Frequency down conversion through Bose condensation of light

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We propose an experimental setup allowing to convert an input light of wavelengths about 1–2 µm into an output light of a lower frequency. The basic principle of operating relies on the nonlinear optical properties exhibited by a microcavity filled with glass. The light inside this material behaves like a 2D interacting Bose gas susceptible to thermalise and create a quasi-condensate. Extension of this setup to a photonic bandgap material (fiber grating) allows the light to behave like a 3D Bose gas leading, after thermalisation, to the formation of a Bose condensate. Theoretical estimations show that a conversion of 1 µm into 1.5 µm is achieved with an input pulse of about 1 ns with a peak power of 10^5 W, using a fiber grating containing an integrated cavity of size about 500 µm × 100µm^2.

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The recent discovery of the Bose-Einstein condensate in atomic gas demonstrated experimentally the possibility of creating a macroscopic state where a majority of particles are in the same quantum state [1]. This new property is of considerable interest, since it offers the possibility to use an atom laser. By analogy with the laser, the particles move collectively to form a coherent wave. The success of this discovery is due, in part, to the ability of atoms to cool evaporatively in a trap. By eliminating the hottest atoms, some cold atoms are heated and get their energy from the other cold ones ensuring a thermal equilibrium. The net results are a cooling leading to a macroscopic population in the lowest ground state.

One question arises then: if such a thermalisation process is succesful for atoms, why not use it for photons? Suppose that a photon gas has an initial non equilibrium spectrum of frequency in the optical domain. If it evolves in a nonlinear medium to allow strong enough collision interactions between photons, then the thermal equilibrium spectral distribution can be reached.

Based on simple dimensional analysis, we show in this paper that thermalisation of optical intense light is indeed possible, provided the following conditions are satisfied: 1) the light evolves in glass medium, having a sufficiently low absorption, in such a way to observe strong enough nonlinear interactions; 2) the glass is inside a high finesse cavity in order to confine the photons during a time greater than the relaxation time towards thermal equilibrium; 3) high intensity light is required to stimulate the collision process by means of Bose enhancement.

Furthermore, in order to realise the (quasi-)condensation of light, the photon must acquire an effective mass. A two-dimensional (2D) massive Bose gas is created in a cavity where only the fundamental is considered among longitudinal modes [2]. Concerning the three-dimensional (3D) Bose gas, we propose that the glass is a photonic bandgap material having a band with a dispersion relation resembling to that of a massive particle.

Using the process of thermalisation or of evaporative cooling, two important applications using the process of thermalisation or of evaporative cooling are interesting to study: first, the conversion of a high frequency pulse of light to a pulse of lower frequencies given by the Bose-Einstein distribution; second, the generation of a multicolor light out of a monochromatic one through an appropriate redistribution of the spectrum. We explore the possibility to use dielectric structures and multimode optical fiber gratings to realise the first application.

We start with the description of the 2D setup. It consists of a planar microcavity surrounded by two semi mirrors that ensure reflexion of the longitudinal component of the electromagnetic field. The microcavity has a transversal size much larger than the longitudinal one $L \sim \mu m$ and is filled with a material exhibiting signficative third order optical non linearity together with a small absorption coefficient. Typically, let us take the case of a glass with a linear and nonlinear refractive indexes $n_0 = 1.44$ and $n_2 \sim 3 \times 10^{-20} m^2/W$ and a linear absorption coefficient $\alpha \sim 5 \times 10^{-3} m^{-1}$. If the semi mirrors are perfect mirrors, then the longitudinal components $\vec{k}_0$ of the wave vector $\vec{k}$ take discrete values. The fundamental frequency corresponds to the lowest value $\omega_0 = 2\pi c/\lambda_0 = c/n_0|\vec{k}_0| = c\pi/n_0L$ where $c$ is the velocity of light. Selecting only this lowest value and continuous values for the transverse components $\vec{k}_\perp$, the energy spectrum $\epsilon(\vec{k})$ of the photon moving in the cavity is that of a relativistic 2D Bose gas with a mass $m = h\pi n_0/(Lc) \sim eV$ [3]:

$$\epsilon_{\vec{k}_\perp} = \hbar \omega_{\vec{k}} = \frac{\hbar c}{n_0} \sqrt{\left(\frac{\pi}{L}\right)^2 + k_{\perp}^2} \approx \hbar \omega_0 + \frac{\hbar^2 k_{\perp}^2}{2m} \quad (1)$$

The perfection of the cavity is limited by the quality of the semi mirror. High finesse of the order $\mathcal{F} \sim 10^6$ has been reported with a dielectric having a thin layer structure made of $SiO_2$ and $Ta_2O_5$.

When an electric field propagates inside the microcavity, a polarisation is created and is decomposed in three terms:}

$$\vec{P}(x,t) = \vec{P}_{\text{cons}}(x,t) + \vec{P}_{\text{dissip}}(x,t) - \vec{P}_{\text{gain}}(x,t) \quad (2)$$
The first term conserves the electromagnetic energy
\[ \bar{P}_{cons}(\vec{x}, t) = (\chi^{(1)} + \chi^{(3)} \vec{E}^2(\vec{x}, t)) \vec{E}(\vec{x}, t) \] (3)
and describes the linear and nonlinear polarisations that contribute to the total effective energy Hamiltonian:
\[ H = \int_V d^3\vec{x} \left[ \frac{1}{2} \vec{E}^2(\vec{x}, t) + \frac{\mu}{2} \vec{H}^2(\vec{x}, t) + \frac{3\chi^{(3)}}{4} \vec{E}^4(\vec{x}, t) \right] \] (4)

The susceptibility coefficients are related to those of the refractive index \( n = n_0 + n_2 I \) through \( \epsilon/\epsilon_0 = n_0^2 \) and \( \chi^{(3)} = \frac{\epsilon_0}{\epsilon} (1 + \chi^{(1)}) c n_2 \). The second term in (3) describes the dissipative losses \( \bar{P}_{dissip}(\vec{x}, t) = \alpha \vec{E}(\vec{x}, t) \). Finally, the third term is present in the case photons are created by an active medium inside the cavity, like a semiconductor. In classical physics, the presence of these terms allows to establish the following balance energy equation:
\[ \frac{dH}{dt} = \int_V d^3\vec{x} \left( \bar{P}_{gain}(\vec{x}, t) - \bar{P}_{dissip}(\vec{x}, t) \right) \cdot \vec{E}(\vec{x}, t) \] (5)

The quantification of the hamiltonian amounts to replacing the coefficients coming from each mode of the fields by the corresponding creation and annihilation operators \( a^\dagger \) and \( \hat{a} \). Each indice \( i \) labels one mode by its wavevector components \( \vec{k} \) and \( \alpha \) which represents the polarisation state that distinguishes between TE and TM waves. Developing the hamiltonian in terms of these operations, we make the rotating wave approximation that amounts to eliminating contribution which does not conserve the photon number. One obtains (6):
\[ \hat{H} = \sum_i \hbar \omega_i a^\dagger_i \hat{a}_i + \sum_{i,j,l,m} \frac{V_{i,j,l,m}}{2V} a^\dagger_i a^\dagger_j a_j a_m \] (6)
The effective potential is assumed to be constant in a first approximation and is related to the scattering length \( a \) through the relation \( V_{i,j,l,m} = \frac{4\pi a^2 \hbar^2}{\lambda} \). A comparison between the two expressions (6) and (3) for the hamiltonian allows to identify:
\[ a \simeq \frac{\hbar c^2 n_2 n_0^2}{4\pi} \left( \frac{2\pi}{\lambda_0} \right)^3 \] (7)

For glass and \( \lambda_0 = 1.5 \mu m \), we obtain \( a \simeq 3 \times 10^{-18} \) much lower than the value for atoms \( \left( a_{\text{atoms}} \sim 10^{-10} \right) \).

The frequency conversion inside the cavity proceeds in the following way. An initial photon pulse with a distribution centered in a frequency \( \omega_p \) higher than \( \omega_0 \) is created inside the cavity. This pulse could be generated from an active medium or could be injected from outside. Then, the thermalisation process turns the initial distribution into a Bose-Einstein distribution \( f(\omega_{\vec{k},z}) = \exp(\beta \hbar \omega_{\vec{k},z} - \beta \mu) - 1 \)^{-1} presenting a narrow width maximum intensity at the lowest frequency \( \omega_0 \), when the effective temperature \( k_B T_{\text{eff}} = 1/\beta \) is close to zero and the chemical potential \( \mu \) close to \( \omega_0 \). No condensate transition is expected for a 2D free Bose gas but quasicondensation is predicted for an interacting gas.

The energy conservation imposes also that at the same time some higher energy photons are distributed in the tail. If these photons escape from the cavity, then the process of evaporative cooling increases the population in the lowest levels. This process takes place provided that a lower refractive index of the coating allows the high transverse components of the light to escape from the edge of the cavity. Assuming the minimum refractive index for the coating (air), only modes with a wavelength greater than \( \lambda = \lambda_0/n_0 \) \( (1 \mu m \text{ for } \lambda = 1.5 \mu m) \) remain confined inside the cavity. But, as seen below, evaporative cooling is not necessary to convert light frequency.

Theorematisation of the light requires that the photon gas remains energetically isolated. Therefore, the average confining time of the photon inside the cavity is much larger than the relaxation time towards equilibrium \( \tau_{\text{relax}} \), which is the average time between two collisions of photons. The confining time is limited by the absorption inside the cavity \( \tau_{\text{abs}} = 1/(\alpha c) \) and by the cavity finesse \( \tau_{\text{cav}} \sim \frac{L}{c \lambda} \). For the case of a glass, the second effect is dominant since \( \alpha^{-1} \gg \frac{\lambda}{L} \) and therefore the realised condition becomes \( \tau_{\text{cav}} \gg \tau_{\text{relax}} \).

The relaxation time can be estimated approximatively from the kinetic theory to be:
\[ \frac{1}{\tau_{\text{relax}}} = \rho c n_0 \sigma (1 + F) \] (8)
and depends directly on the average photon density \( \rho \), the velocity inside the medium \( n_0 c \), the cross section \( \sigma = 4\pi a^2 \) assumed to be constant- and the degeneracy factor \( F \) which is the average photon per mode. Note that if \( \hbar \to 0 \) then \( a \to 0 \) and no relaxation exists since in a classical field system the photon does not exist.

The factor \( F \) is introduced to take into account the Bose enhancement which stimulates the collision process and diminishes the time for thermalisation. It originates from the Uehling-Uhlenbeck quantum kinetic equation establishing the balance of modes population. Third order terms in the mode population distribution appear in this equation and correspond precisely to a contribution to quadratic in the photon density. Indeed, we estimate this factor by dividing the density \( \rho \) by the mode density, assuming that the photon density is, in average, uniformly distributed for any frequency between \( \omega_0 \) and \( \omega_p \) and zero otherwise. In this approximation, the mode density is:
\[ \frac{1}{L} \int_S \frac{d^2 \vec{k}_z}{(2\pi)^2} = \frac{\pi}{L^3} \left[ \left( \frac{\lambda_0}{\lambda_p} \right)^2 - 1 \right] \] (9)
where \( S \) is the area of the wavevector satisfying
\[ |\vec{k}_\perp|^2 < \left( \frac{2\pi}{\lambda_p} \right)^2 - \left( \frac{2\pi}{\lambda_0} \right)^2. \]

We obtain
\[ F = \frac{L^2\rho}{\alpha} \left( \frac{\lambda_0}{\lambda_p} \right)^2 \left( 1 - \frac{1}{F} \right)^{-1} \quad (10) \]

Important deviation from this uniformity occurs if the initial/final distribution is narrowly peaked around the initial/final frequency \( \omega_p / \omega_0 \).

A second realisation condition is that the photon number or power absorbed inside the medium should not exceed a certain amount in order not to increase considerably the temperature of the glass. If we authorise an increase of 1\( K \) per pulse and since the specific heat of glass is about 2\( J/\text{m}^3\text{K}, \) the photon density absorbed must not be greater than \( \rho_t = 2 \times 10^{25} \text{photons/m}^3. \) If \( \tau_{\text{cav}} \ll \tau_{\text{life}} \) then the fraction absorbed is
\[ \rho(1 - \exp(-\tau_{\text{cav}}/\tau_{\text{abs}})) \simeq \rho \tau_{\text{cav}}/\tau_{\text{abs}} \leq \rho_t \quad (11) \]

The two realisation conditions combined together impose the following constraints on the density:
\[ \left[ \left( \frac{\lambda_0}{\lambda_p} \right)^2 - 1 \right]^{1/2} \frac{1}{2L\lambda_0 a \sqrt{F}} \ll \rho \leq \frac{\rho_t}{n_0 a L F} \quad (12) \]

To satisfy these inequalities for a ratio \( \lambda_0/\lambda_p = 1.5 \) the fidelity factor must obey \( F \leq 10^{14} \). In the realistic case of a dielectric with \( F \leq 10^6 \) the minimum density corresponds to \( 10^{26} \text{photons/m}^3 \) or \( 2 \times 10^{16} \text{W/m}^2. \) If the input light pulse time is equal to \( \tau_{\text{cav}} = 2.5 \times 10^{-9} \text{s}, \) we need a peak power of \( 10^{9} \text{W/cm}^2. \)

The effective temperature and the chemical potential are estimated by expressing the conservation of the photon number and energy before and after thermalisation, neglecting the much smaller losses. We deduce the following balance equations:
\[ \rho = \frac{1}{L} \int_S \frac{d^2 \vec{k}}{(2\pi)^2} f(\omega_{\vec{k}_\perp}) = \frac{2}{\lambda_B^2 L} \left( g_1(\kappa) + \frac{g_2(\kappa)}{\beta h \omega_0} \right) \quad (13) \]

\[
\begin{align*}
h \omega_p \rho & = \frac{2}{L} \int_S \frac{d^2 \vec{k}}{(2\pi)^2} h \omega_{\vec{k}_\perp} f(\omega_{\vec{k}_\perp}) \\ & = h \omega_p \rho + \frac{2}{\lambda_B^2 L} \left( g_2(\kappa) + \frac{g_3(\kappa)}{\beta h \omega} \right) \quad (14)
\end{align*}
\]

where \( \lambda_B = h / \sqrt{2\pi \hbar n k_B T} \) is the thermal wavelength and \( \kappa = \beta (h \omega_0 - \mu). \) \( g_k(z) = \sum_{j=1}^{\infty} e^{-j^2 / j^2} \) are the Bose-Einstein functions. A factor 2 takes into account the two states of polarisation. The resulting thermalized pulse has an effective temperature of \( T_{\text{eff}} \sim 10^6 \text{K} \) and a chemical potential \( \mu \rightarrow h \omega_0. \) In that situation, most of the light is converted in the low frequency region. If evaporative cooling takes place, then the temperature is lowered but some high energy photons are lost, limiting the conversion rate.

In the case of multimode fiber gratings, an all-in-one integrated structure is composed of two semi-mirrors and one microcavity (Fig.1). Three typical value of parameters are \( F = 10^3, \alpha = 5 \times 10^{-2} \text{m}^{-1} \) and the transverse core section of \( 10^3 \mu\text{m}^2, \) allowing to populate high transverse modes if the cladding index is close to one [6, 7]. This corresponds to a minimum density of \( 10^{18} \text{W/m}^2 \) and \( \tau_{\text{cav}} = 2.5 \times 10^{-12} \text{s} \) and a peak power of \( 10^9 \text{W}. \) The use of such a power has been reported in fiber grating experiments [8].

In order to increase the photon population in the fundamental level leading to condensation, a 3D cavity is needed and must allow more longitudinal values of the wave vector component. One possibility is that the cavity is itself a photonic bandgap material made of periodic layers, such that \( \omega_0 \) is the minimum frequency of an allowed band. In general, the interband interaction between particles is less frequent than the intraband interaction which, due to the difficulty to satisfy momentum-energy conservation, has much less probability to occur. Therefore, the thermalisation process is mainly achieved inside the band and generates a real condensate. If we assume that within the band the energy spectrum is of the form:
\[ \omega(\vec{k}) = \omega_0 + \frac{\hbar (\vec{k} - \vec{k}_0)}{2m} \quad (15) \]

then the density of modes is \( 4\pi \omega^2 |\vec{k}(\omega_p - \omega_0)|^{3/2}. \) For a longitudinal size \( L \) of the photonic bandgap material, the constraints on the density become:
\[ \left[ \left( \frac{\lambda_0}{\lambda_p} \right)^2 - 1 \right]^{3/4} \frac{1}{\lambda_0^{3/2} a \sqrt{F L}} \ll \rho \leq \frac{\rho_t}{n_0 a L F} \quad (16) \]

For a photonic bandgap integrated inside the fiber grating with \( L = 10^6 \text{L} \) and \( F = 10^4, \) we obtain a lower photon density of \( 2 \times 10^{16} \text{W/m}^2, \) a much higher \( \tau_{\text{cav}} = 2.5 \times 10^{-9} \text{s} \) and a lower peak power of \( 10^8 \text{W} \) than in the case of 2D. The effective temperature and the condensate population \( \rho_0 \) are estimated from the following photon number and energy balance equations:
\[ \rho = \rho_0 + 2\zeta \left( \frac{3}{2} \right) \frac{1}{\lambda_B^2} \quad (17) \]

\[ h (\omega_p - \omega_0) \rho = 2\zeta \left( \frac{5}{2} \right) \frac{k_B T_{\text{eff}}}{\lambda_B^2} \quad (18) \]

where \( \rho_0 \) is the condensate density. We find \( T_{\text{eff}} \sim 10^6 \text{K} \) much lower than the critical temperature \( T_c \sim 10^8 \text{K} \) which means that \( \rho_0 \approx \rho. \) The dispersion frequency in the resulting condensate is limited by \( \Delta \omega = 1/\tau_{\text{cav}}. \)

Strictly speaking, these estimations need to be validated by a more detailed description of the dynamical process. But at present, we prefer a model based on a simple estimation for two reasons. First, adequate kinetic equations must be formulated which take into account the presence of the condensate. Various models.
for atomic particle exists in the literature but no one is commonly accepted\cite{11}. In this context, there is no advantage to use one more sophisticated model if it is not justified by experiment. Second, the particular non-linear properties of a fiber grating at high light intensity are not yet well known to allow a complete characterisation\cite{9,10}.

Other important non-linear effects are the Raman and the Brillouin scatterings which can compete to convert the frequencies and can increase the absorption loss inside the cavity\cite{6}. The rate at which these phenomena occur in an ordinary glass are estimated to be linearly proportional to the density with the constants $10^{-13}m/W$ and $6 \times 10^{-11}m/W$ for the peak Raman-gain and peak Brillouin gain respectively\cite{6}. For the light power considered, this corresponds to relevant rates of $10^{12} - 10^{14}Hz$ which contribute to broaden the initial spectrum.

Another important question is how to inject the light inside the cavity. If we exclude the use of an active medium, the light can be introduced transversaly on the edge, by coupling the cavity medium with optical fiber or with a prism\cite{8}. In the case of fiber gratings, the spatial profile of the pulse should be designed in order to inject a maximum percentage of the initial intensity. This profile is determined by noticing the time reversal process describes an initial density distribution of light inside the cavity, evolving towards a final density distribution outside the cavity. This final distribution is simulated according to the dynamical model used which, in the case of low intensity, is governed by the linear classical Maxwell equations. Since the dynamic is time reversal invariant, this final density distribution gives precisely the initial profile to be used in order to confine all the light inside the cavity and has a size of the order of $\tau_{cav}/c/n_0$. Fig. 1 depicts a possible setup. Two initial pulses of light generated by a laser are modulated to get the initial profile and enter coherently in the fiber grating. After the frequency conversion takes place, the resulting light distribution leaves the cavity and gets out by means of the circulators.

In conclusion, we show the feasibility to convert the frequency of a light inside an optical nonlinear cavity. The analysis is based on an estimation of the collision rate between photons in a glass, leading to a thermalisation and a high condensed population in the fundamental mode of the cavity.

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![FIG. 1: Setup for frequency down conversion using fiber grating](image)

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