A strong $\ddot{\nu} - \dot{\nu}$ correlation in radio pulsars with implications for torque variations

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Submitted, 28 March, 2006; accepted 16 May, 2006

ABSTRACT

We present an analysis of the spin-down parameters for 131 radio pulsars for which $\dot{\nu}$ has been well determined. These pulsars have characteristic ages ranging from $10^3 - 10^8$ yr and spin periods in the range 0.4–30 s; nearly equal numbers of pulsars have $\ddot{\nu} > 0$ as $\ddot{\nu} < 0$. We find a strong correlation of $\ddot{\nu}$ with $\dot{\nu}$, independent of the sign of $\ddot{\nu}$. We suggest that this trend can be accounted for by small, stochastic deviations in the spin-down torque that are directly proportional (in magnitude) to the spin-down torque.

Key words: pulsars: general – stars: neutron

1 INTRODUCTION

Isolated pulsars exhibit fascinatingly rich timing behavior. The most dramatic variations are glitches, sudden increases in spin rate $\nu$, that are usually accompanied by increases in the magnitude of the spin-down rate (for a review of glitch properties see, e.g., Lyne, Shemar & Smith 2000, Krawczyk et al. 2003, and references therein). While large glitches have a well-defined signature, (\Delta \nu, \Delta \dot{\nu}) \approx (+, -), microglitches exhibit all possible signatures [Cordes, Downs & Krause-Polstorff 1988]. There is evidence that some isolated pulsars undergo “precession” (mutation) of their spin vectors (D’Alessandro & McCulloch 1997; Stairs, Lyne & Shemar 2000; Shabanov et al. 2001; Link & Epstein 2001; Akgun, Link & Wasserman 2006). Smaller, long-term variations in pulse phase, timing noise, are seen in all pulsars. Timing noise has been largely attributed to high frequency random walk in the pulse phase, spin frequency or spin-down rate (see, for example, Boynton et al. 1972; Cordes & Helfand 1981). Later analysis showed that the timing noise in some pulsars cannot be explained entirely in terms of high-frequency random walks, but that the spin behavior is due to discrete jumps in one or more of the spin parameters, possibly superimposed on a high frequency random walk [Cordes & Downs 1985]. Some pulsars show quasi-periodic phase variations over periods of years (for examples see, e.g., Downs & Reichl 1983; D’Alessandro et al. 1993). The term “timing noise” is perhaps a misnomer; to the extent that timing noise represents physical processes intrinsic to the neutron star, it is part of the signal.

Upon construction of the pulse phase history of a pulsar, $\phi(t)$, higher order spin parameters (spin rate $\nu$ and its derivatives $\dot{\nu}$ and $\ddot{\nu}$) are extracted from a Taylor series fit to the observed pulse phase,

$$\phi(t) = \phi_0 + \nu(t - t_0) + \frac{1}{2} \dot{\nu}(t - t_0)^2 + \frac{1}{6} \ddot{\nu}(t - t_0)^3 + \ldots,$$  \hspace{1cm} (1)

where $\phi_0$ is the phase at time $t_0$. Values of $\ddot{\nu}$ determined in this way are typically orders of magnitude larger than the prediction of the vacuum dipole model and often differ in sign; such large values of $\ddot{\nu}$ are thought to be highly noise-contaminated. [For example, Baykal et al. 1999, in a study of four pulsars that show significant quadratic trends in spin rate history, found instability in $\ddot{\nu}$ consistent with a noise process]. In this connection, we note that determination of $\ddot{\nu}$ is a difficult task since it is not a stationary quantity; values deduced for a given pulsar generally depend on the origin of time for the fitting, the length of the fit and the particular data set used (see, e.g., Hobbs et al. 2004, Fig. 7). As a result, there is considerable uncertainty in many measurements of $\ddot{\nu}$, and so most quoted values should be regarded as estimates. However, to the extent that $\ddot{\nu}$ is determined by an underlying noise process that we would like to understand, reliable estimates of $\ddot{\nu}$ constitute a useful statistic with which to study the nature of timing noise in the pulsar population as a whole. [We describe what we mean by
“reliable” below]. The idea behind the work we present here is to regard an estimate of \( \dot{\nu} \) at a given time for a given pulsar as one outcome of the intrinsic “noise” process. We consider estimates of \( \dot{\nu} \) in other pulsars as other realizations of a (possibly) similar underlying process. In this way we can quantify the strength of timing noise as a function of other spin variables: the spin rate, its derivative and the spin-down age.

A complete characterization of timing noise has not yet been achieved, though trends of the strength of timing noise with various spin-down parameters have been identified. Much previous work has used \( \dot{\nu} \) as the basic parameter with which to estimate the noise strength. Cordes & Helfand (1980) quantified timing irregularities by defining an activity parameter:

\[
A = \log \left( \frac{\sigma_R(m,T)}{\sigma_R(m,T)_{\text{Crab}}} \right),
\]

where \( \sigma_R(m,T) \) is the residual phase from a least-squares polynomial fit of order \( m \) over an interval of length \( T \) (all logarithms in this paper are base 10). To obtain \( \sigma_R(m,T) \), the measurement error to the rms residual is subtracted quadratically from the observed rms residual. Cordes & Helfand (1980) applied their definition of \( A \) for \( m = 2 \); in an analysis of 50 pulsars, they found evidence for correlations of \( A \) with spin period, period derivative, and spin-down age. More recently, Arzoumanian et al. (1994) introduced a timing noise parameter, characterizing the pulsar clock error caused by stochastic timing noise as

\[
\Delta(t) = \log \left( \frac{1}{b_\nu} |\dot{\nu}| t^3 \right).
\]

The statistic used by Arzoumanian et al. (1994) to quantify the strength of timing noise is \( \Delta_8 \equiv \Delta(t) = 10^8 \) s. The noise parameter is related to the activity parameter, \( A \), by \( A = \Delta_8 + 0.42 \). In a study of 104 pulsars, Arzoumanian et al. (1994) found a correlation between \( \Delta_8 \) and \( \dot{p} \) given roughly by

\[
\Delta_8 = 6.6 + 0.6 \log \dot{p}
\]

with a characteristic spread in \( \Delta_8 \) of a factor of \( \sim 3 \), independent of \( \dot{p} \).

In this Letter we demonstrate the existence of a strong correlation between measured values of \( \dot{\nu} \) and \( \dot{\nu} \). Nearly equal numbers of pulsars have \( \dot{\nu} > 0 \) as have \( \dot{\nu} < 0 \). Interestingly, we find that the correlations are independent of the sign of \( \dot{\nu} \). Because changes in the sign of \( \dot{\nu} \) with time have been seen in some pulsars, we attribute the nearly equal numbers of pulsars with \( \dot{\nu} > 0 \) and \( \dot{\nu} < 0 \) as reflecting stochastic variations in \( \dot{\nu} \) between two extremes over some time scale. In this interpretation, our results imply a correlation of the magnitude of timing noise (as measured by the characteristic magnitude of \( \dot{\nu} \)) with the spin-down torque on the star (as measured by the observed \( \dot{\nu} \)). We suggest that the observed trend can be accounted for by small, stochastic deviations in the spin-down torque that are directly proportional (in magnitude) to the spin-down torque.

2 AN ANALYSIS OF SPIN-DOWN PARAMETERS

For this analysis, we have used the most reliable published values of the measured frequency second derivative \( \dot{\nu} \). These include the recently published ephemerides for 374 pulsars (Hobbs et al. 2004) which have individual data spans of up to 34 yr and employ a new method to mitigate the effects of timing noise by whitening the timing residuals; the rotational parameters, particularly \( \dot{\nu} \), obtained in this way are significantly more precise than those obtained in earlier work in the sense that they give more stable timing solutions. While these \( \dot{\nu} \) values can be used in a timing model to predict the pulse phase at an arbitrary time, Hobbs et al. (2004) warn that they are orders of magnitude larger than the values predicted by the dipole braking model and so may not reflect the physics of the pulsar braking mechanism (we offer an alternative point of view below). In general, the smaller \( |\dot{\nu}| \) are less stationary than larger values, and the relative errors can exceed 100%; we have therefore selected the subset of 127 pulsars for which the quoted errors in \( \dot{\nu} \) are less than 10%. We also include four young pulsars whose \( \dot{\nu} \) have been precisely measured, so that the Hobbs et al. (2004) de-whitening process was not needed. These are B0531 + 21 (Lyne, Pritchard & Smith 1988), B0540 – 69 (Cusumano, Massaro & Mineo 2003), J1119 – 6127 (Camilo et al. 2000) and B1509 – 58 (Kaspi et al. 1994). Livingstone et al. (2003). The data set shows a very large range in pulsar spin parameters. The characteristic ages are in the range \( \sim 10^7 \) – \( 10^9 \) yr and the period 0.4–30 s. We have no millisecond pulsars in our sample.

We consider correlations among the following spin parameters: \( \nu, \dot{\nu}, \ddot{\nu} \), the spin-down age \( \tau_c \equiv -\nu/2\omega \) and \( \ddot{\nu}_{\text{dip}} \equiv 3\dot{\omega}^2/\nu \). The definitions of \( \tau_c \) and \( \ddot{\nu}_{\text{dip}} \) follow from the vacuum dipole model of secular spin evolution: \( \dot{\nu} \propto \nu^3 \). Our results are plotted in Fig. 1, and the cross-correlation coefficients given in Table 1. The highest correlation (91%) is between \( |\dot{\nu}| \) and \( \dot{\nu} \), with similarly strong correlations with \( \ddot{\nu}_{\text{dip}} \) and \( \tau_c \), and a weaker correlation with \( \nu \). The trends of Fig. 1 can be summarized by the following fits:

\[
\log |\dot{\nu}| = -11 + \log |\dot{\nu}|,
\]

\[
\log |\dot{\nu}| = -9.9 + 0.53 \log \dot{\nu}_{\text{dip}},
\]

\[
\log |\dot{\nu}| = -8.9 - 1.2 \log \tau_c.
\]

This system of linear equations is self-consistent through the definitions of \( \tau_c \) and \( \dot{\nu}_{\text{dip}} \). We have applied the analysis above to the full Hobbs et al. (2004) ephemerides of nearly 400 pulsars, and we find the same trends found above for the sub-sample of 131. We note that:

(i) The above trends are independent of the sign of \( \dot{\nu} \). The number of pulsars with \( \dot{\nu} > 0 \) is nearly equal to the number with \( \dot{\nu} < 0 \). Both groups follow the same trends in magnitude of \( \dot{\nu} \) (Fig. 1, second panel).

(ii) \( |\dot{\nu}| \) is always greater than or equal to \( \dot{\nu}_{\text{dip}} \); in many cases \( |\dot{\nu}| \) exceeds \( \dot{\nu}_{\text{dip}} \) by factor of \( \sim 10^6 \) (Fig. 1, bottom panel). Only four pulsars fall on the line \( \dot{\nu} = \dot{\nu}_{\text{dip}} \) : B0531+21, B0540-69, J1119-6127 and B1509-58. These pulsars appear on the upper right of Fig. 1d.

(iii) The correlation between \( |\dot{\nu}| \) and \( \dot{\nu} \) shows generally less scatter than found by Arzoumanian et al. (1994) for \( \Delta_8 \) versus \( \dot{p} \).
values of \( \dot{\nu} \) are consistent with the notion that \( \nu \) signs, accompanied by a marked deviation from the dipole variations with magnetospheric changes that, in turn, affect hybrid models that combine external torque fluctuations could arise from, for example, variations in currents in the pulser magnetosphere (Cheng 1987a,b). Internal torque variations could result from vortex dynamics in the stellar crust (e.g., Alpar et al. 1986; Jones 1990).

First, we point out that variability in the emission region (through movement of the region above the star, for example) cannot, by itself, account for timing noise in pulsars with relatively large values of \( |\dot{\nu}| \). A value of \( |\dot{\nu}| = 10^{-24} \) s\(^{-3}\) would lead to a phase variation of \( \sim 5 \) periods over 10 years. For movement of the emission region to change the phase to this extent, the region would have to move many times around the star over time scales of years, without producing large changes in the pulse profile, an extremely unlikely scenario. In these pulsars, at least, it seems that the cumulative effects of timing noise over years must represent true variations in the rotational phase. [Short time scale variations, however, such as phase jitter, are likely due to processes in the emission region]. On the other hand, for pulsars with relatively small \( |\dot{\nu}| \), we cannot rule out movement of the emission region as the dominant contributor to timing noise. A value of \( |\dot{\nu}| = 10^{-28} \) s\(^{-3}\) would lead to phase variations of only \( \lesssim 10^{-3} \) periods (0.4\(^\circ\)) over 10 years. Since this phase difference is small compared to the typical beam width for these pulsars, the timing noise could be produced by variations within the emission region. However, given that the correlation we have found between \( \nu \) and \( \dot{\nu} \) holds over the nearly eight orders of magnitude in \( \dot{\nu} \) that we have considered, a single process could be at work. We henceforth assume that timing noise represents variations in the spin rate of the star, and that these spin variations represent variations of the torque on the crust. We could be seeing variations in the spin-down torque on the star, or, in principle, variations in the internal torque exerted on the crust by the liquid interior. We consider the first process to be more promising.

Let us write the total torque on the crust as

\[
N_{\text{tot}} = N_{\text{ad}} + N_{\text{noise}},
\]

where \( N_{\text{ad}} \) represents the average spin-down torque and \( N_{\text{noise}} \) is a stochastic (or, at least, largely non-deterministic) additional torque arising from fluctuations in the external torque, the internal torque, or a combination of the two. The observed \( \dot{\nu} \) is

\[
\dot{\nu} = \dot{\nu}_{\text{ad}} + \dot{\nu}_{\text{noise}}.
\]

We expect that \( |\dot{\nu}_{\text{noise}}| \) is much smaller than \( |\dot{\nu}_{\text{ad}}| \), while observations require that \( |\dot{\nu}_{\text{noise}}| \) is much larger than \( |\dot{\nu}_{\text{ad}}| \). This can happen if the characteristic time scale \( T_{\nu} \) over

### Table 1. Cross correlation coefficients of pulsar rotational parameters

| \( \nu \) | \( |\dot{\nu}| \) | \( \dot{\nu}_{\text{dip}} \) | \( \tau_{\epsilon} \) |
|-----------|------------|----------------|----------------|
| \( |\dot{\nu}| \) | 0.69 | 0.67 | -0.91 |
| \( |\dot{\nu}| \) | 0.91 | -0.91 | \( |\dot{\nu}| \) |
| \( |\dot{\nu}| \) | 0.90 | -0.87 | |
The torque parameter $\tilde{N} = |\ddot{\nu}|T_N/|\dot{\nu}|$ versus spin-down rate. which the noise torque varies is short compared to the star’s spin-down age. We can compare the characteristic noise torque to the spin-down torque with a dimensionless torque parameter, defined as:

$$\tilde{N} = \frac{|\ddot{\nu}|T_N}{|\dot{\nu}|}. \quad (10)$$

The torque parameter is shown in Fig. 2 versus $|\dot{\nu}|$, where we take $T_N = 10$ yr as a characteristic noise time scale for illustration (the exact value of $T_N$ is unimportant and, in any case, varies among pulsars). We point out two features from Fig. 2:

(i) The torque ratio $\tilde{N}$ shows no clear trend in $\dot{\nu}$. Torque ratios of $\tilde{N} \lesssim 10^{-2}$ can account for the observed $|\ddot{\nu}|$ in most pulsars.

(ii) Because $\tilde{N}$ is $<< 1$, only small deviations from the spin-down torque are required to produce the observed values of $\ddot{\nu}$. The magnitude of the noise torque is directly proportional to the external spin-down torque. [The scaling with the spin-down torque can be seen in eq. (10)].

The stellar magnetosphere is unlikely to be completely static, and so external torque variations are a natural explanation for our results. Whether or not internal torque variations applied to the crust by the liquid interior are sufficient to account for the range in $\ddot{\nu}$ values presented here, is an interesting question.

While completing this work, we became aware of a similar analysis by Beskin et al. (2006), who find the same correlation of $\ddot{\nu}$ with $\dot{\nu}$ that we find. Our results are in agreement with theirs, but our interpretation is very different. Beskin et al. (2006) attribute the nearly equal numbers of pulsars with $\dot{\nu} > 0$ and $\ddot{\nu} < 0$ to cyclic evolution of pulsar spin rates over time scales of hundreds of years. They suggest that their assumed spin variations could represent free-body precession over very long periods, similar to that observed in PSR B1828-11 over a much shorter period (Stairs, Lyne & Shemar 2000). We emphasize, however, that the phase evolution over decades is essentially stochastic in many cases and quasi-periodic at best in other cases. Only a few pulsars show strongly periodic behavior.

ACKNOWLEDGMENTS

Part of this work was done while JOU was visiting HartRAO and he is very grateful to HartRAO, and the staff of Johannesgurg Planetarium, for support and hospitality. BL acknowledges the support of the National Science Foundation under grant AST-0009872 and thanks the University of Pisa for their hospitality, where much of this work was completed. JMW acknowledges financial support from U.S. National Science Foundation grant AST-0406832.

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