\[ D_{s0}^+(2317) \] as an Iso-Triplet Four-Quark Meson and Production of Its Neutral and Doubly Charged Partners

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By studying \( D_{s0}^+(2317) \to D_s^+ \pi^0 \) and \( D_{s0}^+(2317) \to D_s^+ \gamma \) decays, it is shown that assigning \( D_{s0}^+(2317) \) to the iso-triplet four-quark meson \( \hat{F}_I \) is favored. Productions of its partners \( \hat{F}_0^I \) and \( \hat{F}_{++}^I \) are also studied. As the result, it is concluded that they could be observed in \( B_0^d \to (D_s^+ \pi^-) \bar{D}_0 \) and \( B_0^+ \to (D_s^+ \pi^+) \bar{D}^- \). Their iso-singlet partner \( \hat{F}_0^+ \) might have been observed in the radiative \( B \to (D_s^+ \gamma) \bar{D} \) decays by the BELLE collaboration.

§1. Introduction

Inclusive \( e^+e^- \) annihilation experiments\(^1,2\) have observed a narrow (\(< 4.6\) MeV\(^3\)) scalar resonance [denoted by \( D_{s0}^+(2317) \)] in the \( D_s^+ \pi^0 \) channel. However, no evidence for it has been observed in the \( D_s^+ \gamma \) channel, so that a severe constraint,\(^2\)

\[ R(D_{s0}^+(2317)) = \frac{\Gamma(D_{s0}^+(2317) \to D_s^+ \gamma)}{\Gamma(D_{s0}^+(2317) \to D_s^+ \pi^0)} < 0.059, \]  

(1.1)

has been provided. In addition, we here list the measured ratio of decay rates\(^3\)

\[ R(D_s^+) = \frac{\Gamma(D_s^+ \to D_s^+ \pi^0)}{\Gamma(D_s^+ \to D_s^+ \gamma)} = 0.062 \pm 0.008. \]  

(1.2)

Equation (1.2) implies that the isospin non-conserving interaction is much weaker than the electromagnetic interaction. Therefore, Eq. (1.1) means that the underlying interaction of the decay \( D_{s0}^+(2317) \to D_s^+ \pi^0 \) is much stronger than the electromagnetic interaction, i.e., it is the ordinary strong interaction as is well known. In this case, \( D_{s0}^+(2317) \) should be an iso-triplet meson which can be realized by a four-quark state.

To confirm the above conjecture, we shortly visit scalar four-quark mesons and discuss that charm-strange scalar four-quark mesons can be narrow, in §2, and study their radiative decays and isospin non-conserving decays in §3. Productions of charm-strange scalar mesons in \( e^+e^- \) annihilation and in hadronic \( B \) decays are investigated in §4. A brief summary is given in the final section.

§2. Charmed scalar four-quark mesons

Observed low lying scalar mesons,\(^3\) \( a_0(980), f_0(980), K_0^*(800) \) and \( f_0(600) \), can be well understood by the \([qq][\bar{q}q] \) states, \( \delta^s \sim [ns][\bar{n}s] I=1, \sigma^s \sim [ns][\bar{n}s] I=0, \kappa \sim [ud][\bar{n}s], \chi \sim [ud][\bar{u}d], (n = u, d) \), which are dominantly of \( 3_c \times 3_c \) of color \( SU_c(3) \), as
suggested long time ago\textsuperscript{4} and supported at this workshop.\textsuperscript{5} (However, for simplicity, a possible small mixing of $6_c \times 6_c$ is ignored in this talk.)

With this in mind, we replace one of light quarks in $[q\bar{q}][\bar{q}\bar{q}]$ by the charm quark $c$. Then we have the charmed scalar $[cq][\bar{q}\bar{q}]$ mesons, $F_I \sim [cn][\bar{n}\bar{s}]_I=1$, $F^0_I \sim [cn][\bar{n}\bar{s}]_I=0$, $D^s \sim [cs][\bar{n}\bar{s}]$, $\bar{D} \sim [cn][\bar{u}\bar{d}]$ and $E^0 \sim [cs][\bar{u}\bar{d}]$. However, we here study only $F^0_I$, $F^+_I$ and $F^+_I$. (For the other components, see Refs. 6)–8.) When we assign\textsuperscript{6} $D^+_s(2317)$ to $F^+_I$ as conjectured in §1, one might wonder if it can be so narrow. However, its narrow width can be understood by a small rate for the dominant decay $F^+_I \rightarrow D^+_s\pi^0$ which is given by a small overlap of (color and spin) wave functions. Such a small overlap can be seen by decomposing a color-singlet scalar four-quark state of $\bar{3}_c \times 3_c$ into a sum of products of $\{q\bar{q}\}$ pairs. The coefficient of the product of two color- and spin-less $\{q\bar{q}\}$ pairs in the decomposition provides the overlap under consideration.

Therefore, the parameters describing the overlaps between a charm-strange scalar four-quark meson and two pseudoscalar mesons, for example, $F^+_I$ (or $F^+_I$) and $D^+_s\pi^0$ (or $D^+_s\eta$) is given by $|\beta_0|^2 = 1/12$, and the corresponding one between $F^+_I$ (or $F^+_I$) and $D^+_s\rho^0$ (or $\omega$, $\phi$, $\psi$) is provided by $|\beta_1|^2 = 1/4$. (However, in the case of the conventional mesons, the corresponding overlap is unity, because their color and spin configuration is unique.) For more details, see Refs. 7), 9) and 10). To see numerically that $F^+_I$ is narrow, we use a hard pion technique in the infinite momentum frame.\textsuperscript{11}

In this approximation, the amplitude for two body decay $A(p) \rightarrow B(q)\pi(k)$ is given by

\begin{equation}
M(A \rightarrow B\pi) \simeq \left(\frac{m_A^2 - m_B^2}{f_\pi}\right) \langle B|A_\pi|A \rangle, \tag{2.1}
\end{equation}

where the asymptotic matrix element $\langle B|A_\pi|A \rangle$ has been evaluated in the infinite momentum frame. Then, by assigning $a_0(980)$ to $\delta^s$ and using $\Gamma(a_0(980) \rightarrow \eta\pi)_{\text{exp}} \simeq 60$ MeV from the measured peak width\textsuperscript{9} as the input data, a rather small rate $\Gamma(F^+_I \rightarrow D^+_s\pi^0)_{SU_f(4)} \simeq 8$ MeV can be obtained, where the $\eta$-$\eta'$ mixing with the mixing angle $\theta_\rho \simeq -20^\circ$ has been taken. Because the spatial wave-function overlap is in the $SU_f(4)$ symmetry limit at this stage, however, it is expected that the amplitude is overestimated by about 20–30\%. It can be seen\textsuperscript{7} by comparing the measured rates for the $D^* \rightarrow D\pi$ decays with the estimated ones in which the measured $\Gamma(\rho \rightarrow \pi\pi)_{\text{exp}} = 149.4 \pm 1.0$ MeV\textsuperscript{3} is adopted as the input data. Taking account for the above symmetry breaking, we can get $\Gamma(F^+_I \rightarrow D^+_s\pi^0) \sim 3$–5 MeV.

This leads to a sufficiently narrow width of $F^+_I = D^+_s(2317),\textsuperscript{7,10}$

\section{Radiative decays and isospin non-conserving decays}

Since it has been known that the vector meson dominance (VMD) with the ideal $\omega$-$\phi$ mixing and the flavor $SU_f(3)$ symmetry for the strong vertices work fairly well in the radiative decays of light vector mesons,\textsuperscript{12} we will extend it to the system containing charm quark(s) below. Under the VMD, the amplitude $A(V \rightarrow P\gamma)$ can
be approximated by

\[ A(V \rightarrow P\gamma) \simeq \sum_{V'=\rho^0,\omega,\psi} \left[ \frac{X_{V'}(0)}{m_{V'}^2} \right] A(V \rightarrow PV') , \tag{3.1} \]

where \( X_{V}(0) \) is the \( \gamma V \) coupling strength on the photon mass-shell. \( X_{V} \) is dependent on photon-momentum-square,\(^{12}\) and the values of \( X_{V}(0) \) have been estimated from the analyses in photoproductions of vector mesons on various nuclei.\(^{13}\) The results are \( X_{\rho}(0) = 0.033\pm0.003 \text{ GeV}^2, X_{\omega}(0) = 0.011\pm0.001 \text{ GeV}^2, X_{\phi}(0) = -0.018\pm0.004 \text{ GeV}^2 \) and \( X_{\psi}(0) \sim 0.054 \text{ GeV}^2 \), where the last one has been obtained from \( d\sigma(\gamma N \rightarrow \psi N)/dt|_{t=0} \simeq 20 \text{ nb}/\text{GeV}^2 \) and \( \sigma_T(\psi N) = 3.5 \pm 0.8 \text{ mb}^{14} \) for the \( \psi N \) total cross section. \( (N \text{ denotes a nucleon).} \) The \( VP' \) coupling strength can be estimated as

\[ |A(\omega \to \pi^0\rho^0)| \simeq 18 \text{ GeV}^{-1} , \tag{3.2} \]

from the measured rate\(^3\) \( \Gamma(\omega \to \pi^0\gamma)_{\text{exp}} = 0.757 \pm 0.024 \text{ MeV} \) by putting \( V = \omega, P = \pi^0 \) and \( V' = \rho^0 \) in Eq. (3.1) and by inserting the above \( X_{\rho}(0) \) into it, because the \( \omega \rightarrow \pi^0\gamma \) amplitude is dominated by the \( \rho^0 \) pole. The OZI-rule allowed poles for the amplitude \( A(D^{*+} \to D^+\gamma) \) are given by the \( \rho^0, \omega \) and \( \psi \) mesons. The relevant \( SU_f(4) \) relation \( -2A(D^{*+} \to D^+\rho^0) = 2A(D^{*+} \to D^+\omega) = \sqrt{2}A(D^{*+} \to D^+\psi) = \cdots = A(\omega \to \pi^0\rho^0) \) with Eq. (3.2) leads to \( \Gamma(D^{*+} \to D^+\gamma)_{SU_f(4)} \simeq 2.4 \text{ keV} \). By comparing the above rate with the measured one\(^3\) \( \Gamma(D^{*+} \to D^+\gamma)_{\text{exp}} \simeq 1.5 \text{ keV} \) (with \( \sim 50\% \) errors), it is seen\(^7\) that (the VMD with) the \( SU_f(4) \) symmetry (of spatial wave-function overlap) again overestimates the rate by \( \sim 50\% \), as in §2.

Now we study radiative decays of charm-strange mesons. The amplitude for \( D^{*+}_{s0} \to D^*_{s} \gamma \) is dominated by \( \phi \) and \( \psi \) poles. Taking the \( SU_f(4) \) symmetry relation, \( \sqrt{2}A(D^{*+}_{s0} \to D^*_s\phi) = \sqrt{2}A(D^{*+}_{s} \to D^*_s\psi) = \cdots = A(\omega \to \pi^0\rho^0) \), and Eq. (3.2), we can obtain the rate for the \( D^{*+}_{s} \to D^*_s \gamma \) listed in Table I. For radiative decays of scalar mesons, we consider typical three cases, (i) \( S = D^{*+}_{s0} \sim \{ c\bar{s} \} \), (ii) \( S = \tilde{F}^+_0 \) and (iii) \( S = \tilde{F}^+_1 \). Under the VMD, the amplitude is obtained by replacing \( (V,P) \) in Eq. (3.1) in terms of \( (S,V) \). In the case (i), the amplitude \( A(D^{*+}_{s0} \to D^*_s\gamma) \) is dominated by the \( \phi \) and \( \psi \) poles. Using the \( SU_f(4) \) relation, \( 2A(D^{*+}_{s0} \to D^*_s\phi) = 2A(D^{*+}_{s0} \to D^*_s\psi) = \cdots = A(\chi_{c0} \to \psi\gamma) \), and the input data, \( \Gamma(\chi_{c0} \to \psi\gamma)_{\text{exp}} = 135 \pm 15 \text{ keV}^{3} \) we have the rate for the decay \( D^{*+}_{s0} \to D^*_s\gamma \) listed in Table I. The amplitudes \( A(\tilde{F}^+_0 \to D^*_s\gamma) \) and \( A(\tilde{F}^+_1 \to D^*_s\gamma) \) in the cases (ii) and (iii) are dominated by the \( \omega \) pole and the \( \rho^0 \) pole, respectively. Taking the \( SU_f(4) \) relation,
A(\hat{F}_0^+ \to D_s^{*+}\omega) = A(\hat{F}_I^+ \to D_s^{*+}\rho^0) = \cdots = A(\phi \to \hat{\delta}^{s0}\rho^0)\beta_1$, with the overlap parameter $\beta_1$ given in §2 and the input data, $\Gamma(\phi \to a_0(980)\gamma)_{\text{exp}} = 0.32 \pm 0.03$ keV,\(^3\) we have the rates for radiative decays of charm-strange mesons listed in Table I, where the spatial wave-function overlap is still in the $SU_f(4)$ symmetry limit. Then, the ratio of the rate $\Gamma(\hat{F}_I^+ \to D_s^{*+}\gamma)_{SU_f(4)}$ in Table I to $\Gamma(\hat{F}_I^+ \to D_s^{*+}\pi^0)_{SU_f(4)}$ estimated in §2,

$$\frac{\Gamma(\hat{F}_I^+ \to D_s^{*+}\gamma)_{SU_f(4)}}{\Gamma(\hat{F}_I^+ \to D_s^{*+}\pi^0)_{SU_f(4)}} \sim 0.005,$$

satisfies well the constraint Eq. (1.1).

Isospin non-conserving decays are now in order. The amplitude for the $D_s^{*+} \to D_s^+\pi^0$ decay can be obtained by putting $A = D_s^{*+}$ and $B = D_s^+$ in Eq. (2-1). Here we assume\(^{15}\) that the isospin non-conservation in decays of charm-strange mesons is caused by the $\eta$-$\pi^0$ mixing whose mixing parameter $\epsilon$ has been estimated to be\(^{16}\)

$$\epsilon = 0.0105 \pm 0.0013.$$

It is very small and of the order of the fine structure constant $\alpha$. This implies that the $SU_f(4)$ symmetry of asymptotic matrix elements and the $\eta$-$\eta'$ mixing lead to $2\langle D_s^+|A_{\pi^0}|D_s^{*+}\rangle = -\epsilon \sin\Omega \cdot \langle \pi^+|A_{\pi^+}|\rho^0 \rangle$, where $\Omega \approx 35^\circ$ for the usual $\eta$-$\eta'$ mixing angle $\theta_P = -20^\circ$. The size of $\langle \pi^+|A_{\pi^+}|\rho^0 \rangle$ can be estimated to be $|\langle \pi^+|A_{\pi^+}|\rho^0 \rangle| \approx 1.0(11)$ from the measured rate\(^3\) $\Gamma(\rho \to \pi\pi)_{\text{exp}} = 149.4 \pm 1.0$ MeV. In this way, we are lead to $\Gamma(D_s^{*+} \to D_s^+\pi^0)_{SU_f(4)} \approx 0.05$ keV. Comparing this result with $\Gamma(D_s^{*+} \to D_s^+\gamma)_{SU_f(4)}$ in Table I, we obtain $R(D_s^{*+})^{-1} \approx 0.06$. This is much smaller than unity, as conjectured in §1, and reproduces well the measurement Eq. (1-2). Therefore, the present approach seems to be reliable.

With this in mind, we consider two cases of the isospin non-conserving decays of scalar mesons, (i) $S^+ = D_s^{*+}$ and (ii) $S^+ = \hat{F}_0^+$. The amplitude for the $S^+ \to D_s^{*+}\pi^0$ decay is obtained by putting $A = S^+$, $B = D_s^{*+}$ and $\pi = \pi^0$ in Eq. (2-1). Since this decay is assumed to proceed through the $\eta$-$\pi^0$ mixing as discussed above, we replace the matrix elements, $\langle D_s^+|A_{\pi^0}|D_s^{*+}\rangle$ and $\langle D_s^+|A_{\pi^0}|\hat{F}_0^+\rangle$, by the OZI-rule allowed $-\epsilon \sin\Theta \cdot \langle D_s^+|A_{\pi^+}|\rho^0 \rangle$ and $\epsilon \cos\Theta \cdot \langle D_s^+|A_{\eta^+}|\hat{F}_0^+\rangle$, respectively. The $SU_f(4)$ symmetry of asymptotic matrix elements leads to $\langle D_s^+|A_{\eta^+}|D_s^{*+}\rangle = \langle K^+|A_{\pi^+}|K_0^0(1430)\rangle$ in the case (i) and $2\langle D_s^+|A_{\eta^+}|\hat{F}_0^+\rangle = \langle \pi^+|A_{\eta^+}|\delta^{*+}\rangle \beta_0$ in the case (ii). The size of the former is estimated to be $|\langle K^+|A_{\pi^+}|K_0^0(1430)\rangle| \approx 0.29$ from the experimental data,\(^3\) $\Gamma(K_0^0(1430) \to K\pi)_{\text{exp}} = 270 \pm 24$ MeV, and the isospin $SU_f(2)$ symmetry, where it has been assumed that $K_0^0(1430)$ is the conventional $3^0 P_0 \{ds\}$ state.\(^3\) The latter has already been obtained as $|\langle \pi^+|A_{\eta^+}|\delta^{*+}\rangle| = \sqrt{\frac{1}{2}}|\langle \eta^+|A_{\pi^+}|\delta^{*+}\rangle| \approx 0.6$ in §2. Using the above results on the asymptotic matrix elements, the value of $\epsilon$ in Eq. (3-4) and $\theta_P = -20^\circ$, we have the rates for the isospin non-conserving decays,

$$\Gamma(D_s^{*+} \to D_s^+\pi^0)_{SU_f(4)} \approx \Gamma(\hat{F}_0^+ \to D_s^+\pi^0)_{SU_f(4)} \approx 0.6 \text{ keV}.$$

These results are much smaller than the rates for the radiative decays of the charm-strange scalar mesons listed in Table I, as conjectured in §1. Eventually, the ratios of decay rates under consideration can be obtained as (i) $R(\hat{F}_0^+) \approx 60$, (ii) $R(\hat{F}_0^+) \approx 7$ and (iii)
Figs. 1. Productions of charm-strange scalar mesons through $e^+e^- \rightarrow c\bar{c}$ within the minimal $q\bar{q}$-pair creation. (a) and (b) describe productions of $D_s^+\pi^-$, $D_s^+\pi^-$, $D_s^+\rho^-$, etc., and $D_s^+D_s^+$, $D_s^+D_s^-$, $D_s^+D_s^-$, etc., respectively. Productions of $\hat{F}_I^+\pi^+$ and $(\hat{F}_I^+\pi^-)$ are given by (c) and (d), respectively.

$R(\hat{F}_I^+) \simeq 0.005$ in Eq. (3.3). In this way, it is seen that the experimental constraint Eq. (1.1) can be satisfied only in the case (iii). (For more details, see Refs. 7 and 10.) Its assignment to an isosinglet $DK$ molecule has already been rejected because it leads to $R\{DK\} \gg R(D_{s0}^+(2317))_{\text{exp}}$ as in (ii). Thus we conclude that assigning $D_{s0}^+(2317)$ to $\hat{F}_I^+$ is favored by the experiments while its assignment to the $I = 0$ state, the conventional scalar $D_{s0}^+$ or the scalar four-quark $\hat{F}_0^+$ (or the $DK$ molecule), is not favored.

§4. Production of charm-strange scalar mesons

Although assigning $D_{s0}^+(2317)$ to $\hat{F}_I^+$ is favored by experiments as seen above, its neutral and doubly charged partners, $\hat{F}_0^+$ and $\hat{F}_0^{++}$, have not yet been observed by inclusive $e^+e^-$ annihilation experiment. Therefore, we now study productions of charm-strange scalar four-quark mesons ($\hat{F}_I^{++,0}$ and $\hat{F}_0^+$). To this aim, we consider their production through weak interactions, as a possible candidate, because OZI-rule violating creations of multiple $q\bar{q}$-pairs and their recombinations into four-quark meson states are expected to be strongly suppressed at high energies. We, first, recall the so-called BSW Hamiltonian as the effective weak Hamiltonian,

$$H_w^{\text{BSW}} \propto a_1 Q_1 + a_2 Q_2 + \cdots + H'_w + h.c., \quad (4.1)$$

where $Q_1$ and $Q_2$ are four-quark operators given by products of neutral and charged currents, respectively, and provide amplitudes for color suppressed and color favored decays, respectively, under the factorization prescription. The extra term $H'_w$ is automatically induced when the BSW Hamiltonian is obtained. It is given by a sum of products of colored currents and provides a non-factorizable amplitude, so that it is usually taken away. However, in this talk, it is left intact because it can play an important role in production of charm-strange scalar four-quark mesons.

Next, we draw quark-line diagrams within the minimal $q\bar{q}$-pair creation, because multiple $q\bar{q}$-pair creation is expected to be suppressed due to the OZI rule. In this approximation, the quark-line diagrams related to production of charm-strange scalar four-quark mesons in $e^+e^- \rightarrow c\bar{c}$ annihilation are given in Fig. 1. Because there is no diagram to describe production of $\hat{F}_I^{++}$, it is understood why the $e^+e^- \rightarrow c\bar{c}$ experiment found no evidence for it. Productions of $\hat{F}_0^+$, $\hat{F}_0^+$ and $\hat{F}_1^+$ mesons are described by Figs. 1(c) and (d).
Fig. 2. Productions of charm-strange scalar mesons in weak decays of $B_u$ meson. (a) describes a production of $\hat{F}_I^+$ and $\hat{F}_s^+$ with $D_s^0$ (or $D_s^{*0}$), (b) a production of $\hat{F}_I^{++}$ with $D^-$ (or $D^{*-}$), and (c) and (d) productions of $D_s^+\pi^+$ with $D^-$ and $D_s^+\pi^0$ with $\bar{D}_s^0$, respectively.

The diagrams Figs. 1(a) and (b) in which the weak vertices are given by the color favored spectator diagrams describe productions of $D_s^+\pi^-$, $D_s^{*+}\pi^-$, $D_s^+\rho^-$, etc., and $D_s^+D_s^0$, $D_s^{*+}D_s^0$, $D_s^{*+}D_s^-$, etc., respectively. By the way, it is known that color favored spectator decays are much stronger than color mismatched decays under the factorization prescription (i.e., $|a_1/a_2|^2 \approx 6.8 \times 10^{-3}$ at the scale of charm mass\(^{23}\)). In addition, non-factorizable contributions are actually small in hadronic weak decays of $B$ mesons,\(^{21}\) and they will be much smaller at higher energies. As seen in Fig. 1, productions of $\hat{F}_I^{+0}$ and $\hat{F}_0^+$ involve rearrangements of colors and their amplitudes are non-factorizable, so that they will be much more strongly suppressed than the color favored processes. Therefore, it is not very easy to extract the $\hat{F}_0^+ \rightarrow D_s^{+}\pi^-$ signals in inclusive $e^+e^- \rightarrow c\bar{c}$ experiments. In the case of $\hat{F}_I^+$, however, one does not need to worry about large numbers of background events from Figs. 1(a) and (b) because its main decay is $\hat{F}_I^+ \rightarrow D_s^{+}\pi^0$. Nevertheless, its evidence has not been observed in the radiative channel, because its decay into $D_s^{*+}\gamma$ is strongly suppressed as seen in §3. As for $\hat{F}_0^+$, it can decay much more strongly into $D_s^{*+}\gamma$ than $D_s^{+}\pi^0$ as seen in §3, although its production is depicted by the same diagram Fig. 1(d) as the production of $\hat{F}_I^+$. Therefore, reconstruction of $\hat{F}_0^+ \rightarrow D_s^{*+}\gamma$ might be suspected to be efficient to search for $\hat{F}_0^+$. However, very large numbers of $D_s^{*+}$ and $\gamma$ (from $D_s^{*-} \rightarrow D_s^{*-}\gamma$) produced through the spectator diagrams Figs. 1(a) and (b) (and in $e^+e^- \rightarrow c\bar{c} \rightarrow D_s^{(*)+}D_s^{(*)-}$, etc., without weak interactions) obscure the above signal $D_s^{*+}\gamma$. In this way, it will be understood that whether each of charm-strange scalar mesons can be observed or not depends on its production mechanism, and, therefore, it seems that no evidence for $\hat{F}_I^0$ and $\hat{F}_I^{++}$ in inclusive $e^+e^- \rightarrow c\bar{c}$ annihilation experiments does not necessarily imply their non-existence.

Because it is difficult to observe $\hat{F}_I^{+0}$, $\hat{F}_I^0$ and $\hat{F}_0^+$ in inclusive $e^+e^- \rightarrow c\bar{c}$ experiments as seen above, we now study productions of charm-strange scalar four-quark mesons in $B$ decays. For this purpose, we again draw quark-line diagrams describing their productions within the minimal $q\bar{q}$-pair creation. As expected in the quark-line diagrams of Figs. 2 and 3, resonance peaks which are approximately degenerate with $D_{s0}^{*+}(2317)$ have been observed in the following hadronic weak decays of $B$ mesons: $B_u^+ \rightarrow \bar{D}_s^0\bar{D}_{s0}^{*+}(2317)$, $D_s^{+}\pi^0$, $D_s^{*+}\gamma$] and $B_d^0 \rightarrow D^-\bar{D}_{s0}^{*+}(2317)$, $D_{s0}^{*+}\pi^0$, $D_{s0}^{*+}\gamma]$ in the BELLE experiment,\(^{24}\) and $B_u^+ \rightarrow \bar{D}_s^0(\bar{D}_s^{*0})\bar{D}_{s0}^{*+}(2317)$, $D_{s0}^{*+}\pi^0$, $D_{s0}^{*+}\gamma]$ and $B_d^0 \rightarrow D^-(\bar{D}_s^{*+})\bar{D}_{s0}^{*+}(2317)$, $D_{s0}^{*+}\pi^0$, $D_{s0}^{*+}\gamma]$ in the BABAR experiment.\(^{25}\) It should be noted that indications of new resonances have been observed in the $D_s^{*+}\gamma$ channel. It is quite
different from the case of inclusive $e^+e^- \rightarrow c\bar{c}$. Therefore, the new resonances have been denoted by $\hat{D}_{s0}^+(2317)$ [observed channel(s)] to distinguish them from the previous $D_{s0}^+(2317)$. Because Figs. 2(a) and 3(b) involve both $\hat{F}_I^+$ and $\hat{F}_0^+$ and their main decays are quite different from each other, the new resonance can be assigned to $\hat{F}_I^+$ when it is observed in the $D^+_s\pi^0$ channel, while it might be assigned to $\hat{F}_0^+$ when it is observed in the $D^+_s\pi^0$ channel. Observations of $\hat{F}_I^{++}$ and $\hat{F}_I^0$ are expected in the process $B^+_u \rightarrow D^- (or D^{*-}) \hat{F}_I^{++}[D^+_s\pi^+]$ as depicted in Fig. 2(b), and in the process $B^0_{d} \rightarrow \bar{D}^0 \hat{F}_I^0[D_s^+\pi^-]$ as depicted in Fig. 3(a), respectively. Because the diagrams Figs. 2(a), 2(b), 3(a) and 3(b) are of the same type, rates for production of $\hat{F}_I^{++}$ and $\hat{F}_I^0$ are expected to be not very far from that for $\hat{D}_{s0}^+(2317)[D^+_s\pi^0]$, i.e.,

$$B(B^+_u \rightarrow D^- (or D^{*-}) \hat{F}_I^{++}[D^+_s\pi^+]) \sim B(B^0_{d} \rightarrow \bar{D}^0 (or D^{*0}) \hat{F}_I^0[D^+_s\pi^-])$$

$$\sim B(B \rightarrow \bar{D} (or D^*) \hat{D}_{s0}^+(2317)[D^+_s\pi^0])_{exp} \sim 10^{-3}.$$

(4.2)

Therefore, $\hat{F}_I^{++}$ and $\hat{F}_I^0$ could be observed in $B \rightarrow D (or D^*) D^+_s\pi$ decays.

§5. Summary

By studying the $D^+_{s0}(2317) \rightarrow D^+_s\pi^0$ and $D^0_{s0}(2317) \rightarrow D^*_s\pi\gamma$ decays, we have seen that assigning $D_{s0}^+(2317)$ to $\hat{F}_I^+$ is favored by experiments. To search for its partners $\hat{F}_I^0$ and $\hat{F}_I^{++}$, we have investigated productions of these four-quark mesons through hadronic weak interactions. As the results, we have found that detecting them in inclusive $e^+e^- \rightarrow c\bar{c}$ is likely quite difficult, although $D_{s0}^+(2317)$ itself has already been observed. Taking these points into consideration, we have estimated the branching fractions for decays of $B$ mesons producing $\hat{F}_I^{++}$ and $\hat{F}_I^0$ as $B(B^+_u \rightarrow D^- \hat{F}_I^{++}) \sim B(B^0_{d} \rightarrow \bar{D}^0 \hat{F}_I^0) \sim 10^{-3}$. As for observation of $\hat{F}_I^+$ and $\hat{F}_I^0$, we conclude that they could have been observed as resonances with approximately equal masses in two different channels, $D^+_s\pi^0$ and $D^*_s\pi^0$, as the BELLE collaboration observed.

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