Dark matter cores and cusps: the case of multiple stellar populations in dwarf spheroidals

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ABSTRACT
A number of dwarf spheroidal (dSph) galaxies are known to contain a more extended, metal-poor population with a flattish velocity dispersion profile, and a more concentrated, metal-rich population with a velocity dispersion declining with radius. The two populations can be modelled with Michie–King distribution functions (DFs) in the isothermal and in the sharply truncated limits, respectively. We argue that the truncation of the metal-rich population can be traced back to the spatial distribution of the star-forming gas. Suppose $\delta$ is the exponent of the first non-constant term in the Taylor expansion of the total potential at the centre ($\delta = 1$ for Navarro–Frenk–White or NFW haloes, $\delta = 2$ for cored haloes). Then, we show that the ratio of the half-light radii of the populations $R^{1/2}_{\text{h},2}/R^{1/2}_{\text{h},1}$ must be smaller than the ratio of the line-of-sight velocity dispersions $\sigma_{\text{los},2}/\sigma_{\text{los},1}$.

Specializing to the case of the Sculptor dSph, we develop a technique to fit simultaneously both populations with Michie–King DFs. This enables us to determine the mass profile of the Sculptor dSph with unprecedented accuracy in the radial range $0.2 < r < 1.2$ kpc. We show that cored halo models are preferred over cusped halo models, with a likelihood ratio test rejecting NFW models at any significance level higher than 0.05 per cent. Even more worryingly, the best-fitting NFW models require concentrations with $c \ll 20$, which is not in the cosmologically preferred range for dwarf galaxies. We conclude that the kinematics of multiple populations in dSphs provides a substantial new challenge for theories of galaxy formation, with the weight of available evidence strongly against dark matter cusps at the centre.

Key words: galaxies: dwarf – galaxies: individual: Sculptor dSph – galaxies: kinematics and dynamics – Local Group – dark matter.

1 INTRODUCTION
The population of dwarf spheroidal satellites (dSphs) of the Galaxy furnishes an unprecedented opportunity to test the physics underlying the most extreme dark matter dominated systems. In particular, the prediction of a cusped density profile for dark matter haloes (Navarro, Frenk & White 1996) has naturally focused attention on the structure of the innermost parts. However, even though reasonable photometric and kinematic profiles of single stellar populations in several of the most luminous dwarfs are available, the only clear conclusion is that the methods of stellar dynamics have so far been unable to prove whether the central regions of such haloes are cored or cusped (e.g. Kleyna et al. 2002; Koch et al. 2007; Walker et al. 2009; Amorisco & Evans 2011).

The last decade has also seen the growing realization that the stellar content of dSphs is complex (e.g. Majewski et al. 1999; Kleyna et al. 2004; Tolstoy et al. 2004; Battaglia et al. 2008a; Koch et al. 2008; Ural et al. 2010). Carina, Fornax, Sculptor and Sextans all display evidence for the coexistence of at least two stellar populations with different metallicities – usually an older, more spatially extended and metal-poor population and a younger, more spatially concentrated, metal-rich population. In all cases save Carina, the different stellar populations clearly have different kinematics: while the older component displays a fairly flattish velocity dispersion profile, the dispersion profile of the younger component often shows signs of a quite sharp decline with radius. As a consequence, the metal-rich population is concentrated in the central regions of the system only. This picture is shared by both dSphs – Sculptor and Sextans – where such kinematic profiles have been completely disentangled (Battaglia et al. 2008a, 2011).

This is consistent with a picture in which the dSph has undergone mainly two different star formation epochs, with an underlying evolution driven by tidal stirring in the potential of the Galaxy. The metal-poor population may have undergone thermalization with the help of impulsive tidal stirring near the pericentres of orbits and of
dynamical instabilities driven by the tidal field near the apocentres (e.g. Read et al. 2006; Klimentowski et al. 2007). This can lead to a nearly isothermal distribution of velocities. By contrast, the metal-rich population may originate from a centrally concentrated distribution of gas, with the gas in the outskirts of the dSph being removed during the interval between the two star formation events. The spatial distribution of the gas may enforce an approximate cut in energy in the phase-space distribution function (DF) of the younger stellar population.

The coexistence of two different stellar populations in the same potential provides a stronger instrument to probe the properties of the dark matter halo in dSphs. A first approach is already provided by Battaglia et al. (2008a) in their analysis of the two populations in the Sculptor dSph. They found that, though a Navarro–Frenk–White (NFW) halo is still statistically compatible with the kinematical properties of the two stellar populations of the Sculptor dSph, a cored halo is a better fit in the context of the Jeans equations. Note, however, that the theorem on the central velocity dispersion proved by An & Evans (2009) shows that an isotropic cored stellar density profile cannot be embedded in a cusped dark matter halo, so that the solutions of Battaglia et al. to the spherical Jeans equations are not all physical (see also the recent result provided by Ciotti & Morganti 2010). Very recently, Walker & Peñarrubia (2011) have used an ingenious argument to conclude that the populations in both the Fornax and Sculptor dSphs strongly disfavour cusped dark matter haloes. However, their result is based on the assumption that the mass estimator formula presented in Walker et al. (2009) (see also Wolf et al. 2010) can be applied separately to both metal-poor and metal-rich stellar populations in a dSph, a hypothesis which remains to be demonstrated. Although the velocity dispersion profile of the metal-poor population is consistent with a quasi-isothermal system, and thus with such a formula, that of the metal-rich is probably not (cf. Battaglia et al. 2008a).

At the price of assuming a functional form for the DF of the stellar populations, we can significantly strengthen the analysis by requiring the models to reproduce at the same time the luminosity and kinematical profiles for both populations. In doing this we aim to measure a mass profile (at least in some radial interval), rather than a single total mass measure at some radius. Both Jeans analyses (e.g. Walker et al. 2009; Wolf et al. 2010) and simple phase-space modelling (Amorisco & Evans 2011) are able to measure the total dark halo mass inside some specific radius, typically related to the half-light radius of the stellar population, but have failed to reconstruct the entire mass profile. This is because, though they are able to link the two dimensional scales of the dark matter density profile (the scalelength \( r_0 \) and the characteristic density \( \rho_0 \) for example), they are unable to fix either of these numbers. This is very typical when the velocity dispersion profile only is fitted, as it is naturally degenerate in these two dimensional scales. However, the surface brightness profile can put independent constraints on the scalelength of the halo.

We re-analyse the data for the two stellar populations in the Sculptor dSph by using phase-space models, instead of the less complete Jeans analysis, adopting the general family of Michie–King DFs (King 1962; Michie 1963). This represents a fairly flexible family of centrally isotropic DFs with an adjustable radial anisotropy in the outer regions. The two stellar populations are embedded in a dark halo whose density profile can be shaped at will, cored or cusped.

In Section 2, we introduce the observational data; in Section 3 we describe the motivations and the basic details of the models we develop in the paper. Section 4 sets out the properties of our models and establishes a compatibility criterion between the observed properties of a two populations dSph and the density profile of the dark matter halo. Finally, the results are presented in Section 5.

## 2 SCULPTOR DATA

Although a number of our results on multiple populations in dSphs are general, it is none the less useful to have a particular example in mind. For this, we choose the Sculptor dSph which is located at a distance \( D = 79.3 \) kpc from the Sun. Majewski et al. (1999) already provided evidence for a bimodality in Sculptor’s metallicity distribution based on a properties of the colour–magnitude diagram. This result has been revised and extended by a number of later investigations (e.g. Babusiaux, Gilmore & Irwin 2005; Battaglia et al. 2006, 2008a; Walker & Peñarrubia 2011).

Battaglia et al. (2008b) used the FLAMES spectroscope on the VLT in the Ca II triplet region to obtain high-quality radial velocities for Sculptor members. To divide the stars into metal-rich and metal-poor populations, Battaglia et al. (2008a) assumed a hard cut in metallicity. The metal-rich population has \([Fe/H] > -1.5\) and traces the red horizontal branch stars. The metal-poor population has \([Fe/H] < -1.7\) and traces the blue horizontal branch stars. Those stars in the intervening band of metallicities are not classified. The line-of-sight velocity dispersion profiles associated with the red and blue horizontal branch populations are displayed in Fig. 1. The procedure of Battaglia et al. is open to questioning, as the metallicity is correlated with the kinematics, and the procedure may introduce biases that are not easy to quantify. We explored this issue by re-analysing the entire radial velocity data set, looking for correlations between the position of the hard cut in metallicity and the derived kinematics. We find that the kinematics of the metal-poor population is robust against changes in the cut position, as well as against foreground contamination at large radii (different ways to estimate the number of contaminants – see equation 1 in Battaglia et al. 2008b – do not result in significant variations of the velocity dispersion profile). The metal-rich population is more prone to variations, though we find that binning size is more of an issue when compared to the position of the metallicity cut itself, which leaves the main features of the velocity dispersion profile unchanged if varied within 0.1 in \([Fe/H]\).

Battaglia et al. (2008a) also provide surface brightness profiles for the red and blue horizontal branch stars, which are also shown in Fig. 1. A photometric profile is usually characterized by the half-light radius, namely the (projected) radius that encloses half the light. Battaglia et al. find the metal-poor population is well fitted by a Plummer profile with half-light radius \( 15.1 \pm 0.5 \) arcmin (348 \pm 12 pc), whilst the metal-rich prefers a slightly cusplier Sersic profile with half-light radius \( 8.6 \pm 1 \) arcmin (198 \pm 25 pc). We recompute the two half-light radii directly from the surface brightness profiles in Fig. 1 using a model-independent technique, which does not require the assumption of any precise functional form for the surface brightness profiles themselves. We assume a Gaussian distribution for the observational uncertainties of the surface brightness profiles and use them to generate synthetic surface brightness profiles through a Monte Carlo technique. By direct integration of these profiles we infer the half-light radii of the two stellar populations as well as their uncertainties (variations due to the order of the interpolation of the synthetic profiles are accounted for in the final uncertainty itself). We obtain \( R_{0.1} = 341 \pm 9 \) pc, \( R_{0.2} = 217 \pm 15 \) pc, (1) for the half-light radii of the metal-poor and metal-rich populations, respectively.
3 MODELS

3.1 The dark halo

Both stellar populations are assumed to be embedded in a massive, extended, spherical dark matter halo, the density profiles of which can be shaped at will. The dark halo is truncated at the tidal radius of the metal-poor stellar population. Given the high mass-to-light ratios of dSphs, we assume that the entire gravitational field is determined by the dark matter only and that the stellar components are just tracers.

As a flexible family of dark matter haloes, we use

\[ \rho(r) = \begin{cases} \rho_0 \left( \frac{r}{r_0} \right)^2 & r < r_t, \\ \rho_0 \left( \frac{r}{r_0} \right)^{-3} & r > r_t. \end{cases} \]  

Here, \( r_0 \) and \( \rho_0 \) are the dimensional scalelength and characteristic density of the halo. In our models, the tidal radius \( r_t \) of the dark matter halo is the same as that of the metal-poor stellar population. Even if, in nature, the truncation radius of the dark matter halo is larger than the stellar tidal radius, the potential experienced by the stars remains unchanged. The metal-rich population is allowed to have a smaller tidal radius.

Throughout the paper, we will concentrate on the following two choices:

\( (a, b, c) = (1, 1, 2), \)

\( (a, b, c) = (0, 2, 3). \)

The first choice yields the cosmologically motivated NFW model. The second is a standard example of cored model, with the same asymptotic density fall-off as the NFW profile (\( \rho \sim r^{-3} \)).

3.2 The stellar populations

We are inspired by the fact that, as a result of relaxation processes, the final DF of the stars in phase space tends to near-isothermality, at least in the very central parts. This idea is supported by the universal flatness of the velocity dispersion profiles of the dSphs at least close to the centres (Evans, An & Walker 2009), as well as numerical simulations of tidal stirring of dSphs (Mayer et al. 2001). Fig. 1 shows that for the Sculptor dSph the metal-poor population has a flat dispersion profile out to at least \( \sim 500 \) pc. The metal-rich profile is perhaps more debatable, though consistent with a flattish profile within \( \sim 200 \) pc.

Deviations from flatness of the velocity dispersion profiles can be ascribed to either anisotropy or tidal effects. In principle, since the two populations reside in the same potential and experience the same tidal perturbation, they should share the same cut in energy too if this is tidal in origin. However, we are motivated by the idea that the metal-rich population has indeed been formed by gas which was concentrated in the central and more protected regions of the dSph. Thus, its cut-off in radius may be much smaller than the formal tidal radius. In what follows, we therefore assume that the more extended metal-poor population has the same tidal radius as the dark halo, but allows the radial truncation of the metal-rich population to be set by the data.

The Michie–King DF (King 1962; Michie 1963) is a function of two integrals of the motion, the energy \( E \) (per unit mass) and the angular momentum \( L \) (per unit mass), namely

\[ f_{MK}(E, L) = \frac{\rho_{*,0}}{(2\pi\sigma^2)^{3/2}} \exp \left( \frac{-L^2}{2r_0^2\sigma^2} \right) f_K(E), \]

where

\[ f_K(E) = \exp \left( \frac{\Phi(r_t) - E}{\sigma^2} \right) - 1. \]

The gravitational potential \( \Phi \) is, in general, the total gravitational potential of the system, but here we assume that the stars can be treated as simple tracers.

The dimensional free parameters of the Michie–King DF are as follows:

(1) the velocity dispersion parameter \( \sigma \), which, for a fixed gravitational potential, tunes the scalelength of the associated luminous component;
(2) the tidal radius $r_t$, which is the external edge of the system, truncated by the tidal effects of the Galaxy, or by the original energy distribution of the star-forming gas;

(3) the anisotropy radius $r_a$, which is approximately the characteristic length-scale associated with variations in the anisotropy profile;

(4) the characteristic stellar density $\rho_{*,0}$. As long as the stars are just tracers in the potential of the dark halo, this parameter is an overall normalization, and has no other effect on the model.

To our set of dimensional parameters, we can associate a useful set of dimensionless parameters. We use the scalelength of the halo $r_0$ and its characteristic density $\rho_0$ as dimensional references, and denote dimensionless quantities by a superscript hat, so that the dimensionless half-light radius, anisotropy radius and tidal radius are $\hat{R}_h = R_h/r_0$, $\hat{r}_a = r_a/r_0$ and $\hat{r}_t = r_t/r_0$, respectively. The dimensionless velocity dispersion is

$$\hat{\sigma}^2 \equiv \frac{\sigma^2}{\Phi_0}; \quad \hat{\sigma}_{los}^2(r) \equiv \frac{\sigma_{los}^2(r/r_0)}{\Phi_0},$$

where, respectively, for an NFW and a cored halo, we have (see also Amorisco & Evans 2011)

$$\Phi_0^{\text{NFW}} = 2\pi G \rho_0 r_0^2; \quad \Phi_0^{\text{core}} = 2\pi G \rho_0 r_0^2 / 3.$$  

A Michie–King model for a single stellar population in a dSph is defined by the three numbers ($\hat{\sigma}$, $\hat{r}_a$, $\hat{r}_t$) and two scalings set by the dark halo ($r_0$, $\rho_0$). A two-population model for a dSph therefore has $3 + 3 = 2 + 8$ free parameters.

Michie–King DFs are never tangentially anisotropic, and range from perfectly isotropic to highly radial. This does not seem unduly restrictive on examining the velocity dispersion profiles in Fig. 1, both of which decline with radius. This fact does not suggest any great preponderance of tangential bias in the orbits. The anisotropy profiles $\beta(r) = 1 - \sigma_t^2/(2\sigma_r^2)$ (where $\sigma_t^2 = \sigma_\theta^2 + \sigma_\phi^2$ is the tangential velocity dispersion) of Michie–King DFs are always isotropic at the centre, but can be adjusted at larger radii through the anisotropy radius $\hat{r}_a$. However, Michie–King models with the same anisotropy radius may have completely different degrees of anisotropy (depending on the values of the other two free parameters). Thus, in the following, we prefer to parametrize the anisotropy of our models using the value of the anisotropy profile at the half-light radius

$$\bar{\beta} = \beta(R_h).$$

Also, in order to avoid systems subject to radial orbit instability (e.g. Barnes, Hut & Goodman 1986), we only consider $\bar{\beta} \leq 1/2$.

### 3.3 The isothermal and the strongly truncated limits

To understand the properties of Michie–King models, it is helpful to distinguish two opposite physical regimes. The DFs can vary from exponential in the energy, typical of isothermal systems, to a power-law dependence in the ‘reduced’ energy $E - \Phi(r)$, typical of strongly truncated systems. Note that we are exploring a much wider range of Michie–King models than is usual in their most common application, the study of globular clusters.

In the isothermal limit, most of the stars are unaffected by the tidal cut in King’s DF (5), which is thus dominated by the exponential term

$$[\Phi(r_t) - \Phi(r)]/\sigma^2 \gg 1.$$  

An equivalent way to state this inequality is $\hat{R}_h \ll \hat{r}_t$. When this holds true, the dependence of the Michie–King DFs on energy $E$ and angular momentum $L$ becomes close to the linear combination $E + L^2/(2\sigma_r^2)$, which describes the Osipkov–Merritt case (Osipkov 1979; Merritt 1985). Thus, in this instance, the anisotropy asymptotes to the profile

$$\beta_{\text{OM}}(r) \sim -\frac{r^2}{r^2 + r_a^2}.$$  

This describes systems which are isotropic in the centre ($r < r_a$) and become more radially anisotropic at larger radii ($r > r_a$). The isotropic, and therefore fully isothermal limit, is equivalent to additionally imposing

$$\hat{R}_h / \hat{r}_a \ll 1.$$  

This corresponds to the case in which the half-light radius of the stars is smaller than the other length-scales in the problem. In this regime, isothermality ensures a flat velocity dispersion profile. Furthermore, if the stars are deeply embedded in their dark matter halo (i.e. $\hat{R}_h \ll 1$), then the stellar profile has a Gaussian density law if embedded in a cored halo, and an exponential one if embedded in an NFW halo. Amorisco & Evans (2011) show that these systems satisfy

$$R_h/r_0 \propto (\sigma_t^2/\Phi_0)^{1/\delta} \approx (\sigma_{los}^2/\Phi_0)^{1/\delta},$$  

where $\delta$ is the exponent of the first non-constant term in the Taylor expansion of the total potential at the centre. So, $\delta = 1$ for the NFW case, $\delta = 2$ for a cored dark halo.

The opposite of the isothermal regime is the case in which the exponential term in King’s DF (5) is of the order of unity. This limit describes strongly truncated systems, in which the dimensionless half-light radius $\hat{R}_h$ and the dimensionless tidal radius $\hat{r}_t$ are of the same order. The asymptotic expression for the anisotropy is

$$\beta_{\text{trunc}}(r) \sim \frac{2}{9} \frac{r^2}{r_t^2} \frac{\Phi(r) - \Phi(r_t)}{\sigma^2}.$$  

This is true for any Michie–King model close to its tidal edge. So, any Michie–King model becomes isotropic on approaching its tidal radius. This is evident from the anisotropy profiles plotted in Fig. 2.

Strongly truncated isotropic models are essentially scale-free models, whose properties depend only on the gravitational potential $\Phi(r)$ and the tidal radius $r_t$. The radial velocity dispersion is

$$\sigma_t^2(r) = \frac{2}{7}(\Phi(r_t) - \Phi(r)).$$  

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Figure 2. Anisotropy profiles of Michie–King models embedded in an NFW halo, all having $R_h = 0.2$ and $\bar{\beta} = 1/2$, but with different truncations. The dashed profile shows the Osipkov–Merritt limit (10).
which generates cut-offs in the line-of-sight velocity dispersion profiles. For the stellar density distribution, we have

\[ \rho_*(r) \propto (\Phi(r) - \Phi(r'))^{5/2}. \]

For fixed half-light radius \( \bar{R}_h \) and degree of radial anisotropy, the tidal radius realized by strongly truncated systems is the smallest in the entire family of Michie–King models with a given gravitational potential. Thus, we use this limit to define the function

\[ \bar{r}_t^{\text{min}}(\bar{R}_h, \beta) \equiv \lim_{\bar{\sigma} \to \infty} \bar{r}_t(\bar{\sigma}, \bar{R}_h, \beta). \]

It is often convenient to express the tidal radius as multiples of the minimum value defined by equation (16), that is

\[ n_t \equiv \frac{\bar{r}_t}{\bar{r}_t^{\text{min}}} \geq 1, \]

where \( n_t \) is the tidal number. This is useful in presenting our results, as \( 1/n_t \) lies between 0 and 1.

The Michie–King model for a single population embedded in a dark halo can be described by the natural set of three parameters, namely dimensionless velocity dispersion \( \bar{\sigma} \), tidal radius \( \bar{r}_t \) and anisotropy radius \( \bar{\rho}_a \). An alternative, but equivalent set, is the dimensionless half-light radius \( \bar{R}_h \), the tidal number \( n_t \) and the anisotropy parameter at the half-light radius \( \beta \). It is this set that we will use in the rest of the paper.

### 4 PROPERTIES OF THE MODELS

#### 4.1 Dark halo mass estimators

A main aim in studying stellar populations in dSphs is to map the underlying dark halo. A number of authors have recently suggested that the mass of the dark halo within a characteristic radius related to the half-light radius is well constrained (e.g. Walker et al. 2009; Wolf et al. 2010; Amorisco & Evans 2011). These mass estimators all take the general form (cf. Illingworth 1976)

\[ M(\lambda R_h) = \frac{K}{G} R_h \langle \sigma_{\text{los}}^2 \rangle, \]

where \( K \) and \( \lambda \) are two dimensionless constants and \( \langle \sigma_{\text{los}}^2 \rangle \) is an appropriate ‘average’ of the square of the line-of-sight velocity dispersion. A successful mass estimator needs to fix each of these three ingredients.

Let us begin with the average \( \langle \sigma_{\text{los}}^2 \rangle \). In the isothermal limit, the profile is almost flat by definition, so we do not need to worry at which radius we pick the line-of-sight velocity dispersion. When the dispersion profile falls with radius, it is a moot point as to how best to provide the ‘average’. This topic assumes some importance as it is the heart of the argument provided by Walker & Peñarrubia (2011) in their analysis.

To bring the problem of the ‘average’ into sharp focus, consider two stellar systems embedded in the same dark matter halo and with the same half-light radius, but characterized by a different DF in phase space, and thus having different kinematics and density profiles. For example, consider a radially and tangentially biased system. A proper recipe for an efficient mass estimator should pick the same ‘average’ velocity dispersion for both systems, though they will in general differ both in their velocity dispersion profile \( \sigma_{\text{los}}(R) \) and their surface brightness profile \( \Sigma(R) \). Wolf et al. (2010) suggested that a good choice is the luminosity-weighted average line-of-sight velocity dispersion \( \langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)} \). However, a radially biased model will have both higher velocity dispersion profile and higher surface brightness profile in the central regions and lower in the outer regions, as compared to the tangential model, which militates against the usefulness of a luminosity-weighted average.

Fig. 3 illustrates the dimensionless line-of-sight velocity dispersion profiles for models that span the entire physical range of Michie–King models, for NFW and cored haloes (upper and lower panels, respectively). The panels show models with different half-light radii, tidal radii and anisotropy. In each panel, all the models have the same dimensionless half-light radius \( \bar{R}_h \) and should then provide the same mass estimate and ‘average’. A quick look makes it evident that the easiest choice is the line-of-sight velocity dispersion at one \( \bar{R}_h \), where all the profiles roughly cross. The effectiveness of this choice is readily quantified. For three different values of the dimensionless half-light radius \( \bar{R}_h = 0.1, 0.3 \) and 1, Table 1 collates the relative uncertainty obtained by comparing the line-of-sight velocity dispersion at one half-light radius \( \delta_{\text{los}}(\bar{R}_h) \) of all possible Michie–King models (0 < \( \beta \) < 0.5, 0 < 1/n_t < 1). The same uncertainty is calculated for the luminosity averaged velocity dispersion \( \sqrt{\langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)}} \). Our recipe always performs better, with the luminosity-weighted average \( \sqrt{\langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)}} \) usually showing discrepancies higher than 4 per cent.

![Figure 3](https://example.com/figure3.png)

**Table 1.** The relative uncertainty obtained using the luminosity-weighted average or the value at \( \bar{R}_h \) in the mass estimator. In each instance, the upper line refers to NFW haloes, the lower to cored haloes.

| \( \bar{R}_h \) (per cent) | \( \bar{R}_h = 0.1 \) | \( \bar{R}_h = 0.3 \) | \( \bar{R}_h = 1 \) |
|---------------------------|-----------------|-----------------|-----------------|
| \( \delta_{\text{los}}(\bar{R}_h) \) | 1.6 | 1.5 | 1.7 |
| \( \sqrt{\langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)}} \) | 4.0 | 6.1 | 4.5 |
| \( \delta_{\text{los}}(\bar{R}_h) \) | 2.0 | 1.5 | 1.2 |
| \( \sqrt{\langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)}} \) | 5.9 | 4.6 | 3.7 |
Multiple stellar population in dSphs

4.2 A consistency criterion for two populations

As the two stellar populations reside in the same dark matter halo, their dimensionless half light radius \( R_h \) and line-of-sight velocity dispersion \( \sigma_{\text{los}}(R_h) \) must be picked from the same functional relation. In Fig. 4, this means that the ratios of the half-light radii and the velocity dispersion

\[
 k_R \equiv \frac{R_{h,2}}{R_{h,1}}, \quad k_\sigma \equiv \frac{\sigma_{\text{los},2}(R_{h,2})}{\sigma_{\text{los},1}(R_{h,1}}).
\]

are not independent quantities. They are related to one another through the density profile of the dark matter halo itself. In general, we must have

\[
\left( \frac{R_{h,2}}{R_{h,1}} \right)^4 \leq \left( \frac{\sigma_{\text{los},2}(R_{h,2})}{\sigma_{\text{los},1}(R_{h,1})} \right)^2 \Rightarrow k_R^4 \leq k_\sigma^2,
\]

with equality attained in the isothermal limit. This can be understood from Fig. 4, as the ratio \( R_{h,i}/\sigma_{\text{los},i}(R_{h,i}) \) is an increasing function of \( R_{h,i} \) and attains unity in the isothermal limit for strongly embedded systems.

Let us now specialize to the case of Sculptor. For Battaglia’s two-population dissection, using equation (1), we find that

\[
k_R = 0.64 \pm 0.05.
\]

This measure (with its uncertainty) and the relations in Fig. 4 (with their uncertainties) generate the black line (and grey shaded areas) in Fig. 5. All the pairs \( (R_{h,1}, k_\sigma) \) in the grey areas are compatible with an NFW halo (upper panel) or with a cored halo (lower panel) for some value of the halo’s scalelength \( R_0 \). Also displayed in the figure as the asymptotic dashed line is the isothermal limit.

Measurement of the ratio \( k_\sigma \) remains a challenge despite substantial improvements in the quality of the photometric and spectroscopic data in recent years. We interpolate both the velocity dispersion profiles, and the profiles defined by the uncertainties,
and obtain
\[ k_0 = 0.55 \pm 0.12. \] (24)
which is displayed in Fig. 5 as the yellow shaded area.

It is evident that the cored halo provides a much better interpretation of the available data, as there is a formal agreement between models and data, which is not the case for the NFW halo. Furthermore, we can extrapolate this result and state that any other cusped dark matter density profile with a cusp steeper than the NFW prescription is going to deliver even worse results.

In passing, we note that our consistency criterion (22) can be reformulated to provide the one used by Walker & Peharrubia (2011). In fact, if two populations fail to satisfy the inequality (22) for a given \( \delta \), then the logarithmic ‘slope’ \( \Gamma \), estimated using equation (18), will satisfy the inequality:
\[ \Gamma = \frac{\ln \left[ M(\chi, R_{\chi,2})/M(\chi, R_{\chi,1}) \right]}{\ln \left( R_{\chi,2}/R_{\chi,1} \right)} = 1 + 2 \ln k_0 / \ln k_\chi \geq \delta + 1, \] (25)
thus confirming that such a system is inconsistent with the density profile \( \rho \sim r^{-2} \).

### 5 PHASE-SPACE ANALYSIS: SCULPTOR

To perform a complete maximum likelihood analysis of the Sculptor data using Michie–King DFs necessitates an exploration of an eight-dimensional parameter space. In fact, the main body of this analysis can be successfully reduced to the parallel and independent studies of the two metal-poor and metal-rich stellar populations, ignoring at first that they reside in the same dark matter halo. The combination of the evidence coming from the two stellar populations then constrains the range of feasible dark matter halos.

The details of the fitting of the Michie–King models to the kinematics, the photometry and – in the case of the metal-poor population – the tidal radius are given in Appendix A. Here, we note that the fit to the metal-poor population yields a \( \chi^2_L \), \( \chi^2_r \) and \( \chi^2_0 \), whilst the metal-rich yields has only a \( \chi^2_L \) and \( \chi^2_0 \). This is because the tidal radius of the metal-rich population is free to vary, whilst that for the more extended metal-poor population must be the same as the underlying dark halo.

The combination of all the available evidence for the metal-poor stellar population is provided by the likelihood and associated \( \chi^2 \) values:
\[ L_{\text{MP}}(R_0, n, \beta, r_0, \rho_0) = L_X L_{\sigma} L_\rho, \]
\[ \chi^2_{\text{MP}} = \chi^2_L + \chi^2_r + \chi^2_0, \] (26)
However, for the sake of clarity, we will also consider the reduced likelihood
\[ L_{\text{MP}}^{\text{red}}(R_0, n, \beta, r_0, \rho_0) = L_X L_{\sigma}, \] (27)
partly to check that the fitting of the tidal radius does not have an undue and distorting effect on our results. Similarly, for the metal-rich population, we can define
\[ L_{\text{MR}}(R_0, n, \beta, r_0, \rho_0) = L_X L_\rho, \]
\[ \chi^2_{\text{MR}} = \chi^2_L + \chi^2_0. \] (28)

#### 5.1 Independent analysis of the two populations

Table 2 collects the results pertaining the metal-poor and metal-rich stellar populations when fitted separately, thus allowing each of them to have different dimensional scales \( r_0 \) and \( \rho_0 \). For each of

| Halo Type | \( \chi^2_L \) | \( \chi^2_r \) | \( \chi^2_0 \) |
|----------|----------------|----------------|----------------|
| NFW      | 39.3, 41.5, 45.7 | 48.2, 66.9, 68.3 | 49.0, 67.8, 69.7 |
| Cored    | 5.6, 9.1        | 3.4, 11.1, 13.8 | –              |
| NFW      | 4.0, 5.5, 6.0   | 0.9, 1.2, 1.39  | 0.4, 1.2, 1.38  |
| Cored    | 1.1, 3.3, 4.9   | 2.1, 6.8, 9.9   | –              |

The probability distributions of the free parameters of the metal-poor stellar population are displayed.

A comparison between the values for an NFW and cored halo makes clear that the main difference is in the reproduction of the stellar surface brightness distribution, with the cored halo performing substantially better. Also, the fact that the increase in \( \chi^2_r \) is very similar for an NFW and cored halo shows that the shape of the velocity dispersion profile is largely insensitive to the dark halo properties.

For the metal-poor (metal-rich) population, Fig. 6 displays in blue (red) the probability distributions of the free parameters of the Michie–King models, as deduced from the projection of the total likelihood. First, while the cored halo selects a Michie–King model with a similar scalelength (\( R_0 \approx 1 \)), an NFW halo needs to be several times more extended (\( R_0 \leq 0.2 \)) than the distribution of stars in order to fit (yet with difficulties) the surface brightness profile of the metal-poor stars. Also, in the cored case, an almost isotropic metal-poor stellar population is preferred (\( \beta \approx 0 \)), whilst an NFW halo favours a mild radial velocity bias (\( \beta \approx 0.25 \)). The metal-rich population requires a high degree of radial anisotropy (\( \beta \gtrsim 0.4 \)), irrespective of choice of halo. Note also the necessity of different tidal radii in the phase-space distributions is supported by the results. In both cases, \( n_i \gtrsim 1.4 \), suggesting that the metal-rich stellar population is indeed in a different physical regime compared to the metal-poor one, which has \( n_i \lesssim 3 \). Finally, the probability distributions for the dimensionless half-light radius \( R_0 \) of the metal-rich population show that little help in determining the characteristic radius of the halo can be expected from a population in this heavily truncated regime.

#### 5.2 Two populations in the same halo

Figs 7 and 8 display the projections of our likelihoods on to the halo plane (\( r_0, \rho_0 \)), for an NFW and a cored halo, respectively. In both figures, the upper panel shows the 68 and 95 per cent confidence regions associated with the separate analysis of the two stellar populations with blue (red) representing the metal poor (metal rich), respectively. The full thick blue and red lines in the upper panels of both figures show the trends of the \( \rho_0 (r_0) \) functional relations obtained, respectively, for the metal-poor and metal-rich stellar populations.

---

Table 2. Results of the independent analysis of the metal-poor (upper) and metal-rich (lower) stellar component. The table gives the values of the \( \chi^2 \)-quantities referring, in order, to the best-fitting models, to the 68 and 95 per cent confidence regions, which have been calculated using the likelihoods in equations (A3, A5, A9). Also the corresponding reduced-\( \chi^2 \)-quantities are reported in the second line for each halo model. Uniform priors have been assumed in the range defined by \( 0 < \beta < 2, 0 \leq n_i < 1, 0 < \lambda R_\text{t} < 1.0 \), as for \( r_0 \) and \( \log \rho_0 \).
The probability distributions associated with the likelihoods $L_{\text{MP}}$ and $L_{\text{MR}}$ for the free parameters of the model when embedded in a NFW (left) and cored (right) halo. The blue (red) profiles are associated with the metal-poor (metal-rich) stellar component. Dashed and full blue profiles show the results deriving from the total likelihood (26) and the ‘reduced’ likelihood (27).

Finally, Table 3 collects, for an NFW and cored halo, the values of $\chi^2_{\text{tot}}$ associated with the best-fitting models, the details of which we report here. For an NFW halo, we find

$$(r_0, \rho_0) = (1.4 \text{ kpc}, 2.3 \times 10^7 M_\odot \text{ kpc}^{-3})$$

$m_1 = (\tilde{R}_h = 0.256, \ n_1 = 10.0, \ \tilde{\beta} = 0.20)$,

$m_2 = (\tilde{R}_h = 0.136, \ n_1 = 1.1, \ \tilde{\beta} = 0.37)$.  

(30)

This means that the Michie–King DF for the metal-poor population has a tidal radius of $r_t$ of 12.2 kpc, an anisotropy radius $r_a$ of 720 pc and a velocity dispersion $\sigma$ of 11.0 km s$^{-1}$. The same quantities for the metal-rich population are $r_t$ of 750 pc, $r_a$ of 140 pc and a velocity dispersion $\sigma$ of 15.6 km s$^{-1}$. For a cored halo, we find $r_t = (0.36 \text{ kpc}, 2.1 \times 10^7 M_\odot \text{ kpc}^{-3})$,

$m_1 = (\tilde{R}_h = 0.98, \ n_1 = 6.7, \ \tilde{\beta} = 0.1)$,

$m_2 = (\tilde{R}_h = 0.54, \ n_1 = 1.1, \ \tilde{\beta} = 0.48)$.  

(31)

In physical units, the Michie–King DF for the metal-poor population has a tidal radius of $r_t$ of 12.2 kpc, an anisotropy radius $r_a$ of 720 pc and a velocity dispersion $\sigma$ of 11.0 km s$^{-1}$. The same quantities for the metal-rich population are $r_t$ of 750 pc, $r_a$ of 140 pc and a velocity dispersion $\sigma$ of 15.6 km s$^{-1}$. For a cored halo, we find $r_t = (0.36 \text{ kpc}, 2.1 \times 10^7 M_\odot \text{ kpc}^{-3})$,

$m_1 = (\tilde{R}_h = 0.98, \ n_1 = 6.7, \ \tilde{\beta} = 0.1)$,

$m_2 = (\tilde{R}_h = 0.54, \ n_1 = 1.1, \ \tilde{\beta} = 0.48)$.  

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$m_1 = (\tilde{R}_h = 0.98, \ n_1 = 6.7, \ \tilde{\beta} = 0.1)$,

$m_2 = (\tilde{R}_h = 0.54, \ n_1 = 1.1, \ \tilde{\beta} = 0.48)$.  

(31)
in Section 2). Furthermore, although not reported here in detail, we investigated the issue of a milder cusp by considering separately the $\lambda = 1/2$ case. A milder cusp performs better than the NFW halo, but we can still reject it in favour of a cored halo at any significance level higher than 1.4 per cent (we find a best-fitting total $\chi^2$ of 52.3). All available indications suggest a monotonic behaviour of the best-fitting total $\chi^2$ with the cusp index.

Furthermore, the best-fitting NFW model (30), together with the parts of the halo ($r_0, \rho_0$) plane supported by the joint evidence $L_{tot}$, sit in a region in discord with the standard expectation of cosmological models. The concentration of the best-fitting NFW halo is in fact as low as $c \approx 17$, whereas the present-day dwarf galaxies are expected to have higher concentrations, $c \approx 35$ according to Colín et al. 2004 (but see also Macciò et al. 2007).

Figs 9 and 10 show the quality of the final fits to the surface brightness profiles and velocity dispersion profiles respectively in the case of an NFW halo and a cored halo. The full profiles are the best joint fits, that is obtained by using the same ($r_0, \rho_0$) halo for both the metal-poor and the metal-rich stellar population. The dashed profiles show instead the best separate fits. It is evident from these plots that the real damage to the prospects of fitting the data with an NFW halo come from two sources: it is the combination of photometry and kinematics that does the damage. First, although the properties of the metal-poor population are reproduced reasonably well, the metal-rich population is predicted to have a higher central surface density than the data. Secondly, the NFW halo requires a smaller difference than the observed one between the values of the metal-poor and metal-rich velocity dispersion in order to reproduce the observed half-light radius. This smaller difference is a rephrasing of the inconsistency found in Fig. 5 through our consistency criterion (22). However, note that the addition of the information about the surface brightness profiles of the two stellar populations – which was not included in the arguments presented in Section 4 – has in fact changed the range of best-fitting cored models. While the consistency criterion alone would select haloes with a larger scale-length ($R_h \lesssim 0.5$, see Fig. 5), the complete maximum likelihood analysis prefers models in which, approximately, mass follows light ($R_h \approx 1$).

### 6 THE MASS PROFILE FOR SCULPTOR

Fig. 11 shows the final mass profiles for the Sculptor dSph. In both upper and lower panel, the coloured shaded areas represent, at each radius, the $1\sigma$ uncertainty obtained for the mass profile using the joint evidence $L_{tot}$, for the case of an NFW halo (in green) and for a cored halo (in yellow).

In the cored case, we expect to be able to measure a mass profile rather than a single mass enclosed within some radius. This is because we are able to fix the best ($r_0, \rho_0$) in the halo plane. What is more surprising is that the same thing is true for the NFW halo, even though we have only a lower limit for the characteristic radius $r_0$. However, this can be understood by noticing that, as anticipated in Amorisco & Evans (2011), the mass profile $M(r, r_0)$ generated by the functional relation $\rho_0(r_0)$ becomes insensitive to the characteristic radius $r_0$ when $R_h \ll 1$.

The points displayed in the upper panel show how our results compare with previous mass measures for Sculptor by Strigari et al. (2007, 2008), Walker et al. (2009) and Amorisco & Evans (2011). The lower panel is a zoom for a smaller range of radii. We draw attention to the fact that, even if the matter of a core or cusp at the centre is considered unresolved, we are still able to measure a mass profile rather than the single enclosed mass data point at a particular

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**Table 3.** The values of $\chi^2_{tot}$ associated with the best-fitting halo, as well as the 68 and 95 per cent confidence regions, determined according to the likelihood (29).

|        | $\chi^2_{tot}$ | $\chi^2_{bf}$ |
|--------|----------------|---------------|
| NFW    | 58.5           | 78.1, 80.2    |
| Cored  | 46.3           | 62.2, 68.0    |

In both Figs 7 and 8, only marginal agreement can be found between the 68 per cent confidence region associated with the metal-poor and the metal-rich stellar populations. However, the importance of this is minimal in the case of a cored halo, since it does not cause any significant increase in the area of the confidence regions (or in the $\chi^2$ values) associated with the joint likelihood. Note, in fact, that the values of total $\chi^2_{tot}$ associated with the joint evidence $L_{tot}$, do prefer a dark matter density profile with a constant density core.

Following Eadie et al. (1971), we define a set of models with nested parameters, by considering haloes with a $1 - \lambda$ cusp and a density fall-off $\rho(r) \sim r^{-\lambda}$ at large radii. This obviously includes the cusped NFW density profile ($\lambda = 0$) and the cored profile ($\lambda = 1$). Since the difference in best-fitting total $\chi^2$ values between the NFW case and the cored case is $58.5 - 46.3 = 12.2$, we can reject a pure NFW density profile at any significance level higher than 0.05 per cent. Completely analogous results have been obtained when the original kinematic profiles taken from Battaglia et al. (2008) have been replaced with kinematic profiles derived directly from the radial velocity data set using slightly different metallicity cuts (within 0.1 [Fe/H]) or membership criteria (see also discussion of 52.3).
Figure 9. The photometric and kinematic profiles for the two Sculptor populations in an NFW halo. Thick profiles: final fits provided by the best-fitting Michie–King models associated with the black dot in Fig. 7. Dashed profiles: best fits provided by the Michie–King models, but when \((r_0, \rho_0)\) is allowed to vary separately for the two stellar components. Surface brightness profiles are measured in stars arcmin\(^{-2}\).

Figure 10. As Fig. 9, but for a cored halo.

radius. The mass profiles pertaining to the cored and NFW halo, in fact, agree within their 1σ uncertainty over the entire interval \(200 \lesssim r \lesssim 1.2\) kpc, thus proving a wide and unprecedented radial coverage. The pink shaded area represents the best-fitting cored halo as reported in Battaglia et al. (2008a).

Also displayed in the lower panel of Fig. 11 are the two mass estimates obtained using the formula suggested by Wolf et al. (2010), and applied separately to the metal-poor and metal-rich stellar populations. We calculate each of the two luminosity-averaged velocity dispersions \(\sqrt{\langle \sigma_{\text{los}}(R)^2 \rangle_{\Sigma(R)}}\) using the observed profiles and a Monte Carlo technique which assumes Gaussian uncertainties. Note that the logarithmic slope of the mass profile we would deduce from such a technique is steeper than the one we actually measure.

7 CONCLUSIONS

The suggestion that stellar kinematics in dwarf spheroidals (dSphs) may enable the structure of the dark matter halo to be mapped out is at least a decade old. It motivated programmes to gather radial velocities in the bright dSphs with 4- and 8-m class telescope and multi-object spectrographs. Despite a lot of work, it has proved hard to distinguish the structure of the dark matter halo, especially at small radii, where cold dark matter (CDM) theories predict 1/r density cusps (e.g. Navarro, Frenk & White 1996).

For example, Gilmore et al. (2007) provided a succinct summary of the then available observational evidence. They noted that analyses based on the Jeans equations for six dSphs (Draco, Ursa Minor, Carina, Sextans, Leo I and Leo II) tended to suggest that the dark matter density was shallower than the 1/r density cusp. However, they acknowledged that the observed velocity dispersion profiles and cored light distributions were likely to be consistent with dark matter haloes with both central cores and cusps.

A thorough examination of the evidence using the Jeans equations for the eight brightest dSphs (the six studied by Gilmore et al. together with Fornax and Sculptor) was carried out by Walker et al. (2009) using data sets of thousands of stars. They found that the most meaningful constraint provided by a Jeans analysis was on the total mass enclosed within the half-light radius (see also Wolf et al. 2010). In particular, the Jeans equations were not sufficiently powerful to distinguish between cored and cusped dark matter haloes. A companion theoretical investigation by Evans et al. (2009) made explicit the limitations of the Jeans equations. They showed how assumptions as to the surface brightness profile and velocity anisotropy could allow the misleading identification of a core or a cusp. None the less, this is a consequence of the interlocking assumptions made in the Jeans analysis, and not the data, which are consistent with both cores and cusps.

However, in at least two dSphs, there appears to be direct evidence favouring dark haloes with constant density cores. First, Kleya...
et al. (2003) discovered kinematically cold substructure in the Ursa Minor dSph. They argued from numerical simulations that the persistence of such cold substructure is incompatible with the cusped profiles of CDM cosmogonies. They provided an interpretation in terms of a stellar cluster dissolving in a cored dark matter halo. Secondly, Goerdt et al. (2006) and Sánchez-Salcedo, Reyes-Iturbide & Hernandez (2006) studied the five globular clusters present in the Fornax dSph, which orbit at projected radii \( \sim 1 \) kpc from the centre. In a cored dark matter halo, they would sink to the centre under the action of dynamical friction in a few Gyr. In a cored dark matter halo, the dynamical friction time-scale is much longer and so this provides a natural resolution of the survival problem of Fornax’s globular clusters.

The exploration of multiple stellar populations in dwarf spheroidals is just the beginning. But there are now clear hints that it might provide a resolution of this problem. The fact that both populations reside in the same dark matter halo significantly reduces the degeneracies. To date, there have been three different attempts at modelling – each with their own strengths and limitations. It is striking that all three have come to the conclusion that the data are much more consistent with cored dark matter haloes than cusped one.

Battaglia et al. (2008) investigation of the case of Sculptor used the Jeans equations and came to the conclusion that the data are statistically consistent with both cored and cusped haloes, though the NFW profile yields a poorer fit for the metal-rich stars. The drawback to this analysis is that it is based on the Jeans equations alone, and there is no guarantee that the underlying models exist. In fact, the central velocity dispersion proved by An & Evans (2009) shows that an isotropic cored stellar density profile cannot be embedded in an NFW halo, and so there is no solution for the phase-space DF for the metal-poor population, despite the existence of the Jeans solution. None the less, the basic conclusion of Battaglia et al. (2008) – that a cored halo model provides a better fit than an NFW halo model – seems robust and is recovered by our analysis.

Walker & Peñarrubia (2011) developed a sophisticated statistical method for assigning probabilities as to whether dSph stars belong to metal-rich and metal-poor subpopulations. This is a significant improvement on the simple cut in metallicity used by Battaglia et al. (2008), which can introduce biases when metallicity is correlated with kinematics. As output from their statistical analysis, they obtain a measure of the average velocity dispersion of the metal-poor and metal-rich populations. This is used with a mass estimator of the form (18) to obtain the mass enclosed at two points, namely the half-light radius of the metal-poor and the metal-rich populations. From this, the gradient of the dark halo mass profile is calculated. For the Sculptor and Fornax dSphs, they conclude that cusped NFW profiles are ruled out. The modelling is then simpler than a Jeans analysis, but Walker & Peñarrubia (2011) argue that it is more robust. One qualm is that the method assumes that the two components are drawn from constant velocity dispersion populations, both in the statistical analysis itself and in the application of the mass estimator formula. Whilst this seems reasonable enough for the metal-poor populations in dSphs, it is not at all clear that the metal-rich population can be so described. In fact, the combined or total velocity dispersion of the total population must then decline outwards, which is certainly not the case in, at least, Fornax. None the less, Walker & Peñarrubia have tested their reasoning against synthetic data sets generated from a wide range of phase-space DFs, and it performs very well.

Finally, in this paper, we have used the Sculptor data set provided by Battaglia et al. (2008) to build phase-space DFs for both the metal-poor and the metal-rich populations under the assumption of cored and cusped dark matter haloes. Although both can fit the data, cored dark matter haloes are preferred. There is clear evidence of discrepancies in the surface brightness profile of the metal-rich stars near the centre for cusped haloes. Note that it is the combination of surface photometry with the stellar kinematics – rather than the kinematics alone – that is providing the decisive evidence against cusped haloes. Even more worryingly, the NFW models that are the best fits are much less concentrated than the cosmological predictions from CDM theories. Typically, concentrations of \( c \approx 35 \) are predicted from simulations of galaxy formation (e.g. Colin et al. 2004), as compared against \( c \approx 17 \) found in our analysis.

The fact that all three analyses – which make different assumptions and have different strengths and weaknesses – come to the same conclusion is very striking. Further modelling of other dSphs with multiple populations is needed to confirm the results. However, the evidence that the dark haloes of the dSphs around the Milky Way do not have cusped form, but have cores, is now beginning to look rather strong. Understanding the origin of these constant density dark matter cores could be one of the most exciting challenges facing astronomy in the next few years.

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Multiple stellar population in dSphs

Wolf J., Martínez G. D., Bullock J. S., Kaplinghat M., Geha M., Muñoz R. R., Simon J. D., Avedo F. F., 2010, MNRAS, 406, 1220

APPENDIX A: FULL PHASE-SPACE ANALYSIS: THE TECHNIQUE

A1 The surface brightness profile

For any population defined by a Michie–King model \((\tilde{R}_0, n, \tilde{\beta})\) in a dark halo characteristic radius \(r_0\), we can define the quantity

\[
X^2 = \frac{\sum_i \chi_i}{\text{Var} \chi_i} = \frac{\sum_i \left[ \chi_i^2 - \chi_i \right]}{\text{Var} \chi_i}.
\]

(A1)

Here, the collection of points \((R_i, \Sigma_i^m(R_i))\) is the observed surface brightness profile, each of the points having its variance \(\text{Var} \Sigma_i^m(R_i)\). The function \(\Sigma_i^m(R_i)\) is dimensionless surface brightness of the Michie–King model with parameters \((\tilde{R}_0, n, \tilde{\beta})\) at the dimensionless point \(\tilde{r}\) (measured in units of the halo scalelength). This surface brightness is made comparable with the data by the dimensional coefficient \(\Sigma_i^m\), which is determined in such a way to a give the smallest \(X^2\):

\[
\partial X^2 / \partial \Sigma_i^m = 0.
\]

We associate the quantity \(X^2\) to the standard likelihood

\[
L_\Sigma(\tilde{R}_0, n, \tilde{\beta}, r_0) = \exp \left(-X^2/2\right).
\]

(A3)

A2 The velocity dispersion profile

Similarly, for any triple \((\tilde{R}_0, n, \tilde{\beta})\) and any pair of dimensional scales \(r_0, \rho_0\), we define the quantity

\[
X^\sigma = \frac{\sum_i \sigma_i^2}{\text{Var} \sigma_i} = \frac{\sum_i \left[ \sigma_i^2 - \langle \sigma_i \rangle \right]}{\text{Var} \sigma_i}.
\]

(A4)

Here, the collection of points \((R_i, \sigma_i^m(R_i))\) is the observed projected velocity dispersion profile, each of the points having its variance \(\text{Var} \sigma_i^m(R_i)\). The function \(\sigma_i^m(R_i)\) is the dimensionless projected velocity dispersion profile of the Michie–King model with parameters \((\tilde{R}_0, n, \tilde{\beta})\) at the dimensionless point \(\tilde{r}\) (measured in units of the halo scalelength). This velocity dispersion is made comparable with the data by the square root of the potential term \(\Phi_i\), equation (7), and in which the dimensional scales \(r_0\) and \(\rho_0\) enter. We associate the quantity \(X^\sigma\) to the standard likelihood

\[
L_\sigma(\tilde{R}_0, n, \tilde{\beta}, r_0, \rho_0) = \exp \left(-X^\sigma/2\right).
\]

(A5)

A3 The tidal radius of the metal-poor stellar component

As explained in Section 3.2, the two stellar components are allowed to have different tidal radii. Recall that we interpret the tidal radius of the metal rich as an approximate cut in energy of its phase-space distribution. On the other hand, the tidal radius of the metal-poor stellar component is the true tidal radius of the dSph within the gravitational potential of the Galaxy. At this radius, the density distribution of the halo is truncated too, according to the prescription of equation (2). Hence, such a tidal radius has to satisfy, at least approximately, the Roche criterion

\[
M_{\text{dSph}}(r_t) = \frac{M_{\text{MW}}(D - r_t)}{(D - r_t)^3}.
\]

(A6)
where \( \mathcal{M}_{\text{dSph}}(r) \) is the mass profile of the dwarf, \( \mathcal{M}_{\text{MW}}(r) \) is the mass profile of the Galaxy and \( D \) is their separation.

Suppose we have fixed a Michie–King model \((\tilde{R}_h, n_t, \tilde{\beta})\) and the pair of dimensional scales \( r_0, \rho_0 \). Then the tidal radius of the model is

\[
r_t^m(\tilde{R}_h, n_t, \tilde{\beta}, r_0) = n_t r_t^{\text{min}}(\tilde{R}_h, \tilde{\beta}) r_0.
\]  

(A7)

We denote this by \( r_t^m(\tilde{R}_h, n_t, \tilde{\beta}, r_0, \rho_0) \) and quantify the discrepancy using

\[
\chi^2_t = \frac{(r_t^m - r_t^K)^2}{(\delta r_t^K)^2}.
\]  

(A8)

This compares the two different determinations of the tidal radius \( r_t^m \) and \( r_t^K \). The uncertainty \( \delta r_t^K \) takes into account the uncertainty on the mass profile of the Galaxy, for which we allow the interval \( 2 \gtrsim \mathcal{M}_{\text{MW}}/(10^{12} \, M_\odot) \gtrsim 0.5 \). We finally associate the quantity \( \chi^2_t \) to the standard likelihood via

\[
L_t(\tilde{R}_h, n_t, \tilde{\beta}, r_0, \rho_0) = \exp \left( -\frac{\chi^2_t}{2} \right).
\]  

(A9)

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