Azimuthal asymmetries and the emergence of “collectivity” from multi-particle correlations in high-energy pA collisions

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We show how angular asymmetries $\sim \cos 2\phi$ can arise in dipole scattering at high energies. We illustrate the effects due to anisotropic fluctuations of the saturation momentum of the target with a finite correlation length in the transverse impact parameter plane, i.e. from a domain-like structure. We compute the two-particle azimuthal cumulant in this model including both one-particle factorizable as well as genuine two-particle non-factorizable contributions to the two-particle cross section. We also compute the full BBGKY hierarchy for the four-particle azimuthal cumulant and find that only the fully factorizable contribution to $c_2\{4\}$ is negative while all contributions from genuine two, three and four-particle correlations are positive. Our results may provide some qualitative insight into the origin of azimuthal asymmetries in p+Pb collisions at the LHC which reveal a change of sign of $c_2\{4\}$ in high-multiplicity events.

I. INTRODUCTION

Large azimuthal asymmetries have been observed in p+Pb collisions at the LHC [1–4] and in d+Au collisions at RHIC [5]. These asymmetries are usually measured via multi-particle angular correlations (see below) and were found to extend over a long range in rapidity. Causality then requires that the correlations originate from the earliest times of the collision [6]. Furthermore, the data shows that the asymmetries persist up to rather high transverse momenta, well beyond $p_T \sim 1$ GeV. Recent data by the ATLAS collaboration, for example, shows that large “elliptic” ($v_2$) asymmetries in p+Pb collisions at $\sqrt{s} = 5$ TeV persist up to $p_T = 10$ GeV [7]. Therefore, it is important to develop an understanding of their origin in terms of semi-hard (short distance) QCD dynamics [8–15].

The ALICE collaboration has measured the two- and four-particle $v_2$ cumulants in p+Pb collisions at 5 TeV as a function of multiplicity, see Figs. 1 and 4 in Ref. [2]. These cumulants are defined as [16]

\[
c_2\{2\} = \langle \exp 2i(\phi_1 - \phi_2) \rangle ,
\]

\[
c_2\{4\} = \langle \exp 2i(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle - 2 \langle \exp 2i(\phi_1 - \phi_3) \rangle \langle \exp 2i(\phi_2 - \phi_4) \rangle .
\]

Here, $\langle \cdot \rangle$ denotes an average over the corresponding azimuthal angles weighted by the two- or four-particle distribution, respectively. The two-particle cumulant with a rapidity gap suppresses contributions from resonance decays and jet fragmentation; it depends weakly on multiplicity and is positive over the entire range of multiplicity. On the other hand the four-particle cumulant, $c_2\{4\}$, decreases monotonically and changes sign to become negative in high multiplicity events, an effect also seen by the CMS collaboration (see second paper in [4]). As shown below, this requires an anisotropy of the single-particle angular distribution. In the soft, long wavelength regime, $c_2\{4\}$ is negative when hydrodynamic flow dominates over “non-flow” correlations [17]. In this paper we perform a first computation of all connected and disconnected contributions to the cumulants in the short distance regime using a model that allows for anisotropic “domains” of the color-electric fields $\vec{E}$ of the target [18].
II. CALCULATION

Our discussion is based on the dipole model of high-energy interactions [19]. We consider scattering of a dipole of size $r \sim 1/p_\perpendicular$ from the target described by a particular configuration of the (color) electric field $E^i \sim F^{ij}$. For a small dipole $\vec{r} \equiv \vec{x} - \vec{y}$ the leading C-even interaction with the target is given by

$$S - 1 = \frac{1}{2N_c} \text{tr} (ig \vec{r} \cdot \vec{E})^2,$$

with a C-odd correction at order $(ig\vec{r})^3$ which is not considered here because it does not contribute to $\sim \cos 2\phi$ asymmetries [18]. Equation (3) arises from an expansion of the S-matrix, $\text{tr} V(\vec{x})V^i(\vec{y})/N_c$, in powers of $\vec{r}$, where

$$V(\vec{x}) = \mathcal{P} \exp \left( ig \int dx^- A^+(x^-, \vec{x}) \right)$$

is the path-ordered Wilson line describing the propagation of a charge in the field of the (right-moving) target. We focus on the S-matrix for a fundamental charge though the calculation could be repeated for a charge in the adjoint representation yielding the same results for $c_2\{2\}$ and $c_2\{4\}$.

To obtain the cross section the scattering matrix is averaged over the configurations of the $\vec{E}$ field of the target. Averaging over all such configurations leads to

$$\langle S \rangle - 1 = \frac{(ig)^2}{2N_c} r^i r^j \langle \text{tr} E^i(\vec{b})E^j(\vec{b}) \rangle = -\frac{1}{4} r^2 Q_s^2(\vec{b}) \log \frac{1}{r\Lambda}$$

in the leading log approximation, $\log 1/r\Lambda \gg 1$. Here, $Q_s(\vec{b})$ denotes the saturation scale below which non-linear effects become significant. In what follows we shall assume a very large nucleus and drop the dependence of the average saturation momentum on $\vec{b}$.

Equation (5) corresponds to the single-particle cross section averaged over all configurations of $\vec{E}(\vec{b})$ in the target and is, of course, isotropic. On the other hand, for any particular configuration the S-matrix does exhibit an angular dependence, c.f. for example Fig. 7 in Ref. [20]. The idea that anisotropic fluctuations of the saturation momentum would induce $v_n \neq 0$ has been presented previously in Refs. [10, 18, 21]. Hence, to evaluate the amplitude of the angular modulation of the S-matrix we perform the average subject to the constraint

$$\frac{(ig)^2}{2N_c} r^i r^j \langle \text{tr} E^i(\vec{b})E^j(\vec{b}) \rangle = -\frac{1}{4} r^2 Q_s^2 \log \frac{1}{r\Lambda} \Delta(\vec{b}_1 - \vec{b}_2) (1 - A + 2A (\hat{r} \cdot \hat{a})^2)$$.

That is, we divide the target ensemble into classes such that for a given class the anisotropic part of the electric field correlator in the vicinity of $\vec{b}$ (within a given “domain”) points in a specific direction. The summation over all classes, which corresponds to an integration over the directions $\hat{a}$, is performed only after the m-particle angular cumulant has been evaluated. The quantity $A$ in eq. (6) is the amplitude of anisotropy of the electric field correlator.

For simplicity, as we mentioned above, in our current analysis we singled out only fluctuations of $\hat{a}$ while possible fluctuations of $Q_s$ and $A$ are averaged out in Eq. (6). The results could be extended to account for fluctuations of $Q_s$ and $A$ in the future.

The domain structure of the field is described by the two-point correlation function

$$\Delta(\vec{b}_1 - \vec{b}_2) = \exp \left( -\frac{|\vec{b}_1 - \vec{b}_2|^2}{\xi^2} \right),$$

where $\xi$ denotes the correlation length. We assume a Gaussian correlation function, other options do not change our results qualitatively. To simplify the notation we introduce

$$\frac{1}{N_D} = \frac{1}{S_\perpendicular} \int d^2 b_1 d^2 b_2 \Delta(\vec{b}_1 - \vec{b}_2) = \frac{\pi \xi^2}{S_\perpendicular},$$

which is the area of a domain divided by the area of the collision zone, in other words, the inverse number of domains. Equation (7) essentially describes the correlations of the saturation momentum $Q_s$ in the transverse plane.

We can now compute the angular distribution for scattering of a single dipole, for a fixed $\hat{a}$. Using Eqs. (6) and performing a Fourier transform to momentum space, as well as an average over the impact parameter, we arrive at

$$\left( \frac{1}{\pi} \frac{dN}{dk^2} \right)^{-1} \frac{dN}{d^2 k} = 1 - 2A + 4A (\hat{k} \cdot \hat{a})^2.$$
Hence, the one-particle $v_2$ cumulant

$$v_2 \{1\} \equiv \left\langle e^{2i(\phi_k - \phi_a)} \right\rangle_{\hat{a}} = A. \quad (10)$$

To avoid confusion let us stress that here $\langle \cdot \rangle$ refers to a different average than the average over $\vec{E}$-field configurations from above; it is simply an average over the azimuthal angle $\phi_k$ weighted by the distribution $|\tilde{E}|^2$. We now proceed to two-particle distributions. The averages over $\vec{E}$-field configurations shall be performed assuming a Gaussian action $\frac{1}{2} \int \vec{r}^2$ and a color diagonal four-point function although in general additional contributions could appear $\frac{1}{2} \int \vec{r}^2$. Then the two-particle S-matrix for fixed $\hat{a}$ is given by

$$\langle S_2 \rangle - 1 = \left( \frac{ig}{2N_c} \right)^2 \frac{1}{\hat{a}} \left\langle \text{tr} \left( \frac{dr}{\hat{E}(\hat{b}_1)} \right)^2 \text{tr} \left( \frac{dr}{\hat{E}(\hat{b}_2)} \right)^2 \right\rangle_{\hat{a}} \quad (11)$$

$$= \frac{ig^4}{4N_c^2} \int \frac{d\phi_a'}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (12)$$

$$+ \frac{ig^4}{4N_c^2} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}} \text{conn.} \quad (13)$$

The factorizable (disconnected) contribution involves the correlations of the directions of $\hat{E}(\hat{b})$ in the impact parameter plane; we employ $C(\hat{a}, \hat{a}') = 2\pi \delta(\phi_a - \phi_a') \Delta(\hat{b}_1 - \hat{b}_2)$. Averaging over impact parameters gives

$$\left\langle e^{2i(\phi_k - \phi_a')} \right\rangle_{\hat{a}'} = \frac{1}{Nd} \int \frac{d\phi_a'}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (14)$$

$$= \frac{1}{Nd} \int \frac{d\phi_a'}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (15)$$

$$= \frac{1}{Nd} \int \frac{d\phi_a'}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (16)$$

In this expression the prefactor $1/N_D$ arises due to the fact that the orientation of the electric field is approximately constant only over distance scales of order the correlation length $\xi$. Multiplying the Fourier transform of this expression by $\exp(2i(\phi_1 - \phi_2))$ and averaging over the azimuthal angles leads to the disconnected (single-particle factorizable) contribution to $(v_2 \{2\} )$:

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}} \text{disc.} = \frac{1}{Nd} + \frac{1}{12(N_c^2 - 1)(1 + A^2)} \left( v_2 \{1\} \right)^2. \quad (17)$$

Note that this is independent of the global direction $\hat{a}$ relative to which we define $\phi_1$ and $\phi_2$ and so the final average over $\hat{a}$ is trivial. The additional term in the denominator originates from the connected contribution to the normalization.

The connected contribution from Eq. (13) is

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}} \text{conn.} = \frac{1}{Nd} \int \frac{d\phi_a}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}} C(\hat{a}, \hat{a}') \quad (18)$$

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}} \text{conn.} = \frac{1}{Nd} \int \frac{d\phi_a}{2\pi} \left\langle \text{tr} \left( \hat{r}_1 \cdot \hat{E}(\hat{b}_1) \right)^2 \right\rangle_{\hat{a}} \left\langle \text{tr} \left( \hat{r}_2 \cdot \hat{E}(\hat{b}_2) \right)^2 \right\rangle_{\hat{a}} C(\hat{a}, \hat{a}') \quad (19)$$

Averaging over impact parameters produces a factor

$$\frac{1}{Nd} \int d^2b_1 d^2b_2 \Delta^2(\hat{b}_1 - \hat{b}_2) = \frac{1}{2Nd}, \quad (20)$$

so that the connected contribution to the two-particle cumulant becomes

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}} \text{conn.} = \frac{1}{Nd} \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} e^{2i(\phi_1 - \phi_2)} \left[ \frac{dN_2(\hat{a})}{d^2k_1 d^2k_2} - \frac{dN_1(\hat{a})}{d^2k_1} \frac{dN_1(\hat{a})}{d^2k_2} \right] \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{dN_2(\hat{a})}{d^2k_1 d^2k_2} + \frac{1}{Nd} \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{dN_2(\hat{a})}{d^2k_1 d^2k_2} \quad (21)$$

$$= \frac{1}{Nd} \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} e^{2i(\phi_1 - \phi_2)} \left[ \frac{dN_2(\hat{a})}{d^2k_1 d^2k_2} - \frac{dN_1(\hat{a})}{d^2k_1} \frac{dN_1(\hat{a})}{d^2k_2} \right] \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{dN_2(\hat{a})}{d^2k_1 d^2k_2} \quad (22)$$
As before, here the average \( \langle \cdot \rangle \) on the l.h.s. is an average over \( \phi_1 \) and \( \phi_2 \) but does not involve averaging over \( \vec{E} \)-field configurations since the one- and two-particle distributions have already been averaged over all such configurations corresponding to a given \( \vec{a} \). However, the r.h.s. is independent of \( \vec{a} \) so that the final average over its direction is trivial. Also, for \( A = O(1/N_c) \) the first factor on the r.h.s. of Eqs. \( 22 \) can be approximated by \( 1/N_D \) so that in all, \( v_2 \{ 2 \} \) is then given by

\[
(v_2 \{ 2 \})^2 = \frac{1}{N_D} \left( A^2 + \frac{1}{4(N_c^2 - 1)} \right) .
\]

The first term is the square of the single-particle \( v_2 \{ 1 \} \); it is scaled by \( 1/N_D \) since both particles have to scatter from the same domain. The second contribution corresponds to genuine non-factorizable two-particle correlations. Both contributions are positive; nonetheless Eq. \( 23 \) reveals the existence of two distinct regimes. For

\[
A \gg \frac{1}{N_c} \tag{24}
\]

the ellipticity is mainly due to the asymmetry of the single-particle distribution induced by the \( \vec{E} \)-field domains. In the opposite limit

\[
A \ll \frac{1}{N_c} \tag{25}
\]

\( v_2 \{ 2 \} \) is mainly due to genuine two-particle correlations.

Expression \( 23 \) applies when both particles have sufficiently high transverse momenta as we have approximated both of their S-matrices by their leading small-\( r \) behavior \( \sim \text{tr} (\vec{r} \cdot \vec{E})^2 \). On the other hand, experimentally one typically considers angular correlations of a hard with a softer particle. Recent numerical computations \( 24 \) of \( c_2 \{ 2 \} \) which do not expand the S-matrices show that hard-soft correlations exhibit a fall-off with the transverse momentum of the hard particle. This is due to a decorrelation of the anisotropy axis in a high-\( p_T \) bin with that of the bulk.

The four particle cumulant exhibits qualitatively different behavior in the regimes of “small” vs. “large” \( A \). For general \( A \), \( c_2 \{ 4 \} \) is given by

\[
c_2 \{ 4 \} = \langle \text{exp} (2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle - 2 \langle \text{exp} (2i(\phi_1 - \phi_3)) \rangle \langle \text{exp} (2i(\phi_2 - \phi_4)) \rangle \tag{26}
\]

\[
= -\frac{1}{N_D} \langle v_2 \{ 1 \} \rangle^4 + \langle \text{exp} (2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle_{\text{conn}} . \tag{27}
\]

\[
+ \frac{1}{N_D} \langle \text{exp} (2i(\phi_1 + \phi_2)) \rangle_{\text{conn}} \langle \text{exp} (-2i(\phi_3 + \phi_4)) \rangle_{\text{conn}} + \frac{4}{N_D} v_2 \{ 1 \} \langle \text{exp} (2i(\phi_1 + \phi_2 - \phi_3)) \rangle_{\text{conn}} . \tag{28}
\]

\[
+ \frac{1}{N_D} \langle v_2 \{ 1 \} \rangle^2 \langle \text{exp} (-2i(\phi_3 + \phi_4)) \rangle_{\text{conn}} . \tag{29}
\]

which determines the azimuthal anisotropy from four particle correlations: \( v_2 \{ 4 \} = (c_2 \{ 4 \})^{1/4} \). Before addressing the corrections written in Eqs. \( 28-29 \) we compute the fully connected contribution and show that it is positive.

The fully connected contribution to the S-matrix is given by

\[
\langle N_c^2 - 1 \rangle \prod_{i=1}^{4} \frac{-Q_i^2}{4(N_c^2 - 1)} (\vec{r}_i \cdot \vec{r}_{i+1}) \Delta(\vec{b}_i - \vec{b}_{i+1}) \log \frac{1}{r_i A} + \text{permutations} , \tag{30}
\]

where \( i + 1 \) is defined modulo 4. Averaging over impact parameters generates a factor of \( 1/(4N_D^4) \). We may now perform the Fourier transform and sum the 48 contractions of the amplitudes / conjugate amplitudes of dipoles 1 to 4. This leads to

\[
\langle \text{exp} (2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle_{\text{conn}} = \frac{1}{4N_D^4} \frac{1}{(N_c^2 - 1)^3} (1 + 8A^2) . \tag{31}
\]

Here, corrections of order \( \sim 1/(N_c^2 - 1) \) to the normalization have been neglected, see related discussion for \( v_2 \{ 2 \} \) above. As promised, the fully connected contribution to \( c_2 \{ 4 \} \) is positive; thus if the anisotropy \( A \) is zero, the elliptic harmonic \( v_2 \{ 4 \} \) would be complex. Furthermore, the magnitude of the fully connected contribution relative to \( v_2 \{ 1 \}^4 \) is \( \sim 1/(A^4 N_c^6) \). Hence, parametrically \( c_2 \{ 4 \} \) crosses zero when \( A \sim 1/N_c^{3/2} \).
The terms from Eqs. (28-29), to leading order in $N_c$, are given by

$$\frac{1}{N_D^2} (v_2\{1\})^2 \langle \exp -2 i (\phi_3 + \phi_4) \rangle_{\text{conn}} = \frac{1}{N_D^3} \frac{\mathcal{A}^4}{N_c^2 - 1},$$  \hspace{1cm} (32)

$$\frac{1}{N_D} \langle \exp 2 i (\phi_1 + \phi_2) \rangle_{\text{conn}} \langle \exp -2 i (\phi_3 + \phi_4) \rangle_{\text{conn}} = \frac{1}{N_D^3} \frac{\mathcal{A}^4}{(N_c^2 - 1)^2},$$  \hspace{1cm} (33)

$$\frac{4}{N_D} v_2\{1\} \langle \exp 2 i (\phi_1 + \phi_2 - \phi_3) \rangle_{\text{conn}} = \frac{8}{3N_D^3} \frac{\mathcal{A}^4}{(N_c^2 - 1)^2}.$$  \hspace{1cm} (34)

They provide manifestly positive contributions to $c_2\{4\}$. When $\mathcal{A}$ is of order of $N_c^{-3/2}$, which is the regime where $c_2\{4\}$ changes sign, we can write our final result in the form

$$c_2\{4\} = -\frac{1}{N_D} \left( \frac{\mathcal{A}^4}{4(N_c^2 - 1)^3} \right),$$  \hspace{1cm} (35)

Here the additional terms listed in Eqs. (28-29) are suppressed by additional powers of $N_c^{-2}$.

III. DISCUSSION

An anisotropic single-particle distribution, $v_2\{1\} \neq 0$, requires an angular dependence of the dipole S-matrix $\sim \text{tr}(\vec{r} \cdot \vec{E})^2$ for individual configurations of $\vec{E}$. We describe this by the term $\sim \mathcal{A}(\vec{r} \cdot \vec{a})^2$ in Eq. (4).

Our main results are as follows. The two-particle elliptic asymmetry $c_2\{2\} \equiv (v_2\{2\})^2$ is given by

$$c_2\{2\} = \frac{1}{N_D} \left( \mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right) = \frac{1}{N_D} \left( (v_2\{1\})^2 + \frac{1}{4(N_c^2 - 1)} \right).$$  \hspace{1cm} (36)

The first term corresponds to the square of the asymmetry of the one-particle distribution while the second term is due to non-factorizable, genuine two-particle correlations. The transition between the two regimes occurs at $\mathcal{A} \sim 1/N_c$.

In practice, using $N_c = 3$ and the estimate $\mathcal{A} \simeq 0.2$ from Ref. [18] we conclude that the magnitudes of both terms are comparable.

The elliptic asymmetry from four-particle correlations, $c_2\{4\} \equiv -(v_2\{4\})^4$, is

$$c_2\{4\} = -\frac{1}{N_D^2} \left[ (v_2\{1\})^4 - \frac{1}{4(N_c^2 - 1)^3} \right].$$  \hspace{1cm} (37)

This expression applies when $v_2\{1\} = \mathcal{O}(N_c^{-3/2})$, where $c_2\{4\}$ changes sign. The first term on the r.h.s. corresponds to the fully factorized distribution and is the only negative contribution to $c_2\{4\}$. Thus, parametrically this transition to $c_2\{4\} < 0$ occurs before the one-particle factorizable contribution dominates $c_2\{2\}$. That is, in the vicinity of $c_2\{4\} = 0$ the two-particle cumulant $c_2\{2\}$ is dominated at leading order in $1/N_c^2$ by connected diagrams. We repeat, also, that all contributions in eqs. (36-37) computed within small-x QCD are long range in rapidity.

Our analysis naturally raises a question about the magnitude of the $\vec{E}$-field polarization amplitude $\mathcal{A}$ and its dependence on multiplicity. Averaging over all target configurations without a multiplicity bias gives $\mathcal{A} \sim 0.1 - 0.15$ at small $x$ [23]. In fact, $\mathcal{A}(r)$ exhibits a (weak) dependence on $r$ at small $r$ and this function has been found [25] to coincide with the distribution of linearly polarized gluons (for the MV model) obtained in refs. [26]. The effect of a multiplicity bias remains to be investigated. In order for the disconnected contribution to dominate in high multiplicity events, $\mathcal{A}$ would have to grow with multiplicity.

Although our present discussion is restricted to high-$p_T$ particles, i.e. small dipoles, it suggests that the measurement by the ALICE and CMS collaborations of a sign change of $c_2\{4\}$ corresponds to the fully factorizable contribution becoming dominant. The emergence of “collectivity” in pA collisions could be viewed as multi-particle correlation functions becoming dominated by fully disconnected diagrams, analogous to the BBGKY hierarchy. It will be important to understand specifically how this emerges from small-x QCD dynamics.

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