Analysis of MHD pulsatile flow of Jeffrey fluid in a diseased inclined tapered porous artery exposed to an inclined magnetic field

R.Padma, R. Tamil Selvi and R. Ponalagusamy
Department of Mathematics, National Institute of Technology, Tiruchirappalli, India
E-mail: padmaram.nitt15@gmail.com, tamil@nitt.edu and rpalagu@nitt.edu

Abstract. In this analysis, a theoretical model is proposed to examine the collective effect of slip velocity, magnetic field, and inclination angles on an unsteady non-Newtonian particulate suspension flow in an inclined diseased tapered tube with a porous medium by applying an external inclined magnetic field. By deploying integral transform methods, analytical expressions are obtained for the flow characteristics such as velocity profiles of fluid and particles, wall shear stress, flow rate, and flow resistance. With the aid of numerical computations, the significance of inclination angle, porous medium, and magnetic intensity are analyzed and illustrated graphically. Further, various physiological parameters affecting the flow characteristics are discussed which would facilitate the rheological functions of blood in the field of biology, biomedicine, and engineering sciences.

1. Introduction
Atherosclerosis (an intimal thickening of lumen artery) is the primary reason for the obstruction in the flow of blood. It is caused by the agglomeration of fatty substances, cholesterol, calcium, cellular waste, and fibrin on the arterial wall. These deposits reduce the diameter of the tube and form a blockage known as plaque. These plaques disrupt the flow of blood in the vascular network and affect the physiological characteristics of the fluid flow. Similarly, at times, these plaques breakdown into smaller fragments and get clogged in some capillaries of the vascular system. Either way, plaques cause a malfunction in blood transmission and decrease the supply of oxygen and nutrients to the organs and tissues resulting in cardiac aneurysm diseases. Depending upon the location of stenosis the individual may suffer from a mild constriction in the carotid artery that leads to fibromuscular dysplasia or can experience stroke from a constriction in the arteries of the brain. Worse still, these constrictions in some cases might even cause death. Therefore it is necessary to observe the symptoms of the stenotic condition and analyze the blood flow through a stenosed artery to facilitate the diagnosis and treatment procedures in the healthcare system. Texon [1] mentioned that hemodynamics is the primary factor that causes atherosclerosis and argues that complications such as intimal ulceration, intramural hemorrhage, and dissecting hemorrhage occurs during the pathogenesis of atherosclerosis. Later, Young [2] analyzed the time-dependent growth in the lumen artery during the steady flow of blood and found that stenosis projection in vessel wall triggers biological mechanisms such as vessel caliber variation and intimal cell proliferation. Further, [3] studied the blood flow through stenosed arteries and delineated the impact of stenosis on flow characteristics. Additionally, the effect of hematocrit and stenosis height on flow characteristics of streaming fluid in an overlapping
stenosed artery was analyzed by [4]. They determined the analytical expressions for the axial flow velocity, pressure gradient, pressure drop, impedance, and shear stress at both throats and critical height.

The systemic pulsative motion of heart effectuates the ventricular action of ejection and absorption of blood into the vascular system and thereby causes the blood circulation. Many researchers reviewed the aids of pulsatile flow, which are now well known and more prominent in the literature of blood rheology. Observing the movements of heart, the function of valves, and the pulsative forces in the aorta, Womersley [5] affirmed that the blood flow is no longer steady but pulsatile in nature. Sanyal and Maji [6] have explored the influence of stenosis on the wall shear stress and pressure gradient on the unsteady flow of blood in a mild stenosed artery. Likewise, various physiological factors concerning time variations of blood flow in a stenosed tube have been studied by [7], and the pulsatile flow of blood flow in the catheterized artery has been examined by [8]. In all the aforementioned literature blood is considered as a Newtonian fluid.

Blood is a multi-component mixture that comprises of plasma, red blood cells, white cells, platelets, and many other components that guarantees the presence of yield stress and relative velocity in particulate suspension. This phenomenon enables us to study blood as non-Newtonian fluid. The presence of yield stress in blood assists us to consider it as Newtonian fluid only when it passes through wider arteries at a higher shear rate. Similarly, when blood flows through small arteries at a lower shear rate it behaves like a non-Newtonian model as proved by [9]. The steady flow of Herschel-Bulkley fluid is investigated by [10]. Wherein they explicated the effects of fluid behavior index, yield stress, and non-linear viscoelasticity of the fluid flow in a stenosed artery. Misra et al. [11] observed that the existence of micropolar effect in viscous fluid amplifies the magnitude of micro particles oscillation. Several other non-Newtonian fluids are also discussed by numerous authors. Amongst the non-Newtonian fluid model, the most prominent one is the Jeffrey fluid model that has fascinated many investigators due to its simplistic linear tendencies that serve as a better example for physiological fluids. The Jeffrey model is a generalized fluid flow model that contains relaxation and retardation time parameters. When the value of both these parameters is set as zero, the Jeffrey model is reduced to the common Newtonian model. Akbar et al. [12] have studied the Jeffrey fluid flow in a tapered stenosed artery and reported that the Jeffrey parameter influences the fluid speed considerably. Hussain et al. [13] have examined the effects of heat transfer and inclined magnetic field on the peristaltic flow of Jeffrey fluid in the asymmetric, flexible, and non-conducting channel. They have also analyzed the temperature dependent thermal conductivity and heat transfer and asserted that the heat transfer coefficient decreases when the Newtonian fluid is altered to Jeffrey fluid. Later, Ponalagusamy [14] studied the particulate Jeffrey fluid suspension in a stenosed artery and observed that when the fluid behaves like a Jeffrey fluid model the shear stress and flow resistance is less compared to the Newtonian model. Similarly, [15] has treated blood as a two-layered model consisting of a cell-free layer called plasma and a cell-rich Jeffrey fluid suspension in the core region for the steady flow of blood in a catheterized stenosed artery. Upon accounting the slip velocity at the wall, it is clear that the slip parameter improves the flow resistance relatively compared to the Darcy number. Furthermore, [16] have acknowledged the effects of variable viscosity and hematocrit on the Jeffrey fluid flow through a tapered stenosed tube.

Blood contains more than 45% of RBCs that are ironized particles which proves the presence of magnetic field effect on the flow area. The flow of electrically conducting fluid across the magnetic field has an extensive application in biology and biomedical engineering. Gold [17] pioneered the concept of induced magnetic field under constant pressure gradient and obtained the velocity profile expression. Noting the sinusoidal pressure gradient [18] developed a mathematical model
for the pulsatile flow of blood and found that the magnetic field with sufficient intensity will reduce the blood speed considerably. The coupled effect of the magnetic field and hematocrit on the Newtonian fluid flow through a stenosed artery was discussed by [19]. They asserted that the effect of the magnetic field and hematocrit concentration are inversely proportional to each other. Ponalagusamy and Tamil Selvi [20] studied the effect of the magnetic field on the two-layered blood flow in a stenosed artery and found the connection between phase lag and the flow characteristics. Also, they mentioned the significance of hematocrit, Hartmann number, and boundary layer thickness on the physiological characteristics.

The arterial system is highly connected by various capillaries. For instance, capillaries in the area of kidney and small intestine are known as fenestrated capillaries. Capillaries in the area of spleen, bone marrow, and liver are termed as sinusoidal capillaries. In addition to the gap between the cells, these capillaries have pores that enable the exchange of molecules between fluid and tissues. Hence, the study of blood flow with the permeability concept is essential. Additionally, the deposit of cholesterol, fatty substances, and clogging blood clots on the lumen artery serves as a porous medium. Chakravarty et al. [21] have proposed a mathematical model for the steady and unsteady flow of Newtonian fluid in a porous irregular stenosed artery with body acceleration and obtained analytical solution for flow velocity, wall shear stress, flow flux, and flow resistance. Ogulu and Abbey [22] simulated a mathematical model for the oscillatory flow of blood in a stenosed artery along with heat transfer and external magnetic field. Their studies showcased that in addition to magnetic field impact, the effects of heat transfer also significantly influence the flow characteristics. Treating blood as a cross model, Zaman et al. [23] applied a modified form of Darcy law for the pulsatile flow of blood through a porous saturated overlapping stenosed artery. In their study, it is found that the increased permeability parameter is directly proportional to the flow velocity, flow flux, and wall shear stress. The development of porous depletes bolus growth and results in the decrease of flow resistance and trapped bolus.

Although research on the flow of non-Newtonian fluid in the stenosed arterial system has evolved, the study regarding the inclined magnetic field on Jeffrey fluid with wall slip velocity is a nascent field within blood rheology. The current study attempts to bridge this research lacuna. This mathematical study presents a theoretical model that reveals essential aspects of Jeffrey fluid which experience particles spin at the wall. Obtained flow governing non-linear equations are transformed into linear equations by using suitable transformation and assumptions. These equations are resolved by appropriate integral transforms and exact solutions are derived for fluid and particle velocity, flow flux, wall shear stress, and flow resistivity. Validations of these results have been done through numerical simulation and graphical illustrations.

2. Mathematical model formulation
Consider an axisymmetric, fully developed, laminar flow of Jeffrey fluid in a rigid tube. Also, the flow of blood is pulsative in nature with density $\rho$ and constant viscosity $\mu$ in a cylindrical tapered porous artery. Flowing blood is usually a non-Newtonian electrically conducting fluid suspended with magnetic nanoparticles. The flow medium is a cylindrical tube of length $L$ inclined at an angle $\alpha_1$ is denoted in figure 1. An external magnetic field inclined at an angle $\alpha_2$ is applied to the inclined vessel wall. Assuming the cylindrical polar coordinates $(x, \theta, z)$ in the radial, circumferential, and axial direction respectively with $x = 0$ as the axis of symmetry of the arterial tube. Also, the flow field $V$ is a function of velocity components $(V_x, V_\theta, V_z)$. The stenosis geometry [24] is defined as

$$R(z) = \begin{cases} 
    d_1(z) [1 - \eta_l \{L_l^{m-1}(z - d_0) - (z - d_0)^m\}], & d_0 \leq z \leq d_0 + L_0 \\
    d_1(z), & \text{otherwise}
\end{cases}$$

(1)
where \( d_1(z) = R_0 + \eta z \), \( R_0 \) is the arterial radius in the normal arterial region, \( \eta = \tan \phi \) is the tapering measure, \( \phi \) is the tapering angle with \( \phi = 0 \), \(< 0 \) and \( > 0 \) denoting the non-tapered stenosed region, converging region and diverging region respectively. Further, \( d_0 \) is the spot of stenosis, \( L_0 \) is the length of stenosis, \( m(\geq 2) \) is a shape parameter representing the constriction structure and in particular \( m = 2 \) defines the stenosis is symmetric in shape. The parameter \( \eta_1 = \frac{\delta \eta m \eta^m}{R_0 L_0^{(m-1)^2}} \), where the crest of the stenosis is \( \delta \) and it is at \( z = d_0 + \frac{L_0}{m^{1/m-1}} \) as shown in fig.1 & 2 [25].

![Arterial geometry](image)

**Figure 1.** Arterial geometry (a) tapered and stenosed (b) Inclined stenosed and tapered

### 2.1. Momentum equations

As the fluid flow is axisymmetric, the change in Jeffrey fluid blood characteristic is independent of an azimuthal angle. Hence the flow governing equation of streaming pulsative Jeffrey fluid in an inclined tube is given by

\[
\rho \frac{DV}{Dt} = -\nabla P + \nabla \cdot \frac{\mu}{1 + \lambda_1} (\dot{\gamma} + \lambda_2 \ddot{\gamma}) - \frac{\mu}{K_0} V + \sigma (E + V \times (B_0 + B_1)) - \rho F
\]

(2)

together with the continuity equation

\[
\nabla \cdot V = 0
\]

(3)

where \( \frac{DV}{Dt} \) is the material derivative, \( V \) is the velocity distribution, \( P \) is the pressure gradient, \( \lambda_2 \) is the retardation time, \( \lambda_1 \) is the ratio of relaxation and retardation time, \( \sigma \) is the electrical conductivity, \( E \) is the electric field intensity, \( B_0 \) is the applied magnetic field intensity and \( B_1 \) is the induced magnetic field intensity which is negligible in comparision with \( B_0 \). \( K_0 \) is the porous medium permeability and \( \dot{\gamma} \) is the strain rate.

Since the fluidic flow is axisymmetric and incompressible, the fluid velocity in the tangential direction is zero. Here, the flow field velocity constituents are independent of the circumferential ordinate \( \theta \). In addition to the above assumptions, we include the following constraints to derive the momentum equation:

1. \( \frac{Re \delta \eta m^{1/m}}{L_0} << 1 \)
2. \( \frac{Rm m^{1/m}}{L_0} \sim O(1) \)
Incorporating the concept of interaction force between the moving particles and the streaming blood, the momentum equation governing the fluid flow motion is derived as [26]

$$\rho \frac{\partial u(x,t)}{\partial t} = -\frac{\partial P}{\partial z} + \frac{\mu}{1 + \lambda_1} \left[ \frac{\partial^2 u(x,t)}{\partial x^2} + \frac{1}{x} \frac{\partial u(x,t)}{\partial x} \right] + K_s N [u_1(x,t) - u(x,t)]$$

$$- \sigma B_0^2 \cos^2 \alpha_2 u(x,t) - \frac{\mu}{K_0} u(x,t) - \rho g \sin \alpha_1$$

(4)

The initial and boundary conditions for the fluidic motion are

$$u(x,0) = 0, \quad u_1(x,0) = 0$$

$$u(R(z),t) = u_s, \quad \frac{\partial u}{\partial z}(0,t) = 0$$

(5)

Based on the Newton’s second law of motion,

$$M_p \frac{\partial u_1(x,t)}{\partial t} = K_s \{u(x,t) - u_1(x,t)\}$$

(6)

is the momentum equation governing the particles mobility where \(M_p\) is the average mass of suspended particles and \(K_s\) is the Stoke’s constant.

We further assume the pulsatile pressure gradient having \(\omega_0\) as the pulsative frequency which induces the motion of fluid as

$$-\frac{\partial P}{\partial z} = a + b \cos \omega_0 t$$

(7)

where \(a\) and \(b\) are the steady and unsteady pressure gradient part resulting due to systolic pressure and diastolic pressure. In order to get the involved physiological variables as non-dimensional quantities, presume the mean velocity of fluid as \(u_0\) and have the following transformation of variables:

$$u^* = \frac{u}{u_0}, \quad x^* = \frac{x}{R_0}, \quad d_0^* = \frac{d_0}{L_0}, \quad a^* = \frac{R_0^2 a}{\mu u_0}, \quad Re = \frac{\rho u_0 R_0}{\mu}, \quad M^2 = \frac{\sigma R_0^2 B_0^2}{\mu}$$

$$u_1^* = \frac{u_1}{u_0}, \quad R^*(z) = \frac{R(z)}{R_0}, \quad L_0^* = \frac{L_0}{R_0}, \quad b^* = \frac{R_0^2 B_0}{\mu u_0}, \quad Fr = \frac{u_0}{\sqrt{g R_0}}, \quad Da = \frac{K_0 R_0^2}{\mu u_0}$$

The parameters \(Re, Fr, Da\) and \(M^2\) represent the Reynolds number, Froude number, Darcy number and Magnetic number respectively. With the help of transformation of variables and after ignoring *, we obtain the following non-dimensional form of motion equation for fluid and particle as

$$\alpha^2 \frac{\partial u(x,t)}{\partial t} = a + b \cos t - \frac{1}{Da} u(x,t) - \frac{Re}{Fr} \sin \alpha_1 - M^2 \cos^2 \alpha_2 u(x,t) +$$

$$\frac{1}{1 + \lambda_1} \left[ \frac{1}{x} \frac{\partial u(x,t)}{\partial x} \right] + R_1 (u_1(x,t) - u(x,t))$$

(8)

$$\alpha^2 G \frac{\partial u_1(x,t)}{\partial t} = u(x,t) - u_1(x,t)$$

(9)

where the Womersley parameter is \(\alpha^2 = \frac{\rho R_0^2 \omega}{\mu}\), the particle mass parameter is \(G = \frac{M_p \mu}{\rho R_0^2 K_s}\), the particle concentration parameter is \(R_1 = \frac{KN R_0^2}{\mu}\). Also we get the stenosis geometry equation in the non-dimensional form as

$$R(z) = \begin{cases} 
   d_1(z) \left[ 1 - \eta_1 \{(z - d_0) - (z - d_0)^m\} \right] & \text{for} \ d_0 \leq z \leq d_0 + 1 \\
   d_1(z) & \text{for otherwise}
\end{cases}$$

5
3. Solution method

By considering \( \frac{x}{R(z)} = \xi \), equation of fluid and particles motion together with the constraints:

\[
\alpha^2 \frac{\partial u(\xi, t)}{\partial t} = a + b \cos t - \frac{1}{D_a} u(\xi, t) - \frac{Re}{Fr} \sin \alpha_1 - M^2 \cos^2 \alpha_2 u(\xi, t) + \frac{1}{(1 + \lambda_1) R^2(z)} \left[ \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial u(\xi, t)}{\partial \xi} \right) \right] + R_1 (u_1(\xi, t) - u(\xi, t)) \tag{10}
\]

\[
\alpha^2 G \frac{\partial u_1(\xi, t)}{\partial t} = u(\xi, t) - u_1(\xi, t) \tag{11}
\]

\[
u(\xi, 0) = 0, \quad u_1(\xi, 0) = 0
\]

\[
u(1, t) = u_s, \quad \frac{\partial u}{\partial \xi}(0, t) = 0 \tag{12}
\]

Taking Laplace transform with respect to the pulsatile coordinate \( t \) and finite Hankel transform with respect to the spacial ordinate \( \xi \) on equations (10) and (11) and using initial and the boundary conditions, we get

\[
(s^2 + A_{1n} s + A_{2n}) \mathfrak{H}_H(\beta_n, s) = \left[ \frac{a}{s} + b - \frac{s}{s^2 + 1} - \frac{Re \sin \alpha_1}{s} + h_1 \frac{u_s \beta_n^2}{s} \right] \frac{J_1(\beta_n)}{\beta_n \alpha^2 G} \tag{13}
\]

where \( h_1 = \frac{1}{(1+\lambda_1)R^2(z)} \), \( A_{1n} = \frac{1}{\alpha^2 G} [1 + G(A_{2n} + R_1)] \) and \( A_{2n} = h_1 \beta_n^2 + H a^2 \) with \( Ha^2 = \frac{1}{D_a} + M^2 \cos^2 \alpha_2 \).

Here, the Laplace transform is defined as

\[
L[f(x, t)] = \int_0^\infty f(x, t) e^{-st} dt = \mathcal{F}(x, s), s > 0 \tag{14}
\]

The Hankel transform is represented as

\[
\mathfrak{H}_H(\beta_n, s) = \int_0^1 \xi f(\xi, s) J_0(\xi \beta_n) d\xi \tag{15}
\]

where \( \beta_n \) are the roots of the zeroth order Bessel function of first kind such that \( J_0(\xi \beta_n) = 0 \).

After making inverse Laplace and Hankel transform operations on equation (13) we derive the formula for the fluid velocity as

\[
u(\xi, t) = u_s - 2 u_s \sum_{n=1}^\infty \frac{J_0(\xi \beta_n)}{\beta_n J_1(\beta_n)} \times \left\{ 1 - \frac{h_1 \beta_n^2}{\alpha^2 G} \left[ \frac{\alpha^2 G q_{1n} + 1}{q_{1n} - q_{2n}} - \frac{\alpha^2 G q_{2n} + 1}{q_{1n} - q_{2n}} \right] \right\} \]

\[
+ \sum_{n=1}^\infty \frac{J_0(\xi \beta_n)}{\beta_n J_1(\beta_n)} \alpha^2 G \times \left\{ \left( a - \frac{Re}{Fr} \sin \alpha_1 \right) \left[ \frac{\alpha^2 G q_{1n} + 1}{q_{1n} - q_{2n}} - \frac{\alpha^2 G q_{2n} + 1}{q_{1n} - q_{2n}} \right] \right\}
\]

\[
+ b \left[ \frac{\alpha^2 G q_{1n}}{q_{1n} - q_{2n}} + 1 \right] \frac{1}{q_{1n}^2 + 1} \left[ q_{1n} (e^{q_{1n}t} - \cos t) + \sin t \right] - \frac{\alpha^2 G q_{2n} + 1}{q_{1n} - q_{2n}} \frac{1}{q_{2n}^2 + 1} \left[ q_{2n} (e^{q_{2n}t} - \cos t) + \sin t \right] \tag{16}
\]
where \( q_{1n}, q_{2n} = \frac{A_{1n} \pm \sqrt{A_{1n}^2 - 4A_{2n}}}{2} \).

Using the equation (16), the velocity of suspended particles can be evaluated by the following equation:

\[
    u_1(\xi, t) = \int_{0}^{1} \beta_1 u(\xi, (t - x)) e^{-\beta_1 x} dx
\]

(17)

\( \beta_1 = \frac{1}{\alpha_2 G} \).

The shear stress at the arterial wall can be calculated using the following equation:

\[
    \tau_w(\xi, t) = \frac{1}{R(z)(1 + \lambda_1)} \frac{\partial u(\xi, t)}{\partial \xi}
\]

(18)

The volumetric flow rate of the fluid in the medium is given by

\[
    Vol(z, t) = R^2 \int_{0}^{1} \xi u(\xi, t) d\xi
\]

(19)

Using the volumetric flow rate value, the fluidic resistivity in the tube can be determined by the following equation

\[
    \Gamma = \int_{0}^{L} \frac{a + bcost}{Q(z, t)} dz
\]

(20)

4. Numerical computations and discussion

In order to understand the salient features of blood flow, this paper has studied the impact of key parameters of blood flow geometry and flow physiology. By investigating the effect of inclination angles of artery (\( \alpha_1 \)) and magnetic field (\( \alpha_2 \)), Hartmann number (\( M_2 \)), Jeffrey number (\( \lambda_1 \)), Darcy number (\( Da \)), Reynolds number (\( Re \)), Froude number (\( Fr \)), Particles concentration number (\( R_1 \)) and Particles mass number (\( G \)) on the flow physiognomies the present study explicates significance of inclination angles and magnetic field. Flow characteristics such as velocity distribution of fluid and particles, shear stress at the wall, and the flow resistance are calculated with the help of analytical expressions (22-25) by using MATLAB programming and the results are illustrated through graphical representation. The fluid flow is influenced by an external inclined magnetic field and a pulsating pressure gradient. A closed-form solution is obtained for the non-dimensional axial velocity of the fluid as well as the moving particles. Figures 2-6 describe the velocity profiles against \( \xi = x/R(z) \) for different pertinent parameters.

Figure 2(a) explains the fluid velocity versus the tube radius at different values of Hartmann number, arterial inclination angle, and magnetic field inclination angle. While the advancement of the magnetic field inclination angle enhances the flow speed, the velocity decreases with the increase of magnetic field strength. The increase of Hartmann number [ratio of Lorentz force and viscous force] increases drag force which in turn, decelerates the blood speed. Further, the enhancement of the arterial inclination reduces the flow rapidity.

With the same set of above parameters, figure 2(b) describes the velocity distribution of magnetic particles in the flow medium. Again, the magnetic number and arterial inclination affect the velocity inversely and the magnetic field inclination affects directly. The percentage of reduction of velocity with respect to the parameters is relatively less when compared with fluidic speed because the magnetic particles oppose the respective forces. Figure 3 elucidates the effect of Jeffrey number along with inclination angles on the velocity profiles of fluid and particle. On one hand, the velocity increases with the increase of Jeffrey number and magnetic field inclination angle. On the other hand, the raise of the arterial inclination angle diminishes the velocity. The significance of slip velocity at the wall has been acknowledged along with the
leaning magnitude of the artery and magnetic field in figure 4. Interestingly, the speed of both fluid and particles is more in the absence of arterial tilting and wall slip velocity. In particular, the percentage of variation is more in the particle velocity distribution.

Figure 5 explicates the significance of Reynolds number, Froude number, and Darcy number on the velocity distributions. The increase of Reynolds number reduces the flow speed; however, the increase of permeability and Froude number advances the flow velocity. The presence of particles in the fluid is displayed in fig. 6 by noting the impact of particle mass and concentration numbers along with the variation of inclination amplitude on the flow velocity distribution. The development of particles with respect to its mass and concentration in the flow stream escalates the drag force. Therefore, the velocity function is inversely proportional to the value of $R_1$ and $G$. Even though the inclination angles react significantly on the velocity profiles, their impact is negligible in comparison to the effect of particle mass and concentration level. It is observed that in all the above cases, the particle velocity distribution curve appears to be the same as fluid velocity. Here, the particle velocity is less compared to the fluid velocity due to the Lorentz force experienced by particles. Both the velocities of particle and flowing blood attains its maximum value in $\xi = 0$ and decreases smoothly to wall slip velocity on the boundary. Thus, the wall slip velocity of particles can be estimated by equation (17).

It is understood that the shear stress at the wall plays a crucial role in the formation and development of atherogenesis. Therefore, it is important to study wall shear stress in blood flow analysis. Figures 7-9 depict the variation of wall shear stress for different parameters such as Hartmann number, inclination angles, Jeffrey number, Reynolds number, and Froude number. The influence of variation of magnetic field strength along with inclination angle is described in fig. 7(a). It is found that the increase of magnetic field intensity invariably increases the shear stress at the wall to the point that the exerted magnetic force on the electrically conducting fluid interacts with the Lorentz force. Also, the inclination of the magnetic field angle is inversely related to the wall shear stress. Furthermore, the raise of arterial inclination angle increases the wall shear stress. Figure 7(b) explains the increase of Jeffrey number as well as the magnitude of the inclination of the magnetic field on wall shear stress distribution. While the Jeffrey number produces positive results, the arterial leaning parameter adversely affects the wall shear stress distribution.

The impact of arterial inclination angle on the stress distribution for the case of a converging tapering, non-tapered, and diverging tapering arteries are displayed in fig. 8(a). It is observed that the wall shear stress is directly proportional to the inclination angle of the artery that is, with the increase of arterial inclination angle the stress will reach a higher magnitude. Through fig. 8(b), it is found that an increase in the inclination angle of the artery increases the stress value. However, from fig. 9(a), it can be deduced that the inclination angle of the magnetic field reverses the concept that the increase in the magnetic field leaning parameter reduces the shear stress amplitude. Figure 9(b) illustrates that the stress at the stenotic throat is inversely proportional to the Froude number. Interestingly, in all the above cases of the tapering region, the stress distribution attains higher values for the case of the converging tapered artery than the case of a diverging and non-tapered artery.

Finally, figures 10-12 illustrates the importance of pertinent parameters on the flow resistivity of the fluid. The combined effect of inclination angles and magnetic field on the flow resistance is discussed in fig. 10(a). Higher magnetic field intensity increases the viscosity of fluid along with the interactive forces which in turn, increases the flow resistance. While the raise of inclination angles of the artery increases the flow impedance, the increase of magnetic field inclination
reduces the flow resistance. Figure 10(b) highlights the influence of Jeffrey number on the flow resistance and showcases that the increase of Jeffrey number will diminish the flow resistance. Similarly, the inclination angle of the artery influences the flow resistance. For instance, its increment will increase the flow resistance is shown in fig.11 (a). The variation of flow resistance for different inclination angles of the magnetic field is shown in fig.12 (b). Again, the increase in the magnitude of magnetic field inclination reduces the flow resistance. The impact of Reynolds number on flow resistance is shown in fig.12 (a). Here, the Reynolds number and the flow resistance increase simultaneously. It is noteworthy that the increase of the Froude number reduces the flow resistance substantially in fig.12 (b). However, figures 11 and 12 narrate the influence of variation of flow impedance on the converging tapered, non-tapered, and diverging tapered arteries.

5. Conclusion
This study investigates the analysis of flow characteristics of Jeffrey fluid streaming through a tapered inclined porous artery with mild stenosis subjected to an applied inclined magnetic field. Current study sheds light on the effects of inclination angles of both arterial tube and applied magnetic field on the flow characteristics. Furthermore, the analytical findings of this study help in determining the velocity profiles of particle and fluid, shear stress, and flow resistance in certain conditions. The study of blood flow under the influence of the magnetic field has significant application in the field of biomedicine and bioengineering. The present study can be applied to magnetic and electromagnetic therapy which helps in treating cancer, pain relief, and mitigating blood wastage during surgery. By highlighting, the role of porosity in flow behavior this work showcases the positive effect of porous in the blood flow. The existing computational analysis is given below.

(i) The axial velocity distribution function of both fluid and magnetic particles is inversely proportional to the magnetic field intensity (Hartmann number) and directly proportional to the Jeffrey number.

(ii) The amplitude of the inclination of the artery and magnetic field shows substantial changes in the axial velocity distribution of both fluid and particles.

(iii) Particle mass and concentration numbers decrease the blood velocity and magnetic nanoparticle velocity.

(iv) Increment in Reynolds number decreases the velocity profiles of both fluid and magnetic particles. However, the rise of the Froude number enhances the velocity profiles.

(v) While the behavior of shear stress at the wall is directly proportional to the Darcy number and Froude number, it is inversely proportional to the Reynolds number.

(vi) Wall shear stress surges with the rise of the tapering parameter and the stress increases in the upstream and decreases in the downstream of the stenotic region. Also, the stress reaches its maximum value near the end of stenosis.

(vii) The flow resistance rises with the increase of Hartmann number and declines with the increase of Jeffrey number.

(viii) The flow impedance decreases with the increase of the magnetic field inclination angle for both diverging and converging region. Yet, the amount of impedance is less for the diverging region compared to the converging region. However, the flow resistivity escalates with the increase of arterial inclination angle.
Figure 2. Effect of $Ha$, $\alpha_1$ and $\alpha_2$ on velocity distribution of (a) Fluid and (b) Particle

Figure 3. Effect of $\lambda_1$, $\alpha_1$ and $\alpha_2$ on velocity distribution of (a) Fluid and (b) Particle

Figure 4. Effect of $\alpha_1$, $\alpha_2$ and slip velocity $u_s$ on velocity distribution of (a) Fluid and (b) Particle
Figure 5. Effect of $Re$, $Fr$ and Permeability $Da$ on velocity distribution of (a) Fluid and (b) Particle

Figure 6. Effect of $R_1$, $G$, $\alpha_1$ and $\alpha_2$ on velocity distribution of (a) Fluid and (b) Particle

Figure 7. Wall shear stress distribution profile

(a) Variation of $\tau_w$ with $Ha$, $\alpha_1$ and $\alpha_2$  (b) Variation of $\tau_w$ with $\lambda_1$, $\alpha_1$ and $\alpha_2$
Figure 8. Wall shear stress distribution profile on converging, non-tapered and diverging tube with different inclination angles

(a) Effect of arterial inclination angle $\alpha_1$  
(b) Effect of magnetic field inclination angle $\alpha_2$

Figure 9. Wall shear stress distribution profile on converging, non-tapered and diverging tube with different Re and Fr

(a) Effect of Reynolds number $Re$  
(b) Effect of Froude number $Fr$

Figure 10. Variation of flow impedance distribution

(a) Variation of $\tau_w$ with $Ha$, $\alpha_1$ and $\alpha_2$  
(b) Variation of $\tau_w$ with $\lambda_1$, $\alpha_1$ and $\alpha_2$
Figure 11. Variation of wall shear stress distribution on converging, non-tapered and diverging tube with different inclination angles

(a) Effect of inclination angle of artery $\alpha_1$

(b) Effect of magnetic field inclination angle $\alpha_2$

---

Figure 12. Variation of flow impedance distribution on converging, non-tapered and diverging tube with different $Re$ and $Fr$

(a) Effect of Reynolds number $Re$

(b) Effect of Froude number $Fr$
References

[1] Texon M 1957 A hemodynamic concept of atherosclerosis, with particular reference to coronary occlusion A.M.A. Archives of Internal Medicine 99 418-27.
[2] Young D. F 1968 Effects of a time-dependent stenosis on flow through a tube J. Eng. Ind. 90 248-54.
[3] Chaturani P and Ponalagusamy R 1983 Blood flow through stenosed arteries. Proc. Of 1st International Conf. on physiological fluid mech. 63-67.
[4] Riahi D N , Roy R and Cavazos S 2011 On arterial blood flow in the presence of an overlapping stenosis Mathematical and computer Modelling 54 2999-3006.
[5] Womersley J R 1955 Method for the calculation of velocity, rate of flow and viscous drag in the arteries when the pressure gradient is known J. Physical. 127 553-63.
[6] Sanyal D C and Maji N K 1999 Unsteady blood flow through an indented tube with atherosclerosis Indian J. of Pure and Appl. Math. 30 951-59.
[7] Mazumdar H P, Ganguly (Habishyai) U N, Ghorai S and Dalal D C 1996 On the distribution of axial velocity pressure gradient in a pulsatile flow of blood through a constricted artery Indian J. of Pure and Appl. Math. 27 1137-50.
[8] Biswas D and Chakraborty U S 2010 Pulsatile blood flow through a catheterized artery with an axially nonsymmetrical stenosis Applied Mathematical Sciences 4 2865-80.
[9] Casson N A 1959 Flow equation for pigment oil suspensions of the printing ink type In rheology of disperse systems, Ed. Mdl, C.C., London 84-102.
[10] Chaturani P and Ponalagusamy R 1985 Non-Newtonian aspects of blood flow through stenosed arteries Bio rheology 22 521-31.
[11] Misra J C, Chandra S and Herwig H 2015 Flow of a Micropolar fluid in a micro-channel under the action of an alternated electric field. Estimates of flow in bio-fluidic devices J. of Hemodynamics 27 350-58.
[12] Akbar N S, Nadeem S, Hayat T and Hendi A A 2012 Analytical and numerical analysis of Vogel model of viscosity on the peristaltic flow of Jeffrey fluid J. Aerospace Engg. 25 1.
[13] Hussain Q, Asghar S, Hayat T and Alsaedi A 2015 Heat transfer analysis in peristaltic flow of MHD Jeffrey fluid with variable thermal conductivity Appl. Math. And Medicines 36 499-516.
[14] Ponalagusamy R 2016 Particulate Suspension Jeffrey Fluid flow in a Stenosed artery with a Particle-free Plasma Layer near the Wall Korea-Australia Rheology Journal 28 217-27.
[15] Ponalagusamy R 2017 A Two-Layered Suspension(Particle-Fluid) Model for Non-Newtonian Fluid Flow in a Catheterized Arterial Stenosis with Slip Condition at the wall of Stenosed Artery Korea-Australia Rheology Journal 29 87-100.
[16] Priyadharshini S and Ponalagusamy R 2017 Computational model on pulsatile flow of blood through a tapered arterial stenosis with radially variable viscosity and magnetic field Sadhana 42 1901-13.
[17] Gold R 1962 Magnetohydrodynamic pipe flow J. of Fluid Mechanics 13 146-56.
[18] Sud V K, Suri P K and Misra R K 1974 Effect of magnetic field on oscillating blood in arteries Studia Biophysica 47 35-52.
[19] Sharma M K, Bansal K and Bansal S 2012 Pulsatile unsteady flow of blood through porous medium in a stenotic artery under the influence of transverse magnetic field Korea-Australia Rheology Journal 24 181-89.
[20] Ponalagusamy R and Tamil Selvi 2013 Blood flow in stenosed arteries with radially variable viscosity, peripheral plasma layer thickness and magnetic field Meccanica 48 2427-38.
[21] Chakravarty S and Sanniigrahi A K 1998 An analytical estimate of the flow-field in a porous stenotic artery subject to body acceleration Int. J. of engg. science 36 1083-1102.
[22] Ogulu A and Abbey M 2005 Simulation of heat transfer on an oscillatory blood flow in an indented porous artery Int. Commun. Heat Mass Transf. 32 983-89.
[23] Zaman A, Ali N and Sajid M 2017 Numerical simulation of pulsatile flow of blood in a porous-saturated overlapping stenosed artery Math. and Comp. in simulation 134 1-16.
[24] Padma R, Ponalagusamy R and Tamil Selvi R 2019 Mathematical modeling of electro hydrodynamic non-Newtonian fluid flow through tapered arterial stenosis with periodic acceleration and applied magnetic field App.Math. and Comput. 362 124453.
[25] Srivastava N 2014 Analysis of low characteristics of the blood flowing through an inclined tapered porous artery with mild stenosis under the influence of an inclined magnetic field J. Biophysics 2014.
[26] Padma R, Tamil Selvi R and Ponalagusamy R 2019 Effects of slip and magnetic field on the pulsatile flow of Jeffrey fluid with magnetic nanoparticles in a stenosed artery Europ. Phy. J. Plus 134 221.