Quark to $\Lambda$-hyperon spin transfers in the current-fragmentation region

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Abstract

We perform a study on the struck quark to the $\Lambda$-hyperon fragmentation processes by taking into account the anti-quark fragmentations and intermediate decays from other hyperons. We concentrate on how the longitudinally polarized quark fragments to the longitudinally polarized $\Lambda$, how unpolarized quark and anti-quark fragment to the unpolarized $\Lambda$, and how quark and anti-quark fragment to the $\Lambda$ through the intermediate decay processes. We calculate the effective fragmentation functions in the light-cone SU(6) quark-spectator-diquark model via the Gribov-Lipatov relation, with the Melosh-Wigner rotation effect also included. The calculated results are in reasonable agreement with the HERMES semi-inclusive $ep$ experimental data and the OPAL and ALEPH $e^+e^-$ annihilation experimental data.

In high energy physics, the current-fragmentation (CF) region in lepton-hadron semi-inclusive deep inelastic scattering (SIDIS) is sensitive to the quark distributions and fragmentations. In this kind of processes, the colored parton inside the target is struck with great momentum and then quickly fragments into final hadrons. If we make cross section measurement of one of the final hadrons, both the target parton distribution functions (PDFs) and the quark to final hadron fragmentation functions (FFs) can be extracted. In the study of the proton spin substructure, the $\Lambda$-hyperon among the produced hadrons is suggested to be studied $^1$. This is mainly due to the facts that the $\Lambda$-hyperon has relatively large production cross section and that its polarization is self-analyzing owing to its characteristic decay mode $\Lambda \to p\pi^-$ with a large branching ratio of 64%.

In the naive quark model, the spin of the $\Lambda$-hyperon is carried by the $s$ quark and the $u,d$ quarks inside the $\Lambda$ formulate a spin and isospin zero state. If this correlation conserves in the current-fragmentation process, it is reasonable to speculate that the total spin transfer from the $u,d$ quarks to the $\Lambda$-hyperon is zero. However, the data from the deep inelastic scattering experiment imply that the spin transfers are none zero from the struck $u,d$ quarks to the produced $\Lambda$-hyperon. If this property can be carried

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over to the quark distributions inside the $\Lambda$-hyperon, it means that the quark distributions of the $\Lambda$-hyperon are more interesting than the naive quark model predicted. Previous studies discussed this issue and made various PDF and FF predictions for the $\Lambda$-hyperon [7–20]. Experimental data also indicate that in the current-fragmentation region, the $\Lambda$-hyperon may be produced through the intermediate decay processes of other hyperons. Besides, we know that in the small $x$ region the sea quark distributions are dominating over the valence quark distributions inside the proton. If this correlation keeps in the fragmentation process, the small $z$ region of the $\Lambda$ production cross section should be sensitive to the probability of the anti-quark distribution in the target particle and the probability of the anti-quark fragmentation into the $\Lambda$. There have been relevant discussions concerning anti-quark fragmentations [21, 22] and intermediate decays [23].

In this Letter, we provide a first study combining both the intermediate decay processes and the anti-quark fragmentation processes in the $\Lambda$ fragmentation process. From QCD factorization theorem, the high energy collision cross section can be calculated by using the perturbation theory complemented with the soft QCD effects embedded in quark distributions and fragmentation functions, which are process insensitive and universal. If we take the $eP \to e\Lambda X$ process to extract the fragmentation functions (FFs) using the factorization theorem, the same FFs should be applicable to the $e^+e^-$ annihilation process.

For a general $eP \to eP_h X$ process, the differential scattering cross section at the tree level can be effectively expanded as

$$d\sigma = \frac{1}{4e^4 P \cdot q} \frac{d^3 P}{(2\pi)^3 2E_x} \delta^4(P + \ell - P - P_h - \ell') \times \left\{ \frac{e^4}{Q^4} \left[ \sigma_{\ell'}(\ell', s_{\ell'}) \gamma_{\mu}u_{\ell}(\ell, s_{\ell'}) \right] \left[ \sigma_{\ell}(\ell, s_{\ell}) \gamma_{\nu}u_{\ell'}(\ell, s_{\ell'}) \right] \times \langle X, P_h S_h | J^\mu(0) | PS \rangle \langle X, P_h S_h | J^\nu(0) | PS \rangle \right\} \frac{d^3 \ell'}{(2\pi)^3 2E'} \frac{d^3 P_h}{(2\pi)^3 2E_h}, \tag{1}$$

where $L_{\mu\nu}$ and $W_{\mu\nu}$ are the leptonic tensor and the hadronic tensor respectively.

Defining three Lorentz invariants

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot \ell'}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \tag{2}$$

we rewrite the cross section (1) as

$$\frac{d\sigma}{dx dy dz d^2 P_{h,\perp}} = \frac{\pi a_{em}^2 y}{2Q^4 z L_{\mu\nu} W_{\mu\nu}}, \tag{3}$$

where $P_{h,\perp}$ is the transverse momentum of the produced hadron.
Then in the tree level, the leptonic tensor can be decomposed into a symmetric and an antisymmetric part as

\[ L_{\mu\nu} = \sum_{s_{\ell'}} \left[ \Pi_{\ell'}(\ell', s_{\ell'}) \gamma_{\mu} u_{\ell}(\ell, s_{\ell}) \right] \left[ \Pi_{\ell'}(\ell', s_{\ell'}) \gamma_{\nu} u_{\ell}(\ell, s_{\ell}) \right] = 2(\ell_{\mu} \ell'_{\nu} + \ell_{\nu} \ell'_{\mu} - g_{\mu\nu} \ell \cdot \ell') + 2i\epsilon_{\mu\nu\rho\sigma} \ell^\rho q^\sigma. \] (4)

In the parton model, the hadronic tensor is a convolution of PDFs and FFs. At the twist two level, if we use a polarized electron beam to hit an unpolarized proton target, both the unpolarized quark fragmentation function and the helicity-dependent quark twist two level, if we use a polarized electron beam to hit an unpolarized proton target, an integration over the longitudinal spin transfer factor. Previous work reduced the common factor in Eq. (6) and

\[ W_{\mu\nu} = \frac{1}{(2\pi)^2} \sum_X \frac{d^3 \tilde{P}_X}{(2\pi)^2 2E_X} \delta^4(P + \ell - P_X - P_h - \ell') \langle X, P_h S_h | P(0) \rangle \langle X, P_h S_h | P'(0) \rangle PS \]

\[ = \sum_a e_a^2 \int \frac{dk^- d^2 \vec{k}_T}{(2\pi)^3} \int \frac{dk^+ d^2 \vec{k}_T}{(2\pi)^3} \delta^4(k_T - \vec{q}_T - \vec{k}_T) \]

\[ \text{Tr} \left[ \frac{1}{2} f_a(x, \vec{k}_T^2) \gamma_{\mu} \frac{1}{2}(D_a(z, \vec{k}_T^2)B_h + \lambda_h \Delta D_a(z, \vec{k}_T^2)\gamma_5 B_h) \right]_{\lambda^- = xP^-, x = P_h'}, \] (5)

where \( f_a(x, \vec{k}_T^2) \) is the unpolarized quark distribution in the proton, and \( D_a(z, \vec{k}_T^2) \) and \( \Delta D_a(z, \vec{k}_T^2) \) indicate the probability of an unpolarized quark fragments into an unpolarized hadron and the probability of a longitudinally polarized quark into a longitudinally polarized hadron respectively.

The helicity asymmetry cross section is then obtained as

\[ A(x, y, z) = \frac{d\sigma_{\parallel} - d\sigma_{\perp}}{d\sigma_{\parallel} + d\sigma_{\perp}} = \frac{4m_{\pi}^2 x y (1 - y/2) f_a(x, Q^2) \Delta D_a(z, Q^2)}{4m_{\pi}^2 x y (1 - y/2) f_a(x, Q^2) D_a(z, Q^2)} = \frac{y(2 - y) \sum_a e_a^2 x f_a(x, Q^2) \Delta D_a(z, Q^2)}{1 + (1 - y)^2 \sum_a e_a^2 x f_a(x, Q^2) D_a(z, Q^2)}, \] (6)

From Eq. (6), we take the part

\[ A(x, z) = \frac{\sum_a e_a^2 x f_a(x, Q^2) \Delta D_a(z, Q^2)}{\sum_a e_a^2 x f_a(x, Q^2) D_a(z, Q^2)} \] (7)

as the longitudinal spin transfer factor. Previous work reduced the common factor \( x \) in Eq. (7), which is reasonable in present experimental region. However, if the experimental measurement can make bins with broad \( x \), theoretical calculation should make an integration over \( x \). This means that the \( x \) factor in Eq. (7) should not be neglected.
A Monte Carlo calculation using the LEPTO generator indicates that only about 40\%-50\% of $\Lambda$‘s are produced directly, 30\%-40\% originate from $\Sigma^*(1385)$ decay and about 20\% are decay products of the $\Sigma^0$. The COMPASS Collaboration measured the relative weights of the $\Sigma^*$ and the $\Xi$-hyperon decaying to the $\Lambda\pi$. The results are about 20\% smaller than the Monte Carlo calculation \cite{24}.

Effectively, we can rewrite the helicity-dependent fragmentation function $\Delta D_q(z, Q^2)$ and the unpolarized fragmentation function $D_q(z, Q^2)$ of the $\Lambda$ as

$$\Delta D^\Lambda_q(z, Q^2) = a_1 \Delta D^\Lambda_q^{\text{direct}}(z, Q^2) + a_2 \Delta D^\Sigma_q(z, Q^2)\alpha_{\Sigma\Lambda} + a_3 \Delta D^\Xi_q(z, Q^2)\alpha_{\Xi\Lambda},$$

(8)

and

$$D^\Lambda_q(z, Q^2) = a_1 D^\Lambda_q^{\text{direct}}(z, Q^2) + a_2 D^\Sigma_q(z, Q^2) + a_3 D^\Xi_q(z, Q^2) + a_4 D^\Xi_q(z, Q^2).$$

(9)

Here, the weight coefficients are adjusted as

$$a_1 = 0.4, \ a_2 = 0.2, \ a_3 = 0.3, \ a_4 = 0.1,$$

(10)

based on the spirit of the Monte Carlo prediction.

In the specific calculation, the weight coefficients of the $\Sigma^*$ is divided into three types of particles, that is $\Sigma^*(1385)$, $\Sigma^0(1385)$ and $\Sigma^-(1385)$. So the contribution to the spin transfer from the $\Sigma^*$ is actually a mixture of these three hyperon decays. To simplify the issue, we take 10\% of each branch for an average. The same treatment is done to the $\Xi$, which contains the contribution from the $\Xi^0$ and $\Xi^-$, and 5\% of each branch is taken into consideration.

The $\alpha$‘s are decay parameters, representing the polarization transfer from the decay hyperon to the $\Lambda$. In our study, these parameters are set as

$$\alpha_{\Sigma\Lambda} = -0.333, \ \alpha_{\Sigma\Lambda} = 0.6, \ \alpha_{\Xi\Lambda} = -0.406, \ \alpha_{\Xi\Lambda} = -0.458,$$

(11)

where $\alpha_{\Sigma\Lambda}$ is the decay parameter of the process $\Sigma^0 \rightarrow \Lambda\gamma$ discussed in ref. \cite{23}, $\alpha_{\Sigma\Lambda}$ and $\alpha_{\Xi\Lambda}$ are decay parameters measured in experiments and their specific values are taken from \cite{23}, and $\alpha_{\Xi\Lambda}$ is an estimated parameter by us. The choice of an $\alpha_{\Sigma\Lambda} = 0.6$ is due to the facts that the spin of $\Sigma^*$ (being 3/2) should be almost total positively correlated with $\Lambda$ spin (being 1/2) in the decay process corresponding to the $(s, s_z) = (3/2, 3/2)$ components of $\Sigma^*$ and that there should be a suppression for the spin transfer corresponding to the $(s, s_z) = (3/2, 1/2)$ components of $\Sigma^*$.

In the intermediate decay process, the longitudinal momentum fraction of the $\Lambda$ to the splitting quark should be less than the longitudinal fraction of the decay hyperon to the splitting quark. In the light-cone formalism, the momentum fraction $z$ is defined as $z = \frac{P_L}{Q}$. This effect is taken into account by redefining $\frac{P_L}{Q} = 1.1 + \frac{P_L}{Q}$.

In the year 1989, the polarized deeply inelastic scattering (DIS) experiment carried by the European Muon Collaboration revealed that the sum of the helicity of the quarks inside the proton is much smaller than the spin of the proton \cite{27,28}. This discovery is against the naive SU(6) quark model prediction, causing the so-called “proton spin crisis” or “proton spin puzzle”. One possible explanation to understand this puzzle \cite{29}. 


is to take into account the relativistic effect of the quark transversal motions, i.e., the Melosh-Wigner rotation effect. Based on this spirit, one can construct the light-cone SU(6) quark-spectator-diquark model to calculate the valence quark spin distributions in the light-cone formalism.

We can also consider the Melosh-Wigner rotation effect in the fragmentation process, and apply the light-cone SU(6) quark-spectator-diquark model to estimate the probability of a valence quark directly fragmenting to a hadron. This correlation can be realized through the phenomenology Gribov-Lipatov relation

\[ D_h^q(z) \sim z q_h(z), \]  

(12)

where the fragmentation function \( D_h^q(z) \) indicates a quark \( q \) splitting into a hadron \( h \) with longitudinal momentum fraction \( z \), and the distribution function \( q_h(z) \) presents the probability of finding the same quark \( q \) carrying longitudinal momentum fraction \( z \) inside the same hadron \( h \).

The main idea of the light-cone SU(6) quark-spectator-diquark model is to start from the naive SU(6) wave function of the hadron and then if any one of the quarks is probed, to reorganize the other two quarks in terms of two quark wave functions with spins 0 or 1 (scalar and vector diquarks), i.e., the diquark being served as an effective particle which is called the spectator.

The unpolarized quark distribution for a quark with flavor \( q \) inside a hadron \( h \) is expressed as

\[ q(x) = c_S^q a_S(x) + c_V^q a_V(x), \]  

(13)

where \( c_S^q \) and \( c_V^q \) are the weight coefficients determined by the SU(6) wave function, and \( a_D(x) \) \((D = S \text{ for scalar spectator or } V \text{ for axial vector spectator})\) denotes the amplitude for quark \( q \) to be scattered while the spectator is in the diquark state \( D \).

When expressed in terms of the light-cone momentum space wave function \( \varphi_D(x, k_\perp) \), \( a_D(x) \) reads

\[ a_D(x) \propto \int |\varphi_D(x, k_\perp)|^2, \]  

(14)

and the normalization satisfies \( \int_0^1 dx a_D(x) = 3 \). To obtain a practical formalism of the \( a_D(x) \), we employ the Brodsky-Huang-Lepage (BHL) prescription of the light-cone momentum space wave function

\[ \varphi_D(x, k_\perp) = A_D \exp \left\{ -\frac{1}{8 \alpha_D^2} \left[ \frac{m_q^2 + k_\perp^2}{x} + \frac{m_D^2 + k_\perp^2}{1 - x} \right] \right\}, \]  

(15)

with the parameter \( \alpha_D = 330 \text{ MeV} \). We set \( \alpha_S = \alpha_V \) for \( \alpha_D \)’s in our discussion because the non-perturbative physical effects can be effectively reflected in the scalar and vector diquark masses. More detailed study should consider the difference in \( a_D \)’s between scalar and vector diquarks. The parameter of the quark mass \( m_q \) is the constituent quark mass and the scalar (vector) diquark mass \( m_D \) \((D = S, V)\) is just an estimation from the constituent quark masses and the baryon masses. This parametrization can reduce the free parameters to only a few, which are listed in Table 1.
The polarized quark distributions are obtained by introducing the Melosh-Wigner correction factor 

\[ \Delta q(x) = \hat{c}_q^S \tilde{a}_S(x) + \hat{c}_q^V \tilde{a}_V(x), \]  

(16)

where the coefficients \( \hat{c}_q^S \) and \( \hat{c}_q^V \) are also determined by the SU(6) quark-diquark wave function, and \( \tilde{a}(x) \) is expressed as

\[ \tilde{a}_D(x) = \int [d^2k_\perp] W_D(x, k_\perp)|\varphi_D(x, k_\perp)|^2, \quad (D = S \text{ or } V), \]  

(17)

where

\[ W_D(x, k_\perp) = \frac{(k^+ + m_q)^2 - k_{\perp}^2}{(k^+ + m_q)^2 + k_{\perp}^2}. \]  

(18)

with \( k^+ = xM \) and \( M^2 = \frac{m_q^2 + k_{\perp}^2}{1-x} \). The weight coefficients are also listed in Table 1. In this model, though the mass difference between different quarks and diquarks breaks the SU(3) symmetry explicitly, the SU(3) symmetry between the octet baryons is in principle maintained in formalism.

| Baryon | q | \( \Delta q \) | \( m_d \) (MeV) | \( m_s \) (MeV) | \( m_s \) (MeV) |
|--------|---|----------------|-------------|-------------|-------------|
| p (uud) | u | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} d + \frac{1}{2} \bar{u} s \) | 330 | 800 | 600 |
| n (uud) | d | \( \frac{1}{2} \bar{d} \gamma_5 u + \frac{1}{2} \bar{d} \gamma_5 s \) | \( \frac{1}{2} \bar{d} d + \frac{1}{2} \bar{d} s \) | 330 | 800 | 600 |
| \( \Sigma^+ \) (uus) | s | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} u + \frac{1}{2} \bar{u} s \) | 330 | 950 | 750 |
| \( \Sigma^0 \) (uds) | u | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} d + \frac{1}{2} \bar{u} s \) | 330 | 950 | 750 |
| \( \Sigma^- \) (dds) | d | \( \frac{1}{2} \bar{d} \gamma_5 u + \frac{1}{2} \bar{d} \gamma_5 s \) | \( \frac{1}{2} \bar{d} u + \frac{1}{2} \bar{d} s \) | 480 | 800 | 600 |
| \( \Lambda^+ \) (uds) | s | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} d + \frac{1}{2} \bar{u} s \) | 480 | 800 | 600 |
| \( \Xi^0 \) (dss) | u | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} d + \frac{1}{2} \bar{u} s \) | 480 | 1100 | 900 |
| \( \Xi^- \) (uss) | s | \( \frac{1}{2} \bar{u} \gamma_5 u + \frac{1}{2} \bar{u} \gamma_5 s \) | \( \frac{1}{2} \bar{u} d + \frac{1}{2} \bar{u} s \) | 480 | 950 | 750 |

Based on the same spirit, we give the distribution functions for the \( \Sigma^+ \)-hyperon, which in the naive quark model is a member of the SU(3) decuplet with the total spin of 3/2. Here, we try to use the same parameters to estimate both the helicity and quark distribution functions in the light-cone SU(6) quark-spectator-diquark model based on the following reasons: (1) the mass of \( \Sigma^+ \) (which is about 1385 MeV) is similar to that of \( \Xi^- \) (which is about 1321 MeV), so we can use the same effective quark mass
parameters; (2) the total quark orbital angular momentum of $\Sigma^*$ is 0, so to form a spin $3/2$ particle, the diquark can only be in the vector state. The specific helicity-dependent and unpolarized quark distribution functions for the $\Sigma^*$’s in the quark-spectator-diquark model are shown in Table 2.

### Table 2: The quark distribution functions of $\Sigma(1385)$’s $(s, s_z) = (3/2, \pm 3/2)$ components in the light-cone SU(6) quark-spectator-diquark model

| Baryon  | $q$ | $\Delta q$ | $m_q$ (MeV) | $m_{\Delta q}$ (MeV) |
|---------|-----|------------|-------------|---------------------|
| $\Sigma^*(1385)$ (uss) | $u$ | $\Delta u$ | 330 | 950 |
|         | $s$ | $\Delta s$ | 480 | 800 |
| $\Sigma^*(1385)$ (uds) | $d$ | $\Delta d$ | 330 | 950 |
|         | $s$ | $\Delta s$ | 480 | 800 |

We know that in the naive quark model, there is a SU(3) flavor symmetry relation between octet baryons. We consider the anti-quark distribution inside the octet baryons in the same way. To compare with the experimental data, the CTEQ5 parametrization (ctq5l) for proton is used as an input:

$$ u_{v}^p(x) = u_{v}^{ctq}(x), $$

$$ d_{v}^p(x) = u_{v}^{\Lambda}(x) = \frac{u_{v}^{\Lambda, th}(x)}{u_{v}^{\Lambda, th}(x)} * u_{v}^{ctq}(x), $$

$$ s_{v}^p(x) = \frac{s_{v}^{\Lambda, th}(x)}{u_{v}^{\Lambda, th}(x)} * u_{v}^{ctq}(x), $$

$$ \Delta d_{v}^p(x) = \frac{\Delta u_{v}^{\Lambda, th}(x)}{u_{v}^{\Lambda, th}(x)} * u_{v}^{ctq}(x), $$

$$ \Delta s_{v}^p(x) = \frac{\Delta s_{v}^{\Lambda, th}(x)}{u_{v}^{\Lambda, th}(x)} * u_{v}^{ctq}(x), $$

$$ d_{v}^{\Lambda}(x) = u_{v}^{\Lambda}(x) = \frac{1}{2}(\bar{u}^{ctq}(x) + s^{ctq}(x)), $$

$$ s_{v}^{\Lambda}(x) = \bar{s}^{\Lambda}(x) = \bar{\bar{u}}^{ctq}(x), $$

where the $u_{v}^{ctq}(x)$ means the PDF for the valence $u$ quark inside the proton from the CTEQ5 parametrization, and the $u_{v}^{\Lambda, th}(x)$ is the PDF for the valence $u$ quark inside the $\Lambda$ given by the light-cone SU(6) quark-diquark model, so as other flavors. For the other hyperons, the same spirit is followed. Apply the Gribov-Lipatov relation again, we can obtain the anti-quark FFs to the same hyperon.

Using all these equations from (10) to (12), (19), and Tables 1 and 2, we can obtain the effective $\Lambda$ fragmentation functions expressed in Eqs. (8) and (9), and the results are shown in Fig. 1. The ratios of $\Delta D^p / D^p$ and $\Delta D^p / D^p$ with contributions from different channels are also plotted in Fig. 2. It is interesting that in the $z$ region we
calculated, both $u$ and $s$ quarks contribute a positive spin transfer (where the FFs of the $d$ quark is the same as that of the $u$ quark), when the direct fragmentation process and the intermediate decay process are all considered. The anti-quark contribute a large FFs at the small $z$ region, as predicted. If we start from the struck quark, and end with the $\Lambda$-hyperon, the FFs obtained from our method can be taken as an effective input to extract the PDFs of the target particle.

For the semi-inclusive $eP \to e\Lambda X$ process, where the electron is longitudinally polarized and the target is unpolarized, the spin transfer function extracted from the
QCD factorization theorem is

\[ A^\Lambda(z) = \frac{\sum_q c_q^2 f^q_p(x, Q^2) \Delta D^\Lambda_q(z, Q^2) + (q \rightarrow \bar{q})}{\sum_q c_q^2 f^q_p(x, Q^2) D^\Lambda_q(z, Q^2) + (q \rightarrow \bar{q})} \]  

(20)

By using our effective \( \Lambda \) fragmentation functions and the CTEQ5 parametrization (ctq5l) for the proton, at the point of \( x = 0.08 \), the longitudinal spin transfer distributing with \( z \) is shown in Fig. 3. As is shown with the thick solid line in Fig. 3, our calculation is well consistent with the data from the HERMES Collaboration [40, 41]. To make a comparison, the pure valence quark fragmentation process is calculated using the light-cone SU(6) quark-diquark model, as shown by the thin solid line. The pure quark and anti-quark fragmentation process is calculated based on the light-cone SU(6) quark-diquark model and Eq. (19), as shown by the dashed line. We can see that the anti-quark fragmentation process slightly enhances the spin transfer at the small \( z \) region, while the intermediate decay processes greatly improve the spin transfer in the whole \( z \) region.

Furthermore, to look at the detailed contribution to this result from different channel, we write the separate longitudinal spin transfer as

\[ A^H_i(z) = \frac{\sum_q c_q^2 f^q_p(x, Q^2) a_i \Delta D^{H_q}_i(z, Q^2) + (q \rightarrow \bar{q})}{\sum_q c_q^2 f^q_p(x, Q^2) \sum_j a_j D^{H_q}_j(z, Q^2) + (q \rightarrow \bar{q})}, \]  

(21)

where \( H_i \) represents the \( \Lambda \) fragmentation contributions from direct fragmentation, the intermediate \( \Sigma^0, \Xi, \) and \( \Sigma^* \) decaying processes respectively. The results are shown in Fig. 4 in which the \( \Sigma^0 \) contributes a slightly negative polarization transfer in our plotted region along \( z \), but \( \Sigma^* \)’s provide a higher positive polarization transfer at low and medium \( z \) region, while the influence from the \( \Xi \) is small. We notice that the positive spin transfer at low and medium \( z \) region mainly comes from the \( \Sigma^* \) contribution. This can be easily understood from the following intuitive picture: \( \Sigma^* \) is a spin 3/2 particle composed with three or two positively polarized valence quarks, therefore both the quark to \( \Sigma^* \) fragmentation and the \( \Sigma^* \) to \( \Lambda \) decay process should keep positive spin correlations.

We also examine the longitudinal spin transfer on the \( x \) and the Feynman variable \( x_F \) dependence. As for the \( x_F \), it can be related to the \( x, y, z \) variables through a kinematical transformation.

We know in the target rest frame, the four momentum of the proton and the virtual photon are

\[ P^\mu = (M, 0, 0, 0), \quad q^\mu = (\nu, 0, 0, -\sqrt{\nu^2 + Q^2}). \]  

(22)

With a Lorentz transformation, we get the four momentum in the \( \gamma^* P \) center of mass frame as

\[ P^\mu = (\gamma M, 0, 0, -\gamma \beta M), \]  

\[ q^\mu = (\gamma \nu + \gamma \beta \sqrt{\nu^2 + Q^2}, 0, 0, -\gamma \sqrt{\nu^2 + Q^2} - \gamma \beta \nu). \]  

(23)

where \( \beta \) is determined by

\[ -\gamma \beta M + (-\gamma \sqrt{\nu^2 + Q^2} - \gamma \beta \nu) = 0 \]  

(24)
as $\beta = -\frac{\sqrt{\nu^2 + Q^2}}{M+\nu}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. So the invariant mass of the $\gamma^* P$ system is

$$W^2 = (M + \nu)^2 - (\nu^2 + Q^2).$$  \hfill (25)

The momentum of $P_h$ of the produced hadron $h$ can be parametrized as

$$P_h^\mu \approx zq^\mu + xzP^\mu + P^\mu_{h\perp}. \hfill (26)$$

Then the Feynman variable $x_F$ can be obtained as

$$x_F = \frac{2P_{hl}/W}{2z(1-x)\sqrt{\nu^2 + Q^2}M} = \frac{2z(1-x)\sqrt{\nu^2 + Q^2}M}{M^2 + 2M\nu - Q^2} = \frac{2(1-x)\sqrt{\nu^2 + Q^2}M}{M^2 + Q^2 - Q^2}. \hfill (27)$$

In our study, as for the $x_F$ bins measured in the HERMES experiment [41], we take an average of $Q^2$ as $\bar{Q}^2 = 4$ (GeV)$^2$ and an average of $x$ as $\bar{x} = 0.09$, so a collinear transformation of $z$ to $x_F$ can be obtained from Eq. (27). The calculated result of the $x_F$-dependent longitudinal spin transfer is then shown in Fig. 5. As is shown, the result is in good agreement with the experimental data.

As for the $x$-bins, the calculation is performed at the average of $x_F = 0.22$. The result is shown in Fig. 6. It is known that the cross section of the final hadron produced in the current-fragmentation region is non-sensitive to the $x$-variable. As is shown in Fig. 6, our result is well consistent with this property and the experimental data in the intermediate $x$ region also prove its validity.

The study of the hadronic state produced in the current-fragmentation region of the lepton-lepton annihilation process can also give information of the quark fragmentations. In this Letter, we consider the process of $e^+e^-$ annihilation at the Z pole. In this

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{Figure3.png}
\caption{The results of the $z$-dependent longitudinal spin transfer in polarized charged lepton DIS process for the $\Lambda$-hyperon.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{Figure4.png}
\caption{The results of the $z$-dependent longitudinal spin transfer from different channels in polarized charged lepton DIS process for the $\Lambda$-hyperon.}
\end{figure}
process, the current $q\bar{q}$ pairs are produced through weak interactions and then fragment into final hadrons. Though the initial $e^+e^-$ states are unpolarized, the weak decays can produce polarized current $q\bar{q}$ pairs and these quark pairs can fragment into polarized $\Lambda$-hyperon. The test of the $\Lambda$ polarization is then sensitive to the helicity-dependent fragmentations and the specific calculation formula for this process is

$$P_\Lambda = \frac{\sum_q A_q \left[ \Delta D^H_q(z) - \Delta D^\bar{H}_q(z) \right]}{\sum_q C_q \left[ \sum_j a_j D^H_q(z) + (q \to \bar{q}) \right]}$$  \hspace{1cm} (28)$$

where the $A_q$ and $C_q$ are parameters as shown in Ref. \[16\].

Our calculation results are shown in Fig. 7. In this figure, the thick solid line is the result of our model, while the thin solid line is the result from the light-cone SU(6) quark-diquark model with only valence quarks and the dashed line is by including the sea quark content on the basis of the thin solid line calculation. The result from our effective fragmentation functions agrees with the experimental data.

Detailed contributions to this polarization result from different channels are also considered. The separate contribution is written as

$$P^{H_i}(z) = \frac{\sum_q A_q \left[ a_i \Delta D^H_q(z) a_{H_i\Lambda} - (q \to \bar{q}) \right]}{\sum_q C_q \left[ \sum_j \alpha_j D^H_q(z) + (q \to \bar{q}) \right]},$$  \hspace{1cm} (29)$$

where $H_i$ represents the $\Lambda$ fragmentation contributions from direct fragmentation, the intermediate $\Sigma^0$, $\Xi$, and $\Sigma^*$ decaying processes respectively. The results are shown in Fig. 8. As is shown, the main correction to the $\Lambda$ polarization is from the fragmentation through the $\Sigma^*$ decay channel, while the $\Sigma^0$ and $\Xi$ contribute slightly.

In summary, we studied the quark to the $\Lambda$ fragmentation properties in the current-fragmentation region by taking various fragmentation processes into account. These
processes include the intermediate decay process and the anti-quark fragmentation process. By using the light-cone SU(6) quark-diquark model and the Gribov-Lipatov relation, the effective helicity-dependent fragmentation functions and unpolarized fragmentation functions are obtained. These effective fragmentation functions are applied to several experimental processes and the obtained results are in reasonable agreement with experimental data. We thus suggest that the $\Lambda$-hyperon fragmentation processes are effective to study the structure of the $\Lambda$-hyperon and also the parton distribution functions of the target particle in the semi-inclusive deep inelastic scattering processes.

This work is partially supported by National Natural Science Foundation of China (Grants Nos. 11021092, 10975003, 11035003, and 11120101004), by the Research Fund for the Doctoral Program of Higher Education (China).

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