Transport properties of Heavy Quarks: anisotropic flows $v_n$ and their correlations to the bulk dynamics and initial Electromagnetic field

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Abstract. We study the correlations between light flavor and heavy quarks (HQs) flow harmonics at LHC energy within a transport approach. We have investigated the role of transport coefficient in developing these anisotropic flows correlations. We suggest $v_{n}^{\text{heavy}} - v_{n}^{\text{light}}$ correlation and the relative fluctuations of anisotropic flows $\sigma_{v_{n}}/\langle v_{n} \rangle$ as novel observables to constrain the HQ transport coefficients in quark gluon plasma. Finally, very strong electromagnetic (E.M.) fields are created in Ultra-relativistic Heavy-Ion Collision (HIC). We show within relativistic Boltzmann transport approach coupled with E.M. field that the strong e.m. field is responsible for a splitting of directed flow $v_1$ of D and anti-D mesons of few percent, i.e. much larger compared to the observed charged particles. Moreover, we discuss the role played by the initial large bulk vorticity on the build up of rapidity odd HQs directed flow $v_1$.

1. Introduction

Heavy quarks (HQs), mainly charm and bottom quarks, are initially produced off-equilibrium in hard binary processes at early stages in ultra-Relativistic Heavy Ion Collisions (uRHIC). Their formation time is very small compared to the light quarks one, and due to their large masses they have a large thermalization time compared to the one of life time of the Quark-Gluon Plasma (QGP) phase. Hence before hadronizing and being detected through their final states like heavy meson and baryon (i.e. D mesons and Λc) they can probe both for the initial stages of uRHIC and the thermalized QGP evolution. The first observables studied in HQ sector were the heavy mesons nuclear suppression factor $R_{AA}$ and the elliptic flow $v_2(pT)$. Several theoretical efforts have been made to study both these observables to understand heavy quark dynamics in QGP [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. A comprehensive study of the various theoretical uncertainties arising from the heavy-quark initial conditions and hadronization was performed in [12].
The recent measurements of odd harmonics of heavy flavour can permit to probe the dynamics of relativistic heavy-ion collisions into a part that is transverse to the beam direction and another one that is longitudinal (parallel to the beam axis). The first part through the triangular flow $v_3$ measurement provide information about event-by-event fluctuations of the nucleon positions in the transverse plane. A more realistic modelling requires to take into account initial state fluctuations. Recent theoretical studies, including event-by-event fluctuations in the initial geometry in the transverse plane, have shown that the triangular flow $v_3(p_T)$ of D mesons is not vanishing (see [13, 14, 15]) and have the potential to provide further constraint on the interaction of the heavy quarks with the medium. The second part through the directed flow measurement can probes to the initial conditions of the initial system after the collision and providing information about its tilted profile in the reaction plane and the initial e.m. field. Within a relativistic Langevin approach was shown that the relative tilt between the bulk and HF distribution in the initial state leads to HF $v_1$ that results order of magnitude larger than the charged particle $v_1$ [16]. Finally, as recently recognized, very strong initial electro-magnetic (e.m.) fields are created in Ultra-relativistic Heavy-Ion Collision (HIC) that induce a vorticity in the reaction plane. Another source for a finite directed flow is the initial e.m. field. Recently, within a relativistic Langevin approach was shown that the strong initial e.m. field entails a transverse motion of HQs, resulting in a splitting of directed flow $v_1$ of neutral $D^0$ and $D^0$ mesons that is much larger compared to the observed light charged particles [17].

In this contribution we will discuss these aspects in the framework of a transport calculation.

2. Transport equation for charm quarks

The momentum evolution of the charm quark distribution function in QGP is obtained by solving the relativistic Boltzmann transport equations [18, 10]. The charm quarks interacts with a bulk medium of light quarks and gluons as described by the following eq.s

\begin{align}
\frac{d}{d\tau} f_i(x,p) &= C[f_q,f_g,f_Q](x,p) \\
\frac{d}{d\tau} f_j(x,p) &= C[f_q,f_g](x,p) 
\end{align}

(1)

where $f_i(x,p)$ is the on-shell phase space distribution function for the $i$ parton and $C[f_q,f_g,f_Q](x,p)$ is the relativistic Boltzmann-like collision integral. The the phase-space distribution function of the bulk medium consists of quarks and gluons entering the equation for charm quarks as an external quantities in $C[f_q,f_g,f_Q]$. This imply that the evolution of $f_q$ and $f_g$ are independent of $f_Q(x,p)$ and discard collisions between charm quarks which is by far a solid approximation. The evolution of the bulk of quarks and gluons is instead given by the solution of the other two transport equations where the $C[f_q,f_g]$ is tuned to a fixed $\eta/s(T)$, for details see [19, 20]. For the heavy quark bulk interaction we consider a quasi-particle model (QPM) [21]. This allows to describe the evolution of a system that dynamically has approximatively the iQCD equation of state [22]. In our approach the scattering matrix $M_{(q,g)\leftrightarrow Q\leftrightarrow (q,g)+Q}$ have been evaluated considering the leading-order diagram with the effective coupling $g(T)$ that leads to effective vertices and a dressed massive gluon propagator for $qQ \leftrightarrow gQ$ and massive quark propagator for $gQ \leftrightarrow gQ$ scatterings. For charm quark hadronization in to D mesons, we consider an hybrid approach of hadronization by coalescence plus fragmentation, for details about the hadronization model see our earlier work in Ref. [23].

3. Anisotropic flow $v_n(p_T)$ and heavy-light flavour correlations

In order to extend the analysis to high order harmonic flows $v_n(p_T)$ we need to include the initial state fluctuations. Recently, we have developed an event-by-event transport approach for the bulk in order to study the role of finite $\eta/s$ on the anisotropic flows $v_n(p_T)$ (see [24, 25]). We have used this event-by-event transport approach to extend our analysis to $v_n(p_T)$ of D.
meson anisotropic. In order to set the initial geometry the nucleons within the two nuclei have been distributed according to a Woods-Saxon distribution. The geometrical method is used to determine if the two nucleons are colliding, two nucleons collide if the relative distance in the transverse plane is $d_T \leq \sigma_{NN}/\pi$. A nucleon-nucleon cross section $\sigma_{NN} = 7.0 fm^2$ was employed in our calculations. The discrete distribution for the nucleons is converted into a smooth one by assuming for each nucleon a gaussian distribution centered in the nucleon position. In our calculation we have assumed initially a longitudinal boost invariant distribution from $y = -2.5$ to $y = 2.5$. For more details about the implementation see Ref.s [24, 25] Finally, we initialize the charm quark distribution in the coordinate space in accordance with the number of binary nucleon-nucleon collisions, $N_{coll}$, from the Monte-carlo Glauber model while for the momentum distribution we use charm quark production in Fixed Order + Next-to-Leading Log (FONLL) [26]. In recent years, the correlation between integrated anisotropic flows $v_n(p_T)$ for light hadrons with the initial asymmetry in coordinate space $\epsilon_n$ have been studied within event-by-event hydrodynamics and transport framework [27, 28, 24] The novelty of this contribution is the extension of these studies to the heavy quark sector where we study the correlations between charm quarks and light quarks. In Fig. 1, we show the two-dimensional plots of the integrated flow coefficients $v^\text{light}_n$ for light quarks as a function of the corresponding final integrated flow coefficients $v^\text{heavy}_n$ for heavy quarks. The results are for $Pb+Pb$ collisions at $\sqrt{s_{NN}} = 5.02 TeV$ for two centralities. The viscosity has been fixed to $4\pi\eta/s = 1$ plus a kinetic f.o. realized by the increase in $\eta/s(T)$ for more details see [24, 25]. As shown in the upper panel we observe a strong linear correlation between $v^\text{heavy}_2$ and $v^\text{light}_2$ from central to peripheral collisions while for higher harmonics we observe a reduction of the linear correlation in comparison with the second harmonic. A measure of the linear correlation is given by the correlation coefficient $C(n, m)$:

$$C(n, m) = \frac{\sum_i (v^\text{light}_{n,i} - \langle v^\text{light}_n \rangle)(v^\text{heavy}_{m,i} - \langle v^\text{heavy}_m \rangle)}{\sqrt{\sum_i (v^\text{light}_{n,i} - \langle v^\text{light}_n \rangle)^2 \sum_i (v^\text{heavy}_{m,i} - \langle v^\text{heavy}_m \rangle)^2}}$$

where $v^\text{light}_{n,i}$ and $v^\text{heavy}_{m,i}$ are the values of anisotropic flows corresponding to the event $i$ and respectively for light and heavy quarks.

In this paper we will address the impact of the temperature dependence of the interaction in
Figure 2. Left panel: correlation coefficient between heavy quarks $v_n$ and bulk $v_n$ as a function of the order of the harmonic $n$ obtained within QPM and pQCD. Right panel: $\sigma_{v_n}/\langle v_n \rangle$ as a function of the order of harmonics $n$. The black circles refer to the case with an interaction from QPM model while the red squares to the case corresponding to a pQCD interaction. The orange dashed lines indicate the value $\sqrt{4/\pi} - 1$ expected for a 2D Gaussian distribution. These results are for $Pb + Pb$ at $\sqrt{s_{NN}} = 5.02$ TeV.

particular the T dependence of the drag coefficient on both correlation and $v_n$ distribution. For this scope we are considering two different models having different T dependent drag coefficients: one within the framework of pQCD with constant $\alpha_s = 0.4$ and another one with an interaction coming from the QPM model. For the pQCD interaction we rescale the interaction in order to reproduce the same $R_{AA}(pT)$ obtained in the QPM. This leads to a weakly T dependent drag coefficient for QPM at variance with pQCD with a constant $\alpha_s$ which predict a $T^2$ dependence similar to AdS/CFT approach. As shown the specific T dependence of the drag can strongly modify the soft-hard anisotropic flows correlations, even if the models are tuned to reproduced the same experimental D meson $R_{AA}(p_T)$. In the left panel of Fig. 2 we show the correlation coefficient $C(n, n)$ as a function of the order of the harmonic $n$. As shown, the correlation coefficient decrease with respect to the order of harmonics for both pQCD and QPM and the correlation is stronger for QPM than pQCD for all the harmonics considered. Comparison of the theoretical results with the upcoming experimental results will help to constrain heavy quark transport coefficients and disentangle difference energy loss model. Interesting properties of heavy quarks $v_n$ distributions can be inferred by studying the relative fluctuations $\sigma_{v_n}/\langle v_n \rangle$ where $\sigma_{v_n}$ are the standard deviation for $v_n$. The $\sigma_{v_n}/\langle v_n \rangle$ is an increasing function with the order of the harmonics. As shown in the right panel of Fig. 2 $\sigma_{v_n}/\langle v_n \rangle$ are sensitive observables to the temperature dependence of the transport coefficients for $n \geq 3$.

4. Heavy flavour directed flow $v_1$

There are two sources for a finite directed flow of HQs: i) the initial large vorticity ii) the initial strong Electromagnetic field produced in a HIC. In a recent paper was shown within the relativistic Langevin approach that a tilted initial distribution in the reaction plane, produce a finite $v_1$ of D meson several times larger than that of charged particle [16]. On the other hand, in another recent paper was shown within the relativistic Langevin approach coupled with Lorentz force that a sizeable $v_1$ for charm (anti-charm) quarks is produced and it is odd respect to the charge [17]. We have studied these two aspects within the relativistic Boltzmann transport approach. In order to take in to account the effect of the e.m. field on the propagation of HQs we have solved relativistic Boltzmann equation coupled to the e.m. field.

$$\left[p^\mu \partial_\mu + q F_{\mu \nu}(x)p^\nu \partial_p^\mu\right] f_Q(x, p) = C[f_q, f_g, f_Q](x, p)$$
The time evolution of E.M. field produce in a HIC is calculated by solving Maxwell equations for a single charge and then they are folded with the nuclear transverse density $\rho(x_\perp, \phi)$ and summed over forward ($\eta$) and backward ($-\eta$) rapidity [30]. In our simulation we have assumed constant electric conductivity of the QGP with $\sigma_{el} = 0.023 f m^{-1}$ which is the predicted value by lattice QCD (lQCD) around $T \approx 2T_c$ [31]. In this calculation we neglect the bulk modification due to E.M. currents. In the left panel of Fig.3 is shown the time evolution for the E.M. field. Due to the collision geometry and choosing by convention the impact parameter $b$ along the $x$-axis, the generated magnetic field $\vec{B}$ is dominated by the $y$-component this results in a Lorentz force that acts on the expanding medium along the direction orthogonal to $\vec{B}$ in the $xz$ plane. On the other hand, time variation of $\vec{B}$ induces an electric field $\vec{E}$, whose dominant component is $E_x$, and results in a Faraday current which drifts charged particles in the $xz$ plane. The net combination of the two effects leads to the formation of a finite direct flow $v_1 = \langle p_x/p_T \rangle$.

In realistic simulations for RHIC energies we distribute charm quarks in momentum space according to Fixed Order + Next-to-Leading-Order (FONLL) $pp$-spectra [32]. For bulk partons we employ thermal distribution plus minijet tail at high $p_T$ while in coordinate space we provide initial conditions through standard Glauber model with a slight modification. The initial conditions are a modification of longitudinal boost invariant to take into account that the initial longitudinal energy density profile is no longer symmetric respect to $\eta \rightarrow -\eta$. This come from the ansatz that a participant preferably deposits entropy along its direction of motion and results in an initial tilt of the fireball in the reaction plane. These initial condition have been implemented by using the same parametrization in ref. [33]. The initial tilted fireball develops a rapidity-odd directed flow $v_1$ of charged particles and the parameters that set the initial tilted distribution have been fixed in order to reproduce the experimental data for the $v_1$ of charged particles. In Fig. 3 we show our predictions for the $v_1$ under the combined effect of the initial tilt and e.m. field. Since the Lorentz force acts in opposite directions for oppositely charged particles, the $v_1$ from the e.m. field is expected to generate a non-zero split in the $v_1$ of $D$ and $\bar{D}$ mesons. On the other hand, the tilt mechanism can generate only finite $v_1$ without splitting. As shown we predict a large $v_1$ consistent with the recent experimental data and splitting between particle and anti-particle of about 1%. Notice that these results correspond to an angular momentum of the fireball which is consistent to the value simulated in viscous hydrodynamics calculations [34]. A systematic study of both effect for different e.m. field configuration and initial tilted distribution will be discussed in a forthcoming paper.

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\left[ p^\mu_j \partial_\mu + qF_{\mu\nu}(x)p^\nu \partial_\mu p \right] f_j(x,p) = C[f_q, f_g](x, p) \quad j = q, g
\]
5. conclusion
We studied the dynamics of HQs in the QGP within an event-by-event Boltzmann transport approach. We have studied the integrated heavy-light $v_2^{HF} - v_2^{LF}$ correlation event-by-event and the relative fluctuations of anisotropic flows $\sigma_{v_n}/\langle v_n \rangle$. In this study we show that the linear correlation coefficient $C(n,n)$ is a decreasing function with the order of the harmonic and we observe a strong correlation for the second and third harmonic. Moreover, we have studied the role of QCD interaction in developing correlations between the light and the heavy flavour anisotropic flows. This study shows how the temperature dependence of the heavy quarks - bulk interaction affect the heavy-light event by event $v_n^{light} - v_n^{heavy}$ correlations and the $\sigma_{v_n}/\langle v_n \rangle$. The present study suggests $C(n,n)$ and $\sigma_{v_n}/\langle v_n \rangle$ are very sensitive observables to temperature dependence of transport coefficients. Finally, we have studied the $v_1$ of $D$ and $\bar{D}$ mesons driven by the soft tilted bulk and by the large EM fields produced in these collisions. We find in agreement with other calculation performed within Langevin approach that the dominant contribution to the $v_1$ is due to the initial tilted distribution for the bulk which results in large charm flow. The effect of the EM field is to generate a split in the $v_1$, which is of the order of 1%. Within our approach we are able to describe the recent STAR measurement.

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