Spin-dependent electron transport in waveguide with continuous shape

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We study effects of the shape of a two-dimensional waveguide on the spin-dependent electron transport in the presence of spin-orbit coupling. The transition from classical motion to the tunneling regime can be controlled there by modulating the strength of spin-orbit coupling if the waveguide has a constriction. The spin precession strongly depends on the shape of the waveguide.

The spin-dependent transport of ballistic electrons and the spin manipulation in nanostructures are the core components of spintronics. Two main mechanisms of spin-orbit coupling (SOC), the structure inversion asymmetry described by the Rashba term and the bulk inversion asymmetry described by the Dresselhaus term are the key elements in these studies. Electron transmission in a variety of nanostructures including quantum wire with short-ranged irregularities at the boundary, waveguides containing impurities or attached to a cavity or a quantum dot was studied thoroughly for designing mesoscopic devices. It was shown recently that tuning of the gate voltage in a symmetric quantum point contact can form topological edge states with spin polarization robust against the local potential scatterings. The fact that the shape of a nanostructure determines the wavefunctions of the carriers localized there, and as a result, is responsible for its charge and spin transport properties, motivates the investigation of transport in waveguides with a nontrivial shape.

Electron propagation in nonuniform structures in the presence of SOC provides a promising mean to generate and manipulate the spin-polarized electrons. For example, an electron stub waveguide with SOC was proposed to realize spin filtering and accumulation and the importance of the lateral geometry and pattern of the structure and time-dependent SOC are well understood. The spin scattering with an effective attractive potential produced by the Rashba interaction and with periodically modulated strength of the spin-orbit interaction were also addressed. The concept analysis of this type of devices can be found in Ref. 14.

In this Letter, we propose a theory of spin transport in a quasi-one dimensional semiconductor waveguide with a continuous constriction, and the Dresselhaus and Rashba SOCs. By combining these two factors, geometrical shape and the SOC, we show the effect on the scattering induced by the shape variation and explore the spin polarization by tuning Rashba coupling strength which can be well controlled by applying a bias across the structure. It is also shown that the spin states of the transmitted electrons are determined by the coordinate-dependent SOC. Once designed and produced, this structure has well-documented measured properties and can be used as a hardware element for a spin transport device.

We consider a two-dimensional waveguide [001] grown sketched in Fig. 1 whose width in the $z$-direction is described by the function of $w(x) = (\tanh[(x - a)/L] - \tanh[(x + a)/L])w_1 + w_0$, where $a$, $L$, $w_1$, and $w_0$ are the geometry parameters. The waveguide experiences a continuous change in the width, with $w(|x| \to \infty) = w_0$. We assume the boundary conditions, where the wavefunction vanishes at the boundaries. As a result, the electron propagates along the $x$-axis and is confined in the $z$-direction. In the $y$-direction, we assume that the wave function is a free plane wave $\exp(ik_yy)$ with $k_y = 0$. As shown in Fig. 1, the constriction-like waveguide can be described as a repulsive potential since the bottom of the first subband is higher than in the outside regions. The total Hamiltonian in the presence of Rashba and Dresselhaus SOCs is $\hat{H} = \hat{H}_0 + \hat{H}_R + \hat{H}_D$. The kinetic part is $\hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} + \frac{\hat{\mathbf{p}}_z^2}{2m} + \frac{\hat{\mathbf{p}}_y^2}{2m}$. Rashba term is $\hat{H}_R = \alpha_R \hat{s}_y \hat{k}_x$, where $\alpha_R$ is the Rashba coupling constant. The bulk Dresselhaus term is $\hat{H}_D^{\text{bulk}} = \alpha \hat{s}_z \hat{k}_x (k_x^2 - k_y^2) + \hat{s}_y \hat{k}_x (k_x^2 - k_z^2) + \hat{s}_z \hat{k}_x (k_y^2 - k_z^2)$ and $\alpha$ is a parameter for SOC. The effect of the Dresselhaus term $\hat{H}_D$ on the electron in the waveguide is obtained by orientation-dependent averaging the bulk Hamiltonian on the electron wavefunction, leading, in general, to the orientation-dependent spin transport. For convenience, we introduce material and structure related Dresselhaus coefficient as $\alpha_D = \alpha \pi^2 / w_0^2$, while the Rashba coupling can be modulated by applying the

![Diagram](attachment:waveguide.png)

**FIG. 1.** (Color online) Schematic diagram of waveguide with continuous shape, which experiences the constriction around $x = 0$ and tends to a constant $w_0$ asymptotically. The corresponding spin-split energy bands are shown in the lower panel, demonstrating that the effective potential is strongly spin-dependent.
bias in the z-direction. For simplicity, we use dimensionless units, choose $\hbar = m = w_0 = 1$ and introduce constant $r = \alpha_R/\alpha_D$. The wave function corresponding to the Hamiltonian $\hat{H}$ can be expressed as the superposition of eigenstates of all size quantization sub-branches ($n = 1, 2, 3, ...$) in the “adiabatic” basis

$$\psi(x, z) = \sum_n \sqrt{\frac{2}{w(x)}} C_n(x) \sin \left( \frac{n \pi z}{w(x)} \right), \quad (1)$$

where $C_n(x)$ are two-component spinors. However, if the energy of the particle is lower than the minimum of the second quantization subband, the contribution of the second state $|C_2(x)|^2$ is less than 0.05 for the structures we consider. Therefore, we come to the single mode description ($n = 1$, $C(x) \equiv C_1(x)$) if the incident energy satisfies this condition, as it will be assumed below.

In the limit of $|x| \to \infty$ where the width of the waveguide is a constant $w_0$, Hamiltonian can be simplified as $\hat{H} = \hat{h}_0 + (\alpha_R \hat{\sigma}_y - \alpha_D \hat{\sigma}_z) k_x$. We take the spinor $C(x)$ at $x \to -\infty$ as the superposition of the incident and reflected waves,

$$C(-\infty) = A_1^{[+]} \exp(i k_+ x) \gamma_+ + A_r^{[+]} \exp(-i k_- x) \gamma_+ + A_1^{[-]} \exp(i k_- x) \gamma_- + A_r^{[-]} \exp(-i k_+ x) \gamma_-,$$

while at $x \to \infty$ it is the transmitted waves,

$$C(+\infty) = A_1^{[+]} \exp(i k_+ x) \gamma_+ + A_r^{[+]} \exp(-i k_- x) \gamma_+ + A_1^{[-]} \exp(i k_- x) \gamma_- + A_r^{[-]} \exp(-i k_+ x) \gamma_-.$$

where the spinors $\gamma_+$ and $\gamma_-$ are the two eigenstates parallel and antiparallel to the direction of the effective spin-orbit field, $\gamma_{\pm} = 1/\sqrt{2} \left[ (1 + ir)/\sqrt{1 + r^2}, 1 \right]^T$, and $A_1$, $A_r$, and $A_2$ represent the amplitude of incident, reflected, and transmitted waves, respectively. Due to the SOC, the energy band is split into two subbands “+” and “−” one, corresponding to the spinors $\gamma_+$ and $\gamma_-$ respectively. Corresponding energies $E_+$ and $E_-$ can be expressed at the incidence momentum $k$ as:

$$E_\pm = \frac{k^2}{2} + \frac{\pi^2}{2w_0^2} + \alpha_D \sqrt{1 + r^2} k. \quad (4)$$

The spin splitting $\Delta E = 2 \alpha_D \sqrt{1 + r^2} k$ can be enlarged by increasing the tunable parameter $r$.

To obtain the wavefunction, we solve the Schrödinger equation by using Eq. (1) and total Hamiltonian. By integrating over $0 < z < w(x)$ to eliminate $z$-dependence, one can rewrite the Schrödinger equation as: $C'' + K_1 C' + K_2 C + S_1 C' + S_2 C = 0$, where the matrices are

$$K_1 = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}, \quad K_2 = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \quad S_1 = 2\alpha_D \begin{bmatrix} -i B w' & -i B + r \\ -i B - r & i B w' \end{bmatrix}. \quad (5)$$

For brevity, we use notations $w \equiv w(x)$, $w' \equiv w'(x)$, $w'' \equiv w''(x)$, $M \equiv M(x) = w'/w$, $G \equiv G(x) = 2E - \pi^2/w^2-M^2(1+2\pi^2)/4+w'/2w$, and $B \equiv B(x) = w_0^2/w^2$. The matrices $S_1$ and $S_2$ determine the spin evolution due to the coupling between the spin states and the shape of the waveguide.

In the constriction, as shown in Fig. 11 the ground state energy increases so that the classical motion or the tunneling, as determined by the spin-dependent electron energy can occur. The minimal energy in the constriction, expressed by following Eq. (5), is the threshold energy to open the classically propagating mode in it:

$$E_b \approx \frac{\pi^2}{2w_0^2(0)} - \frac{\alpha_D^2 r^2 + B^2(0)}{2}. \quad (6)$$

By tuning the Rashba coupling $\alpha_{RD}$, one can decrease $E_b$ from the value larger than $E_b$ to a smaller one, realizing transition from the classical motion to the tunneling. Figure 2a shows the dependence of the spin energies $E_+$, $E_-$, and the minimum of the energy band in the constriction $E_b$ on the controllable ratio $r$, provided by the same incidence momentum $k$. The crossing point of $E_-$ and $E_b$ separates the motion through the potential into the classical (left) and the tunneling (right from the point). This observation opens a possibility to manipulate electron transmission in the waveguide by a transverse electric field, solely modulating the SOC. The feasibility of this approach is seen in Figs. 2b-d, demonstrating the spin-dependent transmission $T_{b\sigma}$ versus $r$, where $\sigma'$ and $\delta$ represent the incidence and transmission channels, respectively, for different width $L$ of the boundaries of the constriction. As one can see in the Figure 2 not only the modulation in the width, but also the details of the shape are important. As a result, by choosing the shape of the waveguide, one can achieve its required spin transport properties.

To characterize the spin polarization in the transmission, we use the angle $\theta$ between the direction of spin at any $x$, and the one of the initial state at $x \to -\infty$: $\cos \theta = \langle \sigma(-\infty) \cdot (\sigma(x)) \rangle$, where $\sigma$ consists of the Pauli matrices, and $\langle \sigma_j(x) \rangle$, $j = x, y, z$ is the expectation value of spin component at position $x$. Suppose the incident electrons are in the given channels, spin “+” (parallel) or “−” (antiparallel) to the spin-orbit field, respectively, with energies in the form of Eq. (6), given by the same incidence momentum $k$. When the electron in any spin channel is propagating in the constant width region, the angle $\theta$ is zero as the direction of spin keeps unchanged. However, in the region where $w(x)$ starts to decrease, the direction of the spin-orbit field starts to alter due to the change in the Dresselhaus coupling. As a result, the spin begins to rotate around the axis determined by the local direction of the spin-orbit field. As the momenta for channel “+” and “−” in the constriction are different,
the $\theta$-angles experience different changes. As shown in Fig. 3, the spin angles for the incident electron with spin “+” and “−” begin to separate at the entrance ($x = −4$) and separate by approximately 0.1 at the exit.

In conclusion, we have investigated the spin transport in a waveguide with a narrowing, whose variation in the width plays an important role in charge and spin transmission. The transmission can be strongly modified by modulating the magnitude of the Rashba coupling and can be changed with waveguides oriented along different crystallographic directions. Changeable geometry will provide an alternative way to control the charge and spin transport. The produced structures can be applicable to manipulate charge transport by changing electric field across the electron propagation direction and are promising hardware elements for designing spin transport devices.

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Fig. 3. (Color online) Cosine of the spin angle $\theta$ as a function of $x$ coordinate, with the incidence energies $E_{+}$ (solid line, spin +) and $E_{−}$ (dashed line, spin −), given by $k = 4.35$ and $r = 0.8$. Other parameters are the same as those in Fig. 2(a).