Study on Strong Decays of $D_{sJ}(2632)$

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Abstract
The resonance $D_{sJ}(2632)$ observed by SELEX, has attracted great interests and meanwhile brought up serious dispute. Its spin-parity, so far has not finally determined and if it exists, its quark-structure might be exotic. Following the previous literature where $D_{sJ}(2632)$ is assumed to be a radial-excited state of $1^-$, we consider the possibilities that it might be a $qar{q}$ ground state of $2^+$ or the first radial-excited state of $0^+$ $D_{sJ}(2317)$ and re-calculate its strong decay widths in terms of the Bethe-Salpeter equation. Our results indicate that there still is a sharp discrepancy between the theoretical evaluation and data.

1 Introduction

In 2004, the SELEX collaboration reported that a charmed meson $D_{sJ}(2632)$ with narrow width was observed [1], its mass is $2632.5 \pm 1.7 MeV$ at 90% CL. This state was observed in two channels $D_{sJ}^+\eta$ and $D^0K^+(D^+K^0)$ which are OZI allowed processes. The measured ratio of their branching ratios $R = \Gamma(D_{sJ}^+ \rightarrow D^0K^+)/\Gamma(D_{sJ}^+ \rightarrow D_s^+\eta) = 0.14 \pm 0.06$ shows a strange pattern. Because the phase space available for $D^0K^+$ is almost 1.5 times larger than that for $D_s^+\eta$, but the final products possess the same quark contents, therefore one is tempted to believe that this discrepancy implies that the quark structure of $D_{sJ}(2632)$ might be exotic. To eventually confirm the allegation, one needs to carefully seek for solutions in the traditional theoretical framework. In fact, the spin-parity of the resonance $D_{sJ}(2632)$ has not finally determined yet. According to the final states, it may be $0^+$,
or even $2^+$ which can decay into two pseudoscalar mesons via S-, P- and D-waves respectively.

Actually, before considering the exotic structure, such as hybrid, four-quark state etc., one should seriously investigate if it can be embedded in the frame of regular mesons which contain only one quark and one anti-quark. Chang et al. assumed that $D_{sJ}(2632)$ is the first radial excited state of $D_s^*$ and has spin-parity as $1^-$, and then they evaluated its decay widths in terms of the Bethe-Salpeter equation. Here we consider alternative possibilities that $D_{sJ}(2632)$ may be $0^+$ or $2^+$ resonances of $c\bar{s}$. Namely, we suppose that $D_{sJ}(2632)$ is the first radial excited state of $D_{sJ}(2317)$ whose spin-parity is well determined by several collaborations, or the $2^+$ radial ground state of $c\bar{s}$. Then following Chang et al., we also evaluate the decay widths of $D_{sJ}(2632)$ by assuming it to be of a radial excited state of $0^+$ or a radial ground state of $2^+$ with only $c\bar{s}$ quark contents.

In the derivation, we have also considered two approaches for the $0^+$ case. First, we assume that it is the first radial excited state of the observed $D_{sJ}(2317)$ whose mass is 2317 MeV, and then we obtain a mass of the excited state as $2700 \pm 20MeV$ (see the following text for details). Alternatively, if we assume 2632 MeV as the mass of an $0^+$ radial excited state, then we obtain the mass of its corresponding radial ground state of $0^+$ as $2245 \pm 25MeV$. That is in analog to the approach of ref. where the authors calculated the mass of the first radial excited state of $D_s^*$ as 2658 \pm 15 MeV. In the second approach, the newly obtained mass 2245 \pm 25MeV obviously deviates from the observed 2317 MeV which is well measured, therefore unless a new resonance $D_{sJ}$ of $0^+$ were experimentally observed, this scenario is not favored by the present data. However, considering the experimental errors, it may still be possible, and we will further discuss it in the last section.

This work is organized as follows, after this introduction, we discuss all the aforementioned possibilities and by solving the B-S equation, we obtain the mass spectrum and the OZI-allowed decay widths. In Sec.II, we deal with the case where $D_{sJ}(2632)$ is assumed to be the radial ground state of $2^+$, while in Sec.III, we assume that it is a radial excited state of $0^+$. All the numerical results along with all the input parameters are presented in the sections. The last section is devoted to the discussions and a brief conclusion.

## 2 $D_{sJ}(2632)$ as the ground state of $2^+ c\bar{s}$

The B-S equation with instantaneous approximation about the $0^-, 1^-$ mesons has been thoroughly studied. Following the method and technical details introduced in ref., we solve the B-S equation for the mesons of $2^+$ under the instantaneous approximation.

Generally, the B-S wavefunction for a $2^+$ meson can be written as:

$$\varphi_{\mu}(q) = \epsilon_{ij} q_{\perp}^i \left\{ \varphi_{\perp}(q) + \gamma_{\perp} \varphi_{\parallel}(q) + \gamma_0 \varphi_5(q) + \gamma_0 \gamma_{\perp} \varphi_7(q) + i \epsilon^{0i} q_{\parallel} \gamma_{\parallel} \varphi_6(q) \right\}. \quad (1)$$

where $\varphi_i(q)$ is the component function, $q_{\perp} = (0, q)$, and $q$ is the relative three-momentum of the quark-anti-quark in the meson, $\epsilon_{ij}$ is the fully antisymmetric tensor and $\epsilon_{ij}$ is the
polarization tensor of $2^+$. For the convenience, we redefine $\psi_1 = \varphi_1$, $\psi_2 = \varphi_2$, $\psi_3 = q^2 \varphi_3$, $\psi_4 = q^2 \varphi_4$, $\psi_5 = \varphi_5$, $\psi_6 = -\varphi_6$, $\psi_7 = \varphi_7$, $\psi_8 = \varphi_8$.

By the well-known constraint conditions for the projected wavefunctions $\varphi_{p_i}^0 = 0$ and $\varphi_{p_i}^- = 0$, one has

$$\psi_1(q) = -\frac{((\omega_1 + \omega_2)\psi_3(q) - 2\omega_2\psi_5(q))}{\omega_2 m_1 + \omega m_2},$$

$$\psi_2(q) = -\frac{(\omega_1 - \omega_2)(\psi_4(q) - \psi_6(q))}{\omega_2 m_1 + \omega m_2},$$

$$\psi_3(q) = \frac{(\omega_1 - \omega_2)\psi_5(q)}{\omega_2 m_1 + \omega m_2},$$

$$\psi_4(q) = -\frac{\omega_1 + \omega_2}{\omega_2 m_1 + \omega m_2},$$

where $\omega_1 = \sqrt{m_1^2 + q^2}$, $\omega_2 = \sqrt{m_2^2 + q^2}$, and in this text $m_1$ and $m_2$ stand as $m_c$ and $m_q$, which are masses of charm quark and light flavor $q$ ($q = u, d, s$).

Thus the wavefunction of a $2^+$ meson can be further written as

$$\psi_{p_i}(q) = \epsilon_{ij} q^j \{ \psi_3(q)\left[ \frac{\hat{q} \times q}{q^2} - \frac{\omega_1 + \omega_2}{m_2 \omega_1 + m_1 \omega_2} \right] + \psi_4(q) \gamma_0 \left[ \frac{\hat{q} \times q}{q^2} - \frac{\omega_1 - \omega_2}{m_2 \omega_1 + m_1 \omega_2} \right] + \psi_5(q) \frac{2 \omega_2}{m_2 \omega_1 + m_1 \omega_2} + \psi_6(q) \gamma_0 \left[ \frac{\hat{q} \times q}{q^2} - \frac{\omega_1 - \omega_2}{m_2 \omega_1 + m_1 \omega_2} \right] + \gamma_5 [\psi_5(q) (1 + \frac{\hat{q} \times q}{m_2 \omega_1 + m_1 \omega_2}) - \gamma_0 \psi_6(q)] - ie^{0_0} q_{\perp} \gamma_5 \psi_6(q) \left[ \frac{\omega_1 + \omega_2}{m_2 \omega_1 + m_1 \omega_2} \right].$$

(3)

Then, we obtain an equation group which contains four mutually coupled equations, the detailed expressions are collected in appendix.

In this work we adopt the values given in [6] for the concerned parameters, but only change $V_0$ to obtain the mass of 2632 ± 16 MeV for the ground state of $2^+$. By solving the equation group, numerical solutions for the component functions $\psi_3$, $\psi_4$, $\psi_5$, $\psi_6$ are achieved, these functions are shown in Fig.(1). Actually, $\psi_3 \approx \psi_4$, and $\psi_5 \approx \psi_6$, therefore, in the figure they seem to overlap together.

Now, we can use the formula given by ref.[6] to evaluate the widths of the strong decays.

$$\Gamma = \frac{|P_{f1}|}{8 \pi M^2} |T|^2,$$

(4)
where $M$ is the mass of the initial meson $D_{sJ}(2632)$, $P_{f 1}$ is the three momentum of the produced mesons $D$ (or $D_s$) in the center of mass frame of $D_{sJ}(2632)$. For $D_{sJ}^+ \rightarrow D^0 K^+$ and $D_{sJ}^+ \rightarrow D^+ K^0$, the matrix element $T$ is

$$T = \frac{P_{f 2}^\mu}{f_K} \int \frac{dq}{(2\pi)^3} \text{Tr}[\varphi^+_{P_{f 1}}(q - \frac{m_1}{m_1 + m_2}P_{f 1}) \frac{P_i}{M} \psi^{++}(q) \gamma_\mu \gamma_5].$$  \hfill (5)$$

For $D_{sJ}^+ \rightarrow D_{s}^+ \eta$, it is

$$T = \frac{P_{f 2}^\mu}{f_\eta} \int \frac{dq}{(2\pi)^3} \text{Tr}[\varphi^+_{P_{f 1}}(q - \frac{m_1}{m_1 + m_2}P_{f 1}) \frac{P_i}{M} \psi^{++}(q) \gamma_\mu \gamma_5].$$  \hfill (6)$$

Here $q$ is the inner relative three-momentum in the initial meson $D_{sJ}(2632)$, $P_{f 2}$ is the four-momentum of the produced meson $K$ (or $\eta$), $P_i$ is the four-momentum of $D_{sJ}(2632)$, $M_\eta$ is the mass of $\eta$. $\varphi^{++}_{P_{f 1}}$ and $\psi^{++}$ is the positive-energy wavefunction of the initial or final meson, and $\varphi^+_{P_{f 1}} = -\gamma_0(\varphi^{++}_{P_{f 1}})^+ + \gamma_0$. The factor in eq. (6) $\frac{-2M_\eta^2 \cos\theta}{\sqrt{6}M_\eta^2 f_\eta} + \frac{M_\eta^2 \sin\theta}{\sqrt{3}M_\eta^2 f_\eta}$ takes into account the $\eta - \eta'$ mixing, the readers are recommended to refer to ref.[6] for some details. $f_K$ is the decay constant of $K$ meson, $f_{\eta_8}$, $f_{\eta_0}$ are the decay constants of $\eta_8$ and $\eta_0$ respectively.

As the decay products are pseudoscalar mesons, their positive-energy wavefunctions are\[8\]

$$\varphi^+_{P_{f 1}}(q) = \frac{M_{f 1}}{2}(\varphi_1(q) + \varphi_2(q) \frac{m_1 m_2}{\omega_1 \omega_2})[\frac{m_1 \omega_2 + \omega_1 m_2}{m_1 + m_2}] + \gamma_0 \hat{q} \frac{(m_1 - m_2)}{m_2 \omega_1 + m_1 \omega_2} + \gamma_0 \hat{q} \frac{(m_1 - m_2)}{m_2 \omega_1 + m_1 \omega_2} \gamma_5.$$

$\hfill (7)$

$\hfill (8)$

---

**Figure 1:** The component functions of $D_{sJ}(2632)$ which is assumed to be the radial ground state of $2^+ c\bar{q}, \psi_3, \psi_4, \psi_5, \psi_6$. 
The relation of the positive-energy wavefunction of the initial meson and its wavefunction \( \varphi_{P_i}(q) \) reads \[6\]
\[
\varphi_{P_i}^{++}(q) = \frac{1}{2\omega_1}(\omega_1\gamma_0 + m_1 + \gamma_0\gamma_0\varphi_{P_i}(q)\gamma_0 - \omega_2\gamma_0 - m_2 - \gamma_0). \tag{9}
\]

Using the wavefunctions obtained by solving the equations, we evaluate the partial widths as
\[
\Gamma(D_{sJ}^+ \to D^0 K^+) = 2.10 \pm 0.30 \text{MeV}, \tag{10}
\]
\[
\Gamma(D_{sJ}^+ \to D^+ K^0) = 2.22 \pm 0.31 \text{MeV}, \tag{11}
\]
\[
\Gamma(D_{sJ}^+ \to D_s^+ \eta) = 0.23 \pm 0.02 \text{MeV}. \tag{12}
\]

The corresponding ratio of the branching ratios is
\[
\frac{\Gamma(D_{sJ}^+ \to D^0 K^+)/\Gamma(D_{sJ}^+ \to D_s^+ \eta)}{\Gamma(D_{sJ}^+ \to D^+ K^0)/\Gamma(D_{sJ}^+ \to D_s^+ \eta)} \approx 9.2 \pm 0.9. \tag{13}
\]

If we assume that the observed \( D_{sJ}(2632) \) is a radial ground state of \( 2^+ \), the obtained total width is consistent with the data, but the ratio of the branching ratios obviously differs from the observation.

In the numerical computations, we choose the input parameters as ref.\[6\], \( m_c = 1755.3 \text{MeV}, m_s = 487 \text{MeV}, m_d = 311 \text{MeV}, m_u = 305 \text{MeV}, f_\pi = 130.7 \text{MeV}, f_\eta = 1.26 f_\pi, f_\eta_8 = 1.07 f_\pi, f_K = 159.8 \text{MeV}, M_\eta_8 = 564.3 \text{MeV}, M_\eta_0 = 948.1 \text{MeV} \) and \( \theta = -9.95^\circ \). The theoretical uncertainties are estimated by varying all the parameters simultaneously within \( \pm 5\% \).

### 3 \( D_{sJ}(2632) \) as the first radial excited state of \( 0^+ c\bar{s} \)

In our earlier work, we obtained the expression of the wavefunction for \( 0^+ \) diquark \[12\], for a \( 0^+ \) meson which is composed of a quark and an antiquark, the B-S equation is the same, but the integration kernel is different.

The wavefunction of a \( 0^+ \) scalar meson is written as
\[
\varphi_{P_i}(q) = [\varphi_1(q) + \gamma_0\varphi_2(q) + \gamma_0\gamma_0\varphi_3(q) + \gamma_0\gamma_0\varphi_4(q)]. \tag{14}
\]

For the convenience, we redefine \( \psi_1 = \varphi_1, \psi_2 = \varphi_2, \psi_3 = |q|\varphi_3, \psi_4 = |q|\varphi_4 \)

By the constraint condition of projected wave function, one has
\[
\psi_1(q) = -\frac{(\omega_1 - \omega_2)\psi_4(q)|q|}{\omega_2m_1 + \omega_1m_2},
\]
\[
\psi_3(q) = \frac{(q^2 - m_1m_2 - \omega_1\omega_2)\psi_2(q)}{(m_1 + m_2)|q|}. \tag{15}
\]
Figure 2: Case 1. we adjust parameters when 2317 MeV serves as the basic input parameter, the curves correspond to the component wavefunctions of the $0^+$ ground state and first radial excited state $\psi_2, \psi_4$ respectively.

With the constraints, one can further write the wavefunction as

$$\varphi_{\rho_i}(q) = \psi_2(q)[\gamma_0 + \frac{q^2 - m_1 m_2 - \omega_1 \omega_2}{(m_1 + m_2)q^2}] + \psi_4(q)[\gamma_0 \frac{q}{|q|} \frac{(\omega_1 - \omega_2)|q|}{m_1 \omega_2 + m_2 \omega_1}].$$ (16)

Substituting this wavefunction into (9), we obtain the positive-energy wavefunction of $0^+$ scalar meson.

We can simplify the B-S equation and obtain an equation group which only contains two coupled equations, the detailed expressions are presented in the appendix.

As we indicated in the introduction, we take two approaches.

1. Adjusting parameters while 2317 MeV is taken as the basic input parameter, namely the observed meson (may correspond to $D_{sJ}(2632)$) is supposed to be the first excited state of the well measured $D_{sJ}(2317)$:

Adopting the parameter given in the ref. [4], but varying $V_0$ to fit the mass of the ground state of $0^+$ which is set as 2317 MeV, we obtain the mass of the first radial excited state as $2700 \pm 20 MeV$, and the corresponding component wavefunctions. $\psi_2, \psi_4$ are shown in Fig.2

Substituting the wavefunctions into eq. (14), we obtain the numerical values

$$\Gamma(D_{sJ}^{++} \to D^0 K^+) = 9.58 \pm 1.33 MeV,$$ (17)
$$\Gamma(D_{sJ}^{+} \to D^+ K^0) = 9.36 \pm 1.30 MeV,$$ (18)
$$\Gamma(D_{sJ}^{+} \to D_{s}^{+} \eta) = 3.64 \pm 0.31 MeV.$$ (19)
The corresponding ratio of the branching ratios would be
\[
\Gamma(D_{sJ}^+ \to D^0 K^+) / \Gamma(D_{sJ}^+ \to D_s^+ \eta) \approx \frac{\Gamma(D_{sJ}^+ \to D^+ K^0) / \Gamma(D_{sJ}^+ \to D_s^+ \eta)}{\Gamma(D_{sJ}^+ \to D_s^+ \eta)} \approx 2.6 \pm 0.3. \quad (20)
\]

2. Numerical results while 2632 MeV is taken as the input parameter for the mass of the first radial excited state of an undiscovered 0\(^+\) meson (it might be the measured \(D_{sJ}(2317)\) if the experimental errors are indeed large).

While we adjust \(V_0\) to fit 2632 MeV which is taken as the input parameter for the mass of the first radial excited state, we obtain the mass of the ground state of 0\(^+\) as 2245 \(\pm\) 25 MeV. The corresponding component wavefunctions \(\phi_2, \phi_4\) are shown in Fig. 3.

Substituting the wavefunctions into eq. (11), we obtain
\[
\begin{align*}
\Gamma(D_{sJ}^+ \to D^0 K^+) & = 7.58 \pm 1.06 \text{MeV}, \quad (21) \\
\Gamma(D_{sJ}^+ \to D^+ K^0) & = 7.25 \pm 1.01 \text{MeV}, \quad (22) \\
\Gamma(D_{sJ}^+ \to D_s^+ \eta) & = 2.37 \pm 0.20 \text{MeV}. \quad (23)
\end{align*}
\]

and the corresponding ratio of the branching ratios
\[
\Gamma(D_{sJ}^+ \to D^0 K^+) / \Gamma(D_{sJ}^+ \to D_s^+ \eta) \approx \frac{\Gamma(D_{sJ}^+ \to D^+ K^0) / \Gamma(D_{sJ}^+ \to D_s^+ \eta)}{\Gamma(D_{sJ}^+ \to D_s^+ \eta)} \approx 3.1 \pm 0.4. \quad (24)
\]

4 Discussions and conclusion

In terms of the B-S equation, we evaluate the mass spectra and the decay rates when assuming \(D_{sJ}(2632)\) observed by the SELEX collaboration to be a 0\(^+\) or 2\(^+\) meson of \(c\bar{s}\).

Our observation is that if \(D_{sJ}(2632)\) is a 2\(^+\) \(c\bar{s}\) state, the total width obtained in terms of the B-S equation can be consistent with data, but the predicted branching ratios
obviously conflict with data. If it is a $0^+ c\bar{s}$ state, our results can be summarized as follows. In approach (1), we adjust $V_0$ to fit the mass 2317 MeV which is the mass of the $0^+$ ground state of $c\bar{s}$, and obtain the mass of the first radial excited state as $2700 \pm 20 MeV$. Instead, in approach (2), we adjust $V_0$ to fit 2632 MeV which is taken as the mass of the first radial excited state of $0^+ c\bar{s}$, then we get the mass of the ground state as $2245 \pm 25 MeV$. It is noted that the wavefunctions obtained in the two cases are very close as shown in Figs.2 and 3. The decay rates calculated in approach (1) are a bit larger than that in approach (2) due to a larger phase space. Thus in approach (2), the obtained total width is consistent with data, but not the ratio of branching ratios within the experimental tolerance range, whereas in approach (1), both the total width and the ratio do not coincide with the present data.

Moreover, if $D_{sJ}(2632)$ is a $1^- c\bar{s}$ vector meson and is the first radial excited state of $D_s^*$, as Chang et al. evaluated [6], the ratio of branching ratios is also inconsistent with data.

Definitely, the calculations are model-dependent, especially the linear confinement part of the kernel in the B-S equation is phenomenologically introduced and the coefficient $\kappa$ is determined by fitting data. Thus the numerical results obtained in this theoretical framework cannot be very accurate, a factor of as large as $2 \sim 3$ may be expected, however, the order of magnitude and the qualitative behavior of the solution do not change.

Therefore, there may be some possible explanations for obvious discrepancy between data and theoretical result. First, from the theoretical aspect, the discrepancy may indicate that $D_{sJ}(2632)$ possess a large exotic component, namely it may be a four-quark state[13], molecular state, or a hybrid[14, 15, 16].

Another possibility is, as Swanson et al. suggested[17, 18], it could be the "artefact" of experiments. Moreover, the other prestigious experimental groups[17, 19] do not see evidence of $D_{sJ}(2632)$, therefore, its existence is still suspicious. Further and more precise measurements are necessary to clarify this mystery, great efforts from both sides of theory and experiments must be made.

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Appendix

A \quad 2^+

normalization condition

$$\int \frac{dq}{2\pi^3} \frac{16 q^2 (2\psi_3 \psi_4 + 2\psi_3 \psi_6 - 2\psi_4 \psi_5 - 5\psi_5 \psi_6) \omega_1 \omega_2}{3(\omega_1 m_2 + \omega_2 m_1)} = 2 P_{00}$$  \quad (25)
Coupled equations

\[
\begin{align*}
(M - \omega_1 - \omega_2)4q^4((m_1 + m_2)\psi_4(q) + (\omega_1 + \omega_2)\psi_3(q) - (m_1 + m_2)\psi_6(q) - (\omega_1 + \omega_2)\psi_5(q)) \\
= \int \frac{dk}{(2\pi)^3} \{ -3(V_s - V_v)[(m_1 + m_2)\psi_4(q) + (\omega_1 + \omega_2)\psi_3(q)](m_2\omega_{1k} + m_1\omega_{2k})(k \cdot q)^3/k^2 \\
-3(V_s + V_v)[-\psi_6(q)(m_2\omega_1\omega_{1k} - m_1\omega_2\omega_{1k} - m_2\omega_1\omega_{2k} - m_1\omega_2\omega_{2k}) + \\
\psi_5(q)(\omega_1kq^2 + \omega_2k^2q - m_1m_2\omega_{1k} + \omega_1\omega_2\omega_{1k} - m_1m_2\omega_{2k}) + \\
\psi_4(q)(m_1\omega_2 - m_2\omega_1)(\omega_{2k} - \omega_{1k}) + \\
\psi_3(q)(-q^2 + m_1m_2 - \omega_1\omega_2)(\omega_{1k} + \omega_{2k})]/[3\omega_1\omega_2(m_2\omega_{1k} + m_1\omega_{2k})], \\
\end{align*}
\]

(26)

\[
\begin{align*}
(M + \omega_1 + \omega_2)4q^4((m_1 + m_2)\psi_4(q) - (\omega_1 + \omega_2)\psi_3(q) - (m_1 + m_2)\psi_6(q) + (\omega_1 + \omega_2)\psi_5(q)) \\
= \int \frac{dk}{(2\pi)^3} \{ -3(V_s - V_v)[(m_1 + m_2)\psi_4(q) - (\omega_1 + \omega_2)\psi_3(q)](m_2\omega_{1k} + m_1\omega_{2k})(k \cdot q)^3/k^2 \\
-3(V_s + V_v)[-\psi_6(q)(m_2\omega_1\omega_{1k} - m_1\omega_2\omega_{1k} - m_2\omega_1\omega_{2k} - m_1\omega_2\omega_{2k}) + \\
\psi_5(q)(\omega_1kq^2 + \omega_2k^2q - m_1m_2\omega_{1k} + \omega_1\omega_2\omega_{1k} - m_1m_2\omega_{2k}) + \\
\psi_4(q)(m_1\omega_2 - m_2\omega_1)(\omega_{2k} - \omega_{1k}) + \\
\psi_3(q)(-q^2 + m_1m_2 - \omega_1\omega_2)(\omega_{1k} + \omega_{2k})]/[3\omega_1\omega_2(m_2\omega_{1k} + m_1\omega_{2k})], \\
\end{align*}
\]

(27)
\[+ 2\psi_3(\mathbf{q})(2\mathbf{q}^2 + m_1 m_2 + \omega_1 \omega_2)(m_2 \omega_{1k} + m_1 \omega_{2k})\mathbf{k} \cdot \mathbf{q} - \]
\[k^2 \mathbf{q}^2 (V_s + V_v)(- \psi_6(\mathbf{q})(5\omega_1 \omega_{1k} + \omega_2 \omega_{1k} + \omega_1 \omega_{2k} + \omega_2 \omega_{2k}) + \]
\[\psi_5(\mathbf{q})(-5m_1 \omega_{1k} + m_2 \omega_{1k} + m_1 \omega_{2k} - 5m_2 \omega_{2k}) + 2\psi_4(\mathbf{q})(\omega_1 - \omega_2)(\omega_{1k} - \omega_{2k}) + \]
\[\psi_3(\mathbf{q})(-\mathbf{q}^2 + m_1 m_2 - \omega_1 \omega_2)(\omega_{1k} + \omega_{2k})]/[6\omega_1 \omega_2 (m_2 \omega_{1k} + m_1 \omega_{2k})], \quad (28)\]

\[\frac{(M + \omega_1 + \omega_2)}{3(m_2 \omega_1 + m_1 \omega_2)} 2\mathbf{q}^2[- 5\psi_5(\mathbf{q})(m_1 \omega_2 + m_2 \omega_1) + 2\psi_3(\mathbf{m}_1 \omega_2 + m_2 \omega_1) + \]
\[\psi_6(\mathbf{q})(5m_1 \omega_1 + \omega_2 \omega_1 + \omega_1 \omega_2 + \omega_2 \omega_2) + \psi_5(\mathbf{q})(-5m_1 \omega_{1k} + m_2 \omega_{1k} + m_1 \omega_{2k} - 5m_2 \omega_{2k}) - \]
\[2\psi_4(\mathbf{q})(\omega_1 - \omega_2)(\omega_{1k} - \omega_{2k}) + \psi_3(\mathbf{q})(-\mathbf{q}^2 + m_1 m_2 - \omega_1 \omega_2)(\omega_{1k} + \omega_{2k})](\mathbf{k} \cdot \mathbf{q})^2 + \]
\[2(V_s - V_v)\psi_3(\mathbf{q})(5m_2 \omega_{1k} + m_1 \omega_2) - 2\psi_4(\mathbf{q})(m_2 \omega_1 + m_1 \omega_2) + \psi_5(\mathbf{q})(-\mathbf{q}^2 - 5m_1 m_2 - 5\omega_1 \omega_2) + \]
\[2\psi_3(\mathbf{q})(2\mathbf{q}^2 + m_1 m_2 + \omega_1 \omega_2)(m_2 \omega_{1k} + m_1 \omega_{2k})\mathbf{k} \cdot \mathbf{q} - \]
\[k^2 \mathbf{q}^2 (V_s + V_v)(\psi_6(\mathbf{q})(5\omega_1 \omega_{1k} + \omega_2 \omega_{1k} + \omega_1 \omega_{2k} + \omega_2 \omega_{2k}) + \]
\[\psi_5(\mathbf{q})(-5m_1 \omega_{1k} + m_2 \omega_{1k} + m_1 \omega_{2k} - 5m_2 \omega_{2k}) - 2\psi_4(\mathbf{q})(\omega_1 - \omega_2)(\omega_{1k} - \omega_{2k}) + \]
\[\psi_3(\mathbf{q})(-\mathbf{q}^2 + m_1 m_2 - \omega_1 \omega_2)(\omega_{1k} + \omega_{2k})]/[6\omega_1 \omega_2 (m_2 \omega_{1k} + m_1 \omega_{2k})]. \quad (29)\]

**B 0⁺ normalization condition**

\[
\int \frac{d\mathbf{q}}{2\pi^3} \frac{16\psi_2 \psi_4 |\omega_1 \omega_2(\mathbf{q}^2 + m_1 m_2 + \omega_1 \omega_2)|}{(m_1 + m_2)(\omega_1 \omega_2 + m_2 \omega_1)} = 2P_{00} \quad (30)
\]

**Coupled equations**

\[
\frac{2[\psi_2(\mathbf{q})(m_2 \omega_1 + m_1 \omega_2) - \psi_4(\mathbf{q})(m_2 + m_1)]|\mathbf{q}|}{m_2 \omega_1 + m_1 \omega_2} = \int \frac{dk}{(2\pi)^3} ((V_s + V_v) \]
\[\psi_2(\mathbf{k})(-\mathbf{q}^2 + m_2 m_1 - \omega_1 \omega_2)(m_2 \omega_{1k} + m_1 \omega_{2k}) - \psi_4(\mathbf{k})(m_2 \omega_e - m_1 \omega_s)(\omega_{1k} - \omega_{2k})(\mathbf{k}^2 + \]
\[+(V_s - V_v)\psi_2(\mathbf{k})(m_1 - m_2)(\omega_{1k} - \omega_{2k})\mathbf{k}^2 + 2m_1 m_2)(m_2 \omega_{1k} + m_1 \omega_{2k}) - \]
\[\psi_4(\mathbf{k})(m_1 - m_2)(\omega_{1k} + \omega_{2k})(m_2 \omega_{1k} + m_1 \omega_{2k})(\mathbf{k} \cdot \mathbf{q})^2]/[k^2 \omega_1 \omega_2 (m_2 \omega_{1k} + m_1 \omega_{2k})], \quad (31)
\]

\[
\frac{2[\psi_2(\mathbf{q})(m_2+1 \omega_1 + m_1 \omega_2) + \psi_4(\mathbf{q})(m_2 + m_1)]|\mathbf{q}|}{m_2 \omega_1 + m_1 \omega_2} = \int \frac{dk}{(2\pi)^3} ((V_s + V_v) \]
\[\psi_2(\mathbf{k})(-\mathbf{q}^2 + m_2 m_1 - \omega_1 \omega_2)(m_2 \omega_{1k} + m_1 \omega_{2k}) + \psi_4(\mathbf{k})(m_2 \omega_e - m_1 \omega_s)(\omega_{1k} - \omega_{2k})(\mathbf{k}^2 + \]
\[+(V_s - V_v)\psi_2(\mathbf{k})(m_1 - m_2)(\omega_{1k} - \omega_{2k})\mathbf{k}^2 + 2m_1 m_2)(m_2 \omega_{1k} + m_1 \omega_{2k}) + \]
\[\psi_4(\mathbf{k})(m_1 - m_2)(\omega_{1k} + \omega_{2k})(m_2 \omega_{1k} + m_1 \omega_{2k})(\mathbf{k} \cdot \mathbf{q})^2]/[k^2 \omega_1 \omega_2 (m_2 \omega_{1k} + m_1 \omega_{2k})]. \quad (32)
\]

where \(\omega_{1k} = \sqrt{m_1^2 + k^2}\) and \(\omega_{2k} = \sqrt{m_2^2 + k^2}\); \(V_v\) and \(V_s\) were given by ref.\(^8\).