Fluorescence photons produced in air by extensive air showers

Vitor de Souza*, Gustavo Medina-Tanco and Jeferson A. Ortiz

Instituto de Astronomia, Geofísica e Ciências Atmosféricas
Universidade de São Paulo
Caixa Postal 9638 - CEP 05508-900, São Paulo - SP, Brasil

Abstract

The air fluorescence technique has long been used to detect extensive air showers and to reconstruct its geometry and energy. The fluorescence photon yield of an electron in air is of main importance in the reconstruction procedure. Historically, the fluorescence yield used in the reconstruction of the showers is approximated at all energies by that of an electron with kinetic energy of 80 MeV, because these are the most abundant in a shower. In this paper, we calculate the fluorescence yield taking into account the energy spectrum of electrons in showers initiated by proton and iron nuclei as a function of height. We compare our results with previous calculations based on mono energetic electrons (80 MeV) and a difference in excess of 8% is found. Finally, the influence of a more realistic fluorescence yield in the shower energy reconstruction is also discussed.

Key words: cosmic rays, extensive air showers, air fluorescence yield, energy reconstruction
PACS: 96.40-z,96.40.Pq,96.40.De

1 Introduction

Fluorescence telescopes have been successfully used to measure extensive air shower since the Fly’s Eye [1] experiment proved its efficiency. Presently, the HiRes [2], Auger [3], EUSO [4], OWL [5] and Telescope Array [6]) Experiments are using or planning to use the fluorescence technique to study the cosmic ray with energy above $10^{19}$ eV.

* vitor@astro.iag.usp.br
The detection of the shower by a fluorescence detector is performed by measuring the amount of fluorescence light produced in air by the excitation of the nitrogen molecules as particles (essentially electrons and positrons) travel along the atmosphere. The number of photons detected by the telescope can be converted to the number of charged particles in the shower at a given height taking into account the absorption and scattering of the fluorescence light along the path from the emission to the telescope and knowing the fluorescence yield of the particle.

From the number of particles as a function of height one is able to reconstruct the energy of the shower following the arguments in reference [7]. Therefore, the fluorescence yield has a direct influence in the energy estimation of a shower.

The importance of knowing the fluorescence yield of the electron as a function of energy is underlined by the number of experiments presently under operation to measure the yield as can be seen in references [8,9,10,11].

From the measurements of the yield in the experiments cited above to the use of the fluorescence yield in cosmic rays showers a number of assumptions are made. Specially, the fluorescence yield of the particles in a shower are considered to be the yield of electrons with kinetic energy of 80 MeV.

The contribution of other particles, like muons and hadrons and particles with energy below the simulation energy cut, to the total productions of fluorescence light in a shower has been extensively studied in a recent paper by H. Barbosa et al. [12]. However, the mono energetic approximation has survived widely unchallenged.

The scope of the present work is to evaluate the approximation of the energy spectrum of electron by single energy (80 MeV) in the evaluation of the fluorescence yield. We calculate the fluorescence yield taking into account the true energy spectrum of electrons (= electrons + positrons) in showers initiated by proton and iron nuclei with energy $10^{18}$, $10^{19}$ and $10^{20}$ eV. Our results show that the approximation of mono energetic electrons (80 MeV) leads to an over estimation of the shower energy by a factor of $\sim 8\%$.

2 Fluorescence Yield Measurements and Theory

The fluorescence yield of electrons in air is determined by a competition of two factors: the probability of excitation of a nitrogen molecule and the probability of its de-excitation via collisions with mainly oxygen molecules (quenching).
Based on this assumption Bunner [13] suggested that the fluorescence yield (FY) should be proportional to the energy deposit \((dE/dx)\) and can be parametrized as:

\[
FY_{\text{Bunner}}(K, \rho, T) \propto \frac{dE}{dx} \times \frac{\rho}{1 + \rho B \sqrt{T}}
\]

where \(K\) is the kinetic energy of the particle, \(\rho\) is the air density, \(B\) is a constant and \(T\) is temperature. The \(dE/dx\) is the energy loss rate per unit path which is dominated by ionization losses.

More recently, two measurements have been done to determine the fluorescence yield as a function of the kinetic energy of the electron. Kamimoto et al. [14] measured the fluorescence yield for electrons with energy 1.4, 300, 650 and 1000 MeV and confirmed the proportionality of the yield with the energy loss rate \((dE/dx)\) as suggested by Bunner [13]. Another experiment has been done by Nagano et al. [10] and the same proportionality between the yield and the \(dE/dx\) was verified despite the fact that this experiment was able to make measurements at a greater number of wavelength bandwidths than Kakimoto et al. did.

Both measurements suggest that the relation between yield and \(dE/dx\) can be given by:

\[
FY(K, \rho, T) = \frac{(dE)}{(dE/K_c)} \times \rho \left( \frac{A_1}{1 + \rho B_1 \sqrt{T}} + \frac{A_2}{1 + \rho B_2 \sqrt{T}} \right)
\]

where \(\rho\) is the density in kg m\(^{-3}\) and \(T\) is the temperature in Kelvin. The constants \(A_1, A_2, B_1, B_2\) and \((dE/dx)_{K_c}\) is the energy loss rate calculated at \(K_c\) as given in Table 1. At a given pressure and temperature Kakimoto et al. and Nagano et al. differ in the relation between the energy loss rate and the fluorescence yield only by a constant as can be seen in figure 1 where equation 2 is plotted for \(T = 300\) K and \(\rho = 760\) mm Hg.

Taking into account the variation of the density and temperature of the atmosphere as a function of height, it is possible to determine at a given energy how the yield varies with height. Figure 2 shows the fluorescence yield as a function of height for electrons with kinetic energy of 80 MeV.

This energy is normally chosen because it is the energy at which the probability of an electron undergoing pair-production is equal to the probability of absorption by ionization and therefore the number of particles in a shower reaches its maximum. However, the energy of the electrons in a shower has a wide energy distribution. Furthermore, the energy distribution is different at
different heights and for distinct primary particles.

Nevertheless, the fluorescence yield is a highly dependent function on the kinetic energy (through $dE/dx$) and, since the energy distribution of the electrons varies with height, the fluorescence yield must vary with height in a manner different from that one shown in figure 2.

Figure 2 shows that, for 80 MeV electrons, the fluorescence yield is a slowly varying function of the height in the atmosphere. The later has been accepted as one of the basic features on which rests the fluorescence technique. However, as explained above, the characteristic value of the distribution of yields as a function of kinetic energy at a given height is not equal to the yield calculated at the characteristic value of the energy distribution at a given height.

In the next sections we are going to convolute the energy distribution of electrons for heights varying from the sea level up to 7.5 km and for each height we are going to calculate the characteristic energy and fluorescence yield. A distribution of fluorescence yields for each height was determined allowing us to estimate the confidence level of the distribution of yields at each height.

2.1 The influence of different atmospheric profiles in the yield

In applications of the fluorescence yield to measure cosmic rays showers, a precise knowledge of the fluorescence yield and its dependences with the kinetic energy of the particle are useless if one does not have a good measurement of the atmospheric parameters.

This effect has been shown by B. Keilhauer et al. [15] and we are going to stress its importance here by explicitly calculating the fluorescence yield.

B. Keilhauer et al. have measured the atmospheric profile in the Pierre Auger Observatory site during an entire year. They have used meteorological radio soundings to determine the temperature and pressure variation as a function of height. Their results are published in [16] and we are going to use them in the following calculation.

Knowing the pressure and temperature variation as a function of height it is possible to calculate the variation of the fluorescence yield as a function of height by using equation 2. Figure 3 shows the value of the fluorescence yield for three atmospheric models according to Kakimoto et al.. Seasons effects can change the fluorescence yield by as much as 4 % [15].
3 The fluorescence yield according to the electron energy spectrum

In order to evaluate the influence of the distribution of the electron’s kinetic energy we have done Monte Carlo simulations with the program CORSIKA [17] linked to the QGSJET [18] interaction model. We have defined observational levels varying from sea level up to 7.5 km and determined the energy spectrum of the electrons for each level.

The simulations were performed with the thinning factor of $10^{-6}$ and the energy cuts for hadrons, muons, electron and photons were respectively set to 50 MeV, 50 MeV, 50 keV and 50 keV. Ten vertical showers were simulated for each configuration of energy and primary particle.

Figure 4 shows an example of the kinetic energy distribution of electrons of ten showers initiated by proton with energy $10^{19}$ eV at 0.5, 3.5 and 7.5 km above sea level. It is clear in this figure that the energy spectrum changes significantly with height and that this must affect the dependence of the yield with height.

In order to define a characteristic energy for which we could get a fluorescence yield that represents the energy spectrum of electrons we should solve the following equation:

$$\int S(K) \times FY(K, \rho, T) \, dK = FY(K_{ch}, \rho, T) \times \int S(K) \, dK$$

(3)

where in the left hand side we have the energy spectrum ($S(K)$) convoluted with the fluorescence yield ($FY(K, \rho, T)$) and in the right hand side we have the yield calculated at a characteristic kinetic energy ($K_{ch}$) times an average energy. Using the proportionality of the fluorescence yield with the $dE/dx$ we can solve the equation and find the values of the characteristic energy which we can use in order to calculate a representative fluorescence yield.

Figure 5 shows an example of the numeric solution for equation 3 for the energy spectrum of electrons in proton showers at 1.5 km a.s.l. (above sea level) supposing the correspondence of the yield with the energy deposit suggested by Kakimoto et. al.

Equation 3 has two solutions with the same representative yield. We are going to quote always the higher kinetic energy. In case of figure 5 the intersection occurs at $K_{ch} = 25.8$ MeV what leads to a representative yield of 4.36 photons/m/ electron.

The same calculation was repeated for all heights, different primary particles and energies.
3.1 Varying primary particles type and energy

For different primary particle types and energies the electron energy spectrum at a given height may be different and therefore the characteristic energy could also change. In order to check the later, we have simulated sets of ten showers initiated by proton and iron nuclei with primary energies of $10^{18}$, $10^{19}$ and $10^{20}$ eV.

Figure 6 shows the electron energy spectrum for proton showers at different primary energies at 1.5 km a.s.l.. This figure shows that the number of electron at a given energy interval increases with energy independently of the kinetic energy. In other words, the overall shape of the spectrum is the same for different primary energies what directly implies that the fluorescence yield is not a strong function of the energy for primary protons. Exactly the same feature happens for primary iron showers where the fluorescence yield does not change very much with energy.

Despite that, small changes in the characteristic energy are seen for different primary energies. For example, at 4.5 km for primary proton the characteristic energy is 28.6, 31.7 and 32.5 MeV for primary energies of $10^{18}$, $10^{19}$ and $10^{20}$ eV, respectively. However, since the $dE/dx$ curve vary slowly with energy at this energy range, the correspondent yield changes are within 0.1 photons/m/electron. The $dE/dx$ values are 4.52, 4.56 and 4.58 respectively. Figure 7 shows how the fluorescence yield vary with height for proton primary showers with different energies taking into account the energy spectrum as explained above for the Kakimoto et al. correspondence between yield and $dE/dx$. We remember that the Nagano et al. correspondence between yield and $dE/dx$ is different only a constant factor.

On average we notice a tendency of the fluorescence yield to get higher with increasing primary energy but we emphasize that the differences are always smaller than 0.1 photons/m/electron.

For iron showers we get the same results and conclusion except that the differences between the fluorescence yield for the primary energies are still smaller than the ones for proton showers. Figure 8 shows the small differences for the yield calculated for primaries protons and iron nuclei with energy $10^{19}$ eV.

3.2 The fluorescence yield for inclined showers

The development of a shower is proportional to the amount of matter traversed in the atmosphere. Therefore, the energy spectrum of the electrons in a vertical shower at 1.5 km is not equal to the electrons energy spectrum in an inclined
shower at 1.5 km. The average electrons energy spectrum in showers with different inclinations is only equal at the same slant depth.

Figure 9 shows the fluorescence yield as a function of height for proton showers with energy $10^{19}$ eV for three inclinations.

The calculation regarding the energy spectrum has followed the same procedure given in section 3. To reproduce the inclination of the shower we took the slant depth in the vertical shower and transformed it to equivalent height in the inclined shower. This is equivalent to suppose the shower has the same energy spectrum at a given slant depth, what would lead to the same characteristic energy.

The characteristic energy, as can be seen in equation 3, is independent of pressure and temperature and can be given as a function of slant depth. However, the fluorescence yield calculated for a given characteristic energy is a function of pressure and temperature and hence must be calculated at a given height for each shower inclination.

Figure 9 shows that for the same height the fluorescence yield changes as much as 0.1 photons/m/electron both from 0 to 37 and from 37 to 53 degrees.

### 3.3 Evaluating the fluorescence yield dispersion

Another important feature of the fluorescence yield of electrons in an air shower is their distribution. Historically, experiments have used an average of the yield of all electrons at a given height without considering the spread of the yield around this average value.

At each height, we can drawn single electron energies following the energy distribution probability as shown in figure 4. We have drawn $10^5$ electron energies and for each one we calculated the correspondent yield. Figure 10 shows an example of the distribution of the yield for a proton shower with energy $10^{19}$ eV following Kakimoto et al. suggestion.

As expected, the distribution has a pronounced peak but also has a long tail with high yield values calculated for low energy electrons which is normally neglected. There is an artificial maximum value for the fluorescence yield which corresponds to the minimum electron energy in the simulation (50 keV).
4 Final Results and Conclusion

In the reconstruction of cosmic ray showers with the fluorescence technique it is impossible to know the energy and the identity of the primary particle before the application of the fluorescence yield. Therefore, the safest procedure would be to use the average fluorescence yield corresponding to the energies and primary particles we expect to measure.

Figures 11 and 12 show the average fluorescence yield as a function of height taking into account the electron energy spectrum. We have averaged over the 60 showers initiated by proton and iron with primary energies of $10^{18}$, $10^{19}$ and $10^{20}$ eV. The hatched region shows the 68% confidence level of the distribution of the yields for each height.

We also show in the same plot the fluorescence yield calculated for a mono energetic electron (80 MeV) for three different atmospheres: US Standard, Argentine Winter and Argentine Summer as measured by the Auger Collaboration and published at [16].

Only the US Standard atmosphere was used for these calculations. The difference from the fixed energy to the spectrum calculations varies with height as shown in figure 13 and the average difference is about 8%.

The exact value the energy reconstructed using the mono energetic model (80 MeV) has been overestimated varies from shower to shower but our results suggests that fluorescence experiments should reevaluate their energy estimative reducing them by $\sim 8\%$.

The shower-to-shower and particle-to-particle fluctuation of the yield due to the proper consideration of the kinetic energy distribution function of the electrons is larger than 10% which at least the double of the variation due to atmospheric changes. The distribution of the yield due to the energy distribution is so big that it would make the measurements of Kakimoto et al. and Nagano et al. agree at the 68% C.L. as can be seen in figure 14.

The fluorescence yield to be applied to a cosmic ray shower depends on the inclination of the axis of the shower. We define in equation 3 the concept of characteristic energy which is the kinetic energy for which the yield can be calculate to take into account the electron energy spectrum in the shower. The average characteristic energy for showers initiated by protons and irons with energy $10^{18}$, $10^{19}$ and $10^{20}$ eV is shown in table 2. The characteristic energy is the natural value to be used in the reconstruction of the shower as a function of slant depth as it is also shown in figure 15.

Given the geometry of the axis of the shower, at a given slant depth the
characteristic energy should be used to calculate the fluorescence yield as a function of height. Figure 15 shows the points calculated for each height and a fit to these points. According to this simple fit the characteristic energy can be given as a function of slant depth ($\chi$) by the equation:

$$K_{ch}(\chi) = 52.028 - 0.04617 \chi + 1.6968 \times 10^{-5} \chi^2$$  (4)

We therefore conclude that the electron kinetic energy spectrum cannot be neglected in order to calculate the proper fluorescence yield to be used for energy reconstruction of cosmic ray showers otherwise an over estimation of 8% is artificially introduced.

5 Acknowledgments

This work was supported by the Brazilian population via the science foundations FAPESP and CNPq to which we are grateful. V. de Souza is supported by FAPESP and J.A. Ortiz is supported by CNPq by Post-Doc Fellowships.

References

[1] R. M. Baltrusaitis, et al., The Utah Fly’s Eye Detector, Nucl. Instr. Meth. A (240) (1985) 410.
[2] T. Abu-Zayyad, et al., The prototype high-resolution Fly’s Eye cosmic ray detector, Nucl. Instr. Meth. A (450) (2000) 253.
[3] The Auger Collaboration, Properties and performance of the prototype instrument for the Pierre Auger Observatory, Nuclear Instruments and Methods in Physics Research A, 523 (2004) 50-95.
[4] M. Teshima, et al., EUSO (The Extreme Universe Space Observatory) - Scientific Objectives, in: T. Kajita, Y. Asaoka, A. Kawachi, Y. Matsubara, M. Sasaki (Eds.), 28th Int. Cosmic Ray Conf., Universal Academy Press, Tsukuba, 2003, p. 1069.
[5] P. Dierickx, et al., in: R. G. T. Andersen, A. Ardeberg (Ed.), Proceedings Bckaskog Workshop on Extremely Large Telescopes, 2000, p. 43.
[6] Y. Arai, et al., The Telescope Array Experiment: an overview and physics aims, in: 28th Int. Cosmic Ray Conference, 2003, p. 1025.
[7] R. M. Baltrusaitis, G. L. Cassiday, R. Cooper, J. W. Elbert, P. R. Gerhardy, S. Ko, E. C. Loh, Y. Mizumoto, P. Sokolsky, D. Steck, Energy Calibration of the Fly’s Eye Detector, in: Proc. 19th Int. Cosmic Ray Conf., Vol. 7, La Jolla, 1985, pp. 155–158.
[8] E. Kemp, et al., Study of the fluorescence yield for electrons between 0.5 - 2.2 MeV, in: Proc. 28th Int. Cosmic Ray Conf., 2003, p. 853.

[9] F. Arciprete, et al., AIRFLY: Air fluorescence induced by electrons in a wide energy range, in: Proc. 28th Int. Cosmic Ray Conf., 2003, p. 837.

[10] M. Nagano, K. Kobayakawa, N. Sakaki, K. Ando, Photon yields from nitrogen gas and dry air excited by electrons, Astroparticle Physics 20 (2003) 293–309.

[11] M. Nagano, K. Kobayakawa, N. Sakaki, K. Ando, New measurement on photon yields from air and the application to the energy estimation of primary cosmic rays, astro-ph/0406474, submitted to Astroparticle Physics.

[12] H. Barbosa, F. Catalani, J. A. Chinellato, C. Dobrigkeit, Indirect determination of the missing energy content in extensive air showers, astro-ph/0310234, in press by Astroparticle Physics (2004).

[13] A. N. Bunner, Cosmic ray detection by atmospheric fluorescence, Ph.D. thesis, Cornell University (1967).

[14] F. Kakimoto, E. C. Loh, M. Nagano, H. Okuno, M. Teshima, S. Ueno, A measurement of the air fluorescence yield, Nucl. Instr. Meth. A (372) (1996) 527–533.

[15] H. K. B. Keilhauer, M. Risse, Results fo the first balloon measurements above the pampa amarilla, Tech. Rep. GAP2003-009, The Pierre Auger Observatory, www.auger.org (2003).

[16] B. Keilhauer, J. Blümer, R. Engel, H. Klages, M. Risse, Impact of varying atmospheric profiles on extensive air shower observation: atmospheric density and primary mass reconstruction, arXiv:astro-ph/0405048, submitted to Astroparticle Physics (2004).

[17] D. Heck, J. Knapp, J. Capdevielle, G. Schatz, T. Thouw, A Monte-Carlo code to simulate extensive air showers - report FZKA 6019, Tech. rep., Forschungszentrum Karlsruhe (1998).

[18] N. N. Kalmykov, S. S. Ostapchenko, A. I. Pavlov, Nucl. Phys. B (Proc. Suppl.) 52B (1997) 17.
Table 1
Constants used by Kakimoto et al. and Nagano et al. in equation 2.

| Constants | Kakimoto et al. | Nagano et al. |
|-----------|-----------------|---------------|
| $A_1$ (m$^2$ kg$^{-1}$) | 89.0 ± 1.7 | 147.4 ± 4.3 |
| $A_2$ (m$^2$ kg$^{-1}$) | 55.0 ± 2.2 | 69.8 ± 12.2 |
| $B_1$ (m$^3$ kg$^{-1}$ K$^{-0.5}$) | 1.85 ± 0.04 | 2.40 ± 0.18 |
| $B_2$ (m$^3$ kg$^{-1}$ K$^{-0.5}$) | 6.50 ± 0.33 | 20.1 ± 6.9 |
| $K_c$ (MeV) | 1.4 | 0.85 |

Table 2
Characterist energy as a function of slant depth. The characterist energy is the average over 60 showers initiated by proton and iron nuclei with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV.

| Slant Depth (g/cm$^2$) | $K_{ch}$ (MeV) |
|------------------------|---------------|
| 1036                   | 22.2 ± 0.3     |
| 976                    | 23.4 ± 0.5     |
| 864                    | 25.1 ± 0.5     |
| 764                    | 26.7 ± 0.6     |
| 673                    | 28.3 ± 0.6     |
| 590                    | 30.6 ± 0.9     |
| 517                    | 32.1 ± 0.6     |
| 451                    | 34.7 ± 0.7     |
| 392                    | 37.5 ± 0.9     |
Fig. 1. The $dE/dx$ of an electron in air as a function of its kinetic energy and the corresponding fluorescence yield as suggested by Kakimoto et al. and Nagano et al. We have used $T = 300$ K and $\rho = 760$ mm Hg.

Fig. 2. The fluorescence yield as a function of height in the atmosphere for electrons with kinetic energy equal to 80 MeV. Pressure and temperature varies with height according to the US Standard Atmospheric Model [16].
Fig. 3. The fluorescence yield as a function of height in the atmosphere for electrons with kinetic energy equal to 80 MeV according to Kakimoto et al. suggestion. Pressure and temperature varies with height according to the models published by B. Keilhauer et al. [16]. Arg. Winter and Summer are the average measurements for the Pierre Auger Southern site.
Fig. 4. Examples of the electron kinetic energy distribution for ten proton vertical showers with energy $10^{19}$ eV at 0.5, 3.5 and 7.5 km above sea level.

Fig. 5. Numeric solution of equation 3 for the electron spectrum for showers initiated by protons with $10^{19}$ eV at 1.5 km a.s.l.
Fig. 6. Kinetic energy spectrum for electrons at 1.5 km in ten vertical showers initiated by proton with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV.

Fig. 7. Fluorescence yield according to Kakimoto et al. suggestion as a function of height calculated taking into account the energy spectrum as explained above for vertical showers initiated by proton with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV.
Fig. 8. Fluorescence yield as a function of height calculated taking into account the energy spectrum as explained in section 3 according to Kakimoto et al. suggestion for vertical showers initiated by proton and iron with energy $10^{19}$ eV.

Fig. 9. Fluorescence yield as a function of height calculated taking into account the energy spectrum as explained in section 3.2 according to Kakimoto et al. suggestion for showers initiated by proton with energy $10^{19}$ eV and zenith angle of 0, 37 and 53 degrees.
Fig. 10. Distribution of the fluorescence yield calculated taking into account the energy spectrum as explained above according to Kakimoto et al. suggestion for vertical showers initiated by proton with energy $10^{19}$ eV.
Fig. 11. Average fluorescence yield as a function of height. The average was performed over 30 proton and 30 iron showers with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV. It is shown the yield calculated according to the Kakimoto et al. equation for the US Standard, Argentine Winter and Argentine Summer for 80 MeV electron and for comparison it is also shown the same calculation for the US Standard taking into account the electron energy spectrum. Hatched region corresponds to 68% C.L..
Fig. 12. Average fluorescence yield as a function of height. The average was performed over 30 proton and 30 iron showers with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV. It is shown the yield calculated according to the Nagano et al. equation for the US Standard, Argentine Winter and Argentine Summer for 80 MeV electron and for comparison it is also shown the same calculation for the US Standard taking into account the electron energy spectrum. Hatched region corresponds to 68% C.L..

Fig. 13. Difference between the fluorescence yield calculated according to the energy spectrum and calculated for a fixed energy (80 MeV) as a function of height.
Fig. 14. Average fluorescence yield as a function of height. It is shown the yield calculated according to the Nagano et al. and Kakimoto et al. measurements. We have used the US Standard atmosphere and the calculation was done taking into account the electron energy spectrum.

Fig. 15. Average characteristic kinetic energy as a function of slant depth. The characteristic energy is the average over 60 showers initiated by proton and iron nuclei with energies $10^{18}$, $10^{19}$ and $10^{20}$ eV.