Identifying influential spreaders in complex networks based on gravity formula

Ling-ling Ma\textsuperscript{a} Chuang Ma\textsuperscript{a} Hai-Feng Zhang\textsuperscript{a,b,c,1} Bing-Hong Wang\textsuperscript{d}

\textsuperscript{a} School of Mathematical Science, Anhui University, Hefei 230601, P. R. China
\textsuperscript{b} Key Laboratory of Computer Network and Information Integration (Southeast University), Ministry of Education, Nanjing 211189, P. R. China
\textsuperscript{c} Department of Communication Engineering, North University of China, Taiyuan, Shan’xi 030051, P. R. China
\textsuperscript{d} Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China

Abstract

How to identify the influential spreaders in social networks is crucial for accelerating/hindering information diffusion, increasing product exposure, controlling diseases and rumors, and so on. In this paper, by viewing the k-shell value of each node as its mass and the shortest path distance between two nodes as their distance, then inspired by the idea of the gravity formula, we propose a gravity centrality index to identify the influential spreaders in complex networks. The comparison between the gravity centrality index and some well-known centralities, such as degree centrality, betweenness centrality, closeness centrality, and k-shell centrality, and so forth, indicates that our method can effectively identify the influential spreaders in real networks as well as synthetic networks. We also use the classical Susceptible-Infected-Recovered (SIR) epidemic model to verify the good performance of our method.

Key words: Complex networks, Influential spreader, Gravity formula.

1 Introduction

To effectively identify influential spreaders in social networks is of theoretical and practical significance \textsuperscript{1,2,3,4,5,6,7,8,9,10,11}, since it is crucial for

\textsuperscript{1} Corresponding author: haifengzhang1978@gmail.com

Preprint submitted to Elsevier
developing efficient strategies to control epidemic spreading, accelerate information diffusion, promote new products, and so on. In view of this, many centrality indices have been proposed to address this problem, including degree centrality [12], betweenness centrality [13], neighborhood centrality [14] and closeness centrality [15], etc. In particular, Kitsak et al. proposed a $k$-shell decomposition method to identify the most influential spreaders based on the assumption that nodes in the same shell have similar influence and nodes in higher shells are likely to infect more nodes. $k$-shell method is found to be better than the degree centrality index in many real networks [1]. However, recent researches have demonstrated that the nodes within the same shell often have distinct influences, and this method may fail in some networks without core-like structure, e.g., Barašási-Albert network [16]. Thus, after this, some methods were proposed to further improve the performance of the $k$-shell method. For example, Zeng et al. proposed a mixed degree decomposition method by incorporating the residual degree and the exhausted degree [17]; Liu et al. have demonstrated that the existence of the core-like groups can result in the invalidation of $k$-shell method [18], and then they showed that the accuracy of $k$-shell method can be improved once the redundant links in networks are removed [19]. Chen et al. designed a semi-local index by considering the next nearest neighborhood [20]; Lin et al. presented an improved ranking method by taking into account the shortest path distance between a target node and the node set with the highest $k$-core value [21]; Recently, Bae et al. defined a novel measure–coreness centrality index, which is given by summing all neighbors’ $k$-shell values [22].

In general, a node’s influence is not only dependent on its nearest neighbors but also on the nodes who are not the nearest neighbors [23,24], meanwhile, their interaction influence commonly decreases with their shortest path distance. If the $k$-shell value of each node is viewed as its mass, and the shortest path distance between two nodes is defined as their distance, then we can use the idea of gravity formula proposed by Isaac Newton to measure the influence of nodes. Inspired by these factors, in the work, we propose a new centrality index to measure the influence of nodes, which is called gravity centrality index. We apply the susceptible-infectious-recovered (SIR) spreading dynamics to evaluate the effectiveness of our proposed method, the experimental results indicate that gravity centrality index can better evaluate the influence of nodes than the ones generated by degree centrality, betweenness centrality, $k$-shell centrality, closeness centrality, and so on.

The layout of the paper is as follows: In Sec. 2, we first briefly review several typical centrality indices which are used to compare in this work, and the description of our method is presented. Then the experimental results are presented in Sec. 3. Finally, Conclusions and discussions are summarized in Sec. 4.
2 Method

An undirected network is represented by $G = (N, M)$ with $N$ nodes and $M$ edges, and its structure can be described by an adjacent matrix $A = (a_{ij})_{N \times N}$ where $a_{ij} = 1$ if node $i$ is connected to node $j$, and $a_{ij} = 0$ otherwise.

Here we briefly review the definitions of several centrality indices that will be discussed in this work.

The degree centrality (DC) of a node is defined as the number of nearest neighbors. The betweenness centrality (BC) of a node is defined as the fraction of all shortest paths travel through the node. The closeness centrality (CC) of a node is defined as the reciprocal of the sum of the lengths of the geodesic distance to every other node. The k-shell decomposition method (ks) is implemented by the following steps: Firstly, remove all nodes with degree one, and keep deleting the existing nodes until all nodes’ degrees are larger than one. All of these removed nodes are assigned 1-shell. Then recursively remove the nodes with degree no larger than two (i.e., remove all nodes with degree two, and keep deleting the existing nodes until all nodes’ degrees are larger than two.) and include them to 2-shell. This procedure continues until all nodes have been assigned to one of the shells [17].

To improve the exactness of k-shell method, the mixed degree decomposition (MDD) method was proposed by Zeng et al. [17]. The mixed degree $k_m(i)$ for a node $i$ is defined by considering the residual degree $k_r(i)$ and the exhausted degree $k_e(i)$ simultaneously, which is written as:

$$k_m(i) = k_r(i) + \lambda \cdot k_e(i).$$  

(1)

At each step of the MDD procedure, the nodes are removed according to the mixed degree, and the mixed degrees of remaining nodes are also updated. Where $\lambda$ is a tunable parameter between 0 and 1. As in Ref. [17], we take $\lambda = 0.7$ in this work.

Recently, Baus et al. designed a ranking method–neighborhood coreness $C_{nc}$ by considering the degree and the coreness of a node simultaneously, the $C_{nc}(i)$ for a node $i$ is defined as [22]

$$C_{nc}(i) = \sum_{j \in \Lambda_i} ks(j),$$  

(2)

where $\Lambda_i$ is the neighbor node set of node $i$. They further developed an ex-
tended neighborhood coreness $C_{nc+}$, which is described as:

$$C_{nc+}(i) = \sum_{j \in \Lambda_i} C_{nc}(j).$$ \hspace{1cm} (3)

Chen et al. proposed a semi-local centrality measure as a tradeoff between low-relevant degree centrality and other time-consuming measures (labeled as SL index). It considers both the nearest and the next nearest neighbors. The semi-local centrality $SL(i)$ of node $i$ is defined as [20]

$$Q(s) = \sum_{j \in \Lambda_s} N(j),$$ \hspace{1cm} (4)

$$SL(i) = \sum_{s \in \Lambda_i} Q(s),$$ \hspace{1cm} (5)

where $\Lambda_i$ is the neighbor node set of node $i$. $N(j)$ is the number of the nearest and the next nearest neighbors of node $j$.

It is fact that, on one hand, the influence of a node is increased if its neighbors (here the neighbors of a node do not just includes its nearest neighbors, which may also include next nearest neighbors, next-next nearest neighbors, etc.) have higher value of $ks$; on the other hand, the interaction effect between two nodes decreases with their distance. Enlighten by the idea of classical gravity formula proposed by Isaac Newton, we can view the k-shell value of node $i$ as its mass, and the shortest path distance between two nodes in network is viewed as their distance. In this way, the influence of node $i$ is measured by (labeled as $G$):

$$G(i) = \sum_{j \in \psi_i} \frac{ks(i)ks(j)}{d_{ij}^2},$$ \hspace{1cm} (6)

where $d_{ij}$ is the shortest path distance between node $i$ and node $j$. $\psi_i$ is the neighborhood set whose distance to node $i$ is less than or equal to a given value $r$. To reduce the algorithm complexity, in the paper, we let $r = 3$, i.e, only nearest neighbors, next nearest neighbors and the next-next nearest neighbors are considered.

An extended gravity index is further developed based on Eq. (6), which is defined as (labeled as $G_+$):

$$G_+(i) = \sum_{j \in \Lambda_i} G(j),$$ \hspace{1cm} (7)

$\Lambda_i$ is the nearest neighborhood of node $i$. 

4
3 Experimental results

In this section, we compare the effectiveness of other indices with our $G$ or $G_+$ index from different aspects and on different networks, including real networks as well as synthetic networks.

We employ the standard susceptible-infected-removed (SIR) model \[25\] to estimate the real spreading influence of the nodes (labeled by $R$). In detail, to check the spreading influence of one given node, we set this node as an infected node and the other nodes are susceptible nodes. At each time step, each infected node can infect its susceptible neighbors with infection probability $\beta$, and then it recovered from the diseases with probability $\mu$. In this paper, we set $\mu = 1.0$. This process repeats until there has no any infected nodes. At last, the number of recovered nodes is used to reflect the real influence of the node.

To guarantee the reliability of the results, all of them are at least averaged over 1000 independent realizations.

First, a small example network with $N = 20$ nodes and $M = 25$ edges is given in Fig. 1 to intuitively compare these indices, the ranking lists from different indices are presented in Table 1. From Table 1, one can see that the $ks$ centrality cannot well distinguish the influence of nodes. Even in the same shell, the nodes’ influence may be totally different. Moreover, the result indicates that, our proposed $G$ and $G_+$ index can effectively identify the influence of nodes, i.e., the ranking list determined from $G$ or $G_+$ index is in good agreement with the ranking list obtained from SIR model in the last row.

| Rank | DC | MDD | $G$ | $G_+$ | $C_{nc+}$ | $ks$ | BC | CC | SL | $R$ |
|------|----|-----|-----|-------|--------|-----|----|----|----|-----|
| 1    | 1, 2 | 1, 2 | 2   | 2     | 0      | 0, 2, 3, 4 | 2   | 6  | 0  | 2   |
| 2    | 0, 4 | 0, 4 | 0   | 0     | 2      | 1, 8, 10, 12, 14-16 | 0   | 2  | 2  | 0   |
| 3    | 3, 8, 10, 14 | 3 | 4   | 4     | 4      | others | 1   | 4  | 4  | 4   |
| 4    | 6, 12, 15, 16 | 8 | 3   | 3     | 3      | —     | 4   | 1  | 3  | 1   |
| 5    | others | 10,14 | 1  | 1     | 1      | —     | 14  | 3  | 1  | 3   |
| 6    | —     | 12, 15, 16 | 8  | 8     | 8      | —     | 6, 8, 10 | 8  | 8  | 8   |
| 7    | —     | 6    | 14  | —     | 10     | —     | 16  | 14 | 10 | 14  |
| 8    | —     | others | 10 | —     | 12, 14 | —     | 15  | 10 | —  | 10  |
| 9    | —     | —    | —   | —     | —      | others | —   | —  | —  | —   |

Table 1
The ranking lists determined by different indices. Degree centrality: DC; mixed Degree decomposition: MDD; gravity centrality: $G$; extended gravity centrality: $G_+$; extended neighborhood coreness defined in Eq. (3): $C_{nc+}$; k-shell decomposition: $ks$; betweenness centrality: BC; closeness centrality: CC; semi-local centrality measure: SL; the node spreading influence evaluated by SIR model: $R$, by taking $\beta = 0.25$.

To validate the effectiveness of the $G$ or $G_+$ index, we apply it to 9 real networks, including Facebook (Slavo Zitnik’s friendship network in Facebook) \[26\], Netsci (collaboration network of network scientists) \[27\], Email (e-mail net-
Fig. 1. An example network with 20 nodes and 25 edges. Nodes with larger degrees have larger size.

work of University at Rovira i Virgili, URV) [28], TAP (yeast protein-protein binding network generated by tandem affinity purification experiments) [17], Y2H (yeast protein-protein binding network generated using yeast two hybridization) [29], Blogs (the communication relationships between owners of blogs on the MSN (Windows Live) Spaces website) [30], Router (the router-level topology of the Internet) [31], HEP (collaboration network of high-energy physicists) [32], PGP (an encrypted communication network) [33]. For simplicity, these networks are treated as undirected and unweighted networks in this work. The detailed information about these 9 real networks are presented in Table 2.

How to improve the resolution is the key issue of an algorithm, for instance, in Ref. [34], Zhou et al. have clarified that the resolution problem is a major reason for the poor performance of common neighbor index subject to the AUC value in link prediction, and then they proposed a local path index to solve the resolution problem. Similarly, a good index in ranking the influences
Table 2
Basic structural properties. $N$ and $M$ are the number of nodes and edges, respectively. $\beta_{th}$ is the epidemic threshold. $H$ is degree heterogeneity, given by $\langle k^2 \rangle / \langle k \rangle^2$. $\bar{r}$ is assortativity coefficient. $C$ is clustering coefficient. $L$ is average shortest path length. $D$ is diameter.

| Network   | N   | M   | $\beta_{th}$ | H    | $\bar{r}$ | C    | L     | D  |
|-----------|-----|-----|---------------|------|----------|------|-------|----|
| Facebook  | 324 | 2218| 0.047         | 1.567| 0.247    | 0.465| 3.054 | 7  |
| Netsci    | 379 | 914 | 0.125         | 1.663| -0.082   | 0.741| 6.042 | 17 |
| Email     | 1133| 5451| 0.053         | 1.942| 0.078    | 0.220| 3.606 | 8  |
| TAP       | 1373| 6833| 0.061         | 1.644| 0.579    | 0.529| 5.224 | 12 |
| Y2H       | 1458| 1948| 0.140         | 2.667| -0.209   | 0.071| 6.812 | 19 |
| Blogs     | 3982| 6803| 0.072         | 4.038| -0.133   | 0.284| 6.252 | 8  |
| Router    | 5022| 6258| 0.072         | 5.503| -0.138   | 0.012| 6.449 | 15 |
| HEP       | 5835| 13815| 0.110         | 1.926| 0.185    | 0.506| 7.026 | 19 |
| PGP       | 10680| 24316| 0.053         | 4.147| 0.238    | 0.266| 7.463 | 24 |

of nodes should also have a high resolution. As illustrated in Table 1, $G$ or $G_+$ index is good at distinguishing the nodes’ difference, which is much better than the $ks$ index. So to quantitatively measure resolution of different indices, a monotonicity index $M(X)$ for a ranking list $X$ is used [22]:

$$M(X) = [1 - \sum_{c \in V} \frac{N_c(N_c - 1)}{N(N - 1)}]^2,$$

(8)

where $N$ is the size of network, and $N_c$ is the number of nodes with the same index value $c$. If $M(X) = 1$, which means that the ranking method is perfectly monotonic and each node is categorized a different index value; otherwise, all nodes are in the same rank as $M(X) = 0$. The monotonicity $M$ for different ranking methods is summarized in Table 3. Generally, the results suggest that $G$ or $G_+$ index can give higher value of $M$. Moreover, $M(G)$ and $M(G_+)$ are very near 1 in some networks. Therefore, gravity method can better distinguish the node's influence than other indices.

The Kendall’s tau rank correlation coefficient $\tau$ is used to measure the correlation one topology-based ranking list and the real spreading capability $R$. Let $(x_i, y_i)$ and $(x_j, y_j)$ be a randomly selected pair of joint observations from ranking lists $X$ and $Y$, respectively. If one has $x_i > x_j$ and $y_i > y_j$ or $x_i < x_j$ and $y_i < y_j$, the observations $(x_i, y_i)$ and $(x_j, y_j)$ are said to be concordant. If $x_i > x_j$ and $y_i < y_j$ or $x_i < x_j$ and $y_i > y_j$, they are said to be discordant. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant [35,34]. $\tau$ is
Table 3
M (.) is the monotonicity of the corresponding measures.

| Network | M(DC) | M(MDD) | M(G)  | M(G+) | M(Cnc+) | M(ks) | M(BC) | M(CC) | M(SL) |
|---------|-------|--------|-------|-------|---------|-------|-------|-------|-------|
| Facebook | 0.9315 | 0.9729 | 0.9999 | 0.9995 | 0.8445  | 0.9855 | 0.9953 | 0.9999 |
| Netsci  | 0.7642 | 0.8215 | 0.9949 | 0.9951 | 0.9893  | 0.6421 | 0.3387 | 0.9928 | 0.9939 |
| Email   | 0.8874 | 0.9229 | 0.9999 | 0.9999 | 0.9991  | 0.8088 | 0.9400 | 0.9988 | 0.9999 |
| TAP     | 0.8991 | 0.9599 | 0.9994 | 0.9994 | 0.9981  | 0.8380 | 0.9238 | 0.9988 | 0.9992 |
| Y2H     | 0.4884 | 0.5304 | 0.9966 | 0.9960 | 0.9633  | 0.2972 | 0.5063 | 0.9957 | 0.9936 |
| Blogs   | 0.5654 | 0.5906 | 0.9976 | 0.9976 | 0.9868  | 0.4670 | 0.4004 | 0.9973 | 0.9971 |
| Router  | 0.2886 | 0.3009 | 0.9967 | 0.9965 | 0.9657  | 0.0691 | 0.2983 | 0.9961 | 0.9953 |
| HEP     | 0.7654 | 0.8314 | 0.9998 | 0.9999 | 0.9917  | 0.6303 | 0.5651 | 0.9998 | 0.9990 |
| PGP     | 0.6193 | 0.6678 | 0.9995 | 0.9997 | 0.9851  | 0.4806 | 0.5099 | 0.9996 | 0.9986 |

defined as
\[ \tau = \frac{N_1 - N_2}{0.5N(N-1)}, \]  

where \( N_1 \) and \( N_2 \) are the number of concordant pairs and discordant pairs, respectively.

When we employ SIR model to check the spreading influence of nodes, the infection probability \( \beta \) should not be too small or too large. The epidemic cannot successfully spread over networks if \( \beta \) is too small, so the spreading capability of each node cannot be measured. On the contrary, if \( \beta \) is too large, the epidemic can easily outbreak over almost whole network, leading to the spreading capability of each node cannot be distinguished too. Thus, in this work, we first obtain the epidemic threshold \( \beta_{th} \) for each network, which is given as \( \beta_{th} \sim \langle k \rangle / \langle k^2 \rangle \), with \( \langle k \rangle \) and \( \langle k^2 \rangle \) be the average degree and the second order average degree respectively. The value of \( \beta_{th} \) for different networks is given in Table 2 too. Then, we choose the value of \( \beta \) to be slightly larger than the threshold \( \beta_{th} \) when computing \( \tau \) for different indices (a new index to measure the influence of nodes was proposed in Ref. [36], which is independent on the parameter \( \beta \)). The results in Table 4 manifest that our method outperforms the other methods in most cases.

To further estimate how the infection probability \( \beta \) affects the effectiveness of different methods, the correlation value \( \tau \) as a function of \( \beta \) for different methods is shown in Fig. 2. As described in Fig. 2, in most cases, \( G \) or \( G_+ \) index provides better performance than the other index when \( \beta > \beta_{th} \) (the values of \( \beta_{th} \) for different networks are illustrated by the dot lines in Fig. 2). However, Fig. 2 clearly indicates that though the global indices, such as betweenness index and closeness index are time-consuming, they are not good at measuring the influence of nodes in these networks. Meanwhile, the performance of MDD method in identifying the node’s influence is almost the same as the degree centrality.
Previously, our results were obtained by setting the value of $r = 3$ (i.e., only the effects of the nearest neighbors, next nearest neighbors and the next-next nearest neighbors are considered). To check the sensitivity of $r$ on our results, the effect of the parameter $r$ on the value of $\tau$ is plotted in Fig. 3. As shown in Fig. 3, in generally, the optimal value of $r$ is about 3-5, and the value of $\tau$ becomes stable when $r$ is further increased. As a result, it is unnecessary to choose too large value of $r$, which just increases the algorithm complexity of our method.

Besides the real networks, we also compare the performance of our method with other methods on the two typical synthetic networks—Barabás-Albert (BA) networks [16] and the Watts-Strogatz (WS) small-world networks [37] with $N = 1000$. Starting from a connected network with $m_0$ nodes to construct a BA network, at each step, a new node is added to the network and connected to $m$ existing nodes according to the preferential attachment mechanism, where $m \leq m_0$ [16]. We set the number of nodes $m_0 = 10$ in this paper. The WS small-world model considers a ring nearest neighbor coupled network with $N$ nodes. Each node symmetrically connects to its 2K nearest neighbors. Starting from it, a fraction $p$ of edges in the network are rewired, by visiting all $K$ clock-wise edges of each node and reconnecting them, with probability $p$, to a randomly chosen node [37]. During the rewiring process, self-connection and reconnection are forbidden.

For BA network (see Fig. 4 (a) and (b)), one can see that the performances of $G$, $G_+$ and $C_{nc+}$ indices are almost the same. The reason is that the three indices are all the improved methods of k-shell method, however, all nodes in BA network are almost classified into the same shell when using the k-shell method (so we do not calculate the case of $ks$ in Fig. 4). Moreover, the results show that the three indices are better than CC index and are much better than DC, BC and MDD indices. For WS network (see Fig. 4 (c) and (d)), whose degree distribution shows Poisson distribution, i.e, their degrees are not so different. In this case, it is difficult for DC index to distinguish the

### Table 4

| Network | $\beta$ | $\tau_{DC}$ | $\tau_{M.D.D}$ | $\tau_G$ | $\tau_{G_{nc+}}$ | $\tau_{ks}$ | $\tau_{BC}$ | $\tau_{CC}$ | $\tau_{SL}$ |
|---------|---------|-------------|----------------|---------|----------------|------------|-----------|-----------|-----------|
| Facebook | 0.050   | 0.767       | 0.796          | 0.861   | 0.913          | 0.916      | 0.735     | 0.364     | 0.720     | 0.940     |
| Netsci   | 0.130   | 0.599       | 0.620          | 0.830   | 0.852          | 0.847      | 0.525     | 0.308     | 0.330     | 0.806     |
| Email    | 0.070   | 0.771       | 0.790          | 0.887   | 0.937          | 0.935      | 0.779     | 0.625     | 0.822     | 0.935     |
| TAP      | 0.065   | 0.725       | 0.746          | 0.870   | 0.899          | 0.873      | 0.690     | 0.273     | 0.527     | 0.886     |
| Y2H      | 0.160   | 0.445       | 0.463          | 0.827   | 0.833          | 0.825      | 0.407     | 0.412     | 0.701     | 0.775     |
| Blogs    | 0.075   | 0.525       | 0.532          | 0.834   | 0.763          | 0.795      | 0.482     | 0.390     | 0.579     | 0.706     |
| Router   | 0.075   | 0.322       | 0.323          | 0.797   | 0.805          | 0.786      | 0.186     | 0.315     | 0.642     | 0.790     |
| HEP      | 0.110   | 0.487       | 0.506          | 0.787   | 0.865          | 0.735      | 0.485     | 0.345     | 0.784     | 0.840     |
| PGP      | 0.055   | 0.479       | 0.490          | 0.784   | 0.770          | 0.756      | 0.439     | 0.313     | 0.636     | 0.747     |
Fig. 2. (Color online) The value of $\tau$ obtained by comparing the ranking list generated by the SIR model and the ranking lists generated by the topology-based method on Facebook, Netsci, Email, TAP, Y2H, Blogs, Router and HEP. The dot lines correspond to the epidemic threshold.

influence of nodes. However, as shown in Fig. 4(c) and (d), as $\beta > \beta_{th}$, the performances of $G$ and $G_+$ indices are still better than the other indices. In particular, for WS network, one can observe that the performances of $G$ and $G_+$ indices are much better than the $C_{nc+}$ index when $\beta > \beta_{th}$. The results in Fig. 4 suggest that our method can not only identify the influential nodes on real networks but also on synthetic networks.

4 Conclusions and discussions

In summary, in this paper, we have proposed a gravity method to identify the influential spreaders in complex networks. In the model, each node’s $k$-shell value is considered as its mass and the shortest path distance between two nodes is viewed as their distance. The idea of the gravity method comes from the well-known gravity formula, which is very dramatic and impressive. What’s more, the gravity model can reflect the facts that, on one hand, the interaction influence between two nodes is proportional to their corresponding $k$-shell values; on the other hand, the influences of the neighbors decreases with their distance. We employed our method on some real networks and synthetic
networks, by calculating the monotonicity index $M$, we found that our method can better distinguish the difference of node influence than other indices. Also, by computing Kendalls tau rank correlation coefficient $\tau$, we have shown that, in most cases, our method has a better performance in evaluating the node’s influence than other indices. Therefore, our method provides an effective way to identify the influential spreaders in social networks.

Some extensions may be made based on this method. For example, by defining the combination of node’s degree and node’s strength as the weighted degree of a node in weighted networks, Garas et al have proposed a new k-shell decomposition method for weighted networks \[38\]. Therefore, once the new k-shell value for each node in weighted network is assigned, our method can be simply generalized to weighted networks \[39\]. Also, if we view the closeness centrality, degree centrality, eigenvector centrality, and so forth as the mass of a node, then the gravity method may be further generalized.

We only investigated the performance of the gravity method in some typical networks, and the classical SIR model was used to mimic the the spreading...
 Acknowledgments

This work is funded by the NSFC (Grant Nos. 61473001, 11331009, 11275186). Partially supported by open fund of Key Laboratory of Computer Network and Information Integration (Southeast University), Ministry of Education (No. K93-9-2015-03B), and by Graduate Student Academic Innovation Foundation.
of Anhui University (Grant No. yfc100015).

References

[1] M. Kitsak, L. K. Gallos, S. Havlin, F. Liljeros, L. Muchnik, H. E. Stanley, H. A. Makse, Identification of influential spreaders in complex networks, Nature Physics 6 (11) (2010) 888–893.

[2] P. Basaras, D. Katsaros, L. Tassiulas, Detecting influential spreaders in complex, dynamic networks, Computer 46 (4) (2013) 0024–29.

[3] L. Lü, Y.-C. Zhang, C. H. Yeung, T. Zhou, Leaders in social networks, the delicious case, PLoS one 6 (6) (2011) e21202.

[4] J. Borge-Holthoefer, Y. Moreno, Absence of influential spreaders in rumor dynamics, Physical Review E 85 (2) (2012) 026116.

[5] K. Klemm, M. Á. Serrano, V. M. Eguíluz, M. San Miguel, A measure of individual role in collective dynamics, Scientific Reports 2 (2012) 292.

[6] D. Wei, X. Deng, X. Zhang, Y. Deng, S. Mahadevan, Identifying influential nodes in weighted networks based on evidence theory, Physica A 392 (10) (2013) 2564–2575.

[7] J.-G. Liu, Z.-M. Ren, Q. Guo, Ranking the spreading influence in complex networks, Physica A 392 (18) (2013) 4154–4159.

[8] D.-B. Chen, R. Xiao, A. Zeng, Y.-C. Zhang, Path diversity improves the identification of influential spreaders, EPL (Europhysics Letters) 104 (6) (2013) 68006.

[9] Z.-M. Ren, A. Zeng, D.-B. Chen, H. Liao, J.-G. Liu, Iterative resource allocation for ranking spreaders in complex networks, EPL (Europhysics Letters) 106 (4) (2014) 48005.

[10] L.-F. Zhong, J.-G. Liu, M.-S. Shang, Iterative resource allocation based on propagation feature of node for identifying the influential nodes, Physics Letters A 379 (2015) 2272–2276.

[11] X.-Y. Zhao, B. Huang, M. Tang, H.-F. Zhang, D.-B. Chen, Identifying effective multiple spreaders by coloring complex networks, EPL (Europhysics Letters) 108 (6) (2014) 68005.

[12] P. Bonacich, Factoring and weighting approaches to status scores and clique identification, Journal of Mathematical Sociology 2 (1) (1972) 113–120.

[13] L. C. Freeman, A set of measures of centrality based on betweenness, Sociometry (1977) 35–41.

[14] S. Maslov, K. Sneppen, Specificity and stability in topology of protein networks, Science 296 (5569) (2002) 910–913.
[15] G. Sabidussi, The centrality index of a graph, Psychometrika 31 (4) (1966) 581–603.

[16] A.-L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (5439) (1999) 509–512.

[17] A. Zeng, C.-J. Zhang, Ranking spreaders by decomposing complex networks, Physics Letters A 377 (14) (2013) 1031–1035.

[18] Y. Liu, M. Tang, T. Zhou, Y. Do, Core-like groups result in invalidation of identifying super-spreader by k-shell decomposition, Scientific Reports 5 (2015) 9602.

[19] Y. Liu, M. Tang, T. Zhou, Y. Do, Improving the accuracy of the k-shell method by removing redundant links: From a perspective of spreading dynamics, Scientific Reports 5 (2015) 13172.

[20] D. Chen, L. Liu, M.-S. Shang, Y.-C. Zhang, T. Zhou, Identifying influential nodes in complex networks, Physica A 391 (4) (2012) 1777–1787.

[21] J.-H. Lin, Q. Guo, W.-Z. Dong, L.-Y. Tang, J.-G. Liu, Identifying the node spreading influence with largest k-core values, Physics Letters A 378 (45) (2014) 3279–3284.

[22] J. Bae, S. Kim, Identifying and ranking influential spreaders in complex networks by neighborhood coreness, Physica A 395 (2014) 549–559.

[23] J. Zhang, X.-K. Xu, P. Li, K. Zhang, M. Small, Node importance for dynamical process on networks: A multiscale characterization, Chaos 21 (1) (2011) 016107.

[24] J. Zhang, K. Zhang, X.-k. Xu, K. T. Chi, M. Small, Seeding the kernels in graphs: Toward multi-resolution community analysis, New Journal of Physics 11 (11) (2009) 113003.

[25] Y. Moreno, R. Pastor-Satorras, A. Vespignani, Epidemic outbreaks in complex heterogeneous networks, The European Physical Journal B 26 (4) (2002) 521–529.

[26] N. Blagus, L. Šubelj, M. Bajec, Self-similar scaling of density in complex real-world networks, Physica A 391 (8) (2012) 2794–2802.

[27] M. E. Newman, Finding community structure in networks using the eigenvectors of matrices, Physical Review E 74 (3) (2006) 036104.

[28] R. Guimera, L. Danon, A. Diaz-Guilera, F. Giralt, A. Arenas, Self-similar community structure in a network of human interactions, Physical Review E 68 (6) (2003) 065103.

[29] A. Kumar, M. Snyder, Proteomics: Protein complexes take the bait, Nature 415 (6868) (2002) 123–124.

[30] N. Xie, Social network analysis of blogs, Ph.D. thesis, MSc Dissertation. University of Bristol (2006).
[31] N. Spring, R. Mahajan, D. Wetherall, T. Anderson, Measuring isp topologies with rocketfuel, Networking, IEEE/ACM Transactions on 12 (1) (2004) 2–16.

[32] J. Leskovec, J. Kleinberg, C. Faloutsos, Graph evolution: Densification and shrinking diameters, ACM Transactions on Knowledge Discovery from Data (TKDD) 1 (1) (2007) 2.

[33] M. Boguñá, R. Pastor-Satorras, A. Díaz-Guilera, A. Arenas, Models of social networks based on social distance attachment, Physical Review E 70 (5) (2004) 056122.

[34] T. Zhou, L. Lü, Y.-C. Zhang, Predicting missing links via local information, The European Physical Journal B 71 (4) (2009) 623–630.

[35] W. R. Knight, A computer method for calculating kendall’s tau with ungrouped data, Journal of the American Statistical Association 61 (314) (1966) 436–439.

[36] J.-H. Lin, Q. Guo, J.-G. Liu, T. Zhou, Locating influential nodes via dynamics-sensitive centrality, arXiv preprint arXiv:1504.06672.

[37] D. J. Watts, S. H. Strogatz, Collective dynamics of small-worldnetworks, Nature 393 (6684) (1998) 440–442.

[38] A. Garas, F. Schweitzer, S. Havlin, A k-shell decomposition method for weighted networks, New Journal of Physics 14 (8) (2012) 083030.

[39] Q. Li, T. Zhou, L. Lü, D. Chen, Identifying influential spreaders by weighted leaderrank, Physica A 404 (2014) 47–55.

[40] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, S. Havlin, Catastrophic cascade of failures in interdependent networks, Nature 464 (7291) (2010) 1025–1028.

[41] S. D. Reis, Y. Hu, A. Babino, J. S. Andrade Jr, S. Canals, M. Sigman, H. A. Makse, Avoiding catastrophic failure in correlated networks of networks, Nature Physics.

[42] L. K. Gallos, H. A. Makse, M. Sigman, A small world of weak ties provides optimal global integration of self-similar modules in functional brain networks, Proceedings of the National Academy of Sciences 109 (8) (2012) 2825–2830.

[43] S. Pei, L. Muchnik, J. S. Andrade Jr, Z. Zheng, H. A. Makse, Searching for superspreaders of information in real-world social media, Scientific Reports 4 (2014) 5547.

[44] S. Pei, H. A. Makse, Spreading dynamics in complex networks, Journal of Statistical Mechanics: Theory and Experiment 2013 (12) (2013) P12002.

[45] Y. Hu, S. Havlin, H. A. Makse, Conditions for viral influence spreading through multiplex correlated social networks, Physical Review X 4 (2) (2014) 021031.