Nuclear geometry and elliptic flow in collisions of deformed nuclei

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I Introduction

Elliptic flow measurement \([1, 2]\) has become a very important method for determining properties of dense strongly interacting matter created in ultra-relativistic heavy ion collisions experiments. Being theoretically predicted in 1992 \([3]\) the elliptic flow has been observed in heavy ion collisions at AGS, SPS and RHIC \([4]\).

The elliptic flow is generated during the expansion of compressed QCD matter into vacuum from the initial asymmetrical volume created in non-central collisions of nuclei. During the process of expansion (which can be described by hydrodynamical \([3]\) or rescattering \([5]\) models) an asymmetry in the azimuthal distribution of transverse momenta of particles is generated.

Experimentally measured azimuthal distribution of particles in non-central heavy-ion collisions exhibits second-order oscillation of the form \(\rho(\phi) = a[1 + 2v_2 \cdot \cos(2\phi)]\), where parameter \(v_2\) characterizes the strength of the elliptic flow effect \([1]\). Quantitative comparisons of the initial spatial eccentricity \(\varepsilon\) and final elliptic flow strength \(v_2\) allow one to make implications on the equation of state of strongly-interacting QCD matter \([6]\).

In this contribution we investigate the elliptic flow behaviour in relativistic collisions of deformed nuclei. Using Optical Glauber Model simulation \([7]\) we predict large fluctuations of the elliptic flow (at given collision centrality) in collisions of deformed nuclei. We also suggest that behaviour of the elliptic flow in the most central collisions of deformed nuclei is different for nuclei with oblate and prolate deformations.

II Elliptic flow in collision of spherical nuclei

Initial eccentricity \(\varepsilon\) of the compressed QCD matter in collisions of spherical nuclei (e.g. \(\text{Pb}^{207}\)) is simply related to the size of impact parameter \(b\) (a distance between centers of colliding nuclei in transversal plane). Collisions of spherical Pb nuclei with fixed impact parameter (e.g. \(b = 3\) fm) will exhibit elliptic flow values fluctuating around some average value \(\langle v_2 \rangle\). Fluctuations of \(v_2\) values in such collisions with fixed impact parameter originate e.g. from the fluctuations of initial participant eccentricity which occur due to varying positions of individual nucleons \([8]\) inside the interaction volume. Thus even in the most central collisions of spherical nuclei \((b = 0\) fm\) the eccentricity \(\varepsilon\) fluctuates and on average \(\langle \varepsilon \rangle > 0\).

These effects can be studied in Monte Carlo Glauber Model simulations \([9]\) where individual positions of nucleons are determined for each collision event. In the optical Glauber model such effects are neglected.
In the next sections we predict event-by-event fluctuations of the elliptic flow $v_2$ due to initial eccentricity $\varepsilon$ fluctuations originating from the deformed shape of colliding nuclei. Such $v_2$ fluctuations (present only in collisions of deformed nuclei) are generated additionally, on top of $v_2$ fluctuations observed in collisions of spherical nuclei.

### III Elliptic flow in collisions of deformed nuclei

In collisions of deformed (oblate or prolate) nuclei the initial eccentricity $\varepsilon$ of the interaction zone strongly depends on the orientation of nuclear spin (axis of the ellipsoid). For example, in central ($b = 0$ fm) collision of two prolate nuclei (e.g. Ho$^{165}$) the eccentricity can be large if spins of Ho$^{165}$ nuclei are parallel to each other and orthogonal to the beam axis, or negligibly small for spins of both Ho$^{165}$ nuclei being parallel to the beam axis. Eccentricity $\varepsilon$ thus depends on azimuthal and polar angles $\phi, \theta$ of nuclei colliding:

$$\varepsilon[b] = f(\theta_1, \phi_1, \theta_2, \phi_2)$$  \hspace{1cm} (1)

Using the optical Glauber simulation [7] one can calculate number of participating nucleons $N_{\text{part}}$, number of nucleon-nucleon collisions $N_{\text{coll}}$ and eccentricity $\varepsilon$ for any given value of impact parameter $b$ and angles $\phi_1, \theta_1, \phi_2, \theta_2$ (see Fig.1).

![Figure 1: Contour-plot of [ $\varepsilon$ ; $N_{\text{part}}$] values (left) obtained with optical Glauber simulation of Ho$^{165}$+Ho$^{165}$ collisions using fixed impact parameters $b = 0, 1, 2, \ldots 12$ fm, and the shape of Ho$^{165}_{7/2-}$ nucleus with $\beta_2 = 0.3$ (right).](image)

#### III.1 Prolate nuclei

Let us consider rare-earth nucleus Ho$^{165}$. This element has a single stable isotope with prolate ground-state deformation $\beta_2 = 0.3$ [10]. Using deformed Woods-Saxon density [11] the eccentricity of the interaction zone

$$\varepsilon = \sqrt{\frac{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}{\sigma_y^2 + \sigma_x^2}}$$  \hspace{1cm} (2)
(here $\sigma_{xy} = \langle x \cdot y \rangle - \langle x \rangle \langle y \rangle$) can be evaluated (e.g. using $N_{coll}[x, y]$ density in transversal plane) for all combinations of independent angles $\theta_1, \theta_2 \in (0, 90)$ and $\phi_1, \phi_2 \in (0, 180)$. This has been done in 15 degree steps for impact parameters $b = 0, 1, 2, \ldots 14$ fm (see Fig.1). Every given collision event with angles $\theta_1, \theta_2$ has been weighted by spherical angle factor $\sin(\theta_1) \sin(\theta_2)$ and impact parameter factor $2\pi \cdot b$ to account for the probability of the collisions as they occur in unpolarized experiments. In the resulting histogram of $(\varepsilon; N_{part})$ values (as shown in Fig.1) one observes large fluctuations of eccentricity $\varepsilon$ for collisions with fixed impact parameter $b$. Also number of participants varies at given impact parameter value due to variation of angles $\theta_1, \theta_2$ and $\phi_1, \phi_2$.

III.2 Oblate nuclei

Almost all simulations of relativistic Au+Au collisions assume the gold nucleus $^{197\text{Au}}_{3/2+}$ to be spherical. However, this nucleus with quadrupole moment $Q \approx 0.55 \cdot 10^{-24}$ cm$^2$ \cite{12} is predicted to have slight oblate deformation in the ground state with $\beta_2 \approx -0.13$ \cite{10}. In order to investigate possible consequences of the predicted oblate deformation of $^{197\text{Au}}$ nucleus in heavy ion collisions we assume it to be deformed with $\beta_2 = -0.131$ \cite{10}. The result of our optical Glauber model simulation is shown in Fig.2. One observes fluctuations of eccentricity and number of participants at given fixed impact parameters $b = 0, 1, 2, \ldots$ fm. For spherical nuclei the optical Glauber simulation would give a single value of the eccentricity and the number of participants for any given impact parameter $b$.

![Figure 2: Contour-plot (left) of $[\varepsilon; N_{part}]$ values obtained for $^{197\text{Au}}+^{197\text{Au}}$ collisions at $b = 0, 1, 2, \ldots 12$ fm assuming $\beta_2 = -0.13$. Error bars in the $\text{Ecc}[N_{part}]$ plot (right) show the width of eccentricity fluctuations at given fixed impact parameter values.](image)

One can conclude that ground-state deformation of nuclei used in heavy-ion collision experiments generates additional initial eccentricity (and consequently $v_2$) fluctuations on top of those existing in the collisions of spherical nuclei.

\footnote{To obtain non-zero contribution from $b = 0$ fm collisions factor $2\pi(b + 0.2)$fm has been used.}
IV Central collisions of deformed nuclei

In the previous section we have shown that oblate and prolate deformations of nuclei generate significant fluctuations of the initial eccentricity at given impact parameter of collisions.

What happens in the most central collisions? To investigate this one has to calculate properly the centrality of collisions using number of participants $N_{\text{part}}$ and number of nucleon-nucleon collisions $N_{\text{coll}}$ obtained from the optical Glauber simulation. We use two-component model \cite{13} to calculate charged-particle multiplicity in the form:

$$dN_{\text{ch}}/d\eta = (1 - x) \cdot n_{\text{pp}} \frac{N_{\text{part}}}{2} + x \cdot n_{\text{pp}} N_{\text{coll}}$$

and quantity $dN_{\text{ch}}/d\eta$ is assumed to be the collision centrality measured experimentally. Results shown in Figure 3 have been obtained using $n_{\text{pp}} = 2.25$ and $x = 0.11$ \cite{13}. Since the number $N_{\text{coll}}$ is significantly larger compared to $N_{\text{part}}$, approximately 40% of secondary particles originate from quantity $N_{\text{coll}}$ and the rest of particle multiplicity is generated due to $N_{\text{part}}$.

One observes (see also Figure 4) that most central collisions of oblate and prolate nuclei exhibit different behaviour. For prolate nuclei the highest multiplicity of secondary particles is predicted to happen when $b = 0$ fm and spins of Ho$^{165}$ nuclei are parallel to the beam axis and to each other. In this case the number of nucleon-nucleon collisions is significantly higher compared to the case when spins of Ho$^{165}$ nuclei colliding at $b = 0$ fm are orthogonal to the beams axis (and parallel to each other). Therefore we predict the elliptic flow values at very-high-multiplicity (VHM) collisions of prolate nuclei to decrease significantly (see Fig.4).

For oblate nuclei our optical Glauber model \cite{7} predicts the opposite effect. The elliptic flow at most central (VHM) collisions of prolate nuclei to decrease significantly (see Fig.4).

For central collisions ($b = 0$ fm) the highest number of nucleon-nucleon collisions $N_{\text{coll}}$ is obtained when spins of two oblate nuclei are parallel to each other and orthogonal to the beam.
axis. In this configuration eccentricity $\varepsilon$ of the interacting volume is non-zero (being proportional to deformation parameter $|\beta|^2$ of the oblate nuclei). Increasing the relative azimuthal angle of nuclear spins from $0 \rightarrow 90$ degrees in this configuration causes the eccentricity to vanish and number of $n - n$ collisions $N_{\text{coll}}$ slightly decreases.

![Contour-plots of $\varepsilon; dN_{\text{ch}}/d\eta$ values obtained for central Au$^{197}$+Au$^{197}$ (left) and Ho$^{165}$+Ho$^{165}$ collisions (right) at $b = 0.0, 0.2, \ldots, 3.2$ fm. Error bars in bottom plots show the width of eccentricity fluctuations in the regions of $dN_{\text{ch}}/d\eta$ indicated in contour-plots (top).]

This explains results shown in Fig.4 obtained for very high multiplicity Au+Au collisions simulated with impact parameters $b = 0.0, 0.2, \ldots, 3.2$ fm. One should keep in mind that eccentricity values and fluctuations obtained by our optical Glauber model are subject to additional fluctuations originating from individual positions of interacting nucleons \cite{8} which are taken into account only in Monte-Carlo Glauber model simulations \cite{9}. Therefore, effects predicted here for Au+Au collisions may require high statistics and precise elliptic flow measurements at RHIC.

V Comparison with RHIC data

In heavy ion collisions experiments one can measure only the elliptic flow strength $v_2$ (not the initial eccentricity $\varepsilon$). However, hydrodynamical scenario \cite{3} for the expansion of QCD matter created in these collisions predicts the elliptic flow to scale with initial eccentricity $v_2 \approx \kappa \cdot \varepsilon$. 
This allows one to compare relative strength of $v_2$ fluctuations $\sigma_{v_2}/\langle v_2 \rangle$ with the relative strength of eccentricity fluctuations $\sigma_{\varepsilon}/\langle \varepsilon \rangle$.

In Figure 5 we compare results of Glauber Monte Carlo simulation (assuming Au$^{197}$ nucleus to be spherical) and experimental data obtained by PHOBOS collaboration [14] with results of our optical Glauber simulation assuming Au$^{197}$ nucleus to be slightly deformed ($\beta_2 = -0.131$).

![Figure 5: Relative fluctuations of eccentricity $\sigma(\varepsilon)/\langle \varepsilon \rangle$ obtained in optical Glauber model ($\beta_2^{\text{Au}} = -0.13$) in comparison with Glauber Monte Carlo simulation [14] (assuming spherical Au$^{197}$ nucleus) and relative elliptic flow fluctuations $\sigma(v_2)/\langle v_2 \rangle$ as measured by PHOBOS collaboration [14].](image)

We observe that elliptic flow fluctuations in the most central Au+Au collisions observed by PHOBOS collaboration can be almost fully accounted for assuming Au$^{197}$ nucleus to have oblate ground-state deformation $|\beta_2| \approx 0.13$. Significantly lower strength of the elliptic flow fluctuations at smaller centralities predicted in our optical Glauber model for Au+Au collisions is a simple consequence of neglecting the eccentricity fluctuations originating from variations of the individual positions of interacting nucleons in the optical Glauber model.

**VI Conclusions**

Based on the results obtained with our simple optical Glauber model simulation we suggest to re-investigate consequences of the possible ground-state nuclear deformation of Au$^{197}$ nucleus in Monte-Carlo Glauber model simulations. This would allow us to clarify the influence of the predicted oblate deformation of Au$^{197}$ nucleus on interpretations of relativistic Au+Au heavy ion collision experiments. We suggest to pay attention also to the most-central Cu+Cu collisions at RHIC since both stable Cu isotopes are predicted to be deformed [10].

We have predicted that precise measurement of the elliptic flow and elliptic-flow fluctuations in most central nucleus-nucleus collisions may allow one to determine ground-state nuclear de-
formation of these nuclei. This should apply also to nuclei with zero spin (e.g. $^{28}\text{Si}$, $^{180}\text{W}$) and unstable/exotic isotopes.

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