Prediction of peak ground acceleration using ε-SVR, ν-SVR and Ls-SVR algorithm

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ABSTRACT

In this paper, a prediction model is developed using support vector machine for forecasting the parameter associated with ground motion of a seismic signal. The prediction model is developed using three learning algorithms, ε-support vector regression, ν-support vector regression and least square-support vector regression (Ls-SVR) for forecasting peak ground acceleration (PGA), a parameter associated with ground motion of a seismic signal. The prediction model is developed for each of the algorithms with different kernel functions, namely linear kernel, polynomial kernel and radial basis function kernel. The ground motion parameter is related to four seismic parameters, namely faulting mechanism, average soil shear wave velocity, earthquake magnitude and source to site distance. The database used for modelling is NGA flatfile released by Pacific Earthquake Engineering Research Center. The experimental results show that the optimal prediction model for forecasting PGA is Ls-SVR with RBF kernel. It is observed that the developed prediction model is better compared to the existing conventional models and benchmark models in the same database. This paper further compares the three variations of SVR algorithm for ground motion parameter prediction model. The learning effectiveness of each algorithm is measured in terms of accuracy, testing error and overfitness measure.

KEYWORDS
PGA; earthquake prediction; support vector regression; seismic risk; Ls-SVR

1. Introduction

Among all the natural calamities earthquakes are the most threatening natural calamity, due its tremendous destructive property. Seismic hazard is anything associated with an earthquake like strong ground shaking, landslides, liquefaction, faulting, etc. that may affect the normal activities of the people. The damage of an earthquake to built-in environment is termed as seismic risk and due to growing urbanization, there is tremendous increase in the population density in earthquake prone areas, which in turn is increasing the demand for earthquake resistant structures. Ground motion parameters are vital for designing earthquake resistant structures and seismic hazard analysis (Giacinto et al. 1997; Gullu et al. 2008; Sehhati et al. 2011; Gullu 2013; Gullu & Pala 2014; Güllü & İyisan 2015). The commonly used ground motion parameter in time domain is peak ground acceleration (PGA).

Conventionally, the ground motion parameters are estimated based on ground motion prediction equations (GMPEs) which are developed using the traditional regression analysis method. These predictive equations relate the ground motion parameter in terms of independent variables such as
earthquake magnitude, source to site distance, site conditions, seismic wave propagation and earthquake source characteristics (Giacinto et al. 1997; Perus & Fajfar 1997). The drawbacks of using regression analysis for the development of predictive equations is that, the high nonlinearity and inhomogeneity among the independent variables directly affect the coefficients of the independent variables in the developed regression equation. Moreover, in regression analysis, the model is developed based on a predefined linear or non-linear equation, with the hypothesis of normality of residuals for testing the developed model. Hence the developed predictive equation based on regression analysis is highly uncertain due to both computational uncertainties and the uncertainties of independent variables (Giacinto et al. 1997). Thus, there is a huge need for the modelling of ground motion parameters using newer techniques so as to reduce the existing errors in the ground motion parameter estimation.

A new method called conditional average estimator (CAE) was applied to the attenuation relationship (Perus & Fajfar 1997). CAE is non-parametric multi dimensional regression approach having a structure similar to neural networks. In this paper, horizontal component of PGA is predicted as a function of two parameters namely earthquake magnitude and distance. The ground motion parameters predicted using CAE method are compared to the existing GMPEs models (Boore & Atkinson, 2007; Campbell & Bozorgnia, 2007; Abrahamson & Silva 2008; Chiou & Youngs 2008; Idriss 2008; Perus & Fajfar 2010). The major drawback of using conventional method is that applying regression analysis for the development of attenuation relationships is a lot more complex because of the high nonlinearity and inhomogeneity among the parameters. Thus, there is a huge need for the modelling of ground motion parameters using newer techniques so as to reduce the existing complexities.

With the recent advances in the field of artificial intelligence and soft computing techniques, the traditional mathematical functions used for big data analyses and other complex analyses are replaced by them. Many researchers recently are working to merge these advanced techniques of AI with the field of earthquake engineering and geosciences. Gullu 2014a used gene expression programming (GEP) for estimating the strength and elasticity of soil. The ground motion parameters are significant for the damage potential of structures. For designing a good structure, the site conditions and structure ability are equally relevant as ground motion parameters (Ansal et al. 2001; Gullu et al. 2008; Gullu 2013; Gullu & Pala 2014; Gullu & İlyısan 2015). During an earthquake, both soil and structures play a vital role. The soil conditions with a necessary ground improvement for enhancement of the resistance of structures against earthquake is important (Gullu & Hazirbaba 2010; Hazirbaba & Gullu 2010; Gullu & Girisken 2013; Gullu & Khudır 2014; Gullu 2014b; Gullu 2015a; Gullu 2015b).

Kerh and Ting (2005) used back propagation neural networks for predicting the PGA in three different directions (vertical, east-west, and north-south) at stations along the high speed railway line in Taiwan using the seismic parameters and the historical earthquake data. The estimated values were compared with the microtremor measurement of the respective station. Gullu and Ercelebi (2007) made a remarkable effort to develop an attenuation relationship based on strong motion data on turkey region using artificial neural networks. Though the correlation between the observed and predicted PGA raised many questions (Gullu & Ercelebi 2008), the attempt made indicated that neural networks could be applied in the field of seismology. An equation discovery approach was used by Markic and Stankovski (2011) for modelling PGA. The equation discovery is a machine learning technique which uses context free grammar for generating equation structures which best describes the given data. In the study, Lagrange equation discovery system is used to obtain equations for predicting PGA. The system gives a fair correlation between the observed and the predicted PGA values. The equations obtained by Lagrange equation discovery system, for estimation of PGA have lesser complexities compared to the existing GMPEs. Gandomi et al. (2011) developed a new GMPE model using a hybrid model of genetic programming (GP) and orthogonal least squares for
prediction of PGA. The model gives a fair correlation value with lower mean square error (MSE) values. The model developed is advantageous as the equations developed by this hybrid model for the prediction of ground motion parameters are comprehensible compared to the equations of GMPE models. Alavi and Gandomi (2011) used a hybrid model ANN/SA (coupling of artificial neural network with simulated annealing) to predict the principal ground motion parameters PGA, PGV and PGD. The model is better than the GMPEs for the same database as it gives good correlation value. The disadvantage of the model is the time taken to achieve acceptable MSE due to the introduction of simulated annealing. Alavi et al. (2011) developed a variant GMPE model for prediction of ground motion parameters using multi expression programming (MEP). This model gives comparatively reasonable prediction accuracy and validates the advantage of MEP over the traditional GMPE equations developed using regression analysis. Although this model develops the ground motion prediction equation considering the complex nature of the ground motion parameters, the model suffers the drawbacks of GP-based models as the functions are formed randomly and not on the physical process. Gullu (2012) made an attempt to predict PGA for the turkey region using a new approach called gene expression programming (GEP) and conventional regression method. GEPs are an extension to GP. The best model is selected by ranking the models using likelihood based estimation. The model is said to have a fair validation when compared to the existing attenuation relationship for the region. Mohammadnejad et al. (2012) developed a novel GMPE model using the hybrid model of GP and SA. Although the prediction accuracy of the developed model is not very high, the developed model is advantageous as it gives a lesser complex prediction equation for prediction of principal ground motion parameters PGA, PGV and PGD.

Support vector machine (SVM) algorithms are gaining much popularity compared to other soft computing techniques such as artificial neural networks. Though artificial neural networks are popular due to its ease of implementation, it suffers a major drawback which prevents it from being an efficient algorithm. Artificial neural network works on the principle of empirical risk minimization and hence the best ANN architecture is the one having minimum training error. This leads to two major issues of overfitting and local minima. ANN also has an overhead as its computational complexities are dependent on the dimension of the input space. Overcoming all these drawbacks, SVM provides global, unique and sparse solution to problems. It is also less prone to problem of overfitting as it works on structural risk minimization. Therefore, this architecture is being applied to almost all domains such as credit scoring (Huang et al. 2007; Zhou et al. 2009), prediction problems and image processing (Zhu et al. 2006; Samui and Kurup 2012), control and power system (Ucak and Gunel 2004; Lin and Yu 2004; Iplikci 2006a, 2006b), geotechnical engineering (Pal 2006; Goh and Goh 2007; Samui 2008), slope stability (Samui 2013), coral reef prediction (Karthikeyan and Samui 2014) and so on. Although SVM were initially used as a tool for pattern classification, it is gaining popularity in function estimation problem too.

In this paper, prediction models are developed using three learning algorithms, namely ε-support vector regression (ε-SVR), ν-support vector regression (ν-SVR) and least square-support vector regression (Ls-SVR) for forecasting PGA. Using three kernel functions, namely linear kernel, polynomial kernel and RBF kernel for each of the three learning algorithms, seven prediction models are developed. All the seven models are compared in terms of prediction accuracy, error percentage and overfitness to obtain the best prediction model. It is observed that best prediction model outperforms all the existing models.

The remaining sections of the paper are organized as follows. Section 2 details the earthquake data used in the study, including data preprocessing for extraction of input parameters. Section 3 details the basics of the three learning algorithms ε-SVR, ν-SVR and Ls-SVR and the experimental environment used for the modelling. The comparison and the analysis of the results obtained from the developed model are described in Section 4. The paper concludes with Section 5 which further explores the advantages and future scope of the work done.
2. Earthquake data

The ground motion parameter, PGA, is modelled as shown in equation (1), in terms of earthquake magnitude, source to site distance, faulting mechanism and site conditions. The other geophysical parameters like rupture length and rupture area of the fault, its direction, stress drop etc., are not included in the modelling as they only reflect the uncertainties

\[
\ln(Y) = f(V, M, F, \ln(D)) \tag{1}
\]

where \(Y\) is PGA (g), \(V\) is the average shear wave velocity, \(M\) is earthquake magnitude, \(F\) is the style of faulting and \(D\) is the distance.

2.1. Data preprocessing

The database used in this study is a subset of strong motion database (NGA flatfile V 7.3) systematized as part of Pacific Earthquake Engineering Research (PEER) NGA project (Power et al. 2006; Boore & Atkinson, 2007) which was used to develop few worldwide existing GMPE models (Boore & Atkinson, 2007; Campbell and Bozorgnia, 2007; Abrahamson & Silva 2008; Chiou & Youngs 2008; Idirss 2008). The database includes a very large set of ground motions recorded worldwide (3551 records), consisting of shallow crustal earthquakes in active tectonic regimes. The database is comprehensive having sets of meta-data, including different distance measure (column 48—53 of NGA flatfile V 7.3), various site characterizations, earthquake source data, etc. The four geophysical variables selected from the 116 geophysical variables of the original database form input variables in the model. The input variables are selected from the knowledge gained by the literature survey of the related work. The input variables hence chosen are as follows.

a. Velocity (\(V\)): in this study, this variable represents the average shear wave velocity in the top 30 m layers (\(V_{30m}\)) at the site. This geophysical parameter is highly significant as it represents the site influence on seismic signal.

b. Earthquake magnitude (\(M\)): in this study, the moment magnitude of the earthquake is represented by this variable as it best represents the energy released during the earthquake.

c. Faulting mechanism (\(F\)): table 1 represents the values for \(F\) used in this study. This variable is used to represent the basic three types of faulting which occur during the earthquake rupture process. The values are taken based on the 11th column of the NGA flatfile database which represent the mechanism based on rake angle. The type of faulting basically denotes the direction of the movement of the fault plane, which is decided based on the values of the rake angle.

d. Distance (\(D\)): in this study, this variable is used to represent distance. In the NGA flatfile, there are six different types of measured distances among which epicentral distance, Joyner–Boore distance and closest distance are relatively more significant. Any of the three distance measures could be considered for modelling as they are approximately of the same range. In this study, the closest distance measure is represented by Alavi and Gandomi (2011), Alavi et al. (2011) and Mohammadnejad et al. (2012) has also taken the same distance measure.

| Table 1. Description for input variable \(F\). |
|-------------------|-----------------|------------------|
| Values of \(F\)   | Value in NGA flatfile | Faulting style   |
| 1                  | 02,03            | Reverse          |
| 2                  | 01,04            | Normal           |
| 3                  | 00               | Strike slip      |

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2.2. Data normalization

After the preprocessing of the database, the number of records used for the modelling is narrowed down to 2815. The data-sets used for the analysis in this paper are normalized. There are several methods for the normalization of the data. In this paper, the equations (2)–(4) represent the method of normalization used.

Let $Z$ be the variable and $Z_{\text{max}}$ and $Z_{\text{min}}$ represent the maximum and minimum values of the variable, respectively. Let the range within which the variable is to be normalized be $[P, Q]$. In this paper, the range of normalized data is set as $[0.05, 0.95]$ (Alavi & Gandomi 2011). Let $Z_n$ be the normalized value for variable and it is defined as follows

$$Z_n = cZ + d \tag{2}$$

where

$$c = (Q - P)/(Z_{\text{max}} - Z_{\text{min}}) \tag{2a}$$

$$d = Q - (c \times Z_{\text{max}}) \tag{2b}$$

2.3. Training and testing data

The database is divided into training and testing data, consisting of 80% and 20% of the data respectively. The entire 2815 records are divided into these two data-sets such that the statistical parameters of the variables used in the analysis are consistent in both data-sets. Thus the training dataset has 2252 instances and testing dataset has 563 instances. Table 2 shows the statistical criteria considered in this study for the segregation database. The statistical criteria considered for segregation of the database are maximum value, minimum values, median and standard deviation. The entire steps of data preprocessing are shown in figure 1.

To further validate the performance and the generalization ability of the model, the models are tested for another set of data, NGA WEST 2. The database used for developing the model is the NGA database, which was compiled in 2003 and is popularly known as NGA WEST 1. NGA WEST 2 (Bozorgnia 2012) is the updated database consisting of 21,539 events, whereas the NGA WEST 1 has 3551 events. Thus out of 17988 events which are not included in the NGA WEST 1 database, 140 events are selected such that the statistical parameters are consistent to the training and testing dataset (Alavi et al. 2011). This data-set is used to validate the efficacy of the developed model.

3. Method

3.1. Learning algorithm

SVM works on principle of VC theory by Vapnik and Chervonenkis (1974), which is purely based on statistical learning. The problem is solved by equating it to a quadratic programming problem
Figure 1. Flowchart for the data preprocessing.
with inequality constraint. Least Squares Support Vector Machine (Ls-SVM) is a variation of SVM, which uses equality constraint. When SVM is applied to a regression problem, it is termed as SVR. The SVR algorithm is explained as follows.

Consider a given set of training data \(\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}\), where \(x_i \in \mathbb{R}^d, y_i \in \mathbb{R}, i = 1, \ldots, n\). The training data \(x_i\) from the input space \(X\) is mapped onto a feature space \(Q\) such as \(u: x_i \mapsto u(x_i)\) using a predefined nonlinear function \(\vartheta(x)\). Let \(f\) be the linear function having the form as in equation (5)

\[
f(x) = w^T x + b = \langle w, x \rangle + b, \quad \text{where } w \in X, \quad b \in \mathbb{R}, \quad \langle \cdot, \cdot \rangle \text{ denote dot product}
\]

The following subsections explain about the three variations of SVR algorithm used in this study.

### 3.1.1. \(\varepsilon\)-support vector regression (\(\varepsilon\)-SVR)

In \(\varepsilon\)-SV regression (Vapnik 1995), the function \(f(x)\) is calculated such that it is flat, but at the same time has a maximum deviation of \(\varepsilon\). Hence the permissible error band for the function is \([-\varepsilon, \varepsilon]\). The function \(f(x)\) attains flatness when the value of \(w\) is small and to obtain the minimum value for \(w\) is to obtain the minimum norm solution which is \(\|w\|^2 = \langle w, w \rangle\). Hence reformulating the problem as a feasible convex optimization problem, we obtain equation (6)

\[
\text{minimize } \frac{1}{2} \|w\|^2 \text{ subject to } \begin{cases} y_i - \langle w, x_i \rangle - b \leq \varepsilon \\ \langle w, x_i \rangle + b - y_i \leq \varepsilon \\ \end{cases}
\]

The above equation could be again reformulated (Vapnik 1995) to include infeasible constraints of the problem by introducing slack variables

\[
\text{minimize } \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n (\varepsilon_i + \varepsilon_i^*) \text{ subject to } \begin{cases} y_i - \langle w, x_i \rangle - b \leq \varepsilon + \varepsilon_i \\ \langle w, x_i \rangle + b - y_i \leq \varepsilon + \varepsilon_i^* \\ \varepsilon_i, \varepsilon_i^* \geq 0 \\ \end{cases}
\]

where \(C\) is the tradeoff between permissible error \(\varepsilon\) and \(\|w\|\). The aim is to minimize the empirical risk value given as \(E = \frac{1}{N} \sum_{i=1}^N \|y_i - \hat{y}_i\|_e\), where \(\|y_i - \hat{y}_i\|_e\) is \(\varepsilon\)-insensitive loss function as shown in figure 2, such that

\[
\|y_i - \hat{y}_i\|_e = \begin{cases} 0, & |y_i - \hat{y}_i| \leq \varepsilon \\ |y_i - \hat{y}_i| - \varepsilon, & \text{otherwise} \end{cases}
\]

where \(y_i\) and \(\hat{y}_i\) are target and predicted values.

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**Figure 2.** Support vector regression with \(\varepsilon\) insensitive loss function.
respectively. This formulation is also known as linear \( \varepsilon \)-insensitive loss regression (Cortes & Vapnik 1995; Vapnik 1998).

The equation (7) could be solved using dual formulation involving Lagrange multipliers (Fletcher 1989). Hence the objective function of equation (7) is replaced by corresponding Lagrange function. Let \( L \) be the Lagrangian function and \( \alpha_i, \alpha^*_i, \beta_i, \beta^*_i \) be the corresponding Lagrange multipliers or dual variables. Hence the equation (7) is replaced as follows:

\[
L = \frac{1}{2} ||w||^2 + C \sum_{i=1}^{n} (e_i + e^*_i) - \sum_{i=1}^{n} \left( \alpha_i e_i + \alpha^*_i e^*_i \right) - p - q
\]  

where \( p = \sum_{i=1}^{n} \beta_i (e_i + e_i - y_i + \langle w, x_i \rangle + b) \), \( q = \sum_{i=1}^{n} \beta^*_i (e_i + e^*_i + y_i - \langle w, x_i \rangle - b) \), and \( \alpha_i, \alpha^*_i, \beta_i, \beta^*_i \geq 0 \).

For all feasible solution of the primal and dual variables for the convex optimization objective function, there exist a saddle point (Mangasarian 1968). At optimality, the gap between the primal and dual objective function decreases (strong duality theorem (theorem 6.4.3, Bazaraa et al. 1993).

Taking the partial derivative of Lagrangian function \( L \) with respect to the primal variables \( \langle w, b, e_i, e^*_i \rangle \), we obtain equation (9)

\[
\frac{\partial L}{\partial w} = w - \sum_{i=1}^{n} (\beta_i - \beta^*_i) x_i, \quad \frac{\partial L}{\partial b} = \sum_{i=1}^{n} (\beta^*_i - \beta_i), \quad \frac{\partial L}{\partial e_i} = C - \beta_i - \alpha_i, \quad \frac{\partial L}{\partial e^*_i} = C - \beta^*_i - \alpha^*_i
\]

Hence, at optimality by theory, \( \frac{\partial L}{\partial w} = 0, \frac{\partial L}{\partial b} = 0, \frac{\partial L}{\partial e_i} = 0, \frac{\partial L}{\partial e^*_i} = 0 \). Thus eliminating dual variables, we obtain

\[
\alpha_i = C - \beta_i, \quad \alpha^*_i = C - \beta^*_i
\]

Substituting equations (9) and (10) in equation (8), the dual optimization problem is formulated as follows

\[
\text{Maximize : } \left\{ -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (\beta_i - \beta^*_i) (\beta_j - \beta^*_j) \langle x_i, x_j \rangle - \varepsilon \sum_{i=1}^{n} (\beta_i + \beta^*_i) + \sum_{i=1}^{n} y_i (\beta_i - \beta^*_i) \right\}
\]

subject to \( \sum_{i=1}^{n} (\beta_i - \beta^*_i) \) and \( \beta_i, \beta^*_i \in [0, C] \).

Rewriting Equation (9), we obtain \( w = \sum_{i=1}^{n} (\beta_i - \beta^*_i) x_i \)  

Substituting in equation (12) in equation (5), we obtain the solution to the function \( f(x) \) as shown in figure 3

\[
f(x) = \sum_{i=1}^{n} (\beta_i - \beta^*_i) \langle x_i, x \rangle + b
\]

\( w \) is linear combination of training inputs \( x_i \) in the input space \( X \). Hence this is called the support vector expansion, where the function complexity is dependent on the number of support vectors.
rather than the dimensionalities of the input space \( X \). \( b \) is computed using KKT conditions (Kuhn and Tucker 1951). At KKT optimality condition (constraint dualvariable = 0). Thus we obtain

\[
(C - \beta_i)e_i = 0, \quad (C - \beta_i^*)e_i^* = 0, \quad \beta_i(e + e_i - y_i + \langle w, x_i \rangle + b) = 0, \quad \beta_i^*(e + e_i - y_i + \langle w, x_i \rangle + b) = 0
\]

(14)

From equation (10), \( \forall (x_i, y_i) \) with \( \beta_i, \beta_i^* = C \), there exists points in space \( X \), that lie outside the \([-\varepsilon, \varepsilon]\) band and there can never be a condition such that the set of dual variables \([\beta_i, \beta_i^*]\) is simultaneous zero as \( \beta_i\beta_i^* = 0 \). Hence the solution for ‘b’ is as follows (keerthi et al. 2001)

\[
b = y_i - \langle w, x_i \rangle - \varepsilon, \quad \forall \beta_i \in (0, C)
\]

(15)

\[
b = y_i - \langle w, x_i \rangle + \varepsilon, \quad \forall \beta_i^* \in (0, C)
\]

(16)

Thus the function estimation for \( \varepsilon \)-SVR model for is

\[
f(x) = \sum_{i=1}^{n} (\beta_i - \beta_i^*)\langle x_i, x \rangle + b, \quad \text{with } b = \left\{ \begin{array}{ll} y_i - \langle w, x_i \rangle - \varepsilon, & \forall \beta_i \in (0, C) \\ y_i - \langle w, x_i \rangle + \varepsilon, & \forall \beta_i^* \in (0, C) \end{array} \right.
\]

(17)

3.1.2. \( \nu \)-support vector regression (\( \nu \)-SVR)

Another version of SVR, \( \nu \)-SVR, was proposed by Scholkopf et al. (2000) which uses parameter \( \nu \) of range \((0, 1]\). It is similar to \( \varepsilon \)-SVR with \( \varepsilon \) itself considered as a parameter to have a control on the count of support vector. The formulation for \( \nu \)-SVR is similar to \( \varepsilon \)-SVR, with a slight change. Thus equation (7) is reformulated for \( \nu \)-SVR as

\[
\text{Minimize} \quad \frac{1}{2}||w||^2 + C\nu e + C \sum_{i=1}^{n} (e_i + e_i^*) \quad \text{subject to} \quad \left\{ \begin{array}{l} y_i - \langle w, x_i \rangle - b \leq \varepsilon + e_i \\ \langle w, x_i \rangle + b - y_i \leq \varepsilon + e_i^* \\ e_i, e_i^* \geq 0, \quad \varepsilon \geq 0 \end{array} \right.
\]

(18)

The dual formulation of \( \nu \)-SVR is similar to \( \varepsilon \)-SVR, as given in equation (11), with only change in the constraint. The new constraint for \( \nu \)-SVR is \( \beta_i, \beta_i^* \in [0, C\nu] \).
3.1.3. Least square support vector regression (Ls-SVR)

The function estimation problem using least squares support vector is formulated in this section. The linear model of Ls-SVR is similar to equation (5). Consider a given set of training data \{(x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)\}, where \(x_i \in \mathbb{R}^d, y_i \in \mathbb{R}, i = 1 \ldots n\). Let \(f\) be the linear function, from equation (5) we have \(f(x) = w^t x + b\). Let \(\delta, \gamma\) denote the error vector and the column vector, respectively. Keeping all the notations in the above section, the regression estimation problem is formulated as

\[
\text{Minimize: } \frac{1}{2} w^t H w + \frac{1}{2} \delta^t \delta \text{ subject to } K(x_i,x_i) w + \gamma b + \delta - y_i = 0
\]  

(19)

The Lagrange multipliers method used in Section 3.1.1 is used to solve convex optimization problem equation (19).

Let \(L\) be the Lagrangian function and \(\beta_i\) be the corresponding Lagrange multiplier. Let \(x_i\) and \(y_i\) be denoted as \(x\) and \(y\). Hence the corresponding Lagrangian function for equation (19) is as follows:

\[
L = \frac{1}{2} w^t H w + \frac{1}{2} \delta^t \delta - \beta^t [K(x,x) w + \gamma b + \delta - y] \ (\delta, \alpha) \in \mathbb{R}^d
\]  

(20)

As per the theory explained in the above section at optimality the constraints are as follows:

\[
\frac{\partial L}{\partial w} = Hw - K^t \beta = 0, \frac{\partial L}{\partial b} = \gamma, \beta = 0, \frac{\partial L}{\partial \delta} = \delta - \beta = 0, \frac{\partial L}{\partial \alpha} = Kw + \gamma b + \delta - y = 0
\]  

(21)

Solving we obtain

\[
w = K^t H^{-1} \beta, \ \delta = \beta
\]  

(22)

Combining equations (20)—(22), we obtain

\[
\begin{bmatrix}
0 & \gamma \\
\gamma & KH^{-1}K^t + I
\end{bmatrix}
\begin{bmatrix}
b \\
\beta
\end{bmatrix} =
\begin{bmatrix}
0 \\
y
\end{bmatrix}
\]  

(23)

Let \(H = K\), assuming that the kernel \(K\) being symmetric positive definite, equation (23) reduces as

\[
\begin{bmatrix}
0 & \gamma \\
\gamma & K + I
\end{bmatrix}
\begin{bmatrix}
b \\
\beta
\end{bmatrix} =
\begin{bmatrix}
0 \\
y
\end{bmatrix}
\]  

(24)

Equation (24) is analogous to standard Ls-SVM (Suykens et al. 2002).

For solving \(K\), the assumption is \(H = I\). Hence the restrictions of symmetry, positive definiteness, semi definiteness and continuity on \(K\) is removed.

Hence, we obtain

\[
w = K^t \beta, \begin{bmatrix}
0 & \gamma \\
\gamma & KK^t + I
\end{bmatrix}
\begin{bmatrix}
b \\
\alpha
\end{bmatrix} =
\begin{bmatrix}
0 \\
y
\end{bmatrix}
\]  

(25)

In equation (25), \(KK^t\) is positive semi definite, with no restrictions on \(K\) thus making equation (26) linearaly solvable.

The \textbf{Ls – SVR model} for function estimation is \(f(x) = KK^t \beta + \gamma b\)

(26)
3.2. Experimental environment and parameters

The algorithms are implemented and tested on C and MATLAB R2012 b platform on a PC with processor Intel(R) core(TM) i3-3220 and 4 GB RAM. The LIBSVM package (Chang 2011) is used to implement $\varepsilon$-SVR and $\nu$-SVR. Ls-SVR is implemented using Ls-SVMlab version 1.8 (Brabanter 2011).

The kernel function $k(x_i, x_j)$ defined in LIBSVM package is as follows:

Linear kernel $k(x_i, x_j) = x_i^T x_j$  \hspace{1cm} (27)

Polynomial Kernel $k(x_i, x_j) = (\text{gamma} \ast x_i^T x_j + r)^\text{degree}$ \hspace{1cm} (28)

RBF kernel $k(x_i, x_j) = e^{-\text{gamma} \ast ||x_i - x_j||^2}$ \hspace{1cm} (29)

The kernel function $k(x_i, x_j)$ defined in Ls-SVMlab package is as follows:

Linear kernel $k(x_i, x_j) = x_i^T x_j$ \hspace{1cm} (30)

RBF kernel $k(x_i, x_j) = e^{-\frac{||x_i - x_j||^2}{2s^2}}$ \hspace{1cm} (31)

$\varepsilon$-SVR uses parameters $C \in [0, \infty)$ and $\varepsilon \in [0, \infty)$ to apply a penalty to the optimization for points which were not correctly predicted. There is no penalty associated with points which are predicted within distance $\varepsilon$ from the actual value. By decreasing $\varepsilon$, closer fitting to the data is obtained.

$\nu$-SVR uses parameters $C \in [0, \infty)$ and $\nu \in [0, 1]$. The $\varepsilon$ penalty parameter was replaced by $\nu$. $\nu$ represents an upper bound on the fraction of training samples which are errors poorly predicted and a lower bound on the fraction of samples which are support vectors. $\varepsilon$ and $\nu$ are versions of the penalty parameter.

The other two parameters used are $C$ (cost) and gamma. The cost represents the penalty associated with errors larger than epsilon. Increasing cost value gives closer fitting to data. Parameter gamma controls the shape of the separating hyperplane. Increasing gamma usually increases number of support vectors.

3.2.1. $\varepsilon$-SVR parameters

The $\varepsilon$-SVR algorithm is implemented with linear and RBF kernels. The tolerance error for termination criteria is set at 0.1. A Grid search algorithm with cross validation is used to find the parameters of RBF kernel. Hence the value of $C$ thus obtained is 128. The value of gamma is set as 0.25(1/number of features) and $\varepsilon$ as 0.1.

3.2.2. $\nu$-SVR parameters

The $\nu$-SVR algorithm is implemented with linear, polynomial and RBF kernels. The parameters such as tolerance error for termination criteria, gamma and $C$ are same to that of $\varepsilon$-SVR algorithm. The value for $\nu$ is set as 0.5. The degree is set as 3 for polynomial kernel.

3.3.3. Ls-SVR parameters

The Ls-SVR algorithm is implemented with linear and RBF kernels. The parameter gamma here is called a regularization parameter, which determines the tradeoff between the minimization of training error and the smoothness of the estimated function. The simulated coupling method is used to obtain the best value for gamma. For linear kernel the value obtained is 1.52 and for RBF kernel gamma is 2.321. For RBF kernel, sig2 is additional parameter which represents the variance of the kernel function. The value obtained is 0.19.
4. Result evaluation and discussion

The model performance is measured in terms of criteria as given in table 3, where \( n \), \( a_i \), \( p_i \), \( \text{avg}(a_i) \) and \( \text{avg}(p_i) \) represent the total number of records, observed output, predicted output, the average value of observed output and average value of the predicted output, respectively. The statistical parameters such as correlation coefficient (\( R \)), mean absolute error (MAE), mean absolute percentage error (MAPE) and MSE are used to test the efficacy of the model (Baykasoğlu et al. 2008; Canakci et al. 2009; Gullu 2014b; Gullu 2015b). The criteria are valid for both training and testing data-sets. The results obtained for each prediction model are given in tables 4—6. There exists a well-reasoned presumption that if the correlation coefficient (\( R \)), \( |R| > 0.8 \) (Smith 1986) and error percentage is minimum; there is a high correlation between the predicted and the observed values.

Tables 4 and 5 show the results obtained from the developed prediction model. Table 6 shows the comparison of the developed model with the four other existing prediction models. Table 7 shows the comparison of the developed model with the existing GMPEs. Table 8 shows the overall performance of all models. For the better justification of the obtained results, this section is further divided into subsections. The subsection 4.1 describes the result obtained by developed model. The subsection 4.2 compares the developed model with other existing models on the same database. The subsection 4.3 further evaluates the learning effectiveness of the algorithms.

### Table 3. Criteria for measuring model performance.

| Criteria | Description | Formula |
|----------|-------------|---------|
| \( R \)  | Correlation coefficient | \[ R = \frac{\sum_{i=1}^{n} (a_i - \text{avg}(a_i))(p_i - \text{avg}(p_i))}{\sqrt{\sum_{i=1}^{n} (a_i - \text{avg}(a_i))^2 \sum_{i=1}^{n} (p_i - \text{avg}(p_i))^2}} \] |
| MAE      | Mean absolute error       | \[ \text{MAE} = \frac{\sum_{i=1}^{n} |a_i - p_i|}{n} \] |
| MAPE     | Mean absolute percentage error | \[ \text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|a_i - p_i|}{a_i} \] |
| MSE      | Mean squared error         | \[ \text{MSE} = \frac{\sum_{i=1}^{n} (a_i - p_i)^2}{n} \] |

### Table 4. \( \varepsilon \)-SVR, \( \nu \)-SVR, Ls-SVR prediction model for PGA.

| \( \varepsilon \)-SVR | \( \nu \)-SVR | Ls-SVR |
|-----------------------|--------------|--------|
| Kernel | Linear | RBF | Linear | Polynomial | RBF | Linear | Train | Test | Train | Test | Train | Test | Train | RBF |
| \( R \) | \( 0.4836 \) | \( 0.5932 \) | \( 0.7643 \) | \( 0.5058 \) | \( 0.5515 \) | \( 0.5364 \) | \( 0.7313 \) | \( 0.7629 \) | \( 0.5194 \) | \( 0.5946 \) | \( 0.8719 \) | \( 0.8700 \) |
| MAE | \( 0.0625 \) | \( 0.0733 \) | \( 0.0580 \) | \( 0.0246 \) | \( 0.0622 \) | \( 0.0247 \) | \( 0.0622 \) | \( 0.0239 \) | \( 0.0315 \) | \( 0.0578 \) | \( 0.0149 \) | \( 0.0316 \) |
| MAPE | \( 0.8223 \) | \( 0.7117 \) | \( 0.5581 \) | \( 0.1956 \) | \( 0.2931 \) | \( 0.1934 \) | \( 0.2978 \) | \( 0.2460 \) | \( 0.2765 \) | \( 0.3424 \) | \( 0.3651 \) | \( 0.1367 \) | \( 0.1899 \) |
| MSE | \( 0.0057 \) | \( 0.0102 \) | \( 0.0064 \) | \( 0.0040 \) | \( 0.0151 \) | \( 0.0040 \) | \( 0.0150 \) | \( 0.0023 \) | \( 0.0079 \) | \( 0.0034 \) | \( 0.0113 \) | \( 0.0011 \) | \( 0.0037 \) |

### Table 5. Prediction model for testing NGA WEST2 data.

| \( \varepsilon \)-SVR | \( \nu \)-SVR | Ls-SVR |
|-----------------------|--------------|--------|
| Kernel | Linear | RBF | Linear | Polynomial | RBF | Linear | Train | Test | Train | Test | Train | RBF |
| \( R \) | \( 0.5715 \) | \( 0.6585 \) | \( 0.7061 \) | \( 0.5058 \) | \( 0.5737 \) | \( 0.7171 \) | \( 0.6803 \) | \( 0.8140 \) |
| MAE | \( 0.0393 \) | \( 0.0537 \) | \( 0.0073 \) | \( 0.0073 \) | \( 0.0076 \) | \( 0.0148 \) | \( 0.0160 \) | \( 0.0060 \) |
| MAPE | \( 0.5867 \) | \( 0.8308 \) | \( 0.1059 \) | \( 0.1056 \) | \( 0.2180 \) | \( 0.2461 \) | \( 0.0853 \) |
| MSE | \( 0.0022 \) | \( 0.0036 \) | \( 0.0001 \) | \( 0.0001 \) | \( 0.0003 \) | \( 0.0004 \) | \( 0.0001 \) |
4.1. Analysis of the results obtained from $\varepsilon$-SVR, $\nu$-SVR and Ls-SVR prediction models

Table 4 shows the result obtained from $\varepsilon$-SVR, $\nu$-SVR and Ls-SVR PGA prediction model for both training and testing data. The performance of the each model is evaluated based on the criteria mentioned in Table 3. Table 5 shows the result obtained from the developed models for NGA WEST 2 data.

It is observed from Table 4 that the best prediction model for forecasting PGA is Ls-SVR RBF kernel prediction model followed by $\nu$-SVR RBF kernel prediction model and $\varepsilon$-SVR RBF kernel prediction model. Though the accuracy of the other models (Table 4) is not in the acceptable range (Smith 1986), it is observed that the error percentages are low. The best prediction model is Ls-SVR RBF kernel prediction model as it gives good prediction accuracy with low error percentage (Table 4) and also gives a fair correlation for NGA WEST 2 data (Table 5), which is a data-set outside the training database.

4.2. Comparison with other existing model

Table 6 gives the comparison of the best developed PGA prediction model with other existing model on the same database. The Ls-SVR RBF kernel prediction model is compared to four models namely ANN/SA (Alavi & Gandomi 2011), GP/OLS [a hybrid model of genetic programming coupled with orthogonal least squares (Gandomi et al. 2011)], MEP (Alavi et al. 2011) and GP/SA [a hybrid model of genetic programming coupled with simulated annealing (Mohammadnejad et al. 2012)].

| Criteria | $\varepsilon$-SVR | $\nu$-SVR | Ls-SVR |
|----------|-------------------|-----------|--------|
| $R^2$    | 0.8719            | 0.8700    | 0.869  |
| MAE      | 0.0149            | 0.0316    | 0.30   |
| MAPE     | 0.1367            | 0.1899    | 0.14   |
| MSE      | 0.0011            | 0.0037    | n/a    |

Table 7. Comparison of developed prediction model with GMPEs.

| Model | MAPE  |
|-------|-------|
| Ls-SVR model (RBF kernel) | 0.18  |
| Campbell and Bozorgnia (2007) | 0.93  |
| Ambraseys et al. (1996) | 0.95  |
| Smith et al. (2000) | 14.58 |

Table 8. Comparison of models for overall performance.

| Model | Kernel | Accuracy | Testing error | Overfitness |
|-------|--------|----------|---------------|-------------|
| $\varepsilon$-SVR | Linear | 0.593 | 0.0733 | 0.173 |
|        | RBF    | 0.764 | 0.058  | 0.092 |
|        | Linear | 0.551 | 0.0622 | 0.49  |
| $\nu$-SVR | Polynomial | 0.531 | 0.0622 | 0.54  |
|        | RBF    | 0.763 | 0.046  | 0.12  |
| Ls-SVR  | Linear | 0.595 | 0.0578 | 0.067 |
|        | RBF    | 0.870 | 0.0316 | 0.38  |
| GP/SA   |        | 0.839 | 0.144  | 0.21  |
| GP/OLS  |        | 0.811 | 0.49   | 0.24  |
| MEP     |        | 0.834 | 0.69   | 0.92  |
| ANN/SA  |        | 0.855 | 0.46   | 0.53  |
In table 6 where the corresponding values is missing is denoted by symbol n/a. It is observed that the prediction accuracy of the developed model is better compared to other existing models. Another important observation is that the error percentage of the developed model is much less compared to all the existing benchmark models.

In table 7, the Ls-SVR RBF kernel prediction model is compared with existing GMPEs. The comparison is done for the testing data based on the criteria MAPE. It is observed that the Ls-SVR RBF kernel prediction model gives the least error percentage, compared to the existing GMPEs. Moreover the developed prediction model uses only 4 geophysical parameters as input, whereas the GMPEs include many input parameters. Hence the developed model has comparatively lesser computational overhead with better precision.

4.3. Comparing the learning effectiveness of the algorithm

Tables 4—6 shows the results and the comparison between the prediction models. The comparison is done based on the criteria as given in table 3. Table 8 compares all the developed models along with the existing benchmark models in terms of accuracy, testing error and overfitness (Zhong et al. 2014). The testing error is indicator of overfitness. The higher measure of overfitness implies that the model gives higher measure of error for testing. The measure of overfitness is calculated as in equation (32). Thus the overall authenticity of prediction precision of the model is collectively given by the accuracy, testing error and the overfitness measure of the testing data

\[
\text{Overfitness} = \left| \frac{|\text{error}(\text{testing}) - \text{error}(\text{training})|}{\text{error}(\text{training})} \right|
\]  

(32)

From table 8, it is observed that the best prediction model is Ls-SVR RBF kernel model as it gives a high prediction accuracy with lesser overfitness measure. It is observed that the ANN/SA model, though having slightly higher precision than GP/OLS and GP/SA model in terms of accuracy (correlation coefficient), the testing error is comparable and the overfitness measure is better for GP based hybrid models. Thus the performances of the three existing models are comparable. The MEP model clearly shows that the overfitness measure is very high. Similarly, though the precision accuracy of \(\nu\)-SVR RBF kernel model in terms of correlation coefficient \((R)\) is slightly less, the testing error is comparable to the 3 existing models ANN/SA, GP/OLS and GP/SA, with much lesser overfitness measure. Thus the overall performance of \(\nu\)-SVR RBF kernel prediction model could be assumed to be satisfactory. Similarly, the overall performance of \(\varepsilon\)-SVR RBF kernel prediction model could also be satisfactory.

Another important inference from table 8 is that it is observed that the measure of overfitness for all models of SVR is much lesser than the hybrid model of ANN coupled with SA (Alavi & Gandomi 2011) and MEP model (Alavi 2011). Hence it further validates the claim of SVM being less prone to overfitting than an artificial neural network with better generalization.

5. Conclusion

In this study, a prediction model based on SVR is proposed for forecasting PGA, a parameter associated with seismic signals. The prediction model is developed on the extensive database compiled in the PEER–NGA project. The following are the conclusions drawn on the results obtained in this study:

a. A prediction model is developed using advanced machine learning technique (SVR) for the prediction of ground motion parameter PGA. Among the seven developed models, the Ls-SVR model with RBF kernel is precise and has a better generalization with a lesser measure of...
overfitting. The overall performance of the developed model is better compared to the existing benchmark models in the same database.

b. The proposed prediction model could be used as a tool for faster and accurate prediction of the ground motion parameter with lesser calculation overhead (comparatively with GMPEs), in all areas such as seismic risk assessment, seismic hazard analysis, earthquake resistant structural engineering, etc., where the principal ground motion parameters are used as a vital input parameter.

c. The proposed prediction model relates the ground motion parameter PGA to four seismic parameters, namely faulting mechanism, average soil shear wave velocity, earthquake magnitude and source to site distance.

d. The proposed prediction model is tested on 563 earthquake data records from NGA WEST1 database (table 4) and further tested, as an external validation, on another set of 140 earthquake data records from NGA WEST 2 database (table 5). Hence the proposed model is tested for 703 events. This validates the precision of the proposed model.

e. The proposed prediction model is compared to four existing models on the same database (table 6). The hybrid models developed on the same database are ANN/SA model, GP/OLS model, MEP model and GP/SA model. It is clearly observed that the proposed Ls-SVR (RBF kernel) prediction model has better precision compared to all these benchmark models.

f. The efficacy of the proposed Ls-SVR(RBF kernel) prediction model is further validated by comparing it to existing GMPE model (Ambraseys et al. 1996; Smit et al. 2000; Campbell & Bozorgnia 2007).

g. The proposed model is developed and tested on the global database so that there exists a common platform for comparison and validation. The proposed Ls-SVR (RBF kernel) prediction model gives better results compared to all existing benchmark and conventional ground motion parameter prediction models.

Disclosure statement

No potential conflict of interest was reported by the authors.

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