Analytical Solutions for Taking into Account Composite Action of Reinforcement and Concrete in the Evaluation of Stresses in the Principal Planes of Girder Web Plates with Prestressed Reinforcement

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Abstract. The article addresses the issue of feasibility of strengthening the girders in road bridge superstructures by intensifying the reinforcement of web plates. The article also discloses the need to consider the composite action of concrete and reinforcement for the principal planes of girder web plates in superstructures. A methodology for calculating the effectiveness of the proposed superstructure reinforcement option is presented in this publication, as well as the results of assessing the stress state of structures according to the standard project of the series 3.503.1-81, reinforced in accordance with the methodology described in this publication. The obtained results confirm the validity and effectiveness of the proposed solutions for strengthening the superstructure, as well as the validity and applicability of the methodology which helps to consider the composite action of concrete and reinforcement in girder web plates in road bridge superstructures.

1. Introduction
Road bridges are considered to be technically complicated and important constructions. Road construction increases every year, so the need in artificial constructions grows. Besides, there is an actual problem of disparity of built bridges to modern demands, dictated by constantly growing traffic. It means that the establishment of the factual vehicle capacity resources of road bridges becomes actual.

2. Relevance
The relevance of the study is predetermined by insufficiency of crack resistance requirements along the trajectory of the principal tensile stresses for a number of inclined sections of superstructure girders 21, 24, 33 meters long with prestressed reinforcement. Such girders account for a significant proportion in all reinforced concrete bridges. At the same time, the calculated provisions of SP 35.13330.2011 do not provide algorithms for controlling the stress state of concrete and changing the spacing, diameter of vertical bars of stirrups and horizontal bars in the web plate.
As an example, in the SP 35.13330.2011 section “Calculation of crack opening” an estimate of stresses in transverse and longitudinal reinforcement according to expression (7.95) would be fair for assessing stresses in reinforcing elements oriented along the trajectory of the principle tensile stresses. In this case, it is necessary to take into account the content that clause 7.107 of SP 35.133.2011 gives to the stresses \( \sigma_{bt} \) and reinforcement factor \( \mu \) included in (7.95) SP 35.13330.2011.

Therefore, it is not entirely correct to use expression (7.95) in assessment of tensile stresses in both transverse and longitudinal bars. The active participation of transverse and longitudinal reinforcement bars in concrete behavior in the zone of formation of inclined cracks depends on the position of the planes where principal stresses occur, which in the general case does not coincide with the direction regulated by clause 7.79 of SP 35.13330.2011. Against this background, the coefficient \( \delta \) in clause 7.107 of SP 35.13330.2011, which takes into account the redistribution of stresses in the zone of formation of inclined cracks and determined by the expression (7.96) of SP 35.13330.2011, seems to be a little simplified option for taking into account composite action of reinforcement and concrete of inclined sections.

Also, this problem was raised more than once by various authors, which can be seen in publications [13, 16]. The need to reinforce the girders was also proved in [14, 15].

3. Formulation of the problem

The following is a solution that reveals the internal static uncertainty of the interaction between reinforcement and concrete along the principal planes of inclined sections, based on the condition of mutual deformation. This condition is justified in the calculation of reinforced concrete structures according to the second group of limit states when the hypotheses of material resistance are accepted valid.

4. Theoretical part

Figure 1 reveals the condition of the balance of forces on the plane of unit width with length \( l \), in the direction of the normal where principal tensile stresses \( \sigma_{mt} \) occur, balanced by the stresses acting along the trajectory \( \sigma_{mt} \) in the concrete \( \sigma_{mt,b} \), the forces in the longitudinal horizontal reinforcement bar \( \bar{N}_{s,x} \) and in transverse vertical reinforcement bar \( \bar{N}_{s,y} \).

Projecting forces \( \sigma_{mt} \), \( \sigma_{mt,b} \), \( \bar{N}_{s,x} \), and \( \bar{N}_{s,y} \) on a normal with an angle \( \alpha_1 \) relative to the \( x \) axis, the equilibrium condition will be as follows:

\[
\sigma_{mt} l = \sigma_{mt,b} l + \bar{N}_{s,x} \cos \alpha_1 + \bar{N}_{s,y} \sin \alpha_1 \tag{1}
\]

As shown in the figure 2.a, the composition of the horizontal reinforcement bar on the vertical projection \( l_y \) of a plane with length \( l \) for a web plate with a section width \( b \) is characterized by the bar spacing \( u_y \), the number of bars in one horizontal plane \( n_x \) and the cross section area of one bar \( A_{s,x} \). Similarly, the composition of vertical bar reinforcement on a horizontal projection \( l_x \) is characterized by a bar spacing \( u_x \), the number of bars in one vertical plane \( n_y \) and the cross section area of one bar \( A_{s,y} \).

The indicated reinforcement compositions on the vertical \( l_x \) and horizontal \( l_y \) projections of the plane of length \( l \) with the corresponding dimensions show:

\[
l_x = l \sin \alpha_1 \tag{2}
\]

\[
l_y = l \cos \alpha_1 \tag{3}
\]

Taking into account the stresses \( \bar{N}_{s,x} \) and \( \sigma_{s,y} \), occurring in the bars of the corresponding orientation, the forces in the horizontal \( \bar{N}_{s,x} \) and vertical \( \bar{N}_{s,y} \) reinforcement bars in the considered two-dimensional problem are determined by the expressions:

\[
\bar{N}_{s,x} = \sigma_{s,x} \frac{n_x A_{s,x}}{u_y b} l_y = \sigma_{s,y} \frac{n_x A_{s,x}}{u_x b} l \cos \alpha_1 \tag{4}
\]
\[
\bar{N}_{s,y} = \sigma_{s,y} \frac{n_y A_{s,y}}{u_y b} l_y = \sigma_{s,y} \frac{n_y A_{s,y}}{u_y b} l \sin \alpha_1
\]  \hspace{1cm} (5)

Stresses in the bars of horizontal \(\sigma_{s,x}\) and vertical \(\sigma_{s,y}\) reinforcement are determined in Figure 2, b where the effect of stress \(\sigma_{mt}\) is characterized by relative strains \(\varepsilon_{mt,b}\), which correspond to strains in the bars of horizontal \(\varepsilon_{s,x}\) and vertical \(\varepsilon_{s,y}\) reinforcement, which in turn link the dependences:

\[
\varepsilon_{s,x} = \varepsilon_{mt,b} \cos \alpha_1 ; \Rightarrow \frac{\varepsilon_{s,x}}{E_x} = \frac{\varepsilon_{mt,b}}{E_b} \cos \alpha_1 ; \Rightarrow \sigma_{s,x} = \frac{E_x}{E_b} \sigma_{mt,b} \cos \alpha_1 \hspace{1cm} (6)
\]

\[
\varepsilon_{s,y} = \varepsilon_{mt,b} \sin \alpha_1 ; \Rightarrow \frac{\varepsilon_{s,y}}{E_y} = \frac{\varepsilon_{mt,b}}{E_b} \sin \alpha_1 ; \Rightarrow \sigma_{s,y} = \frac{E_y}{E_b} \sigma_{mt,b} \sin \alpha_1 \hspace{1cm} (7)
\]

Figure 1. a – equilibrium condition for an plane of unit width, b – scheme for determining the forces in the bars of horizontal and vertical reinforcement.

Substitution of \(\sigma_{s,x}\) according to (6) and \(\sigma_{s,y}\) according to (7) in (4) and (5), respectively, determines the forces \(\bar{N}_{s,x}\), and \(\bar{N}_{s,y}\) as follows:

\[
\bar{N}_{s,x} = \sigma_{mt,b} \frac{E_x}{E_b} \frac{n_y A_{s,x}}{u_y b} l \cos^2 \alpha_1 \hspace{1cm} (8)
\]

\[
\bar{N}_{s,y} = \sigma_{mt,b} \frac{E_y}{E_b} \frac{n_y A_{s,y}}{u_y b} l \sin^2 \alpha_1 \hspace{1cm} (9)
\]

Substitution of the forces \(\bar{N}_{s,x}\) and \(\bar{N}_{s,y}\) by expressions (8), (9) in equilibrium condition (1) and its solution with respect to \(\sigma_{mt,b}\) give:

\[
\sigma_{mt,b} = \frac{\sigma_{mt}}{1 + \left( \frac{E_x}{E_b} \frac{n_y A_{s,x}}{u_y b} \cos^2 \alpha_1 + \frac{E_y}{E_b} \frac{n_y A_{s,y}}{u_y b} \sin^2 \alpha_1 \right)} \hspace{1cm} (10)
\]

In a similar manner, an analysis of the stress-strain state of the web plate in the direction of the principal compressive stresses \(\sigma_{mc}\) was performed as shown in Figure 3. In Figure 3, a structure and
symbolism in the designation and characterization of reinforcement composition is the same as above. On one side, the principle compressive stresses \( \sigma_{mc} \) act in the direction of the plane of unit width and length \( l \). On the other side, the principle stresses are balanced by the stresses in concrete \( \sigma_{mc,b} \) and forces in the longitudinal \( \bar{N}_{s,x} \) and transverse \( \bar{N}_{s,y} \) reinforcement bars. Then the equilibrium condition for the projections of the forces on the normal to the plane of unit width and length \( l \) can be written as:

\[
\sigma_{mc} l = \sigma_{mc,b} l + \bar{N}_{s,x} \cos \alpha_2 + \bar{N}_{s,y} \sin \alpha_2
\]

(11)

The forces in the horizontal reinforcement \( \bar{N}_{s,x} \) at the projection plane \( l_y = l \cos \alpha_1 \) and the forces in the transverse reinforcement \( \bar{N}_{s,y} \) at the projection plane \( l_x = l \sin \alpha_1 \) are equal to:

\[
\bar{N}_{s,x} = \sigma_{s,x} \frac{n_s \lambda_{s,x}}{u_y b} l \cos \alpha_2
\]

(12)

\[
\bar{N}_{s,y} = \sigma_{s,y} \frac{n_s \lambda_{s,y}}{u_y b} l \sin \alpha_2
\]

(13)

\[\text{Figure 2. Scheme for determining the stress-strain state of beam.}\]

Figure 2.b reflects the conditions for composite action of concrete and reinforcing elements and gives the following dependencies:

\[\varepsilon_{s,x} = \varepsilon_{mc,b} \cos \alpha_2; \implies \sigma_{s,x} = \frac{E_s}{E_b} \sigma_{mc,b} \cos \alpha_2; \implies \sigma_{s,x} = \frac{E_s}{E_b} \sigma_{mc,b} \cos \alpha_2 \]

(14)

\[\varepsilon_{s,y} = \varepsilon_{mc,b} \sin \alpha_2; \implies \sigma_{s,y} = \frac{E_s}{E_b} \sigma_{mc,b} \sin \alpha_2; \implies \sigma_{s,y} = \frac{E_s}{E_b} \sigma_{mc,b} \sin \alpha_2 \]

(15)

The corresponding substitution of (14), (15) in (12), (13), and (12), (13) in the equilibrium condition (11), allows determining the stress in the concrete:
Thus, in solving the problem of assessing stresses in concrete of web plate $\sigma_{mt,b}$ and $\sigma_{mc,b}$, the initial data are the main tensile $\sigma_{mt}$ and main compressive $\sigma_{mc}$ stresses, the composition of the transverse ($n_y$, $u_x$, $A_{bg, y}$) and longitudinal ($n_y$, $u_x$, $A_{bg, y}$) reinforcement bars and angles of direction of the main stresses $\alpha_1$, $\alpha_2$. The procedure and regulations for determining $\sigma_{mt}$ and $\sigma_{mc}$, which depend on normal stresses in concrete along the longitudinal axis $\sigma_{bx}$, normal stresses in concrete $\sigma_{by}$ perpendicular to the longitudinal axis, and tangential stresses $\tau_b$, are described in SP 2.05.03-84*.

Direction angles of the principal tensile and compressive stress actions $\alpha_1$ and $\alpha_2$ (Figures 1, 2) are based on the following expressions:

$$\tan \alpha_1 = \frac{\sigma_{mt} - \sigma_{bx}}{\tau_b}$$  \hspace{1cm} (17)

$$\tan \alpha_2 = \frac{\sigma_{mc} - \sigma_{bx}}{\tau_b}$$  \hspace{1cm} (18)

where:

- the principal stresses $\sigma_{mt}$, $\sigma_{mc}$ and normal stresses $\sigma_{bx}$ have a plus sign (+) during tension and a minus sign (-) during compression.
- shear stresses $\tau_b$ have a plus sign (+) when the normal stress vector $\sigma_{bx}$ must be rotated 90° clockwise before being combined with the stress vector $\tau_b$.

The angles $\alpha_1$, $\alpha_2$ calculated from (17), (18) are counted clockwise from the x axis when they have a positive value, and counterclockwise when they have a negative value. In expressions (10) and (16), the values of the angles $\alpha_1$, $\alpha_2$ are used in absolute value.

5. Practical significance

The effectiveness of the obtained solutions was shown in the task of adapting superstructures according to the standard project of the series 3.503.1-81 to temporary loads A14, H14 [8, 9] and in verification calculations [6]. The verification calculations showed that it is possible to skip loads A14, H14 for a 33 m long superstructure with a girder height of 1.53 m and a girder spacing in the cross section of 2.0 m which use beams according to the standard project 3.503.1-81. It is possible by reinforcing the bottom flange by laying two bars Ø32AIII and reinforcing the web plates by installing double layer stirrups and double layer longitudinal bars. Also calculations [7] showed that it is feasible to use girders according to the standard project of the series 3.503.1-81 in a 24 m long superstructure with a girder spacing of 2.1 m when reinforcing the bottom flange by laying 4 bars Ø28AIII and reinforcing web plates by installing double layer stirrups and double longitudinal bars for class B40 concrete.

The option of heavy reinforcement of web plate was obtained on the basis of web plate net according to the standard project of the series 3.503.1-81 issue 7-1, in which double layer stirrup and double horizontal bars were used. Heavy reinforcement of the bottom flange was obtained together with the reinforcement by bars of prestressed reinforcement using K7 ropes and additional laying of the reinforcing bars with 4028AIII composition and bar length equal to half of the design span in the bottom flange oriented symmetrically relative to the middle of the span.

The deficiency of the bearing capacity and crack resistance of the girder normal sections is compensated by placing four additional bars in the bottom flange symmetrically relative to the middle of the span. The bars are made from AIII class steel and have 28 mm diameter and a length equal to half the design span.

Table 1 shows an example of heavy reinforcement of web plate to confirm the effect of the proposed solutions for taking into account composite action of reinforcement and concrete in inclined sections of the girder with prestressed reinforcement. The results presented in Table 1 served as the basis for the use of load-bearing structures according to the standard project of the series 3.503.1-81 in

$$\sigma_{mc,b} = \frac{\sigma_{mc}}{1 + \frac{E_{mc}}{E_b} \frac{\alpha_2}{\alpha_1} \left( \cos^2 \alpha_2 - \cos^2 \alpha_1 \right)}$$  \hspace{1cm} (16)
a 24 meters long superstructure with a dimension of G-10+0.75+2.25 m. The superstructure had seven girders in cross section with the girder spacing of 2.1 m [9] under loads A14, H14 according to GOST 32960-2014 and changes in No. 1 to SP 35.13330.2011 as of December 2016.

**Table 1.** Review of the principal tensile stresses taking into account the composite action of concrete with reinforcement of stirrups and longitudinal bars in web plate.

| Beam position | Data for the center of gravity of section | Limitations at B40 0.85R_{d,ser} |
|---------------|-----------------------------------------|-----------------------------------|
| Intermediate beam | \( \sigma_u^{\text{III}} \) (kgf/cm²) | \( \sigma_x \) (kgf/cm²) | \( \tau \) (kgf/cm²) | \( \sigma_{\text{int}} \) (kgf/cm²) | \( \sigma_{\text{int},b} \) (kgf/cm²) |
| 5.43            | 49.9                                   | 12.5                              | 33.9                              | 31.2                              | -19.1                             | -14.92                             |
| 4.2             | 43.6                                   | -7.3                              | 31.2                              | 34.0                              | -20.8                             | -15.63                             |
| 3.63            | 37.0                                   | 8.2                               | 31.2                              | 35.8                              | -21.5                             | -16.02                             |
| 2.7             | 29.6                                   | 7.9                               | 29.7                              | 36.5                              | -23.0                             | -17.23                             |
| 2.4             | 25.9                                   | 1.1                               | 27.5                              | 32.5                              | -21.1                             | -15.69                             |

1. The stress state parameters were found for the indicated girder sections as a part of a 24 m long span with a dimension of G-10+0.75+2.25 m from the load A14, H14 according to GOST 32960-2014.
2. Stresses \( \sigma_u \) were found according to clause 7.104 of SP 35.13330.2011.
3. Stresses \( \sigma_{\text{int}} \) were found according to the expression (10). They take into account the composite action of concrete and reinforcement along the principal planes of the web plate.

6. Implementation Results

The validity of the obtained solutions for assessing stresses in concrete along the trajectory of the principal tensile stresses \( \sigma_{\text{nt},b} \) according to (10) and along the trajectory of the principal compressive stresses \( \sigma_{mc,b} \) according to (16) is confirmed by the data of testbed trials of a girder with the length \( L = 24 \text{ m} \) and prestressed reinforcement according to the standard project of the series 3.503.1-81 with heavy reinforcement of web plate and bottom flange.

7. Conclusions

As conclusion, it can be noted that the obtained expressions (10), (16) make it possible to purposefully and reasonably change the composition of the transverse and longitudinal reinforcement bars in providing the required value of the main stresses satisfying the conditions of crack resistance in the calculations for the second group of limit states for crack formation. The relevance of this test is obvious in relation to structures which reinforcement scheme does not contain reinforcing elements with a certain orientation along the line of action of the principal tensile stresses, as indicated in the publication [12].

Thus, the materials in this publication emphasize the validity and feasibility of the solutions which CNIIH OJSC and Pacific National University presented in [5] and aimed at strengthening the reinforcement of load-bearing structures according to the standard project of the series 3.503.1-81 for loads A14, H14 according to GOST 32960-2014.

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