Quantum cryptographic ranging

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Abstract. We present a system to measure the distance between two parties that allows only trusted people to access the result. The security of the protocol is guaranteed by the complementarity principle in quantum mechanics. The protocol can be realized with available technology, at least as a proof of principle experiment.

The following touching problem will be addressed in this paper. Alice is lost in the woods. Her friend Bob needs to find her, rescue her, and live happily ever after. On the other hand, the bad wolf Eve also wants to find her, in order to gobble her up. Suppose for simplicity an unidimensional forest, Alice and Bob need to find their relative position (ranging) without giving any hint to Eve. This intent is analogous to the one underlying cryptography, i.e. the exchange of information in a secure fashion. In this respect we will refer to it as crypto-positioning procedure.

In this paper, we show how the quantum mechanical time–energy uncertainty principle can be employed to measure in a quantum cryptographically secure fashion the distance between Alice and Bob. This means that the physical limitations imposed by quantum mechanics do not allow, not even in principle in the ideal case, any kind of eavesdropping on Eve’s part. The secure exchange of information, based on analogous “weird” quantum effects has been shown long ago in [1–3] and technological applications seem to be at hand [4]. Our idea to help Bob in his quest stems by joining Ekert’s quantum cryptographic protocol [5] with the recently proposed quantum positioning protocol [6], that allows one to perform ranging using frequency-entangled states.

In [3,4] we have shown how, by using $N$ frequency-entangled photons, one can obtain an $1/\sqrt{N}$ accuracy enhancement in finding out the distance between Alice and Bob over the case in which the $N$ photons are unentangled. This is a truly quantum effect that arises from the strong photon correlations between photons originating from the entanglement. This same fact, however, makes the loss of a single photon critical: as shown in [6], when one of the entangled photons that travel from Bob to Alice is lost, the remaining photons yield no information at all on the distance between the two. Such an apparent drawback turns out to be the key feature in devising the crypto-positioning protocol.

In what follows we can limit our analysis to the case $N = 2$. In this situation it is possible to use the state generated by cw-pumped spontaneous parametric down-conversion, in which the two generated photons are anti-correlated in frequency, i.e.

$$|\Psi\rangle \equiv \int d\omega \, \phi(\omega)|\omega_0 + \omega\rangle_L |\omega_0 - \omega\rangle_S .$$

In Eq. (1), the notation $|\nu\rangle$ refers to a single photon state of frequency $\nu$, the ket subscripts refer to the two distinct field modes (the signal $S$ and the idler $I$) generated by the crystal, $2\omega_0$ is the pump frequency, and $\phi(\omega)$ is the two-photon spectral function centered in $\omega = 0$ with bandwidth $\Delta\omega$. The state $|\Psi\rangle$ is one of the best known sources of entanglement currently available and an enormous amount of literature both theoretical and experimental is accessible (see for example [7] and citations therein). Another possibility that can be exploited is the recently proposed ‘difference beam state’ [8] that displays frequency correlated photons, whereas $|\Psi\rangle$ displays anti-correlation in frequency.

The properties of the entanglement in the state $|\Psi\rangle$ are such that, if one measures the frequency of the signal photon, he/she would obtain a random value $\omega_0 + \omega$ with probability density $|\phi(\omega)|^2$, but the subsequent measurement on the idler photon will have the predictable outcome $\omega_0 - \omega$ (and vice versa if the measurements are reversed). On the other hand, it is possible to show that if one measures the time of arrival of the first photon on a detector, then he/she will be able to predict (with an accuracy of the order of $\Delta\omega^{-1}$) the time of arrival of the second photon on a second detector at a distance $L$. In fact, the joint probability of measuring the first photon at time $t_1$ and the second at time $t_2$ is given by

$$P_c(t_1,t_2) \propto \left| \int d\omega \, \phi(\omega) e^{-i\omega(t_1-t_2+L/c)} \right|^2,$$

(2)

which exhibits a peak centered in $t_2 - t_1 = L/c$ of width proportional to $\Delta\omega^{-1}$. What happens when one measures the frequency of the first photon and the time of arrival of the second? In this case it is possible to show that the outcome of the time of arrival measurement is completely unpredictable: all the timing information has been “erased” by the frequency measurement. In this respect, the measurement of the frequency on one of the photons has the same effect as the loss of such photon.
FIG. 1. Alice and Bob randomly choose to measure either the time of arrival $T$ or the frequency $\omega$ on each copy of the two-photon state $|\Psi\rangle$ they share. They retain only the copies for which their choices agree, i.e. the checked ($\checkmark$) copies.

The crypto-positioning procedure, depicted in Fig. 1, is the following:

1. Bob produces a certain number of labeled copies of the two-photon state $|\Psi\rangle$. Of each copy he sends Alice one of the two photons (e.g. the signal photon).

2. Of the idler photon he did not send Alice, he randomly measures either the frequency or the time at which it reaches a photodetector placed at a known distance from him.

3. In the thick of the woods, Alice receives Bob’s photon and she also randomly chooses to measure either the frequency or the time of arrival.

4. Alice and Bob broadcast the kind of measurement (frequency or time of arrival) they performed on each of the two-photons copies. They discard all the measurement results of the cases in which their choices did not match.

5. Alice and Bob exchange the results of the frequency measurements and compare them. If the communication channel is perfect and there is no eavesdropper measuring photon transit times, these results are correlated (namely, if Alice had obtained the frequency $\omega_0 + \omega$, Bob must have found $\omega_0 - \omega$). On the contrary, if Eve measured the transit times of the photons, she unavoidably spoiled the frequency entanglement. Alice and Bob find this out when they compare their data, since it will no longer be correlated.

6. Once they have verified that Eve is not tapping on the exchanged photons, Alice broadcasts to Bob all the measurement results of the photon times of arrival. This information is useless to anybody except Bob (who knows the timing information of the other photon of each couple) and it allows him to find the lost Alice through Eq. (2) which yields the distance $L$, given the photon times of arrival $t_1$ and $t_2$.

Notice that Eve can measure the frequency of the traveling photons without being detected. This, however, does not allow her to gain any information on the distance $L$. She will only succeed in disturbing Alice and Bob’s communication.

The scheme can be straightforwardly adapted to much more complicated scenarios. For example, one may tailor the entanglement to situations in which multiple rescuers are present and they can obtain Alice’s position only if they meet and exchange their data in the spirit of quantum secret sharing protocols. A discussion of the quantum crypto-positioning protocol can be also found in [6].

In conclusion, we have presented a protocol that, using the frequency entangled state at the output of a parametric downconversion crystal, allows one to perform quantum crypto-positioning. No matter how many resources the evil Eve devotes to eavesdropping, she will not be able to prevent a happy ending, since only Bob will find Alice!

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