Research Article

Study on Model of Penetration into Thick Metallic Targets with Finite Planar Sizes by Long Rods

Juan Wang,1 Junhai Zhao,2 Jianhua Zhang,1 and Yuan Zhou1

1College of Science, Chang’an University, Xi’an 710064, China
2School of Civil Engineering, Chang’an University, Xi’an 710061, China

Correspondence should be addressed to Juan Wang; wangjuanhao@chd.edu.cn

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A finite cylindrical cavity expansion model for metallic thick targets with finite planar sizes, composed of ideal elastic-plastic materials, with penetration of high-speed long rod is presented by using the unified strength theory. Considering the lateral boundary and mass abrasion of the target, the penetration resistance and depth formulas are proposed, solutions of which are obtained by MATLAB program. Then, a series of different criteria-based analytical solutions are obtained and the ranges of penetration depth of targets with different ratios of target radius to projectile radius ($r_t/r_d$) are predicted. Meanwhile, the numerical simulation is performed using the ANSYS/LS-DYNA finite element code to investigate the variations in residual projectile velocity, length, and mass abrasion. It shows that various parameters have influences on the antipenetration performance of the target, such as the strength coefficient $b$, $r_t/r_d$, the shape of the projectile nose, and the impact velocity of the projectile, among which the penetration depth has increased by 18.95% as $b = 1$ decreases to $b = 0$ and has increased by 32.28% as $r_t/r_d = 19.88$ decreases to $r_t/r_d = 4.9$.

1. Introduction

The cavity expansion theory (CET) is the main theoretical method to study the penetration problem [1–5]. Since a medium-low speed projectile penetrating into a target travels along an approximately straight trajectory, its mass consumption can be ignored, and the projectile can be considered a rigid projectile [6, 7]. However, when a high-speed long rod penetrates into a target, the projectile-target interface stress is greater than the yield strength of the material and is sufficient to deform and erode the projectile [8]. To properly reflect the main physical process of penetration, Tate’s eroding rod model is developed [9]. For years, investigators have successfully applied the combination of Tate’s model and the CET to penetration problems with high-speed projectiles and have achieved many results by adopting different constitutive models to study different targets [10]. To consider all stress components of a unit and make the results be applied to all kinds of materials, the unified strength theory (UST) has been used to analyze the dynamic response of long rods penetrating into different targets [10–12] and to establish the calculation model of penetration depth, whose results are the closest to test results. However, most of the existing theoretical studies on the problems of high-speed impact have assumed that the planar size of the target is infinite, not considering the influence of the lateral free boundary of the target. The few investigations of penetration into thick targets that are finite in radial extent by projectiles at high speed are limited to experiments [13, 14]. Given this, Jiang et al. [15] have proposed the finite cylindrical cavity expansion (FCCE) theory and the penetration model of thick metallic targets with finite planar sizes by long rod systematically. Nevertheless, this model is applied only to materials whose ultimate shear strength is 0.577 times the tension or compression yield strength because it has adopted the von Mises yield criterion, and it has not made comprehensive parametric analysis.

To further study the mass erosion of a long rod at a high-speed impact and the effects of the lateral free boundaries,
expand the scope of application for the solution, and ensure the material antipenetration properties to be played fully, using the UST to consider the effect of the second principal stress, a new FCCE model is built to deduce the radial pressure solution for the cavity wall of a thick metallic target composed of an ideal elastic-plastic material and with finite planar sizes when it is penetrated by a high-speed long rod. Furthermore, the resistance formula and penetration depth formula are obtained, which are also adapted to the semi-infinite metallic target. The penetration process is simulated with the explicit finite element software LS-DYNA to study the variation in the motion behavior of the projectiles. Comparisons of the presented model, the previous test, and the model in other documents show that the presented model in this paper has higher precision and incorporates the case in [15]. In the MATLAB program, the penetration depth ranges of metallic targets with different \( r_{f} / r_{d} \) are predicted. In addition, the factors influencing the terminal ballistic effects are studied. The conclusions of this study can ensure the material properties to be played fully and will be helpful for the design of protective structures.

2. UST

The UST was established in 1991 which considers the effects of all stress components and can be applied to all kinds of materials. If \( \sigma_1, \sigma_2, \), and \( \sigma_3 \) are maximum principal stress, intermediate principal stress, and minimum principal stress of a unit, respectively, the mathematical expression of the UST is as [16]

\[
\sigma_i - \frac{a}{1 + b} (\sigma_2 + \sigma_3) = \sigma_i, \quad \sigma_2 \leq \frac{\sigma_i + a \sigma_3}{1 + a}, \quad (1a)
\]

\[
\frac{1}{1 + b} (\sigma_1 + b \sigma_3) - a \sigma_3 = \sigma_i, \quad \sigma_2 \geq \frac{\sigma_i + a \sigma_3}{1 + a}. \quad (1b)
\]

Here, \( a \) is the tension-compression strength ratio; the influence coefficient \( b \) (0 \( \leq b \leq 1 \)) shows intermediate principal stress which has an influence on material yield, which is called yield criterion coefficient.

3. FCCE Model Based on UST

3.1. Computation Model. The FCCE model [15] is shown in Figure 1. The target has radius \( r_t \), its cavity radius is \( r_c \) at time \( t \) (\( r_c \) is constant), and its elastic-plastic boundary radius is \( r_p \). The whole expansion that \( r_c \) increases from 0 to \( r_c \) (the end value) can be divided into two stages. The first stage is the elastic-plastic stage \( (r_p < r_c) \), its cylinder consists of the cavity region \( (0 < r \leq r_c) \), plastic region \( (r_c < r \leq r_p) \), elastic region \( (r_p < r \leq r_c) \), and undisturbed region \( (r_c < r \leq r_c) \). When \( r_c = r_c \), the undisturbed region disappears, and when \( r_c = r_c \), this stage ends. The second stage is the plastic stage \( (r_p \equiv r_c) \), and the cylinder consists only of the cavity region and plastic region. When there appear cracks on the cylinder, the plastic stage ends.

The FCCE model is an axisymmetric plane strain problem; \( \sigma_z \) \( (\sigma_z = (m/2)(\sigma_r + \sigma_\theta)) \) is the intermediate principal stress, where \( m \) is this stress parameter \( (2v \leq m \leq 1, v \) is Poisson’s ratio of the material) and \( m = 1 \) in the plastic region [17]. Because of \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \), if the pressure is positive, \( \sigma_1 = \sigma_r, \sigma_2 = \sigma_\theta, \) and \( \sigma_3 = \sigma_p \), then \( \sigma_2 \leq (\alpha \sigma_1 + \alpha \sigma_3) / (1 + \alpha) \).

For metallic materials, we generally take \( \alpha = 1, B = (2 + b) / (2 + b) \). With equation (1a), we obtain

\[
\sigma_r - \sigma_\theta = \frac{2(1 + b) \sigma_r}{2 + b} = B. \quad (2)
\]

For ideal elastic-plastic materials, the strain-stress relation can be written as

\[
\sigma = E \epsilon \quad \text{(elastic region)}, \quad (3a)
\]

\[
\sigma = \sigma_{iy} \quad \text{(plastic region)}. \quad (3b)
\]

Combining equations (2) and (3a)·(3b), we can get

\[
\sigma_r - \sigma_\theta = \frac{E}{1 + v} (\epsilon_r - \epsilon_\theta) \quad \text{(elastic region)}, \quad (4a)
\]

\[
\sigma_r - \sigma_\theta = \frac{2(1 + b) \sigma_{iy}}{2 + b} \quad \text{(plastic region)}, \quad (4b)
\]

where \( \sigma_r \) and \( \sigma_\theta \) are the radial and hoop stress, respectively; \( \epsilon_r \) and \( \epsilon_\theta \) are the radial and hoop strain, respectively; and \( E \) is the elastic modulus of the material.

If the displacement of the particle is \( s \), whose spatial coordinates are \( r \) at time \( t \), the geometric relations are as follows [15]:

\[
\epsilon_r = -\frac{\partial s}{\partial r}, \quad (5)
\]

\[
\epsilon_\theta = -\frac{s}{r} \quad \text{(elastic region)}, \quad (6a)
\]

\[
\epsilon_r = -\frac{\partial s}{\partial r} \quad \frac{1}{1 - \frac{\partial s}{\partial r}} \quad (6b)
\]

\[
\epsilon_\theta = -\frac{s}{r - s} \quad \text{(plastic region)}. \quad (6c)
\]

In the elastic zone, with equation (5) and the particle displacement field relation, one obtains [15]
where \( \varepsilon_\tau \) and \( \varepsilon_\theta \) are determined by the continuity conditions and yield condition of the elastic-plastic boundary.

Substituting equations (9) and (8a) into (10), we can get

\[
\sigma_r - \sigma_\theta = \frac{E \varepsilon_\tau}{(1 + \nu)r^2} \quad \text{(elastic region),}
\]

\[
\sigma_r - \sigma_\theta = \frac{2(1 + b)\sigma_{\mu\tau}}{2 + b} \quad \text{(plastic region).}
\]

The velocity field of particles for the FCCE model is as follows [15]:

\[
v = \frac{r_c^2}{2} \varepsilon_c.
\]

Based on the momentum conservation, we can get

\[
\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = -\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right),
\]

(10)

where \( \rho \) is the density of the material.

Equations (5), (6), and (8a)-(10) form the fundamental equations of the FCCE model for ideal elastic-plastic materials based on UST.

3.2. Cavity Wall Radial Stress Calculation

3.2.1. Elastic-Plastic Stage \((r_c < r)\). When \((r_c < r \leq r_p)\), substituting equations (2) and (9) into (10) yields

\[
\frac{\partial \sigma_r}{\partial r} = \frac{B}{r} - \rho r^2 c \left( \frac{1}{r} - \frac{2}{r_c} \right).
\]

(11)

Integrating equation (11) yields

\[
\sigma_r = -B \ln r - \frac{1}{2} \rho r^2 c \left( 2 \ln r + \frac{2}{r_c} \right) + M_1,
\]

(12)

where \( M_1 \) is a constant.

According to the boundary condition \( \sigma_r |_{r=r_c} = \sigma_{rc} \), \( M_1 \) is formulated as follows:

\[
M_1 = \sigma_{rc} + B \ln r_c + \frac{1}{2} \rho r_c^2 \left( 2 \ln r_c + 1 \right).
\]

(13)

Combining equations (13) and (12), the radial stress is as follows:

\[
\sigma_r = \sigma_{rc} + B \ln \frac{r_c}{r} - \frac{1}{2} \rho r_c^2 \left( 1 + \ln \frac{r_c}{r} - \frac{r_c^2}{r^2} \right).
\]

(14)

Substituting the plastic region side condition of the elastic-plastic boundary \( \sigma_r |_{r=r_p} = \sigma_{rp} \) into equation (14), the radial stress on the cavity wall \( \sigma_{rc} \) can be formulated as

\[
\sigma_{rc} = \sigma_{rp} - \frac{B}{2} \ln \frac{r_c^2}{r_p^2} - \frac{1}{2} \rho r_c^2 \left( 1 + \ln \frac{r_c^2}{r_p^2} - \frac{r_c^2}{r_p^2} \right),
\]

(15)

Here, \( \sigma_{rp} \) and \( r_c/r_p \) are determined by the continuity conditions and yield condition of the elastic-plastic boundary.

Combining equations (15) and (17)-(21), the radial stress of the cavity wall in this stage \( \sigma_{rc1} \) is derived through the stress continuity conditions \( \sigma_{r|r_p} = \sigma_{rp} \):

\[
\sigma_{rc1} = \sigma_{rc0} + \Delta \sigma_{rc0} + \frac{1}{2} \rho r_c^2 c \times n.
\]

(22)

Combined equations (15) and (17)-(21), the radial stress of the cavity wall in this stage \( \sigma_{rc1} \) is derived through the stress continuity conditions \( \sigma_{r|r_p} = \sigma_{rp} \):

\[
\sigma_{rc1} = \sigma_{rc0} + \Delta \sigma_{rc0} + \frac{1}{2} \rho r_c^2 c \times n.
\]

(22)

Here, \( \sigma_{rc0} \) and \( r_c/r_p \) are determined by the continuity conditions and yield condition of the elastic-plastic boundary.

Substituting equations (9) and (8a) into (10), we can get

\[
\frac{\partial \sigma_r}{\partial r} = -\frac{E}{1 + \nu} \times \frac{r_c^2}{r^2} - \rho r_c^2 \left( \frac{1 - r_c^2}{r - r_c^2} \right).
\]

(16)

When \( r_c < r_r/r_c, \sigma = r_c, \sigma < r_t \), integrating equation (16) and combining equation (9) and the momentum conservation condition [15] \( \sigma_{r=r_c,r} = p \rho c_v \), we have

\[
\sigma_r = \frac{E}{2(1 + \nu)} \left( \frac{r_c^2 - r_r^2}{r^2} \right) + \frac{1}{2} \rho r_c^2 \times \left( \frac{r_c^2 - r_r^2}{r_c^2 - r_r^2} \right).
\]

(17)

When \( r_c \geq (r_r/r_c), \sigma = r_c, \), integrating equation (17) and considering \( \sigma_{r=r_c} = 0 \), we have

\[
\sigma_r = \frac{E}{2(1 + \nu)} \left( \frac{r_c^2 - r_r^2}{r_r^2 - r^2} \right) + \frac{1}{2} \rho r_c^2 \times \left( \frac{r_c^2 - r_r^2}{r_c^2 - r_r^2} \right).
\]

(18)

Because of the continuity of the stresses and particle velocity at the elastic-plastic boundary [18], according to equations (8a)-(8b), one obtains

\[
\frac{r_c}{r_p} = \sqrt{\frac{B(1 + \nu)}{E}}.
\]

(19)

When \( r_p = r_t \), the cavity radius \( r_c \) at the end of the elastic-plastic stage can be expressed as

\[
\sigma_c = \frac{B}{2} \ln \left( \frac{B(1 + \nu)}{E} \right).
\]

(20)

In the elastic-plastic stage, \( r_c \leq r_c1 \).

For the plane strain problems, the elastic wave velocity \( c \) can be expressed as [18,19],

\[
c = \sqrt{\frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}}.
\]

(21)

Combining equations (15) and (17)-(21), the radial stress of the cavity wall in this stage \( \sigma_{rc1} \) is derived through the stress continuity conditions \( \sigma_{r|r_p} = \sigma_{rp} \):

\[
\sigma_{rc1} = \sigma_{rc0} + \Delta \sigma_{rc0} + \frac{1}{2} \rho r_c^2 c \times n.
\]

(22)

Here, \( \sigma_{rc0} \) and \( r_c/r_p \) are determined by the continuity conditions and yield condition of the elastic-plastic boundary.

Combining equations (15) and (17)-(21), the radial stress of the cavity wall in this stage \( \sigma_{rc1} \) is derived through the stress continuity conditions \( \sigma_{r|r_p} = \sigma_{rp} \):

\[
\sigma_{rc1} = \sigma_{rc0} + \Delta \sigma_{rc0} + \frac{1}{2} \rho r_c^2 c \times n.
\]

(22)
3.2.2. Plastic Stage ($r_p = r_c$). Because of $\sigma_{1} = 0$ and equation (14) (the stress field in the plastic region), we can get the radial stress on the cavity wall in this stage $\sigma_{c2}$:

$$\sigma_{c2} = B \ln \frac{r_c}{r_1} + \frac{1}{2} p^2 \left( \ln \frac{r_2}{r_c} + \frac{r_1^2}{r_2^2} - 1 \right),$$

where $r_c$ is the cavity radius at the end of the plastic stage, which is gotten by the fracture criterion in [20], where $\varepsilon_{eq}$ and $\varepsilon_f$ are the equivalent strain and the uniaxial tension fracture strain, respectively.

Due to the boundary condition $r = r_c$, that is, $\varepsilon_c = 0$, we obtain $\varepsilon_\theta = ((\sigma_\theta + \alpha \sigma_r)/(1 + \alpha))$ from equations (1a)-(1b), and we can get the equivalent stress based on UST $\varepsilon_{eq}$:

$$\sigma_{eq} = \frac{2 + b}{2(1 + b)} \sigma - \frac{2 + b}{2(1 + b)} \sigma_\theta - \frac{2}{2(1 + b)} \sigma_\phi.$$  (24)

For the FCCE problem, on the basis of the plastic wave hypothesis [21, 22] and the stress-strain relationship, we have

$$\varepsilon_{eq} = \varepsilon_c = \sigma_\theta \varepsilon_\theta.$$  (25)

where $\varepsilon_{eq}$ is the equivalent strain based on UST. According to equations (24) and (25), $\varepsilon_{eq}$ can be given as

$$\varepsilon_{eq} = \frac{2(1 + b)}{(2 + b)} \varepsilon_\theta.$$  (26)

Considering equation (26), the strain components $\varepsilon_r = -\varepsilon_\theta \approx (1/2)(r_c/r_f)\\theta [15]$ and $\varepsilon_{eq} = \varepsilon_f$ when $r = r_c$ [20], we can get the cavity radius at the end of the plastic stage $r_{c2}$:

$$r_{c2} = r_c \left( \frac{2 + b}{1 + b} \varepsilon_f \right).$$  (27)

Combining equations (20) and (27) and considering $r_{c2} > r_{c1}$, the conclusion in this paper is applied to materials as

$$\varepsilon_f > \frac{B^2}{2E} \left( 1 + v \right),$$  (28)

which are generally satisfied by metal materials.

The theory quoted by this paper assumes that the elastic-plastic boundary movement velocity is less than the elastic wave velocity, $\hat{r}_p \leq \hat{c}$. According to equations (19) and (21), when $\hat{r}_p = \hat{c}$, the maximum cavity expansion velocity $\hat{r}_{c\text{max}}$ applied for materials in this paper meets the following conditions:

$$\hat{r}_{c\text{max}} = \frac{B(1 - v)}{\sqrt{\rho(1 - 2v)}}.$$  (29)

When $r_{c1} \leq r_d < r_{c2}$, the expansion process experiences the elastic-plastic stage and the plastic stage, but when $r_d < r_{c1}$, the elastic wave has not reached or has just reached the target lateral boundary, and the undisturbed region exists or has just disappeared, at which point the expansion process experiences only the elastic-plastic stage; when $r_{c2} < (r_{c2}/c)$, the target size shows no effect on the expansion process; when $r_{c2} > r_{c2}$, the target has been fractured.

3.3. Energy Consuming by Expanding and Mean Stress on Cavity Wall. According to [15], the energy consumed by expanding $W$ and the work done by the radial pressure on the cavity wall $\sigma_{rc}$ are equal during the whole process of the cavity radius $r_c$ increasing from 0 to $r_{c2}$. Taking a target of unit thickness, we can obtain

$$W = 2\pi \int_0^{r_{c2}} \sigma_{rc} r_c dr_c.$$  (30)

Subposing the mean radial stress of the cavity wall is $\sigma_{rc}$ and considering that the energy consumed by expansion remains unchanged, we obtain [15]

$$\sigma_{rc} = \frac{W}{\pi r_{c2}^2} = \frac{1}{2} \int_0^{r_{c2}} \sigma_{rc} r_c dr_c.$$  (31)

Substituting equations (31) into (18) and (19), one obtains

$$\bar{\sigma}_{rc} = A_1 + B_1 \left[ \frac{1}{2} \rho \sigma_f^2 \right].$$  (32)

where $A_1 = (B/2) \left( 1 + \ln\left( E/B(1 + v) \right) \right)$ and $B_1 = (v(1 - v)) + (((1 + v)(1 - 2v))/E(1 - v)) + \ln\left( (1 + v)/((1 + v)(1 - 2v)\rho \sigma_f^2) \right)$. when $r_{c2} < (r_{c2}/c)$; $A_1 = (B/2) \left( 1 + \ln\left( E/B(1 + v) \right) \right) - \left( (r_{c2}^2)(4 + (1/v)\rho) \right)$ and $B_1 = (r_{c2}/\rho) \sigma_f^2 + (r_{c2}^2/2\rho)$. when $(r_{c2}/c) \leq r_d < r_{c2}$; $A_1 = (B/2) \left( (r_{c2}^2/\rho) + 1 \right)$ and $B_1 = (r_{c2}^2/2\rho) + \ln((r_{c2}/\rho) - (1/2))$. when $r_{c2} \leq r_{c2} < r_{c2}$.

4. Penetration Effect on Metallic Thick

Target with Finite Planar Sizes for Long Rod

4.1. Penetration Model Analysis for Long Rod. Assume that the long rod has a radius of $r_d$, a density of $\rho_d$, a yield stress of $\sigma_d$, a feature strength of $Y_d$, a length of $l$ ($l_0$ is the initial length) at time $t$, and a speed of $v$ ($v_0$ is the impact speed) at time $t$. The target has a penetration velocity of $u$, a penetration depth of $x$, and a penetration resistance of $R$. According to Tate’s model [9],

$$\frac{dx}{dt} = u,$$  (33)

$$\frac{dv}{dt} = -\frac{\sigma_d}{\rho_d},$$  (34)

$$\frac{dl}{dt} = -(v - u),$$  (35)

$$0.5\rho_d(v - u)^2 + Y_d = 0.5\mu u^2 + R.$$  (36)

This paper assumes that $R = A_1$ and $r_{c2}$ is approximately equal to the channel radius as [15]

$$r_{c2} = r_d (1 + 0.287v_0 + 0.148v_0^2).$$  (36)
4.2. Penetration Depth Calculation. If \( Y_d < R \), the total penetration depth \( D \) can be calculated to \( u = 0 \) by substituting into the formula given by [15]:

\[
D = -\frac{\rho_d}{\sigma_d} \int_{u_0}^{0} \exp\left\{ \frac{\rho_d}{\sigma_d} [f(u) - f(u_0)] \right\} \frac{f'(u)}{(\rho/\rho_d) u^2 + v_{c1}^2} \, du, \tag{39}
\]

where \( f(u) = (\rho u^2/2 \rho_d) + (u/2) \sqrt{(\rho/\rho_d) u^2 + v_{c1}^2 + \sqrt{\rho_d^2}} \rho \times (v_{c1}^2/2) \times (u\sqrt{\rho/\rho_d} + \sqrt{(\rho/\rho_d) u^2 + v_{c1}^2}). \)

Substituting \( u = u_0 \) and \( v = v_0 \) into equation (35) and rearranging the equation yield

\[
u_0 = \frac{v_0 \rho_d + \sqrt{v_0^2 \rho_d^2 - (\rho_d - \rho)(2Y_d - 2A_1 + \rho_d v_0^2)} \rho_d - \rho}{\rho_d - \rho}. \tag{40}
\]

If \( Y_d > R \), the total penetration depth \( D \) includes the penetration depth \( x_1 \) of the abrasion penetration stage and the penetration depth \( x_2 \) of the rigid penetration stage; that is,

\[
D = x_1 + x_2. \tag{41}
\]

Considering the ending condition \( u = v = v_{c2} \) of \( x_1 \) and energy conservation, \( x_1, x_2 \), and the residual length of the projectile \( l_1 \) when \( x_1 \) ends can be evaluated by the following equations proposed in [15]:

\[
x_1 = -\frac{\rho_d}{\sigma_d} \frac{\rho_d}{\rho} \int_{u_0}^{v_{c2}} \exp\{[g(u) - g(u_0)]\} \times \frac{g'(u)u}{\sqrt{u^2 - v_{c2}^2}} \, du,
\]

\[
x_2 = \frac{\rho_d}{\sigma_d} A_{v2} \frac{\rho_d^2 v_{c2}^2}{2 \pi R_{f2}^2},
\]

\[
l_1 = l_0 \exp\left[\frac{\rho_d}{\sigma_d} [g(v_{c2}) - g(u_0)]\right],
\]

where \( g(u) = (\rho u^2/2 \rho_d) + (u/2) \sqrt{(\rho/\rho_d) u^2 + v_{c1}^2 - v_{c2}^2} \times (u^2/2) \times \ln(u + \sqrt{u^2 - v_{c2}^2}) \).

Integrals of the penetration depth formula can be calculated in the MATLAB program.

5. Example and Discussion

5.1. Experimental Verification. For comparison, the ballistic test data in [13] were substituted into formulas in this paper to calculate and analyze the total penetration depth and penetration resistance. The targets were machined from 4340 steel with different radii, which had a density of \( \rho = 7850 \text{ kg/m}^3 \) and an elastic modulus of \( E = 200.6 \text{ GPa} \), and its Poisson’s ratio is \( \nu = 0.29 \) and its yield stress is \( \sigma_{0y} = 1.365 \text{ GPa} \).

For comparison, the ballistic penetration resistance of the projectile and target which cannot be got by other ways [23,24], we set up 3D solid models to simulate the penetrations by LS-DYNA. The elastic-plastic hydrodynamic model and Johnson-Cook (J-C) constitutive model are used to describe the penetrator and the target, respectively.

J-C model has simple expression and easy access to data from tests, as follows [25]:

\[
\sigma_y = (A_0 + B_0 \varepsilon_{p}^{\sigma}) (1 + C_0 \ln \dot{\varepsilon}^* ) (1 - T_0^{*} y_{nu}), \tag{43}
\]

where \( A_0, B_0, C_0, m_n, n_n \) are the material parameters; \( \varepsilon_{p}^{\sigma} \) is the effective plastic strain and its rate \( \dot{\varepsilon}^* = \varepsilon_{p}^{\sigma} / \dot{k}_{0} \); \( T_0^{*} = (T_0 - T_r) / (T_m - T_r) \) is relative temperature; \( T_0 \) is absolute temperature; \( T_r \) and \( T_m \) are indoor temperature and melt temperature, respectively.

The damage model is expressed as

\[
e^d = (D_1 + D_2 \exp D_3 \sigma^* ) \left( 1 + D_4 \ln \dot{\varepsilon}^* \right) \left( 1 + D_5 T_0^{*} \right), \tag{44}
\]

where \( \sigma^* = \rho / \sigma_{cd} \) is the stress triaxiality. The fracture occurs when \( D_6 = \sum (\Delta \varepsilon / \varepsilon)^{t} = 1 \).

The elastic-plastic hydrodynamic model is material type 10. If EPS (effective plastic strain) and ES (effective stress) are defined, the yield stress expression is [25]

\[
\sigma_y = \sigma_0 + E_{l} / E_{f} \varepsilon_{p}^{\sigma} + (a_1 + p a_2) \max [\rho, 0], \tag{45}
\]

where \( E_{l} \) is Young’s modulus; \( E_f \) is the tangent modulus; \( \rho \) is the pressure.

\( \varepsilon_{p}^{\sigma} \) is calculated as

\[
\varepsilon_{p}^{\sigma} = \int_0^t (2 D_0^{l} D_0^{p})^{1/2} \, dt. \tag{46}
\]

If EPS and ES are undefined, the yield stress expression is

\[
\sigma = \left( \frac{3}{2} s_{ij} s_{ij} \right)^{1/2}, \tag{47}
\]

where \( \sigma \) is the yield stress.
where $s_{ij}$ and $D^p_{ij}$ are the deviatoric stress tensor and the plastic part of the deviatoric strain tensor, respectively.

If $EH$ (parameter of plastic hardening modulus) is undefined, the yield stress expression is $\sigma_y = g(\tau^p)$, which is gained by linear interpolation.

Referring to the literature [13,26], model parameters for calculation are determined after several trial calculations according to the analysis results of parameters influence in the literature [26]; see Table 2 for related parameters.

A Lagrange solver is applied in the simulation of the projectile and target, and surface-to-surface erosion contact is set between them. Then, a quarter model is established as shown in Figure 2, disassembled evenly by hexahedron elements in the direction of the cross section and thickness. The numbers of elements and nodes for projectile are 7500 and 9191, respectively; and these numbers for target are 144000 and 159221, respectively. Symmetric boundary constraint is imposed on the symmetry surface. Because grid discretization will cause a mass error, four times the mass of the projectile in the test (64.7g), which can be ignored.

Figure 3 shows the comparisons of the maximum penetration depth theoretical calculation in this paper when $b = 0.6$, the simulation results in this paper, the test results in [13], and the formula results in [13–15]. During the calculation, when $r_t \leq 10r_d$, $(r_t/r_d) \leq r_d < r_{c1}$, and the penetration expansion process only experiences the elastic-plastic stage, while when $r_t > 10r_d$, $r_{c1} \leq r_d < r_{c2}$, and the penetration expansion process experiences the elastic-plastic stage and the plastic stage. Figure 3 indicates that the simulation results are in good agreement with the test data, and the effectiveness and correctness of parameter selection and model establishment are proved. Moreover, the average relative error between the theoretical formula calculation in this paper, in [15], and in [13] and the test results are 2.00%, 5.47%, and 8.27%, respectively. The maximum error of calculation is 7.36% in this paper but is 10.26% for [15] and more than 20% for [13]. It is indicated that the theoretically calculated values based on UST are more accurate than others.

5.2. Projectile Erosion Change. The time course of the projectile tail velocity, penetration velocity, projectile length, and projectile mass based on the simulations can be seen from Figures 4 and 5. From these figures, we can intuitively observe that the projectile tail velocity is much faster than the projectile nose velocity at the beginning of the penetration. The projectile begins to erode at $t = 0.01$ ms, and the erosion stops at $t = 0.12$ ms when there are more mass residues of the projectile. Thereafter, the curve of the projectile mass is steady, remaining at approximately 13.863 g, which is 85.71% of the projectile mass in the early stage of penetration (16.175 g), and the projectile length is 3.198 cm, which is 41.05% of the initial projectile length (7.79 cm). Because the length loss is much larger than the mass loss, the projectile becomes a short and thick pestle. The period $t = 0.01$–0.12 ms is the erosion stage, in which the difference between the projectile tail velocity and penetration velocity is 32 m/s. At $t = 0.15$ ms, the projectile length can be considered unchanging because the projectile tail velocity and the penetration velocity are essentially equal; thus, 0.12–0.15 ms is the deformation without erosion stage. The stage 0.15–0.25 ms is the rigid penetration stage, in which the projectile tail velocity and the penetration velocity are essentially equal, and their curves basically coincide. The period after 0.25 ms is the elastic resilience stage.

5.3. Effect of Strength Coefficient. Since different materials take the values of the strength coefficient $b$ differently, we can gain a range of calculations that the UST is applicable to a variety of materials. On this basis, the ranges of penetration depth for metallic targets with different $r_t/r_d$ are predicted, and the results are summarized in Table 3. Calculated by the theoretical formulations in this paper, the relationship between penetration depth $D_{\max}$ and $r_t/r_d$ values for different $b$ is shown in Figure 6. From this, we can see that the coefficient $b$ has a large effect on $D_{\max}$; the larger $b$ is, the greater the effect of $\sigma_y$ is and the smaller $D_{\max}$ is. Therefore, appropriate values of $b$ should be adopted for different materials during the penetration analysis. Meanwhile, $b$ is also called yield criterion coefficient that when $b$ takes a different value, the UST degenerates into different yield criterion. Consequently, UST includes infinity yield criteria that a range of different penetration results based on different strength criteria can work out. For instance, when $(r_t/r_d) \geq 20$, a comparison of $b = 1$ (the Tresca yield criterion) and $b = 0$ (the twin shear

### Table 1: Penetration data for long rod penetration into thick metallic targets with finite planar sizes.

| $r_t$ (mm) | $r_d$ (mm) | $r_t/r_d$ | $R_t$ (MPa) | $D_{\text{hi}}$ (mm) | $D_{\text{max}}$ (mm) | $\delta$ |
|-----------|-----------|-----------|-------------|----------------------|----------------------|---------|
| 19.10     | 3.8980    | 4.90      | 2556.85     | 87.25                | 93.67                | 0.0736  |
| 19.10     | 3.8980    | 4.90      | 2556.85     | 88.49                | 93.67                | 0.0585  |
| 25.40     | 3.8957    | 6.52      | 3036.72     | 86.00                | 86.59                | 0.0068  |
| 25.45     | 3.8974    | 6.53      | 3039.30     | 86.31                | 86.55                | 0.0028  |
| 51.10     | 3.8948    | 13.12     | 4211.49     | 74.86                | 72.16                | -0.0360 |
| 51.05     | 3.8969    | 13.10     | 4208.92     | 70.97                | 72.19                | 0.0173  |
| 77.45     | 3.8959    | 19.88     | 4337.50     | 66.76                | 70.81                | 0.0607  |
| 77.10     | 3.8959    | 19.79     | 4334.71     | 69.72                | 70.84                | 0.0161  |
| 77.25     | 3.8956    | 19.83     | 4335.96     | 69.25                | 70.83                | 0.0227  |
| 111.00    | 3.8906    | 28.53     | 4494.88     | 70.73                | 69.17                | -0.0221 |
| 304.80    | 3.9248    | 77.66     | 4623.37     | -                 | 67.86                | -       |

**Notes:** $R_t$ is the theoretical penetration resistance calculation result in this paper; $D_{\text{max}}$ is the theoretical penetration depth calculation result in this paper; $D_{\text{hi}}$ is the test result of penetration depth in [13]; and $\delta$ is the relative error of $D_{\text{max}}$ and $D_{\text{hi}}$.

### Table 2: Material model parameters.

| Material | $EH$ (MPa) | $C_1$ | $S_1$ | $S_2$ | $S_3$ | GAMAO a |
|----------|------------|-------|-------|-------|-------|----------|
| Tungsten alloy | 8713 | 0.404 | 1.23 | 0 | 0 | 1.54 | 0 |
| Material | $A_0$ (MPa) | $B_0$ (MPa) | $n_0$ | $C_0$ | $m_0$ | a |
| 4340 steel | 735 | 473 | 0.26 | 0.014 | 1.03 | 0.43 | — |

| Material | $EH$ (MPa) | $C_1$ | $S_1$ | $S_2$ | $S_3$ | GAMAO a |
|----------|------------|-------|-------|-------|-------|----------|
| Tungsten alloy | 8713 | 0.404 | 1.23 | 0 | 0 | 1.54 | 0 |
| Material | $A_0$ (MPa) | $B_0$ (MPa) | $n_0$ | $C_0$ | $m_0$ | a |
| 4340 steel | 735 | 473 | 0.26 | 0.014 | 1.03 | 0.43 | — |
yield criterion) shows that $D_{\text{max}}$ is higher by up to 17.09%. When \( \frac{r_t}{r_d} < 20 \), this effect becomes more striking that $D_{\text{max}}$ can be up to 18.95% higher. It indicates that, to make the members’ antipenetration performance be played fully, the effect of the intermediate principal stress ($\sigma_2$) cannot be ignored. The result in [15] (von Mises yield criterion result), which is only applicable for materials with $\tau_s \leq 0$.577 $\sigma_s$, is only a special case of the results in this paper when $b = 0.366$.

5.4. Effect of Target Radius. The relationship between $D_{\text{max}}$ and $r_t/r_d$ is also shown in Figure 6 (in the case of $b = 0.6$): as $r_t/r_d$ increases, $D_{\text{max}}$ decreases. When \( \frac{r_t}{r_d} \geq 20 \), $D_{\text{max}}$ decreases so slowly with the increase of $r_t/r_d$ that it has decreased by only 4.19% between $r_t/r_d = 19.83$ and $(r_t/r_d) = 77.66$. When $(r_t/r_d) \geq 30$, as $r_t \longrightarrow r_\infty$, $D_{\text{max}}$ has decreased by just 2%. However, when $(r_t/r_d) < 20$, $D_{\text{max}}$ decreases dramatically with the increase of $r_t/r_d$. When $r_t/r_d = 4.9$ increases to $(r_t/r_d) = 19.88$, $D_{\text{max}}$ decreases by 32.28%. It indicates that the impact of the target planar size cannot be ignored when $(r_t/r_d) < 20$, according well with the result in [15].

5.5. Effect of Other Parameters. Models of long rod projectiles with flat-shaped, hemisphere-shaped, and cone-
Table 3: Penetration depth ranges.

| $r_i/r_d$ | Penetration depth range (mm) | $r_i/r_d$ | Penetration depth range (mm) |
|----------|-----------------------------|----------|-----------------------------|
| 77.66    | 64.79–75.72                 | 13.12    | 68.48–81.45                 |
| 28.53    | 66.03–77.22                 | 13.1     | 68.50–81.48                 |
| 19.88    | 67.58–79.12                 | 6.52     | 83.17–95.09                 |
| 19.79    | 67.60–79.15                 | 6.53     | 83.13–95.05                 |
| 19.83    | 67.59–79.14                 | 4.9      | 90.42–101.68                |

The cone-shaped long rod projectile
The hemisphere-shaped long rod projectile
The flat-shaped long rod projectile

shaped noses are constructed as shown in Figure 7. Then, penetration models of three nose shapes with $v_0 = 500$ m/s, $v_0 = 700$ m/s, $v_0 = 1500$ m/s, and $v_0 = 2500$ m/s are numerically calculated and the penetration depth curves are shown in Figure 8. Figure 8 clearly shows that the penetration depth of flat-shaped projectiles is 22.3% less than that of hemisphere-shaped projectiles and 25.2% less than that of cone-shaped projectiles when $v_0 = 500$ m/s. Nevertheless, when $v_0 = 1500$ m/s and $v_0 = 2500$ m/s, the disparity of the penetration depth among three projectile shapes is less than 2%, which indicates that the projectile nose shape has little effect on the penetration process at high speeds because the stress produced by a high-speed impact leads to erosional failure of the long rod nose in the initial stages of contact with the target.

It also can be seen from Figure 8 that the striking velocity of projectile $v_0$ has some effect on the penetration: the higher $v_0$ is, the greater $D_{\text{max}}$ is, and the increase of $D_{\text{max}}$ for cone-shaped projectiles is much more obvious than others.

6. Conclusions

(1) Based on the UST to consider the effect of $\sigma_s$, combined with Tate’s eroding rod model, the FCCE model of high-speed penetration by long rods into metallic targets with finite planar sizes is obtained. Additionally, the penetration process under different conditions is simulated by LS-DYNA. These results are compared with test data and other calculations, showing that calculated results in this paper have higher precision than those in other studies. On this basis, the ranges of penetration depth for metallic targets with different $r_i/r_d$ are predicted.

(2) The strength coefficient $b$ has a large effect on the penetration: the larger $b$ is, the greater the effect of $\sigma_s$ is and the smaller $D_{\text{max}}$ is. For metallic materials, there are large differences among the calculated results because of different $b$. For instance, as $b = 1$ decreases to $b = 0$, $D_{\text{max}}$ increases by up to 18.95%. Therefore, appropriate values of $b$ should be adopted for different materials during penetration analysis. The result in [15], which is only applicable for materials with $r_i = 0.577\sigma_s$, is only a special case of the results in this paper when $b = 0.366$.

(3) When $(r_i/r_d) \geq 20$, $D_{\text{max}}$ decreases slowly with the increase of $r_i/r_d$: as $(r_i/r_d) = 19.83$ increases to $(r_i/r_d) = 77.66$, $D_{\text{max}}$ decreases by only 4.19%. When $(r_i/r_d) < 20$, $D_{\text{max}}$ decreases dramatically with the increase of $r_i/r_d$. When $(r_i/r_d) = 4.9$ increases to $(r_i/r_d) = 19.88$, $D_{\text{max}}$ decreases by 32.28%.
Therefore, the influence of the target planar size is obvious when \((r_t/r_d) < 2\), and the target cannot be considered as a semi-infinite target anymore.

(4) The striking velocity of projectile \(v_0\) has some effect on the penetration: the higher \(v_0\) is, the greater \(D_{\text{max}}\) is. The projectile nose shape has little effect on the penetration performance at high speeds because of the erosional failure of the long rod nose.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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