A defect of the electromagnetism and it serves as a hidden variable for quantum mechanics

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Abstract

According to the theory of relativity, the 4-vector force acting on a particle is orthogonal to the 4-vector velocity of the particle, but we found that the electromagnetic force of the Maxwell’s theory can not completely satisfy this orthogonality, this incompletion leads us to find that the electromagnetic force has two independent components in the 4 dimensional space time. A 4-vector force has 4 components, because of the orthogonality of 4-vector force and 4-vector velocity, the number of independent components of the force reduces to 3, while the electromagnetic force can merely provide 2 independent components, this situation means that there is an undefined component accompanying the electromagnetic force. This missing undefined component may serve as a hidden variable which quantum mechanics is eager to find for a long time. The primary purpose of this paper is to strictly proof that the electromagnetic force has two independent components in the 4 dimensional space time. We also briefly discuss the possibility that the undefined component of the electromagnetic force serves as a hidden variable for quantum mechanics.

1 Introduction

In the theory of relativity, the 4-vector force acting on a particle is orthogonal to the 4-vector velocity of the particle, because the magnitude of the 4-vector velocity $u$ keeps constant as

$$|u| = \sqrt{u_{\mu}u_{\mu}} = \sqrt{-c^2} = ic$$

(1)

Any force acting on the particle can never change $u$ in the magnitude but change $u$ in the direction, thus the 4-vector force is orthogonal to the 4-vector velocity. Strictly, the proofing of the orthogonality is given by

$$u_{\mu}f_{\mu} = u_{\mu}m\frac{du_{\mu}}{d\tau} = \frac{md(u_{\mu}u_{\mu})}{2} = 0$$

(2)

The orthogonality does not depend upon any property of 4 vector force, holds for any kinds of forces. Where the frame of reference is a Cartesian coordinate system whose axes are orthogonal to one another, there is no distinction between covariant and contravariant components, only subscripts need to be used, $m$ denotes the mass of the particle, the 4 dimensional space time refers to $(x_1, x_2, x_3, x_4 = ic\tau)$, the index $\mu$ takes over 1, 2, 3, 4.

We found that the electromagnetic force of the Maxwell’s theory can not completely satisfy this orthogonality, this incompletion leads us to find that the electromagnetic force has two independent components in the 4 dimensional space time. A 4-vector force has 4 components, because of the orthogonality of 4-vector force and 4-vector velocity, the number of independent components of the force reduces to 3, while the electromagnetic force can merely provide 2 independent components, this situation means that there is an undefined component accompanying the electromagnetic force — does this missing undefined component is a hidden variable?— quantum mechanics is eager to find it for a long time.

The primary purpose of this paper is to strictly proof that the electromagnetic force has two independent components in the 4 dimensional space time. We also briefly discuss the possibility that the undefined component of the electromagnetic force serves as a hidden variable for quantum mechanics.

2 The relationship between the orthogonality and the Maxwell’s equations

2.1 4-vector Coulomb’s force

Suppose there are two charged particle $q$ and $q'$ locating at the positions $x$ and $x'$ respectively in a Cartesian coordinate system $S$, and moving at the 4-vector velocities $u$ and $u'$ respectively, as shown in Fig.1, where we have used $X$ to denote $x - x'$. The Coulomb’s force $f$ acting on the particle $q$ is orthogonal to the velocity direction of
It follows from the direction of Eq. (5) that the unit vector
plane of $u$ where $A$
Combination of Eq. (6) with Eq. (7), we obtain a modified
nal to the 4-vector velocity
magnetism.
for the moment, the expansion would be used to clarify
the completion of the expansion in the subsection 4, but
chosen as two independent basis vectors, we will discuss
space-time.
Using the orthogonality $f \perp u$, we get
\[ u \cdot f = A(u \cdot u') + B(u \cdot X) = 0 \] (4)
By eliminating the coefficient $B$, we rewrite Eq. (3) as
\[ f = \frac{A}{u \cdot X}((u \cdot X)u' - (u \cdot u')X) \] (5)
It follows from the direction of Eq. (5) that the unit vector
f0 of the Coulomb’s force is given by
\[ f^0 = \frac{1}{c^2 r^3}[(u \cdot X)u' - (u \cdot u')X] \] (6)
because $|f^0| = 1$, where $r = |R|$, $R \perp u'$ as illustrated
in Fig.1. Suppose that the magnitude of the force $f$ has
the classical form
\[ |f| = \frac{kq \cdot q'}{r^2} \] (7)
Combination of Eq. (6) with Eq. (7), we obtain a modified
Coulomb’s force
\[ f = \frac{kq \cdot q'}{c^2 r^3}[(u \cdot X)u' - (u \cdot u')X] \]
\[ = \frac{kq \cdot q'}{c^2 r^3}[(u \cdot R)u' - (u \cdot u')R] \\
\] (8)
Using the relation
\[ \partial_\mu \left( \frac{1}{r} \right) = -\frac{F_\mu}{r^3} \] (9)
we obtain
\[ f_\mu = q[-(u_\nu \partial_\nu \left( \frac{kq'}{c^2 r^3} \right))u'_\mu + (u_\nu u'_\nu)\partial_\nu \left( \frac{kq'}{c^2 r^3} \right)] \\
\] (10)
The force can be rewritten in terms of 4-vector components as
\[ A_\mu = \frac{kq \cdot u'_\mu}{c^2 r^3} \] (11)
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \] (12)
\[ f_\mu = q F_{\mu\nu} u_\nu \] (13)
Thus $A_\mu$ expresses the 4-vector potential of the particle
$q'$. It is easy to find that Eq. (13) contains the Lorentz
force.

**2.2 The Lorentz gauge condition and the Maxwell’s equations**

From Eq. (11), because of $u' \perp R$, i.e. $u'_\mu R_\mu = 0$, we have
\[ \partial_\mu A_\mu = \frac{kq \cdot u'_\mu}{c^2 r^3} \partial_\mu \left( \frac{1}{r} \right) = -\frac{kq \cdot u'_\mu}{c^2} \left( \frac{R_\mu}{r^3} \right) = 0 \] (14)
It is known as the Lorentz gauge condition. To note that $R$ has three degrees of freedom under the condition $R \perp u'$, so we have
\[ \partial_\mu R_\mu = 3 \] (15)
\[ \partial_\mu \partial_\mu \left( \frac{1}{r} \right) = -4\pi \delta(R) \] (16)
From Eq. (12), we have
\[ \partial_\nu F_{\mu\nu} = \partial_\nu \partial_\mu A_\nu - \partial_\nu \partial_\mu A_\mu = -\partial_\nu \partial_\mu A_\mu \]
\[ = -\frac{kq \cdot u'_\mu}{c^2} \partial_\nu \partial_\nu \left( \frac{1}{r} \right) = \frac{kq \cdot u'_\mu}{c^2} 4\pi \delta(R) \]
\[ = \mu_0 \mathcal{J}'_\mu \] (17)
where we define $\mathcal{J}'_\mu = q' \cdot u'_\nu \delta(R)$ as the current density of the source. From Eq. (12), by exchanging the indices and taking the summation of them, we have
\[ \partial_\lambda F_{\mu\nu} + \partial_\nu F_{\mu\lambda} + \partial_\mu F_{\lambda\nu} = 0 \] (18)
The Eq. (17) and (18) are known as the Maxwell’s equations. For continuous media, they are valid as well as.
2.3 The Lienard-Wiechert potential

From the Maxwell’s equations, we know that there is a retardation time for the electromagnetic action to propagate between the two particles, as illustrated in Fig.1, the retardation effect is measured by

\[ r = c \Delta t = c \frac{|PC|}{1c} = c \frac{u^0 \cdot X}{1c} = c \frac{u'_\mu(x'_\nu - x_\nu)}{c} \]  

(19)

where \( u^0 \) is the unit vector of \( u' \). Then

\[ A_\mu = \frac{kq'}{c^2} \frac{u'_\mu}{r} = \frac{kq'}{c} \frac{u'_\mu}{u'_\nu(x'_\nu - x_\nu)} \]  

(20)

Obviously, Eq. (20) is known as the Lienard-Wiechert potential for a moving particle.

2.4 The completion of the electromagnetic force’s basis vectors

The above formalism clearly shows that the Maxwell’s equations can be derived from the classical Coulomb’s force and the orthogonality of 4-vector force and 4-vector velocity. In other words, the orthogonality is hidden in the Maxwell’s equation. Specially, Eq. (3) directly accounts for the geometrical meanings of the curl of vector potential, the curl contains the orthogonality. The orthogonality of 4-vector force and 4-vector velocity is one of consequences from the relativistic mechanics.[2]

But, whether the expansion we have used in Eq. (3) is complete? Obviously, we choose two basis vector \( u' \) and \( X \) to construct the electromagnetism successfully, this choice means that the electromagnetic force has two independent components in its Hilbert space based on the two basis vector \( u' \) and \( X \), we readily recognize that the expansion is incomplete. A 4-vector force has 4 components, because of the orthogonality of 4-vector force and 4-vector velocity, the number of independent components of the force reduces to 3, thus a complete expansion of a 4-vector force needs 3 basis vectors.

Our argument also arises from that if the electromagnetic force of the Maxwell’s theory has two independent components, then there is an undefined component accompanying the Maxwell’s electromagnetic force. This missing undefined component may serve a hidden variable for quantum mechanics. In the next sections, we will strictly and comprehensively proof that the electromagnetic force of the Maxwell’s theory does have two independent components, does have an undefined component in the physics. In order to clarify its aspects as much as possible, we present four methods to proof the theme.

3 The proofs that the electromagnetic force of the Maxwell’s theory has two independent components

3.1 the first proof — by an intuitive method

In usual 3D space, the vector product of two vectors \( a \) and \( b \) is defined as another vector \( c \) given by \( c = a \times b \), or \( c_k = \varepsilon_{kij} a_j b_j \), where the indexes i, j and k take over 1, 2, 3 in the 3D space, \( \varepsilon_{kij} \) defined in usual textbooks; as we know, the vector \( c \) is orthogonal to both the vectors \( a \) and \( b \), i.e. \( c \perp a \) and \( c \perp b \). In the Minkowski’s space, we also introduce a definition for vector product: as shown in Fig.1, imaging that there is a 4-vector \( \Gamma \) which is orthogonal to the plain of \( u' \) and \( X \), then \( \Gamma \) is defined by the vector product as

\[ \Gamma = u' \times X \]  

\[ \Gamma = \varepsilon_{\lambda \mu \nu} u'_\mu X_\nu \]  

(21)

where the indexes take 1,2,3,4. It is easy to obtain their orthogonalities \( \Gamma \perp u' \) and \( \Gamma \perp X \), because

\[ \Gamma \cdot X = \Gamma_\lambda X_\lambda = \varepsilon_{\lambda \mu \nu} u'_\mu X_\nu X_\lambda \]

\[ = \varepsilon_{\lambda \mu \nu} u'_\mu X_\nu X_\lambda |_{\nu < \lambda} \]

\[ = \varepsilon_{\lambda \mu \nu} u'_\mu X_\nu X_\lambda |_{\nu > \lambda} \]

\[ = (\varepsilon_{\lambda \mu \nu} + \varepsilon_{\mu \lambda \nu}) u'_\mu X_\nu X_\lambda |_{\nu < \lambda} = 0 \]  

(22)

\[ \Gamma \cdot u' = 0 \]  

(23)

The vector \( \Gamma \) can be easily imaged out intuitively in Fig.1. The electromagnetic 4-vector force has 4 components, it subjects to the first constraint given by

\[ f \cdot u = 0 \]  

(24)

the second constraint on the 4-vector force is given by

\[ f \cdot \Gamma = (Au' + BX) \cdot \Gamma = 0 \]  

(25)

The first constraint is just the orthogonality of 4-vector force and 4-vector velocity, the second constraint reflects our choice that the electromagnetic force lies in the plain of \( u' \) and \( X \), which the vector \( \Gamma \) is orthogonal. Subjecting to these two constraints, the number of independent components of the electromagnetic force reduces to 2. The two independent components allow the electromagnetic force to change in the plain of \( u' \) and \( X \).

3.2 the second proof — in its Hilbert space

The moving particle \( q' \) exerts an electromagnetic force \( f \) on another moving particle \( q \), the action has been sophisticationally expressed as

\[ f_\mu = q F_{\mu \nu} = qu_\nu \partial_\mu A_\nu - qu_\nu \partial_\nu A_\mu \]  

(26)
To note that in the present stage of human’s knowledge, the above each term can not be resolved into more details, therefore, we conclude that the electromagnetic force contains two terms, each term represents a basis vector of its Hilbert space, the number of independent components of the electromagnetic force is 2.

This provides an insight into electromagnetism that a full electromagnetic force should contain at least tree terms in its formalism.

### 3.3 the third proof — hydrogen atom as an instance

The nucleus of a hydrogen atom provides a spherical Coulomb’s electric potential for its outer electron, this 4-vector force subjects to the first and second constraints, given by

\[ f \cdot u = 0 \]  
\[ f \times r = 0 \]

The second constraint expresses that the torque of the centripetal force is zero, and it arises from the above mentioned \( f \cdot \Gamma = 0 \) for this instance. Because, referring to Fig.1, \( f \cdot \Gamma = f \cdot (u' \times X) = f \cdot (u' \times R) = 0 \) permits the expansion of \( f = Au + BR \), whereas the nucleus is at rest, \( u' = (0, 0, 0, ic), R = (r, 0) \), then \( f = (Br, Ai) \) i.e., \( f \| r \) or \( f \times r = 0 \). Inversely, if \( f \| r \), then we can construct a vector \( \Gamma \) so that \( f \cdot \Gamma = 0 \). You can see, if we consider the two constraints, then the 4-force in hydrogen atom has only two independent components.

In Bohr’s hydrogen atom, the electron of hydrogen moves in a circular orbit, the attractive force subjects to the spheric symmetry as its third constraint, in that case, the number of independent components of the 4-vector force reduces to 1, in consistency with its circular motion.

### 3.4 the fourth proof — outside the electromagnetism

A 4-vector force has 4 components, because of the orthogonality of 4-vector force and 4-vector velocity, the number of independent components reduces to 3, while the electromagnetic force can merely provide 2 independent components, this situation means that there is an undefined component accompanying the electromagnetic force. What is the effect of the electromagnetism-undefined-component (EUC) in the physics? Outside the electromagnetism, a variety of problems may concern with the EUC, for example, quantum mechanics is eager to find a hidden variable for a long time for which the EUC may serve. Since its complex, we put the fourth proof in the next section, where we choose and briefly discuss three basic questions: (1) the properties of the EUC; (2) wave function and 4-vector velocity field in the EUC ensemble space; (3) a successful application to hydrogen’s fine structure.

### 4 The proof outside the electromagnetism

#### 4.1 the properties of the electromagnetism undefined component (EUC)

The properties of the EUC include: (1) EUC is a small quantity which is hardly noted by physicists, if it serves as a hidden variable for quantum mechanics, then it should be responsible to the Planck’s constant; (2) EUC is a fluctuating quantity in the ensemble space of enormous identical experiments, its average value at a given point must be zero, like \( < f_{EUC} > = 0 \); (3) as a nature of independence, EUC must be independent from our electromagnetic field, for example, in a vacuum when any our electromagnetic field has vanished off its matter environment, while the EUC can still alive in the vacuum and cause the indeterminacy of the motion of particles moving in the vacuum; (4) EUC also correlates with our electromagnetic field through the orthogonality \( f \cdot u = 0 \); (5) The effect of EUC on physics must be comprehensive, it is not concern with individual cases, no body can really escape from its fluctuation if the EUC still remains at undefined status.

According to the above mentioned properties of EUC, the proof outside the electromagnetism will carry out on "the path" that leads us from the EUC to the two key points of quantum mechanics: wave function and hydrogen atom.

#### 4.2 wave function and 4-vector velocity field in ensemble space

Consider a particle diffraction experiment in an electromagnetic field where the EUC is active and cause the indeterminacy of quantum mechanics, Each particle in the field subjects to both the electromagnetic force \( f \) and EUC \( f_{EUC} \), then it satisfies the dynamic equation:

\[
\frac{m}{\tau} \frac{du_{\mu}}{d\tau} = f_{\mu} + f_{EUC\mu} = qF_{\mu\nu}u_{\nu} + f_{EUC\mu} \tag{29}
\]

\[
u_{\mu}u_{\mu} = -c^2 \tag{30}\]

In order to eliminate the EUC term by means of statistics, i.e. using \( < f_{EUC} > = 0 \), we turn to study this dynamic equation in the ensemble space which consists of enormous particle paths recorded in enormous identical experiments, in the ensemble space we find

\[
m \frac{d < u_{\mu} >}{d\tau} = qF_{\mu\nu} < u_{\nu} > \tag{31}\]

\[
< u_{\mu} < u_{\mu} > = -c^2 \tag{32}\]
Strictly speaking, Eq. (32) is not derived from Eq. (30), it stands for the property that the magnitude of any 4-vector velocity keeps constant, otherwise this velocity concept collapses. If we regard the enormous recorded particle paths in the ensemble space as a flow, it is easy to find that there a 4-vector velocity field for the flow. We clearly emphasize two points: (1) At every point of the ensemble space, the mean 4-vector velocity satisfies the above mean dynamic equation; (2) the mean 4-velocity in ensemble space, the mean 4-vector velocity satisfies the above equation.

For our convenience, we drop the mean sign <> for the 4-vector velocity $u$ in the followings, simply use $u$ in place of $< u >$. As mentioned above, the 4-vector velocity $u$ can be regarded as a 4-vector velocity field in the ensemble space, then

$$\frac{du_{\mu}}{dt} = \frac{\partial u_{\mu}}{\partial x_{\nu}} \frac{dx_{\nu}}{dt} = u_{\nu} \partial_{\nu} u_{\mu}$$ (33)

$$qF_{\mu \nu} u_{\nu} = q u_{\nu} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})$$ (34)

Beware: Eq. (35) is the most important step in the present work after inducing the concept of 4-vector velocity field in ensemble space. Substituting them back into the dynamic equation, and re-arranging these terms, we obtain

$$u_{\nu} \partial_{\nu} (mu_{\mu} + qA_{\mu}) = u_{\nu} \partial_{\nu} (qA_{\mu}) = u_{\nu} \partial_{\nu} (mu_{\mu} + qA_{\mu}) - u_{\nu} \partial_{\nu} (mu_{\mu})$$

$$= u_{\nu} \partial_{\mu} (mu_{\mu} + qA_{\mu}) - \frac{1}{2} \partial_{\mu} (mu_{\mu} u_{\nu})$$

$$= u_{\nu} \partial_{\mu} (mu_{\mu} + qA_{\mu}) - \frac{1}{2} \partial_{\mu} (-mc^2)$$

$$= u_{\nu} \partial_{\mu} (mu_{\mu} + qA_{\mu})$$ (35)

Using the notation

$$K_{\mu \nu} = \partial_{\mu} (mu_{\mu} + qA_{\mu}) - \partial_{\nu} (mu_{\mu} + qA_{\mu})$$ (36)

Eq. (36) is given by

$$u_{\nu} K_{\mu \nu} = 0$$ (37)

Because $K_{\mu \nu}$ contains the variables $\partial_{\mu} u_{\nu}, \partial_{\mu} A_{\nu}, \partial_{\nu} A_{\mu}$ and $\partial_{\nu} A_{\mu}$, they are independent from $u_{\nu}$, then a solution satisfying Eq. (37) is actually of

$$K_{\mu \nu} = 0$$ (38)

$$\partial_{\mu} (mu_{\mu} + qA_{\mu}) = \partial_{\nu} (mu_{\mu} + qA_{\mu})$$ (39)

The above equation allows us to introduce a potential function $\Phi$ in mathematics, further set $\Phi = -i\hbar \ln \psi$, we obtain a very important equation

$$(mu_{\mu} + qA_{\mu}) \psi = -i\hbar \partial_{\mu} \psi$$ (40)

where $\psi$ may be a complex mathematical function, its physical meanings will be determined from experiments after the introduction of the Planck’s constant $\hbar$, as we have know, it is wave function.

Substituting Eq. (40) into $u_{\mu} u_{\mu} = -c^2$, we obtain a wave equation

$$(-i\hbar \partial_{\mu} \psi - qA_{\mu} \psi) (-i\hbar \partial_{\mu} \psi - qA_{\mu} \psi) = -m^2 c^2 \psi^2$$ (41)

It is a new quantum wave equation. Where the left side corresponds to the product of momentum and momentum itself, does not correspond to the product of momentum operator and momentum operator.

In this subsection, by using the $< f_{EUC}>$ 0 concept and 4-vector velocity field in ensemble space, we construct a relation between wave function and particle momentum, as we know, the relation has been widely used in quantum mechanics.

4.3 an application to hydrogen atom’s fine structure

In the following, we use the Gaussian units, and use $m_e$ to denote the rest mass of electron. In a spherical polar coordinate system $(r, \theta, \varphi, ic\tau)$, the nucleus of a hydrogen atom provides a spherically symmetric potential $V(r) = e^2/r$ for the electron motion. The wave equation (11) for the hydrogen atom in the energy eigenstate $\psi(r, \theta, \varphi)e^{-iEt/\hbar}$ may be written in the spherical coordinates:

$$\frac{m_e^2 c^2}{\hbar^2} \psi^2 = (\frac{\partial \psi}{\partial r})^2 + \frac{1}{r} \frac{\partial \psi}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \varphi}^2 + \frac{1}{\hbar^2 c^2} (E + e^2/r)^2 \psi^2$$ (42)

By substituting $\psi = R(r)X(\theta)\phi(\varphi)$, we separate the above equation into

$$\frac{\partial \phi}{\partial \varphi}^2 + \kappa \phi^2 = 0$$ (43)

$$\frac{\partial X}{\partial \theta}^2 + [\lambda - \kappa \sin^2 \theta] X^2 = 0$$ (44)

$$\frac{\partial R}{\partial \tau}^2 + [\frac{1}{\hbar^2 c^2} (E + e^2/r)^2 - \frac{m_e^2 c^2}{\hbar^2} - \frac{\lambda}{r^2}] R^2 = 0$$ (45)

where $\kappa$ and $\lambda$ are constants introduced for the separation. Eq. (43) can be solved immediately, with the requirement that $\phi(\varphi)$ must be a periodic function, we find its solution given by

$$\phi = C_1 e^{\pm i \sqrt{\kappa} \varphi} = C_1 e^{-i m \varphi} \quad m = \pm \sqrt{\kappa} = 0, \pm 1, \ldots$$ (46)
where $C_1$ is an integral constant. It is easy to find the solution of Eq. (44), it is given by

$$X(\theta) = C_2 \exp(\pm i \int \sqrt{\lambda - m^2 \sin^2 \theta} \, d\theta)$$  \hspace{1cm} (47)$$

where $C_2$ is an integral constant. The requirement of periodic function for $X$ demands

$$\int_0^{2\pi} \sqrt{\lambda - \frac{m^2}{\sin^2 \theta}} \, d\theta = 2\pi k \quad k = 0, 1, 2, \ldots$$  \hspace{1cm} (48)$$

This complex integration has been evaluated (see the appendix of the ref. [4]), we get

$$\int_0^{2\pi} \sqrt{\lambda - \frac{m^2}{\sin^2 \theta}} \, d\theta = 2\pi(\sqrt{\lambda} - |m|)$$  \hspace{1cm} (49)$$

thus, we obtain

$$\sqrt{\lambda} = k + |m|$$  \hspace{1cm} (50)$$

We rename the integer $\lambda$ as $j^2$ for a convenience in the followings, i.e. $\lambda = j^2$. The solution of Eq. (45) is given by

$$R(r) = C_3 \exp(\pm i \frac{\lambda}{\hbar c} \int \sqrt{\left(E + \frac{e^2}{r}\right)^2 - m_e^2 c^4 - \frac{\hbar^2 c^2}{r^2}} \, dr)$$  \hspace{1cm} (51)$$

where $C_3$ is an integral constant. The requirement that the radial wave function forms a "standing wave" in the range from $r = 0$ to $r = \infty$ demands

$$\frac{1}{\hbar c} \int_0^\infty \sqrt{\left(E + \frac{e^2}{r}\right)^2 - m_e^2 c^4 - \frac{\hbar^2 c^2}{r^2}} \, dr = \pi s \quad s = 0, 1, 2, \ldots$$  \hspace{1cm} (52)$$

This complex integration has been evaluated (see the appendix of the ref. [3]), we get

$$\frac{1}{\hbar c} \int_0^\infty \sqrt{\left(E + \frac{e^2}{r}\right)^2 - m_e^2 c^4 - \frac{\hbar^2 c^2}{r^2}} \, dr = \frac{\pi E\alpha}{\sqrt{m_e^2 c^4 - E^2}} - \pi \sqrt{\frac{e^2}{r^2}}$$  \hspace{1cm} (53)$$

where $\alpha = \frac{e^2}{\hbar c}$ is known as the fine structure constant.

From the last Eq. (52) and Eq. (53), we obtain the energy levels given by

$$E = m_e c^2 \left[1 + \frac{\alpha^2}{(\sqrt{j^2 - \alpha^2 + s}^2)^{-1/2}}\right]$$  \hspace{1cm} (54)$$

where $j = \sqrt{\lambda} = k + |m|$. Because the restriction $j \neq 0$ in Eq. (54), we find $j = 1, 2, 3, \ldots$.

The result, Eq. (54), is completely the same as the calculation of the Dirac wave equation for the hydrogen atom [5], it is just the fine structure of hydrogen atom energy.

After we introduce the EUC into quantum mechanics, the quantum mechanics becomes reasonable at its key points, we think that we have indirectly proofed the existence of the EUC outside the electromagnetism, and the electromagnetic force of the Maxwell’s theory has two independent components in the 4 dimensional space-time.

## 5 Conclusions

In this paper, we proof that the electromagnetic force of the Maxwell’s theory has two independent components in the 4 dimensional space-time. A 4-vector force has 4 components, because of the orthogonality of 4-vector force and 4-vector velocity, the number of independent components of the force reduces to 3, while the electromagnetic force can merely provide 2 independent components, this situation means that there is an undefined component accompanying the electromagnetic force. Therefore, we also briefly and confirmedly discuss the possibility that the undefined component of the electromagnetic force serves as a hidden variable for quantum mechanics.

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