Spin isospin responses in nuclei and their unified understanding with Landau-Migdal parameters

Tomotsugu Wakasa
for the RCNP E57, E59, E131, and E149 collaborations
Department of Physics, Kyushu University, Hakozaki 6-10-1, Fukuoka 812-8581, Japan
E-mail: wakasa@phys.kyushu-u.ac.jp

Abstract. Both the Gamow-Teller (GT) and the pionic response functions are investigated in the same framework of the continuum random phase approximation with the $\pi^+ + \rho^+$ model interaction. The Landau-Migdal (LM) parameters, $g'_{NN}$ and $g'_{N\Delta}$, are estimated by comparing these calculations with recent experimental data. The peak of the GT resonance and the pionic response functions below the quasielastic scattering (QES) peak constrain $g'_{NN}$, whereas the quenching of the GT total strength and the enhanced pionic strength around the QES peak provide information about $g'_{N\Delta}$. We obtain a common set of the LM parameters, $g'_{NN}=0.6–0.7$ and $g'_{N\Delta}=0.2–0.4$, which reproduce the peak and quenching of the GT strengths as well as the enhancement of the pionic modes. The $g'_{N\Delta}$ value is significantly smaller than $g'_{NN}$, which means that the universality ansatz, $g'_{NN} = g'_{N\Delta}$, should not be valid.

1. Introduction
Recent $(p, n)$ and $(n, p)$ experiments at intermediate energies ($T_p=200–500$ MeV) have presented reliable information on nuclear spin isospin response functions, especially for Gamow-Teller (GT) transition region and quasielastic scattering (QES) region [1]. In the former the total strength of the GT transition is quenched [2] from its sum rule value $3(N-Z)$ [3], while in the latter the pionic (isovector spin-longitudinal) response functions are enhanced [4–6]. A common key concept to understand these two contrastive phenomena is the Landau-Migdal (LM) parameters, $g'_{NN}$, $g'_{N\Delta}$, and $g'_{\Delta\Delta}$, which specify the particle-hole $(ph)$ and Delta-hole $(\Delta h)$ interactions.

In this paper we analyze the recent experimental data of both the GT strength distribution and the QES spectra in the same theoretical framework of the continuum random phase approximation (RPA) with the $\pi + \rho + g'$ model interaction, and estimate the LM parameters by reproducing the experimental data as well as possible. We find that a common set of the LM parameters can explain the quenching of the GT strength and the enhancement of the pionic modes simultaneously.

2. Theoretical framework
We write the $\beta^\pm$($GT^\pm$) transition operators with the nucleon $(N)$ and the $\Delta$-isobar $(\Delta)$ in the unit of $g_A$ as

$$O_{GT,a}^\pm = \frac{1}{\sqrt{2}} \sum_{k=1}^{A} \left\{ \tau_{k,\pm 1} \sigma_{k,a} + \frac{g'_{N\Delta}}{g_A} (T_{k,\pm 1} S_{k,a} - T_{k,\pm 1}^\dagger S_{k,a}^\dagger) \right\}, \quad (a = x, y, z), \quad (1)$$
with \( \tau_{\pm 1} = \pm \frac{1}{\sqrt{2}} (\tau_x \pm i \tau_y) \) and \( T_{\pm 1} = \pm \frac{1}{\sqrt{2}} (T_x \pm i T_y) \), where \( g_A \) and \( g_A^{N\Delta} \) are the coupling constants of the weak interaction for \( NN \) and \( N\Delta \) transitions, respectively, and \( \sigma \) and \( \tau \) are the Pauli spin and isospin matrix of the nucleon, and \( S \) and \( T \) are the spin and the isospin transition operators from \( N \) to \( \Delta \), respectively. Here we neglect the transitions from \( \Delta \) to \( \Delta \).

Similarly we write the isovector spin-longitudinal transition operators with momentum transfer \( q \) as

\[
O^L_{\lambda}(q) = \sum_{k=1}^A \left\{ \tau_{\lambda,\lambda} \sigma_k \cdot \hat{q} + \frac{f_{\pi NN}}{f_{\pi NN}} \left( T_{k,\lambda} s_k \cdot \hat{q} + (-1)^{\lambda} T_{k,-\lambda} s_k^1 \cdot \hat{q} \right) \right\} e^{iq \cdot r_k} ,
\]

where \( \lambda = 0, \pm 1 \), and \( f_{\pi NN} \) and \( f_{\pi N\Delta} \) are the \( \pi NN \) and \( \pi N\Delta \) coupling constants, respectively.

Correspondingly the GT response functions and the isovector spin-longitudinal response functions for the ground-state \( |\Psi_0\rangle \) with zero spin are respectively defined as

\[
R^\pm_{GT}(\omega) = \sum_{n\neq 0} \sum_{a} |\langle \Psi_n | O^\pm_{GT,a} | \Psi_0 \rangle|^2 \delta(\omega - (E_n - E_0)) ,
\]

\[
R^L_{\lambda}(q, \omega) = \sum_{n \neq 0} |\langle \Psi_n | O^L_{\lambda}(q) | \Psi_0 \rangle|^2 \delta(\omega - (E_n - E_0)) ,
\]

where \( |\Psi_n\rangle \) and \( E_n \) denote the \( n \)-th nuclear state and its energy.

The GT response functions are experimentally extracted from the \( \Delta J^p = 1^+ \) cross section, \( d^2\sigma_{1+}(q, \omega)/d\Omega d\omega \), determined by the multipole decomposition analysis (MDA) as \([7]\)

\[
\frac{d^2\sigma_{1+}(q, \omega)}{d\Omega d\omega} = \delta_{GT} F(q, \omega) R^\pm_{GT}(\omega) ,
\]

with the GT unit cross section \( \delta_{GT} \) and the \( (q, \omega) \) dependence factor \( F(q, \omega) \). The spin-longitudinal response functions are extracted in the eikonal approximation from the the spin-longitudinal polarized cross section \( ID_q \) as \([8]\)

\[
ID_q = K \frac{A_{\text{eff}}}{A} |E|^2 R_L ,
\]

with the kinematical factor \( K \), the effective nucleon number \( A_{\text{eff}} \), and the \( NN \) scattering amplitude \( E \) of \( \sigma \cdot q \) part.

In finite nuclei neither the momentum \( q \) nor the spin directions \( a \) are conserved, and the coordinate representation is preferred in practical calculation. Therefore we introduce the spin-isospin transition densities

\[
O^{NN}_{\lambda,a}(r) = \sum_{k=1}^A \tau_{\lambda,\lambda} \sigma_{k,a} \delta(r - r_k) , \quad O^{NN}_{\lambda,a}(r) = \sum_{k=1}^A T_{k,\lambda} s_{k,a} \delta(r - r_k) ,
\]

and the spin-isospin response functions \( R^{\alpha\beta}(r', r, \omega) \)

\[
R^{NN}_{\lambda,ba}(r', r, \omega) = \sum_n \langle \Psi_0 | O^{NN}_{\lambda,ba}(r') | \Psi_n \rangle \langle \Psi_n | O^{NN}_{\lambda,a}(r) | \Psi_0 \rangle \delta(\omega - (E_n - E_0)) ,
\]

\[
R^{NN}_{\lambda,ba}(r', r, \omega) = \sum_n \langle \Psi_0 | O^{NN}_{\lambda,ba}(r') | \Psi_n \rangle \langle \Psi_n | O^{NN}_{\lambda,a}(r) | \Psi_0 \rangle \delta(\omega - (E_n - E_0)) ,
\]

\[
R^{NN}_{\lambda,ba}(r', r, \omega) = \sum_n \langle \Psi_0 | O^{NN}_{\lambda,ba}(r') | \Psi_n \rangle \langle \Psi_n | O^{NN}_{\lambda,a}(r) | \Psi_0 \rangle \delta(\omega - (E_n - E_0)) ,
\]

and similarly \( R^{N\Delta}_{\lambda,ba}(r', r, \omega) \).
To take into account of nuclear collectivity, we calculate these $R^{\alpha \beta}(r', r, \omega)$ by the continuum RPA with the orthogonality condition [9], which properly treats the nuclear finite size. Then we convert them into those in the momentum representation $R^{\alpha \beta}(q', q, \omega)$. Now the GT$^\pm$ response functions are given by

$$R^{\pm}_{\text{GT}}(\omega) = \frac{1}{2} \sum_a \left\{ R^{\pm}_{\alpha \beta}(\omega) + 2 \frac{g^{N\Delta}_a}{g_A} R^{\pm}_{\alpha \beta}(\omega) + \left( \frac{g^{N\Delta}_a}{g_A} \right)^2 R^{\pm}_{\alpha \beta}(\omega) \right\}, \quad (11)$$

where $R^{\alpha \beta}(\omega) = R^{\alpha \beta}(q' = 0, q = 0, \omega)$, and the isovector spin-longitudinal response functions as

$$R^{\pm}_{\text{GT}}(q, \omega) = \sum_{ab} \left\{ R^{\pm}_{\alpha \beta}(q, \omega) + 2 \frac{f^{\pi N\Delta}_{\alpha \beta}}{f_{\pi NN}} R^{\pm}_{\alpha \beta}(q, \omega) + \frac{f^{\pi N\Delta}_{\alpha \beta}}{f_{\pi NN}} R^{\pm}_{\alpha \beta}(q, \omega) \right\} \hat{q}_a. \quad (12)$$

Here we take the quark model relation $f^{\pi N\Delta}_{\alpha \beta}/f_{\pi NN} = g^{N\Delta}_a/g_A = \sqrt{72/25} \approx 1.70$.

As the effective interaction for the RPA, we adopted the $\pi + \rho + g'$ model interaction

$$V^{\text{eff}}(q, \omega) = V_{\text{LM}} + V_{\pi}(q, \omega) + V_{\rho}(q, \omega), \quad (13)$$

where $V_{\pi}$ and $V_{\rho}$ are the one-pion and the one-rho-meson exchange interactions, respectively. The LM interaction $V_{\text{LM}}$ is written by the LM parameters, $g'_{NN}, g'_{N\Delta}$, and $g'_{\Delta\Delta}$ as

$$V_{\text{LM}} = \frac{f^{\pi N\Delta}_{\pi NN}}{m_\pi^2} g'_{NN}(\tau_1 \cdot \tau_2)(\sigma_1 \cdot \sigma_2)$$

$$+ \frac{f^{\pi NN}_{\pi N\Delta}}{m_\pi^2} g'_{N\Delta} \left\{ (\tau_1 \cdot T_2)(\sigma_1 \cdot S_2) + (\tau_1 \cdot T_2^\dagger)(\sigma_1 \cdot S_2^\dagger) + (1 \leftrightarrow 2) \right\}$$

$$+ \frac{f^{\pi N\Delta}_{\Delta\Delta}}{m_\pi^2} g'_{\Delta\Delta} \left\{ (T_1 \cdot T_2)(S_1 \cdot S_2) + (T_1 \cdot T_2^\dagger)(S_1 \cdot S_2^\dagger) \right\} + \text{H.c.} \quad (14)$$

We treat non-locality of mean fields by a local effective mass approximation [10] in the form of

$$m^*(r) = m_N - \frac{f_{\text{WS}(r)}}{f_{\text{WS}(0)}} (m_N - m^*(0)), \quad (15)$$

where $f_{\text{WS}(r)}$ is the Woods-Saxon radial form factor. In the following analysis we fix $g'_{\Delta\Delta}$ to be 0.5 [11] because the calculated results depend very weakly on this value.

3. Solution of GT quenching problem

The strength distribution of the GT transitions, namely, $R^{\pi N}(\omega)$, from $^{90}\text{Zr}$ to $^{90}\text{Nb}$ was determined from the MDA of the $(p, n)$ data [7, 12]. They cover not only the GT giant resonance (GTGR) region but also the continuum region up to excitation energy 50 MeV. Suzuki [13] and Bertsch, Cha, and Toki [14] determined $g'_{NN}$ by reproducing the position of GTGR by the RPA calculations only with the LM interaction. Both obtained similar values of $g'_{NN}$ =0.60. However, they used pure LM interactions only for the nucleons. The effects of $\Delta$ degree of freedom are as well as finite range effects of the interaction are not included.

In our calculations, the effects of the finite size and the finite range of the effective interaction, which comes from $V_{\pi}$ and $V_{\rho}$, are properly treated by the continuum RPA. In Fig. 1, we show only the $g'_{NN}$ dependence of the GTGR peak position since the $g'_{\Delta\Delta}$ and $m^*$ dependences are so weak. The curves correspond to the results employing $g'_{NN}$=0.0–0.9 with 0.3 steps, where $g'_{NN}$ and $m^*/m_N$ are set to be 0.3 and 0.7, respectively. The result with $g'_{NN}$=0.6 reproduces
the experimental data reasonably well, which is consistent with the previous analyses [13, 14]. The excess of the theoretical value should be redistributed by nuclear configuration mixing such as two-particle two-hole configurations [15]. This is clearly seen in the figure as significant experimental GT$^-$ strength in the high excitation region beyond GTGR. This is the quenching of one kind. It should be distinguished from the quenching due to Δh mixing, which we will discuss.

Recently, using the results of MDA applied to both the $^{90}$Zr$(p, n)$ and $^{90}$Zr$(n, p)$ data, Yako et al. [16] determined the GT quenching factor $Q$ defined as

$$Q = \frac{S_{GT}^- - S_{GT}^+}{3(N - Z)},$$

with

$$S_{GT}^\pm(\omega_{top}) = \int_{\omega_{top} ^\pm}^{\omega_{top} ^\mp} R_{GT}^\pm(\omega)d\omega,$$

where $\omega_{top}^\pm$ are the end energies of the integration of $R_{GT}^\pm$. The $\omega_{top}^-$ value is 57 MeV (50 MeV of $^{90}$Nb excitation) and the relevant $\omega_{top}^+$ is chosen by taking into account for the Coulomb energy shift. The result is $Q = 0.85 \pm 0.07$ [16].

Suzuki and Sakai [17] estimated the $g_{N\Delta}^T$ value from $Q$ by Fermi gas model with pure LM interactions, using an approximate treatment of the finite size effect, and obtained $g_{N\Delta}^T \approx 0.2$. Later Arima et al. [18] pointed out that cooperation of the nuclear finite size and the finite range $\pi$- and $\rho$-exchange interactions would increase the $g_{N\Delta}^T$ value to be $0.27 \pm 0.09$ for the same $Q$.

We evaluated the quenching factor $Q$ by the continuum RPA. The results are almost completely determined by the $g_{N\Delta}^T$ values. Thus we display the $g_{N\Delta}^T$ dependence in Fig. 2 with fixed $g_{NN}^T = 0.6$. With the experimental $Q$ and its uncertainty shown by the horizontal solid line and the horizontal band, respectively, the $g_{N\Delta}^T$ is estimated to be $0.37 \pm 0.16$, which coincides with the result of Arima et al. [18]. The shift from the Suzuki-Sakai’s line is mainly due to the finite range nature of the $NN \rightarrow N\Delta$ parts in $V_\pi$ and $V_\rho$. Note that even for the GT transition non-zero $q$ part of the interaction contributes because of nuclear finite size effect.

4. Search for precursor phenomena of pion condensation

Next we investigate the pionic modes in the QES region. Enhancement of $q \neq 0$ is expected [19] to be large at $q \simeq 1.7$ fm$^{-1}$. The relevant experimental data of the spin-longitudinal polarized cross section $ID_q$ exist for $^{12}$C and $^{40}$Ca at $T_p=346$ MeV [5, 6] and 494 MeV [4] taken at RCNP and LAMPF, respectively.

It has been shown [20] that nuclear distortion effects are so important that extraction of $R_L(q, \omega)$ as a precursor of the pion condensation is expected [19] to be large at $q \simeq 1.7$ fm$^{-1}$. The relevant experimental data of the spin-longitudinal polarized cross section $ID_q$ exist for $^{12}$C and $^{40}$Ca at $T_p=346$ MeV [5, 6] and 494 MeV [4] taken at RCNP and LAMPF, respectively.

We show only the $^{12}$C results since the the results for $^{40}$Ca are very similar. Figure 3 compares the experimental $ID_q$ for $^{12}$C taken at RCNP and LAMPF in the left and right panels, respectively, with the calculations. The top panels show the $g_{NN}^T$ dependence for $g_{NN}^T=0.0$–0.9 with 0.3 steps employing the fixed $g_{N\Delta}^T$ and $m'(0)/m_N$ values of 0.3 and 0.7, respectively. It is found that the calculations depend on $g_{NN}^T$ near and below the quasielastic peak. The experimental data could be reasonably reproduced with $g_{NN}^T=0.7$ whose uncertainty is estimated.
Figure 1. The $g'_{NN}$ dependence of GT strength distributions from $^{90}$Zr to $^{90}$Nb, where $g'_{NN}$ and $m^*(0)/m_N$ are set to 0.3 and 0.7, respectively. The filled circles are the experimental data taken from Ref. [7].

Figure 2. The GT quenching factor $Q$ as a function of $g'_{NN}$. The experimental result of $0.85 \pm 0.07$ [16] is shown by the horizontal solid line and band. The dashed curve is the theoretical prediction by Suzuki and Sakai [17].

to be about 0.1. This result is consistent with the value of $g'_{NN} = 0.6 \pm 0.1$ evaluated from the GTGR peak position.

The middle panels denote the $g'_{NN}$ dependence for $g'_{NN}=0.0$–0.9 with 0.3 steps employing the fixed $g'_{NN}$ and $m^*(0)/m_N$ of 0.7 and 0.7, respectively. The $g'_{NN}$ dependence is evidently seen around the quasielastic peak. The best choices of $g'_{NN}$ are about 0.4 and 0.2 for the RCNP and LAMPF data, respectively. The systematic uncertainties of the data are in the range of 6–8% [4–6], which corresponds to $\sim 0.1$ uncertainty of $g'_{NN}$. Thus the difference between the optimum $g'_{NN}$ values seems to be not significant. From these results, we could estimate $g'_{NN} = 0.3 \pm 0.1$, which is consistent with that from the quenching factor $Q$ of the GT strength.

The bottom panels display the $m^*$ dependence for $m^*(0)/m_N=1.0$–0.6 with 0.2 steps employing the fixed $g'_{NN}$ and $g'_{N\Delta}$ of 0.7 and 0.3, respectively. The peak position and width of QES are well reproduced by using $m^*(0)/m_N=0.7$, which is consistent with other theoretical estimations [22, 23].

5. Summary and conclusion

In summary, we reported the theoretical analyses of the latest $(p,n)$ and $(n,p)$ experimental data in the GT transition and the QES regions. In the former the GT strength is quenched, whereas in the latter the pionic modes are enhanced. To understand the contrastive problem, the quenching and the enhancement, in a unified way, we performed the continuum RPA calculations including the $\Delta$ degree of freedom. The $\pi+\rho+g'$ model was used for the effective interaction whose strength at $q = 0$ was determined by the LM parameters. We found a common set of the LM parameters, $g'_{NN}=0.6$–0.7 and $g'_{N\Delta}=0.2$–0.4, which explain the quenching and the enhancement simultaneously. The $g'_{N\Delta}$ should be significantly smaller than $g'_{NN}$ to explain the experimental data, and the universality ansatz, $g'_{NN} = g'_{N\Delta}$, does not hold.

Acknowledgments

We thank all the members of the RCNP-E57, E59, E131, and E149 collaborations, especially for Munetake Ichimura, Hideyuki Sakai, and Kentaro Yako. This work was supported in part
Figure 3. The spin-longitudinal polarized cross section $I_{Dq}$ for the $^{12}$C reaction at $T_p=346$ MeV [5, 6] (left panels) and $T_p=494$ MeV [4] (right panels). The top, middle, and bottom panels show the $g_{NN}', g_{N\Delta}',$ and $m^*(0)/m_N$ dependences of the calculations. The notations of the curves are the same as those in Figs. 1 and 2 except for $g_{NN}'=0.7$ in the middle and bottom panels.

by the Grands-in-Aid for Scientific Research Nos. 12640294, 12740151, and 14702005 of the Ministry of Education, Culture, Sports, Science, and Technology of Japan.

References
[1] For reviews, see, J. Rapaport and E. Sugarbaker, Annu. Rev. Nucl. Part. Sci. 44, 109 (1994); W. P. Alford and B. M. Spicer, Adv. Nucl. Phys. 24, 1 (1994).
[2] C. Gaarde, Nucl. Phys. A396, 127c (1983).
[3] C. Gaarde, J. S. Larsen, M. N. Harakeh, S. Y. van der Verf, M. Igarashi, and A. Müller-Arnke, Nucl. Phys. A334, 248 (1980).
[4] T. N. Taddeucci et al., Phys. Rev. Lett. 73, 3516 (1994).
[5] T. Wakasa et al., Phys. Rev. C 59, 3177 (1999).
[6] T. Wakasa et al., Phys. Rev. C 69, 054609 (2004).
[7] T. Wakasa et al., Phys. Rev. C 55, 2909 (1997).
[8] M. Ichimura and K. Kawahigashi, Phys. Rev. C 45, 1822 (1992).
[9] K. Nishida and M. Ichimura, Phys. Rev. C 51, 269 (1995).
[10] F. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).
[11] W. H. Dickhoff, A. Faessler, J. Meyer-ter-Vehn, and H. Mütter, Phys. Rev. C 23, 1154 (1981).
[12] T. Suzuki et al., J. Phys. Soc. Jpn. 73, 1611 (2004).
[13] T. Suzuki, Nucl. Phys. A379, 110 (1981).
[14] G. Bertsch, D. Cha, and H. Toki, Phys. Rev. C 24, 533 (1981).
[15] G. F. Bertsch and I. Hamamoto, Phys. Rev. C 26, 1323 (1982).
[16] K. Yako et al., Phys. Lett. B 615, 193 (2005); K. Yako and M. Sasano, private communications.
[17] T. Suzuki and H. Sasaki, Phys. Lett. B 455, 25 (1999).
[18] A. Arima, W. Bentz, T. Suzuki, and T. Suzuki, Phys. Lett. B 499, 104 (2001).
[19] W. M. Alberico, M. Ericson, and A. Molinari, Nucl. Phys. A379, 429 (1982).
[20] K. Kawahigashi, K. Nishida, A. Itabashi, and M. Ichimura, Phys. Rev. C 63, 044609 (2001).
[21] Y. Nakaoka, Phys. Rev. C 65, 064616 (2002).
[22] Nguyen van Giai and Pham van Thieu, Phys. Lett. 126B, 421 (1983).
[23] C. Mahaux and R. Sartor, Nucl. Phys. A481, 381 (1988).