No contextual advantage in non-paradoxical scenarios of two state vector formalism

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The two state vector formalism (TSVF) was proposed by Aharonov, Bergmann, and Lebowitz (ABL) to provide a way for the counteraffictual assignment of the probabilities of outcomes of contemplated but unperformed measurements on quantum systems. This formalism underlies various aspects of foundations of quantum theory and has been used significantly in the development of weak values and several proofs of quantum contextuality. We consider the application of TSVF, with pre- and post-selection (PPS) and the corresponding ABL rule, as a means to unearth quantum contextuality. We use the principle of exclusivity to classify the resultant pre- and post-selection scenarios as either paradoxical or non-paradoxical. In light of this, we find that several previous proofs of the emergence of contextuality in PPS scenarios are only possible if the principle of exclusivity is violated and are therefore classified as paradoxical. We argue that these do not constitute a proper test of contextuality. Furthermore, we provide a numerical analysis for the KCBS scenario as applied in the paradigm of TSVF and find that non-paradoxical scenarios do not offer any contextual advantage. Our approach can be easily generalized for other contextual scenarios as well.

I. INTRODUCTION

The standard quantum theory does not provide a framework for making predictions about the measurements in the past (retrodiction) of a quantum system once it has been measured in a definite state. Aharonov, Bergmann, and Lebowitz (ABL), in their seminal work on the time symmetry in successive quantum measurements, introduced a reformulation of the standard quantum theory where one can meaningfully talk about statistical predictions of a measurement on a pre-and post-measurement theory where one can meaningfully talk about statistical predictions of a measurement on a pre-and post-selected (PPS) ensemble at an intermediate time [1]. The retrodiction formula derived by ABL (ABL rule) is the probability of a measurement outcome conditioned on the outcomes of a preceding and a succeeding measurement.

A generalized framework for PPS ensembles in terms of ‘weak values’ was formulated as two state vector formalism (TSVF) [2–4] in order to experimentally validate the ABL formulation [5, 6]. In TSVF, the complete description of a quantum system is specified by two state vectors, one evolving forward in time and the other one evolving backwards. Here the arrow of time is described by the order of preceding and succeeding measurements. TSVF finds intriguing applications in quantum foundations [7–13] and quantum information processing [14]. The ABL retrodiction, more specifically the TSVF, has resulted in various counter intuitive results commonly called PPS paradoxes [15–23]. In a recent work [24] counterfactual use of ABL retrodiction has been shown to run into a direct contradiction with operational quantum mechanics challenging the completeness of TSVF. Therefore further investigations on the appropriateness of ABL retrodiction in connection with various non-classical aspects of quantum theory is critical in order to pinpoint the exact role of TSVF in the studies of quantum foundations.

Contrary to the Born rule, probabilities assigned by the ABL formula are determined by the specification of the measurement setting of an observable and on pre and post selected states in context of which the observable is being measured. This sort of context dependency of measurements has led to connections between PPS paradoxes and contextuality [25]: since the probabilities assigned to measurement outcomes are explicitly context dependent, there is no motivation to consider a non-contextual hidden variable theory as a realistic extension of operational quantum theory. Nevertheless, this reasoning has been disputed based on the fact that Bell-type correlations can be simulated using post-selections in local hidden variable theories [26]. Therefore, the mere presence of context dependent elements in ABL formula should not be sufficient to prove Bell-Kochen-Specker (BKS) theorem [27, 28] or the various statistical versions of contextuality [29]. It is required to dive deeper in order to establish a valid connection between contextuality and the ABL retrodiction formula.

Mermin [30] showed the existence of a strong connection between the two by illustrating how measurements used in a proof of BKS theorem can give birth to unsolvable PPS paradoxes indicating a kind of impossibility of non-contextual hidden variable theories. Leifer and Spekkens [31] later logically showed that for every PPS paradox with a scenario involving non-orthogonal pre- and post-selection states, there exists an associated proof of BKS theorem. Their proof is based on the fact that ABL probability assignments of certain sets of projectors in a variant of 3-box paradox violate algebraic constraints dictated by classical probability theory. An exhaustive discussion on the same in relation with weak values was...
presented by Tollaksen [32]. Another important contribution in this direction was recently made by establishing a direct connection between anomalous weak values and contextuality where it has been suggested that anomalous weak values can be taken as proofs of contextuality [33, 34]. So far the studies in this avenue of research have been focused on logical proofs of contextuality invoking only the paradoxical nature of ABL probability assignments. In these logical proofs of contextuality one arrives at a contradiction while making assignment of probabilities to various outcomes following the ABL rule. Such proofs generally involve the paradoxes generated by the application of the ABL rule. Therefore, it is natural to ask whether there is any contextual advantage in situations with “non-paradoxical assignments” of ABL probabilities.

In this paper, by analyzing the Klyachko-Can-Binicioglu-Shumovsky (KCBS) scenario [35] (which comprises a statistical proof of contextuality) with the ABL formula, we show that non-paradoxical ABL probability assignments do not give rise to any contextual advantage. Furthermore, in order to produce an advantage one needs to renounce the exclusivity principle which is central to any operational theory [36–38]. Our result raises serious questions about the ABL-contextuality connections that have been advocated by previous authors: is this connection merely an illusion created by post-selection just like the detection efficiency loophole in Bell non-locality tests? Since the paradoxical sector of ABL probabilities requires abandoning the notion of principle of exclusivity can ABL retrodiction be considered an appropriate description of quantum systems at all?

The paper is organized as follows: In Sec. II A we introduce the concept of PPS scenarios and briefly discuss the ABL formula and consequently the TSVF. In Sec. II B we present our main result that the ABL rule is unable to correctly predict the statistics of the KCBS scenario. In Sec. V we offer some concluding remarks.

II. ABL RULE, TSVF AND PPS PARADOXES

In this section we describe the TSVF and ABL retrodiction rule. We define PPS scenarios and introduce the notion of paradoxical and non-paradoxical nature of them. This classification depends on whether the probability assignments are properly conditioned by the exclusivity principle or not.

A. TSVF and ABL retrodiction

In this subsection we illustrate a general pre- and post-selected scenario and elucidate how the ABL rule can be used to assign probabilities to intermediate measurements.

A pre- and post-selection scenario deals with statistical assignment of probabilities to the outcomes of the measurement of an observable A at time t when the system is pre-selected to be in the state |ψ⟩ at some time t₁ < t and post-selected in the state |φ⟩ at a later moment in time t_f > t. Pre-selection is achieved by performing a projective measurement P₁ = { |ψ⟩⟨ψ|, 1 − |ψ⟩⟨ψ|} at time t₁ on the initial state of the system ρ (which can be chosen arbitrarily) and selecting only the outcomes corresponding to |ψ⟩⟨ψ|. Similarly for post-selection one can perform a projective measurement of P₂ = { |φ⟩⟨φ|, 1 − |φ⟩⟨φ|} at time t_f where outcomes corresponding to |φ⟩⟨φ| are filtered (see Fig. 1). It is apparent that such probability assignments are conditioned on PPS states |ψ⟩ and |φ⟩ and therefore time symmetric.

Consider an observable A with outcomes { |a_i⟩⟨a_i| } which is measured after pre-selecting a state |ψ⟩ and afterwards post-selecting a state |φ⟩. The probability of obtaining the outcome |a_i⟩⟨a_i| conditioned on pre-and post-selected outcomes is given as

\[ ζ_i = \frac{|⟨ψ|a_i⟩|^2 |⟨a_i|ψ⟩|^2}{∑_j |⟨ψ|a_j⟩|^2 |⟨a_j|ψ⟩|^2}, \]

which can be simplified as

\[ ζ_i = \frac{|⟨φ|Π_i|ψ⟩|^2}{∑_j |⟨φ|Π_j|ψ⟩|^2}, \]

where Π_i = |a_i⟩⟨a_i|. As one can see, ζ_i for a projector Π_i is dependent on PPS states |ψ⟩ and |φ⟩. A different choice of these will yield different probability assignments. Furthermore, the measurement context of the projector Π_i also plays a major role. If the set of
measurement settings in which \( \Pi_i \) appears is chosen differently, the term \( \sum_j \langle \phi | \Pi_j \psi \rangle^2 \) will also change. All of the aforementioned choices form a context for the projector \( \Pi_i \).

This makes ABL formula given in Eq. (2) inherently context dependent and led Albert et al. to draw a parallel between ABL retrodiction and quantum contextuality.

There are two ways to interpret Eq. (2). In the first case, the observable \( \mathcal{A} \) is actually measured after performing a pre-selection. Post selection is then performed on the state after the measurement of \( \mathcal{A} \). In this case there are a total of three different sequential measurements being performed. This case is known as non-counterfactual assignment of probabilities [39–41].

In the second case, the observable \( \mathcal{A} \) is not actually measured, but rather a probability distribution over its outcomes is assigned counterfactually depending on the PPS states [4]. This case is known as counterfactual assignment of probabilities. It has been argued by Aharonov and collaborators that simultaneous counterfactual probability assignments, in accordance with Eq. (2), to any number of arbitrary observables is possible. This in general produces the various PPS paradoxes, as we will define in next subsection.

For the remainder of this paper we consider only counterfactual measurement setting of the observable \( \mathcal{A} \). This rule (Eq. (2)) is eponymously known as the ABL rule and is the same for both the aforementioned cases. However, the interpretation for both the cases is entirely different and leads to some interesting results, especially when linked to counterfactual assignments of projectors in the KCBS scenario.

B. Paradoxical and non-paradoxical PPS scenarios

In this subsection we provide a classification of PPS scenarios into paradoxical and non-paradoxical. We then proceed to show how the KCBS scenario can be modified to fit within the paradigm of TSVF and whether the latter can help in predicting the correct statistics of the particle being in boxes \( A, B \) and \( C \) respectively. Now consider two possible counterfactual measurement settings \( \mathcal{A} = \{ |A\rangle \langle A|, 1 - |A\rangle \langle A| \} \) and \( \mathcal{B} = \{ |B\rangle \langle B|, 1 - |B\rangle \langle B| \} \). It is easy to visualize \( \mathcal{A} \) and \( \mathcal{B} \) as being the actions of opening the boxes \( A \) and \( B \) respectively at an intermediate time in order to check whether the particle was present there. A counterfactual probability assignment to both the projectors \( |A\rangle \langle A| \) and \( |B\rangle \langle B| \) can be made according to ABL formula (2). However, it can be seen that such an assignment leads to a situation in which the particle is present in box \( A \) with unit probability and box \( B \) with unit probability [39].

The exclusivity principle states that the sum of probabilities of mutually exclusive events cannot be greater than 1. Therefore, such scenarios in which two mutually exclusive events are assigned unit probabilities is paradoxical. This motivates the following definition:

**Definition 2** (Logical PPS paradox). A logical PPS paradox consists of at least two counterfactual PPS scenarios \( \{ \langle \phi | \psi \rangle, \mathcal{M}_1 \} \) and \( \{ \langle \phi | \psi \rangle, \mathcal{M}_2 \} \) where \( \mathcal{M}_i = \{ \Pi_i, 1 - \Pi_i \} \) for \( i = 1, 2 \) and \( \text{tr}(\Pi_1 \Pi_2) = 0 \) such that \( \zeta_1 + \zeta_2 > 1 \) where \( \zeta_i \) is counterfactual probability assigned to \( \Pi_i \).

The logical PPS paradox is defined for PPS scenarios which violate the As shown in Ref. [31], to every corresponding logical PPS paradox there exists a proof of BKS theorem. However, the relation between non-paradoxical PPS scenarios and contextuality is still left unexplored. To that end we make the following definition to distinguish between the paradoxical and non-paradoxical sector of PPS scenarios.

**Definition 3** (Paradoxical and non-paradoxical sector). The set of all two-states that generate logical PPS paradoxes for given two counterfactual measurement settings \( \mathcal{M}_1 = \{ \Pi_1, 1 - \Pi_1 \} \) and \( \mathcal{M}_2 = \{ \Pi_2, 1 - \Pi_2 \} \) such that \( \text{tr}(\Pi_1 \Pi_2) = 0 \) is called the paradoxical sector corresponding to the pair \( \{ \mathcal{M}_1, \mathcal{M}_2 \} \). We term the set of all two-states that are not elements of the above as the non-paradoxical sector corresponding to the pair \( \{ \mathcal{M}_1, \mathcal{M}_2 \} \).

III. KCBS CONSTRUCTION AND NON-CONTEXTUALITY

We now analyze whether the non-paradoxical sector of PPS scenarios can offer a proof of contextuality. We first focus on the minimal proof of state dependent contextuality, namely the KCBS scenario and construct an ontological description of the same via the TSVF.

Consider a scenario consisting of 5 tests \( e_i, i \in \{0, 1, 2, 3, 4\} \). A test is an experiment which yields some statistics for a given preparation. These tests are assumed to be cyclically exclusive, i.e.,

\[
P(e_i) + P(e_{i+1}) \leq 1,
\]
where \( i \oplus 1 \) is taken modulo 5. This scenario is eponymously named as KCBS, in honor of the people who first studied it. The KCBS scenario can be represented on a graph, whose vertices correspond to tests and two vertices are connected by an edge if they are exclusive. This scenario is capable of revealing quantum contextuality if the following inequality is violated,

\[
\mathcal{K} := \sum_{i=0}^{4} P(e_i) \leq 2, \tag{4}
\]

where the underlying ontic probability distribution, \( P(e_i) \) is assumed to be non-contextual. This is the well known KCBS inequality.

A valid construction of the KCBS scenario for the quantum case is as follows. Consider 5 different PVMs: \( \mathcal{M}_i = \{ \Pi_i, \mathbb{1} - \Pi_i \}, i \in \{0, 1, 2, 3, 4\} \) and \( \text{tr}(\Pi_i \Pi_{i\oplus1}) = 0 \). Each projector corresponds to a test in the KCBS scenario and cyclic orthogonality ensures the required exclusivity conditions given in Eq. (3). The maximum quantum value of the KCBS inequality (4) for the aforementioned settings and a state \( |\psi\rangle \) is

\[
\text{Max}(\mathcal{K}) := \text{Max} \left( \sum_{i=0}^{4} P(\Pi_i = 1) \right) = \sqrt{5}, \tag{5}
\]

which is greater than the non-contextual bound. This is an indication of contextual advantage of quantum probability distributions.

It is to be noted that any valid construction of the KCBS scenario in any formalism must necessarily ensure the exclusivity conditions (3).

### IV. ABL RULE AND KCBS INEQUALITY

The foremost requirement to check whether the non-paradoxical sector of ABL formalism offers any contextual advantage is to setup the KCBS scenario with the proper exclusivity conditions given in Eq. (3) by assigning a probability distribution \( \zeta_i \) to the projectors under a PPS scenario. We choose the PPS as \(|\psi\rangle\) and \(|\phi\rangle\) respectively to assign a counterfactual probability distribution to the projector \( \Pi_i \) according to the ABL rule as

\[
\zeta_i = \frac{|\langle\phi|\Pi_i|\psi\rangle|^2}{|\langle\phi|\Pi_i|\psi\rangle|^2 + |\langle\phi|(\mathbb{1} - \Pi_i)|\psi\rangle|^2}, \tag{6}
\]

where we the measurement setting is of the form \( \{ \Pi_i, \mathbb{1} - \Pi_i \} \).

By careful selection of a PPS, such counterfactual assignments can lead to a logical paradox in which \( \Pi_i \) and \( \Pi_{i\oplus1} \) are assigned probabilities leading to \( \zeta_i + \zeta_{i\oplus1} > 1 \). This is a direct violation of the exclusivity conditions (3). Furthermore, in order to analyse the non-paradoxical sector of the KCBS scenario, it is required that \( \zeta_i + \zeta_{i\oplus1} \leq 1 \) for all two-states \( \langle \phi | \psi \rangle \).

We now propose the following setup to test the KCBS inequality using the ABL formalism. Without loss of generality we assume a pre-selected state \(|\psi\rangle\) as

\[
|\psi\rangle = (0, 0, 1)^T, \tag{7}
\]

and a post-selected state as

\[
|\phi\rangle = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)^T, \tag{8}
\]

where \( \theta \in [0, \pi] \) and \( \phi \in [0, 2\pi] \). The projectors \( \Pi_i = |v_i\rangle \langle v_i| \) are of the form,

\[
|v_0\rangle = \left( 1, 0, \sqrt{\cos \pi/5} \right)^T, \\
|v_1\rangle = \left( \cos 4\pi/5, -\sin 4\pi/5, \sqrt{\cos \pi/5} \right)^T, \\
|v_2\rangle = \left( \cos 2\pi/5, \sin 2\pi/5, \sqrt{\cos \pi/5} \right)^T, \\
|v_3\rangle = \left( \cos 2\pi/5, -\sin 2\pi/5, \sqrt{\cos \pi/5} \right)^T, \\
|v_4\rangle = \left( \cos 4\pi/5, \sin 4\pi/5, \sqrt{\cos \pi/5} \right)^T.
\]

We then optimize \(|\phi\rangle\) for maximum value of \( \mathcal{K} \) with the exclusivity conditions (3) imposed on \( \zeta_i \). We evaluate \( \mathcal{K} \) using the rule (6) for the measurement \( \{ \Pi_i, \mathbb{1} - \Pi_i \} \) to assign \( P(\Pi_i = 1) = \zeta_i \) accordingly.

By imposing the exclusivity conditions we found that no post-selection can lead to a violation of the KCBS inequality. In Fig 3 we plot the intersection of the solutions that satisfy the inequalities (3) and (4) for various minimum values of \( \mathcal{K} \) over the entire region of post-selected states. Incidentally we do not find any set of states for which KCBS inequality is violated. This provides a clear evidence of the fact that the ABL formalism does not provide a complete description under the non-paradoxical sector of PPS scenario.

In order to achieve a violation it is necessary to violate at least a single exclusivity constraint. If all the constraints are satisfied, the resultant distribution, even from the ABL retrodiction formula is necessarily non-contextual.
Therefore, any violation of the KCBS inequality observed via the ABL rule, must necessarily arise from the paradoxical sector of PPS scenarios. As a consequence, the maximum violation can even go above the algebraic bound. This is because, the exclusivity conditions are not properly satisfied.

It is a natural consequence of this work that a valid PPS-KCBS scenario can be modelled using a non-contextual ontological model.

Our analysis can be extended to arbitrary contextuality scenarios too [42]. Following our analysis, it is required to firstly identify the proper exclusivity conditions according to Eq. (3). These conditions demarcate the set of non-paradoxical PPS scenarios from the paradoxical ones according to definition 3. Within this set of counterfactual PPS scenario one can then vary the pre-selected and post-selected states for a given set of PVMs (which define the corresponding contextuality scenario) to make counterfactual probability assignments. A violation of the corresponding contextuality inequality would then indicate a contextual advantage of the TSVF.

The KCBS scenario requires a set of 5 PVMs, and imposes 5 exclusivity constraints on the ABL rule. Any $n$-cycle scenario [42] would then consequently impose $n$ such exclusivity constraints. This in turn reduces the non-paradoxical sector of PPS scenarios possible for this contextual inequality. While a solution for all $n$-cycle scenarios with $n$ exclusivity constraints applied to TSVF is not possible, we conjecture with good confidence that no $n$-cycle contextual inequality can exhibit a violation under the TSVF paradigm.

V. CONCLUSION

In this work we have focused on unearthing quantum contextuality as identified by the violation of KCBS inequality in the PPS scenarios where ABL rule provides a way to assign counterfactual probabilities to measurement outcomes. We provide a classification of PPS scenarios into paradoxical and non-paradoxical sectors. We then show that the non-paradoxical sector of the ABL rule to evaluate probability distribution over the outcomes of an observable in a PPS scenario does not provide a contextual violation of the KCBS inequality. Since, the ABL rule is applied in a counterfactual manner, the ABL rule acts as an ontic model of the KCBS inequality. By imposing proper exclusivity conditions on the ABL probabilities, we find that it is not able to reproduce the statistics that are observed in nature.

It should be noted that the KCBS scenario, and the pentagram graph in general, underly many other contextual scenarios as well [43]. Apart from KCBS, our result also implies a non-contextual behavior of these scenarios under the paradigm of TSVF.

Our results show that the ABL rule is essentially non-contextual contrary to recent studies [31, 33, 34]. Most of the recent studies deal with probability assignments which are not properly conditioned and lead to scenarios where the sum of probabilities of exclusive events can be greater than 1 leading to false signatures of contextuality. Any such signature arises from definition 2 and therefore violates the principle of exclusivity which is at the heart of operational theories [36–38]. Therefore, in order to observe a violation, the principle of exclusivity needs to be abandoned.

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