D=10 Dirichlet super–9–brane.

Vladimir Akulov, Igor Bandos\textsuperscript{1}, Wolfgang Kummer\textsuperscript{2} and Vladimir Zima\textsuperscript{3}

\textsuperscript{1} Institute for Theoretical Physics, NSC Kharkov Institute of Physics and Technology, 310108, Kharkov, Ukraine 
e-mail: bandos@kipt.kharkov.ua, bandos@tph32.tuwien.ac.at

\textsuperscript{2} Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien 
e-mail: wkummer@tph.tuwien.ac.at

\textsuperscript{3} Kharkov State University, 310077, Kharkov, Ukraine

e-mail: wkummer@tph.tuwien.ac.at

Abstract

Superfield equations of motion for $D = 10$ type \textit{IIB} Dirichlet super-9-brane are obtained from the generalized action principle.

The geometric equations containing fermionic superembedding equations and constraints on the generalized field strength of Abelian gauge field are separated from the proper dynamical equations and are found to contain these dynamical equations among their consequences.

The set of superfield equations thus obtained involves a $Spin(1,9)$ group valued superfield $h_{\alpha}^{\beta}$ whose leading component appears in the recently obtained simplified expression for the $\kappa$--symmetry projector of the D9-brane. The Cayley image of this superfield coincides (on the mass shell) with the field strength tensor of the world volume gauge field characteristic for the Dirichlet brane.

The superfield description of the super-9-brane obtained in this manner is known to be, on the one hand, the nonlinear (Born-Infeld) generalization of supersymmetric Yang–Mills theory and, on the other hand, the theory of partial spontaneous breaking of $D = 10$ N = \textit{II}B supersymmetry down to $D = 10$ N = 1.

PACS: 11.15-q, 11.17+y

\* Supported in part by the INTAS Grants 96-308, 93-127-ext, 93-493-ext and by the Austrian Science Foundation under the Project P–10221
1 Introduction

The $D = 10$ type $II$ superbranes (Dirichlet superbranes or super-Dp-branes, where $p = d - 1$ is a number of space-like dimensions of the brane world volume) [1]–[4] at present are the object of attention due to exceptional role they play in understanding of string dualities, in particular in the approach related to the Matrix model [3].

Covariant and explicitly $\kappa$–symmetric actions for the super-Dp-branes were obtained recently [5, 6, 9, 8] and used intensively for studying brane intersections [11, 13] as well as for an approach to quantize superbranes [12]. The progress in the latter becomes possible because, as found in [8], the super-Dp-brane admits the covariant gauge fixing conditions for $\kappa$-symmetry [1]. This property was used in [11, 12, 13] to revise the $\kappa$-symmetry description and to present it in a more simple way opening the possibility for many applications.

Even before the covariant action have been constructed, the equations of motion for super-Dp-branes written in terms of world volume superfields had been obtained [16] (in the linearized approximation) in the frame of superembedding approach.

This approach was elaborated for $D = 10$ superstrings and $D = 11$ supermembrane (super-M2-brane) in [17, 18] in the course of development of the doubly supersymmetric geometric approach. The latter can be regarded as the supersymmetric generalization of the classical surface theory of the 19th century (see e.g. [19]) and describes superbranes in terms of extrinsic geometry of the world volume superspace of the brane embedded into the target superspace.

The number of fermionic ‘directions’ of the world volume superspace are considered to be half the fermionic coordinates of target superspace. In such a case the local world volume supersymmetry replaces the $\kappa$–symmetry of ordinary Green–Schwarz formulation, and can be reduced to the kappa symmetry when the auxiliary fields are excluded [20]–[27].

The two approaches to the super-Dp-branes were united in [10], where the generalized action principle [18] has been applied to the generic case of super-D-p-branes, and the general form of the superfield equations (including embedding equations and proper dynamical equations in superfield form) has been obtained from the generalized action functional.

The power of the superembedding approach can be seen in the history of the effective description of the $M$-theory 5-brane [28] as well. Again first the superfield equations had been obtained in Ref. [16] before the covariant action was found in Refs. [29, 30].

Subsequently, the equivalence of the equations of motion following from the superembedding superfield equations with the ones obtained from the covariant action [29, 30] was proved in [31]. Since then both the covariant action and superembedding equations have found quite a number of applications [32, 33, 34, 35].

The superembedding equations for the generic case of super-D-p-branes has been studied in a linearized approximation in Ref. [4] (see also a brief description of results for super-D3-brane in [14]). As noted in [4, 36] (see also [37]), the basic superembedding equation (the so–called

These means that, e.g. super-D0-branes are an analog of massive superparticle with extended supersymmetry [13]. It was known that such superparticles has $\kappa$–symmetry when the central charge $Z$ (it carries due to the presence of some 1-dimensional Wess–Zumino term in the action) coincides with the particle mass $m$. The possibility of covariant gauge fixing for $m \neq 0$ was proved and extensively used in [12].
geometrodynamical condition) is not enough to produce all the dynamical equations of super-p-branes with higher values of \( p \) completely. The constraints for the world volume gauge superfield have to be introduced as additional geometric equations for these cases.

In distinction to the 'classical' superembedding approach based on the geometrodynamical equation only, the generalized action approach \[18\] developed for the case of super-Dp-branes in \[10\] produces all the equations (superembedding ones, gauge field 'constraints' as well as the proper dynamical equations) in superfield form. Then the whole set of equations may be split into the geometrical and proper dynamical part and their interrelations may be studied.

Here we realize such program for the case of \( D = 10 \) type II super-D9-brane \[2\].

It should be noted that the super-9-brane in any case requires a separate study as there are no bosonic directions orthogonal to the world volume and thus the geometrodynamical equation \[20, 23, 25, 27\], being the basis of the superembedding approach in its generic form \[17, 4, 16, 39, 40\], becomes trivial. And thus the problem of superembedding description of super-9-brane remained open till now.

Moreover, the 9–brane assumes a special role in the D-branes scan. With covariant gauge fixing for the \( \kappa \)–symmetry it reduces to supersymmetric generalization of the \( D = 10 \) Born-Infeld model and, as Wess-Zumino terms of all super-Dp-branes vanish in the covariant gauge, all the gauge fixed super-Dp-brane actions can be obtained from the super-D9-brane one by world–volume dimensional reduction \[8\].

On the other hand, this theory can be regarded as one of the embedding of \( D = 10, N = 1 \) superspace into the \( D = 10, N = 2, \) type II super-space. Hence the super-9-brane is the model of partial spontaneous breaking of \( D = 10 \) \( N = 2 \) supersymmetry \[13\].

The partial supersymmetry breaking in \( D = 4, 6 \) attracted much attention recently \[11, 12\]. The general theorem about the impossibility of such partial breaking were overcome for the first time in ref. \[13\], where it was demonstrated that it does not hold in the presence of brane solitons in supersymmetric field theory. It was then recognized that any super-p-brane model describes in particular the partial supersymmetry breaking (see e.g. \[14\]). An approach to superfield description of superbranes based on the partial supersymmetry breaking concept was proposed and elaborated for \( D = 2 \) superparticle and \( D = 4 \) superstring in \[45\].

In \[12\] partial supersymmetry breaking in \( D = 4 \) was studied in the 'classical' framework of nonlinear realizations \[16\]. Different multiplets were found there to be of use for description of Goldstone fermions of the partial supersymmetry breaking and their superpartners.

From our point of view (see also \[13\]) such a multiplicity originates in the fact that all the possibilities presented in \[12\] must be related to different compactifications of super-9-brane model down to \( D = 4, 6 \) dimensions with rigid breaking of the corresponding number of both linearly and nonlinearly realized supersymmetries.

Thus the superfield equations describing the super–9–brane can be regarded as a kind of 'master' model of partial supersymmetry breaking \[1\].

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2 While this paper was being written an alternative way to obtain the complete set of geometric equations has been proposed \[38\]. It consists in the consideration of the fundamental strings whose ends lie in the world volume superspace of D-branes.

3 Recently, the relations between superembedding approach and nonlinear realization one was studied in \[37\].
These superfield equations are obtained here from generalized action principle for $D = 10$ type IIB super-D9-brane [10].

The set of geometric equations are extracted and separated from and the proper dynamical ones. These geometric equations contain fermionic superembedding equation and constraints on the generalized field strength of the abelian world volume gauge superfield.

We investigate their integrability conditions and find that the geometric equations contain all the dynamical equations among their consequences. This is just the situation described in [36, 37]. The presence of gauge field constraints among the necessary geometric equations creates certain problems for the use of superembedding approach without a dynamical basis. However, as we demonstrate, these problems disappear when the superembedding is based on the generalized action [18, 19].

The superfield equations obtained from the generalized action involve a Spin$(1,9)$ group valued superfield $h_{\alpha}^{\beta}$ whose leading component appears in the recently obtained simplified expression for the kappa–symmetry projector of D9-brane [11, 12, 13].

This superfield can be regarded as a ‘nonlinear square root’ of the field strength of the world volume gauge (super)field characteristic for the Dirichlet brane. More precisely, this field strength coincides with Cayley image of the $SO(1,9)$ valued matrix $k$ corresponding to the Spin$(1,9)$ valued spin–tensor $h$ (on the mass shell).

Thus we obtain the superfield description of the super-9-brane which is known to be, on the one hand, the nonlinear (Born-Infeld) generalization of supersymmetric Yang–Mills theory and, on the other hand, the theory of partial spontaneous breaking of $D = 10$ $N = 2$, type IIB supersymmetry down to $D = 10$ $N = 1$.

The paper is organized as follows.

We conclude this section by description of our basic notations and conventions.

In Section 2 we describe the generalized action functional [14] for the super-9-brane in flat $D = 10$, type IIB superspace. The peculiar features of the super-9-brane, allowing to write the generalized action without use of Lorentz harmonic variables, are considered here as well as in Appendix A.

Section 3 is devoted to the variation of the generalized action. After a preliminary discussion how to extract the information about equations of motion and symmetries from the external derivative of the generalized action, in Subsection 3.1. we reproduce the calculation of the external derivative of the Lagrangian form Ref. [14]. We then use it in Subsection 3.3 to recall the justification of the (superfield) $\kappa$–symmetry. In Subsection 3.2 it serves for the derivation of the general form of superfield equations of motion in terms of superforms. All this material can be regarded as a specialization for the super-9-brane case of the consideration presented in [10] for generic case of super-Dp-branes. This is necessary for the next sections as, in particular, it provides us with the expressions for basic variations of different pieces of Lagrangian form and on the example of $D = 7$ super-5-brane.
with the identity for the \( \kappa \)-symmetry projector matrix \( \bar{\Gamma} \), being of extremely importance for the following.

In section 4 we justify the antidiagonal form of the \( \kappa \)-symmetry projector \( \bar{\Gamma} \) in the completely Lorentz covariant way and prove that the \( 16 \times 16 \) matrix \( h^\alpha_\beta \) (denoted by \( e^a \) in \([11, 12, 13]\)), determining completely this projector, takes its values in the \( Spin(1,9) \) group. In the frame of ‘standard’ component formulation \([5, 6, 7, 8, 9]\) this result was obtained by use of the special gauge \([11]\) and it found wide applications \([12, 13]\). We, however find it instructive to present the completely covariant way of the proof based on the identity for \( \bar{\Gamma} \) \([10]\) referred to in the previous section.

As a bonus, we get in such a way a covariant form of the relation between the Lorentz group valued spin–tensor field \( h \) and the auxiliary antisymmetric tensor field \( F_{ab} \). The latter coincides with the components of the generalized field strength \( \mathcal{F} \) of the world volume 1–form gauge field \( A \) on the mass shell. We obtain a set of useful identities between \( h^\alpha_\beta \) and \( F_{ab} \) fields as well as between their derivatives which are used in the next sections.

In Section 5 we rewrite the external derivative of the Lagrangian form using the \( h^\alpha_\beta \) spin-tensor and obtain a simple form of fermionic (super)field equations.

The set of essential superfield equations is analyzed in Section 6. Splitting the set of these equations of motion into geometrical and proper dynamical ones, we prove that the geometric equations contain dynamical ones among their consequences. This is done by studying of their integrability conditions as it was done in \([15]\) for \( D = 10 \) superstrings and the \( D = 11 \) supermembrane (M-theory 2-brane).

As we obtain all the dynamical equations of super-9-brane from geometrical ones, we have found the minimal set of superfield equations describing the super–9–brane model and, hence, the nonlinear generalization of \( D = 10 \) supersymmetric Yang–Mills (SYM) theory (i.e. \( D = 10 \) Goldstone SYM multiplet which describes the partial spontaneous breaking of \( D = 10 \) type IIB supersymmetry).

To clarify the multiplet structure, in Section 6 we study the geometric equations in the linearized approximation. A relation between the main field strength \( W^\alpha \) of \( N = 1 \) SYM multiplet and the Goldstone fermion superfield of partially breaking type IIB supersymmetry is noted here.

In Conclusion we discuss briefly the obtained results. The interdependence of the Bianchi identities and the integrability conditions for the fermionic superembedding equations is studied in the Appendix B.

1.1 Basic notations and conventions

Our notations are close to the ones of \([14]\) up to normalization and choice of the metric signature (mostly minus in our article).
The coordinates of flat target superspace are denoted by

$$Z^M = (X^m, \Theta^\mu) \equiv (X^m, \Theta^1\mu, \Theta^2\mu),$$

$$m = 0, 1, \ldots, 9, \quad \mu = 1, \ldots, 16, \quad \hat{\mu} = (I, \mu), \quad I = 1, 2,$$

$E^A$ is the supervielbein 1-form of the flat $D = 10$ type IIB superspace

$$E^A = (E^a, \hat{E}^\hat{a}) \equiv (E^a, E^{1\alpha}, E^{2\alpha})$$

$$a = 0, 1, \ldots, 9, \quad \alpha = 1, \ldots, 16, \quad \hat{\alpha} = (I, \alpha), I = 1, 2.$$

Here $E^a$ is the bosonic vielbein

$$E^a \equiv \Pi^m \delta^a_m$$

$$\Pi^m = dX^m - id\Theta^1\sigma^m\Theta^1 - id\Theta^2\sigma^m\Theta^2 \equiv dX^m - id\hat{\Theta} \begin{pmatrix} \sigma^m & 0 \\ 0 & \sigma^m \end{pmatrix} \hat{\Theta} =$$

$$\equiv dX^m - id\hat{\Theta}^\mu (I \otimes \sigma^m) \hat{\Theta}^\hat{\mu},$$

and

$$E^{\hat{a}} = d\hat{\Theta}^\hat{\mu} \delta^{\hat{a}}_{\hat{\mu}}, \quad \Leftrightarrow \quad E^{I\alpha} = d\hat{\Theta}^{I\mu} \delta^\alpha_{\mu}, \quad I = 1, 2$$

$$d\hat{\Theta}^\hat{\mu} = (d\Theta^1\mu, d\Theta^2\mu)$$

are fermionic (Grassmann) vielbein forms.

Then the expressions of the torsion forms ('torsion constraints') of the flat $D = 10$, type IIB tangent superspace are

$$T^a \equiv \mathcal{D}E^a = dE^a = -i(E^{1\alpha} \wedge E^{1\beta} + E^{2\alpha} \wedge E^{2\beta})\sigma^a_{\alpha\beta} \equiv$$

$$\equiv -iE^\hat{a} \wedge E^{\hat{b}} (I \otimes \sigma^a)_{\hat{a}\hat{b}} = -iE^\hat{a} \wedge E^{\hat{b}} \begin{pmatrix} \sigma^a_{\alpha\beta} & 0 \\ 0 & \sigma^a_{\alpha\beta} \end{pmatrix},$$

$$T^\hat{a} \equiv \mathcal{D}E^{\hat{a}} = dE^{\hat{a}} = 0.$$

The target superspace supervielbein $E^A$ (8) \(8\), (9), being coincident with the natural supervielbein of flat $D = 10$, type IIB superspace $\Pi^M \equiv (\Pi^m, d\Theta^\mu)$, can be used for the construction of the generalized action only for the super-D9-brane.

In the general case of super-Dp-branes with $p < 9$ the Lorentz harmonic variables (see (8), (9) and refs. therein) shall be included into Eqs. (8), (9) instead of Kronecker symbols. They are necessary to adapt the target space frame to the bosonic world volume (see (8), (9)):

The pull backs of $D - p - 1$ vielbein forms $E^i$ entering the set of $D = 10$ bosonic vielbein forms $E^a = (E^\hat{a}, E^i)$ must vanish on the mass shell, while the pull backs of the remaining $(p + 1)$ forms $E^{\hat{a}}$ will give rise to the set of linearly independent forms which can be used as a vielbein forms of world volume superspace. Such an adaptation appears dynamically as a result of variation with respect to the harmonic variables (8), (9).

For the 9–brane case where there are no bosonic directions orthogonal to the world volume we do not need harmonics to adapt the bosonic frame and this is why the generalized action can be written without harmonics at all.
However we keep the separate notation for the flat target space supervielbein to obtain some simplification of the equations as well as to make a connection clear between our equations and ones from Refs. [10, 4, 16, 31], where the branes in curved supergravity background were considered.

For the same reason, we keep the covariant derivative symbol $\mathcal{D}$ as well in all those places where it should appear in the curved $D = 10$ type IIB superspace and/or for the lower dimensional branes, although for the super–9–brane in the flat $D = 10$ type IIB superspace all the induced connections [17] are trivial and vanish for the natural choice of the supervielbein fixed by (2), (3), (5). Hence $\mathcal{D} = d$.

The basic volume form written in terms of the vielbeine is denoted by

$$E^\wedge 10 \equiv \frac{1}{(10)!} \epsilon_{a_1...a_{10}} E^{a_1} \wedge ... \wedge E^{a_{10}}$$

$$\equiv \frac{1}{(10)!} \epsilon_{m_1...m_{10}} \Pi^{m_1} \wedge ... \wedge \Pi^{m_{10}}.$$  

The 'standard' bosonic 8–form and 9–form are normalized as

$$E^\wedge 8_{ab} \equiv \frac{1}{2 \cdot 8!} \epsilon_{a_1...a_8} E^{a_1} \wedge ... \wedge E^{a_8},$$

$$E^\wedge 9_a \equiv \frac{1}{9!} \epsilon_{a_1...a_9} E^{a_1} \wedge ... \wedge E^{a_9}.$$  

The list of products includes following useful identities

$$E^\wedge 9_a \wedge E^b = -\delta^b_a E^\wedge 10,$$

$$E^\wedge 8_{ab} \wedge E^c = -\delta^c_{[a} E^\wedge 9_{b]}.$$  

The world–volume superspace of the super-D9-brane and its local coordinates are denoted by

$$\Sigma^{(10|16)} = \{z^M\} = \{(\xi^m, \eta^\mu = \eta^\mu(\xi^m))\}, \quad m = 0, 1, ..., 9, \quad \mu = 1, ..., 16$$ (8)

and the intrinsic supervielbein forms on the world volume are

$$e^A = (e^a, e^\alpha) \equiv dz^M e^A_M, \quad a = 0, 1, ..., 9, \quad \alpha = 1, ..., 16.$$ (9)

The pull–backs of target space supervielbein forms onto the world volume superspace are denoted by the same symbols $E^a, E^1\alpha, E^2\alpha$.

2 Generalized action functional for $D = 10$ type IIB super-D9-brane

The super-D-p-brane generalized action [10] for the super-9-brane in $D = 10$ type IIB superspace acquires the form

$$S = \int_{\mathcal{M}^{10}} \mathcal{L} = \int_{\mathcal{M}^{10}} (\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_{WZ})$$ (10)
where

\[ \mathcal{L}_0 = E^{\wedge 10} \sqrt{-\det(\eta_{ab} + F_{ab})}, \]  

(11)

\[ \mathcal{L}_1 = Q_8 \wedge (dA - B_2 - \frac{1}{2} E^a \wedge E^b F_{ba}) \]  

(12)

and the Wess-Zumino Lagrangian form is the same the one as appearing in the by now standard formulation \[5, 6, 7, 8, 9\]

\[ \mathcal{L}_{WZ} = e^F \wedge C|_{10}, \quad C = \bigoplus_{n=0}^5 C_{2n}, \quad e^F = \bigoplus_{n=0}^5 \frac{1}{n!} F^{\wedge n} \]  

(13)

where the formal sum of the RR superforms \( C = C_0 + C_2 + \ldots \) and of the powers of two form \( F \) is used and \( |_{10} \) means the restriction to the 10–superform input \[5, 7, 9\].

The two form \( F \) is

\[ F = dA - B_2 \]  

(14)

where \( B_2 \) is the NS-NS two-form (super-)field whose field strength is just the external derivative of the Wess–Zumino term of the type IIB Green-Schwarz superstring

\[ H_3 \equiv dB_2 = iE^a \wedge (E^{1\alpha} \wedge E^{1\beta} - E^{2\alpha} \wedge E^{2\beta})(\sigma_a)_{\alpha\beta} \]  

(15)

\[ \equiv iE^a \wedge E^{\hat{\alpha}} \wedge E^{\hat{\beta}} (\sigma_3 \otimes \sigma_a)_{\hat{\alpha}\hat{\beta}} \equiv iE^a \wedge E^{\hat{\alpha}} \wedge E^{\hat{\beta}} \begin{pmatrix} 0 & 0 \\ 0 & -(\sigma_a)_{\alpha\beta} \end{pmatrix}. \]

The 'vacuum' values of the RR superform gauge fields \( C_{2n} \) is known to be nonvanishing for the case when super-Dp-branes are present. These 'vacuum' values are expressed in terms of target superspace coordinates \( X \) and \( \Theta \) only. However, explicit expressions (see \[17\]) are rather complicated. Fortunately we really need the curvatures of RR superfields only, whose 'vacuum' values are much simpler. It is convenient to write them using the formal sum notations as well \[5\]–\[9\]

\[ R = e^{-F} \wedge d(e^F \wedge C) = \bigoplus_{n=0}^5 R_{2n+1}, \]  

(16)

\[ R_{2n+1} = \frac{i}{(2n+1)!} E^{a_{2n+1}} \wedge \ldots \wedge E^{a_1} \wedge E^{\hat{\alpha}} \wedge E^{\hat{\beta}} \begin{pmatrix} 0 & (\sigma_{a_1\ldots a_{2n+1}})_{\alpha\beta} \\ (-1)^n (\sigma_{a_1\ldots a_{2n+1}})_{\beta\alpha} & 0 \end{pmatrix} \]  

(17)

where the symmetry properties of \( D = 10 \) sigma matrices are used to obtain the second line.

\( F_{ab} \) as included into Eqs. \[11\], \[12\] is an auxiliary antisymmetric tensor superfield. The Lagrange multiplier term \[12\] provides the identification of the (pure bosonic) 2-form

\[ F \equiv \frac{1}{2} E^b \wedge E^a F_{ab} \]  

(18)

being constructed from this auxiliary field and the pull–backs of the target space bosonic vielbeine \[5\] with the generalized field strength \[13\] of the world volume abelian gauge superfield \( A = dz^M A_M(z) \)

\[ \frac{\delta S}{\delta Q_8} = 0 \quad \Rightarrow F \equiv \frac{1}{2} E^b \wedge E^a F_{ab} = F \equiv dA - B_2. \]  

(19)
The integration in (10) is performed over an arbitrary bosonic surface

\[ \mathcal{M} = (\xi^m, \eta^\mu(\xi^m)) \equiv z^M(\xi^m) \]

in the world volume superspace \( \mathcal{N} \).

Henceforth, the coordinate fields entering (10) shall be regarded as world volume superfields taken on the ten dimensional bosonic surface \( X^m = X^m(\xi^m, \eta^\mu(\xi^m)) \)

\[ \Theta^1\mu = \Theta^1\mu(\xi^m, \eta^\mu(\xi^m)), \quad \Theta^2\mu = \Theta^2\mu(\xi^m, \eta^\mu(\xi^m)) \quad (20) \]

\[ A_M = A_M(\xi^m, \eta^\mu(\xi^m)), \quad A = dz^M A_M = d\xi^m(A_m + \partial_m \eta^\mu A_\mu), \]

\[ F_{ab} = F_{ab}(\xi^m, \eta^\mu(\xi^m)), \]

\[ Q_8 = \frac{1}{8!} dz^M s_8(\xi^m) \wedge \ldots \wedge dz^M_i Q_{M_1 \ldots M_8}(\xi^m, \eta^\mu(\xi^m)). \]

Thus the generalized action (10) can be treated as one included additional 16 fermionic fields \( \eta^\mu(\xi^m) \) in a quite nonlinear manner.

To arrive at the equations of motion one varies with respect to these fermionic fields (i.e. with respect to \( M \)) on the same footing as with respect to \( X \) and \( \Theta \).

Further consideration of the properties of the generalized action can be found in [18] for the superbrane case and (in much more detail) in [48] for the case of supergravity 4.

The key points to obtain the superfield equations are

- It can be proved [18] that, for the functional of the type (10) (i.e. written in terms of differential forms without the use of the Hodge operation) the variation with respect to the surface does not produce independent equations, i.e. the corresponding equations are satisfied identically after the 'field' equations, appearing as a result of variations with respect to proper filed variables \( X, \Theta \ldots \) are taken into account (see [18, 18] for details).

In this sense the generalized action is independent of the integration surface \( \mathcal{M} \) and thus possesses a superdiffeomorphism invariance (see [18]).

- As a result of the arbitrariness of the surface \( \mathcal{M} \) and of the independence of the generalized action on this surface, all the remaining equations, appearing as a result of the generalized action variation with respect to 'proper' field variables (\( X, \Theta \) etc.), can be treated as superfield equations, i.e. the equations for the world volume superfields and superforms

\[ X^m = X^m(\xi^m, \eta^\mu), \quad \Theta^1\mu = \Theta^1\mu(\xi^m, \eta^\mu), \quad \Theta^2\mu = \Theta^2\mu(\xi^m, \eta^\mu) \quad (21) \]

\[ A = dz^M A_M(\xi, \eta), \quad F_{ab} = F_{ab}(\xi^m, \eta^\mu), \]

\[ Q_8 = \frac{1}{8!} dz^M s_8(\xi^m) \wedge \ldots \wedge dz^M_i Q_{M_1 \ldots M_8}(\xi^m, \eta^\mu), \]

which are not restricted to any surface [18].

4In supergravity this approach is known under the names 'group manifold' or 'rheonomic' one. Really it is much more complicated than our ('rheotropic' [18]) approach to superbranes, as the basic objects for supergravity actions are curvature two forms instead of supervielbeine in the superbrane case.
• Assuming the surface \( M \) to be the pure bosonic world volume \( M_0 \)

\[
M = M_0 = (\xi^m, \eta^\mu = 0)
\]

and, thus, reducing all the superfields to their leading components

\[
X^m = X^m(\xi^m, 0), \quad \Theta^{1\mu} = \Theta^{1\mu}(\xi^m, 0) \quad \Theta^{2\mu} = \Theta^{2\mu}(\xi^m, 0),
\]

(22)

we get a component formulation of super-D9-brane. Such a formulation is equivalent to the standard (Dirac–Born–Infeld–like) one \([5]–[9]\) (see \([10]\) for the proof in generic case of super-Dp-branes) and possesses the \( \kappa \)-symmetry, \textit{but in the irreducible form}.

• For the generalized action the transformations giving rise to the \( \kappa \)-symmetry on the component level (superfield \( \kappa \)-symmetry) can be regarded as target space or fiber representation of the superdiffeomorphysm invariance (see \([18]\) and refs. therein).

3 Variation of the generalized action.

As the action is written in terms of differential forms, one can extract the variation from the external derivative of the Lagrangian form using

\[
\delta \mathcal{L} = i_\delta d\mathcal{L} + d(i_\delta \mathcal{L}).
\]

(23)

Then the last term can be dropped for the closed brane.

The application of (24) to variations which are not related to superdiffeomorphisms requires care regarding the definition of the contractions of forms. E.g. to vary the Wess–Zumino term with respect to the \( A_M \) superfield (being involved into \( \delta \mathcal{L} \) through its generalized field strength \( \mathcal{F} \equiv dA - B_2, \quad A = dz^M A_M \) ) we have to define

\[
i_\delta \mathcal{F} = \delta A, \quad i_\delta C = 0, \quad i_\delta A = 0, \quad i_\delta E^A = 0.
\]

(24)

In this way we get, e.g. (cf. with eq. (26) below)

\[
\delta \mathcal{L}_{WZ} = e^{\mathcal{F}} \wedge R|_9 \wedge \delta A = i E^{\hat{\alpha}} \wedge E^{\hat{\beta}} \wedge (\hat{\gamma})_{\hat{\alpha}\hat{\beta}} \wedge e^{\mathcal{F}}|_9 \wedge \delta A
\]

Another important variation is the one for the fermionic coordinate fields \( \Theta^{\hat{\alpha}} = (\Theta^{1\alpha}, \Theta^{2\alpha}) \). It can be obtained from (23) by supposing

\[
i_\delta \mathcal{F} = \delta A - i_\delta B_2 = 0, \quad i_\delta C = 0, \quad i_\delta A = 0, \quad i_\delta E^a = 0
\]

(25)

\[
i_\delta E^{\hat{\alpha}} = \delta \Theta^{\hat{\alpha}}.
\]

10
3.1 External derivative of Lagrangian 10-form.

Here we review the calculation of the external derivative of the Lagrangian form \[10\].

Using (16), (17) the derivative of the Wess–Zumino term (13) becomes

\[ dL_{WZ} = eF \wedge R \big|^{11} \equiv iE^{\hat{\alpha}} \wedge E^{\hat{\beta}} \wedge (\hat{\gamma})_{\hat{\alpha}\hat{\beta}} \wedge eF \] \tag{26}

with

\[ (\hat{\gamma})_{\hat{\alpha}\hat{\beta}} \equiv \bigoplus_{n=0}^{4} \left( \begin{array}{cc} 0 & (-1)^n \hat{\sigma}^{(2n+1)} \\ \hat{\sigma}^{(2n+1)} & 0 \end{array} \right) \equiv \bigoplus_{n=0}^{4} (\sigma_1 \sigma_3)^n \otimes \hat{\sigma}^{(2n+1)} \] \tag{27}

\[ \hat{\sigma}^{(2n+1)} \equiv \frac{1}{(2n+1)!} E^{a_1} \wedge \ldots \wedge E^{a_{2n+1}} \sigma_{a_1 \ldots a_{2n+1}}. \] \tag{28}

It is convenient to write (26) as

\[ dL_{WZ} = iE^{\hat{\alpha}} \wedge E^{\hat{\beta}} \wedge (\hat{\gamma})_{\hat{\alpha}\hat{\beta}} \wedge eF + (F - F) \wedge S, \] \tag{29}

where \( S \) is defined by

\[ (F - F) \wedge S \equiv (eF - eF) \wedge R = (eF - eF) \wedge iE^{\hat{\alpha}} \wedge E^{\hat{\beta}} \wedge (\hat{\gamma})_{\hat{\alpha}\hat{\beta}}, \] \tag{30}

or, formally,

\[ S = \frac{eF - eF}{F - F} \wedge iE^{\hat{\alpha}} \wedge E^{\hat{\beta}} \wedge (\hat{\gamma})_{\hat{\alpha}\hat{\beta}}. \] \tag{31}

Here and below the restriction to the 11-form input in the formal sums is assumed but not written explicitly.

The derivative of the 'kinetic term'

\[ dL_0 = E^{\wedge 9}_a \wedge D E^a \sqrt{-\det(\eta_{ab} + F_{ab})} + E^{\wedge 10} \wedge d \sqrt{-\det(\eta_{ab} + F_{ab})} \] \tag{32}

can be transformed by algebraic manipulations into \[10\]

\[ dL_0 = iE^{\wedge 9}_a \wedge E^{\hat{\alpha}} \wedge E^{\hat{\beta}} (I \otimes \sigma_b)_{\hat{\alpha}\hat{\beta}} \left( (\eta + \sigma_3 F)^{-1} ba \otimes I \right)^{\hat{\gamma}} \sqrt{-\det(\eta_{ab} + F_{ab})} - E^{\wedge 8}_{ab} (\eta + F)^{-1} ab \wedge d(F - F) \sqrt{-\det(\eta_{ab} + F_{ab})}. \] \tag{33}

Then, using the property \( \Gamma^2 = I \) of the \( \kappa \)-symmetry projector matrix \[3\] - \[7\]

\[ \Gamma_{\hat{\alpha}}^{\hat{\beta}} = \frac{1}{\sqrt{-\det(\eta + F)}} \sum_{n=0}^{5} \frac{(-1)^n}{2^{n-n!} F_{a_1 b_1} \ldots F_{a_n b_n}} ((\sigma_3)^n \otimes \sigma^{a_1 b_1 \ldots a_n b_n}) \cdot (\sigma_1 \otimes I) \] \tag{34}

(rewritten in our case in terms of the auxiliary tensor) and the identity \[10\] \[9\]

\[ \sigma^{a_1 b_1 \ldots a_n b_n} \sigma^c = \sigma^{a_1 b_1 \ldots a_n b_n c} + 2 n \sigma^{a_1 b_1 \ldots a_n b_n c}. \]
we can represent (33) as
\[ dL_0 = i E^\alpha \wedge E^\beta \wedge (\tilde{\Gamma}^\gamma)_{\alpha\beta} \wedge e^F - 
\]
\[ -E^{ab}_{\tilde{\alpha}\tilde{\beta}} (\eta + F)^{-1} ab \wedge d(F - F) \sqrt{-\det(\eta_{ab} + F_{ab})}. \]

The first term in (36) coincides with one of Eq. (29) up to the \( \tilde{\Gamma} \). Thus we get
\[ dL_0 + dL_{WZ} = i E^\alpha \wedge E^\beta \wedge ((1 + \tilde{\Gamma})^\gamma)_{\alpha\beta} \wedge e^F - 
\]
\[ -E^{ab}_{\tilde{\alpha}\tilde{\beta}} (\eta + F)^{-1} ab \wedge d(F - F) \sqrt{-\det(\eta_{ab} + F_{ab})} + (F - F) \wedge S. \]

The external derivative of the \( L_1 \) becomes
\[ dL_1 = dQ_8 \wedge (F - F) + Q_8 \wedge d(F - F) \equiv 
\]
\[ \equiv dQ_8 \wedge (F - F) - Q_8 \wedge \left( H_3 + E^b \wedge T^a F_{ab} + \frac{1}{2} E^b \wedge E^a \wedge D F_{ab} \right). \]

Collecting Eqs. (37) and (38) we get
\[ dL \equiv dL_0 + dL_1 + dL_{WZ} = 
\]
\[ = i E^\alpha \wedge E^\beta \wedge ((1 + \tilde{\Gamma})^\gamma)_{\alpha\beta} \wedge e^F - 
\]
\[ (dQ_8 + S) \wedge (dA - B_2 - \frac{1}{2} E^b \wedge E^a F_{ab}) 
\]
\[ - \left( Q_8 - E^{ab}_{\tilde{\alpha}\tilde{\beta}} (\eta + F)^{-1} ab \sqrt{-\det(\eta + F)} \right) \wedge \left( H_3 + E^b \wedge T^a F_{ab} + \frac{1}{2} E^b \wedge E^a \wedge D F_{ab} \right). \]

3.2 Bosonic superfield equations following from the generalized action

To obtain the bosonic superfield equations of motion we consider contractions of (39) with a variation symbol obeying
\[ i_{\delta} E^\alpha = 0. \]

Keeping the only nonvanishing contraction in (21) to be \( i_{\delta} dQ_8 \equiv \delta Q_8 \), we get (cf. Eq. (19))
\[ \frac{\delta S}{\delta Q_8} \equiv \frac{i_{\delta} dL}{\delta Q_8} = 0 \implies F \equiv \frac{1}{2} E^b \wedge E^a F_{ab} = F \equiv dA - B_2. \]

If the only nonvanishing contraction is \( i_{\delta} D F_{ab} = \delta F_{ab} \) we obtain from (39) the expression for the Lagrange multiplier \( Q_8 \)
\[ Q_8 = E^{ab}_{\tilde{\alpha}\tilde{\beta}} (\eta + F)^{-1} ab \sqrt{-\det(\eta + F)}. \]

The equation corresponding to the variation of the world volume gauge superfield appears for the choice of only nonvanishing contraction to be \( i_{\delta} dA = \delta A \) (cf. (24)).
Then the only contribution comes from the second line in (39) producing
\[ \frac{\delta S}{\delta A} \equiv \frac{i_\delta dL}{i_\delta F} = 0 \quad \Rightarrow \quad dQ_8 = S, \]
where the 9-superform \( S \) is defined by (30). Substituting (41) into (42) we arrive at the supersymmetric generalization of the Born-Infeld equation
\[ d(E^8_{ab} (\eta + F)^{-1} e^{ab}) = \frac{e^F - e^F}{F - F} \wedge i E^\alpha \wedge E^\beta \wedge (\tilde{\gamma})_{\alpha\beta} |_{F=F}, \]
where the formal way of writing the contribution from the WZ term (31) and Eq. (40) are used.

Only the first line in (39) provides inputs into other variations after Eqs. (42), (40), (41) are taken into account.

So, supposing the only nonvanishing contraction to be \( \delta X^m \equiv \delta E^a \delta^m_a \), we obtain
\[ \frac{\delta S}{\delta X^m} = 0 \quad \Rightarrow \quad E^\alpha \wedge E^\beta \wedge i E^a \left[ \left( (1 + \tilde{\Gamma}) \tilde{\gamma} \right)_{\alpha\beta} \wedge e^F \right] = 0, \]
which is satisfied identically due to the fermionic equations of motion to be considered in detail in the next section.

Such a dependence of the equations obtained by variation of the \( X^m \) variables is really the Noether identity reflecting the (bosonic) general coordinate invariance of the generalized action. This bosonic general coordinate invariance together with the superfield generalization of the \( \kappa \)–symmetry form the superdiffeomorphism invariance of the generalized action.

### 3.3 On fermionic equations and \( \kappa \)–symmetry of super–9–brane

The fermionic equations
\[ \frac{\delta S}{\delta \Theta^\mu} \equiv \frac{i_\delta dL}{i_\delta E^\mu} = 0, \quad \Rightarrow \quad E^\beta \wedge ((1 + \tilde{\Gamma}) \tilde{\gamma} (\alpha\beta) \wedge e^F = 0 \]
appear for \( i_\delta E^a = 0, i_\delta E^a = \delta \tilde{\Theta}^a \neq 0 \) from the contraction of first line of Eq. (39).

Before turning to a more careful investigation of them let us note that they as well as the first line of (39) itself, can be used to prove the \( \kappa \)–symmetry of the super-D9-brane actions at the component level, as done in [10] for the generic case of super-Dp-brane.

The \( \kappa \)–symmetry is provided by the fact that the projector is present in the fermionic equations (43), and, thus only 16 of 32 fermionic forms \( E^\alpha = (E^{1a}, E^{2a}) \) are involved into the derivative of the Lagrangian form as well as in the fermionic equations.

This means that only half of them are independent. This is just the Noether identity reflecting the fermionic gauge symmetry of the action. When the integration manifold \( \mathcal{M} \) is chosen to be a pure bosonic world volume \( \mathcal{M}_0 \) (i.e. \( \eta^a = 0 \)), this fermionic gauge symmetry is just the \( \kappa \)–symmetry of the component (Lorentz harmonic) formulation. For the generalized action such a symmetry also holds and can be regarded as a tangent space representation of the superdiffeomorphism invariance (see [18] and refs. therein).
4 \( \Gamma \) and Lorentz group valued spin-tensor variables \( h_{\alpha}{}^{\beta} \).

The fermionic superfield equation (45) are complicated. Thus it is extremely important to find variables which provide a significant simplification of (45).

We demonstrate here that such variables really exist and are related to the \( \kappa \)-symmetry projector matrix \( \bar{\Gamma} \).

As first emphasized in [11] and used intensively in [12, 13], the 32 \( \times \) 32 projector matrix of the \( \kappa \)-symmetry transformations \( \bar{\Gamma} \) (34) has the block–antidiagonal form

\[
\bar{\Gamma}_{\hat{\alpha}}{}^{\hat{\beta}} = - \left( \begin{array}{cc}
0 & h_{\alpha}{}^{\beta} \\
h^{-1}_{\alpha}{}^{\beta} & 0
\end{array} \right).
\]

(46)

Here we present a new covariant derivation of this (for the 9-brane case, although the same can be done for the general case of any value of \( p \)) as well as of the fact that the matrix \( h \) takes its values in the fundamental representation of Spin(1, 9) group.

Our constructive proof provides as a bonus a covariant expression for \( h \) in terms of anti-symmetric tensor \( F^{ab} \). These results will be extremely important in the study of the superfield equations of super-9-brane.

4.1 Antidiagonal form of \( \bar{\Gamma} \), Lorentz group valuedness of \( h \) superfield and its relation with \( F^{ab} \)

This fact can be proved easily. Indeed, in accordance with (35), multiplication by \( \bar{\Gamma} \) transforms the diagonal matrix 9-form

\[
E_9 \wedge \left( (\eta + \sigma_3 F)^{-1} \right)_{ba} \otimes \sigma_b \right) = E_9 \wedge \left( \sigma_b (\eta + F)^{-1} {0} \right)_{ba} \begin{pmatrix} 0 \\ \sigma_b (\eta - F)^{-1} {0} \end{pmatrix}
\]

into \( \hat{\gamma} \wedge e^F \), the (block–)off–diagonal one (27). This forces the \( \bar{\Gamma} \) to have off-diagonal. Then the requirement \( \bar{\Gamma}^2 = 1 \) results in the condition for off–diagonal blocks to be inverse matrices. Thus one gets (46).

From the identity (35) for the \( \bar{\Gamma} \) matrix (46) we find

\[
\sqrt{...E_9 \wedge \left( (h \sigma_b)_{\alpha}{}^{\beta} (\eta - F)^{-1} \right)_{ba} = e^F \wedge \otimes^4 \left( \sigma_{\alpha}{}^{2n+1} \right)_{\beta} | g, (47)
\]

\( ^{\text{To prove the } \kappa \text{-symmetry we really need only in the identity (35) the part which is symmetric in the spinor indices. But also the statement above can be proved even from the symmetric part only. Indeed, e.g. for the 1-st left 16 \( \times \) 16 block } A_{\alpha}{}^{\beta} \text{ of the } \Gamma \text{ matrix we then get } (A \sigma_b)_{\alpha}{}^{\beta} (\eta + F)^{-1} \right)_{ba} = 0 \). Decomposing the \( A \) matrix into the complete basis (256 = 1 + 45 + 210)

\[
A_{\alpha}{}^{\beta} = A_0 \delta_{\alpha}{}^{\beta} + A^{ab} (\sigma_{ab})_{\alpha}{}^{\beta} + A^{abcd} (\sigma_{abcd})_{\alpha}{}^{\beta}
\]

and using the gamma matrix algebra we find from (\( A \sigma_b)_{\alpha}{}^{\beta} = 0 \) that all the irreducible parts of the \( A \) matrix vanish

\[
A_0 = 0, \\
A^{ab} = 0, \\
A^{abcd} = 0,
\]

and, hence, \( A_{\alpha}{}^{\beta} = 0 \). Nevertheless, these details are not necessary as (35) is true in general (i.e. not only for its symmetric part), see footnote 5.
\[
\sqrt{\ldots E_a^\Lambda (h^{-1} \sigma_b)_{\alpha \beta} (\eta + F)^{-1}}_{\alpha \beta} ^{\alpha \beta} = e^F \wedge \bigoplus_{n=0}^{4} \hat{\sigma}_{\alpha \beta}^{2n+1} |_9 ,
\]

where (cf. (28))
\[
e^F \wedge \bigoplus_{n=0}^{4} \hat{\sigma}_{\alpha \beta}^{2n+1} |_9 = E_c^\Lambda \sum_{n=0}^{4} (-1)^n \frac{1}{2^n n!} F_{a_1 b_1} \ldots F_{a_n b_n} \sigma_{\alpha \beta}^{a_1 \ldots a_n b_1 \ldots b_n} ,
\]
\[
e^F \wedge \bigoplus_{n=0}^{4} (-)^n \hat{\sigma}_{\alpha \beta}^{2n+1} |_9 = E_c^\Lambda \sum_{n=0}^{4} \frac{1}{2^n n!} F_{a_1 b_1} \ldots F_{a_n b_n} \sigma_{\alpha \beta}^{a_1 \ldots a_n b_1 \ldots b_n} .
\]

Now one can find that in \(D = 10\) the \(16 \times 16\) matrix valued forms \(\hat{\sigma}_{\alpha \beta}^{2n+1}\) entering (47), (48) have the symmetry properties
\[
\hat{\sigma}_{\alpha \beta}^{2n+1} = (-1)^n \hat{\sigma}_{\beta \alpha}^{2n+1} .
\]

In this way one finds that the expression (47) for \((h^{-1} \sigma_b)_{\alpha \beta} (\eta + F)^{-1} \alpha \beta\) coincides with (48) taken for \((h^{-1} \sigma_b)_{\beta \alpha} \equiv (\sigma_b h^{-1})_{\alpha \beta}\)
\[
(h \sigma_b)_{\alpha \beta} (\eta + F)^{-1} \alpha \beta = (h^{-1} \sigma_b)_{\beta \alpha} (\eta + F)^{-1} \beta \alpha .
\]

An equivalent form of Eq. (49) is
\[
(h \sigma \alpha h^T)_{\alpha \beta} = \sigma_{\beta \alpha}^b k^a_b
\]
with
\[
k^a_b = \delta^a_b - 2((\eta + F)^{-1})_b^a \equiv ((\eta + F)^{-1} (\eta - F))_b^a \equiv ((\eta - F)(\eta + F)^{-1})_b^a .
\]

Eq. (51) is the Cayley construction for the pseudoorthogonal (\('\eta\)-orthogonal' or Lorentz group valued) matrix \(k\)
\[
(k^T)_b^a \equiv k^a_b = (k^{-1})_b^a \iff k^a_b \in SO(1,9)
\]
for the antisymmetric Cayley image \(F_{ab}\) (see e.g. (49)).

Hence (51) defines the spin-tensor field \(h_\beta^\alpha\) as taking its values in the double covering of the Lorentz group \(Spin(1,9)\):
\[
h_\beta^\alpha \in Spin(1,9) .
\]

For completeness let us present the expression for the contraction of \(h_\beta^\alpha\) spin tensors with tilde sigma matrices, which is expressed through the \(k\) matrix as well as it can be shown
\[
(h^T \tilde{\sigma}^a h)_{\alpha \beta} = \tilde{\sigma}^a_{\alpha \beta} k^a_b .
\]
4.2 Derivatives of \( h^\alpha_\beta \)

As \( h^\alpha_\beta \) field takes the values in \( \text{Spin}(1,9) \) group and is one of the two element of this group \((\pm h)\) corresponding to the vector rotation \( k \) \([50]\), the derivative of \( h \) is related to the derivative of \( k \) by the standard isomorphism relation between \( \text{spin}(1,9) \) and \( \text{so}(1,9) \) algebras \([50]\)

\[
(h^{-1}dh)^\alpha_\beta = \frac{1}{4}(k^{-1}dk)^{ab} (\sigma_{ab})^\alpha_\beta. \tag{54}
\]

At the same time, the derivatives of \( k \) can be calculated in terms of \( F_{ab} \) directly from the expression \([51]\)

\[
dk^{ab} = -2(\eta + F)^{-1} ac dF_{cd}(\eta + F)^{-1} db. \tag{55}
\]

Thus the direct relation between the derivatives of \( h^\alpha_\beta \) and ones of the \( F_{ab} \) tensor is

\[
dh^\alpha_\beta = -\frac{1}{2}(\eta - F)^{-1} ac dF_{cd}(\eta + F)^{-1} db (h\sigma_{ab})^\alpha_\beta. \tag{56}
\]

5 Spin-tensor \( h^\alpha_\beta \) and fermionic equations of motion.

Using the identity \([55]\) inversely, Eq. \([39]\) can be represented in another equivalent way. For the first line in \([39]\), which is the only essential below, we get

\[
d\mathcal{L} = iE^\hat{\alpha} \wedge E^\hat{\beta} \wedge ((1 + \Gamma)\hat{\gamma})_{\hat{\alpha}\hat{\beta}} + \ldots \equiv \equiv iE^\hat{\alpha 0} \wedge E^\hat{\beta} \wedge E^\hat{\alpha 1} \left( (1 + \bar{\Gamma})(I \otimes \sigma_b)((\eta + \sigma_3 F)^{-1} ba \otimes I) \right)_{\hat{\alpha}\hat{\beta}} \sqrt{-det(\eta_{ab} + F_{ab})} + \ldots
\]

As

\[
(I \otimes \sigma_b)(\eta + \sigma_3 F)^{-1} ba \otimes I \equiv \left( \begin{array}{cc} \sigma_b(\eta + F)^{-1} ba & 0 \\ 0 & \sigma_b(\eta - F)^{-1} ba \end{array} \right),
\]

we have for the r.h.s. of \([57]\)

\[
E^\hat{\alpha} \wedge E^\hat{\beta} \wedge ((1 + \bar{\Gamma})(I \otimes \sigma_b)((\eta + \sigma_3 F)^{-1} ba \otimes I))_{\hat{\alpha}\hat{\beta}} = (E^2 - E^1 h) \wedge (-h^{-1}\sigma_b(\eta + F)^{-1} ba E^1 + \sigma_b(\eta - F)^{-1} ba E^2).
\]

in the gauge \([46]\).

With \([50], \(51)\) we can write \((h^{-1}\sigma^a)(\eta + F)^{-1} = (\sigma^b h^T)(\eta - F)^{-1} a_b^a\), and further transform \([58]\) into

\[
E^\hat{\alpha} \wedge E^\hat{\beta} \wedge ((1 + \bar{\Gamma})(I \otimes \sigma_b)((\eta + \sigma_3 F)^{-1} ba \otimes I)) = (E^2 - E^1 h) \wedge \sigma_b(\eta - F)^{-1} ba (E^2 - E^1 h). \tag{59}
\]

Thus we get for \([57]\)

\[
d\mathcal{L} = iE^\hat{\alpha 0} \wedge (E^2 - E^1 h)^\alpha \wedge (\sigma_b)^\alpha_\beta (E^2 - E^1 h)^\beta (\eta - F)^{-1} ba \sqrt{-det(\eta_{ab} + F_{ab})} + \ldots \tag{60}
\]

\(^*\)This can be obtained straightforwardly by taking the derivative of Eq. \([50]\) and by using the \(D = 10\) Fierz identities (see \([50]\) for more details concerning Lorentz group valued quantities).
where ... denotes the second and third lines in (39).

Hence, the fermionic superfield equations (45) following from the generalized action can be represented as (cf. with the D3–brane case [10, 14], where the h-variables are defined, however, in a completely different way)

\[ E_b^\alpha \wedge (E^2 - E^1 h)^\alpha \eta^{-1} = 0. \]  

(61)

Decomposing (61) into basic forms one gets

\[ E^2 - E^1 h = 0, \]  

(62)

\[ (\sigma_a)_{\alpha \beta} (\eta - F)^{-1} a b \psi_a^\alpha = 0, \]  

(63)

In terms of differential forms (62) becomes

\[ E^2 = E^1 h + E^a \psi^a, \]  

(64)

whereas (63) remains

\[ (\sigma_a)_{\beta \alpha} (\eta - F)^{-1} a b \psi_a^\beta = 0, \]  

(65)

with \( \psi_a^\alpha \equiv (E^2 - E^1 h)^\alpha \) by definition.

6 Geometrical equations and proper equations of motion

Eq. (64) can be regarded as a fermionic superembedding equation. One more equation which can be treated as geometrical one appears as a result of variation with respect to \( Q_8 \). Here we prove that the geometric equations (64), (40) contain the dynamical equations (65) among their consequences.

To this end we study the integrability conditions

\[ I_2 \equiv \frac{1}{2} \epsilon^A \wedge e^B I_{BA} \equiv d(E^2 - E^1 h - E^a \psi_a^\alpha) = 0, \]  

(69)

\[ J_3 \equiv \frac{1}{3!} \epsilon^A \wedge e^B \wedge e^C J_{CBA} \equiv d(F - F) = 0. \]  

(70)

Eq. (70) is the superfield Bianchi identities for the world volume gauge super-1-form.

\[ \tilde{e}_a^\alpha \wedge (E^2 - E^1 h)^\alpha (\sigma^a)_{\alpha \beta} = 0 \]  

similar to one for the type I superbranes in terms of adequate variables \( \tilde{e}^a = 2E^b (\eta - F)^{-1} b^a \) (cf. [14] where, however, a completely different parametrization of the projector matrix related to another variant of the induced fermionic vielbein of the world volume superspace has been considered for the D3-brane).
6.1 Induced world volume geometry

6.1.1 Induced supervielbein and ‘supersymmetric static gauge’

Eqs. (69), (70) contain the decomposition of the 2–form and 3–form integrability conditions on the arbitrary basis of the cotangent world volume superspace \( e^A = (e^a, e^\alpha) \). However we find it convenient to use the world volume supervielbein induced by embedding in the sense of the identification

\[
e^a = E^a \equiv \Pi^m \delta^a_m, \quad (71)\]

\[
e^\alpha = E^{\alpha 1} \equiv d\Theta^{1\mu} \delta^\alpha_\mu. \quad (72)\]

Eq. (72) provides the holonomic representation for Grassmann vielbein forms. This reflects the possibility to identify the Grassmann coordinate of the world volume superspace \( \eta^\mu \) with the coordinate function \( \Theta^{1\mu} \):

\[
\Theta^{1\mu} = \eta^\mu. \quad (73)\]

Such an identification (being a ‘superpartner’ of the so–called static gauge widely used in the description of solitonic branes in supergravity [51]) breaks the general supercoordinate invariance as an independent symmetry, retaining however the invariance under combined transformations including the same local shift of \( \Theta^{1\mu} \). The possibility of the latter transformations is provided by the superfield extension of the (irreducible) \( \kappa \) symmetry which leaves invariant the generalized action.

Taking the leading component of Eq. (73)

\[
\Theta^{1\mu} |_{\eta^\mu=0} = 0
\]

we obtain the covariant gauge fixing condition for the \( \kappa \) symmetry of the component formulation of [8, 11, 12, 13].

Hence the identification (73) can be regarded as a superfield generalization of such a covariant gauge fixing.

6.1.2 Covariant derivatives induced by embedding

The world volume covariant derivatives induced by the embedding form the basis in tangent world volume superspace dual to (71), (72)

\[
d \equiv dz^M \partial_M = e^a D_a + e^\alpha D_\alpha \equiv E^a D_a + E^{1\alpha} D_\alpha. \quad (74)\]

For such a choice we find that the Grassmann field \( \psi^\alpha_a \) appearing in the superembedding equation (66) acquires the form of a vector covariant derivative of the \( \Theta^2 \) supercoordinate function

\[
\psi^\alpha_a = E^{2\alpha}_a \equiv D_a \Theta^{2\alpha}. \quad (75)\]

The commutator of the induced covariant derivatives

\[
[D_A, D_B] = -t_{AB}^C D_C
\]
is expressed in terms of the component of the induced torsion, which is basically the pull–back of the target space torsion forms $\mathcal{H}$.

Thus expression for the torsion of flat target superspace $T^a = dE^a = -i(E^{a1} \wedge E^{b1} + E^{a2} \wedge E^{b2})\sigma_{\alpha\beta}^a, \quad T^\alpha = dE^{1\alpha} = 0$ \hspace{1cm} (76)

and the fermionic superembedding equations Eq. (66), taken together with (50), provide us with the following expressions for the induced world volume torsion:

\[ t^a = de^a = -2ie^\alpha \wedge \epsilon^b \sigma_{\alpha\beta}^b (\delta + F)^{-1}b^a + 2ie^b \wedge \epsilon^\beta (h\sigma^a \psi_b)_\beta - ie^b \wedge \epsilon^c (\psi_b \sigma^a \psi_c), \quad \]
\[ t^\alpha = de^\alpha = dE^{1\alpha} = 0. \]

Then the commutation relations between the covariant derivatives induced by the embedding are

\[ \{D_\alpha, D_\beta\} = 4i\sigma_{\alpha\beta}^b (\delta + F)^{-1}b^a D_a, \]
\[ [D_\alpha, D_b] = -2i(h\sigma^a \psi_b)_\beta D_a, \]
\[ [D_a, D_b] = 2i(\psi_a \sigma^c \psi_b) D_c. \]

### 6.2 Dynamical equations from geometrical ones

Using (77) we find that the lowest dimensional (in inverse length units) nontrivial components of the integrability conditions (69), (70) result respectively in the equations

\[ I_{\beta\gamma}^\alpha = 0, \quad \Rightarrow \quad D_{(\gamma} h_{\beta)}^\alpha = i(\sigma^a)_\beta\gamma (\delta + k)^a_{\alpha\beta} \psi^\alpha_a = 2i(\sigma_b)_\beta\gamma (\eta + F)^{-1}b^a \psi^\alpha_a, \]
\[ J_{\alpha\beta c} = 0, \quad \Rightarrow \quad (h\sigma^a h^T)_{\alpha\beta} (\eta + F)_{ab} = \sigma_{\alpha\beta}^a (\eta - F)_{ab}. \]

It is remarkable that Eq. (80) coincides with (50), (51)

\[ (h\sigma^a h^T)_{\alpha\beta} = \sigma_{\alpha\beta}^b k^a_b, \]
\[ k^a_b = ((\eta + F)^{-1}(\eta - F))_{ab} \equiv ((\eta - F)(\eta + F)^{-1})_{ab} \in SO(1, 9). \]

Hence the Lorentz group valuedness of the spin tensor field $h_{\alpha\beta}^\alpha$

\[ h_{\beta}^\alpha \in Spin(1, 9) \]

and the relation of this field with antisymmetric tensor $F_{ab}$ are contained in the integrability condition of geometric equations.

The restriction which (73) puts on the $h$ superfield can be written in closed form as

\[ \tilde{\sigma}_{\alpha1...\alpha5}^\beta\gamma D_{\gamma} h^\alpha_{\beta} = 0. \]

The components of the integrability conditions (63), (74), carrying dimensions 3/2 and 5/2 respectively produce

\[ I_{h\beta}^\alpha = 0, \quad \Rightarrow \quad D_h h^\alpha_{\beta} = D_{\beta} \psi^\alpha_b + 2i(h\sigma^a \psi_b)_\beta \psi^\alpha_a, \]

19
\[ J_{\alpha bc} = 0, \quad \Rightarrow \quad D_\alpha F_{bc} = -4i(h\sigma^a\psi_b)_\alpha(\eta - F)_{c[a}. \] (83)

With (54) and after some algebraic manipulations (83) becomes an expression for the Grassmann derivative of the \( h \) superfield

\[ D_\beta h^\alpha = -2i(h\sigma^a\psi_c)_\beta(\eta + F)^{-1} e^b(h\sigma_{ba})^\alpha. \] (84)

From (84) we can get the expression for the symmetric part \( D_\gamma h^\alpha_{\beta \gamma} = -2i(h\sigma^a\psi_{bc})_{\beta \gamma}(\eta + F)^{-1} e^a(h\sigma_{ba})^\alpha \) (85)

With (51), (52) the r.h.s. of (85) can be written as

\[ (h\sigma^a h^T)^{\beta \gamma}(\eta - F)^{-1} e^a \psi^\alpha_b. \]

and thus the same expression \( (\eta - F)^{-1} e^a \psi^\alpha_b \) appears on both sides of the equation.

Now it is easy to extract \( h \times h \) factor and thus to arrive at

\[ (\sigma^a)^{\beta \gamma}(\sigma^b)_{\gamma\alpha}(\eta - F)^{-1} e^b \psi^\alpha_c = (\sigma^a)^{\beta \gamma}(\sigma^b)_{\gamma\alpha}(\eta - F)^{-1} e^b \psi^\alpha_c. \] (86)

Contracting (86) with \( (\tilde{\sigma}^a)^{\beta \gamma} \) we obtain after straightforward algebra

\[ (\tilde{\sigma}^a)^{\beta \gamma}(\sigma^b)_{\gamma\alpha}(\eta - F)^{-1} e^b \psi^\alpha_c = 0 \]

which evidently results in the superfield fermionic equations of motion (87)

\[ (\sigma^a)^{\beta \gamma}(\sigma^b)_{\gamma\alpha}(\eta - F)^{-1} e^b \psi^\alpha_c = 0 \] (87)

following from the generalized action.

### 6.3 Other integrability conditions

The higher dimensional integrability conditions, i.e. the ones with all lower indices bosonic, are dependent in the sense that all their consequences can be obtained by applying the covariant Grassmann derivative to the equations (79), (80), (82), (83). We prove this fact in the Appendix B using the 'identity for identity' technique (see (82) and refs therein, and Appendix C in ref. 17).

We, however, present the explicit form of the equations following from these integrability conditions for completeness

\[ I^\alpha_{ab} = 0, \quad \Rightarrow \quad D_\alpha \psi^\alpha_{[b} = -i(\psi^c_{[a} \sigma^c \psi_{b]}) \psi^\alpha_{c]. \] (88)

\[ J_{abc} = 0, \quad \Rightarrow \quad D_\alpha F_{bc} = -2i(\eta + F)_{d[a}(\psi_b \sigma^d \psi_{c]}). \] (89)
Eq. (89) is just the bosonic Bianchi identity, but written in terms of the induced covariant vector derivatives (74).

Thus we have proved that the geometric equations (66), (67) contain all the information about the super-9–brane theory, and hence provide a superfield description of the nonlinear (Born–Infeld) generalization of the $D = 10$ SYM theory, included in the generic model of partial supersymmetry breaking: from $D = 10$, $N = 2$, type IIB to $D = 10$, $N = 1$.

7 Geometric equations in linearized approximation

To make the multiplet structure of the super-D9-brane model more transparent and the interrelation with the consideration from Refs. [4, 38] more explicit, we reproduce here the investigation of the geometric equation in the linearized approximation using the physical (‘static’) gauge

7.1 Physical gauge and linear approximation

The basic condition of the physical gauge is

$$\Theta^1 = \eta.$$  (90)

Taking into account the use of the linearized approximation in fields in what follows, it is convenient to introduce a Grassmann Goldstone fermion superfield $W^\mu$ for partially broken type IIB supersymmetry as the differences between two Grassmann coordinate superfields $\Theta_1$ and $\Theta_2$. Hence in the gauge (90)

$$\Theta^2 = \eta^\mu + W^\mu.$$  (91)

and we can write the bosonic gauge fixing conditions in the linearized approximation as

$$X^m + i\eta^\sigma W^m = \xi^m.$$  (92)

To justify such choice, one can consider the Bianchi identities

$$J_3 \equiv d(F - F) = -H_3 - E^b \wedge T^a F_{ab} - \frac{1}{2} E^b \wedge E^a \wedge dF_{ab} =$$

$$-iE^b \wedge d\eta^a \wedge d\eta^\beta \sigma^a_{\alpha\beta}(\eta - F)_{ab} + iE^b \wedge d\Theta^{2\alpha} \wedge d\Theta^{2\beta} \sigma^a_{\alpha\beta}(\eta + F)_{ab} - \frac{1}{2} E^b \wedge E^a \wedge dF_{ab} = 0.$$  (93)

As the $0^{th}$ order term in $E^b$ should be the flat bosonic vielbein $w^b$ (see below), it can be seen that one loses a consistency in $0^{th}$ order input into the Eq. (93), when assumes that $\Theta^2$ is of the $1^{st}$ order. On the other hand, (91) reflects the presence of the first term with invertible spin tensor $h$ in the r.h.s. of the fermionic superembedding equation (66).

With (91)–(92) we get for a bosonic vielbein in the linearized approximation

$$E^a = w^a - 2i\eta^\sigma W^a.$$  (94)
Here

\[ w^m = d\xi^m - 2id\eta^a \sigma^a \eta \]  

(95)

is a bosonic vielbein of the flat world volume superspace.

The flat Grassmann derivatives appear in the decomposition

\[ d = d\xi^m \partial_m + d\eta^\mu \partial_\mu = w^m \partial_m + d\eta^\mu D_\mu \]

and have the form

\[ D_\mu = \partial_\mu + 2i(\sigma^m \eta)_{\mu} \partial_m. \]  

(96)

The algebra of the flat derivatives is

\[ \{D_\mu, D_\nu\} = 4i\sigma^m_{\mu\nu} \partial_m. \]

In this section we will denote the complete derivatives by \( \mathcal{D}_\alpha \). Their expressions up to first order terms in fields are

\[ \mathcal{D}_\alpha = D_\alpha + 2i(\sigma^m W)_\alpha \partial_m, \]

while for the bosonic derivatives one obtains

\[ \mathcal{D}_a = \delta^m_a \partial_m. \]

### 7.2 Linearized geometric equations

With (90) – (92) the linearized the Bianchi identities (93) are

\[ J_3 = 2i w^b \wedge d\eta^\alpha \wedge d\eta^\beta (\sigma^{a\beta} F_{ab} + 2D_{(a|} W^\gamma \sigma_{b|\beta)}) - \frac{1}{2} w^b \wedge w^a \wedge d\eta^\alpha (D_a F_{ab} + 4i\partial_{[a|} W^\beta \sigma_{b]\beta\alpha}) - \frac{1}{2} w^b \wedge w^a \wedge w^c \partial_{[c} F_{ab]}. \]  

(97)

The most essential lowest dimensional component of (97) is

\[ J_{\alpha\beta\gamma} = 0, \quad \Rightarrow \quad F_{ab} \sigma^b_{\alpha\beta} = 2D_{(a|} W^\gamma \sigma_{a\gamma|\beta)}. \]  

(98)

Substituting the general decomposition for the DW superfield on the spinor indices

\[ D_\alpha W^\beta = a^0_\delta \delta^\beta_\alpha + a^{ab}_\sigma \sigma^{a\beta}_{ab\alpha} + a^{abcd}_\sigma \sigma^{abcd\beta}_{abcd\alpha} \]  

(99)

one can find from (98) that the coefficients \( a^0 \) and \( a^{abcd} \) vanish, i.e.

\[ D_\beta W^\beta = 0, \quad \sigma^{abcd}_\beta D_\alpha W^\beta = 0, \]  

(100)

and \( a^{ab} = -\frac{1}{4} F^{ab} \). Thus Eq. (98) is equivalent to

\[ D_\alpha W^\beta = -\frac{1}{4} F_{ab} \sigma^{ab\beta}_\alpha. \]  

(101)

Thus the fermionic superembedding equation becomes (cf. (91))

\[ dW^\alpha = -\frac{1}{4} d\eta^\beta \sigma^{ab\beta}_\alpha F_{ab} + d\xi^m \partial_m W^\alpha. \]  

(102)

This means that

\[ h^\alpha_\beta = \delta^\alpha_\beta - \frac{1}{4} F_{ab} \sigma^{ab\beta}_\alpha, \quad \psi^\alpha_a = \delta^m_a \partial_m W^\alpha \]  

(103)

in the linear approximation.
7.3 Linearized dynamical equations from the Bianchi identities

Acting on Eq. (101) by Grassmann derivative and taking the symmetric part with respect to the lower spinor indices, one gets

$$-8i\sigma^m_{\alpha\beta} \partial_m W^\gamma = D_{(\alpha} F_{\beta)} \sigma^{ab}_{\gamma}. \tag{104}$$

Contracting (104) with $\delta^\beta_\gamma$ we arrive at

$$-16i\sigma^m_{\alpha\beta} \partial_m W^\beta = \sigma^{ab}_{\alpha} D_\beta F_{ab}. \tag{105}$$

On the other hand, (104) means

$$D_{(\alpha} F_{\beta)} \sigma^{ab}_{\gamma} = \frac{1}{16} \sigma^m_{\alpha\beta} D_\delta F_{ab} (\tilde{\sigma}^m \sigma_{ab})^\delta_\gamma. \tag{106}$$

Contracting (106) with $\delta^\beta_\gamma$ and using the gamma matrix algebra ($\tilde{\sigma}^m \sigma_{ab} \sigma^m = 6\sigma_{ab} = -6\sigma^T_{ab}$) we obtain

$$\sigma^{ab}_{\alpha} D_\beta F_{ab} = 0 \tag{107}$$

which is just the fermionic equation of motion in the linearized approximation (cf. with (105))

$$\sigma^m_{\alpha\beta} \partial_m W^\beta = 0. \tag{108}$$

To get the equations of motion for the gauge fields it is enough to take a divergence from the basic equation (98)

$$\partial^a F_{ab} \sigma^{\gamma b}_{\alpha\beta} = 2D_{(\alpha} \sigma^m_{\beta)\gamma} \partial_m W^\gamma. \tag{109}$$

and use (108) to get the standard Yang-Mills equations

$$\partial^a F_{ab} = 0. \tag{110}$$

The Bianchi identities with the solution $F_{ab} = \partial_a A_b - \partial_b A_a$ are included into Eq. (97).

7.4 Comments

Thus we have analyzed the geometric equations of the super-D9-brane in the linearized approximation and obtained the dynamical equations from the geometrical ones.

These results are not unsuspected as the Eqs. (98), (100), (101) are characteristic for $D = 10$ SYM theory with the main field strength $W^\alpha$ related to the dim 3/2 component $F_{ab}$ of the complete field strength $F_{AB} = D_A A_B - (-)^{AB} D_B A_A$ by

$$F_{ab} \propto (\sigma_b W)_\alpha. \tag{111}$$

(The latter equation follows from the superspace Bianchi identities for $F_{AB}$ restricted by the constraints $F_{\alpha\beta} = 0$).

In this respect it is interesting to note that the covariant (with respect to $N = 1$ supersymmetry and gauge symmetry) field strength $W^\alpha$ of $N = 1$ SYM multiplet originates in the Goldstone fermionic superfield (cf. (91)) in the linearized approximation.
8 Conclusion and discussion

In this paper we have obtained the superfield equations of motion for the $D = 10$, type IIB Dirichlet super-9-brane from the generalized action \[10\].

We justify by a covariant proof the statement from refs. \[11,\ 12\] concerning the antidiagonal form of the $\kappa$–symmetry projector $\bar{\Gamma}$ as well as the Lorentz group valuedness of the spin–tensor (super)field $h_{\beta}^{\alpha}$ which determines the projector $\bar{\Gamma}$ completely. For that we use only the identity for $\bar{\Gamma}$ \[10\].

As our proof is constructive, we obtain, as a bonus, a covariant relation between $h$ superfield and the auxiliary tensor $F_{ab}$ coinciding with the field strength of the world volume gauge field on mass shell. This relation means that $F_{ab}$ is the Cayley image of a Lorentz group valued matrix $k$ whose double covering is provided by the spin-tensor superfield $h$.

The derivation and investigation of the superfield equations of motion are considerably simplified by using the superfield $h$ and its covariant relation with $F_{ab}$.

The prize to pay for this is the Lorentz group valuedness of the spin tensor superfield $h$ which, however, does not create any problem as the technique of dealing with Lorentz group valued variables have been developed already in \[50\].

The set of geometric equations following from the generalized action is extracted and separated from the proper dynamical equations of motion. Studying the integrability conditions for the first ones we show as a consequence, that the geometric equations \[66\], \[67\] contain all the information about the super-9–brane theory and, hence about the nonlinear (Born–Infeld) generalization of the $D = 10$ SYM theory included in the generic model of partial supersymmetry breaking.

The separate studying of the geometric equations in the linear approximation displays an interesting point: The Goldstone superfield $W^{\mu} = \Theta^{2\mu} - \Theta^{1\mu}$ of the nonlinearly realized supersymmetry plays the role of the field strength of the $D=10$ SYM multiplet in this approximation (cf. \(100\), \(101\)) \[13\].

Our results provide further evidence for a power of the superembedding approach based on the generalized action \[18,\ 19\] for studying superbrane physics.

Another description of the super-9-brane model is provided by considering an ‘extrinsic’ geometry of the world volume superspace. This just corresponds to the approach of the classical theory of surfaces (see \[19\] and refs. therein), which supersymmetric generalization has been developed in \[17\].

The key observation is that the set of the integrability conditions \(66\), \(70\) together with the torsion constraints \(77\) contain all the information provided by the original geometric equations \(66\), \(67\).

In accordance with Eq. \(88\), \(78\), the $\psi_2^a$ superfield should be regarded as covariant derivative of some Grassmann spinor superfield \(72\). In such a way the supercoordinate function $\Theta^2$ appears in the extrinsic geometry. At the same time, in this description the superfield $\psi_2^a$ is

\[14\]When this paper was being written we received a copy of \[38\] which shows a certain overlap with the Section 7. In particular, Eqs. \(100\), \(101\) can be extracted from that work as well.
dependent because it is simply the notation for a nonvanishing part of the Grassmann derivative of the $Spin(1,9)$ group valued superfield $h_{\alpha}^{\beta}$ \(^{(79)}\).

Thus the Lorentz group valued spin tensor superfield $h_{\alpha}^{\beta}$ can be considered as the main superfield for the description of the super-9-brane world volume superspace 'extrinsic' geometry. The main equation which extract the super-D9-brane theory, or, equivalently, $D = 10$ Goldstone SYM multiplet from the $h_{\alpha}^{\beta} \in Spin(1,9)$ superfield is \( (81) \)

$$\tilde{\sigma}_{a_1...a_5}^{\beta\gamma}D_{\gamma}h_{\alpha}^{\beta} = 0. \quad (111)$$

The fermionic dynamical equations \( (68) \) has the form

$$D_{\alpha}h^{-1}_{-1}{}^{\alpha} = 0, \quad (112)$$

Eq. \( (111) \) can be used in searching for a nonlinear generalization of the action with infinitely many Lagrange multipliers superfield proposed recently in \( [53] \).

The development of the 'extrinsic' geometry description (or doubly supersymmetric geometric approach) for the super–D9–brane is an interesting task for further study. (Note that it is quite nonlinear as the world volume geometry is characterized by the covariant derivative algebra \( (78) \) involving the $h$ superfield).

A direct application of our results would be to study the dimensional reduction of our equations down to $D = 4$ and to arrive in this way at the equations of the Goldstone multiplets found in \( [42] \). Another way consists in constructing a generalized action for $D = 4$, $N = 2$ super-D3-brane, getting the superfield equations and investigating their relations with ones from \( [42] \).

One more interesting problem for further study consists in the possibility to reformulate the generalized action in terms of $h$ superfields instead of its Cayley image $F_{ab}$, as it was done for D3-brane in \( [14] \). Such investigation can provide an insight in searching for a generalized action for M-theory super-5-brane (see \( [39] \)) and, more generally, for unification of the superembedding approach with the PST technique \( [34] \) widely used for a Lagrangian description of theories with self-dual gauge fields (see \( [29, 30, 34, 52, 53] \)).

9 Acknowledgements

The authors are grateful to D. Sorokin, M. Tonin and E. Sezgin for useful discussions and comments. The work was supported in part by the INTAS Grants 96-308, 93-127-ext, 93-493-ext and by the Austrian Science Foundation under the Project P–10221.
Appendix A: Remarks on harmonics and frames.

In distinction with the generic super-D9-brane case [10], the generalized action for the super-9-brane can be written without spinor harmonics at all. However, a reader can understand (if he like) all the equations as ones including the Lorentz group valued Lorentz harmonic variables [50, 17, 18] (and refs. in [17]) as a bridge between target space and world indices. I.e. he can substitute

\[ E^a = \Pi^m u^a_m, \quad E^{\alpha I} = d\Theta^I_{\mu \nu} \]

in any of our equations (except for ones from section 7).

If the harmonics are retained, the following points are important to remember for the 9–brane case:

• As \( u^a_m \) is the complete \((10 \times 10)\) Lorentz group valued matrix, we have the \( SO(1,9) \) gauge invariance in the action [10] and, thus, the harmonic degrees of freedom are pure gauge ones.

• The natural covariant derivative ([50, 17, 18] and refs. therein)

\[ \mathcal{D} u^a_m \equiv du^a_m - u^b_m \Omega^a_b = 0 \]  

vanishes acting on the harmonics.

• As the connection \( \Omega^a_b \) and, hence, the induced spin connection defined as pull back of the connection form onto the world volume superspace \( \Omega^a_b = dz^M \Omega^a_M b \) is \( \Omega^{ab} = u^{am} du^b_m \) [113] and thus trivial, the induced covariant derivative on the world volume superspace being defined as a pull–back of \( \Omega^{am} \)

\[ \mathcal{D} = dz^M \mathcal{D}_M = e^A \mathcal{D}_A \]

produces no curvature

\[ \mathcal{D} \mathcal{D} = 0. \]

As there are no \( E^i \) (i.e. bosonic directions orthogonal to the 9-brane in \( D = 10 \) space-time), we do not need in harmonics to adapt the bosonic frame to the world volume and we can use them to relate our results to any frame. However, the harmonics can be used to relate a general Lorentz frame with the frame, where the antisymmetric tensor \( F \) (auxiliary in our case) takes the form (cf. [11])

\[ F_{ab} = \begin{pmatrix}
0 & \Lambda_0 & 0 & \ldots & 0 \\
-\Lambda_0 & 0 & \ldots & 0 \\
0 & -\Lambda_1 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & -\Lambda_4
\end{pmatrix}, \]  

\[ \begin{pmatrix}
\Lambda_0 & 0 & \ldots & 0 \\
0 & \Lambda_1 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \Lambda_4
\end{pmatrix}, \]  

\[ (114) \]
This gauge is very important in many respects (see \cite{11,12,13}). So, our covariant relations for $h$ and $F$ could have been obtained in an alternative way by the use of the gauge \cite{14,11}, where the spin-tensor field $h$ acquires the form

$$h_{\alpha}^{\beta} = \frac{(1 - \Lambda_0 \sigma^0)(1 - \Lambda_1 \sigma^1)(1 - \Lambda_2 \sigma^2)(1 + \Lambda_3 \sigma^3)(1 + \Lambda_4 \sigma^4)}{\sqrt{(1 - \Lambda_0^0)(1 + \Lambda_1^1)(1 + \Lambda_2^2)(1 + \Lambda_3^3)(1 + \Lambda_4^4)}}.$$ \hfill (115)

### Appendix B: Interdependence of geometric equations

Here we will study the interdependence of the integrability conditions (69), (70).

It should be stressed that the fermionic superembedding equations (66) considered with unrestricted $h_{\alpha}^{\beta}$ has a conventional character. Indeed, the $\psi$ superfield is equal to the bosonic derivative of the $\Theta^2$ superfield

$$\psi^\alpha_a = D_a \Theta^{2\mu} \delta^\alpha_\mu$$

by definition (cf. (65)). Thus the only nontrivial statement containing in the equation (66) with unrestricted $h$ is the supposition about linear independence of the pull–backs of 16 fermionic forms $E^{1\alpha} = d\Theta^{1\mu} \delta^\alpha_\mu$, which are used as a basis in the fermionic sector of the world volume superspace (72).

On the other hand, as we have seen above, the dynamical equations appear when the integrability conditions for both geometric equations (66) and (67) are taken into account.

**B1. Interdependence of the higher dimensional integrability conditions**

We begin by studying a dependence of some higher dimensional integrability conditions inside the sets (69) and (70).

To this end, following the line realized for the Bianchi identities of supergravity \cite{52}, we investigate the integrability conditions for Eqs. (69) and (70) (‘identities for identities’ or ‘integrability conditions for integrability conditions’)

$$I_3^\alpha \equiv \frac{1}{3!} e^A \wedge e^B \wedge e^C I_{\alpha \mu \nu \lambda}^{ABC} \equiv dI_2^\alpha = 0,$$ \hfill (116)

$$J_4 \equiv \frac{1}{4!} e^A \wedge e^B \wedge e^C \wedge e^D J_{DCBA}^{\alpha} \equiv dJ_3 = 0.$$ \hfill (117)

Their components are

$$\frac{1}{3} I_{ABC}^\alpha \equiv D_{\{A} I_{BC\} \}^\alpha + t_{\{AB} D_{C\} \}^\alpha = 0,$$ \hfill (118)

$$\frac{1}{4} J_{ABCD} \equiv D_{\{A} J_{BCD\} \} + \frac{3}{2} t_{\{AB} E_{C\} \} \equiv 0.$$ \hfill (119)
Supposing that all the integrability conditions \((69)\) of dimensions less than 2 are satisfied

\[
I_{\beta\gamma}^\alpha = 0, \quad I_{\beta c}^\alpha = 0,
\]

we obtain from the component of \((118)\) of dimension 2

\[
t_{\beta\gamma}^b I_{bc}^\alpha = 0.
\] (120)

As for our choice of induced geometry \((71), (72)\) the world volume torsion is determined by Eq. \((77)\)

\[
t_{\beta\gamma}^a = \sigma_{\beta\gamma}^b (\eta + F)^{-1} b^a,
\] (121)

we get from \((120)\)

\[
I_{bc}^\alpha = 0.
\]

This means that all the contents of the dim 2 component of the integrability conditions \((69)\) can be obtained from the corresponding action by covariant Grassmann derivatives on their components of dimensions 1 and 3/2. In other worlds, these components of integrability condition \((69)\) are dependent.

In the same manner, using the dim 3 component \(J_{\alpha\beta\gamma d} = 0\) of Eq. \((119)\), one can prove the dependence of the integrability conditions

\[
J_{bcd} = 0
\]

of Eq. \((70)\) on the low dimensional components

\[
J_{\alpha\beta c} = 0, \quad J_{abc} = 0
\]

of the same equation (remember that the dim 3/2 component \(J_{\alpha\beta\gamma} = 0\) of the Eq. \((70)\) is satisfied identically for the case under consideration).

If one turns to the component \(J_{\alpha\beta\gamma d} = 0\) of dim 5/2 of the Eq. \((119)\) and assumes that the integrability conditions \((74)\) of dimension 2, i.e. \(J_{\beta\gamma c} = 0\), are satisfied identically, one obtains

\[
\sigma_{(\alpha\beta)}^b \Psi_{\gamma ba} = 0, \quad \text{with} \quad \Psi_{\gamma ba} \equiv J_{\gamma cd}(\eta + F)^{-1} b^c(\eta + F)^{-1} d^a.
\] (122)

(Here the expression \((121)\) for the world volume torsion have been used).

The general solution of Eq. \((122)\) (with respect to the lower indices) is

\[
\Psi_{\gamma ba} = \frac{1}{10} \sigma_{\gamma\beta}^a \tilde{\sigma}^c\alpha \Psi_{\alpha ca}
\] (123)

(it can be obtained by contraction of \((122)\) with \(\tilde{\sigma}^{b\beta\gamma}\)). Then, using the antisymmetry property of \(\Psi_{\gamma ba}\) \((122)\) with respect to permutation of vector indices \(b, a\)

\[
\Psi_{\gamma ba} = \frac{1}{10} \sigma_{\gamma\beta}^a \tilde{\sigma}^c\alpha \Psi_{\alpha ca} \equiv \frac{1}{10} \sigma_{\alpha\beta}^a \tilde{\sigma}^c\alpha \Psi_{\alpha bc},
\] (124)

and contracting \((124)\) with \(\tilde{\sigma}^{b\beta\gamma}\), we obtain

\[
\tilde{\sigma}^c\alpha \Psi_{\alpha bc} = 0,
\]

\[
(124)
\]
and, hence (cf. (123))

$$\Psi_{abc} = 0 \Rightarrow J_{abc} = 0.$$  

Thus we have proved that only the lowest (dim 2) component $J_{\alpha\beta c} = 0$ (80) of the integrability conditions (70) for the geometric equation (67) (nonlinear SYM constraints) can be independent. I.e. all the results of the Eqs. (83) ((84)) and (89) can certainly be obtained by acting by the Grassmann derivatives on the Eq. (80).

In contradistinction, the independent set of integrability conditions (89) can contain, in principle, not only the lowest dimensional component

$$I_{\beta\gamma}^\alpha = 0$$  

(79), but some irreducible parts of Eq. (83) as well.

If one consider the dim 3/2 component $J_{\beta c}^\alpha = 0$ of the Eq. (116) in assumption that $I_{\beta\gamma}^\alpha = 0$, one obtains

$$\sigma_{\{\beta\gamma\}}^b \Psi_{b\delta}^\alpha = 0$$  

(125)

with

$$\Psi_{b\delta}^\alpha \equiv (\eta - F)^{-1} b h_{\beta}^{\gamma} I_{a\gamma}^\alpha \equiv (\eta - F)^{-1} b (h_{\beta\gamma} I_{a\gamma}^\alpha - h_{\beta}^{\gamma} D_{\gamma} \psi_{\alpha}^{\delta} - 2i(\sigma^c \psi_{\alpha}^{\delta} \psi_{\alpha}^{c})$$.  

(126)

The general solution of Eq. (125) with respect to lower indicies is (cf. with (123))

$$\Psi_{a\beta}^\alpha = \frac{1}{10} \sigma_{\alpha\beta\gamma\delta} \Psi_{b\delta}^\alpha$$  

(127)

To solve Eq. (125) we have to substitute the general decomposition

$$\Psi_{a\beta}^\alpha \equiv \Psi_{0 \alpha} \sigma_{\beta}^\alpha + \Psi_{2 a \beta} (\sigma_{cd})_{\beta}^\alpha + \Psi_{4 a \beta} (\sigma_{c1...c4})_{\beta}^\alpha$$

(128)

and use the $D = 10$ sigma matrix algebra

$$\sigma^a \ddot{\sigma}^{c1...c4} = \sigma^{ac1...c4} + q \eta^{a[e1...c4]}$$

$$\sigma^{c1...c5} = -\frac{1}{5!} e_{c1...c5} d_{d1...d5} \sigma_{d1...d5}, \quad \sigma^{c1...c6} = -\frac{1}{4!} e_{c1...c6} d_{d1...d4} \sigma_{d1...d4}$$

to decompose Eq. (127) onto the irreducible parts. As a result, we obtain the general solution of (127)

$$\Psi_{a\beta}^\alpha \equiv \Psi_{0 \alpha} \sigma_{\beta}^\alpha + (\eta_{a\beta} \Psi_{0 \alpha} \eta) + 4 \Psi_{3 \alpha \beta} (\sigma_{cd})_{\beta}^\alpha + (\Psi_{5 \alpha \beta} [-]_{ac1...c4} + \eta_{a\beta} \Psi_{3 \alpha c234}) (\sigma^{c1...c4})_{\beta}^\alpha$$

(129)

where the following notations for independent irreducible parts of Eq. (83) are used

$$\Psi_{0 \alpha} \equiv \frac{1}{16} \Psi_{\alpha \alpha}$$

(130)

$$\Psi_{3 \alpha \beta} \equiv \Psi_{a\beta}^\alpha (\sigma_{abcd})_{\alpha}$$

(131)

$$\Psi_{5 \alpha \beta} [-]_{ac1...c4} \equiv \Psi_{b\beta}^\alpha (\sigma_{b1...b4})_{\alpha} (\delta_{\alpha}^{b \beta} \delta_{c1}^{b1}...\delta_{c4}^{b4}) - \frac{1}{5!} e_{ac1...c4} b_{b1...b4}$$

(132)

Eqs. (126), (130) – (132) defines the parts of Eq. (82) which can be independent on Eq. (79).
B2. Gauge field constraints and fermionic superembedding condition

Here we use the established dependence of the higher order components of the integrability conditions (70) to obtain a relation of (70) with the fermionic superembedding equation (66).

The complete explicit form of Eq. (70) is given by

\[ J_3 \equiv d(F - F) = -H_3 - E^b \wedge T^a F_{ab} - \frac{1}{2} E^b \wedge E^a \wedge dF_{ab} = \]

\[ -iE^b \wedge E^{1\alpha} \wedge E^{1\beta} \sigma_{\alpha\beta}(\eta - F)_{ab} + iE^b \wedge E^{2\alpha} \wedge E^{2\beta} \sigma_{\alpha\beta}(\eta + F)_{ab} - \frac{1}{2} E^b \wedge E^a \wedge dF_{ab}. \]

If one uses the induced supervielbeine (71), (72) to decompose the world-volume superspace differential (74)

\[ D = E^a D_a + E^{1\alpha} D_\alpha \]

and extracts the expression

\[ E^\alpha \equiv E^{2\alpha} - E^{1\beta} h^\alpha_{\beta} - E^a \psi^\alpha_a \equiv E^{2\alpha} (E^{2\beta}_{\beta} - h^\alpha_{\beta}), \]

\[ \psi^\alpha_a = D_\alpha \Theta^2 \delta^\alpha_\mu \]

from all the terms in the r.h.s. of Eq. (133) (i.e. substituting here \( E^{2\alpha} = E^\alpha + E^{1\beta} h^\alpha_{\beta} + E^c \psi^\alpha_c \)), one gets

\[ J_3 \equiv d(F - F) = -iE^b \wedge E^{1\alpha} \wedge E^{1\beta} (\sigma^a_{\alpha\beta}(\eta - F))_{ab} - (h \sigma^a h^T)_{\alpha\beta}(\eta + F)_{ab} - \]

\[ -\frac{1}{2} E^b \wedge E^c \wedge E^{1\alpha} J_{abc} - \frac{1}{2} E^b \wedge E^c \wedge E^d J_{dcb} + \]

\[ +iE^b \wedge (E^{2\alpha} + E^{1\beta} h^\alpha_{\beta} + E^c \psi^\alpha_c) \wedge E^{2\beta} \sigma^a_{\alpha\beta}(\eta + F)_{ab} \]

where \( J_{abc} \), \( J_{bcd} \) are defined by Eqs. (83) and (89)

\[ J_{abc} = D_\alpha F_{bc} + 4i(h \sigma^a \psi_{[bc]})(\eta - F)_{\alpha c \beta a}, \]

\[ J_{abc} = D_{[a} F_{bc]} + 2i(\eta + F)_{d[a} (\psi_{b} \sigma^d \psi_{c]}) . \]

Taking into account the fermionic superembedding equations (66) following from the generalized action, one obtains the set of integrability conditions (80), (83), (89) considered in the previous section.

As it was already proved, such integrability conditions can be used to obtain the fact that the spin tensor field \( h \) is Lorentz group valued (50), (52) as well as the relation of \( h \) superfield with the gauge field strength (51).

On the other hand, if we suppose that Eqs. (50), (51) are satisfied, the first term in (133) vanishes identically. The conditions of such vanishing coincides with Eq. (80). Henceforth, in accordance with dependence relations being established in the previous Subsection, the second and third terms in (70) (given by the Eqs. (136) and (137)) vanish as a result of (80).

Thus, in the assumption that Eqs. (50), (51) holds, the integrability condition (70) for the world volume gauge field constraints (67) acquires the form

\[ J_3 \equiv d(F - F) = iE^b \wedge (E^{2\alpha} + E^{1\beta} h^\alpha_{\beta} + E^c \psi^\alpha_c) \wedge E^{2\beta} \sigma^a_{\alpha\beta}(\eta + F)_{ab} = 0, \]

\[ (138) \]
Decomposing Eq. (138) onto the basic 3-forms of the world volume superspace, one can find that it contains two equations

\[
(E^{\alpha'}_{\alpha} - h^{\alpha'}_{\alpha} \sigma^{a}_{\alpha'} \beta (E^{\beta'}_{\beta} + h^{\beta'}_{\beta}) + (E^{2\alpha'}_{\alpha} - h^{2\alpha'}_{\alpha}) \sigma^{a}_{\alpha'} \beta (E^{2\beta'}_{\beta} + h^{\beta'}_{\beta}) = 0 \tag{139}
\]

\[
(E^{\alpha'}_{\alpha} - h^{\alpha'}_{\alpha} \sigma^{a}_{\alpha'} \beta (\eta + F)_{a|b} = 0. \tag{140}
\]

Eq. (139) can be simplified to the form

\[
E^{2\alpha'}_{\alpha} \sigma^{a}_{\alpha'} \beta E^{2\beta'}_{\beta} = h_{\alpha}^{\beta} \sigma^{a}_{\alpha'} \beta h^{\beta'}_{\beta} \equiv \sigma^{b}_{\alpha\beta} ((\eta - F)(\eta + F))^{-1}_{b} a. \tag{141}
\]

As it was noted above, the second equation in (141) defines \( h \) spin-tensor completely up to a sing. Hence, we can conclude that Eq. (141) has two solutions

\[
E^{2\beta}_{\alpha} = h_{\alpha}^{\beta}, \tag{142}
\]

\[
E^{2\beta}_{\alpha} = -h_{\alpha}^{\beta}. \tag{143}
\]

For the first solution, the second equation (140) is satisfied identically, while for the second solution Eq. (140) becomes

\[
h_{\alpha}^{\alpha'} \sigma^{a}_{\alpha'} \beta \psi^{\beta}_{[c] (\eta + F)_{a|b]} \equiv h_{\alpha}^{\alpha'} \sigma^{a}_{\alpha'} \beta \psi^{\beta}_{[c] (\eta - F)_{b|a} = 0, \tag{144}
\]

and provides a nontrivial additional restriction for the bosonic derivative of the \( \Theta^2 \) superfield \( \psi^{a}_{\alpha} = D_{a} \Theta^{a\beta} \delta^{\beta}. \)

Comparing (144) with (the dependent) Eq. (138) one can find that (144) signifies \( D_{\alpha} F_{ab} = 0 \) and, hence, remove the world volume gauge field strength from the consideration. Thus only the solution (142) is nontrivial.

Generalized action (10) selects the solution (142) uniquely.

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