Multi-Graph Transformer for Free-Hand Sketch Recognition

Peng Xu 1  Chaitanya K. Joshi 1  Xavier Bresson 1

Abstract

Learning meaningful representations of free-hand sketches remains a challenging task given the signal sparsity and the high-level abstraction of sketches. Existing techniques have focused on exploiting either the static nature of sketches with Convolutional Neural Networks (CNNs) or the temporal sequential property with Recurrent Neural Networks (RNNs). In this work, we propose a new representation of sketches as multiple sparsely connected graphs. We design a novel Graph Neural Network (GNN), the Multi-Graph Transformer (MGT), for learning representations of sketches from multiple graphs which simultaneously capture global and local geometric stroke structures, as well as temporal information. We report extensive numerical experiments on a sketch recognition task to demonstrate the performance of the proposed approach. Particularly, MGT applied on 414k sketches from Google QuickDraw: (i) achieves small recognition gap to the CNN-based performance upper bound (72.80% vs. 74.22%), and (ii) outperforms all RNN-based models by a significant margin.

To the best of our knowledge, this is the first work proposing to represent sketches as graphs and apply GNNs for sketch recognition. Code and trained models are available at https://github.com/PengBoXiangShang/multigraph_transformer.

1. Introduction

Free-hand sketches are drawings made without the use of any instruments. Sketches are different from traditional images: they are formed of temporal sequences of strokes (Ha & Eck, 2018; Xu et al., 2018), while images are static collections of pixels with dense color and texture patterns. Sketches capture high-level abstraction of visual objects with very sparse information compared to regular images, which makes the modelling of sketches unique and challenging.

The modern prevalence of touchscreen devices has led to a flourishing of sketch-related applications in recent years, including sketch recognition (Liu et al., 2019; Sarvadevabhatla et al., 2016), sketch scene understanding (Ye et al., 2016), sketch hashing (Xu et al., 2018), sketch-based image retrieval (Sangkloy et al., 2016; Liu et al., 2017; Shen et al., 2018; Collomosse et al., 2019; Dutta & Akata, 2019; Dey et al., 2019), and sketch-related generative models (Ha & Eck, 2018; Chen & Hays, 2018; Lu et al., 2018; Liu et al., 2019).

If we assume sketches to be 2D static images, CNNs can be directly applied to sketches, such as “Sketch-a-Net” (Yu et al., 2015). If we now suppose that sketches are ordered sequences of point coordinates, then RNNs can be used to recursively capture the temporal information, e.g., “SketchRNN” (Ha & Eck, 2018).

In this work, we introduce a new representation of sketches...
with *graphs*. We assume that sketches are sets of curves and strokes, which are discretized by a set of points representing the graph nodes. This view offers high flexibility to encode different sketch geometric properties as we can decide different connectivity structures between the node points. We use two types of graphs to represent sketches: intra-stroke graphs and extra-stroke graphs. The first graphs capture the local geometry of strokes, independently to each other, with for example 1-hop or 2-hop connected graphs, see Figure 1. The second graphs encode the global geometry and temporal information of strokes. Another advantage of using graphs is the freedom to choose the node features. For sketches, spatial, temporal and semantic information is available with the stroke point coordinates, the ordering of points, and the pen state information, respectively. In summary, representing sketches with graphs offers a universal representation that can make use of global and local spatial sketch structures, as well as temporal and semantic information.

To exploit these graph structures, we propose a new Transformer (Vaswani et al., 2017) architecture that can use multiple sparsely connected graphs. It is worth reporting that a direct application of the original Transformer model on the input spatio-temporal features provides poor results. We argue that the issue comes from the graph structure in the original Transformer which is a fully connected graph. Although fully-connected word graphs work impressively for Natural Language Processing, where the underlying word representations themselves contain rich information, such dense graph structures provide poor innate priors/inductive bias (Battaglia et al., 2018) for 2D sketch tasks. Transformers require sketch-specific design coming from geometric structures. This led us to naturally extend Transformers to multiple arbitrary graph structures. Moreover, graphs provide more robustness to handle noisy and style-changing sketches as they focus on the geometry of strokes and not on the specific distribution of points.

Another advantage of using domain-specific graphs is to leverage the sparsity property of discretized sketches. Observe that intra-stroke and extra-stroke graphs are *highly sparse* adjacency matrices. In practical sketch-based human-computer interaction scenarios, it is time-consuming to directly transfer the original sketch picture from user touchscreen devices to the back-end servers. To ensure real-time applications, transferring the stroke coordinates as a character string would be more beneficial, see Figure 2.

Our main contributions can be summarised as follows:

(i) We propose to model sketches as sparsely connected graphs, which are flexible to encode local and global geometric sketch structures. To the best of our knowledge, it is the first time that graphs are proposed for representing sketches.

(ii) We introduce a novel Transformer architecture that can handle multiple arbitrary graphs. Using intra-stroke and extra-stroke graphs, the proposed *Multi-Graph Transformer* (MGT) learns both local and global patterns along sub-components of sketches.

(iii) This Multi-Graph Transformer model is agnostic to graph domains, and can be used beyond sketch applications.

(iv) Numerical experiments demonstrate the performances of our model. MGT significantly outperforms RNN-based models, and achieves small recognition gap to CNN-based architectures. This is promising for real-time sketch-based human-computer interaction systems. Note that for sketch recognition, CNNs are the performance upper bound of coordinate-based models that involve truncating coordinate sequences, e.g., RNN or Transformer based architectures.

2. Related Work

**Neural Network Architectures for Sketches** CNNs are a common choice for feature extraction from sketches. “Sketch-a-Net” (Yu et al., 2015) was the first CNN-based model having a sketch-specific architecture. It was directly inspired from AlexNet (Krizhevsky et al., 2012) with larger first layer filters, no layer normalization, larger pooling sizes, and high dropout. Song et al. (2017) further improved Sketch-a-Net by adding spatial-semantic attention layers. “SketchRNN” (Ha & Eck, 2018) was a seminal work to model temporal stroke sequences with RNNs. A CNN-RNN hybrid architecture for sketches was proposed in (Sarvadevabhatla et al., 2016). In this work, we propose a novel Graph Neural Network architecture for learning sketch representations from multiple sparse graphs, combining both stroke geometry and temporal order.

**Graph Neural Networks** Graph Neural Networks (GNNs) (Bruna et al., 2014; Defferrard et al., 2016; Sukhbaatar et al., 2016; Kipf & Welling, 2017; Hamilton et al., 2017; Monti et al., 2017) aim to generalize neural networks to non-Euclidean domains such as graphs and manifolds. GNNs iteratively build representations of graphs through recursive neighborhood aggregation (or message passing), where each graph node gathers features from its neighbors to represent local graph structure.

**Transformers** The Transformer architecture (Vaswani et al., 2017), originally proposed as a powerful and scalable alternative to RNNs, has been widely adopted in the Natural Language Processing community for tasks such as machine translation (Edunov et al., 2018; Wang et al., 2019), language modelling (Radford et al., 2018; Dai et al., 2019), and question-answering (Devlin et al., 2019; Yang et al., 2019).
Transformers for NLP can be regarded as GNNs which use self-attention (Bahdanau et al., 2014; Velčković et al., 2018) for neighborhood aggregation on fully-connected word graphs (Ye et al., 2019). However, GNNs and Transformers perform poorly when sketches are modelled as fully-connected graphs. This work advocates for the injection of inductive bias into Transformers through domain-specific graph structures.

3. Method

3.1. Notation

We assume that the training dataset $D$ consists of $N$ labeled sketches: $D = \{(X_n, z_n)\}_{n=1}^N$. Each sketch $X_n$ has a class label $z_n$, and can be formulated as a $S$-step sequence $[C_n, f_n, p] \in \mathbb{R}^{S \times d}$. $C_n = \{(x_n^i, y_n^i)\}_{i=1}^S \in \mathbb{R}^{S \times 2}$ is the coordinate sequence of the sketch points $X_n$. All sketch point coordinates have been uniformly scaled to $x_n^i, y_n^i \in [0, 256]^2$. If the true length of $C_n$ is shorter than $S$ then the vector $[-1, -1]$ is used for padding. Flag bit vector $f_n \in \{f_1, f_2, f_3\}^S$ is a ternary integer vector that denotes the pen state sequence corresponding to each point of $X_n$. It is defined as follows: $f_1$ if the point $(x_n^i, y_n^i)$ is a starting or ongoing point of a stroke, $f_2$ if it is the ending point of a stroke, and $f_3$ for a padding point. Vector $p = [0, 1, 2, \ldots, S-1]^T$ is a positional encoding vector that represents the temporal position of the points in each sketch $X_n$.

Given $D$, we aim to model $X_n$ as multiple sparsely connected graphs and learn a deep embedding space, where the high-level semantic tasks can be conducted, e.g., sketch recognition.

3.2. Multi-Modal Input Layer

Given a sketch $X_n$, we model its $S$ stroke points as $S$ nodes of a graph. Each node has three features: (i) $C_n^s$ is the spatial positional information of the current stroke point $s$, (ii) $f_n^s$ is the pen state of the current stroke point. This information helps to identify the stroke points belonging to the same stroke, and (iii) $p^s$ is the temporal information of the current stroke point. As sketching is a dynamic process, it is important to use the temporal information.

The complete model architecture for our Multi-Graph Transformer is presented in Figure 3. Let us start by describing the input layer. The final vector at node $s$ of the multi-modal input layer is defined as

$$\bar{h}_n^s(t=0) = \mathcal{C}(E_1(C_n^s), E_2(f_n^s), E_2(p^s)),$$

(1)

where $E_1(C_n^s)$ is the embedding of $C_n^s$ with a linear layer of size $2 \times d$, $E_2(f_n^s)$ and $E_2(p^s)$ are the embeddings of the flag bit $f_n^s$ (3 discrete values) and the position encoding $p^s$ (S discrete values) from an embedding dictionary of size $(S + 3) \times d$, and $\mathcal{C}(\cdot, \cdot)$ is the concatenation operator. The node vector $(\bar{h}_n^s)^{(t=0)}$ has dimension $d = 3d$. The design of the input layer was selected after extensive ablation studies, which are described in subsequent sections.

3.3. Multi-Graph Transformer

The initial node embedding $(\bar{h}_n^s)^{(t=0)}$ is updated by stacking $L$ Multi-Graph Transformer (MGT) layers (7). Let us describe all layers.

Graph Attention Layer Let $A$ be a graph adjacency matrix of size $S \times S$ and $Q \in \mathbb{R}^{S \times d}$, $K \in \mathbb{R}^{S \times d}$, $V \in \mathbb{R}^{S \times d}$. Then the attention vector is calculated as

$$\mathcal{A}(Q, K) = \text{softmax}(QK^T),$$

and the updated node embedding is given by

$$h_n^s(t+1) = \text{MHA}(h_n^s(t), \mathcal{A}(Q, K), V).$$

Figure 3. Multi-Graph Transformer architecture. Each MGT layer is composed of (i) a Multi-Graph Multi-Head Attention (MGMHA) sub-layer and (ii) a position-wise fully connected Feed-Forward (FF) sub-layer. See details in text. “B” denotes batch size.
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![Diagram of Multi-Head Attention Layer](image)

Figure 4. Multi-Head Attention Layer, consisting of several Graph Attention Layers in parallel.

\[ R^{d_q \times d_v} \] be the query, key, and value matrices. We define a graph attention layer as

\[
\text{GraphAttention}(Q, K, V, A) = A \odot \text{softmax} \left( \frac{QK^T}{\sqrt{d_k}}V \right),
\]

where \( \odot \) is the Hadamard product. We simply weight the “Scaled Dot-Product Attention” (Vaswani et al., 2017) with the graph edge weights. We set \( d_q = d_k = d_v = \frac{d}{T} \), where \( T \) is the number of attention heads.

**Multi-Head Attention Layer** We aggregate the graph attentions with multiple heads:

\[
\text{MultiHead}(Q, K, V, A) = C(\text{head}_1, \cdots, \text{head}_T)W^O,
\]

where \( W^O \in R^{T \times d_v \times d} \) and each attention head is computed with the graph attention layer (2):

\[
\text{head}_i = \text{GraphAttention}(QW^i, KW^i, VW^i, A),
\]

where \( W^i \in R^{d \times d_q}, W^i \in R^{d \times d_k}, \) and \( W^i \in R^{d \times d_v} \). We add dropout (Srivastava et al., 2014) before the linear projections of \( Q, K \) and \( V \). An illustration of the Multi-Head Attention Layer is presented in Figure 4.

**Multi-Graph Multi-Head Attention Layer** Given a set of adjacency graph matrices \( \{A_g\}_{g=1}^{G} \), we can concatenate Multi-Head Attention Layers:

\[
\text{MultiGraphMultiHeadAttention}(Q, K, V, \{A_g\}_{g=1}^{G}) = \text{ReLU}(C(\text{head}_1, \cdots, \text{ghead}_G)W^O),
\]

where \( W^O \in R^{G \times d \times d} \) and each Multi-Head Attention Layer is computed with (3):

\[
ghead_g = \text{MultiHead}(Q, K, V, A_g).
\]

**Multi-Graph Transformer Layer** The Multi-Graph Transformer (MGT) at layer \( l \) for node \( s \) is defined as

\[
(h^{s}_{n})^{(l)} = \text{MGT}((h^{s}_{n})^{(l-1)}) = \hat{h}^{s}_{n} + \text{FF}^{(l)}(\hat{h}^{s}_{n}),
\]

where the intermediate feature representation \( \hat{h}^{s}_{n} \) is defined as:

\[
\hat{h}^{s}_{n} = (\text{MGMHA}^{s}_{n})^{(l)}((h^{s}_{n})^{(l-1)}, \cdots, (h^{s}_{n})^{(l-1)}).
\]

The MGT layer is thus composed of (i) a Multi-Graph Multi-Head Attention (MGMHA) sub-layer (5) and (ii) a position-wise fully connected Feed-Forward (FF) sub-layer. Each MHA sub-layer (6) and FF (7) has residual-connection (He et al., 2016) and batch normalization (Ioffe & Szegedy, 2015). See Figure 3 for an illustration.

### 3.4. Sketch Embedding and Classification Layer

Given a sketch \( X_n \) with \( t_n \) key points, its continuous representation \( h_n \) is simply given by the sum over all its node features from the last MGT layer:

\[
h_n = \sum_{s=1}^{t_n} (h^{s}_{n})^{(L)}.
\]

Finally, we use a Multi-Layer Perceptron (MLP) to classify the sketch representation \( h_n \), see Figure 3.

### 3.5. Sketch-Specific Graphs

In this section, we discuss the graph structures we used in our Graph Transformer layers. We considered two types of graphs, which capture local and global geometric sketch structures.

The first class of graphs focus on representing the local geometry of individual strokes. We choose \( K \)-hop graphs to describe the local geometry of strokes. The intra-stroke adjacency matrix is defined as follows:

\[
A_{n,ij}^{K\text{-hop}} = \begin{cases} 
1 & \text{if } j \in N^{K\text{-hop}}_i \text{ and } j \in \text{global}(i), \\
0 & \text{otherwise},
\end{cases}
\]

where \( N^{K\text{-hop}}_i \) is the \( K \)-hop neighborhood of node \( i \) and \( \text{global}(i) \) is the stroke of node \( i \).

The second class of graphs capture the global and temporal relationships between the strokes composing the whole...
Table 1. Summary statistics for our subset of QuickDraw.

| Set      | # Samples | # Truncated (ratio) | # Key Points |       |       |       |       |
|----------|-----------|---------------------|--------------|-------|-------|-------|-------|
|          |           | max | min | mean | std  | max | min | mean | std  |
| Training | 345,000   | 11788 (3.42%) | 100 | 2 | 43.26 | 21.85 |
| Validation | 34,500 | 1218 (3.53%) | 100 | 2 | 43.24 | 21.88 |
| Test     | 34,500    | 1235 (3.58%) | 100 | 2 | 43.20 | 21.93 |

sketch. We define the extra-stroke adjacency matrix as follows:

\[
A_{global}^{n,ij} = \begin{cases} 
1 & \text{if } |i - j| = 1 \text{ and } \text{global}(i) \neq \text{global}(j), \\
0 & \text{otherwise}.
\end{cases}
\]  

(11)

This graph will force the network to pay attention between two points belonging to two distinct strokes but consecutive in time, thus allowing the model to understand the relative arrangement of strokes.

4. Experiments

4.1. Experimental Setting

Dataset and Pre-Processing Google QuickDraw (Ha & Eck, 2018) \(^1\) is the largest available sketch dataset containing 50 Million sketches as simplified stroke key points in temporal order, sampled using the Ramer-Douglas-Peucker algorithm after uniformly scaling image coordinates within 0 to 256. Unlike smaller crowd-sourced sketch datasets, e.g., TU-Berlin (Eitz et al., 2012), QuickDraw samples were collected via an international online game where users have only 20 seconds to sketch objects from 345 classes, such as cats, dogs, clocks, etc. Thus, sketch classification on QuickDraw not only involves a diversity of drawing styles, but can also be highly abstract and noisy, making it a challenging and practical test-bed for comparing the effectiveness of various neural network architectures. Following recent practices (Dey et al., 2019; Xu et al., 2018), we create random training, validation and test sets from the full dataset by sampling 1000, 100 and 100 sketches respectively from each of the 345 categories in QuickDraw. Following (Xu et al., 2018), we truncate or pad all samples to a uniform length of 100 key points/steps to facilitate efficient training of RNN and GNN-based models. We provide summary statistics for our training, validation and test sets in Table 1, and histograms visualizing the key points per sketch are shown in Figure 5.

Evaluation Metrics Our evaluation metric for sketch recognition is “top K accuracy”, the proportion of samples whose true class is in the top K model predictions, for values K = 1, 5, 10. (Note that acc. @k = 1.0 means 100%)

Implementation Details For fair comparison under similar hardware conditions, all experiments were implemented in PyTorch (Paszke et al., 2019) and run on one Nvidia 1080Ti GPU. For Transformer models, we use the following hyperparameter values: \(S = 100\), \(L = 4\), \(d = 128\), \(G = 3\) (\(A^{1-hop}\), \(A^{2-hop}\), \(A^{global}\)), and \(I = 8\) (per graph) for our Base model (and \(d = 256\) for our Large model). Our FF sub-layer is a \(d\)-dimensional linear layer \((d = 3d)\) followed by ReLU (Glorot et al., 2011) and dropout. The MLP Classifier consists of two \(4d\)-dimensional linear layers with ReLU and dropout, followed by a 345-dimensional linear projection representing logits over the 345 categories in QuickDraw. We train all models by minimizing the softmax cross-entropy loss using the Adam (Kingma & Ba, 2014) optimizer for 100 epochs. We use an initial learning rate of \(5e^{-5}\) and multiply by a factor 0.7 every 10 epochs. We use an early stopping strategy (with the hyper-parameter “patience” of 10 epochs) for selecting the final model, and the checkpoint with the highest validation performance is chosen to report test performance.

Baselines (i) From the perspective of coordinate-based sketch recognition, RNN models are a simple-yet-effective baseline. Following Xu et al. (2018), we design several bi-directional LSTM (Hochreiter & Schmidhuber, 1997) and GRU (Cho et al., 2014) models at increasing parameter budgets comparable with MGT. The final RNN states are concatenated and passed to the MLP classifier described previously. We use batch size 256, initial learning rate 1e-4 and multiply by 0.9 every 10 epochs. We train models with both our multi-modal input (Section 3.2) as well as the 4D input from Xu et al. (2018).

(ii) Although converting sketch coordinates to images adds time overhead in practical settings and can be seen as auxiliary information, we compare MGT to various state-of-the-art CNN architectures. It is important to note that sketch sequences were truncated/padded for training both MGT and RNNs, hence image-based CNNs stand as an upper bound in terms of performance. For Inception V3 (Szegedy et al., 2016) and MobileNet V2 (Sandler et al., 2018), initial learning rate is 1e-3 and multiplied by 0.5 every 10 epochs. For other CNN baselines, the initial learning rate and decay are configured following their original papers. For each model, we use the maximum possible batch size. Following standard practice in computer vision (He et al., 2016; Huang

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\(^1\)https://quickdraw.withgoogle.com/data

![Figure 5](image-url)
et al., 2017), we employ early stopping based on observing over-fitting in the validation loss, and select the checkpoint with the highest validation accuracy for evaluation on the test set.

(iii) To evaluate the effectiveness of the proposed Graph Transformer layer, we compare it with popular GNN variants: the Graph Convolutional Network (Kipf & Welling, 2017) and the Graph Attention Network (Veličković et al., 2018). All GNN models follow the same hyperparameter setup as Transformers ($L = 4, d = 256$) and are augmented with residual connections and batch normalization for fair comparison, following (Bresson & Laurent, 2018). Optimal hyper-parameters and learning rate schedules are selected based on validation set performance.

4.2. Results

For fair comparison with RNN and CNN baselines at various parameter budgets, we implement two configurations of MGT: Base (10M parameters) and Large (40M parameters). Additionally, we perform several ablation studies to evaluate the effectiveness of our multi-graph architecture and our sketch-specific input design. Our main results are presented in Table 2.

Comparison with RNN Baselines We trained RNNs at various parameter budgets, and present result for the best performing bi-directional LSTM and GRU models in Table 2: (i) MGT outperforms both LSTM and GRU baselines by a significant margin (by 3% acc.@1 for Base, 5% for Large), indicating that both geometry and temporal order of strokes are important for sketch representation learning. (ii) Training larger RNNs is harder to converge, leading to degrading performance, e.g., GRUs outperform deeper LSTMs by 2%.

These results are not surprising: RNNs are notoriously hard to train at scale (Pascanu et al., 2013), while Transformer performance is known to improve with scale, even with billions of model parameters (Shoeybi et al., 2019).

Comparison with CNN Baselines Table 2 also presents performance of several state-of-the-art CNN architectures for computer vision: (i) Inception V3 (Szegedy et al., 2016) and MobileNet V2 (Sandler et al., 2018) are the best performing CNN architectures. Our MGT Base has competitive or better recognition accuracy than all other baselines: AlexNet (Krizhevsky et al., 2012), VGG-11 (Simonyan & Zisserman, 2014), ResNet models (He et al., 2016), and DenseNet-201 (Huang et al., 2017). (ii) MGT Large has small performance gap to Inception V3 and MobileNet V2 (i.e., 72.80% acc.@1 vs. 74.22%, 72.80% acc.@1 vs. 73.10%) and outperforms all other CNN architectures by almost 2%. (iii) Somewhat counter-intuitively, shallow networks (Inception V3, MobileNet V2) outperform deeper networks (ResNet-152, DenseNet-201) by almost 2%. This result highlights that CNNs designed for images with dense colors and textures are not suitable for sparse sketches.

Note that MobileNet V2 is specifically designed for fast inference on mobile phones and is not directly comparable in terms of model parameters.

Ablations for Multi-Graph Architecture We design several ablation studies to evaluate our sketch-specific multi-graph architecture in Table 3: (i) We evaluate Graph Transformers trained on fully-connected graphs, i.e. vanilla Transformers (GT #1), fully-connected graphs within strokes (GT #2), as well as random graphs with 10%, 20% and 30% connectivity (GT #3, #4, and #5 respectively). We compare their performance with Graph Transformers trained on sketch-specific graphs $A^{1\text{-hop}}$, $A^{2\text{-hop}}$, and $A^{\text{global}}$. We find that vanilla Transformers on fully-connected (52.49% acc.@1) and random graphs (52.71%, 53.52%, 53.22%) perform poorly compared to sketch-specific graph structures determined by domain expertise, such as fully-connected stroke graphs (64.87%) and $A^{1\text{-hop}}$ (70.23%). The superior performance of $K$-hop graphs suggests that Transformers benefit from sparse graphs representing local sketch geometry. We also evaluate a combined sketch-specific graph structure, i.e., $A^{1\text{-hop}}||A^{2\text{-hop}}||A^{\text{global}}$ (GT #10), where the graph connectivity is the logical union set of $A^{1\text{-hop}}$, $A^{2\text{-hop}}$, and $A^{\text{global}}$. However, this structure fails to gain performance improvement over $A^{1\text{-hop}}$, $A^{2\text{-hop}}$, and $A^{\text{global}}$, despite involving more domain knowledge.

(ii) We experiment with various permutations of graphs for multi-graph models (GT #11-#17). We find that using a 3-graph architecture (GT #17) combining local sketch geometry ($A^{1\text{-hop}}$, $A^{2\text{-hop}}$) and global temporal relationships ($A^{\text{global}}$) significantly boosts performance over 2-graph and 1-graph models (72.80% vs. 72.37% for 2-graph and 70.82% for 1-graph). This result is interesting because using global graphs independently (GT #9) leads to a comparatively poor performance (54.88%). Additionally, we found that using diverse graphs (GT #15, #17) is better than using the same graph (GT #14). Comparing MGT #14 and MGT #6 further shows that performance gains are due to the multi-graph architecture as opposed to more model parameters.

(iii) We also repeatedly input the adjacency matrix of GT #10 (i.e., $A^{1\text{-hop}}||A^{2\text{-hop}}||A^{\text{global}}$) three times as the multiple graph structures to train our MGT (see MGT #16 in Table 3). Compared with MGT #17, there is a clear performance gap (71.26% vs. 72.80%). This further validates our idea of learning sketch representations through multiple separate graphs.

Comparison with GNN Baselines In Table 4, we present
Table 2. Test set performance of MGT vs. the state-of-the-art RNN and CNN architectures. The $1^{st}/2^{nd}$ best results per column are indicated in red/blue. 

| Network                        | Configurations                                                                 | Recognition Accuracy                                                                 | Parameter Amount |
|--------------------------------|--------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|------------------|
| Bi-directional LSTM #1        | 4D input, $d = 256$, $L = 4$, Dropout$_{LSTM} = 0.5$, Dropout$_{MLP} = 0.15$ | $0.6665$, $0.8820$, $0.9189$                                                       | $5.555,241$     |
| Bi-directional LSTM #2        | 4D input, $d = 256$, $L = 5$, Dropout$_{LSTM} = 0.5$, Dropout$_{MLP} = 0.15$ | $0.6524$, $0.8697$, $0.9133$                                                       | $7.130,201$     |
| Bi-directional GRU            | 4D input, $d = 256$, $L = 5$, Dropout$_{GRU} = 0.5$, Dropout$_{MLP} = 0.15$   | $0.6768$, $0.8854$, $0.9234$                                                       | $5.419,097$     |
| AlexNet (Krizhevsky et al., 2012) | Standard architecture and configurations                                      | $0.6808$, $0.8847$, $0.9203$                                                       | $58.417,305$    |
| VGG-11 (Simonyan & Zisserman, 2014) |                                                     | $0.6743$, $0.8814$, $0.9191$                                                       | $130.179,801$   |
| Inception V3 (Szegedy et al., 2016) |                                                     | $0.7422$, $0.9189$, $0.9437$                                                       | $25.315,474$    |
| ResNet-18 (He et al., 2016)    |                                                     | $0.7031$, $0.9030$, $0.9351$                                                       | $11.353,497$    |
| ResNet-50 (He et al., 2016)    |                                                     | $0.7009$, $0.9010$, $0.9347$                                                       | $21.461,657$    |
| DenseNet-201 (Huang et al., 2017) |                                                     | $0.6924$, $0.8973$, $0.9312$                                                       | $58.850,713$    |
| MobileNet V2 (Sandler et al., 2018) |                                                     | $0.7050$, $0.9013$, $0.9331$                                                       | $18.755,673$    |
| Vanilla Transformer (Vaswani et al., 2017) |                                                     | $0.7310$, $0.9161$, $0.9429$                                                       | $2,665,817$     |
| **MGT (Base)**                | $d = 256$, $L = 4$, $I = 8$, Dropout = 0.1, Fully-connected graph              | $0.5249$, $0.7802$, $0.8486$                                                       | $14.029,401$    |
| **MGT (Large)**               | $d = 256$, $L = 4$, $I = 24$, Dropout = 0.1, $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}, A_{\text{global}}$ | $0.7070$, $0.9030$, $0.9351$                                                       | $10.096,601$    |

Table 3. Ablation study for multi-graph architecture of MGT. GT denotes single-graph variants of MGT. The $1^{st}/2^{nd}$ best results per column are indicated in red/blue. $|$ denotes the logical union operation.

| Network                        | Configurations                                                                 | Recognition Accuracy                                                                 | Parameter Amount |
|--------------------------------|--------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|------------------|
| **GT #1**                      | 1 Fully-connected *(vanilla)*                                                   | $0.5249$, $0.7802$, $0.8486$                                                       | $14.029,401$     |
| **GT #2**                      | 1 Intra-stroke Fully-connected                                                 | $0.6487$, $0.8697$, $0.9151$                                                       | $14.029,401$     |
| **GT #3**                      | 1 Random (10%)                                                                 | $0.5271$, $0.7890$, $0.8589$                                                       | $14.029,401$     |
| **GT #4**                      | 1 Random (20%)                                                                 | $0.5352$, $0.7945$, $0.8617$                                                       | $14.029,401$     |
| **GT #5**                      | 1 Random (30%)                                                                 | $0.5322$, $0.7917$, $0.8588$                                                       | $14.029,401$     |
| **GT #6**                      | $A_{1}^{\text{1-hop}}$                                                        | $0.7023$, $0.8974$, $0.9303$                                                       | $14.029,401$     |
| **GT #7**                      | $A_{2}^{\text{1-hop}}$                                                        | $0.7082$, $0.8999$, $0.9336$                                                       | $14.029,401$     |
| **GT #8**                      | $A_{\text{global}}$                                                           | $0.7028$, $0.8991$, $0.9327$                                                       | $14.029,401$     |
| **GT #9**                      | $A_{1}^{\text{1-hop}}$, $A_{2}^{\text{1-hop}}$, $A_{\text{global}}$          | $0.5488$, $0.8009$, $0.8659$                                                       | $14.029,401$     |
| **GT #10**                     | $A_{1}^{\text{1-hop}}$, $A_{2}^{\text{1-hop}}$, $A_{\text{global}}$          | $0.7057$, $0.9021$, $0.9346$                                                       | $14.029,401$     |
| **MGT #11**                    | 2 $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}$                                | $0.7149$, $0.9049$, $0.9361$                                                       | $28.188,249$     |
| **MGT #12**                    | 2 $A_{1}^{\text{1-hop}}, A_{\text{global}}$                                   | $0.7111$, $0.9041$, $0.9355$                                                       | $28.188,249$     |
| **MGT #13**                    | 2 $A_{2}^{\text{1-hop}}, A_{\text{global}}$                                   | $0.7237$, $0.9102$, $0.9400$                                                       | $28.188,249$     |
| **MGT #14**                    | 3 $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}, A_{\text{global}}$            | $0.7077$, $0.9020$, $0.9340$                                                       | $39.984,729$     |
| **MGT #15**                    | 3 $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}, A_{\text{global}}$            | $0.7156$, $0.9066$, $0.9365$                                                       | $39.984,729$     |
| **MGT #16**                    | 3 $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}, A_{\text{global}}$            | $0.7126$, $0.9051$, $0.9372$                                                       | $39.984,729$     |
| **MGT #17**                    | 3 $A_{1}^{\text{1-hop}}, A_{2}^{\text{1-hop}}, A_{\text{global}}$            | $0.7280$, $0.9106$, $0.9387$                                                       | $39.984,729$     |

Table 4. Test set performance of Graph Transformer vs. other GNN variants. The $1^{st}/2^{nd}$ best results per column are indicated in red/blue. “fully” denotes fully-connected. “Gra. Stru.” denotes graph structure.

| Network                        | Gra. Stru. | Recognition Accuracy | Parameter Amount |
|--------------------------------|------------|----------------------|------------------|
| **GCN (Kipf & Welling, 2017)** | fully      | $0.7384$, $0.8133$, $0.8213$, $6.948,441$ |                 |
| **GAT (Velicković et al., 2018)** | fully      | $0.7892$, $0.9224$, $0.8859$, $11.660,889$ |                 |
| **GT**                         | fully      | $0.7706$, $0.8465$, $0.7892$, $14.029,401$ |                 |
| Pos.-wise FF                   | None       | $0.7091$, $0.8516$, $0.9311$, $4.586,013$ |                 |

The performance of our Graph Transformer model compared to GCN and GAT, two popular GNN variants: (i) We find that all models perform similarly on fully-connected graphs. Using 1-hop graphs results in significant gains for all models, with Transformer performing the best. (ii) Interestingly, both GNNs on fully-connected graphs are outperformed by a simple position-wise embedding method without any graph structure: each node undergoes 4 feed-forward (FF) layers followed by summation and the MLP classifier. These results further highlights the importance of sketch-specific graph structures for the success of Transformers. Our final models use the Transformer layer, which implicitly includes the FF sub-layer (7).
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Figure 6. Selected attention heads at each layer of MGT for a sample from the test set (labelled ‘alarm clock’). Each layer has \( I = 8 \) attention heads per graph in total. We manually choose the most interesting heads for each graph. Darker reds indicate higher attention values. Best viewed in color.

Table 5. Ablation study for multi-modal input for MGT (Large). Notations: “+” and “\( \mathcal{C}(\cdot, \cdot) \)” denote “sum” and “concatenate”, respectively; “coo.”, “flag”, and “pos.” represent “coordinate”, “flag bit”, and “position encoding”, respectively. The 1st/2nd best results per column are indicated in red/blue.

| Input Permutation         | Recognition Accuracy |
|---------------------------|----------------------|
|                           | acc.@1 | acc.@5 | acc.@10 |
| coo.                      | 0.6512 | 0.8735 | 0.9162  |
| coo. + flag               | 0.6568 | 0.8762 | 0.9176  |
| coo. + flag + pos.        | 0.6600 | 0.8766 | 0.9182  |
| \( \mathcal{C}(\text{coo.}, \text{flag}) \) | 0.7017 | 0.8996 | 0.9321  |
| \( \mathcal{C}(\text{coo.}, \text{flag}, \text{pos.}) \) | 0.7280 | 0.9106 | 0.9387  |
| 4D Input                  | 0.6559 | 0.8758 | 0.9175  |
| 4D Input + pos.           | 0.6606 | 0.8781 | 0.9190  |
| \( \mathcal{C}(4D \text{ Input}, \text{pos.}) \) | 0.7117 | 0.9048 | 0.9366  |

Qualitative Results In Figure 6, we visualize attention heads at each layer of MGT for a sample from the test set (labelled ‘alarm clock’). Attention heads in the initial layers attend very strongly to certain neighbors and very weakly to others, i.e., the model builds local patterns for sketch sub-components (strokes) through message passing along their contours. In penultimate layers, the intensity of neighborhood attention is significantly lower and evenly distributed, indicating that the model is aggregating information from various strokes at each node. Additionally, we believe \( A^\text{global} \) graphs are critical for message passing between strokes, enabling the model to understand their relative arrangement, e.g., the feet of the clock are attached to the bottom of the body, the arms are located inside the body, etc.

5. Conclusion

This paper introduces a novel representation of free-hand sketches as multiple sparsely connected graphs. We design a Multi-Graph Transformer (MGT) for capturing both geometric structure and temporal information from sketch graphs. The intrinsic traits of the MGT architecture include: (i) using graphs as universal representations of sketch geometry, as well as temporal and semantic information, (ii) injecting domain knowledge into Transformers through sketch-specific graphs, and (iii) making full use of multiple intra-stroke and extra-stroke graphs.

We hope MGT can serve as a foundation for future work in
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Sketch applications and network architectures, motivating the community towards sketch representation learning using graphs. Additionally, for the graph neural network (GNN) community, we hope that MGT helps free-hand sketch become a new test-bed for GNNs.

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