Constraints on dark energy from the lookback time versus redshift test

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Abstract

We use lookback time versus redshift data from galaxy clusters (Capozziello et al., 2004) and passively evolving galaxies (Simon et al., 2005), and apply a bayesian prior on the total age of the Universe based on WMAP measurements, to constrain dark energy cosmological model parameters. Current lookback time data provide interesting and moderately restrictive constraints on cosmological parameters. When used jointly with current baryon acoustic peak and Type Ia supernovae apparent magnitude versus redshift data, lookback time data tighten the constraints on parameters and favor slightly smaller values of the nonrelativistic matter energy density.

Keywords: cosmological parameters, galaxies: general, galaxies: clusters: general

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1. Introduction

It is now a well established fact that the expansion of the Universe is accelerating, but the underlying mechanism which gives rise to this cosmic acceleration is still a mystery. Recent cosmological observations including the Hubble diagram of Type Ia supernovae (SNeIa, e.g., Hicken et al., 2009; Shafieloo et al., 2009; Guimarães et al., 2009), combined with cosmic microwave background (CMB) anisotropy measurements (e.g., Dunkley et al., 2009; Komatsu et al., 2009), baryon acoustic peak galaxy power spectrum data (e.g., Percival et al., 2007; Samushia & Ratra, 2009; Gaztañaga et al., 2009; Wang, 2009), and galaxy cluster gas mass fraction measurements (e.g., Allen et al., 2008; Samushia & Ratra, 2008; Ettori et al., 2009) indicate that we live in a spatially-flat universe where nonrelativistic matter contributes about 30% of the critical density. Within the framework of Einstein’s general theory of relativity, the rest of the 70% of the energy density of the Universe is termed dark energy, a mysterious component with negative
effective pressure that is responsible for the observed accelerated expansion. For recent reviews of dark energy see Ratra & Vogelezang (2008), Caldwell & Kamionkowski (2009), Frieman (2009), and Sami (2009).

There are many dark energy candidates. The simplest is Einstein’s cosmological constant $\Lambda$. In addition, there are other options like XCDM, a slowly rolling scalar field, Chaplygin gas, etc., which can also give rise to an accelerated expansion of the Universe. In this paper we constrain the parameters of three different dark energy models. The first model is the cosmological constant dominated cold dark matter ($\Lambda$CDM) model (Peebles, 1984). In this model the energy density of the vacuum (the cosmological constant) does not vary with time and it has a negative pressure characterized by $p_\Lambda = -\rho_\Lambda$, where $\rho_\Lambda$ is the vacuum energy density.

Secondly, we consider the XCDM parameterization of dark energy. In this case dark energy is assumed to be a fluid satisfying the following relation between pressure and the energy density, $p_x = \omega_x \rho_x$, with $\omega_x < 0$; this is not a physically complete model. Lastly, we study the slowly rolling dark energy scalar field $\phi$ model ($\phi$CDM) with an inverse power-law potential energy density for the scalar field, $V \propto \phi^{-\alpha}$ where $\alpha$ is a nonnegative constant (Peebles & Ratra, 1988; Ratra & Peebles, 1988; Peebles & Ratra, 2003).

We only consider the spatially-flat $\phi$CDM and XCDM cases. The $\phi$CDM model with $\alpha = 0$ and the XCDM model with $\omega_x = -1$ are equivalent to the spatially-flat $\Lambda$CDM model with the same matter density. In all three models the nonrelativistic matter density is dominated by cold dark matter.

In this paper we use two sets of lookback time versus redshift measurements, for galaxy clusters (Capozziello et al., 2004) and for passively evolving galaxies (Simon et al., 2005), and apply a bayesian prior on the total age of the Universe based on WMAP estimates (Dunkley et al., 2009), to constrain parameters of these dark energy models. This time-based cosmological test differs from other widely-used distance-based cosmological tests. An important feature of this time-based method is that the age of distant objects are independent of each other. Therefore, it may avoid biases that are present in techniques that use distances of primary or secondary indicators in the cosmic distance ladder method. In the literature a variety of time-based methods have been considered, based on measurements of the absolute age of objects, differential age of objects, and lookback time of objects.

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1For discussions of modification of Einsteinian gravity on cosmological scales that attempt to do away with the need for dark energy, see Ranetti et al. (2009), Bamba & Geni (2009), Capozziello & Salzano (2009), Wu & Chen (2009), Zhou et al. (2009), and references therein.

2In the $\phi$CDM model we consider here, $\phi$ only couples gravitationally to other components. For models where $\phi$ also interacts more directly with other components, see Chen et al. (2009), Baldi et al. (2009), Bento & Gonzalez Felipe (2009), Pettorino et al. (2009), Gavela et al. (2009), La Vacca et al. (2009), and references therein. For other dark energy models, see La et al. (2009), Arbej (2008), Hayek & Savidovskii (2009), Basilakos (2009), Tsujikawa et al. (2009), Dutta & Scherrer (2009), Neupane & Trowland (2009), and references therein.

3Distance-based cosmological tests include those mentioned above that use SNeIa, CMB, baryon acoustic peak, and galaxy cluster gas mass fraction data, as well as radio-galaxy and quasar angular size versus redshift data (e.g., Chen et al. (2009), Podariu et al. (2009), Daly et al. (2009), Santos & Lima (2008), and references therein). For further tests see Lin et al. (2008), Wei & Zhang (2008), Liang & Zhang (2008), Wang (2008), Samushia & Ratra (2008), and references therein.

4A variation of this test uses measurements of the Hubble parameter as a function of redshift (e.g., Samushia & Ratra, 2006, Lin et al. 2008, Dev et al. 2008, Fernandez-Martinez & Verde, 2008, and references therein).
The absolute age method is based on the simple criterion that the age of the Universe at a given redshift is always greater than or equal to the age of the oldest object at that redshift (Alcaniz & Lima, 1999; Lima & Alcaniz, 2000; Jain & Dev, 2006; Wei & Zhang, 2007). The differential age method is based on the measurement of $\Delta z/\Delta t$. $\Delta z$ is the redshift separation between the two passively evolving galaxies having the age difference $\Delta t$ (Jimenez & Loeb, 2002; Jimenez et al., 2003). This method requires a large sample of passively evolving galaxies with high quality spectroscopy and is probably more reliable than the absolute age method as a number of systematic effects are eliminated.

Lookback time as a tool to constrain dark energy models was first used by Capozziello et al. (2004) who compiled a list of galaxy cluster ages and redshifts and used this data to constrain the XCDM dark energy parameterization. This data has been used to constrain brane cosmology and holographic dark energy models (Pires et al., 2006; Yi & Zhang, 2007). The lookback time test has also been applied using passively evolving galaxies data, to constrain parameters of XCDM and ΛCDM (Dantas et al., 2007, 2009). No doubt these time-based methods are subject to some different systematic errors but they offer an independent means to cross-check cosmological constraints obtained using other techniques.

In this paper we take advantage of the fact that the Capozziello et al. (2004) galaxy cluster data and the Simon et al. (2005) passive galaxy data are independent, so it is straightforward to use them simultaneously in a lookback time versus redshift test analysis of dark energy models. Our joint analyses of these data sets allow us to derive the tightest lookback time constraints on dark energy parameters to date. The resulting constraints are moderately restrictive, and these data favor lower matter density values than do some other current data, but are consistent with a spatially-flat ΛCDM model in which nonrelativistic matter contributes 30% of the energy budget at a little less than two standard deviations. To derive tighter constraints, we perform a joint analysis of the lookback time data with current baryon acoustic peak and SNeIa measurements.

In Sec. 2 we describe the lookback time as a function of redshift test. The data and method we use are outlined in Sec. 3. Our results are presented and discussed in Sec. 4.

2. Lookback time versus redshift test

The lookback time is the difference between the present age of the Universe ($t_0$) and its age at redshift $z$, $t(z)$,

$$t_L(z, p) = t_0(p) - t(z) = \frac{1}{H_0} \left[ \int_0^\infty \frac{dz'}{(1+z')\mathcal{H}(z', p)} - \int_z^\infty \frac{dz'}{(1+z')\mathcal{H}(z', p)} \right]$$

$$= \frac{1}{H_0} \int_0^z \frac{dz'}{(1+z')\mathcal{H}(z', p)}. \quad (1)$$

Here $p$ are the parameters of the cosmological model under consideration, $\mathcal{H}(z, p) = H(z, p)/H_0$, $H(z, p)$ is the Hubble parameter at redshift $z$, and the Hubble constant $H_0 = 100 h$ km s$^{-1}$Mpc$^{-1}$.

Following Capozziello et al. (2004), the observed lookback time $t_L^{\text{obs}}(z_i)$, to an object $i$ at redshift $z_i$ is defined as

$$t_L^{\text{obs}}(z_i, t_{\text{inc}}, t_0^{\text{obs}}) = \frac{t_0^{\text{obs}} - t_i(z_i)}{3} - t_{\text{inc}}. \quad (2)$$
Here

- $t_0^{obs}$ is the measured current age of the Universe.
- $t_i(z_i)$ is the age of the object (passively evolving galaxy, cluster, etc.), defined as the difference between the current age of the Universe at redshift $z_i$ and the age of the Universe when the object was born at redshift $z_f$,

$$t_i(z_i) = t(z_i) - t(z_f) = t_L(z_f) - t_L(z_i) = \frac{1}{H_0} \int_{z_i}^{z_f} \frac{dz'}{(1 + z')H(z', p)}.$$  \hspace{1cm} (3)

where we have used Eq. (1).

- $t_{inc} = t_0^{obs} - t_L(z_f)$ is the incubation time of the object. This delay factor encodes our ignorance of the formation redshift $z_f$.

To compute model predictions for the lookback time $t_L(z, p)$, Eq. (1), we need an expression for $H(z, p)$. In the ΛCDM model the Hubble parameter is

$$H(z, p) = H_0 \left[ \Omega_m (1 + z)^3 + (1 - \Omega_m - \Omega_\Lambda)(1 + z)^2 + \Omega_\Lambda \right]^{1/2},$$  \hspace{1cm} (4)

where $p$ are $\Omega_m$ and $\Omega_\Lambda$, the nonrelativistic matter and dark energy density parameters at $z = 0$. For the XCDM parameterization in a spatially-flat cosmological model we have

$$H(z, p) = H_0 \left[ \Omega_m (1 + z)^3 + (1 - \Omega_m)(1 + z)^3(1 + \omega_x) \right]^{1/2},$$  \hspace{1cm} (5)

where $p$ are $\Omega_m$ and $\omega_x$. In the spatially-flat $\phi$CDM model

$$H(z, p) = H_0 \left[ \Omega_m (1 + z)^3 + \Omega_\phi(z) \right]^{1/2},$$  \hspace{1cm} (6)

where the scalar field energy density parameter $\Omega_\phi(z)$ can be evaluated numerically by solving the coupled set of equations of motion,

$$\ddot{\phi} + \frac{3}{a} \dot{a} \phi - \frac{\kappa \alpha}{2} a^2 \phi^{-(\alpha + 1)} = 0,$$  \hspace{1cm} (7)

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3 m_p^2} \Omega_m (1 + z)^3 + \Omega_\phi(z),$$  \hspace{1cm} (8)

$$\Omega_\phi(z) = [\langle \dot{\phi}^2 \rangle + \kappa m_p^2 \phi^{-\alpha}] / 12.$$  \hspace{1cm} (9)

Here $a(t)$ is the scale factor, an overdot denotes a time derivative, $m_p$ is Planck’s mass, and $\kappa$ and $\alpha$ are non-negative constants that characterize the inverse power law potential energy density of the scalar field, $V(\phi) = \kappa \phi^{-\alpha}$. In this case the parameters $p$ are $\Omega_m$ and $\alpha$.

3. Data and computation

In order to constrain cosmological parameters of ΛCDM, XCDM, and $\phi$CDM, we use two age data sets. One is the [Simon et al. (2005)](2005) ages of 32 passively evolving galaxies (Table 1, R. Jimenez, private communication 2007) in the redshift interval $0.117 \leq z \leq 1.845$. For this sample we assume a 12% one standard deviation uncertainty on the age measurements (R. Jimenez, private communication 2007). The other is the [Capozziello et al. (2004)](2004) Table 1) ages of 6 galaxy clusters in the redshift
where $\sigma$ values in the model under consideration. From $\chi^2$ range 0 \leq \chi \leq 1.27$. This sample has a 1 Gyr one standard deviation uncertainty on the age measurements. In all, we have 38 measurements of $t_L^{\text{obs}}(z_i)$ with uncorrelated uncertainties $\sigma_i$.

For each model and parameter value set $(p)$ we compute the $\chi^2$ function
\[
\chi^2(p, H_0, t_{\text{inc}}, t_0^{\text{obs}}) = \sum_{i=1}^{38} \frac{(t_L(z_i, p, H_0) - t_L^{\text{obs}}(z_i, t_{\text{inc}}, t_0^{\text{obs}}))^2}{\sigma_i^2 + \sigma_{t_0}^2} + \frac{(t_0(p, H_0) - t_0^{\text{obs}})^2}{\sigma_{t_0}^2},
\]
where $\sigma_{t_0}^{\text{obs}}$ is the uncertainty in the estimate of $t_0$ and $t_L(z_i, p)$ and $t_0(p)$ are the predicted values in the model under consideration. From $\chi^2$ we construct a likelihood function $L'(p, H_0, t_{\text{inc}}) \propto \exp\left(-\chi^2/2\right)$.

The likelihood function $L'(p, H_0, t_{\text{inc}}, t_0^{\text{obs}})$ depends on the total age of the Universe $t_0^{\text{obs}}$, incubation time $t_{\text{inc}}$ and the Hubble parameter $H_0$. We do not know $t_{\text{inc}}$ and so treat it as a nuisance parameter and analytically marginalize $L'$ over it as in Capozziello et al. (2004); Dantas et al. (2007). We treat $H_0$ as a nuisance parameter and marginalize over it with a Gaussian prior with $h = 0.742 \pm 0.036$ (Riess et al., 2009), a little higher than, but still consistent with, the earlier summary value of $h = 0.68 \pm 0.04$ (Chen et al., 2003). We also also apply a bayesian prior as a Gaussian function with central value and variances based on the WMAP estimate of the total age of the Universe, which is $t_0^{\text{obs}} = (13.75 \pm 0.13)$ Gyr for the $\Lambda$CDM model and $t_0^{\text{obs}} = (13.75 \pm 0.26)$ Gyr for the XCDM model (Dunkley et al., 2009). For the $\phi$CDM model we assume the same central value as the other two models and conservatively inflate the error bar to $t_0^{\text{obs}} = (13.75 \pm 0.5)$ Gyr. The resulting lookback time likelihood function depends only on the two cosmological parameters $p$, $L(p)$. The best fit parameters are the pair $p^*$ that maximize the likelihood function and the 1, 2, and 3$\sigma$ confidence level contours are defined as the sets of cosmological parameters $p_\sigma$ at which the likelihood $L(p_\sigma)$ is $\exp(-2.30/2)$, $\exp(-6.18/2)$, and $\exp(-11.83/2)$ times smaller than the maximum likelihood $L(p^*)$.

To check our method we used the Capozziello et al. (2004) galaxy cluster data and the earlier $t_0^{\text{obs}}$ result they used and computed the constraints on the XCDM parameterization. Our contours are consistent with those shown in Fig. 2 of Capozziello et al. (2004). We also used the Simon et al. (2005) passively evolving galaxy ages and the $t_0^{\text{obs}}$ value Dantas et al. (2007) used to constrain the $\Lambda$CDM model. We find that if we pick $h = 0.72$ we are able to accurately reproduce the central and right panels of Fig. 2 of Dantas et al. (2007).

The lookback time versus redshift data constraints on $\Lambda$CDM, XCDM, and $\phi$CDM are shown in Figs. 1–3.

4. Results and discussions

Figure 1 shows the constraints on the $\Lambda$CDM model from the lookback time and age of the Universe measurements. The data favor low values of both $\Omega_m$ and $\Omega_\Lambda$ with the best-fit values being $\Omega_m = 0.01$ and $\Omega_\Lambda = 0.19$. These data prefer spatially-open models, however a spatially-flat $\Lambda$CDM model with $\Omega_m = 0.3$ is less than 3$\sigma$ from the best fit model. The data constrains $\Omega_m$ to be less than 0.45 on 3$\sigma$ confidence level.

\footnote{The numbers are taken from \url{http://lambda.gsfc.nasa.gov/}}
Figure 2 presents the constraints on the XCDM parametrization of the equation of state. The nonrelativistic matter density parameter is constrained to be less than 0.5 at 3σ confidence. Low values of $\Omega_m$ are favored with the best-fit values being $\Omega_m = 0.03$ and $\omega_x = -0.41$ and a spatially-flat model with $\Omega_m = 0.3$ is about 2σ from the best fit model.

Figure 3 shows the constraints on the $\phi$CDM model of dark energy. In this model the nonrelativistic matter density parameter is less than 0.5 at 3σ confidence. The $\alpha$ parameter on the other hand is not well constrained. The best-fit parameter value is $\alpha = 10$, but the likelihood is very flat in the direction of $\alpha$ and the difference between the best-fit value and $\alpha = 0$ (which is the spatially-flat $\Lambda$CDM case) is slightly less than 2σ.

Current lookback time data by themselves are unable to tightly constrain cosmological parameters. Constraints from galaxy cluster gas mass fraction versus redshift data (e.g., Chen & Ratra, 2004), SNeIa apparent magnitude versus redshift measurements (e.g., Wilson et al., 2006), and baryon acoustic peak data (e.g., Samushia & Ratra, 2009) are more restrictive than the lookback time constraints. However, the constraints from lookback time data are somewhat tighter than the constraints from strong gravitational lensing data (e.g., Chae et al., 2004), measurements of the Hubble parameter as a function of redshift (e.g., Samushia et al., 2007), radio galaxy angular size versus redshift data (e.g., Daly et al., 2009), and gamma-ray burst luminosity distance versus redshift data (e.g., Samushia & Ratra, 2010).

To get tighter constraints on cosmological parameters we combine the lookback time data and the measurement of the age of the Universe with baryon acoustic peak data (Percival et al., 2007) and SNeIa “Union” apparent magnitude versus redshift measurements (Kowalski et al., 2008). Since these data sets are independent we compute a joint likelihood function that is a product of individual likelihood functions

$$L_{\text{joint}} = L_L L_{\text{BAO}} L_{\text{SNe}},$$

and define the best-fit parameters and confidence level contours as discussed above.

The constraints on the three dark energy models from a joint analysis of these data are shown in Figs. 4–6. Currently available lookback time data do not significantly change the results derived using BAO peak measurements and SNeIa apparent magnitude data. In all three dark energy models when lookback time data are added to the mix the confidence level regions favor slightly smaller values of nonrelativistic matter density parameter $\Omega_m$.

Overall, current data is a good fit to all three dark energy models. For $\phi$CDM and XCDM they slightly favor time-dependent dark energy, but the time-independent cosmological constant is also a good fit.

We anticipate that a new, improved data set of lookback times will soon be available (R. Jimenez, private communication, 2009.) With more and better data we expect significantly tighter constraints on dark energy parameters. The lookback time versus redshift test, either by itself or at least in combination with other cosmological probes, could prove very useful in detecting or constraining dark energy time evolution.
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Figure 1: 1, 2, and 3σ confidence level contours for the ΛCDM model from the lookback time data and measurement of the age of the Universe. The dashed line corresponds to spatially-flat models. The cross indicates the best-fit parameters $\Omega_m = 0.01$ and $\Omega_{\Lambda} = 0.19$ with $\chi^2 = 33$ for 37 degrees of freedom.
Figure 2: 1, 2, and 3σ confidence level contours for the XCDM parameterization of dark energy in a spatially-flat cosmological model, from the lookback time data and measurement of the age of the Universe. The dashed $\omega_x = -1$ line corresponds to spatially-flat ΛCDM models. The cross indicates the best-fit parameters $\Omega_m = 0.03$ and $\omega_x = -0.41$ with $\chi^2 = 28$ for 37 degrees of freedom.
Figure 3: 1, 2, and 3σ confidence level contours for the spatially-flat $\phi$CDM model from the lookback time data and measurement of the age of the Universe. The $\alpha = 0$ horizontal axis corresponds to spatially-flat $\Lambda$CDM models. The cross indicates the best-fit parameters $\Omega_m = 0.04$ and $\alpha = 10$ with $\chi^2 = 22$ for 37 degrees of freedom.
Figure 4: 1, 2, and 3σ confidence level contours for the $\Lambda$CDM model. Numerical noise is responsible for the jaggedness of parts of the contours. The dashed line demarcates spatially-flat models. Dotted lines (circle denotes the best-fit point at $\Omega_m = 0.30$ and $\Omega_\Lambda = 0.78$ with $\chi^2 = 359$ for 346 degrees of freedom) are derived using the lookback time data, measurement of the age of the Universe, SNeIa Union data, and BAO peak measurements, while solid lines (cross denotes the best-fit point at $\Omega_m = 0.32$ and $\Omega_\Lambda = 0.78$ with $\chi^2 = 318$ for 307 degrees of freedom) are derived using SNeIa and BAO data. The dashed line corresponds to spatially-flat models.
Figure 5: 1, 2, and 3σ confidence level contours for the XCDM parameterization of dark energy in a spatially-flat cosmological model. The dashed line demarcates spatially-flat ΛCDM models. Dotted lines (circle denotes the best-fit point at $\Omega_m = 0.19$ and $\omega_x = -0.80$ with $\chi^2 = 352$ for 346 degrees of freedom) are derived using the lookback time data, measurement of the age of the Universe, SNeIa Union data, and BAO peak measurements, while solid lines (cross denotes the best-fit point at $\Omega_m = 0.19$ and $\omega_x = -0.81$ with $\chi^2 = 321$ for 307 degrees of freedom) are derived using only SNeIa and BAO data. The dashed $\omega_x$ line corresponds to spatially-flat ΛCDM models.
Figure 6: 1, 2, and 3σ confidence level contours for the spatially-flat $\phi$CDM model. The $\alpha = 0$ horizontal axis corresponds to spatially-flat $\Lambda$CDM models. Dotted lines (circle denotes the best-fit point at $\Omega_m = 0.215$ and $\alpha = 0.0$ with $\chi^2 = 359$ for 346 degrees of freedom) are derived using the lookback time data, measurement of the age of the Universe, SNeIa Union data, and BAO peak measurements, while solid lines (cross denotes the best-fit point at $\Omega_m = 0.22$ and $\alpha = 0.0$ with $\chi^2 = 329$ for 307 degrees of freedom) are derived using only SNeIa and BAO data.
| $z_i$ | $t_i(z_i)$ (Gyr) |
|------|-----------------|
| 0.1171 | 10.2 |
| 0.1174 | 10.0 |
| 0.2220 | 9.0 |
| 0.2311 | 9.0 |
| 0.3559 | 7.6 |
| 0.4520 | 6.8 |
| 0.5750 | 7.0 |
| 0.6440 | 6.0 |
| 0.6760 | 6.0 |
| 0.8330 | 6.0 |
| 0.8360 | 5.8 |
| 0.9220 | 5.5 |
| 1.179 | 4.6 |
| 1.222 | 3.5 |
| 1.224 | 4.3 |
| 1.225 | 3.5 |
| 1.226 | 3.5 |
| 1.340 | 3.4 |
| 1.380 | 3.5 |
| 1.383 | 3.5 |
| 1.396 | 3.6 |
| 1.430 | 3.2 |
| 1.450 | 3.2 |
| 1.488 | 3.0 |
| 1.490 | 3.6 |
| 1.493 | 3.2 |
| 1.510 | 2.8 |
| 1.550 | 3.0 |
| 1.576 | 2.5 |
| 1.642 | 3.0 |
| 1.725 | 2.6 |
| 1.845 | 2.5 |