Ionization waves (striations) in DC discharges in noble gases at low plasma densities obtained with a hybrid kinetic-fluid model

Vladimir I. Kolobov¹,² and Robert R. Arslanbekov²

¹University of Alabama in Huntsville, Huntsville, AL, USA
²CFD Research Corporation, Huntsville, AL, USA

Abstract

A hybrid kinetic-fluid model is used to study ionization waves (striations) in DC discharges in noble gases at low plasma densities. Coupled solutions of a kinetic equation for electrons, a drift-diffusion equation of ions, and a Poisson equation for the electric field are obtained to clarify the nature of plasma stratification in the positive column and near-electrode effects. A simplified two-level excitation-ionization model is used for the conditions when the nonlinear effects due to stepwise ionization, gas heating, and Coulomb interactions among electrons are negligible. It is confirmed that the nonlocal effects are responsible for the formation of moving striations in DC discharges at low plasma densities. The calculated properties of self-excited nonlinear waves of s, p, and r types in Neon and s type in Argon agree with available experimental data. The reason for Helium plasma stability to stratification is clarified. It is shown that sustaining stratified plasma is more efficient than striation-free plasma when the ionization rate is a nonlinear function of the electric field. However, the nonlinear dependence of the ionization rate on the electric field is not required for plasma stratification. Neon's striations of s, p, and r types exist with minimal or no ionization enhancement. The previously discussed electron bunching and resonance effects do not appear critical for the studied striations. The generation of electrons in the ionization events controls the formation of nonlocal electron energy distribution function.

I. Introduction

Weakly-ionized collisional plasma is prone to instabilities and pattern formation. The bright and dark layers along the electric current are called striations. Spatial patterns across the current are associated with constriction, filamentation, and streamer formation. Nonlinear and nonlocal effects are responsible for most of these self-organization processes. The nonlinear effects appear because a small fraction of energetic electrons usually controls the ionization processes in plasma. The number of these electrons depends nonlinearly on the ratio of the electric field to gas density, $E/N$. The ionization rate can also depend on the electron density due to stepwise ionization and Coulomb collisions. Specific nonlocal effects arise due to the peculiarities of electron kinetics in collisional plasma [1, 2]. The electron mean free path and the electron energy relaxation length can differ by two orders of magnitude depending on the gas type and the electron kinetic energy [3]. Identifying and understanding specific mechanisms of plasma self-organization in each case remains challenging.

This paper deals with plasma stratification in noble gases. Striations have been observed in gas discharges long before Langmuir introduced the term plasma. They were described in several reviews [4, 5, 6]. The nature of striations as ionization waves became clear by the end of the 70th.
Several types of ionization waves in noble gases have been identified depending on gas pressure and discharge current \([7,8]\). With the advent of computers, studies of electron kinetics in uniform and spatially modulated electric fields were performed using numerical solutions of the Boltzmann equation and particle-based (PIC) methods \([9, 10]\). These studies contributed to understanding the origin of plasma stratification in noble gases and helped interpret the famous Frank Hertz experiments. However, the interplay of the nonlinear and kinetic effects responsible for plasma stratification in different gases and discharge types remained unclear.

The most studied are moving striations in DC discharges in noble gases. Several types of striations have been observed depending on the gas pressure \(p\), the discharge current \(i\), and the tube radius \(R\). Similarity laws \([11]\) allowed producing experimental maps of discharge types in the \((pR, i/R)\) plane shown in Figure 1 for Neon and Argon. Self-excited moving striations have been observed in the white areas of the map. No discharge can be maintained in the dark area of the map at low values of \(pR\) and \(i/R\). The Debye radius is equal to the tube radius at the curve (1). The green area corresponds to a striation-free plasma controlled by ambipolar diffusion to the wall. Volume recombination dominates over surface recombination above the line (2). In this area, artificial striations have been excited to study the dispersion characteristics of the ionization waves. The blue area corresponds to a striation-free, radially constricted arc column. Line (3), which is also called the Pupp boundary, corresponds to the transition from glow to arc discharge. No discharge can be maintained below the curve (4) because the ionization cross section has a finite value. The two dashed lines indicate gas pressure \(p\) at which the electron mean free path \(\lambda\) and the electron energy relaxation length \(\lambda_u\) is equal to the tube radius \(R\).

Depending on the gas pressure and discharge current, several striation types have been identified. The most studied are striations in diffuse and constricted discharges near the Pupp boundary. They appear due to a nonlinear dependence of the ionization rate on the electron density. Three types of waves were observed at lower discharge currents at gas pressures below the dashed line \(\lambda_u = R\) in Figure 1 are due to nonlocal electron kinetics. Striations in diffuse discharges at medium pressures above the dashed line \(\lambda_u = R\) are often irregular and less studied.

![Figure 1: Discharge forms in Neon and Argon for different currents \(i\) and gas pressures \(p\) (after \([11, 12, 13]\)).](image-url)
Standing striations were observed in RF discharges [14]. In molecular gases, both moving and standing striations were detected [15]. Recently, standing striations have appeared in computer simulations of CCP in CF₄ [16], and DC discharges in Nitrogen [17]. In addition to electron kinetics, they may require analysis of complicated chemistry involving negative ions and the kinetics of vibrationally excited states of the molecules.

Recent progress in understanding plasma stratification in noble gases has been achieved using computer simulations with fluid [18, 19], hybrid [20], and Particle-in-Cell (PIC) models [21]. Two-dimensional fluid models have been applied to study ionization waves in DC [18] and RF [19] discharges in Argon at relatively high currents when the nonlinear dependence of the ionization rate on electron density is responsible for plasma stratification. A hybrid model using a Fokker-Planck kinetic solver for electrons and a fluid model for ions has been developed [20] for standing and moving striations in Argon discharges at low currents when nonlocal effects are the leading cause of stratification. Boeuf [21] used a PIC model to study moving striations in DC discharges in Argon and Neon in the absence of metastable atoms and Coulomb collisions among electrons.

The present paper aims to further clarify the nature of plasma stratification in noble gases at low ionization degrees (plasma densities) when the nonlinear dependencies of the ionization rate on plasma density and gas heating are insignificant. The plan of the paper is as follows. We first briefly describe the computational model and boundary conditions. The central part of the paper is devoted to analyzing plasma stratification in a positive column of DC discharges in Argon, Neon, and Helium with periodic boundary conditions. Finally, we briefly discuss the impact of near-electrode effects on plasma stratification and identify a few open questions for future research.

II. Computational model

The hybrid model used in the present paper was first described in [22] and later adapted in [20] to study striations in DC Argon discharges. Here, we extend this model by implementing periodic boundary conditions similar to [21]. The enhanced model is applied to study the stratification of a long positive column in DC discharges in noble gases.

1. Electron kinetics

The electrons are described via a Fokker-Planck (FP) kinetic equation for the EEDF \( f_0(r,u,t) \) in the \((r,u)\) phase space [23, 24]:

\[
\frac{\partial f_0}{\partial t} - \left( \mathbf{v} - \mathbf{E} \frac{\partial}{\partial u} \right) \cdot \mathbf{D}_r \left( \mathbf{v} - \mathbf{E} \frac{\partial}{\partial u} \right) f_0 - \frac{1}{\sqrt{\nu u}} \frac{\partial}{\partial u} \left( \sqrt{\nu u} \Gamma_u \right) = C_0 \tag{1}
\]

Here, \( \mathbf{E} \) is the electric field, \( u = m v^2/(2e) \) is the volt-equivalent of the electron kinetic energy, \( D_r = v^2/(3\nu) \) is a diffusion coefficient in the configuration (physical) space, and \( \nu(\nu) \) is the transport collision frequency. The energy flux density, \( \Gamma_u \), describes processes associated with small energy changes due to quasi-elastic collisions of electrons with atoms, Coulomb interactions,
etc. At \( I_u = 0 \), the left-hand side of Eq. (1) describes electron diffusion in the phase space \((r, u)\) over surfaces of constant total energy \( \epsilon = u - \varphi(r) \), where \( \varphi(r) \) is the electrostatic potential [20].

The right-hand side of Eq. (1) contains inelastic collisions. The excitation of an atomic state with the energy threshold \( \epsilon_1 \) is described by

\[
C_0 = -\nu^*(u)f_0(u) + \frac{\sqrt{u + \epsilon_1}}{\sqrt{u}}\nu^*(u + \epsilon_1)f_0(u + \epsilon_1) \tag{2}
\]

where \( \nu^*(u) \) is the corresponding inelastic collision frequency. The direct ionization by electron impact is described as:

\[
C_{\text{ion}} = -\nu^\text{ion}(u)f_0(u) + 4\frac{\sqrt{2u + \epsilon_\text{ion}}}{\sqrt{u}}\nu^\text{ion}(2u + \epsilon_\text{ion})f_0(2u + \epsilon_\text{ion}) \tag{3}
\]

where \( \nu^\text{ion}(u) \) is the ionization frequency, and \( \epsilon_\text{ion} \) is the ionization threshold. This ionization model assumes that the kinetic energy is evenly distributed between the primary and secondary electrons after an ionization event.

The FP kinetic equation (1) is valid when the EDF and the electric field \( E \) change slowly on the electron mean free path \( \lambda \), during the time \( 1/\nu \) between collisions [25]. This condition corresponds to \( pR \) values above the dashed lines in the maps shown in Fig. 1. Below this line, \( \lambda \) is comparable to the tube radius \( R \). Under these conditions, the two-term spherical harmonics expansion is not valid, and the full Boltzmann equation must be used instead of the FP kinetic equation (1) for the analysis of plasma stratification.

Below, we assume that plasma is contained in a long cylindrical tube with a length \( L >> R \). We reduce the kinetic equation (1) for a 2d phase space \((x, u)\):

\[
\frac{\partial f_0}{\partial t} - \nabla \cdot (D_2 \nabla f_0) - \frac{1}{\sqrt{u}} \frac{\partial}{\partial u} \left( \sqrt{u} I_u \right) = C_0 - C_{\text{wall}} \tag{4}
\]

where the diffusion tensor \( D_2 \) is defined as [20]:

\[
D_2 = D_1 \begin{pmatrix} 1 & E \\ E & E^2 \end{pmatrix},
\]

and \( E(x) \) is the axial electric field.

The radial loss of electrons to the wall in this 1d1u model can be included in the form [26]:

\[
C_{\text{wall}} = -\frac{1}{3} \left( \frac{u}{R} \right) f_0(u) \Theta(u - \Phi_w) \tag{6}
\]

where \( \Phi_w(x) \) is the wall potential with respect to the plasma axis, and \( \Theta(x) \) is the step function.

In our present work, we used a simpler expression:

\[
C_{\text{wall}} = -f_0/\langle \tau_{\text{wall}} \rangle = -f_0/\langle \nu_1 \rangle \tag{7}
\]
where \( \langle \nu_i \rangle \) is the space-averaged ionization frequency and \( \tau_{wall} \) is the time of electron diffusion to the wall. This model assumes that all electrons are lost radially at the same rate they are created along the striation [21]. Our tests showed that the choice of the radial loss model does not significantly affect the results. More advanced models, such as Eq. (9), may be used in future studies.

For the numerical solution of the FP kinetic equation, we introduce a computational grid in phase space. The energy \( u_{max} \) is selected about 2-3 \( \varepsilon_1 \). The boundary condition is specified as \( f_0(x, u = u_{max}) = 0 \). The boundary condition at \( u = 0 \)

\[
\frac{\partial f_0}{\partial x} - E \frac{\partial f_0}{\partial u} \to 0
\]  

ensures the absence of electron flux from the boundary at \( u = 0 \). The boundary conditions in space for simulations of a positive column are discussed below.

Ions are described using a drift-diffusion model:

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} \left( \mu_i n_i E(x) - D_i \frac{\partial n_i}{\partial x} \right) = I - \frac{n_i}{\tau}
\]  

where \( \mu_i \) and \( D_i \) are the ion mobility and diffusion coefficients, and \( I \) is the ionization rate by electron impact. The ion loss term matches the electron loss, \( \langle \nu_i \rangle \tau = 1 \), where \( \langle \nu_i \rangle \) is the ionization frequency averaged over striation length. This condition ensures charge conservation over striation length. The electric field is calculated from the Poisson equation. We assume that the initial EEDF is Maxwellian, and the initial ion density equals the electron density.

The coupled set of the FP kinetic equation for electrons (1), the drift-diffusion for ions (12), and Poisson equation for the electric field was solved using COMSOL with an implicit (BDF) time-stepping method. A direct (MUMPS) solver was employed at each time step for all quantities. We have not used the logarithmic transformation of the ion drift-diffusion and the FP kinetic equations, which was used in our previous paper [20]. We found that the convergence of the coupled equations with different dimensionality, which are solved simultaneously, is substantially better without this transformation (which is particularly important for long transient simulations), and small negative values of \( f_0 \) and \( n_i \) do not affect the results.

2. The boundary conditions for the positive column plasma

The positive column is usually considered an autonomous system weakly influenced by near-electrode processes. Without striations, the positive column is axially uniform with a local balance of the ionization and the particle loss to the wall due to ambipolar diffusion or volume recombination. To analyze stratification of the positive column, we apply periodic boundary conditions for the EEDF and the particle fluxes:

\[
(f_0)_{left} = (f_0)_{right}
\]

\[
-(D_2 \nabla f_0 \cdot \hat{n})_{left} = (D_2 \nabla f_0 \cdot \hat{n})_{right}
\]
where \( \hat{n} \) denotes the unit normal to the boundary. This BC ensures the continuity of the EEDF and the electron flux in phase space.

We also apply periodic boundary conditions for ions by making the ion density and the ion flux equal at the boundaries:

\[
\left( n_i \right)_{\text{left}} = \left( n_i \right)_{\text{right}} - \left( \mu_i n_i E(x) - D_i \frac{\partial n_i}{\partial x} \right)_{\text{left}} = \left( \mu_i n_i E(x) - D_i \frac{\partial n_i}{\partial x} \right)_{\text{right}}
\]

Using periodic BCs for electrons and ions implies that an integer number of waves fits the positive column. Also, such boundary conditions ensure that the total number of electrons and ions remains the same in the plasma (provided that matching electron and ion losses are used). As the total space charge remains zero (initially, a quasineutral plasma is assumed with \( n_e = n_i \)) along the striation length, the periodic BC for the electric field \( E(\text{left}) = E(\text{right}) \) is automatically satisfied. Therefore, we can prescribe the electric potential at the boundaries to specify the potential drop \( U \) over the length \( L \).

The radius of the tube \( R \) was computed from:

\[
R = 2.4 \frac{D_a}{\langle \nu_i \rangle} \tag{10}
\]

The averaged electron temperature \( \langle T_e \rangle \) was used to evaluate the ambipolar diffusion coefficient, \( D_a \).

### III. Moving Striations in DC discharges in noble gases

Simulations of the positive column were performed in Argon, Neon, and Helium at pressures 0.1, 0.2, 0.4, and 1 Torr, and plasma densities \( \langle n \rangle \) from \( 10^{14} \) to \( 10^{16} \) m\(^{-3} \). We have varied the applied voltage \( U \) to change the average electric field \( \langle E \rangle \). The potential drop over striation length \( \Delta \phi_A \) and the average electric field \( \langle E \rangle \) were calculated as:

\[
\Delta \phi_A = \int_0^A dx E(x, t) = v_s \int_0^T dt E(x, t) = \langle E \rangle A \tag{11}
\]

Here, \( v_s \) denotes the speed of the ionization wave, \( T = 1/\nu_s \) is its period, and \( \nu_s \) is the frequency of the wave. The tube radius \( R \) was found self-consistently from Eq. (12) for each value of \( \langle E \rangle \).

Figure 2 shows the dynamics of the plasma stratification process in the positive column of length 15 cm in Neon at 1 Torr with the applied voltage of 100 V. The left part of Figure 2 shows the contours of electron density \( n_e(x, t) \), and the right part shows the time evolution of the electron density and the electric field in the middle of the gap. The dynamics of plasma stratification in a long positive column (described by the periodic BC model) are quite different from the striation development between the electrodes previously studied in Argon [20]. In the case of the long
positive column, striations appear uniformly within the computational domain after \( \sim 150 \mu s \) and propagate to the cathode side with velocities of about 100 m/s. The calculated radius of the discharge tube is \( \sim 5 \text{ mm} \). Plasma stratification occurs at the time scale of the order of the ambipolar diffusion time.

Figure 2: The contours of electron density \( n_e(x,t) \) during striation development in the positive column plasma in Neon (left) and the corresponding time evolution of the plasma density and the electric field in the middle of the computational domain (right).

Below, we describe the properties of established moving striations in the positive column obtained in our simulations for Neon, Argon, and Helium.

1. Three types of moving striations in Neon

Simulations of Neon plasma were performed for gas pressure \( p = 1 \text{ Torr} \) and plasma density \( 10^{15} \text{ m}^{-3} \). By varying the applied voltage, we obtained three types of waves.

a. S waves

Figure 3 shows an example of S waves obtained for \( L = 3 \text{ cm}, U = 48 \text{ V} \). These conditions correspond to capillary discharges with \( R = 2 \text{ mm}, E = 16 \text{ V/cm} \). There are two striations with the potential drop \( \Delta \varphi_A = 24 \text{ V} \) over striation length. The Debye length \( \sim 0.65 \text{ mm} \) is comparable to the tube radius, and there are notable deviations from quasineutrality. The electron temperature is relatively high, \( T_e \sim 6.6 - 8.3 \text{ eV} \) (compared to \( r \) and \( p \) waves described below), and the ionization and excitation rates are of the same order of magnitude. These conditions are close to boundary one on the map in Figure 1, where the electric field modulation is small. The EEDF shape is similar to that observed in a nearly uniform electric field (see results of [20] for Argon).
Figure 3: Linear s waves in Neon. The instantaneous EEDF and the corresponding macro-parameters. The dashed lines show Neon's excitation and ionization thresholds of 16.62 and 21.6 eV.

Figure 4 shows an example of nonlinear s waves obtained for $L = 6 \text{ cm}$, $U = 120 \text{ V}$. These conditions correspond to $E = 20 \text{ V/cm}$ and $R \sim 1.6 \text{ mm}$. The potential drop over striation length is 24 V again, but the electric field reversals take place. The variations of the electron temperature are more significant (from 5.4 to 9 eV), and the ionization and excitation rates are even closer compared to Fig. 3. The EEDF appears closer to “local”: intense electron heating occurs in the areas of large electric fields.
b. **R waves**

The R waves were obtained for the electric fields $E$ in the range 2.6 - 10 V/cm, Figure 5 shows an example of calculation results for 1 Torr, $L = 15$ cm, $U = 38$V, $E = 2.53$ V/cm, wavelength 5 cm, potential drop 12.7 V, $R \sim 1.4$ cm, Debye length $\sim 1$ mm.

![Figure 5: An example of R waves with a potential drop of 12.7 V.](image)

---

**c. **P waves**

The $p$ waves were obtained for the electric fields in the range 1.87 – 2.6 V/cm. Figure 6 shows an example of $p$ waves for $L = 15$ cm, $U = 28$V, 30 eV, which correspond to $E = 1.87$ V/cm and $R \sim 2.2$ cm. The wavelength is 5 cm, and the potential drop over striation length is 9.3 V. The Debye length of $\sim 1$ mm is much smaller than the tube radius, and there are no noticeable deviations from
quasineutrality. The waves are highly nonlinear with strong electric field reversals. The electron temperatures are the lowest, in the range $T_e \sim 4.1$-5.2 eV, compared to the s and r waves. The excitation and ionization rates are strongly modulated. The maximal value of the ionization rate is lower than the maximal value of the excitation rate by a factor of 5.

Figure 6: The $p$ waves with a potential drop of 9.3 V.

Figure 7 summarizes the calculated dependence of the striation length $\Lambda$ on the electric field $\langle E \rangle$, and the dependence of the electric field $\langle E \rangle$ on tube radius $R$. The calculated potential drops for the three types of striations are close to the values observed by Novak [27].
Figure 7: Dependence of striation length $\Lambda$ on the electric field $\langle E \rangle$ (left) and the dependence of the electric field $\langle E \rangle$ on tube radius $R$ (right) in Neon at 1 Torr. Points are our simulations, and solid curves express Novak’s law.

Similarity laws [11] dictate that for a given gas, the properties of positive column plasma are similar for the same values of $pR$, $i/R$, and the wall temperature. Coulomb collisions, stepwise ionization, volume recombination (by pair collisions), and gas heating do not destroy the similarity laws. However, the deviation from quasineutrality and three-particle collisions do. Similarity laws are also valid in the presence of waves in the positive column. According to these laws, the waves with the same reduced frequencies $fR$ (or $f/p$) should have the same reduced length $\Lambda/R$ (or $\Lambda p$). Figure 8 shows the reduced wavelengths $\Lambda/R$, obtained in our simulations. There are three wave types: $s$ waves with $\Lambda/R \sim 7.5$, $r$ waves with $\Lambda/R \sim 4$, and $p$ waves with $\Lambda/R \sim 2.5$. These $\Lambda/R$ values are in good agreement with the experiment [28] where $\Lambda/R \sim 6$ were obtained for $s$ waves, $\Lambda/R \sim 3$ for $r$ waves, and $\Lambda/R \sim 2 \div 3$ for $p$ waves. The experiments were conducted in tubes of radius $R = 1.5$ and $3$ cm over a range of discharge currents.

Figure 8: Calculated dependence of the striation length $\Lambda/R$ (left) and frequency $fR$ (right) on $pR$ for $s$, $p$, and $r$ waves in Neon at 1 Torr at plasma density $10^{15}$ m$^{-3}$.

The right part of Figure 8 shows the calculated reduced frequencies $fR$ for the $s$, $p$, and $r$ waves and the experimental data obtained in capillary tubes near the boundary of the striation existence at low discharge currents [29]. The violation of the similarity law in the experiments (the dependence of $fR$ on $R$) was attributed [29] to deviations from quasineutrality and to the transition from the ambipolar to free diffusion in the radial direction, which is expected to occur near this boundary. Our simulations (see above) confirm this assumption.

It is seen in Figure 8 that the frequency of the waves increases with decreasing $pR$, as discussed in [29]. The waves in capillary tubes with a radius $R \sim 1$ mm could have frequencies up to 1 MHz. Our simulated frequencies for the $r$ waves agree with the experiment [29]. Only the $r$ waves were observed in these experiments; no $s$ waves could be excited due to experimental limitations. As discussed above, our model is not valid for small $pR$, which corresponds to the conditions when the particle mean free path $\lambda$ exceeds the tube radius $R$. The calculated frequencies for $pR \sim 1$ are about an order of magnitude higher than the experimentally observed in [Ilinski]. This discrepancy can be attributed to the neglect of stepwise ionization in our simulations. Our previous work
showed that the inclusion of stepwise ionization could decrease the wave speed by two orders of magnitude due to the transition from ion-guided to metastable guided waves.

2. The $S$ waves in Argon

Similar calculations have been performed for Argon at gas pressures 0.25, 0.5, and 1 Torr and plasma density $10^{15}$ m$^{-3}$ for different electric field values. However, only one wave type was found in our simulations. Figure 9 shows results for 1 Torr, 5 cm, 60V, which corresponds to $E \sim 12$ V/cm. The values of $R \sim 2.4$ mm, and Debye length $\sim 6$ mm were obtained. The $s$ waves with the length of 1.25 cm were found.

![Figure 9: Striations in Argon at 1 Torr. The dashed lines show Argon's excitation and ionization thresholds of 11.55 and 15.8 eV](image)

Figure 10 summarises the calculated dependencies of the striation length on the electric field (left) and the electric field on tube radius (right) for Argon at different pressures. The wave frequency increases with increasing electric field from 0.1 MHz at 10 V/cm to 1.3 MHz at 40 V/cm at 1 Torr.
Figure 10: Dependence of striation length on the electric field (left) and the electric field on tube radius (right) for Argon at different pressures.

We expect that stepwise ionization plays a more critical role in Argon than in Neon and contributes to the formation of p and r striations observed in experiments.

3. Peculiarities of plasma stratification in Helium

We could not obtain striations in Helium under discharge conditions similar to those in Neon and Argon: the plasma appeared stable against ionization waves. However, by reducing the ion mobility compared to actual values, we could obtain robust striations for up to 1/3 of the actual mobility value ($\mu_{i0}$) at $p = 1$ Torr. Figure 11 shows the instantaneous profiles of plasma density, electron temperature, and the electric field in a gap $L = 8$ cm and $U = 50$ V. The calculated tube radius $R = 1.1$ cm, the Debye length is $\sim 1$ mm, and the potential drop per striation length is 16.7 V.

No field reversals are observed in this case due to the low amplitude of the striations. Striations become stronger with decreasing ion mobility, and the tube radius $R$ decreases. The speed of the striations increases with increasing ion mobility from 50 m/c to 80 m/c as $\mu_i/\mu_{i0}$ increases from 0.1 to 0.33.
Figure 11: Striations in Helium with ion mobility reduced by 3. The dashed lines show Helium's excitation and ionization thresholds of 19.82 and 24.6 eV.

Based on these results, we conclude that the leading cause of the experimentally observed stability of Helium plasma to stratification is due to the lowest mass ratio of ions to electrons, $M/m$. The lowest mass ratio results in high ion mobility of Helium compared to other noble gases. It enhances the rate of the EEDF relaxation in elastic collisions of electrons with atoms and the gas heating by electrons. It should also be noted [7] that peculiarities of the energy dependence of the elastic collision cross-section in Helium result in a near-Maxwellian EEDF in the elastic energy range at higher gas pressures and, therefore, in the absence of the EEDF Maxwellization by Coulomb collisions. All these facts make Helium plasma stratification quite distinct from other gases.

4. The Minimum Power Principle

According to Raizer [30], ionization waves appear because maintaining a striated plasma state is more efficient than striation-free. Figure 12 compares calculated ionization and loss rates in the striated and striation-free positive column in Argon at $p = 1$ Torr, for the averaged plasma density $\sim 10^{15}$ m$^{-3}$, and the averaged electric field $\langle E \rangle = 17$ V/cm. In these simulations, the electric field $\langle E \rangle$ was the same in the striated and striation-free plasma. However, the average value of the ionization frequency in the striated case is 3.3 times higher than in the striation-free case. Therefore, maintaining the striated plasma is 3.3 times more efficient than the striation-free case.
Since the ionization must be balanced by loss, the calculated radius of the tube for the striated case ($R = 2.5$ mm) is much smaller than for the non-striated case ($R = 6.8$ mm).

The calculated ionization enhancement factor is shown in Figure 12 (on the right) as a function of the electric field $\langle E \rangle$ in Argon and Neon. The ionization enhancement factor is more significant in striated Argon plasma and decreases with increasing $\langle E \rangle$. In Neon, the enhancement factor drops quickly with an increasing electric field. At $\langle E \rangle = 5$ V/cm, there is almost no enhancement, which means that the ionization rate is a linear function of the electric field at these conditions.

![Figure 12: Comparison of the ionization and loss rates in stratified and striation-free Argon plasma (a) and calculated ionization enhancement factors for Argon and Neon as functions of the average electric field $\langle E \rangle$ (b).](image)

These simulations confirm that the minimal power principle applies to moving striations in DC discharges of noble gases at low discharge currents. Sustaining stratified plasma is more efficient than striation-free plasma when the ionization rate is a nonlinear function of the electric field. However, the nonlinear dependence of the ionization rate on the electric field appears unnecessary for plasma stratification. The case of Neon shows that striations of $s$, $p$, and $r$ types could exist with minimal or no ionization enhancement.

### IV. Discussion

Our simulations have confirmed that plasma stratification in DC discharges in noble gases at low values of $pR$ and $i/R$ is due to nonlocal electron kinetics. The spatial non-locality of the EEDF is associated with the presence of the intrinsic spatial scale $\lambda_e = \varepsilon_i / (eE)$ over which the electrons gain kinetic energy equal to the excitation threshold of atoms $\varepsilon_i$. When the spatial scale of the EEDF relaxation is large compared to $\lambda_e$, electrons diffuse in phase space with conservation of their total energy, and nonlocal kinetic effects dominate. The EEDF relaxation in noble gases occurs via energy loss of electrons in elastic collisions with atoms, excitation of several atomic levels, and generation of secondary electrons in the ionization events [31]. The dominant
mechanism depends on the gas type and the discharge conditions (gas pressure, plasma density, and discharge geometry).

Our one-dimensional model has a simplified treatment for radial electron transport. In the actual 2D discharges, only electrons with kinetic energy exceeding the local value of the radial trapping potential can escape to the wall. As the axial transport is controlled by the (fast) free-electron diffusion and the radial loss is due to the (slow) ambipolar diffusion, only small electron multiplication is needed to compensate for the radial loss of electrons during their lifetime between the electrodes. No surprise, we have found that variation of the electron loss term (for the fast electrons) from $\tau_a$ to $\tau_e$ had no effect on the striation properties.

In DC discharges, the frequency of ionization waves is low compared to the characteristic temporal scales, and the EEDF is quasi-static. At the same time, the velocity of the waves is large compared to the ion drift velocity. Therefore, the ion motion in the axial direction has little effect on the wave properties. However, ion mobility controls the rate of ambipolar loss of charged particles to the wall. This effect explains the impact of the ion mobility of plasma stratification in Helium.

The FP kinetic equation (1) used in this paper is valid under conditions when the electric field changes slowly over the electron mean free path $\lambda$ during the time between collisions. When the mean free path $\lambda$ becomes comparable to the tube radius $R$, the solution of the complete Boltzmann equation is required. Simulations of striations at low values of $pR$ (below dashed lines in Fig. 1) require such a treatment.

V. Conclusions

We have obtained moving striations in DC discharges in noble gases (Ne, Ar, He) using a self-consistent hybrid model of collisional discharge plasma. The numerical solution of a 2D kinetic equation for electrons in the (x,u) phase space was obtained self-consistently with the numerical solution of the drift-diffusion equation for ion transport and the Poisson equation for the electric field. Using the grid-based, non-statistical method for solving the electron kinetic equation, we could detect striations at much lower values of $E/p$ compared to the PIC simulations [21]. Large-amplitude striations have been obtained for $E/p$ as low as 9.5 V/cm/Torr in Argon. Our striation onset values for $E/p$ are thus in closer agreement with the linear theory [21].

Our 2-level excitation-ionization model is applicable for the noble gas plasma at low plasma densities. The nonlinear effects caused by gas heating, stepwise ionization, and Coulomb interactions among electrons were neglected. The periodic boundary conditions have been applied to simulate moving striations in DC positive column. Moving striations of the s, p, and r types were obtained in Neon for plasma density $10^{15}$ m$^{-3}$, depending on the $pR$. Only one type (s striations) was obtained in Argon for similar discharge conditions. No striations appeared for Helium for the realistic value of the ion mobility. However, s striations have been obtained in Helium by artificially decreasing the ion mobility by a factor of three. Therefore, the reasons for the experimentally observed stability of Helium plasma to ionization waves have been clarified.

We have shown that in Argon, lower average electric fields can maintain stratified plasma compared to striation-free plasma. This substantiates the principle of minimal power for the kinetic
striations in DC discharges in Argon. However, the nonlinear dependence of the ionization rate on the electric field appears unnecessary for plasma stratification. In our simulations, striations of $s$, $p$, and $r$ types in Neon appeared with minimal or no ionization enhancement.

The previously discussed electron bunching [32] and resonance effects [33,34] in spatially modulated electric fields do not appear critical for the formation of our striations. No Gaussian-like peaks of EEDFs along resonance trajectories have been observed in our simulations with periodic BCs, which correspond to an infinitely long positive column. The generation of electrons in the ionization events controls the formation of nonlocal EEDF in our 2-level excitation-ionization model.

Further development of the model will include metastable atoms and stepwise ionization to explain the two types of experimentally observed ion-guided (fast) and metastable-guided (slow) ionization waves. We also plan to add Coulomb collisions to study the transition between the wave types with increasing the plasma density (discharge current).

Acknowledgments

This work was supported by NSF projects OIA 1655280 and OIA-2148653, and DOE project DE-SC0021391.

References

1. L. D. Tsendin, Nonlocal electron kinetics in gas-discharge plasma, Phys.-Usp. 53 (2010) 133.

2. V I. Kolobov and VA Godyak, Electron kinetics in low-temperature plasmas, Physics of Plasmas 26, 060601 (2019)

3. A A Kudryavtsev, A S Smirnov and L D Tsendin, Physics of Glow Discharge, St Petersburg Lan (2010) 512 p [in Russian]

4. A.V. Nedospasov, Striations, Sov. Phys. Uspekhi 11 (1968) 174.

5. L. Pekarek, Ionization waves (striations) in a discharge plasma. Sov. Phys. Uspekhi 11 (1968) 188.

6. N.L Oleson, and A.W. Cooper, Moving striations. Adv. Electron. El. Phys. 24 (1968) 155–278.

7. V.I. Kolobov, Striations in rare gas plasmas, J. Phys. D: Appl. Phys. 39 (2006) R487

8. Yu B Golubovskii, V O Nekuchaev, A Yu Skoblo, Advances in the study of striations in inert gases, Technical Physics 59 (2014) 1787

9. Y. B. Golubovskii, V. I. Kolobov, and V. O. Nekuchaev, On electron bunching and stratification of glow discharges, Phys. Plasmas 20, 101602 (2013)
10 N A Dyatko, I V Kochetov and V N Ochkin, Influence of the ionization process on characteristics of spatial relaxation of the average electron energy in inert gases in a uniform electric field, Phys. Rev. E 104 (2021) 065204

11 S. Pfau, A. Rutscher, K. Wojaczeck, Beiträge aus der Plasmaphysik 9 (1969) 333; https://doi.org/10.1002/ctpp.19690090406

12 D. Venzke, E. Hayess, K. Wojaczeck, Ähnlichkeitsbeziehungen für Entladungssäulen in Edelgasen bei mittleren Drücken, Beiträge aus der Plasmaphysik 6 (1966) 299; https://doi.org/10.1002/ctpp.19660060507

13 L. Pekarek, Ion waves and ionization waves, Tenth International Conference on Phenomena in Ionized Gases, (1971), Invited papers, edited by P. A. Davenport. Published by Donald Parsons &Co. Ltd., Oxford, England, p.365

14 A V Nedospasov and V D Haim, Fundamentals of the physics of processes in devices with low-temperature plasma, Moscow Energoatomizdat (1991) 224 p [in Russian]

15 A. Garscadden, Ionization Waves in Glow Discharges, in Gaseous Electronics (1978)

16 Y.-X. Liu, E. Schungel, I. Korolov, Z. Donko, Y.-N. Wang and J. Schulze, Experimental observation and computational analysis of striations in electronegative capacitively coupled radio-frequency plasmas, Phys. Rev. Lett. 116 (2016) 255002

17 M. Tahiyat, J. Stephens, V. Kolobov and T. Farouk, Striations in moderate pressure dc driven nitrogen glow discharge, J. Phys. D: Appl. Phys. 55 (2022) 085201

18 R R. Arslanbekov and V I. Kolobov, Advances in simulations of moving striations in DC discharges of noble gases, Physics of Plasmas 26, 104501 (2019)

19 V I Kolobov, R R Arslanbekov, D Levko and V A Godyak, Plasma stratification in radio-frequency discharges in argon gas, J. Phys. D: Appl. Phys. 53 (2020) 25LT01

20 V I Kolobov, J A Guzman and R R Arslanbekov, A self-consistent hybrid model of kinetic striations in low-current argon discharges, Plasma Sources Sci. Technol. 31 (2022) 035020

21 J-P Boeuf, Ionization waves (striations) in a low-current plasma column revisited with kinetic and fluid models, Physics of Plasmas 29, 022105 (2022)

22 C. Yuan, E. A. Bogdanov, S. I. Eliseev, and A. A. Kudryavtsev, 1D kinetic simulations of a short glow discharge in helium, Physics of Plasmas 24, 073507 (2017)

23 U. Kortshagen, C. Busch and L. D. Tsendin, On simplifying approaches to the solution of the Boltzmann equation in spatially inhomogeneous plasmas, Plasma Sources Science and Technology 5 (1996) 1-17
24 V.I. Kolobov; R.R. Arslanbekov, Simulation of electron kinetics in gas discharges, IEEE Transactions on Plasma Science 34 (2006) 895 – 909

25 Gurevich, A.V. and Shwarzburg, A.B., Nonlinear Theory of Radiowave Propagation in the Ionosphere, Moscow: Science, 1973.

26 D. W. Swain and S. C. Brown, Moving Striations in a Low-Pressure Argon Discharge, Physics of Fluids 14, 1383 (1971)

27 M Novák, Spatial period of moving striations as function of electric field strength in glow discharge. Czech. J. Phys. 10, 954 (1960). https://doi.org/10.1007/BF01688343

28 Il’insky V V, PhD Thesis, Moscow State University (1979)

29 H. Amemija, Characteristics of striations of He and Ne plasmas in small-diameter discharge tubes, J. Phys. D: Appl. Phys. 17 (1984) 2387-2398

30 Yu.P. Raizer, Gas discharge physics, Springer-Verlag. (1991) 449 pp; Third edition, Intellect Publishing House, Moscow (2009) 736 pp [in Russian].

31 N A Dyatko, I V Kochetov and V N Ochkin, Influence of the ionization process on characteristics of spatial relaxation of the average electron energy in inert gases in a uniform electric field, Phys. Rev. E 104 (2021) 065204

32 Y. B. Golubovskii, V. I. Kolobov, and V. O. Nekuchaev, On electron bunching and stratification of glow discharges, Phys. Plasmas 20, 101602 (2013)

33 A. Albert et al., Monte Carlo simulation of resonance effects of electron transport in a spatially modulated electric field in Ar, N2 and their mixtures, J. Phys. D: Appl. Phys. 54 (2020)

34 Yu Golubovskii et al, Resonant behavior of the electron component of the plasma and stratification of the positive column, Plasma Sources Sci. Technol. 30 (2021) 115001