Abstract: Consider a retrial queue with VoIP calls and two kinds of heterogeneous services such as essential and optional services. The multiple vacation policy, retrial policy, customer's impatience and the concept of catastrophe are adopted to derive the required solutions. The steady state system size distribution and probability generating function under different level have been obtained. Based on some assumptions, special and particular cases are discussed.

Keywords: VoIP, Catastrophe, Impatience, Optional service, Vacation and Retrial policies.

I. INTRODUCTION

The concepts of queueing theory have been applied in various fields like phone systems, communication network systems, computer systems, industrial sectors and so on. The techniques adopted in the queueing models produce remarkable solutions. In this paper, Voice and Internet Protocol (VoIP), Vacation policies, Catastrophe, retrial queue, optional service and impatience have been utilized in proper place. Generally, in any queueing models, the arriving units are considered as human beings, but, here, VoIP calls are taken as arriving units.

Voice over IP (VoIP) is a typical method for the delivering voice communications and multimedia sessions through Internet Protocol (IP) networks. VoIP is a toll-free long distance voice, fax calls on IP data networks other than public switched telephone network (PSTN). It saves the long distance costs between two or more locations. Earlier people depend on the PSTN for communicating voice. The connection is dedicated to only two parties between two locations when the call is made. Although there is an availability of many bandwidth, none of the information can pass over the call. Internet Protocol and internet and broadband telephone and broadband telephone services are generally associated with VoIP. The principles of VoIP telephone calls are same as digital telephone calls. The digital information is gathered and transmitted as IP packets.

VoIP services begins with providing solutions for the business and technical problems. In the second generation era, like Skype with closed networks offered free calls. The federated VoIP concept was adopted in the Google Talk, a third generation service providers. All these solutions provide a dynamic interconnection between the users on the internet.

VoIP systems transmit audio streams over IP networks with media delivery protocols. Some of the popular codecs are mu-law and a u-law versions of G.711, G.722, an open source voice codec known as iLBC, and others. Recently, with the rapid usage of IP, a low cost transport mechanism which can be used for both voice and data. Making use of the IP infrastructure and hardware the voice traffic on top of the data network.

The arriving calls are served by a one server in two non-homogeneous modes such as the first one is essential but the second is optional. If the server is busy either with essential or with optional service, the arriving call balks from the system or enters into an orbit to make trials to get desired service. After end of a service of a call, if there is no arrival in the system, the server avails vacation.

Vacation models of server’s are applicable for the systems in which the server wants to utilize the idle time for various purposes. Vacation of the server is one of the concepts of queueing theory and that leads to study new results. After the completion of service, if the queue is empty, the server leaves and engages other work but notice the new arrivals to the system. The period of unavailable of the server in the counter is known as vacation period. On returning from vacation, if there were units in the system the server do the service otherwise he avails another vacation. These type of vacations are called single and multiple vacations.

In communication networks, the system may failure due to various factors, in particular, the occurrence of catastrophe is one of the factors. The concept of catastrophe has played a vital role in the areas of Science and Technology. It occurs at random leading to extinction of all the units and activate the service facility until a new arrival is not unusual in most of the real life situations. The catastrophe may exists from outside or within the system. In computer based systems, if a task is infected, this infected task may transfer the virus to the other processors. These infected network of tasks may be imitated by the catastrophes which leads to construct queueing models with catastrophes.

II. REVIEW OF LITERATURE

Falin and Templeton (1997) have summarized many contributions relating to the queueing systems with retrial queues. Artalejo and Choudhury (2004) have used...
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embedded Markov chain of retrial queues consisting of services with two phases. Also they derived steady state solution, average number of customers and other related measures.

Doshi (1985) has discussed the system with infinite number of vacation types. After completing all the services to customers, the server avails a type-1 vacation. Coming back from type-1 vacation, if there are no customers, the server starts a type-2 vacation and so on. Choudhury and Madhuchanda (2004) have dealt with a queuing system of bulk arrival with heterogeneous services which are governed by a single server. They derived the solution for the stationary queue size distribution, waiting time distribution and related performance measures. Armuganathan et al., (2008) have analysed steady state solutions of a bulk size system of non-Markovian queue with N-policy and various types of vacation.

Senthilkumar & Arumuganathan (2009) have analysed the retrial queue with impatient subscribers under steady state conditions and heterogeneous services of two phases and distinct vacation policies. Voice over IP calls are taken as arrival units. The average number of customers, average waiting duration in retrial queue and some particular cases have been analysed. Jeyakumar and Arumuganathan (2011) have analysed a single server non- Markovian queue with more than one vacations and re-service as optional of the steady state conditions. They mathematically derived the mean queue size, busy period, idle period and cost function and obtained numerical results. Ponnammal et al., (2013) have considered a customer’s arrival follow Hyper Poisson distribution and two types of services. The principles of balking, reneging and impatience are applied to estimate mean queue length for different positions of the server.

Jain and Bura (2010) have analysed the effect of catastrophe intensity of Markovian queue with restoration for a single server with finite capacity. The intensity of catastrophes follows uniform distribution. The steady state probabilities, the expected number of units in both queue and system were derived. Sudshe (2010) has studied transient probes for single server Markovian queue by using the concepts of balking, reneging and catastrophe.

Dhanesh Garg (2013) has discussed Markovian queue of a single server with finite capacity through catastrophe and restoration. He has derived the p.g.f. for the number of times the system reaches its capacity and the number of units in the systems. The corresponding expected values have been obtained. Ayyapppan et el (2013) have analysed the effect of catastrophe in an M/M/I queue when the server is idle and busy. The probabilities under steady state conditions, average length of a queue and the variance have been explicitly derived. The derived results are justified by using the numerical values.

Arul and Vidhya (2014) have discussed a Markovian queue of multi-server with system disaster and impatient customers. They have derived transient probes of the model and justified some special cases. Balasubramanian et al., (2015) have studied a single server bulk service Markovian queue with system disaster. The expressions for the average number of times the system reaches to the capacity, the average number of customers in the system and their respective factorial moments have been derived under different conditions.

III. DESCRIPTION AND ASSUMPTION OF MODEL

In this study a queue of single server with the VoIP calls is considered. Here the arrival times of the calls were assumed to follow Poisson distribution with mean arrival rate of ‘λ’. The calls which were arrived may balk due to impatience or make retrial to receive service again. The server has two services of heterogeneous types in which the first one is essential for all calls and second one is optional. For both the services the times of service were assumed to have general distributions with probability density functions $s_1 (x)$ and $s_2 (x)$ respectively.

If a call, on arrival, sees the server busy, it balks the system with probability $(1 - \alpha)$ or it enters into an orbit with probability $\alpha$ in order to enter the system again. The time gap between successive try of every call follows exponential distribution with mean retrial rate ‘γ’.

At the time of completion of a service of a call, if there is no call in the system, the server goes for $j^{th}$ $(j = 1, 2, ..., M)$ vacation with probability $\beta_j$ or retains in the system with probability $\beta_0$ and $\sum_{j=0}^{M} \beta_j = 1$.

When the system is functioning well, unfortunately, the catastrophe occurs either during the essential or optional service and the system becomes down at once. The time at which catastrophe occurs is exponentially distributed with mean catastrophe rate $\xi$. After recovered from the system is down, the process of the system is going on.

For the above stated model, form a two dimensional Markov chain as $\{N(t), C(t)\}$.

Here, $N(t) = n$, $(n \geq 0)$ is the no. of calls at time ‘t’ and $C(t) = 0, 1, 2, 3$ are the states of the server.

Let $P_{i,n}(t)$, $(i=0,1,2,3; n=0,1,2,3,...)$ be the probability of the system with $n(\geq 0)$ calls at time $t$, when, the server is idle, on essential service, on optional service and on vacation for $i = 0, 1, 2, 3$.

IV. EQUATIONS RELATED TO STEADY STATE OF THE MODEL

On the basis of the above section, the required steady-state equations are framed.
Apply the LST in the equations (2), (3) and (4) and using the relation (5), we have,

\[
\begin{align*}
\Phi_{n+1}(x) &= \Phi_n(x) + \lambda x \Phi_n(x) - \lambda \Phi_n(x - \lambda) + \lambda x \Phi_n(x - \lambda) \\
\Phi_{n+1}(x) &= \Phi_n(x) - \lambda x \Phi_n(x) + \lambda \Phi_n(x - \lambda) - \lambda x \Phi_n(x - \lambda) \\
\phi(x) &= \int_0^\infty e^{-xu} \Phi_n(u) du
\end{align*}
\]

Now, define probability generating functions.

\[
P(z) = \sum_{n=0}^\infty P_n(z) z^n
\]

In this stage, define the probability generation function as

\[
P_1^* (z, 0) = \sum_{n=0}^\infty P_n^* (0) z^n, \quad i = 1, 2, 3.
\]

Now, using the relation (22) in the equations (19), (20) and (21) and they become respectively as

\[
P_i (z) = P_i^* (z, 0) + P_i^* (z, 0) + P_i^* (z, 0)
\]

Substitute the equations (23), (24) and (25) in the condition (26) and get,

\[
\phi(x) = \int_0^\infty e^{-xu} \Phi_n(u) du = \int_0^\infty e^{-xu} \sum_{n=0}^\infty \lambda^n \Phi_n(u) du
\]

where

\[
\Phi_n(u) = \sum_{n=0}^\infty \lambda^n \Phi_n(u)
\]

and

\[
\Phi_n(u) = \sum_{n=0}^\infty \lambda^n \Phi_n(u)
\]

VI. SPECIAL CASES & RESULTS

1. Erlangian Vacation time:

In this case, vacation times are distributed according to Erlang distribution with \( k^k \) phases.

The p.d.f. of k-Erlang is

\[
v_j (x) = \frac{(kv_j)^k x^{k-1} e^{-kv_j x}}{(k-1)!}, \quad j = 1, 2, 3, \ldots, M
\]

Its Laplace Stieltjes Transform is

\[
V^k (s) = \frac{\lambda + s}{\lambda + s - \lambda x}
\]

On considering the vacation times follow Erlang distribution, the p.g.f. of the number of calls \( P(z) \) shown in (27), after applying the equation (29), becomes
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\[ P(z) = \frac{(\lambda \alpha z - \lambda \alpha - \xi (z - D_1(z)) + (\lambda z - \lambda - \xi)(K_1(z) - 1)P_0(z) + \xi(1-z)(K_1(z) - 1))}{\lambda \alpha z - \lambda \alpha - \xi (z - D_1(z))} \]

where

\[ \sum_{j=0}^{\infty} \beta_j z^j \left( \frac{\beta_j}{\alpha \alpha} \right) (k \mu + z_{\alpha - \lambda \alpha - \xi j(z - D_1(z))}) \]

and

\[ z_{\alpha - \lambda \alpha - \xi j(z - D_1(z))} \left( \frac{\beta_j}{\alpha \alpha} \right) \sum_{j=0}^{\infty} \beta_j z^j \left( \frac{\beta_j}{\alpha \alpha} \right) (k \mu + z_{\alpha - \lambda \alpha - \xi j(z - D_1(z))}) \]

It is noted that the equation (18) is re-written for \( P_0(z) \), after replacing \( D(u) \) by \( D(u) \).

2. Erlangian Essential Service time:

Essential and optional service times follow Erlang with \( (k, \mu_1) \) for essential service is

\[ s_1(x) = \frac{(k \mu_1)^k}{(k-1)!} e^{-k \mu_1 x} \]

Its Laplace Stieljes Transform is

\[ \sum_{j=0}^{\infty} \frac{k \mu_1}{(k+1)^j} \]

Similarly, the p.d.f. of Exponential \( \mu_2 \) for optional service is

\[ s_2(x) = \mu_2 e^{-\mu_2 x} \]

and its Laplace Stieljes Transform is

\[ S_2(x) = \frac{\mu_2}{\mu_2 + \xi (z - D_1(z))} \]

Now apply the expressions (32) and (34) in the equation (27) and get

\[ P(z) = \frac{(\lambda \alpha z - \lambda \alpha - \xi (z - D_1(z)) + (\lambda z - \lambda - \xi)(K_1(z) - 1)P_0(z) + \xi(1-z)(K_1(z) - 1))}{\lambda \alpha z - \lambda \alpha - \xi (z - D_1(z))} \]

Where

\[ \sum_{j=0}^{\infty} \frac{k \mu_1}{(k+1)^j} \left( \frac{\beta_j}{\alpha \alpha} \right) (k \mu + \xi (z - D_1(z)) \]

and

\[ \sum_{j=0}^{\infty} \frac{k \mu_1}{(k+1)^j} \left( \frac{\beta_j}{\alpha \alpha} \right) (k \mu + \xi (z - D_1(z)) \]

As in the case of (1), the equation (18) is re-written for \( P_0(z) \) after replacing \( D(u) \) by \( D_x(u) \).

II. Particular cases

1. No Catastrophe in the system:

Suppose the occurrence of catastrophe is not allowed \( (\xi = 0) \), then the probability generating function (27) reduces to

\[ P(z) = \frac{1}{1 - (\lambda \alpha z - \lambda \alpha - (z - D_1(z) + (\lambda z - \lambda - (z - D_1(z) + (\lambda z - \lambda - (z - D_1(z) + \xi (z - D_1(z)) \}

where

\[ D_1(z) = S_1(\lambda \alpha z - \lambda \alpha - (z - D_1(z) + (\lambda z - \lambda - (z - D_1(z) + \xi (z - D_1(z)) \}

and

\[ P_0(z) = P_0(z) \exp \left( \frac{1}{D_1(z) - D_x(z)} \right) \]

The result given in (36) is identical with the result of Senthilkumar and Arumuganathan (2009).
\[ r(z) = P_0(1) \exp \left\{ -\gamma \int \frac{D(u) - u}{D(u)} \, du \right\} \]

and

\[ D_4(u) = S_1 \ast (\lambda \alpha - \lambda \alpha u) S_2 \ast (\lambda \alpha - \lambda \alpha u). \]

### VII. CONCLUSION AND SUGGESTION

A single server queue is studied with some concepts like VoIP, Catastrophe, essential and optional services, impatience, multiple vacations and retrial policy. The steady state system size distribution and probability generating functions under different level have been obtained. Some special and particular cases are discussed and the coincidence of the present work with some previous research works are justified. It is suggested that there are some open problems to be solved by introducing bulk size rules in arrival or services or both in this existing system.

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