Simultaneous Projectile-Target Excitation in Heavy Ion Collisions

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Abstract

We calculate the lowest-order contribution to the cross section for simultaneous excitation of projectile and target nuclei in relativistic heavy ion collisions. This process is, to leading order, non-classical and adds incoherently to the well-studied semi-classical Weizsäcker-Williams cross section. While the leading contribution to the cross section is down by only $1/Z_p$ from the semiclassical process, and consequently of potential importance for understanding data from light projectiles, we find that phase space considerations render the cross section utterly negligible.
Increasingly, heavy ion beams are being exploited as a tool for measuring electromagnetic cross sections for use in studies of nuclear structure\[1\] and astrophysics\[2\]. Central to this effort is the use of the semi-classical Weizsäcker-Williams method\[3\] to relate the cross section measured in heavy ion collisions to those obtained with real photons. In this report, we continue a program of examining the the corrections to the semiclassical approach\[4\] by calculating the leading-order contribution to the cross section for simultaneous excitation of both the target and projectile nucleus.

Before embarking on the detailed calculation, it is useful to perform a simplistic analysis of the processes shown in Fig. 1. In Figs. 1a and b, the leading order Feynman diagrams for target excitation with and without an accompanying excitation of the projectile are shown. In the latter case, the projectile form factor is normalized by the charge of the projectile, $Z_p$, while in 1a, the transition form factors for the projectile can be thought of as being normalized by an appropriately chosen energy-weighted sum rule($\propto Z_p^{1/2}$). Since the final states in Figs. 1a and b are distinct, the simultaneous excitation amplitude adds incoherently to that for single excitation, leading to the expectation that the semi-classical cross section for single excitation will lie below the measured cross sections. (This effect is seen in single neutron removal data taken with low-$Z$ projectiles\[\text{I}\], but is most likely due to the difficulties inherent in estimating the strong-interaction contribution to the measured cross sections\[\text{II}\].) Since the cross section for simultaneous
excitation is down from that for single excitation by a full power of \( Z_P \), the simultaneous excitation process is not relevant for high-\( Z \) projectiles, but, at least at first glance, may play a nonnegligible role for low-\( Z \) projectiles.

In Fig. 1b, the projectile, of mass \( M_P \) and momentum \( P_i^\mu = (E_i, \vec{P}_i) \), scatters elastically from the target by exchanging a virtual photon of momentum \( q^\mu \). In the process, the target, of mass \( M_T \), is excited from its ground state to an excited state of mass \( M_T + \omega_T \). In Fig. 1a, the picture is essentially the same, except that the projectile is excited to a state of mass \( M_P + \omega_P \). Kinematically, this requires that

\[
q^2 - 2P_i \cdot q = 2M_P \omega_P + \omega_P^2. \tag{1}
\]

For nuclear transitions, the momentum transfers and excitation energies are negligible compared to the masses, and we obtain

\[
- \omega_P = q_A^0 = \gamma(q_L^0 - \vec{\beta} \cdot \vec{q}_L), \tag{2}
\]

where the subscript \( L(A) \) denotes the momentum transfer evaluated in the target(projectile) rest frame, \( \vec{\beta} = \vec{P}_i / E_i \), and \( \gamma = 1/\sqrt{1 - \vec{\beta}^2} \). Similarly, for the target we obtain

\[
\omega_T = q_L^0 = \gamma(q_A^0 + \vec{\beta} \cdot \vec{q}_A). \tag{3}
\]

Thus, the minimum three- and four-momentum transfers are given by

\[
|q_L^{\text{min}}| = q_L^\parallel = (\gamma \omega_T + \omega_P) / \beta \gamma, \tag{4}
\]

\[
|q_A^{\text{min}}| = q_A^\parallel = (\gamma \omega_P + \omega_T) / \beta \gamma, \tag{5}
\]

\[
q_{\text{min}}^2 = -(\omega_T^2 + \omega_P^2 + 2\gamma \omega_T \omega_P) / \beta^2 \gamma^2, \tag{6}
\]
where \( q^\parallel \) refers to the component of \( \vec{q} \) along \( \beta \). In contrast to the case of single excitation, where \( q^2_{\text{min}} \) varies like \( \gamma^{-2} \), the minimum value of \( q^2 \) drops only like \( \gamma^{-1} \), so that the cross section for simultaneous excitation is less sensitive to the pole in the photon propagator. Consequently, the Coulomb contribution to the cross section will be more important than in the single excitation case.

Evaluating the contribution to the spin averaged/summed cross section from Fig. 1a, we obtain

\[
d\sigma_{SE} = \frac{e^4}{4((P_i \cdot K_i)^2 - M_P^2 M_T^2)^{1/2}} \frac{d^3 \tilde{P}_f d^3 \tilde{K}_f}{q^4} (2\pi)^4 \delta(P_f + K_f - P_i - K_i) \\
\times \sum_{M_P^f, M_P^i} \frac{\langle P_f M_P^f | J_\mu(0) | P_i M_P^i \rangle \langle P_i M_P^i | J_\nu(0) | P_f M_P^f \rangle}{(2J_f^P + 1)} \\
\times \sum_{M_T^f, M_T^i} \frac{\langle K_f M_T^f | J_\nu(0) | K_i M_T^i \rangle \langle K_i M_T^i | J_\mu(0) | K_f M_T^f \rangle}{(2J_i^T + 1)},
\]

(7)

where \( J_i^{T(P)} \) is the initial state target projectile spin and \( M_i^{T(P)} \) is the third component of the initial final state spin. Following reference 4, we replace the spin sums by Lorentz covariant structure tensors and rewrite the cross section in terms of matrix elements defined in the projectile and target rest frames. The result is

\[
\sigma_{SE} = \frac{e^4}{4\beta \gamma M_P M_T} \int \frac{d^4 q}{(2\pi)^4 q^4} \int d^3 \tilde{P}_f d^3 \tilde{K}_f (2\pi)^4 \delta^4(P_f - P_i + q) \\
\times (2\pi)^4 \delta^4(K_f - K_i - q) \left[ \frac{(\rho \rho)_P (\rho \rho)_T}{4} \left( \frac{9}{4} \frac{(\gamma q^2 + \omega_P \omega_T)^2 q^4}{q_A^4 q_L^4} - \frac{3}{4} \frac{q^4}{q_A^4 q_L^4} \right) \right]
\]

4
\[ + \overline{(\rho \rho)}_P (\rho \rho - \vec{J} \cdot \vec{J})_T \left( -\frac{1}{4} \frac{q^2}{q_A^2} + \frac{3}{4} \frac{\omega_p \omega_T^2}{q_A^2 q_L^2} \right) \]
\[ + \overline{(\rho \rho)}_T (\rho \rho - \vec{J} \cdot \vec{J})_P \left( -\frac{1}{4} \frac{q^2}{q_L^2} + \frac{3}{4} \frac{\omega_q \omega_T^2}{q_L^2 q_A^2} \right) \]
\[ + (\rho \rho - \vec{J} \cdot \vec{J})_P (\rho \rho - \vec{J} \cdot \vec{J})_T \left( \frac{1}{4} + \frac{1}{4} \frac{\omega^2}{q_L^2 q_A^2} \right) \],

(8)

where \( \overline{\rho(\vec{J}) \rho(\vec{J})}_{P(T)} \) indicates the spin-averaged matrix element of projectile(target) transition charge(current) density evaluated in the projectile(target) rest frame.

Assuming a model for the transition densities, the remaining integrations may be carried out numerically or, alternatively, a useful estimate of the cross section can be obtained in a model-independent manner by considering the limit of large \( \gamma \) and low transition energies for both the target and projectile. Reexpressing the spatial delta functions by an integral over complex exponentials and using translational invariance, the integrals over the components of \( \vec{q} \) orthogonal to \( \vec{\beta} \) can be done analytically, yielding

\[
\sigma_{SE} = \frac{e^4}{8\pi M_P M_T} \int d^3P_f d^3K_f \int d^3x d^3y e^{i(q_A^\parallel - q_L^\parallel)} \]
\[ \times \left[ \overline{\rho(\vec{x}) \rho(0)}_P \overline{\rho(\vec{y}) \rho(0)}_T F_{\rho \rho}(\vec{z}_L^\parallel, q_L^\parallel, q_A^\parallel, \delta \omega^2) \right. \]
\[ + \overline{\rho(\vec{x}) \rho(0)}_P \overline{\rho(\vec{y}) \rho(0)}_T \overline{\vec{J}(\vec{y}) \cdot \vec{J}(0)}_T F_{\rho \gamma}(\vec{z}_L^\parallel, q_L^\parallel, q_A^\parallel, \sqrt{-q_{min}^2}, \delta \omega^2) \]
\[ + \overline{\rho(\vec{y}) \rho(0)}_T \overline{\rho(\vec{x}) \rho(0)}_P \overline{\vec{J}(\vec{x}) \cdot \vec{J}(0)}_P F_{\rho \gamma}(\vec{z}_L^\parallel, q_L^\parallel, q_A^\parallel, \sqrt{-q_{min}^2}, \delta \omega^2) \]
\[ + \overline{\rho(\vec{x}) \rho(0)}_T \overline{\rho(\vec{y}) \rho(0)}_T \overline{\vec{J}(\vec{x}) \cdot \vec{J}(0)}_T F_{\gamma \gamma}(\vec{z}_L^\parallel, q_L^\parallel, \sqrt{-q_{min}^2}, \delta \omega^2) \left. \right], \quad (9) \]
where $\bar{x}, \bar{y}$ are the arguments of the matrix elements appearing in the spin averages, $\vec{z}_\perp$ is the component of $\vec{x} - \vec{y}$ orthogonal to $\vec{\beta}$, $x_\parallel$ and $y_\parallel$ are the components of $\vec{x}$ and $\vec{y}$ parallel to $\vec{\beta}$, and

$$F_{pp}(\vec{z}_\perp, q_\parallel, q_L, \delta \omega^2) = \frac{9}{8} \left[ \frac{\omega_\parallel^2}{(\delta \omega^2)^2} q_\parallel^2 K_1(q_L^2 \delta \omega^2) + \frac{\omega_\parallel^2}{(\delta \omega^2)^2} q_L^2 \delta \omega^2 K_1(q_L^2 \delta \omega^2) \right] - \frac{3(\omega_\parallel^2 + \omega_\perp^4 + (6\gamma^2 + 4)\omega_\parallel^2 \delta \omega^2 + 6\gamma \omega_\parallel^2 \delta \omega^2)}{4(\delta \omega^2)^3 \beta^2 \gamma^2} \times \left[ K_0(q_L^2 \delta \omega^2) - K_0(q_L^2 \delta \omega^2) \right],$$

$$F_{p\gamma}(\vec{z}_\perp, q_\parallel, q_L, \sqrt{q_{\min}^2}, \delta \omega^2) = \frac{3q_\parallel^2 K_1(q_L^2 \delta \omega^2)}{8 \delta \omega^2} + \frac{1}{2 \omega_\parallel^2 \beta^2 \gamma^2} \left[ K_0(q_L^2 \delta \omega^2) - K_0(q_L^2 \delta \omega^2) \right] + \frac{3q_\parallel^2}{4(\delta \omega^2)^2} \left[ K_0(\sqrt{-q_{\min}^2} \delta \omega^2) - K_0(q_L^2 \delta \omega^2) \right],$$

$$F_{\gamma\gamma}(\vec{z}_\perp, q_\parallel, q_L, \sqrt{-q_{\min}^2}, \delta \omega^2) = \frac{1}{4} \left[ \frac{-q_{\min}^2 \delta \omega^2 K_1(\sqrt{-q_{\min}^2} \delta \omega^2)}{-q_{\min}^2 \beta^2 \gamma^2} + \frac{q_{\min}^2 K_0(\sqrt{-q_{\min}^2} \delta \omega^2)}{4 \omega_\parallel^2 \omega_\perp^2} \right] + \frac{q_\parallel^2 K_0(q_L^2 \delta \omega^2)}{4 \delta \omega^2 \omega_\perp^2} - \frac{q_\parallel^2 K_0(q_L^2 \delta \omega^2)}{4 \delta \omega^2 \omega_\perp^2},$$

where $K_0, K_1$ are modified Bessel functions, and $\delta \omega^2 = \omega_\parallel^2 - \omega_\perp^2$. The strategy at this point is to expand the Bessel functions around zero frequency, assuming that the logarithms of $|\vec{z}_\perp|, |\vec{x}|$, and $|\vec{y}|$ vary slowly enough to be treated as constants. For dipole-dipole excitations, we obtain, after much algebra

$$\sigma_{E1E1} = \frac{3}{8\pi^3} \int d\omega_\parallel d\omega_T \frac{\sigma_{E1}(\omega_\parallel) \sigma_{E1}(\omega_T)}{\omega_\parallel \omega_T} [(\omega_\parallel^2 + \omega_T^2)\xi_1 - \delta \omega^2 \xi_2] + O(1/\gamma),$$

(12)
where $\xi_1$ and $\xi_2$ are averages of logarithms of projectile/target coordinates over transition densities and are both of order one numerically. Remarkably, all of the dependence on logarithms of the transition frequencies cancels out of expression 12, so that $\xi_1$ and $\xi_2$ have no explicit dependence on the transition frequencies. In addition, we note that the photon-pole terms are all of order $1/\gamma$, so that the high-$\gamma$ limit of the cross section is dominated by the off-shell response functions of the target and projectile.

Assuming that the dipole cross sections are sharply peaked at the giant resonance energy, and that $\xi_1 \approx 1$, we estimate the simultaneous dipole-dipole excitation cross section to be

$$\sigma_{E1E1} = \sigma_0 \frac{N_P Z_P N_T Z_T}{A_P A_T} \left( \frac{A_P^{1/3}}{A_T^{1/3}} + \frac{A_T^{1/3}}{A_P^{1/3}} \right),$$

(13)

with $\sigma_0 \approx 1.1 \times 10^{-4}$ mb. For $^{12}$C projectiles on $^{197}$Au, this gives a cross section of .05 mb for simultaneous excitation, compared to a measured single excitation cross section of $50\pm7$ mb at 2.1 GeV/nucleon. Numerical integration of the cross section using the Goldhaber-Teller model\cite{6} confirms the order of magnitude of this estimate and indicates that the projectile-energy dependence of the cross section is equally negligible.

To leading order, then, the simultaneous excitation cross section is quite negligible, not only because of its $Z_P$ dependence, but also as a result of the factor of $1/8\pi^3$ coming from phase space. At higher order in $\alpha$, this situation may change, however, since the diagram shown in Fig. 2, in which the target and projectile excite one another via the exchange of two photons,
is a classically allowed process. The resulting cross section will be larger by a factor of $Z_f^2 Z_p^2 \alpha^2$ from the leading-order result and may therefore be significant.

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References

[1] J.C. Hill, F.K. Wohn, J.A. Winger, M.Khayat, M.T. Mercier, and A.R. Smith, *Phys. Rev.* **C39**, 524(1989),
M. Ishihara, *Nucl. Phys.* **A538**, 309(1992),
W.J. Llope and P. Braun-Munzinger, *Phys. Rev.* **C45**, 799(1992).

[2] K.E. Kettner, H.U. Becker, L. Buchmann, J. Gorres, H. Kräwinkel, C. Rolfs, P. Schmalbrock, H.P. Trauvetter and A. Vlieks, *Z. Phys.* **A308** 73(1982).

[3] E. Fermi, *Z. Phys.* **29**, 315(1924),
C.F. Weizsäcker, *Z. Phys.* **88**, 612(1934),
E.J. Williams, *Phys. Rev.* **45**, 729(1934).

[4] C.J. Benesh and J.L. Friar, *Phys. Rev.* **48**, 1285 (1993).

[5] C.J. Benesh, B.C. Cook, and J.P. Vary, *Phys. Rev.* **C40**, 1198(1989).
[6] T. de Forest Jr. and J. D. Walecka. *Advances in Physics* **15**, 1(1966).
Figure Captions

- Fig. 1 Feynman diagrams for target excitation (a) with and (b) without simultaneous excitation of the projectile. A square indicates an elastic form factor, while circles represent the inelastic transition form factors.

- Fig. 2 Feynman diagram for simultaneous excitation of projectile and target by exchange of two photons.
This figure "fig1-1.png" is available in "png" format from:

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