On the nature of the S stars in the Galactic Center

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Abstract

Davies and King have suggested that the bright stars observed on short-period orbits about Sgr A* ("S stars") are old, low-mass stripped AGB stars rather than young, high-mass main-sequence stars. If the observationally inferred effective temperatures and luminosities of these stars are correct, however, then DK have grossly overestimated the post-AGB lifetimes and hence underestimated the production rate in steady state. In fact, the total mass in stars stripped over the age of the Galaxy would exceed that of the stellar cusp bound to Sgr A*.

We take issue also with Davies & King’s estimates of the energetics involved in capturing the stars onto their present orbits.

1 Introduction

The S stars with detectable orbital proper motions lie within \( \lesssim 0.5 \sim 4000 \) au of Sgr A*, and the brighter ones have \( K = 14 - 15 \) mag (Schödel et al., 2003). Infrared absorption lines of HI and HeI in their spectra are consistent with early main-sequence B stars (Ghez et al., 2003; Eisenhauer et al., 2005), with implied masses \( \sim 15 M_{\odot} \), luminosities \( \gtrsim 10^4 L_{\odot} \), and effective temperatures \( T_{\text{eff}} \sim 35,000 \) K. How young massive stars might form or be captured so close to the black hole is a fascinating question, but not one addressed here.

(Davies & King, 2005, henceforth DK) have suggested that the S stars are in fact old low-mass stars that were stripped of their envelopes after evolving onto the asymptotic giant branch (AGB). This proposal has several apparent advantages. (i)
Theoretically, the luminosities of AGB stars depend on the masses of their cores rather than their envelopes and exceed $10^4 L_\odot$ for $M_c \gtrsim 0.7 M_\odot$ (Paczyński, 1970a), so that, if the envelopes were sufficiently thoroughly stripped, they could have the required $L \& T_{\text{eff}}$. Indeed, the central stars of planetary nebulae are just such objects. 

(ii) The challenge of explaining young, short-lived stars on tightly-bound orbits would be circumvented. (iii) DK argue that the energy required to strip the AGB stars of their envelopes would come at the expense of the orbital energy of their cores, and that this accounts for the strongly bound orbits.

Unfortunately, the lifetimes of DK’s stars would not be $\sim 10^6$ yr, as they suggest, but more like $\sim 10^3 - 10^4$ yr. The short lifetime undercuts advantages (i) & (ii), while (iii) has its own problems (§4).

2 Observational constraints

The lifetimes of the S stars are sensitive to their luminosities, so it is important to understand the extent to which these luminosities are known.

At $K$ ($\bar{\lambda} = 2.157 \, \mu m$), one is observing these stars well within the Rayleigh-Jeans regime:

$$\frac{hc}{\lambda k_b T_*} \approx 0.22 \left( \frac{3 \times 10^4 K}{T_*} \right).$$

Consequently, the shape of the continuum is almost independent of the brightness temperature $T_*$ and its amplitude varies only linearly:

$$S_\nu \approx \frac{2\pi R_*^2 k_b T_*}{\lambda^2 d_{\text{GC}}^2}. \quad (1)$$

From the spectra given by Eisenhauer et al. (2005), $S_\nu \sim 5 - 30 \, mJy$ at $K$; S2, which has the shortest and best-determined orbit (15.24 ± 0.36 yr), is among the brightest: $S_\nu(S2) \approx 25 \, mJy$. Eisenhauer et al. (2005) have corrected their spectra for an assumed extinction of 2.8 mag at $K$. The luminosity is of course $L \equiv 4\pi \sigma T_{\text{eff}}^4 R_*^2$, hence very sensitive to the actual value of $T_*$ if we may equate that to $T_{\text{eff}}$. The estimates for $T_{\text{eff}}$ rests not upon the shape of the continuum but upon the infrared HI and HeI lines. The implied radii and luminosities of the S stars are

$$R_* = 4.6 \left( \frac{S_K}{10 \, mJy} \right)^{1/2} \left( \frac{3 \times 10^4 K}{T_*} \right)^{1/2} R_\odot,$$

$$L_* = 1.5 \times 10^4 \left( \frac{S_K}{10 \, mJy} \right) \left( \frac{T_*}{3 \times 10^4 K} \right)^3 \left( \frac{T_{\text{eff}}}{T_*} \right)^4 L_\odot. \quad (2)$$
We have used Eisenhauer et al. (2005)’s value for the distance to the Galactic Center: $d_{GC} = 7.62 \pm 0.36$ kpc, which is based mainly on the orbit of S2.

It is clear that the main uncertainty in determining $L_\ast$ is the inference of $T_{\text{eff}}$ from the absorption lines. If $T_{\text{eff}} \sim 15,000$ K instead of $30,000$ K, the lifetime estimated in §3 could be very much longer: by almost two orders of magnitude according to the analytic scalings, which neglect post-AGB winds.

### 3 Theoretical lifetimes of shell-burning stars

It is well established that the luminosity of AGB and post-AGB stars is solely a function of the mass of the degenerate carbon-oxygen core, $M_c$, until the envelope is exhausted. Fundamentally, this is because the CNO and triple-alpha reactions are very sensitive to temperature, while hydrostatic equilibrium demands that the temperature at the base of a radially extended envelope is proportional to $GM_c/R_c$. Paczyński (1970a) quotes

$$\frac{L}{L_\odot} \approx 5.925 \times 10^4 \left( \frac{M_c}{M_\odot} - 0.522 \right) \quad M_c \geq 0.57 M_\odot. \quad (3)$$

The more recent calculations of Vassiliadis & Wood (1994) yield a similar result,

$$\frac{L}{L_\odot} \approx 5.6694 \times 10^4 \left( \frac{M_c}{M_\odot} - 0.5 \right) \quad (4)$$

On the other hand, given $L$ and hence $M_c$ and $R_c$, the photospheric radius ($R$) and effective temperature $T_{\text{eff}}$ clearly depend only on the mass and composition of the envelope. Thus, the structure and evolution of DK’s stripped stars is the same as that of the central stars of planetary nebulae (henceforth CSPN) at the same $L$ and $T_{\text{eff}}$, notwithstanding differences in how the bulk of the AGB envelope was lost.

Using either of relations (3) & (4), the core mass corresponding to $L = 10^4 L_\odot$ is $M_c \approx 0.7 M_\odot$. Interpolating between Paczyński (1970a)’s models for $M_c = 0.6 M_\odot$ and $M_c = 0.8 M_\odot$, we find that the time required to evolve from $T_{\text{eff}} = 10^4$ K to $T_{\text{eff}} = 10^5$ K is $\approx 1300$ yr. The corresponding time in Vassiliadis & Wood (1994)’s tracks is also $1000 - 2000$ yr (depending on metallicity) for hydrogen-burning CNSP and $\approx 3000$ yr for helium burning ones, but requires progenitor masses $M_{\text{init}} > 1 M_\odot$ and total ages significantly less than the age of the Galaxy because of VW’s prescriptions for mass loss in the AGB phase.
Although computer models are needed for accurate results, the strength of the theoretical constraints is best appreciated from simple analytical arguments. There is nothing new in these except the context; see, e.g., Paczyński (1970b).

Energy transport through the envelopes of stripped or post-AGB stars is radiative rather than convective (the outer convective zones of main-sequence stars, which have higher surface gravities than those hypothesized here, disappear at \( T_{\text{eff}} \gtrsim 7000 \text{ K} \)). From the equations of hydrostatic and radiative equilibrium, it follows that

\[
\frac{dP_{\text{rad}}}{dP} = \frac{\kappa L_r}{4\pi G M_r c} \equiv 1 - \beta ,
\]

in which all symbols have their usual meaning. Since the mass of the envelope is negligible compared to that of the core, as will be seen, and since the nuclear reactions are concentrated in narrow shells, \( M_r \approx M_c \) and \( L \approx L \) at \( r > R_c \). Therefore, if the opacity is constant (a reasonable approximation for the low values of \( \rho/T^3 \) relevant here), then \( \beta = 1 - L/L_E \) is constant, where the Eddington luminosity \( L_E \equiv 4\pi G M_c / \kappa \). Integrating (5), one has \( P_{\text{rad}} = (1 - \beta)(P - P_0) \). In the Eddington approximation, the constant of integration is \( P_0 \approx \beta a T_{\text{eff}}^4 / 6 \). At the photosphere \( \tau = 2/3 \), \( P_{\text{rad}} = 2P_0 \), and in a hydrogen shell source where \( T \sim 10^{7.5} \text{ K} \), \( P_{\text{rad}}/P_0 \sim 10^{12} \).

Thus \( P_0 \) can be ignored for the purposes of estimating the envelope mass, so that the gas pressure \( P_{\text{gas}} = k_B T \rho / \mu m \) and radiation pressure \( P_{\text{rad}} = a T^4 / 3 \) are in the constant ratio \( \beta/(1 - \beta) \). The envelope is therefore polytropic:

\[
\rho \approx \frac{\beta \mu m_a a}{3k_B T^3}.
\]

Using these relations to eliminate \( \rho \) and \( P \) in favor of \( T \) in the hydrostatic equation \( dP/dr = -GM \rho / r^2 \) leads to

\[
T \approx \frac{GM_c \beta}{4k_B} \left( \frac{1}{r} - \frac{1}{R} \right),
\]

where \( R \) is another constant of integration and is comparable to the photospheric radius. Combining (6) & (7) and integrating from \( R_c \) to \( R \) yields the envelope mass:

\[
M_{\text{env}} \approx \frac{\pi a}{48} \left( \frac{\mu m_a}{k_B} \right)^4 (GM_c)^3 \frac{L_E}{L_*} \left( 1 - \frac{L_s}{L_E} \right)^4 f \left( \frac{R}{R_c} \right),
\]

where

\[
f(x) \equiv \int_1^x \left( 1 - \frac{t}{x} \right)^4 \frac{dt}{t} \approx \begin{cases} \frac{1}{4}(x-1)^4 & \text{ln } x \ll 1, \\ \ln x & \text{ln } x \gg 1. \end{cases}
\]
$R_c \approx 10^{-2} R_\odot$ as for a cold carbon-oxygen white dwarf, and $R \approx 5 R_c$ to match eq. 2, so \(\ln(R/R_c) \approx 6\). For solar composition and electron-scattering opacity (larger $\kappa$ implies smaller $M_{\text{env}}$), $\kappa \approx 0.34 \text{ cm}^2 \text{ g}^{-1}$ and $\mu \approx 0.62$. Taking $M_c = 0.7 M_\odot$, $L_E \approx 2.7 \times 10^4 L_\odot$, $L = 10^4 L_\odot$, \(\text{(2)}\) predicts $M_{\text{env}} \lesssim 1.3 \times 10^{-3} M_\odot$. Hydrogen burning will consume all of this in 4600 yr, somewhat longer than the times cited for the computer models above. However, as the fuel is consumed, eqs. (5) & (9) imply that \(R/R_c\) must decrease exponentially with time. The corresponding e-folding time of $T_{\text{eff}}$ is $\tau \approx 1500$ yr. If one applies the same formulae to a pure helium envelope at the same $L$, the increased molecular weight (4/3) and decreased opacity (0.2 cm$^2$ g$^{-1}$) give $M_{\text{env}} \approx 0.14 M_\odot$, i.e. two orders of magnitude larger. But since the energetic yield per gram of helium fusion is about a tenth that of hydrogen, the timescale $\tau$ increases roughly tenfold. For smaller luminosities, $M_{\text{env}}$ and $\tau$ vary approximately as $L^{-1}$ and $L^{-2}$, respectively.

Mass loss by winds would further shorten the evolutionary timescale.

4 Discussion

Eisenhauer et al. (2003) tally $\approx 10$ stars with measured velocities $\geq 500 \text{ km s}^{-1}$ and $K = 14 - 16$ mag within 0.7 of Sgr A*. Adopting their extinction $A_K = 2.8$ mag, one finds that the median flux density of these stars is $S_K \approx 9 \text{ mJy}$, which corresponds following eq. (2) to $L = 1.3 \times 10^4 L_\odot$ if $T_{\text{eff}} = 3 \times 10^4 \text{ K}$. The corresponding median lifetime $\bar{\tau} \lesssim 10^4 \text{ yr}$. Thus, if the current epoch is typical, the putative AGB stars would have to be stripped and captured by the black hole at a rate $\gtrsim 10^{-3} \text{ yr}^{-1}$; this would consume $\gtrsim 10^7 M_\odot$ over the age of the Galaxy. However, the entire stellar mass within 1.9 pc (approximately the cusp radius $GM_{\text{bh}}/\sigma_\ast^2$) is only $\approx 3 \times 10^6 M_\odot$ (Genzel et al., 2003).

Although this is reason enough to reject DK’s proposal, it has other problems. There is nothing special about $T_{\text{eff}} = 3 \times 10^4 \text{ K}$1 Depending on the severity of stripping, one would expect stars of a given core mass to be stripped down to a range of initial $T_{\text{eff}}$ extending to $< 3 \times 10^4 \text{ K}$. Since $L$ is fixed by $M_c$, lower $T_{\text{eff}}$ produces brighter fluxes at $K$, so that the coolest stripped stars would would outshine the observed S stars. In other words, fine tuning is required to explain the uniformity of the observed temperatures.

There are also dynamical difficulties. DK equate the loss of orbital energy by the core to the energy required to strip the envelope. To leading order in the ratio

\[^1\text{Other that this being the minimum temperature for a CSPN to ionize its nebula. But that is not relevant to the S stars.}\]
$R/p$ of stellar to pericentral radius, however, the tidal field and the distribution of stripped material are symmetric (quadrupolar) with respect to the core, so that the energy required to unbind the material comes mainly at the expense of its own orbit rather than that of the core. One may perhaps avoid this difficulty if the stripping is preceded by pericentral passages in which less violent tidal interactions agitate the envelope without removing it. But then there is a further difficulty. Assuming that the star is much less tightly bound initially, DK’s argument implies a characteristic semimajor axis after stripping

$$a \approx \frac{M_{bh}}{M_{env}} R_{env},$$

where $M_{env}$ is the initial mass of the envelope before stripping and $R_{env}$ is its virial radius, while $M \approx 3.6 \times 10^6 M_\odot$ is the mass of the black hole. Taking $M_{env} \sim M_\odot$ and $R_{env} \sim R_c$, DK are gratified to find $a \sim 10^{-3} \text{pc}$, in agreement with the observed semimajor axes of the S stars. However, these values of $M_{env}$ and $R_{env}$ are inconsistent. The convective part of the envelope of the red giant might indeed have a mass $\gtrsim 0.1 M_\odot$, but it is a polytrope of index $3/2$ rather than 3, so that its virial radius is comparable to its outer radius, which is much larger than the core radius. The characteristic value of $a$ is larger by the same factor.

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