Influence of soil properties on amplification of saturated inhomogeneous poroviscoelastic soil profile

The influence of local geotechnical site conditions on the soil profile amplification is analysed in this paper by assuming that the incoming seismic waves consist of inclined P-SV waves. The inhomogeneous soil profile is idealized as a multi-layered saturated poroviscoelastic medium and characterized by the Biot’s theory with a shear modulus varying continuously in depth according to the Wichtmann’s model. The results are presented in terms of horizontal and vertical soil profile amplification with linear and nonlinear soil behaviour, for different values of incidence angle, pore-water saturation, porosity, permeability, coefficient of uniformity, and non-cohesive fines content.

Key words:
Biot’s theory, amplification function, nonlinear soil behaviour, inhomogeneous soil profile

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Utjecaj svojstava tla na amplifikaciju zasićenog nehomogenog porovisko-elastičnog profila tla

U radu se analizira utjecaj lokalnih geotehničkih uvjeta tla na amplifikaciju profila tla uz pretpostavku da se dolazni seizmički valovi sastoje od kosih P-SV valova. Nехомогени profil tla idealizira se kao višeslojni zasićeni poroviskoelastični medij karakteriziran Biotovom teorijom, pri čemu modul posmika kontinuirano varira po dubini u skladu s Wichtmannovim modelom. Rezultati su iskazani u vidu horizontalne i vertikalne amplifikacije profila tla pri linearnom i nelinearnom ponašanju tla, za razne vrijednosti upadnog kuta, stupnja zasićenosti, poroznosti, propusnosti, koeficijenta jednolikosti i udjela sitnih nekoherentnih čestica.

Ključne riječi:
Biotova teorija, funkcija amplifikacije, nelinearno ponašanje tla, nehomogeni profil tla

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Einfluss der Bodeneigenschaften auf die Verstärkung des gesättigten inhomogenen porös-elastischen Bodenprofils

Die Abhandlung analysiert den Einfluss lokaler geotechnischer Bedingungen auf die Verstärkung des Bodenprofils, unter der Voraussetzung, dass die ankommenden seismischen Wellen aus schrägen P-SV-Wellen bestehen. Das inhomogene Bodenprofil wird als mehrschichtiges gesättigtes porös-elastisches Medium idealisiert, das durch die Biot-Theorie charakterisiert ist, wobei das Schermodul gemäß dem Wichtmann-Modell kontinuierlich in der Tiefe variiert. Die Ergebnisse werden in Form einer horizontalen und vertikalen Verstärkung des Bodenprofils bei linearem und nicht linearem Bodenverhalten für verschiedene Werte des Einfallswinkels, Sättigungsgrads, Porosität, Durchlässigkeit, des Gleichmäßigkeitsskoeffizienten und Anteile kleiner in Kohärenter Partikel dargestellt.

Schlüsselwörter:
Biot-Theorie, Verstärkungsfunktion, nicht lineares Bodenverhalten, inhomogenes Bodenprofil

Authors:

Toufiq Ouzandja, Mohamed Hadid
1. Introduction

Local site conditions play an important role in the earthquake response of structures supported by soft soil [1]. Therefore, a rigorous earthquake-resistant design of structures should take into account structural complexity of underground soil (stratification, porosity, inhomogeneity of shear modulus, saturation degree, permeability...), which is responsible for multiple alterations that affect seismic waves during their propagation from the source to the top surface.

In most natural soil deposits, mechanical properties and stiffness increase with their distance from the surface [2–8]. The experience has shown that in such soil the shear modulus values vary exponentially with depth due to the confining effect of overburden. Several researchers have evaluated the seismic response of soil deposits to incident plane waves, with the focus on continuous variation of mechanical properties of soil. The soil profile is called an inhomogeneous soil profile and it is assumed that its shear modulus increases with some power exponent of depth [9–19]. However, all these studies are applicable to the one phase solid media only, and cannot be applied for more realistic soils which are often present in the form of fluid-saturated poroviscoelastic media. Due to substantial computational effort required to obtain the response of an inhomogeneous saturated poroviscoelastic soil profile, very few studies applying the Biot’s theory have so far been written [20–22]. Based on this theory, the complex dispersion equation for Love waves is derived for transversely isotropic fluid-saturated porous media with exponential and linear variation of shear modulus [23–24]. The dispersion behaviour of plane wave [25], Rayleigh wave [26] and transient response of viscoelastic porous medium [27], are analysed in the inhomogeneous saturated soil with gradient variation of mechanical properties along the depth. However, it should be noted that none of these studies examine the problem of amplification of the porous soil response. In addition, the formulas proposed in these studies for describing the inhomogeneous shear modulus of the saturated porous soil profile are not realistic, and they do not take into account geotechnical site conditions, such as soil classification (sand, clay ...), degree of saturation, grain size distribution, fines contents, etc., which can play a significant role in the soil profile amplification.

The effect of grain size distribution in terms of coefficient of uniformity, degree of saturation, porosity, permeability, and fines content, on both horizontal and vertical soil amplification with linear and nonlinear soil behaviour, is presented in this paper. The soil profile is idealized as a multi-layered saturated poroviscoelastic medium and characterized by Biot’s theory in conjunction with the Witchman model [6–8] which consider the shear modulus varying continuously with depth and take into consideration the effect of the grain size distribution, porosity and fines content on dynamic properties of sandy soil. The analysis of seismic response is carried out via the exact stiffness matrix method in frequency domain [28], assuming that the incoming seismic waves consist of inclined P-SV waves. The stiffness matrix method used in this paper appears to be well suited for treating poroviscoelastic soil amplification problems. The method is highly beneficial as it provides a significant benefit in reducing the number of calculations operations compared to the numerical methods, while automatically accounting for the radiation condition.

2. Soil model description

The motion in soil is assumed to be governed by the Biot’s two-phase theory of wave propagation in a saturated poroelastic homogeneous material, which is based on the assumption that the motion of the solid matrix is a wave motion, whereas, that of the fluid relative to the solid is a diffusion process described by Darcy’s law. According to Biot’s theory, the constitutive relations of a homogeneous poroelastic medium in a Cartesian coordinate system can be expressed as:

\[\sigma_{ij} = \lambda \delta_{ij} + 2G\varepsilon_{ij} - \alpha \rho \delta_{ij} \quad i, j = x, z\]  
\[\rho = -\alpha M \varepsilon + M \zeta\]  

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In the above equations, \(\sigma_{ij}\) is the total stress tensor of solid skeleton and \(\varepsilon_{ij}\) is the strain tensor; \(e = \text{div } u\) and \(\zeta = -\text{div } w\) denote, respectively, the dilatation components of the solid frame and the increment of fluid content; \(\rho\) is the pore fluid pressure, \(\lambda\) and \(G\) are Lame constants of the bulk material, \(\delta_{ij}\) is the Kronecker delta, \(\alpha\) and \(M\) are the Biot’s parameters accounting for the compressibility of grains and fluid (0 ≤ \(\alpha\) ≤ 1 and 0 ≤ \(M\) ≤ \(\infty\)), and can be given as:

\[K_b = \lambda + \frac{2}{3}G\]  
\[\alpha = 1 - \frac{K_b}{K_s}\]  
\[\frac{1}{M} = \frac{\alpha - n}{K_s} + \frac{n}{K_f}\]  
\[\frac{1}{K_f} = \frac{1}{K_w} + \frac{1 - S_f}{P_a}\]

\[K_b, K_s, K_f\] and \(K_w\) are the bulk modulus of solid grains, skeleton, pore water and pore fluid, respectively. \(P_a\) is the absolute fluid pressure, \(n\) is the porosity, and \(S_f\) is the degree of saturation (0.95 ≤ \(S_f\) ≤ 1).

Therefore, in terms of the displacement of the solid frame, \(u\), and the fluid relative to the solid frame, \(w\), the wave motion equations can be written in the following form [29]:

For the solid skeleton:

\[G\nabla^2 u + (\lambda + \alpha^2 M + G)\nabla \varepsilon - \alpha M \nabla \zeta = \rho \ddot{u} + \rho \dot{w}\]  

For the pore fluid:

\[\alpha M \nabla \varepsilon - M \nabla \zeta = \rho \ddot{u} + m \ddot{w} + b \dot{w}\]
$\rho$ is the density of the porous matrix and is expressed by

$$\rho = (1-n) \rho_s + n \rho_f,$$

where $\rho_s$ and $\rho_f$ are the densities of the solid phase and liquid phase, respectively. $b = \eta/k$ is the parameter accounting for the internal friction due to relative motion between the solid and the pore fluid ($b = 0$, the flow damping is neglected) where $\eta$ is the fluid viscosity and $k$ is the permeability ($m^2$). $n = \tau' / n$ is a parameter related to the mass density of the pore fluid and the pore geometrical features, and $\tau$ is the static tortuosity coefficient (geometrical sinuosity of porous medium) for the inertia induced by solid-fluid interaction, depending on the shape of solid grain particles [30, 31].

$$\tau = 1 + \frac{1}{r} \left( \frac{1}{n} - 1 \right)$$

in the case of spherical solid grains, $r = 0.5$ \(9\)

Moreover, the velocities of the three types of waves existing in a saturated porous medium ($V_{sv}$, $V_{p1}$, and $V_{p2}$) can be computed using the following expressions [28]:

$$V_{sv} = \sqrt{\frac{G(\sigma' - \rho m)}{\omega (\rho f - \rho m) + l \rho b}}$$

$$V_{p1} = \frac{\omega}{k_{p1}}$$

$$V_{p2} = \frac{\sigma_1 - \sqrt{\sigma_1^2 - 4 \sigma_2}}{2}, k_{p2} = \frac{-\sigma_1 - \sqrt{\sigma_1^2 - 4 \sigma_2}}{2}$$

with an angular frequency expressed as $\omega$.

On the other hand, taking into account the inhomogeneity of mechanical properties of soil profile considered in this study, the site is considered as an inhomogeneous sandy soil profile. The shear modulus increases in the depth according to the Witchman’s model, in which the Hardin’s equations [2] are extended to include the influence of the grain size distribution [6] in terms of the coefficient of uniformity $C_u$, as well as the non-cohesive fines content effect $FC$ [8]. Then, the small strain shear modulus is estimated from the following simple equation:

$$G_{\text{max}} = A \left( \frac{a - a_r}{1 + e_1} \right) \left( \frac{\sigma_0}{a \rho f} \right)^{n'}$$

where

$$a = 1.94 \exp(-0.066 C_u) \exp(0.065 FC)$$

$$n' = 0.40 \ C_u^{0.18} \left[ 1 + 0.116 \ ln(1 + FC) \right]$$

$$A = (1563 + 3.13 \ C_u^{2.98}) \left( \frac{1}{2} \left( \exp(-0.3 FC^1.1) + \exp(-0.28 FC^{0.85}) \right) \right)$$

$$\sigma_0 = \frac{\sigma_r}{3} (3 - 2 \sin \phi')$$

and where the following symbols are used: void ratio $e$, $\sigma_r = (1-n)$, friction angle $\phi'$, vertical stress $\sigma'$, mean pressure $\sigma'$, and atmospheric pressure $p_{atm} = 100$ kPa.

It is assumed in this study that the soil profile has mechanical properties (stiffness, shear modulus,) that increase with some power exponent of depth ($z$), according to the increase of confining effect of overburden, which is presented by the vertical stress $\sigma_r'$. Moreover, to additionally explain the shear modulus-dependent depth, the equation (15) of $G_{\text{max}}$ has been rewritten in the form dependant on depth ($z$):

$$G_{\text{max}}(z) = G_0 \left( \frac{z}{H} \right)^{n'}$$

where

$$G_0 = A \left( \frac{a - a_r}{1 + e_1} \right) \left( \frac{\gamma' H (3 - 2 \sin \phi')}{3 p_{atm}} \right)$$

Here $H$ is the soil profile thickness and $G_0$ is the shear modulus at the bottom (Figure 1).

In addition, the Poisson ratio can be expressed as follows [7, 8, 32]:

$$\nu = \frac{\alpha'}{4(1-\alpha') + \sqrt{\frac{\alpha'}{4(1-\alpha')}}^2 - \frac{\alpha'}{2(1-\alpha')}}$$

Figure 1. Variation of shear modulus $G_{\text{max}}$ with the depth.

$\alpha'$ and $M_{\text{max}}/G_{\text{max}}$. In addition, the Poisson ratio can be expressed as follows [7, 8, 32]:

$$\nu = \frac{\alpha'}{4(1-\alpha') + \sqrt{\frac{\alpha'}{4(1-\alpha')}}^2 - \frac{\alpha'}{2(1-\alpha')}}$$
with the constrained elastic modulus $M_{\text{max}}$ expressed for sands by the following equation (23):

$$M_{\text{max}} = A_e \left( \frac{\sigma_0}{P_{\text{atm}}} \right)^{\lambda_e} \frac{1}{1 + e_e}$$  \hspace{1cm} (23)

where

$$a_e = 2.16 \exp(-0.055 \ C_e \ (1 + 0.116 \ FC))$$  \hspace{1cm} (24)

$$n_e' = 0.344 \ C_e^{0.126} \ (1 + 0.125 \ \ln(1 + FC))$$  \hspace{1cm} (25)

$$A_e = (3655 + 26.7 \ C_e^{0.42}) \ \left[ \frac{1}{2} \left( \exp(-0.42 \ FC^{1.10}) + \exp(-0.52 \ FC^{0.85}) \right) \right]$$  \hspace{1cm} (26)

Furthermore, it should be noted that the damping effects considered in the above constitutive equations are caused only by the interaction of the viscous fluid and the elastic solid, but the material damping due to the friction between solid grains always exists. The poroviscoelastic and relaxation properties are assumed by replacing the elastic coefficients $\lambda$, $G$, and $M$ by complex moduli $\lambda^*$, $G^*$, and $M^*$ [33].

$$G^* = G(1 + 2i\xi_p)$$  \hspace{1cm} (27)

$$\lambda^* = \lambda(1 + 2i\xi_p)$$  \hspace{1cm} (28)

$$M^* = M(1 + 2i\xi_p)$$  \hspace{1cm} (29)

$\xi_p$ is the hysteretic damping ratio and

$$\xi_p = \xi_p \left( \frac{\alpha - n}{\alpha - n + n / K_s} \right) , \text{ which can be derived from equation (5).}$$

### 3. Validation and reliability of the method used

It is already known that the vertical propagation of $SV$ wave exhibits an identical behaviour to that of $SH$ vertical propagation case. We can therefore make comparisons with the corresponding results of the common $1-D$ upwards shear waves method. For this purpose, we consider a poroviscoelastic saturated single layer soil profile overlying a half space and subjected to the vertical $SV$ wave propagation. The soil profile properties are summarized in Table 1. The results of the proposed method (stiffness matrix method) in terms of horizontal amplification at the surface are compared with those evaluated by the DEEPSOIL software, which assumes a vertical propagation of $SH$ wave.

Horizontal amplification curves given in Figure 2 show that the results obtained by means of the proposed method (vertical $SV$ wave propagation) are in perfect agreement with those obtained by DEEPSOIL.

To verify the benefits of the proposed method, the vertical response of Kobe’s Port Island in the 1995 Kobe earthquake was also simulated, using the soil properties and wave velocities for different layers given in Table 2 [34, 35], and assuming a vertical incident of plane $P$ wave. Moreover, the up–down component seismic accelerograms recorded at the port island site at a depth of 83 m were selected as input motions (Figure 3). The results obtained by the stiffness matrix method were also compared with the results recorded in situ.

It can be observed that variation in the shear wave velocities profile ($V_s$) is not significant, which implies that the soil stiffness gradually decreases from the base to the surface. However, a significant decrease of $P$ wave velocities (from 1480 m/s to 780 m/s) appears approximately at a depth of 12.6 m. This decrease can be explained by the effect of pore water saturation on the $P$ wave velocity (section 4.1.2). For this purpose, the soil is assumed in this simulation to be fully saturated below the depth of 12.6 m and partly saturated above that level.

### Table 1. Soil properties of fluid-saturated porous soil profile-bedrock system

| Properties                          | Sand  | Half space |
|-------------------------------------|-------|------------|
| Thickness $H$ [m]                   | 30    | /          |
| Shear modulus $G$ [MPa]             | 1000  | /          |
| Permeability $k$ [m$^2$]            | $10^{-10}$ | $10^{-13}$ |
| Porosity $n$                        | 0.3   | 0.1        |
| Solid grains density $p_s$ [kg/m$^3$] | 2600  | 2600       |
| Bulk modulus of solid grains $K_s$ [GPa] | 36    | 36         |
| Bulk modulus of pore water $K_w$ [GPa] | 2.2   | 2.2        |
| Fluid viscosity $\eta$ [Ns/m$^2$]   | $10^{-3}$ | $10^{-3}$ |
| Pore-water saturation [%]           | 100   | 100        |
| Damping $\xi_p$ [%]                 | 5     | 1          |
| Poisson's ratio                     | 0.3   | 0.3        |

![Figure 2. Comparison of horizontal amplification at the surface between the proposed method and DEEPSOIL results](image-url)
Influence of soil properties on amplification of saturated inhomogeneous poroviscoelastic soil profile

Table 2. Soil properties and wave velocities (Kobe’s Port island site)

| Depth [m] | Thickness [m] | Density [kN/m³] | $V_p$ [m/s] | Damping factor [%] | $V_s$ [m/s] |
|----------|---------------|-----------------|-------------|--------------------|-------------|
| 0-2      | 2             | 1.8             | 260         | 17                 | 170         |
| 2-5      | 3             | 1.8             | 330         | 17                 | 170         |
| 5-12.6   | 7.6           | 1.8             | 780         | 17                 | 210         |
| 12.6-19  | 6.4           | 1.8             | 1480        | 10                 | 210         |
| 19-27    | 8             | 1.5             | 1180        | 10                 | 180         |
| 27-23    | 6             | 1.85            | 1330        | 10                 | 245         |
| 33-50    | 17            | 1.85            | 1530        | 10                 | 305         |
| 50-61    | 11            | 1.85            | 1610        | 10                 | 350         |
| 61-79    | 18            | 1.8             | 1610        | 10                 | 303         |
| 79-83    | 4             | 1.9             | 2000        | 10                 | 320         |

The comparison between the evaluated response and the recorded one is presented in terms of spectral ratios of accelerations at the surface and at a depth of 83 m (Figure 4). The comparison between vertical acceleration at ground surface and vertical peak ground acceleration profile is presented in Figure 5. It can be seen that the results obtained by vertical response analysis are in good agreement with those recorded at Port Island, Kobe.

Figure 4. Comparison of spectral ratios of vertical accelerations at the surface and at a depth of 83 m

4. Parametric study

4.1. Linear analysis

A parametric study is presented in this section in order to analyse the effect of variation in incidence angle, pore-water saturation, coefficient of uniformity, fines content, porosity, and permeability on the bi-directional

Figure 5. Comparison of: a) vertical acceleration at ground surface; b) vertical peak ground acceleration profile
amplification response of poroviscoelastic soil profile, whereas, this amplification is given by the ratio of motion at the ground surface and at an outcropping half space. For this propose, it is assumed that the soil profile is an inhomogeneous single 30m sandy soil layer overlying a rock formation, and it is discretized into multiple layers that exhibit constant properties within each sub-layer (Figure 6). The excitation consists of incident plane $P$ and $SV$ waves having different incidence angles and coming from a half space ($\theta_0$ and $\phi_0$ are the incidence angles of plane $P$ and $SV$ waves, respectively). It is also assumed that the horizontal and vertical responses are due only to the propagation of $SV$ and the $P$ waves, respectively. In this paper, both soil profile and half space are considered as a poroviscoelastic material and are characterized by its degree of saturation $S_r$, porosity $n$, permeability $k$, damping $\xi_m$ and $\xi_p$ (the hysteretic damping ratio, assumed to be the same for both $SV$ and $P$ waves), the compressibility of solid and fluid constituents. The properties of the sand and rock used in this computation are summarized in Table 3.

### Table 3. Soil properties of fluid-saturated porous soil-bedrock system

| Properties                          | Sand                  | Half space          |
|-------------------------------------|-----------------------|---------------------|
| Thickness $H$ [m]                   | 30                    | /                   |
| Shear modulus $G$ [GPa]             | /                     | 1.5                 |
| Permeability $k$ [m$^2$]            | $10^{-10}$-$10^{-13}$ | $10^{-13}$          |
| Porosity $n$                        | 0.2-0.4               | 0.05                |
| Solid grains density $\rho_s$ [kg/m$^3$] | 2600                | 2600                |
| Bulk modulus of solid grains $K_s$ [GPa] | 36                   | 36                  |
| Bulk modulus of pore water $K_w$ [GPa] | 2.2                  | 2.2                 |
| Fluid viscosity $\eta$ [Ns/m$^2$]   | $10^{-3}$             | $10^{-3}$           |
| Damping $\xi_p$ [%]                 | 5                     | 1                   |
| Friction angle $\phi'$              | 35°                   | /                   |

4.1.1. Effect of incidence angle

The effect of incidence angles ($\theta_0$ and $\phi_0$) on both horizontal and vertical soil amplification of a poroviscoelastic soil profile is first considered (Figure 6). It is assumed that the incident waves impinge the soil profile-half space interface at $0^\circ$, $10^\circ$ and $20^\circ$ with respect to the vertical axis $z$.

The analysis of amplification functions shown in Figure 7 indicates that the effect of variation in incident angle $\phi_0$ on the vertical amplification for a plane $P$-wave incoming in fully saturated porous medium is negligible. In the same media, for a plane $SV$-wave incident, the horizontal amplification is influenced by incidence angle $\theta_0$ (Figure 8.a). This influence is more pronounced in the case of the porous medium than in the solid medium (Figure 8b). It was also observed that the two horizontal soil amplifications in porous and solid media are identical for vertical $SV$-wave propagation (Figures 8a and 8b). Moreover, the increase of the $SV$-wave incidence angle $\theta_0$ has reduced the amplification. Thus, the amplification peak is equal to 3.71 with a fundamental frequency equal to 1.8 Hz when $\theta_0 = 0^\circ$. Then it drops to 3.6 and 3.23 in the single-phase medium, and also to 3.52 and 2.7 in the fluid-saturated porous medium, when $\theta_0$ becomes $10^\circ$ and $20^\circ$ respectively. This difference could be explained by the fact that the $P-SV$ conversion is not the same for the solid and saturated porous media (Figure 9). When the $SV$-wave presents an angle of incidence different from zero in the saturated porous medium, it generates the fast compressional wave ($P_1$ wave) and the shear wave ($SV$ wave), similar to the corresponding ones in the single-phase medium, and also the slow compressional wave ($P_2$ wave) associated with the out-of-phase movement between pore fluid and solid phase [36].

In the following parametric study, it is assumed that the incidence angles ($\phi_0$ and $\phi_1$) are equal to $10^\circ$ with respect to the vertical axis for both $P$ and $SV$ waves.
4.1.2. Effect of pore-water saturation

In geotechnical analyses, most studies assume that the soil below the groundwater table is fully saturated. However, under some conditions affecting the location of water table, the subsoil becomes incompletely saturated. This difference is due to the existence of air embedded in pore fluid for a partly saturated case. This air proportion influences largely the bulk modulus of the pore fluid (Equation 6). As shown in Figure 10, the pore fluid modulus $K_f$ is equal to the pore water modulus (2200 Mpa) for $S_r = 100\%$, but it decreases by 99.9% when $S_r$ becomes 95%. As the partial saturation reduces the bulk modulus of the pore fluid significantly, the assumption of the incompressibility of the fluid, generally adopted in soil mechanics, is quite inappropriate in this case.

In order to account for the effect of saturation on the site response, both seismic wave’s velocities and site amplification are analysed. It should be noted that only the velocity of $P$ waves ($V_p$) and $V_{p'}$ (is affected by the degree of saturation.

Figure 8. Effect of incidence angle on horizontal amplification: a) fully saturated porous soil ($n = 0.3, C_u = 5, F C = 5 \%, S_r = 100 \%$); b) solid medium

Figure 9. Reflection and transmission of SV waves: a) in solid; b) fluid saturated porous medium

Figure 10. Fluid modulus variation with saturation $S_r$
with the saturation-dependent pores fluid modulus. The $P_2$ wave is highly dispersive and tends to attenuate at low frequencies (Figure 11b). Furthermore, the $SV$ wave velocity is not influenced by the degree of saturation (Figure 11a).

On the other hand, the results for horizontal and vertical amplification in these four cases of saturation indicate that the influence of water saturation on the horizontal amplification is negligible (Figure 12). This is completely logical since only the compressional waves propagate in the interstitial fluid, but its effect is very notable on the vertical amplification (Figure 13). For a weak decrease below complete saturation (99.9 %), the vertical amplification increases strongly from 1.7 to 3, with a significant shifting of the frequency of the first peak to the low- frequencies from 14.84 Hz to 4.66 Hz.

4.1.3. Effect of grain size distribution

In terms of the coefficient of uniformity, the grain size distribution plays a key role in soil mechanics and geotechnical engineering. It constitutes an important factor for evaluating the shear modulus of soil. For small $Cu$-values, the soil is considered to be poorly graded or uniformly graded, as the size of the soil particles are nearly identical. However, with an increase in $C_u$ – values, the shear modulus decreases and the soil become well graded.

On the other hand, the amplification functions are presented for different $C_u$-values, $C_u = 2, 5$ and 8; it has been shown that, as the coefficient of uniformity $C_u$ increases, the amplitude of peaks increases with a shift to low frequencies for horizontal amplification, and also for the vertical one in the case of a partly saturated soil profile (Figure 14). Yet, the vertical component of amplification is not affected in a fully saturated case (Figure 15).
Influence of soil properties on amplification of saturated inhomogeneous poroviscoelastic soil profile

To further investigate this effect, seismic wave velocities are illustrated as a function of \( C_u \)-values. It can be noticed that when \( C_u \) increases from 2 to 8, \( P_1 \) wave velocity is reduced by 8.5 % and less than 1 % for partly and fully saturated medium, respectively (Figure 16), which may explain the small \( C_u \)-effect on the vertical amplification when \( S_r = 100 \% \). However, the \( SV \) wave velocity also reduces in the same manner in partly and fully saturated cases (Figure 17).

![Figure 14. Effect of \( C_u \)-values on: a) the horizontal; b) vertical amplification (\( S_r = 95 \%, FC = 0 \%, n = 0.3 \)).](image1.png)

![Figure 15. Effect of \( C_u \) on the vertical amplification (\( FC = 0 \%, n = 0.3, S_r = 100 \%) \).](image2.png)

![Figure 16. Effect of \( C_u \)-values on \( P_1 \) wave velocity in: a) partly; b) fully saturated porous soil (\( \sigma'_0 = 100 \text{ kPa}, FC = 0 \%, n = 0.3 \)).](image3.png)

![Figure 17. Effect of \( C_u \)-values on \( SV \) wave velocity (\( \sigma'_0 = 100 \text{ kPa}, FC = 0 \%, n = 0.3 \)).](image4.png)
4.1.4. Effect of non-cohesive fines content

Several investigations on the presence of plastic or non-plastic fine content in the sandy soil are focused on the soil liquefaction resistance \([37, 38]\) and often neglect the impact of this fine content on the amplification of soil response. For this purpose, it is assumed in this study, that the shear modulus depends on the non-cohesive fines content according to the Wichtmann’s model \([8]\).

Both horizontal and vertical amplification with \(S_r = 95\%\) exhibit variations similar to those obtained in \(C_u\)-analysis, in which the increase of FC-values induces an increase of amplification peaks with a shift to low frequencies (Figure 18). However, the vertical amplification in the completely saturated soil is slightly influenced by the fines content values (Figure 19).

Furthermore, it should be mentioned that the variation of the fine content, \(FC\) from 0 % to 10 % produces a decrease by 33.6 % and less than 2 % in \(P_1\) wave velocity in partly and fully saturated cases, respectively (Figure 20a and 20b).

Which confirms the little effect of stiffness variation on the vertical amplification in fully saturated soil. Moreover, the \(SV\) wave velocity also exhibits the same variation in the two cases of saturation (Figure 21).
4.1.5. Effect of porosity

Porosity is an important property that characterizes the porous media and often affects the flow conditions, as well as the physical properties of the solid skeleton such as shear modulus. In this study, we consider that the shear modulus depends on porosity as described in the Wichtmann’s model for sandy soil.

Figure 22 shows horizontal amplification for different porosity values \( n = 0.2, 0.3 \) and 0.4. It is clear that as the porosity

\[
\begin{align*}
\text{Figure 21. Effect of FC-values on SV wave velocity (} & \sigma'_{0} = 100 \text{ kPa, } C_{u} = 5, n = 0.3) \\
\text{Figure 22. Effect of porosity } n \text{ on horizontal amplification (} & C_{u} = 5, FC = 0 \%) \\
\text{Figure 23. Effect of porosity } n \text{ on vertical amplification in the cases of: a) partial saturation; b) full saturation (} & C_{u} = 5, FC = 0 \%)
\end{align*}
\]

\[
\begin{align*}
\text{Figure 24. Effect of } n \text{-values on the P}_{1} \text{ wave velocity in: a) partly; b) fully saturated porous soil (} & \sigma'_{0} = 100 \text{ kPa, } C_{u} = 5, FC = 0 \%)
\end{align*}
\]
increases the amplitude of peaks increases with a shift to low frequencies; the same comments are found for vertical amplification. This vertical amplification is more pronounced for the partly saturated soil compared to the fully saturated soil (Figure 23).

The phenomenon explained above can be presented by variation of waves velocities. It is indicated in Figure 24 that the increase in porosity produces a significant influence on $P$ wave velocities, which decrease by 34.43% and 19.2% for the partly and completely saturated case, respectively. The $SV$ wave velocity is also influenced, and the same trend is observed in the two cases of saturation (Figure 25).

![Figure 25. Effect of n-values on SV wave velocity](image)

4.1.6. Effect of permeability

It is obvious that permeability is an important property in energy dissipation of saturated porous medium where the coefficient of dissipation ($b = \eta/k$) depends on the intrinsic permeability of the skeleton and on fluid viscosity; this dissipation is due to the dynamic interaction between the solid and fluid phases [22].

In order to present the effect of permeability $k (m^2)$, both vertical and horizontal amplification in fully water saturated soil are illustrated for $k = 10^{-10}, 10^{-11}, 10^{-12}$ and $10^{-13} m^2$ in Figure 26. As can be noticed, the bi-amplification is not influenced by the variation of permeability in this low range of $k$-values, because of the small relative pore fluid flow at this low permeability.

![Figure 26. Effect of permeability k on: a) horizontal; b) vertical amplification](image)

4.2. Equivalent-linear (EQL) site response analysis

The previous section focused on the linear analysis of a poroviscoelastic soil profile amplification, which assumed that the profile stiffness and damping are constants. However, the soil can exhibit a nonlinear behaviour [5, 39, 40], where dynamic properties of soil (shear modulus, $G$, and damping ratio, $D$) vary with shear strain.

In this study, the nonlinear site response is accounted for through an equivalent linear approach which uses an iterative analysis to estimate strain-compatible soil properties for each soil layer. This approach is only an approximation of the nonlinear hysteretic stress-strain behaviour of cyclically loaded soils.

In the equivalent linear approach, the response of soil profile is first calculated using the small strain stiffness and damping as outlined above for linear analysis. From this initial estimate, shear strain ($\gamma$) histories for each layer are computed, and then the value corresponding to 65% of the peak shear strain of an earthquake motion is considered as the effective shear strain. Next, an iterative procedure is used to determine the degraded soil properties using the shear modulus reduction $G/G_{max}$ and damping ($D$) curves for sand, prescribed in Figures 27-29, which are defined by the following modified Hardin and Drnevich equations [39] in Wichmann et al. [7, 8] analysis:
The curve-fitting parameter \( a \) and the reference shear strain \( \gamma_r \) are defined as

\[
G = \frac{1}{1 + \frac{\gamma}{\gamma_r} \left[ 1 + a_2 \exp\left( -\frac{\gamma}{\gamma_r} \right) \right]}
\]

(30)

\[
D = 0.006 + 0.314 \frac{\gamma}{\gamma_r} \left[ 1 - 0.64 \exp\left( -\frac{\gamma}{\gamma_r} \right) \right]
\]

(31)

For cohesionless soils, the maximum shear stress, \( \tau_{\text{max}} \), can be calculated from:

\[
\tau_{\text{max}} = \sigma' \left[ \frac{1 + K_0}{2} \sin \phi' \right]^2 \left[ \frac{1 - K_0}{2} \right]^2
\]

(34)

with the lateral stress coefficient, \( K_0 = 1 - \sin \phi' \).

It can be noted that the shear modulus reduction \( G/G_{\text{max}} \) and damping \( D \) curves depend on confining pressure \( \sigma_0' \) (Figure 27), the grain size distribution, \( C_u \) (Figure 28), and the non-cohesive fines content, \( FC \) (Figure 29).

During an earthquake, the motion is subjected to simultaneous seismic loads in both horizontal and vertical directions. This means that the soil profile is induced by incident \( P \) and \( SV \) waves at the same time, which implies that the \( P_1 \) waves propagate through a soil profile having the same degraded shear modulus due to shear strain produced by the \( SV \) wave propagation. On the basis of this assumption, and considering the fact that the damping ratio of \( P_1 \) wave is not yet clearly discussed, the same site conditions for the two cases of wave propagation (damping and shear modulus) are assumed in this paper.

![Figure 27. Effect of confining pressure \( \sigma_0' \) on: a) shear modulus reduction curves; b) damping curves](image)

![Figure 28. Effect of \( C_u \) values on: a) shear modulus reduction curves; b) damping curves](image)
4.2.1. Input earthquake motion

To account for the effect of soil properties on nonlinear amplification of the saturated porous soil profile in horizontal and vertical directions, two components seismic accelerograms (North-South and up–down directions) are selected in this study as a rock outcropping motion to excite the soil profile. These motions were recorded during the El Centro (1940) earthquake, where the values of PGA are 0.28g and 0.178g for the horizontal and vertical motions, respectively (Figure 30).

4.2.2. Analysis of amplification functions

In order to approximately estimate the nonlinear response of the soil profile, an equivalent-linear site response analysis is used in this section. It can be seen from figures 31 and 32 that the increase of $C_u$, $FC$ and $n$-values amplifies the peaks amplitudes with a shift to low frequencies for the horizontal amplification, as well as for the vertical one in the partly saturated soil profile case. For instance, the increase of $C_u$-values from $C_u = 2$ to $C_u = 8$ (Figure 31a and 32a) increases the amplitude of the first peak by 34.3 %...
and 23.87 % with a shift to low frequencies by 0.97 Hz and 1.43 Hz for both horizontal and vertical amplifications, respectively. Moreover, in the case of fully saturated soil profile the vertical response is not influenced by all \( C_u \) and FC values (Figure 33a and 33b). Thus, the increase of porosity induced an increase of amplitude with a shift to low frequencies (Figure 33c). In addition, it is interesting to mention that the vertical response exhibits an identical behaviour to that exhibited in linear analysis, which implies that vertical amplification is not influenced by nonlinear behaviour in the fully saturated soil profile.

5. Conclusions

The effect of soil properties on linear and nonlinear amplification function of an inhomogeneous poroviscoelastic soil profile in both horizontal and vertical directions is investigated in this paper, by applying the Biot's two-phase media theory and using an analytical approach based on the exact stiffness matrix method, and by assuming that the incoming seismic wave consists of inclined \( P-SV \) waves. It is taken into consideration that mechanical properties increase with depth, according to the Wichtmann’s model which considers the effect of grain size distribution, porosity and fines content on the small strain shear modulus, the shear modulus reduction, and damping curves. The main conclusions of this study are as follows:

- When the \( SV \) wave presents an angle of incidence different from zero in saturated porous media, the horizontal amplification function exhibits an amplitude less than calculated in the single-phase medium, which can be explained by generation of the \( P_2 \)-wave. The effect of incidence angle on vertical response is negligible.

- Furthermore, saturation did not influence the horizontal amplification, but it strongly affected the vertical amplification, where a slight decrease below complete saturation induced a significant amplification and shift to low frequencies. This behaviour is due to the strong variation of the pore fluid modulus, \( K_f \), with saturation.

- The bi-amplification is not influenced by variation of permeability in the low range of \( k \)-values.

- The coefficient of uniformity, \( C_u \), and fines content, FC, exert an important influence on vertical amplification in partly saturated soil profile in case of both linear and nonlinear soil behaviour, but their effect is negligible when \( S_r = 100 \% \).

- The porosity also has a strong effect on vertical response in the case of partial saturation; yet, its impact is less significant in the case of complete saturation.

- These results indicate that variation of the vertical response and \( P \)-wave velocity in the case of partly saturated soil is related to mechanical properties; this variation is due to the flow conditions in case of complete saturation. The nonlinear vertical response exhibits an identical behaviour to that in linear analysis when \( S_r = 100 \% \), which implies that vertical response is not influenced by nonlinear behaviour of the fully saturated soil profile.

- The coefficient of uniformity, fines content and porosity also have a significant effect on horizontal amplification in both linear and nonlinear soil behaviour.

The results of this study indicate that there is a significant effect of local site properties on the poroviscoelastic soil amplification in both vertical and horizontal direction. Some improvements can be introduced to this research through further study of the response spectral ratio (\( V/H \)) and the use of the soil properties as spatial random fields.
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Influence of soil properties on amplification of saturated inhomogeneous poroviscoelastic soil profile

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