TIME DOMAIN ANALYSIS OF VARIABILITY IN CYGNOUS X-1:
CONSTRAINTS ON THE EMISSION MODELS

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ABSTRACT

We use time domain analysis techniques to investigate the rapid variability of Cygnus X-1. We show that the cross-correlation functions between hard and soft energy bands reach values very close to unity and peak at a lag of less than 2 millisecond for energies separated by a factor of 10. This confirms that the process that produces X-ray photons at different energies is extremely coherent on short time scales and strongly constrains emission models proposed to explain Fourier-frequency-dependent time lags. We present autocorrelation functions at different energies, and note their widths decrease with increasing energy. We show that the extended Compton corona model produces auto-correlation functions whose widths increase with increasing energy, that the model of cylindrical waves moving inward through a transition disk has too large a peak lag in the cross-correlation function. Models of magnetic flaring and of drifting blobs in a hot corona can qualitatively fit the observations.

Subject headings: accretion, accretion disks – black hole physics – methods: data analysis – stars: individual (Cygnus X-1) – X-rays: stars

1. INTRODUCTION

The X-ray/γ-ray spectrum of an accreting black hole such as Cygnus X-1 in its hard state can be represented as the sum of a few components: a soft component associated with the emission from a cold accretion disk, a hard tail extending up to a few hundred keV associated with a hot „corona”, and a Compton reflection bump produced when hard X-rays are reflected from cold material in the accretion disk (see, e.g., Zdziarski et al. 1997; Gierliński et al. 1997; Poutanen 1998). Spectral data suggest thermal Comptonization by a medium with a temperature of about 100 keV as the origin of the hard tail. The observed X-ray spectral slopes and the spectrum of Compton reflection can be used to determine the geometry of the system. However, a variety of models fit the spectra (see Poutanen 1998; Beloborodov 1999a; Zdziarski 2000), so the parameters of the accretion flow are not be well constrained by spectral data alone. Most of the spectral models, however, are applied to time-averaged spectra despite of the fact that the sources show rapid spectral variability (see, e.g., Nolan et al. 1981; Negoro, Miyamoto, & Kitamoto 1994; Feng, Li, & Chen 1999) implying rapid changes in the physical conditions. Examining temporal characteristics should help break the degeneracy among spectral models.

Time domain techniques were used in the early days of X-ray astronomy before the statistical samples of data were sufficient to make use of Fourier domain analyses (see, e.g., Weisskopf, Kahn, & Sutherland 1975; Sutherland, Weisskopf, & Kahn 1978; Friedhorsky et al. 1979; Nolan et al. 1981). The asymmetry of the cross-correlation function (CCF) of Cygnus X-1 was discovered in ~ 150 sec observations by Friedhorsky et al. (1979) and Nolan et al. (1981). They showed that the CCF peaks at a lag ~ 10 – 40 ms. Using data from EXOSAT, Page (1985) confirmed these results and claimed a ~ 6 ms shift of the peak of the CCF between the 5-14 keV and the 2-5 keV bands. These are the last papers, to our knowledge, that present the CCFs despite immense advances in temporal resolution, photon statistics, and duration of observations. Aside from attempts to model individual shots (Lochner, Swank, & Szmytkowiak 1991; Negoro et al. 1994; Focke 1998; Feng et al. 1999), recent analyses have concentrated on Fourier domain techniques.

The CCF asymmetry is related to the Fourier-frequency-dependent hard time lags between different spectral bands discovered by Ginga (Miyamoto et al. 1988). These data gave new strong constraints on spectral models. However, in the Fourier domain it is difficult to measure time lags at frequencies above ~ 30 Hz (see, e.g., Nowak et al. 1999a) corresponding to the light travel time in a region of the main energy dissipation. On the other hand, in the time domain, the CCFs can be measured accurately down to the lags as short as 2 ms. Furthermore, time domain functions (containing, in principle, the same information as their Fourier domain companions) highlight different information which can be used to constrain emission models further.

We present the results of time domain analyses of Cygnus X-1 as observed by the Rossi X-Ray Timing Explorer (RXTE) and compare our observational results with model predictions.

2. OBSERVATIONS AND RESULTS

2.1. Observation Log

Cygnus X-1 was observed in its hard (low) state 12 times by RXTE during December of 1997, for a total of about 30 ksec. Here we present results only from the Proportional Counter Array, using the standard screening criteria of earth elevation greater than 10°, offset from source less than 0.01°, all 5 proportional counter units on, and the standard time since the last South Atlantic Anomaly passage. For the lowest energies (below 8 keV), we have data recorded in single bit (SB) modes, where RXTE counts photons with no spectral information other than whether they fall within the given range of channels. For the higher photon energies, we use the full Event Mode data with all the available spectral information. The use of SB modes and the subsequent loss of some spectral information were required because of telemetry limitations of RXTE.

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We make no background subtractions for our data because the background counts are a very small fraction of the total counts and because current background models for RXTE do not give estimates on time scales shorter than 16 seconds. RXTE deadtime for Cygnus X-1 is about 1% and should affect only zero lag bins in the correlation functions.

2.2. Light Curves

Using the standard FTOOLS 4.1 software, we extract light curves with 2-20 second resolution (~2 ms) for each of the SB modes (for energy channels 2-5 keV, 5-6 keV, and 6-8 keV), plus the event mode data binned into three additional energy bands (channels 24-40, 41-71, and 72-132, or 8-13 keV, 13-24 keV, and 24-40 keV respectively). We do not analyze any higher energy data because the background count rates become large above 40 keV. We attempted to analyze the data with a ~1 ms bin size. For the low energy (and hence higher count rate) bands the 1 ms computations allowed us to produce slightly stronger constraints (discussed below). We plot only the 2 ms binning results in order to make the plots clearer, since the results are essentially the same in either case.

2.3. Cross-correlation Functions

We compute the CCF of Cygnus X-1, comparing each energy band with the lowest energy band. The CCFs are asymmetric with peaks at lags less than 2 ms in all cases (see the inset of Fig. 1), and less than 1 ms between the 3 keV band and bands below 13 keV. The rising part of the CCF (soft lags) becomes narrower with energy substantially faster than the decaying part (hard lags). These results agree qualitatively with past results for the CCF (Priedhorsky et al. 1979; Nolan et al. 1981). The CCFs reach values very close to unity, showing that the signal at all the energies is extremely well synchronized. The asymmetry of the CCFs in the time domain has a direct relation to the Fourier-frequency-dependent time lags.

2.4. Autocorrelation Functions

The autocorrelation functions (ACFs) of Cygnus X-1 are shown in Figure 1 and Figure 2a. The width of the ACF decreases with photon energy approximately as $E^{-0.21\pm0.01}$ at lags smaller than ~0.3 sec. (At larger lags the ACFs at different energies are not self-similar.) This strongly constrains the origin of the spectral variability, since it requires that the pulses producing the variability last longer at low energies than at higher energies. Similar energy dependence is observed also in the ACF of the peak aligned average shot profiles (Feng et al. 1999). Our results extend their work by demonstrating the trend of width of the ACF versus energy across more energy bands. More importantly, we prove that this trend is always present in the data, and is not subject to the selection effects of a shot-fitting algorithm.

3. CONSTRAINTS ON THE EMISSION MODELS

3.1. Implications on Shot Shapes for Shot Noise Models

One can attempt to explain the observed ACFs and CCFs in terms of a simple shot noise model (Terrell 1972). Since the CCFs peak at a lag $\lesssim$2 ms, the shots at different energies also should reach maxima within 2 ms from each other. We can develop some intuition about the typical shot profiles by looking at the analytic form of the CCF for shots which have exponential rise and decay and peak at the same time for different energies. The CCF is

\[
\alpha \frac{\tau_{hr}^2 (\tau_{sr} + \tau_{hd}) e^{\tau_{sr}/\tau_{hd}}}{(\tau_{sr} + \tau_{hd}) (\tau_{sr} - \tau_{hd})} - \frac{\tau_{sr}^2 (\tau_{sr} + \tau_{hd}) e^{\tau_{sr}/\tau_{hr}}}{(\tau_{sr} + \tau_{hr}) (\tau_{sr} - \tau_{hr})} \tag{1}
\]

for negative $\tau$, and

\[
\alpha \frac{\tau_{sr}^2 (\tau_{sr} + \tau_{hd}) e^{-\tau_{sr}/\tau_{hd}}}{(\tau_{sr} + \tau_{hd}) (\tau_{sr} - \tau_{hd})} - \frac{\tau_{hr}^2 (\tau_{hr} + \tau_{hd}) e^{-\tau_{hr}/\tau_{sr}}}{(\tau_{sr} + \tau_{hr}) (\tau_{sr} - \tau_{hr})} \tag{2}
\]
for positive $\tau$, where $\tau_{sr}$, $\tau_{sd}$, and $\tau_{hr}$, $\tau_{hd}$ are the rise and decay time constants of the shots in soft and hard energy bands, respectively. The proportionality coefficient is $2/\sqrt{(\tau_{sr} + \tau_{sd})(\tau_{hr} + \tau_{hd})}$. The energy dependence of the ACF requires the shots at higher energies to be shorter, i.e. $\tau_{hr} < \tau_{sr} + \tau_{sd}$. Since the observed CCF (see Fig. 1) is roughly equal to the ACF of the softer energy band for positive $\tau$ and the ACF of the harder energy band for negative $\tau$, and the CCF rises faster than it decays, one gets $\max(\tau_{sd}, \tau_{hr}) \approx \max(\tau_{hr}, \tau_{hd}) < \max(\tau_{sr}, \tau_{hr}) \approx \max(\tau_{sr}, \tau_{sd})$. This means that the soft rise time scale is the longest, $\tau_{sr} > \tau_{sd}$, $\tau_{hr}$, $\tau_{hd}$, and that the decay times at different energies are very close to one another (i.e. $\tau_{sd} \approx \tau_{hd}$) or are so small that the only relevant parameters are the rise times ($\tau_{hr} \gg \tau_{sd}, \tau_{hd}$). The energy dependence of the rise time then produces the time lags and the asymmetry of the CCF (see also Miyamoto & Kitamoto 1989 who arrived at similar conclusions).

In modified shot noise models (see, e.g., Lochner et al. 1991), there is a broad distribution of shot time scales. The shots at different energies should be perfectly synchronized in order to achieve high values of the CCFs. However, constraints on the shape of the shots are not so strong as for the simplest shot noise model. The shots at different energies can have similar properties as described above, or they can be shifted relative to each other (e.g., Poutanen & Fabian 1999a,b). If the shift depends on the shot time scale as $\propto \tau^\alpha$, one obtains Fourier time lags $\delta(f) \propto f^{-\alpha}$. In such a case, the CCF peaks at a lag equal to the delay corresponding to the shortest time scale. The data then constrain the minimum shot time scale to be $< 1$ ms.

### 3.2. Extended Corona Models

Comptonization in a uniform electron cloud produces time lags which are frequency independent in the range we can probe with current X-Ray instruments (Miyamoto et al. 1988). This inspired Kazanas, Hua, & Titarchuk (1997) to propose an extended corona model with a $r^{-1}$ radial density distribution and a size of a few light seconds. The long lags observed at low Fourier frequencies are produced here by photons travelling and scattering over large radii, while the shorter time lags are produced in the central small core of the cloud. The model fits much of the data, but has the physical problem of having too much energy input at large radii.

We simulated the light curves for this model and computed the auto-correlation functions at different energies. As an example, we consider a set of parameters from Hua, Kazanas, & Cui (1999) ($kT_e = 0.2$ keV, $kT_T = 100$ keV, $p = 1$, $n_i = n_1 = 10^{16}$ cm$^{-3}$, $r_1 = 10^{-3}$ lt-s, $r_2 = 10^4 r_1$, for details see discussion around their Fig. 1). The shots are assumed to be produced by modulations in the soft flux. Because this model produces delays by having larger light travel times for higher energy photons, it always produces a wider shot and hence a wider ACF at higher energies (see Fig. 2b). We note that the ACF at 30 keV in this model is a factor of ten broader than the ACF at 3 keV, while in the observations it is 50% narrower. The fact that the ACFs become broader with energy is the intrinsic property of the model and there is no way to resolve this problem by changing the parameters of the system. We conclude that models where the time lags are produced by light travel delays can be ruled out.

### 3.3. Magnetic Flare Models

Magnetic flares on the surface of the cold accretion disk were shown recently to produce X/$\gamma$-ray spectra in agreement with observations of Cyg X-1 (Beloborodov 1999b). The observed time lags can correspond to the time scale of the evolution of magnetic structures (Poutanen & Fabian 1999a,b). The magnetic field lines twist due to differential rotation in the accretion disk and elevate to the corona releasing magnetic energy and heating the corona. The flare time scale is then of the order of the Keplerian time scale at the relevant distance from the central black hole. Changes in the energy dissipation rate and in the geometry of the flare (e.g., distance from the disk) produce soft-to-hard spectral evolution which is the cause of the hard time lags. A broad distribution of the flare time scale assures that the time lags are inversely proportional to the Fourier frequency $\delta(f) \propto f^{-1/(2\pi f)}$, where $\tau_f$ is the shot time scale.
giving contribution to the power spectrum at frequency $f$.

The model of Poutanen & Fabian (1999b) reproduces well the time lags observed in Cyg X-1, while producing a somewhat wider ACF at higher energies due to the assumption that the energy dissipation rate rises faster than it decays. In the opposite situation, when the dissipation rate rises more slowly than it decays, the ACF and CCF energy dependences can be reproduced easily (see, e.g., Miyamoto & Kitamoto 1989; Poutanen 2000).

3.4. Drifting Blob Models

Böttcher & Liang (1999) proposed a model in which spectral variability is produced by a cool blob drifting inward through an inhomogeneous hot inner disk. This model qualitatively matches the data. However, the parameters presented in that work, produce significant quantitative deviations from the observations in both the Fourier and time domains. While changing the parameters of the system could allow the model to fit the data, a larger problem for this model is that it drives the variability through modulations of the soft photon flux. If the energy dissipation rate in the corona does not change with time, then an increase of the soft photon flux would lead to a softer spectrum and spectral pivoting around $\sim 10$ keV. The amplitude of the variability would be then a strong function of photon energy (larger at higher energies) and the variability above and below the pivoting point would anti-correlate, contrary to what is observed (see, e.g., Nowak et al. 1999a and § 2.3). In order to reproduce the observed correlated variability at different energies without violating the energy balance, one has to assume that the inward drift of cool blobs is perfectly correlated with the increase of the energy dissipation in the hot corona. It remains to be seen whether such a requirement is physically realistic and the model can indeed fit the data. One cannot avoid this constraint by proposing an ADAF-type solution where the coronal cooling rate is dominated by cyclo-/synchrotron radiation rather than by Compton cooling. Such a model would require a cyclo-/synchrotron luminosity (observed in IR/optical light) far in excess of the X-Ray luminosity of $\sim 10^{37}$ ergs/sec. The observed optical luminosity is only about $5 \times 10^{36}$ ergs/sec and is dominated by the companion star.

3.5. Cylindrical Wave Models

Cylindrical waves propagating through the accretion disk from the region where the soft X-rays are emitted to the region where the hard X-rays are emitted (e.g., Miyamoto et al. 1988; Kato 1989; Nowak et al. 1999b) have also been proposed as a mechanism for producing hard time lags. The dispersion of wave velocities results in the frequency dependence of the time lags. A recent and relatively well developed version of such a model (a “transition disk” model) designed to explained the spectrum of Cyg X-1 and the Fourier time lags was recently considered by Misra (2000). However, in this model the peak of the emission at 30 keV is delayed by $\sim 0.015$ s from the peak at 3 keV. In such a case, the CCF between the 30 keV and 3 keV photons would peak at that lag strongly contradicting the data. One can assume that the low frequency signal propagates slowly producing larger time lags, while high frequency signal propagates faster producing smaller time lags. (This is basically a modified shot noise model.) The transition disk model then requires a propagation speed of $\gtrsim 2c$ in order to fit $\lesssim 2$ ms lag in the CCF.

4. CONCLUSIONS

We present the results of cross-correlation and autocorrelation analysis of the light curve of Cygnus X-1 in several energy bands. The width of the ACF was found to become narrower with photon energy as $\propto E^{-0.2}$. The corresponding CCFs are asymmetric, but all peak at lags less than 2 ms.

We compare these results to model calculations for an extended Compton corona model, a magnetic flare model, a drifting blob model, and a cylindrical wave model. The extended corona model inherently produces longer shots at higher energies (and, therefore, broader autocorrelation function), and it cannot fit the data even qualitatively. We find that a magnetic flare model can fit all the data if one requires that the energy dissipation rate rises slower than it decays. We also find that while the drifting blob model fits the data qualitatively, it requires the inward drift of cool blobs to be perfectly correlated with the increase of the energy dissipation in the hot corona. A transition disk model predicts a large shift of the peak of the cross-correlation function from zero lag contradicting the data.

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