Some phase peculiarity in reflection of EM wave from boundaries between two isotropic media

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Abstract. A method of interferometric registration of the phase change of a polarized EM wave reflected from the boundaries between dielectric media is proposed. The method is suitable for determining how the phases of the waves reflected from the boundaries between isotropic media are affected by changes in the polarization plane and the angle of incidence of the wave. The interference pattern from two types of linearly polarized waves, one polarized in the plane of incidence (p-wave), another in a perpendicular plane (s-wave), was observed as interference of thin films in reflected light. This observation showed that the phase of the reflected s-wave from an optically denser medium was changed to the opposite for all tested angles of incidence. On the contrary, the phase of the reflected p-wave depends on the angle of incidence. The phase of reflected p-wave does not change for the range of the incidence angles from 0 to the Brewster angle ($\alpha_B$) and becomes opposite for the range between $\alpha_B$ and $\pi/2$.

1. Introduction

Serious milestones in development of Physics to its modern state were associated with establishing the wave theory of light by Thomas Young and his demonstration of idea of interference with a double-slit experiment. His ideas and experiments were further extended by Augustin-Jean Fresnel, who described aberration and diffraction of light and constructed so called Fresnel lenses. Fresnel equations on light waves and reflectivity are important for many applications to date. Some of his papers were not printed until many years after his death Fresnel did not provide to public how his equations were derived and a number of outstanding physicists tried to re-infer these equations, see e.g. Lorentz [1,2]. Lorentz studied reflection and refraction of EM wave on a boundary of media with different orientation of polarization plane relatively to the plane of incidence (Fresnel equations). He showed that the phases of reflected and refracted waves depend on the polarization plane, angle of the incidence, and refractive indices.

Fresnel himself substantially extended understanding on how the light waves penetrate through the boundaries of dielectric media and how the waves propagate through the media. For example, using his equations he derived reflective and refractive indices on the boundaries of two media for the case of polarized s and p waves, when the polarization plane is perpendicular to the plane of incidence or is coincided, correspondingly. Fresnel proved the existence of the Brewster angle (the angle of incidence at which the light is perfectly transmitted through a dielectric with no reflection) and existence of elliptically-polarized light. Fresnel equations remains in the investigation field and the subject of further theoretical [3] and experimental [4,5] works.

One of the interesting outcomes from analysis of Fresnel equations is a change of the phase of a wave on the boundary between two dielectrics. It is known, that a reflection from an optically denser
medium \( (n_2 > n_1) \), e.g. in the case of the use of a plane-parallel plate, causes some optical path difference to be equal to odd number of \( \lambda/2 \). However, such definition is correct for some narrow conditions. For example, in case of reflection of polarized light, the orientation of its polarization plane and the angle of incidence affects the phase of the reflected wave. Some of the conditions, which are necessary for a reflection EM wave from a dielectric surface [6] according to Fresnel equations, were simulated using Heaviside step function. The interpretation of EM wave with a Heaviside step function is a useful from both experimental and theoretical points of view to provide an alternative inputs to the traditionally used monochromatic waves [8,9]. Detailed analysis of reflections using s- and p- EM waves with different angles of incidence was reported in [7]. A peculiar interest in analysis of Fresnel equations provides detection of phase changes on the boundary of two dielectric media and a penetration of light into another medium after complete internal reflection.

Ratio between electric field intensity for the refracted and incidence waves shows [6] that the phase of the refracted wave is the same as the phase of the incidence wave independently on the angle of incidence and the refractive index. The same analysis also shows that in case of reflection from a denser medium the phase of reflected s-wave is with a \( \pi \)-difference, while the phase for the p-wave is either the same for the incidence angle smaller than the Brewster angle or shows a \( \pi \)-difference for the angles between the Brewster angle and the \( \pi/2 \). For a reflection from a less-denser medium the results for s-wave show no change and for p-wave they demonstrate a \( \pi \)-difference for the incidence angle smaller than the Brewster angle and no change for the angles between the Brewster angle and \( \pi/2 \).

These results based on the analysis presented in [6] are shown in Table 1.

| Ratio of refractive indices at the boundary | \( n_2 > n_1 \) | \( n_2 < n_1 \) |
|-------------------------------------------|----------------|----------------|
| Waves type                               | \( p \)       | \( s \)       |
| \( \alpha < \alpha_B \)                  | 0             | \( \pi \)     |
| \( \alpha > \alpha_B \)                  | \( \pi \)     | 0             |

The goal of this work is to propose a method for experimental verification of the theoretical results presented in [6] (see also Table 1) using practical measurements on proposed setup.

2. Experiments

The results of studying reflection of polarized waves with different angles of incidence and ratios of refractive indices are presented for two types of EM waves: one with a polarization plane, which coincides with the plane of incidence (p-wave), another with a perpendicular direction (s-wave).

The changes of the phases presented in Table 1 were studied using a setup originally designed for observations of Newton’s interference rings.

Figure 1 shows the experimental setup used for investigation of reflection of s- and p-waves from the boundary between two dielectric media with different test incidence angles.

The reflection of the EM waves occurs from the air gap between two glass parts. The task was to determine the changes of the phase for p- and s-waves.
The angle of incidence ($\alpha_1 < \alpha_B$; Figure 1, $n_1/n_2 > 1$) to the first (top) boundary is chosen to provide the angle of incidence to the bottom boundary ($\alpha_2 > \alpha_B$). This is possible with the $n_1$ being slightly larger than the $n_2$. Figure 2 shows the changes of the phase for $p$-wave (a) and $s$-wave (b).

![Figure 1. A schematic of the optical layout used for the experimental study of the reflected beams. Mark 1 and 2 is a reflected ray from the top boundary (1) and a reflected ray from the bottom boundary (2); $n_1$, $n_2$, and $n_3$ are refractive indices for glasses ($n_1$ and $n_3$) and air gap ($n_2$); $\alpha_1 = \alpha$ - incidence angle $\alpha_2 = \beta$ – refraction angle.](image)

![Figure 2. A change of phases for $p$-wave (a) and for $s$-wave (b) for the angle of incidence ($\alpha_1 < \alpha_B$) and $n_1/n_2 > 1$.](image)

Figure 2 shows that the change of the polarization angle for the reflected rays is accompanied by the different phases and the different optical path difference: $\lambda/2$ for the $s$-wave and 0 for the $p$-wave. These conditions may produce an interference pattern (Newton’s rings) with a minimum in the center for the $s$-wave and a maximum for the $p$-wave created by a thin-film air gap.

To verify theoretical results presented in [6], a goniometer equipped with an optical layout suitable to observe the Newton’s rings in the reflected light for $s$- and $p$-waves is shown in Figure 3.
Figure 3 shows experimental setup which consists of a source of light (1), condenser (2), interference filter (3), entrance slit (4), collimator objective (5), polarizer (6), optical combination for interferometry (7), polarizer (8), lens (9). The interference filter (3) is adjusted to transmit the light with $\lambda = 540$ nm.

The optical parts for interferometry (7) are placed on a rotating table (II) which provides different angles of incidence (Figures 1, 2 and Table 1). A rotation of the analyzer (III) may be adjusted to observe a resulting interference (Newton’s rings) (Figure 4) in the monochromatic reflected light.

The interference pattern shown in Figure 4, b with a minimum intensity in the center is presented for s-wave.
When the polarizer (6 on Figure 3) is rotated to produce a $p$-wave, it changes the interference minimum intensity to interference maximum. This change is associated with the change of $\Delta \lambda$ by $\lambda/2$ or the phase by $-\pi$. The interference pattern remains the same for a change of the incidence angle for either $s$- or $p$-wave.

Thus, the results of this experiment are confirming the theoretical [6] analysis.

3. Discussing the results of the experiment

The experimental results confirm the presented in [6] theory and extend our understanding of the reflection and refraction of polarized EM waves on the boundary of dielectric media for different angles of incidence and refractive indices. It was shown that for either $s$- or $p$-wave chosen by the rotation of the polarizer (6 in Figure 3) the interference pattern was the same in a wide range of incidence angles smaller than the Brewster angle. The pattern was changing (minimum to maximum and vice versa) for the angles of incidence close to the Brewster angle. These evidences about the change of $\Delta \lambda$ by $\lambda/2$ or the phase by $-\pi$, are consistent with our theoretical calculations [6].

Fresnel equations for $s$- and $p$-waves in part of ratios of amplitudes of reflected and refracted waves to the input wave amplitude [6] may be written as:

\[
\begin{align*}
\text{s-wave} & \quad \frac{E_1'}{E_1} = -\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)}, \quad \frac{E_2}{E_1} = \frac{2\sin \beta \cos \alpha}{\sin(\alpha + \beta)}; \\
\text{p-wave} & \quad \frac{E_1'}{E_1} = \frac{\tan(\alpha - \beta)}{\tan(\alpha + \beta)}, \quad \frac{E_2}{E_1} = \frac{2\sin \beta \cos \alpha}{\sin(\alpha + \beta)\cos(\alpha - \beta)}.
\end{align*}
\]

Where $E_1$ - electric field intensity for the incident wave, $E_2$ - electric field intensity for the refracted wave, $E_1'$ - electric field intensity for the reflection wave.

The ratio of electric field intensity for the refracted wave $E_2$ to the input wave intensity $E_1$ shows that the phases of these waves are independent on the optical density of the media.

Reflection from optically denser medium ($n_1/n_2 < 1$) with $\alpha > \beta$, where $\beta$ is the refraction angle, the phase for reflected $E_1'$ $s$-wave is changing by $\pi$ and for $p$-wave it is not changing for angles below the Brewster angle and is changing by $\pi$ for angles from $\alpha$ to $\pi/2$. 

Figure 4. The Newton’s rings: a) для $p$ – wave и b) для $s$ – wave, angle of incidence $\alpha_1 < \alpha_B$ and $n_1/n_2 > 1$
For a reflection from optically less dense medium \((n_1/n_2 > 1\) and \(\alpha < \beta\)) the phase for \(s\)-wave is not changing but the phase of \(p\)-wave is changing by \(\pi\) for the angles of incidence below the Brewster angle and is not changing for the angles between \(\alpha\) and \(\pi/2\).

4. Conclusions

The interpretation of EM wave with a Heaviside step function, which was modeled in [6], is a fruitful approach from both experimental and theoretical points of view to provide alternative inputs to the traditionally used monochromatic waves. It allowed us to build a bridge between such theoretical model and real experimental measurements and verify the results of the model. The proposed method may be used to analyze some astronomical observations, where one needs a knowledge of the phase relations as a function of the polarization plane and the angle of incidence. Thus, proposed method of detection of phase variations of the reflected polarized wave on the boundary of dielectric media may be used for observation of different space objects, such as interstellar dust clouds. Obtained phase data may be used to derive the media characteristics.

The proposed interferometric scheme for the phase-relation measurements is easy-realized and reliable. This may provide accurate measurements. The method may be used in a wide spectral window, including both short and long-wavelengths.

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