A General Circuit Analysis and Simulation Method for Superconducting Quantum Interference Devices

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Abstract

The Superconducting Quantum Interference Device (SQUID) is an extremely sensitive flux-to-voltage converter widely used in weak magnetic signal detection systems. It is the superconducting circuit composed of Josephson junctions and superconducting loops. However, the analysis is usually based on superconducting physics rather than the conventional circuit theorems. This article presents a general circuit analysis method using only the conventional circuit variables and laws to simplify the analysis and simulation of SQUID circuits. The method unifies the descriptions of Josephson junctions and non-superconducting elements with a general non-linear inductance concept; and derives the uniform SQUID circuit equations with the common circuit laws used for both superconducting and normal circuits. The uniform circuit equation and dynamic model show that the only element making the SQUID distinct from the non-superconducting circuits is the cosine potential introduced by the Josephson current. This general analysis method bridges the gap between the superconductive SQUID circuits and the conventional normal circuits for the electronics engineers trained with the conventional circuit theory.

Keywords: SQUID, Josephson-junction circuit, loop-analysis method, numerical simulation.
1. Introduction

The Josephson-junction-based Superconducting Quantum Interference Devices (SQUIDs) [1] have been widely used as the ultra-sensitive sensor in many magnetic field measurement applications, such as biomagnetism [2] and geophysical detections [3]. For more than a half-century, different types of SQUIDs have been developed, ranging from the radio-frequency SQUID (rf-SQUID) with single junction to the SQUID-array [4] with multiple junctions and loops, as illustrated in Fig. 1. They are superconducting Josephson junction circuits, or the hybrids with the Josephson junctions and the normal resistor such as the double-relaxation-oscillation SQUID (dro-SQUID) as shown in Fig. 1d.

Like the semiconductor integrated circuits, the SQUID circuit design also needs analysis and simulation tools to guide the parameter optimization. However, due to the different physics principles of SQUIDs [5], it is still a challenge for electronic engineers without knowledge of superconducting physics to design and analyze SQUID circuits compared to conventional normal circuits. First, the current-voltage relationship of the Josephson junction is defined with the macroscopic quantum phase of the superconducting cooper-pairs. Second, the circuit loops comply with the superconducting flux-quantization law (FQL) [6] rather than the Kirchhoff’s voltage law (KVL). The early SQUID analysis method drives the circuit equation directly from the physics principles and is only suitable for the few simple SQUID circuits. Afterward, researchers utilize the SPICE (simulation program with integrated circuit emphasis) tools [7-9] to model the Josephson junction and simulate SQUID circuits with the circuit equations automatically generated with the nodal voltage analysis method. In practical applications, we find that the nodal-voltage method has to keep all the circuit loops comply with the FQL by using the self-inductances and transformers in the SQUID equivalent circuit to describe the flux couplings between loops [10]. Thus, the simulation method with SPICE tools is still an approximation approach for SQUID circuits.

This article introduced a general circuit analysis method to adapt the quantum-phase-based SQUID circuits into the conventional circuit domain. In this method, the Josephson junctions and normal non-superconducting elements inside loops are modeled as the non-linear inductors; the SQUID circuit is abstracted as a group of inductive loops; the uniform circuit equations and dynamic model are derived for any SQUID circuits using only the conventional electric variables and circuit laws. This method is successfully demonstrated in the analyses of several typical SQUID circuits, such as rf-SQUID, dc-SQUID, bi-SQUID, and dro-SQUID. It shows that the only difference between the Josephson junction circuits and the conventional normal circuits is the unique potential introduced by the Josephson current. This understanding enables the electronic engineers to implement the SQUID circuit analysis and simulation with only the conventional circuit methodologies.
Fig. 1. Different types of SQUID circuits: (a) rf-SQUID; (b) dc-SQUID; (c) bi-SQUID; (d) dro-SQUID; (e) superconducting quantum interference filter (SQIF); (f) SQUID-array. The crosses “×” in those circuits are Josephson junctions.

2. Method

2.1 Concept of QPD element

In the SQUID circuits shown in Fig. 1, the macroscopic quantum phase of cooper pairs is continuous along the superconducting wire until the Josephson junctions and resistors discontinue it. We thereby call those circuit elements the Quantum-Phase-Discontinuity (QPD) element.

Josephson junction is the typical QPD element in SQUID circuits. Its equivalent circuit based on the resistively-capacitively-shunted-junction model [11] is shown in Fig. 2a, where the QPD element is biased with the current source $I_b$. With the current $i_\theta$ flowing into, the QPD element creates the quantum phase difference $\theta$ between its two terminals. If we introduce a virtual flux $\Phi_\theta$ to rewrite Josephson current and voltage instead of $\theta$, i.e., $\Phi_\theta = \theta \times \Phi_0/2\pi$ ($\Phi_0 = 2.07 \times 10^{-15}$ Wb), we can treat the QPD element as a non-linear inductance, as shown in Fig. 2b. Accordingly, the current ($i_\theta$)-flux ($\Phi_\theta$) function of the current-biased Josephson junction is written as

$$i_\theta = C \cdot \frac{d^2 \Phi_\theta}{dt^2} + \frac{1}{R} \cdot \frac{d \Phi_\theta}{dt} + I_0 \cdot \sin \frac{2\pi \Phi_\theta}{\Phi_0} + I_n \cdot \delta(t) - I_b$$

$$\equiv J(\Phi_\theta, C, R, I_0, I_n, I_b)$$

Here, we introduce a gaussian-white-noise function with unity standard deviation $\delta(t)$.
and the amplitude $I_r$ to describe the current fluctuations.

The other types of QPD elements [10] are the simplified version of the current-biased RCSJ. For example, in the dro-SQUID shown in Fig. 1d, the shunt resistor $R_a$ is a QPD element with $C=0$ and $I_0=0$. Thus, in SQUID circuit analyses, we unify the descriptions of all types of QPD elements using the same current-flux function defined in (1) and the non-linear inductance symbol shown in Fig. 2b.

![Fig. 2. (a) The general equivalent circuit of QPD elements; (b) The circuit symbol for QPD elements.](image)

![Fig. 3. (a) the dynamics of non-superconducting QPD elements; (b) the dynamics of Josephson junctions.](image)
The critical current $I_0$ tells the difference between the Josephson junction and the non-superconducting QPD (NS-QPD) element since the Josephson junction is the only QPD element containing the Josephson current. To illustrate the difference introduced by the Josephson current, we use a moving particle to simulate the dynamics of the QPD element, where the flux $\Phi_\theta$ is analogous to the trajectory of the particle, the currents are equivalent to the driving forces, the capacitance $C$ is similar with the mass of the particle, and the conductance $1/R$ works like the moving frictional resistance.

Fig. 3 illustrates the NS-QPD and Josephson junction dynamics. It shows that the difference between the two QPD elements is the potential type. The potential for NS-QPD elements is a slope, as shown in Fig. 3a, while the one for Josephson junctions is a wash-board style [5] generated by the Josephson current, as shown in Fig. 3b.

2.2 Circuit abstraction

For any given SQUID circuit, we first extract the circuit elements according to the QPD element concept, then define each of them as a non-linear inductance with the current ($i_\theta$) and flux ($\Phi_\theta$), as shown in Fig 4a.

Assuming that there are $Q$ QPD elements in the given SQUID circuit, the states of the QPD elements are defined with the current [$i_\theta$] and flux [$\Phi_\theta$] as

$$
\begin{bmatrix}
i_\theta \\
\Phi_\theta 
\end{bmatrix} = \begin{bmatrix} i_{\theta_1} & i_{\theta_2} & \ldots & i_{\theta_Q} \\
\Phi_{\theta_1} & \Phi_{\theta_2} & \ldots & \Phi_{\theta_Q}
\end{bmatrix}^T
$$

(2)

The current-flux relations of all the QPD elements are expressed as

Fig. 4. (a) the QPD elements extracted from the SQUID circuit. (b) the loops contained in the SQUID circuit.
\[
\begin{bmatrix}
i_{q1} \\
\vdots \\
i_{qQ}
\end{bmatrix} =
\begin{bmatrix}
J(\Phi_{q1}, C_1, R, I_{q1}, I_{n1}, I_{h1}) \\
\vdots \\
J(\Phi_{qQ}, C_Q, R_Q, I_{qQ}, I_{nQ}, I_{hQ})
\end{bmatrix}
\quad (3)
\]

Behind all the QPD elements, we can draw the topology of the SQUID circuit and define circuit loops according to the loop-analysis method [12]. Assuming that the SQUID circuit is composed of \( P \) loops, as shown in Fig. (b), we use three variables to define the state of each circuit loop. The first is the loop-current \( i_m \) circulating within the loop; the second is the externally applied flux \( \Phi_e \) to the loop; the third is the total coupled flux \( \Phi_m \) of the loop. The states of \( P \) circuit loops are expressed with three variable vectors, i.e., \([i_m]\), \([\Phi_m]\), and \([\Phi_m]\), as

\[
\begin{bmatrix}
i_m \\
\Phi_e \\
\Phi_m
\end{bmatrix} =
\begin{bmatrix}
i_{m1} & i_{m2} & \cdots & i_{nP}
\end{bmatrix}^T
\quad (4)
\]

Since the total coupled flux \( \Phi_m \) comes from the external flux \( \Phi_e \) and the induced fluxes by the loop-currents \( i_m \), we can have the current-flux relations inside the circuit loops expressed as

\[
[\Phi_m] = [\Phi_e] - [L_m][i_m].
\quad (5)
\]

Where \([L_m]\) is the inductance matrix. It depicts the mutual inductive couplings between loops as

\[
[L_m] =
\begin{bmatrix}
L_{11} & \cdots & L_{1P} \\
\vdots & \ddots & \vdots \\
L_{P1} & \cdots & L_{PP}
\end{bmatrix}
\quad (6)
\]

In this matrix, the element \( l_{ij} \) (\( 1 \leq i,j \leq P \)) is defined as

\[
l_{ij} = \begin{cases} 
L_i, & i = j \\
-M_{ij}, & i \neq j 
\end{cases}
\quad (7)
\]

Here, \( L_i \) is the self-inductance of Loop-\( i \); \( M_{ij} \) is the mutual-inductance between Loop-\( i \) and Loop-\( j \); the minus sign of \( M_{ij} \) indicates that the flux contribution by the mutual-inductance \( M_{ij} \) and the one by the self-inductance \( L_i \) are opposite.

2.3 Circuit equations

After deconstructing the given SQUID into QPD elements and circuit loops, we introduce a matrix \([\sigma]\) to describe connections between the QPD elements and circuit loops. The matrix \([\sigma]\) associated with \( Q \) elements and \( P \) loops is defined as
Loop 1

\[
\begin{bmatrix}
\sigma_{11} & \cdots & \sigma_{1Q} \\
\vdots & \ddots & \vdots \\
\sigma_{P1} & \cdots & \sigma_{PQ}
\end{bmatrix}
\] (8)

In this matrix, the parameter \( \sigma_{ij} \) (1 \( \leq \) i \( \leq \) P; 1 \( \leq \) j \( \leq \) Q) is defined according to the connection between the loop-current \( i_m \) and the flux \( \Phi_\theta \) of QPD element as

\[
\sigma_{ij} = \begin{cases} 
+1, & \text{if } i_m \text{ flows into } \Phi_{\theta j} \text{ from "+"} \\
-1, & \text{if } i_m \text{ flows into } \Phi_{\theta j} \text{ from "-"} \\
0, & \text{otherwise}
\end{cases}
\] (9)

In other words, \( \sigma_{ij} \) is the polarity of \( \Phi_{\theta j} \) relative to the defined direction of the loop-current \( i_m \), when the QPD element-\( j \) is connected in the Loop-\( i \).

Using this topology parameter [\( \sigma \)], we can derive two circuit equations according to the circuit laws for nodes and loops.

First, the QPD element defined in Fig. 4a is a two-terminal node; the Kirchhoff’s current law (KCL) states that the current \( i_\theta \) of the QPD element equals the algebraic sum of \( i_m \), namely the loop currents that flow through this QPD element. Thus, the relation between the loop-currents \([i_m]\) and the currents \([i_\theta]\) is derived as

\[
[\sigma]^T[i_m] = [i_\theta]
\] (10)

Second, all the circuit loops defined in Fig. 4b comply with the FQL. The FQL is derived from the KVL by expressing the voltage of QPD elements with the virtual flux \( \Phi_\theta \) and is applied for both the superconducting and normal circuit loops. [10]. It states that the total magnetic flux \( \Phi_m \) of a loop equals the algebraic sum of the virtual flux \( \Phi_\theta \) of all the QPD elements connected in this loop. Thus, the relation between \([\Phi_m]\) and \([\Phi_\theta]\) is expressed as

\[
[\sigma][\Phi_\theta] = [\Phi_m]
\] (11)

2.4 Dynamic model

By synthesizing all the circuit equations, we can depict the dynamics of SQUID circuits with a closed-loop signal flow chart, as shown in Fig. 5. In the flow chart, the current-flux functions of loops combined with the circuit equations by KCL and FQL implement the linear flux-to-current feedback to the QPD elements.

Considering that the QPD element behaves like a classical particle, the SQUID circuit is equivalent to a multi-particle system moving in one-dimension space, where the circuit parameters [\( \sigma \)] and [\( L_m \)] define the linear constraints between the particles interfering with each other.

Therefore, we can simulate the motion of the particles interfering with each other in the wash-board type potentials. From the trajectories of the particles, which are equivalent to the [\( \Phi_\theta \)] of the QPD elements, we can calculate the current-voltage and flux-voltage characteristics of the SQUID circuit.
3. Application Examples

3.1 rf-SQUID

The readout circuit of the rf-SQUID using a tank circuit is shown in Fig. 6a. The tank-circuit is driven by a radio-frequency current source $i_{rf}(t)$, i.e., $i_{rf}(t) = I_{rf}\sin(2\pi f_{rf} t)$, where $I_{rf}$ is the amplitude and $f_{rf}$ is the oscillation frequency.

Based on the nonlinear-inductance model defined in Fig. 2b, we draw the equivalent circuit of the rf-SQUID readout circuit as shown in Fig. 6b, where the rf current source, the 50Ω impedance, and tank capacitor are composed into one QPD element, namely QPD element-2 defined with $\Phi_{\theta 2}$. The tank circuit is modeled as a loop with one QPD element.

Thus, there are two QPD elements and two circuit loops for the rf-SQUID readout circuit, i.e., $Q = 2$ and $P = 2$. The Loop-1 is applied with an external flux $\Phi_{in}$, $\Phi_{e1} = \Phi_{in}$. We extract the topology parameter $[\sigma]$ and the inductance matrix $[L_m]$ from the equivalent circuit as

$$
\begin{align}
[\sigma] &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\
[L_m] &= \begin{bmatrix} L & -M_{12} \\ -M_{12} & L_r \end{bmatrix}
\end{align}
$$

(12)
Fig. 6. (a) The rf-SQUID modulated by the rf current; (b) the equivalent circuit of rf-SQUID readout circuit.

Fig. 7. (a) a typical dc-SQUID circuit; (b) the equivalent circuit of the dc-SQUID.

3.2 dc-SQUID

A typical dc-SQUID biased with a current source $I_b$ is shown in Fig. 7a. Assuming that the bias current $I_b$ is assigned equally into two junctions, i.e., $I_{b1} = I_{b2} = I_b/2$, the equivalent circuit of dc-SQUID is drawn as shown in Fig. 7b. It is a single loop with two QPD elements. Thus, $Q = 2$ and $P = 1$ for dc-SQUID circuit equations. We extract
the parameter $[\sigma]$ and the inductance matrix $[L_m]$ from the equivalent circuit as

$$
\begin{bmatrix}
[\sigma] &= \begin{bmatrix} 1 & -1 \\
L_m &= L_s
\end{bmatrix}
\end{bmatrix}
(13)
$$

3.3 bi-SQUID

A current-biased bi-SQUID circuit is shown in Fig. 8a. It looks like a combination of the rf-SQUID and dc-SQUID. We define three QPD elements for three Josephson junctions and draw the equivalent circuit of bi-SQUID, as shown in Fig. 8b. Here, we assume that the external bias current $I_b$ are assigned equally into $J_1$ and $J_2$, i.e., $I_{b1} = I_{b2} = I_b/2$ and $I_{b3} = 0$.

![Diagram](image)

Fig. 8. (a) a typical bi-SQUID circuit; (b) the equivalent circuit of the bi-SQUID.

For the bi-SQUID circuit, $Q = 3$ and $P = 2$ since there are three QPD elements and two circuit loops. The topology parameter $[\sigma]$ and the inductance matrix $[L_m]$ are

$$
\begin{bmatrix}
[\sigma] &= \begin{bmatrix} 0 & 0 & 1 \\
L_m &= \begin{bmatrix} L_{s1} & -M_{i2} \\
-M_{i2} & L_{s2}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
(14)
3.4 dro-SQUID

A current-biased dro-SQUID is shown in Fig. 9a. It consists of three Josephson junctions and a shunt resistor. Based on the concept of the QPD element, we draw the equivalent circuit as shown in Fig. 9b, where the shunt resistor \( R_a \) driven by the current \( I_b \) is regarded as a QPD element.

![Fig. 9. (a) a typical dro-SQUID circuit; (b) the equivalent circuit of the dro-SQUID.](image)

From the equivalent circuit, we find four QPD elements and two circuit loops. Thus, \( Q = 4 \) and \( P = 2 \), for the dro-SQUID. The topology parameter \([\sigma]\) and the inductance matrix \([L_m]\) are expressed as

\[
\begin{align*}
[\sigma] &= 
\begin{bmatrix}
1 & -1 & 0 & 0 \\
0 & 1 & 1 & -1
\end{bmatrix} \\
[L_m] &= 
\begin{bmatrix}
L_s & -M_{12} \\
-M_{12} & L_a
\end{bmatrix}
\end{align*}
\]

(15)

3.5 SQIF and SQUID array

The superconducting quantum interference filter (SQIF) and SQUID array, as shown in Fig. 1e and f, are regarded as the multiple dc-SQUID cell circuits connected in parallel or serial. We can draw their equivalent circuits simply by repeating the cell
circuit shown in Fig. 7b. Thus, by increasing the dimension of circuit variables and parameters based on the dc-SQUID, our method can simplify their circuit equations derivations significantly. Moreover, in our method, the inductance matrix $[L_m]$ provides the complete parameters to describe all the mutual couplings between loops, including the indirectly connected loops.

4. Simulation Results and Discussion

The SQUID circuit simulation is implemented by finding the numerical solutions of the circuit equations shown in Fig. 5, with the input and the parameters defined in the equivalent circuit. For simplicity in the simulation, the capacitances and inductances are normalized with a reference resistor $R_0$ and a reference critical current $I_0$. For example, an inductance $L_x$ is expressed with the normalized $\beta_{Lx}$, $\beta_{Lx} = 2\pi I_0 L_x / \Phi_0$; a capacitance $C_x$ is normalized as $\beta_{Cx}$, $\beta_{Cx} = 2\pi I_0 R_0^2 C_x / \Phi_0$.

Fig. 10 shows the simulation results of the rf-SQUID circuit shown in Fig. 6a. The typical flux-modulated current-voltage characteristics[13] and the periodical flux-to-voltage conversion properties are demonstrated, where the voltage $V_{rf}$ is the root-mean-square (RMS) value of the voltage of $\Phi_{\theta_2}$.

![Fig. 10. The simulation results of rf-SQUID readout circuit: (a) the current-voltage characteristics; (b) the flux-to-voltage characteristics. In the simulation, $I_0=I_{01}$; $R_0=R_{01}=1\Omega$; $I_{in}=0.3 I_0$.](image)

Fig. 11 exhibits the simulation results of the dc-SQUID circuit shown in Fig. 7a. We obtained the typical flux-modulated current-voltage characteristics as shown in Fig. 11a, and the periodical flux-voltage characteristics varied by the bias currents [1, 14], as shown in Fig. 11b. Here, the voltage $V_s$ is the average value of the voltage of $\Phi_{\theta_1}$ or $\Phi_{\theta_2}$.
Fig. 11. Simulation results of dc-SQUID: (a) current-voltage characteristics; (b) flux-to-voltage characteristics. In the simulation, $I_0 = I_{01} = I_{02}; R_0 = R_{01} = R_{02}; I_{01} = I_{02} = 0.3 I_0$.

Fig. 12. Simulation results of the flux-to-voltage characteristics of the bi-SQUID. In the simulation, $I_0 = I_{01} = I_{02}; R_0 = R_{01} = R_{02}; I_{01} = I_{02} = 0.3 I_0$.

Fig. 12 illustrates the flux-voltage characteristics of the bi-SQUID under different
bias currents, where the average voltage $V_s$ is calculated from the $\Phi_{\theta_1}$ or $\Phi_{\theta_2}$. We can see that the flux-voltage characteristic under a bias current $I_b = 2.0I_0$ is linearized. This characteristic agrees with the results measured in the practical bi-SQUIDs [15].

Fig. 13 depicts flux-voltage characteristics of dro-SQUID simulated with a group of experimental parameters, where the voltage $V_s$ is the average voltage of the real-time response of $\Phi_{\theta_3}$. The flux-voltage curves are square wave-shaped, as found in the typical experimental results [16].

$$\beta_{C_1} = \beta_{C_2} = 0.2; \beta_{C_3} = 0.3; \beta_{L_s} = 1.7; \beta_{M_{12}} = 0.8; \beta_{L_a} = 2.0$$

$$I_{03} / I_0 = 1.5; \quad R_a = R_0; \quad R_1 = R_2 = R_3 = 5R_0$$

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**Fig. 13.** Simulation results of the flux-to-voltage characteristics of the dro-SQUID. In the simulation, $I_0^* = I_{01} = I_{02}; I_{a1} = I_{a2} = I_{a3} = 0.3I_0$.

Our method exhibits two merits in the analysis and simulation of several SQUID circuits.

First, it unifies the definitions of both the superconducting and normal elements with the QPD element concept, using only the conventional current and flux instead of the superconducting quantum phase as the variables. According to this concept, the only element that makes SQUID circuits distinct from the conventional normal circuits is the cosine-function potential generated by the Josephson current.

Second, it significantly simplifies the SQUID circuit analysis and simulation, using the uniform circuit equations and dynamic model. The dynamic model reveals that the SQUID circuit is analog to a multi-particle system, in which the QPD element is equivalent to a classic particle rolling down the current-determined potential.

Therefore, this general circuit analysis method bridges the gap between superconducting and non-superconducting circuits; it improves the understandings of SQUID circuits for electronic engineers who are unfamiliar with superconducting physics.
4. Conclusion

This article introduces a general analysis and simulation method for different SQUIDs and their hybrids with non-superconducting normal circuits. In the method, Josephson junction and normal circuit elements are treated equally as the non-linear inductance and described with the uniform current-flux function. By this means, we can use only conventional current and flux as variables to derive the uniform circuit equations and dynamic model for both superconducting and normal circuits. It shows that the Josephson current is the only element that makes SQUID distinct from the non-superconducting normal circuit. This method removes the obstacles in superconducting circuits analysis and simulation for conventional electronic engineers.

Acknowledgements

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