Research Article

Consensus of the Distributed Multiagent System with the Framework of the Small-World Network

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This paper investigates the consensus problem of the distributed multiagent system (MAS) with the small-world framework. A distributed consensus protocol is provided for the node-to-node communication. According to an error between every two neighbor agents, several consensus criteria among the agents are obtained firstly. Then, consensus criteria are obtained via the diameter of the graph of the MAS. Finally, based on the small-world framework, the consensus criteria are obtained; also, the relations among the consensus, the diameter of the path in the small-world framework, and the errors of agents are disclosed. Finally, one numerical example shows the reliability of the proposed methods.

1. Introduction

In recent years, the coordination problem of the MAS has been drawing considerable attention due to its wide range of application domains [1, 2], such as swarming [3, 4], clustering [5, 6], and flocking [7, 8]. Consensus refers to the fact that the state of each agent converges to a common value.

To solve the consensus problem, the main task is to design the distributed protocol for each agent, and all agents achieve the consensus via the local communications. Hence, the well-designed protocol and optimized topology are usually considered when we investigate the problem.

For different circumstances, we need an effective consensus protocol to ensure the system keeps the coordinative behave, and there are many related results about it, such as the first-order MAS [9–12], the second-order MAS [13, 14], the high-order MAS [15, 16], and the fractional-order MAS [17–21]. Combined with weights and inequalities, sufficient conditions for consistency between the leader node and the following node are studied for the first-order MAS in [9]. H. J. LeBlanc and X. Koutsoukos studied the standards of the first-order MAS and the higher-order MAS consistency [10]. Based on the conditions of spanning trees and strong connectivity in the orientation diagram, the timing consistency of the first-order nonlinear MAS is studied in [12], and the main conditions for MAS consistency are obtained: in the leader-following case, the topology should have a generated tree; in the absence of a reader node, the MAS topology is strongly connected. Based on the eigenvalues of the matrix, the controllability of the second-order MAS is studied in [13], and some equivalent conditions for the second-order MAS controllability are obtained. In [14], the tracking problem of the second-order nonlinear MAS with time-variable and multiple leader nodes is studied, and the proposed control protocol can effectively deal with the problems such as the disturbance and the unknown input of the reader node. Using self-triggering control and dynamic output feedback control methods, the consensus of the high-order MAS is studied, and the obtained results can improve the communication efficiency of the MAS [15].

In many applications, the topology of the MAS should be optimized; hence, the fixed topology and switching topology are considered frequently. For example, in [20], based on the switching topology, quasi-consistency of the MAS with competition and collaboration is studied, and the results show that, as long as the collaboration time is long enough, the system can achieve the consensus. In addition, with the Lipschitz conditions, the adaptive consensus of the MAS under fixed topology is studied in [21]. Additionally, state estimation problems with Markovian switching are
considered in [22, 23], and the fixed-time coordination problems are considered in [24–26].

Furthermore, by optimizing the topology of the MAS, the consensus problems of the MAS are further studied. Considering that the power system is affected by actuator saturation, the event trigger and self-trigger strategy are proposed, and the obtained results can save the resources of the network effectively [27].

Notice that most of the existed results about the consensus problems of the MAS are based on the fixed node set, and when the consensus is considered, the states of all agents are taken into account. In fact, in some applications, it is difficult to obtain all the information of the MAS, such as the BA model of scale-free network (SFN) [28], and the newly added node is connected to the related original node with a certain probability. Therefore, by comparing the error between every two nodes on a route of the communication graph, the related problems have been considered. Based on the balance between the in-degree and the out-degree, the consensus problem has been studied in [29]. In the noise environment, the consensus problem of the MAS under the SFN structure has been studied in [30], and the results show that there exists information flow between agents $v_i$ and $v_j$; or else, $a_{ij} = 0$.

Let $\mathcal{E} = \{ (i, j) | a_{ij} = 1, i \neq j \}$ be the edge set, where $(i, j)$ refers to an edge between $v_i$ and $v_j$. $N_i$ is the neighbor set of $v_i$, where $N_i = \{ j | (v_i, v_j) \in \mathcal{E} \}$. Then, the undirected weighted graph of a multiagent system (MAS) can be shown by $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$.

$\forall v_i \in \mathcal{V}$, let $x_i(t) \in \mathbb{R}^n$ be the state of agent $v_i$, where $t \in \mathbb{R}$, and the consensus of the MAS can be interpreted as follows.

$$\lim_{t \to \infty} \| x_i(t) - x_j(t) \| = 0,$$

then the system achieves the consensus, where $\| x_i(t) - x_j(t) \|$ is the norm of function $x_i(t) - x_j(t)$.

### 3. Problem Statement

Consider a MAS with the node set $\mathcal{V}$; the state of each node satisfies

$$x_i'(t) = u_i(t), \quad i \in \mathcal{N},$$

where $u_i(t)$ is the consensus protocol of node $v_i$. If $u_i(t)$ is implemented as

$$u_i(t) = \sum_{j \in \mathcal{N}, \|x_i(t)\| > 0} \frac{a_{ij}}{d_i} \left[ c_i(t) \| x_i(t) \| x_j(t) \right] - \sum_{j \in \mathcal{N}, \|x_i(t)\| > 0} \frac{a_{ij}}{d_i} x_i(t), \quad v_i \in \mathcal{V},$$

where $d_i = \sum_{j \in \mathcal{N}} a_{ij}$ and $c_i(t)$ is a function, then we have the following system:
or in a compact form,

\[ X'(t) = [A(t) - I]X(t), \]

where \( X(t) = [x_1^T(t), x_2^T(t), \ldots]^T, I = \text{diag}(1, 1, \ldots, 1), \) and

\[ A(t) = \begin{bmatrix} b_{11}(t) & b_{12}(t) & b_{13}(t) & \cdots \\ b_{21}(t) & b_{22}(t) & b_{23}(t) & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \end{bmatrix}, \]

\[ d_i^1 = \sum_{j \in N_i, \|x_j(t)\| > 0} a_{ij}, \]

\[ d_i^2 = \sum_{j \in N_i, \|x_j(t)\| = 0} a_{ij}, \]

\[ d_i = d_i^1 + d_i^2, \]

\[ b_{ij}(t) = \begin{cases} \frac{1}{d_i} \frac{\|x_i(t)\|}{\|x_j(t)\|}, & \|x_j(t)\| \neq 0 \text{ and } i \neq j, \\ 0, & i = j. \end{cases} \]

Remark 1. If \( \|x_i(t)\| = \|x_q(t)\| = 0, \) then the error between \( v_i(t) \) and \( v_q(t) \) is zero, and they achieve the consensus. If one of \( \|x_i(t)\| \) and \( \|x_q(t)\| \) is zero, then their angle of intersection is an arbitrary angle, and we define \( \cos \theta_{iq} = 1. \)

It shows that the real networks have the following property; for any two nodes in a network, the distance between them is relatively small, while, at the same time, the level of transitivity or clustering is relatively high [40]. This property, which is shared by many real-world networks, is called the small-world phenomenon. In the following sections, based on the small-world framework of graph \( \mathcal{G} \), the consensus problems of system (5) will be studied.

### 4. The Main Results

In this section, the consensus problem of the MAS is studied. Firstly, we propose the state error between every two neighbor agents. Then, in the connected topology, the state error between every two agents is provided via the route which associates with two agents. Finally, according to the diameter of a graph, consensus criteria are obtained;
Theorem 1. If \( q \in N_i \), then under protocol (5),
\[
\lim_{t \to \infty} [x_i(t) - x_q(t)] = 0,
\]
where \( \cos \theta_{i,q} = ((x_i(t) - x_q(t)) / \|x_i(t)\| \|x_q(t)\|) \).

**Proof.** First, we estimate the error between \( x_i(t) \) and \( x_q(t) \) under the condition \( \|x_i(t)\| \cdot \|x_q(t)\| \neq 0 \). Since

\[
x'_i(t) = \sum_{j \in N_i} \left[ \frac{a_{ij}}{d_i} \left( \frac{c_j(t)}{\|x_j(t)\|} - \frac{c_i(t)}{\|x_i(t)\|} \right) \right] - \sum_{j \in N_i} \frac{a_{ij}}{d_i} \|x_j(t)\|,
\]

\[
x'_q(t) = \sum_{v \in N_q, \|x_v(t)\| > 0} \left[ \frac{a_{qv}}{d_q} \left( \frac{c_q(t)}{\|x_q(t)\|} - \frac{c_v(t)}{\|x_v(t)\|} \right) \right] - \sum_{v \in N_q, \|x_v(t)\| > 0} \frac{a_{qv}}{d_i} \|x_q(t)\|,
\]

it holds that
\[
x'_i(t) - x'_q(t) = \sum_{j \in N_i} \left[ \frac{1}{d_i} c_j(t) \|x_j(t)\| \frac{1}{\|x_j(t)\|} - \frac{1}{d_q} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|} \right] - \sum_{j \in N_i} \frac{1}{d_i} \|x_j(t)\| + \sum_{v \in N_q, \|x_v(t)\| > 0} \left[ \frac{1}{d_i} c_j(t) \|x_j(t)\| \frac{1}{\|x_j(t)\|} - \frac{1}{d_q} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|} \right].
\]

For convenience, denote
\[
\delta_{i,q}(t) = x_i(t) - x_q(t),
\]

\[
f_{i,q}(t) = \sum_{j \in N_i} \frac{1}{d_i} c_j(t) \|x_j(t)\| \frac{1}{\|x_j(t)\|} - \sum_{v \in N_q} \frac{1}{d_q} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|}.
\]

We have
\[
\|f_{i,q}(t)\| \leq \sum_{j \in N_i} \frac{1}{d_i} c_j(t) \|x_j(t)\| \frac{1}{\|x_j(t)\|} - \sum_{v \in N_q} \frac{1}{d_q} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|},
\]

\[
= \sum_{j \in N_i, v \in N_q} \left[ \frac{1}{d_i d_q} c_j(t) \|x_j(t)\| \frac{1}{\|x_i(t)\|} - \frac{1}{d_q d_i} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|} \right],
\]

\[
\leq \sum_{j \in N_i, v \in N_q} \frac{1}{d_i d_q} c_j(t) \|x_j(t)\| \frac{1}{\|x_j(t)\|} - \frac{1}{d_q d_i} c_q(t) \|x_q(t)\| \frac{1}{\|x_v(t)\|}.
\]

Let \( c(t) = \max\{c_j(t), c_q(t)\} \); it follows that
\[
\left\| f_{iq}(t) \right\| \leq \sum_{j \in N_{i}, v \in N_{q}} \frac{c_{i}(t)}{d_{jq}^{2} d_{i}} \left( \sqrt{\left\| x_{j}(t) \right\|^{2} - 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \left\| x_{q}(t) \right\|} \right)^{2}
\]
\[
\leq \sum_{j \in N_{i}, v \in N_{q}} \frac{c_{i}(t)}{d_{jq}^{2} d_{i}} \sqrt{\left\| x_{j}(t) \right\|^{2} - 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \left\| x_{q}(t) \right\| + \left\| x_{q}(t) \right\|^{2}}
\]
\[
= \sum_{j \in N_{i}, v \in N_{q}} \frac{1}{d_{jq}^{2} d_{i}} \sqrt{c_{i}(t)^{2} \left[ \left\| x_{j}(t) \right\|^{2} - 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \cos \theta_{jq} + \left\| x_{q}(t) \right\|^{2} \right]}.
\]

From Assumption 1, it holds that
\[
\frac{e_{iq} - c(t)^{2}}{e_{iq} - c(t)^{2} \cos \theta_{iq}} > \frac{2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\|}{\left\| x_{j}(t) \right\|^{2} + \left\| x_{q}(t) \right\|^{2}}
\]
and it follows that
\[
\frac{e_{iq} \left[ \left\| x_{j}(t) \right\|^{2} + 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| + \left\| x_{q}(t) \right\|^{2} \right]}{e_{iq} \left[ \left\| x_{j}(t) \right\|^{2} + 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \cos \theta_{jq} + \left\| x_{q}(t) \right\|^{2} \right]} > c_{i}(t)^{2} \left( \left\| x_{j}(t) \right\|^{2} + 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \cos \theta_{jq} + \left\| x_{q}(t) \right\|^{2} \right).
\]

Hence, we have
\[
\left\| f_{iq}(t) \right\| < \sum_{j \in N_{i}, v \in N_{q}} \frac{c_{i}(t)}{d_{jq}^{2} d_{i}} \left( \sqrt{\left\| x_{j}(t) \right\|^{2} - 2 \left\| x_{j}(t) \right\| \left\| x_{q}(t) \right\| \left\| x_{q}(t) \right\| + \left\| x_{q}(t) \right\|^{2}} \right)^{2},
\]
\[
\leq e_{iq} \sum_{j \in N_{i}, v \in N_{q}} \frac{1}{d_{jq}^{2} d_{i}} \left\| x_{j}(t) - x_{q}(t) \right\|
\]
\[
\leq e_{iq} \sum_{j \in N_{i}, v \in N_{q}} \frac{1}{d_{jq}^{2} d_{i}} \left\| x_{j}(t) - x_{q}(t) \right\|
\]
\[
\leq e_{iq} \left\| x_{j}(t) - x_{q}(t) \right\|.
\]

Let the Lyapunov candidate be
\[
V_{iq}(t) = (1/2)\det_{iq}(t)^{2} \delta_{iq}(t); \text{ then, we get}
\]
\[
V'_{iq}(t) = \frac{1}{2} \left[ \delta_{iq}(t) \delta_{iq}(t) + \delta_{iq}(t) \delta_{iq}(t) \right],
\]
\[
\leq \frac{1}{2} \left[ -\delta_{iq}(t) + f_{iq}(t) \right] \delta_{iq}(t) + \delta_{iq}(t) \left[ -\delta_{iq}(t) + f_{iq}(t) \right],
\]
\[
\leq - \delta_{iq}(t) \delta_{iq}(t) + \frac{1}{2} \left[ f_{iq}(t) \right] \delta_{iq}(t) + \frac{1}{2} \left[ f_{iq}(t) \right] \delta_{iq}(t),
\]
\[
\leq - \delta_{iq}(t) \delta_{iq}(t) + \left. \epsilon_{iq} \right| x_{j}(t) - x_{q}(t) \left. \delta_{iq}(t) \right|,
\]
\[
= \left\| \delta_{iq}(t) \right\|^{2} + \epsilon_{iq} \left\| x_{j}(t) - x_{q}(t) \right\| \left\| \delta_{iq}(t) \right\|,
\]
\[
= 2(-1 + \epsilon_{iq}) V_{iq}(t),
\]
\[
< 0,
\]
and \( \lim_{t \to \infty} \delta_{iq}(t) = 0 \), namely, \( \lim_{t \to \infty} \left[ x_{j}(t) - x_{q}(t) \right] = 0 \). If \( \left\| x_{j}(t) \right\| \cdot \left\| x_{q}(t) \right\| = 0 \), then at least one of \( \left\| x_{j}(t) \right\| \) and \( \left\| x_{q}(t) \right\| \) is 0. Without loss of generality, we suppose
\[ x_q(t) = 0, \text{ namely, } x_q(t) \text{ is the zero vector, and we can get the same result. This completes the proof.} \]

**Theorem 2.** If \( G \) is connected, then under protocol (5), system (4) achieves the consensus.

\[
\lim_{t \to \infty} \left\| x_i(t) - x_q(t) \right\| \leq \lim_{t \to \infty} \left\| x_i(t) - x_i(t) \right\| + \lim_{t \to \infty} \left\| x_i(t) - x_q(t) \right\| = 0,
\]

and on the contrary, \( \lim_{t \to \infty} \left\| x_i(t) - x_q(t) \right\| \geq 0; \) then, we have \( \lim_{t \to \infty} \left\| x_i(t) - x_q(t) \right\| = 0, \) and this completes the proof. \( \Box \)

**Remark 2.** The consensus criterion of Theorem 2 is a rough result; it has not disclosed the relations among the scale of the network, the bound of tolerate errors, and the diameter of the graph \( G \). In the following results, the relations will be discussed.

**Assumption 2.** For each diameter \( P = v_i, v_{i_1}, \ldots, v_{i_L}, v_m \) of graph \( G \), we assume that there exist two functions \( V_{iq} \) and \( V_{im} \) such that

\[
V_{iq} \geq V_{iq}(t) \geq V_{im},
\]

\[
V_{iq} = (1/2)\delta_{iq}^T \delta_{iq}(t) + (1/2)\delta_{iq}^T \delta_{iq}(t) + (1/2)\delta_{iq}^T \delta_{iq}(t) + (1/2)\delta_{iq}^T \delta_{iq}(t),
\]

where \( V_{iq} = (1/2)\delta_{iq}^T \delta_{iq}(t) \), \( i, q \in \{v_i, v_{i_1}, \ldots, v_{i_L}, v_m\} \), and \( V_{im} \) and \( V_{im} \) are non-negative constants.

**Theorem 3.** If \( P = v_1, v_2, \ldots, v_{(L+1)} \) is the diameter of the graph \( G \) and \( G \) is connected, then under the control of protocol (5), system (4) achieves the consensus if

\[
\sum_{i=1}^{L+1} \epsilon_i \epsilon_{iq} < 1,
\]

where \( L \) is the length of path \( i_i, i_2, \ldots, i_{L+1} \) in the small-world network of the MAS.

**Proof.** Let \( V_{iq} = (1/2)\delta_{iq}^T \delta_{iq}(t) + (1/2)\delta_{iq}^T \delta_{iq}(t) + (1/2)\delta_{iq}^T \delta_{iq}(t) \); then, it holds that

\[
0 < \epsilon_{iq} \leq \frac{m}{LM},
\]

and it follows that \( \forall v_i, v_q \in \mathcal{G} \), \( \lim_{t \to \infty} \left\| x_i(t) - x_q(t) \right\| = 0. \) This completes the proof. \( \Box \)

**Corollary 1.** If \( P = v_1, v_2, \ldots, v_{(L+1)} \) is the diameter of a small-world graph \( G \), then under the control of protocol (5), system (4) achieves the consensus if \( \forall v_i, v_q \in \mathcal{G} \),

\[
0 < \epsilon_{iq} < \frac{m}{LM},
\]

where \( \epsilon_{iq} \) represents the tolerable error of the node \( i \) with respect to the node \( q \).
5. A Numerical Example

Example 1. Consider a connected MAS (4) with seven agents, where \( \mathcal{V} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7 \} \), and the topology is shown in Figure 1. Suppose \( x_i(t) \in \mathbb{R} \), let

\[
\epsilon_{i,q} = \frac{1}{4} c_i(t) \left( \left\| x_i(t) \right\| - \left\| x_q(t) \right\| \right) \cos \theta_{iq} + \left\| x_i(t) \right\|,
\]

(24)

where \( i, q = 1, 2, \ldots, 7 \), and according to Corollary 1, under the control of protocol (3), MAS (5) achieves the consensus. The simulation result is shown in Figures 2 and 3, where \( c(t) = \max |c_i(t)|, i = 1, 2, \ldots, 7 \).

6. Conclusion

In this paper, the consensus problem of the MAS has been investigated via the node-to-node communications. A kind of distributed protocol has been designed firstly. Then, we have obtained a system with scale-free topology. By the theoretical analysis, the results have shown that based on the protocols, all agents can achieve the consensus; meanwhile, a criterion on the small-world framework has been obtained. Finally, the numerical example has shown the reliability of the proposed methods.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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