Sensorless rotor position estimation by PWM-induced signal injection

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Abstract—We demonstrate how the rotor position of a PWM-controlled PMSM can be recovered from the measured currents, by suitably using the excitation provided by the PWM itself. This provides the benefits of signal injection, in particular the ability to operate even at low velocity, without the drawbacks of an external probing signal. We illustrate the relevance of the approach by simulations and experimental results.

Index Terms—Sensorless control, PMSM, signal injection, PWM-induced ripple.

Nomenclature

| Symbol | Description |
|--------|-------------|
| PWM | Pulse Width Modulation |
| $x^{dq}$ | Vector $(x^{d}, x^{q})^T$ in the $dq$ frame |
| $x^{a\beta}$ | Vector $(x^{a}, x^{\beta})^T$ in the $a\beta$ frame |
| $x^{abc}$ | Vector $(x^{a}, x^{b}, x^{c})^T$ in the $abc$ frame |
| $R_s$ | Stator resistance |
| $\mathcal{J}$ | Rotation matrix with angle $\pi/2$: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ |
| $J$ | Moment of inertia |
| $n$ | Number of pole pairs |
| $\omega$ | Rotor speed |
| $T_l$ | Load torque |
| $\theta, \hat{\theta}$ | Actual, estimated rotor position |
| $\phi_m$ | Permanent magnet flux |
| $L_{d}, L_{q}$ | d and q-axis inductances |
| $C$ | Clarke transformation: $\frac{2}{3} \begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{pmatrix}$ |
| $\mathcal{R}(\theta)$ | Rotation matrix with angle $\theta$: $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ |
| $\varepsilon$ | PWM period |
| $u_m$ | PWM amplitude |
| $\mathcal{S}(\theta)$ | Saliency matrix |
| $O$ | “Big O” symbol of analysis: $k(z, \varepsilon) = O(\varepsilon)$ means $\|k(z, \varepsilon)\| \leq C\varepsilon$, for some $C$ independent of $z$ and $\varepsilon$. |

I. INTRODUCTION

Sensorless control of AC motors in the low-speed range is a challenging task. Indeed, the observability of the system from the measurements of the currents degenerates at standstill, which limits the performance at low speed of any fundamental-model-based control law.

One now widespread method to overcome this issue is the so-called signal injection technique. It consists in superimposing a fast-varying signal to the control law. This injection creates ripple on the current measurements which carries information on the rotor position if properly decoded. Nonetheless, introducing a fast-varying signal increases acoustic noise and may excite mechanical resonances. For systems controlled through Pulse Width Modulation (PWM), the injection frequency is moreover inherently limited by the modulation frequency. That said, inverter-friendly waveforms can also be injected to produce the same effect, as in the so-called INFORM method \cite{1, 2}. For PWM-fed Permanent Magnet Synchronous Motors (PMSM), the oscillatory nature of the input may be seen as a kind of generalised rectangular injection on the three input voltages, which provides the benefits of signal injection, in particular the ability to operate even at low velocity, without the drawbacks of an external probing signal.

We build on the quantitative analysis developed in \cite{3} to demonstrate how the rotor position of a PWM-controlled PMSM can be recovered from the measured currents, by suitably using the excitation provided by the PWM itself. No modification of the PWM stage nor injection a high-frequency signal as in \cite{4} is required.

The paper runs as follows: we describe in section II the effect of PWM on the current measurements along the lines of \cite{3}, slightly generalizing to the multiple-input multiple output framework. In section III we show how the rotor position can be recovered for two PWM schemes schemes, namely standard single-carrier PWM and interleaved PWM. The relevance of the approach is illustrated in section IV with numerical and experimental results.

II. VIRTUAL MEASUREMENT INDUCED BY PWM

Consider the state-space model of a PMSM in the $dq$ frame

\begin{align}
\frac{d\phi_s^{dq}}{dt} &= u_s^{dq} - R_s i_s^{dq} - \omega \mathcal{J} \phi_s^{dq}, \\
\frac{J}{n} \frac{d\omega}{dt} &= n_s^{dq} \mathcal{J} \phi_s^{dq} - T_l, \\
\frac{d\theta}{dt} &= \omega,
\end{align}

where $\phi_s^{dq}$ is the stator flux linkage, $\omega$ the rotor speed, $\theta$ the rotor position, $i_s^{dq}$ the stator current, $u_s^{dq}$ the stator voltage, and $T_l$ the load torque; $R_s$, $J$, and $n$ are constant parameters (see nomenclature for notations). For simplicity we assume no magnetic saturation, i.e. linear current-flux relations

\begin{align}
L_d i_s^d &= \phi_s^d - \phi_m, \\
L_q i_s^q &= \phi_s^q,
\end{align}

with $\phi_m$ the permanent magnet flux; see \cite{5} for a detailed discussion of magnetic saturation in the context of signal injection. The input is the voltage $u_s^{abc}$ through the relation

\begin{equation}
\begin{split}
u_s^{dq} &= \mathcal{R}(-\theta) C u_s^{abc}.
\end{split}
\end{equation}

In an industrial drive, the voltage actually impressed is not directly $u_s^{abc}$, but its PWM encoding $M(u_s^{abc}, \frac{\varepsilon}{\tau})$, with $\varepsilon$ the PWM period. The function $\mathcal{M}$ describing the PWM is 1-periodic and mean $u_s^{abc}$ in the second argument, i.e. $\mathcal{M}(u_s^{abc}, \tau + 1) = \mathcal{M}(u_s^{abc}, \tau)$ and $\int_0^1 \mathcal{M}(u_s^{abc}, \tau) d\tau = u_s^{abc}$.

\begin{align}
\end{align}
its expression is given in section III Setting \( s^0_{abc}(u_{abc}, \sigma) := M(u_{abc}, \sigma) - u_{abc} \), the impressed voltage thus reads
\[
u_{abc}^{\text{pwm}} = u_{abc}^{\text{pwm}} + s^0_{abc}\left(u_{abc}^{\text{pwm}}, \frac{t}{\epsilon}\right),
\]
where \( s^0_{abc} \) is 1-periodic and zero mean in the second argument; \( s^0_{abc} \) can be seen as a PWM-induced rectangular probing signal, which creates ripple but has otherwise no effect. Finally, as we are concerned with sensorless control, the only measurement is the current \( i_1 \). For the PMSM (1)–(3), where \( u, \epsilon \) are sufficiently different, the rotor position \( \theta \) can be extracted from \( y_v \), as explained in section III. When geometric saliency is small, information on \( \theta \) is usually still present when magnetic saturation is taken into account, see [5].

III. Extracting \( \theta \) from the virtual measurement

Extracting the rotor position \( \theta \) from \( y_v \) depends on the rank of the \( 2 \times 3 \) matrix \( CA_{abc}(u_{abc}) \). The structure of this matrix, hence its rank, depends on the specifics of the PWM employed. After recalling the basics of single-phase PWM, we study two cases: standard three-phase PWM with a single carrier, and three-phase PWM with interleaved carriers.

Before that, we notice that \( CA_{abc}(u_{abc}) \) has the same rank as the \( 2 \times 2 \) matrix
\[
A_{\theta}(u_{abc}) := CA_{abc}(u_{abc})C^T = \int_0^1 s_1^{\theta}(u_{abc}, \tau)s_1^{\theta T}(u_{abc}, \tau) \, d\tau,
\]
where \( s_1^{\theta}(u_{abc}, \tau) := Cs_1^{abc}(u_{abc}, \tau) \). Indeed,
\[
A_{\theta}(u_{abc})A_{\theta}^{\tau T}(u_{abc}) = CA_{abc}(u_{abc})C^TCA_{abc}^{T}(u_{abc})C^T = CA_{abc}(u_{abc})(C_{\theta}A_{abc}(u_{abc}))^T,
\]
which means that \( A_{\theta}(u_{abc}) \) and \( CA_{abc}(u_{abc}) \) have the same singular values, hence the same rank. There is thus no loss of information when considering \( S(\theta)A_{\theta}(u_{abc}) \) instead of the original virtual measurement \( y_v \).

A. Single-phase PWM

In “natural” PWM with period \( \epsilon \) and range \([-u_m, u_m]\), the input signal \( u \) is compared to the \( \epsilon \)-periodic triangular carrier
\[
e(t) := \begin{cases} u_m + 4 w\left(\frac{t}{\epsilon}\right) & \text{if } -\frac{u_m}{2} \leq w\left(\frac{t}{\epsilon}\right) \leq 0 \\ u_m - 4 w\left(\frac{t}{\epsilon}\right) & \text{if } 0 \leq w\left(\frac{t}{\epsilon}\right) \leq \frac{u_m}{2} \end{cases},
\]
the 1-periodic function \( w(\sigma) := u_m \text{ mod}(\sigma + \frac{1}{2}, 1) - \frac{u_m}{2} \) wraps the normalized time \( \sigma = \frac{t}{\epsilon} \) to \([-\frac{u_m}{2}, \frac{u_m}{2}]\). If \( u \) varies slowly enough, it crosses the carrier \( c \) exactly once on each rising and falling ramp, at times \( t_1^u \leq t_2^u \) such that
\[
u(t_1^u) = u_m + 4 w\left(\frac{t_1^u}{\epsilon}\right)
\]
\[
u(t_2^u) = u_m - 4 w\left(\frac{t_2^u}{\epsilon}\right).
\]
The PWM-encoded signal is therefore given by

\[

\text{Figure 1. PWM: } u \text{ is compared to } c \text{ to produce } u_{\text{pwm}}
\]
The signals $s_0(u, \sigma)$ and $s_1(u, \sigma)$ are displayed in Fig. 2. Notice that by construction $s_0(\pm u_m, \sigma) = s_1(\pm u_m, \sigma) = 0$, so there is no ripple, hence no usable information, at the PWM limits.

### B. Three-phase PWM with single carrier

In three-phase PWM with single carrier, each component $u_k^a$, $k = a, b, c$, of $u_s^{abc}$ is compared to the same carrier, yielding

$$s_k^0(u_s^{abc}, \sigma) := s_0(u_k^a, \sigma)$$

and

$$s_k^1(u_s^{abc}, \sigma) := s_1(u_k^a, \sigma),$$

with $s_0$ and $s_1$ as in single-phase PWM. This is the most common PWM in industrial drives as it is easy to implement.

Notice that if exactly two components of $u_s^{abc}$ are equal, for instance $u_k^a = u_b^b 
eq u_c^c$, then

$$s_k^1(u_s^{abc}, \sigma) = s_b^1(u_s^{abc}, \sigma) 
eq s_i^1(u_s^{abc}, \sigma),$$

which implies in turn that $A^{\alpha \beta}(u^{abc})$ has rank 1 (its determinant vanishes), and it can be shown this is the only situation that results in rank 1. If all three components of $u_s^{abc}$ are equal, then $A^{\alpha \beta}(u^{abc})$ has rank 0 (i.e. all its entries are zero); this is a rather exceptional condition that we rule out here. Otherwise $A^{\alpha \beta}(u^{abc})$ has rank 2 (i.e. is invertible). Fig. 3 displays examples of the shape of $s_1^{\alpha \beta}$, in the rank 2 case (top), and in the rank 1 case where $u_k^a = u_b^b 
eq u_c^c$.

As the rank 1 situation very often occurs, it must be handled by the procedure for extracting $\theta$ from $S(\theta) A^{\alpha \beta}(u^{abc})$. This can be done by linear least squares, thanks to the particular structure of $S(\theta)$. Setting

$$A = \begin{pmatrix} \alpha & \mu & \nu \\ \mu & \nu & \lambda \\ -\nu & -\lambda & \mu \end{pmatrix}$$

and

$$L := \begin{pmatrix} L_{dq} + L_{d} + L_{q} \\ L_{d} \\ L_{q} \end{pmatrix},$$

we can rewrite

$$y_v = S(\theta) A^{\alpha \beta}(u^{abc})$$

as

$$\begin{pmatrix} \alpha & \mu & \nu \\ \mu & \nu & \lambda \\ -\nu & -\lambda & \mu \end{pmatrix} \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} = L \begin{pmatrix} y_{11} - \alpha \\ y_{12} - \mu \\ y_{21} - \mu \\ y_{22} - \nu \end{pmatrix}.$$

The least-square solution of this (consistent) overdetermined linear system is

$$L [P^T P]^{-1} P^T d = L \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}.$$
Now, even when two, or even three, components of $u^{abc}_s$ are equal, $A^{αβ}(u^{abc})$ remains invertible (except of course at the PWM limits), since each component has, because of the interleaving, a different PWM pattern. It is therefore possible to recover all four entries of the saliency matrix $S(θ)$ by

$$S(θ) := \hat{y}_v \cdot \left[ A^{αβ}(u^{abc}) \right]^{-1} = S(θ) + O(ε).$$

Notice now that thanks to the structure of $S(θ) = (s_ij)_{ij}$, the rotor angle $θ$ can be computed from the matrix entries by

$$s_{12} + s_{21} = \frac{L_q - L_d}{L_d L_q} \sin 2θ,$$

$$s_{11} - s_{22} = \frac{L_q - L_d}{L_d L_q} \cos 2θ,$$

$$θ = \frac{1}{2} \tan 2(s_{12} + s_{21}, s_{11} - s_{22} + kπ),$$

where $k \in \mathbb{N}$ is the number of turns. An estimate $\hat{θ}$ of $θ$ can therefore be computed from the entries $(s_{ij})_{ij}$ of $S(θ)$ by

$$\hat{θ} = \frac{1}{2} \tan 2(s_{12} + s_{21}, s_{11} - s_{22}) + kπ = θ + O(ε),$$

without requiring the knowledge the magnetic parameters $L_d$ and $L_q$, which is indeed a nice practical feature.

**IV. Simulations and experimental results**

The demodulation procedure is tested both in simulation and experimentally. All the tests, numerical and experimental, use the rather salient PMSM with parameters listed in Table I. The PWM frequency is 4 kHz.

The test scenario is the following: starting from rest at $t=0 \text{ s}$, the motor remains there for 5 Hz (electrical), and finally stays at 5 Hz from $t = 8.5 \text{ s}$; during all the experiment, it undergoes a constant load torque of about 40% of the rated torque. As this paper is only concerned with the estimation of the rotor angle $θ$, the control law driving the motor is allowed to use the measured angle. Besides, we are not yet able to process the data in real-time, hence the data are recorded and processed offline.

**A. Single carrier PWM.**

The results obtained in simulation by the reconstruction procedure of section III-B for $\cos 2θ$, $\sin 2θ$, and $θ$, are shown in Fig. 6 and Fig. 7. The agreement between the estimates and the actual values is excellent.

The corresponding results on experimental data are shown in Fig. 10 and Fig. 11. Though of course not as good as...
Figure 6. Reconstruction of cos 2θ and sin 2θ (simulation).

Figure 7. Reconstruction of θ in rad (simulation).

Figure 8. \( s_{1}^{αβ} \) for single-carrier PWM (experimental data): nondegenerate (top), degenerate (bottom).

Figure 9. Measured current \( i_{a}^{s} \) and its filtered version (experimental data).

in simulation, the agreement between the estimates and the ground truth is still very satisfying. The influence of magnetic saturation may account for part of the discrepancies. Fig. 8 displays a close view of the ripple envelope \( s_{1}^{αβ} \) in approximately the same conditions as in Fig. 3 when the rank of \( A^{αβ}(u^{abc}) \) is 2 case (top), and when the rank is 1 case with \( u_{s}^{c} = u_{s}^{b} \neq u_{s}^{a} \). They illustrate that though the experimental signals are distorted, they are nevertheless usable for demodulation.

Finally, we point out an important difference between the simulation and experimental data. In the experimental measurements, we notice periodic spikes in the current measurement, see figure 9; these are due to the discharges of the parasitic capacitors in the inverter transistors each time a PWM commutation occurs. As it might hinder the demodulation procedure of [3], [6], the measured currents were first preprocessed by a zero-phase (non-causal) moving average with a short window length of 0.01ε. We are currently working on an improved demodulation procedure not requiring prefiltering.

B. Interleaved PWM (simulation)

The results obtained in simulation by the reconstruction procedure of section III-C for the saliency matrix and \( S(θ) \) and for \( θ \) are shown in Fig. 12 and Fig. 13. The agreement between the estimates and the actual values is excellent. We insist that the reconstruction does not require the knowledge of the magnetic parameters.

V. Conclusion

This paper provides an analytic approach for the extraction of the rotor position of a PWM-fed PSMM, with signal injection provided by the PWM itself. Experimental and simulations results illustrate the effectiveness of this technique.
Further work includes a demodulation strategy not requiring prefiltering of the measured currents, and suitable for real-time processing. The ultimate goal is of course to be able to use the estimated rotor position inside a feedback loop.

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