Stepwise modelling method for post necking characterisation of anisotropic sheet metal

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Abstract

Modelling and simulation are important tools during design and development processes. For accurate predictions of, e.g. manufacturing processes or final product performance, reliable material data is needed. Usually, the applied material models are calibrated by utilising direct methods such as conventional uniaxial tensile/compression tests but also inverse methods are occasionally applied. Recently, an effective inverse method, the stepwise modelling method (SMM), was presented. By using SMM, the flow stress from initial yielding, beyond necking to final fracture, can be determined. However, the method is developed for sheet materials having isotropic von Mises hardening. In this paper the SMM is extended for post necking characterisation of anisotropic sheet metals using the Barlat yield 2000 criterion. The novel method was applied to analyse the post necking plasticity of the widely used aluminium alloy AA6016 in T4 condition and the aluminium alloy AA5754 in H111 condition. The latter alloy has reported to show serrated yielding, also known as the Portevin–Le Chatelier effect. The obtained flow stress curves agree well with the curves form conventional uniaxial tensile tests up to the point of necking and show credible post necking predictions to final fracture. Furthermore, SMM showed that it could handle the effect of serrated yielding for AA5754-H111. Hence, the novel approach can be used to characterise the post necking.
hardening of a variety of anisotropic sheet metals and thereby contributes to efficient and reliable material model calibration.

Keywords: anisotropy, post necking, Barlat yield 2000, stepwise modelling method, SMM, AA6016 T4, AA5754 H111

(Some figures may appear in colour only in the online journal)

1. Introduction

The industry faces regularly long-term goals regarding the reduction of greenhouse gas emissions. A lot of effort is therefore put into finding innovative design solutions and material selections, which can reduce the lifecycle ecological footprint through energy efficient production and reduced weight of for example vehicles. Advanced sheet metal alloys are often very interesting candidates in product design under lightweight aspects. Hence, the introduced sheet metals in innovative design has to fulfil the demands regarding for example formability as well as final product performance in terms of structural integrity. Modelling and simulation are powerful and efficient tools to study manufacturing processes such as forming and to predict final properties of produced parts. Especially, in the automotive and aerospace sector, industries are using commercial modelling software to simulate processes like e.g. sheet metal forming, and to predict performance such as crashworthiness and structural stiffness. However, the quality of modelling outcome is highly dependent on accurate mechanical descriptions. In for example sheet metal forming and crash situations, large deformations often occur and there might be a risk of unwanted failure.

The mechanical properties are commonly obtained by conventional uniaxial tension tests. However, the fundamental assumption of a homogeneous uniaxial stress and strain state ceases at necking and consequently the flow stress behaviour beyond this point remain unknown. Other biaxial techniques such as bulge tests are commonly applied to extend the flow stress curves to strains beyond necking, but the test has its limitation when it comes to tracking localization and measuring the fracture strain. Many indirect methods based on inverse modelling to obtain flow stress behaviour at large deformations have been proposed over the years. Zhan and Li (1994) introduced an inverse modelling method by using the experimental force and displacement relation to estimate the flow stress behaviour by matching the results from finite element simulations and experiments. In order to measure the high strains obtained after necking, Kajberg and Lindkvist (2004) complemented the tensile tests with digital speckle photography and digital image correlation (DIC) to their inverse modelling procedure based on finite elements simulations. A similar approach was later proposed to obtain the material response at high strain rates in Kajberg and Wikman (2007). Recently, Abedini et al (2020) presented a method, where simple shear tests are used to predict the stress--strain relation for high strains which is used to calibrate the Barlat yield 2000 criterion (Barlat et al 2003). However, computation in inverse modelling may be very costly (Avril and Pierron 2007) and requires always an evolution model based on parameter, which are fitted to experimental data (Tarantola 2013). An overview of different inverse modelling methods is presented in Ponthot and Kleinermann (2006).

A time-efficient and reliable method to obtain the flow stress including post-necking behaviour without computational intense inverse modelling with finite element simulations was proposed by Marth et al (2016). The so-called stepwise modelling method (SMM) has evolved from prior development by the group of Solid Mechanics at Luleå University of Technology, e.g. (Eman 2007, paper B), (Häggblad et al 2009) and (Östlund 2015, p. 15ff). Contrary to the
previous mentioned methods, SMM is a direct method not relying on time-consuming inverse modelling (Marth et al 2017). It uses a radial return algorithm based on von Mises plasticity and was used to predict reliable material models for forming as well as fracture simulations for steel sheet metals (Marth et al 2018). SMM has been applied to obtain the post-necking behaviour of the nickel based super-alloy alloy 718 for aerospace applications (Perez Caro et al 2019).

Furthermore, Sjöberg et al (2018) calibrated a viscoplastic hardening model and three fracture criteria for alloy 718 at elevated temperatures and high strain-rates using SMM (Sjöberg et al 2017). The utilization of SMM to characterise the anisotropic alloy, alloy 718, indicated that its applicability would improve if an anisotropic plasticity algorithm is implemented.

One early attempt to describe directional dependence of mechanical properties was performed by Lankford et al (1950), who proposed experimental ratios to characterize plastic anisotropy. Furthermore, Hill and Orowan (1948), Hill (1952) extended the classic isotropic von Mises model to describe anisotropic materials. Several models specifically intended for aluminium alloy sheets have been developed since these early propositions, e.g. (Barlat and Lian 1989, Karafillis and Boyce 1993, Barlat et al 2003). In Granum et al (2021b) three artificially aged 6XXX series aluminium alloys, were studied, where the post-necking behaviour was obtained by extrapolating calibrated stress–strain curves up to necking. Frodal et al (2020) studied the effect of plastic anisotropy, strength and work hardening on the tensile ductility of aluminium alloys. Zhang et al (2020) presented an experimental and numerical study of Al6016-T4, where the ductile failure was described by a ductile fracture criterion.

In this paper an extension of the SMM to characterise anisotropic materials is presented using the Barlat yield 2000 criterion (Barlat et al 2003). The novel method is applied and evaluated for two aluminium alloys, AA6016-T4 and AA5754-H111. The extended SMM is verified by using finite element simulations to replicate the responses in the directions needed to determine the anisotropic properties.

2. Materials and methods

This article aims to present and verify the SMM for anisotropic sheet metals. First, the used materials and experimental testing methods are presented. The numerical methods, including a brief description of SMM, the anisotropic yield model and its implementation in the SMM are introduced. Finally, the FEM simulations, used to validate the resulting post necking behaviour, are presented.

2.1. Materials

Two different aluminium alloys were investigated: AA6016 in T4 condition and AA5754 in H111 condition. The aluminium alloy AA6016 in T4 condition has been recently studied in many publications e.g. (Zillmann et al 2012, Granum et al 2021a) and was used in this study to evaluate the presented methods. The material belongs to the wrought aluminium–magnesium–silicon 6XXX family having good heat treatment properties and these high strength aluminium alloys show good formability, strong corrosion resistance and good
paint-bake strengthening effect. The chemical composition of the alloy AA6016 is shown in table 1. The received sheets had a nominal thickness of \( t = 1.2 \) mm.

In order to demonstrate that SMM can describe post necking behaviour of anisotropic materials showing serrated yielding during plastic deformation, the aluminium alloy AA5754 in H111 condition was considered. The alloy belongs to the wrought aluminium–magnesium 5XXX family and is a non-heat-treatable aluminium alloy most largely used in automotive and transportation industries having a moderate strength and weldability. These properties have made it attractive in applications as e.g. inner trunk panels, fenders, heat shields and air cleaning covers (Durmuş and Yüksel Çömez 2017).

The aluminium alloy AA5754 in H111 condition shows extensive serrations associated with the strain hardening, known as Portevin–Le Chatelier (PLC) effect (Rodriguez 1984), as earlier reported for AA5754-H111 by Park and Niewczas (2008). The chemical composition of the alloy AA5754 is shown in table 1. The received sheets had a nominal thickness of \( t = 1.5 \) mm.

2.2. Experimental methods

In this section the experimental methods and following evaluation are described to obtain flow stress curves from initial yielding, beyond necking to final fracture for anisotropic materials. Firstly, the tensile testing setup and utilised specimen geometries are presented. Secondly, the evaluation method to determine the typical anisotropic parameters are described.

In order to characterise the anisotropy and the post necking behaviour of the aluminium alloys two sets of tensile tests were conducted. Conventional uniaxial tensile tests were performed to get typical properties needed in anisotropic material models such as uniaxial yield stresses \((\sigma_0, \sigma_{45}, \sigma_{90})\) in the longitudinal, diagonal (DD) and transverse (TD) to the rolling direction (RD), respectively, as well as the Lankford coefficients \((R_0, R_{45}, R_{90})\) in corresponding directions. Since the obtained stress vs strain curves only are valid up to necking, a set of notched specimens were used to obtain the post necking behaviour using SMM. The proposed extension of SMM for characterisation of anisotropic sheet materials relies on the yield stresses and Lankford coefficients determined from the uniaxial tests. All details are presented in the following section 2.3. The flow stress curves from the uniaxial tensile tests, as long as they are valid, i.e. up to necking, were used to confirm that SMM provides similar true stress vs true plastic strain curves up to this point.

In the conventional uniaxial tensile tests, straight A50 specimens were tested in each direction. The A50 specimen were cut by using wire electrical discharge machining from blank in RD, TD and DD (0°, 45°, 90°) direction. The notched specimens used for SMM to get the post-necking behaviour consisted of R30 specimen. These specimens were only cut in the RD. The two specimen geometries are shown in figure 1. The specimen were tested in the servo-hydraulic loading machine INSTRON 1272 under quasi static conditions with cross-head speeds of 2 and 1 mm min \(^{-1}\) for the A50 and R30 specimen, respectively. Each specimen geometry and tensile direction were tested with at least 3 to 5 valid repetitions.

An essential input to SMM is deformation field measurements. As described below, DIC is also suggested and used to obtain the Lankford coefficients. A random speckle pattern was therefore applied with black and white spray colour on one surface of the specimens to enable DIC (GOM ARAMIS software) for evaluation of the local strain field. Images were captured at frame rates between 0.25 to 2 fps to get approx. 100 to 150 frames during the tests. The full field data was also used to create a virtual extensiometers with a gauge length of 50 mm. The measured elongations were used to compare with corresponding values obtained from finite element simulations explained in subsection 2.3.4.
In this study, the Lankford coefficients were determined directly from the DIC data obtained from the A50 tensile testing. This enables to obtain the $R_\phi$-value as a function of the effective plastic strain. The Lankford-coefficient $R_\phi$ is defined as the ratio between the plastic strain in width, $\varepsilon_{w,p}$, and thickness direction, $\varepsilon_{t,p}$, where $\phi$ is the angle from the RD:

$$R_\phi = \frac{\varepsilon_{w,p}}{\varepsilon_{t,p}},$$

(1)

where the plastic strain in thickness direction was calculated by using the in-plane plastic strain components and the plastic incompressibility assumption. The Lankford coefficients and the effective plastic strain values were calculated for all points in the evaluated DIC region. Then the value pairs from all time steps were sorted in ascending order of the effective plastic strain and the mean value for all points in intervals of $\Delta \varepsilon_p = 0.002$ were calculated. By this the Lankford coefficient histories, $R_\phi(\varepsilon_p)$, were obtained for each test. However, usually the Lankford coefficients are given as constants in most anisotropic materials models. Thus, the $R_\phi$-constants were set to the values where they stabilised.

Worth mentioning, is the possibility to use the full-field DIC data obtained from the straight A50 specimens to determine the post necking behaviour by SMM. However, notched R30 specimens triggers the strain localisation to the middle of the specimen right from the beginning. This simplifies the selection of the region of interest for the DIC evaluation. Additionally, the fracture on the notched R30 specimen starts in the centre of the specimen ensuring good reliability of the data obtained just before fracture.
2.3. Numerical methods

In this section the basic idea of the SMM is briefly summarised, followed by a description of the anisotropic material model and its implementation in the extended SMM. Finally, the finite element simulations used to validate the SMM results are described.

2.3.1. Stepwise modelling method. The SMM presented by Marth et al (2016) is based on full field measurements of tensile tests to identify the flow curve from yielding beyond necking until fracture. The flow curve is represented by a piece-wise linear continuous function of the yield stress, \( \sigma_Y \), vs the effective plastic strain, \( \bar{\varepsilon}_p \),

\[
\sigma_Y^k = \sigma_Y^{k-1} + H^k (\bar{\varepsilon}_p^k - \bar{\varepsilon}_p^{k-1}), \quad k = 1, 2, 3, \ldots n_k, \tag{2}
\]

containing of \( n_k \) points, where \( H \) is the hardening modulus. The SMM estimates for each time increment the unknown values of \( H^k \) and \( \bar{\varepsilon}_p^k \) is a step-by-step procedure, presented in algorithm 1. The SMM evaluates the material behaviour during deformation along an integration path. This integration path, \( S \), follows the highest deformed region of the sample to ensure high strain values for the estimated hardening curve. A visualisation for an easier understanding of the SMM is presented as a flowchart in figure 2.

The SMM presented by Marth et al (2016) uses for the stress computation (item 2 in figure 2) a radial return method based on the isotropic von Mises plasticity assumption. By using the von Mises plasticity assumption the SMM is limited to metals with an isotropic elasto-plastic behaviour. Therefore, in this article an improvement to enable the SMM to metals with anisotropic elasto-plastic behaviour is suggested. In this article the Barlat yield 2000 criterion (Barlat et al 2003) is used for the constitutive update in the SMM computation procedure (item 2 in figure 2).
Figure 2. Flowchart describing the SMM procedure from experimental DIC data to the final hardening curve. (Item 1) A suitable integration path, $S$, perpendicular to the tensile direction is chosen. (Item 2) For each time step, the stresses for all points on the integration path based on the measured deformation field are computed. Where the yield stress is taken from the flow curve determined at the earlier time step. (Item 3) The resulting force over the integration path is calculated and compared with the experimentally measured force. (Item 4) The hardening modulus $H_k$ that minimises the residual between the measured force and calculated force along the integration path, $S$, is found. When the residual $F_{res}$ is within a prescribed tolerance ($\delta$), $H_k$ and $\bar{\varepsilon}_p^k$ are updated and the procedure continues with the next time increment at step 2. After all time increments the final hardening curve is determined.

2.3.2. Barlat yield 2000 criterion. Barlat et al (2003) proposed an anisotropic yield function for aluminium alloy sheets which is based on two convex functions proposed by Hershey (2021)
and Hosford (1972). The yield function can be written

\[ \phi = \phi' + \phi'' = 2\bar{\sigma}^a \]  

(3)

with

\[ \phi' = |X'_1 - X'_2|^a; \quad \phi'' = |2X''_2 + X''_1|^a + |2X''_1 + X''_2|^a, \]  

(4)

where \( \bar{\sigma} \) is the effective stress. The exponent \( a \) is mainly associated with the crystal structure, where the recommended value for fcc materials is \( a = 8 \) and \( a = 6 \) for materials with bcc structures (Yoon et al 2004). \( X'_k \) and \( X''_k \) are the principal values of \( X' \) and \( X'' \) and are defined as a linearly transformation of the stress tensor \( \sigma \)

\[ X' = L' \sigma \]

\[ X'' = L'' \sigma. \]  

(5)

The coefficients of the linear transformation tensors \( L' \) and \( L'' \) can be expressed as

\[
\begin{bmatrix}
L'_{11} \\
L'_{12} \\
L'_{21} \\
L'_{22} \\
L'_{66}
\end{bmatrix}
= \begin{bmatrix}
2/3 & 0 & 0 \\
-1/3 & 0 & 0 \\
0 & -1/3 & 0 \\
0 & 2/3 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_7
\end{bmatrix},
\]

\[
\begin{bmatrix}
L''_{11} \\
L''_{12} \\
L''_{21} \\
L''_{22} \\
L''_{66}
\end{bmatrix}
= \frac{1}{9}
\begin{bmatrix}
-2 & 8 & -2 & 0 \\
1 & -4 & -4 & 4 & 0 \\
4 & -4 & 4 & 1 & 0 \\
-2 & 8 & 2 & -2 & 0 \\
0 & 0 & 0 & 0 & 9
\end{bmatrix}
\begin{bmatrix}
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
\alpha_6 \\
\alpha_8
\end{bmatrix},
\]

where \( \alpha_k \) (for \( k \) from 1 to 8) are the independent material coefficients. The parameters \( \alpha_1 \) to \( \alpha_8 \) can be calculated using a Newton–Raphson non-linear solver, shown in Barlat et al (2003).

The experimental input data needed, are the uniaxial yield stresses in the longitudinal, DD and TD to the RD (\( \sigma_0, \sigma_45, \sigma_90 \)), the bi-axial yield stress (\( \sigma_b \)) as well as the Lankford-coefficients in three directions (\( R_0, R_{45}, R_{90} \)) and the Lankford-coefficient \( R_b \). The Lankford-coefficient \( R_b \) can either be determined by the compression of circular disks or calculated by using a polycrystal model (Barlat et al 2003). Finally, the effective stress \( \bar{\sigma} \) in equation (3) is calculated by:

\[ \bar{\sigma} = \left\{ \frac{1}{2}(|X'_1 - X'_2|^a + |2X''_2 + X''_1|^a + |2X''_1 + X''_2|^a) \right\}^{\frac{1}{a}}. \]

(7)

2.3.3 Stress computation. A new stress integration scheme for the SMM based on the stress integration for elasto-plasticity with anisotropic yield functions presented by Yoon et al (2004) is implemented.

The co-rotational rate of deformation tensor, \( \dot{d}_{ij} \), over the time increment, \( \Delta t = t_{n+1} - t_n \), is calculated from the deformation gradient components obtained by DIC, as described in Marth et al (2016).

First, the elastic trial stress is calculated:

\[ \dot{\sigma}'_{ij} = \dot{\sigma}''_{ij} + D'_{ijkl} \dot{d}_{kl} \Delta t. \]

(8)
If the trial effective stress, $\dot{\sigma}$, is less than the current yield stress the solution is advanced to the next time increment with $\dot{\sigma}_{ij} = \dot{\sigma}_{ij}^{t}$. Else the yield condition, shown in equation (9), must be fulfilled by solving the plastic multiplier $\gamma$.

$$F(\gamma) = \ddot{\sigma} - \dot{\sigma}_{ij}^{t} - \gamma D_{ijkl} m_{ji} - \sigma_{Y} (\ddot{\epsilon}_{p} + \gamma) = 0,$$  \hspace{5em} (9)

where

$$m_{ij} = \frac{\partial \dot{\sigma}}{\partial \sigma_{ij}}$$ \hspace{5em} (10)

and $\ddot{\epsilon}_{p}^{(a)}$ is the effective plastic strain at the previous time step. The normal direction of the stress tensor on the yield surface $\frac{\partial \ddot{\sigma}}{\partial \ddot{\sigma}_{ij}}$ is obtained by applying the chain rules; here shown in Voigt-notation ($\tilde{\eta}_{k}$):

$$\frac{\partial \ddot{\sigma}}{\partial \ddot{\sigma}_{k}} = \left(2a \ddot{\epsilon}^{(a-1)} \right)^{-1} \frac{\partial \psi}{\partial \ddot{\sigma}_{k}} = \left(2a \ddot{\epsilon}^{(a-1)} \right)^{-1} \sum_{a}^{2} \sum_{b}^{3} \left( \frac{\partial \psi}{\partial \tilde{\eta}_{a}} \frac{\partial \tilde{\eta}_{a}}{\partial \ddot{\sigma}_{a}} \frac{\partial \tilde{X}_{a}}{\partial \ddot{\sigma}_{k}} + \frac{\partial \psi}{\partial \tilde{\eta}_{b}} \frac{\partial \tilde{\eta}_{b}}{\partial \ddot{\sigma}_{b}} \frac{\partial \tilde{X}_{b}}{\partial \ddot{\sigma}_{k}} \right)$$ \hspace{5em} (for $k = 1$ to $3$),

(11)

where $\tilde{\eta}_{b}$ are the principal values of $\tilde{X}$. Even though, equation (9) has a solution, it is difficult to obtain the numerical solution if the strain increment is not small enough (Yoon et al. 2004). Since the strain increments in the SMM are given by the frame-rate from the experimental DIC, this work uses a multi-stage return mapping procedure to ensure convergence even for large time steps, as suggested by Yoon et al. (1999). A schematic view for the multi-stage return mapping method is shown in figure 3.

In this multi-stage return mapping procedure the nonlinear equation (9) is modified with given residuals to $k$ sub-steps:

$$F(\gamma(k)) = \ddot{\sigma} - \dot{\sigma}_{ij}^{t} - \gamma D_{ijkl} m_{ji} - \sigma_{Y} (\ddot{\epsilon}_{p}^{(a)} + \gamma) = F(k),$$ \hspace{5em} (12)

where

$$F(\gamma(0)) = F(0), \{F(\delta) | F(0) > F(\delta) > \ldots > F(\delta) > \ldots > F(N)\}$$ \hspace{5em} (13)
\(F_{k=1-N-1}\) are prescribed values and the increment \(\Delta F = F_{(k-1)} - F_{(k)}\) should not exceed the nominal yield stress \(\sigma_Y\) (Yoon et al 2004).

The nonlinear equation that needs to be solved for the \(k\)th sub-step is given as the following relationship:

\[
\Phi(\gamma_{(k)}) = \tilde{\sigma}(\sigma_{(k)})^T - \sigma_Y(\tilde{\varepsilon}^{(n)}_p + \gamma_{(k)}) - F_{(k)} = 0. \tag{14}
\]

Equation (14) is solved iterative in \(i\) iterations. Therefore, three equations are defined:

\[
g_1\left(\gamma_{(i)}^{(k)}\right) = \tilde{\sigma}(\sigma_{(k)}) - \sigma_Y(\tilde{\varepsilon}^{(n)}_p + \gamma_{(k)}) - F_{(k)} \tag{15}
\]

\[
g_2\left(\gamma_{(i)}^{(k)}\right) = D^r^{-1}(\sigma_{(k)} - \sigma_f) + \gamma_{(k)}^{(i)} m_{(k)}^{(i)} \tag{16}
\]

\[
g_3\left(\gamma_{(i)}^{(k)}\right) = H(\gamma_{(i)}^{(k)}) = \tilde{\sigma}(\tilde{\varepsilon}^{(n)}_p) - \sigma_Y(\tilde{\varepsilon}^{(n)}_p) - \gamma_{(k)}^{(i)} \tag{17}
\]

where \(H\) is the current hardening modulus in the stress–strain curve.

To solve equation (14), \(\Delta \gamma_{(i)}^{(k)}\) at sub-step \(k\) and iteration \(i\) becomes

\[
\Delta \gamma_{(i)}^{(k)} = \frac{g_1\left(\gamma_{(i)}^{(k)}\right) - m_{(k)}^{(i)} F_{(i)}^{-1} g_2\left(\gamma_{(i)}^{(k)}\right) + g_3\left(\gamma_{(i)}^{(k)}\right) H^{(i)}}{m_{(k)}^{(i)} F_{(i)}^{-1} m_{(k)}^{(i)} + H^{(i)}}. \tag{18}
\]

The second derivative of the yield-function \(\frac{\partial^2 \Phi}{\partial \sigma^2}\) is obtained by the following chain rule; here shown in Voigt-notation (\(\gamma_{(k)}\)):

\[
\frac{\partial^2 \tilde{\sigma}}{\partial \sigma_i \partial \sigma_j} = \frac{\partial}{\partial \sigma_k} \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_i} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_j} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_k} \right); \quad \text{for } i, j = 1 \text{ to } 3, \tag{19}
\]

where

\[
\frac{\partial^2 \tilde{\varepsilon}}{\partial \sigma_i \partial \sigma_j} = 2 \sum_{r} \sum_{s} \sum_{a} \sum_{b} \left\{ \frac{\partial^2 \tilde{\varepsilon}}{\partial \sigma_i \partial \sigma_j} \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_k} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_l} \right) \right\} + \left( \frac{\partial^2 \tilde{\varepsilon}}{\partial \sigma_i \partial \sigma_j} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_k} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_l} \right) + \left( \frac{\partial^2 \tilde{\varepsilon}}{\partial \sigma_i \partial \sigma_j} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_k} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_l} \right) + \left( \frac{\partial^2 \tilde{\varepsilon}}{\partial \sigma_i \partial \sigma_j} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_k} \right) \left( \frac{\partial \tilde{\sigma}}{\partial \sigma_l} \right) \tag{20}
\]

Knowing the plastic multiplier $\gamma(N)$ and the normal direction of the stress tensor on the yield surface $m(N)$ at the last sub-step $(N)$, the final stress and equivalent plastic strain at the time $t_{n+1}$ can be calculated by

$$
\hat{\sigma}_{ij}^{(n+1)} = \hat{\sigma}_{ij} - \gamma(N) D_{ijkl} m_{ikl(N)}.
$$

$$
\varepsilon_p^{(n+1)} = \varepsilon_p^{(n)} + \gamma(N).
$$

The plastic strain increment is perpendicular to the yield surface at the last sub-step $(N)$ and is calculated by,

$$
\Delta \varepsilon_{ij,p} = \Delta \varepsilon_{ij} \frac{\partial \sigma}{\partial \sigma_{ij}} = \gamma(N) m_{ij(N)}.
$$

Using the final stress components, calculated from equation (21), the strain in thickness direction is updated based on plastic incompressibility using the following relationship:

$$
\varepsilon_{zz} = -\frac{\nu}{E} (\sigma_{xx} + \sigma_{yy}) - (\varepsilon_{xx,p} + \varepsilon_{yy,p}).
$$

The presented stress computation procedure replaces in the presented SMM the radial return algorithm, (item 2) in figure 2.

2.3.4. Finite element simulations. The post necking hardening behaviour for the aluminium alloys were obtained by using the force history and the deformation field data only in the RD. Hence, it is of great interest to verify if the calibrated anisotropic material model also could replicate the responses in the other two directions. Therefore, numerical FEM simulations of straight A50 specimens in the three different directions, i.e. in RD, DD and TD, were performed using an explicit LS-Dyna solver. The A50 specimen were discretised with approx. 1 mm Belytschko–Tsay shell elements (Belytschko et al 1988). The material model MAT133 Barlat yield 2000 (Hallquist 2006) was used and calibrated by using the input parameters ($\sigma_0$, $\sigma_{45}$, $\sigma_{90}$), as well as the Lankford-coefficients in three directions ($R_0$, $R_{45}$, $R_{90}$), which were obtained from the tensile tests. The hardening relation was defined as a piece-wise linear curve obtained from the notched R30 specimen RD by the presented SMM. The simulated results were validated by comparing the simulated and experimental force vs elongation relation. The simulated elongation was taken from the distance of two nodes which had a nominal distance of 50 mm.

3. Results and discussion

The aim of this study is to present and verify the stepwise-modelling-method using the Barlat yield 2000 material model to enable the post-necking characterisation of anisotropic sheet metals, to show that the calibrated FEM material model give valid simulation results and to investigate the influence of the PLC effect on the SMM results. The main results are the stress–strain relations for the investigated aluminium alloys, which are presented together with the mechanical properties and the anisotropy coefficient calibration results.

3.1. Tensile testing results

The mechanical properties were obtained by conducting conventional tensile testing of straight A50 tensile specimen in RD, DD and TD. The material showed an isotropic elastic behaviour where the Young’s modulus $E$ and the Poisson’s ratio $\nu$ are similar in all three directions.
Table 2. Mechanical properties of AA6016 and AA5754 from tensile tests.

|               | Young’s modulus $E$, (GPa) | Poisson’s ratio $\nu$, (—) | Yield strength $\sigma_Y$, (MPa) | Fracture elongation $\Delta_50$, (%) |
|---------------|-----------------------------|-----------------------------|----------------------------------|----------------------------------|
| AA6016        | 68                          | 0.33                        | 123–128                         | 24–26                            |
| AA5457        | 69                          | 0.375                       | 134–142                         | 16–24                            |

Figure 4. Lankford coefficient history during deformation based on DIC measurements for three repetitions in 0°, 45° and 90° direction. (a) AA6016 T4; (b) AA5754 H111.

The yield strength and the elongation at fracture showed a variation between the three tested directions. The mechanical properties of the aluminium alloys are shown in table 2.

3.2. Experimental anisotropy coefficient calibration

To calibrate the eight material parameters $\alpha_k$ the results of the straight A50 tensile tests in 0°, 45° and 90° direction were used, evaluating the local strain field during deformation by using the ARAMIS DIC software. The initial yield stress, $\sigma_0$, for each direction was taken from the yield point obtained from the experimentally measured force–elongation. Lankford coefficients, $R_\phi$, were obtained from the local strain field during deformation and their histories for both materials AA6016 and AA5754 are shown in figure 4.

The obtained Lankford coefficient history during deformation based on DIC measurements for three repetitions in each direction of the aluminium alloy AA6016 are presented in figure 4(a). It can be seen that the $R_\phi$-values become stable after $\bar{\varepsilon}_p = 0.003$ and contain almost constant for the samples in RD. The $R_\phi$-value histories for the DD and TD have a slight positive gradient. In this study the $R_\phi$-values are taken as a constant value obtained at a region where they reach an overall stable value at $R_\phi(\bar{\varepsilon}_p = 0.15)$. The obtained Lankford coefficient history for the aluminium alloy AA5754 based on DIC measurements for three repetitions in each direction are presented in figure 4(b). The influence of the PLC effect on the Lankford coefficient history is clearly seen for all test. However, the $R_\phi$-values in the regions between $\bar{\varepsilon}_p = 0.05$ and $\bar{\varepsilon}_p = 0.2$ are quite constant, therefore they are taken as a constant value obtained at $R_\phi(\bar{\varepsilon}_p = 0.15)$.

In table 3 the results used to calibrate the material parameters $\alpha_k$ for AA6016 and AA5754 are summarised. Since the biaxial yield stress was not available for these materials the assumption $\sigma_b = \sigma_0$ was employed. Barlat et al (2003) suggested to assume $L''_{12} = L''_{21}$ to solve
Table 3. Calibration data of AA6016 and AA5754.

|         | $\sigma_0$ (MPa) | $\sigma_{45}$ (MPa) | $\sigma_{90}$ (MPa) | $R_{0}$ (—)   | $R_{45}$ (—)   | $R_{90}$ (—)   |
|---------|-----------------|---------------------|---------------------|---------------|---------------|---------------|
| AA6016  | 128             | 124                 | 123                 | 0.693         | 0.457         | 0.662         |
| AA5754  | 142             | 136                 | 134                 | 0.605         | 0.874         | 0.649         |

Table 4. Barlat yield 2000 parameter for AA6016 and AA5754.

|         | $a$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ |
|---------|-----|------------|------------|------------|------------|------------|------------|------------|
| AA6016  | 8   | 0.9325     | 1.054      | 1.031      | 1.055      | 1.031      | 1.055      | 0.9644     |
| AA5754  | 8   | 0.8861     | 1.109      | 1.041      | 1.071      | 1.041      | 1.071      | 1.035      |

the eight parameter. This leads to the fact that four parameters have the dependency $\alpha_3 = \alpha_5$ and $\alpha_4 = \alpha_6$. Finally, based on these assumptions and the experimental data presented in Table 3 the eight material parameters $\alpha_k$ were calibrated and presented in Table 4. The exponent $a$ is set to 8 as recommended by Logan and Hosford (1980).

3.3. Post necking hardening characterisation

The stress vs effective plastic strain relation for AA6016 is presented in Figure 5. In this figure the results of several methods are compared. A conventional true stress vs true strain curve is included as a reference up to the point of necking. Furthermore, the curves evaluated by SMM and Barlat yield 2000 for the two specimen geometries, notched R30 and straight A50, are presented. Finally, the corresponding curves using the von Mises yield criterion are included to show the importance to employ a proper yield criterion. The presented stress–strain relations based on the presented anisotropic version of SMM were using the eight material anisotropy parameters $\alpha_k$, exponent $a$, Young’s modulus $E$, the Poisson’s ratio $\nu$ and an initial yield strength $\sigma_0$ as input parameters (presented in tables 4 and 2). The conventional true stress vs strain curve is calculated from the experimental force, $F$, and the elongation, $\delta$, measured over the initial length, $L_0$, of the straight A50 specimen (Davis 2004):

$$\bar{\sigma}_{\text{true}} = \frac{F}{A_0} \left(1 + \frac{\delta}{L_0}\right);$$

$$\bar{\varepsilon}_{\text{p}} = \ln \left(\frac{\delta}{L_0} + 1\right) - \frac{\bar{\sigma}_{\text{true}}}{E},$$

where $A_0$ is the initial cross section area and $E$ is the Young’s modulus. The resulting yield stress vs effective plastic strain relation from the presented anisotropic SMM for the studied aluminium alloy AA6016 is predicting a similar hardening behaviour until necking as obtained by the conventional method. However, the results obtained by using the SMM for the A50 specimens show serrations in the stress–strain curves. These are mainly caused by the diffuse necking during the deformation of the straight specimen and the fact that the strain increments are not always concentrated to the integration path, $S$. Therefore the notched specimens are the preferred choice for SMM evaluations.

The stress–strain relations obtained by SMM based on the von Mises plasticity show significantly lower values for both specimen geometries (R30 and A50) compared to the corresponding relations based on Barlat 2000. These discrepancies can be explained by considering the principal stresses determined for the uniaxial tensile tests (A50) using the two yield criteria.
Figure 5. Yield stress vs effective plastic strain relation for AA6016 in RD (three repetitions). Comparing hardening curves in RD, where the black lines are obtained from A50 specimen force–elongation relation; the green and red from SMM based on DIC measurements using the presented anisotropic model from A50 and R30 specimen; the cyan and blue lines from A50 and R30 specimen using the SMM with von Mises plasticity.

According to algorithm 1 the strain increments $\Delta \varepsilon_{ij}$ are calculated along the path $S$ for each time step using the full-field displacement data. When applying the von Mises criterion for this anisotropic material, a positive unrealistic and false transverse stress (width direction), i.e. a positive second principal stress, is induced to reproduce the correct relation between the incremental strain components in the longitudinal and the TDs, respectively. Although, SMM gives a first principal stress agreeing well with the flow curve obtained in the conventional test, the second false non-zero principal stress around 50 MPa reduces the equivalent stress according to von Mises, which is depicted in figure 5. However, by applying the Barlat 2000 criterion for the uniaxial case the correct principal stresses are determined with a nearly vanishing second principal stress. The equivalent stress according to Barlat 2000 agrees therefore well with the conventional flow curve as shown in figure 5. As expected, the comparison of the obtained flow curves based on the two criteria clearly show the importance to use a proper anisotropic plasticity model and that SMM based on the von Mises radial return method cannot be used for sheet metals with anisotropic behaviour.

Figure 6 shows the stress vs effective plastic strain relation for AA5754. In this figure the results of several methods are compared. The true stress vs effective plastic strain relation obtained from A50 specimen force–elongation relation calculated based on the initial cross section area is presented. The shown stress–strain relations based on the presented SMM were
Figure 6. True stress vs effective plastic strain relation for AA5754 in RD (three repetitions). Comparing hardening curves in RD, where the black lines are obtained from A50 specimen force–elongation relation; the green and red from SMM based on DIC measurements using the presented anisotropic model from A50 and R30 specimen.

using the input parameters for AA5754 (presented in tables 4 and 2). The results based on the experimental full field DIC data obtained from the straight A50 tensile specimen follow the results by the conventional method until necking as the results obtained from the notched R30 tensile specimen do. The results obtained from the notched specimen are continuing on the post necking hardening curve, while the curves from the straight A50 tensile specimen are diverting. It is most likely that the influence of the PLC effect causes this due to the strain bands translating through the specimen tensile direction. This will be further discussed in the upcoming section.

Some general observations concerning the prediction of the post-necking behaviour of sheet metals with anisotropic behaviour can be made. The resulting yield stress vs effective plastic strain relation from the presented SMM method for the studied aluminium alloys is predicting a similar hardening behaviour until necking as obtained by the conventional method. In figures 5 and 6 that the graphs from both specimen types are following the same path as the true stress–true plastic strain curves. For the studied alloy AA6016 the SMM results from both the straight and notched specimen (figure 5) are following the same curves, but it can be seen that the results based on the notched R30 specimen have a more stable solution and therefore is much smoother than the curves obtained from A50 specimen. This motivates to perform the SMM evaluation on the notched R30 specimen since the strain field localises much earlier during the deformation than for the straight A50 specimen. The influence of the serrated
yielding or PLC-effect on the SMM results for the different specimen types, shown in figure 6, is another argument to use the R30 specimen for the post-necking behaviour characterisation. Overall the authors recommend to use notched specimens for SMM evaluations.

### 3.4. Influence of the Portevin–Le Chatelier effect

The aluminium alloy AA5754 is used to study the influence of the PLC effect on the SMM results and its computational stability. Having strain bands translating through the specimen during the tensile deformation challenges the SMM calculation stability, since sudden increases in the local strain values and the high oscillation of the tracked force makes it harder to achieve an converging solution at each time step. The translation of strain bands observed in straight A50 specimen can be visualised by the local strain rate during deformation using the full-field measurements, shown in figure 7. It can be clearly seen that a region with an increased strain rate migrates through the presented region during deformation.

The influence of the PLC effect on the SMM results can be reduced by using the notched R30 specimen to evaluate the hardening relation, where the strain localisation concentrates early during tensile testing in the notched area. However, the resulting hardening relation for AA5754, shown in figure 6, has still some oscillation, but still less than the results obtained directly from the force–elongation relation with the conventional method. However, in such appearance of the PLC-effect both the conventional and SMM results need some kind of filtering or curve smoothing before they can be used for FEM simulations.

### 3.5. Numerical evaluation simulations

To evaluate the validity of the overall material behaviour, the results presented in the previous two subsections, were used to perform numerical FEM simulations of straight tensile specimen. The simulations are performed in three different tensile directions, the 0, 45 and 90 degree direction to compare the force–elongation relation with experimental measurements. The material model MAT133 Barlat yield 2000 (Hallquist 2006) is calibrated by using the input parameters presented in tables 4 and 2. The hardening relation is defined as a piece-wise linear curve obtained by evaluating a representative R30 specimen using the presented SMM based
Figure 8. Comparison of the force versus elongation from simulations end experiment of AA6016 and AA5754 using straight A50 specimen. The experimental results are presented in dashed curves and the simulated results are presented in solid curves, while the colour shows the three different test directions (RD, DD, TD).
on Barlat yield 2000 plasticity. These are shown in figure 5 and in figure 6. The hardening relation used for AA5754 was filtered to ensure stability in the simulation.

A comparison of the force versus elongation from simulations and experiment is shown in figure 8, where the colours show the three different test directions. In figure 8(a) the results for the aluminium alloy AA6016-T4 are shown, while figure 8(b) shows the results for the aluminium alloy AA5754-H111. The simulated results of AA5754 are smoother than the experimental data, which is caused by the fact that the PLC-effect was not included in the simulations and that a filtered hardening relation, based on the obtained SMM results, was used. Based on the simulation results for the straight tensile tests of both aluminium alloys, it can be stated that the SMM results are giving sufficient data for the material model even for other directions than the RD. However, for a material model that can handle specific hardening relations in different directions, as for example LS-Dyna’s material model MAT33 (Hallquist et al 2007), the SMM can be also used to evaluate the hardening behaviour of the directions of interest.

4. Conclusions

A novel extension of the SMM to characterise anisotropic sheet materials is presented. The extended SMM is based on the Barlat yield 2000 criterion. Two aluminium alloys, AA6016-T4 and AA5754-H111 were characterised using SMM. Following is concluded concerning the extended SMM for anisotropic sheet materials:

- SMM generates true stress vs true plastic strain curves following curves obtained by conventional tensile testing up to necking
- SMM describes the post necking behaviour to final failure credibly
- By using a notched specimen the PLC, i.e. serrated yielding, can be handled.
- It is sufficient to determine the post necking behaviour hardening in one direction, e.g. RD, if the Lankford coefficient are known or determined by conventional tensile tests in 0, 45 and 90° to the RD.

Hence, the proposed extension of SMM novel can be used to characterise the post necking hardening of a variety of anisotropic sheet metals and thereby contributes to efficient and reliable material model calibration.

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Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.
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