An Investigation on CCT and Ra Optimization for Trichromatic White LEDs Using a Dual-Weight-Coefficient-Based Algorithm

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Abstract: Spectral optimization is applied as an effective tool in designing solid-state lighting devices. Optimization speed, however, has been seldomly discussed in previous reports as regards designing an algorithm for white light-emitting diodes (WLEDs). In this study, we propose a method for trichromatic WLEDs to obtain the optimal Ra under target correlated color temperatures (CCTs). Blue-, yellow-, and red-color monochromatic spectra, produced by the GaN LED chip, YAG:Ce 3+ phosphors, and CdSe/ZnSe quantum dots, respectively, are adopted to synthesize white light. To improve the effectiveness of our method, the concept of dual weight coefficients is proposed, to maintain a numerical gap between the proposed floating CCT and the target CCT. This gap can effectively guarantee that Ra and CCT ultimately move toward the targeting value simultaneously. Mechanisms of interaction between CCT, Ra, and dual-weight coefficients are investigated and discussed in detail. Particularly, a fitting curve is drawn to reveal the linear relationship between weight coefficients and target CCTs. This finding effectively maintains the accuracy and accelerates the optimization process in comparison with other methods with global searching ability. As an example, we only use 29 iterations to achieve the highest Ra of 96.1 under the target CCT of 4000 K. It is hoped that this study facilitates technology development in illumination-related areas such as residential intelligent lighting and smart planting LED systems.

Keywords: light-emitting diode; spectral optimization; correlated color temperature (CCT); general color rendering index (Ra)

1. Introduction

In comparison with RGB LEDs, light-conversion-material-based white light-emitting diodes (WLEDs) have played leading roles in solid-state illumination, due to their high light-conversion efficiencies in specific wavebands, stability under various junction temperatures, low cost, and feasibility in color tunability [1–3]. Conventionally, the color performance of WLEDs is evaluated from two aspects: the emitting color from the WLED and the releasing color of objects exposed under the WLED. Theoretically, the former is characterized by correlated color temperature (CCT), and the latter is characterized by color rendering property, in which the concept of general color rendering index (Ra) is conventionally adopted for evaluation by the International Commission on Illumination [4]. CCT expresses a warm or cold feeling when we observe the light beam, while Ra reveals the ability of a light source to express the real color of an object. Ra represents the average value of the color rendering index (CRI) of eight general colors in a WLED system. CCT and Ra are functions of monochromatic spectral power distributions (SPDs) of different colors [5]. Therefore,
if we intend to adjust the circadian rhythm of humans and plantings or reduce driver’s fatigue in a specific scenario, modulating the SPD of the WLED system is effective and indispensable [6]. In trichromatic WLEDs, the white-light SPD (SPD$_W(\lambda)$) is conventionally generated by downconverting light-conversion materials with blue LED chips, in which light in short wavebands (such as blue light) can be effectively converted to light in long wavebands (such as red light) under the stokes effect.

In the last few decades, SPD optimization technologies have been widely investigated to facilitate illumination in areas of residential lighting [7], agriculture [8], rehabilitation therapy [9], and visible light communication [10,11]. These technologies can be mainly divided into two catalogs: the first type is to optimize the color rendering property and energy consumption by adjusting peak wavelengths, spectral bandwidth, and intensity by using Gaussian functions; the other is to optimize the color rendering property and energy consumption by adjusting the density of real light in different colors. For the first type, Guo et al. [12] conducted comprehensive numerical simulations of three-hump and four-hump SPDs in WLEDs. The changes in Ra and CCT values with the shifting peak wavelengths of full width at half maximums (FWHMs) were analyzed under different operating temperatures. The relationship between scotopic–photopic ratio and CCT was investigated as well. Wei et al. [13] proposed six-channel-based LEDs to synthesize daylight with high quality by using a genetic algorithm and Gaussian spectral model. For the second type, Zhu et al. [14] conducted a comprehensive study on illumination performances of the perovskite-based LED with four humps. Titkov et al. [15] proposed a semi-hybrid device, which combined monolithic blue-cyan LED with green-red phosphor mixture, exhibiting the highest Ra of 98.6 at CCT of 3400 K. Yuan et al. [16] manufactured a trichromatic WLED, which constitutes of blue-pump carbon dots and phosphor glass, realizing the highest Ra of 92.9 at CCT of 3610 K. Among these studies, a variety of methods are conventionally used, such as the multiple Gaussian function method [17], least-squares method [18], and iterative method of gradient descent [19]. However, these methods focus on improving the accuracy and the feasibility, as well as developing light-conversion material species with superior chromaticity; few of them discuss the improvement strategy of optimization speed for WLEDs.

With the development of the Internet of Things (IoT) and 5G technologies, the intelligent control technology of illumination lamps becomes imperative for saving energy and increasing productivity [20,21]. Therefore, improving the effectiveness of spectral optimization becomes a key issue in intelligent control. In this study, we propose a convenient method to optimize CCT and Ra values simultaneously for trichromatic WLEDs by using dual-weight coefficients. These coefficients can effectively control the variation range of CCT while searching for the optimal Ra value. Key steps to realize the proposed method is analyzed comprehensively. Compared with other conventional methods used for spectral optimization, the proposed method can greatly accelerate the calculation process while maintaining accuracy.

2. Monochromatic Spectra Preparation and Theory of Algorithms

As shown in Figure 1a, the blue-emissive LED chip (302 × 198 μm$^2$, Hualian Co., Ltd., Xiamen, China) was selected as the excitation source, and yellow-emissive cerium-doped yttrium aluminum garnet phosphors (YAG:Ce$^{3+}$, Youyan Rare Earth Co., Ltd., Beijing, China) and red-emissive CdSe/ZnSe quantum dots (Poly OptoElectronics Co., Ltd., Jiangmen, China) were selected as light-conversion materials to fabricate white light. YAG:Ce$^{3+}$ phosphors can greatly broaden the white-light spectrum in the visible-light regime, while CdSe/ZnSe quantum dots are able to provide pure red emission with high stability and high quantum yields (QYs). Recently, QYs of YAG:Ce$^{3+}$ phosphors and CdSe/ZnSe quantum dots can reach up to 90% and ~100%, respectively [22,23]. Packaging technology of the WLED has been given in [24]. To facilitate our study, spectra of monochromatic blue, yellow, and red light are referred to as SPD$_B(\lambda)$, SPD$_Y(\lambda)$, and SPD$_R(\lambda)$, respectively.
(1) First, we initialize the procedure and load original data, such as the spectra of different phosphors. For instance, phosphors produce a spectrum with FWHM of 125 nm that covers a wide range of visible light, including yellow and red light, produced by LED chips. YAG:Ce\(^{3+}\) phosphors generate narrow blue and red peaks with FWHM of 54 nm and 56 nm, respectively. On the other hand, YAG:Ce\(^{3+}\) phosphors produce a spectrum with FWHM of 125 nm that covers a wide range of visible light. Here, we assumed that the peak wavelength and the FWHM of these monochromatic spectra are independent of the driven current, so SPD\(_B\)\((\lambda)\) can be described as a linear combination of SPD\(_B\)\((\lambda)\), SPD\(_Y\)\((\lambda)\), and SPD\(_R\)\((\lambda)\), as described by

\[
SPD_W(\lambda) = A_B \cdot SPD_B(\lambda) + A_Y \cdot SPD_Y(\lambda) + A_R \cdot SPD_R(\lambda) \tag{1}
\]

where \(A_B\), \(A_Y\), and \(A_R\) are the proportions of the radiant power of blue, yellow, and red light, respectively.

Before calculation, target CCT, test CCT, and test Ra values are defined as CCT\(_{\text{tar}}\), CCT\(_{\text{test}}\), and Ra\(_{\text{test}}\), respectively. CCT\(_{\text{test}}\) and Ra\(_{\text{test}}\) represent current CCT and Ra values in the calculation. Figure 2a illustrates the steps for spectral optimization using the proposed method. For comparison, conventional methods I and II used for spectral optimization are illustrated in Figure 2b, c. Among these three methods, method I directly considers all the possibilities of \(A_B\), \(A_Y\), and \(A_R\) under CCT\(_{\text{tar}}\), while method II randomly selects values of \(A_B\), \(A_Y\), and \(A_R\) until fulfilling the cycle index, which is set as 1000 for methods I and II. Both methods I and II use the bubbling method to obtain the highest Ra within the error range of CCT\(_{\text{tar}}\). For the proposed method, the calculation steps are described as follows:

1. First, we initialize the procedure and load original data, such as the spectra of monochromatic light, step lengths for iteration, error ranges of Ra and CCT, and initial values of \(A_B\), \(A_Y\), and \(A_R\);

2. Two key problems for CCT optimization are how to adjust CCT\(_{\text{test}}\) and how to optimize Ra in the meantime. According to the relationship between CCT and components of different colors, we first set a floating parameter between the initial CCT and CCT\(_{\text{tar}}\), which is named CCT\(_{\text{m}}\). The relationship between CCT\(_{\text{m}}\) and CCT\(_{\text{test}}\) can be expressed as CCT\(_{\text{test}} = \delta_1 + \text{CCT}_{\text{m}}\), where \(\delta_1\) is the first weight coefficient in our algorithm. To realize CCT\(_{\text{m}}\), we only need to modulate the parameter of \(A_B\);

3. Before realizing CCT\(_{\text{tar}}\), we optimize Ra\(_{\text{test}}\) by using the bubbling method. Keeping the proportion of \(A_B\) and \(A_Y\) unchanged, we attempt to modulate \(A_R\) with a small step to observe the change of Ra\(_{\text{test}}\). If the small step helps to increase the value of Ra, we conduct a similar iteration until Ra\(_{\text{test}}\) reaches the highest value; otherwise, we...
modulate $A_R$ in the negative direction. The relationship between $CCT_m$ and $CCT_{tar}$ can be expressed as $CCT_{tar} = \delta_2 + CCT_m$, where $\delta_2$ is the second weight coefficient in our algorithm.

(4) When the calculation result meets the required conditions, we export the optimized proposal of $CCT$ process: We first impel $CCT_{test}$ to move toward $CCT_m$ and then optimize $Ra_{test}$ and $CCT_{test}$ simultaneously, to reach the optimum $Ra$ and $CCT_{tar}$. If we directly search for $CCT_{tar}$, the variation range of $CCT_{test}$ is very limited, due to the interaction effect of $CCT$ and $Ra$. The proposal of $CCT_{tar}$ can effectively solve this problem, rendering $CCT_{test}$ reach a position near $CCT_{tar}$ before the optimization of $Ra_{test}$.

Below is the design philosophy of the proposed algorithm. Particularly, we propose a floating $CCT$ value named $CCT_m$ and two weight coefficients, named $\delta_1$ and $\delta_2$, to control the variation range of the $CCT_{test}$. There exist two main stages in the optimization process: We first impel $CCT_{test}$ to move toward $CCT_m$ and then optimize $Ra_{test}$ and $CCT_{test}$ simultaneously, to reach the optimum $Ra$ and $CCT_{tar}$. If we directly search for $CCT_{tar}$, the variation space of $Ra_{test}$ is very limited, due to the interaction effect of $CCT$ and $Ra$. The proposal of $CCT_m$ can effectively solve this problem, rendering $CCT_{test}$ reach a position near $CCT_{tar}$ before the optimization of $Ra_{test}$.

Weight coefficients of $\delta_1$ and $\delta_2$, which determine the value of $CCT_m$ and the shifting range of $CCT_{test}$, are key for the optimization result. If $\delta_2$ is too small, $Ra_{test}$ will not reach the highest value due to the limited shifting space; on the other hand, if $\delta_2$ is too large, $Ra_{test}$ can reach the highest value soon but at the expense of the error between $CCT_{test}$ and $CCT_{tar}$. Another problem is how to guarantee that $CCT_{test}$ moves toward $CCT_{tar}$ instead of the reverse direction while optimizing $Ra_{test}$. To solve this problem, we set the original $A_B$, $A_Y$, and $A_R$ values as $0.1$, $0.3$, and $0.5$, respectively, in which $A_R$ is large enough to guarantee the decreasing trend of $A_R$ while optimizing $Ra_{test}$.

3. Results and Discussion

3.1. Relationship between CCT, Ra, and Other Parameters

Figure 3 illustrates the variation of $CCT_{test}$, $Ra_{test}$, and $\delta_1$ under different $\delta_2$ in a 3D coordinate diagram, when $CCT_{tar}$ is set as 8000 K. To facilitate discussion, these results are separately presented in Figure 3a,b at different view angles. In Figure 3a, $CCT_{test}$ is decreasing with the increase in $\delta_1$ when $\delta_2$ equals 3000 K. The reason is that $\delta_1$ directly influences the difference between $CCT_{test}$ and $CCT_m$. If $\delta_1$ is too large, $CCT_{test}$ becomes
much smaller than CCT_m, increasing the difficulty of reaching CCT_tar while optimizing Ra_test. According to the methodology of the algorithm, optimization will finally stop when we obtain the optimal Ra. By then, the final CCT_test obtained may fail to reach CCT_tar.

![Figure 3](image-url)

Figure 3. Shifting trend of CCT_test, Ra_test, and δ_1 under different δ_2 in a 3D coordinate diagram with CCT_tar of 8000 K. (a,b) are the same 3D figure at different view angles.

Secondly, Ra_test increases with the increase in δ_2. Since δ_2 represents the difference between CCT_m and CCT_tar, when we increase δ_2, CCT_m becomes smaller. Thus, larger δ_2 provides a wider range for Ra optimization, extending the shifting area of Ra_test within the permitted range of CCT_test.

From Figure 3b, we observe the variation of CCT_test, Ra_test, and δ_1 under different δ_2 at the other view angle. When δ_2 is set as 600 K, 1200 K, and 1800 K, respectively, the curves almost lie in a similar plane with CCT_tar, equal to 8000 K. However, for δ_2 = 2400 K and δ_2 = 3000 K, corresponding curves stretch out of this plane. In other words, their CCT_test become much smaller than CCT_tar of 8000 K. Below are the explanation for this phenomenon. For δ_2 = 2400 K and δ_2 = 3000 K, we reserve a large variation range of CCT to support the optimization for CCT_m and Ra_test, causing the inaccessibility of CCT_tar when the optimization process of Ra is ending. This explains the phenomenon that those points on the curve are almost far away from 8000 K when δ_2 = 3000 K. When δ_2 decreases from 3000 K to 2400 K, a portion of points on the curve return to the plane of 8000 K. To summarize, those points staying near the plane of 8000 K are constrained by CCT_tar; those points that stretch out from the plane of 8000 K are constrained by optimization conditions of Ra.

As shown in Figure 4a, Ra_test increases with an increase in δ_1 and δ_2. If δ_1 remains unchanged and δ_2 decreases, it would cost more iteration steps to increase A_B in order to improve CCT_test. Hence, even CCT_test reaches CCT_m; however, the value of A_B already becomes very large, which limits the highest Ra the WLED can realize. As lower A_B helps the prompt enhancement of Ra_test, overlarge A_B hinders the improvement of Ra_test, despite the adjustment of A_R. On the other hand, if we keep δ_2 unchanged and decrease δ_1, CCT_test will reach CCT_m sooner; however, Ra_test cannot be fully optimized. Therefore, the value of optimal A_B is influenced by CCT_m and is finally determined by δ_1 and δ_2. Increasing δ_1 and δ_2 under high CCT_tar can greatly decrease CCT_m, enlarging the optimization range of Ra. In Figure 4b, CCT_test slightly decreases when we increase δ_1 or δ_2, indicating that CCT_test has a reverse shifting trend, compared with Ra_test under different δ_1 and δ_2 values.
The WLED spectrum is increasingly close to the spectrum of the reference source (black body 1400 K, and 1800 K, respectively; \( \delta_2 \)) variation in CCT tested under different \( \delta_1 \) values.

These analyses reveal the significance of \( \delta_1 \) and \( \delta_2 \) for the optimization result. The WLED with different CCTtar values has different reactions under similar \( \delta_1 \) and \( \delta_2 \). Thus, balancing the relationship between \( \delta_1 \), \( \delta_2 \), and CCTtar is the next step to accelerate the optimization process.

Figure 5a illustrates the optimized spectra of the trichromatic WLED under conditions of CCTtar = 8000 K and \( \delta_1 = 200 \) K. When \( \delta_2 \) increases from 600 K to 3000 K, peaks of blue and red light slightly decrease, while Ra atest increases from 66.9 to 89.7. This is because the WLED spectrum is increasingly close to the spectrum of the reference source (black body source) [14]. The shifting trend of Ra atest matches well with the analysis results of Figure 3a.

Figure 5b describes the shifting trend of CCT tested and RA tested in the iteration process under various CCTtar values. When CCTtar ranges from 4000 K to 8000 K, Ra atest declines slowly at first, as shown in step 1; thereafter, Ra atest abruptly decreases in a small step, as shown in step 2; finally, Ra atest increases severely until reaching the top point before finishing the optimization steps, as shown in step 3. With the increase in CCTtar, the highest value of Ra that can be achieved decreases. A similar phenomenon has been observed in [24,25].

When CCTtar ranges from 4000 K to 8000 K, CCT tested also shows three steps to reach CCTtar; however, the shifting trend of CCT tested during the optimization process is different from that of Ra tested. Ra atest decreases slowly at first and then increases drastically; on the
other hand, $\text{CCT}_{\text{test}}$ increases slowly at first and then increases drastically. A comparison of Figure 5a with Figure 5b indicates that the increase in $\text{CCT}_{\text{test}}$ in step1 sacrifices the improvement of $\text{Ra}_{\text{test}}$ in the initial time. In step 3, different from the optimization aim of $\text{Ra}_{\text{test}}$, we only need to find a CCT value near $\text{CCT}_{\text{tar}}$ instead of finding the local optimal value of $\text{CCT}_{\text{test}}$. It is worth noting that only three iterations are used for optimization when $\text{CCT}_{\text{tar}}$ equals $3000 \text{ K}$, and the value of iterations increases with the increase in $\text{CCT}_{\text{tar}}$. This is because the initial values of $A_B$, $A_Y$, and $A_R$ are very close to optimized values of $A_B$, $A_Y$, and $A_R$ under low $\text{CCT}_{\text{tar}}$.

As we mentioned in Figure 3, different optimization results can be obtained under different $\text{CCT}_{\text{tar}}$ with similar $\delta_1$ and $\delta_2$ values. To guarantee the achievement of the optimal $\text{Ra}_{\text{test}}$ in all cases, we should initially manage to acquire optimized values for $\delta_1$ and $\delta_2$ ($\delta_{\text{Opt1}}$ and $\delta_{\text{Opt2}}$) under different $\text{CCT}_{\text{tar}}$ values. It is worth noting that there exists a strong relationship between the sum of $\delta_{\text{Opt1}}$ and $\delta_{\text{Opt2}}$ ($\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}})$) and $\text{CCT}_{\text{tar}}$. As shown in Figure 6, $\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}})$ is plotted and fitted using a linear function under different $\text{CCT}_{\text{tar}}$ values, which ranges from $3000 \text{ K}$ to $12,000 \text{ K}$. It is evident that $\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}})$ presents a perfect linear increasing trend with the increase in $\text{CCT}_{\text{tar}}$. The slope of this curve is calculated by using the linear interpolation method, and the curve can be described as

$$\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}}) = \alpha \cdot \text{CCT}_{\text{test}}$$

where $\alpha$ is calculated to be $0.63$ for the proposed WLED. For WLEDs combined with different light-conversion materials, the numerical value of $\alpha$ should be different. Additionally, the measured data slightly deviates from the fitting curve when $\text{CCT}_{\text{tar}}$ equals $3000 \text{ K}$, which is probably because changes in $\delta_1$ and $\delta_2$ do not have visible effects on the optimization result when $\text{CCT}_{\text{tar}}$ is low.

![Figure 6. Data and fitting curve of $\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}})$ under different $\text{CCT}_{\text{tar}}$ values.](image)

Once the law between $\alpha$ and light-conversion materials is identified, it is necessary to select the optimal $\delta_1$ and $\delta_2$ under different $\text{CCT}_{\text{tar}}$ values before optimization. According to Figure 4, the optimal $\text{Ra}_{\text{test}}$ corresponds to the largest $\delta_1$ and $\delta_2$ within the allowed range of $\text{CCT}_{\text{tar}}$. Except for the linear relationship between $\sum(\delta_{\text{Opt1}}, \delta_{\text{Opt2}})$ and $\text{CCT}_{\text{tar}}$, the value of $\delta_2$ should be larger than $\delta_1$, to guarantee the operation of the calculation procedure. Therefore, we had better select larger $\delta_2$ and smaller $\delta_1$ to satisfy Equation (2). This principle provides us with an effective way to accelerate the spectral optimization speed.

3.2. Comparison between the Proposed Method, Method I, and Method II

Tables 1–3 present calculation parameters of spectral optimization with the proposed method, method I, and method II. In Table 1, optimized $\text{CCT}_{\text{test}}$ values are very close to $\text{CCT}_{\text{tar}}$. Among all results, the highest $\text{Ra}_{\text{test}}$ reaches up to 96.1, with $\text{CCT}_{\text{test}}$ of 4013 K. $A_B$, $A_Y$, and $A_R$ exhibit a regular shifting trend in which $A_B$ increases, and $A_R$ reduces.

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\text{Method} & \text{Optimized $\text{CCT}_{\text{test}}$} & \text{Optimized $\text{Ra}_{\text{test}}$} & \text{CCT test result} & \text{Calculation time} \\
\hline
\text{Proposed} & 4013 K & 96.1 & \text{Optimized} & 8.5 s \\
\text{Method I} & 3893 K & 92.3 & \text{Almost the same} & 7.8 s \\
\text{Method II} & 3773 K & 88.5 & \text{Optimized} & 7.2 s \\
\hline
\end{tabular}
\caption{Comparison of calculation parameters for spectral optimization.}
\end{table}
with the increase in CCT\textsubscript{tar}. The sum of \(\delta_1\) and \(\delta_2\) increases with the increasing CCT\textsubscript{tar}, which is consistent with the discussion and results in Figure 6. Optimization results of the proposed method and method I, in terms of CCT\textsubscript{test}, Ra\textsubscript{test}, \(A_B\), \(A_Y\), and \(A_R\) values, are highly coincident with each other. This coincidence verifies the correctness of the proposed method.

Table 1. Calculation parameters of spectral optimization using the proposed method.

| CCT\textsubscript{tar} (K) | CCT\textsubscript{test} (K) | Ra         | \(\delta_{\text{Opt1}}\) | \(\delta_{\text{Opt2}}\) | \(A_B\) | \(A_Y\) | \(A_R\) |
|-----------------------------|-----------------------------|------------|--------------------------|--------------------------|--------|--------|--------|
| 3000                        | 2924                        | 95.1       | 200                      | 200                      | 0.11   | 0.30   | 0.47   |
| 4000                        | 4013                        | 96.1       | 400                      | 300                      | 0.33   | 0.30   | 0.30   |
| 5000                        | 5011                        | 94.2       | 1000                     | 300                      | 0.48   | 0.30   | 0.23   |
| 6000                        | 6036                        | 92.4       | 1600                     | 300                      | 0.60   | 0.30   | 0.20   |
| 7000                        | 7018                        | 90.8       | 2200                     | 300                      | 0.69   | 0.30   | 0.18   |
| 8000                        | 8044                        | 89.7       | 3000                     | 200                      | 0.75   | 0.30   | 0.15   |

Table 2. Calculation parameters of spectral optimization using method I.

| CCT\textsubscript{tar} (K) | CCT\textsubscript{test} (K) | Ra         | \(A_B\) | \(A_Y\) | \(A_R\) |
|-----------------------------|-----------------------------|------------|--------|--------|--------|
| 3000                        | 3059                        | 95.3       | 0.12   | 0.30   | 0.46   |
| 4000                        | 4013                        | 96.1       | 0.33   | 0.30   | 0.30   |
| 5000                        | 4908                        | 94.3       | 0.46   | 0.30   | 0.23   |
| 6000                        | 5927                        | 92.7       | 0.57   | 0.30   | 0.18   |
| 7000                        | 7011                        | 90.8       | 0.69   | 0.30   | 0.17   |
| 8000                        | 7950                        | 89.8       | 0.72   | 0.30   | 0.13   |

Table 3. Calculation parameters of spectral optimization using method II.

| CCT\textsubscript{tar} (K) | CCT\textsubscript{test} (K) | Ra         | \(A_B\) | \(A_Y\) | \(A_R\) |
|-----------------------------|-----------------------------|------------|--------|--------|--------|
| 3000                        | 3013                        | 95.5       | 0.13   | 0.30   | 0.47   |
| 4000                        | 4041                        | 96.0       | 0.34   | 0.30   | 0.30   |
| 5000                        | 5007                        | 94.1       | 0.49   | 0.30   | 0.24   |
| 6000                        | 6047                        | 91.8       | 0.62   | 0.30   | 0.21   |
| 7000                        | 6924                        | 87.2       | 0.58   | 0.30   | 0.07   |
| 8000                        | 7917                        | 85.6       | 0.65   | 0.30   | 0.05   |

In Table 3, calculation results of method II under low CCT\textsubscript{tar} well match those of method I. Compared with the proposed method and method I, we can even obtain better optimization results of Ra\textsubscript{test} under 3000 K by using method II. However, with the increase in CCT\textsubscript{tar}, method II fails to effectively improve Ra\textsubscript{test} to obtain the optimal value. Due to the random selection rule of method II, calculation results of \(A_B\), \(A_Y\), and \(A_R\) listed in Table 3 do not show a similar trend as in Tables 1 and 2. These results reveal that method II cannot effectively optimize WLED spectra under high CCTs. To apply \(A_B\), \(A_Y\), and \(A_R\) in a real scenario for realizing target illumination effects, ref. [24] presented the implementation method in detail.

By using Equation (2) to find \(\delta_{\text{Opt1}}\) and \(\delta_{\text{Opt2}}\) under different values of CCT\textsubscript{tar}, we accelerate the optimization process. In Figure 7a, the number of iterations of these three methods under different values of CCT\textsubscript{tar} is compared. Obviously, the number of iterations of the proposed method is much less than that of the other two methods. For the proposed method, the number of iterations increases with the increase in CCT\textsubscript{tar}. The accuracy of CCT\textsubscript{test} and Ra\textsubscript{test} for these three methods can be evaluated by using the error range concept. Error ranges of CCT\textsubscript{test} and Ra\textsubscript{test} (\(\varepsilon_C\) and \(\varepsilon_R\)) are calculated by \(|\text{CCT\textsubscript{tar}} - \text{CCT\textsubscript{test}}| / \text{CCT\textsubscript{tar}}\) and \(|100 - \text{Ra\textsubscript{test}}| / 100\), respectively. \(\varepsilon_C\) and \(\varepsilon_R\) under different values of CCT\textsubscript{tar} are given in Figure 7b,c for comparison. The \(\varepsilon_C\) values of these three methods are comparable under different values of CCT\textsubscript{tar}. The \(\varepsilon_R\) values of the proposed method and method I are similar,
and they are relatively smaller than that of method II under high CCT values. These results verify the effectiveness and accuracy of the proposed method.

Figure 7. (a) Comparison between iteration times of the proposed method, method I, and method II, respectively, under different values of CCT_{tar}; (b, c) comparison between errors of CCT_{test} and Ra_{test} for the proposed method, method I, and method II, respectively, under different values of CCT_{tar}.

4. Conclusions

In this study, we propose an effective method to optimize the Ra of trichromatic WLEDs under different CCTs. Compared with conventional methods I and II, the proposed method exhibits superior searching ability to find the optimal Ra under target CCTs. Specifically, the highest Ra of 96.1 under 4013 K can be obtained after only 29 iterations. Three main mechanisms were investigated and analyzed for the proposed method: (1) the influence of δ₁ and δ₂ on the calculation results of CCT_{m}, CCT_{test}, and Ra_{test}; (2) the relationship between δ₁, δ₂, CCT_{m}, A_{B}, A_{Y}, and A_{R}; (3) the shifting rule of δ_{Opt1} and δ_{Opt2} under different CCT_{tar} values. Particularly, the fitting linear curve that describes the relationship between ∑(δ_{Opt1}, δ_{Opt2}) and CCT_{tar} can provide an effective way to greatly accelerate the optimization process under different CCT_{tar} values. This study reveals the shifting mechanism of CCT and Ra values with dual-weight coefficients and greatly enhances the effectiveness of spectral optimization for WLEDs. Our method is hopefully applied in related areas such as residential intelligent lighting and smart planting LED systems.

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References
1. Su, Z.; Zhao, B.; Gong, Z.; Peng, Y.; Bai, F.; Zheng, H.; Joo, S.W. Color-tunable white LEDs with single chip realized through phosphor pattern and thermal-modulating optical film. *Micromachines* **2021**, *12*, 421. [CrossRef]
2. Zhao, S.; Cai, W.; Wang, H.; Zang, Z. Inorganic lead-free cesium copper chloride nanocrystal for highly efficient and stable warm white light-emitting diodes. *Photons Res.* **2020**, *9*, 187–192. [CrossRef]
3. Wang, H.; Xing, Y.H.; Li, J.X.; Tan, J.; Li, Z.T.; Song, C.H.; Li, J.S. Highly efficient liquid-quantum dot/melamine-modified urea-formaldehyde microcapsules for white light-emitting diodes. *IEEE Electron Device Lett.* **2021**, *42*, 533–536. [CrossRef]
4. Chiang, C.H.; Gong, S.J.; Zhan, T.S.; Cheng, K.C.; Chu, S.Y. White light-emitting diodes with high color rendering index and tunable color temperature fabricated using separated phosphor layer structure. *IEEE Electron Device Lett.* **2016**, *37*, 898–901. [CrossRef]
5. Zhong, P.; He, G.; Zhang, M. Spectral optimization of the color temperature tunable white light-emitting diode (LED) cluster consisting of direct-emission blue and red LEDs and a diphasor conversion LED. *Opt. Express* **2012**, *20*, A684–A693. [CrossRef]
6. Wu, T.; Lin, Y.; Zhu, H.; Guo, Z.; Zheng, L.; Lu, Y.; Shih, T.M.; Chen, Z. Multi-function indoor light sources based on light-emitting diodes—A solution for healthy lighting. *Opt. Express* **2016**, *24*, 24401–24412. [CrossRef]
7. Arellano-Morales, A.; Molina-González, J.; Desirena, H.; Bujud-Jerez, P.M.; Calixto, S. High CRI in phosphor-in-doped glass under near-ultraviolet excitation for warm white light-emitting diode. *J. Lumin.* **2021**, *229*, 117684. [CrossRef]
8. Murad, M.A.; Razi, K.; Jeong, B.R.; Samy, P.M.A.; Muneer, S. Light emitting diodes (LEDs) as agricultural lighting: Impact and its potential on improving physiology, flowering, and secondary metabolites of crops. *Sustainability* **2021**, *13*, 1985. [CrossRef]
9. Nii, K.; Okada, H.; Itoh, S.; Kusaka, T. Characteristics of bilirubin photochemical changes under green light-emitting diodes. *Micromachines* **2020**, *11*, 6391. [CrossRef]
10. Petr, C.; Andrew, B.; Petr, P.; Li, X. Visible light communications: Increasing data rates with polarization division multiplexing. *Opt. Lett.* **2020**, *45*, 2977–2980.
11. Alatari, A.A.; Holguin-Lerma, J.A.; Kang, C.H.; Shen, C.; Subedi, R.C.; Albadri, A.M.; Allyamani, A.Y.; Ng, T.K.; Ooi, B.S. High-power blue super luminescent diode for high CRI lighting and high-speed visible light communication. *Opt. Express* **2018**, *26*, 26355–26364. [CrossRef]
12. Guo, Z.; Shih, T.M.; Lu, Y.; Gao, Y.; Zhu, L.; Chen, G.; Zhang, J.; Lin, S.; Chen, Z. Studies of scotopic/photopic ratios for color-tunable white light-emitting diodes. *IEEE Photonics J.* **2013**, *5*, 8200409. [CrossRef]
13. Wei, M.; Yang, B.; Lin, Y. Optimization of a spectrally tunable LED daylight simulator. *Color Res. Appl.* **2017**, *42*, 419–423. [CrossRef]
14. Zhu, P.; Zhu, H.; Adhikari, G.C.; Thapa, S. Spectral optimization of white light from hybrid metal halide perovskites. *OSA Contin. J.* **2019**, *6*, 1880–1888. [CrossRef]
15. Titkov, I.E.; Yadav, A.; Karpov, S.Y.; Sakharov, A.V.; Tsatsulnikov, A.F.; Slight, T.J.; Gorodetsky, A.; Rafailov, E.U. Superior color rendering with a phosphor-converted blue-cyan monolithic light-emitting diode. *Laser Photonics Rev.* **2016**, *10*, 1031–1038. [CrossRef]
16. Yuan, B.; Guan, S.; Sun, X.; Li, X.; Zeng, H.; Xie, Z.; Chen, P.; Zhou, S. Highly efficient carbon dots with reversibly switchable red emissions for trichromatic white light-emitting diodes. *ACS Appl. Mater. Interfaces* **2018**, *10*, 16005–16014. [CrossRef]
17. Song, B.M.; Han, B. Spectral power distribution deconvolution scheme for phosphor-converted white light-emitting diode using multiple Gaussian functions. *Appl. Opt.* **2013**, *52*, 1016–1024. [CrossRef]
18. Carli, N.; Sperling, A.; Bizjak, G. Optimization methods for spectral synthesizing of a tuneable colour light source. *Light Eng.* **2018**, *26*, 99–108. [CrossRef]
19. Birgin, E.G.; Martinez, J.M.; Raydan, M. Spectral projected gradient methods: Review and perspectives. *J. Stat. Softw.* **2016**, *60*, 1–21. [CrossRef]
20. Zielonka, A.; Sikora, A.; Wozniak, M.; Wei, W.; Ke, Q.; Bai, Z. Intelligent internet of things system for smart home optimal convection. *IEEE Trans. Ind. Inform.* **2020**, *17*, 4308–4317. [CrossRef]
21. Blanco, J.; García, A.; Morenas, J.D.L. Design and implementation of a wireless sensor and actuator network to support the intelligent control of efficient energy usage. *Sensors* **2018**, *18*, 1892. [CrossRef] [PubMed]
22. Qiang, Y.; Yu, Y.; Chen, G.; Fang, J. A flux-free method for synthesis of Ce3+-doped YAG phosphor for white LEDs. *Mater. Res. Bull.* **2016**, *74*, 353–359. [CrossRef]
23. Pu, C.; Peng, X. To battle surface traps on CdSe/CdS core/shell nanocrystals: Shell isolation versus surface treatment. *J. Am. Chem. Soc.* **2016**, *138*, 8134–8142. [CrossRef] [PubMed]
24. Xiao, H.; Xiao, X.; Wang, K.; Wang, R.; Xie, B.; Chiang, K.S. Optimization of illumination performance of trichromatic white light-emitting diode and characterization of its modulation bandwidth for communication applications. *IEEE Photonics J.* 2018, 10, 8201511. [CrossRef]

25. Erdem, T.; Nizamoglu, S.; Sun, X.W.; Demir, H.V. A photometric investigation of ultra-efficient LEDs with high color rendering index and high luminous efficacy employing nanocrystal quantum dot luminophores. *Opt. Express* 2010, 18, 340–347. [CrossRef]