Activation mechanisms for charge noise

Martin V. Gustafsson,1 Arsalan Pourkabirian,1 Göran Johansson,1 John Clarke,1,2 and Per Delsing1

1Microtechnology and Nanoscience, Chalmers University of Technology, SE-41296, Göteborg, Sweden
2Department of Physics, University of California, Berkeley, California 94720, USA

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Measurements of the temperature and bias dependence of Single Electron Transistors (SETs) show that charge noise increases linearly with temperature above a voltage-dependent threshold temperature, and that its low temperature saturation is due to self-heating. We show further that the two-level fluctuators responsible for charge noise are in strong thermal contact with hot electrons on the SET island, and at a temperature significantly higher than that of the substrate. We suggest that the noise is caused by electrons tunneling between the island and nearby potential wells.

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Low-frequency noise with a power spectral density \( S_Q(f) \propto 1/f^\alpha \) is observed in all charge sensitive devices \((f \text{ is frequency and } \alpha \approx 1)\). Apart from limiting the sensitivity of electrometers, such as the Single Electron Transistor (SET) 1, 2, charge noise is a primary source of decoherence in qubits 3, 4, and gives rise to errors in metrological quantum standards 5, 6. The noise is usually attributed to a superposition of Lorentzian spectra, each generated by a charged particle moving stochastically between two potential wells, known as a Two-Level Fluctuator (TLF) 7, 8. In spite of extensive studies, both experimental and theoretical 8, 11, the sources and locations of the TLFs remain unknown.

The SET consists of a small metallic island connected via tunnel junctions to source and drain electrodes [Fig.1(a)]. Its current-voltage characteristic depends strongly on the charge induced on the island by externally applied electric fields or charges moving in its surroundings, and is periodic in the electron charge \(e\). Because of its unmatched charge sensitivity 20, the SET is an important building block in electrical nanoscale applications and an ideal tool for investigating the properties of charge noise. Several authors have found that the charge noise measured in SETs decreases as the temperature is lowered, saturating to a constant level at low temperature 13–19. This saturation has been attributed to self-heating of the SET but, since there is no obvious model for thermalization between the TLFs and the SET electron gas, the issue has remained open. Kenyon et al. 18 proposed two alternative theories, namely that the TLFs are excited by shot noise from the SET 14, or by photons from the environment 21. The influence of SET bias parameters on the charge noise has also been studied 16, 17, but no firm conclusions have been drawn. This is largely due to the small range of useful bias voltages in SETs with modest charging energy, and the tendency of noise data to suffer from scattering and drift.

In this Letter, we present three experiments on charge noise in SETs. Using SETs with relatively high charging energy and recording an extensive amount of data, we can clearly resolve the dependence of the charge noise on both refrigerator temperature, \(T_R\), and SET bias parameters. Our results demonstrate that charge noise scales with temperature, and that the saturation of the noise at low temperature is predominantly due to power dissipation in the electron gas of the SET island. Furthermore, we determine the temperature of the TLF ensemble, and find that it is thermally coupled to the electrons in the SET more strongly than to the phonons in the substrate. These two results imply that the TLFs are situated at the interface between the SET island and the substrate and/or the interface between the island and its native surface oxide.

Sample and measurement setup. We fabricated several SETs using two-angle evaporation on a single crystalline silicon substrate covered with 400 nm of thermal oxide [Fig.1(a)], and included two of them, S1 and S2, in this study. The chip was cooled in a dilution refrigerator, with a base temperature of 20 mK, fitted with extensive low-temperature filtering of the measurement lines. Before the first set of measurements began, the sample had been cold \((\lesssim 4\text{ K})\) for several weeks. A magnetic field of 0.6 T quenched superconductivity in the aluminum, and we voltage-biased the SET symmetrically with respect to the refrigerator ground using two nominally identical, home-built transimpedance amplifiers. Typical I–V characteristics for S1 are shown in Fig.1(b). By fitting \(I_{SET}(V_b)\) and \(I_{SET}(V_b)\) curves to numerical simulations we find that the SETs have charging energies of \(E_{C,1}/k_B = 10.9\text{ K}\) and \(E_{C,2}/k_B = 6.0\text{ K}\), and total resistances \((\text{sum of the two junction resistances at high bias})\) of \(R_{S,1} = 368\text{ k}\Omega\) and \(R_{S,2} = 147\text{ k}\Omega\).

We acquired noise spectra in a frequency range from 1 Hz to 401 Hz with a 1 Hz linewidth. After subtracting the calculated shot noise and amplifier noise, we normalize each spectrum using the observed amplitude of an oscillating "pilot" signal applied to the SET gate at \(f_p = 377\text{ Hz}\) to null out variations and drift in the gain.
of the SET. We use the formula $S_{\text{shot}} = eI_{\text{SET}}$ for the shot noise throughout the calculations [22, 23], since it is valid in the high-current regime where shot noise may impact the results. As $T_0$ is increased, the contribution from an individual TLF in S1 became more pronounced at low to intermediate frequencies [Fig. 1(c)]. To exclude the influence of these deviations from $S_Q(f) \propto 1/f^\alpha$, we extract a single value $\tilde{S}_Q$ to represent the noise level of each acquired spectrum by averaging $S_Q$ over frequencies between 383 Hz and 401 Hz (above $f_p$). Studying the noise at this relatively high frequency minimizes error due to the limited measurement time of each spectrum, and produces a low spread between neighboring temperature and bias points.

**Experimental results.** In the first part of the study, Experiment (Exp.) A, we measured $\tilde{S}_Q$ for devices S1 and S2 (in separate runs) while increasing $T_0$ from 50 mK to around 4 K over a period of 18 to 19 hours. We adjusted $I_{\text{SET}}$ to the midpoint of the gate modulation curve before the start of each spectrum acquisition. The data, which are displayed in Fig. 2(a), clearly show a linear increase in $\tilde{S}_Q$ with temperature. This result is in contrast with the $S_Q \propto T_0^\beta$ dependence presented by Kenyon et al. [18] and Astafiev et al. [19]. The latter group, however, has observed $S_Q \propto T$ in other devices [24]. We have consistently found a linear dependence also in measurements on a different chip [25]. At temperatures below about 0.2 K we observe a saturation of the noise, as reported also by previous authors [18, 19].

In Exp. B, we repeated the temperature sweep of Exp. A for device S1 from $T_0 = 50$ mK to $T_0 = 1.5$ K over a period of 8 hours, while alternating the bias voltage between the three values shown with colored symbols in Fig. 1(d). The noise data, plotted in Fig. 1(b), clearly show that the saturation levels at low temperature depend on the bias voltage. This result is fully consistent with a picture of TLFs activated by SET self-heating. We can rule out the possibility that saturation in this temperature range is caused by quantum tunneling between TLF states [18] or activation from external photons [21], since neither of those processes would depend strongly on SET bias. At high temperatures, the noise is independent of bias and increases as $\tilde{S}_Q = \beta T_0$. From the

**FIG. 1.** Single Electron Transistor. (a) Scanning electron microscope image showing the SET design. The gate electrode is located outside the picture, 600 nm to the right of the island. (b) $I$-$V$ characteristics of device S1, each taken at a different gate voltage. Inset: $I_{\text{SET}}$ vs. $Q_g$, the charge induced from the gate. Colored points correspond to those in the main plot. (c) Solid blue curve: Typical power spectrum $S_Q(f)$ acquired at $T_0 = 50$ mK. The slope of this spectrum is $\alpha = 1.24 \pm 0.01$. The pilot signal, used for gain calibration, appears as a strong peak at $f_p = 377$ Hz. The red dot marks $\tilde{S}_Q$, the average of $S_Q$ between 383 Hz and 401 Hz. The solid black curve and the dashed green line are the calculated charge-equivalent amplifier noise and shot noise of the SET for this particular value of $I_{\text{SET}}$ and amplifier gain, respectively. The dashed red curve is an example of a spectrum (at $T_0 = 1.5$ K) with substantial contribution from a single TLF at intermediate frequencies, see text. (d) Solid lines: Maximum and minimum SET current as a function of bias voltage. Blue square, red cross and green triangle: Bias points used for the temperature sweep of Exp. B. Dots: Measurement points for the mapping of charge noise at base temperature in Exp. C. Color online.
regime \( T_0 \geq 1 \, \text{K} \), we extract the proportionality constant \( \beta = (8.28 \pm 0.02) \times 10^{-9} \, \text{e}^2 \text{Hz}^{-1} \text{K}^{-1} \).

To investigate in greater detail the connection between charge noise and bias conditions indicated by Exp. B, we performed Exp. C in which we measured the low-temperature saturation level of the charge noise at 315 bias points, each with different values of \( V_b \) and \( I_{SET} \) [Fig. 1(d)]. The total measurement time was 12.6 hours, and the bias points were applied in random order to avoid any influence of drift on the measurement parameters.

Figure 3(a) shows the charge noise as a function of SET bias voltage \( V_b \), with lines connecting points with the same bias current \( I_{SET} \). It is clear from this plot that \( \tilde{S}_Q \) increases with \( V_b \) and not only with \( I_{SET} \). Calculating the SET shot noise \([26] \) in the absence and presence of self heating (see discussion below), we see in both cases that the integrated shot noise power decreases with increasing \( V_b \) for moderate values of \( I_{SET} \). Since this is the opposite of what we observe, we conclude that the TLFs are not primarily excited by shot noise from the SET. Furthermore, if the TLFs were directly excited by inelastic scattering of electrons as they tunnel through the SET junctions \([18] \), we would expect the number of TLFs that can be excited by a tunneling electron to be proportional to \( V_b \) (assuming a uniform distribution of TLF activation energies \([8] \)). For each of these active TLFs, we expect the excitation rate to be proportional to \( I_{SET} \). With TLF relaxation rates dominating over excitation rates \([4] \), the charge noise should increase in proportion to the power dissipated in the SET, \( P_{SET} = V_b I_{SET}/2 \). Plotting the noise data of Fig. 3(a) against \( P_{SET} \) instead of \( V_b \) [Fig. 3(b)], we see a smooth and monotonic increase in the noise with \( P_{SET} \), but with a dependence much weaker than linear. This weak power law is characteristic of electron-phonon thermalization, indicating that the TLFs are activated by the temperature of the thermalized electron gas of the SET island, rather than by direct interaction with tunneling electrons.

In Fig. 3(c), we have used the proportionality constant \( \beta \) determined in Exp. B to calculate the equivalent temperature of the TLFs, \( T_{TLF} \). We use the same noise data as in Figs. 3(a) and (b), along with the three saturation temperatures extracted from Exp. B. Since Exps. B and C were carried out more than one week apart, we expect some drift between the two. The three saturation temperatures of Exp. B fit well to a power function \( T_{TLF} \propto P_{SET}^{0.24} \), which is close to the power dependence expected for the electron temperature \([27] \). The calculated temperatures of the SET electron gas and the substrate surface are also shown in this diagram (see discussion below).

**Thermal modeling.** The current flowing through an SET dissipates power in the island electron gas, which generally relaxes rapidly to a Fermi distribution. These hot electrons may subsequently thermalize via preferential tunneling from the island, and via emission of energy as phonons and possibly photons \([28] \). Widely used models predict the electron-phonon thermalization power \( P \) to follow \( P = \Omega (T_{el}^{5} - T_{ph}^{5}) \), with the exponent \( n = 5 \) \([29] \). In this equation, \( \Omega \) is a material-dependent electron-phonon coupling coefficient, \( \Omega \) is the volume of the electron gas, and \( T_{el} \) and \( T_{ph} \) are the temperatures of the electrons and the phonons, respectively. Using \( \Sigma = 0.4 \, \text{nW} \text{K}^{-4} \) for Al \([30] \), we obtain an estimate of the electron temperature \( T_{el} \) on the SET island as a function of applied power, plotted in Fig. 3(c).

The thermal power flowing from a SET well above the substrate temperature can produce a measurable increase in temperature of devices deposited nearby on the same

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**FIG. 3.** Noise and temperature vs. SET bias (Exp. C). (a) Charge noise \( \tilde{S}_Q \) vs. SET bias voltage. Points grouped by color and connected with lines were acquired with the same nominal value of \( I_{SET} \). Inset: Data for the lowest four values of \( I_{SET} \) with linear least-squares fits. For low values of \( V_b \) (traces with the lowest \( \tilde{S}_Q \)), there is a clear increase in noise level with bias voltage, indicating the the TLFs are not excited by SET shot noise. (b) Values of \( \tilde{S}_Q \) in (a) vs. power dissipated in the SET, \( P_{SET} = V_b I_{SET}/2 \). The weak power dependence indicates that the charge noise is not excited by inelastic scattering by electrons tunneling in the SET. (c) TLF temperature \( T_{TLF} \) (points) vs. \( P_{SET} \). The square, cross and triangle are the noise saturation levels for the three different bias points of Exp. B. The dashed line (green) is a power function without offset, fitted to the three saturation points, \( T_{TLF} \propto P_{SET}^{0.24} \). The upper solid line (black) is the electron temperature of the SET island, and the lower solid line (red) the phonon temperature underneath the SET; see text.
TABLE I. Possible mechanisms for the activation of TLFs.

| TLF activation process                      | Likely? | Reason                     |
|---------------------------------------------|---------|----------------------------|
| Tunneling between TLF states                | No      | Bias dep. of $S_Q$         |
| External photons                            | No      | Bias dep. of $S_Q$         |
| Electron-TLF scattering in tunnel barrier   | No      | $S_Q$ not $\propto P_{SET}$|
| Shot noise from the SET                     | No      | $S_Q$ increases with $V_b$ |
| Heating by SET electrons                    | Yes     | $T_{TLF}$ follows $T_{el}$ |

substance [32, 33]. We use a finite element model implemented in Comsol Multiphysics to evaluate the temperature distribution in the substrate as a function of $P_{SET}$. The model is axially symmetric, with the SET defined as a disc at the surface with the same area as the actual device, transmitting the full power $P_{SET}$ into the substrate, which has the same layer structure as the physical device. We assume that phonons are excited at the interface between Al and SiO$_2$ and propagate isotropically through the materials, which are treated as bulk media. Since the sound velocities in SiO$_2$ and Si are within a factor two of each other, we are justified in neglecting any thermal interface resistance between the two. We assume a refrigeration temperature of $T_0 = 20 \text{ mK}$ and use temperature-dependent thermal conductivities of $5.0T^3 \text{ W/Km}$ for Si and $0.03T^2 \text{ W/Km}$ for SiO$_2$, respectively [34, 35].

These simulations lead to the striking result that the phonon temperature $T_{ph}$ everywhere in the substrate is too low to account for the charge noise, and that $T_{TLF}$ is closer to $T_{el}$ than to $T_{ph}$. This is illustrated in Fig. 3(c), where we plot $T_{sub}$ against $P_{SET}$, with $T_{sub}$ defined as the phonon temperature of the substrate surface directly under the center of the SET.

Discussion and conclusions. Analysis of our detailed noise data yields new information on the processes responsible for charge noise in mesoscopic devices (summarized in Table I).

(i) Quantum tunneling between TLF states may set a lower bound on the charge noise, but the strong bias dependence on the saturation temperature shows that such processes are not responsible for the saturation observed under normal SET operating conditions. Although we cannot exclude the possibility that energy from tunneling events and shot noise in the SET contributes to the excitation of TLFs at some level, we can say that these are not the dominant activation mechanisms, as evidenced by the dependence of the charge noise on bias voltage and current. Rather, our data indicate that heating of the SET electron gas dominates the excitation of charge noise, and that the temperature of the TLFs has a dependence on the power dissipation similar to that of the SET electron temperature.

(ii) Comparing the different temperatures in Fig. 3(c), we see some notable features: The temperature of the TLFs is approximately five times higher than the local surface temperature of the substrate, and three times lower than the electron temperature of the SET island, which indicates that the TLFs are in stronger thermal contact with the electrons on the SET island than with the phonons in the substrate. This qualitative feature cannot be explained by uncertainties in the noise data or the models for electron-phonon thermalization, although these are not accurate enough for us to draw quantitative conclusions.

(iii) The thermal interaction between the TLFs and the SET electrons provides guidance to the microscopic modeling of charge noise and its origins. To account for the data, the TLFs must reside in the immediate vicinity of the device, and we can rule out that they are located in the interface between Si and SiO$_2$ or in the bulk dielectric. In particular, processes where the noise is generated by electrons tunneling between the SET island and local defects would account naturally for the thermal coupling to the SET electron gas. Such possible defects include the residue of Al grains formed around the perimeter of the SET island and leads during two-angle evaporation [12], as well as interface states between the metal of the SET and its surrounding oxides [30].

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