Bubble nucleation in disordered Landau-Ginzburg model

R. Acosta Diaz 1 and N. F. Svaiter 2
Centro Brasileiro de Pesquisas Físicas - CBPF
Rua Dr. Xavier Sigaud, 150, Rio de Janeiro - RJ, 22290-180, Brazil

C.A.D. Zarro 3
Universidade Federal do Rio de Janeiro, Instituto de Física
Av. Athos da Silveira Ramos, 149, Rio de Janeiro - RJ, 21941-909, Brazil

Abstract

In this paper we investigate bubble nucleation in a disordered Landau-Ginzburg model. First we adopt the standard procedure to average over the disordered free energy. This quantity is represented as a series of the replica partition functions of the system. Using the saddle-point equations in each replica partition function, we discuss the presence of a spontaneous symmetry breaking mechanism. The leading term of the series is given by a large-\( N \) Euclidean replica field theory. Next, we consider finite temperature effects. Below some critical temperature, there are \( N \) real instantons-like solutions in the model. The transition from the false to the true vacuum for each replica field is given by the nucleation of a bubble of the true vacuum. In order to describe these irreversible processes of multiple nucleation, going beyond the diluted instanton approximation, an effective model is constructed, with one single mode of a bosonic field interacting with a reservoir of \( N \) identical two-level systems.

KEYWORDS: disordered systems; free energy; bubble nucleation.

PACS numbers: 05.20.-y, 75.10.Nr

1e-mail: racosta@cbpf.br
2e-mail: nfuxsvai@cbpf.br
3e-mail: carlos.zarro@if.ufrj.br
1 Introduction

The critical behavior of disordered systems has been discussed since the 70’s in the literature. Two concepts that are of fundamental importance in such systems are respectively frustration and quenched disorder. Frustration was introduced to describe properties of spin-glasses with many different ground states [1]. The free energy landscape of these systems have a multivalley structure. In quenched disordered systems, the disorder is in static equilibrium and therefore these systems are spatially random. The study of quenched disordered systems leads to new universality classes in critical regions and also the possibility of a large number of metastable states in free energy landscape. In such systems defined in the continuum with quenched disorder, it is a hard task to perform a perturbative expansion in any model, since these systems are intrinsically inhomogeneous. One way to circumvent such problem is to average over the ensemble of all realizations of the disorder quantities of interest. For example, average the free energy functional with respect to the probability distribution of the disordered field. In these disordered systems, the replica symmetry breaking with its physical consequences, has been intensely discussed by the physical community [2–8].

Recently, it was proposed a new method to average the disorder dependent free energy [9, 10]. Physical consequences of this approach were investigated in Refs. [11, 12]. The motivation of this paper are the following. First is to stress the main differences between perturbative expansions in field theories without or in the presence of disorder fields, discussing cluster properties of disordered average $n$-point correlation functions. The second one is to discuss the physical consequences of the results obtained in Ref. [12]. Finally, going beyond the above discussed results, we introduce an effective model to describe false-true vacuum transitions of replica fields. Specifically, we are interested to describe the phase transitions present in the continuous version of the $d$-dimensional random field Ising model [13–15]. Using the approach discussed above, the structure of the replica space is investigated using the saddle-point equations obtained from each replica field theory. Assuming the replica symmetric ansatz, we investigate the spontaneous symmetry breaking mechanism in some replica partition functions. Our approach reveals the existence of replica instantons-like solutions (real or complex) in this model [16–18]. For the case of real instantons-like solutions our methodology produced the following scenario.

Vacuum decay in this theory with $N$ replicas can be described in the following way. For low temperatures, there is a critical temperature where each replica field has two non-degenerate vacuum states. Consequently, for each replica field there will be a transition from the false vacuum to the true one with nucleation of a bubble of the true vacuum. This first-order phase transition, in the low temperature limit, was investigated in Refs. [19,20]. The crucial question here is which tools we can use to describe these irreversible processes, i.e., the nucleation of bubbles of true vacuum in the false vacuum environment.

We shall now be concerning with the description of the collective behavior of the $N$ replica fields. The point of departure is given by Ref. [21]. There, the authors emphasize that it is possible to represent $N$ structures with a false and a true vacuum using two-level systems. The situation where the nucleation of bubbles occurs, decreasing the free energy of the system is
characteristic of an open system. To go further describing this multiple nucleation, i.e., the collective nucleation of bubbles in the disordered system, an effective model is constructed using the functional integral formalism developed to study phase transitions in quantum optics systems by Popov and Fedotov [22–24]. In the large $N$ limit, the functional integral describes an ensemble of $N$ two-level systems interacting with one single bosonic mode, instead of the usual situation of a the countable infinitely set of field modes.

The justifications for introducing the bosonic mode are the following: this bosonic mode is connecting the two-level systems and also it makes possible the decay for each replica field from the false vacuum to the true one. In this scenario, it is possible to show the existence of a temperature where the free energy is non-analytic. The equivalence between these two quite distinct physical models can be justified using the following argument. In the disordered Landau-Ginzburg model, the leading replica partition function in the series representation for the free energy shows that all the replica fields, with a false and true vacuum states are strongly correlated. This is exactly the situation discussed by Popov and Fetotov, where only one single mode is resonant with the two-level atoms. All the two-level atoms interact coherently with this single mode.

The organization of this paper is the following. In Sec. 2 we discuss a $d$-dimensional disordered Landau-Ginzburg model. In Sec. 3 in a generic replica partition function we discuss the structure of the replica space using the saddle-point equations of the model. In Sec. 4 we demonstrate at low temperatures the emergence of $N$ instantons-like solutions in the leading replica partition function of the model. In Sec. 5 to describe the bubble nucleation in the disordered model, an effective model is constructed using the formalism developed by Popov and Fedotov. Conclusions are given in Sec. 6. We use the units $\hbar = c = k_B = 1$.

## 2 A disordered Landau-Ginzburg model

In magnetic materials with disorder, in principle there are two kind of systems. The first set is one where the disorder is related to the local spin interaction. In this case the disorder generates multiple disordered ground states, the spin-glass phase. The second, is one where the disorder is a random external perturbation. One disordered model that belongs to this second set is the random field Ising model. This model has been studied intensively from the theoretical and experimental point of view and used to describe many systems in nature. One is the case of diluted antiferromagnetic in a homogeneous external field [25, 26] and also binary fluids in porous media. For instance, in order to model binary fluids confined in porous media, when the pore surfaces couple differently to the two components of a phase-separating mixture, the random field has been used by the literature. These systems can develop a second or a first-order phase transition [27–29]. The random field Ising model in a hypercubic lattice in $d$-dimensions is described by the Hamiltonian

$$H = -J \sum_{(i,j)} S_i S_j - \sum_i h_i S_i,$$

(1)
where \((i, j)\) indicates that the sum is performed over nearest neighbour pairs and \(S_i = \pm 1\). In the above equation \(N\) is the total number of Ising spins. Periodic boundary conditions can be used and the thermodynamic limit must be used in the end. The partition function is \(Z = Tr e^{-\beta H}\). In Eq. (11) the \(h_i\)'s are the quenched random variables totally uncorrelated on different sites. The average free energy is defined by \[F = -\frac{1}{\beta} \mathbb{E}[\ln Z],\] where \(\mathbb{E}[...]\) means the average over the ensemble of all the realizations of the quenched disorder. Here we consider a Gaussian distribution defined by \[P(h_i) = \frac{1}{\sqrt{2\pi h_0^2}} \exp\left(-\frac{h_i^2}{2h_0^2}\right).\] (2)

The probability distribution of such quenched random variables has zero mean-value, \(\mathbb{E}[h_i] = 0\), and correlation functions given by \(\mathbb{E}[h_i h_j] = h_0^2 \delta_{ij}\). Here we are interested in the small disordered limit, i.e., \(h_0^2 \ll 1\).

The properties of the phase transition of the random field Ising model is still under debate [30–37]. The question of the lower critical dimension, bellow which long-range order is absence and the upper critical dimension, above which the model presents mean-field behavior independent of the dimension has been a matter of controversy. Imry and Ma obtained that the model with nearest neighbor interaction presents spontaneous magnetization only for \(d \geq 3\) [38]. This result is in contradiction with the dimensional reduction argument [39, 40]. The controversy was solved by Bricmont and Kupiainen, who proved that there is a phase transition in the random field Ising model for \(d \geq 3\) [41, 42], and Aizenman and Wehr, that showed the absence of phase transition for \(d = 2\) in the model [43].

The behavior of systems defined in a lattice near the critical point can be modeled by continuous statistical field theories. This can be achieved replacing the lattice structure by a continuum where the order parameter can be obtained averaging with respect to a statistical weight a random continuous field. For instance, the effective \(O(n)\) Landau-Ginzburg model is defined by \(\varphi_i(x)\), a \(n\)-component field. This model is able to describe several universality classes. For the case \(n \to 0\) can describe self-avoided polymers [41]. For \(n = 1\) it describes the critical behavior of the Ising model. For \(n = 2\) the critical behavior of the XY model and also the two-dimensional Coulomb gas are described. For \(n = 3\) the Heisenberg model [47] and low energy dynamics of QCD can be modeled by \(n = 4\). Finally for \(n \to \infty\) it is possible to solve exactly the model. In this paper we study the critical properties of the random field Ising model, by means of a continuous scalar field theory defined in \(\mathbb{R}^d\) with symmetry \(Z_2\) (the \(n = 1\) case).

We assume that the critical behavior of the random field Ising model in \(\mathbb{Z}^d\) can be described by a continuous disordered Landau-Ginzburg model. Firstly, let us briefly discuss the model without disorder. We are following Ref. [48]. The Landau-Ginzburg functional, i.e., the Hamiltonian \(H(\varphi)\) for the scalar field is given by

\[H = \int d^d x \left( \frac{1}{2} \varphi(x) (-\Delta + m_0^2) \varphi(x) + \frac{\lambda_0}{4!} \varphi^4(x) \right),\] (3)
where the symbol $\Delta$ denotes the Laplacian in $\mathbb{R}^d$ and $\lambda_0$ and $m^2_0$ are analytic functions of the temperature. Actually, $m^2_0$ is the inverse of the correlation length. Doing a parallel with Euclidean field theory we call them respectively the bare coupling constant and the squared mass of the model. For high temperatures, away from the critical point, the correlation functions of the model are short-ranged. Near the critical point, the correlation functions becomes long ranged, where the characteristic length scale, the correlation length $\xi$ has a power law behavior, exactly as in Euclidean field theory. The partition function of the model is defined by the functional integral

$$Z = \int_{\partial\Omega} [d\varphi] \exp(-H(\varphi)), \quad (4)$$

where $[d\varphi]$ is a formal Lebesgue measure, given by $[d\varphi] = \prod_x d\varphi(x)$, and $\partial\Omega$ in the functional integral means that the field $\varphi(x)$ satisfies some boundary condition in the boundary $\partial\Omega$ of some bounded domain, i.e., connected open set $\Omega \subset \mathbb{R}^d$. Periodic boundary conditions can be imposed to preserve translational invariance, replacing $\mathbb{R}^d$ by the torus $\mathbb{T}^d$. To remove the ultraviolet divergences, in the Fourier decomposition of the field a cut-off must be introduced. This cut-off is related with a elementary length scale, the lattice spacing of the original model. Since in all the discussions of this paper we need no more than the tree-level calculations, this technical remark is immaterial for the results presented in the paper.

The question now arises is the cluster properties of correlation functions for the model with or without disorder. Therefore, let us start briefly discussing these quantities. Averaging with respect to the Boltzmann weight we get the $n$-point correlation functions of the model

$$\langle \varphi(x_1)\ldots\varphi(x_n) \rangle = \frac{1}{Z} \int [d\varphi] \prod_{i=1}^n \varphi(x_i) \exp(-H(\varphi)). \quad (5)$$

Introducing a fictitious source $j(x)$ we can define $Z(j)$, the generating functional of all $n$-point correlation functions as $[49, 50]$

$$Z(j) = \int_{\partial\Omega} [d\varphi] \exp\left(-H(\varphi) + \int d^d x \, j(x)\varphi(x)\right). \quad (6)$$

Taking functional derivatives with respect to the source and setting to zero in the end, we obtain the $n$-point correlations functions of the model

$$\langle \varphi(x_1)\ldots\varphi(x_k) \rangle = Z^{-1}(j) \left. \frac{\delta^k Z(j)}{\delta j(x_1)\ldots\delta j(x_k)} \right|_{j=0}. \quad (7)$$

Notice that these $n$-point correlation functions are given by the sum of all diagrams with $n$-external legs, including the disconnected ones. Next, using the linked cluster theorem, it is possible to define the generating functional of connected correlation functions given by $W(j) = \ln Z(j)$. The order parameter of the model without disorder $\langle \varphi(x) \rangle$ is given by
\[ \langle \varphi(x) \rangle = Z^{-1}(j) \frac{\delta Z(j)}{\delta j(x)} \bigg|_{j=0}. \] 

Before continue, we would like to clarify terminology. Although streaking speaking, the order parameter is defined by the above equation, throughout this paper we may call local order parameter for the continuous field \( \varphi(x) \) defined for \( x \in \mathbb{R}^d \). Applying two functional derivatives on the generating functional of connected correlation functions we get

\[ \langle \varphi(x_1)\varphi(x_2) \rangle_{\text{connected}} = \left[ \frac{1}{Z(j)} \frac{\delta^2 Z(j)}{\delta j(x_1)\delta j(x_2)} \right]_{j=0} - \left[ \frac{1}{Z(j)^2} \frac{\delta Z(j)}{\delta j(x_1)} \frac{\delta Z(j)}{\delta j(x_2)} \right]_{j=0}. \] 

The large distance decay properties of these connected correlation functions are called cluster properties. These correlation functions goes to zero for \( |x_1 - x_2| \to \infty \). In an Euclidean field theory the cluster properties of the Schwinger functions are equivalent to the uniqueness of the vacuum.

We briefly present the basic tools that we need to discuss disordered systems \(^{51}\). The continuum version for the \( d \)-dimensional random field Ising model, is given by a \( d \)-dimensional Landau-Ginzburg scalar \( \lambda \varphi^4 \) model in the presence of a disorder field linearly coupled to the scalar field. The Hamiltonian in the presence of disorder is given by

\[ H(\varphi, h) = H(\varphi) + \int d^d x \ h(x)\varphi(x), \] 

where \( H(\varphi) \) is the Landau-Ginzburg Hamiltonian, defined in Eq. \(^{53}\), and \( h(x) \) is a quenched disorder field. The disordered functional integral \( Z(h) \) is defined by

\[ Z(h) = \int_{\partial \Omega} [d\varphi] \ \exp(-H(\varphi, h)). \] 

Eq. \(^{51}\) defines the partition function associated with the scalar field for a given disorder configuration. The \( n \)-point correlation functions for one specific realization of the disorder field reads

\[ \langle \varphi(x_1)\ldots\varphi(x_n) \rangle_h = \frac{1}{Z(h)} \int [d\varphi] \prod_{i=1}^{n} \varphi(x_i) \exp(-H(\varphi, h)). \] 

To introduce a generating functional for one realization of the disorder field, \( Z(h; j) \), we again use a fictitious source \( j(x) \):

\[ Z(h; j) = \int_{\partial \Omega} [d\varphi] \ \exp\left(-H(\varphi, h) + \int d^d x \ j(x)\varphi(x)\right). \]
For a particular realization of the disorder field, \( Z(h; j) \) can be used to obtain the \( n \)-point correlation function given by Eq. (12) by means of functional derivatives. With these correlation functions, one can compute the disorder-averaged correlation functions given by

\[
\mathbb{E}\left[ \langle \varphi(x_1) \ldots \varphi(x_n) \rangle_h \right] = \int [dh] P(h) \langle \varphi(x_1) \ldots \varphi(x_n) \rangle_h,
\]

where \( \langle \varphi(x_1) \ldots \varphi(x_n) \rangle_h \) is given by Eq. (12) and \([dh] = \prod_x dh(x)\) is again a formal Lebesgue measure. As in the pure system case, one can define a generating functional for one disorder realization, \( W_1(h; j) = \ln Z(h; j) \). We take the disorder-average of this generating functional, \( W_2(j) = \mathbb{E}[W_1(h; j)] \). We have

\[
W_2(j) = \int [dh] P(h) \ln Z(h; j).
\]

Taking the functional derivative of \( W_2(j) \) with respect to \( j(x) \), we get

\[
\frac{\delta W_2(j)}{\delta j(x)} \bigg|_{j=0} = \int [dh] P(h) \left[ \frac{1}{Z(h; j)} \frac{\delta Z(h; j)}{\delta j(x)} \right] \bigg|_{j=0}.
\]

Since \( \langle \varphi(x) \rangle_h \) is the average of the field for a given configuration of the disorder in the disordered Landau-Ginzburg model the above quantity \( \mathbb{E}\left[ \langle \varphi(x_1) \rangle_h \right] \) is the order parameter of the model [40]. The second functional derivative of \( W_2(j) \) with respect to \( j(x) \) gives \( G(x_1 - x_2) \). We have

\[
\frac{\delta^2 W_2(j)}{\delta j(x_1) \delta j(x_2)} \bigg|_{j=0} = \mathbb{E}\left[ \langle \varphi(x_1) \varphi(x_2) \rangle_h \right] - \mathbb{E}\left[ \langle \varphi(x_1) \rangle_h \langle \varphi(x_2) \rangle_h \right].
\]

Notice that, in general, the following quantities are not equal, i.e.,

\[
\mathbb{E}\left[ \langle \varphi(x_1) \rangle_h \langle \varphi(x_2) \rangle_h \right] \neq \mathbb{E}\left[ \langle \varphi(x_1) \rangle_h \right] \mathbb{E}\left[ \langle \varphi(x_2) \rangle_h \right].
\]

Therefore the Eq. (17) is not the disordered average two-point connected correlation function. To proceed let us define the following averaged quantity

\[
\chi(x_1 - x_2) = \mathbb{E}\left[ \langle \varphi(x_1) \rangle_h \langle \varphi(x_2) \rangle_h \right].
\]

This above disconnected correlation function can be different from zero even if the order parameter of the model is zero. The decay of these two-point correlation functions \( G(x_1 - x_2) \) and \( \chi(x_1 - x_2) \) at critical region defines two critical exponents \( \eta \) and \( \eta' \) [52]. We have

\[
G(x_1 - x_2) \approx |x_1 - x_2|^{-(d-2+\eta)}.
\]
\[
\chi(x_1 - x_2) \approx |x_1 - x_2|^{-(d-4+\eta')}. 
\]

(21)

In a pure system, taking functional derivatives of \(W(j)\) we get the connected correlation functions, that satisfies clustering property. Applying two functional derivatives, the disordered average functional \(W_2(j) = \mathbb{E}[W_1(h; j)]\) does not generate the disordered average two-point connected correlation functions of the model. This can be generalized to the \(n\)-point correlation functions. The fundamental problem is the fact that since there are many minima \([53][55]\) in these systems, we can not expand around only one specific minimum, hence a non-perturbative scenario emerges. The non-perturbative scenario can not be studied neither using the renormalization group equations nor the composite operator formalism \([56][58]\). Composite operator formalism is a way to use resummation methods (sum of infinite series of diagrams) to avoid the infrared divergences of a massless theory. These methods can not reveal the vacuum structure of the disordered system.

One possible way to proceed is the following. In the presence of these metastable states one must identify clustering states, i.e., the states where the connected correlation functions vanishes for large distances, and introduce an order parameter that characterize such domain \([59]\). We do not expect that this program can be implemented in a straightforward way. To deal with this above discussed problem, the first step is to identify the metastable states, i.e., show the presence of many local minima in the free energy landscape. In other words, this fundamental difficulty may point that a local approach of field theory based in the correlation functions must be substituted, at least in the beginning by another more promising procedure. Instead of concentrate our efforts to define local objects, we may study only global quantities, such as, the averaged free energy. As we expected, here we will show the presence of a large number of metastable states in the disordered system.

For instance, for free fields without disorder the spectral zeta-function technique \([60][64]\), which is a way to regularize the determinant of Laplace operator, can be used to compute the free energy of this pure system. In the next section, we show how this approach can be used to access the non-perturbative landscape of the disordered system. Here, we proceed as follows. We are interested to compute \(W_2(j)|_{j=0} = \mathbb{E}[W_1(h; j)]|_{j=0}\), namely the disorder-averaged free energy.

3 Distributional zeta-function approach

In order to circumvented the problem of many local minima that the perturbative expansion fail to take into account, Lancaster et al. \([65]\) discussed a model where many solutions of the mean field equations obtained from each realization of the disorder are weighted by Boltzmann factors. In the following we show that it is possible to investigate a non-perturbative scenario using the distributional zeta-function approach \([9][10]\). This approach has similarities with the above discussed method.
Here, we do not give details of the derivation but only the essential steps of the mathematical rigorous procedure that allow to use the replica partition functions in order to compute the disorder-averaged free energy. For a given probability distribution of the disorder, one is mainly interested in averaging the disorder dependent free energy functional which reads

\[ F = -\frac{1}{\beta} \int [dh] P(h) \ln Z(h), \]

(22)

where \( \beta^{-1} = T \), where \( T \) is the temperature of the system. This averaged free energy represents, in an Euclidean field theory, the connected vacuum-to-vacuum diagrams in the disordered system. For a general disorder probability distribution, using the disordered functional integral \( Z(h) \) given by Eq. (11), the distributional zeta-function, \( \Phi(s) \), is defined as

\[ \Phi(s) = \int [dh] P(h) \frac{1}{Z(h)^s}, \]

(23)

for \( s \in \mathbb{C} \), this function being defined in the region where the above integral converges. The above equation is a natural generalization of the families of zeta-functions [66–72]. The average free energy can be written as

\[ F = (d/ds)\Phi(s)|_{s=0^+}, \quad \Re(s) \geq 0, \]

(24)

where one defines the complex exponential \( n^{-s} = \exp(-s \log n) \), with \( \log n \in \mathbb{R} \). Using analytic tools, the average free energy can be represented as

\[ F = \frac{1}{\beta} \left[ \sum_{k=1}^{\infty} \frac{(-1)^k a^k}{k} \mathbb{E}[Z^k] + \ln(a) + \gamma - R(a) \right] \]

(25)

where \( a \) is a dimensionless arbitrary constant, \( \gamma \) is the Euler-Mascheroni constant, and, for large \( a \), \( |R(a)| \) is quite small, therefore, the dominant contribution to the average free energy is given by the replica partition functions of the model. For simplicity we write \( \mathbb{E}[Z(h)^k] \equiv \mathbb{E}[Z^k] \). Note that a \( \frac{1}{k!} \) factor was absorbed in \( \mathbb{E}[Z^k] \). To proceed, we assume that the probability distribution of the disorder is written as \([dh] P(h)\), where

\[ P(h) = p \exp\left(-\frac{1}{2\sigma} \int d^d x(h(x))^2\right). \]

(26)

The quantity \( \sigma \) is a positive parameter associated with the disorder and \( p \) is a normalization constant. In this case we have a delta correlated disorder field, i.e., \( \mathbb{E}[h(x)h(y)] = \sigma \delta^d(x - y) \). As it was stressed by many authors, it is important to clarify the behavior of the model for small values of \( \sigma \). After integrating over the disorder we get that each replica partition function \( \mathbb{E}[Z^k] \) can be written as
where the effective Hamiltonian $H_{\text{eff}}(\varphi_i)$ describing the field theory with $k$-replica field components is given by

$$H_{\text{eff}}(\varphi_i) = \int d^d x \left[ \sum_{i=1}^k \left( \frac{1}{2} \varphi_i(x) (\Delta + m_0^2) \varphi_i(x) + \frac{\lambda_0}{4!} \varphi_i(x)^4 - \frac{\sigma}{2} \sum_{i,j=1}^k \varphi_i(x) \varphi_j(x) \right) - \frac{\lambda_0}{3!} \varphi_i(x)^3 \right].$$

In the original Landau mean-field theory to discuss second-order phase transitions, an expansion for the free energy near the critical temperature as a power series of the order parameter is introduced. It is important to keep in mind that in the framework discussed by us the same idea is introduced. Nevertheless, by the presence of the disorder field, instead of a series in the order parameter we get a series in the replica partition functions of the model to define the averaged free energy.

The mean-field theory corresponds to a saddle-point approximation in each replica partition function. A perturbative approach gives us the fluctuation corrections to mean-field theory. Hence, to implement a perturbative scheme, it is necessary to investigate fluctuations around the mean-field equations. From each replica field theory, let us investigate the solutions of the saddle-point equations which are given by

$$\left( -\Delta + m_0^2 \right) \varphi_i(x) + \frac{\lambda_0}{3!} \varphi_i(x)^3 = \sigma \sum_{j=1}^k \varphi_j(x).$$

Imposing the replica symmetric ansatz, i.e., $\varphi_i(x) = \varphi(x)$, the saddle-point equation, in each replica partition function, reads

$$\left( -\Delta + m_0^2 - k\sigma \right) \varphi(x) + \frac{\lambda_0}{3!} \varphi^3(x) = 0.$$

At this stage it is easy to understand why the original replica method has problems, at least in this model. In this method, the average free energy is obtained using the formula

$$\mathbb{E} [\ln Z(h)] = \lim_{n \to 0} \frac{\partial}{\partial n} \mathbb{E} [Z(h)^n].$$

The $n \to 0$ limit in Eqs. (27), (28) is translated to a field theory with the dimension of the order parameter going to zero. Therefore, we would like to briefly discuss the limit $n \to 0$ in
the $O(n)$ Landau-Ginzburg model. It is well known that the self-avoiding random walk can be used as a mathematical model for polymers chains, where effects of excluded volume must be modeled [73, 74]. Since it represents a non-Markovian stochastic process, there are many open questions in the literature, as, for instance, how many walks there are between two points. In the case of the self-avoiding random walk problem, the probability of finding the particle at $y$ at time $t$ if the particle was released in point $x$ at $t = 0$, is a sum of diagrams that are exactly those for the correlation function of the $O(n)$ Landau-Ginzburg model for $n \to 0$.

In the original replica method although one work with a replica field theory where the number of replicas must go to zero, the situation is quite different from the above discussed cases. The average free energy involves derivation of the integer moments of the partition function. One consequence of this fact is that using the simplest possible replica symmetric ansatz in each replica partition function reduce the equations to the saddle-point equations of systems without disorder. Therefore, the replica symmetry breaking is introduced as a necessary condition to recover information from the disorder field in the theory.

Using the distributional zeta-function method we can go further, since we have obtained analytic expression for the average free energy that does not involve derivation of such integer moments. Notice that, in principle, we have to consider all terms in Eq. (25), since all values of $k$ are allowed. However, we have a constraint as the squared mass, $m_0^2 - k\sigma$, must be positive definite to describe a well-defined physical theory. In this case, one has a critical value of $k$, namely, $k_c = [m_0^2/\sigma]$, above which one would obtain a negative squared mass, where $[x]$ means the integer part of $x$. For $k < k_c$, the replica fields fluctuate around the zero value. For $k > k_c$, we have to shift these replica fields since the zero value is not a stable equilibrium state. The last situation represents a spontaneous symmetry breaking mechanism.

In the framework of distributional zeta-function method, defining $v = (\frac{6(\sigma N - m_0^2)}{\lambda_0})^{1/2}$, the simplest choice of the replica space is given by

$$
\begin{align*}
\varphi_{l,i}^{(0)}(x) &= \varphi(x) \quad \text{for } l = 1, \ldots, k_c \text{ and } i = 1, \ldots, l \\
\varphi_{l,i}^{(1)}(x) &= \phi(x) + v \quad \text{for } l = k_c + 1, \ldots, N \text{ and } i = 1, \ldots, l \\
\varphi_{l,i}^{(l)}(x) &= 0 \quad \text{for } l > N.
\end{align*}
$$

(32)

Notice that we find a positive squared mass with self-interactions terms $\phi(x)^3$ and $\phi(x)^4$. From Eq. (32) and for $a$ and $N$ very large, the average free energy can be written as

$$
F = \frac{1}{\beta} \sum_{k=1}^{N} \frac{(-1)^k a^k}{k} \mathbb{E}[Z^k],
$$

(33)

which has its leading term for $k = N$. Therefore, in the large-$N$ limit, the expression for disorder-averaged free energy is reduced to the contribution of only one replica partition function, consisting in a large $N$-component replica fields. In the context of a large-$N$ scenario, we introduce two ’t Hooft couplings, namely, $f_0 = \sigma N$ and $g_0 = \lambda_0 N$. These parameters are finite in $N \to \infty$ although $\lambda \to 0$ and $\sigma \to 0$. 

10
4 Replica instantons-like solutions in the disordered system

The mean-field approach is used to analyze the phase diagram of our model. First, we consider that $m_0^2$ is a regular function of temperature. This situation is more complex than in an ordered system. We find three regions of interest. The first occurs for $m_0^2 \geq \sigma N$. In this case, all the replica fields oscillate around $\varphi = 0$, the trivial vacuum. For $a \gg N$, a very large $N$ limit is represented by only one replica partition function with $N$ ($N$ even) replica fields $\phi_i$. The $N$ replica fields has the symmetry $[Z_2 \times Z_2 \cdots \times Z_2]$. There is also a critical temperature $T_c^{(1)}$, where $m_0^2 = N\sigma$. The $[Z_2 \times Z_2 \cdots \times Z_2]$ symmetry is broken below $T_c^{(1)}$. For the second region, $\sigma \leq m_0^2 < \sigma N$, replica fields in some partition functions oscillates around the trivial vacuum, whereas fields in other replica partition functions now oscillates around the non-trivial vacuum. We are not interested in these ranges of $m_0^2$, for more details see Ref. [12]. For $m_0^2 < \sigma$, all the replica fields in each replica partition functions are oscillating around the non-trivial vacuum. In this case, for $a \gg N$ and for a very large-$N$ limit ($N$ even), the average free energy reads

$$F = \frac{1}{\beta} \mathbb{E}[Z^N], \quad \text{(34)}$$

where $a$ is absorbed in normalization of the functional integration and $\mathbb{E}[Z^N]$ is written as

$$\mathbb{E}[Z^N] = \frac{1}{N!} \int \prod_{i=1}^{N} [d\phi_j] \exp \left( -H_{\text{eff}}(\phi_j) \right), \quad \text{(35)}$$

and the effective Hamiltonian $H_{\text{eff}}(\phi_i)$ is given by

$$H_{\text{eff}}(\phi_i) = \int d^d x \left[ \sum_{i=1}^{N} \left( \frac{1}{2} \phi_i(x) \left( -\Delta + 3f_0 - 2m_0^2 \right) \phi_i(x) \right) + \left( \frac{f_0 g_0}{3! N} \right)^{\frac{1}{2}} \left( 1 - \frac{m_0^2}{f_0} \right)^{\frac{1}{2}} \phi_i^3(x) + \frac{g_0}{4! N} \phi_i^4(x) \right] - \frac{f_0}{2N} \sum_{i,j=1}^{N} \phi_i(x) \phi_j(x) \quad \text{(36)}$$

Notice that the symmetry $[Z_2 \times Z_2 \cdots \times Z_2]$ for $N$ replica fields is broken. A relevant question in the random field Ising model concerns the existence of an upper critical dimension, which, above it, the mean field approximation is exact. Since we have a cubic term in the action, the upper critical dimension is obtained from the relation $\frac{3}{2}(d-2) = d$, where the coupling constant becomes
dimensionless, therefore the critical dimension is \( d = 6 \). This result was discussed by Imry and Ma [38] and more recently in Ref. [75].

Our fundamental result is the following. To describe critical phenomena for systems without disorder it is introduced an order parameter that describes second-order phase transition where for low temperatures a state of reduced symmetry appears. In the disordered system the order parameter is now a \( N \)-vector field. Our aim is to describe bubble nucleation in the disordered model at low temperatures. A representation similar to the strong-coupling expansion in field theory [76–79] or the linked cluster expansion [80–84] can be used to represent a replica field theory. Rather than the usual case, which relies upon a gradient-free action, now the replicas become connected after applying a functional differential operator on a well-defined replica partition function. Here we would like to stress that the use of the linked cluster expansion in the Ising model was introduced in the literature by Englert [85].

To proceed, an external source \( J_i(x) \) in replica space linearly coupled with each replica is introduced. Defining \( R(x - y) = \sigma \delta^d(x - y) \), each replica partition function, \( \mathbb{E}[Z^N] = Z(\mathcal{J}) \), is written as a functional differential operator applied on \( Q_0(J) \). Hence

\[
Z(J) = 
\exp \left[ -\frac{1}{2} \sum_{i,j=1}^{N} \int d^d x \, d^d y \frac{\delta}{\delta J_i(x)} \frac{\delta}{\delta J_j(y)} R \right] Q_0(J).
\]

In the above equation, \( Q_0(\mathcal{J}) \), a modified replica partition function, is written as

\[
Q_0(\mathcal{J}) = \frac{1}{N!} \int \prod_{j=1}^{N} [d\phi_j] \exp \left( -H_{\text{eff}}^{(0)}(\phi_j, J_i) \right),
\]

where \( H_{\text{eff}}^{(0)}(\phi, J_i) \) is given by

\[
H_{\text{eff}}^{(0)}(\phi, J_i) = \int d^d x \sum_{i=1}^{N} \left[ \frac{1}{2} \phi_i(x) \left( -\Delta + 3f_0 - 2m_0^2 \right) \phi_i(x) \right.
\]

\[
\left. \left( \frac{f_0 g_0}{3! N} \right)^\frac{1}{2} \left( 1 - \frac{m_0^2}{f_0} \right)^\frac{1}{2} \phi_i^3(x) + \frac{g_0}{4! N} \phi_i^4(x) + J_i(x) \phi_i(x) \right].
\]

Notice that the above equation does not contain interaction terms between replica fields. It is important to notice that Eqs. [38] and [39] fixes all ultraviolet divergences of our model that can be regularized by standard analytic regularization procedures [86–90]. The main idea is that in the \( \epsilon = (4 - d) \) expansion all the primitively divergent correlation functions contain
poles. The principal part of the Laurent expansion defines the counterterms that we have to introduce to cancel such polar contributions. Introducing the renormalization constants $Z_{\phi}$, $Z_{\lambda}$ and $Z_m$ the theory becomes finite. This perturbative expansion program with the regularization and renormalization procedures can be straightforwardly implemented. However, we will not follow it further in this analysis. Instead, we will study the vacuum structure of the first factor of Eq. (37), i.e., $Z(J) = Q_0(J)$. It is possible to define the generating functional of connected correlation functions $\mathcal{W}(J) = \ln Z(J)$. For simplicity we assume that we have one replica field. The generating functional of one-particle irreducible correlations (vertex functions), $\Gamma[\phi]$, is gotten by taking the Legendre transform of $\mathcal{W}(J)$ [91]

$$\Gamma[\phi] + \mathcal{W}(J) = \int d^d x \left( J(x) \phi(x) \right), \quad (40)$$

where

$$\phi(x) = \frac{\delta \mathcal{W}(J)}{\delta J} \bigg|_{J=0}. \quad (41)$$

Now, we assume that the field $\phi(x) = \phi$, is uniform. In this case, we can write the effective potential, $V(\phi)$, as

$$\Gamma[\phi] = \int d^d x \, V(\phi), \quad (42)$$

where $V(\phi)$ takes into account the fluctuations in the model. From above discussion it is possible to write the tree-level effective potential for each replica field in the leading replica partition function. We have $V_{\text{tree}}(\phi) = U(\phi)$ where

$$U(\phi) = \frac{1}{2} (3f_0 - 2m_0^2) \phi^2 + \frac{\lambda_0 v}{3!} \phi^3 + \frac{\lambda_0}{4!} \phi^4, \quad (43)$$

where $v = \sqrt{6(f_0 - m_0^2)/\lambda_0}$ and the replica symmetric ansatz was evoked. The false and the true vacuum states $\phi(\pm)$ can be obtained

$$\phi(\pm) = -\frac{3v}{2} \pm 3 \sqrt{-\frac{f_0}{2\lambda_0} - \frac{m_0^2}{6\lambda_0}}. \quad (44)$$

Therefore, we get the following interesting result: there are instantons-like solutions in our model. The first term in the series representation for the functional differential operator is the diluted instanton approximation, i.e., $N$ non-interacting instantons-like solutions. For $f_0 > m_0^2 > -3f_0$, the system develops a spontaneous symmetry breaking in the leading replica partition function. In this case, all $N$ instantons-like solutions are complex. On the other hand, for $m_0^2 < -3f_0$ we get a similar situation as before, however all the instantons-like solutions are real.
Vacuum transition in this theory with $N$ replicas can be described in the following way. Lowering the temperature each replica field has two non-degenerate vacuum states. The transition from the false vacuum to the true one will nucleate bubbles of the true vacuum. This first-order phase transition, in the low temperature limit, was investigated in Refs. [19, 20]. The crucial question here is which tools we can use to describe the nucleation of bubbles.

5 Bubble nucleation and the fermionic Dicke model

In this section, we introduce a quite simple model to study the collective nucleation of bubbles in the disordered system. Our aim is to transform the original problem substituting by one that is technically treatable where the physical essence of the original problem is maintained. Let us remind the reader that one fundamental problem in quantum optics is the description of spontaneous emission of atoms [92–94]. In fluorescence situation, in the decay by spontaneous emission the atoms tend to decay independently. However, other regime also happens when the atoms act together. Superradiance is exactly this collective behavior when $N$ excited atoms in a cavity or in the free space where they are close together, with some characteristic length, decay spontaneously [95, 96]. The Dicke model was introduced to describe such collective behavior [97–99]. In this model it is assumed that the system is composed by an ensemble of two-level atoms, all of them in the excited state initially. Furthermore one assume that the two-level atoms are trapped in a high-Q cavity, then effectively one single mode in the countable infinitely set of field modes trapped by the cavity interact with the atoms. Other possibility is to assume that the two-level atoms interact with the free space continuum of field modes, but all the atoms are confined in a region with a characteristic length small compare with the wavelength of the resonant field mode. Both situations can describe a collective effect of emission, the superradiance, although irreversibility occurs only in the second situation, since the high-Q cavity makes the first situation time-invertible. In conclusion, this spin-boson model, even in the case of a single mode, is able to describes a phase transition from the fluorescent to superradiant phase, characterized by the fact that atoms in quite special conditions behaves cooperatively. They start to radiate spontaneously much faster and strongly than the emission of independent atoms.

From the multimode Dicke model, with spatially varying coupling between the two-level atoms and the bosonic modes, a spin-glass behavior is obtained after integrating out the bosonic field [100–103]. What firstly comes to mind is the feasibility of the reverse situation, i.e., starting from the random field Landau-Ginzburg model, a particular disordered statistical field theory model defined in the continuum, to use the Dicke model to describe the phase transitions of the system.

Let us start, discussing first the decay of one replica field from the false to the true vacuum state. Suppose that each replica field $\phi_i(x)$ is in the metastable state $\phi_i(+)$. Let us assume that the free energy gap per unit volume between the metastable state $\phi_i(+)$ and the state $\phi_i(-)$ is $\omega_i$. With the bubble formation of radius $R_i$ the free energy decreases by $\frac{4}{3} \pi R_i^3 \omega_i$ inside the bubble. The interface makes the free energy increases by $4 \pi R_i^2 \eta_i$ where $\eta_i$ is the interface free energy per unit area for each replica field. The contribution for each replica field to the free energy $\Delta F_i$ is
4\pi R_i^2 \eta_i - \frac{4\pi}{3} R_i^3 \omega_i$. There is a critical radius $R_c$ where for $R > R_c$ the nucleation of bubbles occurs. For finite temperature we have thermal nucleation of bubbles. In the case where $\beta \rightarrow \infty$ there is a quantum nucleation of bubbles. There is a standard procedure to find the decay rate in a Euclidean scalar theory \cite{104,106}. This formalism is not able to describe the collective behavior, i.e., the nucleation of $N$ bubbles. Since we would like to describe the nucleation of $N$ bubbles, we discuss here an alternative approach where the description of a cooperative behavior of two-level systems was presented.

Going back to the disordered model, lowering the temperature, each replica field has two non-degenerate vacuum states. The transition from the false vacuum to the true one will nucleate bubbles of the true vacuum. Our aim is to obtain an collective effective model to deal with a gas of $N$ real interacting instantons-like solutions (see, e.g. Eq. (36)). We claim that the qualitative features of the disordered system at very low temperatures can be described by the generalized Dicke model with only one single bosonic mode. In the Dicke model there is a mean-field type phase transition with a critical temperature below which the system is in a superradiant state. Some seminal papers discussing the phase transition in such model are Refs. \cite{107,110}.

Following Ref. \cite{21}, it is possible to represent $N$ structures with a false and true vacuum by $N$ two-level systems. Referring to Eqs. (37), (38) and (39), we are modeling the effect of considering more terms of the series, i.e., going beyond the diluted instanton approximation, as a bosonic mode interacting with all the two-level systems. The effective bosonic mode was introduced to play a two-fold role: is an effective mode that allows the interactions between the two-level systems and also to make the decay $\phi_{(+)}^{i} \rightarrow \phi_{(-)}^{i}$, possible. Note that we have actually an open system. In conclusion, the situations where nucleation of bubbles occurs, decreasing the free energy of the system will be substituted by an effective model. It is important to point out that we have assumed that going beyond the diluted instanton approximation, the vacuum structure associated to each replica field is not modified. If the inclusion of more terms of the series defined by Eq. (37) increase number of false vacuum states for each replica field, it is necessary to generalize the Dicke model using intermediate statistics \cite{111,112}.

In order to achieve the effective description discussed above, let us introduce, following Popov and Fedotov, the fermionic generalized Dicke model. See also Refs. \cite{113,114}. To proceed, let us define an auxiliary model to be called the fermionic full Dicke model in terms of fermionic raising and lowering operators $\alpha^+_i$, $\alpha_i$, $\beta^+_i$ and $\beta_i$, that satisfy the anti-commutator relations $\alpha_i \alpha_j^+ + \alpha_j^+ \alpha_i = \delta_{ij}$ and $\beta_i \beta_j^+ + \beta_j^+ \beta_i = \delta_{ij}$. We can also define the following bilinear combination of fermionic operators, $\alpha_i^\dagger \alpha_i - \beta_i^\dagger \beta_i$, $\alpha_i^\dagger \beta_i$ and $\beta_i^\dagger \alpha_i$ which obey the same commutation relations as the pseudo-spin operators $\sigma^-_{(i)}$, $\sigma^\dagger_{(i)}$ and $\sigma^z_{(i)}$.

\begin{equation}
\sigma^z_i \longrightarrow \alpha_i^\dagger \alpha_i - \beta_i^\dagger \beta_i,
\end{equation}

\begin{equation}
\sigma^+_i \longrightarrow \alpha_i^\dagger \beta_i,
\end{equation}

and finally

15
\[ \sigma_i^- \rightarrow \beta_i^\dagger \alpha_i. \] (47)

The Hamiltonian \( H_F \) of the auxiliary fermionic full Dicke model is

\[
H_F = \frac{\Omega}{2} \sum_{i=1}^{N} (\alpha_i^\dagger \alpha_i - \beta_i^\dagger \beta_i) + \omega_0 b^\dagger b + \frac{g_1}{\sqrt{N}} \sum_{i=1}^{N} \left( b \alpha_i^\dagger \beta_i + b^\dagger \beta_i^\dagger \alpha_i \right) + \frac{g_2}{\sqrt{N}} \sum_{i=1}^{N} \left( b \beta_i^\dagger \alpha_i + b^\dagger \alpha_i^\dagger \beta_i \right),
\] (48)

where \( \Omega \) is a known function of \( m_0, \lambda_0 \) and \( f_0 \). It is related to the energy gap between the false and the true vacuum for each replica field. See Eq. (44). On the other hand, \( \omega_0, g_1 \) and \( g_2 \) are phenomenological quantities that are related to the physical parameters \( m_0, \lambda_0 \) and \( f_0 \) of the disordered model. In this situation, the Euclidean action \( S \) associated to the fermionic Dicke model is given by

\[
S = \int_{0}^{\beta} d\tau \left( b^*(\tau) \frac{\partial}{\partial \tau} b(\tau) + \sum_{i=1}^{N} \left( \alpha_i^*(\tau) \frac{\partial}{\partial \tau} \alpha_i(\tau) + \beta_i^*(\tau) \frac{\partial}{\partial \tau} \beta_i(\tau) \right) \right) - \int_{0}^{\beta} d\tau H_F(\tau),
\] (49)

where the Hamiltonian density \( H_F(x) \) is obtained from Eq. (48). In order to define the partition function, the functional integrals have to be done in the space of complex functions \( b^* (\tau) \) and \( b (\tau) \) and Grassmann variables \( \alpha_i^* (\tau), \alpha_i (\tau), \beta_i^* (\tau) \) and \( \beta_i (\tau) \). Since we use thermal equilibrium boundary conditions in the Euclidean time, the integration variables obey periodic boundary conditions for the Bose field, i.e., \( b(0) = b(\beta) \) and anti-periodic boundary conditions for Grassmann variables, i.e., \( \alpha_i(\beta) = -\alpha_i(0) \) and \( \beta_i(\beta) = -\beta_i(0) \). [117, 118]

To proceed, let us define the formal quotient of two functional integrals, i.e., the partition function of the generalized fermionic Dicke model and the partition function of the free fermionic Dicke model. Therefore we are interested in calculating the following quantity

\[
\frac{Z_F}{Z_{F_0}} = \frac{\int [d\eta] e^S}{\int [d\eta] e^{S_0}},
\] (50)

where \( S \) is the Euclidean action of the generalized fermionic Dicke model given by Eq. (49), \( S_0 \) is the free Euclidean action for the free single bosonic mode and the free two-level systems. In the above equation \([d\eta]\) is the standard functional measure for the fermionic and bosonic degrees of freedom. The free action for the single mode bosonic field \( S_0(b) \) is given by

\[
S_0 (b, b^* ) = \int_{0}^{\beta} d\tau \left( b^*(\tau) \frac{\partial b(\tau)}{\partial \tau} - \omega_0 b^*(\tau)b(\tau) \right).
\] (51)

Then we can write the action \( S \) of the generalized fermionic Dicke model, given by Eq. (49), using the free action for the single mode bosonic field \( S_0(b, b^* ) \), given by Eq. (51), plus an additional
term that can be expressed in a matrix form. For more details see the Refs. [113,114]. Performing straightforward calculations it is possible to show that the critical temperature $T_c$ where $T^{-1} = \beta_c$, is

$$\beta_c = \frac{2}{\Omega} \arctanh \left[ \frac{\omega_0 \Omega}{(g_1 + g_2)^2} \right]. \quad (52)$$

Notice that it is possible to have a quantum phase transition when $\omega_0 \Omega = (g_1 + g_2)^2$. The experimental realization of the Dicke superradiance in cold atoms in optical cavities was presented in Ref. [119].

In the disordered system, this situation discussed above corresponds to the quantum nucleation of bubbles. We would like to stress that this scenario, where these bubble nucleations are a collective effect in the system, is a oversimplification of the exact full model. At this point we would like to comment the similarities between these two physical systems, the $N$ two-level systems trapped in a cavity and the random field Landau-Ginzburg model. In the first case, the ensemble of two-level atoms interacts effectively with one bosonic field mode present in the cavity. There are strong correlations between the two-level systems. In the disordered Landau-Ginzburg model the Gaussian disorder is able to make the same effect of the cavity. All the replicas are strongly correlated. See Eq. (36). All the replicas are under the effect of the background generated by the other replicas.

6 Conclusions

In this work we discuss the phase transitions in the continuous version of the $d$-dimensional random field Ising model. First we adopt the general strategy to average over the disordered free energy. Recently it was proposed a new method to average the disorder dependent free energy in systems defined in the continuum. Using this technique, the free energy is represented as a series of the replica partition functions of the system. The structure of the replica space was investigated using the saddle-point equations obtained from each replica field theory. We discuss the presence of a spontaneous symmetry breaking mechanism in some replica partition functions. For very low temperatures there are $N$ replica instantons-like solutions (real or complex) in this model. For the case of real instantons-like solutions, each replica field has two non-degenerate vacua. The transition from the false vacuum to the true one for each replica field corresponds to the nucleation of bubble of the true vacuum.

As we discussed, it is possible to obtain a spin-glass behavior from the multimode Dicke model of quantum optics, integrating out the bosonic field. This spin-boson model describes a phase transition from the fluorescent to superradiant phase. We show that the reverse situation is also feasible. To describe the phase transition in the disordered statistical field theory model we use the one mode Dicke model. The similarities between these two physical systems, the $N$ two-level systems trapped in a cavity and the random field Landau-Ginzburg model are evident.
The ensemble of two-level atoms interact effectively with one bosonic field mode present in the cavity. This fact generates strong correlations between the two-level atoms. In the disordered Landau-Ginzburg model, the Gaussian disorder is able to make the same effect, since all the replicas are strongly correlated. All the replicas are under the effect of the background generated by the other replicas.

Using the formalism developed by Popov and Fedotov the critical temperature is found. This temperature can be characterized by a non-analytical behavior of the thermodynamic quantities as a function of the temperature. At this temperature the free energy of the system is non-analytic, and the system present a transition to the normal to the superradiant phase.

A crucial question is the size of the bubbles in the disordered model. In scalar models in field theory with compactification in one spatial direction, the mass can depend upon the periodicity length in the compact direction [120–124]. This situation allow that topological effects play a role in the breaking and restoration of symmetries in different models. We believe that using the formalism discussed in this section and the above discussed mechanism, it is possible to to predict the size of the nucleating bubbles.

Another natural continuation of our investigations still using the distributional zeta-function method in disordered field theory models, consists in studying the nature of phase transitions in the disordered (random temperature) d-dimensional Ising ferromagnet, which can be described by a statistical field theory model with quenched disorder, i.e., the d-dimensional random temperature Landau-Ginzburg model.

As we discussed in Sec. 1, two concepts that are of fundamental importance in disordered systems are respectively quenched disorder and frustration. The presence of frustration in some disordered systems, as for example the spin glasses suggests that there are many different ground states in such systems. At low temperatures, in the spin-glass there are domains where the spins becomes frozen in space. This randomness in space that characterize the spin-glass phase corresponds to the fact that the free energy landscape of the system has a multivalley structure. Some authors discussed the possibility of a existence of this multivalley structure of the spin-glass phase in the random temperature Landau-Ginzburg model [125,126].

Our aim is to investigate the possibility of found a multivalley structure in the average free energy of the random temperature Landau-Ginzburg model using the distributional zeta-function approach. This subject is under investigation by the authors.

### 7 Acknowledgments

We would like to thank G. Krein, S. Queirós G. Menezes and M. Aparicio Alcalde for useful discussions. This paper was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil).

### References

[1] P. W. Anderson, Journal of Les-Common Metals, 62, 291 (1978).
S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

T. R. Kirkpatrick and D. Sherrington, Phys. Rev. B 17, 4384 (1978).

G. Parisi, Phys. Rev. Lett. 43, 1754 (1979).

G. Parisi, Jour. Phys. A 13, L115 (1980).

M. Mezard, G. Parisi and M. Virasoro, “Spin-Glass Theory and Beyond”, World Scientific, (1987).

V. Dotsenko, “Introduction to the Replica Theory in Disordered Statistical Systems”, Cambridge University Press (2001).

C. De Dominicis and I. Giardina, “Random Fields and Spin Glass”, Cambridge University Press (2006).

B. F. Svaiter and N. F. Svaiter, Int. Jour. Mod. Phys. A 31, 1650144 (2016).

B. F. Svaiter and N. F. Svaiter, arXiv:math-phys 1606.04854 (2016).

R. Acosta Diaz, C. D. Rodríguez-Camargo and N. F. Svaiter, arXiv: cond-mat 1609.07084 (2016).

R. Acosta Diaz, G. Menezes, N. F. Svaiter and C. A. D. Zarro, Phys. Rev. D 96, 065012 (2017).

A. Larkin, Sov. Phys. JETP 31, 784 (1970).

T. Nattterman, in “Spin-Glasses and Random Fields”, A. P. Young (Editor), World Scientific (1988).

T. Nattermann and P. Rujan, Int. Jour. Mod. Phys. B 3, 1597 (1989).

G. Parisi and V. Dotsenko, J. Phys. A 25, 3143 (1992).

M. Dzero, J. Schmalian and P. G. Wolynes, Phys. Rev. B 72, 100201 (2005).

A. Adams, T. Anous, J. Lee and S. Yaida, Phys. Rev. E 91, 032148 (2015).

A. Aharony, Phys. Rev. B 18, 3318 (1977).

M. Gofman, J. Adler, A. Aharony, A. B. Harris and M. Schwartz, Phys. Rev. B 53, 6362 (1996).

A. J. Leggett, C. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. 59, 1 (1987).
[22] V. N. Popov and S. A. Fedotov, Theo. Math. Phys. 51, 363 (1982).
[23] V. N. Popov and S. A. Fedotov, Sov. Phys.-JETP 67, 535 (1988).
[24] V. N. Popov, Functional Integrals and Collective Excitations (Cambridge University Press, Cambridge, 1987).
[25] S. Fishman and A. Aharony, J. Phys. C, 12, L729 (1979).
[26] J. Cardy, Phys. Rev. B, 24 505 (1985).
[27] P. G. de Gennes, J. Phys. Chem 88, 6469 (1984).
[28] T. MacFarland, G. T. Barkema and J. F. Marko, Phys. Rev. B 53, 148 (1996).
[29] R. L. C. Vink, K. Binder and H. Löwen, Phys. Rev. Lett. 97, 230603 (2006).
[30] M. Mézard and A. P. Young, Europhys. Lett. 18, 653 (1992).
[31] M. Mezard and R. Monasson, Phys. Rev. B50, 7199 (1994).
[32] V. Dotsenko, A. B. Harris, D. Sherington and R. B. Stinchcombe, J. Phys. A 28, 3093 (1995).
[33] C. De Dominicis, H. Orland and T. Tenesvari, J. Phys. I 5, 987 (1995).
[34] V. Dotsenko and M. Mézard, J. Phys. A 30, 3363 (1997).
[35] V. Dotsenko, J. Phys. A 32, 2949 (1999).
[36] F. Krzakala, F. Ricci-Tersengui and L. Zdeboravá, Phys. Rev. Lett. 104, 207208 (2010).
[37] F. Krzakala, F. Ricci-Tersengui, D. Sherington and L. Zdeboravá, J. Phys. A 14, 042003 (2011).
[38] Y. Imry and S.-K. Ma, Phys. Rev. Lett. 35, 1399 (1975).
[39] G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979).
[40] G. Parisi, “An introduction to the statistical mechanics of amorphous systems”, in “Field Theory, Disorder and Simulations”, Word Scientific, Singapore (1992).
[41] J. Bricmont and A. Kupiainen, Phys. Rev. Lett. 59, 1829 (1987).
[42] J. Bricmont and A. Kupiainen, Comm. Math. Phys. 116, 539 (1988).
[43] M. Aizenman and T. Wehr, Phys. Rev. Lett. 62, 2503, (1989).
[44] P. G. de Gennes, Phys. Lett. A 38, 339 (1972).
[45] G. Gaspari and J. Rudnick, Phys. Rev. B 33, 3295 (1986).
[46] L. Schäfer, Phys. Rev. B 35, 100201 (1987).
[47] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (Oxford Science Publications, Oxford, 1993).
[48] J. Iliopoulos, C. Itzykson and A. Martin, Rev. Mod. Phys. 47, 165 (1975).
[49] R. J. Rivers, Path Integral Methods in Quantum Field Theory (Cambridge University Press, Cambridge, 1988).
[50] J. R. Klauder, Beyond Conventional Quantization (Cambridge University Press, Cambridge, 2000).
[51] S. -K. Ma, Modern Theory of Critical Phenomena (Perseus Publishing, Cambridge, 1976).
[52] H. Rieger, Phys. Rev. B 52, 6659 (1995).
[53] G. Parisi, in I. A. Batalin (Ed.) et al.: Quantum Field Theory and Quantum Statistics, Vol. 1, 381-392 (1988).
[54] O. Aharony, Z. Komargodski and S. Yankielowicz, JHEP 04 (2016).
[55] N. G. Fytas, V. Martin-Mayor, P. Picco and S. Sourlas, Phys. Rev. E 95, 042117 (2017).
[56] J. M. Corwall, R. Jackiw and E. Tomboulis, Phys. Rev. D 15, 2428 (1974).
[57] G. N. J. Añaños, A. P. C. Malbouisson and N. F. Svaiter, Nucl. Phys. B 547, 221 (1999).
[58] N. F. Svaiter, Physica A 285, 493 (2000).
[59] P. Contucci, C. Giardinà, C. Gilberti, G. Parisi and C. Vernia, Phys. Rev. Lett. 103, 017201 (2009).
[60] R. T. Seeley, Proc. Symp. Pure Math. Chicago III, 288, Am. Math. Soc. (1966).
[61] D. B. Ray and I. M. Singer, Advances in Math. 7, 145 (1971).
[62] S. W. Hawking, Comm. Math. Phys. 55, 133 (1974).
[63] J. S. Dowker and R. Crichley, Phys. Rev. D 13, 224 (1976).
[64] S. A. Fulling, J. Phys. A 36, 6857 (2003).
[65] D. Lancaster, E. Marinari and G. Parisi, J. Phys. A 28, 3359 (1995).

[66] B. Riemann, Monatsberichte der Berliner Akademie, 671 (1859).

[67] A. E. Ingham, “The Distribution of Prime Numbers”, Cambridge University Press, Cambridge (1990).

[68] V. E. Landau and A. Walfisz, Rend. Circ. Mat. Palermo, 44, 82 (1920).

[69] C. E. Fröberg, BIT 8, 187 (1968).

[70] G. Menezes and N. F. Svaiter, arXiv: 1211.5198, “Quantum field theory and prime numbers spectrum”, (2011).

[71] G. Menezes, B. F. Svaiter and N. F. Svaiter, Int. Jour. Mod. Phys. A 28, 1350128 (2013).

[72] A. Voros, “Zeta Functions over Zeros of Zeta Functions”, Springer Verlag, Berlin, Heidelberg (2010).

[73] M. E. Fisher and M. F. Sykes, Phys. Rev. 114, 45 (1959).

[74] D. Dhar, J. Math. Phys. 19, 5 (1978).

[75] B. Ahrens and A. K. Hartmann, Phys. Rev. B 83, 014205 (2011).

[76] S. Köves-Domokos, IL Nouvo Cimento 33, 769 (1976).

[77] R. Menikoff and D. R. Sharp, Jour. Math. Phys. 19, 135 (1977).

[78] C. M. Bender, F. Cooper G. S. Guralnik and D. H. Sharp, Phys. Rev. D 19, 1865 (1979).

[79] N. F. Svaiter, Physica A 345, 517 (2005).

[80] M. Lüscher and P. Weisz, Nucl. Phys. B 300 [FS22] (1988) 325.

[81] M. Lüscher and P. Weisz, Nucl. Phys. B 290 [FS20I] (1987) 25.

[82] M. Lüscher and P. Weisz, Nucl. Phys. B 295 [FS21] (1988) 65.

[83] T. Reisz, Nucl. Phys. B 450 [FS], 569 (1995).

[84] H. Meyer-Ortmanns and T. Reisz, Eur. Phys. Jour. B 27, 549 (2002).

[85] F. Englert, Phys. Rev. 129, 567 (1963).

[86] C. G. Bollini, J. J. Giambiagi and A. Gonzáles Dominguez, Il Nuovo Cim. 31, 550 (1964).
[87] C. G. Bollini and J. J. Giambiagi, Nuovo Cim. B 12, 20 (1972).
[88] J. F. Ashmore, Nuovo Cim. Lett. 4, 289 (1972).
[89] G. ’t Hooft and M. Veltman, Nucl. Phys. B 44, 189 (1972).
[90] G. Leibrandt, Rev. Mod. Phys. 47, 849 (1975).
[91] D. Amit, Field Theory, the Renormalization Group and Critical Phenomena (McGraw-Hill, New York, 1978).
[92] A. Einstein, Physikalische Zeitschrift 18, 121 (1917).
[93] G. S. Agarwal, “Quantum Statistical Theories of Spontaneous Emission and their Relation to Other Approaches” (Springer-Verlag, Berlin, 1974).
[94] L. Fonda, G. C. Ghirardi and A. Rimini, Rep. Prog. Phys. 41, 587 (1978).
[95] A. V. Andreev, V. I. Emel’yanov and Y. A. Il’inskii, Sov. Phys. Usp. 23, 493 (1980).
[96] M. Gross and S. Haroche, Phys. Rep. 93, 301 (1982).
[97] R. H. Dicke, Phys. Rev. 93, 99 (1954).
[98] T. Brandes, Phys. Rep. 408, 315 (2005).
[99] B. M. Garraway, Phil. Trans. R. Soc. A 369, 1137 (2011).
[100] S. Gopalakrishnan, B. L. Lev and P. M. Gouldbart, Phys. Rev. Lett. 107, 277201 (2011).
[101] P. Stack and S. Sachdev, Phys. Rev. Lett. 107, 277202 (2011).
[102] P. Rotondo, E. Tesio and S. Cracciolo, Phys. Rev. B 91, 014415 (2015).
[103] P. Rotondo, M. C. Lagomarsino and G. Viola, Phys. Rev. Lett. 114, 143601 (2015).
[104] S. Coleman, Phys. Rev. D 15, 2929 (1977).
[105] S. Coleman and F. De Luccia, Phys. Rev. D 21, 3305 (1980).
[106] G. H. Flores, R. Ramos and N. F. Svaiter, Int.Jour. Mod. Phys. A 14, 3715 (1999).
[107] K. Hepp and E. H. Lieb, Ann. Phys. 76, 360 (1973).
[108] K. Hepp and E. H. Lieb, Phys. Rev. A 8, 2517 (1973).
[109] Y. K. Wang and F. T. Hioe, Phys. Rev. A 7, 931 (1973).

23
[110] F. T. Hioe, Phys. Rev. A 8, 1440 (1973).
[111] O. W. Greenberg and A. M. L. Messiah, Phys. Rev. 138, B1155 (1965).
[112] O. W. Greenberg, Phys. Rev. Lett. 64, 705 (1990).
[113] M. A. Alcalde, A. L. L. de Lemos and N. F. Svaiter, J. Phys. A 40, 11961 (2007).
[114] M. A. Alcalde, R. Kullock and N. F. Svaiter, J. Math. Phys. 50, 013511 (2009).
[115] M. A. Alcalde, A. H. Cardenas, N. F. Svaiter and V. B. Bezerra, Phys. Rev. A 81, 032335 (2010).
[116] M. A. Alcalde, J. Stephany and N. F. Svaiter, J. Phys. A 44, 505301 (2011).
[117] R. Kubo, J. Phys. Soc. Jap. 12, 570 (1957).
[118] P. Martin and J. Schwinger, Phys. Rev. 115, 1342 (1959).
[119] K. Bauman, R. Mottl, F. Brennecke and T. Esslinger, Phys. Rev. Lett. 107, 140402 (2011).
[120] L. H. Ford and T. Yoshimura, Phys. Lett. A 70, 89 (1979).
[121] D. J. Toms, Phys. Rev. D 21, 928 (1980).
[122] D. J. Toms, Phys. Rev. D 21, 2805 (1980).
[123] G. Denardo and E. Spalucci, Nucl. Phys. B 169, 514 (1980).
[124] L. H. Ford and N. F. Svaiter, Phys. Rev. D 51, 6981 (1995).
[125] S. K. Ma and I. Rudnick, Phys. Rev. Lett. 40, 589 (1978).
[126] G. Targus and V. Dotsenko, J. Phys. A35, 1627 (2001).