Coupled hydro-mechanical modelling on hydraulic fracturing using DDA

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Abstract:
Hydraulic fracturing is often used to increase the permeability of tight reservoir rocks. In this paper, a coupled hydro-mechanical module has been developed and coded into the Discontinuous Deformation Analysis (DDA) program to simulate the hydraulic fracturing progress. The governing equations of the couple hydro-mechanical module were introduced in the DDA framework. Parametric studies were conducted to investigate the effects of the sub-block sizes, sub-block shapes and strength heterogeneity on the development of the hydraulic fractures. It was found that bi-wing fracture was induced due to the increase of injection pressure and its orientation is approximately perpendicular to the direction of the minor principal stress. The increase of sub-block numbers could increase the simulated breakdown pressure. The simulations using quadrilateral sub-blocks would slightly increase the scales of the breakdown pressures compared with the triangular sub-blocks.

Keywords:
Discontinuous Deformation Analysis; hydraulic fracturing; fracture network
1. Introduction

Hydraulic fracturing is a process of fracture initiation, propagation, and branching due to the hydraulic loading applied by a high-pressure fluid [1, 2]. This technique has been widely used to enhance the permeability of reservoir rocks, such as shale and hot dry rock, for exploiting natural resources. Most of the available analytical models, i.e., PK model, PKN model, KGD model, Penny-shaped and Radial model, are based on continuum analysis [1]. Although these models have shown some success in solving practical issues, there are limitations in the assessment of fracturing domains [3]. Due to the complexity of hydraulic fracturing process, numerical tools have been widely used to probe the mechanisms underlying the reservoir stimulation treatment. The coupled hydro-mechanical modeling of reservoir rocks mainly involves the following three stages, i.e. (1) deformation of rock matrix and pre-existing fractures (if any); (2) fluid flow and evolution of fluid pressure along the fractures as a result of the fracture deformation; and (3) generation of newly-created fractures. Based on continuum approaches, the numerical investigations of hydraulic fracturing that are including FEM [4, 5], FDM [6, 7], XFEM [8-10], etc. The simulation results show that limited joint elements could be considered within the continuum-based simulation schemes to model the interaction between newly-generated and pre-existing fractures [5, 11].

Discontinuum-based numerical approaches have the capability to capture the fracture propagation and the rock interactions within fractured reservoir rocks, such as UDEC [12], PFC [13-15] and 3DEC [11]. The simulation results show the influences of the rock mass properties (such as modulus, cohesion and friction angle), the injection properties (such as injection rate and fluid viscosity), the field conditions (such as in-situ stress and existing natural fractures) on the hydraulic fracturing in rock mass. These methods are complex in the generation of the natural fractures. Apart from the above discontinuum-based codes, the Discontinuous Deformation Analysis (DDA) can be another promising approach in the investigation of hydraulic fracture initiation and propagation [16-20].

In this paper, a coupled hydro-mechanical module under the transient flow algorithm is developed in the framework of DDA to explore the hydraulic fracturing process in rock masses. The governing equations of the module are derived. A lab-scaled model is simulated to test the accuracy of the proposed model. Parametric studies are conducted to investigate the influences of the sub-block sizes and the sub-block shapes on the development of the hydraulic fractures.

2. Development of a coupled hydro-mechanical module with DDA

2.1. Assumptions and fluid flow scheme

When modeling hydraulic fracturing of rocks with the DDA, the fracturing fluid is assumed to be flowing along fractures, while the rock matrix is assumed to be impermeable. The cubic law, which has been proven to be appropriate to define the steady laminar flow for incompressible fluids through smooth parallel-plate fractures, is followed in the development of the hydro-mechanical module. For a relatively smaller fracture, the hydraulic pressure is normal to the fracture plane and the flow rate $Q$ could be calculated as [21]:

$$ Q = \frac{\rho g w e^3 \Delta H}{12 \mu L} $$

where $\rho$ is the fluid density; $g$ is the gravity acceleration; $w$ is the fracture width; $e$ is the fracture aperture; $\mu$ is the dynamic viscosity; $\Delta H / L$ is the hydraulic head gradient along the fracture.
After hydraulic pressure is applied on the boundary of an injection hole in fractured rocks, the hydraulic pressure propagates along fractures. In response to the increased hydraulic pressure in each fracture with time, the transient flow algorithm is introduced to calculate the hydraulic pressure on the affected fractures at each stage during the fracturing process [11, 17]. Assuming that there are \( n \) fractures connected through one intersection point \( i \). Based on the mass conservation on each intersection point, if the sink/source term is not considered, the accumulated fluid mass at the intersection point \( i \) should be equal to the total fluid flux through the fracture segments connected with this point. The fluid flow formula for the interaction point \( i \) is given by\(^{(16)}\):

\[
\left( \sum_{j=1}^{n} q_j \right)_i = - d_i \frac{dH_i}{dt} \tag{2}
\]

in which:

\[
\left( \sum_{j=1}^{n} q_j \right)_i = \sum_{j=1}^{n} \rho_w g \frac{w_j e_j^3 \Delta H_{ij}}{12 \mu L} \tag{3}
\]

\[
d = \frac{S_i}{2} \sum_{j=1}^{n} e_j L_j w_j \tag{4}
\]

where \( q_j \) (\( j = 1, 2, \ldots n \)) is the inflow/outflow from the intersection point \( j \), which is connected with the intersection point \( i \); \( S_i \) is the storage; \( e_j \), \( L_j \), and \( w_j \) are the equilibrium fracture aperture, fracture length and fracture width of the fracture through joint \( ij \), respectively.

For the hydraulic pressure applies linearly along a fracture defined by two end points \( i \) and \( j \) as shown in Figure 1, the hydraulic pressure on a point \((x, y)\) along the fracture is given as follows:

\[
F_x(t) = (F_{x2} - F_{x1}) t + F_{x1} \\
F_y(t) = (F_{y2} - F_{y1}) t + F_{y1} \tag{5}
\]

where \( t \) is the relative distance along the fracture segment with a variable range of \( 0 \leq t \leq 1 \). \( F_x \) and \( F_y \) are the hydraulic pressure components in the \( x \) and \( y \) directions for the moving point \((x, y)\) along the fracture length; \( F_{x1} \) and \( F_{y1} \) are the hydraulic pressure components in the \( x \) and \( y \) directions on point \( i \); \( F_{x2} \) and \( F_{y2} \) are the hydraulic pressure components in the \( x \) and \( y \) directions on point \( j \).

![Figure 1. Linearized hydraulic pressure distribution along the joint \( ij \).](image)

The integral expression for the hydraulic pressure along the fracture length can be yielded through the minimization of the potential energy, which is added into the sub-matrix \( \{ F_i \} \) and rewritten as:
where $L$ is the length of the fracture segment.

### 2.2. Fracture initiation and propagation

To simulate the crack propagation in rock in DDA, the artificial joint method was introduced to divide the continuous domain into small blocks \(^{[22-26]}\). The flat joint contact model \(^{[27, 28]}\) is used to simulate the force versus displacement behavior of the artificial joint. As shown in Figure 2a, blocks A and B are co-edged by an artificial joint with the length of $L$. There are forces and moments arising at the co-edge once a relative deformation occurs between the two blocks. By using the normal spring and shear spring to represent the contacts as shown in Figure 2b, the force and moment at the joint can be calculated as:

\[
\begin{align*}
\Delta R_n &= k_n L \Delta d_n \\
\Delta R_s &= -k_s L \Delta d_s \\
\Delta M_s &= -k N I \Delta \theta_s
\end{align*}
\]

where $\Delta R_n$ and $\Delta R_s$ are the increments of normal and shear forces at the artificial joint, respectively; $k_n$ and $k_s$ are the normal and shear stiffness of the contact, respectively; $\Delta d_n$ and $\Delta d_s$ are the increments of normal and shear displacements, respectively; $\Delta \theta_n$ and $\Delta \theta_s$ are the increments of normal and shear rotation angles, respectively, and $I$ and $L$ are the moment of inertia and length of the artificial joint.

![Figure 2](image_url)

**Figure 2.** Contacts at an artificial joint between two sub-blocks in DDA (a) sub-blocks and (b) contacts.
The tensile stress $\sigma_n$ and shear stress $\tau_s$ acting at an artificial joint could be calculated as:

$$\sigma_n = -k_n d_n + k_n |\theta_s L|$$  \hspace{1cm} (10)$$

$$T_s = |k_n d_s|$$  \hspace{1cm} (11)

The fracturing of artificial joint is determined by Mohr-Coulomb failure criteria. The artificial joint opens to being a real one if $\sigma_n > \sigma$ or $\tau_s > \tau_{s,\text{max}}$ where $\sigma$ and $\tau_{s,\text{max}}$ are the tensile strength and shear strength, respectively. The properties of an artificial joint are assigned initially with the friction angle $\varphi$, cohesion $c$ and tensile strength $\sigma$. Once the artificial joint opens, the artificial joint becomes a real joint whose properties are resigned with the residual friction angle $\varphi'$, residual cohesion $c'$ and residual tensile strength $\sigma'$, and usually $c' = 0$, $\sigma' = 0$.

3. Integration with DDA

All the artificial joints with non-zero aperture are assumed to model the inherent hydraulic conductivity of rock to simplify the hydraulic fracturing modeling. The artificial joints are full of water at the very beginning. Moreover, the aperture of fractures including the artificial joints and the real joints will be updated within each time step in the DDA computation. As shown in Figure 3, the framework of the multi-time step calculation in DDA are summarized as follows, 1) discretize the sub-blocks within simulation domains using the finite element meshing method to generate the initial fracture network connected by the artificial joints; 2) determine and mark the failed artificial joints or the real joints as cracks when all the sub-blocks become steady under the hydraulic pressure in each time step; 3) resign the cohesion and tensile strength on these artificial joints using the residual joint properties; 4) update the equilibrium aperture for each joint based on the change of the block positions; 5) calculate and transfer the hydraulic pressure on each intersection point within the fracture network to the next time step with the newly updated apertures; 6) continue the calculation until the hydraulic fracture reaches to the edge of the simulation domain.

![Figure 3. Framework of the multi-time step calculation in DDA](image)

4. Numerical simulation on the hydraulic fracturing

Figure 4 shows the DDA model of a lab-scale hydraulic fracturing model in homogeneous rock blocks with size of $300 \text{ mm} \times 300 \text{ mm}$. The rock domain is divided into 1714 triangular sub-blocks. A
borehole with a diameter of 20 mm is created at the center of the model to simulate the hydraulic injection hole. The four edges of the rock blocks are confined by four rigid plates. The bottom plate is fixed in both horizontal and vertical directions, and the rest three edges are used to apply stresses. The applied vertical and horizontal stresses are 6 MPa and 3 MPa, respectively. The basic material properties are summarized in Table 1.

![DDA model for hydraulic fracturing analysis using 1714 triangular sub-blocks.](image)

**Table 1.** Parameter settings for the model of hydraulic fracturing in homogeneous rock domain.

| Block properties         | Artificial joint properties (before /after fracturing) |
|--------------------------|--------------------------------------------------------|
| Density (kg/m³)          | 2600                                                   |
| Friction angle (°)       | 31.65 / 31.65                                          |
| Young’s modulus (GPa)    | 25                                                     |
| Cohesion (MPa)           | 12.70 / 0                                              |
| Passion ratio            | 0.324                                                  |
| Tensile (MPa)            | 3.74 / 0                                               |
| Contact stiffness (N/m)  | 2.0×10¹¹                                               |
| Initial aperture (m)     | 1.0×10⁻⁴ / 1.0×10⁻⁴                                    |

The numerical analysis was divided into two steps. The initial stresses are firstly applied on the model until the equilibrium balance is reached, followed by an increasing hydraulic pressure applying on the boundary of the injection hole. A stepwise loading scheme is adopted to model the gradually increment of the hydraulic pressure. Theoretically, the number of the time steps in each hydraulic pressure increment should be as large as possible to make sure that all the sub-blocks become steady in the stage. In the current calculation, the number of the required time steps to reach the hydraulic pressure target is determined using trial and error process. A 2500-step period is used with each step pressure of 0.2 MPa (see Figure 5). The loading continues in the simulation until the induced fractures propagated at the upper or lower boundaries of the block model. Once the artificial fracture falls into a real fracture, the program will draw the relative fracture in colors.
To examine the effects of sub block size on hydraulic fracturing numerical simulation, the fracturing progress and the final breakdown pressure in the simulations are monitored. Eight subdivision cases of the rock domain with the number of sub-blocks ranged from 1462 to 4260 are analyzed and listed in Table 2. The induced bi-wing fractures in the eight examples are all perpendicular to the directions of the minor principal stresses. This phenomenon is consistent with the studies in literature \cite{1, 29}. The magnitudes of breakdown pressure, \( p \), are slightly increased when more sub-blocks are used to divide rock domain. For example, the breakdown pressure is 6.6 MPa when 1462 sub-blocks are used in the model, while the breakdown pressure increases to 7.2 MPa when 4260 sub-blocks are used. Similar conclusions have been drawn by the numerical studies using the Particle Flow Codes (PFC)\cite{30}.

**Table 2.** Effects of the numbers of the sub-blocks in the case of triangular sub-blocks division (\( N \) is the numbers of sub-blocks and \( p \) is the breakdown pressure in MPa).

| \( N \)   | \( p \) |
|----------|--------|
| 1462     | 6.6    |
| 1714     | 6.6    |
| 1984     | 6.6    |
| 2320     | 6.8    |
| 2518     | 7.0    |
| 2888     | 7.0    |
| 3260     | 7.0    |
| 4260     | 7.2    |
To examine the effects of the sub-division shapes, the rock domain as shown in Fig. 4 is meshed using quadrilateral sub-blocks. The results of the eight models simulated using different numbers of quadrilateral sub-blocks at the breakdown pressure are presented in Table 3. Similar orientations of the bio-wing fractures are also observed as those from Table 2. The breakdown pressures of these models are also slightly increased from 6.8 MPa when there are 1484 sub-blocks to 7.2 MPa when there are 3852 sub-blocks. Figure 6 plots the breakdown pressures versus numbers of sub-blocks curves. It is found that the breakdown pressures from the quadrilateral are slightly higher than that from triangular sub-blocks. The efficiency of simulation is quite low for the model with sub-block numbers above 3000 as the fracture network was updated at every time step.

Figure 6. Effects of the numbers of sub-blocks on the breakdown pressure.

Table 3. Effects of the numbers of the sub-blocks in the case of quadrilateral sub-blocks division (\(N\) is the numbers of sub-blocks and \(p\) is the breakdown pressure in MPa).
5. Discussions
A coupled hydro-mechanical model is developed and coded into the Discontinuous Deformation Analysis (DDA) program to investigate the hydraulic fracturing in tight reservoir rocks. The governing equations of the module are introduced. A lab-scaled model is simulated to examine the executions of the model. Parametric studies are conducted to investigate the influences of sub-block sizes and sub-block shapes, as well as the joint heterogeneity. It is found through this study that the developed model can simulate the hydraulically induced bi-wing fractures propagation perpendicular to the direction of the minor principal stress in the lab-scaled models. The influence of sub-block size on the breakdown pressure for both models with triangular and quadrilateral sub-blocks is limited. However, the breakdown pressure of the model filled with quadrilateral sub-blocks is slightly higher compared with that of the model filled with triangular sub-blocks in case that the numbers of sub-blocks are similar.

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Reference
[1] Adachi J, Siebirts E, Peirce A and Desroches J. Computer simulation of hydraulic fractures. *International Journal of Rock Mechanics and Mining Sciences.* 2007;44(5):739-57.
[2] Deng S, Li H, Ma G, Huang H and Li X. Simulation of shale–proppant interaction in hydraulic fracturing by the discrete element method. *International Journal of Rock Mechanics and Mining Sciences.* 2014;70(219-28.
[3] Jing L. A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering. *International Journal of Rock Mechanics and Mining Sciences.* 2003;40(3):283-353.
[4] Wangen M. Finite element modeling of hydraulic fracturing on a reservoir scale in 2D. *Journal of Petroleum Science and Engineering.* 2011;77(3):274-85.
[5] Rueda Corder JA, Mejia Sanchez EC, Roehl D and Pereira LC. Hydro-mechanical modeling of hydraulic fracture propagation and its interactions with frictional natural fractures. *Computers and Geotechnics.* 2019;111(290-300.
[6] Zhou L and Hou MZ. A new numerical 3D-model for simulation of hydraulic fracturing in consideration of hydro-mechanical coupling effects. *International Journal of Rock Mechanics and Mining Sciences.* 2013;60(370-80.
[7] Kresse O, Weng X, Gu H and Wu R. Numerical Modeling of Hydraulic Fractures Interaction in Complex Naturally Fractured Formations. *Rock Mechanics and Rock Engineering.* 2013;46(3):555-68.
[8] Gordeliy E, Abbas S and Peirce A. Modeling nonplanar hydraulic fracture propagation using the XFEM: An implicit level-set algorithm and fracture tip asymptotics. *International Journal of Solids and Structures.* 2019;159(135-55.
[9] Saberhosseini SE, Ahangari K and Mohammadrezaei H. Optimization of the horizontal-well multiple hydraulic fracturing operation in a low-permeability carbonate reservoir using fully coupled XFEM model. *International Journal of Rock Mechanics and Mining Sciences.* 2019;114(33-45.)
[10] Sheng M, Li G, Sutula D, Tian S and Bordas SPA. XFEM modeling of multistage hydraulic fracturing in anisotropic shale formations. *Journal of Petroleum Science and Engineering*. 2018;162(801-12).

[11] Hamidi F and Mortazavi A. A new three dimensional approach to numerically model hydraulic fracturing process. *Journal of Petroleum Science and Engineering*. 2014;124(451-67).

[12] Nasehi MJ and Mortazavi A. Effects of in-situ stress regime and intact rock strength parameters on the hydraulic fracturing. *Journal of Petroleum Science and Engineering*. 2013;108(211-21).

[13] Zhang Q, Zhang X-P and Ji P-Q. Numerical study of interaction between a hydraulic fracture and a weak plane using the bonded-particle model based on moment tensors. *Computers and Geotechnics*. 2019;105(79-93).

[14] Fatahi H, Hossain MM and Sarmadivaleh M. Numerical and experimental investigation of the interaction of natural and propagating hydraulic fracture. *Journal of Natural Gas Science and Engineering*. 2017;37(409-24).

[15] Wang T, Hu W, Elsworth D, Zhou W, Zhou W and Zhao X. The effect of natural fractures on hydraulic fracturing propagation in coal seams. *Journal of Petroleum Science and Engineering*. 2017;150(180-90).

[16] Jing L, Ma Y and Fang Z. Modeling of fluid flow and solid deformation for fractured rocks with discontinuous deformation analysis (DDA) method. *International Journal of Rock Mechanics and Mining Sciences*. 2001;38(3):343-55.

[17] Ben YX, Wang Y and Shi GH. Development of A Model for Simulating Hydraulic Fracturing with DDA. In: G. C, Y. O, L. Z, T. S, editors. Frontiers of Discontinuous Numerical Methods and Practical Simulations in Engineering and Disaster Prevention. London: Taylor & Francis Group, 2013.

[18] Fan LF, Yi XW and Ma GW. Numerical manifold method (NMM) simulation of stress wave propagation through fractured rock mass. *International Journal of Applied Mechanics*. 2013;5(02), 1350022.

[19] Chen HM, Zhao ZY, Choo LQ and Sun JP. Rock Cavern Stability Analysis Under Different Hydro-Geological Conditions Using the Coupled Hydro-Mechanical Model. *Rock Mechanics and Rock Engineering*. 2016;49(2):555-72.

[20] Shi GH. Discontinuous Deformation Analysis-a New Numerical Model for the Statics and Dynamics of Block Systems: University of California at Berkeley, Berkeley, 1988.

[21] Hubbert MK and Willis DG. Mechanics Of Hydraulic Fracturing. *Society of Petroleum Engineers*. 1957. p. 16.

[22] Ning Y, Yang J, An X and Ma G. Modelling rock fracturing and blast-induced rock mass failure via advanced discretisation within the discontinuous deformation analysis framework. *Computers and Geotechnics*. 2011;38(1):40-9.

[23] Jiao YY, Zhang XL and Zhao J. Two-Dimensional DDA Contact Constitutive Model for Simulating Rock Fragmentation. *Journal of Engineering Mechanics*. 2012;138(2):199-209.

[24] Nie W, Zhao ZY, Guo W, Shang J and Wu C. Bond-slip modeling of a CMC rockbolt element using 2D-DDA method. *Tunnelling and Underground Space Technology*. 2019;85(340-53).

[25] Cai Y and Wu J. A robust algorithm for the generation of integration cells in Numerical Manifold Method. *International Journal of Impact Engineering*. 2016;90(165-76).

[26] Wu J and Cai Y. Generation Algorithm of Cover System in Manifold Method with Quadrangular Meshes [J]. *Journal of Tongji University (Natural Science)*. 2013;5

[27] Potyondy DO and Cundall PA. A bonded-particle model for rock. *International Journal of Rock Mechanics and Mining Sciences*. 2004;41(8):1329-64.

[28] Zhang XP and Wong LNY. Cracking Processes in Rock-Like Material Containing a Single Flaw Under Uniaxial Compression: A Numerical Study Based on Parallel Bonded-Particle Model Approach. *Rock Mechanics and Rock Engineering*. 2012;45(5):711-37.
[29] Rahman MM and Rahman MK. A Review of Hydraulic Fracture Models and Development of an Improved Pseudo-3D Model for Stimulating Tight Oil/Gas Sand. Energy Sources, Part A: Recovery, Utilization, and Environmental Effects. 2010;32(15):1416-36.

[30] Shimizu H, Murata S and Ishida T. The distinct element analysis for hydraulic fracturing in hard rock considering fluid viscosity and particle size distribution. International Journal of Rock Mechanics and Mining Sciences. 2011;48(5):712-27.

[31] Liu B, Schieber J, Mastalerz M and Teng J. Variability of rock mechanical properties in the sequence stratigraphic context of the Upper Devonian New Albany Shale, Illinois Basin. Marine and Petroleum Geology. 2020;112(104068.

[32] Xu J. Experimental study on the hydraulic fracturing of shale. Wuhan, China: Chinese Academy of Science. 2012.

[33] Nie W. Reinforcement mechanism of rockbolt system for underground excavation. Ph.D., Nanyang Technological University, Singapore. 2019.