A Spectral Approach to Transit Timing Variations

Avi Ofir1, Ji-Wei Xie2,3, Chao-Feng Jiang2,3, Re‘em Sari4, and Oded Aharonson1

1 Department of Earth and Planetary Sciences, Weizmann Institute of Science, Rehovot, 76100, Israel; avirofi@weizmann.ac.il
2 School of Astronomy and Space Science, Nanjing University, Nanjing 210093, People’s Republic of China
3 Key Laboratory of Modern Astronomy and Astrophysics in Ministry of Education, Nanjing University, Nanjing 210093, People’s Republic of China
4 Racah Institute of Physics, the Hebrew University, 91904, Jerusalem, Israel

Received 2017 July 18; revised 2017 October 24; accepted 2017 October 25; published 2018 January 15

Abstract

The high planetary multiplicity revealed by Kepler implies that transit timing variations (TTVs) are intrinsically common. The usual procedure for detecting these TTVs is biased to long-period, deep transit planets, whereas most transiting planets have short periods and shallow transits. Here we introduce the Spectral Approach technique to TTVs that allows expanding the TTV catalog toward lower TTV amplitude, shorter orbital period, and shallower transit depth. In the spectral approach, we assume that a sinusoidal TTV exists in the data and then calculate the improvement to $\chi^2$ that this model allows over that of the linear-ephemeris model. This enables detection of TTVs even in cases where the transits are too shallow, so that individual transits cannot be timed. The spectral approach is more sensitive because it has fewer free parameters in its model. Using the spectral approach, we (a) detect 129 new periodic TTVs in Kepler data (an increase of $\sim2/3$ over a previous TTV catalog); (b) constrain the TTV periods of 34 long-period TTVs and reduce amplitude errors of known TTVs; and (c) identify cases of multi-periodic TTVs, for which absolute planetary mass determination may be possible. We further extend our analysis by using perturbation theory assuming a small TTV amplitude at the detection stage, which greatly speeds up our detection (to a level of few seconds per star). Our extended TTV sample shows no deficit of short-period or low-amplitude transits, in contrast to previous surveys, in which the detection schemes were significantly biased against such systems.

Key words: catalogs – methods: numerical – planetary systems

Supporting material: machine-readable table

1. Introduction

A transiting exoplanet’s Keplerian orbits impart precisely evenly spaced transit events. Any deviation from this regularity indicates that other forces are at play. Importantly, if there are other massive bodies in the system, they would interact with the transiting planet and perturb its orbit, imparting deviations from strict periodicity that are known as transit timing variations (TTVs). This important effect was predicted (Agol et al. 2005; Holman & Murray 2005) and later observed (Kepler-9 system, Holman et al. 2010; Dreizler & Ofir 2014). TTVs are stronger and more easier to detect in systems near mean motion resonances (MMRs) and generally appear as sine-like deviations. Inverting the observed TTVs back for the parameters of the physical system that produced them is difficult (Lithwick et al. 2012), primarily because the important parameters (e.g., planetary mass and eccentricity) are degenerate when higher-order effects are not significant (i.e., departures from exactly sine-shaped TTVs).

In fortiutous cases where higher-order effects are observed in multi-transiting systems, the TTVs may yield the absolute masses of the planets (e.g., Kepler-9, Kepler-11, Kepler-87; Holman et al. 2010; Lissauer et al. 2011; Ofir et al. 2014, and others) without the use of high-precision radial velocity (RV) measurements, allowing the determination of planetary masses even beyond current RV capabilities in some cases (e.g., Kipping et al. 2014). TTVs can be useful without higher-order effects, especially in multi-transiting systems, since anticorrelated TTVs constitute dynamical confirmation of the candidates as true planets in the same system, undergoing angular momentum exchange (e.g., Steffen et al. 2013; Xie 2013, 2014)—although with weak mass constraints. Even TTVs that are observed in only one transiting planet in a system are still useful since they may reveal the presence of additional planets in the system that may not be transiting at all (Nesvorný et al. 2012, 2013). Finally, the fact that TTVs are observed mostly for planets near MMRs allows using TTVs to address questions related to planet formation and migration—and the means by which planets are captured in MMRs (e.g., Xie 2014; Mills et al. 2016).

For these reasons, a significant amount of work has gone into identifying TTVs, particularly in the Kepler data set (e.g., Ford et al. 2011; Mazeh et al. 2013; Xie 2013; Holczer et al. 2016). In this work, we develop and apply a new technique—the spectral approach—to detect TTVs. We focus on detecting low-amplitude TTVs (defined in Section 3.3), with the assumption that high-amplitude TTVs were already identified (at least in the Kepler data). We further increase the sensitivity of TTV detection—both to the fundamental signal and to its higher-order components. To demonstrate this improvement, we compare our work to the recent Holczer et al. (2016) results, hereafter H16, using both their “long-term” and “short-term” TTVs combined (TTV periods longer and shorter than 100 days, respectively), and a treating multi-periodic TTV as separate signals.

Below we present the spectral approach in Section 2 and its more computationally efficient perturbative approximation in Section 3. We then describe the details of applying the spectral approach to Kepler data in Section 4, the resulting TTV catalog in Section 5, and conclude in Section 6.
2. A Spectral Approach to TTVs

2.1. Motivation and Description

This paper is concerned with low-amplitude TTVs, so we assume that a candidate transiting planet signal was already identified using simple linear ephemeris. When described using the Mandel & Agol (2002) formalism (hereafter MA02), the transit model requires explicitly just two parameters, the normalized planetary radius $r$ and the normalized distance between the star and planet $d_0$, to calculate the observed normalized flux at the $i$th point along the orbit. At a given time, $d_i$ is a function of four orbital parameters in the case of circular orbits: $P$, $T_{max}$, $a$, and $b$—for the planetary orbital period, time of mid-transit, normalized semimajor axis, and normalized impact parameter—and two additional parameters for eccentric orbits (where all normalized parameters are relative to the stellar radius). Limb-darkening parameters are also needed but are usually not fitted (except in cases with the highest signal-to-noise ratio (S/N)) and depend on the selected limb-darkening law.

In the usual procedure, the TTV search begins by allowing the individual times of mid-transit to deviate from their linear ephemeris. The individual timings are then searched for excess scatter or periodic signals. We note that in this case, there are a total of $N_\nu + 4$ fitted parameters, where $N_\nu$ is the number of individual transit events in the data, and there are four other circular-orbit parameters. Since $N_\nu$ can be several hundreds or more in the case of short-period planets observed throughout the four years of *Kepler*’s normal operation, the large number of fitted parameters reduces the sensitivity of this technique.

To first order, planetary TTVs close to first-order resonances are theoretically expected to have sine-like shapes with a super-period significantly longer than the orbital period of the transiting planets: $P_{sup} = P_{out} / (\sqrt{\Delta})$, where $\Delta = d_{in} - 1$ is the normalized distance to resonance (Lithwicke et al. 2012). Indeed, many of the detected TTVs (e.g., H16) are actually observed to be mainly sine-like, sometimes exhibiting other frequencies due to, for example, resonances of higher order or terms of higher order of a given resonance (e.g., Deck et al. 2014; Agol & Deck 2016), but all are still well described by sine functions. We therefore do not attempt to measure the individual times of mid-transit and then fit these timings, but rather assume that a sinusoidal TTVs exists in the data and then calculate the improvement of the model under this assumption relative to the linear model. We refer to this procedure as the spectral approach to TTVs.

We wish to scan all possible sine-like TTVs. This requires a three-dimensional search grid of TTV frequency $f$, TTV amplitude $A$, and TTV phase $\phi$ (to avoid ambiguity, the last is translated below to the time $T_0$ at which $\phi = 0$). On one hand, the frequency search axis can be well defined: the minimum frequency $f_{min} = 1/2s$, where $s$ is the span of the data. The maximum frequency is set by the Nyquist frequency $f_{max} = 1/2p$. The critical frequency spacing scale is as usual for sine fitting $\Delta f_{crit} = 1/s$, and one may wish to oversample this critical frequency spacing by a factor of a few to avoid missing local peaks. On the other hand, there is no natural upper limit to $A$ (other than $A < P$, which is not very informative) and no natural resolution to $A$ or $\phi$—leaving the search grid undefined and the processing slow (the MA02 model function needs to be invoked for each transit event separately on each test position on the search grid). The procedure therefore appears plausible but numerically inefficient.

Signal detection can be significantly accelerated, however, if one limits the search to low- or medium-amplitude TTVs (see Section 3.3 for definition). This improvement is made by using perturbation theory, as described in Section 3, and is here called the perturbative approximation (PA) to the spectral approach to TTVs, as opposed to the full fit. We stress that PA is used for detection only. Indeed, and as done in Section 3.9, a full non-perturbative model should be used for the determination of the final TTV signal parameters and their associated errors.

2.2. Benefits of Spectral Approach

The spectral approach to TTVs offers several advantages. These include that (i) it encapsulates all the TTV information in three parameters—the sinusoid period, amplitude, and phase—regardless of $N_\nu$. (ii) Since the entire light curve is fitted with a single TTV model, the TTV detection sensitivity scales closely with the precision of the linear $T_{max}$. The reason is that of the four linear-ephemeris parameters, $T_{max}$ is the only one affected by adding small TTVs. This results in elimination of the bias to long periods since the absolute precision on $T_{max}$ of the linear ephemeris (and thus the TTV detection sensitivity) actually improves with decreasing orbital period. (iii) Short-period signals (with periods of a few days or shorter) have only few points in transit in each individual event, making transit time fitting nearly impossible. The extreme case is KOI 1843.03 (Ofir & Dreizler 2013), which has an orbital period of ~4.25 hr and total transit duration not much longer than a single *Kepler* long cadence. Ultra-short-period planet candidates are also typically too shallow to be detected at all as single-transit events. Searching for TTVs using the spectral approach presents no such limitations. (iv) The resulting TTV spectra are compatible with later stages of system interpretation: theoretical considerations can predict the TTV frequencies and their amplitude ratios (e.g., Lithwick et al. 2012; Deck et al. 2014; Agol & Deck 2016). Therefore this information is useful even when no single-transit event is above the noise.

The classical approach to TTV detection is biased to long-period planets and deep transits (Steffen 2016), whereas most transiting planets have short periods and shallow transits. Indeed, in the the H16 catalog of *Kepler* TTVs, the median orbital period for planets that exhibit TTVs is 29.8 days, while the median period of all *Kepler* candidates is shorter than a third of that number—about 9.5 days. This bias is therefore relevant to a large number of objects, and the proposed spectral approach can all but remove it.

We note that some TTVs are not sine-shaped: TTVs that originate from non-gravitational processes (e.g., instrumental effects, starspots) may have different morphologies, and their description as sinusoids or the sum of a few sinusoids may be inadequate.

2.3. Theoretical Comparison with the Classical Technique

The probability density function (PDF) of the timing of an individual transit event is a delta function at the true time, which is then broadened by observational noise. In data with high S/N, the PDF of transit timing fits is close to a Gaussian centered on the true transit time, and with a width that depends on the S/N. As the S/N decreases, the PDF of the transit time fits reduces in amplitude, broadens, and no longer resembles
the true PDF. Instead, there is an almost uniform probability of finding the transit at any time in the search interval, with only a slight increase at the true transit time. Thus fitting these individual transit PDFs with individual Gaussians, as in past work, results in poor fits that cannot approximate the true PDF when the S/N is low. In other words, when the S/N is low, the PDF is so wide and shallow that many of the individual transits are detected far from the true signal. Thus they do not contribute to the coherent addition that allows searching for a periodic variation. Using a single global fit avoids this problem.

In order to illustrate this effect, we generated \( N_r \) segments of duration unity on either side of a Gaussian-shaped transit-like signal of height \( \epsilon \), each segment with \( N_r \) points including normally distributed noise of unit amplitude. The simulated mid-transit time was chosen as a random (uniform) location within the central half of the domain. We then determined the best-fitting mid-transit time in two ways. We first fit a periodic Gaussian shape to all the segments simultaneously. We also took the average of individual times measured by fitting each segment separately. The difference in time between the best-fitting and the actual mid-transit location is \( \delta t_c \). We repeated this experiment \( N_r \) times and averaged the results to produce smooth curves. Figure 1 clearly shows that the a global fit outperforms the average of individual fits, i.e., the rms of the timing error, \( \langle \delta t_c^2 \rangle^{1/2} \), remains small up to lower values of the signal amplitude, \( \epsilon \), relative to the noise. The mean timing error \( \langle \delta t_c^2 \rangle^{1/2} \) rises sharply and exceeds the transit duration at signal amplitudes much lower (an order of magnitude for the representative parameters chosen) for the global fit than the individual fits. This again demonstrates the general effect that should be unsurprising: when the S/N is low enough to mask the individual transit such that its typical timing error exceeds the event duration, averaging many such fits does not improve the detection. However, a global fit remains sensitive to lower S/N.

### 3. Perturbative Approximation to the Spectral Approach

#### 3.1. Basic Perturbation Theory

To some general data set of \( N \) points \( \{ t_i, F_i \} \) and \( F \)-errors \( \sigma_i \), we fit a model \( m_i = m(t_i) \). We wish to examine the sensitivity of the model to perturbation \( g_i \) in the independent parameter \( t_i \) assuming the perturbation’s functional shape is known (linear trend, periodic variation, etc.). This perturbing function is known and normalized, only its scaling parameter \( S \) is sought. We therefore define the effective independent parameter

\[
t_i' = t_i + Sg_i.
\]

The perturbations are assumed to be small in the sense defined in Section 3.3, and in this limit, a linear approximation is possible:

\[
m(t_i') \approx m(t_i) + Sg_i m_i',
\]

\[
m_i' = \frac{dm}{dt}(t_i), \quad m_i \equiv m(t_i).
\]

Since we are interested in TTVs, the independent parameter \( t_i \) is the time, \( F_i \) is the normalized flux, and \( m_i \) is the Mandel & Agol (2002) transit model. The derivative of the transit model \( m_i' \) is computed with respect to time, but application to sparse data (such as Kepler’s long-cadence light curves) should be done with care (see Section 3.2). Thus the amplitude \( S \) has the desired meaning of TTV amplitude, and \( m_i(t_i') \) is the unknown TTV-inclusive model for which we solve perturbatively. We use a least-squares to fit the new model:

\[
\chi^2 = \sum_i \frac{(F_i - m_i(t_i'))^2}{\sigma_i^2}.
\]

Differentiating with respect to \( S \), we obtain the best-fitting amplitude for the perturbation \( g_i \):

\[
S = \frac{\sum_i (F_i - m_i) g_i m_i'}{\sum_i (m_i')^2}.
\]

It is useful to generalize this result to two-parameter optimization: now two perturbations \( g_i \) and \( h_i \) perturb the model function \( m(x) \), so that the effective independent parameter is now

\[
t_i' = t_i + Sg_i + Sh_i,
\]

giving the set of line equations \( M X = V \) where

\[
M = \begin{bmatrix}
\sum_i (g_i m_i')^2 / \sigma_i^2 & \sum_i (g_i h_i m_i') / \sigma_i^2 \\
\sum_i (h_i m_i')^2 / \sigma_i^2 & \sum_i (h_i m_i')^2 / \sigma_i^2
\end{bmatrix},
\]

\[
X = \begin{bmatrix}
S_i \\
S_j
\end{bmatrix},
\]

\[
V = \begin{bmatrix}
\sum_i (F_i - m_i) m_i' g_i / \sigma_i^2 \\
\sum_i (F_i - m_i) m_i' h_i / \sigma_i^2
\end{bmatrix}.
\]
These linear equations may be solved for $X$, the amplitudes of the two perturbations.

### 3.2. Application to TTVs

We now proceed to apply the analytical method above to TTVs. In this context, the model function $m$ is the linear-ephemeris MA02 model, and we assume that a linear model was already fitted to the light curve. At this point, each data point has an assigned time from mid-transit $t_i$ that corresponds to a normalized distance $d_i$ for the MA02 model. We note that finding the true global minimum $\chi^2_{\text{linear}}$ is of special importance since the TTV analysis depends on the exact shape of the linear signal. We took great care in performing this step, as described in Section 4. Furthermore, Kepler’s long cadence means that $\frac{dm}{dt}$ cannot be computed simply from the time series as $m_i(x) = (m(t_i) - m(t_{i-1}))/t_i - t_{i-1}$ since the details of the ingress/egress are all but erased from the individual transits. To correct for this effect, $m'$ can be estimated either from a densely resampled flux model or from the folded model light curve (in time modulo the period and not in phase, in order to be consistent with the TTV units of time and Equation (2)). In the folded light curve, the ingress/egress region is much better sampled than in individual events, enabling a more precise evaluation of $\frac{dm}{dt}$.

We then add a perturbation—a sine in time, which has a frequency $f$, amplitude $A$, and phase zero at time $T_0$. The effective independent parameter is therefore

$$t'_i = t_i + A \cdot \sin(2\pi f(t_i - T_0))$$

$$= t_i + A \sin(2\pi f t_i) \cos(2\pi f T_0)$$

$$- \cos(2\pi f t_i) \sin(2\pi f T_0)$$

$$= t_i + S_1 \sin(2\pi f t_i) + S_2 \cos(2\pi f t_i),$$

where $\{A, T_0\}$ in $\{S_1, S_2\}$ in the form described by Equation (5). This linearization allows us to analytically solve for the best-fitting amplitude and phase of a sine-shaped TTV, removing the problems identified above and leaving only the TTV frequency as a single searched parameter (which remains well defined with clear boundaries and resolution, as explained in Section 2.1). The resulting perturbed model (Equation (9)) appears similar to a true TTV-shifted signal, but it is only a mathematical construct and not a physical model (e.g., it may have flux values greater than unity unless they are clipped, see Section 3.4).

Figure 2 visualizes the process: the folded linear-ephemeris model is cropped to just a region surrounding the linear-ephemeris transit, and the model derivative $\frac{dm}{dt}$ is calculated. The perturbed model is generated by adding $\frac{dm}{dt}$, scaled by some amplitude, to the linear model above—to mimic a TTV-affected light curve. While the perturbed model usually appears just as a time-shifted model, some points can obtain non-physical values in the perturbed model: above unity and below $\min(m)$. Model clipping (see Section 3.4) can be applied to counter this effect.

### 3.3. Domain of Validity

The PA approximates the light curve of TTV-affected planets assuming that these TTVs are small. Indeed, the ability of the PA to find a better model than the linear model arises from the nonzero derivative of the model $m'_i$ in Equation (4), which allows adjustment of the linear model to better fit the data. Transit light curves are roughly trapezoid-shaped, so nonzero derivatives are significant mostly during ingress and egress. We can therefore divide the possible TTV amplitudes into three broad regimes of validity within the PA:

(i) Low-amplitude TTVs are those with amplitudes lower than the ingress/egress duration. In this regime, the TTVs are small enough for $m'_i$ to be able to approximate the true TTV signal well. We therefore expect here that the PA will both find the true TTV frequency and reproduce the TTV amplitude with optimal sensitivity.

(ii) Medium-amplitude TTVs have amplitudes higher than the ingress/egress duration, but lower than the full transit duration. In this regime, the PA-detected amplitude saturates as $(F_i - m_i)m'_i$ the factor is nonzero but nearly independent of the TTV amplitude—so the PA can only compensate for the ingress/egress duration part of the actual TTV amplitude. Here the correct TTV frequency would likely be identified, but with a lower amplitude than the correct one.

(iii) High-amplitude TTVs have amplitudes comparable to the transit duration or longer. In this case, either $m'_i$ or $y_i - m_i$ in Equation (4) average to zero at all times, and so that the perturbative approximation fails and is unable to detect the TTVs. Note the full search without the linear approximation would still apply well.
These domains are simulated and shown in Section 3.8. The PA approximation is valid for the majority of TTVs—especially those undetected as yet—since the median TTV amplitude in the H16 catalog is <23 minutes, while the median transit duration of the same objects is >300 minutes. In other TTV catalogs, such as those of Xie (2013, 2014), the case is even clearer with a median amplitude of <14 minutes and a median duration of 270 minutes. While rare high-amplitude TTVs do exist with amplitudes that may even exceed the duration of the transit itself, these are better detected by other techniques, e.g., QATS (Carter & Agol 2013), human eyes (Schmitt et al. 2014), or single-event detection schemes (e.g., Osborn et al. 2016).

3.4. Model Clipping

The PA produces a perturbed model that is a mathematical construct (not a physical model). This construct can be made nearly indistinguishable from a physical model by clipping the perturbed model on both the upper and lower ends, to zero flux decrement and to \( \min(m(x)) \), respectively, before \( \chi^2 \) is calculated (see Figure 2). Model clipping is not used here to refine the PA-detected amplitude or signal frequency, but improves the reliability of the \( \chi^2 \) values in discriminating true from spurious TTV signals.

3.5. Correction to the Mandel & Agol (2002) Model

We use the standard MA02 model to calculate the size of the flux decrement during transit. This model is a set of a few mathematical formulae, each valid in its own section of the parameter space and with mathematically accurate transitions between these regions. While the computational implementation of these formulae has finite precision and thus some discontinuities in the transition between sections exist, the effect is small enough (10^{-4} times the transit depth itself) to be largely ignored.

However, in the PA the MA02 derivative with respect to the planet-star separation \( z \) is needed—and here the finite precision can no longer be neglected for points close to two problematic transitions at \( z \approx r \) and \( z \approx 1 - r \). These numerical errors are small in amplitude, but they also occur over a small range of \( z \) values, causing their \( dm/dz \) amplitude to rival that of the main signal (and actually diverge at the limit). This is demonstrated in Figure 3: we sample normalized distances of an \( r = 0.1 \) planet in the MA02 model at a resolution of \( dz \approx 10^{-4} \). The small miscalculation of the model around the \( z \approx r \) transition is observed with an amplitude \( <10^{-3} \) (see inset on the top panel). At this resolution, the anomaly near \( z \approx 1 - r \) is \( \approx 100 \) times smaller still (and thus not visible). The model’s derivative has a large anomaly at \( z \approx r \) (the one near \( z \approx 1 - r \) is still small), however. As the discontinuity is approached, this becomes more severe: at higher resolution (e.g., \( dz = 10^{-5} \)), the anomaly near \( z \approx 1 - r \) becomes dominant—reaching amplitudes several orders of magnitude higher than the main \( dm/dz \) signal.

These numerical errors are inherent to the MA02 model and will persist even if the numerical convergence criteria used in the MA02 are made stricter. We therefore propose an ad hoc correction to the model. The \( z \approx r \) anomaly has continuous slopes on either side (there is no physical change at \( z = r \)), while the \( z \approx 1 - r \) anomaly does reflect a physical change at \( z = 1 - r \) (the second and third contacts). We therefore correct the former by polynomial interpolation from both sides, and the latter by polynomial extrapolation from either side, separately. In both cases, the corrected region is \( \Delta z = 2 \cdot 10^{-4} \) from the anomaly, and the polynomials are fitted using a region up to \( \Delta z = 10^{-3} \) from it. The polynomials are of second order unless \( r > 0.5 \), in which case they are of third order. While satisfactory, this procedure is imperfect: each transition from the MA02 model to polynomial interpolation/extrapolation involves a “stitching” point that produces an outlier point in the model derivative—but these are points (versus finite regions in the uncorrected model) and with far reduced amplitude, regardless of the proximity to the transition points.

3.6. Model Sampling

A further consequence of using the model derivative concerns the model oversampling needed due to Kepler’s finite integration time. The commonly used criterion for computing the needed oversampling is given by Equation (40) of Kipping (2010), which calculates the inaccuracy caused by sampling the instantaneous model at cadence \( I \) relative to integrating it over intervals with cadence \( I \) (the latter is of course closer to a real measurement). If the signal has a depth of \( \delta \) and ingress/egress duration of \( \tau \), that inaccuracy is found to be \( \sigma(F_{\text{resampling}}) \propto \delta I / \tau \) (the \( \sigma \) letter is used since the inaccuracy can be viewed as a type of modeling noise). However, this computation is not valid in the case of PA since now the sampling noise must be compared not with the model flux, but with the model flux derivative. The slope that Kipping (2010) sampled was \( \delta / \tau \). Analogously, the slope that needs to be well sampled in PA is approximately \( \delta / \tau^2 \). We therefore update Equation (40) of Kipping (2010) for use in PA by
dividing it with yet another ingress/egress duration, or

$$\sigma(F_{\text{Resampling}}) = \frac{\delta}{\tau^2} \frac{\mathcal{F}}{8N^2}, \quad (10)$$

where $N$ is the number of subsamples computed. In practice, we impose a lower limit of $N = 10$ to ensure adequate subsampling.

3.7. Speed

Linearly fitting two of the three search dimensions allows a significant speedup in calculating TTVs. This part of the calculation for a typical system now takes 10 s or less on a single-threaded CPU (a far larger amount of time is required to fit the linear model of MA02) as zero calls to the MA02 model function are needed.

3.8. Detection of Simulated Signals

We simulated Kepler-like data that included a transiting planet with sinusoidal TTVs with a TTV period shorter than the time baseline of the simulated data, and tried to detect these TTVs using PA. We considered a detection successful when the frequency of the most significant peak on the TTV spectrum was close to the simulated one (i.e., $|\Delta f| < (\text{time baseline})^{-1}$), and calculated the detection efficiency of the algorithm while scanning along various parameters. Notably, we also recorded the amplitude of the best-detected peak, i.e., the $\chi^2$ improvement (as a positive number) over the best-fit linear model: $\Delta \chi^2 \equiv \chi^2_{\text{linear}} - \chi^2_{\text{PA}}$.

In Figure 4 we show the results of three such parameter scans—each along the transit S/N axis, but at different TTV amplitude. Here, the $S/N$ is defined per transit event, i.e., the depth of the transit divided by the per-point uncertainty and the square root of the average number of points in a transit event. As expected, at a given transit S/N (left column of panels), detection becomes easier when the TTV amplitude increases. Scans of other parameters show similar and expected trends. Detection efficiency is higher when the planet is larger or when the noise or planetary orbital period are smaller. These different dependencies can make it challenging to establish an exact TTV detection limit of a given system. On the other hand, the panels in the right column of Figure 4 show the same data sorted by the $\Delta \chi^2$ score—and the red line is an empirical function (using the error function $f(x) = \frac{1}{2}[1 + \text{erf}(x - x_0)/\sigma]$ with $x_0 = 11, \sigma = 8$). The three panels show similar behavior across significantly different parameter values. Importantly, nearly identical $f(x)$ is obtained when varying all other parameters of the problem. We conclude that the spectral approach and its PA approximation allow TTVs to become uniformly detectable—always becoming detectable with high efficiency when $\Delta \chi^2 \gtrsim 20$ (which is a plausible threshold: $4 - 5\sigma$ detections are usually regarded as reliable). This uniformity allows the spectral approach to detect TTVs using a full data set even if no single transit is detectable, as is often the case for small planets.

As expected, higher TTV amplitude does not always increase the detection efficiency: as explained in Section 3.3, Figure 5 shows that high-amplitude TTV have low detection efficiencies, if at all detected, using this technique. We note that the parameters chosen for the above test, especially the single-event S/N $< 2.5$, is such that this planet would not even qualify for analysis by some catalogs (Mazeh et al. 2013)—but here PA obtains a high detection efficiency over a wide range of values.

3.9. The Full Spectral Approach Fit

The PA is a fast approximation to the full, but slower, spectral approach. Therefore, a full spectral approach model is computed based on the PA results for cases where significant TTVs are detected by PA. The full spectral approach model fits the non-periodic planet similarly to classical techniques of fitting the transit parameters simultaneously with the times of mid-transit (e.g., H16), except that the series of times of mid-transit is given by a sine function and not by a numerical
vector. Grid-searching, as presented in Section 2, is sub-optimal. We therefore use a well-tested Markov chain Monte Carlo (MCMC) code (e.g., Ofir et al. 2014) to find the best-fit solution and the error ranges of eight parameters: four linear-ephemeris orbital parameters, normalized planet radius, and three sine-TTV parameters. The results of the full fit usually agree well with the PA results and have somewhat better \( \Delta \chi^2 \) than PA (see also Figure 11). Mismatches between the PA and the full fit are common when the TTV period is longer than the baseline of the data—causing the solutions range to be highly correlated (see Figure 10)—something that is not captured by the PA model. The adopted values are therefore the result of the full spectral approach fit.

### 4. Application to Kepler Candidates

#### 4.1. Data and Preprocessing

We used *Kepler* Data Release 24 as the source data and the *Kepler* Objects of Interest (KOIs) table downloaded from the NEExSci archive\(^5\) on 2015 December 25 as the source of list of Kepler the PA model. The adopted values are therefore the result of the correlated Carlo optimal. We therefore use a well-tested Markov chain Monte Carlo MCMC code. Grid-searching, as presented in Section 2, is sub-optimal. The PA model after accounting for the stellar limb-darkening coefficients \( u_1 \) and \( u_2 \) for planets with NEExSci-reported model \( S/N > 100 \). After this fit, we rejected in-transit outliers and then calculated the \( \chi^2 \) of every transit event. Events that had \( \chi^2 \) larger by more than 4\( \sigma \) from the median (where \( \sigma \) was assessed ignoring the event with the highest \( \chi^2 \)) were rejected. If indeed either in-transit points or transit events were rejected, then we refitted the remaining data.

We note that multi-planet systems were modeled such that before analyzing a given KOI, all the other transit signals on the same host star were modeled out using the NEExSci linear-ephemeris parameters. This has the effect that systems may be inaccurately modeled if more than one TTV-bearing planet exist and some of the transits are nearly overlapping in time. In such cases, the TTVs of the other planet (not the KOI of interest) are imperfectly modeled out by our pipeline, potentially hampering the analysis of the KOI of interest.

#### 4.2. Selection of Significant TTVs

In order to be selected as a significant TTV, a candidate signal should pass a number of statistical tests: the bootstrap analysis confidence test, and a set of tests based on cumulative \( \Delta \chi^2 \) (below).

##### 4.2.1. Bootstrap Analysis Confidence Test

For each KOI we performed a bootstrap analysis confidence test (Press et al. 1992): we generated \( 10^3 \) artificial light curves, each by sampling with replacement the residuals to the linear model and then adding these resampled residuals back to the linear model. Each artificial light curve was then analyzed using PA, and the peak of its TTV \( \Delta \chi^2 \) spectrum was recorded. The confidence metric, equivalent to 1 minus the false-alarm probability, is the fraction of those random artificial light curves that had a lower peak \( \Delta \chi^2 \) than the real data. Indeed, previously detected TTV-bearing KOIs clearly cluster around high-confidence metric (\( \gtrsim 0.999 \)), with most of the exceptions easily understood as either high-amplitude TTVs (for which PA is not suitable in the first place) or variable stars with special characteristics (e.g., at timescales similar to the transit duration, making the polynomial background estimation inadequate). We therefore use this confidence metric as the main threshold for the selection of significant TTVs.

Note that this test checks for the reality of the most significant peak. Since the degeneracy in TTV inversion can be broken in cases where additional super-frequencies are detected, we also use this test as a rough guide to the significance of other peaks in the PA spectrum.

##### 4.2.2. \( \Delta \chi^2 \) Test

As shown in Section 3.8, high-efficiency detection is expected for \( \Delta \chi^2 \approx 20 \) or larger. We therefore checked if any of the high-confidence signals had lower \( \Delta \chi^2 \). We found only one such case, with an only slightly lower \( \Delta \chi^2 \), confirming that \( \Delta \chi^2 \approx 20 \) is a good discriminant value.

##### 4.2.3. Cumulative \( \Delta \chi^2 \) Definition and Tests

It can be difficult to judge whether a \( \Delta \chi^2 \) for a given system is due to a real signal or just some outliers as \( \Delta \chi^2 \) is only summary total of the difference between the linear and the

\(^{5}\) http://exoplanetarchive.ipac.caltech.edu/
Figure 6. Top row: expected (red) and observed (blue) accumulation of \( \Delta \chi^2 \) relative to the linear-ephemeris solution along the light curve of (a) KOI 103.01 and (b) KOI 4341.01. Real TTV and a correct TTV model are manifested by slow accumulation of \( \Delta \chi^2 \) and shared common behavior of the two curves—as is shown for KOI 103.01. On the other hand, on KOI 4341.01, the observed and expected \( \Delta \chi^2 \) curves are very different, creating a large area between them—although the signal has high observed \( \Delta \chi^2 \) of \( \approx 150 \) and high confidence. Bottom row: visualization of the (c) KOI 103.01 and (d) KOI 4341.01 TTV signals. For both planets, the respective best fits are shown, but for KOI 4341.01, this best fit does not correspond to a real TTV signal. The linear-ephemeris residuals divided by \( \sigma_i \) are plotted in blue, and the linear-ephemeris model derivative is shown in red. The size of the points is scaled by \(|\chi_1|\). If the TTV model is correct, then \( y \rightarrow m_T \) and \( \text{Obs} \) reduces to \( \text{Exp} \). On the other hand, if there are no TTVs in the data, then \( y \rightarrow m_L \) and \( \text{Obs} \) will differ from \( \text{Exp} \). Generally, a wrong TTV model would manifest itself by exhibiting significantly different observed/expected cumulative \( \Delta \chi^2 \) curves. In Figure 6 we demonstrate this metric on the well-known TTV-bearing star KOI 103.01, and on another signal, apparently with high confidence and high-\( \Delta \chi^2 \), that fails some of the cumulative \( \chi^2 \) tests. The cumulative \( \Delta \chi^2 \) curves are information-rich, and we use them to design the following tests for the reality of a PA-detected TTV signals:

(i) Normalized area between curves: the normalized area between the \( \text{Obs}_i \) and \( \text{Exp}_i \) can be defined as \( \frac{\sum \text{Obs}_i - \text{Exp}_i}{\sum \text{Exp}_i} \). False-positive detections are expected to have a large normalized area between the two curves.

(ii) Maximum single-point \( \Delta \chi^2 \): a real TTV signal would accumulate \( \Delta \chi^2 \) slowly over time, while an attempt of the PA to minimize \( \chi^2 \) due to some outliers would accumulate most of the \( \Delta \chi^2 \) in a small region. By recording the highest single-point \( \Delta \chi^2 \) gain (absolute value) for each KOI, one may detect irregular behavior. We note that this test bears similarity to the Kolmogorov-Smirnov (KS) test.

(iii) rms of difference: the rms of the difference between the \( \text{Obs}_i \) and \( \text{Exp}_i \) curves is computed. True TTV signals should have a low value for this rms, so we required that all new TTV signals will have a lower rms value than those found on the H16 objects (that are not high amplitude).

(iv) Correlation coefficient: The correlation coefficient between the \( \text{Obs}_i \) and \( \text{Exp}_i \) curves is computed. True TTV signals should have a high correlation between \( \text{Obs}_i \) and \( \text{Exp}_i \), so we required that the correlation coefficient for all new TTV signals be higher than those found on the H16 objects (that are not high amplitude).
We computed these statistical measures above for all KOIs, and required that all new PA detections be in the range spanned by the H16 high-significance candidates. We found eight KOIs that passed all other criteria but failed the normalized area test, and indeed, all of them were found to be false positives by visual inspection (seven strongly pulsating stars and one EB). The remaining signals passed all the additional tests.

4.2.4. Visualization of the TTV Signal

By rearranging Equation (2) to express the model derivative and applying it to the measurement set \( F_i \), one obtains an observational estimation of the flux derivative. This can be used to visualize the PA-detected TTV signal by comparing the observed shape to the expected one, i.e., a transit derivative,

\[
F'_i = (F_i - m_i)/S_g.
\]

The lower panels of Figure 6 depict this residual. Note that this is not the quantity that is fitted, but is only used for visualization. Since the oscillating function \( g_i \) is in the denominator, some points diverge on the plot.

5. Results of Application to Kepler Candidates

5.1. General Statistics and Comparison with H16

We analyzed 4629 KOIs that met the rather minimum requirements of not being marked as false positives by the NExScI archive, and including at least three usable transits after preprocessing. We found 528 objects with confidence \( > 0.999 \), all of them but one also with \( \Delta \chi^2 > 20.7 \) (see Figure 7), similar to the expected threshold of \( \sim 20 \) from the tests on simulated data. False positives comprise 201 of these object, arising due to the following reasons: (i) they appear in the current KEBC; (ii) they have depth \( > 10 \% \); (c) they have high-amplitude TTVs (making the PA-detected value likely wrong); (d) the TTV frequency was the maximal one (see discussion in Section 5.2); (e) they failed any of the cumulative \( \Delta \chi^2 \) tests (Section 4.2.3); (f) they had a scatter-to-error ratio \( > 50 \). Collectively, these objects are termed the Ebs/FPs (eclipsing binaries and false positives) sample, while the sample of objects with confidence \( \geq 0.999 \) that are not Ebs/FPs is termed the high-confidence TTVs.

The list of 327 remaining KOIs with high-confidence TTVs was cross-referenced to the TTV catalogs of H16, Xie (2013, Xie (2014), Hadden & Lithwick (2014, 2016), hereafter HL14, HL16), Jonot-Hutter et al. (2016) (hereafter JH16), and Van Eylen & Albrecht (2015). In all cases, we required that the reported TTV amplitude in these catalogs be significant to at least 3\( \sigma \) as determined by each catalog, in order to be considered as a significant TTV detection by these catalogs, i.e., a “known object.” After removing the known objects, 163 remaining KOIs with new periodic TTVs were identified, of which 34 are present in the H16 catalog with long-term and unconstrained periods (dubbed “polynomial TTVs”). This means that 129 TTV signals found here are considered new. Our results are given in Table 1.

The power of the spectral approach is seen by comparing the distribution of TTV-bearing KOIs detected using PA and the classical techniques (e.g., H16 in Figure 8). As predicted, we see that PA can detect TTVs for planets with shorter orbital periods, lower amplitude, and smaller size than was previously possible, and the higher object count indicates that PA is also more sensitive. Numerically, as mentioned above, the median orbital period for planets that exhibit TTVs in the H16 catalog is more than three times the \( \sim 9.5 \) days median period of all Kepler candidates, while the median period of the spectral approach new TTV detections is \( \sim 10.4 \) days—eliminating the period bias. Similarly, the median transit depth of the H16 objects is \( \sim 1075 \) ppm, while the same number of all Kepler candidates and the spectral approach detections is \( \sim 428 \) ppm and \( \sim 520 \) ppm, respectively—eliminating the depth bias. Additionally, we see in Figure 9 that the precision in the amplitude of the detected TTVs (and thus, the sensitivity to them) scales exactly as the precision on the linear \( T_{\text{mid}} \) over more than five decades of amplitude.

During this work, we noted a few issues with the H16 catalog to which we compare: (i) the TTV detection and final fits are separate steps, and their results are occasionally incompatible. In particular, 22 of the reportedly significant TTVs (\( p \)-values \( < 10^{-4} \), see H16) have low-amplitude significance \( A/\sigma_A < 3 \). These 22 TTV signals are questionable: we only detected high-confidence TTVs on two of these objects (KOIs 282.01 and 1783.01), and only the first one is consistent with H16. We therefore term the remaining 20 signals low-significance H16 KOIs and do not include them in the following comparison with H16. (ii) The uncertainty of the linear-ephemeris orbital period was found to be systematically too low in H16: while we obtained similar uncertainties for short-period KOIs, the H16 uncertainties were progressively smaller than ours toward longer period, typically about an order of magnitude smaller and approaching three orders of magnitude smaller at the long-period end. Examination of the individual signals confirms that these uncertainties are underestimated in H16. (iii) The H16 uncertainty on the linear-ephemeris time of mid-transit is almost always larger than our own. The difference is far smaller than in point (ii) above (median factor of 2.35) and thus is probably unrelated to it.

Many of the TTV-bearing KOIs we found were previously identified in the literature. Indeed, of the 143 objects in common with the H16 catalog, 137 have consistent TTV frequencies, i.e., \( < 3\sigma \) between the catalogs, demonstrating the compatibility of the spectral approach with the classical TTV identification techniques used in H16. It is noteworthy that due to the lower number of free parameters, the spectral approach allows improving the precision of the determination of the TTV parameters. The uncertainty on the TTV period was reduced by a median factor of \( \sim 10 \% \) and the uncertainty on the TTV amplitude was reduced by a median factor of \( \sim 40 \% \).

There are 30 stars with periodic and significant TTVs identified by H16 that were not detected by the PA. We evaluated each of these stars manually and found that about half of them reside in active stars with high-frequency variability (scale of transit duration or shorter), making TTV detection a more subtle issue. Nearly all the rest were either low significance to begin with (amplitude significant to less than \( 4\sigma - 5\sigma \) according to H16), or the same TTV frequency was detected by the PA and H16, but different confidence levels were attributed to the signals. Indeed, only in two cases (KOIs 1581.02 and 4519.01) was no good explanation possible for the PA, missing robust H16 signals.

5.2. Discussion

Below, we list a number of recurring phenomena that can be useful for understanding the results given in Table 1.
| KOI | $f_{\text{TTV}}$ | $P_{\text{TTV}}$ | $\Delta \chi^2$ | Cumulative $\Delta \chi^2$ Tests | $A$ | $T_0$ | STD/ Med (Linear Model) | Frequencies | Strob. Freq. | Previous | Comments |
|-----|-----------------|-----------------|----------------|----------------|-----|-----|----------------|------------|-----------|----------|----------|
|     | (Days $\times 10^{-4}$) | (Days) | | Area | Single | rms | Corr. | (Minutes) | (KBJD) | $\times 10^{-4}$ (days$^{-1}$) | $\times 10^{-4}$ (days$^{-1}$) | References | |
| 12.01 | 7.29$^{+0.25}_{-0.18}$ | 1371 | 1932 | 0.338 | −121.947 | 249.643 | 0.885 | 1.162$^{+0.021}_{-0.022}$ | 1068.5$^{+5.3}_{-3.3}$ | 1 | 4.02 | [161.1, 24.5, 200.7] ± 3.1 | 459.2 | 1 | Kepler-448 b |
| 13.01 | 1746.12$^{+0.25}_{-0.18}$ | 6 | 12659 | 0.312 | −15.347 | 46.904 | 0.961 | 0.1070$^{+0.0075}_{-0.0077}$ | 134.143$^{+0.129}_{-0.094}$ | 1 | 2.27 | [1718.7, 1771.5] ± 2.7 | 1746.5 | 1 | Kepler-13 b |
| 41.01 | 115.93$^{+0.60}_{-0.34}$ | 86 | 48 | 0.215 | −7.584 | 5.721 | 0.940 | 5.28$^{+0.48}_{-0.69}$ | 190.6$^{+1.5}_{-1.7}$ | 0.994 | 1.62 | ... | 152.2 | ... | Kepler-100 c |
| 42.01 | 9.913$^{+0.05}_{-0.067}$ | 1009 | 3246 | 0.165 | −83.669 | 272.222 | 0.987 | 14.955$^{+0.25}_{-0.19}$ | 523.9$^{+3.6}_{-3.8}$ | 1 | 2.46 | [19.4, 33.9, 143.2] ± 3.1 | 428.4 | 1 | Kepler-410 A b |
| 46.01* | 699.69$^{+0.75}_{-0.71}$ | 14 | 45 | 0.258 | −2.094 | 3.791 | 0.973 | 1.35$^{+0.20}_{-0.20}$ | 134.70$^{+0.75}_{-0.72}$ | 1 | 1.15 | ... | 1960.3 | ... | Kepler-101 b |
| 49.01 | 244.98$^{+0.66}_{-0.71}$ | 41 | 42 | 0.217 | −2.765 | 3.514 | 0.950 | 2.37$^{+0.35}_{-0.38}$ | 170.8$^{+2.3}_{-2.3}$ | 0.997 | 1.33 | ... | 1042.8 | ... | Kepler-63 b, Active star |
| 63.01* | 265.79$^{+0.31}_{-0.29}$ | 38 | 116 | 0.244 | −16.446 | 14.878 | 0.959 | 0.441$^{+0.032}_{-0.033}$ | 160.24$^{+0.39}_{-0.41}$ | 1 | 2.27 | 306.0 ± 3.1 | 738.1 | ... | Kepler-63 b |
| 64.01* | 3.2$^{+1.2}_{-1.3}$ | 3087 | 240 | 0.151 | −4.605 | 14.783 | 0.985 | 4.4$^{+1.7}_{-1.7}$ | 120$^{+172}_{-168}$ | 1 | 1.26 | ... | 2478.0 | ... | Kepler-23 c |
| 70.01 | 335.07$^{+0.58}_{-0.53}$ | 30 | 46 | 0.539 | −3.314 | 6.149 | 0.749 | 1.44$^{+0.20}_{-0.24}$ | 157.3$^{+3.8}_{-3.8}$ | 0.995 | 1.28 | ... | 174.1 | ... | Kepler-20 c |
| 70.02 | 725.66$^{+0.59}_{-0.51}$ | 14 | 14 | 0.456 | −2.296 | 3.983 | 0.880 | 2.23$^{+0.39}_{-0.39}$ | 141.9$^{+0.10}_{-0.12}$ | 0.993 | 1.22 | ... | 2391.1 | ... | Kepler-20 b |
| 72.01 | 667.46$^{+0.52}_{-0.58}$ | 15 | 23 | 0.241 | −1.439 | 5.141 | 0.916 | 1.10$^{+0.20}_{-0.20}$ | 145.12$^{+0.87}_{-0.88}$ | 0.994 | 1.32 | ... | 11773.0 | ... | Kepler-10 b |
| 75.01* | 3.62$^{+0.40}_{-0.29}$ | 2759 | 2308 | 0.106 | −192.179 | 124.444 | 0.975 | 37.2$^{+5.5}_{-5.7}$ | 116$^{+91.1}_{-96}$ | 1 | 3.16 | [36.9, 47.1] ± 2.5 | 65.7 | ... | Kepler-63 b, Active star |
| 82.01* | 214.94$^{+0.46}_{-0.72}$ | 47 | 47 | 0.426 | −4.175 | 4.717 | 0.948 | 1.61$^{+0.19}_{-0.24}$ | 174.9$^{+1.4}_{-1.6}$ | 1 | 1.49 | ... | 94.2 | ... | Kepler-102 e |
| 84.01 | 31.80$^{+0.19}_{-0.18}$ | 314 | 317 | 0.187 | −10.790 | 13.228 | 0.990 | 5.40$^{+0.28}_{-0.28}$ | 291.3$^{+2.3}_{-2.3}$ | 1 | 1.20 | ... | 532.6 | ... | Kepler-19 b |
| 89.02 | 5.39$^{+0.67}_{-0.74}$ | 1854 | 182 | 0.483 | −22.188 | 13.741 | 0.954 | 8.64$^{+0.20}_{-0.20}$ | 10$^{+104}_{-104}$ | 1 | 1.36 | 24.1 ± 2.8 | 43.5 | 1 | Kepler-89 d, A mutual event system |
| 94.01* | 80.06$^{+0.49}_{-0.32}$ | 125 | 72 | 0.252 | −9.305 | 8.650 | 0.937 | 0.817$^{+0.073}_{-0.086}$ | 198.8$^{+5.8}_{-5.9}$ | 1 | 1.54 | [65.0, 117.4, 161.4] ± 2.8 | 180.1 | ... | Kepler-89 e |

Note. Shown here are 449 objects that are of some interest (confidence > 0.99) and that are not EBs or likely false positives as defined in Section 5.1 (i.e., including objects for which the confidence in their TTV is still lower than the adopted threshold for new detections of confidence > 0.999, but excluding high-amplitude objects). In all cases, results are from the full nonlinear spectral approach fit, except for the count of significant frequencies, which is based on the PA spectrum. We refer to Section 5.1 for a general description and possible caveats. The columns are KOI numbers (new detections are marked with an asterisk, new determination of the TTV period to a previously polynomial TTV is marked by two asterisks); frequency of TTV signal—the fitted parameter; period of the TTV signal ($=f_{TTV}$, rounded value); $\Delta \chi^2$ of the TTV signal over the linear model (rounded value); the four cumulative $\Delta \chi^2$ tests (Section 4.2.3); the TTV signal amplitude; the TTV signal reference time; bootstrap test confidence estimation from 1000 runs; ratio of the scatter around the linear model to the median error; other PA-detected independent TTV frequencies ($\Delta \chi^2 > = 0.999$ threshold (or $>5\sigma$ if so many exist); the expected stroboscopic frequency; and previous references indicated by (1) H16, (2) Xie13’+Xie14, (3) HL14 (high significance), (4) HL16, (5) JH16, and (6) Van Eylen & Albrecht 2015 (see Section 5.1 for details and references). (This table is available in its entirety in machine-readable form.)
Figure 7. Distribution of false-alarm (FAP) probability vs. $\Delta \chi^2$ for all analyzed KOIs (blue dots). Higher confidence objects appear at the bottom of the plot. The axes were chosen to highlight the distribution near the transition from low- to high-confidence objects—most high-confidence TTVs have $\Delta \chi^2 > 100$, i.e., beyond the scale of this figure.

The chosen cutoff levels are somewhat arbitrary. We provide in Table 1 information on more than just the high-confidence targets (>0.999) to help identify those targets that may, with relatively little additional input data or detailed analysis, become high-confidence targets.

The best-fit TTV frequency is the maximum frequency: if an EB with two similar eclipses is misidentified as a transiting planet, any difference between the odd and even eclipses (that may actually stem from nonzero eccentricity, different surface brightness of the stars, etc.) may cause the PA fit to incorrectly add high-frequency TTV at twice the “planetary” orbital period. We found 75 high-confidence TTV signals that were close to the maximum one, and these KOIs are suspected to be EBs or otherwise false positives. Indeed, 60 of them were already in the EBs/FPs sample, and the rest (but one) appear in H16’s suspected false-positives list (their Table 1). In practice, the maximum frequency at which we searched for TTVs was limited by the orbital period $P$ of the transiting planet: $f_{\text{max}}=1/(2P)$. Since the frequency resolution scales with the time span of the data, we labeled systems with a best-fit TTV frequency within $s^{-1}$ of $f_{\text{max}}$ as suspect. We note that sometimes stellar pulsations that survived filtering also caused such high-frequency apparent TTVs. To summarize, in cases where the best-fit TTV frequency is consistent with the maximum frequency, it is a strong sign of a misidentified EB, and these objects are labeled likely false positives. However, they are given in Table 1 for completeness of the high-confidence signals.

Very long TTV periods: the error ranges are usually symmetrical in frequency space. However, on very long periods $P_{\text{TTV}} > s$ where $s$ is the span of the data, the period error range is highly correlated with TTV amplitude. This is expected at such long TTV periods as the data do not allow seeing even a single complete TTV period, making the observed amplitude either close to the total real amplitude (if $f_{\text{TTV,true}} \ll s^{-1}$) or just a fraction of it (if $f_{\text{TTV,true}} \ll s^{-1}$). Importantly, in such cases, the best constraint is on some function of $f_{\text{TTV}}$ and $A_{\text{TTV}}$ and not on each of them individually (see Figure 10). For this reason, some objects in Table 1 appear to have low significance to either $f_{\text{TTV}}$ or $A_{\text{TTV}}$, but these objects all have a very long TTV period, and so the detection of some long-period TTVs on these objects is correct, but the exact amplitude and frequency of these TTVs are more poorly constrained.

Stroboscopic frequency: Kepler’s finite exposure time may also produce a stroboscopic effect, exhibiting apparent TTVs on strictly periodic transiting planets, when the orbital planet’s...
period incidentally is close to an integer multiple of the exposure time (Mazeh et al. 2013; Szabó et al. 2013). We therefore also provide in Table 1 the expected stroboscopic frequency and note that it was detected many times in practice.

Unexplained significant TTV signals: TTV signals with no apparent connection to any other known object in the system were detected many times. Such TTVs are very likely due to interaction with additional planets in the system that are not transiting. Such systems are well suited for RV surveys to connect the inner and outer parts of multi-planet systems.

Multi-periodic TTVs: more than half (277 objects) of all stars with high-confidence TTVs were found to have more than one significant TTV frequency. Additional TTV frequencies allow breaking the degeneracy in the TTV inversion back to absolute masses and are therefore very useful—provided they are real. Indeed, pulsating/variable stars can create spurious TTV signals (see also the following paragraph). On the other hand, a high number of apparently significant TTV frequencies (we adopted the >5 threshold) is likely a sign of imperfect filtering of a variable star and not of multiple dynamical phenomena—and 91 stars exhibit it.

Pulsating/variable stars: these are more difficult to filter, and sometimes residuals of the variability signal remain, especially when the variability time is close to or shorter than the transit duration itself. Such stars frequently exhibit TTVs, but these are difficult to judge for reliability without individually tailored filtering, spot or pulsation modeling, etc. To indicate this as well as cases that may not have been filtered and/or modeled well, we provide in Table 1 the ratio of scatter around the linear model to the median error. As a general rule, ratios lower than two usually mean good filtering and modeling (unity being the white-noise limit), and ratios higher than three should be reviewed on a case-by-case basis because unmodeled phenomena likely exist. None of the objects with a scatter-to-error ratio >50 seemed to be actually reasonably modeled with a linear ephemeris, and thus these objects were removed from further analysis as strongly pulsating. We also visually inspected all high-confidence signals and commented on systems that appeared to be affected by such effects.

When computing the bootstrap analysis, we saved its entire PA spectrum in addition to the best $\chi^2$ of each mock data. This in turn allowed us to build a smaller bootstrap test for each frequency individually, following the question how often the test frequency $f$ had higher $\Delta \chi^2$ than our final $\Delta \chi^2$ cutoff. By counting these frequencies, one can easily identify systems that likely include multi-frequency information (and thus possibly enable inversion for masses) even if this is not visible by eye. We note that poorly modeled systems that exhibit a high ratio of scatter to the median error (also reported in Table 1) are prone to exhibiting unrealistically large numbers of apparently significant frequencies.

5.3. Specific Systems

Here we discuss some of systems for which new information was gained by applying the above analysis. Our goal was to flag interesting systems and not to fully characterize them in depth (given the scope of this paper), and our main tool for analyzing the systems are plots as shown in Figure 11: in each such figure, we superimpose the PA spectra of all transit signals in given system as well as all expected and TTV frequencies, which are of four types: (1) all possible super-frequencies expected from all MMRs with $j\cdot j - N$ period ratio up to $j = 9$ (arbitrarily) and $N = j - 1$ using equation from Section 5 of Deck & Agol 2016. (2) The orbital frequencies. (3) The so-called “chopping frequencies”, which are the frequency of conjunctions between any planet pair: $f_C = f_{in} - f_{out}$. (4) The expected stroboscopic frequencies of individual planets. This rather dense figure allowed us to quickly assign an observed PA peak with a possible physical meaning, even in high-multiplicity systems. Plots like Figure 11 show only those expected frequencies that are found to be relevant to a given system and discussed in the text.

1. KOI 89/Kepler-462: (Figure 12) KOI 89.02 was detected by the PA to have significant TTVs with
The nonlinear fit revised this value to $f_{TTV} = (5.39^{+0.67}_{-0.74}) \cdot 10^{-4}$—consistent with the predicted 5:2 MMR with KOI 89.01 at $f_{\text{Sup}} = 4.704 \cdot 10^{-4}$ days$^{-1}$, while previous analyses (e.g., H16) had a $>4\sigma$ discrepancy. This, together with the high confidence of the TTV detection, confirms KOI 89.02, hitherto just a candidate, as a bona fide planet.

2. KOI 108/Kepler-103: (Figure 13) The PA approach detects two significant frequencies in KOI 108.02, and no TTVs in KOI 108.01. The frequencies detected do not correspond to any known interaction frequency between the known planets and hence may suggest the presence of additional objects in the system. These TTVs are consistent with those reported by H16, but are inconsistent with those first identified by Van Eylen & Albrecht (2015), possibly owing to their employing only part of the data used here.

3. KOI 185: (Figure 14) This system presents the longest TTV period we were able to constrain (at $>3\sigma$). A TTV frequency of $f_{TTV} = (1.75^{+0.79}_{-0.45}) \cdot 10^{-4}$ (TTV period of 15.7 ± 4.1 years). Note, however, that KOI-185.01 may not be due to a planet: with a $\sim3\%$ deep grazing transit...
on a $\sim 0.8 R_\odot$ star, the occulting object may be too large for a planet.

4. KOI 209/Kepler-117: (Figure 11) This system is included for illustrative purposes. There are three distinct frequencies in the inner planet’s TTVs (KOI 209.02): one matches the 2:1 MMR, another matches the 3:1 MMR, and the last matches the orbital period of the outer KOI 209.01 (Bruno et al. 2015).

5. KOI 262/Kepler-50: (Figure 15) Steffen et al. (2013) confirmed this two-planet system based on anticorrelated TTVs spanning $\approx 700$ days. The data available today clearly show that the two planets have TTVs of different frequencies: $f_{TTV,01} = (9.74^{+0.12}_{-0.13}) \cdot 10^{-4}$ and $f_{TTV,02} = (15.42^{+0.14}_{-0.13}) \cdot 10^{-4}$. Interpreting these TTV signals is not trivial: their period ratio is very close to the 6:5 MMR ($\Delta \approx 0.000131$), but the expected super-period is too long and we cannot resolve it using current data. The two different observed TTV periods could be a sign of separate interactions of each of the observed planets with yet another non-transiting planet in the system. For example, a planet on a $\sim 11.7$-day orbital period could explain both observed TTV periods. We conclude that the confirmation above of the two planets by connecting the observed TTVs to mutual interaction between them is not correct, and speculate on the cause of the observed TTVs.

6. KOI 271/Kepler-127: (Figure 16) This system reveals dynamical interactions of KOI 271.02 with both KOI 271.01 and KOI 271.03. The main peak of KOI 271.02 is consistent with the expect 5:3 resonant frequency among 271.01 and 271.02. Additionally, the 2:1 between 271.02 and 271.03 appears as the most significant peak of 271.03, and possibly contributes a shoulder to the spectral peak of 271.02. All three planets were hitherto only validated statistically.

7. KOI 282/Kepler-136: (Figure 17) KOIs 282.01 and 282.03 are close to 3:2 period commensurability $\Delta = -0.056$, and both have significant TTVs, yet the TTVs are not close either the expected super-frequency or the expected chopping frequency. The system is not yet understood, and an additional undetected perturbing planet is suspected.

8. KOI 312/Kepler-136: (Figure 18) KOIs 312.01 and 312.02 are close to a 3:2 period commensurability $\Delta = -0.056$, and both have significant TTVs, yet the TTVs are not close either the expected super-frequency or the expected chopping frequency. The system is not yet understood, and an additional undetected perturbing planet is suspected.

9. KOI 464/Kepler-561: (Figure 19) The two planets in the system are widely separated (periods of 5 and 58 days for KOIs 464.02 and 464.01, respectively) and thus likely do not interact. The outer transiting planet KOI 464.01 exhibits TTVs with two distinct and significant spectral frequencies, consistent with the predicted $f_{	ext{Sup}} = 20.73 \cdot 10^{-4}$ days$^{-1}$ for that second-order resonance, confirming both planets (which were only validated statistically thus far, but had no dynamical confirmation).

5 And also Ofir et al. (2014) during the “The Space Photometry Revolution CoRoT Symposium 3” conference, Toulouse, France—see: https://corot-kasc7.sciencesconf.org/33656.
peaks that may be explained by one or more additional undetected planets.

10. KOI 523/Kepler-177: (Figure 20) This system shows that in addition to the known (Xie 2014) TTV signal of KOIs 523.01 (at \( f_{\text{TTV,01}} = (4.3310^8) \cdot 10^{-4}\text{days}^{-1} \), which is consistent with the super-frequency of the 4:3 MMR with 523.02, there exists an additional significant spectral peak at \( f_{\text{TTV,02}} \approx 523.01 \) that may be related to the expected “chopping” frequency of \( f_{\text{chop}} = 68.94 \cdot 10^{-4}\text{days}^{-1} \). Furthermore, hints of both of these peaks are seen in the PA spectrum of KOI 523.02. These new identifications may be used to improve on the current mass determination (Xie 2014).

11. KOI 775/Kepler-52: (Figure 21): In addition to the previously known (e.g., H16, HL14) TTV peak of KOI 775.02 at a frequency close to the super-period associated with the 2:1 MMR, we detect a new peak in the spectrum of KOI 775.01 at nearly the same frequency \( (f_{\text{TTV}} = 48.81 \cdot 10^{-4}\text{days}^{-1}) \), although at a confidence of 0.998. This may allow an improved mass determination for both planets.

12. KOI 841/Kepler-27: (Figure 22) We detect two previously unidentified frequencies in the PA spectrum of KOI 841.02, in addition to the known primary frequency (Steffen et al. 2012, HL14). These peaks may allow improved constraints on the masses.

13. KOI 870/Kepler-28: (Figure 23) KOI 870.01 and 870.02 are close to the 3:2 MMR, and both exhibit significant TTVs at a frequency consistent with the expected \( f_{\text{sup}} = 44.21 \cdot 10^{-4}\text{days}^{-1} \). Upper limits to the masses were given by Steffen et al. (2013) and a low-significance \( (m/\Delta m \lesssim 3) \) detection of masses by HL14, but only using the most significant TTV frequency. Here we detect secondary frequencies that are just below the high-significance threshold at bootstrap confidences of 0.983 and 0.998 for KOIs 870.01 and 870.02, respectively.
which may allow a better mass determination for both planets.

14. KOI 877/Kepler-81: (Figure 24): The TTVs on KOI 877.02 and KOI 877.01 are both consistent with the expected 2:1 MMR between them at $f_{\text{super}} = 18.1465 \cdot 10^{-4}$ days$^{-1}$, although only the former is high confidence, while the latter has near-threshold confidence of 0.996. HL14 analyzed the system and constrained the component’s masses of KOI 877.01 and KOI 877.02 to $3\sigma$ or less, but used only the most significant TTV frequency, while KOI 841.02 exhibits a few more near-threshold frequencies.

15. KOI 880/Kepler-82: (Figure 25): There are prominent and well-known TTVs (e.g., HL14, H16) on both KOI 880.01 and KOI 880.02, where the primary peak of KOI 880.01 at the super-frequency corresponding to 2:1 MMR between them. Here we detect one additional significant PA spectral peak for KOI 880.01 and three additional peaks in KOI 880.02, and note that the most prominent TTV frequency of the KOI 880.02 at $f_{\text{TTV}} = (8.15 \pm 0.12) \cdot 10^{-4}$ days$^{-1}$ is offset from both the predicted super-frequency and from the observed TTV frequency of KOI 880.01 by a significant margin. Finally, we find that the most significant TTV frequency of KOI 880.04 is just below the threshold in PA (confidence = 0.996), but its $\Delta \chi^2$ increases to above-threshold values in the full fit. This observed TTV frequency does not correspond to any expected frequency.

16. KOI 886/Kepler-54: (Figure 26) The most significant TTV frequency on both KOI 886.01 and KOI 886.02 is well known (e.g., H16). Here we find additional peaks in the PA spectrum of KOI 886.01 at frequencies that are harmonics of the main peak. In addition, the primary peaks of both KOI 886.01 and KOI 886.02 are shifted by
4 − 6σ from the expected super-frequency of the 3:2 MMR between them.

17. KOI 935/Kepler-31: (Figure 27) HL14 identified the main TTV frequency of KOIs 935.01 and 935.02 as a 2:1 MMR at the expected $f_{\text{Sup}} = 10.27 \cdot 10^{-4}$ days$^{-1}$ and provided a determination of the mass of KOI 935.02. Here we find the 935.01 also has a second high-significance peak close to the expected $f_{\text{Sup}} = 23.01 \cdot 10^{-4}$ days$^{-1}$ of the 4:1 MMR with 935.03. The PA peak of KOI 935.02 is also wider than expected (and wider than the peak of KOI 935.01)—close to the expected super-frequency of the 2:1 MMR between the 935.02 and 935.03. We therefore suspect that the PA spectrum of KOI 935.02 includes the sum of two blended and similar peaks that tie the three outer planets in a 1:2:4 resonance chain. This may allow determination of other masses in the system.

18. KOI 952/Kepler-32: (Figure 28) We find a significant peak in the PA spectrum of KOI 952.02 at a low frequency, in addition to the known peak near the 3:2 MMR super-frequency ($f_{\text{Sup}} = 38.649 \cdot 10^{-4}$ days$^{-1}$) between it and KOI 952.01, previously identified by HL14.

19. KOI 1102/Kepler-24: (Figure 29) KOI 1102.01 and KOI 1102.02 have both well-known TTVs (e.g., HL14, H16) with a TTV frequency consistent with the 3:2 MMR between them. Here we identify another significant TTV frequency for KOI 1102.01—so a more precise determination of the masses seems possible.

20. KOI 1236/Kepler-279: (Figure 30) Xie (2014) identified the main TTV frequency of KOIs 1236.01 and 1236.03 as a 3:2 MMR at the expected $f_{\text{Sup}} = 8.4773 \cdot 10^{-4}$ days$^{-1}$ and provided a determination of their mass ($9.4 \pm 5.9$ Å and $7.5 \pm 4.5$ Å for KOI 1102.01 and KOI 1102.03, respectively). However, Xie (2014) did not use the fact that there are more significant frequencies for both planets, however—including one peak for KOI 1236.03 near the expected 1:4 MMR super-frequency with KOI 1236.02, which thus far has no mass constraints, and a peaks for KOI 1236.01 near the expected “chopping” frequency with KOI 1236.03. A more precise determination of the masses seems possible.

21. KOI 1258/Kepler-281: (Figure 31) Here we detect a PA spectral peak for 1258.01 that is not consistent with any expected super-frequency of the previously known members of the system. The transit signals of KOIs 1258.01 and 1258.02 were statistically validated (Morton et al. 2016). One of the secondary (low-confidence) peaks of the PA spectrum is close to the orbital frequency of KOI 1258.03, possibly confirming the latter (currently classified as a candidate).

22. KOI 1366/Kepler-293: (Figure 32) We find a significant TTV signal for 1366.01 at $f_{\text{TTV}} = (33.43^{+0.65}_{-0.63}) \cdot 10^{-4}$ days$^{-1}$, consistent with the expected frequency for second-order 3:1 MMR with KOI 1366.02 at $f_{\text{Sup}} = 34.58 \cdot 10^{-4}$ days$^{-1}$. This frequency also
appears as a smaller peak in the PA spectrum of KOI 1366.02. This confirms both planets (hitherto only having only statistical validation).

23. KOI 1426/Kepler 297: (Figure 33) The system is close to 1:2:4 MMR chain with normalized distance from resonances of $\Delta_{01,02} = -0.03748$ and $\Delta_{02,03} = 0.001097$. As previously noted (HL14), the PA spectrum of 1426.02 exhibits evidence for the interaction between it and 1426.01 at the expected super-frequency and its first harmonic. The additional peaks of 1426.02 may be attributed to the orbital period of 1426.03 or the “chopping” frequency between 1426.02 and 1426.03. Moreover, the observed TTV frequency of 1426.03 and 1426.01 at $f_{\text{TTV}} = (8.91^{+0.33}_{-0.28}) \cdot 10^{-4}$ days$^{-1}$ is consistent with the 4:1 MMR between these objects. We note 1426.03 is currently a still a candidate, that no RV variation was detected in the system (Santerne et al. 2016). The system has a high S/N, so it is attractive for further analysis (previous analysis by Diamond-Lowe et al. 2015 is not publicly available).

24. KOI 1529/Kepler-59: (Figure 34) We detect a significant peak in 1529.02 at the same frequency as the known peak in 1529.01, both consistent with the 3:2 MMR super-frequency. Moreover, there are peaks in the PA spectrum of KOI 1529.01 that appear to be just below the adopted significance threshold—possibly enabling absolute mass determinations for these small planets (both have radii $R < 2R_{\oplus}$).

25. KOI 1599: (Figure 35) KOI-1599.02 is found to have TTVs with $f_{\text{TTV}} = (6.00 \pm 0.67) \cdot 10^{-4}$ days$^{-1}$, which is approximately consistent with that of KOI-1599.01, but both are not consistent with the expected 3:2 MMR super-frequency. This, together with the strong ($\Delta = -0.00081$) resonance, suggests that the system is
in resonance (and not just near resonance), rendering the usual expression for the super-frequency irrelevant. In this case, the TTV frequency, in addition to the amplitude, can constrain the planetary masses. It is noteworthy that there are a few more significant frequencies in the PA spectrum of KOI-1599.01, but the signal has a high amplitude. Both KOIs are currently still neither confirmed nor validated.

26. KOI 1783: (Figure 36) TTVs of both candidates were detected by H16: a low TTV frequency of \( f_{\text{H16}} = 7.19 \pm 0.67 \times 10^{-4} \text{ days}^{-1} \), and long-term ("polynomial") TTVs for KOIs 1783.01 and 1783.02, respectively. We find (and indeed see clearly in the figures presented by H16) that the most significant TTV frequency for KOI 1783.01 is much higher at \( f_{\text{TV}} = (35.95^{+10.64}_{-10.02}) \times 10^{-4} \text{ days}^{-1} \)—which is consistent with all three of the orbital frequency of 1783.02, the "chopping" frequency between the two candidates, and the maximum TTV frequency of KOI 1783.01 itself. In addition, we constrain the TTV frequency of KOI 1783.02 to be \( f_{\text{TV}} = (10.60^{+1.55}_{-0.77}) \times 10^{-4} \text{ days}^{-1} \), and the second-most significant TTV frequency for KOI 1783.01 (at low confidence) is virtually identical to this frequency—but not close to any expected super-frequency. The two candidates therefore appear to be interacting, but further study is needed.

27. KOI 1831/Kepler 324: (Figure 37) The known TTVs on KOIs 1831.01 and 1831.03 are anticorrelated but only polynomial [H16], and only the former was statistically validated, while the latter is still a candidate. Here we detect the TTV frequency of KOI 1831.01 at \( f_{\text{TV}} = (4.24^{+1.35}_{-0.67}) \times 10^{-4} \text{ days}^{-1} \) and find it to be in agreement with the predicted 3:2 MMR super-frequency with KOI 1831.03, but we do not detect the very
significant TTVs on KOI 1831.03, as expected, since it is high amplitude. Additionally, there are a few more significant frequencies in the PA spectrum of KOI 1831.01. This dynamically confirms KOI 1831.03, hitherto just a candidate, and possibly allows probing the absolute masses of the planets.

28. KOI 1955/Kepler-342: (Figure 38) We find two significant TTVs in the PA spectrum of KOI 1955.02, also present in KOI 1955.04 (partly also seen by H16). The frequencies are away from the expected 3:2 MMR super-frequency, possibly due to the system being deep in resonance ($\Delta = 0.0027$).

29. KOI 2038/Kepler-85: (Figure 39) This well-studied system (Xie 2013, H16, Hadden & Lithwick 2016) shows significant TTV frequencies on KOIs 2038.01 and 2038.02 that are consistent with the expected 3:2 MMR between them. Here we find that the system may be more interconnected, with a possible blended peak on the 2038.02 PA spectrum related to a 2:1 MMR with KOI 2038.04, and two additional low-significance peaks in the PA spectrum of 2038.01. One occurs at the expected super-frequencies of the 2:1 MMR with 2038.04 ($f_{\text{sup}} = 410 \cdot 10^{-4} \text{ days}^{-1}$), and another occurs near the unusual 9:4 MMR with 2038.03.

30. KOI 2092/Kepler-359: (Figure 40) All three planets in the system were only statistically validated, and weak ($<2\sigma$) mass limits were subsequently given by Hadden & Lithwick (2016). Here we find significant low-frequency TTVs for all three planets, which may arise either from the system residing near or within resonance.

6. Conclusions

We introduced the technique of spectral approach to TTVs for the detection of transit timing variations. The spectral approach is more sensitive because of the fewer free parameters in its model; it is not limited by short or low S/N single events because it uses one global fit and not multiple event-by-event fits; and it is not biased since only the improvement over the linear model matters, and not the properties of linear model itself. New TTV candidates were found, and the overall set has no significant period or depth biases relative to the general Kepler candidate population—unlike catalogs resulting from the classical approach to TTV detection. Consequently, the spectral approach is more sensitive to TTVs of lower amplitude, around smaller- and shorter-period planets, than the classical TTV measurement technique. We also presented the perturbative approximation (PA) to the spectral approach, a linear approximation that is much faster than the full model, although less sensitive to higher-amplitude TTVs, allowing us to quickly identify candidates for a more computationally intensive full spectral approach fit. The PA can also be used for other types of variations, such as impact parameter variations, and this will be further explored in future work.

Applying these techniques to Kepler data, we were able to detect 129 new TTV-bearing stars. The fact that so many new TTVs were detected is interpreted as stemming from the high...
The Astrophysical Journal Supplement Series, 234:9 (21pp), 2018 January
Ofr et al.

planetary multiplicity uncovered by Kepler. TTVs are not the exception, but rather the rule. Of particular importance are (a) stars that exhibit multiple sets of transits, which sometimes allow us to link the observed TTVs to a specific planet–planet interaction, and to place constraints on the masses of the planets; (b) stars that exhibit multiple significant TTV frequencies; and (c) planets that have TTVs that cannot be linked to other planets in the system: these planets are likely affected by other as yet unknown objects in the system. We note that the use of the full PA spectrum, and not just the most significant frequency, was found to be useful.

The sensitivity and generality of PA for all targets that exhibit three or more transits, paired with its short execution time, make it highly suitable for current and future large-scale surveys (e.g., space-based K2, TESS, and PLATO as well as ground-based SuperWASP and HATNet). Moreover, the execution time of PA is so short that applying it to all 150,000 Kepler stars is possible—even those with currently no threshold crossing event at all. The speed will make searching for TTVs on all stars feasible. This may be useful since small, low-mass planets and easier to deflect by gravitational interaction with other bodies in the system, and thus planets may (and probably do) exist that have escaped detection because their transit signals have been blurred by TTVs. Such planets may become detectable once TTVs are accounted for using PA.

This project was supported by the Helen Kimmel Center for Planetary Science, the Minerva Center for Life Under Extreme Planetary Conditions, and by the I-CORE Program of the PBC and ISF (Center No. 1829/12). A.O. acknowledges the support of the Koshland Foundation and McDonald-Leapman grant J.-W.X. acknowledges support from the NSFC Grants (11403012, 11333002, 11661161014) and a Foundation for the Author of National Excellent Doctoral Dissertation of People’s Republic of China. R.S. acknowledges the support of the ISF. This paper includes data collected by the Kepler mission. Funding for the Kepler mission is provided by the NASA Science Mission directorate. Some/all of the data presented in this paper were obtained from the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555. Support for MAST for non-HST data is provided by the NASA Office of Space Science via grant NNX09AF08G and by other grants and contracts. This work has made use of services produced by the NASA Exoplanet Science Institute at the California Institute of Technology.

ORCID iDs
Aviv Ofr https://orcid.org/0000-0002-9152-5042
Ji-Wei Xie https://orcid.org/0000-0002-6472-5348
Re’em Sari https://orcid.org/0000-0002-1084-3656
Oded Aharonson https://orcid.org/0000-0001-9930-2495

References
Agol, E., & Deck, K. 2016, ApJ, 818, 177
Agol, E., Steffen, J., Sari, R., & Clarkson, W. 2005, MNRRAS, 359, 567
Borkovits, T., Hajdu, T., Szatkovszky, J., et al. 2016, MNRRAS, 455, 4136
Bruno, G., Almenara, J.-M., Barros, S. C. C., et al. 2015, A&A, 573, A124
Carter, J. A., & Agol, E. 2013, ApJ, 765, 132
Carter, J. A., Fabrycky, D. C., Ragozzine, D., et al. 2011, Sci, 331, 562
Deck, K. M., & Agol, E. 2016, ApJ, 821, 96
Deck, K. M., Agol, E., Holman, M. J., & Nesvorný, D. 2014, ApJ, 787, 132
Diamond-Lowe, H., Stevenson, K. B., Fabrycky, D., et al. 2015, in American Astronomical Society Meeting Abstracts 225, 438.01
Dreizler, S., & Ofr, A. 2014, arXiv:1403.1372
Ford, E. B., Rowe, J. F., Fabrycky, D. C., et al. 2011, ApJS, 197, 2
Hadden, S., & Lithwick, Y. 2014, ApJ, 787, 80
Hadden, S., & Lithwick, Y. 2016, ApJ, 828, 44
Holczer, T., Mazeh, T., Nachmani, G., et al. 2016, ApJS, 225, 9
Holman, M. J., Fabrycky, D. C., Ragozzine, D., et al. 2010, Sci, 330, 51
Holman, M. J., & Murray, N. W. 2005, Sci, 307, 1288
Jontof-Hutter, D., Ford, E. B., Rowe, J. F., et al. 2016, ApJ, 820, 39
Kipping, D. M. 2010, MNRRAS, 408, 1758
Kipping, D. M., Nesvorný, D., Buchhave, L. A., et al. 2014, ApJ, 784, 28
Kirk, B., Conroy, K., Prša, A., et al. 2016, AJ, 151, 68
Kovács, G., Zucker, S., & Mazeh, T. 2002, A&A, 391, 369
Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011, Natur, 470, 53
Lithwick, Y., Xie, J., & Wu, Y. 2012, ApJ, 761, 122
Mandel, K., & Agol, E. 2002, ApJL, 580, L171
Marcy, G. W., Isaacson, H., Howard, A. W., et al. 2014, ApJS, 210, 20
Mazeh, T., Nachmani, G., Holczer, T., et al. 2013, ApJS, 208, 16
Mills, S. M., Fabrycky, D. C., Majewski, C., et al. 2016, Natur, 533, 509
Morton, T. D., Bryson, S. T., Couglin, J. L., et al. 2016, ApJ, 822, 86
Nesvorný, D., Kipping, D. M., Terrell, D., et al. 2013, ApJ, 777, 3
Nesvorný, D., Kipping, D. M., Buchhave, L. A., et al. 2012, Sci, 336, 1133
Ofr, A., & Dreizler, S. 2013, A&A, 555, A58
Ofr, A., Dreizler, S., Zechmeister, M., & Husser, T.-O. 2014, A&A, 561, A103
Osborn, H. P., Armstrong, D. J., Brown, D. J. A., et al. 2016, MNRRAS, 457, 2273
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical recipes in FORTRAN. The art of scientific computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Rowe, J. F., Bryson, S. T., Marcy, G. W., et al. 2014, ApJ, 784, 45
Santerne, A., Diaz, F. R., Moutou, C., et al. 2012, A&A, 545, A76
Santerne, A., Moutou, C., Tsantaki, M., et al. 2016, A&A, 587, A64
Schmitt, J. R., Wang, J., Fischer, D. A., et al. 2014, AJ, 148, 28
Steffen, J. H. 2016, MNRRAS, 457, 4384
Steffen, J. H., Fabrycky, D. C., Agol, E., et al. 2013, MNRRAS, 428, 1077
Steffen, J. H., Fabrycky, D. C., Ford, E. B., et al. 2012, MNRRAS, 421, 2342
Szabó, R., Szabó, G. M., Dálya, G., et al. 2013, A&A, 553, A17
Van Eylen, V., & Albrecht, S. 2015, ApJ, 808, 126
Xie, J.-W. 2013, ApJS, 208, 22
Xie, J.-W. 2014, ApJS, 215, 45
Xie, J.-W., Wu, Y., & Lithwick, Y. 2014, ApJ, 789, 165