An Effective Maximal-Length Sequence Design for System Identification of a Continuous-Time Linear Aircraft Model

Atsushi FUJIMORI and Shinsuke OH-HARA

Department of Mechanical Engineering, University of Yamanashi, Kofu, Yamanashi 400–8511, Japan

This paper presents an effective maximal-length sequence design for system identification of a continuous-time linear aircraft model. The maximum-length sequence is used as the exciting signal because it is a realistic signal for identifying the aircraft model. This paper proposes two design parameters for the maximum-length sequence, which are related to the dynamical modes of aircraft. According to the identification procedures using the proposed design parameters and the subspace identification method, a continuous-time linear aircraft model in longitudinal motion is well identified in a numerical simulation.

Key Words: Aircraft Models, Maximum-Length Sequence, Bandwidth, Number of Data, Subspace Identification

1. Introduction

The modeling of aircraft is one of the important design processes for constructing flight control systems. There are, in general, two kinds of approaches for constructing an aircraft model. One is to derive the equations of motion based on the theory of flight dynamics with stability and control derivatives which are called “aerodynamic derivatives” in this paper.1) Although the aerodynamic derivatives are obtained by specification and structural parameters of the aircraft, it is necessary to calibrate the derivatives through flight and/or wind tunnel tests. Another is to apply system identification techniques.2–5) The responses of motion are measured by exciting the control surfaces. The model parameters are then estimated using identification calculation. This paper presents a modeling technique that uses system identification.

The authors have studied identification techniques for a continuous-time linear aircraft model using subspace identification.6–8) Subspace identification is used to estimate the state-space matrices of the system based on the realization theory, and is one of the powerful system identification approaches for multi-input and multi-output (MIMO) systems.4,5) In our previous works,6–8) the following points remained to be solved. In Fujimori9) and Sakashita,7) white random signals were used as the exciting inputs. Since those signals required aircraft to activate the control surfaces quickly, it may be impossible to realize the excitation during practical application. Moreover, the long-period mode in longitudinal motion was not appropriately identified.7,8) In particular, it was sometimes estimated as an unstable model.

This paper presents an improved technique for estimating a continuous-time linear aircraft model using subspace identification. The maximum-length sequence (MLS)9–12) is used as an exciting signal form a practical points of view. In particular, this paper proposes two design parameters for the MLS that are related to the dynamical modes of aircraft. One is the bandwidth ratio for the dynamical mode of aircraft. The other is the ratio of data for the dynamical mode. This paper numerically examines the correlation and the causality between the design parameters of the MLS and identification performance. According to the identification procedures using the proposed design parameters, a linear aircraft model in longitudinal motion is well identified using numerical simulation.

The rest of this paper is organized as follows. Section 2 describes the state-space equations of a linear aircraft model in longitudinal motion including the aerodynamic derivatives. Section 3 presents the procedures for identifying the continuous-time linear system using subspace identification. Section 4 explains the MLS from the frequency characteristic point of view and proposes two design parameters that are related to the dynamical modes of aircraft. Numerical verification of the proposed design parameters is examined in Section 5. The correlation between the design parameters and identification performance is presented. Section 6 demonstrates the system identification for aircraft models. Concluding remarks are given in Section 7.

2. Linear Aircraft Model

The model to be identified in this paper is a linear aircraft model in longitudinal motion. This section briefly presents a state-space equation with aerodynamic derivatives. The longitudinal motion of an aircraft is described using the following state-space equation.1)

\[ x_{\text{lon}}(t) = A_{\text{lon}}x_{\text{lon}}(t) + B_{\text{lon}}u_{\text{lon}}(t) \]  (1)

The state vector and the input vector, denoted as \( x_{\text{lon}} \) and \( u_{\text{lon}} \), are defined as

\[ x_{\text{lon}} \triangleq [u \quad \alpha \quad q \quad \theta]^T, \quad u_{\text{lon}} \triangleq [\delta_e \quad \delta_i]^T. \]  (2)
where \( u \) is the \( x \)-axis component of the flight velocity, \( \alpha \) is the angle-of-attack, \( \dot{q} \) is the pitch rate, and \( \theta \) is the pitch angle. \( \delta_e \) and \( \delta_t \) are the deflection angles of the elevator and throttle. Matrices \( A_{lon} \) and \( B_{lon} \) are given by

\[
A_{lon} = E_{lon}^{-1}M_{lon}, \quad B_{lon} = E_{lon}^{-1}L_{lon}.
\]

where

\[
M_{lon} \triangleq \begin{bmatrix}
X_{a} & X_{a} & 0 & -g \cos(\theta_0) \\
Z_{a} & Z_{a} & U_{0} + Z_{q} & -g \sin(\theta_0) \\
M_{a} & M_{a} & M_{q} & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad
L_{lon} \triangleq \begin{bmatrix}
X_{b} & X_{b} \\
Z_{b} & Z_{b} \\
M_{b} & M_{b} \\
0 & 0
\end{bmatrix}, \quad
E_{lon} \triangleq \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & U_{0} & 0 & 0 \\
0 & -M_{a} & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

\( X_{a}, Z_{a}, \ldots \) are the aerodynamic derivatives. \( U_{0} \) is the flight velocity, \( \theta_0 \) is the pitch angle in the steady, and \( g \) is the acceleration of gravity.

The characteristic equation of the longitudinal dynamical modes is written as

\[
|sI - A_{lon}| = (s - \lambda_{sp})(s - \lambda_{pp}) \times (s - \lambda_{lp})(s - \lambda_{lp}) = 0,
\]

where \( \lambda_{sp} \) and \( \lambda_{pp} \) are complex eigenvalues of the short-period and the long-period modes, respectively. \( \lambda_{lp} \) and \( \lambda_{lp} \) are their conjugate eigenvalues.

3. Identification of Continuous-Time LTI System

This section describes procedures for identifying the continuous-time linear time-invariant (LTI) system using subspace identification. Since the subspace identification method used in this paper is a standard method such as N4SID or MOESP, the details can be referenced in Verhaegen and Verdult \(^6\) and Overschee and Moor \(^7\).

The system to be identified is given by the following LTI system

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t),
\end{align*}
\]

where \( t \) is the continuous time. \( u \in \mathbb{R}^{m} \) is the input, \( y \in \mathbb{R}^{p} \) is the output, and \( x \in \mathbb{R}^{n} \) is the state vector. The measurable output is given by

\[
y(t) = y(t) + \nu(t),
\]

where \( \nu \in \mathbb{R}^{p} \) is the measurement noise (see Fig. 1). Let \( T \) be the constant sampling interval and \( N \) be the number of data used for identification. To distinguish the discrete-time signal from the continuous-time signal, the discrete-time input obtained by sampling at \( t = kT \) is denoted as

\[
u[k] \triangleq u(kT) \quad (k = 0, 1, 2, \ldots).
\]

Other discrete-time signals are similarly denoted. The objective of the identification in this paper is to estimate the continuous-time LTI system Eq. (6) using \( N + 1 \) paired discrete-time input and the measured output data

\[
\{ u[k], y[k] \} \quad (k = 0, 1, 2, \ldots, N).
\]

The rest of this section briefly shows the identification procedures presented in Fujimori \(^6\), Sakashita \(^7\), and Sugimoto \(^8\). Using the \( N + 1 \) paired input and the measured output signals of Eq. (9), the following discrete-time LTI model is estimated using a subspace identification method.

\[
\begin{align*}
z[k+1] &= P_dz[k] + Q_du[k] \\
y[k] &= R_zz[k] + S_du[k],
\end{align*}
\]

where \( z \) is a state vector used in the estimated model. Then, a continuous-time LTI model is obtained by inversely transforming Eq. (10) with the zero-th order hold; \(^9\) that is,

\[
\begin{align*}
\dot{z}(t) &= Pz(t) + Qu(t) \\
y(t) &= Rzz(t) + Su(t),
\end{align*}
\]

where

\[
[ P \quad Q ] = \frac{1}{T_d} \ln \left[ \begin{array}{cc} P_d & Q_d \\ 0 & I_m \end{array} \right]
\]

\[
R = R_d, \quad S = S_d.
\]

Equation (11) is the estimated model of Eq. (6). In general, \( z \) in Eq. (11) does not coincide with \( x \) in Eq. (6). However, if system identification is successful, the characteristics of the estimated model using Eq. (11) are equivalent to those of Eq. (6), such as the time histories, frequency responses and dynamical modes. While the objective in Fujimori \(^6\) was to estimate matrices \( A \) and \( B \) in Eq. (6), the objective in this paper is to estimate an equivalent model from the dynamical characteristic point of view.

When the system to be identified, Eq. (6), is strictly proper; that is, \( D = 0 \), it is possible in the subspace identification calculation that Eq. (10) is restricted into the strictly proper; \(^4\) that is, we have \( S_d = 0 \) in Eq. (10).

Furthermore, if all state variables of Eq. (6) are measurable; that is, \( y(t) = x(t) \), this means

\[
C = I_n, \quad D = 0.
\]

Then, we have the following relation.

\[
y(t) = x(t) = Rz(t), \quad S = 0
\]

When \( R \) is nonsingular, \( z \) in Eq. (11) is able to be changed to \( x \) used in Eq. (6). Equation (11) is then transformed into
Eq. (6).

That is, the state vector of Eq. (16) coincides with that of Eq. (6). $RPR^{-1}$ and $RQ$ in Eq. (16) are the estimates of $A$ and $B$ in Eq. (6), respectively. If $R$ is singular, a discrete-time model, Eq. (10), is re-estimated using subspace identification. In the identification simulation, which will be given in the later sections, there were few cases that $R$ was singular.

4. Design Parameters of MLS

This section proposes two design parameters for MLS that are related to the dynamical modes of aircraft. One is the bandwidth ratio for the dynamical mode. The other is the ratio of data for the dynamical mode. Before describing the design parameters, the MLS is explained from the frequency characteristic point of view.

4.1. MLS

MLS $^{9,12}$ is a binary signal having values of $\{0, 1\}$ or $\{-1, 1\}$. Since the period of the MLS can be given long, its frequency characteristic is white. Therefore, the persistency excitation condition holds; $^{21}$ that is, the MLS can be used as an exciting signal. In general, the effective frequency range of the discrete-time signal with the sampling interval $T_s$ is given by $[0, \omega_N]$, where $\omega_N$ is the Nyquist frequency defined as $\omega_N = \pi/T_s$. However, it may be impossible to realize the excitation using the control surfaces of aircraft because of their frequency characteristic limits. In this paper, the required activities of the control surfaces are specified by the bandwidth ratio of the MLS, denoted $BM$. It is defined as

$$B_M \triangleq \frac{T_s}{T_1},$$

(17)

where $T_1$ is the minimum time length where the value of the MLS is not changed. The range of $B_M$ is given as $0 \leq B_M \leq 1$ because of $T_s \leq T_1$. The smaller $T_1$ becomes, the more frequently the value of the MLS is changed; that is, it indicates that the control surfaces have to be activated quickly. Furthermore, $1/B_M$ means the minimum sampling number of times keeping the same signal value. Conversely, it means the maximum sampling number of times when the signal value is changed. For example, $B_M = 0.2 (1/B_M = 5)$ indicates that the value is changed at least once per five samplings. $B_M = 1 (1/B_M = 1)$ indicates that the value changes every sampling.

4.2. Design parameters based on dynamical modes

This section proposes two design parameters that are related to the dynamical modes of aircraft. One is associated with the bandwidth, while the other is the number of data used for identification. In the following sections, the subscript ‘$z$’ corresponds to the longitudinal dynamical modes of aircraft; $sp$: short-period mode and $lp$: long-period mode.

4.2.1. Bandwidth ratio for dynamical mode

The bandwidth of the MLS with the bandwidth ratio $B_M$ is given in the range of $[0, \omega_N B_M]$. Then, the bandwidth ratio for the dynamical mode is defined as

$$B_z \triangleq \frac{\omega_N BM}{|\bar{\alpha}_z|},$$

(18)

where $|\bar{\alpha}_z|$ means the natural frequency for $\bar{\alpha} \in \{sp, lp\}$. $B_z \geq 1$ indicates that the bandwidth of the MLS is covered beyond that of the dynamical mode.

Since $|\bar{\alpha}_{lp}| > |\bar{\alpha}_{sp}|$ in the longitudinal motion, we have $B_{sp} < B_{lp}$. Then, if $B_{lp}$ is greater than one, the MLS is able to excite both the short- and long-period modes.

4.2.2. Ratio of data for dynamical mode

Letting $k_z$ be the number of data needed for a period of the dynamical mode $\bar{\alpha}$, we have $|\alpha_z|k_z T_s = 2\pi$; that is,

$$k_z = \frac{2\pi}{|\bar{\alpha}_z| T_s} = \frac{2\omega_N}{|\bar{\alpha}_z|}.$$  

(19)

Then, the ratio of data for the dynamical mode is defined as

$$R_z \triangleq \frac{N}{k_z} = \frac{|\bar{\alpha}_z| N}{2\omega_N}.$$ 

(20)

$R_z \geq 1$ indicates that the number of data for identification is greater than the period of the dynamical mode.

In longitudinal motion, we have $R_{sp} > R_{lp}$. Then, if $R_{lp}$ is greater than one, the number of data used for identification is greater than that of data needed for the period of the longitudinal dynamical modes.

5. Numerical Verification of Design Parameters

The design parameters of the MLS proposed in the previous section are numerically verified using a numerical aircraft model. It will be clarified that the design parameters have a direct relation to identification performance. The following subsection first describes the evaluation methods adopted in this paper. A numerical verification is then shown.

5.1. Evaluation method of identification results

Identification results are generally evaluated from both global and local points of view. The time and frequency responses, variance accounted for (VAF) $^4$ and $\nu$-gap metric $^{14}$ belong to the global evaluation; while the natural frequency, damping factor, coefficients of the transfer function, and elements in the state-space equation belong to the local evaluation. This paper adopts the following three evaluation methods for aircraft model identification.

5.1.1. $\nu$-Gap metric

Let the transfer functions of the true and estimated continuous-time linear models be $P(s)$ and $\hat{P}(s)$, respectively, where $s$ is the Laplace operator. The $\nu$-gap metric is defined as

$$\delta_{\nu} \triangleq \sup_{\omega} \kappa(\hat{P}(j\omega), P^*(j\omega))$$ 

(21)

where

$$\kappa(X, Y) \triangleq \bar{\sigma}[\bar{\alpha}(I + Y\bar{X})^{-1/2}(Y - X)(I + X\bar{X})^{-1/2}].$$

$\bar{X}(s)$ is the conjugate transfer function of $X(s)$ and $\bar{\sigma}[-]$ is the maximum singular value.$^{14}$
δₚ is one of the global evaluation methods and is the model error in the frequency domain. δₚ is originally derived from the robust stability condition based on the normalized coprime factorization. It is therefore desirable in system identification that δₚ should be as small as possible. The range of δₚ is normalized as 0 ≤ δₚ ≤ 1.14

5.1.2. Pole error of dynamical mode

Letting the true and estimated eigenvalues of the dynamical mode λₜ and λₑ, respectively, the pole error of dynamical mode is defined as

\[ \epsilon(λₜ) = \left| \frac{λₜ - λₑ}{λₑ} \right| \]  \hspace{1cm} (22)

5.1.3. Multiplicative error of aerodynamic derivative

As mentioned in Section 3, the matrices of the state-space form, A and B in Eq. (6) respectively are estimated as RPR⁻¹ and RQ in Eq. (16) when all state variables are measurable. In this case, the aerodynamic derivative included in the matrices can be considered as a local evaluation. Let the true and estimated aerodynamic derivative \( X_ₜ \) and \( Xₑ \), respectively. If \( Xₜ ≠ 0 \), the multiplicative error of aerodynamic derivative \( Xₜ \) is defined as

\[ Er(Xₜ) = \left| \frac{Xₑ - Xₜ}{Xₜ} \right| \times 100\% \]  \hspace{1cm} (23)

The multiplicative errors of other derivatives are similarly defined.

Since the aerodynamic derivatives are elements constructing aircraft models, \( Er(Xₜ) \) is a local evaluation method. The influence of an aerodynamic derivative to global evaluation is individually different for each derivative. Therefore, this paper evaluates the degree of influence using the absolute sensitivities of the aerodynamic derivative for the \( v \)-gap metric and pole error. Specifically, denoting the perturbation as \( \Delta(λₜ) \), the absolute sensitivities of aerodynamic derivative \( Xₜ \) for the \( v \)-gap metric δₚ and pole error of dynamical mode \( \epsilon(λₜ) \) are defined as

\[ S₁(Xₜ) = \frac{Δ(δₚ)}{Δ(λₜ)} \]  \hspace{1cm} (24)

\[ S₂(Xₜ) = \frac{Δ(ε(λₜ))}{Δ(λₜ)} \]  \hspace{1cm} (25)

The multiplicative errors of aerodynamic derivatives whose absolute sensitivities are large are considered for evaluating the identification results.

5.2. Numerical aircraft model

The aircraft to be identified is referenced from Isozaki et al. 15) The flight conditions were given for the following steady straight flight: altitude \( H = 4000 \) [m], flight velocity \( U₀ = 100 \) [m/s], and steady pitch angle \( θ₀ = 0 \) [deg]. Table 1 shows the true value of aerodynamic derivatives in longitudinal model and their absolute sensitivities. The eigenvalues of the dynamical modes were \( λₜ₁ = -0.9994 ± j2.0052 \) and \( λₑ₁ = -0.0076 ± j0.1366 \).

This paper considers the case that all state variables of \( Xₑ \) are measurable. Then, the multiplicative errors of aerodynamic derivatives with large absolute sensitivities are discussed in Section 5.4.

5.3. Simulation conditions

The subspace identification method used was N4SID. 4,5) The sampling interval was given as \( T_s = 0.1 \) [s]. The MLS was used for exciting the control surfaces of the aircraft, where the amplitudes of the control surfaces were given as \( δ_x, δ_y = ±3 \) [deg]. The measurement noise \( v(t) \) was given as the white noise which magnitude was defined using the noise-signal ratio (NSR)

\[ NSR = \frac{1}{\frac{1}{N} \sum_{k=1}^{N} \|v[k]\|^2} \times 100\% \]  \hspace{1cm} (26)

That is, the NSR indicates the averaged amplitude ratio between the measurement noise and true output. Generally speaking, identification performance deteriorates as the NSR increases. The purpose of this section is to clarify the effectiveness of the design parameters \( B_sp \) and \( Rlp \) in identification performance. Then, the identification results were evaluated by averaging 10 times trials where the NSR was given as NSR=[10, 20, 40, ..., 100] (%).

As described in Section 4.2, it is sufficient to evaluate the identification results with respect to \( B_sp \) and \( Rlp \) in the longitudinal model because \( B_sp < B lp \) and \( Rlp > Rlp \); respectively. In the identification simulation with respect to the bandwidth ratio (Sim-1), the range of \( B_sp \) was given as 0.1 ≤ \( B lp \) ≤ 2.0, where the ratio of data was fixed at \( Rlp = 1.0 \). In the identification simulation with respect to the ratio of data (Sim-2), the range of \( Rlp \) was given as 0.3 ≤ \( R lp \) ≤ 2.0, where the bandwidth ratio was fixed at \( B_sp = 1.0 \).

5.4. Numerical verification

5.4.1. Correlation between \( δₚ \) and \( ε(λₜ) \)

Table 2 shows the correlation coefficients between the \( v \)-gap metric \( δₚ \) and pole errors of dynamical modes \( ε(λₜ) \) for \( ε ∈ \{sp, lp\} \). \( ε(λₜ) \) showed a strong correlation with \( δₚ \) for both Sim-1 and -2; that is, these dynamical modes

| \( ε \) | \( S₁ \) | \( S₂ \) | \( S₃ \) | \( S₄ \) |
|-------|-------|-------|-------|-------|
| \( X_u \) | -0.0216 | 0.0037 | 0.0000 | 0.0394 |
| \( X_a \) | 6.5147  | 0.4089 | 0.0014 | 0.0551 |
| \( Z_u \) | -0.2202 | 0.0064 | 0.0008 | 0.2510 |
| \( Zₚ \) | -73.4524 | 0.4429 | 0.1643 | 0.0357 |
| \( Zₚ \) | -1.5334 | 0.0104 | 0.0007 | 0.0034 |
| \( M₀ \) | 0.0000  | 0.0000 | 0.0000 | 0.0000 |
| \( Mₚ \) | -4.3706 | 0.4796 | 0.4780 | 0.0359 |
| \( Mₚ \) | -0.9647 | 0.2156 | 0.2170 | 0.0357 |
| \( Mₚ \) | -0.2976 | 0.0644 | 0.0730 | 0.0003 |
| \( Xₚ \) | 0.0000  | 0.0000 | 0.0000 | 0.0000 |
| \( Xₚ \) | 4.9899  | 0.5003 | —— | —— |
| \( Zₚ \) | -4.5176 | 0.0460 | —— | —— |
| \( Zₚ \) | 0.3174  | 0.0082 | —— | —— |
| \( Mₚ \) | -2.8421 | 0.5036 | —— | —— |
| \( Mₚ \) | 0.0000  | 0.0000 | —— | —— |
strongly influenced $\delta_c$. The correlation coefficients of $\varepsilon(\lambda_{lp})$ were small in Sim-1, but large in Sim-2; that is, $\varepsilon(\lambda_{lp})$ was related to the ratio of data $R_{lp}$.

5.4.2. $\varepsilon(\lambda_t)$ with respect to $B_{sp}$

Figure 2 shows the pole error of dynamical mode $\varepsilon(\lambda_t)$ with respect to the bandwidth ratio $B_{sp}$. $\varepsilon(\lambda_{lp})$ monotonously decreased as $B_{sp}$ increased, and almost converged for $B_{sp} > 1.4$ (see Fig. 2(a)). $\varepsilon(\lambda_{lp})$ was approximately constant regardless of the increase in $B_{sp}$ (see Fig. 2(b)). The bandwidth of the MLS was desirable for identification if $B_{sp}$ was greater than 1.4 for numerical verification.

5.4.3. $\varepsilon(\lambda_t)$ with respect to $R_{lp}$

Figure 3 shows the pole error of dynamical mode $\varepsilon(\lambda_t)$ with respect to the ratio of data $R_{lp}$. All pole errors, especially $\varepsilon(\lambda_{lp})$ decreased as $R_{lp}$ increased. The number of data was sufficient for identification if $R_{lp}$ was greater than 1.2.

5.4.4. $Er(\cdot)$ with respect to $B_{sp}$ and $R_{lp}$

Figures 4 and 5 show the multiplicative errors of longitudinal aerodynamic derivatives with large absolute sensitivities, as shown in Table 1. The multiplicative errors of derivatives $M_{11}$, $Z_{11}$ and $M_{14}$, for which $S_{sp}$ was large, decreased as $B_{sp}$ increased (see Fig. 4(a)). The error of almost all derivatives shown in Fig. 5 decreased as $R_{lp}$ increased.

6. Demonstration Using Design Parameters of MLS

This section demonstrates the identification of a linear aircraft model using the MLS specified by the design parameters proposed in Section 4. The procedures for identification are given as follows.

Step-1: The upper bound of the MLS bandwidth is determined using the performance of actuators for the control surfaces of an aircraft.

Step-2: The natural frequencies of the dynamical modes are estimated.

Step-3: The bandwidth ratio for the short-period $B_{sp}$ is determined using the desired identification accuracy. The bandwidth ratio of the MLS is then obtained.

Step-4: The ratio of data for the long-period $R_{lp}$ is determined using the desired identification accuracy.

| Table 2. Correlation coefficients between $\bar{v}$-gap metric and pole error of dynamical mode. |
|-----------------------------------------------|
| $\varepsilon(\lambda_{lp})$ | $\varepsilon(\lambda_{sp})$ |
| Sim-1 | 0.962 | 0.340 |
| Sim-2 | 0.925 | 0.853 |

Fig. 2. Pole error of longitudinal dynamical mode with respect to $B_{sp}$.

Fig. 3. Pole error of longitudinal dynamical mode with respect to $R_{lp}$.

Fig. 4. Multiplicative errors of longitudinal aerodynamic derivatives with large absolute sensitivities.

Fig. 5. Multiplicative errors of longitudinal aerodynamic derivatives with large absolute sensitivities.
1. Multiplicative error of longitudinal aerodynamic derivatives with respect to $R_\text{a}$. Fig. 4. Multiplicative error of longitudinal aerodynamic derivatives with respect to $R_\text{a}$.

The number of data $N$ is then obtained. The following subsections present the details of the procedures using the same numerical aircraft model described in Section 5.2.

6.1. Upper bound of bandwidth of MLS (Step-1)

The upper bound of the MLS bandwidth is determined using the frequency characteristics of actuators activating the control surfaces of an aircraft. It is assumed that the actuator is given by the following second-order model.\(^{16}\)

$$G_a(s) = \frac{\omega_a^2}{s^2 + 2\xi_a\omega_a s + \omega_a^2}$$

(27)

where $\omega_a$ is the natural frequency and $\xi_a$ is the damping factor of the actuator. When $\xi_a = 1/\sqrt{2}$ is given as a typical damping factor, the overshoot time of the step response, denoted as $t_\text{p}$, is obtained as

$$t_\text{p} = \frac{\pi}{\sqrt{1 - \xi_a^2\omega_a}} = \frac{\sqrt{2\pi}}{\omega_a}.$$  

(28)

The response at $t = t_\text{p}$, denoted as $y_{\text{step}}(t_\text{p})$, is given by

$$y_{\text{step}}(t_\text{p}) = 1 + e^{-\xi_a\omega_a t_\text{p}} = 1 + \sqrt{2}e^{-\pi} = 1.0432.$$  

(29)

That is, the response achieves 104.32% of the reference. This indicates that the MLS is approximately realized by the actuator where $T_i$ is given by $T_i = t_\text{p}$. Then, using $T_i = t_\text{p}$ in Eq. (17), the upper bound of the bandwidth ratio, denoted as $\tilde{B}_M$, is given as

$$\tilde{B}_M = \frac{T_i\omega_a}{\sqrt{2\pi}} = \frac{\omega_a}{\sqrt{2\omega_N}}.$$  

(30)

Referring to actuators used in practical aircraft,\(^{16}\) $T_i$ was calculated as $T_i = t_\text{p} = 0.1415$ [s] for the case of $\omega_a = 10\pi$ [rad/s] (= 5 [Hz]). Then, the upper bound of the bandwidth ratio was obtained as $\tilde{B}_M = 0.7071$.

6.2. Estimation of natural frequencies (Step-2)

The natural frequencies of the dynamical modes $|\lambda_{\text{l0}}|$ and $|\lambda_{\text{l1}}|$ are needed to obtain the design parameters of the MLS as given by Eqs. (18) and (20). They are not, in general, known in advance. Therefore, this paper estimates them using the spectra of the responses in an excitation experiment.

Figure 6 shows MLS of $\delta_e$ and $\delta_i$ used for the excitation experiment, where the bandwidth ratio of the MLS was given by $B_M = 0.5$, which was less than the upper bound $\tilde{B}_M$. Figure 7 shows the spectra of the longitudinal state variables excited by $\delta_e$ and $\delta_i$ shown in Fig. 6, where the NSR of the measurement noise was given as NSR = 30%. It is seen from Fig. 7 that the large spectra of $\alpha$ and $q$ appeared over the true
short-period natural frequency, while those of $u$ and $\theta$ appeared near the true long-period natural frequency. “High range” in these figures indicates the range where the spectrum is greater than 60% of the peak. Accordingly, the upper bound of the “high range” may be an estimate of the natural frequency. Table 3 shows the variance and mean of 10 time trials in longitudinal motion. The smaller the variance, the more confident the mean. Then, the upper bounds of $\alpha$ and $u$ were adopted for the estimates of the short- and long-period modes, respectively. That is, the estimates were obtained as

$$|\hat{A}_{sp}| = 2.266, \quad |\hat{A}_{lp}| = 0.2396 \, \text{[rad/s]}.$$  

(31)

6.3. Bandwidth ratio (Step-3)

Based on the discussion associated with Fig. 2 in Section 5.4.2, $\epsilon(|A_{lp}|)$ almost converged for $B_{lp} > 1.4$. The bandwidth ratio for the short-period mode was then given as $B_{sp} = 1.6$. Instead of $|A_{lp}|$, the estimate $|\hat{A}_{lp}|$ was used in Eq. (18), the MLS bandwidth ratio for identifying the longitudinal model was obtained as

$$B_{M} = \frac{|\hat{A}_{lp}|B_{lp}}{\omega_N} = 0.1154.$$  

(32)

$B_{M}$ obtained using Eq. (32) was sufficiently smaller than the upper bound $\tilde{B}_{M} = 0.7071$; that is, the MLS was applicable to aircraft model identification. Figure 8 shows the spectra of $\delta_{s}$ and $\delta_{l}$, where $B_{M}$ is obtained using Eq. (32). It is seen that the designed input signals contain sufficient power for exciting both the short- and long-period modes.

6.4. Number of data (Step-4)

Based on the discussion associated with Fig. 3 in Section 5.4.3, the ratio of data for the long-period mode was given as $R_{lp} = 1.6$. Instead of $|A_{lp}|$, the estimate $|\hat{A}_{lp}|$ was used in Eq. (20), and the number of data for identifying the longitudinal model was obtained as

$$N = \frac{2\omega_{N}R_{lp} |\hat{A}_{lp}|}{|\hat{A}_{lp}|} \approx 420.$$  

(33)

6.5. Identification results

Using the MLS bandwidth ratio and the number of data obtained using Eqs. (32) and (33), a longitudinal aircraft model was identified where the measurement noise was given as $\text{NSR} = 30\%$. The identification results are shown in Figs. 9–11. It is seen from these figures that the identified model was comparable with the true longitudinal model in terms of time, frequency responses, and pole location of the dynamical modes.

Summarizing the above mentioned so far, the proposed design parameters for the MLS, $B_{sp}$ and $R_{lp}$, are surely effective for identifying a longitudinal aircraft model from the simulation results.

7. Concluding Remarks

This paper has presented an effective MLS design for system identification of a continuous-time linear aircraft model. The MLS was used as the exciting signal because it has a realistic signal for identifying the aircraft model. This paper proposed two design parameters for the MLS that are related to the dynamical modes of aircraft. According to the identification procedures using the proposed design parameters and the subspace identification method, a continuous-time linear aircraft model in longitudinal motion was well identified in a numerical simulation.

The numerical simulation presented in this paper was the case that all state variables were measurable. When all state variables are not measurable, the aerodynamic derivatives
are not estimated and a local evaluation such as the multiplicative error of the aerodynamic derivative is not available. Even in this case, the proposed design parameters are effective for designing the MLS for identifying a continuous-time aircraft model by evaluating the global evaluation such as the $\nu$-gap metric and pole error of the dynamical mode. This is because global evaluation is more important than local evaluation from the viewpoint of system identification. Although this paper presented the identification of an aircraft model in longitudinal motion, the proposed design parameters are also applicable to models in lateral motion and other systems; especially for those that require estimating dynamical modes.

References

1) Schmidt, L. V.: Introduction to Aircraft Flight Dynamics, AIAA, Reston, 1998, pp. 333–354.
2) Ljung, L.: System Identification—Theory for the User, 2nd ed., Prentice Hall, Upper Saddle River, NJ, 1999, pp. 1–567.
3) Söderström, T. and Stoica, P.: System Identification, Prentice Hall, Upper Saddle River, NJ, 1989, pp. 1–352.
4) Verhaegen, M. and Verdult, V.: Filtering and System Identification—A Least Square Approach—, Cambridge University Press, Cambridge, 2007, pp. 1–394.
5) Overschee, P. and Moor, B.: Subspace Identification for Linear Systems, Kluwer Academic Publishers, Massachusetts, 1996, pp. 1–254.
6) Fujimori, A.: Identification of Continuous-Time Linear Aircraft Models Using Subspace Identification, J. Aerospace Science Technology, 2 (2016), pp. 7–12.
7) Sakashita, K.: Continuous-Time Linear Modeling for Aircraft Using Subspace Identification, Graduation Thesis, University of Yamanashi, 2016 (in Japanese).
8) Sugimoto, T.: Continuous-Time Identification of Linear Aircraft Model Using Maximum-Length Sequence, Graduation Thesis, University of Yamanashi, 2017 (in Japanese).
9) Golomb, S. W. and Gong, G.: Signal Design for Good Correlation: For Wireless Communication, Cryptography, and Radar, Cambridge University Press, Cambridge, 2005, pp. 1–367.
10) Buracas, G. T. and Boynton, G. M.: Efficient Design of Event-Related fMRI Experiments Using M-Sequences, NeuroImage, 16 (2002), pp. 801–813.
11) Poudel, K. N. and Robertson, W. M.: Maximum Length Sequence Digital Multilayer Reflector, OSA Continuum, 1 (2018), pp. 358–372.
12) Kondo, K.: A Survey of Maximum-Length Sequences for Measuring Frequency Characteristics of Linear System, Rept. Nara National College of Technology, 40 (2004), pp. 59–64.
13) Feuer, A. and Goodwin, G. C.: Sampling in Digital Signal Processing and Control, Birkhäuser, Boston, 1996, pp. 78–79.
14) Vinnicombe, G.: Uncertainty and Feedback (H∞, Loop-shaping and the $\nu$-Gap Metric), Imperial College Press, London, 2001, pp. 104–166.
15) Isozaki, K., Masuda, K., Tanuchi, A., and Watari, M.: Flight Test Evaluation of Variable Stability Airplane, KHI Tech. Rev., 75 (1980), pp. 50–58.
16) Adachi, T., Awaysa, I., Nakata, M., and Ichikawa, T.: Stabilization of Motion Control System with Rate Limited Actuator, Mitsubishi Heavy Industries Technical Review, 52 (2015), pp. 109–117.

Masayuki Sato
Associate Editor