An Alternate Method for Finding Optimal Solution to Solid Transportation Problem under Fuzzy Environment

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Abstract. In this paper we develop a new algorithm for the initial fuzzy basic feasible solution to a solid transportation problem with imprecise parameters. All the decision variables are assumed to be triangular fuzzy numbers. Using parametric form of triangular fuzzy numbers such as left fuzziness index, right fuzziness index, modal value and by using proposed algorithm we obtained the initial basic fuzzy feasible solution to the problem without changing to its crisp equivalent form. We further discuss the optimality by applying the modified distribution method. A numerical example is solved to show the effectiveness of the proposed algorithm.

1. Introduction
A solid transportation problem is a transportation problem with three parameters that is source, destination and mode of transport which was introduced by Haley [1] in 1962. In many real life circumstances a homogeneous unit is shifted from an sources to destinations by methods for various kinds of transportations, for example ships, trucks, merchandise trains, load flights which are called conveyance. Recently solid transportation problem has reached more attention to the several researchers for modelling. But in real cases the parameters related to a solid transportation problem may not be crisp; it can be imprecise because of some uncontrollable factors. For this case the fuzzy set is suitable for the situation. The fuzzy set theory was presented by Zadeh[2] and has applied it effectively in different fields to manage unsure information in making decision. After the foremost work performed by Bellman and Zadeh[3] the applications of fuzzy set theory become very rapid in the optimization field. Many researchers have proposed numerous algorithms to obtain the solution of solid transportation problem under fuzzy environment. Pandian and Anuradha [4] obtained optimal solution of solid transportation problem by applying principle of zero point method. Sharmistha Halder et.al[5] solved fuzzy solid transportation problem using vogels approximation method and made comparisons by using fuzzy ranking method. Sobana and Anuradha [6] obtained an optimal solution for solid transportation problem using \(\alpha\) -cut under imprecise environment. DeepikaRani et.al[7] suggested a method which changes an unbalanced problem to a balanced one and applied fuzzy programming techniques to determine the optimal compromise solution. Senthilkumar [8] obtained fuzzy optimal solution for fuzzy solid transportation problem by using PSK method. Shashi Aggarwal and Chavi Gupta [9] suggested new ranking techniques dependent on signed distance to determine the solution of intuitionistic fuzzy solid transportation problem. Syed Aqib Jaliilet.al [10] converted multi
objective fuzzy solid transportation problem into single objective fuzzy solid transportation problem by using weighted sum approach and obtain the optimal compromise solution. In this paper we developed a new method dependent on fuzzy version of vogels approximation method to determine the initial basic feasible solution for fuzzy solid transportation problem and test the optimality by modified distribution method without converted to its corresponding crisp form.

Rest of paper is sorted out as follows: Section 2 deals with some definitions, section 3 provides the definition of Solid fuzzy transportation problem, section 4 consists of the proposed algorithm and section 5 gives the numerical example, results and discussion. Finally the conclusion is given in section 6.

2. Preliminaries

Definition 2.1: A fuzzy number \( \tilde{A} \) is convex normalized fuzzy set \( \tilde{A} \) of the real line \( \mathbb{R} \) such that \( \{\mu_{\tilde{A}}(x): \mathbb{R} \rightarrow [0,1], \forall x \in \mathbb{R}\} \) where \( \mu_{\tilde{A}}(x) \) is called the membership function of the fuzzy set and it is piece-wise continuous.

Definition 2.2: A fuzzy number \( \tilde{A} \) on \( \mathbb{R} \) is said to be a triangular fuzzy number if its membership function: \( \mu_{\tilde{A}}: \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & \text{if } x \leq a \\
\frac{x-a}{b-a} & \text{if } a \leq x \leq b \\
\frac{c-x}{c-b} & \text{if } b \leq x \leq c \\
0 & \text{otherwise}
\end{cases}
\]

Definition 2.3: A triangular fuzzy number \( \tilde{A} = (a,b,c) \) can be represented with an ordered pair of functions through \( \rho \)-cut approach as \( [\underline{a}(\rho), \overline{a}(\rho)] = [(b-a)\rho + a, c - (c-b)\rho] \) where \( \rho \in [0,1] \).

The \( \rho \)-cut form is known as parametric form of fuzzy numbers. It may be noted that the lower and upper bounds of the fuzzy numbers satisfies the following requirements.

(i) \( \underline{a}(\rho) \) is a bounded left continuous non decreasing function over \( [0,1] \).

(ii) \( \overline{a}(\rho) \) is a bounded right continuous non increasing function over \( [0,1] \).

(iii) \( \underline{a}(\rho) \leq \overline{a}(\rho) \), where \( 0 \leq \rho \leq 1 \).

Moreover the triangular fuzzy number \( \tilde{A} = (a,b,c) \) can also be represented in parametric form as \( \tilde{A} = (\underline{a}, a^*, \overline{a}) \) where \( \underline{a} = (a_0 - \underline{a}), a^* = (\overline{a} - a_0) \) are called the left fuzziness index function and the right fuzziness index function respectively. For an arbitrary triangular fuzzy number \( \tilde{A} = (\underline{a}, \overline{a}) \) the number \( a_0 = \left( \frac{\underline{a}(1)+\overline{a}(1)}{2} \right) \) is said to be a location index number of \( \tilde{A} \).

2.1 Ranking of Triangular Fuzzy Number

For an arbitrary triangular fuzzy number \( \tilde{A} = (a,b,c) = (a_0, a^*, \overline{a}) \) with parametric form\( \tilde{A} = [\underline{a}(\rho), \overline{a}(\rho)] \), we define the magnitude of the triangular fuzzy number \( \tilde{A} \) by
\[ \text{Mag}(\tilde{A}) = \frac{1}{2} \int_{0}^{1} \left( (a + \bar{a} + a_{0}) \right) f(\rho) \, d\rho \]

\[ = \frac{1}{2} \int_{0}^{1} \left( (a^{*} + 4a_{0} - a_{0}) \right) f(\rho) \, d\rho \]

The function \( f(\rho) \) is a non-negative and increasing function on \([0,1]\) with \( f(0) = 0 \), \( f(1) = 1 \).

It can be considered as a weighting function according to the situation the function \( f(r) \) can be chosen by the decision maker. In this paper for convenience we use \( f(\rho) = \rho \).

Hence \( \text{Mag}(\tilde{A}) = \left( \frac{a^{*} + 4a_{0} - a_{0}}{4} \right) = \left( \frac{a + \bar{a} + a_{0}}{4} \right) \).

For any two triangular fuzzy numbers \( \tilde{A} = (a_{0}, a_{-}, a^{*}) \) and \( \tilde{B} = (b_{0}, b_{-}, b^{*}) \) in \( F(R) \) we define the ranking of \( \tilde{A} \) and \( \tilde{B} \) by comparing the \( \text{Mag}(\tilde{A}) \) and \( \text{Mag}(\tilde{B}) \)

(i) \( \tilde{A} \geq \tilde{B} \) iff \( \text{Mag}(\tilde{A}) \geq \text{Mag}(\tilde{B}) \).

(ii) \( \tilde{A} \leq \tilde{B} \) iff \( \text{Mag}(\tilde{A}) \leq \text{Mag}(\tilde{B}) \).

(iii) \( \tilde{A} \approx \tilde{B} \) iff \( \text{Mag}(\tilde{A}) = \text{Mag}(\tilde{B}) \).

2.2 Arithmetic Operations on Triangular Fuzzy Number

As discussed above triangular fuzzy number \( \tilde{A} = (a, b, c) \) may be transformed into parametric form so for any arbitrary fuzzy number \( \tilde{A} = (a_{0}, a_{-}, a^{*}) \) and \( \tilde{B} = (b_{0}, b_{-}, b^{*}) \). Arithmetic operations on the triangular fuzzy numbers are \( \tilde{A} \ast \tilde{B} = (a_{0} \ast b_{0}, a_{-} \vee b_{-}, a^{*} \vee b^{*}) \).

That is it is based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \).

That is for \( a, b \in L \) we define \( a \vee b = \max \{a, b\} \) and \( a \wedge b = \min \{a, b\} \).

In particular for any two fuzzy numbers \( \tilde{A} = (a_{0}, a_{-}, a^{*}) \) and \( \tilde{B} = (b_{0}, b_{-}, b^{*}) \), we define:

(i) Addition: \( \tilde{A} + \tilde{B} = (a_{0} + b_{0}, \max \{a_{-}, b_{-}\}, \max \{a^{*}, b^{*}\}) \)

(ii) Subtraction: \( \tilde{A} - \tilde{B} = (a_{0} - b_{0}, \max \{a_{-}, b_{-}\}, \max \{a^{*}, b^{*}\}) \)

(iii) Multiplication: \( \tilde{A} \times \tilde{B} = (a_{0} \times b_{0}, \max \{a_{-}, b_{-}\}, \max \{a^{*}, b^{*}\}) \)

(iv) Division: \( \tilde{A} \div \tilde{B} = (a_{0} \div b_{0}, \max \{a_{-}, b_{-}\}, \max \{a^{*}, b^{*}\}) \)

3. Mathematical formulation of solid transportation problem

A mathematical formulation is given by

\[ \min \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{jk} \tilde{x}_{jk} \]

Subject to

\[ \sum_{j=1}^{n} \tilde{x}_{jk} \leq di \quad (i = 1, 2, \ldots, m) \]

\[ \sum_{k=1}^{l} \tilde{x}_{jk} \geq bj \quad (j = 1, 2, \ldots, n) \]

\[ \tilde{x}_{jk} \leq s_{jk} \quad (j = 1, 2, \ldots, n) \]

\[ \tilde{x}_{jk} \geq 0 \quad (j = 1, 2, \ldots, n) \]

Where \( \tilde{z} \) is the objective function, \( c_{jk} \) is the cost of transporting one unit from origin \( i \) to destination \( j \), \( d_{i} \) is the demand at destination \( i \), \( b_{j} \) is the supply at origin \( j \), \( s_{jk} \) is the supply at origin \( j \) and \( \tilde{x}_{jk} \) is the fuzzy quantity of transportation from origin \( i \) to destination \( j \).
\begin{align*}
\sum_{j=1}^{q} \sum_{k=1}^{r} \tilde{x}_{ijk} & \approx \tilde{a}_i (i = 1, 2, \ldots, p) \\
\sum_{i=1}^{p} \sum_{k=1}^{r} \tilde{x}_{ijk} & \approx \tilde{b}_j (j = 1, 2, \ldots, q) \\
\sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{x}_{ijk} & \approx \tilde{c}_k (k = 1, 2, \ldots, r) \\
\tilde{x}_{ijk} & \geq 0 \text{ for all } i, j, k
\end{align*}

Where \( \tilde{c}_{jk} \) is the cost of shifting one unit of the product from supply node \( i \) to demand node \( j \) by using conveyance \( k \). \( p \) denotes the number of supply nodes, \( q \) denotes the number of demand nodes and \( r \) denotes the number of conveyance nodes. Let \( \tilde{a}_i \) represent the amount of item accessible at supply node \( i \) and \( \tilde{b}_j \) represent the amount of item needed at a demand node \( j \). \( \tilde{c}_k \) represent the amount of item transported by conveyance \( k \). \( \tilde{x}_{ijk} \) represent the number of units transported from source node \( i \) to demand node \( j \) by conveyance \( k \) in order to reduce the total transportation cost.

4. Algorithm
An algorithm to obtain an optimal solution to a fuzzy solid transportation problem is as follows:

(i) Express each supply/demand/conveyance in the parametric form \((a_i, a_j, a_k)\).

(ii) Find the difference of smallest and next smallest transportation cost of conveyance corresponding to both row and column.

(iii) Choose the maximum penalty and do the maximum allocation to the cell which contains minimum cost of transportation. Choose arbitrarily when more than one occurs.

(iv) Find the reduced table after removing the fulfilment source or destination or conveyance according to having maximum penalty.

(v) Repeat the above steps until all supply and conveyance points are totally utilized and all demand points are totally received and obtain the initial basic feasible solution to the problem.

(vi) To check the optimality find the set of \((m+n+k)\) Modi indices \( \tilde{u}_i (i = 1, 2, 3, \ldots, p) \) , \( \tilde{v}_j (j = 1, 2, 3, \ldots, q) \) , \( \tilde{w}_k (k = 1, 2, 3, \ldots, r) \) using the relation \( \tilde{C}_{ijk} = \tilde{u}_i + \tilde{v}_j + \tilde{w}_k \) for allocated cell and calculate the required \( \tilde{u}_i, \tilde{v}_j \) and \( \tilde{w}_k \).

(vii) Compute penalties using the formula \( \tilde{P}_{ijk} = \tilde{C}_{ijk} - (\tilde{u}_i + \tilde{v}_j + \tilde{w}_k) \) for unallocated cells. If all \( \tilde{P}_{ijk} > 0 \) then solution is optimal. If all \( \tilde{P}_{ijk} > 0 \) and atleast one \( \tilde{P}_{ijk} = 0 \) then alternative solution exists, with different set allocation and same transportation cost. If at least one \( \tilde{P}_{ijk} < 0 \) go to step (viii).

(viii) Draw a closed path consisting of horizontal and vertical lines starting and end at the cell for which \( \tilde{P}_{ijk} \) has most negative opportunity cost and having other corners at some allocated cell. Assign ‘+θ’ (or ‘-θ’) sign alternatively at the corners and choose the minimum of the allocations from the cells having ‘-θ’ sign. Add the minimum allocation to the cells with‘+θ’ and subtract to the cell with ‘-θ’ sign.

(ix) Repeat the above steps for optimality till you find all \( \tilde{P}_{ijk} \geq 0 \).
5. Numerical Example

A fuzzy solid transportation problem containing three origin, three requirement and three types of conveyance is given in table1. Shipping cost of one unit from each origin to each destination by each mode of conveyance is given by triangular fuzzy number.

| Origin | $O_1$ | $O_2$ | $O_3$ | Capacity | Demand |
|--------|-------|-------|-------|----------|--------|
|        | (3,4,5) | (3,4,5) | (7,8,9) | (10,11,12) | (5,6,7) |
|        | (6,7,8) | (1,2,3) | (5,6,7) | (13,14,15) | (15,16,17) |
|        | (7,8,9) | (0,1,2) | (2,3,4) | (8,9,10) | (11,12,13) |

Table 1. Solid fuzzy transportation problem.

| Origin | $O_1$ | $O_2$ | $O_3$ | Capacity | Demand |
|--------|-------|-------|-------|----------|--------|
|        | (4,1-p,1-p) | (4,1-p,1-p) | (8,1-p,1-p) | (11,1-p,1-p) | (6,1-p,1-p) |
|        | (7,1-p,1-p) | (3,1-p,1-p) | (7,1-p,1-p) | (7,1-p,1-p) | (7,1-p,1-p) |
|        | (1,1-p,1-p) | (3,1-p,1-p) | (8,1-p,1-p) | (5,1-p,1-p) | (4,1-p,1-p) |
|        | (2,1-p,1-p) | (1,1-p,1-p) | (8,1-p,1-p) | (5,1-p,1-p) | (5,1-p,1-p) |
|        | (6,1-p,1-p) | (6,1-p,1-p) | (8,1-p,1-p) | (6,1-p,1-p) | (6,1-p,1-p) |

Table 2. The parametric form of triangular fuzzy numbers in solid transportation problem.

| Origin | $O_1$ | $O_2$ | $O_3$ | Capacity | Demand |
|--------|-------|-------|-------|----------|--------|
|        | (11,1-p,1-p) | (11,1-p,1-p) | (9,1-p,1-p) | (11,1-p,1-p) | (16,1-p,1-p) |
|        | (14,1-p,1-p) | (14,1-p,1-p) | (9,1-p,1-p) | (14,1-p,1-p) | (12,1-p,1-p) |
Table 3. Initial basic feasible solution of fuzzy solid transportation problem.

|   | D₁ |   | D₂ |   | D₃ |   |
|---|----|---|----|---|----|---|
|   | E₁ | E₂ | E₃ | E₁ | E₂ | E₃ |
| O₁ | (4, 1-ρ, 1- ρ) (7, 1-ρ, 1- ρ) | (3, 1-ρ, 1- ρ) (9, 1- ρ, 1- ρ) | (6, 1-ρ, 1- ρ) (7, 1- ρ, 1- ρ) (2, 1-ρ, 1- ρ) |
| O₂ | (4, 1- ρ, 1- ρ) (2, 1- ρ, 1- ρ) | (1, 1- ρ, 1- ρ) (3, 1-ρ, 1- ρ) | (8, 1- ρ, 1- ρ) (4, 1- ρ, 1- ρ) |
| O₃ | (8, 1- ρ, 1- ρ) (1, 1- ρ, 1- ρ) | (4, 1- ρ, 1- ρ) (7, 1- ρ, 1- ρ) (3, 1- ρ, 1- ρ) | (5, 1- ρ, 1- ρ) (6, 1- ρ, 1- ρ) (4, 1- ρ, 1- ρ) |

Hence the initial basic fuzzy solution is

\(= (3, 1- ρ, 1- ρ) (2, 1- ρ, 1- ρ) + (2, 1- ρ, 1- ρ) (9, 1- ρ, 1- ρ) + (1, 1- ρ, 1- ρ) (8, 1- ρ, 1- ρ)\)

\(+ (3, 1- ρ, 1- ρ) (5, 1- ρ, 1- ρ) + (1, 1- ρ, 1- ρ) (6, 1- ρ, 1- ρ) + (4, 1- ρ, 1- ρ) (1, 1- ρ, 1- ρ)\)

\(+ (6, 1- ρ, 1- ρ) (3, 1- ρ, 1- ρ) = (75, 1- ρ, 1- ρ)\)

Table 4. Fuzzy Modi index table.

|   | D₁ |   | D₂ |   | D₃ |   | uᵢ |
|---|----|---|----|---|----|---|----|
|   | E₁ | E₂ | E₃ | uᵢ |   | E₁ | E₂ | E₃ |   | uᵢ |   |
| O₁ | (8, 1- ρ, 1- ρ) (7, 1- ρ, 1- ρ) (11, 1- ρ, 1- ρ) | (0, 1- ρ, 1- ρ) (4, 1- ρ, 1- ρ) (5, 1- ρ, 1- ρ) | (3, 1- ρ, 1- ρ) (2, 1- ρ, 1- ρ) (0, 1- ρ, 1- ρ) | (-1, 1- ρ, 1- ρ) |
| O₂ | (4, 1- ρ, 1- ρ) (7, 1- ρ, 1- ρ) (8, 1- ρ, 1- ρ) | (3, 1- ρ, 1- ρ) (9, 1- ρ, 1- ρ) (7, 1- ρ, 1- ρ) | (6, 1- ρ, 1- ρ) (7, 1- ρ, 1- ρ) (2, 1- ρ, 1- ρ) | (-3, 1- ρ, 1- ρ) |
| O₃ | (8, 1- ρ, 1- ρ) (4, 1- ρ, 1- ρ) (11, 1- ρ, 1- ρ) | (0, 1- ρ, 1- ρ) (0, 1- ρ, 1- ρ) (8, 1- ρ, 1- ρ) | (7, 1- ρ, 1- ρ) (1, 1- ρ, 1- ρ) (5, 1- ρ, 1- ρ) | (0, 1- ρ, 1- ρ) |
| O₄ | (4, 1- ρ, 1- ρ) (2, 1- ρ, 1- ρ) (6, 1- ρ, 1- ρ) | (1, 1- ρ, 1- ρ) (3, 1- ρ, 1- ρ) (8, 1- ρ, 1- ρ) | (8, 1- ρ, 1- ρ) (4, 1- ρ, 1- ρ) (5, 1- ρ, 1- ρ) | (0, 1- ρ, 1- ρ) |

It is clear that the solution is non-degenerate because the number basic feasible solution

\(= p + q + r - 2 = 3 + 3 + 3 - 2 = 7\)

Then we can check the optimality by using the fuzzy version of Modified distribution method
In respect of location index and fuzziness index the fuzzy optimal solution is given by

\[ \tilde{x}_{21} = (2, 1 - \rho, 1 - \rho), \tilde{x}_{33} = (9, 1 - \rho, 1 - \rho), \tilde{x}_{221} = (8, 1 - \rho, 1 - \rho), \tilde{x}_{222} = (5, 1 - \rho, 1 - \rho), \]
\[ \tilde{x}_{312} = (6, 1 - \rho, 1 - \rho), \tilde{x}_{321} = (1, 1 - \rho, 1 - \rho), \tilde{x}_{332} = (3, 1 - \rho, 1 - \rho) \]

Fuzzy optimal transportation cost is given by

\[ \min \tilde{z} = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} \tilde{c}_{ijk} \tilde{x}_{ijk} = (75, 1 - \rho, 1 - \rho) \] where \( 0 \leq \rho \leq 1 \) and it can be appropriately chosen by the decision maker.

The corresponding fuzzy optimal solution to the given problem in the original form is

\[ \min \tilde{z} = (74, 75, 76) \text{ (when } \rho = 0 \text{)} \]
whose crisp value is equal to 75.

6. Conclusion

A new simple methodology is used for the fuzzy initial basic feasible solution to the given solid fuzzy transportation problem whose parameters are the type of triangular fuzzy numbers without changing over into corresponding classical transportation problem. Additionally we checked that the obtained solution is optimal by using the fuzzy version of modified distribution method.

7. References

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