Forgetting Data from Pre-trained GANs

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Abstract

Large pre-trained generative models are known to occasionally provide samples that may be undesirable for various reasons. The standard way to mitigate this is to re-train the models differently. In this work, we take a different, more compute-friendly approach and investigate how to post-edit a model after training so that it forgets certain kinds of samples. We provide three different algorithms for GANs that differ on how the samples to be forgotten are described. Extensive evaluations on real-world image datasets show that our algorithms are capable of forgetting data while retaining high generation quality at a fraction of the cost of full re-training.

1 Introduction

Generative adversarial networks (GANs) are a class of deep generative models that aim to learn a complicated probability distribution from data and generate samples from it [Goodfellow et al., 2014a]. These models have been immensely successful in many large scale tasks from multiple domains [de Masson d’Autume et al., 2019, Tulyakov et al., 2018, Karras et al., 2020, Kong et al., 2020, Zhu et al., 2020, Zhang et al., 2021]. However, many generative models may suffer from undesirable artifacts after training. For example, image models may generate blurred samples [Kaneko and Harada, 2021] or checkerboard artifacts [Odena et al., 2016, Zhang et al., 2019, Wang et al., 2020, Schwarz et al., 2021], speech models may produce unnatural sound [Donahue et al., 2018, Thiem et al., 2020], and language models may emit offensive text [Abid et al., 2021, Perez et al., 2022]. An important question is how to mitigate these artifacts.

Prior work has looked into mitigation by re-designing the pipeline including data augmentation, model architecture, loss functions, and then re-training the model [Isola et al., 2017, Aitken et al., 2017, Kaneko and Harada, 2021]. However, this may involve a lot of computation resources as modern GANs are extremely large and expensive to train. In addition, problems may become apparent after training, and resolving them may require multiple re-trainings. To address this challenge, we consider post-editing, which means modifying a pre-trained model in a certain way rather than training from scratch. This is a much more efficient process. It has shown empirical success in many supervised learning tasks [Frankle and Carbin, 2018, Zhou et al., 2021, Taha et al., 2021], but has not been studied as much for unsupervised learning. In particular, we propose a post-editing framework to forget undesirable samples that might be generated by a GAN, which we call data forgetting.

A naive solution to removing undesirable samples is using a classifier to filter them after generation. This approach, however, has several drawbacks. First, classifiers can take a significant amount of space and time after deployment, especially when there are multiple classifiers. Second, in cases where we provide a generative model to a third party, we have no control over whether they will use the filter. Finally, in the online or continual learning paradigm, we may combine data forgetting and fitting new samples in order to learn a dynamic distribution [Zhou et al., 2021]. If we use classifiers, a sample rejected at the beginning cannot be learned later. Data forgetting via post-editing, on the other hand, poses a cleaner solution which does not suffer from these limitations.
There are two challenges in data forgetting. The first is how to describe the samples to be forgotten. This is important as data forgetting algorithms need to be tailored to specific descriptions. The second challenge is that we need to carefully balance between data forgetting and keeping good generation quality, which means the latent space and the networks must be carefully manipulated.

In this work, we propose a systematic framework for forgetting data from pre-trained generative models (see Section 2). We model data forgetting as learning the data distribution restricted to the complement of a forgetting set \( \Omega \). We then formalize three ways of describing forgetting sets, namely data-based (where a pre-specified set is given), validity-based (where there is a validity checker), and classifier-based (where there is a differentiable classifier).

Then, we introduce three data forgetting algorithms, one for each description (see Section 3). Prior works have looked at avoiding negative samples in the re-training setting with different descriptions and purposes [Sinha et al., 2020, Asokan and Seelamantula, 2020]. They introduce fake distributions to penalize the generation of negative samples. We extend this idea to data forgetting by defining the fake distribution as a mixture of the generative distribution and a forgetting distribution supported on \( \Omega \). We then prove the optimal generator can recover the target distribution when label smoothing [Salimans et al., 2016, Szegedy et al., 2016, Warde-Farley and Goodfellow, 2016] is used. Based on the theory, we introduce the data-based forgetting algorithm (Alg. 1). We then combine this algorithm with an improper active learning algorithm by Hanneke et al. [2018] and introduce the validity-based forgetting algorithm (Alg. 2). Finally, we propose to use a guide function to guide the discriminator via a classifier, and introduce the classifier-based forgetting algorithm (Alg. 3).

We evaluate these forgetting algorithms via experiments on real-world image datasets (see Section 4). We first show these algorithms can forget quickly while keeping high generation quality. We find they are generally comparable. We then investigate applications of data forgetting. We use these algorithms to remove different biases that may not exist in the training set but are learned by the pre-trained model. By data forgetting we can reduce such biases and improve generation quality. We also use data forgetting to understand training data. We investigate which samples are easy or hard to forget through forgetting scores. We finally discuss a potential definition of on-manifold adversarial samples via a variant of the classifier-based forgetting algorithm.

1.1 Related Work

Generative adversarial networks (GANs) [Goodfellow et al., 2014a] have achieved great success in a wide range of challenging, large-scale tasks. Examples include image generation [Karras et al., 2020], point cloud completion [Zhang et al., 2021], speech synthesis [Kong et al., 2020], video generation [Tulyakov et al., 2018], and language models [de Masson d’Autume et al., 2019].

Generative models may have different types of artifacts, such as blurred samples [Kaneko and Harada, 2021], fairness issues [Tan et al., 2020, Karakas et al., 2022], and checkerboard artifacts [Odena et al., 2016, Zhang et al., 2019, Wang et al., 2020, Schwarz et al., 2021] in image generation, offensive text in language models [Abid et al., 2021, Perez et al., 2022], and unnatural sound in speech models [Donahue et al., 2018, Thiem et al., 2020]. Many papers use post-editing to remove artifacts and improve GANs. Examples include improving fairness [Tan et al., 2020, Karakas et al., 2022], rule rewriting [Bau et al., 2020], discovering interpretability [Härkönen et al., 2020], and fine-tuning [Mo et al., 2020, Li et al., 2020, Zhao et al., 2020]. The purposes, use cases, and editing methods of these papers are different from data forgetting in our paper.

The most related papers to our paper are NDA [Sinha et al., 2020], Rumi-GAN [Asokan and Seelamantula, 2020], and Hanneke et al. [2018]. The first two papers look at how to avoid generating negative samples in the re-training setting. This is done by defining new fake distributions to penalize the generation of these samples. However, their purposes are different from us: NDA is used to characterize the boundary of the support of the generative distribution more precisely, and Rumi-GAN is used to handle unbalanced data. We extend their idea and theory to data forgetting in Section 3. ¹ Hanneke et al. [2018] propose an active learning approach to avoid generating invalid samples, also in the re-training setting. However, they only carry theoretical study for discrete distributions. In our paper, the validity-based forgetting algorithm (Alg. 2) is based on a simplified version of their algorithm. We also use their definition of invalidity as an evaluation method.

¹The loss functions in NDA and Rumi-GAN are similar.
There is a related concept called data deletion or machine unlearning [Cao and Yang, 2015]. It is different from data forgetting because these methods aim to approximate the re-trained model when some training samples are removed – mostly due to privacy reasons – while in data forgetting we penalize the model from knowing samples that should be forgotten. In addition, most data deletion techniques are for supervised learning or clustering, and is much less studied for generative models.

There is also a line of work on catastrophic forgetting in supervised learning [Kirkpatrick et al., 2017] and generative models [Thanh-Tung and Tran, 2020]. This concept is different from data forgetting in that we would like the generative model to forget certain data after training, while catastrophic forgetting means knowledge learned in previous tasks is destroyed during continual learning.

2 A Formal Framework for Data Forgetting

Let \( p_{\text{data}} \) be the data distribution on \( \mathbb{R}^d \) and \( X \sim p_{\text{data}} \) be i.i.d. training samples. Let \( \mathcal{A} \) be a generative model and \( \mathcal{M} = \mathcal{A}(X) \) be the pre-trained model on \( X \), which learns \( p_{\text{data}} \). In this paper, we consider \( \mathcal{A} \) to be a GAN learning algorithm [Goodfellow et al., 2014a], and \( \mathcal{M} \) contains two networks, \( D \) (discriminator) and \( G \) (generator), which are jointly trained to optimize

\[
\min_G \max_D \mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{z \sim \mathcal{N}(0, I)} \log(1 - D(G(z))).
\] (1)

Let the forgetting set \( \Omega \subset \mathbb{R}^d \) be the set of samples we would like the model to forget. Formally, the goal is to develop a forgetting algorithm \( D \) such that \( \mathcal{M}' = D(\mathcal{M}, \Omega) \) learns the data distribution restricted on the complement \( \Omega' = \mathbb{R}^d \setminus \Omega \), i.e. \( p_{\text{data}}|_{\Omega'} \). Examples of \( \Omega \) include inconsistent, blurred, unrealistic, or banned samples that are possibly generated by the model.

The forgetting set \( \Omega \), in addition to the pre-trained model, is considered as an input to the forgetting algorithm. We consider three kinds of \( \Omega \), namely data-based, validity-based, and classifier-based. First, the data-based \( \Omega \) is a pre-defined set of samples in \( \mathbb{R}^d \), such as a transformation applied on all training samples [Sinha et al., 2020]. Second, the validity-based \( \Omega \) is defined as all invalid samples according to a validity function \( v : \mathbb{R}^d \to \{0, 1\} \), where 0 means invalid and 1 means valid. This is similar to the setting in Hanneke et al. [2018]. Finally, let \( f : \mathbb{R}^d \to [0, 1] \) be a soft classifier that outputs the probability that a sample belongs to a certain binary class, and \( \tau \in (0, 1) \) be a threshold. Then, the classifier-based \( \Omega \) is defined as \( \{ x : f(x) < \tau \} \). For example, \( f \) can be an offensive text classifier in language generation tasks [Pitsilis et al., 2018]. Our framework is general and applies to any kind of generative model besides GANs.

The way data forgetting differs from data deletion is that in data forgetting, we want the model to assign zero likelihood to the forgetting set \( \Omega \) in order to avoid generating samples from this region; however, in data deletion, there is a set \( X' \subset X \) to be deleted, and the goal is to approximate the re-trained model \( \mathcal{A}(X \setminus X') \).

3 Methods

In this section, we describe algorithms for each kind of forgetting set described in Section 2. We also provide theory on the optimality of the generator and the discriminator. Finally, we generalize the algorithms to situations where we would like the model to forget the union of multiple forgetting sets.

3.1 Data-based Forgetting Set

The data-based forgetting set \( \Omega \) is a pre-defined set of samples we would like the model to forget. One example is a transformation function \( \text{NegAug} \) applied to all training samples, where \( \text{NegAug} \) makes realistic images unrealistic or inconsistent [Sinha et al., 2020]. Another example can be visually nice samples outside data manifold when the training set is small [Asokan and Seelamantula, 2020].

In our framework, the forgetting set \( \Omega \) can be any set of carefully designed or selected samples depending on the purpose of forgetting them – which includes but does not limit to improving the generation quality of the model. For example, we expect the model to improve on fairness, bias, ethics or privacy when \( \Omega \) is properly constructed with unfair, biased, unethical, or atypical samples.

To forget \( \Omega \), we regard both generated samples and \( \Omega \) to be fake samples, and all training samples that are not in \( \Omega \) to be real samples [Sinha et al., 2020, Asokan and Seelamantula, 2020]. Let \( p_{\Omega} \) be
We assume $\Omega$ and provide the proof and theoretical extension to the more general $\lambda$ where

Forgetting Algorithm for Classifier-based Forgetting Set

Algorithm 1

**Inputs:** Pre-trained model $\mathcal{M} = (G_0, D_0)$, train set $X$, forgetting set $\Omega$.
Initialize $G = G_0$, $D = D_0$.
Define the fake data distribution $p_{\text{fake}}$ according to (2) with $p_\Omega = U(\Omega)$.
Train $G, D$ to optimize (3): $\min_G \max_D L(G, D)$.
**return** $\mathcal{M}' = (G, D)$.

Forgetting Algorithm for Validity-based Forgetting Set

Algorithm 2

**Inputs:** Pre-trained model $\mathcal{M} = (G_0, D_0)$, train set $X$, validity function $v$.
Initialize $\Omega' = \{x \in X : v(x) = 0\}$; $\mathcal{M}_0 = \mathcal{M}$.
for $i = 0, \cdots, R - 1$ do
  Initialize $G = G_i$, $D = D_i$. Draw $T$ samples $X^{(i)}_\text{gen}$ from $G$.
  Query $v$ and add invalid samples to $\Omega'$: $\Omega' \leftarrow \Omega' \cup \{x \in X^{(i)}_\text{gen} : v(x) = 0\}$.
  Define the fake data distribution $p_{\text{fake}}$ according to (2) with $p_\Omega = U(\Omega')$.
  Let $\mathcal{M}_{i+1} = (G_{i+1}, D_{i+1})$ optimize (3): $\min_G \max_D L(G, D)$.
end for
**return** $\mathcal{M}' = (G_R, D_R)$.

Forgetting Algorithm for Data-based Forgetting Set

Algorithm 3

**Inputs:** Pre-trained model $\mathcal{M} = (G_0, D_0)$, train set $X$, differentiable classifier $f$.
Initialize $G = G_0$, $D = D_0$.
Define the fake data distribution $p_{\text{fake}}$ according to (2) with $p_\Omega = U(\{x \in X : f(x) < \tau\})$.
Train $G, D$ to optimize (3): $\min_G \max_D L(G, \text{guide}(D, f))$, where $\text{guide}(. , .)$ is defined in (6).
**return** $\mathcal{M}' = (G, D)$.

a distribution such that $\text{supp}(p_\Omega) = \Omega$. Then, the fake data distribution $p_{\text{fake}}$ is a mixture of the generative distribution $p_G = G\#\mathcal{N}(0, I)$ and $p_\Omega$:

$$p_{\text{fake}} = \lambda \cdot p_G + (1 - \lambda) \cdot p_\Omega,$$

where $\lambda \in (0, 1)$ is a hyperparameter. We also apply label smoothing [Salimans et al., 2016, Szegedy et al., 2016, Warde-Farley and Goodfellow, 2016] techniques to the minimax loss function in order to improve robustness. Let $\alpha_+ \in (\frac{1}{2}, 1]$ be the positive target (such as 0.9) and $\alpha_- \in [0, \frac{1}{2})$ be the negative target (such as 0.1). Then, the loss function is

$$L(G, D) = E_{x \sim p_{\text{data}|\Omega}}[\alpha_+ \log D(x) + (1 - \alpha_+)(1 - D(x))] + E_{x \sim p_{\text{fake}}}[\alpha_- \log D(x) + (1 - \alpha_-)(1 - D(x))].$$

**Theorem 1.** The optimal solution to $\min_G \max_D L(G, D)$ is

$$D^* = \frac{\alpha_+ p_{\text{data}|\Omega} + \alpha_- \lambda p_G + (1 - \lambda) p_\Omega}{p_{\text{data}|\Omega} + \lambda p_G + (1 - \lambda) p_\Omega}; \quad p_G^* = p_{\text{data}|\Omega}.$$

We provide the proof and theoretical extension to the more general $f$-GAN [Nowozin et al., 2016] setting in Appendix A. In the data-based setting, we let $p_\Omega = U(\Omega)$, the uniform distribution on $\Omega$. We assume $\Omega$ has positive, finite Lebesgue measure in $\mathbb{R}^d$ so that $U(\Omega)$ is well-defined. The proposed method is summarized in Alg. 1.

Our objective function is connected to Sinha et al. [2020] and Asokan and Seelamantula [2020] in the sense that $p_\Omega$ is an instance of the negative distribution described in their frameworks. However, there are several significant differences between our method and theirs: (1) we start from a pre-trained model, (2) we aim to learn $p_{\text{data}|\Omega}$ rather than $p_{\text{data}}$ and therefore do not require $\Omega \cup \text{supp}(p_{\text{data}})$ to be the empty set, and (3) we use label smoothing techniques to improve robustness and provide theory for this setting. These differences are also true in the following sections.

### 3.2 Validity-based Forgetting Set

Let $v : \mathbb{R}^d \to \{0, 1\}$ be a validity function that indicates whether a sample is valid. Then, validity-based forgetting set $\Omega$ is the set of all invalid samples $\{x : v(x) = 0\}$. For example, $\mathcal{M}$ is a
code generation model, and \( v \) is a compiler that indicates whether the code is free of syntax errors [Hanneke et al., 2018]. Different from the data-based setting, the validity-based \( \Omega \) may have infinite Lebesgue measure, such as a halfspace, and consequently \( \mathcal{U}(\Omega) \) may not be well-defined.

To forget \( \Omega \), we let \( p_\Omega \) in (2) to be a mixture of \( p_{\text{data}}|\Omega \) and \( p_G|\Omega \). This corresponds to a simplified version of the improper active learning algorithm introduced by Hanneke et al. [2018] with our Alg. 1 as their optimization oracle. The idea is to apply Alg. 1 for \( R \) rounds. After each round, we query the validity of \( T \) newly generated samples and use invalid samples to form a data-based forgetting set \( \Omega' \). In contrast to the data-based approach, this active algorithm focuses on invalid samples that are more likely to be generated, and therefore efficiently penalizes generation of invalid samples. The proposed method is summarized in Alg. 2.

The total number of queries to the validity function \( v \) is \( |X| + T \times R \). In case \( v \) is expensive to run, we would like to achieve better data forgetting within a limited number of queries. From the data-driven point of view, we hope to collect as many invalid samples as possible. This is done by setting \( R = 1 \) and \( T \) maximized if we assume less invalid samples are generated after each iteration. However, this may not be the case in practice. We hypothesize some samples are easier to forget while others harder. By setting \( R > 1 \), we expect an increasing fraction of invalid generated samples to be hard to forget after each iteration. Focusing on these hard samples can potentially help the generator forget them. Since it is hard to directly analyze neural networks, we leave the rigorous study to future work. In Appendix B, we study a much simplified dynamical system corresponding to Alg. 2, where we show the invalidity (the mass of \( p_G \) on \( \Omega \)) converges to zero, and provide optimal \( T \) and \( R \) values.

### 3.3 Classifier-based Forgetting Set

We would like the model to forget samples with certain (potentially undesirable) property. Let \( f : \mathbb{R}^d \to [0, 1] \) be a soft binary classifier on the property (0 means having the property and 1 means not having it), and \( \tau \in (0, 1) \) be a threshold. The classifier-based forgetting set \( \Omega \) is then defined as \( \{ x : f(x) < \tau \} \). For example, the property can be being offensive in language generation, containing no speech in speech synthesis, or visual inconsistency in image generation. We consider \( f \) to be a trained machine learning model that is fully accessible and differentiable.

To forget \( \Omega \), we let \( p_\Omega \) be a mixture of \( p_{\text{data}}|\Omega \) and \( p_G|\Omega \), similar to the validity-based approach. We use \( f \) to guide the discriminator and make it able to easily detect samples from \( \Omega \). Let \( \text{guide}(D, f) \) be a guided discriminator that assigns small values to \( x \) when \( f(x) < \tau \) or \( D(x) \) is small (i.e. \( x \sim p_{\text{fake}} \)), and large values to \( x \) when \( f(x) > \tau \) and \( D(x) \) is large (i.e. \( x \sim p_{\text{data}}|\Omega \)). Instead of optimizing \( L(G, D) \) in (3), we optimize \( L(G, \text{guide}(D, f)) \). This will effectively update \( G \) by preventing it from generating samples in \( \Omega \). According to Theorem 1, the optimal discriminator is the solution to

\[
\text{guide}(D^*, f) = \frac{\alpha + p_{\text{data}}|\Omega}{{\alpha}(\lambda + \lambda p_G + (1 - \lambda)p_\Omega)}.
\]

Therefore, the design of the guide function must make (5) feasible. In this paper, we let

\[
\text{guide}(D, f)(x) = D(x) \text{ if } f(x) \geq \tau, \text{ otherwise } \alpha - \tau \alpha \text{ if } f(x) \leq \tau.
\]

The feasibility of (5) is discussed in Appendix C. The proposed method is summarized in Alg. 3. The classifier-based \( \Omega \) generalizes the validity-based \( \Omega \). First, any validity-based \( \Omega \) can be represented by a classifier-based \( \Omega \) if we let \( f = v \) and \( \tau = \frac{1}{2} \). Next, we note there is a trivial way to deal with classifier-based \( \Omega \) via the validity-based approach – by setting \( v(x) = 1 \{ f(x) < \tau \} \). However, potentially useful information such as values and gradients of \( f \) are lost, and we will evaluate this effect in experiments. In addition, the classifier-based approach does not maintain the potentially large set of invalid generated samples, as this step is automatically done in the \( \text{guide} \) function.

### 3.4 Generalization to Multiple Forgetting Sets

Let \( \{ \Omega_k \}_{k=1}^K \) be disjoint sets in \( \mathbb{R}^d \), and we would like the model to forget \( \Omega = \bigcup_{k=1}^K \Omega_k \). In the data-based setting, we let \( p_{\Omega} = \mathcal{U}(\Omega) = \bigcup_{k=1}^K \Omega_k \). In the validity-based setting, each \( \Omega_k \) is associated with a validity function \( v_k \). In the classifier-based setting, each \( \Omega_k \) is associated with a classifier \( f_k \). Similar to the validity-based setting, we let the overall \( f \) to be \( f(x) = \min_k f_k(x) \).
Table 1: Invalidity and generation quality of different forgetting algorithms on forgetting label zero within different datasets. Mean and standard errors are reported for five random seeds. Note that quality measure after data forgetting is not directly comparable with the pre-trained model. The invalidity drops in magnitude after data forgetting. Different forgetting algorithms are highly comparable to each other.

| Dataset   | Evaluation | Pre-trained | Data-based | Validity-based | Classifier-based |
|-----------|------------|-------------|------------|----------------|------------------|
| MNIST (8 epochs) | IS($\uparrow$) ($\times 10^{-3}$) | 7.82 | 8.0 ± 2.2 | 7.20 ± 0.08 | 5.2 ± 3.7 |
| CIFAR-10 (30 epochs) | FID($\downarrow$) ($\times 10^{-3}$) | 1.3 × 10^4 | 7.5 ± 1.1 | 34.8 ± 1.5 | 33.2 ± 0.6 |
| STL-10 (40 epochs) | FID($\downarrow$) ($\times 10^{-1}$) | 6.2 × 10^2 | 8.8 ± 4.5 | 7.7 ± 1.3 | 11.6 ± 3.6 |

Figure 1: Invalidity during data forgetting when forgetting label zero. Mean and standard errors are plotted for five random seeds. Standard errors may be too small to spot. Invalidity drops quickly at the beginning of data forgetting, and different algorithms are highly comparable to each other.

4 Experiments

In this section, we aim to answer the following questions.

- How well can the algorithms in Section 3 forget samples in practice?
- Can these algorithms be used to de-bias pre-trained models?
- Can these algorithms be used to understand training data?

In this section, we examine these questions by focusing on several real-world image datasets, including MNIST (28 × 28) [LeCun et al., 2010], CIFAR (32 × 32) [Krizhevsky et al., 2009], CelebA (64 × 64) [Liu et al., 2015] and STL-10 (96 × 96) [Coates et al., 2011] datasets. In Section 4.1, we investigate how well these algorithms can forget samples with a specific label. In Section 4.2, we investigate how well these algorithms can de-bias pre-trained models and improve generation quality. In Section 4.3, we use these algorithms to understand training data through the lens of data forgetting.

The pre-trained model for each dataset is a DCGAN [Radford et al., 2015] trained for 200 epochs (see details in Appendix D). We use one NVIDIA 3080 GPU to train these models and run experiments.

Evaluation Metrics: invalidity and generation quality. The invalidity is defined as the mass of the generation distribution on the forgetting set. In practice, we measure invalidity by generating 50K samples and computing the fraction of these samples that fall into \( \Omega \).

The generation quality is measured in Inception Score (IS) [Salimans et al., 2016] and Frechet Inception Distance (FID) [Heusel et al., 2017]. Higher IS or lower FID indicates better quality. We compute IS for grey-scale images and FID for RGB images. When measuring quality, we compute IS or FID between 50K generated samples and \( X \cap \Omega \). Therefore, this score is not comparable with the score w.r.t. the pre-trained model if the forgetting set includes samples in the training set, such as samples with a specific label in Section 4.1.

4.1 Forgetting Labels

Question. How well can the algorithms in Section 3 forget samples in practice?

Methodology. We investigate how well the proposed algorithms can forget samples with a specific label \( y \). In the data-based setting (Alg. 1), we express this as \( \Omega = \{ x \in X : \text{label}(x) = y \} \). In the validity-based setting (Alg. 2), we express this by setting \( v(x) = 1 \{ \arg \max_i \log \text{it}(x), \neq y \} \),
We visualize $G$ when those training samples to be forgotten (i.e., when we forget label 0). We plot invalidity during data forgetting in Fig. 1. We also compare invalidity when the total number of queries is fixed. Results indicate that a large $T$ may lead to worse invalidity. Therefore, we conclude that the simplest data-based algorithm is good enough to forget samples after one epoch of data forgetting in Appendix E.1.1. Mean and standard errors for 5 random runs are reported. Results for different hyper-parameters and forgetting other labels are in Appendix E.

In Table 1, we compare invalidity and generation quality among different algorithms and datasets when we forget label 0. We plot invalidity during data forgetting in Fig. 1. We also compare invalidity after one epoch of data forgetting in Appendix E.1.1. Mean and standard errors for 5 random runs are reported. Results for different hyper-parameters and forgetting other labels are in Appendix E.

**Results.** We find all the algorithms in Section 3 work quite well with a much fewer number of epochs used for training the pre-trained model (which is 200). These algorithms are generally comparable. Therefore, we conclude that the simplest data-based algorithm is good enough to forget samples when those training samples to be forgotten $(X \cap \Omega)$ can characterize the forgetting set $(\Omega)$ well.

We also find invalidity rapidly drops after only one epoch of data forgetting, indicating these algorithms are very efficient in penalizing invalidity. While different algorithms perform better on different datasets, they are highly comparable with each other. The reason why the classifier-based algorithm performs the best on MNIST is possibly that the label classifier on MNIST is almost perfect so its gradient information is accurate.

**Visualization.** We sample latents $z \sim N(0, I)$ and choose those corresponding to invalid samples, i.e. $G_0(z) \in \Omega$ where $G_0$ is the pre-trained generator. We select visually good $G_0(z)$ for demonstration. We visualize $G(z)$ during data forgetting in Fig. 2, and more visualizations are in Appendix E.3. This demonstrates how the latent space is manipulated: the label to be forgotten is gradually pushed to other labels, and there is high-level visual similarity between the final $G(z)$ and the original $G_0(z)$.

**Effects of other hyper-parameters.** In Table 2, we compare different $T$ (#queries after each epoch) in the validity-based forgetting algorithm (Alg. 2). We fix the total number of queries by setting $T \times \#epochs$ to be a constant. Results indicate that a large $T$ may lead to worse invalidity, and there is trade-off between invalidity and quality when setting $T$ to be small or moderate.

In Appendix E.1.3, we compare different $\lambda$ (hyperparameter in (2)) in the classifier-based forgetting algorithm (Alg. 3). We find there exists a clear trade-off between invalidity and quality when alternating $\lambda$: a larger $\lambda$ tends to produce better quality, and a smaller $\lambda$ tends to have better invalidity.

**Forgetting multiple sets.** We then investigate how well the proposed algorithms can generalize to multiple forgetting sets with methods in Section 3.4. We focus on the CelebA dataset [Liu et al.,

| $T$ | MNIST |   | C100 |   | STL-10 |   |
|-----|-------|---|------|---|--------|---|
|     | $R$   | Inv(\downarrow) | IS(\uparrow) | $R$ | Inv(\downarrow) | FID(\downarrow) | $R$ | Inv(\downarrow) | FID(\downarrow) |
| 400 | 20    | $0.0 \times 10^{-4}$ | 7.10 | 75 | $0.45 \times 10^{-2}$ | 35.1 | 100 | $1.0 \times 10^{-3}$ | 75.1 |
| 1000| 8     | $0.6 \times 10^{-4}$ | 7.19 | 30 | $0.76 \times 10^{-2}$ | 34.8 | 40  | $0.8 \times 10^{-3}$ | 77.0 |
| 2000| 4     | $2.8 \times 10^{-4}$ | 7.11 | 15 | $1.00 \times 10^{-2}$ | 31.9 | 20  | $1.0 \times 10^{-3}$ | 75.1 |

Figure 2: Visualization of the data forgetting process of invalid samples when forgetting label zero. The first column is generated by the pre-trained generator, and the i-th column is generated after $k \cdot (i - 1)$ epochs of data forgetting. Left: MNIST with $k = 1$. Right: top is CIFAR-10 and bottom is STL-10, both with $k = 4$ and label zero being airplanes. We can see samples associated with invalid labels are gradually pushed to other labels, but a high-level visual similarity is kept.
The classifier is defined as

There can be different artifacts in GAN generated samples, and these could harm the overall generation quality. Table 4: Invalidity after de-biasing boundary artifacts of generated MNIST samples. We run the classifier-based algorithm (Alg. 3) to de-bias label biases. We call this phenomenon the model de-biasing. In this section, we investigate how well Alg. 2 and Alg. 3 apply to this task. We assume training samples are not biased so Alg. 1 does not apply to de-biasing.

To use these algorithms for de-biasing, we assume the target artifact or bias can be automatically detected by a classifier $f$ or a validity function $v$. Specifically, we survey two kinds of biases: boundary artifacts and label biases.

**Boundary artifacts.** A GAN trained on MNIST might generate samples that have numerous white pixels on the boundary (see Appendix F.1). We call this phenomenon the boundary artifact. We use the validity-based algorithm (Alg. 2) to de-bias boundary artifacts. The validity function is defined as $v(x) = \frac{1}{\sum_{i,j \in \text{boundary pixels}} x_{ij} < \tau_b}$, where boundary pixels are those within a certain margin to the boundary, and threshold $\tau_b$ satisfies no training image is invalid.

Results are reported in Table 4. It is clear that the invalidity reduces in order after data forgetting, indicating boundary artifacts are largely removed. Consistent with Table 2, a small or moderate $T$ leads to slightly lower (better) invalidity.

Table 5: Invalidity and generation quality after de-biasing label biases of generated samples from different datasets. We use the classifier-based forgetting algorithm (Alg. 3). The arrow means improvement from the pre-trained model to after data forgetting. There is a clear improvement of generation quality, indicating the proposed algorithm can help GANs generate better samples.

4.2 Model De-biasing

There can be different artifacts in GAN generated samples, and these could harm the overall generation quality. These artifacts may not exist in training samples, but are caused by inductive biases of the model, and become obvious after training. We can post-edit a pre-trained model to remove these artifacts, which we call model de-biasing. In this section, we investigate how well Alg. 2 and Alg. 3 apply to this task. We assume training samples are not biased so Alg. 1 does not apply to de-biasing.

To use these algorithms for de-biasing, we assume the target artifact or bias can be automatically detected by a classifier $f$ or a validity function $v$. Specifically, we survey two kinds of biases: boundary artifacts and label biases.

**Boundary artifacts.** A GAN trained on MNIST might generate samples that have numerous white pixels on the boundary (see Appendix F.1). We call this phenomenon the boundary artifact. We use the validity-based algorithm (Alg. 2) to de-bias boundary artifacts. The validity function is defined as $v(x) = \frac{1}{\sum_{i,j \in \text{boundary pixels}} x_{ij} < \tau_b}$, where boundary pixels are those within a certain margin to the boundary, and threshold $\tau_b$ satisfies no training image is invalid.

Results are reported in Table 4. It is clear that the invalidity reduces in order after data forgetting, indicating boundary artifacts are largely removed. Consistent with Table 2, a small or moderate $T$ leads to better results. We visualize samples before and after de-biasing in Appendix F.1.

**Label biases.** Neural networks may generate visually smooth but labelally ambiguous samples [Kirichenko et al., 2020], e.g. samples that look like multiple objects (see Appendix F.2). We call this phenomenon the label bias. We use the classifier-based algorithm (Alg. 3) to de-bias label biases. The classifier is defined as $f(x) = 1 - \text{Entropy} (\logit(x))/\log (#\text{classes})$, where the logit

2015], which has 40 labeled attributes. We use proposed algorithms to forget a combination of these attributes: $\Omega_1 = \{\text{Black_hair and Blurry}\}$, $\Omega_2 = \{\text{Brown_hair and Wear_eyeglasses}\}$, and $\Omega = \Omega_1 \cup \Omega_2$. These attributes are randomly selected from those easy to capture. See detailed setup in Appendix E.4. Results after 1 or 5 epochs are reported in Table 3. Consistent with results on forgetting just one label, all algorithms can reduce invalidity and retain generation quality and are comparable, while the classifier-based algorithm achieves the best invalidity after one epoch.

Table 5: Invalidity and generation quality after de-biasing label biases of generated samples from different datasets. We use the classifier-based forgetting algorithm (Alg. 3). The arrow means improvement from the pre-trained model to after data forgetting. There is a clear improvement of generation quality, indicating the proposed algorithm can help GANs generate better samples.

| $\tau$ | MNIST (8 epochs, $\lambda = 0.8$) | CIFAR-10 (30 epochs, $\lambda = 0.9$) |
|--------|----------------------------------|----------------------------------------|
|        | Inv(↓)                           | Inv(↓)                                 |
| 0.3    | $8.19 \times 10^{-4}$ → $8.10$    | $5.79 \times 10^{-4}$ → $2.20 \times 10^{-4}$ |
| 0.5    | $2.07 \times 10^{-2}$ → $7.82$    | $2.28 \times 10^{-2}$ → $1.67 \times 10^{-2}$ |
| 0.7    | $1.35 \times 10^{-1}$ → $1.22$    | $1.72 \times 10^{-1}$ → $1.49 \times 10^{-1}$ |

|        | IS(↑)                            | FID(↓)                                 |
|        |                                  |                                        |
| 0.3    | $8.19 \times 10^{-4}$ → $8.10$    | $36.2 \rightarrow 27.1$               |
| 0.5    | $2.07 \times 10^{-2}$ → $7.82$    | $36.2 \rightarrow 26.6$               |
| 0.7    | $1.35 \times 10^{-1}$ → $1.22$    | $36.2 \rightarrow 26.8$               |
In this paper, we propose a systematic framework for forgetting data from pre-trained generative models. We provide three different algorithms for GANs that differ on how the samples to be forgotten are described. We provide theoretical results that data forgetting can be achieved. We then empirically investigate data forgetting on real-world image datasets, and show that our algorithms are capable of forgetting data while retaining high generation quality at a fraction of the cost of full re-training. One limitation of our paper is that the proposed framework only applies to unconditional generative models. It is an important future direction to define data forgetting and propose algorithms for conditional generative models, which are more widely used in downstream deep learning applications.
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A Proof of Theorem 1 and Extension to f-GAN

Background of f-GAN [Nowozin et al., 2016]. Let $\phi$ be a convex, lower-semicontinuous function such that $\phi(1) = 0$. In f-GAN, the following $\phi$-divergence is minimized:

$$D_\phi(P||Q) = \int_{x \in \mathbb{R}^d} Q(x) \phi \left( \frac{P(x)}{Q(x)} \right) dx.$$  

According to the variational characterization of $\phi$-divergence [Nguyen et al., 2010],

$$D_\phi(P||Q) = \sup_T [E_{x \sim P} T(x) - E_{x \sim Q} \phi^*(T(x))],$$

where the optimal $T$ is obtained by $T = \phi' \left( \frac{P}{Q} \right)$.

The objective function (3) corresponds to an f-GAN. Let $\alpha = \alpha_+ + \alpha_-$. We can rewrite (3) as

$$L(G, D) = \alpha \cdot E_{x \sim P} \log D(x) + (2 - \alpha) \cdot E_{x \sim Q} \log (1 - D(x)),$$

where

$$P = \frac{\alpha_+}{\alpha} p_{\text{data}}|\Omega + \frac{\alpha_-}{\alpha} p_{\text{fake}}; \quad Q = \frac{1 - \alpha_+}{2 - \alpha} p_{\text{data}}|\Omega + \frac{1 - \alpha_-}{2 - \alpha} p_{\text{fake}}.$$

Let

$$C = \alpha \log \alpha + (2 - \alpha) \log (2 - \alpha) - 2 \log 2,$$

$$\phi(u) = (\alpha u) \log (\alpha u) - (\alpha u - \alpha + 2) \log (\alpha u - \alpha + 2) + (2 - \alpha) \log (2 - \alpha) - C.$$  

Then, $\phi(1) = 0$, and $\phi''(u) = \frac{\alpha(2 - \alpha)}{u(\alpha u - \alpha + 2)} > 0$ so $\phi$ is convex. Its convex conjugate function $\phi^*$ is

$$\phi^*(t) := \sup_u (ut - \phi(u)) = -(2 - \alpha) \log \left( 1 - e^\frac{t}{\alpha} \right) + C.$$  

Let $T(x) = \alpha \log D(x)$. Then,

$$\max_D L(G, D) = \sup_T [E_{x \sim P} T(x) - E_{x \sim Q} \phi^*(T(x))] + C = D_\phi(P||Q) + C.$$

Optimal $D$. We have

$$\phi'(u) = \alpha \log \frac{\alpha u}{\alpha u - \alpha + 2}.$$

Therefore, the optimal discriminator is

$$\alpha \log D = \phi' \left( \frac{P}{Q} \right),$$

or

$$D = \frac{\alpha P}{\alpha P + (2 - \alpha) Q} = \frac{\alpha p_{\text{data}}|\Omega + \alpha - p_{\text{fake}}}{p_{\text{data}}|\Omega + p_{\text{fake}}}.$$

Finally, the optimal discriminator in (4) is obtained by inserting (2) into the above equation.

Optimal $G$. For conciseness, we let

$$P_1 = p_{\text{data}}|\Omega, \quad P_2 = p_G, \quad P_3 = p_{\Omega},$$

$$\beta_1 = \frac{\alpha_+}{\alpha}, \quad \beta_2 = \frac{\alpha_-}{\alpha}, \quad \beta_3 = \frac{\alpha_-(1 - \lambda)}{\alpha},$$

$$\gamma_1 = \frac{1 - \alpha_+}{2 - \alpha}, \quad \gamma_2 = \frac{(1 - \alpha_-)\lambda}{2 - \alpha}, \quad \gamma_3 = \frac{(1 - \alpha_-)(1 - \lambda)}{2 - \alpha}.$$

Then, we have

$$P = \sum_{i=1}^3 \beta_i P_i, \quad Q = \sum_{i=1}^3 \gamma_i P_i.$$

We also have

$$\frac{\beta_1}{\gamma_1} > \frac{\beta_2}{\gamma_2} = \frac{\beta_3}{\gamma_3}.$$
Because \( \text{supp}(P_1) \cap \text{supp}(P_3) \) is the empty set, we have

\[
D_\phi(P \| Q) = \int_{x \in \mathbb{R}^d} \left( \sum_{i=1}^{3} \gamma_i P_i \right) \phi \left( \frac{\sum_{i=1}^{3} \beta_i P_i}{\sum_{i=1}^{3} \gamma_i P_i} \right) \, dx
\]

\[
= \int_{x \notin \Omega} (\gamma_1 P_1 + \gamma_2 P_2) \phi \left( \frac{\beta_1 P_1 + \beta_2 P_2}{\gamma_1 P_1 + \gamma_2 P_2} \right) \, dx
\]

\[
+ \int_{x \in \Omega} (\gamma_2 P_2 + \gamma_3 P_3) \phi \left( \frac{\beta_2 P_2 + \beta_3 P_3}{\gamma_2 P_2 + \gamma_3 P_3} \right) \, dx
\]

Let

\[
\int_{x \in \Omega} P_2 \, dx = \eta.
\]

We have

\[
\int_{x \in \Omega} (\gamma_2 P_2 + \gamma_3 P_3) \phi \left( \frac{\beta_2 P_2 + \beta_3 P_3}{\gamma_2 P_2 + \gamma_3 P_3} \right) \, dx = (\gamma_2 \eta + \gamma_3) \phi \left( \frac{\beta_2}{\gamma_3} \right).
\]

Let

\[
\zeta = \frac{\beta_2 (\gamma_1 + \gamma_2)}{\gamma_2 (\beta_1 + \beta_2)}.
\]

According to Jensen’s inequality,

\[
\int_{x \notin \Omega} (\gamma_1 P_1 + \gamma_2 P_2) \phi \left( \frac{\beta_1 P_1 + \beta_2 P_2}{\gamma_1 P_1 + \gamma_2 P_2} \right) \, dx
\]

\[
= (\gamma_1 + \gamma_2 (1 - \zeta)) \int_{x \notin \Omega} \left( \frac{\gamma_1 P_1 + \gamma_2 P_2}{\gamma_1 + \gamma_2 (1 - \zeta)} \right) \phi \left( \frac{\beta_1 P_1 + \beta_2 P_2}{\gamma_1 P_1 + \gamma_2 P_2} \right) \, dx
\]

\[
\geq (\gamma_1 + \gamma_2 (1 - \zeta)) \phi \left( \frac{\beta_1 + \beta_2 (1 - \zeta)}{\gamma_1 + \gamma_2 (1 - \zeta)} \right)
\]

\[
= (\gamma_1 + \gamma_2 (1 - \zeta)) \phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right)
\]

Therefore, we have

\[
D_\phi(P \| Q) \geq (\gamma_1 + \gamma_2) \phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) + \gamma_3 \phi \left( \frac{\beta_3}{\gamma_3} \right) + \left[ \gamma_2 \phi \left( \frac{\beta_3}{\gamma_3} \right) - \frac{\beta_2 (\gamma_1 + \gamma_2)}{\gamma_1 + \gamma_2} \phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) \right] \eta.
\]

Now, we show the \( \eta \) term is non-negative. We write

\[
\gamma_2 \phi \left( \frac{\beta_3}{\gamma_3} \right) - \frac{\beta_2 (\gamma_1 + \gamma_2)}{\beta_1 + \beta_2} \phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) = \beta_2 \left( \frac{\gamma_2}{\beta_2} \phi \left( \frac{\beta_3}{\gamma_3} \right) - (\gamma_1 + \gamma_2) \phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) \right)
\]

\[
= \beta_2 \left( \frac{\gamma_2}{\beta_2} \phi \left( \frac{\beta_3}{\gamma_3} \right) - \frac{1 - \beta_3}{1 - \beta_3} \phi \left( \frac{1 - \beta_3}{1 - \beta_3} \right) \right).
\]

It suffices to prove the function \( \psi(u) = \phi(u)/u \) satisfies

\[
\psi \left( \frac{\beta_3}{\gamma_3} \right) \geq \psi \left( \frac{1 - \beta_3}{1 - \gamma_3} \right).
\]

We use the Mathematica software [Inc.] to compute the difference:

\[
\psi \left( \frac{\beta_3}{\gamma_3} \right) - \psi \left( \frac{1 - \beta_3}{1 - \gamma_3} \right) = -\frac{\alpha}{\alpha -} \log \frac{2 - \alpha}{1 - \alpha -} + \alpha \log \frac{\alpha - (2 - \alpha)}{1 - \alpha -} + \frac{\alpha (1 - \alpha -)}{2 - \alpha} (\log 4 - \alpha \log \alpha)
\]

\[
- \frac{\alpha}{2 - \alpha} \left( \frac{\lambda + 1}{\lambda \alpha - + \alpha +} - 1 \right) (\log 4 - \alpha \log \alpha)
\]

\[
- \alpha \log \frac{(2 - \alpha)(\lambda \alpha - + \alpha +)}{\lambda(1 - \alpha -) + 1 - \alpha +} + \frac{\alpha (\lambda + 1)}{\lambda \alpha - + \alpha +} \log \frac{(\lambda + 1)(2 - \alpha)}{\lambda(1 - \alpha -) + 1 - \alpha +}.
\]
The minimum value of the above difference for \( \alpha - \in [0, \frac{1}{2}] \), \( \alpha + \in [0, \frac{1}{2}] \), and \( \lambda \in [0, 1] \) is obtained at \( \alpha = \alpha_+ = \frac{1}{2} \), where the difference equals zero. This makes us able to conclude
\[
D_\phi(P||Q) \geq (\gamma_1 + \gamma_2)\phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) + \gamma_3\phi \left( \frac{\beta_3}{\gamma_3} \right).
\]
Finally, we let \( P_2 = P_1 \). In this case,
\[
D_\phi(P||Q) = \int_{x \in \mathbb{R}^d} \left( \sum_{i=1}^{3} \gamma_i P_i \right) \phi \left( \frac{\sum_{i=1}^{3} \beta_i P_i}{\sum_{i=1}^{3} \gamma_i P_i} \right) dx
\]
\[
= \int_{x \notin \Omega} (\gamma_1 P_1 + \gamma_2 P_2)\phi \left( \frac{\beta_1 P_1 + \beta_2 P_2}{\gamma_1 P_1 + \gamma_2 P_2} \right) dx
\]
\[
+ \int_{x \in \Omega} \gamma_3 P_3\phi \left( \frac{\beta_3 P_3}{\gamma_3 P_3} \right) dx
\]
\[
= (\gamma_1 + \gamma_2)\phi \left( \frac{\beta_1 + \beta_2}{\gamma_1 + \gamma_2} \right) + \gamma_3\phi \left( \frac{\beta_3}{\gamma_3} \right).
\]
Therefore, the optimal generator is \( p_G = p_{\text{data}}|\bar{\Omega} \).

**Extension to \( f \)-GAN.** We can extend the objective (3) to any type of \( f \)-GAN. Let \( \phi \) be a convex, lower-semicontinuous function such that \( \phi(1) = 0 \). Let
\[
P = \frac{\alpha_+}{\alpha} p_{\text{data}}|\bar{\Omega} + \frac{\alpha_-}{\alpha} p_{\text{fake}}; \quad Q = \frac{1 - \alpha_+}{2 - \alpha} p_{\text{data}}|\bar{\Omega} + \frac{1 - \alpha_-}{2 - \alpha} p_{\text{fake}}.
\]
We jointly optimize
\[
\min_{G} \max_{D} L(G, D) = \mathbb{E}_{x \sim P} D(x) - \mathbb{E}_{x \sim Q} \phi^*(D(x)).
\]
Then, the optimal discriminator is \( D = \phi^* \left( \frac{P}{Q} \right) \). If \( \psi \left( \frac{\beta_3}{\gamma_3} \right) \geq \psi \left( \frac{1 - \beta_3}{1 - \gamma_3} \right) \), then the optimal generator is \( p_G = p_{\text{data}}|\bar{\Omega} \).

**Remark 1.** When \( \alpha = 0 \) and \( \alpha_+ = 1 \) (i.e. there is no label smoothing), Theorem 1 in Sinha et al. [2020] implies the above optimal generator. Our theorem also extends their theorem to the label smoothing setting.
B Theoretical Analysis of a Simplified Dynamical System on Invalidity

In this section, we provide theoretical analysis to a simplified, ideal dynamical system that corresponds to Alg. 2 and Section 3.2. In this dynamical system, we assume there are only two types of invalid samples: those easy to forget, and those hard to forget. We assume after each iteration, the generator will generate a less but positive fraction of invalid samples. Formally, let \( \Omega_{\text{easy}}, \Omega_{\text{hard}} \) be a split of \( \Omega \), where \( \Omega_{\text{easy}} \) is the set of invalid samples that are easy to forget, and \( \Omega_{\text{hard}} \) is the set of invalid samples that are hard to forget. We let

\[
\begin{align*}
    m_{\text{easy}} &= \int_{\Omega_{\text{easy}}} p_G(x) dx, \\
    m_{\text{hard}} &= \int_{\Omega_{\text{hard}}} p_G(x) dx, \\
    m_{\text{ratio}} &= \frac{m_{\text{easy}}}{m_{\text{easy}} + m_{\text{hard}}},
\end{align*}
\]

Then, \( m_{\text{easy}} \) is the fraction of invalid generated samples that are easy to forget, and \( m_{\text{hard}} \) is the fraction of invalid generated samples that are hard to forget. \( m_{\text{easy}} + m_{\text{hard}} \) is the fraction of invalid generated samples over all generated ones, which we call invalidity. We use superscript to represent each iteration. We consider the following dynamical system:

\[
\begin{align*}
    m_{\text{easy}}^{i+1} &= m_{\text{easy}}^i \cdot \eta_{\text{easy}}(m_{\text{ratio}}^i, T), \\
    m_{\text{hard}}^{i+1} &= m_{\text{hard}}^i \cdot \eta_{\text{hard}}(m_{\text{ratio}}^i, T).
\end{align*}
\]

In other words, the improvement of \( m_{\text{easy}} \) and \( m_{\text{hard}} \) (in terms of multiplication factor) is only affected by \( m_{\text{ratio}} \) and \( T \). We make this assumption because in practice, the number of invalid samples to optimize the loss function is always fixed. As for boundary conditions, we assume \( m_{\text{easy}}^0 > m_{\text{hard}}^0 \). We assume for \( \eta \in \{ \eta_{\text{easy}}, \eta_{\text{hard}} \}, 0 < \eta(m, T) \leq 1 \), where equality holds only in these situations:

\[
\eta(m, 0) = 1, \quad \eta_{\text{easy}}(0, T) = 1, \quad \eta_{\text{hard}}(1, T) = 1.
\]

We also assume a larger \( T \) leads to smaller \( \eta \), but this effect degrades as \( T \) increases:

\[
\frac{\partial}{\partial T} \eta(m, T) < 0, \quad \frac{\partial^2}{\partial T^2} \eta(m, T) > 0.
\]

To distinguish between samples that are easy or hard to forget, we assume

\[
\frac{1}{m} \cdot \frac{\partial}{\partial T} \eta_{\text{easy}}(m, T) < \frac{1}{1-m} \cdot \frac{\partial}{\partial T} \eta_{\text{hard}}(m, T) < 0.
\]

We can now draw some conclusions below.

As \( i \to \infty \), invalidity converges to 0. Because \( \eta_{\text{easy}}(T) < 1 \) and \( \eta_{\text{hard}}(T) < 1 \) when \( T > 0 \), we have \( m_{\text{easy}}^{i+1} \leq m_{\text{easy}}^i \) and \( m_{\text{hard}}^{i+1} \leq m_{\text{hard}}^i \). According to the monotone convergence theorem, there exists \( m_{\text{easy}}^\infty \geq 0 \) and \( m_{\text{hard}}^\infty \geq 0 \) such that

\[
\lim_{i \to \infty} m_{\text{easy}}^i = m_{\text{easy}}^\infty, \quad \lim_{i \to \infty} m_{\text{hard}}^i = m_{\text{hard}}^\infty.
\]

We now prove \( m_{\text{easy}}^\infty = m_{\text{hard}}^\infty = 0 \). If otherwise, there exists \( m_{\text{ratio}}^\infty = \frac{m_{\text{easy}}^\infty}{m_{\text{easy}}^\infty + m_{\text{hard}}^\infty} \) such that \( m_{\text{ratio}}^i \to m_{\text{ratio}}^\infty \). We then have

\[
\begin{align*}
    m_{\text{easy}}^\infty &= m_{\text{easy}}^\infty \cdot \eta_{\text{easy}}(m_{\text{ratio}}^\infty, T), \\
    m_{\text{hard}}^\infty &= m_{\text{hard}}^\infty \cdot \eta_{\text{hard}}(m_{\text{ratio}}^\infty, T).
\end{align*}
\]

If \( m_{\text{easy}}^\infty > 0 \), then \( m_{\text{ratio}}^\infty > 0 \), and \( \eta_{\text{easy}}(m_{\text{ratio}}^\infty, T) < 1 \), contradiction. Similarly, if \( m_{\text{hard}}^\infty > 0 \), then \( m_{\text{ratio}}^\infty < 1 \), and \( \eta_{\text{hard}}(m_{\text{ratio}}^\infty, T) < 1 \), contradiction. Therefore, we conclude both \( m_{\text{easy}}^i \) and \( m_{\text{hard}}^i \) converge to 0. This indicates the invalidity converges to zero.
Simplifying the dynamical system. To further simplify the problem, we make a strong assumption that \( \eta \) is linear in \( m \). Then, we must have

\[
\begin{align*}
\eta_{\text{easy}}(m, T) &= 1 - \xi_{\text{easy}}(T) \cdot m, \\
\eta_{\text{hard}}(m, T) &= 1 - \xi_{\text{hard}}(T) \cdot (1 - m),
\end{align*}
\]

where \( \xi \in [0, 1], \xi(0) = 0, \xi' > 0, \xi'' < 0 \) for \( \xi \in [\xi_{\text{easy}}, \xi_{\text{hard}}] \). We also have \( \xi'_{\text{easy}} > \xi'_{\text{hard}} \) and therefore \( \xi_{\text{easy}} > \xi_{\text{hard}} \).

Optimal \( T \) and \( R \) from bounds. We have

\[
m_{\text{easy}}^{i+1} + m_{\text{hard}}^{i+1} = m_{\text{easy}}^i + m_{\text{hard}}^i - \frac{\xi_{\text{easy}}(T)(m_{\text{easy}}^i)^2 + \xi_{\text{hard}}(T)(m_{\text{hard}}^i)^2}{m_{\text{easy}}^i + m_{\text{hard}}^i}.
\]

Because \( \xi_{\text{easy}}(T) \geq \xi_{\text{hard}}(T) \), we have

\[
\frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)} (m_{\text{easy}}^i + m_{\text{hard}}^i) \leq \frac{\xi_{\text{easy}}(T)(m_{\text{easy}}^i)^2 + \xi_{\text{hard}}(T)(m_{\text{hard}}^i)^2}{m_{\text{easy}}^i + m_{\text{hard}}^i} \leq \xi_{\text{easy}}(T)(m_{\text{easy}}^i + m_{\text{hard}}^i).
\]

This leads to

\[
1 - \xi_{\text{easy}}(T) \leq \frac{m_{\text{easy}}^{i+1} + m_{\text{hard}}^{i+1}}{m_{\text{easy}}^i + m_{\text{hard}}^i} \leq 1 - \frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)},
\]

and therefore

\[
(1 - \xi_{\text{easy}}(T))^R \leq \frac{m_{\text{easy}}^R + m_{\text{hard}}^R}{m_{\text{easy}}^0 + m_{\text{hard}}^0} \leq \left(1 - \frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)}\right)^R.
\]

Assume the number of queries, \( T \times R \), is fixed. Then, the optimal \( T \) from the lower bound is

\[
T_{\text{low}}^* = \arg\min_T \frac{1}{T} \log(1 - \xi_{\text{easy}}(T)).
\]

By setting the derivative to be zero, we have \( T_{\text{low}}^* \) is the solution to

\[
-T\xi'_{\text{easy}}(T) = (1 - \xi_{\text{easy}}(T)) \log(1 - \xi_{\text{easy}}(T)).
\]

Similarly, the optimal \( T \) from the upper bound is

\[
T_{\text{upp}}^* = \arg\min_T \frac{1}{T} \log \left(1 - \frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)}\right).
\]

By setting the derivative to be zero, we have \( T_{\text{upp}}^* \) is the solution to

\[
-T\frac{\xi'_{\text{easy}}(T)\xi_{\text{hard}}(T)^2 + \xi'_{\text{hard}}(T)\xi_{\text{easy}}(T)^2}{(\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T))^2} = \left(1 - \frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)}\right) \log \left(1 - \frac{\xi_{\text{easy}}(T)\xi_{\text{hard}}(T)}{\xi_{\text{easy}}(T) + \xi_{\text{hard}}(T)}\right).
\]

C Feasibility of Discriminator in the Classifier-based Setting

The solution to (5) and (6) is:

\[
D^*(x) = \begin{cases} 
\frac{\alpha_+ p_{\text{data}} + \alpha_- (1 - \lambda) p_{\text{true}} + \lambda p_{\text{fake}} + (1 - \lambda) |p_{\text{true}}|}{p_{\text{data}} + \lambda p_{\text{true}} + (1 - \lambda) |p_{\text{true}}|} & \text{if } f(x) \geq \tau \\
\alpha_- & \text{if } f(x) < \tau
\end{cases}
\]

which satisfies \( D^* \in [0, 1] \). Therefore, (5) is feasible with the guide function defined in (6).
D Experimental Setup

Pre-training. We use DCGAN [Radford et al., 2015] with latent dimension = 128 as the model. The pre-trained model is trained with label smoothing ($\alpha_+ = 0.9$, $\alpha_- = 0.1$):

$$\min_G \max_D \mathbb{E}_{x \sim X} [\alpha_+ \log D(x) + (1 - \alpha_+) \log(1 - D(x))]$$

$$\quad + \mathbb{E}_{z \sim \mathcal{N}(0, I)} [\alpha_- \log D(G(z)) + (1 - \alpha_-) \log(1 - D(G(z)))]$$

We use Adam optimizer with learning rate $= 2 \times 10^{-4}$, $\beta_1 = 0.5$, $\beta_2 = 0.999$ to optimize both the generator and the discriminator. The networks are trained for 200 epochs with a batch size of 64. For each iteration over one mini-batch, we let $K_D$ be the number of times to update the discriminator, and $K_G$ the number of times to update the generator. We use $K_D = 1$ and $K_G = 5$ to train.

Data forgetting. The setup is similar to the pre-training except for two differences. The number of epochs is much smaller: 8 for MNIST, 30 for CIFAR, and 40 for STL-10. We let $K_G = 1$ for MNIST and CIFAR and $K_G = 5$ for STL-10.

Evaluation. To measure invalidity, we generate 50K samples, and compute the fraction of these samples that are not valid (e.g., classified as the label to be forgotten, or with pre-defined biases). It is the lower the better. The invalidity for forgetting labels is measured based on label classifiers. We use pre-trained classifiers on these datasets.\footnote{https://github.com/aaron-xichen/pytorch-playground (MIT license)}

The other evaluation metric is generation quality. The inception score (IS) [Salimans et al., 2016] is computed based on logit distributions from the above pre-trained classifiers. It is the higher the better. The Frechet Inception Distance (FID) [Heusel et al., 2017] is computed based on an open-sourced PyTorch implementation.\footnote{https://github.com/mseitzer/pytorch-fid (Apache-2.0 license)} It is the lower the better.

When computing these quality metrics, we generate 50K samples, and compare to the set of valid training samples: \{x \in X : x \notin \Omega\}. Therefore, when $X \cap \Omega$ is not the empty set (such as forgetting labels in Section 4.1), the quality measure of the model after data forgetting is not directly comparable to the pre-trained model, but these scores among different forgetting algorithms are comparable and give intuition to the generation quality. When $X \cap \Omega$ is the empty set (such as de-biasing in Section 4.2), the quality measures of the pre-trained model and the model after data forgetting are directly comparable.
E Forgetting Labels

E.1 Forgetting Label 0

We include results for forgetting label 0 in this section. We look at MNIST, CIFAR-10, and STL-10 datasets with different sets of hyper-parameters. With the base set of hyper-parameters, while different forgetting algorithms perform better on different datasets, they are highly comparable with each other. We find the results are worse when there is no label smoothing ($\alpha_+ = 1$, $\alpha_- = 0$), indicating label smoothing is important for data forgetting. We discuss results after one epoch in Appendix E.1.1, the effect of $\lambda$ in Appendix E.1.3, and the effect of $T$ in Table 2.

Results for MNIST.

| Model       | Epochs | Inv(↓) | IS(↑) |
|-------------|--------|--------|--------|
| pre-trained | 200    | $1.095 \times 10^{-1}$ | 7.82   |
| Base: $\alpha_+ = 0.95, \alpha_- = 0.05, \lambda = 0.85$ | 8 | $2 \times 10^{-5}$ | 7.19   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 8 | $4 \times 10^{-5}$ | 6.97   |
| $\alpha_+ = 1.0, \alpha_- = 0.0$ | 8 | $2 \times 10^{-5}$ | 7.02   |
| $\lambda = 0.8$ | 8 | $2 \times 10^{-5}$ | 7.18   |
| $\lambda = 0.9$ | 8 | $4 \times 10^{-5}$ | 7.16   |
| $\lambda = 0.95$ | 8 | $5.2 \times 10^{-4}$ | 7.19   |
| $\lambda = 0.8$ | 8 | $2.98 \times 10^{-3}$ | 7.09   |

| Model       | Epochs | Inv(↓) | IS(↑) |
|-------------|--------|--------|--------|
| pre-trained | 200    | $1.095 \times 10^{-1}$ | 7.82   |
| Base: $\alpha_+ = 0.95, \alpha_- = 0.05, \lambda = 0.85, T = 1000$ | 8 | $3.4 \times 10^{-3}$ | 7.17   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 8 | $3.72 \times 10^{-3}$ | 4.81   |
| $\lambda = 0.8$ | 8 | $2 \times 10^{-4}$ | 7.25   |
| $\lambda = 0.9$ | 8 | $2.2 \times 10^{-4}$ | 7.07   |
| $\lambda = 0.95$ | 8 | $8.8 \times 10^{-4}$ | 7.12   |
| $T = 400$ | 8 | $2.8 \times 10^{-4}$ | 7.16   |
| $T = 2000$ | 4 | $2.8 \times 10^{-4}$ | 7.11   |
| $\lambda = 0.8$ | 1 | $2.8 \times 10^{-3}$ | 6.99   |

| Model       | Epochs | Inv(↓) | IS(↑) |
|-------------|--------|--------|--------|
| pre-trained | 200    | $1.095 \times 10^{-1}$ | 7.82   |
| Base: $\alpha_+ = 0.95, \alpha_- = 0.05, \lambda = 0.85, \tau = 0.5$ | 8 | $1.4 \times 10^{-5}$ | 7.09   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 8 | $2.06 \times 10^{-3}$ | 6.08   |
| $\lambda = 0.8$ | 8 | $6 \times 10^{-5}$ | 7.15   |
| $\lambda = 0.9$ | 8 | $8 \times 10^{-5}$ | 7.18   |
| $\lambda = 0.95$ | 8 | $7.2 \times 10^{-4}$ | 7.24   |
| $\tau = 0.3$ | 8 | $1.2 \times 10^{-5}$ | 7.12   |
| $\tau = 0.7$ | 8 | $6 \times 10^{-5}$ | 7.22   |
| $\lambda = 0.8$ | 1 | $2.54 \times 10^{-3}$ | 7.11   |
Results for CIFAR-10.

Table 9: Data-based forgetting algorithm.

| Model   | Epochs | Inv(↓)     | FID(↓) |
|---------|--------|------------|--------|
| pre-trained | 200    | 1.291 × 10⁻¹ | 36.2   |
| Base: $\alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8$ | 30    | 7.4 × 10⁻³  | 35.8   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 30    | 8.0 × 10⁻³  | 34.4   |
| $\alpha_+ = 0.9, \alpha_- = 0.0$ | 30    | 8.9 × 10⁻³  | 34.2   |
| $\lambda = 0.9$ | 30    | 2.10 × 10⁻²  | 29.2   |
| $\lambda = 0.95$ | 30    | 4.21 × 10⁻²  | 26.2   |
| Base  | 1     | 3.99 × 10⁻²  | 35.7   |

Table 10: Validity-based forgetting algorithm.

| Model   | Epochs | Inv(↓)     | FID(↓) |
|---------|--------|------------|--------|
| pre-trained | 200    | 1.291 × 10⁻¹ | 36.2   |
| Base: $\alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8, T = 1000$ | 30    | 7.9 × 10⁻³  | 35.3   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 30    | 8.1 × 10⁻³  | 35.3   |
| $\alpha_+ = 0.9, \alpha_- = 0.0$ | 30    | 8.1 × 10⁻³  | 34.1   |
| $\lambda = 0.9$ | 30    | 2.54 × 10⁻²  | 28.1   |
| $\lambda = 0.95$ | 30    | 3.57 × 10⁻²  | 27.8   |
| $T = 400$ | 75     | 4.5 × 10⁻³  | 35.1   |
| $T = 2000$ | 15    | 1.00 × 10⁻²  | 31.9   |
| Base  | 1     | 3.85 × 10⁻²  | 36.2   |

Table 11: Classifier-based forgetting algorithm.

| Model   | Epochs | Inv(↓)     | FID(↓) |
|---------|--------|------------|--------|
| pre-trained | 200    | 1.291 × 10⁻¹ | 36.2   |
| Base: $\alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8, \tau = 0.5$ | 30    | 1.28 × 10⁻²  | 33.7   |
| $\alpha_+ = 0.9, \alpha_- = 0.1$ | 30    | 1.04 × 10⁻²  | 32.9   |
| $\alpha_+ = 0.9, \alpha_- = 0.0$ | 30    | 6.3 × 10⁻³  | 32.8   |
| $\lambda = 0.9$ | 30    | 2.25 × 10⁻²  | 28.6   |
| $\lambda = 0.95$ | 30    | 4.26 × 10⁻²  | 26.9   |
| $\tau = 0.3$ | 30    | 9.6 × 10⁻³  | 34.8   |
| $\tau = 0.7$ | 30    | 1.05 × 10⁻²  | 35.2   |
| Base  | 1     | 3.47 × 10⁻²  | 37.8   |
Results for STL-10.

Table 12: Data-based forgetting algorithm.

| Model       | Epochs | Inv (↓)  | FID (↓) |
|-------------|--------|----------|---------|
| pre-trained | 200    | 6.23 × 10^{-2} | 79.1    |
| Base: \( \alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8 \) | 40 | 7.8 × 10^{-2} | 74.3 |
| \( \alpha_+ = 0.9, \alpha_- = 0.1 \) | 40 | 7.6 × 10^{-2} | 75.8 |
| \( \alpha_+ = 0.9, \alpha_- = 0.0 \) | 40 | 1.42 × 10^{-3} | 82.7 |
| \( \lambda = 0.9 \) | 40 | 2.88 × 10^{-3} | 76.9 |
| \( \lambda = 0.95 \) | 40 | 6.71 × 10^{-3} | 78.2 |
| Base | 1 | 6.397 × 10^{-3} | 75.1 |

Table 13: Validity-based forgetting algorithm.

| Model       | Epochs | Inv (↓)  | FID (↓) |
|-------------|--------|----------|---------|
| pre-trained | 200    | 6.23 × 10^{-2} | 79.1    |
| Base: \( \alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8, T = 1000 \) | 40 | 4.8 × 10^{-2} | 79.3 |
| \( \alpha_+ = 0.9, \alpha_- = 0.1 \) | 40 | 8.2 × 10^{-2} | 76.5 |
| \( \alpha_+ = 0.9, \alpha_- = 0.0 \) | 40 | 1.44 × 10^{-3} | 77.0 |
| \( \lambda = 0.9 \) | 40 | 4.32 × 10^{-3} | 75.9 |
| \( \lambda = 0.95 \) | 40 | 8.95 × 10^{-3} | 75.3 |
| \( T = 400 \) | 100 | 1.00 × 10^{-3} | 75.1 |
| \( T = 2000 \) | 20 | 1.00 × 10^{-3} | 75.1 |
| Base | 1 | 8.997 × 10^{-3} | 79.5 |

Table 14: Classifier-based forgetting algorithm.

| Model       | Epochs | Inv (↓)  | FID (↓) |
|-------------|--------|----------|---------|
| pre-trained | 200    | 6.23 × 10^{-2} | 79.1    |
| Base: \( \alpha_+ = 0.9, \alpha_- = 0.05, \lambda = 0.8, \tau = 0.5 \) | 40 | 8.6 × 10^{-2} | 75.4 |
| \( \alpha_+ = 0.9, \alpha_- = 0.1 \) | 40 | 9.5 × 10^{-2} | 74.8 |
| \( \alpha_+ = 0.9, \alpha_- = 0.0 \) | 40 | 1.62 × 10^{-3} | 82.0 |
| \( \lambda = 0.9 \) | 40 | 3.10 × 10^{-3} | 77.2 |
| \( \lambda = 0.95 \) | 40 | 6.89 × 10^{-3} | 76.2 |
| \( \tau = 0.3 \) | 40 | 8.8 × 10^{-3} | 73.8 |
| \( \tau = 0.7 \) | 40 | 1.34 × 10^{-3} | 76.1 |
| Base | 1 | 6.81 × 10^{-3} | 75.6 |
E.1.1 Invalidity after one epoch.

We compare invalidity after only one epoch of data forgetting. These forgetting algorithms are highly comparable to each other. We hypothesize that the classifier-based algorithm performs the best on MNIST because a label classifier on MNIST (and its gradient information) can be very accurate, while this may not be true for CIFAR-10 and STL-10.

Table 15: Invalidity after one epoch of data forgetting.

| Dataset | Scale | Pre-trained | Data-based | Validity-based | Classifier-based |
|---------|-------|-------------|------------|---------------|-----------------|
| MNIST   | \(10^{-3}\) | \(1.1 \times 10^2\) | \(4.7 \pm 0.8\) | \(5.6 \pm 0.9\) | \(3.9 \pm 0.9\) |
| CIFAR-10| \(10^{-2}\) | \(1.3 \times 10^1\) | \(3.7 \pm 0.5\) | \(3.7 \pm 0.8\) | \(3.8 \pm 0.3\) |
| STL-10  | \(10^{-3}\) | \(6.2 \times 10^1\) | \(9.1 \pm 0.9\) | \(8.6 \pm 0.9\) | \(10.6 \pm 1.2\) |

E.1.2 Quality during data forgetting

We plot quality measure of different data forgetting algorithms on different datasets during the forgetting process, complementary to the invalidity in Fig. 1. We find the variances of quality measure is higher than the invalidity, but different forgetting algorithms are generally comparable.

Figure 4: Quality measure during data forgetting. Mean and standard errors are plotted for five random seeds.

E.1.3 Trade-off by alternating \(\lambda\).

We study the effect of \(\lambda\) (hyper-parameter in (2)) in Table 16 and Fig. 5. There is a trade-off by alternating \(\lambda\): a larger \(\lambda\) (less fake data from the forgetting set) leads to better quality measure, and a smaller \(\lambda\) (more fake data from the forgetting set) leads to better invalidity.

Table 16: Invalidity after data forgetting for different \(\lambda\) in the classifier-based forgetting algorithm.

| \(\lambda\) | MNIST \(\text{Inv}(↑)\) | IS(↑) | CIFAR-10 \(\text{Inv}(↓)\) | FID(↓) | STL-10 \(\text{Inv}(↓)\) | FID(↓) |
|------------|----------------|-------|----------------|-------|----------------|-------|
| 0.8        | \(0.6 \times 10^{-4}\) | 7.15  | \(1.28 \times 10^{-2}\) | 33.7  | \(0.86 \times 10^{-3}\) | 75.4  |
| 0.9        | \(0.8 \times 10^{-4}\) | 7.18  | \(2.25 \times 10^{-2}\) | 28.6  | \(3.10 \times 10^{-3}\) | 77.2  |
| 0.95       | \(7.2 \times 10^{-4}\) | 7.24  | \(4.26 \times 10^{-2}\) | 26.9  | \(6.89 \times 10^{-3}\) | 76.2  |

Figure 5: Invalidity during data forgetting for different \(\lambda\) in the classifier-based forgetting algorithm.
### E.2 Forgetting Other Labels

We also demonstrate results for forgetting other labels with our data forgetting algorithms. We use the base set of hyper-parameters in Appendix E.1. Similar to forgetting label 0, all forgetting algorithms can largely reduce invalidity, and they are highly comparable to each other. The classifier-based forgetting algorithm achieves slightly better generation quality on MNIST and CIFAR-10. In terms of different labels, we find some labels are harder to forget in the sense that the invalidity scores for these labels are higher than other scores, such as label 9 in MNIST, and label 3 in CIFAR-10 and STL-10.

#### Table 17: Forgetting other labels on MNIST.

| Label | Pre-trained Inv(↓) | Data-based Inv(↓) | Validity-based Inv(↓) | Classifier-based Inv(↓) |
|-------|--------------------|------------------|----------------------|------------------------|
| 1     | 10.2% 7.81         | 0.002% 7.01      | 0.000% 7.21          | 0.008% 7.13            |
| 2     | 8.6% 7.81          | 0.022% 7.22      | 0.012% 7.20          | 0.028% 7.28            |
| 3     | 11.5% 7.81         | 0.126% 7.20      | 0.136% 7.24          | 0.134% 7.19            |
| 4     | 9.9% 7.81          | 0.138% 7.19      | 0.092% 7.21          | 0.104% 7.26            |
| 5     | 8.7% 7.81          | 0.048% 7.22      | 0.046% 7.21          | 0.056% 7.24            |
| 6     | 9.0% 7.81          | 0.020% 7.04      | 0.022% 7.07          | 0.010% 7.12            |
| 7     | 11.4% 7.81         | 0.114% 7.24      | 0.124% 7.34          | 0.088% 7.32            |
| 8     | 9.1% 7.81          | 0.198% 7.48      | 0.248% 7.35          | 0.302% 7.51            |
| 9     | 10.7% 7.81         | 0.486% 7.30      | 0.414% 7.36          | 0.545% 7.26            |

#### Table 18: Forgetting other labels on CIFAR-10.

| Label | Pre-trained Inv(↓) | Data-based Inv(↓) | Validity-based Inv(↓) | Classifier-based Inv(↓) |
|-------|--------------------|------------------|----------------------|------------------------|
| 1     | 1.5% 36.24         | 0.032% 35.06     | 0.014% 35.23         | 0.082% 33.40           |
| 2     | 11.0% 36.24        | 1.311% 31.67     | 1.537% 31.65         | 1.564% 28.34           |
| 3     | 15.8% 36.24        | 3.013% 30.10     | 3.491% 31.01         | 2.534% 28.06           |
| 4     | 16.8% 36.24        | 1.752% 30.36     | 1.754% 31.26         | 1.590% 29.72           |
| 5     | 6.7% 36.24         | 0.799% 30.76     | 0.985% 30.90         | 1.461% 31.36           |
| 6     | 9.3% 36.24         | 0.797% 29.81     | 1.071% 31.65         | 0.755% 29.64           |
| 7     | 8.6% 36.24         | 0.789% 33.48     | 0.496% 33.40         | 1.325% 34.15           |
| 8     | 10.3% 36.24        | 0.218% 38.96     | 1.451% 38.59         | 0.496% 34.56           |
| 9     | 7.1% 36.24         | 0.138% 38.13     | 0.186% 37.74         | 0.216% 36.85           |

#### Table 19: Forgetting other labels on STL-10.

| Label | Pre-trained Inv(↓) | Data-based Inv(↓) | Validity-based Inv(↓) | Classifier-based Inv(↓) |
|-------|--------------------|------------------|----------------------|------------------------|
| 1     | 9.0% 79.00         | 1.273% 74.89     | 2.168% 73.91         | 1.900% 75.34           |
| 2     | 6.2% 79.00         | 0.158% 72.22     | 0.132% 72.39         | 0.176% 75.75           |
| 3     | 14.9% 79.00        | 3.772% 77.24     | 3.732% 76.80         | 4.412% 75.19           |
| 4     | 8.2% 79.00         | 1.634% 81.91     | 1.345% 82.82         | 1.425% 83.25           |
| 5     | 15.1% 79.00        | 2.072% 76.85     | 3.383% 80.40         | 5.041% 77.74           |
| 6     | 8.7% 79.00         | 0.462% 80.82     | 0.518% 78.17         | 0.745% 79.63           |
| 7     | 10.7% 79.00        | 2.973% 77.53     | 1.838% 78.57         | 2.180% 77.58           |
| 8     | 9.5% 79.00         | 0.304% 79.56     | 0.272% 78.06         | 0.352% 77.07           |
| 9     | 11.6% 79.00        | 0.817% 76.70     | 0.947% 78.37         | 0.941% 76.37           |
E.3 Visualization

Figure 6: Visualization of the data forgetting process of invalid samples when forgetting labels. The first column is generated by the pre-trained generator, and the $i$-th column is generated after $k \cdot (i - 1)$ epochs of data forgetting. Left: MNIST with $k = 1$. Right: top is CIFAR-10 and bottom is STL-10, both with $k = 4$. We can see samples associated with invalid labels are gradually pushed to other labels, but a high-level visual similarity is kept.
E.4 Detailed Setup of Forgetting Multiple Sets

We use 30K images from CelebA-64 as the training set. All other hyper-parameters are the same as the base set for STL-10 in Appendix E.1, except that we run data forgetting algorithms for only 5 epochs. We train attribute classifiers for each attribute separately. The attribute classifiers are fine-tuned from open-sourced pre-trained ResNet [He et al., 2016]. \(^4\) We fine-tune the network for 20 epochs using the SGD optimizer with learning rate $= 1 \times 10^{-3}$, momentum $= 0.9$, and a batch size of 64.

\(^4\)https://pytorch.org/vision/stable/models.html
F Model De-biasing

F.1 Boundary Artifacts

Let the image size be $W \times H$ (the number of channels is 1 for MNIST). For an integer margin, the boundary pixels are defined as

$\{(i, j) : 1 \leq i \leq \text{margin} \text{ or } W - \text{margin} < i \leq W, 1 \leq j \leq \text{margin} \text{ or } H - \text{margin} < j \leq H\}$.

Then, the validity function for boundary artifacts is defined as

$$v(x) = 1 \sum_{(i,j) \in \text{boundary pixels}} x_{ij} < \tau_b,$$

where $\tau_b = 4.25$ for margin = 1 and 10.0 for margin = 2. For these values, no training data has the boundary artifact. Quantitative results are in Table 4. We visualize some samples with boundary artifacts in Fig. 7a. We run the validity-based forgetting algorithm with $\lambda = 0.98, \alpha_+ = 0.95, \alpha_- = 0.05$ for 4 epochs. After de-biasing via data forgetting, these samples have less boundary pixels, as shown in Fig. 7b.

![Figure 7](image)

(a) Samples with boundary artifacts.  
(b) Samples after de-biasing via data forgetting.

Figure 7: De-biasing boundary artifacts with the validity-based data forgetting algorithm. Margin = 1 and $T = 40K$.

F.2 Label Biases

We use classifier-based forgetting algorithm to de-bias label biases. For MNIST, we use $\lambda = 0.8, \alpha_+ = 0.95, \alpha_- = 0.05$ and run for 8 epochs. For CIFAR-10, we use $\lambda = 0.0, \alpha_+ = 0.9, \alpha_- = 0.05$ and run for 30 epochs. Quantitative results are in Table 5. We visualize semantically ambiguous samples generated by the pre-trained model in Fig. 8a. After de-biasing via data forgetting, these samples become less semantically ambiguous, as shown in Fig. 8b.

![Figure 8](image)

(a) Samples with label-biases.  
(b) Samples after de-biasing via data forgetting.

Figure 8: De-biasing label biases with the classifier-based data forgetting algorithm ($\tau = 0.7$).
Sample-level forgetting difficulty. We visualize some most and least difficult-to-forget samples according to the forgetting scores in Fig. 9. We find the most difficult-to-forget samples are visually atypical, while the least difficult-to-forget samples are visually more common.

Figure 9: Samples that are most and least difficult-to-forget in MNIST and CIFAR-100.
**Label-level forgetting difficulty.** We sort all labels according to their average forgetting scores. This tells us which labels are easier or harder to forget. The results for MNIST are in Fig. 10a. Consistent with Table 17, label 9 is the most difficult label to forget. The least and most difficult-to-forget labels for CIFAR-100 are shown in Fig. 10c and 10b.

(a) Label-level forgetting difficulty for MNIST. Top: the most difficult to forget. Bottom: the least difficult to forget.

(b) Label-level forgetting difficulty for CIFAR-100 (10 most difficult-to-forget labels). Top: the most difficult to forget. Bottom: the least difficult to forget.

(c) Label-level forgetting difficulty for CIFAR-100 (10 least difficult-to-forget labels). Top: the most difficult to forget. Bottom: the least difficult to forget.

Figure 10: Label-level forgetting difficulty for MNIST and CIFAR-100. A large forgetting score means a label is easier to be forgotten. We find some labels are more difficult to forget than others.
Relationship to Adversarial Samples

Consider the classifier-based forgetting algorithm. We fix the discriminator and only update the generator. Then, the generator is trained to fool both the discriminator and the classifier at the same time. We may define generated samples from this generator as on-manifold adversarial samples to \((D, f)\). Note that on-manifold samples are not necessarily visually clear samples; instead, they could be high likelihood samples according to the inductive bias of the generative model.

When training, we use the classifier-based forgetting algorithm with \(\tau = 0.5, \alpha_+ = 0.95, \alpha_- = 0.05, \lambda = 0.85\), and a batch size of 64. We “forget” one label at a time, similar to experiments in Appendix E.2. After each iteration over one mini-batch, we generate samples with the same latents. The visualization is shown below. There are several interesting findings shown in the figures. First, the generated samples tend to have less pixels. Second, the generated samples tend to be dis-connected. Third, there are some general patterns across these generated samples (for each label): for example, there are pixels in the middle of zeroes, the bottom of sevens vanish, and nines are split from the middle. We conjecture that these observations correspond to the inductive bias of the discriminator and adversarial samples of the classifier.
Figure 11: Generated samples when we only train the generator and fix the discriminator and the classifier with the classifier-based forgetting algorithm. The first column is generated by the pre-trained generator, and the $i$-th column is generated after $i-1$ iterations (up to 20 iterations).