Cosmic Inflation from Emergent Spacetime Picture

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We argue that the emergent spacetime picture admits a background-independent formulation of cosmic inflation. The inflation in this picture corresponds to the dynamical emergence of spacetime while the conventional inflation is simply an (exponential) expansion of a preexisting spacetime owing to the vacuum energy carried by an inflaton field. We show that the cosmic inflation arises as a time-dependent solution of the matrix quantum mechanics describing the dynamical process of Planck energy condensate in vacuum without introducing any inflaton field as well as an ad hoc inflation potential. Thus the emergent spacetime picture realizes a background-independent description of the inflationary universe which has a sufficiently elegant and explanatory power to defend the integrity of physics against the multiverse hypothesis.

Keywords: Emergent spacetime; Inflationary universe; Quantum cosmology

1. Why Is Emergent Spacetime Necessary for Cosmic Inflation?

Like black holes, the big bang in the very early universe involves extreme conditions that neither relativity nor quantum theory can explain on its own. If we trace back the history of our universe, we will meet an initial singularity in which matter reached almost infinite density and then, according to the theory of general relativity, the space was contracted to a point and the time flow nearly stopped. Thus the big bang suffers the initial singularity in which space and time cease to exist. This implies that the big bang must be a cosmological event generating space and time as well as matters. The initial singularity cannot be avoided in the inflationary cosmology either because there has to be a definite beginning to an inflationary universe. This means that the inflation is incomplete to describe the very beginning of our universe and some new physics is needed to probe the past boundary of the inflating regions. One possibility is that there must have been some sort of quantum creation event as a beginning of the universe.

The inflation scenario so far has been formulated in the context of effective field theory coupled to general relativity. Hence, in this scenario, the existence of space and time is a priori assumed from the beginning and the scenario only describes what happens in a given spacetime. In other words, the inflationary scenario does not describe any generation (or creation) of spacetime but simply characterizes an expansion of a preexisting spacetime. It never addresses the (dynamical) origin of spacetime. Moreover, in most inflationary models, once inflation happens, it never stops and produces not just one universe, but an infinite number of universes. Therefore the conventional inflation inevitably leads to the multiverse which is not falsifiable since the multiverse cannot be tested experimentally.
In consequence, we need a consistent quantum theory to describe how spacetime was generated through the big bang in order to correctly understand the origin of spacetime and our universe. In particular, we need a background-independent theory which does not assume the prior existence of spacetime background but instead provides a mechanism of spacetime generation such that any spacetime structure including flat spacetime arises as a solution of the theory itself. In other words, we need a background-independent formulation of cosmic inflation in which the inflation corresponds to the dynamical emergence of spacetime. Recently such a background independent formulation was proposed so that the cosmic inflation arises as a time-dependent solution of the matrix quantum mechanics describing the dynamical process of Planck energy condensate in vacuum without introducing any inflaton field as well as an ad hoc inflation potential. The underlying mathematical principle is the well-known duality between geometry and algebra. In this scheme, the noncommutative (NC) algebra plays a more fundamental role from which the spacetime geometry is derived and the emergent gravity from NC $U(1)$ gauge theory is basically the large $N$ duality, as depicted in Fig. 1:

![Flowchart for large N duality](image)

Since the concept of the multiverse raises deep conceptual issues even to require to change our view of science itself, it should be important to ponder on the real status of the multiverse whether it is simply a mirage developed from an incomplete physics like the ether in the late 19th century or it is of vital importance even in more complete theories. In next section, we will illuminated how the emergent spacetime picture brings about radical changes of physics, especially, regarding to physical cosmology.
2. Inflationary Universe from Emergent Spacetime

Let us consider the matrix quantum mechanics (MQM) to address the background-independent formulation of cosmic inflation. The action in this case is given by

\[ S = \frac{1}{g^2} \int dt \, \text{Tr} \left( \frac{1}{2} (D_0 \phi_a)^2 + \frac{1}{4} [\phi_a, \phi_b]^2 \right) \]

\[ = \frac{1}{4g^2} \int dt \, \eta^{AC} \eta^{BD} \text{Tr} [\phi_A, \phi_B] [\phi_C, \phi_D], \tag{1} \]

where \( \phi_0 \equiv iD_0 = i \frac{\partial}{\partial t} + A_0(t), \phi_A(t) = (\phi_0, \phi_a)(t) \) and \( \eta^{AB} = \text{diag}(-1, 1, \cdots, 1) \), \( A, B = 0, 1, \cdots, 2n \). With the notation of the symbol \( \eta^{AB} \), it is easy to see that the matrix action \( (1) \) has a global automorphism given by

\[ \phi_A \rightarrow \phi'_A = \Lambda_A^B \phi_B + c_A \tag{2} \]

if \( \Lambda_A^B \) is a rotation in \( SO(2n, 1) \) and \( c_A \) are constants proportional to the identity matrix. It turns out \( (2) \) that the global symmetry \( (2) \) is responsible for the Poincaré symmetry of flat spacetime emergent from a vacuum in the Coulomb branch of MQM and so will be called the Poincaré automorphism.

We remark that the time \( t \) in the action \( (1) \) is not a dynamical variable but a parameter. The concept of time has been introduced in Ref. 3 by considering a one-parameter family of deformations of zero-dimensional matrices which is parametrized by the coordinate \( t \). Then the one-parameter family of deformations can be regarded as the time-evolution of a dynamical system. A close analogy with quantum mechanics implies that the concept of emergent time is connected with the time-evolution of the dynamical system. In this context, the one-dimensional matrix model \( (1) \) can be interpreted as a Hamiltonian system of a zero-dimensional (e.g., IKKT) matrix model \( (2) \).

The equations of motion for the matrix action \( (1) \) are given by

\[ D_0^2 \phi_a + [\phi_b, [\phi_a, \phi_b]] = 0, \tag{3} \]

which must be supplemented with the Gauss constraint

\[ [\phi_a, D_0 \phi_a] = 0. \tag{4} \]

Now we want to apply the large \( N \) duality to MQM that is the \( d = 1 \) case in Fig. \( 1 \). It is important to notice that there are two different kinds of vacua in the Coulomb branch if we consider the \( N \rightarrow \infty \) limit. In addition to the conventional commutative vacuum obeying the property \( [\phi_a, \phi_b]|_{\text{vac}} = 0 \), there exists a novel coherent vacuum, the so-called NC Coulomb branch \( (2) \), defined by

\[ [\phi_a, \phi_b]|_{\text{vac}} = -i B_{ab} \quad \Rightarrow \quad \langle \phi_a \rangle_{\text{vac}} = p_a \equiv B_{ab} y^b \tag{5} \]

where the vacuum moduli \( y^a \) satisfy the Moyal-Heisenberg algebra given by

\[ [y^a, y^b] = i \theta^{ab}, \quad a, b = 1, \cdots, 2n \tag{6} \]

and \( (\theta)^{ab} = (B^{-1})^{ab} = \alpha'(1_n \otimes i \sigma^2) \) is a \( 2n \times 2n \) constant symplectic matrix. We emphasize that the NC Coulomb branch \( (5) \) together with a constant vacuum energy
density given by \( \mathcal{E} \equiv \langle A_0(t) \rangle_{\text{vac}} \) is a consistent vacuum solution of MQM since it satisfies the equations of motion \( \mathcal{E} \) as well as the Gauss constraint \( \mathcal{E} \). Since \( \mathcal{E} \) is proportional to the identity matrix, it plays no role in the temporal covariant derivative \( D_0 \) and so it can be dropped without loss of generality. Also note that the coherent vacuum \( \mathcal{E} \) saves the NC nature of matrices while the conventional vacuum dismisses the property.

If we consider fluctuations around the NC vacuum \( \mathcal{E} \) given by

\[
D_0 = \frac{\partial}{\partial t} - i \tilde{A}_0(t, y), \quad \phi_a = p_a + \tilde{A}_a(t, y),
\]

(7)

the action for the fluctuations is exactly mapped to a \((2n + 1)\)-dimensional NC \( U(1) \) gauge theory as indicated in Fig. 1. If the conventional commutative vacuum were chosen, we would have failed to realize the higher-dimensional NC \( U(1) \) gauge theory from MQM. Indeed it turns out \( \mathcal{E} \) that the NC Coulomb branch is crucial to realize the emergent gravity from matrix models or large \( N \) gauge theories as summarized in Fig. 1. Furthermore the NC Coulomb branch \( \mathcal{E} \) is very different from the conventional commutative vacuum because the former carries a nonzero vacuum energy density in contrast to the latter for which the vacuum energy density identically vanishes. Using the higher-dimensional NC \( U(1) \) gauge theory, we can calculate the energy density for the vacuum condensate in the NC Coulomb branch defined by

\[
\langle \phi_A \rangle_{\text{vac}} = p_A = \left( i \frac{\partial}{\partial t}, p_a \right).
\]

(8)

The result is given by

\[
\rho_{\text{vac}} = \frac{1}{4G_{YM}^2 |B_{ab}|^2}
\]

(9)

where \( G_{YM}^2 = (2\pi)^n |\text{Pf} \theta|^2 \) is the \((2n + 1)\)-dimensional gauge coupling constant.

It was shown in Ref. 3 that quantum gravity can be derived from the electromagnetism on NC spacetime by realizing the large \( N \) duality in Fig. 1 via the duality chain given by

\[
\mathcal{A}_N^d \implies \mathcal{A}_0^d \implies \mathcal{D}^d.
\]

(10)

For the \( d = 1 \) case in Fig. 1 the dynamical variables in MQM take values in \( \mathcal{A}_N^1 \) while those in \( D = (2n + 1)\)-dimensional NC \( U(1) \) gauge theory take values in \( \mathcal{A}_0^1 \). These two NC algebras \( \mathcal{A}_N^1 \) and \( \mathcal{A}_0^1 \) are related to each other by considering the NC Coulomb branch \( \mathcal{E} \). The module of derivations \( \mathcal{D}^1 \) is a direct sum of the submodules of horizontal and inner derivations:

\[
\mathcal{D}^1 = \text{Hor}(\mathcal{A}_N^1) \oplus \mathcal{D}(\mathcal{A}_N^1) \cong \text{Hor}(\mathcal{A}_0^1) \oplus \mathcal{D}(\mathcal{A}_0^1),
\]

(11)

where horizontal derivation is locally generated by a vector field

\[
k(t, y) \frac{\partial}{\partial t} \in \text{Hor}(\mathcal{A}_0^1).
\]

(12)
In particular we are interested in the derivation algebra generated by the dynamical variables in Eq. (7). It is defined by

\[ \hat{V}_A = \{ \text{ad}_\phi, \cdot \} | \phi_A(t, y) = (iD_0, \phi_a)(t, y) \} \in D^1. \]  

(13)

The large \( N \) duality in Fig. 1 says that the gravitational variables such as vielbeins in general relativity arise from the commutative limit of NC \( U(1) \) gauge fields via the map (13). Then one may ask where flat Minkowski spacetime comes from. It turns out that the (2\( n + 1 \))-dimensional flat Minkowski spacetime is emergent from the vacuum condensate since the corresponding vielbeins and the metric are given by

\[ E^{(0)}_A = \hat{V}^{(0)}_A = \left( \frac{\partial}{\partial \sigma}, \frac{\partial}{\partial \sigma^a} \right) \]  

and

\[ ds^2 = -dt^2 + dy \cdot dy. \]  

A striking fact is that the flat Minkowski spacetime is emergent from the Moyal-Heisenberg algebra whose energy density is given by Eq. (9). Thus the flat spacetime was originated from the uniform vacuum energy known as the cosmological constant in general relativity. This is a tangible difference from Einstein gravity, in which the energy-momentum tensor identically vanishes, i.e. \( T_{AB} = 0 \), for the flat spacetime. However, since we have started with the matrix model in which any spacetime structure has not been assumed in advance, the spacetime was not existent at the beginning but simply emergent from the vacuum condensate. Therefore the Planck energy condensation into vacuum must be regarded as a dynamical process.

It is not difficult to show that the dynamical process for the vacuum condensate is described by the time-dependent vacuum configuration given by

\[ \langle \phi_a(t) \rangle_{\text{vac}} = p_a(t) = e^{\frac{H}{2}t} p_a, \quad \langle \hat{A}_0(t, y) \rangle_{\text{vac}} = \hat{a}_0(t, y), \]  

(14)

where the temporal gauge field is given by an open Wilson line

\[ \hat{a}_0(t, y) = \frac{\kappa}{2} \int_0^t d\sigma \frac{dy^a(\sigma)}{d\sigma} p_a(\sigma) \]  

(15)

along a path parameterized by the curve \( y^a(\sigma) = y_0^a + \zeta^a(\sigma) \) where \( \zeta^a(\sigma) = \theta^{ab} k_b \sigma \) with \( 0 \leq \sigma \leq 1 \) and \( y^a(\sigma = 0) \equiv y_0^a \) and \( y^a(\sigma = 1) \equiv y^a \). The constant \( H \equiv (n-1)\kappa \) is identified with the inflationary Hubble constant. The (2\( n + 1 \))-dimensional basis for the time-dependent vacuum (14) can easily be calculated using the map (13):

\[ V_0 = \frac{\partial}{\partial t} - \frac{\kappa}{2} y^a \frac{\partial}{\partial y^a}, \quad V_a = e^{\frac{H}{2}t} \frac{\partial}{\partial y^a}. \]  

(16)

And the dual orthonormal one-forms are given by

\[ e^0 = dt, \quad e^a = e^{Ht} dy^a \]  

(17)

where \( y^a_t = e^{\frac{H}{2}t} y^a \). Here we used the relation \( V_A = (E_0, \lambda E_a) \), and \( \lambda = e^{\kappa t} \) for the vacuum configuration (14). In the end, the time-dependent metric for the inflating background is given by

\[ ds^2 = -dt^2 + e^{2Ht} dy_1 \cdot dy_1. \]  

(18)

Note that the temporal gauge field (15) is crucial to satisfy Eqs. (6) and (4) and to get a geodesically complete inflationary spacetime. Moreover the metric (18) is
conformally flat, i.e., the corresponding Weyl tensors identically vanish and so describes a homogeneous and isotropic inflationary universe known as the Friedmann-Robertson-Walker metric in physical cosmology.

It is also easy to get a general Lorentzian metric describing \((2n+1)\)-dimensional inflating spacetime by considering arbitrary fluctuations around the inflationary background \((14)\). They form a time-dependent NC algebra given by

\[
\mathcal{A}^1_0 \equiv \left\{ \hat{\phi}_0(t, y) = i \frac{\partial}{\partial t} + \hat{A}_0(t, y), \quad \hat{\phi}_a(t, y) = e^{\frac{2G}{\kappa}} (p_a + \hat{A}_a(t, y)) \right\}.
\]

We denote the corresponding time-dependent matrix algebra by \(\mathcal{A}^1_N\) which consists of a time-dependent solution of the action \((1)\). Then the general Lorentzian metric describing a \((2n+1)\)-dimensional inflationary universe can be obtained by the following duality chain:

\[
\mathcal{A}^1_N \Rightarrow \mathcal{A}^1_0 \Rightarrow \mathcal{}^1{D}^1. \tag{20}
\]

The module \(\mathcal{}^1{D}^1\) of derivations of the NC algebra \(\mathcal{A}^1_0\) is given by

\[
\mathcal{}^1{D}^1 = \left\{ \hat{V}_A(t) = (\hat{V}_0(t), \hat{V}_a(t)) = \frac{\partial}{\partial t} + \text{ad}_{\hat{A}_0}, \quad \hat{V}_a(t) = e^{\frac{2G}{\kappa}} \text{ad}_{\hat{\phi}_a} \right\}, \tag{21}
\]

where the adjoint operations are defined by Eq. \((13)\). In the classical limit of the module \((21)\), we get a general inflationary universe described by

\[
ds^2 = -dt^2 + \lambda^2 e^{2Ht} \epsilon_i^\alpha \epsilon_j^\beta (dy_i^\alpha - A^\alpha_i)(dy_j^\beta - A^\beta_j) \tag{22}
\]

where the conformal factor \(\lambda = 1 + \delta \lambda\) is determined by the volume-preserving condition.

In conclusion, the cosmic inflation arises as a time-dependent solution of a background-independent theory describing the dynamical process of Planck energy condensate in vacuum without introducing any inflaton field as well as an \textit{ad hoc} inflation potential. The large \(N\) duality in Fig. \(\|\) also implies that cosmic inflation triggered by the Planck energy condensate into vacuum must be a single event. Thus the emergent spacetime is a completely new paradigm so that the multiverse debate in physics circles has to seriously take it into account.

References

1. A. Borde, A. H. Guth and A. Vilenkin, Phys. Rev. Lett. \textbf{90} (2003) 151301.
2. Universe or multiverse?, edited by B. Carr (Cambridge Univ. Press, Cambridge, 2007).
3. H. S. Yang, \textit{Emergent spacetime and cosmic inflation I & II}, [arXiv:1503.00712].
4. W. Taylor, Rev. Mod. Phys. \textbf{73} (2001) 419.
5. H. S. Yang, Int. J. Mod. Phys. A \textbf{30} (2015) 1550016.
6. S. Azam, Commun. Alg. \textbf{36} (2008) 905.
7. H. S. Yang, J. High Energy Phys. \textbf{05} (2009) 012.
8. J. Lee and H. S. Yang, J. Korean Phys. Soc. \textbf{65} (2014) 1754.
9. B. Carr and G. Ellis, Astronomy \\& Geophysics \textbf{49} (2008) 2.29.