A demonstration of spectral level reconstruction of intrinsic $B$-mode power

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Abstract. We investigate the prospects and consequences of the spectral level reconstruction of primordial $B$-mode power by solving the system of linear equation. We find that this reconstruction technique may be very useful to estimate the amplitude of primordial gravity waves or more specifically, the value of tensor-to-scalar ratio. We also see that one may have very accurate reconstruction of the intrinsic $B$-mode power up to a few hundred multipoles which is more than sufficient to estimate the tensor-to-scalar ratio. The small-scale cosmic microwave background (CMB) anisotropies are not sourced by the primordial gravity waves generated during inflation. We have also demonstrated the efficiency of our algorithm by incorporating the impact of instrumental noise considering CMB-Bharat-like futuristic space mission complemented with Planck 2018 cosmology. We shall see that spectral delensing is capable of reducing the variance in lensed CMB. We do not make any conclusive comments on this as the true non-Gaussian structure and errors in the observed CMB along with the lensing by gravity waves have been ignored.

Keywords. Cosmology; inflationary Universe; cosmic microwave background radiation; gravitational lensing.

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1. Introduction

The existence of gravity waves was predicted long ago by none other than Albert Einstein. After a century’s intermission, finally we have detected the gravity waves first from a binary black hole merger [1] and later from a 22-Solar-Mass binary black hole coalescence [2] jointly by LIGO (https://www.ligo.caltech.edu/) and Virgo (http://www.virgo-gw.eu/) scientific collaborations. As a consequence, people are now eagerly waiting for the primordial gravity waves which are believed to be produced during cosmic inflation [3,4] through the tensor perturbations along with the primordial density perturbations. As the size of the co-moving horizon decreases during inflation, all the modes leave the Hubble radius. The tensor modes remain constant after horizon exit just like the scalar modes [5]. Once inflation is over, the co-moving size of the horizon starts growing again and eventually all the modes re-enter the Hubble radius. Tensor modes with smaller wavelengths make horizon re-entry early and decay very quickly ahead of recombination. As a result, small-scale cosmic microwave background (CMB) anisotropies are not affected by the tensor perturbations, and only the large-scale anisotropies get contribution from the gravity waves. The large-scale CMB $B$-mode polarisation is believed to be sourced solely by the primordial gravity waves [6]. Accordingly, the intrinsic $B$-mode has very little power on the smaller scale ($\ell \geq 200$) and we only concentrate on the large-scale CMB polarisations for primordial gravity waves.

The measurement of large-scale $B$-mode amplitude will help us to determine the energy scale of inflation as it is directly related to the primordial gravity waves. But, during their journey from last scattering surface to the present day detectors, CMB photons encounter several over and under dense regions which perturb their paths—a phenomenon known as gravitational lensing of CMB [7–11]. As a result, the CMB power spectra are modified which is the worst in the case of $B$-modes. So, the subtraction of lensing contribution from the $B$-mode signal is very much necessary. The large-scale CMB $B$-mode is the primary source of primordial gravity waves. But the transfer of power from larger to smaller scales due to gravitational lensing eludes intrinsic CMB [10,12]. Subtraction of lensing confusion from the large-scale $B$-mode will certainly improve our current understanding of cosmic inflation [13,14]. So, to get rid of the lensing artifacts, delensing is a bare necessity.
Many delensing techniques are available in the literature [15,16]. Most of them rely on the reconstruction of the lensing potential [17–23]. Some of them utilise external data set for delensing [24–27]. Very recently, cosmic infrared background has been utilised to reverse the gravitational lensing of CMB [28]. In this work, we shall discuss about the pros and cons of the reconstruction of intrinsic B-mode power through direct matrix inversion technique [29], assuming that the lensing potential and lensed polarisation spectra are already in hand. In this process, we first evaluate the lensing kernels for polarisation spectra and invert them by applying Gauss–Jordan elimination technique [30].

With the help of the inverted kernel matrices, we solve for intrinsic CMB polarisation spectra. To estimate the reconstruction noise, thousand realisations of unlensed CMB for four different values of tensor-to-scalar ratios, \( r = 0, 0.001, 0.01 \) and \( 0.1 \), have been generated along with the lensing potential, which are then passed through the lensing kernels to get the lensed polarisation spectra. We then apply our delensing algorithm. The mean and variance in the reconstructed B-mode power are then calculated from these delensed realisations. One may have very accurate reconstruction of intrinsic \( C^B_ℓ \) even for tensor-to-scalar ratio three orders of magnitude below unity. Though we are far from detecting primordial gravity waves for \( r = 0.001 \), but the future like instrumental noise anticipating CMB-Bharat-like futuristic space mission may not have much impact on our reconstruction procedure. But, we do not make definitive remarks on this as the proper non-Gaussian structure of the lensed CMB, the lensing by gravity waves and the errors on the cosmological parameters have been neglected along with the fact that current observations covers only 70–80% of the sky.

For our entire analysis, we have used the following Planck-2018 best-fit ΛCDM cosmology [31,32]. We have used the publicly available code CAMB (http://camb.info/) to compute the theoretical CMB spectra for the above cosmology. In our analysis, we did not take into account lensing due to the gravity waves as they are expected to be very small. For the instrumental noise, we have considered 175 GHz frequency channel of the upcoming satellite mission CMB-Bharat.

The paper is organised as follows. In §2 we have reviewed the delensing through direct matrix inversion technique. In §3 we have presented our findings for a single realisation of lensed CMB along with the delensing bias and results for simulation in the absence of instrumental noise. In §4 we have discussed the prospect of B-mode delensing in the presence of CMB-Bharat-like instrumental noise. We summarise our findings and discuss future prospects in a brief concluding section.

2. Brief review of delensing through matrix inversion

The path of a CMB photon is perturbed due to the gravitational lensing by the potential gradients transverse to the line of sight. As a result, a point \( \hat{n} \) in the last scattering surface (LSS) appears to be in a deflected position \( \hat{n}' \). The (lensed) temperature \( \hat{T} \) that we measure as coming from a direction \( \hat{n} \) in the sky, actually corresponds to the intrinsic temperature \( T(\hat{n}') \) from a different direction \( \hat{n}' \), where \( \hat{n} \) and \( \hat{n}' \) are related through the deflection angle \( \alpha \) with \( \hat{n}' = \hat{n} + \alpha \). Similarly, the polarisation field is remapped according to \( \hat{P}(\hat{n}) = P(\hat{n}') \).

2.1 Lensed CMB power spectra

The lensed temperature power spectrum in the full spherical sky limit using correlation function technique can be written as [8,33]

\[
\tilde{C}^T_\ell = \sum_{\ell'} K^T_{\ell \ell'} C^T_{\ell'},
\]

where \( K^T_{\ell \ell'} \) is the lensing kernel associated with the temperature field given by the following expression:

\[
K^T_{\ell \ell'} = \delta_{\ell \ell'} + \frac{2\ell' + 1}{2} \int_0^\pi \sin \beta \, d\beta \, d_{00}(\beta) \times \left\{ [X^2_{00} - 1]d_{00}(\beta) \right.
\]
\[
+ \frac{8}{\ell' + 1} A_2(\beta) X^2_{00} d^2_{01-1}(\beta) + A_2(\beta)^2
\]
\[
\times \left( X^2_{00} d^2_{00}(\beta) + X^2_{00} d^2_{02}(\beta) \right) \right\}.
\]

Here \( \cos \beta = \hat{n}_1 \cdot \hat{n}_2 \), \( d^\ell_{mn} \)'s are the standard Wigner rotation matrices and we have defined,

\[
A_0(\beta) \equiv \sum_\ell \frac{2\ell + 1}{4\pi} \ell (\ell + 1) C^\phi_\ell d^\ell_{11}(\beta);
\]

\[
A_2(\beta) \equiv \sum_\ell \frac{2\ell + 1}{4\pi} \ell (\ell + 1) C^\phi_\ell d^\ell_{-11}(\beta);
\]

\[
\sigma^2(\beta) \equiv A_0(0) - A_0(\beta);
\]

\[
X_{imn} \equiv \int_0^\infty \frac{2\alpha}{\sigma^2(\beta)} \left( \frac{\alpha}{\sigma^2(\beta)} \right)^i \times e^{-\alpha^2/\sigma^2(\beta)} d\epsilon_{mn}(\alpha) d\alpha
\] (3)
with the prime denoting derivative with respect to $\sigma^2(\beta)$. Similarly, the lensed polarisation spectra can be obtained from the following equations:

$$
\tilde{C}_\ell^+ = \sum_{\ell'} K_{\ell'\ell}^+ \tilde{C}_{\ell'}^+, \quad \tilde{C}_\ell^- = \sum_{\ell'} K_{\ell'\ell}^- \tilde{C}_{\ell'}^-,
$$

(4)

where $\tilde{C}_\ell^{\pm} \equiv \tilde{C}_\ell^E \pm \tilde{C}_\ell^B$, $C_\ell^{\pm} \equiv C_\ell^E \pm C_\ell^B$ and $K_{\ell'\ell}^{\pm}$ are the associated lensing kernels for polarisation given by

$$
K_{\ell'\ell}^+ = \delta_{\ell\ell'} + \frac{2\ell' + 1}{2} \int_0^\pi \sin \beta \, d\beta \, d_{22}^e(\beta) \times \left\{ [X_{022}^2 - 1]d_{22}^e(\beta) + 2A_2(\beta)X_{132}X_{121}d_{11}^e(\beta) + A_2(\beta)^2[X_{022}^2d_{22}^e(\beta) + X_{242}X_{220}d_{10}^e(\beta)] \right\}
$$

(5)

$$
K_{\ell'\ell}^- = \delta_{\ell\ell'} + \frac{2\ell' + 1}{2} \int_0^\pi \sin \beta \, d\beta \, d_{2-2}^{\prime e}(\beta) \times \left\{ [X_{022}^2 - 1]d_{2-2}^{\prime e}(\beta) + A_2(\beta)[X_{132}^2d_{1-1}^{\prime e}(\beta) + X_{132}^2d_{3-3}^{\prime e}(\beta)] + \frac{1}{2}A_2(\beta)^2[2X_{022}^2d_{2-2}^{\prime e}(\beta) + X_{242}X_{220}d_{10}^{\prime e}(\beta) + X_{242}X_{220}d_{10}^{\prime e}(\beta)] \right\}.
$$

(6)

So, given the lensing spectrum, $C_{\ell}^\phi$, one can completely determine the lensing kernels.

### 2.2 Delensed CMB power spectra

From eqs (1) and (4) we see that the lensed CMB spectra are linear combinations of intrinsic spectra with coefficient matrices completely determined by the lensing potential, $C_{\ell}^\phi$. So, for a given lensing potential one can, in principle, invert the lensing kernels and solve for the intrinsic CMB power as shown in [29]. The delensed spectra obtained by direct matrix inversion can then be written as

$$
C_{\ell}^T = \sum_{\ell_1}(K_{\ell\ell_1}^T)^{-1}\tilde{C}_{\ell_1}^T,
$$

$$
C_{\ell}^E = \frac{1}{2}[C_{\ell}^+ + C_{\ell}^-],
$$

$$
C_{\ell}^E = \frac{1}{2}[C_{\ell}^+ - C_{\ell}^-],
$$

$$
C_{\ell}^B = \frac{1}{2}[C_{\ell}^+ - C_{\ell}^-]
$$

(7)

So, for a given lensing potential and lensed spectra of CMB $E$ and $B$ modes, one can reconstruct the intrinsic polarisation in a very simple manner. Also, instead of direct inversion, one can make use of various numerical techniques available in the literature to solve the system of linear equations. Throughout this work, we have used Gauss–Jordan elimination technique. But here, one has to be extremely careful about how to deal with a truncated system of linear equations – as $\ell$ may run from zero to infinity and we have access to very few multipoles only. We shall come back to this while discussing the delensing bias in the following section.

### 3. Delensing in the absence of instrumental noise

In our following analyses, we delens the lensed polarisation by solving linear system of equations (eq. (7)) for three different values of tensor-to-scalar ratio, $r = 0.1, 0.01, 0.001$. Though recent analysis has set an upper-bound on tensor-to-scalar ratio $r < 0.036$ [34], we have kept the analysis for $r = 0.1$ for demonstration purpose only. We have assumed, in our entire analysis, that power spectrum of the lensing potential, $C_{\ell}^\phi$, along with the lensed polarisation spectra are completely known to us. For delensing, we first evaluate the lensing kernels $K_{\ell\ell'}$, then we calculate the inverses of the lensing kernels employing Gauss–Jordan elimination technique and solve for the intrinsic polarisation spectra.

#### 3.1 Simulating lensed polarisation power spectra

To begin with, we generate a set of unlensed $C_E^\ell$ and $C_B^\ell$ for four different values of tensor-to-scalar ratio, $r = 0, 0.001, 0.01$ and 0.1 along with the lensing potential $C_{\ell}^\phi$ for the best-fit Planck-2018 cosmology using CAMB. With the help of these intrinsic spectra and lensing spectrum, we create 1000 Gaussian realisations for each $C_E^\ell, C_B^\ell$ and $C_{\ell}^\phi$ for $r = 0.0, 0.001, 0.01$ and 0.1. Since we are not considering lensing by gravity waves, $C_{\ell}^\phi$ is the same for all values of $r$. Now, employing lensing algorithm eq. (4) on the unlensed realisations for $C_E^\ell$ and $C_B^\ell$, we generate the corresponding lensed spectra for four different values of tensor-to-scalar ratio.

#### 3.2 Delensing bias – delensed $C_B^\ell$ in the absence of intrinsic $B$-mode power

For delensing, first we estimate the mean $C_{\ell}^\phi$ from the 1000 Gaussian realisation of the lensing potential. Using this mean we calculate the lensing kernels $K_{\ell\ell'}$, and invert them employing Gauss–Jordan elimination technique. The inverted kernels are then used to solve the linear system eq. (7) to get the delensed polarisation spectra.
We now apply our delensing algorithm to the lensed realisations for \( r = 0 \). In ideal situation one expects the reconstructed \( C^B_{\ell} \) to be exactly zero. By ideal situation we mean, where we have \( \ell \) equations in \( \ell \) unknowns, with \( \ell \to \infty \). So, solving the system with finite \( \ell_{\text{max}} \) makes the solution biased. As a result, one gets non-zero values for delensed \( C^B_{\ell} \). To obtain the true solution we need to take into account this bias. So, in the absence of instrumental noise, the delensed \( B \)-mode power may be read as

\[
C^B_{\ell_{\text{del}}} = C^B_{\ell_{\text{intr}}} + C^B_{\ell_{\text{bias}}}. \tag{8}
\]

In this case since \( r = 0 \) we have \( C^B_{\ell_{\text{intr}}} = 0 \), and therefore \( C^B_{\ell_{\text{bias}}} \equiv C^B_{\ell_{\text{del}}} \big|_{r=0} \). So, eq. (8) can be written as

\[
C^B_{\ell_{\text{intr}}} = C^B_{\ell_{\text{del}}} - C^B_{\ell_{\text{bias}}} = C^B_{\ell_{\text{del}}} - C^B_{\ell_{\text{del}}} \big|_{r=0}. \tag{9}
\]

Therefore, the actual \( B \)-mode power may be obtained only after subtracting the bias from the reconstructed \( C^B_{\ell} \). Not only that, the actual error has to be estimated from the following equation:

\[
\text{Var}(C^B_{\ell_{\text{intr}}}) = \text{Var}(C^B_{\ell_{\text{del}}}) + \text{Var}(C^B_{\ell_{\text{bias}}}) - 2 \text{Cov}(C^B_{\ell_{\text{del}}}, C^B_{\ell_{\text{bias}}}). \tag{10}
\]

In figure 1 we have plotted the mean delensing bias (dash–dot dark slate gray) along with the associated standard deviation (pink dotted line). The mean bias is estimated from 1000 delensed realisations of CMB \( B \)-mode power for \( r = 0 \). The standard deviation (SD henceforth) of the bias has been estimated from those delensed spectra. The lensed \( C^B_{\ell} \) (dashed black) along with the corresponding SD (cyan point) have been plotted. The variance in the delensing bias represents the typical scatter in the delensed \( B \) spectrum when the algorithm is applied to the realisations of lensed \( E \) and \( B \) spectra for \( r = 0 \). From figure 1, it is clear that this variance lies much below the cosmic variance curve of the lensed \( C^B_{\ell} \) when the tensor-to-scalar ratio is zero. The implication of this finding is important; spectral delensing has the capability to reduce the variance in the lensed CMB.

3.3 The ideal reconstruction

Before going into the details, we first demonstrate how well one can reconstruct the intrinsic \( B \)-mode power using our technique for a single realisation of lensed CMB polarisation spectra. In order to do so, we generate the lensing potential along with a set of intrinsic \( E \) and \( B \) spectra for \( r = 0.0, 0.001, 0.01 \) and 0.1 using CAMB. We then calculate the lensing kernels using the lensing potential and following our algorithm we find the lensed \( E \) and \( B \) spectra for \( r = 0.0, 0.001, 0.001 \) and 0.1. Then Gauss–Jordan elimination technique has been used to invert those lensing kernels. Finally, we apply our delensing algorithm to the lensed CMB polarisation spectra to get the intrinsic \( B \)-mode power. To bypass the delensing bias-related issues, we have used the same \( \ell_{\text{max}} \) for both forward and backward lensing. As a result, we get a closed system of equations. We have shown our results in figure 2. The blue dots are the reconstructed \( B \)-mode power for \( r = 0 \), not the delensing bias, which may be seen as numerical artifacts. We see that one can have a very good estimate of the intrinsic \( B \)-mode power once the lensing potential is known. So, we find that solving the system of linear equations to reconstruct the intrinsic \( B \)-mode power might be a very useful tool.

3.4 Delensed \( C^B_{\ell} \) for three different values of tensor-to-scalar ratio

We now employ our delensing algorithm to the lensed polarisation for \( r = 0.001 \). From the set of delensed spectra, we first estimate the mean and subtract the mean delensing bias to get the intrinsic \( C^B_{\ell_{\text{intr}}} \). In figure 3, we have plotted the bias-subtracted mean of the recovered \( C^B_{\ell} \) (red dash-dotted line) along with the SD reconstruction (solid purple line). We can see that bias-subtracted mean of the recovered \( C^B_{\ell} \) matches very well with the injected \( B \)-mode power (solid green line). Also figure 3 reveals that the reconstruction noise overcomes the intrinsic signal for \( \ell \sim 200 \). But if we use eq. (10) to estimate the SD for the reconstructed intrinsic \( C^B_{\ell} \), one may have cosmic variance limited (CVL henceforth) reconstruction of the intrinsic \( C^B_{\ell} \). Delensed \( C^B_{\ell} \) without bias correction (blue dotted line) is also shown.

Next, we repeat the above procedure to the lensed realisations for \( r = 0.01 \). We again subtract the mean delensing bias from the mean of the delensed spectra to get the intrinsic \( C^B_{\ell_{\text{intr}}} \). In figure 4, we have plotted the bias-corrected mean of the recovered \( C^B_{\ell} \) (red dash-dotted line) along with the SD (solid purple line) and injected \( C^B_{\ell} \) (solid green line). From figure 4, we see that one can have CVL reconstruction of the intrinsic \( C^B_{\ell} \) up to a few hundred multipoles.

Finally, we employ our technique for \( r = 0.1 \). From the set of delensed spectra, we calculate the mean and subtract the mean delensing bias to get the intrinsic \( C^B_{\ell_{\text{intr}}} \). As before, to get the actual SD, we have used eq. (10). In figure 5, we have plotted the bias-subtracted mean of the recovered \( C^B_{\ell} \) along with the bias-corrected SD and injected \( C^B_{\ell} \). From figure 5, we see that it is possible to have CVL reconstruction of the intrinsic \( C^B_{\ell} \) up to few hundred multipoles.
Figure 1. The mean delensing bias (dash-dotted dark slate gray line) along with the corresponding SD (pink dotted line) are plotted. The solid red, green and blue lines represent intrinsic $C^B_\ell$ for $r = 0.1, 0.01, 0.001$, respectively. For a comparison, the lensed $B$ spectrum (black dashed line) and the corresponding SD (cyan dotted line) for $r = 0$ are plotted.

Figure 2. The delensed and intrinsic $C^B_\ell$ for $r = 0.0, 0.001, 0.001$ and 0.1. For delensing, we have used $\ell_{\text{max}} = 1024$.

Thus, our analysis unveils that delensing by solving the system of linear equations may be a very powerful tool, in the presence of the lensing potential. We see that in the absence of instrumental noise, it may be possible to have CVL reconstruction of intrinsic $B$-mode power which is indeed fascinating. But, we do not make any conclusive comments on this as the non-Gaussian structure and errors in the observed CMB have not been taken into account properly. So, this reconstruction algorithm may be applied for a wide range of values of tensor-to-scalar ratio in any noise-free experiment.
Figure 3. Bias-corrected reconstructed $C_{\ell}^B$ for $r = 0.001$ (dash-dotted red line) and the SD of the reconstruction (solid purple line) are shown. Intrinsic $C_{\ell}^B$ (lawn-green solid line) for $r = 0.001$ and the SD (cyan solid line) are also plotted. The bias-corrected reconstruction error (dotted black line) can also be seen.

Figure 4. Reconstructed $C_{\ell}^B$ after subtracting the mean delensing bias for $r = 0.01$ (red dash-dotted line) along with the SD of the reconstruction (purple solid line) are shown. Intrinsic $C_{\ell}^B$ (solid lawn-green line) for $r = 0.01$ and the corresponding SD (cyan solid line) are also plotted. The bias-corrected reconstruction error (black dotted line) can also be seen.

4. Delensing in the presence of the instrumental noise

In this section, we repeat the above exercise by incorporating instrumental noise anticipating CMB-Bharat-like futuristic space mission. The noise spectrum is given by [35–38]

$$N_\ell = \sigma^2 \theta_{\text{fwhm}}^2 \exp \left[ \ell (\ell + 1) \frac{\theta_{\text{fwhm}}^2}{8 \ln 2} \right],$$  

(11)

where $\theta_{\text{fwhm}}$ is the full-width half-maximum (in radian) of the Gaussian beam and $\sigma$ is the root mean square
of the detector noise for polarisation having unit $\mu K$. For estimating the noise power spectrum, we have considered CMB-Bharat’s 175 GHz channel with specifications as follows: $\theta_{\text{FWHM}} = 5.16$ arcmin, $\sigma = 5.1 \mu K$ for polarisation and number of detectors = 160. We first evaluate the noise power spectrum using $\ell_{\text{max}} = 1024$ for CMB-Bharat’s 175 GHz frequency channel and then generate 1000 Gaussian realisations of the noise spectra. The observed $B$-mode power in the presence of the instrumental noise then may be written as

$$ C_{\ell}^{B_{\text{obs}}} = C_{\ell}^{B_{\text{intr}}} + C_{\ell}^{B_{\text{len}}} + N_{\ell}^{B_{\text{noise}}} . $$

When the observed spectrum is delensed, that $C_{\ell}^{B_{\text{del}}}$ will also contain, apart from the intrinsic signal, the contribution from the instrumental noise as well as the bias. Therefore, the delensed $B$-mode power may now be written as

$$ C_{\ell}^{B_{\text{del}}} = C_{\ell}^{B_{\text{intr}}} + C_{\ell}^{B_{\text{bias}}} + N_{\ell}^{B_{\text{noise}}} . $$

So, to get the true $B$-mode power, we now need to subtract the delensing bias along with instrumental noise from $C_{\ell}^{B_{\text{del}}}$. The detailed procedure has been discussed in the following.

Before we proceed, one natural question arise – how to remove the delensing bias in the mock sky observations in the presence of non-zero gravitational waves. One can only measure the sum of lensing and primordial $B$-mode signals in the mock sky observation. This may be addressed within our current context as follows: Since we are not considering gravitational lensing by gravity waves, we can generate lensing potential using the latest values of the cosmological parameters using CAMB. That potential may be used to generate the lensing kernels. The theoretical lensed polarisation spectra can also be found using CAMB for $r = 0$. With the help of these, we might be able to calculate the noise power spectrum for CMB-Bharat mission. We can then add noise power spectrum to the lensed polarisation spectra and using our delensing technique we may find the delensed $B$-mode power, which will contain delensing bias as well as the instrumental noise. This delensed $B$-mode power may be used for bias correction.

4.1 Delensing bias in the presence of instrumental noise

As we delens the observed $C_{\ell}^{B_{\text{obs}}}$ with $r = 0$ in the presence of non-zero instrumental noise, the delensed spectra is again non-zero. So, the delensing bias in the presence of instrumental noise may be obtained by delensing the lensed spectra (after adding instrumental noise, assuming noise and signal to be uncorrelated) for $r = 0$. As a result, the intrinsic $B$-mode power may be found after subtracting bias and instrumental noise from the delensed $B$-mode power, i.e.,

$$ C_{\ell}^{B_{\text{intr}}} = C_{\ell}^{B_{\text{del}}} - (C_{\ell}^{B_{\text{bias}}} + N_{\ell}^{B_{\text{noise}}}) $$

$$ = C_{\ell}^{B_{\text{del}}} - C_{\ell}^{B_{\text{BN}}} , $$

(14)
Figure 6. Plot of mean delensing bias in the presence of instrumental noise. Intrinsic \( C_B^\ell \) for \( r = 0.001, 0.01 \) and 0.1 along with the mean delensing bias (slate grey dash-dotted line) and the corresponding SD (pink dotted line) are plotted. The lensed \( B \) spectrum (black dashed line) along with its SD (cyan point line) for \( r = 0 \) can also be seen.

Figure 7. Bias-corrected reconstructed \( C_B^\ell \) for \( r = 0.001 \) (red dash-dotted line) and the SD of the reconstruction (purple solid line) are shown. Intrinsic \( C_B^\ell \) (lawn-green solid line) for \( r = 0.001 \) and the SD (cyan solid line) are also plotted. The bias-corrected reconstruction error (dotted black line) can also be seen.

where \( C_B^{\text{BN}} = C_B^{\text{bias}} + N_B^{\text{noise}} = C_B^{\text{del}} | r=0 \) is the delensing bias in the presence of instrumental noise. So, the actual error in the reconstructed \( B \)-mode power may be written as

\[
\text{Var}(C_B^\ell) = \text{Var}(C_B^{\text{del}}) + \text{Var}(C_B^{\text{BN}}) - 2 \text{Cov}(C_B^{\text{del}}, C_B^{\text{BN}}).
\]
In figure 6, we have presented the delensing bias in the presence of instrumental noise, $C^{BN}_{\ell}$. The mean delensing bias and the SD were estimated from 1000 delensed $C^{\ell}$ for zero tensor-to-scalar ratio in the presence of instrumental noise anticipating CMB-Bharat-like futuristic space mission.

4.2 Delensed $C^{\ell}_{\ell}$ for $r = 0.001$, $r = 0.01$ and $r = 0.1$ with CMB-Bharat-like instrumental noise

Finally, we employ our delensing algorithm to the lensed realisations anticipating CMB-Bharat like futuristic satellite mission for non-zero tensor-to-scalar ratio with $\ell_{\text{max}} = 1024$. We first add the noise to the lensed CMB $B$ and $E$ realisations separately. Then, we delens those noisy lensed realisations and analyse the result. In order to get hold of intrinsic $C^{\text{Intr}}_{\ell}$ we subtract the mean delensing bias from the mean of the delensed $C^{\ell}_{\ell}$. The actual error of the intrinsic $C^{\text{Intr}}_{\ell}$ was estimated using eq. (15). The results for $r = 0.001, 0.01$ and 0.1 are shown in figures 7, 8 and 9, respectively. From these figures, we find that the bias-corrected mean and SDs of the recovered $B$-mode power are in good agreement with those of the intrinsic $C^{B}_{\ell}$ for all three values of tensor-to-scalar ratio.

5. Conclusion

In this article, we have presented a simple algorithm for the spectral level reconstruction of intrinsic $B$-mode power. The algorithm consists of solving the simultaneous system of linear equations by finding the inverses of lensing kernels using Gauss–Jordan elimination technique. We find that one may have very accurate reconstruction of intrinsic $B$-mode power even for $r$ as low as $10^{-3}$. Though our current observations are far away to make a detection of $r = 0.001$, the futuristic space missions such as CMB-Bharat may be able to detect signals of primordial gravity waves with tensor-to-scalar ratio as low as $10^{-3}$ or even an order lower. Accordingly, our methodology may be applied to those futuristic space missions as well. We also find that the incorporation of CMB-Bharat-like instrumental noise does not affect our reconstruction. Though we do not make any hefty comments on this, as the non-Gaussian structure of the lensed CMB has not been properly taken into account along with the lensing by gravity waves, still we are optimistic. The errors on the cosmological parameters were not considered as well. But still one may assert that this method of spectral level reconstruction has the potential to serve our purposes in uncovering the primordial gravity waves.
Figure 9. Bias-corrected reconstructed $C_{\ell}^B$ for $r = 0.1$ (red dash-dotted line) and the SD of the reconstruction (purple solid line) are shown. Intrinsic $C_{\ell}^B$ (lawn-green solid line) for $r = 0.1$ and the SD (cyan solid line) are also plotted. The bias-corrected reconstruction error (dotted black) can also be seen.

The lensing induced by gravity waves may not be significant as it is expected to be very small. So it can be safely neglected. We do not consider the fact that the observations only cover around 70–80% of the sky as well. And that partial sky coverage will bring in more noise in the reconstructed intrinsic spectrum. A more rigorous analysis can also be carried out by taking into account the non-zero correlation between lensed E and B-modes which seeks simulation of the lensed CMB sky. But at a first go, realising the simplicity of the method, the assumptions made therein are justified for the time being. We hope to come back with more sophisticated analysis incorporating the non-Gaussian structure of the lensed CMB along with the effect of partial sky coverage in the near future.

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References

[1] B P Abbott et al, Phys. Rev. Lett. 116, 061102 (2016)
[2] B P Abbott et al, Phys. Rev. Lett. 116, 241103 (2016)
[3] A A Starobinsky, J. Exp. Theor. Phys. Lett. 30, 682 (1979)
[4] A H Guth, Phys. Rev. D 23, 247 (1981)
[5] D H Lyth, K A Malik and M Sasaki, J. Cosmol. Astropart. Phys. 05, 004 (2005)
[6] U Seljak and M Zaldarriaga, Phys. Rev. Lett. 78, 2054 (1997)
[7] E Linder, Mon. Not. R. Astron. Soc. 243, 353 (1990)
[8] U Seljak, Astrophys. J. 463, 1 (1996)
[9] R B Metcalf and J Silk, Astrophys. J. 489, 1 (1997)
[10] M Zaldarriaga and U Seljak, Phys. Rev. D 58, 023003 (1998)
[11] A Lewis and A Challinor, Phys. Rep. 429, 1 (2006)
[12] U Seljak and C M Hirata, Phys. Rev. D 69, 043005 (2004)
[13] L Knox and Y S Song, Phys. Rev. Lett. 89, 011303 (2002)
[14] G Simard, D Hanson and G Holder, Astrophys. J. 807(2), 166 (2015)
[15] J Carron, A Lewis and A Challinor, J. Cosmol. Astropart. Phys. 05, 035 (2017)
[16] P A R Ade et al, Phys. Rev. D 103(2), 022004 (2021)
[17] T Okamoto and W Hu, Phys. Rev. D 67, 083002 (2003)
[18] M H Kesden, A Cooray and M Kamionkowski, Phys. Rev. D 67, 123507 (2003)
[19] C M Hirata and U Seljak, Phys. Rev. D 68, 083002 (2003)
[20] D Hanson et al, Phys. Rev. D 83(4), 043005 (2011)
[21] T Namikawa, D Yamauchi and A Taruya, J. Cosmol. Astropart. Phys. 01, 007 (2012)
[22] T Namikawa and R Nagata, *J. Cosmol. Astropart. Phys.* **09**, 009 (2014)
[23] R Pearson, B Sherwin and A Lewis, *Phys. Rev. D* **90**, 023539 (2014)
[24] K Sigurdson and A Cooray, *Phys. Rev. Lett.* **95**(21), 211303 (2005)
[25] K M Smith *et al.*, *J. Cosmol. Astropart. Phys.* **06**, 014 (2012)
[26] B D Sherwin and M Schmittfull, *Phys. Rev. D* **92**, 043005 (2015)
[27] A Manzotti *et al.*, *Astrophys. J.* **846**(1), 45 (2017)
[28] P Larsen *et al.*, *Phys. Rev. Lett.* **17**, 151102 (2016)
[29] B K Pal, H Padmanabhan and S Pal, *Mont. Not. R. Astron. Soc.* **439**, 3022 (2014)
[30] W H Press, S A Teukolsky, W T Vetterling and B P Flannery, *Numerical recipes: The art of scientific computing* (Cambridge University Press, 2007)
[31] N Aghanim *et al.*, *Astron. Astrophys.* **641**, A6 (2020)
[32] Y Akrami *et al.*, *Astron. Astrophys.* **641**, A10 (2020)
[33] A Challinor and A Lewis, *Phys. Rev. D* **71**, 103010 (2005)
[34] P A R Ade *et al.*, *Phys. Rev. Lett.* **127**(15), 151301 (2021)
[35] L Knox, *Phys. Rev. D* **52**, 4307 (1995)
[36] M Tegmark, *Phys. Rev. D* **56**, 4514 (1997)
[37] L Perotto *et al.*, *J. Cosmol. Astropart. Phys.* **10**, 013 (2006)
[38] S Galli *et al.*, *Phys. Rev. D* **90**, 063504 (2014)