Shot noise as a tool to probe an electron energy distribution

O. M. Bulashenko *, J. M. Rubí

Departament de Física Fonamental, Universitat de Barcelona, Av. Diagonal 647, E-08028 Barcelona, Spain

Abstract

We discuss the possibility to employ the shot-noise measurements for the analysis of the energy resolved ballistic currents. Coulomb interactions play an essential role in this technique, since they lead to the shot-noise-suppression level which depends on the details of the energy profile.

Key words: Shot-noise suppression, Space-charge-limited ballistic transport, Long-range Coulomb correlations, Self-consistent electric field, Child law

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1 Introduction

Recently, shot-noise measurements are emerging as an important tool to obtain information on the transport properties and interactions among carriers in quantum structures [1,2]. Since Coulomb repulsion between electrons and their fermionic nature can regulate their motion, this effect may be detected in the shot-noise reduction (in respect to the Poissonian value), but cannot be deduced from time-averaged dc measurements. In particular, shot noise has been used as a tool to probe: fractional charge [3], effective superconducting charge [4], quantum transmission modes in atomic-size contacts [5], mechanisms of tunneling [6], etc. (see, e.g., recent review [2]).

In this contribution, we discuss the possibility to employ the shot-noise measurements to obtain information on the energy distribution of nonequilibrium carriers injected from a contact emitter. Using ballistic electrons to study

* Corresponding author.

Email address: oleg@ffn.ub.es (O. M. Bulashenko).
nanoscale structures has recently been a very active research area. In a usual technique called “hot-electron spectroscopy” [7,8], carriers injected from an emitter contact are analyzed in a collector contact by means of a barrier which is transparent only for the carriers having energy greater than the barrier height. By changing the bias on the collector barrier, the electron energy profile can be analyzed. This technique requires the design of a special collector filter for getting information on the electron energies. Here, we propose a simpler method which does not require a design of the filter, rather it employs a “natural” filter: the potential barrier that appears due to an injected space charge. This space charge limits the current producing the resistance effect by means of a barrier, which reflects a part of the injected carriers back to the emitter. The height of the barrier depends on the screening parameter of the material, and it varies with the external bias. The essential difference with the case of a fixed barrier is that the space-charge barrier fluctuates in time and produces the long-range Coulomb correlations between the transmitted electrons, that leads to the significant suppression of shot noise registered at the collector contact. The level of suppression depends drastically on the energy profile of the injected carriers [9], while the time-averaged quantities (the mean current, conductance, etc.) do not.

![Diagram](image)

**Fig. 1.** Band-energy diagram for a ballistic conductor under a space-charge-limited regime. At equilibrium, the potential space-charge barrier is symmetrical, while under the applied bias $U$ it is closer to the emitter contact, and its magnitude $\Phi_b$ decreases with bias. The electron density (shadowed regions) increases near the emitter contact when the bias is enhanced.
2 Shot-noise suppression by Coulomb correlations

We consider the current injection into a ballistic conductor at high biases (Fig. 1). To adequately describe the transport and noise under the nonlinear far-from-equilibrium conditions, one has to solve the transport equation coupled self-consistently with a Poisson equation [9]. The self-consistent built-in field determines the potential barrier \( \Phi_b \), at which electrons are either reflected or transmitted depending on their energy. The steady-state ballistic current \( I \) is determined by the electrons passed over the barrier (tunneling is neglected):

\[
I = \frac{q}{2\pi\hbar} \int_{\Phi_b}^{\infty} N(\varepsilon) \, d\varepsilon,
\]

where \( q \) is the electron charge, \( N(\varepsilon) = A \int f(\varepsilon, k_\perp) d k_\perp / (2\pi)^{d-1} \) is the number of occupied transversal modes, \( A \) is the cross-sectional area, \( d \) is the dimension of a momentum space and \( f(\varepsilon, k_\perp) \) is the occupation number of a quantum state of a longitudinal energy \( \varepsilon \) and a transverse momentum \( k_\perp \) at the emitter. Note that the differential conductance for this kind of emitter is given by \( G = c \left( q^2/2\pi\hbar \right) N(\Phi_b) \), where \( c(U, \ell) \) is the Coulomb interaction factor dependent on the bias \( U \) and the length of the ballistic region \( \ell \).

Since there is no scattering, the occupation of transversal modes is conserved during the motion, and the only source of noise is the fluctuations of the occupation numbers at the emitter modulated by the potential-barrier fluctuations \( \delta \Phi_b \). The barrier fluctuations induce the long-range Coulomb correlations, and they are found from the Poisson equation [9]. The result may be summarized by introducing the current-fluctuation transfer function \( \gamma(\varepsilon) \) as follows:

\[
\delta I = \frac{q}{2\pi\hbar} \left[ \int_{\Phi_b}^{\infty} \delta N(\varepsilon) \, d\varepsilon - N(\Phi_b) \, \delta \Phi_b \right] = \frac{q}{2\pi\hbar} \int_{\Phi_b}^{\infty} \gamma(\varepsilon) \, \delta N(\varepsilon) \, d\varepsilon.
\]

As a result of the long-range Coulomb interactions, the transmission for different energies differs by the factor \( \gamma(\varepsilon) \). Thus, the low-frequency current-noise spectral density is

\[
S_I = \int_{\Phi_b}^{\infty} \gamma^2(\varepsilon) \, K(\varepsilon) \, d\varepsilon,
\]

where \( K \) is given through the correlator \( \langle \delta N(\varepsilon) \delta N(\varepsilon') \rangle = (q/2\pi\hbar)^2 K(\varepsilon) \, (\Delta f) \, \delta(\varepsilon - \varepsilon') \), with \( \Delta f \) the frequency bandwidth. At high biases \( qU \gg \Phi_b \), we find the asymptotic formula for the function \( \gamma \)
\[ \gamma(\varepsilon) = \frac{3}{\sqrt{qU}} \left[ \sqrt{\varepsilon - \Phi_b - v_\Delta} \right], \]  

(4)

where

\[ v_\Delta = \frac{1}{N(\Phi_b)_{\Phi_b}} \int_{\Phi_b}^{\infty} \left( -\frac{\partial N}{\partial \varepsilon} \right) \sqrt{\varepsilon - \Phi_b} \, d\varepsilon. \]  

(5)

It is important to highlight that the current noise \( (3) \) depends on the details of the distribution function \( N(\varepsilon) \) at the emitter through the parameter \( v_\Delta \). The function \( \gamma(\varepsilon) \) plays the role of the energy-resolved shot-noise suppression factor.

### 3 Example

To illustrate the implementation of the results, we consider the emitter in which the injection energy profile has a peak at energy \( \varepsilon_0 \) superimposed on a wide background of 3D Fermi-Dirac (FD) electrons (see Inset in Fig. 2). Under the assumption that the width of the peak is narrow on the scale of the temperature \( T \), the number of the occupied modes for each longitudinal energy \( \varepsilon \) can be written as

\[ N(\varepsilon) = \frac{mk_B T}{2\pi \hbar^2} \left[ \ln\left\{ 1 + \exp[(\varepsilon_F - \varepsilon)/k_B T] \right\} + \tilde{a} k_B T \, \delta(\varepsilon - \varepsilon_0) \right] \]  

(6)

where \( \varepsilon_F \) is the Fermi energy, and \( \tilde{a} \) is the dimensionless peak magnitude. It is known that the mean current for ballistic injection in the limit of high biases is described by the Child law independently of the distribution function [9]. For the distribution (6), the steady-state current (1) is obtained as \( (\varepsilon_0 > \Phi_b \) is assumed)

\[ I = \frac{q}{2\pi \hbar} N_S \frac{k_B T}{\xi} \left[ F_1(\xi - \Phi_b/k_B T) + \tilde{a} \right], \]  

(7)

where \( N_S = \frac{k_F^2 A}{(4\pi)} \), \( k_F^2 = 2m\varepsilon_F/\hbar^2 \), \( \xi = \varepsilon_F/k_B T \), and we denote the Fermi-Dirac integrals of index \( j \) by \( F_j(\alpha) = \int_0^\infty x^j [1 + \exp(x - \alpha)]^{-1} dx / \Gamma(j + 1) \). It is seen that the information on the peak position \( \varepsilon_0 \) is lost in the time-averaged current \( I \). In contrast, the noise is sensitive to both the peak position and its magnitude, as will be shown below.

Assuming that the peak electrons are uncorrelated (Poissonian), while the FD electrons are correlated by the Pauli exclusion, we obtain the correlation
function for the injected carriers [10]

\[ K(\xi) = \frac{2G_S}{\xi} \left[ F_{-1}(\xi - \xi/k_BT) + \tilde{a} k_BT \delta(\xi - \xi_0) \right], \tag{8} \]

where \( G_S = (q^2/2\pi \hbar)N_S \) is the Sharvin conductance. Thus for the current noise we obtain

\[ S_I = 2qI_{bg} \frac{9k_BT}{qU} \left[ 1 - \sqrt{\pi}w g_1 + w^2 g_2 + a (\sqrt{\xi - w})^2 \right], \tag{9} \]

where \( I_{bg} = I(\tilde{a} = 0) \) is the background current, \( w \equiv v_\Delta/\sqrt{k_BT} = (\sqrt{\pi}g_1 + a/\sqrt{\xi})/(2g_2) \). \( a = a/F_1(\alpha) \) is the ratio between the current of the peak and the current of the background, \( g_1 = F_1(\alpha)/F_1(\alpha) \) and \( g_2 = F_0(\alpha)/F_1(\alpha) \) are the coefficients dependent on the degree of degeneracy of injected electrons \( (g_1 = g_2 = 1 \text{ at high temperatures for nondegenerate electrons}) \), and \( \alpha = \xi - \Phi_b/k_BT \) and \( \xi_\delta = (\xi_0 - \Phi_b)/k_BT \) are the positions of the Fermi energy and the \( \delta \) peak with respect to the potential barrier. The current noise (9) has a form of a shot noise suppressed by Coulomb interactions. The suppression is enhanced with the bias \( U \) and depends on the details of the injected distribution, in particular on the peak parameters \( a \) and \( \xi_\delta \). Note that for noninteracting carriers \( (\gamma = 1) \), one gets \( S_I = 2qI_{bg}(g_2 + a) \), that means the information on the peak position is lost, in a similar way as for the average current (7). For this case the noise is a sum of two shot-noise terms: the first one is multiplied by the factor \( g_2 \) (Fermi suppressed Poissonian shot noise [9]), and the second one is the Poissonian noise from the \( \delta \) peak. Thus, the role of the Coulomb interactions is crucial in providing an additional information on the injected carriers.

In the experiment, if one is interested in getting information on the peak electrons, one can register the noise for two cases: with and without the \( \delta \) peak, and then find the ratio

\[ \beta = \frac{S_I}{S_{I_{bg}}} = 1 + a \frac{(\sqrt{\xi_\delta - w})^2 + a/(4g_2\xi_\delta)}{1 - (\pi g_1^2)/(4g_2)}. \tag{10} \]

It is seen that the parameter \( \beta \) is a nonmonotonic function of the peak position. We have studied the behavior of \( \beta \) when the peak position \( \xi_\delta \) and the peak current \( a \) are varied. The results are illustrated in Fig. 2. For a fixed \( a \), each curve displays a minimum at some energy correspondent approximately to the energy \( \varepsilon_0^* = \Phi_b + v_\Delta^2 \), where the current-fluctuation transfer function vanishes: \( \gamma(\varepsilon_0^*) = 0 \). Since the barrier height \( \Phi_b \) is a function of the applied bias, one can tune by \( U \) the \( \delta \) peak position to \( \varepsilon_0^* \) where the noise is minimal, thereby revealing the peak position. In the opposite case, when the peak position is
Fig. 2. Current noise produced by Fermi-Dirac electrons with an additional δ peak at $\varepsilon=\varepsilon_0$ (the distribution is shown in the inset) with respect to the case when no peak is present. The ratio $S_I/S_{Ibg}$ is shown as a function of the peak position $(\varepsilon_0-\Phi_b)/(k_B T)$ for $\alpha=10$ and different magnitudes of the peak current $a$. For each curve, the point near the minimum indicate the energy at which the energy-resolved shot-noise suppression factor vanishes $\gamma=0$.

given, one may get information on the screening parameter at the space-charge barrier.

Summarizing, we have shown that Coulomb interactions in ballistic structures are of interest from several points of view: On the one hand, they lead to the shot-noise suppression that may be important for applications. On the other hand, they give the possibility to use the shot-noise measurements as a tool to deduce an important information on the properties of nonequilibrium carriers in nanoscale structures with hot-electron emitters, resonant-tunneling-diode emitters, superlattice emitters, etc., not otherwise available from dc measurements.

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