Case study of leak detection based on Gaussian function in experimental viscoelastic water pipeline
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ABSTRACT
Leakage in transmission pipelines and water distribution networks causes water and energy loss and reduces water quality. The accuracy of leakage detection using transient-based methods depends on several factors. This study investigated the sensitivity of location and size of leaks in simple polyethylene transmission pipelines to dynamic parameters, flow regime, sample size, spatial-step increment, and leak size and location. For this purpose, a hydraulic transient solver was first developed to take into account the dynamic effects of unsteady friction, viscoelasticity of the pipe wall, and the leak. The leakage was assumed to function with a quasi-normal distribution around its real location to reduce the problem dimensionality and unnecessary computations. This approach was evaluated based on experimental transient data in which leaks were simulated in different sizes and locations. Results revealed that the hydraulic transient model that includes only viscoelasticity effects could pinpoint leakage characteristics. The sample size evaluation indicated that half and a single period of the pressure signal are sufficient to determine the leakage location and size in simple viscoelastic transmission pipelines, respectively. The optimal ratio of the spatial-step to pipe length ($\Delta x/L$) was 0.025.

Key words | inverse transient analysis, leak detection, sensitivity analysis, time domain, viscoelastic pipeline

HIGHLIGHTS
- Sensitivity analysis of leak detection and sizing in the viscoelastic pipeline.
- Leakage simulate using a new approach with a quasi-normal distribution function.
- Development of an experimental transient model to evaluate the proposed approach.

INTRODUCTION
Recently, climate change and drought, population growth and, consequently, the increasing domestic and industrial demand for water consumption have led to water shortages in many parts of the world. Moreover, the inevitable aging of pipeline infrastructures in urban water supplies leads to an undesirable increase in leakage and burst frequencies. Increasing pressure, inadequate design, improper construction and operation, and pipe corrosion intensify this problem. Water losses from source to consumers account for 15% to 40% of the total water supply and, in some cases, up to 80%, as reported by Maksimovic et al. (2001). As a result, leak control and reduction and demand management are high priorities for managers and decision-makers in the urban water industry. Therefore, water companies in
different countries investigate and diagnose problems to offer better water supply systems (Ghazali 2012).

In recent years, the application of polymer pipes (such as polyethylene (PE) and polyvinyl chloride (PVC)) has been increasing day by day because of their technical and economic advantages over pipes of other materials, such as steel, cast iron, concrete, and asbestos. Transient flow modeling and analysis in polymer pipes have some fundamental differences compared to non-polymer pipes, mainly related to the interaction of fluid oscillations with the pipe wall. Polymers generally exhibit a viscoelastic mechanical behavior, affecting the intensity, formation, and damping of pressure fluctuations in transient flow.

In the last three decades, various fault detection methods in water pipelines have been developed based on steady or transient flow hydraulics. By generating a transient event in transient-based fault detection, a periodical pressure wave propagates throughout the system, affected by its characteristics and possible faults (e.g., leakage, blockage, or air pocket). The system defects can then be identified by collecting transient pressure waves at several points of the system and processing the relevant signals.

Generally, studies related to transient-based defects detection can be classified in two categories: the frequency-domain techniques (Kim 2005; Duan et al. 2011; Gong et al. 2013; Duan 2016; Kim 2018; Al-Tofan et al. 2020) and time-domain techniques (Covas 2003; Vitkovsky et al. 2007; Rahmanshahi et al. 2018; Sarkamaryan et al. 2018; Keramat & Zanganeh 2019; Keramat et al. 2019). Inverse transient analysis (ITA) is a time-domain defect detection method that is applied mostly to simple water supply systems. Covas & Ramos (2010) used transient-based techniques for leakage detection in viscoelastic water supply systems, for which they employed ITA in a step-wise manner. In the first step, the leakage candidates were spread along the entire length of the pipe. In the next steps, they began to gradually concentrate around the significant leakage points from the previous steps. The results of said study showed that leakage candidates have a normal-like distribution around the actual leakage. Sarkamaryan et al. (2018) applied ITA for leak detection in benchmark elastic pipe networks. They modeled the leakage as a quasi-normal distribution using the Gaussian function. Through this approach, the number of decision variables, multimodality, and ITA complexity was reduced. To better understand this method’s capabilities, they suggested evaluating it based on real or laboratory pipe systems. So far, this method has not been evaluated based on laboratory or field results.

The current study aimed to evaluate the Gaussian function in estimating leakage characteristics in viscoelastic pipelines based on a suitable laboratory model. For this purpose, a sensitivity analysis was conducted to determine the location and size of leaks relative to the dynamic parameters, flow regime, sample size, space-step increment, and leak size and location. This method’s capability in estimating single leakage in several laboratory samples and one sample with two leaks was evaluated.

### MATERIALS AND METHODS

#### Equations governing the transient flow

The equations governing transient flow in closed conduits include the equations of conservation of mass and momentum. Assuming a control volume and using the Reynold transport theorem for a differential component of the fluid motion and considering the two dynamic terms of the unsteady friction and the pipe wall viscoelasticity effects, these two partial differential equations can be deduced as follows (Soares et al. 2008; Evangelista et al. 2015; Keramat et al. 2020):

\[
\frac{dH}{dt} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} + \frac{2a^2}{g} \frac{d\varepsilon_t}{dt} = 0
\]

(1)

\[
\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{dQ}{dt} + (h_{f_l} + h_{f_u}) = 0
\]

(2)

where \(Q\) = instantaneous flow rate, \(H\) = instantaneous piezometric head, \(A\) = pipe cross-sectional area, \(a\) = pressure wave speed, \(g\) = gravity acceleration, \(\varepsilon_t\) = retarded strain, \(x\) = distance along the pipe axis, \(t\) = time, and \(h_{f_l}\) and \(h_{f_u}\) are steady and unsteady friction losses per unit length, respectively.

Laminar and turbulent flow friction losses can be calculated from the analytical equation of Darcy Weisbach \((h_{f_i} = (f/D)(Q/2gA^2))\), where \(D\) is the internal diameter of the pipe and \(f\) is the Darcy-Weisbach friction loss.
Coefficient. For laminar flow conditions (Re < 2000), a steady-state friction factor was calculated using the Hagen-Poiseuille formula (Equation (3)). According to Covas (2003), the flow regime in polyethylene pipes is smooth turbulent. Consequently, for turbulent flows (Re > 4000), a steady-state friction factor is calculated with the Blasius formula (Equation (4)), which is particularly adequate for 4000 ≤ Re ≤ 10^5. Furthermore, according to Axworthy (1997) and Covas (2003), a linear relationship was defined to obtain a continuous friction formula in the critical zone (2000 ≤ Re ≤ 4000):

\[ f = \frac{64}{Re} \]  
\[ f = 0.316 \text{ Re}^{-0.25} \]  

where Re = \(VD/ν\) is Reynolds number, \(V\) is average flow velocity, and \(ν\) is kinematic fluid viscosity. According to Covas et al. (2005), other local head losses at elbows and fittings were assumed to be an additional 5% of total energy losses.

A literature search revealed that to date, various relationships have been extracted to estimate unsteady friction loss oscillations are simulated based on instantaneous local and transition acceleration. Vitkovsky et al. (2000) expressed the improved model of Brunone et al. (1991) as follows:

\[ h_{tu} = \frac{k'}{gA} \left( \frac{\partial Q}{\partial t} + a \text{SGN}(Q) \frac{\partial Q}{\partial x} \right) \]  

where \(k'\) is the decay factor, and SGN is the operator for the sign.

Strain (\(ε\)), due to specific stress, is expressed as a sum of an instantaneous-elastic strain (\(ε_0\)) and retarded-viscous strain (\(ε_r\)). For small strains, by applying continuous stress \(σ(t)\) to the polyethylene pipe, the total strain, according to Boltzmann principle, is equal to:

\[ ε(t) = J_0 σ(t) + \int_{0}^{t} σ(t - t') \frac{∂J(t')}{∂t'} dt' \]  

where \(J_0\) is the instantaneous creep compliance function, and \(J(t')\) is the creep compliance function at time \(t'\).

The creep compliance function for polyethylene pipes as a viscoelastic solid can be obtained using the generalized Kelvin-Voigt model within the linear ranges of the pipe wall’s viscoelastic behavior (Aklonis et al. 1972),

\[ J(t) = J_0 + \sum_{k=1}^{N_{KV}} J_k (1 - e^{-t/\tau_k}) \]  

in which \(J\) is the creep compliance function, \(J_k\) represents the creep of the spring of the Kelvin-Voigt \(k\)-element defined as \(J_k = 1/ε_k\), \(τ_k\) is the retardation time of the dashpot of the \(k\)-element, \(E_k\) is the modulus of elasticity of the spring of the \(k\)-element, \(ν_k\) is the viscosity of the dashpot of the \(k\)-element, and \(N_{KV}\) is the number of Kelvin-Voigt elements.

This set of differential equations was solved using the method of characteristics (Wylie & Streeter 1993; Covas et al. 2005), and to complete the calculations in each time step, the upstream (pressure vessel) and downstream (transient valve) boundary conditions were considered.

**Inverse transient analysis method**

ITA is a well-known procedure for the calibration and defect detection of pipeline systems. In recent years, this method has been widely used for leakage detection in water transmission pipelines (Kapelan et al. 2003; Covas et al. 2005). It uses the collected data to estimate the system’s unknown parameters. In this method, the system’s behaviors are simulated using a hydraulic transient solver as a function of unknown parameters. The difference between the measured and computational values is minimized using the optimization model, and unknown parameters are specified. In this study, the measurement data used in the ITA is the pressure signal upstream of the transient valve. Usually, experimental models’ pressure signals are polluted with some high-frequency noises due to environmental conditions. For this purpose, the measured pressure data was first passed through the Butterworth low-pass filter and then called in the ITA. The general flowchart of the ITA method is shown in Figure 1.

The genetic algorithm (GA) was used as the optimization method in this research. According to Equation (8), the average least-square errors (ALSE) are defined as the
optimization’s objective function:

\[ \text{Min } \text{OF}(p) = \frac{\sum_{i=1}^{M} [q_i^2 - q_i(p)]^2}{M} \]  

where \( \text{OF}(p) \) = objective function, \( p \) = decision variables vector, \( q \) = the observation pressure, \( M \) = number of elements of measured pressure, and \( q(p) \) = predicted system response for a given vector \( p \). In this study, \( p \) includes all decision variables, namely quasi-normal function constants, creep function parameters, and the unsteady friction model’s decay coefficient.

Since the number of leakages is unknown at the beginning of the solution, the leak detection process is carried out in several steps in traditional methods. Initially, a large number of leak candidates were scattered throughout the pipeline in a specific location. In the next steps, the leaks were spread around the previous step’s justifiable values (Covas 2003). Given the multi-step nature of the process and the multitude of leak candidates, this method is more time-consuming and burdensome to solve. According to Covas (2003) and Covas & Ramos (2010), leakage candidates in the traditional method have a quasi-normal distribution around the actual leakage. Hence, in the present study, the leakage candidates are introduced into the numerical model as a quasi-normal Gaussian function (Figure 2), in which \( e \) is Napier’s number, and \( b, c, \) and \( d > 0 \) are Gaussian function constants (GFCs). Thus, each errant leakage introduces three decision variables to the ITA. The parameter \( b \) indicates the leakage area size in each node, \( c \) is the leak focus position, and \( d \) represents the leakage distribution in adjacent nodes.

Unlike traditional methods, this function covers the entire pipe, and all nodes can have a potential leakage. With this trick, the number of decision variables was reduced and, consequently, the speed and precision of the ITA was increased.

The leak discharge \( Q_L \) (\( m^3/s \)) was described by the orifice law, which depends on the piezometric-head at the
leak location and the orifice shape as follows:

\[ Q_L = A_e \sqrt{2gh} \]

(9)

where \( A_e \) = effective orifice area; \( H \) = piezometric-head at leak location; and \( g \) = gravity acceleration.

The criteria for relative errors of the leaks’ location and size are defined according to Equations (10) and (11), respectively. These indicators are defined to evaluate the ITA performance in determining the leaks’ location and size:

\[ \varepsilon_{L(location)} = \frac{|X - X_{true}|}{L}, \quad X = \frac{\sum_{i=1}^{N_L} L_i(A_e)_i}{N_L} \]

(10)

\[ \varepsilon_{L(area)} = \frac{|A_e - A_{e(true)}|}{A_{e(true)}}, \quad A_e = \sum_{i=1}^{N_L} (A_e)_i \]

(11)

in which \( X \) and \( X_{true} \) are estimated and true leak distance from the upstream boundary, respectively. \( L \) is total pipe length, \( A_e \) and \( A_{e(true)} \) are estimated and true effective area of leakage, respectively, and \( N_L \) is the number of leaked nodes.

Experimental setup and collected data

In line with this research’s objectives, a laboratory pipeline model was constructed in Shahid Chamran University of Ahvaz using viscoelastic polyethylene pipes capable of generating transient events and leakage (Figure 3). This transient flow system included a centrifugal single head pump (\( n = 2,900 \text{ rpm} \)) and a pressure tank with a total volume of 650 l at the upstream end, standard high-density polyethylene pipes (HDPE, PE100, length, inner diameter, and thickness were 158 m, 5.05 cm, and 6.5 mm, respectively), a water hammer valve, and a flow regulator valve. The pipeline was looped and fixed along with an elliptical metal frame using metal clamps.

In this research, a globe valve regulated the flow rate, and a ball valve generated transient flow. In all experiments, transient events were generated by the full closure of the downstream ball valve, which was equipped with a timer with an accuracy of 0.001 seconds to measure the closing time. Discharge at the end of the pipeline and leakage were measured by the volumetric method. The pressure signal was collected in four locations (in the pressurized tank, at both leak locations, and upstream of the ball valve) using WIKA pressure transmitters (with a pressure range of 0.0 to 16.0 bars and a full-scale accuracy of 0.1%). The sampling frequency during transient flow was 1,000 Hz.

RESULTS AND DISCUSSION

Numerical model calibration

In simple intact viscoelastic pipeline systems (reservoir-pipe-valve), by defining the upstream and downstream boundary conditions, including reservoir level and valve maneuvers, the ITA usually involves estimating steady and unsteady friction coefficients, pressure wave speed, and creep function. In this study, the inside air pressure was measured by a transducer, and the inside water level was introduced to the numerical model as the upstream boundary condition. The downstream boundary condition was a ball valve that was manually closed to generate a transient event. Considering that the valve maneuver depended on various factors, its closure-time and curve-shape were associated with uncertainties. Consequently, the collected pressure head next to the transient valve was introduced to the model during the valve closure-time. In all experiments, valve maneuvers were fast with \( T_c < 2L/a \) (\( T_c = \text{valve closer time} \)).

In this study, the steady-state friction coefficients were estimated based on steady-state flow conditions. As mentioned earlier, the steady friction coefficient was directly calculated from the quasi-steady friction using the Hagen-Poiseuille relation for laminar flow and the Blasius relationship for turbulent flow.

Unsteady friction, pressure wave speed, and pipe wall viscoelastic effects have overlapping effects in polymer pipes. Moreover, the wave speed and the pipe wall’s viscoelasticity depend highly on the pipeline’s axial and circumferential constraints and its stress-time history. Therefore, the calibration of the overlapping parameters needs further discussion.
Sensitivity analysis of leakage detection

As mentioned earlier, the accuracy of transient-based leak detection in pipeline systems depends on several parameters, including (i) transient solver accuracy, (ii) sample size (i.e. simulation-time), (iii) spatial-step of the characteristic lines method, (iv) the leak location and size, (v) pipeline flow-rate, (vi) uncertainty associated with system's characteristics and collected data, and (vii) optimization algorithm accuracy. The current research focused on the analysis of the first five. Based on the preliminary sensitivity analysis, first, the number of Kelvin-Voigt elements and their relaxation times were determined. According to this analysis, a total of three Kelvin-Voigt elements is optimal for modeling, and the values of the two initial relaxation times were 0.05 and 0.5, respectively. Gally et al. (1979) and Covas (2003) also used three Kelvin-Voigt elements for modeling viscoelastic pipelines. The third relaxation time $\tau_3$ value is calibrated simultaneously with other decision variables depending on the tested sample size ($\tau_2 < \tau_3 \leq \text{samplesize}$).

This section is organized into three main subsections. First, the hydraulic transient solver accuracy and the appropriate approach for estimating decision parameters were evaluated. Secondly, the appropriate sample size for the pressure signal was assessed to accurately determine the leak location and size. Finally, a suitable spatial-step was determined for numerical simulation. In the sensitivity analysis stage, the inverse solution model was implemented with one leak, and the scenario with the least errors in leak location and size was selected as the appropriate model. All scenarios were carried out for two sets of experimental data with leak location at 117.4 m from the upstream boundary. The $A_r$ first and second leak parameters were set at 1.98 E-05 ($Q_L = 0.57 \text{ l/s}$, $Q_L$ is leak discharge) and 1.99 E-05


\( Q_L = 0.58 \text{ l/s} \), and the downstream flow-rates were 1.0 and 0.5 l/s, respectively.

**Analysis of transient solver accuracy**

Hydraulic transient solver calibration is usually understood as the definition of boundary conditions, and the estimation of other unknown parameters includes steady and unsteady friction, pressure wave speed, creep function, and water leaks. As mentioned earlier, boundary conditions are measured and steady-state friction coefficients are calculated directly. The major challenge is the calibration of unsteady friction, pressure wave speed, and creep function due to their overlapping effects on the transient pressure response. Another challenge is whether to simultaneously or separately model the leakage with other unknown parameters. For this, assuming constant dynamic parameters for the hydraulic transient solver, these parameters were calibrated for experimental data without a simulated leak for the HTS with and without unsteady friction. Figure 4(a) compares experimental and numerical pressure signals, and Figure 4(b) and 4(c) show calibrated creep coefficients and creep function, respectively. Ten different approaches were used herein to calibrate these unknown parameters with overlapping effects to analyze transient solver accuracy.

In the first scenario, leak detection was performed using the classical (elastic) model of HTS (hydraulic transient solver), in which the unknown parameters include the Gaussian function coefficients (GFCs) and pressure wave speed \( a \). As expected, classic HTS led to significant discrepancies between numerical and real characteristics of leaks (Figure 5(a)). In the next two scenarios, the intact system’s calibrated parameters were used in the corresponding HTS with and without unsteady friction, and the only unknowns of the ITA were the GFCs. Like the first scenario, the numerical results obtained from these scenarios had large discrepancies with the real leaks, and the leak location was determined at the downstream end of the pipeline (Figure 5(b) and 5(c)).

In scenarios 4-6 and 7-8, HTS was considered with and without unsteady friction, respectively. In these scenarios, each decision variable’s effect of separate calibration on the ITA accuracy was investigated by calibrating a single decision variable with GFCs in each scenario. The value of other parameters was set from the calibration of the intact pipeline. The results showed that the HTS is more sensitive to the creep function and pressure wave speed, because incorrect estimation of these parameters dramatically affects the pressure signal’s shape and phase shift (Figure 5(d)–5(h)).

In the last two scenarios, all decision parameters were calibrated simultaneously in the leak’s presence using the HTS model with and without unsteady friction. Figure 5(i) and 5(j) illustrate that both scenarios’ results are in good agreement with the real leak characteristics. Thus, the developed HTS model incorporates only viscoelasticity to accurately predict transient pressure signals in polymer pipes. According to Duan et al. (2010), the results for unsteady friction are quite expected, as its importance increases with time, and in a short period of analysis, its relevance is small.

Figure 4 | (a) The numerical and measured pressure heads at TR4, (b) calibrated creep coefficients, and (c) creep function for the HTS with and without unsteady friction.
Figure 5: (a–j) The detected leakages’ Gaussian functions, and (k) leak’s location and size errors for various HTS model.
The collected transient signals usually contain high-frequency noise due to environmental conditions. In this study, a Butterworth low-pass frequency filter was used to remove these noises. Butterworth is a frequency filter that breaks down a signal into subsignals by transmitting a time-domain signal to the frequency domain. Then, depending on the physics of the problem, high/low-noise signals can be removed. However, to investigate the effects of noise on ITA’s failure, the last two scenarios were also modeled based on unfiltered data. The results of these models are plotted on their corresponding diagrams. Based on the calculated error, leakage detection using noisy signals showed some disagreement with actual data, especially in the HTS involving unsteady friction. Details of calibrated decision variables for various scenarios are shown in Table 1.

The creep function is an inherent feature of viscoelastic materials and can be obtained by mechanical tests at different temperatures, or alternatively, assuming similar isentropic materials. Creep compliance, however, is a function with several sources of uncertainty, such as the stress-time history and the axial and circumferential constraints. Hence, similar to the last scenario, the creep compliance function should be calibrated using an accurate HTS and ITA coupling in an operation stage.

### Sample size assessment

A sensitivity analysis was carried out in this subsection to determine the optimal sample size for leakage location and size. For this purpose, the last two scenarios of the previous section were used for four sample sizes that were analyzed corresponding to multiples of the theoretical

### Table 1 | Calibrated decision variable for various HTS model

| Scenario No. | Leak No. | $T_c$ (s) | $b$ | $c$ | $d$ | $\tau_2$ | $\tau_3$ | $\tau_4$ | $\tau_5$ | $a$ | $k^*$ |
|--------------|---------|-----------|-----|-----|-----|----------|----------|----------|----------|-----|-------|
| a            | I       | 0.1       | 3.25 | 151.84 | 1.17 | -        | -        | -        | -        | 385 | -     |
|              | II      | 0.07      | 2.98 | 151.88 | 1.17 | -        | -        | -        | -        | 390 | -     |
| b            | I       | 0.1       | 2.91 | 151.79 | 1.13 | -        | -        | -        | -        | 385 | -     |
|              | II      | 0.07      | 2.62 | 151.68 | 1.01 | -        | -        | -        | -        | 390 | -     |
| c            | I       | 0.1       | 2.63 | 152.32 | 1.29 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 2.50 | 151.71 | 0.98 | -        | -        | -        | -        | 390 | -     |
| d            | I       | 0.1       | 3.19 | 119.48 | 0.93 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 2.62 | 120.26 | 0.71 | -        | -        | -        | -        | 390 | -     |
| e            | I       | 0.1       | 2.46 | 114.09 | 0.89 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 2.24 | 114.12 | 0.66 | -        | -        | -        | -        | 390 | -     |
| f            | I       | 0.1       | 1.67 | 102.41 | 0.67 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 1.63 | 104.61 | 0.62 | -        | -        | -        | -        | 390 | -     |
| g            | I       | 0.1       | 3.85 | 125.45 | 0.41 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 2.58 | 128.25 | 0.91 | -        | -        | -        | -        | 390 | -     |
| h            | I       | 0.1       | 2.38 | 114.14 | 0.70 | -        | -        | -        | -        | 390 | -     |
|              | II      | 0.07      | 2.58 | 116.65 | 0.51 | -        | -        | -        | -        | 390 | -     |
| i            | I       | 0.1       | 2.03 | 122.77 | 0.76 | -        | -        | -        | -        | 390 | 0.002 |
|              | I (UF)  | 0.1       | 3.79 | 117.20 | 0.93 | -        | -        | -        | -        | 411 | 0.04  |
|              | II      | 0.07      | 1.94 | 119.80 | 0.55 | -        | -        | -        | -        | 404 | 0.002 |
|              | II (UF) | 0.07      | 3.22 | 103.04 | 0.57 | -        | -        | -        | -        | 408 | 0.04  |
| j            | I       | 0.1       | 2.15 | 119.87 | 0.49 | -        | -        | -        | -        | 390 | -     |
|              | I (UF)  | 0.1       | 2.51 | 119.81 | 0.74 | -        | -        | -        | -        | 408 | -     |
|              | II      | 0.07      | 1.97 | 120.25 | 0.67 | -        | -        | -        | -        | 402 | -     |
|              | II (UF) | 0.07      | 2.12 | 119.39 | 0.68 | -        | -        | -        | -        | 407 | -     |

*Calibrated based on the intact pipeline.
period of the pressure wave \( T \) (i.e. 0.5\( T \), \( T \), 2\( T \), and 3\( T \)). This analysis was also performed for the two leaks used in the previous step.

Different sample size modeling results are shown in Figure 6(a)–6(d). Moreover, errors in leak locations and sizes are shown in Figure 6(e) and 6(f), respectively. The results showed that the smallest leak location error corresponds to the half period. Due to the wave’s intense damping in polymer pipes, this reflection effect decreases over time from the onset of transient flow. As a result, the uncertainty of detecting the location of the leak increases. Moreover, sounds that estimate leak size depend on the overall damping of the wave. Therefore, the optimal sample size for estimating leak size is one period of the pressure signal. It is noteworthy that in the larger sample size, because of severe wave attenuation and increasing uncertainties of the numerical and experimental model over time, the accuracy of the leak size estimation decreased. Details of calibrated decision variables for various sample sizes are shown in Table 2.

**Evaluating spatial step**

This subsection highlights the importance of the spatial step to the successful application of ITA. Thus, to assess the spatial-step effect on leak detection accuracy, six different spatial-steps, ranging between 0.006 and 0.053 of the total pipe length, were selected. Similar to the previous steps, the model with the viscoelastic effect only (last scenario) gave the best results, so in this part, this model was used. This analysis was also performed for the two leaks used in the previous steps. It should be noted that the spatial-step affects the accuracy of solving the MOC and the leak modeling function. Errors in leak locations and sizes for both sets of experimental data are shown in Figure 7(a) and 7(b), respectively. As shown, the accuracy of the ITA results was increased by selecting the
spatial-step size between 0.019 and 0.032 of the pipe length. Selecting a smaller spatial-step increase the ITA runtime and reduces the accuracy of detection of leak characteristics. Details of calibrated decision variables for different spatial steps are shown in Table 3.

### Validation of calibrated and analyzed model

The results showed that despite the pipe wall’s viscoelastic effects, the HTS simulated the pressure wave and detected leakages in polymer pipes in the previous sensitivity analysis stages. Thus, the unsteady friction can be ignored. Therefore, in the validation phase, the last scenario of Figure 5 was used in which the numerical model included only the dynamic effect of the pipe wall viscoelasticity, and all the design variables were calibrated simultaneously. In the sensitivity analysis stage, the purpose was to analyze the sensitivity of a particular parameter. Thus, knowing that there is a leak in the system, a leak candidate (Gaussian function) was considered in the inverse solution model. In this study, similar to real systems, it was assumed that the number of leaks in the system is unknown, and ITA was

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**Table 2** Calibrated decision variable at different sample size

| Scenario No. | Leak No. | $T_c$ (s) | $b$ | $c$ | $d$ | $\tau_1$ | $j_1$ | $j_2$ | $j_3$ | $a$ | $k'$ |
|--------------|---------|----------|-----|-----|-----|---------|-------|-------|-------|-----|-----|
| a (0.5T)    | I       | 0.1      | 2.07| 117.89 | 0.79 | 0.61    | 1.21  | 1.52  | 0.90  | 400 | –   |
|             | I       | 0.1      | 1.85| 119.94 | 0.59 | 0.62    | 1.00  | 1.80  | 1.44  | 400 | 0.033 |
|             | II      | 0.07     | 1.77| 117.04 | 0.55 | 0.52    | 1.28  | 1.07  | 1.08  | 405 | –   |
|             | II      | 0.07     | 1.66| 119.98 | 0.77 | 0.78    | 1.30  | 1.43  | 1.18  | 406 | 0.013 |
| b (T)       | I       | 0.1      | 2.15| 119.87 | 0.49 | 0.78    | 0.85  | 1.33  | 1.32  | 400 | –   |
|             | I       | 0.1      | 2.05| 122.77 | 0.76 | 0.77    | 0.75  | 1.52  | 1.40  | 398 | 0.0001 |
|             | II      | 0.07     | 1.97| 120.25 | 0.67 | 0.73    | 0.86  | 1.27  | 1.31  | 402 | –   |
|             | II      | 0.07     | 1.94| 119.80 | 0.55 | 0.89    | 0.99  | 0.82  | 1.86  | 404 | 0.002 |
| c (2T)      | I       | 0.1      | 2.05| 122.45 | 0.96 | 1.95    | 0.76  | 1.51  | 1.67  | 399 | –   |
|             | I       | 0.1      | 1.80| 119.65 | 0.98 | 1.77    | 0.78  | 1.66  | 1.86  | 400 | 0.0001 |
|             | II      | 0.07     | 1.76| 122.53 | 0.90 | 2.28    | 0.90  | 1.47  | 1.67  | 403 | –   |
|             | II      | 0.07     | 1.72| 119.54 | 0.68 | 1.7     | 0.82  | 1.84  | 1.65  | 403 | 0.003 |
| d (3T)      | I       | 0.1      | 2.23| 119.42 | 0.98 | 2.14    | 1.04  | 1.13  | 1.64  | 404 | –   |
|             | I       | 0.1      | 1.76| 116.71 | 0.57 | 1.75    | 0.66  | 1.87  | 2.10  | 400 | 0.009 |
|             | II      | 0.07     | 1.67| 128.47 | 0.98 | 2.99    | 0.87  | 1.42  | 1.71  | 403 | –   |
|             | II      | 0.07     | 1.63| 116.78 | 0.94 | 2.8     | 0.90  | 1.83  | 1.04  | 405 | 0.004 |

**Figure 7** (a) Leak’s location errors, and (b) leak’s size errors at different spatial-step used in ITA for both experimental tests.
run with at least two leak candidates. As mentioned earlier, if the estimated leak locations and sizes were significantly different, the number of candidate leaks would increase in each step over time. This procedure continues until one of the leaks is small or two leaks are adjacent. In the first step, if the model has only one leak, one of the two candidate leaks will have a small amount, or they will fall in the vicinity. Once the number of leaks is specified, the error is calculated based on that number.

**Single leak with different sizes and locations**

In this section, the ITA was performed for 11 new experimental datasets to investigate the effects of leakage location and size on leakage detection accuracy. The modeling results are shown in Table 4. Results indicated that ITA could locate and quantify leakages in most of the analyzed tests, although leak location and size errors depended on leak size and location. In most cases, these errors were less than 5%, which corresponds to 4.74 m in the pipeline. In conclusion, ITA is very promising for use in identifying the leakage range in water pipeline systems and can be combined with other local leak detection techniques such as acoustic equipment.

### Multiple leaks detection

Transient-based ITA was tested for the detection of two simultaneous leaks in the pipeline. According to Figure 3, the first leak was located at \( X = 56.3 \, \text{m} \) (from the upstream) with \( A_e = 1.15 \, \text{E-05 m}^2 \) (\( Q_L = 0.57 \, \text{l/s} \)), and the second at \( X = 117.4 \, \text{m} \) with \( A_e = 1.94 \, \text{E-05 m}^2 \) (\( Q_L = 0.33 \, \text{l/s} \)). The pipeline end flow-rate was 0.95 l/s. Similar to what was described earlier, ITA was run in a step-wise manner for both 0.5T and T sample sizes. In both sample sizes, ITA was implemented in the first step with two leak candidates. Because of the significant values of leakages in the first step, the models were run with three leak candidates in the second step. Figure 8 depicts the optimal solutions of both sample sizes in the two steps, and the respective errors are presented in Table 5. ITA points to two leakage locations

| Leak No. \( \Delta x \) (m) | \( T_c \) (s) | Sample size | \( b \) | \( c \) | \( d \) | \( r_2 \) | \( J_1 \) | \( J_2 \) | \( J_3 \) | \( a \) |
|-----------------------------|-------------|-------------|-----|-----|-----|-----|-----|-----|-----|-----|
| I                           | 8.32        | 0.1         | 0.5T| 2.01| 116.30| 1.76| 0.63| 1.55| 1.56| 1.10| 407 |
|                             | T           | 2.25        | 124.77| 1.09| 0.85| 0.85| 0.95| 1.54| 404 |
|                             | 5.10        | 0.5T        | 2.04| 117.17| 1.05| 0.59| 1.32| 1.36| 1.29| 403 |
|                             | T           | 2.00        | 122.35| 1.08| 1.03| 0.71| 1.75| 1.55| 400 |
|                             | 4.05        | 0.5T        | 2.03| 117.50| 0.95| 0.65| 1.61| 1.38| 0.91| 405 |
|                             | T           | 2.12        | 121.35| 0.82| 1.34| 0.78| 1.65| 1.68| 400 |
|                             | 2.93        | 0.5T        | 2.07| 117.09| 0.79| 0.61| 1.21| 1.52| 0.90| 400 |
|                             | T           | 2.15        | 119.87| 0.49| 0.78| 0.85| 1.33| 1.32| 400 |
|                             | 2.00        | 0.5T        | 1.83| 117.98| 0.39| 0.57| 1.23| 1.55| 1.32| 399 |
|                             | T           | 2.32        | 120.06| 0.43| 0.88| 0.96| 1.10| 1.18| 400 |
|                             | 1.00        | 0.5T        | 1.68| 114.71| 0.22| 0.63| 1.29| 1.62| 1.49| 397 |
|                             | T           | 2.08        | 121.76| 0.31| 1.07| 0.87| 1.41| 1.50| 397 |
| II                          | 8.32        | 0.07        | 0.5T| 1.83| 116.49| 1.68| 0.5 | 1.44| 1.36| 0.70| 410 |
|                             | T           | 1.90        | 124.71| 2.31| 1.2 | 0.84| 1.57| 0.90| 406 |
|                             | 5.10        | 0.5T        | 2.15| 117.77| 1.13| 0.59| 1.34| 1.22| 0.64| 407 |
|                             | T           | 2.01        | 122.59| 1.02| 0.92| 0.82| 1.58| 0.46| 406 |
|                             | 4.05        | 0.5T        | 1.85| 116.91| 0.65| 0.6 | 1.19| 1.15| 1.03| 404 |
|                             | T           | 1.90        | 119.92| 0.69| 0.74| 0.97| 1.17| 1.13| 403 |
|                             | 2.93        | 0.5T        | 1.77| 117.04| 0.55| 0.52| 1.28| 1.07| 1.08| 405 |
|                             | T           | 1.97        | 120.25| 0.67| 0.73| 0.86| 1.27| 1.31| 402 |
|                             | 2.00        | 0.5T        | 1.82| 117.13| 0.84| 0.65| 1.36| 1.09| 0.95| 405 |
|                             | T           | 2.01        | 119.81| 0.42| 0.86| 0.99| 1.08| 1.39| 404 |
|                             | 1.00        | 0.5T        | 1.84| 116.90| 0.80| 0.62| 1.34| 1.12| 0.91| 405 |
|                             | T           | 1.97        | 118.64| 0.29| 0.84| 1.02| 1.43| 0.88| 402 |
and sizes with normal-like distributions around the real leakages. The results revealed that the candidate’s additional leak with a small value was adjacent to the leak farther from the second step’s transient valve. In general, the estimated leak values were less than their real values.

**CONCLUSIONS**

The transient-based ITA method is a well-known approach for the calibration and defect detection of water pipeline systems. Although this model-based approach seems relatively simple to apply in pipelines, its accuracy depends on various parameters. This study investigated the sensitivity of leak location and size in viscoelastic pipelines relative to the dynamic parameters, flow regime, sample size, spatial-step increment, noisy data, and leak size and location. This was approached by developing a hydraulic transient solver, including the dynamic effects of unsteady friction, viscoelastic effects of the pipe wall, the leak, and the collection of transient data for the sensitivity analysis and validation of this model. The leakage was assumed to function with a quasi-normal distribution around its real location to reduce the problem dimensionality and unnecessary computations. The ITA approach determined the function coefficients and other unknown parameters as the decision variables. The results showed that ITA successfully detected leakages’ locations and sizes when the leak and creep function were simultaneously calibrated. HTS incorporates viscoelasticity effects that can reasonably describe the transient pressure response of polyethylene pipes, and therefore, unsteady friction can be neglected for defect detection. Optimal sample size assessment showed that leak location and size could be accurately estimated using a sample size equal to a half and a single period of the pressure signal, respectively. The spatial-step

**Table 4** | Validation’s results of the single leak with different sizes and locations by the ITA method

| Leak | $X_{true}$ (m) | $Q$ (l/s) | $A_{eq}$ (m²) | Sample size | $\varepsilon_L$ (location) | $\varepsilon_L$ (area) |
|------|----------------|-----------|---------------|-------------|---------------------------|---------------------|
| Leak 1 | 56.3 | 0.909 | 1.520E-5 | 0.5T | 0.17% | 19% |
|       |     | 1.004 | 1.907E-5 | 0.5T | 0.92% | 25% |
|       |     | 1.071 | 2.605E-5 | 0.5T | 0.79% | 27% |
|       |     | 0.905 | 3.376E-5 | 0.5T | 1.70% | 26% |
| Leak 2 | 117.4 | 1.003 | 0.141E-5 | 0.5T | Leak not detectable | |
|       |     | 0.846 | 0.261E-5 | 0.5T | Leak not detectable | |
|       |     | 1.000 | 0.813E-5 | 0.5T | 3.28% | 113% |
|       |     | 1.071 | 1.185E-5 | 0.5T | 9.98% | 77% |
|       |     | 1.019 | 1.991E-5 | 0.5T | 2.15% | 26% |
|       |     | 1.100 | 2.731E-5 | 0.5T | 6.94% | 4% |
|       |     | 0.850 | 3.550E-5 | 0.5T | 0.47% | 31% |

**Table 5** | Two leaks locations and sizes results based on ITA

| Sample size | Leak No. | $X_{true}$ (m) | $Q$ (l/s) | $A_{eq}$ (m²) | $\varepsilon_L$ (location) | $\varepsilon_L$ (area) |
|-------------|----------|----------------|-----------|---------------|---------------------------|---------------------|
| 0.5T | 1 | 56.3 | 0.57 | 1.98E-5 | 1.46 | 19.35 |
|     | 2 | 117.4 | 0.33 | 1.15E-5 | 2.18 | 36.93 |
| T | 1 | 56.3 | 0.57 | 1.98E-5 | 3.94 | 22.16 |
|     | 2 | 117.4 | 0.33 | 1.15E-5 | 2.09 | 23.79 |

**Figure 8** | The detected two leakages’ Gaussian functions for 0.5T and T sample sizes.
analysis achieved acceptable results with a spatial step-to-length ratio between 0.019 and 0.052. The ITA was tested for a system with two leaks, and it was found that this approach could also detect multiple leaks.

**DATA AVAILABILITY STATEMENT**

All relevant data are included in the paper or its Supplementary Information.

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First received 18 November 2020; accepted in revised form 7 May 2021. Available online 17 May 2021