Limitation of parallel assumption in repeat-pass InSAR using nonparallel orbits

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Abstract: In this report, we show the critical condition of the slope angle between orbits in InSAR (interferometric synthetic aperture radar) analysis. InSAR analysis, where the phase difference between two SAR observation data is measured, has been used for terrain height monitoring, and ground deformation detection, for example. However, InSAR data observed from nonparallel orbits degrades the performance of analysis. In the case of repeat-pass InSAR using airborne SAR, it is difficult to realize sufficiently parallel orbits. The purpose of this report is to formulate the nonparallel component of repeat-pass InSAR using an airborne SAR system and clarify the limitation of its parallel assumption. Then, we show the critical slope angle for use in InSAR analysis.

Keywords: InSAR, SAR interferometry, airborne SAR, orbital error, nonparallel orbit, critical baseline

Classification: Antennas and propagation

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1 Introduction

InSAR (interferometric synthetic aperture radar) [1, 2] analysis, where the phase difference between two SAR [3, 4, 5] observation data is measured, has been used for terrain height monitoring, and ground deformation detection, for example. Since an interferogram consists of some phases corresponding to the desired geometric information and the flat-earth phase generated by the observation baseline, preprocessing to remove the flat-earth phase is necessary in InSAR analysis. The flat-earth phase is a function of the baseline parameters and a reference surface, so it can be removed by calculating the baseline using flight path information. Therefore, accurate orbit information is required for the removal process.

In this report, we show the critical condition of the slope angle between orbits in InSAR analysis. To simplify the problem in InSAR, we often assume that two orbits are perfectly parallel. Since the baseline is independent of the flight direction under the parallel assumption, the flat-earth phase is a function of only the slant range and it can simplify the analysis. This assumption is not always valid for repeat-pass InSAR using airborne SAR, because guaranteeing parallelism between the orbits is more difficult than in single-pass InSAR or spaceborne SAR. In fact, we confirmed the fluctuation of flat-earth phase residuals fluctuated along the azimuth direction [6, 7], but it cannot be explained under the parallel assumption. Clarifying the mathematical model of the phase residuals is important for orbital error estimation, and phase residuals have been studied in satellite InSAR [8, 9, 10]. However, the slope of the orbit has not been focused on yet, and the limitation of this assumption has not been clarified. The purpose of this report is to formulate the nonparallel component of repeat-pass InSAR using an airborne SAR system and clarify the limitation of its parallel assumption.
2 Repeat-pass InSAR

Let us consider a SAR dataset observed by a linear orbit whose altitude from the ground surface is \( H \), and define a cylindrical coordinate system whose cylindrical axis is the observation orbit of the SAR data, as shown in Fig. 1(a). Under this definition, any point in the coordinate system can be represented as \((u, r, \theta)\) with azimuth \( u \), slant range \( r \), and elevation angle \( \theta \). In this letter, this SAR dataset and the corresponding orbit/coordinate system are called the master dataset and the master orbit/coordinate system, respectively.

We assume that we have another SAR data whose observation orbit is approximately parallel to the master orbit, where the data and orbit are called the slave data/orbit, respectively. Let us define four parameters to represent the slave orbit as follows (also see Fig. 1):

1. \( B \): The distance between master/slave orbits at the slow-time center \( t = 0 \).
2. \( \alpha \): The elevation angle of the slave orbit at \( t = 0 \).
3. \( b \): The velocity of the slave orbit projected to the polar plane. If \( b = 0 \), then the two observation orbits are perfectly parallel.
4. \( \beta \): The movement direction on the polar plane. If \( \beta = 0 \), then the movement is vertical; if \( \beta = 90^\circ \), then the movement is horizontal.

These parameters represent two straight orbits gradually approaching (or leaving) each other. Note that what these parameters represent is the relative movement of the slave orbit normalized by that of the master orbit.

In addition, we define the angle \( \delta \) between the master and slave orbits; it can be expressed as

\[
\tan \delta = \frac{b}{v},
\]

where \( v \) is the platform velocity. Also, azimuth \( u \) and slow-time \( t \) satisfy the linear relationship \( u = vt, t \in [-\frac{T}{2}, \frac{T}{2}] \), where \( T \) is the time width of the slow-time.

In this letter, we do not discuss dependence of the nonlinear error on the slow-time \( t \), or other factors such as acceleration, vibration, or rotation.

The propagation distance of the slave orbit \( \rho(u, r, \theta) \) observing a scatterer \((u, r, \theta)\) can be expressed as

\[
\rho(u, r, \theta|t) = \sqrt{(r \cos \theta - B \cos \alpha - bt \cos \beta)^2 + (r \sin \theta - B \sin \alpha - bt \sin \beta)^2} \\
\approx r - B \cos (\theta - \alpha) - bt \cos (\theta - \beta) \\
\approx r - B \cos (\theta - \alpha) - u \delta \cos (\theta - \beta),
\]

where \( B, bt \ll r \), and \( bt = b \frac{u}{v} = u \tan \delta \approx u \delta \). In this situation, the interferometric phase \( \phi(u, r, \theta) \) can be expressed as

\[
\phi(u, r, \theta) = 2k \{\rho(u, r, \theta) - r\}
\]
Fig. 1. Geometry of repeat-pass InSAR observation system.

\[ t = -\frac{T}{2} \]

\[ t = 0 \]

\[ t = \frac{T}{2} \]

where

\[ k = 2\pi \frac{f_c}{c} \]

is the spatial frequency corresponding to the wavelength \( \lambda \) and center frequency \( f_c \). Equations (4) and (5) are the parallel and nonparallel components, respectively, and the interferometric phase \( \phi(u, r, \theta) \) is their sum. Obviously, if \( \delta \to 0 \), then \( \tan \delta \to 0 \) and \( \phi_{\parallel}(u, \theta) \to 0 \).
Fig. 2. Examples of Eq. (5), where azimuth \( u \in [-1000m, 1000m] \), elevation angle \( \theta \in [30^\circ, 65^\circ] \), and \( \beta = 0 \). The title of each subimage \((\delta, f_c)\) denotes the orbit slope angle \((\delta)\) and the center frequency \((f_c)\) of SAR systems.

3 Discussion

3.1 Computer simulation

We show some examples of \( \phi_\delta(u, \theta) \) in Fig. 2. From these results, we can see that a small \( \delta \) generates a phase shift over one cycle. If we use an InSAR dataset that is not sufficiently parallel for interferometric analysis, such a phase residual may occur in the analysis results. Therefore, we should not use the parallel assumption unless the dataset satisfies the condition described below.
3.2 Parallelism condition (a): perfect parallel

Obviously, \( \phi_{\parallel}(u, \theta) \) can be ignored under the condition

\[
\max |\phi_{\parallel}(u, \theta)| = kD|\delta \cos (\theta - \beta)| \ll 2\pi,
\]

\[\rightarrow |\delta \cos (\theta - \beta)| = |\delta_{\parallel}| \ll \frac{\lambda}{D}, \tag{7}\]

where \( D = u_{\max} - u_{\min} \) is the azimuth image width and \( \delta_{\parallel} = \delta \cos (\theta - \beta) \) is the component of \( \delta \) parallel to the line of sight. This condition indicates that even a small angle of the wavelength \( \lambda \) relative to the azimuth image width \( D \) cannot be ignored. Since the azimuth width \( D \) of airborne SAR is several kilometers whereas the wavelength \( \lambda \) is about a few centimeters to 1m, there is a difference of at least three orders of magnitude.

3.3 Parallelism condition (b): azimuth direction

The second condition is derived by the sampling theorem along the azimuth direction as follows:

\[
\max \left| \frac{\partial \phi_{\parallel}}{\partial u} \right| \leq \frac{2\pi}{\Delta u}, \tag{8}\]

where \( \Delta u \) is the sampling interval along the azimuth direction, which is equal to the azimuth pixel size. As long as the condition Eq. (8) is satisfied, the
vibration of $\phi_\parallel(u, \theta)$ along the azimuth direction is observable and can be corrected. By substituting Eq. (5) into Eq. (8), we can obtain the second condition

$$\frac{\partial \phi_\parallel}{\partial u} = 2k \delta \cos(\theta - \beta),$$

$$\rightarrow |\delta_\parallel| \leq \frac{\lambda}{2\Delta u}.$$  \hspace{1cm} (9)

The ratio $\frac{\lambda}{\Delta u}$ is greater than 0.1 in most SAR systems, as shown in Fig. 3(a), and 0.1 rad $\approx 6^\circ$, so the limit of $\delta_\parallel$ is about $3^\circ$. Therefore, satisfying the condition Eq. (9) is much easier than satisfying the condition Eq. (7).

### 3.4 Parallelism condition (c): slant range direction

The third condition is derived by the sampling theorem for the flat-earth surface as follows:

$$\max \left| \frac{\partial \phi_\parallel}{\partial r_{\text{ref}}} \right| \leq \frac{2\pi}{\Delta r},$$  \hspace{1cm} (10)

where

$$r_{\text{ref}} = \frac{H}{\cos \theta}$$  \hspace{1cm} (11)

is a dependent slant range $r$ constrained on the reference surface and $\Delta r$ is the slant range sampling interval of the SAR system. By solving Eq. (10) for $\delta$, Eq. (12) can be obtained:

$$\frac{\partial \phi_\parallel}{\partial r_{\text{ref}}} = \frac{2k\delta \sin(\theta - \beta)}{r_{\text{ref}} \tan \theta},$$

$$\rightarrow |\delta_\perp| \leq \frac{\lambda}{\Delta r} \frac{r_{\text{ref}} \tan \theta}{D} \tan \theta = \frac{\lambda}{\Delta r} \frac{H \tan \theta}{D \cos \theta},$$  \hspace{1cm} (12)

where $\delta_\perp = \delta \sin(\theta - \beta)$ is the perpendicular component of $\delta$. In Eq. (12), $\frac{\lambda}{\Delta r}$ is generally about 0.1, as shown in Fig. 3(b), and the function $\frac{\tan \theta}{\cos \theta}$ is greater than 1.0 for almost all $\theta$, as shown in Fig. 3(c). Therefore, the condition Eq. (12) is not severe as long as we choose a suitable $D$ less than or equal to $H$.

### 4 Conclusion

In this report, we clarified the limitation of the parallel condition of repeat-pass InSAR using nonparallel orbits.

The first condition Eq. (7) indicates that even a small slope of one wavelength cannot be approximated as parallel. Therefore, we should consider the nonparallel component of the interferometric phase for repeat-pass InSAR analysis unless the orbits of the data are perfectly parallel.

The second and third conditions, Eq. (9) and Eq. (12), show the limitation of the critical slope angle to observe and correct the nonparallel component of the flat-earth phase. The nonparallel component $\phi_\parallel$ will be observable and correctable only if the conditions, Eq. (9) and Eq. (12), are satisfied.