On the circular polarization of pulsar radiation

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Abstract

We consider the polarization behaviour of radio waves propagating through an ultrarelativistic highly magnetized electron-positron plasma in a pulsar magnetosphere. The rotation of magnetosphere gives rise to the wave mode coupling in the polarization-limiting region. The process is shown to cause considerable circular polarization in the linearly polarized normal waves. Thus, the circular polarization observed for a number of pulsars, despite the linear polarization of the emitted normal waves, can be attributed to the limiting-polarization effect.

1 Introduction

Pulsar radiation is known to have a high linear polarization. Although the circular polarization of the average pulse profiles is typically less than the linear one, for the most of pulsars the circular polarization is also significant (for recent review, see Han et al. 1998 and references therein). The circular polarization tends to peak near the centre of the profile. In many pulsars the circular polarization reverses its sense near the pulse centre, while in the others the sense remains constant throughout the pulse (Radhakrishnan & Rankin 1990). The observed circular polarization of pulsar radiation is commonly attributed either to the propagation effects or to the emission mechanism.

The magnetic field pattern of a pulsar is generally considered to be that of a rotating dipole. The magnetosphere contains an electron–positron plasma outflowing along the open magnetic field lines with Lorentz-factors $\gamma \sim 100$. So pulsar radiation, believed to originate in the open field line tube, propagates through the highly magnetized relativistic plasma. The propagation of radio waves in a relativistically moving medium was analyzed by Lee & Lerche (1975), Harding & Tademaru (1981) However, they considered rather abstract model of a medium, with the spatial dispersion being ignored. A qualitative consideration of wave polarization behaviour in application to a more realistic pulsar model allowed to attribute a number of characteristic features of the observed polarization pattern to the propagation effects inside the magnetosphere (Cheng & Ruderman 1979, Stinebring 1982, Barnard 1986, von Hoensbroech et
Given that the plasma is embedded in an infinitely strong magnetic field, the normal wave modes are linearly polarized. One of them, namely the ordinary mode, is polarized within the same plane as the wave vector, $\mathbf{k}$, and the magnetic field, $\mathbf{b}$; the other one, called the extraordinary mode, is polarized perpendicularly to this plane. In the emission region, the scale length for beats between the wave modes is much less than the scale length for change in the plasma parameters. So in the vicinity of the emission origin the normal waves propagate independently, their polarization plane being adjusted to the local orientation of the magnetic field. Along the trajectory, the angle $\theta$ between the wave vector and the magnetic field increases due to magnetic line curvature and the plasma density decreases due to open field line tube widening, the scale length for beats increasing. Eventually the plasma density decreases enough and the medium does not influence the waves; their polarization remains now fixed. Thus, the emergent polarization is set up by the processes in the so-called polarization–limiting region, where the geometrical optics approximation is violated (Budden 1952). The polarization–limiting radius, $r_p$, can be found from the relation:

$$\frac{\omega}{c} \Delta n(r_p) s(r_p) \sim 1,$$

where $\Delta n$ is the difference between the refractive indices of the wave modes considered, $\omega$ the wave frequency, $s$ the scale length for change in the plasma parameters, $s \sim r_p$. In the highly magnetized relativistic plasma of number density $N$ and Lorentz-factor $\gamma$, the refractive indices for the ordinary and extraordinary modes are known to be, respectively (Melrose & Stoneham 1977):

$$n_o = 1 - \frac{\omega_p^2 \sin^2 \theta}{2\omega^2 \gamma^2 (1 - \beta \cos \theta)^2}, \quad n_e = 1,$$

with $\omega_p \equiv \sqrt{4\pi Ne^2/m}$ being the plasma frequency, $\beta$ the plasma velocity in units of $c$.

In application to pulsars, the polarization-limiting radius was estimated by Cheng & Ruderman (1979), Stinebring (1982) and Barnard (1986). Note that if a normal wave were emitted in the field line plane of a non-rotating dipolar magnetic field, the polarization plane would not vary along the trajectory at all; the normal modes then would propagate independently preserving their initial polarization states. As the wave propagates in the rotating magnetosphere, the wave vector acquires a tilt to the ambient magnetic line planes, so that the $\mathbf{k} \times \mathbf{b}$–plane turns along the trajectory. Another consequence of rotation, namely the magnetic line sweepback, also leads to rotation of the $\mathbf{k} \times \mathbf{b}$–plane, however, this effect is less significant. As the scale length for the wave mode beats increases sufficiently, the polarization of a normal wave has no time to follow the variations of the ambient magnetic field and the wave mode coupling holds. Cheng & Ruderman (1979) pointed out that the propagation of the linearly polarized
normal waves in the region where the geometrical optics fails, results in elliptical polarization of the emergent radiation. The effect is likely to account for the circular polarization observed in pulsars that exhibit predominantly one sense of circular polarization across the pulse (Radhakrishnan & Rankin 1990).

The influence of pulsar rotation on the observed polarization was also considered by Blaskiewicz et al. (1991). These authors investigated the contribution of rotation to the velocity of the particles emitting curvature radiation, with the propagation effects being neglected. However, it is the wave propagation through the magnetosphere that sets up the characteristics of outgoing radiation. Indeed, whatever the emission mechanism, only the waves corresponding to the modes allowed by the magnetospheric plasma can propagate and ultimately escape from pulsars.

Provided that the plasma with the various distribution functions for electrons and positrons is embedded in the finite magnetic field, the normal waves propagating at the small angle to the field should be circularly polarized. Proceeding from this, Cheng & Ruderman (1979), von Hoensbroech et al. (1998) proposed to explain the observed circular polarization of pulsar radiation by the dispersive properties of the magnetospheric plasma. However, well within the pulsar magnetosphere the magnetic field strength is so high that the critical angle for the circularly polarized normal modes appears to be too small.

The present paper deals with a more detailed consideration of wave polarization behaviour in the polarization-limiting region. In Sect. 2 we treat the equations describing the waves which propagate through an ultrarelativistic highly magnetized plasma rotating together with the magnetic field. In Sect. 3 these equations are applied to the pulsar magnetosphere. The wave mode coupling in the polarization-limiting region is found to produce a significant circular polarization in the initially linear normal waves. The results are summarized in Sect. 4.

2 Basic equations

Consider an ultrarelativistic electron-positron plasma rotating together with an infinitely strong inhomogeneous magnetic field. For the waves of frequency $\omega$ the wave fields $\mathbf{E}$, $\mathbf{B}$ are described by Maxwell’s equations:

$$\nabla \times \mathbf{B} = -\frac{i\omega}{c} \mathbf{E} + \frac{4\pi}{c} \mathbf{j}_1,$$

$$\nabla \times \mathbf{E} = \frac{i\omega}{c} \mathbf{B},$$

$$-i\omega e(n_1^+ - n_1^-) + \text{div} \mathbf{j}_1 = 0.$$  \hspace{1cm} (2.1)

Here $\mathbf{j}_1$ is the linearized current density caused by the waves,

$$\mathbf{j}_1 \equiv e[v_0(n_1^+ - n_1^-) + n_0(v_1^+ - v_1^-)],$$

\hspace{1cm} (2.2)
with \( v_0, n_0 \) being the unperturbed particle velocity and number density, respectively, \( v_1^\pm, n_1^\pm \) the small perturbations of these quantities for electrons and positrons. Although the distribution functions for electrons and positrons are generally believed to be slightly different, the difference is insignificant in our case. Therefore we assume that \( v_0 \) and \( n_0 \) are equal for both the particle species.

In the infinitely strong magnetic field, the particle motion can be treated in terms of the mechanical bead-on-a-wire model. The particle moves along the magnetic field line which rotates at the angular velocity \( \Omega \). The particle velocity in the laboratory frame may be written as

\[
v = \Omega \times r + v_b b,
\]

where \( b \) is the unit vector along the magnetic field, \( v_b \) the velocity along the field line. Substituting Eq. (2.3) into the Lagrangian

\[
L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{e}{c} (A \cdot v) - e\varphi,
\]

with \( A \) and \( \varphi \) being, respectively, the vector and scalar potentials of the wave, one can get the equation of motion. To the first order in \( \frac{\Omega \times r}{c} \) we obtain:

\[
m\gamma^3 \frac{dv^\pm_b}{dt} = \pm e \left[ E + \frac{(\Omega \times r) \times B}{c} \right] \cdot b.
\]

(2.4)

Here the curvature radius of the magnetic field lines was assumed to be much higher than the wavelength; indeed, this is a good approximation in our case. The equation of motion (2.4) implies that the inertial forces introduced by the rotation are small as \( O\left(\frac{\Omega \times r}{c^2}\right) \). Note that the left-hand side of Eq. (2.4) contains the total derivative, \( \frac{d}{dt} \equiv -i\omega + v_0 \cdot \nabla \).

Equations (2.1), (2.2) and (2.4) yield the self-consistent description of the wave fields and the plasma particle motion in these fields. The refractive indices for the waves in the polarization-limiting region equal unity to within \( \frac{2\Omega r}{c^2} \ll 1 \). Therefore the waves propagate almost straight-line and one can choose the three-dimensional Cartesian system with the \( z \)-axis aligned with the wave vector. Then all the perturbed quantities should depend only on the \( z \)-coordinate. On the basis of Eqs. (2.1) and (2.4) one can write the component equations:

\[
\frac{d^2 E_x}{dz^2} + \frac{\omega^2}{c^2} E_x + \frac{4\pi i\omega}{c^2} j_{1x} = 0,
\]

\[
\frac{d^2 E_y}{dz^2} + \frac{\omega^2}{c^2} E_y + \frac{4\pi i\omega}{c^2} j_{1y} = 0,
\]

\[
\frac{\omega^2}{c^2} E_z + \frac{4\pi i\omega}{c^2} j_{1z} = 0,
\]

(2.5)

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\[-i\omega e(n_1^+ - n_1^-) + \frac{dj_1^-}{dz},\]

\[m\gamma^3 \left( -i\omega v_1^+ + v_0^+ \frac{dv_1^+}{dz} \right)\]

\[= \pm e \left[ E - i(\Omega \times r) \times (\nabla \times E) \right] \cdot \mathbf{b}.\]

We consider the waves propagating along the z-axis in positive direction. Since the refractive indices of the waves considered are very close to unity, the spatial dependence should be close to $\exp \left( i\omega c z \right)$. Then one can present the field components in the form:

\[E_\mu = a_\mu \exp \left( i\frac{\omega}{c} z \right), \quad (2.6)\]

with the amplitudes $a_\mu$ varying slowly:

\[\frac{da_\mu}{dz} \ll \frac{a_\mu \omega}{c}, \quad (2.7)\]

Of course, the rest of the perturbed quantities can be presented similarly. The scale length for change in the medium parameters, namely in $\mathbf{b}$, $v_0$ and $n_0$, also exceeds the wavelength essentially. Then the set of equations (2.5) is reduced to the form:

\[\frac{da_x}{dz} = -iR[(b_x + q_y)^2a_x + (b_x + q_y)(b_y - q_x)a_y], \quad \frac{da_y}{dz} = -iR[(b_x + q_y)(b_y - q_x)a_x + (b_y - q_x)^2a_y], \quad (2.8)\]

where

\[q = \frac{\mathbf{b} \times (\Omega \times r)}{c}, \quad R = \frac{\omega_p^2}{2\omega c^3(1 - \beta_z)^2},\]

with $\omega_p \equiv \sqrt{\frac{8\pi n_0 e^2}{m}}$ being the plasma frequency, $\beta_z$ the z-component of the plasma velocity $v_0$ in units of $c$. Above we took into account that the waves considered are quasi-transverse ones, $1 - n \ll 1$, and, correspondingly, the z-component of the wave electric field is small, $a_z \ll a_x, a_y$. With $n$ given by Eq. (1.2), this is surely valid at the condition (1.1).

Neglecting the rotation and the z-dependence of unperturbed plasma parameters we come to the homogeneous problem. Then the solutions are easy to be found in the form:

\[a_\mu(z) \propto \exp \left( -i\frac{\omega}{c}(1 - n)z \right). \quad (2.9)\]

Setting the determinant of the system equal to zero immediately yields the customary refractive indices (1.2) for the ordinary and extraordinary modes in the homogeneous highly magnetized ultrarelativistic plasma.
3  Polarization transfer in the rotating magnetosphere

3.1 The main features of wave propagation

First we examine the set of equations (2.8) to outline the characteristic features of wave polarization behaviour along the trajectory in pulsar magnetosphere. As long as the plasma density is sufficiently high, so that \( Rz \gg 1 \), geometrical optics approximation is valid. Then the wave field amplitudes can be presented in the form:

\[
a_\mu = a_\mu^{(0)} \exp[G(z)],
\]

(3.1)

with \( G(z) \) being as large as \( Rz \). Substituting Eq. (3.1) into Eq. (2.8) and setting the determinant of the system equal to zero one can find, to the first order in \( (Rz)^{-1} \),

\[
\left( \frac{dG}{dz} \right)_o = -iR[(b_x + q_y)^2 + (b_y - q_x)^2],
\]

\[
\left( \frac{dG}{dz} \right)_e = 0,
\]

(3.2)

with the subscripts \( o \) and \( e \) referring to the ordinary and extraordinary modes, respectively. Note that the terms \( q_x \) and \( q_y \) are introduced by the magnetosphere rotation, so that in the corotating frame \( q_x = q_y \equiv 0 \). Comparison of Eqs. (3.1), (3.2) with Eqs. (1.2), (2.9) corresponding to the homogeneous plasma then leads to a well-known result: the wave vector in a weakly inhomogeneous medium equals, in each point of the trajectory, to that in the homogeneous medium with the same parameters, so that \( a_\mu \propto \exp[-i\omega \int (1 - n(z))dz] \). With Eq. (3.2) we obtain:

\[
\left( \frac{a_x^{(0)}}{a_y^{(0)}} \right)_o = \frac{b_x + q_y}{b_y - q_x}, \quad \left( \frac{a_x^{(0)}}{a_y^{(0)}} \right)_e = -\frac{b_y - q_x}{b_x + q_y}.
\]

(3.3)

Hence, in the corotating frame the polarization of normal waves follows the slow variations of the ambient magnetic field.

Since the plasma density decreases along the trajectory, \( N \propto z^{-3} \), geometrical optics ultimately fails. In the opposite limit, \( Rz \ll 1 \), wave propagation is no longer influenced by the plasma and, obviously, wave polarization remains fixed (cf. Eq. (2.8)). Thus, the emergent polarizatation is formed in the so-called polarization-limiting region, where \( Rz \sim 1 \). In this region, the scale length for wave mode beats becomes so large that the polarization of a normal wave has no time to follow the variation of the ambient magnetic field, so that the wave mode coupling holds.

As follows straightly from Eq. (2.8), if the magnetosphere were non-rotating one, the normal waves emitted in the plane containing the field lines would propagate independently throughout the trajectory, the polarization plane remaining constant. Now we are interested in the wave mode coupling introduced
by the slow magnetosphere rotation. The rotation causes the wave vector tilt of \( \sim z/r_L \) to the ambient field line planes along the trajectory; here \( r_L \equiv c/\Omega \) is the light cylinder radius. In addition, the dipolar magnetic field structure should be distorted. Indeed, the poloidal current flow, \( j \sim \Omega B_p^2/2\pi \), with \( B_p \) being the poloidal magnetic field strength (Goldreich & Julian 1969), gives rise to the azimuthal component of the magnetic field, \( B_\phi \): \( (\text{rot}B)_p \sim B_p^2/4\pi \), where \( d \) is the open field line tube diameter, \( d = z^{3/2}/r_L^{1/2} \). The field line twist then can be estimated as \( B_\phi \sim (z/r_L)^{3/2} \). Thus, the sweepback of magnetic lines appears to be less significant than the rotation effect. Therefore below we consider the rotation of purely dipolar magnetic field.

Let the open field line tube be narrow, so that \( |\alpha - \xi| \leq \delta \ll 1 \). To the first order in \( \Omega t \), the direction cosines of the non-rotating dipolar magnetic field far from the emission point are

\[
\begin{align*}
b_x &= b_{x0} = \frac{\delta}{2}, \quad b_y = 0, \quad b_z = 1 - b_{x0}^2/2. \tag{3.4}
\end{align*}
\]

Now let the magnetic field rotate at the angular velocity \( \Omega \), the rotational axis making the angles \( \alpha \) and \( \xi \) with the magnetic axis and the wave vector, respectively. The open field line tube is believed to be narrow, so that \( |\alpha - \xi| \leq \delta \ll 1 \). To the first order in \( \Omega t \), the direction cosines of the rotating field are then given by

\[
\begin{align*}
b_x &= b_{x0} + \Omega t \sin \xi \sqrt{\delta^2 - (\alpha - \xi)^2}/\delta, \\
b_y &= \pm \Omega t \sin \xi (\alpha - \xi)/\delta, \\
b_z &= 1 - b_{x0}^2 + b_{y0}^2/2,
\end{align*}
\]

with \( t \equiv z/c \). The sign of the component \( b_y \) is that of the production \( b_d \cdot (k \times \Omega) \); here \( b_d \) is aligned with the magnetic axis at the moment of emission. Using Eq. (3.5) and taking into account that the waves are emitted close to the magnetic axis, \( x, y \ll z \), one can obtain the components of the vector \( q \equiv b \times (\Omega \times r) \):

\[
\begin{align*}
q_x &= \pm \frac{\Omega z}{c} \sin \xi (\alpha - \xi)/\delta, \\
q_y &= \frac{\Omega z}{c} \sin \xi \sqrt{\delta^2 - (\alpha - \xi)^2}/\delta. \tag{3.6}
\end{align*}
\]

The sign of \( q_x \) is opposite to the sign of \( b_d \cdot (k \times \Omega) \).
3.2 Wave mode coupling in case $\frac{z_p}{r_L \delta} \ll 1$

First we assume that $\frac{z_p}{r_L \delta} \ll 1$. Then taking into account that $\omega_p \propto N \propto z^{-3}$ and substituting Eqs.(3.5), (3.6), one can reduce Eq.(2.8) to the form

$$\frac{da_x}{du} - iua_x = i \frac{z_p}{r_L \delta} \eta (b_1 ua_x \pm b_2 a_y),$$

$$\frac{da_y}{du} = \pm i \frac{z_p}{r_L \delta} \eta b_2 a_x,$$  \hspace{1cm} (3.7)

where $u = z_p/z$, with $z_p$ referring to the polarization–limiting radius, $\eta = 2 \sin \xi$, $b_1 = \sqrt{\delta^2 - (\alpha - \xi)^2}$, $b_2 = \frac{\alpha - \xi}{\delta}$. Above we used the definition of the polarization-limiting radius (Eqs. (1.1), (1.2)) implying that

$$\frac{8 \omega_p^2 (z_p) z_p}{\omega c \gamma^3 \delta^2} = 1.$$

The wave mode coupling caused by the magnetosphere rotation is described by the right-hand sides in Eq. (3.7), which are small as $\frac{z_p}{r_L \delta}$. It is easy to find the solutions of Eq. (3.7) to the leading order in $\frac{z_p}{r_L \delta}$. If, say, the ordinary mode is emitted, that is at the emission origin $a_x = C_x$, $a_y = 0$, the wave polarization can be found to vary along the trajectory as follows:

$$a_x = C_x \exp(iu^2/2),$$

$$a_y = \pm i \frac{z_p}{r_L \delta} \eta b_2 C_x \int_u^0 \exp(iu^2/2) du'.$$  \hspace{1cm} (3.8)

Setting $z \rightarrow \infty (u \rightarrow 0)$ yields the limiting polarization of the emitted ordinary wave:

$$a_x = C_x,$$

$$a_y = \mp i C_x \frac{z_p}{r_L \delta} \eta b_2 \sqrt{\frac{\pi}{2}} \exp(i\pi/4).$$  \hspace{1cm} (3.9)

In case of extraordinary wave (at emission $a_x = 0$, $a_y = C_y$), the evolution of polarization can be treated similarly. The limiting polarization is then given by

$$a_x = \mp i C_y \frac{z_p}{r_L \delta} \eta b_2 \sqrt{\frac{\pi}{2}} \exp(-i\pi/4),$$

$$a_y = C_y.$$  \hspace{1cm} (3.10)
So the wave mode coupling in the polarization-limiting region causes the elliptical polarization of the escaping normal waves. The contribution of circular polarization is characterized by the normalized Stokes parameter \( V \) defined as
\[
V = \frac{i(a_y a_x^* - a_x a_y^*)}{a_x a_x^* + a_y a_y^*}.
\] (3.11)

Involving Eqs. (3.9), (3.10) one can obtain
\[
|V| = \frac{4 z_p}{r_L \delta} \sin \xi |\alpha - \xi| \delta.
\] (3.12)

It is easy to see that the handedness of the circular polarization resulting from the limiting polarization effect remains constant throughout the pulse. Note that, to the first order in \( \frac{z_p}{r_L \delta} \), the polarization profile appears to be symmetrical, with the peak at the centre. This agrees qualitatively with the observational data for a number of pulsars (Han et al. 1998). According to Eq. (3.12), the degree of circular polarization is \( \sim \frac{z_p}{r_L \delta} \). Although this quantity was regarded as a small parameter, it proved to be not very small at pulsar conditions, \( \frac{z_p}{r_L \delta} \geq 0.1 \) (Cheng & Ruderman 1979). Thus, the linearly polarized waves can acquire considerable circular polarization because of the wave mode coupling in the polarization-limiting region. In particular, the highest observed circular polarization, \( \sim 60\% \) for PSR 1702-19 (Biggs et al. 1988), is likely to be explained by this effect.

### 3.3 The case \( \frac{z_p}{r_L \delta} \gg 1 \)

Now we turn to the wave mode coupling in the opposite limit, \( \frac{z_p}{r_L \delta} \gg 1 \). The latter inequality corresponds to short-period pulsars (Barnard 1986). Substituting Eqs. (3.5) and (3.6) into Eq. (2.8) we have, to the first order in \( \frac{r_L \delta}{z_p} \):
\[
\frac{da_1}{dw} = \frac{i}{4} a_1 = \mp \frac{i}{16 z_p \sin \xi} b_2 w^{1/4} a_2 - \frac{3i}{8} \frac{r_L \delta}{z_p \sin \xi} b_1 w^{1/4} a_1,
\]
\[
\frac{da_2}{dw} = \pm \frac{i}{16 z_p \sin \xi} b_2 w^{1/4} a_1,
\] (3.13)

with
\[
a_1 \equiv b_1 a_x - b_2 a_y, \quad a_2 \equiv b_2 a_x + b_1 a_y, \quad w \equiv (z_p/z)^4.
\] (3.14)

Here the polarization-limiting radius, \( z_p \), is given by the relation:
\[
\frac{8 \omega_p^2 (z_p) r_L^2}{\omega c \gamma^3 z_p \sin^2 \xi} = 1.
\]
Solving Eq. (3.13) to the leading order in $\frac{rL}{z_p}$, one can trace the evolution of the polarization of the ordinary wave (initially $a_1 = C_1$, $a_2 = 0$):

\[ a_1 = C_1 \exp(iw/4), \]
\[ a_2 = \mp C_1 \frac{rL \delta b_2}{16z_p \sin \xi} \left[ 4w^{1/4} \exp(iw/4) - \int^w w'^{-3/4} \exp(iw'/4) dw' \right]. \quad (3.15) \]

Setting $w \to 0$ we find the limiting polarization:

\[ a_1 = C_1, \]
\[ a_2 = \mp C_1 \frac{rL \delta b_2}{16z_p \sin \xi} \frac{\Gamma(1/4)}{(1/4)^{1/4}} \exp(i\pi/8). \quad (3.16) \]

Correspondingly, the limiting polarization of the extraordinary wave (initially polarized as $a_1 = 0$, $a_2 = C_2$) can be found to be

\[ a_1 = \pm C_2 \frac{rL \delta b_2}{16z_p \sin \xi} \frac{\Gamma(1/4)}{(1/4)^{1/4}} \exp(-i\pi/8), \]
\[ a_2 = C_2. \quad (3.17) \]

With Eqs. (3.16) and (3.17), the normalized Stokes parameter becomes

\[ |V| = 0.2 \frac{rL \delta}{z_p \sin \xi} \frac{|\alpha - \xi|}{\delta}. \quad (3.18) \]

Now the circular polarization is constant along the pulse, with $|V| \leq 10\%$. Such values of $V$ are observed for a number of pulsars. Note that in this case the handedness of circular polarization also remains constant throughout the pulse.

\section{Conclusion}

We investigated the evolution of normal wave polarization along the trajectory in the ultrarelativistic highly magnetized electron-positron plasma filling the magnetosphere of a pulsar. The plasma allows two normal waves which can ultimately escape from the magnetosphere. In the emission region, they are linearly polarized, the radiation emitted being also linearly polarized or containing an incoherent mixture of radiation with the two linear polarizations. While propagating inside the magnetosphere, the waves are influenced by the plasma. So the emergent polarization should be set up by the propagation effects.

The polarization behaviour was described proceeding from Maxwell’s equations together with the equation of particle motion in the rotating magnetosphere. For simplicity we examined the wave mode coupling introduced by the rotation of a dipolar magnetic field in the limits $\frac{z_p}{rL \delta} \ll 1$ and $\frac{z_p}{rL \delta} \gg 1$, where
$z_p$ is the polarization-limiting radius, $r_L$ the light cylinder radius, $\delta$ the beam width. As a result of this effect, the emergent radiation should be elliptically polarized, with the sense of the circular component being the same throughout the pulse. Given that $r_L\delta \ll z_p < r_L$, the degree of circular polarization is found to be of a few per cents and to remain constant along the pulse. In case $z_p \leq r_L\delta$ the wave mode coupling can cause considerably higher circular polarization. In this limit, the polarization profile is symmetrical and peaks at the center of the pulse. Such behaviour is compatible with the observed V-profiles in many pulsars. So the observations testify to the case $z_p < r_L\delta$ implying that the polarization-limiting radius should lie not too far from the emission region. This is also supported by the absence of essential decrease in the position angle swing as a result of the limiting polarization effect (Lyne & Manchester 1988).

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