Acceleration of dust particles by vortex ring

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\textbf{Abstract:} It is shown that nonlinear interaction between large amplitude circularly polarized EM wave and dusty plasma leads to a nonstationary ponderomotive force which in turn produces a vortex ring, and magnetic field. Then the ensuing vortex ring in the direction of propagation of the pump wave can accelerate the micron-size dust particles which are initially at rest and eventually form a non relativistic dust jet. This effect is purely nonstationary and unlike linear vortices, dust particles do not rotate here. Specifically, it is pointed out that the vortex ring or closed filament can become potential candidate for the acceleration of dust in tokamak plasmas.

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In the past there have been fairly extensive investigations in the field of nonlinear interaction of high frequency EM waves or short pulse laser beams with an electron-ion plasma \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\]. Such interactions can produce various types of phenomena, like self focusing, Brillouin or Raman scattering, filamentation or modulational instability, collisionless shock waves or solitons, wave breaking, absorption of EM waves, generation of vortex rings, quasi-static magnetic fields etc. Spontaneously generated magnetic fields are not only important for laboratory produced plasmas, but in many cosmic environments, in our Universe as well as in galactic and intergalactic spaces\[19, 20, 21, 22, 23, 24, 25\]. A mechanism for the generation of large quasi-static magnetic fields and vortex ring by a non-potential ponderomotive force resulting from the time dependent amplitude of EM waves in electron ion plasma was discussed in Ref. \[5\]. This way production of magnetic field can essentially affects the transport phenomena, as well as the absorption of an EM field in inertial confinement fusion (ICF) schemes. Indeed, the vortex ring phenomena can have profound influence on laser experimental research, and at the same time its behavior can be described within the main equations of a continuous medium \[13, 14, 15\]. Most significantly, once this vortex is generated, it develops only under the effect of its own dynamics. Unlike straight vortex filament that is a static phenomenon of plasma, the vortex ring moves relative to the plasma, and in the same time it expands noticeably in the process.

Recently, research on dust in tokamak plasmas has attracted a lot of interest, although existence of these micron-size particles has been known for a long time. Besides some safety threats and engineering issues, the hereness of these such heavier particles in fusion devices can influence the plasma operation and performance effectively. Thus it is expected that dust in plasmas is going to play requisite/essential role in the next generation ITER-like fusion devices\[29, 30, 31, 32, 33, 34, 35\].

It has been predicted theoretically that dust particles in tokamak plasma can gain very high velocity and their presence can significantly affects when these collide the tokamak wall, the ejecta far exceed the projectile masses. This not only supplies fresh particles but releases neutral gas and a runaway effect for wall erosion and plasma contamination that could be a potential hazard for maintaining fusion conditions\[33\]. Also if such fast particles exist in the scrape-off layer (SOL), dust impact ionization could be used as a diagnostic. Different mechanisms have been proposed which can cause the acceleration of dust to such velocities. Among them, ion/dust drag force is mostly believed to play
dominant role the values of which strongly depend on plasma parameters as well as on
dust size, shape, temperature, and electric charge, it could speed up the dust up to ion
flow velocities, which are found to be tens of km/s for toroidal plasma flow near the last
closed magnetic surface (LCMS)\textsuperscript{[33]}.

deAnglis et al proposed a mechanism for the acceleration of dust based on stochastic
heating\textsuperscript{[34]}. Shukla and Tsintsadze quite recently showed that the normal component of
the space charge electric field may accelerate dust particles in the scrape-off layer close to
the tokamak chamber wall\textsuperscript{[35]}.

In this article we propose a mechanism for the acceleration of dust particles by closed
filaments or vortex rings, which are produced by the action of nonstationary ponderomo-
tive force occurring due to the time dependence of the amplitude of EM wave, which may
have effects on the physics of tokamak plasmas.

Following standard technique for the interaction of a high-frequency electromagnetic
radiation with some accidental and relatively low-frequency plasma perturbations, we
write down the equation of motion for dust grains and ions,

\[
\frac{\partial \mathbf{v}_d}{\partial t} + (\mathbf{v}_d \cdot \nabla)\mathbf{v}_d = -\frac{Z_d e}{m_d} \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_d \times \mathbf{B} \right),
\]

\[
m_i \left[ \frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i \right] = e \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_i \times \mathbf{B} \right) + \mathbf{F}_{NL} - \frac{1}{n_i} \nabla P_i. \tag{2}
\]

The coupling with the intense EM field is described by the time dependent ponderomotive
force \textsuperscript{[4, 6, 16, 37]} expressed in terms of the vector potential

\[
\mathbf{F}_{NL} = -m_i c^2 \left[ \frac{\omega_o}{\omega_o - \omega_{ci}} \nabla \left( \frac{e^2 |A|^2}{m_i^2 c^4} \right) - \frac{\omega_{ci} k_o}{(\omega_o - \omega_{ci})^2} \frac{\partial}{\partial t} \left( \frac{e^2 |A|^2}{m_i^2 c^4} \right) \right].
\]

It is to be noted that above ponderomotive force expression contains two parts, space
dependant and time dependent, where the later is due to magnetic field. In the absence of
static magnetic field ($B_0 = 0$) the ponderomotive force expression reduces to the ordinary
one\textsuperscript{[38]}. In an electron-depleted plasma, or equivalently for oscillations well below the
electron Debye scale, the ions play a vital role for the excitation of a very low frequency
wave. Hence, the quasi-neutrality condition which we use reads $\delta n_i \simeq Z_d \delta n_d$. Neglecting
the inertial term in Eq. (2), and combining the remaining equation with Eq. (1) in the
limit $v_d, v_i \ll c$, we get

\[
\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{1}{2 m_d} \nabla \frac{v_d^2}{2} + \mathbf{v}_d \times (\nabla \times \mathbf{v}_d) - \frac{Z_d m_i c^2}{m_d} \left[ \frac{\omega_o}{\omega_o - \omega_{ci}} \nabla \Psi^2 - \frac{\omega_{ci} k_o}{(\omega_o - \omega_{ci})^2} \frac{\partial \Psi^2}{\partial t} \right] - \frac{Z_d}{n_i m_d} \nabla P_i. \tag{3}
\]
where $\Psi = eA/(m_c c^2)$ is a dimensionless vector potential. Taking the curl of both sides, we obtain

$$\frac{\partial \Omega}{\partial t} = \nabla \times \mathbf{v}_d \times \mathbf{\Omega} + \frac{Z_d \omega_{ci} m_c c^2}{m_d (\omega_o - \omega_{ci})^2} \nabla \frac{\partial \Psi^2}{\partial t} \times \mathbf{k}_o. \quad (4)$$

here $\mathbf{\Omega} = \nabla \times \mathbf{v}_d$ represents the vorticity of the dust velocity, and $\mathbf{k}_o$ is the wave vector which is directed along the external magnetic field [$\mathbf{k}_o = (0, 0, k_o e_z)$]. Considering $l > v_d t$, where $l$ and $t$ represent the characteristic spatial and time scale lengths, respectively, will allow us to neglect the first term on the right-hand side (r.h.s) in comparison with the term on the left-hand side (l.h.s) in Eq. (4). As a result, we obtain a simple relation between the vorticity and the source

$$\mathbf{\Omega} = \frac{Z_d \omega_{ci} m_c c^2}{m_d (\omega_o - \omega_{ci})^2} \nabla \Psi^2 \times \mathbf{k}_o. \quad (5)$$

For a circularly polarized EM wave propagating along the $z$-axis, the vorticity has only two components, $\Omega_x$ and $\Omega_y$. Thus vortices are produced in the plane perpendicular to the propagation of the pump wave, i.e., we have the formation of vortex rings or close filaments.

Now we investigate Eq. (4) in the cylindrical coordinates $r, \theta, z$ at the center of the ring, which clearly shows that the vorticity has only one component, $\mathbf{\Omega} = \Omega_\theta \mathbf{e}_\theta$, and the particle velocity is $\mathbf{v}_d = v_r \mathbf{e}_r + v_z \mathbf{e}_z$, where $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z)$ are the unit vectors. Expressing $(\mathbf{\Omega} \cdot \nabla) = (\mathbf{\Omega}_\theta / r) (\partial / \partial \theta)$, $\partial v_r / \partial \theta = v_r e_\theta$, and $v_r = dr / dt$ in Eq. (4), we obtain

$$\frac{d}{dt} \left( \frac{\Omega_\theta}{n_d r} \right) = \frac{Z_d \omega_{ci} m_c c^2}{m_d (\omega_o - \omega_{ci})^2} \frac{k_o}{n_d r} \frac{\partial}{\partial r} \left( \frac{\partial \Psi^2}{\partial t} \right). \quad (6)$$

From Eqs. (3)-(6) it can be noticed that the time dependence of the ponderomotive force does not conserve the velocity circulation, $\Gamma_d = \oint \mathbf{v}_d \, d\mathbf{r} = \int_s \oint \mathbf{\Omega} \, ds$. The same follows for frozen-in condition which does not take place but only for the wake vorticity ($\partial \Psi^2 / \partial t = 0$)

$$\frac{\Omega_\theta}{n_d r} = \text{const.} \quad (7)$$

Eqs. (4)-(6) indicate that if at the initial instant of time there are no vortices of the dust fluid flow at a given point, they will be generated by the circularly polarized EM field. It is important to note that if there are $N$ point-like vortices (filaments), then these filaments can interact with each other, leading eventually to the merging of vortices, to the decaying of a vortex into two other vortices, to the annihilation of vortices, etc. Another interesting feature is that here unlike linear vortices, dust particles do not rotate.
We have shown above how a circularly polarized EM wave generates closed filaments, values of which can be defined by the intensity of the pump. We now demonstrate that there is a mechanism of the acceleration of the dust particles up to a particular velocity due to the vortex ring. For this considering the inverse problem, i.e., by a given vortex we can define the velocity of the dust grains \((v_d)\) at any point in the plasma. To this end, we assume that the plasma is at rest until the vortex is generated, \(\Omega = \nabla \times v_d\). Let us introduce the vector \(P\) such that \(\nabla \cdot P = 0\), and \(v_d = \nabla \times P\). For \(P\) we can write down the following equation

\[
\nabla^2 P = -\Omega, \tag{8}
\]

which has solution of the form

\[
P = \frac{1}{4\pi} \int \frac{\Omega(r')dr'}{R}, \tag{9}
\]

where \(dr' = dx'dy'dz'\), \(R = [(x-x')^2 + (y-y')^2 + (\xi-z')^2]^{1/2}\), \(\xi = \zeta - u_g t\) and \(u_g = \frac{2k_o \omega_{cd}^2}{\omega_{pd}^2}\) is the group velocity of the pump wave.

By knowing the vector \(P\), we can define the velocity \(v_d\) at any point in the plasma

\[
v_d(r, t) = \nabla \times \frac{1}{4\pi} \int \frac{\Omega(r')dr'}{R}. \tag{10}
\]

Let us now consider a vortex ring with radius \(\rho_o\), and characterize the position of the variable on the vortex ring by the angle \(\alpha\). Any point on the vortex filament in the Cartesian coordinates is determined by \(x' = \rho_o \cos \alpha, y' = \rho_o \sin \alpha, z' = 0\).

In order to calculate the integral in Eq. (9), we assume that \(\Psi^2 = \Psi_0^2 u_g \tau \delta(\zeta' - u_g \tau)\theta(r' - \rho_o)\), and \(P\) has only one component, \(P_\theta = P_\theta(r, z)\) [39]. From Eq. (9), we have

\[
P_\theta(r, \xi) = \frac{\omega_{ci} k_o u_g \tau \rho_o \Psi^2}{m_d \pi (\omega_o - \omega_{ci})^2} \oint \frac{d\alpha \cdot \cos \alpha}{R}, \tag{11}
\]

here \(R = (r^2 + \xi^2 + \rho_o^2 - 2r \rho_o \cos \alpha)^{1/2}\), \(\tau\) is the duration of EM field and the integral is

\[
\oint \frac{\cos \alpha d\alpha}{R} = \frac{4}{\eta} \sqrt{1 - \frac{\eta^2}{2}} \left[ K(\eta) - E(\eta) \right], \tag{12}
\]

where \(\eta^2 = 4r \rho_o / [(r + \rho_o)^2 + \xi^2]\), \(K\) and \(E\) are the elliptical integrals of the first and second kind,

\[
K(\eta) = \int_0^{2\pi} \frac{d\beta}{\sqrt{1 - \eta^2 \sin^2 \beta}},
\]

\[
E(\eta) = \int_0^{2\pi} \sqrt{1 - \eta^2 \sin^2 \beta} d\beta.
\]
and $\beta = (\alpha - \pi)/2$. As a result we have

$$P_{\theta}(r, \xi) = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d (\omega_o - \omega_{ci})^2 \eta} \times \sqrt{\frac{\rho_o}{r}} \left[ \left( 1 - \frac{\eta^2}{2} \right) K(\eta) - E(\eta) \right].$$ \hspace{1cm} (13)

The components of the velocity can be written as

$$v_{dr} = -\frac{\partial P_{\theta}}{\partial z}, \quad v_{dz} = \frac{1}{r} \frac{\partial}{\partial r}(r P_{\theta}),$$ \hspace{1cm} (14)

or

$$v_{dr} = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d (\omega_o - \omega_{ci})^2 2\pi r \sqrt{(r + \rho_o)^2 + \zeta^2}} \times \left[ -K(\eta) + \frac{r^2 + \rho_o^2 + \zeta^2}{(\rho_o - r)^2 + \zeta^2} E(\eta) \right],$$ \hspace{1cm} (15)

$$v_{dz} = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d (\omega_o - \omega_{ci})^2 2\pi \sqrt{(r + \rho_o)^2 + \zeta^2}} \times \left[ K(\eta) + \frac{r^2 + \rho_o^2 + \zeta^2}{(\rho_o - r)^2 + \zeta^2} E(\eta) \right].$$ \hspace{1cm} (16)

Now we consider two cases, for the first, assuming $r \to 0$ i.e., on the axis

$$v_{dr} = 0, \quad v_{dz} = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d (\omega_o - \omega_{ci})^2 (\rho_o^2 + \zeta^2)^{3/2}}.$$ \hspace{1cm} (17)

Recalling that $\xi - \zeta - u_g t = 0$, and for the maximum, we write

$$v_{dz,\text{max}} = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d \rho_o (\omega_o - \omega_{ci})^2}.$$ \hspace{1cm} (18)

Now we consider the case at the points near the filament, i.e., $r = \rho_o$ and $\xi = \zeta = u_g \tau = 0$. In this case $\eta^2 = 1$, $E = \pi/2$, but $K(1)$ is logarithmically divergent at the lower limit as

$$K = \frac{1}{2} \int_0^\pi \frac{d\alpha}{\sin \alpha/2} \approx \int_0^\pi \frac{d\alpha}{\alpha}.$$ \hspace{1cm} (19)

In reality, the closed filament as a ring has a finite size denoted by $r_o$ ($r_o$ is the core radius of the thin vortex ring), so there must be a cut off at a value $\alpha \sim r_o/\rho_o$, and $K(1) = \ln \rho_o/r_o$. The $z$-component of the velocity now becomes

$$v_{dz} = \frac{\omega_{ci} k_0 u_g \tau \Psi_0^2}{m_d \rho_o (\omega_o - \omega_{ci})^2} \ln \frac{\rho_o}{r_o}.$$ \hspace{1cm} (20)

Using the simple relation between vorticity and source we have obtained expressions of dust speed [Eqs.(12) and (13)] which particles gain from the filaments and have shown vortex ring can generate a collimated dust jet along the propagation of EM wave. We emphasis that above consideration can be applied to the acceleration of dust particles in the scrape-off layer of the Tokamak.
Now we manifest how the nonstationary ponderomotive force of the EM wave which creates slowly varying electric fields and vector potentials can generate magnetic fields.

A simple expression for this can be obtained by taking the curl of the momentum equation for inertialess ions

$$\nabla \times E = -\frac{1}{e} \nabla \times F_{NL}. \quad (21)$$

Using the Faraday law $\nabla \times E = -(\partial B/\partial t)/c$, Eq. (21) becomes

$$\frac{\partial B}{\partial t} = \frac{c}{e} \nabla \times F_{NL} = \frac{c}{e} \left[ \frac{\omega_{ci}m_i c^2}{(\omega_o - \omega_{ci})^2} \nabla \frac{\partial \Psi^2}{\partial t} \times k_o \right] . \quad (22)$$

Integrating both sides we get

$$B = \frac{\omega_{ci}m_i c^3}{e (\omega_o - \omega_{ci})^2} \nabla \Psi^2 \times k_o. \quad (23)$$

The $x$ and $y$ components of the above equation are:

$$eB_x/m_i c = \frac{c^2 \omega_{ci} k_o}{(\omega_o - \omega_{ci})^2} \frac{\partial \Psi^2}{\partial y},$$

$$eB_y/m_i c = -\frac{c^2 \omega_{ci} k_o}{(\omega_o - \omega_{ci})^2} \frac{\partial \Psi^2}{\partial x}. \quad (24, 25)$$

Squaring and then addition of Eqs. (24) and (25) gives us generation of magnetic field

$$\frac{eB}{m_i c} = \pm \frac{k_o \omega_{ci}}{m_i (\omega_o - \omega_{ci})^2} \frac{\partial \Psi^2}{\partial r}, \quad r = (x^2 + y^2)^{1/2}. \quad (26)$$

Eqs. (24)-(26) show that magnetic field is generated in the plane perpendicular to the starting magnetic field $B_o e_z$, and the components of the generated magnetic field are determined by both, the parameters of the plasma, and the intensity of the pump wave.

To summarize, the main idea here is to identify a new mechanism for the generation of vortex ring which can accelerate dust particles to very high velocity resulting in the formation of a non relativistic dust jet by employing the nonlinear interaction of circularly polarized EM wave with dusty plasma. Specifically, this interaction leads to a nonstationary ponderomotive force which pushes the ions locally and creates slowly varying electric fields and vector potentials. The latter, in turn, generates vortex ring and quasi stationary magnetic fields in the perpendicular direction. Then, we considered an interesting inverse problem that the ensuing vortex ring becomes a means for the formation of dust jets (dust particles were initially in an equilibrium state), which are emitted out after gaining acceleration. The relevance of the above analysis for tokamak plasmas can be assessed by
making some simple estimates. In addition, generation of magnetic field is a ubiquitous phenomenon relevant to astrophysics and this can also affect energy transport in inertial confinement fusion schemes.

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