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Big Bounce and inhomogeneities

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Abstract
The dynamics of an inhomogeneous universe is studied with the methods of loop quantum cosmology, via a so-called hybrid quantization, as an example of the quantization of vacuum cosmological spacetimes containing gravitational waves (Gowdy spacetimes). The analysis of this model with an infinite number of degrees of freedom, performed at the effective level, shows that (i) the initial Big Bang singularity is replaced (as in the case of homogeneous cosmological models) by a Big Bounce, joining deterministically two large universes, (ii) the universe size at the bounce is at least of the same order of magnitude as that of the background homogeneous universe and (iii) for each gravitational wave mode, the difference in amplitude at very early and very late times has a vanishing statistical average when the bounce dynamics is strongly dominated by the inhomogeneities, whereas this average is positive when the dynamics is in a near-vacuum regime, so that statistically the inhomogeneities are amplified.

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(Some figures in this article are in colour only in the electronic version)

The cosmological models based on classical general relativity (GR) predict that the Big Bang singularity is the true beginning of the universe (boundary of the spacetime). This prediction, however, is believed not to be physical since, when one describes the early universe, GR is applied beyond its domain of validity. The quantum effects which dominate in this epoch are expected to resolve the singularity.

This issue was studied recently in the context of loop quantum cosmology (LQC) [1]. The analysis of a simple model of a homogeneous and isotropic universe revealed surprising results [2]: a large classical expanding universe was preceded by a (also large and classical) contracting one, which bounced (deterministically) and started to expand once the energy density of its matter content reached the Planck scale. The robustness of this result in more general situations was confirmed later [3, 4], including the case of anisotropic universes.

However, up to now these studies were mostly restricted to homogeneous spacetimes. Although a preliminary analysis of the effects of inhomogeneities was presented in [5], it was not known whether the modifications to the universe dynamics predicted by LQC...
survive in the presence of inhomogeneities. The possibility of answering this question arose when an LQC quantization scheme was formulated [6] for a class of cosmological spacetimes known as Gowdy universes [7]. These spacetimes, while still symmetric (they admit two spatial symmetries), include inhomogeneities that can be interpreted as (linearly polarized) gravitational waves, thus having local degrees of freedom. To describe them, a hybrid quantization scheme was applied: first the geometry was represented as the Fourier modes of a gravitational field (the linearly polarized wave) propagating in a homogeneous (Bianchi I) spacetime, next this Bianchi geometry was quantized using loop techniques, while for the gravitational wave modes standard Fock quantization methods were employed after introducing a suitable time-dependent scaling.

This construction paved the way to analyse the quantum dynamics of this inhomogeneous system; nonetheless, the (field-like) complexity of the quantum configuration space makes the investigation of the genuine quantum evolution extremely difficult. This forces one to resort to the so-called effective dynamics [8, 9]—a classical theory which incorporates the main effects of spacetime discreteness and which has been shown to accurately mimic the quantum evolution in the cases where it has been tested on so far. Here, we apply this technique to analyse the quantum dynamical behaviour of the Gowdy universe, answering in particular the following questions: (i) Does the Big Bounce persist in the presence of inhomogeneities? (ii) If the answer is in the affirmative, does the Big Bounce occur in similar conditions as in homogeneous models? (iii) How does the structure of the gravitational wave modes change through the bounce?

Let us start by specifying in more detail the physical system which we consider here. The Gowdy universes are vacuum spacetimes with compact sections of constant time which, in spite of possessing considerable symmetry, still contain local degrees of freedom. Namely, these spacetimes possess two spatial isometries (two commuting spacelike Killing vectors). The Gowdy universes can be classified by their spatial topology [7]. The best studied case, on which we will concentrate, is that with the topology of a three-torus. This family of spacetimes provides a generalization of the homogeneous and anisotropic Kasner solution (for spatially flat topology) to include inhomogeneities which depend only on one spatial coordinate [10]. The spacetime inhomogeneities can be interpreted as gravitational waves propagating in a homogeneous background spacetime. We will consider exclusively the simplest case of linearly polarized waves, in which, after a suitable gauge fixing, all the inhomogeneities can be described by a single metric field and the metric adopts a diagonal form globally [6, 11]. That field can be expanded in Fourier series exploiting the periodicity in the only spatial coordinate on which it depends (this coordinate is cyclic for the studied topology). Strictly speaking, the inhomogeneities are determined by the nonzero modes of this decomposition. The rest of gravitational degrees of freedom of the system describe a homogeneous universe on which the gravitational waves propagate. Specifically, this is a Bianchi I spacetime with three-torus topology. The classical solutions of this cosmological model are known in exact form, and generically present a Big Bang singularity [12].

In order to quantize the system we follow the prescription used in [2], where the geometry (homogeneous) degrees of freedom were quantized via loop techniques, whereas for the ‘matter’ ones (in this case the gravitational waves with a convenient field parametrization) standard (Schroedinger-like) methods are employed [11, 13]. The classical metric of the Bianchi I background in the adopted gauge is diagonal and determined by a triple of scale factors $a_i(t)$, where $i = 1, 2, 3$. In the formalism used for quantization, the phase space for this background is coordinatized by the $SU(2)$ (Ashtekar) connection $A^a_\mu = c^\mu \delta^a_\mu / (2\pi)$ and the densitized triads $E^a_\mu = p_\mu \delta^a_\mu / (4\pi^2)$ [14]. All the information about the system is encoded in the canonically conjugate variables $(c^j, \rho_i)$, where $|\rho_i| = |(\epsilon^{ijk} a_j a_k) / (8\pi)|$ ($\epsilon^{ijk}$ is the completely
antisymmetric unit tensor). These variables are physically meaningful quantities, without ambiguities introduced by choices of fiducial structures, since the system under consideration is spatially compact. Among the constraints that GR imposes on the system, only two are not automatically satisfied in the introduced gauge: the generator of $S^1$ translations on the inhomogeneous spatial direction (which affects exclusively the inhomogeneities described by the gravitational waves) and the spatial average of the Hamiltonian constraint. The latter can be written (up to a global constant) as $C = C_{\text{II}} + C_F$ (where $C_{\text{II}}$ and $C_F$ stand for the homogeneous background part and the ‘matter’ correction encoding inhomogeneities, respectively). Specifically, the background part equals $C_{\text{II}} = \int_\Sigma d^3x |\det(E)|^{-1/2} e^{abc} E_a^i F_{ij}^c$, where $F_{ij}$ is the curvature (field strength) of the connection $A_i^c$. The quantization methods parallel those of LQG [15]. In a first step, the constraint is ignored. The basic objects promoted to operators are the integrals of $A_i^c$ along straight lines (holonomies) and those of $E_a^i$ along square surfaces (fluxes). The resulting kinematical Hilbert space is a product $\mathcal{H}_{\text{kin}} = \bigotimes_{\mu=1}^{3} L^2(\mathbb{R}_B, d\mu_B) \otimes \mathcal{H}^{\text{kin}}_\pi$ where $\mathbb{R}_B$ is a Bohr compactification of the real line and $\mathcal{H}^{\text{kin}}_\pi$ is a Hilbert space for the inhomogeneous degrees of freedom. In the next step, the constraint $C$ is promoted to (and solved as) a quantum operator. For this, $C_{\text{II}}$ has to be expressed in terms of holonomies and fluxes. In particular, the term $F_{ij}^c$ entering $C_{\text{II}}$ is approximated by holonomies along small rectangular loops. Since the limit of the loop shrinking to zero does not exist in LQC, the rectangular loop is fixed (following [4]) by the requirement that its physical area equals the lowest nonzero eigenvalue of an operator defined in LQG. In the matter part of $C$, the degrees of freedom corresponding to gravitational wave modes (conveniently scaled by a time-dependent function) are represented via fields and momenta operators combined into creation and annihilation operators $(\hat{a}_m^\dagger, \hat{a}_m)$, $m = \pm 1, \pm 2, \ldots$, with standard commutation relations [11]. Therefore, they form the standard Fock space $\mathcal{H}^{\text{phy}}_\pi = \mathcal{F}$. The representation of these degrees of freedom is uniquely fixed by the requirement of a unitary implementation of the physical evolution parametrized by the areas of the Killing orbits, together with the invariance of the vacuum under the gauge group of $S^1$-translations [13].

The final quantum constraint $\hat{C}$ is a difference operator in all three coefficients $p_i$. In principle, one can find the (generalized) states annihilated by it, thus identifying the physical Hilbert space $\mathcal{H}^{\text{phy}}$. This is indeed done in [6]. However, the representation of any state of physical interest is complicated to the extreme by the presence of an infinite number of degrees of freedom. To be able to extract interesting physics out of the system, we appeal here to the so-called effective dynamics.

To arrive to this effective description (see [8] for details), one uses the heuristic method of replacing the basic operators (holonomies and fluxes) in the Hamiltonian constraint by their expectation values, thus building back a classical Hamiltonian. Nonetheless, some aspects of the quantum theory are preserved by leaving the lengths of holonomies as determined by the minimal area requirement discussed above.

The final form of the constraint (after proper densitization) reads

\begin{align}
C_G &= -\frac{2}{\gamma^4} (\Theta_1 \sqrt{p_1} \Theta_2 + \Theta_1 \Theta_3 + \Theta_2 \Theta_3) + \frac{G}{\gamma^2} (\Theta_2 + \Theta_3)^2 + \frac{1}{|p_1|} H^{\ell}_{\text{int}} + 32 \pi^2 G |p_1| H^{\ell}_\phi, \\
\Theta_i &= \frac{p_i \sqrt{|p_i|}}{M} \sin \left( \frac{M c_i \ell}{\sqrt{|p_i|}} \right), \quad H^\ell_{\phi} = \sum_{m=1}^{\infty} m [a^*_m a_m + a^*_m a_m], \tag{2a}
\intertext{and}
H^{\ell}_{\text{int}} &= \sum_{m=1}^{\infty} m [a^*_m a_m + a^*_m a_m + a^*_m a_m + a^*_m a_m]. \tag{2b}
\end{align}
Here, \( i = 1 \) denotes the direction in which there exists spatial dependence, \( G \) is Newton’s constant and \( M = \sqrt{2/3\pi y l_p} \) (\( y \) is the Immirzi parameter and \( l_p = \sqrt{\hbar G} \) the Planck length). We note that the variables \( \Theta_2 \) and \( \Theta_3 \) are constants of motion, a fact which will considerably simplify the discussion of the effective dynamics. We also notice that the inhomogeneous modes interact with the ‘background’ homogeneous geometry only through the ‘energy’ terms \( H_0^\xi \) and \( H_0^\eta \). As required by consistency with their vacuum expectation values in the introduced Fock quantization, these terms vanish when the inhomogeneous modes have zero amplitudes.

Given this effective constraint, it is straightforward to derive the complete (countable) set of equations of motion (EOM’s), namely the Hamilton–Jacobi equations. By integration, one then obtains the time evolution of the system from any initial point in the phase space.

At this stage, it is worth pointing out that the above procedure does not take into account state-dependent parameters (such as dispersions and higher order moments) which might significantly affect the dynamics. However, comparison of this effective approximation against the full quantum dynamics has been carried out in simpler models (i.e. homogeneous cosmologies with both massless and massive scalar fields), showing that the corrections arising from such parameters are negligible for the states of physical interest (semiclassical at late times). This fact has been confirmed analytically in the simplest cosmological model [9]. Although the validity of the adopted effective description has not been tested in the model considered here, the commented results (and the observation that modes with different values of \( m \) interact only through their backreaction on the Bianchi I geometry) strongly support the ability of this description to reproduce the behaviour of the quantum system accurately.

The effective system specified above was employed to analyse the evolution of inhomogeneous universes. The study was focused on two issues: (a) the existence of the bounce and (b) the changes in the structure of the inhomogeneities through the bounce.

In order to address the first of these issues, the evolution of the inhomogeneous universe was compared against the evolution of its homogeneous counterpart, i.e. the universe determined by the same initial data for the homogeneous degrees of freedom and all inhomogeneities set to vanish. Our study confirms the presence of the bounce in all three spatial directions (see figure 1). Thus, the evolution picture is qualitatively the same as in homogeneous models. Remarkably, the form of the EOM’s allows one to extract qualitative features of this bounce analytically, therefore ensuring that such features are valid for general solutions (in the region of physical interest). One of these features comes from the observation that the values of \( p_2 \) and \( p_3 \) when they bounce do not depend on the \( a_m \)’s; hence, the bounces in the homogeneous directions occur at the same values of \( p_i \) as for homogeneous universes. On the other hand, to check the behaviour of the bounce in the inhomogeneous direction, more extensive analytical/numerical studies were performed. Exploiting the properties of the EOM’s, the investigation was carried out separately (and using different methods) in the two domains: \( \Theta_2\Theta_3 < 0 \) and \( \Theta_2\Theta_3 > 0 \).

In the first case, it follows straightforwardly from (1) that the presence of inhomogeneities can only push the bounce away, that is, the bounce happens at a larger value of \( p_1 \). Indeed, recalling that \( p_1 = \{p_1, C\} \) and taking into account that \( \dot{p}_1 = 0 \) at the bounce, one gets the constraint satisfied at that point,

\[
|p_1|^3 = M^2 \frac{F\left(p_1, \Theta_2, \Theta_3; H_0^\xi, H_0^\eta\right) - \Theta_2\Theta_3}{(\Theta_2 + \Theta_3)^2},
\]

(3)

where \( F \) is a positive definite function such that \( F(p_1, \Theta_2, \Theta_3; 0, 0) = 0 \). The form of (3) implies immediately that, when \( \Theta_2\Theta_3 < 0 \), the contribution of \( F \) increases the value of \( p_1 \) (provided that \( \Theta_2 \) and \( \Theta_3 \) remain the same). As a consequence, the bounce in the inhomogeneous case always happens at larger universe sizes than in the homogeneous scenario.
Figure 1. A dynamical trajectory for the case \( m_{\text{max}} = 5 \) (black/thicker line) compared against the classical trajectories to which it converges in the distant future and past (red/thinner lines). Here, the values of \( H_{\xi o} \) and \( H_{\xi \text{int}} \) at the initial point \( (p_1 = p_2 = p_3 = 180 l_P^2) \) are 0.657\( h \) and 0.0507\( h \), respectively. The constants of motion \( \Theta_2 \) and \( \Theta_3 \) are chosen to be 100\( l_P^2 \) and the Immirzi parameter is set to its standard value, \( \gamma = 0.2375 \ldots \).

Figure 2. The set of points satisfying relation (2) for the case \( \Theta_2 = 3750 l_P^2 \), \( \Theta_3 = 2500 l_P^2 \) and \( H_{\text{int}} / H_o = 2 \times 10^{-4} \). They form reflective boundaries for the dynamical trajectories.

The case \( \Theta_2 \Theta_3 > 0 \) required a detailed numerical analysis because the two terms in brackets in the numerator of (3) then have opposite signs. Nevertheless, since the inhomogeneities enter (3) only through the terms \( H_{\xi o} \) and \( H_{\xi \text{int}} \), it was possible to analyse the entire space of parameters thoroughly. In particular, for those parameters which run over unbounded intervals, appropriate asymptotic behaviours were verified in order to ensure the robustness of the results. The dependence of the points satisfying (3) with respect to the magnitude of the inhomogeneities (specifically, on \( H_{\xi o}^2 \)) is shown in figure 2. A feature which is worth noting is the existence of the throat which allows, in principle, that a dynamical trajectory may go ‘down it’ through an infinite sequence of bounces and recollapses, reaching the singularity. However, an extensive numerical analysis of the dynamical trajectories shows that this would be a critical trajectory (the set of initial data leading to such evolution has zero measure in the phase space). Thus, generically the universe will bounce at finite \( p_1 \). Besides, except perhaps for a very small subset of initial conditions (near the critical case), the bounce
happens at a value of \( p_1 \) above certain bound, which is of similar order to the value found in the absence of inhomogeneities, \( p_1^c \). The bound provided by our analysis is \( \approx 0.13 p_1^c \).

Our study of the inhomogeneities was focused on discussing how their energy distribution changes through the bounce. This information is encoded in \( |a_m| (t) \). Since, for large \( p_1 \), these quantities converge to well defined limits, it is particularly interesting to investigate the corresponding amount of asymptotic change for large universes, \( \Delta |a_m| = \lim_{t \to +\infty} |a_m| - \lim_{t \to -\infty} |a_m| \). Actually, once the full set of initial data is specified, the evolution of the universe (including the inhomogeneities) is deterministic, so that the value of \( \Delta |a_m| \) is fixed. Nonetheless, \( \Delta |a_m| \) acquires a stochastic nature if one restricts its attention to energies instead of amplitudes for each of the gravitational wave modes, thus ignoring the initial phases of the \( a_m \)'s.

To investigate the behaviour of the \( a_m \)'s, the dynamical trajectories were studied numerically. Provided that \( H_0^I \) (which may be interpreted as a total energy of the inhomogeneities) be finite in the asymptotic past, the amplitudes of the inhomogeneous modes must satisfy certain fall-off conditions for large \( m \)'s. This and a careful analysis of the EOM’s allow us to show that \( H_0^I \) generically stays finite also in the asymptotic future. Given the form of the equations, that implies in turn that the numerical solutions computed by taking a sufficiently large, but still finite number of modes, \( m \leq m_{\text{max}} \), converge to the true trajectories as \( m_{\text{max}} \to \infty \). This convergence allows us to employ the standard numerical recipe of considering just a finite number of modes, obtaining the numerical trajectories with a good precision when this number becomes large enough. Furthermore, it turns out that those features of the inhomogeneous modes behaviour which are relevant to our study remain unchanged as the number of modes increases, so that the results of our investigation are indeed robust.

We probed the dynamics of the effective system using Monte-Carlo methods. First, a large population (about \( 10^4 \) points) of initial points was selected randomly in the phase space. In particular, apart from the restriction to satisfy the \( S^1 \)-translation constraint, the values of the \( a_m \)'s were generated randomly with Gaussian probability distribution (centred at the origin).

Owing to technical limitations, simulations were restricted to a finite number of nonvanishing modes (specifically, the cases considered were \( m \leq m_{\text{max}} = 5, 10, 20 \)). To find the dynamical trajectories, the initial value problem consisting of the full set of EOM’s plus the chosen initial data was integrated via built-in adaptive methods of \textit{Mathematica}.

In the light of the simulations, one can distinguish between two distinct regimes within the space of possible trajectories: \( (i) \) \textit{inhomogeneities dominated}, for which the dynamics around the bounce is dominated by the content of gravitational waves, and \( (ii) \) \textit{near-vacuum}, for which those waves introduce only small corrections to the vacuum Bianchi I dynamics around the bounce. In case \( (i) \), our numerical analysis of a large population of universes shows that, generically, \( \Delta |a_m| \) is an antisymmetric function of the initial phase of \( a_m \). Owing to this antisymmetry, the expectation value of \( \Delta |a_m| \) (namely, its average over the dependence on initial phases) is ensured to vanish. In case \( (ii) \), a similar analysis shows that the antisymmetric behaviour of \( \Delta |a_m| \) is generically lost and its average becomes strictly positive. Therefore, in the near-vacuum case the quantum geometry effects around the bounce \textit{pump energy} into the inhomogeneities\(^1\).

It is worth emphasizing that, although our numerical analysis has been performed in all cases for a finite number of nonvanishing modes \( m_{\text{max}} \) (which plays the role of an UV cutoff on the gravitational waves), the conclusions reached here remain valid when the full Fock space of inhomogeneities is considered, as we discussed above. The results explained above

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\(^1\) Bouncing models in LQC are free from problems owing to entropy growth since the entropy clock is reset just before the bounce by the appearance of a dynamical horizon [16].
constitute the first systematic analysis of the dynamics of inhomogeneous spacetimes in LQC and, moreover, in the presence of an infinite number of degrees of freedom. In particular, they provide further support to the bounce picture found earlier in homogeneous scenarios. The reported study, and the methodology developed for it, paves the way for future analyses of the effective physics derived from LQG/LQC in more general spacetimes, opening an avenue, e.g. for the discussion of the effects of quantum geometry in the process of gravitational collapse and black hole formation.

Let us conclude clarifying that the presented analysis should not be treated as final. First, our inhomogeneous system has been investigated by means of an effective theory which does not take into account many quantum effects. To confirm the reliability of the results, one ought to repeat the analysis in the full quantum setting. Secondly, the method of construction of the minimal area loop used to define the quantum constraint $\hat{C}$ is not unique. There exist other prescriptions for the construction, giving different quantitative predictions of the dynamics (more specifically, see [17]). In particular, the prescription used here does not provide physically reliable results when applied to the noncompact cases. Furthermore, on the basis of the effective description, it has been argued [18] that it does not distinguish a unique scale of quantum phenomena. Therefore, an investigation of the system constructed with any other such prescription (and specifically that of [17]) seems necessary. Finally, one has to remember that, in the studied system, there is only one inhomogeneous direction; therefore, the results regarding change of inhomogeneities through the bounce cannot be directly applied to the kind of models considered in observational cosmology. To obtain reliable results verifiable against observations, one has to extend the analysis to models which have the energy level degeneracy characteristic of a spherical harmonics decomposition. This is the task that we plan to accomplish in future work.

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