New angles on D-branes

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Abstract

A low-energy background field solution is presented which describes several D-membranes oriented at angles with respect to one another. The mass and charge densities for this configuration are computed and found to saturate the BPS bound, implying the preservation of one-quarter of the supersymmetries. T-duality is exploited to construct new solutions with nontrivial angles from the basic one.

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1 Introduction

Understanding of non-perturbative aspects of string theory has advanced rapidly during the past two years \cite{1}. For example, all five consistent superstring theories can now be related through the use of various string dualities. These connections suggest that all of these string theories are really perturbative expansions about different points in the phase space of a more fundamental framework, commonly called M-theory. The development of these string dualities has brought with it the realization that extended objects beyond simply strings play a crucial role in these theories. In the case of the Type II (and I) superstrings, of particular interest are Dirichlet branes (D-branes) which carry charges of the Ramond-Ramond (RR) potentials\cite{2}.

D-branes have also proven their worth from a calculational standpoint. For example, bound states of D-branes have recently facilitated the computation of the entropy of black holes from a counting of the underlying microscopic degrees of freedom\cite{3}. In these analyses, the bound state configurations must be supersymmetric (or nearly so — see, however, \cite{4}) in order to protect the counting of states from loop corrections by BPS saturation. In this case, the microscopic counting which is made at weak coupling can be compared with the expected degeneracy of states at strong coupling at which the bound state has formed a black hole. This is one of the reasons for which supersymmetric D-brane configurations are of particular interest.

A great deal of effort has gone into generating the low-energy background field solutions corresponding to various D-brane bound states\cite{5}. These solutions are restricted to those describing p-branes which are either parallel or intersect orthogonally. It has been shown\cite{6}, however, that there exist supersymmetric configurations where the angles between the D-branes are other than zero or $\pi/2$. Preserving supersymmetry in such multiple D-brane configurations requires that the angles are restricted to lie in an $SU(N)$ subgroup of rotations. The corresponding background field configurations remain largely unexplored, but in this paper, we will present one such class of solutions. Our basic solution describes any number of D-membranes whose relative orientations are given by certain $SU(2)$ rotations.

The paper is organized as follows: section 2 presents our solution and calculation of the mass and charge densities for this system of angled D-branes. With the latter, we demonstrate that the BPS bound saturated by this configuration. In section 3, we exploit T-duality to create solutions involving angled D3- and D4-branes, as well as some more exotic configurations, arrived at by considering T-duality along world-volume coordinates of D-membranes in our original solution. Finally, a brief discussion follows in section 4. Our notation and conventions follow those established in \cite{7}.
2 Membranes at angles

We begin by writing down the solution describing an arbitrary number \( n \) of D-membranes, each of which is rotated by certain \( SU(2) \) angle, in the type IIA low energy effective string theory. The solution contains only a nontrivial (string-frame) metric, three-form RR potential and dilaton:

\[
\frac{ds^2}{1 + X} = \sqrt{1 + X} \left[ \frac{1}{1 + X} \left( -dt^2 + \sum_{j=1}^{4} (dy^j)^2 \right) + \sum_{a=1}^{n} X_a \left\{ [(Ra)^1_i dy^i]^2 + [(Ra)^3_j dy^j]^2 \right\} + \sum_{i=5}^{9} (dx^i)^2 \right] \\
A^{(3)} = \frac{dt}{1 + X} \wedge \left\{ \sum_{a=1}^{n} X_a (Ra)^2_i dy^i \wedge (Ra)^4_j dy^j - \sum_{a<b} X_a X_b \sin^2(\alpha_a - \alpha_b) (dy^1 \wedge dy^3 - dy^2 \wedge dy^4) \right\} \\
e^{2\phi_a} = \sqrt{1 + X} 
\]

where

\[
X = \sum_{a=1}^{n} X_a + \sum_{a<b} X_a X_b \sin^2(\alpha_a - \alpha_b) .
\]

Above, the rotation matrix \( R_a \) associated with the \( a \)'th D-membrane is given by

\[
R_a = \begin{pmatrix}
\cos \alpha_a & -\sin \alpha_a & 0 \\
\sin \alpha_a & \cos \alpha_a & 0 \\
0 & 0 & \cos \alpha_a \sin \alpha_a
\end{pmatrix}
\]

The matrices acting in the space of \( y^i \)'s are easily recognized as \( SU(2) \) rotations as follows: one defines the complex coordinates \( z^1 = y^1 + iy^2 \) and \( z^2 = y^3 + iy^4 \). Then the above rotations are given by \( (z^1, z^2) \rightarrow (e^{i\alpha_a} z^1, e^{-i\alpha_a} z^2) \), or \( z^i \rightarrow [\exp(i\alpha_a \sigma_3)]_i^j z^j \). One expects from \( \Box \) that restricting the relative orientation of the membranes in this way will preserve some of the supersymmetry, and we confirm this fact in the following.

The functions \( X_a \) are harmonic functions in the transverse space of \( x^i \)'s. That is, they solve the flat-space Poisson’s equation in the transverse space, \( e.g. \)

\[
\delta_{ij} \partial_i \partial_j X_a = -\ell_a^3 A_4 \prod_{k=5}^{9} \delta(x^k - x^k_a) .
\]

yielding the solutions

\[
X_a(\vec{x}) = \frac{1}{3} \left( \frac{\ell_a}{|\vec{x} - \vec{x}_a|} \right)^3 .
\]
Above, $\ell_a$ are arbitrary positive parameters which have the dimension of length, and we use $A_4$ to denote the volume of a unit four-sphere. In general, one has $A_{n-1} = 2\pi^{n/2}/\Gamma(n/2)$.

In fact, one may introduce any number of delta-function sources at arbitrary positions on the right hand side of eq. (4), and the corresponding solution would describe a system of parallel branes.

A few words are in order as to the origin of this solution. It is in effect an interpolation between the known solutions for parallel D-membranes, and that for orthogonal D-membranes intersecting over a point. It is straightforward to verify that when the angles are all set to $\alpha_a = 0$, the solution reduces to that of $n$ parallel branes lying in the $(y^2, y^4)$ plane. Note that in this case the membranes have also been delocalized or smeared out in the $y^1$ and $y^3$ directions. One may also verify that choosing all $\alpha_a = \alpha_o$ simply corresponds to an overall $SU(2)$ rotation of the previous solution. Similarly the known configuration of orthogonally oriented membranes is reproduced by choosing $\alpha_a$’s to be either zero or $\pi/2$. Further with the $\alpha_a$ set to either $\alpha_o$ and $\pi/2 + \alpha_o$, eq. (4) corresponds to a rotation of this solution. Finally, one may verify that making a further $SU(2)$ rotation of the entire solution simply corresponds shifting all of the angles $\alpha_a$ by the same constant. For this to work, it is important that the second term in $A^{(3)}$ is proportional to $dt \wedge \text{Re}(dz^1 \wedge dz^2)$, which is invariant under $SU(2)$ rotations. Verifying that eq. (1) solves the low-energy field equations of type IIA string theory was only done with the aid of a computer.

One final comment on our notation: we refer to $x^i$ and $y^i$ as transverse and world-volume coordinates, respectively. For a given brane, however, a particular (linear combination of) $y^i$ may actually still correspond to a transverse direction, although it will be one in which the brane is delocalized. Hence in the next section, when we smear out the solution in some $x^i$ making the solution independent of this coordinate, the designation for the coordinate is changed to $y^i$. We will also assume that the $y^i$ coordinates are all compact with a range of $2\pi L_i$.

### 2.1 Mass and Charge Relations

In this section, we consider some of the physical characteristics of the above configuration (1). In particular, we calculate the mass and charge densities of our solution. The latter densities are calculated using asymptotic flux integrals, and so they are completely determined by the leading-order behavior of the asymptotic fields. In examining the solution, one sees that these leading order fields are simply linear superpositions of the asymptotic fields generated by the individual rotated membranes. Hence we generalize the rotation appearing in these linearized fields by replacing $\alpha_a$ by an independent angle $\beta_a$ in the lower two-by-two block of the rotation matrices (3). Such a configuration would only solve the linearized asymptotic equations of motion, and not the full nonlinear supergravity equations, but this generalization does yield some interesting insight when examining the BPS bound.
For a $p$-brane, the ADM mass per unit $p$-volume is defined as

$$m = \frac{1}{2\kappa^2} \oint \sum_{i=1}^{9-p} n^i \left[ \sum_{j=1}^{9-p} (\partial_j h_{ij} - \partial_i h_{jj}) - \sum_{a=1}^{p} \partial_i h_{aa} \right] r^{8-p} d\Omega$$

(6)

where $n^i$ is a radial unit vector in the transverse space and $h_{\mu\nu}$ is deformation of the Einstein-frame metric

$$h_{\mu\nu} = g^E_{\mu\nu} - \eta_{\mu\nu}$$

(7)

from flat space in the asymptotic region. Calculating the mass per unit four-volume (of the internal space of $y^i$'s) for our angled system by means of (6) gives us the result

$$m = \frac{A_4}{2\kappa^2} \sum_{a=1}^{n} \ell_a^3.$$

(8)

Thus the mass density is simply the sum of the mass densities of the constituent branes, which was to be entirely expected. Note then that this result is completely independent of the rotation angles.

The membranes carry an electric RR four-form field strength and the corresponding physical charge density is given by

$$q = \frac{1}{\sqrt{2\kappa}} \oint * F^{(4)}.$$

(9)

Hodge duality produces a six-form which is then integrated over the asymptotic four-sphere in the transverse space and some two-torus in $(y^1, y^2, y^3, y^4)$. Thus given, the three-form potential in eq. (4), in applying (4) we obtain a number of independent charges related to the choice of asymptotic surface over which one integrates. For example the term in $A^{(3)}$ proportional $dt \wedge dy^2 \wedge dy^4$ yields a term in $* F^{(4)}$ to be integrated over the compact coordinates $y^1$ and $y^3$ as well as the four-sphere at infinity. We use the following notation to write the resulting charge

$$q_{13} = -q_{31} = \frac{A_4}{\sqrt{2\kappa}} (4\pi^2 L_1 L_3) \sum_{a=1}^{n} \mu_a \ell_a^3 \cos \alpha_a \cos \beta_a.$$

(10)

where the antisymmetric matrix notation will be useful later on. This result gives the charge per unit area in the $(y^2, y^4)$ plane, i.e., the plane in which the branes lie for $\alpha_a = \beta_a = 0$. In order to compare the charges, however, we should divide out the area of the orthogonal $(y^1, y^3)$ torus in order to produce a charge per four-volume in the entire compact space. Hence we define $\tilde{q}_{13} = q_{13} / (4\pi^2 L_1 L_3)$. In a like manner all the charge densities $\tilde{q}_{ij}$ can be calculated and we list the nonvanishing contributions

$$\tilde{q}_{13} = -\frac{A_4}{\sqrt{2\kappa}} \sum_{a=1}^{n} \ell_a^3 \cos \alpha_a \cos \beta_a$$
\[ \tilde{q}_{14} = -\frac{A_4}{\sqrt{2\kappa}} \sum_{a=1}^{n} \ell_a^3 \cos \alpha_a \sin \beta_a \]
\[ \tilde{q}_{23} = \frac{A_4}{\sqrt{2\kappa}} \sum_{a=1}^{n} \ell_a^3 \sin \alpha_a \cos \beta_a \]
\[ \tilde{q}_{24} = \frac{A_4}{\sqrt{2\kappa}} \sum_{a=1}^{n} \ell_a^3 \sin \alpha_a \sin \beta_a . \] (11)

Of course these charge densities are dependent on the rotation angles which orient the various D-membranes. Note that if \( \alpha_a = \beta_a = 0 \) we recover the expected charge configuration of a collection of parallel membranes lying in the \((y^2, y^4)\) plane, i.e.,
\[ \tilde{q}_{13} = -\frac{A_4}{\sqrt{2\kappa}} \sum_{a=1}^{n} \ell_a^3 \quad \tilde{q}_{14} = \tilde{q}_{23} = \tilde{q}_{24} = 0 \] (12)

where the single nonvanishing charge density is simply the sum of that for the individual branes.

Having calculated these physical characteristics of our configuration of D-membranes with angles, we would like to examine the BPS bound. The latter may be determined from the eigenvalues of the Bogomol’nyi matrix, which is derived using both the supersymmetry algebra and the asymptotic form of the background fields\[11\]. Unbroken supersymmetries arise when this matrix has eigenspinors with a vanishing eigenvalue. In the present problem, the Bogomol’nyi matrix is\[12\]
\[ \mathcal{M} = m + \frac{1}{\sqrt{2\kappa}} \tilde{q}_{ij} \Gamma_{0ij} \] (13)

for which the distinct eigenvalues are
\[ m \pm \frac{1}{\sqrt{2\kappa}} \sqrt{\tilde{q}_{ij} \tilde{q}_{ij} \pm \frac{1}{2} \epsilon_{ijkl} \tilde{q}_{ij} \tilde{q}_{kl}} . \] (14)

In these formulae, the implicit sums all run from 1 to 4, and we use the antisymmetric notation \( \tilde{q}_{ij} = -\tilde{q}_{ji} \) introduced above. Also note that the two signs in the eigenvalues are chosen independently. Since the mass is positive, the eigenvalues for which the first sign is positive cannot vanish, and hence at least half of the supersymmetries are broken by our solution. The vanishing of the remaining eigenvalues can be expressed in terms of a BPS mass limit
\[ m_{\perp}^2 = \frac{1}{2\kappa^2} \left( \tilde{q}_{ij} \tilde{q}_{ij} \pm \frac{1}{2} \epsilon_{ijkl} \tilde{q}_{ij} \tilde{q}_{kl} \right) . \] (15)

Substituting our values for the charge densities (11) results in
\[ m_{\perp}^2 = \left( \frac{A_4}{2\kappa^2} \right)^2 \left[ \left( \sum_{a=1}^{n} \ell_a^3 \cos \alpha_a \pm \beta_a \right)^2 + \left( \sum_{a=1}^{n} \ell_a^3 \sin \alpha_a \pm \beta_a \right)^2 \right] . \] (16)
In comparing these BPS bounds (16) with the mass (8), we find that in general the mass exceeds the former bounds. To make this more apparent, one may introduce complex variables $Z_{\pm,a} = (A_a/2\kappa^2)\ell_a^3 \exp[i(\alpha_a \pm \beta_a)]$. Now it is clear that generically $m^2 = (\sum_a |Z_{\pm,a}|^2)$ exceeds $m^2_\pm = |\sum_a Z_{\pm,a}|^2$. It is also clear that the only way to lower the mass to one of the bounds is to choose all of the phases to be equal, i.e., $\alpha_a - \beta_a = 2\theta$ or $\alpha_a + \beta_a = 2\theta'$. There are only two distinct choices here up to an overall rotation. If we set $\alpha_1 = \beta_1 = 0$ to fix the overall orientation of the configuration, we must choose the remaining angles with $\beta_a = \alpha_a$ or $\beta_a = -\alpha_a$. The former corresponds to the choice made in our solution (1), and for which we then have $m = m_+$ and one-quarter of the supersymmetries being preserved. The latter choice, for which $m = m_-$, would yield a slightly different configuration. Complex $SU(2)$ rotations are again relevant in this case, but now the $SU(2)$ acts on $(z^1, z^2) = (y^1 + iy^2, y^3 - iy^4)$. Our solution would be modified by changing the sign of $\alpha_a$ in the lower two-by-two block of the rotation matrices (3), and the sign of $dy^2 \wedge dy^4$ would be reversed in the last term in $A^{(3)}$. As expected, our results here are entirely consistent with the analysis of [6] mentioned earlier which is formulated at the level of the string world-sheet and provide an independent confirmation of their results when applied to D-membranes.

3 T-Duality

The ten-dimensional T-duality map between the type IIA and IIB string theories was given in ref. [13] — see [7] for the transformation using the present conventions. In the next subsection, we consider the effect of T-duality along coordinates that are in the transverse space. The effect of these transformations is to extend the dimension of the D-branes. The results then are new solutions describing $Dp$-branes with relative $SU(2)$ angles and so remaining parallel over a $(p-2)$-brane. In subsection 3.2, we consider the effect of T-duality transformations along world-volume coordinates. The results here involve more exotic bound state configurations of D-branes, as found in [7].

3.1 Transverse directions

In order to apply T-duality along one of the transverse coordinates, e.g., $x^5$, we must first delocalize the solution in this direction, which we then denote as $y^5$. This amounts to replacing the sources in eq. (14) by four-dimensional delta-functions, producing solutions of the form

$$X_a(\vec{x}) = \frac{1}{2} \left( \frac{\ell_a}{|\vec{x} - \vec{x}_a|} \right)^2$$

(17)

where now $\vec{x} = (x^6, x^7, x^8, x^9)$. A straightforward application of the T-duality map from the type IIA to the type IIB theory along $y^5$ in this smeared out solution yields
\[ ds^2 = \sqrt{1+X} \left[ \frac{1}{1+X} \left( -dt^2 + \sum_{i=1}^{5} (dy^i)^2 \right) + \sum_{a=1}^{n} X_a \left\{ [(R_a)^1_i dy^i]^2 + [(R_a)^3_j dy^j]^2 \right\} + \sum_{i=7}^{9} (dx^i)^2 \right] \]

\[ F^{(5)} = dt \wedge dy^5 \wedge dx^k \wedge \partial_k \left\{ \frac{1}{1+X} \left[ \sum_{a=1}^{n} X_a (R_a)^{2_i} dy^i \wedge (R_a)^{4_j} dy^j \right. \right. \]

\[ \left. \left. - \sum_{a<b} X_a X_b \sin^2(\alpha_a - \alpha_b) \left( dy^1 \wedge dy^3 - dy^2 \wedge dy^4 \right) \right]\right) \right) \}

\[ + dx^h \wedge dx^i \wedge dx^j \wedge \epsilon_{hijk} \partial_k \left\{ \sum_{a=1}^{n} X_a (R_a)^{1_l} dy^l \wedge (R_a)^{3_m} dy^m \right\} \]

\[ e^{2\phi_b} = 1 . \]  

(18)

This solution obviously describes a system of angled D3-branes, as indicated by the presence of the nontrivial five-form RR field strength. We have written the solution in terms of the self-dual field strength, rather than the potential \( A^{(4)} \), because the magnetic part of the latter is rather unwieldy when the D3-branes are centered at arbitrary positions \( \vec{x}_a \). If one sets \( \vec{x}_a = 0 \), the potential can be given in a fairly compact form using polar coordinates on the transverse space. Note also that \( \epsilon_{hijk} \) is the antisymmetric Levi-Civita symbol on the transverse space with \( h,i,j,k = 6 \ldots 9 \) and \( \epsilon_{6789} = +1 \).

One can carry this process further by delocalizing the above solution in another transverse coordinate \( x^6 \) (which we then denote \( y^6 \) — also, note that one now has \( X_a = \ell_a / |\vec{x} - \vec{x}_a| \)), and applying T-duality along this direction to produce a system of D4-branes with \( SU(2) \) angles. Here, the T-duality map from type IIB to type IIA generates a magnetic three-form potential through \( A^{(3)}_{\mu \nu \rho} = A^{(4)}_{\mu \nu \rho 6} \) (the remaining terms in this relation vanish in the present case). This part of the transformation is equivalent to mapping the field strengths \( F^{(4)}_{\mu \nu \rho} = F^{(5)}_{\mu \nu \rho 6} \), since the delocalized solution is independent of \( y^6 \). Hence the T-dual solution may be expressed as

\[ ds^2 = \sqrt{1+X} \left[ \frac{1}{1+X} \left( -dt^2 + \sum_{i=1}^{6} (dy^i)^2 \right) + \sum_{a=1}^{n} X_a \left\{ [(R_a)^1_i dy^i]^2 + [(R_a)^3_j dy^j]^2 \right\} + \sum_{i=7}^{9} (dx^i)^2 \right] \]

\[ F^{(4)} = dx^i \wedge dx^j \wedge \epsilon_{ij} \partial_k \left\{ \sum_{a=1}^{n} X_a (R_a)^{1_i} dy^i \wedge (R_a)^{3_m} dy^m \right\} \]

\[ e^{2\phi_b} = \frac{1}{\sqrt{1+X}} . \]  

(19)

Again the magnetic field strength takes a much more compact form than the corresponding
potential for the multi-center solution. One sees that this solution obviously describes a
system of D4-branes since the magnetic $F^{(4)}$ is the only nontrivial RR field.

Of course, this procedure of T-dualizing in the transverse space can be continued to pro-
duce configurations of higher dimensional D-branes with angles. Since the $SU(2)$ rotations
effectively extend the dimension of the world-volume by two, the remaining solutions will
have a transverse space of dimension lower than three, and hence will not be asymptotically
flat. For example, the solution describing angled D6-branes would have a transverse space
of dimension one, and thus would have the appearance of an anisotropic domain wall.

3.2 World-volume directions

An alternative to the above procedure is to apply T-duality in the world volume directions
of the original solution (1). Since the membranes are rotated in these directions, T-dual
configurations will involve D-brane bound states for which the difference in dimension is
two, as discussed in [7]. To simplify the procedure we specialize the general solution to the
case of two D-membranes and also set the rotation angles $(\alpha_1, \alpha_2) = (0, \alpha)$. With these
simplifications, eq. (1) reduces to

\[
\begin{align*}
\frac{ds^2}{\sqrt{1 + X}} &= \frac{1}{1 + X} \left( -dt^2 + (1 + X_1)(dy^1)^2 + (dy^3)^2 + (dy^4)^2 \\
&\hspace{1cm} + X_2 \left[(\cos \alpha dy^1 - \sin \alpha dy^2)^2 + (\cos \alpha dy^3 + \sin \alpha dy^4)^2\right] + \sum_{i=5}^9 (dx^i)^2 \right) \\
A^{(3)} &= \frac{dt}{1 + X} \wedge \left\{- (X_2 + X_1X_2) \sin^2 \alpha dy^1 \wedge dy^3 + X_2 \sin \alpha \cos \alpha dy^1 \wedge dy^4 \\
&\hspace{1cm} - X_2 \cos \alpha \sin \alpha dy^2 \wedge dy^3 + (X_1 + X_2 \cos^2 \alpha + X_1X_2 \sin^2 \alpha) dy^2 \wedge dy^4 \right\} \\
e^{2\phi_a} &= \sqrt{1 + X} \\
\text{and } X \text{ is given by} \\
X &= X_1 + X_2 + X_1X_2 \sin^2 \alpha. 
\end{align*}
\]

We also simplify the following results by positioning the second membrane at the origin,
i.e., we set $x_2 = 0$, but leave $x_1$ arbitrary.

As the first example, we apply T-duality along the $y^4$ direction — note that this direction
is tangent to the world-volume of the a=1 membrane, but is angled with respect to
the second. We find that

\[
\begin{align*}
\frac{ds^2}{\sqrt{1 + X}} &= \frac{1}{1 + X} \left( -dt^2 + (1 + X_1)(dy^1)^2 + (dy^2)^2 \\
&\hspace{1cm} + X_2(\cos \alpha dy^1 - \sin \alpha dy^2)^2 \right) + \frac{(dy^3)^2 + (dy^4)^2}{1 + X_2 \sin^2 \alpha} \\
&\hspace{1cm} + dr^2 + r^2 (d\theta^2 + \sin^2 \theta (d\phi_1^2 + \sin^2 \phi_1 (d\phi_2^2 + \sin^2 \phi_2 d\phi_3^2)))) \right) \\
\end{align*}
\]
\[ A^{(4)} = -\frac{1}{2} X_2 \sin^2 \alpha \left\{ \frac{1 + X_1}{1 + X} + \frac{1}{1 + X_2 \sin^2 \alpha} \right\} dt \wedge dy^1 \wedge dy^3 \wedge dy^4 \]

\[ -\frac{1}{2} X_2 \cos \alpha \sin \alpha \left\{ \frac{1}{1 + X_2 \sin^2 \alpha} + \frac{1}{1 + X} \right\} dt \wedge dy^2 \wedge dy^3 \wedge dy^4 \]

\[ + \ell_3^2 \sin \alpha \sin^3 \theta \sin^2 \phi_1 \cos \phi_2 (\cos \alpha dy^1 - \sin \alpha dy^2) \wedge d\theta \wedge d\phi_1 \wedge d\phi_3 \]

\[ A^{(2)} = \frac{dt}{1 + X} \wedge \left\{ X_2 \cos \alpha \sin \alpha dy^1 + (X - X_2 \sin^2 \alpha) dy^2 \right\} \]

\[ B^{(b)} = \frac{X_2 \cos \alpha \sin \alpha}{1 + X_2 \sin^2 \alpha} dy^3 \wedge dy^4 \]

\[ e^{2\phi_b} = \frac{1 + X}{1 + X_2 \sin^2 \alpha} \]

\[ A^{(1)} = \left\{ \frac{1 + X_2 \sin^2 \alpha}{1 + X} - 1 \right\} dt \]

\[ B^{(a)} = \frac{X_2 \cos \alpha \sin \alpha}{1 + X_2 \sin^2 \alpha} (dy^3 \wedge dy^4 - dy^1 \wedge dy^2) \]

\[ e^{2\phi_a} = \frac{(1 + X)^{1/2}}{(1 + X_2 \sin^2 \alpha)^{1/2}}. \]

where we have transformed the coordinates transverse to the system into spherical coordinates to facilitate the computations of the four-form RR potential. Setting \( X_2 = 0 \), one can verify that this solution reduces to that of a D-string lying parallel to \( y^2 \) and at the same time delocalized in \( y^1, y^3 \) and \( y^4 \). Setting \( X_1 = 0 \) and comparing with the solutions of \([7]\), one finds that the solution is precisely that of a D(3,1)-brane bound state. There has been a rotation of this bound state so that it lies in \( \cos \alpha y^1 - \sin \alpha y^2 \) direction. The angle \( \alpha \) also determines the relative charge densities of the D-strings and D3-branes — in \([7]\), \( \phi = \pi/2 - \alpha \).

Next we continue by applying T-duality in the \( y^2 \) direction producing a solution of the form

\[ ds^2 = \sqrt{1 + X} \left\{ -\frac{dt^2}{1 + X} + \frac{(dy^1)^2 + (dy^2)^2 + (dy^3)^2 + (dy^4)^2}{1 + X_2 \sin^2 \alpha} + \sum_{i=5}^{9} (dx^i)^2 \right\} \]

\[ A^{(3)} = \frac{X_2 \cos \alpha \sin \alpha}{1 + X_2 \sin^2 \alpha} dt \wedge (dy^1 \wedge dy^2 - dy^3 \wedge dy^4) \]

\[ + \ell_3^2 \sin^2 \alpha \sin^3 \theta \sin^2 \phi_1 \cos \phi_2 d\theta \wedge d\phi_1 \wedge d\phi_3 \]

\[ A^{(1)} = \left\{ \frac{1 + X_2 \sin^2 \alpha}{1 + X} - 1 \right\} dt \]

\[ B^{(a)} = \frac{X_2 \cos \alpha \sin \alpha}{1 + X_2 \sin^2 \alpha} (dy^3 \wedge dy^4 - dy^1 \wedge dy^2) \]

\[ e^{2\phi_a} = \frac{(1 + X)^{1/2}}{(1 + X_2 \sin^2 \alpha)^{1/2}}. \]

In this case setting \( X_2 = 0 \) reduces the solution to that of a D-particle positioned at \( \vec{x}_1 \) and delocalized in the \( y^i \) directions. Setting \( X_1 = 0 \) reproduces a special case of the D(4,2,2,0)-brane bound state given in \([7]\). Here the two angles of that solution are related, \( i.e., \phi = -\psi = \pi/2 - \alpha \). The full solution then describes the configuration conjectured by Lifschytz\([14]\) from the consideration of the D-brane scattering processes.
As a final example, we perform T-duality along $y^3$ in the two membrane solution (20) with the resulting solution

$$ds^2 = \sqrt{1 + X} \left\{ -\frac{dt^2}{1 + X} + \frac{(1 + X_1 + X_2 \cos^2 \alpha)\, (dy^1)^2 + (1 + X_2 \sin^2 \alpha)\, (dy^2)^2}{1 + X} 
- \frac{2X_2 \cos \alpha \sin \alpha \, d\alpha \, dy^2}{1 + X} + \frac{(dy^3)^2 + (dy^4)^2}{1 + X_1 + X_2 \cos^2 \alpha} + \sum_{i=5}^9 (d\alpha_i)^2 \right\}$$

$$A^{(4)} = -\frac{X_2 \cos \alpha \sin \alpha}{2} \left\{ \frac{1}{1 + X} + \frac{1}{1 + X_1 + X_2 \cos^2 \alpha} \right\} dt \wedge dy^1 \wedge dy^3 \wedge dy^4$$

$$+ \left\{ \frac{1 - 2X_1}{2X_1} + \frac{1}{2(1 + X_1 + X_2 \cos^2 \alpha)} - \frac{1 + X_2}{2X_1(1 + X)} \right\} dt \wedge dy^2 \wedge dy^3 \wedge dy^4$$

$$+ \ell^3_1 \sin^3 \theta \sin^2 \phi_1 \cos \phi_2 \, dy^1 \wedge d\theta \wedge d\phi_1 \wedge d\phi_3$$

$$+ \ell^3_2 \cos \sin^2 \sin^2 \phi_1 \cos \phi_2 \, (\cos \alpha \, dy^1 - \sin \alpha \, dy^2) \wedge d\theta \wedge d\phi_1 \wedge d\phi_2$$

$$A^{(2)} = -\frac{X_2 \sin \alpha}{1 + X} \left\{ \sin \alpha (1 + X_1) \, dy^1 + \cos \alpha \, dy^2 \right\}$$

$$B^{(b)} = -\frac{X_2 \cos \alpha \sin \alpha}{1 + X_1 + X_2 \cos^2 \alpha} \, dy^3 \wedge dy^4$$

$$e^{2\phi_b} = \frac{1 + X}{1 + X_1 + X_2 \cos^2 \alpha}. \quad (24)$$

where we have also put $\vec{x}_1 = 0$ here for simplicity. With $X_2 = 0$, we have a single D3-brane filling $(y^2, y^3, y^4)$ and delocalized in $y^1$. With $X_1 = 0$, one may verify that the result describes a D(3,1)-brane bound state parallel to $(\sin \alpha \, y^1 + \cos \alpha \, y^2, y^3, y^4)$ with the D1-branes lying in the first of these directions. Again the relative charge densities of the bound state are determined by the rotation angle.

## 4 Discussion

In this paper, we presented a new low-energy solution (1) describing an arbitrary number $n$ of D-membranes oriented at angles with respect to one another. We were also able to show that this configuration saturated the BPS bound because the relative rotations between the membranes are in an $SU(2)$ subgroup. As a result, the system preserves one-quarter of the supersymmetries.

Our solution provides the most general supersymmetric configuration containing (only) two D-membranes. One might think of extending the rotations considered here to an arbitrary $SU(2)$ rotation, but this generalization would only change the overall orientation of our solution. Following the analysis of [1], with three D-membranes one might make $SU(3)$ rotations while still preserving one-eighth of the supersymmetries. This would extend the space in which the rotations act to produce an effective seven-dimensional world volume. It would be interesting to find the corresponding background field solution.
For general $n$, one might consider $SU(n)$ rotations\[1\], however, in practice one would be limited to $SU(4)$ by the fact that the spacetime is ten-dimensional.

By applying T-duality to the membrane solution (1), we produced solutions describing systems of higher dimensional D-branes oriented at angles, and also configurations involving D$(p+1,p–1)$-brane bound states. Since supersymmetry is preserved by T-duality, these other new solutions also preserve one-quarter of the supersymmetries. These configurations may be useful in trying to understand the microscopic counting of states of new four-dimensional black holes in Type II string theories. By explicit construction, we have confirmed the existence of a supersymmetric configuration including D0-branes and D(4,2,2,0)-bound states. These supersymmetric solutions were conjectured in \[14\], where it was shown that the interaction potential precisely vanished between these two objects.

After this research was carried out, two new papers\[15, 16\] appeared which discuss branes oriented at angles in different contexts than considered in the present paper. In \[15\], a construction is presented of a configuration of D4-branes tilted by a real $SO(2)$ rotation and held in static equilibrium by the presence of D-membranes and fundamental strings. In \[16\], a novel new configuration of angled D5-branes which preserve 3/16 of the supersymmetries. At present there is no obvious connection between these solutions and those presented here, however, it will be interesting to explore this question in more detail.

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