$B_s$-$\bar{B}_s$ Mixing in $Z'$ Models with Flavor-Changing Neutral Currents

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Abstract

In models with an extra $U(1)'$ gauge boson family non-universal couplings to the weak eigenstates of the standard model fermions generally induce flavor-changing neutral currents. This phenomenon leads to interesting results in various $B$ meson decays, for which recent data indicate hints of new physics involving significant contributions from $b \to s$ transitions. We analyze the $B_s$ system, emphasizing the effects of a $Z'$ on the mass difference and $CP$ asymmetries.

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I. INTRODUCTION

The study of $B$ physics and the associated CP violating observables has been suggested as a good means to extract information on new physics at low energy scales [1, 2, 3, 4, 5, 6, 7]. Since $B$-$\overline{B}$ mixing is a loop-mediated process within the standard model (SM), it offers an opportunity to see the footprints of physics beyond the SM. The currently observed $\Delta M_d = 0.489 \pm 0.008 \text{ ps}^{-1}$ [8] and its mixing phase $\sin 2\beta = 0.736 \pm 0.049$ extracted from the $B_d \rightarrow J/\psi K_S$ mode [9] agree well with constraints obtained from other experiments [10]. However, no such information other than a lower bound $\Delta M_s > 14.5 \text{ ps}^{-1}$ [11] is available for the $B_s$ meson yet.

Based upon SM predictions, $\Delta M_{B_s}$ is expected to be about 18 ps$^{-1}$ and its mixing phase $\phi_s$ only a couple of degrees. In contrast to the $B_d$ system, the more than 25 times larger oscillation frequency and a factor of four lower hadronization rate from $b$ quarks pose the primary challenges in the study of $B_s$ oscillation and CP asymmetries. Since the $B_s \rightarrow J/\psi \phi$ decay is dominated by a Cabibbo-Kobayashi-Maskawa (CKM) favored tree-level process, $b \rightarrow c \bar{c}s$, that does not involve a different weak phase in the SM, its asymmetry provides the most reliable information about the mixing phase $\phi_s$.

Although new physics contributions may not compete with the SM processes in most of the $b \rightarrow c \bar{c}s$ decays, they can play an important role in $B_s$-$\overline{B}_s$ mixing because of its loop nature in the SM [12]. In particular, the mixing can be significantly modified in models in which a tree-level $b \rightarrow s$ transition is present. Thus, measurement of the properties of $B_s$ meson mixing is of high interest in future $B$ physics studies as a means to reveal new physics [13, 14]. Since the current $B$ factories do not run at the $\Upsilon(5S)$ resonance to produce $B_s$ mesons, it is one of the primary objectives of hadronic colliders to study $B_s$ oscillation and decay in the coming years [15, 16].

Flavor changing neutral currents (FCNC) coupled to an extra $U(1)'$ gauge boson arise when the $Z'$ couplings to physical fermion eigenstates are non-diagonal. One way for this to happen is by the introduction of exotic fermions with different $U(1)'$ charges that mix with the SM fermions [17, 18, 19, 20, 21] as occurs in $E_6$ models. In the $E_6$ case, mixing of the right-handed ordinary and exotic quarks, all $SU(2)_L$ singlets, induces FCNC mediated by a heavy $Z'$ or by (small) $Z-Z'$ mixing, so the quark mixing can be large. Mixing between ordinary (doublet) and exotic (singlet) left-handed quarks induces FCNC mediated by the
SM Z boson \[21\]. We will also allow for this possibility, but in this case the quark mixing must be very small.

Another possibility involves family non-universal couplings. It is well-known that string models naturally give extra $U(1)'$ groups, at least one of which has family non-universal couplings to the SM fermions \[24, 25, 26, 27\]. Generically, the physical and gauge eigenstates do not coincide. Here, unlike the above-mentioned $E_6$ case, off-diagonal couplings of fermions to the $Z'$ boson can be obtained without mixing with additional fermion states. In these types of models, both left-handed and right-handed fermions can have family non-diagonal couplings with the $Z'$, while couplings to the $Z$ are family diagonal (up to small effects from $Z - Z'$ mixing).

The $Z'$ contributions to $B_s - \overline{B}_s$ mixing are related to those for hadronic, semileptonic, and leptonic $B$ decays in specific models in which the diagonal $Z'$ couplings to $q\bar{q}$, $\ell^+\ell^-$, etc. are known, but are independent in general \[46\]. We have found that in specific models, $B_s - \overline{B}_s$ mixing effects can be significant while being consistent with the other constraints; these results will be presented elsewhere. In the present paper, we will treat the mixing in a model-independent way.

Recently, we have studied the implications of a sizeable off-diagonal $Z'$ coupling between the bottom and strange quark in the indirect CP asymmetry of $B \to \phi K_S$ decay \[28\], which appears to show a significant deviation from the SM prediction \[5, 6, 29, 30\]. Here we extend our analysis to $B_s - \overline{B}_s$ mixing where the $Z'$ contributions also enter at the tree level. Applications to the $B \to \pi K$ anomaly are under investigation \[31\].

The paper is organized as follows. In Section II, we review the basic formalism of $B_s - \overline{B}_s$ mixing. In Section III, we evaluate $\Delta M_s$ in the SM. In Section IV, we include the $Z'$ contributions, allowing both left-handed and right-handed couplings in the mixing, and study their effects on observables. Our main results are summarized in Section V.

II. $B_s - \overline{B}_s$ MIXING

In the conventional decomposition of the heavy and light eigenstates

$$|B_s\rangle_L = p|B_s^0\rangle + q|\overline{B}_s^0\rangle,$$

$$|B_s\rangle_H = p|B_s^0\rangle - q|\overline{B}_s^0\rangle,$$ (1)
the mixing factor
\[
\left( \begin{array}{c} q \\ p \end{array} \right)_{SM} \simeq \sqrt{\frac{M_{12}^{SM*}}{M_{12}^{SM}}} , \tag{2}
\]
has a phase
\[
\phi_{s}^{SM} = 2 \arg(V_{tb}V_{ts}^{*}) = -2\lambda^{2}\bar{\eta} \simeq -2^\circ , \quad \sin 2\phi_{s}^{SM} \simeq -0.07 , \tag{3}
\]
where the theoretical expectation \( \Gamma_{12}^{SM} \ll M_{12}^{SM} \) is used. The approximate formula Eq. (2) receives a small correction once \( \Gamma_{12}^{SM} \) is included. Model independently, this only shifts \( \phi_{s} \) is at the few percent level. With errors on \( \lambda \) and \( \bar{\eta} \) included, we have the SM expectation that \( \sin 2\phi_{s}^{SM} \simeq -0.07 \pm 0.01 \).

The off-diagonal element of the decay matrix, \( \Gamma_{12}^{SM} \), is evaluated by considering decay channels that are common to both \( B_{s} \) and \( \bar{B}_{s} \) mesons, and \( M_{12} \) is the off-diagonal element of the mass matrix. Due to the CKM enhancement, \( \Gamma_{12}^{SM} \) is dominated by the charm-quark contributions over the up-quark contribution in a box diagram. Unlike the Kaon system, \( \Gamma_{12}^{SM} \) is much smaller than \( M_{12}^{SM} \) for \( B \) mesons because the former is related to the \( B \) meson decays and set by the scale of its mass, whereas the latter is proportional to \( m_{t}^{2} \). We can safely assume that \( \Gamma_{12} \) is not significantly modified by new physics because \( \Gamma_{12} \) receives major contributions from CKM favored \( b \to c\bar{c}s \) decays in the SM, and the SM result \( \Gamma_{12} \ll M_{12} \) is unlikely to change.

The mass difference of the two physical states is
\[
\Delta M_{s} \equiv M_{H} - M_{L} \simeq 2|M_{12}| . \tag{4}
\]
The width difference is
\[
\Delta \Gamma \equiv \Gamma_{H} - \Gamma_{L} = \frac{2\text{Re}(M_{12}^{*}\Gamma_{12})}{|M_{12}|} = 2|\Gamma_{12}| \cos \theta , \tag{5}
\]
where the relative phase is \( \theta = \arg(M_{12}/\Gamma_{12}) \). Since \( \Gamma_{12} \) is dominated by the contributions from CKM favored \( b \to c\bar{c}s \) decays, we have \( \theta = \arg((V_{tb}V_{ts}^{*})/(V_{cb}V_{cs}^{*})) \simeq \pi \), and thus \( \Delta \Gamma \simeq -2\Gamma_{12} \) is negative in the SM. Although \( \Gamma_{12} \) is unlikely to be affected by new physics, the width difference always increases as long as the weak phase of \( M_{12} \) gets modified [32].

The observability of \( B_{s}\bar{B}_{s} \) oscillations is often indicated by the parameter
\[
x_{s} \equiv \frac{\Delta M_{s}}{\Gamma_{s}} , \tag{6}
\]
where $\Gamma_s = (4.51 \pm 0.18) \times 10^{-13}$ GeV, converted from the world average lifetime $\tau_s = 1.461 \pm 0.057$ ps \cite{8}. The expected large value of $x_s$ is a challenge for experimental searches. Currently, the result from all ALEPH \cite{33}, CDF \cite{34}, DELPHI \cite{35}, OPAL \cite{36}, and SLD \cite{37} studies of $\Delta M_s$ with a combined 95% confidence level (CL) sensitivity on $\Delta M_s$ of 18.3 ps$^{-1}$ gives \cite{11}

$$\Delta M_s > 14.5 \text{ ps}^{-1}, \quad \text{and} \quad x_s > 20.8.$$ \hspace{1cm} (7)

It is also measured that $m_{B_s} = 5369.6 \pm 2.4$ MeV \cite{8} and $\Delta \Gamma_s/\Gamma_s = -0.16^{+0.15}_{-0.16}$ (with the 95% CL upper bound given in parentheses \cite{11}) consistent with recent next-to-leading-order (NLO) QCD estimates \cite{38}. In comparison, the $B_d$ system has $m_{B_d} = 5279.4 \pm 0.5$ MeV, $\Delta M_d = (0.489 \pm 0.008)$ ps$^{-1}, x_d = 0.755 \pm 0.015$, and $\tau_{B_d} = 1.542 \pm 0.076$ ps \cite{8}.

III. $\Delta M_s$ IN THE SM

The $|\Delta B| = 2$ and $|\Delta S| = 2$ operators relevant for our discussions are:

$$O_{LL}^{\text{SM}} = [\bar{s}\gamma_\mu(1 - \gamma_5)b][\bar{s}\gamma^\mu(1 - \gamma_5)b],$$
$$O_1^{LR} = [\bar{s}\gamma_\mu(1 - \gamma_5)b][\bar{s}\gamma^\mu(1 + \gamma_5)b],$$
$$O_2^{LR} = [\bar{s}(1 - \gamma_5)b][\bar{s}(1 + \gamma_5)b],$$
$$O_{RR}^{\text{SM}} = [\bar{s}\gamma_\mu(1 + \gamma_5)b][\bar{s}\gamma^\mu(1 + \gamma_5)b].$$ \hspace{1cm} (8)

Because of the $V - A$ structure, only the operator $O_{LL}^{\text{SM}}$ contributes to $B_s\bar{B}_s$ mixing in the SM. The other three operators appear in the $Z'$ models because of the right-handed couplings and operator mixing through renormalization, as considered in the next section.

In the SM the contributions to

$$M_{12}^{\text{SM}} \simeq \frac{1}{2m_{B_s}} \langle B_s^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | B_s^0 \rangle$$ \hspace{1cm} (9)

are dominated by the top quark loop. The result, accurate to NLO in QCD, is given by \cite{39}

$$M_{12}^{\text{SM}} = \frac{G_F^2}{12\pi^2} M_W^2 m_{B_s} f_{B_s}^2 (V_{tb}V_{ts}^*)^2 \eta_{2B_s} S_0(x_t) [\alpha_s(m_b)]^{-6/23} \left[1 + \frac{\alpha_s(m_b)}{4\pi} J_5 \right] B^{LL}(m_b),$$ \hspace{1cm} (10)

where $x_t = (m_t/m_t)/M_W^2$ and

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3x^3 \ln x}{2(1 - x)^3}.$$ \hspace{1cm} (11)
Using $m_t(m_t) = 170 \pm 5$ GeV, we find $S_0(x_t) = 2.463$. The NLO short-distance QCD corrections are encoded in the parameters $\eta_{2B} \simeq 0.551$ and $J_5 \simeq 1.627$. The bag parameter $B^{LL}(\mu)$ is defined through the relation

$$
\langle B_s | O^{LL} | B_s \rangle \equiv \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B^{LL}(\mu) .
$$

(12)

In the following numerical analysis, we will use $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ and $M_W = 80.423 \pm 0.039$ GeV [8], and write the SM part of $\Delta M_s$ as

$$
\Delta M_s^{SM} = 1.19 \left| \frac{V_{tb} V_{ts}^*}{0.04} \right|^2 \left( \frac{f_{B_s}}{230 \text{ MeV}} \right)^2 \left( \frac{B^{LL}(m_b)}{0.872} \right) \times 10^{-11} \text{ GeV} .
$$

(13)

Current lattice calculations still show quite large errors on the hadronic parameters $f_{B_s} = 230 \pm 30$ MeV and $B^{LL}(m_b) = 0.872 \pm 0.005$ [40, 41, 42]. However, the ratio

$$
\xi \equiv \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}
$$

(14)

can be determined with a much smaller theoretical error, where $\hat{B}_{B_q}$ is the renomalization-independent bag parameter for the $B_q$ meson ($q = d, s$). Therefore, the error on $\Delta M_s$ within the SM can be evaluated by comparing with $\Delta M_d$, i.e.,

$$
\Delta M_s^{SM} = \Delta M_d^{SM} \xi \frac{m_{B_s}}{m_{B_d}} \frac{(1 - \lambda^2)^2}{\lambda^2 [1 - (1 - \eta^2)][1 - \eta^2]} .
$$

(15)

Using the measured values of the Wolfenstein parameters $\lambda = 0.2265 \pm 0.0024$, $A = 0.801 \pm 0.025$, $\tilde{\rho} = 0.189 \pm 0.079$, and $\tilde{\eta} = 0.358 \pm 0.044$ [10], $\xi = 1.24 \pm 0.07$ [44], and the mass parameters quoted above, we obtain the SM predictions

$$
\Delta M_s^{SM} = (1.19 \pm 0.24) \times 10^{-11} \text{ GeV} = 18.0 \pm 3.7 \text{ ps}^{-1},
$$

$$
x_s^{SM} = 26.3 \pm 5.5 .
$$

(16)

As noted above, the central value of $x_s$ is slightly larger than the current sensitivity based upon the world average. Recent LHC studies show that with one year of data, $\Delta M_s$ can be explored up to 30 ps$^{-1}$ (ATLAS), 26 ps$^{-1}$ (CMS), and 48 ps$^{-1}$ (LHCb) (corresponding to $x_s$ up to 46, 42, and 75); the LHCb result is based on exclusive hadronic decay modes [16]. The sensitivity of both CDF and BTeV on $x_s$ can also reach up to 75 using the same modes [15], for a luminosity of 2 fb$^{-1}$. The sensitivity on $\sin 2\phi_s$ is correlated with the value of $x_s$, and it becomes worse as $x_s$ increases. A statistical error of a few times $10^{-2}$ can be reached at CMS and LHCb for moderate $x_s \simeq 40$ [16].
IV. \( Z' \) CONTRIBUTIONS

For simplicity, we assume that there is no mixing between the SM \( Z \) and the \( Z' \) (small mixing effects can be easily incorporated [17]). A purely left-handed off-diagonal \( Z' \) coupling to \( b \) and \( s \) quarks results in an effective \(|\Delta B| = 2, |\Delta S| = 2\) Hamiltonian at the \( M_W \) scale of

\[
\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} B_{sb} \right)^2 \frac{e^{2i\phi_L}}{\rho_L^2} O^{LL}(m_b) ,
\]

where \( g_2 \) is the \( U(1)' \) gauge coupling, \( g_1 = e/(\sin \theta_W \cos \theta_W) \), \( M_{Z'} \) is the mass of the \( Z' \), and \( B_{sb} \) is the FCNC \( Z' \) coupling to the bottom and strange quarks. The parameters \( \rho_L \) and the weak phase \( \phi_L \) in the \( Z' \) model are defined by the second equality. Generically, we expect that \( g_2/g_1 \sim 1 \) if both \( U(1) \) groups have the same origin from some grand unified theory, and \( M_Z/M_{Z'} \sim 0.1 \) for a TeV-scale \( Z' \). If \( |B_{sb}^L| \sim |V_{tb}V_{ts}^*| \), then an order-of-magnitude estimate gives us \( \rho_L \sim O(10^{-3}) \), which is in the ballpark of giving significant contributions to the \( B_s - \bar{B}_s \) mixing. The \( Z' \) does not contribute to \( \Gamma_{12} \) at tree level because the intermediate \( Z' \) cannot be on shell. After evolving from the \( M_W \) scale to \( m_b \), the effective Hamiltonian becomes [39]

\[
\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(M_W)}{4\pi} J_5 \right] R^{\delta/23} \rho_L^2 e^{2i\phi_L} O^{LL}(m_b) ,
\]

where \( R = \alpha_s(M_W)/\alpha_s(m_b) \). Although the above effective Hamiltonian is largely suppressed by the ratio \((g_2 M_Z)/(g_1 M_{Z'})\), it contains only one power of \( G_F \) in comparison with the corresponding quadratic dependence in the SM because the \( Z' \)-mediated process occurs at tree level.

The full description of the running of the Wilson coefficient from the \( M_W \) scale to \( m_b \) can be found in [39]. We only repeat the directly relevant steps here. The renormalization group equation for the Wilson coefficients \( \tilde{C} \),

\[
\frac{d}{d \ln \mu} \tilde{C} = \gamma^T(g) \tilde{C}(\mu) ,
\]

can be solved with the help of the \( U \) matrix

\[
\tilde{C}(\mu) = U(\mu, M_W) \tilde{C}(M_W) ,
\]

in which \( \gamma^T(g) \) is the transpose of the anomalous dimension matrix \( \gamma(g) \). With the help of \( dg/d \ln \mu = \beta(g) \), \( U \) obeys the same equation as \( \tilde{C}(\mu) \). We expand \( \gamma(g) \) to the first two
terms in the perturbative expansion,

\[ \gamma(\alpha_s) = \gamma^{(0)} \frac{\alpha_s}{4\pi} + \gamma^{(1)} \left( \frac{\alpha_s}{4\pi} \right)^2. \]  

(21)

To this order the evolution matrix \( U(\mu, m) \) is given by

\[ U(\mu, m) = \left( 1 + \frac{\alpha_s(\mu)}{4\pi} J \right) U^{(0)}(\mu, m) \left( 1 - \frac{\alpha_s(m)}{4\pi} J \right), \]  

(22)

where \( U^{(0)} \) is the evolution matrix in leading logarithmic approximation and the matrix \( J \) expresses the next-to-leading corrections. We have

\[ U^{(0)}(\mu, m) = V \left( \frac{\alpha_s(m)}{\alpha_s(\mu)} \right)^{\frac{\gamma^{(0)}(\mu)}{2\beta_0}} V^{-1}, \]  

(23)

where \( V \) diagonalizes \( \gamma^{(0)T} \), i.e., \( \gamma^{(0)}_D = V^{-1}\gamma^{(0)T}V \), and \( \gamma^{(0)} \) is the vector containing the diagonal elements of the diagonal matrix \( \gamma^{(0)}_D \). In terms of \( G = V^{-1}\gamma^{(1)T}V \) and a matrix \( H \) whose elements are

\[ H_{ij} = \delta_{ij} \gamma^{(0)}_i \frac{\beta_1}{2\beta_0} - \frac{G_{ij}}{2\beta_0 + \gamma^{(0)}_i - \gamma^{(0)}_j}, \]  

(24)

the matrix \( J \) is given by \( J = VH V^{-1} \).

The operators \( O^{LL} \) and \( O^{RR} \) do not mix with others under renormalization. Their Wilson coefficients follow exactly the same RGEs, where the above-mentioned matrices are all simple numbers. The factor

\[ \left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(M_W)}{4\pi} J_5 \right] R^{6/23} \]  

(25)

in Eq. (18) reflects the RGE running. On the other hand, \( O_1^{LR} \) and \( O_2^{LR} \) form a sector that is mixed under RG running. Although the \( Z' \) boson only induces the operator \( O_1^{LR} \) at high energy scales, \( O_2^{LR} \) is generated after evolution down to low energy scales and, in particular, its Wilson coefficient \( C_2^{LR} \) is strongly enhanced by the RG effects [45].

With contributions from both the SM and the \( Z' \) boson with only left-handed FCNC couplings included, the \( B_s \) mass difference is

\[ \Delta M_s = \Delta M_{s}^{SM} \left( 1 + \frac{\Delta M_s^{Z'}}{\Delta M_{s}^{SM}} \right) = 18.0 \left| 1 + 3.858 \times 10^5 \rho^2 L e^{2i\phi_L} \right| \text{ps}^{-1}, \]  

(26)

The corresponding result for the oscillation parameter is

\[ x_s = 26.3 \left| 1 + 3.858 \times 10^5 \rho^2 L e^{2i\phi_L} \right|. \]  

(27)
FIG. 1: Three-dimensional plot of $x_s$ (a) and $\sin 2\phi_s$ (b) versus $\rho_L$ and $\phi_L$ with a $Z'$-mediated FCNC for left-handed $b$ and $s$ quarks. The color shadings in both plots have no specific physical meaning.

With couplings of only one chirality, the physical observables $\Delta M_s$, $x_s$, and $\sin 2\phi_s$ are periodic functions of the new weak phase $\phi_L$ with a period of 180°.

Fig. 1 (a) shows the effects of including a $Z'$ with left-handed coupling. We see that if $\rho_L$ is small, $x_s$ is dominated by the SM contribution and has a value $\sim 26$. For $\phi_L$ around 90° and $\rho_L$ between 0.001 and 0.002, the $Z'$ contribution tends to cancel that of the SM and reduces $x_s$ to be smaller than the SM value of 26.3. In Eq. (27) and Fig. 1(a), we see that the $Z'$ has a comparable contribution to the SM if $\rho_L \gtrsim 0.002$, independent of the actual value of $\phi_L$. The planned resolution of Fermilab Run II and LHCb are both about $x_s \lesssim 75$ [15, 16]. Thus, a $\rho_L$ greater than about 0.003 will result in an $x_s$ beyond the planned sensitivity. If $x_s$ is measured to fall within a range, one can read from the plot what the allowed region is for the chiral $Z'$-model parameters. The same discussion can easily be applied to a $Z'$ model with only right-handed couplings. Fig. 1(b) shows $\sin 2\phi_s$ as a function of $\rho_L$ and $\phi_L$. As $\rho_L$ increases, $\sin 2\phi_s$ goes through more oscillations when $\phi_L$ varies from 0 to $\pi$.

In Fig. 2 we show the overlayed plot of the contours of fixed $x_s$ and those of fixed $\sin 2\phi_s$. The shaded region in the center shows the experimentally excluded points in the $\phi_L-\rho_L$ plane.
FIG. 2: Contour plot of $x_s$ and $\sin 2\phi_s$ with a $Z'$-mediated FCNC for left-handed $b$ and $s$ quarks. The shaded region is for $x_s < 20.6$, which is excluded at 95% CL by experiments. The hatched region corresponds to $1\sigma$ ranges around the SM value $x_s = 26.3 \pm 5.5$ (black curve). The solid curves open to the left are contours for $x_s = 50$ (red), 70 (green) and 90 (blue) from left to right. The curves open to the right are contours for $\sin 2\phi_s = 0.5$ (dotted), $-0.5$ (solid) and the SM value $-0.07 \pm 0.01$ (dashed).

that induce $x_s$ values smaller than 20.6. The hatched area corresponds to the parameter space points that produce $x_s$ values falling within the $1\sigma$ range of the SM value of 26.3. Contours for higher values of $x_s$ are also shown. The SM predicted $\sin 2\phi_s \simeq -0.07 \pm 0.01$ would appear as narrow bands around the $\sin 2\phi_s = -0.07$ curves. Note that even if the $x_s$ measurement turns out to be consistent with the SM expectation, it is still possible that the new physics contributions, such as the $Z'$ model considered here, can alter the $\sin 2\phi_s$ value significantly. It is therefore important to have a clean determination of both quantities simultaneously. Once $x_s$ and $\sin 2\phi_s$ are extracted from $B_s$ decays, one can determine $\rho_L$ up to a two-fold ambiguity and $\phi_L$ up to a four-fold ambiguity, except for the special case with $\sin 2\phi_s \simeq 0$.

$\Delta \Gamma_s$ can be determined with a high sensitivity by measuring the lifetime difference between decays into $CP$-specific states and into flavor-specific states. Using the $J/\psi\phi$ mode, simulations determine [16] that the LHC can measure the ratio $\Delta \Gamma_s/\Gamma_s$ with a relative error
\[ \lesssim 10\% \text{ for an actual value around } -0.15. \] Tevatron simulations show that \( \Delta \Gamma_s/\Gamma_s \) can be measured with a statistical error of \( \sim 0.02 \). For a sufficiently large \( \rho_L \), the cos \( \theta \) factor in Eq. (5) increases from \( -1 \) at \( \phi_L = 0^\circ \) (mod 180°) to the maximum 1 at \( \phi_L = 90^\circ \) (mod 180°).

We are left with the phase ranges \( 0^\circ \lesssim \phi_L \lesssim 30^\circ \), \( 60^\circ \lesssim \phi_L \lesssim 120^\circ \), and \( 150^\circ \lesssim \phi_L \lesssim 180^\circ \) (mod 180°) where a 3σ determination of \( \Delta \Gamma_s \) can be made.

Once the right-handed \( Z' \) couplings are introduced, we also have to include the new \( |\Delta B| = 2 \) operators \( O_1^{LR} \), \( O_2^{LR} \), and \( O^{RR} \) defined in Eq. (8) into the effective Hamiltonian that contributes to the \( B_s \bar{B}_s \) mixing. The matrix element of \( O^{RR} \) is the same as that of \( O^{LL} \), while those of \( O_1^{LR} \) and \( O_2^{LR} \) are

\[
\langle \bar{B}_s | O_1^{LR} | B_s \rangle = -\frac{4}{3} \left( \frac{m_{B_s}}{m_b(m_b) + m_s(m_b)} \right)^2 m_{B_s} f_{B_s}^2 B_1^{LR}(m_b) \tag{28}
\]

\[
\langle \bar{B}_s | O_2^{LR} | B_s \rangle = 2 \left( \frac{m_{B_s}}{m_b(m_b) + m_s(m_b)} \right)^2 m_{B_s} f_{B_s}^2 B_2^{LR}(m_b) \tag{29}
\]

For the \( Z' \) coupling to right-handed currents, we define new parameters \( \rho_R \) and the associated weak phase \( \phi_R \):

\[
\rho_R e^{i\phi_R} \equiv \frac{g_2 M_Z}{g_1 M_{Z'}} B_{sb}^R. \tag{30}
\]

At the \( M_W \) scale, we have additional contributions to the effective Hamiltonian due to the right-handed currents, similar to Eq. (17). The terms due to the left-right mixing are

\[
\mathcal{H}_{\text{eff}}^Z' \supset \frac{G_F}{\sqrt{2}} \rho_L \rho_R e^{i(\phi_L + \phi_R)} (O_1^{LR} + O_1^{RL}, O_2^{LR} + O_2^{RL}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{31}
\]

In the RGE running, the Wilson coefficient of \( O_1^{LR} \) mixes with that of \( O_2^{LR} \); the relevant anomalous dimension matrices are

\[
\gamma^{(0)} = \begin{pmatrix} \frac{6}{N_c} & 12 \\ 0 & -6N_c + \frac{6}{N_c} \end{pmatrix}, \tag{32}
\]

\[
\gamma^{(1)} = \begin{pmatrix} \frac{137}{6} + \frac{15}{2N_c} - \frac{22}{3N_c} f & \frac{200}{3} N_c - \frac{6}{N_c} - \frac{44}{3} f \\ \frac{71}{4} + \frac{9}{N_c} - 2f & -\frac{203}{6} N_c^2 + \frac{479}{6} + \frac{15}{2N_c} + \frac{10}{3} N_c f - \frac{22}{3N_c} f \end{pmatrix}, \tag{33}
\]

where \( N_c \) is the number of colors and \( f \) is the number of active quarks. At the scale of the \( B \) meson masses, the value of \( f \) is 5.

We take \( m_b(m_b) = 4.4 \text{ GeV} \), \( m_s(m_b) = 0.2 \text{ GeV} \), and \( \Lambda_{MS}^{(5)} = 225 \text{ MeV} \). Following
Eqs. (34, 29), we find the effective Hamiltonian terms for the operators $O_{1,2}^{LR}$ at $m_b$ to be

$$
H_{\text{eff}}^{Z'} \supset \frac{G_F}{\sqrt{2}} \rho_R \rho_L e^{i(\phi_L + \phi_R)} (O_1^{LR} + O_1^{RL}, O_2^{LR} + O_2^{RL}) \begin{pmatrix} 0.930 \\ -0.711 \end{pmatrix}.
$$

(34)

Note that there is no contribution of the operator $O_2^{LR}$ at the $M_W$ scale. It is induced through the operator mixing in RGE running and actually has an important effect at the $m_b$ scale, as one can see from its Wilson coefficient in Eq. (33).

In the numerical analysis, we use the central values of $B_1^{LR}(m_b) = 1.753 \pm 0.021$ and $B_2^{LR}(m_b) = 1.162 \pm 0.007$ given in Ref. [41] with the decay constant $f_{B_s}$ the same as before. The predicted mass difference with all the $Z'$ contributions included is then

$$
\Delta M_s = 18.0 \left| 1 + 3.858 \times 10^5 \left( \rho_L e^{2i\phi_L} + \rho_R e^{2i\phi_R} \right) - 2.003 \times 10^6 \rho_L \rho_R e^{i(\phi_L + \phi_R)} \right| \text{ ps}^{-1}.
$$

(35)

The overall contribution to $x_s$ from the SM and $Z'$ is

$$
x_s = 26.3 \left| 1 + 3.858 \times 10^5 \left( \rho_L e^{2i\phi_L} + \rho_R e^{2i\phi_R} \right) - 2.003 \times 10^6 \rho_L \rho_R e^{i(\phi_L + \phi_R)} \right|.
$$

(36)

To illustrate the interference among different contributions, we set $\rho_L = \rho_R = 0.001$ and plot $x_s$ and $\sin 2\phi_s$ versus the weak phases $\phi_L$ and $\phi_R$ in Fig. 3(a) and (b), respectively.

First, we note that after the RGE running the operators $O_1^{LR}$ and $O_2^{LR}$ interfere constructively. This can be seen from the relative minus sign between the Wilson coefficients in Eq. (34) and a corresponding relative minus sign in the hadronic matrix elements given in Eqs. (28) and (29). Because of the constructive interference and the fact that the bag parameters $B_1^{LR}$ and $B_2^{LR}$ are twice as large as $B_1^{LL}$, the LR and RL operators together become the dominant contributions. The interference of all the terms makes $x_s$ reach its maximum when one of the weak phases is $180^\circ$ and the other is $0^\circ$ (mod $360^\circ$). If $\rho_L$ and $\rho_R$ are both much smaller than $10^{-3}$, the variation in $x_s$ in the $\phi_L-\phi_R$ space will be indistinguishable from the predicted SM range. Compared to Fig. 1(a) for $Z'$ with only $LL$ couplings, Fig. 3(a) shows that even for large values of $\rho_L$ and $\rho_R$, $x_s$ can be smaller than 20.6 due to the interference among all the terms in Eq. (36). The current $x_s \geq 20.6$ bound excludes the regions with $\phi_L + \phi_R \approx 0^\circ$ (mod $360^\circ$). Because of the assumed equal values of $\rho_L$ and $\rho_R$, the parameter space points with the same $\sin 2\phi$ output lie along directions that are roughly parallel to the $\phi_L + \phi_R = 360^\circ$ line. For the more general cases of different $\rho_L$ and $\rho_R$ values, the crests and troughs in Fig. 3(b) are no longer parallel to the $\phi_L + \phi_R = 360^\circ$ line.
FIG. 3: $x_s$ (a) and $\sin 2\phi_s$ (b) as functions of $\phi_L$ and $\phi_R$ for $\rho_L = \rho_R = 0.001$. The color shadings have no specific physical meaning.

V. CONCLUSIONS

In this paper we discuss the effects of a $Z'$ gauge boson with FCNC couplings to quarks on the $B_s-\overline{B_s}$ mixing. We show how the mass difference and $CP$ asymmetry are modified by the left-handed and right-handed $b-s-Z'$ couplings that may involve new weak phases $\phi_L$ and $\phi_R$. In the particular case of a left-chiral (right-chiral) $Z'$ model, one can combine the measurements of $\Delta M_s$ (or $x_s$) and $\sin 2\phi_s$ to determine the coupling strength $\rho_L$ ($\rho_R$) and the weak phase $\phi_L$ ($\phi_R$) up to discrete ambiguities. Once comparable left- and right-chiral couplings are considered at the same time, we find the left-right interference terms dominate over the purely left- or right-handed terms, partly because of the renormalization running effects and partly because of the larger bag parameters.

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