Multicomponent binary spreading process

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I investigate numerically the phase transitions of two-component generalizations of binary spreading processes in one dimension. In these models pair annihilation: $AA \rightarrow \emptyset$, $BB \rightarrow \emptyset$, explicit particle diffusion and binary pair production processes compete with each other. Several versions with spatially different productions have been explored and shown that for the cases: $2A \rightarrow 3A$, $2B \rightarrow 3B$ and $2A \rightarrow 2AB$, $2B \rightarrow 2BA$ a phase transition occurs at zero production rate ($\sigma = 0$), that belongs to the class of N-component, asymmetric branching and annihilating random walks, characterized by the order parameter exponent $\beta = 2$. In the model with particle production: $AB \rightarrow ABA$, $BA \rightarrow BAB$ a phase transition point can be located at $\sigma_c = 0.3253$ that belongs to the class of the one-component binary spreading processes.

One-dimensional, non-equilibrium phase transitions have been found to belong to a few universality classes, the most robust of them is the directed percolation (DP) class [2]. According to the hypothesis of [4] all continuous phase transitions to single absorbing states in homogeneous, single component systems with short ranged interactions belong to this class provided there is no additional symmetry and quenched randomness present. The most well known exception from the robust DP class is the parity conserving (PC) class [6], where a mod 2 conservation of particles happens (example in even offspring the parity conserving (PC) class [5], where a mod 2 conditional symmetry and quenched randomness present. The interactions belong to this class provided there is no additional symmetry and quenched randomness present. 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In this paper I extend the investigations to coupled binary production spreading processes, where new universal behavior has recently been reported. Studies on the annihilation fission process $2A \rightarrow \emptyset$, $2A \rightarrow 3A$, $A\emptyset \leftrightarrow \emptyset A$ found evidence that there is a phase transition in this model that does not belong to any previously known universality classes. This model without the single particle diffusion term – the so called pair contact process (PCP), where pairs of particles can annihilate or create new pairs – was introduced originally by Jensen [23] and while the static exponents were found to belong to DP class the spreading ones show non-universal behavior. By adding explicit single particle diffusion [24] Carlon et al. introduced the so called PCPD particle model. The renormalization group analysis of the corresponding bosonic field theory was given by Howard and Täuber [24]. This study predicted a non-DP class transition, but it could not tell to which universality class this transition really belongs. An explanation based on symmetry arguments are still missing but numerical simulations suggest [28,30] that the behavior of this system can be well described (at least for strong diffusion) by coupled sub-systems: single particles performing annihilating random walk coupled to pairs $(B)$ following DP process: $B \rightarrow 2B$, $B \rightarrow \emptyset$. The model has two non-symmetric absorbing states: one is completely empty, in the other a single particle walks randomly. Owing to this fluctuating absorbing state this model does not oppose the conditions of the DP hypothesis.

In the low diffusion region $(d < \sim 0.4)$ some exponents of the PCPD model are close to those of the PC class but the order parameter exponent $(\beta)$ has been found to be very far away from both of the DP and PC class values [25]. In fact this system does not exhibit neither a $Z_2$ symmetry nor a parity conservation that appear in models with PC class transition. In the high diffusion region the critical exponents seem to be different [28,29,30] suggesting an other universality class there [28]. This is also supported by the pair mean-field results [25]. A recent universal finite size scaling amplitude study [31] suggests however that a single universality class with strong corrections to scaling may also be possible.

It is conjectured by Henkel and Hinrichsen [32] that this kind of phase transition appears in models where (i) solitary particles diffuse, (ii) particle creation requires two particles and (iii) particle removal requires at least two particles to meet. Very recently Park et al. [33] have investigated the parity conserving version of the PCPD model $(2A \rightarrow 4A, 2A \rightarrow \emptyset, A\emptyset \leftrightarrow \emptyset A)$ and contrary the apparent conservation law they have found similar scaling behavior that led them to the assumption that the binary nature of the offspring production is a necessary condition for this class. Other conditions that would influence the occurrence of this class should be clarified too. In this paper I address the question whether the particle exclusion effects are relevant like in the case of BARW processes and whether the hypothesis set up for N-BARW systems [21] could be extended.

One site update step of the applied algorithms consist of the following processes. A particle is selected randomly. A left or right nearest neighbor is chosen with probability $0.5$. With probability $\sigma$ a pair production is attempted in case of an appropriate neighbor. Otherwise (with probability $d = \lambda = 1 - \sigma$) a hopping is attempted if the neighbor is empty or if the neighbor if filled with a particle of same type they are annihilated. The following models with the same diffusion and annihilation terms as (4) and different production processes will be investigated here.

a) Production and annihilation random walk model (2-PARW):

$$AA \xrightarrow{\sigma/2} AAB, \quad AA \xrightarrow{\sigma/2} BAA, \quad (3)$$

$$BB \xrightarrow{\sigma/2} BBA, \quad BB \xrightarrow{\sigma/2} ABB. \quad (4)$$

b) Symmetric production and annihilation random walk model (2-PARWS):

$$AA \xrightarrow{\sigma} AAA, \quad (5)$$

$$BB \xrightarrow{\sigma} BBB. \quad (6)$$

c) Asymmetric production and annihilation random walk model (2-PARWA):

$$AB \xrightarrow{\sigma/2} ABB, \quad AB \xrightarrow{\sigma/2} AAB, \quad (7)$$

$$BA \xrightarrow{\sigma/2} BAA, \quad BA \xrightarrow{\sigma/2} BBA. \quad (8)$$

d) Asymmetric production and annihilation random walk model with spatially symmetric creation (2-PARWAS):

$$AB \xrightarrow{\sigma/2} ABA, \quad AB \xrightarrow{\sigma/2} BAB, \quad (9)$$

$$BA \xrightarrow{\sigma/2} BAB, \quad BA \xrightarrow{\sigma/2} ABA. \quad (10)$$

The evolution of particle densities were followed by Monte-Carlo simulations started from randomly distributed $A, B, \emptyset$ sites in systems of sizes $L = 10^5$ and periodic boundary conditions.

The 2-PARWA (c) model does not have an active steady state. The $AA$ and $BB$ pairs annihilate themselves on contact, while if an $A$ and $B$ particle meets an $AB \rightarrow ABB \rightarrow A$ process reduces out blockades therefore the densities decay with the $\rho \propto t^{-1/2}$ law for $\sigma > 0$. This was confirmed by my simulations. Note that for $\sigma = 0$ the blockades persist and in case of random initial state a $\rho \propto t^{-1/4}$ decay can be observed [15].

The 2-PARW (a) and the 2-PARWS (b) models exhibit active steady states for $\sigma > 0$ with a continuous phase transition at $\sigma = 0$. Therefore the exponents at the critical point will be those of the ARW-2 model. The convergence to the steady state is very slow. For $\sigma = 0.1$
it was longer than $10^9$ Monte Carlo steps (MCS). This has limited the simulations by approaching the critical point at $\sigma = 0$. However as Figure 1 shows a rather good scaling behavior of the density versus $\sigma$ can be observed.

FIG. 1. Steady state densities as the function of $\sigma$ in the 2-PARW (squares) and in the 2-PARWS (circles) models.

The local slope analysis shows that the scaling behavior extrapolates to $\beta = 2.1(2)$ in case of the 2-PARWS model and to $\beta = 1.9(2)$ in case of the 2-PARW model. These values are in agreement with those of the 2-BARW2a class ($\beta = 2$), where production is such that pair annihilation is enhanced.

In case of the 2-PARWAS (d) model the $AB$ blockades proliferate by production events. As the consequence of this an active steady state appears for $\sigma > 0.3253(1)$ with a continuous phase transition. The space-time evolution from random initial state shows (Fig. 2) that compact domains of alternating ..$ABAB$.. sequences separated by lonely wandering particles are formed. This is very similar to what was seen in case of one-component binary spreading processes [3]: compact domains within a cloud of lonely random walkers, except that now domains are built up from alternating sequences only. This means that ..$AAAA$.. and ..$BBBB$.. domains decay at this annihilation rate and particle blocking is responsible for the compact clusters. In the language of coupled DP + ARW model [33] the pairs following DP process are the $AB$ pairs now, which cannot decay spontaneously but through an annihilation process: $AB + BA \rightarrow \emptyset$. They interact with two types of particles executing annihilating random walk with exclusion.

Simulations from random initial state were run up to $10^6$ MCS. The local slopes of the particle density decay

\[
\alpha_{\text{eff}}(t) = -\frac{\ln[\rho(t)/\rho(t/\ln m)]}{\ln(m)} \quad (11)
\]

(where $m = 8$ is used) at the critical point go to exponent $\alpha$ by a straight line asymptotically, while in sub(super)-critical cases they veer down(up) respectively.

FIG. 2. Space-time evolution from random initial state of the 2-PARW model at the critical point. Black dots correspond to $A$ particles, red dots to $B$-s.

FIG. 3. Local slopes of the density decay in the PARWAS model. Different curves correspond to $\sigma = 0.325, 0.3252, 0.3253, 0.3254, 0.3255, 0.326$ (from bottom to top).

At the critical point ($\sigma_c = 0.3253(1)$) one can estimate that the effective exponent tends to $\alpha = 0.19(1)$, which is higher than the the exponent of the 1+1 dimensional directed percolation $0.1595(1)$ [33] and in fairly good agreement with that of the PCPD model in the high diffusion rate region $0.20(1)$ [28].

In the supercritical region the steady states have been determined for different $\epsilon = \sigma - \sigma_c$ values. Following level-off the densities were averaged over $10^4$ MCS and 1000 samples. By looking at the effective exponent defined as

\[
\beta_{\text{eff}}(\epsilon_i) = \frac{\ln[\rho(\epsilon_i) - \ln \rho(\epsilon_{i-1})]}{\ln \epsilon_i - \ln \epsilon_{i-1}} \quad (12)
\]
one can read-off: $\beta_{\text{eff}} \rightarrow \beta \simeq 0.37(2)$, which is again higher than that of the 1+1 dimensional DP value 0.27649(4) \cite{37}, and agrees with that of the PCPD model in the high diffusion rate region (0.39(2)) \cite{28}.

Finally the survival probability ($P(t)$) of systems started from random initial condition was measured for sizes: $L = 50, 100, 200, 400, 800, 1600$. The characteristic time $\tau(L)$ to decay to $P(\tau) = 0.9$ was determined and shown on Figure 5. At criticality one expects the finite size scaling

$$\tau(L) \propto L^Z ,$$

where $Z$ is the dynamical exponent. The power-law fitting resulted in $Z = 1.81(2)$, which is far away from the DP value $Z = 1.580740(34)$ \cite{36} but close to various estimates for the PCPD value $Z = 1.75(10)$ \cite{36,27}.

In conclusion I have shown in this work that the hypothesis that I made for N-BARW models with exclusion \cite{21} may be extended for coupled binary production

annihilation models. The critical point in the 2-PARW and 2-PARWS models, where $AA$ and $BB$ pairs create offsprings, continuous phase transition occurs at $\sigma = 0$ production rate therefore the on-critical exponents coincide with those of the the 2-ARW model. The simulations for the off-critical behavior of the order parameter have shown that the transition belongs to the 2-BARW2a class. The robustness of this class is striking especially in case of the 2-PARWS model where in principle two copies of PCPD models are superimposed and coupled with the exclusion interaction only.

If the production is generated by different types of particles ($AB$) such that alternating sequences are generated (2-PARWAS model) the space-time evolution will resemble to the of the PCPD model with alternating frozen sequences inside the compact domains. This system exhibits a continuous phase transition at $\sigma = 0.3253(1)$ with exponents in fairly good agreement with those of the PCPD model in the high diffusion region. In the model where $AB$ pairs create offsprings in a such a way that prompt annihilation is possible active steady states are not formed for any $\sigma$ and the density decays without blockades for $\sigma > 0$ as $\rho \propto t^{-0.25}$ but a crossover to 2-ARW model scaling $\rho \propto t^{-0.5}$ occurs at $\sigma = 0$.

Acknowledgements:

The author thanks H. Chaté and P. Grassberger for their comments. Support from Hungarian research funds OTKA (No. T-25286) and Bolyai (No. BO/00142/99) is acknowledged. The simulations were performed on the parallel cluster of SZTAKI and on the supercomputer of NIIF Hungary.

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