High-temperature superconductivity: the explanation

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Abstract
Soon after the discovery of the first high-temperature superconductor by Georg Bednorz and Alex Müller in 1986, the late Sir Nevill Mott in answering his own question ‘Is there an explanation?’ (1987 Nature 327 185) expressed the view that the Bose–Einstein condensation (BEC) of small bipolarons, predicted by us in 1981, could be the one. Several authors then contemplated BEC of real-space tightly bound pairs, but with a purely electronic mechanism of pairing rather than with an electron–phonon interaction (EPI). However, a number of other researchers criticized the bipolaron (or any real-space pairing) scenario as incompatible with some angle-resolved photoemission spectra, with experimentally determined effective masses of carriers and unconventional symmetry of the superconducting order parameter in cuprates. Since then, the controversial issue of whether EPI is crucial for high-temperature superconductivity or is weak and inessential has been one of the most challenging problems of contemporary condensed matter physics. Here I outline some developments in the bipolaron theory suggesting that the true origin of high-temperature superconductivity is found in a proper combination of strong electron–electron correlations with a significant finite-range (Fröhlich) EPI, and that the theory is fully compatible with key experiments.

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(Some figures in this article are in colour only in the electronic version.)

1. Real space and Cooper pairs

There is still little consensus on the origin of high-temperature superconductivity in cuprates [1] and other related compounds. The only consensus that exists is that charge carriers are bound into pairs with an integer spin. Pairing of two fermionic particles has been evidenced in cuprate superconductors [2] from the quantization of magnetic flux in units of the flux quantum \( \phi_0 = \hbar / 2e \).

A long time ago, London suggested that the remarkable superfluid properties of \(^4\)He were intimately linked to the Bose–Einstein condensation (BEC) of the entire assembly of Bose particles [3]. The crucial demonstration that superfluidity was linked to the Bose particles and the BEC came after experiments on liquid \(^3\)He, whose atoms were fermions, which failed to show the characteristic superfluid transition within a reasonable wide-temperature interval around the critical temperature for the onset of superfluidity in \(^4\)He. In sharp contrast, \(^3\)He becomes a superfluid only below a very low temperature of some 0.0026 K. Here, we have a superfluid formed from pairs of two \(^3\)He fermions below this temperature.

The three orders of magnitude difference between the critical superfluidity temperatures of \(^4\)He and \(^3\)He kindles the view that the BEC might represent the ‘smoking gun’ of high-temperature superconductivity [4]. Unfortunately electrons are fermions. Therefore, it is not surprising at all that the first proposal for high-temperature superconductivity, made by Ogg Jr in 1946 [5], was the pairing of individual electrons. If two electrons are chemically coupled together, the resulting combination is a boson with the total spin \( S = 0 \) or \( S = 1 \). Thus, an ensemble of such two-electron entities can, in principle, be condensed into the Bose–Einstein superconducting condensate. This idea was further developed as a natural explanation of superconductivity by Schafroth [6] and by Blatt and Butler in 1955 [7].
However, with one or two exceptions [8], the Ogg–Schafroth picture was condemned and practically forgotten because it neither accounted quantitatively for the critical parameters of ‘old’ (i.e. low $T_c$) superconductors, nor did it explain the microscopic nature of the attractive force that could overcome the natural Coulomb repulsion between two electrons that constitute a Bose pair. The same model that yields a rather precise estimate of the critical temperature of $^4$He leads to an utterly unrealistic result for superconductors, namely $T_c = 10^4$ K with the atomic density of electron pairs of about $10^{22}$ per cm$^3$ and with the effective mass of each boson twice the electron mass, $m^{**} = 2m_e$.

The failure of this ‘bosonic’ picture of individual electron pairs became fully transparent when Bardeen et al [9] proposed that two electrons in a superconductor were indeed correlated in real space, but on a very large (practically macroscopic) coherence length $\xi$ of about $10^4$ times the average inter-electron spacing. Bardeen–Cooper–Schrieffer (BCS) theory was derived from an early demonstration by Fröhlich [10] that conduction electrons in states near the Fermi energy attract each other on account of their weak interaction with vibrating ions of a crystal lattice. Cooper then showed that any two electrons were paired in the momentum space due to their quantum interaction (i.e. the Pauli exclusion principle) with all other electrons in the Fermi surface. These Cooper pairs strongly overlap in the real space, in sharp contrast with the model of non-overlapping real-space pairs discussed in [5–7]. Highly successful for metals and alloys with a low $T_c$, the BCS theory led some theorists (e.g. [11]) to the conclusion that there should be no superconductivity above 30 K. Whereas the Ogg–Schafroth phenomenology predicted unrealistically high values of $T_c$, the BCS theory left perhaps only a limited hope for the discovery of new materials that could superconduct at liquid-nitrogen or higher temperatures.

2. Strong-coupling superconductivity beyond the BCS–Migdal–Eliashberg approximation

It became clear now that the Ogg–Schafroth and BCS descriptions are actually two opposite extremes of the electron–phonon interaction (EPI). On a phenomenological level, Eagles [12] proposed pairing without superconductivity in some temperature range solving simultaneous equations for the BCS gap and for the Fermi energy when an electron–electron attraction becomes greater than some critical value, and that superconductivity sets in at a lower temperature, of the order of the BEC temperature of the pairs in some low-carrier-density compounds like SrTiO$_3$. Later on, extending the BCS theory to the strong interaction between electrons and ion vibrations, a Bose liquid of tightly bound electron pairs surrounded by the lattice deformation (i.e. of small bipolarons) was predicted [13, 14]. A further prediction was that high-temperature superconductivity should exist in the crossover region of the EPI strength from the BCS-like to bipolaronic superconductivity [15] (figure 1).

Compared with the early Ogg–Schafroth view, two fermions (now small polarons) are bound into a small bipolaron by the lattice deformation. Such bipolaronic states, at first sight, have a mass too large to be mobile. Actually, earlier studies [16, 17] considered small bipolarons as entirely localized objects. However, it has been shown analytically [13, 15, 18, 19] and using different numerical techniques [20, 21] that small bipolarons are itinerant quasi-particles existing in the Bloch states at temperatures below the characteristic phonon frequency which coherently...
tunnel through the lattice with a reasonable effective mass in particular if the EPI is finite-range. As a result, the superconducting critical temperature, proportional to the inverse mass of a bipolaron, is reduced in comparison with the ‘ultra-hot’ local-pair Ogg–Schafroth superconductivity, but turns out to be much higher than the BCS prediction (figure 1). A strong enhancement of $T_c$ in the crossover region from BCS-like polaronic to BEC-like bipolaronic superconductivity is entirely due to a sharp increase of the density of states in a narrow polaronic band [15], which is missing in the so-called negative Hubbard $U$ model [22, 23]. Quite remarkably Bednorz and Müller noted in their Nobel Prize lecture that in their ground-breaking search for high-$T_c$ superconductivity, they were stimulated and guided by the polaron model. Their expectation was that if ‘an electron and a surrounding lattice distortion with a high effective mass can travel through the lattice as a whole, and a strong electron–lattice coupling exists, an insulator could be turned into a high-temperature superconductor’ [24].

After we showed [15]—unexpectedly for many researchers—that the BCS–Migdal–Eliashberg (BCS–ME) theory breaks down at the EPI coupling $\lambda \gtrsim 0.5$ for any adiabatic ratio $\hbar \omega_0/\varepsilon_F$, multi-polaron physics gained particular attention [25]. The parameter $\lambda \hbar \omega_0/\varepsilon_F$, which is supposed to be small in the BCS–ME theory [26, 27], becomes in fact large at $\lambda \gtrsim 0.5$ since the electron bandwidth is narrowed and the Fermi energy, $\varepsilon_F$, is renormalized down exponentially below the characteristic phonon energy, $\hbar \omega_0$ (figure 1) [4]. Nevertheless, as noted in the unbiased comment by Jorge Hirsch [28], in order to explain the increasingly higher $T_c$’s found in supposedly ‘conventional’ materials, values of the electron–phonon coupling constant $\lambda$ larger than 1 have been used in the conventional BCS–ME formalism. This formalism completely ignores the polaronic collapse of the bandwidth, but regrettably continues to be used by some researchers irrespective of whether $\lambda$ is small or large.

3. Key pairing interaction and unconventional symmetry of the order parameter

In general, the pairing mechanism of carriers could be not only ‘phononic’ as in the BCS theory or its strong-coupling bipolaronic extension [4] but also ‘excitonic’, ‘plasmonic’, ‘magnetic’, ‘kinetic’ or due to purely repulsive Coulomb interaction combined with an unconventional pairing symmetry of the order parameter [4]. Actually, following the original proposal by Anderson, many authors ([29] and references therein) assumed that the electron–electron interaction in novel superconductors was very strong but repulsive and provided high $T_c$ without phonons via e.g. superexchange, spin fluctuations, excitons or any other non-phononic mechanism. A motivation for this concept can be found in the earlier work by Kohn and Luttinger (KL) [30], who showed that the Cooper pairing of fermions with a weak hard-core repulsion was possible in a finite orbital momentum state. However, the same work showed that $T_c$ of hard-core repulsive fermions was well below the mK scale, and more importantly the KL pairing with moderate values of angular momenta ($p$ or $d$) was impossible for charged fermions with the realistic finite-range Coulomb repulsion [31, 32] in disagreement with some recent claims [33]. Also advanced simulations with a (projected) BCS-type trial wave function [34], using the sign-problem-free Gaussian-Basis Monte Carlo algorithm (GBMC), showed that the simplest repulsive Hubbard model did not account for high-temperature superconductivity in the intermediate and strong-coupling regimes either.

On the other hand, some density functional theory (DFT) calculations [35, 36] found small EPI insufficient to explain high critical temperatures within the BCS–ME framework, while other first-principles studies found large EPI in cuprates [37] and in recently discovered iron-based compounds [38]. It is commonplace that DFT underestimates the role of the Coulomb correlations and nonadiabatic effects, predicting an anisotropy of electron-response functions much smaller than that experimentally observed in the layered high-$T_c$ superconductors. Adiabatic DFT calculations could not explain the optical infrared c-axis spectra and the corresponding electron–phonon coupling in the metallic state of the cuprates. On the other hand, these spectra are well described within the nonadiabatic response approach of [37]. There is a strong nonlocal polar EPI along the c-axis in the cuprates together with optical conductivity as in an ionic insulator even in the well-doped ‘metallic’ state [37]. The inclusion of a short-range repulsion (Hubbard $U$) via the LDA + U algorithm [39] also significantly enhances the EPI strength due to a poor screening of some particular phonons. Substantial isotope effects on the carrier mass and a number of other independent observations (see e.g. [40] and references therein) unambiguously show that lattice vibrations play a significant although unconventional role in high-temperature superconductors. Overall, it seems plausible that the true origin of high-temperature superconductivity should be found in a proper combination of strong electron–electron correlations with a significant EPI [41].

We have recently calculated the EPI strength, the phonon-induced electron–electron attraction, and the carrier mass renormalization in layered superconductors at different dopings using a continuum approximation for the renormalized carrier energy spectrum and the RPA dielectric response function [42].

If, for instance, we start with a parent insulator as $\text{La}_2\text{CuO}_4$, the magnitude of the Fröhlich EPI is unambiguously estimated using the static, $\epsilon_s$, and high-frequency, $\epsilon_\infty$, dielectric constants [18, 43]. To assess its strength, one can apply an expression for the polaron binding energy (polaronic level shift) $E_p$, which depends only on the measured $\epsilon_s$ and $\epsilon_\infty$:

$$E_p = \frac{e^2}{2\epsilon_0k} \int_{\mathbb{BZ}} \frac{d^3q}{(2\pi)^3} q^2.$$

Here, the integration is over the Brillouin zone (BZ), $\epsilon_0 \approx 8.85 \times 10^{-12}$ F m$^{-1}$ is the vacuum permittivity and $k = \epsilon_s \epsilon_\infty/(\epsilon_s - \epsilon_\infty)$. In the parent insulator, the Fröhlich interaction alone provides the binding energy of two holes, $2E_p$, an order of magnitude larger than any magnetic interaction ($E_0 = 0.647$ eV in $\text{La}_2\text{CuO}_4$ [43]). Actually, equation (1) underestimates the polaron binding energy,
since the deformation potential and/or molecular-type (e.g. Jahn–Teller [44]) EPIs are not included.

It was argued earlier [18] that the interaction with $c$-axis polarized phonons in cuprates would also remain strong at finite doping due to a poor screening of high-frequency electric forces as confirmed in some pump–probe [45, 46] and photoemission [47, 48] experiments. However, a quantitative analysis of the doping-dependent EPI remained elusive because the dynamic dielectric response function, $\epsilon(\omega, \mathbf{q})$, was unknown. Recent observations of the quantum magnetic oscillations in some underdoped [49] and overdoped [50] cuprate superconductors are opening up the possibility for a quantitative assessment of EPI in these and related doped ionic lattices with the quasi two-dimensional (2D) carrier energy spectrum. The oscillations revealed cylindrical Fermi surfaces, enhanced effective masses of carriers (ranging from $2m_e$ to $6m_e$) and the astonishingly low Fermi energy, $E_F$, which appears to be well below 40 meV in underdoped Y–Ba–Cu–O [49] and less than or about 400 meV in heavily overdoped TI2201 [50]. Photoemission spectroscopies ([47, 48] and references therein) do not show small Fermi-surface pockets and there are alternative interpretations of slow magnetic oscillations in underdoped cuprates unrelated to Landau quantization, for example [51]. However, a poorly screened strong EPI [42] is not sensitive to particular band structures and Fermi surfaces, but originates in the low Fermi energy, which is supported by other independent experiments [52]. Such low Fermi energies make the Migdal–Eliashberg (ME) adiabatic approach to EPI inapplicable in these compounds. Indeed, the ME non-crossing approximation breaks down at $\lambda\hbar\omega_0/E_F > 1$ when the crossing diagrams become important. The characteristic oxygen vibration energy is about $\hbar\omega_0 = 80$ meV in oxides; therefore the ME theory cannot be applied for a weak EPI with the coupling constant $\lambda < 0.5$. In the strong coupling regime, $\lambda \gtrsim 0.5$, the effective parameter $\lambda\hbar\omega_0/E_F$ becomes large irrespective of the adiabatic ratio, $\hbar\omega_0/E_F$, because the Fermi energy shrinks exponentially due to the polaron narrowing of the band [15]. Since carriers in cuprates are in the non-adiabatic (underdoped) or near-adiabatic (overdoped) regimes, $E_F \lesssim \hbar\omega_0$, their energy spectrum renormalizes by EPI and the polaron–polaron interactions can be found with the familiar small-polaron canonical transformation at any coupling $\lambda$ [53].

With doping the attraction and the polaron mass drop [42]. Nevertheless, on-site and inter-site attractions induced by EPI remain well above the superexchange (magnetic) interaction $J$ (about 100 meV) at any doping since the non-adiabatic carriers cannot fully screen high-frequency electric fields. The polaron mass [42] agrees quite well with the experimental masses [49, 50]. Decreasing the phonon frequency lowers the attraction and increases the polaron mass in underdoped compounds with little effect on both quantities at overdoping. Hence, the Fröhlich EPI with high-frequency optical phonons turns out to be the key pairing interaction in underdoped cuprates and remains the essential player at overdoping. What is more surprising is that EPI is clearly beyond the BCS–ME approximation since its magnitude is larger than or comparable with the Fermi energy and the carriers are in the non-adiabatic or near-adiabatic regimes.

Together with the deformation potential and Jahn–Teller EPIs, the Fröhlich EPI overcomes the direct Coulomb repulsion at distances comparable with the lattice constant even without any retardation [15]. Since EPI is not local in the nonadiabatic electron system with poor screening, it can provide the d-wave symmetry of the pairing state [54]. Remarkably, the internal symmetry of an individual bipolaron in the underdoped regime can be different from the symmetry of the BEC if the pairing takes place with nonzero centre-of-mass momentum [55]. All these conditions point to a crossover from bipolaronic to polaronic superconductivity [15] in cuprates with doping.

4. Pseudogap, superconducting gap, angle-resolved photoemission spectroscopy and tunnelling spectra of cuprate superconductors

A detailed microscopic theory capable of describing unusual angle-resolved photoemission spectroscopy (ARPES) and tunnelling data has so far remained elusive. Soon after the discovery of high-$T_c$ superconductivity [1], a number of tunnelling, photoemission, optical, nuclear spin relaxation and electron-energy-loss spectroscopies revealed an anomalously large gap in cuprate superconductors existing well above the superconducting critical temperature, $T_c$. The gap, now known as the pseudogap, was originally assigned [56] to the binding energy of real-space preformed hole pairs—small bipolarons—bound by a strong EPI. Since then, many alternative explanations of the pseudogap have been proposed.

The present-day scanning tunnelling spectroscopy (STS) [57–59], intrinsic tunnelling spectroscopy [60] and ARPES [61, 62] have offered a tremendous advance in the understanding of the pseudogap phenomenon in cuprates and some related compounds. Both extrinsic (see [57, 59] and references therein) and intrinsic [60] tunnelling as well as high-resolution ARPES [61] have found another energy scale, reminiscent of a BCS-like ‘superconducting’ gap that opens at $T_c$ accompanied by the appearance of Bogoliubov-like quasi-particles [61] around the node. Earlier experiments with a time-resolved pump–probe demonstrated two distinct gaps, one a temperature-independent pseudogap and the other a BCS-like gap [63]. Another remarkable observation is the spatial nanoscale inhomogeneity of the pseudogap observed with STS [57–59] which is presumably related to an unavoidable disorder in doped cuprates. Essentially, the doping and magnetic field dependence of the superconducting gap compared with the pseudogap and their different real-space profiles have prompted the opinion that the pseudogap is detrimental to superconductivity [60].

Without a detailed microscopic theory that could describe highly unusual tunnelling and ARPES spectra, the relationship between the pseudogap and the superconducting gap has remained a mystery [61]. Recently, we have developed the bipolaron theory of ARPES [64] and tunnelling [65, 66] by taking into account real-space pairing, coherence effects in a single-particle excitation spectrum and disorder. Our theory accounts for major peculiarities in extrinsic and intrinsic tunnelling in cuprate superconductors and in ARPES.
Real-space pairs, whatever the pairing interaction, can be described as a charged Bose liquid on a lattice if the carrier density is relatively small avoiding their overlap [4]. The superfluid state of such a liquid is the true BEC, rather than a coherent state of overlapping Cooper pairs. Single-particle excitations of the liquid are thermally excited within the single-particle band-tail within the charge-transfer gap, $\Delta_\nu$, both in the superconducting and normal states [4]. Here, in the superconducting state ($T < T_c$), following [67] one takes into account that polarons interact with the condensate via the same potential that binds the carriers. As in the BCS case, the superfluid state of such a liquid is the true BEC, the chemical potential, $\mu$, is found outside the band by about half of the bipolaron binding energy, $\Delta_p$, both in the superconducting and normal states [4]. Here, in the superconducting state ($T < T_c$), following [67] one takes into account that polarons interact with the condensate via the same potential that binds the carriers.

To calculate ARPES and the tunnelling conductance, we adopted the first-principle ‘LDA + GTB’ band structure [68] amended with impurity band-tails [64] (figure 2). It explains the charge-transfer gap, $E_{ct}$, sharp ‘quasi-particle’ peaks near $(\pi/2, \pi/2)$ of the Brillouin zone and a high-energy ‘waterfall’ observed by ARPES in underdoped cuprates [64]. The chemical potential is found in the single-particle band-tail within the charge-transfer gap at the bipolaron mobility edge (figure 2(b)), in agreement with the S-N-S tunnelling experiments [69]. Such a band structure explains an insulating-like low-temperature normal-state resistivity as well as many other unusual normal-state properties of underdoped cuprates [4].

The bipolaron theory captures key unusual signatures of the experimental tunnelling conductance in cuprates, such as the low-energy coherent gap, the high-energy pseudogap and the asymmetry [65]. In the case of atomically resolved STS, one should use a local band-tail DOS $\rho(E, \mathbf{r})$, which depends on different points of the scan area $\mathbf{r}$ due to a nonuniform dopant distribution, rather than an averaged DOS or spectral functions measured with ARPES. As a result the pseudogap shows nanoscale inhomogeneity, while the low-energy coherent gap is spatially uniform, as observed in [59]. Increasing the doping level tends to diminish the bipolaron binding energy, $\Delta_p$, since the pairing potential becomes weaker due to a partial screening of EPI with low-frequency phonons [42, 70]. However, the coherent gap, $\Delta_\nu$, which is the product of the pairing potential and the square root of the carrier density [67], can remain almost a constant or even increase with doping, as also observed in [59].

5. Concluding remarks on lattice (bi)polarons in high-temperature superconductors

A growing number of observations tell us that high-$T_c$ cuprate superconductors [1] are not the conventional BCS superconductors [9], but represent a realization of strong-coupling non-adiabatic polaronic and bipolaronic superconductivity [4]. The fundamental origin of such a strong departure of superconducting cuprates from conventional BCS behaviour stems from the poorly screened Fröhlich EPI of the order of 1 eV, routinely neglected in the Hubbard $U$ and $t-J$ models [29]. This interaction with optical phonons is poorly screened because the charge carriers are found in the non-adiabatic or near adiabatic regime with their plasmon frequency below or near the characteristic frequency of optical phonons. Since screening is poor, the magnetic interaction remains small compared with the Fröhlich EPI at any doping of cuprates. Consequently, to generate an adequate theory of high-temperature superconductivity, finite-range Coulomb repulsion and the Fröhlich EPI must be treated on an equal footing. When both the interactions are strong compared with the kinetic energy of carriers, our theory predicts the low-energy state in the form of mobile inter-site bipolarons at underdoping and mobile small polarons at overdoping [4].

There is abundant independent evidence in favour of (bi)polarons [25] and 3D BEC in cuprate superconductors [71, 72]. The substantial isotope effect on the carrier mass [73–76] predicted for (bi)polaronic conductors in [77] is the most compelling evidence for (bi)polaronic carriers in cuprate superconductors. High-resolution ARPES [47, 48, 78] provides another piece of evidence for the strong EPI in cuprates and related doped layered compounds apparently with c-axis-polarized optical phonons. These, as well as tunnelling spectroscopies of cuprates [79, 80] and recent pump–probe experiments [45, 46], unambiguously show that the Fröhlich EPI is important in highly polarizable ionic lattices.
A parameter-free estimate of the Fermi energy using the magnetic-field penetration depth [52] and the magnetic quantum oscillations [49] found its very low value, $\epsilon_F \lesssim 50$ meV, supporting the real-space pairing in underdoped cuprate superconductors.

Magnetotransport and thermal magnetotransport data strongly support preformed bosons in cuprates. In particular, many high-magnetic-field studies revealed a non-BCS upward curvature of the upper critical field $H_{c2}(T)$ (see [81]) for a review of experimental data), predicted for the BEC of charged bosons in the magnetic field [82]. The Lorentz number, $L = \kappa_e/T\sigma$, differs significantly from the Sommerfeld value $L_s$ of the standard Fermi-liquid theory if carriers are double-charged bosons [83]. Here $\kappa_e$ and $\sigma$ are electron thermal and electrical conductivities, respectively. Alexandrov and Mott [83] predicted a rather low Lorentz number for bipolarons, $L \approx 0.15L_s$, due to the double elementary charge of bipolarons and also due to their nearly classical distribution function above $T_c$. Direct measurements of the Lorenz number using the thermal Hall effect [84] produced a value of $L$ just above $T_c$, almost the same as predicted by the bipolaron model.

Single polarons, localized within an impurity band-tail, coexist with bipolarons in the charge-transfer doped Mott–Hubbard insulator. They account for sharp ‘quasi-particle’ peaks near $(\pi/2, \pi/2)$ of the Brillouin zone and high-energy ‘waterfall’ effects observed with ARPES in cuprate superconductors [64]. This ‘band-tail’ model also accounts for two energy scales in ARPES and in the extrinsic and intrinsic tunneling, their temperature and doping dependence, and for the asymmetry and inhomogeneity of extrinsic tunneling spectra of cuprates [65]. On the other hand, essentially different doping and magnetic field dependence of the superconducting gap compared with the pseudogap and their different real-space profiles have prompted the opinion [60] that the pseudogap is not connected with the so-called ‘preformed’ Cooper pairs advocated by Emery and Kivelson [85] as an alternative to the BEC of real-space pairs. The unusual normal state diamagnetism uncovered by torque magnetometry has also been convincingly explained as the normal state (Landau) diamagnetism of charged bosons [86].

Overall the real-space pairing seems to be a remarkable feature of cuprates no matter what the microscopic pairing mechanism is. A lattice disorder introduces additional complexity to the problem since an interference of impurity potential with the lattice distortion, which accompanies the polaron movement, contributes to polaron and bipolaron localization [87]. Self-organized discrete dopant networks [88] lead to multiscale complexity for key materials as well. However, the detailed microscopic physics of the bosonic many-body state seems to be irrelevant for fitting their electrodynamic properties [89].

As was emphasized in a number of early qualitative [71] (1990s) and more recent numerical studies [90, 91] of strongly correlated electrons with a significant EPI, the anti-ferromagnetic spin fluctuations facilitate a doping-induced lattice polaron formation profoundly lowering the required strength of the bare EPI, but playing virtually no role in pairing compared with the EPI [42, 92].

It is quite surprising that despite clear evidence for lattice polarons in cuprate and related superconductors, there are still opinions (e.g. [29]) suggesting that EPI is inessential or that polaron formation does not help, but hinders the pairing instability, very much in contrast to the notion advanced by this and other authors that in fact lattice polarons explain high-$T_c$ superconductivity.

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