Gribov horizon beyond the Landau gauge

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Abstract

Gribov and Zwanziger proposed a modification of Yang-Mills theory in order to cure the Gribov copy problem. We employ field-dependent BRST transformations to generalize the Gribov-Zwanziger model from the Landau gauge to general $R_\xi$ gauges. The Gribov horizon functional is presented in explicit form, in both the non-local and local variants. Finally, we show how to reach any given gauge from the Landau one.

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1 Introduction and summary

It is long known that the covariant quantization of Yang-Mills theory is beset by the Gribov problem: the existence of infinitely many discrete gauge copies even after gauge fixing [1]. A natural remedy, suppressing the field integration outside the Gribov horizon, is accomplished by adding to the action a Gribov horizon functional [1]–[5]. The latter, however, is not BRST invariant and usually chosen in the Landau gauge. For a better understanding of its effect on the gauge variance of Greens functions, a knowledge of the horizon functional in other gauges is desirable [6].

Recently, we have discovered an explicit way to change the gauge in Faddeev-Popov quantization by effecting a suitable field-dependent BRST transformation [7]. Here, we utilize this strategy to define horizon functionals for the non-local and local forms of the Gribov-Zwanziger model in any $R_\xi$ gauge. At the end of the paper, we present the horizon functional in an arbitrary gauge.

2 Yang-Mills theory with Gribov horizon

Yang-Mills theory with gauge group $SU(n)$ in $d$ spacetime dimensions features gauge potentials $A^a_\mu(x)$ with $a = 1, \ldots, n^2 - 1$ and $\mu = 0, 1, \ldots, d - 1$. The classical action has the standard form

$$ S_0(A) = -\frac{1}{4} \int d^d x \, F^a_{\mu\nu} F^{\mu\nu a} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu, \quad (2.1) $$

where $f^{abc}$ denote the (totally antisymmetric) structure constants of the Lie algebra $su(n)$. The action (2.1) is invariant under the gauge transformations

$$ \delta A^a_\mu = D^a_{\mu \lambda} \xi^b \quad \text{with} \quad D^a_{\mu \lambda} = \delta^{a b} \partial_\mu + f^{a c b} A^c_\mu. \quad (2.2) $$

The BRST formulation of the quantum theory extends the field content to

$$ \{ \phi^A \} = \{ A^a_\mu, B^a, C^a, \bar{C}^a \} \quad (2.3) $$

by adding the Nakanishi-Lautrup auxiliary fields as well as the Faddeev-Popov ghost and antighost fields, in the order above. The Grassmann parities $\varepsilon$ and ghost numbers $gh$ are

$$ \varepsilon(C^a) = \varepsilon(\bar{C})^a = 1, \quad \varepsilon(A^a_\mu) = \varepsilon(B^a) = 0, \quad gh(A^a_\mu) = gh(B^a) = 0, \quad gh(C^a) = -gh(\bar{C}^a) = 1. \quad (2.4) $$

In DeWitt notation [8], the quantum action à la Faddeev and Popov [9] takes the form

$$ S(\phi) = S_0(A) + \bar{C}^a K^{ab}(A) C^b + \chi^a(A) B^a, \quad (2.5) $$
with the Faddeev-Popov operator

$$K^{ab}(A) = \frac{\delta \chi^a(A)}{\delta A^b_\mu} D^{cb}_\mu = \partial^\mu D^{ab}_\mu = \delta^{ab} \partial^\mu \partial_\mu + f^{abc} A^c_\mu \partial^\mu$$

(2.6)

for the gauge-fixing functions \(\chi^a\) of the Landau gauge,

$$\chi^a(A) = \partial^\mu A^a_\mu .$$

(2.7)

The action (2.5) is invariant under the BRST transformation \([10, 11]\)

$$\delta \lambda A^a_\mu = D^{ab}_\mu C^b \lambda , \quad \delta \lambda C^a = B^a \lambda , \quad \delta \lambda B^a = 0 , \quad \delta \lambda C^a = \frac{1}{2} f^{abc} C^b C^c \lambda$$

(2.8)

where \(\lambda\) is an odd constant Grassmann parameter. Introducing the Slavnov variation \(sX\) of any functional \(X(\phi)\) via

$$\delta \lambda X(\phi) = (sX(\phi)) \lambda \quad \text{so that} \quad sX(\phi) = \frac{\delta X(\phi)}{\delta \phi^A} R^A(\phi)$$

(2.9)

with the notation

$$\{R^A(\phi)\} = \{D^{ab}_\mu C^b, 0 , \frac{1}{2} f^{abc} C^b C^c, B^a\} \quad \text{and} \quad \varepsilon(R^A(\phi)) = \varepsilon_A + 1 ,$$

(2.10)

the action (2.5) can be written in the compact form

$$S(\phi) = S_0(A) + s\psi(\phi) ,$$

(2.11)

where \(\psi(\phi)\) denotes the the associated fermionic gauge-fixing functional (in the Landau gauge),

$$\psi(\phi) = \bar{C}^a \chi^a(A) = \bar{C}^a \partial^\mu A^a_\mu .$$

(2.12)

The Gribov horizon \([1]\) in the Landau gauge can be taken into account by adding to the action (2.11) the non-local horizon functional

$$M(A) = \gamma^2 f^{abc} A^b_\mu (K^{-1})^{ad} f^{dec} A^{e\mu} + \gamma^4 d(n^2 - 1) ,$$

(2.13)

where \(K^{-1}\) inverts the (matrix-valued) Faddeev-Popov operator \(K^{ab}(A)\) of (2.6) and \(\gamma \in \mathbb{R}\) is the so-called thermodynamic or Gribov parameter \([2, 3]\). The effective action of the Gribov-Zwanziger model,

$$S_M(\phi) = S(\phi) + M(A) = S_0(A) + s\psi(\phi) + M(A) ,$$

(2.14)

is not BRST invariant because

$$sM(A) = \gamma^2 f^{abc} f^{cde} [2D^{ba}_\mu C^q(K^{-1})^{ad} - f^{mpn} A^b_\mu (K^{-1})^{am} K^p q C^q(K^{-1})^{nd}] A^{e\mu} \neq 0 .$$

(2.15)

In \([6]\), we have investigated the resulting gauge dependence of the vacuum functional, assuming the existence of a horizon functional beyond the Landau gauge. With the help of recent results \([7]\), we now verify this assumption and propose an explicit form for such a functional in general \(R_\xi\) gauges.
3 Gribov horizon in $R_\xi$ gauges

The vacuum functional for the Gribov-Zwanziger model is given by a functional integral,

$$Z = \int \mathcal{D}\phi \exp \left\{ \frac{i}{\hbar} (S_0(A) + s\psi(\phi) + M(A)) \right\} .$$

(3.1)

Let us perform a change of variables which amounts to a particular field-dependent BRST transformation,

$$\phi^A \rightarrow \phi^A + (s\phi^A)\Lambda_\xi(\phi) \quad \text{with} \quad \Lambda_\xi(\phi) = \bar{C}^a B^a (B^2)^{-1} \left( \exp \left\{ \frac{\xi}{2\hbar} B^2 \right\} - 1 \right),$$

(3.2)

where $B^2 = B^a B^a$. Taking into account the Jacobian and using $\ln(1 + s\Lambda_\xi) = \frac{\xi}{2\hbar} B^2$, the vacuum functional then reads [7]

$$Z = \int \mathcal{D}\phi \exp \left\{ \frac{i}{\hbar} (S_0(A) + s\psi_\xi(\phi) + M_\xi(\phi)) \right\} ,$$

(3.3)

with a shifted fermionic gauge-fixing functional and a modified horizon functional,

$$\psi_\xi(\phi) = \bar{C}^a (\partial^a A^a + \frac{\xi}{2} B^a) \quad \text{and} \quad M_\xi(\phi) = M(A) + (sM(A))\Lambda_\xi(\phi) ,$$

(3.4)

respectively. The explicit expression for $sM(A)$ is given in (2.15).

We have moved away from the Landau gauge and reached a general $R_\xi$ gauge. Therefore, we propose

$$M_\xi(\phi) = \gamma^2 f^{abc} A^b_\mu (K^{-1})^{ad} f^{dec} A^a_{\mu} + \gamma^4 d(n^2 - 1)$$

$$+ \gamma^2 f^{abc} f^{cde} [2D^{pq} C^q (K^{-1})^{ad} - f^{mpn} A^b_\mu (K^{-1})^{am} K^{pq} C^q (K^{-1})^{nd}] A^a_{\mu} C^d B^\ell (B^2)^{-1} (e^{\frac{\xi}{2\hbar} B^2} - 1)$$

(3.5)

as the explicit form for the horizon functional in a general $R_\xi$ gauge. Under further BRST transformations, its Slavnov variation is

$$sM_\xi = sM(A) \left[ 1 - s\Lambda_\xi(\phi) \right] .$$

(3.6)

In linear approximation in $\xi$ we have $\Lambda_\xi(\phi) = \frac{\xi}{2\hbar} C^a B^a$ and get

$$M_\xi = M(A) + \frac{\xi^2}{2\hbar} f^{abc} f^{cde} [2D^{pq} C^q (K^{-1})^{ad} - f^{mpn} A^b_\mu (K^{-1})^{am} K^{pq} C^q (K^{-1})^{nd}] A^a_{\mu} C^d B^\ell$$

(3.7)

still depending on all field variables. For $\xi=0$, it smoothly reduces to the Landau-gauge functional, $M_0 = M(A)$.
4 Gribov-Zwanziger action

Originally, the Gribov-Zwanziger model was presented in the non-local form (2.13) and (2.14) [1, 2]. Later, the non-locality was ‘resolved’ by adding auxiliary field variables [3, 4, 5]. The resulting local action is referred to as the Gribov-Zwanziger action and takes the form (for details, see [12])

\[ S_{GZ}(\Phi) = S_0(A) + s\psi(\phi) + S_\gamma(A, \varphi, \bar{\varphi}, \omega, \bar{\omega}) \] (4.1)

where

\[ S_\gamma = \bar{\varphi}_\mu^{ac} K^{ab} \varphi^{b\mu} - \bar{\omega}_\mu^{ac} K^{ab} \omega^{b\mu} + 2i\gamma f^{abc} A^b_\mu (\varphi^{\mu ac} + \bar{\varphi}^{\mu ac}) + \gamma^4 d(n^2 - 1) \] (4.2)

represents the horizon functional written in local form for the Landau gauge. The set of fields has been further enlarged to

\[ \{\Phi^A\} = \{\phi^A, \varphi^{ac}_\mu, \bar{\varphi}^{ac}_\mu, \omega^{ac}_\mu, \bar{\omega}^{ac}_\mu\} . \] (4.3)

The fields \(\varphi^{ac}_\mu\) and \(\bar{\varphi}^{ac}_\mu\) are commuting while \(\omega^{ac}_\mu\) and \(\bar{\omega}^{ac}_\mu\) are anticommuting. The additional fields form BRST doublets [13],

\[
\begin{align*}
\delta_\lambda \varphi^{ac}_\mu &= \omega^{ac}_\mu \lambda , & \delta_\lambda \bar{\varphi}^{ac}_\mu &= 0 , \\
\delta_\lambda \omega^{ac}_\mu &= 0 , & \delta_\lambda \bar{\omega}^{ac}_\mu &= -\bar{\varphi}^{ac}_\mu \lambda .
\end{align*}
\] (4.4)

The local horizon functional \(S_\gamma\) is not BRST invariant,

\[ sS_\gamma = f^{ab} \left[ \bar{\varphi}_\mu^{ac} K^{de} C^e_\lambda \varphi^{b\mu} + \omega^{ac}_\mu K^{de} C^e_\lambda \omega^{b\mu} + 2i\gamma \left( D^{de}_\mu C^e_\lambda (\varphi^{\mu ac} + \bar{\varphi}^{\mu ac}) + A^d_\mu \omega^{\mu ab} \right) \right] \neq 0 . \] (4.5)

Like in the previous section, we may move to a general \(R_\xi\) gauge by performing the specific field-dependent BRST transformation (3.2) in the vacuum functional integral of the Gribov-Zwanziger model based on the local action (4.1). As a result, the action gets modified,

\[ S_{GZ}(\Phi) \mathrel{\mapsto} S_0(A) + s\psi_\xi(\phi) + S_{\gamma\xi}(\Phi) \] (4.6)

where

\[ \psi_\xi(\phi) = \bar{C}^a \left( \partial^\mu A^a_\mu + \frac{\xi}{2} B^a \right) \quad \text{and} \quad S_{\gamma\xi}(\Phi) = S_\gamma(A, \xi, \bar{\xi}, \omega, \bar{\omega}) + (sS_\gamma(A, C, \xi, \bar{\xi}, \omega, \bar{\omega})) \Lambda_\xi(\phi) \] (4.7)

We propose this \(S_{\gamma\xi}\) together with (3.2), (4.2) and (4.5) as the proper extension of the local horizon functional to a general \(R_\xi\) gauge. Its Slavnov variation reads

\[ sS_{\gamma\xi} = sS_\gamma(A, C, \xi, \bar{\xi}, \omega, \bar{\omega}) \left[ 1 - s\Lambda_\xi(\phi) \right] . \] (4.8)

With this information, we may revisit the gauge dependence of Greens functions proposed in [6]. For the Gribov-Zwanziger model based on (4.1) one can find the gauge dependence of the effective action even on shell.
5 Horizon functional in an arbitrary gauge

Although the \( R_\xi \) gauges were easy to reach, they are not the only ones accessible by our method. In fact, [7] provides a general formula for connecting any two gauges in terms of their fermionic gauge-fixing functionals \( \psi \): To get from a reference gauge \( \psi_0 \) to a desired gauge \( \psi \), change the variables inside the generating functional \( Z(J) \) by a BRST transformation with a field-dependent parameter

\[
\Lambda_\psi(\phi) = (\psi - \psi_0)(s(\psi - \psi_0))^{-1}(\exp\left\{ \frac{1}{i\hbar} s(\psi - \psi_0) \right\} - 1)
\]

\[
= \frac{1}{i\hbar}(\psi - \psi_0) \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \left( \frac{1}{i\hbar} s(\psi - \psi_0) \right)^n . \tag{5.1}
\]

The corresponding change of the horizon functional reads

\[
M_\psi(\phi) - M_0(\phi) = (sM_0(\phi))\Lambda_\psi(\phi) . \tag{5.2}
\]

The gauge variation of the Gribov-Zwanziger model can now be studied explicitly.

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