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The monotonicity of the apsidal angle in power-law potential systems. (English)

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Summary: In a central force system the apsidal angle is the angle at the centre of force between two consecutive apsides and measures the precession rate of the line of apsis. The apsidal angle has applications in different fields and Newton’s apsidal precession theorem has been extensively studied by astronomers, physicist and mathematicians. The perihelion precession of Mercury, the dynamics of galaxies, the vortex dynamics, the JWKB quantisation condition are some examples where the apsidal angle is of interest. In case of eccentric orbits and forces far from inverse square, numerical investigations provide evidence of the monotonicity of the apsidal angle with respect to the orbit parameters, such as the orbit eccentricity. However, no proof of this statement is available. In this paper central force systems with \( f(r) \sim \mu r^{-\alpha-1} \) are considered. We prove that for any \(-2 < \alpha < 1\) the apsidal angle is a monotonic function of the orbital eccentricity, or equivalently of the angular momentum. As a corollary, the conjecture stating the absence of isolated cases of zero precession is proved.

MSC:
70F15 Celestial mechanics

Keywords:
central force systems; homogeneous potential; precession rate; monotonicity of the apsidal angle; two-body problem

Software:
INTLAB

Full Text: DOI arXiv

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