Mathematical modelling of a search and rescue robot with TriSTAR locomotion units

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Abstract. In this paper the authors present the mathematical model of a search and rescue robot with hybrid locomotion system. The robot uses a locomotion system with two TriSTAR units and two conventional wheels. Details are also presented regarding the design of the robot and the TriSTAR locomotion unit, the robot locomotion and the robot actuation and control.

1. Introduction

Urban search and rescue (USAR) tasks may endanger the lives of the human operators, so using search and rescue robots can be a solution. Rescue robots can be used to explore the unknown environment, which is dangerous to the human operator, by taking and sending sensor data, or thermal and colour video.

The application of search and rescue robots is increasingly used in emergency situations. This domain remains a challenging field, with many problems such as: mobility, autonomy, sensors, control, communication and human-robot interaction [1].

Key capabilities of fully autonomous rescue robots include map building while detecting the location, determining the next exploration objective, planning of the route, following the route by controlling the movement, and detecting and locating victims.

Due to the importance of search and rescue field, research in this area is fully justified. In this sense, there are many researches in the specialized literature [1], [2], [3], [4], [5], [6], [7], [8], [9], [10].

An extremely important role in increasing the mobility of these robots is the locomotion system. The locomotion system with tracks is one of the most widespread [2], [3]. Another solution could be the snake type robots (articulated robots), robots which can change their shape and are manageable in limited spaces [4], [5], [6]. However, these robots have several actuators, which can make controlling the robot quite difficult. Search and rescue robots with hybrid locomotion systems can successfully reach hard-to-reach locations [7]. From the hybrid systems, the systems that combine the wheels and the legs can be a solution, because these combine the advantages of the two singular systems with wheels and legs [8], [9], [10].

The TriSTAR locomotion units proposed by the authors have a great advantage in the fact that they can move on the stairs. Of course, the proposed robot with locomotion system that combines classic wheels with TriSTAR locomotion units may have limitations in various operating environments, but nevertheless it may be a useful solution for different types of terrain in the field of search and rescue.
The paper is further structured as follows. The second chapter presents the structure of the search and rescue robot, the mathematical model is presented in the third chapter, and the fourth chapter presents the research directions and finally the conclusions of the paper.

2. The structure of the mobile search and rescue robot

The proposed search and rescue robot is realized by using a central platform, to which is connected a hybrid locomotion system that includes TriSTAR locomotion units and classic wheels (Figure 1).

The central platform of the robot (Figure 1a) contains a module with a video camera at 180° on the top side, video cameras positioned on the front and the rear side. Included are also control board and four DC motors [11] with GHM-01 reducer, lighting LEDs and battery for the autonomy of the robot.

As the design has been done modularly, the top camera can be disassembled and the serial robotic arm can be connected for manipulation (Figure 1b).

Because the size of the robot is an important feature of the search and rescue robots field, the proposed robot is designed with the dimensions of 400 x 380 x 180 mm, to have the possibility to transport it in a backpack, to don’t need special transport to reach the intervention site.

![Figure 1. The proposed search and rescue robot a), b) 3D model of the robot with robotic arm c) design parameters of the TriSTAR locomotion unit d) exploded view of the TriSTAR locomotion unit.](image-url)

The geometry of a TriSTAR type locomotion unit allows switching between wheeled locomotion (advancing mode) and locomotion with legs (automatic climbing mode) [12], [13], [14]. In this type of design, we use a classic wheel with radius $r_w$ attached to the end of each leg, the legs being disposed at an angle of 120°. On each leg, there are gear wheel transmissions (Figure 1c).

TriSTAR locomotion units have the great advantage of using a single motor for movement. The authors connected the motor to the central axis of the locomotion unit.
The operating diagram of the robot is shown in Figure 2. In this the main parts necessary for the robot's operation can be identified.

The robot will be moving with the help of four DC motors which are actuating the locomotion units and the conventional wheels. The motors are controlled through two drivers, which are connected to the Arduino control board.

Ultrasonic sensors will be connected to the control board to detect obstacles and to measure the distance from different obstacles. Also, gas and temperature sensors will be mounted to identify the characteristics of the area in which the robot operates. Video and sound sensors will be connected through the Arduino control board to the PC to communicate real-time data to the rescue team. The Arduino control board will be connected to a computer, but it can also be controlled through a joystick or Wi-Fi to ensure remote operations of the robot.

We consider the robot only for inspection without using the serial arm. The mathematical modelling of the proposed robot will be presented below.

3. Mathematical model of the search and rescue robot with hybrid locomotion system

Using Lagrange's equation, the motion equation of the mobile robot with hybrid locomotion system composed of TriSTAR units and conventional wheels will be determined [15], [16].

The Langrangian of robot L is given by the relation:

\[ L = K - P, \]

where:

- \( K \) - the kinetic energy of the robot,
- \( P \) - the potential energy of the robot.
Lagrange equation is [17]:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q'_i \quad (i = 1, 2, \ldots, n),
\]

where:
- \( q_i \) - the generalized coordinate,
- \( Q'_i \) - the generalized force not derivable from a potential function.

Because the robot has to move on different type of surfaces we will consider for our study the movement on a ramp with the angle of inclination \( \alpha \) as shown in Figure 3.

![Figure 3. Movement of the robot on and angled plan with \( \alpha \) angle](image)

The following notations are introduced in Figure 3:
- \( F_f \) - frictional forces for the wheels of the TriSTAR units,
- \( F_{fr} \) - frictional forces of the conventional wheels on the rear side of the robot,
- \( M_{mTS} \) - the torque of the axis of the TriSTAR locomotion unit,
- \( M_m \) - the torque of the wheels of TriSTAR locomotion unit,
- \( M_{mw} \) - the torque of the conventional wheels,
- \( G \) - weight force of the robot,
- \( \ddot{x} \) - the linear acceleration of the mobile robot.

To describe the robot's motion a single generalized coordinate \( x \) will be considered \((q_i = x)\).

The kinetic energy \( K \) for the robot with hybrid locomotion system is determined with the equation:

\[
K = \frac{1}{2} m_c \ddot{x}^2 + 2 \left( \frac{1}{2} m_{TS} \ddot{x}^2 \right) + 6 \left( \frac{1}{2} I_w \omega_w^2 \right) + 2 \left( \frac{1}{2} m_{wc} \ddot{\omega}_{wc}^2 \right) + 2 \left( \frac{1}{2} I_{wc} \omega_{wc}^2 \right),
\]

where:
- \( m_c \) - the mass of the robot chassis,
- \( m_{TS} \) - the total mass of a TriSTAR locomotion unit (this mass includes the mass of the outer wheels \( w \), the mass of the star-shaped frame and the mass of the gear wheels),
- \( m_{wc} \) - the mass of the conventional wheel,
- \( I_w \) - the moment of inertia of the wheel the TriSTAR unit,
- \( I_{wc} \) - the moment of inertia of the conventional wheel,
- \( \omega_w \) - angular velocity of the wheels of the TriSTAR unit,
\( \omega_{W_c} \) - angular velocity of the conventional wheels,
\( \dot{x} \) - the linear velocity of the mobile search and rescue robot.

In relation (3) the kinetic energy was included in the translational motion of the robot chassis, conventional wheels and TriSTAR units and in the rotation motion of the two conventional wheels and the six wheels of the three TriSTAR locomotion units. In this relation (3) the movement of the gear wheels was neglected because the kinetic energy in their rotation motion is insignificant (the moments of inertia of the gear wheels are small).

Angular velocity can be determined:

\[
\dot{x} = r_w \omega_w = \omega_w = \frac{\dot{x}}{r_w},
\]

\[
\dot{x} = r_{W_c} \omega_{W_c} = \omega_{W_c} = \frac{\dot{x}}{r_{W_c}}.
\]

in relations (4), (5) the meaning of the terms is:
\( r_w \) - radius of the driving wheels of the TriSTAR unit,
\( r_{W_c} \) - radius of the conventional wheels.

The kinetic energy will have the expression:

\[
K = \frac{1}{2} m_c \dot{x}^2 + 2 \frac{1}{2} m_{TS} \dot{x}^2 + 6 \frac{1}{2} \left( I_w \left( \frac{\dot{x}}{r_w} \right)^2 \right) + 2 \frac{1}{2} m_{W_c} \dot{x}^2 + 2 \frac{1}{2} \left( I_{W_c} \left( \frac{\dot{x}}{r_{W_c}} \right)^2 \right).
\]

The total mass of the robot \( m_R \) is obtained using the equation:

\[
m_R = m_c + 2 m_{TS} + 2 m_{W_c}.
\]

The potential energy \( P \) of the robot with hybrid locomotion system is obtained with the relation:

\[
P = \left( m_c + 2 m_{TS} + 2 m_{W_c} \right) g x \sin \alpha.
\]

Using the relation (1) Lagrangian \( L \) will have the expression:

\[
L = \left[ \frac{1}{2} m_s \dot{x}^2 + 2 \frac{1}{2} m_{TS} \dot{x}^2 + 6 \frac{1}{2} \left( I_r \left( \frac{\dot{x}}{r_r} \right)^2 \right) + 2 \frac{1}{2} m_{R_c} \dot{x}^2 + 2 \frac{1}{2} \left( I_{R_c} \left( \frac{\dot{x}}{r_{R_c}} \right)^2 \right) \right] - \left[ \left( m_s + 2 m_{TS} + 2 m_{R_c} \right) g x \sin \alpha \right].
\]

By introducing the generalized coordinate \( q_i = x \) in the relation (2), we get:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = Q_i.
\]

By introducing relations (11), (12), (13), into relation (10) we obtain:
\[
m_C \dddot{x} + 2 m_{TS} \dddot{x} + 6 \left[ I_w \left( \frac{\dddot{x}}{r_w^2} \right) \right] + 2 m_{WC} \dddot{x} + 2 \left[ I_{Ec} \left( \frac{\dddot{x}}{r_{WC}^2} \right) \right] + (m_C + 2 m_{TS} + 2 m_{WC}) g \sin \alpha = Q'.
\]

(14)

### 3.1. Determination of generalized force \(Q'\)

In order to determine the generalized force, the torque and the rolling friction force from each wheel will be taken into consideration, using the principle of virtual mechanical work.

The virtual mechanical work generated by the wheels of the locomotion unit and the conventional wheel can be determined as follows:

- for the frontal wheels of the TriSTAR unit:
  \[
  \delta W_{TS} = M_m \delta. \tag{15}
  \]
  
  We mention that in the case of TriSTAR units, the torque of the motor will be distributed equally to the two wheels that are in contact with the locomotion surface. So, in the case of the proposed robot that has two TriSTAR units each actuated by a motor, there are four loaded wheels that receive half of the input torque from the corresponding drive motor through the gear transmissions that connects the motor input to the wheel output.

**Figure 4.** Distribution of moments on TriSTAR unit’s wheels

\(M_m\) can be determined according to \(M_{mTS}\) from the power relation, thus:

\[
M_{mTS} \omega_m = M_m \omega_w + M_m \omega_w = M_m \omega_w = \frac{M_{mTS} \omega_m}{2}, \tag{16}
\]

\[
M_m = \frac{M_{mTS}}{2} \omega_m = \frac{M_{mTS}}{2} i. \tag{17}
\]

where:
- \(i\) is the transmission ratio of the gear transmission,
- \(\omega_m\) - angular velocity of the TriSTAR motor.

- for the conventional wheels arranged in the rear the virtual mechanical work is determined with the relation:
  \[
  \delta W_{Wc} = M_{mWc} \delta, \tag{18}
  \]
  
  where:
  - \(\delta W_w\) - the virtual mechanical work of the wheels of TriSTAR locomotion unit,
\[ \delta W_{Wc} \] - the virtual mechanical work of the conventional wheels.

The movements of the wheels of the TriSTAR unit and of the conventional wheel are:

\[ x = r_w \theta, \quad (19) \]
\[ x = r_{wc} \theta, \quad (20) \]

and the virtual movements are:

\[ \delta x = r_w \delta \theta, \quad (21) \]
\[ \delta x = r_{wc} \delta \theta, \quad (22) \]

so:

\[ W_{TS} = \frac{m_{TS}}{r_w} \delta x, \quad (23) \]

\[ \delta W_{Wc} = \frac{M_{w_{wc}}}{r_{wc}} \delta x. \quad (24) \]

The robot mass will be considered equally distributed on the TriSTAR locomotion units (mass distributed on the front wheels \( m_{RF} \)) and on the conventional wheels (mass distributed on the rear wheels \( m_{RR} \)) and will be expressed as follows:

\[ m_R = m_{RF} + m_{RR}, \quad (25) \]
\[ m_{RF} = m_{RR}, \quad (26) \]

where:

\[ m_{RF} \] – is the mass of the robot on the front side,
\[ m_{RR} \] – is the mass of the robot on the rear side.

The mass distributed on the front wheels can be obtained using the expression:

\[ m_{RF} = \frac{m_c}{2} + 2 m_{TS}. \quad (27) \]

The mass distributed on the rear wheels can be obtained using the expression:

\[ m_{RR} = \frac{m_c}{2} + 2 m_{Wc}. \quad (28) \]

We note with \( N_F, N_R \) the reaction forces from between the wheels and the locomotion surface (Figure 5 and 6).

\[ N = N_F + N_R, \quad (29) \]

where:

\[ N_F \] - the normal reaction force to the wheels from the TriSTAR unit placed on the frontal side of the robot,
\[ N_R \] - the normal reaction force to the conventional wheels place on the rear side of the robot,

from the equilibrium equation:

\[ \sum F_y = ma_y, \quad (30) \]

results (Figure 5):

\[ N_F \frac{m_{RF}}{4} g \cos \alpha = 0. \quad (31) \]
\[ N_F = \frac{m_{RF}}{4} g \cos \alpha. \]  

(32)

In order to correctly highlight the process that takes place in the case of moving the wheels on the locomotion surface, the rolling friction force between the wheels and the locomotion surface will be considered. The rolling friction is usually much smaller than the slipping friction.

The virtual mechanic work of the rolling friction force can be obtained with:

\[ \delta W_w = [s_F N_F] \delta x, \]  

where it was noted with \( s_F \) - the coefficient of rolling friction for the wheels of the TriSTAR unit,

and

\[ \delta W_w = \left[ s_F \left( \frac{m_{RF}}{4} g \cos \alpha \right) \right] \delta x. \]  

(34)

The virtual mechanic work is obtained:

\[ \delta W_w = \left[ s_F \left( \frac{m_{RF}}{4} g \cos \alpha \right) \right] \delta x. \]  

The distribution of forces on the wheels of the TriSTAR unit and on the conventional wheel is presented in Figures 5 and 6 [18].

**Figure 5.** Distribution of forces and moments on the wheels of the TriSTAR unit

In figure 5 it was noted with \( M_{rf} \) - the moment of rolling friction and \( F_{TS} \) traction force on the wheels of TriSTAR units.

Similarly for the classic rear wheels we will obtain:

\[ N_R - \frac{m_{RG}}{2} g \cos \alpha = 0, \]  

(35)

\[ N_R = \frac{m_{RG}}{2} g \cos \alpha. \]  

(36)

The virtual mechanic work is obtained:

\[ \delta W_{Wc} = [s_R N_R] \delta x, \]  

(37)

where with \( s_R \) it was noted the coefficient of rolling friction on conventional wheels, so:

\[ \delta W_{Wc} = \left[ s_R \left( \frac{m_{RG}}{2} g \cos \alpha \right) \right] \delta x. \]  

(38)
In figure 6 it was noted with $F_{i,w_c}$ - the traction force on the conventional wheels.

The generalized force $Q'$ results from the equation:

$$Q' = \left\{ 4 \frac{M_{mRS}}{2r_w} \cdot 4 \left[ s_F \left( \frac{m_{Rg}}{4} g \cos \alpha \right) \right] + \frac{2m_{w_c}}{F_{Wc}} \cdot 2 \left[ s_R \left( \frac{m_{Rg}}{2} g \cos \alpha \right) \right] \right\}.$$  (39)

and the law of motion of the mobile robot is:

$$m_c \ddot{x} + 2 m_{TS} \ddot{x} + 6 \left[ I_w \left( \frac{1}{r_w} \right) \right] + 2 m_{w_c} \ddot{x} + 2 \left[ I_{w_c} \left( \frac{1}{r_{wc}} \right) \right] + (m_c + 2 m_{TS} + 2 m_{w_c}) g \sin \alpha = \left\{ 2 \frac{M_{mRS}}{r_w} \cdot i-4 \left[ s_F \left( \frac{m_{Rg}}{4} g \cos \alpha \right) \right] + \frac{2m_{w_c}}{F_{Wc}} \cdot 2 \left[ s_R \left( \frac{m_{Rg}}{2} g \cos \alpha \right) \right] \right\}. \quad \text{(40)}$$

or

$$\left[ m_c + 2 m_{TS} + 6 \left[ I_w \left( \frac{1}{r_w} \right) \right] + 2 m_{w_c} + 2 \left[ I_{w_c} \left( \frac{1}{r_{wc}} \right) \right] \right] \ddot{x} + (m_c + 2 m_{TS} + 2 m_{w_c}) g \sin \alpha = \left\{ 2 \frac{M_{mRS}}{r_w} \cdot i-4 \left[ s_F \left( \frac{m_{Rg}}{4} g \cos \alpha \right) \right] + \frac{2m_{w_c}}{F_{Wc}} \cdot 2 \left[ s_R \left( \frac{m_{Rg}}{2} g \cos \alpha \right) \right] \right\}. \quad \text{(41)}$$

Because for the drive we use the same motor then $M_{mTS} = M_{mW_c} = M$ equation (41) becomes:

$$\left[ m_c + 2 m_{TS} + 6 \left[ I_w \left( \frac{1}{r_w} \right) \right] + 2 m_{w_c} + 2 \left[ I_{w_c} \left( \frac{1}{r_{wc}} \right) \right] \right] \ddot{x} + (m_c + 2 m_{TS} + 2 m_{w_c}) g \sin \alpha = \left\{ 2 \frac{M_i}{r_w} \cdot i-4 \left[ s_F \left( \frac{m_{Rg}}{4} g \cos \alpha \right) \right] + \frac{2M}{F_{Wc}} \cdot 2 \left[ s_R \left( \frac{m_{Rg}}{2} g \cos \alpha \right) \right] \right\}. \quad \text{(42)}$$

By replacing the masses $m_{RF}, m_{RR}$ in the relation 42 results:

$$\left[ m_c + 2 m_{TS} + 6 \left[ I_w \left( \frac{1}{r_w} \right) \right] + 2 m_{w_c} + 2 \left[ I_{w_c} \left( \frac{1}{r_{wc}} \right) \right] \right] \ddot{x} + (m_c + 2 m_{TS} + 2 m_{w_c}) g \sin \alpha = 2 M \left[ \frac{1}{r_w} + \frac{1}{r_{WC}} \right] - \left[ s_F m_{RF} + s_R m_{RR} \right] g \cos \alpha = 2 M \left[ \frac{1}{r_w} + \frac{1}{r_{WC}} \right] - \left[ s_F m_{RF} + s_R m_{RR} \right] g \cos \alpha. \quad \text{(43)}$$

This relation (43) is the law of motion of the mobile robot. Using the relation (43) we can determine $M$ if the acceleration $\ddot{x}$ of the robot is required, or imposing torque $M$ we can determine the acceleration with which the robot will move.
3.2. Rolling conditions without sliding of the wheels

During the locomotion of the robot, it is necessary that its wheels roll without slipping on the locomotion surface. We note the friction coefficient of sliding with $\mu$ and consider it the same for all 6 wheels of the robot (2 classic wheels and the 4 wheels of the TriSTAR unit).

The condition that the wheels to do not slide on the locomotion surface is:

- for the wheels of the TriSTAR locomotion unit:
  \[
  \frac{M_m}{2r_w} < \mu N_F, 
  \]  
  (44)
  or
  \[
  \frac{M_m}{2r_w} < \mu \frac{m_{RF}}{4} g. 
  \]  
  (45)

- for the classic wheels from the rear side of the robot:
  \[
  \frac{M_{cw}}{r_{wc}} < \mu N_R, 
  \]  
  (46)
  or
  \[
  \frac{M_{cw}}{r_{wc}} < \mu \frac{m_{RR}}{2} g. 
  \]  
  (47)

The conditions for the wheels to roll on the locomotion surface are:

- for the wheels of the TriSTAR locomotion unit:
  \[
  M_r > s_F N_F, 
  \]  
  (48)
  where $M_r$ it is rolling friction torque.

  \[
  M_r > s_F \frac{m_{RF}}{4} g. 
  \]  
  (49)

- for the classic wheels from the rear side of the robot:
  \[
  M_r > s_R N_R, 
  \]  
  (50)
  \[
  M_r > s_R \frac{m_{RR}}{2} g. 
  \]  
  (51)

In figure 7 is presented the movement of the robot in various situations which can be found in the operating field.
4. Development directions
During the next period, simulations will be performed and the obtained results will allow the structures to be improved. For example, the proposed robot does not have a suspension system, so in this sense an improvement could be done. Also, the robot will be physically realized and tested under laboratory and field conditions.

5. Conclusions
In this paper, the authors propose a search and rescue robot with hybrid locomotion system that combines TriSTAR units and conventional wheels. The proposed robot was 3D modelled and mathematical modelling was performed using Lagrange's methodology.

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