Newtonian Gravitomagnetism and analysis of Earth Satellite Results

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Abstract

The possibility of a Newtonian gravitomagnetic field is considered here with its immediate and far-reaching implications for the interpretation of 2004 LAGEOS experimental results confirming the general relativistic prediction of Lense-Thirring effect.

PACS: 04.80; 04.80Cc

Keywords: Gravitation; Newtonian Gravitomagnetism; Gravitomagnetism; General Relativity, Lense-Thirring Effect; LAGEOS satellites; Gravity Probe B experiment

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1 Introduction

The gravitomagnetic field [1] is one of the most important predictions of general relativity, which is believed to have no Newtonian counterpart [1, 2, 3], that emerges as a consequence of mass currents, analogous to the generation of magnetic field by electric (charge) currents. Working within the framework of general relativity, Lense and Thirring in 1918 [4] predicted that a spinning massive body like Earth would generate a gravitomagnetic field, which in turn would cause a precession of the orbits of its satellites. This effect, called Lense-Thirring (LT) effect, is also known as “dragging of inertial frames” or more simply “frame-dragging” as Einstein named it. Recently, frame-dragging or LT-effect is reported to have been detected [2, 5] by Ciufolini et al. [2, 5] with experimental results differing from the general relativistic predictions by about 20% during 1998-2002 [2] and by 10% in 2004 [5]. Gravity probe-B (an ongoing space mission of NASA to test general relativity using cryogenic gyroscopes in orbit) was launched in April 2004 and aims at measurement of Lense-Thirring effect to about 1% error. The important fact is that the analysis of the LAGEOS experimental results [2, 5] and the ongoing data analysis of Gravity Probe B experiment [6] are based on the assumption that Newton’s theory of gravitation has no phenomena analogous to magnetism although Newton’s law of gravitation has a formal counterpart in Coulomb’s law of electrostatics. In view of the above facts, the possibility of prediction of a Newtonian gravitomagnetic field is of immediate practical importance in the context of the gravitomagnetic effects are now becoming amenable to experimental observation. By noting the possible existence of a gravitomagnetic field first speculated by J. C. Maxwell in 1865 [7] and later pursued by Oliver Heaviside in 1893 [8], well before the advent of the relativity theories of Einstein, here we report the possibility of making a “natural” prediction of gravitomagnetic field within the framework of Newtonian Physics. The full set of Faraday-Maxwell-type field equations of gravity describing Newtonian gravitodynamics has been derived here within the framework of Newtonian physics in a most natural way. The results represent a substantial advance in understanding Newton’s theory of gravity in its entirety and also the important problem of the gravitomagnetic phenomena in physics. The conclusions seem to have immediate and far-reaching implications for the current research on the theory and experiments on gravitomagnetic effects in particular and gravitation in general.

2 History of the Gravitomagnetic field concept

Recognizing the striking formal analogy between Newton’s law of gravitostatics and Coulomb’s law of electrostatics, J. C. Maxwell [7], in one of his fundamental works on electromagnetism, turned his attention to the possibility of formulating the theory of gravity
in a form corresponding to the electromagnetic equations. In this attempt, Maxwell considered that the potential energy of a static gravitational configuration is always negative but he guessed that this should be re-expressible as an integral over field energy density, which being the square of the gravitational field (by electromagnetic analogy) is positive. Because of this puzzle, Maxwell did not work further on the topic. However, we will see a solution to Maxwell’s puzzle in this work.

Nonetheless, in 1893 O. Heaviside pursued Maxwell’s attempt further and wrote down the full set of Lorentz-Maxwell type equations for gravity, very much analogous to the corresponding equations in classical electrodynamics, by virtue of his power of speculative thought. Heaviside’s field equations implied the existence of gravitational waves in vacuum, so he considered that the propagation velocity of gravitational waves in vacuum might well be the speed of light in vacuum. He also explained the propagation of energy in a gravitational field, in terms of gravitoelectromagnetic Poynting vector, even though he (as Maxwell did) considered the nature of gravitational energy is a mystery. Lacking experimental evidence of gravitomagnetic effect and for some other reasons, he did not work further. As per a report by McDonald (see McDonald in [8]), surprisingly Heaviside seemed to be unaware of the long history of measurement of the precession of Mercury’s orbit.

The formal analogy was then explored by Einstein, in the framework of General Relativity. Any theory that combines Newtonian gravity together with Lorentz invariance in a consistent way, must include a gravitomagnetic field, which is generated by mass current. This is the case, of course, of General Relativity. May be Einstein had not seen Heaviside’s field equations when he was working on his relativistic theory of gravity. Had Einstein seen Heaviside’s field equations, his remark on Newton’s theory of gravity would have been different than what he made before the 1913 congress of natural scientists in Vienna, viz.,

“After the untenability of the theory of action at distance had thus been proved in the domain of electrodynamics, confidence in the correctness of Newton’s action-at-a-distance theory of gravitation was shaken. One had to believe that Newton’s law of gravity could not embrace the phenomena of gravity in their entirety, any more than Coulomb’s law of electrostatics embraced the theory of electromagnetic processes.”
3 Derivation of the field equations of Newtonian Grav- itodynamics

In Galileo-Newtonian physics, the source of gravitational field is mass and mass is a conserved quantity - any flow of mass must come from some supply, which we know to be true. The generation of gravitational field is expressed by Gauss’s law of gravitostatics:

\[ \nabla \cdot \vec{E}_g = -4\pi G \rho \quad (1) \]

where \( \vec{E}_g \) is the gravitational field intensity, \( \rho \) (= mass density) is the source of \( \vec{E}_g \) and \( G \) is Newton’s gravitational constant. The law of conservation of mass is expressed by the continuity equation:

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{j} \quad (2) \]

where \( \vec{j} \) is mass current density. Assuming the validity of the two laws (1) and (2), we now proceed to find the gravitational effects associated with mass currents. We know that there exist physical systems where the two laws (1) and (2) are at play simultaneously or co-work peacefully. To explore the gravitational behavior of such systems, we take the time derivative of (1) to obtain the equation

\[ \nabla \cdot \left( \frac{1}{4\pi G} \frac{\partial \vec{E}_g}{\partial t} \right) = 0 \quad (3) \]

Now by using (2) in (3), we obtain the equation:

\[ \nabla \cdot \left( \frac{1}{4\pi G} \frac{\partial \vec{E}_g}{\partial t} - \vec{j} \right) = 0 \quad (4) \]

The quantity inside the parenthesis of (4) is a vector whose divergence is zero. Since \( \nabla \cdot (\nabla \times \vec{A}) = 0 \) is true for any vector \( \vec{A} \), the vector inside the parenthesis of (4) can be expressed as the curl of some other vector, say \( \vec{H}_g \). Mathematically speaking, the eq.(4) admits of two solutions, viz.,

\[ \nabla \times \vec{H}_g = \pm \left( -\vec{j} + \frac{1}{4\pi G} \frac{\partial \vec{E}_g}{\partial t} \right) \quad (5) \]

which is formally analogous to Ampere-Maxwell’s Law of classical electrodynamics, viz.,

\[ \nabla \times \vec{H} = \vec{j}_e + \frac{\partial \vec{D}}{\partial t} \quad \text{(in S.I. units)} \quad (6) \]
where \( \vec{H} \) is the magnetic field, \( \vec{j}_e \) is electric current density and \( \vec{D} \) is the displacement vector. In empty space \( \vec{D} = \epsilon_0 \vec{E} \), where \( \epsilon_0 \) is the permittivity of empty space and \( \vec{E} \) is the electric field. One of the solutions (5) would correspond to the reality and the other might be a mathematical possibility having no or new physical significance in which we are not presently interested in. For our present purpose, we have to choose one of these two solutions that may correspond to the reality. Which one to choose? We can answer this question by following the rule of ‘study by analogy’. To follow this rule, let us write down the Gauss’s law of electrostatics in the form:

\[
\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}
\]  

(7)

where \( \rho_e \) is the charge density, \( \vec{E} \) is electric field intensity, \( \epsilon_0 \) is the permittivity of empty space. Eq.(7) is analogous to Eq.(1), with only a difference in the sign of the source functions. From (6) and (7) we find that \( \rho_e \) and \( \vec{j}_e \) have got the same sign. So by analogy we infer that in case of gravity \( \rho \) and \( \vec{j} \) should have the same sign and the signs of \( \vec{j}_e \) and \( \vec{j} \) should be opposite. So by this logic, our most natural choice for the real solution of (4) is

\[
\vec{\nabla} \times \vec{H}_g = -\vec{j} + \frac{1}{4\pi G} \frac{\partial \vec{E}_g}{\partial t}
\]

(8)

The comparisons of (1) with (7) and (6) with (8) suggest us to introduce or define the following terms in gravitation, viz., \( \vec{E}_g \) as the gravitoelectric (GE) field, \( \epsilon_{0g} = \frac{1}{4\pi G} \) as the gravitoelectric permittivity of empty space, \( \vec{D}_g = \epsilon_{0g} \vec{E}_g \) as the gravitational displacement vector in empty space, \( \vec{H}_g \) as the gravitomagnetic field, and name the Eq.(8) as the gravitational Ampere-Maxwell Law of Newtonian gravitoelectromagnetic (GEM) theory or Newtonian gravitodynamics. The appearance of the sign difference of the corresponding source functions in (1) and (7), (6) and (8) implies the characteristic dissimilarity between the two fundamental interactions of nature - which are opposite in nature. For instance, in electromagnetism like charges repel and unlike charges attract under static conditions, but under dynamic condition the nature of the interaction gets reversed - like electric currents (i.e. parallel currents) attract and unlike (i.e. anti-parallel) currents repel. In case of gravitation, we have the opposite situation, viz., like masses attract [ and if negative mass \[11\] exists, the unlike masses should repel ] under static condition, and by the nature analogy between gravitational and electrical phenomena, we would have a reversed situation in the dynamic condition, viz., like (i.e. parallel) mass currents would repel (as a form of anti-gravity \[12\]) and unlike (i.e. anti-parallel) mass currents would attract. The deep analogy between electrical and gravitational phenomena in the framework of general relativity has earlier been discussed by R. L. Forword \[12\], R. Wald \[13\], V. B. Braginsky, C.M. Caves and K.S. Thorne \[14\]. One of the implications or predictions
of gravitational Ampere-Maxwell Law (8) is that mass currents would generate gravito-
magnetic field in accordance with what we may call the gravitational Ampere’s Law of
gravitomagnetostatics, viz.,
\[ \nabla \times \vec{H}_g = -\vec{j} \quad (\text{when } \frac{\partial \vec{E}_g}{\partial t} = 0 \text{ in (8)} ) \] (9)
which is the gravitational analogue of Ampere’s Law in magnetostatics, viz.,
\[ \nabla \times \vec{H} = \vec{j}_e \quad (\text{when } \frac{\partial \vec{D}}{\partial t} = 0 \text{ in (6)} ) \] (10)
To understand the significance and the implications of the 2nd term on the right hand side of (8), let us note the significance and the implications of the term \( \frac{\partial \vec{D}}{\partial t} (= \varepsilon_0 \frac{\partial \vec{E}}{\partial t} ) \) in Ampere-
Maxwell’s Law (6). This term was added to Ampere’s Law (10) by Maxwell who called it the displacement current. The displacement current is of crucial importance for rapidly fluctuating fields. Without it there would be no electromagnetic wave [15]. Analogously the gravitational displacement current \( \frac{\partial \vec{D}_g}{\partial t} (= \varepsilon_0 \varepsilon_g \frac{\partial \vec{E}_g}{\partial t} ) \) is of crucial importance for rapidly fluctuating gravitational fields. Without it there would be no gravitational wave. Because of these implications of gravitational Ampere-Maxwell Law (8), it is natural to assume the existence of gravitational waves in empty space where the law still stands even if \( \vec{j} = 0 \). Thus if gravitational waves exist in empty space, then the electromagnetic analogy we are uncovering suggests that the wave equation for the fields \( \vec{E}_g \) and \( \vec{H}_g \) in empty space should stand as
\[ \nabla^2 \cdot \vec{H}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{H}_g}{\partial t^2} = 0 \] (11)
\[ \nabla^2 \cdot \vec{E}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{E}_g}{\partial t^2} = 0 \] (12)
where \( c_g \) is some finite speed of propagation of gravitational waves, which is to be determined by experiments. To explore this possibility, let us take the curl of (8) to obtain the equation
\[ \nabla^2 \vec{H}_g + \frac{1}{4\pi G} \frac{\partial}{\partial t}(\nabla \times \vec{E}_g) = \nabla \times \vec{j} + \nabla \cdot (\nabla \cdot \vec{H}_g) \] (13)
In empty space (13) reduces to the wave equation (11), provided the following relations
\[ \nabla \times \vec{E}_g = -\frac{4\pi G}{c_g^2} \frac{\partial \vec{H}_g}{\partial t} \] (14)
\[ \nabla \cdot \vec{H}_g = 0 \] (15)
\[ \vec{\nabla} \times \vec{j} = 0 \quad (16) \]

hold good in empty space. The relation (16) is certainly true in empty space where \( \vec{j} = 0 \). But the relations (14) and (15) have to be accepted if the existence of gravitational waves in empty space is accepted. We now recognize that we have arrived at the full set of Faraday-Maxwell-type field equations of gravity which may be represented by

\[ \vec{\nabla} \cdot \vec{E}_g = -4\pi G \rho = -\rho / \epsilon_0 g, \quad \text{by defining} \quad \epsilon_0 g = 1/4\pi G \quad (17) \]

\[ \vec{\nabla} \times \vec{B}_g = -\mu_0 g \vec{j} + (1/c_g^2)(\partial \vec{E}_g / \partial t), \quad \text{by defining} \quad \mu_0 g = 4\pi G / c_g^2 \quad (18) \]

\[ \vec{\nabla} \cdot \vec{B}_g = 0 \quad (19) \]

\[ \vec{\nabla} \times \vec{E}_g = -\partial \vec{B}_g / \partial t \quad (20) \]

where we have defined \( \vec{B}_g = (4\pi G / c_g^2) \vec{H}_g = \mu_0 g \vec{H}_g \) as the gravitomagnetic induction field in empty space and \( \mu_0 g \) as the gravitomagnetic permeability of empty space, in analogy with the electromagnetic case where magnetic induction \( \vec{B} \) in empty space is defined by \( \vec{B} = \mu_0 \vec{H} \), \( \mu_0 \) is the magnetic permeability of empty space. This definition of \( \mu_0 g \) ensures the relation \( c_g = (\epsilon_0 g \mu_0 g)^{-1/2} \) in complete analogy with its electromagnetic counterpart : \( c = (\epsilon_0 \mu_0)^{-1/2} \). As in electrodynamics, \( \mu_0 g \) would be the coefficient that would determine the strength of gravitomagnetic interaction or effects. Assuming \( c_g = c \), a typical value of \( \mu_0 g \) may be estimated at \( \mu_0 g = 9.33 \times 10^{-27} N.s^2/kg^2 \). This gives us a glimpse of the order of magnitude of \( \mu_0 g \), for we know not yet the exact value of \( c_g \) to calculate \( \mu_0 g \). It is this order of smallness of the the value of \( \mu_0 g \) that makes the strength of gravitomagnetic effects or interaction very negligible. From (18) it is now clear that very large mass currents or very rapidly fluctuating fields are required for production of gravitomagnetic field \( \vec{B}_g \) of appreciable strength. Therefore any search for gravitomagnetic effects should search for physical systems or processes where the mass current density is very large or the gravitational field fluctuation is very rapid. Such conditions or situations are available in certain astrophysical systems or events. The smallness of the value of \( \mu_0 g \) also explains why the existence of gravitomagnetic effects had escaped the human detection in spite of the long history of its study. By the way, it is to be noted that in 1893 Heaviside [8] speculated exactly these equations (17-20) with different notations and definitions of the various quantities involved.

To complete the dynamical picture we ask : What replaces the equation \( \vec{F} = m\vec{E}_g \) to describe the force on a particle of mass \( m \), when that particle moves with some velocity \( \vec{v} \) in given gravitoelectric and gravitomagnetic fields \( \vec{E}_g \) and \( \vec{B}_g \)? Because of the rich and detailed correspondence between electrical and gravitomagnetic phenomena uncovered above, one may at once suggest ( as Heaviside did ) the force on mass \( m \) is now

\[ \vec{F} = m\vec{E}_g + m\vec{v} \times \vec{B}_g \quad (21) \]
where $\vec{v}$ is the velocity of mass $m$ in this expression, in analogy with the Lorentz force Law of electrodynamics:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

(22)

where the symbols have their usual meanings. It is to be noted that the gravitational Lorentz force law (21) can also be derived from other Newtonian assumptions chosen by Feynman and Dyson [16] and following the path of Feynman’s proof of Maxwell’s equations as reported by Dyson [16] with his editorial comment on the derivations. The other way to arrive at the above Lorentz-Maxwell-type equations of gravity may be that of the Newtonian way Schwinger et al. [17] have chosen to infer the Lorentz-Maxwell equations of classical electrodynamics. From all these variant approaches which lead to the same equations as detailed above, we are convinced that the set of four equations (17)-(20) would form the basis of all Newtonian gravitoelectromagnetic phenomena. When combined with the gravitational Lorentz force equation (21) and Newton’s 2nd law of motion, these equations would provide a complete description of the dynamics of the interacting massive particles and gravitomagnetic fields.

To compare the field equations (17)-(20) with those predicted by general relativity (GR), we note the following approximations to the Maxwell-type field equations of GR in the parametrized-post-Newtonian (PPN) formalism [14], which as per the present definitions and notations may be written as

$$\nabla \cdot \vec{E}_g \approx -4\pi G \rho$$

(23)

$$\nabla \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

(24)

$$\nabla \cdot \vec{B}_g = 0$$

(25)

$$\nabla \times \vec{B}_g = \left(\frac{7}{2}\Delta_1 + \frac{1}{2}\Delta_2\right)\left(-\frac{4\pi G}{c^2} \rho \vec{v} + \frac{1}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t}\right)$$

(26)

where $\Delta_1$ and $\Delta_2$ are PPN parameters, $\rho$ is the density of rest masses in the local frame of the matter, $\vec{v}$ is the ordinary (co-ordinate) velocity of the rest mass relative to the PPN co-ordinate frame. In general relativity $\left(\frac{7}{2}\Delta_1 + \frac{1}{2}\Delta_2\right) \approx 4$ and so Eq.(26) can be rewritten as

$$\nabla \times \vec{B}_g \approx -\frac{16\pi G}{c^2} \rho \vec{v} + \frac{4}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t}$$

(27)

In empty space (where $\rho = 0$), these field equations reduce to the following equations:

$$\nabla \cdot \vec{E}_g = 0$$

(28)
\[ \vec{\nabla} \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t} \]  
(29)

\[ \vec{\nabla} \cdot \vec{B}_g = 0 \]  
(30)

\[ \vec{\nabla} \times \vec{B}_g = \frac{4}{c^2} \cdot \frac{\partial \vec{E}_g}{\partial t} \]  
(31)

As per our previous analysis, these Maxwell-type equations of GR imply that

\[ \epsilon_{0g,GR} = \frac{1}{4\pi G}, \quad \mu_{0g,GR} = \frac{16\pi G}{c^2}, \]  
(32)

which in turn imply that in the low velocity and weak field approximation of GR the speed of gravitational wave in empty space is \( c_{g,GR} = (\epsilon_{0g,GR}\mu_{0g,GR})^{-1/2} = c/2 \), a result not expected from a Lorentz covariant theory of gravitation. This value of \( c_g = c/2 \) can also be inferred from the wave equations that follow from the above equations (28)-(31) by taking the curl of (29) and utilizing Eqs.(28) and (31) we get the wave equation for the field \( \vec{E}_g \) in empty space as

\[ \vec{\nabla}^2 \cdot \vec{E}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{E}_g}{\partial t^2} = 0 \]  
(33)

where \( c_g = c/2 \). Similarly the wave equation for the field \( \vec{B}_g \) can be obtained by taking the curl of Eq.(31) and utilizing Eqs.(29) and (30) :

\[ \vec{\nabla}^2 \cdot \vec{B}_g - \frac{1}{c_g^2} \cdot \frac{\partial^2 \vec{B}_g}{\partial t^2} = 0 \]  
(34)

where again we get \( c_g = c/2 \). However, Peng in [18] has discussed a set of Maxwell-like equations that arise in the slow motion, weak field limits of Einstein’s field equations which in the present notation agrees with Newtonian gravitodynamic equations (17-20) with \( c_g = c \) [19, 20]. In [20] Peng’s equations have been utilized to predict the quantization of planetary revolution speeds which matches with the experimental data. It is to be noted that Ciufolini’s theoretical analysis of LAGEOS data is based on the gravitomagnetic field \( \vec{B}_g \) that can be obtained from Eq.(27) which differs from Peng’s corresponding equation by a factor of 4 [18, 19, 20] which is responsible for the above result \( c_g = c/2 \).

### 4 Immediate theoretical consequences of Interest

Newtonian gravitodynamics is very much analogous to Maxwell’s electrodynamics as revealed by the form of its equations. Therefore gravitational phenomena very much analogous to those of electromagnetic theory are not surprising to be revealed by this theory.
However few concepts and results of unconventional nature and importance may be discussed as under. Historians tell us that Newton was quite unhappy over the fact that his law of gravitation implies an action-at-a-distance interaction over very large distances such as that between the Sun and the Earth. But he was unable to resolve this problem [21]. To resolve Newton’s problem within the framework of Newtonian physics and in terms of potential functions of Newtonian gravitodynamics, we note that the homogeneous equations (19) and (20) admit of the solutions:

\[ \vec{B}_g = \vec{\nabla} \times \vec{A}_g, \quad \vec{E}_g = -\vec{\nabla} \cdot \Phi_g - \partial \vec{A}_g / \partial t \]  

(35)

where \( \Phi_g \) and \( \vec{A}_g \) represents respectively the gravitational scalar and vector potential of this theory. These potentials satisfy the inhomogeneous wave equations:

\[ \nabla^2 \cdot \Phi_g - \frac{1}{c_g^2} \frac{\partial^2 \Phi_g}{\partial t^2} = 4\pi G \rho = \rho / \epsilon_0 \]  

(36)

\[ \nabla^2 \cdot \vec{A}_g - \frac{1}{c_g^2} \frac{\partial^2 \vec{A}_g}{\partial t^2} = \frac{4\pi G}{c_g^2} \vec{j} = \mu_0 \vec{j} \]  

(37)

if the gravitational Lorentz [22] gauge condition

\[ \vec{\nabla} \cdot \vec{A}_g + \frac{1}{c_g^2} \frac{\partial \Phi_g}{\partial t} = 0 \]  

(38)

is imposed. These will determine the generation of gravitational waves by prescribed mass and mass current distributions. Particular solutions (in vacuum) are

\[ \Phi_g (\vec{r}, t) = -G \int \frac{\rho(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d\nu' \]  

(39)

\[ \vec{A}_g (\vec{r}, t) = -G \frac{c_g^2}{c^2} \int \frac{\vec{j}(\vec{r}', t')}{|\vec{r} - \vec{r}'|} d\nu' \]  

(40)

where \( t' = t - |\vec{r} - \vec{r}'| / c_g \) is the retarded time. These are called the retarded potentials. Thus we saw that retardation in Newtonian gravity is possible in flat space and time in the same procedure as we adopt in electrodynamics. Had Einstein seen these possibilities, his confidence in the correctness of Newton’s theory would not have been shaken and his approach to relativistic gravity would have been different with the implication that the history and the character of gravitation might have been different from what we know today. Now let us come to Maxwell’s puzzle over the gravitational field energy density. Actually this puzzle is not a real puzzle, but a guess-work of Maxwell, since the actual calculation done by electromagnetic procedure yields the energy density of gravito-electric
(i.e. the electric-type component of gravity) and gravitomagnetic (i.e. the magnetic-type component) field, respectively as

\[(i) \quad u_{ge} = -\frac{1}{2} \epsilon_{0,g} \vec{E}_g \cdot \vec{E}_g, \quad (ii) \quad u_{gm} = -\frac{1}{2\mu_{0,g}} \vec{B}_g \cdot \vec{B}_g \]

(41)

where \(\epsilon_{0,g} = 1/4 \pi G\) and \(\mu_{0,g} = 4 \pi G/c_g^2\) and the total field energy density is given by a sum of the above two, i.e.

\[u_{field} = u_{ge} + u_{gm} \]

(42)

For a particle at rest, i.e., in gravitostatics, the only contribution to its gravitational field energy is that due to the gravito-electric field. In gravitostatics, it is easy to compute the gravitational or gravito-electric self energy of a sphere of radius \(R\) and mass \(M\) with uniform mass density by using Eq.(41i), which comes out as:

\[U_g = -\frac{1}{2} \epsilon_{0,g} \int_0^\infty E_g^2 4\pi r^2 dr = -\frac{3GM^2}{5R} \]

(43)

The result (43) is in complete agreement with the Newtonian result. It is to be noted that Visser [23] used exactly this definition of gravitational field energy density in his classical model for the electron. Such a definition of the field energy of gravity has the advantage of describing the correct nature of gravitation on quantization because in analogy with electromagnetic theory, the present theory will eventually lead to a gauge field of spin 1 and the spin 1 gauge fields having positive and definite field energy, on quantization, as we know lead to a repulsive force field for identical charges of such fields. It is due to this reason Gupta [24] and Feynman [25] suggested rejection of any spin 1 gauge theory of gravity with field energy being positive and definite as such fields do not account for the observed nature of gravitational interaction. Since general relativity speaks of spin 2 tensor theory of gravity and Newtonian theory as developed here implying spin 1 vector theory of gravity very much analogous to the electromagnetic theory, one of the fundamental questions raised in the Dicke Framework (see in [26]):

“What types of fields, if any, are associated with gravitation - scalar fields, vector fields, tensor fields, ..........?”

requires immediate attention to understand the riddle of gravitation better. The very possibility of a Newtonian gravitoelectromagnetic (GEM) theory also raises another important question: “Does general relativity obey the correspondence principle?” When it does, (as per a comparison of Peng’s [18] work with the present one assuming \(\epsilon_g = c\)) do the predictions of gravitomagnetic phenomena agree with experimental results [2, 5]?
5 Analysis of Earth Satellite (LAGEOS) Data

As an immediate application of Newtonian gravitodynamics, let us consider the analysis of Earth satellite (LAGEOS and LAGEOS2) data \[2, 5\]. In the slow motion and weak field limit of GR, the gravitomagnetic induction field \(\vec{B}_{g,GR}\) associated with the spin-angular momentum \(\vec{J}\) of a spherical spinning body (such as the Earth) can be estimated from (27) following the standard electromagnetic procedure of estimation of magnetic field generated by localized current distributions \[15\]. The result is that

\[
\vec{B}_{g,GR} = 2G[\vec{J}r^2 - 3(\vec{J} \cdot \vec{r})\cdot \vec{r}]/(c^2r^5) \tag{44}
\]

where \(\vec{r}\) is the position vector of the field point from the center of the Earth. For a satellite orbiting the Earth having a gravitomagnetic field presumed at that given by (44), the Lense-Thirring (LT) nodal precession is estimated \[1\] at

\[
\dot{\Omega}_{LT}^{GR} = \frac{2G\vec{J}}{c^2a^3(1 - e^2)^{3/2}} \tag{45}
\]

where \(a\) and \(e\) respectively represents the semi-major axis and the eccentricity of the satellite orbit. In 2004 Ciufolini and Pavlis \[5\] measured the Lense-Thirring nodal precession at

\[
\dot{\Omega}_{LT}^{experiment} = (0.99 \pm 0.10)\dot{\Omega}_{LT}^{GR} \tag{46}
\]

But the above interpretation has not taken care of the contribution arising out of the Newtonian gravitomagnetic field, which as we saw here a theoretical possibility. By the same procedure, one can estimate the Newtonian gravitomagnetic field of Earth at

\[
\vec{B}_{g,New} = G[\vec{J}r^2 - 3(\vec{J} \cdot \vec{r})\cdot \vec{r}]/(2c_g^2r^5) \tag{47}
\]

and this field would then cause an LT-type nodal precession at

\[
\dot{\Omega}_{LT}^{New} = \frac{G\vec{J}}{2c_g^2a^3(1 - e^2)^{3/2}} = \left(\frac{c}{2c_g}\right)^2 \dot{\Omega}_{LT}^{GR} \tag{48}
\]

To see the implications of (48) we consider the following two cases of interest, viz.,

(a) if \(c_g = c\), then \(\dot{\Omega}_{LT}^{New} = \frac{1}{4} \dot{\Omega}_{LT}^{GR} = 25\%\) of \(\dot{\Omega}_{LT}^{GR}\), and

(b) if \(c_g = c/2\), then \(\dot{\Omega}_{LT}^{New} = \dot{\Omega}_{LT}^{GR} = 100\%\) of \(\dot{\Omega}_{LT}^{GR}\).

It is to be noted that the exact value of \(c_g\) in the slow motion and weak field approximation of GR is not unique as seen in sec.3 of this paper. Because of the uncertainties in the exact value of \(c_g\) and the possible existence of a Newtonian gravitomagnetic field,
the interpretation of the LAGEOS experimental result [5] as a pure general relativistic effect with 99% claimed accuracy of GR is doubtful. It may be noted that the 2004 claim of Ciufolini and Pavlis [5] had been earlier suspected by Iorio [27, 28] from other possible sources of gravitational error which of course have already been addressed in the authors’ 2005 work [5] as completely unfounded. Nonetheless, Iorio [29] still suspects the correctness of the latest results [5]. However the most likely Newtonian gravitomagnetic error introduced by the result (a) considered above for the case $c_g = c$ seems to be large enough to be of some concern let alone the possibility $c_g = c/2$ leading to the result (b) which appears to be more serious one.

6 Concluding Remarks

The possibility of a Newtonian gravitomagnetic field can not be ruled out in principle and therefore should be taken care of in the current and future measurements of gravitomagnetic phenomena. This possibility adds to our understanding of Newtonian gravity in its entirety and raises some fundamental questions of importance in the description of gravitational phenomena which may deserve certain attention. The analysis and the interpretation of the earth satellite (LAGEOS and LAGEOS2) data [5] which ignore(s) the Newtonian gravitomagnetic contribution to the observed effects may be reconsidered or re-examined for an unambiguous explanation of the experimental observation.

Acknowledgments The author is very much grateful to Prof. Lorenzo Iorio, Viale Unità di Italia, Bari, Italy for his interest in this work, for providing some valuable references on the topic, for encouraging and valuable suggestions and for permitting the author to mention him in the acknowledgements. The author is also very much indebted to Prof. N. Barik, Prof. L. P. Singh, Prof. N. C. Mishra, Prof. S. Jena, Dr. P. Khare, Dr. K. Moharana all of Department of Physics, Utkal University, Bhubaneswar, India and N. K. Behera of U. N. College, Soro Balasore, India, for discussions and helpful suggestions. The help received from the Institute of Physics, Bhubaneswar for using the library and computer facility for this work is gratefully acknowledged by the author.

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