A work done approach on analyzing the effects of densification parameters on tapered screw extruders

Addisu Kidanemariam Tadese

Abstract

Densification is the process of compacting bulk material to increase its physical and energy density. One way of densification is by using screw extruders. Yet, sufficient theoretical studies have not been carried out on the densification parameters for a tapered screw. In this study, a mathematical model for investigating the effect of the operational parameters on a tapered screw has been studied by analyzing the work balance on the plug along the tapered screw channel. By using the taper angle, screw pitch, and barrel friction coefficients as control variables, a mathematical model for the pressure gradient, volumetric throughput, and output density was established. The velocity profile of the plug along the screw channel was measured to determine the volumetric throughput. The final model for the pressure gradient along the axial length of the screw has been approximated by Runge-Kutta fourth order approximation model in the python programing environment. From the results, an increase in screw length, screw taper angle, and barrel friction coefficient has been attributed to an increase in compaction pressure. This parameter has also changed for a 0° taper angle, which is primarily due to centrifugal, gravity, and frictional forces acting on the plug. Further investigation into the volumetric throughput and output density has the same effect. According to the analysis, the optimum taper angle has been identified to be between 3° and 5°, which yields the optimum output density without compromising the combustion property of the compacted biomass.

Keywords

Densification, tapered screw, compaction Pressure, compaction density, Python programing, Runge-Kutta fourth order approximation

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Introduction

Because of their abundance and carbon-neutral raw material, biomass materials have been used as an alternative renewable energy source. However, due to their low bulk density, high moisture content, storage difficulty, and low energy density (typically 80–150 kg/m³ and 150–200 kg/m³ for herbaceous and woody biomass respectively), their application as a major source of energy production is limited.1 To extract the most energy from such loose biomass, their physical and energy density must be increased via any compaction mechanisms.2–4 Evidence suggests that compacting loses biomass by gradually increasing the applied pressure, increases the output density. In the absence of a binder material, having an output density greater than 1200 kg/m³ stabilizes the material for a durability of greater than 90%.5,6

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The most often utilized compacting technologies are hydraulic press and tapered screw press compaction.  

Experimentally, the compaction mechanisms of 15 hydraulic press and 15 tapered screw press machines were compared to determine the most efficient compaction mechanism. According to their research, the screw press had a higher output density of 1122 kg/m$^3$ than the hydraulic press, which had a density of 902 kg/m$^3$ for the same power consumption.

Solid compaction with a variable screw pitch can also be utilized to densify a biomass with a constant screw diameter, resulting in a significant increase in output density. In the past, several models for solid conveying and compaction in extruders were discussed. Their research focuses mostly on uniaxial compaction models.

According to Tadmor, in order to investigate solid conveying in a standard screw, they separately investigated the three extrusion processes (solid conveying, melting, and melt conveying). They hypothesized that the initial filling pressure $P_1$ is affected by the weight of the material in the hopper instead of the frictional and gravitational forces, resulting in the following equation:

$$P = P_1 e^{Ax}$$

Where: $P_1$ is the initial filling pressure at $x = 0$, $A$ is a constant for a given screw geometry, which have the form

$$A = r_h \rho g \left[ 1 - e^{-\frac{2u_m^2}{u_c}} \right]$$

Lovegrove and Williams developed an equation for the rate of pressure change (equation (2)) in a standard screw by assuming a constant pressure along the screw shaft, barrel surface, and screw surface, as well as incorporating the width of the flight ($w$), helix angle ($\phi$), conveying angle ($\theta$), and different frictional coefficients $\mu$.

$$\frac{dP}{dx} = \frac{P_A}{B}$$

Where: $A = \mu_k w_h (\cos \alpha - K \sin \alpha) - 2h \mu_m (d_h (d_h - h) \cos \psi_{av} + K \sin \psi_{av}) - w \mu_m (\cos \theta_{av} - 2h \cos \theta_{av} + K \sin \theta_{av})$

$$B = wh (d_h (d_h - h) \cos \theta_{av} + K \sin \theta_{av})$$

By studying the forces and torque balance on the plug for a conventional screw, an improved model with a non-isotropic pressure distribution in the material was developed by Broyer and Tadmor. They established an equation for pressure rises in an exponential form (equation (3)), by adjusting the channel depth and neglecting the acceleration down the channel.

$$P = P_0 e^{R Z_h}$$

Where the term R is a function of several sub-functions such as screw width, barrel and screw flight friction coefficients, screw helix angle and solid conveying angle, $P_0$ is initial filling pressure, $Z_h$ is down channel distance at the barrel surface.

Based on the developed friction forces, a traction and retardation mechanism was used to develop a theoretical model for pressure rise. To evaluate the pressure distribution in a tapered screw, a plug flow model was assumed, which presupposes no internal shearing occurs during compaction. It was assumed that the pressure is non-isotropic. The taper angle (mainly between $20^\circ$ and $40^\circ$) has a major effect on the pressure developed depending on the length of the compaction region of a tapered screw.

It is difficult to develop an accurate mathematical model for identifying the output density in a biomass, since it is dependent on a variety of non-linear characteristics. Using genetic algorithms and a neural network, a theoretical model was developed by analyzing the effects of the variables on the output density of compacted biomass. Their artificial neural network makes use of node-to-node connections with numerical dependence. They constructed a quadratic equation (equation (4)) based on their experimental findings that predicts the output density with a mean square error of 0.01732. 

$$\rho = + 126.22 + 46.5 X_1 - 16.63 X_2 - 27.92 X_4 - 1.17 X_2 X_3 - 0.59 X_1^2 - 60.72 X_4^2$$

Where the dependent variables $X_1$, $X_2$, $X_3$, and $X_4$ are moisture content, piston speed, die length, and particle size respectively.

From their result, a moisture content above 40%, a particle size above 0.8 mm, and a piston speed above 2 mm/s will negatively affect the output density while increasing the die length shows a positive effect on increasing the density.

The effect of taper angle on final density through a conical die was investigated, and a simplified mathematical model for back pressure (equation (5)) was developed.

$$P_{back} = P_{axial} e^{-\frac{4H}{D_k} \sin \phi}$$

Where $P_{back}$ is the counter pressure in the chamber, $P_{axial}$ is the axial pressure of the hydraulic press, $k$ is the ratio of the main strains, $\mu$ is the friction coefficient, $H$ is the length of a pressed briquette, and $D_k$ is the diameter of the pressing chamber.

They used a $1^0$ and $2^0$ taper angle pressing chamber, as well as an external heating source, to investigate the effect of the external heating source on density and pressing pressure. From their result, an internal pressures of 88 Mpa with 1.216 kg/m$^3$ 101 Mpa with
1.224 kg/m³ for 1⁰ and 2⁰ tapper angle at 120⁰C die temperature respectively and 106 Mpa with 1.221 Kg/m³ 144 Mpa with 1.236 Kg/m³ for 1⁰ and 2⁰ tapper angle at 80⁰C die temperature were obtained. It may be extrapolated from their findings that, while increasing the pressure increases the final output density, increasing the temperature decreases it.

The main gaps in the above studies are that they focus on the change in pressure without taking into account the effects of all possible acting forces on the plug, the effect of friction coefficients on the plug by the screw, the barrel, and the shaft, and the effect of the tapered screw’s complex geometrical parameters, which will impede their ability to generate an accurate model. On the other hand, other studies are based on experimental investigations with specific materials, which, according to the authors perspective, renders their work non-universal and inapplicable to a variety of materials.

The goal of this study is to develop a mathematical model that predicts the pressure rise in the compaction zones of the tapered screw, the volumetric throughput, and the output density of the biomass by considering all three dimensional forces, making the work more universal and applicable in the fields of powder technology, energy, food, and agriculture.

**Methods**

**Mathematical models**

Based on the assumption of plug flow theory, which has been adopted by Broyer and Tadmor, Weert, a mathematical model for analyzing the compaction pressure and theoretical flow rate was constructed in this study.

The equation derived for compaction pressure follows the principle of Broyer and Tadmor. They devised a model for straight screw extruder that links the ratio of the exit pressure to the entrance pressure using simple kinetics considerations. Force balances over a differential part of the plug are then used to calculate the pressure ratio.

The equation obtained in this study is similar to the approach taken by Broyer and Tadmor, yet it modifies and incorporates all forces for a tapered screw while examining the work done on the differential element. Furthermore, using a kinematic method, an equation correlating the volumetric flow rate to the conveying angle, helix angle, screw tapper angle, and other geometrical parameters of the tapered screw was constructed.

In addition to the plug flow assumption, the solid compaction process adopted here are based on the following assumptions:

- The solid particle in the channel is assumed to be a rigid body.
- The solid particles occupy all of the channel’s available space.
- The screw’s geometry changes as it moves along its axis.
- The distance between the screw and the barrel is regarded as insignificant.
- It is expected that the compacted biomass is in contact with the barrel surface, the screw surface, and the screw shaft.
- The pressure distribution varies from section to section along the channel. But the pressure on the screw $P_{sc}$, barrel $P_b$ and screw shaft $P_{sh}$ are proportional to each other. $P_b = P_{sc} = P_{sh} = kP$

The stress transmission coefficient, also known as the Janseen coefficient “$k$”, is the ratio of pressure acting in the lateral direction to pressure acting in the vertical direction and is calculated using the equation (6)\textsuperscript{17}:

$$k = \frac{1 - \sin \phi}{1 + \sin \phi} \tag{6}$$

where $\phi$ internal friction angle. This angle is close to the angle of repose.\textsuperscript{18}

**Geometrical representation.** Figure 1 depicts a sectional view of the plug’s geometry (middle segment) as well as the pushing and trailing sides of the plug (top and bottom sections respectively). We can derive the following geometrical relationships among the parameters from Figure 1, assuming that the screw taper angle is equivalent to the barrel taper angle and the space between them is insignificant.

$$dx_b = r_b d\Omega \quad dx = r_{av} d\Omega \quad dx_s = r_s d\Omega \tag{7}$$

From equation (7) a relationship between the radius and the angular distance.

$$\frac{dx_b}{r_b} = \frac{dx}{r_{av}} = \frac{dx_s}{r_s} \tag{8}$$

The helix angle at the barrel $\psi_b$, the screw root $\psi_s$, and the mid line $\psi_{av}$ of the plug will be:

$$\tan \psi_b = \frac{p}{2\pi r_b}, \quad \tan \psi_s = \frac{p}{2\pi r_s}, \quad \tan \psi_{av} = \frac{p}{2\pi r_{av}} \tag{9}$$

We can find the link between differential angular distances and helix angles by combining and rearranging equations (8) and (9).
The radius at any position along the x axis can be expressed as a function of the starting radius $r_o$ and the channel taper angle $\theta$. This can be regarded as the barrel radius $r_b$.

$$r_x = r_o - x \tan \theta$$

$$x = 0 \text{ at } l = 0$$

(11)

The plug radius $r_1$ and $r_2$ at the trailing and pushing side of the screw will be:

$$r_2 = r_o - x \tan \theta \quad r_1 = r_2 - p \tan \theta$$

(12)

The average radius of the plug:

$$r_{av} = \frac{r_1 + r_2}{2} = r_o - \left(x + \frac{p}{2}\right) \tan \theta$$

(13)

The height of the plug $h_1$ and $h_2$ on the trailing and pushing sides of the screw, respectively, varies with channel length $x$.

$$h_1 = r_1 - r_s = r_o - \tan \theta(x + p) - r_s$$

$$h_2 = r_2 - r_s = r_o - x \tan \theta - r_s$$

(14)

(15)

The average plug height $h_{av}$ will be:

$$h_{av} = \frac{h_1 + h_2}{2} = r_o - r_s - \left(x + \frac{p}{2}\right) \tan \theta$$

(16)

The width of the plug at the barrel $w_b$, the screw shaft $w_s$, and the mid line $w_{av}$, are as follows:

$$w_b = \frac{p - t}{\cos \theta}, \quad w_s = p - t$$

(17)

$$w_{av} = \frac{w_b + w_s}{2} = \frac{p(1 + \cos \theta) - t}{2 \cos \theta}$$

(18)

**Volumetric throughput of the extruder.** The first equation for estimating the theoretical volumetric output of a straight screw was proposed by AM Samus (19th century). Which is merely the screw’s sweep volume. It is defined as the sum of the screw cross sectional area and the axial velocity of material movement. The equation is as follows:

$$Q_t = \pi(r_b^2 - r_s^2)pN$$

(19)

Considering the screw thickness $t$ and axial velocity of the material $V_a$, a more accurate expression can be developed:

$$Q_t = V_a\pi(r_b^2 - r_s^2)\left(\frac{p - t}{p}\right)$$

(20)

The theoretical volumetric throughput of the plug down the screw channel $Q_t$ and the axial velocity of the plug along the tapper channel $V_{pa}$ can be expressed in terms of the cross-sectional area of the plug down the screw channel $A_{ps}$ and the axial velocity of the plug along the tapper channel ($V_{pa}$).

$$Q_t = V_{pa}A_{ps}$$

(21)

The area of the plug $A_{ps}$, which is represented in Figure 2, is the sum of the areas of each section which decreases as it goes down the screw channel.

$$dA_{ps} = \int r_c^r \left[2\pi r - \frac{t}{\sin \psi_{av}}\right] dr$$

(22)

Integrating the above equation gives:
The portion of the cross-sectional area occupied by the screw wing is represented by the last term in equation (23). The axial velocity of the plug is independent of channel depth and axial position. From Figure 3 the axial velocity of the plug along the screw axis is given by:

\[ V_a = \frac{V_{pa}}{\cos \theta} \tag{24} \]

Its expression can also be derived from the velocity diagram of Figure 4 in terms of the velocity of the plug along the channel \( V_{pc} \):

\[ V_a = \frac{V_{pc}}{\sin \psi} \tag{25} \]

Substituting this two equations in to equation (26):

\[ V_i = V_{t1} + V_{t2} \]

\[ V_{t1} = \frac{V_a}{\tan \alpha} \quad V_{t2} = \frac{V_a}{\tan \psi} \tag{26} \]

Substituting for \( V_a \) from equation (24) and substituting it to equation (27) the tangential velocity can be expressed in terms of the axial velocity of the plug.

\[ V_i = V_{pa} \left[ \frac{\tan \psi + \tan \alpha}{\cos \theta \tan \alpha \tan \psi} \right] \tag{28} \]

From equation (28) we can derive for the axial velocity of the plug parallel to the screw axis \( V_{ap} \):

\[ V_{pa} = V_i \left[ \frac{\cos \theta \tan \alpha \tan \psi}{\tan \psi + \tan \alpha} \right] \tag{29} \]

Since the tangential velocity \( V_i \) can be expressed in terms of the screw rotational speed \( N \) and diameter \( D \) as \( \frac{\pi D N}{60} \) the axial velocity of the plug will be:

\[ V_{pa} = \frac{\pi D N}{60} \left[ \cos \theta \tan \alpha \tan \psi \right] \tag{30} \]

Substituting equations (23) and (30) in to equation (21) gives the final expression for the theoretical volumetric throughput of the tapered extruder as:

\[ Q_t = \frac{\pi^2 N}{30} \left[ \frac{\cos \theta \tan \alpha \cos \psi}{\tan \psi + \tan \alpha} \right] (r_b^2 - r_s^2)(r_b - r_s) \tag{31} \]

From the above equation the theoretical volumetric throughput of the extruder is directly related to the
rotational speed of the screw, the diameter of the barrel and the screw, the solid conveying angle, the screw helix angle, the channel tapper angle, the pitch, and thickness of the screw.

The conveying angle \( \alpha \), which is the angle of the relative motion between the solid plug and the barrel wall, can be equated by rearranging equation (31) and can be written as a function of the theoretical volumetric throughput \( Q_t \), screw speed \( N \), and the screw geometry parameters.

\[
\alpha = \tan^{-1} \left[ \frac{Q_t \tan \psi}{\frac{\pi N}{30} \left( r_b^2 - r_s^2 \right) (r_b - r_s) \cos \theta \cos \psi - Q_t} \right]
\]  

(32)

It’s worth mentioning that if the volumetric flow rate is known, the conveying angle can be calculated using equation (32). To determine the flow rate with preset screw speeds, an experimental investigation is required. According to Peter and Matus,\(^{16} \) the solid conveying angle, in most experimental studies, is less than 50\(^0 \), regardless of the material being carried.

**Density variation across the screw channel.** This work is made on the basis the model proposed by Faborode and O’Callaghan\(^ {19} \). The equation (equation (33)) is a function of initial density \( \rho_0 \), compaction pressure \( P \), the barrel friction coefficient \( \mu_b \), the tapper angle \( \theta \), and other geometrical variables, as well as the physical property and material compressibility constants. In comparison to other research, this approach is a near approximation of output density because it includes the crucial parameters that the output density most possesses.

\[
\rho = \rho_0 + \frac{P}{B_0} \left( \frac{PB_0}{C_0} + 1 \right)
\]

(33)

Where:

\[
\beta = \frac{h}{h - x \tan \theta} \left( \frac{1 + \left( \frac{w_m}{h_0} \right) \tan \theta}{\frac{w_m}{h_0}} \right) k \left( \sin \theta + \mu_b \cos \theta \right)
\]

\[
h = r_0 - r_s
\]

The plug is driven by a friction force \( F_5 \) which moves the solid along the channel.

\[
F_5 = \mu_s \rho w_b r_h d\Omega
\]

(39)

The weight of the compacted material \( G \) acts at the center of the element in the downward direction.

\[
G = \rho gh_0 w_m r_m d\Omega
\]

(40)
Since the angular position $\vartheta$ of the plug changes as the screw rotates, the weight of the material will have a tangential ($F_6$) and radial ($F_7$) components.

\[
F_6 = G \sin \vartheta = \rho g h_{av} w_{av} r_{av} d\Omega \sin \vartheta \quad (41)
\]
\[
F_7 = G \cos \vartheta = \rho g h_{av} w_{av} r_{av} d\Omega \cos \vartheta \quad (42)
\]

The angular position of the plug $\vartheta$ has an initial position $\vartheta_1$ which is assumed to be zero and a variable component that varies with the channel length $x$.

\[
\vartheta = \vartheta_1 + \vartheta_x \quad (43)
\]

Consider an arc length that the plug creates as it moves down the screw channel which is the product of the radius and the angle it creates, using similar analogy the change in radius along the screw length in the x-direction is the difference of the circumference of the initial and the desired position divided by the angular plug position. Mathematically:

\[
\Delta R = \frac{2\pi (r_0 - r_x)}{\vartheta_x} \quad (44)
\]

Simplifying:

\[
\vartheta_x = \frac{2\pi x}{p} \quad (45)
\]

The final equation for the tangential ($F_6$) and radial ($F_7$) components of the plug weight will be,

\[
F_6 = \rho g h_{av} w_{av} r_{av} d\Omega \sin (\vartheta) \quad (46)
\]
\[
F_7 = \rho g h_{av} w_{av} r_{av} d\Omega \cos (\vartheta) \quad (47)
\]

The forces $F_6$ and $F_7$ vary with angular position of the plug that is, if $\vartheta = 0^\circ$, $F_7 = G$ and if $\vartheta = 90^\circ$, $F_6 = G$.

The force exerted by the pushing flight of the screw ($F_8$) with the friction force ($F_9$) is given by:

\[
F_8 = -P \omega \frac{r_{av} d\Omega}{2 \cos \psi_{av}} h_2 \quad (48)
\]
\[
F_9 = \mu \omega F_8 = \mu \omega P \frac{r_{av} d\Omega}{2 \cos \psi_{av}} h_2 \quad (49)
\]

The force exerted by the plug as it move along the channel is given by:

\[
F_{10} = P \beta r_{av} d\Omega w_{av} \cos \theta \quad (50)
\]

As the screw rotates the plug will generate a centrifugal force ($F_{11}$) acting upward perpendicular to the barrel surface.

\[
F_{11} = m \omega^2 r_b = G \omega^2 r_b \quad (51)
\]

Substituting appropriate parameters,

\[
F_{11} = \rho h_{av} w_{av} r_b d\Omega \omega^2 r_{av} \quad (52)
\]

Where the angular velocity ($\omega$) can be expressed as $\omega = \frac{2\pi n}{60}$.

($F_{12}$) and ($F_{13}$) represent forces that act on the opposite side of the differential plug with due to the change in pressure.

\[
F_{12} = P w_{av} h_{av} \quad (53)
\]
\[
F_{13} = (P + dp) w_{av} h_{av} \quad (54)
\]

The above forces acts in opposite to each other and the net force will be:

\[
F_{14} = dp w_{av} h_{av} \quad (55)
\]

**Analysis of work done.** Because the material is moving at a constant speed while retaining its kinetic and potential energy, the total work done by acting on the element as it travel from one point to another is zero.

\[
\sum_{i=1}^{n} W_i = 0 \quad (56)
\]

The forces acting on the plug are shown in Figure 5. They can be divided in to tangential ($F_{ni}$), axial($F_{ai}$), and radial ($F_{ri}$) components which is in the direction of global axis $x, z$ and $y$ respectively. The total work done is the sum product of the forces ($F_{ni}$ and $F_{ai}$) by the displacement the element travels ($S_{ni}$ and $S_{ai}$) inside the screw and can be expressed as:

\[
\sum_{i=1}^{n} W_i = \sum_{i=1}^{n} W_a + \sum_{i=1}^{n} W_ai \quad (57)
\]
\[
\sum_{i=1}^{n} W_i = \sum_{i=1}^{n} F_{ni} S_{ni} + \sum_{i=1}^{n} F_{ai} S_{ai} \quad (58)
\]

As the compacted material travels through the screw, the work done due to solid displacement in the axial ($S_{ai}$) and tangential ($S_{ni}$) components can be represented as a unit length multiplied $sin \alpha$ and $\frac{d\theta}{cos \alpha}$ respectively, as presented in Figure 6. Depending on the application of the forces, the radius $r$ will take the shape of $r_b$, $r_{av}$ and $r_s$.

Since there is no acceleration in the axial direction the sum of all the forces are zero which consequently implies the work done in the axial direction can also be equated to zero.
The axial and tangential work done by this differential forces are:

\[
F_{a3} = F_3 \sin \psi_r = \mu_s \rho_{sc} r \Omega \sin \psi_r \tag{69}
\]

\[
F_{t3} = F_3 \cos \psi_r = \mu_s \rho_{sc} r \Omega \cos \psi_r \tag{70}
\]

The work done associated with this forces are:

\[
W_{a3} = F_{a3} \alpha = \mu_s \rho_{sc} \rho \Omega \sin \alpha \sin \alpha \tag{71}
\]

\[
W_{t3} = F_{t3} r \cos \alpha = \mu_s \rho_{sc} \rho \Omega \cos \alpha \cos \alpha \tag{72}
\]

The friction force at the screw’s root \((F_3)\) is directed in the direction of the screw’s channel, with the helix angle at the screw’s root \(\psi_r\). As a result, the axial and tangential components of this force are as follows:

The differential frictional force \(F_t\) have an axial and tangential components:

\[
F_{a1} = F_1 \cos \psi = P_{sc} r_{av} \frac{d\Omega}{2} h_1 \tag{61}
\]

\[
F_{t1} = -F_1 \sin \psi = P_{sc} r_{av} \frac{d\Omega}{2} h_1 \tan \psi \tag{62}
\]

The force acting on the trailing side of the screw \(F_1\) can be divided in to axial \(F_{a1}\) and \(F_{t1}\):

\[
\sum F_{ai} = 0 \Rightarrow \sum W_{ai} = 0 \tag{59}
\]

\[
\sum W_{ai} = -\sum W_{ai} \tag{60}
\]

The friction force at the screw’s root \((F_3)\) is directed in the direction of the screw’s channel, with the helix angle at the screw’s root \(\psi_r\). As a result, the axial and tangential components of this force are:

\[
F_{a3} = F_3 \sin \psi_r = \mu_s \rho_{sc} \rho \Omega \sin \psi_r \tag{69}
\]

\[
F_{t3} = F_3 \cos \psi_r = \mu_s \rho_{sc} \rho \Omega \cos \psi_r \tag{70}
\]

The work done associated with this forces are:

\[
W_{a3} = F_{a3} \alpha = \mu_s \rho_{sc} \rho \Omega \sin \alpha \sin \alpha \tag{71}
\]

\[
W_{t3} = F_{t3} r \cos \alpha = \mu_s \rho_{sc} \rho \Omega \cos \alpha \cos \alpha \tag{72}
\]

The friction force \(F_a\) acts in a direction \(\theta\) to the plane normal to the screw shaft axis and \(\alpha\) to the plane parallel to the screw axis. Consequently, the axial and tangential components of this force are:

\[
F_{a5} = F_5 \sin \alpha \cos \theta = \mu_s \rho_{wp} \rho \Omega \sin \alpha \cos \theta \tag{73}
\]

\[
F_{t5} = F_5 \cos \alpha \cos \theta = \mu_s \rho_{wp} \rho \Omega \cos \alpha \cos \theta \tag{74}
\]

The work done associated with these forces are:

\[
W_{a5} = F_{a5} \alpha = \mu_s \rho_{wp} \rho \Omega \sin^2 \alpha \cos \theta \tag{75}
\]

\[
W_{t5} = F_{t5} \cos \alpha \cos \theta = \mu_s \rho_{wp} \rho \Omega \cos^2 \alpha \cos \theta \tag{76}
\]

The work done by force \(F_6\), which is the tangential component of the weight of the plug is:

\[
W_{f6} = \rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \sin (\theta) \tag{77}
\]

\[
W_{f6} = \rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \tan \psi \tag{78}
\]

The axial and tangential components \(F_7\) forces will be

\[
F_{a7} = -F_7 \sin \alpha = -\rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \cos \alpha \sin \alpha \tag{79}
\]

\[
F_{t7} = F_7 \cos \alpha = \rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \cos \alpha \cos \alpha \tag{80}
\]

The work done by force \(F_7\) will be

\[
W_{a7} = F_{a7} \alpha = -\rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \cos \alpha \sin^2 \alpha \tag{81}
\]

\[
W_{t7} = F_{t7} \cos \alpha = \rho g h_{sw} r_{av} \frac{d\Omega}{2} h_1 \cos \alpha \cos^2 \alpha \tag{82}
\]

The forces on the pushing flight of the screw (equation (48)) can be divided in to the axial and tangential components:

\[
F_{a8} = F_8 \sin \alpha = -P_{sc} r_{av} \frac{d\Omega}{2} h_1 \tan \psi \tag{83}
\]

\[
F_{t8} = F_8 \cos \alpha = -P_{sc} r_{av} \frac{d\Omega}{2} h_2 \tan \psi \tag{84}
\]
The work done by the forces (equation (48)) on the pushing side of the screw can be obtained as:

$$ W_{d8} = F_{d8} \sin \alpha = -P_{sc} \frac{r_{av} d\Omega}{2} h_2 \tan \psi_{av} \sin \alpha $$  \hspace{1cm} (85)

$$ W_{s8} = F_{s8} \frac{r_{av}}{r_b} \cos \alpha = -P_{sc} \frac{r_{av} d\Omega}{2} h_2 \frac{r_{av}}{r_b} \cos \alpha $$  \hspace{1cm} (86)

The friction force on the pushing side of the screw by the friction force $F_F$ (equation (49)) can be broken into axial and tangential components.

$$ F_{d9} = F_9 \sin \psi_{av} = -\mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 \tan \psi_{av} h_2 $$  \hspace{1cm} (87)

$$ F_{d9} = F_9 \cos \psi_{av} = \mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 $$  \hspace{1cm} (88)

The axial and tangential components of the work done by these forces are:

$$ W_{d9} = F_{d9} \sin \alpha = -\mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 \tan \psi_{av} \sin \alpha $$  \hspace{1cm} (89)

$$ W_{d9} = F_{d9} \frac{r_{av}}{r_b} \cos \alpha = \mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 \frac{r_{av}}{r_b} \cos \alpha $$  \hspace{1cm} (90)

As the material undergoes compaction the normal force created $F_{10}$ (equation (50)) has an axial and radial components. Here we will only concern ourselves with the axial force only. Therefore:

$$ F_{a10} = F_{10} \sin \theta = -P_{b} \frac{r_{av} d\Omega}{2} \cos \theta \sin \theta $$  \hspace{1cm} (91)

Consequently, the work done by this force is:

$$ W_{a10} = F_{a10} \sin \alpha = -P_{b} \frac{r_{av} d\Omega}{2} \cos \theta \sin \alpha $$  \hspace{1cm} (92)

The centrifugal force $F_{11}$ (equation (52)) acting perpendicular to the barrel surface is in the same direction as $F_{10}$ therefore, the axial force will be:

$$ F_{a11} = -F_{11} \sin \theta = -\rho \frac{r_{av} d\Omega}{2} \cos \theta \sin \alpha $$  \hspace{1cm} (93)

Consequently, the work done by this force is:

$$ W_{a11} = -F_{a11} \sin \alpha = -\rho \frac{r_{av} d\Omega}{2} \cos \theta \sin \alpha $$  \hspace{1cm} (94)

There exist a net force $F_{14}$ created by the change in pressure along the plug flow which acts in the axial and tangential directions.

$$ F_{d14} = F_{14} \sin \psi_{av} = dpw_{av} h_{av} \sin \psi_{av} $$  \hspace{1cm} (95)

$$ F_{t14} = F_{14} \sin \psi_{av} = dpw_{av} h_{av} \cos \psi_{av} $$  \hspace{1cm} (96)

Consequently, the work done for a unit distance traveled in the axial direction is:

$$ W_{a14} = F_{a14} \sin \alpha = dpw_{av} h_{av} \sin \psi_{av} $$  \hspace{1cm} (97)

$$ W_{t14} = F_{t14} \frac{r_{av}}{r_b} \cos \alpha = dpw_{av} h_{av} \sin \psi_{av} \frac{r_{av}}{r_b} \cos \alpha $$  \hspace{1cm} (98)

**Analysis of pressure distribution across the screw channel.** By equating the axial and tangential work done components to zero, the pressure across the screw channel can be calculated: Equating the axial work done (equations (63), (67), (71), (75), (81), (85), (89), (92), (94), and (97)) with the tangential work done (equations (64), (68), (72), (76), (78), (86), (90), and (98)) as indicated in equation (60) we get:

$$ P_{sc} \frac{d\Omega}{2} h_1 \sin \psi_{av} \frac{r_{av}}{r_b} \cos \alpha = \mu_{sc} P_{sc} \frac{d\Omega}{2} h_1 \frac{r_{av}}{r_b} \cos \alpha $$

$$ + \mu_{ba} P_{sc} W_{av} r_{av} \cos \theta \sin \psi_{av} \frac{r_{av}}{r_b} \cos \alpha $$

$$ + \rho h_{av} \frac{r_{av} d\Omega}{2} \cos \theta \sin \psi_{av} \sin \alpha $$

$$ - P_{sc} \frac{r_{av} d\Omega}{2} h_2 \frac{r_{av}}{r_b} \cos \alpha + \mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 \frac{r_{av}}{r_b} \cos \alpha $$

$$ + dpw_{av} \sin \psi_{av} \frac{r_{av}}{r_b} \cos \alpha $$

$$ = -P_{sc} \frac{r_{av} d\Omega}{2} h_1 \sin \alpha - \mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_1 \tan \psi_{av} \sin \alpha $$

$$ - \mu_{ba} P_{sc} W_{av} r_{av} \sin \psi_{av} \sin \alpha $$

$$ + \mu_{ba} P_{sc} W_{av} r_{av} \sin \psi_{av} \sin \alpha $$

$$ + P_{sc} \frac{r_{av} d\Omega}{2} \tan \psi_{av} h_2 \sin \alpha $$

$$ + \mu_{sc} P_{sc} \frac{r_{av} d\Omega}{2} h_2 \tan \psi_{av} \sin \alpha $$

$$ + k P_{sc} \frac{r_{av} d\Omega}{2} \cos \theta \sin \psi_{av} \sin \alpha $$

$$ + \rho h_{av} \frac{r_{av} d\Omega}{2} \frac{r_{av}}{r_b} \sin \alpha - dpw_{av} \sin \psi_{av} \sin \alpha $$  \hspace{1cm} (99)

Substituting equations (14), (15), (10), (13), and (16) in to equation (99) and rearranging we get:

$$ P_{sc} \frac{dx}{x} \left[ 4 \left( x^2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_3 C_4 \right) \right] $$

$$ + A_2 \left[ C_1 - x \tan \theta \right] + P_{sc} dx A_3 + P_{sc} dx A_4 $$

$$ + A_4 \left( x^2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_3 C_4 \right) $$

$$ + \frac{A_5}{x^2} \frac{dx}{x} \left[ A_7 (C_2 - x \tan \theta) \right] $$

$$ + A_6 \left( x^2 \tan^2 \theta - x \tan \theta (C_2 + C_4) + C_2 C_4 \right) \left[ 1 + \mu_{sc} \right] $$

$$ + dp \frac{A_9 (C_3 - x \tan \theta) \right] $$

$$ + A_{10} \left[ x^2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_3 C_4 \right] $$  \hspace{1cm} (100)
Where:

\[ A_1 = \tan \psi_{av} \cos \alpha - \mu_{sc} \cos \alpha \]
\[ A_2 = \sin \alpha(1 - \mu_{sc} \tan \psi_{av}) \]
\[ A_3 = \mu_{sc} w_t \tan \psi_{av} \cos \psi_r \frac{p}{r_b} \cos \alpha - \sin \alpha \]
\[ A_4 = \frac{w_t}{\sin \psi_{av}} \frac{\mu_h \cos \theta \cos 2\alpha - \sin 2\alpha}{\tan \psi_{av}} \]
\[ A_5 = \rho w_{av} \left[ \frac{\cos^2 \alpha \cos \theta}{r_b} - \alpha^2 \sin \theta \sin \alpha \right] \]
\[ A_6 = \rho w_{av} \left[ \sin \theta - \cos \theta \cos \psi_{av} \sin \alpha \right] \]
\[ A_7 = \tan \psi_{av} \sin \alpha, \quad A_8 = \frac{\cos \alpha}{r_h} \]
\[ A_9 = w_{av} \sin \psi_{av} \sin \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_10 = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ A_{10} = w_{av} \sin \psi_{av} \cos \alpha \]
\[ C_1 = r_0 - r_s - p \tan \theta \]
\[ C_2 = r_0 - r_s \]
\[ C_3 = r_0 - r_s - \frac{p}{2} \tan \theta \]
\[ C_4 = r_0 - \frac{p}{2} \tan \theta \]

The pressure along the screw \((P_{sc})\), the barrel \((P_b)\), and the shaft \((P_{sh})\) can be expressed in terms of the compaction pressure using the Janssen coefficient as \(kP = P_b = P_{sc} = P_{sh}\).

Equation (100) can be further simplified and written in the form of a first order differential equation of the form:

\[
\frac{dP}{dx} = \frac{PB_1(x) + B_2(x)}{B_3(x)} \quad (101)
\]

Where:

\[ B_1 = \frac{A_1}{2} \left[ x^2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_2 C_4 \right] + \frac{A_2}{2} \left[ C_1 - x \tan \theta \right] \]
\[ + \mu_{sc} w_t \frac{\tan \psi_{av}}{\tan \psi_{av}} \frac{r_t}{r_b} \cos \alpha - \sin \alpha \]
\[ + w_h \frac{\tan \psi_{av}}{\tan \psi_{av}} \left[ \mu_h \cos \theta (\cos^2 \alpha - \sin^2 \alpha) - \sin \alpha \cos \alpha \right] \]
\[ - \left[ \frac{A_7}{2}(C_2 - x \tan \theta) + \frac{A_8}{2}(x^2 \tan^2 \theta \right] \]
\[ - x \tan \theta (C_2 + C_4) + C_2 C_4 \] \[1 + \mu_{sc}] \]
\[ B_2 = A_5 \left[ x_2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_3 C_4 \right] + \mu_{sc} C_3 - A_6 \tan \theta \]
\[ B_3 = A_6 (C_3 - x \tan \theta) + A_{10} x^2 \tan^2 \theta - x \tan \theta (C_3 + C_4) + C_3 C_4 \]

Equation (101) is a first order - first degree ordinary differential equation that can be evaluated analytically using the more accurate Runge-Kutta method.

**Numerical solution and simulation process**

A flow chart of the simulation process is shown in Figure 7. The pressure gradient was numerically integrated using the Runge-Kutta 4th order approximation method. The model equations for compaction pressure, volumetric flow rate, and output density were solved in the Python programming environment using the symbolic mathematics operator module, SymPy, by initializing the pressure and setting the variable length across the tapered screw channel as constants. The pressure distribution, output density, and volumetric flow rate were then calculated and graphically represented using the NumPy module by varying the tapper angle and barrel friction coefficient.

**Results and discussion**

The results obtained below are direct computations of equations (31), (33), and (101). For analyzing these
parameters a shaft radius of 20 mm, exit screw radius of 45 mm, a screw length of 200 mm, with an initial entry pressure of 0.01325 MPa is used. The biomass used for this analysis is sawdust with material property of a bulk modulus of 0.62 MPa, porosity index of 2.05, and an angle of repose of 22° for a particle size of 1.3 mm. The change in pressure throughout the screw channel was first investigated using equation (101). The coding for Python programming is presented in the Appendix section.

Effect of tapper angle on compaction pressure across the length of the screw

The tapper angle on the screw has the primary impact of increasing compaction pressure, which in turn increases output density. Based on the theoretical prediction, the simulated results showed that the compaction pressure increases exponentially (from 0.25 (MPa) to 618.8 (MPa)) with an increase in tapper angle from (0°) to (60°) respectively, assuming that the barrel friction coefficients $\mu_b = 0.25$ and screw shaft friction coefficients $\mu_sc = 0.2$ are constant. Figure 8 shows the effect of the tapper angle on the pressure buildup for screw length of 200 mm. The model calculates the values of tapper angle from 0° to 60° by 1° increment.

Effect of barrel friction coefficient on compaction pressure

Figure 9 illustrates the effect of the barrel friction coefficient between the biomass and the barrel. With a tapper angle of 3°, the barrel friction is controlled between 0 and 0.8, resulting in compaction pressures of 76 and 350 MPa, respectively. From the result the pressure built up more quickly with an increase in barrel friction due to an increased resistance created between the barrel surface and the compacted biomass as the friction increases. (Ojolo13) analyzed for a screw compaction length of 1 m and barrel friction coefficient of 0.3 the compaction pressure was 101 MPa. As shown in

Effect of tapper angle on compaction pressure and output density

For analyzing the change in output density across the length of the screw with a change in tapper angle, an initial bulk density of 260 kg/m³, $\mu_b = 0.25$, and $\mu_sc = 0.2$ was used. According to studies, the physical upper limit of compactness for lignocellulosic materials is around 1500 Kg/m³, and densifying above this limit affects the compacted biomass’s combustion characteristics.20,21 From Figure 10 to 16, the influence of tapper angle on compaction pressure and output density is shown. As
the tapper angle is steadily increased from $0^\circ$ to $6^\circ$ across the length of the screw, the output density continues to build in an S-shaped logistic curve. As per the predicted results, a higher tapper angle causes the pressure to build up more quickly. A compaction pressure between 108.3–389.15 MPa corresponding to $30^\circ–50^\circ$ tapper angle is sufficient to compact the bulk material to an output density of 1200–1400 kg/m$^3$. Peter and Matus$^6$ discovered that a compaction pressure of 58 Mpa for a $10^\circ$ tapper angle and 62 Mpa for a $20^\circ$ tapper angle was sufficient to drive the compacted material out of the chamber for the same material. The corresponding output density was 1221 kg/m$^3$ and 1236 kg/m$^3$ respectively.

**Effect of tapper angle on volumetric flow rate**

The curves of the volumetric throughput, obtained from equation (31), versus the screw pitch, the screw speed, and conveying angle with different screw taper angle is presented in Figures 17 to 19 respectively. Where a screw radius $r_0 = 90$ mm, screw thickness $(t) = 5$ mm, and shaft radius $r_s = 20$ mm was used. These curves show that volumetric throughput increases with an increase of screw pitch (from 30 to 65 mm), screw speed (from 30 to 135 rpm) and conveying angle (from $10^\circ$ to $50^\circ$). From an experimental study of Abdul and Anjum,$^6$ a screw speed below 150 rpm was recommended for a durability of the compacted
material to be above 90% which agrees with the initial condition of this work.

**Conclusion**

The process of solid compaction in a tapered screw is dynamic and complex. The screw geometries, property of the compacted material, friction coefficients, frictional and non-frictional forces, and compaction pressure all play a significant role in the compaction process.

The pressure distribution, output density, and volumetric flow rate of solid compaction in a tapered screw extruder were investigated using a mathematical model developed in this study. The cumulative work done by the frictional, gravitational, and centrifugal forces created during the solid compaction process was studied in order to construct a mathematical model for pressure distribution across the tapered screw. The discharge pressure, rotational speed, friction coefficients, and screw dimensional parameters significantly affect those forces. The compaction process was evaluated following the concept of Broyer and Tadmor, but modified to the tapered screw by incorporating the geometries and three-dimensional forces in the plugs differential element. By assuming constant values for the screw geometrical parameters, equation (101) yields the expression for pressure change across a small channel length $dx$. Using the calculated pressure, equation (33) can be used to analyze the output density.

From the analysis and the simulation results the following conclusion can be drawn:

1. The compressibility and density change of the compacted material were investigated through pressure-density relationship.
2. The method of analysis can be used to compute the volumetric throughput if the solid conveying angle is known, or the solid conveying angle if the volumetric throughput is known, given the screw dimensions.

3. Increasing the taper angle, solid conveying angle, screw pitch, and screw speed individually or concurrently has resulted in an increase in the tapered screw’s output capacity.

4. The screw taper angle and friction coefficients between the compacted material and the barrel surface have a big impact on the change in compaction pressure of a tapered screw. This demonstrates a high friction coefficient or a rough barrel surface are required to achieve a higher compaction pressure. It was also discovered that adjusting the taper angle while keeping the barrel friction coefficient constant had a comparable effect on the compaction pressure as keeping the barrel friction constant while varying the taper angle.

5. To simulate the results of compression pressure and output density, sawdust was chosen as an input material, and a taper angle of $30^\circ / 50^\circ$ was discovered to be sufficient to compact the output density from $1302 \text{ kg/m}^3$ to $1401 \text{ kg/m}^3$ without affecting its combustion property.

6. The developed mathematical model for volumetric flow rate and compaction pressure can be applied to other ligno-cellulosic biomass materials and chemical powders by specifying the appropriate material and physical properties to obtain an approximate result for compaction pressure and volumetric flow rate.

7. The results and methods utilized to formulate the mathematical models in this study can be used as a useful inputs for future research to improve the accuracy of tapered screw extruder predictions.

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Appendices

Appendix 1: Python programming for analyzing the pressure build up along the tapered screw

```python
stringstyle
 ***********************************************
 * This program evaluates the general equation of the *
 * pressure build change as a function of the channel *
 * length, and friction coefficients *
 ***********************************************

# Definition of terms
# ri, rs, ro : inlet, shaft, and outlet radius respectively
# l, p, x, rpm : length, pitch, variable position, and angular speed of the screw respectively
# CO, BO, rho, rhoi, mue, alpha : bulk modulus, porosity index, initial density, output density, internal friction angle, and conveying angle of the material respectively
# theta, psib, psir, psiav : taper angle, helix angle at the barrel, helix angle at the screw root, and average helix angle of the screw respectively
# wb, ws, wav : width at the barrel, screw shaft, and average width of the plug respectively
# r1, r2, rav : plug radius at the trailing side of the screw, plug radius at the pushing side of the screw, and average radius of the plug respectively
# h1, h2, hav, ho : plug height at the trailing side of the screw, plug height at the pushing side of the screw, average, and inlet height of the plug
# mub, mus : friction coefficient at the barrel and screw surfaces
# rb : barrel radius
# k : stress transmission coefficient
# beta : tapering factor
# vartheta : angular position of the plug
# Pcr : compaction pressure

from math import *rom numpy import log as ln
from sympy import *

x,Y,mus,mub= symbols(’x Y mus mub’) 
rs = float(input("Enter shaft radius (mm): "))
ro = float(input("Enter outlet die radius (mm):"))
l = float(input("Enter the die length (mm):"))
p = float(input("Enter the pitch of the screw (mm):"))
t = float(input("Enter the screw thickness (mm):"))
rpm = float(input("Enter the speed of the screw (rpm):"))
CO = float(input("Enter Initial bulk modulus of the material (MPa):"))
BO = float(input("Enter the porosity index of the material :"))
rhoi = float(input("Enter the initial density of the material (g/mm3):"))
mue = float(input("Enter the internal friction angle:"))
alpha = float(input("Enter the solid conveying angle:"))
h0=ro-rs
pi=3.14159265

# gravitational acceleration (mm/s^2)
g=9810
# k=(1-sin(mue*3.14/180))/(1+sin(mue*3.14/180))
# for i in range (0, len(theta)):
theta=arange(0,1,1)
for i in range (0, len(theta)):
    print(‘\N{greek small letter theta}={’,theta[i],’}’)
    rl=ln(tan(theta[i]*3.14/180))+ro
    print(‘r_i=’,rl)
    rb=rl-(x+(tan(theta[i]+3.14/180))
    r1=rb-(x+(tan(theta[i]+3.14/180))
    wb=(p/(cos(theta[i]+pi/180)))-t
    print(‘wb=’,wb)
    ws=p-t
    wav=(ws+wb)/2
    Ci=ro-ra-p*atan(theta[i]*pi/180)
    C2=ro-rs
```
C3 = (p2/2) + tan(\theta[i]*\pi/180)
C4 = -p2/2 * tan(\theta[i]*\pi/180)
# print(C4)

rb = r1 - x*(tan(\theta[i]*3.14/180))
r1 = rb - p*(tan(\theta[i]*3.14/180))
rav = (rb + r1)/2

h1 = r1 - rs
h2 = r0 - rs = x*(tan(\theta[i]*3.14/180))
hav = (h1 + h2)/2

psib = atan(p/(2*pi + rb)) * 180/pi
psir = atan(p/(2*pi - rs)) * 180/pi
psiax = atan(p/(2*pi + zav)) * 180/pi
vartheta = (2*pi/(p + tan(\theta[i]*\pi/180)) + tan(psiax + pi/180))
y1 = hav/k

k = (1+y1/(y1 + tan(\theta[i]*3.14/180))) * (k = (sin(\theta[i]*3.14/180)) + mub*cos(\theta[i]*3.14/180))

beta = (ho + kc) / (ho - x*tan(\theta[i]*3.14/180))
rho = rhoi + (((rhoi/20) - ln((1 + BO + beta*CO) + 1))

A1 = tan(psiax + 3.14/180) * cos(alpha + 3.14/180) - mub*acos(alpha + 3.14/180)
A2 = sin(alpha + 3.14/180) - (1 - mub*tan(psiax + 3.14/180))
A3 = mub*ws*tan(psiax + pi/180) * cos(alpha + 3.14/180) * sin((ws + those[i] + pi/180) * cos(alpha + 3.14/180))
A4 = wb * ((tan(psiax + pi/180) + tan(psib + pi/180)) * mub*cos(\theta[i]*pi/180)
A5 = r0i*wav * (((cos(alpha + pi/180) + 3.14/180) * x*(tan(\theta[i]*pi/180))/r1 - ((omegax + 2*sin(\theta[i]*pi/180)) * sin(alpha + pi/180))
A6 = r0i*wav * sin(vartheta - pi/180) - sin(alpha + pi/180) * cos(psiax + pi/180) * sin(alpha + pi/180)
A7 = tan(psiax + 3.14/180) * cos(alpha + 3.14/180)
A8 = cos(alpha + 3.14/180)/r1
A9 = wav + sin(pi - psiax + pi/180) * sin(alpha + pi/180)
A10 = (psiax + 3.14/180) * cos(alpha + 3.14/180) / r1

B11 = A1 + 0.5 * (x + 2) * (tan(\theta[i]*3.14/180)) * 2
B12 = A2 + 0.5 * (C1 - x*(tan(\theta[i]*3.14/180)) * mus = ws * (tan(\theta[i]*3.14/180)) * sin(psiax + 3.14/180) * (\theta[i]*pi/180) * (\theta[i]*pi/180) + (\theta[i]*pi/180) * sin(alpha + 3.14/180)
B13 = (wb + tan(\theta[i]*3.14/180) * tan(psiax + 3.14/180)) * mub*cos(\theta[i]*3.14/180) * cos(2*alpha + 3.14/180) - 0.5*sin(alpha + 3.14/180)
B14 = A7 + 0.5 * (1 - mus) * (C2 - x*(tan(\theta[i]*3.14/180))
B15 = A8 + 0.5 * (x + 2) * (tan(\theta[i]*3.14/180)) * 2 - x*(tan(\theta[i]*3.14/180) * (C2 + C4) + C2 + C4) * (1 - mus)

print()

B1 = B11 - B12 - B13 - B14 - B15
B2 = A5 + (x + 2) * (tan(\theta[i]*3.14/180)) * 2 - x*(tan(\theta[i]*3.14/180) * (C3 + C4)
 + C3 + C4) + A6 * (C3 - A6*tan(\theta[i]*3.14/180))
B3 = A10 + (x + 2) * (tan(\theta[i]*3.14/180)) * 2 + x*(tan(\theta[i]*3.14/180) * (C3 + C4)
 + A9 * (C3 - x*tan(\theta[i]*3.14/180)) + C3 + C4

dPdx = (x + Y + B1) / B2 / B3
print('dPdx ^'= dPdx)
Appendix 2: Python programming for calculating the volumetric throughput

```python
# Subroutine Runge-Kutta 4th order approximation

xo=0; h=0.001; l=200; n=int((l-xo)/h)
for x in range(0,n):
    k1=h*(x+Y)
    k2=h*(x+h/2, Y+k1/2)
    k3=h*(x+h/2, Y+k2/2)
    k4=h*(x+h, Y+k3)
    Y=Y+k*(k1+2*k2+2*k3+k4)
    X.append(x)
    Prsr.append(Y)
print('X= ', X)
print('Prsr= ', Prsr)
```

**Appendix 3: Python programming for calculating the output density**

```python
rpm=range(30,150,15)
for i in range (0, len(rpm)):
    for j in range (0, len(thetal)):
        rb=(1+(tan(thetal[j]+3.14/180)+3.14/180)*cos(thetal[j]+3.14/180))/(tan(psi+b*3.14/180))
        cons=(tan(alpha+3.14/180)*cos(psi+b*3.14/180)+sin(alpha+3.14/180)+tan(psi+b*3.14/180))
        print('t=', t)
print('\n')
```

```python
x= range (0, 1)
RHO=[]
for i in range (0, len(x)):
    z=((1+(w/v*hav)*tan(theta+3.14/180))*k+(sin(theta+3.14/180)+mu*b*cos(theta+3.14/180)))*psi+b
    rho=rho+((rho/B)*exp(Prsr[i]+BO*((ho+ko)/(ho-(x[i]*tan(theta+3.14/180)))+z)/CD)+1))
RHO.append(rho)
print('Rho=', RHO)
```
**Abbreviations**

| Symbol | Description                                      | Subscript | Description          |
|--------|--------------------------------------------------|------------|----------------------|
| r      | Radius (mm)                                      |            |                      |
| p      | Pitch (mm)                                       |            |                      |
| r₁, r₂ | Plug radius at pushing and trailing side of the screw (mm) |            |                      |
| l      | Length of the screw (mm)                         |            |                      |
| x      | Screw length at any point of the screw           |            |                      |
| h₁, h₂ | Plug height at pushing and trailing side of the screw (mm) |            |                      |
| w      | Width of the plug (mm)                           |            |                      |
| v      | Velocity of the plug (m/s)                       |            |                      |
| P      | Pressure (Mpa)                                   |            |                      |
| W      | Work done                                        |            |                      |
| β      | Tapering factor                                  |            |                      |
| Bo     | Material porosity index                          |            |                      |
| Co     | Initial bulk modulus (Mpa)                       |            |                      |
| υ      | Angular position of the plug                     |            |                      |
| θ      | Taper angle                                      |            |                      |
| α      | Conveying angle                                  |            |                      |
| ρ      | Output density (kg/m³)                           |            |                      |
| ρ₀     | Bulk density (kg/m³)                             |            |                      |
| μ      | Friction coefficient                             |            |                      |
| ω      | Angular speed                                    |            |                      |

**Subscripts**

- b: barrel
- sc: screw
- sh: shaft
- av: average
- o: outside
- a: axial
- t: tangential