MCMC-based Voigt Profile Fitting to a Mini-BAL System in the Quasar UM 675∗†

Dai Ishita1, Toru Misawa2, Daisuke Itoh1, Jane C. Charlton3, and Michael Eracleous3,4

1 Department of Physics, Faculty of Science, Shinshu University, 3-1-1 Asahi, Matsumoto, Nagano 390-8621, Japan
2 School of General Education, Shinshu University, 3-1-1 Asahi, Matsumoto, Nagano 390-8621, Japan; misawat@shinshu-u.ac.jp
3 Department of Astronomy & Astrophysics, Pennsylvania State University, 525 Davey Lab, University Park, PA 16802, USA
4 Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, PA 16802, USA

Received 2021 May 13; revised 2021 July 4; accepted 2021 July 13; published 2021 November 8

Abstract

We introduce a Bayesian approach coupled with a Markov Chain Monte Carlo method and the maximum-likelihood statistic for fitting the profiles of narrow absorption lines (NALs) in quasar spectra. This method also incorporates the overlap between different absorbers. We illustrate and test this method by fitting models to a “mini-broad” (mini-BAL) and six NAL profiles in four spectra of the quasar UM 675 taken over a rest-frame interval of 4.24 yr. Our fitting results are consistent with past results for the mini-BAL system in this quasar by Hamann et al. We also measure covering factors (Cf) for two narrow components in the CIV and NV mini-BALs and their overlap covering factor with the broad component. We find that Cf(NV) is always larger than Cf(CIV) for the broad component, while the opposite is true for the narrow components in the mini-BAL system. This could be explained if the broad and narrow components originated in gas at different radial distances, but it seems more likely to be due to being produced by gas at the same distance but with different gas densities (i.e., ionization states). The variability detected only in the broad absorption component in the mini-BAL system is probably due to gas motion, since both Cf(CIV) and Cf(NV) vary. We determine for the first time that multiple absorbing clouds (i.e., a broad and two narrow components) overlap along our line of sight. We conclude that the new method improves fitting results considerably compared to previous methods.

Unified Astronomy Thesaurus concepts: Quasar absorption line spectroscopy (1317); Quasars (1319); Astronomy data analysis (1858); Broad-absorption line quasar (183)

Supporting material: figure set

1. Introduction

Astronomical objects along our line of sight to distant quasars will produce absorption lines in the spectra of those quasars (hereafter quasar absorption lines, QALs). The absorbers include not only cosmologically intervening objects, like foreground galaxies and the intergalactic medium (IGM; hereafter intervening QALs), but also gas clouds that are physically associated with the quasars themselves, such as active galactic nucleus outflows (hereafter intrinsic QALs). These absorption lines have historically been studied by counting their numbers as a function of redshift (dN/dz) for various transitions to place constraints on their physical sizes and comoving number densities (e.g., Steidel 1990; Bechtold 1994; Lanzetta et al. 1995). With the advent of 8–10 m class telescopes and their high-dispersion spectrographs, such as the Keck telescope with the High Resolution Echelle Spectrometer (HIRES), the Very Large Telescope with the Ultraviolet and Visual Echelle Spectrograph, and the Subaru telescope with the High Dispersion Spectrograph (HDS), we are now able to measure physical parameters in addition to the absorption redshift (zabs), e.g., the column density (N), and the Doppler parameter (b) by fitting Voigt profiles (VPs) to the absorption lines.

The intrinsic QALs are generally blueshifted from the quasar emission redshift by up to ∼0.2–0.3c (Hamann et al. 2018), which suggests that they are accelerated away from the quasar by several possible mechanisms, including radiative pressure (Murray et al. 1995; Proga et al. 2000), magnetocentrifugal force (e.g., Everett 2005), and thermal pressure (e.g., Chelouche & Netzer 2005). They are usually classified into three categories according to their line widths: broad absorption lines (BALs) with total FWHM ≥ 2000 km s−1, narrow absorption lines (NALs) with FWHM ≤ 500 km s−1, and an intermediate subclass (mini-BALs). The difference in line width is often explained by the inclination angle of our line of sight relative to the direction of motion of the wind (e.g., Elvis 2000; Ganguly et al. 2001; Itoh et al. 2020) or the stage of quasar evolution (e.g., Farrah et al. 2007), although it may also depend on the emission line outflow properties and/or the hardness of the ionizing spectral energy distribution (e.g., Rankine et al. 2020). Intrinsic QALs may play an important role in providing energy and momentum feedback to the interstellar medium and circumgalactic medium of their host galaxies and the surrounding IGM and may play a role in regulating star formation activity in their host galaxies (e.g., Springel et al. 2005).

In past studies of QALs, several χ2-based VP fitting codes have been used, including VPFIT (Carswell et al. 1991), autoVP (Dave et al. 1997), and MINFIT (Churchill 1997; Churchill et al. 2003). Recently, Liang & Kravtsov (2017) employed a Bayesian approach for VP fitting with a Markov Chain Monte Carlo (MCMC) method (BayesVP) because the traditional χ2-based fitting codes have several weaknesses; for example, (a) they cannot place strong constraints on the parameters of undetected or saturated QALs, (b) they can provide incorrect results depending on the initial conditions if the parameters have a multimodal probability distribution, and (c) a grid search is computationally expensive when the number of fitting parameters is large. Using
$f_{\text{model}}(\lambda) = (C_{f(1)} - C_{\text{fov12}}) \times \exp(-\tau(\lambda; z_{\text{abs}(1)}, N_{(1)}, b_{(1)})) + C_{\text{fov12}} \times \exp(-\tau(\lambda; z_{\text{abs}(1)}, N_{(1)}, b_{(1)}) - \tau(\lambda; z_{\text{abs}(2)}, N_{(2)}, b_{(2)})) + (C_{f(2)} - C_{\text{fov12}}) \times \exp(-\tau(\lambda; z_{\text{abs}(2)}, N_{(2)}, b_{(2)})) + 1 - C_{f(1)} - C_{f(2)} + C_{\text{fov12}}$

Figure 1. Sketch of an absorption system with two components. If multiple clouds overlap along our line of sight to the background source, the flux from the source is absorbed more than once. The number of parameters needed to fit the spectrum is $\left[\Sigma_{i=1}^{n} 4n\right] + 4n$ and increases quickly with the number of components $n$.

BayesVP, Liang et al. (2018) successfully determined or constrained the physical parameters of absorption lines detected at low significance, as well as nondetected or saturated ones. Sameer et al. (2021) also introduced Bayesian methods for fitting photoionization models to intervening absorbers and were able to place stringent constraints on their physical parameters.

The Bayesian approach with MCMC methods has another powerful property: it allows us to set upper and/or lower limits in advance for each fitting parameter as needed. When we apply VP fits to intrinsic QALs, we always need a fourth parameter, the covering factor ($C_t$; the fraction of the flux from background sources that passes through a foreground absorber along our line of sight; Barlow 1995), in addition to the other three ($z_{\text{abs}}, \log N$, and $b$) because the size of the corresponding absorber in the vicinity of the flux source can be smaller than the source itself (i.e., the continuum source or broad emission line region, BELR). By its definition, $C_t$ should be between zero and 1.

One of the $\chi^2$-based VP fit codes, MINFIT, enables us to perform VP fits to intrinsic QALs with $C_t$ as a fourth fit parameter; however, it sometimes gives unphysical values such as $C_t < 0$ or $> 1$. Misawa et al. (2005) showed that the measurement of the $C_t$ value is very sensitive to continuum-level errors, especially for very weak absorption lines whose real $C_t$ values are close to 1. As long as the $C_t$ value is unphysical, the other fit parameters (i.e., $z_{\text{abs}}, \log N$, and $b$) have no physical meaning. In previous studies, this problem was handled by assuming $C_t = 1$ for those components and refitting to solve for their $z_{\text{abs}}, \log N$, and $b$ values, but this procedure does not adequately cover all physical possibilities.

In this study, we employ a Bayesian approach with an MCMC-based method for fitting VPs to intrinsic QALs, as Liang & Kravtsov (2017) did for intervening QALs. In Section 2, we introduce the MCMC method, describe the fitting procedure in detail, and compare it to the traditional $\chi^2$-based method. As a test case, we apply the method to the C IV and N V mini-BALs and intervening C IV NALs in the spectrum of the quasar UM 675 in Section 3 and present our fitting results in Section 4. We summarize our conclusions in Section 5. We use a cosmology with $H_0 = 69.6$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.29$, and $\Omega_{\Lambda} = 0.71$ throughout the paper.

2. Method: MCMC-based VP Fitting

We constrain the posterior distribution of the fit parameters, $p(\theta|\text{data})$, in the Bayesian approach,

$$p(\theta|\text{data}) \propto \mathcal{L}(\text{data}|\theta)p(\theta),$$  

(1)

where $\theta = \{z, \log N, b, C_t\}$ is the vector of the fit parameters, $p(\theta)$ is the prior distribution, and $\mathcal{L}(\text{data}|\theta)$ is the total likelihood.

The function $\mathcal{L}(\text{data}|\theta)$ is expressed as the product of the Gaussian likelihood for each pixel by

$$\mathcal{L}(\text{data}|\theta) = \prod_{i=1}^{n} l_i$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(f_{\text{obs}_i} - f_{\text{mod}_i})^2}{2\sigma_i^2}\right),$$  

(2)

where $l_i$ is the likelihood of the individual pixel, and $f_{\text{obs}_i}$, $f_{\text{mod}_i}$, and $\sigma_i$ are the observed flux, model flux, and uncertainty of the observed flux in the $i$th pixel of the spectrum.

2.1. Overlap Covering Factor

In addition to the physical parameters noted above, we also add the overlap covering factor ($C_{\text{fov}}$) that is the covering factor by multiple absorbers simultaneously along our line of sight, as illustrated in Figure 1. For example, if we detect two absorbing components 1 and 2 with covering factors of $C_{(1)} = 0.4$ and $C_{(2)} = 0.8$ in a single system, the maximum and minimum values of $C_{\text{fov}}$ are max($C_{\text{fov}}$) = $\max(C_{(1)}, C_{(2)}) = 0.4$ and $\min(C_{\text{fov}})$ = $\min(0, C_{(1)} + C_{(2)} - 1) = 0.2$, respectively. Thus, the permitted range of $C_{\text{fov}}$ depends on the $C_t$ of each component. To avoid any possible biases for the overlap covering factors, we assume a uniform distribution for the normalized covering factor,

$$C_{\text{fov}}^\text{norm} = \frac{C_{\text{fov}} - \min(C_{\text{fov}})}{\max(C_{\text{fov}}) - \min(C_{\text{fov}})},$$  

(3)

where $\min(C_{\text{fov}})$ and $\max(C_{\text{fov}})$ are determined ahead of time as described in Section 2.2. Using these boundary values, we also introduce the overlap covering factor ratio ($C_{\text{fov}}^\text{ratio}$), defined by

$$C_{\text{fov}}^\text{ratio} = \frac{C_{\text{fov}}}{\max(C_{\text{fov}})}.$$  

(4)

The traditional $\chi^2$-based MINFIT fits models to the observed spectra by

$$f_{\text{mod}}(\lambda) = \prod_{j=1}^{n} [C_{(j)}$$

$$\times \exp\{-\tau(\lambda; z_{\text{abs}(j)}, N_{(j)}, b_{(j)})\} + 1 - C_{(j)}],$$  

(5)

where $C_{(j)}$, $z_{\text{abs}(j)}$, $N_{(j)}$, and $b_{(j)}$ are the parameters of component $j$. In Equation (5), the overlap covering factor $C_{\text{fov}}$ is not considered; instead, the brightness distribution of the background source, as seen by all absorbers along the cylinder of sight, is always uniform and the same, regardless of whether the absorbers overlap. This assumption is incorrect except for intervening absorbers where
\( C_f = 1 \). Once we introduce \( C_{\text{fov}} \), the residual flux after being attenuated by two absorbing components, for example, would be written as

\[
f'_{\text{mod}} (\lambda) = (C_{f(1)} - C_{\text{fov}(12)}) \times \exp(-\tau(\lambda; z_{\text{abs}(1)}, N_{(1)}, b_{(1)})) + (C_{f(2)} - C_{\text{fov}(12)}) \times \exp(-\tau(\lambda; z_{\text{abs}(2)}, N_{(2)}, b_{(2)})) + C_{\text{fov}(12)} \times \exp(-\tau(\lambda; z_{\text{abs}(1)}, N_{(1)}, b_{(1)})) - \tau(\lambda; z_{\text{abs}(2)}, N_{(2)}, b_{(2)})) + 1 - C_{f(1)} - C_{f(2)} + C_{\text{fov}(12)},
\]

where \( C_{\text{fov}(12)} \) is the overlap covering factor between components 1 and 2. Since our code can accept any number of components, the second term returns the minimum number of necessary components for fitting the absorption profile. From these, we determine \( \min(C_{\text{fov}}) \) and \( \max(C_{\text{fov}}) \), used in Equation (3).

2. We start the MCMC sampling using the affine-invariant ensemble sampler proposed by Goodman & Weare (2010) in the model parameter space with a number of dimensions of \( \left[ \sum_{i=1}^{n} \binom{n}{i} \right] + 4n \). When the sampling points (i.e., walkers; the default number is 256 in our calculations) move from the position of the roughly estimated values, we demand they should fulfill the detailed balance.\(^7\)

3. Once we confirm that the autocorrelation time has converged (i.e., the MCMC algorithm has also converged), we continue sampling until the effective sample size becomes \( \geq 100,000 \).\(^8\)

\( C_{\text{fov}} \), except for the overlap covering factor \( C_{\text{fov}} \), using the \( \chi^2 \)-based VP fit code MINFIT. The code automatically returns the minimum number of necessary components for fitting the absorption profile. From these, we determine \( \min(C_{\text{fov}}) \) and \( \max(C_{\text{fov}}) \), used in Equation (3).

2.2. Fitting Procedure

Before using our MCMC-based VP fitting code (hereafter \texttt{mc2fit}), we need the prior distribution of the model parameters. Since no information is available in advance, we assume a uniform distribution between the appropriate lower and upper limits of each parameter as the prior. We then apply \texttt{mc2fit} to the absorption lines following the procedure below.

1. We make an initial estimate of the values of the parameters for each absorption component (i.e., \( z_{\text{abs}}, \log N, b, \) and \( C_i \)), except for the overlap covering factor \( C_{\text{fov}} \), using the \( \chi^2 \)-based VP fit code MINFIT. The code automatically returns the minimum number of necessary components for fitting the absorption profile. From these, we determine \( \min(C_{\text{fov}}) \) and \( \max(C_{\text{fov}}) \), used in Equation (3).

2. We start the MCMC sampling using the affine-invariant ensemble sampler proposed by Goodman & Weare (2010) in the model parameter space with a number of dimensions of \( \left[ \sum_{i=1}^{n} \binom{n}{i} \right] + 4n \). When the sampling points (i.e., walkers; the default number is 256 in our calculations) move from the position of the roughly estimated values, we demand they should fulfill the detailed balance.\(^7\)

3. Once we confirm that the autocorrelation time has converged (i.e., the MCMC algorithm has also converged), we continue sampling until the effective sample size becomes \( \geq 100,000 \).\(^8\)

4. We finally obtain a probability distribution and its mode (i.e., the best-fit value) for each parameter by projecting the total probability density distribution on each parameter axis.

2.3. Comparison of MCMC and \( \chi^2 \) Methods

We compare the fitting efficiencies of the MCMC method and the traditional \( \chi^2 \) method as follows. For the latter, we use MINFIT. First, we synthesize spectra with two \( \text{C IV} \) absorption components whose line widths and column densities are close to the broad and narrow components in the mini-BAL system that we will discuss in the next section, \( b = 200 \text{ km s}^{-1} \) and \( \log(N/cm^2) = 15 \) for the former and \( b = 20 \text{ km s}^{-1} \) and \( \log(N/cm^2) = 14 \) for the latter, but both components have the same absorption redshift and covering factor \( z_{\text{abs}} = 2.0 \) and \( C_i = 0.5 \). These parameters are kept fixed. But we do change the overlap conditions as \( C_{\text{fov,norm}} = 0, 0.25, 0.5, 0.75, \) and 1. A spectrum is synthesized from 4634 to 4661 Å with a pixel size of 0.03 Å pixel\(^{-1}\). As shown in Figure 2, the total absorption depth is deeper when \( C_{\text{fov,norm}} \) is smaller. Next, we convolve the spectrum with the appropriate line-spread function to simulate a spectral resolution of \( \lambda / \Delta \lambda = 36,000 \), which is a typical value for observational data as presented in Section 3. In fact, the convolution does not significantly affect the absorption profiles, since both the broad and the narrow components are much broader than the spectral resolution element (\( \Delta \nu \sim 8.3 \text{ km s}^{-1} \)). To examine how much the fitting accuracy would improve as the signal-to-noise ratio (S/N) increases, we synthesize spectra with different noise realizations with S/Ns from 10 to 100 pixel\(^{-1}\) in steps of 10 and reproduce the fit parameters repeatedly.

Once all of the spectra are synthesized, we fit them with VP components using \texttt{mc2fit} and MINFIT. We repeat this analysis 10 times by changing the seed value of the random number generator (used for adding noise). Finally, we calculate the average and standard deviation of the best-fit (i.e., mode) values for each line parameter.

Figure 3 shows a comparison of fit parameters (\( \log N, b, \) and \( C_i \)) that are returned by \texttt{mc2fit} and MINFIT as a function of...
In this demonstration, we adopt $C_{\text{fov,norm}} = 0.5$ (i.e., $C_{\text{fov}} = 0.25$ and $C_{\text{fov, ratio}} = 0.5$). The code mc2fit always provides the correct values within $1\sigma$ uncertainty, while MINFIT underestimates $\log N$ and $b$ for the broad component even in high-quality spectrum with $S/N = 100$ pixel$^{-1}$. The discrepancy is probably a result of not allowing for an overlap covering factor in MINFIT. Thus, for cases where multiple absorbers overlap along the line of sight, the MCMC method is required to derive the correct line parameters.

We also summarize the fit results of the overlap covering factor ($C_{\text{fov}}$) and the overlap covering factor ratio ($C_{\text{fov, ratio}}$) by mc2fit in Figure 4 for some selected values: $C_{\text{fov}} = 0.0, 0.125, 0.25, 0.375$, and $0.5$ and $C_{\text{fov, ratio}} = 0.0, 0.25, 0.5, 0.75$, and $1.0$. An example of a corner plot is also shown in Figure 5. The code recovers $C_{\text{fov, ratio}}$ generally well, while $C_{\text{fov}}$ is always underestimated even in spectra with $S/N \sim 100$ pixel$^{-1}$ if the correct $C_{\text{fov}}$ value is 0.5. Therefore, we will use $C_{\text{fov, ratio}}$ to examine whether absorbers overlap, since it is always determined reliably (at least the correct value is within $1\sigma$ uncertainty) if the data quality is high enough, $S/N \geq 30$ pixel$^{-1}$.

### 3. Application to Observed Spectra

To test mc2fit further, we apply it to a mini-BAL system whose line profile is easy to separate into multiple absorption components (i.e., an ideal target to test the code) that are detected in an optically bright quasar. There also exist several NAL systems in the same quasar spectrum.

#### 3.1. Mini-BAL Quasar UM 675

In the spectrum of the radio-loud luminous quasar UM 675 at $\zeta_{\text{em}} = 2.147$, Sargent et al. (1988) detected three C IV NALs

---

**Figure 3.** Comparison of fit parameters for the synthesized spectra in Figure 2 (logN, $b$, and $C_f$ from top to bottom) using mc2fit (red filled triangles) and MINFIT (blue open triangles) with $1\sigma$ uncertainties (vertical solid lines) as a function of $S/N$. The normalized covering factor is fixed to $C_{\text{fov,norm}} = 0.5$. To facilitate comparison, the points are intentionally shifted horizontally. Horizontal dotted lines denote correct values. The left and right panels are the results for the broad ($b$) and narrow ($n$) components, respectively.

---

9 Here $\min(C_{\text{fov}}) = 0$ and $\max(C_{\text{fov}}) = 0.5$, since $C_{\text{fov, 1}} = C_{\text{fov, 2}} = 0.5$. By substituting these into Equations (3) and (4), we obtain $C_{\text{fov, norm}} = C_{\text{fov, ratio}} = C_{\text{fov}}/0.5$. Therefore, $C_{\text{fov}} = 0.25$ and $C_{\text{fov, ratio}} = 0.5$ when $C_{\text{fov, norm}} = 0.5$.

10 Here we emphasize that $C_f$ is reliable even if we use MINFIT. Therefore, the past results of classifying absorption lines into intrinsic and intervening ones based on values of $C_f$ determined by $\chi^2$ methods are still reliable.

---

11 We classify UM 675 as a radio-loud quasar with a radio-loudness parameter of $\cal{R} = 351$ using an optical magnitude ($V = 17.4$) and a radio flux ($f_\nu = 122$ mJy at 4.85 GHz) from Griffith et al. (1994), while it was classified as a radio-quiet quasar in Hamann et al. (1995, 1997b).
at \(z_{\text{abs}} \approx 1.7666, 1.9288, \) and 2.0083. Hamann et al. (1995) also discovered CIV, \( \text{N V}, \) and \( \text{Ly}\alpha \) mini-BALs with FWHM \( \approx 500 \text{ km s}^{-1} \) with a high metallicity \((Z > 2 \ Z_\odot)\) at \(z_{\text{abs}} \approx 2.134, \) corresponding to an offset velocity of \(v_{\text{off}} \approx 1500 \text{ km s}^{-1}\) from the quasar emission redshift. The mini-BAL system is optically thin because the Lyman-continuum edge is absent. The system also has a wide variety of absorption lines with a full range of ionization from NeVIII (ionization potential \(\text{IP} = 239 \text{ eV}\); Beaver et al. 1991; Hamann et al. 1995), which suggests that the gas density and/or radial distance from the flux source spans a factor of \(\approx 100\) or \(\approx 10,\) respectively.

Using intermediate-resolution spectra \((R \approx 1500),\) Hamann et al. (1995) discovered time variability in the absorption strength of the CIV and N V mini-BALs in \(\Delta t_{\text{rest}} \sim 2.9 \text{ yr}\) in the quasar rest frame and placed a lower limit on the electron density of \(n_e \gtrsim 4000 \text{ cm}^{-3}\) and an upper limit on the radial distance from the source of \(r \lesssim 200 \text{ pc} ,\) assuming the absorber is in ionization equilibrium. There is clear evidence for partial coverage of the background flux source \((C_f = 0.63^{+0.2}_{-0.1} \text{ for N V and } 0.34^{+0.1}_{-0.0} \text{ for C IV at } v_{\text{off}} \approx 1500 \text{ km s}^{-1};\) Hamann et al. 1997b) based on high-resolution spectra taken with Keck/HIRES, which means the mini-BAL system is physically associated with the quasar (i.e., intrinsic QAL). Hamann et al. (1997b) also detected three narrow components with FWHM \(\approx 30–50 \text{ km s}^{-1}\) inside the smooth mini-BAL profiles of CIV and N V, as shown in Figure 2 of their paper.

To date, high-resolution spectra \((R \approx 35,000–45,000)\) of the quasar have been obtained three times with Keck/HIRES, in 1994 September (hereafter epoch E1), 1998 September (E2), and 2008 January (E4), and once with Subaru/HDS in 2005 August (E3), as summarized in Table 1. The spectrum in epoch E1 is shown here in Figure 6 as an example. Using the high-resolution spectra in epochs E1 and E3, Misawa et al. (2014) confirmed again that the absorption strengths (i.e., equivalent width) of Ly\(\alpha,\) CIV, and N V show an obvious time variability with \(>5\sigma\) significance but no discernible changes in the narrow components inside it. In this study, we use all of the above that are available from the Keck Observatory Archive (KO\(A\))\(^{12}\) or obtained by us with Subaru/HDS, although the N V mini-BAL is not covered by the spectrum in epoch E2. We

---

\(^{12}\) http://www2.keck.hawaii.edu/koa/public/koa.php
reduced the data ourselves using a data reduction pipeline for Keck/HIRES data (Mauna Kea Echelle Extraction, MAKEE).\(^\text{13}\)

After normalizing the spectra, we searched for absorption lines whose depth is greater than five times the corresponding noise level. We identified six C\textsc{i}v NALs at \(z_{\text{abs}} = 2.0569, 2.0083, 1.9288, 1.7663, 1.6769, \) and \(1.6387\) between the Ly\(\alpha\) and C\textsc{i}v emission lines (hereafter systems B, C, D, E, F, and G) in addition to the mini-BAL system at \(z_{\text{abs}} \sim 2.134\) (hereafter system A), as summarized in Table 2. Among these, systems C, D, and E were already reported in Sargent et al. (1988).

\(^{13}\) http://www.astro.caltech.edu/~tb/makee/index.html

### 3.2. MCMC Fit to a Mini-BAL and Six NALs

We use \texttt{mc2fit} to fit the C\textsc{i}v and N\textsc{v} mini-BALs and six C\textsc{i}v NALs in the spectra of UM 675 in the four epochs. As a prior, we assume a uniform distribution between the lower and upper limits of each parameter: \([1.469, 2.147]\) (i.e., between the redshift of the C\textsc{i}v absorption line corresponding to the Ly\(\alpha\) emission line and the quasar emission redshift) for \(z_{\text{abs}}\), \([0, 16]\) for \(\log(N/\text{cm}^{-2})\), \([0, 2000]\) (i.e., the boundary between BAL and mini-BAL) for \(b\) [\(\text{km s}^{-1}\)], \([0, 1]\) for \(C_f\), and \([0, 1]\) for \(C_{\text{fov, norm}}\) where parentheses \((a, b)\) and square brackets \([a, b]\) denote an interval from a to b, but only the latter includes the

![Figure 5. Example of posterior probability distribution function of covering factors (\(C_f\); correct values are 0.5) of both broad and narrow components, as well as their overlap covering factor (\(C_{\text{fov}}\); 0.25) and overlap covering factor ratio (\(C_{\text{fov, ratio}}\); 0.5).](image-url)
Figure 6. Normalized spectrum of UM 675 after rebinning to 0.1 Å per pixel. The C IV, N V, and Si IV doublets and other detected single lines are marked. The positions of the quasar emission lines of Lyα, N V, Si IV, and C IV are marked with downward arrows. The regions blueward of the Lyα absorption in system A and redward of the C IV emission line are not shown. The lower solid line shows the 1σ error spectrum.

Table 1
Log of Monitoring Observations

| Epoch | Obs. Date     | Instrument   | λ Coverage (Å) | Exp. Time (sec.) | λ/Δλ | S/N (N V) | S/N (C IV) | Source of Data |
|-------|---------------|--------------|----------------|------------------|------|-----------|------------|----------------|
| E1    | 1994 Sep 24–25| Keck+HIRES   | 3750–6100      | 18,000           | 34,000 | 47        | 9          | A              |
| E2    | 1998 Sep 22   | Keck+HIRES   | 4150–6520      | 3600             | 47,800 | 8         | 16         | B              |
| E3    | 2005 Aug 19–20| Subaru+HDS   | 3600–5980      | 23,000           | 36,000 | 48        | 28         | C              |
| E4    | 2008 Jan 15   | Keck+HIRES   | 3200–5990      | 840              | 47,750 | 12        | 9          | B              |

Notes.

a Spectral resolution.

b S/N on the red side of N V (∼3950 Å) and C IV (∼4900 Å) mini-BALs.

c (A) Provided by Fred Hamann, (B) from KOA, and (C) taken by us (proposal ID: S05A-041).

d N V is not covered by the observed spectrum.
(narrow 1; at longer wavelength), and n2 (narrow 2; at shorter wavelength), respectively, as shown in Figure 9, where the best-fit model is superposed for comparison.

We find significant variability at a $>3\sigma$ confidence level in the $C_t$ of component b in both the C IV and N V mini-BALs, as well as in the b of component b only in the N V mini-BAL (see Table 3 and Figure 10). In contrast, all parameters of components n1 and n2 are stable over all epochs at the $3\sigma$ level.

We also study the overlap covering factor ratio ($C_{fov\_ratio}$) between components b and n1/n2. We placed the strongest constraints on $C_{fov\_ratio}$ in the E3 spectrum. As shown in Figure 11, components b ($C_{fov\_ratio} = 0.33 \pm 0.01$) and n1 ($C_{fov\_ratio} = 0.52 \pm 0.05$) in the C IV mini-BAL are found to overlap by at least 50% of the projected size of the smaller component (i.e., component b), since the $3\sigma$ lower limit on $C_{fov\_ratio}$ is $\sim 0.54$. Thus, for the first time, we find with high confidence that multiple absorbing clouds along our line of sight overlap with each other. However, there need not be overlap between b and n1 for N V, since we found a $3\sigma$ upper limit on $C_{fov\_ratio}$ of $\sim 0.69$ for the N V mini-BAL (see Figure 11). That is, in the case of Figure 1, the two C IV absorbers should overlap along our line of sight by at least 50% of their size, while the two N V absorbers do not necessarily overlap. Here we should emphasize that the C IV and N V absorbers do not necessarily have an identical overlap covering factor ratio because they may represent different layers in the absorber (i.e., they have different covering factors).

### 3.4. System B

We detect only a weak C IV NAL with $EW_{rest} \sim 0.03$ Å in the system. The covering factor is $C_t = 1$, which suggests an intervening absorber. A weak absorption profile on the left side of the blue component is not part of C IV $\lambda 1548$, since we do not find any absorption features in the C IV $\lambda 1551$ window. Therefore, we ignore this region when carrying out the fit. The best-fit model is shown in Figure 7.

### 3.5. System C

There are five components in C IV and two in Si IV in this system, some of which are completely separated from each other by unabsorbed spectral regions. However, we regard these components as a single NAL system, since their overall velocity separation is $<200$ km s$^{-1}$ from each other, following Misawa et al. (2007a). The best-fit model is presented in Figure 7. The fit to component 1 of the Si IV NAL implies partial coverage ($C_t \sim 0.69$) with a $3\sigma$ confidence level. However, this result should be regarded with caution because (i) there exist narrow spikes in both the blue and red members of the Si IV doublet that could be due to data defects, and (ii) the best-fit model appears to overestimate the depth of the red component, which would underestimate $C_t$. The $C_{fov\_ratio}$ between components 4 and 5 in the C IV NAL is consistent with unity, which is reasonable, since both components have full coverage.

### 3.6. System D

We detect C IV and Si IV NALs in this system. We fit models with two components to the C IV NAL because of its asymmetric profile, while a single-component model is acceptable for the Si IV NAL (see Figure 7). All components are consistent with

---

**Table 2**

Absorption Line Systems

| System | $z_{abs}$ | $v_{abs}$ (km s$^{-1}$) | Class $^b$ | Ions $^i$ | Variability $^d$ |
|--------|------------|------------------------|------------|----------|------------------|
| A      | $\sim 2.134$ | $\sim 1240$ | Mini-BAL | C IV, N V, (Ly$\alpha$) | Y |
| B      | 2.0569     | 8710       | NAL      | C IV    | N |
| C      | 2.0083$^f$ | 13,510     | NAL      | C IV, Si IV, (C II) | N |
| D      | 1.9288     | 21,520     | NAL      | C IV    | N |
| E      | 1.7663     | 38,470     | NAL      | C IV    | N |
| F      | 1.6769     | 48,120     | NAL      | C IV    | N |
| G      | 1.6387     | 52,310     | NAL      | C IV    | N |

Notes.

$^a$ Offset velocity from the quasar emission redshift ($z_{em} = 2.147$). Positive values denote blueshifted from the quasar.

$^b$ Absorption class: mini-BAL or NAL.

$^c$ Detected ions in each absorption system. Ions in parentheses represent transitions that are not fitted because they are not doublets.

$^d$ Absorption line is variable (Y) or not (N).

$^e$ Multiple components are separated from each other by unabsorbed spectral regions, but we regard them as a single system, since their velocity separation is $<200$ km s$^{-1}$, following Misawa et al. (2007a).

$^f$ Redshift of the middle of five components.

---

14 We also confirm that no NAL systems show variability in their equivalent width with a $<3\sigma$ level during the monitoring period, which also suggests that these are intervening NALs.

15 In this study, we do not discuss the variability of $z_{abs}$ because it could be affected by small systematic linear/nonlinear offsets of the spectra in each epoch from an uncertainty in wavelength calibration.
full coverage at the 1σ confidence level. It is reasonable that $C_{\text{fov.ratio}} \sim 1$ between the two components of the C IV NAL, since both have full coverage.

### 3.7. System E

We detect only a C IV NAL that consists of two components. Both components are consistent with full coverage at the 3σ level. There could exist a third, very weak component at $\Delta v \sim 60 \text{ km s}^{-1}$ in Figure 7, but we ignore it, since it is not detected with $>5\sigma$ confidence (i.e., it does not satisfy our detection criterion).

### 3.8. System F

A shallow and broad C IV NAL with a Doppler parameter of $b \sim 23 \text{ km s}^{-1}$ is detected in the system, as shown in Figure 7.

The system is probably an intervening absorber, since $C_t \sim 1$ at the 3σ confidence level.

### 3.9. System G

Only a C IV NAL is detected in this system. Although we fit a model with a single component to this NAL, there could be two components, since the C IV $\lambda 1548$ profile is slightly asymmetric. For multicomponent fitting, we would need a higher-S/N spectrum. The covering factor is consistent with full coverage. The best-fit models are presented in Figure 7.

### 4. Discussion

#### 4.1. Possible Geometry

Using the MCMC-based method for fitting the mini-BALs in the spectrum of UM 675, we find for the first time that multiple
absorbing clouds (i.e., components b and n1) overlap along our line of sight to the background flux source, which could not have been inferred with the χ²-based method. Thus, the projected distribution of the integrated total column density from multiple absorbing clouds along the line of sight is much more complex than previously thought. By synthesizing artificial spectra, Sabra & Hamann (2005) studied how the column density that we derive from the observed spectrum depends on inhomogeneous partial coverage. They found that homogeneous and inhomogeneous models have little difference (apparent optical depths within 50%) as long as the column density distribution does not contain spatially narrow peaks (i.e., large enhancements in the optical depth over small coverage areas).

The fits to the C IV and N V mini-BALs in epoch E3 provide the strongest constraints on partial coverage among the four epochs because of the high S/N in that spectrum: \( C_I(b) = 0.33 - 0.01, C_I(n_1) = 0.52 \pm 0.04, \) and \( C_I(n_2) = 0.38 \pm 0.04 \) for components b, n1, and n2 of the C IV mini-BAL and \( C_I(b) = 0.49 - 0.02, C_I(n_1) = 0.16 \pm 0.02, \) and \( C_I(n_2) = 0.83 \pm 0.06 \) for those of the N V mini-BAL, respectively. All components (except for component n2 of the N V mini-BAL) show partial coverage with a confidence >3σ. The width of component b (FWHM ∼ 200 km s⁻¹) is much larger than those of the narrow components (FWHM ∼ 10–30 km s⁻¹), which suggests that only component b has a large velocity dispersion, possibly due to mechanisms such as internal turbulence or continuous acceleration, but that its transverse scale is comparable to those of the narrow components.

We also noticed that \( C_I(N \ V) \) is always larger than \( C_I(C \ IV) \) in all epochs for component b. This trend (i.e., higher ions tend to have larger covering factors) is consistent with other reports in the literature (e.g., Pettijean & Srianand 1999; Srianand & Petitjean 2000; Misawa et al. 2007a; Muzahid et al. 2016), suggesting that the size of the N V absorbers is larger than that of the C IV absorbers and/or the effective size of the flux source behind the N V absorber is smaller than that behind the C IV absorber (see Figure 1). Interestingly, the opposite is true for components n1 and n2; i.e., \( C_I(N \ V) \) tends to be smaller than \( C_I(C \ IV) \), as shown in Figure 12. This discrepancy between the broad and narrow components suggests that the size (or flux amplitude) of the background flux source is different between N V and C IV absorbers. This could happen for the narrow components if the N V absorption line is more diluted than the C IV absorption line by the flux from the BELR whose scale (∼0.1 pc; Hamann et al. 1997b and references therein) is ∼2 orders of magnitude larger than the size of the continuum source, as we will discuss later. Because both the C IV and N V mini-BALs are located on the blue wing of the corresponding broad emission lines (see Figure 1 of Hamann et al. 1995), their depths can be diluted by the broad emission line flux (e.g., Arav et al. 1999).

Considering contributions from both the continuum source and the BELR, we can calculate the total covering factor \( C_I \) by

\[
C_I = \frac{C_I(\text{cont}) + W \times C_I(\text{BELR})}{1 + W},
\]

where \( C_I(\text{cont}) \) and \( C_I(\text{BELR}) \) are covering factors of the continuum source and BELR, and \( W = F_{\text{BELR}} / F_c \) is defined as
Table 4
Fitting Parameters of NAL Systems in Epoch E1

| Ion  | Comp. | z_{abs} | log N^a (cm^{-2}) | b_{rest} (km s^{-1}) | C_l^a | C_{ion_ratio}^{a,b} | EW_{rest}^c (Å) |
|------|-------|---------|-------------------|------------------|-------|-------------------|----------------|
| C IV | 1     | 2.0569  | 13.00^{+0.15}_{-0.04} | 13.2^{+1.0}_{-0.9} | 1.00^{+0.00}_{-0.30} & 0.03 ± 0.01 |

System B

C IV 1 2.0063 13.39^{+0.05}_{-0.02} 7.5^{+0.3}_{-0.1} 1.00^{+0.00}_{-0.22} 0.06 ± 0.00
C IV 2 2.0071 13.09^{+0.11}_{-0.03} 6.8^{+0.5}_{-1.5} 0.99^{+0.01}_{-0.46} 0.04 ± 0.00
C IV 3 2.0083 14.00^{+0.03}_{-0.01} 9.0^{+0.2}_{-0.6} 0.96^{+0.02}_{-0.05} 0.13 ± 0.01
C IV 4 2.0099 13.80^{+0.04}_{-0.02} 23.6^{+5.5}_{-1.7} 1.00^{+0.01}_{-0.22} 0.17 ± 0.01
Si IV 5 2.0100 13.32^{+0.12}_{-0.06} 2.7^{+0.6}_{-1.7} 1.00^{+0.00}_{-0.36} 1.00^{+0.00}_{-0.22} 0.05 ± 0.01
Si IV 1 2.0083 13.40^{+0.18}_{-0.19} 5.9^{+0.5}_{-1.4} 0.69^{+0.04}_{-0.06} 0.04 ± 0.01
Si IV 2 2.0099 13.04^{+0.11}_{-0.12} 6.3^{+0.7}_{-1.1} 0.74^{+0.13}_{-0.18} 0.04 ± 0.01

System C

C IV 1 1.9287 13.74^{+0.03}_{-0.02} 14.7^{+0.5}_{-1.6} 1.00^{+0.00}_{-0.14} 0.14 ± 0.01
C IV 2 1.9290 12.62^{+0.32}_{-0.18} (0.00–12.14) 0.80^{+0.20}_{-0.65} 1.00^{+0.00}_{-0.31} 0.02 ± 0.01
Si IV 1 1.9287 12.52^{+0.31}_{-0.12} 11.3^{+2.1}_{-2.2} 1.00^{+0.00}_{-0.67} 0.02 ± 0.01

System D

C IV 1 1.7663 13.47^{+0.08}_{-0.01} 15.5^{+0.7}_{-1.8} 1.00^{+0.00}_{-0.39} 0.09 ± 0.01
C IV 2 1.7668 13.10^{+0.24}_{-0.07} 12.7^{+1.5}_{-3.3} 0.69^{+0.28}_{-0.42} 1.00^{+0.00}_{-0.64} 0.04 ± 0.01

System F

C IV 1 1.6769 14.00^{+0.17}_{-0.21} 22.5^{+2.5}_{-2.3} 0.28^{+0.08}_{-0.04} 0.07 ± 0.01

System G

C IV 1 1.6387 13.40^{+0.09}_{-0.10} 9.8^{+0.8}_{-0.7} 1.00^{+0.00}_{-0.38} 0.08 ± 0.01

Notes.

^a 1σ upper/lower error bars in superscript/subscript. The 3σ uncertainties are in parentheses.
^b Overlap covering factor ratio between components 4 and 5 in system C or components 1 and 2 in systems D and E.
^c Rest-frame equivalent width of blue component.
^d Total rest-frame equivalent widths of components 4 and 5.
^e The best-fit model overestimates the depth of the red component, which could underestimate C_l.
^f This is the 3σ distribution range, since the 1σ distribution is multimodal.

as the ratio of the fluxes from the BELR (F_{BELR}) and the continuum source (F_c; Ganguly et al. 1999). Indeed, the flux ratio (W) around the N\alpha mini-BAL due to the contribution from both Ly\alpha and N\alpha broad emission lines is obviously larger than that around the CIV mini-BAL (see Figure 1 of Hamann et al. 1995). Thus, the opposite trend of the ratio of C_l(CIV) and C_l(N\alpha) in components b and n1/n2 can be due to a difference in the contribution from the background flux source, as discussed in Wu et al. (2010).

We speculate that component b does not absorb light from the BELR (it may be at a smaller radial distance than the BELR, r ≤ 0.1 pc, or embedded within it), while components n1 and n2 also absorb light from the BELR (possibly because they are located at a larger radial distance); we refer to this scenario as model A. The very high ionization state of the gas in component b, showing Ne\alpha and O\alpha, also supports its small radial distance from the continuum source. In other words, components b and n1/n2 are not cospatial but happen to be located along the same cylinder of sight with a similar offset velocity, perhaps due to line locking. Just as in the UM 675 spectrum, narrow kinematic components are sometimes detected near the centers of mini-BALs in other quasars (e.g., HE 1341–1020, Q1157+014, and Q2343+125), and they usually show no variability, while broader components vary (Hamann et al. 1997a; Misawa et al. 2014). Possible origins of these narrow components include (i) dense clumpy clouds with large volume density (i.e., less affected by fluctuation of incident flux) embedded in the mini-BAL flow (Misawa et al. 2014) and (ii) gas clouds in the quasar host galaxy or foreground galaxies that are physically unrelated to the quasar (Hamann et al. 1997a). In addition to these, we propose a third possible origin: (iii) dense clumpy clouds with large volume density whose radial distance is a few orders of magnitude larger than the mini-BAL flow (e.g., Itoh et al. 2020) but with
their radial velocity coinciding. The variety of absorption lines with a wide range of ionization potential (Beaver et al. 1991; Hamann et al. 1995) also supports this idea.

Although the above scenario is possible, it is unlikely that the broad and narrow absorbers at very different distances have similar velocities relative to the quasar by chance. In that spirit, we present an alternative model, model B, in which the broad and narrow absorbers coexist at the same radial distance. In this model, most of the absorber has a relatively low density and high ionization state and large velocity gradient (corresponding to component b). Within the large gas parcel, there are dense clumps with a low ionization state and small velocity gradient (corresponding to components n1 and n2). The entire absorber is moving in both the transverse and radial directions as it orbits around the flux sources. The size of component b is comparable to the size of the continuum source so that it can partially cover both the continuum source and the BELR. Within the large gas parcel, there are dense clumps with a low ionization state and small velocity gradient (corresponding to components n1 and n2). The entire absorber is moving in both the transverse and radial directions as it orbits around the flux sources. The size of component b is comparable to the size of the continuum source so that it can partially cover both the continuum source and the BELR. Within the large gas parcel, there are dense clumps with a low ionization state and small velocity gradient (corresponding to components n1 and n2). The entire absorber is moving in both the transverse and radial directions as it orbits around the flux sources. The size of component b is comparable to the size of the continuum source so that it can partially cover both the continuum source and the BELR.

In either model, the geometry of the mini-BAL system in UM 675 requires two distinct flux sources (i.e., the continuum source and BELR), while mc2fit (as well as other χ²-based codes) assumes a single background flux source. To verify this geometry, we need to introduce covering factors (C_i) and their overlap covering factor ratios (C_{flux,ratio}) as free parameters for both background flux sources individually, introduce a model for the brightness distribution of the background flux source(s) and then adopt specific geometrical arrangements for the absorbers, or both. However, the current version of mc2fit does not have these features. These are improvements that we will introduce in a future version of the code.

### 4.2. Origin of Time Variability

In addition to the variability in absorption strength (Hamann et al. 1995; Misawa et al. 2014), we also find variability in the...
Figure 10. Variation of fit parameters with time (logN, b, and Cf, from top to bottom) of component b in C IV (left) and N V (right) mini-BALs. The horizontal axis gives the relative time from the first observing epoch (i.e., epoch E1) in the quasar rest frame. Black dots show the mode of the fit parameter posterior distributions, while black and red error bars denote 1σ and 3σ uncertainties. Only Cf shows >3σ variability in the CIV mini-BAL, while Cf and b vary in the NV mini-BAL.

Figure 11. Probability distribution histogram of Cf$_{\text{fov}}$(b and n1) between components b and n1 in epoch E3 for C IV (top) and N V (bottom). Vertical dotted lines are modes of the Cf$_{\text{fov}}$(b and n1). Horizontal black and red error bars denote 1σ and 3σ uncertainties.

Figure 12. Difference between Cf (N V) and Cf (C IV) of components b (blue) and n1 (red) as a function of relative time from the first epoch in the quasar rest frame. Black dots are the modes of the difference. Black and blue/red error bars denote 1σ and 3σ uncertainties of the difference. The dots with error bars are intentionally shifted by ±0.05 yr from the observed epoch in the horizontal direction for display purposes.
line parameters of the mini-BAL system; component b shows $>3\sigma$ variability in $C_I$ of the C IV mini-BAL and in both $C_I$ and $b$ of the N V mini-BAL in $\sim$4.24 yr in the quasar’s rest frame. Since the time separation between epochs E3 and E4 is the smallest ($\Delta t_{\text{rest}} \sim 0.77$ yr) among the four observing epochs, we concentrate on the variability pattern on that short time interval, since it can place the most stringent constraints on the physical conditions of the absorbers.

There are two major scenarios for the origin of time variability that have often been discussed in the literature: (1) a change of the ionization state of the gas clouds (e.g., Misawa et al. 2007b; Hamann et al. 2011; Filiz et al. 2013; Horiuchi et al. 2016) and (2) the motion of the absorbing clouds across our line of sight to the background flux source (e.g., Gibson et al. 2008; Hamann et al. 2008; Vivek et al. 2016; Krongold et al. 2017). Neither situation is applicable to intervening absorbers, unless they have very large gas densities and/or sharp edges (Narayanan et al. 2004).

The column densities of component b in the C IV and N V mini-BALs are almost constant while their $C_I$ values vary (see Figure 10), which supports the gas motion scenario. To examine this scenario, we first need to estimate the sizes of the background flux sources (i.e., the continuum source and BELR). Following Misawa et al. (2005), we take five times the gravitational radius as the continuum source size using a black hole mass of $\log(M_\text{BH}/M_\odot) = 9.52$ (Horiuchi et al. 2016). For the size of the portion of the BELR that emits the C IV line, we use Equation (1) of Lira et al. (2018) with a Galactic extinction of $A_V = 0.44$ and assume a power-law index of $\alpha = 0.61$ (Lusso et al. 2015). We obtain $R_{\text{con}} \sim 2 \times 10^{-3}$ and $R_{\text{BELR}} \sim 0.2$ pc as the sizes of the continuum source and BELR, respectively.

Since component b of the C IV mini-BAL shows a variability in the covering factor from $C_I = 0.33^{+0.01}_{-0.01}$ to $0.42^{+0.02}_{-0.02}$ in $\Delta t_{\text{rest}} = 0.77$ yr, we can place a lower limit on the crossing velocity $v_{\text{cross}} \geq d_s \times \Delta C_I/\Delta t_{\text{rest}}$, where $d_s$ is the size of the continuum source $d_{\text{cont}}$ (or the size of the absorber $d_{\text{abs}}$) if $d_{\text{abs}}$ is larger (or smaller) than $d_s$. We adopt $\Delta C_I = 0.09$ as the variability amplitude of $C_I$ and take $\Delta t_{\text{rest}}$ to be the time interval between observing epochs in the quasar’s rest frame. Since we want to place a lower limit on $v_{\text{cross}}$, we should use a smaller size between $d_{\text{abs}}$ and $d_{\text{cont}}$ (i.e., $d_s = \min(d_{\text{abs}}, d_{\text{cont}})$). However, these sizes should be almost the same, since the absorber shows partial coverage, which leads us to use $d_s \sim d_{\text{cont}}$, whose value we have been able to estimate. Substituting the values above into $d_{\text{cont}} \times \Delta C_I/\Delta t_{\text{rest}}$ we obtain $v_{\text{cross}} \geq 366$ km s$^{-1}$. If the crossing velocity is equal to the Keplerian orbital velocity around the central black hole $^{18}$ we can also place a weak constraint on the absorber’s radial distance from the flux source as $\leq 106$ pc for component b. We also note that in the context of model A, this radial distance is probably smaller than the size of the BELR ($< R_{\text{BELR}} \sim 0.2$ pc), as discussed above.

In comparison, components n1 and n2 do not show significant (i.e., $>3\sigma$) variability during our monitoring campaign in $\Delta t_{\text{rest}} = 4.24$ yr. Therefore, we cannot place any constraints on either crossing velocity or radial distance.

$^{17}$ Here we assume that a homogeneous absorber moves with a constant crossing velocity.

$^{18}$ The enclosed mass is much larger than the black hole mass if the absorber’s radial distance is as large as the size of the host galaxy.

5. Summary

In this study, we introduce a Bayesian approach combined with an MCMC method for fitting a mini-BAL and six NAL systems in the spectrum of the radio-loud quasar UM 675 taken with Keck/HIRES and Subaru/HDS in four epochs. Our methodology is implemented in a new code, mc2fit, which we test here using synthetic and real spectra. Our main results are as follows.

1. Using mc2fit, we restrict the range of the covering factor ($C_I$) from zero to 1 as is physically possible and determine the fit parameters more accurately than the traditional $\chi^2$-based methods. Our fitting results for synthetic spectra are fairly consistent with those obtained by the $\chi^2$-based method described in Misawa et al. (2007a), but the latter method slightly underestimates $\log N$ and $b$ compared to the correct values.

2. In addition to the original fit parameters (i.e., $z_{\text{abs}}$, $\log N$, $b$, and $C_I$), we introduce the overlap covering factor ratio by multiple absorbers along our line of sight ($C_{\text{fov\_ratio}}$). Because of the large number of fit parameters any observed spectrum to which we apply mc2fit must have a very high S/N. Otherwise, the code tends to overestimate $C_{\text{fov\_ratio}}$ especially when the correct value is small.

3. Among seven absorption systems (systems A–G) in the quasar UM 675, only the C IV and N V mini-BALs at $z_{\text{abs}} \sim 2.1341$ (system A) show clear simultaneous variability in their $C_I$ and $b$ at a $>3\sigma$ confidence level. We also determine for the first time that multiple absorbing clouds (i.e., the broad and narrow components) overlap each other along our line of sight.

4. For component b of the mini-BAL system (system A), $C_I$(N V) is always larger than $C_I$(C IV) at all epochs, while the opposite trend holds for components n1 and n2. These trends suggest that the broad and narrow components originate in gas at different radial distances or that they are produced by gas at the same distance but with different gas densities.

5. The column densities of component b in the C IV and N V mini-BALs are almost constant, while their $C_I$ values vary, which supports the gas motion scenario for their variability. If this is the case, the broad component must be at $r < 106$ pc (and could be closer than the size of $R_{\text{BELR}} \sim 0.2$ pc) with a rotational velocity of $v_{\text{rot}} > 366$ km s$^{-1}$ assuming Keplerian motion (we cannot place any meaningful constraints for the narrow components).

The MCMC-based approach improves the fitting results significantly compared to the traditional $\chi^2$-based methods, especially for high-quality spectra with $S/N \geq 30$ pixel$^{-1}$. It also provides the overlap covering factor ratio ($C_{\text{fov\_ratio}}$) if multiple components overlap in the spectrum, which conveys important information on the absorber’s geometry. More detailed information can be obtained if we introduce the fit parameters for the continuum source and BELR separately. To capitalize on the MCMC-based technique, we plan to monitor several mini-BAL systems whose absorption profiles are easy to deblend using high-quality spectra like that in UM 675.
We are honored and grateful for the opportunity of observing the universe from Maunakea, which has cultural, historical, and natural significance in Hawaii. We would like to thank the anonymous referee for very useful comments and suggestions. We also would like to thank Fred Hamann for providing us with his Keck/HIRES data, Christopher Churchill for providing us with the MINFIT software package, and Takashi Horiuchi for valuable comments. This work was supported by JSPS KAKENHI grant No. 21H01126.

ORCID iDs
Toru Misawa @ https://orcid.org/0000-0002-5464-9943
Jane C. Charlton @ https://orcid.org/0000-0003-4877-9116
Michael Eracleous @ https://orcid.org/0000-0002-3719-940X

References
Arav, N., Becker, R. H., Laurent-Muehleisen, S. A., et al. 1999, ApJ, 524, 566
Beaver, E. A., Burbidge, E. M., Cohen, R. D., et al. 1991, ApJL, 377, L1
Bechtold, J. 1994, ApJS, 91, 1
Bowler, R. A. A., Hewett, P. C., Allen, J. T., et al. 2014, MNRAS, 445, 359
Carswell, R. F., Lanzetta, K. M., Parnell, H. C., et al. 1991, ApJ, 371, 36
Chelouche, D., & Netzer, H. 2005, ApJ, 625, 95
Churchill, C. W. 1997, PhD Thesis, Univ. California
Churchill, C., Vogt, S. S., & Charlton, J. C. 2003, AJ, 125, 98
Davé, R., Hernquist, L., Weinberg, D. H., et al. 1997, ApJ, 477, 21
Elvis, M. 2000, ApJ, 545, 63
Everett, J. E. 2005, ApJ, 631, 689
Farrah, D., Lacy, M., Priddey, R., et al. 2007, ApJL, 662, L59
Filiz, Ak, N., Brandt, W. N., Hall, P. B., et al. 2013, ApJ, 777, 168
Ganguly, R., Bond, N. A., Charlton, J. C., et al. 2001, ApJ, 549, 133
Ganguly, R., Eracleous, M., Charlton, J. C., et al. 1999, AJ, 117, 2594
Gibson, R. R., Brandt, W. N., Schneider, D. P., et al. 2008, ApJ, 675, 985
Goodman, J., & Weare, J. 2010, CMAPS, 5, 65
Griffith, M. R., Wright, A. E., Burke, B. F., et al. 1994, ApJS, 90, 179
Hall, P. B., Anderson, S. F., Strauss, M. A., et al. 2002, ApJ, 141, 267
Hamann, F., Barlow, T. A., Beaver, E. A., et al. 1995, ApJ, 443, 606
Hamann, F., Barlow, T. A., & Junkkarinen, V. 1997a, ApJ, 478, 87
Hamann, F., Barlow, T. A., Junkkarinen, V., et al. 1997b, ApJ, 478, 80
Hamann, F., Chartas, G., Reeves, J., et al. 2018, MNRAS, 476, 943
Hamann, F., Kanekar, N., Prochaska, J. X., et al. 2011, MNRAS, 410, 1957
Hamann, F., Kaplan, K. F., Rodríguez Hidalgo, P., et al. 2008, MNRAS, 391, L39
Horiuchi, T., Misawa, T., Morokuma, T., et al. 2016, PASJ, 68, 48
Itoh, D., Misawa, T., Horiuchi, T., et al. 2020, MNRAS, 499, 3094
Krongold, Y., Binette, L., Bohlin, R., et al. 2017, MNRAS, 468, 3607
Lanzetta, K. M., Wolfe, A. M., & Turnshek, D. A. 1995, ApJ, 440, 435
Liang, C., & Kravtsov, A. 2017, BayesVP: Full Bayesian Voigt profile fitting, Astrophysics Source Code Library, ascl:1711.004
Liang, C. J., Kravtsov, A. V., & Agertz, O. 2018, MNRAS, 479, 1822
Lira, P., Kaspi, S., Netzer, H., et al. 2018, ApJ, 865, 56
Lusso, E., Worseck, G., Hennawi, J. F., et al. 2015, MNRAS, 449, 4204
Misawa, T., Charlton, J. C., Eracleous, M., et al. 2007a, ApJS, 171, 1
Misawa, T., Charlton, J. C., & Eracleous, M. 2014, ApJ, 792, 77
Misawa, T., Eracleous, M., Charlton, J. C., & Kashikawa, N. 2007b, ApJ, 660, 152
Misawa, T., Eracleous, M., Charlton, J. C., & Tajitsu, A. 2005, ApJ, 629, 115
Murray, N., Chiang, J., Grossman, S. A., & Voit, G. M. 1995, ApJ, 451, 498
Muzahid, S., Srianand, R., Charlton, J., et al. 2018, MNRAS, 475, 2665
Narayanan, D., Hamann, F., Barlow, T., et al. 2004, ApJ, 601, 715
Petitjean, P., & Srianand, R. 1999, A&A, 345, 73
Proga, D., Stone, J. M., & Kallman, T. R. 2000, ApJ, 543, 686
Rankine, A. L., Hewett, P. C., Banerji, M., et al. 2020, MNRAS, 492, 4553
Sabra, B. M., & Hamann, F. 2005, arXiv:astro-ph/0509421
Sameer, Charlton, J. C., Norris, J. M., et al. 2021, MNRAS, 501, 2112
Sargent, W. L. W., Boksenberg, A., & Steidel, C. C. 1988, ApJS, 68, 539
Springel, V., Di Matteo, T., & Hernquist, L. 2005, ApJL, 620, L79
Srianand, R., & Petitjean, P. 2000, A&A, 357, 414
Steidel, C. C. 1990, ApJS, 72, 1
Vivek, M., Srianand, R., & Gupta, N. 2016, MNRAS, 455, 136
Wu, J., Charlton, J. C., Misawa, T., et al. 2010, ApJ, 722, 997