Photo-actuated bending of chromatic liquid crystal polymer strips and laminates

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Photochromic liquid crystal polymer (LCP) films can undergo spontaneous mechanical deformations upon light illuminations. The light-induced strain is due to photoinimerization and the nematic–isotropic phase transition. A gradient strain exists due to absorption decay of the light intensity and will produce bending of LCP strips. Beam bending models are studied for single layered and bilayered LCP strips with substrate. The results show that the multilayered structures can have much larger bending curvature due to the utilization of the average light-induced contractions which were mostly not active for single-layered LCPs. Some structural optimizations for bilayered laminate are also discussed.

Keywords: liquid crystal polymer (LCP); photochromic film; light-induced bending; laminate

1. Introduction

Liquid crystal polymers (LCPs) are a class of ordered polymers with liquid crystal (LC) moieties. Crosslinked LCPs include weakly linked liquid crystal elastomers (LCEs) and heavily linked liquid crystal glasses (LCGs). These LCPs exhibit many entirely new effects that are not simply enhancements of native liquid crystals or polymers [1]. This type of synthetic new material can undergo spontaneous mechanical deformation in response to some external stimulus such as thermal, electric, magnetic, chemical, and optical fields. LCPs are emerging as attractive candidates for many applications including remote controllable implementations and especially micro-mechanical devices such as actuators and sensors [2].

Currently, most studies have concentrated on monodomain nematic LCPs that possess a uniaxial orientation with the director n associated with aligned rod-like LC molecules. A nematic order parameter $Q$ is introduced to measure the molecular alignments. Chromatic ordered nematic LCPs that are blended with light-sensitive chromophores (such as azobenzene or stilbene moieties) can undergo reversible mechanical deformation in response to a specific light wavelength [3]. In fact, the idea of photo-induced mechanical responses in polymeric materials was first proposed by Lovrien in 1967 [4]. Most of the early work focused on the synthesis of azobenzene-based polymeric materials. Despite
the considerable effort, the maximum light-induced strain was relatively limited (∼1%). A breakthrough came in 2001 when Finkelmann et al. synthesized liquid crystal elastomers (LCEs) with an azobenzene that led to substantial photo-induced contraction strains (∼20%) [3]. These light-driven LCP actuators offer several advantages over their thermal or electric counterparts and allow for operation in some extreme environments (both wet and dry) [5,6]. Thus far, a number of experimental outputs have been reported in chromatic LCP films [3,7–10]. Recovery stress up to 2 kPa can be reached for clamped samples [7]. Various photochromic LCP systems have been developed with opto-mechanical effects [11–14], some with very high speed, remotely accessed oscillations [15]. Yu et al. prepared high-modulus liquid crystal glass (LCG) and also found it could produce large deformation under light illuminations [16,17]. Notably, these fast and large amplitude oscillations were demonstrated using either UV or sun light, the cheapest and most readily available actinic source [18].

A variety of theoretical studies have been conducted on exploring the light-induced uniaxial contraction and gradient bending in chromatic nematic LCPs [5,19–24]. With an assumption of the light intensity following an exponential attenuation through the thickness, Warner and Mahadevan [20], van Oosten et al. [5] and Jin et al. [21,22] analyzed the curvature and nonlinear stress distribution of monodomain nematic azo-LCP beams irradiated by uniform diffused UV light. Dunn et al. [23,24] investigated curvatures of chromatic nematic LCP plates without any constrained boundaries by simple analytical estimates and nonlinear finite element simulation. Some significant studies focus on nematic LCP cantilevers with nonlinear photoisomerization and the fundamental understanding of the complex photo-mechanics of these photomechanical systems [25].

Recently, LCP composite multilayered films were prepared by Yu et al. to make a variety of simple devices. The devices can walk in one direction like an ‘inchworm’ or move like a ‘robotic arm’ by light [11,26]. They found there are some benefits to use the multilayered films instead of the single-layered LCP film. First, the substrate could protect the LCP film. Second, the amount of LCP film can be saved by not using it to make the whole device. Last, and more important, the bending direction of LCP film can be controlled by sticking the substrate on the up or downside of it.

To understand the mechanism of the devices moved by light, we did some primary research about the bending behavior of the basic structure of LCP laminate films. The bending directions of the LCP predicted from our model agreed with the experimental observations. Then our calculations showed that a composite structure exhibits larger bending than the simple LCP film [27,28]. In this paper, we model the LCP laminate structure. Then we analyze its bending behavior under the light thoroughly to find out the reason for its larger bending. Furthermore, we study the properties of it systematically to obtain structural optimization for the bilayered laminate.

Light-induced bending is a typical motion of LCP films and sometimes of great use. Light-induced strain originates from a multi-scale process from the molecular structures of the polymer, through the liquid crystal phase transition to the macroscopic deformation of the crosslinked LCPs. The gradient optical strain needed for bending is due to the absorption decay of light intensity. In this work, we shall first use some simple equations to clarify the process of producing the light-induced strain and its gradient. Small and large beam deflection models of single-layered LCPs will be briefly presented. The bending behavior of a bilayered nematic LCP with a substrate will be studied in some detail.
2. Photoisomerization and gradient strains induced by light

The key of the opto-mechanical effect in crosslinked LCPs is the photoisomerization process of the azobenzene and its subsequent influence on the nematic–isotropic phase transition. Here, we shall briefly review the relevant equations needed for our modeling and refer to [3,29] for more detailed discussions.

2.1. Dynamics of photoisomerization of azobenzene

As shown schematically in Figure 1, there are two different configurations (isomers) of azobenzene; one is the rod-like trans configuration with a lower potential energy and the other is the kinked cis configuration with a higher energy. The meta-stable cis isomer can be attained mainly by photo-excitation at a suitable wavelength, such as UV light. Considering the thermal activation and relaxation, the following kinetic equation has been proposed to calculate the number fraction $\phi$ of the cis isomers,

$$\frac{\partial \phi}{\partial t} = \eta I (1 - \phi) - \phi / \tau,$$

where $\tau = \tau_0 \exp(\Delta / k_B T)$ is the thermal relaxation with $T$ the absolute temperature, $\tau_0, \Delta$ positive constants and $k_B$ the Boltzmann constant. $\eta$ is the absorption term, which can depend on many factors including the polarization and the nematic order. For diffused light irradiation at normal incidence, it is reasonable to take $\eta$ as a positive constant. For fixed light intensity and temperature, the solution reads,

$$\phi = \phi^\infty \left(1 - \exp \left(-\frac{1 + \tau \eta I}{\tau}t\right)\right)$$

with $\phi^\infty = \tau \eta I / (1 + \tau \eta I)$. Obviously, under light irradiation, the cis isomer $\phi$ increases with time and eventually reaches the steady solution $\phi^\infty$ at $t \to \infty$. More cis isomer will be finally induced by a stronger light and at lower temperatures.

![Figure 1](image-url)  
**Figure 1.** Schematic depiction of the potential energy profiles for trans–cis transition of azobenzene.
2.2. Light induced strain

As the rod-like trans isomer is transformed into the kinked cis isomer, the nematic order parameter of the liquid crystal phase will be changed due to the disordered nature of the cis isomer, as shown schematically in Figure 2. The change may be approximately calculated by the following simple relation,

\[ Q(\phi) = \rho (T_{ni}^0 - \beta \phi - T)^\xi \]  

with \( \beta \) and \( \rho \) positive constants. \( T_{ni}^0 \) is the nematic–isotropic transition temperature when there is no cis isomer.

The anisotropic shape of the nematic polymers depends on the nematic order. Thus, light-induced order change will result in a light-induced strain, a contraction along the nematic director and extensions on the perpendicular plane. For one-dimensional beam bending models in this paper, we consider only samples with the nematic director along the length direction and the light-induced contraction strain in that direction. It is generally a very hard task to exactly calculate the macroscopic strain resulting from the photoisomerization process. Several approximate formulae have been proposed, notably the two linear relations in Equation (4). The more sophisticated nonlinear formula, Equation (4.3), was studied by the present authors [21,22]:

\[ \varepsilon^p = \begin{cases} 
\varepsilon_1^p I \\
\varepsilon_2^p \phi \\
\varepsilon_3^p(\phi) 
\end{cases} \]  

where \( \varepsilon_1^p \) and \( \varepsilon_2^p \) are the proportional constants that are negative to represent the contraction. \( \varepsilon_3^p \) satisfies the following expression

\[ \varepsilon_3^p(\phi) = \frac{(\lambda_\parallel^p)^3 - 1}{(\lambda_\parallel^p)^3 + 2}, \quad \text{with} \quad \lambda_\parallel^p = \frac{1 + \alpha Q(\phi)}{1 + \alpha Q(0)}, \]  

where \( \alpha \) is a positive constant and \( Q(\phi) \) satisfies the Equation (3).

![Figure 2. Schematic depiction of light-induced nematic order change and contraction along the nematic director \( n \) in LCPs containing azo dyes resulting from a rod-like trans to a kinked cis conformation under irradiation.](image-url)
It can be shown [21,22] that the linear relations, Equation (4.1–4.2), can be obtained from the nonlinear one, Equation (4.3) for infinitesimal strains. For the small deflections considered here, all three formulae can be used and the results are all similar. This means that the constitutive equations are suitable for both LCGs and LCEs. More discussion can be found in [21].

2.3. Decay of light due to absorption and the gradient strain

When the UV light is irradiated on a sample, its intensity will decay along the propagation because of the light absorption by azobenzene, since the energy of the light was used to transform the lower energy trans isomer to the higher energy cis isomer. Considering only normal incident light, as shown in Figure 3, the decay along the thickness direction, z-axis, can be calculated by the following differential equation,

\[
\frac{\partial I}{\partial z} = -\gamma \eta I, \quad (6)
\]

where \(\gamma\) is the material absorption parameter. If \(\gamma\) is taken as a constant, the following Beer’s exponential law of attenuation can be easily obtained,

\[
\frac{I}{I_0} = \exp(-z/d) \quad (7)
\]

with \(I_0\) the light intensity on the surface at \(z = 0\) and \(d = \gamma \eta\) the attenuation distance due to the light absorption. By Equation (2), the cis isomer will decrease along the thickness as well and consequently, the light-induced contraction of Equation (4) will decrease with \(z\). Such a gradient strain along the thickness will certainly induce a bending of the beam toward the light.

Recently, Corbett et al. [25] suggested that for very strong light intensity and heavily azo-dyed systems, the material absorption parameter \(\gamma\) should depend on the amount of remaining trans isomers, namely,

\[
\gamma = \gamma_0 (1 - \phi). \quad (8)
\]

In this case, Equation (1) and Equation (6) are generally nonlinear and coupled with each other. Only at the steady state with \(\partial \phi / \partial t = 0\) in Equation (1), can the cis fraction \(\phi^\infty\) be inserted into Equation (6) and the light decay be obtained as [25].

![Figure 3](image-url)
\[ I/I_0 = \frac{1}{i_0} W_L(i_0 \exp(i_0 - z/d)), \] (9)

where \( W_L(\xi) \) is the Lambert-W (product logarithm) function and \( i_0 = \tau \eta I_0 \).

For non-steady states, we have derived the following expressions for the solutions,

\[ t(z, \hat{I})/\tau = \int_{z/d}^{\hat{I}} \frac{d\xi}{f(z, \xi)} \quad \text{and} \quad \phi(z, \hat{I}) = 1 - \frac{f(z, \hat{I})}{f(z, z/d)} - f(z, \hat{I}) \int_{z/d}^{\hat{I}} \frac{d\xi}{f(z, \xi)^2}, \] (10)

where \( \hat{I} = -\log(I/I_0) \) and \( f(z, \xi) = z/d - \tau \eta I_0 - \xi + \tau \eta I_0 e^{-\xi} \). \( d = \gamma_0 \eta \) is the characteristic attenuation distance. From Equation (10.1), we can resolve \( \hat{I}(t, z) \) and then by Equation (10.2) the cis fraction \( \phi(z, t) \). Finally from Equation (4), we obtain the gradient strain induced by light, \( \epsilon^\rho(z, t) \).

### 3. Beam bending model of crosslinked chromatic LCP strips under light illumination

When a strip of crosslinked LCP is under light illumination, as in Figure 3, it will bend toward the light due to the light-induced gradient strains. The bending behavior of such a beam depends on the material properties of the LCP, the geometry and the boundary conditions of the beam, and the incident light. In this section, we shall consider both small and large deflection bending models and they will help to understand the laminate bending in the next section. In general, the elastic modulus will change as a result of light-induced nematic order change. However, its effect on the bending behavior is rather limited, as discussed in [20,21]. Thus, we shall not consider this effect and give a much simpler version of the deflection equations.

#### 3.1. Small-deflection Euler–Bernoulli LCP beam

Under the assumptions of small deformations, we have Hooke’s law as

\[ \sigma_{xx}(x, z) = E(\epsilon_{xx}(x, z) - \epsilon^\rho(x, z)). \] (11)

For a small-deflection Euler–Bernoulli beam, the following plane cross-section assumption holds,

\[ \epsilon_{xx}(x, z) = -(z - h/2)w''(x) + \epsilon_{xx}(x, h/2) \] (12)

with \( w(x) \) the deflection of the beam. Since the force and the moment along each cross-section should vanish, we obtain the following equations for the deflection,

\[ D \frac{\partial^2 w}{\partial x^2} = -M^p_{xx} \] (13)

with \( D = Ebh^3/12 \) the bending stiffness and \( M^p_{xx} \) the effective optical moment defined as

\[ M^p_{xx}(x) = Eb \int_0^h (\epsilon^\rho(x, z) - \bar{\epsilon}^\rho(x))zdz, \] (14)
where \( \bar{\varepsilon}(x) = \int_0^h \varepsilon^p(x,z)dz/h \) is the average light-induced contraction at the cross-section.

Many examples have been studied in [20,21]. Here, we shall just give a cantilever solution to demonstrate the light-induced bending effect. The following boundary conditions should be satisfied for a cantilever with one end \((z = 0)\) fixed and the other \((z = a)\) free,

\[
(w)_{x=0} = \frac{\partial w}{\partial x}_{x=0} = 0, \quad \frac{\partial^2 w}{\partial x^2}_{x=a} = -\frac{1}{D} M^p_{xx}, \quad \frac{\partial^3 w}{\partial x^3}_{x=a} = -\frac{1}{D} \frac{\partial M^p_{xx}}{\partial x}. \tag{15}
\]

Upon uniform illumination, the solution can be easily obtained as

\[
w = -\frac{M^p_{xx}}{2D} x^2. \tag{16}
\]

As shown in Figure 4, the LCP cantilevers will bend towards the light source under illumination. However, the maximal deflection and the beam curvature are not necessarily larger for stronger light, although the light-induced strain is larger. This nonlinear effect is shown in Figure 5 where the bending curvature quickly increases with the characteristic light intensity, \(i_0 = \tau \eta I_0\), and then decreases. The reason is that the effective optical bending moment, Equation (13), depends only on the difference of the light-induced strains from their average values. Thus, more intense illumination induces more cis isomers and larger strains, but too larger light intensity will reduce the gradient of light strain and correspondingly causes a smaller bending.

The thickness ratio \(h/d\) also heavily affects the bending behavior, as can be seen from Figure 5. Thus, when it comes to practical applications, we should choose adequate light intensity and thickness ratio to optimize the bending. In the numerical simulation, we used weakly crosslinked LCE model, i.e. Equation (4.3), to investigate bending behavior. But in fact, for those strongly crosslinking glassy LCPs, we can obtain similar behavior. We choose the constants as measured in [25]: \(\rho = 0.4, \alpha = 0.22, \xi = 0.19, \beta = 11.8 \text{ K}, T_{ni}^0 = 340 \text{ K}\). The other parameters are \(h/d = 1, i_0 = 1, a/h = 10\).

![Figure 4](image-url)  
Figure 4. Steady deflection curve of a chromatic nematic LCP cantilever, which was uniformly illuminated.
3.2. Large-deflection Euler–Bernoulli LCP beam

In the above model, the average light-induced contraction $\bar{\varepsilon}_p = \int_0^h \varepsilon_p dz / h$ can be very large but has no contribution to the bending behavior due to the small deflection assumption. However, it can affect the beam bending rather strongly, as will be shown here by using the large-deflection beam bending model. The main difference between the two theories is the following strain equation,

$$
\varepsilon_{xx}^{(l)}(x, h/2) = \frac{\partial u(x, t)}{\partial x} + \frac{1}{2} \left( \frac{\partial w(x, h/2, t)}{\partial x} \right)^2 
$$

(17)

with $u(x, t)$ the horizontal displacement at $z = h/2$. Then, again by the force and moment balance at each cross-section, we have the deflection equation,

$$
D \frac{d^2 w}{dx^2} - bF_x w = -M_{xx}^p, 
$$

(18)

where the membrane force $F_x$ due to the contraction is defined as

$$
F_x = \int_0^h \sigma_{xx}(x, z) dz = hE \left[ \varepsilon_0(t) - \varepsilon_0^p(t) + \frac{1}{2a} \int_0^a w_0^2(x, t) dx \right]. 
$$

(19)

The average strains are

$$
\varepsilon_0(t) = (u(a, t) - u(0, t)) / a \quad \text{and} \quad \varepsilon_0^p(t) = \frac{1}{a} \int_0^a \bar{\varepsilon}_p(x, t) dx. 
$$

(20)

We consider a beam with two simple supported edges and assume that the distance between the two clamps is constant during illumination. Thus, the corresponding boundary conditions are

Figure 5. The bending magnitude $M_{xx}^p / D_{xx}$ versus the characteristic light intensity $\alpha$, with various thickness ratio $h/d$. 
(w)_{x=0,a} = 0, (\partial^2 w/ \partial x^2)_{x=0,a} = -M_{xx}^p / D \quad \text{and} \quad (u)_{x=a} - (u)_{x=0} = 0. \quad (21)

Under uniform illumination, the deflection from Equation (18) reads

\[
w(x) = \frac{M_{xx}^p}{D} \frac{\chi a - 1}{\sinh \chi a} \left( \frac{\sinh \chi x - \cosh \chi x + 1}{\sinh \chi a} \right),
\]

Where \( \chi = \sqrt{bF_x/D} \) and the membrane force from Equation (19),

\[
F_x = hE \left[ -\bar{\varepsilon}^p + \frac{1}{2a} \int_0^a w_x^2(x) dx \right]. \quad (23)
\]

The solution is shown in Figure 6. Due to the boundary constrain imposed on the horizontal displacement, Equation (21.3), no contraction was allowed; thus there will build up a very large membrane force. The deflection shown by the solid line in Figure 6 is much smaller than it might be without considering this force, as shown by the dashed line. All the parameters are the same as before.

More examples and discussion about large-deflection bending are presented in [30]. Here, we just want to demonstrate the striking effect of the average contraction \( \bar{\varepsilon}^p(x) \) induced by light that was not present in the bending moment, Equation (14). Thus, from the energetic point of view, a large part of the energy of the incoming light was not utilized in the bending of the LCP beam. It would be more desirable to design a structure that can utilize both the bending moment given by the gradient of light-induced strain and the membrane force coming from the average light-induced strain. A laminated structure, to be discussed in the next section, is exactly the type of design that will exhibit a much larger bending than single-layered LCPs.

Figure 6. The steady state deflection of simple supported beams of fixed length for the large deflection model (solid line) and the small deflection model (dashed line).
4. Beam bending of LCP laminated with substrates

Multilayered thin films in the micrometer range composed of LCP and some stronger substrate such as polyethylene can protect the relative soft LCP films and enlarge the bending effect, as will be shown in this section. Consider the bilayered model shown in Figure 7; two different illumination directions will be considered. In order to have a consistent light decay solution, the coordinate systems are both set such that \( z = 0 \) is at the light incoming surface of the LCP and the \( z \)-axis has the same direction as the light propagation. The elastic and geometric parameters are also shown there.

For such a composite beam, the bending equation can be obtained in the same way as before. For simplicity, we consider only the small deflection model. Equation (11) is the same only with the following modifications for the elastic modulus and the light-induced strains:

\[
E(z) = \begin{cases} 
E_L & \text{in LCP} \\
E_b & \text{in Substrate}
\end{cases} \quad \varepsilon^p(z) = \begin{cases} 
\varepsilon_L^p & \text{in LCP} \\
0 & \text{in Substrate}
\end{cases}
\]

The plane cross-section assumption of simple beam theory is valid but the bending axis is no longer at \( z = h/2 \). Equation (12) should be rewritten as

\[
\varepsilon_{xx}(x, z) = -(z - \bar{z})w''(x) + \varepsilon_{xx}(x, \bar{z})
\]

with \( \bar{z} \) the bending axis such that \( \int_{h_0}^{h_1} E(z - \bar{z})dz = 0 \), where \( h_0 = 0 \) and \( h_1 = h = h_L + h_b \) for Model I and \( h_0 = -h_b \) and \( h_1 = h_L \) for Model II.

From the force and moment balances, we can obtain the deflection equation for laminate in the same form as Equation (13) with the bending stiffness \( D \) and the effective optical moment \( M_{xx}^p \) defined as

\[
D = b \int_{h_0}^{h_1} E(z)(z - \bar{z})^2 dz \quad \text{and} \quad M_{xx}^p = b \int_{h_0}^{h_1} E[\varepsilon^p(z) - \bar{\varepsilon}^p]zdz.
\]

The average light-induced contraction at the cross-section is now \( \bar{\varepsilon}^p = \int_{h_0}^{h_1} E\varepsilon^p(z)dz/\int_{h_0}^{h_1} Edz \). By inserting Equation (24) into the above integrations, the deflection equation can be simplified as

\[
\frac{h}{R} = \frac{\partial^2 w}{\partial x^2} = -\frac{hM_{xx}^p}{D} = -\frac{m_{xx}^p \pm \xi(\hat{e}, \hat{h})|\bar{\varepsilon}_L^p|}{\delta_D(\hat{e}, \hat{h})},
\]

\( \delta_D(\hat{e}, \hat{h}) \)

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Figure 7. Bilayered laminate with LCP and a polymer substrate. (a) Model I: light comes from LCP side; (b) Model II: light comes from substrate side.
where the plus sign is for Model I and the minus sign for Model II. $m^p_L = M^p_L/E_L b = \int_0^{h_L} (e^p_L(z) - \bar{\varepsilon}^p_L) z dz$ is the effective photo-induced moment of the LCP layer, i.e. the same as Equation (14). $\bar{\varepsilon}^p_L = \int_0^{h_L} \varepsilon^p z dz/h_L$ is the average photo-induced strain of the LCP layer. $\hat{e} = E_L/(E_L + E_p)$ and $\hat{h} = h_L/(h_L + h_p) = h_L/h$ are the ratios of the elastic modules and the thicknesses of the bi-layered structure. $\zeta(\hat{e}, \hat{h})$ and $\delta_D(\hat{e}, \hat{h})$ are two dimensionless parameters defined as

$$\zeta(a, b) = \frac{(1 - \hat{e})(1 - \hat{h})}{2\hat{h}[\hat{e} + (1 - \hat{e})(1 - \hat{h})]}$$

$$\delta_D(a, b) = \frac{1}{3\hat{e}\hat{h}^2} \left( \hat{e}\hat{h}^3 \left( \zeta + \frac{1}{2} \right)^3 + (1 - \hat{e}) \left( 1 - \hat{h} \left( \zeta + \frac{1}{2} \right) \right)^3 \right) + (2\hat{e} - 1) \left( \frac{1}{2} - \zeta \right)^3 \hat{h}^3$$

From Equation (27), it is obvious that in addition to the optical moment $m^p_L = M^p_L/E_L b$ of Equation (14), there is a bilayer moment $m^b_L = \zeta |\bar{\varepsilon}^p_L|$ very similar to the well-known bimetal bending moment. This additional moment is due to the presence of an elastic substrate and is proportional to the average light-induced strain of the LCP layer. Moreover, when the illumination is from the LCP side of Model I, the bilayer moment always adds up to the optical moment. But when the light comes from the substrate side of Model II, these two moments have different sign. This means that the laminate could bend in the other direction, as shown in Figure 8 for a cantilever.

Figure 8 also suggests that the laminates may have much larger maximal deflections and bending curvatures than the single-layered LCPs. Indeed, this is true for most laminates, as can be seen from Figure 9, where the curvatures $\frac{1}{R} = \frac{\partial^2 w}{\partial x^2}$ of the two laminate models are compared with the single-layered LCPs. It is clear from Figure 9 that the Model I curvature

![Figure 8](image-url)
is always positive and the Model II curvature is negative for not very thin substrates. Both are generally much bigger than the single layered LCPs except for very thin and very thick substrates. The bending directions predicted from our model agree with experimental results presented in [11,26].

The reason for the large enhancement of the bending of laminated structures is that the bilayer moment $m_p^b = \zeta |\bar{\varepsilon}_L^p|$ can be much larger than the optical one $m_p^L$. Their ratio is shown in Figure 10. It is obvious from Figure 10 that the bilayer moment is generally several magnitudes larger than the optical moment, especially for not very thin substrates ($h_b/h_L \geq 1/\hat{h}_{10} - 1 \approx 0.25$).

![Figure 9](image1.png)

Figure 9. Ratio of curvatures of Model I and II to the single layered LCPs as a function of the relative thickness $\hat{h}$ for $\hat{\varepsilon} = 1/3, i_0 = 5, h_L/d = 5$.

![Figure 10](image2.png)

Figure 10. Ratio of the two moments $m_p^b/m_p^L$ as a function of the relative thickness $\hat{h}$ for $\hat{\varepsilon} = 1/3, i_0 = 5, h_L/d = 5$. 
Based on the above observations, it is possible to simplify the deflection, Equation (27), by neglecting the optical moment for \( \hat{h} \) not very close to 1 as

\[
\frac{h}{R} = h \frac{\partial^2 w}{\partial x^2} \approx \frac{\zeta(\hat{e}, \hat{h})|\bar{\varepsilon}_p|}{\delta_D(\hat{e}, \hat{h})}.
\]  

(29)

It is obvious that \( \zeta/\delta_D \) depends only on the structure parameters \( \hat{e} \) and \( \hat{h} \) and the average optical strain \( \bar{\varepsilon}_p \) is determined by the light intensity \( i_0 \) and the absorption constant \( h_L/d \). These dependences are plotted in Figure 11, which shows clearly that the magnitude of \( \bar{\varepsilon}_p \) increases monotonically with the light intensity. Thus, the bilayer moment is also an increasing function of the light intensity. This bending behavior is rather different from the single layered one shown in Figure 5.

The coefficient \( \zeta/\delta_D \) depends strongly on the thickness ratio \( \hat{h} \), as shown in Figure 12(a). Obviously, for any given substrate material, there is an optimal thickness ratio \( \hat{h}_M(\hat{e}) \) at which the bending curvature of Equation (29) will be the largest. As shown in Figure 12(b), the optimal thickness of the substrate is larger for softer materials.

5. Conclusions

Photochromic LCPs are currently being developed as smart materials that can contract significantly and bend under illumination. Multilayered structures have been proposed for some applications. In this paper, we begin with the beam bending model of single layered LCP film to clarify the effects of optical moment and the optical membrane force on the bending. It is clear that a single-layered LCP film could only use the optical moment, the small contribution of the opto effect to bend. Then we proposed the bilayered laminate structures model and demonstrated that it can take advantage of average contraction (optical membrane force) to make much stronger bending. As a result, it should be considered as a better candidate than single-layered LCPs in applications. To give the structural optimizations for bilayered laminate, we also systematically study the properties of the LCP laminate structure. The main results are

![Figure 11](image_url). The average optical strain \( \bar{\varepsilon}_p \) as a function of the light intensity \( i_0 \) with \( h_L/d = 2, 4, 6, 8, 10 \).
Large contraction strains can be induced in chromatic LCPs by UV light through the photoisomerization between trans and cis isomers of the azobenzene moiety, and the resulting nematic–isotropic transitions.

(2) A gradient of the optical strain exists inside the LCP samples along the light propagation direction due to absorption. This gradient is responsible for the observed bending of LCP films.

(3) A small deflection beam bending model was developed and the examples show that an optical moment is produced by the light through the inhomogeneous contraction. The average contraction contributes neither to the optical moment nor to the bending behavior.

(4) Considering the large deflection model, the average light-induced contraction will affect the bending behavior when there are some boundary constrains.
(5) In a bilayered laminate of LCP and a substrate, the average light-induced contraction is utilized for the bilayer moment similar to the bimetal bending. This bilayer moment is generally much larger than the optical moment. Thus, the bending of laminate is much stronger than single-layered LCPs.

(6) The bending directions can be reversed for substrates attached on the upper and lower surface of LCPs.

(7) Given the substrate material, there is an optimal thickness to maximize the bending curvature.

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