HARD THERMAL LOOPS NEAR THE LIGHT-CONE

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ABSTRACT

In hot gauge theories, perturbation theory at the scale of the Debye screening mass requires the resummation of the so-called hard thermal loops, which corresponds to using an effective action obtained by integrating out the modes with momentum of order of the temperature. As is well-known, quantities which are sensitive to the nonperturbative magnetic screening mass still remain incalculable in this resummed perturbation theory. A different breakdown of the latter occurs whenever external momenta are light-like, because the hard thermal loops themselves develop collinear singularities. However, taking into account asymptotic thermal masses for the hard modes regulates the hard thermal loops without spoiling their gauge invariance.

1. Introduction

At the scale of the order of the Debye and plasmon masses, hot gauge theories are no longer adequately described by ordinary thermal perturbation theory. A consistent perturbative approach requires (at least) the resummation of the so-called hard thermal loops (HTL). The latter can be summarized by compact gauge-invariant effective actions which have been found by Taylor and Wong, and by Braaten and Pisarski, to wit

\[ S_{\text{eff}} = -\frac{3m^2}{4} \int d^4xF^\mu_{\alpha}(x) \left( \frac{Y_\alpha Y_\beta}{(YD)^2} \right) F^\beta_{\mu}(x) - M_f^2 \int d^4x \bar{\psi}(x) \frac{Y^\mu}{iYD} \gamma^\mu\psi(x) \]  

(1)

where \( m \) and \( M_f \) are the long-wavelength plasma masses of the gauge bosons and the fermions, resp., and \( Y = (1, \vec{e}) \) is a normalized light-like vector whose spatial direction is averaged over in \( \langle ... \rangle \).

This approach has been applied successfully to a number of problems, and even when the quantities under consideration are sensitive to the magnetic mass scale, it is usually possible to extract a leading logarithmic correction. A quite different shortcoming of the by now standard HTL resummation has been encountered recently in the attempt to calculate the production rate of soft real photons from a

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quark-gluon-plasma\textsuperscript{5}. This requires to use hard thermal loops with external momenta on the light-cone, for which collinear singularities appear.

In a somewhat different context, similar difficulties can be observed already in the simple case of hot scalar electrodynamics\textsuperscript{8}.

2. Longitudinal plasmons in scalar electrodynamics

In hot scalar electrodynamics, the matter part of the HTL effective action (\textsuperscript{4}) is replaced by a simple mass term,

\[ S_{\text{eff}} = -m_{\text{sc}}^2 \int d^4 x |\phi|^2, \quad m_{\text{sc}} = \frac{eT}{2} \]  

(2)

which makes it possible to calculate analytically the complete next-to-leading order corrections to the dispersion law of the photonic plasma excitations. It turns out that for both, the transverse and the longitudinal branches, the frequencies \( \omega_{t,\ell}(q) \) associated with a given wave-vector \( \vec{q} \) are diminished by corrections \( \sim e \). However, in the case of the longitudinal plasmons, the dispersion curve approaches the light-cone exponentially at leading order, and the negative correction appears to make this curve pierce through the light-cone. While there is nothing wrong with that (the group velocity remains smaller than the speed of light), it certainly constitutes a qualitative change in the dispersion laws which a merely perturbative treatment never can decide.

Indeed, by inspecting the behaviour of \( \Pi_{00}(Q) \) as \( Q^2 \to 0 \), one finds that the correction term goes like

\[ \delta \Pi_{00}(Q) \sim \frac{e m^2 q}{\sqrt{Q^2}} \]  

(3)

whereas the HTL piece is only logarithmically singular. So for \( Q^2/q^2 \ll e^2 \), \( \delta \Pi_{00} \gg \Pi_{00}^{\text{HTL}} \), and perturbation theory clearly cannot be trusted any longer.

The difficulty is in fact rooted already in the singular behaviour of the HTL part of \( \Pi_{00} \). Universally (i.e. up to different normalizations in different theories), it reads \textsuperscript{9}

\[ \Pi_{00}(Q) = 4e^2 \int \frac{d^3 p}{(2\pi)^3} n(p) \left\{ 1 - \frac{Q_0}{Q_0 - \vec{p} \cdot \vec{q}/p} \right\} \]  
\[ = 3m^2 \left( 1 - \frac{Q_0}{2q} \ln \frac{Q_0 + q}{Q_0 - q} \right). \]  

(4)

In this form, the logarithmic singularity for light-like momenta is seen to come from a collinear singularity appearing when the hard momentum \( \vec{p} \) becomes parallel to the soft \( \vec{q} \). Any mass for the hard modes would cut off this singularity.

In fact, the scalar particles in the plasma always have a thermal mass associated with them, for the latter is a constant, independent of momentum, cf. \textsuperscript{2}. Including this thermal mass of the scalars already at the HTL level indeed removes
the behaviour, and modifies the leading order piece such that it is finite up to and including the light-cone. This gives a finite value of $|\vec{q}|$, where the longitudinal plasmon dispersion curve hits the light-cone,

$$q^2_e = -\Pi_{00}(Q_0 = q = q_e) = \frac{e^2 T^2}{3} \left( \ln \frac{4}{e} + \frac{1}{2} - \gamma + \frac{\zeta'(2)}{\zeta(2)} + O(e) \right)$$

where $\ln(4/e)$ comes from $\ln(2T/m_{sc})$.

There is still a logarithmic branch cut for $|Q_0| < q$ and a large imaginary part for space-like momenta, which overdamps any excitations far from the light-cone. However, since the singularity at the light-cone has gone, this imaginary part sets in smoothly, so that there is a small but finite range just across the light-cone in the space-like region with weakly damped plasmons.

### 3. Improved hard thermal loops for QED and QCD

In spinor QED and QCD, the dispersion laws are much more complicated than the one of hot scalars with the simple mass term. There are branches corresponding to collective modes that have no analogue in the vacuum theory—the longitudinal plasmons and the fermionic plasminos. These have effective thermal masses which approach zero as the momentum increases, but at the same time the residues of the corresponding poles in the propagator go to zero as well, and they do so exponentially fast. On the other hand, the branches connecting to the ordinary degrees of freedom retain nonvanishing thermal masses, which approach a constant value and read

$$m^2_\infty = \frac{3}{2} m^2, \quad M^2_\infty = 2M_f^2$$

for gauge bosons and fermions, respectively.

These asymptotic thermal masses are generated by the self-interactions of the hard modes among themselves. In the effective (HTL) action obtained by integrating out the hard modes, these asymptotic masses are negligible everywhere except close to the light-cone. There they enter as a cutoff to the collinear singularities of the conventional hard thermal loops.

While there was obviously no problem with gauge invariance with the constant mass term in hot scalar electrodynamics, one might expect problems with gauge invariance through the inclusion of an asymptotic thermal mass for photons and gluons. However, it turns out that the improved HTL effective action which is regular at the light-cone is still gauge invariant. In fact, the compact expressions of (1) can be taken over with such modifications that manifest gauge invariance is retained. In the gluonic part these modifications merely amount to redefining the light-like vector $\vec{Y}$ into a time-like one and extending the averaging prescription to a certain integral over $Y_0$. In the fermionic part, there is instead a slight change in the functional form, but which does not spoil manifest gauge invariance either.

The gauge invariance of the improved effective action is certainly encouraging for setting up a likewise improved resummed perturbation theory. However, the
question whether it is already sufficient for having a well-behaved perturbation series in the vicinity of the light-cone is still open.

Another candidate for the removal of the collinear singularities would be the inclusion of damping of the hard modes. In the process of integrating out the hard modes, this should be negligible compared to the effect of thermal masses because the damping produced by hard self-interactions is \( \gamma \sim g^4 T \). However, through the interactions with soft modes, there is an anomalously large damping \( \gamma \) for the hard modes of the order \( g^2 T \ln(1/g) \). Although formally still of higher order, such a damping would be even more effective in cutting off the collinear singularities than the asymptotic mass, since (roughly)

\[
\frac{Q^2}{q^2} \to \frac{Q_0 \gamma}{q^2} \sim g \gg \frac{m^2_\infty}{T^2} \sim g^2.
\]  

(With \( \gamma \sim g^4 T \) the asymptotic thermal mass term would have won.)

However, in contrast to a resummation of asymptotic thermal masses, it is generally not sufficient to modify only the propagators, when damping is to be taken into account. Including damping only into the propagators violates the Ward identities. In Abelian theories the latter imply \( \Pi_{\mu \nu} Q^\nu \equiv 0 \), but one finds e.g.

\[
\Pi_{0 \nu}(Q_0, 0) Q^\nu \bigg|_{q=0} = \frac{2i\gamma Q_0}{Q_0 + 2i\gamma} \frac{e^2 T^2}{3}.
\]  

Correcting also the vertices by sort of a ladder resummation restores gauge invariance, but it turns out that \( \gamma \) gets eliminated not only from \( \Pi_{\mu \nu} Q^\nu \), but from all the components of \( \Pi_{\mu \nu} \)

If this holds true more generally, then one may expect that the resummation of the asymptotic thermal masses is indeed the relevant mechanism to screen the collinear singularities.

On the other hand, there is recent work on the soft-real-photon production rate which seems to indicate that certain formally higher-order contributions dominate over lower-order when regularizing the light-cone singularities by resumming the asymptotic thermal masses. By the same token, we have found similar difficulties in the imaginary part of the plasmon polarization tensor at one-loop-resummed order. We expect that a careful study of the light-cone properties of the resummed plasmon polarization tensor will elucidate the question of how one has to improve thermal perturbation theory when light-like external momenta are involved.

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References

1. M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 1996) and references therein.
2. E. Braaten and R. D. Pisarski, *Nucl. Phys. B* 337 (1990), 569.
3. J. C. Taylor and S. M. Wong, *Nucl. Phys. B* 346 (1990), 115.
4. R. D. Pisarski, in *From fundamental fields to nuclear phenomena*, eds. J. A. McNeil and C. E. Price, (World Scientific Publ. Co., Singapore, 1991).
5. E. Braaten and R. D. Pisarski, *Phys. Rev. D* 45 (1992) R1827.
6. V.P. Nair, preprint CCNY-HEP 94/10 ([hep-th/9411220](https://arxiv.org/abs/hep-th/9411220)) and references therein.
7. R. Baier, S. Peigne and D. Schiff, *Z. Phys. C* 62 (1994) 337;
   P. Aurenche, T. Becherrawy and E. Petitgirard, preprint ENSLAPP-A-452-93 ([hep-ph/9403320](https://arxiv.org/abs/hep-ph/9403320));
   A. Niegawa, preprints OCU-PHYS-153 ([hep-th/9408117](https://arxiv.org/abs/hep-th/9408117)), OCU-PHYS-160 ([hep-th/9607150](https://arxiv.org/abs/hep-th/9607150)).
8. U. Kraemmer, A. K. Rebhan and H. Schulz, *Ann. Phys. (NY)* 238 (1995) 286.
9. O. K. Kalashnikov and V. V. Klimov, *Sov. J. Nucl. Phys.* 31 (1980) 699;
   H. A. Weldon, *Phys. Rev. D* 26 (1982) 1394.
10. V. P. Silin and V. N. Ursov, *Sov. Phys. – Lebedev Inst. Rep.* 5 (1988) 43;
   V. V. Lebedev and A. V. Smilga, *Ann. Phys. (NY)* 202 (1990) 229.
11. R. D. Pisarski, *Physica A* 158 (1989), 146.
12. F. Flechsig and A. K. Rebhan, *Nucl. Phys. B* 464 (1996) 279.
13. V. V. Lebedev and A. V. Smilga, *Physica A* 181 (1992) 187.
14. C. P. Burgess and A. L. Marini, *Phys. Rev. D* 45 (1992) R17;
   A. Rebhan, *Phys. Rev. D* 46 (1992) 482;
   T. Altherr, E. Petitgirard and T. del Rio Gaztelurrutia, *Phys. Rev. D* 47 (1993) 703;
   R. D. Pisarski, *Phys. Rev. D* 47 (1993) 5589;
   F. Flechsig, A. K. Rebhan and H. Schulz, *Phys. Rev. D* 52 (1995) 2994;
   J.-P. Blaizot and E. Iancu, *Phys. Rev. Lett.* 76 (1996) 3080; preprint SACLAY-T96-085 ([hep-ph/9607303](https://arxiv.org/abs/hep-ph/9607303)).
15. P. Aurenche, F. Gelis, R. Kobes and E. Petitgirard, preprint ENSLAPP-A-586/96 ([hep-ph/9604308](https://arxiv.org/abs/hep-ph/9604308)).
16. F. Flechsig and A. Rebhan, in preparation.