Multiple-Scattering Series For Color Transparency

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Abstract

Color transparency CT depends on the formation of a wavepacket of small spatial extent. It is useful to interpret experimental searches for CT with a multiple scattering scattering series based on wavepacket-nucleon scattering instead of the standard one using nucleon-nucleon scattering. We develop several new techniques which are valid for differing ranges of energy. These techniques are applied to verify some early approximations; study new forms of the wave-packet-nucleon interaction; examine effects of treating wave packets of non-zero size; and predict the production of $N^*$'s in electron scattering experiments.

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I. INTRODUCTION

Color Transparency (CT) is the postulated [1,2] absence of final (or initial) state interactions caused by the cancellation of color fields of a system of quarks and gluons with small spatial separation. For example, suppose an electron impinges on a nucleus knocking out a proton at high momentum transfer. The consequence of color transparency is that there is no exponential loss of flux as the ejected particle propagates through the nucleus. Thus, the usually “black” nucleus becomes transparent. We restrict our attention to processes for which the fundamental reaction is elastic, or at least a two-body reaction. This requires that the nuclear excitation energy be known well enough to ensure that no extra pions are created. This subject is under active experimental investigation [3-5].

The existence of color transparency depends on: (1) the formation of a small-sized wave-packet in a high momentum transfer reaction. (2) the interaction between such a small object and nucleons being suppressed (color neutrality or screening) and (3) the wave-packet escaping the nucleus while still small [1,2]. That color neutrality (screening) causes the cross section of small-sized color singlet configurations with hadrons to be small was found in Ref. [6], and is well-reviewed in Refs. [7-9]. So we take item (2) as given. The truth of item (1), for experimentally available energies, is an interesting issue. It is discussed in Refs. [9-11], and is not probed in depth here.

It is also true that at experimentally available energies, the small object does expand as it moves through the nucleus. Thus the final state interactions are suppressed but not zero. The importance of this expansion was found by Farrar et al. [12], and by Jennings and Miller [13]. See also Ref. [14]. The purpose of this paper is to determine improved methods to calculate the evolution of a wave packet as it moves through a nucleus. In Ref. [12] the uncertainty principle is used to argue that the wave-packet-nucleon interaction can be treated by using an effective cross section, \( \sigma_{eff}(Z) \propto Z \).
where $Z$ is the propagation length. In Ref. [13], the wave-packet-nucleon interactions are computed using a hadronic basis. An approximation to the full multiple scattering series is made in which the first-order term is exponentiated. The work of Ref. [14] uses an exact Green’s function for the evolution of the wave-packet in a model in which the wave-packet-nucleon interaction is proportional to the sum of the squares of the transverse separations between the quarks in the ejected wave-packet, $b^2$. Moreover, the baryon states are treated as quarks bound in a two-dimensional harmonic oscillator. The exact solution depends on using this particular interaction and model space.

The aim of the present work is to develop techniques and approximations that allow the use of more general models of the wavepacket-nucleon interaction and baryon space. A general formal expression for the necessary multiple scattering series is derived in Section II. Different approximation schemes are also defined in that section. Numerical studies of the different approximations are made, using the $(e, e'p)$ reaction as an example, in Sect III. (The initial wave packet is assumed to be of zero size.) Approximations that work best in different regions of energy are identified. The applications in Sect. III result from using a wave-packet-nucleon interaction proportional to $b^2$. Sect. IV shows how different interactions, e.g. that of Ref. [15], can be used. This demonstrates the general nature of our methods. Sect. V displays how our techniques are used in situations in which the wavepacket is allowed to have a non-zero size. Another application is to $N^*$ production, Sect. VI. The final section is reserved for a few brief summary remarks.

II. FORMALISM AND APPROXIMATIONS

Our technique is to treat the wavepacket, formed in high momentum transfer processes, as a coherent sum of baryon states. The means that the relevant wave equation should be a matrix equation. Hence the starting point for our analysis is the relativistic
wave equation

\[(\nabla^2 + \hat{p}^2) |\Psi_{N,p}\rangle = U |\Psi_{N,p}\rangle. \tag{2.1}\]

The quantity \(|\Psi_{N,p}\rangle\) represents a vector in internal quark and nuclear (center of mass position) spaces. The subscripts refer to the asymptotic boundary condition that a free nucleon (N) has a momentum \(\vec{p}\). The operator \(\hat{p}\) is

\[\hat{p} = \sqrt{p^2 + M^2 - \hat{M}^2}, \tag{2.2}\]

\[\approx p + \frac{M^2 - \hat{M}^2}{2p}, \tag{2.3}\]

where the energy eigenstates of the quark “Hamiltonian”, \(\hat{M}^2\), are such that

\[\hat{M}^2 |n\rangle = |n\rangle M_n^2. \tag{2.4}\]

In this notation, the nucleon is the ground state, with \(n = N\) with eigenvalue \(M^2\). The matrix operator \(U\) acts in both quark and nuclear space. It has matrix elements between the nucleon and its excited states and also depends on the nuclear density \(\rho(\vec{R})\).

We wish to simplify Eq. (2.1). To this end, let \(\vec{R}\) be the operator denoting the position of the center of mass of the struck nucleon. Also, we take the final nucleon momentum, \(\vec{p}\), to be large and in the \(\hat{Z}\) direction. Define another column vector \(\Psi_p(\vec{R})\) as

\[\Psi_p(\vec{R}) \equiv \langle \vec{R} |\Psi_{N,p}\rangle. \tag{2.5}\]

It is convenient to factor out a plane wave factor and write,

\[\Psi_p(\vec{R}) = e^{i\hat{p}Z} \hat{\varphi}(\vec{R}) |N\rangle. \tag{2.6}\]

This is the defining equation for \(\hat{\varphi}(\vec{R})\), a matrix in baryon space. Then, as an operator equation, assuming the momentum is large compared with any gradient in the problem,

\[2i\hat{p} \frac{\partial \hat{\varphi}(\vec{R})}{\partial Z} = \hat{U}(\vec{R}) \hat{\varphi}(\vec{R}), \tag{2.7}\]

4
where

\[ \hat{U}(\vec{R}) \equiv e^{-i\hat{p}Z} U(\vec{R}) e^{i\hat{p}Z}. \] (2.8)

The solution to Eq. (2.7) is a path ordered exponential,

\[ \hat{\varphi}(\vec{R}) = \mathcal{P} \exp \left( \int_{-\infty}^{\infty} \frac{dZ_1}{2i \hat{p}} \hat{U}(\vec{B}, Z_1) \right). \] (2.9)

Here, \( \mathcal{P} \) is the path ordering symbol. Notice that we have chosen to use outgoing boundary conditions and that \( \vec{R} = \vec{B} + Z \hat{Z} \). We often suppress the index \( \vec{B} \) in evaluating the path integral.

Before proceeding it is worthwhile to discuss the nuclear interaction operator \( U(\vec{R}) \).

This is the product of the wave-packet-nucleon interaction \( 4\pi \hat{f} \) by the target nuclear density \( \rho(\vec{R}) \)

\[ U(\vec{R}) = 4\pi \hat{f} \rho(\vec{R}). \] (2.10)

In the forward scattering approximation used here, the operator \( \hat{f} \) acts on the internal quark space only. We write \( \hat{f} = f(b^2) \) where \( b_{ij} \) is the transverse (perpendicular to \( \vec{p} \)) separation operator between quarks \( i \) and \( j \) in the wave packet and \( b^2 \equiv \sum_{ij} b_{ij}^2 \). That the interaction does not depend on the longitudinal coordinates is suggested by the work of Ref. [6]; see also Ref. [8]. Although there is no detailed theory for \( \hat{f} \), some general things are known. For small wave packets with \( b^2 \ll b_H^2 \) where \( b_H^2 = \langle N|b^2|N \rangle \), \( \hat{f} \) should vanish. Interactions do occur for larger wavepackets. The expectation value of \( \hat{f} \) in a normal baryon should be the experimentally measured forward scattering amplitude. Such are dominated by their imaginary part so we keep only \( \text{Im} \hat{f} \). Thus we write

\[ 4\pi \hat{f} = -i \frac{2\hat{p}}{\sigma} f(b^2), \] (2.11a)

where \( \sigma \) is the proton-nucleon total cross section with

\[ f(b^2 = 0) = 0, \] (2.11b)
\[ \langle N | f(b^2) | N \rangle = 1. \]  
\hspace{1cm} (2.11c)

In the two gluon exchange approximation,
\[ \lim_{b^2 \to 0} f(b^2) \approx \frac{b^2}{b_H^2}. \]  
\hspace{1cm} (2.11d)

We shall use a general function \( f(b^2) \), subject only to the constraints of Eqs. (2.11). The presence of the factor \( \hat{p} \) allows the high energy baryon-nuclear cross section to be independent of the beam energy. Replacing \( \hat{p} \) by \( p \) in Eq. (2.8) is of the same order of approximation as those already made and produces no essential changes in our results.

To be specific, we examine the \((e, e'p)\) reaction [4] and compute amplitudes for processes in which a proton is knocked out of some shell model bound state, \( \alpha \). With semi-exclusive kinematics, as in the experiment done at SLAC, we look for an exiting proton which has momentum, \( \vec{p} \), very close to that of the virtual photon, whose momentum we label by \( \vec{q} \). By close we mean that no extra pions are produced in the collision. Then, we define the “knockout” amplitude and cross section by the overlap
\[
\mathcal{M}_\alpha = \langle N, \alpha | T_H(Q^2) | \Psi_{N,p} \rangle, \]  
\hspace{1cm} (2.12a)

\[
\sigma \propto \sum_\alpha |M_\alpha|^2, \]  
\hspace{1cm} (2.12b)

where the sum is over all the occupied shells \( \alpha \). The cross section of Eq. (2.12) is obtained by integrating over the angle of the outgoing proton, corresponding to the SLAC experiment. We shall be concerned with ratios \( \sigma/\sigma^B \) in which \( \sigma^B \) is obtained by replacing \( \langle \vec{R} | \Psi_{N,p} \rangle \) of Eq. (2.12), as given by Eqs. (2.5), (2.6), and (2.9), by a proton plane wave function.

The operator \( T_H(Q^2) \) accounts for the absorption of a high momentum photon by a nucleon. Ignoring the effects of spin, the matrix elements of the hard scattering operator are
\[
\langle \vec{R}, N | T_H(Q^2) | \vec{R}, m \rangle = e^{-i \vec{q} \cdot \vec{Z}} \delta^{(3)}(\vec{R} - \vec{R}') F_{N,m}(Q^2), \]  
\hspace{1cm} (2.13)
where $F_{N,m}(Q^2)$ is the inelastic transition form factor. The term $e^{-iqZ} \sum_m F_{N,m}(Q^2)\langle m|$ represents a high momentum wave packet propagating through the nucleus.

After inserting a complete set of nuclear position states, the scattering amplitude $M_\alpha$ has the expression:

$$M_\alpha = \int d^3R \Phi_\alpha(\vec{R}) \langle N|T_H(Q^2) e^{i\vec{p}Z} \mathcal{P} \exp \left( \int_{-\infty}^{Z} dZ_1 \hat{V}(\vec{B}, Z_1) \right) |N\rangle,$$

(2.14)

where $\Phi_\alpha(\vec{R})$ is the wavefunction of a nucleon bound to the nucleus in shell $\alpha$, and $\vec{q}$ is the three momentum of the incoming photon. We take the shell model parameter $\hbar\omega = 41 \text{MeV}$. The operator $\hat{V}$ is given by

$$\hat{V}(\vec{R}) \equiv \frac{1}{2i\rho} \hat{U}(\vec{R}),$$

(2.15a)

$$\hat{V}(\vec{R}) = -e^{-i\vec{p}Z} \frac{\sigma}{2} \rho(\vec{R}) f(b^2) e^{i\vec{p}Z}.$$  

(2.15b)

The “hat” over $V$ is meant to symbolize that $\hat{V}(\vec{R})$ also depends on $b^2$. We note that in general, $[\hat{V}(\vec{R}), \hat{V}(\vec{R}')] \neq 0$ since the operators $\hat{p}$ and $b^2$ do not commute. All of the nuclear interactions are contained in the scattering wave function $|\Psi_{N,p}\rangle$. At initial stages the wave packet is small and so are interactions. If the wave packet expands, the influence of interactions is more important. Thus, an accurate approximation must apply for both small and large sized wave packets. The aim of this work is to develop approximation techniques for Eq. (2.9) or Eq. (2.14). This involves handling the evolution of an initially small wave packet as it moves through the nucleus.

At this stage there are several things that one can do to obtain $\hat{\varphi}$ and $\Psi_p$. We will consider what we call the “Order By Order” calculation, the “Low Energy Expansion”, then the “High Energy Expansion” and the “Equal Spaced Momentum” replacement, and finally the “Exponential Approximation".
A. The Order by Order Calculation (OBO)

This means that we simply expand the exponential in Eq. (2.9) order by order.

\[
\mathcal{P} \exp \left( \int_{-\infty}^{Z_1} dZ_1 \hat{V}(Z_1) \right) = 1 + \int_{-\infty}^{Z_1} dZ_1 \hat{V}(Z_1) + \int_{-\infty}^{Z_1} dZ_1 \hat{V}(Z_1) \int_{-\infty}^{Z_2} dZ_2 \hat{V}(Z_2) + \cdots .
\]

This approximation scheme should be accurate when \( \hat{V} \) acts as a small number. This occurs at high enough energies such that wave packet expansion effects are minimal.

B. The Low Energy Expansion (LEE)

Before considering the low energy expansion in detail, we first obtain a general result. We write \( \hat{V} \) as a sum of two terms,

\[
\hat{V}(\vec{B}, Z) = V_L(\vec{B}, Z) + \Delta V_L(\vec{B}, Z),
\]

with

\[
V_L \equiv \langle N | \hat{V} | N \rangle, \quad (2.18a)
\]

\[
\Delta V_L = \hat{V} - V_L. \quad (2.18b)
\]

Notice that with the above definitions, \( \langle N | \Delta V_L | N \rangle = 0 \). Further, if \( V_L \) and \( \hat{V} \) are local operators in nuclear space, as in Eq. (2.15b), then

\[
[V_L, \Delta V_L] = 0 . \quad (2.19)
\]

Let us suppress the transverse nuclear coordinate, \( \vec{B} \), since it is the same for all terms. Then, in general,

\[
\mathcal{P} e^{\int_{-\infty}^{Z} dZ_1 (V_L(Z_1) + \Delta V_L(Z_1))} = e^{\int_{-\infty}^{Z} dZ_1 V_L(Z_1)}
\]
The low energy expansion is obtained by using Eq. (2.19) to simplify the above equation:

\[
\mathcal{P} e^{\int_{-\infty}^{Z} dZ_1 \bigl(V_L(Z_1) + \Delta V_L(Z_1)\bigr)} = e^{\lambda(Z)} \left(1 + \int_{-\infty}^{Z} dZ_1 \Delta V_L(Z_1) + \int_{-\infty}^{Z} dZ_1 \Delta V_L(Z_1) \int_{-\infty}^{Z_1} dZ_2 \Delta V_L(Z_2) + \cdots\right)
\]

(2.21)

where

\[
\lambda(\vec{B}, Z) \equiv \int_{-\infty}^{Z} dZ_1 V_L(\vec{B}, Z_1).
\]

(2.22)

This is called a low energy expansion, since at low energies the wave packet size increases and \( \hat{V} \approx V_L \). Then, \( \Delta V_L \) is small and one recovers the results of low energy nuclear physics.

C. The High Energy Expansion (HEE)

Here we also define a large \( V_H \) and small \( \Delta V_H \) potentials and treat \( \Delta V_H \) with perturbation theory. The separation of terms is now based on \( \Delta V_H \) acting as small at high energies. Thus we define

\[
\hat{V} = V_H + \Delta V_H,
\]

(2.23a)

\[
\Delta V_H = e^{i\hat{p}Z} \hat{V} e^{-i\hat{p}Z} - \hat{V}.
\]

(2.23b)

This approximation is expected to be valid at high energies because, in the closure limit, all momenta are equal and \( \Delta V_H = 0 \). The operators \( V_H \) and \( \Delta V_H \) do not commute,
so we must use the more difficult form, Eq. (2.20), of the path ordered exponential in Eqs. (2.9) and (2.14).

D. The Exponential Approximation (EA)

This is an approximation first made by Miller and Jennings which turns out to be useful to examine. These authors make the following approximation:

\[
\sum_{n} |n\rangle \langle n| e^{i \hat{p} \hat{Z}} \mathcal{P} \exp \int_{-\infty}^{Z} \hat{V}(Z_1) dZ_1 |N\rangle \\
\approx e^{i \hat{p} \hat{Z}} \exp \left[ \sum_{n} |n\rangle \langle n| \int_{-\infty}^{Z} dZ_1 \hat{V}(Z_1) \right] |N\rangle.
\]

(2.24)

At high energies \( \hat{V} \) is small and the exponential is well approximated by its first-order expansion. The approximation is also accurate at low energies because the exponential accounts for an exact treatment of the usual distorted wave function.

III. MODEL EVALUATIONS

The aim here is to determine the regions of accuracy for the various approximations of Sect. II. We do this by using tractable models for the baryon space interaction operator \( \hat{V} \) and initial wave packet. The baryonic states are chosen as those of a two dimensional (transverse) harmonic oscillator that binds two quarks. We take \( \hbar \omega = 500 \text{ MeV} \). The interaction is specified as

\[ f(b^2) = b^2 / b_H^2, \]

in the notation of Eqs. (2.11). This is a simple function with the correct general properties. A more elaborate form is studied in Sect. IV. Now consider the initial wave packet. We assume that the hard scattering, represented by \( T_H \), creates a wavepacket of zero-size. Such an object is called a Point-Like configuration, or PLC. Writing down only the internal (quark) degrees of freedom, we have

\[
\langle N | T_H (Q^2) = \sum_{n} F_{N,n} (Q^2) \langle n |, \tag{3.2a}
\]

\[ = C(Q^2) \langle \bar{b} | = 0 |. \tag{3.2b}
\]
So, $F_{N,n}(Q^2) = \langle \vec{b} = 0 | n \rangle C(Q^2)$. For the two dimensional oscillator, the matrix element $\langle \vec{b} = 0 | n \rangle$ has the same value for all $n$, $\left( \frac{1}{bH\sqrt{\pi}} \right)$, and all form factors are equal in this model.

We now turn to the evaluation of Eq. (2.14). Inserting a complete set of oscillator states gives

$$
\mathcal{M}_\alpha = \int d^3R e^{-iqZ} \Phi_\alpha(\vec{R}) \sum_{m=0}^{\infty} F_{N,2m}(Q^2) e^{ipz2m} \langle 2m | P \exp \left( \int_{-\infty}^{Z} dZ_1 \hat{V}(\vec{B}, Z_1) \right) | N \rangle
$$

where only the isotropic state $n = 2m$ need be kept. The quantum number $m$ represents the number of nodes in the $b$-space baryon wavefunctions. With these preliminaries out of the way, we turn to the evaluation of the different approximations.

A. Order by Order Calculation

The order by order calculation is to evaluate Eq. (2.16). The result is

$$
\mathcal{M}_\alpha = F_{NN} \int d^3R \Phi_\alpha(\vec{R}) e^{-iqZ} \Psi(\vec{R}) ,
$$

where

$$
\Psi(\vec{R}) = e^{ipz} \sum_{n=0} \Psi_{n}^{BO}(\vec{R}) ,
$$

and the index $n$ is the order of the term in $\hat{V}$. We find

$$
\Psi_{0}^{BO}(\vec{R}) = 1 ,
$$

$$
\Psi_{1}^{BO}(\vec{R}) = -\frac{\sigma}{2} \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) \left( 1 - e^{ip_2-p}(Z-Z_1) \right) ,
$$

$$
\Psi_{2}^{BO}(\vec{R}) = \left( \frac{\sigma}{2} \right)^2 \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) \int_{-\infty}^{Z_1} dZ_2 \rho(\vec{B}, Z_2) \left[ 1 - e^{ip_2-p}(Z-Z_1) \right. \\
+ e^{ip_2-p}(Z_1-Z_2) - 3e^{ip_2-p}(Z-Z_2) + 2e^{ip_2-p}(Z_1-Z_2)+ip_4-p)(Z-Z_1) \right] ,
$$
\[ \Psi_{3}^{OBO}(\vec{R}) = -\left(\frac{\sigma}{2}\right)^{3} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) \int_{-\infty}^{Z_{1}} dZ_{2} \rho(\vec{B}, Z_{2}) \int_{-\infty}^{Z_{2}} dZ_{3} \rho(\vec{B}, Z_{3}) \]
\[ \times \left[ 1 - e^{i(p_{2}-p)(Z-Z_{1})} + e^{i(p_{2}-p)(Z-Z_{2})} - 3e^{i(p_{2}-p)(Z-Z_{2})} \right] \]
\[ + 2e^{i(p_{2}-p)(Z-Z_{2})+i(p_{4}-p)(Z-Z_{1})} + e^{i(p_{2}-p)(Z-Z_{3})} - e^{i(p_{2}-p)(Z-Z_{1}+Z_{2}-Z_{3})} \]
\[ + 3e^{i(p_{2}-p)(Z-Z_{2})} - 9e^{i(p_{2}-p)(Z-Z_{3})} + 6e^{i(p_{2}-p)(Z-Z_{3})+i(p_{4}-p)(Z-Z_{1})} \]
\[ - 4e^{i(p_{2}-p)(Z-Z_{3})+i(p_{4}-p_{2})(Z-Z_{1})} + 10e^{i(p_{4}-p)(Z-Z_{2})+i(p_{2}-p)(Z-Z_{3})} \]
\[ - 6e^{i(p_{6}-p)(Z-Z_{1})+i(p_{4}-p_{2})(Z-Z_{2})+i(p_{2}-p)(Z-Z_{3})} \] \[ (3.9) \]
where \( p \) is the momentum of the outgoing nucleon and \( p_{2m} = p^{2} + M^{2} - M_{2m}^{2} \). Note that the above wavefunctions, \( \Psi_{1}^{OBO} \), \( \Psi_{2}^{OBO} \), and \( \Psi_{3}^{OBO} \) all vanish in the closure limit. The order by order curves are shown in Fig. 1. The curves labelled by \( OBO_{n} \) are defined by using \( \Psi_{OBO_{n}}^{OBO} \equiv \sum_{m=0}^{n} \Psi_{m}^{OBO} \).

The distorted wave Born approximation DWBA is obtained by using a scattering wave function approximated by the neglect of excited baryon states. Then, since there are no quark space operators, the exact answer is obtained by simply exponentiating the first order result without excited states, \( \Psi = e^{ipZ} \left( 1 - \frac{\sigma}{2} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) \right) \). That is,
\[ \Psi_{DWBA} = e^{ipZ} e^{-\frac{\sigma}{2} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1})} \equiv e^{ipZ} e^{\lambda(\vec{B}, Z)} \]
\[ (3.10) \]
where
\[ \lambda(\vec{B}, Z) = -\frac{\sigma}{2} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) \].
\[ (3.11) \]
The name DWBA arises from the notion that the plane wave factor of Eq. (3.10) is “distorted” by a factor, here an exponential damping factor. The DWBA curve is denoted by the straight solid line on all graphs. We see that for \( -Q^{2} \approx 30 GeV^{2} \) the second and third order calculations are converging. At lower energies, there is poor convergence. This is expected, however, since at low energies we expect there to be many (more than two or three) scatterings.
B. The Low Energy Expansion (LEE)

Now let us proceed to describe the low energy expansion. With the present $b^2$ scattering operator, the operators of Eqs. (2.17) and (2.18) become

$$V_L(\vec{B}, Z) = -\frac{\sigma}{2} \rho(\vec{B}, Z), \quad (3.12)$$

$$\Delta V_L(\vec{B}, Z) = -\frac{\sigma}{2} \rho(\vec{B}, Z)e^{-i\hat{p}Z} \Delta b^2 e^{i\hat{p}Z}. \quad (3.13)$$

where $\Delta b^2 = \frac{b^2}{b_H^2} - 1$. Here too, we have gone to third order in $\Delta V_L$. Using previous notation, we obtain

$$\Psi_{0}^{\text{LEE}}(\vec{R}) = e^{\lambda(Z)}, \quad (3.14)$$

$$\Psi_{1}^{\text{LEE}}(\vec{R}) = \frac{\sigma}{2} e^{\lambda(Z)} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) e^{i(p_{2} - p)(Z - Z_{1})}, \quad (3.15)$$

$$\Psi_{2}^{\text{LEE}}(\vec{R}) = \left(\frac{\sigma}{2}\right)^{2} e^{\lambda(Z)} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) \int_{-\infty}^{Z_{1}} dZ_{2} \rho(\vec{B}, Z_{2}) \left[e^{i(p_{2} - p)(Z_{1} - Z_{2})} - 2e^{i(p_{2} - p)(Z - Z_{2})} + 2e^{i(p_{2} - p)(Z_{1} - Z_{2}) + i(p_{4} - p)(Z - Z_{1})}\right], \quad (3.16)$$

$$\Psi_{3}^{\text{LEE}}(\vec{R}) = \left(\frac{\sigma}{2}\right)^{3} e^{\lambda(Z)} \int_{-\infty}^{Z} dZ_{1} \rho(\vec{B}, Z_{1}) \int_{-\infty}^{Z_{1}} dZ_{2} \rho(\vec{B}, Z_{2}) \int_{-\infty}^{Z_{2}} dZ_{3} \rho(\vec{B}, Z_{3}) \times \left[e^{i(p_{2} - p)(Z - Z_{1} + Z_{2} - Z_{3})} - 2e^{i(p_{2} - p)(Z_{1} - Z_{3})} + 4e^{i(p_{2} - p)(Z - Z_{3})} - 4e^{i(p_{4} - p)(Z - Z_{1}) + i(p_{2} - p)(Z_{1} - Z_{3})} - 8e^{i(p_{4} - p)(Z - Z_{2}) + i(p_{2} - p)(Z_{2} - Z_{3})} + 4e^{i(p_{2} - p)(Z - Z_{1} + Z_{2} - Z_{3}) + i(p_{4} - p)(Z_{1} - Z_{2})} + 6e^{i(p_{6} - p)(Z - Z_{1}) + i(p_{4} - p)(Z_{1} - Z_{2}) + i(p_{2} - p)(Z_{2} - Z_{3})}\right]. \quad (3.17)$$

In the closure limit, the wavefunctions given by Eqs. (3.14 - 3.17) do not vanish but rather conspire to form the first three terms in the expansion of $e^{-\lambda(Z)}$, times $e^{\lambda(Z)}$. 

13
Therefore, to all orders, the full wavefunction converges to the Born wavefunction, corresponding to transparency. Plots of cross sections using the above distorted waves appear on Fig. 2. The curves labelled by $\text{LEE}_n$ are defined by using $\Psi_{\text{LEE}}^n \equiv \sum_{m=0}^{n} \Psi_m^{\text{LEE}}$. The DWBA curve is $\Psi = e^{ipZ} \Psi_0^{\text{LEE}}$. As before, the Born term is given by $\Psi(\vec{R}) = e^{ipZ}$.

The fact that the $\text{LEE}$ does not join smoothly up to the DWBA is evidence that the $m = 1$ “Roper” has influence below threshold.

C. The High Energy Expansion (HEE)

In the present model, the evaluation of Eqs. (2.23a) and (2.23b) leads to

$$V_H(\vec{B}, Z) = -\frac{\sigma}{2} \rho(\vec{B}, Z) \frac{b^2}{b_H^2},$$  \hspace{1cm} (3.18)

$$\Delta V_H(\vec{B}, Z) = -\frac{\sigma}{2} \rho(\vec{B}, Z) \left[ e^{-ipZ} \frac{b^2}{b_H^2} e^{ipZ} - \frac{b^2}{b_H^2} \right].$$  \hspace{1cm} (3.19)

Because $V_H$ and $\Delta V_H$ no longer commute, we must use the path ordered exponential shown in Eq. (2.20). The result is

$$\Psi_1^{\text{HEE}}(\vec{R}) = e^{ipZ} \left( \sum_m x^m(Z) e^{i(p_{2m}-p)Z} + \sum_{m,n} x^{n+1}(Z) e^{i(p_{2m}-p)Z} \langle 2m \mid \frac{b^2}{b_H^2} \mid 2n \rangle \right)$$

$$- \frac{\sigma}{2} \int_{-\infty}^{Z} dZ_1 \frac{\rho(Z_1)}{1 - \lambda(Z_1)} \sum_{m,n,l} e^{i(p_{2m}-p)Z} e^{i(p_{2n}-p_{2l})Z_1} x^n(Z_1)$$

$$\times \langle 2m \mid e^{-\overline{\lambda}(Z_1) \frac{b^2}{b_H^2}} \mid 2l \rangle \langle 2l \mid \frac{b^2}{b_H^2} \mid 2n \rangle \right),$$  \hspace{1cm} (3.20)

where $\lambda(Z)$ is defined in Eq. (3.11) above,

$$\overline{\lambda}(Z, Z_1) = -\frac{\sigma}{2} \int_{Z_1}^{Z} dZ \rho(Z),$$  \hspace{1cm} (3.21)

$$x(Z_1) = \frac{-\lambda(Z_1)}{1 - \lambda(Z_1)},$$  \hspace{1cm} (3.22)

and

$$\langle 2m \mid e^{-\overline{\lambda} \frac{b^2}{b_H^2}} \mid 2l \rangle = -\overline{\lambda} \left( \frac{1}{\lambda(-1 + \lambda)} \right)^{l+m+1} \min(l, m) \sum_{i=0}^{\min(l, m)} \binom{m}{i} \binom{l}{i} (\overline{\lambda})^{l+m-i}.$$  \hspace{1cm} (3.23)
The result (3.23) is obtained from
\[ \langle b | 2m \rangle = \frac{1}{b_H \sqrt{\pi}} \sum_{i=0}^{m} \frac{(-1)^i}{i!} \left( \frac{m}{i} \right) e^{-\frac{b^2}{2b_H^2}} \left( \frac{b^2}{b_H^2} \right)^i. \] (3.24)

Although it is not immediately obvious, \( \Psi_1^{HEE}(\vec{R}) \), given by Eq. (3.20), in the closure limit, does indeed approach unity and color transparency is obtained. Notice that we have suppressed the transverse nuclear distance, \( \vec{B} \), in \( \lambda, x \), and \( \Psi \) for clarity.

At this stage, a further approximation suggests itself. At very high energies, considering the form of \( \hat{p} \) in Eq. (2.3), it seems reasonable to replace
\[ p_{2m} - p \rightarrow m(p_2 - p). \] (3.25)

This is what we call the “Equal Spaced Momentum” (ESP) replacement. With the above replacement, we can now do the sum on \( l \) in Eq. (3.20) with the result that
\[ \Psi_1^{HEE}(\vec{R}) = e^{ipZ} \left( \frac{1}{(1 - \lambda(Z))(1 - y(Z, Z))} + \frac{\left( x(Z) - y(Z, Z) \right)(1 - x(Z))}{(1 - y(Z, Z))^2} \right) \]
\[ - \frac{\sigma}{2} \int_{-\infty}^{\infty} dZ_1 \frac{\rho(Z_1)}{1 - \lambda(Z_1)} \sum_{m,n} e^{i(p_2 - p)mZ_1} x^n(Z_1) \times \]
\[ \left( (2n + 1)(2m) | e^{i(Z_1, Z_1)\frac{b_2^2}{b_H^2}} | 2n \right) \]
\[ - (n + 1)e^{i(p_2 - p)Z_1} \left( 2m | e^{i(Z_1, Z_1)\frac{b_2^2}{b_H^2}} | 2n + 2 \right) \]
\[ - ne^{i(p_2 - p)Z_1} \left( 2m | e^{i(Z_1, Z_1)\frac{b_2^2}{b_H^2}} | 2n - 2 \right) \). \] (3.26)

The new function \( y \) is given by
\[ y(Z, Z_1) = x(Z_1)e^{i(p_2 - p)Z}. \] (3.27)

Intermediate states corresponding to \( m = n = 9 \) must be included to accurately obtain numerical results for expressions (3.20) and (3.26). On Fig. 3, we see this first order term of the HEE (solid) compared with the OBO_3 (dotdashed). As expected, the two
curves agree at higher energies. The ESP replacement is very accurate at all but the very lowest energies.

D. The Exponential Approximation (EA)

If one looks at only the first order term in Section IIIA, we see that the only thing which has changed between this wave function calculation and the traditional Glauber calculation is that the cross section $\sigma$ has been replaced by an effective cross section $\sigma(Z, Z_1) \equiv \sigma(1 - e^{i(p_2 - p)(Z - Z_1)})$. Then, it is quite tempting to simply exponentiate this first order result. This so-called exponential approximation is not correct, since only the second excited state enters whereas we know that higher states enter in higher orders. However, because we have learned that the ESP replacement is a safe one, we can cast all higher excitations in terms of excitations of the second state. In fact, if we examine the difference $\Delta_2$ between the exact second order OBO calculation and the second order term of the EA, we find that,

$$
\Delta_2 = \left( \frac{\sigma}{2} \right)^2 \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) \int_{-\infty}^{Z_1} dZ_2 \rho(\vec{B}, Z_2) e^{i(p_2 - p)(Z_1 - Z_2)} \left( 1 - e^{2i(p_2 - p)(Z - Z_1)} \right)^2,
$$

where we have used the ESP replacement. Therefore, we see why the EA works as well as it does. The exponentiation of the first OBO contains many more of the higher order terms than one would naively think. Thus, at high energies, the difference between the EA and the true answer is rapidly suppressed, as shown above. The EA (dotted) is shown along with the first order OBO calculation on Fig. 4.

E. Summary of Approximation Schemes

We have presented many different approximations and graphs. Here, we compare and contrast the results of the above calculations. In particular, we want to see which approximation method or methods work best. To do this, we study the convergence properties among all curves. Looking at the Figures we see that:
1. The curves $OBO_2$ and $OBO_3$ converge at about $-Q^2 \approx 30 GeV^2$. This leads us to believe that above $-Q^2 \approx 30 GeV^2$, the second/third order result is the true answer in this model. At low energy, we see no convergence. This is not surprising since at low energies, we expect there to be many (more than two or three) scatterings.

2. The curves $LEE_2$ and $LEE_3$ follow each other closely up until $-Q^2 \approx 20 GeV^2$. This indicates that $LEE_3$ is the correct answer at least until $-Q^2 \approx 20 GeV^2$, and probably further, depending on how big the fourth order contribution is. Comparing Figs. 1 and 2, we see that $LEE_3$ and $OBO_{2,3}$ line up at about $-Q^2 \approx 26 GeV^2$. This gives us confidence that $LEE_3$ is the correct answer for $-Q^2 \approx 26 - 35 GeV^2$. Therefore, we can conclude that $LEE_3$ is the correct answer all the way up to $-Q^2 \approx 35 GeV^2$. Of course, at higher energies, we expect $LEE_3$ to be inaccurate, as did $LEE_1$, and higher order terms would be needed. Not bad, however, for a low energy expansion.

3. Fig. 3 shows the results of the high energy expansion including the ESP replacement. Also shown for comparison is the $OBO_3$. The two curves are shown to line up for energies higher than $-Q^@ \approx 20 GeV^@$ indicating that the true answer in this model is well approximated by the $HEE$.

4. Fig. 4 shows the exponential approximation and the $OBO_1$ for comparison. The $EA$ matches the $HEE$ all the way to $-Q^2 \approx 6 GeV^2$, indicating that the $EA$ is a simple and fast way to calculate the distorted wave for high energies.

5. The most successful of these approximations are the $LEE_3$, the $HEE$ and the $EA$. They are compared in Fig. 5.

IV. MORE GENERAL INTERACTIONS

The advantage of the present formalism is its applicability to general forms of the interaction, $f(b^2)$ of Eqs. (2.11). The tests of Sect. III were performed with $f(b^2) = b^2 / b^2_H$. Here we use the form of Nikolaev and Zakharov [15], which is well approximated [16]
by
\[ f(b^2) = \frac{1 + \gamma}{\gamma} \left( 1 - e^{-\gamma b^2 / b_H^2} \right). \] (4.1)

This is derived for \( q\bar{q} \) color singlet wave packets, but we use it here as a general representative of Eqs. (2.11). The parameter \( \gamma = 0.762 \).

We use the LEE to determine the cross sections predicted by this interaction. Then the result Eq. (2.18a) is unchanged and the new form of \( \Delta V_L \) is
\[ \Delta V_L = -\frac{\sigma}{2} \rho(R) \frac{1}{\gamma} \left[ 1 - (\gamma + 1) e^{-\gamma b^2 / b_H^2} \right]. \] (4.2)

A straightforward application leads to the results of Fig. 6. At high energies the OBO2 and LEE2 approximations are in agreement. At low energies the results of LEE2 agree with those of the EA. Thus the LEE2 provides a good representation of the correct answer for this model. We see that the results are qualitatively similar to those of Ref. [13], but the predicted ratios of \( \sigma / \sigma_B \) are reduced by about 10%.

V. FINITE SIZE EFFECTS

So far we have assumed that a PLC was made in the hard scattering. Another consequence of this assumption, also used, is that the elastic and inelastic form factors, \( F_{NN} \) and \( F_{2m,N} \), are equal. Yet another way to restate Eqs. (3.2) is to assert that \( T_H(\vec{b}) \propto \delta^{(2)}(\vec{b}) \), where \( \vec{b} \) is the transverse distance between the two quarks in the struck nucleon. We now examine the effects of including a nonzero size for the PLC. In this section, we use the simpler form (\( \sim b^2 \)) for the wavepacket-nucleon interaction.

We have incorporated the effects of finite size in two different ways. In the first case, we examine the effect which letting \( T_H(\vec{b}) \) have a nonzero range has on the limit of \( \frac{\sigma}{\sigma_B} \). Here, we assume
\[ T_H(\vec{b}) = \frac{1}{\Lambda^2 \pi} e^{-\frac{\vec{b}^2}{\Lambda^2}}. \] (5.1)
It is then simple to calculate the form factors. In fact, a calculation similar to that for deriving Eq. (3.24) enables us to calculate all of the form factors at once. The result is

\[
F_{2m,N} = \frac{1}{\pi(\Lambda^2 + b_H^2)} \left( \frac{b_H^2}{\Lambda^2 + b_H^2} \right)^m, \tag{5.2}
\]

where we take \( b_H = 1 \text{fm} \). In order to illustrate the effects of finite size, we examine the OBO\(_1\) case for simplicity. In the case of unequal form factors, the distorted OBO\(_1\) wave becomes

\[
\Psi^{OBO_1}(\vec{R}) = 1 - \frac{\sigma}{2} \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) \left( 1 - \frac{F_{2,N}}{F_{N,N}} e^{i(p_2-p)(Z-Z_1)} \right). \tag{5.3}
\]

This is to be compared with Eqs. (3.6) and (3.7). Fig. 7 shows the ratio of \( \sigma \) to \( \sigma_B \) for a very large value of \( -Q^2 \). As \( \Lambda^2 \) approaches zero, \( T_H(\vec{b}) \) approaches a delta function, and the transparency is unity. As \( \Lambda^2 \) goes to infinity, the ratio of the form factors goes to zero, and the first order term of the standard DWBA results. This shows that the predicted cross sections are controlled by the existence of a PLC.

The immediate question is, then, what is \( \Lambda^2 \)? Above, it is an arbitrary parameter. But, if we believe that a small sized configuration is produced in the hard interaction, then \( \Lambda^2 \) should be related to the momentum transfer of the incoming photon to the ejectile. Suppose we have a nucleon of three quarks which absorbs a photon of three momentum \( \vec{q} \). Each quark, then, gets approximately \( q/3 \) of the photon’s momentum. So, we use a form [17] for the hard scattering operator

\[
T_H(\vec{b}) = e^{-\frac{b_H \vec{q}}{3}}. \tag{5.4}
\]

This form is suggested by caricatures of pQCD calculations; see Eq. (1) of Ref. [13]. With Eq. (5.4) in hand, the form factors can be obtained in closed form. The results are

\[
F_{NN} = 1 - e^{x^2} \sqrt{\pi} x \text{ Erfc}(x), \tag{5.5}
\]

\[
F_{2,N} = -x^2 + \frac{1}{2} e^{x^2} \sqrt{\pi} x \text{ Erfc}(x) + e^{x^2} \sqrt{\pi} x^3 \text{ Erfc}(x). \tag{5.6}
\]
where \( x = \frac{1}{2} \frac{\partial \mu}{\partial \nu} \) and Erfc(x) is the complementary error function. We use Eq. (5.3) to calculate the cross section. Fig. 8 shows the finite sized result obtained using Eqs. (5.5) and (5.6), labelled by \( OBO_{FS} \), compared with the zero-size \( OBO_1 \). We see that including finite size, via Eq. (5.4) has a very small effect.

VI. \( N^* \) PRODUCTION

We can also apply our machinery to examine \( N^* \) production. Very little changes in the formalism if a PLC is formed and if the \( b^2 \) form of the wave-packet nucleon interaction is used.

Let us first consider the DWBA for \( N^* \)'s. The \( N^* \) experiences its own optical potential as it moves through the nucleus. We can obtain the result for the distorted wave by neglecting states other than the \( N^* \), e.g. make \( \hat{U} \) diagonal in the quark space. The result is

\[
\Psi_{DWBA}(\vec{R}) = e^{ip_2 Z} e^{-\frac{3\sigma}{2} \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1)} \equiv e^{ip_2 Z} e^{\lambda^*(\vec{B}, Z)},
\]

where \( p_2 \) the momentum of the outgoing excited state is close to the photon momentum \( \vec{q} \). Note the appearance of the factor 3 in the exponent of Eq. (6.1). This factor has a large effect, causing terms in which the \( N^* \) decays to be more important than those in which the \( N^* \) propagates.

Using previous notation, we have calculated \( OBO_1^*, OBO_2^*, LEE_1^*, LEE_2^* \), and \( EA^* \). The superscripts serve to remind us that we are calculating \( N^* \) terms.

We first show the \( OBO^* \) results. Recall, \( \Psi(\vec{R}) = e^{ip_2 Z} \left[ \Psi_0^{OBO^*}(\vec{R}) + \Psi_1^{OBO^*}(\vec{R}) + \Psi_2^{OBO^*}(\vec{R}) + \cdots \right] \).

\[
\Psi_0^{OBO^*}(\vec{R}) = 1,
\]

\[
\Psi_1^{OBO^*}(\vec{R}) = -\frac{\sigma}{2} \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) (3 - e^{i(p_0 - p_2)(Z - Z_1)} - 2 e^{i(p_1 - p_2)(Z - Z_1)}),
\]

(6.3)
\[ \Psi_2^{OBO^*}(\vec{R}) = \left( \frac{\sigma}{2} \right)^2 \int_{-\infty}^{\infty} dZ_1 \rho(\vec{B}, Z_1) \int_{-\infty}^{Z_1} dZ_2 \rho(\vec{B}, Z_2) \]
\[ + 4e^{i(p_4-p_2)(Z_1-Z_2)} + e^{i(p_0-p_2)(Z_1-Z_2)} - e^{i(p_0-p_2)(Z-Z_2)} \]
\[ - 3e^{i(p_0-p_2)(Z-Z_1)} - 6e^{i(p_4-p_2)(Z-Z_1)} - 10e^{i(p_4-p_2)(Z-Z_2)} \]
\[ + 6e^{i(p_6-p_4)(Z-Z_1)} + i(p_4-p_2)(Z-Z_2) \]. \tag{6.4} \]

The \( OBO^* \) curves are shown on Fig. 9 with the same notation as before. The lower solid line is the full distorted wave Born approximation, denoted by \( DWBA^* \); the upper solid line is the cross section calculated using only the first order term of the DWBA and is labelled \( DWBA_1^* \). The difference between those two curves is much bigger than in the nucleon case (the relevant lines there would correspond to \( DWBA = .54, DWBA_1 = .48 \) ) This difference is simply due to the presence of the extra factor of 3 in Eq. (6.1). The resonance-like bump at lower energies is a false effect. Because the first order term starts out so big, which we see is because of the large value produced by the first order DWBA, the tendency of the phases in Eqs. (6.2–6.4) is to increase the cross section at the lower energies. It is not surprising that the order by order calculation fails here. Indeed, we learned from our experience with the nucleon that the \( OBO \) curves were unreliable at low energies. The fact that the exponent of the DWBA is large in magnitude only accentuates that failure. On these grounds, we expect the \( LEE^* \) and the \( EA^* \) curves, which have many more higher order effects, to do much better.

Of course, the \( EA^* \) is calculated by simply exponentiating the \( OBO_1^* \), Eq. (6.3). The \( LEE^* \) results for the distorted waves are shown below.

\[ \Psi_0^{LEE^*}(\vec{R}) = e^{\lambda(\vec{B}, Z)}, \tag{6.5} \]
\[ \Psi_1^{LEE^*}(\vec{R}) = -\frac{\sigma}{2} e^{\lambda(\vec{B}, Z)} \int_{-\infty}^{\infty} dZ_1 \rho(\vec{B}, Z_1) \left( 2 - e^{i(p_0-p_2)(Z-Z_1)} \right) \]
\[ -2e^{i(p_4 - p_2)(Z - Z_1)} \), \]  

(6.6)

\[
\Psi^{LLE} (\vec{R}) = \left( \frac{\sigma}{2} \right)^2 e^{\lambda(\vec{B}, Z)} \int_{-\infty}^{Z} dZ_1 \rho(\vec{B}, Z_1) \int_{-\infty}^{Z_1} dZ_2 \rho(\vec{B}, Z_2) \left( 4 + 4e^{i(p_4 - p_2)(Z_1 - Z_2)} \right. \\
+ e^{i(p_0 - p_2)(Z_1 - Z_2)} - 2e^{i(p_0 - p_2)(Z - Z_1)} - 4e^{i(p_4 - p_2)(Z - Z_1)} \\
\left. - 8e^{i(p_4 - p_2)(Z - Z_2)} + 6e^{i(p_0 - p_4)(Z - Z_1) + i(p_4 - p_2)(Z - Z_2)} \right). \]  

(6.7)

Here, \( \lambda(\vec{B}, Z) \) is defined in Eq. (3.11). The \( LEE^*_1, LEE^*_2, \) and \( EA^* \) are shown on Fig. 10 as the dotted, dashed, and solid lines, respectively. As expected, there is good agreement among all three curves, especially below \(-Q^2 \sim 20 \text{ GeV}^2\).

It is curious that the \( LEE^* \) works as well as it does since now the 0th order term does not reproduce the \( DWBA^* \). Therefore, one might think that instead of defining \( \frac{b^2}{\pi_H} = 1 + \Delta b^2 \) it would be more appropriate to define the operator \( \bar{\Delta}b^2 \) by the assignment \( \frac{b^2}{\pi_H} = 3 + \bar{\Delta}b^2 \) and then use Eqs. (3.12 – 3.13). Calculations using this “bar” method show that the that convergence is better in the \( LEE^* \) scheme than in the \( LEE^* \). This is due to the preference, noted above, that \( N^* \) decay is preferred to \( N^* \) propagation.

Note that in this \( N^* \) work, we have taken the ejected wavepacket to be in a PLC. Qualitative changes in the predictions could occur if this is assumption is not correct [11].

We now summarize our results for the \( N^* \) case.

1. The curves \( OBO^*_1 \) and \( OBO^*_2 \) do not really start to converge well at the energies shown. However, starting at about \(-Q^2 \approx 16 \text{GeV}^2\), the two curves are not far apart.

2. The curves \( LEE^*_1 \) and \( LEE^*_2 \) show good agreement for energies less than \(-Q^2 \approx 20 \text{GeV}^2\). Above that, the \( LEE^*_1 \) is not accurate. The fact that the \( LEE^*_2 \) agrees so well with the \( EA^* \) starting from \(-Q^2 \approx 8 \text{GeV}^2\) suggests that the correct answer is given by these two curves above this energy.

3. Below \(-Q^2 \approx 8 \text{GeV}^2\) the \( LEE^*_2 \) agrees extremely well with the \( EA^* \). This indicates
that these two curves give the correct answer for this energy range. Also, this leads us to the conclusion that the shoulder which appears at low energies in the $LEE_2^*$ at low energies is not real.

4. Our experience in the nucleon sector taught us that the $LEE$ and the $EA$ were the best approximations. This conclusion remains true in the case of $N^*$’s too.

VII. SUMMARY

We have derived approximation schemes that are accurate at low (LEE, Sect. IIB) and high (HEE, Sect. IIC) energies. Their regimes of accuracy overlap, so that methods for calculations at any energy exist. The applications here use simplified interactions and baryon wavefunctions. The methods are more general and allow calculations using more realistic interactions and wavefunctions.

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FIGURE CAPTIONS

Fig. 1. Order-by-order approximation scheme. Ratios of cross sections for the $^{12}C(e, e'p)$ reaction. Dotted- OBO$_1$. Dashed-OBO$_2$. Solid- OBO$_3$.

Fig. 2. The low energy expansion. Dotted-LEE$_1$. Dashed-LEE$_2$. Solid-LEE$_3$.

Fig. 3. The high energy expansion.

Fig. 4. The exponential approximation.

Fig. 5. Comparing different approximation schemes.

Fig. 6. Use of the interaction of Eq. (4.1), Ref. [15].

Fig. 7. The effect of a finite sized wave-packet at very high $Q^2$. Size $\sim \frac{1}{\Lambda}$.

Fig. 8. Energy dependence of the finite-size effects, $\Lambda \sim Q$.

Fig. 9. Order-by-order approximation for quasi-elastic $N^*$ production.

Fig. 10. Low energy expansion for quasi-elastic $N^*$ production.