Observation of Weak Collapse in a Bose-Einstein Condensate

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We study the collapse of an attractive atomic Bose-Einstein condensate prepared in the uniform potential of an optical-box trap. We characterise the critical point for collapse and the collapse dynamics, observing universal behaviour in agreement with theoretical expectations. Most importantly, we observe a clear experimental signature of the counterintuitive weak collapse, namely that making the system more unstable can result in a smaller particle loss. We experimentally determine the scaling laws that govern the weak-collapse atom loss, providing a benchmark for the general theories of nonlinear wave phenomena.

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I. INTRODUCTION

Wave collapse occurs in a wide range of physical contexts, including optics, atomic and condensed-matter physics. Generally, collapse occurs if an attractive nonlinearity exceeds a critical value. If the collapse is triggered at time $t = 0$, the wave amplitude asymptotically diverges at some point in space as the collapse time $t_c$ is approached. In practice, the amplitude divergence results in dissipation of wave energy (or particle loss).

The unifying theoretical framework for understanding different collapse phenomena is provided by the nonlinear Schrödinger equation, which has been extensively studied for various forms of nonlinearity [1, 2]. This general formalism is applied to self-focusing of light [3–7], collapse of Langmuir waves [8, 9] and Bose-Einstein condensates (BECs) [10–14], and even surface water waves [15, 16].

In this framework, wave collapse is classified as either strong or weak (see Fig. 1). In a strong collapse, a finite portion of the wave (here 100%, for simplicity) collapses towards the singularity. In weak collapse, as time progresses, a diminishing fraction of the wave approaches the singularity, with long tails left behind.

FIG. 1. Strong versus weak collapse (cartoon). In strong collapse, a finite portion of the wave (here 100%, for simplicity) collapses towards the singularity. In weak collapse, as time progresses, a diminishing fraction of the wave approaches the singularity, with long tails left behind.

In Sections II-IV we address in turn: (i) the critical point

Previous collapse experiments with atomic BECs [22–27] (see also [28]) were performed in the traditional setting of a harmonic trap. The critical point [23] and collapse times [24, 26] were in general agreement with theoretical expectations [10, 36–43], but no evidence of weak collapse was observed; the atom loss was only seen to grow with $|a|$ [25].

In this article we study BEC collapse in a new experimental setting, using a $^{40}$K condensate [44, 45] prepared in the uniform potential of an optical-box trap [46]; for details of our setup see Appendix A. The combination of large system size (up to 41 µm) and fine tuning of the scattering length (with a resolution of 0.03 $a_0$, where $a_0$ is the Bohr radius) gives us a very large dynamic range: we observe metastable attractive BECs with up to $2 \times 10^5$ atoms, and collapse times that vary between 3 and 400 ms. We demonstrate the expected scaling of the critical scattering length $a_c$ with the BEC atom number $N$ and the system size $L$, and show that the collapse time can be expressed as a universal function of the dimensionless interaction strength $a N/L$. Most importantly, we observe conclusive evidence for weak collapse, namely the counterintuitive decrease of the atom loss with increasing $|a|$, and experimentally determine the scaling laws that govern the weak-collapse atom loss. The weak nature of the collapse is directly revealed only by resolving single collapse events, and is obscured in the multiple collapse regime, which has been seen in previous cold atom experiments.

In Sections II-IV we address in turn: (i) the critical point...
for the collapse, (ii) the collapse dynamics in a system that is suddenly made unstable by an interaction quench, and (iii) the aftermath of the collapse, which reveals its weak nature.

II. CRITICAL POINT

The starting point for our discussion is the GP equation for a homogeneous box potential, with a heuristically added three-body loss term [13]:

\[
\frac{i\hbar}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{4\pi\hbar^2a}{m} |\psi|^2 \psi - i\frac{\hbar K_3}{2} |\psi|^4 \psi, \tag{1}
\]

where \(m\) is the atom mass, \(K_3\) is the three-body loss coefficient, \(\psi\) is normalised to the atom number \(N\), and the boundary condition is \(\psi = 0\) at the trap walls.

We use a cylindrical box trap [46] of variable length \(L\) and radius \(R\), and always set \(R = L/2\), so \(L\) is the only length-scale characterising the system size. We may thus rewrite Eq. (1) in a dimensionless form, defining \(\tilde{r} = r/L\) and \(\tilde{t} = t/\tau_0\), with characteristic time \(\tau_0 = 2mL^2/\hbar^2\):

\[
\frac{\partial \tilde{\psi}}{\partial \tilde{t}} = -\nabla^2 \tilde{\psi} + \alpha|\tilde{\psi}|^2 \tilde{\psi} - i\eta |\tilde{\psi}|^4 \tilde{\psi}, \tag{2}
\]

where

\[
\alpha = \frac{8\pi aN}{L} \quad \text{and} \quad \eta = \frac{N^2 m K_3}{\hbar L^2}, \tag{3}
\]

and \(\tilde{\psi}\) is initially normalised to unity. For the range of scattering lengths that we study, we assume that \(K_3\) is constant [47, 48], with the value \(1.3(5) \times 10^{-41} \text{ m}^6 \text{s}^{-1}\) [49]. The corresponding value of \(\eta\), for all our \(L\) and \(N\) values, is very small (\(< 6 \times 10^{-4}\) and thus three-body loss is negligible in our (meta)stable condensates. However, if the BEC collapses, significant loss occurs, providing the primary experimental signature of the collapse.

Neglecting the atom loss in a metastable BEC, based on Eq. (2) the critical interaction strength, \(a_c\), can depend only on the boundary conditions, i.e. the box shape. For a family of self-similar boxes \((R/L = \text{const.})\) it should be a universal constant, so \(a_c \propto L/N\).

To experimentally study the critical point for collapse, we prepare a stable BEC at \(4a_0\), then over \(1\) s ramp the scattering length to a variable \(a < 0\), wait for \(2\) s before turning off the trap and imaging the atoms after \(100\) ms of time-of-flight (ToF) expansion. We image the cloud along the axial direction of our cylindrical trap, and for ToF we jump the scattering length to \(20 a_0\).

In Fig. 2(a) we show how, for a given initial \(N\), the final atom number depends on the negative \(a\). A well defined \(a_c\) is signaled by a sharp drop in the atom number. As shown in Fig. 2(b), the atom loss is accompanied by a qualitative change in the appearance of the cloud in ToF.

In Fig. 2(c) we plot \(a_c\) for \(L = 30 \mu\text{m}\) and a wide range of \(N\) values, from \(10^4\) to \(2 \times 10^5\). We clearly observe the expected scaling \(a_c \propto 1/N\) (see also Appendix B). In Fig. 2(d) we plot the measured \(Na_c\) versus box size and confirm the scaling \(Na_c \propto L\). We find that the dimensionless critical interaction strength is \(a_c = -4(1)\), where the error includes the systematic uncertainties in box size and absolute atom number calibration. For comparison, numerical simulations of the GP equation for our box geometry give \(a_c = -4.3\).

III. COLLAPSE DYNAMICS

To study the collapse dynamics, we perform interaction-quench experiments [24]. We prepare a BEC just above \(a_c\) and then quench the scattering length to a variable \(a < a_c\) to initiate the collapse (see Appendix B for more details). After a variable hold time \(t\) we jump the scattering length to \(20 a_0\), switch off the trap, and observe the cloud in ToF.

As shown in the left panel of Fig. 3(a), for quenches close to the critical point (small \(|a - a_c|\)), at \(t_c\) the atom number suddenly drops to a stable lower value. We understand this as a single collapse event. On the other hand, for large quenches [right panel of Fig. 3(a)], the atom number appears to gradually decay until it stabilises. Such behaviour, also seen in [24, 26], is understood as arising from a series of multiple (experimentally unresolved) collapses [13, 20, 36, 37, 50–54], and we accordingly associate \(t_c\) with the onset of the atom loss [55]. (For further evidence for the occurrence of single and multiple discrete collapse events see Appendix C.)

In Fig. 3(b) we show typical ToF images for different times after the quench. At \(t < t_c\), before any change in the atom number occurs, the swelling of the cloud in ToF reveals the shrinking of the wavefunction in-trap. Right after \(t_c\), within the first \(\approx 10\) ms, we observe that the remnant cloud consists of a lower-energy central part and a higher-energy shell, reminiscent of the atom bursts generated during collapse in [24]. At longer times we observe more irregular patterns. We see a similar shell structure in images taken along a perpendicular direction, which implies that the outgoing atom shell is spher-
FIG. 3. Collapse dynamics. (a) Atom number versus time after quenches to $a = -0.86 a_0$ (left) and $-2.19 a_0$ (right); here $L = 30 \mu m$ and $N = 11.4 \times 10^5$, corresponding to $a_c = -0.79 a_0$. Green bands indicate $t_c$ and its uncertainty. (b) Typical ToF images at various stages after the quench (here for $a = -1.02 a_0$). (c) Collapse time versus $a$ for six data sets taken for various $L$ and $N$; see legend in (d). The shaded bands indicate $a_c$ values. (d) Universal collapse dynamics. We plot the dimensionless collapse time versus the reduced distance from the critical point, for all six data sets. The solid line shows the results of lossless GP simulations without any free parameters.

IV. WEAK COLLAPSE

We now turn to the aftermath of the collapse. Since $\tilde{\psi}$ is initially normalised to unity, the fractional atom loss, $\Delta N/N$, should be some universal function of $\alpha$ and $\eta$; here $\Delta N = N - N_t$ is the difference between the initial (pre-collapse) and the final (time-dependent) atom number. The counterintuitive implication of the weak-collapse theory is that $\Delta N/N$ decreases if the BEC is made more unstable, by quenching $a$ to a more negative value.

In Fig. 4 we focus on one data set, for fixed $L = 30 \mu m$ and $N = 20.3 \times 10^4$. As we illustrate in the left panel of Fig. 4(a), close to the critical point, where we observe only single-collapse events, the atom loss indeed decreases with $\eta$, between $4 \times 10^{-5}$ and $4 \times 10^{-4}$. The solid line in Fig. 3(d) shows results of lossless GP simulations, without any free parameters; we reproduce a very similar dependence of $t_c$ on $\alpha$, although the numerical values are systematically slightly below the experimental ones.
increasing $|a|$, indicating weak collapse. On the other hand, as shown in the right panel of Fig. 4(a), in the regime of large quenches and multiple collapse, the atom loss in the long-time limit shows the opposite trend; only this type of behaviour was seen in harmonic-trap experiments [24, 25].

In Fig. 4(b) we present a consistent picture of the atom-loss trends for all $a < a_c$, from $a/a_c \approx 1$ to $a/a_c \to \infty$. Here we plot $\Delta N/N$ versus $a_c/|a|$, and for each $a$ show $\Delta N/N$ values observed for all $t$; the points clustered around $\Delta N \approx 0$ correspond to $t \approx t_c$.

The single-collapse regime, $a_c/|a| < -0.6$, is clearly identified by the small spread of the non-zero $\Delta N$ values. The single-collapse atom loss clearly decreases with increasing $|a|$, and extrapolates to zero for $a_c/|a| \to 0$. This is the unambiguous signature of a weak collapse. The dot-dashed black line shows a linear extrapolation, which gives $\Delta N/N = -0.02(2)$ for $a_c/|a| = 0$, while the (almost indistinguishable) solid black line shows a power-law fit, $\Delta N/N \propto |a|^{-1.05(7)}$.

For $a_c/|a| > -0.6$, multiple collapse occurs, because the diminishing single-collapse atom loss does not re-stabilise the system. However, we see that even in this regime the minimal loss we observe at each $a$ still follows the weak-collapse trend (solid black line). It is also instructive to plot the function $\Delta N/N = 1 - a_c/a$ (dashed purple line); this is atom loss such that, if a quench to a given $a < a_c$, the atom number drops to the new critical value $N_c(a) = a_c L/(8 \pi a) = N a_c/a$ [see Eq. (3)]. This equilibrium stability criterion is not obviously applicable in the non-equilibrium situation after the first collapse [25]. Still, it provides a good estimate of both the point, $a_c/|a| \approx -0.6$, beyond which the single-collapse loss is insufficient to re-stabilise the system (see also Appendix C), and the long-time loss at large $a/a_c$.

We now extend the study of the weak-collapse atom loss to other $L$ and $N$ values [see Fig. 5]. In this analysis we include all $a$ values for which only single collapse occurs, and also those where clearly resolved single and double collapses occur (see Appendix C).

Writing $\Delta N/N \propto |a|^{-\gamma}$ for each data set with fixed $L$ and $N$, as in Fig. 4(b), we always get $\gamma$ consistent with unity [see Fig. 5(a)]; averaging over all data sets gives $\bar{\gamma} = 1.02(2)$. We then assume the form $\Delta N/N = C/|a|$ and study the dependence of $C$ on $L$ and $N$. As shown in Fig. 5(b), on a log-log plot, we find $C \propto N^{-0.51(2)}$, with no clear dependence on $L$; the two points taken with $L = 16 \mu m$ and $41 \mu m$ fall onto the same line as the four points taken with $L = 30 \mu m$.

Thus we experimentally find that weak-collapse atom loss is described very well by $\Delta N/N \propto 1/(\sqrt{N}|a|)$. From Eq. (3), this corresponds to $\Delta N/N \propto \eta^{1/4}/|a|$, which is indeed independent of $L$, and vanishes in the limit of infinitely strong attraction, $|a| \to \infty$. We note that while the weak collapse atom loss does not depend on $L$ (the overall size of the box) it may depend on the box shape; this is an interesting question for future research.

In Fig. 5(c) we plot all our single-collapse data versus $\eta^{1/4}/|a|$ and confirm that it falls onto a single universal curve [56]. These experimentally obtained scaling laws should provide useful input for further theoretical work.

V. CONCLUSIONS AND OUTLOOK

In conclusion, we have performed a comprehensive study of the collapse of an attractive BEC confined in the homogeneous potential of a 3D box trap. We have fully characterised the critical point for collapse, and the collapse dynamics of an interaction-quenched BEC, finding universal behaviour in agreement with the theoretical expectations. Most importantly, we have provided conclusive experimental evidence for the counterintuitive weak collapse, and have experimentally determined weak-collapse scaling laws that should provide a useful reference point for the general theories of nonlinear wave phenomena.

Our work also points to many avenues for further research. It would be very interesting to explore quenches from a large positive $a$, where the BEC is initially deep in the Thomas-Fermi regime, and in the case of a box potential the density is uniform. In this case it is not obvious how the condensate would spontaneously ‘choose’ the position at which to collapse, or whether many local collapses would occur instead of a global one. Additionally, since the fractional atom loss cannot exceed 100%, the linear trend seen in Fig. 5(c) cannot extend to the regime of strong dissipation (large $\eta$). It would be interesting to explore that regime using a different geometry, a different Feshbach resonance, or a different atomic species. Finally, a major extension would be to perform similar experiments with 2D gases, for which a strong collapse and hence fundamentally different behaviour is expected.
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APPENDIX A - EXPERIMENTAL SETUP

Our setup is the first 3D BEC box experiment with tunable interactions. The setup for producing harmonically trapped $^{39}$K condensates is similar to our previous apparatus [45]. The main difference is that here we employ the gray molasses technique [57–59] and directly cool $^{39}$K without the need for sympathetic cooling with rubidium atoms (see also [60, 61]). We load the laser-cooled atoms directly into a crossed optical dipole trap (using a 20-W 1070-nm laser) and achieve efficient evaporative cooling using the Feshbach resonance in the $|F, m_F⟩ = |1, 1⟩$ state at 402.70(3) G [62]. This results in a quasi-pure BEC of $≈ 2 \times 10^5$ atoms. We then load the atoms into a cylindrical optical box formed by blue-detuned (552 nm) laser light, and cancel out gravity with a magnetic field gradient, as in [46]. The loading procedure is essentially 100% efficient and results in a quasi-pure box-trapped BEC of $≈ 2 \times 10^5$ atoms.

The Feshbach resonance in the $|1, 1⟩$ state has a width of $ΔB = 52$ G and the background scattering length is $a_{bg} = −29 a_0$ [64]. Hence, near the zero-crossing of $a$, at $≈ 350$ G, the variation of the scattering length with the magnetic field is $a_{bg}/ΔB ≈ 0.6 a_0/G$. We tune $B$ in steps of 50 mG, corresponding to a scattering length resolution of 0.03 $a_0$.

APPENDIX B - SCATTERING LENGTH CALIBRATION

The exact magnetic field at which the scattering length in the $|1, 1⟩$ state vanishes was independently measured in Ref. [63] to be $B_{a=0} = 350.4(1)$ G. For Fig. 2(a–c) we calculate our $a$ values assuming $B_{a=0} = 350.4$ G. Fitting the data in Fig. 2(c) with a free intercept gives an intercept $a_c(1/N = 0) = 0.03(1) a_0$, which is consistent with zero within the systematic 0.06 $a_0$ error due to the uncertainty in $B_{a=0}$. We take this to be an unbiased confirmation of the zero intercept and the expected scaling $a_c ∝ 1/N$, and use this scaling to slightly refine the value of the zero-crossing field, to $B_{a=0} = 350.45(3)$ G. The remaining 30 mG uncertainty in $B_{a=0}$ corresponds to a systematic uncertainty in our $a$ values of $≈ 0.02 a_0$.

For our interaction quenches we have determined, using radio-frequency spectroscopy, that the magnetic field takes 4 ms to change (from 20 to 80 % of the jump). We account for this delay in our determination of the collapse time, and also include an additional 2 ms uncertainty in all the reported $t_c$ values.

APPENDIX C - FROM SINGLE TO DOUBLE COLLAPSE

In Fig. 6 we present evidence for a gradual transition between single- and double-collapse events, which strongly supports the interpretation that an increasing number of discrete collapse events occur as $|α|$ is increased. This data was taken with $L = 30$ μm and initial $N = 11.4 \times 10^4$.

![Figure 6](image_url)

**Figure 6.** Transition from one to two collapse events. (a) Atom number versus $t$ for $L = 30$ μm, initial $N = 11.4 \times 10^4$, and various closely spaced $a$ values; here $a_c = −0.79 a_0$. On the right we show histograms of $N_f$ values. For $t > t_c$ we see two clearly resolved $N_f$ branches, corresponding to one (upper branch) and two (lower branch) collapse events. The probability of a double collapse gradually increases with $|α|$. (b) Fractional atom loss versus $|α|$ on a log-log plot. The transparent black circles show the raw data. The coloured circles and diamonds show, respectively, the average values for the single- and double-collapse events. The colour code is the same as in (a). The error bars show the standard deviations. The purple dashed line shows the BEC stability criterion as in Fig. 4(b) and the shading shows its uncertainty. The solid black line shows the (single-event) weak-collapse scaling, $ΔN/N ≈ 1/|α|$. 
In Fig. 6(a) we show the evolution of $N_f$ after a quench to various $a < a_c$. A fine scan of $a$ resolves a striking bifurcation of the collapse outcome. We interpret the upper and lower branch as the result of, respectively, one and two collapse events. As $|a|$ is increased, the probability of a double collapse gradually increases. This crossover is highlighted in the histograms shown on the right.

In Fig. 6(b) we show the fractional atom loss versus $|a|$ on a log-log plot. In the regime where a double collapse occurs, the single-collapse branch still clearly follows the weak-collapse scaling $\Delta N/N \propto 1/|a|$. Note that in this data set the double collapse occurs slightly closer to $a_c$ than expected from the simple equilibrium stability criterion (purple band).

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[55] Note that in general $t_c$ is best defined as the time when the maximum density is reached and the atom loss rate is the highest. In all our data sets we observe the highest loss rate at the onset of atom loss.

[56] Here each point and its error bar show the average and standard error for a cluster of single-collapse points taken for same $\{a, N, L\}$ and just different $t > t_c$, such as seen in Fig. 4(b) for $a_c/|a| < -0.6$.

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