Effect of the Flavor Changing Neutral Current on Rare B decays

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Abstract

We study the effect of the FCNC on rare B decays in the beyond standard model with vector like singlet quarks. It has been shown that $b \rightarrow s\gamma$ does not receive sizable contribution compared with that of the standard model while $b \rightarrow d\gamma$, $b \rightarrow s(d)l^+l^-$, can be changed from the predictions of the standard model.

1 Introduction

It has been known that the Flavor Changing process in down quark sector is sensitive to the mass difference among up-type quark in the standard model (SM). GIM mechanism tells us that the FCNC in one-loop level of SM vanishes when up type quarks are degenerate. In the real world, there is large mass gap among up quark sector ($M_u \ll M_c \ll M_t$). Then the Flavor Changing Process in down quark sector like $b \rightarrow s\gamma$ and $b \rightarrow s(l^+l^-)$ are enhanced. If we extend the fermion sector beyond the standard model, there are several different possibilities. If we just add the chiral fermions as fourth generation in sequential way, GIM mechanism still works and the new contribution comes from up-type quark ($t'$) in the fourth generation. Therefore we can get constraints for the mass of $t'$ and its Kobayashi Maskawa mixing to light down-type quarks ($V'_{CKM}, i = d, s, b$) However if we extend the fermion sector in non-sequential way, the different aspect arises (refs. [1], [2], [3], [6] and [7]). The tree level FCNC arises both in Z and neutral Higgs sector. Furthermore, the size of the FCNC depends on the structure of the down quark mass matrix rather than up type quark masses. Therefore we may study the structure of down type quark mass matrix by studying the flavor changing process in down quark sector.

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2 Non-sequential extention of fermion sector

We study the Standard Model (SM) with extended quark sector. In addition to the three standard generations of quarks, $N_d - 3$ down-type and $N_u - 3$ up-type vector like singlet quarks are introduced (refs. [1], [2] and [3]).

\[ I_W = \frac{1}{2} : \left( \begin{array}{ccc} u & c & t \\ d & s & b \end{array} \right)_L, \quad I_W = 0 : \left( \begin{array}{ccc} u_R & c_R & t_R \ t'_{L+R} \\ d_R & s_R & b_R \ b'_{L+R} \ \end{array} \right). \]

(1)

In this model, there is tree level FCNC in $Z$ and Higgs sector. In order to illustrate this point and explain the relation between the size of FCNC and down-type quark mass matrix, let us introduce a toy model. This is the so called "top-prime less" model

\[ I_W = \frac{1}{2} : \left( \begin{array}{c} t^0_L \\ b^0_L \end{array} \right), \quad I_W = 0 : \left( \begin{array}{c} t^0_R \\ b^0_R \end{array} \right). \]

(2)

The most general mass matrix for down quark sector $M_d$ is given by,

\[ L_{mass} = -(b^0_L b^0_{L'}) \left[ \begin{array}{cc} m & 0 \\ J & m_4 \end{array} \right] \left( \begin{array}{c} b^0_R \\ b^0_{R'} \end{array} \right). \]

(3)

where $J$ is complex and $m$ and $m_4$ are real numbers. $m$ comes from the vacuum expectation value of Higgs doublet. $J$ leads to the mixing between left handed singlet quarks and right handed ordinary quarks. In the limit of $J = 0$, the vector like quark decouples from the ordinary quark. $M_d M_d^\dagger$ can be diagonalized by the following unitary transformation :

\[ \left( \begin{array}{c} b^0_L \\ b^0_{L'} \end{array} \right) = \left( \begin{array}{cc} \cos \theta & -\sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & \cos \theta \end{array} \right) \left( \begin{array}{c} b_L \\ b'_{L} \end{array} \right), \]

(4)

where $b^0_L$ and $b^0_{L'}$ indicate the weak basis. When $m$ is much smaller than $M = \sqrt{|J|^2 + m_4^2}$, the elements of the unitary matrix are given by the following formulae approximately :

\[ \cos \theta \approx 1, \]

\[ \sin \theta e^{-i\phi} \approx \frac{mJ}{M^2}. \]

(5)

(6)

Correspondingly, the mass eigenvalues for heavy quark ($M_H$) and light quark ($M_L$) are given by :

\[ M_L = m \frac{m_4}{M}, \]

\[ M_H = M. \]

(7)

(8)

There are two interesting limits :

1. $J \ll m_4$
2. $J \gg m_4$

In the case of (1), the mixing between ordinary quark and vector like quark is suppressed. The physical masses are given by the diagonal elements of the mass matrix:

$$M_L = m, \quad M_H = m_4.$$  \hspace{1cm} (9)

In the case of (2), the diagonal elements no longer reflect the physical masses:

$$M_L = m \frac{m_4}{|J|}, \quad M_H = |J|.$$  \hspace{1cm} (10)

Note that the light quark mass vanishes in the limit $m_4 = 0$. On the other hand, the neutral current is written in terms of physical basis,

$$L_Z = -\frac{g}{2 \cos \theta_W} b_L^0 \gamma_\mu b_L^0 Z^\mu$$

$$= -\frac{g}{2 \cos \theta_W} \left[ \cos^2 \theta b_L^0 \gamma_\mu b_L - \cos \theta \sin \theta e^{-i\phi} b_L^0 \gamma_\mu b_L' + h.c. \right] Z^\mu.$$  \hspace{1cm} (11)

Thus the flavor diagonal coupling for neutral current ($Z^{bb}$) and the FCNC coupling ($Z^{bb'}$) are given by:

$$Z^{bb} = \cos^2 \theta = 1 - \sin^2 \theta \simeq 1 - O \left( \frac{mJ}{M^2} \right)^2,$$  \hspace{1cm} (12)

$$Z^{bb'} = -\cos \theta \sin \theta e^{-i\phi} \simeq -\frac{mJ}{M^2}.$$  \hspace{1cm} (13)

Depending on the two cases mentioned above, the enhancement and suppression of the FCNC occur respectively.

1. $J \ll m_4$ (suppression)

$$M_L = m,$$  \hspace{1cm} (14)

$$M_H = m_4,$$  \hspace{1cm} (15)

$$|Z^{bb'}| = \frac{m|J|}{m_4^2} \ll \frac{M_L}{M_H}.$$  \hspace{1cm} (16)

2. $J \gg m_4$ (enhancement)

$$M_L = m \frac{m_4}{|J|},$$  \hspace{1cm} (17)

$$M_H = |J|,$$  \hspace{1cm} (18)

$$|Z^{bb'}| = \frac{m}{|J|} \gg \frac{M_L}{M_H}.$$  \hspace{1cm} (19)
For instance, if the physical mass of the vector-like quark mass is $M_H = 500$ (GeV), the following mass matrices realize the same mass eigenvalues for heavy and light quark masses ($M_H = 500$ (GeV), $M_L = M_b = 5$ (GeV)) while giving rise to the different size of FCNC.

1. $M_d = \begin{bmatrix} 5 & 0 \\ 50 & 500 \end{bmatrix}$ (GeV), $Z^{bb'} = 10^{-3}$,
2. $M_d = \begin{bmatrix} 50 & 0 \\ 500 & 50 \end{bmatrix}$ (GeV), $Z^{bb'} = 10^{-1}$.

This exercise tells us that the FCNC between singlet quark and ordinary quark is enhanced when the off-diagonal element in the mass matrix is larger than the diagonal element.

This kind of analysis can be extended to the general case with arbitrary numbers of isosinglet up-type quarks and down-type quarks. Here we just record the full lagrangian for the model (ref. [7]).

$$L = L_{W^\pm} + L_{\chi^\pm} + L_A + L_Z + L_H + L_{\chi^0}, \quad (20)$$

where,

$$L_{W^\pm} = \frac{g}{\sqrt{2}} V_{CKM}^{\alpha\beta} \bar{u}^\alpha \gamma^\mu L d^\beta W^{\mu +} + h.c., \quad (21)$$

$$L_{\chi^\pm} = \frac{g}{\sqrt{2} M_W} V_{CKM}^{\alpha\beta} \bar{u}^\alpha (m_{u^\alpha} L - m_{d^\beta} R) d^\beta \chi^\pm + h.c., \quad (22)$$

$$L_A = \frac{e}{3} \left( 2 \bar{u}^\alpha \gamma^\mu u^\alpha - \bar{d}^\alpha \gamma^\mu d^\alpha \right) A_\mu, \quad (23)$$

$$L_Z = \frac{g}{2 \cos \theta_W} \left\{ \bar{u}^\alpha \gamma^\mu \left[ \left( Z_u^{\alpha\beta} - \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} \right) L - \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right] u^\beta \\
+ \bar{d}^\alpha \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} - Z_d^{\alpha\beta} \right) L + \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right] d^\beta \right\} Z_\mu, \quad (24)$$

$$L_H = \frac{-g}{2 M_W} \left[ Z_u^{\alpha\beta} \bar{u}^\alpha (m_{u^\alpha} L + m_{u^\beta} R) u^\beta \\
+ Z_d^{\alpha\beta} \bar{d}^\alpha (m_{d^\alpha} L + m_{d^\beta} R) d^\beta \right] H, \quad (25)$$

$$L_{\chi^0} = \frac{-ig}{2 M_W} \left[ Z_u^{\alpha\beta} \bar{u}^\alpha (m_{u^\alpha} L - m_{u^\beta} R) u^\beta \\
- Z_d^{\alpha\beta} \bar{d}^\alpha (m_{d^\alpha} L - m_{d^\beta} R) d^\beta \right] \chi^0. \quad (26)$$

For $N_u = 3$ and $N_d = 4$, the diagonal elements and off-diagonal elements of FCNC are given by the following equations:

$$Z^{bb} = 1 - \left| V_L^{4b} \right|^2, \quad (27)$$

$$Z^{bs} = -V_L^{4b} V_L^{4s}, \quad (28)$$

$$Z^{bb'} = -V_L^{4b} V_L^{44}, \quad (29)$$
where $V_L$ is a unitary matrix which diagonalizes the down type quark mass matrix

$$d_L^{0a} = V_L^{ab} d_L^b.$$  \hspace{1cm} (30)

In the basis in which up-type quark is diagonalized, 3 by 4 part of $V_L$ is just CKM matrix,

$$V_L^{ia} = V_{CKM}^{ia} \quad (i = 1, 2, 3, \quad a = 1, 2, 3, 4).$$ \hspace{1cm} (31)

This leads to the relation between the FCNC and the deviation of unitarity of CKM matrix

$$Z^{bs} = -V_L^{4b} V_L^{4s} = \sum_{i=1}^{3} V_{CKM}^{ib} V_{CKM}^{is}.$$ \hspace{1cm} (32)

Therefore the unitarity of the CKM matrix no longer holds. This non-unitarity “quadrangle” relation is used to constrain the FCNC coupling, e.g. $Z^{bs}$, in later section. However, the deviation is suppressed when the diagonal element of singlet quark mass is infinite and keeping off-diagonal element finite.

3 Rare decays and Effect of FCNC

3.1 New Physics v.s. Standard Model in $b \to sZ \to sl^+l^-$

In the present beyond standard model, there is FCNC which leads to tree level coupling $Z^{bs}$, $Z^{bd}$ and $Z^{sd}$. In the standard model, the same vertex comes from top quark one loop diagrams. Therefore the condition that the $Z$ FCNC dominates over the standard model contribution in $Z$ sector is:

$$\left| \frac{Z^{bs}}{V_{CKM}^{tb} V_{CKM}^{ts}} \right| > O(\alpha) \simeq 0.012.$$ \hspace{1cm} (33)

Then, even tiny coupling for $Z$ FCNC, it can easily dominates over the standard model contribution. Experimental constraints and quadrangle constraints for the FCNC in $bs$, $bd$ and $sd$ sectors are given in Table (1). From Table (1) (refs. [4], [3] and [7]), $\left| Z^{bs}/V_{CKM}^{tb} V_{CKM}^{ts} \right| \leq 0.05$ and $\left| Z^{bd}/V_{CKM}^{tb} V_{CKM}^{td} \right| \leq 0.79$. Therefore, there are allowed regions for $Z^{bs}$ and $Z^{bd}$ couplings where the tree level $Z$ FCNC can dominate over the 1-loop top quark diagrams in the standard model. We can expect the drastic change of the differential decay rates in the present beyond standard model in $b \to sl^+l^-$ and $b \to dl^+l^-$ process.

3.2 New Physics v.s. Standard Model in $b \to s\gamma$ and $b \to sg$ process

There is no flavor changing neutral currents for $b \to s\gamma$ and $b \to sg$ ($g$: gluon) processes in the beyond standard model and standard model. Therefore, in order that the new physics contribution dominates over the standard model contribution, the FCNC coupling constant must be the same order as that of the CKM matrix elements.
| Decay process | Experiment constraint | Quadrangle constraint |
|----------------|-----------------------|-----------------------|
| $B \to X_s \mu^+ \mu^-$ | $\frac{Z_{bs}^{bs}}{V_{CBKM}^{th} V_{CBKM}^{ts}} \leq 0.047$ | $\frac{V_{CBKM}^{th} V_{CBKM}^{ts}}{V_{CBKM}^{th} V_{CBKM}^{ts}} \geq 0.94$ |
| $B \to X_d \mu^+ \mu^-$ | $\frac{Z_{bd}^{bd}}{V_{CBKM}^{ed} V_{CBKM}^{cs}} \leq 0.23$ | $\frac{V_{CBKM}^{ed} V_{CBKM}^{cs}}{V_{CBKM}^{ed} V_{CBKM}^{cs}} \geq 0.29$ |
| $K^+ \to \pi^+ \nu \bar{\nu}$ | $\frac{Z_{sd}^{sd}}{V_{CBKM}^{cd} V_{CBKM}^{cs}} \leq 2.9 \times 10^{-4}$ | $\frac{V_{CBKM}^{cd} V_{CBKM}^{cs}}{V_{CBKM}^{cd} V_{CBKM}^{cs}} \geq 0$ |

Table 1: Upper bound for $|Z^{a\beta}|$ from experiments, with assuming $Z$ exchange tree diagrams are dominant.

The condition for the new physics contribution dominates over the standard model contribution is:

$$|\frac{Z_{bs}^{bs}}{V_{CBKM}^{th} V_{CBKM}^{ts}}| > O(1).$$  \hfill (34)

From Table (1) (refs. [2], [7]) we can see that there will be no significant contribution to $b \to s \gamma(g)$ process because $|\frac{Z_{bs}^{bs}}{V_{CBKM}^{th} V_{CBKM}^{ts}}| < 0.05$. For $b \to d \gamma(g)$, new physics contribution can be significant because $|\frac{Z_{bd}^{bd}}{V_{CBKM}^{ed} V_{CBKM}^{cs}}| < 0.79$.

Let us summarize the computation of $b \to d \gamma$ briefly. The amplitude is proportional to the magnetic moment interaction (refs. [4], [6] and [7]),

$$T = \frac{G_F e}{4\sqrt{2} \pi^2} d_L \sigma_{ll} b_R e^\mu q^\nu F,$$  \hfill (35)

where,

$$F = Q_u V_{CBKM}^{lb} V_{CBKM}^{td} F^{cc} \left( \frac{m_t}{M_W} \right) + Q_d Z_{db}^{db} \left( \frac{2}{3} \sin^2 \theta_W F_1^{NC} (0) + 2 F_3^{NC} (0) \right) + Q_d Z'_{db} \left( F_3^{NC} \left( \frac{m_H}{M_W} \right) + F_2^{NC} \left( \frac{m_H}{m_H} \right) \right)$$  \hfill (36)

where $Q_u = 2/3$ and $Q_d = -1/3$, and $m_H$ is the neutral Higgs mass. The first term comes from the top quark loop. The second and the third terms come from down type quark loop. The second term represents the light down type quarks ($d, b$) loop and the third term represents the heavy down type quark ($b'$). The details of the functions and their behavior are given in ref. [7].

4 Summary

We study the rare $B$ decays in the non-sequential extension of fermion sector. The effect of vector like down type singlet quark is studied. It has been shown that the
tree level FCNC can contribute to $b \to s(d)l^+l^-$ processes, while $b \to s\gamma(g)$ does not receive the significant contribution due to the New Physics. In $b \to d\gamma(g)$, the effect of FCNC ($Z^bd$, $Z^{db}'$, $Z^{bb}'$) may be seen. **Acknowledgments**

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**Q & A**

**Q :I. Bigi** I think that it is too optimistic to say that the 15 percentage deviation from the SM can be seen in $b \to d\gamma$ process because of $b \to u\bar{u}d\gamma$ process.

**A :T. M.** I did not consider this point.