Search for a weak coupling between gravity and electromagnetic field with Lorentz symmetry breakdown at low energy scales

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This research aims to develop a new approach toward a consistent coupling of electromagnetic and gravitational fields, by using an electron that couples with a gravitational potential by means of its electromagnetic field. To accomplish this we must first build a new model which provides the electromagnetic nature of both the mass and the energy of the electron, and which is implemented with the idea of the $\gamma$-photon decay into a electron-positron pair. After this, we place the electron (or positron) in the presence of a gravitational potential so that its electromagnetic field undergoes a very small perturbation, leading to a slight increase in the field's electromagnetic energy density. This perturbation takes place by means of a tiny coupling constant since gravity is an extremely weak interaction. We compute such a coupling constant $\xi$ given in terms of fundamental constants like $G$, $\hbar$, $c$, the charge $e$ and the mass $m_e$. Thus we realize that $\xi$ is a new dimensionless universal constant, which reminds us of the fine structure constant $\alpha = e^2/\hbar c \simeq 1/137$; however, it is much smaller than $\alpha$, since it includes gravity, leading to the emergence of a background field that breaks down Lorentz symmetry. We find $\xi = V/c \simeq 3.57 \times 10^{-24}$, where $c$ is the speed of light and $V(\sim 10^{-15}$ m/s) is a universal minimum speed, i.e., the lowest limit of speed for any particle with lower energy, leading to a doubly (symmetrical) special relativity with two invariant speeds $c$ and $V$ for higher and lower energies.

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I. INTRODUCTION

Theoretical physicists have been attempting for decades to find a satisfactory marriage of quantum field theory and gravity. While the full theory of quantum gravity is far from being achieved, a great deal of work has been done, and some efforts to couple the electron minimally with gravity have also been made by using the Dirac equation[1]. In particular, through the formalism of quantum field theory in curved space-time, most of the classical theory is adequately understood.

The current research also attempts to combine quantum effects of electromagnetic fields with gravitational effects; however, we use an alternative (heuristic) approach in order to extend, by means of the idea of complementarity, the classical concept of electromagnetic fields (vectorial fields to represent waves) to a new quantum aspect of such fields (scalar fields to represent corpuscles) and, thus, explain the nature of relativistic mass as being of electromagnetic origin, given in a scalar form (corpuscular aspect of matter). With this new approach, it becomes possible to establish a new form of coupling between electromagnetic and gravitational fields in order to emphasize the fact that gravity is the weakest interaction in Nature (the hierarchical problem), since such a new coupling is given by a tiny pure number with direct dependence on the small constant of gravitation ($G$). So, the present approach will allow us to realize that a minuscule influence of gravity on electromagnetic fields leads to a violation of Lorentz symmetry, which should be thoroughly investigated. Such minuscule apparent violations of Lorentz invariance might be observable in Nature[2]. The basic idea is that the violations would arise as suppressed effects from a more fundamental theory of space-time with quantum-gravity effects.

In the present work, a theory with Lorentz symmetry breakdown will be investigated by searching for a new kind of coupling, i.e., a dimensionless coupling constant, working like a “fine structure” between electromagnetic and gravitational fields in such a way as to extend our notion of flat space-time by means of the emergence of a weak background field related to a zero-point energy (a vacuum energy), that is associated to a minimum speed ($V$) as an invariant and universal constant for lower energy scales, thus leading to a violation of Lorentz symmetry, since $V$ establishes the existence of a preferential reference frame for the background field (a vacuum
During the last 30 years of Einstein’s life, he attempted to bring the principles of Quantum Mechanics (QM) and Electromagnetism (EM) into his gravity theory or General Relativity (GR) by means of a unified field theory[3]. Unfortunately his effort was not successful in establishing a consistent theory between QM, EM and GR, from where the uncertainty principle should naturally emerge.

Currently string theories inspired by an old idea of Kaluza[4] and Klein[5] regarding extra dimensions in space-time have been prevailing in the scenario of attempts to find a unified theory[6].

In the next section, a new model will be built to describe the electromagnetic nature of the electron mass. It is based on Maxwell theory used for investigating the electromagnetic nature of a photon when the amplitudes of the fields of a certain electromagnetic wave are normalized just for one single photon with energy $\hbar \omega$[7]. Thus, due to reasonings of reciprocity and symmetry that give support to our heuristic approach, we should extend such an alternative model of the photon to be applied to matter (electron), implemented with the idea of pair materialization ($e^-/e^+$), after $\gamma$-photon decay. So we define an electromagnetic mass for the pair $e^-/e^+$ in order to compute the energy of the pair, such that it must be equivalent to its own energy $m_e^2/c^2 = 0.51$MeV.

In section 3, we consider the electron (or positron) moving in the presence of a gravitational potential, so that the electromagnetic field created in the space around it undergoes a very small perturbation due to gravity, thus leading to a slight increase in the electromagnetic energy density. Such an increase occurs by means of a dimensionless coupling constant whose value is infinitesimal, since it has its origin in gravity, the weakest interaction. This coupling constant is called the fine adjustment constant $\xi$, which behaves like a kind of fine structure. Nevertheless, unlike the usual fine structure constant of quantum electrodynamics, the constant $\xi$ plays a coupling role of gravitational and electromagnetic origin rather than simply being an electromagnetic interaction between two electronic charges.

In section 4, we will obtain the value of the tiny dimensionless coupling constant $\xi$, where we find that $\xi = V/c = \sqrt{G/4\pi \epsilon_0} m_e q_e/\hbar c \cong 3.57 \times 10^{-24}$, being $V(\sim 10^{-15}\text{m/s})$ interpreted as a universal minimum speed attainable for any particle at lower energies, and of the same status as the speed of light $c$ (maximum speed) for higher energies, in the sense that both speeds are invariant. Thus, we have a new kind of Doubly Special Relativity (DSR), that was denominated as Symmetrical Special Relativity (SSR)[9][10][11].

II. ELECTROMAGNETIC NATURE OF THE PHOTON AND ELECTRON (OR POSITRON)

A. Electromagnetic nature of the photon

In accordance with some laws of Quantum Electrodynamics introduced by Feynman[7], we may assume the electric field of a plane electromagnetic wave, whose amplitude is normalized for just one single photon[7]. To do this, consider the vector potential of a plane electromagnetic wave, as follows:

$$\vec{A} = a \cos(\omega t - \vec{k} \vec{r}) \vec{e},$$

where $\vec{k} \vec{r} = k z$, admitting that the wave propagates in the direction of $z$, $\vec{e}$ being the unitary vector of polarization. Since we are in the vacuum, we have

$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} = \left(\frac{\hbar a}{c}\right) \sin(\omega t - k z) \vec{e}.$$  

(2)

In the Gaussian System of units, we have $|\vec{E}| = |\vec{B}|$. So the average electromagnetic energy density of this wave shall be

$$\langle \rho_{\text{electromagnetic}} \rangle = \langle \rho_{\text{em}} \rangle = \frac{1}{8\pi} \left( |\vec{E}|^2 + |\vec{B}|^2 \right) = \frac{1}{4\pi} \left( \langle \vec{E}|\vec{B} \rangle \right),$$

(3)

where $|\vec{E}|^2 = |\vec{B}|^2 = |\vec{B}|^2$. 

Inserting Eq.(2) into Eq.(3), we obtain
\[ \langle \rho_{em} \rangle = \frac{1}{8\pi} \frac{w^2 a^2}{c^2}, \] (4)
where \( a \) is an amplitude which depends upon the number of photons.

We wish to obtain the plane wave of one single photon. So, imposing this condition \((\hbar w)\) in Eq.(4) and considering a unitary volume for the photon \((v_{ph} = 1)\), we find
\[ a = \sqrt{\frac{8\pi \hbar c^2}{w}}. \] (5)

Inserting Eq.(5) into Eq.(2), we get
\[ \vec{E}(z, t) = \frac{w}{c} \sqrt{\frac{8\pi \hbar c^2}{w}} \sin(\omega t - k z) \hat{e}, \] (6)
from where we deduce
\[ e_0 = \frac{w}{c} \sqrt{\frac{8\pi \hbar c^2}{w}} = \sqrt{8\pi \hbar w}, \] (7)
where \( e_0 \) could be thought of as an electric field amplitude normalized for one single photon, with \( b_0 = e_0 \) (Gaussian system), the magnetic field amplitude normalized for one single photon. So we may write
\[ \vec{E}(z, t) = e_0 \sin(\omega t - k z) \hat{e}. \] (8)

Inserting Eq.(8) into Eq.(3) and considering the unitary volume \((v_{ph} = 1)\), we find
\[ \langle E_{em} \rangle = \frac{1}{8\pi} e_0^2 \equiv \hbar w. \] (9)

Now, following the classical theory of Maxwell for the electromagnetic wave, let us consider an average quadratic electric field normalized for one single photon, namely \( e_m = e_0/\sqrt{2} = \sqrt{\langle |\vec{E}|^2 \rangle} \) [see Eq.(8)]. So, by doing this, we may write Eq.(9) in the following alternative way:
\[ \langle E_{em} \rangle = \frac{1}{4\pi} e_m^2 \equiv \hbar w, \] (10)
where
\[ e_m = \frac{e_0}{\sqrt{2}} = \frac{w}{c} \sqrt{\frac{4\pi \hbar c^2}{w}} = \sqrt{4\pi \hbar w}. \] (11)

Here it is important to emphasize that although the field given in Eq.(8) is normalized for only one photon, it is still a classical field of Maxwell in the sense that its value oscillates like a classical wave [Eq.(8)]; the only difference here is that we have considered a small amplitude field for just one photon.

Actually, the amplitude of the field \((e_0)\) cannot be measured directly. Only in the classical approximation (macroscopic case) where we have a very large number of photons \((N \to \infty)\), can we somehow measure the macroscopic field \( \vec{E} \) of the wave. Therefore, although we could idealize the case of one single photon as if it
were a Maxwell electromagnetic wave of small amplitude, Eq.(8) is still a classical solution since the field $\vec{E}$ presents oscillation.

On the other hand, we already know that the photon wave is a quantum wave, i.e., it is a de-Broglie wave, where its wavelength ($\lambda = h/p$) is not interpreted classically as the oscillation frequency (wavelength due to oscillation) of a classical field because, if it were so, using the classical solution in Eq.(8), we would have

$$E_{em} = \frac{1}{4\pi} |\vec{E}(z, t)|^2 = \frac{1}{4\pi} e_0^2 \sin^2(wt - kz). \quad (12)$$

If the wave of a photon were really a classical wave, then its energy would not have a fixed value according to Eq.(12). Consequently, its energy $\hbar w$ would be only an average value [see Eq.(10)]. Hence, in order to achieve consistency between the result in Eq.(10) and the quantum wave (de-Broglie wave), we must interpret Eq.(10) as being related to the de-Broglie wave of the photon with a discrete and fixed value of energy $\hbar w$ instead of an average energy value, since we should consider the wave of one single photon as being a non-classical wave, namely a de-Broglie (quantum) wave. Thus we simply rewrite Eq.(10), as follows:

$$E_{em} = E = pc = \frac{hc}{\lambda} = \hbar w \equiv \frac{1}{4\pi} e_{ph}^2, \quad (13)$$

from where we conclude

$$\lambda \equiv \frac{4\pi hc}{e_{ph}^2}, \quad (14)$$

where we replace $e_m$ by $e_{ph}$, $\lambda$ being the de-Broglie wavelength.

According to Eq.(14), the single photon field $e_{ph}$ should not be assumed as a mean value for an oscillating classical (vectorial) field, and so now we shall preserve it in order to be interpreted as a quantum electric field, i.e., a microscopic scalar field of one single photon. Thus, let us also call $e_{ph}$ a scalar electric field to represent the corpuscular (or quantum-mechanical) aspect of the field of one single photon. So, the quantity $e_{ph}$ is taken just as the magnitude (scalar) of its mean electric field ($e_m$), being simply $e_m = e_{ph}$.

As the scalar field $e_{ph}$ is responsible for the energy of the photon ($E \propto e_{ph}^2$), with $w \propto e_{ph}^2$ and $\lambda \propto 1/e_{ph}^2$, we realize that $e_{ph}$ presents quantum behavior since it provides the dual aspect (wave-particle) of the photon, so that its mechanical momentum may be written as $p = \hbar k = 2\pi\hbar/\lambda = \hbar e_{ph}^2/2hc$ [refer to Eq.(14)], or simply $p = e_{ph}^2/4\pi c$.

**B. Electromagnetic nature of the electron mass (or positron)**

Our goal is to extend the idea of photon electromagnetic energy [Eq.(13)] to matter. By doing this, we shall provide heuristic arguments that rely directly on the de-Broglie reciprocity postulate, which has extended the idea of wave (photon wave) to matter (electron), which also behaves like wave. Thus, Eq.(14) for the photon by using its scalar (quantum) field $e_{ph}$, which is based on de-Broglie relation ($p = h/\lambda = e_{ph}^2/4\pi c$), may also be extended to matter (electron) in accordance with the very idea of de-Broglie reciprocity. In order to strengthen such an argument, besides this, we are going to assume the phenomenon of pair formation, where the $\gamma$-photon decays into two charged massive particles, namely the electron ($e^-$) and its anti-particle, the positron ($e^+$).

Such an example will enable us to understand better the need of extending, by means of the idea of reciprocity, the concept of scalar field of a photon and its electromagnetic mass (relativistic energy) [Eq.(13)] also to be introduced into the matter (massive particles like $e^-$ and $e^+$). So, we use the heuristic assumption about scalar electromagnetic fields to simply represent the magnitudes of such fields for matter, such magnitudes being associated with the corpuscular aspect of the fields, so that these magnitudes are now given by scalar quantities related to the mass-energy [see Eq.(19)].

In short, we intend to extend the idea of de-Broglie reciprocity to be applied specifically to electromagnetic fields, and to expand our notion of wave-field-matter so that, whereas the well-known vectorial (classical) fields
represent waves, we introduce the heuristic concept of scalar (quantum) fields that provides the corpuscular aspect for matter. Thus, the electromagnetic field itself should also have an aspect of duality, namely the corpuscular-wave field.

Now consider the phenomenon of pair formation, i.e., $\gamma \rightarrow e^- + e^+$. Taking into account the conservation of energy for $\gamma$-decay, we write the following energy equation:

$$E_{\gamma} = \hbar \nu = m_{\gamma}c^2 = m_0^e c^2 + m_0^+ c^2 + K^- + K^+,$$

(15)

being $m_0^e c^2$ (or $m_0^+ c^2$) the electron (or positron) mass, where $m_0^e c^2 + m_0^+ c^2 = 2m_0 c^2$, since the electron and positron have the same mass $(m_0)$. $K^-$ and $K^+$ represent the kinetic energies of the electron and positron respectively. We have $E_0 = m_0^e c^2 = m_0^+ c^2 \cong 0.51\text{MeV}$.

Since the electromagnetic energy of the $\gamma$-photon is $E_{\gamma} = \hbar \nu = m_{\gamma}c^2 = \frac{1}{4\pi}e^2 \gamma = \frac{1}{4\pi}e\gamma b_\gamma$, or likewise, in SI (International System) of units, we have $E_{\gamma} = \varepsilon_0 e^2_\gamma$, and also knowing that $e_\gamma = c b_\gamma$ (in SI), where $b_\gamma$ is the magnetic scalar field of the $\gamma$-photon, we may also write

$$E_{\gamma} = c \varepsilon_0 (e_\gamma)(b_\gamma).$$

(16)

The photon has no charge, however, when the $\gamma$-photon is materialized into an electron-positron pair, its electromagnetic content (the scalar fields $e_\gamma$ and $b_\gamma$) given in Eq.(16) ceases to be free or purely kinetic (pure relativistic mass) to become massive due to the materialization of the pair. Since such massive particles ($u_{(+,-)} < c$) also behave like waves in accordance with the de-Broglie idea, it would be natural to extend Eq.(14) of the photon to represent now the wavelengths of matter (electron or positron) after $\gamma$-decay, namely:

$$\lambda_{(+,-)} \propto \frac{hc}{\varepsilon_0 e^{(+,-)}_e^2} = \frac{h}{\varepsilon_0 e^{(+,-)}_e [b^{(+,-)}_s]},$$

(17)

where the fields $e^{(+,-)}_s$ and $b^{(+,-)}_s$ play the role of the electromagnetic content (scalar electromagnetic fields) that provides the total mass (energy) of the particle (electron or positron), its mass being essentially of electromagnetic origin, as follows:

$$m \equiv m_{em} \propto e_s b_s,$$

(18)

where $E = mc^2 \equiv m_{em} c^2$.

Using Eq.(16) and Eq.(17) as a basis, we may write Eq.(15) in the following way:

$$E_{\gamma} = c c_0 e_\gamma b_\gamma = c c_0 e_\zeta b_\zeta^+ v^+_e + c c_0 e_\xi b_\xi^+ v^+_e,$$

(19)

with $v_e$ being the volume of the electron (or positron), where $c c_0 e_\zeta b_\zeta^+ v^-_e = (c c_0 e^-_0 b_\zeta^+ b_\xi^+ v^+_e + K^-) = (m_0^- c^2 + K^-)$ and $c c_0 e_\xi b_\xi^+ v^+_e = (c c_0 e^0 b_\zeta^0 b_\zeta v^+ + K^+) = (m_0^+ c^2 + K^+)$. The quantities $e^{(+,-)}_{\zeta 0}$ and $b^{(+,-)}_{\zeta 0}$ represent the scalar electromagnetic fields that provide the mass ($m_0$) or energy ($E_0$) of the electron or positron.

A fundamental point which the present heuristic model challenges is that, in accordance with Eq.(19), we realize that the electron is not necessarily an exact point-like particle. Quantum Electrodynamics, based on Special Relativity (SR), deals with the electron as a point-like particle. The well-known classical theory of the electron foresees for the electron radius the same order of magnitude as the proton radius, i.e., $R_e \sim 10^{-15}m$.

Some experimental evidences of the scattering of electrons by electrons at very high kinetic energies, indicate that the electron can be considered approximately as a point-like particle. Actually, electrons have an extent smaller than collision distance, which is about $R_e \sim 10^{-16}m$. Of course, such an extent is negligible in comparison to the dimensions of an atom ($\sim 10^{-10}m$) or even the dimensions of a nucleus ($\sim 10^{-14}m$), but the
extent is not exactly a point. For this reason, the present model can provide a very small non-null volume \( v_e \) of the electron. But, if we just consider \( v_e = 0 \) according to Eq.(19), we would have an absurd result, i.e., divergent scalar fields (\( e_{s0} = b_{s0} \rightarrow \infty \)). However, for instance, if we can consider \( R_e \sim 10^{-16} \text{m} \) (\( v_e \propto R_e^3 \sim 10^{-48} \text{m}^3 \)) in our model, we know that \( m_0 c^2 \simeq 0.51 \text{MeV}(\sim 10^{-13} \text{J}) \); hence, in this case [see Eq.(19)], we would obtain \( e_{s0} \sim 10^{23} \text{V/m} \). This value is extremely high and therefore we may conclude that the electron is extraordinarily compact, having a high mass (energy) density. If we imagine over the “surface” of the electron or even inside it, we would detect a constant and finite scalar field \( e_{s0} \sim 10^{23} \text{V/m} \) instead of a divergent value. So, according to the present model, the quantum scalar field \( e_{s0} \) inside the non-classical electron that is almost point-like with a small radius (\( \sim 10^{-16} \text{m} \)), is finite and constant (\( \sim 10^{23} \text{V/m} \)) instead of a function like \( 1/r^2 \) with divergent behavior. Of course, for \( r > R_e \), we have the external vectorial (classical) field \( \vec{E} \), decreasing with \( 1/r^2 \) (see Fig.1).

The next section will be dedicated to the investigation of the electron (or positron) coupled to a gravitational field according to the present model.

III. ELECTRON (OR POSITRON) COUPLED TO GRAVITY

A. Photon in a gravitational potential

When a photon with energy \( h \nu \) is subjected to a gravitational potential \( \phi \), its energy \( E \) and frequency \( \nu \) increase to \( E' = h \nu' \), being

\[
E' = h \nu' = h \nu \left(1 + \frac{\phi}{c^2}\right).
\]

As, by convention, we have defined \( \phi > 0 \) for an attractive potential, we get \( \nu' > \nu \). Considering Eq.(16) given for any photon and inserting Eq.(16) into Eq.(20), we write

\[
E' = c \epsilon_0 e_{ph}' b_{ph}' = c \epsilon_0 e_{ph} b_{ph} \sqrt{g_{00}},
\]

where \( g_{00} \) is the first component of the metric tensor, being \( \sqrt{g_{00}} = \left(1 + \frac{\phi}{c^2}\right) \) and \( e_{ph} = c b_{ph} \).

From Eq.(21), we can extract the following relations, namely:

\[
e_{ph}' = e_{ph} \sqrt{g_{00}}, \quad b_{ph}' = b_{ph} \sqrt{g_{00}}.
\]

Due to the presence of gravity, the scalar fields \( e_{ph} \) and \( b_{ph} \) of the photon increase according to Eq.(22), leading to the increase of the photon frequency or energy according to Eq.(20). Thus, we may think about the increments of scalar fields in the presence of gravity, namely:

\[
\Delta e_{ph} = e_{ph}' - e_{ph}, \quad \Delta b_{ph} = b_{ph}' - b_{ph}.
\]

B. Electron (or positron) in a gravitational potential

When a massive particle with mass \( m_0 \) moves in a gravitational potential \( \phi \), its total energy \( E \) is

\[
E = mc^2 = m_0 c^2 \sqrt{g_{00}} + K,
\]

\[
K = m_0 \frac{\phi c^2}{2} \ln \left(1 + \frac{\phi c^2}{2m_0 c^2}\right).
\]
where we can think that \( m_0(= m'_0) \) represents the mass of the electron (or positron) emerging from \( \gamma \)-photon decay in the presence of a gravitational potential \( \phi \).

In order to facilitate the understanding of what we are proposing, let us consider \( K << m_0c^2 \) \((v << c)\) since we are interested only in the influence of the potential \( \phi \). Therefore, we simply write

\[
E = m_0c^2\sqrt{g_{00}}. \tag{25}
\]

Since we already know that \( E_0 = m_0^{(+,-)}c^2 = cc_0e_0^{(+,-)}b_0^{(+,-)}v_e \), we can also write the total energy \( E \) as follows:

\[
E = cc_0e_0^{(+,-)}b_0^{(+,-)}v_e = cc_0e_0^{(+,-)}b_0^{(+,-)}v_e\sqrt{g_{00}}, \tag{26}
\]

from where, we get

\[
e_s^{(+,-)} = e_s^{(+,-)}\sqrt{g_{00}}, \quad b_s^{(+,-)} = b_s^{(+,-)}\sqrt{g_{00}}, \tag{27}
\]

analogously to Eq.(22), since Eq.(27) is an extension of Eq.(22) \((\text{photon})\) for matter \((\text{electron-positron})\) in a gravitational potential.

So we obtain the following increments:

\[
\Delta e_s = e_s^{(+,-)}(\sqrt{g_{00}} - 1), \quad \Delta b_s = b_s^{(+,-)}(\sqrt{g_{00}} - 1), \tag{28}
\]

where \( \Delta e_s = c\Delta b_s \).

As the energy of the particle \((\text{electron or positron})\) can be obtained from the scalar electromagnetic fields with magnitudes \( e_s \) and \( b_s \) that undergo an increase in the presence of gravity, this heuristic model is capable of assisting us to imagine that the external fields \( \vec{E} \) and \( \vec{B} \) created by the moving charge, by storing an energy density \((\propto |\vec{E}|^2 + |\vec{B}|^2)\), should also undergo perturbations (shifts) due to the presence of gravity (Fig.1).

We know that any kind of energy is also a source of a gravitational field. This non-linearity that is inherent in a gravitational field leads us to think that the classical \((\text{external})\) fields \( \vec{E} \) and \( \vec{B} \) should undergo tiny shifts like \( \delta \vec{E} \) and \( \delta \vec{B} \) in the presence of a gravitational potential \( \phi \). As such small shifts have positive magnitudes, having the same direction of \( \vec{E} \) and \( \vec{B} \), they should lead to a slight increase of the electromagnetic energy density around the particle. And, since the internal energy of the particle \((\text{scalar fields})\) also increases in the presence of \( \phi \) according to Eq.(26), we expect that the magnitudes of the external shifts \( \delta \vec{E} \) and \( \delta \vec{B} \) should be proportional to the increments of the internal \((\text{scalar})\) fields of the particle \((\Delta e_s \text{ and } \Delta b_s)\), as follows:

\[
\delta E \propto \Delta e_s[= (e_s - e_{s0})], \quad \delta B \propto \Delta b_s[= (b_s - b_{s0})], \tag{29}
\]

being \( \delta E = \delta E(\phi) = (E' - E) > 0 \) and \( \delta B = \delta B(\phi) = (B' - B) > 0 \), where \( \phi \) is the gravitational potential. Here we have omitted the signs \((+, -)\) just for the purpose of simplifying the notation.

In accordance with Eq.(29), we may conclude that there is a constant of proportionality that couples the external electromagnetic fields \( \vec{E} \) and \( \vec{B} \) of the moving particle \((\text{electron})\) with gravity by means of the external small shifts \( \delta \vec{E} \) and \( \delta \vec{B} \) proportional to the internal increments \( \Delta e_s \text{ and } \Delta b_s \). So we write Eq.(29), as follows:

\[
\delta \vec{E} = \vec{\epsilon}\xi \Delta e_s, \quad \delta \vec{B} = \vec{\epsilon}\xi \Delta b_s, \tag{30}
\]

where \( \vec{\epsilon} \) is the unitary vector given in the same direction of \( \vec{E} \) \((\text{or } \vec{B})\). So the small shift \( \delta \vec{E} \text{ (or } \delta \vec{B}) \) has the same direction as \( \vec{E} \text{ (or } \vec{B}) \) (Fig.1).
Electric energy density around the charge increases in the presence of quantum fields of the electron, i.e., they represent the magnitudes of internal fields, where $\xi$ being a tiny dimensionless coupling constant of gravitational origin. The fields $\delta B$ and $\delta E$ depend only on gravitational potential ($g_{00}$) on the electron (Fig.1).

From Eq.(30), it is easy to conclude that the coupling $\xi$ is a dimensionless proportionality constant. We expect that $\xi << 1$ due to the fact that the gravitational interaction is much weaker than the electromagnetic one. The external shifts $\delta E$ and $\delta B$ depend only on gravitational potential ($g_{00}$) on the electron (Fig.1).

Inserting Eq.(28) into Eq.(30), we obtain

$$\delta E = \xi e_s(\sqrt{g_{00}} - 1), \quad \delta B = \xi b_s(\sqrt{g_{00}} - 1).$$

Due to the tiny positive shifts with magnitudes $\delta E$ and $\delta B$ in the presence of a gravitational potential $\phi$, the total electromagnetic energy density in the space around the charged particle (electron or positron) is slightly increased, as follows:

$$\rho_{em}^{total} = \frac{1}{2} \epsilon_0[E + \delta E(\phi)]^2 + \frac{1}{2\mu_0}[B + \delta B(\phi)]^2. \tag{32}$$

Inserting the magnitudes $\delta E$ and $\delta B$ from Eq.(31) into Eq.(32) and performing the calculations, we finally obtain the total electromagnetic energy density in view of the very weak coupling with gravity, namely:

$$\rho_{em}^{total} = \frac{1}{2} \left[ \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right] + \xi \left[ \epsilon_0 E e_s + \frac{1}{\mu_0} B b_s \right] \left( \sqrt{g_{00}} - 1 \right) + \frac{1}{2} \xi^2 \left[ \epsilon_0 (e_s)^2 + \frac{1}{\mu_0} (b_s)^2 \right] \left( \sqrt{g_{00}} - 1 \right)^2. \tag{33}$$

We may assume that $\rho_{em}^{total} = \rho_{em}^{(0)} + \rho_{em}^{(1)} + \rho_{em}^{(2)}$ to represent Eq.(33), where $\rho_{em}^{(0)}$ is the free electromagnetic energy density (of zero order) for the ideal case of a charged particle uncoupled to gravity ($\xi = 0$), i.e., the ideal case of a free charge. We have $\rho_{em}^{(0)} \propto 1/r^3$ (coulombian term).

The coupling term $\rho_{em}^{(1)}$ represents an electromagnetic energy density of first order, since it contains a dependence on $\delta E$ and $\delta B$, i.e., it is proportional to $\delta E$ and $\delta B$ due to the influence of gravity. Therefore, it is a mixed term that behaves essentially like a radiation term to represent a density of radiant energy. So we find $\rho_{em}^{(1)} \propto 1/r^2$ as we have $E \propto 1/r^2$ and $e_s \propto b_s \propto 1/r^2 \sim constant$. It is interesting to notice that this radiation term has its origin in the non-inertial aspect of gravity that couples weakly with the electromagnetic field of the moving charge, since, according to classical electromagnetic theory, it would already be expected that such a coupling term, acting like a radiation field ($\propto 1/r$), comes from an accelerated charge. In fact, the term $\rho_{em}^{(1)}$ reflects a quite weak non-inertial effect of the charged particle, since it is weakly coupled to gravity.
The last coupling term ($\rho_{\text{em}}^{(2)}$) is purely interactive due to the presence of gravity only, namely it is a 2nd order interactive electromagnetic energy density term, since it is proportional to $(\delta E)^2$ and $(\delta B)^2$. And, as $e_a b \propto 1/r^0 \sim \text{constant}$, we find $\rho_{\text{em}}^{(2)} \propto 1/r^0 \sim \text{constant}$, where we can also write $\rho_{\text{em}}^{(2)} = \frac{1}{2} \varepsilon_0 (\delta E)^2 + \frac{1}{2 \mu_0} (\delta B)^2$, which depends only on the gravitational potential ($\phi$) [see Eq.(31)].

As we have $\rho_{\text{em}}^{(2)} \propto 1/r^0$, this term has non-locality behavior. Non-locality behavior means that $\rho_{\text{em}}^{(2)}$ behaves like a kind of non-local field inherent to space (a background field). This term $\rho_{\text{em}}^{(2)}$ is purely of gravitational origin. It does not depend on the distance $r$ from the charged particle. Therefore $\rho_{\text{em}}^{(2)}$ is a uniform energy density for a given gravitational potential $\phi$ on the particle.

In reality, we generally have $\rho_{\text{em}}^{(0)} >\rho_{\text{em}}^{(1)} >\rho_{\text{em}}^{(2)}$. For a weak gravitational field, we can make a good practical approximation as $\rho_{\text{em}}^{\text{total}} \approx \rho_{\text{em}}^{(0)}$; however, from a fundamental viewpoint, we cannot neglect the coupling terms $\rho_{\text{em}}^{(1)}$ and $\rho_{\text{em}}^{(2)}$, especially the latter ($\rho_{\text{em}}^{(2)}$) for large distances, since $\rho_{\text{em}}^{(2)}(\propto 1/r^0 \sim \text{constant})$ has a vital importance in this work, allowing us to understand the constant energy density of the background field.

The term $\rho_{\text{em}}^{(2)}$ does not have $r$-dependence, since $e_a b \propto b_a \sim \text{constant}$, and thus still remains constant when $r \to \infty$. That is the reason why it represents a uniform density of background energy (vacuum energy).

The last term $\rho_{\text{em}}^{(2)}$ has profound implications in our understanding of space-time structure at very large scales of length. In a previous paper\[9], we had the opportunity to investigate the implications of such a uniform energy density $\rho_{\text{em}}^{(2)}$ in cosmology, i.e., a vacuum energy density of a background field that breaks down Lorentz symmetry and plays the role of a cosmological constant.

In the next section, we will estimate the minuscule value of the coupling constant $\xi$, whose value depends on fundamental constants like $G$, $c$, $h$, $q_e$ and $m_e$.

IV. THE FINE ADJUSTMENT CONSTANT $\xi$ AND SPACE-TIME WITH AN INVARIANT MINIMUM SPEED $v$ THAT BREAKS DOWN LORENTZ SYMMETRY

A. The fine structure constant $\alpha$ and the minuscule coupling constant $\xi$

Let us begin this section by considering the well-known problem that deals with the electron in the bound state of a coulombian potential of a proton (Hydrogen atom). We have started from this issue because it poses an important analogy with the present model of the electron coupled to a gravitational field.

We know that the fine structure constant ($\alpha \approx 1/137$) plays an important role for obtaining the energy levels that bind the electron to the nucleus (proton) in the Hydrogen atom. Therefore, similar to this idea, we plan to extend it in order to see that the fine coupling constant $\xi$ plays an even more fundamental role than the fine structure $\alpha$, since the constant $\xi$ couples gravity with the electromagnetic field, breaking down Lorentz symmetry as will be seen later.

We notice that the electron spin is not considered here. Thus, in this model, the magnetic field of the electron has its origin only in its translational motion.

Let’s initially consider the energy that binds the electron to the proton in the fundamental state of the Hydrogen atom, as follows:

$$\Delta E = \frac{1}{2} \alpha^2 m_0 c^2,$$  \hspace{1cm} (34)

where $\Delta E$ is assumed as module. We have $\Delta E \ll m_0 c^2$, where $m_0 (= m_e)$ is the electron mass.

We have $\alpha = e^2/hc = q_e^2/4\pi \varepsilon_0 hc \approx 1/137$ (fine structure constant). Since $m_e c^2 = m_0 c^2 \approx 0.51$ MeV, from Eq.(34) we get $\Delta E \approx 13.6$ eV.

As we already know that $E_0 = m_0 c^2 = \varepsilon_0 e_b b_a v_e$, we may write Eq.(34) in the following alternative way:

$$\Delta E = \frac{1}{2} \alpha^2 e\varepsilon_0 e_b b_a v_e = \frac{1}{2} \varepsilon_0 (\Delta e_a) (\Delta b_a) v_e,$$  \hspace{1cm} (35)

from where we extract
\[ \Delta e_s \equiv \alpha e_{s0} = \frac{e^2}{\hbar c} e_{s0}; \quad \Delta b_s \equiv \alpha b_{s0} = \frac{e^2}{\hbar c} b_{s0}. \quad (36) \]

It is interesting to observe that Eq.(36) maintains a certain analogy with Eq.(30); however, first of all we must emphasize that the variations (increments) \( \Delta e_s \) and \( \Delta b_s \) of the electron scalar fields, given in Eq.(36), are of purely coulombian origin, since the fine structure constant \( \alpha \) depends solely on the electron charge. Thus, we could express the electric force between two electrons in the following way:

\[ F_e = \frac{e^2}{r^2} = \frac{q_e^2}{4\pi \epsilon_0 r^2} = \frac{\alpha hc}{r^2}. \quad (37) \]

If we now consider a gravitational interaction between two electrons, thus, in a similar way to Eq.(37), we have

\[ F_g = G \frac{m_e^2}{r^2} = \frac{\beta hc}{r^2}, \quad (38) \]

where we extract

\[ \beta = G \frac{m_e^2}{hc}. \quad (39) \]

We have \( \beta \ll \alpha \) due to the fact that the gravitational interaction is very weak when compared to the electrical interaction, so that \( F_g/F_e = \beta/\alpha \sim 10^{-42} \), where \( \beta \equiv 1.75 \times 10^{-45} \). Therefore, we shall denominate \( \beta \) as the superfine structure constant, since the gravitational interaction creates a bonding energy extremely smaller than the coulombian interaction given for the fundamental state (\( \Delta E \)) of the Hydrogen atom.

To sum up, we say that, whereas \( \alpha (e^2) \) provides the adjustment for the coulombian bonding energies between two electronic charges, \( \beta (m_e^2) \) gives the adjustment for the gravitational bonding energies between two electronic masses. Such bonding energies of electrical or gravitational origin lead to an increment of the energy of the particle by means of variations like \( \Delta e_s \) and \( \Delta b_s \).

Following the above reasoning, we realize that the present model enables us to consider \( \xi \) as the fine tuning constant (coupling) between a gravitational potential created by the electron mass \( m_e \) and the electrical field (electrical energy density) created by another charge \( q_e \) at a certain distance. Hence, in this more fundamental case, we have a kind of bind of the type “\( m_e q_e \)” (mass-charge) represented by the tiny coupling constant \( \xi \).

The way we have followed in obtaining \( \xi \) starts from important analogies with the ideas of the fine structure constant \( \alpha = \alpha (e^2) \) (electromagnetic interaction) and the superfine structure constant \( \beta = \beta (m_e^2) \) (gravitational interaction), so that it is easy to conclude that the kind of mixing coupling we are proposing here, of the type “\( m_e q_e \)”, plays the role of the gravi-electrical coupling constant \( \xi \), namely \( \xi \) is of the form \( \xi = \xi (m_e q_e) \). So, the only way to satisfy this condition is to write

\[ \xi = \sqrt{\alpha \beta}. \quad (40) \]

As we have \( \alpha \) and \( \beta \) [Eq.(39)], from Eq.(40) we finally obtain

\[ \xi = \sqrt{\frac{G}{4\pi \epsilon_0}} \frac{m_e q_e}{\hbar c} = \frac{\sqrt{G} m_e}{\hbar c}, \quad (41) \]

where, indeed, we verify the previous condition, i.e., \( \xi \propto m_e q_e \). So, from Eq.(41), we find \( \xi \approx 3.57 \times 10^{-24} \). Let us denominate \( \xi \) as the fine adjustment constant. We have \( e = q_e/\sqrt{4\pi \epsilon_0} \). The quantity \( \sqrt{G} m_e \) can be thought of as if it were a gravitational charge \( e_g \), i.e., \( e_g = \sqrt{G} m_e \), such that we write \( \xi = e_g e/\hbar c \) and functions like a minuscule fine tuning constant of gravi-electromagnetic origin.
Finally, from Eq.(41), we rewrite Eq.(33) as follows:

$$
\rho = \frac{1}{2} \left[ \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right] + \frac{e_0 e}{\hbar c} \left[ \epsilon_0 E e_0 + \frac{1}{\mu_0} B b_0 \right] \left( \sqrt{g_{00}} - 1 \right) + \frac{\epsilon_0 e^2}{\hbar^2 c^2} \left[ \epsilon_0 (e_0)^2 + \frac{1}{\mu_0} (b_0)^2 \right] \left( \sqrt{g_{00}} - 1 \right)^2. \quad (42)
$$

In the Hydrogen atom, we have the fine structure constant \( \alpha = e^2/\hbar c = v_B/c \), where \( v_B = e^2/\hbar \cong c/137 \). This is the speed of the electron at the fundamental atomic level (Bohr velocity). At this level, the electron does not radiate because it is in a kind of balanced state in spite of its electrostatic interaction with the nucleus (centripetal force), that is to say, it acts as if it were effectively an inertial system. With analogous reasoning applied to the case of the electron coupled to a gravitational field, i.e., the gravi-electrical coupling constant \( \xi \), we may write Eq.(41), as follows:

$$
\xi = \frac{V}{c}, \quad (43)
$$

from where we get a very low speed, namely:

$$
V = \sqrt{\frac{G m_e q_e}{4\pi\epsilon_0 \hbar}} = \frac{e_0 e}{\hbar}, \quad (44)
$$

being \( V \cong 1.071 \times 10^{-15} \text{m/s} \). This seems to be a fundamental constant of Nature with the same status of the speed of light \( c = e^2/\alpha\hbar \); however \( V \) plays the role of an invariant minimum speed.

Similarly to the Bohr velocity \( v_B(= \alpha c = e^2/\hbar) \), the speed \( V(= \xi c = e_0 e/\hbar) \) is also a universal constant; however the crucial difference between \( v_B \) and \( V \) is that the minimum speed \( V \) should be related to the most fundamental bound state (the universal background energy), since gravity \( (G) \), the weakest interaction, comes into play now.

In the previous papers \[9\][10][11], we have postulated \( V \) as an unattainable universal minimum speed for all particles in the subatomic world, but before this, we have provided a better justification of why we should consider electron mass and its charge to calculate the universal minimum speed \( V \) \( (V \propto m_e q_e) \), instead of the masses and charges of other particles. Although there are fractional electrical charges such as is the case of quarks, their charges are not free in Nature. They are strongly bonded by the strong force (gluons). Therefore, the charge of the electron is the smallest free charge in Nature. On the other hand, the electron is the elementary charged particle with the smallest mass. Consequently, the product \( m_e q_e \) assumes a minimum value. In addition, the electron is completely stable. Other charged particles such as for instance \( \pi^+ \) and \( \pi^- \) have masses that are much greater than the electron mass and so they are unstable, decaying very quickly. Although neutrinos have masses much smaller than the electron mass, they cannot be considered here, since they do not have a charge, and so they do not interact with electromagnetic fields.

Here we should emphasize that the present model does not take into account nuclear interactions, since, according to this new approach, we are working only with the electromagnetic nature of the electron coupled to gravity.

In summary, we have shown that there is a tiny dimensionless coupling constant \( \xi = V/c \sim 10^{-24} \) that couples gravity with electromagnetic fields, where \( V(\sim 10^{-15} \text{m/s}) \) is a universal minimum speed related to a preferred background frame, that is associated to a weak and uniform background field (a vacuum energy), and which has been investigated in previous works \[9\][10][11]. So, we were led to build a deformed relativity with two invariant speeds \( (c \text{ and } V) \) by breaking down Lorentz symmetry.

**B. Flat space-time with a uniform low energy density of background field and minimum speed \( V \)**

As the last term in Eq.(33) \( (\rho_{em}^{(2)}) \) does not depend on the distance from the particle, \( \rho_{em}^{(2)} \) represents a uniform background field energy density. Alternatively, we can write this term as follows:
\[ \rho_{em}^{(2)} = \frac{V^2}{2c^2} \left[ \epsilon_0 (c \alpha)^2 + \frac{1}{\mu_0} (b \alpha)^2 \right] \left( \sqrt{\gamma_{00}} - 1 \right)^2, \]

where \( V/c = \xi (\sim 10^{-24}) \) according to Eq.(43). \( V (\sim 10^{-15}\text{m/s}) \) is the minimum speed. Thus, we conclude that the background energy density is extremely low, since, from Eq.(43), we get \( \xi^2 = V^2/c^2 \sim 10^{-47} \), unless the gravitational potential is so high that it overcomes the extremely weak effects of the tiny coupling squared \( (\xi^2) \). Since the value of \( \xi^2 \) is almost negligible, however, this overcoming does not happen for a large range of gravitational potential, including high potentials.

In a perfectly flat space-time of Special Relativity (SR), we would have \( g_{00} = 1 (\phi = 0) \) and, thus, we would get \( \rho_{em}^{(2)} = 0 \); however, in physical reality, the existence of a background field, even if it is so weak or almost negligible, should be taken into account, since there is no exact zero gravity anywhere.

The minuscule gravitational effects that arise by means of a background field related to a vacuum energy density \( \rho_{em}^{(2)} \), in quasi-flat space-time, lead to a very small zero-point energy at cosmological scales, since we should remember that our universe has quasi-flat space-time with a very low vacuum energy density.[1][2][3][4][5]. Such a small zero-point energy of gravitational origin, i.e., a vacuum energy that fills the whole universe, leads to the existence of the universal minimum speed \( V \), which prevents the space-time from being perfectly flat. This effect is still consistent with the fact that the minimum speed \( V \) has direct dependence on the gravitational constant in accordance with Eq.(44), where \( V \propto \sqrt{\gamma} \).

The background field energy density \( \rho_{em}^{(2)} \) brings a new aspect of gravitation that needs to be thoroughly investigated. This new feature brings with it a change in our notion of space-time and, thus, also the concept of inertial frames (galilean frameworks) with regard to the motion of subatomic particles.[11].

Our attention here must be given to the existence of the minimum speed \( V \) of gravitational origin, which produces a symmetrization of space-time together with the speed of light \( c \) of electromagnetic origin, in the sense that, while \( c \) is the upper limit of speed (photon), unattainable by any massive particle, \( V \) is the lowest limit of speed, being unattainable and, hence, invariant; however, the point is that no particle is found at \( V \), because this speed is associated with a privileged reference frame coinciding with a cosmic background frame, which is inaccessible and presents non-locality.[11].

It is very curious to notice that the idea of a universal background field was sought in vain by Einstein[10] during the last four decades of his life. Einstein has coined the term ultra-referential as the fundamental aspect of space to represent a universal background field,[17]. although he had abandoned the older concept of ether in 1905. Inspired by this new concept of space, let us simply call ultra-referential \( S_V \) the universal background field \( (\rho_{em}^{(2)}) \) of the preferred reference frame associated to \( V \).

The dynamics of particles in the presence of the universal background reference frame associated to \( V \) is within a context of the ideas of Scamah[18], Schrödinger[19] and Mach[20], where there should be an “absolute” inertial reference frame in relation to which we have the speeds and the inertia of all bodies. However, we must emphasize that the approach used here is not classical like Mach’s idea, since the lowest (unattainable) limit of speed \( V \) plays the role of a privileged reference frame of background field instead of the “inertial” frame of fixed stars. Thus, in this space-time, all the speeds \( (v) \) are given in relation to the ultra-referential \( S_V \) as the absolute background frame, where we have the interval of speeds with \( V < v < c \).

The doubly special relativity with an invariant minimum speed \( V \) emerging from space-time with a very low vacuum energy density \( (\rho_{em}^{(2)}) \) was denominated Symmetrical Special Relativity (SSR)[9][10][11], which is a new kind of Deformed Special Relativity (DSR) with two invariant scales of speed \( (c \text{ and } V) \). Here it is important to mention that DSR-theory was first proposed by Camelia[21][22][23][24], which contains two invariant scales, namely the speed of light \( c \) and the minimum length scale (Planck length \( l_P \)). An alternate approach to DSR-theory, inspired by that of Camelia, was proposed later by Smolin and Magueijo[25][26][27]. There is also another extension of SR, which is known as triple special relativity, being characterized by three invariant scales, namely the speed of light \( c \), a mass \( k \) and a length \( R \). Still another generalization of SR is the quantizing of speeds[28], where the Barrett–Crane spin foam model for quantum gravity with a positive cosmological constant was used to aid the authors search for a discrete spectrum of velocities and the physical implications of this effect, namely an effective deformed Poincaré symmetry.
C. Coordinate and velocity transformations in space-time with the ultra-referential $S_V$

The classical notion we have of inertial (galilean) reference frames, where the system at rest exists, is eliminated in SSR, where $v > V$ (Fig.2). However, if we consider classical systems composed of macroscopic bodies, the minimum speed $V$ is naturally neglected ($V = 0$) and, thus, we can reach a vanishing velocity ($v = 0$), i.e., in the classical approximation ($V \to 0$), the ultra-referential $S_V$ (background frame) is eliminated and simply replaced by the galilean reference frame $S$ connected to a system at rest. So, before we deal with the implications of the ultra-referential $S_V$, let us make a brief presentation of the meaning of the galilean reference frame (reference space), well-known in SR-theory. In accordance with SR, when an observer assumes an infinite number of points at rest in relation to himself, he introduces his own reference space $S$. Thus, for another observer $S'$ who is moving with a speed $v$ in relation to $S$, there should also exist an infinite number of points at rest in $S'$’s reference space. Therefore, for observer $S'$, the reference space $S$ is not standing still and has its points moving with a speed $-v$. It is for this reason that there is no privileged galilean reference frame at absolute rest according to the principle of relativity, since the reference space at rest of a given observer becomes motion to another one.

The absolute space of pre-Einsteinian physics, connected to the ether in the old sense, also constituted by itself a reference space. Such a space was assumed to be a privileged reference space of the absolute rest. However, since it was also essentially a galilean reference space like any other, comprised of a set of points at rest, actually it was also subjected to the notion of movement. So, the idea of relative movement could also be applied to “absolute space” when, for example, we assume an observer on Earth moving with a speed $v$ in relation to such a space. In this case, for an observer at rest on Earth, the points that would constitute the newtonian absolute space of reference would be moving with a speed of $-v$. Since the absolute space was connected to the old ether, the Earth-bound observer should detect a flow of ether $-v$; however, the Michelson-Morley experiment has not substantiated the luminiferous ether, that was denied by Einstein.

Since we cannot consider a reference system made up of a set of infinite points at rest in quantum space-time with an invariant minimum speed, then we should define a new status of referentials, namely a non-galilean reference system, which is given essentially as a set of all the particles having the same state of movement (speed $v$) with respect to the ultra-referential $S_V$ (preferred reference frame of the background field), so that $v > V$, $V$ being unapproachable and connected to $S_V$. So, a set of particles with the same speed $v$ with respect to the ultra-referential $S_V$ provides a given non-galilean framework. Hence, SSR should contain three postulates, namely:

1)-the non-equivalence (asymmetry) of the non-galilean reference frames due to the presence of the background frame $S_V$ that breaks down Lorentz symmetry, i.e., we cannot exchange $v$ for $-v$ by means of inverse transformations, since we cannot achieve a rest state ($v = 0$) for a certain non-galilean reference frame in order to reverse the direction of its velocity;

2)-the invariance of the speed of light ($c$);

3)-the covariance of the ultra-referential $S_V$ (background framework) connected to an invariant and unattainable minimum limit of speed $V$, i.e., all the non-galilean reference frames with speeds $V < v \leq c$ experience the same background frame $S_V$, in the sense that the background energy (vacuum energy) at $S_V$ does not produce a flow $-v$ at any of these referentials. Thus $S_V$ does not work like the newtonian absolute space filled by luminiferous (galilean) ether in the old (classical) sense, in spite of $S_V$’s being linked to the background energy ($\rho^{(2)}_{em}$) that works like a non-galilean “ether” of gravitational origin.

Of course if we consider $V = 0$, we recover the well-known two postulates of SR, i.e., the equivalence of inertial reference frames, where one can exchange $v$ for $-v$ with appropriate transformations and, consequently, this leads to the absence of such a background field ($S_V$), but the constancy of the speed of light is preserved.

Let us assume a reference frame $S'(x', y', z', t')$ with a speed $v$ in relation to the ultra-referential $S_V(X, Y, Z, t)$, according to Fig.2.

According to Fig.2, we consider the motion in one spatial dimension, namely $(1+1)D$ space-time with background field-$S_V$. So we write the following transformations for $(1+1)D$ in differential form:

$$dx' = \Psi(dX - \beta_c dt) = \Psi(dX - vdt + Vdt) = \Psi(dX - vdt + \xi dt), \quad (46)$$
where $V = \xi c$ [Eq.(43)]; $\beta_* = \beta \epsilon = \beta (1 - \alpha)$, being $\beta = v/c$ and $\alpha = V/v$, so that $\beta_* \to 0$ for $v \to V$ or $\alpha \to 1$.

$$dt' = \Psi \left( dt - \beta \epsilon dX/c \right) = \Psi \left( dt - v \epsilon dX/c^2 + V dX/c^2 \right) = \Psi \left( dt - v \epsilon dX/c^2 + \xi \epsilon dX/c \right),$$

being $\bar{v} = v \epsilon x$. We have $\Psi = \sqrt{1 - \epsilon^2}$. If we make $V \to 0$ ($\alpha \to 0$), we recover Lorentz transformations.

The general transformations for $(3 + 1)D$ space-time in SSR were treated in a previous work[10].

In order to get the transformations of Eq.(46) and Eq.(47), let us consider the following more general transformations: $dx' = \theta \gamma (dX - \epsilon_1 v \epsilon dt)$ and $dt' = \theta \gamma (dt - \frac{2 \epsilon_2 vdX}{c^2})$, where $\theta$, $\epsilon_1$ and $\epsilon_2$ are factors (functions) to be determined. We hope all these factors depend on $\alpha$, such that, for $\alpha \to 0$ ($V \to 0$), we recover Lorentz transformations as a particular case ($\theta = 1$, $\epsilon_1 = 1$ and $\epsilon_2 = 1$). By using those transformations to perform $[c^2 dt'^2 - dx'^2]$, we find the identity: $[c^2 dt'^2 - dx'^2] = \theta^2 \gamma^2 [c^2 dt^2 - 2 \epsilon_1 v \epsilon dt dX + 2 \epsilon_2 \epsilon v \epsilon dt dX - \epsilon_1^2 v^2 dt^2 + \frac{\epsilon_2^2 \gamma^2 dX^2}{c^2} - dX^2]$.

The metric tensor of such space-time with a uniform and very low energy density remains in a diagonal form. Thus, the crossed terms must vanish and so we are assured that $\epsilon_1 = \epsilon_2 = \epsilon$. Due to this fact, the crossed terms $(2 \epsilon_1 \epsilon_2 v \epsilon dt dX)$ cancel themselves out and finally we obtain $[c^2 dt'^2 - dx'^2] = \theta^2 \gamma^2 (1 - \frac{\epsilon^2}{c^2}) [c^2 dt^2 - dX^2]$. For $\alpha \to 0$ ($\epsilon = 1$ and $\theta = 1$), we reinstate $[c^2 dt'^2 - dx'^2] = [c^2 dt^2 - dx^2]$ of SR. Now, we write the following transformations: $dx' = \theta \gamma (dX - \epsilon_1 v \epsilon dt) = \theta \gamma (dX - v \epsilon dt + \delta)$ and $dt' = \theta \gamma (dt - \frac{2 \epsilon_2 vdX}{c^2}) = \theta \gamma (dt - \frac{2 \epsilon_2 \epsilon v \epsilon dt dX}{c^2} + \Delta)$, where we assume $\delta = \Delta (V)$, so that $\delta = \Delta = 0$ for $V \to 0$, which implies $\epsilon = 1$. So, from such transformations we extract: $-v \epsilon dt + \Delta (V) = -v \epsilon dt$ and $-v \epsilon \epsilon \epsilon dX + \Delta (V) = -v \epsilon \epsilon \epsilon dX$ from where we obtain $\epsilon = (1 - \frac{\delta (V)}{v \epsilon dt}) = (1 - \frac{\epsilon_2 \Delta (V)}{v \epsilon dt dX})$. As $\epsilon$ is a dimensionless factor, we immediately conclude that $\delta (V) = V dt$ and $\Delta (V) = \frac{\epsilon_2 \epsilon dX}{c^2}$, so that we find $\epsilon = (1 - \frac{V dt}{c^2}) = (1 - \alpha)$. On the other hand, we can determine $\theta$ as follows: $\theta$ is a function of $\alpha$ ($\theta (\alpha)$), such that $\theta = 1$ for $\alpha = 0$, which also leads to $\epsilon = 1$ in order to recover Lorentz transformations. So, as $\epsilon$ depends on $\alpha$, we conclude that $\theta$ can also be expressed in terms of $\epsilon$, namely $\theta = \theta (\epsilon) = \theta [(1 - \alpha)]$, where $\epsilon = (1 - \alpha)$. Therefore, we can write $\theta = \theta [(1 - \alpha)] = [f (\alpha) (1 - \alpha)]^k$, where the exponent $k > 0$.

The function $f (\alpha)$ and $k$ will be estimated by satisfying the following conditions:

i) as $\theta = 1$ for $\alpha = 0$ ($V = 0$), this implies $f (0) = 1$.

ii) the function $\theta \gamma = \frac{[f (\alpha) (1 - \alpha)]^k}{(1 - \beta)^k} = \frac{[f (\alpha) (1 - \alpha)]^k}{(1 + \beta) (1 - \beta)^k}$ should have symmetrical behavior, that is to say it approaches to zero when closer to $V$ ($\alpha \to 1$) and, in the same way, to the infinite when closer to $c$ ($\beta \to 1$). In other words, this means that the numerator of the function $\theta \gamma$, which depends on $\alpha$ should have the same shape of its denominator, which depends on $\beta$. Due to such conditions, we naturally conclude that $k = 1/2$ and $f (\alpha) = (1 + \alpha)$, so that $\theta \gamma = \frac{[1 + \alpha] (1 - \alpha)^{1/2}}{(1 + \beta) (1 - \beta)^{1/2}} = \frac{(1 - \alpha^2)^{1/2}}{(1 - \beta^2)^{1/2}} = \frac{\sqrt{1 - V^2/c^2}}{\sqrt{1 - \epsilon^2 c^2}} = \Psi$, where $\theta = (1 - \alpha^2)^{1/2} = (1 - V^2/c^2)^{1/2}$.

The transformations in Eq.(46) and Eq.(47) are the direct transformations from $S_V [X^\mu = (X, ic\epsilon t)]$ to $S' [x'^\nu = (x', ic\epsilon t')]$, where we have $x'^\nu = \Omega^\nu_\mu X^\mu$ ($x' = \Omega X$), so that we obtain the following matrix of transformation:

$$\Omega = \begin{pmatrix} \Psi & i\beta (1 - \alpha) \Psi \\ -i\beta (1 - \alpha) \Psi & \Psi \end{pmatrix},$$

FIG. 2: $S'$ moves with a speed $v$ with respect to the ultra-referential $S_V$. If $V = 0$, $S_V$ is eliminated (no background field) and, thus, the galilean frame $S$ takes place, recovering Lorentz symmetry.
such that $\Omega \rightarrow L$ (Lorentz matrix of rotation) for $\alpha \rightarrow 0$ ($\Psi \rightarrow \gamma$). We should investigate whether the transformations in Eq.(48) form a group. However, such an investigation can form the basis of a further work.

We obtain $det\Omega = \frac{(1-\alpha^2)}{(1-\beta^2)}[1-\beta^2(1-\alpha)^2]$, where $0 < det\Omega < 1$. Since $V(S_V)$ is unattainable ($v > V$), this assures us that $\alpha = V/v < 1$ and, therefore, the matrix $\Omega$ admits inverse ($det\Omega \neq 0 (> 0)$). However $\Omega$ is a non-orthogonal matrix ($det\Omega \neq \pm 1$) and so it does not represent a matrix of rotation ($det\Omega \neq 1$) in such a space-time due to the presence of the privileged frame of background field $\Omega$, namely $\Omega_R$.

In the approximation for $v >> V$ or $\alpha \approx 0$, i.e., far from the background field effects, we obtain $det\Omega \approx 1$ and so we practically reinstate the rotational behavior of the Lorentz matrix (Lorentz symmetry) as a particular regime for higher energies. Alternatively, if we make $V = 0$ ($\alpha = 0$), we recover $det\Omega = 1$ and, thus, the Lorentz symmetry. The inverse transformations (from $S'$ to $S_V$) in differential form are

$$dX = \Psi'(dx' + \beta_c c dt') = \Psi'(dx' + vdt' - V dt') = \Psi'(dx' + vdt' - \xi c dt')$$

and

$$dt = \Psi'\left(\frac{dt'}{\beta(1-\alpha)} - \frac{vdx'}{c^2} - \frac{V dt'}{c^2}\right) \Rightarrow \frac{dt'}{\beta(1-\alpha)} = \Psi'\left(dX - \frac{v dx'}{c^2} - \frac{V dt'}{c^2}\right).$$

In matrix form, we have the inverse transformation $X' = \Omega'^{\mu}_{\nu}x'^{\nu}$ ($X = \Omega^{-1}x'$), so that the inverse is

$$\Omega^{-1} = \begin{pmatrix} \Psi' & -i\beta(1-\alpha)\Psi' \\ i\beta(1-\alpha)\Psi' & \Psi' \end{pmatrix},$$

where we can show that $\Psi' = \Psi^{-1}/[1-\beta^2(1-\alpha)^2]$, so that we must satisfy $\Omega^{-1}\Omega = I$.

Indeed we have $\Psi' \neq \Psi$ and, thus, $\Omega^{-1} \neq \Omega^T$. This non-orthogonal aspect of $\Omega$ has an important physical implication. In order to understand such an implication, let us first consider the orthogonal (e.g. rotation) aspect of Lorentz matrix in SR-theory. Under SR, we have $\alpha = 0$, so that $\Psi' \rightarrow \gamma' = \gamma = (1-\beta^2)^{-1/2}$. This symmetry ($\gamma' = \gamma$, $L^{-1} = L^T$) happens since galilean reference frames allow us to exchange the speed $v$ (of $S'$) for $-v$ (of $S$) when we are at rest at $S'$. However, under SSR, since there is no rest at $S'$, we cannot exchange $v$ (of $S'$) for $-v$ (of $S_V$) due to the asymmetry ($\Psi' \neq \Psi$, $\Omega^{-1} \neq \Omega^T$). Due to this fact, $S_V$ must be covariant, namely $V$ remains invariant for any change of reference frame in such a space-time. Thus we can notice that the paradox of twins, which appears due to the symmetry by exchange of $v$ for $-v$ in SR, should be naturally eliminated in SSR.[10], where only the reference frame $S'$ can move with respect to $S_V$. So, $S_V$ remains covariant (invariant for any change of reference frame). We have $det\Omega = \Psi^{-2}[1-\beta^2(1-\alpha)^2] \Rightarrow [(det\Omega)\Psi^{-2}] = [1-\beta^2(1-\alpha)^2]$. So, we can alternatively write: $\Psi' = \Psi^{-1}/[1-\beta^2(1-\alpha)^2]$.

By dividing Eq.(46) by Eq.(47), we obtain the following speed transformation:

$$v_{rel} = \frac{v' - v + V}{1 - \frac{vu}{c^2} + \frac{V}{c^2}},$$

where we have considered $v_{relative} = v_{rel} \equiv dx'/dt'$ and $v' \equiv dX/dt$. The speeds $v'$ and $v$ are given with respect to $S_V$, $v_{rel}$ being the relative velocity between $v'$ and $v$. Let us consider $v' \geq v$ (see Fig.3).

If we make $V \rightarrow 0$, the transformation in Eq.(52) recovers the Lorentz velocity transformation, where $v'(S')$ and $v(S)$ are given in relation to a certain galilean frame $S_0$ at rest, instead of $S_V$. But, since Eq.(52) implements
the ultra-referential $S_V$, the speeds $v'$ and $v$ are in fact given with respect to $S_V$. The covariance of $S_V$ is verified, for instance, if we assume $v' = v = V$ in Eq.(52). Thus, for this case, we obtain $v_{rel} = "V - V" = V$. Let us also consider the following cases in Eq.(52):

a) $v' = c$ and $v \leq c \Rightarrow v_{rel} = c$. This just verifies the well-known invariance of $c$.

b) if $v' > v (= V) \Rightarrow v_{rel} = "v' - V" = v'$. For example, if $v' = 2V$ and $v = V \Rightarrow v_{rel} = "2V - V" = 2V$. This means that $V$ really has no influence on the speed of a particle. So, $V$ works as if it were an “absolute zero of motion”, being invariant and having the same value in all directions of space in the isotropic background field.

c) if $v' = v \Rightarrow v_{rel} = "v - v"(\neq 0) = \frac{V}{1 - \frac{v}{c}}$. From (c) let us consider two specific cases, namely:

- $c_1$ assuming $v = V \Rightarrow v_{rel} = "V - V" = V$ as verified before.
- $c_2$ if $v = c \Rightarrow v_{rel} = c$, where we have the interval $V \leq v_{rel} \leq c$ for $V \leq v \leq c$.

This last case (c) shows us in fact that it is impossible to find the rest state for the particle on its own (the absolute speed of the particle is $c$). However, if we make $V \rightarrow 0$, then we would have $v_{rel} \equiv \Delta v = 0$ and, thus, it would be possible to find the rest state for $S'$, which would become simply a galilean reference frame of SR.

By dividing Eq.(49) by Eq.(50), we obtain

$$v_{rel} = \frac{v' + v - V}{1 + \frac{v}{c} - \frac{v'}{c}}.$$  \hspace{1cm} (53)

In Eq.(53), if $v' = v = V \Rightarrow "V + V" = V$. Indeed $V$ is invariant, working like an absolute zero of motion. If $v' = c$ and $v \leq c$, this implies $v_{rel} = c$. For $v' > V$ and considering $v = V$, this leads to $v_{rel} = v'$. As a specific example, if $v' = 2V$ and assuming $v = V$, we would have $v_{rel} = "2V + V" = 2V$. And if $v' = v \Rightarrow v_{rel} = "v + v" = \frac{2v - V}{1 + \frac{v}{c}}$. In a classical (newtonian) regime ($V << v << c$), we recover $v_{rel} = "v + v" = 2v$. In a relativistic (einsteinian) regime ($v \rightarrow c$), we reinstate the Lorentz transformation for this case ($v' = v$), i.e., $v_{rel} = "v + v" = 2v/(1 + v^2/c^2)$.

By joining both transformations in Eq.(52) and Eq.(53) into only one equation, we write the following compact form:

$$v_{Rel} = \frac{v' \mp \epsilon v}{1 \mp \frac{\epsilon v}{c}} = \frac{v' \mp v(1 - \alpha)}{1 \mp \frac{\epsilon v(1 - \alpha)}{c}} = \frac{v' \mp v \pm V}{1 \mp \frac{\epsilon v}{c} \pm \frac{v}{c}},$$  \hspace{1cm} (54)

being $\alpha = V/v$ and $\epsilon = (1 - \alpha)$. For $\alpha = 0$ ($V = 0$) or $\epsilon = 1$, we recover the Lorentz speed transformations.

A new group algebra for SSR will be investigated elsewhere.

D. The metric of flat space-time with the ultra-referential $S_V$ and the dynamics of particles in SSR

Let us consider the ultra-referential $S_V$ as a uniform background field that fills the whole flat space-time as a perfect fluid, playing the role of a kind of de-Sitter (dS) space-time\cite{30}. So, let us define the following metric:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{$S_V$ is the covariant ultra-referential of the background field. $S$ represents the reference frame for a massive particle with speed $v$ in relation to $S_V$, where $V < v < c$. $S'$ represents the reference frame for a massive particle with speed $v'$ in relation to $S_V$. In this case, we consider $V < v \leq v' \leq c$.}
\end{figure}
FIG. 4: $v_0$ represents a low speed $v(<< c)$ with respect to $S_V$, from where we get the own energy ($m_0c^2$), being $\Psi_0 = \Psi(v_0) = 1$, where $v_0 = \sqrt{cV} \sim 10^{-4}$m/s. For $v << v_0$ or closer to $S_V (v \rightarrow V)$, a new relativistic correction of energy arises, so that $E \rightarrow 0$.

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \Theta g_{\mu\nu}dx^\mu dx^\nu,$$

where $G_{\mu\nu} = \Theta g_{\mu\nu}$, $g_{\mu\nu}$ being the well-known Minkowski metric. $\Theta$ is a scale factor that increases for very large wavelengths (cosmological scales) governed by vacuum (dS), that is to say, $\Theta$ increases for much lower energy scales, where we have $\Theta \rightarrow \infty$. On the other hand, $\Theta$ decreases to 1 for smaller scales of length, namely for higher energy scales, so that the dS space-time reduces to the Minkowski metric as a special case for $\Theta \rightarrow 1$.

We should understand that $\Theta$ breaks strongly the invariance of interval $ds$ only for very large distances (very low energy scales), which are governed by vacuum of the ultra-referential $S_V$. For smaller scales of length governed by matter (higher energy scales), we see that Lorentz symmetry and the invariance of $ds$ are restored. Following such considerations, let us consider $\Theta$ to be a function of the speed $v$ with respect to $S_V$, namely:

$$\Theta = \Theta(v) = \theta(v)^{-2} = \frac{1}{(1 - \frac{V^2}{v^2})},$$

such that $\Theta \approx 1$ for $v >> V$, i.e., the Lorentz symmetry regime for higher energy scales. For $\Theta \rightarrow \infty$ when $v \rightarrow V$, we get the regime of ultra-referential $S_V$ for lower energy scales, breaking down strongly the $ds$ invariance, so that $ds \rightarrow \infty$. In such a regime, we have a Lorentz symmetry breakdown, but, for $v >> V$ (higher energies), we recover Lorentz symmetry.

As the metric $G_{\mu\nu}$ depends on velocity, it seems to be a kind of Finsler’s metric, namely a Finslerian space with a metric that depends on the position and velocity, i.e., $g_{\mu\nu}(x, \dot{x})$.

The energy $E$ of a particle that moves with respect to the ultra-referential $S_V$ is as follows:

$$E = \theta(\gamma m_0c^2) = \Psi m_0c^2 = m_0c^2 \sqrt{1 - \frac{V^2}{v^2}} \sqrt{1 - \frac{\alpha^2}{\beta^2}},$$

where we have $\theta = \Theta^{-1/2} = \sqrt{1 - \alpha^2}$, the factor that breaks down Lorentz symmetry due to the background field-$S_V$, and $\gamma = 1/\sqrt{1 - \beta^2}$ (the Lorentz factor), being $\alpha = V/v$ and $\beta = v/c$.

In Eq.(57), we see that $E \rightarrow 0$ for $v \rightarrow V (S_V)$. For $v = v_0 = \sqrt{cV}$, we obtain $\theta\gamma = \Psi(v_0) = 1 \Rightarrow E = E_0 = m_0c^2$. Actually, as a massive particle has speed $v > V$ with respect to the unattainable ultra-referential $S_V$, its energy $m_0c^2$ requires a non-zero motion $v(= v_0)$ in relation to $S_V$ (see Fig.4).

The momentum of the particle in relation to $S_V$ is as follows:

$$\vec{P} = m_0\vec{v} \sqrt{1 - \frac{V^2}{v^2}} \sqrt{1 - \frac{\alpha^2}{\beta^2}}.$$
From Eq.(57) and Eq.(58), we get the following energy-momentum relation: 
\[ c^2 \vec{P}^2 = E^2 - m^2 c^4 (1 - \frac{V^2}{v^2}) \],
which leads to a deformed dispersion relation that should be extensively investigated within a new quantum field theory, emerging from the space-time of such DSR (SSR) to be more explored in future works.

Now, the de-Broglie wavelength of the particle is due to its speed \( v \) with respect to \( S_V \), namely:
\[ \lambda = \frac{\hbar}{P} = \frac{\hbar}{m_0 v} \sqrt{1 - \frac{v^2}{c^2}} \sqrt{1 - \frac{V^2}{v^2}}, \] (59)
from where we have used the momentum in Eq.(58) given with respect to \( S_V \).

If \( v \to c \Rightarrow \lambda \to 0 \) (spatial contraction). But, if \( v \to V (S_V) \Rightarrow \lambda \to \infty \) (spatial dilation), this limit leads to a strong violation of Lorentz symmetry, which means we get very large wavelengths. So we have \( \Theta \to \infty \).

E. Covariance of the Maxwell wave equation in the presence of the ultra-referential \( S_V \)

Let us assume a light ray emitted from the non-galilean frame \( S' \) (see Fig.2). The equation of its electrical wave in this reference frame \( (S') \) is given as follows:
\[ \frac{\partial^2 \vec{E}(x', t')}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}(x', t')}{\partial t'^2} = 0. \] (60)

As is well-known, when we make the exchange by conjugation on the spatial and temporal coordinates, we obtain respectively the following operators: \( X \to \partial/\partial t \) and \( t \to \partial/\partial X \); also \( x' \to \partial/\partial t' \) and \( t' \to \partial/\partial x' \). Thus, the transformations in Eq.(46) and Eq.(47), given for such differential operators are the following:
\[ \frac{\partial}{\partial t'} = \Psi \left[ \frac{\partial}{\partial t} - \beta c (1 - \alpha) \frac{\partial}{\partial X} \right], \] (61)
and
\[ \frac{\partial}{\partial x'} = \Psi \left[ \frac{\partial}{\partial X} - \frac{\beta}{c} (1 - \alpha) \frac{\partial}{\partial t} \right], \] (62)
where \( \beta = v/c \) and \( \alpha = V/v \).

If we square Eq.(61) and Eq.(62) and insert into Eq.(60), we can calculate and finally obtain
\[ \text{det} \Omega \left( \frac{\partial^2 \vec{E}}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t'^2} \right) = 0, \] (63)
where \( \text{det} \Omega = \Psi^2 [1 - \beta^2 (1 - \alpha)^2] \) [see Eq.(48)].

As the ultra-referential \( S_V \) is definitely inaccessible for any particle, we always have \( \alpha < 1 \) (or \( v > V \)), which always implies \( \text{det} \Omega = \Psi^2 [1 - \beta^2 (1 - \alpha)^2] > 0 \). And, as we must have \( \text{det} \Omega > 0 \), this always assures that
\[ \frac{\partial^2 \vec{E}}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t'^2} = 0. \] (64)

By comparing Eq.(64) with Eq.(60), we verify the covariance of the equation of the electromagnetic wave propagating in the presence of the background field of the ultra-referential \( S_V \). So, indeed we can conclude that the space-time transformations in SSR also preserve the covariance of the Maxwell equations in the vacuum-\( S_V \) as well as the Lorentz transformations preserve the covariance of the wave equations in SR. This leads us to think that \( S_V \) works like a relativistic background field in the sense that it is compatible with electromagnetism, i.e., the presence of \( S_V \) still preserves the covariance of the Maxwell equations in the exchanges of non-galilean reference frames.
V. PROSPECTS

The present research has various implications which shall be investigated in coming articles. Let us firstly stress that we should search for a new group algebra that is more general than the Lorentz group. Besides this, we need to explore many important consequences of SSR in quantum field theories (QFT), since a QFT should be built on such an almost flat space-time, due to the presence of the weak background field of gravitational origin, i.e., the ultra-referential $S_V$.

As the last term of Eq.(33) ($\rho_{em}^{(2)}$) provides an electromagnetic effect of purely gravitational origin, such an effect being capable of increasing significantly the magnitude of the electric force between two charges immersed only in a strong gravitational field, the idea of reciprocity by replacing a strong gravitational field by a strong static electric field, in which two masses are immersed, can lead to a very small change in the gravitational force acting on the test mass. To achieve such a slight deviation on gravity, it would be feasible to search for such an effect using a torsional balance with a discharged test mass immersed in a very intense electrostatic field. This could be a possible experiment to test any crosstalk between gravitation and electromagnetism.

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