NLO QCD corrections to the production of Higgs plus two jets at the LHC

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Abstract

We present the calculation of the NLO QCD corrections to the associated production of a Higgs boson and two jets, in the infinite top-mass limit. We discuss the technical details of the computation and we show the numerical impact of the radiative corrections on several observables at the LHC. The results are obtained by using a fully automated framework for fixed order NLO QCD calculations based on the interplay of the packages GoSAM and SHERPA. The evaluation of the virtual corrections constitutes an application of the $d$-dimensional integrand-level reduction to theories with higher dimensional operators. We also present first results for the one-loop matrix elements of the partonic processes with a quark-pair in the final state, which enter the hadronic production of a Higgs boson together with three jets in the infinite top-mass approximation.

Keywords: QCD, Jets, NLO Computations, LHC

1. Introduction

The recent discovery reported by ATLAS and CMS\textsuperscript{[1, 2]} indicates the existence of a new neutral boson with mass of about 125 GeV and spin different from one. At present, all measurements are consistent with the hypothesis that the new particle is the Standard Model Higgs boson. Nevertheless, further studies concerning its CP properties, spin, and couplings are mandatory to confirm its nature.

The vector boson fusion (VBF) processes can be used to study the CP properties of the new particle, and to extract its couplings with the heavy gauge bosons. The dominant background to VBF comes from Higgs plus two jet production \cite{12}. The signal-over-background ratio can be improved by imposing stringent cuts on the Higgs decay products, and by requiring large rapidity separation between the two forward jets. A veto on the jet activity in the central region further reduces the impact of the background. Indeed, VBF processes are characterized by low hadronic activity, owing to the exchange of a color-singlet in the $t$-channel. An accurate estimation of the efficiency of the central-jet veto requires the inclusion of the next-to-leading order (NLO) corrections to $Hjj$ production \cite{13, 14}.

The leading order (LO) contribution to $Hjj$ production has been computed in Refs.\textsuperscript{[3, 4]}. The calculation was performed retaining the full top-mass dependence, and it showed the validity of the large top-mass approximation ($m_t \to \infty$) whenever the mass of the Higgs particle and the $p_T$ of the jets are not sensibly larger than the mass of the top quark. In the $m_t \to \infty$ limit, the Higgs coupling to two gluons, which at LO is mediated by a top-quark loop, becomes independent of $m_t$. Hence, it can be described by an effective operator obtained by integrating out the top quark degrees of freedom \cite{7}. In this approximation, the number of loops of the virtual diagrams that need to be computed is reduced by one.

In the heavy top quark limit, the inclusive Higgs boson production cross section has been computed at NLO\textsuperscript{[5, 6]} and at next-to-next-to-leading order (NNLO)\textsuperscript{[7, 12]}, showing a reduced sensitivity of the perturbative prediction to scale variations.

The implementation of final state cuts, to reduce the impact of the Standard Model background for the identification of Higgs production demands fully exclusive calculations of the theory predictions. In the $m_t \to \infty$ limit, the NNLO corrections to Higgs production via gluon fusion have been computed fully exclusively\textsuperscript{[13, 16]}. These
corrections also included the contributions of $H + 1j$ final states to NLO \[17,20\], and of the $H + 2j$ final states to LO \[21,22\].

The impact of the NLO QCD corrections to the $Hjj$ production rate has been studied in Ref. \[3\], using the real corrections presented in Refs. \[23,24\] and the semi-analytic virtual corrections computed in Refs. \[26,27\]. A great effort has been devoted to the analytic computation of the one-loop helicity amplitudes involving Higgs plus four partons, using on-shell and generalized unitarity methods \[28–34\]. The resulting compact expressions have been implemented in MCFM \[1\], which has been used to obtain matched NLO plus shower predictions within the POWHEG box framework \[35\].

In this letter we present an independent computation of the NLO contributions to Higgs plus two jets production at the LHC in the large top-mass limit. These results have been obtained by using a fully automated framework for fixed order NLO QCD calculations, which interfaces via the Binoth Les Houches Accord (BLHA) \[36\] the GoSAM package \[37\], for the generation and computation of the virtual amplitudes, with the SHERPA package \[38\], for the computation of the real amplitudes and the Monte Carlo integration over phase space. Details of the GoSAM - SHERPA interface, together with a selection of ready-to-use process packages, can be found in \[39,40\]. Moreover, the evaluation of the virtual corrections for a model described by an effective Lagrangian constitutes an application of the $d$-dimensional integrand-level reduction techniques \[41–43,52–54\]. More specifically, the virtual corrections are evaluated using the $d$-dimensional integrand-level decomposition implemented in the SAMURAI library \[45\], which allows for the combined determination of both cut-constructible and rational terms at once. Moreover, the presence of effective couplings in the Lagrangian requires an extended version \[47\] of the integrand-level reduction, of which the present calculation is a first application. After the reduction, all relevant scalar (master) integrals are computed by means of QCDLoop \[55,56\], OneLoop \[57\], or GOLEM95C \[58\].

For the calculation of tree-level contributions we use SHERPA \[38\], which computes the LO and the real radiation matrix elements \[59\], regularizes the IR and collinear singularities using the Catani-Seymour dipole formalism \[60\], and carries out the phase space integrations as well.

The code that evaluates the virtual corrections is generated by GoSAM and linked to SHERPA via the Binoth-Les-Houches Accord (BLHA) \[36\] interface. This interface allows to generate the code in a fully automated way by a system of order and contract files containing the amplitudes requested by SHERPA. Furthermore, it allows for a direct communication between the two codes at running time, when SHERPA steers the integration by calling the external code which computes the virtual amplitude. A detailed description of this interface is beyond the scope of this paper and will be presented elsewhere \[39\].

For $Hjj$ production, the partonic processes in the contract file are:

\[
\begin{align*}
q q & \rightarrow H q q, & q \bar{q} & \rightarrow H q \bar{q}, \\
q \bar{q} & \rightarrow H g g, & q q & \rightarrow H q \bar{q}, \\
q q' & \rightarrow H q q', & q g & \rightarrow H g q, \\
\bar{q} q & \rightarrow H q \bar{q}, & q q' & \rightarrow H q \bar{q}, \\
g q & \rightarrow H g g, & g g & \rightarrow H q \bar{q}, \\
g g & \rightarrow H g g.
\end{align*}
\]

(3)

In the $\overline{\text{MS}}$ scheme, the coefficient $g_{\text{ct}}$ reads \[8,9\]

\[
 g_{\text{ct}} = - \frac{\alpha_s}{3\pi v} \left(1 + \frac{11}{4\pi\alpha_s}\right) + \mathcal{O}(\alpha_s^2),
\]

(2)
in terms of the Higgs vacuum expectation value $v$. The operator $\Pi$ leads to new Feynman rules involving the Higgs field and up to four gluons. They are collected in Appendix A.

Next-to-leading order corrections to cross sections require the evaluation of virtual and real emission contributions. For the computation of the virtual corrections we use a code generated by the program package GoSAM, which combines automated diagram generation and algebraic manipulation \[48–51\] with integrand-level reduction techniques \[41,42,52–54\]. More specifically, the virtual corrections are evaluated using the $d$-dimensional integrand-level reduction, of which the present calculation is a first application. After the reduction, all relevant scalar (master) integrals are computed by means of QCDLoop \[55,56\], OneLoop \[57\], or GOLEM95C \[58\].

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q \bar{q} & \rightarrow H g g, & q q & \rightarrow H q \bar{q}, \\
q q' & \rightarrow H q q', & q g & \rightarrow H g q, \\
\bar{q} q & \rightarrow H q \bar{q}, & q q' & \rightarrow H q \bar{q}, \\
g q & \rightarrow H g g, & g g & \rightarrow H q \bar{q}, \\
g g & \rightarrow H g g.
\end{align*}
\]

(3)

These processes are not independent, as they can be related by crossing and/or by relabeling. GoSAM identifies
and generates the following minimal set of processes

\[ gg \rightarrow H q \bar{q}, \quad gg \rightarrow H q \bar{q}, \quad gq \rightarrow H q^\prime \bar{q}^\prime. \]  

The other processes are obtained by performing the appropriate symmetry transformation.

The ultraviolet (UV), the infrared, and the collinear singularities are regularized using dimensional reduction (DRED). UV divergences have been renormalized in the MS scheme. In the case of LO [NLO] contributions we describe the running of the strong coupling constant with one-loop [two-loop] accuracy, decoupling the top quark from the running.

The effective \( Hgg \) coupling, see Appendix A, leads to integrands that may exhibit numerators with rank \( r \) larger than the number \( n \) of the denominators, i.e. \( r \leq n + 1 \). In general, for these cases, the parametrization of the residues at the multiple-cut has to be extended as discussed in Ref. [47]. As a consequence, the decomposition of any one-loop \( n \)-point amplitude in terms of master integrals (MIs) acquires new contributions, reading as,

\[ \mathcal{M}_n^{\text{one-loop}} = A_n + \delta A_n. \]  

The term \( A_n \) corresponds to the standard decomposition for the case of a renormalizable theory \( (r \leq n) \), while the additional contribution \( \delta A_n \) enters whenever \( r \leq n + 1 \). Their actual expressions can be found in Eqs. (2.16) and (6.11) of [47].

The extended integrand decomposition has been implemented in the SAMURAI library. In particular, the coefficients multiplying the MIs appearing in \( A_n \) and \( \delta A_n \) are computed by using the discrete Fourier transform as described in Refs. [45, 53].

In the case of Higgs plus jets production, higher rank numerators arise from diagrams where the Higgs boson is attached to a pure gluonic loop. However, as shown in Appendix B, the rank-(\( n + 1 \)) terms of an \( n \)-point integrand are proportional to the loop momentum squared, \( q^2 \), which simplifies against a denominator. Therefore, they generate \( (n-1) \)-point integrands with rank \( r = n-1 \). Consequently, the coefficients of the MIs in \( \delta A_n \) have to vanish identically, as explicitly verified. Since \( \delta A_n \) in Eq. 48 does not play any role, the integrand reduction can be also performed with the current public version of SAMURAI, which does not contain the extended decomposition - hence, implying a lighter reduction, with fewer coefficients involved.

We remark that, within the integrand reduction algorithm, it is possible to benefit immediately from the presence of powers of \( q^2 \) in the numerators, without any algebraic cost: the contribution of those terms is automatically taken into account by the integrand reconstruction of the subdiagrams (because they give no contribution on the corresponding massless cut). On the contrary, within a tensor reduction algorithm, these terms would cancel only after the algebraic manipulation of the integrand.

The numerical values of the one-loop amplitudes of the processes (4) in a non-exceptional phase space point are collected in Appendix C. The values of the double and the single poles conform to the universal singular behavior of dimensionally regulated one-loop amplitudes [61–65]. After appropriate crossing to the \( H \rightarrow 4 \)-parton decay kinematics, we compared our results with the ones presented in Table I of Ref. [26], finding excellent agreement. Furthermore, converting our results for the \( Hjj \)-production channels from DRED to the 't Hooft-Veltman scheme, we are in perfect agreement with the most recent version of MCFM (v6.4).
3. Numerical results for $pp \rightarrow Hjj$

In this section we present a selection of phenomenological results for proton-proton collisions at the LHC at $\sqrt{s} = 8$ TeV, as a sample of the results that can be easily obtained with the GoSam-SHERPA automated setup [37–40]. A more complete analysis of Higgs production in gluon fusion, which merges several multiplicities [66] and employs the code for the virtual matrix elements of $Hjj$ presented here, is going to be discussed in [67].

The results shown in this section are obtained using the parameters listed below:

\begin{align*}
M_H &= 125 \text{ GeV}, \\
G_F &= 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}, \\
\alpha_s^{\text{LO}}(M_Z) &= 0.129783, \\
\alpha_s^{\text{NLO}}(M_Z) &= 0.117981, \\
\varepsilon^2 &= \frac{1}{\sqrt{2}G_F}.
\end{align*}

We use the CTEQ6L1 and CTEQ6mE [68] parton distribution functions (PDF) for the LO and NLO, respectively. The value of the strong coupling at the scale $\mu$ is taken from the PDF set starting from the initial values in Eq. (6). The jets are clustered by using the anti-$k_T$ algorithm provided by the FASTJET package [69–71] with the following setup:

\begin{align*}
p_{t,j} &\geq 20 \text{ GeV}, \\
|\eta_j| &\leq 4.0, \\
R &= 0.5.
\end{align*}

The Higgs boson is treated as a stable on-shell particle, without including any decay mode. To fix the factorization and the renormalization scale we define

\begin{equation}
\hat{H}_t = \sqrt{M_H^2 + p_{t,H}^2 + \sum_j p_{t,j}^2},
\end{equation}

where $p_{t,H}$ and $p_{t,j}$ are the transverse momenta of the Higgs boson and the jets. The nominal value for the two
The missing channel $gg \rightarrow Hgg$, together with the phase space integration, will be discussed in a successive study.

4. Virtual corrections to $pp \rightarrow Hjjj$

We explore the possibility of extending our framework to the production of a Higgs boson plus three jets at NLO. The independent partonic processes contributing to $Hjjj$ can be obtained by adding one extra gluon to the final state of the processes listed in Eq. (11). Accordingly, we generate the codes for the virtual corrections to the partonic processes with a quark-pair in the final state,

$$gg \rightarrow Hqg , \quad qg \rightarrow Hqg , \quad q\bar{q} \rightarrow Hq'\bar{q}'g .$$  \hspace{1cm} (11)

The missing channel $gg \rightarrow Hgg$, together with the phase space integration, will be discussed in a successive study.

We compute, for the first time, the virtual matrix elements for the three subprocesses listed above, and show their results along a certain one-dimensional curve in the space of final state momenta. We take the initial partons to have momentum $p_1$ and $p_2$, whose 3-momenta lie along the $z$-axis, and choose an arbitrary point for the final state momenta $\{p_3, p_4, p_5, p_6\}$. For simplicity, we start with the same phase space point used in the Appendix D (see Table D.4). Then, we create new momentum configurations by rotating the final state through an angle $\theta$ about the $y$-axis. Figure 7 displays the behavior of the finite part $a_0$ of the individual $2 \rightarrow 4$ amplitudes defined as

$$\frac{2\Re \{ \mathcal{M}^{\text{tree-level}} \mathcal{M}^{\text{one-loop}} \}}{(4\pi\alpha_s) |\mathcal{M}^{\text{tree-level}}|^2} \equiv \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0 ,$$  \hspace{1cm} (12)

when the final external momenta are rotated from $\theta = 0$ to $\theta = 2\pi$. The plots are obtained by sampling over 100 points.

Numerical values for the one-loop amplitudes of the processes listed in (11) are collected in Appendix D. Also in this case we verify that the values of the double and the single poles conform to the universal singular behavior of dimensionally regulated one-loop amplitudes.65.

5. Conclusions

We presented the calculation of the associated production of a Higgs boson and two jets, $pp \rightarrow Hjj$, at NLO in QCD, employing the infinite top-mass approximation. The results were obtained by using a fully automated framework for fixed order NLO QCD calculations based on the interplay of the packages GoSam and SHERPA, interfaced through the BLHA standards. We discussed the technical aspects of the computation, and showed the numerical impact of the radiative corrections on the distribution of the transverse momentum of the Higgs boson and its pseudorapidity, as well as of the transverse momentum and pseudorapidity of the leading and second leading jet. All plots show a K-factor between the LO and the NLO scales is defined as

$$\mu = \mu_R = \mu_F = \hat{H}_t ,$$  \hspace{1cm} (9)

whereas theoretical uncertainties are assessed by varying simultaneously the factorization and renormalization scales in the range

$$\frac{1}{2} \hat{H}_t < \mu < 2\hat{H}_t .$$  \hspace{1cm} (10)

The error is estimated by taking the envelope of the resulting distributions at the different scales.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7.png}
\caption{Finite-term of the virtual matrix-elements for $gg \rightarrow Hqg$ (red), $qg \rightarrow Hqg$ (green), $q\bar{q} \rightarrow Hq'\bar{q}'g$ (blue).}
\end{figure}
distributions of about 1.5, over a large fraction of kinematical range, and a decrease of the scale uncertainty of about 50%.

The evaluation of the virtual corrections constitutes an application of the $d$-dimensional integrand reduction to theories with higher dimensional operators.

Finally, as an initial step towards the evaluation of $pp \to Hjjj$ at NLO, we presented first results for the one-loop matrix elements of the partonic processes with a quark-pair in the final state.

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The Feynman diagrams present in this paper are drawn using FENNYARTS \cite{hep-ph/1008.2690}.

Appendix A. Effective Higgs-gluon vertices

The operator $\mathcal{L}$ in Eq. \ref{eq:L} describes the gluon-Higgs interaction in the large top-mass limit and leads to the following set of Feynman rules:

\[
\begin{align*}
\left\langle \begin{array}{c}
g_1 \\
H \\
g_2 \\
\end{array} \right\rangle &= -i g \alpha_s F_{c_1,c_2}^{\mu_1 \mu_2} \\
\left\langle \begin{array}{c}
g_1 \\
H \\
g_3 \\
\end{array} \right\rangle &= g_3 g_\alpha F_{c_1,c_2,c_3}^{\mu_1 \mu_2 \mu_3} \\
\left\langle \begin{array}{c}
g_1 \\
H \\
g_4 \\
\end{array} \right\rangle &= i g_4 g_\alpha F_{c_1,c_2,c_3,c_4}^{\mu_1 \mu_2 \mu_3 \mu_4}
\end{align*}
\]

where we define

\[
\begin{align*}
F_{c_1,c_2}^{\mu_1 \mu_2} &= \delta_{c_1,c_2} (p_1^{\mu_1} p_2^{\mu_2} - p_1 \cdot p_2 g^{\mu_1 \mu_2}) \\
F_{c_1,c_2,c_3}^{\mu_1 \mu_2 \mu_3} &= f_{c_1,c_2,c_3}^{\mu_1 \mu_2 \mu_3} [g^{\mu_1 \mu_2} (p_1^{\mu_3} - p_2^{\mu_3}) + g^{\mu_1 \mu_3} (p_2^{\mu_2} - p_1^{\mu_2}) + g^{\mu_2 \mu_3} (p_3^{\mu_1} - p_4^{\mu_1})] \\
F_{c_1,c_2,c_3,c_4}^{\mu_1 \mu_2 \mu_3 \mu_4} &= f_{c_1,c_2,c_3,c_4}^{\mu_1 \mu_2 \mu_3 \mu_4} [g^{\mu_1 \mu_4} g^{\mu_2 \mu_3} - g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} + g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} - g^{\mu_1 \mu_4} g^{\mu_2 \mu_3}]
\end{align*}
\]

In Eq. \ref{eq:A1} sum over repeated indices is understood.

Appendix B. Higher-rank integrands

Higher-rank integrands, i.e. integrands where the powers of loop momenta in the numerator is higher than the numbers of denominators, are present in diagrams with a Higgs boson coupled to a purely gluonic loop involving only three-gluon vertices. The generic numerator $\Gamma_{\varepsilon_1 \cdots \varepsilon_n}$ of a $(n+1)$-denominator one-loop diagram can be written as

\[
\Gamma_{\varepsilon_1 \cdots \varepsilon_n} \equiv F_{\mu_1 \mu_2} G_{\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n}^{\mu_1 \mu_2},
\]

where $F_{\mu_1 \mu_2}$ is the $Hgg$ vertex defined in Eq. \ref{eq:A1}, and $G_{\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n}$ is the numerator of an $(n+2)$-gluon tree-level diagram, which can be represented by

\[
\begin{array}{c}
\varepsilon_1 \\
\epsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_n \\
\mu_1 \\
\mu_2 \\
\mu_2 \\
\varepsilon_1
\end{array}
\hspace{1cm}
\left\langle \begin{array}{c}
q \\
\mu_1 \\
\mu_2 \\
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_1 \\
\varepsilon_3 \\
\varepsilon_4 \\
\mu_1 \\
\varepsilon_5 \\
\varepsilon_6 \\
\mu_2 \\
\varepsilon_1
\end{array} \right\rangle
\equiv
\left\langle \begin{array}{c}
q \\
\mu_1 \\
\mu_2 \\
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_1 \\
\varepsilon_3 \\
\varepsilon_4 \\
\mu_1 \\
\varepsilon_5 \\
\varepsilon_6 \\
\mu_2 \\
\varepsilon_1
\end{array} \right\rangle
\]

We are interested in the leading behaviour in $q$ of $\Gamma_{\varepsilon_1 \cdots \varepsilon_n}$, and we want to show that the highest-rank terms, with rank $r = n + 2$, are proportional to the loop momentum squared, $q^2$. In order to show it, we neglect all external momenta and all the terms proportional to $q^2$.

From Eq. \ref{eq:A1}, one trivially has

\[
F_{\mu_1 \mu_2} = q_{\mu_1} q_{\mu_2} + O(q^2),
\]

while the generic tensor structure of $G_{\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n}$ is

\[
G_{\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n} = q^{\mu_1} T_1^{\mu_2 \varepsilon_1 \varepsilon_2 \cdots \varepsilon_n} + q^{\mu_2} T_2^{\mu_1 \varepsilon_1 \varepsilon_2 \cdots \varepsilon_n} +
\]

\[
+ g^{\mu_1 \mu_2} T_3^{\varepsilon_1 \varepsilon_2 \cdots \varepsilon_n} + O(q^2),
\]

where $T_1, T_2,$ and $T_3$ are tensors which may depend on $q$ as well. Indeed Eq. \ref{eq:B3} is fulfilled for $n = 0, 1$,

\[
G_{\varepsilon_1 \varepsilon_2} = g^{\varepsilon_1 \varepsilon_2}
\]

\[
G_{\varepsilon_1 \varepsilon_2 \varepsilon_3} = g^{\varepsilon_1 \varepsilon_2} q^{\varepsilon_3} + g^{\varepsilon_1 \varepsilon_3} q^{\varepsilon_2} - 2 g^{\varepsilon_2 \varepsilon_3} q^{\varepsilon_1}, \quad \text{(B.4)}
\]
while for \( n > 1 \) it can be proven by induction over \( n \) by using

\[
G^{\mu_2 \varepsilon_1 \ldots \varepsilon_n} = G^{\mu_2 \varepsilon_1 \ldots \varepsilon_{n-1}} G^{\mu_1 \varepsilon_n},
\]

that is

\[
\begin{array}{cccc}
\varepsilon_1 & \varepsilon_2 & \varepsilon_n & \varepsilon_1 & \varepsilon_2 & \varepsilon_{n-1} & \varepsilon_n \\
\mu_2 & q & \mu_1 & \mu_2 & q & \mu_1 & \mu_1
\end{array}
\] \quad \equiv \left( \begin{array}{c}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_n \\
\mu_2 \\
q \\
\mu_1
\end{array} \right)

Combining Eq. (B.2) and Eq. (B.3), it is easy to realize that each rank-(\( n+2 \)) term of an \( n+1 \)-denominator diagram \( \Gamma^{\varepsilon_1 \ldots \varepsilon_n} \) is proportional to \( q^2 \). The factor \( q^2 \) simplifies against one denominator leading to a rank \( n \) numerator of an \( n \)-denominator integrand.

**Appendix C. Benchmark points for \( pp \rightarrow Hjj \)**

In this appendix we provide numerical results for the renormalized virtual contributions to the processes \((4)\), in correspondence with the phase space point in Table C.1. The parameters can be read from Eqs. (6), while the renormalization and factorization scales are set to the Higgs mass value. The assignment of the momenta proceeds as follows

\[
\begin{align*}
g(p_1) g(p_2) & \rightarrow H(p_3) g(p_4) g(p_5), \\
g(p_1) \bar{q}(p_2) & \rightarrow H(p_3) q(p_4) \bar{q}(p_5), \\
q(p_1) \bar{q}(p_2) & \rightarrow H(p_3) q(p_4) \bar{q}(p_5), \\
q(p_1) \bar{q}(p_2) & \rightarrow H(p_3) q'(p_4) \bar{q}'(p_5).
\end{align*}
\]

The results are collected in Table C.2 and are computed using DRED. In the second column of the table we provide the LO squared amplitude,

\[
c_0 \equiv \frac{|\mathcal{M}_{\text{tree-lo}}|^2}{(4\pi \alpha_s)^2 g_s},
\]

and the coefficients \( a_i \) defined in Eq. (12). As a check on the reconstruction of the renormalized poles, in the last column we show the values of \( a_{-1} \) and \( a_{-2} \) obtained by the universal singular behavior of the dimensionally regularized one-loop amplitudes [52].

**Appendix D. Benchmark points for \( pp \rightarrow Hjjj \)**

In this appendix we collect first numerical results for the renormalized virtual contributions to

\[
\begin{align*}
g(p_1) g(p_2) & \rightarrow H(p_3) g(p_4) \bar{q}(p_5) g(p_6), \\
q(p_1) \bar{q}(p_2) & \rightarrow H(p_3) q(p_4) \bar{q}(p_5) g(p_6), \\
q(p_1) \bar{q}(p_2) & \rightarrow H(p_3) q'(p_4) \bar{q}'(p_5) g(p_6).
\end{align*}
\]

The results, collected in Table D.3, have been computed using the parameters in Eqs. (6), with the renormalization and factorization scales set to the Higgs mass value, and choosing the phase space point given in Table D.4. In particular, in the second column of Table D.3 we provide...
the universal singular behavior of one-loop amplitudes.

\[ \equiv |M_{\text{tree-level}}|^2 \frac{1}{(4\pi\alpha_s)^2 g^4 s_t^2}, \tag{D.2} \]

and the coefficients \( a_i \) defined in Eq. \( \text{(12)} \). In the third column we show the values of \( a_{-1} \) and \( a_{-2} \) obtained from the universal singular behavior of one-loop amplitudes.

Table C.1: Benchmark phase space point for Higgs plus two jets production

| particle \( |E| \) | \( p_x \) | \( p_y \) | \( p_z \) |
|-----------------|-----------------|-----------------|-----------------|
| \( p_1 \) | 250.00000000000000 | 0.00000000000000 | 0.00000000000000 | 250.00000000000000 |
| \( p_2 \) | 250.00000000000000 | 0.00000000000000 | 0.00000000000000 | 250.00000000000000 |
| \( p_3 \) | 143.67785140616080 | 51.66334918413812 | -22.54431400012950 | 42.90518245000000 |
| \( p_4 \) | 190.20318663787611 | -153.36110830475005 | -108.2375850696623 | -30.70241157719542 |
| \( p_5 \) | 166.1189603051594 | 101.6977438633616 | 130.7829199128202 | -12.2069719578783 |

Table D.4: Benchmark phase space point for Higgs plus three jets production

| particle \( |E| \) | \( p_x \) | \( p_y \) | \( p_z \) |
|-----------------|-----------------|-----------------|-----------------|
| \( p_1 \) | 250.00000000000000 | 0.00000000000000 | 0.00000000000000 | 250.00000000000000 |
| \( p_2 \) | 250.00000000000000 | 0.00000000000000 | 0.00000000000000 | 250.00000000000000 |
| \( p_3 \) | 131.06896655823209 | 27.70726481472267 | -13.23548290039414 | 24.72252947259168 |
| \( p_4 \) | 117.0295363273803 | 54.48051662427356 | 97.99054664150677 | -33.55065837062932 |
| \( p_5 \) | 87.15729570855642 | 47.18805954775526 | -5.35761276804790 | 73.06871134996961 |

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