NONLINEAR CURRENT DENSITY IN A COMPOSITIONAL SEMICONDUCTOR SUPERLATTICE UNDER CROSSED ELECTRIC AND MAGNETIC FIELDS

Bui Dinh Hoi*, Le Thi Thu Phuong
College of Education, Hue University

Abstract: The dc electrical transport in a compositional semiconductor superlattice subjected to a crossed electric field and a magnetic field is studied theoretically. The electron-optical phonon interaction is taken into account at high temperature and strong magnetic field. We obtain the expression for nonlinear current density involving external (electric and magnetic) fields and characteristic parameters of the superlattice. The analytical result is numerically evaluated and graphed for GaAs/Al$_{0.3}$Ga$_{0.7}$As superlattice. The intra- and inter-subband magnetophonon resonance effect is observed when the dc electric field is absent. Resonance peaks shift to higher cyclotron energy region when the dc electric field is switched on. The current density increases linearly with small value region and nonlinearly with large value region of the dc electric field.

Keywords: Nonlinear current density; compositional superlattices; electron-phonon interaction

1 Introduction

In the late seventies and early eighties of the twentieth century, Carolyne M. van Vliet together with her coworkers in Montreal developed a many-body master equation, using the interaction-picture and projection operators in the Liouville space, thus extending earlier work by van Hove, Zwanzig, Kubo, and others [1-4]. The so-obtained Pauli-van Hove-van Vliet master equation has diagonal as well as non-diagonal terms, with irreversibility vested in the diagonal part. Using the formalism of second quantization for an electron gas in interaction with impurities or phonons, these researchers obtained a full quantum transport equation, applicable to both extended and localized electronic states. This quantum transport equation has proven to be a major tool for transport and conductivity calculations in modern submicron devices with extreme quantum confinement, such as quantum wells, quantum wires and quantum dots. With several coworkers (Vasilopoulos, Charbonneau, et al.) she has successfully formulated quantitative theories for the quantum Hall effect, magnetophonon resonance (MPR), cyclotron resonance, and the Aharonov-Bohm effect, among other phenomena [5-9].

For the square wells with an infinite potential, Vasilopoulos et al. [9] presented the nonlinear electrical conduction in the presence of the strong electric and magnetic field where electrons are scattered by optical phonons and impurities. The authors replaced delta functions by Poisson summations in the expression of nonlinear current density to avoid the divergence. They also clearly showed the conditions for the appearance of the MPR effect. However, the results were calculated for the intrasubband transitions ($n = n'$) only, and a graphical consideration was not carried out. In a recent work [10], we have applied the above-mentioned
theory to study the nonlinear electrical conduction in a parabolic quantum well under crossed dc electric and magnetic fields. We derived the $x$-component of the nonlinear current density (NCD) taking into account the scattering of electrons by optical phonons at high temperatures. The analytical expression of the NCD was calculated for the arbitrary values of subband indices (both intra-subband and inter-subband transitions), and the numerical results were obtained for a specific parabolic quantum well with the help of a computational program.

It is well-known that the studying of the MPR effect is a powerful tool to understand transport phenomena in semiconductors, such as the determination of effective mass, effective charge, energy levels of carriers, etc. [11-14]. To our knowledge, the application of above-mentioned theory to MPR effect problems in the low-dimensional electron systems has still been open for studying up to date. So, in the present work we calculate the NCD in a compositional semiconductor superlattice (CSSL) subjected to a crossed electric field and a magnetic field. The magnetic field is applied along to the growth direction of the CSSL. The present paper is organized as follows. In Sec. 2, we briefly describe the simple model of a CSSL in the crossed electric and magnetic fields. In the next section, we calculate analytically the NCD for electron-optical phonon interaction. Numerical results and discussion are given in Sec. 4. Finally, remarks and conclusions are shown briefly in Sec. 5.

2 A compositional semiconductor superlattice in crossed electric and magnetic fields

We consider the transport of an electron gas in a compositional semiconductor superlattice which is composed of $N_0$ layers of GaAs (the layer thickness is $d_1$) and $N_0$ layers of AlGa$_{1-c}$As (the layer thickness is $d_2$) arranged alternatively. The energy-gap difference between these two materials is $U$, then the motions of electron gas along the growth direction (assumed the $z$-direction) are considered to be governed by the periodic superlattice potential, and the motions are nearly free along two other directions (assumed the $x$-direction and the $y$-direction). We assume that the difference of the electron effective mass between GaAs and AlGa$_{1-c}$As can be neglected ($m_e \approx m_o = m$). If a static magnetic field $B$ is applied in the $z$-direction, and a dc electric field $E$ is applied in the $x$-direction, then the one-electron Hamiltonian ($h_0$), its normalized eigenfunctions ($\psi$), and the eigenvalues ($\varepsilon_\psi$) in the Landau gauge for the vector potential $\vec{A} = (0, B, 0)$ are, respectively, given by [9, 15, 16]

$$h_0 \equiv \left[ \frac{\vec{p} + e \vec{A}}{2m} \right] + U + eEx,$$

$$|\psi\rangle = \left| N, n, k_x, k_z \right\rangle = \frac{1}{\sqrt{L_z}} \exp(ik_z y) \phi_n(x - x_0) \otimes |n, k_z\rangle,$$

$$\varepsilon_\psi = \varepsilon_{N, n, k_x, k_z} = \left( N + 1/2 \right) \hbar \omega_z + \varepsilon_{n, k_z} - \hbar v_d k_x + \frac{1}{2} m_v^2; \quad N = 0, 1, 2, \ldots.$$
where $e$ and $m$ are the charge and the effective mass of a conduction electron, respectively; $\vec{p}$ is its momentum operator; $N$ is the Landau level index and $n$ denotes level quantization in the $z$-direction; \( v_s = E / B \) being the drift velocity, and $\omega_c = eB / m$ is the cyclotron frequency. $k_1$ and $L_y$ are the wave vector and normalization length in the $y$-direction, respectively. Also, $\phi_y$ represents harmonic oscillator wave functions, centred at $x_i = -\ell^2 \left( k_1 - m v_s / \hbar \right)$ where $\ell = \sqrt{\hbar / (m \omega_c)}$ is the radius of the Landau orbit in the $x$-$y$ plane; $\{ n, k_c \}$ is the wave function in the $z$-direction. Also, in the tight-binding approximation, we have [16, 17]

$$
\varepsilon_{n,k_c} = \varepsilon_n - t_c \cos(k_c d). \quad n = 0, 1, 2, \ldots
$$

where $d = d_i + d_u$ is the superlattice period, and $\varepsilon_n = \hbar^2 \pi^2 (n + 1)^2 / (2md_i^2)$, $t_c$ is the half-width of the $n$th mini-band given by [18, 19]

$$
t_c = -4 \left( -1 \right)^n \frac{d_i}{d - d_i} \frac{\exp(-2\sqrt{2md_i^2 U / \hbar^2})}{\sqrt{2md_i^2 U / \hbar^2}}.
$$

For the calculation of this paper, the following matrix elements will be used [10, 15]

$$
\langle \xi | \phi_{n',k'} \rangle = \frac{\left( \ell / \sqrt{\hbar} \right) \omega_c}{\sqrt{N N' / N' - N} \delta_{N' N} - \sqrt{N N' / N' - N} \delta_{N' N}} \langle n,k | n',k' \rangle \delta_{\alpha \beta}.
$$

$$
\langle \xi | \phi_{n',k'} | \rangle = -i \ell \left( \sqrt{N N' / N' - N} \delta_{N' N} + \sqrt{N N' / N' - N} \delta_{N' N} \right) \langle n,k | n',k' \rangle \delta_{\alpha \beta},
$$

$$
\langle \xi | \phi_{n',k'} | \rangle = \left( \ell / \sqrt{\hbar} \right) \omega_c \langle n,k | n',k' \rangle \delta_{\alpha \beta}.
$$

where $\ell = \left( \ell / \sqrt{\hbar} \right) \omega_c$; $\delta_{\alpha \beta} = \delta_{k_c k_c} \delta_{k_z k_z}$; $\tilde{a} = \tilde{r} - \langle \tilde{r} \rangle_{eq}$, with $\tilde{r}$ and $\langle \tilde{r} \rangle_{eq}$ are, respectively, the position of carrier and its equilibrium position prior to the switching on of the external fields; $x, y$ are the Cartesian components of $\tilde{r}$. Also,

$$
\left| \langle \xi | e^{i\phi} | \xi' \rangle \right|^2 = \left| I_{n,n'}(k_c,k_c,q_z) \right|^2 \left| J_{N,N'}(u) \right|^2 \delta_{k_c,k_c,q_z},
$$

$$
\left| J_{N,N'}(u) \right|^2 = (N' y N') e^{-u / \hbar} \left[ 1_{N-N}^{N-N} \right]^2,
$$

$$
I_{n,n'}(k_c,k_c,q_z) = \langle n,k_c | e^{i\phi} | n',k_c \rangle,
$$

with $u = \ell \left( q_z^2 + q_z^2 \right) / 2 = \ell q_z^2 / 2$, and $1_{N}^{N'} (u)$ is the associated Laguerre polynomial. The matrix elements of the form factor $I_{n,n'}(k_c,k_c,q_z)$ are assumed to be independent on the number of periods, and in this study, we will deal with the intrawell form factor only (neglecting the interwell form factor), then [20]

$$
I_{n,n'}(k_c,k_c,q_z) = \frac{1}{2} \frac{\sin \left[ \left| q_z \pm \left( k_c - k_z \right) \right| d / 2 \right]}{\left| q_z \pm \left( k_c - k_z \right) \right| d / 2} \exp \left[ -\left| q_z \pm \left( k_c - k_z \right) \right| d / 2 \right],
$$

where $k_z = \sqrt{2me_{n',k_c}} / \hbar$. 

37
3 Expression for the nonlinear current density

For the above-mentioned model of CSSL, we consider the strong applied dc electric field and high lattice temperature, then the expression of the NCD is written as [9]

$$\langle j_\mu \rangle = \frac{e}{2V_0}\sum_{\xi,\bar{\xi}} (a_{\mu\xi} - a_{\mu\bar{\xi}}) \left[ f_\xi (1 - f_{\bar{\xi}}) w_{\xi\bar{\xi}} - f_{\bar{\xi}} (1 - f_\xi) w_{\xi\bar{\xi}} \right].$$  \hspace{1cm} (13)

where $\mu = x, y, z$; $V_0$ is the normalization volume of the sample; $a_{\mu\xi} = \langle \xi | \mu \xi \rangle$; $f_\xi$ and $f_{\bar{\xi}}$ are the occupancies of the eigenstates $|\xi\rangle$ and $|\bar{\xi}\rangle$, respectively, given by Fermi-Dirac distribution functions; $w_{\xi\bar{\xi}}$ is the binary transition rate (probability of transition) between two above eigenstates, given by the “golden rule”,

$$w_{\xi\bar{\xi}} = \sum_q \left[ Q(\xi, q \rightarrow \bar{\xi}) \langle N_q \rangle_{eq} + Q(\bar{\xi} \rightarrow \xi, q) \left(1 + \langle N_q \rangle_{eq}\right) \right].$$  \hspace{1cm} (14)

Where

$$Q(\xi, q \rightarrow \bar{\xi}) = \frac{2\pi}{\hbar} |C(q)|^2 \langle \xi | e^{iq\xi} | \bar{\xi} \rangle \delta(\xi_q - \epsilon_{\xi} + \hbar \omega_q),$$  \hspace{1cm} (15)

$$Q(\bar{\xi} \rightarrow \xi, q) = \frac{2\pi}{\hbar} |C(q)|^2 \langle \bar{\xi} | e^{-iq\xi} | \xi \rangle \delta(\epsilon_{\xi} - \epsilon_{\bar{\xi}} - \hbar \omega_q).$$  \hspace{1cm} (16)

The first and the second terms of Eq. (14) stand for the absorption and emission of a phonon of wave vector $q$ with energy $\hbar \omega_q$, respectively. The notation $(\xi, q \rightarrow \bar{\xi})$ ($\bar{\xi} \rightarrow \xi, q$) in the first term (second term) means that an electron at state $|\xi\rangle$ absorbs (emits) a phonon of wave vector $q$ to change its state to $|\bar{\xi}\rangle$. $\langle N_q \rangle_{eq}$ is the average number of phonons, given by Bose-Einstein distribution function.

At high temperatures, the electrons system is non-degenerate and assumed to obey the Boltzmann’s distribution function, then $1 - f_\xi \approx 1 - f_{\bar{\xi}} \approx 1$. Also, optical phonons are important in this case, so we consider the electrons scattering by optical phonons with the assumption that phonons are dispersionless, i.e., $\hbar \omega_q \approx \hbar \omega_{LO}$, where $\omega_{LO}$ is the frequency of a longitudinal optical phonon, assumed to be constant.

With above mentions, from Eqs. (6)-(14), the x-component of NCD can be written as

$$j_x = \frac{e}{2V_0} \sum_{a_{xx}} \sum_{K_1 K_2} \sum_{n_1 n_2} \exp \left[ \beta (e_{K_1} - e_{n_1}) \right] \left| J_{a_{xx}}(K_1, K_2, q) \right|^2 \left| C(q) \right|^2 \left| J_{n_1 n_2}(a) \right|^2 \ell^2 q_x,$$

$$\times \left\{ N_\delta \left[ (N-N') \hbar \omega_x + e_{n_1} - e_{n_2} + \hbar v_x q_x + \hbar \omega_{LO} \right] - (1+ N_\delta) \delta \left[ (N'-N) \hbar \omega_x + e_{n_1} - e_{n_2} - \hbar v_x q_x - \hbar \omega_{LO} \right] \right\}.$$  \hspace{1cm} (17)
in which \( \varepsilon \) is the Fermi energy, \( \varepsilon_{N,n,k} = (N+1/2)\hbar\omega_{\varepsilon} + \varepsilon_{n,k} + m_{\varepsilon}/2 \). \( N_q = \langle N_{1/n} \rangle \approx 1/(\beta\hbar\omega_0) \) [5-8], \( \beta = 1/(k_B T) \) with \( k_B \) being Boltzmann’s constant, and \( T \) the temperature. For electron - optical phonon interaction, \(|C(\tilde{q})|^2 = e^2\hbar\omega_0/(1/X_0 - 1/X_0)/2V_{kq}^2 \) [21], where \( \kappa \) is the electric constant (vacuum permittivity), \( X_0 \) and \( X_0 \) are the static and high-frequency dielectric constants, respectively. The second term of \( j_{1}, j_{2}^{(2)} \), can be obtained from \( j_{1}^{(i)} \) by interchanging \( \xi \) with \( \xi' \).

We now change the summations over \( k_{\parallel} \) and \( q_{\parallel} \) as follows

\[
\sum_{k_{\parallel}} \rightarrow \frac{L_{\perp}}{2} \int_{L_{\perp}/2}^{L_{\perp}/2} dk_{\parallel} = \frac{L_{\perp}L_{\parallel}}{2\ell^2} = \frac{A_0}{2\ell^2},
\]

\[
\sum_{q_{\parallel}} \rightarrow \frac{V_0}{4\pi^2} \int_{0}^{2\pi} q_{\parallel} dq_{\parallel}, \int_{-\infty}^{\infty} dq_{\parallel} = \frac{V_0}{4\pi^2} \int_{0}^{2\pi} du \int_{-\infty}^{\infty} du dq_{\parallel},
\]

where \( A_0 \) is the cross section of CSSL. For simplicity in performing the integral over \( q_{\parallel} \), we replace \( q_{\parallel} \) by \( eB\Delta\tilde{x}/\hbar \), where \( \Delta\tilde{x} \) is a constant of the order of \( \ell \). This has been done in Ref. 9 and is equivalent to assuming an effective phonon momentum: \( eV_{q}q_{\parallel} = eE\Delta\tilde{x} \), \( \Delta\tilde{x} = (\sqrt{N+1/2} + \sqrt{N+1+1/2})\ell/2 \). Eq. (17) becomes

\[
j_{\parallel} = \frac{e^2B\omega_0}{16\pi^2\kappa\hbar L}(1 - \frac{1}{Z_0})\sum_{N,q,N,q,k,k'} \sum_{n,n,n,n} \text{exp} \left[ \beta(\varepsilon_{n,k} - \varepsilon_{N,n,k'}) \right] \left[ \sum_{n,n,n,n} (N - N')\hbar\omega_{\varepsilon} + \varepsilon_{n,k} - \varepsilon_{N,n,k'} + eE\Delta\tilde{x} + \hbar\omega_{0} \right]
\times \int_{0}^{2\pi} \left[ I_{N,N}(u) \right]^{2} u^{1/2} du \left[ (N - N')\hbar\omega_{\varepsilon} + \varepsilon_{n,k} - \varepsilon_{N,n,k'} - eE\Delta\tilde{x} - \hbar\omega_{0} \right],
\]

where \( I_{N,N}(u) = \frac{N!}{2} \left[ L_{N}^{(N)}(u) \right]^{2} du \), which will be numerically evaluated by the computational program, and

\[
\int_{0}^{2\pi} \left[ I_{N,N}(u) \right]^{2} u^{1/2} du = \frac{N!}{2} \int_{0}^{2\pi} e^{-u^{N}} \left[ L_{N}^{(N)}(u) \right]^{2} du.
\]

This integral can be calculated immediately by using the formulas [7-9]

\[
\int_{0}^{2\pi} e^{-u^{M}} \left[ L_{N}^{(N)}(u) \right]^{2} du = \frac{1}{M} \left( \frac{N + M)!}{N!} \right), \quad M \neq 0.
\]

The appearance of \( k_{\parallel} \) and \( k'_{\parallel} \) in delta functions in Eq. (21) causes the difficulty in performing the summations over \( k_{\parallel} \) and \( k'_{\parallel} \). To reduce this difficulty, we now take \( k_{\parallel} = 0 \) and \( k'_{\parallel} = \pi/d \). This means that only the processes at the centre and the boundary of the first mini-Brillouin zone are included. We also replace the delta functions by Lorentzians to avoid the divergence as [6]

\[
\delta(\varepsilon_{n,k} - \varepsilon_{N,n,k'}) = \frac{1}{\pi} \frac{\Gamma_{N,N}}{(\varepsilon_{n,k} - \varepsilon_{N,n,k'})^{2} + \Gamma_{N,N}^{2}}.
\]
where $\Gamma_{N,N'}$ is the damping factor associated with the momentum relaxation time (assumed to be constant in this calculation) by $\Gamma_{N,N'} = \hbar / \tau$. After some manipulation, we have the following expression for the $x$-component of the current density

$$j_x = \frac{e^4 B_0 \omega_0}{16\pi^2 \kappa L} \left( \frac{1}{Z_e} - \frac{1}{Z_0} \right) \sum_{n,n',n''} \exp\left[ \beta\left( \varepsilon_n - \varepsilon_{N,n,0} \right) \right] R_{n,n'}(0,\pi/d) \Delta \left[ N_{n'} \left( N_{n'}^2 \right)^2 \right. $$

$$\times \left( \frac{1}{N-N'} \right)^{-} \left[ (N-N')h\omega_0 + \varepsilon_{n,0} - \varepsilon_{n',0} + eE\Delta \tau + h\omega_{LO} \right] + \frac{\Gamma_{N,N'}}{N_{n}^2} \left( \frac{1}{N-N'} \right)^{+} \left[ (N-N')h\omega_0 + \varepsilon_{n,0} - \varepsilon_{n',0} - eE\Delta \tau - h\omega_{LO} \right] + \frac{\Gamma_{N,N'}}{N_{n}^2} \right].$$

(25) Expression (25) shows the complicated dependence of the NCD on the dc electric field. It is obtained for arbitrary values of indices $n$ and $n'$. However, it contains the term $I_{n,n}(0,\pi/d)$ which is difficult to find out the exact analytical result. This term will be numerically evaluated by the computational method in the next section where we give a deeper insight for the dependence of the NCD on the external fields and other parameters.

4 Numerical results and discussion

In this section, we present detailed numerical calculations of the $x$-component of the NCD, $j_x$ for a CSSL subjected to the uniform crossed magnetic and dc electric fields. For the numerical evaluation, we consider the GaAs/Al$_0.3$Ga$_{0.7}$As superlattice with the following parameters [17, 22]: $e = 1.6 \times 10^{-19} \text{C}$, $\varepsilon_r = 5.2 \text{meV}$, $U = 300 \text{meV}$, $h\omega_{LO} = 36.6 \text{meV}$, $\tau = 10^{-12} \text{s}$, $m = 0.067 \times m_e$ ($m_e$ is the mass of the free electron).

Figure 1 shows the dependence of the NCD on the cyclotron energy $h\omega_c$ for the transitions $N = 0, N' = 1, n = 0, n' = 0 \pm 1$ (the lowest and the first-excited levels) in the case of absence of the dc electric field ($E = 0$). We can very clearly see the appearance of three maximum peaks in the NCD. Using the computational method, we can conclude that all the peaks correspond to the condition $(N-N')h\omega_c = h\omega_{LO} \pm \Delta \varepsilon_{n,n'}$ where $\Delta \varepsilon_{n,n'} = \varepsilon_{n,0} - \varepsilon_{n',0}$. Recall that $\varepsilon_{n,k} = \varepsilon_n - t_n \cos(k,d)$, so if we take $N = 0, N' = 1, n = 0, n' = 0$ (there is no transition between the subbands $n$ and $n'$) then $\Delta \varepsilon_{0,0} = -2t_0$. For the above-mentioned parameters, $t_0 \approx -3.4 \times 10^{-14} \text{meV} \ll h\omega_{LO}$ and the condition becomes $h\omega_c = h\omega_{LO}$. This is the magnetophonon resonance (MPR) condition and corresponds to the second peak (from the left to the right) as we can see in the figure at $h\omega_c = 36.6 \text{meV}$.

Similarly, it is easy to show that the first and the third peaks satisfy the conditions $h\omega_c = h\omega_{LO} + \Delta \varepsilon_{0,1}$ and $h\omega_c = h\omega_{LO} - \Delta \varepsilon_{0,1}$, respectively. They arise from the intersubband transition $N = 0, N' = 1, n = 0, n' = 1$ ($\Delta \varepsilon_{n,n'} = \Delta \varepsilon_{0,1} \approx -3h^2 \pi^2 / (2md^2) \approx 27 \text{meV}$) and are symmetrical to the second one.
Fig. 1. The NCD as a function of the cyclotron energy for \( N = 0, N' = 1, n = 0, n' = 0 \pm 1 \) when the dc electric field is absent. Here, \( T = 270 \text{K}, \; N_0 = 50, \; d_i = 25 \text{nm}, \; d_{\|} = 30 \text{nm} \)

\begin{align*}
(a) & \quad \text{NCD} [\text{arb. units}] \\
& \quad \text{Cyclotron energy (meV)}
\end{align*}

Fig. 2. (Color online) (a, b, c) The NCD as a function of the cyclotron energy for \( N = 0, N' = 1, n = 0, n' = 0 \pm 1 \) at \( E = 0 \text{V/m} \) (the solid, red curve) and \( E = 3 \times 10^4 \text{V/m} \) (the dashed, blue curve).

Here, \( T = 270 \text{K}, \; N_0 = 50, \; d_i = 25 \text{nm}, \; d_{\|} = 30 \text{nm} \)

In figure 2, we show the NCD versus cyclotron energy for two cases: absence (red-dashed curve) and presence (blue-dashed curve) of the dc electric field. The appearance of the peaks in the former case can be explained similarly as for figure 1. However, for the latter case the condition for the resonant peaks becomes

\begin{equation}
(N' - N)\hbar \omega_i = \hbar \omega_{i0} + eE\Delta \tau \pm \Delta \epsilon_{n,n'},
\end{equation}

where, \( N = 0, N' = 1, n = 0, n' = 0 \) for the second peak, and \( N = 0, N' = 1, n = 0, n' = 1 \) for the others. The condition (26) is generally called the intersubband MPR condition in the presence of a dc electric field. It was obtained in Refs. 5 - 8 for square quantum wells without the dc electric field and \( n = n' \). It is easily seen that the presence of the dc electric field shifts the resonant peaks to the right, and also the larger values of the magnetic field the smaller distance of the shifting. To explain this, we consider the condition (26) and recall that \( \Delta \tau = \left( \sqrt{N + 1/2} + \sqrt{N + 1 + 1/2} \right) \ell / 2 \), i.e., \( \Delta \tau = \ell / B \). Hence, for a fixed value of the dc electric field \( E \), the term \( eE\Delta \tau \) decreases with the increase of the magnetic field. Namely, in the region of the strong magnetic field, the influence of the dc electric field on the peak shifting is weak. With the presence of the dc electric field, we
also see that the first and the third peaks are not symmetrical to the second one. This is different compared with the case of absence of the electric field. In fact, if we denote $\hbar \omega_{\xi}$ and $\hbar \omega_{\xi'}$ the cyclotron energies, respectively, at the first and the third peaks, the conditions for these peaks are

$$\hbar \omega_{\xi} = \hbar \omega_{\xi} + eE\Delta \varepsilon \pm \Delta \varepsilon_{\xi},$$

and

$$\hbar \omega_{\xi'} = \hbar \omega_{\xi} + eE\Delta \varepsilon \pm \Delta \varepsilon_{\xi'},$$

so the distances (in the unit of energy) from the first and the third peaks to the second peak are determined, respectively,

$$\Delta \varepsilon_{\xi} = \hbar \omega_{\xi} - \hbar \omega_{\xi} = \Delta \varepsilon_{\xi} - eE\Delta \varepsilon$$

and

$$\Delta \varepsilon_{\xi'} = \hbar \omega_{\xi'} - \hbar \omega_{\xi} = \Delta \varepsilon_{\xi'} + eE\Delta \varepsilon.$$

Evidently, these two peaks are not symmetrical to the second one (at $\hbar \omega_{\xi} = \hbar \omega_{\xi'}$) if $E$ is nonzero.

![Fig. 3. The NCD as a function of the dc electric field for $N = 0, N' = 1, n = 0, n' = 1$ at different values of the temperature: $T = 100$ K (solid curve), $T = 200$ K (dashed curve), and $T = 300$ K (dotted curve). Here, $N_0 = 50$, $B = 12$ T, $d_1 = 25$ nm, $d_2 = 30$ nm]

To show the dependence of the NCD on the dc electric field, in figure 3 we plot the NCD versus the dc electric field for transition $N = 0, N' = 1, n = 0, n' = 1$ at different values of the temperature. Two ranges of the electric field are considered: small value (figure 3a) and large value (figure 3b). We can see that the NCD increases linearly in the small value range of the electric field. However, in the large value range of the electric field (larger than $5 \times 10^4$ V.m$^{-1}$), the NCD increases nonlinearly. Moreover, the NCD increases with increasing the temperature. This is a typical property of semiconductors.

5 Conclusions

In this work, we study the nonlinear electrical conduction in a CSSL in the presence of crossed electric and magnetic fields with the assumption that electron - optical phonon interaction is dominant at high temperatures. We obtain the analytical expression for the NCD which nonlinearily depends on the dc electric field. Numerical calculations are performed for a specific CSSL to clarify the theoretical results. We can summarize some main features of the obtained results as follows. All the resonant peaks arise under the condition $M \hbar \omega_{\xi} = \hbar \omega_{\xi} + eE\Delta \varepsilon \pm \Delta \varepsilon_{\xi,n}$ ($M = 1, 2, 3, \ldots$). This condition resulted from the law of energy conservation which is
mathematically interpreted by delta functions in the expression (25) of the NCD. In the case of absence of the dc electric field \((E = 0)\) and no intersubband transition \((n = n')\), we have the usual MPR effect represented by \(M\hbar\omega = \hbar\omega_{\text{c}}\), which occurs whenever the energy of an optical phonon is equal to an integer multiple \(M\) of the cyclotron energy. In the case of \(E \neq 0\) and \(n \neq n'\), we have the intersubband MPR effect in the presence of the dc electric field.

References

1. Van Vliet K M (1978), Linear response theory revisited. I. The many-body van Hove limit, J. Math. Phys., Vol. 19, 1345 - 1370.
2. Van Vliet K M (1979), Linear response theory revisited. II. The master equation approach, J. Math. Phys., Vol. 20, 2573 - 2595.
3. Charbonneau M, Van Vliet K M and Vasilopoulos P (1982), Linear response theory revisited III: One-body response formulas and generalized Boltzmann equations, J. Math. Phys., Vol. 23, 318 - 336.
4. Vasilopoulos P and Van Vliet K M (1984), Linear response theory revisited. IV. Applications, J. Math. Phys., Vol. 25, 1391 - 1403.
5. Vasilopoulos P (1985), Finite temperature aspects of the quantum Hall effect: A Boltzmann-equation approach, Phys. Rev. B, Vol. 32, 771 - 776.
6. Chaubey M P and Van Vliet K M (1986), Transverse magnetoconductivity of quasi-two-dimensional semiconductor layers in the presence of phonon scattering, Phys. Rev. B, Vol. 33, 5617 - 5622.
7. Vasilopoulos P. (1986), Magnetophonon resonance in quasi-two-dimensional quantum wells, Phys. Rev. B, Vol. 33, 8587 - 8594.
8. Vasilopoulos P. (1986), Integral quantum Hall effect in superlattices, Phys. Rev. B, Vol. 34, 3019 - 3022.
9. Vasilopoulos P. (1987), Charbonneau M and Van Vliet C M, Linear and nonlinear electrical conduction in quasi-two-dimensional quantum wells, Phys. Rev. B, Vol. 35, 1334 - 1344.
10. Hoi B. D., Phong T. C. (2012), Nonlinear current density in quantum wells with parabolic potential under crossed electric and magnetic fields, Int. J. Comput. Mater. Sci. Engin., Vol. 1, 1250021 (11 pages).
11. Gurevich V. L. and Firsov Y. A. (1961), On the theory of the electrical conductivity of semiconductors in a magnetic field, I. Soviet Physics JETP-USSR, Vol. 13, 137 - 146.
12. Puri S. M. and Geballe T. H. (1963), Bulletin of the American Physical Society, Vol. 8, 309.
13. Firsov Y. A., Gurevich V. L., Parfeniev R. V., and Shalyt S. S. (1964), Investigation of a new type of oscillations in the magnetoresistance, Phys. Rev. Lett., Vol. 12, 660 - 662.
14. Tsui D. C., Englert T., Cho A. Y. and Gossard A. C. (1980), Observation of magnetophonon resonances in a two-dimensional electron system, Phys. Rev. Lett., Vol. 44, 341 - 344.
15. Kahn A H and Frederikse H P R (1959), Oscillatory behaviour of magnetic susceptibility and electronic conductivity, Sol. Stat. Phys., Vol. 9, 257 - 291.
16. Mitra B. and Ghatak K. P. (1991), Effect of Crossed Electric and Quantizing Magnetic Fields on the Einstein Relation in Semiconductor Superlattices, Phys. Stat. Sol. (b), Vol. 164, K13 – K18.
17. Ploog K. and Dohler G. H. (1983), Compositional and doping superlattices in III-V semiconductors, Adv. Phys., Vol. 32, 285 - 359.
18. Esaki L., Tsu R. (1970), Superlattice and negative differential conductivity in semiconductors, *IBM. J. Res. Develop.*, Vol. 14, 61 - 65.

19. Silin A. P. (1985), Semiconductor superlattices, *Sov. Phys. Usp.*, Vol. 28, 972 - 993.

20. Friedman L. (1985), Electron-phonon scattering in superlattices, *Phys. Rev. B*, Vol. 32, 955 - 961

21. Lee S. C. (2007), Optically Detected Magnetophonon Resonances in Quantum Wells, *J. Korean Phys. Soc.*, Vol. 51, 1979 - 1986.

22. Smrcka L et al (2006), Magnetoresistance oscillations in GaAs/AlGaAs superlattices subject to in-plane magnetic fields, *Physica E*, Vol. 34, 632 - 635.