The Highest Weight property for the $SU_q(n)$ invariant spin chains.

H.J. de Vega †
A. González-Ruiz ‡*
†L.P.T.H.E.

Tour 16, 1er étage, Université Paris VI
4 Place Jussieu, 75252 Paris cedex 05, FRANCE

‡Departamento de Física Teórica
Universidad Complutense, 28040 Madrid, Spain

Abstract

The $SU_q(n)$ generators are obtained as large spectral parameter limit of the Yang-Baxter operators in the integrable $SU_q(n)$ invariant vertex model. The commutation relations, including Serre relations, are obtained as limits of the Yang-Baxter equations. The recently found eigenvectors of the $SU_q(n)$ invariant spin chains are shown to be Highest Weight vectors of the corresponding quantum group.

*Work supported by the Spanish M.E.C. under grant AP90 02620085
1 Introduction.

Quantum groups arose from investigations on integrable lattice models which provide a natural arena for its representations. Recently the $SU_q(n)$ invariant spin chains [1] have been solved [2]. To achieve this, a non-trivial generalization of the nested Bethe ansatz to open boundary conditions was obtained. A Yang-Baxter (YB) algebra of the reflection type naturally appears in the $SU_q(n)$ invariant models [2, 3, 5]. We show in this note how the $SU_q(n)$ generators for N-sites arise from such YB generators in the limit when the spectral parameter tends to infinity. Then, we prove that all Bethe states (constructed through the nested Bethe Ansatz) are highest weight vectors. Therefore, physical states appear as $SU_q(n)$ multiplets, obtained from the Bethe states through the action of the appropriate generators for N-sites. The highest weight property is interesting in relation with the organization of eigenstates and also in the study of the associated RSOS models [4].

We will make some comments on the solution in order to fix the notation and to give the necessary equations. The reader is referred to [2] for details in the notation and some of the operators appearing in this article.

The transfer matrix of the $SU_q(n)$ invariant chains is given by:

$$ t(\theta) = \sum_{a=1}^{n} K^+(\theta)U_{aa}(\theta). $$

In the previous formula $K^+(\theta)$ is given in [2], and $U_{ab}(\theta)$ is the doubled transfer matrix given by:

$$ U_{ab}(\theta) = \sum_{c=1}^{n} T_{ac}(\theta)\tilde{T}_{cb}(\theta), $$

with $T$, $\tilde{T}$ Yang-Baxter operators. The doubled transfer matrix can be written as an operator matrix with elements $U_{11} = A$, $U_{1j} = B_j$, $U_{j1} = C_j$, $U_{jk} = D_{jk}; j, k = 2, \ldots, n$. In the solution of the eigenvalue problem for the transfer matrix (1) the following operators turn out to be useful and will appear in section 3:

$$ \hat{D}_{bd}(\theta) = \frac{1}{\sinh 2\theta}\left[e^{2\theta} \sinh(2\theta + \gamma)D_{bd}(\theta) - \sinh \gamma \delta_{bd}A(\theta)\right] $$

$$ \hat{B}_c(\theta) = \frac{\sinh(2\theta + \gamma)}{\sinh 2\theta}B_c(\theta). $$
As Bethe ansatz for the eigenstates of the transfer matrix (3) we use:

\[ \Psi = \sum_{2 \leq i_j \leq n} X^{i_1 \ldots i_p_1} \hat{B}_{i_1}(\mu_1) \ldots \hat{B}_{i_p_1}(\mu_{p_1}) \| 1 > = \hat{B}(\mu_1) \otimes \ldots \otimes \hat{B}(\mu_{p_1}) \| 1 > X. \]  

(3)

Parallel to the case of periodic boundary conditions the system is solved by giving a recursion relation between the original eigenvalue and the one of a reduced problem with one less state per link. This relation is given by:

\[ \Lambda^{(k)}(\theta, \tilde{\mu}^{(k-1)}) = \prod_{j=1}^{p_k} \frac{\sinh[\theta + \mu_j^{(k)} + (k-1)\gamma] \sinh(\theta - \mu_j^{(k)} - \gamma)}{\sinh(\theta + \mu_j^{(k)} + k\gamma) \sinh(\theta - \mu_j^{(k)})} \]

\[ + \frac{\sinh[2\theta + (k-1)\gamma]}{\sinh[2\theta + (k+1)\gamma]} \prod_{j=1}^{p_k-1} \frac{\sinh[\theta + \mu_j^{(k-1)} + (k-1)\gamma] \sinh(\theta - \mu_j^{(k-1)} - \gamma)}{\sinh(\theta + \mu_j^{(k-1)} + k\gamma) \sinh(\theta - \mu_j^{(k-1)} + \gamma)} \]

\[ \prod_{j=1}^{p_k} \frac{\sinh[\theta + \mu_j^{(k)} + (k+1)\gamma] \sinh(\theta - \mu_j^{(k)} + \gamma)}{\sinh(\theta + \mu_j^{(k)} + k\gamma) \sinh(\theta - \mu_j^{(k)})} \Lambda^{(k+1)}(\theta, \mu^{(k)}), \]  

(4)

\[ 1 \leq k \leq n - 1, \mu_j^{(0)} = 0, \Lambda^{(n)}(\theta, \mu^{(n-1)}) = 1, \ p_0 = N. \]

The roots \( \mu_i^{(k)} \) have to obey the Bethe ansatz equations:

\[ \Lambda^{(k+1)}(\mu_i^{(k)}, \tilde{\mu}^{(k)}) = \]

\[ \prod_{j=1}^{p_{k+1}} \frac{\sinh(\mu_i^{(k)} + \mu_j^{(k-1)} + \gamma)}{\sinh(\mu_i^{(k)} + \mu_j^{(k-1)} + (k-1)\gamma) \sinh(\mu_i^{(k)} - \mu_j^{(k-1)} + \gamma)} \]

\[ \prod_{j \neq i} \frac{\sinh[\mu_i^{(k)} + \mu_j^{(k)} + (k-1)\gamma] \sinh(\mu_i^{(k)} - \mu_j^{(k)} - \gamma)}{\sinh[\mu_i^{(k)} + \mu_j^{(k)} + (k+1)\gamma] \sinh(\mu_i^{(k)} - \mu_j^{(k)} + \gamma)}, \]  

(5)

where we have set the inhomogeneities at the first level equal to zero for simplicity (notice also the change \( \theta \rightarrow \theta + \gamma/2 \) with respect to reference (3)).

Using the recursion formula (3) the eigenvalue problem is solved.

2 The \( SU_q(n) \) generators and its relations.

In this section we show how in certain limits of the spectral parameter \( \theta \) the Yang–Baxter algebra leads to the quantum group \( SU_q(n) \).
First we obtain the generators of the quantum algebra as the leading terms of the transfer matrices $T, \tilde{T}$ in the limits $\theta \to \pm \infty$. We find ($q = e^\gamma$):

$$T_{ab}(\infty) := T^+ =
\begin{cases}
  a > b; & 0, \\
  a = b; & q^{-L}q^{W_a}, \\
  a < b; & \begin{cases}
    b = a + 1; & q^{-L}(q - q^{-1})q^{-1/2}J_a^-q^{W_{a+1}/2}q^{W_a/2}, \\
    b = a + j, j > 1; & T^+_a,
  \end{cases}
\end{cases}$$

(6)

$$T_{ab}(-\infty) := T^- =
\begin{cases}
  a > b; & \begin{cases}
    a = b + 1; & -q^L(q - q^{-1})q^{1/2}J_{a-1}^-q^{-W_{a-1}/2}q^{-W_a/2}, \\
    a = b + j, j > 1; & T^-_a,
  \end{cases} \\
  a = b; & q^Lq^{-W_a}, \\
  a < b; & 0,
\end{cases}$$

(7)

$$\tilde{T}_{ab}(\infty) =
\begin{cases}
  a > b; & \begin{cases}
    a = b + 1; & q^{-L}(q - q^{-1})q^{-1/2}q^{W_{a+1}/2}q^{W_{a-1}/2}J^+_a, \\
    a = b + j, j > 1; & \tilde{T}^+_a,
  \end{cases} \\
  a = b; & q^{-L}q^{W_a}, \\
  a < b; & 0,
\end{cases}$$

(8)

$$\tilde{T}_{ab}(-\infty) =
\begin{cases}
  a > b; & 0, \\
  a = b; & q^Lq^{-W_a}, \\
  a < b; & \begin{cases}
    b = a + 1; & -q^L(q - q^{-1})q^{1/2}q^{-W_{a+1}/2}q^{-W_{a+1}/2}J^-_a, \\
    b = a + j, j > 1; & \tilde{T}^-_a.
  \end{cases}
\end{cases}$$

(9)

In the previous formulas the following operators have been introduced:

$$q^{\pm W_a} = q^{\pm E_{aa}} \otimes q^{\pm E_{aa}} \otimes \cdots \otimes q^{\pm E_{aa}},$$

$$J^+_a = \sum_{i=1}^L q^{-h_a/2} \cdots q^{-h_a/2} \otimes E_{aa+1}^{ith} \otimes q^{h_a/2} \otimes \cdots \otimes q^{h_a/2},$$

$$J^-_a = \sum_{i=1}^L q^{-h_a/2} \cdots q^{-h_a/2} \otimes E_{a+1a}^{ith} \otimes q^{h_a/2} \otimes \cdots \otimes q^{h_a/2},$$

(10)

where $[E_{ab}]_{ij} = \delta_{ia}\delta_{jb}$ and $h_a = E_{aa} - E_{a+1a+1}$ are the $su(n)$ generators in the fundamental representation. The operators $T^\gamma_{\pm}$, corresponding to the limits of the $T_{ab}$ operator, are
polynomials of order \( j \) on the generators \( J^+_l, l = b, b \pm 1, \ldots, a \mp 1 \). Formula (10) gives the coproduct of the quantum group generators to the power \( L - 1 \) \([7]\), we will see that this gives in fact a representation of the quantum group on the lattice of \( L \) sites.

The operators \( T_{ab}(\theta) \) obey the Yang-Baxter relation:

\[
M_{cd}^{ab} = R(\theta - \theta')^{ab}_{ef} T(\theta)_{ec} T(\theta')_{fd} = T(\theta')_{ae} T(\theta)_{bf} R(\theta - \theta')^{ef}_{cd} = N_{cd}^{ab}.
\]  

(11)

By taking the limit \( \theta \to -\infty, \theta' \to \infty \), in the previous equations we get the spectral parameter independent commutation relations:

\[
R_{-ef} T_{-ec} T_{+fd} = T_{+ae} T_{-bf} R_{-cd}^{ef},
\]

where \( R_- = \lim_{\theta \to -\infty} R(\theta) \). If we use the equations \( M_{ba+1}^{ab+1} = N_{ba+1}^{ab+1} \) the following commutation relations are obtained:

\[
[J_i^+, J_j^-] = \delta_{ij} q^{H_i - q^{-H_i}} / (q - q^{-1}),
\]

\[
q^{H_i} = q^{h_i} \otimes \ldots \otimes q^{h_i}.
\]

(12)

Using \( M_{ba+1}^{ab+1} = N_{ba+1}^{ab+1} \) and \( M_{b+1}^{ab} = N_{b+1}^{ab} \) the result is:

\[
q^{H_i} J_j^\pm q^{-H_i} = q^{\pm a_{ij}} J_j^\pm,
\]

(13)

where \( (a_{ij})_{1 \leq i,j \leq n-1} \) denotes the Cartan matrix of type \( A_{n-1} \), i.e., \( a_{ii} = 2 \), \( a_{ij} = -1 \) (\( i, j \) \( \pm 1 \), 0 (otherwise).

In the limit \( \theta \to -\infty, \theta' \to -\infty \), the spectral parameter independent Yang-Baxter relation is:

\[
R_{-ef} T_{-ec} T_{+fd} = T_{+ae} T_{-bf} R_{-cd}^{ef}.
\]

Using the equalities \( M_{ba+1}^{a+1b+1} = N_{ba}^{a+1b+1}, b \neq a, a \pm 1 \) the result is:

\[
J_a^+ J_b^+ = J_b^+ J_a^+, \quad |a - b| \geq 2.
\]

(14)
When \( b = a + 1 \) in the previous equality:

\[
T(-\infty)_{a+2a} = (q - q^{-1}) q^{L/2} [J^+_a J^+_a - q^{-1} J^+_a J^-_{a+1}] q^{-W_a/2} q^{-W_{a+2}/2}, \tag{15}
\]

and using \( M_{aa}^{a+1a+2} = N_{aa}^{a+1a+2} \) we obtain the Serre relation:

\[
(J_a^+) \cdot \left( J_{a+1}^+ \right) - (q + q^{-1}) J_a^+ J_{a+1}^+ J_a^+ + J_{a+1}^+ (J_a^+) = 0, \quad 1 \leq a, a + 1 \leq n - 1. \tag{16}
\]

Making \( a \to a - 1 \) in (13) and using \( M_{a-1a}^{a+1a+1} = N_{a-1a}^{a+1a+1} \) a second Serre relation is obtained:

\[
(J_a^+) \cdot \left( J_{a-1}^+ \right) - (q + q^{-1}) J_a^+ J_{a-1}^+ J_a^+ + J_{a-1}^+ (J_a^+) = 0, \quad 1 \leq a, a - 1 \leq n - 1. \tag{17}
\]

Proceeding in a parallel way with the limit \( \theta \to \infty, \theta' \to \infty \) of relation (11) given by:

\[
R_{+\epsilon f} T_{ec}^+ T_{jd}^+ = T_{ae}^+ T_{bf}^+ R_{+\epsilon f},
\]

where \( R_+ = \lim_{\theta \to \infty} R(\theta) \), the rest of relations are obtained:

\[
q^{H_a} q^{H_b} = q^{H_b} q^{H_a}, \tag{18}
\]

\[
J_a^- J_b^- = J_b^- J_a^-, \quad |a - b| \geq 2 \tag{19}
\]

\[
(J_a^-) \cdot \left( J_{a+1}^- \right) - (q + q^{-1}) J_a^- J_{a+1}^- J_a^- + J_{a+1}^- (J_a^-) = 0, \quad 1 \leq a, a \pm 1 \leq n - 1. \tag{20}
\]

Equations (12-20) give the \( SU_q(n) \) quantum group commutation relations [7].

We pass at this point to the study of the large spectral parameter limits of the doubled monodromy matrix. Using equations (13-14) and the definition of the doubled monodromy matrix we have:

\[
U_{ab}(\infty) = \sum_{l \geq \max(a,b)} T_{al}(\infty) \tilde{T}_{lb}(\infty),
\]

\[
U_{ab}(-\infty) = \sum_{l \leq \min(a,b)} T_{al}(-\infty) \tilde{T}_{lb}(-\infty). \tag{21}
\]
From these formulas we see that not all the quantum group generators can be obtained cleanly from the limits of the doubled monodromy matrix. We will have in general that these limits are formed by polynomials in the generators. Nevertheless for some special cases is possible to obtain cleanly the quantum group generators, these are:

\[
U_{nn}(\infty) = q^{-2L} q^{2W_n} \\
U_{n-1n}(\infty) = q^{-2L} (q - q^{-1}) q^{-1/2} J_{n-1}^- q^{3W_n/2} q^{W_{n-1}/2} \\
U_{nn-1}(\infty) = q^{-2L} (q - q^{-1}) q^{-1/2} q^{3W_n/2} q^{W_{n-1}/2} J_n^+ \\
U_{11}(-\infty) = q^{2L} q^{-2W_1} \\
U_{12}(-\infty) = -q^{2L} (q - q^{-1}) q^{1/2} q^{-3W_1/2} q^{-W_2/2} J_1^- \\
U_{21}(-\infty) = -q^{2L} (q - q^{-1}) q^{1/2} q^{-3W_1/2} q^{-W_2/2} J_2^- \\
\]

\[(22)\]

3 The Highest Weight property.

In this section we prove the highest weight property for the Bethe eigenstates. This property has been shown to hold in the case of open spin chains for the \(SU_q(2)\) invariant case, for the \(SO(4)\) Hubbard model and for the \(sp_l q(2, 1)\) invariant t-J model \([4, 6, 11, 8]\). This is the first proof for an algebra of arbitrary rank.

We need to obtain the commutation relation between the infinite spectral parameter limits of the doubled monodromy matrix and the operators \(\hat{\mathcal{B}}\). For that we use the “reflection” relation \([2, 3]\):

\[
M_{cd}^{ab} \equiv R(\theta - \theta')_{ef}^{cd} U_{ef}(\theta) R(\theta + \theta')_{bd}^{gf} U_{bc}(\theta') = \\
N_{cd}^{ab} \equiv U_{ac}(\theta') R(\theta + \theta')_{fg}^{cd} U_{fg}(\theta) R(\theta - \theta')_{cd}^{hf} ,
\]

which in some special values of \(a, b, c, d\) and limits will give the necessary relations. It is necessary to prove that \(J_a^+ \Psi = 0\), \(a = 1, \ldots, n - 1\).

We begin proving the case \(J_1^+ \Psi = 0\).

Take the equation \(M_{11}^{21} = N_{11}^{21}\) in the limit \(\theta' \to -\infty\) to obtain:

\[
J_1^+ q^{-3W_1/2} q^{-W_2/2} \hat{B}_1(\theta) = R_{12}^{21} \hat{B}_1(\theta) J_1^+ q^{-3W_1/2} q^{-W_2/2} \\
+ q^{3/2} [\delta_{12} A(\theta) - e^{-2\theta} \hat{D}_2(\theta)] q^{-2W_1}. \quad (23)
\]

6
Using $M_{ii}^{11} = N_{ii}^{11}$ in the $\theta' \to -\infty$ limit:

$$q^{-2W_i} \hat{B}_i(\theta) = q^2 \hat{B}_i(\theta) q^{-2W_i}. \quad (24)$$

As $J_i^+ \parallel 1 \geq 0$, with the help of commutation relations (23, 24) we find:

$$J_i^+ q^{-3W_1/2} q^{-W_2/2} \Psi = q^{2(p_1 - l - 1/4)} \sum_{k=1}^{p_1} \delta_{jk2} \left[ \prod_{j \neq k}^{p_1} \frac{\sinh(\mu_k^{(1)} + \mu_j^{(1)} + 2\gamma) \sinh(\mu_k^{(1)} - \mu_j^{(1)} + \gamma)}{\sinh(\mu_k^{(1)} - \mu_j^{(1)})} \right] \Lambda(2)_{\mu_k^{(1)}; \mu_j^{(1)}}.$$

$$\hat{B}_{j_{k+1}}(\mu_{k+1}^{(1)} \ldots \hat{B}_{j_{k-1}}(\mu_{k-1}^{(1)} \parallel 1 > M^{(j)}_{(i)} X^{(i)} = 0, \quad (25)$$

where $M^{(j)}_{(i)}$ takes count of the reordering of the $\hat{B}$ operators [2, 3]. The last equality in the previous equation holds by virtue of the first level Bethe ansatz equations. Using that $J_i^+ q^{-3W_1/2} q^{-W_2/2} = qq^{-3W_1/2} q^{-W_2/2} J_i^+$ and that $q^{W_i}$ are invertible operators, this prove that $J_i^+ \Psi = 0$.

For the rest of the generators things are not so easy. First we will prove that:

$$U_{bd}(\infty) \Psi = 0, \quad b > d, \quad b, d > 1 \quad (26)$$

Using the equations $M_{cd}^{bb} = N_{cd}^{bb}$ and taking the limit $\theta \to \infty$ (see appendix A of [2] for details):

$$U_{bd}(\infty) \Psi = q^{2p_1} \hat{B}(\mu_1^{(1)}) \otimes \ldots \otimes \hat{B}(\mu_{p_1}^{(1)}) \parallel 1 > U_{bd}^{(2)}(\infty) X. \quad (27)$$

In the previous formula $U_{bd}^{(2)}(\infty)$ is the $\theta \to \infty$ limit of the $U_{bd}(\theta + \gamma/2)$ operator of a problem with local weights $R_{km}^{ij}(\theta + \gamma/2)$, $i, j, k, m = 2, \ldots, n$. It can be seen also as a $U_{b-d-1}(\theta + \gamma/2)$ operator of an $SU_q(n - 1)$ chain with local weights $R_{km}^{ij}(\theta + \gamma/2)$, $i, j, k, m = 1, \ldots, n - 1$. We can follow this process $l$ times up to the moment when $d - l = 1$:

$$U_{bd}(\infty) \Psi = q^{2(p_1 + p_2 + \ldots + p_{d-1})} \hat{B}(\mu_1^{(1)}) \otimes \ldots \otimes \hat{B}(\mu_{p_{d-1}}^{(1)}) \parallel 1^{(d)} > U_{bd}^{(d)}(\infty) X^{(d-1)}. \quad (27)$$
Now $U_{bd}(\infty)$ can be seen as $U_{b-l1}(\theta+\gamma/2) = C_{b-l}(\theta+\gamma/2)$ operator for an $SU_q(n-l)$ chain with local weights $R^j_{km}(\theta+\gamma/2)$, $i,j,k,m = 1,\ldots,n-l$. We arrive with this process always to an operator of the type $C_{b-l}(\infty)$ at level $l+1$ acting on $X^{(l)}$. We need to now the commutation relations between operators $C_{b}(\infty)$ and $\hat{B}_c(\theta)$ to evaluate this action. The commutation relations are obtained in the $\theta \to \infty$ limit of the relation $M_{c1}^{1b} = N_{c1}^{1b}$, the result is:

$$C_{b}(\infty)\hat{B}_c(\theta) = R_{+c3}^{\alpha}R_{\alpha c}^{\beta}C_{\beta}(\infty)$$

$$+ q(q - q^{-1})D_{bg}(\infty)[\delta_{ge}\mathcal{A}(\theta) - e^{-2\theta}\hat{D}_{ge}(\theta)],$$

where a summation in $g$ is understood and we have used $[D_{bg}(\infty), \mathcal{A}(\theta)] = 0$. Using this formula at an arbitrary level:

$$U_{bd}(\infty)X^{(d-1)} = q(q - q^{-1})\sum_{k=1}^{p_d} D_{bk}(\infty)$$

$$\prod_{j \neq k} \frac{\sinh[\mu_k^{(d)} + \mu_j^{(d)} + (d-1)\gamma] \sinh(\mu_k^{(d)} - \mu_j^{(d)} - \gamma)}{\sinh(\mu_k^{(d)} + \mu_j^{(d)} + d\gamma) \sinh(\mu_k^{(d)} - \mu_j^{(d)})}$$

$$\prod_{j \neq k} \frac{\sinh[\mu_k^{(d)} + \mu_j^{(d)} + (d+1)\gamma] \sinh(\mu_k^{(d)} - \mu_j^{(d)} + \gamma)}{\sinh(\mu_k^{(d)} + \mu_j^{(d)} + d\gamma) \sinh(\mu_k^{(d)} - \mu_j^{(d)})}$$

$$\prod_{i=1}^{p_d} \frac{\sinh[\mu_k^{(d)} + \mu_i^{(d-1)} + (d-1)\gamma] \sinh(\mu_k^{(d)} - \mu_i^{(d-1)} - \gamma)}{\sinh(\mu_k^{(d)} + \mu_i^{(d-1)} + d\gamma) \sinh(\mu_k^{(d)} - \mu_i^{(d-1)} + \gamma)} \Lambda^{(d+1)}(\mu_k^{(d)}; \mu_i^{(d)})$$

$$\hat{B}_{j k+1}^{(d)}(\mu_k^{(d)}; \mu_k^{(d)}) \parallel 1^{(d)} > M_{c1}^{(j)}X^{(d)(i)}$$

$$= 0,$$

where the last equality holds by virtue of the Bethe ansatz equations at level $d$. In conclusion this shows the desired result $U_{bd}(\infty)\Psi = 0$, $b > d$, $b, d > 1$. This shows directly with (22) that $J_{n-1}^{+}\Psi = 0$. Using this result $U_{nn-2}(\infty)\Psi = T_{nn-2}(\infty)T_{nn-2}(\infty)\Psi = 0$, as $T_{nn}(\infty)$ is an invertible operator then $T_{nn-2}(\infty)\Psi = 0$. Now using $U_{n-1n-2}(\infty)\Psi = [T_{n-1n-1}(\infty)\tilde{T}_{n-1n-2}(\infty) + T_{n-1n}(\infty)\tilde{T}_{n-2n-2}(\infty)]\Psi = 0$, which implies $\tilde{T}_{n-1n-2}(\infty)\Psi = 0$, and the relations (8) we have $J_{n-2}^{+}\Psi = 0$. In this way it is possible to prove the highest weight property for the rest of the generators.

This ends the proof of the highest weight property $J_{l}^{+}\Psi = 0$, $l = 1,\ldots,n-1$

We have seen that the highest weight property of the Bethe states is maintained in the open quantum group invariant case for an algebra of arbitrary rank. The quantum
group generators and relations are obtained in the usual way. This proof is interesting in
the study of the associated RSOS models and is currently under investigation. It would
be interesting to generalize this property to the cases where an elliptic symmetry of the
model is present as in [10].

References

[1] H.J. de Vega and A. Gonzalez Ruiz, J. Phys. A 26 (1993) L519-524.

[2] H. J. de Vega and A. González-Ruiz, Nucl. Phys. B 417, 553 (1994).

[3] E.K.Sklyanin, J. Phys A, 21, 2375 (1988).

[4] C.Destri and H.J. de Vega, Nucl.Phys B 374 (1992) 692 and Nucl.Phys B 385 (1992) 361.

[5] L. Mezincescu and R. Nepomechie, Int. J. Mod. Phys. A7 (1992) 5657

[6] L. Mezincescu and R. Nepomechie, Mod. Phys. Lett. A6 (1991) 2497

[7] M. Jimbo, Lett. Math. Phys. 10 (1985)63.

[8] A. González-Ruiz, Integrable open boundary conditions for the supersymmetric t-J
model. The quantum group invariant case. Preprint F.T/U.C.M-94/1. hep-th 9401118

[9] H.J. de Vega, Int.J.Mod.Phys. A4, 2371 (1989).

[10] R. Cuerno and A. González–Ruiz, J. Phys. A26 (1993) L605

[11] F. Essler, V. Korepin and K.Schoutens, Nucl Phys. B 372, 559, (1992).