A critical analysis of Popper’s experiment

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An experiment which could decide against the Copenhagen interpretation of quantum mechanics has been proposed by K. Popper and, subsequently, it has been criticized by M.J. Collett and R. Loudon. Here we show that both the above mentioned arguments are not correct because they are based on a misuse of basic quantum rules.

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I. INTRODUCTION

An experiment involving a couple of entangled particles emitted in opposite directions from a common source has been proposed by K. Popper as a crucial test for the Copenhagen interpretation of quantum mechanics. We begin by presenting the problem in the terms used by Popper and by Collett and Loudon, and only subsequently we will be more precise about it. The two particles are assumed to propagate (see Fig. 1) along the x-axis towards two arrays of detectors wired to operate in coincidence and placed at left (L) and right (R) of the emitting source, at equal distances from it.

Two slits orthogonal to the flight direction are placed, along the y-axis, before an array of detectors and, initially, their width ∆ is wide enough so that only the counters which are placed directly behind them get activated. In fact, if the wave function of the particles is

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suitably prepared, only the detectors placed at small angles with respect to the flight direction $x$ will register the arrival of a particle with a non-negligible probability. Subsequently, the slit at R is narrowed so as to produce an uncertainty-principle scatter which appreciably increases the set of counters behind the slit which may be activated with a non-negligible probability. Popper’s argument \[1\] goes then as follows: if the Copenhagen interpretation of quantum mechanics is correct, any increase in the precision of the knowledge of the $y$-position of the particle at R should correspond to an analogous increase of the knowledge concerning the $y$-coordinate of the particle at L, as a consequence of the assumed strict correlations implied by the entanglement. Hence, also the scatter of the particle at L should instantaneously increase, even if the width of the slit at this side has not been modified. This prediction is testable, since new counters would be activated with an appreciable probability, giving rise to a detectable form of superluminal influence at odds with the postulates of special relativity.

To save the principle of causality, Popper concludes, no wider scatter at the left hand side L will occur by narrowing the slit at R (see Fig. 2). As a consequence, the Copenhagen interpretation of quantum mechanics which, it has to be stressed, in Popper’s view predicts a wider scatter at L induced by the “mere knowledge” \[1\] of the position of the particle at L which has been caused by the narrowing of the slit at R, would be falsified. In brief, either the Copenhagen interpretation or special relativity must be abandoned, and Popper definitively favors the first option.

Subsequently, a criticism of Popper’s proposal by Collett and Loudon appeared \[2\] (along
FIG. 2: The analogous of Fig. 1 for the case in which the slit at R has been remarkably narrowed. After the slit the scatter in position increases remarkably.

with an exchange of views [3], claiming that the experiment cannot represent a crucial test for the Copenhagen interpretation of quantum mechanics. The reason for this, in the authors’ opinion, is that the non-negligible indeterminacy about the state of the source would make undetectable the effects which the Copenhagen interpretation predicts and Collett and Loudon fully agree with Popper about such an interpretation. In fact, in Ref. [2] these authors pretend to prove that the limitations imposed by the uncertainty principle on the accuracy with which the state of the source can be specified, unavoidably remove the increase in scatter induced by the (indirect) more precise knowledge of the position of the particle at L. Even more, the authors surprisingly claim (as it is put into clear evidence also by Eq. (9) and by Fig. 2 of their paper [2]) that both the width of the slit at R and the amplitude of the scatter at L are expected to decrease together. To conclude, Collett and Loudon agree with Popper’s statement about the alleged implications of the Copenhagen interpretation of the proposed test, and address their criticism solely to the unphysical assumption that the source is stationary at a fixed and known position.

However, as we are going to prove here, the situation is radically different from the one described in the above mentioned papers: the argument developed by Popper [1] is invalid not because of some unjustified assumption about the source but, rather, because the (universally accepted) rules of quantum mechanics, even in the Copenhagen interpretation, do not entail what Popper claims them to entail.

In this paper we will resort to two (equivalent) arguments showing clearly the fallacy of Popper’s idea. The first one descends from a well-known theorem on the impossibility of any form of superluminal signaling using the quantum correlations [4]. In fact, if one could
induce an instantaneous larger scatter of a particle at L by simply narrowing a slit at place R (as described in Popper’s test) taking advantage of the peculiar spatial correlations of entangled wave functions, a superluminal signaling protocol between two arbitrarily distant parties would have been achieved. However, as we will see, quantum formalism forbids in complete generality such a possibility, guaranteeing the peaceful coexistence with the causality principle dictated by special relativity. The second argument, which we include for pedagogical reasons, shows by explicit calculations that, given a (particular and simple) entangled quantum state which correctly describes the kind of spatial correlations envisaged by Popper, it is impossible to induce a larger scatter at position L by narrowing the slit at R. It is almost amusing to observe that our precise analysis makes absolutely clear that a correct use of the quantum rules implies exactly what Popper was waiting for (i.e., no larger scatter at L).

Both our arguments, the first one being completely general while the second being based on direct calculations performed on a particular, though representative, quantum entangled state, will definitely show the reasons why Popper’s argument, and the ensuing discussions, cannot be considered crucial neither for the Copenhagen interpretation of quantum mechanics nor for the causality principle of special relativity. Moreover we hope that our analysis will contribute to stop the investigations and even the proposals of actually performing the experiment which, from time to time, reappear in the literature.

II. NO SIGNALING CONDITION

As already stated, it is a remarkable feature of quantum mechanics its peaceful coexistence with special relativity. In fact, within a quantum mechanical framework, it is impossible to devise a superluminal signaling protocol between two distant parties by resorting to the nonlocal correlations exhibited by entangled states. This is due to the fact that (i) arbitrary unitary operations applied locally on a particle cannot alter the probability distributions of measurement processes performed onto the other particle, (ii) the genuine stochasticity of the reduction process prevents one to control the measurement outcomes, independently both from the kind of measurements one actually performs (it holds true both for projection valued and positive operator valued measurements) and from the choice of the specific reduction mechanism which is assumed to govern the measurement process (wave packet
reduction, spontaneous localization or any other conceivable reduction process). This result is commonly referred to as *no superluminal signaling condition* in quantum mechanics. For the sake of brevity in this section we will present a simplified version of the no signaling theorem by limiting our consideration only to the case of local unitary operations and of measurements described by projection operators within standard quantum mechanics, and address the reader to Ref. [7] for the most general scenario.

To this end, consider a composed bipartite quantum system described by an arbitrary (normalized) statistical operator \( \rho_{12} \) acting on the finite dimensional Hilbert space \( \mathcal{H}_1 \otimes \mathcal{H}_2 \). Now, suppose that an observer performs locally an arbitrary unitary operation, represented by the operator \( U_2 \), onto particle 2. Accordingly, the initial state of the system \( \rho_{12} \) becomes \( \tilde{\rho}_{12} = I_1 \otimes U_2 \rho_{12} I_1 \otimes U_2^\dagger \). This local action does not modify the reduced statistical operator associated to particle 1 since

\[
\text{Tr}^{(2)}[\tilde{\rho}_{12}] = \text{Tr}^{(2)}[I_1 \otimes U_2 \rho_{12} I_1 \otimes U_2^\dagger] = \text{Tr}^{(2)}[\rho_{12} I_1 \otimes U_2^\dagger U_2] = \text{Tr}^{(2)}[\rho_{12}] \tag{1}
\]

where the cyclic property of the (partial) trace operation has been used. Thus, no unitary operation applied locally to one of the two particles can alter the probability distributions of measurements performed onto the other particle. Now, it remains to be proven that also a local nonselective projective measurement cannot be effective for sending a superluminal message. To this end, let us consider two Hermitian operators \( A = \sum_i a_i P_{a_i} \) and \( B = \sum_j b_j Q_{b_j} \), referring to the first and to the second component subsystems respectively. For simplicity, we suppose that the spectrum \( \{a_i\} \) and \( \{b_j\} \) of both observables is nondegenerate. In the previous expressions of the observables we have denoted as \( \{P_{a_i}\} \) and \( \{Q_{b_j}\} \) the associated family of (one-dimensional) orthogonal projection operators. Now, suppose that initially an observer performs a (nonselective) measurement of the observable \( B \) on his particle 2 obtaining the outcome \( b_j \); this causes the state \( \rho_{12} \) of the system to collapse onto the (trace one) state \( I \otimes Q_{b_j} \rho_{12} I \otimes Q_{b_j} / \text{Tr}[I \otimes Q_{b_j} \rho_{12}] \) with probability \( Pr(B = b_j) = \text{Tr}[I \otimes Q_{b_j} \rho_{12}] \). Subsequently, a measurement of the observable \( A \) is performed onto the (arbitrarily distant) particle 1 and we calculate the (nonselective) probability distribution \( Pr(A = a_r | B) \), ignoring the outcome which has been obtained in the previous measurement.
process of \( B \), as follows

\[
Pr(A = a_r|B) = \sum_j Pr(A = a_r|B = b_j)Pr(B = b_j)
\]  

(2)

\[
= \sum_j \text{Tr}\left[ P_{a_r} \otimes Q_{b_j} \rho_{12} P_{a_r} \otimes Q_{b_j} \right] \text{Tr}[I \otimes Q_{b_j} \rho_{12}]
\]  

(3)

\[
= \sum_j \text{Tr}[P_{a_r} \otimes Q_{b_j} \rho_{12}] = \text{Tr}[P_{a_r} \otimes I \rho_{12}]
\]  

(4)

where we have used the linearity and the cyclic property of the trace, the fact that operators belonging to different spaces commute and that the family of projectors \( \{Q_{b_j}\} \) constitutes a resolution of the identity. Now, if we calculate the outcome probability distribution \( Pr(A = a_r) \) of the observable \( A \) when no previous measurement of \( B \) is performed, we easily notice that it coincides with the expression in Eq. (4), that is, \( Pr(A = a_r|B) = Pr(A = a_r) \). This proves that no projective measurement procedure performed on particle 2 may affect the probability distributions of measurements performed subsequently on a distant particle 1, and this holds true irrespective of the state \( \rho_{12} \), which might be entangled or not.

To summarize, the previous arguments (and the more general result of [7]) imply that local operations performed on particle 2 (such as unitary operations or projective measurements) cannot be used to transmit any information whatsoever in the region where particle 1 is located, in spite of the fact that quantum correlations are genuinely nonlocal and the reduction mechanism is assumed to change instantaneously at a distance the state vector of particle 2.

It should now be absolutely clear that the kind of experiment proposed by Popper [1], where an observer placed at R might affect, at his will, the probability distribution of the \( y_1 \)-component of the momentum of the particle at L by simply narrowing a slit at R (i.e., by performing a \( y_2 \)-position measurement of the particle at R), actually contradicts the quantum mechanical rules. In fact, the unfolding of the process hypothesized by Popper violates the theorem we have proved above [11].

Before concluding this section, a further remark is at order. We recall that, both in the original proposal [1] and in its subsequent analysis [2], the authors assume that the counters used for the positions measurements of the particles are wired to operate in coincidence; in other words, only those events in which two counters fire simultaneously at R and at L are considered for the test, while events in which one of the particle gets absorbed by the slit (while the other does not) are discarded. Obviously, in this specific situation, no observer
at L could ever (not even hypothetically) think to get evidence of a (superluminal) signal as a consequence of a (simultaneous) action performed at R (e.g., the narrowing of the slit), simply because the two observers should know which events have to be recorded and which ones discarded. Therefore, as it has been presented in Ref. [1] and [2], Popper’s test is, from its starting hypothesis, completely meaningless for what concerns the possibility of exhibiting a violation of special relativity.

III. A PEDAGOGICAL EXAMPLE

In order to stress once again the fallacy of Popper’s proposal without relying on general no-go theorems of the kind exhibited in the previous section but by making explicit calculations, we shall resort, for pedagogical reasons, to a mathematically unsophisticated modelization of Popper’s experimental set up, which retains and enlightens the distinctive features of the original proposal. These features comprise (i) the existence of strict spatial correlations between the two particles, (ii) the fact that the wave function is initially chosen so that only a limited fraction of counters are activated with a non-negligible probability when the slits are wide-opened, and (iii) the occurrence of a detectable increase in scatter of a particle whenever it is passed through a very narrow slit, implying that it will activate counters at larger angles.

It is worth stressing that all these requests are indeed crucial in order that the test could be significative and meaningful but, unfortunately, Popper’s paper [1] lacks of the needed clarity concerning precisely these points. In fact, he does not make precise the mathematical form of the initial wave function of the particle, limiting himself to speak of an Einstein-Podoloski-Rosen-like state [8] which should imply perfect correlations between the positions $y_1$ and $y_2$ of the two particles. It is important to stress that if this would really be the case, the wave function, at time $t=0$ at which they reach the region where the slits are placed, would be $\delta(y_1 + y_2)$, which is a non-normalizable state which does not fulfill either of the requests (ii) and (iii). In fact it is easy to prove that the evolved wave function of such a function, at any subsequent time, gives rise to a constant spatial probability distribution for both coordinates $y_1$ and $y_2$. This amounts to say that perfect correlations in position necessarily imply that all counters at both sides are activated with the same probability independently of the widths of the slits and, as a consequence, that no further scatter will
be ever detected when one narrows a slit. In accordance with these remarks, one must release the condition of perfect correlations in order that the spread of the wave function be not infinite and Popper’s test be meaningful.

To correctly account for an experiment which embodies all fundamental aspects of the one under consideration, one can choose a wave function which exhibits entanglement of the $y_1$ and $y_2$ coordinates (we neglect the $x$-coordinates which play no role for the argument), in which the superposed factorized wave functions (any finite number of them) should describe particles which, contrary to the tacit assumption made by Popper, are not perfectly correlated in position but only rather accurately. In order to keep at minimum the level of mathematical sophistication, we will consider the following initial wave function (see Fig. 3):

$$\psi(y_1, y_2, t = 0) = \frac{1}{\sqrt{3}} (\psi_{\alpha, \sigma}(y_1)\psi_{-\alpha, \sigma}(y_2) + \psi_{0, \sigma}(y_1)\psi_{0, \sigma}(y_2) + \psi_{-\alpha, \sigma}(y_1)\psi_{0, \sigma}(y_2))$$  \hspace{1cm} (5)

where $\psi_{\beta, \sigma}(y) = (2\pi \sigma^2)^{-1/4} \exp\left(-\frac{(y-\beta)^2}{4\sigma^2}\right)$ is a Gaussian state with mean value $\beta$ and standard deviation $\sigma$. We will also assume that $\alpha$ is much larger than $\sigma$ and that, in turn, the initial width $\Delta$ of both slits is larger than $2(\alpha + \sigma)$. Eq. (5) describes a pair of particles whose corresponding $y_i$-positions ($i = 1, 2$) are almost perfectly correlated (e.g., when the first particle is found to be within the interval $[\alpha - 2\sigma, \alpha + 2\sigma]$ the other one will be found, almost with certainty, within $[-\alpha - 2\sigma, -\alpha + 2\sigma]$) and whose one-particle wave functions $\psi_{-\alpha, \sigma}, \psi_{0, \sigma}$ and $\psi_{0, \sigma}$ are almost orthogonal, since their centers are much more far away from each other than their width, i.e., $\sigma \ll \alpha$. Of course, the parameter $\sigma$, which quantifies the degree of localization, cannot be vanishingly small in order to satisfy the (above) request (ii), but, at the same time, it is assumed to be sufficiently small to guarantee the desired quite accurate correlations in position of the two particles. At this point, if one narrows the slit at $R$ changing its width from $\Delta$ to $\delta$, a quantity of the order, let us say, of $6\sigma$, essentially only the product state $\psi_{0, \sigma}(y_1)\psi_{0, \sigma}(y_2)$ will contribute to the activation of the detectors placed behind the slit, since the other parts of the wave function $\psi(y_1, y_2, t = 0)$ will be almost completely absorbed by the walls of the slit, yielding a negligible contribution to the probability of activating some counters. One then has to take into account that, given the wave function $\psi_{0, \sigma}(y_1)\psi_{0, \sigma}(y_2)$, the particle at $R$ has a probability approximately equal to 99.7% of being found within the interval $[-3\sigma, 3\sigma]$ of the $y_2$-axis which corresponds to the opening of the slit. So, narrowing the slit at $R$ from $\Delta$ to $\delta$ and considering only those events in which detectors on both sides are activated, amounts to assume that the wave
FIG. 3: The explicit example of a correct Popper-like experiment we are discussing. We have represented explicitly the three correlated Gaussians appearing in the wave function. One has to keep in mind that we have assumed that the width $\sigma$ of these Gaussians is much smaller than their separation $\alpha$, a fact which cannot be represented correctly in the figure.

function of both particles is practically equal to $\psi_{0,\sigma}(y_1)\psi_{0,\sigma}(y_2)$.

We come now to show once more that this test cannot disprove the Copenhagen interpretation of quantum mechanics. In order to reach such a conclusion, we let the wave function $\psi_{0,\sigma}(y_1)\psi_{0,\sigma}(y_2)$ evolve freely for the time interval $(0, t)$, which is the one needed to the particles moving along the $x$-axis to cover the distance from the slits to the detectors. The evolved wave function remains a product of two gaussian states, having the same mean value $y_1 = y_2 = 0$ but with a larger standard deviation $\bar{\sigma} > \sigma$ equal to:

$$\bar{\sigma} = \sigma \sqrt{1 + \frac{\hbar^2 t^2}{4m^2\sigma^4}},$$

where $m$ is the mass of the particles. The standard deviation $\bar{\sigma}$ of Eq. (6) is a decreasing function of $\sigma$ as long as $\sigma \in (0, \sqrt{\frac{\hbar t}{2m}})$; therefore, for $\sigma$ belonging to such an interval, the more localized the initial gaussian state is, the more it will spread in the time interval $(0, t)$, as expected. So, in what follows, we will set $\sigma \simeq \sqrt{\frac{\hbar t}{2m}}$ in order to get the minimum of $\bar{\sigma}$ and we assume that the physical parameters $m$ and $t$ are such that the evolved wave function can activate (with significant probability) only a limited fraction of all the detectors (actually only those in front of the interval $[-\delta/2, \delta/2]$) at both sides. Now, in order to reach the core of Popper’s proposal, let us consider (with him) a different situation in which the width of the slit at R is reduced to a value $d$ remarkably smaller than $\delta$ ($d \ll \delta = 6\sigma$). This procedure requires to change the wave function of the particles passing through the slit making it equal to $N\psi_{0,\sigma}(y_1)\psi_{0,\sigma}(y_2)\chi_{d/2}(y_2)$, where $\chi_{d/2}(y_2)$ is the normalized characteristic function of the
interval \([-\frac{d}{2}, \frac{d}{2}]\) and \(N\) is a normalization factor. As it is evident, this narrowing of the slit has reduced to \(d\) the support \(\delta\) of the wave function at \(R\) but it has left unchanged and equal to \(\delta\) the one of the wave function at \(L\). If we let the whole system evolve for a time \(t\), the standard deviation of the gaussian state at \(L\) remains that of Eq. (6) (that is, the particle at \(L\) will not increase its scatter with respect to the previously considered case of an opening \(\delta = 6\sigma\)) while the corresponding particle at \(R\) will exhibit a considerably increased scatter.

To analyze this point in an extremely simplified but essentially correct way, we suppose that \(N\psi_{0,\sigma}(y_2)\chi_{d/2}(y_2)\) be approximately equivalent to the (normalized) characteristic function \(\chi_{d/2}(y_2)\) (this assumption is certainly correct since \(d \ll 6\sigma\)). If we denote as \(\phi(y_2, t)\) the wave function which is the evolved, in the time interval \((0, t)\), of the characteristic function \(\chi_{d/2}(y_2)\), we obtain the following position probability density \([9]\) for the \(R\) particle:

\[
|\phi(y_2, t)|^2 = \frac{1}{2d}[(C(u + v) - C(u - v))^2 + (S(u + v) - S(u - v))^2]
\]  

(7)

where \(u = y_2\sqrt{\frac{\pi}{m\hbar}}\), \(v = \frac{d}{2}\sqrt{\frac{\pi}{m\hbar}}\) and \(C(\theta) = \int_0^\theta dz \cos(\frac{\pi z^2}{2})\) and \(S(\theta) = \int_0^\theta dz \sin(\frac{\pi z^2}{2})\) are the Fresnel integrals. The probability density of Eq. (7) exhibits the following simpler expression

\[
|\phi(y_2, t)|^2 = \frac{2\hbar t}{md\pi} \frac{1}{y_2^2} \sin^2\left(\frac{md}{2\hbar t} y_2\right)
\]  

(8)

in the limiting case \(v \ll 1\) (which amounts to \(d \ll 2\sqrt{\frac{\pi\hbar}{m}}\) or, equivalently, to \(d \ll 5\sigma\)). The function of Eq. (8) is the very well-known Fraunhofer diffraction pattern which describes the intensity of a monochromatic light beam diffracted by a single slit \([10]\). The relevant width \(\Delta y_2\) of such a curve is given by the distance between the two first minima (surrounding the absolute maximum) and it equals:

\[
\Delta y_2 = \frac{4\pi\hbar t}{md}.
\]  

(9)

Now, if we choose \(d = \frac{\sigma}{n} \approx \frac{1}{n}\sqrt{\frac{m\hbar}{2\pi}}\), with \(n\) integer arbitrarily large, we see that the width of the Fraunhofer probability distribution \(\Delta y_2\) increases arbitrarily and, in particular, it becomes much greater than \(6\sigma\), the interval which defines the range of counters which are activated with high probability (> 97%) whenever the slit at \(R\) is large enough to let the whole wave function \(\psi_{0,\sigma}\) to pass through. More precisely, one can prove that \(\frac{\Delta y_2}{6\sigma} \approx O(n)\).

Thus, as expected by Popper and as implied by the quantum rules, by narrowing the slit at \(R\) (i.e., by choosing any \(n > 2\)) we may considerably increase the \(y\)-scatter of the corresponding particle of a factor \(n\) but, contrary to Popper’s expectations and in full agreement
with the quantum rules about wave packet reduction, we leave unaffected the wave function at L, which will then not exhibit an increased scatter.

Finally, if we look at Eq. (1) of Ref. [2] we easily understand why the formula given by the authors conflicts (instead of being a consequence of) with the standard interpretation of quantum mechanics. In fact, the considered formula pretends to relate the standard deviation $\Delta_L$ of the particle at L with the width $s_R$ of the slit at R in the physical situation where the slit at R has been narrowed while the slit at L is left wide open, as follows:

$$\Delta_L^2 = \left( \frac{d + r}{d} s_R \right)^2 + \left( \frac{r \lambda}{4\pi s_R} \right)^2$$

(10)

where, according to the notation of Ref. [2], $\lambda$ is the particle wavelength, $d$ is the distance from the source to the slits and $r$ is the distance from the slits to the detectors. This incorrect formula implies that, whenever $s_R < \left[ r/(d + r) \right]^{1/2}$, the second term dominates over the first and, consequently, yields a larger scatter $\Delta_L$ for a smaller $s_R$.

But, as it emerges clearly from our analysis, these two quantities are completely uncorrelated and, consequently, all calculations performed in Ref. [2] are irrelevant for Popper’s proposal, curiously enough just because the authors explicitly agree with Popper’s (wrong) belief of what quantum mechanics would predict for such an experiment.

**IV. CONCLUSIONS**

In this paper we have illustrated the reasons why Popper’s proposal [1] cannot be considered neither a crucial test for the Copenhagen interpretation of quantum mechanics nor for the principle of causality of special relativity. More precisely, exhaustive arguments have been exhibited which explicitly show why one cannot obtain a larger scatter of a particle at L by local actions (such as by narrowing a slit) on a particle at R, even when the two particles are described by a (quite strictly spatially correlated) wave function.

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