Theoretical aspects on internal cylindrical gears with small tooth number difference

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Abstract. Despite of some drawbacks, internal-external gear pairs with small tooth number difference have some advantages, especially when they are used in planetary or differentially gear mechanisms, as high transmission ratio and high torque at small size. In order to study the limits of these gears, regarding the minimum number of teeth of internal gear, the minimum tooth difference of the gear pair, the center distance, the addendum modification coefficients, the influence of the different geometric parameters on interference phenomena, an algorithm was designed that allowed to do that. In this paper, some numerical results based on a study on the theoretical aspects on such internal-external gear pairs will be reported.

1. Introduction
Internal-external gear pairs with small tooth number difference are commonly used in planetary or differential gear mechanisms and compact gear boxes [1], [2], [3], [4]. They have some advantages, like: compactness, high transmission ratio and high torque at small size, lower contact stresses comparing to external gear pairs, both the pinion and gear rotate in the same direction, lower relative sliding between teeth, and a greater length of contact possible between mating teeth. Due to that, many research works are doing in recent years [5]-[15].

But, despite these advantages, these transmissions also have some drawbacks as: interference (involute, trochoid and trimming interference) and small contact ratio when the tooth number difference is small. When the tooth number difference is decreasing the contact ratio is also decreasing [9], [10], [11].

In planetary or differential gear mechanisms with small tooth number difference, addendum modification coefficient is necessary to avoid possible teeth interference. Due to that, the design and cutting processes become complicated [15].

In this paper, some numerical results based on a study on the theoretical aspects on internal-external gear pairs with small tooth number difference will be reported.

2. Some theoretical aspects on internal-external gear pairs with small tooth number difference

2.1. General aspects
By appropriately choosing the center distance between the internal and external gears and the addendum modification coefficients, the tooth difference of an internal-external involute gear pair can be reduced to one (Figure 1) and even to zero. These gear pairs can be used both as usual geared mechanisms, but especially as planetary mechanisms, in which case they have a number of
advantages, including the possibility of obtaining large gear ratios with simple, compact and small in size constructive solutions [2]-[6], [14].

![Figure 1. Internal-external involute gear pair with small tooth number difference (z₂ - z₁ = 1), [16].](image)

In order to study the limits of these gears, regarding the minimum number of teeth of internal gear, the minimum tooth difference of the gear pair, the center distance, the addendum modification coefficients, the influence of the different geometric parameters on interference phenomena, an algorithm was designed that allowed to do that. Some of the numerical results based on this study will be reported in this work.

### 2.2. Minimum number of teeth of the internal gear

Ignoring the manufacturing process of the internal gear, in this paragraph the limits of the minimum number of teeth, \( z_{2\text{min}} \), of the internal gear will be presented, depending on the addendum modification coefficient, \( x_{n2} \), the tooth angle, \( \beta \), and the reference pressure angle, \( \alpha_0 \).

As is well known, in order that the entire inner tooth of the internal gear to have an involute profile, it is necessary that the tip circle diameter, \( d_{a2} \), to be greater than or equal to the base circle diameter, \( d_{b2} \), which means:

\[
m_t \cdot z_2 - 2\left(h_{an}^* - x_{n2}\right) \cdot m_t \cdot \cos \beta \geq m_t \cdot z_2 \cdot \cos \alpha_t,
\]

meaning

\[
z_2 \geq \frac{2\left(h_{an}^* - x_{n2}\right) \cdot \cos \beta}{1 - \cos \alpha_t},
\]

where: \( m_t \) is transversal module; \( h_{an}^* \) is addendum coefficient.

Thus, the minimum number of teeth that can be adopted for the internal gear can be calculated from the relation (2). For straight teeth (\( \beta = 0^\circ \)), not modified tooth profile (\( x_{n2} = 0 \)) and standardized tool (\( h_{an}^* = 1, \alpha_0 = 20^\circ \)), \( z_{2\text{min}} \) will be:

\[
z_{2\text{min}} \geq \frac{2\left(1 - 0\right) \cdot \cos 0}{1 - \cos 20^\circ} = 33.
\]

The number of teeth could be decreased when \( \alpha_0, \beta \) and/or \( x_{n2} \) are different from these values. In Figure 2, the variation of \( z_{2\text{min}} = f(\beta, \alpha_0) \) for \( x_{n2} = 0 \ldots 0.8 \) are presented. Analyzing these surfaces, it can be seen that when one of the parameters in question is increasing (\( \alpha_0, \beta, x_{n2} \)) there is a decrease in the minimum number of teeth of the internal gear and it can be lowered even below 10. As can be
seen, the greatest influence has the addendum modification coefficient, $x_{n2}$, then the value of the reference pressure angle, $\alpha_0$, and, finally, the value of $\beta$ angle.

Figure 2. The influence of the parameters $\alpha_0$, $\beta$, $x_{n2}$ on the minimum number of teeth, $z_{2\min}$: a) $x_{n2} = 0$; b) $x_{n2} = 0.2$; c) $x_{n2} = 0.4$; d) $x_{n2} = 0.6$; e) $x_{n2} = 0.8$; f) $x_{n2} = 0 \ldots 0.8$. 
2.3. Some aspects on the internal-external gear pair

Based on the above mentioned algorithm, internal-external gear pairs with \( z_2 - z_1 = 1 \) were discussed, having the teeth number of the external gear, \( z_1 = 20 \ldots 100 (z_2 = 21 \ldots 101) \), the reference pressure angle, \( \alpha_0 = 20^\circ \), and the tooth angle, \( \beta = 0^\circ \). For \( z_1 = 60 \), the influence of the \( \beta \) angle (\( \beta = 0^\circ \ldots 15^\circ \)), the influence of the reference pressure angle, \( \alpha_0 = 15^\circ \ldots 25^\circ \), as well as the influence of the tooth number difference, \( z_2 - z_1 = 1 \ldots 4 \), were also studied.

This study led to the following results regarding the limits of the centre distance between the internal gear and the external gear, \( a_w \), and results on the limits of the addendum modification coefficients, \( x_{n1}, x_{n2} \).

![Figure 3](image1)

**Figure 3.** The center distance, \( a_w \), relative to the addendum modification coefficient, \( x_{n2} \), for: \( z_1 = 20 \), \( z_2 = 21 \), \( \alpha_0 = 20^\circ \), \( \beta = 0^\circ \), \( m_n = 1 \).

![Figure 4](image2)

**Figure 4.** The center distance, \( a_w \), relative to the addendum modification coefficient, \( x_{n2} \), for: \( z_1 = 40 \), \( z_2 = 41 \), \( \alpha_0 = 20^\circ \), \( \beta = 0^\circ \), \( m_n = 1 \).
Some of the results are presented in Figures 3 … 9 and they can be very useful to all those who want to calculate the geometric parameters of such a gear pair. Other results will be presented in future work.

Analyzing the diagrams presented in Figures 3 … 9, it can be observed that, for the same values of the $\alpha_0$ and $\beta$ angles, when the numbers of teeth $z_1, z_2$ are increasing, there is an increase of the interval $[x_{n1min}, x_{n1max}]$, an also of the absolute values of $x_{n1min}, x_{n1max}$, an increase that becomes insignificant at large values of the pinion teeth number ($z_1 > 100$).
Also, there is an increase in the centre distance, for the same value of the coefficient \( x_{n1} \), when increasing the numbers of teeth \( z_1 \), and \( z_2 \). The variations of the angles \( \alpha_0 \) and \( \beta \), as well as the variation of the difference \( z_2 - z_1 \), have the same effects, with the observation that the influence of the angle \( \beta \) is small in comparison with the other two parameters under discussion.

**Figure 7.** The center distance, \( a_w \), relative to the addendum modification coefficient, \( x_{n2} \), for: \( z_1 = 100, \ z_2 = 101, \ \alpha_0 = 20^\circ, \ \beta = 0^\circ, \ m_n = 1 \).

**Figure 8.** The center distance, \( a_w \), relative to the addendum modification coefficient, \( x_{n2} \), for: \( z_1 = 60, \ z_2 = 61, \ \alpha_0 = 20^\circ, \ x_{n1} = 0, \ m_n = 1 \).

By choosing the centre distance and the addendum modification coefficients in the presented fields and using the proposed algorithm, the geometric parameters of the internal-external gear pair can be obtained, avoiding all the negative phenomena that can occur during manufacturing or gearing.
Figure 9. The center distance, $a_w$, relative to the addendum modification coefficient, $x_{n2}$, for: $z_1 = 60$, $z_2 = 61$, $\beta = 0^\circ$, $x_{n1} = 0$, $m_n = 1$.

Figure 10. The center distance, $a_w$, relative to the addendum modification coefficient, $x_{n2}$, for: $\alpha_0 = 20^\circ$, $\beta = 0^\circ$, $x_{n1} = 0$, $m_n = 1$.

3. Conclusion

Internal-external gear pairs with small tooth number difference are commonly used in planetary or differential gear mechanisms and compact gear boxes. They have some advantages, like: compactness, high transmission ratio and high torque at small size, lower contact stresses comparing to external gear pairs, both the pinion and gear rotate in the same direction, lower relative sliding between teeth, and a greater length of contact possible between mating teeth. Despite these advantages, these transmissions also have some drawbacks, as interference and small contact ratio when the tooth number difference is small. In order to study the limits of these gears, regarding the minimum number of teeth of internal gear, the minimum tooth difference of the gear pair, the center distance, the addendum modification coefficients, the influence of the different geometric parameters on interference phenomena, an
algorithm was designed that allowed to do this study. In this paper, some numerical results based on a study on the theoretical aspects on such internal-external gear pairs have been reported.

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