Entanglement transfer in a double-cavity optomechanical system

Tiantian Huan,1,2 Rigui Zhou,1 and Hou Ian2

1College of Information Engineering, East China JiaoTong University, Nanchang, China
2Institute of Applied Physics and Materials Engineering, FST, University of Macau, Macau

We give a theoretical study of a double-cavity system in which a mechanical resonator beam is coupled to two cavity fields on both side through radiation pressures. The indirect coupling between the cavities via the resonator sets up a correlation between optomechanical entanglements in the two cavities to the common resonator. This correlation translates into an entanglement transfer from one side of the cavities to the other, initiating an indirect entanglement of the cavities.

I. INTRODUCTION

Cavity optomechanical systems [1] arise from the classical Fabry-Perot interferometer [2] by replacing one of the fixed sidewalls with a cantilever or double-clamped beam [3–5]. The one-dimensional degree of freedom introduced by the movable mechanical element adds a free resonator mode to the cavity system and allows this mode to interact with the cavity field through radiation pressure on the reflectively coated mechanical resonator. Regarded as a micromirror, this resonator can be feedback-controlled through the cavity field, on which numerous cooling protocols have been conceived and experimentally demonstrated in the last decade [6–10].

The degree of control in this hybrid cavity-micromirror system can be further enhanced when the micromirror is replaced by a double-face reflective membrane [11,12]. If a second optical cavity is coupled to it on the opposite side of the existing cavity, a two-mode or double-cavity optomechanical system with enhanced nonlinearity is formed [13–16]. Entanglement-wise, though it is predicted that the enhanced squeezing resulted from the nonlinear coupling helps generate static entangled state of distant mirrors [13], the dynamic property of entanglement between the two cavities is less well-understood.

Recent studies reveal that the dynamics of phonon-photon entanglement plays an important role in controlling the system characteristics, such as the transitions between distinct types of oscillating motions [17,18], robustness against the noise environment [19], sudden death and revival of states [20,22], and laser cooling for optimal entanglement [23]. In this article, we study the dynamics of the entanglements in a double-cavity optomechanical system where each photon mode in the two opposite cavities is symmetrically coupled to a common mechanical resonator mode via radiation pressure, albeit assuming asymmetric coupling strengths and asymmetric driving input. Our main concern is to determine how the entanglement between the resonator and one of the cavities [24] can be transferred to that between the resonator and the other cavity over time.

We find that the entanglement can be transferred from one side of the resonator to the other side of the resonator, thereby facilitating a mechanism for entanglement transfer between cascaded cavities although the cavities are physically indirectly coupled. Such a mechanism would be useful to quantum information processing, especially in terms of non-adiabatic quantum state transfer [25,26], and would provide a physical means to realize cavity arrays or resonator waveguides for transmitting information encoded in a quantum state [27,28].

In particular, the entanglement is here measured in logarithmic negativity, which is computed through determining the symplectic eigenvalues of a covariance matrix that relates the fluctuations of all six quadratures of the system’s main components. This method is standard in the literature of dynamic entanglement but we have generalized it to apply on a 6 × 6 covariance matrix. We observe that, lacking a driving field, the entanglement between the resonator and the right cavity will not achieve a high value compared to that between the resonator and the left cavity. Nonetheless, the indirect entanglement between the left and the right cavity is apparent and follows a saturating pattern similar to that between the resonator and the left cavity. In addition, all the entanglements measured exhibit a delay between the initial moment and the moment when oscillations subside and the entanglement begins to build up. The turning point signifies the motions of the indirectly coupled cavities are progressing towards a resonance at which their entanglement saturates to a maximum value.

In Sec. III we give a detailed description of the double-cavity model. The equations of motions are derived in Sec. III under the Heisenberg picture, from which the covariance matrix of the fluctuations is introduced. The direct and indirect entanglements are computed numerically and their significances are discussed in Sec. IV. The conclusions are given in Sec. V at the end.

II. DOUBLE OPTOMECHANICAL CAVITY

The proposed double-cavity optomechanical system is illustrated in Fig. 1 in which a mechanical resonator with reflective coatings on both sides receives the radiation pressures from both the cavity on the left side (L) and the cavity on the right side (R). The total Hamiltonian \( H = H_0 + H_R + H_D \) thus consists of three parts, which reads, respectively,

\[
H_0 = \omega_L a_L^{\dagger} a_L + \omega_R a_R^{\dagger} a_R + \frac{p^2}{2m} + \frac{1}{2} m \Omega^2 \dot{q}^2, \tag{1}
\]

\[
H_{\text{rad}} = \left( \eta_L a_L^{\dagger} a_L - \eta_R a_R^{\dagger} a_R \right) q, \tag{2}
\]

\[
H_{\text{ext}} = i \varepsilon_L (a_L^{\dagger} e^{-i \omega_C t} - a_L e^{i \omega_C t}) + i \varepsilon_R \times (a_R^{\dagger} e^{-i \omega_D t} - a_R e^{i \omega_D t}), \tag{3}
\]
The part $H_0$ accounts for the free Hamiltonians of the resonator and the cavities, the latter being regarded as bosonic modes of frequencies $\omega_L$ and $\omega_R$. The part $H_{\text{rad}}$ accounts for the phonon-photon interactions derived from the radiation pressures. The part $H_{\text{ext}}$ accounts for the two external driving lasers with frequency $\omega_C$ and frequency $\omega_D$. Note that we assume an asymmetric setting for the double-cavity system: a mechanical element with reflective coatings on both sides serves as a double-face mirror that experiences radiation through deriving a set of nonlinear Langevin equations.

III. DYNAMICS AND ENTANGLEMENT

To eventually study the indirect entanglement across the two cavity modes, we begin with the dynamics of the quantum fluctuations of the middle resonator mode and the right cavity mode. Consequently, there is no direct coupling between the two cavity modes.

The Hamiltonian $H_R$ quantifies the radiation pressures the mirror resonator receives from both its left side and its right side. The radiation pressure will deform the volume of an optomechanical system: a mechanical element with reflective coatings on both sides serves as a double-face mirror that experiences radiation through deriving a set of nonlinear Langevin equations.

We carry out this step by finding the Heisenberg equations of the operators from the Hamiltonian in Eq. (1) and introducing phenomenologically the relaxation terms and their associative Brownian noise terms. The Langevin equations under the rotating frames of reference read

$$\dot{q} = \frac{p}{m},$$
$$\dot{p} = -m\Omega_M^2 q - \Gamma_M p - \eta_L a_L^\dagger a_L + \eta_R a_R^\dagger a_R + \xi,$$
$$a_L^\dagger = -(\kappa_L + i\Delta_L) a_L - \eta_L a_L^\dagger q + \varepsilon_L + \sqrt{2\kappa_L} a_L^{in},$$
$$a_R^\dagger = -(\kappa_R + i\Delta_R) a_R + \eta_R a_R^\dagger q + \varepsilon_R + \sqrt{2\kappa_R} a_R^{in},$$

where $\Delta_L = \omega_L - \omega_C$ ($\Delta_R = \omega_R - \omega_D$) is the detuning of the left (right) cavity field from the left (right) driving laser. The radiation input noise $a^{in}$ has zero mean values, whose only nonzero correlation function is

$$\langle a^{in}(t) a^{in\dagger}(t') \rangle = \delta(t - t').$$

Describing the dissipation of the system, the cavity leakages at each side is expressed as $\kappa_\sigma = \pi c/(2 F_\sigma L_\sigma)$. $\varepsilon_\sigma$ is related to the input laser power $P$ by $|\varepsilon_\sigma|^2 = 2k_\sigma P/\hbar\omega_D$.

The mechanical mode is under the influence of stochastic Brownian noise that satisfies the non-Markovian auto-correlation relation $\langle \xi(t)\xi(t') \rangle = (\Gamma_M/\Omega_M) \int 2\pi e^{-i \omega (t-t') \omega_C} \text{coth} [\hbar \omega / 2k_B T] + 1$, where $k_B$ is the Boltzmann constant and $T$ is the temperature of the mechanical bath. However, the mirror Brownian noise $\xi(t)$ is not delta-correlated so that it does not describe a Markovian process. Furthermore, quantum effect is so significant that it needs to apply oscillators with a large mechanical quality factor $Q = \Omega_M/\Gamma_M \gg 1$. In this regime, we have the following Markovian delta-correlated relation:

$$\langle \xi(t)\xi(t') + \xi(t')\xi(t) \rangle / 2 = \Gamma_M (2\bar{n} + 1) \delta(t - t'),$$

where $\bar{n} = 1/(\exp[\hbar \Omega_M/(k_B T)] - 1)$ is the mean occupation number of the mechanical mode.

We focus on the dynamics of the quantum fluctuations around the steady state of the system. Replacing each operator by a c-number steady-state value plus an additional fluctuation operator with zero-mean value [32], the operators can be rewritten as $O(t) = \langle O(t) \rangle + \delta O(t)$, where $O \equiv q, p, a_L, a_L^\dagger, a_R, a_R^\dagger$. These expressions are inserted into the nonlinear Langevin equations for the initial conditions. In this reference frame, only when $|\langle a \rangle| \gg 1$, the usual linearization approximation $|\langle a \rangle| \simeq |\langle a \rangle|^2, \langle a q \rangle \simeq \langle a \rangle \langle q \rangle$ to Eqs. (4) can be achieved. A final simplification can be made by neglecting the fluctuations in the strong optical mode. Thus, we can safely ignore the dynamics of those nonlinear terms and linearize the Langevin equations successfully. We define the quadratures of the two cavity fields as

$$X_\gamma = (a_\gamma + a_\gamma^\dagger)/\sqrt{2}, Y_\gamma = (a_\gamma - a_\gamma^\dagger)/i\sqrt{2},$$

the corresponding Hermitian input noise operators $X^{in}_\gamma = (a^{in}_\gamma + a^{in\dagger}_\gamma)/\sqrt{2}, Y^{in}_\gamma = (a^{in}_\gamma - a^{in\dagger}_\gamma)/i\sqrt{2}$ with $\gamma \in \{L, R\}$.

From this approximate analysis of Langevin equations, it is clearly suggested that in such a double-cavity optomechanical system, the various of the dimensionless position operator of the mirror will change the annihilation and creation operators of the cavity at either two sides. That means there may...

![FIG. 1. (Color online) Model schematic of the double-cavity optomechanical system: a mechanical element with reflective coatings on both sides serves as a double-face mirror that experiences radiation pressures from both the left-side cavity and the right-side cavity. An incident driving laser enters the double-cavity system from the left side.](image-url)
exist entanglement between the mirror and the cavity at the right side. What we are interested in is whether the quantum fluctuations of the two cavity fields and the mechanical resonator are coupled so that the generation of quantum entanglement between the two optical modes becomes possible. Therefore, we calculate the quantum entanglement value of each two modes as follows. In order to show the fluctuations, we have the 6-component vector over all quadratures $u = (\delta q, \delta p, \delta X_L, \delta Y_L, \delta X_R, \delta Y_R)$ and define the vector of input noises as $n = (0, \xi, \sqrt{2\kappa_L}X_L^{in}, \sqrt{2\kappa_L}Y_L^{in}, \sqrt{2\kappa_R}X_R^{in}, \sqrt{2\kappa_R}Y_R^{in})$. Then the time-dependent inhomogeneous equations of motion can be written as $u(t) = A(t)u(t) + n(t)$, with $A(t) =$

$$
\begin{pmatrix}
0 & 1/m & 0 & 0 & 0 & 0 \\
-m\Omega_M^2 & -\Gamma & -G_{xL}(t) & -G_{yL}(t) & G_{xR}(t) & G_{yR}(t) \\
G_{yL}(t) & 0 & -\kappa_L & \Delta_L(t) & 0 & 0 \\
-G_{xL}(t) & 0 & -\kappa_L & \Delta_L(t) & 0 & 0 \\
-G_{yR}(t) & 0 & 0 & -\kappa_R & \Delta_R(t) & 0 \\
G_{xR}(t) & 0 & 0 & 0 & -\kappa_R & \Delta_R(t)
\end{pmatrix}
$$

(8)

where the real elements contain the optomechanical time-dependent coupling constants $G_{x}(t), G_{y}(t)$, which are described by

$$
G_{x}(t) = g_{\gamma}(x(t)), G_{y}(t) = g_{\gamma}(y(t)).
$$

(9)

with $\gamma \in \{L, R\}$ and the detuning as $G_{\gamma}(t) = G_{x\gamma}(t) + ig_{y\gamma}(t)$.

$$
G_{\gamma}(t) = \sqrt{2}(a(t))\eta_{\gamma}, \\
\Delta_L(t) = \Delta_{L0} + \eta_L(q(t)), \\
\Delta_R(t) = \Delta_{R0} - \eta_R(q(t)).
$$

(10)

When the three-mode system is stable, it reaches a unique steady state, independently from the initial condition. Since the quantum noises $\xi, X_L^{in}, Y_L^{in}, X_R^{in}$ and $Y_R^{in}$ are interpreted by zero-mean quantum Gaussian random process and the dynamics of the system have been linearized, the quantum steady state for the fluctuations is fully characterized by its $6 \times 6$ covariance matrix that obeys $\dot{V}(t) = A(t)V(t) + V(t)A^T(t) + D$, where the elements of the covariance matrix are the pairwise correlations of all quadratures. The diagonal elements of the covariance matrix are auto-correlations of the three modes. $D$ is the matrix that describes damping and decay rate in the system, which is given as $D = \text{diag}(0, \Gamma(2\bar{\eta} + 1), \kappa_L, \kappa_L, \kappa_R, \kappa_R)$. This comes from $\langle n_i(t)n_j(t') + n_j(t)n_i(t') \rangle / 2 = \delta(t - t')D_{ij}$, characterizing the magnitudes of the noisy terms. The corresponding sub-matrix $V_{ij}$ of the covariance matrix $V$, which consists of $V_{ij}$, will reveal different meanings if we abstract different elements from the full matrix to the sub-matrix.

We now have one mechanical mode and two optical modes interacting in pairs. Thus, there are following three cases: if $i, j \in \{1, 2, 3, 4\}$, $V_{ij}$ becomes a $4 \times 4$ matrix, which is formed by the first four rows and columns of $V$. That means the covariance matrix of the mechanical mode and the optical mode at the left side of the mechanical mode; Similarly, if $i, j \in \{1, 2, 5, 6\}$, $V_S$ means the covariance matrix of the mechanical mode and the optical mode at the right side; If $i, j \in \{3, 4, 5, 6\}$, $V_S$ means the covariance matrix of the two opposite optical modes. For convenience, we can express $V_S$ as

$$
V_S = \begin{pmatrix}
V_{\alpha\alpha} & V_{\alpha\beta} \\
V_{\alpha\beta} & V_{\beta\beta}
\end{pmatrix},
$$

(11)

where $\alpha$ and $\beta$ designate the side of the cavities to show the mentioned three cases respectively. The non-diagonal elements correspond to the correlations of each two different quadratures, which describe the entanglement of the three modes in the system. We bring $V_S$ into a standard form by a process known as symplectic diagonalization and get a set of symplectic eigenvalues of the covariance matrix. The obtained symplectic eigenvalues, which are expressed as the symplectic diagonalization $v = \text{diag}(v_{-}, v_{-}, v^{+}, v^{+})$ of $V_S$, read

$$
v_F = \sqrt{\frac{\Sigma(V_S) + \sqrt{\Sigma(V_S)^2 - 4\det V_S}}{2}},
$$

(12)

where $\Sigma(V_S) = \det(V_{\alpha\alpha}) + \det(V_{\beta\beta}) + \det(V_{\alpha\beta})$. The symplectic eigenvalues encode essential information on a two-mode quantum state $\rho$. The determinants $\det(V_{\alpha\alpha}), \det(V_{\beta\beta}), \det(V_{\alpha\beta})$ and $V_S$ are a set of local symplectic invariants for $V_S$ so that the standard form corresponding to the covariance matrix remains unchanged.

We now interpret that for a two-mode Gaussian state $\rho$, the negativity is defined as

$$
N(\rho) = \frac{\|\rho^T\|_1 - 1}{2},
$$

(13)

where $\|\rho^T\|_1$ indicates the trace norm of the partial transposition of the bipartite quantum state $\rho$. The negativity is also a decreasing function of $v^-$, which means that $\|\rho^T\|_1 = 1/2v^- \Rightarrow N(\rho) = \text{max}[0, \frac{1}{2\sqrt{2}}]$, when $v^-$ is the minimum symplectic eigenvalue of the covariance matrix. In order to establish direct estimation of the entanglement, we measure it by the logarithmic negativity, defined as

$$
E_N = \ln\|\rho^T\|_1.
$$

Strictly related to $N$, the logarithmic negativity can be written as

$$
E_N = \text{max}[0, -\ln(2v^-)].
$$

(14)

The symplectic eigenvalue $v^-$ thus completely quantifies the quantum entanglement of each of the two modes in the system. Hence, a Gaussian state is entangled if and only if $v^- < 1/2$, which is equivalent to Simon’s necessary and sufficient entanglement nonpositive partial transpose criterion for Gaussian states. Then, we can write as $4\det V_S < \Sigma(V_S) - 1/4$.

IV. ENTANGLEMENT TRANSFER

To study the whole transformation of the entanglement between the three modes in the double-cavity system, the developments of the entanglement value of different two modes are
plotted in three figures, respectively. We have made an estimation of a parameter region and found a set of parameters close to that of the experiments [36]. For the reasonable parameter setting, we assume a higher mechanical quality factor $Q = 20000$, $\omega_M = 2\pi$ MHz, and $m = 10$ ng. The cavity length is $L = 25$ mm with finesse $F = 4 \times 10^4$. The dumping of the mechanical mode is related to the quality factor $\Gamma_M = \omega_M/Q$. The cavity is assumed to be driven by a laser with value of detuning $\Delta = 6.5\omega_M$. The full time evolution of the system is given by the application of the operators in the appropriate range of value.

From now on, we make a detailed discussion about the quantum entanglement of each of the two modes in the system. Fig. 2 shows the quantum entanglement values of the mechanical mode and the optical mode at the left side, which is measured by $E_N$ and evolves over time. With the same decay rate and coupling constants, indicated by the blue curve, there exists a strong fluctuation at the first part of the evolution. From the time point where the entanglement value grows steadily, it is observed that the fluctuation is instead by oscillation motion, in which the maximum value increasing as linear growth and gets a fixed value after a certain time. According to the red curve, we can find that for a double-cavity system that has asymmetric cavities from the two sides, the starting point of the steady growth of entanglement appears earlier with different decay rates. And then, the entanglement reaches the maximum at a finite time and keep a periodic oscillation around the maximum value. Furthermore, with larger coupling constants, the strength of oscillation in the evolution reduces and the plot becomes flattening out.

After that the mirror is moved by the cavity at the left side, the motion of the mirror will impact on the dynamics of the cavity at the right side. Hence, in order to reveal the interaction of the mirror and the cavity at the right side clearly, we plot the logarithmic negativity under the condition that has the same parameters with the red curve in Fig. 2. According to the figure, we can see that there are two phases in the evolution. The former is a regular oscillation whose least value exhibits a linear growth and the latter is an unstable fluctuation around the maximum value after the quantum entanglement reaches a saturation point. Actually, compared with the red curve in Fig. 2, the entanglement of the mirror and the cavity at the right side is already increasing when the driving from the cavity at the left side just started. Afterward, in the wake of the time point where the entanglement showed by the red curve in Fig. 2 grows steadily, the evolution of the entanglement in Fig. 3 gets a saturation and this status is persisted over time.

Proving that the transmission of the quantum entanglement between the two cavities is possible, we plot the time evolution of $E_N$ between the two cavities for input noises on three different orders of magnitude, as shown in Fig. 4. We can see that there exists quantum entanglement between the two cavities and it is obvious that there are two evolutionary stages in each of the three curves. The entanglement value increases linearly first and reaches a saturation with an unstable periodic oscillation in each curve. The curves shows that the quantum entanglement of the two cavities appears at $14.4\mu s$ for the asymmetric case and at $38\mu s$ for the symmetric case, which coincides with the time points where the entanglement begin to grow steadily in Fig. 2. Besides, the value of entanglement shown by the red curve in Fig. 2 and the value of entanglement shown by the red curve in Fig. 4 almost reach the maximum at the same time. We can see the evolution of $E_N$ between the two cavities evolves synchronously with $E_N$ between the mechanical mode and the optical mode of the cavity at the left side. In other words, the quantum entanglement
is transferred from the cavity at the left side of the mirror to the cavity at the other side. The blue curve and the yellow one, which are plotted with smaller input noise, show that the entanglement appears earlier when the input noise is smaller. This phenomenon means that the entanglement will be influenced by a large noise at the beginning of the evolution. The smaller the input noise is, the larger maximum value of quantum entanglement can be reached.

According to these plots, a noticeable feature is that, given the same system parameters, the effective optomechanical coupling at the right side is enhanced and the interaction of the two cavities is also generated with the increase of the entanglement intensity between the mirror and the cavity at the left side. We observe a strong correlation between the two cavities that follows the time evolution of entanglement between the mirror and the cavity at the left side. Hence, we denote that the mechanical deformation of the mirror establishes the connection between the two cavities and couples the two cavities. As a transmission medium, the mirror delivers the entanglement from the left side to the right side.

V. CONCLUSIONS

To summarize, we have studied the transfer of quantum entanglement between each of coupled two modes in the double-cavity system. We first introduce the dynamics in the whole system by using the quantum Heisenberg-Langevin equation. We show that with a realistic set of parameters, the quantum entanglement in each two modes of the system can be generated. And then, we exhibit the time evolution of entanglement value in every plot and analyze the connections among these figures. Above all, we have interpreted that the two cavities are entangled through the mirror and the quantum entanglement is conveyed from the cavity at the left side of the mirror to the cavity at the other side. By comparing the entanglement evolution for three different magnitude of input noises, we show that how the noise plays an important role in the development of entanglement. Furthermore, this indirect coupled can be employed in many significant problems in both classical optical communication and quantum information processing.

ACKNOWLEDGMENTS

R.Z. is supported by the National Natural Science Foundation of China under Grant No. 61463016 and 61340029, Program for New Century Excellent Talents in University under Grant No. NCET-13-0795, Landing project of science and technique of colleges and universities of Jiangxi Province under Grant No. KJLD14037, Project of International Cooperation and Exchanges of Jiangxi Province under Grant No. 20141BDH80007. H. I is supported by the FDCT of Macau under grant 013/2013/A1, University of Macau under grants MRG022/13/FST and MYRG2014-00052-FST, and National Natural Science Foundation of China under Grant No. 11404415.

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