An Adaptive Multivariable Smooth Second-Order Sliding Mode Approach

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Abstract—This paper presents a novel adaptive multivariable smooth second-order sliding mode approach with the features of fast finite-time convergence, adaptation to disturbances and smooth. This approach can be directly applied to the controller design of multi-input and multi-output (MIMO) systems. In addition, a novel adaptive multivariable smooth disturbance observer is proposed based on this structure. In terms of the types of disturbances, the fast finite-time convergence and the fast finite-time uniformly ultimately boundedness of the systems are proved with the corresponding fast finite-time Lyapunov stability theory. Finally, the effectiveness of the proposed approach is validated by comparative numerical simulations.

Index Terms—Adaptive multivariable smooth second-order sliding mode control (AMSSOSMC), Adaptive multivariable smooth disturbance observer (AMSDO), Fast finite-time convergence, Fast finite-time uniformly ultimately boundedness.

I. INTRODUCTION

In recent years, sliding mode control has attracted much attention because of its insensitivity and strong robustness to the parameter and external disturbance uncertainty. However, the chattering phenomenon existing in traditional sliding mode control restricts its application in practice. To diminish the chattering effect, the concept of high order sliding mode (HOSM) is proposed. In the present HOSM algorithms, the super-twisting algorithm is very popular and own practical application value due to the characteristics of finite-time convergence, strong robustness and solely requiring the information of sliding mode variables [1]. Numerous modified super-twisting methods have been proposed to further improve the control performance [2]–[7].

However, the above-mentioned control methods can only be utilized in single-input single-output (SISO) system. When applied in the MIMO system, the system needs to be decoupled into multi-SISO systems, which constrains the application of super-twisting sliding mode control. In [8], a multivariable super-twisting sliding mode approach is present to directly design the controller for MIMO systems, which obtains better control performance than that of the single super-twisting sliding mode control. In [9], an adaptive multivariable super-twisting sliding mode control method is proposed, which can adapt to the disturbance of unknown boundary.

In this paper, inspired by [9], we extend our previous work [7] to multivariate form and propose a new proposition called the adaptive multivariable smooth second-order sliding mode control (AMSSOSMC) approach, which integrates the merits of fast finite-time convergence, adaptation to the disturbances and smooth. This new method can be directly applied to the controller design of MIMO systems. Moreover, a novel adaptive multivariable smooth disturbance observer (AMSDO) is also proposed based on this new method, which can adjust the parameters automatically without a priori knowledge of the upper bound of disturbances derivative and have smoother output than that of adaptive multivariable disturbance observer (AMDO) proposed in [9].

In terms of the types of disturbances, the fast finite-time convergence and the fast finite-time uniformly ultimately boundedness of the systems will be proved with the corresponding finite-time Lyapunov stability theory. The effectiveness and superiority of the proposed method is verified by comparative simulation experiments.

The rest of this paper is organized as follows. In Section II, some necessary lemmas are given. The adaptive multivariable smooth second-order sliding mode approach is provided in Section III. Contrastive numerical simulations are executed to verify the effectiveness of the proposed approach in Section IV. Section VI concludes this paper.

Notation: In this paper, we use $\| \cdot \|$ for the Euclidean norm of vectors and $\otimes$ for the Kronecker product. $\lambda_{\text{max}}(\cdot)$ and $\lambda_{\text{min}}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix, respectively. $I_n$ and $I_n$ denote the $n \times n$ identity matrix and $n$ dimension unit column vector, respectively. $0_{nn}$ and $0_n$ denote $n \times n$ zero matrix and $n$ dimension zero column vector, respectively.

II. PRELIMINARIES

To better describe the following Lemmas, we consider a general system

$$\dot{x} = h(x(t)), x_0 = x(0)$$  \hspace{1cm} (1)

where $h : U_1 \to \mathbb{R}^n$ is continuous on an open neighborhood $U_1 \subset \mathbb{R}^n$ of the origin and assume that $h(0) = 0$. The solution of (5) is denoted as $x(t, x_0)$, which is understood in the sense of Filippov [10].

Lemma 1 (fast finite-time stability [11]): Suppose there exists a continuous and positive-definite function $V : U_1 \to \mathbb{R}$ such that the following condition holds:

$$\dot{V}(x) \leq -c_1 V(x)^p - c_2 V(x)$$  \hspace{1cm} (2)
where $c_1 > 0, c_2 > 0, p \in (0, 1)$, then the trajectory of (5) is fast finite-time stable, and the settling time is given by:

$$T \leq \frac{\ln[1 + c_2 V(x_0)^{1-p}/c_1]}{c_2(1-p)}$$  (3)

**Lemma 2** (fast finite-time uniformly ultimately boundedness [6]): Suppose there exists a continuous and positive-definite function $V: U_1 \to R$ such that the following condition holds:

$$\dot{V}(x) \leq -c_1 V(x)^{p_1} - c_2 V(x) + c_3 V(x)^{p_2}$$  (4)

where $c_1 > 0, c_2 > 0, c_3 > 0, p_1 \in (0, 1), p_2 \in (0, p_1)$, then the trajectory of (5) is fast finite-time uniformly ultimately boundedness, and the settling time is given by:

$$T \leq \ln[1 + (c_2 - \theta_2) V(x_0)^{1-p}/(c_1 - \theta_1)]/(c_2 - \theta_2)(1-p)$$  (5)

where $\theta_1$ and $\theta_2$ are arbitrary positive constants holding $\theta_1 \in (0, c_1), \theta_2 \in (0, c_2)$, then $x(t, x_0)$ can converge to a region of equilibrium point in a finite time $T$. In addition, the residual set of solution of (5) can be given by:

$$D = \left\{ x : \theta_1 V(x)^{p_1 - p_2} + \theta_2 V(x)^{1 - p_2} < c_3 \right\}$$  (6)

Define an auxiliary variable $\theta_3 \in (0, 1)$. If $\theta_3$ is selected satisfying

$$\theta_3^{1-p_2} \theta_2^{p_1 - p_2} c_3^{1-p_2} = \theta_1^{1-p_2}(1 - \theta_3)^{p_1 - p_2}$$  (7)

then (10) can be reduced to $D = D_1 = D_2$, where

$$D_1 = \left\{ x : V(x)^{p_1 - p_2} < \theta_3^3 / \theta_1 \right\}$$
$$D_2 = \left\{ x : V(x)^{1 - p_2} < (1 - \theta_3) c_3 / \theta_2 \right\}$$  (8)

which means the state $x$ can converge to $D_1 = D_2$ in finite time $T$.

**III. AN ADAPTIVE MULTIVARIABLE SMOOTH SUPER-TWISTING SLIDING MODE APPROACH**

Motivated by [9], we extend our previous work [7] to multivariate form and obtain the following new proposition.

**Proposition 1**: Considering the following system

$$\dot{x}_1 = -L_1(t) \frac{x_1}{\|x_1\|^m} - L_2(t)x_1 + x_2$$
$$\dot{x}_2 = -L_3(t) \frac{x_2}{\|x_2\|^m} - L_4(t)x_1 + d$$  (9)

where $x_1, x_2 \in \mathbb{R}^n$ and $\|d\| \leq \delta, \delta$ is an unknown non negative constant. The adaptive gains $L_1(t), L_2(t), L_3(t), L_4(t)$ are formulated as

$$L_1(t) = k_1 L_0^{m-1}(t), L_2(t) = k_2 L_0(t)$$
$$L_3(t) = k_3 L_0^{2m-2}(t), L_4(t) = k_4 L_0^2(t)$$  (10)

where $k_1, k_2, k_3, k_4, m$ are positive constants satisfying

$$m^2 k_3 k_4 > \left( \frac{m^3 k_3}{m-1} + (4m^2 - 4m + 1) k_2 \right)^2, m > 2$$  (11)

$L_0(t)$ is a positive, time-varying, and scalar function. The $L_0(t)$ satisfies

$$L_0(t) = \begin{cases} \kappa, & \text{if } \|x_1\| \geq \varepsilon \\ 0, & \text{else} \end{cases}$$  (12)

where $\kappa$ is a positive constant, $\varepsilon$ is an arbitrary small positive value. Then the following statements hold

(i) If $d = 0$, then $x_1, \dot{x}_1$ can fast converge to the origin in finite time.

(ii) If $d \neq 0$ and $d$ is a bounded disturbance, then $x_1, \dot{x}_1$ can fast converge to a region of the origin in finite time.

**Proof**: To facilitate the analysis, define the following new state vectors

$$\xi_1 = \frac{x_1}{\|x_1\|^m}, \xi_2 = \frac{x_2}{\|x_2\|^m}, \xi_3 = x_3$$

where $\xi_i \in \mathbb{R}^n (i = 1, 2, 3)$.

Taking the derivative of (13) yields

$$\dot{\xi} = \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} \frac{m-1}{m} \frac{L_0 x_1}{\|x_1\|^m} + \frac{L_0 m^{-1}}{\|x_1\|^m} \left[ I_{nn} - \frac{1}{m} \frac{x_1 x_1^T}{\|x_1\|^2} \right] \dot{x}_1 \\ -k_3 \frac{L_0 m^{-2}}{\|x_1\|^m} x_1 - k_4 L_0^2 \dot{x}_1 + d \end{bmatrix} - k_3 L_0 m^{-2} \frac{x_1}{\|x_1\|^m}$$  (14)

In view of the definition in (13), one has

$$\frac{x_1}{\|x_1\|} = \frac{\xi_1}{\|\xi_1\|} = \frac{\xi_2}{\|\xi_2\|}$$  (15)

Then

$$\begin{bmatrix} I_{nn} - \frac{1}{m} \frac{x_1 x_1^T}{\|x_1\|^2} \end{bmatrix} \dot{x}_1 = \begin{bmatrix} \xi_3 - \frac{\xi_1 \xi_1^T \xi_3}{m \|\xi_1\|^2} - \sum_{i=1}^2 k_i \left[ I_{nn} - \frac{1}{m} \frac{\xi_i \xi_i^T}{\|\xi_i\|^2} \right] \xi_i \\ -k_3 \xi_1 k_2 \xi_3 - m^{-1} (k_1 \xi_1 + k_2 \xi_2) \end{bmatrix}$$

Substituting (16) into (14) results in

$$\dot{\xi} = \begin{bmatrix} \frac{m-1}{m} k_1 \frac{m^{-1} k_2}{m} - 1 \end{bmatrix} \odot I_{nn} \in \mathbb{R}^{3n \times 3n}$$

$$A = \begin{bmatrix} \frac{m-1}{m} k_1 \frac{m^{-1} k_2}{m} - 1 \\ 0 \\ 0 \end{bmatrix} \odot I_{nn} \in \mathbb{R}^{3n \times 3n}$$

$$B = \begin{bmatrix} k_1 k_2 - 1 \\ k_3 \end{bmatrix} \odot I_{nn} \in \mathbb{R}^{3n \times 3n}$$

$$C = C_1 + C_2 + C_3 \in \mathbb{R}^{3n}$$
For the system (9), select a positive definite Lyapunov function as $V (\xi) = \xi^T P \xi$, where

$$P = \frac{1}{2} \begin{bmatrix} \frac{2m}{m-1} k_3 + k_1^2 & k_1 k_2 & -k_1 \\ k_1 k_2 & 2k_4 + k_2^2 & -k_2 \\ -k_1 & -k_2 & 2 \end{bmatrix} \otimes I_{nn} \in \mathbb{R}^{3n \times 3n}$$

where $P$ is symmetric positive definite.

Taking the derivative of $V (\xi)$ along the trajectories of system (9) yields

$$\dot{V} = -\frac{L_0}{\|\xi\|^{\frac{m-1}{m}}} \xi^T \Omega_1 \xi - L_0 \xi^T \Omega_2 \xi + \tilde{V}$$

where $\Omega_1 = A^T P + PA$, $\Omega_2 = B^T P + PB$ and $\tilde{V} = 2\xi^T PC$

Denoting $\tilde{V}_i = 2\xi^T PC_i (i = 1, 2, 3)$, $\tilde{V}$ can be rewritten as

$$\tilde{V} = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3$$

The $\tilde{V}_1$ and $\tilde{V}_2$ in (22) satisfy the following inequalities

$$\tilde{V}_1 = 2\xi^T PC_1 \leq \frac{m-1}{m} \frac{L_0}{L_0} \xi^T Q \xi$$

$$\tilde{V}_2 = 2\xi^T PC_2 \leq \sqrt{k_1^4 + k_2^4 + 4 \|\xi\| \delta}$$

where $Q = \text{diag}[q_1, q_2, q_3] \otimes I_{nn}$ is a diagonal matrix with positive diagonal elements, which are expressed as follows

$$q_1 = \frac{2m}{m-1} k_3 + k_1^2 + \frac{(2m-1) k_1 k_2}{2(m-1)} + \frac{k_2}{2}$$

$$q_2 = \frac{m}{2(m-1)} \left(4k_4 + 2k_2^2 + k_2 \right) + \frac{(2m-1) k_1 k_2}{2(m-1)}$$

$$q_3 = \frac{k_1}{2} + \frac{mk_2}{2(m-1)}$$

In addition, it can be observed that

$$\frac{\xi_3^T \xi_1 \xi_1^T \xi_3}{\|\xi_1\|^2} = \frac{(\xi_1^T \xi_3)^2}{\|\xi_1\|^2} \leq \|\xi_3\|^2$$

Thus, the $\tilde{V}_3$ satisfies the following inequality

$$\tilde{V}_3 = 2\xi^T PC_3$$

$$\leq \frac{1}{m} \frac{L_0}{\|\xi_1\|^{\frac{m-1}{m}}} \left( \frac{2m}{m-1} k_3 + k_1^2 \right) \xi_1^T \xi_1 - \frac{k_1}{m} \|\xi_3\|^2 - \frac{L_0 k_1 k_2}{m} \xi_1^T \xi_3$$

Incorporating (26) into (21), (21) can be rewritten as

$$\dot{V} \leq -\frac{L_0}{\|\xi_1\|^{\frac{m-1}{m}}} \dot{\xi}^T \Omega_1 \dot{\xi} - L_0 \dot{\xi}_3^T \Omega_2 \dot{\xi} + \tilde{V}_1 + \tilde{V}_2$$

where

$$\Omega_1 = \begin{bmatrix} k_3 m + k_1^2 & 0 & 0 \\ 0 & k_1 m + k_2^2 & -k_1 \\ 0 & -k_1 & k_2 (m-1) & -k_1 \end{bmatrix} \otimes I_{nn}$$

$$\Omega_2 = k_2 \begin{bmatrix} k_3 + k_1 (3m - 2)/m & 0 & 0 \\ 0 & k_4 + k_2^2 & -k_2 \\ 0 & -k_2 & 1 \end{bmatrix} \otimes I_{nn}$$

It is easy to prove that the matrices $\Omega_1$ and $\Omega_2$ both are positive definite with (11). By using

$$\lambda_{\min} (P) \|\xi\|^2 \leq V \leq \lambda_{\max} (P) \|\xi\|^2$$

(27) can be further rewritten as

$$\dot{V} \leq -L_0 (t) n_1 V^{p_1} + n_2 \frac{\dot{V}}{2} - \left( L_0 (t) n_3 - \frac{2m - 2}{m} n_4 \frac{\dot{L}_0}{L_0} \right) V$$

where $p_1 = (2m - 3) / (2m - 2)$,

$$n_1 = \frac{\lambda_{\min} (\Omega_1)}{\lambda_{\max} (P)}$$

$$n_2 = \lambda_{\min} (\Omega_2) \left( \lambda_{\max} (P) \right)$$

$$n_3 = \lambda_{\min} (Q) \left( 2 \lambda_{\min} (P) \right)$$

(i) If $d(t) = 0$, then $\delta = 0$, (30) will become

$$\dot{V} \leq -L_0 (t) n_1 V^{p_1} - \left( L_0 (t) n_3 - \frac{2m - 2}{m} n_4 \frac{\dot{L}_0}{L_0} \right) V$$

Due to $\dot{L}_0 (t) \geq 0$, $L_0 (t) n_3 - (2m - 2) n_4 \dot{L}_0 / (L_0 m)$ is positive in finite time. It follows from (32) that

$$\dot{V} \leq -c_1 V^{p_1} - c_2 V$$

where $c_1$ and $c_2$ are positive constants, $p_1 \in (0.5, 1)$. By using Lemma I, $\xi$ can converge to origin in fast finite time, then $x_1, \dot{x}_1$ can fast converge to the origin in finite time and the proof of (i) is completed.

(ii) If $d(t) \neq 0$, with the same analysis of (i), it follows from (30) that

$$\dot{V} \leq -c_3 V^{p_1} - c_5 \dot{V} + c_3 V^{\frac{1}{2}}$$

(34) where $c_3, c_4$ and $c_5$ are positive constants, $p_1 \in (0.5, 1)$. By using Lemma 2, $\xi$ can converge to a region of origin in fast finite time. In addition, using (9), (10) and (13), the $x_1$ and $\dot{x}_1$ converge to a region of the origin in fast finite time. The proof of (ii) is completed.

Remark I: By adopting different values of $m$ in (9), a series of adaptive multivariable smooth second-order sliding mode control methods can be obtained. The proposed methods will be used in the design of controller and observer for MIMO systems and the superiority of the proposed control methods will be validated in the next section.
IV. NUMERICAL SIMULATIONS

To facilitate validation of the effectiveness of the proposed approach, the following simplified multivariable perturbed control system is considered

\[ \dot{x}_1 = u + d_1 \]  

(35)

Where states \( x_1 = [x_{11} \ x_{12} \ x_{13}]^T \in \mathbb{R}^3 \), \( u \in \mathbb{R}^3 \) and disturbance \( d_1 = [d_{11} \ d_{12} \ d_{13}]^T \in \mathbb{R}^3 \) satisfies \( \left\| \dot{d}_1 \right\| \leq \delta_1 \)

where \( \delta_1 \) is an unknown non-negative constant. The initial values of the states are set as \( x_1(0) = [1 \ 3 \ 2]^T \).

A. Experiment I: Comparison with constant disturbance

In experiment I, the constant disturbance is set as \( d_1 = [0.1 \ 0.2 \ 0.2]^T \). Based on Proposition 1, the controller is expressed as follows

\[ u = -L_{c3}(t) \frac{x_1}{\|x_1\|} - L_{c2}(t)x_1 \]

\[-\int_0^t \left[ L_{c3}(t) \frac{x_1}{\|x_1\|} + L_{c4}(t)x_1 \right] d\tau \]

(36)

where \( L_{c3}(t), L_{c2}(t), L_{c3}(t), L_{c4}(t) \) are formulated the same as (15) and the parameters are set as: \( m = 3, k_1 = 2, k_2 = 2.5, k_3 = 4, k_4 = 30, \kappa = 10. \)

The adaptive multivariable super-twisting sliding mode control (AMSTSMC) method proposed in [9] is implemented as the comparison method, which adopts the same parameters except that \( m \) is set as 2.

The results of experiment I are given in Fig.1: (a)-(f). Fig. 1: (a), (c) and (e) illustrate the responses of state variables by utilizing AMSTSMC and the proposed method, respectively. Fig. 1: (b), (d) and (f) present the local magnification to show the steady-state response more clearly. Fig. 1: (a) shows that the state variable converges to the origin in finite time by using the proposed control method as fast as the method in [9], which means the newly proposed method also possesses the characteristic of fast finite-time convergence. In addition, Fig. 1: (b) demonstrates that the proposed method can diminish the chattering existing in the AMSTSMC method. A similar conclusion can be drawn from the Fig. 1: (c), (d) and (e), (f).

B. Experiment II: Comparison with time-varying disturbance

In experiment II, the time-varying disturbance is set as \( d_1 = [0.1 \sin(t) \ 0.2 \cos(4t) \ 0.2 \cos(2t)]^T \). The controllers and parameters are selected the same as those in the experiment I.

The results of experiment II are shown in Fig. 2: (a)-(f). Fig. 2: (a), (c) and (e) demonstrate the responses of state variables by using AMSTSMC and the proposed method, respectively. Fig. 2: (b), (d) and (f) present the local magnification corresponding state variable to show the steady-state response more clearly. From Fig. 2: (a) and (b), one can see that the proposed method can guarantee the fast finite-time uniformly ultimately boundedness coupled with enormous chattering suppression. A similar conclusion can be drawn from the Fig. 2: (c), (d) and (e), (f).

C. Experiment III: Observer comparison

Based on Proposition 1, the observer for estimating the disturbance \( d_1 \) is designed as follows

\[ \dot{d}_1 = L_{d1}(t) \frac{e_1}{\|e_1\|} + L_{d2}(t)e_1 \]

\[ + \int_0^t \left[ L_{d3}(t) \frac{e_1}{\|e_1\|^2} + L_{d4}(t)e_1 \right] d\tau \]

(37)

where \( e_1 = z_1 - x_1, \dot{z}_1 = u + \dot{d}_1 \). The adaptive gains \( L_{di}(t)(i = 1, 2, 3, 4) \) are formulated the same as (15) where the parameters are set as: \( m = 3, k_1 = 2, k_2 = 2.5, k_3 = 4, k_4 = 30, \kappa = 10. \)

The purpose of the experiment III is to compare the performance of the proposed observer with that of the adaptive multivariable disturbance observer (AMDO) in [9], which adopts the same parameters except that \( m \) is set as 2. To facilitate the comparison, the disturbance is set as \( d_1 = [\sin(t) \ 2 \cos(4t) \ 2 \cos(2t)]^T \). The results of experiment III are given in Fig. 3: (a)-(f). Fig. 3: (a), (c) and (e) indicate the responses of observer estimation error components by using AMDO and the proposed observer. Fig. 3: (b), (d) and (f) present the local magnification to show the responses more clearly. From Fig. 3: (a) and (b), one can
see that the estimation error can fast converge to a region of the origin in finite time by using the proposed observer. In addition, the proposed observer can effectively alleviate chattering effect existing in the AMDO and provide smoother output for the disturbance estimation. A similar conclusion can be drawn from the Fig. 3: (c), (d) and (e), (f).

V. CONCLUSION

In this paper, a novel adaptive multivariable smooth second-order sliding mode approach has been proposed. This newly proposed approach is utilized in the design of controller and observer for MIMO systems. According to the types of disturbances, the fast finite-time convergence and the fast finite-time uniformly ultimately boundedness of the systems are proved with the corresponding finite-time Lyapunov stability theory. The comparative numerical simulations are performed to demonstrate the effectiveness and superiority of the proposed approach with fast finite-time convergence, adaptation to disturbances, and chattering suppression for the MIMO system.

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