Prediction of repulsive potential for high-energy heavy-ion scatterings

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Abstract. The complex $G$-matrix folding model predicts that the real part of the potential becomes repulsive for high-energy heavy-ion systems. We investigate the energy evolution of the potential and discuss a possible method for giving evidence for the repulsive nature of heavy-ion optical potentials. We propose to measure the characteristic evolution of the diffraction pattern in differential cross sections, for the first time, to find clear evidence of the repulsive optical potential for heavy-ion systems.

1. Introduction
The self-consistent nuclear meanfield for finite nuclear systems is an attractive potential as a whole and nucleons in a nucleus are bound in such an attractive single-particle potential. The attractive nature of the nuclear potentials is smoothly connected to the positive-energy region of the nucleon scattering states [1]. The optical potentials for low-energy nucleons scattered by finite nuclei are known to have an attractive real part together with an absorptive imaginary part. However, the optical potential for nucleons is highly dependent on the incident energy and the depth of the attractive real part is known to become shallower as the energy increases. The strength of the interior part decreases more rapidly than that of the surface part, leading to the so-called wine-bottle-bottom (WBB) shape for $E_{\text{in}} \geq 200 \sim 300$ MeV, and finally the potential changes its sign from negative (attractive) to positive (repulsive), first in the interior and then at the surface at around $E_{\text{in}} \approx 500 \sim 800$ MeV [2, 3, 4, 5, 6, 7, 8]. The origin of this transition of the optical potentials from attractive to repulsive has been studied at the microscopic level of nucleon-nucleon interactions both in the relativistic and non-relativistic frameworks.
A similar attractive-to-repulsive transition of optical potentials was found for intermediate-energy deuteron-nucleus scattering [9, 10, 11, 12]. The similarity of proton scattering and deuteron scattering is quite natural for high scattering energies because of the small binding energy of deuteron. The characteristic feature of deuteron scattering is known to be well understood in the picture of the impulse or sudden approximation, in which the deuteron-nucleus scattering amplitude is written to a good approximation as the coherent sum of the proton-nucleus scattering with the neutron being a spectator and the neutron-nucleus scattering with the proton being a spectator [13]. However, no such attractive-to-repulsive transition of optical potentials has been studied theoretically nor proved experimentally for scattering of composite projectiles other than the loosely-bound deuteron. The only exception was a report on the possible repulsive optical potentials for intermediate-energy $^{16}$O-nucleus scattering [14] but it was far from conclusive.

Recently, the present authors have proposed a theoretical model for constructing the complex optical potential for any composite projectile through the double-folding-model (DFM) procedure using a newly proposed complex $G$-matrix $NN$ interaction called CEG07 [15, 16, 17] and demonstrated that the folding model provides quite reliable complex optical potentials for intermediate-energy scattering of light heavy ions, such as $^{16}$O and $^{12}$C, by various target nuclei. Moreover, the folding model predicted a repulsive optical potential for an incident energy per nucleon of $E/A \approx 400$ MeV [17, 18] and the attractive-to-repulsive transition was found to occur below this energy, say around $E/A \approx 200 \sim 300$ MeV. These energies are apparently lower than the energies ($E/A \approx 300 \sim 400$ MeV) at which the proton and deuteron optical potentials start to change their sign from negative (attractive) to positive (repulsive).

The purpose of the present paper is to analyze the characteristic behaviour of the elastic-scattering cross sections with the increase of the incident energy in this energy region and to study its close relation to the attractive-to-repulsive transition of the optical potential predicted by the folding model. This will give us an experimental signature for identifying evidence of the attractive-to-repulsive transition of optical potentials for nucleus-nucleus systems. To this end, we decompose the calculated differential cross sections into the nearside and farside ($N/F$) components [19] in order to clarify the attractive or repulsive contribution of the optical potential to the differential cross sections.

### 2. Double folding model potential

Now, we apply the complex $G$-matrix interaction, CEG07, to investigate the repulsive optical potential in heavy-ion scattering through the folding procedure. The CEG07 complex $G$-matrix interaction is derived from an $NN$ interaction in free space by solving the Bethe-Goldstone equation in nuclear matter with the scattering boundary conditions. We adopted the extended soft-core model (ESC04) [20, 21] as the free-space $NN$ interaction. In this paper, we apply two-types of interaction, CEG07a and CEG07b [15], the latter including effects of the three-body force (TBF).

The optical model potential (OMP) is composed of the real and imaginary parts as,

$$U_{\text{OMP}}(R) = V(R) + iW(R).$$

We then give the optical potential in the framework of the double folding model for heavy-ion scattering systems. Because we use the complex $G$-matrix interaction, the folding model potential also becomes complex. The complex optical potential for heavy-ion scattering is derived in the same manner as given in Refs. [17, 18]. The optical model potential based on the folding model is written as,

$$U_{\text{OMP}}(R) = V_{\text{DF}}(R) + iW_{\text{DF}}(R),$$

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where $V_{DF}$ and $W_{DF}$ denote the real and imaginary components of the central part of the double folding model potential.

3. Result and Discussion

We now apply the DFM potential with the CEG07 $G$-matrix interaction to $^{12}\text{C} + ^{12}\text{C}$ elastic scattering at four incident energies, $E/A = 100, 200, 300, \text{and} 400$ MeV, and analyze the energy dependence of the calculated DFM potentials and its close relation to the elastic scattering observables. For the nucleon density of $^{12}\text{C}$, we adopt the density deduced from the charge density [22] extracted from electron-scattering experiments by unfolding the finite-size effect of the proton charge in the standard way [23]. We perform the calculations using both types of the CEG07 interactions, CEG07a (without the TBF effect) and CEG07b (with the TBF effect).

![Figure 1](image1.png)

**Figure 1.** Real (upper) and imaginary (lower) parts of the DFM potentials calculated with CEG07a (left) and CEG07b (right). This figure is taken from [18].

![Figure 2](image2.png)

**Figure 2.** Rutherford ratio of the differential cross sections, displayed as the functions of the momentum transfer $q$. This figure is taken from [18].

First, we compare the DFM potentials calculated at the four energies with the CEG07a and CEG07b interactions. Figure 1 shows the real part ($V_{DF}(R)$) and the imaginary part ($W_{DF}(R)$) of the DFM potentials calculated with CEG07a and CEG07b. The imaginary part has a similar shape at all energies for both interactions and its strength becomes larger as the energy increases, whereas the real part becomes shallower with increasing energy and starts to change its sign from negative (attractive) to positive (repulsive) around $E/A = 300$ MeV for CEG07a and 200 MeV for CEG07b, although this depends on the radial position. It is seen that the DFM potentials at the origin ($R = 0$) with CEG07b are more repulsive by about 80 ~ 100 MeV than that with CEG07a at all energies, which is mainly due to the contribution of the repulsive component of the TBF [17].

Figure 2 shows the angular distributions of the $^{12}\text{C} + ^{12}\text{C}$ elastic-scattering cross sections calculated at the four incident energies. The relativistic-kinematics correction has been made when we solve the Schrödinger equation in all the scattering calculations presented in this paper. The calculated cross section with CEG07a is larger than the cross section with CEG07b at the middle and backward angles at $E/A = 100$ MeV, whereas both potentials give almost identical angular distributions at 200 MeV and, as we go to higher energies, the CEG07b cross section
becomes that larger of the two. At $E/A = 400$ MeV, the situation becomes completely opposite to that of 100 MeV. In addition, the cross sections show a strong diffractive oscillation pattern at 300 MeV for CEG07b and at 400 MeV for CEG07a, respectively, while they show no strong oscillation at other energies.

Figure 3. Schematic representations of the outgoing spherical waves (lower) and semi-classical schematic representations of the $N/F$ cross sections (upper) in the cases of the attractive (a) and repulsive (b) potentials.

Here, we decompose the elastic-scattering cross sections calculated with the present DFM potentials into the $N/F$ components in order to understand the characteristic behaviour of the cross sections with the increase of the incident energy, shown in Fig. 2, and to clarify its close relation to the attractive-to-repulsive transition of the optical potentials, shown in Fig. 1. When the real part of the inter-nucleus potential is strongly attractive ($V < 0$), the incident particle on or inside the grazing trajectory is deflected to negative (farside) scattering angles. This causes the enhancement of the scattering into the negative (farside) angles that is compensated by the reduction of the scattering to positive (nearside) angles, which leads to a crossover between the nearside and farside amplitude at a certain crossover angle, as shown in Fig. 3 (a). However, the situation will completely be changed if the nuclear potential is repulsive ($V > 0$), not attractive. In such a case, there exists no attraction that deflects the incident particle to the negative (farside) angles. Namely, in the semi-classical picture, the farside component of the scattered wave is reduced by the repulsive nuclear potential while the nearside component is enhanced and, hence, no crossover between the nearside and farside amplitudes occurs, as shown in Fig. 3 (b).

Now, we have seen that the present folding model predicts the attractive to repulsive transition around $E/A = 200 \sim 300$ MeV region, the transition energy being dependent on the interaction used, and both the CEG07a and CEG07b interactions give repulsive potentials ($V > 0$) at $E/A = 400$ MeV. Therefore, one may naturally expect that the effect of the drastic change of the calculated real potentials with the increase of the incident energy should appear in the evolution of the calculated angular distribution of the elastic scattering. In order to understand the situation more quantitatively, we decompose the elastic-scattering cross sections into the $N/F$ components in the quantum-mechanical way as given in Ref. [19].

Figure 4 shows the $N/F$ cross sections together with the full cross sections calculated by DFM potentials with CEG07a and CEG07b at $E/A = 100 \sim 400$ MeV, respectively. At $E/A$
$E/A = 100$ MeV, the attraction of the real potential is very strong, as already seen in Fig. 1, which largely enhances the farside component and reduces the nearside one and, hence, the farside component dominates over the nearside one at most scattering angles except for very forward angles. This is particularly clear in the case of CEG07a interaction. The farside dominance persists up to $E/A = 300$ MeV in the case of the CEG07a interaction, while it does up to 200 MeV in the case of CEG07b. The difference comes, of course, from the less attraction (see Fig. 1) in the case of the CEG07b interaction due to the repulsive TBF effect [15, 17]. At $E/A = 400$ MeV (300 MeV) in the case of the CEG07a (CEG07b) interaction, the nearside and farside components are seen to have comparable magnitudes and strongly interfere with each other showing a strong $N/F$ diffraction pattern in the angular distribution. The strong diffraction pattern implies that the contribution of the real potential becomes effectively minimum at the corresponding energy. In the case of the CEG07b interaction, one can clearly see the nearside dominance with the reduced $N/F$ interference at $E/A = 400$ MeV, which is a clear signature generated by the repulsive nature of the real potential.

These results strongly suggest a possible experimental confirmation for the attractive-to-repulsive transition of the nucleus-nucleus optical potentials can be obtained by measuring the characteristic energy evolution of the elastic-scattering angular distribution in this energy region. Namely, the evidence would be obtained by observing the farside dominance at the lower energies and the nearside dominance at the higher energies both with weak diffraction patterns due to the reduced $N/F$ interference, together with the strong diffraction around a certain transition energy caused by the strong $N/F$ interference due to the comparable magnitudes of the both components, although the predicted transition energy depends on the effective interaction used ranging from $E/A = 300$ MeV $\sim 400$ MeV.

Figure 4. Nearside and farside ($N/F$) decomposition of the elastic-scattering cross sections calculated from the CEG07a (left) and CEG07b (right) interactions.

4. Summary
The real part of heavy-ion optical potential is believed to be attractive in the low energy region below $E/A \approx 200$ MeV where experimental data for the elastic scattering exist. However, the microscopic double-folding model (DFM) potentials based on the complex $G$-matrix interaction CEG07 are predicted to change their character from attractive to repulsive around the energies of $E/A \approx 200 \sim 300$ MeV and to become strongly repulsive above $E/A \approx 400$ MeV [15, 16, 17]. It is also shown that the nearside and farside ($N/F$) decomposition of the elastic-scattering cross sections clarifies the close relation between the attractive-to-repulsive transition of the calculated DFM potentials and the characteristic evolution of the elastic-scattering angular distributions.
with the increase of the incident energy in the range of \( E/A = 100 \sim 400 \) MeV.

No experimental evidence exists so far for the repulsive nature of heavy-ion optical potentials in this energy range because of the lack of experimental data above \( E/A = 200 \) MeV. Therefore, on the basis of the present analyses, we here propose experimental measurements of elastic-scattering angular distributions in the \(^{12}\text{C} + ^{12}\text{C}\) system, particularly the careful measurements of the characteristic evolution of the diffraction pattern with the increase of the incident energy in the range of \( E/A = 200 \sim 400 \) MeV. More explicitly, we propose the observation of a strong diffraction pattern caused by the interference between the nearside and farside amplitudes, both having comparable magnitudes, around the attractive-to-repulsive transition energy, together with the observation of less diffractive patterns in the farside-dominant regime (i.e. the attractive-potential regime) on the lower energy side as well as in the nearside-dominant regime (the repulsive-potential regime) on the higher energy side, that we have seen in Fig. 4. Such experiments will provide decisive evidence for the repulsive nature of heavy-ion optical potentials as well as information about the energy region where the attractive-to-repulsive transition occurs.

5. Acknowledgment

This work was supported by the Grant-in-Aid for the Global COE Program ⊗ The Next Generation of Physics, Spun from Universality and Emergence ⊗ from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan.

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