Dynamical modelling of disc vertical structure in superthin galaxy ‘UGC 7321’ in braneworld gravity: An MCMC study

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Abstract

Low surface brightness (LSBs) superthins constitute classic examples of very late-type galaxies, with their disc dynamics strongly regulated by their dark matter halos. In this work we consider a gravitational origin of dark matter in the brane world scenario, where the higher dimensional Weyl stress term projected onto the 3-brane acts as the source of dark matter. In the context of the braneworld model, this dark matter is referred to as the ‘dark mass’. This model has been successful in reproducing the rotation curves of several low surface brightness and high surface brightness galaxies. Therefore it is interesting to study the prospect of this model in explaining the vertical structure of galaxies which has not been explored in the literature so far. Using our 2-component model of gravitationally-coupled stars and gas in the external force field of this dark mass, we fit the observed scale heights of stellar and atomic hydrogen (HI) gas of superthin galaxy ‘UGC7321’ using the Markov Chain Monte Carlo approach. We find that the observed scaleheights of ‘UGC7321’ can be successfully modelled in the context of the braneworld scenario. In addition, the model predicted rotation curve also matches the observed one. The implications on the model parameters are discussed.

1 Introduction

Historically, the concept of dark matter was invoked to address the missing mass problem in spiral galaxies (Rubin et al., 1979) as well as to explain the mass discrepancy in galaxy clusters (Zwicky, 1933, 1937). The observed rotation curves of galaxies, as determined by optical tracers or by HI 21cm radio-synthesis studies, are flat or tend to be asymptotically flat. Interestingly, however, the observed distribution of visible matter predicts a Keplerian fall-off beyond the visible galactic disc (Binney & Tremaine, 2008). Basic physics suggests, the flatness of the rotation curve requires the total galactic mass to be increasing linearly with galacto-centric radius even beyond the baryonic disc of the galaxy. This hinted at the presence of non-luminous matter in the discs of galaxies, the “dark matter”, which was invoked to explain the mass discrepancy and hence the flat rotation curves of spiral galaxies. In fact, the “dark matter” hypothesis

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has been successful in explaining the observed rotation curves of a wide range of spiral galaxies including massive high surface brightness galaxies (HSBs), intermediate-mass low surface brightness galaxies (LSBs) to dwarf irregulars (Gentile et al., 2004; Kranz et al., 2003; McGaugh et al., 2001; Oh et al., 2015; Sofue & Rubin, 2001; de Blok, 2005; de Blok et al., 2008). Similarly, the concept of “dark matter” came as a rescue to address the mass discrepancy problem in galaxy clusters. The mass of a galaxy cluster estimated by summing up the masses of its individual member galaxies is much lower than the virial mass of the galaxy-cluster determined using the observed line of sight velocity dispersion values of the member galaxies (Carlberg et al., 1997). This again hinted at the presence of non-luminous matter at cluster scales, which could be explained by invoking the concept of “dark matter”.

Although the ‘dark matter hypothesis’ has been successful in resolving a host of astrophysical and cosmological problems, the fundamental particles constituting the dark matter have evaded detection in dark matter search experiments (See, for example, Cryogenic Dark Matter Search (CDMS), Agnese et al. (2013)). The lack of detectional evidence for dark matter particles opens up the possibility for a gravitational origin of dark matter wherein Newtonian gravity is modified to explain the ‘missing mass’ problem. One of the earliest attempts to modify the Newton’s laws for explaining the galactic rotation curves was Modified Newtonian dynamics (MOND) (Milgrom, 1983), which is well-tested in the context of the Milky Way (Famaey & Binney, 2005) and also on a large sample of spiral galaxies (Sanders & Noordermeer, 2007; Sanders & Verheijen, 1998; de Blok & McGaugh, 1998). Besides, extra dimensional models or brane-worlds can also lead to alternative gravitational origin of “dark matter” (Binetruy et al., 2000; Csaki et al., 1999; Hagnani et al., 2012; Koyama, 2003; Maartens, 2000, 2004a; Mazumdar, 2001) where the Standard Model particles and fields are confined in a 3-brane while gravity enters into the bulk (Antoniadis, 1990; Antoniadis et al., 1998; Arkani-Hamed et al., 1998; Csaki et al., 2000; Garriga & Tanaka, 2000; Randall & Sundrum, 1999a,b). Higher dimensional models or brane-worlds were mainly introduced to provide a scheme to unify all the known forces of nature thereby giving birth to string theory and eventually M-theory (Horava & Witten, 1996; Kaluza, 2018; Klein, 1926; Polchinski, 1998). The huge difference between the Planck scale and the electroweak scale led to the gauge hierarchy problem in particle physics which could also be addressed by introducing higher dimensions (Antoniadis, 1990; Antoniadis et al., 1998; Arkani-Hamed et al., 1998; Csaki et al., 2000; Randall & Sundrum, 1999a). Further, extra-dimensional models have interesting phenomenological (Arkani-Hamed et al., 1999; Chakrabory & SenGupta, 2014; Davoudiasl et al., 2000a,b, 2001; Hundi & SenGupta, 2013) and cosmological implications (Arkani-Hamed et al., 2000; Banerjee & Paul, 2017; Banerjee et al., 2019; Chakrabory & Sengupta, 2014; Dienes et al., 1999; Lukas et al., 2000; Mazumdar & Wang, 2000). Moreover, since the nature of gravity in the high energy regime is unknown, it is often believed that the deviations from Einstein gravity may manifest itself through the existence of extra dimensions.

The brane world model of Maartens (2004b) considers a single 3-brane embedded in a five dimensional bulk. The modifications in the Einstein’s equations arise mainly due to the non-local effects of the bulk Weyl tensor. A brane observer perceives this Weyl stress term like a fluid, known as the Weyl fluid, with its own energy density and pressure. It was shown by Mak & Harko (2004), Harko & Cheng (2006), Boehmer & Harko (2007), Rahaman et al. (2008) and Gergely et al. (2011) that such a model successfully complies with observed rotation curves of galaxies. The aim of this work is to explore the prospect of the brane world model in explaining the observed scale height of stars and neutral hydrogen gas (HI) in galaxies. Towards this end, we consider the prototypical low surface brightness superthin galaxy, UGC7321, which was found to be dark matter dominated at all radii (Banerjee et al., 2010). In fact, Banerjee & Jog (2013) have shown that the superthin vertical structure of the stellar disc crucially depends on its dense and compact dark matter halo. Therefore, in this work, we intend to constrain its dark matter density profile in the brane world scenario using the observed stellar and HI scale heights of UGC 7321. We derive the
density profile of the Weyl fluid which arises in the brane world scenario to resemble the cored dark matter halo profile consistent with mass models of the low surface brightness galaxies (de Blok, 2005). We use this derived density profile of the Weyl fluid to mimic the dark matter density profile in the 2-component model of gravitationally-coupled stars and gas in the external force field of the dark matter halo (Narayan & Jog, 2002). The observed stellar and HI scale heights are used to constrain the vertical stellar velocity dispersion and vertical HI velocity dispersion of the galactic disc in addition to the Weyl parameters. Finally, we check the consistency of the Weyl model with the observed rotation curve of UGC 7321.

The paper is organised as follows: in §2, we describe the brane-world model and present the density profile derived from such a model mimicking the characteristics of the dark matter. In §3, we present the 2-component model of the baryonic disc in the force field of the dark matter halo such that the Weyl fluid plays the role of the dark matter. We present our results in the context of the the LSB super thin galaxy UGC 7321 in §4 and conclude with a summary of our findings in §5.

2 Braneworld gravity: A possible proxy to dark matter

We consider a single 3-brane embedded in a 5-D spacetime (the bulk). We assume that the Standard Model particles and fields are confined in the brane while gravity permeates the extra dimensions. The coordinates of the bulk are denoted by capitalized latin indices $x^A$, $A = 0, 1, ... 4$ while the brane coordinates are denoted by Greek indices $x^\mu$, $\mu = 0, 1, 2, 3$.

Such a system is described by the action,

$$S_{\text{bulk}} = \int d^5x \sqrt{-G} \left[ \frac{R}{2\kappa_5^2} + \Lambda_5 + \delta(\phi)\mathcal{L}_m \right]$$

(1)

where $G_{AB}$ is the bulk metric, $R$ is the bulk Ricci scalar, $\kappa_5^2 = 8\pi G_5$ is the five dimensional gravitational constant, $\Lambda_5$ is the bulk cosmological constant and $\mathcal{L}_m$ is the matter Lagrangian in the brane representing standard model fields.

From the action (given by Eq. (1)) the gravitational field equations in the bulk are given by,

$$R_{IJ} - \frac{1}{2} G_{IJ} R = \kappa_5^2 T_{IJ}$$

(2)

where $R_{IJ}$ is the bulk Ricci tensor and $T_{IJ}$ the energy-momentum in the bulk. The bulk energy-momentum tensor can be written as,

$$T_{AB} = -\Lambda_5 G_{AB} + \delta(\phi)(-\lambda_T g_{\mu\nu} + \tau_{\mu\nu}) e_A^\mu e_B^\nu$$

(3)

where $\phi$ refers to the extra coordinate, the brane being located at $\phi = 0$ and $e_A^\mu$ projects the quantities of the bulk onto the brane. The induced metric on the $\phi = 0$ hypersurface is denoted by $g_{\mu\nu}$. The negative vacuum energy density on the bulk $\Lambda_5$, the brane tension $\lambda_T$ and the brane energy-momentum tensor $\tau_{\mu\nu}$ are the sources of the gravitational field on the bulk.

Gauss-Codazzi equation which relates the Riemann tensor of the bulk to that of the brane is used to arrive at the effective four dimensional gravitational field equations. The extrinsic curvature $K_{\mu\nu}$, which encodes the embedding of the brane into the bulk is associated with the covariant derivative of the normalized normals to the brane $n^A$. If the brane has an energy momentum tensor, the extrinsic curvature $K_{\mu\nu}$ exhibits a discontinuity across the brane. This discontinuity in the extrinsic curvature is related to the brane energy momentum tensor by means of Israel junction conditions and a $Z_2$ orbifold symmetry.
With the above considerations the effective four-dimensional gravitational field equations on the brane assume the form,

\[ \mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G_4 \tau_{\mu\nu} + \kappa_4^4 \pi_{\mu\nu} - E_{\mu\nu} \]  

(4)

where

\[ \Lambda_4 = \frac{1}{2} \kappa_4^4 \left[ \Lambda_5 + \frac{1}{6} \kappa_5^2 \lambda_T^2 \right] \]  

(5)

\[ G_4 = \frac{\kappa_4^2 \lambda_T}{48\pi} \]  

(6)

\[ \pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\nu} \tau^\alpha + \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} \tau^2 \]  

(7)

\[ E_{\mu\nu} = C_{ABCD} e^A_\mu n^B e^C_\nu n^D \]  

(8)

In Eq. (4), \( \Lambda_4 \) and \( G_4 \) represent the 4-dimensional cosmological constant and gravitational constant respectively while \( \mathcal{R}_{\mu\nu} \) and \( \mathcal{R} \) refer to the Ricci tensor and Ricci scalar on the brane. The local effects of the bulk on the brane is encoded in the term \( \pi_{\mu\nu} \) while \( E_{\mu\nu} \), the electric part of the bulk Weyl tensor \( C_{ABCD} \) captures the non-local effect from the free bulk gravitational field. In Eq. (5) the brane tension can be adjusted with the bulk cosmological constant to yield de-Sitter, anti de-Sitter or flat branes such that it serves as the fine balancing relation of the Randall-Sundrum single brane model (Randall & Sundrum, 1999b; Shiromizu et al., 2000).

The conservation of the matter energy-momentum tensor on the brane enables us to constrain \( E_{\mu\nu} \) and \( \pi_{\mu\nu} \) as \( D_\nu E_{\mu}^{\nu} - \kappa_4^4 D_\nu \pi_{\mu}^{\nu} = 0 \), (where \( D_\nu \) represents the brane covariant derivative).

The symmetry properties of \( E_{\mu\nu} \) allow an irreducible decomposition of the tensor in terms of a given 4-velocity field \( u^\nu \) (Harko & Mak, 2004; Maartens, 2001),

\[ E_{\mu\nu} = -k^4 \left[ U(r)(u_\mu u_\nu + \frac{1}{3} \zeta_{\mu\nu}) + 2Q_{(\mu} u_{\nu)} + P_{\mu\nu} \right] \]  

(9)

where \( \zeta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \) is the projector orthogonal to \( u^\mu \), \( k = \frac{\kappa_4^4}{\kappa_5^2} \) and \( \kappa_5^2 = 8\pi G_4 \). It is important to note that \( \kappa_5^2 = \kappa_4^2 \lambda_T / 6 \) and we retrieve general relativity in the limit \( \lambda_T \rightarrow 0 \) (Harko & Mak (2004)). The scalar \( U(r) = -\frac{1}{12} E_{\mu\nu} u^\mu u^\nu \) in Eq. (9) is often known as the “Dark Radiation” term, while \( Q_{\mu} = \frac{1}{12} \zeta_{\mu}^{\alpha\beta} E_{\alpha\beta} u^\beta \) represents a spatial vector and \( P_{\mu\nu} = -\frac{1}{12} \left[ \zeta^{\alpha\beta} E_{\alpha\beta} u^\beta \right] \) consists of a spatial, tracefree, symmetric tensor.

Since we are interested in deriving vacuum solutions on the brane, we consider \( \tau_{\mu\nu} = \pi_{\mu\nu} = 0 \) such that the gravitational field equations on the brane reduce to,

\[ \mathcal{R}_{\mu\nu} - \frac{1}{2} \mathcal{R} g_{\mu\nu} = -\Lambda_4 g_{\mu\nu} - E_{\mu\nu} \]  

(10)

The conservation of energy-momentum tensor on the brane assumes the form \( D_\nu E_{\mu}^{\nu} = 0 \). Further, if the solutions are static \( Q_{\mu} = 0 \) in Eq. (9) such that \( D_\nu E_{\mu}^{\nu} = 0 \) leads to,

\[ \frac{1}{3} \bar{D}_\mu U + \frac{4}{3} U A_\mu + \bar{D}^\nu P_{\nu\mu} + A^\nu P_{\nu\mu} = 0 \]  

(11)

where \( \bar{D}_\mu = u^\nu D_\nu u_\mu \) is the 4-acceleration and \( \bar{D} \) denotes covariant derivative on the space-like hypersurface orthonormal to \( u_\mu \). Further, assumption of spherical symmetry, enables us to write \( A_\mu = A(r)r_\mu \), while
the term $P_{\mu\nu}$ takes the form,

$$P_{\mu\nu} = P(r) \left( r_{\mu} r_{\nu} - \frac{1}{3} \eta_{\mu\nu} \right) \quad (12)$$

where $A(r)$ and $P(r)$ (also known as the “Dark Pressure”) are scalar functions of the radial coordinate $r$ and $r_{\mu}$ is the unit radial vector.

### 2.1 Motion of test particles in the braneworld model

In this work we are interested to explore the properties of low surface brightness galaxies (LSBs) which are dark matter dominated with presumably a spherically symmetric mass distribution. Such a mass distribution is expected to give rise to a static and spherically symmetric spacetime given by the metric ansatz,

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (13)$$

We solve for $\nu(r)$, $\lambda(r)$, $U(r)$ and $P(r)$ from the fact that Eq. (13) satisfies Eq. (10) and Eq. (11).

By studying the motion of test particles in the above spacetime it can be shown that the tangential velocity or the circular velocity of motion is given by

$$v_c^2 = \frac{r\nu'}{2} \quad (14)$$

(Gergely et al., 2011)

With Eq. (13) the gravitational field equations and the energy momentum tensor conservation in the brane gives us,

$$-e^{-\lambda} \left( \frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2} = 3\alpha U \quad (15)$$

$$e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{2} \right) - \frac{1}{r^2} = \alpha (U + 2P) \quad (16)$$

$$\frac{e^{-\lambda}}{2} \left( \nu'' + \frac{\nu'}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2} \right) = \alpha (U - P) \quad (17)$$

$$\nu' = -\frac{U' + 2P'}{2U + P} - \frac{6P}{(2U + P)r} \quad (18)$$

where prime denotes derivative with respect to $r$ and $\alpha = \frac{1}{4\pi G_4 \Lambda T}$. One can show that the solution of these equations lead to the following form for $e^{-\lambda}$,

$$e^{-\lambda} = 1 - \frac{\Lambda_4}{3} r^2 - \frac{Q(r)}{r} - \frac{C}{r} \quad (19)$$

where $C$ is an arbitrary integration constant and $Q(r)$ is defined as,

$$Q(r) = \frac{3}{4\pi G_4 \lambda T} \int r^2 U(r) dr \quad (20)$$
From the form of $e^{-\lambda}$ it can be inferred that $Q(r)$ is the gravitational mass originating from the dark radiation and can be interpreted as the “dark mass” term.

Further, one can show that for a static, spherically symmetric spacetime the ordinary differential equations for dark radiation $U(r)$ and dark pressure $P(r)$ satisfy

$$
\frac{dU}{dr} = -2 \frac{dP}{dr} - 6 \frac{P}{r} - \frac{(2U + P)[2G_4M + Q + \{\alpha(U + 2P + \frac{2}{3}\chi)\}r^3]}{r^2(1 - 2G_4M - \frac{Q(r)}{r} - \frac{2}{3}r^2)} \tag{21}
$$

and

$$
\frac{dQ}{dr} = 3\alpha r^2U. \tag{22}
$$

where $\alpha = \frac{1}{4\pi G_4 \chi T}$ and $\chi = -\Lambda_4$ (Gergely et al., 2011). In the subsequent calculations we will neglect the effect of the cosmological constant $\Lambda_4$ (Gergely et al., 2011) on the vertical scale height of the galaxies, i.e. we will take $\chi = -\Lambda_4 = 0$.

Eq. (21) and Eq. (22) can be recast into a more convenient form namely,

$$
\frac{d\mu}{d\theta} = -(2\mu + p) \frac{\bar{q} + \frac{1}{3}(\mu + 2p)}{1 - \bar{q}} - 2 \frac{dp}{d\theta} \tag{23}
$$

$$
\frac{d\bar{q}}{d\theta} = \mu - \bar{q} \tag{24}
$$

by defining the variables,

$$
\bar{q} = \frac{2G_4M + Q}{r}; \quad \mu = 3\alpha r^2U; \quad p = 3\alpha r^2P; \quad \theta = \ln r; \tag{25}
$$

Eq. (23) and Eq. (24) can be referred to as the differential equations governing the source terms on the brane, while the circular velocity of the test particle $v_c$ assumes the form,

$$
v_c^2 = \frac{1}{2} \frac{\bar{q} + \frac{1}{3}(\mu + 2p)}{1 - \bar{q}} \tag{26}
$$

### 2.2 Choice of the equation of state of the Weyl fluid

The brane observer perceives the extra dimensions through the term $E_{\mu\nu}$ which in turn can be written in terms of the dark radiation $U(r)$ and the dark pressure $P(r)$. These are like the energy density and the pressure of the stress energy tensor of the Weyl fluid whose origin is attributed to extra dimensions. An equation of state connecting the dark radiation $U$ and dark pressure $P$ is therefore necessary since the source equations Eq. (23) and Eq. (24) cannot be solved simultaneously until we impose some further conditions on them. Hence, we choose some specific relations between dark radiation $U$ and dark pressure $P$, necessarily defining the various equations of state in the framework of the brane world model.

In order to close the system of field equations we adopt the 3+1+1 covariant approach developed by Keresztes & Gergely (2010a,b). Assuming (i) cosmological vacuum in the bulk, (ii) vanishing of the brane cosmological constant, (iii) symmetrical brane embedding, (iv) absence of matter on the brane and
(v) the brane is static and spherically symmetric, the Weyl fluid can be characterized by five variables: (a) the dark radiation \( U(r) \), (b) the dark pressure \( P(r) \), (c) the electric part of the 4-d Weyl tensor \( E(r) = C_{ABCD} n^A u^B n^C u^D \), (d) the acceleration \( A^a = A(r) r^a \) and (e) the expansion \( \Theta \) of the radial geodesics \((3)D^a r_a = \tilde{\Theta}\). From the 4-d gravitational field equations and the conservation of the brane energy momentum tensor one can show that the aforesaid variables obey the following two algebraic equations:

\[
k_4^4 U(r) = \tilde{\Theta} \left( A - \frac{\tilde{\Theta}}{4} \right) + \frac{4\tilde{E}}{3} + \frac{1}{r^2}
\]

\[
k_4^4 P(r) = \tilde{\Theta} \left( A + \frac{\tilde{\Theta}}{2} \right) - \frac{2\tilde{E}}{3} - \frac{2}{r^2}
\]

For a detailed discussion on the relations connecting these variables, one is referred to Keresztes & Gergely (2010a,b). We note that in the absence of the Weyl fluid the metric is described by the Schwarzschild spacetime in which case \( A(r) \), \( \tilde{\Theta} \) and \( \tilde{E}(r) \) are connected by the equation,

\[
\frac{2\tilde{E}}{3} + A \tilde{\Theta} = \tilde{\Theta} \left( \frac{\tilde{\Theta}}{4} + A \right) - \frac{1}{r^2} = 0
\]

In the presence of the Weyl fluid, the above relation modifies to,

\[
\frac{2\tilde{E}}{3} + A \tilde{\Theta} = M_0 \tilde{\Theta} \left( \frac{\tilde{\Theta}}{4} + A \right) - \frac{N_0}{r^2}
\]

where \( M_0 \) and \( N_0 \) are two constants characterizing the Weyl fluid and reducing to 1 if the spherically symmetric brane is described by the Schwarzschild solution. From Eq. (27) and Eq. (28) one can show that the Weyl fluid obeys the following equation of state,

\[
P(r) = (a - 2)U(r) - \frac{B}{k_4^4 r^2}
\]

where, \( M_0 = a/(2a - 3) \) and \( N_0 = (a - B)/(2a - 3) \) such that in the Schwarzschild limit \( a = 3 \) and \( B = 0 \).

In terms of the reduced variables defined in Eq. (25), the equation of state can be rewritten as,

\[
p(\mu) = (a - 2)\mu - B
\]

Eq. (32) is akin to the boundary condition imposed on the brane at some arbitrary time \( t \) which is preserved during the extra dimensional evolution (Keresztes & Gergely, 2010a,b) due to the static character of the problem. The equation of state can also be thought of as equivalent to imposing ‘initial condition’ while studying evolution of an off-brane parameter along the extra dimensions such that the constraint Eq. (11), representing conservation of brane energy momentum tensor, is obeyed.

### 2.3 Density profile for the Weyl fluid

Using the equation of state discussed in the last section and ignoring the effect of the cosmological constant, the equation for the dark pressure Eq. (23) can be simplified to,

\[
(2a - 3) \frac{d\mu}{d\tilde{\theta}} = \frac{(a\mu - B)(\tilde{\theta} + (2a - 3)\mu/3 - 2B/3)}{1 - \tilde{\theta}} + 2\mu(3 - a) + 2B
\]
while the reduced dark radiation assumes the form,

\[
\mu(\theta) = \theta^{2(3-a)/(2a-3)} \exp \left[ -\frac{2a}{2a-3} \int v_\theta^2(\theta) d\theta \right] \times \\
\left\{ C - \frac{3B}{2a-3} \int [1 + v_\theta^2(\theta)] \theta^{-2(3-a)/(2a-3)} \times \\
\exp \left[ \frac{2a}{2a-3} \int v_\theta^2(\theta) d\theta \right] \right\}
\]

(34)

where \( C \) is an arbitrary integration constant (Gergely et al., 2011).

In what follows, we will consider the situation where \( a \neq 3/2 \) and \( \tilde{q} << 1 \). The justification for \( \tilde{q} << 1 \) arises from the fact that a typical galactic dark matter halo has mass \( M \sim 10^{12} M_\odot \) and radius \( R \sim 100 \text{ kpc} \), such that \( \tilde{q} \approx \frac{GM_u}{R} \sim 10^{-7} << 1 \). Since observations reveal that in a galaxy mass is directly proportional to the radius, this ratio remains roughly constant for all galactic radii. The smallness of \( \tilde{q} \) further enables us to neglect higher order terms in \( \tilde{q} \), such that Eq. (24) assumes the form,

\[
\frac{d^2 \tilde{q}}{d\theta^2} + m \frac{d\tilde{q}}{d\theta} - n \tilde{q} = b
\]

(35)

\[
m = 1 - \frac{B}{3} - \frac{2}{3} \frac{a(B - 3) + 9}{2a - 3} \quad a \neq 3/2
\]

(36)

\[
n = \frac{2}{3} \frac{a(2B - 3) + 9}{2a - 3} \quad a \neq 3/2
\]

(37)

\[
b = \frac{2}{3} \frac{B(B - 3)}{3 - 2a} \quad a \neq 3/2
\]

(38)

The general solution of Eq. (35) is,

\[
\tilde{q}(r) = q_0 + C_1 r^{l_1} + C_2 r^{l_2}
\]

(39)

where \( C_1 \) and \( C_2 \) are constants of integration and \( q_0 \) is given by,

\[
q_0 = -\frac{b}{n} = \frac{B(B - 3)}{a(2B - 3) + 9}
\]

(40)

while

\[
l_{1,2} = \frac{-m \pm \sqrt{m^2 + 4n}}{2}
\]

(41)

The solution for reduced dark radiation is given by,

\[
\mu(r) = q_0 + C_1 (1 + l_1) r^{l_1} + C_2 (1 + l_2) r^{l_2}
\]

(42)
In the original radial coordinate $r$ the solution for dark radiation $U(r)$ is,

$$\rho_h(r) = 3\alpha U(r) = \frac{q_0}{r^2} + C_1(1 + l_1)r^{l_1-2} + C_2(1 + l_2)r^{l_2-2}$$

which serves as the proxy for the density profile of dark matter.

The dark mass profile is given by,

$$Q(r) = r(q_0 + C_1 r^{l_1} + C_2 r^{l_2}) - 2GM$$

where $M$ is the baryonic mass.

For completeness we also mention that the tangential velocity of a test particle in the ‘dark matter’ dominated region is given by,

$$v_c^2 \approx v_{c\infty}^2 + \gamma r^{l_1} + \eta r^{l_2}$$

where,

$$v_{c\infty}^2 = \frac{1}{3}(aq_0 - B)$$

$$\gamma = \frac{C_1}{2} \left[ 1 + \frac{(2a - 3)}{3(1 + l_1)} \right]$$

$$\eta = \frac{C_2}{2} \left[ 1 + \frac{(2a - 3)}{3(1 + l_2)} \right]$$

(Gergely et al., 2011). In order to obtain a flat rotation curve at large distances $l_1$ and $l_2$ should be negative. The constrain on the Weyl parameters from the rotation curve has been derived in Gergely et al. (2011). The goal of this work is to constrain the Weyl parameters from the vertical scale height data of the Low Surface Brightness galaxies (LSBs).

One can show that when $\tilde{q} << 1$ and $a \neq 3/2$ the parameters $m$, $n$, $q_0$ and $v_{c\infty}^2$ can be further simplified such that,

$$m \approx \frac{4a - 9}{2a - 3},$$

$$n \approx -2 \frac{a - 3}{2a - 3},$$

$$q_0 \approx \frac{B}{a - 3}$$

and

$$v_{c\infty}^2 \approx \frac{a}{3} \left( q_0 - \frac{B}{a} \right)$$
Eq. (49) and Eq. (50) implies that

$$l_1 \approx -1 \quad \text{and} \quad l_2 \approx -1 + \frac{3}{2a - 3}$$

(53)

which further ensures that $a$ cannot assume values between $3/2$ to $3$. The positivity of $v_{c,\infty}^2$ requires that when $a < 3/2$, $B \leq 0$ while when $a > 3$, $B > 0$. The density profile for the Weyl fluid assumes the form,

$$\rho_h(r) \approx \frac{c_0}{r^2} + \frac{3C_2}{2a - 3} r^{-3(1 - \frac{1}{2a - 3})}$$

(54)

while the rotation curve is given by,

$$v_c^2 \approx \frac{B}{a - 3} + \frac{C_1}{2} r^{-1} + C_2 r^{-1 + \frac{1}{2a - 3}}$$

(55)

By taking $C_1 = \frac{2G(M_b + M_U)}{c^2}$, $C_2 = Cc^2 R_{c(DM)}^{1-\alpha_{DM}} = -\beta_{DM}$ and by defining $\alpha_{DM} = 3/(2a - 3)$ and $\beta_{DM} = B/(a - 3)$, the final expressions for the rotation curve and the density profile are given by,

$$\left(\frac{v_c(r)}{c}\right)^2 \approx \frac{G(M_b + M_U)}{c^2 r} + \beta_{DM} \left[1 - \left(\frac{R_{c(DM)}}{r}\right)^{1-\alpha_{DM}}\right]$$

and

$$\rho_h(r) \approx \frac{c^2 \beta_{DM}}{Gr^2} \left[1 - \alpha_{DM} \left(\frac{R_{c(DM)}}{r}\right)^{1-\alpha_{DM}}\right]$$

(56)

(57)

Due to the constraints on the parameters $a$ and $B$, it can be shown that either $\alpha_{DM} < 0$ or $0 < \alpha_{DM} < 1$ and $0 < \beta_{DM} << 1$ (Gergely et al., 2011).

3 Can Weyl fluid act as a proxy for dark matter for LSB galaxies?

The low surface brightness galaxies (LSBs) are dark matter dominated with negligible baryonic mass. In this work however, we do not assume dark matter, but attribute its origin to higher dimensional gravity. Such LSB galaxies have a constant mass density core with core radius of a few kpc (de Blok, 2005). Unlike High Surface Brightness (HSB) galaxies, for LSBs we ignore baryonic contribution even within the core radius. Assuming the core radius to be $R_{c(DM)}$ and the mass of the core to be $M_{DM}$, the density profile describing the dark matter in low surface brightness galaxies is given by

$$\rho_{DM}(r) = \frac{3M_{DM}}{4\pi R_{c(DM)}^3} (1 - H_{kDM}(r)) + H_{kDM}(r) \left\{\frac{c^2 \beta_{DM}}{Gr^2} \left(1 - \alpha_{DM} \left(\frac{R_{c(DM)}}{r}\right)^{1-\alpha_{DM}}\right)\right\}$$

(58)

where, $H(k)$ is a smoothing function which ensures smooth transition from the region of constant density core to the constant rotation curve regime where the mass is proportional to the radius. The smoothing function is given by,

$$H_{kDM}(r) = \frac{1}{1 + \exp(-2k_{DM}(r - R_{c(DM)})$$

(59)

such that it smoothly approaches the Heaviside step function as $k_{DM}$ tends to infinity, i.e.,

$$H(R_{c(DM)}) = \lim_{k_{DM} \to \infty} H_{kDM}(R_{c(DM)}) = \begin{cases} 0 & r < R_{c(DM)} \\ 1 & r \geq R_{c(DM)} \end{cases}$$
### 3.1 2-component model of the baryonic disc

We model the galactic baryonic disc as composed of two concentric, co-planar, axi-symmetric discs of stars and atomic hydrogen (HI) gas, which are gravitationally coupled to each other and also in the external force field of a rigid halo of the dark mass. Molecular hydrogen gas $\text{H}_2$ has been neglected in this dynamical model as it is hardly traced in LSBs. In fact, the $\text{H}_2$ to HI mass ratio in late-type LSB spirals is reported to be $\sim 10^{-3}$ (Matthews et al., 2005) and hence can be considered to be a dynamically insignificant component in UGC7321 without significant errors.

The joint Poisson equation in terms of the galactic cylindrical coordinates $(R, \phi, z)$ is given by

$$\frac{\partial^2 \Phi_{\text{total}}}{\partial z^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Phi_{\text{total}}}{\partial R} \right) = 4\pi G \left( \sum_{i=1}^{2} \rho_i + \rho_{\text{DM}} \right) \tag{60}$$

Here, $\Phi_{\text{total}}$ is the total gravitational potential due to the baryonic disc (stars + gas) and the halo of the dark mass. $\rho_i$ represents the density of the $i^{th}$ component ($i = \text{s(tars), HI}$) and $\rho_{\text{DM}}$ is the effective density of the dark mass as given by Eq. (58). We note that $\rho_{\text{DM}}$ is characterized by five free parameters. Here, the angular term drops off due to the assumed azimuthal symmetry and, similarly, the radial term, for a flat rotation curve. Thus the joint Poisson’s equation reduces to

$$\frac{\partial^2 \Phi_{\text{total}}}{\partial z^2} = 4\pi G \left( \sum_{i=1}^{2} \rho_i + \rho_{\text{DM}} \right) \tag{61}$$

We further assume that the stars and HI gas are in vertical hydrostatic equilibrium and also their vertical velocity dispersions remain constant in the $z$-direction. At a given galacto-centric radius $R$, the equation of vertical hydrostatic equilibrium for the $i^{th}$ component of the disc (i=s(tars), gas) is given by

$$\frac{\sigma_{z,i}^2}{\rho_i} \frac{\partial \rho_i}{\partial z} + \frac{\partial \Phi_{\text{total}}}{\partial z} = 0 \tag{62}$$

where $\sigma_{z,i}$ the vertical velocity dispersion of the $i^{th}$ component (Rohlfs, 1977).

The radial profile of the vertical velocity dispersion of the stars is modelled as

$$\sigma_{z,i}(R) = \sigma_{0s} \exp\left(-\frac{R}{\alpha_s R_D}\right) \tag{63}$$

where $\sigma_{0s}$ and $\alpha_s$ are free parameters. Here $\sigma_{0s}$ denotes the vertical velocity dispersion of the stars at the centre of the galactic disc; $\alpha_s$ represents the scale length (in units of the exponential disc scale length $R_D$) with which the vertical velocity dispersion falls off with $R$. This is closely following van der Kruit (1989) who found that a vertical velocity dispersion of an isothermal, self-gravitating stellar disc should fall off exponentially with radius with a scale length of 2 $R_D$ to match the observed constant stellar scale height in edge-on galaxies.

Similarly, the vertical velocity dispersion of HI has in general been found to either remain constant, or to linearly vary with radius. See, for example, Narayan & Jog (2002). Therefore, we parametrize the vertical velocity dispersion of HI as a polynomial as follows:

$$\sigma_{z,\text{HI}}(R) = \sigma_{0\text{HI}} + \alpha_{\text{HI}} R + \beta_{\text{HI}} R^2 \tag{64}$$

where $\sigma_{0\text{HI}}$, $\alpha_{\text{HI}}$ and $\beta_{\text{HI}}$ are free parameters. Here $\sigma_{0\text{HI}}$ is the HI vertical velocity dispersion at the galactic centre.
Therefore, at a given galacto-centric radius $R$, combining the joint Poisson’s equation and the equation for vertical hydrostatic equilibrium for the $i$th component we get

$$\frac{\partial^2 \rho_i}{\partial z^2} = -4\pi G \frac{\rho_i}{\sigma_{z,i}^2} (\rho_i + \rho_{DM}) + \left( \frac{\partial \rho_i}{\partial z} \right)^2 \frac{1}{\rho_i}; \quad (65)$$

For a given set of free parameters ($M_{DM}$, $R_{c(DM)}$, $\alpha_{DM}$, $\beta_{DM}$, $k_{DM}$, $\sigma_{0s}$, $\alpha_s$, $\sigma_{0HI}$, $\alpha_{HI}$, $\beta_{HI}$), the above set of two coupled, non-linear, ordinary differential equations in the variables $\rho_s$ and $\rho_{HI}$ is solved iteratively using Runge-Kutta method with initial conditions at mid plane $z = 0$. See, for example, Narayan & Jog (2002) for details. The half-width-at-half-maxima of the density distribution $\rho_i$ at a given $R$ thus obtained is taken to be the scaleheight $h_z$ at that $R$ for that $i$th component. The set of parameters which gives the best match to the observed stellar and HI scaleheight versus $R$ data defines our best-fitting model.

We use the Markov Chain Monte Carlo (MCMC) method for determining the best-fitting set of parameters of our model. We use the task modMCMC from the publicly available R package FME (Soetaert et al., 2010), which implements MCMC using adaptive Metropolis procedure (Haario et al., 2006).

### 3.2 Input Parameters

UGC 7321 is a prototypical superthin galaxy with a radial-to-vertical axis ratio of 10.3. It is observed at an inclination of 88° and is at a distance of 10 Mpc (Matthews & van Driel, 2000; Matthews et al., 1999). The galaxy has a steeply rising rotation curve with an asymptotic velocity of about 110 km s$^{-1}$ (Uson & Matthews, 2003).

| Parameters | UGC7321$^B$ |
|------------|------------|
| $\mu_0$(magarcsec$^{-2}$) $^a$ | 23.5 |
| $\Sigma_0(M_\odot pc^{-2})$ $^b$ | 34.7 |
| $R_D$(kpc) $^c$ | 2.1 |
| $h_z$(kpc) $^d$ | 0.105 |

$^a$Central surface brightness of the stellar disk  
$^b$Central surface density of the stellar disk  
$^c$Exponential disc scalelength of the stellar disk  
$^d$Scaleheight (HWHM) of the stellar disk

Table 1: Stellar parameters of UGC 7321 in B-band.

The de-projected central surface brightness in B-band is $\mu_0 \sim 23.5$ magarcsec$^{-2}$ (Matthews et al., 1999). It also has high values of the dynamical mass-to-light ratio with $M_{dyn}/M_{HI} = 31$ and $M_{dyn}/M_{L_B} = 29$, where $M_{L_B}$ is the B-band luminosity, and $M_{HI}$ the total HI mass is the galaxy, $M_{dyn}$ being the dynamical mass of the galaxy. This indicates that the galaxy is dark matter rich as was also corroborated by Banerjee & Jog (2013), who found dark matter dominates the disc dynamics at all radii in this galaxy. Earlier studies have shown that in the optical i.e., B-band, the surface density profile is well-fitted with an exponential. The same trend holds good for UGC7321 and hence $\Sigma_s(R)$ is given by

$$\Sigma_s(R) = \Sigma_0 \exp\left(-R/R_D\right)$$
where $\Sigma_0$ is the central stellar surface density and $R_D$ the exponential stellar disc scalelength. The structural parameters corresponding to the exponential stellar disc of UGC 7321 in B-band were either directly taken or derived from Uson & Matthews (2003).

| Parameters                      | UGC7321 |
|--------------------------------|---------|
| $\Sigma_{01} (M_\odot pc^{-2})$ | 4.912   |
| $\Sigma_{02} (M_\odot pc^{-2})$ | 2.50    |
| $a_1$ (kpc)                    | 3.85    |
| $a_2$ (kpc)                    | 0.485   |
| $r_{01}$ (kpc)                 | 2.85    |
| $r_{02}$ (kpc)                 | 1.51    |

*Central surface density of the first HI gaussian disk
*Central surface density of the second HI gaussian disk
Centre of the first HI gaussian disk
Centre of the second HI gaussian disk
Scalelength of the first HI gaussian disk
Scalelength of the second HI gaussian disk

Table 2: Input parameters for HI

The structural parameters corresponding to the exponential stellar disc of UGC 7321 in B-band were taken or derived from Uson & Matthews (2003).

The HI surface density and the HI scaleheight data for UGC 7321 were obtained from Uson & Matthews (2003) and O’Brien et al. (2010) respectively. Earlier work indicated that the radial profiles of HI surface density could be well-fitted with double-gaussians profiles. See, for example Begum et al. (2005), Patra et al. (2014), possibly signifying the presence of two HI discs. Also, galaxies with the HI surface density peaking away from the centre are common, which indicates the presence of an HI hole at the centre. The HI surface density profiles could be fitted well with off-centred double Gaussians given by

$$
\Sigma_{HI}(R) = \Sigma_{01} \exp\left[-\frac{(r-a_1)^2}{2r_{01}^2}\right] + \Sigma_{02} \exp\left[-\frac{(r-a_2)^2}{2r_{02}^2}\right]
$$

where $\Sigma_{01}$ is the central surface density, $a_1$ the centre and $r_{01}$ the scalelength of gas disc 1 and so on.

The parameters corresponding to stellar disc and the HI disc are summarized in Tables 1 and 2 respectively.

4 Results

Our dynamical model of UGC 7321 in B-band consists of ten free parameters. Out of these, five free parameters correspond to the baryonic disc: $\sigma_{0s}$ and $\alpha_s$ corresponding to the stellar vertical velocity dispersion profile and $\sigma_{HI}$, $\alpha_{HI}$ and $\beta_{HI}$ corresponding to the HI vertical velocity dispersion profile. The remaining five free parameters are related to the dark mass profile (Eq. (58)) derived from the brane world model: $M_{DM}$, $R_{c(DM)}$, $\alpha_{DM}$, $\beta_{DM}$ uniquely describing the Weyl fluid while $k_{DM}$ is associated with the
Figure 1: In the Left Panel, we present the vertical velocity dispersion of stars and HI as a function of galacto-centric radius $R$ using 'stars' and 'dots' respectively, as determined from the 2-component model of the baryonic disc embedded in a halo of dark mass as constrained by the observed stellar and HI scaleheight data. In the Middle Panel, we plot the observed stellar and HI scaleheight with black 'dash-dot' and black 'solid line' respectively. Overlaid on them is the modelled stellar and HI scaleheight represented by grey 'stars' and 'dots' respectively. In the Right Panel, we present the rotation curve of UGC 7321 using the best-fit parameters describing the dark matter density profile in the Brane-world model and superpose it on the observed rotation curve.

In Figure 1 [Left Panel], we present the vertical velocity dispersion of stars and HI as a function of galacto-centric radius from the 2-component model. We find that the central stellar velocity dispersion is (13.4±0.6) km/s, which falls off exponentially with a scalelength $(2.1±0.4)R_D$, $R_D$ being the exponential stellar disc scale length. The central HI vertical velocity dispersion is given by $(15.4±0.5)$ km/s with $\alpha_{HI} = (-1.3 ± 0.2)$ km/s/kpc$^{-1}$ and $\beta_{HI} = 0.04 ± 0.02$ km/s/kpc$^{-2}$, indicating that it falls off almost linearly with radius. We note that the velocity dispersion of the stars is lower than the HI velocity dispersion. It is, in general, not possible for the stars to have lower dispersion than the gas clouds in which they are formed, as stars are collision-less and hence cannot dissipate energy via collisions. This possibly indicates that the thin disc stars were born in an underlying cold component of the gas with lower values of velocity dispersion (Patra et al., 2014).
Figure 2: We present the correlation plots and the posterior distributions of the free parameters characterizing the dynamical model of UGC7321 consisting of the 2-component baryonic disc embedded in a halo of dark mass as constrained by the observed stellar and HI scaleheight data using the MCMC method.

The values of dark matter mass obtained from the model $M_{DM} = (1.3 \pm 0.2) \times 10^9 M_\odot$, core radius $R_{c(DM)} = (1.3 \pm 0.2)\, \text{kpc}$, the parameters describing the Weyl fluid $\alpha_{DM} = -0.4 \pm 0.1$, and $\beta_{DM} = (1.4 \pm 0.2) \times 10^{-7}$, the term associated with smoothing function is given by $k_{DM} = 2.7 \pm 0.3$. The results are summarized in Table 3. We note the the estimated dark mass $M_{DM}$ is consistent with the typical mass of the dark matter in low surface brightness superthin galaxies. The value of $M_{DM}$ is of the order of $10^9$, which is in agreement with the order of magnitude of dark masses of LSBs reported in Gergely et al. (2011). The core radius $R_{c(DM)}$ is 1.3 kpc ($0.6 \, R_D$), possibly indicating that the Weyl fluid has a dense and compact structure. This complies with a previous study by Banerjee et al. (2010) showing UGC7321 has dense and compact pseudo-isothermal dark matter halo. The values of $\alpha_{DM}$ and $\beta_{DM}$, which refer to the deviations of the spherically symmetric metric from the Schwarzschild scenario in general relativity, satisfy the condition $\alpha_{DM} < 0$ and $0 < \beta_{DM} < 1$ and are also comparable with the values of the parameter set obtained by Gergely et al. (2011) for a sample of nine LSBs using the observed rotation curve. However, we note that the value of $k_{DM}$ is of the order of 15-150 kpc$^{-1}$, for the sample LSBs studied by Gergely et al. (2011), but for UGC 7321 we find that the value of $k_{DM}$ equals 2.66kpc$^{-1}$. This could be possibly attributed to the fact that UGC7321, like some other superthins, has a relatively steeply rising rotation
curve compared to LSBs in general (Banerjee & Bapat, 2016).

In Figure 1 [Middle Panel] we have overlaid the scale heights predicted by the 2-component model on the observed scale heights. This confirms that our best-fitting model matches well with the observations. In Figure 1 [Right Panel] we compare the theoretical rotation curve corresponding to the best-fitting 2-component model with the observed rotation curve of UGC 7321. We note that the theoretical rotation curve obtained by using the best-fitting parameters from the scaleheight constraint on two-component model mostly agrees with the observed rotation curve within error-bars. In Figure (2), we present the correlation plots and the posterior distributions obtained from the MCMC fitting of the 2-component model. Except for a few cases, we do not note strong correlations between the free parameters of the model.

| Parameters | UGC7321 |
|------------|---------|
| $\sigma_{0s}$ (kms$^{-1}$) | 13.4 ± 0.6 |
| $\alpha_s$ (kpc$^{-1}$) | 2.0 ± 0.4 |
| $\sigma_{HI}$ (kms$^{-1}$) | 15.4 ± 0.5 |
| $\alpha_{HI}$ (kms$^{-1}$kpc$^{-1}$) | −1.3 ± 0.2 |
| $\beta_{HI}$ (kms$^{-1}$kpc$^{-2}$) | 0.04 ± 0.02 |
| $M_{DM}$ ($M_\odot$) | $(1.3 \pm 0.2) \times 10^9$ |
| $R_{c(DM)}$ (kpc) | 1.3 ± 0.2 |
| $\alpha_{DM}$ | −0.4 ± 0.1 |
| $\beta_{DM}$ | $(1.4 \pm 0.2) \times 10^{-7}$ |
| $k_{DM}$ (kpc$^{-1}$) | 2.7 ± 0.3 |

*aCentral stellar vertical velocity dispersion
*bExponential radial scale length (in units of $R_D$) of the stellar vertical velocity dispersion
*cCentral HI vertical velocity dispersion
*dRadial gradient of HI vertical velocity dispersion
*eGradient of the radial gradient of HI vertical velocity dispersion
*fDark mass
*gCore-radius of the dark mass density profile
*hWeyl-fluid parameter
*iWeyl-fluid parameter
*jSmoothing term associated with dark mass density profile

Table 3: Best-fit parameters found after optimizing the model

5 Summary and Conclusions

In this work, we explore the possibility of higher dimensional gravity in explaining the vertical scaleheight structure of the dark matter dominated LSB galaxy UGC 7321, where the role of dark matter is played by five dimensional Einstein gravity. The five dimensional Einstein’s equations are projected onto the 3-brane where our visible universe resides such that the effective 4-dimensional gravitational field equations inherits a source term originating from the bulk. The source term owes its origin to the electric part of the bulk.
Weyl tensor and captures the non-local effects of the bulk onto the brane. For a brane observer therefore, the bulk effectively behaves like a fluid (the so called Weyl fluid) possessing an energy density and pressure, viz, the dark radiation and the dark pressure terms respectively. Due to the presence of the Weyl term, the static, spherically symmetric and asymptotically flat solution of the 4-d effective gravitational field equations deviates from the Schwarzschild spacetime. The degree of deviation from the Schwarzschild scenario is determined by the parameters connecting the dark radiation and the dark pressure terms in the equation of state of the Weyl fluid. The equation of state essentially assigns some initial conditions to the evolution equations of the off-brane quantities (e.g. the normalized normals) along the extra dimensions. In the context of galaxies, it turns out that the appropriate equation of state is \( p = (a - 2)\mu - B \), where \( a = 3 \) and \( B = 0 \) corresponds to the Schwarzschild scenario. With this equation of state and by employing \( \tilde{\mu} = GM/R \sim 10^{-7} \ll 1 \) (which holds true in the galactic situation), one can obtain the density profile and the rotation curve of the LSB galaxies in terms of the equation of state parameters. These are fitted with the available rotation curve and scale height data of UGC 7321 to constrain the Weyl parameters as well as the stellar and HI vertical velocity dispersion profiles.

The Weyl model has been successfully employed in explaining the rotation curves of LSB as well as HSB galaxies (Chakraborty & SenGupta, 2016; Gergely et al., 2011) which has been much explored in the past. This motivates us to confront this model with the stellar and HI vertical scale height data of dark matter dominated LSB galaxies, e.g. UGC 7321 which is a very well studied object with a dynamical mass as large as \( M_{\text{dyn}}/M_{\text{HI}} = 31 \) and \( M_{\text{dyn}}/M_{\text{LB}} = 29 \). Our analysis reveals that the Weyl model can not only address the observations related to the rotation curve but can also successfully explain the vertical scaleheight data of UGC 7321 within the error bars. Therefore apart from the rotation curve, this work opens up a new observational avenue which can be utilized in understanding the role of extra dimensions in the galactic scale.

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References

Agnese R., et al., 2013, Physical review letters, 111, 251301
Antoniadis I., 1990, Phys.Lett., B246, 377
Antoniadis I., Arkani-Hamed N., Dimopoulos S., Dvali G., 1998, Phys.Lett., B436, 257
Arkani-Hamed N., Dimopoulos S., Dvali G., 1998, Phys.Lett., B429, 263
Arkani-Hamed N., Dimopoulos S., Dvali G., 1999, Phys. Rev. D, 59, 086004
Arkani-Hamed N., Dimopoulos S., Kaloper N., March-Russell J., 2000, Nucl. Phys. B, 567, 189
Banerjee A., Bapat D., 2016, Monthly Notices of the Royal Astronomical Society, 466, 3753
Banerjee A., Jog C. J., 2013, Monthly Notices of the Royal Astronomical Society, 431, 582
Banerjee N., Paul T., 2017, Eur. Phys. J. C, 77, 672
Banerjee A., Matthews L. D., Jog C. J., 2010, New Astronomy, 15, 89
Banerjee I., Chakraborty S., SenGupta S., 2019, Phys. Rev. D, 99, 023515
Begum A., Chengalur J. N., Karachentsev I., 2005, Astronomy & Astrophysics, 433, L1
Binetruy P., Deffayet C., Langlois D., 2000, Nucl.Phys., B565, 269
Binney J., Tremaine S., 2008, Galactic dynamics. Princeton university press
Boehmer C., Harko T., 2007, Classical and Quantum Gravity, 24, 3191
Carlberg R., et al., 1997, The Astrophysical Journal Letters, 485, L13
Chakraborty S., SenGupta S., 2014, Phys. Rev. D, 90, 047901
Chakraborty S., SenGupta S., 2016, Eur. Phys. J., C76, 648
Chakraborty S., Sengupta S., 2014, Eur. Phys. J. C, 74, 3045
Csaki C., Graesser M., Kolda C. F., Terning J., 1999, Phys.Lett., B462, 34
Csaki C., Graesser M., Randall L., Terning J., 2000, Phys.Rev., D62, 045015
Davoudiasl H., Hewett J., Rizzo T., 2000a, Phys. Rev. Lett., 84, 2080
Davoudiasl H., Hewett J., Rizzo T., 2000b, Phys.Lett., B473, 43
Davoudiasl H., Hewett J., Rizzo T., 2001, Phys.Rev., D63, 075004
Dienes K. R., Dudas E., Gherghetta T., Riotto A., 1999, Nucl. Phys. B, 543, 387
Famaey B., Binney J., 2005, Monthly Notices of the Royal Astronomical Society, 363, 603
Garriga J., Tanaka T., 2000, Phys.Rev.Lett., 84, 2778
Gentile G., Salucci P., Klein U., Vergani D., Kalberla P., 2004, Monthly Notices of the Royal Astronomical Society, 351, 903
Oh S.-H., et al., 2015, The Astronomical Journal, 149, 180
Patra N. N., Banerjee A., Chengalur J. N., Begum A., 2014, Monthly Notices of the Royal Astronomical Society, 445, 1424
Polchinski J., 1998
Rahaman F., Kalam M., DeBenedictis A., Usmani A., Ray S., 2008, Monthly Notices of the Royal Astronomical Society, 389, 27
Randall L., Sundrum R., 1999a, Phys.Rev.Lett., 83, 3370
Randall L., Sundrum R., 1999b, Phys.Rev.Lett., 83, 4690
Rohlf K., 1977
Rubin V., Ford Jr W., Roberts M., 1979, The Astrophysical Journal, 230, 35
Sanders R., Noordermeer E., 2007, Monthly Notices of the Royal Astronomical Society, 379, 702
Sanders R. H., Verheijen M., 1998, The Astrophysical Journal, 503, 97
Shiromizu T., Maeda K.-i., Sasaki M., 2000, Phys.Rev., D62, 024012
Soetaert K., Petzoldt T., et al., 2010, Journal of Statistical Software, 33, 1
Sofue Y., Rubin V., 2001, Annual Review of Astronomy and Astrophysics, 39, 137
Uson J. M., Matthews L., 2003, The Astronomical Journal, 125, 2455
Zwicky F., 1933, Helvetica physica acta, 6, 110
Zwicky F., 1937, The Astrophysical Journal, 86, 217
de Blok W., 2005, The Astrophysical Journal, 634, 227
de Blok W., McGaugh S., 1998, The Astrophysical Journal, 508, 132
de Blok W., Walter F., Brinks E., Trachternach C., Oh S., Kennicutt Jr R., 2008, The Astronomical Journal, 136, 2648
van der Kruit P., 1989, in , The World of Galaxies. Springer, pp 256–275