EFFECT OF GRAVITATIONAL LENSING ON MEASUREMENTS OF THE SUNYAEV-ZELDOVICH EFFECT

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ABSTRACT

The Sunyaev-Zeldovich (SZ) effect describes the distortion introduced to the cosmic microwave background (CMB) spectrum due to its Compton scattering off free electrons, which are either hot (the thermal effect) or possess a bulk peculiar velocity (the kinematic effect; see reviews of both effects in Sunyaev & Zeldovich 1980 and Rephaeli 1995). The thermal SZ effect provides an important diagnostic of the hot gas in clusters of galaxies and by now has been measured in a number of systems (see Table 1 in Rephaeli 1995). The kinematic effect has an amplitude that is typically an order of magnitude smaller and has not yet been definitively detected (see Rephaeli & Lahav 1991 and Haehnelt & Tegmark 1995 regarding prospects for a future detection).

It has long been realized that a measurement of the thermal SZ effect, combined with X-ray observations, can be used to estimate the distance to the cluster and hence the Hubble constant, H0, under the assumption that the cluster is spherical (Cavaliere, Danese, & De Zotti 1977; Gunn 1978; Silk & White 1978; Birkinshaw 1979). The inferred value of the Hubble constant is inversely proportional to the square of the SZ temperature decrement. This approach had led to values of the Hubble constant that are typically on the low side of the range inferred from other methods (see, e.g., Table 2 in Rephaeli 1995). An often-cited systematic effect that could account for this bias is elongation of the selected (high surface brightness) clusters along the line of sight. The inferred value of H0 is then underestimated by a factor equal to the ratio between the core radii of the cluster, parallel and perpendicular to the line of sight. The associated bias can be avoided by considering a sample of clusters that are selected based on their total X-ray flux (Birkinshaw, Hughes, & Arnaud 1991). In this Letter, we explore a different effect that leads to a systematic bias toward low H0 values, even if these clusters are perfectly spherical. The effect results from gravitational lensing by the cluster potentials.

Measurements of the decrement in the Rayleigh-Jeans (RJ) temperature of the microwave background due to the thermal SZ effect are routinely accompanied by the removal of background radio sources down to some flux threshold (see, e.g., Birkinshaw et al. 1991). In this process, it is implicitly assumed that the flux threshold for the removal of sources behind the cluster core is the same as in a control field far from the cluster center. However, this assumption is not strictly true because of the inevitable magnification bias that is introduced by the gravitational lensing effect of the cluster potential. In reality, the cluster acts as a lens that magnifies, and thus resolves, sources that are otherwise below the detection threshold. The residual intensity of unresolved sources is therefore systematically lower behind the cluster core, as compared to that in the control field. Lensing artificially increases the flux deficit behind the cluster core, and thus leads to a systematic underestimate of the Hubble constant.

In this Letter we calculate the effect of lensing on SZ measurements of the Hubble constant. Our discussion of lensing follows closely the approach developed in an earlier paper (Refregier & Loeb 1997, hereafter RL) that focused on lensing of the X-ray background by galaxy clusters; the interested reader should consult this earlier paper for more details. Here, we describe our models for the background population of radio sources and for the cluster potential in §2. We then show in §3 how the lensing effect leads to a systematic decrement in the intensity of unresolved sources. In §4 we present numerical results for different values of our model parameters and for the specific example of A2218. Finally, §5 summarizes the main conclusions of this work.

1. INTRODUCTION

The Sunyaev-Zeldovich (SZ) effect describes the distortion of the microwave background (CMB) spectrum due to its Compton scattering off free electrons, which are either hot (the thermal effect) or possess a bulk peculiar velocity (the kinematic effect; see reviews of both effects in Sunyaev & Zeldovich 1980 and Rephaeli 1995). The thermal SZ effect provides an important diagnostic of the hot gas in clusters of galaxies and by now has been measured in a number of systems (see Table 1 in Rephaeli 1995). The kinematic effect has an amplitude that is typically an order of magnitude smaller and has not yet been definitively detected (see Rephaeli & Lahav 1991 and Haehnelt & Tegmark 1995 regarding prospects for a future detection).

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2. MODEL

We model the gravitational potential of the cluster as a singular isothermal sphere (SIS) (e.g., Schneider, Ehlers, & Falco 1992). This model provides a good first-order approximation to the projected mass distribution of known cluster lenses (Tyson & Fischer 1995; Narayan & Bartelmann 1996; Squires et al. 1996a, 1996b). The SIS potential causes back-
ground sources to appear brighter but diluted on the sky by the magnification factor

$$\mu(\theta) = \left| 1 - \frac{\theta_\text{ls}}{\theta} \right|^{-1},$$  

where $\theta$ is the angle between the image of the source and the lens center, and $\theta_\text{ls}$ is the Einstein angle. For a SIS with a line-of-sight velocity dispersion $\sigma_v$, $\theta_\text{ls} = 4\pi(\sigma_v^2/c^2) D_\text{ls}/D_\text{co}$, where $D_\text{ls}$ and $D_\text{co}$ are the lens-source and the observer-source angular diameter distances, respectively.

In general, the Einstein angle $\theta_\text{ls}$ depends on the redshifts of the lens, $z_\text{ls}$, and of the source, $z$. However, the dependence on the source redshift is weak if $z \approx z_\text{ls}$ (see Fig. 1 in RL). Most measurements of the SZ effect are performed with nearby clusters ($z \approx 0.2$), while sub-mJy radio sources have median redshifts in the range $0.5 \approx z \approx 0.75$ (Windhorst et al. 1993). We therefore take $\theta_\text{ls}$ to be independent of $z$, and simply consider the two-dimensional distribution of radio sources on the sky.

We model the flux distribution of background radio sources according to the observed number-flux relation at 4.86 GHz (Windhorst et al. 1993). The circles with error bars in Figure 1 show the observed differential counts, $dn/dS$, normalized by $S^{-2}$. The dotted line corresponds to the number count limits inferred from fluctuations of the unresolved background (Fomalont et al. 1991) from a fluctuation analysis of the unresolved background (Fomalont et al. 1991). The solid line shows our model with its six power-law components.

![Number-flux relation for radio sources at 4.86 GHz. The counts were normalized by $S^{-2}$, the relation that remains invariant under lensing. The observed counts summarized by Windhorst et al. (1993) are shown as circles with error bars. The dotted line corresponds to the limits inferred from a fluctuation analysis of the unresolved background (Fomalont et al. 1991). The solid line shows our model with its six power-law components.](image)

**Fig. 1.—Number-flux relation for radio sources at 4.86 GHz.** The counts were normalized by $S^{-2}$, the relation that remains invariant under lensing. The observed counts summarized by Windhorst et al. (1993) are shown as circles with error bars. The dotted line corresponds to the limits inferred from a fluctuation analysis of the unresolved background (Fomalont et al. 1991). The solid line shows our model with its six power-law components.

3. THE LENSING EFFECT

In a region of the sky where the magnification factor is $\mu$, the apparent differential counts of sources are given by $(dn/dS)|_x = \mu^{-2}(dn/dS)|_{\mu x}$, where the caret denotes unlensed quantities. In particular, for a power-law differential count relation, $(dn/dS)|_x \propto S^{-\gamma}$, the observed differential counts are $(dn/dS)|_x \propto \mu^{-2}\cdot S^{-\gamma}$. The differential count therefore increases (decreases) as $\mu$ increases if $\gamma$ is above (below) the critical slope $\gamma_\text{crit} = 2$. When $\gamma > \gamma_\text{crit}$, lensing has no effect on the apparent differential count. Interestingly, Figure 1 shows that the actual radio count slope oscillates around $\gamma_\text{crit}$, with crossovers at $S_\text{co} \approx 10^{-2}$, $10^{-4}$, and possibly $10^{-6}$ Jy.

In measurements of the SZ effect, discrete sources are typically removed down to a given detection flux threshold, $S_d$. The mean residual intensity $\bar{i}(S_d)$ due to the superposition of all undetected discrete sources with fluxes below $S_d$ is then assumed to be equal to its sky-averaged value,

$$\bar{i}(S_d) = \int_0^{S_d} dS \frac{\partial i}{\partial S} \left. \frac{d\hat{i}}{d\hat{S}} \right|_{\hat{S} = S_d}.$$  

However, the magnification due to lensing lowers the unresolved intensity systematically relative to its sky-averaged value (by raising faint sources above the detection threshold) and changes it to

$$i(S_d) = \int_0^{S_d} dS S \mu^{-2} \frac{\partial i}{\partial S} \left. \frac{d\hat{i}}{d\hat{S}} \right|_{\hat{S} = S_d/\mu},$$  

where $\mu = \mu(\theta)$ is given by equation (1). Lensing conserves the total intensity of the radio source background and merely reduces the effective flux threshold for resolving sources by a factor $\mu$. The intensity offset due to lensing is then $\Delta \bar{i}_{\text{th}} = i(S_d) - \bar{i}(S_d)$. In the RJ regime, this can be expressed more conveniently in terms of the brightness temperature difference, $\Delta T_\text{th} = (c^2/2\nu k_\text{B})\Delta \bar{i}_{\text{th}}$, where $k_\text{B}$ is Boltzmann’s constant.
constant. For \( \mu > 1 \) (i.e., \( \theta > \theta_k/2 \)), the unresolved intensity is decreased, implying a negative \( \Delta T_{\text{cls}} \), and so the SZ decrement in the RJ regime, \( \Delta T_{\text{SZ}} \), is overestimated due to lensing. Note that for \( \theta >> \theta_k \), equation (1) yields \( \mu \approx 1 + \theta_k/\theta \) and \( \Delta T_{\text{cls}} \propto \theta^{-1} \). The effect of lensing on estimates of \( H_0 \) can be easily found from the scaling, \( H_0 \approx (\Delta T_{\text{SZ}})^{-1} \), where \( \Delta T_{\text{SZ}} \) is the temperature offset produced by the SZ effect. The small systematic correction \( (\Delta H_0)_{\text{true}} - H_0(\text{observed}) \), which must be incorporated in order to compensate for the lensing effect, is, to leading order,

\[
\frac{(\Delta H_0)_{\text{true}}}{H_0} \approx 2 \frac{\Delta T_{\text{cls}}}{\Delta T_{\text{SZ}}}.
\]

For \( \theta \approx \theta_k/2 \) and the RJ spectral regime, both \( \Delta T_{\text{cls}} \) and \( \Delta T_{\text{SZ}} \) are negative, and so \( (\Delta H_0)_{\text{true}} \) is positive. The lensing correction will then tend to increase the estimated value of the Hubble constant.

4. RESULTS

Figure 2 shows \( \Delta T_{\text{cls}} \) as a function of angular separation from the cluster center, \( \theta \), for several values of the detection threshold \( S_0 \). The values for \( \Delta T_{\text{cls}} \) and \( S_0 \) correspond to a frequency \( \nu = 4.86 \text{ GHz} \). The dependence of \( \Delta T_{\text{cls}} \) on the Einstein angle \( \theta_0 \) and on frequency \( \nu \) were factored out. The Einstein angles for clusters with observed optical arcs are in the range of \( 10'' - 50'' \) (Le Fèvre et al. 1994).

The lensing decrement, \( \Delta T_{\text{cls}} \), shows a sharp peak near the Einstein angle. For \( \theta \approx \theta_k \), \( \Delta T_{\text{cls}} \) is first weakened and then enhanced as \( S_0 \) varies from \( 10^{-3} \) to \( 10^{-5} \) Jy. This is because the count slope \( \gamma \) crosses the critical value \( \gamma_{\text{crit}} = 2 \) around \( S_0 \approx 5 \times 10^{-3} \) Jy (see Fig. 1). The enhancement in \( \Delta T_{\text{cls}} \) as \( S_0 \) decreases below \( 10^{-3} \) Jy occurs in spite of the decrease in the unlensed intensity \( i(<S_0) \) there. At these fluxes, the removal of fainter radio sources paradoxically makes the lensing decrement more pronounced, because of the associated deviation of \( \gamma \) from \( \gamma_{\text{crit}} \). Note that because of the large shot noise in the source counts (with an rms of \( \sigma/i \approx 0.5 \) in a 1 arcmin\(^2\) cell for \( S_0(4.86 \text{ GHz}) = 10^{-3} \text{ Jy} \), \( \Delta T_{\text{cls}} \) will not necessarily be realized in each individual cluster. The lensing-induced decrement should be regarded as a systematic effect that must be corrected for statistically when a large sample of clusters is considered.

For observations with a large field of view, the weak lensing signature in the outer part of each individual cluster will be contaminated by the imprint of source clustering. The clustering of mJy radio sources was recently detected by Cress et al. (1996) using the FIRST survey data (White et al. 1997). The inferred amplitude of the autocorrelation function is \( \approx 20\% \) on arcminute scales. The fluctuations in the counts of sub-mJy sources have a correlation amplitude of \( \approx 15\% - 20\% \) on sub-degree scales resulting from the existence of sheets and superclusters in the distribution of radio sources (Windhorst, Mathis, & Nechaeva 1990). While source clustering could affect the error budget of the SZ decrement in each individual cluster, it does not shift the SZ decrement systematically, and its significance relative to lensing can be reduced by using a large statistical sample of clusters.

As a specific example we consider A2218, an Abell richness class 4 cluster at a redshift \( z = 0.175 \), which shows several optical arcs (Pelló et al. 1992; Le Borgne, Pelló, & Sanahuja 1992). Arc 359 in Pelló et al. (1992) is separated by 20'8 from the central CD galaxy and has a measured redshift of 0.702, close to the probable median redshift of sub-mJy sources (5. \( \approx 0.5 - 0.75 \); cf. Windhorst et al. 1993). We therefore model the cluster potential as an SIS with an Einstein angle of \( \theta_0 = 20'8 \) for our radio sources (see also Miralda-Escudé & Babul 1995). Interferometric imaging of the SZ effect in this cluster was performed by Jones et al. (1993) at 15 GHz, after the removal of point sources with fluxes above \( 1 \text{ mJy} \). The restoring beam for their short baseline image had an FWHM of 129' \( \times \) 120'. The observed angular dependence of \( \Delta T_{\text{SZ}} \) was fitted by a \( b \) model,

\[
\Delta T_{\text{SZ}}(\theta) = \Delta T_0 \left( 1 + \frac{\theta_0}{\theta} \right)^{-1/2 - 3/2 \beta},
\]

Acceptable \( \chi^2 \) values were obtained for different sets of parameters ranging from \( \beta \approx 0.6, \theta_k \approx 0.9 \), and \( \Delta T_0 \approx 1.1 \text{ mK} \), to \( \beta \approx 1.5, \theta_k \approx 2.0 \), and \( \Delta T_0 \approx 0.6 \text{ mK} \).

Figure 3 shows the expected ratio \( \Delta T_{\text{cls}}/\Delta T_{\text{SZ}} \) for A2218 at 15 GHz, assuming a source spectral index of \( \alpha = 0.35 \). The ratio is shown for two values of \( S_0(4.86 \text{ GHz}) \) and for the two extreme sets of fit parameters for \( \Delta T_{\text{SZ}}(\theta) \). The sharp peak at \( \theta = 20'8 \) reflects the enhancement in \( \Delta T_{\text{cls}} \) around the Einstein radius (see Fig. 2). For the \( \beta = 1.5 \) model, \( \Delta T_{\text{cls}}/\Delta T_{\text{SZ}} \) diverges at \( \theta \approx 2'. \) If the mass distribution follows the SIS profile, \( \Delta T_{\text{SZ}} \propto \theta^{-1} \) at \( \theta >> \theta_k \). Since \( \Delta T_{\text{SZ}} \propto \theta^{-3/2} \) for \( \theta >> \theta_k \) (eq. [5]), the ratio \( (\Delta T_{\text{cls}}/\Delta T_{\text{SZ}}) \propto \theta^{3/2} \) diverges at large radii if \( \beta > 2/3 \). The values of \( \beta \) derived from X-ray observations of clusters have a large scatter around a mean value of \( \approx 0.65 \) (Sarazin 1988; Jones & Forman 1984; Bahcall & Lubin 1994). Weak lensing studies in the optical band could be used in conjunction with X-ray observations to predict the relative radial behavior of the lensing and SZ decrements in each individual cluster.

It is convenient to average the temperature offset over a circular “top hat” beam of radius \( \theta_0 \) centered on the cluster.
center, \((\Delta T(\theta)) = 2 \theta^2 \int \phi \Delta T(\theta) \, d\theta\). For the above model of A2218 with \(S_\delta(4.86 \text{ GHz}) = 10^4 \text{ Jy}\), the 15 GHz mean temperature offsets due to lensing are \((\Delta T_{\text{rms}}) \approx -38, -16, -0.2 \mu \text{K}\), for \(\theta_l = 0.4, 1\), and 60', respectively. The corresponding decrement ratios are \((\Delta T_{\text{rms}})/\Delta T_{\text{Sz}}) \approx 0.07, 0.03, 0.19\) for the \(\beta = 1.5\) fit, and 0.04, 0.02, 0.004 for the \(\beta = 0.6\) fit. The fractional correction to the Hubble constant (eq. [4]) is then \(\Delta H_l/H_l \approx 7\% - 13\%, 4\% - 7\%,\) and \(0.8\% - 38\%\) for \(\theta_l = 0.4, 1\), and 60', respectively, where the ranges reflect the ambiguity in the fit parameters of \(\Delta T_{\text{Sz}}(\theta)\) and in particular the value of \(\beta\).

5. CONCLUSIONS

We have shown that gravitational lensing of unresolved radio sources leads to a systematic overestimate of the SZ temperature decrement at angles \(\theta > \theta_{\text{E}}/2\). The amplitude of the lensing effect peaks close to the Einstein angle of the cluster, \(\theta_{\text{E}} \sim 30\) (Fig. 2). While \(\Delta T_{\text{Sz}}\) is independent of frequency in the RJ regime, the lensing decrement \(\Delta T_{\text{lens}} \approx \nu^{-2}\) (with \(\alpha \approx 0.35\)) is significant only at frequencies \(\nu \approx 30\) GHz. In clusters where the radial profile of the gas pressure is steeper than that of the dark matter density (e.g., due to a gradient in the gas temperature), the ratio of the lensing to the SZ decrement increases at large projected radii. For observations of A2218 at 15 GHz with a source removal threshold of \(S_\delta(4.86 \text{ GHz}) = 10^4 \text{ Jy}\), \(H_l\) could be overestimated by \(\sim 1\% - 40\%\), for a beam radius in the range of 0.4–60' (see Fig. 3). The overestimate might still be larger if a significant fraction of all sub-mJy sources have inverted spectra. The importance of the lensing effect will be enhanced in future observations (including attempts to detect the kinematic SZ effect) with greater sensitivity, higher angular resolution, and a fainter source removal threshold.

Lensing should also affect the power spectrum of microwave background anisotropies on arcminute scales behind the cluster. These anisotropies are expected to originate primarily from the Ostriker-Vishniac effect (Hu & White 1996) and the cumulative SZ effect of other background clusters (Colafrancesco et al. 1994; Rephaeli 1995). Future SZ experiments might be contaminated by noise from these clusters (Colafrancesco et al. 1994; Rephaeli 1995). Hence these diffuse fluctuations will not be removed, lensing will conserve their net intensity and will not systematically offset the SZ decrement as it does in the case of discrete sources.

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