Numerical investigation of the cavitating flow for constant water hammer number

K Urbanowicz¹, A Bergant², U Karadžić⁴, H Jing⁵ and A Kodura⁶

¹ West Pomeranian University of Technology, Faculty of Mechatronics and Mechanical Engineering, Szczecin, Poland
² Litostroj Power d.o.o., Ljubljana, Slovenia
³ University of Ljubljana, Faculty of Mechanical Engineering, Slovenia
⁴ University of Montenegro, Faculty of Mechanical Engineering, Podgorica, Montenegro
⁵ Xi’an University of Technology, Institute of Water Resources and Hydro-electric Engineering, Xi’an, China
⁶ Warsaw University of Technology, Faculty of Building Services, Hydro and Environmental Engineering, Warsaw, Poland

kamil.urbanowicz@zut.edu.pl

Abstract. Several comparative studies in this work were carried out with the help of the method of characteristics. This numerical method is the most effective for solving a system of partial equations (hyperbolic type) describing a complex problem associated with the water hammer phenomenon. The numerical tests have been performed for the selected constant value of the introduced dimensionless water hammer number. The presented comparisons showed that the unsteady flows without column separation are analogous in various hydraulic systems when the value of this number has been preserved. Besides, cavitating flows with such a constant value of this number were also tested. These studies indicated the existence of another dimensionless number which was called a cavitation number. Maintaining the fixed values of both dimensionless numbers guaranteed similarity of flows in different examined systems in which cavitation occurred.

1. Introduction

Water hammer has been the subject of research for at least 120 years. This phenomenon occurs in many systems (hydraulic, water supply, cooling, etc.) when the flow parameters (velocity and pressure) change rapidly. The fluctuation of the basic parameters can be forced by rapid flow shutdown, distributor overload, power failure, pump inertia, etc. Currently, intensive research of this flow and associated phenomena i.e. unsteady friction [1, 2], fluid structure interaction [3, 4], cavitation [5, 6], the influence of the pipe material properties [7, 8] is still ongoing. The system of equations discussed in this work, when considering unsteady hydraulic resistance and assuming elastic pipe material behaviour still does not have an analytical solution. The above means the need to use numerical methods to approximate the behaviour of the flow. Authors who study this flow usually point two dimensionless numbers in their works that influence this flow, they are the Reynolds number \( Re = (2vR)/\nu \) and shear wavenumber \( s = R\sqrt{\omega/\nu} \) [9, 10]. However, our recent unpublished comparative studies show that the most important number will be the water hammer number, whose origin will now be discussed.
Cohen and Tu [11] studied the effects of fluid viscosity and wall interference on wave motion in a simple hydraulic device. They were probably the pioneers that discovered that damping is controlled by the ratio \((vL/(\pi R^2 c))^{1/2}\). So, the tube radius and the kinematic viscosity are therefore related in this manner. They noted that \(L/(\pi c)\) has the dimension of time relating travel of an acoustic wave through the cylinder, whereas \(R^2/\nu\) is a time that has to do with cross-tube interference to wave travel. Gerlach in his Ph.D. thesis [12] introduced two dimensionless damping numbers. The first one was a radial damping number \(D_{nr} = v/(Rc)\) and the second one was an axial damping number \(D_{nz} = (vL)/(cR^2)\) (our water hammer number). Gerlach did not discuss this number [13]. In Goodson and Leonard paper [14] about a survey of modelling techniques for fluid line transients, the authors define it as a dissipation number \(D_n\) (which is our analysed water hammer number). They have explained that the value of this number indicates the attenuation and distortion of pressure waves along a line. This number is important for both transient and frequency response. It is the ratio of the wave travel time along a line to a term indicative of the transient decay. Streeter with Wylie [15] distinguished two dimensionless numbers that characterise the dynamic response of a fluid line. First is the dissipation number \(D_n\) which is the ratio of the wave travel time to a term that represents viscous decay, which is for laminar flow our analysed water hammer number. The second dimensionless number was defined similarly:

\[
D_n = \frac{\text{fr}_{n}}{2cR}
\]

but used for a turbulent flow which is characterised by the use of the Darcy-Weisbach equation.

When we look at contemporary works, such as the work of Manhartsgruber [16] or Johnston [17], it can be seen that this number is of key importance because it appears in the equation for hydraulic impedance. Perhaps the poor recognition of this number and the complete lack of discussion about it in contemporary works is caused by its omission in review works, including the excellent works of Stecki-Davis [9, 18], whose authors focused their attention on another dimensionless number, namely shear wave number. The water hammer number analysed in this work also plays an important role in the approximate analytical solution of water hammer equations presented by Mei and Jing [19, 20], derived with the use of the asymptotic method of multiple-scale expansions. The authors of this analytical solution use a similar form of this number to the earliest found in the literature [11], except that in their final form \(\pi\) is missing. They describe this number as a ratio of a characteristic thickness of the Stokes oscillatory boundary layer \(\delta = (vL/c)^{1/2}\) to the pipe inner radius \(R\).

In the present work, formula\(\text{s}\) will be derived for the dimensionless form of the continuity and motion equation describing the transients flow of liquids in pressure lines. It is from them that our dimensionless number will emerge, whose usability will be discussed. The impact of this dimensionless number will be investigated not only for laminar homogeneous single-phase flows but also for two-phase flows in which liquid column separation occurs (commonly referred to as cavitation) as a result of the pressure drop to the saturated vapor pressure [21-24].

2. The Water Hammer Number

Neglecting the convective terms in the equation of continuity (2) and motion (3), a simplified form of two basic equations describing the transient flow of slightly compressible liquid in a horizontal metal pipe is obtained:

\[
\frac{\partial p}{\partial t} + \rho c^2 \frac{\partial v}{\partial x} = 0 \tag{2}
\]

\[
\rho \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{g}{c^2} \tau = 0 \tag{3}
\]

where: \(v = v(x,t)\) – mean liquid flow velocity in the pipe cross-section, \(p = p(x,t)\) – pressure in the pipe cross-section, \(R\) – internal diameter of the pipe, \(\tau\) – wall shear stress, \(\rho\) – liquid density, \(g\) – acceleration due to gravity, \(c\) – pressure wave speed, \(t\) – time, \(x\) – axial coordinate of the pipe. Herein we assume constant liquid density and wave speed in our investigations.
By entering dimensionless variables in the continuity equation (2):

\[ x = L \hat{x}; \quad t = \frac{L}{c} \hat{t}; \quad p = \rho c v_0 \hat{p}; \quad v = v_0 \hat{v} \]  

(4)
yields the following dimensionless form:

\[ \frac{\rho c v_0 \frac{\partial \hat{p}}{\partial \hat{t}}}{L} + \frac{\rho c^2 v_0 \frac{\partial \hat{p}}{\partial \hat{x}}}{L} = 0 \]  

(5)

Now multiplying by \( L/(\rho c^2 v_0) \) we get the final dimensionless form of the continuity equation:

\[ \frac{\partial \hat{p}}{\partial \hat{t}} + \frac{\partial \hat{v}}{\partial \hat{x}} = 0 \]  

(6)

The above form of the continuity equation, which ultimately does not contain any of the known dimensionless numbers, guarantees that the above equation can be used regardless of the scale of the analysed system. Applying in addition to the above dimensionless variables in the equation of motion (3) the following relation:

\[ \tau = \frac{\mu v_0}{R} \hat{t} \]  

(7)
gives a dimensionless form of the equation of motion:

\[ \frac{\rho c v_0 \frac{\partial \hat{v}}{\partial \hat{t}}}{L} + \frac{\rho c v_0 \frac{\partial \hat{p}}{\partial \hat{x}}}{L} + 2 \frac{\mu v_0}{R} \frac{\partial \hat{v}}{\partial \hat{x}} \hat{t} = 0 \]  

(8)

Dimensionless wall shear stress can be generally presented as a sum of the quasi-steady and unsteady component: \( \hat{\tau} = \hat{\tau}_{qs} + \hat{\tau}_u \).

In laminar flow:

\[ \hat{\tau} = 4 \hat{v} + 2 \int_0^{\hat{t}} \hat{\omega}_t (\hat{t} - \hat{u}) \frac{\partial \hat{\rho}}{\partial \hat{t}} d \hat{u} \]  

(9)

In turbulent flow:

\[ \hat{\tau} = \frac{Re_0}{16} \hat{\rho} |\hat{u}| f_t + 2 \int_0^{\hat{t}} \hat{\omega}_t (\hat{t} - \hat{u}) \frac{\partial \hat{\rho}}{\partial \hat{t}} d \hat{u} \]  

(10)

where: \( Re_0 \) – is an initial Reynolds number and \( f_t \) – is the Darcy-Weisbach friction factor for turbulent flow.

Further multiplying the equation (8) by \( L/(c \rho v_0) \), we get the following equation after rearrangement

\[ \frac{\partial \hat{p}}{\partial \hat{t}} + \frac{\partial \hat{v}}{\partial \hat{x}} + 2 \frac{v_0}{c R} \hat{\tau} = 0 \]  

(11)

In the above equation, a dimensionless relationship appears, which is defined as the water hammer number \( Wh = (\nu L)/(c R^2) \). If we apply the continuity equation in the above form to flows in two different scales then we obtain, according to Buckingham theorem, the full similarity of phenomena while maintaining the equal value of water hammer number. The found dimensionless number can be considered as the product of the radial damping number \( D_{nr} = \nu/(c R) \) which was introduced by Gerlach [12] and pipe scale factor \( S = L/R \). However, if we multiply this number by four, we will get another interesting form of this dimensionless number \( Wh' = (4 \nu L)/(c R^2) \), which can be considered as the ratio of two timescales, namely water hammer timescale \( T_{Wh} = 4 L/c \), being the period of water hammer and radial diffusion timescale \( T_{rad} = R^2/\nu \) [25].

What is more, this number also appears when normalising the Navier dynamic equation [26]:

\[ \frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right) \]  

(12)

Using dimensionless variables previously presented (4) and a new one:

\[ r = R \hat{r} \]  

(13)
gives the following form of equation (12):

\[ c \frac{v_0}{L} \frac{\partial \rho}{\partial t} + \frac{\rho c v_0}{L} \frac{\partial \rho}{\partial x} = \nu \left( \frac{v_0}{R^2} \frac{\partial^2 \rho}{\partial t^2} + \frac{v_0}{R^2 R} \frac{\partial \rho}{\partial x} \right) \]  

(14)

Then multiplying equation (14) by \( L/(c v_0) \) yields:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} = \frac{L v}{c R^2} \left( \frac{\partial^2 \rho}{\partial x^2} + \frac{1}{R} \frac{\partial \rho}{\partial x} \right) \Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} = Wh \left( \frac{\partial^2 \rho}{\partial x^2} + \frac{1}{R} \frac{\partial \rho}{\partial x} \right) \]  

(15)

Does the maintenance of the constant value of the derived dimensionless number \( Wh \) truly guarantee the comparability of results? Completed numerical simulations in the next two sections of this work will answer this question.

Computational Fluid Dynamics (CFD) is a branch of fluid mechanics that uses numerical methods to solve fluid flow problems. Due to the numerical solution and discretisation of partial differential equations describing the flow, it is possible to calculate approximate values of velocity, pressure, temperature, and other parameters in the flow. Modern CFD codes allow solving complex flows considering the influence of viscosity and compressibility. With their help, one can simulate and analyse several flow situations: multi-phase, with chemical reactions or combustion processes, through porous structures in which the fluid is Newtonian or non-Newtonian, and recently also flows with a fluid-solid interaction. Most modern CFD codes are based on the Navier-Stokes equations (the mass, momentum, and energy conservation equations for a fluid) and discretize them using the finite volume method, the finite difference method, or the finite element method. Unfortunately, the disadvantage of most of the available CFD codes is that the user often has little influence on the solution method (Black Box). Therefore, a proprietary in-house computer program was developed in the Matlab programming language and used in all simulation runs in this paper. The solution is based on the method of characteristics (MOC) [27]. The MOC transforms the system of basic partial differential equations (2) and (3) into an equivalent system of algebraic equations, whose solutions are found in discrete time-space sections (nodes) within the grid of characteristics. Unsteady hydraulic resistance was determined using the improved effective approach discussed in detail in [2].

3. Comparisons of flows without cavitation

From the works about water hammer in which experimental research was carried out in the field of laminar flow (Table 1), it can be seen that the analysed systems were characterised by a relatively low value of the water hammer number. The highest value can be noted in the Holmboe and Rouleau system [28], while the lowest value in the Bergant and Simpson system [29]. According to the discussion conducted in the work of Godson and Leonard [14] in systems in which the value of this analysed dimensionless number is low \( Wh < 0.0001 \) [–], the initial phase of the water hammer can be described correctly using only the quasi-steady model of hydraulic resistance.

| System          | Holmboe & Rouleau [28] | Bergant & Simpson [29] | Adamkowski & Lewandowski [30] |
|-----------------|------------------------|------------------------|-------------------------------|
| \( L \) [m]     | 36.09                  | 37.22                  | 98.11                         |
| \( R \) [m]     | 0.0127                 | 0.01105                | 0.008                         |
| \( v \) [m²/s]  | 4·10⁻⁵                 | 1.13·10⁻⁶              | 9.5·10⁻⁷                     |
| \( c \) [m/s]   | 1335                   | 1302                   | 1308                          |
| \( Wh \) [–]    | 0.0067                 | 2.65·10⁻⁴              | 0.0011                        |

In an unpublished work by the first author, an oil system with a small diameter pipe was analysed in which the \( Wh < 0.145 \) [–]. In such experiments characterised by a large value of \( Wh \) number, the diffusive effect of viscosity starts to prevail the transport effect.
In this section let us examine three theoretical systems that have all the parameters that define the different test stands as well the parameters that define the liquid (see Table 2) but are represented by the same water hammer number \( Wh = 0.01 \) [–].

The first Case A presents a long transport line in which the oil is delivered from one point to another on a long distance (larger than 1 km). The second Case B represents a typical test pipe that is used in many recent experiments and here the flowing liquid is an emulsion of water and oil. The last simulated test stand (Case C) can be considered as a typical cooling or medical equipment system with a small micro steel pipe with water as flowing liquid. The received simulation results of a water hammer case initiated with an instantaneous valve closure in the reservoir-pipe-valve R-P-V systems are presented in Figure 1a. In this figure in the ordinate axis, the pressure rise is presented in a dimensionless form \( \hat{p} = (p - p_R)/(\rho c v_0) \), while on the abscissa axis the dimensionless time is presented \( \hat{t} = (c \cdot t)/L \). From what we see in Figure 1a all the dimensionless pressure surges histories for the three analysed scenarios are identical. The performed comparison, therefore, confirms that regardless of the values of kinematic viscosity \( v \), pipe length \( L \), the pressure wave speed \( c \) and pipe inner radius \( R \), if the calculated dimensionless water hammer number \( Wh \) has a constant value, the flows can be treated as similar – characterised by the same course of pressure surges in the dimensionless form. The water hammer number \( Wh \) should be treated as a criterion number during transient flows in which the water hammer is analysed. It should be noted that a similar dimensionless analysis has been performed by Bergant and Karadžić [31] with the objective to investigate the effects of unsteady friction and actual valve closure on the attenuation, shape, and timing of pressure pulses. As an example, the use of this number can help us to build a small size hydropower stand, in which the large-scale water hammer effects can be checked and verified experimentally in the laboratory. The effect of the dimensionless number on the pressure surges are presented in Figure 1b where the simulation was done for a few selected values of this important number \( (Wh_1 = 0.0001; Wh_2 = 0.001; Wh_3 = 0.01 \ and \ Wh_4 = 0.1) \).

From comparisons presented in Figure 1b, it can be seen that the value of the dimensionless number being analysed is of key importance for dissipation and dispersion of the dimensionless pressure waves. The larger its value, the greater is the dissipation and dispersion. The dispersion is associated mainly with the moment of reflection of the wave in the reservoir and the cross-section of the valve location, it is manifested by the fact that the number of \( Wh \) analysed in this work increases, the duration of subsequent amplitudes becomes longer. Whereas dissipation is closely related to fluid friction during transient flow. The friction effect also increases as the analysed dimensionless number increases. What we cannot see in Figure 1b are the initial values of dimensionless pressure. The higher the water hammer number the lower value of initial dimensionless pressure (see a difference of starting pressure for \( Wh = 0.01 \) [–] on Figure 1a and \( Wh = 0.1 \) [–] on Figure 1b). This difference results from the pressure drop over the pipe length during steady motion proceeding the transient flow. From Figure 1b it may be seen that the dimensionless duration of the water hammer decreases as the number of water hammer increases.

| Test stand                      | Case A | Case B | Case C |
|--------------------------------|--------|--------|--------|
| Pipe length \( L \) [m]        | 1100   | 120    | 13     |
| Pipe inner radius \( R \) [m]  | 0.1    | 0.01   | 0.001  |
| Pressure wave speed \( c \) [m/s] | 1100  | 1200   | 1300   |
| Kinematic viscosity \( v \) [m²/s] | 1·10⁻⁴| 1·10⁻⁵| 1·10⁻⁶|
| Density of liquid \( \rho \) [kg/m³] | 850  | 925    | 1000   |
| Initial flow velocity \( v_0 \) [m/s] | 0.6  | 0.4    | 0.5    |
| Reservoir pressure \( p_R \) [Pa] | 1·10⁶ | 3·10⁵ | 2·10⁴ |
| Reynolds number \( Re \) [-]    | 1200   | 800    | 1000   |
| Water hammer number \( Wh \) [-] | 0.01  | 0.01   | 0.01   |

The first Case A presents a long transport line in which the oil is delivered from one point to another on a long distance (larger than 1 km). The second Case B represents a typical test pipe that is used in many recent experiments and here the flowing liquid is an emulsion of water and oil. The last simulated test stand (Case C) can be considered as a typical cooling or medical equipment system with a small micro steel pipe with water as flowing liquid. The received simulation results of a water hammer case initiated with an instantaneous valve closure in the reservoir-pipe-valve R-P-V systems are presented in Figure 1a. In this figure in the ordinate axis, the pressure rise is presented in a dimensionless form \( \hat{p} = (p - p_R)/(\rho c v_0) \), while on the abscissa axis the dimensionless time is presented \( \hat{t} = (c \cdot t)/L \). From what we see in Figure 1a all the dimensionless pressure surges histories for the three analysed scenarios are identical. The performed comparison, therefore, confirms that regardless of the values of kinematic viscosity \( v \), pipe length \( L \), the pressure wave speed \( c \) and pipe inner radius \( R \), if the calculated dimensionless water hammer number \( Wh \) has a constant value, the flows can be treated as similar – characterised by the same course of pressure surges in the dimensionless form. The water hammer number \( Wh \) should be treated as a criterion number during transient flows in which the water hammer is analysed. It should be noted that a similar dimensionless analysis has been performed by Bergant and Karadžić [31] with the objective to investigate the effects of unsteady friction and actual valve closure on the attenuation, shape, and timing of pressure pulses. As an example, the use of this number can help us to build a small size hydropower stand, in which the large-scale water hammer effects can be checked and verified experimentally in the laboratory. The effect of the dimensionless number on the pressure surges are presented in Figure 1b where the simulation was done for a few selected values of this important number \( (Wh_1 = 0.0001; Wh_2 = 0.001; Wh_3 = 0.01 \ and \ Wh_4 = 0.1) \).

From comparisons presented in Figure 1b, it can be seen that the value of the dimensionless number being analysed is of key importance for dissipation and dispersion of the dimensionless pressure waves. The larger its value, the greater is the dissipation and dispersion. The dispersion is associated mainly with the moment of reflection of the wave in the reservoir and the cross-section of the valve location, it is manifested by the fact that the number of \( Wh \) analysed in this work increases, the duration of subsequent amplitudes becomes longer. Whereas dissipation is closely related to fluid friction during transient flow. The friction effect also increases as the analysed dimensionless number increases. What we cannot see in Figure 1b are the initial values of dimensionless pressure. The higher the water hammer number the lower value of initial dimensionless pressure (see a difference of starting pressure for \( Wh = 0.01 \) [–] on Figure 1a and \( Wh = 0.1 \) [–] on Figure 1b). This difference results from the pressure drop over the pipe length during steady motion proceeding the transient flow. From Figure 1b it may be seen that the dimensionless duration of the water hammer decreases as the number of water hammer increases.
Figure 1. Dimensionless pressure surges for a) constant $Wh = 0.01 \, [-]$; b) selected values of $Wh$.

4. Comparisons of flows with cavitation

To perform a simulation in which column separation of the liquid stream occurred, the commonly known DVCM (discrete vapor cavitation model) improved by Adamkowski and Lewandowski [32] will be used. The principle of this model is to transfer locally determined vapor areas to one collecting node (in this work, the node at the valve) while meeting the continuity equation describing the dynamics of the cavitation area. The pressure in this model as well its prototype cannot fall below a certain imposed value, the so-called saturated vapor pressure $p_v$. The value of this pressure strictly depends on the properties of the flowing liquid as well as the current temperature value. For example, for water at room temperature ($T = 21.1 \, ^\circ C$) the value of this pressure is $p_v = 2.5 \cdot 10^3 \, [Pa]$.

Preliminary simulations have shown that during cavitation flows, to keep the flow similarity, not only the analysed dimensionless water hammer number must have the same value, but also the following dimensionless cavitation number must be constant:

$$Cn = \frac{p_v - p_R}{\rho c v_0} \quad (16)$$

The above number will take negative values in real conditions. Let us try to determine its maximum value now. For this purpose, let us assume that we are dealing with a system in which the pressure during the steady motion along the length of the pipe cannot fall to the saturated vapor pressure. But let assume that the pressure at the valve can already be a criterion pressure equal to the saturated vapor pressure. In such a system, steady flow takes place from the initial to the final cross-section where the valve is located, while after the valve closes abruptly, pressure drops can reach the initial pressure (in the cross-section next to the valve). Each such decrease of the pressure would result in the creation of cavitation areas. In the above case, the constant pressure in the reservoir can be determined as follows $p_R = p_v + \Delta p$, where $\Delta p$ is the pressure drop along the length of the pipe. In this work, we will focus only on laminar flow, and in it according to the Darcy-Weisbach equation:

$$\Delta p = f \frac{L \rho v_0^2}{D} \frac{v_0^2}{2} = \frac{64 \, L \, \rho \, v_0^2}{Re \, D} = 8 \frac{\mu c v_0}{R^2} \quad (17)$$

When the above pressure drop is inserted into the formula (16), we get the condition for the maximum possible value of cavitation number:

$$Cn_{max} = -8 \frac{v_0}{cR^2} = -8 \cdot Wh \quad (18)$$
As may be seen from formula (18), there is a close relationship between two dimensionless numbers discussed in this work. In general, for laminar flow, we can write the formula for cavitation number also making its value dependent on the water hammer number:

\[ Cn = \frac{p_v - p_R}{\rho c v_0} = \frac{p_v - p_{valve,t=0} - \frac{\mu L}{R}}{\rho c v_0} = \hat{p}_v - \hat{p}_{valve,t=0} - 8 \cdot Wh \]  

(19)

Having the \( Cn \) value determined for one real system (e.g. 1:1 scale), it is possible to determine the pressure in the reservoir necessary to maintain similarity in another experimental system:

\[ p_R = p_v - Cn \cdot \rho c v_0 \]  

(20)

The above condition may be particularly useful when testing the effect of temperature on the dynamics of cavitating flows. The temperature changes the basic parameters of the flowing liquid and modifies the saturated vapor pressure. Moreover, a change in the value of basic parameters also has an impact on the change in the value of the pressure wave speed.

Let us now carry out an example simulation assuming a constant value of the water hammer number \( Wh = 0.01 \) [\(-\)] and using other data summarised earlier in Table 2 except the pressure in the reservoir \( p_R \). For this value of the water hammer number according to formula (18), the maximum possible value of the cavitation number is \( Cn_{max} = -0.08 \). In the example comparison, let us assume that we will perform simulations for the following number of cavitation: \( Cn = 4Cn_{max} = -0.32 \) [\(-\)]. After determining the above number, to maintain the similarity of flows in three different systems (Case A'', Case B'' and Case C''), it is necessary to calculate the necessary pressure in the reservoirs using the formula (20). Assuming for the simplification the same value in all cases of saturated vapor pressure \( p_v = 2.5 \cdot 10^3 \) [\( Pa \)], the following values of pressure in the reservoirs were obtained for the compared numerical cases: Case A'' – \( p_R = 182020 \) [\( Pa \)], Case B'' – \( p_R = 144580 \) [\( Pa \)] and Case C'' – \( p_R = 210500 \) [\( Pa \)]. The simulation results obtained (while maintaining the constant value of the two dimensionless numbers analysed: \( Wh = 0.01 \) [\(-\)] and \( Cn = -0.32 \) [\(-\)]) are presented in Figure 2a.

![Figure 2a](image1.png)

**Figure 2a.** Dimensionless pressure surges of cavitating flows for a) constant \( Wh = 0.01 \) [\(-\)] and \( Cn = -0.32 \) [\(-\)]; b) selected values of \( Cn (Wh = 0.005 \) [\(-\)]).

It may be seen from Figure 2a that keeping the identical values of dimensionless numbers (\( Wh \) and \( Cn \)) for the three analysed theoretical systems guaranteed that the simulation similarity was maintained – similarly by keeping constant only water hammer number \( Wh \) for a single-phase flow. Figure 2b shows the influence of the cavitation number for a selected fixed value of the water hammer number \( Wh = 0.005 \) [\(-\)]. Comparisons reveal that the cavitation number indicates:

a) duration of transient flows with cavitation – the higher the \( Cn \) value, the longer the duration of transient runs.

![Figure 2b](image2.png)
b) pressure values at subsequent (after the first) pressure amplitudes – the lower the $Cn$ value, the higher the pressure values simulated after subsequent closing of the cavitation areas.

With the help of dimensionless analysis, it was also possible to determine a certain criterion value of water hammer number $Wh_{cr} = 0.0725 [-]$, which delimits systems in which there is a chance of cavitation with those that are safe and in which there is no possibility for cavitation flow separation (column separation). For this specific criterion number, the pressure drop after the first amplitude does not exceed the initial pressure - see Figure 3a. Therefore, any simple R-P-V system (with a discrete vapour cavity lumped at the valve only) represented by water hammer number $Wh \geq 0.0725 [-]$ can be regarded as cavitation free.

![Figure 3a](image1.png)  
![Figure 3b](image2.png)

**Figure 3.** Pressure surges for the critical water hammer numbers: a) $Wh_{cr} = 0.0725 [-]$, b) $Wh = 0.0387 [-]$.

Figure 3b shows the result obtained for $Wh = 0.0387 [-]$ and $Cn_{max}$. In the range of $0.0387 \leq Wh < 0.0725$, only a single vapour area may appear in the analysed valve cross-section of the examined system during the water hammer event, which means that the pressure cannot drop after the second visible amplitude to the saturated vapor pressure. For the water hammer number $Wh = 0.0387 [-]$, the range of the cavitation numbers in which there is a single separation of the liquid stream was also analysed. Studies have shown that this range is as follows: $-0.76 \leq Cn \leq -0.3094$. It means that if in a system characterised by the value of the water hammer number $Wh = 0.0387 [-]$ the value of cavitation numbers will be kept $Cn < -0.76$; in such system, there will be no cavitation separation of the liquid stream.

5. Conclusions

Completed studies have shown the great importance of the dimensionless numbers that define the transient flows resulting from the rapid closure of the downstream end valve in a horizontal pipe. Maintaining the constant water hammer number $Wh$ guarantees similarity of flows in different R-P-V systems. The advantages of the proposed dimensionless analysis of water hammer are, for example, the possibility of testing systems characterised by large dimensions (e.g. hydroelectric power plants) on a micro-scale. In order not to duplicate experimental research in the future, it is necessary to calculate the value of this number before building new test rigs. This work also showed that recognising cavitating flows as similar requires maintaining the same value of two dimensionless numbers, apart from the number $Wh$, also a certain cavitation number $Cn$. The cavitation number is the ratio of the pressure difference between the saturated vapor pressure and the real pressure and the product of liquid density, pressure wave speed, and the initial velocity of flow.

The dimensionless numbers discussed in this work are used to accurately analyse the pressure pulsations during the water hammer event in which the flow cut-off valve was closed immediately (rapid
water hammer). In real conditions, we often have to deal with complex water hammer, then the size of the pressure increase is also affected by the closing characteristics of the valve, type of valve, and its closing time. At the next stage of this research, the usefulness of these dimensionless numbers will be examined for turbulent pipe flow. Also, it is necessary to thoroughly analyse the values of cavitation numbers in known experimental studies of cavitating flows, and determining the impact of the above proposed dimensionless numbers on transient flows (with and without cavitation) in plastic pipes, which are commonly used in water supply systems.

Acknowledgments

Bergant gratefully acknowledges the support of the Slovenian Research Agency (ARRS) conducted through the project L2-1825 and the programme P2-0126.

References

[1] Ioriatti M, Dumbser M and Iben U 2017 Z. Angew. Math. Mech. 97 1358
[2] Urbanowicz K 2018 Z. Angew. Math. Mech. 98 802
[3] Aliaibadi H K, Ahmadi A and Keramat A 2020 Mech. Syst. Signal Proc. 144 106848
[4] Henclík S 2018 J. Fluids Struct. 76 469
[5] Tang X, Duan X, Gao H, Li X and Shi X 2020 Water 12 448
[6] Jiang D, Ren C, Zhao T and Cao W 2018 Appl. Sci. 8 388
[7] Pan B, Duan H F, Meniconi S, Urbanowicz K, Che T C and Brunone B 2020 J. Hydraul. Eng.-ASCE 146 04019068
[8] Urbanowicz K, Duan H F and Bergant A 2020 Strojniški vestn.-Mech. Eng. 66 77
[9] Stecki J S and Davis D C 1986 Proc. Inst. Mech. Eng. Part A-J. Power Energy 200 215
[10] Tijdeman H 1975 J. Sound Vibr. 39 1
[11] Cohen H and Tu Y 1962 J. Basic Eng. 84 593
[12] Gerlach C R 1966 The Dynamics of Viscous Fluid Transmission-Lines with Particular Emphasis on Higher Mode Propagation, Phd Thesis, Oklahoma State University
[13] Gerlach C R and Parker J D 1967 J. Basic Eng. 89 782
[14] Goodson R E and Leonard R G 1972 J. Basic Eng. 94 474
[15] Streeter V L and Wylie E B 1974 Annu. Rev. Fluid Mech. 6 57
[16] Manhartsguber B 2008 J. Fluids Eng.-Trans. ASME 130 121402
[17] Johnston N 2012 Proc. Inst. Mech. Eng. Part I-J Syst Control Eng. 226 586
[18] Stecki J S and Davis D C 1986 Proc. Inst. Mech. Eng. Part A-J. Power Energy 200 229
[19] Mei C C and Jing H 2018 Eur. J. Mech. B-Fluids 69 62
[20] Mei C C and Jing H 2016 Math. Biosci. 280 62
[21] Szala M and Awtoniuk M 2019 IOP Conf. Ser.: Mater. Sci. Eng. 710 012016
[22] Urbanowicz K, Bergant A and Duan H F 2019 IOP Conf. Ser.: Mater. Sci. Eng. 710 012013
[23] Jasonowski R and Kostrzewa W 2017 ITM Web Conf. 15 07011
[24] Szala M 2017 ITM Web Conf. 15 06003
[25] Duan H F, Ghidaoui M S, Lee P J and Tung Y K 2012 J. Hydraul. Eng.-ASCE 138 154
[26] Navier M 1827 Sur les Lois des Mouvements des Fluides, Mémoires del’ Académie des sciences, sciences mathématiques et physiques tome VI (1823) 389
[27] Wylie E B and Streeter V L 1993 Fluid Transients in Systems (Englewood Cliffs, New Jersey: Prentice-Hall Inc.)
[28] Holmboe E L and Rouleau W T 1967 J. Basic Eng. 89 174
[29] Bergant A and Simpson A R 1999 J. Hydraul. Eng.-ASCE 125 835
[30] Adamkowsi A and Lewandowski M 2006 J. Fluids Eng.-Trans. ASME 128 1351
[31] Bergant A and Karadžić U 2015 Proc. of 12th Int. Conf. on Pressure Surges (Dublin: BHR Group Press) 639
[32] Adamkowsi A and Lewandowski M 2009 J. Fluids Eng.-Trans. ASME 131 071302