**Optimal design of fracturing initiation position and breakdown pressure in horizontal well fracturing**

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**Abstract.** The fracture initiation mechanism of horizontal wells is significantly different from that of vertical wells and conventional directional wells. Therefore, the study of fracture initiation mechanism and fracture initiation pressure in horizontal wells is of great significance to the optimal design of hydraulic fracturing in horizontal wells. In order to determine the fracturing initiation pressure, the fracturing position and the initiation direction, the stress state and stress distribution of borehole are analyzed. The optimal model of fracture initiation position and breakthrough pressure is established and the calculation process is proposed.

1. **Introduction**

In horizontal wells fracturing, the breakdown pressure of horizontal wells is much higher than that of vertical wells, so the fracture cannot be opened, leading to the failure of fracturing. These phenomena indicate that the fracture initiation mechanism of horizontal wells is significantly different from that of vertical wells and conventional directional wells. Therefore, the study of fracture initiation mechanism and fracture initiation pressure in horizontal wells is of great significance to the optimal design of hydraulic fracturing in horizontal wells.

2. **Stress distribution around horizontal wellbore**

The formation of borehole breaks the equilibrium state of in-situ stress, and the stress redistributes in the rock around the borehole, causing stress concentration. During fracturing, the formation begins to fracture when the circumferential stress on the rock exceeds the tensile strength of the rock due to the increasing pressure in the well. Therefore, the fracturing initiation pressure, the fracturing position and the initiation direction all depend on the stress state of the wellbore. In order to study initiation, the stress state and stress distribution of borehole should be analyzed.
2.1. Change of well shaft coordinate and original stress coordinate

Since there is a difference in azimuth between horizontal shaft and local triaxial principal stress, the original stress should be converted into coordinates in order to facilitate the analysis of the stress around the horizontal shaft. The coordinate system \((1, 2, 3)\) is consistent with the direction of the original ground stress \(\sigma_H\), \(\sigma_h\) and \(\sigma_v\) respectively. Since it is convenient to analyze the stress distribution around the wellbore with column coordinates, the rectangular coordinate system \((x, y, z)\) and column coordinate system \((r, \theta, z)\) are established in this paper, where the \(oz\) axis corresponds to the horizontal well shaft, and the \(oy\) axis is located in the plane perpendicular to the shaft, as shown in Fig. 1.

![Figure 1. Change of well shaft coordinate and original stress coordinate](image)

The normal stress and shear stress components around the horizontal shaft in the coordinate system \((x, y, z)\) are:

\[
\begin{align*}
\sigma_{xx} &= \sigma_v \\
\sigma_{yy} &= \sigma_H \sin^2 \beta + \sigma_h \cos^2 \beta \\
\sigma_{zz} &= \sigma_H \cos^2 \beta + \sigma_h \sin^2 \beta \\
\tau_{xy} &= 0 \\
\tau_{yz} &= (\sigma_h - \sigma_H) \sin \beta \cos \beta \\
\tau_{xz} &= 0
\end{align*}
\]

(1)

2.2. Mathematical model of stress distribution in horizontal well wall

In this work, the stress near the perforation hole is determined in a small range by the method which is similar to that of the open hole wellbore stress solution. Different from the calculation of wellbore stress in open hole, the original triaxial stress around perforation hole is no longer the result of the original stress, but the result of the combination of the existence of borehole, fracturing and the original stress.

We assume that there is no perforation friction between the wellbore formation and perforation, that is the bottom hole pressure is equal to the borehole pressure. We also assume that the cement ring and the formation are well consolidated, the percolation and filtration of the fracturing fluid to the formation will not occur through the borehole wall, but only from the perforation hole wall. The perforated hole can be regarded as two orthogonal cylindrical holes with different diameters. Perforating wellbore geometric model and stress redistribution model are shown in Fig. 2.
Figure 2. Geometry and stress redistribution of perforating wellbore

The stress distribution around the hole can be obtained by substituting the stress \( \sigma_r \), \( \sigma_{\theta} \), and \( \sigma_z \) around the wellbore into the stress formula around the hole:

\[
\sigma'_r = P_i - \alpha p_p + \delta \phi (p_i - p_p)
\]

\[
\sigma'_{\theta} = \sigma_{xx} + \sigma_{yy} + 2 \sigma_{xx} - \sigma_{yy} \cos 2\theta - 2 \sigma_{xy} \sin 2\theta - P_i - \alpha P_p + \sigma_{zz} - 2\mu(\sigma_{xx} - \sigma_{yy}) \cos 2\theta - 4\mu \sigma_{xy} \sin 2\theta - \alpha P_p
\]

\[
-2 \left[ \sigma_{xx} + \sigma_{yy} + 2 \sigma_{xx} - \sigma_{yy} \cos 2\theta - 2 \sigma_{xy} \sin 2\theta - P_i - \alpha P_p \right] \cos 2\theta
\]

\[
-2 \left[ \sigma_{xx} + \sigma_{yy} + 2 \sigma_{xx} - \sigma_{yy} \cos 2\theta - 2 \sigma_{xy} \sin 2\theta - P_i - \alpha P_p \right] \cos 2\theta
\]

\[
\tau'_{r\theta} = 0
\]

\[
\tau'_{r\theta} = 2(-\sigma_{xz} \sin \theta + \sigma_{zz} \cos \theta)
\]

\[
\tau'_{rz} = 0
\]

2.3. Calculation model of fracture initiation position and breakdown pressure

According to the tensile fracture criterion, when any of the three stresses exceeds the tensile strength of the rock, the fracture will initiate at the wall of the wellbore. According to the theory of fracture mechanics, the initial fracture should be in the \( z - \theta \) plane, so \( \sigma_2 \) and \( \sigma_3 \) play a leading role in the initiation of fracture. The maximum tensile stress can be expressed as:

\[
\sigma_{\max}(\theta') = \sigma_3 = \frac{1}{2} \left[ \sigma'_r + \sigma'_{\theta} - \sqrt{(\sigma'_r - \sigma'_{\theta})^2 + 4\tau'^2_{r\theta}} \right]
\]

Obviously, the fracture initiation point and fracture pressure are related to the maximum tensile stress in the \( z - \theta \) plane, and the maximum tensile stress is a function of the azimuth angle of the horizontal
well $\beta$, mechanical properties of the rock $\mu$, porosity $\phi$, permeability coefficient $\delta$, porous elasticity coefficient $\alpha$, initial pore pressure in the formation $P_p$, and perforation orientation $\theta$, that is:

$$\sigma_{\text{max}}(\theta') = F(\beta, \mu, \phi, \delta, \alpha, P_p, \theta)$$ (9)

3. Optimal model of fracture initiation position and breakdown pressure

According to the above discussion, the optimal model of fracture initiation position and breakdown pressure can be written as:

$$\min \sigma_{\text{max}}(\theta')$$ (10)

The constrained variable scale method is applied to solve the optimization model of fracture point and fracture pressure. When solving, Lagrange function should be defined:

$$L(\theta', \lambda) = J(\theta') - \lambda^T C(\theta')$$ (11)

where $\lambda$ represents Lagrange multiplier. $\theta'_k$ and $\theta'_{k+1}$ represents the $k$ th and the $k+1$ th approximation of the minimum point respectively. At the same time, the descending direction of the $k$ th is $d_k = \theta'_{k+1} - \theta'_k$. $L(\theta', \lambda) = J(\theta') - \lambda^T C(\theta')$ is expanded in accordance with Taylor series at $\theta'_k$. If we omit the higher order terms that are greater than the second order, the following equation are obtained:

$$L(\theta'_{k+1}, \lambda_k) = L(\theta'_k, \lambda_k) + \nabla_\theta L(\theta'_k, \lambda_k)^T d_k + \frac{1}{2}(d_k)^T B_k d_k$$ (12)

where $B_k = \nabla^2_\theta L(\theta'_k, \lambda_k)$ is the Hesse matrix and $\nabla_\theta L(\theta'_k, \lambda_k)^T$ is the gradient vector. Since the constraint condition is linear and the objective function is non-linear, the constraint function $C(\theta')$ expanded at $\theta'_k$ is:

$$C(\theta'_{k+1}) = C(\theta'_k) + \nabla_\theta C(\theta'_k) d_k$$ (13)

Therefore, the algorithm of nonlinear constraint optimization can be described as:

In the first step, the initial iteration value $\theta'_0$, the positive definite matrix $B_0$ (which can be taken as the unit matrix), the required accuracy $\varepsilon$ are given in advance and $k = 0$.

In the second step, the following quadratic programming subproblems are constructed, and $d_k$ and $\lambda_k$ are obtained:

$$\min \left\{ \nabla_\theta J(\theta'_k)^T d_k + \frac{1}{2}(d_k)^T B_k d_k \right\}$$ (14)

s.t. $$C(\theta'_k) + \nabla_\theta C(\theta'_k) d_k \geq 0$$ (15)

In the third step, one-dimensional search is used to solve the new minimum point $\theta'_{k+1} = \theta'_k + \alpha_k d_k$. In the fourth step, $B_{k+1}$ can be obtained by revising $B_k$:

$$B_{k+1} = B_k + \frac{y_k y_k^T}{s_k^T y_k} - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k}$$ (16)

where $y_k = \nabla_\theta L(\theta'_{k+1}, \lambda_k) - \nabla_\theta L(\theta'_k, \lambda_k)$ and $s_k = \theta'_{k+1} - \theta'_k$. 

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This text document contains mathematical equations and variables related to mechanics and optimization, particularly focusing on fracture mechanics and the optimization of fracture initiation positions and breakdown pressures. The equations are structured to illustrate the process of solving optimization problems in the context of rock mechanics, including the use of Lagrange multipliers and quadratic programming techniques. The document highlights the importance of variables such as mechanical properties, porosity, permeability, initial pore pressure, and perforation orientation in fracture mechanics problems.
In the fifth step, if \( \| d_k \| \leq \varepsilon \), then the calculation is stopped, else \( B_k \) is revised and \( k = k + 1 \). Jump to step 2 and iterate again.

4. Conclusion

(1) The study of fracture initiation mechanism and fracture initiation pressure in horizontal wells is of great significance to the optimal design of hydraulic fracturing in horizontal wells.

(2) In order to determine the fracturing initiation pressure, the fracturing position and the initiation direction, the stress state and stress distribution of borehole are analyzed.

(3) The optimal model of fracture initiation position and breakdown pressure is established and the calculation process is proposed.

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Reference

[1] Perkins Jr, T. K., & Kern, L. R. (1961). Widths of Hydraulic Fractures. JPT: 937-49. Trans.

[2] Zheltov, A. K. (1955, January). 3. Formation of vertical fractures by means of highly viscous liquid. In 4th world petroleum congress. World Petroleum Congress.

[3] Geertsma, J., & De Klerk, F. (1969). A rapid method of predicting width and extent of hydraulically induced fractures. Journal of petroleum technology, 21(12), 1-571.