 1980; Iversen & White 1982; Greeley et al. 1984; Burr et al. 2015, 2020; Swann et al. 2020), nonequilibrium cessation (Andreotti et al. 2021), and equilibrium cessation threshold measurements for aeolian transport of quartz (Bagnold 1937; Martin & Kok 2018; Zhu et al. 2019), clay loam (Chepil 1945), and snow at sea level (Sugiura et al. 1998) and high altitude (Clifton et al. 2006, HA). Symbols that do not appear in the legend correspond to numerical simulations for various combinations of $s$ and the Galileo number $Ga$ (see table 1 and figure 1), with open and filled symbols indicating bedload and saltation conditions, respectively (see figure 1 for definition). The solid line corresponds to $U_{st} \propto s^{1/3}$ and represents the lower bound for cessation and initiation thresholds of saltation.

To physically explain the lower-bound scaling of saltation, $U_{st} \propto s^{1/3}$, one needs to describe the equilibrium saltation state associated with the cessation threshold. This is a difficult task because bed randomness causes a continuous distribution of saltation trajectories even in the absence of turbulent fluctuations. For this reason, most existing cessation threshold models represent the entire grain motion by identical periodic saltation trajectories (Claudin & Andreotti 2006; Berzi et al. 2016, 2017; Pähtz et al. 2021) or by an average motion behavior that is mathematically equivalent to an identical periodic trajectory representation (Kok 2010b). However, as we will see, important physical insights into the scaling of $U_{st}$ can be gained from comparably mild simplifications of the flow and fluid-grain interactions that allow us to treat the grain motion in a much more general manner:

(i) We consider only buoyancy and Stokes drag as fluid-grain interactions, neglecting form drag contributions.

(ii) We consider a smooth inner turbulent boundary layer mean flow velocity profile $u_x(z)$, neglecting hydrodynamically rough contributions (and turbulent fluctuations, which are also neglected in the numerical simulations).

(iii) Since vanishingly few grains are in motion sufficiently close to the cessation threshold, we neglect the feedback of the grain motion on the flow.

(iv) Since saltation trajectories are typically much larger than the grain size, we consider an idealized flat bed and assume that the zero-level of the flow velocity coincides with the grain elevation at grain-bed impact ($z = 0$), neglecting the effect of the flow very near the bed surface to the overall grain motion.
Under these simplifications, the equations of motion for a given grain are (Pähtz et al. 2021)

\[ v_x^+ = -1 - \frac{v_x^+}{v_s^+}, \]  
\[ v_s^+ = \frac{u_x^+ - v_s^+}{v_s^+}, \]  
\[ u_x^+ = u_x f_u(u_x^+ z^+), \]

where \( v^+ \) and \( v_s^+ = s d^2/18 \) are the rescaled grain velocity and Stokes settling velocity, respectively, and \( f_u(X) \) denotes a function describing \( u_x / u_s \) for an undisturbed smooth inner turbulent boundary layer. It obeys \( f_u(X) = X \) within the viscous sublayer of the turbulent boundary layer \( (X \leq 5) \) and \( f_u(X) \sim k^{-1} \ln(9X) \), with \( k = 0.4 \) the von Kármán constant, within its log-layer \( (X \geq 30) \). Extrapolated into the transitional buffer layer in between, both profiles would intersect at about \( X = 11 \), which is why \( \delta_v = 11v/u_s \) is termed viscous-sublayer thickness.

Parametrized by \( v_s^+ \) and \( u_x^+ \), (3.1)-(3.3) map the distribution \( f_1(v_x^+) \) of rescaled lift-off velocities \( v_x^+ \) to the distribution \( f_1(v_x^+) \) of rescaled impact velocities \( v_x^+ \) of grains. The boundary conditions in turn, describing the distribution of outcomes of grain-bed collisions, map \( f_1(v_x^+) \) back to \( f_1(v_x^+) \). Previous studies provided different kinds of evidence that the boundary conditions relevant for the cessation threshold are scale-free rebound laws (Pähtz & Durán 2018a; Pähtz et al. 2020, 2021). Combined with (3.1)-(3.3), they describe a quasiconstant motion of the ensemble of saltating grains parametrized by \( v_s^+ \) and \( u_x^+ \). For a given \( v_s^+ \), the cessation threshold \( u_x^+ \) then corresponds to the smallest value of \( u_x^+ \) for which a nontrivial solution of the combined system exists (Pähtz et al. 2021), implying that there is a function \( f \) mapping \( v_s^+ \) to \( u_x^+ \):

\[ u_x^+ = f(v_s^+). \]

The numerical simulations of saltation confirm the functional relationship in (3.4), supporting its underlying simplifications (figure 3(a)). The fact that a dimensional analysis based on Stokes drag interactions is sufficient to collapse the numerical saltation data, even though the numerical transport model accounts also for form drag effects, suggests that relatively fast saltating grains dominate the near-threshold grain dynamics, since comparably faster grains exhibit comparably lower fluid-particle-velocity differences and thus comparably less form drag relative to Stokes drag. The simulations also reveal that \( u_x^+ \) exhibits global minimum \( u_x^+ \min = 1.6 \) for saltation at \( v_s^+ \min = 14.2 \) (figure 3(a)). Since \( U_x^+ = (s - 1)^{1/3} u_x^+ \), this minimum corresponds exactly to the lower-bound scaling \( U_x^+ \propto s^{1/3} \) in figure 2.

3.1.2. Hydrodynamic origin of the minimum of the cessation threshold for saltation

The minimum \( u_x^+ \min \) for saltation occurs when the hop height \( \bar{v}^+_{zt}/\bar{g} \) is about equal to the viscous-sublayer thickness \( \delta_v = 11v/u_x \) near the cessation threshold (figure 3(b)). This can be explained using the empirical, yet physically reasonable, transport simulation-supported proportionality between the average fluid velocity \( \bar{u}_x \) and \( (\bar{v}^+_{zt})^{1/2} \) near the cessation threshold (figure 3(c)). In fact, averaging (3.3) over all grain trajectories and the transport layer, using the approximation \( \bar{f}_u(u_x z^+) = f_u(u_x z^+) \), and inserting this proportionality approximately yields for saltation \( (\bar{z}^+ = v_{zt}^{1/2} \), see figure 1):

\[ u_x^+ \propto \left[ \frac{u_x^+ \bar{v}^+_{zt}}{f_u \left( u_x^+ \bar{v}^+_{zt} \right)} \right]^{1/3}. \]
Figure 3: (a) Rescaled threshold shear velocity $u_{st}^+$ versus rescaled settling velocity $v_s^-$. (b) $u_{st}^+$ versus ratio between hop height $v_{zt}^2/\bar{g}$ and viscous-sublayer thickness $\delta_{vt} = 11v/u_{st}$ near the cessation threshold. (c) Rescaled transport layer-averaged fluid velocity $u_{xt}^+$ versus $(v_{zt}^2)^{1/2}$ near the cessation threshold. (d) $u_{st}^+v_{zt}^2$ versus $v_s^-$. Symbols correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 1), with open and filled symbols indicating bedload and saltation conditions, respectively (see figure 1 for definition). The solid lines in (b), (d), (a), and (c) correspond to (3.6), (3.7), (3.8), and $u_{xt}^+ = 6(v_{zt}^2)^{1/2}$, respectively.

Within the viscous sublayer ($u_{st}^+v_{zt}^2 \leq 5$), this relation simplifies to $u_{st}^+ \propto (u_{st}^+v_{zt}^2)^{-1/3}$ and within the log-layer approximately to $u_{st}^+ \propto (u_{st}^+v_{zt}^2)^{1/3}$, neglecting the logarithmic term. The crossover between the two power laws occurs about at $u_{st}^+v_{zt}^2 = 11$, that is, when the
hop height exceeds the viscous-sublayer thickness \( \left( \frac{\nu^2}{\kappa t} / \hat{g} = \delta_{vt} \right) \). Hence, the parabolic law

\[
u^+_{sl} = u^{+\text{min}}_{sl} \max \left[ \left( \frac{\nu_s^+}{v^{+\text{min}}_s} \right)^{-1/4}, \left( \frac{\nu_s^+}{v^{+\text{min}}_s} \right)^{1/4} \right]
\]

fits the saltation data reasonably well (solid line in figure 3(b)).

Following from the dimensional analysis we have used to deduce (3.4), the grain kinematics near the cessation threshold, and thus \( \sqrt{\nu^2_{sl} t} \), should be controlled by \( u^+_{sl} \) or \( v^+ \). Indeed, the numerical simulations of saltation suggest the empirical relation (figure 3(c))

\[
u^+_{sl} v^{+2}_{sl} = 1.5 v^{+3/4}_s,
\]

which leads to the parabolic law (solid line in figure 3(a))

\[
u^+_{sl} = u^{+\text{min}}_{sl} \max \left[ \left( \frac{v^+_s}{v^{+\text{min}}_s} \right)^{-1/4}, \left( \frac{v^+_s}{v^{+\text{min}}_s} \right)^{1/4} \right],
\]

with \( v^{+\text{min}}_s = (11/1.5)^{4/3} \approx 14.2 \).

3.1.3. Simple expression for the cessation threshold across saltation and turbulent bedload

It is possible to semiempirically generalize (3.8) to also include turbulent bedload conditions, defined by \( s \lesssim 10 \) and \( v^+_s \geq v^{+\text{min}}_s \) (equivalent to \( G a s^{1/4} \gtrsim 64 \), which ensures that transported grains significantly penetrate the log-layer, Pähtz & Durán 2020). Turbulent bedload does not only include saltating grains but also rolling grains. The threshold shear velocity required to sustain a pure, very slow rolling motion along the bed surface scales as \( u_{sl} \propto \sqrt{\hat{g} d} \) (Pähtz et al. 2021), which corresponds to \( u^+_{sl} \propto s^{1/4} \) at the cessation threshold minimum \( v^+_s = v^{+\text{min}}_s \).

We find that the empirical relation \( u^+_{sl} = \sqrt{f_s u^{+\text{min}}_{sl}} \), with \( f_s \equiv (1 + \sqrt{10/s})^{-1} \), captures the transition from \( u^+_{sl} \propto s^{1/4} \) for \( s \ll 10 \) to \( u^+_{sl} = u^{+\text{min}}_{sl} \) for \( s \gg 10 \) at \( v^+_s = v^{+\text{min}}_s \). The resulting generalized cessation threshold equation, written in terms of the normalized median grain diameter \( D^* \equiv 18v^{+\text{min}}_s = \sqrt{\hat{g} d}^* \) and \( D^{+\text{min}}_s = \sqrt{18v^{+\text{min}}_s} \approx 16 \), is given by

\[
u^+_{sl} = \sqrt{f_s u^{+\text{min}}_{sl}} \max \left[ \left( \frac{D^*_s}{D^{+\text{min}}_s} \right)^{-1/2}, \left( \frac{D^*_s}{D^{+\text{min}}_s} \right)^{1/2} \right].
\]

It is consistent with numerical transport simulations (figure 4(a)) and experiments (figure 4(b)) across aeolian and fluvial transport conditions with \( G a \sqrt{s} \gtrsim 81 \) (which excludes viscous bedload, Pähtz et al. 2021).

Note that, for the experimental conditions in figure 4(b), we changed the location and magnitude of the cessation threshold minimum from \( (D^{+\text{min}}_s, u^{+\text{min}}_{sl}) = (16, 1.6) \) to \( (D^{+\text{min}}_s, u^{+\text{min}}_{sl}) = (18, 2.3) \). The relative change of 2.3/1.6 \approx 1.4 is well within the typical systematic uncertainty of cessation threshold measurements. For example, Creyssels et al. (2009) reported \( \Theta_t = 0.009 \) for their terrestrial wind tunnel experiments (\( d = 242 \mu m \)), obtained from extrapolating transport rate measurements to vanishing transport using the transport rate model by Ungar & Haff (1987), while Pähtz & Durán (2020) reported \( \Theta_t = 0.0035 \) for the very same data using a different transport rate model for the extrapolation, resulting in a relative change of \( \sqrt{0.009/0.0035} \approx 1.6 \).

The grain size scaling of the cessation threshold in (3.9) is mathematically equivalent to the well-known cohesive \( (u_{sl} \sim d^{-1/2}, \text{left branch}) \) and cohesionless \( (u_{sl} \sim d^{1/2}, \text{right branch}) \) limits of the saltation initiation threshold (Shao & Lu 2000). However, the scaling of the left branch and the most mobile normalized grain size \( D^{+\text{min}}_s \approx 18 \) corresponding to
Figure 4: Bedload-corrected rescaled cessation threshold shear velocity $u_{st}^+/\sqrt{f_s}$ versus normalized median grain diameter $D_*$, $D_* \equiv \sqrt{s}d^+$. Symbols in (a) correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 1) with $Ga\sqrt{s} > 81$ (excluding viscous bedload), where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). Symbols in (b) correspond to experimental cessation threshold data (see legend) for terrestrial aeolian saltation of quartz (Bagnold 1937; Martin & Kok 2018; Zhu et al. 2019), clay loam (Chepil 1945), and snow at sea level (Sugiura et al. 1998) and high altitude (HA) (Clifton et al. 2006), and a compilation of experimental threshold data for subaqueous bedload (Buffington & Montgomery 1997). Only data with $Ga\sqrt{s} > 81$ are shown. The lines correspond to (3.9), with $(D_s^{min}, u_{st}^{min}) = (16, 1.6)$ in (a) and $(D_s^{min}, u_{st}^{max}) = (18, 2.3)$ in (b). Note that the left branch of this analytical prediction is a signature of viscous sublayer-driven saltation and entirely unrelated to increasing cohesion with decreasing $D_*$. The dashed line in (b) indicates a below-critical Rouse number $(Ro_t < 1)$ for aeolian conditions $(s \gg 10)$, associated with turbulent suspension in natural environments, while the solid line in (b) indicates nonsuspended transport $(Ro_t \geq 1)$.

the cessation threshold minimum $u_{st}^{max,min} \approx 2.3$ follow purely from hydrodynamics rather than the onset of cohesion at small grain sizes.

The relation between $u_{st}^+$ and $v_s^+$ or $D_*$ also has implications for the scaling of the transition between saltation and turbulent suspensions near the cessation threshold, as defined by the critical condition $Ro_t = 1$ (Ho 2012), with $Ro_t \equiv \tilde{v}_s/u_{st}$ the Rouse number at the cessation threshold. The grain settling velocity $\tilde{v}_s$, which includes both Stokes drag and form drag, can be approximated as the Stokes settling velocity $v_s$, since cessation threshold conditions near the saltation-suspension transition are dominated by Stokes drag. Hence, near the saltation-suspension transition, $Ro_t$ is mainly a function of $v_s^+$ and thus of $D_*$. Therefore, like $D^{min}_s$ the normalized grain size at the saltation-suspension transition $D_s^{sus} \approx 7.9$ is also constant (figure 4(b)).

3.2. Equilibrium sediment transport rate

3.2.1. Scaling of the equilibrium sediment transport rate for saltation

For saltation, the transport rate data of the numerical transport simulations reasonably collapse on the master curve (filled symbols in figure 5)

$$ Q^+/d^{3/2} = 1.7s^{1/3}(\Theta - \Theta_r) + 13s^{1/3}(\Theta - \Theta_r)^2. $$

(3.10)
Figure 5: Normalized sediment transport rate $s^{-1/3}Q^+/d^{+3/2}$ versus Shields number in excess of the cessation threshold $\Theta - \Theta_t$. Symbols correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1) with $Ga\sqrt{s} > 81$ (excluding viscous bedload), and Shields number $\Theta$, where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). The solid line corresponds to (3.10).

In contrast, the saltation simulation data do not or not as well obey functional relationships associated with the two most widely used physically-based expressions for the equilibrium aeolian transport rate by Ungar & Haff (1987) and others (e.g., Jenkins & Valance 2014; Berzi et al. 2016), $Q^+/d^{+3/2} = f_1(\Theta - \Theta_t)$ (figure 6(a)), and by Durán et al. (2011) and others (Kok et al. 2012; Pähzt & Durán 2020), $Q^+/d^+u_{st}^+ = f_2(\Theta - \Theta_t)$ (figure 6(b)). Note that, for most conditions relevant in nature, the wind is sufficiently weak so that the term quadratic in the excess Shields number $\Theta - \Theta_t$ on the right-hand side of (3.10) can be neglected. For such conditions, (3.10) resembles the scaling law by Ungar & Haff (1987) but with a nonconstant scaling coefficient proportional to $s^{1/3}$. Hence, the simulation data suggest that, when applying the scaling law by Ungar & Haff (1987), fitted to transport rate data for Earth’s atmospheric conditions ($s \approx 2000$), to the Martian atmosphere ($s \approx 200000$) or atmospheres with even lower surface pressure, as sometimes done in the literature (Andreotti et al. 2021), one would underestimate the sediment transport rate by more than a factor of $100^{1/3} \approx 5$.

3.2.2. Simple expression for the sediment transport rate across saltation and turbulent bedload

Equation (3.10) does not capture intense turbulent bedload conditions ($s \leq 10$ and $v^+_s > v_s^{+\min}$ open symbols in figure 5), since for bedload, $Q^+/d^{+3/2}$ asymptotically scales as $s^{1/3}(\Theta - \Theta_t)^2$ rather than $s^{1/3}(\Theta - \Theta_t)^2$. In particular, the alternative relationship

$$Q^+/d^{+3/2} = 1.7s^{1/3}(\Theta - \Theta_t) + 3.7s^{1/2}(\Theta - \Theta_t)^2 \quad (3.11)$$

reasonably collapses simulation data with $s \leq 1000$ (figure 7(a)). Furthermore, when using again the previously introduced correction function $f_s \equiv (1 + \sqrt{10/s})^{-1}$, one can merge (3.10) with (3.11) through (figure 7(b))

$$Q^+/d^{+3/2} = 1.7s^{1/3}(\Theta - \Theta_t) + 13f_ss^{1/3}(\Theta - \Theta_t)^2. \quad (3.12)$$
Figure 6: Evaluation of the physically-based functional relationships for the sediment transport rate by Ungar & Haff (1987) and Durán et al. (2011). Normalized sediment transport rate (a) $Q^+/(d^+^{3/2})$ and (b) $Q^+/(d^+u_{t}^*)$ versus Shields number in excess of the cessation threshold $\Theta - \Theta_t$. Symbols correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga\sqrt{s} > 81$ (excluding viscous bedload), and Shields number $\Theta$, where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition).

When combined with $\Theta_t = u_{t}^2/(sd^*)$ and (3.9) to calculate $\Theta_t$, (3.12) is consistent with measurements of the equilibrium transport rate for terrestrial conditions in air and water (figure 7(c)). For the comparison with bedload data, we corrected the raw laboratory data for sidewall drag using the method described in section 2.3 of Guo (2015) and for steep bed slopes using $u_{t}^*|_{\text{corrected}} = u_{t}^*/f_\alpha$, with $f_\alpha \equiv 1 - \tan \alpha/0.63$ (Pähtz et al. 2021).

3.2.3. Physical origin of saltation transport rate scaling

Analytical, physical equilibrium models of the transport rate $Q$ for aeolian saltation typically separate it into the mass of transported sediment per unit area of the bed $M$ and its average streamwise velocity $V$ through $Q = MV$. In most models, it is reasoned that the scaling of $V$ is in one way or another linked to grain-bed collisions, and since the average outcome of grain-bed collisions should be roughly independent of the wind speed at equilibrium, $V$ is taken as equal to its near-threshold value $V_t$ (Ungar & Haff 1987; Durán et al. 2011; Kok et al. 2012; Berzi et al. 2016). However, it has been shown that, for sufficiently intense saltation, midair collisions significantly disturb grain trajectories (Carneiro et al. 2013; Ralaiarisoa et al. 2020), leading to an additional additive term increasing as $M^+/d^+$ (Pähtz & Durán 2020):

$$Q^+ = M^+V^+_t (1 + c_M M^+/d^+),$$

(3.13)

where $c_M$ is a constant parameter. Existing models agree on the scaling $M^+/d^+ \propto \Theta - \Theta_t$ (Ungar & Haff 1987; Durán et al. 2011; Berzi et al. 2016; Pähtz & Durán 2018b), in which case (3.13) with $V^+_t \propto s^{1/3} \sqrt{d^*}$ would be equivalent to (3.10). However, this scaling for $V^+_t$ substantially differs from the previous propositions $V^+_t \propto \sqrt{d^*}$ (Ungar & Haff 1987; Berzi et al. 2016) and $V^+_t \propto u_{t}^*$ (Durán et al. 2011; Kok et al. 2012; Pähtz & Durán 2020), as indirectly already shown in figure 6. The former proposition is a rescaled writing of
Figure 7: (a) Normalized sediment transport rate $s^{-1/6} Q^+/d^{3/2}$ versus rescaled excess Shields number $s^{1/6}(\Theta - \Theta_t)$. (b) and (c) $f_s s^{-1/3} Q^+/d^{3/2}$ versus $f_s (\Theta - \Theta_t)$. Symbols in (a) and (b) correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga \sqrt{s} > 81$ (excluding viscous bedload), and Shields number $\Theta$, where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). Symbols in (c) correspond to measurements for different grain sizes (indicated in the legend) for terrestrial aeolian saltation of minerals (Creyssels et al. 2009; Ho et al. 2011; Ho 2012; Martin & Kok 2017; Ralaiarisoa et al. 2020) and snow (Sugiura et al. 1998), and subaqueous bedload (Meyer-Peter & Müller 1948; Smart & Jaeggi 1983; Capart & Fraccarollo 2011). Note that Ralaiarisoa et al. (2020) reported that transport may not have been completely in equilibrium in their experiments. Simulated values of the cessation threshold Shields number $\Theta_t$ have been used in (a) and (b). Values of $\Theta_t$ calculated using $\Theta_t = u_{t}^2/(s d^2)$ and (3.9) with $(D_{min}^2, u_{min}^2) = (18, 2.3)$ have been used in (c). The solid line in (a) corresponds to (3.11). The solid lines in (b) and (c) correspond to (3.12).
the assumption that \( V_t \) scales with the velocity needed to escape the trapping potential of bed surface pockets, \( V_t \propto \sqrt{g d} \), whereas the latter proposition is a rescaled writing of the assumption that \( V_t \) scales with characteristic flow velocity in the transport layer, \( V_t \propto u_{st} \).

It is not trivial to evaluate the scalings of \( M^+ \) and \( V_t^+ \) with the simulation data, since extracting \( M \) and \( V \) from DEM-based numerical transport simulations is ambiguous (Durán et al. 2012; Pähtz & Durán 2018b). One possible way is to define \( M \) as the mass \( M_0 \) of grains moving above the bed surface (\( z = 0 \)) per unit bed area and \( V \) as their average streamwise velocity (Pähtz & Durán 2018b):

\[
M \equiv \int_0^\infty \rho \, dz = M_0, \quad (3.14)
\]

\[
V \equiv \frac{\int_0^\infty \rho \langle v_x \rangle \, dz}{\int_0^\infty \rho \, dz} = \bar{v}_x, \quad (3.15)
\]

This definition uses that most (but not all) sediment transport occurs at elevations \( z > 0 \), especially for saltation, and therefore \( M_0 \bar{v}_x = \int_0^\infty \rho \langle v_x \rangle \, dz \approx \int_0^\infty \rho \langle v_x \rangle \, dz = Q \) (Pähtz & Durán 2018b). Alternatively, one can define \( V \) as the mass flux-weighted average \( \bar{v}_x^q \) of the streamwise velocity of all grains and \( M_q \), the associated value of \( M \), as \( M_q \equiv \int Q/\bar{v}_x^q \) (Durán et al. 2012):

\[
M \equiv \frac{\left( \int_{-\infty}^\infty \rho \langle v_x \rangle \, dz \right)^2}{\int_{-\infty}^\infty \rho \langle v_x^2 \rangle \, dz} = M_q, \quad (3.16)
\]

\[
V \equiv \frac{\int_{-\infty}^\infty \rho \langle v_x^2 \rangle \, dz}{\int_{-\infty}^\infty \rho \langle v_x \rangle \, dz} = \bar{v}_x^q, \quad (3.17)
\]

where \( \bar{v}_x^q \equiv \frac{1}{Q} \int_{-\infty}^\infty \rho \langle v_x \rangle \, dz \).

For the above two definitions of \( M \) and \( V \), the data from the numerical transport simulations are roughly described by scaling laws in which a comparably small part of the \( s^{1/3} \)-scaling factor in (3.10) goes into \( M^+/d^+ \) and a comparably large part into \( V_t^+/\sqrt{d^+} \) (figure 8). However, the exact partitioning of \( s^{1/3} \) depends on the chosen definition (figures 8(a) and 8(b) versus figures 8(c) and 8(d)):

\[
M_0^+ \propto s^{1/12} d^+ (\Theta - \Theta_t), \quad \bar{v}_x^q \propto s^{1/4} \sqrt{d^+}, \quad (3.18)
\]

\[
M_q^+ \propto d^+ (\Theta - \Theta_t), \quad \bar{v}_x^{*q} \propto s^{1/3} \sqrt{d^+}. \quad (3.19)
\]

The above scaling laws for \( M^+ \) (i.e., \( M_0^+ \) and \( M_q^+ \)) are consistent with the prediction \( M^+ \propto d^+ (\Theta - \Theta_t) \) from physical models, though \( M_0^+ \) exhibits a slight additional trend with \( s \). However, the above scaling laws for \( V_t^+ \) (i.e., \( \bar{v}_x^q \) and \( \bar{v}_x^{*q} \)) are highly unusual and we currently do not have a reasonable explanation for them. On first glance, it seems that such scaling laws may represent some kind of mix between the previously proposed rescaled bed escape velocity scale \( \sqrt{d^+} \) and rescaled transport layer velocity scale \( u_{st}^+ \), the latter of which can be replaced by the rescaled Stokes settling velocity \( \bar{v}_s^+ \) because of (3.4) or (3.8). For example, on first glance, it seems that \( \bar{v}_x^+ \propto s^{1/4} \sqrt{d^+} \) is equivalent to \( \bar{v}_x^+ \propto v_s^{+1/4} \). However, we have excluded this equivalency through additional simulations in which we artificially varied the law describing the drag force \( F_d \) applied by the fluid on the grains. The standard simulations, the data of which we have shown so far, use \( F_d = \frac{\pi}{8} \rho_f d^2 C_d |v_r| v_r \), with \( v_r \) the
Figure 8: (a) and (c) Normalized transport loads $s^{-1/12}M_0^+/d^+$ and $M_+^+/d^+$, using the definitions (3.14) and (3.16), respectively, of $M$; and (b) and (d) normalized average streamwise grain velocities $s^{-1/4}v_x^+/\sqrt{d^+}$ and $s^{-1/3}v_x^q/\sqrt{d^+}$, using the definition (3.15) and (3.17), respectively, of $V$ versus Shields number in excess of the cessation threshold $\Theta - \Theta_t$. Symbols correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga \sqrt{s} > 81$ (excluding viscous bedload), and Shields number $\Theta$, where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). The solid lines in (a) and (b) correspond to the left equations in (3.18) and (3.19), respectively.

fluid-grain-velocity difference and

$$C_d = \left( \frac{Re_c}{|v_r|d/\nu} + \sqrt{C_d^\infty} \right)^2$$  \hspace{1cm} (3.20)

the drag coefficient, where $Re_c = 24$ and $C_d^\infty = 0.5$. The additional simulations use $C_d^\infty = 0$ and $Re_c = [6, 24, 96]$ for a few saltation conditions, corresponding to a rescaled Stokes settling velocity $v_s^+ = 4sd^{+2}/(3Re_c)$. However, these artificial changes do not have a significant effect on the numerical validity of (3.9) and (3.12), which are the here reported scaling laws for $u_+^{*t}$ and $Q^+$ (yellow symbols in figure 9). In fact, while an artificial change of the drag law affects $u_+^{*t}$ via its change of $v_s^+$, as predicted by (3.9), it has no significant affect on $Q^+$, as predicted by (3.12), which means that $v_s^+$ is indeed not much affected by $v_s^+$. Also, note that the fact that a change of $C_d^\infty$ does not significantly affect $u_+^{*t}$ supports the assumption that Stokes drag dominates form drag near the cessation threshold, which we have employed to deduce (3.4).

The fact that $v_s^+ = 4sd^{+2}/(3Re_c)$ is not much affected by $v_s^+$ probably means that there is a crucial difference between the streamwise grain motion near the cessation threshold and at the cessation threshold. In fact, at the cessation threshold, we would expect that the dimensional analysis
we have used to deduce (3.4) holds, which would imply that $\overline{v_{x,t}^+}$ depends only on $v_s^+$. However, we previously provided evidence that steady transport states, though they theoretically exist, become unstable against small perturbations (i.e., they cannot be sustained indefinitely) when the Shields number $\Theta$ is too close to $\Theta_t$ (Pähtz & Durán 2018a; Pähtz et al. 2020, 2021). This is because a second cessation threshold $\Theta_{\text{ImE}}$ associated with the entrainment of bed grains by grain-bed impacts, which is strictly larger than $\Theta_t$, must be exceeded for a stable steady state to exist. That is, there may be a fundamental physical difference between the stable steady transport state that is closest to $\Theta_t$ (i.e., $\Theta = \Theta_{\text{ImE}}$), which is the near-threshold state that can be approached in the numerical transport simulations, and the unstable steady transport state at $\Theta = \Theta_t$ to which the dimensional analysis applies. In particular, when $\Theta = \Theta_{\text{ImE}}$, the transport rate is strictly larger than zero, which may imply that the feedback of the grain motion on the flow cannot be neglected as done in the dimensional analysis. Additionally, impact entrainment on its own may cause the bed boundary conditions, which we assumed to be scale-free in the dimensional analysis, to become significantly dependent on the ratio between $v_s$ and the bed escape velocity scale $\sqrt{\tilde{g}d}$. Each of these factors, or both together, may be responsible for why $\overline{v_{x,t}^+}$ does not much depend on $v_s^+$.

Since the grain-flow feedback does not affect the vertical grain motion in the Stokes drag limit, it is interesting to take a closer look at the scaling of the rescaled hop height $\overline{v_{z,t}^{+2}}$ near the cessation threshold. The simulation data suggest that the bedload-corrected version of (3.7),

$$\overline{v_{z,t}^{+2}} u_s^{+3/4} = 1.5 f_s^{3/2} v_s^{+3/4},$$

roughly holds within a factor of 1.8 (figure 10(a)). In particular, the additional data points corresponding to the artificially changed drag laws with decreased or increased $Re_c$ (yellow square and yellow up-pointing triangle, respectively) fall within the spread of the standard simulation data ($Re_c = 24$). In contrast, when replacing $v_s^+ = 4sd^{+2}/(3Re_c)$ by $sd^{+2}/18$ (assuming $Re_c = 24$ for all simulations), these additional data points exceed the spread of the standard simulation data (figure 10(b)). That is, for saltation, $\overline{v_{z,t}^{+2}}$ seems to be consistent
with the dimensional analysis we used to deduce (3.4). Hence, in contrast to the streamwise grain motion, for the vertical grain motion, there is no evidence in the simulation data that conditions near the cessation threshold differ from conditions at the cessation threshold. This indirectly suggests that impact entrainment via its effect on the grain-flow feedback, which affects the streamwise but not the vertical grain motion in the Stokes drag limit, is behind the strangeness of the scaling of $v_\xi / x^t$.

Note that (3.21) may be useful by itself as it, in combination with (3.9), can be used to estimate the hop height $v_\xi^2/\bar{g}$, more precisely, the contribution of hopping grains to the characteristic transport height of all transported grains $\bar{z}$.

### 3.2.4. Combined cessation threshold and equilibrium transport rate model

In combination, (3.9) and (3.12), using $\Theta_t = u_{st}^2/(sd^*)$, predict the equilibrium sediment transport rate for arbitrary environmental conditions ($s, Ga, \Theta$) with $Ga\sqrt{\gamma} > 81$ (excluding viscous bedload) in agreement with the numerical transport simulations (figure 11(a)) and terrestrial experiments (figure 11(b)).

It was previously shown that the combined analytical, physical threshold and transport rate model by Pähtz et al. (2021) is consistent with numerical and experimental data for density ratios corresponding to terrestrial conditions in water and air and between (i.e., $s \in (1, 2200)$). This model calculates $\Theta_t$ using an identical periodic trajectory representation of grain motion, which also yields expressions for $V_0^*$ and a surface friction coefficient $\mu_{bt}$ at the cessation threshold. These are then used to calculate $Q^*$ via $M^* = d^* (\Theta - \Theta_t)/\mu_{bt}$ and (3.13). The comparison with our extended numerical data set reveals that this physical model does not only capture data for $s \in (1, 2200)$ but also data for larger values of $s$.
Figure 11: Rescaled sediment transport rate $Q^+$ predicted by (3.9) and (3.12) versus (a) simulated $Q^+$ and (b) measured $Q^+$. Symbols in (a) correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga \sqrt{s} > 81$ (excluding viscous bedload), where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). Symbols in (b) correspond to measurements for terrestrial aeolian saltation of minerals (Creyssels et al. 2009; Ho et al. 2011; Ho 2012; Martin & Kok 2017; Ralaiarisoa et al. 2020) and snow (Sugiura et al. 1998), and subaqueous bedload (Meyer-Peter & Müller 1948; Smart & Jaeggi 1983; Capart & Fraccarollo 2011). For figure legend in (b), see figure 7. Note that Ralaiarisoa et al. (2020) reported that transport may not have been completely in equilibrium in their experiments. For the calculation of $u_{t}^{+}$ in (3.9), $(D_{s}^{\text{min}}, u_{t}^{\text{min}}) = (16, 1.6)$ is used in (a) and $(D_{s}^{\text{min}}, u_{t}^{+\text{min}}) = (18, 2.3)$ in (b). The solid lines indicate perfect agreement. The dashed lines indicate a deviation by a factor of 2.

within a factor of about 2 (figure 12(a)), though the agreement is substantially worse than the combined semiempirical model based on (3.9) and (3.12) which we have presented here (cf. figure 11(a)). The reason why the model by Pähtz et al. (2021) captures the general trend in the simulation data is that, for saltation conditions at and around the threshold minimum $(D_{s}^{\text{min}}, u_{t}^{+\text{min}})$, it roughly predicts $V_{t}^{+} = \text{const}$ and thus

$$Q^+ / d^{3/2} = c_1 \sqrt{d^s}^{-1} (\Theta - \Theta_t) + c_2 \sqrt{d^s}^{-1} (\Theta - \Theta_t)^2,$$

(3.22)

with some constants $c_1$ and $c_2$. In units of the particle density $\rho_p$, gravity $g$, and dynamic fluid viscosity $\mu$ (all of which are independent of the air density $\rho_f$),

$$\sqrt{d^s}^{-1} = \frac{(g \mu / \rho_p)^{1/3}}{\sqrt{gd}} \frac{s^{1/2}}{(s-1)^{1/6}},$$

(3.23)

which increases as $s^{1/3}$ for large $s$. That is, $Q^+ / d^{3/2}$ in (3.22) exhibits the same trend with the density ratio $s$ as in (3.12). However, (3.22) also predicts a superimposed scaling with $(g \mu / \rho_p)^{1/3} \sqrt{gd}^{-1}$, which is not present in the simulation data. In terms of existing terrestrial transport rate measurements, (3.12) and (3.22) perform similarly well (see figure 11(b) and figure 12(b), respectively).
Figure 12: Rescaled sediment transport rate $Q^+$ predicted using the model by Pähtz et al. (2021) versus (a) simulated $Q^+$ and (b) measured $Q^+$. Symbols in (a) correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga\sqrt{s} > 81$ (excluding viscous bedload), where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). Symbols in (b) correspond to measurements for terrestrial aeolian saltation of minerals (Creyssels et al. 2009; Ho et al. 2011; Ho 2012; Martin & Kok 2017; Ralaiarisoa et al. 2020) and snow (Sugiura et al. 1998), and subaqueous bedload (Meyer-Peter & Müller 1948; Smart & Jaeggi 1983; Capart & Fraccarollo 2011). For the figure legend in (b), see figure 7. Note that Ralaiarisoa et al. (2020) reported that transport may not have been completely in equilibrium in their experiments. The solid lines indicate perfect agreement. The dashed lines indicate a deviation by a factor of 2.

### 3.3. Fluctuation energy dissipation and grain energy impact flux

Pähtz & Durán (2020) showed that, apart from separating the sediment transport rate $Q$ in $M$ and $V$ (section 3.2.3), one can also unravel new physical insights into the scaling of $Q$ by exploiting that $Q$ is linked to the global particle fluctuation energy balance by first physical principles. In particular, the balance

$$-\frac{1}{2} Q \langle a_z^q \rangle^q = D^c_{(xz)} + D^d_{(xz)}, \quad (3.24)$$

is an exact consequence of Newton’s axioms in the steady state. Here $-\frac{1}{2} Q \langle a_z^q \rangle^q$ corresponds to the production rate and $D^c_{(xz)}$ and $D^d_{(xz)}$ to the dissipation rates of $-\frac{1}{2\Delta} \sum_p m_p c_{px} c_{pz}$, with $c_p = v_p - \langle v \rangle$ the particle fluctuation velocity, by grain-grain contacts (superscript $c$) and fluid drag (superscript $d$), respectively. The physical meaning of $-\frac{1}{2\Delta} \sum_p m_p c_{px} c_{pz}$ is similar to that of $\frac{1}{2\Delta} \sum_p m_p c_p^2$, which is the actual total particle fluctuation energy per unit area of the bed ($\Delta$). The only difference is the formal replacement $c_p^2 \to -c_{px} c_{pz}$.

Using the fluctuation energy balance, $D^c_{(xz)}$ and $D^d_{(xz)}$ are calculated from the numerical transport simulation data through $D^c_{(xz)} \equiv -\int_{-\infty}^{\infty} \rho \langle a_{(x)v_z} \rangle dz$ and $D^d_{(xz)} \equiv \int_{-\infty}^{\infty} \rho \langle a_{(x)c_z} \rangle dz$, respectively, where the parentheses denote the symmetrization of the indices.

The negative flux-weighted average of the vertical grain acceleration by noncontact forces $-\langle a_z^q \rangle^q$ in (3.24) can be well approximated by the buoyancy-reduced gravity $\tilde{g}$ (this approximation is exact in the Stokes drag limit in the steady state). Hence, rescaling (3.24)
Figure 13: (a) Bedload-corrected normalized collisional dissipation rate $f_s s^{-5/12} D^{c+}_{(xz)} / d^{+3/2}$ versus bedload-corrected normalized excess Shields number $f_s s^{-1/12} (\Theta - \Theta_t)$. (b) $f_s s^{-1/3} D^{c+} / d^{+3/2}$ versus $f_s (\Theta - \Theta_t)$. Symbols correspond to numerical simulations for various combinations of the density ratio $s$ and Galileo number $Ga$ (see table 1 and figure 5) with $Ga \sqrt{s} > 81$ (excluding viscous bedload), where open and filled symbols indicate bedload and saltation conditions, respectively (see figure 1 for definition). The solid lines in (a) and (b) correspond to (3.26) and (3.27), respectively.

leads to (Pähtz & Durán 2020)

\[
\frac{1}{2} Q^+ \simeq D^{c+}_{(xz)} + D^{d+}_{(xz)}. \tag{3.25}
\]

Most of the dissipation of fluctuation energy occurs during grain-grain contacts, including collisional contacts during collisions between transported grains and quasistatic contacts within the bed, which dissipate energy transferred from transported grains to the bed via grain-bed collisions (Pähtz & Durán 2020). We therefore expect that $D^{c+}_{(xz)}$ carries much of the scaling of $Q^+$ in (3.12). However, while this expectation is confirmed for the term that is linear in $\Theta - \Theta_t$ in (3.12), the numerical transport simulations suggest a slightly different scaling of $D^{c+}_{(xz)} / d^{+3/2}$ as $s^{1/4}$ for the term that is quadratic in $\Theta - \Theta_t$ (figure 13(a)) when compared with the $s^{1/3}$-scaling of $Q^+ / d^{+3/2}$ in (3.12):

\[
D^{c+}_{(xz)} / d^{+3/2} = 0.7 s^{1/3} (\Theta - \Theta_t) + 7 f_s s^{1/4} (\Theta - \Theta_t)^2. \tag{3.26}
\]

In contrast, $D^{c+}$, which is based on the actual rescaled collisional energy dissipation rate $D^c = \int_{-\infty}^{\infty} \rho \langle \mathbf{a} \cdot \mathbf{v} \rangle dz$, exhibits the same scaling behavior as $Q^+$ (figure 13(b)):

\[
D^{c+} / d^{+3/2} = 0.8 s^{1/3} (\Theta - \Theta_t) + 5.5 f_s s^{1/3} (\Theta - \Theta_t)^2. \tag{3.27}
\]

Hence, understanding the physical origin of (3.27) may represent an alternative route toward understanding (3.12).

The difference between the particle energy produced by fluid drag and dissipated via collisions above the bed surface is described by the grain impact energy flux $q_{E \perp}$ (section 2.2). Modeling this quantity is crucial for estimating the rate at which dust is emitted into a planetary body’s atmosphere due to the bombardment of the regolith during saltation (Kok et al. 2012). The numerical simulations of saltation indicate that $q_{E \perp}$ scales roughly as
4. Conclusions

We have used sediment transport simulations with a well-established DEM-based numerical model (Durán et al. 2012) to determine the scaling behaviors of important equilibrium transport properties across almost seven orders of the particle-fluid-density ratio $s$, ranging from sediment transport driven by water ($s \approx 2.65$) to extremely rarefied air on Pluto ($s \approx 10^7$). For saltation transport, we have found that the cessation threshold shear velocity $u_+^*$, rescaled using units of the particle density $\rho_p$, buoyancy-reduced gravity $\bar{g}$, and kinematic fluid viscosity $\nu$, is only a function of the rescaled Stokes settling velocity $v_+^*$. We have shown that this behavior can be explained when assuming a quasicontinuous saltation motion driven by Stokes drag in a smooth inner turbulent boundary layer mean velocity profile. A slight semiempirical modification of this law that accounts for rolling grains has led to a general equation predicting $u_+^*$ across arbitrary saltation and turbulent bedload conditions. Unlike for the cessation threshold, the general scaling laws we have found for the sediment transport rate, collisional energy dissipation rates, and grain impact energy flux are currently not well supported by theoretical reasoning, though we have provided some clues hinting at their physical origin. Importantly, the scaling laws for the equilibrium sediment transport rate by Ungar & Haff (1987) and Durán et al. (2011), currently favored by theoreticians, do not well describe the numerical data. For example, the law by Ungar & Haff (1987), which has been
adjusted to Earth conditions, underestimates the sediment transport rate for the simulated Martian conditions by a factor of about 5. The reason is that the scaling factor in this law is not a constant but actually increases as $s^{1/3}$ according to the simulation data. What adds to the mystery of this finding is that this $s^{1/3}$-scaling factor is insensitive to artificial changes of the drag law and thus $v_s^*$, implying that there is no obvious way how this scaling factor can emerge even from a simple dimensional analysis.

The equations for the cessation threshold (3.9), equilibrium sediment transport rate (3.12), and grain impact energy flux (3.28), supported by their agreement with terrestrial measurements in air and water, provide a simple means to make predictions of aeolian processes across a large range of planetary conditions. However, to be reliable, they need to be put on a more solid theoretical footing, which we plan to do in the future.

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