Resonant $CP$ violation in rare $\tau^{\pm}$ decays

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Abstract: In this work, we study the lepton number violating tau decays via intermediate on-shell Majorana neutrinos $N_j$ into two scalar mesons and a lepton $\tau^{\pm} \rightarrow M_1^{\pm} N_j \rightarrow M_1^{\pm} M_2^{\pm} \ell^{\mp}$. We calculate the Branching ratios $Br(\tau^{\pm})$ and the CP asymmetry $(\Gamma(\tau^{+}) - \Gamma(\tau^{-}))/\left(\Gamma(\tau^{+}) + \Gamma(\tau^{-})\right)$ for such decays, in a scenario that contains at least two heavy Majorana neutrinos. The results show that the CP asymmetry is small, but becomes comparable with the branching ratio $Br(\tau^{\pm})$ when their mass difference is similar with their decay width $\Delta M_N \sim \Gamma_N$. We also present regions of the heavy-light neutrino mixing elements, in which the $CP$ asymmetry could be explored in future tau factories.

Keywords: Heavy Neutrinos, CP violation, Lepton Number Violation, Tau Decay, Tau Factory.
1 Introduction

During the last decades, neutrino experiments that have shown that neutrinos have non-zero masses [1, 2], also suggest that the first three mass eigenstates are very light with masses $\sim 1$ eV, and the mixing between flavour and mass eigenstates is characterized by the Pontecorvo-Maki-Nakagawa-Sakata Matrix, $U_{\text{PMNS}}$ [3]. Therefore, if these light masses are produced by means of some see-saw mechanism [4, 5], the existence of one or more heavier neutrinos is needed. The current experimental uncertainties in the $U_{\text{PMNS}}$ matrix elements allow introduce these new heavy neutral leptons called sterile neutrinos (SN) [6–10], however the small values of these uncertainties imply a strongly suppressed interaction between standard model (SM) particles and SN. In addition, due to the fact that neutrinos are massive particles, a fundamental question arises: are neutrinos Dirac or Majorana particles?, If neutrinos are Dirac particles, the reactions in which they participate must preserve the lepton number ($\Delta L = 0$). On the contrary, if neutrinos are Majorana particles, they are indistinguishable from their antiparticles, and the lepton number can be violated in two units ($\Delta L = 2$). On the other hand, Neutrino oscillations (NOs) experiments have confirmed that $\theta_{13}$ angle of $U_{\text{PMNS}}$ is non zero [11, 12], thus, the possibility of $CP$ violation in the light neutrino sector is still open; nevertheless, extra sources of $CP$ violation are needed in order to explain Baryogenesis via Leptogenesis [13]. Recent studies explored the $CP$ violation and the phenomenology of SN neutrinos in the context of rare meson decays [14–21], however, in this work we will focus in the phenomenology of the rare
tau decays \cite{22-24} in the framework of tau factories, such as Super Charm-Tau Factory (CTF) in the Budker Institute of Nuclear Physics (Novosibirsk, Russia), \cite{25, 26} making it possible to extend the SN searches to tau decay processes. In this letter we focus in the rare decays of tau leptons into two scalar mesons and one charged lepton ($\ell = e, \mu$), via two on-shell intermediate neutrinos $N_j$, and look for the possibility of detection of CP asymmetries in such decays. The relevant processes are the lepton number violating channels $\tau^\pm \rightarrow M_1^\pm M_2^\pm \ell^\mp$ where $M_1, M_2 = \pi, K$ and $\ell = e, \mu$. We also show that the branching ratios are very small\(^1\), but could be appreciable enough and could be measured in future $\tau$ factories where huge numbers of taus will be produced \cite{26, 27}, if the heavy-light neutrino mixing elements are sufficiently large but still lower than the present upper bounds.

The program of this paper is the following: in Section 2 we present the notation and formalism for the rare tau decay; in Sections 3 we present the relevant expression for the branching ratio calculations; in Sections 4 we present the relevant expression for the $CP$ asymmetries calculations; in Sections 5 we present the results of the relevant parameters for the future searches; finally, in section 6 we present the summary and conclusions.

2 Process and Formalism

As we stated above, we are interested in studying the $\Delta L = 2$ rare tau decays mediated by two on-shell heavy ($0.140 \leq M_N \leq 1.638$ GeV) Majorana neutrinos with the expectation of obtaining $CP$ violating signal in the neutrino sector. The relevant Feynman diagrams of the studied processes are presented in Fig. 1 and Fig. 2 for $\tau^+ \rightarrow M_1^+ M_2^+ \ell^-$ and $\tau^- \rightarrow M_1^- M_2^- \ell^+$, respectively.

\textbf{Figure 1.} Feynmann diagrams for the process $\tau^+ \rightarrow M_1^+ M_2^+ \ell^-$. Left side: Direct channel $D$. Right side: Crossed channel $C$.

\(^1\)Both the branching ratio as $CP$ asymmetries are proportional to the product of square mixing elements $|B_{\tau N}|^2 |B_{\ell N}|^2$
Figure 2. Feynmann diagrams for the process $\tau^- \rightarrow M_1^- M_2^- \ell^+$. Left side: Direct channel $D$. Right side: Crossed channel $C$.

In order to write down the amplitude and all the relevant quantities, we first define the neutrino flavor state as:

$$\nu_\ell = \sum_{i=1}^{3} B_{\ell i} \nu_i + \sum_{j=1}^{n} B_{\ell N_j} N_j ,$$  \hspace{1cm} (2.1)

where $B_{\ell N_j}$ are the elements of the $PMNS$ matrix\(^2\) (heavy-light neutrino mixings elements) which are defined as follow

$$B_{\ell N_j} = |B_{\ell N_j}| e^{i \phi_{\ell N_j}} ,$$  \hspace{1cm} (2.2)

the left side of Eq. (2.1) stands for light neutrino sector and the right side for the heavy neutrino sector. The amplitude for a general process involving $n$ sterile neutrinos is\(^3\)

$$iM_+ \equiv iM(\tau^- \rightarrow M_1^+ M_2^- \ell^-) = M_+^D + M_+^C = \frac{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B_{\tau N_j} P_j(D) L_+^D}{M_+^C} + \frac{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B_{\tau N_j} P_j(C) L_+^C}{M_+^C} ,$$  \hspace{1cm} (2.3a)

$$iM_- \equiv iM(\tau^- \rightarrow M_1^- M_2^+ \ell^-) = M_-^D + M_-^C = \frac{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B_{\tau N_j} P_j(D) L_-^D}{M_-^C} + \frac{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B_{\tau N_j} P_j(C) L_-^C}{M_-^C} ,$$  \hspace{1cm} (2.3c)

where $f_1$ and $f_2$ are the meson decay constants of $M_1^\pm$ and $M_2^\pm$, and $V_{M_1}$, $V_{M_2}$ are the mixings elements of CKM matrix corresponding to mesons $M_1$ and $M_2$, respectively. The factors $L_\pm^D$ and $L_\pm^C$ contain the information related to the kinematics and are given by

$$L_+^D = \bar{u}(p_\ell) \gamma_5 p_1 P_j(D)(1 + \gamma_5)u(p_\tau) ; \hspace{1cm} L_+^C = \bar{u}(p_\ell) \gamma_5 p_2 P_j(C)(1 + \gamma_5)u(p_\tau) ,$$  \hspace{1cm} (2.4)

$$L_-^D = \bar{u}(p_\ell) \gamma_5 p_2 P_j(D)(1 + \gamma_5)u(p_\tau) ; \hspace{1cm} L_-^C = \bar{u}(p_\ell) \gamma_5 p_1 P_j(C)(1 + \gamma_5)u(p_\tau) ,$$  \hspace{1cm} (2.5)

and finally the factors $P_j(D)$ and $P_j(C)$ are the heavy Majorana neutrino propagators

$$P_j(D) = \sum_{j=1}^{n} \frac{M_{N_j}}{(p_\tau - p_1)^2 - M_{N_j}^2 + i \Gamma_{N_j} M_{N_j}} ; \hspace{1cm} P_j(C) = \sum_{j=1}^{n} \frac{M_{N_j}}{(p_\tau - p_2)^2 - M_{N_j}^2 + i \Gamma_{N_j} M_{N_j}} ,$$  \hspace{1cm} (2.6)

\(^2\)Experimental limits for $|B_{\ell N_j}|^2$ in our mass range of interest are presented in figure Fig. 7.

\(^3\)The definitions $M_\pm^D$ and $M_\pm^C$ can be understood as the amplitude for the direct channel and for the crossed one, respectively. Furthermore, the squared amplitude probability for the process will be $|M_\pm|^2 = |M_\pm^D|^2 + |M_\pm^C|^2 + M_\pm^D M_\pm^C + M_\pm^D M_\pm^C$.
here $\Gamma_{N_j}$ is the total decay width of the intermediate neutrinos, and can be approximated as follow

$$\Gamma_{N_j} \approx \mathcal{K}_{j}^{M_{N_j}} \frac{G_F^2 M_{N_j}^5}{96\pi^3}, \quad (2.7)$$

where

$$\mathcal{K}_{j}^{M_{N_j}} \equiv \mathcal{K}_j (M_{N_j}) = N_{e_j} |B_{eN_j}|^2 + N_{\mu_j} |B_{\mu N_j}|^2 + N_{\tau_j} |B_{\tau N_j}|^2, \quad (2.8)$$

the factors $N_{ij}$ being effective mixing coefficients and are presented in Fig. 3 for our mass range of interest.

| $N_{ij}$ | $N_{e_j}$ | $N_{\mu_j}$ | $N_{\tau_j}$ |
|----------|----------|------------|------------|
|          |          |            |            |

**Figure 3.** Effective mixing coefficients. The dashed line (online red) is for $N_{e_j}$, solid line (online blue) for $N_{\mu_j}$ and the dotted one (online black) for $N_{\tau_j}$.

The decay width of the process is given as follow

$$\Gamma(\tau^\pm \rightarrow M_1^\pm M_2^\pm \ell^\mp) \equiv \Gamma(\tau^\pm) = \frac{1}{2!} (2 - \delta_{M_1 M_2}) \frac{1}{2M_\tau} \int \frac{d^4 p_\tau}{(2\pi)^4} \frac{d^3 \vec{p}_1}{2E_1(\vec{p}_1)} \frac{d^3 \vec{p}_2}{2E_2(\vec{p}_2)} \frac{d^3 \vec{p}_\ell}{2E_\ell(\vec{p}_\ell)} \delta^{(4)}(p_\tau - p_1 - p_2 - p_\ell), \quad (2.9)$$

where $\frac{1}{2!} (2 - \delta_{M_1 M_2})$ is the symmetry factor that counts for identical particles in the final states, $d_3$ denotes the number of states available per unit of energy in the 3-body final state\(^4\).

$$d_3 \equiv \frac{d^3 \vec{p}_1}{2E_1(\vec{p}_1)} \frac{d^3 \vec{p}_2}{2E_2(\vec{p}_2)} \frac{d^3 \vec{p}_\ell}{2E_\ell(\vec{p}_\ell)} \delta^{(4)}(p_\tau - p_1 - p_2 - p_\ell), \quad (2.10)$$

here, $p_1$ and $p_2$ denote the momenta of $M_1$ and $M_2$ respectively, and $p_\ell$ the momentum of the charged lepton (see Fig. 1 and Fig. 2).

3 **Branching ratio of $\tau^\pm \rightarrow M_1^\pm N_j \rightarrow M_1^\pm M_2^\pm \ell^\mp$ decays**

In a scenario with $n = 2$ sterile neutrinos, the decay widths presented in Eq.(2.9) can be written as the double sum of the contributions of $N_i$ and $N_j$ ($i, j = 1, 2$), with the mixing

\[^4\]The decomposition of the 3-body phase space is presented in Appendix B.
elements factored out

\[ \Gamma(\tau^\pm) = \frac{1}{2i}(2 - \delta_{M_1 M_2}) \sum_{i=1}^{2} \sum_{j=1}^{2} k_i^{(\pm)} k_j^{(\pm)*} \times \left[ \tilde{\Gamma}_{\tau}(DD^*)_{ij} + \tilde{\Gamma}_{\tau}(CC^*)_{ij} + \tilde{\Gamma}_{\tau\pm}(DC^*)_{ij} + \tilde{\Gamma}_{\tau\pm}(CD^*)_{ij} \right], \tag{3.1} \]

here \( \tilde{\Gamma}'s \) are the canonical decay widths (without heavy-light explicit mixing), and \( k_j^{(\pm)} \) are parameters which contain the corresponding mixing factors and are presented in Eq. (3.2).

\[ k_j^{(\pm)} = B_{tN_j} B_{\tau N_j}, \quad k_j^{(-)} = (k_j^{(\pm)})^*. \tag{3.2} \]

Due to the fact that \( |L^D_+|^2 = |L^D_-|^2 \) and \( |L^C_+|^2 = |L^C_-|^2 \), we can omit the subscripts \( \pm \) in the contribution terms \( \tilde{\Gamma}_{\tau}(DD^*)_{ij} \) and \( \tilde{\Gamma}_{\tau}(CC^*)_{ij} \) in Eq. (3.1). The canonical decay widths \( \tilde{\Gamma}_{\tau\pm}(XY^*)_{ij} \), where \( X, Y \) stand for direct and crossed channel \( (X, Y = C, D) \) and \((i, j = 1, 2)\), are given by

\[ \tilde{\Gamma}_{\tau\pm}(XY^*)_{ij} \equiv K^2_\tau \frac{1}{2M_\tau} \int d_3 P_i(X) P_j(Y)^* L^X_\pm L^{Y\dagger}_\pm, \tag{3.3} \]

where

\[ K^2_\tau = G^4_F f^2_M f^2_{M_\tau} V^2_{ud} V^2_{ts}. \tag{3.4} \]

From now on, we will pay our attention in a scenario where both mesons are equal, then \( M_1 = M_2 \equiv M \) and the constant \( K^2_\tau \equiv K^2_M \) presented in Eq. (3.4) becomes \( K^2_\tau = G^4_F f^4_M V^4_{ud} \) when the mesons are pions and \( K^2_\tau = G^4_F f^4_K V^4_{us} \) when they are kaons. The canonical decay width has been evaluated numerically by means of Monte-Carlo integrations using Vegas algorithm [28]\(^5\). Furthermore, the evaluation were implemented using small \( \Gamma_{N_j} = 10^{-3} \) in the heavy neutrino propagators. The numerical results can be summarized as follows:

i) The contribution of \( (DD^*)_{jj} \) and \( (CC^*)_{jj} \) channels are approximately equal, thus \( \tilde{\Gamma}_{\tau}(DD^*)_{jj} \approx \tilde{\Gamma}_{\tau}(CC^*)_{jj} \).

ii) The contribution of \( (DC^*)_{ij} \) and \( (CD^*)_{ij} \) channels are approximately equal, thus \( \tilde{\Gamma}_{\tau}(DC^*)_{ij} \approx \tilde{\Gamma}_{\tau}(CD^*)_{ij} \).

iii) The terms \( \tilde{\Gamma}_{\tau}(DD^*)_{jj} \propto 1/\Gamma_{N_j} \)\(^6\), while \( \tilde{\Gamma}_{\tau}(DC^*)_{jj} \) and \( \tilde{\Gamma}_{\tau}(CD^*)_{ij} \) are approximately independent of \( \Gamma_{N_j} \).

iv) When \( \Gamma_{N_j} = 10^{-3} \), the terms \( \tilde{\Gamma}_{\tau\pm}(DC^*)_{ii} \) and \( \tilde{\Gamma}_{\tau\pm}(CD^*)_{ii} \) are suppressed by a factor \( \sim 10^{-3} \), besides taking into account the latter point iii), the terms \( \tilde{\Gamma}_{\tau\pm}(DC^*)_{jj} \) and \( \tilde{\Gamma}_{\tau\pm}(CD^*)_{ij} \) are negligible in all cases, in comparison with \( \tilde{\Gamma}_{\tau}(DD^*)_{jj} \) and \( \tilde{\Gamma}_{\tau}(CC^*)_{jj} \).

\(^5\)The integration were performed in two different languages Python and Fortran in order to reduce the uncertainties.

\(^6\)It is important to note that the dependence \( \tilde{\Gamma}_{\tau}(DD^*)_{ij} \propto 1/\Gamma_{N_j} \) is in agreement with the fact that sterile neutrino are weakly interacting particles and therefore the narrow width approximation \( (p_{N_j}^2 - M_{N_j}^2)^2 \rightarrow \frac{\Gamma_{N_j}}{p_{N_j}^2} \) is valid.
v) The contribution of \((DD^*)_{ij}\) and \((CC^*)_{ij}\) channels are approximately equal, and can reach the same order of magnitude than the \((DD^*)_{jj}\) and \((CC^*)_{jj}\) contributions\(^7\).

Thus, under the above considerations and taking into account that \(M_1 = M_2 = M_\tau, M_K\), we rewrite the Eq. (3.1) only in terms of the dominant contributions, as follows

\[
\Gamma(\tau^\pm) = \frac{1}{2!} \sum_{i=1}^{2} \sum_{j=1}^{2} k_i^{(\pm)} k_j^{(\pm)*} \times [\tilde{\Gamma}_r(DD^*)_{ij} + \tilde{\Gamma}_r(CC^*)_{ij}] \tag{3.5a}
\]

\[
= |B_{\ell N_1}|^2 |B_{\tau N_1}|^3 \tilde{\Gamma}_r(DD^*)_{11} + |B_{\ell N_2}|^2 |B_{\tau N_2}|^3 \tilde{\Gamma}_r(DD^*)_{22} + 2|B_{\ell N_1}||B_{\ell N_2}| |B_{\tau N_1}| |B_{\tau N_2}| \tilde{\Gamma}_r(DD^*)_{11} \cos(\theta_{12}) \delta_{12} + 2|B_{\ell N_1}||B_{\ell N_2}| |B_{\tau N_1}| |B_{\tau N_2}| \tilde{\Gamma}_r(DD^*)_{11} \frac{\eta(y)}{y} \sin(\theta_{12}) , \tag{3.5b}
\]

Here \(\delta_{12} = \frac{3|\tilde{\Gamma}_r(DD^*)_{11}|}{\tilde{\Gamma}_r(DD^*)_{11}}\) measures the effect of \(N_1 - N_2\) overlap\(^8\), the factor \(\frac{\eta(y)}{y}\) will be discussed later, however, their values are presented in Fig. 4 and \(\theta_{12} = \phi_{\ell N_1} - \phi_{\ell N_2} + \phi_{\tau N_2} - \phi_{\tau N_1}\). The diagonal canonical decay widths, presented in Eq. (3.5b), can be implemented by means of the narrow width approximation

\[
\tilde{\Gamma}_r(DD^*)_{jj} = \frac{K_i^2}{128\pi^2 M_j^2 M_N \Gamma_{N_j}} \times \lambda^{1/2} \left( 1, \frac{M_j^2}{M_N^2}, \frac{M_N^2}{M_j^2} \right) \times Z(M_\tau, M_N, M_M, M_\ell) , \tag{3.6}
\]

where the functions \(Z(a, b, c, d)\) and \(\lambda(x, y, z)\) are kinematical functions, which are defined in Appendix B. The branching ratio for the process \(\tau^\pm \to M_1^\pm M_2^\mp \ell^\mp\) is

\[
Br(\tau^\pm) = \frac{\Gamma(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})} , \tag{3.7}
\]

where \(\Gamma(\tau^\pm \to \text{all})\) is the total decay width for \(\tau^\pm\) lepton and is given by

\[
\Gamma(\tau^\pm \to \text{all}) = \frac{G_F^2 M_\tau^5}{192\pi^3} . \tag{3.8}
\]

In order to have a more realistic discussion, we must consider the acceptance factor, which is defined as the probability of the neutrino \(N_j\) decay inside of a detector of length \(L\)

\[
P_{N_j} \approx \frac{L}{\gamma_{N_j} \tau_{N_j} \beta_{N_j}} \approx \frac{L \Gamma_{N_j}}{\gamma_{N_j} \beta_{N_j}} \tag{3.9}
\]

where \(\gamma_{N_j}\) is the Lorentz time dilation factor in the Laboratory frame and \(\beta\) is the neutrino speed\(^9\). Therefore, the effective branching ratio\(^{10}\) is

\[
Br^{\text{eff}}(\tau^\pm) = P_{N_j} Br(\tau^\pm) = \frac{\Gamma^{\text{eff}}(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})} = P_{N_j} \frac{\Gamma(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})} . \tag{3.10}
\]

\(^7\)The effect of this kind of interference will be studied later in detail.

\(^8\)\(\Re\) stand for the real part.

\(^9\)In this work, we will provide \(\gamma_{N_j} \sim 2, \beta \sim 1\) and \(L = 1\) mts.

\(^{10}\)The \(Br^{\text{eff}}(\tau^\pm)\) correspond to the real branching ratio, while \(\Gamma^{\text{eff}}(\tau^\pm)\) correspond to the effective decay with, whose can be measured in an experiment.
4  CP Asymmetry of $\tau^{\pm} \to M_1^{\pm} N_j \to M_1^{\pm} M_2^{\pm} \ell^{\mp}$ decays

In this section we will calculate the size of CP asymmetry $A_{CP}$, which is defined as follows

$$A_{CP} = \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{\Gamma(\tau^+) + \Gamma(\tau^-)},$$

(4.1)

The CP violation comes from the complex phases in the transition amplitudes Eq. (2.3a), and the observable effects only arise due to interference of at least two amplitudes. The CP-odd phases are those that come from the Lagrangian of the theory, in other words from the heavy-light mixing elements $(B_{jN})$; these phases change sign between a process and its conjugate. On the other hand, the CP-even phases appear as absorptive parts in the propagators Eq. (2.6) and do not change sign for the conjugate process. In order to have a more phenomenological discussion about CP violation, it is useful define a new quantity $A_{CP} Br^{\text{eff}}(\tau^+)$ which is the corresponding branching ratio for the CP-violating asymmetry\(^{11}\)

$$A_{CP} Br^{\text{eff}}(\tau^+) = \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{2\Gamma(\tau^+ \to \text{all})}$$

(4.2)

The CP-violating difference $\Gamma(\tau^+) - \Gamma(\tau^-)$ is proportional to the imaginary part of $\bar{\Gamma}_\tau(DD^*)_{12}$ and can be written as\(^{12}\)

$$\Gamma(\tau^+) - \Gamma(\tau^-) \approx 4|B_{tN_1}||B_{tN_2}||B_{\tau N_1}||B_{\tau N_2}| \sin \theta_{12} \Im \left[ \bar{\Gamma}_\tau(DD^*)_{12} \right]$$

(4.3)

where we have neglected all the $(DC^*)$ and $(CD^*)$ interference contributions, due to fact that numerical simulation shows that they are strongly suppressed in comparison with $(DD^*)$ and $(CC^*)$. The imaginary part of Eq. (4.3) correspond to the imaginary part of the off-diagonal elements in Eq. (3.5)

$$\Im \left[ \bar{\Gamma}_\tau(DD^*)_{12} \right] = \frac{1}{2M_\tau} \int d_3 P_{1}(D)P_{2}(D)^* |\mathcal{J}^{D}_+|^2.$$

(4.4)

The imaginary part of the product of propagators (see Eq. (A.7b) in Appendix. A) can be expressed using the narrow width approximation as

$$\Im (P_{1}(D)P_{2}(D)^*) = \left( p_{N_1}^2 - M_{N_1}^2 \right) \Gamma_{N_2} M_{N_2} - \Gamma_{N_1} M_{N_1} \left( p_{N_2}^2 - M_{N_2}^2 \right) + \frac{\pi}{M_{N_2}^2 - M_{N_1}^2} \left[ \delta(p_{N_2}^2 - M_{N_2}^2) + \delta(p_{N_1}^2 - M_{N_1}^2) \right]$$

\hspace{1.5cm} (4.5a)

$$\approx \frac{\Delta M_{N_1}}{\Delta M_{N_2}} \frac{\Delta M_{N_2}}{\Delta M_{N_1}} \left[ \delta(p_{N_2}^2 - M_{N_2}^2) + \delta(p_{N_1}^2 - M_{N_1}^2) \right];$$

\hspace{1.5cm} (4.5b)

the validity of Eq. (4.5b) strongly depends on the assumption $\Gamma_{N_j} \ll |\Delta M_N| \equiv M_{N_2} - M_{N_1}$. However, it is useful introduce the parameter $\eta(y)$ where $y \equiv \frac{\Delta M_{N_1}}{\Gamma_{N_j}} = \frac{\Delta M_{N_1}}{\frac{1}{2} (\Gamma_{N_1} + \Gamma_{N_2})}$, which parametrizes any deviation of Eq. (4.5a) when $\Gamma_{N_j} \ll |\Delta M_N|

$$\eta(y) = \frac{\Im \left[ \bar{\Gamma}_\tau(DD^*)_{12} \right]_{\text{NWA}}}{\Im \left[ \bar{\Gamma}_\tau(DD^*)_{12} \right]_{\text{NUM}}}$$

(4.6)

\(^{11}\)In Eq. (4.2) we have used $\Gamma(\tau^+) + \Gamma(\tau^-) \approx 2\Gamma(\tau^+)$. \(^{12}\)Here we assumed the fact that $\Im \left[ \bar{\Gamma}_\tau(DD^*)_{12} \right] \approx \Im \left[ \bar{\Gamma}_\tau(CC^*)_{12} \right]$. 

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In Eq. (4.6) the subscripts NWA and NUM stand for "Narrow Width Approximation" and "Numerical", respectively. The values of $\eta(y)$ were evaluated numerically using finite $\Delta M_N$ and their values are presented in Fig. 4 as a function of $y \equiv \Delta M_N / \Gamma_N$. The general expression of Eq. (4.4) including the $\eta(y)$ parameter and under the assumptions $M_{N_1} + M_{N_2} \approx 2 M_N$ is given by\(^{13}\)

$$
\Im \left[ \hat{\Gamma}_\tau (DD^*)_{12} \right] \approx \eta(y) \frac{K_3^2}{128 \pi^2 M_N^2 \Delta M_N} \times \lambda^{1/2} \left( 1, \frac{M^2}{M^2_N}, \frac{M^2_\tau}{M^2_N} \right) \times Z(M_\tau, M_N, M_M, M_\ell)
$$

(4.7)

finally, the CP-violating difference becomes

$$
\Gamma(\tau^+) - \Gamma(\tau^-) \approx \eta(y) \frac{K_2^2 |B_{\ell N_1}| |B_{\ell N_2}| |B_{\tau N_1}| |B_{\tau N_2}|}{32 \pi^2 M_N^5 M_N \Delta M_N} \sin \theta_{12}
$$

$$
\times \lambda^{1/2} \left( 1, \frac{M^2_\tau}{M^2_N}, \frac{M^2_M}{M^2_N} \right) \times Z(M_\tau, M_N, M_M, M_\ell)
$$

(4.8)

Figure 4. Solid line (online red) overlap function $\delta_{12}$. Dashed line (online blue) $\eta(y)$ function. Dotted line (online black) $\eta(y)/y$ function.

From Eq. (4.1), Eq. (4.8) and Fig. 4 we can conclude that the best scenario for simultaneous maximization of $ACP$ and $Br(\tau)$, occurs when $y = 1$. From now on, we will focus in a scenario where heavy neutrinos are almost degenerate $\Delta M_N \sim \Gamma_N$; within this context we have assumed $|B_{\ell N_1}| \approx |B_{\ell N_2}| \equiv |B_{\ell N}|$, where $\ell = e, \mu, \tau$ and the mixing elements are $K_{1 M}^a \approx K_{2 M}^a \equiv K_{M}^a$, therefore, the CP asymmetry becomes

$$
A_{CP} \approx \eta(y) \frac{\Gamma_N}{\Delta M_N} \sin \theta_{12} \equiv \frac{\eta(y)}{y} \frac{\sin \theta_{12}}{1 + \delta_{12} \cos \theta_{12}},
$$

(4.9)

\(^{13}\)Due to the fact that $\Gamma_N \sim K_{M}^a \sim |B_{\ell N}|^2$ the mass difference becomes $\Delta M_N \ll 1$, hence the assumption $M_{N_1} + M_{N_2} \approx 2 M_N$ is reasonable in Eq. (4.7).
consequently

\[
A_{\text{CP}} \times \text{Br}^{\text{eff}}(\tau^+) \approx \frac{\eta(y)}{y} \frac{L}{\gamma_N} |B_{\ell N}|^2 |B_{\tau N}|^2 \sin \theta_{12} \times \frac{3\pi K^2_M}{2G^2_F M^5_N M^3_N} \\
\times \chi^{1/2} \left(1, \frac{M^2_F}{M^2_N}, \frac{M^2_M}{M^2_N}, \frac{M^2_F}{M^2_M} \right) \times Z(M_{\tau}, M_N, M_M, M_\ell). \tag{4.10}
\]

There is just one caveat in the expressions above: we have disregarded the effect of \( N_1 - N_2 \) oscillation, these type of oscillations have been studied in detail in Ref. [19] and it is straightforward to show that the \( L \) dependent effective differential decay width is\(^{14}\)

\[
\frac{d}{dL} \Gamma_{\text{eff}}^{(\text{osc})}(\tau^+ \to \pi^+ \pi^+ \mu^-; L) \approx \frac{1}{\gamma_N \beta_N} \Gamma(\tau^+ \to \pi^+ N) \Gamma(N \to \pi^+ \mu^-) \\
\times \left\{ \sum_{j=1}^{2} |B_{\mu N_j}|^2 |B_{\tau N_j}|^2 + 2 |B_{\mu N_1}| |B_{\tau N_1}| |B_{\mu N_2}| |B_{\tau N_2}| \cos \left(\frac{L \Delta M_N}{\beta_N \gamma_N} + \theta_{12}\right) \right\} \tag{4.11}
\]

where \( \Gamma(\tau^+ \to \pi^+ N) \) and \( \Gamma(N \to \pi^+ \mu^-) \) are kinematical functions presented in appendix A. In Eq. (4.11) it is also possible to notice that the oscillation length is \( L_{\text{osc}} = \frac{2\pi \beta_N \gamma_N}{\Delta M_N} \).

Then, the argument of cosine in Eq. 4.11 can be written as \( 2\pi \frac{L_{\text{osc}}}{\gamma_N} \theta_{12} \), therefore, in order to integrate out there are two possible scenarios:

1. \( L \gg L_{\text{osc}} \): In this regime we recover the main contributions of the \( L \)-independent effective decay width (Eq. (3.10)), because the oscillation term \( \sim \cos(f(L) + \theta_{12}) \) gives a relatively negligible contribution when integrated over several \( L_{\text{osc}} \).

2. \( L \approx L_{\text{osc}} \): In this scenario the integration of expression 4.11 is

\[
\Gamma_{\text{eff}}^{(\text{osc})}(\tau^+ \to \pi^+ \pi^+ \mu^-; L) \approx \frac{L}{\gamma_N \beta_N} \Gamma(\tau^+ \to \pi^+ N) \Gamma(N \to \pi^+ \mu^-) \times \left( \sum_{j=1}^{2} |B_{\mu N_j}|^2 |B_{\tau N_j}|^2 \\
+ \frac{L_{\text{osc}}}{\pi L} |B_{\mu N_1}| |B_{\tau N_1}| |B_{\mu N_2}| |B_{\tau N_2}| \left( \sin \left(\frac{2\pi \frac{L}{L_{\text{osc}}} + \theta_{12}}{\gamma_N} \right) - \sin \left(\theta_{12}\right) \right) \right), \tag{4.12}
\]

in 4.12 we can see, immediately, that when \( L_{\text{osc}} \gg L \) and \( L_{\text{osc}} = L \) the oscillation effect disappear and we recover the \( L \)-independent main contributions of the Eq. (3.10). On the other hand, when \( L \sim L_{\text{osc}} \) neutrinos have traveled enough to have a well-defined oscillation, which means that neutrinos have not decayed yet (i.e. \( \frac{P_N}{\gamma_N} \ll 1 \)). Moreover, \( L \sim L_{\text{osc}} \) means \( y \equiv \frac{\Delta M_N}{\gamma_N} \approx \frac{2\pi}{P_N} \gg 1 \) and then from Fig. 4 we notice that \( y \gg 1 \) destroy the effect of resonant CP violation. Therefore, the fact that disregard the \( N_1 - N_2 \) oscillation when we have chosen \( \eta(y) \sim 1 \) is valid.

It is important to note that the oscillation effect is present when \( L \sim L_{\text{osc}} \), therefore, in general CP violating scenarios (i.e. when we are off CP resonant region) this must be taken into account.

\(^{14}\)In Eq. (4.11) \( L \) is the distance between production vertex and detector; the quantities \( \gamma_N \) and \( \beta_N \) are: \( \gamma_N = \frac{1}{2} (\gamma_{N_1} + \gamma_{N_2}) \) and \( \beta_N = \frac{1}{2} (\beta_{N_1} + \beta_{N_2}) \), respectively.
5 Results

In this section the main results obtained in this work will be applied in order to provide a clue for future searches in tau factories. The result for the effective branching ratios presented in Eq. (3.10) are shown in Fig. 5 and Fig. 6.

![Figure 5](image)

**Figure 5.** Effective branching ratios per unit of $|B_{eN}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\cos \theta_{12} = 1/\sqrt{2}$, overlap factor $\delta_{12} = 0.5$, detector length $L = 1$ mts, neutrino speed $\beta = 1$ and Lorentz factor $\gamma_N = 2$.

![Figure 6](image)

**Figure 6.** Effective branching ratios per unit of $|B_{\mu N}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\cos \theta_{12} = 1/\sqrt{2}$, overlap factor $\delta_{12} = 0.5$, detector length $L = 1$ mts, neutrino speed $\beta = 1$ and Lorentz factor $\gamma_N = 2$.

The difference between the cases with $M_M = \pi$ and $M_M = K$ in the final states is mainly due to the elements of $CKM$ matrix, whereas for pions $V_\pi \approx 0.97$ and $V_K \approx 0.22$, respectively. Moreover, the values of meson decay constant are $f_\pi \approx 0.13$ GeV and $f_K \approx 0.15$ GeV, therefore $K_\pi^2/K_\pi^2 \approx 2 \times 10^2$. In order to estimate the region of heavy-light mixings elements $|B_{eN}|^2|B_{\tau N}|^2$ which can be explored in future experiment\textsuperscript{15} we define the

\textsuperscript{15}The Eq. (5.1) is presented in order to detect at least 1 event of difference between $Br(\tau^+)$ and $Br(\tau^-)$, here we have chosen $\eta(y)/y = 1/2$. 

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following relation

\[ A_{CP} B_{\tau^+} (\tau^+) \times N_{\tau} \geq 1 \Rightarrow |B_{\ell N}|^2 |B_{\tau N}|^2 \geq \frac{\gamma_N}{L_N \sin \theta_{12} S(M_N)} , \]  

(5.1)

here \( N_{\tau} \) is the number of \( \tau \) lepton produced in an experiment and \( S(M_N) \) is given by

\[ S(M_N) = \frac{3\pi K_M^2}{4G_F M^8_N} \lambda^{1/2} \left( 1, \frac{M^2_M}{M^2_N}, \frac{M^2_M}{M^2_N} \right) \times Z(M_\tau, M_N, M_M, M_\ell) . \]  

(5.2)

The actual experimental limits for heavy-light mixing elements are given in Ref. [29], and we have summarized them in Fig. 7(a) for the range of mass of interest. On the other hand, and due to the fact that our results depend on \( |B_{\tau N}|^2 |B_{\ell N}|^2 \), we present in Fig. 7(b) the product of the experimental limits of interest.

The CTF in Novosibirsk, Russia is expected to collect \( 10^{10} \) pairs of \( \tau^\pm \) leptons after few years of operation [26], therefore under the latter considerations we can estimate the mixing region that can be explored in such experiment, this region is presented in Fig. 8.
Figure 8. The shaded region (online green) show the limits over the mixings parameter which could be reached in the future $\tau^\pm$ factory [26]. Right side: Limits for $|B_{eN}|^2|B_{\tau N}|^2$. Left side: Limits for $|B_{\mu N}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\eta(y)/y = 0.5$, $N_\tau = 10^{10}$, $\cos \theta_{12} = 1/\sqrt{2}$, $L = 1$ mts, $\beta = 1$ and $\gamma_N = 2$.

It is important to point out that due to the CKM elements suppresion only channels with pions in the final state offer real possibilities to constrain the heavy-light mixings parameters.

6 Summary and Conclusions

In this letter we studied the ($\Delta L = 2$) rare tau decays $\tau^\pm \to M_1^\pm M_2^\pm \ell^\mp$, where $M_1$ and $M_2$ are pseudo scalar mesons ($M_1$, $M_2 = \pi, K$) and the charged lepton can be $\ell = e, \mu$, also we studied the possibility of $CP$ violation detection in future tau factories. We have assumed that the decays occur via the exchange of two on-shell sterile neutrinos $N_j$ at tree level, and we have shown that the amplitude of these processes is suppressed by the mixing elements of the PMNS matrix $|B_{\tau N}|^2|B_{\ell N}|^2$. The aforementioned $CP$ violation effects come from the interference between the $N_1$ and $N_2$ propagators and the complex phases ($CP$-odd phases $\phi_{\ell N_j}$, see Eq. (2.2)) in the PMNS mixing matrix. Our results shows that these signals of $CP$ violation could be detected in future tau factories for $\tau^\pm \to \pi^\pm \pi^\mp \ell^\mp$ tau decays, where $\ell = e, \mu$ if there exist, at least, two sterile neutrinos in the on-shell mass range, their masses are almost degenerate $\Delta M_N \sim \Gamma_N$, the $CP$ odd phases $\sin \theta_{12} \ll 1$ and the mixing parameters are in the allowed region of Fig. 7. In such a case, the CP-violating difference $\Gamma(\tau^+) - \Gamma(\tau^-)$ becomes large and comparable with $\Gamma(\tau^+) + \Gamma(\tau^-)$ and the corresponding CP asymmetry $A_{CP}$ becomes $A_{CP} \sim 1$. In addition, there exist several models with quasi-degeneracy $\Delta M_N \sim \Gamma_N$, between them it is worth to mention the well-know $\nu$MSM model [30, 31], where the quasi-degeneracy of the two heavy neutrinos (with mass $M_{N_j} \sim 1$ GeV) is fundamental in order to get a successful dark matter candidate. However, our results can be framed in the context of the $\nu$MSM model or more general models [21, 32] with at least two quasi-degenerate neutrinos.
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A Amplitude and kinematic relations for $\tau^\pm \rightarrow M_1^\pm M_2^\pm \ell^\pm$

The amplitude for the process via two on-shell intermediate heavy neutrino is

$$|M_+|^2 = K_+^2 \left[ |B_{11}|^2 |B_{11}|^2 \left( |P_1(D)|^2 |L_+^D|^2 + |P_1(C)|^2 |L_+^C|^2 \right) 
   + |B_{21}|^2 |B_{21}|^2 \left( |P_2(D)|^2 |L_+^D|^2 + |P_2(C)|^2 |L_+^C|^2 \right) 
   + 2|B_{11}| |B_{12}| |B_{21}| |B_{22}| \cos \theta_2 \left( \Re[P_1(D)P_2(D)^*] |L_+^D|^2 + \Re[P_1(C)P_2(C)^*] |L_+^C|^2 \right) 
   + 2|B_{11}|^2 |B_{21}|^2 \Re[P_1(D)P_2(C)^*] L_+^D L_+^C 
   + B_{11} B_{21}^* B_{22} B_{21} \left( P_2(D) P_2(C)^* L_+^D L_+^C + P_2(C) P_2(D)^* L_+^C L_+^D \right) \right] \quad (A.1)$$

$$|M_-|^2 = K_-^2 \left[ |B_{11}|^2 |B_{11}|^2 \left( |P_1(D)|^2 |L_-^D|^2 + |P_1(C)|^2 |L_-^C|^2 \right) 
   + |B_{21}|^2 |B_{21}|^2 \left( |P_2(D)|^2 |L_-^D|^2 + |P_2(C)|^2 |L_-^C|^2 \right) 
   + 2|B_{11}| |B_{12}| |B_{21}| |B_{22}| \cos \theta_2 \left( \Re[P_1(D)P_2(D)^*] |L_-^D|^2 + \Re[P_1(C)P_2(C)^*] |L_-^C|^2 \right) 
   + 2|B_{11}|^2 |B_{21}|^2 \Re[P_1(D)P_2(C)^*] L_-^D L_-^C 
   + B_{11} B_{21}^* B_{22} B_{21} \left( P_2(D) P_2(C)^* L_-^D L_-^C + P_2(C) P_2(D)^* L_-^C L_-^D \right) \right]. \quad (A.2)$$

The kinematical factors presented in Eq. (2.3a), Eq. (A.1) and Eq. (A.2) are given by

$$|L_+^D|^2 = |L_-^D|^2 = 32(p_1 \cdot p_2)(p_2 \cdot p_3)(p_3 \cdot p_1) - 16M_2^2(p_1 \cdot p_3)(p_1 \cdot p_2) - 16M_2^2(p_1 \cdot p_2)(p_2 \cdot p_3) + 8M_2^4(p_3 \cdot p_1) \quad (A.3)$$

$$|L_+^C|^2 = |L_-^C|^2 = 32(p_1 \cdot p_2)(p_1 \cdot p_3)(p_2 \cdot p_3) - 16M_2^2(p_2 \cdot p_3)(p_2 \cdot p_1) - 16M_2^2(p_1 \cdot p_3)(p_1 \cdot p_2) + 8M_2^4(p_2 \cdot p_1) \quad (A.4)$$

$$L_+^D L_+^C = \mp i\epsilon_{p_1 p_2 p_3 p_4} (p_1 \cdot p_2) + 16M_2^2(p_1 \cdot p_4)(p_1 \cdot p_2) + 16M_2^2(p_2 \cdot p_3)(p_2 \cdot p_4) + 16(p_1 \cdot p_2)^2(p_2 \cdot p_4) - 16(p_1 \cdot p_2)(p_2 \cdot p_3)(p_2 \cdot p_1) - 16(p_1 \cdot p_2)(p_1 \cdot p_3)(p_2 \cdot p_4) \quad (A.5)$$

$$L_+^D L_+^C = \left( L_+^D L_+^C \right)^* \quad (A.6)$$
The product of propagators \( P_1(X)P_2(X)^\ast \) (where \( X = D, C \)) can be expressed as the sum of the real and imaginary parts

\[
P_1(X)P_2(X)^\ast = M_{N_1}M_{N_2} \left( \frac{(P_{N_1}^2(X) - M_{N_1}^2) + \Gamma_{N_1} \Gamma_{N_2} M_{N_1} M_{N_2}}{(P_{N_1}^2(X) - M_{N_1}^2)^2 + \Gamma_{N_1}^2 M_{N_1}^2} \right)
\]

\[
- i M_{N_1} M_{N_2} \left( \frac{(P_{N_1}^2(X) - M_{N_1}^2) \Gamma_{N_1} - (P_{N_2}^2(X) - M_{N_2}^2) \Gamma_{N_2}}{(P_{N_1}^2(X) - M_{N_1}^2)^2 + \Gamma_{N_1}^2 M_{N_1}^2} \right)
\]

The partial decay widths presented in Eq. (4.11) are:

\[
\Gamma(\tau^\pm \to \pi^\pm N) = \frac{1}{8\pi} G_f^2 f_\pi^2 |V_{\pi}|^2 \frac{1}{M_\tau} \lambda^{1/2} \left( 1, \frac{M_\pi^2}{M_N^2} \frac{M_N^2}{M_\tau^2} \right) \times \left[ \left( M_\pi^2 - M_N^2 \right)^2 - M_\pi^2 \left( M_\pi^2 + M_N^2 \right) \right], \quad (A.8a)
\]

\[
\Gamma(N \to \mu^+\pi^-) = \frac{1}{16\pi} G_f^2 f_\pi^2 |V_{\pi}|^2 \frac{1}{M_N} \lambda^{1/2} \left( 1, \frac{M_\pi^2}{M_N^2} \frac{M_\mu^2}{M_N^2} \right) \times \left[ \left( M_N^2 + M_\mu^2 \right) \left( M_N^2 - M_\pi^2 + M_\mu^2 \right) - 4M_N^2 M_\mu^2 \right]. \quad (A.8b)
\]

**B Phase space relations**

The integration presented in Eq. (2.9) can be performed in the following way:

\[
\Gamma(\tau^\pm) = \frac{1}{2!} (2 - \delta_{M_1 M_2}) \frac{1}{64\pi^3 M_\tau} \int |M_\pm|^2 dE_1 dE_2;
\]

the integration limits over \( E_2 \) and \( E_1 \) for the \((DD^\ast)\) channel are

\[
E_2 \geq \frac{1}{2m_{23}^2} \left( (M_\tau - E_1)(m_{23}^2 + M_2^2 - M_3^2) - \sqrt{(E_1^2 - M_2^2)\lambda(m_{23}^2, M_2^2, M_3^2)} \right), \quad (B.2)
\]

\[
E_2 \leq \frac{1}{2m_{23}^2} \left( (M_\tau - E_1)(m_{23}^2 + M_2^2 - M_3^2) + \sqrt{(E_1^2 - M_2^2)\lambda(m_{23}^2, M_2^2, M_3^2)} \right), \quad (B.3)
\]

\[
M_1 \leq E_1 \leq \frac{M_\tau^2 + M_2^2 - (M_2 + M_3)^2}{2M_\tau} \quad \text{,} \quad (B.4)
\]

where

\[
m_{23}^2 = M_\tau^2 + M_1^2 - 2M_\tau E_1 \quad \text{.} \quad (B.5)
\]

Finally, the kinematical functions \( \lambda(x, y, z) \) and \( Z(a, b, c, d) \) are
\[ \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \]  \hspace{1cm} (B.6)

\[
Z(a, b, c, d) = \left( (b^2 - d^2)^2 - c^2(d^2 + b^2) \right) \left( (a^2 - b^2)^2 - c^2(b^2 + a^2) \right) \\
\times \sqrt{\left( a^2 - (b - c)^2 \right) \left( a^2 - (b + c)^2 \right)} \]  \hspace{1cm} (B.7)
References

[1] Y. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 81, 1562 (1998), arXiv:hep-ex/9807003 [hep-ex] .

[2] K. Eguchi et al. (KamLAND), Phys. Rev. Lett. 90, 021802 (2003), arXiv:hep-ex/0212021 [hep-ex] .

[3] Z. Maki, M. Nakagawa, and S. Sakata, Prog.Theor.Phys. 28, 870 (1962).

[4] R. N. Mohapatra et al., Rept. Prog. Phys. 70, 1757 (2007), arXiv:hep-ph/0510213 [hep-ph] .

[5] R. N. Mohapatra and A. Y. Smirnov, Elementary particle physics. Proceedings, Corfu Summer Institute, CORFU2005, Corfu, Greece, September 4-26, 2005, Ann. Rev. Nucl. Part. Sci. 56, 569 (2006), arXiv:hep-ph/0603118 [hep-ph] .

[6] S. Antusch, J. P. Baumann, and E. Fernandez-Martinez, Nucl. Phys. B810, 369 (2009), arXiv:0807.1003 [hep-ph] .

[7] M. Malinsky, T. Ohlsson, and H. Zhang, Phys. Rev. D79, 073009 (2009), arXiv:0903.1961 [hep-ph] .

[8] P. B. Dev and R. Mohapatra, Phys.Rev. D81, 013001 (2010), arXiv:0910.3924 [hep-ph] .

[9] D. Forero, S. Morisi, M. Tortola, and J. Valle, JHEP 1109, 142 (2011), arXiv:1107.6009 [hep-ph] .

[10] A. Das and N. Okada, (2017), arXiv:1702.04668 [hep-ph] .

[11] J. Beringer et al. (Particle Data Group), Phys.Rev. D86, 010001 (2012).

[12] M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, JHEP 1212, 123 (2012), arXiv:1209.3023 [hep-ph] .

[13] A. Strumia, in Particle physics beyond the standard model. Proceedings, Summer School on Theoretical Physics, 84th Session, Les Houches, France, August 1-26, 2005 (2006) pp. 655–680, arXiv:hep-ph/0608347 [hep-ph] .

[14] C. Dib, V. Gribanov, S. Kovalenko, and I. Schmidt, Phys. Lett. B493, 82 (2000), arXiv:hep-ph/0006277 [hep-ph] .

[15] G. Cvetic, C. Dib, and C. S. Kim, JHEP 06, 149 (2012), arXiv:1203.0573 [hep-ph] .

[16] G. Cvetic, C. Kim, and J. Zamora-Saa, J.Phys. G41, 075004 (2014), arXiv:1311.7554 [hep-ph] .

[17] G. Cvetic, C. Kim, and J. Zamora-Saa, Phys.Rev. D89, 093012 (2014), arXiv:1403.2555 [hep-ph] .

[18] G. Cvetic, C. Dib, C. S. Kim, and J. Zamora-Saa, Symmetry 7, 726 (2015), arXiv:1503.01358 [hep-ph] .

[19] G. Cvetic, C. S. Kim, R. Kogerler, and J. Zamora-Saa, Phys. Rev. D92, 013015 (2015), arXiv:1505.04749 [hep-ph] .

[20] C. O. Dib, M. Campos, and C. Kim, JHEP 1502, 108 (2015), arXiv:1403.8009 [hep-ph] .

[21] G. Moreno and J. Zamora-Saa, Phys. Rev. D94, 093005 (2016), arXiv:1606.08820 [hep-ph] .

[22] J. C. Helo, S. Kovalenko, and I. Schmidt, Nucl. Phys. B853, 80 (2011), arXiv:1005.1607 [hep-ph] .
[23] V. Gribanov, S. Kovalenko, and I. Schmidt, Nucl. Phys. B607, 355 (2001), arXiv:hep-ph/0102155 [hep-ph].

[24] J. C. Helo, S. Kovalenko, and I. Schmidt, Phys. Rev. D84, 053008 (2011), arXiv:1105.3019 [hep-ph].

[25] E. Levichev, 7th International Scientific Workshop to the memory of Prof. V.P. Sarantsev: Problems of Charged Particle Accelerators: Electron-Positron Colliders Alushta, Crimea, Ukraine, September 2-8, 2007, Phys. Part. Nucl. Lett. 5, 554 (2008).

[26] S. Eidelman, Proceedings, 13th International Workshop on Tau Lepton Physics (TAU 2014): Aachen, Germany, September 15-19, 2014, Nucl. Part. Phys. Proc. 260, 238 (2015).

[27] A. E. Bondar, Physics of Atomic Nuclei 76, 1072 (2013).

[28] G. P. Lepage, (1980).

[29] A. Atre, T. Han, S. Pascoli, and B. Zhang, JHEP 0905, 030 (2009), arXiv:0901.3589 [hep-ph].

[30] T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys.Lett. B631, 151 (2005), arXiv:hep-ph/0503065 [hep-ph].

[31] T. Asaka and M. Shaposhnikov, Phys.Lett. B620, 17 (2005), arXiv:hep-ph/0505013 [hep-ph].

[32] M. Drewes and B. Garbrecht, (2015), arXiv:1502.00477 [hep-ph].