Accelerating Universe and Pioneer anomaly as manifestation of conformal time inhomogeneity

L.M. Tomilchik

1B.I. Stepanov Institute of Physics, 68 Nezalezhnasci Avenue, 220072 Minsk, Belarus

The description of the cosmological expansion and its possible local manifestations via treating the proper conformal transformations as a coordinate transformation from a comoving Lorentz reference frame (RF) to an uniformly accelerated RF is given. The explicit form of the conformal deformation of time is established. The expression defining the location cosmological distance in the form of simple function on the red shift is obtained. By coupling it with the well known relativistic formula defining the relative velocity of the mutually moving apart source and receiver of the signal, the explicit analytic expression for the Hubble law is obtained. The connection between acceleration and the Hubble constant follows therefrom immediately. The expression for the conformal time deformation in the small time limit leads to the quadratic time nonlinearity. Being applied to describe the location-type experiments, this predicts the existence of the uniformly changing blue-shifted frequency drift. Phenomenon of the Pioneer Anomaly (PA) is treated as the first of such a kind of effects discovered experimentally. The obtained formulae reproduce the PA experimental data. The expression generalizing the conventional Hubble law reproduces the experimentally observed phenomenon which in the frame of the conventional cosmological paradigm is treated as the transition from the decelerated expansion of the Universe to the accelerated one.

PACS numbers: 03.30.+p, 04.80.Cc, 11.25.Hf

I. INTRODUCTION

The experimental detection of the Pioneer Anomaly (PA) [1–4] indicates on the existence, in the outer of the solar system at the distances 20–70 AU, an observable effect unaccountable via contemporary celestial mechanics including all the necessary General Relativity (GR) corrections.

The experimentally observed phenomenon is the anomalous blue frequency drift with a numerical value \( \frac{\Delta f}{f} = (5.99 \pm 0.01) \times 10^{-9} \) Hz/s detected in the signals retransmitted by Pioneer 10 and 11 satellites. It is interpreted as the Doppler shift, that is, due to the uniform acceleration directed (within the limits of experimental error) towards the Sun amounting, after all corrections, to \( a_p = (8.74 \pm 1.33) \times 10^{-8} \) cm/s\(^2\) (see [1–4]).

The fact that the observable \( a_p \) is close to the quantity \( W_0 = cH_0 \) (\( c \) is the speed of light, \( H_0 \) is the Hubble constant) and therefore can point to the cosmological origin of the PA effect, was noted by several authors (see, for example, [2], sec. XI, point C). However, any unambiguous theoretical arguments in favor of the possible existence of such a connection are absent up to now.

Moreover, the rigorous consideration of this subject on the basis of the precise Friedmann and Schwarzschild solutions of the GR equations gives the quadratic dependence of the effect on the parameter \( H_0 \) [5], so that the predicted magnitude of \( a_p \) turns out to be much less than the observable one and, in addition, the effect was found to have the opposite sign [2]. Nevertheless, an interpretation of PA as a local manifestation of the cosmological expansion seems to be acceptable when using the possibilities inherent in the general symmetry group of the Minkowski space.

Most recent experimental data of astrophysics clearly indicate an infinitesimal space curvature of the observable Universe. Because of this, it seems natural to select the conformally Minkowski space as a model for space-time manifold of the modern Metagalaxy. As is well known, its most wide symmetry group is the group \( SO(4,2) \) which, apart from the Poincaré group isometric transformations, includes a subgroup of special conformal transformations (SCT) and dilatations changing the space and time scales [6]. Naturally, it may be expected that an expansion of the space-time symmetry by the inclusion of this subgroup transformations will result in some generalization of the standard kinematics of the Special Relativity (SR).

Such an attempt was undertaken by us before. In the paper [7], on the basis of the nonisometric transformations subgroup of the \( SO(4,2) \) group, the nonlinear time inhomogeneity one-parameter conformal transformations were constructed. After postulating the connection between the group parameter and the Hubble constant it is shown that the existence of an anomalous blue-shifted frequency drift is the pure kinematic manifestation of the time

*Electronic address: lmt@dragon.bas-net.by
in the Universe expansion. The obtained formulae reproduce the observable Pioneer Anomaly effect. According to the proposed approach, the anomalous blue-shifted drift is universal, does not depend on the presence of gravitating centers and can be, in principle, observed at any frequencies under suitable experimental conditions. The explicit analytic expression for the speed of recession — intergalactic distance ratio is obtained in the form of a simple function of the red shift $z$, valid in the whole range of its variation. In the small $z$ limit this expression exactly reproduces the Hubble law. The existence of maximum of this function at $z = 0.475$ quantitatively corresponds to the experimentally found value $z_{	ext{exp}} = 0.46 \pm 0.13$ of the transition from the decelerated to the accelerated expansion of the Universe.

As it is known, the special conformal transformations (SCT) can be treated as a coordinate transformation from an inertial reference frame (RF) to the uniformly accelerated one. Such a consideration (the "new relativity") was proposed in due time in the papers [8], and was subsequently discussed by a number of authors (see [9, 10] and references therein). But in the paper [7] this interpretation was not employed in the explicit and successive form.

In the present paper the description of the cosmological expansion and its possible local manifestations is given via treating the proper conformal transformations as a coordinate transformation from a comoving Lorentz RF to an uniformly accelerated RF. Such an approach permit us to derive the explicit analytic expression for the Hubble law, which allows us to connect the acceleration parameter with the Hubble constant as well as to reproduce the accelerating Universe effect in exact correspondence with the observations.

In so doing these effects, once dictated by the conformal time inhomogeneity, can be interpreted as the observable manifestations of the background acceleration existence, i.e., manifestations of the noninertial character of any physical frame of reference which is locally coupled to an arbitrary point of the modern expanding Metagalaxy.

The content of the present paper is as follows. In section 2 we consider the proper conformal transformations in the two-dimensional subspace of the Minkowski space treated as a coordinate transformations from the local comoving Lorenz reference frame to the noninertial (uniformly accelerated) one including the transfer to Galilei-Newton (GN) kinematic limit.

The general expression for the velocity transformation is obtained in section 3. In the GN limit, the relation brings into existence the blue Doppler shift which is to be observed as a manifestation of the noninertialty of the observer reference frame.

In section 4, the general formula for the acceleration transformation is obtained. It is shown that the parameter of the conformal transformations defines the background acceleration which is to be observed in the absence of any real sources of the dynamical subjection.

In section 5, it is shown that SCT, which is preserving the light cone equation, nevertheless brings into existence the nonlinear transformation of its generating lines (conformal deformation of time or the time inhomogeneity). We establish the explicit form of this transformation. The parameter having the dimension of time (the limiting time $t_{\text{lim}}$) is connected with the acceleration parameter $w$ ($t_{\text{lim}} = c/w$, $c$ is the speed of light).

Then in section 6, the explicit expression defining the location cosmological distance $R(z)$ in the form of simple function on the red shift $z$ is obtained as a direct outcome of the formula defining the conformal time deformation. Coupling it with the well known relativistic formula defining the relative velocity $V(z)$ of the mutually moving apart source and receiver of the signal, we obtain the explicit analytic expression for the ratio $\phi(z) = V(z)/R(z)$. The function $\phi(z)$ reproduces, in the limit $z \to 0$, the conventional form of the Hubble law. It follows herefrom immediately the connection between acceleration parameter $w$ and the Hubble constant $H_0$: $w = \frac{1}{2} c H_0$.

The expression describing the conformal time deformation in the small time limit ($t/t_{\text{lim}} \ll 1$) leads to the quadratic time nonlinearity. Such a time nonuniformity, being applied to describe the location-type experiments, predicts the existence of the uniformly changing blue-shifted frequency drift. Its magnitude $\Delta \nu_{\text{obs}} = \nu_0 H_0 t$ (where $\nu_0$ is the frequency of the signal emitted, $t$ is the time of the signal propagation from the emitter to the receiver). Phenomenon of the PA is treated as the first of such a kind of effects discovered experimentally. The obtained formulae reproduce the PA experimental data (section 7).

The formula for the function $\phi(z) = V(z)/R(z)$ generalizing the conventional expression for the Hubble law is derived by a pure kinematical considerations. But, owing to the availability of a maximum of this function at the point $z_0 \simeq 0.475$, this formula, in fact, reproduces the experimentally observed phenomenon which, in the frame of the conventional cosmological paradigm, is treated as the transition from the decelerated expansion of the Universe to the accelerated one.

II. CONFORMAL TRANSFORMATIONS AND THE ACCELERATED FRAME OF REFERENCE

It is well known that the special conformal transformations (SCT)

$$x'^{\mu} = \frac{x^{\mu} + a^{\mu}(x^{a} x_{a})}{1 + 2(a^{a} x_{a}) + (a^{a} a_{a})(x^{b} x_{b})},$$

(1)
where $a^\mu$ is the four-vector parameter, $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, can be interpreted as the transformations between the Lorentz (inertial) frame of reference $S(x^\mu)$ and the noninertial (accelerated) frame of reference $S'(x'^\mu)$ (see, for example, [10]). Following [10] we shall for simplicity consider a two-dimensional subspace $\{t, x\}$, i.e. we put

$$x^\mu = (ct, x, 0, 0), \ a^\mu = (0, -\frac{w}{2c^2}, 0, 0),$$

(2)

where $w$ is a constant, having the dimension of acceleration.

Let us write the transformations (1) in the following noncovariant form:

$$x' = x \left(1 + \frac{wx}{2c^2} - \frac{w^2}{2} \right),$$

$$t' = \frac{t}{\left(1 + \frac{wx}{2c^2} - \frac{w^2}{2c^2} \right)^2}.$$  

(3)

In the case when $\frac{wx}{2c^2}$ and $\frac{wt}{2c^2}$ are negligible from (3) we have

$$x' = x - \frac{wt^2}{2}, \quad t' = t,$$

(4)

which corresponds to Galilei-Newton kinematics. It should be noted that the identification of $w$ with a constant three-dimensional newtonian acceleration is essentially based on this correspondence. It is also clear from (3) that sign ($-$) of the vector’s $a^\mu$ x-component describes a positive direction of $S'$ acceleration along the $x$-axis of the inertial reference frame (IRF) $S$.

The following note deals with the identification of the RF $S$. It is well known that the parameter $a^\mu$ is related with a constant 4-acceleration $W^\mu$ as follows:

$$a^\mu = \frac{1}{2c^2} W^\mu.$$  

From the other side, generally, the 4-vector of acceleration is

$$W^\mu = (W^0, \vec{W}) = \gamma^2 \left(\frac{\vec{v} \cdot \vec{w}}{c^2}, \frac{\vec{v} \cdot \vec{w}}{c^2} \vec{v} + \vec{w} \right),$$

(5)

where $\vec{v} = \frac{d\vec{r}}{dt}$, $\vec{w} = \frac{d\vec{v}}{dt}$, $\gamma = \left(1 - \frac{\vec{v}^2}{c^2}\right)^{-\frac{1}{2}}$.

From relation (5) it follows that in the local comoving reference frame (CRF), i.e., in the frame where $\vec{v} = 0$, time component of the 4-acceleration is exactly null, and its space part is $\vec{W} = \vec{w}$:

$$W^\mu = (0, \vec{w}).$$

Because of this, the parameter $a^\mu$ of the form (2) determines the 4-acceleration in CRF. Therefore, the 4-vector $x^\mu$ in the transformations (1) is supposed to be determined in the CRF.

We also emphasize that, due to (3), the transformations (1) are singular when

$$\left(1 + \frac{wx}{2c^2}\right)^2 = \left(\frac{wt}{2c}\right)^2,$$

(6)

i.e. on the lines

$$x_\pm = -\frac{2c^2}{w} \pm ct,$$

(7)

corresponding to the null cone of the $(0, -2r_0)$ point (see Fig. 1).

It is well known that under the transformations (3) the world lines $\{x_0, x_{fix}\}$ in the $(x_0, x)$ plane, where $x_{fix}$ is fixed, convert to hyperbolae in the $(x'_0, x')$ plane. Again following [10], we denote the world lines $\{x_0, x_{fix}\}$ as $\{x_0, \alpha\}$, where $\alpha = 1 + \frac{x_{fix}}{2r_0}$, such that $x_{fix}$ is defined as a function of non-zero real parameter $\alpha$:

$$x = 2r_0(\alpha - 1), \quad (\alpha \geq 0).$$

(8)
Then the world lines \( \{x_0, x_{fix}\} \) transform into hyperbolas

\[
(x' - \lambda(\alpha))^2 - x'^2 = \left(\frac{r_0}{\alpha}\right)^2
\]

in the \((x'_0, x')\) plane. Here

\[
\lambda(\alpha) = \frac{r_0}{c}(1 - 2\alpha)
\]

defines the hyperbola center position \((x'_0 = 0, x' = -\lambda)\). Vertex of hyperbola is determined from (9) with \(x'_0 = 0\)

\[
x'_{\pm}(\alpha) = -\frac{r_0}{\alpha} + 2r_0 \pm \frac{r_0}{\alpha} = \begin{cases} 2r_0, \\ 2r_0 \frac{\alpha - 1}{\alpha}. \end{cases}
\]

Two such lines are showed on Fig. 2.

The meaning of the transformation is clear: every point fixed in Lorentz RF \(S\) is moving hyperbolically in non-inertial RF \(S'\).

Let us now consider the velocity and acceleration transformation laws.

III. VELOCITY TRANSFORMATION AND THE BLUE-SHIFTED DOPPLER DRIFT

The transformation of coordinate differentials is convenient to express by (3) in the following symmetric form:

\[
\begin{align*}
dx' &= \frac{(\xi^2 + \eta^2)dx - 2\xi\eta dx_0}{\xi^2 - \eta^2}, \\
dx'_0 &= \frac{(\xi^2 + \eta^2)dx_0 - 2\xi\eta dx}{\xi^2 - \eta^2}.
\end{align*}
\]

Here

\[
\xi = 1 + \frac{x}{2r_0}, \quad \eta = \frac{x}{2r_0}, \quad x_0 = ct, \quad x'_0 = ct', \quad r_0 = \frac{c^2}{w}.
\]

This results in the following expression for velocity transformations

\[
V_x' = \frac{dx'}{dt'}, \quad V_x = \frac{dx}{dt}.
\]

\[
V_x' = \frac{V_x - V_w}{1 - \frac{V_x V_w}{c^2}},
\]

This equation can be used to calculate the velocity of an observer moving with constant velocity \(V_w\) with respect to the inertial frame. The components of the velocity in the \(x\)-direction are

\[
V_x' = \begin{cases} V_x, & w < c, \\ V_x - \frac{V_x V_w}{c^2}, & w > c. \end{cases}
\]
where the "effective relative velocity" of $S$ and $S'$ RF $V_w$ is
\[ V_w = c \frac{2\xi\eta}{\xi^2 + \eta^2} \leq c. \] (15)

The value of $V_w$ is a function of space and time coordinates. It is notable that form of (14) is exactly the same as of usual velocity transformations in Special Relativity. In the non-relativistic approximation $\xi \approx 1$, $\eta \ll 1$ we obtain
\[ V'_x = V_x - wt, \] (16)
corresponding to Galilei-Newton kinematics. The maximum of $V_w$ is determined by $|\xi| = |\eta|$, and is equal to $c$.

From (16), the existence of blue Doppler drift immediately follows. Really, due to (16) the longitudinal component of a point velocity $V'_x$ measured by an observer fixed in non-inertial RF $S'$ will be less then that measured by an observer fixed in the inertial comoving RF $S$ by $\Delta V_x = wt$.

Clearly, in this case an observer with fixed space coordinates in RF $S'$ measures blue frequency shift as compared to an observer fixed in $S$. In fact, measured frequency of the signal of a moving away emitter in the non-relativistic limit will be
\[ \nu' = \nu_0 \left( 1 - \frac{V'_x}{c} \right) \quad \text{in } RF \ S', \]
\[ \nu' = \nu_0 \left( 1 - \frac{V_x}{c} \right) = \nu_{mod} \quad \text{in } RF \ S, \]
where $\nu_0$ is the signal frequency emitted by a source fixed in $S$.

Therefore, for the observed blue shift we have
\[ \Delta \nu_{obs} = \nu' - \nu_{mod} = \nu_0 \frac{wt}{c}, \] (17)
and $\nu_{mod}$ is the frequency defined by neglecting non-inertiality of $S'$. In the approximation considered the shift is linear in time. The rate of shift $\nu_{obs} = \frac{d \nu_{obs}}{dt}$ is defined by the following simple relation
\[ \nu_{obs} = \nu_0 \frac{w}{c}. \] (18)

This result, in principle, is well known. Here we have to do, in fact, with the effect of the gravitational (Einstein) frequency drift described on the basis of the equivalence principle. The shift is blue because the non-inertial observer and the source of signal approach each other, and therefore the magnitude of the equivalent gravitational potential at the observation point is above its magnitude at the point of the signal emission.
IV. TRANSFORMATION OF ACCELERATIONS. THE BACKGROUND ACCELERATION

The transformation law of the acceleration’s $x$-component $W'_x = \frac{d^2x'_x}{dt'^2}$ can be derived directly from (14). Elementary calculus gives

$W'_x = \left(\frac{\xi^2 - \eta^2}{(\xi^2 + \eta^2)^3}\right) \left(1 - \frac{V_x V_0}{c^2} \left(1 + \frac{1}{2\gamma_0}(x - V_xt)\right)\right). \tag{19}$

In the comoving RF ($V_x = 0$), the above formula becomes

$W'_x = \left(\frac{\xi^2 - \eta^2}{(\xi^2 + \eta^2)^3}\right)(W_x - \xi w).$

In the approximation of $\xi \approx 1, \eta \ll 1$ we have therefrom

$W'_x = W_x - w. \tag{20}$

This relation holds in every comoving RF as long as in this frame $\xi \approx 1, \eta \ll 1$ approximation is valid. A probe particle free of dynamical influence in the comoving RF, i.e., with $W_x = 0$, in accordance with (16) will be uniformly accelerated in the non-inertial RF $S'$ with the constant acceleration of $-w$. Such an acceleration can be registered by any observer fixed in any point of this non-inertial RF $S'$. By the equivalence principle the non-inertial observer is entitled to identify this acceleration with an existence of a constant (background) gravitational field which results in acceleration $w$.

V. CONFORMAL DEFORMATION OF THE LIGHT CONE AND TIME INHOMOGENEITY

Now we consider the transformations of the light cone generatrices under SCT (1). Since $x'^\mu x'_\mu = x^\mu x_\mu (1 + 2(ax) + (a)^2(x)^2)^{-1}$, transformations (1) leave the light cone equation invariant, i.e., from $x^\mu x_\mu = 0$ follows $x'^\mu x'_\mu = 0$. However the light cone surface is deformed non-linearly. From (1), it generally follows

$x'^\mu = \frac{x^\mu}{1 + 2(ax)}, \tag{21}$

when additionally $x'^\mu x'_\mu = x'^\mu x'_\mu = 0$.

In the two-dimensional case considered we have the relation

$t'_{\pm} = \frac{t}{1 \pm t_{lim}}, \tag{22}$

where $t_{lim} = c/w$.

The choice of sign corresponds to signal propagation in the forward and backward directions, respectively. We are reminded that the symbol $t$ ($t'$) represents time of a light signal propagation between two spatially separated points in the space of Lorentz RF $S$ (in the non-inertial RF $S'$). So the quantities $R = ct$ and $R' = ct'$ define location distances in both of these RFs.

Obviously a semi-infinite time interval $0 \leq t < \infty$ corresponding to the positive (forward) direction of signal propagation maps onto a finite time interval $0 \leq t'_{+} \leq t_{lim}$. For the backward direction, on the contrary, a finite interval $0 \leq t \leq t_{lim}$ maps onto a semi-infinite time interval $0 \leq t'_{-} < \infty$ (Fig. 3).

The non-linear time transformation (22) will further be referred to as the conformal deformation of time, or conformal time inhomogeneity.

VI. THE DEPENDENCE OF DISTANCE ON RED SHIFT. THE HUBBLE LAW

First and foremost we show that transformations (22) allow us to obtain an explicit expression for location distance as a simple function of red shift $z$.

Let us consider, on the basis of the formula (22), the case of a signal propagation from the deep past to the point of the observer position. That means that we choose the lower sign in the formula (22):

$t' = \frac{t}{1 - \frac{t}{t_{lim}}}. \tag{23}$
where \( t_{\text{lim}} = c/w \), and \( t \) is the time of the signal propagation to the point of observation.

First of all, from (23) we obtain the explicit expression for the time interval of the signal propagation as a function of the red shift \( z \).

From (23), we have, for small time increments \( \Delta t' \) and \( \Delta t \), the following expression

\[
\Delta t' = \Delta t (1 - \frac{t}{t_{\text{lim}}})^2. \tag{24}
\]

If \( \Delta t \) and \( \Delta t' \) are the periods of oscillations of the emitted \( (\Delta t = T_{\text{emitted}}) \) and received \( (\Delta t' = T_{\text{observable}}) \) signals, respectively, then, using the standard definition of the red shift

\[
\frac{\lambda_{\text{observable}}}{\lambda_{\text{emitted}}} = z + 1, \tag{25}
\]

where \( \lambda_{\text{observable}} = cT_{\text{observable}} \) and \( \lambda_{\text{emitted}} = cT_{\text{emitted}} \), we find, from (24), the expression

\[
\frac{\lambda_{\text{observable}}}{\lambda_{\text{emitted}}} = (1 - \frac{t}{t_{\max}})^{-2} = z + 1, \tag{26}
\]

which gives

\[
t(z) = t_{\max} \left( \frac{z + 1}{(z + 1)^{1/2}} \right). \tag{27}
\]

Here \( t(z) \) represents the time interval between the moments of emitting and receiving the light (electromagnetic) signal. So, assuming that the speed of light is constant and does not depend on the velocity of the emitter, the quantity \( R = ct \) can be regarded as the distance covered by the signal.

From the formula (27), we obtain an expression which determines the explicit form of dependence of \( R \) on the red shift \( z \):

\[
R(z) = R_u \left( \frac{z + 1}{(z + 1)^{1/2}} \right)^{1/2} = R_u \left( 1 - \frac{1}{(z + 1)^{1/2}} \right). \tag{28}
\]

Here \( R_u = ct_{\text{lim}} \) is a parameter, which, within the model suggested, has the sense of the limit (maximal) distance.

The quantity \( R(z) \) defined by (28) corresponds to the distance, which in cosmology is referred to as a location distance. In principle, the relation (28) allows for a direct experimental verification in the whole range of \( z \) variation, and can be confirmed or refused by observations.
Now, we can obtain the explicit expression for the Hubble law. For this purpose, we make use of the well-known formulae describing the Doppler effect in Special Relativity. In the context of our approach, to find the explicit expression for the longitudinal Doppler effect, it is convenient to apply the formulae which immediately follow from the Lorentz boosts written in the light-cone variables:

\[ u = \frac{1}{2}(x_0 + x), \quad v = \frac{1}{2}(x_0 - x). \]

These expressions are (see, for example, [12]):

\[ u' = k(\beta)u, \quad v = k^{-1}(\beta)v, \]  \hspace{1cm} (29)

where \( k(\beta) = \left(1 + \frac{\beta}{1 + \beta} \right)^{1/2} \), \( \beta = V/c \), \( V \) is the velocity which can be identified with the radial component of relative velocity of emitter and receiver motion.

Clearly, in terms of \( u \) and \( v \), the Lorentz boosts have the form of dilatations. For the description of the Doppler effect, we need to use the equation of light cone \( 4uv = 0 \). Then, for the case of the signal traveling in positive \((v = 0)\) and negative \((u = 0)\), directions we have in coordinates \((x, t)\):

\[ t' = \frac{1 + \beta}{1 - \beta}^{1/2} t, \]  \hspace{1cm} (30)

In the case of emitter and receiver moving away from each other, we find, from (30), for small time increments \( \Delta t' \) and \( \Delta t \):

\[ \Delta t' = \Delta t \left( \frac{1 + \beta}{1 - \beta} \right)^{1/2}, \]

whence, taking into account (30), the known expression for the function \( V(z) \) follows:

\[ \frac{V(z)}{c} = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}. \]  \hspace{1cm} (31)

Equation (31) is valid in the whole range of the velocity \( V \) variation.

We again emphasize the essentially kinematic nature of the relation (28). It is the manifestation of the nonlinear conformal time deformation (22) which follows from Special Conformal Transformations exactly in the same manner as the Doppler effect, and the dependence \( V(z) \) follows from the linear time deformation (30) arising from the Lorentz boosts leaving the equation of light cone unaltered.

Now we can find, using (28) and (31), the following expression for the ratio \( V/R \):

\[ \frac{V(z)}{R(z)} = cR_u^{-1} f(z), \]  \hspace{1cm} (32)

\[ f(z) = \frac{(z + 1)^{1/2}}{(z + 1)^2 + 1} \cdot \frac{(z + 1)^2 - 1}{(z + 1)^{1/2} - 1}. \]  \hspace{1cm} (33)

It is easy to see that \( \lim_{z \to 0} f(z) = 2 \).

In this limit we obtain the conventional expression for the Hubble law:

\[ V = H_0 R, \]  \hspace{1cm} (34)

where \( H_0 \) is the Hubble constant.

By comparing this formula with \( \lim_{z \to 0} \frac{V(z)}{R(z)} \) from formula (32), we can establish the following connection between the acceleration \( w \) and the Hubble constant \( H_0 \):

\[ 2cR_u^{-1} = 2w = cH_0. \]  \hspace{1cm} (35)

It is seen that the relation (22) defining the conformal deformation of time (the time inhomogeneity) allows us to establish the following simple connection between the parameter \( w \) defining the background acceleration and the Hubble constant \( H_0 \)

\[ w = \frac{1}{2} cH_0. \]  \hspace{1cm} (36)

Hence, in the considered approach the constant acceleration \( w \) intrinsic to non-inertial RF \( S' \) can naturally be connected to the Hubble constant \( H_0 \), defining space expansion.
VII. TIME INHOMOGENEITY AND THE BLUE-SHIFTED FREQUENCY DRIFT. THE PIONEER ANOMALY

Now let us consider, on the basis of the formula (22), the location-type experiments. The conventional scheme of such an experiment is as follows:

1. the signal is emitted from the point of the observer location at the time instant \( t_A^0 \),
2. the signal is arrived and reemitted at the time instant \( t_B \),
3. the signal is returned to the observer at the time instant \( t_A \).

Under the assumption of the coincidence of the forward \((t_B - t_A^0)\) and backward \((t_A - t_B)\) time intervals, one can obtain the formula for the signal traveling time \( t = \frac{1}{2}(t_A - t_A^0) \), and then accept the formula \( R = ct \) for the corresponding location distance.

The time inhomogeneity (22) changes situation such that the forward and backward time intervals do not coincide. The latter time interval is larger then the former one (Fig. 4).

In application to real experiment analysis one should use (22) for the small time intervals, i.e., when \( \Delta t/t_{lim} = (t_A - t_A^0)/t_{lim} \ll 1 \). In this case, formula (22) gives, to the second order of \( t/t_{lim} \)

\[
t'_\pm = t \mp \frac{t^2}{t_{lim}}. \tag{37}
\]

In accordance with the location distance definition, the signal travelling distances in forward and backward directions, respectively, will be

\[
x'_\pm = ct'_\pm = x \mp \Delta x = x \mp \frac{W_0}{2}t^2, \tag{38}
\]

where

\[
W_0 = \frac{2c}{t_{lim}} = 2w. \tag{39}
\]

Therefore in the \( t/t_{lim} \ll 1 \) approximation (\( t \) is the signal propagation time) the forward and backward location distances \( x'_\pm \) differ from \( R = ct \) by \( \Delta x = \frac{W_0}{2}t^2 \). From the usual point of view, it appears that the emitter fixed in any space point is subjected to constant acceleration \( W_0 = 2w \) directed to the observer.
Clearly, \( t/t_{lim} \ll 1 \) condition is equivalent to the condition \( \frac{\Delta t}{t} \ll 1 \), \( \frac{\Delta \nu}{\nu} \ll 1 \) (see Sec. 1), which formally corresponds to Galilei-Newton kinematics. So, the predicted effect in the location-type experiments will be the blue frequency shift, which value will be defined by the formula similar to (17), i.e.

\[
\Delta \nu_{\text{obs}} = \nu_0 \frac{W_0 t}{c}.
\]

Due to (39) the value of this shift is twice as large as predicted by (17).

For the constant rate of frequency shift \( \dot{\nu}_{\text{obs}} = \frac{d(\Delta \nu_{\text{obs}})}{dt} \), we find the following relation analogous to (18):

\[
\dot{\nu}_{\text{obs}} = \frac{1}{c} \nu_0 W_0.
\]

Since by (35), \( W_0 = cH_0 \), where \( H_0 \) is the Hubble constant, from (41) we have

\[
\dot{\nu}_{\text{obs}} = \nu_0 H_0.
\]

This relation defines the frequency drift as a function of the fixed emitter frequency \( \nu_0 \).

According to the approach under consideration the anomalous blue-shifted drift is the consequence of the background acceleration existence (i.e. non-inertiality of the observers RF). It can be observed in principle under suitable conditions (in the absence of any gravitating sources) on any frequency even in the case of mutually fixed emitter and receiver. From this point of view PA should be treated as the first clearly observed effect of that kind. The experimental data of Pioneer tracking can be used for determination (or at least for estimation) of the parameter \( \nu_{\text{lim}} \) and corresponding background acceleration.

On the other hand, the uniform blue-shifted drift \( \dot{\nu}_{\text{obs}} \) is measured experimentally with a great accuracy \( \dot{\nu}_{\text{obs}} = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz/s} \) [1-4]. Therefore it can form a basis for the new (alternative to the cosmological observations) high precision experimental estimation of the numerical value of the Hubble constant.

For that goal, we make use of (42), and recall that frequency of Pioneer tracking is \( (\nu_0) = 2.29 \cdot 10^{9} \text{ Hz} \), such that

\[
H_0 = \frac{(\dot{\nu}_{\text{obs}})_P}{(\nu_{\text{obs}})_P} \approx 2.62 \cdot 10^{-18} \text{ s}^{-1},
\]

what is consistent with generally accepted value of \( H_0 \approx 2.4 \cdot 10^{-18} \text{ s}^{-1} \) obtained from cosmological observations.

For the ”acceleration” \( a_P \) we have

\[
a_P = cH_0 = 7.85 \cdot 10^{-8} \text{ cm/s}^2,
\]

what is in the range of uncertainty of PA data \( a^{exp}_{PP} = (8.74 \pm 1.33) \cdot 10^{-8} \text{ cm/s}^2 \).

The numerical coincidence of the results can be considered as experimental evidence of anomalous blue-shifted drift as a kinematical manifestation of the conformal time inhomogeneity. In other words, from the view point of the considered approach, the quantities measured in experiments of electromagnetic wave propagation favor relation (40) (but not (17)) for the anomalous frequency shift.

It should be stressed that the physical meaning of the relations (17) and (40) is fundamentally different in spite of their visual similarity.

Equation (17), defining blue frequency shift by the non-inertiality of RF \( S' \), in fact was obtained in the Galilei-Newton kinematics. There time transformation under transition from RF \( S \) to \( S' \) has the form \( t = t' \) (see (4)), and the velocity transformations (16) as well as the acceleration \( w \) are defined as usually in the Galilei-Newton kinematics.

On the other side, formula (40) was obtained from the exact non-linear time transformation (22) defining the time inhomogeneity, that is, beyond the Galilei-Newton kinematics. “Constant acceleration” \( W_0 = 2w \) appears due to the quadratic character of the first non-linear term in the power series expansion of \( t'(t) \) in terms of small parameter \( t/t_{lim} \) in (22), while the location distance is defined as \( R' = ct' \).

Hence the “acceleration” \( W_0 \) is not a “truly” acceleration (i.e. its origin is not a force or a dynamical source) but rather a “mimic” acceleration. This imitation appears because of the effects originated from non-linearity of time course (time inhomogeneity), and interpreted in terms of traditional paradigm of time homogeneity.

The possibility of assigning the anomalous frequency drift observed in the signals transmitted by Pioneer 10/11 to the quadratic time inhomogeneity was noted in the first comprehensive works on PA [1] and [2]. However no theoretical reasons were mentioned for such a phenomenological approach.

Concerning the acceleration parameter \( w = \frac{1}{2}cH_0 \) we note that it seems to be a strictly natural candidate for the “minimal acceleration” of Milgrom, which is a fundamental dynamical parameter of the Modified Newton Dynamics (MOND) (see [13, 14]), which in turn is a phenomenological alternative for the Cold Dark Matter approach. This question will be subject of a separate publication.
VIII. THE POSSIBLE KINEMATIC ORIGIN OF THE ACCELERATING UNIVERSE PHENOMENON

Now we analyze the general relation (32) for $V(z)/R(z)$

$$
V(z) = cR_u^{-1} \frac{(z + 1)^{1/2}}{(z + 1)^2 + 1} \cdot \frac{(z + 1)^2 - 1}{(z + 1)^{1/2} - 1}.
$$

Taking into account the connection (35) between $R_u$ and the Hubble constant $H_0$, we rewrite this equation in the dimensionless form as

$$
\phi(z) = \frac{V(z)}{H_0R(z)} = \frac{1}{2} \frac{(z + 1)^{1/2}}{(z + 1)^2 + 1} \cdot \frac{(z + 1)^2 - 1}{(z + 1)^{1/2} - 1}.
$$

The function $\phi(z)$ and its derivative $\phi'(z) = \frac{d\phi(z)}{dz}$ are shown in Fig. 5 and Fig. 6. Horizontal line in Fig. 5 represents strict Hubble law (34). We see that this function possesses a maximum at $z_0 \approx 0.475$ (see Fig. 5 and Fig. 6). Overall variation of $\phi(z)$ demonstrates that in the interval $0 \leq z < z_0$ the distance $R(z)$ increases with $z$ more slowly, and in the interval $z_0 < z < \infty$ approaches its limit value $R_u$ more rapidly, than the velocity $V(z)$ approaches its limit $c$ (Fig. 7).

As regards a possible treatment of the behavior of the function $\phi(z)$ in terms of the standard dynamical GR approach using the deceleration parameter, we are to notice the following. According to the pure kinematic approach proposed in our paper, the source of the effects induced by the cosmologic expansion is the conformal time inhomogeneity. The “acceleration” attributed to the emitting source arises because of treating the actual nonlinear time dependence $t'(t)$ in terms of the traditional theoretical paradigm based on the time homogeneity concept. The uniform “acceleration” $W_0 = cH_0$, which appears in the formula (38) in the $t/t_{lim} \ll 1$ approximation, is not a “true” but the “effective” acceleration in reality. Such an “acceleration” in the general case must be treated as time-dependent one. Purely
formally, it can be determined as the second time derivative of the function

$$R(t) = ct' = ct \left(1 - \frac{t}{t_{lim}}\right)^{-1}$$

(45)

and it takes the following form

$$W(t) = \frac{2c}{t_{lim}} \left(1 - \frac{t}{t_{lim}}\right)^{-3}.$$  

(46)

Such a "time-dependent effective acceleration" is directed to the point of observation and coincides in the first order in $t/t_{lim}$ with $W_0 = cH_0$. This "acceleration" can be presented, according to (26), in the form

$$W(z) = W_0 (1 + z)^{3/2}.$$  

Thus, one can say that the numerical value of such an "acceleration" decreases during the process of the Universe expansion, starting from the very large magnitude ($z \gg 1$).

Evidently, the interpretation of $\phi(z)$ behavior from the point of view of common treatment seems as follows. In the interval $\infty > z > z_0$, there is deceleration $(\frac{d\phi}{dz} < 0)$ of cosmological expansion, which turns to acceleration $(\frac{d\phi}{dz} < 0)$ at $z_0 \approx 0.475$ it changes to the. Numerical value of $z_0$ agrees quite well with experimentally founded "point of change" $z_{exp} = 0.46 \pm 0.13$.

It should be emphasized that the basic formula (22) for the conformal transformations of the time, as well as all its consequences, are valid on the assumption that the Hubble parameter $H_0$ is constant. Hopefully this assumption is reasonable as applied to at least later stages of the Universe evolution. In this case the proposed formulae (28) and (32) can be valid for the experimentally obtained values of the read shift having the order of several units.

**IX. SUMMARY AND OUTLOOK**

The main results of the proposed approach consist in derivation of kinematical consequences of the conformal deformation of the time, which have not yet received the proper attention so far. The first of these consequences is the possibility to express the propagation time of the electromagnetic signal traveling from the remoting source to the point of observation as an explicit function on the red shift. That gives the possibility to obtain a simple general expression defining dependence of the cosmological distance on the red shift.

It would be of interest to correlate the obtained formula with the cosmological observation data in the range of the red shift values accessible experimentally. We stress that the obtained expression for the cosmological distance dependence on the red shift follows directly from the conformal transformations. Likewise, the well known relativistic formula for the source — receiver relative velocity dependence on the frequency shift (Doppler effect) follows from the Lorenz transformations in a pure kinematical way. In doing so, the light-cone equation is used as an additional condition in both cases.

The proposed derivation of the expression for the speed of recession/intergalactic distance ratio is therefore a proof that a Hubble law can be derived as a direct kinematical outcome of the conformal group transformations.
The obtained expression can be verified experimentally, at least in the accessible range of the red shift. It is remarkable that the correspondence function in the red shift possesses a maximum at the point $z_0 = 0.475$. This numerical value corresponds surprisingly well to the experimentally registered value $z_{exp} = 0.46 \pm 0.13$.

The proof of existence of the direct connection between the acceleration parameter of the conformal transformations and the Hubble constant is the all-important conclusion of the proposed derivation of the generalized expression for the Hubble law. The existence of such a connection gives the theoretical foundation for treatment of the observable local effects of the cosmological expansion as a manifestation of the non-inertial (non-Lorentz) character of the observer RF.

According to the proposed treatment such an effect can be detected in the experiments dealing with signal propagation along the closed path. The location-type experiment is the most simple one from viewpoint of the possibility of its adequate description in the frame of the spatially one-dimensional model.

The parameter $t_{lim}$ defining the upper limit of the duration of signal propagation coincides by an order of magnitude with the age of the Universe. It is much longer than the duration of any really feasible location-type experiment, therefore it is sufficient to restrict ourself with the first order approximation in $t/t_{lim} \sim H_0 t$ in the general formula for the conformal time deformation. In such an approximation, the time inhomogeneity becomes quadratic, and, consequently, the corresponding blue shift gets the linear dependence on time. As it is known, the experimentally observed PA-effect presents precisely such a shift.

In the framework of the conventional consideration assuming time homogeneity, such a dependence allows for the only possible interpretation as some additional uniform acceleration experienced by signal source and directed towards observer.

According to the proposed approach, this background acceleration is responsible for the PA which is manifestation of the non-inertial character of the observer RF caused by acceleration. One can say that the PA-effect has revealed the non-inertial character of the "expanding" RF in the way analogous to the well-known Sagnac effect which is a direct observed manifestation of the non-inertial character of the rotating RF.

By the proposed interpretation of PA, the measured frequency shift can be considered as a new independent high-precision measurement of the Hubble constant. Moreover, the concept as a whole can be directly tested in the experiment. The use of the appropriate sources and monochromatic detectors at required experimental conditions enables observation of the described anomalous blue shift in its "pure form", when the source and detector are mutually motionless, practically at all frequencies. As the effect is linearly growing with the frequency, it is expedient to use high-frequency radiation. In principle, this allows for a considerable decrease in the observation time. To study this effect, it is required to provide

(i) maximum immobility of the source and detector,

(ii) elimination of the gravitational field effect of massive bodies,

(iii) minimization of thermal fluctuations and mechanical deformations.

One should not rule out a possibility of providing all these conditions in zero gravity at the satellites orbiting along the circumterrestrial orbits. This proposal has been put forward in [15].

As it was shown, the value of the mimic "acceleration" $W_0 = cH_0$ defined from the experiments dealing with the electromagnetic signal propagation, is double of the value of relative acceleration $w$ playing the role of the conformal transformation parameter. At the same time, the quantity $w = \frac{1}{2} cH_0$ coincides with the acceleration of any test particle defined by the non-inertial observer in the absence of any real force sources. This quantity can be correlated with the "minimal acceleration" in the MOND concept. One can notice that in the distinct papers devoted to the MOND concept, different numerical values of the Milgrom parameter appear, but in any case they are defined as a fraction of $cH_0$.

All the conclusions of the present paper were obtained by the assumption that the Hubble constant is time-independent. Needless to say that such an assumption does not mean that the observable magnitude of the Hubble constant is its lower limit. Any rigorous theoretical arguments in favour of such an assumption are absent today. Only some reasoning which are noting more than a suggestive ones may be cited. They are connected with so-called Mass Dependent Maximal Acceleration (MDMA) concept (see [16] and references within) together with the Maximal Tension (or Maximal Force) hypothesis firstly proposed in [17] and independently in [16]. If we define, following Gibbons [17], the Maximal Force $F_0$ as $F_0 = \frac{c^4}{8\pi G} \rho_c$ (G being the Newton gravitational constant) the corresponding MDMA for the mass $M$ will be defined as $W(M) = \frac{c^4}{4G\rho_c}$. Putting the Universe "diameter" equal to $R_u = 2cH_0^{-1}$ and defining the Universe "mass" $M_u$ as a product of the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$ and the Universe "volume" $V_u = \frac{4\pi}{3} \left( \frac{2cH_0}{3} \right)^3$ we obtain $W(M_u) = \frac{1}{2}cH_0$ i.e. the value coinciding with the background acceleration magnitude.
It is evident that such an intriguing coincidence is to be confirmed in the framework of some consistent theoretical scheme. The problem connected with the maximum tension hypothesis as applied to the cosmological problems call for special considerations in further publications of the author.

It should be noted that the existence of the background acceleration resulting from the Methagalaxy expansion opens the nontrivial alternative possibility to treat the dark matter phenomenon (first of all - the dark energy one) as a peculiar kind of the inertial forces manifestation. In this case the future of the dark matter concept would become similar to the destiny of the substantional ether model abolished in due time by the Special Relativity kinematics.

The proposed approach, in itself, is a certain natural (and minimal) extension of Special Relativity concept, with evident adherence to the correspondence principle (the availability of the correct limiting transfer to the conventional physics when the dimensionless parameter $tH_0$ trends to zero).

In the present paper we restrict ourself to the use of spatially one-dimensional model. But the results obtained are applicable to the description of the effects associated with the longitudinal components of the relative motion only. The location-type experiments and the observation of signals from the cosmological distant objects (i.e. from the past) are experiments of that kind. The most simple line of taking into account the uniformity and isotropy of the cosmological expansion in the framework of the proposed approach is to choose the four-vector-parameter $a^\mu$ in the following form $a^\mu = (0, -w^2c^2\vec{r})$. The acceleration $\vec{W} = w\vec{r}$, of course, unlike the one-dimensional case, cannot be connected with either of known physical forces. Nevertheless such a form of parameter $a^\mu$ seems to be quite acceptable for the phenomenological description of the cosmological expansion. This question is now under investigation.

It goes without saying that the proposed approach must be driven in conformity with a General Relativity concept, in any case, in the part of the GR using the conventional model of inertial (Lorentz) observer. The matter is that the existence of nonzero background acceleration caused by Universe expansion means non-adequateness of such a model to physical reality. Really any observer carrying out local measurements at any point of Universe can detect background acceleration existence. As such he comes to an unavoidable alternative: either his RF is non-inertial, or the RF is inertial but there is some external constant gravitational field. In any case the observer is not a Lorentz one.

It seems to be possible that the needed modification of traditional gravitation theory can be accomplished by replacement of the geometry of tangential space from Poincare to conformal one.

X. ACKNOWLEDGMENTS

The author would like to acknowledge E.V. Doktorov, V.V. Kudryashov, A.E. Shalyt-Margolin, Ya.M. Shnir, I.A. Siutsou and Yu.P. Vyblly for their invaluable critical remarks and for the fruitful discussions. The paper is in part supported by the Belarus Foundation for Fundamental Research (BFFR), Project F06-129.

[1] J.D. Anderson et al., Phys. Rev. Lett. 81, 2858-2861 (1998); arXiv:gr-qc/9808081.
[2] J.D. Anderson et al., Phys. Rev. D. 65, 082004/1–50 (2002); arXiv:gr-qc/0104064.
[3] S.G. Turyshhev, M.M. Nieto, and J.D. Anderson, arXiv:gr-qc/0503021.
[4] S.G. Turyshhev, M.M. Nieto, and J.D. Anderson, arXiv:gr-qc/0510081.
[5] M. Mizony and M. Lachieze-Rey, arXiv:gr-qc/0412084.
[6] A.O. Barut and R. Raczka. Theory of Group Representations and Applications (PWN - Polish Scientific Publishers, Warszawa, 1977).
[7] L.M. Tomilchik, arXiv:gr-qc/0704.2745.
[8] L. Page, Phys. Rev. 49, 254, 946 (1936); L. Page and N.I. Adams, ibid. 49, 466 (1936).
[9] T. Fulton, F. Rohrlich, and L. Witten, Rev. Mod. Phys. 34, 442 (1962).
[10] T. Fulton, F. Rohrlich, and L. Witten, Nuovo Cim. 34, 652 (1962).
[11] R.L. Ingraham, Nuovo Cim. 12, 825 (1954).
[12] W.L. Burke, Spacetime, Geometry, Cosmology (University Science Books, Mill Valley, California, 1980).
[13] M. Milgrom, Astrophys. J. 270, 365 (1983).
[14] J. Bekenstein and M. Milgrom, Astrophys. J. 286, 7 (1984).
[15] L.M. Tomilchik, Optics Spectrosc. 103, 237 (2007).
[16] L.M. Tomilchik and V.V. Kudryashov, Proc. Int. School-Sem. "Actual Problems of Microworld Physics", V 1, JINR, Dubna (2004); arXiv:gr-qc/0507090.
[17] G.W. Gibbons, Found. Phys. 32, 1891 (2002); arXiv:hep-th/0210109.
[18] Relation of the type $R(z) = \text{const} \cdot z$, that connects cosmological distance $R$ with the red shift, seems to be first obtained from the conformal symmetry arguments by Ingraham [11] in 1954.