Simulation of flow-induced vibration of a cylinder in an expansion tube

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Abstract
In this study, a two-dimensional simulation is carried out to understand the motion of a small cylinder in an expansion tube. The simulation is performed on the FLUENT platform by using the Overset function. The collision between the cylinder and the tube wall is considered as a positive collision of two rigid bodies, and there is no energy loss. Two key parameters, dimensionless gravity ($Mg^*$) and Reynolds number ($Re$), are focused. $Mg^*$ and $Re$ are taken into account for 4.9-79.4 and 1-300, respectively. Two types of inflow are considered: the regular inflow or superimposed sinusoidal periodic fluctuating incoming flow. For regular inflow, three patterns of motion are found in the phase diagrams of ($Mg^*$, $Re$), i.e., flow outside, vibrate and hover in the tube. The phase diagrams of ($Mg^*$, $Re$) can be divided into five regimes. Under the parameter of $Re = 300$ and $Mg^* = 39.25$, superimposed sinusoidal periodic fluctuating incoming flow is tested. The cylinder can vibrate violently in this way instead of hovering in the tube for regular inflow. If the three-dimensional motion of the sphere is not considered, the two-dimensional cylinder can be regarded as the simplification of the sphere. Our research may be helpful to understand this ancient problem of mobility.

Key words: cylinder, expansion tube, vibration, collision, simulation.

1. Introduction

Flow induced vibration (FIV) is a classical fluid-structure coupling problem. Because of its complexity and application background, it has attracted wide attention, such as structural fatigue failure (Xu et al., 2020), vibration noise (Hattori et al., 2017), and enhanced heat dissipation (Lu et al., 2010, Kumar et al., 2020). FIV usually simplifies the vibration system into a mass spring system. In addition, only the rigid body system of gravity can also induce vibration, such as the vibration of a single ball pendulum system, small

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ball in a dilution tube, etc. A bottle in an expansion pipe is a simple system. However, it is difficult to understand the movement of the bottle due to the wake vortex shedding, variable blocking rate, vibration and collision, etc. In this work, a small cylinder in a dilution tube is simulated in two dimensions, as shown in Fig. 1.

![Figure 1. A small cylinder in an expansion tube.](image)

The flow-induced vibration is caused by the interaction between the cylinder motion and the separation vortex shedding (Williamson & Govardhan, 2004). Therefore, wake vortex shedding is quite important to understand the movement of the cylinder. Reynolds number (Re) affects the morphology of wake vortex shedding. The critical Reynolds number for cylinder’s initial vortex shedding in a free domain is about 47 (Lashgari et al., 2012). The initial Re of the wake vortex shedding is affected by various factors such as the vibration of a cylinder (Kou et al., 2017), rotation (Kang et al., 1999; Rao et al., 2015), and blocking rate (Sahin and Owens, 2004), etc. The critical Reynolds number can be dramatically reduced (Kou et al., 2017) if the cylinder vibrates. The cylinder's blockage rate also affects the critical Re. Sahin and Owens (2004) consistently examined the influence of the blocking rate on critical Reynolds number (Re_c). For our system, the critical Reynolds number of the cylinder is affected not only by the vibrations, but also by the blockage rate gradient.

In this system, the collision can not be ignored. For the simulation of collision, the empirical formula of lubrication force when two wall collide closely (Izard et al., 2014) or reflective boundary (Ardekani & Rangel, 2008) are often used to make the walls separate quickly without causing non-physical penetration. Fluid-solid coupling often uses mesh reconstruction method (Xiong et al., 2019), immersed boundary
method (Izard et al., 2014), or Overset method (Chandar et al., 2018). The mesh reconstruction method is not suitable for dealing with topological changes or large motion problems. The immersed boundary method or Overset method may be used to solve the collision problem better. Compared with the immersed boundary method, the Overset method can maintain the quality of the mesh near the wall. The Overset method employs two series of grids, one series of background grids and the other series of component grids. The background grid does not move, but the component grid moves and interpolates to the background grid. Another thing to note is that the collision between particle and the walls is also potentially important to other fields of fluid mechanics, such as the enhancement of heat transfer due to convection of fluid towards and away from a surface, and the development of improved multiphase models including the wall effects.

The report about the vibration of the two-dimensional cylinder or three-dimensional sphere in the expansion nozzle is incomplete. Masmoudi et al. (1998) theoretically and experimentally studied the final stage of sedimentation of a spherical particle moving along the axis of a conical vessel containing a viscous incompressible fluid. They found that the particle settling velocity varies like \( d^{5/2} \), where \( d \) is the gap. They also claimed that the results of lubrication theory and experiment were very consistent. Lecoq et al. (2007) studied the creeping motion of a sphere along the axis of a closed axisymmetric container. They used the numerical technique to solve the Stokes equation by using the classical Sampson expansion and experimental technique to get vertical displacement by using laser interferometry with an accuracy of 50\( \text{nm} \). Both studies were set at a very low \( Re \) and do not consider the incoming flow at the bottom.

In this work, we use the Overset method for the first time to simulate a cylinder in an expansion tube. In the present work, simulation of the moving cylinder in a special expansion tube is explored. The trajectory, amplitude and motion forms are studied and classified.

2. Mathematical formulation

2.1 Model equations and specific problem

In this study, the Arbitrary Lagrangian–Eulerian (ALE) scheme is applied to simulate the movement of
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The boundaries due to the motion of the cylinder. The hydrodynamics of incompressible fluids can be described by the following set of equations (Zhao, 2020):

\[ \nabla \cdot (u - u_c) = 0, \]  
\[ \rho_f \frac{\partial u}{\partial t} + \rho_f \left[ (u - u_c) \cdot \nabla \right] u = -\nabla p + \mu \Delta u. \]  

Where \( u \) is the two-dimensional velocity vector. \( t \) is the time. \( u_c \) is the velocity vector of the moving grid. \( p \) is pressure. \( \mu \) is the viscosity. \( \rho_f \) is the density of fluid. The Reynolds number is defined as \( Re = \rho_f DU_c/\mu \), where \( D \) is the cylinder’s diameter. The gravity of fluid is not considered in this equation.

The generalized solid movement for two degrees of freedom can be written as follows:

\[ F^*(t) = m \frac{d^2 x}{dt^2} + (0, mg), \]  

where \( x = (x, y) \), \( x, y \) are the horizontal and transversal displacement, respectively, \( F^*(t) = (F^*_x(t), F^*_y(t)) \), \( F^*_x(t), F^*_y(t) \) are the horizontal and transversal forces acting on the cylinder, respectively, \( m \) represents the mass of the cylinder, expressed as:

\[ m = \rho_s \pi \left( \frac{D}{2} \right)^2, \]  

where \( \rho_s \) is density of the cylinder.

The force of fluid on the cylinder consists of two parts: differential pressure force and viscous force, which can be written as follows:

\[ F^* = \oint (-p^* I + \tau) \cdot dS, \]  

where \( p^* \) and \( \tau \) are the pressure and shear stress acting at cylinder, respectively. \( I \) is second-order two-dimensional unit tensor. \( S \) is the surface normal vector of the cylinder. The pressure \( p^* \) is different to \( p \) for the existence of fluid’s gravity. In fact, the following conversion relationship exists between these two variables:

\[ p^* = p - \rho_f g y. \]
The force of the fluid on the cylinder also can be written as follow:

\[ \mathbf{F} = \oint \left[ -p \mathbf{I} + \mathbf{\tau} \right] \cdot d\mathbf{S} + \left( 0, \rho_f g \pi D^2 / 4 \right). \] (7)

The force of the fluid on the cylinder can be written as follow if not consider gravity of the fluid:

\[ \mathbf{F} = \oint \left[ -p \mathbf{I} + \mathbf{\tau} \right] \cdot d\mathbf{S}. \] (8)

The force coefficient \( C(Re, u_c / \bar{U}) = \left( C_x(Re, u_c / \bar{U}), C_y(Re, u_c / \bar{U}) \right) \), can be written as follow:

\[ C(Re, u_c / \bar{U}) = 8F / \rho_f \pi D^2. \] (9)

The force coefficient is a function of Reynolds number and the moving speed of the cylinder.

Combined Eqns (3), (4), (7), (8) and (9), we have

\[ \frac{1}{\bar{U}^2 / D} \left( 1 - \frac{\rho_f}{\rho_s} \right) \left( 0, g \right) - \frac{2 \rho_f C(Re, u_c / \bar{U})}{\rho_s \pi} = \frac{D}{\bar{U}^2} \frac{d^2 x}{dt^2}. \] (10)

Let

\[ g^* = \frac{g}{\bar{U}^2 / D}, \] (11-1)

\[ M = \frac{\rho_s}{\rho_f} - 1, \] (11-2)

\[ x' = x / D, \] (11-3)

\[ t' = \frac{t}{D / \bar{U}}. \] (11-4)

We have

\[ \left( 0, Mg^* \right) - (M+1) \frac{d^2 x'}{dt'^2} = \frac{2C(Re, u_c / \bar{U})}{\pi}. \] (12)

If the cylinder finally stop somewhere, Eqn. (12) can be simplified to

\[ C(Re) = \frac{\pi}{2} \left( 0, Mg^* \right). \] (13)

This means that there is such a balanced relationship between the force coefficient and dimensionless gravity. We consider Eqn. (12), if the cylinder always vibrates. There exits two dimensionless parameters \( Mg^* \) and \( u_c / \bar{U} \). Actually, the speed of the solid body depends on \( Re \) and \( Mg^* \).
2.2 Simulation method description

In order to couple the structural displacement, loose fluid structure coupling is used to simulate nonlinear FSI problem. Firstly, the flow field is obtained according to the instantaneous geometry and control equation of fluid. Then, the hydrodynamic load is assigned to the structure motion Eqn. (3). Next, based on the initial value problem of the first order ordinary differential equation, the displacement of structure is solved in the same physical time step by the so-called state space method. The pressure velocity coupling algorithm is adopted to solve the mass and momentum equations to get the result of velocity and pressure. In each time step, when the residual error converges, the iteration stops. By the iteration of the above-mentioned time step we can get the load of the flow field on the cylinder. Then, the velocity of the cylinder is updated, and the displacement equations of the cylinder is solved by the fourth-order Runge-Kutta method in the same step, so as to obtain the position and velocity of the cylinder.

The numerical solution to the governing equations is obtained via a control volume method in which partial differential equations are converted to a set of discrete algebraic equations with conservative property. The convection term in the momentum equations is discretized by the second order upwind scheme. The diffusion term in the momentum equation is discretized by the second order central difference scheme. Then update the grid to prepare for the next time-step. Through the above description, we get a new combination of flow field calculation and the grid.

Moving grids are handled using the Overset method. There are two sets of grids, as shown in Fig. 2. One is the constant background grid during the calculation process, as shown in Fig. 2(b) and red region in Fig. 2(a). Another is the component mesh surrounding the cylinder, as shown in Fig. 2(c) and green region in Fig. 2(a). This set of mesh also does not deform and only moves with the cylinder. The moving component mesh is interpolated with the background grid. We select the grid at two moments, as shown in Fig. 3. The cylinder may collide with the outer wall surface. The dry coefficient of the collision recovery between the cylinder and outer wall is set as zero. For a frictionless collision, the tangential force is zero, and the same normal force is applied to the cylinder in opposite directions, as shown in Fig. 4. The reflected circular
motion speed can be calculated as follows:

\[ \text{motion speed} = \frac{\text{distance traveled}}{\text{time taken}} \]

Figure 2. The grid of component and background.

Figure 3. The grid at two moments.

Figure 4. Reflection model.
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\[ u_{at} = \frac{2u_x (x_1 - x_0)(y_1 - y_0) + u_x (x_1 - x_0)^2 - u_x (y_1 - y_0)^2}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \]

\[ u_{ot} = \frac{2u_x (x_1 - x_0)(y_1 - y_0) - u_x (x_1 - x_0)^2 + u_x (y_1 - y_0)^2}{(x_1 - x_0)^2 + (y_1 - y_0)^2} \]

(14)

The calculated spatial domain and geometrical dimensions are shown in Fig. 2(a). At the initial moment, the cylinder is placed at the coordinates of the center point \((0, 0.5D)\). The outer wall surface of the cylinder is set as the boundary of the non-slip wall. The outlet pressure is set as zero. The width of the entrance is set as \(H_2 = 4D\) and the length of the exit is set to \(L_2 = 5D\) in y-direction. The angle \(\alpha\) satisfies \(\tan(\alpha) = 3.3333\), that is, \(\alpha\) equals to 73.3\(^\circ\). The width of the entrance is set as \(H_1 = D\). The inlet length is set as \(L_1 = 4.5D\). The inlet velocity is set as parabolic distribution:

\[ u_y (x) = 1.5U \left( 1 - \frac{y^2}{H_1^2} \right) \]

(15)

Or the inlet velocity is set as an incoming stream with a periodic time change;

\[ u_y (x) = 1.5 \left[ 1 + \beta \sin \left( 2\pi f_i t \right) \right] U \left( 1 - \frac{y^2}{H_1^2} \right) \]

(16)

where \(\beta\) is the amplitude of the incoming wave, and \(f_i\) is the frequency of the incoming wave.

We can use reduced velocity to describe the frequency of the change of flow velocity:

\[ U_{red} = \frac{U}{f_i D} \]

(17)

When the average velocity of incoming flow and the diameter of the cylinder are constant, the reduced velocity is inversely proportional to the fluctuation frequency of incoming flow velocity.

2.3 Numerical Validation (Vortex-induced vibration)

There is less literature to simulate the flow induced vibration of a cylinder in an expansion tube. We consider a uniform flow past a vibrating cylinder. The movement of the cylinder is controlled by the following equation:

\[ m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F_i(t), \]

(18)
where $x_i$ is the lateral displacement, $t$ is time, $F_i(t)$ is the transversal force acting on the cylinder, $m$, $c$ and $k$ represent the mass of the cylinder, damping and spring constants respectively. As the displacement and time are normalized by $D$ and $D/U_\infty$ respectively, the structural motion equation can be written as follows:

$$\frac{d^2y^*}{dt^2} + \frac{4\pi \xi}{U_{red}} \frac{dy^*}{dt} + \frac{4\pi^2}{U_{red}^2} y^* = \frac{2C_i}{\pi m},$$

(19)

where $y^*$ and $t^*$ are the corresponding dimensionless transversal displacement and time. $U_{red}$ is the so-called reduced velocity which measures the root of the ratio of inertial force to spring force of the cylinder. It is defined by $U_{red} = U_\infty / (f_n D)$, where $f_n = 1 / 2\pi \sqrt{k/m}$ is the natural frequency of the cylinder. $m^* = 4\pi \rho D^2$ is the mass ratio of solid to fluid in the same volume, $C_i(t) = 2F_i(t)/(\rho U_\infty^2 D)$ and $\xi = c / 2\sqrt{k/m}$ are the corresponding lift coefficient and structural damping coefficient, respectively. The Reynolds number ($Re$) is set as 150. The reduced velocity ($U_{red}$) is from 3 to 12. The structural damping ratio ($\xi$) is set to zero. The Mass ratio ($M^*$) is set to 2.0.

![Figure 5. The amplitude curve of $A_y^*$ and $U_{red}$.](image)

The curve of the dimensionless maximum amplitude $A_y^*$ with $U_{red}$ is shown in Fig. 5. The maximum value obtained by us is about 0.578$D$, which is close to the simulation result of Xiong et al.
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(2019), Ahn & Kallinderis (2006) and Borazjani & Sotiropoulos (2009). The phase diagrams $A_y^* \sim C_l$ for different reduced velocity ($U_{red}$) are shown in Fig. 6. Although there are some slight burrs at high $U_{red}$. The phase of $A_y^*$ and $C_l$ changes from a positive phase at low $U_{red}$ (such as $U_{red} = 3.0$) to an opposite phase at high $U_{red}$ (such as $U_{red} = 8.0$). We can still capture this feature in our simulation.

![Figure 6. The phase diagram of $A_y^* \sim C_l$ for vortex-induced vibration.](image)

3. Results and discussions

3.1 Regular incoming flow

A regular incoming flow is considered firstly. Two key parameters: dimensionless gravity ($Mg^*$) and Reynolds number ($Re$) are discussed. $Mg^*$ and $Re$ are considered to be from 4.9 to 79.4 and 1 to 300, respectively. Fig. 7 shows the flow map for main motion form in this parameter space. Three kinds of motion patterns are found in the ($Mg^*$, $Re$) phase diagram. According to the movement of the cylinder, the phase diagrams can be divided into five areas. We will discuss this in detail below.
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The contrast relationship between thrust and cylinder gravity at different Reynolds numbers influences the flow pattern of the cylinder. The thrust force ($F_y^*$) acting on the cylinder is negatively related to $Re$. At low $Re$, the thrust force may be larger than the gravity of the cylinder. For example, when $Re = 1$ and $Mg^* = 4.9$, the cylinder may move outward when the dimensionless time $t^*$ is about 10, as shown in Fig.8(b). The component grid and background grid are also displayed.

With increase of $Re$ or decrease of dimensionless gravity ($Mg^*$), the thrust may be equal to the gravity of the cylinder. At $Re = 1$, $Mg^* = 79.4$, $Re = 3$, $Mg^* = 79.4$ and $Re = 3$, $Mg^* = 4.9$, the cylinder finally be fixed at a point in the tube. For a fixed $Mg^*$ at 78.4, as $Re$ increases, the cylinder may move towards the bottom of the expansion tube, as shown in Fig. 9(a) and Fig. 9(b), due to the decrease of thrust. For a fixed $Re$, with the decrease of $Mg^*$, the cylinder may move upwards, as shown in Fig. 9(b) and Fig. 9(c). Fig. 9 shows the velocity distribution of these three conditions at the equilibrium point. The closer the equilibrium point of the cylinder is to the bottom, the higher the blocking rate of the cylinder on the tube. It leads to a higher speed on both sides of the cylinder.

Figure 7. Flow mode map.
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Figure 8. The component grid and the background grid for (a) $t^* = 0$, (b) $t^* = 10$ for $Re = 1$, $Mg^* = 4.9$.

Figure 9. Equilibrium velocity distribution.

In Fig. 7, there is an interval in which $Re$ is about 4 to 200, and the cylinder vibrates. The initial $Re$ of vortex shedding of flow over a fixed cylinder in a free domain appears around 47.5. However, when the cylinder vibrates in the expansion tube, the initial Reynolds number of vortex shedding is obviously advanced. For example, when $Mg^* = 9.8$, the cylinder may vibrate when $Re$ is about 4. The vibration of the cylinder makes the flow more unstable.
We select four moments, \( Re = 10 \) and \( Mg^* = 19.6 \), as shown in Fig. 10. Obviously, the cylinder vibrates in the expansion tube. It should be pointed out that at \( t^* = 100 \), the cylinder collides with the wall of the expansion tube, and then the cylinder leaves quickly.

\[ t^* = 100 \quad \text{and} \quad t^* = 110 \]

\[ t^* = 120 \quad \text{and} \quad t^* = 130 \]

**Figure 10.** The motion for \( Re =10, Mg^*=19.6 \).

At high Reynolds numbers, two different equilibrium states are found at low or high \( Mg^* \). At low \( Mg^* \), the cylinder may balance on the central axis. However, at high \( Mg^* \), the cylinder will eventually deviate from the center axis. At the equilibrium moment, the velocity distribution diagram of \( Re = 300, Mg^* = 4.9, Re = 200, Mg^* = 39.25 \) and \( Re = 300, Mg^* = 39.25 \) is shown in Fig. 11. The fluid is symmetrically ejected from the grooves on both sides of the cylinder for \( Re = 300, Mg^* = 4.9 \). However, the fluid is asymmetrically sprayed in the grooves on both sides of the cylinder for \( Re =200, Mg^* =39.25 \) or \( Re =300, Mg^* =39.25 \). It is possible that the central axis is not a stable equilibrium point.
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towards the bottom, which may need further research.

\[ Re = 300, Mg^* = 4.9 \]
\[ Re = 200, Mg^* = 39.25 \]
\[ Re = 300, Mg^* = 39.25 \]

**Figure 11.** The cylinder balance at high Reynolds numbers.

The time-averaged y-coordinates of the cylinder centers are listed in Tab. 1. Obviously, the time-averaged y-coordinate are negatively related to Reynolds number \((Re)\) and dimensionless gravity \((Mg^*)\). This means that the heavier the cylinder or the faster the inlet flow speed, the more the cylinder tends to balance or vibrate at the bottom of the tube.

| \(Mg^*/Re\) | 1   | 2   | 3   | 4   | 5   | 10  | 20  | 40  | 100 | 200 | 300 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 4.9        | -   | -   | 3.254 | 2.804 | 2.497 | 1.763 | 1.051 | 0.934 | 0.700 | 0.615 | 0.582 |
| 9.8        | 3.943 | 2.810 | 2.285 | 1.980 | 1.561 | 1.118 | 0.805 | 0.642 | 0.539 | 0.479 | 0.455 |
| 19.6       | 2.816 | 1.988 | 1.638 | 1.423 | 1.145 | 0.860 | 0.630 | 0.503 | 0.423 | 0.381 | 0.362 |
| 39.25      | 1.992 | 1.439 | 1.201 | 0.992 | 0.855 | 0.644 | 0.487 | 0.398 | 0.337 | 0.304 | 0.291 |
| 78.4       | 1.443 | 1.065 | 0.899 | 0.751 | 0.665 | 0.495 | 0.395 | 0.301 | 0.272 | 0.246 | 0.236 |

The evolution diagrams of \(y/D\) and \(x/D\) for different \(Re\) and \(Mg^*\) are drawn in Fig. 12. First, we consider the cases of \(Mg^* = 4.9\). For lower \(Re\), \(y/D\) becomes larger over time and tends to be a platform. When the cylinder moves outward, the weaker blocking effect leads to a smoother flow, which may cause the cylinder to produce a lower driving force. If the cylinder doesn't reach the platform before it
flows out, the cylinder may eventually run out, as shown in Fig. 12(a) of $Re = 1$ and $Re = 2$. With the increase of $Re$, the driving force of the cylinder may be lower than the gravity of the cylinder. Eventually, the cylinder will be pulled back into the field, as shown in Fig. 12(a) of $Re = 3$, $Re = 4$ and $Re = 5$. As Reynolds number continues to increase, such as $Re = 10$, the cylinder may not stop in the flow field, but vibrate. This may be because the wall shear layer of the cylinder becomes unstable due to vibration. The vibrate amplitude reaches the maximum amplitude when $Re = 10$ compared with other Reynolds numbers, while the $y/D$ and $x/D$ shows obvious periodic motion of sin and cosine. As the increase of Reynolds number, the time-averaged equilibrium point of the cylinder moves to the bottom of the expansion tube. It can be seen that when $Re = 200$, the vibration performance is no longer obvious sin-cosine periodic motion, which may be due to the frequent collisions between cylinder and expansion tube. When $Re = 300$, the vibration may eventually stop due to the limitation of the expansion tube. The rest of $Mg^*$, behave like $Mg^* = 4.9$. For larger $Mg^*$, even when $Re = 1$, the cylinder will not go outside. With the increase of $Mg^*$, the critical Reynolds number ($Re_c$) of the cylinder's initial vibration also decreases. For example, for $Mg^* = 4.9$, the $Re_c$ is between 5 and 10. On the contrary, for $Mg^* = 78.4$, it is between 3 and 4. For all $Mg^*$, the maximum amplitude appears at Reynolds number around 10. Under the condition of high Reynolds numbers, the vibration tends to weaken.
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(b) $Mg^* = 9.8$;

(c) $Mg^* = 19.6$;
Figure 12. Movements over time.
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(a) $Mg^* = 4.9$;

(b) $Mg^* = 9.8$;

(c) $Mg^* = 19.6$;

(d) $Mg^* = 39.25$;

(e) $Mg^* = 78.4$

(f)

(g)

Figure 13. Trajectory of $y/D - x/D$. 
The trajectory of $y/D \sim x/D$ for different cases are shown in Fig. 13. Most trajectories behave like an "8" shape, with the expanding tube as an asymptote. There is an obvious periodic track on the trajectory curve. However, there are some exceptions, such as $Mg^* = 39.25$, $Re = 40$ and $Mg^* = 78.4$, $Re = 40$, the trajectory of the vibration is no longer like the "8" shape. They no longer show periodicity, but chaotic trajectory instead, which means that these vibrations have no stable periodic orbit. At large $Re$ or $Mg^*$, the vibration amplitude of the cylinder decreases as it approaches the bottom of the tube.

### 3.2 Superimposed periodic fluctuation incoming flow

We choose the case of $Re = 300$, $Mg^* = 39.25$, and superimpose a sinusoidal periodic fluctuation incoming flow, which is expressed by Eqn. (16). At this parameter, the cylinder will eventually stop at a point off the central axis for a regular incoming flow. However, as shown in Fig. 14, it indicates that the cylinder may always vibrate in a complicated orbit under our given alternating incoming flow. It is worth pointing out that the center point of the trajectory is almost all in the same $y/D$ (about 0.291), which seems to be only related to the time-averaged incoming flow. Two dimensionless parameters are used to describe fluctuation of non-uniform inflow: $\beta$ and $U_{red}$, which affect fluctuation amplitude and frequency respectively. The larger $\beta$ is, the greater the fluctuation range is. For $\beta = 0.5$, $\beta = 0.9$ and 0.99, the actual $Re$ floats from 150 to 450, 27 to 570 and 3 to 597 for transient flow. A higher $U_{red}$ means a longer period of fluctuation.

For $\beta = 0.5$, $U_{red} = 8.0$, the geometrical center point of trajectory deviates significantly from the position of $x = 0$, which seems to the case of regular incoming flow. The cylinder is no longer ultimately stationary, but periodically vibrates in the dilation tube. For other cases, the geometrical center point of trajectory all at $x \approx 0$.

For higher $U_{red}$ or $\beta$, the cylinder tends to have a larger vibration amplitude in the $y$ direction. The cylinder is more likely to appear at the bottom of the dilation tube (The trajectory is like an awl). For higher $\beta$, the incoming flow has a wide range within a wide fluctuation range of incoming flow velocity, which leads to a wide range of equilibrium point, which results in a higher amplitude fluctuation. For higher $U_{red}$, the fluctuation period is longer. In this way, the duration of the same Reynolds number interval will be longer, which also leads to greater wide vibration.
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(a) \( Re =300, \, Mg^* = 39.25, \, \beta = 0.5; \)

(b) \( Re = 300, \, Mg^* = 39.25, \, \beta = 0.9; \)
4. Conclusion

Through numerical simulation, we studied the motion of a small cylinder in an expansion tube. The simulation is performed on the FLUENT platform by using the Overset function. The collision between the cylinder and the tube wall is considered as a positive collision of two rigid bodies, and there is no energy loss. Two key parameters, dimensionless gravity ($Mg^*$) and Reynolds number ($Re$), are focused. $Mg^*$ and $Re$ are taken into account for 4.9-79.4 and 1-300, respectively. Two types of inflow are considered: the regular inflow or superimposed sinusoidal periodic fluctuating incoming flow. For regular inflow, three patterns of motion are found in the phase diagrams of ($Mg^*$, $Re$), which can be divided into five regimes. At very low $Mg^*$ and $Re$, such as $Mg^* = 4.9$, $Re = 1$, the cylinder may move outward bound due to high thrust. At low $Mg^*$ and $Re$, such as $Mg^* = 4.9$, $Re = 4$ or $Mg^* = 19.6$, $Re = 3$, due to the balance of gravity and thrust, the cylinder may eventually be fixed in a certain position. At high $Mg^*$ and $Re$, for example, $Mg^* = 9.8$, $Re = 5$, the cylinder may vibrate in the expansion tube due to the wake vortex shedding. At very high $Mg^*$ and $Re$, such as $Mg^* = 39.25$, $Re = 300$, a strange phenomenon is discovered that the cylinder may stop at a point.
where it deviates from the central axis. In the range of $Re$ from 5 to 40, high-amplitude vibration of the cylinder is common for regular incoming flow. Under the parameter of $Re = 300$ and $Mg^* = 39.25$, superimposed sinusoidal periodic fluctuating incoming flow is also tested. Different from the regular inflow, the cylinder can vibrate violently in this way.

If the three-dimensional motion of the sphere is not considered, the two-dimensional cylinder can be regarded as the simplification of the sphere. Our research may be helpful to understand this ancient problem of mobility. Our simulation is performed at two-dimensional; However, the sphere is a three-dimensional case. It can be considered that in the case of three dimensions, the motion of the ball has a more complex trajectory. In addition, the rotation of the cylinder is not considered in this simulation. It is necessary to evaluate the influence on the trajectory of a rotating cylinder or sphere. In a word, this paper is a preliminary discussion on this complicated problem.

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Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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