Empirical Analysis of Carbon Price Based on EGARCH

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Abstract. The carbon trading market has been increasingly heated for more than a decade, and its market participants and market rules are not exactly the same as traditional stock or exchange rate markets. Based on the mature and efficient GARCH series model, this thesis analyzes the rules of the carbon trading market. Starting from the actual data, it studies the conclusions and models applicable to carbon trading market from both qualitative and quantitative perspectives.

1. Introduction
With the development of finance, there have been highly diverse theories and models to illustrate the laws of the market. Different theories frequently correspond to different assumptions. It is often difficult to directly verify these assumptions and deduce the applicable conclusions. In the context of the increasingly popular carbon trading market, the study of the rules of the carbon trading market has become more significant. In order to clarify the market rules applicable to the carbon trading market, while avoiding the complex process from the assumptions of various theories, we consider using mathematical models to analyze the time series of representative carbon price formation in the carbon trading market. The mathematical model obtained by simulation is used to reverse the relevant laws of the carbon price market.

In financial markets, there are all kinds of uncertainties, of which some are closely linked to time. In order to better simulate conditions of the underlying price changes, Engle [5] in 1982 proposed ARCH (Autoregressive conditional heteroscedasticity) model. This model combines the various currently available information to obtain the results of the fitting through autoregressive forms. Bollerslev promoted Engle’s ARCH model in 1986 and obtained GARCH model, namely, the general ARCH model. The form of GARCH (p, q) model is as follows [3]:

\[ y_t = X_t^T b + \epsilon_t \]
\[ \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \]
\[ \epsilon_t \sim N(0,1) \]

where \( X_t \) is a set of given time series, \( \epsilon_t \) are white noises, \( p \) is the maximum lagged term of ARCH item, \( q \) is the maximum lagged term of GARCH item, and \( p \geq 0, q \geq 0, \omega \geq 0, \alpha_i \geq 0, \beta_i \geq 0, i = 0,1,..., p ; \sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{p} \beta_i < 1 \) is required for guaranteed wide stationarity of \( \{ y_t \} \). As the GARCH (1, 1) model is efficient, simple and convenient in fitting, while better reflecting the heteroscedasticity of financial sequences under common circumstances, it retains the long-term memory...
of actual data[2]. On the basis of GARCH model, a series of models such as EGARCH and TGARCH have been derived, which have high similarity with the GARCH model in subject, but their application has its own unique advantages and disadvantages. Consequently, they should be selected flexibly according to the specific situation.

2. Establishment and analysis of the EGARCH model

In 2005, the European Union Emissions Trading System (EU ETS) was established. In the same year, the Kyoto Protocol came into force, and carbon trading entered a period of rapid development. After the “6+1” pilot from 2012 to 2016, China also established a national unified carbon trading market in 2016. GARCH series model is one of the widely used models in the financial community. This thesis selects the daily average value change of the EUA primary market price from May 3, 2016 to April 30, 2018 (data from EXX) as a sample. By fitting the changes of carbon price, the applicability of EGARCH model to the study of carbon price is obtained. It also uses quantitative models to explain the applicable financial laws in the carbon price market. The empirical analysis results of this thesis are obtained by Stata15 software.

2.1 Establishment of mean value model

The sequence formed by the daily carbon price mean is a financial time series. In order to determine by which fitting method it is suitable for, we first test its stationarity. Therefore, we need to establish an appropriate AR model for this sequence, and then judge the stationarity of the sequence by using this model [3]. The lag intervals for endogenous of the sequence is determined, and the results in table 1 are obtained.

| Lag | LL   | LR    | df | p       | FPE  | AIC  | HQIC | SBIC  |
|-----|------|-------|----|---------|------|------|------|-------|
| 0   | 25.1366 | 0.0518 | -0.1231 | -0.1191 | -0.1130 |
| 1   | 25.4001 | 0.5270 | 0.0520 | -0.1194 | -0.1114 | -0.0991 |
| 2   | 27.5071 | 4.2140 | 0.0517 | -0.1250 | -0.1130 | -0.0946 |
| 3   | 27.6341 | 0.2540 | 0.0519 | -0.1206 | -0.1045 | -0.0801 |
| 4   | 28.0984 | 0.9286 | 0.0520 | -0.1178 | -0.0978 | -0.0672 |

Note: "*" represents that the corresponding statistic is of the most significance under the lag intervals for endogenous of that line.

According to the results in table 1, it can be observed that the significance of commonly used statistics is concentrated in the case of lag intervals for endogenous Lag=2. Therefore, AR(2) model is selected as the mean value model of this time series, and the results obtained after the establishment are shown in table 2.

| Coef. | Std. Err. | z    | P>|z| |
|-------|-----------|------|------|
| L1.   | -0.0267   | 0.0499 | 0.53 | 0.593 |
| L2.   | -0.1063   | 0.0499 | 2.13 | 0.033 |
| _cons | 0.0213    | 0.0116 | 1.83 | 0.067 |

With this model as a referential mean value model, we can make sequence diagram, autocorrelogram and partial autocorrelogram to determine the stability of the time series. Only the sequence diagram shown in figure 1 is demonstrated here.
Figure 1. Sequence diagram of differential time series of carbon price

From the above results, we can observe and discover that the stability of the sequence is not obvious, so it is not suitable to use the ARAM model for processing. However, we can consider using the ARCH series models. In order to determine whether the sequence can be applied to the ARCH series models, we need to detect its ARCH effects.

2.2 Detection of ARCH effects
Aiming to test whether a sequence can be applied to an ARCH series model, we should judge whether the sequence has heteroscedasticity effect. From the sequence diagram in Figure 1, we can directly find out through ocular observation that fluctuations have a certain cluster effect, but the conclusion obtained by intuitionistic observation alone is not convincing enough [1]. Therefore, the method of Q-test can be used for quantitative judgment, and then the existence of heteroscedasticity effects can be reversed by simply observing the significance of various coefficients after the establishment of the ARCH series model.

Table 3. Q-test of residual sequence

| Lag | AC    | PAC   | Q   | P>Q |
|-----|-------|-------|-----|-----|
| 1   | 0.090 | 0.090 | 3.239 | 0.072 |
| 2   | 0.055 | 0.048 | 4.461 | 0.108 |
| 3   | 0.051 | 0.042 | 5.501 | 0.139 |
| 4   | 0.137 | 0.129 | 13.122 | 0.011 |
| 5   | 0.353 | 0.338 | 63.679 | 0.000 |
| 6   | 0.063 | 0.008 | 65.268 | 0.000 |
| 7   | 0.043 | 0.000 | 66.019 | 0.000 |
| 8   | 0.044 | 0.004 | 66.807 | 0.000 |

According to the results of Q-test in Table 3, it can be concluded that the square sequence autocorrelation \( \{e_t^2\} \) of OLS residuals exists. Correspondingly, this time series has significant conditional heteroscedasticity effect, which means it can be analyzed by using the ARCH series model.

2.3 Establishment of EGARCH (1, 1) model
Since the corresponding time series has passed ARCH effect detection, we can consider the ARCH series model as the fitting orientation. The concise and effective EGARCH(1,1) model is adopted to deal with it, and its specific form is as follows [4]:

\[
\ln(\sigma_t^2) = \omega + g(\eta_{t-1}) + \beta \ln(\sigma_{t-1}^2) \quad \text{where} \quad g(\eta) = \phi\eta + \gamma(|\eta| - E|\eta|)
\]
The exponential type GARCH model is of many advantages. For instance, we no longer have requirements on the sign of the parameters. The above EGARCH(1,1) model is used for fitting, and the results in Table 4 are obtained.

| Coef. | Std. Err. | z   | P>|z| |
|-------|-----------|-----|-----|
| y     | L1.       | -0.0618 | 0.0446 | -1.39 | 0.165 |
|       | L2.       | -0.0470 | 0.0365 | -1.29 | 0.198 |
| _cons |           | 0.0232  | 0.0121 | 1.92  | 0.055* |
| ARCH  |           |         |       |       |       |
| egrarch | L1.    | -0.8095 | 0.0774 | -10.46 | 0.000*** |
| arch   | L1.      | 1.0240  | 0.3832 | 2.67  | 0.008*** |
| _cons  |          | -5.3824 | 0.2540 | -21.19 | 0.000*** |

Note: “***” means significant at 1% level, “*” means significant at 10% level.

As is shown in the fitting results in Table 4, taking $1 - \alpha = 0.99$ as the confidence coefficient, it is believed that each corresponding ARCH coefficient in EGARCH model has good significance, which also confirms that the time series does have prominent ARCH effects.

2.4 Model interpretation and corresponding market rules
Since all coefficients obtained by the EGARCH (1,1) model have good significance, we have sufficient reasons to use the conclusions given by this model to indicate and explain the suitable market rules.

First of all, due to the significance of egarch coefficient, we can conclude that the previous prices do somehow have influences on the future price trend. From the perspective of the market itself, this conclusion is also close to actual situation. The carbon price is determined by the supply and demand relationship of carbon emission indicators, and the demand of carbon emissions is not a short-term thing. For example, if a steel-making enterprise plans to increase steel production this year, it is bound that the carbon emission credits required by this enterprise will rise due to the rising steel production, and the increase in steel production will usually be allocated to each quarter or month. As a result, the enterprise will have higher demand for carbon emission indicators in a longer period of time. This means that since there was a strong demand for carbon emission yesterday, and then today’s carbon price is likely to still be continuously influenced by the yesterday’s demand. Therefore, the previous carbon price will indicate the fluctuation of today’s carbon price from some aspects. Although real markets often fail to meet the strict assumptions of efficient market theory, we can still use this kind of ideas to quantify the fluctuation of the underlying price in the market.

Secondly, it can be seen from the results that the coefficient of the egarch term is negative, which actually means that when the price falls, the price will have a stronger impact on that of the next day than when the price rises. Similarly, we also verify the results predicted by this model from actual market conditions. We treat this issue from the perspective of behavioral finance. Robert J. Shiller, a well-known behavioral finance economist, mentioned the effect of “fear trampling” in the book Irrational Exuberance[6]. After the market enters the downlink channel, even if the market fundamentals have tended to be stable, the downtrend will continue for a while, due to the fact that the market participants are affected by the previous market downlink, actively selling the holding underlying or choosing to be bears. In carbon market, we can follow this concept, believing that the “fear trampling” effect caused by price decline is relatively obvious in carbon trading market.

3. Summary
By using the EGARCH model to fit the carbon price, we obtain a model with quite good significance. The coefficients of the model also have consistent financial interpretation, and the model is highly explanatory. Thus it can be seen that ARCH series model is indeed applicable to the current carbon trading market. Via the use of such models, we can effectively simulate the market situation and make reasonable explanations or predictions.
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