New Insights into Uniformly Accelerated Detector in a Quantum Field

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We obtained an exact solution for a uniformly accelerated Unruh-DeWitt detector interacting with a massless scalar field in (3+1) dimensions which enables us to study the entire evolution of the total system, from the initial transient to late-time steady state. We find that the Unruh effect as derived from time-dependent perturbation theory is valid only in the transient stage and is totally invalid for cases with proper acceleration smaller than the damping constant. We also found that, unlike in (1+1)D results, the (3+1)D uniformly accelerated Unruh-DeWitt detector in a steady state does emit a positive radiated power of quantum nature at late-times, but it is not connected to the thermal radiance experienced by the detector in the Unruh effect proper.

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I. INTRODUCTION

A uniformly accelerated detector (UAD) moving in Minkowski vacuum experiences a thermal bath at the temperature $T_U = \frac{\hbar a}{2\pi c k_B}$ with the proper acceleration $a$ [1]. This effect is called the Unruh effect and the temperature $T_U$ is called the Unruh temperature.

The Unruh effect was originally derived and is usually demonstrated under the framework of time-dependent perturbation theory [1, 2, 3]. Consider a point-like quantum mechanical object, the “detector”, with internal degree of freedom $Q$ coupling to a quantum field $\Phi$ through the interacting Hamiltonian $H_I = \lambda_0 Q(\tau) \Phi(z^{\mu}(\tau))$, where $\lambda_0$ is the coupling constant, $\tau$ is the proper time of the detector and $z^{\mu}(\tau)$ is the trajectory the uniformly accelerated detector is going along. Suppose at the initial moment $\tau_0$ the initial state for the detector-field system can be factorized into

$$|\psi_0\rangle = |E_0\rangle \otimes |0_M\rangle$$

where $|E_0\rangle$ is the ground state of the free detector and $|0_M\rangle$ is the Minkowski vacuum of the free field. Then, from time-dependent perturbation theory in quantum mechanics, to first order in $\gamma \sim \lambda_0^2$, the transition probability from the ground state to the n-th excited state of the detector is given by [3]

$$P_{0 \rightarrow n} = \frac{\lambda_0^2}{2\pi \hbar^2} \int_{-\infty}^{\infty} d\tau \left(\frac{E_n - E_0}{e^{2\pi(E_n-E_0)/a\hbar} - 1}\right)^2.$$  \hspace{1cm} (2)

which is non-zero. In particular, for a simple harmonic oscillator detector with natural frequency $\Omega_r$, all the transition probabilities with $n > 1$ are $O(\gamma^2)$, and the only non-vanishing $P$ of $O(\gamma)$ is

$$P_{0 \rightarrow 1} = \frac{\lambda_0^2}{4\pi m_0} \frac{\eta}{e^{2\pi \Omega_r/a} - 1}.$$  \hspace{1cm} (3)

where $\eta \equiv \int_{-\infty}^{\infty} d\tau$ is the duration of interaction in the detector’s proper time. Accordingly one claims that a uniformly accelerated detector moving in Minkowski vacuum experiences the same effect as does an inertial detector immersed...
in a thermal bath at the Unruh temperature (which can be read off from the Planck factors in \(^2\) and \(^3\)). When \(a = 0\), the transition probability per unit time \(P_{0 \rightarrow 1}/\eta\) vanishes, which implies that there is no excitation in an inertial detector initially prepared in its ground state \(^2\).

Recently we studied an Unruh-DeWitt detector theory in \((3+1)D\) and obtained a complete description of the combined system with exact expressions for the evolution of the detector and field correlations \(^4\). With these non-perturbative results, we found that some long-held beliefs based on the perturbation theory are not true. Furthermore, some intuitions gained from \((1+1)D\) results cannot be applied to \((3+1)D\) case, though the spacetime dimension does not matter in the above arguments focused on the response of the detector. We summarize these points in the following.

II. THE MODEL

We consider the combined system of a Unruh-DeWitt (UD) detector interacting with a massless scalar field in \((3+1)D\) Minkowski space, described by the action \(^1\) \(^2\) \(^4\) \(^5\)

\[
S = \int d^4\tau \frac{m_0}{2} \left[ \dot{\Phi}^2 - \Omega_0^2 \Phi^2 \right] - \int d^4x \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \lambda_0 \int d\tau \int d^4x Q(\tau) \Phi(x) \delta^4(x^\mu - z^\mu(\tau)). \tag{4}
\]

Here \(Q\) is the internal degree of freedom of the detector, assumed to be a harmonic oscillator with mass \(m_0\) and natural frequency \(\Omega_0\), \(\tau\) is the detector’s proper time, and \(Q \equiv dQ(\tau)/d\tau\). \(\Phi\) is the massless scalar field, and \(\lambda_0\) is the coupling constant.

For simplicity, we do not consider the trajectory of the detector \(z^\mu\) as a dynamical variable (for a discussion of the case where the trajectory is determined by its interplay with the quantum field, see \(^6\) \(^7\)), but gauged by an external agent. We assume the UD detector is moving in a prescribed trajectory in uniform acceleration: \(z^\mu(\tau) = (a^{-1} \sinh \sigma \tau, a^{-1} \cosh \sigma \tau, 0, 0)\) with \(x^0 - x^1 = 0\) being the event horizon for the detector.

When \(a = 0\), the UD detector theory is a special case of the harmonic oscillator quantum Brownian motion (QBM) model, studied before by many authors (see references in e.g., \(^8\)). The relation between these two models becomes clear when we make the substitutions \(Q \rightarrow x\), \(\int d^3k \rightarrow \sum_n\), \(\Phi_k \rightarrow q_n\) and \(-\lambda_0 e^{ikz}\rightarrow C_n\) in \(^8\). The QBM model incorporates the effect of the environment (the quantum field) on the system (harmonic oscillator) with dissipative and stochastic dynamics. This shows that even for the \(a = 0\) case the detector is not just laying idle but has interesting physical behaviors due to its interaction with the fluctuations in the quantum field.

III. EXACT EVOLUTION OF OPERATORS

We start at the initial time \(\tau = \tau_0\) with the same initial state \(^1\) and assume the initial operators are those for free theories. Suppose the coupling \(\lambda_0\) is not turned on until \(\tau_0\), when all the dynamical variables are allowed to interact and evolve. By virtue of the linear coupling between \(\dot{Q}\) and \(\dot{\Phi}\) in \(^4\), the time evolution of \(\Phi(x)\) and \(\dot{Q}(\tau)\) from the Heisenberg equations of motion is simply a linear transformation in the phase space spanned by \(\{\dot{\Phi}(x), \ddot{\Phi}(x), \dot{Q}(\tau), \ddot{Q}(\tau)\}\). Thus \(\dot{\Phi}(x)\) and \(\dot{Q}(\tau)\) can be expressed in the form

\[
\dot{\Phi}(x) \sim \int d^3k \left[ f^{(+)}(x; k) \hat{b}_k + f^{(-)}(x; k) \hat{b}_k^\dagger \right] + f^a(x) \hat{a} + f^{a*}(x) \hat{a}^\dagger, \tag{5}
\]

\[
\dot{Q}(\tau) \sim \int d^3k \left[ q^{(+)}(\tau; k) \hat{b}_k + q^{(-)}(\tau; k) \hat{b}_k^\dagger \right] + q^a(\tau) \hat{a} + q^{a*}(\tau) \hat{a}^\dagger, \tag{6}
\]

where \(\{\hat{b}_k, \hat{a}\}\) and \(\{\hat{a}^\dagger, \hat{a}\}\) are the creation and annihilation operators defined in free theories for the scalar field and the detector, respectively. The whole problem is now transformed from solving the Heisenberg equations of motion for the operators into one for solving for the c-number functions \(f^a(x)\) and \(q^a(\tau)\) with suitable initial conditions.

After the regularization and renormalization of the retarded Green’s function, similar in spirit to that in deriving the Abraham-Lorentz-Dirac equation for moving electrons in classical electrodynamics \(^6\) \(^7\) \(^9\) \(^10\), the back reaction of the quantum field is incorporated into the equation of motion for \(q^{(+)}\), which reads

\[
(\partial_\tau^2 + 2\gamma \partial_\tau + \Omega_r^2) q^{(+)}(\tau; k) = \frac{\lambda_0}{m_0} f^{(+)}(z(\tau); k), \tag{7}
\]

where \(f^{(+)}(x; k) \equiv \exp(-i\omega t + ik \cdot x)\) is the free field solution in Minkowski coordinate, \(\Omega_r\) is the renormalized natural frequency and \(\gamma \equiv \alpha_0^2/8\pi m_0\) is the damping constant resulting from the interaction with the field. We see that \(q^{(+)}\)
behaves like a driven damped harmonic oscillator with dissipation induced by the vacuum fluctuations of the scalar field. Eq. (4) is causal and local in \( \tau \). Once the form of \( f_0 (+) \) is given, \( q (+) (\tau) \) in (4) is totally determined by the motion of the detector from \( \tau_0 \) to \( \tau \). In other words, the response of \( q (+) \) here is purely kinematic.

As for the \( q^a \) coefficient of \( \dot{a} \), its equation of motion including the back reaction of the field looks similar. \( q^a \) acts like a damped harmonic oscillator with the renormalized natural frequency \( \Omega_r \) and the damping constant \( \gamma \) but without the driving force.

IV. INTERNAL ACTIVITIES OF THE DETECTOR

For the detector-field system initially prepared in the factorized initial state (4), the two-point functions of \( Q \) will split into two parts, \( \langle Q (\tau) Q (\tau') \rangle \) = \( \langle E_0 | E_0 \rangle \langle Q (\tau) Q (\tau') \rangle + \langle Q (\tau) Q (\tau') \rangle _\alpha \langle 0_M | 0_M \rangle \). Here \( \langle Q (\tau) Q (\tau') \rangle _\alpha \) can be interpreted as accounting for the response to the vacuum fluctuations of the quantum field, while \( \langle Q (\tau) Q (\tau') \rangle _\alpha \) corresponds to the intrinsic quantum fluctuations in the detector.

The two-point functions of the detector with respect to the vacuum, \( \langle Q (\eta) Q (\eta') \rangle _\nu \sim \int d^3 k q (\tau; k) q (-1) (\tau; k) \) with \( \eta \equiv \tau - \tau_0 \) during the duration of interaction, can be explicitly obtained from the solution of \( q (+) \). The coincidence limit of it looks like

\[
\langle Q (\eta)^2 \rangle _\nu = \frac{\hbar^4 \lambda_0^2 \theta (\eta)}{(2 \pi m_0 \Omega)^2} \left[ \Lambda_0 e^{-2 \gamma \eta} \sin^2 \Omega \eta + \text{(regular terms)} \right],
\]

where \( \Lambda_0 = - \ln |\tau_0 - \tau_0| \) is finite in real processes because \( |\tau_0 - \tau_0| \) characterizes the time scale that the interaction is turned on. (This means that \( \Lambda_0 \) would not be important at late times: for every finite value of \( \Lambda_0 \), the \( \Lambda_0 \)-term vanishes as \( \gamma \eta \to \infty \).) In Ref. [4], the evolution of the regular part of \( \langle Q (\eta)^2 \rangle _\nu \) has been shown. Roughly speaking it saturates exponentially in the detector’s proper time to a positive number.

The coincidence limit of the two-point function \( \langle Q (\eta) Q (\eta') \rangle _\nu \) reads

\[
\langle \dot{Q} (\eta)^2 \rangle _\nu = \frac{\hbar^4 \lambda_0^2 \theta (\eta)}{(2 \pi m_0 \Omega)^2} \left[ \Lambda_1 \Omega^2 + \Lambda_0 e^{-2 \gamma \eta} (\Omega \cos \Omega \eta - \gamma \sin \Omega \eta)^2 + \cdots \right],
\]

where \( \Lambda_1 = - \ln |\tau - \tau'| \) corresponds to the time-resolution of this theory. The regular part of \( \langle \dot{Q} (\eta)^2 \rangle _\nu \) acts quite similarly to those for \( \langle Q (\tau)^2 \rangle _\nu \).

For the expectation values of the detector two-point functions with respect to the ground state, the coincidence limits of them are straightforward and independent of the proper acceleration \( a \). The quantity \( \langle Q (\eta)^2 \rangle _\lambda \sim (q^a)^* q^a \) decays exponentially due to the dissipation of its zero-point energy to the field. As \( \langle Q (\eta)^2 \rangle _\lambda \) decays, \( \langle Q (\eta)^2 \rangle _\nu \) grows in such a way that \( \langle Q^2 \rangle _\lambda = \langle Q^2 \rangle _\lambda + \langle Q^2 \rangle _\nu \) saturates at late times. For \( \langle \dot{Q} (\eta)^2 \rangle _\lambda \) and \( \langle \Delta \dot{Q} (\eta)^2 \rangle \), their behavior are similar.

One can express the reduced density matrix \( \rho^R (Q, Q') \) for the detector in terms of the above two-point functions of the detector to study the statistical properties of the detector such as entropy of entanglement and purity relevant to quantum information processing and teleportation issues [5]. Here we use the reduced density matrix to compare our results obtained from exact solutions with those from conventional perturbation theory.

The initial state (4) implies that \( \rho^R (Q, Q') \) is a Gaussian function of \( Q \) and \( Q' \). Transforming \( \rho^R (Q, Q') \) to the representation in the basis of energy-eigenstate for the free harmonic oscillator \( Q \),

\[
\rho^R (Q, Q') = \sum_{n \geq 0} \rho^R_{m, n} \phi_m (Q) \phi_n (Q')
\]

where \( \phi_n (Q) \) is the wave function for the \( n \)-th excited state, then the transition probability from the initial ground state to the first excited state is given by the \( m = n = 1 \) component,

\[
\rho^R_{1, 1} = \frac{\hbar}{m_0^2 (\langle \dot{Q}^2 \rangle (Q^2) - m_0^2 \langle \{ \dot{Q}, Q \} \rangle^2 - \frac{h^2}{4})} \left[ \left( \frac{m_0^2}{\pi \alpha} (\langle \dot{Q}^2 \rangle + \frac{h^2}{4}) \right) (\langle Q^2 \rangle h \alpha^2 + \frac{h^2}{4}) - m_0^2 \langle \{ \dot{Q}, Q \} \rangle^2 \right]^{3/2},
\]

with \( \alpha = \sqrt{m_0 \Omega_r / \hbar} \) and \( \{ A, B \} \equiv (A B + B A) / 2 \). When \( \eta \equiv \tau - \tau_0 \gg a^{-1} \), expanding \( \langle \cdots \rangle \) in terms of \( \gamma \equiv \lambda_0^2 / 8 \pi m_0 \), the approximate value to first order in \( \gamma \) is

\[
\rho^R_{1, 1} |_{\gamma \ll 1} \sim \frac{\lambda_0^2}{4 \pi m_0} \left[ \frac{\eta}{e^{2 \pi \Omega_r / \alpha} - 1} + \frac{\Lambda_1 + \Lambda_0}{2 \pi \Omega_r} \right].
\]
from the results in Ref. [4]. We see that the first term of (12) gives the transition probability [3] but emphatically it is not in a steady state situation. The approximation used in obtaining (12) is valid only at \( a^{-1} \ll \eta \ll \gamma^{-1} \), when the system is still in transient. If \( a < \gamma \), no perturbative regime exists at all. So the \( a = 0 \) case is beyond the reach of perturbation theory, and the conventional wisdom from perturbation theory that no transition occurs in an inertial detector is simply untenable. In contrast, \( \rho^{S,1}_{\|} \) at \( a = 0 \) behaves quite similarly to those cases with nonzero acceleration [3]. This agrees with our expectation when we observed that the UD detector theory with \( a = 0 \) is a special case of the model of the quantum Brownian motion [8], where there is a great deal of interplay between the oscillator and the quantum field.

Note further that the two additional (divergent) constants \( \Lambda_0 \) and \( \Lambda_1 \) present in (12) need be kept throughout the calculation because, if \( \Lambda_1 \) was subtracted naively, the uncertainty principle will be violated or \( \sqrt{\langle \bar{P}^2 \rangle \langle \bar{Q}^2 \rangle}_{\Lambda_1=0} < h/2 \) at late times for small enough \( a \). With these two divergent constants, of course, the scenario of the transition process will be quite different from the conventional ones. Further exposition of these new results are contained in [3].

V. CLASSICAL AND QUANTUM RADIATION

It is common knowledge that an accelerated point-charge coupled with electromagnetic(EM) field give rise to EM radiation [3, 10, 11]. Since our accelerated detector is also a point-like object coupled with a quantum field, it is natural to ask whether there is radiation emitted from a UAD, even under steady state conditions, as opposed to the thermal radiance experienced by the detector. Some even view the radiation emitted from a UAD as evidence of the Unruh effect [12, 13, 14]. (For a critique of this view and explanation, see, e.g., [15, 16].)

Prior work in (1+1) dimensions shows that there is no emitted radiation from a uniformly accelerated oscillator under equilibrium conditions (steady state and uniform acceleration) [17]. Nevertheless, most experimental proposals on the detection of Unruh effect are designed for the physical four dimensional spacetime, so one needs to examine the question for (3+1) dimensions. We have performed such an analysis which offer some new insights on this question.

Following a similar argument in classical theory [10], the radiation power emitted by the UD detector in (3+1)D is given by

\[
\langle \frac{dW^{\text{rad}}}{d\tau_-} \rangle = - \lim_{r \to \infty} \int r^2 d\Omega_{\text{II}} \, u^\mu \langle T_{\mu\nu} \rangle_{\text{ren}} v^\nu.
\]  

(13)

Here \( \tau_- (x) \) is the detector proper time at the moment when the spacetime point \( x \) (where the retarded field is measured) lies on the future lightcone with origin located at \( z^\mu (\tau_-) \) (see Eq. (34) in Ref. [4]). The quantum expectation value of the renormalized stress-energy tensor \( \langle T_{\mu\nu} \rangle_{\text{ren}} \) is obtained by calculating

\[
\langle T_{\mu\nu} [\Phi(x)] \rangle_{\text{ren}} = \lim_{x' \to x} \left[ \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} - \frac{1}{2} g_{\mu\nu} g^{\sigma\rho} \frac{\partial}{\partial x^\sigma} \frac{\partial}{\partial x'^\rho} \right] G_{\text{ren}}(x, x'),
\]

(14)

where \( G_{\text{ren}} \) is the renormalized two-point function of the field, defined by \( G_{\text{ren}}(x, x') \equiv \langle \Phi(x) \Phi(x') \rangle - G^{(0)}(x, x') \) with the Green’s function for free fields \( G^{(0)} \) subtracted. After some algebra, it turns out that \( r^2 u^\mu \langle T_{\mu\nu} \rangle_{\text{ren}} v^\nu \) is regular and non-vanishing at the null infinity of Minkowski space \( (r \to \infty) \) even in steady state [4], when the radiation power [15] can be written as

\[
\langle \frac{dW^{\text{rad}}}{d\tau_-} \rangle = \frac{\lambda^2}{8\pi} \int_0^\pi d\theta \sin \theta \left\{ \langle \dot{Q}^2 \rangle_{\text{tot}} - \frac{h\Theta_{\pm}}{\pi m_0} + a^2 \cos^2 \theta \langle Q^2 \rangle_{\text{tot}} + a \cos \theta \left\{ \langle Q, \dot{Q} \rangle_{\text{tot}} - \frac{h\Theta_x}{\pi m_0} \right\} \right\}.
\]

(15)

This is the quantum (massless) scalar field radiation emitted by the UAD in (3+1)D spacetime. The quantities in this formula are defined in Eqs. (103), (104) and Appendix A of Ref. [4].

The first term in (15), \( \langle \dot{Q}^2 \rangle_{\text{tot}} \), goes to zero at late times [4], so the corresponding monopole radiation ceases after the transient. The interference between the quantum radiation induced by the vacuum fluctuations and the vacuum fluctuations themselves totally obliterate any information pertaining to the appearance of the Unruh effect in this part of the radiation. Its behavior is analogous to that in (1+1)D: emitted radiation from UAD is only associated with nonequilibrium process [16].

The total screening of the monopole radiation corresponding to \( \langle \dot{Q}^2 \rangle_{\text{tot}} \) is actually a consequence of energy conservation. We found that the energy of the “dressed” detector

\[
E(\eta) = (m_0/2) \langle \dot{Q}^2(\eta) \rangle + \Omega_\epsilon^2 \langle Q^2(\eta) \rangle
\]

(16)
changes in time as

\[- \dot{E}(\eta) = \frac{\lambda_0^2}{4\pi} \langle \dot{Q}^2(\eta) \rangle_{\text{tot}}, \tag{17}\]

for all \( \eta > 0 \). This relation says that the rate of energy-loss of the dressed detector is equal to the radiated power via the monopole radiation corresponding to \( \langle \dot{Q}^2(\eta) \rangle_{\text{tot}} \). Thus Eq. (17) is simply a statement of energy conservation between the detector and the field. The external agent which drives the detector along the trajectory \( z^\mu(\tau) \) has no additional influence in this channel.

At late times, while \( \langle \dot{Q}^2 \rangle_{\text{tot}} \) ceases, a positive radiated power flow to the null infinity of Minkowski space still remains:

\[
\left\langle \frac{dW^{\text{rad}}}{d\tau_-} \right\rangle \to \frac{\hbar \lambda_0^2}{8\pi^2 m_0} \left\{ a^2 - a - \frac{2}{3} \left[ \frac{a^3}{3\Omega_r^2} - a + 2\gamma + \text{Re} \left[ \frac{i(\gamma + i\Omega)}{a\Omega} \right] ((\gamma + i\Omega)^2 - a^2) \psi^{(1)} \left( \frac{\gamma + i\Omega}{a} \right) \right] \right\}. \tag{18}\]

Thus we conclude that there exists a steady, positive radiated power of quantum nature emitted by the detector even when the detector is in a steady state. For large \( a \), the first term in (18) dominates, and the radiated power is approximately

\[
\left\langle \frac{dW^{\text{rad}}}{d\tau_-} \right\rangle \approx \frac{\lambda_0^2}{4\pi} \frac{a^2}{3} \frac{\hbar a}{2\pi m_0 \Omega_r^2} \propto a^2 T_U, \tag{19}\]

where \( T_U \) is the Unruh temperature. This could be interpreted as a hint of the Unruh effect.

The steady radiated power flow (18) does not originate from the thermal radiance that the detector experiences as in the Unruh effect, since the internal energy of the dressed detector is conserved only in relation to the radiated energy of a monopole radiation corresponding to \( \langle \dot{Q}^2 \rangle_{\text{tot}} \), which contributes nothing to (18). Learning from the EM radiation emitted by a uniformly accelerated charge [10, 11], we expect that the above non-vanishing radiated energy of quantum origin is supplied by the external agent driving the motion of the detector.

VI. SUMMARY

Our exact solution indicates that the conventional time-dependent perturbation theory in demonstrating Unruh effect is valid only in transient, with the duration of interaction timed between \( 1/a \) and \( 1/\gamma \). For the cases with proper acceleration \( a \) smaller than the damping constant \( \gamma \), time-dependent perturbation theory is invalid. Moreover, even with \( a = 0 \) there still exists non-trivial behavior in the detector when coupled with the quantum field. We also found new divergent constants present in the transition probability from the initial ground state of the detector to the excited states. They alter the scenario about the evolution of the system.

Going outside of the detector we found that unlike the (1+1)D case, the (3+1)D uniformly accelerated UD detector in a steady state does emit a positive radiated power of quantum nature. When the proper acceleration \( a \) is large, this flux is approximately proportional to the Unruh temperature \( T_U \), so it could be interpreted as a hint of the Unruh effect. Nevertheless, since the total energy of the dressed detector is conserved only with the radiated energy of a monopole radiation which ceases in steady state, the \textit{hint} of the Unruh effect in the late-time radiated power in (3+1)D is not connected to the thermal radiance experienced by the detector in the Unruh effect \textit{proper}. The experiments proposed so far [13] for the detection of Unruh radiation were not meant to be nor are they sensitive enough for this quantum radiation.

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[1] W. G. Unruh, Phys. Rev. D\textbf{14}, 870 (1976).
[2] B. S. DeWitt, in \textit{General Relativity: an Einstein Centenary Survey}, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).
[3] N. D. Birrell and P. C. W. Davies, \textit{Quantum Fields in Curved Space} (Cambridge University Press, Cambridge, 1982).
[4] S.-Y. Lin and B. L. Hu, Phys. Rev. D \textbf{73}, 124018 (2006) [gr-qc/0507054].
[5] S.-Y. Lin and B. L. Hu, in preparation.
[6] Philip R. Johnson and B. L. Hu, Phys. Rev. D\textbf{65} (2002) 065015
[7] Philip R. Johnson and B. L. Hu, Found. Phys. 35 (2005) 1117-1147 [gr-qc/0501029]
[8] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D 45, 2843 (1992).
[9] J. D. Jackson, Classical Electrodynamics (Wiley, New York, 1983).
[10] F. Rohrlich, Classical Charged Particles (Addison-Wesley, Redwood, 1965).
[11] D. G. Boulware, Ann. Phys. (N.Y.) 124, 169 (1980).
[12] P. Chen and T. Tajima, Phys. Rev. Lett. 83, 256 (1999).
[13] P. Chen (ed.), Quantum Aspects of Beam Physics, 18th Advanced IFCA Beam Dynamics Workshop, (World Scientific, Singapore, 2001).
[14] J. M. Leinaas, in [13] quant-ph/0012135.
[15] B. L. Hu and A. Raval, in [13] quant-ph/0012132.
[16] B. L. Hu and P. R. Johnson, in [13] quant-ph/0012132.
[17] P. G. Grove, Class. Quan. Grav. 3, 801 (1986); D. J. Raine, D. W. Sciama, and P. G. Grove, Proc. Roy. Soc. Lond. A435, 205 (1991); W. G. Unruh, Phys. Rev. D46, 3271 (1992); S. Massar, R. Parentani and R. Brout, Class. Quantum Grav. 10, 385 (1993).
[18] Since the UD detector considered here is a quantum mechanical object, there is a natural cutoff on the frequency at the energy threshold for the creation of detectors. Thus it is justified to assume that the detector has a small but finite extent in spacetime.