Measuring $\gamma$ Cleanly with CP-Tagged $B_s$ and $B_d$ Decays

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We propose a new method for measuring the CKM phase $\gamma$ using the partial rates for CP-tagged $B_s$ decays. Such an experiment could be performed at a very high luminosity symmetric $e^+ e^-$ collider operating at the $\Upsilon(5S)$ resonance, where the $B_s \overline{B_s}$ state is produced in a state of definite CP. We also discuss CP-tagging in the $B_d$ system at the $\Upsilon(4S)$, where a time-dependent analysis is required to compensate for the anticipated large CP violation in $B_d - \overline{B_d}$ mixing.

The accurate determination of the Unitarity Triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is one of the most important problems of experimental $B$ physics. Reasonably precise measurements of some of its parameters—three sides, determined by $|V_{ub}|$, $|V_{cb}|$ and $|V_{td}|$, and one angle $\sin 2 \beta$—will soon be performed at the $B$ Factories operating at the $\Upsilon(4S)$ and at Run II of the Tevatron. Although this is enough to fix the triangle up to discrete ambiguities, one really wants to overconstrain the system and thereby be sensitive to deviations from the CKM description of flavor-changing processes. In view of this goal, it is important to measure the angles $\alpha$ and $\gamma$ as well.

The situation with these other angles is more problematic. A number of methods have been proposed to measure or constrain $\alpha$ and $\gamma$, but unfortunately they each suffer to some degree from either theoretical or experimental difficulties [1]. In what follows we investigate a new proposal to constrain $\gamma$ in the decays of two $B_s$ mesons produced in a coherent state. This can be achieved if the pair comes from the decay of a $b \overline{b}$ meson such as the $\Upsilon(5S)$. In this case one has not only the option of tagging the flavor of the initial $B_s$’s, but the alternative of tagging it as an eigenstate of CP. The advantage of the $B_s$ in this regard is that CP violation in its mixing is small in the Standard Model. We will also discuss an analogous proposal for the $B_d$ system at the $\Upsilon(4S)$, where large CP violation in mixing complicates the situation.

The CKM angle $\gamma$ is defined to be

$$\gamma = \arg \left[ -V_{ub} V_{cb}^* / V_{cd} V_{cb}^* \right].$$

(1)

Here we consider the possibility of extracting $\gamma$ from the $B_s$ decays to the final states $D_s^+ K^-$ (or the analogous $D_s^{(*)} K^{(*)}$ combinations). The fact that $D_s^- K^+$ and $D_s^+ K^-$ can be reached from both the $B_s$ and its CP conjugate $\overline{B_s}$ already has been exploited in a proposal to extract $\gamma$ from a time-dependent study [2]. The two transition amplitudes have similar magnitudes,

$$|A(B_s \to D_s^- K^+)| \sim |A(\overline{B_s} \to D_s^- K^+)| \sim \lambda^3,$$

where $\lambda = \sin \theta_C \approx 0.22$ is the small parameter which controls the hierarchy of the CKM matrix. Hence triangles built from these amplitudes need not suffer from being “squashed”. Given sufficient statistics, the time-dependent analysis eventually should yield $\sin \gamma$ at some level of accuracy.

By contrast, our proposal allows one to measure $\sin \gamma$ using branching ratios only, with no need to determine the time at which the decay occurs. What will be necessary, instead, is to measure not only flavor-tagged but also CP-tagged $B_s$ decays.

We begin by defining amplitudes for $B_s$ and $\overline{B_s}$ decay to the final state $D_s^+ K^-$,

$$A_1 = A(B_s \to D_s^+ K^+) = a_1 e^{i \delta_1},$$
$$A_2 = A(\overline{B_s} \to D_s^- K^+) = a_2 e^{-i \delta_2} e^{i \delta_2}.$$  

(2)

The amplitude $A_1$ arises from the quark transition $\bar{b} \to c \bar{u} s$ and is real (in the Wolfenstein parameterization), while $A_2$ arises from $b \to u \bar{c} s$ and carries the relative weak phase $e^{-i \xi}$. There are no penguin contributions. The amplitudes $A_i$ also have strong phases $e^{i \delta_i}$. The CP conjugated amplitudes are given by

$$\overline{A}_1 = A(\overline{B_s} \to D_s^+ K^-) = a_1 e^{i \delta_1} e^{-2i \xi},$$
$$\overline{A}_2 = A(\overline{B_s} \to D_s^- K^+) = a_2 e^{i \delta_2} e^{2i \xi}.$$  

(3)

where the phase $\xi$ depends on the convention for CP transformations of the $B_s$ states,

$$\text{CP} |B_s\rangle = e^{2i \xi} |\overline{B_s}\rangle, \quad \text{CP} |\overline{B_s}\rangle = e^{-2i \xi} |B_s\rangle.$$  

(4)

Any physical observable must be independent of $\xi$. We also define a set of amplitudes for the CP eigenstates of the $B_s$ meson,

$$|B_s^\pm\rangle = \frac{1}{\sqrt{2}} \left[ |B_s\rangle \pm e^{2i \xi} |\overline{B_s}\rangle \right],$$

(5)

to decay into the same $D_s K$ final states,
\[ A_\pm = A(B_s^\pm \to D_s^- K^+) \]
\[ \overline{A}_\pm = A(B_s^\pm \to D_s^+ K^-) \]

These amplitudes satisfy simple triangle relations,
\[ \sqrt{2} A_\pm = A_1 \pm e^{2i\xi} A_2 = (a_1 \mp a_2 e^{-i\gamma} e^{i\delta}) e^{i\delta_1}, \]
\[ \sqrt{2} \overline{A}_\pm = \overline{A}_2 \pm e^{2i\xi} \overline{A}_1 = (\pm a_1 \pm a_2 e^{i\gamma} e^{i\delta}) e^{i\delta_1}, \]
where \( \delta = \delta_2 - \delta_1 + 2\xi \) is the convention-independent (and observable) strong phase difference. It is clear that any construction which is insensitive to \( \delta \) will also be insensitive to the unphysical phase \( \xi \). It is also clear that by changing \( \xi \), it is possible to take \( B_s^\pm \) to be a linear combination of \( B_s \) and \( \overline{B}_s \) with any relative phase. We will derive a relation for \( \sin 2\gamma \) involving the magnitudes of the amplitudes \( A_1 \) and \( \overline{A}_1 \). From the freedom to choose \( \xi \) in Eq. (5), it is clear that the CP-even and CP-odd amplitudes will yield triangle relations which contain the same information about \( \gamma \).

For any CP eigenstate \( B_s^{CP} \), then, it is possible to choose \( \xi \) so that
\[ A_{CP} = A(B_s^{CP} \to D_s^- K^+) = (A_1 + A_2)/\sqrt{2}, \]
\[ \overline{A}_{CP} = A(B_s^{CP} \to D_s^+ K^-) = (\overline{A}_1 + \overline{A}_2)/\sqrt{2}. \]

Without loss of generality, we also choose a convention in which \( \delta_1 = 0 \), in which case the triangle relations are very simple. They are illustrated graphically in Fig. 1, where the amplitudes may be interpreted as vectors in the complex plane. As drawn, the angle between \( A_2 \) and \( \overline{A}_2 \) is \( 2\gamma \). For an analytical solution, it is convenient to define
\[ \alpha = \frac{2|A_{CP}|^2 - |A_1|^2 - |A_2|^2}{2|A_1||A_2|}, \]
\[ \overline{\alpha} = \frac{2|\overline{A}_{CP}|^2 - |\overline{A}_1|^2 - |\overline{A}_2|^2}{2|\overline{A}_1||\overline{A}_2|}, \]
in terms of which we find
\[ \sin 2\gamma = \pm \left( \alpha \sqrt{1 - \alpha^2} - \overline{\alpha} \sqrt{1 - \overline{\alpha}^2} \right). \]

The determination of \( \gamma \) itself then has an eightfold ambiguity. This construction is quite analogous to that of Ref. [3], in which the extraction of \( \sin \gamma \) from \( B_d \) decay to a CP eigenstate \( D_{CP} \) was studied. However, the triangles are “squashed” in that analysis.) Note that if \( \Delta \Gamma_s / T_s \) is significant, the squared amplitudes \( |A_i|^2 \) and \( |\overline{A}_i|^2 \) are proportional to partial rates, e.g.,
\[ |A_1|^2 \propto \Gamma(B_s \to D_s^- K^+), \]
rather than to branching ratios.

The measurement of \( |A_{CP}| \) requires that one tag the initial \( B_s \) state as a CP eigenstate \( B_s^{CP} \). This is possible if the \( B_s \) is produced in the decay \( \Upsilon(5S) \to B_s \overline{B}_s \). Since the \( \Upsilon(5S) \) is a CP even state and the \( B_s \) and \( \overline{B}_s \) are emitted in a relative \( p \)-wave, the CP eigenvalues of the \( B_s/\overline{B}_s \) mixtures are anti-correlated. Hence if the “tagging” \( B_s \) decays to a CP eigenstate such as \( D_s^0 D_s^- \), then the other \( B_s \) is constrained to be a CP eigenstate as well. It is crucial that the tagging decay be one in which direct CP violation is expected to be small. Tagging modes with spin one particles, such as \( B_s^{\pm} \to \psi_3 \) and \( D_s^{*}D_s^- \), can be used if an angular analysis is performed to select a final state of definite CP. So long as the amplitudes \( |A_{CP}| \) and \( |\overline{A}_{CP}| \) are measured with the same CP tagging mode on the opposite side, it is unimportant whether a CP even or CP odd tag is employed.

This simple method of CP tagging relies on the Standard Model expectation that CP violation in \( B_s \) mixing is not significant. As \( B_s \) mixing is generated dominantly by \( tW \) box diagrams, CP violating effects are proportional to \( \sin 2\beta_s \), where
\[ \beta_s = \arg \left[ -V_{ts}V_{cb}^* \over \sqrt{V_{ts}V_{cb}} \right] \sim \lambda^2 \]
is small. Furthermore, we assume that CP violation in \( B_s \) mixing will have been constrained experimentally by the time the analysis proposed here is performed.

The method is not appropriate to a hadronic production environment such as the Tevatron or the LHC, since in this case the \( B_s \) and \( \overline{B}_s \) do not arise from an initial CP eigenstate. Nor, of course, can this analysis be performed at the \( B \) Factories as presently configured to operate at the \( \Upsilon(4S) \). To our knowledge, this is the first proposal for a clean measurement of a CKM phase which is unique to an \( e^+e^- \) collider operating at the \( \Upsilon(5S) \).

We now make a crude estimate of the number of \( B_s \overline{B}_s \) pairs required to measure \( \sin \gamma \) with a precision of 0.1, for which approximately \( 10^2 \) reconstructed events would be needed. To be concrete, we take the tagging mode \( B_s^{CP} \to D_s^+ D_s^- \). With order-of-magnitude estimates of the relevant branching ratios, \( B(B_s^{CP} \to D_s^+ D_s^-) \sim 10^{-2} \) and \( B(B_s^{CP} \to D_s K) \sim 2 \times 10^{-4} \), and assuming that the \( D_s \) can be reconstructed efficiently by combining a number of decay modes, we find a combined CP-tagged branching fraction \( B_{tot} \sim 10^{-6} \). Hence approximately \( 10^6 B_s \overline{B}_s \) events would be needed for this measurement.

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**FIG. 1.** Amplitude triangle relations for \( B_s \to D_s K \).
The decays of the $\Upsilon(5S)$ to $B_s$ flavored mesons produce primarily the combination $B_s^+\bar{B}_s^-$, as well as $B_s^0\bar{B}_s^0$, and $B_s\bar{B}_s$. The relative rates have been computed in a variety of models, yielding the estimates

$$\sigma(B_s^+\bar{B}_s^-)/\sigma(B_s^0\bar{B}_s^0) \approx 0.1 - 0.2 \quad (13)$$

and

$$\sigma(B_s^+\bar{B}_s^- + B_s^0\bar{B}_s^0)/\sigma(B_s^0\bar{B}_s^0) \approx 0.05 - 0.5 \quad . \quad (14)$$

A $B_s^+$ produced in this way decays to a $B_s$ and a very soft photon, so the other combinations will also be seen as $B_s\bar{B}_s$. In fully reconstructed $B_s$ decays the combinations can be separated by measuring the boost of the $B_s$ [4]. From the ratio $\sigma(B_s^+\bar{B}_s^-)/\sigma(\Upsilon(4S)) \approx 0.1$ of production cross sections and the fact that an $e^+e^-$ collider with luminosity $L = 10^{33} \text{cm}^{-2}\text{sec}^{-1}$ produces $3 \times 10^7 \Upsilon(4S)$ events per year [5], we see that $10^5$ CP-tagged $B_s\bar{B}_s$ events, decaying in this mode, are within the reach of a $B$ Factory upgraded to operate at $L \approx 10^{35} \text{cm}^{-2}\text{sec}^{-1}$.

Since it is not necessary to measure the time-dependence of the decay, the experiment could be performed in a future high luminosity run of the Cornell $e^+e^-$ storage ring.

Moreover, there are ways to increase substantially the number of usable events. First, one may repeat the analysis with the final states $D_s^+K^-$, $D_s^0K^-$ and $D_s^0K^+$. Note that no angular analysis is necessary here, since one is not isolating a CP eigenstate on the decay side. Second, one can add additional CP-tagging modes such as $B_s^{CP} \to \psi\phi$ or $D_s^{CP}$. Although in this case an angular analysis would be required to separate final states of definite CP, studies in the $B_s$ system indicate that this can be done without a large cost in tagging efficiency [5]. This gain in efficiency will be offset in part by the cost of fully reconstructing the $D_s$ states, a penalty which we have not explicitly included.

Finally, it also may be possible to use the $B_s^+\bar{B}_s^-$ and $B_s\bar{B}_s^0$ combinations for CP tagging. Parity conservation requires that the pair be produced in a relative $p$-wave. Therefore the initial state is of the form

$$\frac{1}{\sqrt{2}} [B_s^+B_s^- + B_s^-B_s^+] \quad , \quad (15)$$

where $B_s^{\pm}$ are the CP eigenstate mixtures of $B_s^+$ and $\bar{B}_s^-$, in analogy with the $B_u$ combinations [3]. After the transition $B_s^* \to B_s\gamma$, in which the magnetic photon carries CP $= -1$, the CP eigenvalues of the $B_s/\bar{B}_s$ mixtures on the two sides are correlated (rather than anti-correlated, as in direct $\Upsilon(5S) \to B_s\bar{B}_s$). As we have shown, our analysis is equivalent for correlated and anti-correlated states. Unfortunately, it is not possible to CP tag using the dominant $B_s^+\bar{B}_s^-$ combination, since the total spin quantum number is not fixed at production.

Taken together, the use of additional modes on the tagging and decay sides and of the $B_s^+\bar{B}_s^-$ and $B_s\bar{B}_s^0$ initial states should allow one to relax considerably the luminosity requirement estimated above. Alternatively, for a given integrated luminosity these enhancements would allow a statistically more precise measurement of $\sin \gamma$.

Generally speaking, we believe that our proposal is feasible within many of the scenarios under discussion for future luminosity upgrades of the $B$ Factories now operating at the $\Upsilon(4S)$.

As shown above, the measurement of $\gamma$ from this analysis is insensitive to the strong phase difference $\delta$ between the amplitudes $A_1$ and $A_2$. In fact, $\delta$ could be extracted simultaneously with $\gamma$ from the amplitude triangles shown in Fig. [6]. Nevertheless, it is useful to have some idea of whether $\delta$ may be expected to be large. It is clear that elastic rescattering of the $D_sK$ final state will be the same for $B_s$ and $\bar{B}_s$ transitions and will not lead to nonzero $\delta$. Instead, what is needed to generate $\delta \neq 0$ is rescattering through an intermediate state $f$ which is produced differently by $B_s$ and $\bar{B}_s$. Then we may write

$$\frac{A(B_s \to D_s^-K^+)}{A(\bar{B}_s \to D_s^-K^+)} = \frac{A_1^f + \epsilon_f A_1^b}{A_2^f + \epsilon_f A_2^b} \approx \frac{A_1^f}{A_2^f} = \frac{1 + \epsilon_f A_1^f/|A_1^b|}{1 + \epsilon_f A_2^f/|A_2^b|} , \quad (16)$$

where $A_i^f$ are the amplitudes for the direct production of $D_s^-K^+$, $A_i^b$ are the amplitudes for the production of the intermediate state $f$, and $\epsilon_f$ is the amplitude for the rescattering $f \to D_s^-K^+$. While $A_1^f$ and $A_2^f$ have the same strong phase, a strong phase difference can be generated if $A_1^f/A_1^b \neq A_2^f/A_2^b$. For a similar discussion in the context of $D$ decays, see Ref. [7].

To estimate the size of $\delta$ which could be generated, we consider a model in which $f$ is a two body intermediate state. We employ the formalism of Ref. [8], based on Regge phenomenology and naive factorization of the $B_s$ decay matrix elements. With $f = D_s^*K^*$ and rescattering to $D_sK$ via exchange of the Pomeron and $\phi$ trajectories, we find

$$\delta < 5^\circ . \quad (17)$$

Although our estimate is extremely model-dependent, it does provide some evidence that this mechanism is unlikely to produce a large value of $\delta$. We note that a perturbative factorization formalism is not applicable to the decay $B_s \to D_sK$.

Finally, we turn to the issue of CP tagging in $B_d$ decays. Here the situation is complicated by the fact that the Standard Model predicts large CP violation in $B_d$ mixing. Therefore a state which is tagged at time $t = 0$ as being in a CP eigenstate will evolve by time $t$ into a linear combination $B_d^\pm(t)$ of the CP even ($B_d^0$) and CP odd ($B_d^\mp$) states. The evolution is given by

$$B_d^\pm(t) = e^{-i(M_d + \Gamma/2)t} \left[ a_\pm(t)B_d^\mp + b_\pm(t)B_d^\pm \right] , \quad (18)$$
and ∆m_{d} is the mass splitting between B_H and B_L. Here

\[ \beta = \arg \left[ -\frac{V_{ut} V_{ub}^*}{V_{td} V_{tb}} \right] \]  

(20)

is an angle which is expected to be large. (If B_d mixing receives a significant contribution from new physics, then the CP violating phase “sin 2γ” extracted from the asymmetry in B_d → J/ψK_S is actually what governs the time evolution \((19)\).) Note that in the CP conserving case, the final states ρ_d → D^±π^± are “squashed”, with one side shorter than the other two by a factor of order λ^2. (They would not be squashed, however, if the analysis were performed instead for a mode such as B_d → D^{(*)}_s K_S.)

In summary, we have presented a new approach to extracting the CKM angle γ, employing an analysis which depends on tagging an initial B_s or B_d as a CP eigenstate. This theoretically clean method is free from dependence on unknown strong phases. In the B_s case, the analysis is unique to an experiment performed at a very high luminosity e^+e^- collider operating at the \(\Upsilon(5S)\) resonance. While there are no definite plans to upgrade any of the existing symmetric or asymmetric B Factories to operate in this mode, we hope that the proposal outlined here will help rekindle interest in this possibility.

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The necessity of determining the decay time \(t\) means that such a measurement of γ in the B_d system would have to be performed at an asymmetric B Factory operating at the \(\Upsilon(4S)\). Although in principle the analysis could be performed by the BaBar or BELLE Collaborations, there are several difficulties. First, an accurate independent determination of sin 2β must be available. Second, it is necessary to collect sufficient statistics to construct the amplitude triangles for individual bins in \(t\). Third, in the case of B_d → D^{±}_s π^±, D^±π^± or D^{(*)}_s ρ^± the amplitude triangles are “squashed”, with one side shorter than the other two by a factor of order λ^2. (They would not be squashed, however, if the analysis were performed instead for a mode such as B_d → D^{(*)}_s K_S.)

The procedure may be repeated to give an independent determination of the decay time \(t\) means that such a measurement of γ in the B_d system would have to be performed at an asymmetric B Factory operating at the \(\Upsilon(4S)\). Although in principle the analysis could be performed by the BaBar or BELLE Collaborations, there are several difficulties. First, an accurate independent determination of sin 2β must be available. Second, it is necessary to collect sufficient statistics to construct the amplitude triangles for individual bins in \(t\). Third, in the case of B_d → D^{±}_s π^±, D^±π^± or D^{(*)}_s ρ^± the amplitude triangles are “squashed”, with one side shorter than the other two by a factor of order λ^2. (They would not be squashed, however, if the analysis were performed instead for a mode such as B_d → D^{(*)}_s K_S.)

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