COULD JUPITER OR SATURN HAVE EJECTED A FIFTH GIANT PLANET?

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ABSTRACT

Models of the dynamical evolution of the early solar system that follow the dispersal of the gaseous protoplanetary disk have been widely successful in reconstructing the current orbital configuration of the giant planets. Statistically, some of the most successful dynamical evolution simulations have initially included a hypothetical fifth giant planet, of ice giant (IG) mass, which is ejected by a gas giant during the early solar system’s proposed instability phase. We investigate the likelihood of an IG ejection (IGE) event by either Jupiter or Saturn through constraints imposed by the current orbits of their wide-separation regular satellites Callisto and Iapetus, respectively. We show that planetary encounters that are sufficient to eject an IG often provide excessive perturbations to the orbits of Callisto and Iapetus, making it difficult to reconcile a planet ejection event with the current orbit of either satellite. Quantitatively, we compute the likelihood of reconciling a regular Jovian satellite orbit with the current orbit of Callisto following an IGE by Jupiter of \( \sim 42\% \), and conclude that such a large likelihood supports the hypothesis of a fifth giant planet’s existence. A similar calculation for Iapetus reveals that it is much more difficult for Saturn to have ejected an IG and reconciled a Kronian satellite orbit with that of Iapetus (likelihood \( \sim 1\% \)), although uncertainties regarding the formation of Iapetus, with its unusual orbit, complicates the interpretation of this result.

Key words: methods: numerical – planets and satellites: dynamical evolution and stability

1. INTRODUCTION

Various solar system formation models argue that the giant planets underwent planetesimal driven migration (e.g., Fernández & Ip 1984; Malhotra 1995; Hahn & Malhotra 1999; Tsiganis et al. 2005) at early times (\( \lesssim 1\) Gyr) following a dynamical instability. The Nice model, originally presented by Gomes et al. (2005), Morbidelli et al. (2005), and Tsiganis et al. (2005), with subsequent variants under the same name, has been the most successful in reproducing the settling of the four giant planets into their present orbital configuration (Tsiganis et al. 2005; Morbidelli et al. 2007; Levison et al. 2011), the Late Heavy Bombardment at \( \sim 700 \) Myr (Gomes et al. 2005), the capture of Jupiter’s Trojan asteroids (Morbidelli et al. 2005), the capture of gas giant irregular satellites (Nesvorný et al. 2007), as well as the structure of the Kuiper Belt (Levison et al. 2008) and how its dynamical evolution led to the contamination of the outer asteroid belt by primitive trans-Neptunian objects (Levison et al. 2009).

The precise nature of giant planet migration in the early solar system remains uncertain due to our lack of knowledge regarding each body’s initial conditions following their formation out of the solar nebula and the chaotic nature of the migration process. However, Morbidelli et al. (2009) argued that smooth divergent migration of the gas giants is unable to sufficiently excite their orbital eccentricities and inclinations to their observed values. Additionally, Brasser et al. (2009) showed that such migration from an initial resonant configuration following the dispersal of the gaseous disk leads to excessive orbital eccentricities in the previously formed terrestrial bodies via sweeping secular resonances. A proposed solution, known as the jumping-Jupiter scenario (Brasser et al. 2009), invokes close encounters between the gas giants and an ice giant (IG), resulting in the step-wise migration of Jupiter and Saturn from their initial mean-motion resonance. This can sufficiently excite giant planet eccentricities and inclinations while jumping over the problematic secular frequencies of the terrestrial planets. In addition, the jumping-Jupiter scenario does not disrupt the asteroid belt’s observed morphology (Morbidelli et al. 2010).

A statistical study by Nesvorný (2011) of the dynamical evolution of the solar system during such a phase of frequent planetary encounters showed that the likelihood of reconstructing the current orbital configuration of the four giant planets is increased when a fifth giant planet of approximately Uranian mass is included in the early solar system. The instability, which gives rise to multiple planetary encounters, results in the ejection of the hypothetical fifth giant planet, reconstructing the outer solar system while preserving the orbits of the inner terrestrial bodies over long timescales (Batygin et al. 2012; Nesvorný & Morbidelli 2012). Such planet scattering events (Rasio & Ford 1996; Weidenschilling & Marzari 1996), applicable to any multi-body system, provide a potential explanation for the existence of the recently detected “free-floating” planets (e.g., Delorme et al. 2012; Liu et al. 2013; Luhan & Esplin 2014).

In addition to reconstructing the current orbital configuration of both large and small bodies in the solar system, models attempting to achieve a full description of the solar system’s early dynamical evolution must require the survival of the giant planets’ regular satellites (e.g., Deienno et al. 2014). Regular satellites that are thought to form via accretion processes in circumplanetary disks (Canup & Ward 2002; Mosqueira & Estrada 2003) are expected to form on prograde, low-eccentricity orbits that are nearly coplanar with the host’s equatorial plane and have relatively small semimajor axes. The current deviations of Callisto and Iapetus’ orbits from circular, uninclined orbits therefore limit how close an IG could have come to the gas giants in the early solar system.
In general, outer satellites, which are less tightly bound to the planet, will suffer larger perturbations during a close approach with an IG. In addition, the orbital eccentricities of these outer moons are only marginally damped through tides (which could otherwise mask the effects of early encounters). It is therefore the outermost regular satellites (Callisto around Jupiter and Iapetus around Saturn) that provide the most stringent constraints.

Specifically, Deienno et al. (2014) investigated whether the close encounters in the particular simulations of Nesvorný & Morbidelli (2012, hereafter referred to as NM12), that best reproduced the giant planets’ orbital architecture, would excessively excite Callisto’s orbit. Given the interest in additional planets in the early solar system, we generalize this question to ask how likely is it that Callisto (Iapetus) would be retained at its observed orbit following an ejection of an IG by Jupiter (Saturn)? Although in the jumping-Jupiter scenario only Jupiter may be responsible for ejecting an IG, we include an analysis of close IG encounters with Saturn, as we are more generally interested in early solar system instability models in which either gas giant could undergo close encounters with the solar system’s IGs. It also permits the direct comparison of the likelihood of retaining a Callisto-like satellite orbit with the likelihood of retaining an Iapetus-like satellite orbit following the ejection of an IG (see Section 6).

In Section 2 we discuss the relevant properties of the satellites of interest. Section 3 summarizes our methods of investigation. Sections 4 and 5 present our study’s results, and Section 6 presents our calculation of the likelihood of reconciling satellite orbits following planetary encounters with present-day orbits. We conclude with a detailed discussion in Section 7 and a summary in Section 8.

2. SUMMARY OF SATELLITES: CALLISTO AND IAPETUS

Callisto is the outermost Galilean satellite, moving on a nearly circular ($e = 0.007$) and uninclined ($i < 0.28$) orbit. Callisto’s orbital period is $\sim 16.7$ days and is the only Galilean satellite not locked in a mean-motion resonance (Musotto et al. 2002). Iapetus’ orbit is somewhat more eccentric ($e \sim 0.03$) and circles Saturn every $\sim 79$ days. Curiously, Iapetus exhibits a significantly inclined orbit, possibly due to inclined planetary encounters between Saturn and an IG like those expected in the jumping-Jupiter scenario (Nesvorný et al. 2014). Current satellite orbital elements are summarized in Table 1. These data were obtained from JPL HORIZONS.

While Callisto does not participate in an MMR like the other Galilean satellites, its eccentricity evolution is nevertheless secularly coupled to that of the inner moons (Greenberg & Van Laerhoven 2011). The eccentricities of the inner satellites are more easily damped by tides than Callisto because of their smaller semimajor axes and as a result of the secular coupling. $e_{\text{Callisto}}$ is damped on timescales faster than expected for an isolated planet-satellite system that is tidally locked. We take this into account as described in Section 4.2, following numerical simulations of this effect by Deienno et al. (2014). Conversely, the eccentricity damping of Iapetus since the solar system’s instability phase has been negligible (Castillo-Rogez et al. 2007).

3. METHODS

We model the evolution of an IG heading for a close approach with one of Jupiter or Saturn and investigate the encounter’s effect on the gas giant’s satellites. We first determine viable encounter parameters that lead to the ejection of the IG. Within this set, we investigate if there are any satellites remaining around Jupiter (Saturn) with an orbit consistent with that of Callisto (Iapetus).

3.1. Numerical Model: Simulating Planetary Encounters

We consider a reduced-body subset of the solar system including the Sun, a satellite-hosting gas giant planet, and an IG. The gas giant of mass $M_I$ is initialized on an osculating, circular orbit with semimajor axis $a_I$. By approximating the gas giant’s orbit as circular, we remove any dependence on its orbital phase at the epoch of encounter; $t_{\text{enc}}$. Because the mass of the ejected IG is not well constrained, we select a fiducial value approximately equal to the mass of Uranus ($M_I = 5 \times 10^{-5} M_\odot$), as was used by NM12. A system of Keplerian satellites is placed in orbit around the gas giant (see Section 3.2 for a detailed description of satellite orbits).

To limit the number of encounter parameters and the computational cost of our survey, we assume a coplanar geometry (we present a more detailed discussion of the effect of inclined encounters in Section 7.1). Thus, at its closest approach the IG’s velocity is perpendicular to its separation vector from the gas giant and the encounter is fully specified at this time ($t_{\text{enc}}$) by the impact parameter $b$, the relative planet velocity $v_{\text{rel}}$, and the phase angle $\theta$ (see Figure 1).

We linearly step the impact parameter outward from $b_{\text{min}} = 0.02$ AU until encounters no longer lead to ejections. Deienno et al. (2014) found that close encounters with $b < b_{\text{min}}$ excessively excite the Galilean satellites; our estimates in Section 6 for the likelihood of retaining the observed satellite orbits following an ejection therefore represent a conservative overestimate due to our comparatively gentle ejections. However, we note that values smaller than $b_{\text{min}}$ are less likely due to the reduced encounter cross-section at small radii. We estimate the size of this effect by including a reduced simulation sample with $b < b_{\text{min}}$ and find that our final results (Section 6) changed (fractionally) by $\lesssim 12\%$.

To determine which encounter parameters are capable of ejecting the IG, we uniformly sample 20,000 parameter combinations within $b \in [b_{\text{min}}, 0.1\,\text{AU}]$, $v_{\text{rel}} \in [1, 5]v_{\text{esc}}$, and $\theta \in [0, 2\pi]$, where $v_{\text{esc}} = \sqrt{2GM_I/b}$. We then remove any
Figure 1. Pictorial representations of a close planetary encounter between a gas giant (larger, red planet) and an unspecified ice giant (IG; smaller, blue planet) at the epoch of encounter $t_{\text{enc}}$. The coplanarity of planetary encounters permits an analysis solely in the $xy$-plane ($z = 0$). Left: the heliocentric reference frame with the Sun, at a distance of the gas giant’s semimajor axis, aligned with the $\hat{x}$ axis. The planet’s instantaneous velocity vectors are shown along with their trajectories (dashed lines) in the vicinity of $t_{\text{enc}}$. Right: the reference frame is centered on the gas giant and is zoomed-in to depict the phase angle $\theta$; the angle between the $\hat{x}$ axis and the IG’s position vector ($r_{\text{rel}}$) at $t_{\text{enc}}$, measured in the counter-clockwise direction. Here, $\theta \approx 310^\circ$. At $t_{\text{enc}}$, the magnitude of the IG’s relative position vector is equal to the impact parameter $b$ and is orthogonal to the relative velocity vector $v_{\text{rel}}$. In both diagrams, the dotted gray rings represent the ring of regular satellites in orbit around the gas giant. The scale used here is approximate, as these schematics are not intended to be exact, but instead are included to aid in the reader’s visualization of the experimental setup.

unphysical encounters in which the IG is unbound from the Sun prior to the encounter. In the cases of Jupiter and Saturn we find $N_{\text{sim}} = 278$ and $N_{\text{sim}} = 274$ as valid parameter combinations, respectively.

To simulate an encounter with the aforementioned parameters from Figure 1, we integrate backward in time until the absolute separation of the planets $|r_I - r_p| \geq 2$ AU. At this separation a satellite with a Callisto-like or Iapetus-like orbit feels a force from its host that is $>10^4$ times greater than that felt from the IG. The positions and velocities of all three massive bodies are then used as initial conditions for the forward simulations that include the satellites. These simulations are integrated forward in time toward the encounter at $t = t_{\text{enc}}$ and are halted at $t = 2t_{\text{enc}}$. After $2t_{\text{enc}}$ the influence of the IG on the satellites is again negligible and satellite orbits are no longer perturbed by the IG’s influence. In order for close encounters with $b < 0.1$ AU to be possible, numerous “soft” encounters between the gas giants and IG, prior to $t_{\text{enc}}$, were needed in order to build-up the IG’s eccentricity. These encounters will supply a small perturbation to regular satellite orbits that, on average, increase satellite eccentricity over time (see Nesvorný et al. 2014, Figure 4). However, the effect of encounters on satellite orbits is strongly dependent on $b$, which is much larger than 0.1 AU for “soft” encounters. We are therefore only concerned with the strongest (final) encounter that leads to the ejection of the IG as its effect dominates the final satellite orbits.

Finally, we do not include the perturbations from the satellite host’s oblateness. This should be a good approximation since the encounter timescale with the IG is much shorter than the precession timescales of the satellites due to the non-spherical shape of its host planet.

3.2. Numerical Model: Initializing Satellites

In each encounter simulation, we include a ring of Callisto or Iapetus analog satellites around the gas giant planet. The satellite ring consists of $N = 100$ non-interacting test particles with azimuthal positions randomly sampled from a uniform distribution between 0 and $2\pi$ in order to remove any azimuthal dependence at $t_{\text{enc}}$. Modeling the satellite system as an ensemble of test particles ensures that the orbital evolution of the satellites is governed solely by gravitational interactions with the host planet and perturbations from the IG and the Sun.

The semimajor axes of the satellites $a_s$ are initialized to $\pm 1\%$ of the current orbital radius of Callisto or Iapetus. This fractional deviation is chosen to be on the order of the observed satellite eccentricities $e_s$ (see Table 1). Because changes in $a_s$ and $e_s$ are related through changes in the satellites’ angular momentum, it is unlikely that larger shifts in $a_s$ would be reconcilable with the observed $e_s$, assuming that each satellite formed out of a circumplanetary disk on a circular orbit (Canup & Ward 2002; Mosquera & Estrada 2003). Satellites are initialized on nearly coplanar and circular orbits ($i \ll 0^\circ.1$, $e \ll 10^{-5}$), as expected from circumplanetary formation scenarios. Satellite particles are not allocated physical sizes as we do not account for particle collisions in our simulations.
3.3. Numerical Code

We performed our simulations using the REBOUND N-body numerical code (Rein & Liu 2012). We employ the IAS15 integrator (Integrator and Adaptive Step-size control, 15th order; Rein & Spiegel 2015), whose adaptive timestepping ensures optimal resolution of the short satellite orbital periods and rapid encounter timescales.5

4. RESULTS OF IG EJECTIONS (IGES) BY JUPITER

Here we focus on simulations in which Jupiter is the satellite-hosting gas giant planet that ejects the hypothetical fifth giant planet from the solar system.

4.1. Properties of Encounters

Figure 2 summarizes the properties of the planetary encounters that result in the ejection of the IG planet. We find $N_{\text{sim}} = 278$ such encounters. The greatest impact parameter we find capable of ejecting the IG is $\approx 0.05$ AU.

All successful ejections involve the closest approach in the lower hemisphere of the $xy$-plane in Jupiter’s reference frame ($180^\circ < \theta(\text{enc}) < 360^\circ$; see Figure 1). As familiar from spacecraft gravity assists, an IG trailing Jupiter at $t_{\text{enc}}$ will receive a positive velocity kick via their interaction. At these phase angles, the relative velocity vector is rotated by the encounter such that the IG’s inertial speed accelerates. Depending on $b$ and $v_{\text{rel}}$, the encounter can potentially boost the IG to escape velocity from the solar system. From simple vector diagrams, the maximum increase to the IG velocity can be shown to occur when $\theta(\text{enc}) = 270^\circ$. Figure 2 highlights this as the distribution of $\theta(\text{enc})$ peaks at $\sim270^\circ$, where the encounter geometry is most conducive to ejecting the IG.

5 A short video from REBOUND depicting a close planetary encounter with a ring of regular satellites can be found here: http://www.astro.utoronto.ca/~cloutier/rebound_encounter.mp4.

Figure 3. Evolution of five sample Jovian satellites as Jupiter undergoes a close encounter with an unspecified IG. The encounter parameters at $t_{\text{enc}}$: impact parameter $b$, relative planet velocity $v_{\text{rel}}$ and IG phase angle $\theta$ are shown in the legend of the lower panel. Each satellite’s azimuthal position relative to $\theta(\text{enc})$ is annotated in the upper panel. The vertical and horizontal dotted lines indicate the epoch of encounter and the current values of Callisto’s orbital elements, respectively.

Low $v_{\text{rel}}$ trajectories more often lead to ejection since Jupiter can more effectively deflect the IG’s path. We note that there are many encounters in which the velocity of the IG with respect to Jupiter at $t_{\text{enc}}$ is greater than the escape speed from the solar system at Jupiter’s distance from the Sun. This is due to the IG being accelerated upon approach to the encounter, which occurs deep in Jupiter’s gravitational well. Such cases consist of the IG becoming unbound from the solar system prior to the realization of the impact parameter at $t_{\text{enc}}$ and being thus accelerated to super-escape speeds even before its closest approach to Jupiter.
In addition, the position of the satellite relative to different regions of the parameter space. The survival fractions shown here reveal that Jupiter has a reasonably large probability of ejecting an IG while retaining a Callisto-like satellite (see Section 6).

The average relative velocities necessary require the satellites making their azimuthal position at the subsequent eccentricity evolution due to tidal damping of Callisto (Deienno et al. 2014, see Figure 7) and allows for Callisto to be excited beyond the present $e_{\text{Callisto}}$ at $t_{\text{enc}}$ and consequently settle into its current orbital eccentricity in the 4-Gyr following the solar system’s instability phase.

For each satellite we define a reconcilable orbit to be when the satellite’s final average eccentricity is $\leq 2e_{\text{Callisto}}$ (recall $e_{\text{Callisto}} \approx 0.007$). The factor of 2 in our definition comes from the subsequent eccentricity evolution due to tidal damping of Callisto (Deienno et al. 2014, see Figure 7) and allows for Callisto to be excited beyond the present $e_{\text{Callisto}}$ at $t_{\text{enc}}$ and consequently settle into its current orbital eccentricity in the 4-Gyr following the solar system’s instability phase.

For the remainder of the paper we refer to the event of a simulated satellite’s final orbit being reconcilable with Callisto as RS, for “reconcilable satellite.” The boundary dividing RS from non-RS satellites is depicted in Figure 4 as a dashed horizontal line.

Final average $e_s$ values are never $\geq 0.4$, implying that no Jovian satellite becomes unbound from Jupiter following the ejection of the IG. That is, the vast majority of planetary encounters that are capable of ejecting an IG from the solar system are not sufficiently violent to strip Jupiter’s regular satellites. This favors the possibility of Jupiter being able to eject an IG while retaining a regular satellite whose orbit is Callisto-like. We estimate the likelihood in Section 6.

While the phase angle $\theta(t_{\text{enc}})$ is an important parameter in determining whether the IG is ejected, it has little effect on the fraction of perturbed satellites, due to their uniformly distributed azimuthal positions around Jupiter. Therefore, for each $b$ and $v_{\text{rel}}$ we can marginalize over $\theta$ to show in Figure 5 the fraction of satellites that are reconcilable with the orbit of Callisto after each encounter. Interpolating over the fraction of
reconcilable Jovian satellites in \( (b, v_{rel}) \) space, the resulting high-resolution contours at 10% and 50% are fitted with cubic functions and overplotted as solid and dashed curves, respectively, to aid in the visualization of the different regions of the parameter space.

It is clear that as encounters become closer, the perturbation to \( e_s \) increases and the fraction of RSs shrinks. Similarly, as the duration of encounters becomes longer (smaller \( v_{rel} \)), the timescale over which the perturbation is applied grows, thus increasing the deviation of satellite orbits from circular. Hence forming and maintaining a Jovian satellite with a Callisto-like orbit favors encounters that are wide and fast.

We suggest that researchers simulating solar system formation scenarios can use Figure 5 to estimate whether or not a given Jupiter/IG encounter is consistent with Jupiter’s current Galilean satellite orbits. We note that our requirement that the IG gets ejected only determines what regions of the plot are populated, hence the contours are useful guides regardless of whether or not the IG survives the encounter (and for any \( \theta \)). We caution, however, that these will only be approximate due to our assumption of coplanarity. For a given \( b \) and \( v_{rel} \), inclined encounters will increase the fraction of satellites whose eccentricities are reconcilable with current orbits. However, they will also tend to overly excite the satellite inclinations. We discuss this further in Section 7.1. For particular simulations that lie on the boundary of plausibility in Figure 5, one could re-simulate the close approaches, including inclined encounters, following our setup to more accurately quantify the effect.

5. RESULTS OF IGES BY SATURN

Here we focus on simulations in which Saturn is the satellite-hosting gas giant planet that ejects the hypothetical fifth giant planet from the solar system.

5.1. Properties of Encounters

The properties of planetary encounters between Saturn and an IG are summarized in Figure 6. We find \( N_{sim} = 274 \) simulations that result in the IG being ejected by Saturn. The largest impact parameter that is still capable of ejecting the IG is nearly identical to the Jupiter/IG case; \( \approx 0.05 \) AU. We perform an additional sampling of encounter parameters with \( b > 0.05 \) AU at an increased resolution to confirm that the largest impact parameter capable of ejecting the IG is nearly the same in both the Jupiter and Saturn cases rather than being a statistical anomaly due to the finite sampling of encounter parameters. We find that this limit is real and not an artifact of our sampling procedure. The physical interpretation of this similarity is beyond the scope of this paper.

Similar trends to those shown in Figure 2 are observed. This should be expected because, modulo the variations in the physical parameters \( M_p \) and \( a_p \), the Saturn/IG encounters investigated are fundamentally equivalent to those described in Section 4.1. With the exception of the decreased magnitude of \( v_{rel} \) by a factor of \( \sqrt{M_{Sat}/M_{Jup}} \), on average, the set of successful scattered-by-Saturn simulations are statistically identical to
those in the scattered-by-Jupiter simulations; i.e., the histograms in Figures 2 and 6 exhibit the same behavior.

5.2. Resulting Kronian Satellite Orbits

Similar to the case of Jupiter ejecting the IG from the solar system, the Kronian satellite orbits will be perturbed as a result of the encounter between Saturn and the IG. As a result of the wide separation of Iapetus ($\sim 61 R_S$) and the similar nature of the encounters between an IG and either Jupiter or Saturn (Figures 2 and 6), it is expected that such orbital perturbations will be more destructive for simulated satellite orbits than for those previously explored in Section 4. This notion is supported in Figures 7–9 when compared to their counterparts in the Jupiter/IG case (Figures 3–5), as the fraction of reconcilable Kronian satellites is systematically lower than in the Jupiter case.

The time evolutions of $a_e$ and $e_e$ for five example satellites during a sample planetary encounter are shown in Figure 7. This example is a particularly violent encounter with $b \approx 0.033$ AU and $v_{\text{rel}} \approx 5.87 \text{ km s}^{-1}$, as no satellite in this simulation has a resulting orbit that is reconcilable with Iapetus. One satellite, whose azimuthal position is only $4^\circ 6$ from $\theta (t_{\text{enc}})$, is ejected from Saturn by the encounter. The encounter timescale ($b/v_{\text{rel}} \sim 9.6$ days) is much less than the orbital period of Iapetus, making the effect of the encounter behave approximately as an impulse.

The average final orbital elements of all Kronian satellites after each unique encounter are depicted in Figure 8. The black contours in Figure 8 are representative of the final orbital elements of the Jovian satellites from Figure 4 to aid in visual comparison between the final orbits of Jovian satellites to Kronian satellites following similar close planetary encounters. This shows that the wide separation of the simulated Kronian satellites makes them more susceptible to excessive orbital perturbations.

The event RS in the Saturn/IG case is defined similar to the Jupiter/IG case and is achieved by Kronian satellites whose final average $e_e$ is less than or equal to $e_{\text{Iapetus}}$. As noted earlier, the eccentricity evolution of Iapetus since the solar system’s instability phase is negligible (Castillo-Rogez et al. 2007), thus we assume that resulting orbits from our simulations will go largely unchanged to the present day. The boundary dividing RSs from non-RSs is depicted in Figure 8 as a dashed horizontal line.

As noted in Section 4.2, no Jovian satellite becomes more eccentric than 0.4, whereas a sizable fraction of Kronian satellites become equivalently or excessively excited including a subset of Kronian satellites whose final average $e_e$ is approximately unity. Furthermore, we find that ~6% of sampled Kronian satellites are sufficiently excited to final $e_e > 1$. The orbital elements of these ejected satellites are not included in Figure 8. It is clear that the perturbations to the Kronian satellites resulting from close encounters are, on average, much stronger and capable of stripping Iapetus-like satellites from the Kronian system. The large fraction of satellites perturbed beyond the orbit of Iapetus makes it statistically difficult for Saturn to have ejected an IG while retaining an Iapetus-like regular satellite. We formally estimate the likelihood of reconciling Iapetus’ orbit in Section 6.

The effect of encounter properties on the resulting Kronian satellite orbits is shown in Figure 9. For each encounter with a given impact parameter and relative planet velocity, the fraction of Kronian satellites whose final orbit is reconcilable with the current orbit of Iapetus is shown. Approximate contours of 10% and 50% fractions are overplotted. The 10% contour is computed identically to the contours in Figure 5 (i.e., cubic interpolation). However, the 50% contour lacks a sufficient number of points to perform a robust cubic interpolation. Therefore we opt for a linear fit in its place. It is clear that the region of the $(b, v_{\text{rel}})$ parameter space in which $\geq 50\%$ of the Kronian satellites are reconcilable with Iapetus’ orbit is very small compared to the Jovian satellites with only five unique ejections sampled there.
The effects of $b$ and $v_{rel}$ on resulting satellite orbits are nearly identical to those shown in Figure 5. One interesting difference that is unique to Saturn/IG encounters is that regardless of how close the encounter is, if the encounter is sufficiently long ($v_{rel} \lesssim 8$ km s$^{-1}$), then $<10\%$ of satellites will be reconcilable with Iapetus. This is in contrast to the Jovian satellite case where even the slowest encounters could retain a high fraction of Callisto-like satellites if the encounter’s impact parameter is large. Another important feature to note is that no Saturn/IG encounter resulting in the ejection of the latter is guaranteed to preserve an Iapetus-like satellite. That is, even the least violent encounters leading to ejection are only capable of preserving a maximum of $\sim 70\%$ of the in situ Iapetus-like satellites.

Figure 9 contains information on the likelihood that Iapetus survives an IGE by Saturn with a particular $b(t_{enc})$ and $v_{rel}(t_{enc})$ when the mutual inclination of the encounter is near zero. Similar to the Jupiter/IG encounter case, we suggest that researchers simulating solar system formation scenarios can use Figure 9 to estimate whether or not a given Saturn/IG encounter is consistent with Iapetus’ current orbit in the limit of uninclined planetary encounters.

6. LIKELIHOOD OF RECONCILING SATELLITE ORBITS FOLLOWING IGEs

6.1. Methodology

The resemblance of the final average orbits of the Jovian and Kronian satellites (Figures 4 and 8) to the current orbits of Callisto and Iapetus can in principle be used to compute the likelihood of a fifth giant planet getting ejected by either of the gas giant planets in the early solar system. If such an event were to have occurred, it must be consistent with the satellite orbits presently observed. The likelihood of an IG getting ejected by either gas giant requires the likelihood of the current orbits of Callisto or Iapetus being reconcilable by simulated satellites after an IGE. A successful event in which the resulting orbit of a simulated Jovian (Kronian) satellite is reconcilable with Callisto’s (Iapetus’) current orbit is referred to as RS for “reconcilable satellite.”

Given that an IG gets ejected in all simulations (event IGE; “IG ejected”), for the $i$th satellite in the $j$th simulation, we record whether or not the event RS is achieved using

$$p_{i,j}(RS|IGE) = \begin{cases} 1 & \text{if RS} \\ 0 & \text{if not RS}. \end{cases} \quad (1)$$

However, not all planet orbits in our ejection simulations are statistically relevant. Our adopted methodology for determining which encounter parameters result in an ejection is heavily biased toward initially high-eccentricity orbits of the IG ($e_I \lesssim 1$), which are not long-term stable and therefore are uncommon in nature. This bias naturally arises because it is easier to kick the IG to $e_I > 1$ if $e_I$ is initially very close to unity. Therefore in each simulation $j$, each satellite’s likelihood of an RS must be weighted by the likelihood of the IG having an initially bound orbit with $e_{ij}$, where $e_{ij}$ is the IG’s initial eccentricity in the $j$th simulation.

The corresponding weighting function $W(e_{ij})$ is modeled by the distribution of planet eccentricities observed in a statistically significant number of exoplanetary systems; the eccentricity distribution of solar system bodies is insufficient, as it can only be derived in the limit of small number statistics. These data are recovered from the www.exoplanets.org database (Han et al. 2014) and only include RV exoplanet detections with Doppler variation semi-amplitude $K/\sigma_K > 5$ (neglect low signal-to-noise observations). We find that our results do not sensitively depend on whether we focus on RV detections or the full catalog of exoplanets with orbital solutions. The distribution of planet eccentricities is shown in Figure 10. Due to variations in empirically derived eccentricity distributions we consider three proposed analytical forms of the distribution. Namely a Beta probability density function (PDF; Kipping 2013), a Rayleigh PDF plus decaying exponential (Juric & Tremaine 2008; Steffen et al. 2010), and the model from Shen & Turner (2008). We use a Levenberg–Marquardt least squares algorithm to fit for each PDF’s unique parameters. A summary of the adopted distributions including fitted parameters is given in Table 2 and each fitted weighting function is overplotted in Figure 10. Differences in $W(e_{ij})$ from adopting three unique PDFs result in $\sim 6\%$ variance among computed likelihoods.

While we take the known exoplanet orbital eccentricities as our nominal distribution, this sample is obviously biased by the requirement that systems be stable. If early on the solar system’s IGs moved on significantly more elliptical paths than the known exoplanets, it would be easier to eject planets while keeping Jupiter and Saturn’s satellites on nearly circular orbits, raising the likelihood of reconciling satellite orbits. However, this scenario would require a substantial amount of damping to subsequently recircularize the surviving planets orbits, and if eccentricities reached such high values, one might expect to also lose Uranus and Neptune. These considerations may be interesting directions for future work.

One must additionally weigh $p_{i,j}$ by the simulation’s impact parameter $b_i$, since wider encounters should occur more frequently. Given our coplanar setup, the differential interaction cross-section for a given impact parameter scales linearly with $b_i$ rather than with $b_i^2$, as is true in the full 3D (three-dimensional) case. We find that our results do not sensitively depend on this distinction. We therefore assume $W(x_{ij} \propto b_i)$. The distribution of $e_{ij}$ is not explicitly prescribed and is instead determined from the encounter parameters $(b_i, v_{rel}, \theta)$. The resulting distribution of $e_{ij}$ is not sampled uniformly,
Table 2

| PDF Name | PDF | Parameter 1 | Parameter 2 | Parameter 3 |
|----------|-----|-------------|-------------|-------------|
| Beta     | \( W_{\beta}(x; a, b) = \frac{1}{\beta a b \Gamma(1 + b)} e^{a(1 - x)^b} \) | 0.786 ± 0.055 | 2.764 ± 0.167 | ... |
| Rayleigh + exponential | \( W_{R + \text{exp}}(x; \alpha, \lambda, \sigma) = \frac{1}{\alpha \sigma \sqrt{2 \pi}} \exp\left(\frac{(x - \alpha)^2}{2 \sigma^2}\right) + \alpha \lambda \exp(- \lambda x) \) | 0.782 ± 0.301 | 5.131 ± 2.342 | 0.266 ± 0.061 |
| ST08     | \( W_{\text{ST08}}(x; k, a) = \frac{1}{\Gamma(k + a)} \left(\frac{1}{(1 + x)^k} - \frac{x}{k}\right) \) | 0.305 ± 0.012 | 3.413 ± 0.309 | ... |

Note. Parameter columns are written in the order in which they appear in the functional form \( W(x; ...) \), shown in the PDF column. We write the independent variable of planet eccentricity as \( x \) to make a distinction between eccentricity and Euler’s number \( e \approx 2.7182818 \).

7. DISCUSSION

To recapitulate, the above likelihoods assumed an IGE in a coplanar geometry, considering whether an initially circular Callisto (Iapetus) would have its orbital eccentricity excited beyond values that are reconcilable with its current orbit. We now consider these assumptions in turn, in order to interpret the results from our study and discuss possible implications of our work.

7.1. Effect of Inclined Encounters

To limit the computational cost of our study, we have restricted our analysis to planetary encounters with no mutual inclination. However, if planetary eccentricities are excited enough to permit orbit-crossing and ejections, one might expect comparably large orbital inclinations. During an inclined encounter, some of the applied torque goes into realigning Callisto’s (Iapetus’) orbital plane so that, on average, the perturbations to \( a_s \) and \( e_s \) are reduced as some energy goes into increasing \( i_s \).

A preliminary analysis of inclined encounters with mutual inclination \( i(\text{enc}) = 5^\circ \) effectively revealed no change to the final satellite orbital elements compared to uninclined, but otherwise equivalent encounters. In a more heavily inclined test case with \( i(\text{enc}) = 45^\circ \), we found that, on average, the final \( e_s \) were smaller by \( >0.01 \) than in the uninclined case.

Therefore, by limiting our investigation to coplanar encounters, we are sampling the largest possible perturbations to \( e_s \) without affecting \( i_s \). The introduction of inclined encounters would thus raise the likelihoods quoted in Section 6.2. However, such a 3D case would additionally excite the satellite inclinations. As mentioned by Deienno et al. (2014), these inclinations may provide more rigorous constraints on planetary encounters because even in cases where eccentricity damping is important, the tidal evolution of the inclinations is effectively null. A generalization of this study would therefore consider the combined constraint from the satellites’ orbital eccentricities and inclinations. However, the uncertainties in the likelihoods we compute in Section 6.2 are dominated by the large uncertainties in the planetary orbital eccentricities (and inclinations) early in the solar system. We therefore believe that our simplified analysis considering only the orbital eccentricities captures the correct likelihoods at the approximate level allowable by our current state of knowledge.

As a rough check, in a fully 3D case, the differential cross-section for encounters of a given impact parameter would scale as \( b_f^2 \) rather than as \( b_f \). Using the full data set from our coplanar simulations, but adopting this new scaling for \( W(b_f) \) in Equations (2) and (3), we find that our results do not discernibly change. Specifically, in the Jupiter case our results change from \( \sim 42\% \) to \( 54\% \) and from \( \sim 1.1\% \) to \( 1.3\% \) in the Saturn case. This is certainly within the errors of our uncertain knowledge of the planetary orbits’ initial eccentricities and inclinations and therefore does not appreciably change the interpretation of our results.
7.2. Interpretation of Results

We conclude that Jupiter could plausibly have ejected an IG from the solar system. Nearly half of the hypothetical set of ejections that were modeled in Section 6.2 keep Callisto on an orbit that is reconcilable with the one we observe today. Put another way, we conclude that Callisto’s orbit cannot meaningfully constrain whether or not Jupiter ejected an additional IG in the early solar system. Nevertheless, this is an important test for the fifth giant planet hypothesis to pass, thus providing more stringent evidence for its plausibility.

Interpretation of our results in the Saturn case is more subtle. We showed in Section 6.2 that an initially circular Iapetus orbit gets overly excited by a single ejection event ~99% of the time. This suggests that Saturn is not capable of ejecting an IG mass planet from the solar system.

But did Iapetus originally move on a circular path, as expected if it formed out of a circumplanetary disk? Starting with a circular orbit, all encounters act to raise the eccentricity. But if the moon’s initial path were instead elliptical, some ejection geometries could act to lower the eccentricity, complicating the constraint that we nominally set. Perhaps one reason to doubt that Iapetus formed from a circum-Kronian disk is that it is substantially inclined (~8°) to the local equilibrium plane that one would expect it to follow.

So what caused this aberrant inclination? Hamilton (2013) recently suggested that Iapetus could indeed have formed on a circular orbit from a disk in its equilibrium plane, and subsequent collisions between Saturn’s inner moons could have instead tilted the equilibrium plane for the exterior moons by the requisite amount. In this case, our study implies that Saturn likely did not eject an IG from the solar system because Iapetus’ relatively low-eccentricity orbit cannot be reconciled with IGEs by Saturn. On the other hand, Iapetus’ inclination could be the signature of an ejection event itself. Nesvorný et al. (2014) recently studied such a scenario, trying to simultaneously match Iapetus’ current orbital eccentricity and inclination using simulations of the specific early solar system dynamical instabilities from NM12. They show that some cases are capable of sufficiently exciting Iapetus’ orbital inclination while maintaining a low orbital eccentricity even for encounters as close as those considered in this study but not necessarily leading to an ejection of the IG.

In the Kronian case, therefore, the interpretation depends critically on the formation mechanism for Iapetus, which is currently unknown. If Iapetus’ inclination is the result of collisions between inner moons (Hamilton 2013), our results show that Saturn is unlikely to have ejected an IG from the early solar system. By contrast, if Iapetus’ orbit is fully explainable through close planetary encounters, Nesvorný et al. (2014) showed that this requires many such close approaches that might be capable of ejecting the IG.

7.2.1. Single-encounter Assumption

We argue that the sole consideration of the final gas giant/IG encounter leading to the ejection of the latter is a sufficient measure of how closely planetary encounters will modify regular satellite orbits. Nesvorný et al. (2014) showed for Iapetus, which is more susceptible to dynamical perturbations than Callisto, that numerous “soft” encounters prior to ejection have a fractional effect on satellite eccentricity compared to the eccentricity kick typically felt during the final encounter (Figures 4 and 8). It should be noted that the inclusion of numerous “soft” encounters prior to ejection would be detrimental to the computed likelihoods P(RS|IGE), thus emphasizing that our results represent a conservative, best-case scenario.

7.2.2. Close Encounters without Ejection

Due to the phase angle of an encounter being random, it is possible for a close encounter to have occurred at 0° < θ < 180° and is subsequently not included in our likelihood calculation as all ejection events occurred at 180° ≤ θ ≤ 360° (see Figures 2 and 6). Any close encounter that does not lead to an ejection would still perturb regular satellite orbits, making their orbits prior to an IGE event non-circular. The effect of an increased initial eccentricity reduces P(RS|IGE), again making our calculation a conservative upper limit. However, because the frequency of close encounters with 0° < θ < 180° is equivalent to the frequency of close encounters with 180° ≤ θ ≤ 360°, the effect on P(RS|IGE) of close encounters without an ejection will only differ from our calculated values by a factor of order unity, such that we capture the correct order of magnitude on P(RS|IGE).

7.3. Additional Considerations

One sought after quantity relating to this work is the probability of an IG getting ejected by either gas giant given that the orbit of Callisto or Iapetus can be reconciled; P(IGE|RS). Using Bayes’ theorem, this quantity could in principle be computed with knowledge of the likelihood functions calculated in this paper but also requires the independent probability of the IG getting ejected (P(IGE)); see NM12 and the normalization factor by the probability of a satellite exhibiting the current orbit of one of the wide-separation satellites (P(RS)). Despite the abundance of work done on the latter (e.g., Canup & Ward 2002; Estrada & Mosqueira 2006; Ward & Canup 2010; Crida & Charnoz 2012; Hamilton 2013; Heller et al. 2015), precise constraints on P(RS) are difficult to compute.

Also, a robust calculation of the likelihood of ejecting an IG from the solar system would require the event to be consistent with a vast number of constraints imposed by solar system bodies, of which the orbital constraint RS imposed by Callisto or Iapetus is just one. Therefore, such a calculation is not practical. However, consistency checks of IGEs by a gas giant, such as those presented in this study, help to substantiate the proposed existence of a fifth giant planet. Based on our current understanding, which includes the main results of our study, there is little evidence demonstrating that an additional IG mass planet could not have existed in the early solar system.

8. SUMMARY

Several studies trying to match the solar system’s current orbital architecture argue for an early period of frequent planetary encounters (e.g., Tsiganis et al. 2005; Brasser et al. 2009; Morbidelli et al. 2009; Levison et al. 2011). In addition, Nesvorný (2011) found that adding a fifth giant planet to the solar system, which is subsequently ejected, better matches the current orbits of the remaining giant planets. In this paper, we therefore study whether such an ejection by either Jupiter or Saturn is reconcilable with the current observed orbits of their outermost regular satellites, Callisto and Iapetus. Our main conclusions are as follows.
1. The properties of planetary encounters (i.e., impact parameter, relative planet velocity, and encounter geometry) between Jupiter or Saturn and an unspecified IG, which are sufficiently violent to eject the latter, exhibit similar trends.

2. The current (dynamically cold) orbit of the widest-separation Galilean satellite Callisto has a significant likelihood (∼42%) of being reconciled following the ejection of an IG planet from the solar system by Jupiter.

3. Given the observed difficulty in reconciling the orbit of Iapetus with simulated Kronian satellites following an ejection event, the likelihood of Saturn ejecting an IG from the solar system is determined to be unlikely; the likelihood is ∼1%.

4. However, we note the caveat that this interpretation is heavily dependent on the assumed formation scenario of Iapetus out of the circum-Kronian disk. Currently, the formation of Iapetus is largely uncertain.

We caution that these likelihoods should not be interpreted in an absolute sense. Rather, they are useful for showing that it is much easier for Jupiter to have ejected an IG than it is for Saturn. The evident plausibility of Jupiter being able to eject an IG thus supports the hypothesis of a fifth giant planet’s existence in the early solar system.

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