CONDITIONS OF DYNAMICAL STABILITY FOR THE HD 160691 PLANETARY SYSTEM

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Received 2003 January 24; accepted 2003 July 31

ABSTRACT

In our previous paper we showed that the currently determined orbital parameters placed four recently announced planetary systems (HD 12661, HD 38529, HD 37124, and HD 160691) in very different situations from the point of view of dynamical stability. In the present paper we deal with the last of these systems, whose orbital parameters of the outer planet are yet uncertain. We discover a stabilizing mechanism that could be the key to its existence. The paper is devoted to the study of this mechanism by a global dynamics analysis in the orbital parameter space related to the HD 160691 system. We obtained our results using a new technique called the mean exponential growth factor of nearby orbits (MEGNO), and verified them with the fast Lyapunov indicator technique (FLI). In order to be dynamically stable, the HD 160691 planetary system has to satisfy the following conditions: (1) it should have a 2:1 mean motion resonance, (2) combined with an apsidal secular resonance, (3) in a configuration \( P_c(ap) - S - P_b(ap) \) (which means that the planets c and b may be considered as initially located at their apoastron around the central star S), and (4) it must satisfy specific conditions for the respective sizes of the eccentricities. High eccentricity for the outer orbit \( (e_c > 0.52) \) is the most probable necessary condition, while the eccentricity of the inner orbit \( e_b \) becomes relatively unimportant when \( e_b > 0.7 \). We also show that there is an upper limit for planetary masses (in the interval permitted by the undetermined line-of-sight inclination factor \( \sin i_l \)) due to the dynamical stability mechanism. More generally, in this original orbital topology, where the resonance variables \( \theta_1 \) and \( \theta_3 \) librate about 180° while \( \theta_2 \) librates about 0°, the HD 160691 system and its mechanism have revealed aspects of the 2:1 orbital resonances that have not been observed nor analyzed before. The present topology with antialigned apsidal lines, combined with the 2:1 resonance, is indeed more wide-ranging than the particular case of the HD 160691 planetary system. It is a new theoretical possibility that is suitable for a stable regime despite relatively small semimajor axes with respect to the important masses in interactions.

Subject headings: celestial mechanics — instabilities — planetary systems — stars: individual (HD 160691) — stellar dynamics

1. INTRODUCTION

In our previous paper (Kiseleva-Eggleton et al. 2002) we applied the new technique invented by Cincotta & Simó (2000) and called MEGNO (Mean Exponential Growth factor of Nearby Orbits) to a wide neighborhood of orbital parameters determined using standard two-body Keplerian fits for the recently discovered multiplanetary systems HD 12661, HD 38529, HD 37124, and HD 160691 in order to distinguish between regular and chaotic planetary configurations. We showed that the recently announced orbital parameters place these systems in very different situations from the point of view of dynamical stability. While HD 38529 and HD 37124 are located within large stability zones in the phase space around their determined orbits, the orbital parameters of the HD 12661 planets are located in a border region between stable and unstable dynamical regimes, so while its currently determined orbital parameters produce stable orbits, a minor change within the margin of error of just one parameter may result in a chaotic dynamical system. The orbits in HD 160691 (Jones et al. 2002) at first appeared highly unstable, but using MEGNO we were able to identify a few stability zones in a parameter space that included the parameters not determined from observations, such as the relative inclination \( i_r \) between the two planetary orbits and the longitudes of the ascending nodes \( \Omega \). All these stable configurations are associated with the 2:1 mean motion resonance. The present paper is wholly devoted to a detailed and complete dynamical analysis of the HD 160691 planetary system, taking into account the angular orbital parameters not constrained by observational data \( (i_r, \Omega) \), as well as \( \sin i_l \) (the line-of-sight inclination factor) and the resulting different planetary masses. We have also explored the space of the mean anomalies \( M \) of the two planets, initially taking into account the time of periastron passage \( \tau_{per} \) given by Jones et al. (2002; see Table 1).
In one of our earlier papers (Goździewski, Bois, & Maciejewski 2002) we have clearly identified (using MEGNO) the exact location of the 2:1 mean motion resonance and its width for the Gliese 876 planetary system. A recent study by Hadjidemetriou (2002) of periodic orbits in this resonance predicts stable and unstable configurations of planetary systems, depending on the hierarchy of planetary masses and eccentricities. The Gliese 876 system (where \( m_b < m_c \) and \( e_b > e_c \)) is, according to this study, a stable configuration. In contrast, the hierarchies of the HD 160691 system are inverse, i.e., \( m_b > m_c \) and \( e_b < e_c \). Hadjidemetriou (2002) found that a planetary system at the 2:1 resonance, where the inner planet is much more massive than the outer planet, is unstable for all values of the eccentricities. However, in the HD 160691 system the two planetary masses are comparable, and this leaves open the question of its stability.

In this work we have explored the parameter space available for planets in HD 160691 in order to determine the stability conditions for this system. We note that the orbital parameters of both planets are rather speculative as a result of insufficient observations, and even the existence of the second planetary companion is not yet fully confirmed (Jones et al. 2002; Butler et al. 2001). Nevertheless, the mechanism we present in this paper is probably the key to the existence of planetary systems like HD 160691. We have clearly identified the exact location of the 2:1 mean motion resonance, its width, and the secular resonance in apsidal longitudes preserving the stability related to the mean motion resonance.

2. METHOD

A classical method that allows us to distinguish between regular and chaotic dynamical states is the method of Lyapunov characteristic numbers (LCNs). The estimation of LCNs usually requires computations over long, sometimes much longer than the lifetime of the system studied. Note that “chaotic,” in the Poincaré sense, means that the dynamical behavior is not quasi-periodic (the conventional definition useful for conservative dynamical systems), and does not necessarily mean that the system will disintegrate during any limited period of time. Let us state in addition that we use the property of “stability” in the Poisson sense: stability is related to the preservation of a certain neighborhood relative to the initial position of the trajectory. In conservative systems, quasi-periodic orbits always remain confined within certain limits; in this sense they are stable.

In the present work we use two different methods in order to identify the dynamical state of the HD 160691 system: the MEGNO and FLI (fast Lyapunov indicator) techniques. These two methods converge faster and are more sensitive than the LCN technique.

FLI is the method introduced by Froeschlé, Lega, & Gonczi (1997) that permits us to distinguish qualitatively between regular and chaotic motion in a dynamical system (see for example Pilat-Lohinger & Dvorak 2002). MEGNO is a new method developed by Cincotta & Simó (2000) that we have already successfully applied to the study of dynamical stability of extrasolar planetary systems (see, e.g., Goździewski et al. 2001, 2002; Kiseleva-Eggleton et al. 2002). This method provides relevant information about the global dynamics and the fine structure of the phase space, and it simultaneously yields a good estimate of the LCN with a comparatively small computational effort (Cincotta & Giordano 2000). MEGNO is an alternative technique that proves efficient for investigation of both ordered and stochastic components of phase space (Cincotta, Giordano, & Simó 2002). It provides a clear picture of the resonance structures, location of stable and unstable periodic orbits, and a measure of hyperbolicity in chaotic domains (i.e., the rate of divergence of unstable orbits) that coincides with that given by the LCN.

3. STABILITY CONDITIONS

In order to identify the different dynamical behaviors in the parameter space, we use MEGNO, which provides the exact location of stable and unstable orbits, as well as a measure of hyperbolicity. Figures 1 and 2 show the dynamical state of the HD 160691 system as a function of both orbital semimajor axes \( a_b \) and \( a_c \) (Fig. 1) and eccentricities \( e_b \) and \( e_c \) (Fig. 2). All other orbital parameters were taken from Table 1, with the addition of values of \( \Omega_{b,c} \) and \( i_{b,c} \) undetermined from observations, and with \( \tau_{\text{per}} \) replaced by the corresponding calculated values of mean anomalies \( M = (2\pi/P)(t - \tau_{\text{per}}) \) (with an initial time of reference \( t = \tau_{\text{per}} \), one obtains \( M_b = 156.3^\circ \) and \( M_c = 0^\circ \)). In order to avoid a dynamical behavior bound to the plane, the two initial orbital inclinations \( i_b \) and \( i_c \) are to be taken slightly differently (0° and 1°, respectively); because of gravitational interactions of the three-body problem, the relative inclination \( i_c \) is free to evolve in the three-dimensional space. In Figure 2, as well as in other figures in this paper, \( a_c \), was taken to be 2.381 AU, according to the table footnote. In all our figures the intersection of horizontal and vertical lines indicates the “observational” initial parameters taken from Table 1. By using the MEGNO indicator characteristic

![Table 1](http://www-ma1.upc.es/intas/preprints).

**TABLE 1**

| Planet | \( m_p \sin i_p (M_J) \) | \( a (\text{AU}) \) | \( P (\text{days}) \) | \( e \) | \( \omega (\text{deg}) \) | \( \tau_{\text{per}} \) (HJD) |
|--------|----------------|----------------|----------------|-----|----------------|----------------|
| b      | 1.7 ± 0.2     | 1.5 ± 0.1     | 638 ± 10      | 0.31 ± 0.08 | 320 ± 30      | 50,698 ± 30    |
| c      | 1.0           | 2.3 \(^a\)    | 1300          | 0.8    | 99             | 51,613         |

Note.—Data from Jones et al. 2002, \( M_b = 1.08 M_J \).

\(^a\) In our numerical models we changed the value of \( a_c \) from 2.3 to 2.381 AU (well within the error of its determination). The latter value gives the exact location of the 2:1 resonance (by Kepler equation resolution) and is located in the middle of the stability valley on the \([a_b, a_c]\) parameter space (see Fig. 1a).
Yhi (Cincotta & Simó 2000), filled and open circles in Figure 1 and all other figures plotted in two dimensions in this paper) indicate stable orbits (\(Y_{hi} = 2 \pm 3\%\) and \(Y_{hi} = 2 \pm 5\%, \) respectively), while small dots not surrounded by circles indicate highly unstable orbits. The \(Y_{hi}\) is the MEGNO indicator characteristic value (Cincotta & Simó 2000). Here, (a) is a cross section of (b) in the plane \(Y_{hi} = 2\) with projections of the different points according to their nature. In (b), the peaks indicate the magnitude of instability. The stability strip in (a) corresponds to a stability valley in three dimensions in (b) (however, its fine structure cannot be seen in this three-dimensional graph because of the scale factor in the \(z\)-axis due to the high magnitude of the instability peaks). In each figure of the paper, the parameters used (except the pair of parameters scanned in the particular figure) have the following values (initial conditions for numerical integrations): \(a_b = 1.5, e_b = 0.31, i_b = 1^\circ, \Omega_b = 0^\circ, \omega_b = 320^\circ, M_b = 166.3; a_c = 2.381, e_c = 0.80, i_c = 1^\circ, \Omega_c = 0^\circ, \omega_c = 99^\circ, M_c = 0^\circ\).

Fig. 1.—Stability maps in the \([a_b, a_c]\) parameter space in (a) two dimensions and (b) three dimensions for the HD 160691 planetary system (resolution of the grid is 30 x 30 points). In (a), filled and open circles indicate stable orbits ((\(Y\) = 2 ± 3\%) and (\(Y\) = 2 ± 5\%, respectively), while small dots not surrounded by circles indicate highly unstable orbits. The \(Y\) is the MEGNO indicator characteristic value (Cincotta & Simó 2000). Here, (a) is a cross section of (b) in the plane (\(Y\) = 2) with projections of the different points according to their nature. In (b), the peaks indicate the magnitude of instability. The stability strip in (a) corresponds to a stability valley in three dimensions in (b) (however, its fine structure cannot be seen in this three-dimensional graph because of the scale factor in the \(z\)-axis due to the high magnitude of the instability peaks). In each figure of the paper, the parameters used (except the pair of parameters scanned in the particular figure) have the following values (initial conditions for numerical integrations): \(a_b = 1.5, e_b = 0.31, i_b = 1^\circ, \Omega_b = 0^\circ, \omega_b = 320^\circ, M_b = 166.3; a_c = 2.381, e_c = 0.80, i_c = 1^\circ, \Omega_c = 0^\circ, \omega_c = 99^\circ, M_c = 0^\circ\).

Fig. 2.—Stability map in the \([e_b, e_c]\) parameter space for the HD 160691 planetary system. The symbols and grid resolution are the same as in all figures (see Fig. 1a).
formally preserved, the stability valley is lost.

system nominal elements (cf. Table 1 and Fig. 1) except for the mean anomaly of the b planet: \( M_b \) (computed with periastron, and consequently the dynamical behavior shift only of the \( M_b \) orbits, determined by the particular relative positions of the two planets on their orbits, determined by the \( M \) elements. For example, for a shift only of the \( M_b \) element, without losing the 2:1 mean motion resonance, close approaches eventually occur at the periastron, and consequently the dynamical behavior becomes totally and highly unstable, as shown in Figure 3 (computed with \( M_b = 0^\circ \) instead of 156.3'). The same situation appears with a comparable shift in the \( \omega \) elements. For instance, with \( \omega_b = 0^\circ \), 100', or 200' instead of 320', the stability valley is lost. The whole stabilizing mechanism depends on particular combinations of the elements \( \Omega, \omega, i_r \), and \( M \). We discuss these stability conditions in detail below.

Figure 2 shows two stability regions in the \([e_b, e_c]\) parameter space: (1) a small one in the bottom left-hand corner with a handful of points where both eccentricities are small; and (2) a much larger region for high values of the outer eccentricity \( e_c \). Note that in the latter region, the eccentricity of the inner orbit \( e_b \) is relatively unimportant when \( e_c > 0.7 \). We should emphasize that the large region with a sizeable number of stable points decreases significantly when \( e_c \) is taken to be 2.3 instead of 2.381 AU (see Fig. 3d in our previous paper, Kiseleva-Eggleton et al. 2002). In Figure 2, because of the resolution of the grid, the dynamical behavior of the “observational” point in eccentricities (see Table 1) does not appear. However, with a higher resolution (i.e., more integration points), computations confirm the existence of a stable point at the intersection of the horizontal and vertical lines. In addition, there are a few more unstable points in the close vicinity of the observation point. Recall that some zones (notably on the sides of stability structures) may present a sensitivity to initial conditions for stable versus unstable points in a very close vicinity between them. This situation reminds us of the dynamical state of the HD 12661 planetary system (Kiseleva-Eggleton et al. 2002).

Figure 4 shows how the dynamical state of the HD 160691 system depends on the relative inclination \( i_r \) between the planetary orbits and on the angular elements \( \omega \) and \( \Omega \) for both planets b and c. In the general case of a two-planet system, the relative inclination \( i_r \) is related to the longitudes of ascending nodes \( \Omega_b \) and \( \Omega_c \), as follows:

\[
\cos i_r = \cos i_b \cos i_c + \sin i_b \sin i_c \cos(\Omega_c - \Omega_b)
\]

However, in our computations, for a variation of \( i_b \) or \( i_c \), the inclination of the other orbit is taken equal to zero. As a consequence, \( i_r = i_b \) or \( i_c \), and then the inclinations on the vertical axes shown in Figure 4 can be simply read as a scanning of the relative inclination \( i_r \) between the orbits. It is important to note that the distributions of structures are relatively similar in the pairs \([\omega_b, i_b]\) and \([\omega_c, i_c]\), as well as in \([\Omega_b, i_b]\) and \([\Omega_c, i_c]\), with stable orbits appearing in strips of a comparable width \( \sim 100' \) around the nominal values of \( \omega_b \) and \( \omega_c \) (see Table 1) for all values of \( i_r \). There are also additional stability zones, but very narrow ones, for small values of \( i_r < 20' \) in both maps related to \( \omega \), as well as in the two others related to \( \Omega \). The nominal values for \( \omega_b \) and \( \omega_c \), namely 320' and 99', respectively, are located well inside the stable strips.

Figures 5 and 6 present the stability maps of the \([\hat{\omega}_b, \hat{\omega}_c]\) (where \( \hat{\omega} = \Omega + \omega \)) and \([M_b, M_c]\) parameter spaces, respectively. Figure 5 shows a linear relationship between the longitudes of periastron \( \hat{\omega}_b \) and \( \hat{\omega}_c \) for stable configurations. This means that \( \hat{\omega}_b \) and \( \hat{\omega}_c \) precess at the same rate. Figure 6 shows stable strips in the mean anomaly \([M_b, M_c]\) plane that permit us to identify adequate relative orbital positions for stable configurations at the 2:1 resonance in mean motions.
Figures 5 and 6 also show that the HD 160691 planetary system is dynamically stable only when the corresponding orbital parameters are simultaneously represented by stable points inside the strips in both $\omega_b$; $\omega_c$ and $\Omega_b$; $\Omega_c$ planes. If only one of the two conditions is realized, the map of $\omega_b$; $\omega_c$ exhibits a totally unstable dynamical state without any particular stability structure (see Fig. 3). We have also computed the $\omega_b$; $\omega_c$ stability maps for different relative inclinations (from $0^\circ$ to $90^\circ$; not shown in this paper), and consequently we can point out that the whole mechanism of stabilization can be successfully applied even to high relative inclinations if $\Omega$, $\omega$, and $M$ are determined in an appropriate way: $\omega_b$ and $\omega_c$ on average precess at the same rate, while the planets b and c, in 2:1 mean motion resonance, are located on their orbits in such a way that there are no close approaches at their periastron. The favorable respective positions are defined by the stable strips in the mean anomaly $[M_b, M_c]$ plane (cf. Fig. 6).
in the HD 160691 system the simultaneous librations of the two mean motion resonance variables \( \theta_1 = \lambda_b - 2\lambda_c + \dot{\omega}_b \) and \( \theta_2 = \lambda_b - 2\lambda_c + \dot{\omega}_c \) (where \( \lambda = M + \omega \)), while the secular resonance variable

\[
\theta_3 = \theta_1 - \theta_2 = \dot{\omega}_b - \dot{\omega}_c
\]

librates about 180° with a period of about 2800 yr and an amplitude of 80° (see Fig. 7). As \( \theta_1 \) is only librating (i.e., without a secular term), the two orbital planes on average precess at the same rate. This is a mechanism of secular resonance in periastron that, as a consequence, ensures the maintenance of the orbital topology as well as the respective motions of the planets (or, in the present case, the 2:1 mean motion resonance without close approaches of the planets at their periapses). In other words, this mechanism, when combined with adequate relative positions of the planets on the initial osculating orbits, can be understood as the condition for the preservation of the dynamical stability related to the 2:1 mean motion resonance. Because the two variables \( \theta_1 \) and \( \theta_2 \) librate around 180° and 0°, respectively (see Fig. 7), the lines of apsides are antialigned. This situation is different from the GJ 876 configuration, in which the apsidal lines are aligned, with \( \theta_1, \theta_2, \) and \( \theta_3 \) all librating about 0° (Lee & Peale 2002; Gozdziewski et al. 2002). We should note that in the familiar Io-Europa 2:1 resonance, the very small eccentricities lead to a geometry in which \( \theta_1 \) is librating about 0°, while \( \theta_2 \) and \( \theta_3 \) are librating about 180° (Lee & Peale 2002). In this case, conjunctions occur when Io is near periapse and Europa near apoapse.

The whole stability mechanism for the HD 160691 system allows it to avoid close approaches between planets, especially at their periapses (or near them). If this is not the case, the dynamical behavior is wholly unstable, since the close approaches near periapses occur regularly (see Fig. 3, where \( M_b = M_c = 0° \)). The original orbital topology of the HD 160691 planetary system associated with the 2:1 mean motion resonance can be written as \( P_c(ap)-S-P_b(ap) \), which means that planets c and b can be considered as initially located at their apoastron around the central star S (state 1). Taking into account the 2:1 mean motion resonance, the present topology is equivalent to \( S-P_b(ap) \). After one revolution of planet b, planet c is at its periapse (state 2). This situation corresponds (by taking into account the important amplitude of the secular resonance variable \( \theta_1 \), namely 80°) to the nominal anomalies given in Table 1 (\( M_b \sim 160° \)). After one more revolution of planet b, the planets are again near their apostraon (state 1). Besides, with the second state, we understand that the present topology may drive to a solid stability where the sum of both eccentricities is particularly important (i.e., \( e_b + e_c > 0.7 \)). High \( e_c \) helps; see Fig. 2). Because of the high eccentricities of the orbits, and despite relatively small semimajor axes, the relative distances between the two planets may remain sufficiently large over the whole evolutionary timescale of the system.

We have also tested the robustness of this mechanism when the values of the planetary masses were progressively increased. In Figure 8, which was computed by the FLI

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2 This three-dimensional topology with antialigned apsidal lines is out of the scope of the various 2:1 resonance configurations analyzed by Hadjijemetriou (2002; two types of configuration) and Hadjijemetriou & Psychogiou (in preparation; four types in the planar case).
technique in the $[a_0 \cdot a_c]$ parameter space, the two masses are consecutively multiplied by a factor $k = 1$ (Fig. 8a), 1.3 (Fig. 8b), 1.4 (Fig. 8c), 1.5 (Fig. 8d). The total destruction of the stable structure is reached for $k = 1.5$, which corresponds to a line-of-sight inclination factor $\sin i = 2/3$ (i.e., $i = 41.8^\circ$), as we know only the lower limit of planetary masses $m_0$ from the function $m_0 = m_P \sin i$, where $m_P$ is the real mass. This result is in agreement with a numerical analysis by Kiseleva-Eggleton & Bois (2001) of multiplanetary systems of $v$ And type, which showed that the dynamical regime, and consequently the lifetime of the system, depends strongly on the mass hierarchy, as well as on the absolute values of planetary masses. In other words, assuming that the observed planetary system is dynamically stable, we could probably determine a window for the possible values of line-of-sight inclinations $i$, and as a consequence give upper limits on the planetary masses.

4. CONCLUSION

Using the MEGNO technique of global dynamics analysis, we scanned the most relevant cases of the orbital parameter space for the HD 160691 planetary system. We have found the existence of a stability zone ruled by a mechanism that involves angular elements of the system. This stability zone is in fact due to the $2:1$ mean motion resonance, coupled with adequate relative positions of the planets on their orbits, avoiding close approaches at their periastron, the two apsidal lines being antialigned. The mechanism is not lost during the dynamical evolution of the system, because of an apsidal secular resonance: the mean motion resonance variables are librating, while the longitudes of periapse on average precess at the same rate.

We conclude that in order to be dynamically stable, the HD 160691 planetary system has to satisfy the following conditions: (1) it must have a $2:1$ mean motion resonance, (2) combined with an apsidal secular resonance, (3) in a configuration $P_c(\text{ap}) - S - P_b(\text{ap})$ (i.e., an apsidal antialignment), and (4) it must satisfy specific conditions on the respective sizes of the eccentricities. High eccentricity for the outer orbit ($e_c > 0.52$) is the most probable necessary condition, while the eccentricity of the inner orbit $e_b$ becomes relatively unimportant when $e_c > 0.7$. These four conditions, also taking into account various relative inclinations between the two orbits, determine the dynamical behavior of the system in such a way that the planets are never too close to each other. In the end, the HD 160691 system, where the resonance variables $\theta_1$ and $\theta_3$ librate about $180^\circ$ while $\theta_2$ librates about $0^\circ$, has revealed resources of the $2:1$ orbital resonances that have not been observed nor analyzed before. The present orbital topology $[P_c(\text{ap}) - S - P_b(\text{ap})]$, combined with the $2:1$ orbital resonance, is indeed more wide-ranging than the particular case of the HD 160691 planetary system. It is a new theoretical possibility that is suitable for a stable regime despite relatively small semimajor axes with respect to the important masses in interactions.3

Combining our MEGNO maps (confirmed with the FLI method) for different pairs of parameters, it is possible to converge toward stability conditions related to different combinations of the angular parameters. Since the observational determination of the elements of the HD 160691

3 As we were finishing the present paper, we learned that an apsidal antialignment has been proposed for the dynamical stability of the HD 82943 planetary system related to a $2:1$ orbital resonance (Ji et al. 2003).
system is far from being finalized, we hope that the maps presented here will be useful for testing both future observations and different parameter-fitting techniques. With respect to this, let us mention the fitting method of Laughlin & Chambers (2001), which is suitable for resonant interactions between the planets and cases in which true masses can be determined by eliminating the indeterminacy in \( \sin i \) inherent in fits that assume independent Keplerian motions. We should note here that we have already tested some of the new fits for HD 160691 obtained by Eugenio Rivera (2002, private communication), and found that the only stable systems with \( k < 2 \) that do not meet our stability conditions would be ones with a very massive substellar distant companion \( m_c \sim 50M_J \) on a very large orbit \( a_c \sim 30 \text{ AU} \).

Fig. 8.—Stability maps in the \([a_b, a_c]/\text{parameter spaces for different masses. Here, } k \text{ is the multiple of the determined minimum mass of the planets: (a) } k = 1, \text{ (b) } k = 1.3, \text{ (c) } k = 1.4, \text{ (d) } k = 1.5. \text{ The stable motion is represented by black squares (FLI } < 10^8 \text{), crosses are plotted if } 10^8 < \text{FLI } < 10^{16} \text{, and the white area shows the chaotic regions (FLI } > 10^{16} \text{).}
This type of system is very different from the systems with Jupiter-mass planets on close orbits that we have discussed in this paper.

L. K.-E. thanks the University of Bordeaux for a short-term research fellowship in Bordeaux, and John Hadjidemetriou for the text of his yet unpublished paper. E. B. and L. K.-E. thank Rudolf Dvorak and the Institute of Astronomy at the University of Vienna for his hospitality. E. P.-L. wishes to acknowledge the support of the Austrian FWF (Hertha Firnberg Project T122). The authors thank Eugenio Rivera for providing us with his latest unpublished orbital fits, and for useful discussions. We thank Peter Eggleton for his comments and English corrections, and referee George Contopoulos, whose questions and comments greatly helped to improve the paper.

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