Simulation of satellite bearings loading in planetary cycloid gear

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Abstract. The loading of satellite bearings of planetary cycloid gearbox is a very important characteristic that can determine its operability. The article presents an analysis of changes in the main parameters of gear loading. Initially, the loading analysis of satellite bearings for the starting position was performed, taking into account the manufacturing tolerances. After that, the change in loading during the satellite rotation was considered. The gear was considered with stationary input shaft and rotating pinwheel and satellite. To establish relationships between parameters in the gear, mathematical models of the satellite's force interaction with the pin-rollers and pins of the output shaft were used. The computer experiment used the MathCad package. The results of the analysis showed how the forces on the bearing change with increasing clearances at a constant torque on the output shaft. It was found that the gear operation is accompanied by a continuous change in the forces acting on the bearing.

1. Introduction
The bearing capacity of planetary cycloid reduction gear, especially at high operating times, may be limited by the satellite bearings. In the monography of V.N. Kudryavtsev [1] was studied the static loading of satellite bearings, as well as meshing in gears without a clearance. Subsequent studies [2-7] and others have shown that clearance affect a complex of interrelated variables: backlash and kinematic error of rotation, forces acting in meshing, loss in meshing, torsional stiffness of gear, etc. The influence of calculated parameters on forces and contact stresses was shown by S.K. Malhotra and M.A. Parameswaran in paper [2]. A dynamic model and a method for determining the number of elements that transmit the load in a cycloidal gearbox with a clearance are presented by Li-xin Xu in paper [3]. J.G. Blanche and D.C.H. Yang developed models and investigated the effect of manufacturing inaccuracy on torque ripples on drive wheels with a cycloidal tooth profile [4]. M. Blagojevic presented a method for calculating the deformations of cycloid disk teeth and clearances that arise between the cycloid disk and the ring gear [5]. In papers [6, 7], the distribution of force across the elements of a cycloidal drive and power loss is obtained.

In force analysis, forces in contacts are more often examined, while much less attention is paid to determining the forces acting on bearings. The influence of clearances in meshing and W mechanism on
the force acting on the bearings is still poorly studied. The purpose of this article is to study the influence of the gear manufacturing accuracy on the forces acting on the bearings.

2. Forces transmitted to bearing from meshing in initial position

In gears with cycloidal meshing, one of the toothed wheels makes a planetary movement, and the second is stationary. When determining the force acting on the satellite bearing it is more convenient, however, to study the mechanism in which both toothed wheels rotate. Figure 1, a shows the primal location of the satellite and the pinwheel, at the same time all the initial clearances in the normal direction to the profile are the identical and equal to \( \Delta_z \). Assume \(^1\) that when the pinwheel is stationary, the satellite rotates by a some \( \beta \) angle under the action of the torque applied to it. At the same time in the \( i \)-th contact pair there is rapprochement along the normal to contact surfaces and in some pairs a deformation appears \( \delta_i = \beta l_i - \Delta_z \). The value \( l_i = r_{w1} \sin \theta_i \) – is a distance from the center of the satellite \( O_i \) to the line of action of the force in contact going from the center of the \( i \)-th pin-roller to the pole of meshing \( P_z \); \( r_{w1} \) – is radius of the satellite's initial circle. Therefore

\[
\delta_i = \beta r_{w1} \sin \theta_i - \Delta_z .
\]  

The forces \( F_z \) in meshing for a given position can be determined by the formula \(^8\):

\[
F_z = \frac{4T_i k_z}{z_w} \left( \sin \theta_i - p_{un} (1 - \sin \theta_i) \right) \sin \theta_i - p_{un} (1 - \sin \theta_i) \geq 0 ,
\]  

where \( i \) – pin-roller number, which is counted counterclockwise from the pin-roller located on the vertical axis of symmetry, it is assigned number one (figure1); \( T_i \) – torque on satellite; \( z_w \) – number of the pinwheel teeth; \( k_z \) – coefficient of increased load in meshing due to clearances

\[
k_z = \pi \left( 1 + p_{un} \right) \left( \pi - 2 \arcsin \frac{p_{un}}{1 + p_{un}} - 2 \frac{p_{un}}{1 + p_{un}} \left( 1 - \left( \frac{p_{un}}{1 + p_{un}} \right)^2 \right) \right)^{-1} ;
\]  

\( p_{un} \) – parameter of elastic loading in meshing, equal to the ratio of the initial clearance in meshing to the maximum deformation:

\[
p_{un} = k_z \pi \Delta_z z_w E b r_{w1} \left( 8T_i \left( 1 - \nu^2 \right) \left[ \frac{z_w r_{w1} \pi E b r_{w1} \sqrt{1 - \lambda^2}}{8T_i \left( 1 - \nu^2 \right) + 0.815} \right] \right)^{-1} ;
\]  

\( k_z \) – coefficient taking into account the design features of unit; \( E \) – elastic modulus; \( b \) – width of satellite; \( \nu \) – Poisson’s ratio; \( \lambda \) – coefficient of shortening of the epicycoids, \( \lambda = r_{w2}/r_2 \); \( r_{w2} \) – radius of initial circle of pinwheel in meshing with satellite.

If we assume that all forces in meshing with a clearance are directed to the pole, then the sum of the projections of forces on the \( Ox \) axis in gears with a clearance and in gears without a clearance:

\[
\sum_i F_{zix} = T_k / r_{w1} .
\]
Figure 1. Forces acting in the meshing and W mechanism in the initial position.

Projection of the $F_{xi}$ force on the $Oy$ axis

$$F_{zy} = F_{xi} \cos \theta_i.$$  

The sum of the projections of forces acting in the meshing on the $Oy$ axis,

$$\sum F_{zy} = \sum F_{xi} \cos \theta_i = k_y \frac{4T_i}{z_{2i}r_{cm}} \sum_i [(1 + p_{am}) \sin \theta_j - p_{am}] \cos \theta_i = k_y \frac{T_i}{r_{cm}},$$  

where $k_y$ - radial force coefficient:

$$k_y = k_y \frac{2}{\pi} \left\{ (1 + p_{am}) \lambda_1 + \frac{p_{am}}{2\lambda} \left[ \frac{\lambda^2 - 1}{2\lambda} \right] \lambda_2 + I_3 \right\}.$$  

Integrals $I_1, I_2, I_3$ included in formula (8) have the following form. Integral $I_1$,

$$I_1 = \int_{\tau_{al}}^{\tau_2} \sin \tau_{al} (\lambda - \cos \tau_{al}) d\tau_{al} \frac{1}{1 - 2\lambda \cos \tau_{al} + \lambda^2} = \frac{1}{2\lambda} \ln \frac{\lambda^2 - 1}{2\lambda} + \frac{\cos \tau_{al} - \cos \tau_{al}}{2\lambda}.$$  

Integral $I_2$

$$I_2 = \frac{2}{1 + \lambda} F(k, \varphi_2) - F(k, \varphi_1).$$  

Integral $I_3$:

$$I_3 = 2(1 + \lambda) \left[ E(k, \varphi_2) - E(k, \varphi_1) \right] + 4\lambda \left( \frac{\sin \tau_{al} \sin \tau_{al} - \cos \tau_{al} \cos \tau_{al}}{\sqrt{1 - 2\lambda \cos \tau_{al} + \lambda^2}} - \frac{\sin \tau_{al} \sin \tau_{al} - \cos \tau_{al} \cos \tau_{al}}{\sqrt{1 - 2\lambda \cos \tau_{al} + \lambda^2}} \right).$$
where $F(k, \varphi), \ E(k, \varphi)$ – normal elliptic Legendre integrals of the first and second type respectively, $k$ – modulus of an elliptic integral, $k = 2\sqrt{\lambda}(1+\lambda)^{-1}$; $\varphi_{1,2}$ – amplitude of the integral

$$\varphi_{1,2} = \arcsin \frac{1+\lambda}{\sqrt{2}} \sqrt{\frac{1 - \cos \tau_{u_{1,2}}}{1 + \lambda^2 - 2\lambda \cos \tau_{u_{1,2}}}}.$$

3. Forces transmitted to bearing from $W$ mechanism in initial position

As the initial position of satellite, we take the position in which the radius of the most loaded pin is perpendicular to the inter-axis line of gear (figure 1(b)). The peculiarity of the $W$ mechanism, in comparison with meshing, is that the forces in contact, in the first approximation, are directed parallel to the inter-axis line $O_1O_2$. All clearances in this direction are assumed to be the same and equal to the $\Delta_w$. When the satellite rotates clockwise around its center $O_1$ (or when the output shaft with pins rotates counterclockwise around the center $O_2$) by an $\omega$ angle sufficient to elimination of clearances in at least some contact pairs, in them will arise deformations $\delta_w$

$$\delta_w = \omega r_w \sin \varphi - \Delta_w,$$

where $r_w$ – radius of circles of positions centers of pins and holes in satellites.

Assuming that the stiffness of contact pairs in the direction of the acting forces is constant, we will think that the forces are proportional to these deformations.

$$Q_i = \frac{4T_i k_Q}{n_w r_w} (\sin \varphi_i - P_{anw} (1 - \sin \varphi_i)), \quad \text{при} \sin \varphi_i - P_{anw} (1 - \sin \varphi_i) \geq 0, \quad (12)$$

where $k_Q$ – coefficient of increasing the load on the most loaded pin due to inaccurate manufacturing of the $W$ mechanism; $n_w$ – number of pins of the $W$ mechanism; $P_{anw}$ – elastic loading parameter in the mechanism $W$, similar to the parameter for meshing, $P_{anw} = \Delta_w n_w r_w c_w (4T_i k_Q)^{-1}$, $c_w$ – coefficient of stiffness in the contact pair pin – hole of satellite.

In paper [9] is received a formula for $k_Q$, which can be transformed to the following form:

$$k_Q = \pi \left[ (1 + P_{anw}) \left[ \pi - 2 \arcsin \frac{P_{anw}}{1 + P_{anw}} - 2 \frac{P_{anw}}{1 + P_{anw}} \sqrt{1 - \left( \frac{P_{anw}}{1 + P_{anw}} \right)^2} \right] \right]^{-1}. \quad (13)$$

The sum of forces acting on the pins and transmitted to the satellite bearing:

$$Q_i = \frac{T_i}{r_w} k_{wy}, \quad (14)$$

where $k_{wy}$ – the coefficient of total force.

Based on the continuum model for $k_{wy}$, the following expression is obtained:
\[
  k_{wy} = 2k_Q \left(2 \sin \frac{\alpha}{2} - \alpha \cos \frac{\alpha}{2}\right) \left| \pi \left(1 - \cos \frac{\alpha}{2}\right) \right|^{-1},
\]

where \( \alpha \) is the central angle within which there is contact of the pins with the holes in the satellite.

Based on the discrete model for \( k_{wy} \), the expression is obtained:

\[
  k_{wy} = \left\{4k_Q \sum_{i-k}^{n} \sin \gamma_i + \left(n_w - 4k_Q \sum_{i-k}^{n} \sin^2 \gamma_i \right) \left[\sum_{i-k}^{n} \sin \gamma_i - (n - k + 1) \left(\sum_{i-k}^{n} \sin^2 \gamma_i - \sum_{i-k}^{n} \sin \gamma_i \right)\right]^{-1} \right\} n_w^{-1},
\]

where \( k, n \) are the numbers of the boundary pins on which the load acts, if counted counterclockwise from the contact pair to which the number 1 is assigned (figure 1 (b)).

4. Forces transmitted to the bearing in the turned position of gear

When the pinwheel and the satellite are rotated from the initial position, the load distribution over the contact pairs, the angle of elastic rotation of the satellite, and the total load transmitted to the bearing change. The torsional stiffness of the meshing and \( W \) mechanism depends on the position of the elements. When the satellite rotates to the angle \( \frac{\phi_1}{z_1} \) around the center of \( O_1 \) counterclockwise (figure 2), the pinwheel rotates around the center of \( O_2 \) in the same direction to the angle \( \frac{\phi_2}{z_2} = \frac{\phi_1}{z_1} \). When the satellite is rotated by one angular step, the system return to its initial position, with the only difference that in the meshing the contact pair 1 moves to the position 2, and the contact pair 2 moves to the position 3, and so on. To study the forces arising in the system, an iteration calculation algorithm is proposed in paper [9].

![Figure 2. Forces acting in the meshing (a) and W mechanism (b) in the rotated position.](image)

Similarly, the forces that arise in the \( W \) mechanism when it turns are studied. The system discretely, with the selected angular step, turns by an angle \( 2\pi/n_w \) until contact pair 1 moves to the position that was
occupied by contact pair 2 at the beginning, and pair 2 moves to the position that was occupied by contact pair 3 at the beginning, and so on. For each intermediate position, an approximate value $\omega = \omega_0$ is taken at the beginning of the calculation, and then it is refined, ensuring that the satellite’s equilibrium condition is met

$$T_s = \sum_i Q_i r_W \sin(\gamma_i + \varphi_i).$$

(17)

5. Interpretation of the decision

The force acting on the satellite bearing in static, without taking into account the friction forces in the contacts, in accordance with figures 1 and 2 and formulas (5), (7) and (14) can be determined by the formula

$$R = \frac{T_s}{r_{w1}} \sqrt{1 + \left(\frac{r_{w1}k_{w_i}}{r_W} - k_y\right)^2} = k_R \frac{T_s}{r_{w1}},$$

(18)

where $k_R = \sqrt{1 + \left(\frac{r_{w1}k_{w_i}r_W}{k_y}\right)^2}$ – coefficient of radial force.

The ratio $r_{w1}/r_W$ substantially depends on the value of the coefficient of shortening of the epicycloid. Figure 3 shows the results of calculations performed in Mathcad for the gear with $\lambda = 0.85$ and $\lambda = 0.55$. The calculated torque on one satellite, reduction ratio, initial clearances are shown in the figure. The calculated torque on one satellite $T_s = 360\text{Nm}$, reduction ratio $u = 33$, initial clearances are shown in the figure.

Figure 3 shows that the clearances in the $W$ mechanism and meshing can significantly change the coefficient of the radial force. The maximum values of the coefficient of the radial force take place at the zero-clearance $W$ mechanism and great clearances in the meshing. The nature of the changes of values $k_R$ in the direction of the axis of the clearance in meshing is determined by the change of the coefficient $k_y$ (figure 4), and in the direction of the clearances in the $W$ mechanism it is determined by the change of the coefficient $k_{w_i}$. The values $k_R$ decrease significantly when the coefficient of shortening of the epicycloid

Figure 3. Values of coefficient $k_R$ on bearing.

Figure 4. Values of coefficient $k_y$. 
λ decrease. When estimating the value of the radial force acting on the satellite bearings, the change of the value \( r_{w1} \) should be taken into account. When the value \( λ \) decreases, the value \( r_{w1} \) also decreases. When the gear is rotated from the initial position, the forces acting in the meshing and \( W \) mechanism change significantly. Figures 5 and 6 show the change of the forces in the meshing and coefficient \( k_y \) when the gear is rotated by one angular step.

Figures 7 and 8 show the change in forces in the \( W \) mechanism, the angle of elastic rotation \( \omega_{elw} \) and the coefficient \( k_{WY} \), when turning the gear within the angular step of the \( W \) mechanism. Figure 7 shows that the graphs of forces consist of three sections, with the points of change in the behavior of forces corresponding to changes in the number of pins that perceive the load, i.e., the moments of change the structure of the contact zone.

The angle of elastic rotation, which turns the output shaft when loading it with a torque, changes periodically. As the clearance in the \( W \) mechanism increases, the shape of the curve changes, the angle of elastic rotation increases significantly, and the torsional stiffness of the satellite – output shaft system decreases significantly. When the \( \Delta_w \) of presented gear increased from 0 to 0.3 mm, the torsional stiffness decreases by ≈1.77 times. The smallest angle of rotation corresponds to the position when the
loaded pins 3 and 4 (figure 8) are located symmetrically relative to the position where the radius of the pin is perpendicular to the inter-axis line. In this position, the mechanism has the highest angular stiffness. The sum of forces acting on the pins changes in the working mechanism with a period equal to the angular step $2\pi/n_w$. The highest value corresponds to the initial position. The angular step of the satellite teeth $2\pi/z_i$ does not coincide with the angular step of the holes in the $W$ mechanism $2\pi/n_w$. Therefore, the total force acting on the satellite bearing changes in the operating mechanism with a period equal to the smallest common multiple for angular steps in the engagement and $W$ mechanism. In planetary motion, the initial position of the system: the pinwheel – satellite – output shaft is repeated when the input shaft (carrier) is rotated by an angle $2\pi z_i n_w z_i^{-1}$.

6. Conclusions
The clearances in the meshing and $W$ mechanism of the planetary cycloid gear affect the forces transmitted to the satellite bearing. The gear operation is followed by a continuous change in the forces acting in contact pairs, accompanied by a change of the contact zones at certain angles of rotation. The dependence of the angle of elastic rotation of the mechanism, which occurs under the action of a moment applied to the output shaft, on the angular position of the output shaft is determined. The law of change of forces acting on the pins of the $W$ mechanism in the operating gear, taking into account the inaccuracy of its manufacture, is established. The increase in the clearances in the $W$ mechanism leads to a decrease in torsional stiffness of the cycloid gearbox and the force transmitted to the bearing.

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