RPA Puzzle in $^{12}C$ Weak Decay Processes

F. Krmpotić, A. Mariano and A. Samana

Departamento de Física, Facultad de Ciencias Exactas,
Universidad Nacional de La Plata,
C.C. 67, 1900 La Plata, Argentina

February 8, 2008

Abstract

We explain the origin of the difficulties that appear in a straightforward application of the QRPA in $^{12}C$, and we demonstrate that it is imperative to use the projected QRPA (PQRP A). Satisfactory results, not only for the weak processes among the ground states of the triad $\{^{12}B, ^{12}C, ^{12}N\}$, but also for the inclusive ones are obtained. We sketch as well a new formalism for the neutrino-nucleus interaction that furnishes very simple final formulae for the muon capture rate and neutrino induced cross sections.

Pacs: 13.75.Gx, 14.20.Gk, 13.40.Em

Keywords: neutrino-nucleus cross section, muon capture, beta decay, projected QRPA

New types of nuclear weak processes have been measured in recent years. They are based on neutrino and antineutrino interactions with complex nuclei and, rather than being used to study the corresponding cross sections, they are mainly aimed to inquire on possible exotic properties of neutrino themselves, such as neutrino oscillations and the associate neutrino massiveness, which are not contained in the Standard Model (SM) of elementary particles.

So, in neutrino oscillation experiments with liquid scintillators, the charge-exchange reactions $^{12}C(\nu_e, e^-)^{12}N$ and $^{12}C(\nu_\mu, \mu^-)^{12}N$, both exclusive (to the $1^+$ ground state) and inclusive (to all final states), are just tools. As such, and to be useful, the corresponding cross sections $\sigma_{e,\mu}^{exc}$ and $\sigma_{e,\mu}^{inc}$ must be accurately accounted for by nuclear structure models.

From the recent works [1, 2, 3, 4] we have learned, however, that neither RPA nor QRPA are able to explain the weak processes ($\beta$-decays, $\mu$-capture, and neutrino induced reactions) among the ground states of the triad $\{^{12}B, ^{12}C, ^{12}N\}$. In fact, in the RPA a rescaling factor of the order of 4 is needed to bring the calculations and data in agreement [1], and a subsequent
ad hoc inclusion of partial occupancy of the \( p_{1/2} \) subshell reduces this factor to less than 2 [2, 3]. But, when the RPA is supplemented with the pairing correlations in a self-consistent way, \textit{i.e.}, in the framework of a full QRPA [4], the exclusive cross sections turn out to be again overestimated by a factor of \( \approx 4 \). Moreover, Volpe \textit{et al.} [3] have called attention to “difficulties in choosing the ground state of \( ^{12}N \) because the lowest state is not the most collective one” when the QRPA is used.

In the present paper we explain the origin of the difficulties that appear in a straightforward application of the BCS approximation in a light nucleus such as \( ^{12}C \), and we demonstrate that the problem is circumvented by the employment of the particle number projected BCS (PBCS). We show simultaneously that the proton-neutron QRPA is not a recommended approach, and that the aforementioned RPA puzzle is solved within the projected QRPA (PQRP A) for the charge-exchange excitations [3]. The later approach furnishes satisfactory results not only for the weak processes among the ground states of the triad \( \{^{12}B, ^{12}C, ^{12}N\} \), but also for the inclusive weak processes. For numerical evaluation of the weak decay observables we have found it suitable to develop a new theoretical framework, which is similar to that build up by Barbero \textit{et al.} [6] for the neutrinoless double beta decay. The motivation for that and the complete formulation will be exposed elsewhere. Here we just explain the notation and exhibit the final formulæ.

The weak Hamiltonian is expressed in the form [6, 7, 8]

\[
H_W(r) = \frac{G}{\sqrt{2}} J^\dagger_\mu L^\mu(r) + h.c.,
\]

where

\[
J^\mu = \gamma_0 \left[ g_V \gamma_\mu + \frac{g_M}{2M} i \sigma_{\mu\nu} k_\nu - g_A \gamma_\mu \gamma_5 + \frac{g_P}{m_\ell} k_\mu \gamma_5 \right],
\]

is the hadronic current operator, and

\[
L^\mu_\mu(r) = \pi_{s_\mu}(p, E_\ell) \gamma_\mu (1 - \gamma_5) u_{s_\nu}(q, E_\nu) e^{i r \cdot k}
\]

is the plane waves approximation for the matrix element of the leptonic current; \( G = (3.04545 \pm 0.00006) \times 10^{-12} \) is the Fermi coupling constant (in natural units) [6],

\[
k = P_i - P_f \equiv \{k_0, k\}
\]

is the momentum transfer (\( P_i \) and \( P_f \) are momenta of the initial and final nucleon (nucleus)), \( M \) is the nucleon mass, \( m_\ell \) is the mass of the charged lepton, and \( g_V, g_A, g_M \) and \( g_P \) are, respectively, the vector, axial-vector, weak-magnetism and pseudoscalar effective dimensionless coupling constants. Their numerical values are [7, 8, 9]:

\[
g_V = 1; \quad g_A = 1.26; \quad g_M = \kappa_p - \kappa_n = 3.70; \quad g_P = g_A \frac{2M m_\ell}{k^2 + m_\ell^2}.
\]
The above estimates for $g_M$ and $g_P$ come from the (well tested) conserved vector current (CVC) hypothesis, and from the partially conserved axial vector current (PCAC) hypothesis, respectively. In the numerical calculation we will use an effective axial-vector coupling $g_A = 1$ [10, 11, 12]. The finite nuclear size (FNS) effect is incorporated via the dipole form factor with a cutoff $\Lambda = 850 \text{ MeV}$, i.e., as [13]:

$$g \to g \left( \frac{\Lambda^2}{\Lambda^2 + k^2} \right)^2.$$

To use (1) with the non-relativistic nuclear wave functions, the Foldy-Wouthuysen transformation has to be performed on the hadronic current (2). When the velocity dependent terms are neglected [12], this yields [16]:

$$J_0 = g_V - (g_A + g_{P_1}) \sigma \cdot \hat{k},$$

$$J = g_A \sigma - ig_{W} \sigma \times \hat{k} - g_{V_2}(\sigma \cdot \hat{k}) \hat{k},$$

where the following short notation has been introduced:

$$g_V = \frac{|k|}{2M}; \quad g_A = \frac{|k|}{2M}; \quad g_{W} = (g_V + g_{M}) \frac{|k|}{2M};$$

$$g_{P_1} = g_V \frac{|k|}{2M m_\ell}; \quad g_{V_2} = g_V \frac{|k|}{2M m_\ell}.$$

For the neutrino-nucleus reaction $k = p - q$, with $p \equiv \{E_\ell, p\}$ and $q \equiv \{E_\nu, q\}$, and the corresponding cross section from the initial state $|J_i\rangle$ to the final state $|J_f\rangle$ reads

$$\sigma(E_\ell, J_f) = \frac{|p| E_\ell}{2\pi} F(Z + 1, E_\ell) \int_{-1}^{1} d(\cos \theta) T_\sigma(|k|, J_f),$$

where $F(Z + 1, E_\ell)$ is the Fermi function, $\theta \equiv \hat{q} \cdot \hat{p}$, and

$$T_\sigma(|k|, J_f) \equiv \frac{1}{2J_i + 1} \sum_{s_i, M_i, s_f, M_f} \left| \int d\mathbf{r} \psi_i^*(\mathbf{r}) H_w(\mathbf{r}) \psi_f(\mathbf{r}) \right|^2,$$

with $\psi_i(\mathbf{r}) \equiv \langle \mathbf{r} | J_i M_i \rangle$ and $\psi_f(\mathbf{r}) \equiv \langle \mathbf{r} | J_f M_f \rangle$ being the nuclear wave functions. The transition amplitude can be cast in the form:

$$T_\sigma(|k|, J_f) = G^2 \left( M_V K_V + \sum_{\mu=-1,0,+1} M_\mu A_\mu K_\mu \right),$$

\footnote{The effect of the nucleon-velocity terms is of the order of a few per cent, in both the neutrino-nucleus scattering [13] and in the muon capture [14, 15].}
where

\[ M_V = \frac{4\pi}{2J_i + 1} \sum_J \left| \langle J_f | i^J j_J (|k| r) Y_J(\hat{r}) | J_i \rangle \right|^2, \]

\[ M_A^\mu = \frac{4\pi}{2J_i + 1} \sum_J \left| \sum_L \sqrt{2L + 1} \left( \begin{array}{ccc} L & 1 & J \\ 0 & -\mu & \mu \end{array} \right) \langle J_f | i^L j_L (|k| r) [Y_L(\hat{r}) \otimes \sigma]_J | J_i \rangle \right|^2, \tag{12} \]

are the nuclear matrix elements, and

\[ K_V = g_V^2 L_4 + 2g_V \overline{g}_V L_{40} + \overline{g}_V^2 L_0 \]

\[ K_A^\mu = \left\{ \begin{array}{ll} (g_A - \overline{g}_{F_2})^2 L_0 + 2(g_A - \overline{g}_{F_1})(\overline{g}_A + \overline{g}_{F_1}) L_{40} + (\overline{g}_A + \overline{g}_{F_1})^2 L_4 & ; \text{for } \mu = 0 \\ (g_A + \mu \overline{g}_W)^2 L_\mu & ; \text{for } \mu = \pm 1 \end{array} \right., \tag{13} \]

are the effective coupling constants, which contain the lepton traces

\[ L_4 = 1 + \frac{\hat{p} \cdot \hat{q}}{E_\ell E_\nu}; \quad L_{40} = \left( \frac{q_0}{E_\nu} + \frac{p_0}{E_\ell} \right) \]

\[ L_0 = 1 + \frac{2q_0 p_0 - \hat{p} \cdot \hat{q}}{E_\ell E_\nu}; \quad L_{\pm 1} = 1 - \frac{q_0 p_0}{E_\ell E_\nu} \pm \left( \frac{q_0}{E_\nu} - \frac{p_0}{E_\ell} \right), \tag{14} \]

with

\[ q_0 = \hat{k} \cdot \hat{q} = \frac{E_\nu (|p| \cos \theta - E_\nu)}{|k|}; \quad p_0 = \hat{k} \cdot p = \frac{|p| (|p| - E_\nu \cos \theta)}{|k|}. \tag{15} \]

and the momentum transfer \( k \) is along the \( z \) axis (\( \hat{k} \equiv \hat{z} \equiv \hat{e}_0 \)).

In going from the results for the neutrino-nucleus reaction cross section to that for the muon capture rate one should keep in mind that: i) the roles of \( p \) and \( q \) are interchanged within the matrix element of the leptonic current, which brings in a minus sign in the last term of \( L_{\pm 1} \), ii) the momentum transfer turns out to be \( k = q - p \), and therefore the signs on the right hand sides of (14) have to be changed, and iii) the threshold values (\( p \to 0 : k \to q, k_0 \to E_\nu - m_\ell \)) must be used for the lepton traces. All this yields:

\[ L_4 = L_{40} = L_0 = 1; \quad L_{\pm 1} = 1 \mp 1. \tag{16} \]

Finally, one should remember that instead of summing over the initial lepton spins \( s_\ell \), as done in (11), one has now to average on the same quantum number. The resulting transition amplitude \( T_\Lambda(J_f) \) is again of the form (11) but the effective charges are here:

\[ K_V(p \to 0) = (g_V + \overline{g}_V)^2 \]

\[ K_A^\mu(p \to 0) = \delta_{|\mu|, 1} (g_A - \overline{g}_W)^2 + \delta_{\mu, 0} (g_A + \overline{g}_A - \overline{g}_F)^2, \tag{17} \]
with
\[ \bar{g}_V = \frac{g_V E_\nu}{2M}; \quad \bar{g}_A = \frac{g_A E_\nu}{2M}; \quad \bar{g}_W = (g_V + g_M) \frac{E_\nu}{2M}; \quad \bar{g}_P = \bar{g}_{p2} - \bar{g}_{p1} = g_V \frac{E_\nu}{2M}. \]  

For the capture rate one gets \[ \Lambda(J_f) = \left[ \frac{E^2}{2\pi} \right] |\phi_{1S}|^2 T_\Lambda(J_f), \]

where \( \phi_{1S} \) is the muonic bound state wave function evaluated at the origin. Note that the neutrino energy is fixed by the energy of the final state through the relation:
\[ E_\nu = m_\mu - (m_n - m_p) - E_B^\mu - E_f + E_i, \]
where \( E_B^\mu \) is the binding energy of the muon in the 1S orbit.

Lastly, we mention that the \( B \)-values for the GT beta transitions are defined and related to the \( ft \)-values as \[ |g_A \langle J_f || \sigma || J_i \rangle|^2 \equiv B(\text{GT}) = \frac{6146}{ft} \text{ sec.} \]

To start the discussion on the difficulties found by Volpe et al. \[ 4 \], it should be remembered that, the pn-QRPA yields the same energy spectra for the four \((Z\pm1, N\mp1)\) and \((Z\mp1, N\pm1)\) nuclei, when the BCS equations are solved in the parent \((Z,N)\) nucleus under the constraint
\[ \sum_{k=n,p} (2j_k + 1)v_{jk}^2 = N(Z). \]

This is a physically sound zero order approximation when the nuclei in question are far from the closed shells and possess a significant neutron excess. Yet, as we show below, the use of the QRPA in \( N = Z \) nuclei is not free from care.

Let us define the quasiparticle energies relative to the Fermi levels:
\[ E_{jk}^{(\pm)} = \pm E_{jk} + \lambda_k; \quad k = p, n, \]
where \( E_{jk} \) and \( \lambda_k \) are the BCS quasiparticle energies and chemical potentials, respectively. In the particle-hole limit the energies \( E_{jk}^{(+)}/E_{jk}^{(-)} \) correspond to the single-particle (-hole) excitations for the levels above (below) the Fermi surface \[ 18 \], and to the 2p1h (1p2h) seniority-one excitations for levels below (above) the Fermi surface. In nuclei with large neutron excess \( E_{jp}^{(+)} \) and \( E_{jn}^{(-)} \) are in general quite different, but in \( N = Z \) nuclei the proton and neutron spectra are almost equal, except for the Coulomb energy displacement. As a consequence the unperturbed QRPA energies:
\[ \mathcal{E}_{j_p,j_n} = \begin{cases} E_{j_p}^{(+)} - E_{j_n}^{(-)}, & \text{for: } (Z + 1, N - 1) \\ -E_{j_p}^{(+)} + E_{j_n}^{(-)}, & \text{for: } (Z - 1, N + 1) \\ E_{j_p}^{(+)} + E_{j_n}^{(-)}, & \text{for: } (Z + 1, N + 1) \\ -E_{j_p}^{(+)} - E_{j_n}^{(-)}, & \text{for: } (Z - 1, N - 1) \end{cases} \]
are almost degenerate with \( \mathcal{E}_{j,n} \) i.e., \( \mathcal{E}_{j,n} \cong \mathcal{E}_{j,n} \), for all four odd-odd \((Z \pm 1, N \mp 1)\) nuclei. Moreover, in the case of \(^{12}\text{C}\), both the proton and the neutron Fermi energies are placed almost in the middle between the \(1p_{3/2}\) and \(1p_{1/2}\) shells. This causes an additional degeneracy, namely \( E_{1p_{3/2}} \cong E_{1p_{1/2}} \), resulting in
\[
\mathcal{E}_{3/2,1/2} \cong \mathcal{E}_{1/2,3/2} \cong \mathcal{E}_{3/2,3/2} \cong \mathcal{E}_{1/2,1/2},
\]
for \(^{12}\text{N},^{12}\text{B},^{14}\text{N}\) and \(^{10}\text{B}\). But we know that the physically sound energy sequences are:
\[
\begin{align*}
^{12}\text{N} & : \mathcal{E}_{1/2,3/2}(1p1h) < \mathcal{E}_{3/2,3/2}(2p2h) \approx \mathcal{E}_{1/2,1/2}(2p2h) < \mathcal{E}_{3/2,1/2}(3p3h), \\
^{12}\text{B} & : \mathcal{E}_{3/2,1/2}(1p1h) < \mathcal{E}_{3/2,3/2}(2p2h) \approx \mathcal{E}_{1/2,1/2}(2p2h) < \mathcal{E}_{1/2,3/2}(3p3h), \\
^{14}\text{N} & : \mathcal{E}_{1/2,1/2}(2p) < \mathcal{E}_{3/2,3/2}(3p1h) \approx \mathcal{E}_{3/2,1/2}(3p1h) < \mathcal{E}_{3/2,3/2}(4p2h), \\
^{10}\text{B} & : \mathcal{E}_{3/2,3/2}(2h) < \mathcal{E}_{3/2,1/2}(1p3h) \approx \mathcal{E}_{1/2,3/2}(1p3h) < \mathcal{E}_{1/2,1/2}(2p4h),
\end{align*}
\]
as can be easily seen from the scrutiny of the particle-hole limits for the seniority-two pn-states, which are indicated parenthetically in \((25)\). The RPA correlations are unable to remedy the situation and the degeneracy \((24)\) among four lowest \(\mathcal{E}_{pn}\) is the cause for the problems found in Ref.\(^{[4]}\) regarding the ground state of \(^{12}\text{N}\).

Well aware of all these difficulties, Cha \([19]\), in his study of the Gamow-Teller (GT) resonances, has solved the BCS equations in the daughter nuclei under the constraints
\[
\sum_{k=n(p)} (2j_k + 1)\nu^2_j = N \pm 1 \ (Z \mp 1),
\]
which gives way to the energy orderings \((25)\). Thus, the problem risen by Volpe et al.\([4]\) can, in principle, be solved by using the Cha’s recipe. However, the price to pay is that a different QRPA equation has to be worked out for each nucleus separately, \textit{i.e.}, one for the \((Z + 1, N - 1)\) nucleus and the other for the \((Z - 1, N + 1)\) nucleus, being in each case significant only the positive energy frequencies. This means that we have to abandon the nice properties of the particle-hole charge-exchange RPA, where: (1) only one RPA equation is solved for the \((Z \pm 1, N \mp 1)\) nuclei, and (2) both the positive and negative solutions are physically meaningful, with the \(\beta^+\) spectrum viewed as an extension of the \(\beta^-\) spectrum to negative energies \([20, 21, 22]\). Note also that, in order to fulfill the GT sum rule, Cha has evaluated the transition probabilities with the usual pairing factors \(u’s\) and \(v’s\), obtained from \((21)\). None of the undesirable features of the Cha’s method appear within the charge-exchange PQRPA. This approach has been presented in detail in Ref.\([3]\), and we just mention here that the PBCS quasiparticle energies read:
\[
\begin{align*}
E^{(+)}_j &= \frac{R^K_0(j) + R^K_1(jj)}{I^K(j)} - \frac{R^K_0}{I^K}, \\
E^{(-)}_j &= -\frac{R^{K-2}_0(j) + R^{K-2}_1(jj)}{I^{K-2}(j)} + \frac{R^{K-2}_0}{I^{K-2}},
\end{align*}
\]
for all four odd-odd \((Z \pm 1, N \mp 1)\) nuclei.
where $K = N, Z$ and the quantities $R^K$ and $I^K$ are defined in \[3\].

Table 1: BCS and PBCS results for neutrons. $E_j^{exp}$ stand for the experimental energies used in the fitting procedure, and $e_j$ are the resulting s.p.e. The underlined quasiparticle energies correspond to single-hole excitations (for $j = 1s_{1/2}, 1p_{3/2}$) and to single-particle excitations (for $j = 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2}$). The non-underlined energies are mostly two hole-one particle and two particle-one hole excitations. The fitted values of the pairing strengths $v_s^{\text{pair}}$ in units of MeV-fm$^3$ are also displayed.

| Shell  | $E_j^{exp}$ | $E_j^{(+)}$ | $E_j^{(-)}$ | $e_j$ | $E_j^{(+)}$ | $E_j^{(-)}$ | $e_j$ |
|--------|-------------|-------------|-------------|------|-------------|-------------|------|
| $1s_{1/2}$ | −18.72 | −5.07 | −18.72 | −7.80 | −1.28 | −18.73 | −7.24 |
| $1p_{3/2}$ | −4.94 | −4.94 | −18.85 | −2.07 | −4.95 | −22.33 | −1.51 |
| $1p_{1/2}$ | −1.09 | −1.09 | −22.70 | 2.12 | −1.09 | −26.82 | 2.16 |
| $1d_{5/2}$ | 2.72 | 2.72 | −26.51 | 6.24 | 2.73 | −30.79 | 6.26 |
| $2s_{1/2}$ | 2.72 | 2.72 | −26.51 | 6.24 | 2.73 | −30.79 | 6.26 |
| $1f_{7/2}$ | 5.81 | 5.82 | −29.61 | 8.14 | 5.83 | −33.61 | 8.17 |
| $2p_{3/2}$ | 7.17 | 7.18 | −30.98 | 11.49 | 7.16 | −35.23 | 11.47 |
| $2p_{1/2}$ | 12.89 | 12.89 | −36.69 | 17.30 | 12.89 | −41.01 | 17.32 |
| $1f_{5/2}$ | 16.72 | 16.72 | −40.52 | 19.18 | 16.72 | −44.58 | 19.21 |
| $v_s^{\text{pair}}$ | 23.16 | 23.92 | 23.92 |

The numerical calculations were performed within the $nl = (1s, 1p, 1d, 2s, 1f, 2p)$ configuration space, and for the residual interaction we adopted the delta force

$$V = -4\pi (v_s P_s + v_t P_t) \delta(r), \quad (28)$$

where $v_s$ and $v_t$ are given in units of MeV-fm$^3$.

The bare single-particle energies (s.p.e.) $e_j$ were fixed from the experimental energies of the odd-mass nuclei $^{11}C$, $^{11}B$, $^{13}C$ and $^{13}N$. That is, we assume that the ground states in $^{11}C$ and $^{11}B$ are pure quasi-hole excitations $E_{1p_{3/2}}^{(-)}$, and that the lowest observed $1/2^−, 5/2^+, 1/2^+, 3/2^+, 7/2^−$ and $3/2^−$ states in $^{13}C$ and $^{13}N$ are pure quasi-particle excitations $E_j^{(+)}$ with $j = (1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2})$. This is in essence the idea of the inverse-gap-equation.
(IGE) method, which also fixes the value of the singlet strength within the pairing channel \((v_{s}^{pair})\). We have considered the faraway orbitals 1s\(_{1/2}\), 2p\(_{1/2}\) and 1f\(_{5/2}\) as well. Their s.p.e. were taken to be that of a harmonic oscillator (HO) with standard parametrization. The single-particle wave functions were also approximated with that of the HO with the length parameter \(b = 1.67\) fm, which corresponds to the estimate \(\hbar \omega = 45A^{−1/3}−25A^{−2/3}\) MeV for the oscillator energy.

The BCS and PBCS results for neutrons are displayed in Table 1. The underlined quasiparticle energies correspond to single-hole and single-particle excitations, while the non-underlined ones are basically 2h1p and 2p1h excitations. Note that, while the first ones are fairly similar within the BCS and PBCS approaches, the last ones are quite different. (The resulting s.p.e. are also quite similar.) Analogous results are obtained for protons, with the same value of \(v_{s}^{pair}\).

The unperturbed energies \(E_{j_{p}j'_{n}}\) of lowest four \(pn\) quasiparticle states within the BCS and PBCS approximations are shown in Table 2. For comparison, the results obtained with the Cha’s method are also displayed in the same table. It is easy to see that, while the standard BCS approximation exhibits the degeneracy, the Cha’s recipe and the PBCS approach produce the energy sequences, being the energy separations between the 1p1h, 2p2h and 3p3h-like states significantly larger in the later case. This does not take us by surprise since the role of the number projection is precisely that of restoring the correct number of particles and holes.

### Table 2: Unperturbed energies \(E_{j_{p}j'_{n}}\) (in units of MeV) of lowest four proton-neutron quasiparticle states in the neighborhood of \(^{12}C\), within the approximations: (a) BCS equations are solved in \(^{12}C\) with the condition (21), (b) BCS equations are solved in daughter nuclei, employing (24) as suggested by Cha, and (c) number projected BCS (PBCS) equations are employed. The underlined energies are equal for all three cases, because they are adjusted to the experimental data via the IGP procedure.

| \(j_{p}j'_{n}\) | \(E_{j_{p} + E_{j'_{n}}}\) | \(^{12}N\) | \(^{12}B\) | \(^{14}N\) | \(^{10}B\) |
|---|---|---|---|---|---|
| \(1/2, 3/2\) | 14.0 | 16.8 | 16.8 | 11.2 | -7.0 | 35.0 |
| | | (a) | (b) | (c) | (a) | (b) | (c) | (a) | (b) | (c) |
| \(3/2, 1/2\) | 13.8 | 16.6 | 20.6 | 23.8 | \textbf{11.0} \textbf{11.0} \textbf{11.0} | -7.2 | -5.0 | -3.5 | 34.8 | 36.6 | 38.3 |
| \(1/2, 1/2\) | 14.1 | 16.9 | 18.7 | 20.4 | 11.3 | 13.1 | 14.8 | -6.9 | -6.9 | -6.9 | 35.1 | 38.7 | 42.1 |
| \(3/2, 3/2\) | 13.7 | 16.5 | 18.7 | 20.2 | 10.9 | 13.1 | 14.7 | -7.3 | -2.9 | 0.2 | 34.7 | 34.7 | 34.7 |

After having established truthful unperturbed PQRPA energies we proceed with full calculations, where the values of \(v_{s}\) and \(v_{t}\) within the particle-particle (pp) and particle-hole (ph) channels are needed. In similar calculations of double beta decaying nuclei, which possess significant neutron excess, the following procedure has been pursued: (i) \(v_{s}^{ph}\) and \(v_{t}^{ph}\) were
taken from the study of energetics of the GT resonances done by Nakayama et al.\,[25] (see also ref. \,[11]), and (ii) the \( pp \) strengths were fixed on the basis of the isospin and SU(4) symmetries as: \( v_s^{pp} = v_s^{pair} \), and \( v_t^{pp} \gtrsim v_s^{pp} \). Different to what happens in the \( N > Z \) nuclei, such a parametrization is not suitable for the \( N = Z \) nuclei, and the best agreement with data is obtained when the \( pp \) channel is totally switched off. Thus we will exhibit here only the results for \( v_s^{pp} = v_t^{pp} = 0 \), and the next three sets of \( ph \) parameters:

**Calculation I**: \( v_s^{ph} = v_s^{pair} = 23.92 \text{ MeV-fm}^3 \), and \( v_t^{ph} = v_s^{ph}/0.6 = 39.86 \text{ MeV-fm}^3 \). That is, the singlet \( ph \) strength is taken to be the same as \( v_s^{pair} \) obtained from the gap equation, while the triplet \( ph \) depth is estimated from the relation used by Goswami and Pal\,[26] in the RPA calculation of \( ^{12}C \).

**Calculation II**: \( v_s^{ph} = 27 \text{ MeV-fm}^3 \), and \( v_t^{ph} = 64 \text{ MeV-fm}^3 \). These values were suggested in refs.\,[25, 11] and have been extensively used in the QRPA calculations of \( ^{48}Ca \)\,[6, 24].

**Calculation III**: \( v_s^{ph} = v_t^{ph} = 45 \text{ MeV-fm}^3 \). This parametrization gives fairly good results for the energies of the \( J^\pi = 0^+_1 \) and \( 1^+_1 \) states in \( ^{12}B \) and \( ^{12}N \).

Table 3: Results for the energy of the \( J^\pi = 1^+_1 \) state in \( ^{12}N \) in units of MeV, the average \( B(GT) \)-value for the \( \beta \)-decay from \( ^{12}N \) and \( ^{12}B \), and the \( \mu \)-capture rates to the ground state (\( \Lambda_{\text{exc}} \)) and to all final states (\( \Lambda_{\text{inc}} \)) in \( ^{12}B \) in units of \( 10^3 \text{ sec}^{-1} \). In the upper part of the table the smallest (largest) estimates obtained in previous RPA calculations are shown. As explained in the text three different PQRPA calculations are presented. The lower and upper experimental \( B(GT) \)-value correspond to \( ^{12}N \) and \( ^{12}B \), respectively.

| RPA I | \( E(1^+_1) \) | \( B(GT) \) | \( \Lambda_{\text{exc}} \) | \( \Lambda_{\text{inc}} \) |
|------|----------------|--------------|-----------------|-----------------|
|      |                | 1.94 (2.02)  | 22.8 (25.4)     | 57 (59)         |
|      |                |              | 32.4 (34.8)     | 69 (72)         |
|      |                |              | 4.1 (7.3)       | 31 (36)         |
|      |                | 0.693 (0.776)| 8.5 (9.3)       | 40 (42)         |
|      |                | 13.74        | 2.03            | 25.4            |

| RPA I | \( E(1^+_1) \) | \( B(GT) \) | \( \Lambda_{\text{exc}} \) | \( \Lambda_{\text{inc}} \) |
|-------|----------------|--------------|-----------------|-----------------|
| PBCS  | 16.78          | 1.063        | 15.2            | 66              |
| PQRPA (I) | 17.89 | 0.568 | 7.8 | 46 |
| PQRPA (II) | 18.14 | 0.477 | 6.5 | 40 |
| PQRPA (III) | 18.13 | 0.480 | 6.5 | 42 |

Expt. | 17.34 | 0.466 - 0.526 | 6.2 ± 0.3 | 38 ± 1 |

In Table 3 we confront our PBCS and PQRPA results with previous RPA calculations\,[1, 2, 3, 4], and with experiments\,[27, 28, 29, 30] for: the energy of the \( J^\pi = 1^+_1 \) state in \( ^{12}N \), the \( B(GT) \)-value for the \( \beta \)-decay from \( ^{12}N \) and \( ^{12}B \), and the exclusive and inclusive \( \mu \)-capture...
rates to \(^{12}B\): \(\Lambda(J^\pi_f = 1^+_1)\) and \(\sum_{J^\pi} \Lambda(J^\pi_f)\). We don’t show our QRPA results because of the above mentioned difficulties with the \(J^\pi = 1^+_1\) ground states in \(^{12}N\) and \(^{12}B\). In comparing the calculations of \(B(\Gamma T)\) with data it should be remembered that it is still not clear the origin of the observed 10% difference measured \(ft\) values for the \(\Gamma T\) \(\beta\)-decays from the ground states \(J^\pi = 1^+_1\) in \(^{12}B\) and \(^{12}N\) to the ground state \(J^\pi = 0^+\) in \(^{12}C\): \(ft(\Gamma T) = (1.1669 \pm 0.0037) \times 10^4\)seg, and \(ft(\Gamma T) = (1.3178 \pm 0.0084) \times 10^4\)seg \(^{45}\). In the past this difference has been attributed mostly to the violation of charge symmetry in the involved nuclear states, and occasionally also to the second class current (or induced tensor interaction) which violates the G-parity \(^{34,35}\). As this kind of effects are not considered in the present work the above \(ft\) values will be taken as lower and upper experimental limits. The corresponding \(B\)-values, obtained from \(^{20}\) \((B_B(\Gamma T) = 0.526\) and \(B_N(\Gamma T) = 0.466\)\) are shown in Table 3. Due to the same reason, the small difference \((\lesssim 3\%)\) between the theoretical results for \(B_B(\Gamma T)\) and \(B_N(\Gamma T)\) is not physically relevant and only the mean values \((B_B(\Gamma T) + B_N(\Gamma T))/2\) are exhibited.

Table 4: Results for averaged exclusive and inclusive neutrino-nucleus cross sections \(\langle \sigma_e \rangle\) (in units of \(10^{-42}\) cm\(^2\)) and \(\langle \sigma_\mu \rangle\) (in units of \(10^{-40}\) cm\(^2\)). (See caption to Table 1.)

| Model       | \(\langle \sigma^\text{exc}_e \rangle\) | \(\langle \sigma^\text{inc}_e \rangle\) | \(\langle \sigma^\text{exc}_\mu \rangle\) | \(\langle \sigma^\text{inc}_\mu \rangle\) |
|-------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| RPA         | 36.0 (38.4)                     | 42.3 (44.3)                     | 2.48 (3.11)                     | 21.1 (22.8)                     |
| RPA         | 54.8 (68.2)                     | 63.2 (76.3)                     | 3.35 (3.80)                     | 21.1 (22.4)                     |
| RPA+pair\(^2\) | 7.1 (16.0)                     | 12.9 (22.7)                     | 0.39 (0.77)                     | 13.5 (15.2)                     |
| CRPA        | 12.5 (13.9)                     | 18.15 (19.28)                   | 1.06 (1.06)                     | 17.8 (18.2)                     |
| RPA         | 50.0                            | 55.1                            | 2.09                            | 19.2                            |
| QRPA        | 42.9                            | 52.0                            | 1.97                            | 20.3                            |
| PBCS        | 21.0                            | 41.2                            | 1.67                            | 19.1                            |
| PQRPA (I)   | 9.9                             | 21.6                            | 0.72                            | 14.6                            |
| PQRPA (II)  | 8.0                             | 18.5                            | 0.56                            | 12.8                            |
| PQRPA (III) | 8.1                             | 17.4                            | 0.56                            | 13.4                            |
| Expt.       | \(9.1 \pm 0.4 \pm 0.9\)\(^{44}\) | \(14.8 \pm 0.7 \pm 1.4\)\(^{45}\) | \(0.66 \pm 0.1 \pm 0.1\)\(^{46}\) | \(12.4 \pm 0.3 \pm 1.8\)\(^{47}\) |
|             | \(8.9 \pm 0.3 \pm 0.9\)\(^{48}\) | \(13.2 \pm 0.4 \pm 0.6\)\(^{49}\) | \(0.56 \pm 0.08 \pm 0.10\)\(^{50}\) | \(10.6 \pm 0.3 \pm 1.8\)\(^{51}\) |

Similarly, in Table 4 are given the results for the exclusive and inclusive flux-averaged neutrino scattering cross sections to \(^{12}N\): \(\langle \sigma_\ell(J^\pi_f = 1^+_1)\rangle\), \(\sum_{J^\pi} \langle \sigma_\ell(J^\pi_f)\rangle\) with \(\ell = e, \mu\). They are

\(^2\)Presently, the study of the G-parity irregular weak nucleon current is still of interest \(^{34,35}\).
defined as
\[
\langle \sigma_{\ell}(J_f) \rangle = \int dE_\nu \sigma(E_\ell = E_i - E_f - E_\nu, J_f)f(E_\nu),
\]
(29)
where \(f(E_\nu)\) is the normalized neutrino flux. For electron neutrinos it was approximated by the Michel spectrum, and for the muon neutrinos we used that from ref. [40].

Table 5: Results for the energies (in units of MeV) and the partial muon capture rates (in units of \(10^3\) s\(^{-1}\)) the bound excited states in \(^{12}\)B. In the upper part of the table are shown the previous theoretical calculations based on the RPA [1, 41] (where only the results for \(\Lambda\) are reported) and the on shell model [42].

| Model          | \(J_f^e\) | \(1^+_1\)       | \(2^+_1\)       | \(2^-_1\)       | \(1^-_1\)       |
|----------------|-----------|-----------------|-----------------|-----------------|-----------------|
| RPA [1, 41]   | \(\Lambda\) | 25.4 (22.8)     | \(\leq 10^{-3}\) | 0.04 (0.02)     | 0.22 (0.74)     |
| PBCS          | E         | 0.00            | 0.00            | 3.10            | 3.10            |
|               | \(\Lambda\) | 15.4            | 0.40            | 1.70            | 1.13            |
| PQRPA (I)     | E         | 0.00            | 0.34            | 2.83            | 3.13            |
|               | \(\Lambda\) | 7.83            | 0.21            | 0.34            | 0.66            |
| PQRPA (II)    | E         | 0.00            | 0.50            | 2.82            | 3.31            |
|               | \(\Lambda\) | 6.50            | 0.16            | 0.18            | 0.51            |
| PQRPA (III)   | E         | 0.00            | 0.28            | 2.82            | 2.97            |
|               | \(\Lambda\) | 6.54            | 0.17            | 0.18            | 0.58            |
| Expt [13, 14] | E         | 0.00            | 0.95            | 1.67            | 2.62            |
|               | \(\Lambda\) | 6.00 ± 0.40     | 0.21 ± 0.10     | 0.18 ± 0.10     | 0.62 ± 0.20     |

We wish to restate the ingredients that play a part in the agreement between the data and calculations for the ground state processes within the triad \{\(^{12}\)B, \(^{12}\)C, \(^{12}\)N\}. They are: (a) the pairing short range correlations, which are added to improve the description of the \(^{12}\)C ground state, (b) the RPA-type correlations, which are repulsive in the particle-hole channel, and (c) the effective axial-vector coupling constant \(g_\Lambda = 1\), which in principle simulates the removal of the spin strength due to the coupling to the \(\Delta\) resonance [10, 11, 12, 42]. For instance, these effects reduce the bare single-particle value \(B(GT) = (16/9)g_A^2\) by factors 1.7, 1.8 – 2.2 and 1.6, respectively. It is worthy of note that the PBCS by itself reproduces better the data than the
majority of previous RPA and QRPA calculations \[1, 2, 3, 4\]. We have considered all orbitals from 1s\(\frac{1}{2}\) up to 1f\(\frac{5}{2}\), but the valence \(p\)-shell correlations (both pairing and RPA like) are definitely the most important ones for the quenching of the \(1^+_1 \leftrightarrow 0^+_1\) transition rates. Yet, as discussed by Vogel et al. \[3, 4\], the effect of these correlations on the dipole and quadrupole operators is very tiny.

In addition to the total \(\mu\) capture rates in Table 3, we show the individual rates to the individual bound states of \(^{12}B\) in Table 5. They represent another test for our calculation and have been derived from the intensities of the observed de-excitation \(\gamma\) rays following the \(\mu\) capture \[13, 44\]. The agreement between the experiment and the present PQRPA estimate for the energies of the \(J^\pi = 2^+_1, 2^-_1\) and \(1^-_1\) states is only moderate, but that for the capture rates is as good or even better than in a recent shell model (SM) study \[12\].

In summary, we have shown that to account for the weak decay observables around \(^{12}C\) in the framework of the RPA, besides including the BCS correlations, it is imperative to perform the particle number projection, and this is the way out of the RPA puzzle in \(^{12}C\). More, as far as we are acquainted with, such an important effect of the projected linear response theory for charge-exchange excitations has never before been observed, indicating that it could be more relevant in light \(N = Z\) nuclei than in heavy nuclei with large neutron excess \[3\]. Thus, it could be interesting enough to investigate the consequences of the PQRPA in other light \(N = Z\) and \(N \cong Z\) nuclei. On the other hand, the fact that we have been forced to switch off completely the residual interaction in the particle-particle channel could indicate that some relevant piece of physics is still lacking in our approach. In this sense it would be very illuminating to redo the PQRPA calculations with more realistic forces than the one used here.

The authors acknowledge the support of ANPCyT (Argentina) under grant BID 1201/OC-AR (PICT 03-04296) and of CONICET under grant PIP 463. F.K. and A.M. are fellows of the CONICET Argentina.

References

[1] E. Kolbe, K. Langanke and S. Krewald, Phys. Rev. C 49, 1122 (1994).
[2] N. Auerbach, N. Van Giai, O.K. Vorov, Phys. Rev. C 56, 2368 (1997).
[3] E. Kolbe, K. Langanke and P. Vogel, Nucl. Phys. A 652, 91 (1999).
[4] C. Volpe, N. Auerbach, G. Colò, T. Suzuki, N. Van Giai, Phys. Rev. C 62, 015501 (2000).
[5] F. Krmpotić, A. Mariano, T.T.S. Kuo, and K. Nakayama, Phys. Lett. B 319 393(1993).
[6] C. Barbero, F. Krmpotić, and D. Tadić, Nucl. Phys. A 628, 170 (1998); C. Barbero, F. Krmpotić, A. Mariano and D. Tadić, Nucl. Phys. A 650, 485 (1999).

[7] T. Tomoda, Rep. Prog. Phys. 54 (1991) 53.

[8] M. Doi and T. Kotani, Prog. Theor. Phys. 89, (1993) 139.

[9] I.S. Towner and J.C. Hardy, The Nucleus as a Laboratory for Studying Symmetries and Fundamental Interactions, eds. E.M. Henley and W.C. Haxton, nucl-th/9504013.

[10] B.A. Brown and B.H. Wildenhal, At. Data Nucl. Data Tables 33, 347 (1985).

[11] H. Castillo and F. Krmpotić, Nucl. Phys. A 469, 637 (1987), and references there in.

[12] F. Osterfeld, Rev. Mod. Phys. 64, 491 (1992).

[13] T. Kuramoto, M. Fukigita, Y. Kohyama and K. Kubodera, Nucl. Phys. A 512 (1990) 711.

[14] J.R. Luyten, H.P.C. Rood and H.A. Tolhoek, Nucl. Phys. 41, 236 (1963).

[15] V.A. Kuzmin, T.V. Tetereva, K. Junker, A.A. Ovchinnikova, nucl-th/0104061.

[16] R.J. Blin-Stoyle and S.C.K. Nair, Adv. Phys. 15, 493 (1966).

[17] J.D. Walecka, Theoretical Nuclear and Subnuclear Physics, Oxford University Press, New York, 531 (1995).

[18] C. Conci, V. Klempt and J. Speth, Phys. Lett. 148B (1984) 405; C. Conci, Ph.D. Thesis, unpublished, Jülich 1984.

[19] D. Cha, Phys. Rev. C 27, 2269 (1983).

[20] A. Bohr B. R. Mottelson, Nuclear Structure Vol.II (W.A. Benjamin Inc., New York, Amsterdam, 1975).

[21] A.M. Lane and J. Martorrel, Ann. Phys. 129, 273 (1980).

[22] F. Krmpotić, K. Ebert, W. Wild, Nucl. Phys. A 342, 497 (1980).

[23] R. Alzetta, T. Weber, Y. K. Gambhir, M. Gmitro, J. Sawicki, and A. Rimini Phys. Rev. 182, 1308(1969)

[24] J. Hirsch and F. Krmpotić, Phys. Rev. C 41, 792(1990); F. Krmpotić and Shelly Sharma, Nucl. Phys. A 572, (1994) 329.
[25] K. Nakayama, A. Pio Galeão and F. Krmpotić, Phys. Lett. B 114 (1982) 217.
[26] A. Goswami and M.K. Pal, Nucl. Phys. 35, (1962) 544.
[27] F. Ajzenberg-Selove, Nucl. Phys. A 433, 1(1985).
[28] D. E. Alburger and A.M. Nathan, Phys. Rev. C 17 (1978) 280.
[29] G. H. Miller et al., Phys. Lett. B 41, (1972) 50.
[30] T. Suzuki et al., Phys. Rev. C 35, (1987) 2212.
[31] R.J. Blin-Stoyle and M. Rosina, Nucl. Phys. 70, 321 (1965).
[32] B. Eman, D. Tadić, F. Krmpotić and L. Szybisz, Phys. Rev. C 6, 1 (1972).
[33] B.R. Holstein and S.B. Trieman, Phys. Rev.C 13, 3059 (1978).
[34] H. Shiorui, Nucl. Phys. A 603, 281 (1996).
[35] K. Minamisono, et al. Phys. Rev. C 65, 015501 (2001).
[36] LSND Collaboration, C. Athanassopulus et al., Phys. Rev.C 55, 2078 (1997).
[37] LSND Collaboration, C. Athanassopulus et al., Phys. Rev.C 56, 2806 (1997).
[38] LSND Collaboratio, L-B. Auerbach et al., Phys. Rev.C 64, 065501 (2001).
[39] LSND Collaboratio, L-B. Auerbach et al., arXiv:nucl-ex/0203011.
[40] LSND home page, http://www.neutrino.lanl.gov/LSND/figures.
[41] E. Kolbe, K. Langanke and P. Vogel, Phys. Rev. C 50, 2576 (1994).
[42] N. Auerbach and B.A. Brown, Phys. Rev. C 65, 024322 (2002).
[43] D.F. Maesday, Phys. Rep. 354, 243 (2001).
[44] T.J. Stocki, D.F. Maesday, E. Gete, M.A. Saliba, and T.P. Gorrinde Nucl. Phys. A 697, 55 (2002).
[45] P. Vogel, nucl-th/9901027.