INELASTIC SCATTERING AND SHOT NOISE IN DIFFUSIVE MESOSCOPIC

CONDUCTORS

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A short summary of the drift-diffusion-Langevin formalism for calculating finite-frequency shot noise in diffusive conductors is presented. Two new results are included in this presentation. First, we arrive at a simple (but accurate) phenomenological expression for the semiclassical distribution function of electrons in the presence of electron-electron scattering. Second, it is shown that in thin samples, low-frequency shot noise may be large even if the sample length is much larger than the electron-phonon relaxation length.

1 Introduction

Non-equilibrium (“shot”) noise has been recognized in the past decade as a valuable probe of correlations in mesoscopic electronic systems. It was extensively studied in diverse systems ranging from single-electron devices to quantum point contacts, Josephson junctions, and fractional quantum Hall effect layers (for a review see Ref. [18]).

In diffusive mesoscopic conductors, with elastic mean free path $l$ much shorter than the sample length $L$, spectral density $S_I(\omega)$ of the noise at zero frequency equals $1/3$ of the classical Schottky value $2eI$, with $I$ the average current [19,20], a result obtained in a quantum-mechanical transmission approach [19] and a semiclassical approach [21]. This quantum suppression of the noise is due to Pauli’s exclusion principle which effectively limits the phase space available for electrons emerging from collisions with impurities, thus reducing the randomness of the scattering events and enhancing correlations [22]. However, these correlations may be strongly affected by interactions in the electronic system. Such interactions include the direct Coulomb interaction manifested by short-range electron-electron scattering [23–26] and by long-range screening [27–29] electron-phonon interaction [20] and BCS and Andreev pairing. Furthermore, the frequency dependence of the spectral density may contain additional information on electronic correlations that is not available from the zero-frequency result [27].

It is not clear how to include the effect of interactions or finite frequencies in the quantum transmission formalism for noise [30]. However, in conductors with conductance $G \gg e^2/h$, a full quantum-mechanical calculation is not necessary because the semiclassical suppression of the noise is already of the order of $eI$, while interference effects lead at the most to weak localization corrections, of the order of $(e^2/h)eV \ll eI$, with $V$ the applied voltage. Also, quantum effects due to finite frequency become appreciable only at $\hbar \omega \sim eV$. (Weak localization and quantum frequency corrections to the noise where studied in Refs. [31] and [32], respectively).

Below we summarize the “drift-diffusion-Langevin” formalism [33], which results in a simple recipe for the calculation of finite-frequency noise in degenerate diffusive conductors, generally in the presence of interactions. This formalism is based on the semiclassical Boltzmann-Langevin equation [34]. Its range of validity is the same as the that of the Boltzmann equation, namely, $\lambda_F \ll l$, with $\lambda_F$ the Fermi wavelength, and $\omega \ll eV/\hbar, 1/\tau$ with $\tau$ the elastic mean free time. We also assume $G \gg e^2/h$. We are interested in frequencies comparable to the inverse Thouless time $1/\tau_T = D/L^2 \ll eV/\hbar$, with $D$ the diffusion coefficient.
2 Theory

According to the drift-diffusion-Langevin theory\cite{27,29,27}, the noise spectral density as measured in the electrodes connecting the conductor can be presented as

\[ S_I(\omega) = \frac{2G}{L} \int L^{-\frac{L}{2}} \left| K(x; \omega) \right|^2 C(x) \, dx \]  \hspace{1cm} (1)

where

\[ C(x) = 2 \int f_s(x, E) \left[ 1 - f_s(x, E) \right] \, dE \]  \hspace{1cm} (2)

is the correlator of local fluctuations, and \( f_s(x, E) \) the symmetric part (with respect to momentum) of the local steady state distribution function of electrons at energy \( E \). The response function \( K(x; \omega) \), which is solely responsible for the frequency dispersion of the noise, gives the current generated in the electrodes by a fluctuating unit current source at \( x \). It is dependent upon the specific geometry of the conductor, but its integral over the sample length always equals 1. For example, in a ground-plane geometry it becomes

\[ K(x; \omega) = \frac{\kappa L}{2} \cosh(\kappa x) \sinh(\kappa L/2), \]  \hspace{1cm} (3)

where \( \kappa(\omega) = \sqrt{-i\omega/D'} \) with \( D' = D + GL/C_0 \) and where \( C_0 \) is the (dimensionless) linear capacitance between the conductor and the ground plane.

Once the electrostatic response of the system is known, the only missing ingredient in Eq. (1) is the distribution function \( f_s \). It is found by solving the stationary Boltzmann equation in the diffusion approximation,

\[ -D \frac{d^2 f_s(x, E)}{dx^2} = I(x, E), \]  \hspace{1cm} (4)

with \( I(x, E) \) the collision integral.

As an example of using the above recipe, consider an equilibrium situation where \( f_s \) is a Fermi-Dirac distribution with lattice temperature \( T \). Then \( C(x) = 2T \), and the noise assumes the Johnson-Nyquist value. For typical non-equilibrium distributions \( C(x) \) becomes much larger than the equilibrium correlator, and at \( T = 0 \) the noise given by Eq. (1) is defined as the shot noise.

3 Electron-Electron Scattering

Eqs. (1-4) were solved numerically in Ref. [26] for the case of finite electron-electron scattering, under the assumption of a classical collision integral\cite{30,31,32,33,34,35,36,37}

\[ I(\varepsilon, \xi) = \frac{1}{\tau_{ee}^V} \int d\varepsilon' \int d\omega_0 \left[ (1 - f_s) f'_s f'^+_s (1 - f'^+_s) - f_s f'_s (1 - f^-_s) (1 - f'^+_s) \right]. \]  \hspace{1cm} (5)

Here \( f_s = f_s(\varepsilon), f'_s = f_s(\varepsilon'), f^+_s = f_s(\varepsilon \pm \omega_0), f'^+_s = f_s(\varepsilon' \pm \omega_0), \) and \( \varepsilon = E/eV, \xi = x/L, \tau_{ee}^V \propto V^{-2} \) is the electron-electron energy relaxation time of an electron with excess energy \( eV \). This assumption is controversial at low voltages\cite{34,35,36,37} but is certainly valid at \( eV \gg \hbar/\tau \)\cite{38}.

The strong dependence of the noise on the ratio \( \gamma = L/l_{ee} = L/\sqrt{D' \tau_{ee}^V} \) (at both low and high frequencies), as found in Ref. [26], is well within current experimental resolution. Thus, shot noise measurements can be used as an independent probe of \( l_{ee} \), provided \( \gamma \) is between 1 and 1000 [26]. Such measurements should be viewed as complimentary to regular magnetoresistance measurements, as they are sensitive to the actual scattering length, which may be different from the dephasing length measured in the latter.\cite{38}
However, the numerical solution of Eqs. (4,5) is not simple and requires significant computer time. We therefore present here a phenomenological expression which approximates the exact solution to these equations. The expression is given by

\[ f(\xi, \varepsilon) = \frac{\left( \frac{1}{2} - \xi \right) \exp \left( -4G^+ \varepsilon \right) f_0 \left( \varepsilon - \frac{1}{2} \right) + \left( \frac{1}{2} + \xi \right) \exp \left( -4G^- \varepsilon \right) f_0 \left( \varepsilon + \frac{1}{2} \right) + B \tan^{-1}(\gamma)f_{th}(\xi, \varepsilon)}{\left( \frac{1}{2} - \xi \right) \exp \left( -4G^+ \varepsilon \right) + \left( \frac{1}{2} + \xi \right) \exp \left( -4G^- \varepsilon \right) + B \tan^{-1}(\gamma)} \]  

(6)

Here

\[ G^\pm = \left( \frac{1}{2} \pm \xi \right) \gamma^{1/2}, \]  

(7)

\[ f_0(\varepsilon) = \left[ 1 + \exp \left( \varepsilon/t \right) \right]^{-1} \]  

(8)

and

\[ f_{th}(\xi, \varepsilon) = \left\{ 1 + \exp \left[ (\varepsilon + \xi)/t_h(\xi) \right] \right\}^{-1} \]  

(9)

with \( t = T/eV \), \( t_h(\xi) = \sqrt{t^2 + 3(1 - 4\xi^2)/4\pi^2} \), and \( B \) a parameter to be determined.

Eq. (6) has a simple physical interpretation: At \( \gamma > 1 \) electrons are entering the sample from both electrodes and are keeping their original distribution up to a length scale \( \gamma^{1/2} \) [26]. At the same time the electrons are being thermalized inside the sample, an effect given by the term proportional to the hot-electron distribution \( f_{th}(\xi, \varepsilon) \). \( B \) serves as a mixing constant between these two effects. (It is straightforward to verify that Eq. (6) reduces to the appropriate distributions at \( \gamma \to 0 \) and \( \gamma \to \infty \), and, at any \( \gamma \), to the equilibrium distribution at \( \xi = \pm 1/2 \).

Minimization of the root-mean-square parameter

\[ \Delta = \left\{ \int_{-1/2}^{1/2} d\xi \int_{-\infty}^{\infty} d\varepsilon \left[ f(\xi, \varepsilon) - f_{\text{exact}}(\xi, \varepsilon) \right]^2 \right\}^{1/2} \]  

(10)

gives

\[ B = \frac{5}{8\pi} \]  

(11)

(Root-mean-square minimization of the correlator \( C(\xi) \) gives a slightly different value for \( B \). It is remarkable that the mixing parameter \( B \) is almost universal, \( i.e., \) does not depend strongly on \( \gamma \).

Fig. 1(a) shows the exact distribution function at \( \gamma = 30 \) (see Ref. [26] for similar figures at other values of \( \gamma \).) Fig. 1(b) shows the distribution function given by Eq. (6) at the same \( \gamma \). Fig. 2(c) show the difference between the two. As can be seen, The difference peaks at \( \delta_{\text{max}} = 6\% \) near the step-like singularity of \( f \), and rapidly falls to zero at the bulk of the sample. Fig. 2(d) shows the values of \( \Delta \) and \( \delta_{\text{max}} \) for various values of \( \gamma \). As is clear from the figure, Eq. (6) provides a good approximation for \( f \) only at \( \gamma > 3 \) (at \( \gamma < 3 \) the concept of scattering length \( l_{ee} \) is itself questionable).

4 Electron-Phonon Scattering

The process of electron-electron scattering increases the noise by virtue of adding energy to the electronic system, thus spreading the the distribution of electrons, and increasing the correlator \( C(\xi) \). Electron-phonon scattering does the opposite: It drains energy from the electronic system, thus reducing the spread of \( f \) and decreasing \( C \). This may suggest that when \( L \) is much longer than the electron-phonon scattering length, \( L_c \), and thus the noise, vanish. As we will see below, this is indeed the case at strictly zero frequency, but is not necessarily the case at finite (but small) frequencies.
In order to understand this effect we first note that due to the response function (3), the only current fluctuations in the conductor which are of importance in inducing noise in the electrodes are those which are within a distance \( \lambda_\omega = 1/|\kappa(\omega)| \) from the conductor-electrode interfaces. Therefore, at high enough frequencies, the measured noise is associated with the highly non-equilibrium distribution of electrons near the edges of the conductor, and not necessarily with the nearly-equilibrium distribution at the bulk of the sample. This simple argument means that whenever \( \lambda_\omega \) is smaller than some length scale \( l_S \) (which gives the spatial extent of non-equilibrium electrons in the conductor), the shot noise value should remain large even with increasing \( L \).

To allow for a quantitative description of the above effect, Eq. (4) should be solved with the electron-phonon collision integral. This was done in Ref. [30], where it was shown that the width of the layer in which the electron distribution is far from equilibrium is \( l_S \approx \sqrt{D/\omega} \). Therefore, one should expect large shot noise if \( \lambda_\omega \approx \sqrt{D/\omega} < \sqrt{Dl_{ep}} \), or \( L > L_0(\omega) \) with

\[
L_0(\omega) = \frac{D'}{l_{ep}\omega}. \tag{12}
\]

In what follows we would be interested in relatively long samples and low frequencies. Therefore we assume here that \( l_{ee} \ll L, \lambda_\omega \). Having numerical results for \( f \) (and thus for \( C \)) in this situation, we can find the noise spectral density by combining Equations (1) and (3).

Results for the noise spectral density \( S_I(\omega) \) are presented in Fig. 2 for a specific set of experimental parameters. The upper curves in the figure show the total noise. The lower curves show, on the same scale, the thermal noise. Since the latter is smaller by at least an order of magnitude than the former, the upper curves actually depict the shot noise.

The physical discussion presented above is fully supported by the results shown in Fig. 2. One sees that at each of the three frequencies depicted, the noise initially decreases with \( L \) up to \( L \approx \lambda_\omega \), whereupon it increases, and reaches its mesoscopic value again at \( L \approx L_0(\omega) \). The initial decrease of the noise with increasing \( L \) is due to the electrons being increasingly thermalized in the bulk of the sample, while the subsequent increase is due to the widening of the non-equilibrium surface layer as \( \sqrt{Dl_{ep}} \), and therefore the increasing distance from equilibrium of the noise-inducing electrons within the layer of distance \( \lambda_\omega \) from the interfaces. As expected, at strictly zero frequency the noise reduces monotonically to the thermal value at \( L \to \infty \).
Figure 2: Noise spectral density as a function of sample’s length. Lower curves show the thermal noise and upper curves the full noise which is dominated by the shot noise. Material parameters are $l_{ee} = 10^{-3}$ cm and $D' = 1000$ cm$^2$/s. Temperature-to-voltage ratio is $T/eV = 10^{-5}$. Arrows indicate the positions of $\lambda_\omega$ and $L_0$ for each of the depicted frequencies (at zero frequency these lengths tend to infinity).

5 Discussion

We believe that the phenomenological expression for the distribution function [Eq. (3)] is sufficiently accurate for all types of calculations involving nonequilibrium electrons in diffusive conductors, provided that $L > 3l_{ee}$. In addition to noise calculations, possible applications of Eq. (3) may include superconducting spectroscopy experiments and critical current modulation in dirty metals between superconductor electrodes. It is important to note that because of the singular (step-like) boundary conditions of the distribution function at the edges of the sample, a perturbative solution of the Boltzmann equation in the limit $\gamma \to \infty$ is not a good approximation of $f(x, E)$ even for very large values of $\gamma$.

The collision integral (3) used here to model the electron-electron interaction is strictly valid only at $eV \gg \hbar/\tau$ [36]. In this limit the mixing parameter $B$ in Eq. (3) is universal and equals to $B = 5/8\pi$. At lower voltages weak localization effects become important, and Eq. (3) is no longer valid. However, even though the exact form of the collision integral is not known, it is reasonable to assume that equation (3) may still be valid, but with an exponent in Eq. (7) which scales down as the exponent of the kernel in the collision integral and with possibly a different value of $B$.

In the presence of electron-phonon scattering, the unusual result of shot noise increasing with increasing sample length is essentially due to a competition between two independent physical processes: screening and equilibration. The importance of screening in affecting shot noise was first discussed by Landauer in qualitative terms, and was later studied quantitatively in Refs. [27,29]. Its outcome is summarized by Eq. (3). Equilibration, on the other hand is responsible for the surface layers of non-equilibrium electrons. The fact that the width $l_S$ of these layers grows with $L$ is readily understood since the electron-phonon relaxation time decreases strongly with the energy of the emitted phonon, at large $L$, when the electric field in the conductor is small, an electron entering the sample from the electrode must diffuse elastically for a long distance before being able to emit a phonon.

Parameters chosen in obtaining Fig. 2 correspond to typical experimental setups. At frequencies higher than 1 MHz one sees that shot noise remains of the order of $2eI$ for any length of the conductor. Moreover, even ‘zero frequency’ experiments are invariably performed at an actual frequency of 10 KHz or higher, and should therefore reveal large shot noise when the sample is long. While $D = 1000$ cm$^2$/s is quite realistic, the electrostatic term in $D'$, $GL/C_0 \approx D(td/\Lambda_0^2)$, dominates if the thickness $t$ of the conductor or its distance $d$ from the
ground plane are larger than the static screening length $\Lambda_0$. Thus, the results shown in Fig. 1 are of particular importance when the conductor is very thin, possibly a two-dimensional electron gas. In addition, for $L_0$ to be reasonably small $l_{ep}$ must be large. To maintain $l_{ep} = 10^{-3}$ cm, $V$ cannot be larger than about 100 mV. It is therefore likely that in an actual situation $T/eV$ would not be smaller than $10^{-3}$. Then, at large $L$ and $\omega$, the thermal noise may be as large as the shot noise.

The response function (3), and thus the results for the noise shown in Fig. 2, are not necessarily valid for geometries different from the one studied here. In particular, the question of whether any specific geometry exhibits shot noise when the conductor is long enough reduces to the question whether finite-frequency fluctuations in the bulk of the conductor are sufficiently screened as to not induce current in the electrodes. Theoretically, a detailed answer to this question may involve difficult solutions of the Poisson equation. However, in a charged Fermi system finite-frequency currents are known to be screened beyond some typical length scale $\lambda'$, which does not depend on $L$ [44]. On the other hand, the ‘hot-electron’ length scale $l_S = \sqrt{Ll_{ep}}$ is independent of the geometry. Therefore, it is argued that in sufficiently long samples of an arbitrary geometry $l_S$ is larger than $\lambda'$, so the only important sources of noise are from the non-equilibrium regions near the electrodes. Following the physical discussion above implies that the qualitative features of the results presented in Fig 2 may be of a general nature.

6 Conclusions

The “drift-diffusion-Langevin” formalism for finite-frequency shot noise in the presence of interactions was summarized. A phenomenological equation for the distribution function in a mesoscopic sample with $l_{ee} \leq L \ll l_{ep}$ was presented. In the presence of phonon relaxation, $l_{ee} \ll l_{ep} \leq L$, and for thin conductors, it was shown that shot noise may be large even at $L \gg l_{ep}$. For example, for a two-dimensional electron gas near a ground plane, shot noise at $\omega = 100$ MHz is of the order of the Schottky value for any length of the conductor.

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