The adaptive neural network sliding mode control for angle/force tracking of the dexterous hand

Xing Qin\textsuperscript{1,2}, Heng Shi\textsuperscript{2,3}, Xin Gao\textsuperscript{3} and Xiyu Li\textsuperscript{3}

Abstract
In order to achieve high precision control of the dexterous hand, an adaptive neural network sliding mode control algorithm based on the U-K (Udwadia-Kalaba) equation is proposed. Firstly, based on the U-K equation and considering the ideal and non-ideal constrained force at each link of the dexterous hand, the detailed dynamic equation is derived. Secondly, considering the uncertainty of the non-ideal constrained force (mainly the friction force on each link of the dexterous hand) and the chattering phenomenon when using sliding mode control alone, the adaptive neural network and the sliding mode control algorithm are combined to realize the high-precision tracking and estimation of each link angle trajectory and the non-ideal constrained force. Finally, in order to verify the correctness and rationality of the proposed algorithm, the 3-DOF spatial dexterous hand is taken as the simulated object. The simulation results show that the tracking and estimation errors of each link angle and the non-ideal constrained force are $10^{-2}$ order of magnitude.

Keywords
Dexterous hand, Udwadia-Kalaba equation, dynamic model, the non-ideal constrained force, adaptive neural network sliding mode control

Date received: 24 May 2021; accepted: 13 July 2021

Handling Editor: Chenhui Liang

Introduction
Dexterous hand is the last contact of the interaction between robot and environment, which directly affects the robot’s maneuverability and intelligent level. The single-DOF end effector, which is widely used in the industrial field, has simple structure and convenient control, but its versatility is poor, and it is difficult to arbitrarily grasp objects with complex shapes. Therefore, in order to adapt to various grasping tasks, multi-DOF dexterous hand has become one of the research hotspots in the robot field in recent years. The most important function of dexterous hand is to achieve the task of operation and grasping. The accuracy of constraint solution directly affects the accuracy of dexterous manual mechanical modeling,\textsuperscript{1} and the accuracy of dynamic equation is very important for the high-precision control of grasping and positioning. The dexterous hand has three to five fingers, which can be regarded as a multi-DOF rigid body system. For the dexterous hand, the interference of the restraint and the external environment is uncertain, and the conventional modeling and control methods are no longer applicable, so new theories and methods must be explored.

A lot of academic researches solved the constrained force of the dexterous hand. Lippiello et al.\textsuperscript{2}

1\textsuperscript{University of Chinese Academy of Sciences, Beijing, China}
2\textsuperscript{Xi’an Institute of Optics and Precision Mechanics of CAS, Xi’an, China}
3\textsuperscript{Beijing Institute of Tracking and Telecommunications Technology, Beijing, China}

Corresponding author:
Xin Gao, Beijing Institute of Tracking and Telecommunications Technology, No. 26 Beijing Road, Haidian District, Beijing 100094, China.
Email: gaoxin526@126.com

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en/us/nam/open-access-at-sage).
transformed the constrained force optimization problem of dexterous hand into a convex optimization problem, and proposed a solution suitable for online implementation. By dynamically reducing the number of active torque constraints, the computational load was greatly reduced. Nguyen and Véronique solved the constrained force of the tendon driven dexterous hand, and then realized the strong grasping and operating potential of the dexterous hand by controlling the maximum torque and constrained force. However, the performance of the force control algorithm is still poor when the recoil force of dexterous hand is high. Kaya et al. proposed a force compensation method based on model to complete the estimation calculation of the environmental constrained force of multi-joint dexterous hand. The method considered a variety of constrained reaction forces at the end of the dexterous hand, but lacked the decomposition solution of the viscous friction force, making its dynamic model incomplete. Mavrogiannis et al. proposed an optimal solution to solve the forced closure constraint of a dexterous hand, taking into account the mechanical and geometric constraints imposed by the shape design of the hand and object. Ji and Peng put forward a method to acquire constrained force for three-fingered dexterous hands in operational tasks.

Liu et al. and Lin et al. both approximate the size and direction of the constrained force by covering dense tactile sensors at the fingertips of the dexterous hand. Ma et al. aiming at the uncertainty of the constrained force of dexterous hand, established the environmental attraction region by using the model in the configuration space, and then configured and generated the offline grasping plan, and then eliminated the uncertainty of the constrained force through online dynamic adjustment. All of the above constrained force solutions only consider the constraint of perpendicular constraint reaction force on the surface of the object, while the influence of the tangential friction is not taken into account. Therefore, the problem of constrained force solving has not been completely solved.

The dexterous hand can be regarded as the general mechanical rigid body system. So far, there are many researches on the dynamic modeling of the constrained mechanical system. Lagrange was the first to use Lagrange multiplier method for the motion analysis of the constrained mechanical system, which is difficult to get the Lagrange multiplier. Gibbs and Appell solve the dynamic equation by adding auxiliary variables and using Appell equation, but it is difficult to be used in the research of multi-DOF dexterous hand. Udwadia and Kalaba proposed the constraint equation of multi degree of freedom mechanical system, which is called U-K equation. According to the D’Alembert’s principle, the virtual work done by the ideal constrained force is zero, while the virtual work done by the non-ideal constrained force is not zero. The U-K equation is suitable for ideal and non-ideal constrained force.

At present, some scholars use U-K equation to model and solve different mechanical systems. Liu and Liu and Huang et al. used this equation to obtain the dynamic models of the series and parallel robots respectively. Zhao et al. successfully established the dynamic equation of mechanical fish through the equation. By now, the above researches only consider the ideal constrained force, and lack the modeling and solving of non-ideal constrained force. In addition, the U-K equation has not been used to solve the constrained force of dexterous hand.

Dexterous hand is a typical mechatronics system. In addition to friction, saturation and other nonlinear factors, there is a strong coupling relationship between the links. The characteristics of sliding mode control are suitable for the control of dexterous hand. On the one hand, when designing the sliding mode control system, there is no need to do special decoupling for the internal coupling of the system. It can be designed according to the own independent systems, and the selection of parameters is not strict. On the other hand, the sliding mode control has strong robustness to parameter perturbation and external disturbances, and has fast response speed, small amount of calculation and simple physical implementation.

Although the sliding mode control has the advantage of invariance, the discontinuous switching characteristics of the sliding mode control will cause chattering. A lot of researches have been done on anti-chattering problem of the sliding mode control. Xu et al. divided the control law into three parts: equivalent control, switching control, and continuous control. Eun et al. designed a discrete sliding mode control. By designing an observer to observe the interference at the input end of the control, the interference was effectively compensated and chattering was effectively eliminated. By designing an adaptive integral term to replace the switching term, Wong et al. realized the adaptive adjustment of the gain of the switching term, effectively reduced the gain of the switching term, and thus weakened chattering. Lakhekar et al. combined fuzzy control with sliding mode control to reduce the influence of state switching through the empirical characteristics of fuzzy rules, thus reducing chattering of sliding mode control. Ren et al. developed an adaptive fault-tolerant BC by utilizing strict formula derivations to compensate for unknown composite disturbance, dead zone, and actuator fault in the FS system. Huang et al. proposed a method based on the combination of RBF neural network and sliding mode control, and used the approximation ability of neural network function to load the switching state quantity into the neural network function in the form of input quantity, thus
achieving the purpose of eliminating chattering. Li et al.33 proposed a novel neural network and sliding mode control architecture for the intuitive and unmarked visual remote operation of dexterous hands, and realized the high-precision control of dexterous hands. Liu et al.32 designed a new neural-network-based sliding mode control architecture for the intuitive and unmarked visual remote operation of dexterous hands. Termination of the control algorithm, a 3-DOF spatial dexterous hand is studied. The Udwadia-Kalaba equation

The dexterous hand can be regarded as the multi-body system. To the multi-body system without constraint, the dynamic equation24 of n-DOF system can be established as:

\[
M(\mathbf{\theta})\ddot{\mathbf{\theta}} + C(\mathbf{\theta}, \dot{\mathbf{\theta}})\dot{\mathbf{\theta}} + G(\mathbf{\theta}) = \tau
\]  

(1)

where \( \mathbf{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T \) is n-dimensional generalized coordinates for describing systems. \( M(\mathbf{\theta}) \) is the mass matrix. \( C(\mathbf{\theta}, \dot{\mathbf{\theta}}) \) denotes coriol and centrifugal torques. \( \tau \) represents applied link torques vector. \( G(\mathbf{\theta}) \) is the gravitational torque vector.

When the system has \( m_1 \) ideal constraints, it can be written as

\[
\phi_i(\mathbf{\theta}) = 0 \quad i = 1, 2, \ldots, m_1
\]

(2)

The \( m - m_1 \) non-ideal constraints can be given as:

\[
\phi_j(\mathbf{\theta}, \dot{\mathbf{\theta}}) = 0 \quad j = m_1 + 1, m_2 + 2, \ldots, m
\]

(3)

Equations (2) and (3) are derived for time respectively, and the constrained equation20 of the system is as follows:

\[
A(\mathbf{\theta}, \dot{\mathbf{\theta}})\ddot{\mathbf{\theta}} = b(\mathbf{\theta}, \dot{\mathbf{\theta}})
\]

(4)

Where \( A \) is \( m \times n \) constraint matrix and \( b \) is a \( m \)-vector. Considering the ideal and non-ideal constrained force, the equation (1) can be expressed as:

\[
M(\mathbf{\theta})\ddot{\mathbf{\theta}} + C(\mathbf{\theta}, \dot{\mathbf{\theta}})\dot{\mathbf{\theta}} + G(\mathbf{\theta}) + Q^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) = \tau
\]

(5)

Where \( Q^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) \) donates the additional forces acting on the multi-body system, which is \( n \)-vector.

According to the D’Alembert’s principle, the \( Q^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) \) can be divided into the ideal constrained force and the non-ideal constrained force:12,13

\[
Q^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) = Q_{id}^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) + Q_{nid}^e(\mathbf{\theta}, \dot{\mathbf{\theta}})
\]

(6)

where \( Q_{id}^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) \) represents the ideal constrained force (perpendicular to the action surface of each link) and \( Q_{nid}^e(\mathbf{\theta}, \dot{\mathbf{\theta}}) \) donates the non-ideal constrained force (mainly the friction force on each link of the dexterous hand).

On the basis of the U-K equation, the detailed analytic expressions of the ideal and non-ideal constrained forces are as follows:

\[
Q_{id}^e = M^\frac{1}{2}B^+ (b - AM^{-1}Q)
\]

(7)

and

\[
Q_{nid}^e = M^\frac{1}{2}(I - B^+ B)M^{-\frac{1}{2}}c
\]

(8)

where \( B = AM^{-\frac{1}{2}} \) and the superscript “ + ” denotes the Moore-Penrose generalized inverse matrix. The vector
obtained as follows:

From equations (5) and (6), the dynamic model can be regarded as an independent multi-body.

**Dexterous hand model**

The dexterous hand with multi-sensor and multi-DOF is a highly integrated mechanical system, which integrates the mechanical, driving, sensing, and microprocessor systems. Each finger of the dexterous hand can be regarded as an independent multi-body.

In order to verify the effectiveness and correctness of the dynamic model and control algorithm, a 3-DOF spatial dexterous hand is taken as the research and simulation object, and the model is shown in Figure 1.

Figure 1 shows a simplified model of a three link simulation object, and the model is shown in Figure 1.

The variables of the mass matrix $M(\theta)$ can be written as follows:

\[
\begin{align*}
    m_{11} = l + a_1 \cos^2(\theta_1) + a_2 \cos^2(\theta_2 + \theta_3) + 2a_3 \\
    m_{12} = m_{21} = m_{31} = m_{31} = 0, \\
    m_{22} = l_2 + a_1 + a_2 + 2a_3 \cos(\theta_3) \\
    m_{23} = m_{32} = a_2 + a_3 \cos(\theta_3), \\
    m_{33} = l_3 + a_2 
\end{align*}
\]

The variables of the coriolis and centrifugal torques matrix $C(\theta, \dot{\theta})$ is given by:

\[
\begin{align*}
    c_{11} &= -\frac{1}{2}a_1 \dot{\theta}_1 \sin(2\theta_2) - \frac{1}{2}a_2(\dot{\theta}_2 + \dot{\theta}_3) \sin(2\theta_2 + 2\theta_3) - a_3 \dot{\theta}_2 \sin(2\theta_2 + \theta_3) - a_3 \dot{\theta}_3 \cos(\theta_2) \sin(\theta_2 + \theta_3) \\
    c_{12} &= -\frac{1}{2}a_1 \dot{\theta}_1 \sin(2\theta_2) - \frac{1}{2}a_2 \dot{\theta}_1 \sin(2\theta_2 + 2\theta_3) - a_3 \dot{\theta}_1 \sin(2\theta_2 + \theta_3) \\
    c_{13} &= -\frac{1}{2}a_1 \dot{\theta}_1 \sin(2\theta_2 + 2\theta_3) - a_3 \dot{\theta}_1 \cos(\theta_2) \sin(\theta_2 + \theta_3) \\
    c_{21} &= -c_{12}, \\
    c_{22} &= -a_3 \dot{\theta}_3 \sin(\theta_3), \\
    c_{23} &= -a_3(\dot{\theta}_2 + \dot{\theta}_3) \sin(\theta_3) \\
    c_{31} &= -c_{13}, \\
    c_{32} &= -a_3 \dot{\theta}_2 \sin(\theta_3), \\
    c_{33} &= 0 
\end{align*}
\]

The variables of the gravitational torque vector $G(\theta)$ can be written as follows:

\[
\begin{align*}
    g_1 &= 0 \\
    g_2 &= b_1 \cos(\theta_2) + b_2 \cos(\theta_2 + \theta_3) \\
    g_3 &= b_2 \cos(\theta_2 + \theta_3) 
\end{align*}
\]

where, $a_1 = m_2 r_2^2 + m_3 r_2^2$, $a_2 = m_3 r_2$, $a_3 = m_3 r_3 l_2$, $b_1 = (m_2 r_2 + m_3 l_2) g$, $b_2 = m_3 r_3 g$.

dexterous hand. The first link rotates, and the second and third links pitch. The length of link 2 and link 3 respectively are $l_2$, $l_3$, and the mass of link 2 and link 3 respectively are $m_2$, $m_3$. The distance from the mass center of link 2 and 3 to the rotation axis are $r_2$, $r_3$, respectively. The rotational inertia of three links are $I_1$, $I_2$, $I_3$. From equations (5) and (6), the dynamic model can be obtained as follows:

\[
M(\theta, \dot{\theta}) \ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + Q^c_{id}(\theta, \dot{\theta}) + Q^c_{nd}(\theta, \dot{\theta}) = \tau 
\]
Adaptive neural network sliding mode control for dexterous hand

According to the dynamic model, define the tracking error:

\[ e = \theta - \theta_d \] (13)

The sliding mode surface is designed as follows:

\[ s = \dot{e} + \Lambda e \] (14)

From the above equation, it can be obtained:

\[ \dot{\theta} = -s + \dot{\theta}_d + \Lambda e \] (15)

The equation (15) are multiplied by the matrix \( M \), the following can be obtained:

\[
\begin{align*}
M\dot{s} &= M(\dot{\theta}_d - \dot{\theta} + \Lambda \dot{e}) = M(\dot{\theta}_d + \Lambda \dot{e}) - M\dot{\theta} \\
&= M(\dot{\theta}_d + \Lambda \dot{e}) + C\dot{\theta} + G + \mathcal{Q}'_{\text{id}} + \mathcal{Q}'_{\text{mid}} - \tau \\
&= M(\dot{\theta}_d + \Lambda \dot{e}) + C\dot{\theta} + G + \mathcal{Q}'_{\text{id}} + \mathcal{Q}'_{\text{mid}} - \tau \\
&= f(x) - Cs - \tau \\
\end{align*}
\] (16)

where \( f(x) = M(\dot{\theta}_d + \Lambda \dot{e}) + C(\dot{\theta}_d + \Lambda e) + G + \mathcal{Q}'_{\text{id}} + \mathcal{Q}'_{\text{mid}} \)

\( x \) is the input variable to be designed.

The sliding mode controller is designed as follows:

\[
\tau = M\dot{\theta}_d + \Lambda \dot{e} + C\dot{\theta} + G + \mathcal{Q}'_{\text{id}} + \mathcal{Q}'_{\text{mid}} + Ps + K\text{sgn}(s) \]

(17)

where \( \text{sgn}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases} \)

\( f(x) \) donates the non-ideal constrained force \( \mathcal{Q}'_{\text{mid}} \).

From the previous derivation, \( \mathcal{Q}'_{\text{mid}} \) cannot be obtained the exact analytical expression. Therefore, the RBF neural network is used to approximate \( f(x) \), and then the approximate \( f(x) \) is decomposed to obtain the non-ideal constraint \( \mathcal{Q}'_{\text{mid}} \). Finally, the system control is realized without the precise mathematical model of the controlled object.

According to the expression of \( f(x) \), the input of neural network is as follows:

\[
x = \begin{bmatrix} e^T & \dot{e}^T & \dot{\theta}_d^T & \ddot{\theta}_d^T & \dddot{\theta}_d^T \end{bmatrix} \] (18)

According to the learning process of the RBF neural network, the ideal output of the RBF neural network is as follows:

\[
\psi_i = g \left( \frac{||x - c_i||^2}{\sigma_i^2} \right) \] (19)

\[ f(x) = \mathbf{w}^T \Phi(x) + \xi \]

where, \( \Phi(x) = [\psi_1, \psi_2, \ldots, \psi_n] \) is the output of Gaussian basis function; \( \mathbf{w}^* \) is the weight matrix of ideal neural network; \( \xi = [\xi_1, \xi_2, \ldots, \xi_m]^T \) is the ideal approximation error of neural network.

The actual output of RBF neural network is set as follows:

\[
\tilde{f}(x) = \hat{\mathbf{w}}^T \Phi(x) \] (20)

Make \( \hat{\mathbf{w}} = \mathbf{w}^* - \hat{\mathbf{w}} \), where, \( \hat{\mathbf{w}} \) is the weight of actual approximation, \( \hat{\mathbf{w}} \) is the weight error, and \( \tilde{f}(x) \) is the actual approximation value of RBF neural network to \( f(x) \).

Then equation (20) is substituted into equation (17), and the neural network sliding mode controller is obtained as follows:

\[
\tau = \hat{\mathbf{w}}^T \Phi(x) + Ps + K\text{sgn}(s) \] (21)

Substituting equations (19) and (21) into equation (16), the following results can be obtained:

\[
M\dot{s}_d = \mathbf{w}^*T \Phi(x) + \xi - Cs - [\hat{\mathbf{w}}^T \Phi(x) + Ps + K\text{sgn}(s)] \\
= \hat{\mathbf{w}}^T \Phi(x) - (P + C)s + \xi - K\text{sgn}(s) \]

(22)

In order to verify the stability of the control system, the Lyapunov function is taken as:

\[
V = \frac{1}{2} s^T M s + \frac{1}{2} tr(\hat{\mathbf{w}}^T Q^{-1} \hat{\mathbf{w}}) \] (23)

where, \( tr(\bullet) \) is the trace of a matrix; \( Q \) is a symmetric positive definite matrix with constant coefficients.

The matrix characteristics of the dexterous hand is \( s^T (M - 2C)s = 0 \), the following equation can be obtained after deriving formula (23):

\[
\dot{V} = s^T M \dot{s}_d + \frac{1}{2} s^T M s + tr(\hat{\mathbf{w}}^T Q^{-1} \hat{\mathbf{w}}) \\
= s^T M s + s^T C s + tr(\hat{\mathbf{w}}^T Q^{-1} \hat{\mathbf{w}}) \]

(24)

Substituting equation (22) into equation (24):

\[
\dot{V} = s^T M s + \frac{1}{2} s^T M s + tr(\hat{\mathbf{w}}^T Q^{-1} \hat{\mathbf{w}}) \\
= -s^T P s + s^T \hat{\mathbf{w}}^T \Phi(x) + tr(\hat{\mathbf{w}}^T Q^{-1} \hat{\mathbf{w}}) \\
+ s^T [\xi - K\text{sgn}(s)] \]

(25)

Because of the following formula:

\[
s^T \hat{\mathbf{w}}^T \Phi(x) = tr(\hat{\mathbf{w}}^T \Phi(x) s^T) \] (26)

From equations (25) and (26), it can be obtained that:
The law of RBF neural network weight adjustment is designed to ensure that the system is stable, \( V \) is the constraint condition, and the DOF dexterous hand in Figure 1, the constraint is the angle trajectory constraint of each link, and the constraint condition is:

\[
\dot{V} = - s^T P s + tr \{ \hat{\omega}^T [\Phi(x)s^T + Q^{-1} \hat{\omega} ] \} + s^T [\zeta - K \text{sgn}(s)]
\]  

(27)

Since \( \omega^* \) is the weight of ideal approximation, when the system is stable, \( \omega^* \) keeps the ideal state unchanged, that is \( \dot{\omega}^* = 0 \). The result is as follows:

\[
\dot{\omega} = \dot{\omega}^* - \dot{\omega} = - \dot{\omega}
\]  

(28)

In order to get a stable control system, the adaptive law of RBF neural network weight adjustment is designed:

\[
\dot{\omega} = - \dot{\omega} + Q \Phi(x)s^T
\]  

(29)

Substituting equation (29) into equation (27):

\[
\dot{V} = - s^T P s + s^T [\zeta - K \text{sgn}(s)]
\]  

(30)

When \( K \gg \zeta \), it can be obtained as follows:

\[
\dot{V} \approx 0
\]  

(31)

When \( V = 0 \), \( s = 0 \), according to the principle of LaSalle invariance, the closed-loop system is gradually stable, \( t \to \infty \), \( s \to 0 \).

Simulated example

In order to verify a set of constraints given for the 3-DOF dexterous hand in Figure 1, the constraint is the angle trajectory constraint of each link, and the constraint condition is:

\[
\begin{align*}
\theta_1 &= \sin (0.5 \pi t) \\
\theta_2 &= \cos (0.5 \pi t) \\
\theta_2 + \theta_3 &= \pi
\end{align*}
\]  

(32)

Take the second derivative of the equation (32) with respect to time, and the second-order constraint form can be obtained as follows:

\[
\begin{align*}
\dot{\theta}_1 &= -0.25 \pi^2 \sin (0.5 \pi t) \\
\dot{\theta}_2 &= -0.25 \pi^2 \cos (0.5 \pi t) \\
\dot{\theta}_2 + \dot{\theta}_3 &= 0
\end{align*}
\]  

(33)

According to the constraint equation form of the equation (4), one can get:

\[
A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -0.25 \pi^2 \sin (0.5 \pi t) \\ -0.25 \pi^2 \cos (0.5 \pi t) \\ 0 \end{bmatrix}, \quad \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}
\]  

(34)

In addition, the ideal constrained force at each link of the dexterous hand can be obtained according to equation (4), but the vector \( e \) in the analytical formula of non-ideal constrained force is unknown. In this paper, the high-precision approximation is realized to solve the non-ideal constrained force through the universal approximation property of the neural network. Finally, the high-precision tracking and control of the each link angle trajectory of dexterous hand and the non-ideal constrained force are realized.

The physical parameters of the three-link dexterous hand are set as: \( m_2 = 1.5 \text{kg}, m_3 = 1 \text{kg} \),

\[
r_2 = 0.6 \text{m}, r_3 = 0.5 \text{m}, l_2 = 1.2 \text{m}, l_3 = 1 \text{m},
\]

\[
I_1 = 3 \text{kg} \cdot \text{m}^2, I_2 = 2.5 \text{kg} \cdot \text{m}^2, I_3 = 2 \text{kg} \cdot \text{m}^2,
\]

\[
g = 9.8 \text{m/s}^2
\]

The control parameters of the control algorithm are set to: \( K = \text{diag}(25, 25, 25) \), \( P = \text{diag}(50, 50, 50) \), \( \Lambda = \text{diag}(5, 5, 5) \), and \( Q = \text{diag}(12, 12, 12) \). The initial angle position is set to \( \theta(0) = [0.6, 0.6, \pi - 0.6]^T \). The structure of the adaptive neural network sliding mode control system of the whole three-link dexterous hand is shown in Figure 2:

The simulation results are shown in Figures 3 to 7. Figures 3 to 5 shows the angle trajectory tracking and tracking error of each link, in which the red solid line...
represents the angle trajectory given in theory and the tracking error, and the blue dotted line represents the actual angle tracking trajectory. Figure 6 shows the control torque of each link. Figure 7 shows the input and estimated approximation of the non-ideal constrained force of each link, in which the red solid line represents the input value and the blue dotted line represents the estimated approximation value.

The maximum error values of the angle trajectory tracking and non-ideal constrained force approximation estimation errors of each link in the above figures in the stable stage are shown in the following table:

**Figure 3.** The angle tracking and tracking error of the link 1.

**Figure 4.** The angle tracking and tracking error of the link 2.
From the above simulation results, it can be seen that the adaptive neural network sliding mode control algorithm can realize the high-precision approximation and tracking of the angle trajectory and non-ideal constrained force of the dexterous hand. Three conclusions can be drawn as follows:

(1) In the aspect of angle trajectory tracking, the tracking error of each link in the stable stage is close to 0, and the angle relationship between link 2 and 3 satisfy the constraint conditions, which proves that the dynamic model and control algorithm are effective and meet the control performance requirements.

(2) In terms of control torque, the simulation result shows that the torque curve of each link has no chattering phenomenon, which proves that the universal approximation characteristic of the adaptive neural network algorithm overcomes the chattering hidden trouble of the sliding mode control.

Figure 5. The angle tracking and tracking error of the link 3.

Figure 6. The control torques of the dexterous hand.

Figure 7. The input and estimation of the non-ideal constrained force.
(3) In the non-ideal constrained force approximation estimation, the adaptive neural network algorithm used in this paper achieves the approximation and estimation of the uncertainty of non-ideal constrained force, which makes the dynamic model of dexterous hand more complete in practical application.

Conclusion

This paper presents an adaptive neural network sliding mode control method for dexterous hand. Based on the U-K equation, the complete dynamic equations of dexterous hand under ideal and non-ideal constrained forces are obtained. Aiming at the uncertainty of non-ideal constrained force of each link and chattering phenomenon of the sliding mode control, the adaptive neural network and the sliding mode control method are combined to achieve high-precision control of dexterous hand. The stability of the system is proved by Lyapunov theorem. In order to verify the effectiveness of the control algorithm, a 3-DOF spatial dexterous hand is studied. The simulation results show that the tracking errors of each link angle and the non-ideal constrained force are $10^{-2}$ order of magnitude. The high-precision control for the angle/force control of the dexterous hand is realized, and the chattering phenomenon caused by single sliding mode control is suppressed. In the future, the dynamic modeling method and adaptive neural network sliding mode control algorithm proposed in this paper will be extended to other multi-DOF mechanical systems, and will also be applied to practical engineering applications.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iDs

Heng Shi https://orcid.org/0000-0003-4362-2347
Xin Gao https://orcid.org/0000-0002-6045-9265

References

1. Jia P, Meng QX and Wang LQ. (2007). Dynamic analysis on fingers of the underwater multi-fingered robot hand. In: Proceedings of the 2007 IEEE international conference on mechatronics and automation, Harbin, China, 2007, pp. 757–762. IEEE.
2. Lippiello V, Siciliano B and Villani L. (2012). A grasping force optimization algorithm for dexterous robotic hands. In: IEEE international conference on robotics & automation, Saint Paul, MN, USA, 2012. IEEE.
3. Nguyen KC and Véronique P. (2014). Fingertip force control based on max torque adjustment for dexterous manipulation of an anthropomorphic hand. In: IEEE/RSJ international conference on intelligent robots & systems, Tokyo, Japan, 2014. IEEE.
4. Kaya O, Yıldırım MC, Kuzuluk N, et al. (2015). Environmental force estimation for a robotic hand: compliant contact detection. In: 2015 IEEE-RAS 15th international conference on humanoid robots (humanoids), Seoul, Korea (South), 2015. IEEE.
5. Mavrogiannis CI, Bechlioulis CP, Liarokapis MV, et al. (2014). Task-specific grasp selection for underactuated hands. In: IEEE international conference on robotics & automation, Hong Kong, China, 2014. IEEE.
6. Ji T and Peng X. (2015). Task-based finger force analysis for robotics dexterous hand. In: 10th International conference on computer science & education (ICCSE), Cambridge, UK, 2015. IEEE.
7. Liu H, Nguyen KC, Perdereau V, et al. Finger contact sensing and the application in dexterous hand manipulation. Auton Robots 2015; 39: 25–41.
8. Lin G, Li Z, Liu L, et al. Development of multi-fingered dexterous hand for grasping manipulation. Sci China Inf Sci 2014; 57: 1–10.
9. Ma C, Qiao H, Li R, et al. Flexible robotic grasping strategy with constrained region in environment. Int J Autom Comput 2017; 14: 552–563.
10. Chablat D, Venkateswaran S and Boyer F. Mechanical design optimization of a piping inspection robot. Procedia CIRP 2018; 70: 307–312.
11. Grami S and Fareh R. (eds) Friction compensation in a 2DOF robot manipulator. In: Design and modeling of mechanical systems. Cham: Springer, 2018, pp.157–170.
12. Udwadia FE and Kalaba RE. A new perspective on constrained motion. Proc Math Phys Sci 1992; 439: 407–410.
13. Udwadia FE and Kalaba RE. Explicit equations of motion for mechanical systems with non-ideal constraints. J Appl Mech 2001; 68: 462–467.
14. Udwadia FE and Wanchanont T. A new approach to the tracking control of uncertain nonlinear multi-body mechanical Systems. Nonlinear approaches in engineering applications 2. New York: Springer, 2014, pp.101–136.
15. Udwadia FE and Koganti PB. Optimal stable control for nonlinear dynamical systems: an analytical dynamics based approach. Nonlinear Dyn 2015; 82: 547–562.
16. Udwadia FE and Koganti PB. Dynamics and control of a multi-body planar pendulum. Nonlinear Dyn 2015; 81: 845–866.
17. Udwadia FE and Mylapilli H. Energy control of nonhomogeneous Toda lattices. *Nonlinear Dyn* 2015; 81: 1355–1380.
18. Udwadia FE. Fundamental principles of Lagrangian dynamics: mechanical systems with non-ideal, holonomic, and nonholonomic constraints. *J Math Anal Appl* 2000; 251: 341–355.
19. Liu J and Liu R. Simple method to the dynamic modeling of industrial robot subject to constraint. *Adv Mech Eng* 2016; 8: 168781401664651.
20. Huang J, Chen YH and Zhong Z. Udwadia-Kalaba approach for parallel manipulator dynamics. *J Dyn Syst Meas Control* 2013; 135: 061003.
21. Zhao H, Zhen S and Chen YH. Dynamic modeling and simulation of multi-body systems using the Udwadia-Kalaba theory. *Chin J Mech Eng* 2013; 26: 839–850.
22. Hu F, Chen F and Yu M. (2018). Sliding Mode Control of Three-Links Spatial Robot Based on Low-Pass Filter. In: *International conference on cybernetics*, Chengdu, China, 2018. IEEE.
23. Shi H, Liang Y and Liu Z. An approach to the dynamic modeling and sliding mode control of the constrained robot. *Adv Mech Eng* 2017; 9: 168781401769047–168781401769110.
24. Liang Y, Shi H and Tian G. A reduced-order approach to the adaptive fuzzy sliding mode control of the constrained manipulator. *Adv Mech Eng* 2018; 10: 168781401878679–168781401878712.
25. Xu JX, Pan YJ and Lee TH. A gain scheduled sliding mode control scheme using filtering techniques with applications to multi-link robotic manipulators. *J Dyn Syst Meas Control* 2000; 122: 641–649.
26. Eun Y, Kim JH and Kim K. Discrete-time variable structure controller with a decoupled disturbance compensator and its application to a CNC servomechanism. *IEEE Trans Control Syst Technol* 1999; 7: 414–422.
27. Wong LK, Leung FH and Tam PK. A chatter elimination algorithm for sliding mode control of uncertain nonlinear systems. *Mechatronics* 1998; 8: 765–775.
28. Lakhekar GV, Waghmare LM and Londhe PS. Enhanced dynamic fuzzy sliding mode controller for autonomous underwater vehicles. In: *2015 IEEE underwater technology (UT)*, Chennai, India, 2015. IEEE.
29. Ren Y, Zhu P, Zhao Z, et al. Adaptive fault-tolerant boundary control for a flexible string with unknown dead zone and actuator fault. *IEEE Trans Cybern* 2021; PP: 1–10.
30. Huang SJ, Huang KS and Chiu KC. Development and application of a novel radial basis function sliding mode controller. *Mechatronics* 2003; 13: 313–329.
31. Li S, Ma X, Liang H, et al. Vision-based teleoperation of shadow dexterous hand using end-to-end deep neural network. In: *2019 international conference on robotics and automation (ICRA)*, Montreal, QC, Canada, 2019. IEEE.
32. Liu C, Wen G, Zhao Z, et al. Neural-network-based sliding-mode control of an uncertain robot using dynamic model approximated switching gain. *IEEE Trans Cybern* 2021; 51: 2339–2346.
33. Ren Y, Zhao Z, Zhang C, et al. Adaptive neural-network boundary control for a flexible manipulator with input constraints and model uncertainties. *IEEE Trans Cybern* Published online October 8, 2020. doi: 10.1109/TCYB.2020.3021069