A New Control for the Stochastic Distribution Systems

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Abstract. In our work, the new control for the Stochastic Distribution Systems is study. A novel model transformation is investigated, Achieving the goals that the closed-loop Stochastic Distribution System is progressive stability and a given indicator is not exceeding a specified number for all disturber, and that that the system output PDF is follow the trail of the target PDF.

Introduction

Different form the stochastic control systems, stochastic distribution control aim to control system output PDF to track the target PDF[1-6]. In recent ten years, many methods have also been developed, but the above methods was present, This led to control methods with a large amount of calculation and cannot be guaranteed to be stable for the stochastic distribution systems[3,4,7-12]. To overcome these problems, in our work, a controller which is stability and ensure an adequate level of performance for the stochastic distribution systems.

Model Description

For the stochastic distribution system, denote $x(k)$ as the control system input, $y(k)$ as the stochastic system output and the probability $y(k)$ can be expressed as$^{[4,5,11,12]}$

$$p(a < y(k) < \gamma, x(k)) = \int_a^\gamma p(y, x(k))d\eta$$ (1)

Where $p(y, x(k))$ is the output PDF $x(k)$. This express that the $p(y, x(k))$ is controlled by $x(k)$. For the system output PDF $p(y, x(k))$, the following equation is obtain

$$\sqrt{\gamma(y, x(k))} = \sum_{i=1}^{n} \nu_i(x(k))B_i(y) + \varepsilon$$ (2)

Where $B_i(y)$ are the basis function and $\nu_i(x(k))$ are the matching weight that relay on $x(k)$, $\varepsilon$ represents the approximation error. It can be seen that the PDF should satisfy

$$\int_a^b \gamma(\eta, x(k))d\eta = 1,$$

and n-1 weights are independent. So equation(2) further expressed as follows:

$$\gamma(y, x(k)) = (B_0(y)V_0(k) + v_0B_n + \varepsilon(k))$$ (3)

where $B_0(\eta) = [B_1(\eta) \ B_2(\eta) \ldots \ V_0(\eta) = [v_1(\eta) \ v_2(\eta) \ldots]$

To reduce system error, let $\varepsilon(k) = B_0(y)\omega_0(k)$, $\omega_0(k)$ is disturbance. And the equation (3) was worked as:

$$\sqrt{\gamma(y, x(k))} = [B_0(y) \ B_n(y)]\begin{bmatrix} V(k) \\ v_n \end{bmatrix}$$ (4)

where $V(k) = B_0(k) + \omega_0(k)$

The purpose of the guaranteed cost methods is to choose input $\{u_i(t)\}$ such that the system output PDF track as much as possible a target PDF $\gamma_d(y)$, so as to guarantees stability and an adequate
index level. Then equation (4) are defined:

\[ \gamma(y, x(k)) = B'(y)V'(y) \]  

(5)

Where \[ B'(y) = \begin{bmatrix} B_0(y) & B_n(y) \end{bmatrix} \]

\[ V'(k) = \begin{bmatrix} V_0(k) & h(V(k)) \end{bmatrix}^T. \]

The prespecified PDF \( \gamma_g(y) \) can be expressed as

\[ \sqrt{\gamma_g} = \sqrt{B_0(y)V_g + h(V_g)B_n} = B'(y)\tilde{V}_g \]  

(6)

Where \( \tilde{V}_g = [V_g \ h(V_g)] \).

For system control, the following performance function is given

\[ J = \sum_{k=0}^{\infty} \left[ V''(k)QV''(k) + x^T(k)Rx(k) \right] \]  

(7)

Where \( V''(k) = \tilde{V}(k) - \tilde{V}_g, Q = \tilde{B}''(y)\tilde{B}(y) = \|\tilde{B}(y)\|^2 \)

### Design of the System Controller

In this paper, the state space form

\[
\begin{align*}
\begin{cases}
  x(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k-d) + (B + \Delta B)u(k) \\
  \sqrt{\gamma(y, x(k))} = B'(y)V'(k) = B'(y)(V_g + x(k))
\end{cases}
\end{align*}
\]

(8)

is used to expressed the relationship between state vector and system input. \( A, A_d, B, \) and \( B_h \) are given real constant matrices, \( \Delta A, \Delta A_d, \Delta B \) denote real time-varying matrices, \( d \) and \( h \) represent unknown time-delay integers. In general, Note that \( 0 \leq \sigma \leq \sigma^*, 0 \leq \sigma \leq \sigma^* \) with \( \sigma^*, \sigma^* \) being upper bounds of the \( u(k) \) and \( x(k) \) delay.

The system uncertainties being norm-bound can be further expressed as

\[
\begin{bmatrix}
  \Delta A & \Delta A_d \\
  \Delta B & \Delta B_h
\end{bmatrix} = DF
\begin{bmatrix}
  E_a & E_b \\
  E_d & E_h
\end{bmatrix}
\]

(9)

where \( F \in \mathbb{R}^n \) is an uncertain marix with \( F^TF \leq I, E_a, E_b, E_d, E_h \) and \( D \) are matrices.

Suppose the system is measurable, the purpose of guaranteed cost controller design is develop to design a control method

\[ u(k) = Kx(k) \]  

(10)

such that the system(11)

\[ x(k+1) = A_1x(k) + A_2x(k-d) + B_1Kx(k-h) \]  

(11)

is stable. And the function(7) satisfies \( J \leq J^* \) , \( J^* \) is given constant. Where

\[ A_1 = A + KB + K\Delta B \quad A_2 = A_d + \Delta A_d, B_1 = B_h + \Delta B_h \]

equation (11) can be further shown as

\[ -x(k+1) + A_1x(k) + A_2x(k-d) + B_1Kx(k-h) = 0 \]  

(12)

The following lemmas will be used

**Lemma 1.** For vectors \( a, b \) and positive-definite matrix \( X \), there exists
\[ \pm 2a^T b \leq a^T X^{-1} a + b^T X b \]

**Lemma 2.** matrices D,E is given, and symmetric Y, the following inequality been
\[ Y + D F E + E^T F^T D^T < 0 \]
holds for all F satisfying \( F^T F \leq I \) if and only if there exists a constant \( \varepsilon > 0 \) such that
\[ Y + \varepsilon D^T D + \varepsilon^{-1} E^T E < 0 \]

**Lemma 3.** The linear matrix inequality
\[ S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \]
is equivalent to \( S < 0 \) \( S_{11} < 0, S_{22} < 0, S_{12}^T S_{12}^{-1} S_{12} < 0 \) \( S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12} < 0 \)

**Definition 1.** For the stochastic distribution system (8) and guaranteed cost function (7), if there exist a control method \( u^*(k) \) and a const scalar \( J^* \) such that for F with \( F^T F \leq I \), the system(11) is stable and the value of the equation (7) satisfied \( J \leq J^* \), then \( J^* \) is a const number, and \( u^*(k) \) is control method.

Note that the model transformation of system can obtain a less convervative result. This law can reduce the convervatism for the PDFs system ([18,19]). We now give main results.

**Theorem 1.** There have been positive-definite symmetric matrices P,T and \( N_1,N_2,N_3,N_4 \) such that the matrix inquality (13) hold:
\[
\begin{bmatrix}
P + PT + N_1^T P N_1 & -PA_1 + N_1^T P N_2 & -PA_2 + N_1^T P N_3 & -PB_1 + N_1^T P N_4 \\
* & \Sigma & A_1^T PA_2 + N_1^T P N_3 & A_1^T PB_2 + N_1^T P N_4 \\
* & * & A_2^T PA_2 + N_1^T P N_3 & A_2^T PB_2 + N_1^T P N_4 \\
* & * & * & B_1^T PB_2 + N_1^T P N_4 - T
\end{bmatrix} < 0
\]

* indicates was induced by symmetry.

Where \( \Sigma = Q + K^T TK + K^T PK + N_1^T P N_2 + A_1^T PA_2 - P \). Then, for any admissible uncertain matrix F, \( u(k) = Kx(k) \), is a controller.

**Proof.** Let us assume that there exits positive-definite symmetric matrices P,T and \( N_1,N_2,N_3,N_4 \) such that matrix inquality holds. And then the Lyapunov function
\[
V(k) = x^T(k) P x(k) + \sum_{i=1}^{k-1} x^T(k-i) S x(k-i) + \sum_{i=1}^{h} x^T(k-i) K^T TK x(k-i)
\]
is positive-definite. And the Lyapunov difference is obtained as following
\[
\Delta V(K) = V(k+1) - V(k) = x^T(k+1) P x(k+1) - x^T(k) (K^T TK - P) x(k) - x^T(k-h) K^T TK x(k-h) + 2[x(k+1) N_k + x(k-d) N_k K u(k-h) N_k] - [x(k+1) + A_1 x(k) + A_2 x(k-d) + B_1 K x(k-h)]
\]

Difining vector
\[ \omega = [x(k+1) \quad x(k) \quad x(k-d) \quad K x(k-h)]^T \]

Applying the lemma 1, equation (14) can be further updated as follow
\[ \Delta V(k) \leq \omega^T \left[ \begin{array}{c} P \\ K^T K \end{array} \right] \omega \]

Under condition (15), we can obtain

\[ \Delta V(k) \leq x^T(k)(-Q + K^T RK)x(k) \leq -\lambda_{\text{min}}(-Q + K^T RK)\|x(k)\|^2 \] (16)

Where \( \lambda_{\text{min}}() \) is the minimum eigenvalue of matrix \( Q \).

We conclude that the systems (8) are stable.

Then, Equation (16) can be further expressed as

\[ -\Delta V(k) \geq x^T(k)(Q - K^T RK)x(k) \]

Summing both side of in equation (16) from 0 to \( \infty \), we can obtain

\[ J \leq x^T(0)Px(0) + \sum_{i=1}^{d} x^T(i)Sx(i) + \sum_{j=1}^{h} x^T(j)K^T Kx(j) = J^* \] (17)

We can conclude that the theorem is true from definition 1.

**Theorem 2.** There are positive–definite symmetric matrices \( P,T \). For all admissible uncertainties such that matrices (13) holds if there are \( \varepsilon, \rho_2, \rho_3, \rho_4 \) and matrices \( W, X,M,V \) and \( N \) such that the following LMI holds:

\[
\begin{bmatrix}
-\Gamma & 0 & -I & \Delta \Gamma + BS & A_p M & B_h V & 0 & 0 & 0 & 0 & 0 \\
* & -\Gamma & N & \rho_2 N \Gamma & \rho_2 N M & \rho_2 N V & 0 & 0 & 0 & 0 & 0 \\
* & * & -\Gamma + DD^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\Gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -M & 0 & ME_h & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -V & NE_h & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\rho_1 I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -M & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -V & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -Q^{-1} & 0 \\
* & * & * & * & * & * & * & * & * & * & -R^{-1}
\end{bmatrix} < 0 \] (18)

Furthermore, the \( u(k) = \Sigma^{-1} x(k) \) is a control method, and the value of the guaranteed cost function satisfies

\[ J \leq (1 + h^*) \lambda \max \left( \Psi^T \Gamma^{-1} \Psi \right) + d^* (\Psi^T M^{-1} \Psi) \] (19)

**Proof.** Equation (13) can be work as

\[
\begin{bmatrix}
2P \\
K^T TK - P + Q + K^T RK \\
- Q \\
- T
\end{bmatrix} + \begin{bmatrix}
N_1^T \\
N_2^T \\
N_3^T \\
N_4^T
\end{bmatrix} * P \begin{bmatrix}
N_1 & N_2 & N_3 & N_4 \\
A_p^T & A_p^T & A_p^T & B_h^T
\end{bmatrix} P \begin{bmatrix}
-I & A_p & A_p & B_h
\end{bmatrix} < 0 \] (20)
It follows the lemma 3 (the Schur complement) that the inequality (20) can be further lead to

\[
\begin{bmatrix}
-P^{-1} & 0 & -I & A_1 & A_2 & B_1 \\
-P^{-1} & N_1 & N_2 & N_3 & N_4 \\
P & 0 & 0 & 0 \\
\Omega & 0 & 0 \\
* & -Q & 0 \\
-T
\end{bmatrix} < 0
\]

(21)

\[\Omega = Q + K^T K - P + K^T R K\]

By substitution the representation of matrices \(A_1, A_2, B_1\) in inequality (21), it is equivalent to

\[
\begin{bmatrix}
-P^{-1} & 0 & -I & A + KB & A_d & B_d \\
-P^{-1} & N_1 & N_2 & N_3 & N_4 \\
-2P & 0 & 0 & 0 \\
\Omega & 0 & 0 \\
* & -Q & 0 \\
-T
\end{bmatrix} + \begin{bmatrix}
D \\
F [0 & 0 & 0 & E_a + kE_d & E_b & E_h]
\end{bmatrix} < 0
\]

\[
+ \begin{bmatrix}
0 \\
0 \\
0 \\
(E_a + kE_d)^T
\end{bmatrix} F^T \begin{bmatrix}
D^T & 0 & 0 & 0 & 0
\end{bmatrix} < 0
\]

By lemma 2 and lemma 3, we can get easily

\[
\begin{bmatrix}
-P^{-1} & 0 & -I & A + KB & A_d & B_d & 0 \\
-P^{-1} & N_1 & N_2 & N_3 & N_4 & 0 \\
-P + DD^T & 0 & 0 & 0 & 0 \\
\Omega & 0 & 0 & E_a + kE_d \\
* & -Q & 0 & E_b \\
-T & E_h & & -\varepsilon^3 I
\end{bmatrix} < 0
\]

(22)

By pre-multiplying by \(\text{diag}\{I \ I \ I \ P^{-1} \ Q^{-1} \ T^{-1} \ I\}\) and post-multiplying \(\text{diag}\{I \ I \ I \ P^{-1} \ Q^{-1} \ T^{-1} \ I\}\) on both side of LMI (22). At the same time, letting

\[\Gamma = P^{-1}, \Sigma = K P^{-1}, M = Q^{-1}, V = T^{-1}, N_1 = N, N_2 = \rho_2 N, N_3 = \rho_3 N, N_4 = \rho_4 N\]. And applying lemma 3 yield matrix inequality (18).

If the LMI (18) has a feasible \(\varepsilon, \rho\ (i=1,2,3,4), \Gamma, \Sigma, M\), we can obtain that

\[u(k) = \Sigma \Gamma^{-1} x(k)\]

is a control method for the system (8) and guaranteed cost function (7).
Conclusions

In this paper, because numerical solutions cannot be guaranteed to be stable for the output PDFs systems, we develop the PDF state feedback control law with model transformation. The existence condition for the given control law has been studied, and that has shown that this condition is the feasibility of a LMI. Furthermore, based on the above result, a convex optimization algorithms has been given to optimize guaranteed cost control law with model transformation. It has proved that the method in this paper can sure the good index.

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