The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

J. Cleymans and D. Worku

UCT-CERN Research Centre and Department of Physics, University of Cape Town, Rondebosch 7701, South Africa

Abstract. The Tsallis distribution has been used recently to fit the transverse momentum distributions of identified particles by the STAR [1] and PHENIX [2] collaborations at the Relativistic Heavy Ion Collider and by the ALICE [3] and CMS [4] collaborations at the Large Hadron Collider. Theoretical issues are clarified concerning the thermodynamic consistency of the Tsallis distribution in the particular case of relativistic high energy quantum distributions. An improved form is proposed for describing the transverse momentum distribution and fits are presented together with estimates of the parameter $q$ and the temperature $T$.

PACS numbers: 25.75.Dw, 13.85.Ni

1. Introduction

The Tsallis distribution has gained prominence recently in high energy physics with very high quality fits of the transverse momentum distributions made by the STAR [1] and PHENIX [2] collaborations at the Relativistic Heavy Ion Collider and by the ALICE [3] and CMS [4] collaborations at the Large Hadron Collider.

In the literature there exists more than one version of the Tsallis distribution [5, 6] and we investigate in this paper a version which we consider suited for describing results in high energy particle physics. Our main guiding criterion will be thermodynamic consistency which has not always been implemented correctly (see e.g. [7, 8, 9]). The explicit form which we use is:

$$\frac{dN}{dp_T\, dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2} \left[ 1 + \left( q - 1 \right) \frac{m_T \cosh y - \mu}{T} \right]^{\frac{q}{(1-q)}}$$

(1)

where $p_T$ and $m_T$ are the transverse momentum and mass respectively, $y$ is the rapidity, $T$ and $\mu$ are the temperature and the chemical potential, $V$ is the volume, $g$ is the degeneracy factor. In the limit where the parameter $q$ goes to 1 this reproduces the standard Boltzmann distribution:

$$\lim_{q \to 1} \frac{dN}{dp_T\, dy} =$$
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

$$gV \frac{p_T m_T \cosh y}{(2\pi)^2} \exp \left( -\frac{m_T \cosh y - \mu}{T} \right).$$

(2)

In order to distinguish Eq. (1) from the form used by the STAR, PHENIX, ALICE and CMS collaborations [1, 2, 3, 4]. The motivation for preferring this form is presented in detail in the rest of this paper. The parametrization given in Eq. (1) is close (but different) from the one used by STAR, PHENIX, ALICE and CMS:

$$\frac{d^2N}{dp_T dy} = p_T \frac{dN}{dy} \frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left( 1 + \frac{m_T - m_0}{nC} \right)^{-n}$$

(3)

where $n$, $C$ and $m_0$ are fit parameters. The analytic expression used in Refs. [1, 2, 3, 4] corresponds to identifying

$$n \rightarrow \frac{q}{q-1}$$

(4)

and

$$nC \rightarrow \frac{T}{q-1}$$

(5)

But differences do not allow for the above identification to be made complete due to an additional factor of the transverse mass on the right-hand side and a shift in the transverse mass. They are close but not the same. In particular, no clear pattern emerges for the values of $n$ and $C$ while an interesting regularity is obtained for $q$ and $T$ as seen in Figs. (6) and (7) shown at the end of this paper.

In the next section we review the derivation of the Tsallis distribution by emphasizing the quantum statistical form and the thermodynamic consistency.

2. Tsallis Distribution for Particle Multiplicities.

In the following we discuss the Tsallis form of the Fermi-Dirac distribution proposed in [10, 11, 12, 13, 9] which uses

$$n_T^{FD}(E) \equiv \frac{1}{1 + \exp_q \left( \frac{E - \mu}{T} \right)}.$$  

(6)

where the function $\exp_q(x)$ is defined as

$$\exp_q(x) \equiv \begin{cases} 
[1 + (q - 1)x]^{1/(q-1)} & \text{if } x > 0 \\
[1 + (1 - q)x]^{1/(1-q)} & \text{if } x \leq 0 
\end{cases}$$

(7)

and, in the limit where $q \rightarrow 1$ reduces to the standard exponential:

$$\lim_{q \rightarrow 1} \exp_q(x) \rightarrow \exp(x)$$

The form given in Eq. (6) will be referred to as the Tsallis-FD distribution. The Bose-Einstein version (given below) will be referred to as the Tsallis-BE distribution [14].

All forms of the Tsallis distribution introduce a new parameter $q$. In practice this parameter is always close to 1, e.g. in the results obtained by the ALICE and CMS collaborations typical values for the parameter $q$ can be obtained from fits to the
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

transverse momentum distribution for identified charged particles [3] and are in the range 1.1 to 1.2 (see below). The value of $q$ should thus be considered as never being far from 1, deviating from it by 20% at most. An analysis of the composition of final state particles leads to a similar result [15] for the parameter $q$.

The classical limit will be referred to as Tsallis-B distribution (the B stands for the fact that it reduces to the Boltzmann distribution in the limit where $q \to 1$) and is given by result [5, 6]

$$n_{BT}^B(E) = \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{1}{q-1}}. \quad (8)$$

Note that we do not use the normalized $q$-probabilities which have been proposed in Ref. [6] since we use here mean occupation numbers which do not need to be normalized. In the limit where $q \to 1$ all distributions coincide with the standard statistical distributions:

$$\lim_{q \to 1} n_{BT}^B(E) = n_B^B(E), \quad (9)$$

$$\lim_{q \to 1} n_{FD}^E(E) = n_{FD}^E(E), \quad (10)$$

$$\lim_{q \to 1} n_{BE}^T(E) = n_{BE}^T(E). \quad (11)$$

A derivation of the Tsallis distribution, based on the Boltzmann equation, has been given in Ref. [16, 17]. Numerically the difference between Eq. (6) and the Fermi-Dirac distribution is small, as shown in Fig. (1) for a value of $q = 1.1$.

The Tsallis-B distribution is always larger than the Boltzmann one if $q > 1$. Taking into account the large $p_T$ results for particle production we will only consider this possibility in this paper. As a consequence, in order to keep the particle yields the same, the Tsallis distribution always leads to smaller values of the freeze-out temperature for the same set of particle yields [15].

3. Thermodynamic Consistency

The first and second laws of thermodynamics lead to the following two differential relations [18]

$$d\epsilon = Tds + \mu dn, \quad (12)$$

$$dP = sdT + nd\mu. \quad (13)$$

where $\epsilon = E/V$, $s = S/V$ and $n = N/V$ are the energy, entropy and particle densities respectively. Thermodynamic consistency requires that the following relations be satisfied

$$T = \left. \frac{\partial \epsilon}{\partial s} \right|_n, \quad (14)$$

$$\mu = \left. \frac{\partial \epsilon}{\partial n} \right|_s. \quad (15)$$
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

**Figure 1.** Comparison between the Fermi-Dirac and Tsallis-FD distributions as a function of the energy $E$, keeping the Tsallis parameter $q$ fixed, for various values of the temperature $T$. The chemical potential is kept equal to one in all curves, the units are arbitrary.

\begin{align}
  n &= \frac{\partial P}{\partial \mu} \bigg|_T, \\
  s &= \frac{\partial P}{\partial T} \bigg|_\mu.
\end{align}

The pressure, energy density and entropy density are all given by corresponding integrals over Tsallis distributions and the derivatives have to reproduce the corresponding physical quantities. For completeness, in the next section, we derive Tsallis thermodynamics using the maximal entropy principle and discuss quantum $q$-statistics in particular Bose-Einstein and Fermi-Dirac distribution by maximizing the entropy of the system for quantum distributions. This extends the derivation of Ref. [9]. We will show that the consistency conditions given above are indeed obeyed by the Tsallis-FD distribution.
4. Quantum Statistics

The entropy in standard statistical mechanics for fermions is given in the large volume limit by:

\[
S^{FD} = -gV \int \frac{d^3p}{(2\pi)^3} \left[ n^{FD} \ln n^{FD} + (1 - n^{FD}) \ln(1 - n^{FD}) \right],
\]

where \( g \) is the degeneracy factor and \( V \) the volume of the system. For simplicity Eq. (18) refers to one particle species but can be easily generalized to many. In the limit where momenta are quantized, which is given by:

\[
S^{FD} = -g \sum_i \left[ n_i \ln n_i + (1 - n_i) \ln(1 - n_i) \right],
\]

For convenience we will work with the discrete form in the rest of this section. The large volume limit can be recovered with the standard replacement:

\[
\sum_i \rightarrow V \int \frac{d^3p}{(2\pi)^3}
\]

The generalization, using the Tsallis prescription, leads to [10, 11, 12]

\[
S^{FD}_T = -g \sum_i \left[ n^q_i \ln_q n_i + (1 - n_i) q \ln_q(1 - n_i) \right],
\]

where use has been made of the function

\[
\ln_q(x) \equiv \frac{x^{1-q} - 1}{1-q},
\]

often referred to as q-logarithm. It can be easily shown that in the limit where the Tsallis parameter \( q \) tends to 1 one has:

\[
\lim_{q \rightarrow 1} \ln_q(x) = \ln(x).
\]

In a similar vein, the generalized form of the entropy for bosons is given by

\[
S^{BE}_T = -g \sum_i \left[ n^q_i \ln_q n_i - (1 + n_i) q \ln_q(1 + n_i) \right],
\]

In the limit \( q \rightarrow 1 \) equations 21 and 24 reduce to the standard Fermi-Dirac and Bose-Einstein distributions. Further, as we shall presently explain, the formulation of a variational principle in terms of the above equations allows to prove the validity of the general relations of thermodynamics. One of the relevant constraints is given by the average number of particles,

\[
\sum_i n^q_i = N.
\]

Likewise, the energy of the system gives a constraint,

\[
\sum_i n^q_i E_i = E.
\]
it is necessary to have the power \( q \) on the left-hand side as no thermodynamic consistency would be achieved without it. The maximization of the entropic measure under the constraints Eqs. (25) and (26) leads to the variational equation:

\[
\delta \frac{\delta n_i}{\delta n_i} \left[ S_{FD}^F + \alpha (N - \sum_i n_i^q) + \beta (E - \sum_i n_i^q E_i) \right] = 0,
\]

where \( \alpha \) and \( \beta \) are Lagrange multipliers associated, respectively, with the total number of particles and the total energy. Differentiating each expression in Eq. (27)

\[
\delta \frac{\delta n_i}{\delta n_i} \left( S_{FD}^F \right) = \frac{q}{q-1} \left[ (1 - n_i) \right]^{q-1} - 1, \quad (28)
\]

\[
\delta \frac{\delta n_i}{\delta n_i} \left( N - \sum_i n_i^q \right) = -qn_i^{q-1}, \quad (29)
\]

and

\[
\delta \frac{\delta n_i}{\delta n_i} \left( E - \sum_i n_i^q E_i \right) = -qE_i n_i^{q-1}. \quad (30)
\]

By substituting Eqs. (28), (29) and (30) into (27), we obtain

\[
qn_i^{q-1} \left\{ \frac{1}{q-1} \left[ -1 + \left( \frac{1 - n_i}{n_i} \right)^{q-1} \right] - \beta E_i - \alpha \right\} = 0. \quad (31)
\]

Which can be rewritten as

\[
\frac{1}{q-1} \left[ -1 + \left( \frac{1 - n_i}{n_i} \right)^{q-1} \right] = \beta E_i + \alpha, \quad (32)
\]

and, by rearranging Eq. (32), we get

\[
\frac{1 - n_i}{n_i} = [1 + (q - 1)(\beta E_i + \alpha)]^{\frac{1}{q-1}},
\]

which gives the Tsallis-FD form referred to earlier in this paper as \( [10,11,12] \)

\[
n_i = \frac{1}{[1 + (q - 1)(\beta E_i + \alpha)]^{\frac{1}{q-1}} + 1}, \quad (33)
\]

Using a similar approach one can also determine the tsallis-BE distribution by starting from the extremum of the entropy subject to the same two conditions:

\[
\frac{\delta}{\delta n_i} \left[ S_{BE}^F + \alpha (N - \sum_i n_i^q) + \beta (E - \sum_i n_i^q E_i) \right] = 0, \quad (34)
\]

which leads to

\[
n_i = \frac{1}{[1 + (q - 1)(\beta E_i + \alpha)]^{\frac{1}{q-1}} - 1}, \quad (35)
\]

where the usual identifications \( \alpha = -\mu/T \) and \( \beta = 1/T \) have been made.
5. Proof of Thermodynamical Consistency

In order to use the above expressions it has to be shown that they satisfy the thermodynamic consistency conditions. To show this in detail we use the first law of thermodynamics \[ P = -E + TS + \mu N \] \[ \frac{\partial P}{\partial \mu} \bigg|_T = 1 \frac{\partial E}{\partial T} + \frac{\partial E}{\partial \mu} \frac{d\mu}{dT} + \frac{\partial S}{\partial T} + \frac{\partial S}{\partial \mu} \frac{d\mu}{dT} \] \[ \frac{\partial n}\partial \mu = n T \left[ 1 + (q - 1) \frac{E_i - \mu}{T} \right]^{q-1}, \] \[ (1 - n_i)^{q-1} = n_i^{q-1} \left[ 1 + \frac{(q - 1)(E_i - \mu)}{T} \right]. \] Introducing this into equation 37, yields \[ \frac{\partial P}{\partial \mu} \bigg|_T = n, \] which proves the thermodynamical consistency \[ n \] is kept fixed one has the additional constraint \[ \frac{dn}{dT} \frac{\partial n}{\partial T} + \frac{dn}{\partial \mu} \frac{d\mu}{dT} = 0, \] leading to \[ \frac{d\mu}{dT} = -\frac{\frac{dn}{dT}}{\frac{dn}{\partial \mu}}. \]
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

Now, we rewrite 39 and 40 in terms of the following expressions

\[
\begin{align*}
\frac{\partial E}{\partial T} &= \sum_i q E_i n_i^{q-1} \frac{\partial n_i}{\partial T}, \\
\frac{\partial E}{\partial \mu} &= \sum_i q E_i n_i^{q-1} \frac{\partial n_i}{\partial \mu}, \\
\frac{\partial S}{\partial T} &= \sum_i q \left[ -n_i^{q-1} + (1 - n_i)^{q-1} \right] \frac{\partial n_i}{\partial T}, \\
\frac{\partial S}{\partial \mu} &= \sum_i q \left[ -n_i^{q-1} + (1 - n_i)^{q-1} \right] \frac{\partial n_i}{\partial \mu}, \\
\frac{\partial n}{\partial T} &= \frac{1}{V} \left[ \sum_i q n_i^{q-1} \frac{\partial n_i}{\partial T} \right], \\
\frac{\partial n}{\partial \mu} &= \frac{1}{V} \left[ \sum_i q n_i^{q-1} \frac{\partial n_i}{\partial \mu} \right].
\end{align*}
\]

By introducing the above relations into equation 39, the numerator of equation 39 becomes

\[
\begin{align*}
\frac{\partial E}{\partial T} + \frac{\partial E}{\partial \mu} d\mu dT &= \sum_i q E_i n_i^{q-1} \frac{\partial n_i}{\partial T} \\
&\quad - \sum_{i,j} q^2 E_j (n_i n_j)^{q-1} \frac{\partial n_j}{\partial \mu} \frac{\partial n_i}{\partial T} \\
&\quad \frac{\sum_j q n_j^{q-1} \frac{\partial n_j}{\partial \mu}}{\sum_j n_j^{q-1} \frac{\partial n_j}{\partial \mu}}, \\
&\quad \frac{\sum_{i,j} q E_i (n_i n_j)^{q-1} C_{ij}}{\sum_j n_j^{q-1} \frac{\partial n_j}{\partial \mu}}. \\
\end{align*}
\]

Where the abbreviation

\[
C_{ij} \equiv (n_i n_j)^{q-1} \left[ \frac{\partial n_i}{\partial T} \frac{\partial n_j}{\partial \mu} - \frac{\partial n_j}{\partial T} \frac{\partial n_i}{\partial \mu} \right],
\]

has been introduced. One can rewrite the denominator part of equation 39 as

\[
\begin{align*}
\frac{\partial S}{\partial T} + \frac{\partial S}{\partial \mu} d\mu dT &= q \sum_{i,j} \left[ -n_i^{q-1} + (1 - n_i)^{q-1} \right] n_j^{q-1} C_{i,j} \\
&\quad \frac{(q - 1) \sum_j n_j^{q-1} \frac{\partial n_j}{\partial \mu}}{T \sum_j n_j^{q-1} \frac{\partial n_j}{\partial \mu}}, \\
&\quad \frac{q \sum_{i,j} (E_i - \mu) (n_i n_j)^{q-1} C_{i,j}}{T \sum_j n_j^{q-1} \frac{\partial n_j}{\partial \mu}}, \\
\end{align*}
\]

(41)
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

where

$$\frac{-n_i^q - (1 - n_i)^q - 1}{q - 1} = \frac{(E_i - \mu)}{T} n_i^q - 1,$$

hence, by substituting equation 41 and 43 in to 39, we find

$$\frac{\partial E}{\partial S}_n = T \sum_{i,j} E_i C_{ij} \sum_{i,j} \frac{(E_i - \mu) C_{ij}}{T},$$

(44)

since $\sum_{i,j} C_{ij} = 0$, this finally leads to the desired result

$$\frac{\partial E}{\partial S}_n = T.$$

(45)

Hence thermodynamic consistency is satisfied.

It has thus been shown that the definitions of temperature and pressure within the Tsallis formalism for non-extensive thermostatistics lead to expressions which satisfy consistency with the first and second laws of thermodynamics.

6. Tsallis Fit Details

The total number of particles is given by the integral version of Eq. (25),

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{q/(1-q)}. $$

(46)

The corresponding (invariant) momentum distribution deduced from the equation above is given by

$$E \frac{dN}{d^3p} = gV \frac{E}{(2\pi)^3} \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{q/(1-q)}, $$

(47)

which, in terms of the rapidity and transverse mass variables, becomes

$$\frac{dN}{dy dp_T dp_T} = gV \frac{m_T \cosh y}{(2\pi)^2} \left[ 1 + (q - 1) \frac{m_T \cosh y - \mu}{T} \right]^{q/(1-q)}, $$

(48)

At mid-rapidity $y = 0$ and for zero chemical potential this reduces to the following expression

$$\frac{dN}{dp_T dy} \bigg|_{y=0} = gV \frac{p_T m_T}{(2\pi)^2} \left[ 1 + (q - 1) \frac{m_T}{T} \right]^{q/(1-q)}. $$

(49)

Fits using the above expressions based on the Tsallis-B distribution to experimental measurements published by the CMS collaboration are shown in Figs. (2), (3) and (4) are comparable with those shown by the CMS collaboration. Fits to the results obtained by the ALICE collaboration are shown in Fig. (5) for $\pi^-, K^-, \bar{p}$. The resulting
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

parameters are considerably different from those obtained from Eq. 3 and are collected in Table I. The most striking feature is that the values of the parameter $q$ are fairly stable in the range 1.1 to 1.2 for all particles considered at 0.9 TeV. The temperature $T$ cannot be determined very accurately for all hadrons but they are consistent with a value around 70 MeV.

For clarity we show these results also in Fig. (6) for the values of the parameter $q$ and $T$.

| Particle | $q$               | $T$ (GeV)       |
|----------|-------------------|-----------------|
| $\pi^+$  | 1.154 ±0.036      | 0.0682 ±0.0026  |
| $\pi^-$  | 1.146 ±0.036      | 0.0704 ±0.0027  |
| $K^+$    | 1.158 ±0.142      | 0.0690 ±0.0223  |
| $K^-$    | 1.157 ±0.139      | 0.0681 ±0.0217  |
| $K^0_S$  | 1.134 ±0.079      | 0.0923 ±0.0139  |
| $\Lambda$| 1.107 ±0.147      | 0.0730 ±0.0425  |
| $\bar{p}$| 1.106 ±0.158      | 0.0764 ±0.0464  |
| $\Lambda$| 1.114 ±0.047      | 0.0698 ±0.0148  |
| $\Xi^-$  | 1.110 ±0.218      | 0.0440 ±0.0752  |

Table 1. Fitted values of the $T$ and $q$ parameters for strange particles measured by the ALICE [3] and CMS collaborations [4] using the Tsallis-B form for the momentum distribution.

in Fig. (7) for the values of the Tsallis parameter $T$. The striking feature is that the values of $q$ are consistently between 1.1 and 1.2 for all species of hadrons.

7. Conclusions

In this paper we have presented a detailed derivation of the quantum form of the Tsallis distribution and considered in detail the thermodynamic consistency of the resulting distribution. It was emphasized that an additional power of $q$ is needed to achieve consistency with the laws of thermodynamics [9]. The resulting distribution, called Tsallis-B, was compared with recent measurements from the ALICE [3] and CMS collaborations [4] and good agreement was obtained. The resulting parameter $q$ which is a measure for the deviation from a standard Boltzmann distribution was found to be around 1.11. The resulting values of the temperature are also consistent within the errors and lead to a value of around 70 MeV.

References

[1] B. I. Abelev, et al. (STAR), Phys. Rev. C75, 064901 (2007)
[2] A. Adare et al. (PHENIX), Phys. Rev. C83, 064903 (2011)
[3] K. Aamodt, et al. (ALICE Collaboration), Eur. Phys. J. C71 1655 (2011)
[4] V. Khachatryan, et al. (CMS), JHEP 05, 064 (2011)
[5] C. Tsallis, J. Statist. Phys. 52, 479 (1988)
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

Figure 2. Comparison between the measured transverse momentum distribution for $K_S^0$ as measured by the CMS collaboration [4] and the Tsallis-B distribution. The full line is a fit using the parameterization given in Eq. (49) to the 0.9 TeV data with the parameters listed in Table I.

[6] C. Tsallis, R. S. Mendes, A. R. Plastino, Physica A261, 534 (1998)
[7] F. Pereira, R. Silva, J. Alcaniz, Phys. Rev. C76, 015201 (2007)
[8] F. Pereira, R. Silva, J. Alcaniz, Phys. Lett. A373, 4214 (2009), 0906.2422
[9] J. M. Conroy, H. G. Miller, A. R. Plastino, Phys. Lett. A374, 4581 (2010)
[10] F. Buyukkilic, D. Demirhan, Phys.Lett. A181, 24 (1993)
[11] F. Pennini, A. Plastino, A. R. Plastino, Phys. Lett. A208, 4, 309 (1995)
[12] A. M. Teweldeberhan, A. R. Plastino, H. G. Miller, Phys.Lett. A343, 71 (2004)
[13] J. M. Conroy, H. Miller, Phys. Rev. D78, 054010 (2008)
[14] J. Chen, Z. Zhang, G. Su, et al., Phys. Lett. A 300, 1, 65 (2002)
[15] J. Cleymans, G. Hamar, P. Levai, S. Wheaton, J.Phys. G36, 064018 (2009)
[16] T. Biro, G. Purcsel, Phys. Rev. Lett. 95, 162302 (2005)
[17] G.G. Barnafoldi, K. Urmossy, T.S. Biro, J. Phys. Conf. Ser. 270 (2011) 012008
[18] S. R. de Groot, W. A. van Leeuwen, C. G. van Weert, Relativistic Kinetic Theory (North Holland, 1980)
Figure 3. Comparison between the measured transverse momentum distribution for \( \Lambda \) as measured by the CMS collaboration [4] and the Tsallis-B distribution. The full line is a fit using the parameterization given in Eq. (49) to the 0.9 TeV data with the parameters listed in Table I.
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

Figure 4. Comparison between the measured transverse momentum distribution for $\Xi^-$ as measured by the CMS collaboration [4] and the Tsallis-B distribution. The full line is a fit using the parameterization given in Eq. (49) to the 0.9 TeV data with the parameters listed in Table I.
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

![Graph](image)

Figure 5. Comparison between the measured transverse momentum distribution for $\pi^-$, $K^-$ and $\bar{p}$ as measured by the ALICE collaboration [3] and the Tsallis-B distribution. The lines are fits using the parameterization given in Eq. (49) to the 0.9 TeV data with the parameters listed in Table I. Full line is for $\pi^-$, the dotted line is for $K^-$, the dashed line is for anti-protons.
The Tsallis Distribution in Proton-Proton Collisions at $\sqrt{s} = 0.9$ TeV at the LHC.

Figure 6. Values of the Tsallis parameter $q$ for different species of hadrons.

Figure 7. Values of the Tsallis temperature $T$ for different species of hadrons.