Compressive phase-only filtering at extreme compression rates

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Abstract: We introduce an efficient method for the reconstruction of the correlation between a compressively measured image and a phase-only filter (POF). The proposed method is based on two properties of POF: such filtering is a unitary circulant transform, and the correlation plane it produces is usually sparse. The method can be applied to the measurements from single-pixel detectors, which opens a way for adding some functionalities of an optical correlator to these kind of cameras at no additional computational expense. We show that images measured at extremely high compression rates may still contain sufficient information for target classification and localization.

Keywords: Computational imaging; pattern recognition; image recognition, algorithms and filters

Compressive sensing [1, 2](CS) is a rapidly developing field of mathematics and signal processing with important contributions to the introduction of novel measurement methodologies and information recovery techniques in numerous fields of science and technology. In optics, CS has been initially applied for computational ghost imaging [3, 4, 5, 6] giving rise to novel image acquisition techniques often referred as single-pixel or bucket detectors. The growing range of research directions and applications based on CS now includes 3D imaging [7, 8], lidar imaging [9], joint measurement of distance to the object and its shape [10], development of 3D laser-radar devices [11], fluorescence microscopy [12], medical imaging [13], terahertz imaging [14, 15], Stokes polarimetric imaging [16], compressive holography [17, 18], and imaging through scattering media [19]. A route to super-resolution has been indicated [20]. Metamaterial imaging systems aided by CS have been also recently investigated [21]. At the same time, the shortcomings of CS are now also becoming clear [22, 23, 24]. Attempts to develop adaptive measurement schemes for the use together with CS have been reported [25]. Moreover, other reconstruction methods such as the pseudo-inverse [26] have been shown to be an interesting alternative to $l_1$-norm minimization which is used in CS.

Compressive sensing allows to redesign measurement and processing methods for images that have a sparse representation. In this letter we show that images measured compressively at extremely high compression rates, on the order of $10^3–10^4$, at which their proper reconstruction with a quality allowing for visual examination is no longer possible, may still contain sufficient information for pattern recognition and target localization. We propose a pattern recognition method applicable to compressive measurements based on filtering with a phase-only matched filter (POF) [27] and lasso optimization [28]. This may be regarded as a refined concept of the smashed filter where a matched filter was used for target localization [29, 30, 31]. POF filter is one of the basic filters used in optical correlation-based pattern recognition. We note that POF-filtering is a unitary circulant transform which may be directly included in the
CS framework, and that correlation-based pattern recognition, especially with the use of highly discriminant filters such as POF, tends to transform images containing a target object into a representation with a high sparsity. Small coherence between the noiselet [32] or Walsh-Hadamard (WH) basis and the typical POF-filters makes the measurement and recognition operations independent, hence at the time of compressive measurement one does not need to have the knowledge of the target objects which the images will be later scanned for.

We begin with discussing joint use of linear filtering and CS, when some measured quantity, which in our case would be a target detection signal, is found indirectly by solving an optimization problem such as lasso,

$$\tilde{s} = \arg \min_{s} \|A \cdot s - y\|_2 \quad \text{subject to} \quad \|s\|_1 \leq \tau. \quad (1)$$

Here $\tilde{s}$ is determined as an approximate solution to the underdetermined set of linear equations $A \cdot s = y$ with an imposed constraint on the $l_1$ norm $\|s\|_1 \leq \tau$. Lasso optimization (1) tends to recover a sparse solution $\tilde{s}$ to the underdetermined set of linear equations, i.e. a solution which contains lots of small elements such that $\tilde{s}_i \approx 0$. Efficient numerical algorithms exist for solving the problem (1), when the matrix $A$ is orthonormal or semi-orthogonal (unitary or semi-unitary for complex matrices). A rectangular and right invertible matrix is semi-unitary if its product with its complex conjugate transpose gives an identity matrix $A \cdot A^\dagger = I$.

Lasso, like basis pursuit denoising [28], may be used for the recovery of the compressible signal $x$ from a compressive measurement $y = M \cdot x$ (where the size of the signal is $n$ and the size of the measurement $y$ is $m < n$). The measurement matrix $M$ includes information on the measurement method and its size is $m \times n$. It is assumed that there exists an a priori unknown sparse representation of the signal $s = T^{-1} \cdot x$ with at most $k < m$ non-zero elements, whereas the size of $s$ and $x$ is the same and is equal to $n$. The exact or approximate sparse representation of the signal $x$ may be recovered as the solution $\tilde{s}$ of (1) with $A = M \cdot T$ where the matrices $M$, and $T$ are maximally incoherent. A straightforward way to assure the (semi)orthogonality of $A$ is by selecting (semi)orthogonal matrices $M$ and $T$. We are now going to focus on matrices $T$ that correspond to linear filtering. Such matrices have a circu-
lant form. A $n \times n$ circulant matrix is defined with an $n$ element vector $h$ as $T_{k,l} = h_{(k-l) \mod n}$. A circulant matrix is diagonalized by one-dimensional discrete Fourier transform (DFT) $F$ (vectors and matrices in the Fourier-domain will be shortly denoted with a caret), i.e. $\hat{T} = F \cdot T \cdot F^\dagger$ is a diagonal matrix with diagonal elements $\hat{T}_{k,k} = \sqrt{n} \cdot \hat{h}_k$ consisting of the DFT of the vector $h$, i.e. $\hat{h} = F \cdot h$. Finally, a left-multiplication of a vector $v$ by $T$ corresponds to a circulant convolution between vectors $v$ and $h$ i.e. $v \ast h = h \ast v = T \cdot v$. In two dimensions $F$ is replaced by the two-dimensional DFT $F \otimes F$, where $\otimes$ is the Kronecker product. Images are stored in a vector form, while the circulant (Toeplitz) matrix is now block-circulant (Toeplitz). In a shift-invariant linear system, $h$ is often called the point spread function, and $\hat{h}$ the transfer function. Circulant matrices representing linear filters need not to be orthogonal (or unitary for complex matrices). However, an important class of orthogonal filters exists, namely the phase-only filters (POF). In fact, when the transfer function $[\hat{h}]_{\text{POF}} = \exp(i\phi_k)$, one gets $T_{\text{POF}} \cdot {T_{\text{POF}}}^\dagger = \hat{T}_{\text{POF}} \cdot {\hat{T}_{\text{POF}}}^\dagger = I$. Circulant matrices defined in the Fourier space with phase-only random elements have been used for sampling in CS by Romberg [33]. Yin et al. [34] introduced basis pursuit and related optimization algorithms applicable to several kinds of Toeplitz and circulant matrices, however solving the optimization problem (1) is simpler for unitary matrices and in consequence for POF than for other filters.

As indicated, POF can be used as the compression matrix $T$ with the lasso algorithm, in which $A = M \cdot T$. Needless to say that filters with a phase-only transfer function are of great importance to optics [27]. For instance, Fresnel diffraction is a kind of POF spatial filtering occuring during propagation. In time domain the evolution of a pulse envelope is also within the second-order dispersion approximation described as POF-filtering. More general POF spatial filtering may be realized in numerous correlator-based architectures e.g. [35], with phase-only spatial light modulators (SLM). Phase-only modulation makes use of the total light energy incident on a SLM, and is characterized with a high diffraction efficiency. There exist coding methods for phase diffractive optical elements (DOE) such as iterative Fourier transform algorithm which allow to encode a rather general response in phase-only elements. Finally, POF matched filters have been applied to optical pattern recognition constituting a recognition method with a high discrimination capacity. Phase-only filtering techniques have been successfully used for phase visualization as well as for optical encryption. The use of POF filtering jointly with CS in optics depends on its actual capability to produce a compressed signal representation. Let us examine some of the aforementioned examples. For instance, a sharp image produced by Fresnel diffraction at the image plane of an imaging set-up is likely to be more sparse than the same image out of focus. A pulse envelope becomes sparse at a distance which corresponds to the highest pulse compression. For filtering with the POF matched filter

$$[\hat{h}]_{\text{POF}}[k] = \hat{r}_k/|\hat{r}_k|,$$  

(2)

where $r$ is the target image, the correlation signal with the analyzed input scene $x$ is equal to $s = h_{\text{POF}} \ast x$. The intensity distribution in $|s|^2$ contains image-recognition information with narrow detection-peaks at the locations of the recognized target images. With a dictionary of reference objects $r$, examination of $|s|^2/\|r\|^2$ may be used for target classification. Both $s$ and $|s|^2$ are sparse, provided that the distribution of target objects is sparse and that the cross-correlation with background objects is small. This last example will now be further examined alongside with the pure-phase correlation (PPC) defined as

$$s = h_{\text{POF}} \ast x' \quad \text{with} \quad |x'|_k = \hat{x}_k/|\hat{x}_k|,$$  

(3)

which is also a well established pattern recognition method equivalent to POF-filtering of the whitened input scene. A compressive measurement for the PPC is based on $x'$ rather than on $x$, i.e. $y = M \cdot x'$. Unlike POF, PPC belongs to nonlinear filtering techniques [36,37] with certain adaptive properties. For both POF and PPC, $s$ is sparse and POF is unitary, therefore it is direct to apply lasso optimization (1), which returns the correlation plane $s$. If one needs
to reconstruct the input scene $x$ or $x'$ from the same compressive measurement, conjugate phase-only filtering may be applied to $s$. However, as we will see, target recognition can be still implemented at compression rates at which visual examination of a reconstructed image is no longer possible.

In numerical experiments we found that lasso-based compressive matched POF filtering is particularly efficient when the compressive measurements are taken with measurement matrix $M$ consisting of rows with randomly selected WH basis, while the PPC correlation gives even better results with discrete noiselet basis. The WH transformation matrix whose size is a power of two has the following recursive form

$$H_{2m} = \frac{1}{\sqrt{2}} \begin{bmatrix} H_m & H_m \\ H_m & -H_m \end{bmatrix} \quad \text{with} \quad H_1 = 1. \quad (4)$$

A respective formula for the discrete noiselet transformation is

$$N_{2m} = \frac{1}{2} \begin{bmatrix} (1 - i) \cdot N_m & (1 + i) \cdot N_m \\ (1 + i) \cdot N_m & (1 - i) \cdot N_m \end{bmatrix} \quad \text{with} \quad N_1 = \frac{1}{2} I. \quad (5)$$

Same as for DFT, the two-dimensional transforms are obtained through the Kronecker product of one-dimensional transforms, i.e. $H_m^{2D} = H_m \otimes H_m$ and $N_m^{2D} = N_m \otimes N_m$. A measurement matrix consisting of Fourier basis could be also used with either POF or PPC, since POF-filtering has a minimum coherence with the DFT transform. We include the results obtained with DFT for comparisons, however its optical implementation would be a lot more problematic. Actually, while all these methods can be easily applied numerically, only the POF-WH case can be straightforwardly implemented in an optical single-pixel detector with a binary modulator, with real non-negative images and without input scene whitening [3]. In Fig. 1(a) we present a comparison between the three methods applied to a $1024 \times 1024$ 8-bit input scene showing the surface of sea with two ships: a target object and a false-target object. Both ships have a similar brightness, orientation and size. Correct recognition is possible up to extremely high compression ratios on the order of $\rho = n/k = 10^3 - 10^4$, respectively. Each point in Fig. 1(b) with the percentage of correct detections has been calculated based on 1000 trials with randomly selected subsets of noiselet, WH, or Fourier basis for noise-free and noisy (additive Gaussian white noise with signal-to-noise-ratio SNR=0 dB) compressive measurements. We are using the SPGL1 package for lasso optimization [38]. The algorithm has usually converged in 1–2 Newton iterations with an execution time of 1–2 seconds. At the same time it fails to converge for other filters than POF i.e. for non-unitary circulant operators $T$. The input scene could be approximately reconstructed from the WH-based compressive measurement $y$ directly, or indirectly - from the CS-reconstructed correlation signal $s$ for compression rates $\rho \lesssim 10^2$. However, as we show in Figs. 1(c) such recovery is no longer possible at $\rho = 10^3$, while the target may still be detected. As another example we consider a combined localization and classification problem with a two-element POF dictionary and input scenes with between 1 and 5 randomly located non-overlapping ships of the two kinds. An example of such an input scene is shown in Fig. 1(e). As may be expected, the acceptable compression decreases with the number of objects, as shown in Fig. 1(f).

We have validated the proposed POF-based compressive target recognition method experimentally using an optical single-pixel detector shown in Fig. 2. We have used a $512 \times 512$ pixel area of the TI LightCrafter DLP-4500 micromirror device (DMD).
Figure 3: Experimental results from a single pixel detector: (a) analyzed image with marked target; (b) image reconstruction from a full $512 \times 512$ WH basis; (c) Probability of correct target detection from a compressive measurement vs compression rate obtained theoretically at various SNR levels and experimentally; (d) Signal reconstructed at $\rho = 30$.

with a skew (45°) orientation with respect to the display matrix to project WH basis onto the analyzed image. WH functions have been normalized to take binary values of \{0, 1\}. Then they were displayed on DLP at 120 Hz. Apart from the WH functions, we have also projected their negation. This allowed us to conduct a differential measurement and to eliminate an unknown bias from the measurement. The detector consisted of a photodiode integrated with an operational amplifier, and with a 16-bit A/D converter. Its distance from the object is approximately 5 m. In Fig. 3(a)(b) we show the $512 \times 512$ synthetic image used in the experiment and its reconstruction from a full set of WH basis. Apart from the nonuniform illumination that could be accounted for with simple post-processing, the low SNR on the order of 0 dB is probably the most important limitation of the set-up. In Fig. 3(c) we show the probability of correct target location within the radius of 5 pixels from the exact location obtained experimentally at various compression rates. A smaller resolution than in the theoretical study presented before in Fig. 1 as well as the finite SNR limit the acceptable level of compression. The experimental results agree with the theoretical prediction for the estimated level of SNR=0 dB. Nonetheless the results clearly demonstrate that CS-based recovery of the correlation signal from an incomplete measurement is feasible. A sample reconstruction of the image at $\rho = 30$ when it is still possible to detect the target but not to see it is shown in Fig. 3(d).

In summary, we have discussed the application of circulant matrices as compression operators for CS in optics, and we have demonstrated a compressive pattern recognition technique which allows to detect and localize a target object from a compressive measurement. It encapsulates lasso optimization, a phase-only matched filter or a pure-phase correlation adapted from optical correlation-based pattern recognition methods and a non-adaptive Walsh-Hadamard-based or noiselet-based compressive measurement. We have shown in a numerical experiment that for $1024 \times 1024$ pixel scenes with a relatively sparse distribution of objects in the presence of a nonuniform background it is fast and reliable at extreme compression rates of $\rho \approx 10^3 - 10^4$. The acceptable compression rate is smaller at lower resolutions. Like for any pattern recognition method, convergence and performance are signal-dependent. The study of various noise-models, or generalizations of the algorithm to other transformation invariances than shift invariance [39], while possible, is beyond the scope of this letter. In many areas data inflow grows faster than computational power and CS-based techniques like the one discussed here will be gaining importance. The proposed method can be applied to the compressive measurements in remote sensing, bioinformatics or medical imaging, in particular for high-speed screening of data from small measurement samples.

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