Hydrodynamization Physics from Holography

Jakub Jankowski

Institute of Physics, Jagiellonian University, ul. Lojasiewicza 11, 30-348 Kraków, Poland

1 Introduction

The quest for a better understanding of the properties of nuclear matter under extreme conditions (such as those created in relativistic heavy ion collisions) has led to a number of theoretical challenges. One of them is to explain the success of the hydrodynamic description of quark-gluon plasma (QGP) evolution at time scales of order $\tau \sim 0.5\text{–}1\text{ fm/c}$ [1, 2]. Immediately following the nuclear collision the system is very far from equilibrium, yet rapidly reaches a state where hydrodynamics appears to apply. This process is fast in the sense that it occurs on a time scale less or equal the inverse local temperature, i.e. $\tau_H T_H \leq 1$.

The fact that the hydrodynamic description works well with a very low value of the shear viscosity to entropy ratio ($\eta/s$) and vanishing bulk viscosity suggests that the plasma state may be viewed effectively as a strongly coupled, conformal fluid. This has motivated studies aimed at understanding how such a hydrodynamic description could emerge in a case which is amenable to theoretical studies – the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM). In this example one has the option to apply gauge/gravity duality [3] to model the approach to equilibrium. Such studies were initiated in [4], where the case of Bjorken flow (discussed in more detail below) was considered. Janik and Peschanski showed in that context that the hydrodynamic description does indeed emerge at late proper times. This work was followed by numerous articles devoted to the description of hydrodynamic states in the context of gauge/gravity duality. In particular, second order transport coefficients were computed [5, 6], and many new insights into the meaning of relativistic hydrodynamics were gained [7, 8, 9, 10, 11].

To describe far from equilibrium states of $\mathcal{N} = 4$ SYM it is necessary to resort to numerical calculations. A decisive step opening this field of research was made by Chesler and Yaffe [12] who devised a very effective numerical scheme for solving Einstein equations in asymptotically AdS spaces based on characteristic evolution. This was soon applied to the case of Bjorken flow [13], which is a particularly attractive setting, since it was an important model used to understand basic features of QGP evolution (such as entropy production), and at the same time is simple enough to implement easily in the context of gauge/gravity duality. The results described here [14] follow from a modification of this scheme. A different method was used in [15, 16], where some basic features of the approach to hydrodynamics were studied. Most importantly, it was found there that the system reaches the hydrodynamic
regime quickly (in the sense described earlier). Another physically important conclusion from these papers (earlier noted in [13]) was that hydrodynamics works very well already at a time when pressure gradients are large. Thus the process of reaching this stage of evolution is often referred to as hydrodynamization instead of thermalization.

The numerical studies [15, 16] used a different numerical scheme from [12, 13], which was however limited by the fact that only a few (29) consistent initial states were known. The study [14] reported here adapted the approach of [13] in a way which allows an arbitrary number of initial conditions to be analysed, making it possible to look for generic features. In our work we looked at 600 initial conditions randomly generated on the gravity side of the duality. In the context of gauge/gravity duality there is a natural characteristic of the initial state called the initial entropy [15] (to be defined precisely below). Observables such as hydrodynamization time depend in particular on this quantity.

In this note we will focus on two natural questions

- Is hydrodynamization generically a fast process?
- Are there any universal physical characteristics of hydrodynamization?

The answer to the first question seems to be positive and confirms previous investigations [15, 16]; regardless of the values of the initial entropy chosen the system reaches hydrodynamic description on time scales of the order of the inverse local temperature. The second question is more subtle. In the sample of initial data analysed in our study the hydrodynamization time appears not to be correlated with initial entropy (in contrast to [15], where such a correlation seemed to be present on the basis of a smaller sample of initial states). However, when looking at the energy density at the time when hydrodynamic evolution starts, there appears to be a linear correlation with initial entropy.

One should also mention the work [17, 18] where the process of isotropization was considered in a similar spirit. Recently a technically different (closer to the original formulation of the problem [12]) but conceptually similar studies appeared in [19].

2 Supersymmetric plasma and gravity

As discussed in the introduction, $\mathcal{N} = 4$ SYM is amenable to quantitative studies in the strongly coupled limit. In the absence of methods which could be used in QCD for the study of strongly coupled, real time dynamics, this theory has become a focus of much attention. This theory shares some features of QCD (especially at high temperature), but is rather different from it. It contains, apart from gluons, 6 real massless scalars and 4 Majorana massless fermions, all in the adjoint representation of the $U(N)$ gauge group. The theory is known to be conformal even at the quantum
level. As mentioned in the introduction, we work within the AdS/CFT correspondence [3] which becomes an effective computational tool in the 't Hooft limit $N \to \infty$ and $\lambda = g_{\text{YM}}^2 N \to \infty$, where quantum and stringy corrections on the gravity side can be neglected.

There are important similarities and differences between SYM and QCD which one has to keep in mind. Among the similarities are the existence of the deconfined phase. Also, in the perturbative regime at $T > 0$, both theories have been shown to behave similarly, with the difference coming mostly from the different number of degrees of freedom [20]. The most crucial difference is that $\mathcal{N} = 4$ SYM has a vanishing beta function, which implies that it is not confining, has no finite temperature phase transition and has an exactly conformal equations of state.

The modeling of nuclear collisions is an extremely complex task. To reduce the complexity of the problem we adopt strong symmetry assumptions introduced by Bjorken [21] for the description of matter following a heavy ion collision. The dynamics of the system is assumed to be independent of boosts along the longitudinal (collision) axis and is independent of transverse coordinates. In proper time-rapidity coordinates $t = \tau \cosh y$, $z = \tau \sinh y$ this reduces to the statement that observables dependent only on proper time $\tau$. This approximation becomes exact in the limit of an infinite energy collision of infinitely large nuclei.

Two physical quantities of interest to us will be the energy momentum tensor and entropy. In the present circumstances first one takes the form

$$T_{\mu\nu} = \text{Diag}\{\epsilon(\tau), p_L(\tau), p_T(\tau), p_T(\tau)\}, \quad (1)$$

where $\epsilon(\tau) = p_L(\tau) + 2p_T(\tau)$ as required by conformal symmetry. The energy density defines local effective temperature by the relation

$$\epsilon(\tau) = 3N^2\pi^2T(\tau)^4. \quad (2)$$

This is the temperature of an equilibrium system with the same energy density. It can be shown that for late times the dynamics is governed by the equations of hydrodynamics; up to third order the effective temperature follows [4, 22, 5, 23]

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left\{1 - \frac{1}{6\pi(\Lambda\tau)^{2/3}} - \frac{1 + \log 2}{36\pi^2(\Lambda\tau)^{4/3}} + \frac{-21 + 2\pi^2 + 51\log 2 - 24\log^2 2}{1944\pi^3(\Lambda\tau)^2}\right\}. \quad (3)$$

The energy scale $\Lambda$ appearing in Eq. (3) depends on the initial conditions chosen and it is the only trace of initial state information contained in the hydrodynamic expansion. It also sets the scale for the energy density at hydrodynamization.
The calculation strategy follows usual lore of holography \cite{24}. States in the boundary theory correspond to asymptotically AdS geometries in the bulk. For example an equilibrium, finite temperature, deconfined plasma state corresponds to a static (planar) black hole in the bulk. The Hawking temperature of this black object is interpreted as the temperature in the dual $\mathcal{N} = 4$ SYM. Extending this notion to the out-of-equilibrium situation we assume that non-equilibrium plasma states correspond to geometries with non-static horizons. Such geometries are assumed to pose an event horizon, but there are reasons to believe \cite{25, 26, 27} that the physical notions (such as entropy for instance) should be associated with apparent horizons.

This conjecture allows us to extend the notion of entropy to non-equilibrium states by the Bekenstein-Hawking relation, which with our normalization translates to

$$S = \frac{a_{AH}}{\pi},$$

(4)

where $a_{AH}$ is apparent horizon area \cite{23}. By the area law theorems this quantity is non-decreasing and agrees with thermodynamic definition for late times.

In calculations performed here we took 600 different initial states described by randomly generated initial geometries – with each of these we associate an initial entropy as defined above. We then evolved these geometries according to Einstein equations well into the hydrodynamic regime. From the rules of the holographic correspondence we are able to read of the relevant physical observables i.e. energy density $\epsilon(\tau)$ and entropy $S(\tau)$. An important assumption on the initial conditions is that $\epsilon(0) \neq 0$, which allows us to normalize the initial effective temperature as $T(0) = 1/\pi$. For more technical details of construction of initial geometries and solving for the time evolution we refer to the original paper \cite{14}.

3 Results and conclusions

In order to present quantitative results on the hydrodynamization process we need to give it a precise definition. The approach to hydrodynamics can be observed by monitoring the pressure anisotropy

$$\Delta \equiv \frac{p_T - p_L}{\epsilon}.$$  

(5)

It is convenient to measure time in units of inverse local temperature, that is, to use $w = \tau T(\tau)$ as a parameter of evolution. The pressure anisotropy can then be expressed as $\Delta(w) = 6f(w) - 4$ in terms of the function (first introduced in \cite{15})

$$f(w) = \frac{\tau}{w} \frac{dw}{d\tau}.$$  

(6)
which for dimensional reasons is independent of \( \Lambda \). At large times (large \( w \)) this function attains a universal (hydrodynamic) form \( f_H(w) \), determined by an infinite set of transport coefficients, which up to the third order reads

\[
f_H(w) = \frac{2}{3} + \frac{1}{9\pi w} + \frac{1 - \log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45 \log 2 + 24 \log^2 2}{972\pi^3 w^3}.
\]

The beginning of the hydrodynamic stage might now be defined as the value of \( w \) (hence proper time \( \tau \)) when the difference between the actual \( f(w) \) and the hydrodynamic form \( f_H(w) \) is less than some arbitrary small number; for example

\[
|\frac{f_H(w)}{f(w)} - 1| < 0.05.
\]

While this definition involves some arbitrary choice, varying this criterion within reason leads to no appreciable change in the calculated value of the hydrodynamization time.

Figure 1: Histogram of hydrodynamization times \( w_{th} \) in units of effective hydrodynamization temperature.

Figure 1 shows a histogram of hydrodynamization times obtained this way. Regardless the initial entropy, the hydrodynamization time is of the order if the inverse hydrodynamization temperature. On the average \( w_{av} = 0.57 \). It is instructive to compare it to the estimation from the RHIC data; \( T = 500 \) MeV and \( \tau = 0.25 \) fm/c gives \( w_{RHIC} = 0.63 \) which is very close to theoretical prediction. The pressure anisotropy at hydrodynamization is found to be quite high: \( \Delta \approx 0.35 \).

The other quantity we focus on here is the energy scale \( \Lambda \) which sets the scale for the hydrodynamic cooling. This quantity is obtained by fitting the tail of the data to Eq. (3).
Results for this scale are shown in figure 2. For an intermediate range of entropies below $S \sim 0.3$ a strong linear correlation is observed. For initial entropies larger than 0.3 the correlation is lost and “chaotic” behaviour develops.

In conclusion, through our analysis of the time evolution of a bulk sample of different non-equilibrium initial states we find significant evidence to claim that the early hydrodynamization of [15] is a generic process. Our analysis supports previous findings that hydrodynamization is different from thermalization in the sense of non-negligible pressure gradients being present and well described by hydro. It also suggests that entropy production during the hydrodynamic stage of evolution is not negligible despite the low value of $\eta/s$.

Our simulations suggest that, at least in some regions of initial state parameters, there might exist characteristic regularities, reflecting the nature of collective non-hydrodynamic degrees of freedom [28]. At this point an important question to what extent is this correlation a consequence of the strong symmetry assumptions imposed and to what extent it reflects the true nature of the process.

Acknowledgements

I would like to thank M. Spaliński and G. Plewa for collaboration on this project and S. Mrówczyński for useful comments. I express my thanks to the organizers of the CSQCD IV conference for providing an excellent atmosphere which was the basis for inspiring discussions with all participants. We have greatly benefited from this. This research was supported by a post-doctoral internship grant No. DEC-2013/08/S/ST2/00547.
References

[1] D. Teaney, J. Lauret and E. V. Shuryak, “Flow at the SPS and RHIC as a quark gluon plasma signature,” Phys. Rev. Lett. 86 (2001) 4783 [nucl-th/0011058].

[2] P. F. Kolb and U. W. Heinz, “Hydrodynamic description of ultrarelativistic heavy ion collisions,” In *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 634-714 [nucl-th/0305084].

[3] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231] [hep-th/9711200].

[4] R. A. Janik and R. B. Peschanski, “Asymptotic perfect fluid dynamics as a consequence of Ads/CFT,” Phys. Rev. D 73, 045013 (2006) [hep-th/0512162].

[5] M. P. Heller and R. A. Janik, “Viscous hydrodynamics relaxation time from AdS/CFT,” Phys. Rev. D 76 (2007) 025027 [hep-th/0703243 [HEP-TH]].

[6] S. Bhattacharyya, R. Loganayagam, S. Minwalla, S. Nampuri, S. P. Trivedi and S. R. Wadia, “Forced Fluid Dynamics from Gravity,” JHEP 0902, 018 (2009) [arXiv:0806.0006 [hep-th]].

[7] R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov, “Relativistic viscous hydrodynamics, conformal invariance, and holography,” JHEP 0804, 100 (2008) [arXiv:0712.2451 [hep-th]].

[8] P. Romatschke, “New Developments in Relativistic Viscous Hydrodynamics,” Int. J. Mod. Phys. E 19, 1 (2010) [arXiv:0902.3663 [hep-ph]].

[9] M. P. Heller, R. A. Janik and P. Witczak, “Hydrodynamic Gradient Expansion in Gauge Theory Plasmas,” Phys. Rev. Lett. 110, no. 21, 211602 (2013) [arXiv:1302.0697 [hep-th]].

[10] M. P. Heller, R. A. Janik, M. Spalinski and P. Witczak, “Coupling hydrodynamics to nonequilibrium degrees of freedom in strongly interacting quark-gluon plasma,” Phys. Rev. Lett. 113, no. 26, 261601 (2014) [arXiv:1409.5087 [hep-th]].

[11] M. P. Heller and M. Spalinski, “Hydrodynamics Beyond the Gradient Expansion: Resurgence and Resummation,” [arXiv:1503.07514 [hep-th]].

[12] P. M. Chesler and L. G. Yaffe, “Horizon formation and far-from-equilibrium isotropization in supersymmetric Yang-Mills plasma,” Phys. Rev. Lett. 102 (2009) 211601 [arXiv:0812.2053 [hep-th]].
[13] P. M. Chesler and L. G. Yaffe, “Boost invariant flow, black hole formation, and far-from-equilibrium dynamics in $N = 4$ supersymmetric Yang-Mills theory,” Phys. Rev. D 82, 026006 (2010) [arXiv:0906.4426 [hep-th]].

[14] J. Jankowski, G. Plewa and M. Spalinski, “Statistics of thermalization in Bjorken Flow,” JHEP 1412 (2014) 105 [arXiv:1411.1969 [hep-th]].

[15] M. P. Heller, R. A. Janik and P. Witaszczyk, “The characteristics of thermalization of boost-invariant plasma from holography,” Phys. Rev. Lett. 108 (2012) 201602 [arXiv:1103.3452 [hep-th]].

[16] M. P. Heller, R. A. Janik and P. Witaszczyk, “A numerical relativity approach to the initial value problem in asymptotically Anti-de Sitter spacetime for plasma thermalization - an ADM formulation,” Phys. Rev. D 85, 126002 (2012) [arXiv:1203.0755 [hep-th]].

[17] M. P. Heller, D. Mateos, W. van der Schee and D. Trancanelli, “Strong Coupling Isotropization of Non-Abelian Plasmas Simplified,” Phys. Rev. Lett. 108, 191601 (2012) [arXiv:1202.0981 [hep-th]].

[18] M. P. Heller, D. Mateos, W. van der Schee and M. Triana, “Holographic isotropization linearized,” JHEP 1309, 026 (2013) [arXiv:1304.5172 [hep-th]].

[19] L. Bellantuono, P. Colangelo, F. De Fazio and F. Giannuzzi, “On thermalization and isotropization of a boost-invariant non Abelian plasma,” arXiv:1503.01977 [hep-ph].

[20] A. Czajka and S. Mrowczynski, “$N=4$ Super Yang-Mills Plasma,” Phys. Rev. D 86 (2012) 025017 [arXiv:1203.1856 [hep-th]].

[21] J. D. Bjorken, “Highly Relativistic Nucleus-Nucleus Collisions: The Central Rapidity Region,” Phys. Rev. D 27 (1983) 140.

[22] R. A. Janik, “Viscous plasma evolution from gravity using AdS/CFT,” Phys. Rev. Lett. 98, 022302 (2007) [hep-th/0610144].

[23] I. Booth, M. P. Heller and M. Spalinski, “Black brane entropy and hydrodynamics: The Boost-invariant case,” Phys. Rev. D 80 (2009) 126013 [arXiv:0910.0748 [hep-th]].

[24] R. A. Janik, “The Dynamics of Quark-Gluon Plasma and AdS/CFT,” Lect. Notes Phys. 828 (2011) 147 [arXiv:1003.3291 [hep-th]].

[25] I. Booth, “Black hole boundaries,” Can. J. Phys. 83, 1073 (2005) [gr-qc/0508107].
[26] I. Booth, M. P. Heller and M. Spalinski, “Black Brane Entropy and Hydrodynamics,” Phys. Rev. D 83, 061901 (2011) [arXiv:1010.6301 [hep-th]].

[27] I. Booth, M. P. Heller, G. Plewa and M. Spalinski, “On the apparent horizon in fluid-gravity duality,” Phys. Rev. D 83, 106005 (2011) [arXiv:1102.2885 [hep-th]].

[28] R. Peschanski, “Dynamical entropy of dense QCD states,” Phys. Rev. D 87, no. 3, 034042 (2013) [arXiv:1211.6911 [hep-ph]].