Petri transport networks

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Abstract. This paper presents a new class of Petri nets - Petri transport networks, used to simulate the transport's systems. This class can be attributed to hierarchical temporal colored probabilistic Petri nets. The paper gives a definition, basic concepts, structure and functioning rules of Petri transport networks. The relevance of the topic raised in the article is due to increasing attention to the development of transport systems, their evolution and modernization. Modern management methods, various mechanisms for regulating traffic flows, infrastructural extensions, all this does not meet the challenges posed by modern transport infrastructure. Moreover, the problem is increasing every year. Petri transport networks introduced in the work, are a convenient tool ohms to simulate the transport's systems of any complexity and scale. A new class of Petri transport networks is characterized by a hierarchical structure, in which the first level displays the entire system as a whole, the second - the internal structure of each object. Distinctive features are also the adjustment of the activation of traffic flows by means of a control marker, inter-level synchronization, separation of positions and transitions to types having different functions.

1. Introduction
Petri nets [1] - a modeling tool widely used by engineers. This is due to its functionality and flexibility. It is quite simple to adapt to specific tasks by changing the rules of functioning or by adding new properties and procedures. Therefore, during the development of the Petri net [2-3] underwent many modifications. Such subclasses of networks as probabilistic, temporary, colored, hierarchical, and many others were formed and found their use. Another important advantage of Petri nets [2] is the convenience of combining such subclasses. In addition, Petri nets [2-3] have several types of representation: through matrices, graphs and analytical view, which allows to see the graphical structure, as well as the dynamics of processes and static states of the model.

The class of Petri transport networks (PTN) introduced in the article is planned to be used in the future for modeling transport systems [4] of any complexity and scale. For this, it is necessary to synthesize various extensions of Petri nets. Namely, colorized, temporary, hierarchical, probabilistic. But a simple synthesis of such extensions of Petri nets brings with it some limitations that, when modeling systems of great complexity, can have a critical impact on the speed of processes. Therefore, the logic of the processes in Petri nets was changed. Thus, it is possible to obtain a system [5] that is flexible both to the settings and to the situations that arise in it. In such a system, it is convenient to conduct research on new algorithms, to check the impact on the system of new infrastructure elements that are only planned to be added, to model other various transport tasks.
A large number of articles are devoted to the study area of traffic flow modeling based on Petri nets, and each of them uses its own approach to the problem under consideration. It should be noted that the apparatus of Petri nets in the transport field is often used to model railway systems [6]. In works in which ideas and subject area coincide, the logic of the processes in Petri nets is different. So in [7], the road segment corresponding to the length of the car acts as the nodes of the Petri net of the transport system. In this work, the nodes and transitions of the same type from the point of view of Petri nets differ from each other both in logical purpose and in physical representation. For example, there is a node type that is responsible for allocating lanes of vehicles arriving at the infrastructure element, and a node type corresponding to the entire infrastructure element is determined. This is useful when scaling the system. In addition, a new concept of the environment of an element of infrastructure is introduced. In [8], the Petri net is used to study the problem of controlling the displacement of an emergency vehicle. The proposed work considers regular, non-emergency situations of the distribution of vehicle flows. In [9], Petri nets are used in the design of the controller to prevent collisions in automated vehicle systems. In [10], modeling of transport systems using metagraphs of Petri nets was proposed. This article uses the classic approach of using digraphs of Petri nets.

The aim of the work is the formation of a basic mathematical apparatus, focused on modeling complex transport systems, based on various modifications of Petri nets, modified taking into account the characteristics of the subject area.

2. Basic concepts and general structure of PTN
PTN will consist of several levels. Upper - the first level displays the entire systems of the whole, the next - the internal structure of an individual object. In this regard, PTN is hierarchical. In this paper, we consider a two-level system.

Before we consider the structure of the PTN, we introduce the concept of environment. The environment will be called the set of connections with neighboring objects included in the system under consideration. There are two types of environment relationships: complete and incomplete. The first connections are those that connect the 2 infrastructure elements (IE) of the system to each other. In incomplete communications, at least on one side, there is no IE of the system. The full environment of the IE will be considered the environment, all of which are complete communications. If there is at least one incomplete connection in the environment, it is incomplete. If all environment connections are incomplete, it is considered zero.

Consider PTN level 1. A superscript will be used to indicate the ownership of an element of the structure of the PTN to a certain level. For example, the 2nd position of the 1st level of the PTN will be designated as $P^1_2$. So also on this level, there are transitions, interconnecting the respective positions. In classical Petri nets, their activation depends on the availability of necessary chips (markers). At this level, they are active constantly. In this case, the transitions contain an internal and dynamic vector of delays $Z$. This is due to the fact that vehicles, hereinafter VEH, do not move between transport objects instantly, but spend some time on this. When a marker gets into such a transition, it is given some time for which it remains in the transition. At the end of the set interval, the marker leaves the transition and moves to the appropriate position. Time delays in transitions and directions of movement will be set with some probabilities $W$. In this regard, PTN is probabilistic. It should also be noted the synchronism of all levels. It is understood that a single timeline is used for processes of different levels, and they are not duplicated.

Now we turn to the consideration of PTN level 2. Suppose we have a crossroads (figure 1) with 4 traffic lights group (TLG), 4 exit directions (numbers in rectangles) and 10 input (numbers in ovals), as well as an incomplete environment.
Starting from figure 1, we obtain the following structure PTN level 2 (figure 2). The number of positions will be 5. This number is determined by the number of exit directions $P_1, ..., P_4$ and a distribution position $P_5$, wherein the determined direction of further motion of markers in the system. In turn, the number of transitions $t$ will be more variable for each infrastructure element, in the future IE. This number depends on the number of output directions $t_1, ..., t_{10}$, number TLG $t_1, ..., t_6$, the presence of a generating transition $t_i$ in the absence in the system of one or more relationships with real objects, the type of environment IE $t_j$. Separately, it should be noted the case of the lack of environment,
i.e. when the simulated element is isolated from the system. In this case, vehicles can only be generated. In this case, there is no \( t_2 \).

The next element of the PTN of the 2nd level are markers. At this level, they have colors in accordance with the strip along which VEH moved until the moment of entry directly to the intersection itself. Further, they are discolored, due to the loss of the need for color identification. So at the crossroads under consideration there are 10 colors (col. 1-10) for markers. In PTN there is one more type of marker for each IE. He is designated as manager (MN). Only one marker of this type can exist on one IE at a time. This marker performs a control function. It is he who indicates the activity of the TLG and determines the operating time of the green signal in it. The last is the lifetime of this marker. On the IE under consideration, 4 colors of this type of markers or (col. 11-14) are possible.

3. Mathematical model PTN

Having a hierarchical structure, the PTN at each level may be slightly different, but two presented in the work are enough to derive the concept. So PTN of the 2nd level will be the seven \( PTN^2 = (P^2, T^2, F^2, C^2, D^2, W^2, M^2) \), where the superscript indicates the considered level of PTN.

In turn, \( P^2 = \{ P_{1}^2, ..., P_{\text{vol}}^2, P_{\text{do}}^2 \} \) – a plurality of positions, wherein \( \text{out} \) – the number of output directions for this IE, \( P_{\text{do}}^2 \) – distributing position, designated in figure 2 as \( P_1, P_2, ..., P_{\text{vol}} \) – positions corresponding to the exit directions at the given IE and related to the corresponding exit transitions. Figure 2 is \( P_1, ..., P_k \). \( T^2 = \{ T_1^2, ..., T_{\text{sg}}^2, B_1^2, ..., B_{\text{out}}^2, B_{\text{gen}}^2, B_{\text{ent}}^2 \} \) – a plurality of transitions where \( \text{sg} \) – the number of traffic lights groups in this IE, \( B_{\text{gen}}^2 \) – generating a transition, forming the MN and markers with IE beyond the thinning the considered transport system corresponding \( t_1 \) to figure 2, \( B_{\text{ent}}^2 \) – the inlet passage associated with \( P_{\text{in}}^2 \), provides the entry of markers into the object from other objects of the system and the corresponding \( t_2 \), \( T^2 \) – traffic light crossings, designated \( t_3, ..., t_6 \), \( B^2 \) – distribution transitions, appropriate \( t_1, ..., t_{\text{vol}} \). \( F^2 = (P^2 \times T^2) \cup (T^2 \times P^2) \) – set of arcs. \( C^2 = \{ C_1, ..., C_{\text{in}}, K_1^1, ..., K_{\text{in}}^1 \} \) – set of colors, where \( \text{in} \) – the number of input directions, \( C_1, ..., C_{\text{in}}^2 \) – the colors corresponding to the input directions, \( K_1^2, ..., K_{\text{in}}^2 \) – he colors corresponding to MN TLG.

\( D^2 = \{ D_{\text{do}}^2, D_1^2, ..., D_{\text{sg}}^2 \} \) – set of delay matrices, where there are \( D_{\text{do}}^2 \) – delays in the distribution position by color, \( D_1^2 \) – delays in the corresponding traffic light transition \( T_1^2 (i = 1, ..., \text{sg}) \). \( W^2 = \{ W_{\text{gen}}^2, W_1^2, ..., W_{\text{ent}}^2 \} \) – a set of probability matrices, where the \( W_{\text{gen}}^2 \) – probabilities are in the generating transition, \( W_1^2, ..., W_{\text{ent}}^2 \) – the probabilities in the input transition. \( M^2 = \{ M_{\text{in}}^2, M_{\text{ent}}^2 \} \) – set of markings, \( M_{\text{in}}^2 \) – marking in positions, \( M_{\text{ent}}^2 \) – marking in transitions.

4. Rules for the functioning of the PTN

Consider the functioning of the first level of PTN. IE plays the role of position \( P \). Markers are colorless at this level. Transitions are always active and contain marker delay matrices. The marker is moved from
the 1st level of the PTN to the second at the moment when its delay time in the transition has ended, and it has moved to the position next to the transition.

At the second level, the marker, getting into the input transition, receives a color, thereby identifying on this IE. In the distribution position, the marker is ordered by the delay time, the zeroing of which allows this marker to move to one of the traffic lights transitions, the activation of which is due to the MN. In turn, the MN is formed in the generating transition, depending on the situation on the IE and the internal limitations of the system. The number of colors that can be stained with MN is the same as the amount of TLG. When a marker gets into a traffic light transition, a position corresponding to one of the output directions is set for it. Also, a certain delay is set corresponding to the time required by the vehicle for maneuver. The choice of the direction of movement of the marker from the traffic light transition is determined by the probabilities and parameters of the IE. Next, the marker moves to the output position corresponding to the selected direction. When switching to the output transition, the marker loses color identification. Then he moves to the 1st level of the PTN.

It is worth noting that the operating time of the green signal of traffic lights in an active TLG corresponds to the lifetime of the MN, after which the MN is removed from the system.

5. Conclusion
Thus, the PTN considered in the work is a new class of Petri nets, formed specifically for solving the problems of modeling complex transport systems. Its distinguishing feature is hierarchy, adjustment of traffic flows by means of a control marker, inter-level synchronization, separation of positions and transitions into types having different functions. Such a network will function more flexible, since some rules for the functioning of classical Petri nets, in particular, the rules for triggering a transition, have been changed. PTN, as the heir to Petri nets, remained susceptible to structural changes. The probabilistic component allows for more accurate tuning of both a single IE and the entire system. The hierarchy of the PTN allows you to consider and analyze each level of the transport system separately. As a result, this class becomes a convenient and promising tool for modeling transport systems.

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