Effect of the noise component of the aperture uncertainty on the dynamic range of the sample and hold circuit

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Abstract. In this article, the influence of the noise component of the aperture uncertainty on the dynamic range of the sample-and-hold circuit (S/H circuit) is considered. For this purpose, a mathematical model of the S/H circuit is constructed, which is presented in the form of an ideal sampler, at the input of which a functional block is placed that simulates distortions of the sampled oscillation in the real S/H circuit. An analytical expression is obtained to show that the noise component of the aperture uncertainty generates frequency distortions of the input signal and additional noise, the properties of which are determined by the characteristics of the aperture uncertainty and the sampled oscillation.

Keywords: noise component of aperture uncertainty, dynamic range, sampling, sample-and-hold circuit.

1. Introduction

The problems concerning the converting of an analog signal to a digital signal are the main reason for holding back the improvement in the accuracy of various measurements, especially at ultra-high frequencies (UHF). The conversion accuracy depends on the bit depth of the analog-to-digital converter (ADC), and the conversion speed depends on the speed of the converter. Implementing a high-bit ADC with high performance is not an easy task.

The signals of the microwave range have a high rate of change in their shape, which leads to a difference in the actual moments of taking the report from the nominal ones, and to additional distortions. In multi-bit ADCs, the dynamic error is quite large, so the sampling of the input oscillation is carried out by the sample-and-hold circuit. During the duration of the time strobing pulse, the timer S/H circuit takes a sample and fixes it for the time of the analog-to-digital conversion, which allows the ADC to convert higher-frequency oscillations compared to the absence of S/H circuit.

The dynamic range of the S/H circuit must be greater than the dynamic range of the ADC, which has a structure significantly more complex than the S/H circuit structure. According to the method of obtaining a sample, S/H circuits can be divided into two types – tracking and integrating ones [1]. In [1, 2, 3, and 4], the S/H circuit errors that occur at the moments of sampling, storage, and changing the operating mode are considered. It is advisable to choose the noise immunity of the reception, as well as the resulting distortions in the output signal, as a criterion for evaluating the S/H circuit errors. In addition, it is necessary to take into account the spectral composition of the resulting interference, individual components of which may fall into the frequency band of the input signal.

2. Problem statement

A convenient parameter that evaluates the noise immunity of a digital receiver is its dynamic range [1, 5]:

\[ D = 20 \log_{10} \frac{U_{\text{max}}}{U_{\text{min}}} \]
where $D$ is the dynamic range, dB; $U_{max}$ и $U_{min}$ is the maximum and minimum amplitudes of the voltage at the input of the S/H circuit.

To match the S/H circuit to the selected ADC, it is necessary that $U_{max}$ the nonlinear distortion of the useful signal, the nonlinear noise and cross-modulation do not exceed the required norms. In addition, the power of external noise, aperture uncertainty noise, intrinsic noise, intermodulation noise, and harmonics did not exceed the lower limit of the dynamic range of the ADC, which is set by the quantization noise power and the non-linearity of the quantization characteristic.

In a digital radio receiver, the main selection is performed in the digital filter of the main selection, so the dynamic range of the S/H circuit affects the selectivity of the receiver, both on the main and on the neighboring channels.

In [3, 6, 7], the influence of intermodulation interference and harmonics, as well as aperture uncertainty, on the dynamic range of the S/H circuit and, as a consequence, the entire receiver is considered. Mathematical models of S/H circuit are presented, which makes it possible to reduce the problem of studying circuits with changing parameters to the analysis of models with continuous parameters.

The noise component of the S/H circuit aperture uncertainty manifests itself in the form of random deviations of the actual position of the boundaries of the time strobing interval from the nominal one, which do not depend on the sampled oscillation, which occur due to the phase noise of the reference generator, fluctuation processes in the pulse shapers, the strobing key, and external interference. Since the reasons for the occurrence and the nature of the noise and nonlinear components of the aperture uncertainty are different, it is advisable to analyze their influence on the S/H circuit characteristics separately.

### 3. Theory

To analyze the noise component of the aperture uncertainty, we can neglect the linear distortion of the oscillation due to the aperture shift, as well as the noise caused by minor changes in the shape of the key conductivity function at the moments of closing and opening. Therefore, when constructing S/H circuit models, we will assume that the strobing key from the open state to the closed state and back passes instantly, but at random moments of time. The mathematical model will represent an ideal sampler; at the input of which a circuit is included that simulates the distortion of the sampled oscillation in real S/H circuit. Distortions occur due to the phase noise of the reference generator, fluctuation processes in the pulse shapers, the sampling key, and external interference.

When conditions are met that are easy to implement in practice:

$$\Delta t_c \gg CR_{3.0} \text{ and } \omega C/R_{3.0} \gg 1$$

where $C$ is the storage capacitor capacity;

$R_{3,0}$ is the active resistance of the charge circuit of the storage capacitor.

At the output of the tracking S/H circuit, a sample is formed, which depends only on the value of the input oscillation $u(t)$ at the end of the strobing interval:

$$u_{ot}(m) = u(t_2 + mT - \tau_{a2}),$$  \hspace{1cm} (1)

where $t_2$ is the nominal opening moment of the strobing key;

$\tau_{a2}$ is the aperture uncertainty of the opening moment of the strobing key;

$m$ is the 1, 2, 3, ....

Then, in the tracking S/H circuit, the signal at the input of the ideal sampler has the following form:

$$u_s(t) = u(t - \tau_{a2}).$$

Similarly, a sample is formed at the output of the integrating S/H circuit, which depends on the value of the input oscillation $u(t)$ at the beginning and end of the strobing interval:
Where $\tau_{a1}$ and $\tau_{a2}$ are the values of the aperture uncertainty of the key make and break moments.

Therefore, in the integrating S/H circuit, the signal at the input of the ideal sampler has the form:

$$u_s(t) = u_1(t - \tau_{a2}) - u_1(t - \Delta t_c - \tau_{a1}).$$

The structures of the mathematical models of the tracking and integrating S/H circuit take into account the noise component of the aperture uncertainty, containing time intervals of random delay determined as follows [9]:

$$\tau_{a1} = \xi_1(t - \Delta t_c), \tau_{a2} = \xi_2(t),$$

where $\xi_1(t)$ and $\xi_2(t)$ are stationary centered random processes independent of $u(t)$.

When using such a model, the analysis of the effects associated with the noise component of the aperture uncertainty is reduced to the study of a random process at the output of a linear system with continuous random parameters [8]. The spectral power density $W_s(\omega)$ of the oscillation $u_s(t)$ is determined as follows [9]:

$$W_s(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_e(y, v)W(v)e^{j(\omega-y)t}dydv,$$

where $R_e(y, v) = M[K_{l,g}(j\omega, t)K_{l,g}(-j\omega, t + y)]$ is the correlation function of a linear system in the S/H circuit model; $K_{l,g}(j\omega, t) = e^{-j\omega\xi_2(t)}$ is the transfer function of this system; $W(v)$ is the spectral power density of the oscillation $u(t)$.

For tracking S/H circuit

$$K_{l,g}(j\omega, t) = e^{-j\omega\xi_2(t)},$$

and for integrating S/H circuit

$$K_{l,g}(j\omega, t) = (e^{-j\omega\xi_2(t)} - e^{-j\omega[\xi_1(t-\Delta t_c) + \Delta t_c]})/j\omega\tau_n.$$

Let $\xi_1(t)$ and $\xi_2(t)$ be Gaussian processes for which [9]

$$M\left[e^{-j[\xi_p(t)-\xi_q(t+y)]}\right] = e^{-0.5v^2[\sigma_{\xi_p}^2 + \sigma_{\xi_q}^2 - 2R_{\xi_p\xi_q}(y)]},$$

where $\sigma_{\xi_p}^2$ and $\sigma_{\xi_q}^2$ are the variances of Gaussian processes $\xi_p(t)$ and $\xi_q(t)$, $R_{\xi_p\xi_q}(t)$ is their mutual correlation function.

At $p = q$ we get $R_{\xi_p\xi_p}(t) = R_{\xi_p}(t)$ which is the correlation function of the process $\xi_p(t)$.

Hence, the correlation function for the tracking S/H circuit is as follows:

$$R_e(y, v) = e^{-v^2[\sigma_{\xi_1}^2 - R_{\xi_1\xi_1}(y)]},$$

and for integrating S/H circuit:

$$R_e(y, v) = \frac{1}{(\tau n)^2}\left[e^{-v^2[\sigma_{\xi_1}^2 - R_{\xi_1\xi_1}(y)]} + e^{-v^2[\sigma_{\xi_2}^2 - R_{\xi_2\xi_2}(y)]} - e^{-v^2[\sigma_{\xi_1}^2 + \sigma_{\xi_2}^2]} \times \left(e^{j\Delta t_c - v^2R_{\xi_1\xi_1}(\Delta t_c - y)} + e^{-j\Delta t_c - v^2R_{\xi_1\xi_1}(\Delta t_c + y)}\right)\right].$$

When calculating the integral in (2), it is necessary to take into account that $W(\omega) \approx 0$, if $\omega > \omega_u$, where $\omega_u$ is the upper frequency in the oscillation spectrum $u(t)$.
cases $\sigma_{\xi_1}^2 \omega_n^2 \ll 1$ and $\sigma_{\xi_2}^2 \omega_n^2 \ll 1$, we can, by decomposing the exponents in formulas (3) and (4) into a Taylor series, limit ourselves to the linear approximation for tracking S/H circuit

$$R_c(v, \gamma) \approx 1 - v^2 \left[ \sigma_{\xi_2}^2 - R_{\xi_2}^2(\gamma) \right],$$

for integrating S/H circuit

$$R_c(\gamma, v) \approx \left( 1 - \frac{\sigma_{\xi_1}^2 + \sigma_{\xi_2}^2}{2} v^2 \right) \frac{2 \sin \left( \frac{v \Delta t}{2} \right)}{v t_i^2} - \frac{1}{t_i^2} \left[ R_{\xi_1}(\gamma) + R_{\xi_2}(\gamma) - e^{-jv \Delta t_c} R_{\xi_12}(\Delta t_c + \gamma) \right].$$

Then from (3), (5) and (6) after the transformations, we find that in the tracking S/H circuit

$$W_s(\omega) = \left( 1 - \sigma_{\xi_2}^2 \omega^2 \right) W(\omega) + \frac{1}{2\pi} \int_{-\infty}^{\infty} v^2 W(v) W_{\xi_2}(\omega - v) dv,$$

and in the integrating S/H circuit

$$W_s(\omega) = \left( 1 + \frac{\sigma_{\xi_1}^2 + \sigma_{\xi_2}^2}{2} \right) |K_i(j\omega)|^2 W(\omega) +$$

$$+ \frac{1}{2\pi t_i^2} \int_{-\infty}^{\infty} \left\{ W_{\xi_1}(\omega - v) + W_{\xi_2}(\omega - v) \right\} -$$

$$- 2Re \left[ e^{j(\omega - 2v) \Delta t_c} W_{\xi_12}(\omega - v) \right] W(v) dv,$$

where $W_{\xi_1}(\omega), W_{\xi_2}(\omega), W_{\xi_12}(\omega)$ are the spectral power densities of the processes $\xi_1(t)$ and $\xi_2(t)$ and are their mutual spectral power density.

The value of the cross-correlation of the deviations of the strobing interval boundaries $\xi_1(t)$ and $\xi_2(t)$ depends on the source of the aperture uncertainty in the S/H circuit. If the main cause of the aperture uncertainty is the broadband noise of the reference generator, the strobing key, and the control circuit, the correlation interval of which is much smaller $\Delta t_c$, then $\xi_1(t)$ and $\xi_2(t)$ are practically mutually independent, and $W_{\xi_12}(\omega) = 0$. Hence, $W_{\xi_1}(\omega) \approx W_{\xi_2}(\omega)$ and from (10) we find for the integrating S/H circuit

$$W_s(\omega) = \left( 1 - \sigma_{\xi_2}^2 \omega^2 \right) |K_i(j\omega)|^2 W(\omega) + \frac{1}{2\pi t_i^2} \int_{-\infty}^{\infty} W(v) W_{\xi_2}(\omega - v) dv,$$

If the deviations of both boundaries are mainly caused by one factor, and $\xi_2(t) = \pm \xi_1(t)$, then $\xi_2(t) = \xi_1(t)$, if an external influence equally deviates both boundaries of the strobing interval, but in opposite directions, then $\xi_2(t) = -\xi_1(t)$. Then for integrating S/H circuit is:

$$W_s(\omega) = \left( 1 - \sigma_{\xi_2}^2 \omega^2 \right) |K_i(j\omega)|^2 W(\omega) +$$

$$+ \frac{1}{\pi t_i^2} \int_{-\infty}^{\infty} \left\{ 1 \mp \cos[(\omega - 2v) \Delta t_c] \right\} W(v) W_{\xi_2}(\omega - v) dv,$$
4. Results

The analysis of expressions (7) and (8) shows that due to the noise component of the aperture uncertainty, first, frequency distortions \( u(t) \) occur, and the attenuation increases with frequency growth, and, secondly, additional noise, the spectrum of which is determined by the convolution of the spectra of the aperture uncertainty and its derivative. Frequency distortions are insignificant; they practically do not influence the dynamic range of the S/H circuit. The noise of the aperture uncertainty can significantly worsen the signal-to-noise ratio at the S/H circuit output, thereby limiting the dynamic range of the digital radio receiver not only on the main channel, but also on the adjacent one.

Indeed, the power of this noise is proportional to the power of the oscillation \( u(t) \), and its spectrum is wider than \( W(\omega) \). Therefore, the presence of a powerful interference in the oscillation \( u(t) \), the spectrum of which is located outside the frequency band of the desired signal, increases the noise level of the aperture uncertainty in this band.

We determine the maximum permissible standard deviation of the boundaries of the strobing interval, at which the power of the additional noise does not exceed the power of the quantization noise of the \( n \)-bit ADC. Noise power due to aperture uncertainty in the tracking S/H circuit

\[
\sigma_{g,a}^2 = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} v^2 W(\omega) W_{\xi_2}(\omega - v) dv d\omega \ll (\omega_h \sigma_{\xi_2})^2, \tag{13}
\]

where \( \sigma^2 \) is the maximum power of the sampled oscillation \( u(t) \) (to the limitation in ACD). The quantization noise power of the ADC with a quantization step \( h \) with a peak oscillation factor \( u(t) \) equal to 3,

\[
\sigma_{g,k}^2 = \frac{h^2}{12} = \frac{3\sigma^2}{2^{2n}}, \tag{14}
\]

In order that \( \sigma_{g,a}^2 < \sigma_{g,k}^2 \), in accordance with (13) and (14), it is necessary to provide

\[
\sigma_{\xi_2} < \frac{\sqrt{3}}{2^{n} \omega_h}, \tag{15}
\]

Similarly, for integrating S/H circuit in the absence of mutual correlation of the processes \( \xi_1(t) \) and \( \xi_2(t) \) we obtain

\[
\sigma_{\xi_2} < \frac{\sqrt{6} \sin(\omega_0 \Delta t_c/2)}{2^n \omega_h}. \tag{16}
\]

5. Conclusion

The analysis of expressions (9) and (10) shows that in the absence of a correlation between \( \xi_1(t) \) and \( \xi_2(t) \) and \( \Delta t_c = 1/(2f_0) \), the dispersion of the noise component of the aperture uncertainty is twice as large as in the tracking S/H circuit. With the same aperture uncertainty and the signal-to-noise ratio \( \Delta t_c = 1/(2f_0) \) ratio at the output of the integrating S/H circuit is twice greater than at the output of the tracking S/H circuit, since the errors in the formation of the sample caused by mutually uncorrelated deviations of the boundaries of the strobing interval are added incoherently in the integrating S/H circuit, and their total power is only twice greater than the power of each error, while the gain of the integrating S/H circuit in energy storage in comparison with the tracking S/H circuit is 4. However, the reduction \( \Delta t_c \) worsens the signal-to-noise ratio in the integrating S/H circuit, which \( \Delta t_c < 1/(4f_0) \) decreases compared to the tracking S/H circuit.

If the process \( \xi_1(t) \) is correlated with \( \xi_2(t) \), then, according to (12), due to the filtering action of the integrating S/H circuit, individual spectral components of the noise of the aperture uncertainty can be attenuated. Let us assume that \( \xi_2(t) = \pm \xi_1(t) \), and the oscillation \( u(t) \) is harmonic with frequency \( f_0 \) and power \( \sigma^2 \). At \( |K(j\omega)| = 1 \) in accordance with (1), the integration time constant is equal to
\[ \tau_i = 2 \sin(\omega_0 \Delta t_c/2)/\omega_0, \] (17)

Then from (12) and (17) we obtain the following expression for the spectral power density of the noise of the aperture uncertainty:

\[ W_{\xi_1}(\omega) = \frac{\sigma^2 \omega_0^2}{4 \sin^2(\omega_0 \Delta t_c/2)} \left\{ \left[ 1 \mp (\omega + 2 \omega_0) \Delta t_c \right] \times W_{\xi_2}(\omega + \omega_0) + \left[ 1 \mp \cos((\omega - 2 \omega_0) \Delta t_c) \right] \times W_{\xi_2}(\omega - \omega_0) \right\}, \] (18)

As follows from (18) at \( \xi_2(t) = -\xi_1(t) \) (the opposite direction of deviation of the initial and final boundaries of the strobing interval under the influence of external interference) and \( \Delta t_c = 1/(2f_0) \) the noise of the aperture uncertainty is suppressed around \( f_0 \). However, if the noise of the aperture uncertainty is broadband (the correlation interval of each of the processes \( \xi_1(t) \) and \( \xi_2(t) \) is significantly less than \( T \)), then the filtering effect is hardly noticeable due to the repeated overlap of the noise spectrum during sampling.

6. References

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