Eliminating the LIGO bounds on primordial black hole dark matter

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Primordial black holes (PBHs) in the mass range (30–100) $M_\odot$ are interesting candidates for dark matter, as they sit in a narrow window between microlensing and cosmic microwave background constraints. There are however tight constraints from the binary merger rate observed by the LIGO and Virgo experiments. In deriving these constraints, PBHs were treated as point Schwarzschild masses, while the more careful analysis in an expanding universe we present here, leads to a time-dependent mass. This implies a stricter set of conditions for a black hole binary to form and means that black holes coalesce much more quickly than was previously calculated, namely well before the LIGO/Virgo’s observed mergers. The observed binaries are those coalescing within galactic halos, with a merger rate consistent with data. This reopens the possibility for dark matter in the form of LIGO-mass PBHs.

Introduction—Primordial black holes (PBHs) [1] are one of the oldest and arguably least speculative candidates for the elusive dark matter [2–4]. They attracted a renewed interest recently [5–7] after the discovery by the LIGO/Virgo Collaboration [8] of the mergers of black hole binaries in the mass range $\sim (10–50) M_\odot$. Remarkably, the merger rate of virialized PBH binaries in this mass window is consistent with the rate inferred from the LIGO/Virgo observations [5]. Moreover, black holes with $(30–100) M_\odot$ masses sit in a narrow window between stringent microlensing [9–11] and cosmic microwave background [12, 13] constraints on the PBH dark matter fraction. While there do exist some constraints that overlap this mass range — e.g. from dynamical effects on stellar systems [14–20], or from PBH accretion at early [21–23] and late [24–26] times — these often come with loop-"stellar systems; a problem with a long history dating back to the works of McVittie [42] and Einstein and Straus [43]. For a recent review see, e.g. Ref. [44] and references therein.

Here we focus on the constraints discussed in Refs. [6, 31, 32] which seem to disprove much more convincingly the claim that LIGO/Virgo has observed PBH binaries merging. While the merger rate of virialized PBHs is consistent with the LIGO/Virgo observations [5], it was shown [33, 34] that PBH binaries would be produced at a much higher rate in the early universe. Recent estimates of the binary production rate [6, 31] imply that LIGO/Virgo should observe many more mergers than it does if PBHs constitute a large fraction of the dark matter. These calculations suggest that the PBH dark matter fraction must be $f_{\text{PBH}} \lesssim 10^{-3}$ in the LIGO-mass range.1 Up to date summaries of constraints on PBH dark matter can be found in, for example, Refs. [40, 41].

In this work, we readdress these constraints by questioning the validity of the commonly used notion of the black-hole mass, as well as the condition for a binary to decouple from the Hubble flow. Fundamentally these issues are related to the issue of how cosmological expansion influences local gravitating systems; a problem with a long history dating back to the works of McVittie [42] and Einstein and Straus [43]. For a recent review see, e.g. Ref. [44] and references therein.

Indeed, the concept of mass in General Relativity is not universal but rather depends on the background geometry. The Arnowitt-Deser-Misner (ADM) mass [45] for vacuum black holes used in Refs. [6, 31, 33] is strictly applicable only if spacetimes are asymptotically flat. For example, the ADM mass approximation is good when the typical size or time scale under consideration is much smaller than the scale at which the curvature of the background geometry becomes significant, like the case of stellar black holes. However, PBHs are produced from the gravitational collapse of inhomogeneities at scales comparable to the Hubble radius: a scenario in which the ADM mass approximation is not applicable. Consequently, binary dynamics should be affected by the cosmological expansion, leading to a dramatically different merger rate than has been reported previously [6, 31, 33].

Our analysis is thus based on the description of black holes in a flat expanding Friedmann-Lemaître-Robertson-Walker (FLRW) universe using a spacetime metric first proposed by Thakurta [46]. In terms of the “physical” radial coordinate $R = a(t)r$ and angular coordinates $\theta, \phi$, it can be written as:

$$
\text{d}s^2 = f(R) \left(1 - \frac{H^2 R^2}{f^2(R)}\right) \text{d}t^2 + \frac{2HR}{f(R)} \text{d}t \text{d}R + R^2 \left(\text{d}\theta^2 + \sin^2 \theta \text{d}\phi^2 \right),
$$

(1)

where $f(R) = 1 - 2Gm(a(t))/R = 1 - 2Gm/r$ and $m$ is the physical mass of the PBHs today. The Hubble expansion

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1 Clustering of PBHs may loosen these constraints [35, 36]. In addition, while this paper was in preparation, several e-prints [37–39] appeared on the arXiv, which claim the constraints from the merger rate can be fully relaxed if PBH clustering takes place. This conclusion is complementary to ours but requires additional assumptions. Here we make no assumption about clustering, which is a more conservative option for our argument.
rate reads \( H = \dot{a}/a \), with \( \dot{a} \) standing for the derivative of the scale factor \( a(t) \) with respect to cosmological time \( t \), normalized to \( a(t_0) = 1 \) at the present time \( t_0 \). In accordance with physical intuition, at scales much smaller than the cosmological horizon, \( (2Gma \lesssim 1/H, \text{Eq.}(1)) \) approaches the static Schwarzschild spacetime, while at scales larger than the Schwarzschild radius \( 2Gma \ll R (\lesssim 1/H) \), the metric approaches the FLRW.

The Thakurta metric stands out among other similar proposals [44] since it is the late time attractor solution of the entire class of generalized McVittie geometries [47]. These geometries are sourced by an imperfect fluid, with a radial energy flow which causes black holes to have a time-dependent “Misner-Sharp” mass — as opposed to a static mass. This observation is central to the results of this paper.

In contrast, the standard McVittie geometries do not have a radial energy flow. For realistic FLRW backgrounds, this leads to a spacelike singularity instead of the usual black hole horizon. In the perfect fluid which sources these geometries, a spacelike singularity corresponds to a divergent pressure. While there exist other analytic solutions for black holes embedded in FLRW spacetimes, these tend to be plagued with other physical problems such as negative energy densities [44]. This is why the Thakurta metric is now considered the more adequate description of a non-rotating black hole in an expanding universe.

Armed with a more realistic description of cosmological black holes, it is pertinent to consider recalculating the dynamics of their binaries. We detail these calculations in the following sections, but present our final result in Fig. 1. We show the required coalescence time, \( t_{\text{b}} \), that a binary must have if it decouples from the Hubble flow at some redshift \( z_{\text{dec}} \). This is a dynamical requirement that has not been considered previously because it is a direct consequence of the new description of cosmological black holes that we adopt here. The physical origin behind this dramatic change in the dynamics of decoupling arises from the aforementioned feature of Thakurta black holes: namely, that they should have growing Misner-Sharp masses.

The reason why this new result eliminates the LIGO/Virgo constraints on PBHs can be seen by comparing the gulf between the required coalescence times (blue lines) and the observability window for LIGO mass black hole mergers (red shaded region). This window encloses the coalescence time that a binary decoupling at \( z_{\text{dec}} \) would need to have to be observable by LIGO/Virgo. This window is bounded from below by the requirement that the binary has not merged prior to the highest redshift merger (the very distant GW170729 at \( z \sim 0.49 \) [48, 49]), but also not so long that the binary still has not merged by the present day. The required coalescence times derived in this work fall short of this window by many orders of magnitude, right up until well past \( z \sim 1 \). Beyond this point, galaxies will have formed, and the PBHs will be virialized in their halos. The merger rate in this environment is then controlled by an entirely different set of conditions [5]. This demonstrates that these early-universe PBH binaries would not be able to contribute to the merger rate or stochastic background [50, 51] observed by LIGO/Virgo. Consequently, the bounds set on the PBH dark matter fraction based on this population of binaries can be completely evaded, thus re-opening the possibility that PBHs in the LIGO-mass range can indeed make up 100% of dark matter.

**PBHs in an expanding universe**—To begin, we define the notion of a black-hole mass. For spherically symmetric cosmological black holes, instead of the ADM mass, it is more appropriate to define the quasi-local Misner-Sharp mass [52], which is a measure of active mass in a given volume of the spacetime defined by Eq.(1). This mass can be read-off from the 00-component of the metric:

\[ m_{\text{MS}} = ma(t) + \frac{H^2 R^3}{2Gf(R)}. \]  

(2)

It is worth noting that, although we used the Thakurta metric to compute the Misner-Sharp mass, it is known to be invariant under coordinate re-parametrization [53] and thus is a physically meaningful quantity. After PBHs are produced during the radiation dominated era, \( H \sim 1/a^2 \) and \( R \sim a \), so while the first term in Eq.(2) increases \( \propto a(t) \), the second term decreases \( \propto 1/a(t) \). Therefore we can ignore the latter and simply describe the mass of a PBH as being equal to \( ma(t) \).

Next, let us inspect the radial geodesic motion of a test

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**FIG. 1.** PBH binary coalescence times, as calculated in this work. For a given value of eccentricity, \( e_{\text{dec}} \) (equally spaced between 0 and 1), the coalescence time for binaries decoupling at redshift \( z_{\text{dec}} \) is given by the respective blue line. These coalescence times are what are required for black holes embedded in expanding spacetime to emit enough energy in the form of gravitational waves to remain decoupled from the Hubble flow which tries to pull them apart. In contrast, the coalescence times needed to be observed by LIGO/Virgo are enclosed by the red region. The binary needs to have merged between the farthest lookback time of the LIGO/Virgo (around \( z \sim 0.49 \) and today) All binaries decoupling prior to galaxy formation will have merged well before they could be observed by LIGO/Virgo.
mass in the spacetime of Eq.(1). In the relevant approximation \( Gma \ll R \ll 1/H \) the geodesic equation takes the following form (see also Ref. [54]):

\[
\ddot{R} = -\frac{Gma}{R^2} + \frac{\dot{a}}{a} R. \tag{3}
\]

The first term on the right hand side of this equation is the standard attractive Newtonian force exerted by the cosmological black hole of “active” mass \( ma \equiv m/(1+z) \). The second term in Eq.(3) is entirely due to the cosmological expansion and has a simple heuristic interpretation: it is a cosmological drag force (per unit mass) acting on a test particle with no peculiar motion. Ensuring that the Newtonian force dominates over the cosmological force, we obtain our first “static” decoupling condition:

\[
\frac{ma}{V} \gg \frac{3}{4\pi G} \left| \frac{\dot{a}}{a} \right| = \rho_{\text{ct}} \left[ -\Omega_m(1+z)^3 - 2\Omega_r(1+z)^4 + 2\Omega_\Lambda \right], \tag{4}
\]

where, \( V = (4\pi/3)R^3 \) and \( \Omega_{m,r,\Lambda} = \rho_{m,r,\Lambda}/\rho_{\text{ct}} \) are respectively the cosmological densities of matter, radiation and a cosmological constant relative to the critical density, \( \rho_{\text{ct}} \). We assume \( \Omega_m \approx 0.307, \Omega_\Lambda \approx 0.691 \) and \( \Omega_r \approx 5.4 \times 10^{-5} \) [55].

The decoupling condition, Eq.(4), is very similar to the one used previously [6, 31, 33, 34], but differs in one important aspect: it involves the approximate time-dependent Misner-Sharp mass, \( ma \equiv m/(1+z) \), instead of the static ADM mass \( m \). This has important ramifications for the decoupling condition of binaries. Since the Misner-Sharp mass grows with the cosmological expansion, it allows for binary decoupling at much later times compared to previous calculations, even allowing for decoupling later than matter-radiation equality.

This modification to the decoupling condition alone would already imply a weakening of the LIGO-mass PBH dark matter bounds. However, on closer inspection it turns out that the Misner-Sharp mass assumption allows for another decoupling condition to be written down. As we will see, this condition allows for the bounds to be weakened further.

The argument is as follows: on the one hand, a binary system loses its energy through the emission of gravitational waves; resulting in orbital decay and eventually a merger. On the other hand, cosmological expansion tends to push black holes apart. Hence for a decoupled binary to merge, orbital decay due to the emission of gravitational waves must dominate over the pull of the expansion of the universe. Using the active Misner-Sharp mass, \( ma(t) \), a system of two cosmological black holes separated by a radial distance \( R \) carries the total energy \( E = -GM\mu a^2/(2R) \) (in the Newtonian circular orbit approximation; for elliptical orbits \( R \) refers to the semi-major axis), where \( M = m_1 + m_2 \) and \( \mu = m_1 m_2/M \), are respectively the present-day total and reduced masses of the binary system. Hence, the change in radial separation of black holes is given by \( \ddot{R}/R = -\dot{E}/E + 2H \). Requiring that the binary be coalescing gives us a new “dynamical” condition for decoupling:

\[
\dot{E}/E > 2H = 2H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}, \tag{5}
\]

where \( H_0 = \sqrt{8\pi G \rho_{\text{ct}}}/3 \). At leading order, \( \dot{E} \) is given by the suitably modified quadrupole formulae, which will be discussed in the next section.

**PBH binary formation in the early universe**—Now that we have a new condition that must be met for a black hole binary to decouple and merge, we must consider what this implies for a population of PBHs forming in the early universe.

We will assume a random (as opposed to clustered) initial distribution and a monochromatic mass function at \( m \). Both of these are probably unrealistic simplifications, but in fact they turn out to be the more conservative options for our argument here. A population of PBHs constituting a fraction \( \rho_{PBH} = f_{PBH}\rho_{DM} \) of the cosmological dark matter density, would have an average separation today (at \( t_0 \)) of \(^2\)

\[
\tau_0 = \left( \frac{m}{f_{PBH}\rho_{\text{ct}}\Omega_{DM}} \right)^{1/3} \approx 1.2 \text{ kpc} \left( \frac{m}{30 M_\odot} \right)^{1/3}. \tag{6}
\]

We then consider a pair of neighbouring PBHs separated by a distance \( x = x_0/(1+z) \) at redshift \( z \), where \( x_0 \) is the separation at \( t_0 \). The third nearest PBH to the pair would help them to form an elliptical binary system through tidal effects, with semi-major axis \( a \) and semi-minor axis \( b \), given respectively by:

\[
a = \alpha \frac{x_0}{1+z}, \quad b = \beta \left( \frac{x_0}{y_0} \right)^3 a, \tag{7}
\]

where \( y = y_0/(1+z) \) is the distance to the third nearest black hole. Numerical 3-body calculations (for a static mass) indicate that \( \alpha \approx 0.4 \) and \( \beta \approx 0.8 \) [34] are suitable corrections for the semi-analytic approximation here.

The binary is considered to be formed when the system decouples from the Hubble flow at some \( z_{\text{dec}} \) and evolves according to local dynamics only, independent of the cosmological expansion. This is when both conditions, Eqs.(4) and (5), are satisfied. After decoupling, the binary system with the semi-major axis \( a_{\text{dec}} = a(z_{\text{dec}}) \) and eccentricity \( e_{\text{dec}} = e(z_{\text{dec}}) \) coalesces with a lifetime given by the usual result,

\[
\tau_b = \frac{3}{85} \frac{a_{\text{dec}}^4(1-e_{\text{dec}}^2)^{7/2}}{r_s^3}, \tag{8}
\]

after which the PBHs merge [56]. Here for convenience we define the binary Schwarzschild scale \( r_s = (G^3 M^2 \mu)^{1/3} \).

\(^2\) Note that this quantity in the literature is usually defined at the epoch of matter-radiation equality (\( z = z_{eq} \approx 3000 \)), \( \tau = \tau_0/(1+z_{eq}) \), since PBH binary decoupling in the matter dominated era was considered unfeasible.
Note that the cosmological evolution of binaries we describe here is only relevant before galaxy formation, so certainly not later than \( z \sim 1 \). At later times PBHs are expected to be bound within galactic halos, so that binaries start to form according to the scenario described in Ref. [5]. The cosmic time elapsed from \( z \sim 1 \) until a typical LIGO/Virgo binary merger at \( z \sim 0.1 \) is \( \sim 6.6 \) Gyr. Hence, the binaries formed over the course of cosmological evolution must live long enough to contribute to the merger rate observed by LIGO/Virgo, i.e., \( \tau_b \gtrsim 6.6 \) Gyr.

**Conditions for binary decoupling**—The final step is to apply the conditions written down previously to the population of PBH binaries we have just described. For the second “dynamical” condition Eq. (5) — which is more restrictive than the static condition Eq. (4) — we need to recalculate the binary energy loss following Ref. [56], accounting for the quasi-local Misner-Sharp mass and the Hubble expansion rate. In the leading quadrupole approximation we can ignore some sub-leading terms proportional to the expansion rate and its time derivatives to find,

\[
\dot{E} = -\frac{32}{5} \frac{G^4 M_1^2 \mu^2 a^5}{a^5(1-e^2)^{7/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right). \tag{9}
\]

Hence, Eq. (5) implies for binaries decoupling prior to virialization:

\[
(1+z_{\text{dec}})^3 \frac{H(z_{\text{dec}})}{H(z_{\text{eq}})} < \frac{96}{425} \left( 1 + \frac{73}{24} \frac{e_{\text{dec}}^2}{e_{\text{eq}}^2} + \frac{37}{96} \frac{e_{\text{dec}}^4}{e_{\text{eq}}^4} \right). \tag{10}
\]

The relationship between \( \tau_b \) and \( z_{\text{dec}} \) is shown in Fig. 1. Remarkably, there is no explicit dependence on the PBH mass; hence the assumption of a monochromatic mass function may not be that restrictive. Taking the example of \( e_{\text{dec}} = 1 \) (which gives the longest coalescence times) we find that the dynamical constraint implies rather short-lived binaries, for example,

\[
\tau_b < 135 \text{ s} \quad (z_{\text{dec}} = z_{\text{eq}} \approx 3000), \tag{11}
\]

for binaries that decoupled in the radiation era, and

\[
\tau_b < 1.0 \text{ Gyr} \quad (z_{\text{dec}} = 1), \tag{12}
\]

for binaries that decoupled in the era of matter domination before PBHs are virialized in galactic halos.

For PBH binaries decoupling at some \( z_{\text{dec}} \), the coalescence time, shown by the blue lines in Fig. 1, would need to coincide with the red observability region. We thus conclude that the binaries that decouple in the radiation- and most of the matter-dominated eras merge well before the present day. This renders these early-decoupling PBHs unobservable to LIGO or Virgo. Therefore the previously calculated bounds on the abundance of \( (30–100) \ M_\odot \) PBH dark matter [6, 31] no longer apply.

**Conclusion** —By revisiting the PBH merger rate to account for a more adequate description of black holes in their surrounding spacetime, we have found that the merger rate constraints on the abundance of dark matter PBHs are evaded entirely. The reason for this is that PBH binaries which are able to form before galaxy formation must have had small initial separations and so would have merged well before the present time. The remaining PBHs — which represent the vast majority of the population — will not form binaries until galaxy formation under an alternative set of conditions.

To arrive at this conclusion we adopted the quasi-local Misner-Sharp mass in place of the usually assumed ADM mass, and reconsidered the conditions for a black hole binary to decouple from the Hubble flow. The two conditions, Eqs. (4) and (5) — labeled “static” and “dynamical” respectively — are stronger than previously considered, implying a substantially lower binary merger rate in the window of cosmic time covered by LIGO/Virgo’s observed mergers. This revitalizes the possibility that \( (30–100) \ M_\odot \) mass PBHs could constitute all of the dark matter. The leftover PBHs that have not yet decoupled by \( z \sim 1 \) will have been virialized in galactic halos. The merger rate for these PBHs will then follow the calculation presented in Ref. [5], which is intriguingly consistent with LIGO/VIRGO’s observed merger rate.

The implications of this work may extend beyond the LIGO/Virgo merger rate bounds, and could also invalidate numerous other constraints on PBH dark matter. Following essentially the same arguments, constraints on PBH dark matter from the stochastic gravitational-wave background [50, 51] should similarly be invalidated. Additionally, the non-detection of subsolar-mass black hole mergers places strong constraints on the fraction of dark matter in subsolar-mass PBHs [57, 58]. Since our final result is free of any mass dependence, it is safe to presume that these constraints will also be evaded.

The next step to decisively open up the possibility for dark matter in the form of LIGO-mass PBHs, is to address the remaining astrophysical bounds [15–18, 21, 22, 24–26]. Astrophysical uncertainties aside [13, 15, 27, 28], many of these constraints will also be affected by a time-varying mass. In particular, since a black hole’s accretion rate is proportional to its mass, early-universe constraints on PBH accretion could be weakened substantially. This may ultimately show that \( (30–100) \ M_\odot \) PBHs can constitute a large fraction, or possibly even the totality, of the dark matter in the Universe. We will address this issue in a follow-up study.

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