Geostatistical Modeling by the Ordinary Kriging in the Estimation of Mineral Resources on the Kieselguhr Mine, Algeria

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Abstract. The most important step in a feasibility study of a mining project is the determination of reserves in situ. But the question which arises that no method of estimation can give exact value since there is inevitably an associated error when it is important to know the order of magnitude. Decision-makers need to know if the estimated content is correct or not. Geostatistics can help the mining engineer to obtain from the available information, and thus, enable him to decide if the project deserves more investments. Geostatistics provides not only the estimated value but also with the kriging variance, a measure of the accuracy of the estimate. This is one of the superiorities of geostatistics over traditional methods of estimating reserves. In this article, the Kieselguhr deposit reserve of the Sig mine (western Algeria) was evaluated according to the linear geostatistical method with ordinary Kriging (OK), to know the economic value of this mine that allows the exploitation of this deposit, and to carry out study cartography accompanied by 2D modeling of deposit. Finally, we obtained maps in which we could estimate exploitable and geological reserves, and know the characteristics of the deposit.

1. Introduction
In light of the recent drop in oil prices and the concentration of Algeria's efforts in the mining sector, the government seeks to accelerate the pace of exploitation in all mines, particularly in the kieselguhr mine in Mascara. The increase in production from the exploitation of kieselguhr deposit from 3500t to 80 000t [1] in Mascara has become a necessity. The primary objective of this article is to estimate the kieselguhr reserves of the Sig deposit (Algeria) using the spatial interpolation method known as "Kriging".

Kriging is the optimal method, in the statistical sense of the term, of estimation. It can be used for both interpolation and extrapolation. Here we use 2D spatial interpolation. Kriging is named after its precursor, the South African mining engineer D.G. Krige [2]. The latter has developed a number of empirical statistical methods to determine the spatial distribution of ores from a set of boreholes. However, it is the French Matheron [3] who formalized the approach by using the interrelations between boreholes to estimate their spatial distribution. He named it the "Kriging method". And he was the first to use the term "geostatistics" for statistical modeling of spatial data.

Geostatistics [4], [5] comes from the engineering sciences, born from problems of mineral resource
estimation, and has found in recent decades applications in many fields, especially in the environment, as, for example, that of estimating acoustic exposure in an open environment [6] or radio exposure in urban areas [7].

2. Methodology

In this article, all the variographic calculations, their modeling as well as the ordinary Kriging [2]. Calculations were carried out using the software S-GeMS [8]. This software makes it possible to calculate the histograms, the variograms with the adjustment of the theoretical models, the Kriging maps and their variances. The data processing was performed using an Excel computer tool, after that requires a special form of the file containing the field information studied, extension "txt". A database will be prepared using the software (S-GeMS).

3. Statistical and Variographic Study

The use of a variographic model is necessary in order to be able to calculate or rather to estimate the global estimation variances, and also to be able to determine the kriging weights in the local estimation.

This modeled variogram (theoretical) that will be used. The step of estimating and modeling the variogram from all the available data is traditionally called structural analysis or variographic analysis [9].

3.1. The theoretical semi-variogram

The theoretical variogram is defined as the quadratic expectation of the random variable [(Z (x) - Z (x + h))]:

\[ \gamma(h)_{th} = 1/2 \text{Var}[Z(x) - Z(x + h)] = 1/2 E [(Z(x) - Z(x + h))^2] \quad (1) \]

Where x is the coordinate vector (1,2 or 3 dependsvthe case). h is the distance vector.

This function, usually increasing according to h, synthesizes a lot of information concerning the joint behavior of the random variables and concerning the "continuity" of the mineralization [9].

3.1.1. The spherical model. The spherical model is probably the one most commonly used. It has a simple polynomial expression and its shape corresponds to what is often observed: an almost linear growth to a certain distance, then a stabilization [10], [11].

\[ \gamma(h) = C_0 + C (3h/2a - h^3/2a^3) \quad \ldots \ldots \quad 0 \leq h \leq a \quad (2) \]

\[ C + C_0 \quad \ldots \ldots \ldots \ldots \quad h > a \]

Where: range a, sill C + C0, C0 nugget effect.

4. Ordinary Kriging

Kriging is a stochastic method of spatial interpolation that predicts the value of a natural phenomenon at unsampled sites [12]. Was developed by Matheron. G. In this method the mean is assumed to be unknown but uses the invariant on the estimate point neighborhood. This method only calls for the hypothesis of intrinsic stationarity [2].

\[ \sum^n_j \gamma_j \text{Cov} [Z_i, Z_j] + \mu = \text{Cov} [Z_i, Z_j] \quad \forall i = 1 \ldots n \quad (3) \]

\[ \sum^n_j \gamma_j = 1 \]

\( \mu \): is an unknown constant,

The minimum estimate variance, called kriging variance, is obtained as a substitute.

The kriging equations in the general expression for the estimation variance:

\[ \sigma_k^2 = \sigma_e^2 = \text{Var}[Z_v] - \sum^n_{i=1} \lambda_j \text{Cov}[Z_v, Z_j] - \mu \quad (4) \]

Since the variance of the estimate is also written in terms of variogram, one can also rewrite the kriging system according to the variogram.
\( \sigma^2_e \): The estimation variance
\[
C(h) = \sigma^2 - \gamma(h) \quad \text{And} \quad \sum \lambda_i = 1
\]
\[
\left\{ \sum_i^n \lambda_i y(x_i, x_j) - \mu = \bar{y}(v, x_i) \right\} \forall i = 1 \ldots n
\]
(5)

And, then
\[
\sigma^2_k = \sum_{i=1}^n \lambda_i \bar{y}(v, x_i) - \bar{y}(v, v) - \mu
\]
(6)

It is interesting to visualize the ordinary kriging system and the ordinary kriging variance in matrix form:
\[
K \lambda = \kappa
\]
\[
\sigma^2_k = \sigma^2 - \lambda. \kappa
\]
(7)

When
\[
K = \begin{bmatrix}
\text{Cov}(Z_1, Z_v) \\
\text{Cov}(Z_2, Z_v) \\
\text{Cov}(Z_n, Z_v)
\end{bmatrix}
\quad \text{And} \quad \lambda = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix}
\quad \text{And} \quad \sigma^2_v = \bar{C}(v, v)
\]
(8)

5. Descriptive statistics and histogram
A random variable \((x)\) follows a lognormal law when its logarithm follows a normal law. As in the normal probability plot, the cumulative frequencies will represent a straight line if the given values have a lognormal distribution [12].

5.1. The thicknesses of the layer of diatomite
Statistical analysis of field data from the exploration phase shows that the kieselguhr layer has the largest thickness; where it can reach up to 34.5 m, with an average of 11.52 m. Three quarters of the thicknesses are less than 11.7 m, the minimum thickness is 2.8 m, with a variance of 29.95 which reflects the heterogeneity of the thickness distribution. The results obtained are shown in (Figure 1) below:

5.2. The contents of kieselguhr layer
The average SiO2 content of the Core drill is 53.95%, with a maximum of 60.85% and a minimum of 44.57%. The variance is 11.32, which reflects the homogeneity of the distribution of the contents in this layer. The results obtained are shown in (Figure 2) below:

![Figure 1. Histogram of the thicknesses of the Kieselguhr layer](image-url)
According to the analysis of the frequency curves (thicknesses, contents) of the mineralized layer (kieselguhr), one finds that the distributions have some null values which later can approach neither the log-normal law, nor of the normal law. This drives us to look for other methods of adjustment.

6. Variographic analysis
This model on which kriging is based, largely assumes the knowledge of the spatial dependence structure of the random function $\gamma$ [13]. However, in practice, this is rarely known. The variographic analysis is a preliminary step to the kriging, which allows estimating it. This analysis is, in fact, the study of the spatial behavior of the regionalized variable examined [14].

6.1. The variograms thickness of the layer of Kieselguhr
First of all, the experimental variograms that have been calculated according to the principal directions (0°, 45°, 90°, 135°) as shown in (Table 1). All directions have been presented in order to be able to estimate the parameters of the variograms of the thicknesses and contents. The directional variograms of the thickness models show a nugget effect of 3 and a sill of 29 for different ranges, which explains the presence of geometric anisotropy. The minimum range in the 90° direction variogram is 185. And we can the same directional variograms of the content with the nugget effect of 3, a sill of 9.2 for different ranges, with the minimum range in the 90° direction variogram is 120.
Table 1. Parameters for calculating variograms

| Interdistances | Number of lags | Lag separation | Lag tolerance |
|----------------|----------------|----------------|--------------|
| 500            | 80             | 40             |

| Directions | Number of Directions | 5 |
|------------|----------------------|---|
| Number of Directions | 5 |
| Azimuth (°) | Dip | Tolerance | Bandwidth |
| Omnidirectionnel | 0 | 0 | 90 | 5000 |
| 1 | 0 | 0 | 22.5 | 500 |
| 2 | 45 | 0 | 22.5 | 500 |
| 3 | 90 | 0 | 22.5 | 500 |
| 4 | 135 | 0 | 22.5 | 500 |

6.1.1 Omnidirectional variogram. The omni-directional variogram of the thickness of the Kieselguhr layer was constructed and adjusted by a spherical model with a range (a) is 320 m, C₀ = 3 nugget effect and a sill C (0) = 29 as shown in (Figure 3).

6.2 Study variographic contents

We proceeded, in the same manner, to establish the experimental variograms for the contents with the principal directions (0°, 45°, 90°, 135°) as shown in (Table 1) which respect the azimuthal direction, in order to choose the theoretical model suitable for the estimation.

6.2.1 Omnidirectional variogram. The omni-directional variogram of the SiO2 contents of the Kieselguhr layer was constructed and adjusted by a spherical model with a range = 220 m, and a nugget effect C₀ = 3 and a sill C (0) = 9.2 as shown in (Figure 4).
In conclusion, the variographic analysis is a very important part in extracting the theoretical parameters that will be used later, for kriging estimation on the studied phenomenon. This is the essential step in any geostatistical study and certainly the least amenable to automation.

**Note:** The numbers on the dots represent the number of pairs of samples used to calculate the point in question.

7. Kriging methodology

The estimate consists of kriging the thickness and the contents of the kieselguhr layer. The variogram models of the kieselguhr layer were used for ordinary two-dimensional (2D) kriging of thicknesses and SiO2 contents. We considered a regular horizontal grid (2D) that covers the entire surface area of the mine, with a unit mesh of (5x5 m²). The grid thus constructed comprises a total of 309 x 360 points.

7.1 Results achieved by ordinary kriging

7.1.1 Map of kieselguhr thicknesses. The following map as shown in (Figure 5) shows us the distribution of the thicknesses of the layer of kieselguhr and also that the dominant thicknesses have values varying from (2.8 to 8.09 m) are found in the center and a little on the center direction of the East of 23m.
7.1.2 Variance of kriging for kieselgurh thicknesses. The kriging variance map as shown in (Figure 6) shows the maximum kriging variance of 64 at the ends of all directions. But according to the maximum variance of the variogram of thicknesses does not exceed 29.9. On the prospected area, we notice that variance does not exceed 27, therefore less than 29.9. Here we can say that the thicknesses are acceptable for the exploitation, which gives us clarifications on the quality of the map estimated by kriging.

7.1.3 Map of distribution of SiO2 contents. According to the observation of the map as shown in (Figure 7) that represents the spatial distribution of the contents, it can be said that where the contents have high considered values are found much in the center and southwest and a little on the north-east. With values range from (53 to 58), and as we move towards the center of East, the values gradually begin to decrease.
7.1.4 Kriging variance for contents. According to the maximum variance variogram of the contents is 11.7. The kriging variance map as shown in (Figure 8) shows that the maximum variance of 24.4 at the ends of the North-West and South directions, this can be explained by the lack of information (lack of sampling) in these places. On the prospected zone, we notice that the variance of contents does not exceed 8, therefore less than 11.7. Which shows a very good estimate and this area gives us acceptable contents that justify the possibility of exploitation.

8. Estimate of reserves and classification of SiO2 reserves by categories
The mineral reserves estimated in this section are exclusively those delineated at the current mining exploration concession at Tahalait. Ordinary kriging results are used to calculate the tonnage of kieselguhr layer reserves at the Tahalait deposit (Sig) (Table 2), based on the following formula:

\[ Q_m = \sum V_i \cdot \rho 
\]

Or

\[ V_i = S \cdot E_{pi} \]

Qm: Quantity of kieselguhr ore.
Vi: Volume of the unit block.
\( \rho \): Specific weight of kieselguhr ore = (0.95 t / m3),
S = 25 m2. Epi: Elemental thickness.
Table 2. Summary table of reserves by category

| JORC  | Category of reserves | Layers  | Tonnages (Mt) | SiO2 (%) |
|-------|----------------------|---------|---------------|----------|
| Proven| A                    | Kieselguhr | 623 402.47   | 53.74    |
| Probable| B                   | Kieselguhr  | 5 296 173.57   | 53.97    |
| Possible| C                   | Kieselguhr   | 13 985 128.45   | 54.40    |

8.1 Distribution of contents by category

These maps are estimated by the variance of SiO2 contents. As we approach the point to be estimated, the variance decreases, making the estimate more reliable and the reserves increasing to higher categories. The following maps as shown in (Figure 9, 10, 11) show the geological reserves by category of the kieselguhr layer.

![Figure 9. Proven Reserves (A)](image)
![Figure 10. Probable Reserves (B)](image)
![Figure 11. Possible Reserves (C)](image)

9. Results and discussions

Two-dimensional (2D) geostatistical modeling is a very efficient tool for carrying out very fast studies for the characterization and estimation of reservoirs. With additional advantages of 2D modeling that it allows data posting in 2D space and it gives permission to easily visualize the different properties (thicknesses and contents) of the reservoir.

The distribution of core drilling is irregular, particularly in the northwest, as all maps show, and this slightly hinders the estimate of the reserve accurately. In this work, we used all the data of grill with the coordinates xy, and these grids identified inside zone to study, for the calculation of the reserve.

34 core drill holes were made we can see that are generally found in the central axis of the zone as illustrated in all maps, so it is necessary to make other core drill in a regular way for the precision of calculation reserves and to identify the direction optimal to exploitation, especially in areas that are not yet exploited.
And finally, we find the results of the estimate of exploitable reserves are 5,919 Mt that allows the exploitation given the economic value this ore.

10. Conclusion
The problem we have been concerned with is the evaluation of the SiO2 reserves of the Sig deposit of kieselguhr by the geostatistical method. In this research, we have focused on three essential objectives. First of all, a bibliographic research was carried out to study the bases of the new techniques of spatial interpolation and in particular of ordinary kriging (ok), the latter has been presented from a theoretical and applied point of view. The second step is represented by the translation of digital data (Core drill) of land that will be used to establish the variographic models and which are used in turn for the ordinary kriging (OK). The third step is the estimation of reserves through the kriging maps obtained.

The maps obtained by ordinary 2D kriging are simple to manipulate, which allows a better presentation of the deposit. Kriging ensures the valuation and implementation of maps according to the appropriate methods, as well as the control of contents and thicknesses, which directs the work of exploitation for a rational management. And we finally concluded that the entire geological reserve is 13 985 128.45 Mt contains 5,919 Mt exploitable.

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References
[1] ENOF Internal document of National Company of Non-Ferrous Mining Products, Report about Sig Mine, Algeria, 2004.
[2] D. G. Krige, A statistical approach to some basic mine valuation problems on the Witwatersrand. Journal of the Chemical, Metallurgical and Mining Society of South Africa, 52, 119-139, 1951.
[3] G. Matheron, Principles of Geostatistics. Economic Geol., 58, 1246-1268, 1963.
[4] J. Chilès, P. Delfiner, Geostatistics. Modeling Spatial Uncertainty. New York: Wiley,1999.
[5] H. Wackernagel, Multivariate Geostatistics. an Introduction with Applications. Berlin: Springer-Verlag, 3rd ed, 2003.
[6] O. Baume, B. Gauvreau, M. Bérangier, F. Junker, Junker, and H. Wackernagel, Apport des méthodes géostatistiques à l'estimation de l'impact acoustique d'une source en environnement ouvert, Acoustique & Techniques, vol. 45, pp. 29-33, 2006.
[7] Y. Ould Isselmou, H. Wackernagel, W. Tabbara, J. Wiart, Geostatistical estimation of electromagnetic exposure. In geoENV VI, Geostatistics for Environmental Applications (A. Soares, M. Pereira, and R. Dimitrakopoulos, eds.), pp. 59–69, Springer, 2008.
[8] Stanford geostatistical Modeling Software, [Online] Available at: http://sgems.sourceforge.net/?q=node/77.
[9] E. Xavier, “Géostatistique linéaire”, Linear Geostatistics, Center of Geostatistics, Paris School of Mines, p. 403, 2001 (In French).
[10] M. Armstrong, J. Carignan, “Géostatistique linéaire”. application to the mining field, the presses of the mining school, 115 p, 1997 (In French).
[11] A. G. Journel, C.J. Huijbregts, Mining Geostatistics. Academic Press, 600 p, 1978.
[12] M. Mazari, “Etude géostatistique des ressources minières: exemple d’une mine en cours d’évaluation”, Geostatistical study of mining resources: example of a mine under evaluation, Mémoire de Magister, Ecole Nationale Polytechnique, 2012 (In French).
[13] S. Baillargeon, “Le krigage: revue de la théorie et application à l’interpolation spatiale de données de précipitations, Mémoire de maître és sciences”, Kriging: A Review of the Theory and Application to Spatial Interpolation of Precipitation Data, Master of Science, Université Laval Québec, 2005 (In French).
[14] A. Laurent, Acquisition, “Traitement et restitution des données d’une reconnaissance archéologique: La ville gallo-romaine du vieil-Evreux”, doctoral thesis, University Paris 6, 2003 (In French).