Majorana Neutrinos from Inverse Seesaw in Warped Extra Dimension

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Abstract

We propose the inverse seesaw mechanism as a way to understand small Majorana masses for neutrinos in warped extra dimension models with seesaw scale in the TeV range. The ultra-small lepton number violation needed in implementing inverse seesaw mechanism in 4D models is explained in this model as a consequence of lepton number breaking occurring on the Planck brane. We construct realistic models based on this idea that fit observed neutrino oscillation data for both normal and inverted mass patterns. We compute the corrections to light neutrino masses from the Kaluza-Klein modes and show that they are small in the parameter range of interest. Another feature of the model is that the absence of global parity anomaly implies the existence of at least one light sterile neutrino with sterile and active neutrino mixing in the range suggested by the LSND and MiniBooNE observations.
1 Introduction

Randall-Sundrum (RS) hypothesis of the existence of a fifth space dimension with a warped metric [1] provides an alternative solution to the gauge hierarchy problem, which is distinct from the supersymmetric (SUSY) approach. Different embedding of the Standard Model (SM) into warped extra dimension (WED) have been discussed and there has been considerable attention focused on studying the phenomenological implications and consistencies of the WED models [2]. Understanding the smallness of neutrino masses in WED models however has been quite non-trivial. In contrast, in SUSY framework, a simple extension of the Minimal Supersymmetric Standard Model (MSSM) by the addition of three right-handed neutrinos leads via type I seesaw mechanism [3] to a set of three light neutrinos. The formula for neutrino masses in this case has the form

\[ M_\nu \simeq -m_D m_N^{-1} m_T^0 \] for \( m_D \ll m_N \) where \( m_D \) is the Dirac mass and \( m_N \) the Majorana mass of right-handed (RH) neutrinos. Since \( m_N \) is a new scale unrelated to the SM gauge group, its value can be much higher than the weak scale \( v_w \) whereas \( m_D \) breaks SM gauge group and is of order of \( v_w \), making \( M_\nu \) much smaller than the known quark and lepton masses. Typical values of the seesaw scale \( m_N \) in Grand Unification Theories (GUTs) are of order \( 10^{14} \) GeV. A common theme of all seesaw-like solutions to neutrino masses is that neutrinos are Majorana fermions implying observable lepton number violating processes.

Several ongoing searches for lepton number violating process such as neutrinoless double beta decay of nuclei are under way to test this hypothesis.

There have been several interesting proposals to understand small neutrino masses in WED models [4–11]. A generic prediction of these models (with the exception of [6, 9, 10]) is that neutrinos are Dirac fermions so that total lepton number remains a good symmetry of nature and processes such as neutrinoless double beta decay and \( K^+ \to \pi^- \mu^+ \mu^+ \) etc. that violate lepton number should not be observed. It has also been argued that this kind of approach provides a simple way to understand the flavor structure among neutrinos (a much milder hierarchy for neutrinos compared to charged leptons) [11]. The models [6, 9, 10] that have Majorana neutrinos use type I seesaw for the purpose so that the seesaw scale is in the range of \( 10^{14} \) GeV or higher and not directly accessible at the LHC.

In this paper, we discuss an alternative class of WED models where neutrinos are Majorana fermions and obtain their masses from a different mechanism, known in literature as the inverse seesaw mechanism [12]. Its implementation requires adding two gauge singlet chiral fields \( N \) and \( S \) per family to the SM such that they form a pseudo-Dirac pair with mass in the TeV range. The smallness of neutrino masses is related to the extent of their “pseudo-Dirac-ness” which is governed by a tiny lepton number breaking mass term for the fields \( N, S \) (denoted by \( m_{S,N} \)). The generic mass formula for the neutrino mass matrix is given by

\[ M_\nu \simeq -m_D \left( m_{SN} m_S^{-1} m_{SN}^T \right)^{-1} m_D^T \] with \( m_S \ll m_D \ll m_{SN} \) where \( m_{SN} \) is the Dirac mass that couples \( N \) and \( S \). Unlike the usual four-dimensional inverse seesaw models, where smallness of \( m_S \) requires introducing a tiny parameter by hand, we show here that in the WED models, one can have this smallness dictated by parameters of order one that govern the location of the 5D profile of the \( S \) fields in the bulk. In this sense, the RS framework is ideally suited to the implementation of inverse seesaw. Furthermore, in contrast with the type I embedding in WED, the seesaw scale in this case is in the TeV range so that it is accessible at the LHC.
We implement the inverse seesaw mechanism in WED models in this paper and present realistic examples that fit current neutrino oscillation data. An interesting outcome of our model is that it predicts the existence of an eV mass sterile neutrino in a natural manner due to the fact that absence of global parity anomaly requires that there be an even number of singlet S-fermions: four in our case out of which only three are required for inverse seesaw, the remaining one will become the light sterile neutrino. We note some of the properties of the sterile neutrino predicted in our model.

This paper is organized as follows: in Sec. 2 we review the implementation of type I seesaw mechanism in WED \cite{6}. In Sec. 3 we present our model using the inverse seesaw mechanism. We first illustrate the appearance of light sterile neutrino with a toy model. We then consider mechanism in WED \cite{6}. In Sec. 4, we comment briefly on some phenomenological implication of the model, in particular the effect on the neutrinoless double beta decay.

2 Type I seesaw in warped extra dimension

The Randall-Sundrum (RS) model \cite{1} has the warped metric
\[ ds^2 = G_{AB} dx^A dx^B = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad \sigma(y) = k |y|, \]
where \( k \) is the AdS curvature, \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) and the fifth dimension \(-\pi R \leq y \leq \pi R\) is taken to be a \( S_1/Z_2 \) orbifold.

As discussed in Ref. \cite{6}, one way to implement type I seesaw in WED is to extend the SM by adding three RH neutrinos \( N \), one for each family and including their Yukawa couplings in 5D. The bulk action for this model can be written as follows:
\[ S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G} \left[ N_i E_a^A \gamma^a D_A N - m_{DN}^2 N N - \left( \frac{1}{Z} m_{N_5} \overline{\sigma} + \lambda_{N_5} \overline{\tau} N H + \text{h.c.} \right) \right], \]
where \( \ell \) and \( H \) are respectively the \( SU(2)_L \) lepton and Higgs doublets \cite{1}. In Eq. (2), \( a, b, ... \) and \( A, B, ... \) are respectively the flat and curve indices which run from 0 to 4. We have \( \gamma^a = (\gamma^\mu, i\gamma^5) \) for \( a = 0, 1, 2, 3, 4 \) and the spacetime covariant derivative \( D_A = \partial_A + \omega_A \).

From the warped metric, the inverse vielbein is given by \( E_a^A = \text{diag}(e^\sigma, e^\sigma, e^\sigma, e^\sigma, 1) \) while the spin connection is given by \( \omega_A = \left( \frac{1}{2} \sigma' \gamma^\mu, 0 \right) \) where \( \sigma' = d\sigma/dy = k \text{sgn}(y) \). We then determine \( \sqrt{G} = \sqrt{\text{det} G_{AB}} = e^{-4\sigma} \). For a spinor \( \Psi, \Psi^c = C\gamma^0 \Psi^* \) is the corresponding charge conjugate spinor with \( C = i\gamma^2 \gamma^0 \gamma^5 = \gamma^2 \gamma^0 \gamma^4 \) such that \( \gamma^{aT} = -C \gamma^a C^{-1} \).

If we assign the lepton number \( L \) for both \( N, \ell \) as \( L = 1 \) the only term that violates \( L \) is Majorana masses \( m_{DN} \) in the action \cite{6}. In the standard notation where the warped factor is given by \( e^{-k|y|} \) with the Planck and TeV branes located at \( y = 0 \) and \( y = \pi R \) respectively, the fermion zero modes are given by the 5D profile \( \tilde{f}^{(0)}(y) = \sqrt{\frac{\pi k R (1-2\sigma)}{e^{\pi k R (1-2\sigma)} - 1}} \left( \frac{1}{2} - \sigma \right) \) with Dirac

\(^1\)To avoid clutter, we have suppressed the family indices of \( \ell \) and \( N \).
mass parameter \( c_f = m_f/k \) and \( \sigma \equiv k|y| \). We follow a definition of the profile wave functions whose normalization condition does not include any extra warped factor (i.e. with respect to flat metric) such that

\[
\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \tilde{f}^{(m)}(y) \tilde{f}^{(n)}(y) = \delta_{mn}.
\]

(3)

It is clear from this that if \( c_f > \frac{1}{2} \), the profile peaks near the Planck brane whereas if \( c_f < \frac{1}{2} \), it peaks near the TeV brane. Electroweak precision constraints demand that the 5D profiles of charged leptons peak near the Planck brane due to small wave function overlaps with the KK modes [2]. The RH neutrinos being electroweak singlets do not however have any such constraints. To implement the seesaw mechanism, lepton number is assumed to be broken at the Planck brane via the Majorana mass term

\[
m_N^5 = d_N \delta(y)
\]

where \( d_N \) is a dimensionless number. The zero mode profile of the RH neutrino is chosen to peak near the TeV brane i.e. \( c_N < 1/2 \).

In order to obtain fermion masses, we need to know the Higgs doublet profile. We assume that it is localized on the TeV brane. To estimate the order of magnitude of the model parameters, we work with only one generation of fermion. Denoting the Dirac mass parameters for the lepton doublet, lepton singlet and RH neutrino respectively by \( c_\ell > 1/2 \), \( c_{e_R} > 1/2 \) and \( c_N < 1/2 \), we can write the effective 4D charged lepton mass \( m_\ell \), Dirac mass for the neutrinos \( m_D \) and the Majorana mass \( m_N \) for the RH neutrinos as:

\[
m_\ell \sim k \times e^{-k\pi R(c_\ell+c_{e_R})},
\]

\[
m_D \sim k \times e^{-k\pi R(c_\ell+\frac{1}{2})},
\]

\[
m_N \sim k \times e^{k\pi R(2c_N-1)},
\]

(4)

which leads to light neutrino mass

\[
m_\nu \sim k \times e^{-2k\pi R(c_\ell+c_N)}.
\]

(5)

Here all 5D dimensionless couplings are assumed to be unity. With \( k\pi R \sim 37 \) and \( k = 2.4 \times 10^{18} \text{ GeV} \), for example, in order to get the right charged lepton masses, we choose \( c_\alpha = 0.65 (\alpha = e, \mu, \tau) \), \( c_{e_R} = 0.78 \), \( c_{\mu_R} = 0.61 \) and \( c_{\tau_R} = 0.53 \). Since \( e^{-74} \sim 7 \times 10^{-33} \), to get neutrino masses \( m_\nu \lesssim 1 \text{ eV} \), we will have \( c_N \gtrsim 0.2 \). Here the smallness of the light neutrino mass is attributed to a large \( m_N \) as in the usual seesaw. Since we place the hierarchy in \( c_{\alpha R} \) and fix the \( c_\alpha \) to be the same for all flavors, we will get a non-hierarchical (anarchical) neutrino mass matrix if \( c_N \) is non-hierarchical. On the other hand, if we fix \( c_{\alpha R} \) while having hierarchical \( c_\ell, c_N \) should also be hierarchical in order to get an anarchical neutrino mass matrix.

3 Inverse seesaw in warped extra dimension

As noted earlier, to implement the inverse seesaw mechanism [12] in 4D models, two types of chiral gauge singlet fermions are needed. As before we denote by \( N \) the RH neutrinos used
for seesaw mechanism discussed above and by $S$ the extra singlet fermion fields. They form a pseudo-Dirac pair with splitting given by a tiny parameter that breaks the lepton number. The smallness of this parameter is chosen by hand in the 4D case. We follow this strategy closely in the discussion of WED embedding of inverse seesaw. The bulk action for inverse seesaw in WED can be written as follows:

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G} \left[ N_i \bar{E}_a^a \gamma^a D_A N - m_{DN5} \bar{N}N + \bar{S}i \bar{E}_a^a \gamma^a D_A S - m_{DS5} \bar{SS} \\
- \left( \frac{1}{2} m_{N5} \bar{N}N^c + \frac{1}{2} m_{S5} \bar{SS}^c + \frac{1}{2} m_{SN5} \bar{SN}^c + \lambda_{N5} \bar{t}N \bar{H} + \lambda_{S5} \bar{t}S \bar{c}H + \text{h.c.} \right) \right]. \quad (6)$$

If we assign the lepton number $L$ for $N, S$ respectively as $L = 1, -1$, the only fermion bilinears which violate $L$ are the Majorana masses $m_{N5}$ and $m_{S5}$ in the action (6).

This above action could also arise from an exact gauge symmetry such as $U(1)_{B-L}$ (with the usual definition of quantum numbers) after spontaneous symmetry breaking by Higgs field that transforms as $B - L = +1$. The $S$ field is $B - L$ neutral and therefore its Majorana mass term $m_{S5}$ which is allowed by $B - L$ breaks the global symmetry $L$ (defined above) which persists in gauged $B - L$ version. The $B - L$ model implies that $m_{N5} = 0$; however, since $m_{N5}$ does not play a role in the neutrino masses and mixing, the final results derived in the model without $B - L$ symmetry and presented below remain unchanged.

Note that in 4D, a five dimensional field splits into two chiral pairs and only one chirality remains as a zero mode. So in our model, in 4D, only the left chirality of the lepton doublet $\ell$ and the right chiralities of $S$ and $N$ survive as zero modes.

In odd space-time dimension (i.e. five), the action (6) contains parity anomaly if the total number of bulk fermions that couple to gauge and gravity fields is odd [13,14]. In warped type I seesaw as discussed in Sec. 2, where the lepton doublets also propagate in the extra dimension, cancellation of the parity anomaly naturally requires three generations of RH neutrinos $N$. However, in the warped inverse seesaw, in order not to reintroduce parity anomaly, we have to add an even number of singlet Dirac fermions $S$. The minimal number of $S$ fields required to obtain three active light neutrino is three. Since we cannot have odd number of $S$, the minimal number has to be four. Thus, after the three of the four $S$-fields pair up with the three $N$ fields to make the three pseudo-Dirac fermions, we are left with an extra $S$ field which in the end becomes the sterile neutrino with mass in the eV range.

We again assume that Higgs doublet is localized on the TeV brane and that the $L$-violating Majorana masses are confined to the Planck brane i.e. $m_{N5} = d_N \delta(y)$, $m_{S5} = d_S \delta(y)$ with $d_N, d_S$ dimensionless numbers. For simplicity, we further assume that $m_{SN5} = d_{SN} k$ with $d_{SN}$ dimensionless number and ignore any possible boundary masses. To estimate the order of magnitude of the dimensionless parameters that characterize the model, we consider only one generation for all fermions. Assuming the 5D location of the fields to be $c_\ell > 1/2, c_{e_R} > 1/2, c_N < 1/2$ and $c_S < 1/2$, we find the 4D effective masses to be

$$m_D \sim k \times e^{-k \pi R (c_\ell + 1/2)},$$
$$m_N \sim k \times e^{k \pi R (2c_N - 1)}.$$
\begin{equation}
m_S \sim k \times e^{\pi k R (2c_S - 1)},
m_{SN} \sim k \times e^{-k \pi R}.
\end{equation}

In the equation for \( m_{SN} \) we have also assumed that \( c_N + c_S \leq 0 \) and we will see that this is indeed what we need for the inverse seesaw mechanism to work. This leads to the effective light neutrino mass

\begin{equation}
m_\nu \sim k \times e^{-2\pi k R (c_\ell - c_S)}.
\end{equation}

In contrast to Eq. (5), here the smallness of the light neutrino mass is attributed to small \( m_S \).

For example, if we take \( c_\ell = 0.65 \) and \( c_S \lesssim -0.2 \), we have \( m_\nu \lesssim 1 \text{eV} \) with the seesaw scale \( m_{SN} \sim \mathcal{O}(\text{TeV}) \). Also, if we assume all \( c_\ell \) and \( c_S \) to be of same order for all generation, we get a neutrino mass matrix with non-hierarchical pattern. It is interesting that the final neutrino mass formula is independent of the precise 5D profile of the \( N \) fields.

### 3.1 The appearance of sterile neutrino(s)

To understand the appearance of the sterile neutrino(s), let us consider a toy model with one generation of \( \ell \) and \( N \) and two generations of \( S \) called \( S_1, S_2 \). In this case, since only one \( S \) is needed for the inverse seesaw, the additional one would result in a sterile neutrino. In this model, the mass matrix in the basis \((\nu, N, S_1, S_2)\) is given by

\begin{equation}
M = \begin{pmatrix}
0 & m_D & 0 & 0 \\
m_D & m_N & m_{SN_1} & m_{SN_2} \\
0 & m_{SN_1} & m_{S_{11}} & m_{S_{12}} \\
0 & m_{SN_2} & m_{S_{12}} & m_{S_{22}}
\end{pmatrix}.
\end{equation}

For simplicity, we assume that \( m_{SN_1} = m_{SN_2} = m_{SN} \), \( m_{S_{11}} = m_{S_{22}} = m_S \) and \( m_N = m_{S_{12}} = 0 \). Assuming \( m_S \ll m_D \ll m_{SN} \), we can diagonalize matrix (9) and obtain two heavy and two light states with their respective masses given by

\begin{equation}
m_{\text{heavy}} \simeq \pm \sqrt{2m_{SN}^2 + m_D^2 + m_S^2 \frac{m_{SN}^2}{2m_{SN}^2 + m_D^2}},
\end{equation}

\begin{equation}
m_{\text{light}} \simeq m_S \frac{m_D^2}{2m_{SN}^2 + m_D^2}.
\end{equation}

The two heavy states mix with the light neutrino with \( \sim m_D/m_{SN} \) and can be named the heavy RH neutrinos. The light state with mass \( m_S \) can be identified as sterile neutrino while the other is the light active neutrino. Hence, we obtain an interesting relation between the mass of active and sterile neutrinos as follows

\begin{equation}
m_{\text{active}} \simeq m_{\text{sterile}} \frac{m_D^2}{2m_{SN}^2 + m_D^2}.
\end{equation}

For instance, to have an active neutrino with mass \( m_{\text{active}} \sim 0.05 \text{eV} \) and a sterile neutrino with mass \( m_{\text{sterile}} \sim 1 \text{eV} \), we would require a hierarchy between \( m_D \) and \( m_{SN} \) to be \( m_D/m_{SN} \sim 0.22 \).
On the other hand, if we want $m_{\text{sterile}} \sim 1$ keV which could be a potential dark matter candidate, it would require $m_D/m_{SN} \sim 0.007$.

It should be pointed out that although the result above is obtain by assuming $m_{S_{12}}$ to be vanishing, barring any accidental cancellation, it holds in general even if $m_{S_{12}}$ is of the order of the diagonal elements $m_{S_{11}}$ and $m_{S_{22}}$. The result will also hold if $m_N$ is non-vanishing as long as $m_N \ll m_{SN}$. In order to obtain more than one sterile neutrino, we can extend the number of $S$ in the model.

3.2 Neutrino mixing in warped inverse seesaw

We will now present a realistic warped inverse seesaw model with three $N$ and four $S$ fields to explore the detailed neutrino mixing and mass hierarchy pattern. We first ignore the contributions of the KK modes which will be discussed in a subsequent section, where we will show under what conditions their effects can be safely ignored. Considering for now only the zero modes, we have the leading $10 \times 10$ neutrino mass matrix in the basis $(\nu, N, S)$, which is given as follows

$$M = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & m_N & m_{SN} \\ 0 & m_{SN}^T & m_S \end{pmatrix}. \quad (13)$$

where $m_D$, $m_{SN}$ and $m_S$ are respectively $3 \times 3$, $3 \times 4$ and $4 \times 4$ matrices. Assuming $m_S, m_N \ll m_D \ll m_{SN}$, we can block diagonalize the mass matrix above and obtain the light neutrino mass matrix to be

$$M_\nu \simeq -m_D \left( m_{SN}^{-1} m_{SN}^T \right)^{-1} m_D^T. \quad (14)$$

Note that $m_N$ does not appear in the light neutrino masses. It only affects the mass splitting of the pseudo-Dirac pair $(N, S)$.

3.3 Examples I: Normal hierarchy (NH) mass spectrum

In this section, we address the issue of neutrino mixing. We will search for the parameter space which reproduces the neutrino mixing matrix for the normal hierarchy spectrum ($m_{\nu_3} > m_{\nu_2} > m_{\nu_1}$) while having anachiral pattern for $m_D$ and $m_{SN}$. Notice that $m_{SN}$ is naturally anachiral. However, as far as $m_D$ is concerned, unlike the RH charged leptons since the $N$ is located closer to the TeV brane, whether it is hierarchical or not depends on the profiles of the left-handed charged leptons. By fixing the values of left-handed lepton doublets and attributing the hierarchy to the RH singlet charged leptons, we can have an anachiral $m_D$. For example, we have chosen the bulk mass parameters for charged leptons as follows: $c_\ell_e = c_\ell_\mu = c_\ell_e = 0.65$, $c_{eR} = 0.7770$, $c_{eR} = 0.6099$, $c_{eR} = 0.5271$. This fits the charged lepton mass spectrum. We then choose $c_{N_1} = c_{N_2} = c_{N_3} = -0.340$ and $c_{S_1} = -0.338$, $c_{S_2} = -0.366$, $c_{S_3} = -0.358$, $c_{S_4} = -0.377$, and all the dimensionless couplings having the values in the range $[0.1, 1.0]$. We then obtain the following mass matrices (in GeV)
\[ m_D = \begin{pmatrix} 2.763 & 6.029 & 15.826 \\ 9.294 & 5.778 & 12.058 \\ 12.540 & 16.077 & 7.285 \end{pmatrix}, \]  
\( m_{SN} = \begin{pmatrix} 172.342 & 191.492 & 138.988 & 141.208 \\ 177.903 & 222.665 & 134.505 & 264.764 \\ 82.1093 & 276.105 & 347.918 & 269.177 \end{pmatrix}, \]  
\( m_S = \begin{pmatrix} 5.8522 & 3.2171 & 2.5562 & 1.8856 \\ 3.2171 & 0.7979 & 2.5724 & 1.5791 \\ 2.5562 & 2.5724 & 3.8971 & 0.8128 \\ 1.8856 & 1.5791 & 0.8128 & 1.3833 \end{pmatrix} \times 10^{-9}, \]  
\( m_N = \begin{pmatrix} 0.4072 & 1.018 & 0.6108 \\ 1.018 & 0.8143 & 1.222 \\ 0.6108 & 1.222 & 1.018 \end{pmatrix} \times 10^{-9}. \] 

From the matrices above, we obtain the light neutrino mixing matrix (by diagonalizing the 10 × 10 neutrino mass matrix) as follows

\[ U_{\nu}^{nor} = \begin{pmatrix} -0.8517 & 0.5122 & 0.0962 & -0.0135 \\ 0.3183 & 0.6593 & -0.6694 & 0.1104 \\ -0.4136 & -0.5468 & -0.7182 & 0.1005 \\ 0.0466 & 0.0633 & 0.1638 & 0.9887 \end{pmatrix}, \]  

where the last row and column correspond to the mixing with light sterile neutrino. The top left 3 × 3 submatrix of matrix [19] corresponds to the mixing between active neutrinos and is in good agreement with the one obtained from the approximate formula Eq. (14)\(^2\). The active neutrino mixing matrix [19] is in good agreement with the experimentally measured values [15]

\[ U_{\nu}^{exp} = \begin{pmatrix} -0.8212 & 0.5623 & 0.0976 \\ 0.3598 & 0.6429 & -0.6762 \\ -0.4429 & -0.5202 & -0.7302 \end{pmatrix}. \]  

\(^2\)In this work, we will only use the exact mixing matrix obtained from diagonalizing 10 × 10 neutrino mass matrix.
The masses of three light active neutrinos are \((\nu_1, \nu_2, \nu_3) = (0.00172, 0.00885, 0.0516)\) eV. On the other hand, the sterile neutrino has a mass of 0.848 eV which could potentially explain the anomaly in LSND and MiniBooNE [16, 17]. From the above, we can determine the mass squared differences of the active neutrinos

\[
\Delta m_{21}^2 = 7.54 \times 10^{-5} \text{eV}^2,
\]

\[
\Delta m_{31}^2 = 2.66 \times 10^{-3} \text{eV}^2,
\]

which are within 1\(\sigma\) of the experimental values.

With the existence of an extra light sterile neutrino with significant mixing with the active neutrinos, we have to check if this could be consistent with the experimentally determined number of species of neutrinos. In the SM, the difference of the total width of the \(Z^0\) boson and the width for the decay into all visible channels is attributed only to the light neutrinos that couple to the \(Z^0\) boson. This was determined very precisely from LEP data to be \(N_\nu = 2.9841 \pm 0.0083\) [18]. In our example above, we can calculate the correction due to the existence of the light sterile neutrino which mixes with active neutrinos [19]

\[
N_\nu = \sum_{i,j=1}^{4} \left| \sum_{\alpha=e,\mu,\tau} U_{\alpha i}^* U_{\alpha j} \right|^2 = 2.979 ,
\]

where we have ignored the neutrino masses since \(m_{1,2,3,4} \ll M_{Z^0}\).

Finally we can also write down 4 \(\times\) 4 neutrino mass matrix including three active and one light sterile neutrinos as follows (in GeV)

\[
M_{\nu}^{\text{nor}} = \begin{pmatrix}
0.003890 & 0.0004645 & -0.004287 & 0.01234 \\
0.0004645 & 0.01681 & 0.01198 & -0.09776 \\
-0.004287 & 0.01198 & 0.02098 & -0.09063 \\
0.01234 & -0.09776 & -0.09063 & -0.8271
\end{pmatrix} \times 10^{-9} .
\]

Since the experiment could not determine the sign of \(\Delta m_{31}^2\), we consider below the possibility of inverted neutrino mass spectrum i.e. \(m_{\nu_2} > m_{\nu_1} > m_{\nu_3}\).

### 3.4 Examples II: Inverted hierarchy (IH) mass spectrum

We again choose anarchical \(m_D\) and \(m_{SN}\). The 5D parameters in this case are different. For example, we have chosen the bulk mass parameters as follows: \(c_{\ell e} = c_{\ell \mu} = c_{\ell \tau} = 0.65\), \(c_{eR} = 0.7770\), \(c_{\mu R} = 0.6099\), \(c_{\tau R} = 0.5271\), \(c_{N_1} = c_{N_2} = c_{N_3} = -0.360\), \(c_{S_1} = -0.3869\), \(c_{S_2} = -0.353\), \(c_{S_3} = -0.3029\), \(c_{S_4} = -0.343\), and all the dimensionless couplings having the values in the range \([0.1, 1.0]\) and we obtain the following mass matrices (in GeV)

\[
m_D = \begin{pmatrix}
3.1553 & 5.7058 & 7.4499 \\
8.1511 & 3.9266 & 2.7872 \\
3.8560 & 5.5772 & 0.3675
\end{pmatrix} ,
\]
\[ m_{SN} = \begin{pmatrix} 
105.405 & 101.936 & 107.203 & 70.534 \\
85.528 & 110.527 & 73.175 & 133.853 \\
32.219 & 150.114 & 176.516 & 141.532 
\end{pmatrix}, \quad (25) \]

\[ m_S = \begin{pmatrix} 
0.2383 & 0.9876 & 4.0775 & 1.5672 \\
0.9876 & 2.0009 & 29.796 & 8.5212 \\
4.0775 & 29.796 & 214.205 & 20.529 \\
1.5672 & 8.5212 & 20.529 & 16.493 
\end{pmatrix} \times 10^{-9}, \quad (26) \]

\[ m_N = \begin{pmatrix} 
0.09489 & 0.2372 & 0.1423 \\
0.2372 & 0.1898 & 0.2847 \\
0.1423 & 0.2847 & 0.2372 
\end{pmatrix} \times 10^{-9}. \quad (27) \]

From the matrices above, we obtain the light neutrino mixing matrix

\[ U_{\nu}^{inv} = \begin{pmatrix} 
-0.8131 & -0.5714 & 0.09739 & -0.0358 \\
0.3440 & -0.6076 & -0.7662 & -0.0866 \\
-0.4635 & 0.5353 & -0.6964 & 0.1003 \\
-0.0755 & 0.1336 & 0.0834 & -0.9905 
\end{pmatrix}, \quad (28) \]

where again the last row and column correspond to the mixing with light sterile neutrino. As before, the top left \(3 \times 3\) submatrix of matrix \(28\) corresponds to the mixing between active neutrinos and is in good agreement with the measured value in Eq. \(20\).

The masses of three light active neutrinos are \((\nu_1, \nu_2, \nu_3) = (0.0471, 0.0479, 0.000454)\) eV. On the other hand, the sterile neutrino has a mass of 16.9 eV which could be too large to explain the anomaly in LSND and MiniBooNE. From the above, we can determine the mass squared differences of the active neutrinos

\[ \Delta m^2_{21} = 7.74 \times 10^{-5} \text{eV}^2, \]
\[ \Delta m^2_{31} = -2.22 \times 10^{-3} \text{eV}^2, \quad (29) \]

which are within 1\(\sigma\) of the experimental values for the inverted mass spectrum. In this example, we determine \(N_\nu = 2.9767\) from Eq. \(22\).

We can also write down \(4 \times 4\) neutrino mass matrix including three active and one light sterile neutrinos as follows (in GeV)

\[ M_{\nu}^{inv} = \begin{pmatrix} 
0.03718 & 0.02257 & -0.02833 & 0.6064 \\
0.02257 & 0.1148 & -0.1385 & 1.4528 \\
-0.02833 & -0.1385 & 0.1770 & -1.6816 \\
0.6064 & 1.4528 & -1.6816 & 16.5961 
\end{pmatrix} \times 10^{-9}. \quad (30) \]

\(^3\)The sign differences in the second column of Eq. \(28\) can be accounted for by changing the Majorana phases accordingly.
3.5 Contributions from Kaluza-Klein modes

In this section we would like to estimate the contributions from KK modes. For simplicity, we would consider single generation for each \( N, S \) and \( \ell \) fields. We KK decompose the 5D fermionic fields as

\[
\Psi_{L,R}(x^\mu, y) = \frac{e^{2\sigma}}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x^\mu) \tilde{\Psi}_{L,R}^{(n)}(y),
\]

where \( \Psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\Psi \). For \( N \) and \( S \), we choose \( N_R \) and \( S_R \) to be even under \( Z_2 \) while for \( \ell \), we choose \( \ell_L \) to be even. As before, we will restrict \( H \) to be strictly confined to the TeV brane with \( H(y) = k \delta(y - \pi R) \) and the Majorana masses to be strictly confined to the Planck brane i.e. \( m_{N5} = d_N \delta(y) \) and \( m_{S5} = d_S \delta(y) \). Similarly, we also assume \( m_{SN5} = d_{SN}k \).

Substituting Eq. (31) for \( N, S \) and \( \ell \) fields respectively into the action (6), we can write down the mass matrix in the basis of \( (\nu_L^{(0)}, N_R^{(0)}, S_R^{(0)}, \nu_L^{(1)}, N_R^{(1)}, S_R^{(1)}, \ldots) \) as follows

\[
M_{kk} = \begin{pmatrix}
0 & m_D^{(00)} & 0 & 0 & 0 & 0 & m_D^{(01)} & 0 & 0 & \ldots \\
0 & m_D^{(00)} & m_S^{(00)} & m_D^{(10)} & 0 & 0 & m_D^{(01)} & 0 & m_D^{(11)} & \ldots \\
0 & m_D^{(00)} & m_S^{(00)} & m_D^{(10)} & 0 & 0 & m_D^{(01)} & 0 & m_D^{(11)} & \ldots \\
0 & 0 & m_D^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & m_N^{(11)} & m_S^{(11)} & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & m_N^{(11)} & m_S^{(11)} & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & m_S^{(11)} & m_S^{(11)} & m_S^{(11)} & m_S^{(11)} & m_S^{(11)} & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{pmatrix},
\]

where \( m_D^{(1)} \), \( m_N^{(1)} \) and \( m_{\nu}^{(1)} \) are respectively the first KK masses of \( S, N \) and \( \nu \) and

\[
m_D^{(mn)} = e^{-k\pi R} \lambda_{N4} k \tilde{\nu}_L^{(m)}(\pi R) \tilde{N}_R^{(n)}(\pi R),
\]

\[
m_N^{(mn)} = \frac{d_N}{2\pi R} \tilde{N}_R^{(m)}(0) \tilde{N}_R^{(n)}(0),
\]

\[
m_S^{(mn)} = \frac{d_S}{2\pi R} \tilde{S}_R^{(m)}(0) \tilde{S}_R^{(n)}(0),
\]

\[
m_{SNL,R}^{(mn)} = \int_{-\pi R}^{\pi R} dy \frac{e^{-k|y|}}{2\pi R} d_{SN} k \tilde{S}_L^{(m)}(y) \tilde{N}_R^{(n)}(y).
\]

In the first equation of Eqs. (33), \( \lambda_{N4} = \frac{\lambda_{N4}}{2\pi R} \) is dimensionless.

Assuming \( m_{N_L}^{(mn)} \), \( m_{S_R}^{(mn)} \ll m_D^{(mn)} \ll m_{SNL,R}^{(mn)} \ll KK \) masses, we obtain the light neutrino mass roughly as

\[
m_{\nu} \sim m_{S_R}^{(00)} \frac{m_D^{(00)}}{(m_{SN}^{(00)})^2} \left[ 1 + O(e^2) \right],
\]
where $\epsilon \sim \frac{m_{S_{N_{L,R}}}^{(mn)}}{\text{KK masses}}$. Notice that at leading order, the light neutrino mass is given by the inverse seesaw relation and the contributions from KK modes are suppressed as long as $m_{S_{N_{L,R}}}^{(mn)}$ is less than KK masses.

Including the contributions from the first KK modes by considering the entire $9 \times 9$ mass matrix in Eq. (32), in Figure 1, we numerically plotted the contributions of the first KK modes to the light neutrino mass as a function of $d_{SN} = m_{S_{N5}}/k$. As long as we keep $d_{SN} \lesssim 0.3$ (as we did in the examples in Secs. 3.3 and 3.4), the corrections from the first KK modes are not more than 20%. Hence, we expect the contributions from higher KK modes to be negligible.

![Figure 1: The contributions from the first KK modes to the light neutrino mass as a function of $d_{SN} = m_{S_{N5}}/k$.](image)

**4 Comments**

A few comments are now in order about our model.

(i) In the above discussion, we have added two kinds of lepton number breaking terms on the Planck brane and assumed that these are the only sources of lepton number violation in our model i.e. Majorana mass terms of type $NN$ and $SS$. However, we could just keep only the second of the two terms, in which case in Eq. (13), depicting the inverse seesaw matrix for the zero modes, the term $m_N$ will absent. Similarly in the discussion of KK mode contributions, all Majorana terms involving $N_L, N_R$ (i.e. $m_{N_{L,R}}^{(mn)}$) will be absent. This makes it easier to estimate the KK contributions to the zero mode mass and it confirms our result that they are indeed small. Such a situation can be guaranteed by adding an extra $B - L$ gauge symmetry into the theory under which $S$ is a singlet but $N$ field is not. The $m_{SN}$ is then generated by a Higgs field which breaks the $B - L$ symmetry by one unit. Since $m_N$ and all Majorana masses involving the higher KK modes of the $N$ field break $B - L$ by two units, they will be absent.

(ii) A comment on the phenomenology of our model: A key feature of the model is the presence of a light sterile neutrino, which arises because of the need to guarantee freedom from parity anomalies as noted. Since the number of $S$ field $N_S$ we can add to the model has to be even, the prediction of this model is that we will have an odd number of sterile neutrinos...
\( N_{\text{sterile}} = N_S - 3 \) where the 3 is the number of family in the SM. The sterile neutrino will contribute to neutrinoless double beta decay due to its mixing with \( \nu_e \); however the effective neutrino mass due to this contribution remains in the 3 meV range due to small mixing and eV range sterile mass. This remains far below the reach of the current double beta decay search. This sterile neutrino could also provide a way to understand the recent reactor anomalies as well as the MiniBooNE and LSND results \[20\]. However at LSND and MiniBooNE, it will predict the same oscillation effect for both neutrinos and anti-neutrinos. The sterile neutrino will contribute an extra neutrino species in the analysis of Big Bang nucleosynthesis (BBN). This is consistent with current analyses of the BBN as well as cosmological structure formation and WMAP data \[21\].

(iii) The scenario outlined here leads to a \( \theta_{13} \simeq 0.096 \) for the NH and 0.097 for the IH case. This is however not a prediction since the Dirac neutrino Yukawa coupling, the lepton number violating masses \( m_S, m_N \) as well as the \( m_{SN} \) matrices all involve free parameters.

(iv) The specific model discussed here predicts RH neutrinos with masses of order 100 GeV which are therefore accessible at the LHC via their mixing with the left-handed neutrino. LHC signals for such Dirac neutrino Yukawas have been studied in Ref. \[22\]. Their primary decay signal is the three lepton plus missing energy in \( pp \) collisions. Furthermore, an interesting possibility is the KK excited mode of the electron, if in the TeV range could decay to the RH neutrino and the \( W \). Since the dominant decay mode of the RH neutrino is to two leptons plus missing energy \( (N \rightarrow \ell^+ \ell^- \nu) \), there could be exotic final states such as \( \ell^\pm \ell^\mp \ell^- \nu \).

(v) The TeV scale particle spectrum in the model is similar to an SO(10) model with inverse seesaw discussed in the literature. Extrapolating the discussion of that model \[23\], it appears very likely that it will provide a satisfactory framework for realization of resonant leptogenesis idea to understand the origin of matter.

5 Conclusions

In summary, we have presented a new way to understand the small neutrino masses by embedding the inverse seesaw mechanism into the warped extra dimension models. In the four dimensional implementation of inverse seesaw, a small lepton number violating mass term needs to be put in by hand (of order of or less than a keV) to get sub-eV scale active neutrino masses. In the WED framework on the other hand, locating the lepton number violating mass terms on the Planck brane provides a simple way to understand this smallness without any fine-tuning of parameters. This model differs from the type I seesaw in WED by the presence of sub-TeV scale right-handed neutrinos which may be accessible at the LHC. An interesting prediction of the model is an eV scale sterile neutrino which arises from the requirement of cancellation of parity anomaly in odd number of dimensions. Its small mass is again connected to the small parameter in the inverse seesaw and the lepton number breaking in the Planck brane. We have also worked out numerical examples which give active neutrino masses and mixing in accord with observations for both normal and inverted hierarchy cases, showing that such models can indeed provide realistic description of nature.
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