Evolutionary game model of risk propensity in group decision making

Andrea Civilini,1 Nejat Anbarci,2 and Vito Latora1,3,4,5

1School of Mathematical Sciences, Queen Mary University of London, London E1 4NS, United Kingdom
2Department of Economics and Finance, Durham University, Durham DH1 3LB, United Kingdom
3The Alan Turing Institute, The British Library, London NW1 2DB, United Kingdom
4Complexity Science Hub Vienna, A-1080 Vienna, Austria
5Dipartimento di Fisica ed Astronomia, Università di Catania and INFN, Catania I-95123, Italy

We introduce an evolutionary game on hypergraphs in which decisions between a risky alternative and a safe one are taken in social groups of different sizes. The model naturally reproduces choice shifts, namely the differences between the preference of individual decision makers and the consensual choice of a group, that have been empirically observed in choice dilemmas. In particular, a deviation from the Nash equilibrium towards the risky strategy occurs when the dynamics takes place on heterogeneous hypergraphs. These results can explain the emergence of irrational herd and radical behaviours in social groups.

Choice dilemmas describe the most general situations in which decision makers are faced with an alternative between two possibilities: a riskier strategy, that either brings a high reward with a probability \( w_p \), or a low one with probability \( 1 - w_p \), and a safer strategy with an intermediate reward \([1–3]\). If the probability \( w_p \) is unknown to the decision maker, we talk of decision under uncertainty, if it is known instead we talk of decision under risk \([4, 5]\).

Expected Utility Theory and its strategic version Game Theory, the classical paradigms for the study of decision making under risk and uncertainty, assume that rational decision makers always act to maximize their private utilities/payoffs \([6]\). However, over the years a series of empirical evidences in contrast with the theoretical predictions have clearly pointed out the descriptive limitations of these theories and have undermined their very fundamental assumptions. The alternative explanations and models that have been proposed are able to justify these discrepancies by incorporating ad hoc behavioural mechanisms in the decision making process \([1, 3–5, 7–13]\). In particular, two anomalies of great practical relevance arise when decision makers interact in social groups, influencing each other. The first anomaly is the herd behaviour leading to irrational outcomes, observed for example during financial bubbles, where decision makers can follow the opinion of others, apparently disregarding their own interests \([10–13]\). The second anomaly has been empirically observed when decision makers are organized in groups and decide collectively their strategy \([2–4, 14]\) (a precious source of insights and experimental data is offered by the studies on trial juries conducted in the Seventies \([15–19]\)).

In this Letter, we propose to use evolutionary game theory on hypergraphs to model group choice dilemmas under uncertainty. The model can be seen as a generalization to higher-order interactions of anti-coordination pairwise games, such as the game of Chicken \([23]\), in which the decision makers are organized in groups of different sizes and each decision maker can participate to a variable number of groups. In this way the game dynamics describes, at the same time, how information spreads according to a stochastic imitation process and how the group members aggregate information in order to determine the group strategy. We show that group choice shifts emerge naturally in our model, and we explain how they depend on the mechanisms of preference aggregation and on the structure of the hypergraph. In particular, when implemented on heterogeneously structured populations, our model predicts the spontaneous emergence of irrational herd behaviour towards the riskier strategy, where the adoption of a strategy is followed by a collapse of the average utility.

Model. A population of \( M \) interacting decision makers is modelled as the nodes of a hypergraph, whose \( N \) hyperedges describe the interactions in groups of two or more agents \([24, 25]\). The hypergraph can be represented by a \( M \times N \) adjacency matrix \( A \), whose entry \( a_{ij} \) is equal to 1 if the agent \( i \) is in group \( j \), or is zero otherwise. The hyperdegree of agent \( i \), \( k_i = \sum_{j \in M} a_{ij} \), and the size of group \( j \), \( q_j = \sum_{i \in M} a_{ij} \), represent respectively the number of groups in which agent \( i \) takes part, and the number of members of group \( j \). Let \( Q(q) \) and \( K(k) \) be the arbitrary probability distributions for the group sizes and...
agent hyperdegrees respectively [26]. In particular, we focus on power-law group size distributions \( Q(q) \sim q^{-\lambda} \), with \( 2 < \lambda < 4 \). In fact, many real-world group sizes are distributed according to a power-law distribution: scientific teams [27, 28], firms [29], human settlements/cities [30], and also groups of animals[31, 32]. Moreover, it has been shown that also the number of social activities/groups in which a person is involved (i.e. the hyperdegree) follows a long-tailed distribution [27, 33]. Given \( 0 \leq w_p \leq 1 \) the unknown probability of success of a risky activity, each agent can be in one of two states: \( s \) (safe) or \( t \) (tempted). If in state \( s \), the agent prefers not to participate to the risky activity. Instead if in state \( t \), agent is willing to share with the other group members the cost \( C \) to attempt this risky strategy. The strategy of a group, namely the decision to participate to the risky activity (group strategy \( T \)) or not (strategy \( S \)), depends on the states of the group members and on the way in which these states/personal opinions are aggregated. We have considered different rules of opinion aggregation, namely group decision schemes, ranging from simple majority (where the group adopts the strategy preferred by at least half of the group members + 1), to two-third majority, and proportionality. If we indicate with \( C \) the cost for the group to adopt a risky strategy, and with \( T \) and \( S \) the rewards, respectively in the case of success or failure of the risky activity, then we assume the following group payoffs \( \pi^{(g)} \):

\[
\pi^{(g)}(S) = S
\]

\[
\pi^{(g)}(T) = \begin{cases} 
W := T - C, & \text{with probability } w_p \\
L := S - C, & \text{with probability } 1 - w_p
\end{cases}
\]

associated with the group strategies. Imposing \( 0 \leq C \leq S \), it follows that \( L \leq S \leq W \) and then we are in a classical choice dilemma scenario, where the risky choice \( T \) brings a high payoff \( W \) with probability \( w_p \), otherwise a low payoff \( L \), while the safe route of action \( S \) guarantees an intermediate payoff \( S \). Let us define as \( z = N r / N \) the fraction of groups adopting strategy \( T \). In our model at each time step the \( zN \) groups with strategy \( T \) are in competition among them for a fixed share \( W \) of the total reward \( \theta NW \). As a consequence, the probability of success \( w_p \), of a risky activity is modelled as a non-decreasing function of the total number of shares \( \theta N \), per group with strategy \( T \): \( w_p = f(z) \). Moreover in our model the successful groups are selected uniformly at random among the \( zN \) groups with strategy \( T \). As a consequence, the higher is \( z \) the fraction of player with strategy \( T \), the lower is the chance of high reward (i.e. actions \( T \) are strategic substitutes [34]). We show in the Supplemental Material (SM) that, when played by a population divided in two groups of equal size, our game is equivalent to the pairwise game of Chicken with ties for middle payoffs [35]. To keep the model as general as possible, we allow the cost \( C \) to be a non-increasing function \( C(r^{(g)}) = (r^{(g)})^{-1} \)

of a group resource \( r^{(g)} \in \mathbb{R}^+ \). The idea is that the more resource a group has, the lower the cost for attempting the risky activity and the risk perception are. To define the resource of a group, we assume that each agent \( i \) is given an individual resource \( r_i = \gamma \) that the agent splits equally among the \( k_i \) groups to which it participates. This is a realistic assumption, as the greater is the resource of an agent the larger will be on average the number of activities it is involved. Moreover, this means that agent \( i \) invests in each of its \( k_i \) activities an amount of resource \( r_i^{(g)} = \gamma \), which can be interpreted as the minimum amount of resource required to be accepted in a group. Consequently, the total amount of resource \( r^{(g)} \) of a group \( g \) is a function of its size \( q^{(g)} \):

\[
r^{(g)} = \left( \sum_{i \in g} r^{(g)}_i \right)^\beta = \left( \gamma \right)^\beta
\]

(2)

where the exponent \( \beta \geq 0 \) takes into account possible nonlinear synergistic effects raising from the interaction among group members [36]. In particular, for \( \beta > 1 \) the interaction among agents leads to a superlinear scaling of the group resource with the group size. To determine the payoff of the individual agents, we simply assume that the payoff of a group in Eq. (1) is equally shared among its group members. Then, the payoff of agent \( i \) is defined as the sum of the returns from all the groups in which it is involved: \( \pi_i = \sum_{g \ni i} \pi^{(g)} / (q^{(g)})^\zeta \). The exponent \( 0 \leq \zeta \leq 1 \) allows to tune the way in which group members benefit from the group payoff. For example, in contexts where the payoff represents a material or countable quantity (e.g. a cash prize), \( \zeta = 1 \) implies that the group payoff is equally split among the group members and the total group payoff is conserved. In the limit \( \zeta = 0 \) instead each group member earns the full payoff coming from the group [37]. In the evolutionary dynamics of our model, we assume perfect rationality of the agents, meaning that the agent states are updated in time according to a stochastic dynamics where each agent tries to imitate the fittest neighbour [38]. Namely, at each time step, a focal individual \( i \) is selected at random in the population. A second individual \( j \), that we call reference, is randomly selected among the co-members of the focal individual. Then, the probability \( p_{ij} \) for the decision maker \( i \) to adopt the state of the decision maker \( j \) is defined as a growing function of the payoffs difference between the two individuals: \( p_{ij} = g(\pi_j - \pi_i) \). The co-membership network we consider is obtained as the projection of the hypergraph on the set of nodes representing group members (two nodes are linked if they are co-members in at least one group). We denote as \( h_i \) the degree of node \( i \) in the co-membership network, i.e. the number of co-members of agent \( i \), and as \( H(h) \) its distribution (see SM for details).

Results. We have investigated how the presence of
groups affects decision making through a series of numerical simulations of the model dynamics performed using the quasi-stationary (QS) approach [39, 40] (see SM for details). The numerical simulations on structured population have been compared to an analytically treatable mean-field version of our model dynamics. We describe the mean-field dynamics of the group strategies directly at the group level, using a coarse-grained approach, that is neglecting the microscopic dynamics of decision making that involves the group members. Let us start defining the fraction \( z_q \) of groups of size \( q \) having adopted strategy \( \mathbf{T} \): 

\[
z_q := \frac{N_q}{N}, \quad \text{such that} \ 0 \leq z_q \leq 1.
\]

Following a similar procedure to the one reported in Refs. [41, 42], through a Kramers-Moyal expansion of the Master Equation describing the time evolution of the system, we obtain the Fokker-Planck equation for the probability density of \( z_q \). In the limit \( N \to \infty \), the derived Fokker-Planck equation gives us the following time evolution for \( z_q \) (see SM for the derivation):

\[
\frac{dz_q}{dt} = (1 - z_q) \sum_{q'} Q(q') z_{q'} p_{ST}^{q'q} - z_q \sum_{q'} Q(q') (1 - z_{q'}) p_{TS}^{q'q}
\]  

(3)

where \( p_{ST}^{q'q} \) and \( p_{TS}^{q'q} \) are respectively the transition probabilities from strategy \( \mathbf{S} \) to \( \mathbf{T} \) and vice versa, given a focal group of size \( q \) and a reference group of size \( q' \). Each transition probability can be expressed as functions of the transition probabilities given the group payoffs \( S/q^c \), \( L/q^c \) and \( W/q^c \) [37]:

\[
p_{ST}^{q'q} := (1 - w_p) p_{SL}^{q'q} + w_p q_{SW}^{q'q}
\]

(4)

\[
p_{TS}^{q'q} := (1 - w_p) p_{LS}^{q'q} + w_p q_{WS}^{q'q}
\]

(5)

Under the assumption of statistical independence \( P(q|T) = Q(q) \), we get an equation for the time evolution of \( z := \sum_q Q(q) z_q \) the fraction of groups with strategy \( \mathbf{T} \) in the whole population:

\[
\frac{dz}{dt} = z(1 - z) \mathbb{E} \left[ p_{ST}^{q} - p_{TS}^{q} \right]
\]

(6)

where the expectation value of function \( g(q, q') \) is defined as \( \mathbb{E}[g(q, q')] := \sum_{q, q'} g(q, q') Q(q) Q(q') \). We recognize in Eq. (6) the celebrated Replicator Equation for a pairwise zero-sum symmetric game [43], with a payoff matrix defined by \( \pi(T, S) = \mathbb{E} \left[ p_{ST}^{q} - p_{TS}^{q} \right] \), which depends on this case on \( w_p(z) \). Hence, besides the two absorbing states \( z^* = 0 \) and \( z^* = 1 \), the dynamics described by Eq. (6) has a third non trivial stationary solution, which is obtained equating to zero the expectation value in Eq. (6). We find an explicit expression for the third steady state in the limit of large population \( N \gg 1 \). We can then consider \( q \) continuously distributed according to

\[ Q(q) \sim q^{-\lambda} \]

and replace the sum defining the expectation value in Eq. (6) with an integral. The transition probability from a generic state \( i \) to \( j \) is usually modelled using a Fermi function: 

\[ p_{ij} = \frac{1}{1 + \exp(w_F(\pi_i - \pi_j))} \]

where \( \pi_i \) is the payoff associated with state \( i \). Under weak selection hypothesis (that is, for small \( w_F(\pi_i - \pi_j) \)), the Fermi function can be replaced by its linear approximation [42, 44]:

\[ p_{ij} = \frac{1}{1 + \frac{w_F}{2} (\pi_j - \pi_i)} \]

where \(-1 \leq \frac{w_F}{2} (\pi_j - \pi_i) \leq 1 \). Substituting this linear expression for the transition probability in \( \mathbb{E}[p_{ST}^{q} - p_{TS}^{q}] = 0 \), using the definitions of the group payoffs and \( w_p = f(\theta/2) \), we finally find the nontrivial steady state:

\[
z_{th}^* = \frac{\theta}{f^{-1}(\frac{\zeta + \lambda - 1}{(\zeta + \lambda + \beta - 1)(T - S)r_{\min}})}
\]

(7)

We notice that the nonlinearity introduced with the exponents \( \beta, \zeta \) and \( \lambda \) brings just a scale factor \( m := \frac{\zeta + \lambda + \beta - 1}{\zeta + \lambda - 1} \) in the solution, without changing its functional form. Since \( 0 \leq z_{th}^* \leq 1 \) by definition, this implies that \( T - S \) can be restricted to the range \( 0 \leq T - S \leq 1 \left[ m r_{\min} f(\theta) \right] \). Therefore, we can define a normalized quantity \( 0 \leq (T - S)^{'} \leq 1 \), where \( (T - S)^{'} := (T - S) \left[ m r_{\min} f(\theta) \right] \).
that allows us to rewrite Eq. (7) simply as: $z_{th} = \frac{\theta}{f^{-1}(\frac{f(\theta)}{(T-S)})}$. It can be shown (see SM) that this steady state is an evolutionary stable state, and therefore a mixed strategy Nash Equilibrium (NE), of the pairwise zero-sum game defined by Eq. (6). For a probability of success inversely proportional to $z$, $w_p = \theta/z$, we can solve the Cauchy problem associated with Eq. (6) to get an analytical expression for $z(t)$ which converges to $z_{th}$ for all the initial conditions $0 < z_0 < 1$ (see SM). By comparing $z_{th}$, the analytical expression for the null-model’s nontrivial steady state, to the long term behaviour of the numerical simulations on structured populations, we can evaluate how the social hypergraph’s topology influences the dynamics. Panel (a) in Fig. 1 shows that for hypergraphs characterized by a co-membership degree distribution $H(h)$ with a finite second moment (in particular we show results for power-law distribution with exponent $\gamma = 3.5$ [45]), the numerical simulations are in good agreement with the mean-field solution in Eq. (7), the QS state coinciding with the NE. Instead, when the strategy adoption dynamics takes place on hypergraphs with $H(h) \sim h^{-\gamma}$, $\gamma < 3$ (results shown are for $\gamma = 2.5$ [46]), we found that a phase transition occurs from the absorbing state $z = 0$ to a nontrivial stationary state $0 < z < 1$ that rapidly converges to $z = 1$. To better characterize the phase transition we have computed a susceptibility function that is commonly used in SIS epidemic models [47]: $\chi = N^2 \frac{(z^*)^2-(z)^2}{(z^*)^2}$. Susceptibility functions are specifically designed to peak (diverge in the thermodynamic limit) at the critical value $(T - S)'_{c}$ of the order parameter at which the phase transition occurs. The peak of $\chi$ for $\gamma < 3$ in Fig. 1 panel (b) confirms the occurrence of a phase transition. By comparing the average group payoff in the QS state $\langle \pi_\gamma \rangle$ to the expected average payoff at the NE $\langle \pi_{NE} \rangle$ (see SM for its derivation), we see, Fig. 1 panel (c), that the QS state for $\gamma < 3$ is sub optimal. In fact the relative average income $\langle \pi_{\gamma=2.5} \rangle/\langle \pi_{NE} \rangle$ decreases sharply starting from $(T - S)'_c$ and reaches a minimum when the distance between the QS solution and the NE equilibrium is maximal (for $(T - S)' = 0.5$). Instead for $\gamma > 3$ the average payoff coincides with the NE one. Since the average payoff/utility decreases, the collective adoption of strategy $T$ observed at the phase transition can be regarded as irrational from Expected Utility Theory. Such irrational behaviour is a consequence of the scale-free nature of the network where the strategy adoption dynamics takes place. Simulations for different values of parameters $0 \leq \beta, \zeta \leq 1$ show no appreciable difference with respect to the results in Fig. 1. However, the syn-

**FIG. 2.** Average group income divided by the expected NE payoff as a function of the group size, for values of $\beta = 0$ (panel (a)) and $\beta > 0$ (panel (b)) and for $\gamma > 3$ (red-blue lines) and $\gamma < 3$ (grey lines). Each line refers to a different value of $0 \leq (T - S)' \leq 1$ used in the numerical simulations.

**FIG. 3.** Group polarization under simple majority (a), (b) and two-third majority (c), (d) decision schemes. In (a), (c), the simulated QS distribution $z^*_q$ as a function of group size. The grey dotted lines separate risky and safe group shifts. In (b), (d), $z^*_q$ for groups of size 6, 12 and 24 as a function of $z_{i \in q}$, risk propensity of the members of groups of size $q$. The colored dots in (d) represents empirical data on trial juries from Refs. [15–19]. Shaded areas are the standard deviations.
size 1, $z_q^* = z_{i\in q}$. Therefore, for a given level curve, if $z_q^* > z_{i\in q} \equiv z_{q=1}^*$ (or vice versa $z_q^* < z_{i\in q} \equiv z_{q=1}^*$) the average risk propensity of the groups is higher (lower) than the average individual risk propensity of their members. Thus, the red curves in panels (a) and (c) show group polarization effects towards strategy $T$ (risky shift), while blue curves display a shift towards the safer strategy. The transition between the risky and safe group shift phase occurs at values of $z_q^*$ equal to the fraction of group members needed to agree the group strategy, respectively $z_q^* = 1/2$ and $z_q^* = 2/3$. However, independently from the decision scheme adopted, our model shows that the group polarization effect increases with the group size. This is somewhat counter-intuitive, since one would expect that the larger is the group the less probable are extreme collective decisions. We compared the predictions of our model to data from empirical studies about group choice shifts observed in trial juries [15–19]. To do this, in panel (b) and (d) we plot $z_q^*$ as a function of $z_{i\in q}$, where the curves are level curves for different sizes $q$. The empirical data (dots), obtained with trial juries of size 6, are better reproduced by the model with a two-third majority decision scheme. This is in agreement with the observation that a two-third majority decision scheme is the one that is spontaneously adopted by real-world trial juries [16].

In conclusion, our evolutionary game model on social hypergraphs, which is grounded in recent developments in network science [24], represents a novel approach to the study of choice dilemmas, alternative to standard behavioural approaches that neglect higher-order (group) interactions [49]. The model reproduces, without any ad hoc assumption, the shifts observed both at a local and a global scale in choice dilemmas, and characterizes them in terms of the topological properties of the underlying hypergraph. Such results can also explain how and why radical behaviours can emerge when decisions are taken in groups.

We warmly thank Lucas Lacasa for discussions and suggestions.
[1] J. A. F. Stoner, *A comparison of individual and group decisions involving risk*, Ph.D. thesis, Massachusetts Institute of Technology (1961).
[2] N. Kogan and M. A. Wallach, Journal of Experimental Social Psychology 3, 75 (1967).
[3] K. Eliaz, D. Ray, and R. Razin, American Economic Review 96, 1321 (2006).
[4] J. H. Davis, *Organizational Behavior and Human Decision Processes* 52, 3 (1992), group Decision Making.
[5] C. Starmer, Journal of Economic Literature 38, 332 (2000).
[6] J. Von Neumann and O. Morgenstern, *Theory of games and economic behavior* (Princeton University Press, 1944).
[7] M. J. Machina, Journal of Economic Perspectives 1, 121 (1987).
[8] W. Edwards, The American Journal of Psychology 67, 56 (1954).
[9] J. H. Davis, N. Kerr, M. Sussmann, and A. K. Rissman, Journal of Personality and Social Psychology 30, 248 (1974).
[10] T. Gärling, E. Kirchler, A. Lewis, and F. Van Raaij, Psychological Science in the Public Interest 10, 1 (2009).
[11] S. Bikchandani and S. Sharma, IMF Staff papers 47, 279 (2000).
[12] R. W. Sias, The Review of Financial Studies 17, 165 (2004).
[13] T. Lux, The economic journal 105, 881 (1995).
[14] J. H. Davis, Psychological Review 80, 97 (1973).
[15] J. H. Davis, N. L. Kerr, R. S. Atkin, R. Holt, and D. Meek, Journal of Personality and Social Psychology 32, 1 (1975).
[16] J. H. Davis, N. L. Kerr, G. Stasser, D. Meek, and R. Holt, *Organizational Behavior and Human Performance* 18, 346 (1977).
[17] S. Penrod and R. Hastie, Psychological Review 87, 133 (1980).
[18] C. Nemet, Journal of Applied Social Psychology 7, 38 (1977).
[19] R. M. Bray and A. M. Noble, Journal of Personality and Social Psychology 36, 1424 (1978).
[20] A. Barrat, M. Barthelmy, and A. Vespignani, *Dynamical processes on complex networks* (Cambridge university press, 2008).
[21] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, Reviews of modern physics 87, 925 (2015).
[22] S. N. Dorogovtsev, A. V. Goltsev, and J. F. Mendes, Reviews of Modern Physics 80, 1275 (2008).
[23] A. Rapoport and A. M. Chammah, *American Behavioral Scientist* 10, 10 (1966).
[24] F. Battiston, G. Cencetti, I. Iacopini, V. Latora, M. Lucas, A. Patania, J.-G. Young, and G. Pett, Physics Reports 747, 1 (2020).
[25] E. Estrada and J. A. Rodríguez-Velázquez, *Physica A: Statistical Mechanics and its Applications* 364, 581 (2006).
[26] J.-L. Guillaume and M. Latapy, *Physica A: Statistical Mechanics and its Applications* 371, 795 (2006).
[27] M. E. J. Newman, Physical Review E 64, 016131 (2001).
[28] S. Milojević, Proceedings of the National Academy of Sciences 111, 3984 (2014).
[29] R. L. Axtell, *Science* 293, 1818 (2001).
[30] X. Gabaix and Y. M. Ioannides, in *Handbook of regional and urban economics*, Vol. 4 (Elsevier, 2004) pp. 2341–2378.
[31] P. Minasandra and K. Isvaran, *Behaviour* 157, 541 (2020).
[32] H.-S. Niwa, Journal of Theoretical Biology 224, 451 (2003).
[33] L. Muchnik, S. Pei, L. C. Parra, S. D. Reis, J. S. Andrade Jr, S. Havlin, and H. A. Makse, Scientific reports 3, 1 (2013).
[34] J. I. Bulow, J. D. Geanakoplos, and P. D. Klemperer, *Journal of Political Economy* 93, 488 (1985).
[35] B. R. Bruns, *Games* 6, 495 (2015).
[36] L. M. A. Bettencourt, J. Lobo, D. Helbing, C. Kühnert, and G. B. West, Proceedings of the National Academy of Sciences 104, 7301 (2007).
[37] It is worth to notice that an equivalent formulation of this hypothesis is that the group payoff is a function of the group size:

$$\pi^g(S) = \frac{S}{(q^g)^{\gamma}}$$

where

$$\pi^g(T) = \left\{ \begin{array}{ll}
   \frac{W}{(q^g)^{\gamma}} & \text{with probability } w_p \\
   1 - w_p & \text{with probability } 1 - w_p
\end{array} \right.$$
I. DETAILS ON THE HYPERGRAPHS CONSTRUCTION AND NUMERICAL SIMULATIONS

The hypergraphs describing the population of agents organized in groups were built using a configuration model-like algorithm. Let us consider $M$ nodes, representing agents, and $N$ hyperedges representing groups of two or more agents. The algorithm works as follows. First, for each group member $i$ we draw its hyperdegree $k_i$ from the assigned probability distribution function (pdf) $K(k)$, and to each group $j$ we assign the size $q_j$, drawing it from the pdf $Q(q)$. Then we build two separate lists, for agents and groups respectively, where the label of each agent $i$ and group $j$ is repeated $k_i$ and $q_j$ times. Finally, we pick uniformly at random one index $i$ from the list of agents and one index $j$ from the list of groups, and we add the index $i$ to the hyperedge $j$. The hypergraphs built in this way will consist of different components. We restricted our analysis to the connected component with the largest number of agents. Given the power-law distributions of the group sizes $Q(q) \sim q^{-\lambda}$ and the agent hyperdegrees $K(k) \sim k^{-\nu}$, we have checked that the resulting co-membership degree is long-tailed distributed with distribution $H(h)$. By fitting $H(h)$ with a power-law distribution (i.e. $H(h) \sim h^{-\gamma}$) we derived an exponent $\gamma$ which depends on $Q(q)$ and $K(k)$ in a non trivial way. Hence, to tune $\gamma$ we have adopted the following procedure. Keeping fixed $\lambda$ (i.e. the group size distribution), we changed $\nu$ in small steps checking at each step the resulting $\gamma$ by computing the co-membership degree distribution. In particular, the results shown in the main text correspond to two hypergraphs built using $\lambda = 3.9$ and, respectively, $\nu = 3.5$ and $\nu = 2.5$. By fitting the co-membership degree distribution of the largest connected component (for the fit we used the python package powerlaw [50]) we measured $H(h) \sim h^{-\gamma}$ with exponents respectively equal to $\gamma \approx 3.5$ and $\gamma \approx 2.5$, as shown in Fig. S1.

All the simulations were performed on a population with a largest connected component of cardinality $M \approx 10^4$. For our results we utilized data collected during the last $10^7$ time steps of the dynamics of $10^2$ independent runs, after a thermalisation time of $10^8$ time steps. In order to sample the quasi-stationary (QS) distribution $z^*$ and avoid the two absorbing states of the dynamics $z = 0$ and $z = 1$, we applied the method described in Refs. [39, 40] with a memory capacity of $10^3$ states and an overwriting probability of 0.01. We recall that the transition probability from the state of agent $i$ to the state of agent $j$ is usually modelled using a Fermi function: $p_{ij} = [1 + \exp(w_F(\pi_i - \pi_j))]^{-1}$, where $\pi_i$ and $\pi_j$ are respectively the payoff of the focal and reference individual. Under weak selection hypothesis (that is for small $w_F(\pi_i - \pi_j)$), the Fermi function can be replaced by its linear approximation [42, 44]: $p_{ij} = \frac{1}{2} \left[1 + \frac{w_F}{w_F} (\pi_j - \pi_i)\right]$, where $-1 \leq \frac{w_F}{w_F} (\pi_j - \pi_i) \leq 1$. We obtained consistent results from the numerical simulations for a broad range of values of the strategies adoption strength $w_F$, both using the Fermi function and its linear approximation. In particular, the results shown in the main text were obtained using a linear strategy transition probability with a strength of strategy selection $w_{lin}$ such that $\text{max}_{i,j} |(\pi_j - \pi_i)| w_{lin} = 1$, where $\text{max}_{i,j} |(\pi_j - \pi_i)|$ is the maximum payoffs difference (in absolute value) between two agents in the population. We computed $w_{lin}$ in the first simulation run of each simulations series, initializing $w_{lin}$ to 1 and then updating it every time a new maximum of the payoffs difference $|\pi_j - \pi_i|$ was encountered: $w_{lin} = w_{lin} / |\pi_j - \pi_i|$. This initial run was exclusively used to tune $w_{lin}$, and no data were collected from it.
II. GAME FOR A POPULATION OF TWO GROUPS

We consider a population divided into two groups of equal size \( q \). As for the derivation of the stationary state \( z^* \) in the main text, we describe the model at the group level using a coarse-grained approach. Since the sizes of the two groups are equal and constant (during the dynamics) both the groups cost and payoff functions are equal and constant, even if in principle they can depend on \( q \). Therefore, we neglect the respective scaling factors \( q^{-\beta} \) (for the cost) and \( q^{-\zeta} \) (for the payoffs), defining the cost simply as \( C \) and the payoffs as:

\[
\begin{align*}
\pi(S) &= S \\
\pi(T) &= \begin{cases} 
W := T - C, & \text{with probability } w_p \\
L := S - C, & \text{with probability } 1 - w_p
\end{cases}
\end{align*}
\]

where \( w_p = \theta N / N_T \) is the probability of earning the high reward \( W \) associated to the risky strategy \( T \). In particular, for a population of \( N = 2 \) groups, \( N_T \in \{0, 1, 2\} \). For this population is then possible to write a symmetric payoffs matrix describing the pairwise game:

\[
\begin{array}{ccc}
& T & S \\
T & a & b \\
S & S & S
\end{array}
\]

TABLE S1. Symmetric payoffs matrix for a population of two groups of equal size. The row player represents one group, the column player the other one.

where \( a = \theta(T - C) + (1 - \theta)(S - C) \), \( b = 2\theta(T - C) + (1 - 2\theta)(S - C) \) and \( z = (z, 1 - z) \) defines a mixed strategies profile (i.e. \( 0 < z < 1 \) is the probability of strategy \( T \) adoption). Since the payoffs matrix is symmetric, we omitted the payoffs for the column player. For \( a < S \) and \( S < b \), that is for \( \frac{1}{2} < \frac{\theta(T - S)}{C} < 1 \), this payoffs matrix is the payoffs matrix of the game of Chicken with ties for middle payoffs (i.e. the Volunteer’s Dilemma) \[35\]. Since a pairwise game is completely defined by its payoffs matrix, our game for two groups and the pairwise game of Chicken are equivalent. The game have two pure strategies Nash Equilibria (NE): \((T, S)\) and \((S, T)\). However, these pure strategies NE are asymmetric and require coordination to work (i.e. the two players deciding in advance who is going for \( S \) and who for \( T \)) \[51\]. The only symmetric NE of the game is \( z^* = (z^*, 1 - z^*) \), a mixed strategies profile NE, which can be found equalizing the payoffs on the support of \( z^* \):

\[
z^* a + (1 - z^*) b = z^* S + (1 - z^*) S
\]

Eq. (S2) leads to:

\[
z^* = \frac{b - S}{b - a} = 2 - \frac{C}{\theta(T - S)}
\]

such that \( 0 < z^* < 1 \) for \( \frac{1}{2} < \frac{\theta(T - S)}{C} < 1 \), which means that the mixed NE can assume any meaningful value for a probability (i.e. in range \((0, 1)\)) \[23\].

III. DERIVATION OF EQ. (3)

Eq. (3) in the main text has been derived by adapting the procedure described in Refs. \[41, 42\] to a compartmental approach. We first write the Master Equation describing the dynamics in the sub-population (compartment) of groups of size \( q \). We recall that we are describing the mean-field dynamics for the model using a coarse-grained compartment. It is important to notice that the dynamics of the system is a Markovian process, since the strategy transition probabilities only depend on the current strategies and payoffs of the groups. Therefore, the probability \( P(N_T(q), \tau) \) of being at the
time step $\tau$ in a state characterized by exactly $N_T(q)$ groups of size $q$ with strategy $T$ satisfies the following equation:

$$
P(N_T(q), \tau + 1) - P(N_T(q), \tau) = + P(N_T(q) - 1, \tau) \sum_{q'} T^+_{q'} (N_T(q) - 1) 
+ P(N_T(q) + 1, \tau) \sum_{q'} T^-_{q'} (N_T(q) + 1) 
- P(N_T(q), \tau) \sum_{q'} \left( T^+_{q'} (N_T(q)) + T^-_{q'} (N_T(q)) \right) \tag{S4}
$$

Introducing the quantities $z_q Q(q) = N_T(q)/N$, $t = \tau/N$ and the probability density $\rho(Q(q)z_q, t) = NP(N_T(q), \tau)$ yields:

$$
\rho(Q(q)z_q, t + N^{-1}) - \rho(Q(q)z_q, t) = + \rho(Q(q)z_q - N^{-1}, t) \sum_{q'} T^+_{q'} (Q(q)z_q - N^{-1}) 
+ \rho(Q(q)z_q + N^{-1}, t) \sum_{q'} T^-_{q'} (Q(q)z_q + N^{-1}) 
- \rho(Q(q)z_q, t) \sum_{q'} \left( T^+_{q'} (Q(q)z_q) + T^-_{q'} (Q(q)z_q) \right) \tag{S5}
$$

where $T^+_{q'} (N_T(q))$ and $T^-_{q'} (N_T(q))$ are respectively the probabilities of increasing and decreasing by one the number $N_T(q)$ of groups of size $q$ through the interaction with a group of size $q'$. They can be expressed as:

$$
T^+_{q'} (Q(q)z_q) = [Q(q) (1 - z_q) Q(q') z_{q'}] p^q_{S'T} \tag{S6}
$$

$$
T^-_{q'} (Q(q)z_q) = [Q(q) z_q Q(q') (1 - z_{q'})] p^q_{ST} \tag{S7}
$$

where $z_q = N_T(q)/N(q)$ and $Q(q)$ is the probability distribution of the group sizes. Therefore, the term in square brackets in Eq. (S6) represents the probability of picking at random a group of size $q$ with strategy $S$ and a group of size $q'$ with strategy $T$. $p^q_{S'T}$ is instead the probability of strategy transition from $S$ to $T$ given the group sizes, described in the main text. Eq. (S7) has an analogous interpretation. To simplify the notation, let us define $x := Q(q)z_q$ (it is worth to notice that $Q(q)$ does not change during the dynamics, therefore $x \propto z_q$). For $N \gg 1$, we can perform a Kramers-Moyal expansion of the Master Equation. Using Taylor series expansions of the probability densities and the transition probabilities, and neglecting higher order terms in $N^{-1}$, we get:

$$
\rho(x, t + N^{-1}) - \rho(x, t) \simeq \frac{d\rho(x, t)}{dt} N^{-1} \tag{S8}
$$

$$
\rho(x \pm N^{-1}, t) \simeq \rho(x, t) \pm \frac{d\rho(x, t)}{dx} N^{-1} + \frac{1}{2} \frac{d^2 \rho(x, t)}{dx^2} N^{-2} \tag{S9}
$$

$$
T^+_q(x \pm N^{-1}) \simeq T^+_q (x) \pm \frac{dT^+_q (x)}{dx} N^{-1} + \frac{1}{2} \frac{d^2 T^+_q (x)}{dx^2} N^{-2} \tag{S10}
$$

Substituting in Eq. (S5) and after some manipulation, we obtain:

$$
\frac{d\rho}{dt} = - \rho \sum_{q} \frac{d}{dx} \left( T^+_q - T^-_q \right) \frac{d\rho}{dx} \sum_{q'} \left( T^+_{q'} - T^-_{q'} \right) 
+ \frac{d\rho}{dx} \sum_{q'} \frac{d}{dx} \left( T^+_q + T^-_q \right) N^{-1} + \frac{1}{2} \left\{ \rho \sum_{q'} \left( \frac{d^2 T^+_q}{dx^2} + \frac{d^2 T^-_q}{dx^2} \right) + \frac{d^2 \rho}{dx^2} \left( T^+_q + T^-_q \right) \right\} N^{-1} \tag{S11}
$$

where for a matter of convenience we used a simplified notation for $\rho(x, t)$ and $T^+_q (x)$, omitting the variables $x$ and $t$. The previous equation can be written in a more convenient form as:

$$
\frac{d\rho}{dt} = - \frac{d}{dx} \left[ \rho \sum_{q'} \left( T^+_q - T^-_q \right) \right] + \frac{1}{2} \frac{d^2}{dx^2} \left[ \rho \sum_{q'} \left( T^+_q + T^-_q \right) \right] N^{-1} \tag{S12}
$$
This equation is in the form of a Fokker-Plank equation:

$$\frac{d\rho(x,t)}{dt} = -\frac{d}{dx}(\rho(x,t)a(x)) + \frac{1}{2} \frac{d^2}{dx^2}(\rho b^2(x))$$

(S13)

where $a(x) = \sum_q \left(T_q^+(x) - T_q^-(x)\right)$ is the drift and $\frac{1}{2}b^2(x) = \frac{1}{2} \sum_q \left(T_q^+(x) + T_q^-(x)\right)N^{-1}$ is the diffusion coefficient. Since the internal noise is microscopically uncorrelated in time, as subsequent steps of the dynamics are independent, the Itô calculus applies and we obtain the Langevin equation:

$$\dot{x} = a(x) + b(x)\xi$$

(S14)

where $\xi$ is uncorrelated Gaussian noise. Taking the limit $N \to \infty$, $b(x) \propto N^{-1/2} \to 0$ and we obtain the deterministic equation:

$$\dot{x} = a(x)$$

(S15)

By substituting in $a(x)$ the definition of $T_q^+(x)$, $T_q^-(x)$ and $x$, we finally obtain:

$$\frac{dz_q}{dt} = (1 - z_q)\sum_{q'}Q(q')z_qp_{ST}^{qq'} - z_q\sum_{q'}Q(q')(1 - z_q)p_{TS}^{qq'}$$

(S16)

IV. EVOLUTIONARY STABLE STATE OF THE MEAN-FIELD DYNAMICS

In order to prove that $z_{th}$ (such that the payoffs matrix element $E \left[p_{ST}^{qq'} - p_{TS}^{qq'}\right] = 0$) is an evolutionary stable strategy (ESS), we need to prove that, $\forall z \neq z_{th}, \exists \epsilon_0(z) > 0$ such that $\forall \epsilon < \epsilon_0(z)$, we have [52]:

$$\pi(z_{th}, (1 - \epsilon)z_{th} + \epsilon z) > \pi(z, (1 - \epsilon)z_{th} + \epsilon z)$$

(S17)

That is, in a population where a fraction $1 - \epsilon$ of the agents adopts the mixed strategy $z_{th}$ and a fraction $\epsilon$ the strategy $z$, if $z_{th}$ is an ESS there exists a threshold fraction $\epsilon_0(z_{th})$ below which strategy $z_{th}$ provides an higher expected payoff than $z$. This implies that on average every mutant strategy $z$ is eliminated by $z_{th}$ before reaching the critical fraction $\epsilon_0(z)$. For convenience, let define

$$z' = (1 - \epsilon)z_{th} + \epsilon z$$

(S18)

as the fraction of groups than on average plays strategy $T$ (being $z$ and $z_{th}$ the probabilities of playing $T$ respectively for a fraction $\epsilon$ and $1 - \epsilon$ of the groups population). We recall that the winning probability $w_p$, that appears in the payoffs matrix element is a function of the fraction $z'$ of groups adopting $T$ in the population: $w_p = f \left(\frac{z'}{z}\right)$. As a consequence the payoffs in Eq. (S17) depend implicitly on $z'$. Let us rewrite the payoffs matrix element, by making explicit its dependence on $z'$, as:

$$M_{z'} := \pi(T,S)|_{z'} = E \left[p_{ST}^{qq'} - p_{TS}^{qq'}\right]$$

$$= \left[1 - f \left(\frac{\theta}{z'}\right)\right]R_{SL} + f \left(\frac{\theta}{z'}\right)R_{SW}$$

$$= R_{SL} + f \left(\frac{\theta}{z'}\right)[R_{SW} - R_{SL}]$$

(S19)

where we have used the definitions of $p_{ST}^{qq'}$ and $p_{TS}^{qq'}$ their definitions and we have introduced the quantities

$$R_{SL} := E \left[p_{SL}^{qq'} - p_{LS}^{qq'}\right]$$

(S20)

$$R_{SW} := E \left[p_{SW}^{qq'} - p_{WS}^{qq'}\right]$$

(S21)

We recall that we are assuming the transition probabilities to be linear in the payoffs difference: $p_{ST}^{qq'} = \frac{1}{2} [1 + w_{lin} (\pi_{x2} - \pi_{s1})]$, where $w_{lin}$ is the coefficient of linear proportionality, such that $-1 \leq w_{lin} (\pi_{x2} - \pi_{s1}) \leq 1$. We
can therefore substitute the transition probabilities in Eqs. (S20), (S21) with their linear expressions, to get:

\[
R_{SL} = -\frac{w_{th}}{2} E \left[ \frac{C(q) + C(q')}{q^{\epsilon'}} \right]
\]

\[
R_{SW} = \frac{w_{th}}{2} E \left[ \frac{T - S - C(q)}{q^{\epsilon'}} + \frac{T - S - C(q')}{q^{\epsilon'}} \right]
\]

(S22)

(S23)

Since \(C(q) = (r_{\min} q)^{-\beta} > 0\) and \(T > S\), it follows that \(R_{SL} < 0\) and \(R_{SW} - R_{SL} > 0\). We stress that the payoff matrix of the pairwise zero-sum symmetric game defined by the Replicator Equation is completely determined by \(M_{z'}\). In fact, being the game symmetric zero-sum it implies that the two diagonal elements are 0 an the two off-diagonal elements are respectively \(M_{z'}\) and \(-M_{z'}\). We can rewrite \(z_{th}^*\) as:

\[
z_{th}^* = \frac{\theta}{f^{-1} \left( \frac{-R_{SL}}{R_{SW} - R_{SL}} \right)}
\]

(S24)

We can now substitute in Eq. (S17) the explicit expressions for the expected payoffs given the mixed strategy profiles \((z_{th}^*, z')\) and \((z, z')\), where \(z' = (1 - \epsilon)z_{th} + \epsilon z\), obtaining:

\[
z_{th}^* (1 - z')M_{z'} - (1 - z_{th}^*)z'M_{z'} > z(1 - z')M_{z'} - (1 - z)z'M_{z'}
\]

(S25)

Bringing all the terms on the left of the inequality, and after some simple manipulations, one obtains the condition:

\[
(z_{th}^* - z)M_{z'} > 0
\]

(S26)

It is easy to prove that this condition holds true \(\forall z \neq z_{th}^*\) and \(\forall \epsilon < \epsilon_0(z) \equiv 1\) and hence that \(z_{th}^*\) is an ESS. In fact, given the definition of \(M_{z'}\) in Eq. (S19), it follows immediately that \(M_{z'} > 0\) if:

\[
z' < \frac{\theta}{f^{-1} \left( \frac{-R_{SL}}{R_{SW} - R_{SL}} \right)} \equiv z_{th}^*\]

(S27)

Substituting the definition of \(z'\), Eq. (S18), we obtain:

\[
(1 - \epsilon)z < (1 - \epsilon)z_{th}^*
\]

(S28)

which for \(\epsilon < 1\) implies \(M_{z'} > 0\) for \(z < z_{th}^*\) (i.e. if \(z_{th}^* - z > 0\)), and \(M_{z'} < 0\) for \(z_{th}^* - z < 0\). Hence, if \(z_{th}^* - z > 0\), then \(\forall \epsilon < 1 \equiv \epsilon_0\) we have \(M_{z'} > 0\), and the condition Eq. (S26) holds true. On the other hand, if \(z_{th}^* - z < 0\) from Eq. (S28) it follows that also \(M_{z'} < 0\) \(\forall \epsilon < 1 \equiv \epsilon_0\), and Eq. (S26) is again satisfied. Thus, \(z_{th}^*\) is an ESS.

V. AN ANALYTICAL EXPRESSION FOR \(z(t)\)

The time evolution of \(z\), the fraction of groups with strategy \(T\), for \(N >> 1\) is described by the deterministic equation:

\[
\frac{dz}{dt} = z(1 - z) E \left[ p_{ST}^{\epsilon} - p_{TS}^{\epsilon} \right]
\]

(S29)

In the particular case \(w_p = \frac{\theta}{z}\) (i.e. \(f(x) = x\)), Eq. (S29) takes the form:

\[
\frac{dz}{dt} = z(1 - z) \frac{a - b}{z - b}
\]

(S30)

where \(b > a > 0\), being in our model \(a = \theta (R_{SW} - R_{SL})\) and \(b = -R_{SL}\) (see the definitions of \(R_{SL}\) and \(R_{SW}\) in Eqs (S20),(S21)). Solving Eq. (S30) with initial condition \(0 < z_0 := z(0) < 1\), we find:

\[
z(t) = \frac{a(1 - z_0) - (a - bz_0)e^{(a-b)t}}{b(1 - z_0) - (a - bz_0)e^{(a-b)t}}
\]

(S31)
Since \( a < b \), in the limit \( t \to \infty \) the trajectory converges to \( z^* = a/b \), which for our model coincides with the stationary state found in the main text:

\[
    z_{th}^* = \theta f^{-1} \left( \frac{\zeta + \lambda - 1}{(\zeta + \lambda + \beta - 1)(T - S)_{\min}} \right) \quad (S32)
\]

This happens for all initial conditions \( 0 < z_0 < 1 \), therefore \( z_{th}^* \) is a global attractor of the dynamics. Fig. S2 shows the trajectory \( z(t) \) for different values of the initial condition \( z_0 \).

![Graph showing the trajectory z(t) for different initial conditions z_0. The dotted grey line represents the attractor of the dynamics z^* = a/b, for a = 0.5 and b = 1.](figure)

**VI. NASH EQUILIBRIUM (NE) EXPECTED PAYOFF**

The expected payoff for a group of size \( q \) is:

\[
    \langle \pi_{NE} \rangle_q = z_q \left[ w_p W'_q + (1 - w_p) L'_q \right] + (1 - z_q) S'_q \quad (S33)
\]

where \( w_p = f(\frac{\theta}{2}) \), \( S'_q = \frac{S}{q^\lambda} \), \( W'_q = \frac{W}{q^\lambda} \) and \( L'_q = \frac{L}{q^\lambda} \).

Under the assumption of statistical independence \( P(T|q) = P(T) \), we can approximate \( z_q \sim z \) in Eq. (S33). Replacing \( z \) with the NE solution Eq. (S32), we find the analytical expression for the expected payoff at the NE as a function of group size \( q \).

\[
    \langle \pi_{NE} \rangle_q = \left[ \frac{\theta r_{\min}^\beta}{f^{-1} \left( \frac{f(\theta)}{(T - S)^r} \right)} \left( \frac{1}{m - 1} \right) + S \right] \frac{1}{q^\lambda} \quad (S34)
\]

We can also compute the expected average payoff in the whole population by multiplying this expression by the group sizes distribution \( Q(q) \sim q^{-\lambda} \) and integrating over \( q \):

\[
    \langle \pi_{NE} \rangle = \left[ \frac{\theta r_{\min}^\beta}{f^{-1} \left( \frac{f(\theta)}{(T - S)^r} \right)} \left( \frac{1}{m (\lambda + \zeta - 1)} - \frac{1}{\lambda + \zeta + \beta - 1} \right) + \frac{S}{\lambda + \zeta - 1} \right] (\lambda - 1) \quad (S35)
\]