Stellar-mass black holes in star clusters: implications for gravitational wave radiation

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ABSTRACT
We study the dynamics of stellar-mass black holes (BH) in star clusters with particular attention to the formation of BH–BH binaries, which are interesting as sources of gravitational waves (GW). In the present study, we examine the properties of these BH–BH binaries through direct $N$-body simulations of star clusters using the NBODY6 code on graphical processing unit platforms. We perform simulations for star clusters with $\leq 10^5$ low-mass stars starting from Plummer models with an initial population of BHs, varying the cluster mass and BH-retention fraction. Additionally, we do several calculations of star clusters confined within a reflective boundary mimicking only the core of a massive star cluster which can be performed much faster than the corresponding full cluster integration. We find that stellar-mass BHs with masses $\sim 10 \, M_\odot$ segregate rapidly ($\sim 100$ Myr time-scale) into the cluster core and form a dense subcluster of BHs within typically $0.2–0.5$ pc radius. In such a subcluster, BH–BH binaries can be formed through three-body encounters, the rate of which can become substantial in dense enough BH cores. While most BH binaries are finally ejected from the cluster by recoils received during superelastic encounters with the single BHs, few of them harden sufficiently so that they can merge via GW emission within the cluster. We find that for clusters with $N \gtrsim 5 \times 10^4$, typically 1–2 BH–BH mergers occur per cluster within the first $\sim 4$ Gyr of cluster evolution. Also for each of these clusters, there are a few escaping BH binaries that can merge within a Hubble time, most of the merger times being within a few Gyr. These results indicate that intermediate-age massive clusters constitute the most important class of candidates for producing dynamical BH–BH mergers. Old globular clusters cannot contribute significantly to the present-day BH–BH merger rate since most of the mergers from them would have occurred much earlier. On the other hand, young massive clusters with ages less that 50 Myr are too young to produce significant number of BH–BH mergers. We finally discuss the detection rate of BH–BH inspirals by the ‘Laser Interferometer Gravitational-Wave Observatory’ (LIGO) and ‘Advanced LIGO’ GW detectors. Our results indicate that dynamical BH–BH binaries constitute the dominant channel for BH–BH merger detection.

Key words: black hole physics – gravitational waves – scattering – stellar dynamics – methods: $N$-body simulations – galaxies: star clusters.

1 INTRODUCTION

Star clusters, e.g. globular clusters (GC), young and intermediate-age massive clusters (IMC) and open clusters harbour a large over-density of compact stellar remnants compared to that in the field by virtue of their high density and the mass segregation of the compact remnants towards the cluster core (Hut & Verbunt 1983). These compact stars, which are neutron stars (NS) and black holes (BH), are produced by the stellar evolution of the most massive stars within the first $\sim 50$ Myr after cluster formation. These compact stars, being generally heavier than the remaining low-mass stars of the cluster, segregate quickly (within one or a few half-mass relaxation times) to the cluster core, forming a dense subcluster of compact stars. Such a compact-star subcluster is of broad interest as it efficiently produces compact-star binaries through dynamical encounters (Hut & Verbunt 1983; Hut et al. 1992b), e.g. X-ray binaries and NS–NS and BH–BH binaries, which are of interest for a

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wide range of physical phenomena. For example, X-ray binaries are the primary sources of GC X-ray flux, while mergers of tight NS–NS binaries (through emission of gravitational waves (GWs)) is the most likely scenario for the production of short-duration gamma-ray bursts (GRBs), and both NS–NS and BH–BH mergers are very important sources of GWs detectable by future GW observatories like ‘Advanced LIGO’ (AdLIGO) and ‘LISA’ (Amaro-Seoane et al. 2007). Double-NS systems are also interesting because they could be observable as double-pulsar systems (Ransom 2008), allowing important tests of General Relativity (Kramer 2008). In the present work, we investigate the dynamics of stellar-mass BHs in star clusters, with particular emphasis on dynamically formed BH–BH binaries. Such binaries are strong sources of GWs as they spiral in through GW radiation, a process detectable out to several thousand Mpc distances.

Tight BH binaries that can merge within a Hubble time can also be produced in the galactic field from tight stellar binaries, which result from stellar evolution of the components (e.g. Bulik & Belczynski 2003; Belczynski et al. 2007; O’Shaughnessy, Kalogera & Belczynski 2007; O’Shaughnessy et al. 2008). However, Belczynski et al. (2007) have shown with revised binary-evolution models that the majority of potential BH–BH binary progenitors actually merge because of common-envelope (CE) evolution which occurs when any of the binary members crosses the Hertzsprung gap. They found that this reduces the merger rate by a factor as large as ~500 and the resulting AdLIGO-detection rate of BH–BH binary mergers in the Universe from primordial binaries is only ~2 yr$^{-1}$, subject to the uncertainties of the CE-evolution model that has been incorporated. Note that the corresponding NS–NS merger rate, as the above authors predict, is considerably higher, ~20 yr$^{-1}$. In view of the above result, the majority of merging BH binaries are possibly those that are formed dynamically in star clusters.

As studied by several authors earlier (Merritt et al. 2004; Mackey et al. 2007), BHs, formed through stellar evolution, segregate into the cluster core within ~0.3 pc and form a subcluster of BHs, where the density of BHs is large enough that BH–BH binary formation through three-body encounters becomes important (Heggie & Hut 2003). These dynamically formed BH binaries then ‘harden’ through repeated superelastic encounters with the surrounding BHs (Heggie 1975; Banerjee & Ghosh 2006). The binding energy of the BH binaries released is carried away by the single and binary BHs involved in the encounters. This causes the BHs and the BH binaries to get ejected from the BH core to larger radii of the cluster, and they heat the cluster while sinking back to the core through dynamical friction (Mackey et al. 2007). Most of the energy of the sinking BHs is deposited in the cluster core as the stellar density is much higher there. As the BH binaries harden, the encounter-driven recoil becomes stronger and finally the recoil is large enough that the encountering single BH and/or the BH binary escape from the cluster (see Section 3, also Benacquista 2002). Because of the associated mass loss from the cluster core, this also results in cluster heating. These heating mechanisms result in an expansion of the cluster, as studied in detail by several authors, e.g. Merritt et al. (2004) and Mackey et al. (2007). Mackey et al. (2007) found notable agreement between the core expansion as obtained from their N-body simulations and the observed age–core-radius correlation for star clusters in the Magellanic Clouds.

The dynamics of stellar-mass BHs and formation of BH binaries in star clusters and the resulting rate of GW emission from close enough BH binaries have been studied by several authors using N-body integrations (Portegies Zwart & McMillan 2000), numerical three-body scattering experiments (Gültekin, Miller & Hamilton 2004) or Monte Carlo methods (O’Leary et al. 2006). Portegies Zwart & McMillan (2000) considered the rate of mergers of escaping BH binaries from various stellar systems, e.g. massive GCs, young populous clusters and galactic nuclei. They estimated the merger rates within 15 Gyr (their adopted age of the Universe) by assuming the binding-energy ($E_b$) distribution of the escaping BH binaries to be uniformly distributed in log $E_b$, as inferred from simulations of $N$ ≈ 2000 or 4000 star clusters (see Portegies Zwart & McMillan 2000 for details). Considering the space densities of the different kinds of star clusters, they found that while for LIGO the corresponding total (i.e. contribution from all types of star clusters) detection rate is negligible, it can be as high as ~1 day$^{-1}$ for AdLIGO. Gültekin et al. (2004) performed sequential numerical integrations of BH-binary–single-BH close encounters in a uniform stellar background, where, in between successive encounters, the BH binaries were evolved due to GW emission. From such simulations, these authors studied the growth of BHs through successive BH binary mergers for the first time. In a more self-consistent study of the growth of BHs and the possibility of formation of intermediate mass BHs through successive BH binary mergers, O’Leary et al. (2006) used the Monte Carlo approach and considered ‘pure’ BH clusters that are dynamically detached from their parent clusters which can be expected to form due to mass stratification instability (Spitzer 1987). These authors considered dynamically formed BH binaries from three-body encounters in such BH clusters utilizing theoretical cross-sections of three-body binary formation (see O’Leary et al. 2006, and references therein). Furthermore, they included both binary–single and binary–binary encounters. Considering the subset of BH binaries formed in the BH clusters that merge within a Hubble time, they determined typically a few AdLIGO detections per year for old GCs.

While such results are remarkable and promising, a more detailed study of the dynamics of stellar-mass BHs in star clusters with a realistic number of stars is essential. As the cross-sections of different processes governing the dynamics of BH binaries, viz. three-body binary formation, binary–single star encounters and ionization, have different dependencies on the number of stars $N$ of the cluster, an extrapolation to much different $N$ can be problematic. Hence, it is important to consider clusters with values of $N$ appropriate for massive clusters or GCs. Also, exact treatments of the various dynamical processes are crucial for realistic predictions of BH–BH merger rates. Finally, a more careful study of the different dynamical processes leading to the formation and evolution of BH binaries in star clusters is needed to understand better under which conditions tight inspiralling BH binaries can be formed dynamically from a star cluster.

In the present work, we make a detailed study of the dynamics of BH–BH binaries formed in a BH subcluster, as introduced above. In particular, we investigate whether hard enough BH binaries that can merge via gravitational radiation in a Hubble time within the cluster or after getting ejected from the cluster can be formed in such a subcluster. To that end, we perform direct N-body integrations of concentrated star clusters (half-mass radius $r_h \lesssim 1$ pc) consisting of $N < 10^5$ low-mass stars in which a certain number of stellar-mass BHs is added, representing a star cluster with an evolved stellar population.

The present paper is organized as follows. In Section 2, we describe our simulations in detail. We discuss the various elements and assumptions of the simulations and summarize all the runs that we perform (Section 2.2). We also discuss the use of a reflective boundary to simulate only the core of a star cluster (Section 2.3). In Section 3, we discuss our results with particular emphasis on
the dynamical BH binaries formed during the simulations. We discuss the BH–BH mergers that occur within the clusters and also the merger time-scales of the BH binaries that escape from the clusters (Section 3.2) and obtain the distributions of merger times for both cases (Section 3.3). Finally, in Section 4 we interpret our results in the context of different types of star clusters that are observed and provide estimates of BH–BH merger detection rates.

2 MODELS

To study the dynamics of BHs in star clusters, we perform direct N-body simulations using NBODY6 with star clusters in which a certain number of BHs are added initially. The initial density distribution follows a Plummer model with half-mass radius \( r_h \leq 1 \) pc consisting of \( N \leq 10^5 \) low-mass main-sequence stars in the mass range \( 0.5 M_\odot \leq m \leq 1.0 M_\odot \). Observed half-mass radii for GCs and open clusters are usually larger, but as we expect the clusters to expand considerably due to the heating caused by the encounters in the BH core (see Section 1), we begin with more concentrated clusters.

BHs formed through stellar evolution are typically within the mass range \( 8 M_\odot \lesssim M_{BH} \lesssim 12 M_\odot \) for stellar progenitors with solar-like metallicity (Casares 2007; Belczynski et al. 2009). The exact form of the BH mass function is still debated and depends on the metallicity of the parent stellar population and the stellar wind mass-loss model (Belczynski et al. 2009). In a dense stellar environment, it is further affected by the frequent stellar mergers and binary coalescence (Belczynski et al. 2008). In view of the uncertainty of the BH mass function, we consider only equal-mass BHs, with \( M_{BH} = 10 M_\odot \) in this work. Such a value was also assumed by earlier authors, e.g. Portegies Zwart & McMillan (2000) and Benacquista (2002).

A specified number of BHs are added to each cluster, and we assume that the BHs follow the same spatial distribution as the stars initially. The number of BHs added depends on the BH-retention fraction. We explore both full retention and the case where half of the BHs are ejected from the cluster by natal kicks. With such a cluster, we mimic the epoch at which massive stars have already evolved and produce BHs. While such a cluster is not completely representative of the more realistic case in which the BHs are produced from stellar evolution during the very early phases, it still serves the purpose of studying the dynamics of the BHs since this becomes important only later after segregation of the BHs into the cluster core (see Section 3).

We perform our simulations using NBODY6 code (Aarseth 2003) enabled for use with graphical processing units (GPUs). We have made arrangements in NBODY6 to put a specified number \( N_{BH} \) of BHs of a given mass \( M_{BH} \) by picking stars randomly from the cluster and replacing them by the BHs. Necessary rescalings have been done to compensate for the excess mass gained by the cluster. These runs are performed on NVIDIA 9800 GTX2/GTX 280/GTX 285 GPU platforms, located at the Argelander-Institut für Astronomie (AIfA), University of Bonn, Germany, with the GPU enabled version of NBODY6 (see Sverre Aarseth’s Homepage: http://www.sverre.com).

2.1 BH–BH mergers

To evolve the BH–BH binaries due to GW emission, Peters’ formula (Peters 1964) is utilized in NBODY6 (also see Baumgardt et al. 2006), which provides approximate orbit-averaged rates of evolution of binary semimajor axis \( a \) and eccentricity \( e \) due to GW emission. According to this formula, the merger time \( T_{\text{merge}} \) of an equal-mass BH–BH binary due to GW emission is given by

\[
T_{\text{merge}} = 150 \text{Myr} \left( \frac{M_\odot}{M_{BH}} \right)^3 \left( \frac{a}{R_\odot} \right)^4 (1 - e^2)^{3/2}.
\]

(1)

Peters’ formula is limited up to the mass quadrupole terms of the radiating system. Close to the merger, higher order Post-Newtonian (PN) terms become important, which modifies the above value of the merger time (Blanchet 2006). In the present work, however, we are primarily interested in an overall statistics of the merger events rather than the detailed orbit of a single BH–BH merger, so that the higher order PN corrections are not crucial. Hence, we restrict ourselves to Peters’ formula. Moreover, the orbit of a tight BH–BH binary gets modified by dynamical encounters, which will anyway modify its merger time to a much larger extent compared to that for the unperturbed binary with the higher PN terms.

In NBODY6, the orbital evolution of compact binaries is also considered when the binary is inside a hierarchy. Thus, a tight BH binary will continue to shrink even if it acquires an outer member forming a hierarchical triple, which can often happen due to the strong focusing effect of BH binaries. Also, energy removal due to GW (bursts) during a close hyperbolic passage between two BHs is considered in the code.

Numerical simulations of BH–BH mergers (see Hughes 2009 for an excellent review) indicate that for unequal-mass BHs or even for equal-mass BHs with unequal spins, the merged BH product acquires a velocity kick of typically 100 km s\(^{-1}\) or more due to an asymmetry in momentum outflow from the system, associated with the GW emission. Although we consider equal-mass BHs, the merger kicks associated with the inequality of the spins of the merging BHs would be generally sufficient to eject merged BHs from the cluster. Therefore, in our simulations we provide an arbitrarily large kick of 150 km s\(^{-1}\) immediately after a BH–BH merger, to make sure that the merged BH escapes.

2.2 Computations

To study the rate of BH–BH mergers coming from a star cluster, we perform simulations of isolated star clusters with single low-mass stars and BHs as mentioned above. The formation of the BH core through mass segregation and its dynamics remains largely unaffected by the presence of a tidal field, which mainly affects stars near the tidal boundary. While the enhanced removal of stars accelerates the core-collapse of the cluster (see e.g. Spitzer 1987, chapter 3), the latter is more strongly enhanced by the collapse of the BHs themselves (~100 Myr time-scale, see below), so that the effect of any tidal field is only of the second order. Hence, isolated clusters are good enough for our purposes. Further, for simplicity, we do not take into account primordial binaries in this initial study. The primordial binary fraction in GCs and their period distribution are still widely debated (Ashman & Zepf 1998; Bellazzini et al. 2002; Sommariva et al. 2009). The presence of a primordial binary population can however significantly influence the dynamics of stellar-mass BHs which we defer for a future more detailed study.

For solar-like metallicity, Eggleton’s stellar-evolution model (Eggleton, Fitchett & Tout 1989, adapted in NBODY6) gives about \( N_{BH} \approx 200 \) BHs for a cluster with \( N = 10^5 \) stars following a Kroupa initial mass function (IMF) (Kroupa 2001). The above \( N_{BH} \) (or its proportion with \( N \)) is thus an upper limit to the number of BHs in a GC that corresponds to a full retention (i.e. no or low natal kicks for all BHs).

We perform two simulations with \( N = 4.5 \times 10^4 \) and \( N_{BH} = 80 \), two runs with \( N = 6.5 \times 10^4, N_{BH} = 110 \), i.e. about full BH

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retention. Two of the above runs are repeated with half the above $N_{\text{BHs}}$. Also, one run with $N = 5 \times 10^3$ and excess $N_{\text{BH}} = 200$, appropriate for a top-heavy IMF has been performed. Finally, we do two runs with $N = 10^5$ with $N_{\text{BH}} = 80$ (about 50 per cent retention fraction) and 200 (full retention). All the clusters consist of low-mass stars between 0.5 $M_\odot \lesssim m \lesssim 1.0$ $M_\odot$ with a Kroupa IMF, and the BHs have $M_{\text{BH}} = 10 M_\odot$, as discussed in detail in the beginning of the section.

In addition to these systems, we also study stellar-mass BHs in clusters with smaller $N$ representing open clusters, in order to estimate the lower limit in cluster mass for the occurrence of BH–BH mergers. We perform 10 runs with $N = 5 \times 10^3$, $N = 10^4$ and $N = 2.5 \times 10^4$ each with full BH retention (i.e. $N_{\text{BH}} = 12, 20$ and 50, respectively). Results of all our runs are summarized in Table 1.

2.3 Simulation of a GC core: reflective boundary

We also perform simulations with a smaller number of stars and BHs that are confined within a reflecting spherical boundary. With such a dynamical system one can mimic the core of a massive cluster, where the BHs are concentrated after mass segregation. The advantage of this approach is that one can simulate the evolution of a massive cluster with much fewer stars. We simulate $N = 3000–4000$ stars packed within 0.4 pc. This provides a stellar density of $\sim 10^4 M_\odot$ pc$^{-3}$, appropriate for the core density of a massive cluster. The initial BH population is taken to be $N_{\text{BH}} \approx 100$ or 200, representing half or full BH retention, respectively, of an $N = 10^5$ star cluster. In this way, the simulation of the core of a massive GC can be performed much faster than the simulation of the whole cluster.

To implement the reflective boundary, we simply consider all outgoing stars beyond the reflective sphere $R_s$ and reflect them elastically. This is done in the main integration loop. For these stars, we compute the force polynomials (Aarseth 2003) separately (using the CPU) which makes the code slower by factors of 2–3 depending on the stellar density. While the behaviour of an $N$-body code in presence of a reflective boundary needs to be studied in more detail, the present implementation seems to be fairly stable with single stars and BHs, with energy errors marginally larger than that for a free cluster. (In fact, in the NBODY6 package, some routines are already available for optionally implementing the reflective boundary.) For stars and BHs beyond a pre-assigned speed $v_{\text{esc}}$, representing the escape speed of the host cluster, we do allow them to escape through the reflective boundary.

It is of course important to note that a cluster simulation within a reflective boundary cannot represent the evolution of a free cluster, in particular, since the cluster expands due to the BH heating which continually reduces the stellar density in the core. With the reflective boundary, the stellar density still decreases because of the escape of stars with speed $v > v_{\text{esc}}$, but it generally does not mimic the cluster expansion. The cooling effect of these escaping stars also helps to inhibit the runaway heating of the cluster due to the binding energy released by binary–star/binary–binary encounters. The escaping single BHs and BH binaries are particularly efficient in this mass-loss cooling due to their larger masses. Fig. 1 shows a typical example from one of our runs (R3K180 of Table 1, see below) in which the behaviour of the virial coefficient $Q = V/T$ ($T = $ total kinetic energy of the system, $V = $ total potential energy) is shown. While $Q$ initially increases quickly indicating rapid heating of the cluster, the escape rate of stars and BHs also increases due to the increased velocity dispersion. The latter effect in turn increases

| Model name | $N$ | $N_{\text{sim}}$ | $r_s(0)$ or $R_s$ (pc) | $N_{\text{BH}}(0)$ | $N_{\text{mrg}}$ | $t_{\text{mrg}}$ (Myr) | $N_{\text{esc}}$ | $R_{\text{AdLIGO}}$ |
|------------|-----|-----------------|-----------------------|-------------------|----------------|-----------------|-------------|----------------|
| Isolated clusters |
| C5K12 | 5000 | 10 | 1.0 | 12 | 0 | -- | -- | -- |
| C10K20 | 10000 | 10 | 1.0 | 20 | 0 | -- | -- | -- |
| C25K50 | 25000 | 10 | 1.0 | 50 | 0 | -- | -- | -- |
| C50K80 | 45000 | 1 | 1.0 | 80 | 1 | 698.3 | 3.10 | 28(±14) |
| C50K80.1 | 45000 | 1 | 0.5 | 80 | 2 | 217.1, 236.6 | 3.21 | 35(±15) |
| C50K40.1 | 45000 | 1 | 0.5 | 40 | 0 | -- | -- | -- |
| C50K200 | 50000 | 1 | 1.0 | 200 | 2 | 100.8, 467.8 | 0.00 | 14(±10) |
| C65K110 | 65000 | 1 | 1.0 | 110 | 1 | 314.6 | 4.21 | 35(±15) |
| C65K110.1 | 65000 | 1 | 0.5 | 110 | 0 | -- | -- | -- |
| C65K55.1 | 65000 | 1 | 0.5 | 55 | 1 | 160.5 | 1.00 | 14(±10) |
| C100K80 | 100000 | 1 | 1.0 | 80 | 2 | 219.4, 603.2 | 5.21 | 42(±15) |
| C100K200 | 100000 | 1 | 1.0 | 200 | 0 | -- | -- | -- |
| Reflective boundary |
| R3K180 | 3000 | 1 | 0.4 | 180 | 1 | 1723.9 | 5.31 | 35(±15) |
| R4K180A | 4000 | 1 | 0.4 | 180 | 1 | 3008.8 | 2.21 | 21(±12) |
| R4K180B | 4000 | 1 | 0.4 | 180 | 2 | 100.2, 1966.5 | 2.10 | 28(±14) |
| R3K100 | 3000 | 1 | 0.4 | 100 | 2 | 3052.8, 3645.9 | 1.10 | 18(±10) |
| R4K100A | 4000 | 1 | 0.4 | 100 | 2 | 104.4, 814.2 | 3.31 | 28(±14) |
| R4K100B | 4000 | 1 | 0.4 | 100 | 1 | 1135.3 | 3.33 | 28(±14) |

Note. The meaning of different columns is as follows. Column (1): identity of the particular model – similar values with different names (ending with A, B etc.) imply computations repeated with different random seeds. Column (2): total number of stars $N$. Column (3): number of simulations $N_{\text{sim}}$ with the particular cluster, Column (4): initial half-mass radius of the cluster $r_s(0)$ (isolated cluster) or radius of reflective sphere $R_s$. Column (5): initial number of BHs $N_{\text{BH}}(0)$. Column (6): total number of BH–BH binary mergers within the cluster $N_{\text{mrg}}$. Column (7): the times $t_{\text{mrg}}$ corresponding to the mergers. Column (8): number of escaped BH-pairs $N_{\text{esc}}$ – the three values of $N_{\text{esc}}$ are those with $T_{\text{mrg}} \lesssim 3$ Gyr, 1 Gyr and 100 Myr, respectively. Column (9): BH–BH merger rate $R_{\text{AdLIGO}}$ detected by AdLIGO assuming that the corresponding model cluster has a space density of $\rho_s = 3.5 h^3$ Mpc$^{-3}$ (see Section 4.1).
the cooling rate so that $Q$ finally tends to flatten to a value for which the dynamical heating is balanced by the escape cooling of the cluster. For the reflective boundary clusters used in our simulations (see above), the initial velocity dispersion of the stars within the reflective boundary is chosen to be $\sigma_0 \approx 7\, \text{km s}^{-1}$ and escape speed $v_{\text{esc}} \approx 24\, \text{km s}^{-1}$. Typically, during each run, the virial coefficient $Q$ increases by a factor of about 6.5 and most of the growth takes place during the first few hundred Myr, as in Fig. 1. As $\sigma \sim Q^{1/2}$, it increases to $\approx 17\, \text{km s}^{-1}$, which is typical for the central velocity dispersion of a massive cluster.

In a reflective boundary cluster, the BH binaries start forming immediately after the start of the integration since the system already initiates with a BH core. We perform a set of six runs with reflective boundary clusters (see Table 1). We shall discuss results of our reflective boundary simulations in Section 3.

3 RESULTS: BLACK HOLE BINARY MERGERS AND ESCAPERS

In this section, we discuss the results of the simulations introduced in Section 2.2. As already mentioned, we focus on tight BH–BH binaries both within the cluster and the escaped ones that can merge within a few Gyr. Table 1 provides an overview of the results of our simulations for both isolated and reflective clusters. It shows the number of BH–BH mergers within the cluster for each computation and the corresponding merger times. Also, for each computation, the numbers of escaped BH–BH binaries that merge within 3 Gyr are shown. For convenience of discussion, we utilize one of these models, viz. the one with identity C50K80 (see Table 1) – all others generally possess similar properties.

Fig. 2 (top panel) demonstrates the mass segregation of the BHs in the cluster which takes about 50 Myr. Before the BHs segregate to within about 0.3 pc of the cluster centre, the BH density is small so dynamical encounters among the BHs are not significant and $N_{\text{BH}}$ remains constant. As the BHs segregate within about 0.3 pc of the cluster core, the BH density of this subcluster becomes high enough to form BH–BH binaries through three-body encounters. Once BH–BH binaries start forming, single and BH binaries begin to escape from the BH core by the recoils received due to repeated superelastic encounters between the single and binary BHs (see Section 1). In Fig. 2 (top panel), one can clearly distinguish the two phases of the BH subsystem – the initial segregation phase and the BH-core formation, the radial positions of the BHs in the latter phase being scattered outwards due to the recoils. The decrease of $N_{\text{BH}}$ during this phase is also shown in Fig. 2 (bottom panel).

The time $t_{\text{seg}}$ taken by the BHs to segregate to the cluster core to form the BH subcluster is essentially the core-collapse time for the initial BH cluster. $t_{\text{seg}}$ is given by (see e.g. Spitzer (1987), chapter 3):

$$t_{\text{seg}} = \frac{\langle m \rangle}{M_{\text{BH}}} t_{\text{cc}},$$

where, $t_{\text{cc}}$ is the core-collapse time of the host star cluster itself (i.e. without the BHs) and $\langle m \rangle$ is its mean stellar mass. According to numerical experiments by several authors (e.g. Baumgardt, Hut & Heggie 2002), a Plummer cluster takes about $t_{\text{cc}} \approx 15t_{\text{cc}}(0)$ to reach the core-collapse stage, where $t_{\text{cc}}(0)$ is the initial half-mass relaxation time of the Plummer cluster. For the above example, $t_{\text{cc}}(0) \approx 73 \, \text{Myr}$ (see equations (2-63) of Spitzer 1987) and $\langle m \rangle \approx 0.6$ which gives $t_{\text{seg}} \approx 65 \, \text{Myr}$ from equation (2). This roughly agrees with the formation time of the central dense BH cluster as observed in the $N$-body integration (see Fig. 2).

3.1 Dynamical black hole–black hole binaries

A particular dynamically formed BH–BH binary can initially be very wide, sometimes with semimajor axis $a$ more than $5000\, R_\odot$. However, it then shrinks very rapidly due to repeated superelastic
encounters with the surrounding BHs. The collisional hardening rate is given by \( \dot{a} \propto a^2 \) (see Banerjee & Ghosh 2006, and references therein). The variation in \( a \) and \( e \) is random due to the stochastic nature of the encounters, but the binary hardens on average (see e.g. Heggie & Hut 2003, Chapters 19 and 21) and the overall hardening rate decreases with \( a \). These encounters include both flybys and exchanges with single BHs. When the binary is wide, the recoils it receives are low, so that it usually remains in the cluster. However, as it becomes harder, the recoils become stronger, and the binary is ejected from the BH core, but returns to the BH core due to dynamical friction (see Section 1). Finally, the recoil becomes strong enough that the BH binary escapes from the cluster (see also Benacquista 2002).

For equal-mass binary–single star encounters, 40 percent of the binding energy of the binary is released per close encounter on average [Spitzer 1987, equations (6–25); Hut, McMillan & Romani 1992a]. This leads to the following expression for the average recoil velocity of a BH binary due to encounters with single BHs:

\[
\langle v^{rec}_{esc} \rangle = 6.75 \times 10^{-2} \frac{GM_{BH}}{a},
\]

(3)

Hence, the BH binary can be ejected due to recoil from a cluster with escape velocity \( v_{esc} \) for \( a < a_{esc} \), where \( a_{esc} \) is obtained by setting the left-hand side of equation (3) to \( v^{2}_{esc} \). For the above Plummer cluster, we then get from the value of its central potential (see Heggie & Hut 2003, table 8.1) \( a_{esc} \approx 500 \, R_{\odot} \), which is shown as solid line in Fig. 3. The escaped binaries in our calculations are generally found to be harder than \( a_{esc} \) except for a few systems, which escape within triple BHs.

Fig. 3 provides an overall impression of the hardening of the BH binaries where the values of \( a \) for all BH–BH binaries formed during the computation are plotted. Each newly formed BH pair is represented by a different symbol. From the sequences of points, it can be seen that each of the BH–BH pairs exhibits an overall tendency of rapid hardening and the steep \( a(t) \) dependence tends to flatten as \( a \) decreases. The termination of a particular curve generally indicates escape of the corresponding BH binary from the cluster (or more rarely a merger, see Section 3.2). Typically, a BH binary leaves the cluster with \( a < 200 \, R_{\odot} \) (see Fig. 5).

### 3.2 Mergers and escapers

Can a BH pair be hardened to small enough \( a \) (and eccentric enough at the same time) for it to merge via GW radiation? To investigate this question, we consider the positions of the BH–BH binaries within the cluster in an \( a \) versus \((1-e^2)\) plane (Fig. 4), where each newly formed BH pair is represented by a different symbol, as in Fig. 3. The points are colour coded with the evolution time in Myr, the colour scale being displayed on the right. Although for each BH pair \( a \) and \( e \) fluctuate over the plane, the changes occur on a collision time-scale which is \( \sim \)Myr. Since the orbital period of the binaries corresponding to these points is much shorter (from \( \sim \)days to years), these points generally represent binaries which are stable over many orbits. Overplotted on Fig. 4 are lines of constant GW merger time, \( T_{mrg} \), as given by equation (1). While most of the points have very large merger time, a few of them do lie around the \( T_{mrg} = 10 \, \text{Myr} \) line. This indicates that these binaries are indeed hardened up to small enough \( a \) and/or acquire sufficient eccentricity that if they are left unperturbed, they can merge via GW emission within several Myr. This feature is found to be generally true for all the computations reported here (see Table 1), i.e., each of them does produce BH pairs that are capable of merging within several Myr. It is important to note that the short merger time-scale of these binaries is due to their high eccentricities since they are generally far too wide (\( \sim 100 \, R_{\odot} \)) to merge unless they are highly eccentric (see equation 1). This is also true for the escaping BH binaries (see below).

However, these merging BH pairs, particularly due to their large eccentricity and focusing effect, can still be perturbed by further encounters on time-scales \( \sim \) Myr which can often prevent them from merging. In the particular example shown in Fig. 4, only one of the ‘merging candidates’ (\( T_{mrg} \leq 10 \, \text{Myr} \)) could actually merge.
Stellar-mass black holes in star clusters

Figure 4. Positions of all the BH binaries within the cluster in an $1 - e^2$ versus $a$ plane where different symbols are used to distinguish between different BH pairs (for C50K80). The colour coding of the points, as indicated by the colour scale, represents the evolution time (in Myr) of the cluster at which they appear at a particular location in the above plane. See text for details.

Figure 5. Positions of all escaped BH binaries in an $1 - e^2$ versus $a$ plane for the model C65K110. Lines of constant merger times are plotted as in Fig. 4. Within the cluster (inverted triangles). For all the other candidates, including the one which escaped (filled dots), the eccentricity became much smaller due to encounters so that the merger time-scales increased considerably.

As for the escapers, since they remain unperturbed afterwards, all with GW merger times smaller than a Hubble time are of interest. However, we find that typically for each cluster there are relatively few BH binaries with GW merger times $T_{mrg} > 3$ Gyr among those that merge within a Hubble time (see e.g. Figs 5 and 6). In the above example, i.e. model C50K80, there is one BH–BH escaper with $T_{mrg} \sim 1$ Gyr. A more interesting example is shown in Fig. 5 corresponding to the model C65K110 of Table 1 where two binaries merge in $\approx 3$ Gyr, one in $\approx 1$ Gyr and one in $< 10$ Myr.

The results of the present computations, as summarized in Table 1, indicate that a medium-mass to massive cluster with sufficient BH retention is likely to have at least one BH pair that can merge within the cluster within a time-span of a few Gyr. Each of these clusters also typically eject a few BH binaries that can merge within a Hubble time. In one of the models (C50K200), one BH–BH hyperbolic collision also occurred at $t_{mrg} \approx 820$ Myr. As such hyperbolic mergers did not show up in any of the other models, they must be much rarer than GW-driven mergers.
3.3 Merger-time distribution

As a representation of the merger-time distribution of mergers within the cluster, a combined distribution of $t_{\text{mrg}}$ of all the BH binaries that merged within the clusters is shown in Fig. 6 (top panel). This is justified, as the values of BH–BH binary merger times $t_{\text{mrg}}$ within the cluster, shown in Table 1, apparently do not indicate any correlation of $t_{\text{mrg}}$ with the cluster parameters, in particular the number of stars $N$. Both the isolated clusters and the reflective boundary clusters are included. The $t_{\text{mrg}}$ distribution in Fig. 6 indicates that $N_{\text{mrg}}$ decreases with $t_{\text{mrg}}$. This can be expected, as at later times, $N_{\text{mrg}}$ in a cluster decreases, so the hardening rate of BH binaries also decreases, making their merger less probable. Comparing our results for clusters with different values of $N$, we find that the rate of depletion of BHs from the BH core during the first few Gyr does not depend appreciably on $N$. In our computations, BHs are depleted within a few Gyr for most of the models. Clusters with large $N_{\text{BHs}}$, e.g. C100K200 and C50K200, do indicate longer BH retention, but this is due to the expansion of the cluster (and therefore the BH subcluster itself) because of the heating by the BH core (see Section 1), and hence it is unlikely that tight BH–BH binaries can be formed during the later phases.

Fig. 6 (bottom panel) shows the distribution of the merger times for the escaping BH binaries from all the computations of Table 1. Note that the number of tight escapers in Table 1 also does not indicate any dependence on $N$. For each escaper, $t_{\text{mrg}}$ includes its time of escape $t_{\text{esc}}$, i.e. $t_{\text{mrg}} = t_{\text{esc}} + t_{\text{mrg}}$, where $T_{\text{mrg}}$ is calculated from equation (1) using the values of $a$ and $e$ with which it escaped. The merging rate among escaped binaries also shows a decrease with time. Such a decrease is expected from equation (1). For example, if we consider that the escapers that merge within a Hubble time have a mean radius $\bar{a} \sim 50 \, R_\odot$ and a thermal eccentricity distribution $dN_{\text{mrg}}/de \propto 2e$, it is easy to show from equation (1) that $dN_{\text{mrg}}/dT_{\text{mrg}} \propto T_{\text{mrg}}^{-5/7}$. The above merger-time distribution indicates that significantly more BH–BH mergers occur outside the clusters, i.e. among the escapers, than within the clusters. We shall return to this point in Section 4. The median merger time for BH mergers within the clusters is $t_{\text{mrg}} \approx 1 \, \text{Gyr}$ and for the escaped BH binaries it is $\approx 3 \, \text{Gyr}$.

4 DISCUSSION

Our present study indicates that centrally concentrated star clusters, with $N \gtrsim 4.5 \times 10^4$, are capable of dynamically producing BH binaries that can merge within a few Gyr, provided a significant number of BHs are retained in the clusters after their birth. The results of our simulations (see Table 1) imply that most of the BH–BH mergers occur within the first few Gyr of cluster evolution for both mergers within the cluster and mergers of escaped BH binaries.

The above results imply that an important class of candidates for dynamically forming BH binaries that merge at the present epoch are star clusters with initial mass $M_{\text{cl}} \gtrsim 3 \times 10^4 \, M_\odot$, which are less than few Gyr old. Such clusters represent IMC with initial masses close to the upper limit of the initial cluster mass function (ICMF) in spiral (Wiedner, Kroupa & Larsen 2004; Larsen 2009a) and starburst (Gieles et al. 2006) galaxies. While it is not impossible to obtain BH–BH mergers within a Hubble time from lower mass clusters, the overall BH–BH merger and escape rates strongly decrease with cluster mass as Table 1 indicates. For $M_{\text{cl}} \lesssim 1.5 \times 10^4 \, M_\odot$, mergers already become much rarer (see Table 1). Due to the statistical nature of merger or ejection events, it is ambiguous to set any well-defined limit on the cluster mass beyond which these events become appreciable (such an estimate would also require a much larger number of $N$-body integrations). In view of our results, $M_{\text{cl}} \approx 3 \times 10^4 \, M_\odot$ is a representative lower limit beyond which an appreciable number of mergers and escapers merging within a Hubble time can be obtained.

Old GCs, which can be about 10 times or more massive, are expected to produce mergers or escapers more efficiently. As the time-scale of depletion of BHs from the BH cluster is nearly independent of the parent cluster mass (see Section 3.3), GCs can also be expected to produce BH–BH mergers over similar time-span as the IMCs, i.e. within the first few Gyr of evolution. Since GCs are typically much older ($\sim 10 \, \text{Gyr}$), they do not contribute significantly to the present-day merger rate, since most of the mergers from them would have occurred earlier. Considering the light-travel time of $\approx 4.5 \, \text{Gyr}$ from the maximum distance $D \approx 1500 \, \text{Mpc}$ form which these BH–BH binaries can be detected by ‘AdLIGO’ (see below), only GCs close to the above distance could contribute detectable events, mostly from escaped BH–BH binaries.

On the other hand, young massive clusters with ages less than 50 Myr, representing star clusters near the high-mass end of the ICMF (Larsen 2009b), are generally too young to produce BH–BH mergers. All models in Table 1 produce mergers significantly later than this age (except one escaped BH binary in each of the models C65K110 and C100K200). Hence, IMCs seem to be most likely candidates for producing observable BH–BH mergers dynamically.

4.1 Detection rate

We now make an estimate of the BH–BH merger detection rate from IMCs by ground-based GW observatories like LIGO and AdLIGO. In estimating the overall BH–BH merger rate using the results of our model clusters, one needs to consider the distribution of the cluster parameters that are varied over the models, e.g., cluster mass, half-mass radius and BH-retention fraction. Such distributions are far from being well determined, except for the mass distribution for young clusters in spiral and starburst galaxies (Bastian & Lamers 2003; Bik et al. 2003; Gieles, Bastian & Lamers 2004). Therefore, determination of an overall merger rate considering the distribution of our computed clusters can be ambiguous. Hence, as a useful alternative, we determine the BH binary merger detection rates for each of the cluster models in Table 1 that gives an appreciable number of mergers, for a representative density of IMCs. Such an approach has been considered by earlier authors, e.g. O’Leary et al. (2006), and can provide a reasonable idea of the rate of detection of BH–BH mergers from IMCs.

As an estimate of the space density of IMCs, we adopt that for young populous clusters in Portegies Zwart & McMillan (2000), which has a similar mass range as the IMCs:

$$\rho_{\text{cl}} = 3.5 \, h^3 \, \text{Mpc}^{-3},$$

where $h$ is the Hubble parameter, defined as $H_0/100 \, \text{km s}^{-1} \cdot \text{Mpc}$, being the Hubble constant (Peebles 1993). The above space density has been derived from the space densities of spiral, blue elliptical and starburst galaxies (Heyl et al. 1997) assuming that young...
populous clusters have the same specific frequencies \((S_N)\) as old GCs (van den Bergh 1995; McLaughlin 1999), but in absence of any firm determination of the \(S_N\)s of the former. We compute the detection rate for each model cluster assuming that it has a space density of the above value.

The LIGO/AdLIGO-detection rate of BH–BH mergers from a particular model cluster can be estimated from (Belczynski et al. 2007, and references therein)

\[
R_{\text{LIGO}} = \frac{4}{3} \pi D^3 \rho_\text{cl} R_{\text{esc}} R_{\text{merg}},
\]

where \(R_{\text{merg}}\) is the compact binary merger rate from a cluster and \(D\) is the maximum distance from which the emitted GW from a compact binary inspiral can be detected. \(D\) is given by

\[
D = D_0 \left( \frac{M_{\odot}}{M_{\text{ch, rms}}} \right)^{5/6},
\]

where \(D_0 = 18.4\) and \(300\) Mpc for LIGO and AdLIGO, respectively. The quantity \(M_{\text{ch}}\) is the ‘chirp mass’ of the compact binary with component masses \(m_1\) and \(m_2\), which is given by

\[
M_{\text{ch}} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5},
\]

and \(M_{\text{ch, rms}} = 1.2 M_\odot\) is that for a binary with two \(1.4 M_\odot\) neutron stars.

For a BH binary with \(m_1 = m_2 = 10 M_\odot\), \(M_{\text{ch}} = 8.71 M_\odot\) which gives \(D \approx 1500\) Mpc for AdLIGO. The AdLIGO-detection rates \(\mathcal{R}_{\text{AdLIGO}}\) (mean over 3 Gyr taking into account the time of escape \(t_{\text{esc}}\) of the escaped binaries) for the model clusters are shown in Table 1, where the currently accepted value of the Hubble parameter \(h = 0.73\) is assumed. The error in each detection rate is simply obtained from the Poisson dispersion of the total number of mergers for the corresponding cluster. Note that these detection rates are for clusters with solar-like metallicity which is implied by our assumption of \(10 M_\odot\) BHs for all the clusters.

To obtain a basic estimate of the overall detection rate of BH–BH mergers from IMCs, we consider the subset of our computed models that are isolated clusters with full BH retention (see Table 1). We take the mass function of the IMCs to be a power law with index \(\alpha = -2\) which is the approximate index of the ICDF in spiral and starburst galaxies (see e.g. Gieles et al. 2004; Larsen 2009a). Then the weighted average of the corresponding AdLIGO-detection rates is \(\mathcal{R}_{\text{AdLIGO}} \approx 31 (\pm 7) \text{ yr}^{-1}\), which estimates the total present-day detection rate of BH–BH mergers from IMCs expected for AdLIGO. The corresponding LIGO detection rate is negligible, \(\mathcal{R}_{\text{LIGO}} \approx 7.4 \times 10^{-3} \text{ yr}^{-1}\). Note that these BH–BH detection rates are only lower limits. First, the observed population of star clusters can be underestimated by a factor of 2 owing to their dissolution in the tidal field of their host galaxies (see Portegies Zwart & McMillan 2000, and references therein). Secondly, the above detection rates are only from IMCs and there can be additional contributions from GCs (see above).

In comparing the AdLIGO-detection rates from our computations with those from earlier works, we note that our rates are typically an order of magnitude smaller than those of Portegies Zwart & McMillan (2000), but about one order of magnitude larger than those obtained by O’Leary et al. (2006). The principal origin of the former difference is due to the fact that while we considered only IMCs distinguishing them as the most appropriate candidates for producing present-day BH–BH mergers (see above), Portegies Zwart & McMillan (2000) also included GCs, which have considerably larger spatial density and also larger fraction of BH–BH binaries merging within the Hubble time, as obtained from their analytic extrapolations. Note that the number of escapers as obtained by them for young populous clusters and the fraction of them merging within a Hubble time (see table 1 of Portegies Zwart & McMillan 2000) is similar to that obtained from the present computations, implying qualitative agreement. The above authors apparently did not consider the time-scales of formation and depletion of the BH subsystems in their preliminary study. On the other hand, although O’Leary et al. (2006) considered clusters significantly more massive than ours in their Monte Carlo approach, so that larger merger detection rates can be expected, their clusters were much older (8 and 13 Gyr) than IMCs which, in accordance with our results, accounts for their much lower detection rate.

It is interesting to note that the dynamical BH–BH merger detection rates obtained by us are typically an order of magnitude higher than that from primordial stellar binaries as predicted by Belczynski et al. (2007) based on their revised binary-evolution model, and is similar to that for the isolated NS–NS binaries derived by them. Hence, our results imply that dynamical BH–BH binaries constitute the dominant contribution to the BH–BH merger detection. Thus, the dynamical BH–BH inspirals from star clusters seem to be a promising channel for GW detection by the future AdLIGO, although their estimated detection rate with the present LIGO detector is negligible, in agreement with the hitherto non-detection of GW.

### 4.2 Limitations and outlook

The work presented here is a first step towards a detailed study of the dynamics of stellar-mass BHs in star clusters and the consequences for GW-driven BH mergers, and improvements in several directions are possible. First, we do not consider the initial phase of the cluster here when BHs form through stellar evolution, and a more consistent approach would be to begin with a star cluster with a full stellar spectrum and produce the BHs from stellar evolution. Note however that in the present study, we have inserted numbers of BHs in our clusters of low-mass stars similar to what would have been formed from the stellar evolution of a cluster following a Kroupa IMF (see Section 2.2). Stellar evolution also produces NSs (about twice in number as the BHs) which also segregate to the central region of the cluster. It is interesting to study the dynamics of the NS cluster and how it is affected by the (more concentrated) BH cluster – in particular the formation of tight inspiralling NS–NS and NS–BH binaries, which are important for both GW detection and GRBs.

Another aspect that we do not consider in our present study is the effect of primordial stellar binaries. Tight stellar binaries aid the formation of compact binaries through double exchanges (Grindlay, Portegies Zwart & McMillan 2006), in addition to the three-body mechanism (see Section 1) which can increase the number BH–BH (also BH–NS and NS–NS) binaries formed and hence the merger rate. Thus, the study of BH–BH binaries in star clusters with primordial binaries is an important next step. Such studies are in progress and will be presented in future papers.

Finally, the number of \(N\)-body computations in this initial study is not enough to obtain the BH–BH merger rate as a function of cluster mass and BH-retention fraction with a reasonable accuracy. There is typically one integration per cluster model. To obtain merger rate dependencies with cluster parameters, e.g. mass, half-mass radius, binary fraction and BH retention, many computations are needed within smaller intervals, which involves much larger time and computing capacity than that utilized for the present project.
Such results, combined with improved knowledge of cluster parameter distributions and BH formation through supernova explosions of massive stars, would provide a robust estimate of the BH–BH merger detection rate. Conversely, with such a merger rate function, the detection of BH–BH mergers by GW detectors like AdLIGO in the near future will shed light on the above-mentioned long-standing questions.

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