Identity Method for Particle Number Fluctuations and Correlations

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Abstract

An incomplete particle identification distorts the observed event-by-event fluctuations of the hadron chemical composition in nucleus-nucleus collisions. A new experimental technique called the identity method was recently proposed. It eliminated the misidentification problem for one specific combination of the second moments in a system of two hadron species. In the present paper this method is extended to calculate all the second moments in a system with arbitrary number of hadron species. Special linear combinations of the second moments are introduced. These combinations are presented in terms of single-particle variables and can be found experimentally from the event-by-event averaging. The mathematical problem is then reduced to solving a system of linear equations. The effect of incomplete particle identification is fully eliminated from the final results.

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I. INTRODUCTION

A study of event-by-event (e-by-e) fluctuations in high-energy nucleus-nucleus (A+A) collisions opens new possibilities to investigate properties of strongly interacting matter (see, e.g., review [1] and references therein). Specific fluctuations can signal the onset of deconfinement when the collision energy becomes sufficiently high to create the quark-gluon plasma [2] at the initial stage of A+A collision. By measuring the fluctuations, one may also observe effects caused by dynamical instabilities when the expanding system goes through the 1-st order transition line between the quark-gluon plasma and the hadron gas [3]. Furthermore, the QCD critical point may be signaled by a characteristic fluctuation pattern [4–6]. Fluctuations of the chemical (particle-type) composition of hadronic final states in A+A collisions are expected to be sensitive to the phase transition between hadronic and partonic matter. First data on the e-by-e chemical fluctuations from the CERN SPS [7–9] and BNL RHIC [10] have been already published, and more systematic measurements are in progress. The e-by-e fluctuations of hadron multiplicities have been studied theoretically in statistical models (see, e.g., Ref. [11]) and in dynamical transport models (see, e.g., review [12] and references therein).

Studies of the e-by-e chemical fluctuations assume particle number measurements for different hadron species (e.g., pions, kaons, and protons). The NA49 Collaboration [7–9] has used the measure $\sigma_{\text{dyn}}$, which is defined as the difference between fluctuations observed in real and mixed events. The STAR Collaboration [10] has used, in addition to the $\sigma_{\text{dyn}}$ measure, the quantity $\nu_{\text{dyn}}$ (see, e.g., Ref. [13]). Moreover, it was suggested long ago [14, 15] to quantify chemical fluctuations by the measure $\Phi$ [16]. Note that different fluctuation measures can be presented as specific combinations of the second moments of the multiplicity distribution. Some important features of different measures for the e-by-e fluctuations have been considered in Ref. [17].

A serious experimental problem of the e-by-e measurements of the chemical fluctuations is incomplete particle identification; that is the impossibility to determine uniquely the type of each detected particle. The effect of particle misidentifications distorts the measured fluctuations. For this reason the analysis of chemical fluctuations is usually performed in a small acceptance, where particle identification is relatively reliable. However, an important part of the information on e-by-e fluctuations in full phase space is then lost. Although it is usually impossible to identify each detected particle, one can nevertheless determine with a high accuracy
the average multiplicities (averaged over many events) for different hadron species.

In Ref. [18] a new experimental technique called the identity method was proposed. It solves the misidentification problem for one specific combination of the second moments in a system of two hadron species (‘kaons’ and ‘pions’). In the present study we extend these results in two directions. First, we prove that not only the one specific combination of the second moments but all the second moments themselves can be uniquely reconstructed in spite of the effects of incomplete identification. Second, the identity method is extended to an arbitrary number \( k \geq 2 \) of hadron species. This is important for practical purposes since typically there is an incomplete identification for pions, kaons, and protons, which means \( k = 3 \). The identity method is introduced in Section II. In Section III the main results are presented. We also discuss several examples which illustrate some limiting cases of particle identification. Section IV presents the summary.

II. IDENTITY METHOD

The identity method was proposed in Ref. [18] for the fluctuation measure \( \Psi \). This method is based on the fact that the analysis of chemical fluctuations can be performed within two different but fully equivalent formulations. The first formulation [14] uses the identity variables; that is, the \( \Psi \) measure of chemical fluctuations is calculated using single-particle variables \( z_i \equiv x_i - \overline{x} \), where the over-bar denotes averaging over the single-particle inclusive distribution. The event variable \( Z \), which is a multi-particle analog of \( z \), is defined as \( Z \equiv \sum_{i=1}^{N} (x_i - \overline{x}) \), where the sum runs over the \( N \) particles in a given event. The measure \( \Psi \) is defined as

\[
\Psi = \frac{\langle Z^2 \rangle}{\langle N \rangle} - \overline{z^2},
\]

where the symbol \( \langle \ldots \rangle \) corresponds to the e-by-e averaging. One defines the single-particle variable \( x_i \) as the identity variable \( w_1(i) \) which equals 1 if the \( i \)th particle is of the first type (‘kaon’), and \( w_1(i) = 0 \) if the \( i \)th particle is of the second type (‘pion’). In a real measurement, it is unknown exactly whether a given particle is ‘kaon’ or ‘pion’. As a consequence of this incomplete identification the variable \( w_1(i) \) is not exactly 0 or 1, but becomes a distribution function with possible values in the whole \([0, 1]\) interval. Nevertheless, despite the incomplete particle identification, one can directly use the definition (1) to evaluate \( \Psi \).
In the second formulation, $\Psi$ is calculated in terms of the moments of the multiplicity distribution. In the case of complete particle identification it was found \[15\] that

$$\Psi = \frac{1}{(N)^3} \left[ \langle N_1^2 \rangle \langle N_2 \rangle^2 + \langle N_1 \rangle^2 \langle N_2^2 \rangle - 2 \langle N_1 \rangle \langle N_2 \rangle \langle N_1 N_2 \rangle - \langle N_1 \rangle^2 \langle N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle^2 \right], \quad (2)$$

where indices 1 and 2 correspond to different hadron species (‘kaon’ and ‘pion’), and $N = N_1 + N_2$. Using the presentation \(1\), it was shown \[18\] that the measure $\Psi$ can be factorized into a coefficient that represents the effect of misidentification, and the quantity \(2\), which corresponds to the value that $\Psi$ would have for complete identification.

We follow Ref. \[18\] and assume that particle identification is achieved by measuring the particle mass $m$. Since any measurement is of finite resolution, we deal with continuous distributions of observed masses denoted as $\rho_j(m)$ and normalized as ($j = 1, \ldots, k \geq 2$)

$$\int dm \rho_j(m) = \langle N_j \rangle . \quad (3)$$

Note that the functions $\rho_j(m)$ are found for the different particle species using the values averaged over all particles from all collision events. The identity variables $w_j(m)$ will be defined as

$$w_j(m) \equiv \frac{\rho_j(m)}{\rho(m)} , \quad \rho(m) \equiv \sum_{i=1}^k \rho_i(m) . \quad (4)$$

The complete identification (CI) of particles corresponds to distributions $\rho_j(m)$ which do not overlap. In this case, $w_j = 0$ for all particle species $i \neq j$ and $w_j = 1$ for the $j$th species. When the distributions $\rho_j(m)$ overlap, $w_j(m)$ can take the value of any real number from $[0, 1]$.

We introduce the quantities $W_j^2$, with $j = 1, \cdots, k$, and $W_p W_q$, with $1 \leq p < q \leq k$,

$$W_j^2 \equiv \left( \sum_{i=1}^{N(n)} w_j(m_i) \right)^2 , \quad W_p W_q \equiv \left( \sum_{i=1}^{N(n)} w_p(m_i) \right) \times \left( \sum_{i=1}^{N(n)} w_q(m_i) \right) , \quad (5)$$

and define their event averages as

$$\langle W_j^2 \rangle = \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_j^2 , \quad \langle W_p W_q \rangle = \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_p W_q , \quad (6)$$

where $N_{ev}$ is the number of events, and $N(n) = N_1(n) + \cdots + N_k(n)$ is the total multiplicity in the $n$th event. Each experimental event is characterized by a set of particle masses $\{m_1, m_2, \ldots, m_N\}$, for which one can calculate the full sets of identity variables:
\{w_j(m_1), w_j(m_2), \ldots, w_j(m_N)\}$, with $j = 1, \ldots, k$. Thus, the quantities $W_j^2$ and $W_pW_q$ are completely defined for each event, and their average values can be found experimentally by straightforward e-by-e averaging. In the case of CI, one finds $W_j^2 = N_j^2$ and $W_pW_q = N_pN_q$, thus, Eq. (6) yields

$$\langle W_j^2 \rangle = \langle N_j^2 \rangle, \quad \langle W_pW_q \rangle = \langle N_pN_q \rangle. \quad (7)$$

**III. SECOND MOMENTS OF CHEMICAL FLUCTUATIONS**

The quantities $\langle W_j^2 \rangle$ and $\langle W_pW_q \rangle$ can be calculated as follows

$$
\langle W_j^2 \rangle = \sum_{N_1=0}^{\infty} \cdots \sum_{N_k=0}^{\infty} P(N_1, \ldots, N_k) \int dm_1^1 P_1(m_1^1) \cdots \int dm_k^1 P_1(m_k^1) \\
\times \int dm_2^2 P_2(m_2^2) \cdots \int dm_N^2 P_2(m_N^2) \times \cdots \int dm_1^k P_k(m_1^k) \cdots \int dm_N^k P_k(m_N^k) \\
\times \left[ w_j(m_1^1) + \cdots + w_j(m_1^{N_1}) + w_j(m_2^1) + \cdots + w_j(m_2^{N_2}) + \cdots + w_j(m_N^1) + \cdots + w_j(m_N^{N_2}) \right]^2
$$

$$=
\sum_{i=1}^{k} \langle N_i \rangle \left[ u_{ji}^2 - (u_{ji})^2 \right] + \sum_{i=1}^{k} \langle N_i^2 \rangle (u_{ji})^2 + 2 \sum_{1 \leq i < l \leq k} \langle N_iN_l \rangle u_{ji}u_{jl}, \quad (8)
$$

$$
\langle W_pW_q \rangle = \sum_{N_1=0}^{\infty} \cdots \sum_{N_k=0}^{\infty} P(N_1, \ldots, N_k) \int dm_1^1 P_1(m_1^1) \cdots \int dm_k^1 P_1(m_k^1) \\
\times \int dm_2^2 P_2(m_2^2) \cdots \int dm_N^2 P_2(m_N^2) \times \cdots \int dm_1^k P_k(m_1^k) \cdots \int dm_N^k P_k(m_N^k) \\
\times \left[ w_p(m_1^1) + \cdots + w_p(m_1^{N_1}) + w_p(m_2^1) + \cdots + w_p(m_2^{N_2}) + \cdots + w_p(m_N^1) + \cdots + w_p(m_N^{N_2}) \right] \\
\times \left[ w_q(m_1^1) + \cdots + w_q(m_1^{N_1}) + w_q(m_2^1) + \cdots + w_q(m_2^{N_2}) + \cdots + w_q(m_N^1) + \cdots + w_q(m_N^{N_2}) \right]
$$

$$=
\sum_{i=1}^{k} \langle N_i \rangle \left[ u_{pqi} - u_{pi}u_{qi} \right] + \sum_{i=1}^{k} \langle N_i^2 \rangle u_{pi}u_{ki} + \sum_{1 \leq i < l \leq k} \langle N_iN_l \rangle \left[ u_{pi}u_{ql} + u_{pi}u_{qi} \right]. \quad (9)
$$

In Eqs. (5) and (6), $P(N_1, \ldots, N_k)$ is the multiplicity distribution, $P_s(m) = \rho_s(m)/\langle N_i \rangle$ are the mass probability distributions of the $i$th species, and $(s = 1, 2)$

$$u_{ji}^s = \frac{1}{\langle N_i \rangle} \int dm w_s^j(m) \rho_i(m), \quad u_{pqi} = \frac{1}{\langle N_i \rangle} \int dm w_p(m) w_q(m) \rho_i(m). \quad (10)$$

In the case of CI, when the distributions $\rho_j(m)$ do not overlap, one finds that

$$u_{ji}^s = \delta_{ji}, \quad u_{pqi} = 0, \quad (11)$$
and Eqs. (8) and (9) reduce then to Eq. (7). The incomplete particle identification transforms the second moments $\langle N^2_j \rangle$ and $\langle N_p N_q \rangle$ to the quantities $\langle W^2_j \rangle$ and $\langle W_p W_q \rangle$, respectively. Each of the later quantities contains linear combinations of all the first and second moments, $\langle N_i \rangle$ and $\langle N^2_i \rangle$, as well as all the correlation terms $\langle N_i N_l \rangle$.

Having introduced the notations

$$\langle W^2_j \rangle - \sum_{i=1}^{k} \langle N_i \rangle [u^2_{ji} - (u_{ji})^2] \equiv b_j \ , \quad \langle W_p W_q \rangle - \sum_{i=1}^{k} \langle N_i \rangle [u_{pqi} - u_{pi} u_{qi}] \equiv b_{pq} \ , \quad (12)$$

one can transform Eqs. (8) and (9) to the following form:

$$\sum_{i=1}^{k} \langle N^2_i \rangle \ u^2_{ji} + 2 \sum_{1 \leq i < l \leq k} \langle N_i N_l \rangle \ u_{ji} u_{jl} = b_j \ , \quad j = 1, 2, \ldots, k \ , \quad (13)$$

$$\sum_{i=1}^{k} \langle N^2_i \rangle \ u_{pi} u_{qi} + \sum_{1 \leq i < l \leq k} \langle N_i N_l \rangle \left( u_{pi} u_{ql} + u_{pl} u_{qi} \right) = b_{pq} \ , \quad 1 \leq p < q \leq k \ . \quad (14)$$

The right-hand side of Eqs. (13) and (14) defined by Eq. (12) are experimentally measurable quantities. The same is true for the coefficients $u^s_{ji}$ (with $s = 1$ and 2) entering the left-hand side of Eqs. (13) and (14). Therefore, Eqs. (13) and (14) represent a system of $k + k(k - 1)/2$ linear equations for the $k$ second moments $\langle N^2_j \rangle$ with $j = 1, \ldots, k$ and $k(k - 1)/2$ correlators $\langle N_p N_q \rangle$ with $1 \leq p < q \leq k$.

In order to solve Eqs. (13) and (14) we introduce the $[k + k(k - 1)/2] \times [k + k(k - 1)/2]$ matrix $A$

$$A = \begin{pmatrix}
  a^1_1 & \ldots & a^k_1 & | & a^{12}_1 & \ldots & a^{(k-1)k}_1 \\
  . & \ldots & . & | & . & \ldots & . \\
  . & \ldots & . & | & . & \ldots & . \\
  a^1_k & \ldots & a^k_k & | & a^{12}_k & \ldots & a^{(k-1)k}_k \\
  \ldots & \ldots & \ldots & | & \ldots & \ldots & \ldots \\
  a^1_{12} & \ldots & a^k_{12} & | & a^{12}_{12} & \ldots & a^{(k-1)k}_{12} \\
  . & \ldots & . & | & . & \ldots & . \\
  . & \ldots & . & | & . & \ldots & . \\
  a^1_{1(k-1)k} & \ldots & a^k_{1(k-1)k} & | & a^{12}_{1(k-1)k} & \ldots & a^{(k-1)k}_{1(k-1)k}
\end{pmatrix} \ . \quad (15)$$
where
\[
a^i_j \equiv u^2_{ji}, \quad 1 \leq i, j \leq k; \quad a^p_q \equiv 2u_{ip}u_{iq}, \quad 1 \leq p < q \leq k, \quad i = 1, \ldots, k; \quad (16)
\]
\[
a^{lm}_{pq} \equiv u_{pl}u_{qm} + u_{ql}u_{pm}, \quad 1 \leq p < q \leq k, \quad 1 \leq l < m \leq k. \quad (18)
\]

The solution of Eqs. (13) and (14) can be presented by Cramer’s formulas in terms of the determinants
\[
\langle N^2_j \rangle = \frac{\det A_j}{\det A}, \quad \langle N_p N_q \rangle = \frac{\det A_{pq}}{\det A}, \quad (19)
\]
where the matrices \(A_j\) and \(A_{pq}\) are obtained by substituting in the matrix \(A\) the column
\[
a^1_k, \ldots, a^j_k, a^1_{(k-1)k}, \ldots, a^{(k-1)k}_k\]
and the column \(a^p_k, a^p_{12}, \ldots, a^p_{(k-1)k}\), respectively, for the column
\[
b_1, \ldots, b_k, b_{12}, \ldots, b_{(k-1)k}. \]
Therefore, if \(\det A \neq 0\), the system of linear equations (13) and (14) has a unique solution (19) for all the second moments. In the case of CI (11), one finds \(\det A = 1\), \(\det A_j = b_j\), and \(\det A_{pq} = b_{pq}\). The solution (19) reduces then to Eq. (7).

Introducing the \([k + k(k-1)/2]-\)vectors
\[
\mathcal{N} \equiv \begin{pmatrix}
\langle N^2_1 \rangle \\
\ldots \\
\langle N^2_k \rangle \\
\langle N_1 N_2 \rangle \\
\ldots \\
\langle N_{k-1} N_k \rangle
\end{pmatrix}, \quad \mathcal{B} \equiv \begin{pmatrix}
b_1 \\
\ldots \\
b_k \\
b_{12} \\
\ldots \\
b_{(k-1)k}
\end{pmatrix}, \quad (20)
\]
one can write Eqs. (13) and (14) in the matrix form \(A\mathcal{N} = \mathcal{B}\). The solution (19) can be then rewritten as
\[
\mathcal{N} = A^{-1} \mathcal{B}, \quad (21)
\]
where \(A^{-1}\) is the inverse matrix of \(A\). For two particle species, \(k = 2\), this solution takes the form
\[
\begin{pmatrix}
\langle N^2_1 \rangle \\
\langle N^2_2 \rangle \\
\langle N_1 N_2 \rangle
\end{pmatrix} = \begin{pmatrix}
u_{11}^2 & \nu_{12}^2 & 2\nu_{11}\nu_{12} \\
u_{21}^2 & \nu_{22}^2 & 2\nu_{21}\nu_{22} \\
u_{11}\nu_{21} & \nu_{12}\nu_{22} & \nu_{11}\nu_{22} + \nu_{12}\nu_{21}
\end{pmatrix}^{-1} \begin{pmatrix}
b_1 \\
b_2 \\
b_{12}
\end{pmatrix}. \quad (22)
Then Eq. (22) yields

\[ \langle N_1^2 \rangle = \frac{b_1 u_{22}^2 + b_2 u_{12}^2 - 2b_{12} u_{12} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}, \]  
\[ \langle N_2^2 \rangle = \frac{b_2 u_{11}^2 + b_1 u_{21}^2 - 2b_{12} u_{21} u_{11}}{(u_{11} u_{22} - u_{12} u_{21})^2}, \]  
\[ \langle N_1 N_2 \rangle = \frac{b_{12} (u_{11} u_{22} + u_{12} u_{21}) - b_{1} u_{22} u_{21} - b_2 u_{11} u_{12}}{(u_{11} u_{22} - u_{12} u_{21})^2}. \]  

These results, inserted into Eq. (2), provide an alternative way to evaluate \( \Psi \) directly from the moments of the multiplicity distribution.

In general, the particle-by-particle identification is difficult; that is, it is not known whether a given particle really corresponds to the \( j \)th sort. On the other hand, the statistical identification in terms of the functions \( \rho_j(m) \) is usually reliable. Experimental measurements of the \( \rho_j(m) \) functions give the average numbers of each particle species. In most cases, a unique calculation of the second moments using Eq. (19) is also possible. There is, however, an extreme situation when the only available experimental information consists of the average particle multiplicities. This leads to random identification (RI) which only defines, for each particle, the probabilities \( p_j \) of being of the \( j \)th sort. These probabilities are evidently equal to \( p_j = \langle N_j \rangle / \langle N \rangle \), where \( N = \sum_{i=1}^{k} N_i \). This situation is described by the mass distributions given by

\[ \rho_j(m) = \langle N_j \rangle f(m), \]  
where \( \int dm f(m) = 1 \); that is, all functions \( \rho_j(m) \) have the same shape \( f(m) \) but different normalization \( \langle N_j \rangle \). With these distributions one finds

\[ u_{ji} = \frac{\langle N_j \rangle}{\langle N \rangle} \equiv p_j, \quad j = 1, \ldots, k. \]  
This leads to \( \det A = 0 \), and Eqs. (13) and (14) do not define the second moments in a unique way. In fact, from Eq. (5) follows

\[ \langle W_j^2 \rangle = p_j^2 \langle N^2 \rangle, \quad \langle W_{pq} \rangle = p_p p_q \langle N^2 \rangle, \]  
that is, in the case of RI, the measured values (28) include only the average multiplicities \( \langle N_j \rangle \) and the second moment of the total multiplicity \( \langle N^2 \rangle \). Equations (13) and (14) for \( \langle N_j^2 \rangle \), and \( \langle N_p N_q \rangle \) reduce to a single relation

\[ \sum_{j=1}^{k} \langle N_j^2 \rangle + 2 \sum_{1 \leq p < q \leq k} \langle N_p N_q \rangle = \langle N^2 \rangle, \]  
8
where the right-hand side of Eq. (29) is the only experimentally measured combination of the second moments. Therefore, RI gives only one restriction on \( k + k(k - 1)/2 \) second moments and thus admits an infinite number of solutions for \( \langle N_j^2 \rangle \) and \( \langle N_p N_q \rangle \). Any correctly normalized multiplicity distribution \( P(N_i, \ldots, N_k) \), which reproduces experimental values of the first moments, would reproduce Eqs. (13) and (14); that is, in the case of RI the experimental data do not provide any non-trivial information on chemical fluctuations.

It is also instructive to consider an illustrative example when particle species are divided into two groups: \( j = 1, \ldots, k_R \) with RI (26) and \( j = k_R + 1, \ldots, k \) with CI (11). Equations (13) and (14) are then given by

\[
\begin{align*}
\langle N_{R}^2 \rangle &= \langle W_{1}^2 \rangle = \ldots = \langle W_{k_{R}}^2 \rangle = \langle W_{1} W_{2} \rangle = \ldots = \langle W_{k_{R}-1} W_{k_{R}} \rangle, \quad N_{R} = \sum_{j=1}^{k_{R}} N_{j}; \quad (30) \\
\langle N_{R} N_{q} \rangle &= \langle W_{1} W_{q} \rangle = \ldots = \langle W_{k_{R}} W_{q} \rangle, \quad k_{R}+1 \leq q \leq k; \quad (31) \\
\langle N_j^2 \rangle &= \langle W_j^2 \rangle, \quad j = k_R + 1, \ldots, k; \quad (32) \\
\langle N_p N_q \rangle &= \langle W_p W_q \rangle, \quad j = k_R + 1, \ldots, k; \quad k_R + 1 \leq p < q \leq k. \quad (33)
\end{align*}
\]

Quantities \( \langle W_j^2 \rangle \) and \( \langle W_p W_q \rangle \) can be measured experimentally by using their definitions according to Eq. (5). For the particle species with RI, as follows from Eqs. (30) and (31), the second moments of particle multiplicities include only their total multiplicity \( N_{R} \). Therefore, one knows all individual average multiplicities \( \langle N_j \rangle \), but as far as chemical fluctuations are concerned, all particles in the RI group, \( 1 \leq j \leq k_{R} \), look undistinguishable. On the other hand, this fact does not prevent calculations of the second moments (32) and (33) in the CI group, \( k_{R} + 1 \leq j \leq k \).

In the formulation considered in this paper, the functions \( \rho_j(m) \) are defined as quantities averaged over all particles from all collision events. One can consider the set of events with fixed total multiplicity \( N \). All formulae of this paper straightforwardly apply in this case too. The only modifications are: 1) event averaging \( \langle \ldots \rangle \) over all events is changed to averaging \( \langle \ldots \rangle_N \) over events with fixed \( N \); 2) the functions \( \rho_j(m) \) should be replaced by \( \rho_j(m; N) \) calculated for fixed \( N \). This procedure may open some new possibilities in the studies of chemical fluctuations.
IV. SUMMARY

An incomplete particle identification prevents a straightforward measurement of the second moments $\langle N_j^2 \rangle$ and $\langle N_pN_q \rangle$ of the multiplicity distribution. In this paper we extend the identity method proposed in Ref. [18]. We introduce the quantities $\langle W_j^2 \rangle$ and $\langle W_pW_q \rangle$. Each of these quantities is a specific linear combination of all the first and second moments $\langle N_i \rangle$ and $\langle N_i^2 \rangle$, as well as the correlation terms $\langle N_iN_l \rangle$. The quantities $\langle W_j^2 \rangle$ and $\langle W_pW_q \rangle$ are presented in terms of e-by-e averages of functions depending on the single-particle identity variables according to Eq. (6), and can thus be measured experimentally. Mathematically, the problem of finding all the second moments $\langle N_j^2 \rangle$ and $\langle N_pN_q \rangle$ is then reduced to solving the system of $k + k(k - 1)/2$ linear equations (13) and (14). All coefficients entering the left-hand side of these equations are given in terms of experimentally measurable density functions $\rho_j(m)$. The right-hand side in Eqs. (13) and (14) is defined by Eq. (12) which also includes experimentally measurable quantities. In most cases the determinant of matrix (15) is not equal to zero and, therefore, all second moments of particle number distributions can be uniquely reconstructed by Eq. (19) from event-by-event measurements despite the effects of incomplete identification. This is valid for an arbitrary number $k \geq 2$ of different hadron species. The matrix $A^{-1}$ in Eq. (21) represents the correction of the measured values (12). Such a correction eliminates the effect of misidentification. This provides the values of all the second moments $\langle N_j^2 \rangle$ and $\langle N_pN_q \rangle$ in a model-independent way, as they would be obtained in an experiment in which each particle is uniquely identified. However, all measured quantities entering Eqs. (13) and (14) contain experimental errors. Therefore, the practical applicability of the identity method procedure constructed in this paper requires further studies.

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