Optimization Design of Lens Roll-Pin Flexible Support Structure

Jun Li$^{1,2,3}$ and Yi Chen$^{1,2,*}$

$^1$National Astronomical Observatories/Nanjing Institute of Astronomical Optics & Technology Chinese Academy of Sciences, 188 Bancang Street, Nanjing 210042, P. R. China
$^2$CAS Key Laboratory of Astronomical Optics & Technology, Nanjing Institute of Astronomical Optics & Technology, Nanjing 210042, China
$^3$University of Chinese Academy of Sciences, Beijing 100049, China

*Corresponding author e-mail: ychen@niaot.ac.cn

Abstract. There is a relative contradiction between the stiffness requirement and the size and weight of the lens roll-pin flexible support structure. Aim to solve this contradiction, the mechanical model of the roll-pin flexible element was established, and the radial and tangential stiffness of the roll-pin flexible element were theoretically deduced and verified by finite element method by using cartesian theorem. Then, based on the stiffness formula, the optimization design model with geometric configuration parameters was constructed, and the optimization design model was applied to a lens with a diameter of 400mm. The results showed that the simulation was basically consistent with the theoretical design. Finally, based on the optimized design, the improved design of the circular roll-pin flexible structure was carried out and the finite element analysis was completed. The results show that the weight of the improved design is reduced by 10%, the maximum contact stress is reduced by 10%.

Keywords: roll-pin flexible support structure, mechanical model, optimized design, finite element simulation, improved design.

1. Introduction
The aperture of the optical lens of the spectrograph camera increases with the aperture of the telescope [1]. Then, a key problem has shown in the support of large diameter lens element with maintaining the assurance of shape accuracy. With the development of EDM, Stephen a. Smee et al. developed A roller flexible support structure [2]. The structure determines the radial preload value by precisely controlling the radial outline size without using elastic binder. However, the roller flexible support structure is more complex than other structures, it is difficult to match the optimal solution when designing the optical machine structure, which will lead to the increase of the size of the optical machine system and the waste of materials.

Therefore, through the mechanical modeling of the roller flexible installation structure, the mathematical model of optimal design is established after analyzing the factors affecting the stiffness of the structure and finite element verification. In addition, taking 400mm camera lens as an example, while verifying the model, the roller flexible support structure is further improved to reduce the optimal size and weight and further improve the performance.
2. Mechanical modeling of roller flexible support structure

2.1. Mechanical model and stiffness analysis of flexible element

The roller type flexible support structure and its element unit are shown in Fig. 1. It shows that the optical lens is supported by a uniform roller flexible unit at the edge with a certain interference. Therefore, in this article, the roller flexible structural element is simplified into a cantilever beam mechanical model with shape parameters and designed and analyzed based on it.

As shown in Fig. 2, the roller flexible element is simplified into a cantilever plane curved beam structure. It can be seen from the simplified model that the displacement of the curved cantilever beam is more complicated than that of the single cantilever beam model when the element is subjected to the radial force from the lens. Therefore, the cantilever curved beam can be decomposed into three sections as shown in Fig. 2 (b), (c) and (d), and solved respectively.

![Figure 1. Roll-pin flexure lens mounting.](image-url)

![Figure 2. Mechanical model and decomposition of flexible unit. (a) Simplified cantilever beam of flexible cell; (b) First stage decomposition of flexible cell; (c) Second stage decomposition of flexible cell; (d) Third stage decomposition of flexible cell.](image-url)

Now, the cylinder diameter of the flexible element unit is set as \( D \), the radius is \( R \), the thickness of the beam is \( t \), the width outside the surface is \( L \), and the opening Angle is \( \theta \). Firstly, the radial displacement and tangential displacement of segment 2 (b) at point A are solved.

Assuming that the radial concentrated force acting on point A is \( F_r \) to cause the radial displacement, and the tangential concentrated force acting on point A is \( F_t \) to cause the tangential displacement, then the bending moment and its partial derivative on any section of the curved beam are
\[ M_r = F_r R \cos \theta_A; \quad \frac{\partial M_r}{\partial F_r} = R \cos \theta_A \]  

(1)

\[ M_t = F_t R (1 - \sin \theta_A); \quad \frac{\partial M_t}{\partial F_t} = R (1 - \sin \theta_A) \]  

(2)

According to the cartesian principle in material mechanics, that is, the partial derivative of strain energy with respect to any displacement is equal to the load applied in the direction of the displacement, the radial displacement and the tangential displacement of point A can be obtained as

\[ \delta_A = \int_S \frac{M_r}{EI} \frac{\partial M_r}{\partial F_r} ds = \frac{1}{EI} \int_0^\pi F R \cos \theta_A R \cos \theta_A R d\theta_A = \frac{\pi F R^3}{4 E I} \]  

(3)

\[ \varphi_A = \int_S \frac{M_t}{EI} \frac{\partial M_t}{\partial F_t} ds = \frac{1}{EI} \int_0^\pi F_t R (1 - \sin \theta_A) (1 - \sin \theta_A) R d\theta_A = \frac{(3 \pi - 8) F R^3}{4 E I} \]  

(4)

While \( E \) is the elastic modulus of the element material and \( I \) is the moment of inertia of its section. In this article \( I = L t^3 / 12 \).

According to the same method, the displacement of B and C point can be obtained respectively, so as to obtain the total displacement. Finally, Hooke's law can be used to obtain that the radial stiffness \( K_r \) and tangential stiffness \( K_t \) of the roller flexible element

\[ K_r = \frac{F_r}{\delta_{all}} = \frac{16}{27 \pi - 54} E L \left( \frac{t}{D} \right)^3 \]  

(5)

\[ K_t = \frac{F_t}{\varphi_{all}} = \frac{32}{\left(72\pi + (9\pi + 24\sqrt{2} + 6)\right)} E L \left( \frac{t}{D} \right)^3 \]  

(6)

2.2. Stress analysis of flexible element

Similarly, the flexible element unit is subjected to piecewise stress analysis as shown in Fig. 2. In section AB of Fig. 2 (b), it can be seen from the formula of bending moment that the bending stress at point b should be the maximum. The radial and tangential bending stresses are considered respectively

\[ \sigma_B = \sigma_{RB} - \sigma_{TB} = \frac{M_{rB} Y}{l} - \frac{M_{tB} Y}{l} = \left( \frac{8}{\pi} \delta_A - \frac{8}{3 \pi - B} \varphi_A \right) E \frac{t}{D^2} \]  

(7)

\[ \sigma_C = \sigma_{tC} = \frac{8}{3 \pi + B} \varphi_B E \frac{t}{D^2} \]  

(8)

\[ \sigma_D = \sigma_{rD} + \sigma_{tD} = \left( \frac{8 \sqrt{2}}{\pi - 2} \delta_C + \frac{8}{\pi + 2} \varphi_C \right) E \frac{t}{D^2} \]  

(9)

While \( y \) is the distance from the point on the section to the neutral axis.

It should be noted that the bending stress here is calculated in terms of the small curvature beam, which requires \( t/D < 0.1 \). Otherwise, the large curvature beam formula should be used. In general, the maximum stress of the flexible element occurs at point B or point D, depending on the ratio of the load \( F_r/F_t \) applied by the lens on the element and on the stiffness of the element in both directions.

3. Optimization design model of roller flexible support structure

From the point of precision system structure design method, the design need to satisfy the following conditions: maximum stress of each flexible unit to meet the strength condition, each interface is the on the lens and the contact stress of the birefringence in lens caused by contrast, the face of the lens shape accuracy can meet the requirements, the lens centering accuracy should meet the requirements, and these requirements are in the worst environment conditions. However, from the perspective of practical engineering design, the surface shape precision of the lens is related to the contact stress, and the centering precision of the lens can also be adjusted by micro-adjustment mechanism. Therefore, the optimization model of roll-pin flexible support structure can be expressed as
\[
\begin{align*}
\min & \quad \phi(D, t, L, n) \\
\text{s.t.} & \quad \begin{cases}
K_{\text{all}} \delta_g \geq G \\
\sigma_g \leq \sigma_{\text{max}}
\end{cases}
\end{align*}
\] (10)

While \( \phi \) is the objective function, \( K_{\text{all}} \) is the overall stiffness of the flexible unit. According to the derivation and verification of literature [3], \( K_{\text{all}} = 0.65n(K_t + K_r)/2 \), \( n \) is the number of flexible units, \( \delta_g \) is the vertical displacement of the lens under the action of gravity, \( \sigma_{\text{max}} \) is the maximum allowable stress for structural materials, \( \sigma_w \) and \( \sigma_{\text{wmax}} \) are the stress at the contact surface and the birefringence limit stress of the lens respectively.

In practical engineering design, the most dangerous flexible element is usually used as the basis for optimal design. However, in the uniformly distributed flexible support structure, the most dangerous element is usually the one perpendicular to the gravitational direction of the lens and at the lower end, where the radial load is the largest. Therefore, according to the method of designing flexible support structure for the most dangerous element, the optimization model can be modified to the form of equation (11).

While \( \delta_{\Delta T} \) is the relative displacement between the lens and the supporting structure under the maximum design temperature difference, \( \varepsilon \) is the interference of the preload, \( \delta_{\text{max}} \) is the maximum radial displacement of the flexible element, \( w \) is the length of the contact line between the flexible element and the lens, \( S \) is the safety factor.

It should be noted that due to the relatively small tangential load, the maximum stress is still designed according to point B. However, in the actual situation, the stress value difference between point B and point D is very small, and the maximum stress point will often appear at point D.

4. Simulation and optimization
A 400mm lens is used for example analysis and verification. In optical lenses with flexible support, 6061 aluminum alloy is usually used for easy machining in order to reduce the overall support weight. And lens material here chooses commonly used BK7 optical glass. Detailed parameters of the two materials are shown in Table 1.

| Material | Density/kg \( \cdot \) mm\(^{-3} \) | Modulus of elasticity /Gpa | \( \sigma_s \) /Mpa | CTE/ \( ^{\circ} \text{C}^{-1} \) |
|----------|----------------------------------|-----------------|-----------------|-----------------|
| 6061-T651 | 2.75 \times 10^6                 | 68.9            | 228             | 2.3 \times 10^{-5} |
| BK7      | 2.51 \times 10^6                 | 82              | 3.4             | 7.1 \times 10^{-6} |

According to the optimization design model proposed in equation (11), the relevant parameters should first be obtained by using 3d simulation modeling and finite element simulation, so as to carry out the optimization design. Table 2 shows the values of related parameters after preprocessing. The
initial frame structure parameters are $t/D=1/12$, $L=20\text{mm}$, $n=8$, and the ambient temperature is from $22\, ^\circ\text{C}$ to $-30\, ^\circ\text{C}$.

Table 2. Preprocess related parameter values.

| G/N  | $\delta_T/\text{mm}$ | $\varepsilon/\text{mm}$ | $w/\text{mm}$ |
|------|----------------------|-------------------------|--------------|
| 150.79 | 0.1688               | 0.05                   | 3.69         |

By substituting the values of pre-processed related parameters into the optimal design model of equation (11), the optimal parameter values of roll-pin flexible can be obtained by taking the safety coefficient $S=1.6$ and using the real number programming program of Lingo software. Since the structural parameter variables of roll-pin flexible unit have thickness $t$ and diameter $D$, the optimal parameter value is expressed as a functional relationship between $t$ and $D$, as shown in Fig. 3.

For example, a flexible structural element with a thickness of $t=1\text{mm}$ is selected for thermally coupled finite element analysis. The corresponding cell diameter $D$ is equal to $16.4\text{mm}$ after multiplying under the safety factor of 1.6, and $D$ is equal to $17\text{mm}$. In order to simulate the actual application scene as much as possible, the picture frame adopts the boundary condition of circumferential support and releases its radial freedom. The contact between the lens and the element body adopts the nonlinear frictional contact type, the friction coefficient is 0.15, and the ambient temperature is reduced from $22\, ^\circ\text{C}$ to $-30\, ^\circ\text{C}$. The results of finite element simulation of optimal structural parameters are shown in Fig. 4.
From the results of finite element simulation, it can be seen that the maximum contact stress of the optimized optimal structural parameter lens is about 0.37MPa, and the maximum stress of the flexible element body is 154.58MPa, which all meet the allowable stress requirements and basically conform to the stress range of the safety factor. In addition, in order to verify the accuracy and reliability of the optimization model, the element body D=13mm without multiplying the safety coefficient is analyzed by finite element method. The finite element simulation results under the limit structure parameters are shown in Fig.5. It can be seen that the frame stress is very close to the allowable stress value of 228MPa under the limit structural parameters. Therefore, it can be considered that the optimization model has high precision and is sufficient to meet the requirements of engineering design.

5. Improved design based on optimization model

Now the roll-pin flexible unit is regarded as a generalized ellipse, and the design is improved by reducing the radial element length and increasing the tangential element length. Due to the limitation of allowable stress, the radial and tangential lengths have certain limits.

As can be seen from Fig. 6, with the decrease of radial length and the increase of tangential length, the maximum stress value of the roll-pin circular element tends to decrease first and then increase. When the radial value decreases by 2mm and the tangential value increases by 2mm, the maximum stress of the elliptic element body is basically restored to the previous level of the circular element body. Although the maximum stress value does not change much, after the change from the circular element to the elliptical element, not only the size and weight of the picture frame can be reduced, but also the contact stress of the lens can be reduced to a certain extent, and the ability to resist external disturbance can be improved. According to the calculation, if the picture frame size of 400mm decreases by 2mm in the radial direction and 2mm in the tangential direction, the weight loss ratio can be up to 10%, and the maximum contact stress of the lens can be reduced by 10%.
Figure 6. Improved design of lens and frame stress nephogram. (a) Stress nephogram of lens and frame with radial length reduced by 1mm and tangential length increased by 1mm; (b) Stress nephogram of lens and frame with radial length reduced by 2mm and tangential length increased by 2mm

6. Summary
In this article, aiming at the contradiction between the stiffness requirement and the size and weight of the lens-roller flexible support structure, the physical mechanics model of the flexible element with shape parameters and the mathematical model of the optimal design are established. The radial and tangential stiffness of the flexible element body are derived by simplifying the flexible unit into the cantilever beam. Taking the thickness and diameter of the unit as the objective function, the corresponding relation between the diameter and thickness of the minimum unit is calculated, and the finite element verification is carried out. The contradiction between the stiffness requirement and the size weight in the flexible support structure is preliminarily solved.

In addition, based on the correlation between stiffness and structural parameters, the roller flexible structure is elliptically designed to improve its overall performance. The results of finite element simulation show that the weight of the improved flexible support structure is reduced by 10% and the maximum contact stress of the lens is reduced by 10%.

Acknowledgement
This research was financially supported by the National Natural Science Foundation of China (NSFC). The number of NSFC is 11573048.
References
[1] Zhu Yongtian. High Resolution Spectrographs for 8~10m Class Optical/IR Telescope, J. Progress in Astronomy, 2001, 19(3): 337-345.
[2] Stephen A Smee. A Precision Lens Mount for Large Temperature Excursions, J. Proc of SPIE, 2010, 7739: 77393O.
[3] Cao Yuyan, Wang Zhichen, Zhou Chao, et al. General Modeling and Optical Design of Flexure Supporting Structure for Optical Components, J. Optics and Precision Engineering 2016, 24(11): 2793-2803.
[4] Yoder P R. Opto-Mechanical Systems Design, Third Edition, Macmillan, Lasers Optics & Photonics, 2005.
[5] Farah A, Tejada C, Gonzalez J, et al. OSIRIS camera barrel optomechanical design, J. Proc of SPIE, 2004, 5492: 880-890.