Spherical accretion: the influence of inner boundary and quasi-periodic oscillations

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ABSTRACT
Bondi accretion assumes that there is a sink of mass at the center – which in case of a black hole (BH) corresponds to the advection of matter across the event horizon. Other stars, such as a neutron star (NS), have surfaces and hence the infalling matter has to slow down at the surface. We study the initial value problem in which the matter distribution is uniform and at rest at $t = 0$. We consider different inner boundary conditions for BHs and NSs: outflow boundary condition (mimicking mass sink at the center) valid for BHs; and reflective and steady-shock (allowing gas to cross the inner boundary at subsonic speeds) boundary conditions for NSs. We also obtain a similarity solution for cold accretion on to BHs and NSs. 1-D simulations show the formation of an outward propagating and a standing shock in NSs for reflective and steady-shock boundary conditions, respectively. Entropy is the highest at the bottom of the subsonic region for reflective boundary conditions. In 2-D this profile is convectively unstable. Using steady-shock inner boundary conditions, the flow is unstable to the standing accretion shock instability (SASI) in 2-D, which leads to global shock oscillations and may be responsible for quasi-periodic oscillations (QPOs) seen in the lightcurves of accreting systems. For steady accretion in the quiescent state, spherical accretion rate on to a NS can be suppressed by orders of magnitude compared to that on to a BH.

Key words: accretion, accretion discs – hydrodynamics – instabilities – methods: numerical – X-rays: binaries

1 INTRODUCTION
Spherical accretion of adiabatic gas without angular momentum in steady state is the simplest model of accretion onto a central gravitating object, as proposed by Bondi (1952). The theory of Bondi accretion was studied numerically in a series of papers by Ruffert and his collaborators (Ruffert 1994a; Ruffert & Arnett 1994; Ruffert 1994b). While spherical symmetry and adiabaticity are extreme idealizations – angular momentum, anisotropy due to large scale magnetic fields, entropy generation due to viscous/magnetic dissipation and conduction are almost always going to be important (Shvartsman 1971; Blandford & Begelman 1999; Sharma, Quataert & Stone 2008) – Bondi solution is the starting point for estimating the accretion rate in hot, non-radiative accretion flows on to BHs (Baganoff et al. 2003; Loewenstein et al. 2001; Allen et al. 2006).

We recall that the Bondi solution assumes a uniform, static gas with a specified density and temperature at large distance from the accreting object. The Bondi radius ($\sim 2GM/c_s^2$; $c_s$ is the sound speed in the ambient medium far away from the accreting object; $M$ is the mass of the central accretor) is a measure of the sphere of influence of the central gravitating object. The transonic solution, applicable for a black hole (BH) that allows mass to flow across the event horizon at the speed of light, becomes supersonic inside a critical radius ($r_c$. Neutron stars (NSs), white dwarfs and normal stars, on the other hand, possess a surface and the accretion flow must necessarily attain zero or a very subsonic velocity at the stellar surface (strong cooling can, however, allow matter to fall freely on to the surface). Thus, the mass accretion rate and the gravitational energy released in the same ambient conditions may be vastly different for BHs and NSs.

Observations of BH and NS X-ray binaries (XRBs) show some dissimilarities. First, the BH XRBs are more luminous compared to NS XRBs in the high-soft/thermal state; this is expected as the maximal accretion rate (known as the Eddington rate) is proportional to the compact object mass (BHs are more massive than NSs). Second, the NS XRBs are observed to be more luminous compared to BH XRBs in the quiescent state (Narayan, Garcia & McClintock 1997;
Garcia et al. 2001; see, however, Chen et al. 1998). This difference between the BH and NS systems is often explained as follows. In quiescent state, owing to a low accretion rate, the accretion flow around the central compact object is optically thin and geometrically thick. It is in the hot, radiatively inefficient regime (e.g., Das & Sharma 2013 and references therein). The gravitational power extracted via accretion is given by \( \eta M^2 \), where \( \eta \) is the accretion efficiency and \( M \) is the accretion rate on to the compact object. For a BH accreting in the quiescent state the radiative output (e.g., in X-rays) is sub-dominant, and most of the accretion energy (stored as thermal/kinetic energy) is lost because of advection across the event horizon. However, advection is absent in a NS because of the surface, and all of the energy is thermalized and radiated ultimately (Narayan & Yi 1995; Garcia & McClintock 1997; Mukhopadhyay 2002).

The above argument implicitly assumes that the accretion rate on to the compact object (\( \dot{M} \)) is the same, irrespective of the central object. This assumption is likely to break down because of qualitatively different inner boundary conditions for BHs and NSs. Studying BH and NS binaries with similar orbital periods, Menou et al. (1998) (see also Asai et al. 1998) showed that even if the mass transfer rates are similar (this is assumed for BHs and NSs with similar orbital periods), the accretion rate on to NSs is less compared to that on to BHs (this is inferred from the lower X-ray luminosity in NS XRBs compared to what is expected if all the accretion energy is radiated). They invoked the propeller effect (Illarionov & Sunyaev 1975) due to a magnetosphere to suppress NS accretion relative to a BH in similar ambient conditions. A rotating magnetosphere flings away matter, preventing it from reaching the surface. However, not all quiescent NS binaries have strong enough magnetic fields for this to work (e.g., D’Angelo et al. 2015). A floor in quiescent NS X-ray luminosity, which is larger than the most quiescent BHs, may also be explained if the X-ray emission is not due to current accretion, but say due to thermal radiation from a hot NS (e.g., Cackett et al. 2010), shock driven by an underlying pulsar (Campana et al. 1998), or due to coronal emission of the companion (e.g., Bildsten & Rutledge 2000).

The most prominent difference between NSs and BHs is that, unlike the latter, the former have a hard surface. We want to isolate the effect of a hard surface in NS accretion. Therefore, we consider a set-up without magnetic fields and angular momentum. While this is very idealized, it illustrates the essential (and potentially large) differences between BH and NS accretion in the quiescent state. We show that, in similar ambient conditions, the mass accretion rate on to NSs compared to BHs in the radiatively inefficient regime can be orders of magnitude lower. Thus, one cannot simply use the presence of a hard surface in NSs to explain their larger radiative output in the quiescent state.

An outcome of our idealized simulations is that for some inner boundary conditions we obtain large amplitude coherent oscillations of a standing shock. We identify these oscillations as a result of the standing accretion shock instability or SASI, widely seen in multi-dimensional supernova simulations (e.g., see Hanke et al. 2012 and references therein). We propose that the coherent oscillations seen in our simulations may be responsible for some of the quasi-periodic oscillations (QPOs) observed in the lightcurves of XRBs. The observed QPOs have frequencies ranging from mHz (low-frequency QPOs) to kHz (kHz QPOs). Some models describe the observed variability based on the orbital and epicyclic motions, while some are based on resonance between disk rotation and NS spin. Some models identify QPOs as global disk oscillation modes. Although several models have been proposed over the years, the mechanism(s) responsible for QPOs is still unknown (for a review, see van der Klis 2004).

The paper is organized as follows. In section 2 we describe our physical set-up. In section 3 we describe the similarity solutions for cold spherical accretion with outflow and reflective inner boundary conditions. In section 4 we describe the results from our numerical simulations. In section 5 we discuss the astrophysical implications of our results and summarize in section 6.

### 2 Physical Set-Up

We set up an initial value problem in which a gravitating compact object is embedded in an initially static, spherically-symmetric medium. In section 3.2 we consider similarity solutions for compact objects embedded in static atmospheres with power-law density. We take the central gravitating objects to be either a black hole (BH) or a neutron star (NS). We solve the Euler equations to study the problem of spherical accretion on to NSs and BHs. The only difference between NS and BH accretion in our formalism is that we impose reflective boundary condition (or a steady-shock boundary condition; see section 4.1) for the flow at the inner boundary in former and outflow boundary condition in latter. The Euler equations in presence of gravity of a compact object are given by

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + \rho \mathbf{I}) = -\rho \nabla \Phi, \tag{2}
\]

\[
\frac{\partial \left( \frac{\rho v^2}{2} + e + \rho \Phi \right)}{\partial t} + \nabla \cdot \left( \mathbf{v} \left( \frac{\rho v^2}{2} + e + p + \rho \Phi \right) \right) = 0, \tag{3}
\]

where, \( \rho, \mathbf{v}, p (= [\gamma - 1] e) \), \( e \) are the mass density, velocity, pressure, and internal energy density, respectively, and \( \gamma \) (chosen to be 1.4, unless mentioned otherwise) is the adiabatic index. The Newtonian potential of the central accretor is \( \Phi = -GM/r \), where \( G \) is Newton’s gravitational constant, \( M \) is the mass of the central compact object, and \( r \) is the distance from the center. We use Newtonian potential because the similarity solution that we obtain in section 3 is strictly applicable only for a scale-free potential. Newtonian approximation should not change our results qualitatively; the differences between BH and NS accretion are mostly due to the presence of a hard surface in latter (which we mimic by using a different inner boundary condition from BHs). We have verified that the results from numerical simulations using a pseudo-Newtonian potential mimicking general relativistic effects (Paczynsky & Wiita 1980) are qualitatively similar to the Newtonian simulations. This is essentially because the inner boundary affects the flow far away from the event horizon, where GR effects are subdominant.
3 SIMILARITY SOLUTION

In the limit that the external medium (loosely referred to as interstellar medium or ISM) is cold ($p_{\text{ISM}} = 0$), the only length scale that can be constructed from the parameters of the problem is

$$r_s(t) = \beta(\gamma)(GMt^2)^{1/3},$$

where $\beta(\gamma)$ is a function of the adiabatic index ($\beta \approx 0.21$ for $\gamma = 1.4$ and it increases with an increasing $\gamma$). This scale radius applies to both NSs and BHs but the solutions in these two cases differ because the inner boundary conditions in the two cases are different. While the accretion flow comes to rest at the center ($v = 0$ at $r = 0$) for NSs, the matter is freely-falling ($v = \sqrt{GM/r}$ as $r \to 0$) onto BHs. Since matter falls freely in a cold medium and the dynamical time is shortest at the center, the supersonic matter coming to rest at NS surface ($r = 0$) launches an outward propagating shock. For a BH, matter is removed supersonically from the inner boundary and there is no shock. Eq. 4 can be taken as the shock radius as a function of time for a NS, and as a scale radius for a BH. In both cases, the solutions (density, pressure, velocity profiles) as a function of distance from the center at different times can be scaled to $r_s(t)$. The solutions as a function of the scaled radius (defined in Eq. (6)) lie on top of each other when scaled appropriately (Eq. 7 or 13).

The velocity with which the scale radius (equal to the shock velocity for a NS) moves out is

$$v_s \equiv \frac{dr_s}{dt} = 2r_s \frac{d}{dt} = 2\beta \left(\frac{GM}{t}\right)^{1/3}.$$

We can convert the partial differential equations (PDEs; Eqs. (1)-(3)) into ordinary differential equations (ODEs) if we scale the length scales with $r_s$ and velocities with $v_s$. This is the essence of the similarity method, the well-known application of which is the Sedov-Taylor solution for a point explosion (Sedov 1946; Taylor 1950). We introduce a similarity variable

$$\xi = \frac{r}{r_s(t)},$$

which captures both the spatial and temporal evolution. We discuss the similarity solutions for two different initially static density profiles: i) a uniform density, $\rho = \rho_0$; and ii) a power law density, $\rho = Dr^{-\alpha}$ ($D$, $\alpha$ are parameters).

3.1 Uniform density

The scaled density, velocity and pressure are as follows:

$$\tilde{\rho} = \frac{\rho}{\rho_0}, \quad \tilde{v} = \frac{v}{v_s}, \quad \tilde{p} = \frac{p}{\rho_0 v_s^2},$$

where $\rho_0$ is the initial density of the uniform ambient medium and $v_s$ is the velocity corresponding to the scaling radius. Plugging these scalings in 1-D spherically symmetric form of the mass and momentum equations (Eqs. 1 and 2), we obtain

$$\left(-\xi + \tilde{v}\right) \frac{d\tilde{\rho}}{d\xi} + \frac{\tilde{\rho}}{\xi} \frac{d\tilde{v}}{d\xi} + 2\tilde{v} \xi^{-1} \tilde{\rho} = 0,$$  

$$\left(-\xi + \tilde{v}\right) \frac{d\tilde{\rho}}{d\xi} + \frac{1}{\xi} \frac{d\tilde{p}}{d\xi} = \frac{\tilde{v}}{2} \frac{\gamma - 1}{\gamma + 1} \tilde{\rho}^\gamma,$$  

where $\gamma = 5/3$ for a cold ISM.

Since the entropy of a fluid element is conserved, except when it crosses the shock (which forms in case of a NS), the alternative form of the energy equation (Eq. (3)) is

$$\left(\frac{\partial}{\partial \xi} + \frac{\tilde{v}}{\gamma - 1} \frac{\partial}{\partial\tilde{\rho}}\right) \frac{\tilde{p}}{\tilde{\rho}^{\gamma - 1}} = 0,$$

which, when scaled, becomes

$$\left(-\xi + \tilde{v}\right) \frac{d\tilde{\rho}}{d\xi} + \tilde{p} = 0.$$

Note also that this equation is valid everywhere except at the shock. Also note that Eqs. 8, 9 and 11 are ordinary differential equations in one independent variable ($\xi$), much simpler to solve than the original partial differential equations. The spatial and temporal dependence of physical quantities is obtained by the scaling relations in Eq. (7).

For a cold interstellar medium (ISM), far away from the accreting object ($\xi \to \infty$), $\tilde{p} = 0$ and $\tilde{\rho}$ is unity (scaling the density to the ambient value). Eqs. (8) and (9) as $\xi \to \infty$ give $\tilde{v}\propto -3/2(2\beta^2\xi^2)$. These are chosen as the outer boundary conditions for the ODEs (Eqs. 8, 9, and 11). For BHs, the pressure remains zero throughout; i.e., $\tilde{p} = 0$. Therefore, for BHs we simply integrate Eqs. 8 and 9 inwards. In contrast, for NSs a shock forms, within which pressure roughly balances gravity. The inner boundary condition for NSs is chosen to be $v = 0$ as $\xi \to 0$; i.e., flow comes to rest at the surface. Moreover, Rankine-Hugoniot shock jump conditions are applied across the shock at $\xi = 1$. For a strong shock, the post-shock quantities (denoted by subscript 2) are related to the pre-shock quantities (denoted by subscript 1) as

$$\tilde{v}_2 = \frac{\gamma - 1}{\gamma + 1} \tilde{v}_1 - \frac{2}{\gamma + 1} \tilde{p}_1, \quad \tilde{p}_2 = \frac{2\tilde{p}_1(1 - \tilde{v}_1)^2}{\gamma + 1}.$$

We compare the results that we get from similarity solution (ODE) with the PDE solutions discussed later. Fig. 1 shows a comparison between the ODE and PDE solutions.
for density and velocity at different times. We see that density profiles at different times coincide, so do the velocity profiles at different times. This is a signature of similarity solution which we have stated before also. While the match between the ODE and PDE solutions is perfect for BH, there are small discrepancies for NS solutions because of small errors at the inner boundary. As expected, the NS and BH solutions match outside the shock.

3.2 Power-law density

In this section we consider a power-law initial density profile for the cold ISM, \( \rho_{ini} = Dr^{-\alpha} \). We can obtain the similarity solution following a procedure similar to section 3.1. The scale length, shock velocity and similarity variable are still given by Eqs. 4-6. The scaled density, velocity, and pressure in this case are

\[
\tilde{\rho} = \frac{\rho}{Dr_s^{\alpha}}, \quad \tilde{v} = \frac{v}{v_s}, \quad \tilde{p} = \frac{p}{Dr_s^{\alpha}v_s^2}.
\]

Using these scaled variables and considering spherical symmetry, the 1-D mass, momentum, and entropy equations (Eqs. 1, 2 and 10) take the form

\[
(-\xi + \tilde{v}) \frac{d\tilde{\rho}}{d\xi} + \tilde{\rho} \frac{d\tilde{v}}{d\xi} + \frac{2}{\xi} \tilde{v} \tilde{\rho} - \alpha \tilde{\rho} = 0,
\]

\[
(-\xi + \tilde{v}) \frac{d\tilde{v}}{d\xi} + \frac{1}{\tilde{\rho}} \frac{d\tilde{p}}{d\xi} \equiv \tilde{\rho} = \frac{9}{2} - \frac{1}{4 \beta^2 \xi^2},
\]

\[
(-\xi + \tilde{v}) \frac{d}{d\xi} \left( \frac{\tilde{p}}{\tilde{\rho}} \right) + \left( \gamma - 1 \right) \alpha - 1 \frac{\tilde{\rho}}{\tilde{\rho}^*} = 0.
\]

Using a method similar to section 3.1, we solve Eqs. (14) - (16) to obtain the similarity solution for different density power laws \( (\alpha) \). The boundary conditions as \( \xi \to \infty \) are \( \tilde{\rho}_\infty = \xi^{-\alpha} \) and \( \tilde{v}_\infty = -3/(2\beta^2\xi^2) \). Similarly, for NSs, \( \tilde{v} \to 0 \) as \( \xi \to 0 \) is the additional boundary condition. Moreover, for NSs the shock jump conditions in Eq. 12 apply across the shock at \( \xi = 1 \).

Fig. 2 shows the scaled density and velocity profiles. We also find that these results are in good agreement with the results obtained from the numerical simulations of PDEs. Note that \( \beta \) (see Eq. 4) for a power-law density profile depends on \( \alpha \), the power-law slope, and therefore the velocity at large radii \( (\tilde{v}_\infty = -3/(2\beta^2\xi^2)) \) differ for different \( \alpha \).

We note that Sakashita (1974) and Sakashita & Yokosawa (1974) have also obtained similarity solutions for NSs with shocks. Before these, Bisnovatyi-Kogan, Zel’dovich & Nadezhin (1972) obtained similarity solutions with shocks in a uniform gravitational field. However, there are several differences from our approach. These early papers do not consider the solution outside the shock; specifically, the large \( \xi \) regime in which \( \tilde{v}_\infty = -3/(2\beta^2\xi^2) \) is missing. Also, rather than solving ODEs at all radii, these analytic works assume an asymptotic power-series form of profiles for \( \xi < 1 \). Not only do we obtain the correct solutions for the profiles at all \( \xi \), we also show that similar solutions are applicable for BHs, in which shocks are absent. We have also confirmed our solutions with the numerical solutions of PDEs discussed in section 4.

4 TIME-DEPENDENT NUMERICAL SIMULATIONS

In this section we describe the time dependent numerical simulations of Euler equations (Eqs. 1-3) treated as PDEs. We use the PLUTO hydrodynamic code in 1-D and 2-D (Mignone et al. 2007). The problem set-up is the same as described in section 2, namely a static uniform medium at \( t = 0 \). We consider two kinds of ambient conditions: (i) a cold ISM to check the validity of our similarity solution in section 3; (ii) a warm ISM, in which steady state solutions for BHs and NSs are attained. The steady transonic solution for BHs is the well-known Bondi solution (Frank, King & Raine 2002).

4.1 Initial and boundary conditions

The initial pressure is \( p_0 = \rho_0 c_s^2/\gamma \), where \( c_s = (\gamma p_0/\rho_0)^{1/2} \) is the sound speed in the ambient ISM. As mentioned earlier, we consider initial conditions: (i) a cold ISM (with \( c_s = 0 \)) for comparing with the similarity solution; and (ii) a warm ISM (with \( c_s = 0.002\gamma c_\infty \)) where \( c_\infty \) is the speed of light in vacuum) with the sonic radius \( r_s = (5 - 3\gamma)GM/(4c_\infty^2) \) (the radius at which the radial inflow velocity equals the local sound speed) and the Bondi radius \( r_B = 8r_s/(5 - 3\gamma) = 2GM/c_\infty^2 \approx 714GM/c^2 \) (for our choice of parameters) well inside the simulation domain, for which the PDE solution reaches a steady state.

4.1.1 1-D

The 1-D runs are carried out till \( 10^6r_g/c \), time by which a steady solution is attained by our warm ISM simulations. In all cases computational domain extends from an inner boundary \( r_{in} = 6r_g \) (corresponding to the innermost stable circular orbit of a Schwarzschild black hole and roughly the size of a neutron star) to an outer boundary \( r_{out} = 10^4r_g \), where \( r_g = GM/c^2 \) is the gravitational radius. A logarithmic grid with \( N_r = 1024 \) grid points is used. For both BHs and NSs, the outer boundary conditions are the same; we fix the pressure and density to their initial values \( p_0 \) and \( \rho_0 \), and set the velocity to zero. For BHs we use the outflow boundary condition at the inner boundary, such that mass is allowed to be advected into the BH but cannot come out of it.
For most NS runs reflective boundary condition is applied at the inner radius, such that matter comes to rest at the stellar surface. Another inner boundary condition that we use for some NS runs is what we call the ‘steady-shock’ boundary condition (a similar boundary condition is used by Blondin, Mezzacappa & DeMarino 2003, who call it a leaky boundary condition), in which the velocity of the gas crossing the inner boundary ($v_g$ in the ghost zones) is fixed to a given subsonic value (chosen to be 0.05$c$). The values of pressure and density in the ghost zones are copied from the innermost zone of the computation domain. The importance of the inner boundary condition for NSs is discussed in section 5.

### 4.1.2 2-D

We also carry out 2-D axisymmetric simulations in spherical ($r, \theta, \phi$) coordinates for a warm ambient medium. The 2-D simulations are necessary because multi-dimensional effects such as convection and standing shock instability can qualitatively change the accretion flow. The computational domain extends from an inner radius $r_{in} = 6r_s$ to an outer radius $r_{out} = 10^5r_s$. We use a logarithmic grid along radial direction with the number of grid points $N_r = 512$. Along $\theta$ ($0 \leq \theta \leq \pi$) a uniform grid with $N_\theta = 128$ is used.

Along the radial direction we use the same outer boundary conditions as in 1-D simulations (see section 4.1.1). Additionally, we set all velocity components to zero at the outer boundary. At the inner boundary we fix the tangential velocity ($v_\theta$) to zero for the steady-shock boundary condition; similar results are obtained if $v_\theta$ is copied in the ghost zones instead of being set to zero. The tangential velocity is copied in the ghost zones at the inner radial boundary for both reflective (applicable to NSs) and outflow (applicable to BHs) boundary conditions. In all cases, axisymmetric boundary conditions are used for both BHs and NSs at $\theta = 0$ and $\theta = \pi$.

**Figure 3.** Density profiles at different times for cold ($\tilde{\rho}_c = \rho_c/\rho_0$; top panel) and warm ($\tilde{\rho}_w = \rho_w/\rho_0$; bottom panel) ISM in 1-D BH simulations. Time $t$ is in units of $r_g/c$.

**Figure 4.** Density profiles at different times for cold ($\tilde{\rho}_c = \rho_c/\rho_0$; top panel) and warm ($\tilde{\rho}_w = \rho_w/\rho_0$; bottom panel) ISM in 1-D NS simulations with reflective inner boundary condition. Time $t$ is in units of $r_g/c$.

### 4.2 Simulation results

#### 4.2.1 1-D

Fig. 3 shows the density profiles for BH simulations at different times for an initially cold (top panel) and warm (bottom panel) ISM. The density increases with time for a cold ISM, never reaching a steady state; pressure, being zero, is never able to balance the gravitational pull and the density front keeps propagating out in accordance with Eq. 4. For a warm ISM, on the other hand, the density profile attains a steady state roughly after the density front reaches the Bondi radius (i.e., at $r_B/c_\infty \approx 1.3 \times 10^3$). Velocity takes far longer time ($\sim r/v$) to equilibrate at large radii because the flow is extremely subsonic ($v \propto 1/r^2$) beyond the Bondi radius. In steady state, the warm ISM settles down to the classic Bondi solution.

Fig. 4 shows the scaled density profiles for accretion on to a NS immersed in a cold (top panel) and a warm (bottom panel) ISM; reflective boundary condition is used at the inner radius. For a NS with reflective boundary condition, an outward-propagating shock is launched. For a cold ISM, the shock is always strong as the shock Mach number is infinite, and the solution never reaches a steady state (like in the case of a BH). In contrast, for a warm ISM the shock weakens as its velocity becomes comparable to the upstream sound speed. Eventually the flow attains a steady state and there is no shock; this state is essentially a hydrostatic atmosphere.

Fig. 5 shows the scaled density and Mach number ($M \equiv -v/c_s$; $c_s = \sqrt{\gamma p/\rho}^{1/2}$ is the local sound speed) profiles in steady state for the warm NS run with reflective inner boundary condition. The Mach numbers and velocities are tiny $\sim 10^{-4}$ throughout the box; at some radii the velocity is positive! Strictly speaking, in steady state, the velocity throughout the computational domain should be zero because the mass flux at the inner boundary is set to zero. Small velocities arise because of small numerical errors at the inner boundary (these errors decrease with increasing resolution and at late times).

Fig. 6 shows the scaled density and velocity profiles for accretion on to a NS in a warm medium with the steady-shock inner boundary condition (see section 4.1.1). In this...
Figure 6. Density ($\dot{\rho}_w = \rho_w / \rho_0$; top panel) and velocity profiles ($\dot{v}_w = v_w / c$; bottom panel) at different times for 1-D accretion on to a NS with steady-shock boundary condition.

Fig. 7 also shows the entropy profiles for 1-D NS simulations with steady-shock boundary conditions. The flow in these cases is qualitatively different from the hydrostatic atmosphere obtained with the inner reflective boundary condition. For the steady-shock boundary condition, entropy is constant both outside and inside the shock, with a jump in entropy at shock-crossing. Small oscillations are seen in the post-shock entropy profiles. These are small amplitude entropy modes (in hydrostatic balance) arising because of reflections at the inner boundary. The standing shock is stable to radial perturbations (e.g., see Blondin, Mezzacappa & DeMarino 2003).

The results from 2-D BH simulations (with outflow inner boundary condition) match the 1-D runs, and in steady state (which is attained eventually for a warm ISM) are well-

Figure 7. Entropy ($K \equiv p/\rho^2$) profile in steady state from 1-D NS simulations with reflective and two different steady-shock inner boundary conditions ($v_{in} = 0.05c$ and $v_{in} = 0.07c$).
Figure 8. Snapshots of overdensity \( \delta(r, \theta) \equiv [\rho(r, \theta) - \bar{\rho}(r)] / \bar{\rho}(r) \); where \( \bar{\rho}(r) \) is the \( \theta \)-averaged density) at different times from the 2-D simulation of NS with reflective inner boundary condition. Arrows show the flow velocity; local velocity is along the direction of arrows and the length of the arrow is proportional to the magnitude of velocity. For clarity, the velocity arrows are not placed at all grid points.

described by the standard Bondi solution. Therefore, in this section we only discuss the 2-D NS simulations for a warm ISM, with both reflective and steady-state inner boundary conditions. These 2-D simulations are qualitatively different from 1-D.

**Reflective inner boundary condition:** Fig. 8 shows the snapshots of overdensity \( \delta(r, \theta) \equiv [\rho(r, \theta) - \bar{\rho}(r)] / \bar{\rho}(r) \); where, \( \rho(r, \theta) \) is the density at \((r, \theta)\), \( \bar{\rho}(r) \) is the \( \theta \)-averaged density at \( r \) superposed with velocity arrows at different times for the 2-D NS run with reflective inner boundary conditions. Initially \((t = 0)\) the density is uniform with \( \delta = 0 \) and the velocity vanishes. With time, NS starts accreting matter. Matter with supersonic velocity hits the NS surface and forms a shock which propagates outward in time. The top-right snapshot at \( t = 2 \times 10^5 \) in Fig. 8 shows that the flow is radial before it is shocked, but within the shock spherical symmetry is broken. Also the shock location (most easily located by the sudden change in arrows representing velocities) is not spherically symmetric; at \( t = 2 \times 10^5 \) (top-right panel) the shock extends much further out along poles as compared to the equator. Fig. 7 shows that entropy is maximum toward the center for NS simulations with reflective inner boundary condition. Such an atmosphere is convectively unstable, and indeed the gas within the shock in Fig. 8 shows convective swirling motions, with underdense \((\delta < 0)\) blobs rising and overdense \((\delta > 0)\) blobs sinking with respect to the background gas. Snapshot at \( t = 2 \times 10^4 \) (bottom-left panel) in Fig. 8 shows that convection (as measured by \( \delta \)) becomes weaker with time. As in 1-D (see the bottom panel of Fig. 4 and red dashed line in Fig. 7), the outer shock also becomes weaker with time in 2-D. The snapshot at \( t = 2 \times 10^5 \) (bottom-right panel in Fig. 9) shows that after a sufficiently long time (longer than the sound crossing time across the Bondi radius) the density fluctuations and velocities become negligible. By this time the outer shock vanishes and the radial entropy gradient is erased by convection. In this steady state the system is well described as a polytrope \((p/\rho^\gamma \approx \text{constant})\) in hydrostatic equilibrium.
steady state the mass accretion rate vanishes (because the velocity at the inner boundary is set to zero for a reflective inner boundary), like in 1-D. However, the key difference from 1-D is that entropy ($p/\rho^2$) is approximately constant (red-dashed line in Fig. 7 is the entropy profile in 1-D).

Fig. 9 shows the $\theta$–averaged entropy distribution as a function of radius at different times. Post-shock gas has a higher entropy because of the entropy generated at the shock. With time the shock becomes weaker, with a lower post-shock entropy. The peak entropy at early times ($t = 200$) in Fig. 9 is similar to the entropy at the smallest radii for the 1-D run shown by red dashed line in Fig. 7. However, with time the entropy peak at the center flattens because of convection; higher entropy, underdense blobs rise leaving behind lower entropy gas at the center. In 2-D, not only does the shock front become weaker and moves out with time as in 1-D, the entropy profile within the shock is flatter. Eventually, at $t = 2 \times 10^5$ the entropy profile is perfectly flat and convection (and associated density and velocity perturbations) turns off.

**Steady-shock inner boundary condition:** Fig. 10 shows density snapshots and velocity vectors at various times for the 2-D NS simulation with the steady-shock inner boundary condition.

![Figure 10](https://example.com/figure10.png)

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### Table 1. Results from 2-D SASI simulations in steady state

| $v_{in}/c$ | $r_{sh}$ | $r_{sh, max}$ | $r_{adv}$ (\(r_g/c\)) | $r_{adv}^*$ (\(r_g/c\)) | $T^*(v_r)$ | $T^*(v_\theta)$ |
|-----------|----------|--------------|------------------------|------------------------|-------------|-------------|
| 0.045     | 26.7     | 33.1         | 392                    | 163                    | 400         | 800         |
| 0.05      | 20.8     | 28.8         | 291                    | 145                    | 290         | 580         |
| 0.07      | 11.8     | 17.6         | 162                    | 58                     | 120         | 240         |

* time-averaged in steady state (see the caption of Fig. 13).

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with the same period are seen for the radial velocity in Fig. 12) and is chiefly responsible for density and luminosity (c.f. Fig. 17) fluctuations.

Fig. 11 shows the angle-averaged entropy profiles for the 2-D NS simulation with the inner steady-shock boundary condition ($v_{in} = 0.05c$). Initially (at $t = 2.9 \times 10^3 r_g/c$) the entropy gradient is negative in the post-shock region. Eventually convection and advection through the inner boundary remove the entropy gradient and the post-shock entropy profile becomes almost flat. Therefore, as with the reflective inner boundary condition, convection does not play a significant role after the flow attains a steady state (as also noted by Blondin, Mezzacappa & DeMarino 2003). Note that the angle-averaged post-shock entropy profiles are not perfectly flat and the shock location is smeared because the shock is not spherically symmetric. A comparison of the angle-averaged entropy profile in Fig. 11 with the 1-D profiles in Fig. 7 is instructive. The shock location and the entropy value for the 2-D steady-shock boundary condition run in steady state are similar to the corresponding values in the 1-D run. Unlike with the reflective inner boundary condition, convection is not necessarily required to attain an isentropic post-shock steady state; even the analogous 1-D run (see black solid line in Fig. 7) attains a constant entropy in the post-shock region because of advection of entropy out of the computational domain at the inner boundary. Since energy is advected through the inner boundary (albeit subsonically) for the steady-shock inner boundary condition, the shock does not propagate to as large a distance as with the reflective inner boundary condition (compare Figs. 11 and 9).

The large amplitude global oscillations seen in Fig. 10 result from the vortical-acoustic instability of standing shocks known as the standing accretion shock instability or SASI (Foglizzo & Tagger 2000; Foglizzo et al. 2007, 2012). The instability is thought to arise due to the unstable advection-acoustic cycle in which vorticity/entropy perturbations are advected inwards and sound waves propagate outwards due to reflection at the inner boundary and the shock, respectively (e.g., see left panel in Fig. 1 of Guilet & Foglizzo 2012).

Fig. 12 shows the variation of radial and meridional velocities at a fixed point within the oscillating shock. There are two prominent oscillation time periods, roughly 290$r_g/c$ (clearly seen in $v_r$ oscillations) and 580$r_g/c$ (seen in $v_\theta$ oscillations; also prominent in global density oscillations of Fig. 10). To study the effects of the velocity imposed at the inner boundary, we have run NS simulations using $v_{in} = 0.45, 0.07c$. A smaller velocity at the inner boundary pushes the steady shock outwards, and we expect the radial advection time and hence the SASI oscillation period to be longer. Fig. 13 shows the time variation of advection and
Figure 10. Density snapshots at different times for the 2-D NS simulation with steady-shock boundary condition. Arrows show the flow velocity. Top right and lower panels clearly show the vertically oscillating ($l = 1$ mode) standing shock. The first two panels show the shock developing and moving outward in time. There are signatures of convection in the top-third panel at $t = 7105 \, r_g/c$. The vertical shock oscillation period is roughly $580 \, r_g/c$ (see Fig. 12). One can also see the spherical higher frequency ($T = 290 \, r_g/c$) mode outside the shock in the density snapshots at $t = 123685, 123830, 123975 \, r_g/c$.

sound-crossing times across the shock for different $v_{in}$. As expected, the timescales are longer for a smaller $v_{in}$. Table 1 lists various important quantities (shock location from 1-D simulations, maximum shock location in 2-D, time-averaged advection and sound-crossing times from the shock to the inner boundary, and the time periods for $v_r$ and $v_\theta$ oscillations) from our 2-D steady shock NS simulations with three different $v_{in}$. In all cases, the period of $v_\theta$ oscillations (which coincides with global $l = 1$ density oscillations seen in Fig. 10) is double that of radial velocity oscillations (which oscillates at the same frequency as the radial mode seen in Fig. 10). The time period of the radial velocity oscillation roughly matches the radial advection time for $v_{in} = 0.045c$ and $v_{in} = 0.05c$, but deviates for $v_{in} = 0.07c$. This deviation may be due the smallness of the size of the post-shock region for $v_{in} = 0.07c$. There is still no precise prediction for the oscillation period of SASI, but the advection timescale, which shows a similar trend as the measured shortest oscillation period in all cases, gives a good estimate (Guilet & Foglizzo 2012).

5 DISCUSSION AND IMPLICATIONS

In this section we discuss the astrophysical implications of our results. Although our set up is quite idealized, we can apply some of our results to interpret various observations of radiatively inefficient accretion on to compact objects.
flow becomes isentropic because of convection; see section 4.2.2); (ii) a solution with a steady shock in 1-D with an accretion rate equal to the Bondi value (as we note in section 4.2.2, this solution is unstable to SASI which results in global oscillations in 2-D; the average accretion rate still equals the Bondi value though).

Fig. 14 shows the variation of Mach number ($M \equiv v_{\infty}/c_{s}$) as a function of radius scaled to the sonic radius in 1-D ($r_{s}$). The top-left panel shows all branches of the solution (Holzer & Axford 1970). The transonic branch (a) is the classic accretion solution, which is also shown in the top right panel. The bottom-left panel shows a solution with a steady shock (this shock is unstable to SASI in 2-D and 3-D).

For a small enough inflow velocity at the inner radius, the solution is the subsonic settling/sinking atmosphere shown in the bottom-right panel (also known as the breeze solution; it is branch (d) in the top-left panel).

The steady-shock solution connects the transonic accretion branch (a) to the higher entropy branch (e) with the same accretion rate (McCrea 1956). Other Rankine-Hugoniot jump conditions must also, of course, be satisfied. Branches (a) and (d), however, cannot be connected by a shock because (d) has a lower entropy for the same accretion rate. For obtaining the steady-shock solution we solve the steady state forms of Eqs. (1) and (2) in 1-D (assuming spherical symmetry) with the polytropic equation of state $p = K\rho^{\gamma}$.

We apply the Rankine-Hugoniot shock conditions at the chosen shock location $r_{sh}$ (which must lie between the stellar surface and the sonic point). For a given shock location, the post-shock flow traces out a unique trajectory in the $M = v_{\infty}/r_{s}$ plot, with a non-zero (subsonic) velocity at the NS surface. The shock location is very sensitive to the value of velocity at the inner boundary, $v_{\infty}$.

Fig. 15 shows the variation of the velocity at the inner boundary, $v_{\infty}$.
Spherical accretion on neutron stars and black holes

Figure 14. Mach number as a function of the scaled radius for spherical accretion in 1-D. Top-left panel shows all the possible branches for steady, isentropic accretion. The other panels show the different flow regimes realized for the different values of infall velocity \( v_{\text{in}} \) at the inner boundary \( (r_{\text{in}}) \).

Figure 15. The inflow velocity at the surface of NS (i.e., the inner boundary) \( v_{\text{in}} \) (in units of \( c \)) as a function of the location of the steady shock \( r_{\text{sh}} \) (in units of \( r_g \equiv GM/c^2 \)). The same choice of parameters is made as the rest of the paper. The steady-shock solution is realized only if \( v_{\text{in,min}} \leq v_{\text{in}} \leq v_{\text{in,max}} \).

boundary (at 6\( r_g \)) as a function of the shock location for our choice of parameters (e.g., \( c_{\text{in}} \)). The maximum value of the inner velocity \( (v_{\text{in,max}}) \) occurs when the shock forms just at the surface and the minimum value occurs when the shock occurs just inside the sonic point (\( \approx 71r_g \) for our choice of parameters). The steady-shock solution is not allowed for the inner velocity outside this range. An isentropic, subsonic settling solution occurs for \( v_{\text{in}} < v_{\text{in,min}} \) and a transonic solution (similar to that for a BH) occurs for \( v_{\text{in}} > v_{\text{in,max}} \). It should be noted that mass accretion rates for both the transonic solutions (with and without shocks) are \( \dot{M} = \dot{M}_{\text{B}} \), where \( \dot{M}_{\text{B}} \) is the Bondi accretion rate. But, for the settling solution the accretion rate can be much smaller than the Bondi value (\( \dot{M} < \dot{M}_{\text{B}} \)).

5.2 Cooling and \( v_{\text{in}} \)

Section 5.1 shows that adiabatic spherical accretion is very sensitive to the velocity at the inner boundary, \( v_{\text{in}} \). A key question is how this settling velocity is determined at the surface of an accreting star. Various factors, such as the porosity of the accreting surface and cooling, govern the velocity at the inner boundary. We have carried out 1-D simulations with free-free cooling to assess its role in setting \( v_{\text{in}} \), and consequently, the accretion regime. We do not discuss these simulations in detail here, but just mention the salient features.

If radiative cooling in the dense, inner accretion flow is efficient (i.e., cooling time is shorter than the dynamical time), a cooling layer on top of the stellar surface can form (e.g., see Blondin, Mezzacappa & DeMarino 2003). In steady state, even with a reflective inner boundary condition (valid for a NS), gas can accrete on to the cooling layer, with most of the accretion energy being radiated away (this corresponds to the \( v_{\text{in}} > v_{\text{in,max}} \) transonic solution, except that energy cannot be advected but has to be radiated). If cooling is less efficient, even with reflective/non-absorbing inner boundary condition, we can realize the \( v_{\text{in,min}} < v_{\text{in}} < v_{\text{in,max}} \) regime in which a steady shock is formed. In this case, substantial energy may still be lost ra-
conditions, the mass accretion rate for NSs should be smaller than the corresponding value for BHs. In $v_{\text{in}} > v_{\text{in,min}}$ regime, $M = M_B \propto M^2$, and even the scaled mass accretion rate $\dot{m} \equiv M/M_{\text{Edd}} \propto M$ ($M_{\text{Edd}} \propto M$ is the Eddington accretion rate) is larger for BHs as compared to NSs. For $v_{\text{in}} < v_{\text{in,min}}$ due to the NS surface, the accretion rate is further suppressed ($M \propto v_{\text{in}}$ for $v < v_{\text{min}}$). Therefore, in the radiatively inefficient accretion regime, the NS accretion rate can be orders of magnitude smaller than a BH.

It is instructive to study the flow of mass and energy in spherical accretion in different regimes. In the simplest adiabatic Bondi regime, gravitational energy is converted to the kinetic energy of the infalling supersonic gas, which is advected on to the BH. In steady state, all the gravitational power ($G M M / 2 r_{\text{in}}$) is accreted by the BH. Even in presence of cooling, owing to the low density and temperature in the inner regions (see the top panel of Fig. 16), radiative losses are subdominant for BHs. Since energy cannot be advected through the inner boundary for NSs, all the gravitational power is radiated in this regime ($G M M / r_{\text{in}}$). This up stream of the cooling layer and most within it. If matter cannot radiate efficiently close to the NS surface (likely at low accretion rates), the accretion rate is suppressed by several orders of magnitude compared to the Bondi value. In this case, in steady state, the gravitational power extracted is small but all of it goes into radiative cooling.

We should mention here that any deviation from spherical symmetry (e.g., Stone, Pringle & Begelman 1999; Proga & Begelman 2003), any chance of thermalization of energy (for example due to magnetic fields and their dissipation; e.g., Igumenshchev & Narayan 2002), etc. cause non-radiative accretion on to even BHs to be qualitatively different from the idealized Bondi solution, and in fact, closer to the NS solution. In this case, the mass accretion rate can be orders of magnitude smaller than the Bondi estimate, as is the case for Sgr A* in the Galactic center (Baganoff et al. 2003). One key difference of our simulations from previous BH simulations with angular momentum and magnetic dissipation is that the midplane density is shallower in them (compare Fig. 16 with Fig. 5 in Stone, Pringle & Begelman 1999 and Fig. 13 in Sharma, Quataert & Stone 2008). The density profile in simulations with angular momentum depends on the form of accretion viscosity (Eq. 12 in Das & Sharma 2013). The profiles at inner radii in relativistic simulations ( unlike us, Sharma, Quataert & Stone 2008 use a pseudo-Newtonian potential to capture GR effects) are affected by the presence of the innermost stable circular orbit (ISCO), close to which the density profile flattens. Also, the density in the inner regions is flatter if entropy is larger at the center (e.g., compare the top panel of Fig. 16 with Fig. 5); this may be due to magnetic dissipation close to the center. This discrepancy in the density profile will be resolved in the future.

5.4 QPOs and SASI

The presence of the standing accretion shock instability (SASI) for certain inner boundary conditions leads to the exciting possibility of explaining some of the quasi-periodic oscillations (QPOs; e.g., see Psaltis, Belloni & van der Klis 1999; Remillard & McClintock 2006; Mukhopadhyay et al. 2008)
Figure 17. Free-free luminosity within $r_{\text{sh, max}}$ ($r_{\text{sh, max}} = 1.4 \times 10^{-27} (\rho/m_p)^{2/3} T^{1/2} dV$; Rybicki & Lightman 1986) as a function of time, scaled to a NS mass of $1 M_\odot$; $\rho_\infty = 10^{12} m_p$ ($m_p$ is proton mass) gives $M_{\text{ff}} \approx 9 \times 10^{-13} M_\odot \text{yr}^{-1}$ (Eq. 2.36 in Frank, King & Raine 2002; this is $\approx 4 \times 10^{-5}$ of the Eddington limit). The bottom-right inset shows a zoomed-in version in steady state. The top-left inset shows a power spectrum with the most prominent peak at 713 Hz, which in dimensionless units, corresponds to a time period of 290 $r_s/c$, which matches the $v_\theta$ oscillation frequency in Fig. 12 and Table 1. Peaks are also seen for higher harmonics. The power spectrum is taken in the steady state (from 0.1 to 0.56 s).

2003; Mukhopadhyay 2009) observed in BH and NS XRBS accreting in the hard and intermediate spectral states.

Typically the observed QPO frequency increases with the increasing mass accretion rate. This is expected in the SASI model because a larger accretion rate implies higher density, at which effective velocity at the inner radius ($v_{\text{in}}$) increases and the shock moves in. A shorter advection time leads to higher harmonics. This frequency corresponds to a time period of 290 $r_s/c$ (for $M = 1 M_\odot$), seen for $v_\theta$ oscillations in Fig. 12. The $v_\theta$ and $l = 1$ density oscillations seen in Fig. 10 have double the time period (and half the frequency), but are not prominent in the power spectrum because density displacement does not substantially change the luminosity (radial compression, on the other hand, appreciably changes luminosity). The results are expressed assuming a NS mass of one solar mass, but can be scaled easily with the mass of the compact object. The shock location, and hence the inverse of the QPO frequency (identified with $v_\theta$ oscillations in Fig. 12) is $\propto M$ for the same ambient conditions. We note that the $Q$-value for the oscillations is quite high, but a slow modulation of $v_{\text{in}}$ in time due to cooling can lower it such that it matches observations. Moreover, there will be contributions to the lightcurve from the MHD turbulent accretion disk which can lower the $Q$-factor (Reynolds & Miller 2009).

One of the most important X-ray transients, GRS 1915 + 105, is classified in twelve different temporal classes in terms of its timing properties (lightcurves; Belloni et al. 2000). Interestingly, while some of the classes, e.g., $\chi$ exhibiting the low/hard state, show continuous jets (as measured from persistent radio emission), some others, e.g., $\theta$ corresponding to the intermediate state, show episodic jets. Chakrabarti (1999); Molteni, Sponholz & Chakrabarti (1996); Ryu, Chakrabarti & Molteni (1997) argued that steady jet is produced by the steady shock seen in their 2-D simulations. The steady jet may thus describe the $\chi$ state of GRS 1915+105. We also see a quasi-steady oscillating shock in our 2-D simulations if we fix the velocity at the inner boundary to an intermediate value (section 4.2.2). We speculate on how a transient jet observed in the $\theta$ state may be produced. The spectral state transition from a low-hard ($\chi$) to an intermediate state ($\theta$) happens due to an increase in the mass accretion rate (and the resulting efficient cooling; Das & Sharma 2013). Because of enhanced cooling, the effective velocity at the inner boundary increases beyond the maximum value for which a steady shock can form ($v_{\text{in, max}}$; see Fig. 15; see also section 5.2). In this scenario, the shock becomes stronger for a short time (of order the cooling time) and eventually disappears, plausibly leading to strong episodic jets as seen in the intermediate state. Quantitative comparisons with observations using realistic simulations is left for future.

5.5 Caveats: role of rotation, asymmetry, and magnetic fields

Although for simplicity we have assumed spherical symmetry, and neglected rotation and magnetic fields, they are necessary ingredients for a realistic model of accretion. While spherically symmetric accretion on to a BH does not admit a shock, steady state models of an accretion flow with angular momentum allows a shock. The shock may occur at radii where the pre-shock flow is accelerating or decelerating (e.g., see Fig. 6 in Chakrabarti 1989; the shock outside/inside the O-type sonic point corresponds to the former/latter). For radial perturbations, out of the two possible shock locations, the inner one is unstable due to post-shock decel-
eration (Nakayama 1992; Nobuta & Hanawa 1994). However, even the outer shock is unstable to non-axisymmetric perturbations. These instabilities were invoked to explain time variability in accreting systems (e.g., Molteni, Tóth & Kuznetsov 1999; see Iwakami et al. 2009 for simulations of SASI with rotation). The occurrence of non-axisymmetric Papaloizou-Pringle instability and its interplay with the advective-acoustic cycle is an additional complication with rotation (Gu & Foglizzo 2003). Moreover, MHD turbulence in the accretion flow (Balbus & Hawley 1998) may damp the advection of entropy and vorticity, an essential component of SASI. Foglizzo, Galletti & Ruffert (2005) show that the advective-acoustic cycle also results in shock oscillations for wind accretion. Although shock oscillations appear to be robust, for modeling observations they must be studied in a more realistic set up than ours.

6 SUMMARY

In this paper we study the dependence of spherical accretion on the nature of the central accretor. In particular, we study the influence of the hard surface of a neutron star (NS) which slows down the accreting matter to subsonic speeds. In contrast, spherically infalling matter onto a black hole (BH) accelerates to supersonic speeds and eventually is lost through the event horizon. Following are the key findings of our study:

- We obtain similarity solutions for the flow profiles for accretion on to BHs (matter is allowed to accrete at supersonic speeds) and NSs (matter comes to rest at the surface). While an outward-propagating shock is present for NSs, BH profiles are smooth and transonic. While similar work was done in the past, we have obtained the complete flow profiles, particularly the solutions outside the shock in case of NSs.
- Classical Bondi accretion theory is applicable only for adiabatic spherically symmetric inviscid accretion onto BHs, but not onto NSs. Due to the presence of a surface, matter has to slow down at the surface, which gives rise to a shock. To study the effect of a surface, we study accretion with two different inner boundary conditions – reflective and steady-shock. For the reflective boundary condition, we allow a small subsonic infall velocity at the inner boundary. If the velocity at the inner boundary is within a certain range (see Fig. 15), a steady standing shock is obtained in 1-D. However, in 2-D the standing shock is unstable to the standing accretion shock instability (SASI), giving rise to radial ($l = 0$) and vertical ($l = 1$) oscillations. The effect of radial oscillations is reflected in the luminosity curve (Fig. 17) which shows coherent oscillations. This implies SASI can be a possible mechanism for quasi-periodic oscillations (QPOs) in XRBs (even in BHs in which a sudden deceleration of the infalling flow can be produced by a centrifugal barrier, rather than a surface).
- In our model, the velocity at the inner boundary (physically, this may be governed by cooling close to the inner surface) controls different spherical accretion regimes. If cooling allows for the inflow velocity at the surface to be faster than a limit ($v_\text{in} > v_{\text{in, min}}$), accretion rate on to the NS, $\dot{M}_\text{NS} = \dot{M}_B$. But for $v_\text{in} < v_{\text{in, min}}$, the accretion rate, $\dot{M}_\text{NS} < \dot{M}_B$. In both cases the amount of matter reaching the NS surface is less than that crossing the BH event horizon. This is because for Bondi accretion $\dot{M}_B \propto M^2$ ($M$ is the mass of the central accretor), and BHs are more massive than NSs.
- In the quiescent state, some NS XRBs are observed to be more luminous compared to BX XRBs. This has been interpreted as an evidence for the advection of the majority of energy released due to accretion across the event horizon of BHs. However, this argument assumes that the accretion rate on to BHs and NSs embedded in a similar environment are identical. This assumption is unlikely to be true in the quiescent state with inefficient cooling, as the accretion rate (scaled to the Bondi value) is likely to be much smaller for NSs, which have surfaces and an effective inflow velocity close to the surface $v_\text{in} < v_{\text{in, min}}$ (see Fig. 15). Spherical accretion is an appropriate model for the radiatively inefficient quiescent accretion flow, which is optically thin but geometrically thick. A floor in X-ray luminosity of NS XRBs may be due to radiation not linked to the current accretion rate, but due to other effects such as thermal radiation from the NS surface.

We aim to improve our – admittedly simple – models to include important physical effects such as angular momentum and magnetic fields. Our paper provides a basic foundation for more realistic future simulations, which are needed to directly match with the observations of NS and BH XRBs.

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1While these papers refer to their perturbation analysis as axisymmetric, they only consider radial variations in the perturbations.
