A review on the wavelet methods for sonar image segmentation

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Abstract
The sonar image segmentation is needed such as in underwater object orientation and recognition, in collision prevention and navigation of underwater robots, in underwater investigation and rescue, in seafloor object seeking, in seafloor salvage, and in marine military affairs like torpedo detection. The wavelet-based methods have the ability of multiscale and multiresolution, and they are apt at edge detection and feature extraction of images. The applications of these methods to the sonar image segmentation are increasingly raised. The contents of the article are to classify the sonar image segmentation methods with wavelets and to describe main ideas, advantages, disadvantages, and conditions of use of every method. In the methods for sonar image region (or texture) segmentation, the thought of multiscale (or multiresolution) analysis of the wavelet transform is usually combined with other theories or methods such as the clustering algorithms, the Markov random field, co-occurrence matrix, Bayesian theory, and support vector machine. In the methods for sonar image edge detection, the space–frequency local characteristics of the wavelet transform are usually utilized. The wavelet packet-based and beyond wavelet-based methods can usually reach more precise segmentation. The article also gives 12 directions (or development trends predicted) of the sonar image segmentation methods with wavelets which should be researched deeply in the future. The aim of writing this review is to make the researchers engaged in sonar image segmentation learn about the research works in the field in a short time. Up to now, the similar reviews in this field have not been found.

Keywords
Sonar image, image segmentation, edge detection, wavelet transform

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Introduction
With the rapid growth of the ocean development activity and the urgent need of underwater engineering and military affairs, more and more fields are relating to underwater object positioning and recognition, such as collision prevention and navigation of underwater robots, underwater investigation and rescue, underwater course survey and renovation, seafloor object search, seafloor salvage, seabed hunt for treasure like manganese nodule, seabed hydrothermal solution detection, combustible ice detection, underwater archaeology studies, seabed construction, marine organisms observation and research, fish school detection

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and fish catch, and marine military affairs like torpedo detection. The above needs are motivating the rapid development of ocean detection technologies. There are two important types of underwater detection technologies: optical imaging and acoustical imaging. Optical imaging can obtain the image with better quality than acoustical imaging. Unfortunately, most ocean waters are rather turbid and optical imaging is not applicable in turbid water. Deep ocean water (undisturbed) has 6–15 m visibility, while nearshore waters have typically only 1–6 m visibility. Moreover, optical imaging is not applicable to long-range detection. The detection range for optical imaging is below 100 m (using electromagnetic waves like radars). In contrast, acoustical imaging is applicable to long-range detection even though water is turbid. The detection range for acoustical imaging can be thousands of meters even above 10,000 m, for example, 11,000 m for the full ocean depth multibeam system SeaBeam 3012. In fact, the acoustical signal is currently the only form of energy radiation that can be transmitted over long range in the ocean. Hence, sonars (underwater acoustic devices) have become the main and unique means in the case of turbid water or long-range detection. Imaging sonars, an important class of sonars, have a good prospect in underwater object positioning and recognition.

In the practical application of imaging sonars, sonar image (also called as underwater acoustical image) segmentation is a basis and normally an important step. The aim of the sonar image segmentation is to divide a sonar image into some different parts and obtain the parts of interest. The sonar image segmentation is a nuclear problem in low-level image processing and it is a difficult problem acknowledged in the field of sonar image processing. The methods for sonar image segmentation are researched by the scholars in many countries. There are many sonar image segmentation methods reported such as: (1) the ones based on fuzzy c-means (FCM) clustering; (2) the ones based on the Markov random field (MRF) model; (3) the ones based on the snake model and the level set; (4) the ones based on spectral clustering; (5) the ones based on the watershed transform; (6) the ones based on the expectation maximization; (7) the ones based on the fractal geometry; (8) the ones based on deep learning; (9) the ones based on wavelet (references are discussed in the following); and so on.

Among the above methods, there is a class of the methods for sonar image segmentation that are wavelet-based. The methods are suitable for image segmentation due to their good spatial/frequency localization ability and multi-scale (multiresolution) analysis ability. Gonzalez and Woods also stated “The appeal of such an approach (wavelet-based method, the authors add the note) is obvious-features that might go undetected at one resolution may be easy to detect at another.” Hence, the wavelet-based methods for sonar image segmentation are attractive. The aim of writing this review is to make the researchers engaged in sonar image segmentation learn about the research works in the field in a short time. Up to now, we have not found the similar reviews in this field.

The contents of the article are to classify the wavelet-based methods for sonar image segmentation; describe main ideas, advantages, disadvantages, and conditions of use of every method; and try to give 12 directions (or development trends predicted) of the wavelet-based methods for sonar image segmentation which should be researched deeply in the future. Because different classification results can be obtained according to different classification criteria, the classification of sonar image segmentation methods involved in the article is not absolute and not necessarily reasonable completely. The aim of the classification in the article is only for the sake of the structure requirements and easy description. In addition, owing to our negligence, some important works may not be involved in the article.

The remaining parts are arranged as follows. A brief description of sonar images is given in “Brief description of sonar images.” A comprehensive review of the sonar image segmentation methods with wavelets is given in “Common wavelet-based methods for sonar image segmentation,” in “Wavelet packet-based methods for sonar image segmentation,” and in “Beyond wavelet-based methods for sonar image segmentation.” A summary of the article and 12 directions (or development trends predicted) of the wavelet-based methods for sonar image segmentation are given in “Conclusion and discussion.”

### Brief description of sonar images

There are many varieties of sonar images. They can be divided into different types according to different criteria. According to imaging sonar types, they can be divided into the forward looking sonar image, side scan sonar image, synthetic aperture and inverse synthetic aperture sonar image, underwater acoustic lens image, underwater acoustic holography image, and so on. According to target type, they include many varieties of sonar images of shipwrecks, wrecked aircrafts, frogmen, naval mines, pipelines, telecommunication cables and optic fiber cables, underwater experimental equipment and facilities, seafloor, manganese nodule, hydrothermal solution, combustible ice, and various marine organisms. Figure 1(a) is a shipwreck sonar image. As a comparison, an underwater optical image of the concrete structure surface crack is given in Figure 1(b).

Table 1 shows the general comparison between underwater optical imaging (underwater optical images) and underwater acoustical imaging (sonar images). In Table 1, the “Low,” “High,” and “Bad” are the descriptions compared with the ones of optical imaging (optical images) in the air. The “More low,” “More high,” and “More bad” are the descriptions compared with the ones...
In general, a sonar image from a target lying in seabed contains a bright region, a dark (shadow) region, and a bottom reverberation region, and a sonar image from a target in sea water contains a bright region and a volume reverberation region.

Generally speaking, sonar images commonly have these disadvantages such as serious speckle noise, low contrast, and low resolution. Above reasons do not make the segmentation methods of optical, medical, and radar images applicable to sonar images well.

Common wavelet-based methods for sonar image segmentation

Basic concepts of wavelets

The wavelet transform is a branch of mathematics gradually developed from the 1980s. In the development of wavelet transform, the most important characters are A Haar, J Morlet, Y Meyer, S Mallat, J Daubechies, J Strömberg, A Grossman et al. From the view of mathematics, the wavelet transform is another brilliant example of the perfect combination of pure and applied mathematics after Fourier transform. It enjoys the reputation of the mathematical microscope. From the view of applied science, the wavelet transform is a major breakthrough in methods in computer applications, signal processing, image processing, nonlinear science, and engineering technology in recent years. It is one of the most powerful and the most widely used tools for time–frequency analysis and has widely been used in the field of signal and image processing.\(^\text{61,62}\) Suppose that \(L^2(R)\) is the square integrable space on the real number set \(R\). For the continuous signal (or function) \(f(t)\in L^2(R)\), then

\[
(W_{\psi}f)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi * \left( \frac{t - b}{a} \right) dt
\]

is called the wavelet transform of the signal (or function) \(f(t)\). Here \(\psi(t)\) is a small wave with limited duration and therefore called as a wavelet or the mother wavelet function, \(a > 0\) is the scale factor and \(b \in R\) is the translation factor, * stands for conjugate.\(^\text{63}\)

The above is the basic concept of the wavelet transform for the continuous one-dimensional signal and continuous transform. In practical application, the wavelet transform for the discrete signal and discrete transform (usually written as the DWT) is used. The DWT can be carried out by means of the filter bank with various frequency bands. Therefore, The DWT is also called the wavelet decomposition.

An image can be regarded as a two-dimensional (2-D) signal composed of rows and columns. The wavelet decomposition of an image can translate into the wavelet decomposition of the one-dimensional signals (rows and columns). After the one-layer wavelet decomposition of an image, the low-frequency component in the horizontal
and vertical directions (LL), low frequency in the horizontal direction and high frequency in the vertical direction (LH), high frequency in the horizontal direction and low frequency in the vertical direction (HL), and high-frequency components in horizontal and vertical directions (HH) are obtained. Such wavelet decomposition can continue. For example, the LL can be decomposed into the LLLL, LHLL, HLLL, and HHLL by means of the above way. Such wavelet decomposition is shown in Figure 2. Figure 2(a) is a schematic diagram of the two-layer wavelet decomposition, and Figure 2(b) is the decomposition of Figure 1(a) corresponding to Figure 2(a) based on DB2 wavelet function. Only LL is decomposed and decomposed once in Figure 2(a) and Figure 2(b). If it is needed, the LL can be decomposed repeatedly. There a more detailed introduction to the application of the wavelet transform (decomposition) in image processing, as given in Gonzalez and Woods.58

It is clearly seen in Figure 2(b) that the LLLL reflects the overall information of the image, and the others reflect detailed information of the image.

The wavelet transform is an important tool of the multi-resolution (multiscale) analysis. It can decompose an image by multiresolution (multiscale), expand an image into a specific series, and form layered images with different resolutions (scales). The ability for the multiresolution (multiscale) analysis of the wavelet transform leads to the good space–frequency local characteristics. The good space–frequency local characteristics of the wavelet transform are very suitable for image processing.50–58

**Research status of sonar image region (or texture) segmentation via the wavelet transform**

A given image can be analyzed at various resolution levels via the wavelet transform. For a sonar image, since different textures are recorded with different resolutions, therefore the wavelet coefficients in different sub-bands work efficiently for texture analysis and classification. Different textures have different values of the energy in different detail sub-bands. The method for characterizing the energy by means of the amplitude of the wavelet coefficients in the sub-bands had been proposed by Javidan.64,65 Each high-frequency sub-band was partitioned into the nonoverlapping wavelet coefficient blocks with size $M \times M$. The energy value for each subblock was calculated by

$$E_{\text{block}} = \sqrt{\frac{\sum_{x,y \in \text{block}} f(x,y)^2}{n}}$$

(2)

where $f(x, y)$ represents wavelet coefficient, and $n$ is the total number of coefficients in each $M \times M$ block. The feature vector composed by energy values of the subblocks was used to segment the seabed texture images. After the rough segmentation of wavelet sub-images of each layer, the rough segmentation results were fused into a fine segmented image. The rough segmentation results are mixed with the segmentation results from the fuzzy edge detector together, and final segmented image was obtained.

The fast implementation of sonar images is an important factor for real-time applications. On the basis of previous research, the standard wavelet transform was used, and the energy values of the detail sub-bands are used as features for classification in the same way.65 Following distance evaluation, an algorithm for merging connected regions of two coarse segmented images was used to construct the final segmentation result. This method is also applicable in seabed recognition system with acceptable error rates.

Figure 3 shows the experimental results of the above two methods.64,65 It can be seen that there is almost no difference in the segmentation results between the two methods. However, the latter is fast enough to incorporate in a real-time seabed recognition (or segmentation) systems with low segmentation accuracy.
Williams follows the method for calculating the energy values, in which $2/C^2$ of seabed areas was selected as the data points of the wavelet transform according to the actual situation of the sea floor. The texture information of the sonar image was completely and accurately described by means of the feature vectors which is calculated by five-layer wavelet coefficients. The spectral clustering algorithm was used to classify the feature vectors. In the spectral clustering algorithm, the K-mean clustering algorithm was used. The K-mean clustering algorithm has an inherent disadvantage that it is subjected to fall into local optimum and the clustering effect excessively depends on the choice of clustering centers. In view of this, Williams and Groen improved the above method. The K-mean clustering algorithm was replaced by the variational Bayesian Gaussian mixture model without supervision. The experimental results show that the improved method reaches relatively ideal segmentation results.

In the experiments from Cobb and Principe, the excellent performance of wavelet coefficients in characterizing the texture information of sonar images has also been proven.

Each of seabed types has its unique texture feature. Williams proposed the method for using a unique Gaussian mixture model to express seabed texture types established on wavelet coefficients. The original sonar image was replaced by a vector of wavelet-based features and the seabed was classified by means of the Bayesian theory. The method is a method for seabed classification. However, it can also be applied to seabed sonar image segmentation.

Baussard proposed a seabed sonar image segmentation (and classification) method based on the wavelet transform and the Bayesian framework. The sonar images was transformed by the method based on the 2-D steerable Riesz wavelets, and then the low-frequency approximate sub-band coefficients and the high-frequency detailed sub-band coefficients were obtained. The coefficients were modeled based on the conventional generalized Gaussian distribution (GGD) in the high-frequency sub-band. The coefficients were modeled based on the finite mixture of Gaussian model in the low-frequency sub-band. The method retains the low-frequency approximate sub-band omitted by Karine et al. It can improve the classification accuracy of the seabed with the similar feature such as sand and silt. Hence the seabed sonar images can be segmented more accurately. However, the pixels on the boundary of two regions could be classified falsely.

On the whole, the methods in Williams and Song et al. can be applied to segmenting seabed sonar images with obvious texture feature.

Karine et al. divided the sonar image into tiles using a sliding window, and then operated wavelet transform for each tile, statistically modeled the wavelet sub-band coefficients. The GGD and the $\alpha$-stable distribution

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**Figure 3.** Comparison of the experimental results: (a) original image in Javidan, (b) segmentation result in Javidan and Eghbali, (c) segmentation result in Javidan, (a') original image in Javidan, (b') segmentation result in Javidan and Eghbali, and (c') segmentation result in Javidan.
parameters were used as the features of the sonar images. The feature information was classified by means of the support vector machine (SVM) or k-nearest neighbor (KNN) algorithm, and the seabed sonar image was segmented by means of the Bayesian framework. Karine et al. assigned four combinations: the SVM combined with the GGD, the KNN combined with the GGD, the SVM combined with the $\alpha$-stable distribution parameters, and the KNN combined with the $\alpha$-stable distribution parameters. These four combinations were used to segment or classify seabed sonar images of posidonia, ripples, rock, sand, and silt. The experiments prove that the SVM combined with GGD leads to the highest classification rates, even the rock classification rate of 100\%. This combination is suitable for seabed image segmentation. However, when different combinations are used to classify different types of the seabed, the effects are often different. In practice, sometimes the type of the seabed is not known in advance, hence it is difficult to select the best proper combination.

The method in Karine et al. can reach more precise image segmentation or classification if learning samples are enough and sample selection is reasonable. However, the method needs learning samples. The method can be applied to the seabed sonar images with texture feature and learning samples obtained in advance.

There is the sonar image segmentation method proposed by Collet et al. which combines the multiresolution analysis with the MRF. The MRF is a relatively mature and recognized model for image segmentation. The pixel distribution in an image is considered as an MRF. In the MRF, the interface relationship between the different pixels can be considered fully and the relevance between the pixel and its neighbor is built up. Hence the problem of image segmentation is successfully turned into the predicted and controlled problem. Because the model parameters and spatial constraints of the MRF are less and the MRF has a good robustness for noises, the MRF has obtained wide attentions from the researchers in image segmentation field. However, the conventional MRF cannot meet the need of sonar image segmentation in serious noise contamination. This method builds up the relationship between spatial scales based on the MRF and realizes the multiresolution MRF for image segmentation. However, the description of the inner scale relationships is not enough in this method.

Wu et al. built up an MRF model in low-resolution image scales and caught local weak feature information of sonar images. The connections of father and son nodes between adjacent scales were built up to express the connection of inner scale coefficients. The method effectively resists to noise and the precision of the segmentation results is improved.

The wavelet transform has some limitations. One limitation is that the description of direction property is not enough. The other limitation is that the translation is sensitive. In view of this, Xia et al. introduced the dual-tree complex wavelet transform (DTCWT). To an extent, the method can improve the accuracy of image segmentation. However, the method is more complex.

On the whole, the methods in Collet et al., Wu et al., and Xia et al. can take full advantage of priori knowledge of sonar images and can reaches more accurate segmentation results. However, the methods are more complex. Hence the methods are suitable for low real-time requirements.

There is the method for segmenting textured backscattering strength sonar images that was presented by Karoui et al. The method takes advantage of the multiresolution analysis. In the method, the textures were distinguished by directly measuring the similarity between co-occurrence statistics on the most informative frequency sub-band obtained by the wavelet transform, and the segmentation results from different scales were fused together so as to obtain final segmentation results. The multiscale fusion can improve the quality of the image segmentation. The method can be applied to segmenting seabed sonar images with obvious interregional texture feature. The texture image segmentation method based on the co-occurrence matrix has been recognized widely. The method has also been applied to sonar image segmentation, but the segmentation result is subjected to the speckle noise in sonar images.

Celik and Tjahjadi used the data in a wavelet transform resolution and between wavelet transform resolutions to extract a feature vector of each pixel. In the method, the dimension of the feature vectors was reduced by means of the principal component analysis (PCA), and the feature vector was divided into different types by means of the K-mean clustering algorithm so as to realize the segmentation of the side-scan sonar images. The method can automatically estimate the clustering number, and it has strong resistance to interference. Moreover, the PCA dimension reduction also brings the algorithm speed up to an acceptable range. In addition, the noise need not be reduced before segmenting the sonar images in the method. Hence the method can retain image details well. There are slight but negligible shifts on the boundaries of adjacent regions in the results of the proposed method, which are mainly due to the multiresolution structure of undecimated discrete wavelet transform. The method can be applied to the segmentation of seabed texture images.

Xia et al. proposed a segmentation method in which FCM clustering of the multiscale statistical information and MRF in the wavelet domain are used. In the method, before building up wavelet sub-band MRF, the FCM clustering algorithm for pre-segmentation was used. The method raises convergence rate of subsequent wavelet sub-band MRF so as to realize the stable and accurate segmentation of sonar images. The method requires the FCM clustering number known and it is suitable for the situation with the segmentation area number known.
Research status of sonar image edge detection via the wavelet transform

Targets in images can be represented by their edges. Hence, the image edge detection is equivalent to the image region segmentation. The multiscale characteristics of the wavelet transform can be used for image edge extraction. Jia and Li transformed the sonar image by means of DB4 wavelet. The edges in low-frequency components of images were detected by means of the edge detection operator. The method is easy to implement. The advantage of the method is to reduce the influences of sonar image noise on edge detection. However, because high-frequency components of images are omitted, some edges can be lost and some edges orientation is not accurate.

The morphological wavelet transform comprehensively utilizes nonlinear filtering characteristics of mathematical morphology and multiscale (multiresolution) wavelet analysis, and it has a better ability of resistance to noise and detail preservation. Hence the method is suitable for edge detection. Shen et al. proposed an improved morphological median wavelet transform based on Hong et al. The transform was applied to sonar image edge detection. The experimental results show that the method can do well in detecting sonar image edges.

On the whole, the methods in Jia and Li and Shen et al. can be applied to segmenting sonar images with low speckle noise, obvious and continuous edges.

Wavelet packet-based methods for sonar image segmentation

Basic concepts of wavelet packets

In equation (1), the bigger the values of the scale factor are, the wider the time- (space-) windows become and the lower the center frequencies become. In this situation, wavelets can detect at a high resolution the low-frequency components. The smaller the values of the scale factor are, the narrower the time- (space-) windows become and the higher the center frequencies become. In this situation, wavelets can detect at a low resolution the high-frequency components. Hence, there is a problem that the high frequency is always with a low resolution. For this reason, in the orthogonal wavelet transform, only the low-frequency components in signals (images) are decomposed further, and the high-frequency ones are not. Hence, the wavelet transform only has the ability to represent low-frequency information in signals (images). However, in practice, there are many abrupt changes in nonstationary mechanical vibration signals, seismic ones and biomedical ones, and many edges and textures in some of images. For this reason, the wavelet packets came into being. The wavelet packets were pioneered by RR Coifman, Y Meyer, and MV Wickerhauser. They proposed the concept of the orthogonal wavelet packets on the basis of the study for orthogonal wavelet bases. Wavelet packets are a generalization of orthogonal and compactly supported wavelets. In wavelet packet transform (decomposition), the high-frequency components in signals (images) are further decomposed at a high resolution. Moreover, this decomposition is both nonredundant and omissionless. Hence, wavelet packet transform (decomposition) is more suitable for the time- (space-) frequency localization analysis of the signals (images) with many frequency information. Figure 4 is the wavelet packet transform (decomposition) corresponding to Figure 2 based on DB4 wavelet function. It is seen from Figure 4 that the high-frequency components in the image are also decomposed further, which is different from Figure 2. There a more detailed introduction to application of the wavelet packet transform (decomposition) in image processing in Gonzalez and Woods.
Research status of sonar image segmentation via wavelet packet

The wavelet packet transform (decomposition) is more suitable for image edge detection and texture analysis.\textsuperscript{89–91} We have also seen a report of the texture sonar image segmentation using wavelet packet transform (decomposition). Karoui et al. comprehensively utilized co-occurrence matrices, wavelet frames, and Gabor filters to obtain the texture statistics for application of level-set methods to sonar image segmentation.\textsuperscript{92} In their article, a set of distributions of the energy of the image wavelet packet coefficients were computed for different bands for the Haar wavelet. The wavelet packet transform can provide more detailed decomposition of high-frequency parts as needed, hence it is more suitable for analyzing and the segmenting texture sonar images. However, the calculation amount of the method is bigger. The method is suitable for low real-time requirements and texture sonar images.

Beyond wavelet-based methods for sonar image segmentation

Basic concepts of beyond wavelets

Due to some advantages over the Fourier transform, the wavelet and wavelet packet transforms (decompositions) are widely used in various fields of image processing.\textsuperscript{50–58,93} The wavelet and wavelet packet bases are not the best tools for image representation because they can only express the position and characteristics of the singular points and cannot fully characterize geometrical features such as multidirectional edges and textures in images.\textsuperscript{93} Do and Vetterli proposed that an excellent tool for image representation is supposed to meet the following natures: (1) multiresolution, (2) localization, (3) critical sampling, (4) directionality, and (5) anisotropy.\textsuperscript{94} It is obvious that the 2-D wavelet and wavelet packet transforms (decompositions) only meet some of the above natures. To seek the better tools for image representation and more efficiently represent and process high-dimensional spatial data such as images, the beyond wavelet transform (also named as multiscale geometric analysis) is proposed and quickly became a research hot spot. It can meet all of the above natures for image representation and has achieved great success in image processing.\textsuperscript{93} The beyond wavelet transform is regard as the union of several “wavelet transforms” with geometrical features and an extension of the wavelet transform. The beyond wavelet transform, which includes the ridgelet, curvelet, bandelet, contourlet, beamlet, surfacelet transform, and so on, has caught the attention of the researchers in the field of image segmentation.\textsuperscript{61,93,95} There a more detailed introduction to the beyond wavelet transform in Yan and Qu.\textsuperscript{95}

Research status of sonar image segmentation via beyond wavelets

Due to some excellent natures, the beyond wavelet transform is very suitable for image processing such as denoising, compression, and feature extraction.\textsuperscript{93} We have also seen some reports of the sonar image segmentation using the beyond wavelet transform. The curvelet transform (CVT) has generated increasing interest in the community of applied mathematics and signal processing over the past years. It is a multiscale directional transform which allows an almost optimal non-adaptive sparse representation of image edges.\textsuperscript{96} Therefore, it can represent edge feature and curve singularity much more efficiently than the wavelet and wavelet packet transforms. Yoon and Kim proposed an effective edge enhancement method based on the CVT for object recognition in the sonar image.\textsuperscript{97} In the method, the maximum value was calculated by the ridgelet coefficients at each angular line which is derived from the sub-step of the CVT. The real edge direction was determined by local maxima selection after finding the azimuth of this value.

The non-subsampled contourlet transform (NSCT)\textsuperscript{98} can realize flexible decomposition with multiscale, multidirectional, and translational invariant. It has an ability of better edge capture and expression. Wang et al. quoted the superior wavelet modulus maximum method of optics image edge detection based on the NSCT to obtain the modulus maximum point in each scale directional sub-band.\textsuperscript{99} In their method, each sub-band threshold was adaptively determined by means of the intra-class variance minimization method. After threshold process, the edge image of each scale directional sub-band of an image was obtained. Finally, the edges inner a scale and between scales are fused to obtain the edge image with single-pixel wide. The edges obtained by the method are relatively complete. The number of pseudo-edge points is less. But the algorithm is more complex, it is suitable for low real-time requirements. Li et al. combined the NSCT with the ideas of region segmentation.\textsuperscript{100} In their method, the K-mean clustering algorithm was used to segment the shadow regions, and the modulus maximum position in high frequency was searched to much more accurately determine the image edges. Then the image edges in a scale and between scales were fused. At last, the image was segmented by means of the regional growth method. The operational load of the NSCT is lager. It is suitable for low real-time requirements. Huo et al. combined the NSCT with the gray-level co-occurrence matrix (GLCM).\textsuperscript{101} In their method, the image feature was extracted in the NSCT field to make up the flaws of inadequate detailed texture expression during extracting the GLCM texture feature. These two textures feature were combined to generate a multidimensional eigenvector of each pixel. Their method can improve the accuracy of the image segmentation.

On the whole, the sonar image segmentation methods based on the beyond wavelet transform are with a big
calculation amount, and they are suitable for the low real-time and high-precision segmentation requirements.

Conclusion and discussion

Wavelet methods applied to sonar image segmentation rise people’s extensive interest and attention. The wavelet methods for sonar image segmentation in the past more than 20 years are summarized in the article. In the views of image segmentation, the closed areas and edges of the objects are equivalent. If one is known, then the other is also known. If one of them is known, then the segmentation of the sonar image can be realized. Hence, in the article, the wavelet methods of sonar image segmentation are divided into two classifications: sonar image region segmentation (including texture) and sonar image edge detection. In addition, the wavelet packet and the beyond wavelet transforms have a better performance in sonar image segmentation, hence the wavelet packet-based and beyond wavelet-based methods for sonar image segmentation are briefly overviewed in the article.

The essence of human visual capture objects (targets) is the image segmentation. The process of human visual capture objects (targets) is a multiscale analysis process. The wavelet transform is a multiscale (multiresolution) analysis tool. It can capture the local features of the image in multiscale (multiresolution) and more accurately position the image details. In addition, the wavelet transform can position both in the spatial domain and in the frequency domain of the image. It is convenient to search and position the image details. Hence, the wavelet transform is very suitable for the (sonar) image segmentation. It will make a great of prospects in the (sonar) image segmentation. With regard to the wavelet methods for the sonar image segmentation, we think that the following directions are worthy of further study.

(1) The sonar images contain speckle noises. Hence, the sonar image segmentation is a problem of resistance to speckle noise. De-noising is the strength of the wavelet transform. While using the wavelet transform, de-noising and segmentation of the sonar images can merge together. Therefore, the sonar image segmentation methods of resistance to speckle noise could be obtained if the wavelet transform is used.

(2) Because of speckle noise, the target edges (boundaries) in the sonar images are usually discontinuous. The closed target edges (boundaries) are obtained by means of the image segmentation methods based on deformable contour such as snake model. Combing the wavelet transform with the image segmentation method based on deformable contour expects to generate precise methods for sonar image segmentation.

(3) Due to speckle noise, the target edges (boundaries) in the sonar images are generally discontinuous. It can generate distorted edges and pseudo-edges. The wavelet transform is apt at detecting and recognizing the edges. If only using the wavelet transform, distorted edges and pseudo-edges will also be detect. Besides, the edges detected are also discontinuous. These consequences will lead to an inaccurate segmentation of sonar images. In the view of this, combing the wavelet transform with the simple segmentation methods based on the regions such as entropy threshold methods expects to generate simple and economic methods for sonar image segmentation.

(4) The DTCWT has the properties of translation invariance and direction. It is suitable for image edge detection. We think that the DTCWT has a greater prospect in sonar image segmentation.

(5) If the Ridgelet transform, the CVT, the surfacelet transform, the wedgelet transform, the bandelet transform, and the contourlet transform are creatively applied to the sonar image segmentation, and precise segmentation methods of sonar images will be obtained. Meanwhile, we think that the beyond wavelet transform has a greater prospect in the sonar image segmentation.

(6) The wavelet transform is subjected to multiscale (multiresolution) analysis. A pulse-coupled neural network is an artificial neural network for image processing. Combing the wavelet transform with a pulse coupled neural network expects to generate precise methods for sonar image segmentation.

(7) The wavelet transform has an aptitude for image feature extraction. Deep learning can omnidirectionally percept and memorize the image features so as to precisely classify (or segment) the images if there are a large number of learning samples and sample selection is reasonable. In the cases of a large sample of the images obtained, combination of the two expects to generate good and precise methods for sonar image segmentation.

(8) The wavelet packet transform is subjected to analysis high-frequency components of the images. Therefore, it is more suitable for (sonar) image edge detection. The wavelet packet transform has a greater prospect in the sonar image segmentation.

(9) The sonar images contain speckle noises. De-noising before segmenting is a reasonable scheme. Combing the wavelet transform with some excellent filter methods such as the block-matching 3-D algorithm expects to generate sonar image segmentation methods with resistance to speckle noise.

(10) In the occasion of high real-time requirements, fast algorithms and parallel realization of the wavelet transform for sonar image segmentation are two important research aspects. The progresses in these two aspects will promote the application of the wavelet transform for sonar image segmentation.

(11) In the light of specific problems for sonar image segmentation, constructing the appropriate wavelet basis is a difficult and challenging problem by face to face. The progresses in this aspect will promote the application of the wavelet transform for sonar image segmentation.
(12) In the portable and embedded occasion with high real-time requirements, the hardware realization of the wavelet transform for sonar image segmentation such as digital signal processing and field-programmable gate array is significant and actual. The progresses in this aspect will promote the application of the wavelet transform for sonar image segmentation.

Although the wavelet transform is very suitable for sonar image segmentation, and current works are encouraging, and the better methods for sonar image segmentation based on the wavelet transform will emerge later, we are also aware of the deficiencies of the segmentation methods base on the wavelet transform. In the conceptual of mathematics, the wavelet transform of an image is an infinite series expansion of the image. Infinite polynomials cannot be considered in practical application. Hence, some high-frequency information is inevitably lost during detecting the sonar image edges by means of the wavelet transform. To an extent, the accuracy of edge detection is influenced. In addition, in the light of the specific application, the appropriate construction of the wavelet basis is a difficult and intimidating problem.

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