Dual Branes, Discrete Chain States and the Entropy of the Schwarzschild Black Hole

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Abstract

We review a recent proposal towards a microscopic understanding of the entropy of non-supersymmetric spacetimes – with emphasis on the Schwarzschild black hole. The approach is based at an intermediate step on the description of the non-supersymmetric spacetime in terms of dual Euclidean brane pairs of type-II string-theory or M-theory. By counting specific chain structures on the brane-complex, it is shown that one can reproduce the exact Schwarzschild black hole entropy plus its logarithmic correction.

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1 Introduction

We know since the early work of \[1\],\[2\],\[3\],\[4\],\[5\] that black holes come equipped with an entropy, known as the Bekenstein-Hawking (BH) entropy. It is determined by the area $A_H$ of the hole’s event horizon

$$S_{BH} = \frac{A_H K_B c^3}{4G_4 \hbar}. \quad (1)$$

Since the laws of black hole thermodynamics are in a neat one-to-one correspondence with the conventional laws of thermodynamics, one expects in analogy that the BH-entropy should likewise be explainable statistically mechanically by counting the entropy of an underlying set of microstates in a microcanonical ensemble.

So far string-theory satisfied these expectations for a variety of supersymmetric black holes (see e.g. \[6\] for reviews). Here, supersymmetry is an essential ingredient given that the counting of the microscopic degrees of freedom takes place in a different regime of moduli space than where the black hole actually resides \[7\]. One counts the entropy of specific string excitations at weak string coupling where the string dynamics is under control and by relying on supersymmetry exports the result to the strongly coupled regime where the black hole lives but where up to now the full string-theory dynamics is not sufficiently understood. In another approach which tries to avoid the supersymmetry constraint one maps the spacetime of interest by means of T- and S-Dualities and sometimes also boost transformations to a spacetime whose microscopic entropy counting is under control, e.g. the D=3 BTZ black hole or near-extremal branes \[8\]. Due to lack of our knowledge of the full formulation of string-/M-theory in the strongly coupled regime, both methods rely on an indirect method of counting the relevant degrees of freedom. It is therefore not obvious what the degrees of freedom actually are that make up the black hole in the strongly coupled regime. In particular it is not clear where these are located.

Our aim here is to introduce an alternative approach which does not rely on supersymmetry and therefore might be applied to both non-supersymmetric and supersymmetric spacetimes. Moreover we do not exploit weakly coupled string technology for the entropy counting in order not to be constrained from the outset to this region in moduli space. Instead the intention of this approach is to focus directly on the entropy counting problem in the strongly coupled regime where the string-coupling constant $g_s$ becomes $O(1)$. Of course since we still lack a complete formulation of string-/M-theory in this regime, the idea is to solve the state-counting issue first, thereby extracting a suitable set of microstates for the strongly coupled regime and then in a future second step to
build the dynamics on these states in order to hopefully arrive at a non-perturbative formulation of string-theory. The presentation given here focuses on a proposition and counting of states for a D=4 spacetime with spherical event horizon and is based on [9], [10], [11]. Our strategy will be to reformulate the D=4 BH-entropy purely in terms of geometrical Nambu-Goto actions corresponding to certain dual brane pairs and then to consider suitable microstates related to these branes whose entropy will be compared to the BH-entropy.

2 BH-Entropy and Dual Brane Doublets

Let us consider type II String-Theory on a D=10 Lorentzian spacetime $\mathcal{M}^{1,3} \times \mathcal{M}^{p-1} \times \mathcal{M}^{7-p}$ ($p = 1, \ldots, 5$) with D=4 external part $\mathcal{M}^{1,3}$

$$\text{de}^2 = g^{(1,3)}_{\mu \nu} \text{de}^\mu \text{de}^\nu + g^{(p-1)}_{ab}(x^c) \text{dx}^a \text{dx}^b + g_{kl}(x^m) \text{dx}^k \text{dx}^l. \quad (2)$$

We consider $\mathcal{M}^{p-1}$ and $\mathcal{M}^{7-p}$ as compact such that we have a geometric background describing a compactification from D=10 down to D=4. For such a background the D=4 Newton’s constant is related to the Regge slope $\alpha'$ and $g_s$ through

$$G_4 = \frac{G_{10}}{V_{p-1}V_{7-p}} = \frac{(2\pi)^6 \alpha'^4 g_s^2}{8V_{p-1}V_{7-p}}, \quad (3)$$

where $V_i = \text{vol}(\mathcal{M}^i) \equiv \int_{\mathcal{M}^i} d^i x \sqrt{g^{(i)}}$.

Next, let us choose a sphere $S^2$ within the spacelike part of $\mathcal{M}^{1,3}$ and wrap two mutually orthogonal Euclidean “electric-magnetic” dual branes, $Dp$ and $D(6-p)$, around $S^2 \times \mathcal{M}^{p-1}$ and $\mathcal{M}^{7-p}$, resp. such that together they cover the whole internal space plus the external sphere. For such a dual brane pair it follows from the Dirac-quantization condition that the product of their tensions obeys

$$\tau_{Dp}\tau_{D(6-p)} = \frac{1}{(2\pi)^6 \alpha'^4 g_s^2}. \quad (4)$$

Thus we can write the inverse of the D=4 Newton’s constant as

$$\frac{1}{G_4} = 8(\tau_{Dp}V_{p-1})(\tau_{D(6-p)}V_{7-p}). \quad (5)$$

Notice that the right-hand-side of this expression comes already close to the product of two Nambu-Goto actions for the resp. Euclidean branes except for the fact that we miss the area of the external two-sphere in the first bracket.
Since our aim is here to deal with the BH-entropy of D=4 spacetimes with spherical event horizons $S^2_H$, let us now point out how we incorporate them and what is the role played by the dual branes. In general the branes will act as supergravity sources and therefore will give rise to some particular D=10 geometry of the form

$$(\text{D=4 Spacetime}) \times (\text{Compact Internal Space}) .$$

If we include no other sources than the dual branes, the D=4 geometry will possess spherical symmetry because the D=4 gravitational source is distributed evenly over the $S^2$. So we have to ask whether there exists suitable dual branes (actually we will see soon that what we need is a doublet of dual brane pairs) whose D=10 geometry includes the D=4 spacetime of interest in its external part. For what follows, it will be important moreover to have an identification of the $S^2$ with the event horizon sphere $S^2_H$ of the D=4 spacetime

$$S^2 \equiv S^2_H .$$

Under this hypothesis for which evidence for the Schwarzschild black hole case was given in [10],[11], we will now proceed with the general argument and will comment below more on the specific brane pairs needed for the D=4 Schwarzschild black hole description.

Assuming the identification (7) it is easy to see that by using (8) we can reformulate the D=4 spacetime’s BH-entropy as

$$S_{BH} = \frac{A_H}{4G_4} = 2S_{Dp}S_{D(6-p)},$$

where

$$S_{Dp} = \tau_{Dp} \int_{S^2 \times M^{p-1}} d^{p+1}x \sqrt{\det g}, \quad S_{D(6-p)} = \tau_{D(6-p)} \int_{M^{7-p}} d^{7-p}x \sqrt{\det g}$$

are the respective Nambu-Goto actions of the involved dual branes. Actually, we can also get rid of the factor two by repeating the procedure once more and employing a doublet of Euclidean brane pairs. This doubling of the dual pairs will turn out to be important in the second half when we determine the microscopic entropy. Notice, that there is no a priori reason why the second dual pair has to coincide with the first. Therefore let us wrap a further dual Euclidean brane pair $Dp' - D(6 - p')$, with $Dp'$ and $D(6 - p')$ again mutually orthogonal, in the same manner as before around the $S^2$ plus the internal space. Then by following the previous steps, the D=4 BH-entropy becomes purely expressible in terms of the respective Nambu-Goto actions

$$S_{BH} = S_{Dp}S_{D(6-p)} + S_{Dp'}S_{D(6-p')} .$$
Figure 1: The Euclidean brane anti-brane pairs \((D3, D3), (\overline{D3}, \overline{D3})\) which describe a D=4 Schwarzschild black hole. They are oriented along the directions marked by dots. The coordinates \(t, r, \theta, \phi\) describe the external D=4 spacetime with \(\theta, \phi\) describing the sphere \(S^2 = S^2_H\). The whole configuration is located at some common fixed D=4 radial value \(r = r_H\) describing the event horizon of the black hole in Schwarzschild coordinates.

Furthermore, notice that we are free to exchange any of the appearing branes with its anti-brane and still arrive at the same expression (of course under the premise that the inclusion of the antibrane gives rise to the requested D=4 spacetime in the external part of the ensuing D=10 geometry).

It turns out that this formula works for all doublets of dual brane pairs of string- and M-Theory \([9]\). Thus whenever we find a doublet of dual brane pairs \((E_1, M_1), (E_2, M_2)\) which gives rise to the requested D=4 spacetime with spherical event horizon in the external part of the D=10 metric together with the identification \([7]\), it is possible to rewrite the BH-entropy of the D=4 spacetime as

\[
S_{BH} = \sum_{i=1,2} S_{E_i} S_{M_i},
\]

where the pairs \((E_i, M_i)\) range over all possible dual pairs of type-II string-theory and M-theory

\[
(E_i, M_i) \in \{(Dp_i, D(6-p_i)), (F1, NS5), (NS5, F1), (M2, M5), (M5, M2)\}.
\]

Moreover, any occurring object might also be replaced by its anti-partner as this exchange leaves the Nambu-Goto action invariant.

In particular, for the case of the D=4 Schwarzschild black hole one might take the self-dual \((D3, D3), (\overline{D3}, \overline{D3})\) type-IIB brane anti-brane doublets, distributed along the ten coordinates as depicted in fig.[1]. An equal amount of branes and anti-branes guarantees an uncharged solution of the D=10 supergravity field equations and moreover leads to a non-supersymmetric background. Furthermore, the choice of the non-dilatonic \(D3\)’s
ensures that one finds a D=10 vacuum solution without dilaton matter present. It can then be worked out [10] that indeed the described \((D3, D3), (\overline{D3}, \overline{D3})\) configuration gives rise in the D=4 part of the metric to an exterior Schwarzschild black hole geometry outside the \(S^2\) together with the identification of the spheres \([7]\). Notice that Euclidean branes positioned somewhere in spacetime usually decay because they are localized in time. However, Euclidean branes which are positioned at an event horizon are seen by an outside (for whom \(r\) is bigger than \(r_H\) – the position of the brane configuration) D=4 observer through an infinite redshift. This stretches classically any decay-time infinitely long such that from the observer’s perspective Euclidean branes wrapping an event horizon lead to a stationary spacetime \([10]\).

In this context it might be interesting to note as an aside that brane-antibrane systems in general cannot annihilate and decay into the closed-string vacuum classically \([12]\). Such a decay in which the tachyon would roll down its potential hill and thereby radiating off into the bulk the surplus of energy can occur only quantum mechanically. In weakly coupled string-theory this can be understood from the fact that during this transition open strings on the brane-antibrane system have to transform into closed strings which can enter the bulk. However this is a one-loop process from the open string point of view and thus a quantum mechanical process. On the other hand it is well-known that also the black hole’s Hawking radiation is an intrinsic quantum mechanical phenomenon which is forbidden at the classical level and likewise describes radiation sent into the bulk. It therefore seems natural to conjecture within the D=10 \((D3, D3), (\overline{D3}, \overline{D3})\) description of the D=4 Schwarzschild black hole that its Hawking radiation might be due to tachyon condensation \([10]\). It would certainly be interesting to investigate this further which however we will not do here.

### 3 Chain-States and their Entropy

To proceed further with the analysis of the D=4 BH-entropy, let us reflect briefly upon the tension of a Euclidean brane. For Lorentzian \(Dp\)-branes their tension is usually interpreted as

\[
\tau_{Dp} = \frac{\text{mass}}{\text{unit of p-volume}} \tag{13}
\]

which treats the time and the space directions differently. For a Euclidean brane however there is no time direction on the worldvolume and instead one has to treat all spacelike worldvolume directions on an equal footing. It therefore seems required in this case to
interpret the tension
\[ \tau_{Dp} := \frac{1}{v_{Dp}} = \frac{1}{l_{Dp}^{p+1}} \] (14)
in terms of a smallest fundamental volume unit \( v_{Dp} \) giving a smallest length \( l_{Dp} \). This is a natural generalization of the fact that \( \sqrt{\alpha'} \) constitutes a smallest length for strings. In the strongly coupled regime, however, it will be chains – to be introduced shortly – which cannot resolve distances smaller than \( l_{Dp} \).

Evidence for such a smallest volume unit on the brane’s worldvolume comes from the “worldvolume uncertainty relations for D-branes” [13]. In [13] it was shown that the worldvolume \( X^0, \ldots, X^p \) of a \( Dp \)-brane (which could be Lorentzian or Euclidean) is subject to the following uncertainty relation
\[ Dp : \quad \delta X^0 \delta X^1 \ldots \delta X^p \gtrsim g_s \alpha'^{\frac{p+1}{2}}, \] (15)
where the right-hand-side was determined up to numerical factors. By employing string-dualities it was furthermore shown that similar uncertainty relations hold true for the \( NS5 \)-brane, the fundamental string and the M-theory \( M5 \)- and \( M2 \)-branes
\[ NS5 : \quad \delta X^0 \delta X^1 \ldots \delta X^5 \gtrsim g_s^2 \alpha' \]
\[ M5 : \quad \delta X^0 \delta X^1 \ldots \delta X^5 \gtrsim l_{Pl}^6, \quad M2 : \quad \delta X^0 \delta X^1 \delta X^2 \gtrsim l_{Pl}^3 \] (16)
with \( l_{Pl} \) the D=11 Planck-length. Indeed the relation for the fundamental string had been proposed earlier in [14]. By employing the tensions of these objects
\[ Dp : \quad \tau_{Dp} = \frac{1}{(2\pi)^pg_s \alpha'(p+1)/2}, \quad NS5 : \quad \tau_{NS5} = \frac{1}{(2\pi)^5g_s^2 \alpha'^3}, \quad F1 : \quad \tau_{F1} = \frac{1}{2\pi \alpha'}, \] (18)
\[ M5 : \quad \tau_{M5} = \frac{1}{(2\pi)^5l_{Pl}^6}, \quad M2 : \quad \tau_{M2} = \frac{1}{(2\pi)^2l_{Pl}^3} \] (19)
one sees that all the different worldvolume uncertainty relations can be combined into the statement that the smallest worldvolume allowed by the brane uncertainty principle is given by the inverse of the object’s tension
\[ \delta X^0 \ldots \delta X^p \gtrsim \frac{1}{\tau}. \] (20)
This motivates us to interpret similarly the tension \( \tau_E \) or \( \tau_M \) of any of the dual objects occurring in (12) analogously to (14) in terms of some smallest volume \( v_{E,M} = \tau_{E,M}^{-1} \).

The introduction of a smallest volume unit on the brane’s worldvolume endows the brane with a discrete structure. Namely, we can now perceive the brane as a lattice made
Figure 2: Constructive view of an \((N - 1)\)-chain where we arrange all cells of the lattice in a column and use \(N\) copies of them. We allow each link to connect any cell of a column with any cell of the succeeding column. Horizontal links correspond to loops.

out of a certain number \(N_{Dp}\) of such smallest volume units which we will call cells from on. With the interpretation (14) for the tension, it is precisely the Nambu-Goto action which measures the number of these cells contained in the brane

\[
N_{Dp} := \tau_{Dp} \int d^{p+1}x \sqrt{\det g} = S_{Dp}.
\]  

This implies that the expression (11) which we had found for the \(D=4\) BH-entropy can be rewritten further and now becomes identical to an integer \(N\)

\[
S_{BH} = \sum_{i=1,2} N_{E_i} N_{M_i} =: N
\]  

with \(N\) the total number of cells contained in the joint worldvolume of the doublet of dual brane pairs.

So far we have reformulated the \(D=4\) BH-entropy in terms of string-/M-theory notions under the inclusion of the discrete brane structure coming from a reinterpretation of the brane’s tension. We will now investigate in which way this helps us to propose a set of black hole microstates for the strongly-coupled regime.

To this aim, let us conceive on the combined \((E_1, M_1), (E_2, M_2)\) worldvolume lattice an \((N - 1)\)-chain, which is a chain composed out of \(N - 1\) successive links where we allow all links to start and end democratically on any of the \(N\) cells of the lattice (see fig.2). In particular a link might start and end on the same cell thus creating a loop. Altogether the number of possible chain configurations is \(N^N\). Alternatively, one might consider closed \(N\)-chains (see fig.3) which exhibit the same number \(N^N\) of different chain configurations.
Figure 3: Alternatively one might use closed $N$-chains. They possess the same number of configurations.

Part of the motivation to consider long chains (long enough to connect all cells of the whole lattice instead of just a few) comes from the weakly coupled string at finite temperature. So far, we had argued for a smallest unresolvable volume on the Euclidean branes – or in other words an uncertainty in space-resolution – which leads via the Heisenberg uncertainty principle to an uncertainty in energy (where we assume $l_{Dp}$ small enough so that $\Delta P$ becomes relativistic)

$$\Delta E \simeq \Delta P \simeq \frac{1}{l_{Dp}} = (\tau_{Dp})^{\frac{1}{p+1}} = \frac{1}{\sqrt{\alpha'(g_s(2\pi)^p)^{\frac{1}{p+1}}}}.$$ \hspace{1cm} (23)

Therefore at strong coupling where $g_s \simeq 1$, the temperature associated with the uncertainty $\Delta E$ is of order the Hagedorn-temperature. Now at this temperature we know that in the weakly coupled regime it is entropically favourable to allocate the energy of the system to just one single long string instead to many short ones \cite{13}. This motivates us to consider as well long chains instead of short ones for the strongly coupled regime. Moreover it is known that open and closed strings behave at high excitation levels much like a random walk \cite{14} which furthermore motivated the choice of discrete chain states.

The counting of different chain configurations has up to now been classical in the sense that all cells were regarded as distinguishable. However, in a proper quantum theory the cells should likely be regarded as indistinguishable bosonic degrees of freedom. How to account for this quantum feature is well-known from statistical mechanics – for $N$ indistinguishable cells we have to divide by the Gibbs-correction factor $N!$. This gives
the quantum-mechanically corrected number of different chain states

\[ \Omega(N) = \frac{N^N}{N!}. \]  

To evaluate the entropy of the chain-states in the thermodynamic large \( N \) limit (as adequate for macroscopic black holes) we use Stirling’s approximation, \( \ln(N!) = N \ln N - N + O(\ln N) \), and the identity for the D=4 BH-entropy (22) to obtain

\[ S_c = \ln \Omega(N) = N = S_{BH} \]  

up to corrections of \( O(\ln N) \). Thus we learn that the proposed discrete \((N-1)\)-chains or alternatively the closed \( N \)-chains correctly reproduce the corresponding D=4 BH-entropy and therefore might be considered as viable black hole microstate candidates.

## 4 Corrections to BH-Entropy

Having found agreement between the chain and the D=4 BH-entropy at leading large \( N \) order, one might be curious what happens at subleading order. Corrections to the BH-entropy for D=4 black holes had been determined in supersymmetric cases from String-Theory while results in non-supersymmetric cases came from the Quantum Geometry program [17] or the conformal field theory approach of Carlip [18].

The general result is that there exists a subleading logarithmic correction to the semi-classical BH-entropy of the form

\[ -k \ln S_{BH} \]  

with a positive constant \( k > 0 \) (a negative subleading correction is in accordance with the holographic principle). Though initially a value of \( k = 3/2 \) had been favoured, it seems that recently this has been corrected to \( k = 1/2 \) [19]. The reason being that one obtains a correction of the form

\[ -\frac{3}{2} \ln S_{BH} + \ln c. \]  

However, the central charge \( c \) has been shown in [19] not to be constant but instead given by \( c \propto S_{BH} \) which leads to an entropy correction with \( k = 1/2 \). Moreover, \( k = 1/2 \) has also been found independently by other methods exploiting the AdS/CFT duality [20].

Let us now come to the entropy corrections within the chain state approach. Corrections to the chain entropy come simply from a more accurate approximation of \( N! \) by the
Stirling-series, e.g. including higher-order terms one could take
\[ N! = \sqrt{2\pi NN^N} e^{-N} \left(1 + \frac{1}{12N} + O\left(\frac{1}{N^2}\right)\right). \]  
(28)

Using this corrected Stirling approximation for the evaluation of the chain entropy and once more considering the identity (22), we get a corrected chain-entropy formula
\[ S_c = \ln \Omega(N) = S_{BH} - \frac{1}{2} \ln S_{BH} - \ln \sqrt{2\pi} - \frac{1}{12S_{BH}} + O\left(\frac{1}{S_{BH}^2}\right). \]  
(29)

This shows that the proposed chain configurations can also easily account for the sub-leading logarithmic entropy correction and moreover give the precise numerical coefficient \( k = 1/2 \). We can therefore conclude that the proposed chain states have passed a first non-trivial test and consequently constitute an interesting possible set of black hole microstates whose dynamics should be worth investigating in more detail.

We would like to stress that the mechanism of counting the chain entropy is not restricted to some specific value of \( g_s \). In particular \( N = N(vol(E_i), vol(M_i), \alpha', g_s) \) is a function of \( g_s \) and the entropy counting considerations go through irrespective of how \( g_s \) is chosen. All what changes when we vary \( g_s \) (and keeping \( \alpha' \) plus the volume of the dual branes fixed) is the size of the cells and thus their number. The combined cell volume for one of the dual pairs is given by
\[ \text{cell volume} = (\tau E \tau M)^{-1} \propto \alpha'^4 g_s^2 \]  
(30)

for the string-theory cases and proportional to \( l_P^9 \) for the M-theory case where \( E, M = M2, M5 \). Therefore in the weakly coupled string-theory limit, where \( g_s \rightarrow 0 \), the cell volume becomes infinitesimally small while the number of cells approaches infinity \( N \rightarrow \infty \) when keeping the brane volume fixed. The extreme strongly coupled limit \( g_s \rightarrow \infty \) on the contrary would exhibit huge cells covering big portions of the brane worldvolume.

5 Implications for the Schwarzschild Black Hole

Let us finally come to some consequences for the Schwarzschild black hole. We had obtained that \( S_{BH} \) should equal an integer \( N \). By using the Schwarzschild radius to transform the horizon area into a mass squared, one sees that for the D=4 Schwarzschild black hole this leads directly to a discrete mass-spectrum
\[ M_{BH}(N) = \mathcal{C}\sqrt{N}, \quad \mathcal{C} = \frac{1}{\sqrt{4\pi G_4}} \]  
(31)
of Bekenstein-type. This relation has also been found in many different ways (see e.g. the references in [11]). However, when written in terms of the Schwarzschild radius $r_S$ itself

$$r_S = \frac{l_{pl}}{\sqrt{\pi}} \sqrt{N},$$

(32)
it suggests an effective D=4 picture of the black hole as a random Brownian Walk with step-width $l_{pl}/\sqrt{\pi}$. Such an effective picture in terms of random walks is hard to understand in many alternative approaches which derive (31) but appears to be compatible with the higher-dimensional chain-state description which is likewise a random walk however with variable step-width. This and the exact matching of the chain entropy with the black hole’s BH-entropy suggests to identify the D=4 Schwarzschild black hole at a microscopic level with one of the chain-states and consequently to identify the black hole’s mass with the chain’s energy [11]

$$E_c(N) = M_{BH}(N).$$

(33)
The importance of this identification lies in the fact that it allows us to derive thermodynamical quantities within the microcanonical chain ensemble with given energy $E_c$. Without this identification one would have to know the complete dynamical chain-theory in order to derive such quantities.

One obtains a chain-temperature $T_c$

$$\frac{1}{T_c} = \frac{\partial S_c(N)}{\partial E_c(N)} = \frac{1}{T_H} - \frac{1}{M_{BH}} + \mathcal{O}\left(\frac{M_{Pl}^2}{M_{BH}^3}\right)$$

(34)

where

$$\frac{1}{T_H} = 8\pi G_4 M_{BH} \simeq \frac{M_{BH}}{M_{Pl}^2}$$

(35)
is the inverse Hawking temperature of the black hole. Hence the leading-order chain temperature equals the Hawking temperature but in addition there is a non-trivial correction suppressed by a factor $M_{Pl}^2/M_{BH}^2$. As expected this correction becomes important for small black holes whose Schwarzschild radius approaches the Planck-length. Similarly one can derive the chain’s specific heat as

$$C_c = \frac{\partial E_c(N)}{\partial T_c(N)} = -8\pi G_4 M_{BH}^2 + 3 + \mathcal{O}\left(\frac{M_{Pl}^2}{M_{BH}^2}\right)$$

(36)

where

$$C_{BH} = -8\pi G_4 M_{BH}^2$$

(37)
is the specific heat derived from conventional black hole thermodynamics. Thus, once more the leading order result reproduces the black hole thermodynamics result while in addition there is a correction suppressed by a factor $M_{Pl}^2/M_{BH}^2$. 

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References

[1] D. Christodoulou, Phys.Rev.Lett. 25 (1970) 1596;
[2] S.W. Hawking, Phys.Rev.Lett. 26 (1971) 1344;
[3] J.M. Bardeen, B. Carter and S.W. Hawking, Comm.Math.Phys. 31 (1973) 161;
[4] J. Bekenstein, Phys.Rev. D 7 (1973) 2333; Phys.Rev. D 9 (1974) 3292;
[5] S.W. Hawking, Nature 248 (1974) 30; Comm.Math.Phys. 43 (1975) 199;
[6] T. Mohaupt, Class.Quant.Grav. 17 (2000) 3429, hep-th/0004098;
A.W. Peet, hep-th/0008241;
S.R. Das and S.D. Mathur, Ann.Rev.Nucl.Part.Sci 50 (2000) 153, gr-qc/0105063;
[7] A. Strominger and C. Vafa, Phys.Lett. B 379 (1996) 99, hep-th/9601029;
[8] K. Sfetsos and K. Skenderis, Nucl.Phys. B 517 (1998) 179, hep-th/9711138;
[9] A. Krause, hep-th/0201260;
[10] A. Krause, hep-th/0204206;
[11] A. Krause, hep-th/0205310;
[12] A. Sen, Rolling Tachyon, hep-th/0203211;
[13] C.S. Chu, P.M. Ho and Y.C. Kao, Phys.Rev. D 60 (1999) 126003, hep-th/9904133;
[14] T. Yoneya, hep-th/9707002;
[15] J.J. Atick and E. Witten, Nucl.Phys. B 310 (1988) 291;
G.G. Athanasiu and J.J. Atick, Nucl.Phys.B(Proc.Suppl.) 11 (1989) 304;
[16] D. Mitchell and N. Turok, *Phys.Rev.Lett.* **58** (1987) 1577; *Nucl.Phys.* **B 294** (1987) 1138;

[17] R.K. Kaul and P. Majumdar, *Phys.Rev.Lett.* **84** (2000) 5255, gr-qc/0002040;

[18] S. Carlip, *Class.Quant.Grav.* **17** (2000) 4175, gr-qc/0005017;

[19] J.L. Jing and M.L. Yan, *Phys.Rev.* **D 63** (2001) 024003, gr-qc/0005103;

[20] S. Mukherji and S.S. Pal, *JHEP* **0205** (2002) 026, hep-th/0205164.