Study on Fast Positioning Algorithm for BeiDou Post-Mission Network RTK

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Abstract. Aiming at the GNSS data resolving process and the network RTK positioning algorithm for the multisystem of GPS/BDS/GLONASS, C# and C++ programming were used to develop the resolving software of fast positioning for BeiDou post-mission Network RTK. The software supported self-defined resolving time and could resolve three dimensional user coordinates in a short time. The software was tested with the measured data from the BeiDou high precision reference station network in Guangxi and compared with the TBC+COSA_GPS processing software. And the results show that the proposed software could provide rapid positioning results for users with a centimeter-level precision.

1. Introduction
With the establishment of the national high precision reference station network for BeiDou, the BeiDou network RTK technology has been widely applied. Fast real-time GNSS positioning based on the BeiDou reference station network can enhance the operation efficiency of network RTK and can also provide reliable services for engineering and research projects.

At present, other than delivering mainstream GNSS data processing software such as TBC of Trimble, LGO of Leica and HGO of Hi-Target, many scholars have conducted research on network RTK resolution techniques and have achieved abundant results. Reference [1] researched on long range network RTK algorithms and developed a corresponding software to provide positioning services to moving station users with a centimeter level accuracy. In Ref. [2], aiming at the Kalman filtering function model and the random model with additional ambiguity parameters, a GPS/BDS RTK positioning software was programmed to achieve a centimeter-level positioning accuracy in the circumstance of a short base line. Integrating the PPP data processing issue with the multi-GNSS system, Ref. [3] adopted the platform of MATLAB to develop a PPP resolution software for the multi-GNSS system, which obtained relatively high precision. Based on relevant algorithms such as the integer ambiguity searching for network RTK and user error estimation, Ref. [4] developed the NRTK resolution software to offer single epoch network RTK positioning results with centimeter-level accuracy. And in Ref. [5], a new multi-system network RTK algorithm was proposed which could rapidly resolve three dimensional user coordinates when lacking of communication signals disabled normal operation.

Guangxi locates at the south-eastern edge of the Yunnan-Guizhou Plateau at the second step of China’s terrain and has a landscape feature of plenty mountainous areas and few plains, which makes it common to suffer a loss of communication signals in field operations. Aiming at this issue and drawing lessons from Ref. [5], a fast positioning resolution software for BeiDou post-mission network RTK was
developed by C# and C++ programming in this paper. Also, the measured data from the BeiDou high precision reference station network in Guangxi were adopted for testing and studying the availability of the software.

2. GNSS super-fast network RTK algorithm

2.1. Resolution of integer ambiguity

First, the observed value of the carrier phase and the P code pseudo-range are combined to form the Melbourne-Wübbena (M-W) combination for the calculation of wide-lane integer ambiguity \[6\].

The distance between reference stations in the CORS network is typically tens of kilometers and the ionospheric delay in the base line cannot be removed by difference between stations. Therefore, carrier phase and pseudo-range observations were combined without the influence of the ionosphere to calculate the real number solution of the ionosphere-free ambiguity \[5,6\] and the observation equation is:

\[
\begin{align*}
\lambda \cdot \Delta \psi_{ij}^{pq} &= \Delta \psi_{ij}^{pq} - \lambda \cdot \Delta N_{ij}^{pq} \\
+ \Delta \nabla T \rho_{ij}^{pq} + \epsilon_{ij}^{pq} \\
\Delta \psi_{ij}^{pq} &= \Delta \psi_{ij}^{pq} + \Delta \nabla T \rho_{ij}^{pq} + \gamma_{ij}^{pq}
\end{align*}
\]

where \( \rho \) is the distance from the observation station to the satellite, Trop is the delay error of the troposphere, \( \varepsilon \) is the noise of the observed value for the carrier phase and \( \gamma \) is the noise of the observed value for the pseudo-range.

According to the signal structure of GLONASS frequency division multiple access, the double difference ambiguity is transformed into a double-difference integer ambiguity and a single-difference integer ambiguity \[7,8\], and the observation equation is:

\[
\lambda_{glo} \cdot \Delta \psi_{glo}^{pq} = \Delta \psi_{ij}^{pq} - \lambda_{glo} \cdot \Delta N_{glo}^{pq} + (\lambda_{glo} - \lambda_{glo, q}) \cdot \Delta N_{glo}^{pq} + \Delta \nabla T \rho_{ij}^{pq} + \epsilon_{ij}^{pq}
\]

\[
\Delta \psi_{ij}^{pq} = \Delta \psi_{ij}^{pq} + \Delta \nabla T \rho_{ij}^{pq} + \gamma_{ij}^{pq}
\]

where \( \lambda_{glo} \) is the wavelength of observed value for GLONASS, \( \psi_{glo} \) is the observed phase value, \( \rho_{glo} \) is the observed value of the pseudo-range, \( N_{glo} \) is the integer ambiguity while \( p \) represents the observation satellite and \( q \) represents the reference satellite.

Based on the wide-lane integer ambiguity, the real part of the ionosphere-free ambiguity and the corresponding variance and covariance matrices \[9\], the wide-lane integer phase ambiguity is fixed by using the LAMBDA algorithm \[10\] to obtain the L1 and L2 integer phase ambiguity.

2.2. Error calculation of network RTK

After fixing the integer ambiguity of base lines, the ionospheric delay, the tropospheric delay and the comprehensive bias including the orbit error and the multi-path error are then calculated according to the observed value of each base line, its corresponding variance and covariance matrices. The calculation formula is as follows:

\[
\Delta \nabla \text{Lono} = \frac{f_{z}^2}{f_{z}^2 - f_{z}^2} [(\lambda_1 \Delta \nabla \varnothing_1 - \lambda_2 \Delta \nabla \varnothing_2) + (\lambda_1 \Delta \nabla N_1 - \lambda_2 \Delta \nabla N_2)]
\]

\[
\Delta \nabla \text{Trop} = [M_F^m(e^{h}_{m}) - M_F^m(e^{s}_{m})] \cdot ZW_{D_{m}} - [M_F^m(e^{h}_{n}) - M_F^m(e^{s}_{n})] \cdot ZW_{D_{n}} + [M_F^h(e^{h}_{m}) - M_F^h(e^{s}_{m})] \cdot ZH_{D_{m}} - [M_F^h(e^{h}_{n}) - M_F^h(e^{s}_{n})] \cdot ZH_{D_{n}}
\]

\[
\Delta \nabla \text{Other} = \Delta \nabla \rho - (\lambda_1 \Delta \nabla \varnothing_1 - \lambda_2 \Delta \nabla \varnothing_2) - \Delta \nabla \text{Lono} - \Delta \nabla \text{Trop}
\]

where MF is the mapping function of the troposphere, ZWD is the estimated wet delay of the troposphere by Eq. (1) and ZHD is the hydrostatic delay of the troposphere obtained by interpolation of the GPT2w model \[5\].
2.3. Bias interpolation of network RTK

As the ionospheric delay error, the tropospheric delay error and the comprehensive bias have a relatively high spatial-temporal correlativity, the linear interpolation model for two dimensional plane coordinates was adopted in this paper to respectively interpolate these errors of moving stations and the specific interpolation formula is as follows:

\[
\begin{bmatrix}
\text{Err}_1 \\
\text{Err}_2 \\
\vdots \\
\text{Err}_n
\end{bmatrix} =
\begin{bmatrix}
\Delta X_1 & \Delta Y_1 \\
\Delta X_2 & \Delta Y_2 \\
\vdots & \vdots \\
\Delta X_n & \Delta Y_n
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\text{D}_1} & 0 & \cdots & 0 \\
0 & \frac{1}{\text{D}_2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{1}{\text{D}_n}
\end{bmatrix}
\begin{bmatrix}
\text{a}_1 \\
\text{a}_2 \\
\vdots \\
\text{a}_n
\end{bmatrix}
\] (4)

where \(\text{Err}\) is the base line error, \(\Delta X, \Delta Y\) is the base line vector in the east and north direction, respectively, \(D\) is the length of the base line, and \(\text{a}_1\) and \(\text{a}_2\) are the corresponding vectors of interpolation coefficient which can be obtained by the least squares method. Then the bias correction of the moving station is interpolated using the following equation.

\[
\text{Err}_v = \text{a}_1 \cdot \Delta X_v + \text{a}_2 \cdot \Delta Y_v
\] (5)

where \(\text{Err}_v\) is the interpolation error of the moving station while \(\Delta X_v\) and \(\Delta Y_v\) is the vector from the moving station to the main station baseline in the east and north direction, respectively.

![Flow chart of GNSS super-fast network RTK algorithm](image)

Figure 1. Flow chart of GNSS super-fast network RTK algorithm

3. Analysis of examples

3.1. Source of data

The northwest region of Guangxi was selected as the experiment object in this paper. The region belongs to the southern foot of the Yunnan-Guizhou Plateau and is mainly consisted of mountainous areas with
complex terrains. The experiment region locates at 106°34'E-109°09'E and 23°41'N—25°37'N with an area of 33.5 thousand square kilometers. By using the measured data from the BeiDou high precision reference station network in Guangxi, the data collection time in the moving station was from 15 o’clock to 18 o’clock Beijing Time on August 20th, 2019 (Day of Year 232), a total of three hours with a sampling interval of 15s. The data from reference station adopted the measured data on Day 232 of the year 2019 provided by the BeiDou high precision reference station network in Guangxi and the accurate coordinates of CS01, CS02 and CS03 moving stations were known. The distribution of the stations is shown in Fig. 2.

![Distribution of observation stations](image)

**Figure 2. Distribution of observation stations**

### 3.2 Analysis of accuracy

To validate the accuracy of the resolving software of fast positioning for BeiDou post-mission Network RTK, TEQC was used in this paper to cut out moving station data with an observation period of 5min, 15min, 45min, 60min, 90min, 120min, and 180min, respectively. The proposed software and the TBC+COSA_GPS software were both utilized to resolve the coordinate values of the moving station in difference observation periods. The calculated coordinates of CS01, CS02 and CS03 were compared with their exact coordinates and the deviations between the N, E and U coordinate components and the true values were obtained and shown in Fig. 3. And the positioning results of CS04, CS05 and CS06 using the two kinds of softwares were subtracted and compared as illustrated in Fig. 4.

![Deviation comparison](image)

**Figure 3. Figure for deviation between the true value and the calculated value by the super-fast resolution software and the TBC+COSA_GPS software**

![Deviation comparison](image)

**Figure 4. Calculating time comparison**
As shown in Fig. 3, both of the two kinds of software have good stability for all the periods for CS02 and the calculated values agree well with the true values. And the proposed software had a largest deviation of 2.6cm in the N direction when the observation time was 15min while the largest deviation for the E and U direction was 2.4cm and 4.1cm, respectively. The positioning results of both resolving methods were inferior for CS01 and CS03 with an observation period of 5min, however, the deviation was basically within 0.5m. When the observation time reached 15min, the deviation between the positioning results and the true values in the horizontal direction was basically within 3cm while the elevation deviation was less than 5cm. As the observation time increased, the positioning results gradually converged near the true values. As shown in Fig. 4, the resolving results for different periods in N and E directions of the two resolving methods had good consistency with small differences and the resolving results in the U direction gradually reached convergence when the observation time was 30min.

It is known from Fig. 2 that the difference between the result of the proposed software and that of the TBC+COSA_GPS software was comparatively large for resolution of moving stations when the observation time was 5min. The deviation in this case was removed when counting the RMS of the two resolving methods, and the obtained RMS of N, E and U components are shown in Table 1. It can be seen from Table 1 that the proposed software has better stability and higher accuracy in terms of its positioning results with an accuracy of centimeter level.

### Table 1. Positioning precision statistics of the two kinds of software /m

| Station | RMS of the proposed software | RMS of TBC+COSA_GPS |
|---------|------------------------------|---------------------|
|         | N   | E   | U   | N    | E   | U    |
| CS01    | 0.009 | 0.007 | 0.039 | 0.020 | 0.001 | 0.030 |
| CS02    | 0.017 | 0.011 | 0.028 | 0.003 | 0.002 | 0.015 |
| CS03    | 0.015 | 0.020 | 0.024 | 0.014 | 0.004 | 0.021 |

### 4. Conclusion

The fast positioning algorithm for BeiDou post-mission network RTK was studied in this paper, mainly including base line resolution between reference stations, rapid fixing of double-difference integer ambiguity, error calculation and correction for moving station users, and fast resolution of integer ambiguity for moving station users. The algorithm was implemented with C++ programming and the resolving software of fast positioning for BeiDou post-mission Network RTK was developed. Also, the software was tested by the measured data from the BeiDou high precision reference station network in
Guangxi. The results show that the proposed software could provide users with centimeter-level accuracy positioning results. When the observation time reached 5 min, location services with a positioning accuracy higher than 1 m could be provided and when the observation lasted for 15 min, the plane precision was better than 3 cm while the elevation precision was better than 5 cm.

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