Mathematical modelling of influence of the variable incidence at the supersonic flow around spherically blunted cone on the characteristics of the conjugate heat and mass transfer

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Abstract. Results of a numerical study of the three dimensional supersonic flow over a spherically blunted cone fluctuating in a pitch plane taking into account mass breaking down blowing and heat transfer between body and flow are presented. Influence of the body fluctuation with angular velocity in range of 0–100 deg/s on surface temperature distribution and heat transfer characteristics is considered. The effect of the body fluctuation on characteristics of conjugated heat and mass exchange in a thermal protection material is analyzed.

1. Introduction

The body fluctuation leads to an alteration of the flow conditions over body in contrast to case of its absence [1-6]. In previous studies [1-2] the investigation for influence of variable angles of attack (fluctuation) on aerodynamic characteristics of axisymmetric bodies was considered. In publications [3-6] the study for influence of rotation around body axis on characteristics of conjugate heat and mass transfer is performed. At a flow around a body at constant angle of attack [7, 8], the difference in thermal fluxes on the leeward and windward sides may be very considerable, which leads to a non-uniform heating. To reduce this effect, a hypersonic vehicle may be forced to oscillating motion around the longitudinal axis and in the plane of pitch.

2. Problem state

For a model of the chemically equilibrium air, the system of equations of the spatial boundary layer in the natural coordinate system has the following form within the hypothesis of the “passivity” and the equality of Lewis numbers for all components equal to 1 [3] (see Figure 1).

\[
\frac{\partial}{\partial s} (\rho w w) + \frac{\partial}{\partial n} (\rho w v) + \frac{\partial}{\partial \eta} (\rho w) = 0,
\]

\[
\rho \left( \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} + \frac{w}{r_w} \frac{\partial u}{\partial \eta} - \frac{w^2}{r_w} \frac{\partial r_w}{\partial s} \right) = -\frac{\partial P_s}{\partial s} + \frac{\partial}{\partial n} \left( \mu_s \frac{\partial u}{\partial n} \right),
\]

\[
\rho \left( \frac{\partial w}{\partial s} + v \frac{\partial w}{\partial n} + \frac{w}{r_w} \frac{\partial w}{\partial \eta} + \frac{uw}{r_w} \frac{\partial r_w}{\partial s} \right) = -\frac{1}{r_w} \frac{\partial P_s}{\partial \eta} + \frac{\partial}{\partial n} \left( \mu_s \frac{\partial w}{\partial n} \right),
\]
\[ \rho \left( \frac{\partial H}{\partial s} + v \frac{\partial H}{\partial n} + \frac{w}{r_w} \frac{\partial H}{\partial \eta} \right) = \frac{\partial}{\partial n} \left( \frac{\mu_x}{Pr} \left( \frac{\partial H}{\partial n} + \left( \frac{Pr_T - 1}{Pr_T} \right) \frac{\partial}{\partial \eta} \left( \frac{u^2 + w^2}{2} \right) \right) \right), \]

\[ P = \rho h(\gamma_{ef} - 1)/\gamma_{ef}, \quad P = P_e(s, \eta), \quad H = h + \left( u^2 + w^2 \right)/2, \]

\[ \mu_x = \mu + \Gamma \mu_T \frac{Pr}{Pr_T + \Gamma \mu_T Pr}, \]

where \( u, v, \) and \( w \) are the components of the mean-mass velocity vector in the natural coordinate system \((s, n, \eta)\), \( \rho \) is the density, \( h \) is the enthalpy, \( \Gamma \) is the intermittency coefficient, \( P \) is the pressure, \( r_w \) is the Lamé coefficient, \( H \) is the total enthalpy, \( \mu \) is the coefficient of dynamic viscosity, \( \gamma_{ef} \) is the effective adiabatic exponent, \( Pr \) is the Prandtl number. Here and in the following, the subscripts e, e0, and w correspond to the quantities at the external edge of the boundary layer, at the external boundary at the stagnation point, and on the streamlined body surface, respectively. Subscripts 1 and 2 refer to the characteristics of the sphere and cone in condensed phase, g refers to the gaseous phase on the spherical and conical parts of the body, \( \infty \) refers to the free stream gas flow at infinity; the subscripts T and 0 refer to the characteristics of turbulent transfer and the initial conditions, and the subscript \( k \) refers to the shell peripheral part.

Thermal state for a porous spherical shell \((0 < s_1 < s_2)\) (see Figure 1) is determined from the solution of an unsteady equation for energy conservation and the quasi-stationary one-dimensional equation for the filtration velocity of the cooling gas through pores of body is determined [9].

\[ \frac{\partial (\rho \gamma^{(i)} r_1 H_1 \varphi)}{\partial n_1} = 0, \]

\[ (\rho c_p)_{e1}(1 - \varphi) \frac{\partial T_1}{\partial t} = \frac{1}{r_1 H_1} \left( \frac{\partial}{\partial n_1} \left[ r_1 H_1 \beta_1 (1 - \varphi) \frac{\partial T_1}{\partial n_1} \right] \right) + \]

\[ + \frac{\partial}{\partial s_1} \left( r_1 \beta_1 (1 - \varphi) \frac{\partial T_1}{\partial s_1} \right) + \frac{\partial}{\partial \eta_1} \left[ H_1 \beta_1 (1 - \varphi) \frac{\partial T_1}{\partial \eta_1} \right] + c_{pe}^{(i)}(\rho \gamma^{(i)}) \frac{\eta_w}{r_1 H_1} \frac{\partial T_1}{\partial n_1}, \]

\[ A \mu_1 \gamma^{(i)} + B \rho^{(i)} \phi^{(i)} \psi^{(i)} = - \frac{\partial P}{\partial n_1}, \]

\[ P = \rho^{(i)} RT_1 \frac{M}{M}, \quad H_1 = \frac{R_N - n_i}{R_N}, \quad \bar{s} = \frac{s_i}{R_N}, \]

**Figure 1.** Geometric model configuration:

1 – porous spherically blunted area,
2 – conical part of graphite V-1 body.
For the body conical part \((s_A < s_1 < s_A)\) (Figure 1) the equations for the energy and mass conservation are written in a moving coordinate system by using the mathematical models [10, 11]:

\[
\rho_c c_p \frac{\partial T_2}{\partial t} - \psi^2 \frac{\partial T_2}{\partial n_1} + c_{pg}^2 G \frac{\partial T_2}{\partial n_1} = \frac{\partial}{\partial n_1} \left( \lambda_2 \frac{\partial T_2}{\partial n_1} \right) + \frac{\partial}{\partial s_1} \left( \lambda_2 \frac{\partial T_2}{\partial s_1} \right) + \frac{1}{r_s^2} \frac{\partial}{\partial n_1} \left( \lambda_2 \frac{\partial T_2}{\partial n_1} \right) \quad \text{.........(9)}
\]

\[
r_\perp = (R_N - n_1) \sin \theta + (s_1 - s_A) \sin \theta, \quad l = L_0 - x(t), \quad x(t) = \int_0^t \psi \, d\tau, \quad c_{pg}^{(1)} = b_1 + b_2 T_1,
\]

\[
\rho v_{\perp w} = \varphi_w \rho_w \left[ \frac{m_2}{m_2 - 1} c_{wB_1} + \frac{2 m_2}{m_2 - 1} c_{wB_2} + \left( \frac{m_2}{m_1 - 1} c_{wB_3} + \frac{2 m_2}{m_6 - 1} c_{wB_4} \right) \right], \quad B_i = k_{iw} \exp \left( - \frac{E_{iw}}{RT_w} \right), \quad i = 1, 4,
\]

\[
\rho v_{\perp w} = \varphi_w \sum_{i=1}^4 m_i A_{ci} \left( \frac{P_{ei} - P_{ci}}{2 \pi R T_{2 w} m_i} \right)^{0.5}, \quad P_{ci} = \frac{P_c m_{ci} m_{w}}{m_c}, \quad m_i = \sum_{\alpha=1}^{8} m_{\alpha}, \quad i = 7, 8,
\]

\[
\rho v_{\perp w} = \sum_{i=2}^8 \left( \rho v_{\perp w} \right)_w, \quad \rho v_{\perp w} = \sum_{i=2}^8 \left( \rho v_{\perp w} \right)_w, \quad G = 0, \quad P_{ci} = 10^5 \exp \left( D_i - E_i / T_{2 w} \right).
\]

where \(R_N\) is the radius of spherical nose cap, \(V_w\) is free stream velocity, \(c_{iw}, \quad i = 1, ..., 8\) are mass concentrations of the \(i\)-th component, \(E_{iw}, k_{iw} \quad i = 1, ..., 4\) are the activation energy and pre-exponent of the \(i\)-th heterogeneous reaction of the shell of the conical part of the body, \(m_\alpha, \quad \alpha = 1, ..., 8\) are the molecular weights of the components.

Initial and boundary conditions for gas and condensed phases (1)–(9) are presented in [4].

The kinetic scheme of non-equilibrium chemical reactions for chemical components \(O, O_2, N, N_2, CO, CO_2, C_1, C_3\) was considered at the interface of media at \(s_1 \geq s_A\) [4, 13].

\[
\begin{align*}
C + O_2 &\rightarrow CO_2, \quad 2C + O_2 \rightarrow 2CO, \\
C + O &\rightarrow CO, \quad C + CO_2 &\rightarrow 2CO, \\
2O + C &\rightarrow O_2 + C, \quad 2N + C &\rightarrow N_2 + C, \\
C &\leftrightarrow C_1, \quad C &\leftrightarrow C_3.
\end{align*}
\]

The balance relations for mass concentrations of the components using the Fick law for diffusion fluxes and the analogy of the heat and mass transfer processes were applied [4, 11, 12]. The rupture products are assumed to dilute weakly the aerial mixture in the boundary layer. This enables the use of the above statement for the equations in boundary layer.

Unlike papers [3–6], in this investigation the periodic oscillations of attack angle in pitch plane are considered. This motion is described by the expression:

\[
\beta(t) = \begin{cases} 
\beta_{\max} - |\omega t + \beta_{\max}| & \text{at } (n-1)T_f \leq t < nT_f / 2 \\
|\omega t - 3\beta_{\max}| - \beta_{\max} & \text{at } nT_f / 2 \leq t < nT_f
\end{cases}
\]

where \(t\) is process duration, \(n = 1, 2, ..., \), \(T_f = T_{\max} / \omega_f\) is a fluctuation period, \(\omega_f\) is angular velocity of change of angle of attack \(\beta\), \(\beta_{\max}\) is a maximum angle of attack.

The assumption is accepted that the typical linear velocity of the body fluctuation is much less than the free stream velocity:

\[
\Omega_f = \omega_f R_N / V_w < 1.
\]
3. Numerical method and input data

The system of the equations of the spatial boundary layer with regard for the laminar, transitional, and turbulent flow regions was solved. To describe the turbulent flow a two-layer model of the turbulent boundary layer was employed [13, 14]. The intermittency coefficient and the transition from the laminar flow regime to the turbulent one are described by the Dhawan–Narasimha formula [23]. At the numerical integration, $\text{Pr} = 0.72$, $\text{Pr}_T = 1$. The described model of the boundary layer was tested by comparing with experimental results of the works [16, 17], and this showed its good performance.

To test the processes of the interaction of high-enthalpy air flows with graphite surfaces the results of theoretical [18] and generalized experimental studies [19] were used.

The computations of a chemically equilibrium air flow at the variable angle of attack $-\beta_{\text{max}} < \beta < \beta_{\text{max}}$ around the sphere-blunted cone with the taper angle $\theta = 15^\circ$ were done for the conditions corresponding to the parameters $V_\infty = 3000–6000$ m/s, $\beta_{\text{max}} = 10^\circ$, flight altitude $Z = 2.5\times10^4$ m, and initial shell body width $L_0 = 0.02$ m. The kinetic constants of chemical reactions were defined in the work [12], the graphite enthalpy $h_c$ was calculated by formula [20]. For the carbon material of the conical shell, the thermo-physical coefficients were found in the work [10, 20], for the porous steel in [21].

The relaxation times in the gaseous and condensed phases were estimated according to the works [5]. Using these estimates one finds the characteristics of conjugate heat and mass transfer from the solution of quasi-stationary equations of the spatial boundary layer at different flow regimes.

The results presented below have been obtained at initial body surface $T_0 = 300$ K, $\delta = 100$ W/(m$^2\cdot$K), $M = 29$ kg/kmole, $\sigma = 5.67\times10^{-8}$ W/(m$^2\cdot$K$^4$), $R = 8.314$ J/(mole·K), $E_\gamma = 85715$ K, $E_b = 93227$ K, $D_\gamma = 18.69$, $D_b = 23.93$, $\varepsilon_2 = 0.9$, $\varphi = 0.34$, $b_1 = 965.5$, $b_2 = 0.147$, $A_\gamma = 0.24$, $A_\delta = 0.023$. The thermo-physical characteristics of the porous bluntness corresponded to porous steel: $\varepsilon_1 = 0.8$, $\lambda_1 = 2.92 + 4.5\times10^{-3}\times T_1$ W/(m·K), $\rho\cdot c_{pl} = (1252 + 0.544\times T_1)\times10^3$ J/(K·m$^3$), $A = 2.3\times10^{11}$ 1/m$^2$, $B = 5.7\times10^{-5}$ l/m [21]. The thermo-physical characteristics of the body conical part corresponded to the solid graphite V-1 [22]. The problem (7), (9) was solved numerically locally-one-dimensional splitting method [23].

4. Numerical solution results and their analysis

Figure 2 shows the time dependence of the surface temperature in the section $s_1/R_N = 4.6$ for a body velocity of $V_\infty = 3000$ m/s and for a fixed angle of attack $\beta_{\text{max}} = 10^\circ$ ($\omega_t = 0$ deg/s) (curves 1 and 2) and $\omega_t = 40$ deg/s (curve 3). Curve 1 corresponds to $\eta = 180^\circ$ (the windward side), and curve 2 corresponds to $\eta = 0^\circ$ (the leeward side). In the presence of oscillatory motion (see Figure 2, curve 3), the point with coordinates $s_1/R_N = 4.6$ and $\eta = 180^\circ$ moves from windward side to the leeward side and back. Figure 2 illustrates this effect, leading to a decrease in the temperature difference compared with the case of $\omega_t = 0$ deg/s.

This fact can be similarly demonstrated using the surface temperature distributions along the circumferential coordinate presented in Figure 3, for time $t = 1$ s. Here curves 1, 2, 3, 4 correspond to $\omega_t = 0$, 20, 40, 100 deg/s. From Figure 3 it can be seen that at this moment of time $\eta = 180^\circ$ corresponds to the windward side for curves 1, 2, 4 and the leeward side for curve 3. In the absence of body oscillations, the maximum temperature difference along the circumferential coordinate has reached (see Figure 3, curve 1). During oscillatory motion, the behavior of curves 2, 3, 4 has a similar character with curve 1 ($\omega_t = 0$ deg/s).

However, at certain moments of time, this behavior of the curves presented in Figure 3, changes at $\omega_t \neq 0$ (see Figure 4), since local extremums appear on the lateral surface of the body.
Figure 2. The time dependence of the surface temperature at $s_i/R_N = 4.6$ and $V_\infty = 3000$ m/s. Curve 1 corresponds to $\omega_T = 0$ deg/s and $\eta = 180^\circ$, 2 -- $\omega_T = 0$ and $\eta = 0^\circ$, 3 -- $\omega_T = 40$ deg/s and $\eta = 180$. 

Figure 3. The surface temperature distribution along the circumferential coordinate at $s_i/R_N = 4.6$, $t = 1$ s, $V_\infty = 3000$ m/s. Curves 1, 2, 3, 4 correspond to $\omega_T = 0, 20, 40, 100$ deg/s respectively.

Figure 4. The surface temperature distribution along circumferential coordinate at $s_i/R_N = 4.6$, $\omega_T = 40$ deg/s, $V_\infty = 3000$ m/s. Curves 1, 2, 3 corresponds $t = 1.07, 1.08, 1.09$ s respectively.

Figure 5. The surface temperature distribution along circumferential coordinate at $s_i/R_N = 4.6$, $\omega_T = 40$ deg/s, $V_\infty = 6000$ m/s. Curves 1, 2, 3, 4 corresponds $t = 9.63, 9.67, 9.69, 9.73$ s respectively.

In Figure 4, the calculation results for the moments of time $t = 1.07, 1.08, 1.09$ s (curves 1, 2, 3 respectively) are presented. Local maxima occur 2 times during the period of the body oscillation $T_f$ and exist for several tens of milliseconds. It can be seen from Figure 4, the temperature difference on the
surface in the circumferential direction $\eta$ at such a moment is of the order of several degrees. At later moments of time and at higher body flight speeds ($V_\infty = 6000$ m/s), the temperature difference in the circumferential direction can increase substantially (see Figure 5). Curves 1, 2, 3, 4 in Figure 5 correspond to the moments of time $t = 9.63, 9.67, 9.69, 9.73$ s respectively.

Conclusion
It follows from the study that with a supersonic flow around a body that makes periodic oscillations in the pitch plane, the temperature distribution of the body surface has a smaller distinction along the circumferential coordinate $\eta$ than in case of constant angle of attack. It has been shown that periodic oscillations of a spherically blunted cone lead to the appearance of local temperature extremums on the lateral surface of the body, in contrast to the case of flow around at a fixed angle of attack.

References
[1] Hoffman G H and Platzer M F 1962 AIAA J. 4 370–1
[2] Telionis D and Gupta T 1977 AIAA J. 15 974–83
[3] Efimov K N, Ovchinnikov V A and Yakimov A S 2017 Thermophys. Aeromech. 24 657–69
[4] Efimov K N, Ovchinnikov V A and Yakimov A S 2018 AIAA J. 56 743–51
[5] Efimov K N, Ovchinnikov V A and Yakimov A S 2018 High Temp. 56 239–48
[6] Efimov K N, Ovchinnikov V A and Yakimov A S 2018 J. Eng. Phys. Thermophys. 91 1199–210
[7] Zinchenko V I, Efimov K N and Yakimov A S 2007 J. Eng. Phys. Thermophys. 80 751–9
[8] Zinchenko V I, Efimov K N and Yakimov A S 2011 High Temp. 49 81–91
[9] Zinchenko V I, Efimov K N and Yakimov A S 2007 High Temp. 45 681–7
[10] Grishin A M, Golovanov A N, Zinchenko V I, Efimov K N and Yakimov A S 2011 Mathematical and Physical Modeling of Thermal Protection (Tomsk: Tomsk State University) p 358
[11] Polezhaev Yu V and Yurevich F P 1976 Thermal Protection (Moscow: Energiya) p 392
[12] Grishin A M and Fomin V M 1984 Conjugate and Unsteady Problems of the Mechanics of Reacting Media (Novosibirsk: Nauka) p 320
[13] Patankar S V and Spalding D B 1967 Heat and Mass Transfer in Boundary Layers (London: Morgan–Grampian Press) p 138
[14] Cebecci T 1970 AIAA J. 8 2152–6
[15] Dhawan D and Narasimha R 1958 J. Fluid Mech. 3 418–36
[16] Feldhuhn R N 1976 AIAA Paper 119 pp 1-9
[17] Widhopf G F and Hall R 1972 AIAA J. 10 1318–25
[18] Gofman A G and Grishin A M 1984 J. Appl. Mech. Tech. Phys. 25 598–605
[19] Baker R L 1977 AIAA J. 15 1391–7
[20] Buchnev L M, Smyslov A I, Dmitriev I A 1987 High Temp. 25 1120–5
[21] Alifanov O M, Tryanin A P and Lozhkin A L 1987 J. Eng. Phys. Thermophys. 52 340–6
[22] Nagorny V G, Kotosonov A S., Ostrovsky V S, Dymov B K, Lutrov A I, Anufriev Yu P, Barabanov V N, Belogorskiy V D, Kuteynikov A F, Virgiliev Yu S, Sokker G A 1975 Properties of Carbon-based Construction Materials: Handbook ed V P Sosedov (Moscow: Metallurgiya) p 336
[23] Samarskiy A A 1971 Introduction to the theory of difference schemes (Moscow: Science) p 552