Models of the \textit{in situ} formation of detected extrasolar giant planets\textsuperscript{*,*}\textsuperscript{**}

Peter Bodenheimer\textsuperscript{a}, Olenka Hubickyj\textsuperscript{a,b} and Jack J. Lissauer\textsuperscript{b}

\textsuperscript{a}UCO/Lick Observatory, Board of Studies in Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA
\textsuperscript{b}Space Science Division, NASA-Ames Research Center, Moffett Field, CA 94035, USA

\textbf{ARTICLE INFO}

\textit{Keywords:}
origin of planetary systems
jovian planets, formation
jovian planets, interiors
accretion
extrasolar planets

\textbf{Abstract}

We present numerical simulations of the formation of the planetary companions to 47 UMa, $\rho$ CrB, and 51 Peg. They are assumed to have formed \textit{in situ} according to the basic model that a core formed first by accretion of solid particles, then later it captured substantial amounts of gas from the protoplanetary disk. In most of the calculations we prescribe a constant accretion rate for the solid core. The evolution of the gaseous envelope is calculated according to the following assumptions: (1) it is in quasi-hydrostatic equilibrium, (2) the gas accretion rate is determined by the requirement that the outer radius of the planet is the place at which the thermal velocity of the gas allows it to reach the boundary of the planet’s Hill sphere, (3) the gas accretion rate is limited, moreover, by the prescribed maximum rate at which the nebula can supply the gas, and (4) the growth of the planet stops once it obtains approximately the minimum mass determined from radial velocity measurements (in one case the planet is allowed to grow to twice this limit). Calculations are carried out through an initial phase during which solid accretion dominates, past the point of crossover when the masses of solid and gaseous material are equal, through the phase of rapid gas accretion, and into the final phase of contraction and cooling at constant mass. Alternative calculations are presented for the case of 47 UMa in which the solid accretion rate is calculated, not assumed, and the dissolution of planetesimals within the gaseous envelope is considered. In all cases there is a short phase of high luminosity ($10^{-3} - 10^{-2}$ $L_\odot$) associated with rapid gas accretion. The height and duration of this peak depend on uncertain model parameters. The conclusion is reached that \textit{in situ} formation of all of these companions is possible under some circumstances. However, it is more likely that orbital migration was an important component of the evolution, at least for the planets around $\rho$ CrB and 51 Peg.

\textsuperscript{*}UCO/Lick Observatory Bulletin No. 1389.

\textsuperscript{**}Paper presented at Protostars and Planets IV Conference, Santa Barbara, CA, July 1998.
1. INTRODUCTION

The recent discoveries of giant planets orbiting at relatively close distances around solar-type stars (reviewed by Marcy and Butler 1998) have led to intense speculation regarding their origin. The objects discovered so far have orbits significantly closer to their star than Jupiter’s distance from the Sun, but have masses in the range 0.4 – 10 Jupiter masses (\(M_J\)). Examples of objects with nearly circular orbits, but with distances from the central object great enough so that the circularity of the orbit is unlikely to have been achieved through the influence of the tidal effects of the star, include the companion to 47 UMa (Butler and Marcy 1996) at 2.1 AU and the companion to \(\rho\) CrB (Noyes et al. 1997) at 0.23 AU. Examples of objects very close to the central star, with periods of only a few days, include the companions to 51 Peg (Mayor and Queloz 1995), 55 \(\rho\) 1 Cancri (Butler et al. 1997), \(\tau\) Boo (Butler et al. 1997), and \(\nu\) And (Butler et al. 1997). A few of the newly-discovered objects travel on eccentric orbits. In the cases of HD 114762 (Latham et al. 1989) and 70 Vir (Marcy and Butler 1996), the planets are relatively massive with 9 and 6.6 \(M_J\) as minimum masses, respectively. In the case of the companion to 16 Cyg B (Cochran et al. 1997), the eccentricity is 0.6 but the minimum mass is only 1.7 \(M_J\).

A number of theories have been proposed regarding the origin of these systems. The objects could have formed by fragmentation of a collapsing protostellar cloud and subsequent orbital interactions among the fragments. The companion to Gl 229 (Oppenheimer et al. 1995; Nakajima et al. 1995) almost certainly falls into this class, because of its relatively high mass (\(\approx 40 M_J\)); see Allard et al. 1996; Marley et al. 1996), and Black (1997) argues that most of the objects discovered in radial velocity surveys do also. This mechanism is most likely to produce masses \(> 7 M_J\) (Low and Lynden-Bell 1976; Rees 1976) although it has not been definitively proved that smaller masses are impossible. The second mechanism involves collapse of a molecular cloud core into a protostar and a disk, followed by gravitational instability in the disk leading to fragmentation on a dynamical time scale (Kuiper 1951; Cameron 1978; DeCampli and Cameron 1979). Numerical calculations on gravitationally unstable disks by Adams and Benz (1992) and Boss (1997) suggest that if a disk can be produced with appropriate conditions, fragmentation into objects of \(\sim 10 M_J\) rather than \(\sim 1 M_J\) is likely to occur. Laughlin and Bodenheimer (1994) argue that the collapsing system will not result in a disk that is subject to fragmentation, but rather in one that develops a spiral wave pattern that saturates in amplitude. This second mechanism also has difficulties because, although it is possible to get a gravitationally unstable disk model with a mass of about 0.14 \(M_\odot\) out to 10 AU (Boss 1998), the implied total disk mass is high compared with most observations. Nevertheless, the more massive objects are candidates for this process.

The third process proposed for the formation of giant planets is gradual accretion of small solid particles in a low-mass protoplanetary disk, followed by gravitational capture of gas. The main physical effects are reviewed by Lissauer (1993). The stages that are envisioned are as follows: (1) Accretion of dust particles (Safronov 1969) results in a solid core with mass \(M_Z\) of several \(M_\oplus\), accompanied by a gaseous envelope of very low mass, \(M_{XY}\). (2) Further accretion of gas and solids results in the mass of the envelope increasing faster than that of the core until a crossover mass \(M_{cross} \equiv M_Z = M_{XY}\) is reached (Perri and Cameron 1974; Mizuno 1980). (3) Runaway gas accretion occurs with relatively little accretion of solids, and the peak luminosity reaches \(10^{-3} - 10^{-4} L_\odot\) (Bodenheimer and Pollack 1986). (4) Accretion is terminated by tidal truncation (Lin and Papaloizou 1979, 1985, 1993) or dissipation of the nebula. (5) The planet contracts and cools at constant mass to the present state (Bodenheimer and Pollack 1986; Saumon et al. 1996; Burrows et al. 1997). The resulting orbits are likely to be initially relatively circular and the masses not more than a few \(M_J\), because tidal truncation sets in at that point and stops appreciable further accretion of gas (Lin and Papaloizou 1979). Note that the value of this limiting mass depends on uncertain disk parameters such as viscosity. Moreover, Artymowicz (1998) and Artymowicz and Lubow (1996) have made calculations which indicate that accretion onto a protoplanet can continue even after gap formation at a rate which depends on the disk viscosity; thus, the issue of the limiting mass of a protoplanet formed by this process is controversial.

Evolutionary calculations based on this general scenario have been performed by Bodenheimer and Pollack (1986) through stages (1)–(5), under the assumption of a constant solid accretion rate, and by Pollack et al. (1996) through stages (1)–(3) with more detailed physics, including a (non-constant) solid accretion rate calculated from three-body accretion cross sections. These studies show that the core accretion theory is successful in the sense that it (i) gives a mass in the solid component of the giant planets which agrees with the observations of the giant planets in the Solar System, and (ii) shows that the mass of the solid component is relatively independent of the position of the planet in the solar nebula, again in agreement with the observations of Jupiter and Saturn. As discussed by Pollack et al. (1996), the theory must allow for the partitioning of the solid component into an actual core and as dissolved material into the gaseous envelope; recent observational constraints are discussed by Guillot et al. (1997) and Wuchterl et al. (1999).

An often-discussed problem with the core accretion process is that in a ‘minimum mass’ solar nebula the accretion times for the cores of Jupiter and Saturn are too long, over \(10^7\) yr. The range of observed disk lifetimes, although very uncertain, is quoted to be \(\sim 0.1 - 10\) Myr (Strom et al. 1993). However, if the disk’s surface density is increased by a factor of a few over the minimum value, formation times are reasonable (Lissauer 1987; Pollack et al. 1996) and the corresponding disk masses are still in agreement with those deduced from observations (Beckwith and Sargent 1993). Also, the excess mass could be invoked to explain the Oort cloud. Another problem with the core accretion mechanism has arisen as a result of hydrodynamic calculations by Wuchterl (1991). He found that at the beginning of stage (3), when rapid gas accretion starts, most of the gaseous envelope is ejected, leaving...
only about 1 M⊕ around the solid core. If the nebular density is increased to the point where the planetary gaseous envelope becomes predominantly convective, then the instability is avoided (Wuchterl 1995); however, the required density is about a factor 8 higher than densities in a ‘minimum mass’ nebula at 5 – 10 AU from the central star. On the other hand, Tajima and Nakagawa (1997) find that the protoplanetary envelopes are dynamically stable to small perturbations even in a minimum-mass nebula.

Lin et al. (1996) suggested that the companion to 51 Peg formed by this mechanism at a distance of a few AU from the star and then migrated to its present position through interaction with the disk. At distances of ≈ 0.05 AU (but not for appreciably greater distances) the migration can be stopped by tidal interaction with the star or truncation of the inner disk by the stellar magnetic field. More detailed calculations of post-formation orbital evolution (Trilling et al. 1998; Murray et al. 1998) suggest that several of the orbital positions of extrasolar planets can be explained. Certainly migration is a potentially important element in the formation of all types of giant planets (Goldreich and Tremaine 1980; Lin and Papaloizou 1986a,b; Lin 1997; Takeuchi et al. 1996; Ward 1997a,b). In this context, the planets with eccentric orbits could be explained by various processes of excitation of eccentricity: interaction with the disk (Artymowicz 1992), close encounters and scatterings between two or more giant planets (Lin and Ida 1997; Levison et al. 1998; Weidenschilling and Marzari 1996; see also Rasio and Ford 1996 for a related explanation of 51 Peg type planets), or the presence of a distant binary companion as in the case of 16 Cyg B b (Mazeh et al. 1997; Holman et al. 1997). However, the relatively high masses of 70 Vir b and HD 114762 b cause some difficulty for their formation by the core accretion process unless the standard tidal truncation limit can be exceeded, and they may possibly have formed by fragmentation.

The calculations reported here are based on the third process just described, that is, the standard mechanism for forming giant planets in a disk. However, we do not include post-formation migration of the planets in the calculations, but ask instead whether in situ formation is a viable possibility and then consider the consequences. As examples we consider three objects with nearly circular orbits: 51 Peg b, 47 UMa b, and ρ CrB b. There are several good reasons why it is difficult to form a giant planet close to a star. First, according to many nebular models (e.g., Bell et al. 1997), the temperature is too high at ≈ 0.05 AU from the central star to allow for condensation of solid particles. Second, even if solid particles could condense, there is insufficient mass in the inner region of the nebula to produce a Jupiter mass. Third, even if a Jupiter mass were to form, tidal interactions between it and the disk would rapidly cause it to migrate into the star (Goldreich and Tremaine 1980; Ward 1997a,b). The first objection does not apply to the system of ρ CrB at 0.23 AU; for 51 Peg we use disk models which have relatively low mass accretion rates and therefore are cool enough at 0.05 AU for some condensible material to exist (Bell et al. 1997).

The second objection is overcome under the assumption that solid particles can migrate through the disk as a result of its normal (viscous) evolution or gas drag, and that larger chunks can migrate relative to the disk as a result of tidal effects. Thus, for 51 Peg b and ρ CrB b we actually consider migration in the sense that we assume that solid material is delivered at a constant rate to the formation site. The third objection is still a major problem for all giant planets and we do not consider it here, except that in the case of 51 Peg b we consider an alternative scenario of in situ formation based on a modified disk structure with reduced density in the inner region. This alternative is a modification of and elaboration upon a model for 51 Peg b recently presented by Ward (1997a), which involves the buildup of an inner planet (at 0.05 AU) through collisions of protoplanetary cores in the range 1 – 10 M⊕ which have migrated there from the outer parts of the disk. In the case of 47 UMa b (at 2.1 AU), the assumption of in situ formation is certainly a reasonable one, and we consider two possibilities, first, a model with variable solid accretion rate (as in Pollack et al. 1996), and second, one with constant solid accretion rate (as in Bodenheimer and Pollack 1986). Other in situ models for the formation of 51 Peg b have been presented by Wuchterl (1997) and Ruzmaikina (1998); also, preliminary results of the present authors are found in Bodenheimer (1998).

2. PHYSICAL ASSUMPTIONS AND METHOD

In our model, the protoplanet consists of a solid core and a gaseous envelope, both of which evolve as a consequence of accretion. The computational procedure is described in detail in Pollack et al. (1996) and consists of three main components: (1) the calculation of the rate of accretion of solid material, assumed to be in the form of planetesimals, onto the protoplanet, taking into account the physical cross section as well as the gravitational enhancement factor; (2) the calculation of trajectories and destruction rate of planetesimals as they pass through the gaseous envelope, to determine the radial profile of energy deposition in cases where the planetesimals do not plunge all the way to the core; and (3) a calculation of the evolution and mass accretion rate of the gaseous envelope, under the assumption that the planet is spherical and that the standard equations of stellar structure apply. Table I lists the parameters for each of the cases computed. A glossary of symbols is provided in the Appendix. In Cases U1 and U2 the full procedure is used. In all other cases a simplified procedure is used in which the solid accretion is assumed to occur at a constant, parameterized, rate and in which the accreted solids are assumed to fall directly to the core, releasing all of their gravitational energy there.

For the calculation of the solid accretion rate (Cases U1, U2) the basic assumptions are: the protoplanet is surrounded by a disk with an initial uniform surface density $\sigma_{\text{ini},Z}$ of solid material, and the solid material is in the form of planetesimals of uniform size, 100 km in radius. The rate of growth of the solid core of a protoplanet is given by the
TABLE I. Input Parameters

| Case | $a$ | $M_c$ | $M_{XY}$, max | $\rho_{core}$ | $T_{rub}$ | $\rho_{rub}$ | $\sigma_{init,Z}$ | $\sigma_{init,XY}$ |
|------|-----|-------|---------------|--------------|----------|-------------|------------------|------------------|
|      | (A.U.) | ($M_\odot$/yr) | ($M_\odot$/yr) | (g/cm$^3$) | (K) | (g/cm$^3$) | (g/cm$^2$) | (g/cm$^2$) |
|      |     |       |                |             |          |             |                  |                  |
| U1   | 2.1 | variable | 1.053 x 10$^{-2}$ | 5.0 | 400. | 2.0 x 10$^{-5}$ | 50.0 | 1.25 x 10$^4$ |
| U2   | variable | 1.053 x 10$^{-2}$ | 5.0 | 1000. | 2.0 x 10$^{-5}$ | 57.5 | 1.43 x 10$^4$ |
| U3   | 2.1 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 5.0 | 400. | 3.0 x 10$^{-10}$ |      |                |
| P1   | 0.05 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1500. | 5.0 x 10$^{-8}$ |      |                |
| P2   | 0.05 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1500. | 5.0 x 10$^{-11}$ |      |                |
| P3   | 0.05 | 1.0 x 10$^{-5}$ | 2.107 x 10$^{-5}$ | 6.0 | 1500. | 5.0 x 10$^{-11}$ |      |                |
| P4   | 0.05 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1500. | 5.0 x 10$^{-6}$ |      |                |
| P5   | 0.05 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1500. | 5.0 x 10$^{-6}$ | low opacity |                |
| P6   | 0.05 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1500. | 5.0 x 10$^{-8}$ | Guenther comparison |                |
| C1   | 0.23 | 1.0 x 10$^{-5}$ | 1.053 x 10$^{-2}$ | 6.0 | 1200. | 5.0 x 10$^{-8}$ |      |                |

The third component involves the calculation of the structure and evolution of the envelope; it is included in all cases. The structure is determined from the equations of mass conservation, energy conservation, hydrostatic equilibrium, and radiative or convective energy transport, as given in Bodenheimer and Pollack (1986). The assumption of hydrostatic equilibrium has been verified, at least in the case of a constant accretion rate of solids, by Tajima and Nakagawa (1997). The energy generation rate is calculated from the accretion of planetesimals and quasi-static contraction. If planetesimals land on the core, the energy deposition is smoothed over a region of about one core radius in extent. The grain and molecular opacity in the envelope is based on calculations by Pollack et al. (1985) and Alexander and Ferguson (1994), who used an interstellar size distribution. Justification of this assumption and tests to determine the effect of a large change in the opacity on the outcome of the calculations have been discussed by Bodenheimer and Pollack (1986) and Pollack et al. (1996). The equation of state is non-ideal and has been updated since the calculations of Pollack et al. (1996) were made; it is now based on calculations by Saumon et al. (1995), interpolated to a near-protosolar composition of $X = 0.74$, $Y = 0.243$, $Z = 0.017$.

The calculations are started at $t = 10^4$ yr\(^{-1}\) Note that the starting times used by Pollack et al. (1996) are not $t = 0$, as stated in the text, but 0.21 Myr for Jupiter models, 0.18 Myr for Saturn models, and 1.25 Myr for the standard Uranus case U1, with a core mass of 0.1 $M_\oplus$; the corresponding envelope mass is $10^{-9}$ $M_\oplus$. These initial values were chosen for computational convenience, and our results are completely insensitive to changes in the initial conditions.

Boundary conditions, which are used for all cases de-
scribed below, are provided at the inner and outer edges of the envelope. At the inner edge, the luminosity \( L_r = 0 \) and the radius \( r = R_{\text{core}} \), where \( R_{\text{core}} \) is determined from the current core mass \( M_Z \) and the core density \( \rho_{\text{core}} \). The core is assumed to have a uniform density and to be composed of rock, under the assumption that ices have evaporated at the nebular positions considered here. Thus, for the cases of 51 Peg b and \( \rho \) CrBC b, \( \rho_{\text{core}} = 6 \) g cm\(^{-3} \) and for 47 UMa b \( \rho_{\text{core}} = 5 \) g cm\(^{-3} \).

The outer boundary condition can be applied in three different ways, depending on the evolutionary stage of the planet. During the first, or nebular, stage the outer edge of the planetary structure is in direct contact with the nebula gas. The outer density \( \rho_{\text{neb}} \) and temperature \( T_{\text{neb}} \) are prescribed from nebula models and do not vary with time. The outer radius of the planet is assumed to fall at a modified accretion radius \( R_{\text{a}} \). Let the tidal, or Hill, radius be

\[
R_H = a \left( \frac{M_p}{3M_*} \right)^{1/3},
\]

where \( a \) is the distance to the central star, \( M_p \) is the planet’s mass, and \( M_* \) is the star’s mass. Then \( R_{\text{a}} \) is given by

\[
R_a = \frac{GM_p}{c^2 + \frac{GM_p}{R_H}},
\]

where \( c \) is the sound speed in the nebula. In the limits of large and small \( R_H \), this expression reduces to the accretion radius and the tidal radius, respectively. The gas accretion rate is determined by the requirement that the outer radius of the protoplanet be close to \( R_{\text{a}} \), within a small tolerance. At every time step mass is added at the outer edge so that this requirement is satisfied.

The limiting gas accretion rate is determined by the ability of the nebula to supply gas at the required rate. In a typical nebula the mass transfer rate at a given radius caused, for example, by viscous effects, is \( 3 \times 10^{-8} M_{\odot}/\text{yr} \) or \( \approx 10^{-2} M_{\odot}/\text{yr} \). Once the limiting rate is reached, the planet contracts inside \( R_a \), and the evolution enters the transition stage. Here the planet is still assumed to be in hydrostatic equilibrium, but gas is accreting hydrodynamically onto it at near free-fall velocities. Thus, the luminosity of the protoplanet is given by the sum of two contributions, its normal luminosity \( L_{\text{contr}} \) caused by contraction and accretion of solids, plus a gas accretion luminosity given by

\[
L_{\text{acc}} = GM_p M_{XY,\text{max}} (1/R_p - 1/R_a)
\]

where \( R_p \) is the planetary radius and \( M_{XY,\text{max}} \) is the limiting accretion rate. The density \( \rho_p \) and temperature \( T_p \) at \( R_p \) are also modified. The density is obtained from

\[
\rho_0 = \frac{M_{XY,\text{max}}}{4\pi R_p^2 v_{\text{ff}}},
\]

where \( \rho_0 \) is the density at the inner edge of the infalling flow and \( v_{\text{ff}} \) is the free-fall velocity from \( R_a \) to \( R_p \). Then \( \rho_0 \) is multiplied by the square of the infall Mach number to get the planetary boundary density \( \rho_p \), under the assumption that the boundary shock is isothermal. To get the temperature, the radiative diffusion equation

\[
\frac{dT}{dr} = \frac{3\kappa\rho_0 R_p^{1.5}}{4\pi r^1.5} - \frac{L_r}{4\pi r^2}
\]

is integrated and solved approximately under the assumptions that the Rosseland mean opacity, \( \kappa \), and the total luminosity, \( L_r = L_{\text{acc}} + L_{\text{contr}} \), are constants as a function of distance \( r \). In the limit that the envelope is optically thin the result is simply \( T_r = T_{\text{neb}} \). These approximations have been checked against more detailed solutions of the radiative transfer equation and found to be quite adequate.

The supply of gas to the planet is eventually assumed to be exhausted (e.g., as a result of tidal truncation of the nebula, removal of the gas by effects of the star, and/or the accretion of all nearby gas by the planet), and the planet’s mass levels off to some limiting value, defined by the planetary system that is being modeled (assuming \( \sin i \approx 1 \), except in Case P1a). This process is not modeled in detail, but \( M_{XY} \) onto the planet is assumed to reduce smoothly to zero as the limiting value is approached. The planet then evolves through the isolated stage, during which it remains at constant mass. The boundary conditions are then simply the standard photospheric conditions

\[
L = 4\pi \sigma_{SB} R_p^2 T_p^4,
\]

where \( \sigma_{SB} \) is the Stefan-Boltzmann constant, and

\[
\kappa P = \frac{2}{3} g,
\]

where \( P \) is the photospheric pressure and \( g \) is the acceleration of gravity at \( R_p \). If the planet is close to the central star, insolation is taken into account according to the procedure described by Stringfellow et al. (1990).

### 3. RESULTS

#### 3.1. Formation of the companion to 47 UMa

The companion to 47 UMa, a solar-type star, has a nearly circular orbit, a period of 3 years, an orbital radius of 2.1 AU, and a minimum mass of 2.38 \( M_{\oplus} \) (Marcy and Butler 1998). We perform two types of calculations, the first (Cases U1 and U2) in which the solid accretion rate is calculated in detail according to the procedures described in Pollack et al. (1996) and summarized in the previous section, and the second (Case U3) in which the solid accretion rate is assumed to have a constant value of \( 10^{-5} M_{\oplus} \) yr\(^{-1} \). The work of Bell et al. (1997) provides detailed disk models for a range of values of the assumed steady-state accretion rate of nebular mass \( M_{\text{neb}} \) and viscosity parameter (\( \alpha \)). These models are used to determine boundary conditions in the nebular stage. The results are summarized in Table II.

For Cases U1 and U2 the crucial parameter is the assumed surface density of solid material in the disk, \( \sigma_{\text{int}, Z} \) (Pollack...
Table II

Results

| END OF PHASE 1 | CROSS-OVER POINT | PEAK | FINAL MASS | ENDPOINT |
|----------------|------------------|------|------------|----------|
| run            | Timea            | $M_{\text{XY}}$b | $M_{2}$ | $M_{\text{XY}}$=a$c$ | Time | Log $L^{d}$ | $\Delta$ Time | Time | $M_{\text{XY}}$ | $M_{2}$ | Time | $L^{e}$ |
| U1             | 0.27             | 0.66 | 8.4     | 3.2x10^{-5} | 17.23 | 11.91 | 3.4x10^{-6} | 6.8x10^{-7} | 18.74 | -2.015 | 0.19 | 18.7 | 38.42 | 70.16 | 8.3x10^{7} | -7.7 |
| U2             | 0.23             | 0.35 | 20.4    | 3.2x10^{-5} | 1.85 | 28.76 | 1.8x10^{-5} | 9.2x10^{-7} | 2.165 | -2.128 | 0.165 | 3.251 | 60.15 | 72.24 | 3.5 | -3.82 |
| U3             | 2.74             | 38.3 | 4.22    | 1.0x10^{-5} | 3.208 | 32.05 | 4.4x10^{-3} | 1.0x10^{-5} | 3.256 | -2.028 | 0.133 | 3.355 | 34.21 | 62.27 | 8.3x10^{7} | -8.6 |
| P1             | 4.320           | 41.48 | 82.32   | 1.0x10^{-5} | 4.900 | 45.78 | 1.0x10^{-5} | 1.0x10^{-5} | 4.875 | -2.711 | 0.02 | 4.488 | 64.98 | 113.7 | 6.4x10^{7} | -8.4 |
| P2             | 4.903           | 48.13 | 92.00   | 1.0x10^{-5} | no cross— upper point | 0.11 | -0.18 | — | 0.11 | 80.26 | 77.40 | 1.61x10^{3} | -4.6 |
| P3             | 4.903           | 48.13 | 92.00   | 1.0x10^{-5} | 5.45 | 56.90 | 1.0x10^{-5} | 1.0x10^{-5} | 5.46 | -2.09 | 0.02 | 5.47 | 55.98 | 113.4 | 4.03x10^{7} | -4.52 |
| P4             | 3.273           | 31.03 | 7.334   | 1.0x10^{-5} | 3.743 | 35.06 | 1.0x10^{-5} | 1.0x10^{-5} | 3.766 | -2.980 | 0.02 | 3.766 | 35.80 | 130.8 | 1.05x10^{7} | -4.45 |
| Cl             | 2.59            | 30.7  | 4.64    | 1.0x10^{-5} | 3.404 | 35.30 | 1.0x10^{-5} | 1.0x10^{-5} | 3.472 | -2.338 | 0.054 | 3.506 | 35.87 | 300.6 | 1.55x10^{7} | -5.8 |

a Time is in units of million of years, Myr.
b Mass is in units of Earth's mass, $M_{\oplus}$.
c The accretion rate is in units of Earth masses per year, $M_{\oplus}/yr$.
d Luminosity is in units of solar luminosity, $L_{\odot}$.

e In Fig. 1, during the first phase, at a time of 1.0 x 10^{5} yr, the solid accretion rate peaks at 10^{-4} M_{\oplus} yr^{-1} and the luminosity at 2 x 10^{-5} L_{\odot}. The estimate of the isolation mass (Eq. 14 of Pollack et al. 1996) corresponding to the choice of $\sigma_{\text{init},Z}=50$ g cm^{-2} is approached, the third phase, rapid gas accretion, sets in. The calculations reported by Pollack et al. (1996) were stopped during this third phase; here they are continued into the transition stage and the isolated stage.

The results are shown in Fig. 1. During the first phase, at a time of 1.0 x 10^{5} yr, the solid accretion rate peaks at 10^{-4} M_{\oplus} yr^{-1} and the luminosity at 2 x 10^{-5} L_{\odot}. The estimate of the isolation mass (Eq. 14 of Pollack et al. 1996) corresponding to the choice of $\sigma_{\text{init},Z}=50$ g cm^{-2} is approached, the third phase, rapid gas accretion, sets in. The calculations reported by Pollack et al. (1996) were stopped during this third phase; here they are continued into the transition stage and the isolated stage.

The results are shown in Fig. 1. During the first phase, at a time of 1.0 x 10^{5} yr, the solid accretion rate peaks at 10^{-4} M_{\oplus} yr^{-1} and the luminosity at 2 x 10^{-5} L_{\odot}. The estimate of the isolation mass (Eq. 14 of Pollack et al. 1996) corresponding to the choice of $\sigma_{\text{init},Z}=50$ g cm^{-2} is approached, the third phase, rapid gas accretion, sets in. The calculations reported by Pollack et al. (1996) were stopped during this third phase; here they are continued into the transition stage and the isolated stage.

The results are shown in Fig. 1. During the first phase, at a time of 1.0 x 10^{5} yr, the solid accretion rate peaks at 10^{-4} M_{\oplus} yr^{-1} and the luminosity at 2 x 10^{-5} L_{\odot}. The estimate of the isolation mass (Eq. 14 of Pollack et al. 1996) corresponding to the choice of $\sigma_{\text{init},Z}=50$ g cm^{-2} is approached, the third phase, rapid gas accretion, sets in. The calculations reported by Pollack et al. (1996) were stopped during this third phase; here they are continued into the transition stage and the isolated stage.
Models of the in situ formation of detected extrasolar giant planets

Figure 1: The growth and subsequent evolution of the companion to 47 UMa in a massive protoplanetary disk with the solid accretion rate calculated as a function of time (Case U1). a) Mass of the solid component (solid line), the gaseous component (dotted line), and the total mass (dot-dashed line) as functions of time. b) Luminosity radiated by the planet as a function of time. c) Logarithms of the mass accretion rate of planetesimals (solid line) and the accretion rate of gas (dotted line) as functions of time. d) Radii of the solid core (dashed line) and the entire protoplanet (solid line) as functions of time.

Figure 2: The luminosity radiated by the planet as a function of time for Case U2.

The results show that, compared with case U1, the isolation mass is increased by more than a factor 2 in phase 1, and $\dot{M}_{XY}$ and $\dot{M}_Z$ increase by more than a factor 10 in phase 2. The rapid gas accretion occurs in a manner similar to that in case U1, with a peak of $10^{-2}$ $L_\odot$ and a width (measured at $\log (L/L_\odot) = -3.4$) of $1.65 \times 10^7$ yr (Fig. 2). The final values of $M_Z$ and $M_{XY}$ are 69 and 732 $M_\oplus$, respectively, and the planet forms in just over $1.9 \times 10^6$ yr, about a tenth of the time needed for Case U1.

The results of Case U1 and U2 show that the evolution can be reasonably approximated by an assumed constant rate of solid accretion throughout much of phase 2. Thus for case U3 we assume that $\dot{M}_Z$ is constant at $10^{-5} M_\oplus$ yr$^{-1}$ throughout the entire evolution, a rate slightly less than the mean accretion rate for Case U2. An accretion rate of this order of magnitude is needed to obtain a reasonable formation time. The parameters for the nebular model for Case U3 are taken to be $a = 0.01$, $M_{neb} = 10^{-7} M_\odot$ yr$^{-1}$. At 2.1 AU the model gives $T_{neb} = 400$ K and $\rho_{neb} = 3 \times 10^{-10}$ g cm$^{-3}$, corresponding to a total surface density of about 1000 g cm$^{-2}$. The tidal truncation limit for these conditions is 2.3 $M_J$, very close to the observed minimum mass. The masses as a function of time are plotted in Fig. 3; a formation time of 3.4 million years is obtained and the final values of $M_Z$ and $M_{XY}$
are 34 and 633 M_⊕, respectively. The radiative envelope in these models contains a significant mass fraction; thus the analysis of Stevenson (1982), which assumes a constant M_Z, should apply, and the final core mass should be insensitive to the nebular boundary conditions. The luminosity as a function of time is plotted in Fig. 3b. The peak luminosity reaches 10^{-2} L_☉, essentially the same as that in Case U1, and the full width of the peak, corresponding to L > 10^{-5} L_☉, is 1.3 x 10^5 yr. The isolated phase is calculated up to a time beyond 10^6 yr; the late-phase luminosity decline is exponential. Insolation is not included in this case and the total mass is cut off at 2.1 M_J to facilitate a comparison with an isolated protoplanet of 2 M_J calculated by Burrows et al. (1997). As shown in Fig. 3b, the agreement is very good.

3.2. Formation of 51 Peg b

The companion to 51 Peg has an orbital period of 4.23 days, an orbital separation of 0.05 AU from the star, and a minimum mass of 0.44 M_J = 140 M_⊕ (Marcy and Butler 1998). The first calculation presented here assumes that the planet formed in a standard nebular disk. The Bell et al. (1997) model for M_{neb} = 10^{-8} M_☉ yr^{-1} and a = 0.01 gives a midplane temperature in the disk at 0.05 AU of about 1500 K, so some solid material could survive. We assume that the solid material is supplied to the inner region of the disk by viscous evolution; the above value for M_{neb} corresponds to \approx 10^{-5} M_⊕ yr^{-1} of solid particles that can exist at 1500 K. From the models, \rho_{neb} = 5 x 10^{-8} g cm^{-3} and T_{neb} = 1500 K. All of the solid material that arrives at 0.05 AU is assumed to be collected by the planet. The tidal truncation conditions (Lin and Papaloizou 1993), which give the approximate maximum mass a giant planet can accrete before complete gap formation, are given by

\[ M_{p,max} = \max \frac{4 \nu v}{(\Omega a^2)} M_*, 3 \left( \frac{c}{\Omega a} \right)^3 M_* \]  \hspace{1cm} (9)

Here \nu is the nebular viscosity. In the particular nebular model chosen, the viscosity condition determines the maximum mass, which is about 0.7 M_J. In the calculation, denoted as Case P1, the mass is assumed to level off at 0.52 M_J = 167 M_⊕.

The results (Figs. 4 and 5) show that the final mass of the solid core is 47 M_⊕, that of the gaseous envelope is 120 M_⊕, and the time to reach final mass is 4.70 x 10^6 years. Figs. 4a,b,c,d show the evolution of the mass of the core and envelope, the luminosity, the mass accretion rate, and the radius, respectively. During the early evolution, the envelope mass is very low compared with the core mass; only after 4.1 x 10^6 yr does the envelope mass increase rapidly. The crossover mass (M_Z = M_{XY}) is almost identical to the final M_Z and is reached at 4.7 x 10^6 yr. The transition stage, when the planet first contracts within R_a, begins at about the same time.

The luminosity at early times is dominated by accretion of planetesimals. Between 4.68 and 4.7 Myr, when M_{XY} is comparable to M_Z, it becomes dominated by gravitational contraction as a result of rapid gas accretion. However, because material is added at a surface temperature of 1500 K, the molecular hydrogen dissociation zone lies fairly close to the surface. As the material which is added contracts and is heated to higher temperatures, it undergoes dissociation in the range 3000 – 6000 K. When the gas accretion rate becomes high, more than 90% of the contraction energy released by the entire mass must supply the dissociation energy of the added mass. Thus, the emerging luminosity remains about constant at 10^{-5.5} L_☉. The rapid increase in L corresponds to the transition stage and is almost entirely the accretion luminosity liberated at the planetary surface as material falls from the Roche radius to the surface. After a sharp spike, with maximum log (L/L_☉) = -2.7 when M_{XY} = 119.6 M_⊕, the accretion rate decreases as the planet reaches its terminal mass. The width of the narrow spike in luminosity (Fig. 4b) is only 2 x 10^4 yr, narrower than that in Cases U1, U2, and U3 because of the smaller amount of mass that is accreted. During the isolated phase (t > 4.7 x 10^6 yr), the
Models of the \textit{in situ} formation of detected extrasolar giant planets

Figure 4: The evolution of the companion to 51 Peg in a standard protoplanetary disk (Case P1). a) Mass of the solid component of the planet (solid line), the gaseous component (dotted line), and the total mass (dot--dashed line) as functions of time. b) Luminosity of the protoplanet as a function of time. The luminosity spike corresponds to the brief period of rapid contraction of the planet inside the Hill radius and the hydrodynamic accretion of nebular gas onto it. After about $10^8$ yr the luminosity levels off to log ($L/L_\odot$) = $-4.5$. c) Logarithm of the mass accretion rate of planetesimals (solid line) and the accretion rate of gas (dotted line) as functions of time. The upper limit on the gas accretion rate is assumed to be $10^{-2} M_\oplus$ yr$^{-1}$. This rate is assumed to drop smoothly to zero as the planet approaches its final mass of 0.52 $M_\oplus$. d) Radii of the solid core (dashed line) and the entire protoplanet (solid line) as functions of time. The outer radius corresponds very closely to that of the Hill sphere at times prior to the maximum.

The luminosity is at first produced by contraction and cooling at constant mass, but it soon becomes dominated by the heating of the planetary surface by the star, and it levels off at log $L/L_\odot = -4.5$ for $t > 10^9$ yr.

The radius (Fig. 4d) does not undergo major variation during the evolution. During much of the nebular phase it is close to the Hill radius of 0.2 – 0.3 $R_\odot$, reaching a maximum of 0.35 $R_\odot$ at the transition stage, which is indicated by the sudden drop in Fig. 4d. At the beginning of the isolated stage, the radius is 0.25 $R_\odot$, small enough so that even in the presence of the radiation of the nearby star the Jeans escape rate of material from the outer atmosphere should be negligible. During the isolated stage, the radius decreases slowly to a final value of 0.11 $R_\odot$ (approximately 10% larger than Jupiter’s present radius) after several Gyr. The observable surface temperature is $T_{\text{neb}} = 1500$ K during the nebular and transition phases, and during the isolated phase it soon levels off to the insolation value of 1300 K. The evolution of the interior is illustrated in Fig. 5. Early in the calculation, when $M_{\ast} = 1 M_\oplus$, the central temperature $T_c \approx 2000$ K and the central density $\rho_c \approx 3 \times 10^{-6}$ g cm$^{-3}$. At the end of the transition stage $T_c$ reaches a maximum of $4.85 \times 10^4$ K. Thereafter, it declines slowly as the internal luminosity is supplied mainly by cooling of the interior rather than by contraction. After 5 Gyr, $\rho_c = 2$ g cm$^{-3}$ and $T_c = 1.2 \times 10^4$ K. During the entire evolution, energy transport is by convection through more than 99% of the mass. A thin surface radiative zone exists for $T < 2300$ K.

The calculation was repeated (Case P1a) with a final assumed mass of 1.05 $M_\ast = 333 M_\oplus$. The final value of the core mass was 47.27 $M_\oplus$, only slightly larger than that for Case P1, and the envelope mass was 286 $M_\oplus$, a factor of 2.4 larger. The luminosity curve was essentially the same as that shown in Fig. 4b, but the width of the spike at 4.7 Myr was increased to $4.9 \times 10^4$ yr, a factor of 2.45 larger than in Case P1, because of the accretion of more matter at the same limiting rate as in that case. The peak luminosity also increased by a factor 2.7, because of the higher value of $M_p$. 

P. Bodenheimer, O. Hubickyj, & J. J. Lissauer: \textit{Preprint submitted to Elsevier}
Models of the in situ formation of detected extrasolar giant planets

in the calculation of the accretion luminosity (Eq. 4).

Another test was performed in which the uncertain grain opacity in the range $1500 < T < 1800$ K was reduced by about a factor 10 (Case P110). Otherwise the parameters were the same as in Case P1. The final value of $M_\ast$ was changed (downwards) by only about 3%, and the peak luminosity was slightly reduced (Table II). The grains occupy only a thin surface layer in this model, so evidently it is the deeper molecular layers, just above the convection zone, that are important in determining the luminosity and thereby the evolutionary time scale to reach $M_{\text{cross}}$.

Models of star formation theory suggest that the stellar dipole magnetic field truncates the disk out to about 0.1 AU (Königl 1991; Shu et al. 1994; Ward 1997a) and channels the flow of matter from disk to star. A further calculation (Case P2) was therefore made with a highly reduced disk density in the vicinity of the protoplanet. If the planet is interior to the 2:1 resonance with the inner edge of the disk, the tidal effect is reduced by several orders of magnitude, so planetary migration is less likely than in Case P1. Chunks of rock can, however, migrate through the disk and assemble into a protoplanet at 0.05 AU (Ward 1997a). We assume that a small amount of gas still remains in the cavity. In this case we take $\rho_{\text{nem}} = 5 \times 10^{-11}$ g cm$^{-3}$, a factor of 1000 smaller than that in the previous case. Otherwise, the calculation is identical to Case P1. The values of $M_\ast$ and $M_{XY}$ turn out to be 52.88 $M_\oplus$ and 113.8 $M_\oplus$, respectively, yielding a total mass of 0.52 $M_J$, the same as in Case P1. The formation time to final mass is $5.29 \times 10^6$ yr. Evolution of the radius, $T_{\text{eff}}$, $L$, and the interior conditions are very similar to that in the previous case. The close similarity of the core masses in Cases P1 and P2 indicate that the conclusions of Mizuno (1980) and Stevenson (1982) – namely that the value of $M_{\text{cross}}$ is insensitive to the outer boundary condition – still hold even in this case of a protoplanet with relatively small radius. The small difference that was found is a result of the fact that the present models are largely convective. Wuchterl (1993) has emphasized the fact that in this case the core and envelope masses do depend on the outer boundary condition.

A further test was made (Case P3) which was identical to Case P2 except that the limiting gas accretion rate was also reduced, to a value of $2 \times 10^{-5}$ $M_\oplus$ yr$^{-1}$. In this case the crossover mass was never reached, and the final values of $M_\ast$ and $M_{XY}$ were 89.3 and 77.4 $M_\oplus$, respectively, with the total mass the same as that for Case P1. The formation time to final mass was $8.9 \times 10^6$ yr. Because of the low limiting gas accretion rate, the maximum value of the luminosity was $10^{-4} L_\odot$, more than a factor 10 less than that in the previous case.

Case P4 is identical to Case P1 except that $\rho_{\text{nem}}$ is set to $5 \times 10^{-6}$ g cm$^{-3}$. The limiting gas accretion rate is also the same. Except for relatively small adjustments in the core and envelope mass, which turn out to be 36 and 131 $M_\oplus$, respectively, the results are quite similar to those of Case P1. This result is significantly different from that reported by Wuchterl (1997), who uses the same value of $\rho_{\text{nem}}$ but finds that rapid gas accretion starts with a core mass of 13.5 $M_\oplus$. However his $T_{\text{nem}} = 1252$ K is lower than ours, and his solid accretion rate is not constant but is calculated according to a variant of Eq. (1).

3.3. Formation of the companion to $\rho$ CrB

The solar-type star $\rho$ CrB has a low-mass companion with orbital period of 40 days, distance 0.23 AU, orbital eccentricity consistent with zero, and a minimum mass of 1.1 $M_\oplus = 350 M_\oplus$. To model the formation of the planet (Case C1), we take nebular conditions from the Bell et al. (1997) model with viscosity parameter $a = 0.01$ and $M_{nem} = 10^{-7} M_\oplus$ yr$^{-1}$. At 0.23 AU we have $T_{\text{nem}} = 1200$ K and $\rho_{\text{nem}} = 5 \times 10^{-8}$ g cm$^{-3}$. Again, in this region of a disk there is insufficient mass in the neighborhood of the orbit to supply a Jupiter mass, so we assume that solid material migrates into the region and collects on the protoplanet at a rate of $10^{-5}$ $M_\oplus$ yr$^{-1}$. The maximum rate of gas accretion is taken to be $10^{-2} M_\oplus$ yr$^{-1}$. The tidal truncation limit for these nebular conditions (Eq. 9) is 1.4 $M_J$, consistent with the probable mass of the planet.

The core mass and envelope mass are shown as functions of time in Fig. 6a. The formation time to final mass is $3.59 \times 10^6$ yr, $M_{XY} = 331 M_\oplus$, $M_\ast = 36 M_\oplus$, and the total mass is 1.15 $M_J$. The luminosity as a function of time is shown in Fig. 6b. The peak value is almost $10^{-2} L_\odot$, higher than that in the inner planet case because of the greater difference between the planetary radius and the accretion radius during the transition phase (Eq. 4) and the higher planetary mass. The width of the spike is $5 \times 10^4$ yr. Interior characteristics are similar to those in the 51 Peg b model P1, with a maximum internal temperature of 63,000 K. As a result of insolation in the isolated phase, the luminosity levels off to log $L/L_\odot = -5.8$ and the surface temperature to 580 K.
with observed disk lifetimes under the assumption that (Case C1). a) Mass of the solid component (solid line), the gaseous component (dotted line), and the total mass (dot-dashed line) as functions of time. b) Luminosity radiated by the planet as a function of time.

4. CONCLUSIONS

The main purpose of this paper has been to demonstrate that it is possible to model the formation of the companions to 51 Peg, ρ Cr B, and 47 UMa in situ, given reasonable disk models, but that there are problems associated with this assumption. Formation times of a few million years are obtained. For the companions of 51 Peg and ρ Cr B the mass accretion rate of solid material, \( \dot{M}_Z \), is assumed to be constant. For the companion of 47 UMa, cases with both constant and variable accretion rates are considered. In the cases of constant \( \dot{M}_Z \), the formation times are comparable with observed disk lifetimes under the assumption that \( \dot{M}_Z \approx 10^{-5} M_\oplus \text{ yr}^{-1} \). If that rate were assumed to be an order of magnitude smaller, the formation times would be > 10^7 yr (Bodenheimer and Pollack 1986), greater than typical disk lifetimes. In the case of variable \( \dot{M}_Z \), the crucial parameter is the disk surface density of solid material, \( \sigma_{\text{init},Z} \). The location of 47 UMa b at 2.1 AU corresponds to probable nebula temperatures above the ice evaporation point; thus the available concentration of condensible material is low. To obtain a reasonable formation time for the planet, \( \sigma_{\text{init},Z} \) must be in the range 50 – 100 g cm\(^{-2}\), corresponding in standard disk models to \( T \approx 1000 \) K, and \( \dot{M}_{\text{neb}} = 10^{-5} M_\oplus \text{ yr}^{-1} \). A change in the assumed value of \( \sigma_{\text{init},Z} \) from 50 to 90 g cm\(^{-2}\) results in almost a factor 10 reduction in formation time, from 1.86 \times 10^7 to 1.94 \times 10^6 yr. The instantaneous lifetime of such a nebula is much shorter than the deduced formation time of the planet; however, the parameters of the nebula are uncertain because the viscosity (assumed to correspond to \( \alpha = 10^{-2} \)) is not well constrained and, furthermore, the evolution time of the disk will soon slow down, with probable decoupling of the density of solids from that of the gas. The initial tidal truncation mass (Eq. 9) is 10 M\(_J\) in this hot disk, but as the nebula evolves this limit will decrease, and, furthermore, the dispersal of the nebula is another way to limit the planet’s final mass. The conclusion can be reached that there are still difficulties in explaining the in situ formation of 47 UMa b by the models U1 and U2. The main problem with the model is the length of phase 2, during which both solid and gas accretion rates are low, unless \( \sigma_{\text{init},Z} \) is much larger than typical solar nebula estimates would indicate. Such massive disks may be gravitationally unstable (Boss 1998), indicating possible prior formation of a massive planet at larger distances. Similar problems would have been severe for the cases of 51 Peg and ρ Cr B, had they been calculated with a variable accretion rate; however, the constant accretion rate that was used is consistent with the estimates of Ward (1997b). The evolution of the solid and gaseous components of the disk must be considered along with the planetary formation to make further progress on these questions.

Thus, orbital migration may well be important. For 51 Peg b, the case in which the gas density in the inner part of the disk was highly reduced compared with standard values is more likely to result in the survival of the planet than the case which was calculated in a standard disk. Short-period planets which form in regions of low gas density are likely to have an enhanced solid component for a given total mass; for example, the core mass for the model of 51 Peg b in Case P1 is 47 M\(_\oplus\), while in Case P3 it is 89 M\(_\oplus\) out of a total mass, in both cases, of 167 M\(_\oplus\). In the case of 47 UMa another possible scenario would involve the formation of the planet at a larger distance, say 5 AU, where the models of Pollack et al. (1996) show that it is possible to form Jupiter-mass objects in 5 – 10 \times 10^6 yr in a disk with surface density about 3 times that of the minimum-mass solar nebula. Then migration could take it in to 2.1 AU. In fact one must consider migration as a critical part of the formation process itself, since migration begins long before the planet is massive enough to open up a gap (Goldreich and Tremaine 1980; Ward 1997a,b). When the mass of a protoplanet at Jupiter’s orbit is 3 M\(_\oplus\), the migration time is already as short as 10^2 yr (Ward 1997a,b). As the mass approaches the nominal ‘isolation’ mass, the migration of the planet has carried it into regions of the disk where planetesimals are still plentiful (Ward 1997b), or where
there are other embryos with which it can merge. Thus, the lengthy semi-isolated phase 2 in the Case U1 scenario could be avoided and the formation process could be speeded up considerably. An examination of these effects must be included in future calculations. The problem still remains, as with all scenarios for giant planet formation, of how to stop the migration at the observed locations, especially for those planets farther than a tenth of an AU from the star that they orbit.

**A. GLOSSARY OF SYMBOLS**

- $a$ ≡ distance to the central star
- $c$ ≡ sound speed in the disk
- $F_g$ ≡ gravitational enhancement factor for accretion of planetesimals
- $g$ ≡ acceleration of gravity at $R_p$
- $L$ ≡ radiated luminosity at surface of protoplanet
- $L_{acc}$ ≡ luminosity due to gas accretion
- $L_{contr}$ ≡ luminosity due to contraction and accretion of solids
- $L_r$ ≡ energy per second crossing sphere of radius $r$
- $L_{\odot}$ ≡ solar luminosity = $3.85 \times 10^{33}$ ergs/s
- $M_{\oplus}$ ≡ Earth mass = $6 \times 10^{27}$ g
- $M_J$ ≡ Jupiter mass = $1.899 \times 10^{30}$ g
- $M_{\odot}$ ≡ solar mass = $1.989 \times 10^{33}$ g
- $M_{crz}$ ≡ planet’s $M Z$ at which $M_{XY} = M Z$
- $M_1$ ≡ total mass of protoplanet
- $M_{p,max}$ ≡ maximum mass that a planet can accrete before gap formation
- $M_*$ ≡ mass of central star
- $M_{XY}$ ≡ mass of gaseous envelope of protoplanet
- $M_{Z}$ ≡ mass of solid core of protoplanet
- $M_{neb}$ ≡ accretion rate of disk mass onto central star
- $M_{XY}$ ≡ accretion rate of gas onto envelope
- $M_{Z}$ ≡ accretion rate of planetesimals onto protoplanet
- $M_{XY,max}$ ≡ limiting accretion rate of gaseous material
- $P$ ≡ photospheric pressure
- $r$ ≡ distance from center of protoplanet
- $R_a$ ≡ modified accretion radius, Eq. (3)
- $R_c$ ≡ effective capture radius of the protoplanet
- $R_{core}$ ≡ radius at core/envelope interface
- $R_H$ ≡ protoplanet’s tidal radius
- $R_p$ ≡ actual outer radius of protoplanet
- $R_{\odot}$ ≡ solar radius = $6.96 \times 10^{10}$ cm
- $t$ ≡ time
- $T_c$ ≡ temperature of the gas within protoplanet at core/envelope interface
- $T_{eff}$ ≡ $(L/4\pi R_{\odot}^2 \sigma_{SB})^{1/4}$
- $T_{neb}$ ≡ assumed temperature at $R_a$
- $T_s$ ≡ temperature at $R_p$
- $v_{ff}$ ≡ free-fall velocity from $R_c$ to $R_p$
- $X$ ≡ mass fraction of hydrogen
- $Y$ ≡ mass fraction of helium
- $Z$ ≡ mass fraction of elements heavier than helium
- $\alpha$ ≡ viscosity parameter in disk model
- $\kappa$ ≡ Rosseland mean opacity in units of cm$^2$/g
- $\nu$ ≡ disk viscosity
- $\rho_c$ ≡ density of the gas within the protoplanet at core/envelope interface
- $\rho_{core}$ ≡ assumed density of core of protoplanet
- $\rho_{neb}$ ≡ assumed density at $R_a$
- $\rho_s$ ≡ density at $R_p$
- $\rho_0$ ≡ density at the inner edge of the infalling flow
- $\sigma_{init,XY}$ ≡ initial surface density of gaseous material in the disk
- $\sigma_{init,Z}$ ≡ initial surface density of condensed material in the disk
- $\sigma_{SB}$ ≡ Stefan-Boltzmann constant
- $\sigma_Z$ ≡ surface density of condensed material in the disk
- $\Omega$ ≡ orbital frequency of protoplanet

**ACKNOWLEDGEMENTS**

This work was supported in part through NASA Grant NAG5-4494 from the Origins of Solar Systems Program. We dedicate this paper to the memory of Jim Pollack, in acknowledgment of his vision, wisdom, and hard work in connection with the problem of the origin of giant planets.

**REFERENCES**

Adams, F. C., and W. Benz 1992. Gravitational instabilities in circumstellar disks and the formation of binary companions. In *Complementary Approaches to Double and Multiple Star Research* (H. McAlister and W. Hartkopf, Eds.), pp. 185-194. Astronomical Society of the Pacific, San Francisco.

Alexander, D. R., and J. W. Ferguson 1994. Low-temperature Rosseland opacities. *Astrophys. J.* 437, 879–891.

Allard, F., P. H. Hauschildt, I. Baraffe, and G. Chabrier 1996. Synthetic spectra and mass determination of the brown dwarf Gliese 229B. *Astrophys. J.* 465, L123–L127.

Artymowicz, P. 1992. Dynamics of binary and planetary-system interaction with disks: eccentricity changes. *Publ. Astron. Soc. Pacific.* 104, 769–774.

Artymowicz, P. 1998. On the formation of eccentric superplanets. In *Brown Dwarfs and Extrasolar Planets, ASP Conference Series Vol. 134* (R. Rebolo, E. L. Martin, and M. R. Zapatero Osorio, Eds.), pp. 152–161. Astronomical Society of the Pacific, San Francisco.

Artymowicz, P., and S. H. Lubow 1996. Mass flow through gaps in circumbinary disks. *Astrophys. J.* 467, L77–L80.

Beckwith, S. V. W., and A. I. Sargent 1993. The occurrence and properties of disks around young stars. In *Protostars and Planets III* (E. Levy and J. Lunine, Eds.), pp. 521-541. Univ. of Arizona Press, Tucson.

Bell, K. R., P. M. Cassen, H. H. Klahr, and Th. Henning 1997. The structure and appearance of protostellar accretion disks: limits on disk flaring. *Astrophys. J.* 486, 372–387.
Black, D. C. 1997. Possible observational criteria for distinguishing brown dwarfs from planets. *Astrophys. J.* **490**, L171–L174.

Bodenheimer, P. 1998. Formation of substellar objects orbiting stars. In *Brown Dwarfs and Extrasolar Planets*, ASP Conference Series Vol. 134 (R. Rebolo, E. L. Martin, and M. R. Zapatero Osorio, Eds.), pp. 115–127. Astronomical Society of the Pacific, San Francisco.

Bodenheimer, P., and J. B. Pollack 1986. Calculations of the accretion and evolution of giant planets: The effects of solid cores. *Icarus* **67**, 391–408.

Boss, A. P. 1996. Evolution of the solar nebula. III. Protoplanetary disks undergoing mass accretion. *Astrophys. J.* **469**, 906–920.

Boss, A. P. 1997. Giant planet formation by gravitational instability. *Science* **276**, 1836–1839.

Boss, A. P. 1998. Evolution of the solar nebula. IV. Giant gaseous protoplanet formation. *Astrophys. J.* **503**, 923–937.

Burrows, A., M. Marley, W. B. Hubbard, J. I. Lunine, T. Guillot, D. Saumon, R. Freedman, D. Sudarsky, and C. Sharp 1997. A nongray theory of extrasolar giant planets and brown dwarfs. *Astrophys. J.* **491**, 856–875.

Butler, R. P., and G. W. Marcy 1996. A planet orbiting 47 Ursae Majoris. *Astrophys. J.* **464**, L153–L156.

Butler, R. P., G. W. Marcy, E. Williams, H. Hauser, and P. Shirts 1997. Three new “51 Pegasi-type” planets. *Astrophys. J.* **474**, L115-L118.

Cameron, A. G. W. 1978. Physics of the primitive solar accretion disk. *Moon and Planets* **18**, 5–40.

Cochran, W. D., A. P. Hatzes, R. P. Butler, and G. W. Marcy 1997. The discovery of a planetary companion to 16 Cygni B. *Astrophys. J.* **483**, 457–463.

DeCampli, W. M., and A. G. W. Cameron 1979. Structure and evolution of isolated giant gaseous protoplanets. *Icarus* **38**, 367–391.

Goldreich, P., and S. Tremaine 1980. Disk–satellite interactions. *Astrophys. J.* **241**, 425–441.

Greenzweig, Y., and J. J. Lissauer 1992. Accretion rates of protoplanets. II. Gaussian distributions of planetesimal velocities. *Icarus* **100**, 440–463.

Guillot, T., D. Gautier, and W. B. Hubbard 1997. New constraints on the composition of Jupiter from Galileo measurements and interior models. *Icarus* **130**, 534–539.

Holman, M. J. Touma, and S. Tremaine 1997. Chaotic variations in the eccentricity of the planet orbiting 16 Cygni B. *Nature* **386**, 254–256.

Kary, D. M., and J. J. Lissauer 1994. Numerical simulations of planetary growth. In *Numerical Simulations in Astrophysics* (J. Franco, S. Lizano, L. Aguilar, and E. Daltabuit, Eds.), pp. 364–373. Cambridge Univ. Press, Cambridge.

Königl, A. 1991. Disk accretion onto magnetic T Tauri stars. *Astrophys. J.* **370**, L39–L43.

Kuiper, G. P. 1951. On the origin of the solar system. In *Astrophysics* (J. A. Hynek, Ed.), pp. 357–424. McGraw
Models of the in situ formation of detected extrasolar giant planets

Mazeh, T., Y. Krymolowski, and G. Rosenfeld 1997. The high eccentricity of the planet orbiting 16 Cygni B. Astrophys. J. 477, L103–L106.

Mizuno, H. 1980. Formation of the giant planets. Prog. Theor. Phys. 64, 544–557.

Murray, N., B. Hansen, M. Holman, and S. Tremaine 1998. Migrating planets. Science 279, 69–72.

Nakajima, T., B. R. Oppenheimer, S. R. Kulkarni, D. A. Golimowski, K. Matthews, and S. T. Durrance 1995. Discovery of a cool brown dwarf. Nature 378, 463–465.

Perri, F., and A. G. W. Cameron 1974. Hydrodynamic instability of the solar nebula in the presence of a planetary core. Icarus 22, 416–425.

Podolak, M., J. B. Pollack, and R. T. Reynolds 1988. Interactions of planetesimals with protoplanetary atmospheres. Icarus 73, 163–179.

Pollack, J. B., O. Hubickyj, P. Bodenheimer, J. J. Lissauer, M. Podolak, and Y. Greenzweig 1996. Formation of the giant planets by concurrent accretion of solids and gas. Icarus 124, 62–85.

Pollack, J. B., C. McKay, and B. Christofferson 1985. A calculation of the Rosseland mean opacity of dust grains in primordial Solar System nebulae. Icarus 64, 471–492.

Rasio, F. A., and E. B. Ford 1996. Dynamical instabilities and the formation of extrasolar planetary systems. Science 274, 954–956.

Rees, M. J. 1976. Opacity-limited hierarchical fragmentation and the masses of protostars. Mon. Not. R. Astron. Soc. 176, 483–486.

Ruzmaikina, T. V. 1998. Formation of 51 Peg type systems. Lunar and Planetary Science Abstracts 29, 1873.

Safronov, V. S. 1969. Evolution of the Protoplanetary Cloud and Formation of the Earth and Planets. Nauka Press, Moscow (in Russian). English translation: NASA–TT–F–677, 1972.

Saumon, D., G. Chabrier, and H. M. Van Horn 1995. An equation of state for low-mass stars and giant planets. Astrophys. J. Suppl. 99, 713–741.

Saumon, D., W. B. Hubbard, A. Burrows, T. Guillot, J. I. Lunine, and G. Chabrier 1996. A theory of extrasolar giant planets. Astrophys. J. 460, 993–1018.

Shu, F. H., J. Najita, E. Ostriker, F. Wilkin, S. Ruden, and S. Lizano 1994. Magnetocentrifugally driven flows from young stars and disks. I. A generalized model. Astrophys. J. 429, 781–796.

Stevenson, D. J. 1982. Formation of the giant planets. Planet. Space Sci. 30, 755–764.

Stringfellow, G. S., D. C. Black, and P. Bodenheimer 1990. Brown dwarfs as close companions to white dwarfs. Astrophys. J. 349, L59–L62.

Strom, S. E., S. Edwards, and M. F. Skrutskie 1993. In Protostars and Planets III (E. H. Levy and J. I. Lunine, Eds.), pp. 837–866. Univ. of Arizona Press, Tucson.

Tajima, N., and Y. Nakagawa 1997. Evolution and dynamical stability of the proto-giant-planet envelope. Icarus 126, 282–292.

Takeuchi, T., S. Miyama, and D. N. C. Lin 1996. Gap formation in protoplanetary disks. Astrophys. J. 460, 832–847.

Trilling, D., W. Benz, T. Guillot, J. I. Lunine, W. B. Hubbard, and A. Burrows 1998. Orbital evolution and migration of giant planets: modeling extrasolar planets. Astrophys. J. 500, 428–439.

Ward, W. R. 1997a. Survival of planetary systems. Astrophys. J. 482, L211–L214.

Ward, W. R. 1997b. Protoplanet migration by nebular tides. Icarus 126, 261–281.

Weidenschilling, S. J., and F. Marzari 1996. Gravitational scattering as a possible origin for giant planets at small stellar distances. Nature 384, 619–621.

Wuchterl, G. 1991. Hydrodynamics of giant planet formation. III. Jupiter’s nucleated instability. Icarus 91, 53–64.

Wuchterl, G. 1993. The critical mass for protoplanets revisited: Massive envelopes through convection. Icarus 106, 323–334.

Wuchterl, G. 1995. Giant planet formation. Earth, Moon, and Planets 67, 51–65.

Wuchterl, G. 1997. Giant planet formation and the masses of extrasolar planets. In Science with the VLT Interferometer (F. Paresce, Ed.), pp. 64–71. Springer, Berlin.

Wuchterl, G., T. Guillot, and J. J. Lissauer 1999. In Protostars and Planets IV (V. Mannings, A. P. Boss, and S. Russell, Eds.), in press. Univ. of Arizona Press, Tucson.