Hydrodynamic instability at the interface between colliding inhomogeneous mediums

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Abstract. This paper deals with the analysis of the perturbation growth at the interface
between the colliding mediums, one of them has an initial perturbation field of density. The main
difference of this hydrodynamic instability from the classical Richtmyer–Meshkov instability is
the nature of the initial disturbance. The analyzed hydrodynamic instability is caused by the
initial perturbation of the medium density.

The time evolution of interface between two mediums has been studied analytically in
nonlinear approximation. The problem has been solved for the third-order corrections to
hydrodynamic quantities. Nonlinear interactions cause the appearance of surface sound wave
near the contact discontinuity. This wave decreases the growth of the spike velocity. Spike and
bubble evolutions have a saturation stage.

The numerical calculations confirm the results of the theoretical analysis.

1. Introduction
The perturbation growth at the interface between mediums was studied in the last half century
in numerous theoretical and experimental works [1–11]. Most of them deal with the analysis
of Richtmyer–Meshkov instability (RMI) in the linear approximation [3–5, 9]. The analysis in
the nonlinear approximation is submitted in [6, 7, 10]. The time evolution of the interface is
considered, providing the interface has the curved form at the initial moment.

In the article [11] the interface instability between the mediums called Richtmyer–Meshkov–
like (RMLI) is analyzed. The main difference of this hydrodynamic instability from the classical
RMI is the nature of the initial disturbance. The RMLI is caused by the initial perturbation of
the medium density. The authors of this paper argue that they can describe the development
of this hydrodynamic instability at the interface between the mediums by their theory [12]. But
the description of this development has not been obtained yet.

The problem of the collision of two mediums, one of which has the initial perturbation field
of density, was studied in [13] in the linear approximation. The evolution of hydrodynamic
instabilities in the mediums has been analyzed.

In this paper we deal with the development of the hydrodynamic instability at the interface
between the mediums. This hydrodynamic instability is similar to RMLI. The common origin
of these instabilities is caused by the following. During the impact of two mediums two shock
waves begin to propagate in different directions within them. The interaction of shock wave
with the perturbation of the density is the cause of the entropy-vortex perturbations. These entropy-vortex perturbations cause the perturbation growth at the interface.

In this paper we analyze the perturbation growth at the interface between the mediums in the nonlinear approximation. Further this problem is solved for the first-order, the second-order and the third-order corrections to hydrodynamic quantities. We use the solution of this problem in the linear approximation obtained in [13].

Carrying out nonlinear analyses is caused by the following. The authors of the paper [14,15] suggest that the perturbation growth at the interface between the mediums is valid till a certain time moment. At the last stage the velocity growth reaches saturation and the velocity stops changing. This conclusion is based on experimental and numerical results. In this paper we confirm this conclusion and analytically demonstrate that the velocity saturation has been obtained only if we take into account the third-order correction to hydrodynamic quantities.

2. Theoretical analysis

The collision of two semi-infinite plates one of which is at rest (see Fig. 1) is studied in the paper. A spatially anisotropic density perturbation field initially exists in the incident plate (the impactor).

It is assumed that the velocity of the impactor is rather high (about 5 km/s). For this reason, the elastoplastic properties of the colliding plates can be neglected and the interaction of the shock wave with the density perturbation field in the impactor can be described by the equations of hydrodynamic [16]. The system of the equations of hydrodynamics should be supplemented by the Rankine–Hugoniot conditions at the shock front and equation of the medium state. Equation of the medium state is used in the form of the Mie–Gruneisen equation [17].

As a result of the colliding event we obtain two shock waves diverging in both plates from the contact discontinuity see Fig. 1. At the interaction of the density perturbation field with the shock wave, acoustic and entropy–vortex waves are formed behind the shock front [16]. Our analysis is carried out in the contact discontinuity system of coordinates, $x$ axis being directed towards the movement of the impactor (in the right direction) and orthogonal $y$ axis being directed upwards (see Fig. 1). Let the initial density perturbation field in the impactor be described by the expression:

$$\delta \rho_1(x,y) = A\delta \rho_1 \exp(ik_0x + ik_0y).$$  

(1)

The axis of anisotropy for such perturbation is directed along the vector $k_0 = (k_{0x}, k_{0y})$ (see Fig. 1). The shock wave moves from the right at the velocity $(d_1 - v_1)$ in the negative direction of the $x$ axis $(d_1 - v_1) < 0$. The angle between the vector $k_0$ and the $x$ axis is $\theta_0$ ($\tan \theta_0 = k_{0y}/k_{0x}$).

It was shown [13] that the regime of perturbation propagation in mediums depends on the angle $\theta_0$. In this paper we consider the angle $\theta_0$ in the entire range $(\theta_0, \pi/2)$, where $\theta_0 = \arctan \left( (v_1 - d_1) / \sqrt{c_s^2 - (v_2 - d_1)^2} \right)$. In this case only the entropy-vortex wave is formed near the contact discontinuity.

To solve the system of the hydrodynamic equations for perturbation of hydrodynamic values we need to use the condition of the velocity equality on the contact discontinuity:

$$\delta v_{2x} + \delta'' v_{2x} = -ik_0y\xi \delta v_{2y} + \delta'' v_{2x} - ik_0y\xi \delta'' v_{2y} - 2ik_0y\delta v_{2y} \delta'' \xi =$$

$$= \delta v_{3x} + \delta'' v_{3x} - ik_0y\xi \delta v_{3y} + \delta'' v_{3x} - ik_0y\xi \delta'' v_{3y} - 2ik_0y \delta v_{3y} \delta'' \xi,$$  

(2)
The expression being used is the third-order correction to the velocity:

$$\delta v_{2x} = \frac{\rho_2}{\rho_1 + \rho_2} \left[ -i \left( \frac{\rho_2 - \rho_3}{\rho_2 + \rho_3} k_{0y} - i \left( \frac{\rho_3 (k_{2x} - k_{3x})}{\rho_2 + \rho_3} - k_{2x} \right) \right) (k_{2x} - k_{3x}) (\delta v_{2x})^3 t^2 + \delta v_{2x}^{(se)} - \delta v_{2x}^{(se)} \right] \exp \{ i (2 + \varepsilon) k_{0y} y \},$$

where $$k_{2,3x}^{(3)} = \kappa_{2,3x} + \varepsilon k_{2,3x}$$, $$k_{2,3y}^{(3)} = (2 + \varepsilon) k_{0y}$$, $$\gamma_{2,3x} = (\varepsilon - 2) k_{2,3x} + 2 k_{0y}^2 \varepsilon / k_{2,3x}$$, $$\gamma_{2,3y} = - (k_{2,3x}^2 k_{0y} + 2 k_{2,3x} k_{0x} (2 + \varepsilon)) / k_{2,3x} (2 + \varepsilon)$$, and $$\delta v_{2,3x}^{(se)} = 0.5 \left( \gamma_{2,3y}^{(3)} k_{2,3x}^{(3)} - \gamma_{2,3x}^{(3)} \left( k_{2,3y}^{(3)} \right)^2 \right) / \left( \left( k_{2,3x}^{(3)} \right)^2 + \left( k_{2,3y}^{(3)} \right)^2 \right) \delta v_{2x} \delta v_{2,3x}.$$

2.3. Discussing results

As it follows from the analytical analysis nonlinear interactions cause the appearance of the surface sound wave near the contact discontinuity. This wave has the following dispersive ratio $$\Omega_2 = \kappa_2 c_2$$ and causes to take into account the second-order and the third-order corrections to hydrodynamic quantities. The $$x$$ projection of the velocity perturbation of contact discontinuity
has the following final expression: $v_{CD} = \delta v_x + \delta'' \tilde{v}_x t + \delta''' \tilde{v}_x t^2$, where $v_x$, $\delta'' \tilde{v}_x$, $\delta''' \tilde{v}_x$ from (3), (4), (5), accordingly.

As we take into account the second-order and the third-order corrections to the hydrodynamic quantities the bubble velocity growth is a monotonically increasing function of time till a certain time moment. This time moment coincides with the maximum value of the bubble velocity, $V_{b,max}$. Following [14, 15], we consider that this time moment is the bubble saturation time, $t_b$. It is suggested that the bubble velocity is equal $V_{b,max}$ for $t > t_b$.

The spike velocity growth is decreasing function of time. So, we can not obtain the value of spike saturation time, $t_s$. In this paper we consider two different values of saturation time.

3. Numerical calculations

We analyze the collision of two semi-infinite aluminium and ferrum plates. The ferrum plate has the initial perturbation field of density (1) with $\theta_0 = 1.471$, $A_{\delta \rho_1}/\rho_1 = 0.05$. The numerical analysis has been carried out in the contact discontinuity system of coordinates. The colliding velocity value is 5 km/s. The initial position of contact discontinuity is $x = 0$. This statement of the problem is equivalent to the task in the previous section. The calculations have been performed with the method from [18].

Fig. 2 demonstrates the numerically calculated profile of ferrum density near the interface at different time moments. The color of aluminium is grey. According to Fig. 7 the value of the bubble velocity is twice as many as the value of the spike velocity, approximately.

Fig. 6 shows the analytically calculated time profile of the bubble and the spike velocity obtained for the first-order, the second-order and the third-order corrections to hydrodynamic quantities. Dashed segment of the red curve corresponds to time interval $t > t_b$. The results of the theoretical analysis of the bubble velocity are in good agreement with the results obtained from the numerical calculation (see Fig. 7).

Fig. 7 demonstrates the spike and bubble amplitude as a function of time. The spike development has two stages. At the first stage ($t < t_s$) the spike velocity is decreasing function of time. At the second stage ($t \geq t_s$) spike velocity is slowly decreasing function of time. Assuming
with \(a/\lambda = 0.05\). In this case the density field has a staggered structure. For the case of the RMI we take \(a/\lambda = 0.05\), \(a\) is the perturbation amplitude, \(\lambda\) is the perturbation wavelength.

To compare RMLI and RMI we numerically analyze the case of RMLI with the following initial density perturbation of ferrum plate:

\[ \delta \rho_1(x, y) = A \delta \rho_1 \left( \cos (k_{0x} x + k_{0y} y) + \cos (k_{0x} x - k_{0y} y) \right), \quad (6) \]

with \(A \delta \rho_1/\rho_1 = 0.05\). In this case the density field has a staggered structure. For the case of the RMI we take \(a/\lambda = 0.05\), \(a\) is the perturbation amplitude, \(\lambda\) is the perturbation wavelength.

The analysis of these numerical calculations shows that the development of instability at the interface differs from the classical case of RMI. According to Fig. 2 the direction of the perturbation development is orthogonally to the wave vector of the initial perturbation (1). In the case of the classical RMI the direction of the perturbation development is orthogonally to the interface (see Fig. 3).

To compare RMLI and RMI we numerically analyze the case of RMLI with the following initial density perturbation of ferrum plate:

\[ \delta \rho_1(x, y) = A \delta \rho_1 \left( \cos (k_{0x} x + k_{0y} y) + \cos (k_{0x} x - k_{0y} y) \right), \quad (6) \]

with \(A \delta \rho_1/\rho_1 = 0.05\). In this case the density field has a staggered structure. For the case of the RMI we take \(a/\lambda = 0.05\), \(a\) is the perturbation amplitude, \(\lambda\) is the perturbation wavelength.

The bubble and the spike Figure 4. Mixing width as a function of time amplitude as a function of time for I – \(\theta_0 = \theta_0 = 1.471, \delta \rho_1 - (1)\); II – \(\theta_0 = 1.471, 1.471, \delta \rho_1 - (1)\); II – \(\theta_0 = 1.471, \delta \rho_1 - (6)\); \(\delta \rho_1 - (6)\); III – \(\theta_0 = 0.785, \delta \rho_1 - (6)\); IV – III – \(\theta_0 = 0.785, \delta \rho_1 - (6)\); IV – \(\theta_0 = 0.1, \delta \rho_1 \theta_0 = 0.1, \delta \rho_1 - (6)\); V – RMI.

– (6); V – RMI.

Figure 5. Time dependence of spike and profile of the bubble velocity and the spike bubble amplitude. Black line – \(t_s = t_b\); blue velocity obtained for the first-order, the line – \(t_s = 2.5t_b\), second-order and the third-order corrections to hydrodynamic quantities.

Figure 6. The analytically calculated time Figure 7. The analytically calculated time as a function of time for I – \(\theta_0 = \theta_0 = 1.471, \delta \rho_1 - (1)\); II – \(\theta_0 = 1.471, 1.471, \delta \rho_1 - (1)\); II – \(\theta_0 = 1.471, \delta \rho_1 - (6)\); \(\delta \rho_1 - (6)\); III – \(\theta_0 = 0.785, \delta \rho_1 - (6)\); IV – III – \(\theta_0 = 0.785, \delta \rho_1 - (6)\); IV – \(\theta_0 = 0.1, \delta \rho_1 \theta_0 = 0.1, \delta \rho_1 - (6)\); V – RMI.

– (6); V – RMI.
of contact discontinuity. Interface velocity for RMI and RMLI cases is equal. Fig. 3 shows the numerically calculated profile of density near the interface at the time moment $t = 1.7 \mu s$ for the different angles $\theta_0$. Fig. 4 demonstrates the bubble and the spike amplitude as a function of time. Fig. 5 demonstrates the mixing width as a function of time.

Let we compare results of the initial density distribution for two cases, (1) and (6), ($\theta_0 = 1.471$). The mixing width development is identical at the initial stage. At the later stage in case of (6) the bubble velocity decreases due to interaction between the bubble and the neighbouring entropy cell formed behind the shock wave (see Fig. 4). The spike growth is equivalent in both cases, provided perturbations are absent in aluminium plate ($\theta_0 = 1.471$). We note that the mixing width depends on value of $\theta_0$.

4. Conclusions

The agreement between the analytical result and numerical calculation has been obtained only due to the fact that the third-order correction to hydrodynamic quantities is taken into account. As it follows from the obtained results the second-order and the third-order corrections to the velocity are proportional to $t$ and $t^2$, accordingly. We describe the anisotropy of the process using nonlinear corrections to hydrodynamic quantities. It is shown that bubble amplitude grows with time faster than spike amplitude. The bubble velocity is a monotonous function of time till a certain time moment. This time moment coincides with the maximum value of the bubble velocity. At the last stage the bubble amplitude growth reaches saturation and the bubble velocity stops changing. Taking into account the the second and the third-order correction we can not obtain the value of spike saturation time. It is numerically shown that the spike saturation time is larger than bubble saturation time. Appearance of vortex cells depends on initial conditions, (1) and (6). These cells influence the perturbation growth of contact discontinuity.

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