LATTICE GAUGE THEORY : STATUS REPORT 1994*

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Abstract

We report on recent developments and results from Lattice Gauge Theory. These include determinations of the strong coupling constant $\alpha_s$ and the b-quark mass $M_b$ and an update on the Kaon B parameter $B_K$. Lattice results for heavy meson decay constants and semileptonic B and D meson decay form factors are reviewed. The first attempts to study the Isgur-Wise function and the radiative Penguin decay process, $B \to K^* \gamma$, on the lattice are discussed.

1. Introduction

Lattice Gauge Theory is emerging as a powerful and effective approach to Particle Physics. It allows for theoretical investigations of the Standard Model, that go beyond perturbation theory, and is a practical tool for carrying out much needed calculations. A major part of the effort to date has been on nonperturbative studies of QCD and on calculations of hadronic matrix elements. These latter quantities are necessary in order to compare theory with experiment, and to extract Standard Model parameters.

It has taken many many years and the dedicated work of a large number of physicists to reach this point, but in recent years some of the lattice results are starting to be of direct relevance to experimentalists and phenomenologists. There is growing optimism that the impact of lattice gauge theory on phenomenology will continue to increase. Part of this development is due to the availability of more and more powerful computers and sophisticated algorithms. Just as important, however is the fact that lattice gauge theorists are becoming “smarter” in the types of questions they are asking and how they go about systematically improving their calculations. They are taking a harder look at systematic errors and at how to correct for them. There is also now a better understanding of how to match from a lattice calculation to continuum physics.

In this talk I will be focusing on results of lattice calculations rather than on the theory behind those calculations. Hence I will not be discussing some of the nice theoretical developments taking place that is helping to control lattice calculations and that will allow for significant improvements in the future. Let me just mention that some of this work goes under the name of “improved actions” [1-7] and there has been a rebirth of sorts in this area. Another theoretical development of the last couple of years that has had a huge impact on the field, is the revamping of lattice perturbation theory by Lepage and Mackenzie [8]. These authors have demonstrated that their new improved perturbation theory is applicable for couplings at which numerical simulations are currently been done. Hence it is possible to exploit fruitful interplays between perturbative calculations and nonperturbative simulation results to extract good physics. One is also in a better position to justify using perturbation theory to match between lattice and continuum operators, as one needs to do in hadronic matrix element calculations. Perturbation theory will also play a crucial role in the “improvement program”, and help eliminate lattice artifacts. All this has contributed to strengthening confidence in lattice numbers.

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Let me conclude this Introduction by emphasizing three points,

I. There now exist results from lattice QCD, in which the bulk of errors are under good control and an attempt has been made to correct for them. In this category one would include recent calculations in,

- Light Hadron Spectroscopy
- Heavy Quark Spectroscopy ($c\bar{c}$, $b\bar{b}$)
- Fundamental Parameters of QCD: $\alpha_s$, $M_Q$
- $B_K$

The total error is less than 10% in many instances. In the case of the important fundamental parameters, $\alpha_s$ and the b-quark mass $M_b$, the accuracy is considerably better.

II. Lattice calculations are starting to provide phenomenologically important hadronic matrix elements.

- Meson Decay Constants: $f_D$, $f_{D_s}$, $f_B$, $f_{B_s}$, $f_{B_u\sqrt{B_B}}$
- Semileptonic Decays: $D \to K(K^*) l\nu$
- Isgur-Wise Function
- Rare $B$ Decays: $B \to K^* \gamma$

Current lattice results for these matrix elements have uncertainties that are generally larger than 10% and one is still exploring better ways to approach the problem and reduce statistical and systematic errors. However, prospects are good that in due time accurate results for quantities such as $f_{B_u\sqrt{B_B}}$ will become available.

III. The Lattice continues to be a versatile tool to investigate Standard Model and Quantum Field Theory issues. It provides a general purpose, mathematically well defined regularization scheme for quantum field theories. A partial list of topics under study using lattice techniques includes,

- QCD at finite $T$
- Structure of Hadrons
- Test of the Parton Model
- Electroweak phase transition
- Confinement Mechanism, Monopoles
- Strong CP
- Higgs and Fermion Mass Upper Bounds
- Chiral Gauge Theories
- Quantum Gravity

Due to time constraints this talk will focus only on the first two category of topics. Interesting parallel session talks were presented on several of the items in the third category[8]. Apologies to those speakers for not covering their work. Readers are urged to read their contributions to these proceedings.

2. Light Hadron Spectroscopy

![Light hadron spectrum results from the IBM GF11 group][9]. Masses are measured in terms of the $\rho$ meson mass, $M_\rho$. Experimental numbers are denoted by crosses.

Testing whether QCD, our theory of the strong interactions, contains a low lying spectrum that is consistent with observed light hadrons remains a basic, fundamental problem in nonperturbative QCD. Fig.1. shows results by the IBM GF11 group for a first principles QCD calculation of light hadron masses[9]. Plotted are hadron masses measured in units of the $\rho$ meson mass $M_\rho$. Three parameters of the theory, the inverse lattice spacing, $a^{-1}$, and two quark masses, $M_u = M_d$ and $M_s$, need to be fixed by experiment. The GF11 group uses the $\rho$ meson mass, $M_\rho$, to fix $a^{-1}$. $M_u$ and $M_s$ are determined by the pion and kaon masses respectively. Having done so, there are no adjustable parameters left and the remaining hadron mass values become predictions of the theory. The GF11 group has attempted to correct for three out of the four main sources of systematic errors that contaminate many lattice calculations. The three are finite volume errors, finite lattice spacing errors and quark mass errors (the fact that present technology forces people to work with quark masses larger than up- and down-quark masses). The fourth common error comes from the neglect of light quark vacuum polarization contributions (the “quenched approximation”). The results of Fig.1. have not been corrected for quenching errors. Nevertheless
one sees that, agreement between lattice results and the real world is good. These calculations can and will be improved upon in the future. Eventually calculations going beyond the quenched approximation will become feasible. This is on the one hand a question of algorithmic breakthroughs, or failing that the availability of ever more powerful computers. On the other hand progress in dealing with improved actions, should allow lattice gauge theorists to work on smaller lattices and with larger lattice spacings than many people currently anticipate. So the prospect for having high statistics unquenched results on realistic systems may not be too far off in the future.

3. Quarkonium Physics

Systems involving one or more heavy quarks and/or antiquarks have been the focus of intense investigations on the lattice in recent years. In this section I discuss heavy-heavy (quarkonium) systems and will return to heavy-light systems in later sections. At first sight one may worry about working with quarks with masses larger than the inverse lattice spacing, $M_Q \geq a^{-1}$. Several ways to deal with heavy quarks have been developed in recent years and demonstrate that this does not pose a problem. For instance, one has:

- The Static Approach\[10\]
  Here one works in the $M_Q \rightarrow \infty$ limit. This approach is being applied extensively to heavy-light systems but is not appropriate for heavy-heavy systems.

- The Non-Relativistic Approach (NRQCD)\[3, 16\]
  One works with heavy fermions with a nonrelativistic dispersion relation
  $$E(\vec{p}) = M_1 + \frac{\vec{p}^2}{2M_2} + ....$$
  The rest mass $M_1$ is eliminated as a dynamical variable and one is working with an expansion in $v^2$. For $\Upsilon$ $v^2$ is typically of order $\sim 0.1$ and for charmonium one has $v^2 \sim 0.3$.

- The Heavy Wilson Approach\[14, 15\]
  In this approach a generalized set of operators is used for the fermionic action, the coefficients of which are kept to all orders in the quark mass. This action encompasses as special cases the Wilson and the non-relativistic action. The only expansion parameters left are $pa$ and the gauge coupling.

There are several reasons why studies of quarkonia on the lattice are particularly rewarding.

1 There exists a wealth of experimental data on these systems and more continues to be accumulated. Quarkonia provide good opportunities to test lattice methods and also for the lattice to present experimentalists with predictions.

2 Studies of quarkonia are computationally relatively cheap. High statistics results are now available. They demonstrate what can be achieved when calculations are not statistics limited, so that one can concentrate on systematics.

3 Systematic effects are easier to understand and control. The NRQCD collaboration, for instance, \[3, 16\] includes relativistic corrections through $O(M_Q v^4)$ and finite lattice spacing corrections in their quark action through $O(a^2)$. Finite volume effects are negligible for these “small” mesons. Corrections to counteract quenching errors can often be estimated. Furthermore calculations with $n_f = 2$ light dynamical fermions have now appeared and to date there have been no surprises. Unquenching effects have been observed where expected, and have been demonstrated to be small where they were predicted to be small.

4 Studies of quarkonia on the lattice provide the basis for determining fundamental parameters of QCD such as $\alpha_s$ and heavy quark masses. These important results are discussed in the next section.

5 Experience gained here will be crucial in studies of heavy-light systems, such as the $B$ and $D$ meson systems.
Figure 3 shows the Υ spectrum using non-relativistic fermions [16, 17]. The NRQCD collaboration has results both from quenched configurations ($n_f = 0$) and from configurations with two light dynamical quarks ($n_f = 2$). Charmonium results from the Fermilab [18] and NRQCD [19] collaborations are shown in figure 3. The charmonium results were obtained in the quenched approximation. The zero of energy was adjusted to give the correct Υ or the center of mass of the $J/\Psi - \eta_c$ states. The inverse lattice spacing was obtained from the 1S-1P results both from quenched configurations (fermions [16, 17]. The NRQCD collaboration has 4 investigations of the electroweak theory will be reviewed. Of QCD. In later sections, matrix elements relevant to investigations of the electroweak theory will be reviewed.

4. Fundamental Parameters of QCD

One major goal of the lattice effort is to contribute towards nailing down fundamental parameters of the Standard Model. There has been significant progress in this area. Here we discuss fundamental parameters of QCD. In later sections, matrix elements relevant to investigations of the electroweak theory will be reviewed. Precision calculations of Quarkonium spectra allow good determinations of the inverse lattice spacing, $a^{-1}$, and of the value of the lattice bare quark mass, $M_Q^0$. This information can then be converted into determinations of $\alpha_s$ and heavy quark pole and $\overline{MS}$ masses.

4.1. The Strong Coupling Constant

Lattice simulations give the Υ 1S-1P or 1S-2S splittings measured in units of the inverse lattice spacing, $a^{-1}$ is determined for instance in GeV’s by combining the dimensionless simulation numbers with experimental values for the splittings. The actual value of $\alpha_s(C/a)$ (“C” is a known constant) is obtained by focusing on some short distance quantity, for which both its nonperturbative simulation value and its perturbative expansion are known. A useful candidate is the “average plaquette”, $W_{1,1}$, whose expansion is given by [21, 20, 4],

$$-lnW_{1,1} = \frac{4\pi}{3} \alpha_V(3.41/a)(1 - \alpha_V(3.41/a) [1.185 + 0.070n_f])$$

$\alpha_V$ is a coupling introduced in [24] and defined in terms of the static quark potential. Once $\alpha_V$ at scale 3.41/a GeV is known, one can convert to other definitions such as $\alpha_{\overline{MS}}$ and run the coupling to other scales using the two- or three-loop beta-function.

A. $\alpha_s$ from Quenched Data

The first lattice determinations of $\alpha_s$ following the methods just described [23, 24, 12] used simulations in the quenched approximation, i.e. one initially had $\alpha_V^{(n_f=0)}(3.41/a)$, and the effects of light quarks had to be included a posteriori. This was accomplished by running $\alpha_V^{(0)}$ from scale 3.41/a down to scales typical of momenta exchanged in Υ or $J/\Psi$ mesons. $\alpha_V^{(0)}$ at the lower scale is equated to the coupling of an $n_f = 3$ theory, $\alpha_V^{(3)}$, since it is at this scale that the two couplings must produce the same physics such as splittings. $\alpha^{(3)}$ can then be evolved from the quarkonium scale through the charm and bottom thresholds up to the Z scale. Based on Υ results from two lattice groups, the latest Particle Data Group booklet quotes a lattice result of,

$$\alpha_{\overline{MS}}^{(n_f=5)}(M_Z)_{lattice} = 0.110(6)$$

This is an average over Fermilab’s 0.108(6) and NRQCD’s 0.112(4). Both numbers were obtained from quenched configurations, with dynamical light quark effects incorporated as described above. The conversion from $\alpha_V$ to $\alpha_{\overline{MS}}$ is described below in eq.(4).
B. \( \alpha_s \) using Unquenched Data

Recently, lattice results for \( \alpha_s \) based on calculations with \( n_f = 2 \) dynamical light quarks have appeared. The NRQCD collaboration extrapolates directly to \( n_f = 3 \) at scale 3.41/\( a \) using their \( n_f = 0 \) and \( n_f = 2 \) data. They no longer need to run down to a much lower scale in order to match onto an \( n_f = 3 \) theory. NRQCD also uses two different splittings, the 1S-1P and 1S-2S splittings, to carry out two separate analyses. From the 1S-1P splitting they find,

\[
\alpha_s^{(n_f)}(8.2 \text{GeV}) = \begin{cases} 
0.1548(23) & n_f = 0 \\
0.1800(16) & n_f = 2 
\end{cases}
\]

Extrapolating to \( n_f = 3 \) one gets,

\[
\alpha_s^{(3)}(8.2 \text{GeV}) = 0.1959(34)
\] (3)

A similar analysis based on the 1S-2S splitting gives \( \alpha_s^{(3)}(8.2 \text{GeV}) = 0.1958(46) \).

In order to compare with other non-lattice determinations of \( \alpha_s \), one needs to convert to the commonly used \( \overline{\text{MS}} \) scheme. The \( \alpha_V \) and \( \alpha_{\overline{\text{MS}}} \) schemes are related by,

\[
\alpha_{\overline{\text{MS}}}(Q) = \alpha_V(e^{5/6}Q)(1 + \frac{2}{\pi} \alpha_V + \mathcal{O}(\alpha_V^2))
\] (4)

so that Eq.\((3)\) becomes,

\[
\alpha_{\overline{\text{MS}}}^{(3)}(3.56 \text{GeV}) = 0.2203(84)
\] (5)

Errors are now dominated by higher orders in the \( \alpha_V \) - \( \alpha_{\overline{\text{MS}}} \) conversion formula, rather than by \( a^{-1} \) uncertainties.

Evolving to the Z boson scale, one finds,

\[
\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.115(2)
\] (6)

This should be compared with the world average quoted at this conference of \( \alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.117(5) \). Had one used the old method of evolving down to a typical \( \Upsilon \) scale and matching onto \( n_f = 3 \), the resulting numbers would have been,

\[
\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.112(4) \quad \text{(from } n_f = 0 \text{ data)} \] (7)

\[
\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.115(3) \quad \text{(from } n_f = 2 \text{ data)} \] (8)

i.e. the previous method gives values consistent with direct extrapolation to \( n_f = 3 \), but with larger errors. In the future one would like to see calculations repeated on configurations with \( n_f = 4 \) or \( n_f = 3 \) light dynamical quarks. Knowing the next order in the \( \alpha_V - \alpha_{\overline{\text{MS}}} \) conversion formula, eq.\((6)\), would contribute significantly to further reduction in the errors.

A recent paper describes an independent calculation of \( \alpha_s \) using \( n_f = 2 \) dynamical configurations. The authors employ the same methods but start from the charmonium spectrum obtained using Wilson fermions. They find \( \alpha_s \) at the \( Z \) mass consistent with the above lattice determinations, but with larger errors. Other lattice approaches to \( \alpha_s \) are also being pursued.

In particular, reference\( [29] \) has accurate results for the pure SU(3) gauge theory which agrees with calculations by the groups already mentioned in the same limit.

Once the calculations of reference\( [29] \) have been generalized to include quarks, one will have another independent first principles lattice determination of \( \alpha_s \) that can be compared with Eq.\((6)\).

4.2. Heavy Quark Masses

By fitting to the experimental \( \Upsilon(1S) \) mass one can fix the value of the bare lattice mass, \( M_b^0 \), that appears in the lattice action. \( M_b^0 \) can then be converted into other definitions of quark masses, such as the pole mass, using perturbation theory \( (M_b^{\text{pole}}) = Z_m M_b^0 \). Another approach to, extracting the pole mass, starts from the following relation between the \( \Upsilon \) mass, the b-quark pole mass and the binding energy inside an \( \Upsilon \) meson.

\[
M_T = 2(M_b^{\text{pole}} - E_0) + E_{NR}(\Upsilon)
\] (9)

\( M_T \) is known from experiment, \( E_0 \) can be calculated perturbatively and \( E_{NR} \) can be extracted from simulations (the binding energy corresponds to \( E_{NR} - 2E_0 \)). This allows for another determination of \( M_b^{\text{pole}} \). The NRQCD collaboration has estimated \( M_b^{\text{pole}} \) using both methods and finds that the two methods lead to consistent values. They now have results based on \( n_f = 0 \) and \( n_f = 2 \) configurations.

\[
M_b^{\text{pole}} = (5.0 \pm 0.2) \text{GeV} \quad (n_f = 2)
\]

\[
M_b^{\text{pole}} = (4.94 \pm 0.15) \text{GeV} \quad (n_f = 0)
\]

For several applications the \( \overline{\text{MS}} \) definition of a running quark mass is more appropriate. The bare lattice b-quark mass can also be converted to a \( M_b^{\overline{\text{MS}}} \) value. The two-loop formula relating pole and \( \overline{\text{MS}} \) masses was calculated in reference\( [31] \) in terms of the \( \overline{\text{MS}} \) coupling. Converting to the \( \alpha_V \) coupling one finds,

\[
M_b = M_b^{\overline{\text{MS}}}(M_b)(1 + \frac{4}{3\pi} \alpha_V(0.22 M_b) + 0.164 \alpha_V^2 + ..)
\] (10)
Nonperturbative determination of the Kaon B-parameter, $B$, is one of the important contributions to phenomenology from the lattice. Major progress has taken place in recent years in reducing errors. This has been possible due to,

- Better understanding of perturbative corrections to lattice operators.
- Use of the Lepage-Mackenzie coupling.
- Tests demonstrating that different choices for lattice operators lead to the same physical results.
- Control over extrapolations in “a” (quadratic).
- First tests of effects due to dynamical quarks.

The current best value, using the NDR scheme and with scale fixed at 2GeV gives:

$$B_K(NDR, 2\text{GeV}) = 0.616 \pm 0.020 \pm 0.017$$

Eq. (14) includes statistical and systematic errors coming from the lattice calculation. There is a further systematic error coming from perturbative uncertainties in going to eq. (16) of about 3%. The recent reduction of errors in $B_K$ significantly reduces the allowed region in the $\rho - \eta$ plane of CKM parameters. At the moment the lattice $B_K$ results have the smallest errors among theoretical estimates of this quantity. In the future more detailed studies of quenching effects, of the light quark mass dependence and of the use of degenerate quarks in the above calculations, should further improve the lattice calculations.

### 6. Heavy Mesons Decay Constants

A major challenge to the lattice community is to produce accurate values for the decay constants of D and B mesons. For instance, combinations such as $f_B \sqrt{B_B}$ or $f_B \sqrt{B_B}$ are crucial inputs into analyses of $B^0 - \bar{B}^0$ mixing phenomena. These nonperturbative QCD numbers are necessary in order to determine CKM matrix elements from experimental data.
The pseudoscalar decay constant, \( f_{PS} \), is defined (in Euclidean space) by,

\[
< 0 | A_\mu | PS; p > = p_\mu f_{PS}
\]

(17)

where \( A_\mu \) is the axial vector current and \( |PS; p > \) is the pseudoscalar state with momentum \( p \). A lattice determination of \( f_{PS} \) involves first calculating and fitting the following correlation functions.

\[
\sum_{\vec{x}} < 0 | A_\mu^L(\vec{x}, t) \mathcal{P}(0) | 0 >
\]

\[
t \gg 1 \rightarrow < 0 | A_\mu^L | PS > < PS | \mathcal{P} | 0 > \frac{1}{2 M_{PS} a^3} e^{-M_{PS} t}
\]

\[
\equiv \zeta_{AP} e^{-M_{PS} t}
\]

(18)

\[
\sum_{\vec{x}} < 0 | \mathcal{P}(\vec{x}, t) \mathcal{P}(0) | 0 >
\]

\[
t \gg 1 \rightarrow \frac{1}{2 M_{PS} a^3} | < PS | \mathcal{P} | 0 > |^2 e^{-M_{PS} t}
\]

\[
\equiv \zeta_{PP} e^{-M_{PS} t}
\]

(19)

\( \mathcal{P}(x) \) is an interpolating operator with the quantum numbers of a pseudoscalar meson and \( A_\mu^L \) is the lattice axial vector current. Both \( \mathcal{P} \) and \( A_\mu^L \) are dimensionless. From (18) and (19) one has,

\[
a^{-1} < 0 | A_\mu^L | PS > \equiv a f_{PS} a M_{PS} = \frac{\zeta_{AP} \sqrt{2 a M_{PS}}}{\sqrt{\zeta_{PP}}}
\]

(20)

or,

\[
a f_{PS} = \sqrt{\frac{2}{a M_{PS} \sqrt{\zeta_{PP}}}} \zeta_{AP}
\]

(21)

In order to convert to a dimensionful continuum decay constant one needs the value of \( a^{-1} \). This must come from a separate calculation of some dimensionful quantity (the \( \rho \) mass or the light meson decay constant \( f_\pi \) have been used in the past) which is compared with experiment to determine \( a^{-1} \). Once \( a^{-1} \) is specified one has,

\[
\sqrt{M_{PS}} f_{PS} = \sqrt{M_{PS}} f_{PS}^L Z_A
\]

\[
= Z_A \sqrt{2} \frac{\zeta_{AP}}{\sqrt{\zeta_{PP}}} a^{-3/2}
\]

(22)

\( Z_A \) is the matching constant necessary to relate the lattice axial vector current to its continuum counter part. It can be estimated using perturbation theory. There have also been attempts to extract \( Z_A \) nonperturbatively directly from simulations using Ward identities.

From eq.(21) and eq.(22) one sees that in order to obtain reliable results for \( f_{PS} \) many factors have to be under good control.

- \( a^{-1} \)
- confidence that the ground state has been isolated when extracting \( \frac{\zeta_{AP}}{\sqrt{\zeta_{PP}}} \)
- \( Z_A \) including correct normalization of lattice fermions

In addition a careful look at other systematic errors is necessary.

- finite volume effects
- extrapolation in the light quark mass
- extrapolation in the heavy quark mass. Many calculations have been done in the static limit or using Wilson fermions with masses smaller than \( M_b \).
- finite lattice spacing errors. This is particularly important for Wilson fermions which have \( O(a) \) errors unless an improved action is used.

Many groups have been working on heavy meson decay constants, as can be seen by the following list.

UKQCD [41], BLS [42], APE [43, 50], ELC [44], FNAL [45], PWCD [46], Hashimoto [47], LANL [48], HEMCGC [49].

Fig.4 shows lattice results for the \( D \) meson decay constant from a large number of groups. Although \( f_D \) is less crucial for extracting CKM matrix elements, nevertheless this is an important quantity for the lattice to calculate, since experimental values have now started to appear. Experimental numbers from the CLEO [51] and WA75 [52] collaborations are also shown in Fig.4. and are consistent with the lattice numbers. At this conference another experimental result for \( f_D \) was presented by the BES collaboration [53].

\[
f_D = 434^{+153+35}_{-134-35} MeV,
\]

which is still consistent with the other experimental numbers and with the lattice values. Hopefully it will be possible to reduce the experimental errors, so that lattice calculations can be calibrated and better tested.

Tables 1. and 2. summarize values for \( D \) and \( B \) meson decay constants and for the ratios \( f_D/f_D \) and \( f_B/f_B \). At the moment it is not easy to compare the values for \( f_B \) obtained by different groups, since each group has handled the sources of errors listed above differently. Even for gauge configurations with similar or identical parameters different values for \( a^{-1} \) are being used, although most groups include \( a^{-1} \) uncertainties in their systematic errors. The last two entries in Table 2. use static quarks (\( M_Q \rightarrow \infty \)) whereas the third from last entry uses nonrelativistic quarks. The first four groups
Table 1. D Meson Decay Constants

|        | $f_D$ [MeV] | $f_{D_s}$ [MeV] | $f_{D_s}/f_D$ |
|--------|-------------|----------------|--------------|
| BLS[42] | 208 ± 9.37  | 230 ± 7.35     | 1.11 ± 0.02 ± 0.05 |
| UKQCD[41] | 185^{+4.42}_{-3.7} | 212^{+4.67}_{-4.7} | 1.18 ± 0.02 |
| PWCD[46] | 170 ± 30          | 240 ± 9         | 1.11 ± 0.01   |
| APE[43] | 218 ± 9          | 227 ± 15        | 1.08 ± 0.02   |
| ELC[44] | 210 ± 15        | 266 ± 15        |              |
| LANL[48] | 241 ± 19          | 288 ± 5.64     |              |
| HEMCGC[49] | 215 ± 5.53       | 232 ± 45.52    |              |

Table 2. B Meson Decay Constants

|        | $f_B$ [MeV] | $f_{B_s}$ [MeV] | $f_{B_s}/f_B$ |
|--------|-------------|----------------|--------------|
| BLS[42] | 187 ± 10 ± 37 | 207 ± 9 ± 40   | 1.11 ± 0.02 ± 0.05 |
| UKQCD[41] | 160^{+5.3}_{-6.19} | 194^{+6.62}_{-5.39} | 1.22^{+1.4}_{-3} |
| PWCD[46] | 180 ± 50          |                 |              |
| ELC[44] | 205 ± 40          |                 | 1.08 ± 0.06   |
| Hashimoto[47] (Nonrel.) | 171 ± 22^{+19}_{-45} |                 |              |
| FNAL[45] (Static) | 188 ± 23^{+34}_{-21} |                 | 1.216 ± 0.041 ± 0.016 |
| APE[50] (Static) | 290 ± 15 ± 45     |                 | 1.11 ± 0.03   |

Figure 4. Lattice and experimental results for the $D_s$-meson decay constant $f_{D_s}$.

Figure 5. Static and Wilson fermion results for pseudoscalar meson decay constants (from reference [42]).

These calculations are now being improved with higher statistics. Looking elsewhere, methods that have been developed on heavy-heavy quarkonium systems such as the NRQCD and Fermilab Heavy Wilson approaches are starting to be applied to heavy-light D and B mesons. With these approaches one can work directly at the B meson mass, and there is no need to extrapolate in the heavy quark mass. One can expect a lot of activity here. The issue of accurately determining the appropriate $a^{-1}$ in heavy-light systems as well as how to eliminate finite lattice spacing errors, will have to be resolved by all groups. Only then can one hope to be able to provide experimentalists and

worked with conventional Wilson fermions, normalized with varying degrees of sophistication in order to go smoothly to the large mass limit.

The next couple of years should see a vast improvement in these types of calculations as people learn from the current results and a consensus emerges on the best ways to proceed. Reference [42] was able to interpolate smoothly between D and B meson scales up to the static limit. One figure from their paper is shown in figure 5. Plotted is the combination given in eq. (22) rescaled by $(1 + \alpha_s \ln(aM_P)/\pi)^{-1}$. The fit is to an expression quadratic in $1/M_P$. It was crucial for their good fit that they corrected for large $aM_Q$ errors coming from the use of Wilson fermions.
relevant mass heavy to light meson decays is to go onto charmless semileptonic decays. For instance, the decay $f_+$ different values of $q$ matrix element $V_{cb}$ are close to $1$, which results in a good handle on the CKM matrix element $|V_{cb}|$. Heavy Quark Effective Theory (HQET) and the introduction of heavy quark spin and flavor symmetries and the Isgur-Wise function have had great impact on how to extract information from experimental data. In the large $M_Q$ limit all the form factors, $h_+, h_-, h_V, h_A_1, h_A_2, h_A_3$ are related to the Isgur-Wise function $\xi(\omega)$ ($\omega = v \cdot v'$). Experiments provide the combination $|V_{cb}|^2 \eta_A \xi^2(\omega)$, where $\xi$ means corrections to the heavy quark symmetry limit are included and $\eta_A$ includes short distance loop corrections. Since $\xi(\omega = 1)$ and $\eta_A$ can be calculated, one can determine $|V_{cb}|$. Experiments are not done at $\omega = 1$ so one needs to extrapolate from $\omega > 1$ to $\omega = 1$. An important quantity is the slope with which the Isgur Wise function approaches $\omega = 1$.

$$\rho^2 \equiv -\xi'_{|\omega=1}$$

Several parametrizations of the Isgur-Wise function exist. Using any one of them, close to $\omega = 1$ one has

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + O((\omega - 1)^2)$$

Two lattice groups, BSS and UKQCD, have made the first attempts to obtain the slope parameter $\rho^2$ from lattice QCD. Combining this information with CLEO and ARGUS data they also determine $|V_{cb}|$. Experimentalists (and theorists working with HQET) can, of course just fit their own data to extract $\rho^2$ and determine $|V_{cb}|$ (as they have done) without any input from the lattice. What lattice simulations can provide are more accurate data close to $\omega = 1$ than is possible with current experiments and this should serve to guide fits to experimental data. It is also useful to be able to compare determinations of the slope parameter $\rho^2$ coming from a lattice QCD calculation with other theoretical estimates and with experiment. Lattice calculations will eventually study the individual form factors $h_+, h_A_1$ etc. and investigate $1/M_Q$ corrections. Some work in this direction by the UKQCD collaboration has already appeared.

The most accurate lattice results for the Isgur-Wise function come from studies of Pseudoscalar → Pseudoscalar decays, i.e. using $h_+$. BSS looks at the elastic $D \to D$ process, where current conservation helps to reduce systematic uncertainties. UKQCD considers $B \to D$ (and $B \to D^*$) decays. Using the BSW parametrization of the Isgur-Wise function they obtain,

$$\rho^2 = \begin{cases} 
1.4 \pm 0.2 \pm 0.4 & \text{BSS} \\
0.9^{+2+4}_{-3-2} & \text{UKQCD}
\end{cases}$$

Both groups find that the slope increases with the light quark mass $M_q$. UKQCD finds, for instance that for

|         | $f_+(0)$ | $V(0)$ | $A_1(0)$ | $A_2(0)$ |
|---------|---------|--------|----------|----------|
| BKS[54] | .90(8)(21) | 1.43(45)(49) | .83(14)(28) | .59(14)(24) |
| UKQCD[55] | .67^{+7}_{-5} | .98^{+1}_{-13} | .70^{+5}_{-10} | .68^{+1}_{-17} |
| APE[56] | .72(9) | 1.00(20) | .64(11) | .46(34) |
| ELC[57] | .60(15)(7) | .86(24) | .64(16) | .40(28)(4) |
| LANL[58] | .73(6) | 1.24(8) | .66(3) | .45(19) |
| Exp.[59] (aver.) | .77(4) | 1.16(16) | .61(5) | .45(9) |

Table 3. Semileptonic $D$ Decays

7. Semileptonic $D$ and $B$ Meson Decays

7.1. $D \to K, K^*$ Semileptonic Decays

There is a growing body of results on semi-leptonic form factors coming from the lattice. Table 3. gives a summary for $f_+, V, A_1$ and $A_2$ at $q^2 = 0$ for $D \to K l \bar{\nu}$ or $D \to K^* l \bar{\nu}$ decays. Listed at the end is an average of experimental numbers. One sees that the agreement among different lattice groups and between lattice predictions and experiment is good. The generic form factor, $f(q^2)$, is evaluated at several different values of $q^2$ and a pole dominance ansatz $f(q^2) = f(0)/(1 - q^2/M^2)$ is used to extract $f(0)$. The relevant mass $M$ can either be fit simultaneously with $f(0)$ or obtained from an independent calculation of an appropriate correlation function. The next step in heavy to light meson decays is to go onto charmless B decays, $B \to \rho(\pi) l \bar{\nu}$. This will be a challenge for both experimentalists and lattice gauge theorists. When successful, one will obtain a good handle on the CKM matrix element $V_{ub}$.  

7.2. Isgur Wise Function

There has been a lot of activity, both experimentally and theoretically on heavy meson to heavy meson semileptonic decays. For instance, the decay $B \to D^* l \bar{\nu}$ has led to a determination of the CKM matrix
M_q \sim M_{\text{strange}}$ the slope increases from the above value to $\rho^2 = 1.2 \pm 0.2^{+1.2}_{-1.0}$. This $M_q$ dependence of $\rho^2$ agrees with QCD Sum Rule\cite{63} and Quark Model\cite{67} results reported at this conference, but disagrees with predictions from chiral perturbation theory\cite{68}.

From fits to their own and experimental data the two lattice groups find,

$$\sqrt{\frac{\Gamma_B}{1.55 \text{ps}}} |V_{cb}| = \begin{cases} 0.044 \pm 0.005 \pm 0.007 & \text{BHS} \\ 0.035^{+1.1+2.4}_{-1-2} & \text{UKQCD} \end{cases}$$

Figure 6 is a plot from the UKQCD collaboration, used to extract their value for $|V_{cb}|$. It should be noted that, although radiative corrections at $\omega = 1$ have been taken into account, exact heavy quark symmetry is assumed in relating the lattice simulations of the pseudoscalar $\rightarrow$ pseudoscalar process to experimental data. UKQCD is currently also studying the decay $B \rightarrow D^*$, in order to obtain another estimate of $|V_{cb}|$.

![UKQCD plot](image)

Figure 6. UKQCD plot\cite{63} to extract $|V_{cb}|$ using CLEO II Data.

8. Radiative Penguin Decays, $B \rightarrow K^* \gamma$

The rare decay $B \rightarrow K^* \gamma$ was observed last year by the CLEO collaboration\cite{69} and this year, at this conference, there was considerable excitement over their measurement of the inclusive $b \rightarrow s $ $\gamma$ branching ratio\cite{70}. The experimental numbers are,

$$BR(b \rightarrow s \gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}$$

$$BR(B \rightarrow K^* \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$$

Lattice gauge theorists have also started to investigate rare B decays. For the process $B \rightarrow K^* \gamma$, what is required is a nonperturbative evaluation of the hadronic matrix element,

$$< K^*| \bar{s} \sigma_{\mu\nu} q^\nu b_R | B >$$

$q^\nu$ is the momentum of the emitted photon. This matrix element can be expressed in terms of three form factors, $T_1(q^2), T_2(q^2)$ and $T_3(q^2)$. On-shell one has,

$$T_1(0) = T_2(0)$$

and the “hadronization ratio” can be written as,

$$R_{K^*} = \frac{Br(B \rightarrow K^*\gamma)/Br(b \rightarrow s \gamma)}{4[M_B/M_0]^3 [1 - M_{K^*}^2/M_B^2] |T_1(0)|^2}$$

Calculations are done at $q^2 > 0$ and to date also with masses smaller than the B meson mass. Hence two important extrapolations, $q^2 \rightarrow 0$ and $M_H \rightarrow M_B$ ($M_H$ is the mass of the initial decaying meson) are required. Different groups are still exploring the best way to carry out these extrapolations.

BHS\cite{71} extrapolates $T_2$ in $M_H$ up to $M_H = M_B$ using a form suggested by Heavy Quark Symmetry,

$$\sqrt{M_HT_2(q^2_{\text{max}})} = A_1 + A_2/M_H$$

$q^2_{\text{max}} \equiv (M_H - M_{K^*})^2$. They then use pole dominance to extrapolate from $q^2_{\text{max}}$ to $q^2 = 0$ and then obtain $T_1(0)$ from eq. (23). They were able to check for evidence in favor of pole dominance over a small region in $q^2 > 0$, but not all the way to $q^2_{\text{max}}$ appropriate for $M_H = M_B$. They find,

$$T_1(0) = T_2(0) = 0.101 \pm 0.010 \pm 0.028$$

$$R_{K^*} = (6.0 \pm 1.2 \pm 3.4)\%$$

UKQCD\cite{72} uses pole dominance for $T_1$ to go to $q^2 = 0$ and a modified scaling law, in which $T_1 M_H^{3/2}$ is expanded in powers of $1/M_H$, to extrapolate to $M_B$.

They find,

$$T_1(0) = 0.124^{+0.20}_{-0.18} \pm 0.022$$

$$R_{K^*} = (8.8^{+2.8}_{-3.0} \pm 3.0 \pm 1.0)\%$$

The central values by BHS and UKQCD for $R_{K^*}$ appear to be considerably smaller than what one would calculate from the CLEO results. However within the large errors in both the lattice and the experimental numbers one still finds consistency.
At this conference preliminary results on $B \to K^* \gamma$ were presented by the APE collaboration. Using pole dominance on $T_2$ they obtain $T_2(0) = 0.087(7)$. However, using pole dominance on $T_1$ they find $T_1(0) = 0.20(7)$, a larger number. It appears more work is needed to resolve the issue of how best to carry out the $q^2 \to 0$ and $M_B \to M_B$ extrapolations. Lattice results for $B \to K^* \gamma$ should be viewed as very promising but still exploratory studies at this time. In the future in addition to improving the $B \to K^* \gamma$ calculations one would also want to go onto $B \to \rho \gamma$ processes.

9. Summary

These are exciting times for lattice gauge theory. More and more phenomenologically relevant quantities are now being studied on the lattice and lattice results are starting to contribute towards testing and understanding the Standard Model as we see it today. It is also being used to investigate the Standard Model in extreme environments, such as QCD at finite temperatures and the Electroweak theory undergoing its symmetry breaking phase transition. There has been considerable progress in understanding how to correct for lattice artifact and other systematic errors and extract continuum physics. Precision calculations of $\alpha_s$, $M_b$ and $B_K$ have appeared. There is optimism that significant improvements in the accuracy of many other lattice results are achievable.

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