On the release of binding energy and accretion power in core collapse-like environments

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Abstract. All accretion models of gamma-ray bursts share a common assumption: accretion power and gravitational binding energy is released and then dissipated locally, with the mass of its origin. This is equivalent to the Shakura-Sunyaev 1973 (SS73) prescription for the dissipation of accretion power and subsequent conversion into radiate output. Since their seminal paper, broadband observations of quasars and black hole X-ray binaries insist that the SS73 prescription cannot wholly describe their behavior. In particular, optically thick black hole accretion flows are almost universally accompanied by coronae whose relative power by far exceeds anything seen in studies of stellar chromospheric and coronal activity. In this note, we briefly discuss the possible repercussions of freeing accretion models of GRBs from the SS73 prescription. Our main conclusion is that the efficiency of converting gravitational binding energy into a GRB power can be increased by an order of magnitude or more.

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INTRODUCTION

A parcel of matter in a Keplerian orbit about a dominant central source of gravity has the potential to release some fraction of $\frac{GM}{Rc^2}$, multiplied by its rest mass energy via the act of accretion. The value of this fraction, often referred to as the accretion efficiency $\varepsilon$, depends upon the compactness of the massive central object. For a solar-type star $\varepsilon \sim 10^{-6}$, while for a white dwarf $\varepsilon \sim 10^{-3}$. Neutron stars and black holes – the most compact objects in the Universe – possess the largest values for the accretion efficiency, with $\varepsilon \sim 10^{-1}$.

The total energy release from the act of accretion $\Delta E$ is therefore equal to the total mass accreted $\Delta M$ multiplied by $\varepsilon$ i.e., $\Delta E = \varepsilon \Delta M$. It follows that the total accretion power $L_{\text{acc}}$ is given by $L_{\text{acc}} = \Delta E / \Delta t$, where $\Delta t$ is the characteristic accretion time. These simple considerations, along with realizing that gamma-ray bursts (GRBs) trigger at cosmological distances, provided workers to explain their large and rapid energy release by the accretion of $\Delta M \sim M_\odot$ of matter in time $\Delta t \sim$ a few seconds (Goodman et al. 1987; Narayan et al. 1992; Woosley 1993). The basic idea is that somehow, accretion power is partially converted into outgoing mechanical power of electronically interacting particles such as pairs and baryons (the so-called “fireball” model) or coherent photon states (so-called “Poynting flux dominated” jets).

We have yet to specify in which manner, or through which channels, gravitational
FIGURE 1. Accretion + fireball model of gamma ray bursts. The basic ingredients are a stellar mass black hole and a debris torus of non-negligible mass i.e., the mass of the disk is a significant fraction of a solar mass. The tidal radius of the disk $R_d \sim 10^3$ gravitational radii $R_g$, which is roughly the radius of a stellar mass object supported by relativistic electron degeneracy pressure (e.g., a white dwarf or burnt-out iron core). Under these rather extreme conditions, it is possible to generate $\varepsilon$\,\Delta\,M $\sim$ a few $\times 10^{53}$ ergs worth or accretion energy, where $\Delta\,M$ is given by the mass of debris torus.

binding energy is (A) initially randomized (B) how that randomized gravitational energy is converted into particle energy (C) and finally, the manner in which particle energy is then converted into radiation which ultimately removes binding energy from the system.

In the last 15 years, a tentative consensus – despite the lack of any concrete observational evidence – has been reached with respect (A) (Balbus & Hawley 1991; 1998). However, knowledge of (A) only informs us of the value of $\Delta t$ and $L_{\text{acc}}$ for a given energy reservoir $\Delta E$. Ultimately, the physical processes that determine (B) and (C) allow for theorists to infer the properties of the fraction of $\Delta E$ that goes into powering the highly relativistic (Lorentz factor $\Gamma \sim E/M \sim 100$) GRB outflow.

Admittedly, our conversation has been abstract. In the next few sections we become more concrete as we discuss the the basic features of hyper-Eddington accretion disks (HEDs) and then closely examine they key idea that all workers have assumed when considering their structure and applicability to GRBs.

HYPER-EDDINGTON DISKS AS THE CENTRAL ENGINES OF GRBS AND THE SHAKURA-SUNYAEV (1973) PRESCRIPTION.

Only a few plausible astrophysical scenarios can lead to a HED, as depicted in Figure 1. In recent years, the most popular progenitor model has been the “collapsar” model (Woosley 1993; MacFadyen & Woosley 1999), which is involves the progenitor of a core-collapse supernova (SN) whose degenerate iron core possesses an appreciable amount of angular momentum. Rather than forming a proto-neutron star, which are thought to power “normal” core-collapse SNe, some combination of a stellar mass BH
and a massive debris torus is formed. That is, matter initially lying near the progenitor’s axis rotation axis falls towards the system’s gravitational center, thus making a prompt contribution to the formation of a BH. Whereas, matter originating near the progenitor’s equator forms a disk whose tidal radius is determined by initial specific angular momentum of the debris before the onset of collapse.

The viscous time of the disk $\Delta t$ is given by

$$\Delta t \sim \frac{R_d^2}{\nu},$$

where along with its mass, determines the accretion rate $\dot{M} = \Delta M / \Delta t$. The Shakura-Sunyaev (1973; hereafter SS73) prescription for the kinematic viscosity $\nu$ reads

$$\nu \sim \alpha c_s H,$$

where $\alpha$, $c_s$ and $H$ is the SS73 “$\alpha$- parameter”, the sound speed of the disk and the disk half-thickness, respectively. The parameter $\alpha$ is equal to the ratio of the accretion stress, $\tau_{R\phi}$ to the disk pressure at the midplane $P$. If the tidal radius $R_d$ corresponds to $\sim$ a few $\times 10^3 R_g$ for a stellar mass BH and the disk is only marginally thin such that $H/R \leq 1$, which implies that $c_s \leq R\Omega$, then $\Delta t \sim$ a few seconds. If the mass fuel supply $\Delta M \sim M_\odot$, it follows that the accretion power takes on formidable values upwards of $L_{\text{acc}} \sim$ a few $\times 10^{52}$ erg s$^{-1}$, five orders of magnitude larger than the brightest quasars.

In order to determine the value of $\Delta t$, the value of $\alpha$ had to be specified. For almost two decades, theorists typically assumed that $\alpha \sim 10^{-1} - 10^{-2}$, for almost no particular reason. Now theorists believe MHD turbulence is the source of the accretion stress $\tau_{R\phi}$ (Balbus & Hawley 1991), with calculated values of $\tau_{R\phi}/P \sim 10^{-1} - 10^{-2}$, in a satisfying act of validation. With this, we have a simple, plausible and therefore attractive, working hypothesis for the mechanism responsible for (A) i.e., the initial randomization of gravitational binding energy.

In order to determine the structure of a HED, a condition that enforces conservation of energy, or radiative equilibrium, must be specified. At any given radius, the input of energy is roughly quantified by the rate viscous dissipation, while energy losses result from some combination of the inward advection of heat and radiative output.

The accretion rate $\dot{M}$ and the accretion power $L_{\text{acc}}$ are enormous because the reservoir of mass is proportionally large, hence the term hyper-Eddington accretion disk. Consequently, the Thomson optical depth for these HEDs are so high that the photon diffusion time greatly exceeds the inward advection time and therefore, the emission of photons cannot contribute to cooling the disk. Furthermore, the enormous accretion power $L_{\text{acc}}$ also implies that the characteristic disk temperatures are large as well. In fact, near the hole – where most of $L_{\text{acc}}$ is generated – the temperatures and densities of the flow resemble that of a proto-neutron star’s surface i.e., an object that can can generate

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1 Timing studies of cataclysmic variables indicate that $\alpha \sim 0.1$ (Pringle 1981).
2 It’s straightforward to show that the Thomson optical depth $\tau_{\text{es}} \geq L_{\text{acc}} / L_{\text{Edd}} \geq L_{\text{GRB}} / L_{\text{Edd}}$, where $L_{\text{Edd}}$ is the Eddington luminosity and $L_{\text{GRB}}$ is the luminosity of the GRB. Therefore, $\tau_{\text{es}} \geq 10^{12}$, which implies that the photon diffusion speed $c / \tau_{\text{es}}$ is so small that photon diffusion can safely be ignored.
neutrinos in copious amounts via the reactions

\[ e + p \rightarrow n + \nu_e \quad \text{AND} \quad e^+ + n \rightarrow p + \bar{\nu}_e. \]

(3)

If a significant amount \( L_{\text{acc}} \) is liberated in the form of neutrino radiation, then the condition for radiative equilibrium is approximately given by

\[ H \tau_{R_\phi} \frac{d\Omega}{d\ln R} \sim \sigma T_{\text{eff}}^4 \]

(4)
or in words,

\text{ACCRETION POWER} \sim \text{THERMAL EMISSION.} \hspace{1cm} (5)

The notion encapsulated by eqs. (4) and (5) is the same dissipation prescription adopted by SS73. The supposition that accretion power is matched by thermal radiation implies some knowledge of the mechanisms that account for (B) and (C). That is, randomized binding energy ultimately heats particles that can easily radiate their energy on an accretion time \( \Delta t \). Furthermore, the spatial location of the particles that receive the binding energy must roughly coincide with the particles (baryons) that serve as the source of gravitational and accretion energy. In doing so, the absorption optical depth under which the accretion power is released into heat is relatively large, as long as the absorption opacity increases with density, and the resultant outgoing radiation therefore approaches a black body. This is the meaning of local dissipation of binding energy i.e., the overwhelming majority of the binding energy is dissipated in the overwhelming majority of the matter.

The Shakura-Sunyaev (1973) prescription for accretion, encapsulated by eqs. (2) and (4), has profound consequences for the HED-powered fireball model of GRBs, outlined in Figure 1. Woosley (1993) took note that the most straightforward way of energizing a pair-rich relativistic plasma along the flow’s axis of symmetry is the annihilation of disk neutrinos into \( ee^+ \) pairs. The neutrino annihilation deposition rate is given by (neglecting geometric factors)

\[ Q_{\nu, \bar{\nu}}^+ = \sigma_0 \frac{L_{\nu}^2}{A^2} \langle E_{\nu} \rangle. \]

(6)

Definitions of the various symbols in the above expression are given in Ramirez-Ruiz and Socrates (2005). For a fixed value of \( L_{\text{acc}} \sim L_{\nu} \), the basic thermodynamics of radiating sources informs us that the quantity \( L_{\nu} \langle E_{\nu} \rangle \), which is \( \propto Q_{\nu, \bar{\nu}}^+ \), assumes its smallest value as the source approaches the black body limit. As is well known, black body radiation is “efficient” because it releases a given amount of radiant power at the lowest possible temperature or energy per quanta. The physical reason why the deposition rate takes on the functional dependence of eq. (6) is because the neutrino cross section increases with the square of the neutrino energy. In short, utilization of the Shakura-Sunyaev prescription, which implicitly assumes local dissipation, is the least efficient mode of gravitational energy release in terms of energizing a GRB pair fireball with a HED disk.

Interestingly, stellar mass BH accretion in X-ray binaries (BHXRBs) and quasars i.e, black hole accretion that we can actually observe, does not heed the Shakura-Sunyaev prescription. Even in the “high-soft” state of BHXRBs or quasars, sources in
which a cool optically thick thermal-emitting disk dominates the photon spectral energy distribution (SED), a significant fraction (∼ 10% – 50%) of the accretion power $L_{\text{acc}}$ is released in the form of hard Comptonized X-rays. The source of Compton power is widely believed to originate from a hot diffuse corona, adjacent to the cool optically thick disk. In other words, a disproportionately small amount of matter is responsible for a disproportionately large amount of radiative energy release, the implication being that the assumption of local dissipation is invalid and that black hole accretion flows observed in Nature dissipate their gravitational binding energy non-locally. The net effect on the SED is that the outgoing radiation field may display significant distortions from a thermal black body spectrum, implying that the average energy per quanta in the disk + corona case is larger in comparison to the purely thermal disk situation.

If we now lift the extreme condition that the mode of energy release in HEDs are purely described by the Shakura-Sunyaev (1973) prescription, then we may break the integral $L_{\nu} \langle E_{\nu} \rangle$ into a soft component coming from a cool dense disk and the other from a hot diffuse corona

$$L_{\nu} \langle E_{\nu} \rangle = L_{\nu} \left[ (1 - f) \langle E_{\nu} \rangle_s + f \langle E_{\nu} \rangle_c \right]$$

(7)

where $f$ is the fraction of power released in a corona and $\langle E_{\nu} \rangle_s$ and $\langle E_{\nu} \rangle_c$ is the average neutrino energy emitted from the disk and corona, respectively. So, if $f \sim 1\%$ of $L_{\text{acc}}$ is released in a corona with a temperature that is $1/f \sim 100$ times larger than the disk temperature, then the energy deposition rate resulting from pair annihilation from both regions of the flow are equal. Clearly, the possibility that some fraction of $L_{\text{acc}}$ in HEDs is released non-locally in a corona can have profound effects on the net energetics of the GRB explosion. Consider Figure 2. The black curve is the luminosity SED, which indicates the amount of power radiated per decade in energy, while the red and green curve represents the normalized (with respect to the soft disk contribution) deposition rate for the pair-annihilation process (relevant for HEDs as GRB engine) and capture of electron-type neutrinos (relevant for proto-neutron stars as SN engines), respectively.

The basic question of whether or not a HED scenario can energize a GRB fireball raises an even more fundamental question in a broader astrophysical sense. That is, what are the physical mechanisms at work in determining the (A)(B)(C)s of accretion? If neutrino-emitting hyper-Eddington black hole accretion flows work anything at all like their photon-emitting sub-Eddington counterparts, then the possibility that a significant fraction of the accretion power is released through a corona can no longer be ignored.

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FIGURE 2. Possible neutrino spectral energy distribution from a hyper-Eddington disk. The $y$–axis is the logarithm of the spectral energy distribution and the $x$–axis is the logarithm of the emitted radiation’s energy.

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