Emission of gamma rays by
X-ray electron-nuclear double transitions

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Abstract

The X-ray electron-nuclear double transitions (XENDT) are processes in which a transition
effected by an inner atomic electron takes place simultaneously with a nuclear electromagnetic
transition. We give expressions for the cross sections of electric and magnetic XENDTs of various
multipole orders. We calculate the rate of deexcitation of isomeric nuclei induced by XENDTs
for the case when the holes in the atomic shells are produced by incident ionizing electrons and
find that the induced nuclear deexcitation rate becomes comparable to the natural decay rate for
ionizing fluxes of the order of $10^{14}$ W cm$^{-2}$. We show that for E1 and M1 nuclear processes for
which there is a matching between the electron and the nuclear transition energies, the XENDTs
can be used to produce pulses of Mössbauer radiation, with yields of the order of $10^4$ Bq mA$^{-1}$.

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The electron-nuclear double transitions are a class of processes in which a transition effected by an atomic electron takes place simultaneously with a nuclear electromagnetic transition. There are several types of electron-nuclear double transitions, depending on the range of energies for the electron and nuclear transition energies. Thus, the application of a radiofrequency magnetic field to a sample generates additional lines in the Mössbauer spectra [1]-[8]. In the case of the electron-nuclear double resonance [9]-[11], a microwave magnetic field interacts with the Zeeman sublevels of electrons from incomplete atomic shells while a radiofrequency magnetic field interacts resonantly with the magnetic sublevels of the ground nuclear state. Another possibility [12] is that the electron transition takes place between the Zeeman substates of incomplete electron shells while the nucleus makes a gamma-ray transition. These electron-nuclear double transitions produce changes in the Mössbauer spectra, and may have applications for the research on the amplification of gamma rays without inversion of nuclear population, and for the identification of the position of lines in complex Mössbauer spectra. In the case of the electron-nuclear double transitions at optical frequencies, the optical pumping of atomic hyperfine transitions by a laser induces an anisotropy in the angular distribution of gamma rays emitted by the nucleus [14].

In the case of X-ray electron-nuclear double transitions (XENDTs), the electrons make transitions to fill the holes created in the atomic shells by a beam of incident electrons or by the electrons of a dense hot plasma. In the process of nuclear excitation by electron transition first described by Morita [13]-[23], an electron from a higher electron shell makes a transition to fill a hole in an inner electron shell while at the same time the nucleus makes a transition from the ground nuclear state to an excited nuclear state. The nuclear excitation by electron transition competes with the conventional atomic deexcitation via X-ray emission or Auger emission. The branching ratio for the nuclear excitation by electron transition depends on the energy of interaction between the inner-shell electron and the protons of the nucleus, and since the inner electrons are rather close to the nucleus, this energy of interaction is relatively large. The branching ratio for the nuclear excitation by electron transition is inversely proportional to the square of the difference between the electron transition energy and the nuclear transition energy. Therefore, the nuclear excitation by electron transition becomes important when there is a matching between the transition energy from a populated nuclear state, like the ground state or an isomeric state, and an X-ray transition energy. Typical transition energies are of the order of a few tens of keV’s, and the detuning between
the electron and nuclear transition energies may be of about 1 keV. The nucleus can also make a transition from an excited state to a lower state while an electron is raised from an inner shell to a hole in a higher shell. This process may be called nuclear deexcitation by electron transition. The process of nuclear excitation by electron transition has been observed in $^{189}$Os [16], [17], [21], $^{235}$U [18], $^{237}$Np [19] and $^{197}$Au [22]. Although the cross sections for the nuclear excitation by electron transitions are rather small, these processes are interesting because they represent a way to control a nuclear transition by actions exerted at the atomic level.

In this work we give expressions for the cross sections of electric or magnetic XENDTs, and study the application of XENDTs to the problem of induced gamma emission. If the nucleus is initially in a long-lived isomeric state $|i\rangle$, the XENDTs open a new deexcitation channel for the isomeric state, in addition to the regular gamma-ray emission and internal conversion. One possibility is the direct deexcitation of the isomeric nucleus by a transition from the isomeric state $|i\rangle$ to a lower state $|l\rangle$, while an electron is raised to fill a hole in a higher electron shell. Another possibility is the two-step deexcitation of the isomeric nucleus by a transition from the isomeric state $|i\rangle$ to a higher nuclear state $|h\rangle$ while an electron from a higher shell makes a transition to a hole in an inner electron shell. The nuclear intermediate state $|h\rangle$ then decays by the emission of a gamma-ray photon to the lower nuclear state $|l\rangle$. These alternative nuclear paths for the deexcitation of an isomeric state by electron transition are shown in Fig. 1. The XENDT rate is proportional to the electron flux which produces the holes in the atomic shells, and for sufficiently large ionizing electron fluxes the deexcitation rate induced by XENDT becomes comparable to the natural decay rate of the isomer. Another application of XENDTs is the possibility of producing pulsed Mössbauer gamma radiation for isotopic elements where there is a matching between an electron transition energy and a resonant nuclear transition energy.

We shall calculate the probability of an XENDT for electric or magnetic interactions of arbitrary multipole orders, and shall estimate the cross section for the production of an XENDT for the case when the holes in the atomic shells are produced by incident ionizing electrons. We shall estimate the ionizing electron fluxes for which the XENDT rate becomes equal to the natural decay rate of the isomeric state, both for a direct deexcitation $|i\rangle \rightarrow |l\rangle$ and for a two-step deexcitation $|i\rangle \rightarrow |h\rangle \rightarrow |l\rangle$. Then we shall list the isotopic species for which it is possible to generate resonant
Mössbauer radiation by XENDT, and shall estimate the corresponding yields.

The initial state of the XENDT process is prepared when an electron hole is created in a certain atomic subshell, for example by electron impact. We designate this initial atomic state by \( |a_1\rangle \) and its energy by \( E_1^{(a)} > 0 \), as shown in Fig. 2. Moreover, we designate the initial nuclear state by \( |n_1\rangle \) and the energy of the initial nuclear state by \( E_1^{(n)} \). The initial state of the XENDT process is thus \( |a_1 n_1\rangle \). In the course of the XENDT, an electron from another atomic subshell fills the initial hole, thereby creating a new hole in a different subshell and thus leaving the atom in the final state \( |a_2\rangle \) of energy \( E_2^{(a)} > 0 \). The electron transition \( |a_1\rangle \rightarrow |a_2\rangle \) takes place simultaneously with a nuclear transition \( |n_1\rangle \rightarrow |n_2\rangle \) from the initial nuclear state \( |n_1\rangle \) to the final nuclear state \( |n_2\rangle \) of energy \( E_2^{(n)} \). The final state of the XENDT process is thus \( |a_2 n_2\rangle \). If \( E_1^{(n)} > E_2^{(n)} \), we have a nuclear deexcitation by electron transition, as shown in Fig. 2(a), and if \( E_1^{(n)} < E_2^{(n)} \) we have a nuclear excitation by electron transition, as shown in Fig. 2(b).

The probability of the XENDT depends on the detuning \( \Delta \) between the total initial energy \( E_1^{(n)} + E_1^{(a)} \) and the total final energy \( E_2^{(n)} + E_2^{(a)} \),

\[
\Delta = E_1^{(n)} + E_1^{(a)} - (E_2^{(n)} + E_2^{(a)}).
\]  

The widths \( \Gamma_1, \Gamma_2 \) of the atomic states \( |a_1\rangle, |a_2\rangle \) are determined by the X-ray emission and the Auger and Coster-Kronig emission of electrons. We shall represent these processes by imaginary terms \( -i \Gamma_1/2, -i \Gamma_2/2 \) in the Hamiltonian matrix of the electron-nuclear system. If \( c_1(t), c_2(t) \) are the amplitudes to find the electron-nuclear system in the states \( |a_1 n_1\rangle \) and respectively \( |a_2 n_2\rangle \), the Hamiltonian equations for this system are

\[
i\hbar \frac{dc_1}{dt} = -\frac{i \Gamma_1}{2} c_1 + V_{12} e^{i \Delta t/\hbar} c_2,
\]

\[
i\hbar \frac{dc_2}{dt} = V_{12}^* e^{-i \Delta t/\hbar} c_1 - \frac{i \Gamma_2}{2} c_2,
\]

where \( V_{12} = \langle a_1 n_1|V|a_2 n_2\rangle \) is the matrix element for the interaction between the electrons and the nucleus, and \( V_{12}^* \) is the complex conjugate of \( V_{12} \). The solution of Eqs. (2), (3) with the initial conditions \( c_1 = 1, c_2 = 0 \) at \( t=0 \) is

\[
c_1(t) = e^{i \Delta t/\hbar + (\Gamma_1 + \Gamma_2)t/2\hbar} \left( \cos \Omega t - \frac{i \Delta + (\Gamma_1 - \Gamma_2)/2}{2\hbar \Omega} \sin \Omega t \right)
\]
$$c_2(t) = \frac{iV_{12}}{\hbar \Omega} e^{-i\Delta t/\hbar - (\Gamma_1 + \Gamma_2)t/4\hbar} \sin \Omega t,$$

where

$$\Omega^2 = \frac{1}{4\hbar^2} \left( \Delta - \frac{i}{2} \frac{\Gamma_1 - \Gamma_2}{2} \right)^2 + \frac{|V_{12}|^2}{\hbar^2}. \tag{6}$$

The fact that \(c_1(t) \to 0, c_2(t) \to 0\) as \(t \to \infty\) is due to the presence of the imaginary diagonal terms in the Hamiltonian equations (2), (3).

The probability \(P\) to have the nucleus in the state \(n_2\) at the end of the process can be obtained by multiplying the probability \(|c_2(t)|^2\) of finding the electron-nuclear system in the state \(|a_2n_2\rangle\) by the probability \(\Gamma_2 dt\) of an electron transition from the state \(|a_2n_2\rangle\) to other states \(|a'_2n_2\rangle\),

$$P = \int_0^\infty |c_2(t)|^2 \Gamma_2 dt. \tag{7}$$

Usually we have \(|\Delta| \gg \Gamma_1, \Gamma_2, |V_{12}|\), so that \(\Omega^2 \approx \Delta^2/4\hbar^2\), and then

$$P = \frac{\Gamma_1 + \Gamma_2 |V_{12}|^2}{\Gamma_1 \Delta^2}. \tag{8}$$

This is in agreement with the expression given in [17]. The \(1/\Delta^2\) dependence is characteristic to non-resonant Rabi oscillations in two-state systems. [25] The phase of the state \(|a_1n_1\rangle\) is the sum of the phase of the atomic state \(|a_1\rangle\) and of the phase of the nuclear state \(|n_1\rangle\), and similarly the phase of the state \(|a_2n_2\rangle\) is the sum of the phase of the atomic state \(|a_2\rangle\) and of the phase of the nuclear state \(|n_2\rangle\). The probability \(P\), Eq. (8), is independent of these phases.

We shall assume that in the initial atomic state \(|a_1\rangle\) the hole is in the \(nl_J\) subshell of principal quantum number \(n\), orbital angular momentum \(l\) and total angular momentum \(J\). An atomic electron can make a transition to fill this hole, thereby creating the final state \(|a_2\rangle\) having a hole in the subshell \(n'l'_{J'}\). If the angular momentum and parity of the nuclear states \(|n_1\rangle, |n_2\rangle\) are respectively \(I_{1}^{\pi_1}, I_2^{\pi_2}\), then the angular momentum \(F_1\) of the state \(|nl_J, n_1\rangle\) is such that \(|J - I_1| < F_1 < J + I_1\) and the parity of the state \(|nl_J, n_1\rangle\) is \(\Pi_1 = \pi_1(-1)^l\). The angular momentum \(F_2\) of the state \(|n'l'_{J'}, n_2\rangle\) is such that \(|J' - I_2| < F_2 < J' + I_2\) and the parity of the state \(|n'l'_{J'}, n_2\rangle\) is \(\Pi_2 = \pi_2(-1)^{l'}\).

In Eq. (8) we have assumed that one electron interacts with the nucleus to produce the XENDT. There are however \(2J' + 1\) electrons which can make a transition from the \(n'l'_{J'}\) subshell to fill the
hole in the \(nl_J\) subshell, so that the total transition probability is

\[ P_{\text{tot}} = (2J' + 1)P. \]  \(\text{(9)}\)

We shall estimate the matrix element \(V_{12}\) appearing in Eq. (8) as the energy of interaction between two electric multipoles of the same order, or between two magnetic multipoles of the same order. In the case of the interaction between electric multipoles of order \(L\), the energy of interaction \[\text{[26], [27]}\] can be estimated as

\[ V_{12}^{(EL)} = \frac{3f_E e^2 r_A}{(4\pi)^{1/2} \epsilon_0 (2L + 1)(L + 3)} \langle n'^l' J' | r^{L-1} | nl_J \rangle, \]  \(\text{(10)}\)

where \(r_A\) is the radius of a nucleus of mass number \(A\), \(r_A = r_0 A^{1/3}\), \(r_0 = 1.2 \cdot 10^{-15}\) m, and the dimensionless factor \(f_E\) is of the order of unity. In the independent-particle nuclear model used in this work the energy of interaction \(V_{12}^{(EL)}\) is not proportional to the proton number \(Z\).

The energy of interaction between two magnetic multipoles of order \(L\) can be estimated as

\[ V_{12}^{(ML)} = f_M \left( \frac{\hbar}{m_p c r_A} \right) \frac{3e^2 r_A}{(4\pi)^{1/2} \epsilon_0 (2L + 1)(L + 3)} \langle n'^l' J' | \left( \frac{m_e^2 c^2 r^2}{\hbar^2 + 1} \right)^{-1/2} r^{L-1} | nl_J \rangle, \]  \(\text{(11)}\)

where \(m_e\) is the electron mass, \(m_p\) the proton mass, and the dimensionless factor \(f_M\) has values of a few units. The factor \(\hbar/m_p c r_A\) is of the order of \(v_p/c\), where \(v_p\) is the proton velocity, and gives approximately the ratio between the magnetic and the electric multipole moments. \[\text{[27]}\] and the factor \(\left( m_e^2 c^2 r^2 / \hbar^2 + 1 \right)^{-1/2}\) is of the order of \(v_e(r)/c\), where \(v_e(r)\) is the electron velocity at a distance \(r\) from the origin. For \(r \gg \hbar/m_e c r\) the factor \(m_e^2 c^2 r^2 / \hbar^2 + 1)^{-1/2}\) is approximately equal to \(\hbar/m_e c r\), while for \(r \to 0\) it converges to 1.

The total transition probability in the electric case is then

\[ P_{\text{tot}}^{(EL)} = \frac{\Gamma_1 + \Gamma_2 (2J' + 1)|V_{12}^{(EL)}|^2}{\Delta^2}, \]  \(\text{(12)}\)

and the total transition probability in the magnetic case is

\[ P_{\text{tot}}^{(ML)} = \frac{\Gamma_1 + \Gamma_2 (2J' + 1)|V_{12}^{(ML)}|^2}{\Delta^2}. \]  \(\text{(13)}\)

In this work we shall assume that \(f_E = 1, f_M^2 = 10\), as in the Weisskopf estimate of gamma-ray transition rates. \[\text{[27]}\] As in the case of the Weisskopf estimates, the probabilities in Eqs. (12), (13) may differ from the real transition probabilities by about two orders of magnitude.
The cross section of an XENDT is then
\[ \sigma = \sigma_{\text{hole}} P_{\text{tot}}, \]  
where \( \sigma_{\text{hole}} \) is the cross section for the production of the initial hole in the atomic subshell \( nlJ \).

We shall estimate \( \sigma_{\text{hole}} \) according to Gryziński [28] as
\[ \sigma_{\text{hole}} = \frac{\pi e^4 (2J + 1)}{16 \pi^2 c_0^2 E_{nlJ}^2} g_i(E_{el}/E_{nlJ}), \]  
where \( E_{el} \) is the energy of the ionizing electron, and where the expression of the function \( g_i \) is given in ref. [28]. We shall assume that \( E_{el} = 1.6E_{nlJ} \), and we have \( g_i(1.6) = 0.109 \).

In the case of a direct nuclear transition from state \( |i\rangle \) to state \( |l\rangle \) the XENDT rate is
\[ R_I = \sigma N_{el}, \]  
where \( N_{el} \) is the number of ionizing electrons per unit surface and unit time. The relative deexcitation rate for an isomeric state of half-life \( t_i \) is then
\[ T_I = \sigma N_{el} t_i / \ln 2. \]  
In the case of a two-step nuclear transition \( |i\rangle \rightarrow |h\rangle \rightarrow |l\rangle \), the induced deexcitation rate of the state \( |i\rangle \) is
\[ R_{II} = \sigma N_{el} B, \]  
where \( B \) is the branching ratio for the gamma-ray transition \( |h\rangle \rightarrow |l\rangle \). We have determined \( B \) from the Weisskopf estimate of the radiative widths \( \Gamma_{hi}, \Gamma_{hl} \) and from the internal conversion coefficients \( \alpha_{hi}, \alpha_{hl} \) of the transitions \( |h\rangle \rightarrow |i\rangle, |h\rangle \rightarrow |l\rangle \) as
\[ B = \frac{(1 + \alpha_{hl})\Gamma_{hl}}{(1 + \alpha_{hi})\Gamma_{hi} + (1 + \alpha_{hl})\Gamma_{hl}}. \]  
The relative deexcitation rate for an isomeric half-life \( t_i \) is then
\[ T_{II} = \sigma N_{el} B t_i / \ln 2. \]  
The induced gamma emission becomes significant when the induced and natural decay rates are equal, so that \( T_I = 1 \) or \( T_{II} = 1 \). In the case of a direct deexcitation process the energy flux \( \Phi_I \) of the ionizing electrons for which \( T_I = 1 \) is
\[ \Phi_I = \frac{\ln 2 E_{el}}{\sigma t_i}. \]
In the case of a two-step deexcitation process the energy flux $\Phi_{II}$ of the ionizing electrons for which $T_{II} = 1$ is

$$\Phi_{II} = \frac{\ln 2 E_{el}}{\sigma t_i B}$$

(22)

In Table I we have given the results of calculations on the direct deexcitation of isomeric nuclei induced by XENDTs, for several isomeric nuclides having a half-life $t_i > 10$ minutes, and for which there is a downward transition $|i\rangle \rightarrow |l\rangle$ of energy $E_{il} < 100$ keV. We have used the total atomic level widths as given by Keski-Rahkonen and Krause, [29] which include the radiative width, the Auger width and the Coster-Kronig width. In order to evaluate the matrix elements in Eqs. (10) and (11) we have represented the atomic states $|nlJ\rangle, \ |n'l'p\rangle$ by screened hydrogenic wave functions. The values of the screening constants have been determined according to the rules of Slater. [30]

In Tables I, II and III we have used the values of the gamma-ray transition energies, energy levels and half-lifes adopted by the IAEA Nuclear Data Information System, and for the atomic energy levels we have used the values listed by Lederer et al. [31] We see from Table I that for direct induced emission the lowest values of the ionizing electron energy flux $\Phi_I$ are of the order of $10^{15}$ W cm$^{-2}$, for $^{174m}\text{Lu}$ and $^{99m}\text{Tc}$.

In Table II we have given the results of calculations on the two-step deexcitation of isomeric nuclei induced by XENDTs, for several isomeric nuclides having a half-life $t_i > 10$ minutes, and for which there is an upward transition $|h\rangle \rightarrow |i\rangle$ of energy $E_{hi} < 100$ keV. We have used the theoretical internal conversion coefficients of Band et al. [32] and of Rössel et al. [33] The values of the branching ratio $B$ are calculated according to Eq. (19). We see from Table II that for the two-step induced emission the lowest values of the ionizing electron energy flux $\Phi_{II}$ are of the order of $10^{14}$ W cm$^{-2}$, for $^{174m}\text{Lu}$ and $^{188m}\text{Re}$.

In the case of $^{99m}\text{Tc}$ the energy of the isomeric transition is $E_{il}=2.1$ keV, and the $^{99}\text{Tc}$ nucleus makes a transition from the $|l\rangle$ level to the ground state with the emission of an 140.5 keV photon. In the case of $^{188m}\text{Re}$ the energy of the upward transition is $E_{hi}=10.7$ keV, then the $^{188}\text{Re}$ nucleus emits 26.7 keV, 63 keV, 156 keV photons. In the case of $^{174m}\text{Lu}$ the energy of the upward transition is $E_{hi}=29.1$ keV, then the $^{174}\text{Lu}$ nucleus emits 44.7 keV, 67.1 keV, 88.2 keV photons. Thus, the emission of gamma rays induced by XENDT may be regarded in these cases as an upconver-
sion of the incident electron energy. The efficiency of the upconversion process is however very low.

A preliminary step in the direction of the gamma-ray emission from nuclear isomers induced by XENDTs would be the generation of resonant Mössbauer gamma radiation by XENDTs. In this case, a transition of a nucleus to an excited state is induced by an electron transition to a hole in an inner atomic shell, the hole being produced by incident ionizing electrons. The nucleus thus excited then emits a gamma-ray photon whose energy is narrowly centered on the nuclear transition energy, and can be used for regular Mössbauer experiments. This type of Mössbauer source would be active only as long as it is excited by the incident electrons.

If ionizing electrons of suitable energy are incident on a thin layer of surface $S$ and thickness $d_e$ containing the XENDT nuclei, the number $N_\gamma$ of resonant gamma-ray photons emitted per second by the nuclei in the layer is

$$N_\gamma = \frac{IN_{d_e}}{(1 + \alpha)e}\sigma,$$

where $I$ is the incident electron current, $e > 0$ is the electron charge, $N$ the concentration of XENDT nuclei in the foil, and $\alpha$ the internal conversion coefficient for the transition under study. The number of Mössbauer gamma-ray photons generated per second via XENDTs per unit of incident electron current is then $N_\gamma/I$. We have estimated the thickness $d_e$ as

$$d_e = a\frac{A}{Z\rho}E_{nlJ}^2,$$

where $a = 5.63 \cdot 10^{-3}$ when $d_e$ is in $\mu$m, the density $\rho$ of the target in g cm$^{-3}$ and $E_{nlJ}$ in keV, $A$ and $Z$ being the mass number and the proton number for the nuclei in the target.

In Table III we have listed the stable nuclides and the nuclides having a half-life greater than 100 days for which the detuning is $|\Delta| < 10$ keV. We have also required that the spectral gamma-ray intensity at the center of the Mössbauer line produced by XENDT should be greater than the spectral intensity of the bremsstrahlung generated by the ionizing electrons, and should also be greater than the spectral intensity due to non-resonant X-ray emission. We have used the X-ray emission rates calculated by Scofield. The afore-mentioned conditions are fulfilled by E1 and M1 transitions when there is a matching between the X-ray transition energy and the nuclear transition energy. We have evaluated the induced Mössbauer activity per unit of incident electron
current $N_\gamma/I$ for a target concentration $N = 10^{22}$ nuclides cm$^{-3}$. The nuclides in Table III which have a significant recoilless fraction at room temperature are $^{119}$Sn, $^{161}$Dy, $^{189}$Os, $^{193}$Ir. For the E1 and M1 transitions listed in Table III, the energy of interaction $V_{12}$ is of the order of 1 eV, the probability $P_{tot}$ of the XENDT process is in the $10^{-7} \cdot 10^{-4}$ range, the cross section of the XENDT process with respect to the incident ionizing electrons is in the range $10^{-30} \cdots 10^{-28}$ cm$^2$, and the Mössbauer activity per unit of incident electron current $N_\gamma/I$ is of the order of $10^4$ Bq/mA. This would be a rather weak continuous gamma-ray source, but the interesting property of such a Mössbauer source is that it can produce pulses of Mössbauer radiation.

We have assumed so far that the holes in the atomic shells are produced by electrons. The generation of Mössbauer radiation by XENDTs can be also be studied with incident protons having an energy of several MeV. The smaller cross sections for the generation of the holes in the atomic shells are compensated by the larger proton range in the target.

In this paper we have investigated the possibility of inducing nuclear deexcitation rates comparable to the natural emission rates from nuclear isomeric states by using the relatively large energy of interaction extant between the nuclear protons and the inner atomic electrons. The calculations of the present work show that significant induced-emission rates could be obtained for ionizing energy fluxes of the order of $10^{14}$ W cm$^{-2}$. An application of XENDTs which requires lower electron fluxes is the generation for certain elements of pulses of Mössbauer gamma radiation.

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FIGURE CAPTIONS

Fig. 1. Nuclear paths for the deexcitation of an isomeric state by electron transition. The two possibilities are the direct deexcitation from the isomeric state $|i\rangle$ to a lower state $|l\rangle$ via a nuclear deexcitation by electron transition, and the two-step deexcitation, when the nucleus goes from the isomeric state $|i\rangle$ to a higher state nuclear state $|h\rangle$ via a nuclear excitation by electron transition, then it decays to the lower state $|l\rangle$ by regular gamma-ray emission.

Fig. 2. The two cases of XENDT. (a) Nuclear deexcitation by electron transition (NDET), when an electron from an inner shell makes a transition to fill a hole in a higher electron shell. (b) Nuclear excitation by electron transition (NEET) when an electron from a higher electron shell makes a transition to fill a hole in an inner shell. The subscripts 1, 2 indicate the initial state and respectively the final state. The higher electron shells are represented by the upper lines in the atomic level schemes. The arrows in the atomic schemes suggest the path of the electron which fills the hole in the initial state.
Table I. Direct deexcitation of isomeric nuclei induced by XENDT.

| nuclide  | $E_{el}$, keV | $EL/ML$ | $t_i, s$ | $nI_J$ | $n'J', \nu_J$ | $V_{12}$, eV | $\Delta$, keV | $P_{tot}$ | $\sigma$, cm$^2$ | $E_{el}$, keV | $\Phi_I$, W cm$^{-2}$ |
|----------|---------------|---------|----------|--------|---------------|-------------|-------------|-----------|----------------|--------------|------------------|
| $^{85}$Sr | 6.96          | E3      | 4.06E3   | 3d$_{5/2}$ | 2p$_{3/2}$   | 3.27E-9     | 5.15        | 3.12E-23  | 7.50E-41      | 0.213        | 7.76E19          |
| $^{86}$Y | 10.2          | E3      | 2.88E3   | 3d$_{5/2}$ | 2p$_{3/2}$   | 4.04E-9     | 8.23        | 2.06E-23  | 3.51E-41      | 0.253        | 2.78E20          |
| $^{99}$Tc| 2.17          | E3      | 2.16E4   | 3d$_{5/2}$ | 2p$_{3/2}$   | 9.47E-9     | -0.251      | 6.30E-20  | 4.19E-38      | 0.405        | 4.96E16          |
| $^{162}$Ho| 10            | E3      | 4.02E3   | 4d$_{5/2}$ | 2p$_{3/2}$   | 3.97E-8     | 2.09        | 2.69E-21  | 4.42E-39      | 0.258        | 1.61E18          |
| $^{174}$Lu| 59.1          | M3      | 1.23E7   | 4d$_{5/2}$ | 1s$_{1/2}$   | 9.73E-8     | -4.04       | 6.89E-21  | 7.63E-39      | 0.314        | 3.72E14          |
| $^{188}$Re| 2.63          | M3      | 1.12E3   | 4f$_{5/2}$ | 3p$_{1/2}$   | 4.40E-12    | -0.009      | 6.25E-23  | 1.44E-39      | 0.069        | 4.76E18          |
Table II. Two-step deexcitation of isomeric nuclei induced by XENDT.

| nuclide | $E_{it}$, keV | $E_{hi}$, keV | $E_{hl}$, keV | $B$ | $nI_J$ | $n'I'J'$ | $V_{12}$, eV | $\Delta$, keV | $P_{tot}$, cm$^2$ W cm$^{-2}$ |
|---------|---------------|---------------|---------------|-----|--------|-------------|-------------|--------------|-----------------|
| $^{58}$Co | 24.9 | M3 | 3.29E4 | 28.3 | M1 | 52.9 | E2 | 2.90E-5 | 1s$_{1/2}$ | 2s$_{1/2}$ | 6.52E-2 | -21.5 | 9.16E-11 | 2.1 |
| $^{99}$Tc | 142.6 | M4 | 2.16E4 | 38.4 | M2 | 181.1 | E2 | 1 | 1s$_{1/2}$ | 2p$_{3/2}$ | 1.50E-4 | -20.1 | 2.88E-16 | 9.2 |
| $^{111}$Cd | 150.8 | E3 | 2.91E3 | 20.5 | M2 | 171.3 | M1 | 1 | 1s$_{1/2}$ | 2p$_{3/2}$ | 2.32E-4 | 2.69 | 3.75E-14 | 7.4 |
| $^{133}$Ba | 275.9 | M4 | 1.40E5 | 2.93 | E3 | 278.8 | M1 | 1 | 2p$_{3/2}$ | 3d$_{3/2}$ | 6.78E-8 | 1.52 | 9.97E-21 | 1.0 |
| $^{162}$Ho | 10.0 | E3 | 4.02E3 | 65.7 | M2 | 75.6 | M1 | 1 | 1s$_{1/2}$ | 2p$_{3/2}$ | 8.66E-4 | -18.1 | 1.03E-14 | 4.7 |
| $^{174}$Lu | 59.1 | M3 | 1.23E7 | 29.5 | E2 | 88.5 | M1 | 1 | 2p$_{3/2}$ | 3p$_{3/2}$ | 1.02E-3 | -22.2 | 2.78E-14 | 9.2 |
| $^{186}$Re | 50 | E5 | 6.31E12 | 37 | M2 | 86.6 | M3 | 5.34E-6 | 1s$_{1/2}$ | 2p$_{3/2}$ | 1.34E-3 | 24.1 | 1.37E-14 | 3.7 |
| $^{188}$Re | 15.9 | M3 | 1.12E3 | 10.7 | E2 | 26.7 | M1 | 1 | 2s$_{1/2}$ | 3d$_{5/2}$ | -9.79E-5 | -0.026 | 1.07E-10 | 9.6 |
| $^{191}$Os | 74.4 | M3 | 4.72E4 | 57.6 | M1 | 131.9 | E2 | 2.26E-4 | 1s$_{1/2}$ | 2s$_{1/2}$ | 8.54E-1 | 3.34 | 1.55E-7 | 4.0 |
| $^{202}$Pb | 129.5 | E4 | 1.27E4 | 38.6 | E2 | 168.1 | E2 | 0.829 | 2p$_{3/2}$ | 3p$_{3/2}$ | 1.82E-3 | -28.6 | 4.31E-14 | 7.2 |
Table III. Emission of Mössbauer radiation induced by XENDT.

| nuclide | nat. ab./half-life, s | $E_\gamma$, keV | $E_L$, ML | half-life, s | $n_{lJ}$ | $n'_{l'J}$ | $V_{12}$, eV | $\Delta$, keV | $P_{tot}$ | $\sigma/(1+\alpha)$, cm$^2$ | $E_{el}$, keV | $B$ |
|---------|----------------------|-----------------|-----------|-------------|---------|----------|-------------|----------|---------|----------------|--------------|-----|
| $^{119}$Sn | 8.6 % | 23.9 | M1 | 1.80E-8 | $1s_{1/2}$ | $2s_{1/2}$ | 0.316 | 0.86 | 4.30E-7 | 7.16E-30 | 46.7 | 1 |
| $^{129}$I | 4.95E14 | 27.8 | M1 | 1.68E-8 | $1s_{1/2}$ | $2s_{1/2}$ | 0.364 | 0.17 | 1.16E-5 | 1.49E-28 | 53.1 | 5 |
| $^{152}$Eu | 4.27E8 | 89.8 | E1 | 3.84E-7 | $1s_{1/2}$ | $2p_{1/2}$ | 1.85 | -48.9 | 3.38E-9 | 2.04E-32 | 77.6 | 5 |
| $^{154}$Eu | 2.71E8 | 68.2 | E1 | 2.2E-6 | $1s_{1/2}$ | $2p_{1/2}$ | 1.86 | -27.3 | 1.10E-8 | 6.62E-32 | 77.6 | 5 |
| $^{153}$Gd | 2.09E7 | 41.5 | M1 | 4.08E-9 | $1s_{1/2}$ | $2s_{1/2}$ | 0.571 | 0.30 | 8.64E-6 | 4.85E-29 | 80.4 | 2 |
| $^{157}$Gd | 15.7 % | 63.9 | E1 | 4.6E-7 | $1s_{1/2}$ | $2p_{1/2}$ | 1.94 | -21.6 | 1.10E-2 | 6.06E-31 | 80.4 | 2 |
| $^{161}$Dy | 18.9 % | 25.6 | E1 | 2.91E-8 | $1s_{1/2}$ | $2p_{1/2}$ | 2.08 | 19.6 | 2.64E-8 | 1.29E-31 | 86.1 | 2 |
| $^{161}$Dy | 18.9 % | 43.8 | M1 | 8.3E-10 | $1s_{1/2}$ | $2s_{1/2}$ | 0.614 | 0.92 | 1.08E-6 | 5.28E-30 | 86.1 | 2 |
| $^{179}$Ta | 5.65E7 | 30.7 | E1 | 1.42E-6 | $1s_{1/2}$ | $2p_{1/2}$ | 2.67 | 25.6 | 2.47E-8 | 7.71E-32 | 107.8 | 6 |
| $^{181}$Ta | 99.9 % | 6.24 | E1 | 6.05E-6 | $2p_{3/2}$ | $3s_{1/2}$ | 0.0968 | 0.94 | 1.09E-7 | 3.17E-29 | 15.8 | 5 |
| $^{189}$Os | 16.1 % | 69.6 | M1 | 1.62E-9 | $1s_{1/2}$ | $3s_{1/2}$ | 0.413 | 1.28 | 2.93E-7 | 7.63E-31 | 118.2 | 3 |
| $^{193}$Ir | 62.7 % | 73.0 | M1 | 6.09E-9 | $1s_{1/2}$ | $3s_{1/2}$ | 0.426 | -0.10 | 4.93E-5 | 1.21E-28 | 121.8 | 4 |
| $^{195}$Au | 1.61E7 | 61.5 | M1 | 3.0E-9 | $1s_{1/2}$ | $2s_{1/2}$ | 0.934 | 4.91 | 8.79E-8 | 1.91E-31 | 129.2 | 8 |
| $^{197}$Au | 100 % | 77.3 | M1 | 1.91E-9 | $1s_{1/2}$ | $3s_{1/2}$ | 0.454 | -0.054 | 1.97E-4 | 4.29E-28 | 129.2 | 8 |
| $^{231}$Pa | 1.03E12 | 84.2 | E1 | 4.51E-8 | $1s_{1/2}$ | $2p_{1/2}$ | 4.62 | 8.07 | 7.22E-7 | 8.07E-31 | 180.2 | 4 |
| $^{237}$Np | 6.77E13 | 102.9 | E1 | 8.0E-10 | $1s_{1/2}$ | $2p_{3/2}$ | 4.88 | -1.91 | 2.83E-5 | 2.85E-29 | 189.9 | 3 |
direct deexcitation

two-step deexcitation
(a) NDET

| $n_1 >$ | $E_1^{(n)}$ | $|a_1 >$ | $E_1^{(a)}$
|---------|-------------|---------|-------------
| $n_2 >$ | $E_2^{(n)}$ | $a_2 >$ | $E_2^{(a)}$

nucleus

atomic shell

(b) NEET

| $n_2 >$ | $E_2^{(n)}$ | $a_2 >$ | $E_2^{(a)}$
|---------|-------------|---------|-------------
| $n_1 >$ | $E_1^{(n)}$ | $a_1 >$ | $E_1^{(a)}$

nucleus

atomic shell