RENORMALIZATION GROUP AND RELATIONS BETWEEN SCATTERING AMPLITUDES IN A THEORY WITH DIFFERENT MASS SCALES

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Abstract

In the Yukawa model with two different mass scales the renormalization group equation is used to obtain relations between scattering amplitudes at low energies. Considering fermion-fermion scattering as an example, a basic one-loop renormalization group relation is derived which gives possibility to reduce the problem to the scattering of light particles on the "external field" substituting a heavy virtual state. Applications of the results to problems of searching new physics beyond the Standard Model are discussed.

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1 Introduction

An important problem of nowadays high energy physics is searching for deviation from the Standard Model (SM) of elementary particles which may appear due to heavy virtual states entering the extended models and having the masses much greater than the W-boson mass $m_W$ [1]. One of approaches for the description of such phenomena is the construction of the effective Lagrangians (EL) appearing owing to decoupling of heavy particles. In principle, it is possible to write down a lot of different EL describing effects of new physics beyond the SM. In Ref. [2] the EL generated at a tree level in a general renormalizable gauge theory have been derived. These objects by construction contain a great number of arbitrary parameters responsible for specific processes. But it is well known that a renormalizable theory includes a small number of independent constants due to relations between them. The renormalizability of the theory is resulted in the renormalization group (RG) equations for scattering amplitudes [3]. In Ref. [4] it has been proven that RG equation can be used to obtain a set of relations between the parameters of the EL. Two main observations were used. First, it has been shown that a heavy virtual state may be considered as an external field scattering SM light particles. Second, the renormalization of the vertices, describing scattering on the external field, can be determined by the $\beta$- and $\gamma$- functions calculated with light particles, only. Hence, the relations mentioned above follow. As an example the SM with the heavy Higgs scalar has been investigated. In the decoupling region the RG equations for scattering amplitudes have been reduced to the ones for vertices describing the scattering of light particles on the external field substituting the corresponding virtual heavy field. In Ref.[4] the only scalar field of the theory was taken as the heavy particle, and no mixing between the heavy and the light fields at the one-loop level has been considered. Here, we are going to investigate the Yukawa model with a heavy scalar field $\chi$ and a light scalar field $\varphi$. The purposes of our investigation are two fold: to derive the one-loop RG relation for the four-fermion scattering amplitude in the decoupling region and to find out the possibility of reducing this relation in the equation for vertex describing the scattering of light particles on the external field when the mixing between heavy and light virtual states takes place. In Ref.[4] the specific algebraic identities originated from the RG equation for scattering amplitude have been derived. When the explicit couplings in EL are unknown and represented by the arbitrary parameters, one may treat the identities as the equations dependent on the parameters and appropriate $\beta$- and $\gamma$- functions. If due to a symmetry the number of $\beta$- and $\gamma$- functions is less than the number of RG relations, one can obtain non trivial system of equations for the parameters mentioned. This was shown for the gauge couplings [4]. In present paper we derive RG relations for the EL parameters in the model including one-loop mixing of heavy and light fields.

2 Renormalization group relation for amplitude

The Lagrangian of the model reads

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \varphi)^2 - \frac{m^2}{2} \varphi^2 - \lambda \varphi^4 + \frac{1}{2} (\partial_{\mu} \chi)^2 - \frac{\Lambda^2}{2} \chi^2 - \xi \chi^4$$
\[ \rho \varphi^2 \chi^2 + \bar{\psi} (i \partial_{\mu} \gamma_{\mu} - M - G_{\varphi} \varphi - G_{\chi} \chi) \psi, \]  

(1)

where \( \psi \) is a Dirac spinor field. The \( S \)-matrix element for the four-fermion scattering at the one-loop level is given by

\[ \hat{S} = -\frac{i}{2} \int \frac{dp_1}{(2\pi)^4} \cdots \frac{dp_4}{(2\pi)^4} (2\pi)^4 \delta (p_1 + \cdots + p_4) \mathcal{N} [S_{1PR} + S_{box}], \]

\[ S_{1PR} = \sum_{\phi_1, \phi_2 = \varphi, \chi} G_{\phi_1} G_{\phi_2} \left( \frac{\delta_{\phi_1 \phi_2}}{s - m_{\phi_1}} + \frac{1}{s - m_{\phi_1}} \Pi_{\phi_1 \phi_2} (s) \frac{1}{s - m_{\phi_2}} \right) \times \]

\[ \bar{\psi} (p_1) (1 + 2 \Gamma (p_2, -p_1 - p_2)) \psi (p_2) \times \bar{\psi} (p_4) \psi (p_3), \]

(2)

where \( s = (p_1 + p_2)^2 \), \( S_{1PR} \) is the contribution from the one-particle reducible diagrams shown in the Figs.\# and \( S_{box} \) is the contribution from the box diagram. The one-loop polarization operator of scalar fields \( \Pi_{\phi_1 \phi_2} \) and the one-loop vertex function \( \Gamma \) are usually defined through the Green functions:

\[ D_{\phi_1 \phi_2} (s) = \frac{\delta_{\phi_1 \phi_2}}{s - m_{\phi_1}} + \frac{1}{s - m_{\phi_1}} \Pi_{\phi_1 \phi_2} (s) \frac{1}{s - m_{\phi_2}}, \]

\[ G_{\phi \phi \psi} (p, q) = -\sum_{\phi_1} G_{\phi_1} D_{\phi_1 \phi} \left( q^2 \right) S_{\psi} (p) (1 + \Gamma (p, q)) S_{\psi} (-p - q), \]

(3)

where \( S_{\psi} \) is the spinor propagator in the momentum representation. The renormalized fields, masses and charges are defined as follows:

\[ \psi = Z_{3/2}^{-1/2} \psi_0, \quad \left( \begin{array}{c} \varphi \\ \chi \end{array} \right) = Z_{\phi}^{-1/2} \left( \begin{array}{c} \varphi_0 \\ \chi_0 \end{array} \right), \quad \left( \begin{array}{c} G_{\varphi} \\ G_{\chi} \end{array} \right) = Z_{G}^{-1} \left( \begin{array}{c} G_{\varphi_0} \\ G_{\chi_0} \end{array} \right), \]

\[ M^2 = M_0^2 - \delta M^2, \quad m^2 = m_0^2 - \delta m^2, \quad \Lambda^2 = \Lambda_0^2 - \delta \Lambda^2, \]

(4)

Using the dimensional regularization (the dimension of the momentum space is \( D = 4 - \varepsilon \)) and the \( \overline{MS} \) renormalization scheme \# one can compute the renormalization constants

\[ Z_\psi = 1 - \frac{1}{16 \pi^2 \varepsilon} \left( G_\varphi^2 + G_\chi^2 \right), \quad \delta M^2 = \frac{3}{8 \pi^2 \varepsilon} \left( G_\varphi^2 + G_\chi^2 \right) M^2, \]

\[ Z_{\phi}^{1/2} = 1 - \frac{1}{8 \pi^2 \varepsilon} \left( \frac{G_\varphi^2}{-2 G_\varphi G_\chi \frac{m^2 - 6 M_0^2}{\Lambda^2 - m^2}} + \frac{2 G_\varphi G_\chi \frac{m^2 - 6 M_0^2}{\Lambda^2 - m^2}}{G_\chi^2} \right), \]

\[ \delta m^2 = \frac{1}{4 \pi^2 \varepsilon} \left( \left( G_\varphi^2 + 6 \lambda \right) m^2 - 6 G_\varphi^2 M^2 - \rho \Lambda^2 \right), \]

\[ \delta \Lambda^2 = \frac{1}{4 \pi^2 \varepsilon} \left( \left( G_\chi^2 + 6 \xi \right) \Lambda^2 - 6 G_\chi^2 M^2 - \rho m^2 \right), \]

\[ Z_{G}^{-1} = \left[ 1 - \frac{3}{16 \pi^2 \varepsilon} \left( G_\varphi^2 + G_\chi^2 \right) \right] \left( Z_{\phi}^{1/2} \right)^T. \]

(5)

From Eq.\# we obtain the appropriate \( \beta - \) and \( \gamma - \) functions \# at the one-loop level:

\[ \beta_\varphi = \frac{d G_\varphi}{d \ln \Lambda} = \frac{1}{16 \pi^2} \left( 5 G_\varphi^3 + 3 G_\varphi G_\chi^2 - 4 m^2 - 6 M^2 \frac{\Lambda}{\Lambda^2 - m^2} G_\varphi G_\chi^2 \right), \]

(6)
\[
\beta_\chi = \frac{dG_\chi}{d\ln\kappa} = \frac{1}{16\pi^2} \left( 5G_\chi^3 + 3G_\chi G_\varphi^2 + 4 \frac{\Lambda^2 - 6M^2}{\Lambda^2 - m^2} G_\chi G_\varphi^2 \right),
\]

\[
\gamma_m = -\frac{dlnm^2}{d\ln\kappa} = -\frac{1}{4\pi^2} \left( G_\varphi^2 \frac{m^2}{m^2} - 6M^2 + 6\lambda - \rho \frac{\Lambda^2}{m^2} \right),
\]

\[
\gamma_\Lambda = -\frac{dln\Lambda^2}{d\ln\kappa} = -\frac{1}{4\pi^2} \left( G_\chi^2 \left( 1 - \frac{6M^2}{\Lambda^2} \right) + 6\xi - \frac{m^2}{\Lambda^2} \right),
\]

\[
\gamma_\psi = -\frac{dln\psi}{d\ln\kappa} = \frac{1}{32\pi^2} \left( G_\varphi^2 + G_\chi^2 \right).
\]

Then, the S-matrix element can be expressed in terms of the renormalized quantities \(\Pi^{\text{fin}}\). The contribution from the one-particle reducible diagrams becomes

\[
S_{1PR} = \sum_{\phi_1, \phi_2} G_{\phi_1} G_{\phi_2} \left( \frac{\delta_{\phi_1 \phi_2}}{s-m_{\phi_1}} + \frac{1}{s-m_{\phi_1}} \Pi^{\text{fin}}_{\phi_1 \phi_2} (s) \frac{1}{s-m_{\phi_2}} \right)
\]

\[
\tilde{\psi}(p_1) \left( 1 + 2\Gamma^{\text{fin}}(p_2, -p_1 - p_2) \right) \psi(p_2) \times \tilde{\psi}(p_3) \psi(p_3),
\]

where the functions \(\Pi^{\text{fin}}_{\phi_1 \phi_2}\) and \(\Gamma^{\text{fin}}\) are the expressions \(\Pi_{\phi_1 \phi_2}\) and \(\Gamma\) without the terms proportional to \(1/\epsilon\). Since the quantity \(S_{\text{box}}\) is finite, the renormalization leaves it without changes. Introducing the RG operator at the one-loop level \([6]\)

\[
\mathcal{D} = \frac{d}{d\ln\kappa} = \frac{\partial}{\partial\ln\kappa} + \mathcal{D}^{(1)} = \frac{\partial}{\partial\ln\kappa} + \sum_\phi \beta_\phi \frac{\partial}{\partial G_\phi} - \gamma_m \frac{\partial}{\partial\ln m^2} - \gamma_\Lambda \frac{\partial}{\partial\ln\Lambda^2} - \gamma_\psi \frac{\partial}{\partial\ln\psi} \quad (8)
\]

we determine that the following relation holds for the S-matrix element

\[
\mathcal{D} (S_{1PR} + S_{\text{box}}) = \frac{\partial S_{1PR}^{(1)}}{\partial\ln\kappa} + \mathcal{D}^{(1)} S_{1PR}^{(0)} = 0, \quad (9)
\]

where the \(S_{1PR}^{(0)}\) and the \(S_{1PR}^{(1)}\) are the contributions to the \(S_{1PR}\) at the tree level and at the one-loop level, respectively:

\[
S_{1PR}^{(0)} = \left( \frac{G_\varphi^2}{s-m^2} + \frac{G_\chi^2}{s-\Lambda^2} \right) \tilde{\psi}\psi \times \tilde{\psi}\psi, \quad (10)
\]

\[
\frac{\partial S_{1PR}^{(1)}}{\partial\ln\kappa} = \frac{\tilde{\psi}\psi \times \tilde{\psi}\psi}{4\pi^2} \left( - (G_\varphi^2 + G_\chi^2) \left( \frac{G_\varphi^2}{s-m^2} + \frac{G_\chi^2}{s-\Lambda^2} \right) + \frac{G_\varphi^2 \left( \rho\Lambda^2 - 6\lambda m^2 + G_\varphi^2 (6M^2 - s) \right)}{(s-m^2)^2} + 2G_\varphi^2 G_\chi^2 \left( \frac{6M^2 - s}{(s-m^2)(s-\Lambda^2)} \right) + \frac{G_\chi^2 \left( \rho m^2 - 6\xi\Lambda^2 + G_\chi^2 (6M^2 - s) \right)}{(s-\Lambda^2)^2} \right). \quad (11)
\]

The first term in Eq.\((\text{[11]}))\) is originated from the one-loop correction to the fermion-scalar vertex. The rest terms are connected with the polarization operator of scalars.
The third term describes the one-loop mixing between the scalar fields. It is canceled in the RG relation (3) by the mass-dependent terms in the $\beta-$ functions produced by the non-diagonal elements in $Z_\phi$. Eq.(3) is the consequence of the renormalizability of the model. It insures the leading logarithm terms of the one-loop $S$-matrix element to reproduce the appropriate tree-level structure. In contrast to the familiar treatment we are not going to improve scattering amplitudes by solving Eq.(9). We will use it as an algebraic identity implemented in the renormalizable theory. Naturally if one knows the explicit couplings expressed in terms of the basic set of parameters of the model, this RG relation is trivially fulfilled. But the situation changes when the couplings are represented by unknown arbitrary parameters as it occurs in the EL approach [1],[2]. In this case the RG relations are the algebraic equations dependent on these parameters and appropriate $\beta-$ and $\gamma-$ functions. In the presence of a symmetry the number of $\beta-$ and $\gamma-$ functions is less than the number of RG relations. So, one has non trivial system of equations relating the parameters of EL. Such a scenario is realized for the gauge coupling as it has been demonstrated in [4]. Although the considered simple model has no gauge couplings and no relation between the EL parameters occurs, we are able to demonstrate the general procedure of deriving the RG relations for EL parameters in the theory with one-loop mixing. This is essential for dealing with the EL describing deviations from the SM. At energies $s \ll \Lambda^2$ the heavy scalar field $\chi$ is decoupled. So, the four-fermion scattering amplitude consists of the contribution of the model with no heavy field $\chi$ plus terms of the order $s/\Lambda^2$. The expansion of the heavy scalar propagator

$$\frac{1}{s - \Lambda^2} \rightarrow -\frac{1}{\Lambda^2} \left( 1 + O \left( \frac{s}{\Lambda^2} \right) \right)$$

in Eq.(14) is resulted in the effective contact four-fermion interaction

$$L_{eff} = -\alpha \bar{\psi} \psi \times \bar{\psi} \psi, \quad \alpha = \frac{G^2}{\Lambda^2},$$

and the tree level contribution to the amplitude becomes

$$S_{PR}^{(0)} = \left( \frac{G_\phi^2}{s - m^2} - \alpha + O \left( \frac{s}{\Lambda^4} \right) \right) \bar{\psi} \psi \times \bar{\psi} \psi.$$
One is able to rewrite the differential operator (8) in terms of these new low-energy parameters:

\[ D = \frac{\partial}{\partial \ln \kappa} + \tilde{D}^{(1)} = \frac{\partial}{\partial \ln \kappa} + \sum_\phi \tilde{\beta}_\phi \frac{\partial}{\partial G_\phi} - \tilde{\gamma}_m \frac{\partial}{\partial \ln \tilde{m}^2} - \tilde{\gamma}_\Lambda \frac{\partial}{\partial \ln \tilde{\Lambda}^2} - \tilde{\gamma}_\psi \frac{\partial}{\partial \ln \tilde{\psi}}, \]

where \( \tilde{\beta} \) and \( \tilde{\gamma} \) functions are obtained from the one-loop relations (8) and (15):

\[ \tilde{\beta}_\phi = \frac{1}{16\pi^2} \left( 5\tilde{G}_\phi^3 - 4\frac{\tilde{m}^2 - 6\tilde{M}^2}{\tilde{\Lambda}^2 - \tilde{m}^2} \tilde{G}_\phi \tilde{G}_\phi \right), \]
\[ \tilde{\beta}_\chi = \frac{1}{16\pi^2} \left( 2\tilde{G}_\chi^3 + \left( 3 + 4\frac{\tilde{\Lambda}^2 - 6\tilde{M}^2}{\tilde{\Lambda}^2 - \tilde{m}^2} \right) \tilde{G}_\chi \tilde{G}_\chi \right), \]
\[ \tilde{\gamma}_m = -\frac{1}{4\pi^2} \left( \tilde{G}_\phi^2 \frac{\tilde{m}^2 - 6\tilde{M}^2}{\tilde{m}^2} + 6\tilde{\lambda} \right), \]
\[ \tilde{\gamma}_\Lambda = -\frac{1}{4\pi^2} \left( \tilde{G}_\chi \left( 1 - 6\frac{\tilde{M}^2}{\tilde{\Lambda}^2} \right) - \rho \frac{\tilde{m}^2}{\tilde{\Lambda}^2} \right), \]
\[ \tilde{\gamma}_\psi = \frac{1}{32\pi^2} \tilde{G}_\psi^2. \]

Hence, one immediately notices that \( \tilde{\beta} \) and \( \tilde{\gamma} \) functions contain only the light particle loop contributions, and all the heavy particle loop terms are completely removed from them. The S-matrix element expressed in terms of new parameters satisfies the following RG relation

\[ D (S_{1PR} + S_{box}) = \frac{\partial \tilde{S}_{1PR}^{(1)}}{\partial \ln \kappa} + \tilde{D}^{(1)} \tilde{S}_{1PR}^{(0)} = 0, \]
\[ \tilde{S}_{1PR}^{(0)} = \left( \frac{\tilde{G}_\phi^2}{s - \tilde{m}^2} - \tilde{\alpha} + O \left( \frac{s^2}{\tilde{\Lambda}^4} \right) \right) \tilde{\psi} \tilde{\psi} \times \tilde{\psi} \tilde{\psi}, \]

\[ \frac{\partial \tilde{S}_{1PR}^{(1)}}{\partial \ln \kappa} = \frac{\tilde{\psi} \tilde{\psi} \times \tilde{\psi} \tilde{\psi}}{4\pi^2} \left( -\frac{\tilde{G}_\phi^4}{s - \tilde{m}^2} + \frac{\tilde{G}_\phi^2 \left( -6\tilde{\lambda} \tilde{m}^2 + \tilde{G}_\phi^2 (6\tilde{M}^2 - s) \right)}{(s - \tilde{m}^2)^2} \right) + \tilde{\alpha} \tilde{G}_\phi^2 \frac{2\tilde{G}_\phi^2 \tilde{\alpha} (6\tilde{M}^2 - s)}{s - \tilde{m}^2} + O \left( \frac{s^2}{\tilde{\Lambda}^4} \right), \]

where \( \tilde{\alpha} = \tilde{G}_\chi^2 / \tilde{\Lambda}^2 \) is the redefined effective four-fermion coupling. As one can see, Eq.(20) includes all the terms of Eq.(11) except for the heavy particle loop contributions. It depends on the low energy quantities \( \tilde{\psi}, \tilde{G}_\phi, \tilde{\alpha}, \tilde{\lambda}, \tilde{m}, \tilde{M} \). The first and the second terms in Eq.(20) are just the one-loop amplitude calculated within the model with no heavy particles. The third and the fourth terms describe the light particle loop correction to the effective four-fermion coupling and the mixing of heavy and light virtual fields.

### 3 Elimination of one-loop scalar field mixing

Due to the mixing term it is impossible to split the RG relation (18) for the S-matrix element into the one for vertices. Hence, we are not able to consider Eq.(18) in the
framework of the scattering of light particles on an external field induced by the heavy
virtual scalar as it has been done in [4]. But this is an important step in deriving the
RG relation for EL parameters. Fortunately, there is a simple procedure allowing to
avoid the mixing in Eq.(20). The way is to diagonalize the leading logarithm terms of
the scalar polarization operator in the redefinition of the $\tilde{\varphi}$, $\tilde{\chi}$, $\tilde{G}_\varphi$, $\tilde{G}_\chi$

\[
\left( \begin{array}{c}
\varphi \\
\chi
\end{array} \right) = \zeta^{1/2} \left( \begin{array}{c}
\tilde{\varphi} \\
\tilde{\chi}
\end{array} \right), \quad \left( \begin{array}{c}
G_\varphi \\
G_\chi
\end{array} \right) = \left[ 1 + \frac{3\tilde{G}_\chi^2 \ln \frac{\kappa^2}{\Lambda^2}}{32\pi^2} (\zeta^{-1/2})^T \left( \begin{array}{c}
\tilde{G}_\varphi \\
\tilde{G}_\chi
\end{array} \right) ,
\right.
\]

\[
\zeta^{1/2} = 1 - \frac{\tilde{G}_\varphi \tilde{G}_\chi}{8\pi^2 (\Lambda^2 - \tilde{m}^2)} \ln \frac{\kappa^2}{\Lambda^2} \left( \begin{array}{c}
0 \\
-\tilde{m}^2 - 6\tilde{M}^2
\end{array} \right) .
\]

The appropriate $\tilde{\beta}$– functions

\[
\tilde{\beta}_\varphi = \frac{5\tilde{G}_\varphi^3}{16\pi^2}, \quad \tilde{\beta}_\chi = \frac{1}{16\pi^2} \left( 2\tilde{G}_\chi^3 + 3\tilde{G}_\chi \tilde{G}_\varphi^2 \right)
\]

contain no terms connected with mixing between light and heavy scalars. So, the fourth
term in Eq.(20) is removed, and the RG relation for the S-matrix element becomes

\[
\mathcal{D} \left( S_{1PR} + S_{box} \right) = \frac{\partial \tilde{S}^{(1)}_{1PR}}{\partial \ln \kappa} + \tilde{\mathcal{D}}^{(1)} S^{(0)}_{1PR} = 0,
\]

\[
\tilde{S}^{(0)}_{1PR} = \left( \begin{array}{c}
\tilde{G}_\varphi^2 \\
\tilde{G}_\chi^2 \\
\tilde{G}_\varphi \tilde{G}_\chi
\end{array} \right) \left( \begin{array}{c}
\tilde{s} \\
\tilde{s} \\
\tilde{s}
\end{array} \right) - \tilde{\alpha} + \tilde{O} \left( \begin{array}{c}
s^2 \\
s^2 \\
s^2
\end{array} \right) \frac{\tilde{s} \tilde{\psi} \times \tilde{\psi}}{\tilde{s} - \tilde{m}^2},
\]

\[
\frac{\partial \tilde{S}^{(1)}_{1PR}}{\partial \ln \kappa} = \frac{\tilde{\psi} \tilde{\psi} \times \tilde{\psi}}{4\pi^2} \left( \begin{array}{c}
- \tilde{G}_\varphi^4 \\
\tilde{G}_\varphi^2 \\
\tilde{G}_\varphi^2 (-6\tilde{\lambda} \tilde{m} + \tilde{G}_\chi^2 (6\tilde{M}^2 - \tilde{s}))
\end{array} \right) - \tilde{\alpha} \tilde{G}_\varphi^2 + \tilde{O} \left( \begin{array}{c}
s^2 \\
s^2 \\
s^2
\end{array} \right).\]

At $\tilde{\alpha} = 0$ Eq.(23) is just the RG identity for the scattering amplitude calculated in
the absence of the heavy particles. The terms of order $\tilde{\alpha}$ describe the RG relation for
the effective low-energy four-fermion interaction in the decoupling region. The last one
can be reduced in the RG relation for the vertex describing the scattering of the light
particle (fermion) on the external field $\sqrt{\tilde{\alpha}}$ substituting the virtual heavy scalar:

\[
\mathcal{D} \left( \sqrt{\tilde{\alpha}} \tilde{\psi} \tilde{\psi} \right) = \frac{\tilde{G}_\varphi^2}{8\pi^2} \sqrt{\tilde{\alpha}} \tilde{\psi} \tilde{\psi} + \tilde{\mathcal{D}}^{(1)} \left( \sqrt{\tilde{\alpha}} \tilde{\psi} \tilde{\psi} \right) = 0,
\]

where

\[
\tilde{\mathcal{D}}^{(1)} = \tilde{\beta}_\varphi \frac{\partial}{\partial G_\varphi} - \tilde{\gamma}_\alpha \frac{\partial}{\partial \ln \alpha} - \tilde{\gamma}_m \frac{\partial}{\partial \ln \tilde{m}^2} - \tilde{\gamma}_\psi \frac{\partial}{\partial \ln \psi},
\]

\[
\tilde{\gamma}_\alpha = -\mathcal{D} \tilde{\alpha} = -\frac{1}{8\pi^2} \left( 3\tilde{G}_\varphi^2 + \tilde{O} (\tilde{\alpha}) \right).
\]
Eqs. (23)-(27) is the main result of our investigation. One can derive them with only the knowledge about the EL (13) and the Lagrangian of the model with no heavy particles. One also has to ignore all the heavy particle loop contributions to the RG relation and the one-loop mixing between the heavy and the light fields. Eqs. (23)-(27) depend on the effective low-energy parameters, only. But as the difference between the original set of parameters and the low-energy one is of one-loop order, one may freely substitute them in Eqs. (23)-(26).

4 Discussion

Let us discuss the results obtained. The RG relation for the four-fermion scattering amplitude is derived in the decoupling region $s \ll \Lambda^2$. It was shown that one can redefine the parameters and the fields of the model in order to remove all the heavy particle loop contributions to the RG relation. Then the RG relation becomes dependent on the low-energy physics parameters, only. As the RG operator coefficients and the difference between the original parameters and the redefined ones are of the one-loop order one can substitute one set of parameters by another at the lowest level. Thus, we extend the result of Ref. [4] to the case when mixing terms are present. The additional transformation of fields and charges allows one to diagonalize the leading logarithm terms of the scalar polarization operator and to avoid the contributions to the RG relation originated from the one-loop mixing between heavy and light field. Since the difference between the diagonalized fields and charges and the original ones is of one-loop order, one may simply omit one-loop mixing terms in the RG relation at the lower level. Then it is possible to reduce the RG relation for $S$-matrix element to the one for vertex describing the scattering of light particles on the external field induced by the heavy virtual particle. In fact, this result is independent on the specific features of the considered model, as it was shown in [4].

The RG relations of the considered type may be used in searching for the dependences between the parameters of EL describing physics beyond the SM. For example, let a symmetry requires the same charge structure for some effective Lagrangians. Then the number of unknown $\tilde{\beta}$– and $\tilde{\gamma}$– functions is less than the number of RG relations, and it is possible to derive non-trivial solutions for the parameters. The present results allow to omit the one-loop mixing diagrams in construction of the RG relations for the tree-level EL.
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Figure 1: Tree level contribution to the four-fermion amplitude.
Figure 2: One-loop level contribution to the one-particle reducible four-fermion amplitude.