Numerical models of rotating accretion flows around black holes

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Abstract. Numerical, two-dimensional, time-dependent hydrodynamical models of geometrically thick accretion discs around black holes are presented. Accretion flows with non-effective radiation cooling (ADAFs) can be both convectively stable or unstable depending on the value of the viscosity parameter $\alpha$. The high viscosity flows ($\alpha \simeq 1$) are stable and have a strong equatorial inflow and bipolar outflows. The low viscosity flows ($\alpha \lesssim 0.1$) are convectively unstable and this induces quasi-periodic variability.

1. Introduction

Advection-dominated accretion flows (ADAFs) have recently attracted much attention since they naturally explain the properties of X-ray transients, low luminosity active galactic nuclei and other high energy objects. Most of the information of ADAF structure derives from a simplified vertically integrated approach, which reduces the complicated three-dimensional problem of accretion flow hydrodynamics to a one-dimensional problem (see Narayan & Yi 1995; Chen, Abramowicz & Lasota 1997; Narayan, Kato & Honma 1997; Igumenshchev, Abramowicz & Novikov 1998). In the vertically integrated approach only the radial structure of the disc is studied in a detailed way. Due to significant simplifications introduced by the vertical integration, some important effects, such as convection (Narayan & Yi 1994; Igumenshchev, Chen & Abramowicz 1996) or outflows (see discussion in Narayan & Yi 1994) are not properly treated. Understanding of ADAF properties could be improved by a discussion of two-dimensional time-dependent hydrodynamical models, where one explicitly treats both the radial and vertical structure of the accretion flow.

In this contribution we present preliminary results of two-dimensional numerical simulations of rotating accretion flows around black holes. Complete discussion of the results will be published (Igumenshchev & Abramowicz 1999).

We construct hydrodynamical time-dependent models with viscosity parameter $\alpha \sim 0.1 - 1$ and high geometrical thickness. We assume a simplified model of the radiative cooling: $(1 - \varepsilon)$ of the energy generated by the viscous dissipation is radiated away. We consider a large value of $\varepsilon$ (0.5 $\leq$ $\varepsilon$ $\leq$ 1), when the accretion flow has a low efficiency of the conversion of its internal energy.

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to the escaping radiation. We demonstrate that stability of the flow strongly depends on the value of viscosity. The low viscosity flows ($\alpha \ll 0.1$) are convectively unstable, and the instability produces a quasi-periodic behaviour of the accretion flows and outflows. In the case of high viscosity ($\alpha \sim 1$), the convective instability is suppressed, and the flow is stable.

2. Numerical method

Our numerical technique is based on the solution of the non-relativistic Navier-Stokes equations in the spherical coordinate system ($r$, $\theta$, $\varphi$) in a stationary gravitational field of the black hole. The gravity of the black hole is modeled by the Newtonian potential

$$\Phi(r) = -\frac{c^2 r_g}{2} \frac{r}{r},$$

(1)

where $r_g = 2GM/c^2$ is the gravitational radius of black hole of mass $M$. We assume axial symmetry of the flow with respect to the rotational axis that coincides with the $\theta = 0$ direction. We take into account the contribution of all components of the viscous stress tensor to the equations of motion and the energy equation. Shear viscosity is only considered.

For simplicity, we describe the dynamics of accreting plasma in the framework of the one fluid approximation. We use the equation of state for an ideal gas. The adiabatic index of gas is assumed to be a constant, and takes the value $\gamma = 3/2$. In this case the equation of thermal energy conservation can be written in the form:

$$\rho T \frac{dS}{dt} = Q_{\text{visc}} - Q_{\text{rad}},$$

(2)

where the operation $d/dt$ is the comoving (Lagrangian) time derivative, $S$ is the specific entropy, $Q_{\text{visc}}$ is the dissipation function and $Q_{\text{rad}}$ is the volume cooling rate. The problem of the radiation losses in the high temperature magnetized plasma is a quite difficult one and it is far from the complete solution. We do not address this problem here, but instead assume a simple model for the cooling rate,

$$Q_{\text{rad}} = (1 - \varepsilon)Q_{\text{visc}},$$

(3)

where $\varepsilon$ is a parameter, $\varepsilon \leq 1$. The case $\varepsilon = 1$ corresponds to the non-radiating accretion flow, whereas $\varepsilon = 0$ corresponds to the isentropic flow.

The kinematic viscosity coefficient is described by

$$\nu = \alpha \frac{c_s^2}{\Omega_K},$$

(4)

where $\alpha$ is a constant, $0 < \alpha \ll 1$, $c_s = \sqrt{RT/\mu}$ is the isothermal sound speed, and $\Omega_K = c\sqrt{r_g/2r^3}$ is the Keplerian angular velocity.

To solve the Navier-Stokes equations we split the numerical procedure of one time step $\Delta t$ calculation into two parts, hydrodynamical and viscous. The hydrodynamical part is calculated by using the explicit finite-difference PPM algorithm developed by Colella & Woodward (1984). The viscous part is solved...
by the implicit operator splitting method for the contributions to the equations of motion. The viscous contribution to the thermal energy equation (2) is calculated using an explicit scheme. The time step $\Delta t$ is chosen in accordance with the Courant condition for the hydrodynamical sub-step.

We use the absorbing inner boundary at the radius $3 r_g$, which is the location of the last stable orbit of the Schwarzschild black hole. The outer boundary is located at the radius $\simeq 300 r_g$. At this radius we inject the matter with a fixed rate $\dot{M}_{\text{inj}}$, close to the equatorial plane. Injected matter has zero $r$- and $\theta$-components of velocities, and the angular momentum close to the Keplerian one.

We look for stationary or quasi-stationary solutions, which are obtained in the course of the time-dependent calculations from the initial state. This requires to follow the evolution of flow pattern during a few characteristic accretion times, measured as an average time of motion of the matter from the outer boundary to the inner one.

3. Results

We have calculated a number of evolved models with four different values of $\alpha = 1, 0.3, 0.1$ and 0.03, and different values of $\varepsilon$, which vary in range $0.5 - 1$. The models show a weak dependence on $\varepsilon$, and they strongly depend on $\alpha$.

For large value of $\alpha = 1$ and 0.3 the models are stable. They are symmetric with respect to the equatorial plane and show no time-dependent behaviour. Figure 1 represents the flow pattern of the model with $\alpha = 1$ and $\varepsilon = 1$. In this figure the meridional cross-section of the model is shown, and vertical axis coincides with the axis of rotation. Black hole locates in the origin $(0,0)$. Axes are labeled in the units of $r_g$. Upper left plot shows the contours of density $\rho$. The contour lines are spaced with $\Delta \log \rho = 0.1$. Upper right plot shows the contours of pressure $P$. The lines are spaced with $\Delta \log P = 0.1$. The density and pressure monotonically increase toward the black hole. In lower left plot the arrows with the length in relative units show the momentum vectors $\rho v$. The flow pattern consists of the equatorial inflow and bipolar outflows, which originate very close to the black hole, at radius $8 r_g$. Lower right plot shows the contours of Mach number $\mathcal{M}$. The lines are spaced with $\Delta \mathcal{M} = 0.05$. The maximum value of $\mathcal{M}$ at given radius is reached at the equatorial plane. Two stagnation points, where $\mathcal{M} = 0$, locate at the axis of rotation at radius $8 r_g$.

The flow is subsonic ($\mathcal{M} < 1$) everywhere in our computation domain, which has inner boundary at $r = 3 r_g$.

The model with $\alpha = 0.3$ does not show deep outflows. The stagnation points locate at radius $80 - 90 r_g$ on the axis of rotation. Inside this radius the matter moves to the black hole almost radially. Oppositely to the case of $\alpha = 1$ the distribution of the Mach number in this model has an equatorial minimum at a given radius. This minimum indicates that the inflowing matter is overheated in the inner equatorial parts due to viscous dissipation. In the low viscosity models this overheating is the reason for development of the convective instability.

Low viscosity models with $\alpha \leq 0.1$ are unstable. They demonstrate rich and complicated time-dependent variations of flow pattern. Example snapshot of flow pattern of the convectively unstable model with $\alpha = 0.1$ and $\varepsilon = 1$ is
Figure 1. The flow pattern of stable model with $\alpha = 1$ and $\varepsilon = 1$ (see text for details).
shown in Figure 2. In the upper left plot for density distribution it is clearly seen the reason for the instability — hot convective bubbles, which quasi-periodically originate in the innermost region (4−6rg from the center) of accretion flow and propagate outward.

The bubbles arise from the initial perturbations, which usually appear in the θ-directions, where the maximum mass flux onto the black hole occurs at that moment. As a rule, the bubbles originate near, slightly above or below, the equatorial plane. When a bubble has developed, it forces the direction of the maximum mass inflow to change. In the new direction, a new convective bubble originates when the previous bubble reaches the radial distance 50–100 rg. This cycle repeats quasi-periodically. Typically, convective instability produces sequences of convective bubbles, where the previous and next bubbles originate and move outward in different hemispheres with respect to the equatorial plane. The quasi-periodic behaviour of the convective bubbles, accompanied by the
outflows in polar directions, produces a significant variability of the flow pattern inside $r \sim 100r_g$. The intensity and directions (up or down) of the outflows strongly correlate with the convective activity in the vicinity of the black hole.

The spatial scale of perturbed motion in the accretion flow becomes smaller with decreasing of the viscosity parameter $\alpha$. Accordingly, the flow pattern is more complicated in model with $\alpha = 0.03$. Small scale vortices accompany the convective motion. The vortices exist quite a long time and strongly interacts with convective bubbles.

We have studied the time-dependent behaviour of the unstable models calculating radiative energy losses in the case of proton-electron bremsstrahlung cooling for optically thin plasma,

$$L(t) \propto \int \rho^2 T^{1/2} dV.$$  

(5)

Here the integration is taken over the volume of a sphere with the radius 100 $r_g$. Fourier analysis of time series of the radiative cooling rates shows the presence of a strong feature at frequencies $10 - 20 (M_\odot/M)$ Hz in the power spectrum of model with $\alpha = 0.1$. In the case of $\alpha = 0.03$, a weaker feature is observed at $1 - 2 (M_\odot/M)$ Hz. These oscillations can be explained by quasi-periodic presence of convective bubbles and perturbations of the accretion flow introduced by these bubbles. Note, this variability has typical time scales in the observed quasi-periodic oscillations (QPOs) range of the Galactic black hole candidate X-ray sources.

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