The Strong Multifield Slowroll Condition and Spiral Inflation

I-Sheng Yang

ISCAP and Physics Department
Columbia University, New York, NY, 10027, U.S.A.

ABSTRACT: We point out the existing confusions about the slowroll parameters and conditions for multifield inflation. If one requires the field to roll down the gradient flow, we find that only articles adopting the Hubble slowroll expansion are on the right track and a correct condition can be found in a recent book by Liddle and Lyth. We further analyze this condition and show that the gradient flow requirement is stronger than just asking for a slowly changing, quasi-de Sitter solution. Therefore it is possible to have a multifield slowroll model that does not follow the gradient flow. Consequently, it no longer requires the gradient to be small, so it even bypasses the first slowroll condition and some related no-go theorem from string theory. We provide the “spiral inflation” as a generic blueprint of such inflation model and show that it relies on a monodromy locus—a common structure in string theory effective potentials.

*isheng.yang@gmail.com
Contents

1. Introduction 1

2. Multifield Slowroll Conditions 3
   2.1 The Sasaki-Stewart Condition 4
   2.2 The Hertzberg-Tegmark-Kachru-Shelton-Ozcan Condition 5
   2.3 The Balasubramanian-Berglund-Jimenez-Simon-Verde Condition 6
   2.4 The Strong Second Slowroll Condition 6

3. Spiral Inflation 8

1. Introduction

Multifield inflation models gathered a lot of attention recently, resulting from the combination of top-down and bottom-up motivations. UV considerations like supersymmetry [1] and some specific string theory derived models [2-5] all tend to hint that slowroll inflation can happen at an environment where more than one field is dynamically important. The multifield dynamics provide a rich variety of potential observational signatures like the isocurvature modes [6] and non-gaussianities [7, 8].

It is somewhat surprising that a fundamental question, which some might consider as a prior before those developments, remains to have non-uniform answers. Basically, “What are the conditions for multifield slowroll inflation”? By analogy to the single field slowroll models, one might want to take the “slowroll condition” as a property of a given point in the multi-dimensional field space that is both necessary and sufficient to support a self-consistent slowroll solution which follows the gradient flow. This is therefore an expansion of the potential at this point, and the $n$th order will be related to the $n$th derivative of $V$. It is often truncated at $n = 2$ as we will do in this paper, which shall provide us what commonly known as the first and the second slowroll conditions.

Pioneers on multifield string inflation [3, 4] have attempted similar goals. As far as we can tell, the conditions derived or used in those papers do not agree with each other, and we can show that neither of them meet our explicit criterion of sufficient and necessary conditions. The condition one can find in the classic paper by Sasaki
and Stewart [6] was also not meant to be necessary. In Sec. 2 we will start by analyzing these earlier works and eventually derive the correct condition. Our first milestone is equivalent to an equation given in the recent book by Lyth and Liddle [9].

It is worth noting that parallel to this potential expansion, slowroll conditions can be studied by another well-known method— the Hubble slowroll expansion [10] which expands $H$ as a function of $\phi$. The usual impression of their difference is the following: The potential expansion is useful to high energy physicists for the purpose of studying an effective potential from some complicated UV theory; the Hubble slowroll expansion is useful to astrophysicists as it is naturally related to the observables, which makes it known as the “phenomenological expansion”.

The equation in [9] is kind of a hybrid, that it contains explicitly both $H$ and $V$ as function of time or of the fields. In this paper we try to exploit the advantage for both methods more thoroughly. We first follow the potential expansion to the end so that our condition only involves $V$. Then at each order we remind ourselves the corresponding “phenomenological meaning”. This allows us to easily recognize the correct slowroll parameters as how they enter observables, and the correct slowroll condition that is actually constrainting a slightly different combination of parameters. It then becomes obvious that in the multifield context, following the gradient flow is a stronger condition than just requiring a quasi-de Sitter solution.

This discovery implies the possibility to have a non-standard slowroll inflation, defined as just a slowly changing $H$, but does not require fields following the gradient flow. This possibility was already pointed out in [11] but did not receive a lot of attention. In Sec. 3 we will push the idea further by writing down an explicit potential for it. Since the field is no longer following the gradient flow, this type of models eliminates the need for a small $\nabla V$, which was thought to be the an obstacle of string inflation [12, 13]. It turns out to require a special point which is surrounded by a radially attractive potential and allows multi-value in the angular direction. The radial gradient provides the centripedal force and the angular gradient balances the hubble friction. This combination allows the field to spiral rapidly yet descend slowly. The multi-value property is met by the abundant monodromy loci in the string theory moduli space [14], and recent calculation of the effective potential suggests that some of them are radially attractive [15–19]. Therefore it is very likely that such “spiral inflation” could be realized around those monodromy loci [20].

Finally, it is worth mentioning that as a hindsight, there is another important advantage of the Hubble slowroll expansion which is less appreciated. In the potential expansion, one often uses $3H \dot{\phi} = -\partial_i V$ to relate the field motion and the potential. So it is awkward in the formalism to describe something that does not follow the gradient flow. In Hubble slowroll expansion, $\dot{\phi}$ is related to $H$ by just the Einstein equations
and no approximations involved. Thus, its multifield generalization [21,22] in principle provides the correct conditions already. An alternative way to reach our conclusion is to follow that approach, then at the second order analyze the possible field motion and shapes of the potential.

2. Multifield Slowroll Conditions

We start by reviewing the well known slowroll conditions for a single field inflation and remind ourselves their phenomenological meanings. The input are the field equation of motion,

\[ \ddot{\phi} + 3H\dot{\phi} = -V', \]  

(2.1)

and the assumption that the potential dominates over the kinetic energy in the Einstein equation,

\[ 3M_P^2H^2 = V. \]  

(2.2)

The consistency of a slowroll solution requires two slowroll conditions. We will first assume that the second derivative in Eq. (2.1) could be ignored, which later will turn out to be the second slowroll condition. Given this assumption, we can derive the first slowroll condition,

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3\dot{\phi}^2/2}{V} = \frac{M_P^2V'^2}{2V^2} \ll 1. \]  

(2.3)

Its phenomenological meaning is twofold: the expansion rate \( H \) changes by a small fraction during one hubble time \( H^{-1} \), also the potential energy dominates. They are directly related by virtue of the Einstein equation.

Now, the consistency of ignoring the second derivative term,

\[ \left| \frac{d}{dt} \left( \frac{-V'}{3H} \right) \right| \ll |V'|, \]  

(2.4)

leads us to

\[ \left| \frac{\epsilon}{3} - \frac{M_P^2V''}{3V} \right| \ll 1. \]  

(2.5)

Thus we may define

\[ \eta \equiv \frac{M_P^2V''}{V} \]  

(2.6)

as the second slowroll parameter, and

\[ |\eta| \ll 1 \]  

(2.7)
The second slowroll condition. Again, it has a phenomenological meaning from
\[ \frac{1}{\epsilon H} \frac{d \epsilon}{dt} = 4\epsilon - 2\eta . \] (2.8)

Namely, the first slowroll parameter changes by a small fraction during one hubble time.\(^1\)

In the case of multiple fields, the only difference is in the field equation of motion,
\[ \ddot{\phi}_i + 3H \dot{\phi}_i = -\partial_i V . \] (2.9)

Here we assume canonically normalized kinetic terms. Nontrivial field space metric will promote those partial derivatives to covariant derivatives. Since the slowroll conditions are about a particular point in the field space around which we can always locally canonically normalize, such technicality should not bother us.

The first slowroll condition is about the first derivative of the potential, \( \partial_i V \). Though it is a vector now, obviously we only care about its magnitude and there is no room for confusion.

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = 3 \left( \frac{\dot{\phi}_i^2}{V} \right) = \frac{M_P^2 (\partial_i V)^2}{2V^2} \ll 1 . \] (2.10)

It also retains the same phenomenological implications.

The second slowroll condition will concern the second derivative of \( V \), which is now a matrix, \( \partial_i \partial_j V \). It is not directly clear which part of this matrix really needs to be small.

2.1 The Sasaki-Stewart Condition

In a pioneering study of the multifield slowroll inflation [6], Sasaki and Stewart required that the trace of the square of the matrix \( \partial_i \partial_j V \) to be small,
\[ M_P^2 \sqrt{(\partial_i \partial_j V)(\partial_j \partial_i V)} \ll V . \] (2.11)

This means the curvature in all directions have to be small, which is of course sufficient. Clearly they did not mean it to be necessary as it is straightforward to demonstrate why not. Consider a potential \( V(\phi) \) where the single field slowroll conditions are satisfied

\(^1\)Note that in the Hubble slowroll expansion the second order parameter is naturally defined as something proportional to \( H'' \), which happens to be just \( \dot{\epsilon}/(H\epsilon) \). In this paper we will follow the potential expansion, so we always define the second order parameter as the second derivative of the potential.
at $\phi_*$. We can promote it to a two field potential by simply adding an independent orthogonal direction,

$$V(\phi, \psi) = V(\phi) + \frac{m^2}{2} \psi^2. \quad (2.12)$$

A large $m^2$ immediately ruins the condition in Eq. (2.11), but we know at $\phi = \phi_*$, $\psi = 0$, slowroll inflation will occur just as in the single field potential. Therefore Eq. (2.11) is a sufficient but not necessary condition.

### 2.2 The Hertzberg-Tegmark-Kachru-Shelton-Ozcan Condition

In an important paper connecting string inflation models to astrophysics [3], it was claimed that

$$\eta = \text{min eigenvalue} \left\{ \frac{M_P^2 (\partial_i \partial_j V)}{V} \right\} \quad (2.13)$$

is the second slowroll parameter, and $|\eta| \ll 1$ is the second slowroll condition. It is not necessary nor sufficient. We can understand that through the following example.

$$V(\phi_1, \phi_2) = V_0 - \frac{m_1^2}{2} \phi_1^2 + \frac{m_2^2}{2} \phi_2^2. \quad (2.14)$$

In the neighborhood where $V_0$ dominates, the HTKSO condition means that

$$|\eta| = \frac{M_P^2 m_1^2}{V_0} \ll 1. \quad (2.15)$$

For a point along the $\phi_2$ axis in this region, we have

$$\epsilon = \frac{M_P^2 m_2^4 \phi_2^2}{2V_0^2}, \quad (2.16)$$

but we know choosing a small $\phi_2$ would not mean slowroll even though both conditions are satisfied. Because whether the first slowroll parameter changes slowly,

$$\frac{1}{\epsilon H} \frac{d\epsilon}{dt} = 4\epsilon - 2\frac{M_P^2 m_2^2}{V_0}, \quad (2.17)$$

cares about $m_2$ instead of $m_1$. Actually, it is more appropriate to take

$$\eta = \frac{M_P^2 m_2^2}{V_0} \quad (2.18)$$

here as it is what enters observables like the spectral index, not some minimum eigenvalue which is not along the rolling direction.
2.3 The Balasubramanian-Berglund-Jimenez-Simon-Verde Condition

In another important paper connecting string inflation to cosmology [4], the authors argued that in each vector component of Eq. (2.9), the second derivative needs to be negligible.

\[ |\ddot{\phi}_i| \ll |3H\dot{\phi}_i| \text{ or } |\partial_i V| \text{ for each } i. \tag{2.19} \]

This starting point is fundamentally incorrect since the components do not have a specific physical meaning unless we specify a special frame. For example, if we rotate to a frame that \( \partial_i V \) goes along one of the axis, then in all the orthogonal directions \( \partial_i V \) components are zero and \( \ddot{\phi}_i \) cannot be negligible—which means by definition Eq. (2.19) can never be satisfied.

2.4 The Strong Second Slowroll Condition

To get the correct second slowroll condition as a consistency condition that

\[ 3H\dot{\phi}_i = -\partial_i V \tag{2.20} \]

is a good approximate solution, we should start from Eq. (2.19) but instead of taking its components, treat it like what it is—a vector equation. \( \ddot{\phi}_i \) being negligible just means its magnitude is negligible.

\[ |\ddot{\phi}_i| \ll |\partial_i V|. \tag{2.21} \]

This means the same thing as

\[ \frac{|H^{-1}\ddot{\phi}_i|}{3|\dot{\phi}_i|} \ll 1, \tag{2.22} \]

that the change to the vector \( \dot{\phi}_i \) within one hubble time is negligible. After some algebra, we have

\[ \frac{1}{3} \left[ \frac{M_P^2(\partial_i V)(\partial_j V)(\partial_j \partial_k V)(\partial_k V)}{V^2(\partial_i V)^2} - M_P^4 \frac{(\partial_i V)(\partial_i \partial_j V)(\partial_j V)}{V^3} + \epsilon^2 \right]^{1/2} \ll 1. \tag{2.23} \]

It is useful to introduce the following notations:

\[ \hat{V}_1 \equiv \frac{\partial_i V}{|\partial_i V|} \tag{2.24} \]

is the normalized direction of the first derivative (gradient) vector of \( V \);

\[ \uparrow \! V_2 \equiv \frac{M_P^2(\partial_i \partial_j V)}{V} \tag{2.25} \]
is the unitless second derivative matrix of $V$. We can simplify Eq. (2.23) to
\[ \frac{1}{3} \left[ \dot{V}_1 \cdot \dddot{V}_2 \cdot \dddot{V}_2 \cdot \dot{V}_1 - 2\epsilon \dot{V}_1 \cdot \dddot{V}_2 \cdot \dot{V}_1 + \epsilon^2 \right]^{1/2} \ll 1, \tag{2.26} \]
which is identical to Eq.(20.9) in [9]. On top of the first slowroll parameter, we have two other terms related to the projection of $\dddot{V}_2$ and $(\dddot{V}_2)^2$ along $\dot{V}_1$.

It is easier to understand these two terms by going to the eigenbasis of $\dddot{V}_2$. Since it is symmetric, it will have a diagonal form
\[ \dddot{V}_2 = \text{Diag}\{\lambda_i\}, \tag{2.27} \]
\[ (\dddot{V}_2)^2 = \text{Diag}\{\lambda_i^2\}. \tag{2.28} \]
Note that $\dot{V}_1 = \{v_i\}$ in general will not be an eigenvector, so we should have
\[ \dot{V}_1 \cdot \dddot{V}_2 \cdot \dddot{V}_2 \cdot \dot{V}_1 = \sum v_i^2 \lambda_i^2, \tag{2.29} \]
\[ \dot{V}_1 \cdot \dddot{V}_2 \cdot \dddot{V}_2 = \sum v_i^2 \lambda_i, \tag{2.30} \]
with $\sum v_i^2 = 1$.

One may imagine two ways to satisfy Eq. (2.26). Either the first 2 terms are individually small or they mostly cancel each other. However, since $\epsilon \ll 1$, a cancellation already implies their smallness. So the sufficient and necessary condition is
\[ \xi \equiv \sqrt{\dot{V}_1 \cdot \dddot{V}_2 \cdot \dddot{V}_2 \cdot \dot{V}_1} = \sqrt{\sum v_i^2 \lambda_i^2} \ll 1. \tag{2.31} \]
We call this the \textit{Strong Second Slowroll Condition} because it implies that the second term is small, too.

\[ \eta \equiv \dot{V}_1 \cdot \dddot{V}_2 \cdot \dot{V}_1 = \sum v_i^2 \lambda_i \ll 1. \tag{2.32} \]

Why do we still care about Eq. (2.32)? Apparently by the choice of symbol $\eta$ we do intend to identify it as the analog of the second slowroll parameter in the single field case. That is because if we consider the phenomenological definition that the first slowroll parameter changes slowly,
\[ \frac{1}{\epsilon H} \frac{d\epsilon}{dt} = 4\epsilon - 2\dot{V}_1 \cdot \dddot{V}_2 \cdot \dot{V}_1 = 4\epsilon - 2\eta \ll 1, \tag{2.33} \]
it only cares about $\eta$. Therefore it is $\eta$ instead of $\xi$ that enters observables like the spectral index.

It is intriguing that Eq. (2.20) requires Eq. (2.31), which is stronger than the requirement of Eq. (2.33). This means that with multiple fields, \textit{demanding the field to slowly follow the gradient flow is not the only way to get a slowly changing quasi-de Sitter solution}. This possibility was also noticed in [11].
3. Spiral Inflation

In order to develop a slowroll model in which Eq. (2.20) does not hold, it is not wise to think about conditions for the potential \( V \), as Eq. (2.20) was used almost everywhere to simplify the equations in the previous section. A parallel technique, Hubble slowroll expansion [22], is much more appropriate and we encourage the readers to learn it. Here we will again stop at the second order which is already very informative, so we can just simplifying Eq. (2.33) with the full equation of motion, Eq. (2.9), instead of Eq. (2.20).

\[
\frac{1}{H \epsilon} \frac{d \epsilon}{dt} \approx 2 \epsilon + 2 \frac{\dot{\phi}_i \ddot{\phi}_i}{H \dot{\phi}_i^2}. \tag{3.1}
\]

Now it is obvious that instead of making \( \ddot{\phi}_i \) small, we can satisfy the second slowroll condition by making it almost orthogonal to \( \dot{\phi}_i \). This means the \( \dot{\phi}_i \) vector will be rapidly turning but maintaining roughly constant speed. Such situation is familiar to physicists as a stable circular orbit which is easiest to analyze in the polar coordinate.\(^2\)

\[
\mathcal{L} = \frac{1}{2} \left( r^2 + r^2 \dot{\theta}^2 \right) - V(r, \theta). \tag{3.2}
\]

Here \( r \) is a field with the unit of mass, \( \theta \) is a unitless field. The equations of motion are

\[
\ddot{r} + 3H \dot{r} - r \dot{\theta}^2 + \frac{\partial V}{\partial r} = 0, \tag{3.3}
\]

\[
r^2 \ddot{\theta} + 2r \dot{r} \dot{\theta} + 3Hr^2 \dot{\theta} + \frac{\partial V}{\partial \theta} = 0. \tag{3.4}
\]

A rapid turning slowroll can be realized when both of these equations are dominated by their last two terms. At zeroth order we adopt the following ansatz:

\[
r = \text{const.} = R, \tag{3.5}
\]

\[
3HR^2 \dot{\theta} = -\frac{\partial V}{\partial \theta} = -c, \tag{3.6}
\]

\[
R \dot{\theta}^2 = \frac{\partial V}{\partial r} = \frac{c^2}{9H^2 R^3}. \tag{3.7}
\]

These can be satisfied by a simple choice of potential

\[
V(r, \theta) = V_0 + c \theta + \frac{c^2 r^\alpha}{9\alpha H^2 R^{\alpha+2}}. \tag{3.8}
\]

\(^{2}\)Similar techniques have been used to study sharp turns which the field temporarily leaves the slowroll trajectory [23], or slow turns that still follow the gradient flow [24]. It was not realized that one can stay out of the gradient flow yet maintain slowroll inflation.
More generally $c$ can be a function of $\theta$ and $r$, as long as it does not contribute significantly to the radial derivative and changing slowly enough with $\theta$.

The intuitive way to think about this model is that a radially attractive potential maintains a stable circular orbit, while a slowly descending angular spiral balances the hubble friction.

The first slowroll condition can be derived by its phenomenological meaning of potential energy domination,

$$\epsilon = -\frac{\dot{H}}{H^2} = 3\frac{R^2\dot{\theta}^2/2}{3M_P^2H^2} \ll 1,$$

which leads us to

$$c = 3\sqrt{2}\epsilon M_P R H^2 \ll 3\sqrt{2}M_P R H^2. \tag{3.10}$$

The change of $\theta$ per e-folding is

$$|\Delta \theta| = |H^{-1}\dot{\theta}| = \sqrt{2}\epsilon M_P R. \tag{3.11}$$

If we choose Planckian radius $R \gtrsim M_P$, then $\Delta \theta \ll 1$ and it is actually an usual gradient flow inflation since it is not turning rapidly enough. We would require

$$R \lesssim \sqrt{2}\epsilon M_P, \tag{3.12}$$

such that the field rotates a significant fraction of $2\pi$ per e-folding. That is the spiral slowroll we are looking for.

Of course, this implies that the potential is not singled-value after a $2\pi$ rotation. This is totally ok and actually exciting. In string theory, one can get an effective potential for the moduli fields from Calabi-Yau compactification. The moduli space always have several branch-cuts and multiple layers. The end points of these branch-cuts are monodromy loci, some of which can have attractive potential when the strong warping correction is included [15–19].

Since $|\nabla V|$ is dominated by the radial component that is just supplying the centripetal force, it does not have to be small and can bypass the “first slowroll condition” in the usual sense of a small gradient. In fact, $|\nabla V|$ is bounded from below through the fast spiraling condition Eq. (3.12),

$$\frac{M_P^2|\nabla V|^2}{2V^2} \geq \frac{4}{9}\epsilon^3. \tag{3.13}$$

Near a strongly warped conifold, it is possible to have an attractive potential satisfying the above properties. However it remains unknown whether there is a good chance to sustain a long period of inflation and we will try to addressed that more general problem in [20].
Acknowledgments

We thank Pontus Ahlgvist, Brian Greene, Lam Hui, Shamit Kachru, David Kagan and Eugene Lim for stimulating discussions. We especially thank Andrew Liddle and David Lyth who pointed out several useful references and provided a chapter from their book. This work is supported in part by the US Department of Energy, grant number DE-FG02-11ER41743.

References

[1] D. Baumann and D. Green, “Signatures of Supersymmetry from the Early Universe,” arXiv:1109.0292 [hep-th].

[2] R. Kallosh, “On inflation in string theory,” Lect. Notes Phys. 738 (2008) 119–156, arXiv:hep-th/0702059.

[3] M. P. Hertzberg, M. Tegmark, S. Kachru, J. Shelton, and O. Ozcan, “Searching for Inflation in Simple String Theory Models: An Astrophysical Perspective,” Phys. Rev. D76 (2007) 103521, arXiv:0709.0002 [astro-ph].

[4] V. Balasubramanian, P. Berglund, R. Jimenez, J. Simon, and L. Verde, “Topology from Cosmology,” JHEP 0806 (2008) 025, arXiv:0712.1815 [hep-th].

[5] P. Berglund and G. Ren, “Multi-Field Inflation from String Theory,” arXiv:0912.1397 [hep-th].

[6] M. Sasaki and E. D. Stewart, “A General analytic formula for the spectral index of the density perturbations produced during inflation,” Prog. Theor. Phys. 95 (1996) 71–78, arXiv:astro-ph/9507001 [astro-ph].

[7] D. Seery and J. E. Lidsey, “Primordial non-Gaussianities from multiple-field inflation,” JCAP 0509 (2005) 011, astro-ph/0506056.

[8] X. Chen, “Primordial Non-Gaussianities from Inflation Models,” Adv. Astron. 2010 (2010) 638979, arXiv:1002.1416. 84 pages, invited review for special issue of Advances in Astronomy on 'Testing the Gaussianity and Statistical Isotropy of the Universe'/ v3, various improvement and corrections, especially in Sec.6.4, 8.1 & 9.2.

[9] D. H. Lyth and A. R. Liddle, “The primordial density perturbation: Cosmology, inflation and the origin of structure,”.

[10] A. R. Liddle, P. Parsons, and J. D. Barrow, “Formalizing the slow roll approximation in inflation,” Phys. Rev. D50 (1994) 7222–7232, astro-ph/9408015.
[11] R. Brandenberger, P.-M. Ho, and H.-c. Kao, “Large N cosmology,” *JCAP* **0411** (2004) 011, arXiv:hep-th/0312288 [hep-th].

[12] M. P. Hertzberg, S. Kachru, W. Taylor, and M. Tegmark, “Inflationary Constraints on Type IIA String Theory,” *JHEP* **0712** (2007) 095, arXiv:0711.2512 [hep-th].

[13] M. C. Johnson and M. Larfors, “Field dynamics and tunneling in a flux landscape,” *Phys.Rev.* **D78** (2008) 083534, arXiv:0805.3705 [hep-th].

[14] U. H. Danielsson, N. Johansson, and M. Larfors, “The World next door: Results in landscape topography,” *JHEP* **0703** (2007) 080, arXiv:hep-th/0612222 [hep-th].

[15] M. R. Douglas, J. Shelton, and G. Torroba, “Warping and supersymmetry breaking,” arXiv:0704.4001 [hep-th].

[16] G. Shiu, G. Torroba, B. Underwood, and M. R. Douglas, “Dynamics of Warped Flux Compactifications,” *JHEP* **0806** (2008) 024, arXiv:0803.3068.

[17] M. R. Douglas and G. Torroba, “Kinetic terms in warped compactifications,” *JHEP* **0905** (2009) 013, arXiv:0805.3700.

[18] L. Martucci, “On moduli and effective theory of N=1 warped flux compactifications,” *JHEP* **0905** (2009) 027, arXiv:0902.4031.

[19] P. Ahlqvist, B. R. Greene, D. Kagan, E. A. Lim, S. Sarangi, and I.-S. Yang, “Conifolds and Tunneling in the String Landscape,” *JHEP* **03** (2011) 119, arXiv:1011.6588 [hep-th].

[20] P. Ahlqvist, B. R. Greene, D. Kagan, and I.-S. Yang, *work in progress*.

[21] S. Groot Nibbelink and B. van Tent, “Density perturbations arising from multiple field slow roll inflation,” arXiv:hep-ph/0011325 [hep-ph].

[22] R. Easther and J. T. Giblin, “The Hubble slow roll expansion for multi field inflation,” *Phys.Rev.* **D72** (2005) 103505, arXiv:astro-ph/0505033 [astro-ph].

[23] X. Chen, “Primordial Features as Evidence for Inflation,” *JCAP* **1201** (2012) 038, arXiv:1104.1323.

[24] X. Chen and Y. Wang, “Quasi-Single Field Inflation and Non-Gaussianities,” *JCAP* **1004** (2010) 027, arXiv:0911.3380. 56 pages, v4, minor revision with added comments, JCAP version.