Collective two-boson decay of excitons in Bose-Einstein condensate and generation of coherent photon-phonon radiation

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The collective decay of excitons from initial Bose-Einstein condensate state is investigated theoretically. As practically more interesting case we consider excitons of the yellow series in the semiconductor cuprous oxide where we have collective photon and phonon assisted decay of excitons. It is shown that because of intrinsic instability of recoilless two-boson decay of Bose-Einstein condensate, the spontaneously emitted bosonic pairs are amplified leading to an exponential buildup of a macroscopic population into the certain modes. The collective decay rate has a nonlinear dependence on the excitonic density being comparable or larger than Auger recombination loss rate up to the high densities, which makes obtainable its observation. The considering phenomenon can also be applied for the realization of phonon laser.

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I. INTRODUCTION

Over the past half-century, excitons were considered as notably interesting candidates for Bose–Einstein condensation (BEC), in which collective coherence may lead to intriguing macroscopic quantum phenomena (see, Ref. 1 and references therein). Exciton being a bound state of an electron and a hole in a semiconductor is a unique physical system with a rather small mass comparable to the free electron mass. This is a crucial advantage from the experimental point of view since the BEC critical temperature of an excitonic gas is much higher than that of an atom gas with the same number density.2 However, the BEC was first successfully realized for trapped alkali atoms which are several thousand times heavier than excitons. The latter provided additional stimulus for realization of BEC for various condensed matter physics of quasiparticles. In this context, it is worthy to mention realization of BEC of quasiparticles, known as exciton–polaritons existing even at room-temperature.

Among the variety of bosonic quasiparticles, the excitons of the yellow series in the semiconductor cuprous oxide (Cu$_2$O) are still considered as the most promising candidates for pure excitonic BEC.2,2 Experiments in this direction have been done since 1986,2,2 and continued up to present due to several favorable features of excitons in Cu$_2$O. First, the large binding energy of 0.15 eV which increases the Mott density up to $10^{19}$ cm$^{-3}$. Second, the ground state of this series splits into the threefold degenerate orthoexciton and the non-degenerate paraexciton. The latter is the lowest energetic state lying below the orthoexciton states. Due to the selection rules one photon decay of paraexciton is forbidden. Its decay is only possible via optical phonon and photon resulting in a long lifetime.22,23 The latter is in the microsecond range during which BEC may be reached. To achieve excitonic BEC one should create a dense gas of excitons either in a bulk crystal or in a potential trap. However experiments,2,2,2,12,13,16,24 did not demonstrate conclusively excitonic BEC. The main reason for this failure is connected with the fact that the lifetime of excitons in Cu$_2$O decreases significantly at high gas densities. This effect has been attributed to an Auger recombination process between two excitons resulting in a loss rate $\Gamma_A = a_n$, where $a$ is the Auger constant and $n$ is the exciton gas density. However, there is no general consent on the value of Auger constant. The reported values for $\alpha$ range are from $10^{-20}$ cm$^3$ns$^{-1}$ to $1.8 \times 10^{-16}$ cm$^3$ns$^{-1}$ and differ for ortho- and para-excitons.24,29

As was mentioned above, the isolated exciton in Cu$_2$O is unstable and decays into photon and phonon. Due to BEC coherence, one can expect collective radiative effects at the decay of a large number of excitons. The latter may be a tool that evidences the state of the BEC, as well as, it may significantly reduce the lifetime of the BEC state. Such an effect has been revealed for the positronium atoms,30,31 which in some sense resembles excitons. It has been shown that at the coupling of two coherent ensembles of bosons – the BEC of positronium atoms and photons there is an instability at which, starting from the vacuum state of the photonic field, the expectation value of the photon’s mode occupation grows exponentially for a narrow interval of frequencies. For the excitons in Cu$_2$O one will have coupling between three bosonic fields and it is of interest to investigate how excitonic BEC burst into photons/phonons.

In this paper collective decay of excitons from initial Bose-Einstein condensate state is investigated arising from the second quantized formalism. It is shown that because of intrinsic instability of recoilless two-boson decay of Bose-Einstein condensate, the spontaneously emitted bosonic pairs are amplified, leading to an exponential buildup of a macroscopic population into certain modes. The exponential growth rate has a nonlinear dependence on the BEC density and it is quite large for the experimentally achievable densities. For the elongated condensate, one can reach self-amplification of the end-fire-modes. With the initial monochromatic photonic beam, one can generate the monochromatic phononic beam. Hence, the considered phenomenon may also be applied...
for realization of phonon laser.

The paper is organized as follows. In Sec. II the main Hamiltonian is introduced. In Sec. III spontaneous decay of exciton is analyzed. In Sec. IV we consider intrinsic instability of recoilless collective two-boson decay of excitonic BEC. Finally, conclusions are given in Sec. V.

II. BASIC HAMILTONIAN

We start our study with the construction of the Hamiltonian which governs the quantum dynamics of considered process. The total Hamiltonian consists of four parts:

\[ \hat{H} = \hat{H}_{\text{exc}} + \hat{H}_{\text{phot}} + \hat{H}_{\text{phon}} + \hat{H}_d. \]

Here the first part is the Hamiltonian of free excitons:

\[ \hat{H}_{\text{exc}} = \int d\Phi p \varepsilon_{\text{e}}(p) \hat{c}^+_p \hat{c}_p, \]

where \( \hat{c}_p \) (\( \hat{c}^+_p \)) is the annihilation (creation) operator for an exciton. These operators satisfy the Bosonic commutation rules for a relatively small number density \( n \) of excitons, that is at \( n < n_M \), where \( n_M \) is the Mott density. For the integration in phase-space we have introduced the notation \( d\Phi p = V d^3p / (2\pi)^3 \) (\( V \) is the quantization volume). Then, \( \varepsilon_{\text{e}}(p) = \hbar^2 p^2 / 2m_e + \hbar \omega_{\text{exc}} \) is the total energy of exciton with the momentum \( \hbar p \) of the center-of-mass motion, \( m_e \) is an exciton mass, \( \varepsilon_{\text{in}} \) is the exciton internal energy \( (\hbar \omega_{\text{exc}} = \varepsilon_0 - \varepsilon_b, \text{in terms of the band-gap difference } \varepsilon_0 \) and the binding energy \( \varepsilon_b ) \).

The second term in Eq. (1) is the Hamiltonian of the free photons

\[ \hat{H}_{\text{phot}} = \int d\Phi k \omega(k) \hat{c}^+_k \hat{c}_k, \]

where \( \hat{c}_k \) (\( \hat{c}^+_k \)) is the annihilation (creation) operator of the photon with the momentum \( \hbar k \) and dispersion relation \( \omega = \omega(k) \). The third term in Eq. (1) is the Hamiltonian of the free phonons with annihilation (creation) operator \( \hat{\delta}_q \) (\( \hat{\delta}^+_q \)):

\[ \hat{H}_{\text{phon}} = \int d\Phi q \omega_{\text{phon}}(q) \hat{\delta}^+_q \hat{\delta}_q, \]

The last term in Eq. (1)

\[ \hat{H}_d = \int d\Phi q \int d\Phi p \left[ \frac{\hbar M(q,p)}{V^{1/2}} \hat{c}^+_q \hat{c}_p \hat{c}^+_p \hat{c}_q - \frac{\hbar M^*(q,p)}{V^{1/2}} \hat{c}^+_p \hat{c}_p \hat{c}^+_q \hat{c}_q \right] \]

is the Hamiltonian of the two-boson decay of an exciton. Here we assume that the direct recombination of electrons and holes is very weak and the main decay process is a phonon-assisted recombination process in which an exciton decays emitting an optical phonon, as well as, a photon. The amplitude \( M(q,p) \) for an exciton decay can be calculated by the Feynman diagrams.

III. SPONTANEOUS DECAY OF AN EXCITON

Before considering collective decay of excitons it will be useful to consider spontaneous decay of a single exciton from the quantum dynamic point of view. For the spontaneous decay we consider initial condition in which the photonic and phononic fields begin in the vacuum state, while excitonic field is prepared in a Fock state with one exciton in the rest (\( p = 0 \)). Such state can be represented as \( |\Psi(0)\rangle = |0\rangle_{\text{phon}} \otimes |0\rangle_{\text{phot}} \otimes \hat{E}^+_0 |0\rangle_{\text{exc}} \). Then the state vector for times \( t > 0 \) is just given by the expansion

\[ |\Psi\rangle = C_0 e^{-i\varepsilon_0 t / \hbar} |0\rangle_{\text{phon}} \otimes |0\rangle_{\text{phot}} \otimes \hat{E}^+_0 |0\rangle_{\text{exc}} + \int d\Phi k d\Phi k' \]

\[ \times C_{k,k'}(t) e^{-i[\omega_{\text{ph}}(k')+\omega(k)]t} \hat{c}^+_k |0\rangle_{\text{phon}} \otimes \hat{c}^+_k |0\rangle_{\text{phot}} \otimes |0\rangle_{\text{exc}}, \]

where \( C_{k,k'}(t) \) is the probability amplitude for the photonic and phononic fields to be in the single-particle state, while excitonic field in the vacuum state. From the Schrödinger equation one can obtain evolution equations:

\[ i \frac{\partial C_{k,k'}(t)}{\partial t} = \frac{M(k',0)}{V^{1/2}} C_0(t) \frac{(2\pi)^3}{V^2} \delta(k+k') \]

\[ \times e^{i[\omega_{\text{ph}}(k') + \omega(k) - \omega_{\text{exc}}]t}. \]

(7)

Then, according to perturbation theory we take \( C_0(t) \simeq 1 \), and for the amplitude \( C_{k,k'}(t \to \infty) \) from Eq. (7) we obtain

\[ C_{k,k'} = \frac{M(k',0)}{i V^{1/2}} \frac{(2\pi)^3}{V^2} \]

\[ \times \delta(\omega_{\text{ph}}(k) + \omega(k) - \omega_{\text{exc}}) \delta(k+k'). \]

For the decay of an exciton the modes laying in the narrow interval of wavenumbers are responsible. Hence, for the dispersion relations we assume \( \omega_{\text{ph}}(k) = \text{const} \equiv \omega_{\text{ph}} \) and \( \omega(k) = k c_l \), where \( c_l \) is the light speed in a semiconductor. Then returning to expansion (6), one can write

\[ |\Psi\rangle \simeq C_0 e^{-i\omega_{\text{exc}} t} |0\rangle_{\text{phon}} \otimes |0\rangle_{\text{phot}} \otimes |1\rangle_{\text{exc}} \]

\[ + \frac{V^{1/2} M(k_0,0) k_l^2}{i (2\pi)^2 c_l} e^{-i\omega_{\text{exc}} t} \]

\[ \times \delta(\omega_{\text{ph}}(k_0) + \omega(k_0) - \omega_{\text{exc}}) \delta(k_0+k'). \]
\[ \times |0\rangle_{\text{exc}} \otimes \int d\mathbf{k}|1_{\mathbf{k}}\rangle_{\text{phon}} \otimes |-1_{-\mathbf{k}}\rangle_{\text{phot}}, \]  

where \( \mathbf{k} = \mathbf{k}/|\mathbf{k}|, \) and  

\[ k_0 = \frac{\omega_{\text{exc}} - \omega_{\text{ph}}}{c_1}. \]  

Hear we have taken into account that the decay amplitude does not depend on the direction of \( \mathbf{k} \) and, as a result, the final state (9) resulting from an exciton decay is a superposition of the states of oppositely propagating photon and phonon with the given momentum \( k_0. \) That is, we have recoilless two-boson decay of exciton, which is crucial for the development of instability in BEC where the lowest energy single particle state is occupied. For the decay rate one can write  

\[ \Gamma = \int d\Phi_\mathbf{k} d\Phi_{-\mathbf{k}} |C_{\mathbf{k}'; \mathbf{k}}|^2 / t_{\text{int}}, \]  

where \( t_{\text{int}} \) is the interaction time. With the help of Eq. (8) we obtain the well known result:  

\[ \Gamma = \frac{M^2}{\pi c_1} k_0^2. \]  

The radiative lifetime of an isolated exciton is \( \Gamma^{-1}. \)

### IV. COLLECTIVE DECAY

For analysis of the collective photon-phonon decay of excitons we will use Heisenberg representation, where the evolution operators are given by the following equation  

\[ i \frac{\partial \hat{L}}{\partial t} = \left[L, \hat{H}\right], \]  

and the expectation values are determined by the initial wave function \( \Psi_0: \)  

\[ \langle \hat{L} \rangle = \langle \Psi_0 | \hat{L} | \Psi_0 \rangle. \]

We will assume that the excitonic field starts up in the Bose-Einstein condensate state, while for photonic and phononic fields we will consider both vacuum state and states with nonzero mean number of particles. Taking into account Hamiltonian (11) from Eq. (12) we obtain a set of equations:  

\[ i \frac{\partial \hat{c}_{\mathbf{k}}}{\partial t} = \omega (\mathbf{k}) \hat{c}_{\mathbf{k}} + \int d\mathbf{p} \frac{M(p - \mathbf{k}, \mathbf{p})}{\sqrt{2}} \hat{b}_{p-k}^+ \hat{e}_p, \]  

\[ i \frac{\partial \hat{b}_{\mathbf{k}}}{\partial t} = \omega_{\text{ph}} (\mathbf{k}) \hat{b}_{\mathbf{k}} + \int d\mathbf{p} \frac{M(k, \mathbf{p})}{\sqrt{2}} \hat{c}_{p-k}^+ \hat{e}_p, \]  

\[ i \frac{\partial \hat{e}_p}{\partial t} = \hbar^{-1} \chi (\mathbf{p}) \hat{e}_p + \int d\mathbf{q} \frac{M^*(\mathbf{q}, \mathbf{p})}{\sqrt{2}} \hat{c}_{p-q} \hat{b}_q. \]

These equations are a nonlinear set of equations with the photonic, phononic and excitonic fields defined self-consistently. As we are interested in the quantum dynamics of considered system in the presence of instabilities we can decouple the excitonic field treating the dynamics of photonic and phononic fields. For this propose we just use the Bogolubov approximation. If the lowest energy single particle state has a macroscopic occupation, we can separate the field operators \( \hat{e}_p \) in the condensate term and the non-condensate components, i.e. the operator \( \hat{e}_p \) in Eqs. (13) and (14) is replaced by the c-number as follow  

\[ \hat{e}_p = \sqrt{n_0} (2\pi)^3 \delta (\mathbf{p}) e^{-i\omega_{\text{exc}} t}, \]  

where \( n_0 \) is the number density of excitons in the condensate. Making Bogolubov approximation we arrive at a linear set of the Heisenberg equations  

\[ i \frac{\partial \hat{c}_{\mathbf{k}}}{\partial t} = \omega (\mathbf{k}) \hat{c}_{\mathbf{k}} + \chi (\mathbf{k}) \hat{b}_{\mathbf{k}}^+ e^{-i\omega_{\text{exc}} t}, \]  

\[ i \frac{\partial \hat{b}_{\mathbf{k}}}{\partial t} = \omega_{\text{ph}} (\mathbf{k}) \hat{b}_{\mathbf{k}} + \chi (\mathbf{k}) \hat{c}_{\mathbf{k}}^+ e^{-i\omega_{\text{ph}} t}, \]

which couples photon modes with momentum \( \mathbf{k} \) to the phonons with momentum \(-\mathbf{k}\). The coupling constant is  

\[ \chi (\mathbf{k}) = \sqrt{n_0} M(k, 0). \]

Equations (17) and (18) compose a set of linearly coupled operator equations that can be solved by the method of characteristics whose eigenfrequencies define the temporal dynamics of the bosonic fields. The existence of an eigenfrequency with an imaginary part would indicate the onset of instability at which the initial spontaneously emitted bosonic pairs are amplified leading to an exponential buildup of a macroscopic mode population. Solving Eqs. (17) and (18), we obtain  

\[ \hat{b}_{\mathbf{k}}^+ = e^{i(\omega_{\text{ph}} - \frac{\delta (\mathbf{k})}{2}) t} \left\{ \hat{b}_{\mathbf{k}}^+ (0) \cosh (\sigma (k) t) + \frac{i}{2\sigma (k)} \right\} \]  

\[ \times \left\{ \delta (k) \hat{c}_{\mathbf{k}}^+ (0) + 2\chi (k) \hat{c}_{-\mathbf{k}} (0) \right\} \sinh (\sigma (k) t), \]  

\[ \hat{c}_{-\mathbf{k}} = e^{i(\frac{\delta (\mathbf{k})}{2} - \omega (k)) t} \left\{ \hat{c}_{-\mathbf{k}} (0) \cosh (\sigma (k) t) - \frac{i}{2\sigma (k)} \right\} \]  

\[ \times \left\{ 2\chi (k) \hat{b}_{\mathbf{k}}^+ (0) + \delta (k) \hat{c}_{\mathbf{k}} (0) \right\} \sin (\sigma (k) t), \]

where  

\[ \delta (k) = \omega (k) - \omega_{\text{exc}} + \omega_{\text{ph}} \]  

is the resonance detuning, and  

\[ \sigma (k) = \sqrt{|\chi (k)|^2 - \frac{\delta^2 (k)}{4}}. \]
As is seen from Eqs. (20)-(23), the condition for the dynamic instability is:

$$|\chi(k)| > \frac{|\delta(k)|}{2}$$

leading to the exponential growth of the modes in the narrow interval of wavenumbers

$$\omega_{\text{exc}} - \omega_{\text{ph}} - 2 |\chi(k)| < k c_t < \omega_{\text{exc}} - \omega_{\text{ph}} + 2 |\chi(k)|.$$  \hspace{1cm} (24)

For the interval (24) we find that the expectation value of the photonic and phononic modes occupations grow exponentially:

$$N_{\text{phot}}(k,t) = \langle \Psi_0 | \hat{c}_k^+ \hat{c}_k | \Psi_0 \rangle = N_{\text{phot}}(k,0) \left( \cosh^2 (\sigma(k)t) + \frac{\delta^2(k)}{4 \sigma^2(k)} \sinh^2 (\sigma(k)t) \right) + \frac{|\chi(k)|^2}{\sigma^2(k)} \left( 1 + N_{\text{phon}}(-k,0) \right) \sinh^2 (\sigma(k)t),$$  \hspace{1cm} (25)

$$N_{\text{phon}}(k,t) = \langle \Psi_0 | \hat{b}_k^+ \hat{b}_k | \Psi_0 \rangle = N_{\text{phon}}(k,0) \left( \cosh^2 (\sigma(k)t) + \frac{\delta^2(k)}{4 \sigma^2(k)} \sinh^2 (\sigma(k)t) \right) + \frac{|\chi(k)|^2}{\sigma^2(k)} \left( 1 + N_{\text{phot}}(-k,0) \right) \sinh^2 (\sigma(k)t).$$  \hspace{1cm} (26)

For the central wavenumber ($\delta(k_0) = 0$) the exponential growth rate is

$$G = 2 \chi(k_0) = 2 \sqrt{n_0} M(k_0,0).$$  \hspace{1cm} (27)

Taking into account Eq. (19) and derived expression (11) for the decay rate, we obtain compact expression for the exponential growth rate:

$$G = \sqrt{\frac{4 \pi n_0 c_t \Gamma}{k_0^3}}.$$  \hspace{1cm} (28)

As is seen from Eqs. (25) and (26), we have an exponential buildup of a macroscopic mode population even for the initial vacuum state $N_{\text{phot}}(k,0) = N_{\text{phon}}(k,0) = 0$. In this case from Eqs. (25) and (26) we have

$$N_{\text{phot}}(k,t) = N_{\text{phon}}(k,t) = \frac{4 |\chi(k)|^2}{4 |\chi(k)|^2 - \delta^2(k)} \times \left( e^{\sqrt{4|\chi(k)|^2 - \delta^2(k)t}} + e^{-\sqrt{4|\chi(k)|^2 - \delta^2(k)t}} - 2 \right).$$  \hspace{1cm} (29)

We have solved the issue considering uniform BEC without boundary conditions and, as a consequence, according to Eq. (20) we have an isotropic exponential gain. Due to the BEC coherence, here we have an absolute instability, i.e., the number of photons/phonons grows at every point within a BEC and the gain is scaled as $\sqrt{n_0}$. Here the exciton BEC burst into photons and phonons. Note, that our approximation is valid for the interaction times $t_{\text{int}}$ at which the total number of photons and phonons are much smaller than the number of excitons in BEC: $N_{\text{phot}}, N_{\text{phon}} << N_{\text{exc}}$.

For laserlike action, i.e., for directional radiation, one should take an elongated shape of the BEC. In this case, boundary conditions define interaction time. This can be incorporated into the derived equation (14) and (18) by introducing mode damping. The latter is simply due to the propagation of the bosonic fields, which escapes from the active medium and is inversely proportional to the transit time of a photon in the active medium. This transit time strictly depends on the propagation direction. The latter is equivalent to the finite interaction time strictly depending on the shape of the BEC.

For the directional radiation decay, one can also consider initial photonic or phononic beam. For the initial monochromatic photonic beam, in the result of the collective decay, one will have backscattered monochromatic photonic beam. Thus, one can realize a coherent source of phonons applying resonant laser beam.

Let us make explicit calculations for the initial photonic beam with the distribution:

$$N_{\text{phot}}(k,0) = N_0 \exp \left( -\frac{k_z^2 + k_y^2}{2 \delta^2} \right) \exp \left( -\frac{(k_z - k_0)^2}{2 \delta^2} \right),$$

where $N_0 >> 1$ and $\delta$ is the width of distribution in the momentum space $\delta << k_0$. In this case, for the angular distribution of the phonon number density (we assume $N_{\text{phon}}(k,0) << 1$) we have

$$\frac{dN_{\text{phon}}}{d\vartheta} \approx \frac{N_0}{(2\pi)^3} \int_{k_0 - \Delta k}^{k_0 + \Delta k} dk \frac{k^2 |\chi(k)|^2}{\sigma^2(k)} \sin \vartheta \times \exp \left( -\frac{k^2 \sin^2 \vartheta}{2 \delta^2} - \frac{(k \cos \vartheta + k_0)^2}{2 \delta^2} \right) \sinh^2 (\sigma(k) t_{\text{int}}),$$  \hspace{1cm} (30)

where $t_{\text{int}}$ is the interaction time of the photonic beam with excitonic BEC. As is seen from Eq. (30), phonons are radiated in the opposite to the photonic beam direction and have peak near $\vartheta \approx \pi$. For the phonon number density one should integrate Eq. (30) over $\vartheta$. Taking into account that $G << k c_t$, we obtain:

$$n_{\text{phon}} \approx \frac{\delta^2 G N_0}{2 \pi^2 c_t} F(G t_{\text{int}}),$$  \hspace{1cm} (31)

where

$$F(G t_{\text{int}}) = \int_0^1 dx \frac{\sinh^2 \left( \frac{G t_{\text{int}}}{2} \sqrt{1 - x^2} \right)}{1 - x^2}.$$  \hspace{1cm} (32)
Thus, for the exponential growth rate we have:

\[ \text{gap} \]

estimate possible parameters of coherent phononic beam is the amplification factor. The latter is a rapidly increasing function, displayed in Fig. 1.

Let us consider the parameters required for observation of the considered effect for the excitons of the yellow series in the semiconductor cuprous oxide. In Cu$_2$O, the radiative lifetime of an isolated exciton is \( \Gamma^{-1} \approx 10^{-5}\text{s} \), the refractive index is approximately 3 \( (c_n \approx 10^8\text{cm/s}) \), the energy of the optical phonon is \( 10^{-2} \text{eV} \), the energy gap \( \varepsilon_G \approx 2 \text{eV} \) and the binding energy \( \varepsilon_b \approx 0.15 \text{eV} \).

Thus, for the exponential growth rate we have:

\[ G \simeq \left( \frac{n_0}{10^{18}\text{cm}^{-3}} \right)^{1/2} \times 4 \times 10^{11}\text{s}^{-1}. \]  (33)

As is seen from Eq. (33), the growth rate is quite large \( G \approx 4 \times 10^{11}\text{s}^{-1} \) for the experimentally achievable densities \( n_0 = 10^{18}\text{cm}^{-3} \). Note that collective growth rate is larger than Auger recombination loss rate \( \Gamma_A = \alpha n_0 \) up to high densities \( n_0 < 4 \times 10^{18}\text{cm}^{-3} \). Let us also estimate possible parameters of coherent phononic beam generated by the photon beam. Taking density \( n_0 = 10^{18}\text{cm}^{-3} \) and interaction time \( t_{\text{int}} \approx 50 \text{ps} \) from Fig. 1 one can define \( F(Gt_{\text{int}} \approx 20) \approx 3.5 \times 10^7 \). From Eq. (33) for the phonon number density we have \( n_{\text{ph}} \approx N_0 \times 5.6 \times 10^{12}\text{cm}^{-3} \). Thus, considered phenomenon may be applied for the realization of a phonon laser.

V. CONCLUSION

In conclusion, we have studied the collective two-boson decay of excitons, arising from the second quantized formalism. It was shown that BEC state is unstable because of recoilless two-boson decay. The spontaneously emitted bosonic pairs are amplified leading to an exponential buildup of a macroscopic population into resonant modes. As a practically more interesting case, we have considered the decay of excitons of the yellow series in the semiconductor cuprous oxide, where BEC burst into the photons and phonons with the collective growth rate proportional to the square root of the BEC density. Calculations show that the collective decay rate is comparable or larger than Auger recombination loss rate up to the high densities. Hence, it can be used as a tool that evidences the formation of BEC state in Cu$_2$O. We have also studied another application of considered effect – a possible source for generation of coherent phonon beam. For the latter propose one can take an elongated condensate where self-amplification of the end-fire-modes takes place. Otherwise, applying a resonant photonic beam one can generate backscattered intense coherent phonon beam.

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