Gas-liquid Two Phase Flow Modelling of Incompressible Fluid and Experimental Validation Studies in Vertical Centrifugal Casting

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Abstract. In this paper, Gas-liquid two phase flow mathematic models of incompressible fluid were proposed to explore the feature of fluid under certain centrifugal force in vertical centrifugal casting (VCC). Modified projection-level-set method was introduced to solve the mathematic models. To validate the simulation results, two methods were used in this study. In the first method, the simulation result of basic VCC flow process was compared with its analytic solution. The relationship between the numerical solution and deterministic analytic solution was presented to verify the correctness of numerical algorithms. In the second method, systematic water simulation experiments were developed. In this initial experiment, special experimental vertical centrifugal device and casting shapes were designed to describe typical mold-filling processes in VCC. High speed camera system and data collection devices were used to capture flow shape during the mold-filling process. Moreover, fluid characteristic at different rotation speed (from 40rpm, 60rpm and 80rpm) was discussed to provide comparative resource for simulation results. As compared with the simulation results, the proposed mathematical models could be proven and the experimental design could help us advance the accuracy of simulation and further studies for VCC.

1. Introduction

Vertical centrifugal casting (VCC) is a special casting technique could significantly improve the quality of thin-walled castings. During the process of mold-filling and solidification, the high centrifugal-force and the gravity are applied to the molten metal in the spinning die. The free surface changes become more intense by bi-action of two direction forces, particular under the higher rotation speed condition. So the mold-filling process of VCC is more complex than other casting process, and the numerical simulation is challenged.

In primer research, the numerical simulation of mold-filling in VCC was used to the Solution Algorithm (SOLA)-Volume of Fluid (VOF) method. Jia et al developed analytic method to analyse the relationship between filling velocity and filling distance, and the relationship between the maximum rotational angular velocity and diameter and filling distance¹. Xu D M et al. proposed a mathematical model based on SOLA-VOF method for mold-filling processes under centrifugal force conditions, and

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simulated the free surface developments in a rotating cylindrical container\cite{3}. A foundry CAE system based on the SOLA-VOF method for centrifugal casting was developed by Zeng X W and was applied to the industry \cite{4}. Humphreys et al\cite{5} established numerical models to analyse morphological changes of fluid free surface, and validate it through water simulation test results. Wu S P et al\cite{6} studied the filling process in VCC process by hydraulic experiment. Prasad et al\cite{7} used different viscosity of transparent water and 140EP oil to study the flow shape affected by processes parameters for horizontal centrifugal casting, and also applied the STAR-CD to simulate the fluid behaviour in cold die and verify the result with the experiment\cite{8}. Combined with water simulation and theoretical analysis method, Li et al\cite{9} qualitatively analysed the flow pattern of VCC filling process with advanced high-speed cameras, and systematically discussed impact factors of melt flow pattern and casting defects of titanium alloy by processes parameters in the following practical experiment studies. Mukunda et al\cite{10}, Watanabe et al\cite{11}, Shimizu et al\cite{12}, and etc. studied actual production process problems of centrifugal casting for different casting alloy.

However, above simulation and experiment researches have weakness on exploring complex two phase flow and geometry of VCC. On one hand, the free surface of fluid and gas becomes more complex because of the bi-action of centrifugal and gravity, and the gas pressure would be important influential factors on the shape of fluid in VCC. In this paper, mold-filling process is modelled by the combination of the Projection and the Level Set. The Projection algorithm is applied to calculate the governing equation of the flow filed in VCC, and the Level set method is adopted to deal with the interface transformation. On the other hand, because of hard prospecting and data recording under rotation condition, few primer researches give comprehensive fluid information such as fluid pressure, velocity etc. In this paper, we built specific experiment instruments and plan to create new data recording schemes in the future studying.

2. Mathematical models

2.1. The governing equation of the flow filed in vertical centrifugal

As casting process, the mold-filling process of VCC is combined with the transformation of the free surface. The momentum and mass governing equations, i.e. the incompressible Navier-Stokes equations that describe the motion of the Newtonian fluids are available as well.

The based governing equations of casting mold-filling are written as\cite{13}:

$$\vec{F} = \frac{1}{\rho} \nabla P + \mu \Delta \vec{U} = \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla)\vec{U} \tag{1}$$

$$\nabla \cdot \vec{U} = 0 \tag{2}$$

Where $\vec{F}$ is the acceleration velocity component, $\rho$ is the fluid density, $P$ is the pressure, $\mu$ is the constant viscosity, $\vec{U} <u, v, w>$ is the velocity field in the Cartesian coordinate system, $t$ is time.

**Figure 1.** The acceleration velocity components of a cell in each direction.

The acceleration velocity components, $\vec{F}$, which are affected by the centrifugal force and the Coriolis force\cite{14}, are $a_x, a_y, a_z$ in three directions of the Right-handed coordinate system (see fig.1). The sum of the vector of acceleration velocity in each direction is written as:
\[ a_x = \omega^2 x + 2\omega v_t \]
\[ a_y = \omega^2 y - 2\omega u_t \]
\[ a_z = g \]

Where \( \omega \) is the angular velocity, \( u_t \) and \( v_t \) refer to the relative velocity in the \( X, Y \) direction, \( g \) is the gravity acceleration velocity.

### 2.2. Free Surface Treatment

In this paper, the Level Set method is applied to simulate the interface movement between the gas and the liquid in the mold filling process\(^{[15]}\). As depicted in fig. 2, consider a grid configuration on a domain \( \Omega \) separate into two disjoint subsets \( \Omega_1 \) and \( \Omega_2 \), such that \( \Omega = \Omega_1 \cup \Gamma \cup \Omega_2 \), where \( \Omega_1 \) is filled liquid, \( \Omega_2 \) is gas, and \( \Gamma(t) \) is the interface between \( \Omega_1 \) and \( \Omega_2 \). Assume that the domain \( \Omega \) is represented by a level function \( \phi(x, y, z, t) \), and the interface \( \Gamma(t) \) is the set of function \( \phi(x, y, z, t) \) is equal to zero:

\[
\phi(x, y, z, 0) = \begin{cases}
-d(x, y, z, \Gamma) & \text{if } (x, y, z) \in \Omega_1 \\
0 & \text{if } (x, y, z) \in \Gamma \\
d(x, y, z, \Gamma) & \text{if } (x, y, z) \in \Omega_2
\end{cases}
\]

**Figure 2.** The interface of the computational domain.

Time step \( t_n \), \( \Gamma(t) \) is from calculating the function \( \phi(x, y, z, t) \). By updating the \( \Gamma \), we could track the fluid-gas interface. At the time \( t \), the microelement on the free surface consists with the function listed below:

\[ \phi(x(t), y(t), z(t), t) = 0 \]

Then,

\[ \frac{\partial \phi}{\partial t} + \vec{U} \nabla \phi = 0 \]

For specific fluid field, the Level Set function is written as this format:

\[ \frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = 0 \]

Where \( u, v \) and \( w \) refer to the velocity component in the Cartesian coordinate system.

### 2.3. Projection method

The Projection method states that any vector field \( \vec{U}^* \) could be decomposed into the sum of divergence-free vector \( \vec{U} \) and a weighted gradient field \( \nabla P/\rho \) for scalar function \( P \) and positive function \( \rho \). Considering the Eq. 1 and Eq. 2. The momentum equation is discretized to explicit scheme:

\[
\frac{U^{n+1} - U^n}{\Delta t} + U^n \cdot \nabla U^n = - \frac{1}{\rho} \nabla P^n + \mu \Delta U^n + F
\]

Define tentative velocity \( U^n \). Now the Eq.10 could be written as:
\[
\frac{U^{n+1} - U^n}{\Delta t} + U^n \cdot \nabla U^n = \mu \Delta U^n + F
\] (11)

\[
\frac{U^{n+1} - U^n}{\Delta t} = -\frac{1}{\rho} \nabla P^{n+1}.
\] (12)

Take the divergence on each side, and then apply the mass equation \( \nabla \cdot U_{n+1} = 0 \). The equation becomes a pressure Poisson Equation (PE):

\[
\frac{\nabla \cdot U^n}{\Delta t} = \frac{\nabla \cdot P^{n+1}}{\rho}.
\] (13)

With Neumann boundary conditions on the domain’s boundaries and on the solid objects:

\[
n \cdot \frac{\nabla P}{\rho} = n \cdot (U_{bc}^* - U_{bc}).
\] (14)

During the numerical computation, the Level Set function must be re-initialized so as to keep the definition of level function \( \phi(x, y, z, t) \)\[^{[16]}\]. Hamilton-Jacobi equation would be solved to update the function \( \phi \) in this paper\[^{[17-18]}\]:

\[
\begin{align*}
\frac{\partial \phi}{\partial t} &= \text{sgn}(\phi_0)(1 - |\nabla \phi|) \\
\phi_0 &= \phi(x, y, z, 0)
\end{align*}
\] (15)

Where \( \text{sgn}(\phi_0) \) is signum function, \( \phi_0 \) is the value of the preceding time step. Eq.15 should be iterated to reach a steady state, and \( \phi(x, y, z, t) \) would be updated in this time step.

### 3. Numerical and analytic solutions examples

To verify the mathematical model and numerical algorithm of VCC, an example is carried out in this paper. In fig. 3, the height of the computation casting model is 80mm. We put the emphasis on inspecting the effect of the centrifugal force and the transformation of the free surface. Assume that the molten metal has finished the pouring process before the die spinning, and the height of the liquid is 60mm at the initial time in the cavity. The liquid is aluminium with a purity of 99.999\%, and the density is 2350kg/m\(^3\). The density of the gas is 1.2kg/m\(^3\). The kinematic viscosity of the liquid aluminium and the gas is \( 4.255 \times 10^{-7} \) m\(^2\)/s and \( 1.48 \times 10^{-6} \) m\(^2\)/s respectively. In this paper, the uniform grid is applied, and the spatial step is 1mm.

![Test model and Inwall of the cavity(mm)](image)

**Figure 3.** The Model of VCC.

#### 3.1. The Shape of the Free Surface

Theoretically, the cavity rotates with the angle velocity of \( \omega \) by the vertical axis, i.e. \( Z \) axis in the governing equations, so the liquid in the cavity would rotates with the constant angular velocity \( \omega \) in
the steady state. Assume that the pressure in the cell \((r, z)\) is \(P\), and the weight density is \(\gamma\). Every cell in a steady-state flow field satisfies the equilibrium equation\(^{(19)}\):

\[
dP = \frac{\gamma}{g} \omega r dr - rdz
\]

This equation is just the base pressure equation that we study the state of relative rest. As known, we should consider the pressure gradient \(dP = 0\) in the isobaric face. Then,

\[
dz = \frac{w^2}{g} rdr
\]

Take the integral on each side:

\[
z = \frac{w^2r^2}{2g} + c_i
\]

Where, \(c_i\) is a constant. If we draw a curve by \(r\) and \(z\), which \(r\) is treated as independent variable, while \(z\) as dependent variable, it would be a parabola. It illustrates that the isobaric surface of the free surface in the spinning die is paraboloidal surface of revolution, as shown in fig. 4.

![Figure 4. The free surface in 3D model.](image)

Fig. 5 shows the distribution of free surface at a time (during the pouring process at 0.02s, 0.25s and 0.7s). The height of the molten metal near the mold increases, while the height of the molten metal in the rotational center of the mold decreases. At last, the free surface is formed, and it appears that the side near the wall is high and the side far from the wall is low.

For the restriction of accuracy of mesh, the free surface may have burr in the computation zone with jag. But the distribution of the liquid phase calculated by the Projection-Level Set method coincides with the physical truth.

### 3.2. The Distribution of the Pressure

This paper observes and studies the distribution of pressure in the cross section which the normal direction is the \(z\)-axis. For a designated section, the item \(dz\) is equal to 0 in Eq.16. Then,

\[
dP = \frac{\gamma}{g} \omega r dr
\]
Take the integration on each side \(^{(20)}\), we obtain:

\[
P = \frac{\gamma w^2 r^2}{2g} + c_2
\]

(20)

Where, \(c_2\) is a constant. In this equation, \(p\) is only related to \(r\), and with the increase of \(r\), it shows a quadratic curve change. In the spinning die, it also indicates that, the pressure distribution of the molten metal is parabola. As illustrated in fig.6, the numerical and analytic solutions are presented, and the change regularity of which are homologous and tallied with Eq.20.

Figure 6. The numerical and analytic solutions in the selected section.

Figure 7. The contour of pressure in the selected section.

At flow steady state time, the radical pressure distribution of same investigate points (19 points along with the radial coordinate) is compared. Since the analytic solution (calculated by equation (20)) ignores viscous forces of molecules of the fluid, the pressure gradient would generally be larger than actual phenomenon. Relative error between numerical and analytic solutions: the maximum error is 7.8%; the minimum error is 0, and the average error is 4.1%. The error value is acceptable, and the numerical solution could describe the equation (20). For observation purposes, this paper selects a horizontal section to draw the contour of pressure, which the height is 25mm. Fig.7 shows that the pressure and the axis center are radially symmetric.

4. Water simulation experiment and discussion

This article puts emphases on shape change of the flow during mold-filling. The schematic of the experimental platform is shown in fig. 8. The actual equipment is shown in fig. 9. The major devices of the experiment are die casting mold, high speed camera, the centrifugal workbench, console, acquisition system, motor, etc. Experimental media are water, carbon ink, plotting paper and so on. The carbon ink is used as a marker of water simulation tracer. The die casting mold is transparent organic glass, its three-dimensional size is as shown in fig.10. The high speed camera used in the experiment is professional high speed camera of Motion Pro Y4 - S1 type of the IDT Company.

Figure 8. The schematic of experimental platform.

Figure 9. Actual equipment.
Figure 10. Installation of die casting mold.

The centrifugal workbench is driven by motor through the pulley and the level error is about ±1mm. The motor is three-phase asynchronous motor which is controlled by the frequency converter to make speed stable. The speeds are acquired through laser reflection principle to get signal of centrifugal spinning. This measurement is characterized by high resolution, wide scope of application and high reliability. The console is used to control pouring, motor, centrifugal rotary table and display speed. The centrifugal force is associated with the centrifugal rotation. Therefore, the rotation speed is the main experimental parameter, which is set as 40rpm, 60 rpm, 80 rpm and the according fluid flow during filling is filmed respectively.

Simulation and experiment discussion

0.0375s

0.1000 s

0.1250 s
Figure 11. Top view at cross gate at different filling time

Figure 12. Front view at the mold outer wall at different filling time
With same parameter setting, the proposed models and numerical results (based on Fluent) is presented to compare with experiment results. Due to spatial confined, this paper simply enumerates partial data of experiment and simulation as shown in fig. 11 and fig. 12. Before pouring process, the mold is put on the centrifugal system and is in a stable operation at 80 rpm (because of paper length limit, we just discuss it to present 40rpm and 60 rpm). Fig. 11 shows that the fluid flows through the runner gate and present wall-flow on account of centrifugal force during pouring process. The simulation results are basically meeting the tendency in experimental results. And it results in that most of the fluids only flow to one side, which is just the opposite direction of mold rotation. Meanwhile, gas flows into mold centre along with the radius direction. But at the runner gate, partial fluids hinder the gas escape. It comes into being a liquid gas mixture because of severe turbulence. Besides, the fluid front at initial time is random like turbulence and is more likely to exist in the centre of runner gate, this is the result of interaction between liquid and gas. Eventually, the fluids gather in outer mold. On a negative note, the accurate and precise gas-liquid interface is difficult to trace owing to swift-running fluid.

In fig. 12, the fluids from runner gate crash into the mold outer wall and splash as wave crest. At the same time, from the experiment figure, it can be seen that the fluid deflect from a straight course of runner gate. Combined with the above analysis results from fig. 11, it can be known that the inertial effect cause the fluid to tend to flow along original orientation from runner gate. In addition, the gas-liquid interface is also affected by the fluid flow form, especially in collision between liquid and mold. That will bring about casting defects in modern casting process.

Therefore, in order to acquire high-quality castings, the principle of smooth filling in centrifugal system is very important. And it needs to be able to avoid collisions between liquid and mold. At last, the structure optimization of casting mold must be given enough thought to the centrifugal force.

5. Summary

In this paper, The Projection-Level Set method is proposed to solve the flow filed governing equation to simulate the mold-filling in VCC. Contrast to the analytic solution, the reliability of the proposed method is acceptable. On the other hand, experiment instrument and simple water simulation experiment are designed. By intuitively and qualitatively analysing, the simulation results could reflect the experimental phenomena. The filling sequence and shape of fluid in simulation is basic familiar with the experiment. In the future studying, the systematic experimental schemes and more comprehensive data (fluid pressure, velocity, etc.) recorders should be developed to deeply discuss the simulation results with quantitative compassion.

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