Magnetoresistance of two-dimensional electrons with spin-orbit coupling disorder

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\textbf{Abstract.} We present analysis of random spin-orbit coupling in two-dimensional electron systems. This randomness leads to several interesting order-from-disorder phenomena, where this random interaction having nanoscale spatial size produces or modifies macroscopic spin and charge dynamics. Anisotropic negative magnetoresistance related solely to the spin dynamics of electrons is one of the examples of these effects and will be considered here in detail.

1. Introduction

Spin-dependent transport phenomena in solids are in the basis of spintronics, a rapidly developing field of fundamental and applied physics. One of the spintronics keywords is spin-orbit coupling, usually considered as the coupling between electron spin and momentum. The spin-orbit coupling leads to at least two classes of phenomena \cite{1, 2, 3} such as (1) spin relaxation and (2) ability of spin manipulation by external electric field. In addition, spin-orbit coupling determines two novel transport effects: the spin-Hall- \cite{4, 5} and the anomalous Hall effect \cite{6, 7}. In the former the electric field applied to a sample causes spin current while in the latter it leads to a transversal current in a system with finite magnetization even if the external magnetic field is zero.

On the other hand, an interesting aspect of the influence of magnetic field on electric current is the magnetoresistance, that is the dependence of the system conductivity on the applied magnetic field. This effect is usually attributed to the change in the electron motion in external magnetic field. However, in the systems with spin-orbit coupling one expects that the magnetization of the system influencing electron spin will also somehow influence the orbital motion and, as a result, lead to a nonzero magnetoresistance.
This paper focuses on two major points. First, for completeness, we discuss spin-orbit coupling as a random function of the electron position. Second, we demonstrate that this randomness leads to a very unusual negative magnetoresistance, where the conductivity increases when the magnetic field is applied. We estimate the magnitude of this effect and show that it can be observable in narrow-gap semiconductor structures such as InSb-based quantum wells.

2. Physics and Hamiltonian of random spin-orbit coupling

We begin with the conventional Rashba Hamiltonian of spin-orbit coupling in the form [8]:

$$H_R = \alpha (\sigma_x k_y - \sigma_y k_x), \quad (1)$$

where $\sigma_i$ and $k_j$ are the Cartesian components of spin and momentum (we use $\hbar \equiv 1$), respectively. The coupling constant $\alpha$ is the material and structure-dependent parameter, which appears due to the structure inversion asymmetry with respect to the $z \rightarrow -z$ operation, where $z$ is perpendicular to the structure, and vanishes if the structure is fully symmetric. If the asymmetry with respect to the reflection in the structure plane is caused by the electric field applied across the quantum well, the coupling can be expressed in terms of the field as [9]:

$$\alpha = \xi e E_z, \quad (2)$$

where $\xi$ is a constant, $e$ is the fundamental charge, and $E_z$ is the field component. The value of $\xi$ rapidly increases with the atomic mass of the compound. The numerical examples are: $\xi = 0.05$ nm$^2$ in GaAs, $\xi \sim 1$ nm$^2$ in typical InGaAs alloys, and $\xi = 5$ nm$^2$ for InSb systems. Random spatial variations in the electric field cause random spatial variations in the spin-orbit coupling. The Rashba Hamiltonian requires modification to restore the Hermitian form and becomes [10]:

$$H_R = \frac{1}{2} \left( \sigma_x \{ k_y, \alpha \} - \sigma_y \{ k_x, \alpha \} \right), \quad (3)$$

where $\{ \ldots \}$ stands for anticommutator.

We consider an example of random spin-orbit coupling in semiconductor structure, where the details can be understood in terms of a simple model. The structure consists of a two-dimensional conducting channel and two dopant layers positioned at $z = \pm R_d$ with the two-dimensional concentration of dopants per layer $n_d$ (Fig.1). The average electric field in the well is, therefore, zero. Figure 1 shows, however that inevitable fluctuations in the dopant density break the symmetry locally. As a result, random electric field appears in the system and leads to the random Rashba field. The $z$-component of the Coulomb field of the dopant ions is given by the sum over the dopant layers

$$E_z(\rho) = -\frac{e}{\varepsilon} \sum_j f(\rho, r_j), \quad (4)$$

where the dopant $j$ position $r_j = (r^j_\parallel, z)$ and $\varepsilon$ is the dielectric constant. Function $f(\rho, r_j)$ is the Coulomb field in the form:

$$f(\rho, r_j) = \frac{z}{\left( (\rho - r^j_\parallel)^2 + R_d^2 \right)^{3/2}}. \quad (5)$$

We assume that the correlation function of the dopant concentration is the “white noise”

$$\langle (n(r_1) - n_d)(n(r_2) - n_d) \rangle = n_d \delta(r_1 - r_2), \quad (6)$$
and the fluctuations obey the Gaussian statistics, where the higher-order correlatives can be reduced to the lower-order ones. Under these assumptions the correlation function of random electric field $z$-components, $\langle E_{z,r}(0)E_{z,r}(\rho) \rangle$, where subscript $r$ corresponds to the random contribution, depends on the distance between the points only. By using Eqs.(4)-(6) we obtain the correlation function of the random electric field:

$$\langle E_{z,r}(0)E_{z,r}(\rho) \rangle = \langle E_{z}^2 \rangle F_{\text{corr}}(\rho),$$  \hspace{1cm} (7)

with

$$\langle E_{z}^2 \rangle = 2\pi \left( \frac{e}{\hbar c} \right)^2 \frac{n_d}{R_d^2}. \hspace{1cm} (8)$$

The resulting fluctuation in the Rashba coupling is of the order of

$$\alpha_r = e\xi \sqrt{\langle E_{z}^2 \rangle} = \sqrt{2\pi \xi^2 \frac{e^2 \sqrt{n_d}}{R_d}}. \hspace{1cm} (9)$$

To obtain a numerical estimate for a narrow-gap semiconductor such as InSb, we assume typical parameters $n_d = 5 \times 10^{11} \text{ cm}^{-2}$, $R_d = 20 \text{ nm}$ and obtain $\alpha_r$ of the order of $0.5 - 1 \times 10^{-6} \text{ meV cm}$.

With the increase in the distance to the dopant layers, the random variations in the electric field $E_{z,r}(\rho)$ become weaker and smoother since the electric field becomes only weakly sensitive to the details of the charge distribution. It is interesting to mention that correlation length of the random electric field is solely determined by the distance to the dopant layer rather than by the distance between the dopant ions since the white-noise distribution does not have a characteristic spatial scale and the correlations pick up the scale of the distance to the dopant layer.

This randomness can be crucial for systems where spin dynamics is prohibited otherwise. An important example is given by symmetric GaAs quantum wells grown along the [110]-direction, where the symmetry of the Dresselhaus coupling prohibits evolution of the spin component along the growth direction. On the other hand, the random Rashba coupling in these systems leads to a variety of interesting phenomena, including spin relaxation [11, 12] and nonlinear spin-Hall effect at finite frequency of the electric field [13], which can be seen as the “order-from-disorder” effect, where the macroscopic current is solely due to randomness in the driving spin-orbit field. Another kind of the “order-from-disorder” is the conventional spin-Hall effect, which, in contrast to common wisdom, does not vanish in the presence of disorder [14].

To make the picture of random spin-orbit coupling in low-dimensional systems more complete, we mention several other examples where the randomness of spin-orbit coupling is important.

**Figure 1.** (Upper panel) Single quantum well surrounded by two dopant layers. Filled circles schematically show positions of dopant ions. (Lower panel) A typical profile of a random spin-orbit coupling produced by given distribution of dopants.
First example is given by Si/Ge interfaces in the Si\textsubscript{0.8}Ge\textsubscript{0.2]/Si quantum wells, where random alignment of the bonds leads to the random spin-orbit coupling of the Rashba or Dresselhaus type [15]. Second example is given by graphene, where adatoms and long-range ripples with randomly coordinate-dependent curvature form a short-range and long-range random spin-orbit fields, respectively [16, 17]. Third example is topological insulators, including HgTe quantum wells, where random spin-orbit coupling combined with electron-electron interaction leads to a variety of nontrivial transport phenomena [18]. Fourth, recent developments on the tunneling spectroscopy of the depletion layers in Cs-covered InSb systems clearly demonstrate the random coordinate dependence of the Rashba coupling [19] seen as coordinate-dependent spin splitting of the Landau levels. Finally, we mention recent developments on the physics of thin Bi films showing the random distribution of the spin-orbit coupling there [20].

In the momentum representation Eq. (3) can be rewritten as

$$H^{(so)}_{kk'} = \frac{\alpha_{kk'}}{2} \left[ \sigma_x(k_y + k'_y) - \sigma_y(k_x + k'_x) \right].$$

The Fourier transform of the correlation function of fields in Eq. 10 is given by:

$$\langle \alpha^2 \rangle_q = 2\pi \langle \alpha^2 \rangle F(q)$$

where $F(q)$ is the system-dependent range function. A simple generic approximation for this function can be given by $F(q) = R^2 e^{-qR}$; the result becomes exact in the case of random Rashba field produced by the Coulomb field charged dopants shown in Fig.1 with $R = 2R_d$. This equation demonstrates two most important features of the random spin-orbit coupling: variation $\langle \alpha^2 \rangle$ and correlation length $R$. The knowledge of these two parameters allows one in general to study the related to the random spin-orbit coupling effects.

3. Magnetoresistance due to random spin-orbit coupling

3.1. Hamiltonian for kinetic phenomena in magnetic field.

We describe two-dimensional electron gas of interest by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_R$, where (we use the units with $\hbar \equiv 1$)

$$\hat{H}_0 = -\frac{\nabla^2}{2m} + U_{\text{rnd}}(r) + \sigma_x \beta B,$$

corresponding to the magnetic field along the $x$-axis in the quantum well plane with $m$ being the electron effective mass. The term $U_{\text{rnd}}(r)$ describes the spin-diagonal disorder assumed to be of white-noise type, and $\beta = g\mu_B/2$, where $g$ is the electron Landé factor. Assuming in-plane magnetic field we avoid the diamagnetic orbital effects, and therefore take into account only the Zeeman term. The magnetization leads to splitting of the Fermi surface, and in the limit of small magnetization $\Delta K = 2|\beta|Bm/k_F$, as shown in Fig. 2, where $k_F$ is the Fermi momentum at $B = 0$. Here the weak magnetization condition implies $\Delta K \ll k_F$.

The random spin-orbit coupling leads to spin-flip scattering (Fig.2(a)) between states with opposite spin orientations with the minimum momentum transfer equal to $|\Delta K|$ as well as to spin-conserving scattering (Fig.2(b)) with the zero minimum momentum transfer. Probabilities of both these scatterings depend on the direction of the electron momentum. We will see in the next subsection that these two features of the random spin-orbit coupling lead to the negative anisotropic magnetoresistance.

3.2. Negative anisotropic magnetoresistance.

Conventional dependence of the conductivity of two-dimensional electron gas on magnetic field perpendicular to the structure is due to the orbital motion and can be expressed as:

$$\sigma(B) = \frac{\sigma(0)}{1 + \omega_c^2 \tau^2},$$

\text{(13)}
Figure 2. Spin-split Fermi surface. $1/R$ is the possible range of momentum change, $q = |\mathbf{k}' - \mathbf{k}|$, in the spin-flip scattering process between two Fermi surfaces. In the limit of small Fermi surface splitting, $\Delta \mathbf{K} = 2|\beta|Bm/k_F$ (spin orientation is taken with respect to the $x$-axis). This geometry with $k_{F\uparrow} > k_{F\downarrow}$ corresponds to a negative $g$-factor of electron. Panel (a) shows spin-flip transitions while panel (b) corresponds to the spin-conserving ones.

where $\omega_c = eB/mc$ is the cyclotron frequency, and $\sigma(0)$ is the zero-field conductivity. As can be seen in Eq.13, usually, the conductivity decreases in a magnetic field the Lorenz force prevents straightforward motion on electrons, and, as result, works against the electron propagation necessary to carry electric current. However, the orbital motion in some special kinds of disorder can be more complicated and the magnetic-field dependence of the conductivity can correspond to the increase rather than to the decrease with the magnetic field [21]. Also, quantum effects such as weak antilocalization can lead to negative magnetoresistance in a certain interval of magnetic fields [22, 23]. However, in the presence of spin-orbit coupling, the orbital motion, and, as a result, the conductivity, becomes spin-dependent. Here we explore this possibility for the random spin-orbit coupling and show its unusual consequences.

In magnetic field the conductivity can be expressed as the sum of spin-dependent terms:

$$\sigma(B) = \frac{e^2 (n_\uparrow \tau_\uparrow + n_\downarrow \tau_\downarrow)}{m},$$

where $n_\uparrow$ and $n_\downarrow$ are corresponding concentrations, and $\tau_\uparrow$ and $\tau_\downarrow$ are the momentum relaxation times, dependent on magnetic field. These times can demonstrate anisotropy, which will be of our interest below. To present the analysis in a simple and general form, we use the conventional Boltzmann kinetic equation approach for the momentum- and spin-dependent distribution functions $f_{\mathbf{k}\downarrow}$ and $f_{\mathbf{k}\uparrow}$:

$$\frac{e}{m} (\mathbf{E} \cdot \mathbf{k}) \frac{\partial f_{\mathbf{k}\uparrow}}{\partial \varepsilon_k} = \int \frac{d^2k'}{(2\pi)^2} \left[ W_{\mathbf{k}\mathbf{k}'} (f_{\mathbf{k}\uparrow} - f_{\mathbf{k}'\uparrow}) + W_{\mathbf{k}\mathbf{k}'}^f (f_{\mathbf{k}\uparrow} - f_{\mathbf{k}'\downarrow}) \right],$$

$$\frac{e}{m} (\mathbf{E} \cdot \mathbf{k}) \frac{\partial f_{\mathbf{k}\downarrow}}{\partial \varepsilon_k} = \int \frac{d^2k'}{(2\pi)^2} \left[ W_{\mathbf{k}\mathbf{k}'} (f_{\mathbf{k}\downarrow} - f_{\mathbf{k}'\downarrow}) + W_{\mathbf{k}\mathbf{k}'}^f (f_{\mathbf{k}\downarrow} - f_{\mathbf{k}'\uparrow}) \right],$$

where $\varepsilon_k = k^2/2m$. The scattering probabilities are

$$W_{\mathbf{k}\mathbf{k}'} = 2\pi \left[ w_{\mathbf{k}\mathbf{k}'} + \frac{1}{4} |\alpha_{\mathbf{k}\mathbf{k}'}|^2 (k_y + k'_y)^2 \right] \delta(\varepsilon_k - \varepsilon_{k'}),$$

$$W_{\mathbf{k}\mathbf{k}'}^f = 2\pi \frac{|\alpha_{\mathbf{k}\mathbf{k}'}|^2}{4} (k_x + k'_x)^2 \delta(\varepsilon_k - \varepsilon_{k'} \pm 2\beta B),$$

where $w_{\mathbf{k}\mathbf{k}'}$ is the scattering probability due to spin-orbit interaction.
for spin-conserving \( (W_{kk'}) \) and spin-flip \( (W_{fk}kk') \) processes, respectively. The \( w_{kk'} \) term is due to spin-diagonal disorder described by the \( U_{\text{nd}}(r) \) term in Eq.10 and is assumed to be a constant \( w_0 \) below. Change in the electron momentum:

\[
q = \left| k - k' \right| = \sqrt{(\Delta K)^2 + k_F^2 \gamma^2}
\]

in the spin-flip process, and \( q = k_F |\gamma| \) in the spin-conserving process, where \( \gamma \) is the scattering angle.

Equations 17 and 18 show two features of the spin-dependent scattering: anisotropy and strong dependence on the magnetization. We concentrate on the semiclassical regime \( k_F R \gg 1 \), realizable in experimentally available quantum wells. To get an idea of time scales involved in the spin-dependent scattering, we introduce the spin relaxation time \( \tau_s \) due to the spin-orbit coupling [10]:

\[
\frac{1}{\tau_s} = 4m \langle \alpha^2 \rangle k_F R,
\]

which will be used below for comparison. This expression corresponds to the generalized Dyakonov-Perel’ spin relaxation mechanism due to the random spin precession in the random spin-orbit field.

We begin with the zero magnetic field limit, where the splitting of the Fermi surface is very small, that is \( R \Delta K \ll 1 \). For \( k_F R \gg 1 \), the spin-dependent scattering occurs only at small angles \( \leq 1/k_F R \) and effectively becomes independent of the direction of the electron momentum since the sum \( W_{kk'} + W_{fk}kk' \) does not depend on the direction of \( k \) here. In this approximation we can write

\[
\frac{eE \cos \varphi}{m} \frac{\partial f^0_k}{\partial \varepsilon} = 2\pi \nu \int \frac{d\varphi'}{2\pi} \left( w_0 + 2\pi k^2 R^2 \langle \alpha^2 \rangle e^{-kR|\varphi' - \varphi|} \right) (f_k - f_{k'})
\]

where \( \varphi \) is the angle between \( \mathbf{E} \) and \( \mathbf{k} \) (Fig.3) whereas \( \varphi' \) is the angle between \( \mathbf{E} \) and \( \mathbf{k'} \), \( \nu = m/2\pi \) in the density of states per single spin component, and \( f^0_k \) is the equilibrium distribution function.

Figure 3. Geometry of magnetoresistance. (a) longitudinal magnetoresistance, (b) transversal magnetoresistance.

For \( B = 0 \), where the scattering anisotropy is unimportant, we obtain a conventional Drude conductivity

\[
\sigma(0) = \frac{ne^2 \tau}{m},
\]

with the momentum relaxation time modified by the spin-orbit coupling disorder as:

\[
\frac{1}{\tau} = 1/\tau_0 + 1/\tau_1, \quad \frac{1}{\tau_1} = \frac{2m \langle \alpha^2 \rangle}{k_F R} = \frac{1}{\tau_s} \times \frac{1}{2(k_F R)^2}.
\]
Here \( \tau_0 \) is the relaxation time due to scattering from short-range spin-diagonal disorder. The correction to the conductivity can be expressed in terms of the spin relaxation time and the \( k_F R \) parameter, which suppresses the correction to the conductivity since the spin-flip scattering occurs at small angles only.

Now we can study the effect of strong magnetic field, where spin-flip scattering is almost suppressed, and only spin-conserving terms (Fig.2(b)) remain. Since one of contributions (spin-flip one) in the electron scattering is completely suppressed, and the remaining spin-conserving one is anisotropic, one can expect that the conductivity of the electron gas will increase and become, in turn, anisotropic. In these strong fields |\( \beta B m R/k_F \gg 1 \), the kinetic equation reads

\[
\frac{eE \cos \varphi}{m} \frac{\partial f_k^0}{\partial \varepsilon_k} = 2\pi \nu \int \frac{d\theta'}{2\pi} \left( w_0 + 2\pi k_F^2 R^2 \alpha^2 e^{-k_R|\theta-\theta'|} \sin^2 \theta \right) (f_k - f_k'),
\]

where \( k = k(\cos \theta, \sin \theta) \) and \( k' = k'(\cos \theta', \sin \theta') \). We begin with the longitudinal magnetoresistance for \( E \parallel B \). In this case the angle \( \varphi \) between \( E \) and \( k \) is equal to \( \theta \), as shown in Fig.3(a). To account for the scattering anisotropy, we seek for solution of the Boltzmann equation in the form:

\[
f_k = f_k^0 + kE (g_1 \cos \varphi + g_2 \cos^3 \varphi)
\]

(24)

with \( g_1, g_2 \) being scalar functions of \( k \) and obtain

\[
g_1 = \frac{e\tau}{m} \frac{\partial f_k^0}{\partial \varepsilon_k}, \quad g_2 = \frac{\tau_0}{\tau_1} g_1.
\]

(25)

The term \( g_2 \cos^3 \varphi \) in the distribution function leads to the increase and anisotropy in the conductivity due to suppressed spin-flip scattering. The resulting conductivity is

\[
\sigma_l(B) = \sigma(0) \left( 1 + \frac{3\tau_0}{4\tau_1} \right),
\]

(26)

and the longitudinal magnetoresistance is negative.

Now we calculate the transversal magnetoresistance taking into account that \( \theta + \varphi = \pi/2 \) for \( B \perp E \) (Fig.3(b)). Using the solution similar to Eq.24, we obtain:

\[
\sigma_t(B) = \sigma(0) \left( 1 + \frac{\tau_0}{4\tau_1} \right) < \sigma_l(B).
\]

(27)

The transversal conductivity increases, however, less than the longitudinal one, leading to anisotropic magnetoresistance. This anisotropy is due to the properties of the electron scattering in the random Rashba field, namely due to the \( \sin^2 \theta \) in the scattering probability in Eq.23. This factor vanishes for electron momentum along the \( x \)-axis and has maximum for \( k \) parallel to the \( y \)-axis. To understand the scale of the effect in InSb we estimate \( \tau_1 = 5 \times 10^{-11} \) s. Taking into account that for the mobility \( \mu = 10^5 \) cm\(^2\)/Vs, the relaxation time \( \tau_0 \) is of the order of \( 10^{-12} \) s, the decrease in the resistivity and the corresponding anisotropy can be easily seen. The required magnetic field to split the Fermi surface and observe this effect is of the order of one Tesla, dependent on the system parameters.

Here we mention that this effect is not related to the negative magnetoresistance in the weak antilocalization regime [22, 23] and can be related to the spin-dependent phenomena in the negative magnetoresistance recently observed experimentally [24].
4. Conclusions.
We considered two-dimensional electron gas with a spatially random spin-orbit coupling. This randomness having the correlation length of the order of 10...100 nm arises due to a conventional disorder always present in these systems. The scattering of electrons by the randomly fluctuating spin-orbit coupling is strongly sensitive to the external magnetic field and leads to new interesting “order-from-disorder” kinetic phenomena in the charge and spin channels. As an example, we studied the negative anisotropic magnetoresistance solely related to this random spin-orbit coupling. These effects can be observed experimentally and used in spintronics applications.

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