Parity violating superfluidity in ultra-cold fermions under the influence of artificial non-Abelian gauge fields

Kangjun Seo\textsuperscript{1,2}, Li Han\textsuperscript{1} and C. A. R. Sá de Melo\textsuperscript{1}

\textit{1. School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA and 2. Department of Physics, Clemson University, Clemson, South Carolina 29634, USA}

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We discuss the creation of parity violating Fermi superfluids in the presence of non-Abelian gauge fields involving spin-orbit coupling and crossed Zeeman fields. We focus on spin-orbit coupling with equal Rashba and Dresselhaus (ERD) strengths which has been realized experimentally in ultra-cold atoms, but we also discuss the case of arbitrary mixing of Rashba and Dresselhaus (RD) and of Rashba-only (RO) spin-orbit coupling. To illustrate the emergence of parity violation in the superfluid, we analyze first the excitation spectrum in the normal state and show that the generalized helicity bands do not have inversion symmetry in momentum space when crossed Zeeman fields are present. This is also reflected in the superfluid phase, where the order parameter tensor in the generalized helicity basis violates parity. However, the pairing fields in singlet and triplet channels of the generalized helicity basis are still parity even and odd, respectively. Parity violation is further reflected on ground state properties such as the spin-resolved momentum distribution, and in excitation properties such as the spin-dependent spectral function and density of states.

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Parity violating phenomena are very rare in physics, but a classical example is known from particle physics, where parity violating processes of the weak interaction were proposed \textsuperscript{1} and observed in the decay of \textsuperscript{60}Co several decades ago \textsuperscript{2}. In this case, the weak interactions allow for parity violation, but the particle kinetic energies are parity even, reflecting the inversion symmetry of their space. The Standard Model of particle physics, which is a non-Abelian gauge theory, incorporates parity violations and postulates that for nuclear beta decay parity is maximally violated. Other examples of parity violation exist for instance in condensed matter physics, where parity breaking is associated with crystals without inversion symmetry \textsuperscript{3} or with crystals which have inversion symmetry initially, but can develop spontaneously permanent electric polarization through lattice distortions leading to ferroelectric materials \textsuperscript{4}. However, examples of parity breaking in superfluids, such as those encountered in nuclear, atomic, condensed matter and astrophysics are hard to find, and to our knowledge there seems to be no confirmed example in nature.

Recently, it has been possible to create non-Abelian gauge fields in ultra-cold atoms via artificial spin-orbit (SO) coupling of equal superposition of Rashba \textsuperscript{5} and Dresselhaus \textsuperscript{6} h\textsubscript{R}(k) = v\textsubscript{R}(-k\textsubscript{y} \hat{x} + k\textsubscript{z} \hat{y}) and Dresselhaus \textsuperscript{6} h\textsubscript{D}(k) = v\textsubscript{D}(k\textsubscript{y} \hat{x} + k\textsubscript{z} \hat{y}) terms, leading to the equal-Rashba-Dresselhaus (ERD) form \textsuperscript{7,8} h\textsubscript{ERD}(k) = v\textsubscript{R} k\textsubscript{y} \hat{y}, where v\textsubscript{R} = v\textsubscript{D} = v/2, for which parity preserving superfluidity is possible \textsuperscript{9,11}. Other forms of SO fields, such as the Rashba-only or Dresselhaus-only cases, require additional lasers and create further experimental difficulties \textsuperscript{12}, while several theory groups have investigated the Rashba-only case \textsuperscript{13,14} due to the connection to earlier condensed matter literature \textsuperscript{17,19}.

The current Zeeman-SO Hamiltonian created in the laboratory is

\[ H\textsubscript{ZSO}(k) = -h\textsubscript{z} \sigma\textsubscript{z} - [h\textsubscript{y} + h\textsubscript{ERD}(k)] \sigma\textsubscript{y} \] (1)

for an atom with center-of-mass momentum k and spin basis |↑⟩, |↓⟩. The fields \( h\textsubscript{y} = -\Omega\textsubscript{R}/2, h\textsubscript{y} = -\delta/2, \) and \( h\textsubscript{ERD}(k) = v\textsubscript{R} k\textsubscript{y} \hat{y} \) can be controlled independently. Here, \( \Omega\textsubscript{R} \) is the Raman coupling and \( \delta \) is the detuning, which can be adjusted to explore phase diagrams as achieved in \textsuperscript{87}Rb experiments \textsuperscript{2}, or to study the high-temperature normal phases of Fermi atoms \textsuperscript{20,21}.

In this letter, we show that ultra-cold Fermi superfluids in the presence of non-Abelian gauge fields consisting of artificially created spin-orbit and crossed Zeeman fields described in Eq. (1) can produce a parity violating superfluid state when interactions are included. However, unlike the case of the Standard Model where parity breaking is driven by the weak force, in our case, parity breaking is driven by the effects of the non-Abelian gauge field on the kinetic energy. To illustrate the lack of parity in physical observables, we analyze spectroscopic quantities such as the elementary excitation spectrum, momentum distribution, spectral function and density of states in the superfluid state.

\textbf{Hamiltonian:} To analyze parity violation in ultra-cold Fermi superfluids, we start from the Hamiltonian in momentum space as

\[ H\textsubscript{0} = \sum_{k\textsubscript{s}} \psi\textsuperscript{†}\textsubscript{k}(k) H\textsubscript{0}(k) \psi\textsubscript{k}(k), \] (2)

where \( H\textsubscript{0}(k) = [K(k) \mathbf{1} - \mathbf{h}\textsubscript{eff}(k) \cdot \sigma] \) with \( K(k) = k^2/2m - \mu \) being the single particle kinetic energy relative to the chemical potential \( \mu \); the vector-matrix \( \sigma \) describes the Pauli matrices \( \sigma\textsubscript{x}, \sigma\textsubscript{y}, \sigma\textsubscript{z} \); and \( \mathbf{h}\textsubscript{eff}(k) \) is the effective magnetic field with components \( |h\textsubscript{x}(k), h\textsubscript{y}(k), h\textsubscript{z}(k)| \)
and $\psi_\alpha^\dagger (k)$ is the creation operator for fermions with spin $s$ and momentum $k$. In the ERD case, which is readily available in ultra-cold atoms, the effective magnetic field is simply $h_{\text{ERD}}(k) = [0, h_y + h_{\text{ERD}}(k), h_z]$, where $h_y$ and $h_z$ are Zeeman components corresponding to the detuning $\delta$ and the Raman coupling $\Omega_R$, while $h_{\text{ERD}}(k) = v_F k_x$ is the spin-orbit field. We define the total number of fermions as $N = N_\uparrow + N_\downarrow$, and the induced population imbalance as $P_{\text{ind}} = (N_\uparrow - N_\downarrow)/N$. We choose our scales through the Fermi momentum $k_F$ defined from $N/V = k_F^3/(3\pi^2)$, leading to the Fermi energy $\epsilon_F = k_F^2/2m$ and the Fermi velocity $v_F = k_F/m$.

**Generalized Helicity Basis:** The matrix $H_0(k)$ can be diagonalized in the generalized helicity (GH) basis $| k, \alpha \rangle \equiv \Phi_\alpha (k)|0\rangle$ via a momentum-dependent SU(2) rotation generated by the unitary matrix

$$U_k = \begin{pmatrix} u_k & v_k \\ -v_k & u_k \end{pmatrix},$$

where the normalization condition $|u_k|^2 + |v_k|^2 = 1$ is imposed to satisfy the unitarity condition $U_k^\dagger U_k = 1$. The corresponding eigenvectors are the spinors $\Phi_\alpha (k) = U_k^\dagger \Psi (k)$, where $\Psi (k) = [\psi_\uparrow^\dagger (k), \psi_\downarrow (k)]$ is expressed in terms of $\psi (k) = [\psi_\uparrow^\dagger (k), \psi_\downarrow^\dagger (k)]$ by the relations $\Phi_\alpha (k) = u_k \sqrt{\epsilon_\alpha^+} - v_k \sqrt{\epsilon_\alpha^-} + u_k \epsilon_\alpha^+ + v_k \epsilon_\alpha^-$. The coherence factor $u_k = \sqrt{1 + \frac{h_y}{|h_{\text{eff}}(k)|}}$ is chosen to be real without loss of generality, and $v_k = -e^{i\varphi_k} \sqrt{\frac{1}{2} \left( 1 - \frac{h_y}{|h_{\text{eff}}(k)|} \right)}$ is a complex function with phase $\varphi_k$ defined by $\varphi_k = \text{Arg}[h_{\uparrow \downarrow}(k)]$. The complex field $h_{\uparrow \downarrow}(k) = h_x(k) + ih_y(k)$ has components $h_x(k)$ and $h_y(k)$ along the $x$ and $y$ directions, respectively. The magnitude of the effective field is $|h_{\text{eff}}(k)| = \sqrt{h_x^2 + |h_{\uparrow \downarrow}(k)|^2}$. In the ERD case $h_x(k) = 0$, and the ratio $h_{\uparrow \downarrow}(k)/|h_{\text{eff}}(k)| = e^{i\varphi_k} = -i\text{sgn}[h_y(k)]$, where $h_y(k) = h_y + u_k k_x$.

The generalized helicity spins $\alpha = (\uparrow, \downarrow)$ are aligned or antialigned with respect to the effective magnetic field $h_{\text{eff}}(k)$, and the corresponding eigenvalues of $H_0(k)$ are $\epsilon_\alpha^\pm (k) = \epsilon_\alpha (k) - \mu$ and $\xi_\alpha^\pm (k) = \epsilon_\alpha^\pm (k) - \mu$. Here, the helicity energies are simply $\epsilon_\alpha (k) = K(k) - |h_{\text{eff}}(k)|$ and $\epsilon_\alpha (k) = K(k) + |h_{\text{eff}}(k)|$. In the specific case of ERD coupling with non-zero detuning ($h_y \neq 0$) the effective field is $h_{\text{eff}}(k) = h_y + h_{\text{ERD}}(k) k_x$, with magnitude $|h_{\text{eff}}(k)| = \sqrt{h_y^2 + (h_y + v_F k_x)^2}$ and parity violation occurring along the $x$ axis. This is illustrated in Fig. 1(a) where for finite $h_y$ (non-zero detuning $\delta$) the generalized helicity bands $\epsilon_\alpha^\pm (k)$ do not have well defined parity in momentum space. As seen in Fig. 1(a)-(b), parity is preserved for $v \neq 0$ if $h_y = 0$ (zero detuning). While as noted in Fig. 1(c)-(d), parity is violated for $v \neq 0$, if $h_y \neq 0$ (finite detuning). Similar parity violation along the $x$ axis occurs for other mixtures of Rashba and Dresselhaus terms as long as $h_y \neq 0$.

**Interactions and Order Parameter:** In order to understand the underlying physics of this system, it is important to rewrite the interaction Hamiltonian in the generalized helicity basis. The starting interaction is $\mathcal{H}_I = -g \sum_q b_\alpha^\dagger (q) b_\alpha (q)$, where the pair creation operator with center of mass momentum $q$ is $b_\alpha^\dagger (q) = \sum_k \psi_\alpha^\dagger (k + q/2) \psi_\alpha^\dagger (-k + q/2)$, can be written in the helicity basis as $\mathcal{H}_I = -g \sum_{q, \alpha \beta} Q_{\alpha \beta} B_{\alpha \beta}^\dagger (q) B_{\alpha \beta} (q)$, where the indices $\alpha, \beta, \gamma, \delta$ cover $\uparrow$ and $\downarrow$ pairs. Pairing is now described by the operator

$$B_{\alpha \beta} (q) = \sum_k \Lambda_{\alpha \beta} (k, k_\perp) \Phi_\alpha (k_\perp) \Phi_\beta (k_\perp)$$

and its Hermitian conjugate, with momentum indices $k_\perp = \pm \mathbf{k} + \mathbf{q}/2$. The matrix $\Lambda_{\alpha \beta} (k_\perp, k_\perp)$ is directly related to products of coherence factors $u(k_\perp)$, $v(k_\perp)$ (and their complex conjugates) of the momentum dependent SU(2) rotation matrix $U(k_\perp)$. Seen in the GH basis, the interactions reveal that the center of mass momentum $k_\perp + k_\perp = \mathbf{q}$ and the relative momentum $k_\perp - k_\perp = 2k_\perp$ are coupled and no longer independent, and thus do not obey Galilean invariance. The interaction constant $g$ is related to the scattering length via the Lippmann-Schwinger relation $V/g = -V_{\text{in}}/(4\pi a_s) + \sum_q 1/(2\epsilon_q)$.

From Eq. 3 it is clear that pairing between fermions of momenta $k_\perp$ and $k_\perp$ can occur within the same helicity band (intra-helicity pairing) or between two different helicity bands (inter-helicity pairing). For pairing at zero
center-of-mass momentum \( \mathbf{q} = 0 \), the order parameter for superfluidity is the tensor \( \Delta_{\alpha\beta}(\mathbf{k}) = \Delta_0 \alpha_{\alpha\beta}(\mathbf{k},-\mathbf{k}) \), where \( \Delta_0 = -g \sum_\beta (B_{\alpha\beta}(0)) \), leading to components:

\[
\Delta_{++}(\mathbf{k}) = \Delta_0(u_k u_\alpha - v_{-\alpha}) \text{ for total helicity projection } \lambda = +1, \quad \Delta_{\pm\mp}(\mathbf{k}) = -\Delta_0(v_k u_\alpha + v_{-\alpha}) \quad \text{and} \quad \Delta_{\mp\pm}(\mathbf{k}) = \Delta_0(v_k u_\alpha + v_{-\alpha}) \text{ for total helicity projection } \lambda = 0, \quad \text{and} \quad \Delta_{-\mp}(\mathbf{k}) = -\Delta_0(u_k u_{-\alpha} - v_k v_{\alpha}) \text{ for total helicity projection } \lambda = -1.
\]

Parity is violated in \( \Delta_{\alpha\beta}(\mathbf{k}) \) since they do not have well defined parity for non-zero spin-orbit coupling and crossed Zeeman fields \( h_y \) and \( h_z \).

However, we may still define singlet and triplet sectors in the generalized helicity basis, which are even and odd in momentum space respectively for any value of \( h_y \).

The singlet sector is defined by the scalar order parameter \( \Delta_{S,0}(\mathbf{k}) = [\Delta_{++}(\mathbf{k}) - \Delta_{-\mp}(\mathbf{k})]/2 \) corresponding to \( \lambda = 0 \). While the triplet sector is defined by the vector order parameter \( \Delta_{T,\lambda}(\mathbf{k}) \), by its generalized helicity components \( \Delta_{T,\pm 1}(\mathbf{k}) = [\Delta_{++}(\mathbf{k}) + \Delta_{\mp\pm}(\mathbf{k})]/2 \) corresponding to \( \lambda = 0; \Delta_{T,0}(\mathbf{k}) = [\Delta_{++}(\mathbf{k}) - \Delta_{\mp\pm}(\mathbf{k})]/2 \) corresponding to \( \lambda = \pm 1 \).

**Superfluid Ground State and Elementary Excitations:** The ground state for uniform superfluidity can be expressed in terms of fermion pairs in the GH basis as the many-body wavefunction \( |G\rangle = \prod_k \left\{ \sum_{\alpha\beta} \left[ U_{\alpha\beta}(k) + V_{\alpha\beta}(k) \tilde{\Phi}_{\alpha}(k) \tilde{\Phi}_{\beta}(-k) \right] \right\} |0\rangle \), where \( |0\rangle \) is the vacuum state with no particles.

The Hamiltonian matrix in the GH basis is

\[
\bar{H}_{\text{ex}}(\mathbf{k}) = \begin{pmatrix}
\xi_{\pm\pm}(k) & 0 & \Delta_{++}(k) & \Delta_{-\mp}(k) \\
0 & \xi_{\pm\mp}(k) & -\Delta_{++}(k) & \Delta_{-\mp}(k) \\
\Delta_{++}(k) & -\Delta_{++}(k) & -\xi_{-\pm}(k) & 0 \\
\Delta_{-\mp}(k) & \Delta_{-\mp}(k) & 0 & -\xi_{-\mp}(k)
\end{pmatrix}, \tag{5}
\]

which is traceless, showing that the sum of its eigenvalues is zero. We have obtained analytical solutions for the eigenvalues of \( \bar{H}_{\text{ex}}(\mathbf{k}) \) for arbitrary RD spin-orbit orbit and arbitrary Zeeman fields \( h_y \) and \( h_z \), but we do not list them here, because their expressions are cumbersome. However, for each momentum \( \mathbf{k} \), the determinant \( \text{Det}\ [\omega I - \bar{H}_{\text{ex}}(\mathbf{k})] \), leads to the quartic equation

\[
\omega^4 + a_3(\mathbf{k})\omega^3 + a_2(\mathbf{k})\omega^2 + a_1(\mathbf{k})\omega + a_0(\mathbf{k}) = 0. \tag{6}
\]

In the particular case of ERD spin-orbit coupling with crossed Zeeman fields, the coefficients become \( a_3(\mathbf{k}) = 0 \), the coefficient of the quadratic term takes the form

\[
a_2(\mathbf{k}) = -2 \left( K^2(\mathbf{k}) + |\Delta_0|^2 + |v_k x|^2 + |h_y|^2 + |h_z|^2 \right),
\]

while the coefficient of the linear term is \( a_1(\mathbf{k}) = -8 K(\mathbf{k}) v_k x h_y \), and lastly the coefficient of the zero-th order term is

\[
a_0(\mathbf{k}) = \xi_{\pm\pm}(k)\xi_{\pm\mp}(k)\xi_{-\pm}(k)\xi_{-\mp}(k) + |\Delta_0|^2 a_3^2(\mathbf{k}),
\]

where \( a_3^2(\mathbf{k}) = (2 K^2(\mathbf{k}) + |\Delta_0|^2 + h_y^2(\mathbf{k})) \) with \( h_y^2(\mathbf{k}) = 2|v_k x|^2 - 2|h_y|^2 - 2|h_z|^2 \). Notice that \( a_2(\mathbf{k}) \) and \( a_0(\mathbf{k}) \) have even parity, while \( a_1(\mathbf{k}) \) has odd parity and is thus responsible for the parity violation that occurs in the elementary excitation spectrum. Furthermore, parity violation occurs only when both \( v \) and \( h_y \) are non-zero, since when either \( h_y = 0 \) or \( v = 0 \) the coefficient \( a_1(\mathbf{k}) \) vanishes and parity in the elementary excitation spectrum is fully restored. From the secular equation, it follows that when \( k_z = 0 \), the coefficient \( a_1(\mathbf{k}) \) also vanishes and the excitation energies \( E_1(0, k_y, k_z) \) have the same analytical form as in the case for \( h_y = 0 \), with the simple replacement of \( h_z^2 \to h_y^2 + h_z^2 \). This property is just a consequence of the reflection symmetry of the Hamiltonian through the \( k_x = 0 \) plane. However, parity is violated, because inversion symmetry through the origin of momenta does not exist, that is, \( E_1(-\mathbf{k}) \neq E_1(\mathbf{k}) \). In contrast, quasiparticle-quasihole symmetry is preserved since the corresponding quasiparticle-quasihole energies obey the relations \( E_2(\mathbf{k}) = -E_3(-\mathbf{k}) \) and \( E_1(\mathbf{k}) = -E_4(-\mathbf{k}) \).

A simple inspection shows that gapless and fully gapped phases emerge. A gapless phase with two rings of nodes (US-2) appears when \( h_y^2 + h_z^2 - |\Delta_0|^2 < 0 \) and \( \mu > \sqrt{h_y^2 + h_z^2 - |\Delta_0|^2}^2 \). A gapless phase with one ring of nodes (US-1) occurs for \( h_y^2 + h_z^2 - |\Delta_0|^2 < 0 \) and \( |\mu| < \sqrt{h_y^2 + h_z^2 - |\Delta_0|^2}^2 \). A directly gapped phase (i-US-0) arises for \( h_y^2 + h_z^2 - |\Delta_0|^2 > 0 \) and \( \mu < -\sqrt{h_y^2 + h_z^2 - |\Delta_0|^2}^2 \) while an indirectly gapped phase (i-US-0) emerges for \( h_y^2 + h_z^2 - |\Delta_0|^2 < 0 \) and \( \mu > 0 \). Lastly, the quasiparticle excitation energy \( E_2(\mathbf{k}) \) becomes negative in certain momentum regions when \( h_y^2 > |\Delta_0|^2 \), indicating that the uniform ground state becomes less energetically favorable against the normal state.

**Phase Diagram and Thermodynamic Potential:** From the thermodynamic potential \( \Omega_{US} = -(T/2) \sum_{k,j} [1 + \exp(-E_j(\mathbf{k})/T)] \sum_k K(\mathbf{k}) + |\Delta_0|^2/g \) we obtain self-consistently the zero temperature \( (T = 0) \) phase diagram as a function of crossed Zeeman fields \( h_y \) and \( h_z \) for \( v/\nu_F = 0.4 \) at unitarity \( 1/(k_Fa_0) = 0 \) in Fig. 2(a), and at the BEC regime \( 1/(k_Fa_0) = 2.0 \) in Fig. 2(b), but a stability analysis against non-uniform phases is necessary as in the parity-preserving case \( \Omega_{11} \). At unitarity the uniform superfluid phases i-US-0, US-1, US-2 and the normal (N) phase are present in the range shown, while in the BEC regime only the d-US-0 occurs in the same range of fields. The transitions between different US phases is topological with no change in symmetry as in the parity-preserving case \( \Omega_{11} \). While the transitions from US phases to the N phase involve a change in symmetry, from broken to non-broken U(1), and are discontinuous, as seen in the insets of Fig. 2.

**Detecting parity violation:** A direct measurement of parity violation in the superfluid state can be made through the momentum distributions \( n_s(\mathbf{k}) = \)
In Fig. 2, the $T = 0$ phase diagram in the $h_y$-$h_z$ parameter space showing various uniform superfluid phases US-2, US-1, d-US-0 and i-US-0, and the normal phase for ERD spin-orbit coupling $v/\epsilon_F = 0.4$ and at (a) unitarity $1/(k_F a_s) = 0.0$ and in (b) the BEC regime $1/(k_F a_s) = 2.0$. The insets show $|\Delta_0|$ as a function of $h_y$ for $h_z/\epsilon_F = 0.2$ (dotted line); for $h_z/\epsilon_F = 0.4$ (dot-dashed line); $h_z/\epsilon_F = 0.6$ (dashed line); and $h_z/\epsilon_F = 0.8$ (solid line). In the range shown, $|\Delta_0|$ is essentially independent of $h_y$ and $h_z$ in the BEC regime.\

\[ \langle \psi_t^\dagger(k) \psi_s(k) \rangle. \] They are illustrated in Fig. 3 for US-1 superfluid with spin-orbit $v/\epsilon_F = 0.4$ and interaction $1/(k_F a_s) = 0$, in the parity-preserving case with $h_y/\epsilon_F = 0$ and $h_z/\epsilon_F = 0.7$ in (a)-(d) and in the parity-violating case with $h_y/\epsilon_F = 0.2$ and $h_z/\epsilon_F = 0.7$ in (e)-(h). At finite temperatures, the momentum distributions broaden, but parity violation is still self-evident.\

In Fig. 3, (color online) Momentum distributions ($T = 0$) $n_t(k)$ (two left-most columns) and $n_s(k)$ (two right-most columns) for $1/(k_F a_s) = 0.0$ and $v/\epsilon_F = 0.40$ at the US-1 phase. In (a)-(d) the field values are $h_y/\epsilon_F = 0$, $h_z/\epsilon_F = 0.7$, with $\mu/\epsilon_F = 0.5803$, $|\Delta_0|/\epsilon_F = 0.3592$, and $P_{\text{ind}} = 0.6592$. In (e)-(h) the field values are $h_y/\epsilon_F = 0.2$ and $h_z/\epsilon_F = 0.7$, with $\mu/\epsilon_F = 0.5871$, $|\Delta_0|/\epsilon_F = 0.3157$, and $P_{\text{ind}} = 0.6958$. The blue-dashed and red-solid lines represent cuts of $n_s(k)$ along the directions $|k_y, 0, 0 \rangle$ and $(k_x, 0, 0 \rangle$, respectively.\

Parity violation is also manifested in other momentum resolved properties such as the spectral function $A_s(\omega, k) = -(1/\pi)\text{Im}G_{ss}(i\omega = \omega + i\delta, k)$, where $G_{ss}(i\omega, k) = [i\omega 1 - \tilde{H}_{\text{ex}}(k)]^{-1}$, written in the $s = \uparrow, \downarrow$ basis. Instead, in Fig. 4 we choose to illustrate a manifestation of parity violation in the elementary excitation spectrum for the US-1 superfluid phase, and the corresponding implications for momentum integrated quantities such as the spin-resolved density of states $\rho_s(\omega) = \sum_k A_s(\omega, k)$. The most important point is that for finite spin-orbit coupling $v$ and when $h_y \neq 0$, the excitation energies $E_s(k) \neq E_s(-k)$. This implies that degenerate peaks at $h_y = 0$ (corresponding to minima or maxima of the excitation spectrum) are increasingly split with growing $h_y$. This effect is illustrated in Fig. 4 at the locations indicated by the small black arrows.\

In Fig. 4, (color online) Eigenvalues $E_{s}(k)$ and density of states $\rho_s(\omega)$ (in units of $\epsilon_F$ and $\epsilon_F^{-1}$, respectively) for $1/(k_F a_s) = 0$ and $v/\epsilon_F = 0.4$ in the US-1 phase, but close to the US-1/US-2 boundary, with parameters $h_y/\epsilon_F = 0$, $h_z/\epsilon_F = 0.7$, $\mu/\epsilon_F = 0.5803$, $|\Delta_0|/\epsilon_F = 0.3592$, and $P_{\text{ind}} = 0.6592$ in (a)-(d); and $h_y/\epsilon_F = 0.2$, $h_z/\epsilon_F = 0.7$, $\mu/\epsilon_F = 0.5871$, $|\Delta_0|/\epsilon_F = 0.3157$, and $P_{\text{ind}} = 0.6958$ in (e)-(h). Cuts of $E_s(k)$ along $(k_x, 0, 0 \rangle$ are shown in (a) and (e), and along $(k_y, 0, 0 \rangle$ are shown in (d) and (h). In panels for $\rho_s(\omega)$ a small broadening $\delta/\epsilon_F = 0.01$ is used. The black arrows indicate examples of peaks that split when parity breaking occurs for finite $h_y$.\

Conclusions: We showed that non-Abelian gauge fields consisting of spin-orbit and crossed Zeeman fields lead to parity violating superfluidity in ultra-cold Fermi systems. We derived general relations that can be applied to spin-orbit couplings involving any linear combination of Rashba and Dresselhaus terms. We focused mostly on the case of equal Rashba-Dresselhaus (ERD) spin-orbit coupling. The presence of such fields produce a superfluid order parameter tensor whose components in the generalized helicity basis are neither even nor odd under spatial inversion. Even though the elements of this tensor written in generalized singlet or triplet helicity channels have even or odd parity, respectively, the excitation spectrum does not have well defined parity, but preserves quasiparticle-quasihole symmetry. This parity violation has important experimental signatures leading to momentum distributions without inversion symmetry and to spin-resolved density of states that possess split peaks in frequency.\

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[22] Near the phase boundary between the uniform superfluid (US) and the normal (N) phases the thermodynamic potential can be expanded in even powers of $|\Delta_0|$ leading to

$$
\Omega_{US} = \Omega_N + A(h_y, h_z, v, \mu)|\Delta_0|^2 + B(h_y, h_z, v, \mu)|\Delta_0|^4 + C(h_y, h_z, v, \mu)|\Delta_0|^6,
$$

where most notably the coefficient $B$ is negative, while both $A$ and $C$ are positive. Such properties lead to a discontinuous transition from different US phases to the N phase, where the order parameter $\Delta_0$ jumps discontinuously from a finite value to zero across the phase boundaries and the symmetry changes. Furthermore, parity violation due to the simultaneous presence of $h_y$ and $v k_z$ in the helicity spectrum $\xi_\alpha(k)$ also suggests that pairing with finite center-of-mass momentum is possible and may lead to a stable modulated superfluid, such as generalized Larkin-Ovchinnikov (LO) or Fulde-Ferrell (FO), between the uniform superfluid and the normal phases.