The objective of this paper is to connect the study of self-avoiding walks with the theory of formal languages. For an infinite, locally finite, connected graph without loops or multiple edges, the authors label the oriented edges by elements of a finite alphabet deterministically, i.e., edges with the same initial or terminal vertex have distinct labels. They also consider that the label-preserving automorphism group acts quasi-transitively on the graph, i.e., that there are finitely many orbits on the set of vertices.

In this context, the language of self-avoiding walks for some start vertex $o$ is the formal language that consists of the words read along sequences of edge-connected vertices starting at $o$ for which no vertex is visited twice. The main results of the paper characterize when this language is regular or context-free. Namely, the authors show that for any vertex $o$, this language is regular exactly when the graph has more than one end, and the size of all ends is $1$. Furthermore, they show that this language is context-free exactly when the graph has more than one end and all ends are of size at most $2$.

Ends of a connected graph, a notion introduced by R. Halin [Math. Ann. 157, 125–137 (1964; Zbl 0125.11701)] are equivalence classes of infinite paths that are included, but for finitely many elements, in the same component when finitely many vertices are removed.

The authors use cut-vertex tree decompositions to show that the language is regular and the 3-block tree decompositions of C. Droms et al. [Electron. J. Comb. 2, 271–280 (1995; Zbl 0829.05041)] to show that it is context-free. As their proof is constructive, they finally discuss how the obtained grammar can be used to calculate the generating function of self-avoiding walks and determine related invariants.

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