Calculation of Dynamic Amplification Factor for Railway Concrete and Masonry Arch Bridges Subjected to High-speed Trains

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Abstract
The dynamic amplification factor (DAF) is one of the most important parameters to express the dynamic behavior of bridges under moving loads. This parameter is used in bridge design codes instead of exhaustive dynamic analyses. Therefore, the DAF and the possible derived relationships can be good alternatives to dynamic analyses because of time and computational cost savings. Masonry arch bridges are complex infrastructures due to their geometry and structural behavior, and it is troublesome to prepare an accurate numerical model. To conduct dynamic analyses, due to their multiplicity, calculating the DAF of the bridges imposed by high-speed trains can lead to a rapid assessment of these old railway arch bridges. For this purpose, in the present study, the finite element models of two concrete and masonry arch bridges with small fill material heights, which are completely different in terms of geometric and mechanical characteristics, were prepared. In the next step, by performing 378 dynamic analyses based on the 27 different train models, the DAF has been computed. The results show that the calculated DAFs are in the rational range, and bogies interval, axles interval, and span length are recognized as the most important parameters in the DAF changes.

Keywords
old railway infrastructures, concrete and masonry arch bridges, dynamic amplification factor, high-speed trains, finite element models

1 Introduction
Masonry arch bridges are one of the most vital structures of Iran's railway network since there are about 3300 masonry arch bridges out of 30000 railway bridges. Many of these railway bridges still in service have been designed back when vehicles' speeds were considerably lower as these structures have a history extending back about 90 years. At present, the masonry arch bridges in Iranian railway network are being serviced for trains with a maximum speed of 80 km/h. Design calculations for nearly all masonry arch bridges were based on simplified analysis considering service loads of that time (low-speed trains). The demand for transportation is generally influenced by factors such as time considerations as the main factor that determines whether or not people choose a certain mode of transportation. As transportation systems have developed over time, the speed and performance of these systems have improved drastically. The development of railway networks for high-speed trains is rapidly growing in many countries around the world, to meet the increasing demand for faster transportation. Because of the importance of passenger and freight moving time and increasing the speed of railways, the study of dynamic loads subjected to high-speed trains is more important than the load due to conventional trains. In the last few decades, the use of high-speed trains has greatly expanded all over the world which also affects Iran. Among the special considerations for high-speed railway bridges, the most relevant one is the limitation of dynamic effects when high-speed trains cross. The dynamic effect of trains on bridges is related to some important dynamic characteristics such as trains' speed. The existing masonry arch bridges cannot be replaced by any means because of time, field, and economic constraints. Hence, extensive dynamic analyses have to be taken for updating the current speed limits due to the increasing demand for railway transportation, thus maintaining the safety of the bridges [1]. So, it is necessary to evaluate the dynamic behavior of these bridges under the effect of high-speed trains.
The study of the dynamic behavior of bridges (deck bridges) imposed by moving vehicles dates back to about a century ago. Of course, the investigation of the behavior of bridges subjected to high-speed trains has begun since the first high-speed train was used in Japan in 1964 [2]. A lot of studies have been dedicated to the investigation of the dynamic behavior of railway bridges in different types including masonry arch bridges imposed by moving trains throughout the world [3–5]. The most recent study carried out by Yazdani and Azimi [6] on masonry arch bridges under the moving high-speed trains has shown more sensitivity of the long span bridge to acceleration response notwithstanding the short span bridge to displacement response.

Although the load-carrying capacity is considered the most significant of the design criteria during the bridge design process, the dynamic behavior is also assessed by applying a parameter known as the dynamic amplification factor (DAF) to represent the bridge’s dynamic characteristics. It is now well-known that all sorts of moving vehicles generate a dynamic impact effect on bridges, namely, the increment from the static load effect. Usually, they are indicated by the DAF introduced in numerous research studies and design codes. The DAF, defined as the ratio of the dynamic increment to the corresponding static response, has been used to describe the dynamic effect of moving vehicles on bridge structures [7]. The DAF is an important parameter in the design procedure of both railway and highway bridges and shall be taken into account by the evaluation of the existing bridge load-carrying capacity as well. To investigate the behavior of highway bridges under vehicles, numerous studies have been performed on determining the DAFs [8–12]. The significant role of road surface roughness in calculating DAFs, not enough evidence for a correlation between material characteristics and DAFs, and the relationship between DAFs and vehicle’s speed and weight are numbers of the foremost conclusions which have been drawn [13].

DAF plays a vital role in the practice of railway bridge condition assessment. One of the most recent studies on the dynamic amplification factors (DAFs) of high-speed railway continuous bridges has revealed that the experimental formulas in the Japan Railway Technical Research Institute (JRTRI) code could make a conservative estimation of the dynamic amplification factors of high-speed railway continuous bridges [14]. Accurate evaluation of the factor will provide valuable information for enhancement railway bridges codes. A study on Korean railway steel plate girder bridges aimed to determine the DAFs for fatigue investigation of the bridges has shown that the Korean code overestimates the DAF [15]. Furthermore, there are multiple research projects on different types of railway bridges under trains considering the DAF values [16–18]. According to these studies, different parameters affect the dynamic behavior and DAFs of bridges. Accordingly, a high impact effect has been found at shorter side spans and much lower around the longer spans in a bridge [17]. The insignificant effect of track geometry on the dynamic response of bridge [19], a regular interval of axles at the critical speeds which leads to the strong DAFs [1] and higher correlation between the rise to span ratio and the DAFs in comparison with span length and the DAFs [16] are some of the correlations between parameters. Another study that has been engaged in specifying the effects of various parameters on DAFs has shown that, however, an increase in train speed causes an increase in DAF, greater axle distance to span length ratios generate the smaller DAFs [20].

The dynamic effect has certainly sparked another worldwide debate about how to ensure the efficiency of existing bridges subjected to high-speed trains. Hence, to get a better understanding of the structural behavior of bridges under high-speed trains, and because the bridges are subjected to high impacts under loads of high-speed trains [21], it is necessary to consider the DAF. So, many authors have dealt with the DAF of bridges under high-speed trains in different countries. It has been concluded in an experimental and theoretical project which has been performed on the bridges crossed by the Korean high-speed train that, the DAFs depend on the number of coaches and the intensity of the loading [22]. As another paper’s outcome it is notable that, softer ballast tends to reduce the impact response of the bridges, although the degree of reduction is only marginal [23]. The DAFs have been more affected by the increase in speed for the shortest span, whereas they remained almost constant when the velocity increased to the particular value for the longest spans [24].

In the majority of previous research conducted on the determination of DAFs of bridges, as a common part of the numerical studies, simply supported and continuous beams have been frequently used [25, 26]. On account of the inherent complexity of masonry arch bridges, no study has been reported on the DAF of masonry arch bridges from moving high-speed trains yet. This has made it increasingly important to authors to estimate the dynamic effects of high-speed trains passage on the serviceability of the existing bridge accurately.
**2 Characteristics of investigated high-speed trains**

In the present study, a combination of real trains along with the European standard train load models [27] designated HSLM-A and HSLM-B are imposed on railway bridges. These classifications have been intended to evaluate the dynamic behavior of the bridges applied by high-speed trains using the DAF parameters. The researchers made an effort to perform a comprehensive study of the dynamic behavior of the bridges applied by high-speed trains by extracting a thorough range of load classifications.

Some loading models are derived from real trains configuration currently in service and other loading models are proposed by European standards [27]. The loading models consist of mitigated representations in the form of different sets of concentrated loads. Real train models, types 1–10, represent high-speed trains running on railways in the developed countries of Asia and Europe. In addition, 17 train models which are called Universal Trains comprised of two independent families identified as HSLM-A and HSLM-B.

The procedure of choosing the high-speed trains (both real trains and Universal train models) have been performed with a particular attitude towards maintaining trains’ characteristics. Coach distributions, spacing of axles, and length of the train are some of the mentioned attributes in the current study. The characteristics of real high-speed trains are sorted as No. 1 to 10 and displayed in Table 1. As it is shown in Table 1, B4 and A2 are designated as the shortest and the longest trains of the HSLM category, respectively. Also, Renfe S104 and SKS are investigated as the shortest and the longest real trains, respectively.

| Table 1 | Characteristic of Asian, European and Universal trains |
|---------|---------------------------------------------------|
| **Train number** | **Train name** | **Country** | **Number of axles** | **Total length (m)** | **Average load of axles (kN)** | **Total load (kN)** |
| 1 | RENFE S100 | Spain | 26 | 192.6 | 165 | 4290 |
| 2 | RENFE S102 | Spain | 21 | 191 | 166 | 3486 |
| 3 | RENFE S103 | Spain | 32 | 193.5 | 145 | 4640 |
| 4 | RENFE S104 | Spain | 16 | 99.4 | 153 | 2448 |
| 5 | RENFE S130 | Spain | 20 | 176.3 | 175 | 3500 |
| 6 | China-Star | China | 44 | 263 | 150 | 6600 |
| 7 | KHST | Korea | 46 | 380.2 | 170 | 7820 |
| 8 | SKS | Japan | 64 | 395 | 137.4 | 8793.6 |
| 9 | TGV | France | 26 | 193.2 | 168 | 4368 |
| 10 | ICE | Germany | 48 | 292.7 | 152 | 7296 |
| 11 | HSLM-A1 | Universal Train | 50 | 397.4 | 170 | 8500 |
| 12 | HSLM-A2 | Universal Train | 48 | 398.5 | 200 | 9600 |
| 13 | HSLM-A3 | Universal Train | 46 | 397.4 | 180 | 8280 |
| 14 | HSLM-A4 | Universal Train | 44 | 394.4 | 190 | 8360 |
| 15 | HSLM-A5 | Universal Train | 42 | 389.4 | 170 | 7140 |
| 16 | HSLM-A6 | Universal Train | 40 | 382.4 | 180 | 7200 |
| 17 | HSLM-A7 | Universal Train | 40 | 397.4 | 190 | 7600 |
| 18 | HSLM-A8 | Universal Train | 38 | 387.5 | 190 | 7220 |
| 19 | HSLM-A9 | Universal Train | 36 | 375.4 | 210 | 7560 |
| 20 | HSLM-A10 | Universal Train | 36 | 388.4 | 210 | 7560 |
| 21 | HSLM-B1 | Universal Train | 8 | 24.5 | 170 | 1360 |
| 22 | HSLM-B2 | Universal Train | 10 | 36 | 170 | 1700 |
| 23 | HSLM-B3 | Universal Train | 15 | 70 | 170 | 2550 |
| 24 | HSLM-B4 | Universal Train | 3 | 5 | 170 | 510 |
| 25 | HSLM-B5 | Universal Train | 20 | 114 | 170 | 3400 |
| 26 | HSLM-B6 | Universal Train | 7 | 19.8 | 170 | 1190 |
| 27 | HSLM-B7 | Universal Train | 5 | 12 | 170 | 850 |
As it is depicted in Fig. 1, these 10 train models differ in total length, coach length, number of intermediate passenger coaches, axles and bogies interval, and axle loads. No. 1 to 5 are related to Spain high-speed trains (Renfe S100 to Renfe S130) [28], No. 6 refers to China high-speed train (China Star) [29], No. 7 signifies South Korea high-speed train (KHST) [22], No. 8 symbolizes Japan high-speed train (SKS) [30], No. 9 mentions France high-speed train (TGV) [31], and No. 10 is related to Germany high-speed train (ICE) [32].
The train load models in Eurocode are a theoretical idealization of high-speed trains. According to Eurocode, a dynamic analysis must be performed using the train load characteristic values of the high-speed HSLM load model. The HSLM consists of different train load configurations, with different characteristic axle loads and intervals between axles. These two load models also differ in coach length, the number of intermediate passenger coaches, axle spacing of bogies, and axle loads limited by the range of 170 and 210 kN. HSLM-B load models represent the concentrated loads spacing uniform axle load of 170 kN, and the number of axles is related to the bridge span length. High-speed traffic models of HSLM-A and HSLM-B represent the dynamic load effects of articulated, conventional, and regular high-speed passenger trains. Numbers 11–27 represent all universal trains A1-A10 and B1–B7 (Table 1). As shown, HSLM-B4 and HSLM-A2 are the shortest trains, and the longest ones are represented by number 12 and number 24 with 5 and 398.5 meters long, respectively. Given the different geometries of real high-speed trains and Eurocode HSLM, a total of 27 distinct train geometries are gathered to pursue a comprehensive and detailed study on the behavior of bridges under high-speed trains. The multi-axle-moving-force models are used to perform dynamic analyses.

Research studies have shown that increasing details of vehicle modeling is only impressive in the resolution of calculating vehicle responses and has no remarkable effect on bridge responses [3]. Therefore, it is reasonable to use fewer details to simulate the moving vehicular loads since the aim is to investigate the behavior of bridges in this study. Therefore, concerning the purpose of the present study, investigating the bridges’ responses, the trains axles were crossed over the bridges as concentrated moving forces at certain distances [3]. The intended speeds of the moving loads in the study are limited to 75 and 400 km/h, respectively 20.8 and 111.1 m/s.

Vehicle type and characteristics are vital parameters in the realistic prediction of bridge load-carrying capacity and its dynamic response. While many different models (some very complicated) have been provided before, it is generally agreed that, in the face of complex random loading, none can meet all conceivable probabilities. Using Eurocode HSLM, the dynamic behavior of masonry arch bridges may not be properly assessed. So, an effort has been made to analyze the behavior of these bridges under real high-speed trains to tackle the results with more reliance.

3 DAF description
The DAF is an applicable term to design and analyze the dynamic behavior of bridges. As the response of bridges caused by static vehicular loads increases, the dynamic behavior of structures is considered.

The term DAF increases static loads by applying them then takes the effects of trains’ dynamic loads into consideration. This method has focused on different types of bridges, including highway and railway bridges of guidelines.

The DAF is calculated by employing the values of the dynamic and static responses (deflections, bending moments, or shear forces) of bridges [1]. Earlier studies indicated that the DAF obtained by taking the deflection in the midspan achieved equivalence to the values based on strain [33, 34]. According to the analyses performed in the present study and the outputs obtained, in the current study, the dynamic load effects on the bridge are measured in terms of the maximum dynamic and static deflections. The DAF is described using the allowable dynamic load allowance (DLA) based on the maximum values of dynamic and static responses in the midspan of the bridges and given by Eq. (1) [7]:

\[
DLA = \frac{D_{dyn} - D_{sta}}{D_{sta}} \text{ or } DLA + 1 = \frac{D_{dyn}}{D_{sta}} = DAF,
\]

where \(D_{dyn}\) and \(D_{sta}\) denote, respectively the maximum dynamic and static deflections, in mm, of the bridges calculated in the midspan.

4 Guidelines suggestions for railway bridges
This section describes guideline considerations for computing DAF of railway bridges in regular and high-speed conditions.

4.1 Regular speed vehicles
The DAF equations of different railway bridge standards are described based on diverse main variables [27, 35, 36]. Train speed, bridge type, span length, and the first vibration frequency are some of the mentioned parameters. As an impressive parameter on the dynamic response of bridges, span length \((L)\) has been distinguished in most current bridge design codes. American railway engineering and maintenance of way (AREMA) has defined the DAF as follows [36]:

\[
DAF = \begin{cases} 
\left(40 - \frac{3L^2}{148.6}\right) / 100 & L \leq 24 \\
\left(16 + \frac{182.9}{L - 9.1}\right) / 100 & L > 24 
\end{cases}
\]
Eurocode suggests following relationships for DAF to be calculated in simplified method [27]:

$$DAF = \begin{cases} 
\frac{2.16}{L_{\phi}} + 0.73 & \text{standard maintenance} \\
\frac{1.44}{L_{\phi}} + 0.82 & \text{carefully maintained tracks} 
\end{cases} \quad (3)$$

where $L_{\phi}$ is in meters and the determinant length is equivalent to twice the clear opening for masonry arch bridges.

Iranian standard loads for bridges utilizes two DAF equations for rail application depending on the track maintenance condition as well as the simplified method of Eurocode (Eq. (3)). Where $L_{\phi}$ is in meters and the determinant length is equivalent to half span.

Train speed and first vibration frequency are also main parameters on the dynamic response identified in some other codes. Eurocode detailed method (Eq. (4) to Eq. (8)), and Ontario railway regulations (ORE) (Eq. (9)) fall into this category based on DAFs below [27]:

$$DAF = \begin{cases} 
1 + \phi' + \phi'' & \text{standard maintenance} \\
1 + \phi' + 0.5\phi'' & \text{carefully maintained tracks} 
\end{cases} \quad (4)$$

where $\phi'$ and $\phi''$ are computed by Eq. (5).

$$DAF = \begin{cases} 
\phi' = \frac{K}{1 - K + K^2} & K < 0.76 \\
\phi' = 1.325 & K \geq 0.76 
\end{cases} \quad (5)$$

where

$$K = \frac{v}{2L_{f}n_0}, \quad (6)$$

and

$$\phi'' = \frac{\alpha}{100} \left[ 56e^{\frac{L_{\phi}}{60}} + 50(\frac{L_{\phi}n_0}{80} - 1)e^{\frac{L_{\phi}}{60}} \right], \quad (7)$$

with

$$\begin{align*}
\alpha &= \frac{v}{22} & v \leq 22 \text{ (m/s)} \\
\alpha &= 1 & v > 22 \text{ (m/s)} 
\end{align*} \quad (8)$$

In the aforementioned equations, $v$ denotes train speed in m/s, $n_0$ is the first bending frequency of the structure, $L_{\phi}$ is the determinant length in m, and $\alpha$ is train speed coefficient.

$\phi'$ maintains the rate of loading due to the traffic load speed. It also covers the effects of the passage of successive loads which may excite the structure and cause resonance. $\phi''$ covers the effects of variations in wheel loads resulting from track or vehicle imperfections.

ORE has the same perspective on main parameters on extracted dynamic response as Eurocode does (Eq. (9), Eq. (10)) [35].

$$DAF = \frac{0.65K}{1 - K + K^2}, \quad (9)$$

where

$$K = \frac{v}{2L_{f}}, \quad (10)$$

where $v$ denotes vehicle's speed in m/s, $f$ is the first bending frequency of the structure, and $L$ is span length in m.

4.2 High-speed vehicles

DAF equations used on old railway bridges or highway bridges cannot be used for high-speed railway bridges. Railway regulations of various countries, including Eurocode, consist of specific rules for high speeds on railways and some of which have been represented in Eq. (4) to Eq. (8). The dynamic responses of bridges including stress, deflection, and acceleration caused by high-speed moving loads are greater than low-speed ones [6]. So, the dynamic performance of bridges under high-speed dynamic loads may become more complicated. Furthermore, modern high-speed trains pose serious challenges to bridge design and maintenance in terms of the dynamic load they have to be able to bear. Therefore, there is a great demand for keeping structures subjected to moving loads reliable by considering the effects caused by them.

Hence, more expanded analyses and experimental studies should be performed about it. Moreover, the dynamic response of masonry arch bridges to high-speed trains traveling on these structures can be scientifically stepped into the spotlight through this study. Accordingly, we try to clarify the correlation between the bridges’ dynamic behavior and impressive parameters, comprising bridge length, number of spans, and train length, by calculating the DAF.

There are not several standards giving serious consideration to the debate about calculating the DAF of bridges under high-speed trains. According to Eurocode, For structures imposed by dynamic loads at speeds over 200 km/h, the aforementioned dynamic amplification factor may be insufficient and may need to be determined by a dynamic analysis [27]. So, the dynamic analysis shall be undertaken using characteristic values of the loading from HSLM and the specific Real Trains [27]. However, Eq. (4) to Eq. (8), regarding the Eurocode detailed method, as mentioned before, state how DAF to be calculated at speeds higher than 80 km/h [27]. Iranian regulations for design and inspection of the high-speed railway network,
propose DAFs maintaining track condition and vehicle’s speed. The DAF is equivalent to 1.5 for speeds below 200 km/h and 1.75 for speeds over 200 km/h.

Nevertheless, accurate estimations of DAFs of bridges under high-speed trains, to some extent, have not gone under the heading of the code provisions.

4.3 Guidelines for masonry arch bridges
A common practice of analyzing railway bridges exerted by dynamic effects of a moving train is to apply the DAF defined by design codes. It is shown that the dynamic responses of bridges are expressed by DAF, and the vertical acceleration parameter of the vehicle is used to evaluate passenger comfort. In recent years, extensive studies have been carried out to validate the regulatory equations and compare them with the results of field and numerical results. Despite all the inherent advantages of masonry arch bridges, they create challenging problems for engineers in assessing their DAF for evaluation of the dynamic behavior. Thus, while masonry arch bridges have formed a large number of existing railway bridges, the design codes are rather focused on the rest of the bridge stock of the railway network. Most research studies have concentrated on developing the dynamic behavior assessment of RC and steel railway bridges.

There are a few experimental and analytical research papers performed on investigating the correlation between DAF and train-masonry arch bridges characteristics [3, 16]. However, according to the international union of railways (UIC), the most reliable relations regarding DAF have been expressed for masonry arch bridges in UIC Code [37]. Nevertheless, this issue needs to be further addressed.

5 Numerical modeling
Owing to the deterioration of masonry and the complexity of details, numerical simulation is necessary to evaluate the dynamic behavior of masonry arch bridges. The discrete element method (DEM) and finite element approach (FEM) have been widely used for numerical simulation of the masonry arch bridges in both the macro-modeling and the micro-modeling methods [38–42].

5.1 Finite element analysis
In the present study, the finite element method and macro modeling approach are used as efficient methods to prepare a numerical model. Different components of the structures, including arches, abutment, spandrel, and wing walls, are simulated in accordance with the real condition of the bridges and concerning details. Also, according to the in-plane behavior of masonry arch bridges under vertical load, plane strain analysis is used in the simulation process. The investigated structures are two old bridges (2L20 and 5L06 bridges) depicted in Fig. 2, and their geometric characteristics are presented in Table 2.

The modeling procedure consisted of two parts related to the bridges simulated through the finite element method and the moving load method used to investigate the dynamic effect of the moving trains on the structures. The results of several research papers show that increasing the detail of vehicle modeling is only effective in boosting the accuracy of calculating vehicle responses and has no significant effect on bridge responses. One of these cases is a recent study on the dynamic behavior of masonry arched

![Image](a)

![Image](b)

**Fig. 2** Geometric characteristics of bridges: (a) 2L20 and (b) 5L06 (units are in meter)

| Bridge | No of spans | Span's length (m) | Shape of arch | Thickness of crown (m) | Thickness of arch ends (m) | Arch width (m) | Bridge height (m) | Thickness of spandrel walls (m) |
|--------|-------------|------------------|---------------|-----------------------|---------------------------|----------------|------------------|-------------------------------|
| Km-23  | 2           | 20               | Segment of circle | 1                     | 1.9                       | 3.9            | 12               | 1                             |
| Km-24  | 5           | 6                | Half circle    | 0.7                   | 1.1                       | 3.9            | 8                | 1                             |

Table 2 Geometric characteristic of the bridges
Therefore, since the present study intends to investigate bridges’ responses, it is reasonable to use fewer details to simulate vehicles. Not only does this approach increase the accuracy of the results, but it also reduces computational costs. Accordingly, in the present study, a moving load model is used to simulate the train load crossing the bridges instead of the moving mass model.

Finally, due to the principal two-dimensional behavior displayed by masonry arch bridges under vertical loads and according to Fig. 3, the finite element models have been shown. In the present numerical model, four-node and high-order eight-node elements are utilized. In total, a finite element model of 7505 elements and 16129 nodes (equivalent to 33348 degrees of freedom) is formed for the 5L06 bridge.

The bridges were both built more than 80 years ago, and all parts, including arches, wing walls, spandrels, piers, and foundations are constructed using unreinforced concrete. By taking cylindrical cores from different parts of the bridges, the quality of concrete is determined, and the mechanical properties of the plain concrete are extracted as a result of the test and reported by Marefat et al. [43, 44].

The present study focuses on arches with small fill material heights, because test data show that there is a considerable reduction in the dynamic amplification factor for arch bridges with depths of cover greater than 1. During the bridge simulation process, the materials were assumed to behave non-linearly, and the Drucker-Prager yielding criterion has been implemented to predict possible failures. For this purpose, the mechanical properties of the materials are considered according to Table 3. It is noteworthy that the equation has been used to calculate the tensile strength of concrete in accordance with their compressive strength, and all their nonlinear characteristics have been obtained based on this assumption.

### 5.2 Verification

In order to validate the presented models, static analysis, modal analysis, and dynamic analysis have been conducted, and the results have been compared with the field test outputs. The static experiment was carried out using the weights of 40 kN on the right span of the 2L20 bridge and the midspan of the 5L06 bridge. The static test continued until the load of 7280 kN was applied on the 2L20 bridge and 5000 kN on the 5L06 bridge. The vertical displacement of the bridges was recorded at the crowns under the effect of these loads [44, 45]. Fig. 4 displays the results of nonlinear static analysis of the numerical model compared with the field test results reported by Marefat et al. [43, 44]. In the second step of validation of the proposed models, using modal analysis, the three fundamental frequencies of the numerical model are calculated and compared with the experimental results in Table 4.

According to Fig. 5, the error between non-linear static analysis and the experimental results of the 2L20 and 5L06 bridges equals 8 and 1.5 percent, respectively. Table 4 provides a summarized comparison between the gained frequencies of the first three modes from the natural frequencies of the bridges calculated based on experimental test and finite element method. The average error between the modal analysis of the numerical model and the field test results of 2L20 and 5L06 is equal to 7.4 and 5.9 percent, respectively.

### Table 3 Final properties of materials after calibration

| Item            | Density (kg/m³) | Modulus of elasticity (GPa) | Poisson ratio | Cohesion (KPa) | Friction angle (degree) | Dilatancy angle (degree) |
|-----------------|-----------------|-----------------------------|--------------|----------------|-------------------------|--------------------------|
| Bridge 2L20     | 2380            | 23.00                       | 0.27         | 1210           | 47.5                    | 47.5                     |
| Bridge 5L06     | 2250            | 22.00                       | 0.26         | 1180           | 46.5                    | 46.5                     |
| Fill-material   | 2280            | 22.00                       | 0.27         | 1210           | 47.5                    | 47.5                     |
| Arch            | 2290            | 23.00                       | 0.28         | 1220           | 48.0                    | 48.0                     |
| Ballast         | 2300            | 20.00                       | 0.25         | 1000           | 50.0                    | 50.0                     |

Fig. 3 Finite element model of the bridges: (a) 2L20, (b) 5L06
Finally, in the last calibration level, using non-linear dynamic analysis, the six-axles locomotive, shown in Fig. 5, was passed on the numerical models of 2L20 and 5L06 at a speed of 60 km/h and 80 km/h. The dynamic analysis is conducted using time-history method along with Rayleigh’s damping model implemented to consider the damping ratio. Finally, the time-history dynamic analyses were carried out by implementing the Newmark-Beta technique, and its results are presented in Fig. 6. It is valuable to say that the maximum response is the fundamental parameter in comparison between experimental test and numerical model results, so according to the results obtained from Fig. 6, the error between the maximum numerical model response and the diesel moving load tests of the 2L20 and 5L06 are equal to 1 and 1.4 percent, respectively. It should be noted that several references have reported that the damping ratio of masonry arch bridges is between 1% and 10%, in which the value decreases with increasing the span length [46]. Therefore, for the 2L20 bridge, a damping ratio of 2% and for the 5L06 bridge, a damping ratio of 5% is assumed so that the maximum displacement values are the same in the numerical and experimental outcomes. According to the results obtained from the numerical model, it can be inferred that the numerical model has been successfully validated and it can be concluded that the presented finite element model has the ability to display other dynamic characteristics of the structure.

5.3 Sensitivity analysis

Five key parameters like train length ($L_t$), bogies intervals ($I_b$), axles intervals ($I_a$), bridge length ($L_b$), and span length ($L_s$) were defined to evaluate the DAFs. A sensitivity analysis was conducted first to investigate the effect of
these parameters on the dynamic response of the prototype bridges. Then, four critical concepts included train length to bridge length ratio ($L_t / L_b$), train length to span length ratio ($L_t / L_s$), bogies intervals to span length ratio ($I_b / L_s$), and axles intervals to span ratio ($I_a / L_s$) were extracted. The sensitivity study revealed that the change in the parameters has a significant effect on the DAFs. In the first step, a comprehensive sensitivity analysis is designed and performed to evaluate the effects of the aforementioned parameters covering different classifications of realistic masonry arch bridges in terms of bridge span length which exist nowadays in conventional railway lines. To this end, a couple of masonry arch bridges, with span lengths ranging from 6 to 20 meters, different total length of bridges, and a different number of spans is opted for this study.

Due to the increasing use of high-speed trains, studies concerning the dynamic behavior of high-speed railway bridges have developed recently. Accordingly, although, numerous studies have been conducted to investigate the dynamic behavior of railway bridges under the effect of vehicles traveling at high speeds, no consensus on this subject has been reached yet and many previous studies have been led to different findings. In the second step, the outputs of the presented equations in the regulations can be compared with the values obtained from the sensitivity analyses. Besides, the possibility of using the DAF method for masonry arch bridges can be studied as an alternative method. To do so, the correlation between DAF values and the aforementioned effective parameters on the dynamic behavior of the bridges is depicted in Fig. 7. As it is shown, Fig. 7 displays the effect of the different parameters on DAFs at any speed value due to both real trains and HSLM trains distinctively. Since the consideration of higher speeds does not seem essential, the considered speeds of the moving loads in the present study are restricted by the range of 75 and 400 km/h, corresponding to 20.8 and 111.1 m/s. So, Fig. 7 contains 4 scatter charts in which 378 values of DAF are represented.

Fig. 8 depicts the maximum DAF values of the bridges exerted by real trains and HSLM at every speed. So, it consists of 4 charts with 54 symbols to show how the considered parameters affect the peak values changes. Plus, the peak values of the DAFs are sorted by the trains' categories in Fig. 8.

6 Discussion
The following results can be concluded based on the DAF values of the numerical models displayed in Figs. 7 and 8:

![Fig. 7](image_url)
The minimum and the maximum DAFs are equal to 1 and 3.2, respectively. The maximum DAF value of the bridges is obtained under A10 of the HSLM category, whereas the minimum DAF is calculated under the number of real and Eurocode high-speed trains (See Fig. 7). The peak DAFs of the bridges exposed by the moving real high-speed trains are between 1 and 2.5. As HSLM, type A is larger than type B, the HSLM-A family has a bigger quantity in train length to bridge length ratio and the HSLM-A family reaches the higher DAFs as well. The most critical condition takes place when train length to bridge length ratio is equal to 6.6. As the parameter increases from this value, fewer numbers of DAF are displayed. The minimum value of train length to bridge length ratio, in the HSLM category, is 0.1 related to B4, and the maximum value is 9.2 which is related to A7. These values in real trains are 1.7 and 9.1 produced by Renfe S104 and ICE, respectively. As it is evident, train length to bridge length ratio is restricted by the range of 1.7 to 9.16 for real trains and 0.1 to 9.2 for HSLM. It shows that Eurocode predicts a desirable range of DAF and train length to bridge length ratio parameter which contains any type of real trains even though real trains do not represent a specific trend (See Fig. 7).

The minimum value of train length to span length ratio, in the HSLM category, equals 0.25 related to B4, and the maximum value is 66.2 which is related to A7. These parameters in real trains are 5 and 65.8 related to Renfe S104 and ICE, respectively. Despite the increase in train length to span length ratio, there is no specific visible pattern in DAF changes of real trains. HSLM-A family has larger amounts of train length to span length ratio than HSLM-B due to the larger train models and type A also has a bigger quantity of DAF. It is worth mentioning that Eurocode provides acceptable values comparing real trains in terms of train length to span length ratio parameter. According to the peak values of DAF, it is deduced that when train length to span length ratio is equal to 19.4, the maximum DAF is produced. By passing the maximum DAF and with increasing train length to span length ratio the DAF values decrease considerably (See Fig. 7).
• The minimum value of bogies interval to span length ratio, in the HSLM category, is equal to 0.12 related to B4, and the maximum value is 4.5 related to A10. These parameters in real trains equals 0.7 and 3.2 related to Renfe S130 and Renfe S104, respectively. It is interpretable that Eurocode overestimates this parameter according to the ratios obtained from real trains in comparison with HSLM. When bogies interval to span length ratio equals 1.3, DAF reaches 3.2 as its maximum value. From this point on, the DAFs decrease with increasing bogies interval to span length ratio (See Fig. 7).

• Despite the aforementioned parameters, axles interval to span length ratio represents the different pattern. Bogies interval, however, has a directed correlation with bridge length, although axles interval and bridge length are inversely correlated (See Fig. 7). The most critical condition happens when axles length to span length ratio is equal to 0.1 and the DAFs decrease with increasing axles interval to span length ratio. With increasing axles interval to span length ratio, the DAF values tend to 1.

• Based on the obtained responses as DAF values in the present study, the median of these data is equal to 1.089, their average equals 1.168, and the standard deviation is equal to 0.246. The sum of the mean and the standard deviation values of these data shows the number of 1.414 and 90% of these indicated dynamic amplification factors are less than 1.414.

• Due to the study reports in numerous cases on the dynamic behavior of railway bridges, 99.7% of all DAF values vary from 1 to 2.6. With respect to the obtained responses as DAF values in the study, the maximum DAF value of the bridges subjected to Renfe S100 equals 1.64. This parameter shows different values under real high-speed trains equal to 1.84, 1.7, 2.54, 1.82, 2.4, 1.54, 1.84, 1.62, and 1.97 produced by Renfe S102, Renfe S103, Renfe S104, Renfe S130, China Star, KHST, SKS, TGV, and ICE, respectively (See Fig. 8). According to obtained DAF values, KHST has the lowest peak DAF, and Renfe S104 has the highest peak DAF among the real high-speed trains.

• More than 50% of the obtained DAFs are between 1 and 1.1, and less than 8% of the DAF values are more than 1.5. The frequency of DAF was found to reduce with the increase of the values of this parameter. Accordingly, the frequency of DAF values more than 2.1 equals 5. KHST is chosen as a train with the best performance because it produces the lowest peak DAF value when the train travelled on the 2L20 at 300 km/h and Renfe S104 as a train with the worst performance in this case because of the highest DAF occurred when it travelled on the 2L20 at 400 km/h.

• Based on the calculated DAFs in the present study, and according to Fig. 7, there are 378 indicated values of DAF in each scatter chart, and 92% of these values have dynamic amplification factors of less than 1.5. The median of these data is equal to 1.089, their average equals 1.168, and the standard deviation is equal to 0.246. The sum of the mean and the standard deviation values of these data shows the number of 1.414, and 90% of these indicated dynamic amplification factors are less than 1.414.

• There is no apparent relation between the speed and DAF values. So, DAF does not represent a constant direct or inverse correlation with the speed of moving vehicles.

• The DAFs of the 2L20 have higher values in comparison with the corresponding DAF values of the 5L06 at the same speeds. Accordingly, it is understood that the DAFs are directly correlated with the bridge length.

• As it is depicted in Fig. 8, bogies interval to span ratio and axles interval to span ratio against DAF, display specific smooth trend while two other parameters do not tend to show any particular trend in resulting charts.

• According to the maximum DAF values charts shown in Fig. 8, it is deduced that bogies interval, axles interval, and span length play a crucial role in calculating DAFs while train length and bridge length are not such significant parameters as Eurocode considers axles interval as one of the most important parameters in terms of the dynamic behavior of railway bridges exposed to the movement of trains.

• According to the considerations, the obtained DAF values of HSLM (type A) have a bigger quantity than real trains, thus representing the proper intended safety factor of Eurocode.

• Fig. 9 shows the trend of the maximum DAF values via axles interval to span ratio. It is indicated that the approximated function matches 44 out of 50 data entries.
7 Conclusions

There are about 3300 masonry arch bridges out of 30000 railway bridges in Iran. The existing masonry arch bridges cannot be replaced because of time, field, and economic constraints. That is why dynamic analyses have to be performed to determine the safety and reliability of these bridges. On the other hand, because of the increasing demand for reducing traffic time, higher speeds on railway networks lead us to the maximum use of the existing structures. So, it is necessary to evaluate the dynamic behavior of these bridges under the effect of high-speed trains. Numerical models can be very expensive in relation to practical use in engineering. Hence, in order to be able to perform calculations in a satisfactory amount of time, the simplicity of the model and the size of the model are important considerations. As it is discussed, bogies interval, axles interval, and span length are recognized as the most important parameters in the DAFs changes. Almost all of the determined DAFs vary within 1 and 2.6, however, a majority of these values are less than 1.1 and few cases exceed a value of 1.7. Based on the indicated data in this research, the median of DAFs is equal to 1.089, the average equals 1.168, and the standard deviation is 0.246. The sum of the mean and the standard deviation values of these data shows the number of 1.414, and 90% of these DAF values are less than 1.414. Due to the limit, KHST has the best performance among all the investigated trains and Renfe S104 displays the worst performance.

It is necessary to mention that, there is no high-speed train in Iran right now. So, these conclusions based on mentioned responses can be appropriate but, it seems by implementation some consideration the behavior of masonry arch bridges under high-speed trains in railway networks may be acceptable, and it is recommended to use the Korean high-speed train in Iran railway network. Since the 2L20 and 5L06 bridges are settled in the medium-sized bridges category, as a conclusion, KHST can be deduced as the best high-speed train in terms of the dynamic amplification factor to impose on these bridges. Also, it can be concluded that the maximum DAF values via axles interval to span ratio is the most important parameter for the dynamic assessment of railway masonry arch bridges under high-speed trains.

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