On the Physical Origin of the Semiannual 1 Component of Surface Air Temperature over 2 Mid-latitude and Subpolar Oceans

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On the Physical Origin of the Semiannual Component of Surface Air Temperature over Mid-latitude and Subpolar Oceans

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Abstract

With the understanding that seasonal cycle of the temperature are forced principally by the annually evolving solar irradiance, many previous studies have defined seasonal cycle of surface air temperature (SAT) as the sum of yearly-period sinusoidal component and its harmonics, especially semiannual component. In mid-latitude and subpolar regions, the ratio between the semiannual and annual components of solar irradiance is negligibly small but that of the SAT over oceans is not, which remains to be understood. In this study, a simple energy budget model including main energy sources and sinks of oceanic mixed layer is designed to understand this puzzle. It is revealed that, when the oceanic mixed layer is prescribed as a layer of constant depth, the phase and amplitude of the modeled SAT is not consistent with that of the observation. However, when the annually changing heat capacity of the oceanic mixed layer is included, both the amplitude and phase of the modeled SAT share these of the observed SAT, proving that the semiannual component of SAT over mid-latitude and subpolar oceans is a result of the heat capacity-varying oceanic mixed layer in response to annually evolving solar irradiance.

Keywords: Seasonal cycle, Oceanic mixed layer heat capacity, Semiannual component
1 Introduction

The principal force of the Earth’s climate system is the solar irradiance. Due to the annual revolving of the tilted Earth around the Sun, the solar irradiance received locally at the Earth’s surface contains strong seasonal cycle. Over the large majority of the Earth’s surface, the seasonal change of a climate variable is large and dominates its variability and change on other timescales. With this in mind, the longer term climate variability and change is often quantified as the anomaly with respect to its corresponding climatological seasonal cycle. It is, therefore, evident that the physical understanding of the evolving seasonal cycle of a climate variable may bear significant implication of the physical interpretation of climate variability and change (Wu et al., 2008; Qian et al., 2011).

One of the major characteristics of the seasonal cycle of surface air temperature (SAT) is its asymmetry, as illustrated in Fig. 1, that the long-term averaged seasonal cycle contains different lengths of time from its minimum to maximum and from its maximum to minimum or has faster change during warm or cold seasons. Traditionally, this asymmetry of seasonal cycle is often accounted in terms of the annual sinusoidal component and its harmonics using Fourier transform, i.e.

\[ X(t) = a_0 + \sum_{k=1}^{N} a_k \sin\left(\frac{2\pi k}{\tau_a} t + \phi_k\right), \]  

where \( a_0 \) is a non-negative constant, \( a_k \) non-negative amplitude, \( \phi_k \) the phase ranging from 0 to \( 2\pi \), \( \tau_a \) the length of a year, and \( N \) corresponds to Nyquist frequency.

Previously, many studies have made effort to obtain parameters of SAT in Eq. (1) through analyzing observations or modeled data and to explain the causes or impacts of their changes [e.g. May et al. (1992); Thomson (1995); Wallace and Osborn (2002); Stine et al. (2009); Dwyer et al. (2012); Stine and Huybers (2012)]. Using this approach, the changing phase of seasonal cycle associated with longer term climate variability and change was also found. Based on observed monthly SAT, Wallace and Osborn (2002) found that the yearly-period sinusoidal component of northern hemisphere averaged SAT showed an advance of 0.76 day from 1934 to 1993 in phase. Among this amount of phase advancing, 0.47 day may be explained by the difference between tropical year and year in Gregorian civil calendar. To eliminate such impact, Stine et al. (2009) compared the relative phase change of annual component of SAT with respect to that of the solar irradiance at the top of atmosphere (TOA) and found that the relative phase of SAT turns to earlier by turns to earlier by 1.7 day in land and to later by 1.0 day in ocean over 1954-2007.
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The changes revealed by the previous mentioned studies appear to be negligibly small. However, these revealed changes only reflect the phase change of one component, i.e., the annual sinusoidal component, not the whole seasonal cycle. As we mentioned earlier, the seasonal cycle of SAT contains temporal asymmetry, which cannot be accounted by the annual sinusoidal component. Donohoe et al (2020) showed detailed analysis of such asymmetry by comparing the timing of maximum and minimum between SAT and solar irradiance, but didn’t provide quantitative explains. Thus, other harmonics of annual components, especially the semiannual component, are diagnosed and understood in this study.

To illustrate the importance of the semiannual component of SAT in the climatological seasonal cycle, we displayed SAT seasonal cycles and their semiannual, annual components for selected regions, and their amplitude ratio (the amplitude of the semiannual component divided by that of the annual component; hereafter, amplitude ratio) for the whole globe in Fig. 1. It is indeed the semiannual component being the major source of the asymmetry to seasonal cycle. In the tropics, the amplitude of the semiannual component ranges from a few ten percents of that of the corresponding annual component to a few times. In the North Pacific and Southern ocean, that ratio ranges largely from 0.2 to 0.4. Another noticeable feature is the phase alliance of semiannual and annual components. In the Northern Pacific, the semiannual component is out of phase with the annual component in cold seasons and in phase in warm seasons (Fig. 1a). In contrast, the phase relation is opposite (Fig. 1c) in the Southern Ocean where seasonal ice may play a major role.

As mathematics tells us, when a air/water column of a constant heat capacity is only heated by a periodic forcing of given amplitude, the temperature response of the column tends to have larger amplitude in response to lower frequency forcing (see more details in Section 3). Since solar irradiance can contain both annual and semiannual components, the amplitude ratio of SAT is anticipated to be smaller than that of solar irradiance, making amplitude ratios of SAT and solar irradiance preliminary indicators for whether a region’s SAT variability fits to the model mentioned a few lines ago. For that reason, we plot the semiannual components of solar irradiance and SAT in Fig. 2. In the tropics (Reed, 1962; Qu et al, 2008) and in polar regions (von Loon, 1967; Meehl, 1991; van den Broeke, 1998), since the solar irradiance at these regions contains both significant annual and semiannual components when it is decomposed using Fourier Transform, it is not surprising that a climate variable contains a semiannual component. The diagnosed amplitude ratio of solar irradiance is much larger than that of SAT as anticipated. However, in both northern and southern mid-latitudes, the amplitude ratio of SAT exceeds that of solar irradiance, implying the unfit of the simple model to these regions. Another region of SAT amplitude ratio larger than that of solar irradiance is subpolar oceans. In addition, Fig. 2 shows interesting phase information of
the semiannual components of solar irradiance and SAT. In each hemisphere, the phase of the semiannual component of solar irradiance is largely dividable into two regions of opposite phase, low latitudes and high latitudes, roughly separated by the 40° latitude line. In contrast, the phase of the semiannual component of SAT shows four-zones: tropics, mid-latitudes, subpolar regions, and polar regions, with neighboring zones exhibit strong contrasts in phase.

The observed feature illustrated in both Figs. 1 and 2 raises one question: what is the cause of the semiannual component of SAT in mid-latitude and subpolar oceans? In considering the larger amplitude ratio of SAT than that of solar irradiance in these regions, the semiannual component of observed SAT can only be caused by physical processes other than the direct forcing of the semiannual component of the local solar irradiance. However, the large SAT amplitude ratio centers indicates that such a center is not likely to be a result of heat advection associated with the semiannual component to the region since advection is generally down-gradient; implying that this semiannual component is more a result of local dynamics.

In the following, we will answer this question through constructing a simple energy budget-based model following first principles and comparing model
Fig. 2  The semianual components of (a) zonal mean solar irradiance at TOA and (b) SAT, and (c) amplitude ratios of solar irradiance and SAT. The average is applied only over oceans. The contours in (a) and (b) represent the original values divided by the amplitude of annual component at that latitude, so the maximum value of each latitude is the same as the amplitude ratio shown in (c).

results with observations. We will show that the model captures both amplitude and phase well within the realistic model parameter ranges in comparison to those of the observed. By varying different model settings, we identify principal physical causes of the semianual component of mid-latitude and subpolar oceanic SAT.

The structure of the paper is as follows: Section 2 introduces data and design of model we used in this paper. Section 3 shows analytical solutions to the idealized model with some linear approximations. The nonlinear numerical results are given and compared with observations in Section 4. A discussion and summary is presented in Section 5.
2 Data and Methods

2.1 Data

The observational data being analyzed include solar irradiance at TOA, SAT, and ARGO ocean temperature at different depths. The daily solar irradiance and SAT from 1948 to 2019 are from the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis (Kalnay et al., 1996). To obtain the climatological seasonal cycles of the solar irradiance and SAT, the data of February 29 of any leap year is left out and then a 72-year-mean (1948-2019) is applied to the data for any individual day of a year. The ARGO data are collected and made freely available by the International Argo Program and the national programs that contribute to it (http://www.argo.ucsd.edu, http://argo.jcommops.org). In this study, the original pressure coordinate of ARGO data is converted to depth coordinate by assuming a linear relation between pressure and depth.

2.2 Model design

In this study, an energy budget-based conceptual model is constructed. This model is general enough to include major energy sources and sinks, and also simple enough to be understood analytically. Such type of model was previously used in many studies of the temperature variability of oceanic mixed layer, soil, and firn (Stevenson and Niiler, 1983; Jun et al., 2002; Cronin et al., 2015; Staniec and Nowak, 2016).

We consider the temperature change of an oceanic mixed layer with an unit horizontal area and a depth $D$ being forced by the solar irradiance and other heat sources. This layer is assumed to be well mixed and is homogeneous in the vertical. The temperature variability and change is driven by various energy sources and sinks, including processes such as shortwave and longwave radiation, and gain or loss of energy to atmosphere and surrounding oceans. The governing equation representing this model can be expressed as

$$C_pD \frac{dT}{dt} = (1 - \alpha)\gamma S - (\epsilon - \beta)\sigma T^4 - F,$$

where $C_p$ is the volume specific heat capacity of sea water ($4 \times 10^6 \text{ Jm}^{-3} \text{K}^{-1}$), $\alpha$ the surface albedo, $\gamma$ the atmospheric transmissivity, $S$ the solar irradiance at TOA, $\epsilon$ the emissivity of surface layer, $\beta$ a parameter related to the atmospheric downward longwave radiation as it is assumed that the downward longwave radiation at surface is linearly correlated with the blackbody radiation at SAT, and $\sigma$ the Stefan–Boltzmann constant ($5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4}$). The term $F$ lumps the latent and sensible heat flux to atmosphere, and heat flux due to horizontal advection and vertical diffusion together.
We consider two cases of the depth of the mixed layer. In the first case, $D$ is considered as a constant; and, in the second case, $D$ contains annual component. The first case mimics a constant oceanic mixed layer and the second case takes into account of the annual progression of the mixed layer depth. Adapting the characteristic of oceanic mixed layer to our idealized model layer is justified since the oceanic temperature within the mixed layer often changes proportionally, if not uniformly, in the vertical direction (Holbrook and Bindoff, 1999; Kara et al., 2000b). Since the global meridional heat transport in ocean is often estimated from the energy budget of TOA radiation and atmospheric heat transport (Trenberth et al., 2019), the local horizontal and vertical heat flux in mixed layer, which are essential in this study, are hard to be calculated directly. For convenience, $F$ is assumed to be constant in our analysis. We will discuss the nature of $F$ at due locations and justify our assumption.

3 Some Analytical Solutions to the Model

While solving analytically Eq. (2) for all cases of varying model parameters is beyond our reach, obtaining analytical solutions in some simple cases can help to understand the characteristics of the model and provide insights into which physical processes responsible for the observed feature of seasonal cycles at different regions, especially the semiannual component. In this section, we consider mainly two cases: an oceanic mixed layer with a constant depth and an seasonally varying mixed layer.

3.1 An oceanic mixed layer with a constant depth

Because the annual and semiannual components are the focuses in this study, we retain the annual and semiannual components of solar irradiance $S$ from Fourier Transform but ignore components of higher order, i.e.

$$S \approx s_0 + s_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi_{s1}\right) + s_2 \sin\left(\frac{4\pi}{\tau_a} t + \phi_{s2}\right),$$  \hspace{1cm} (3)$$

where $\tau_a$ is the length of one year, $s_0$ the yearly mean solar irradiance, and $s_1$ and $s_2$ amplitudes of annual and semiannual components. And, to simplify analysis, $T^4$ is linearized using Taylor expansion near a reference temperature $T_0$:

$$T^4 \approx 4T_0^3 T - 3T_0^4.$$  

With these simplifications and assuming $F$ to be constant, Eq. (2) can be rewritten as

$$C_p D \frac{dT}{dt} = (1-\alpha)\gamma[s_0 + s_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi_{s1}\right) + s_2 \sin\left(\frac{4\pi}{\tau_a} t + \phi_{s2}\right)] - (\epsilon - \beta)\sigma(4T_0^3 T - 3T_0^4) - F.$$
After reorganization, we obtained

$$\frac{dT}{dt} + A_4 T = A_1 + A_2 \sin\left(\frac{2\pi}{\tau_a} t + \phi_{s1}\right) + A_3 \sin\left(\frac{4\pi}{\tau_a} t + \phi_{s2}\right), \quad (4)$$

where

$$A_1 = \frac{1}{C_pD} [(1 - \alpha)\gamma s_0 + 3(\epsilon - \beta)\sigma T_0^4 - F],$$

$$A_2 = \frac{1}{C_pD} (1 - \alpha)\gamma s_1,$$

$$A_3 = \frac{1}{C_pD} (1 - \alpha)\gamma s_2,$$

$$A_4 = \frac{4}{C_pD} (\epsilon - \beta)\sigma T_0^3$$

When $t$ is large enough, a stable solution to Eq. (4) is obtained:

$$T = a_0 + a_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi_1\right) + a_2 \sin\left(\frac{4\pi}{\tau_a} t + \phi_2\right), \quad (5)$$

where

$$a_0 = \frac{A_1}{A_4},$$

$$a_1 = \frac{A_2}{\sqrt{A_4^2 + \frac{4\pi^2}{\tau_a^2}}}, \phi_1 - \phi_{s1} = -\arctan\frac{2\pi}{\tau_a A_4},$$

$$a_2 = \frac{A_3}{\sqrt{A_4^2 + \frac{16\pi^2}{\tau_a^2}}}, \phi_2 - \phi_{s2} = -\arctan\frac{4\pi}{\tau_a A_4}$$

and the amplitude ratio of SAT is

$$\frac{a_2}{a_1} = \frac{s_2}{s_1} \sqrt{1 - \frac{3}{4 + m^2}}, \quad m = \frac{\tau_a A_4}{2\pi}. \quad (6)$$

Compared to the phase in solar irradiation, the semiannual component of SAT lags by $\frac{365}{4\pi} \arctan \frac{4\pi}{\tau_a A_4}$ days.

Two extreme cases are helpful to understand the solution: a very large oceanic heat capacity, i.e. $D \to \infty$, and a negligibly small oceanic heat capacity, i.e. $D \to 0$. In the first case with a deep oceanic mixed layer, due to its huge heat capacity, the temperature only has negligible changes under the forcing of seasonally varying solar irradiance, but its slight temperature change corresponds to the gain or loss of a large amount of energy. So, according to Eq. (4), the thermal radiation term, i.e. the second term on the left-hand-side (LHS), is almost a constant and the variation of forcing is balanced by the term of temporal derivative of temperature, i.e. the first term on the LHS. In this case, the amplitude ratio of temperature is half of that of solar irradiance.
and the phase of semiannual component of temperature is behind by about 45 days with respect to that of solar irradiance.

This explains the features of tropical oceanic SAT displayed (see Fig. 2). Near the equator, the oceanic mixed layer has a depth of about 70 m and shows little annual variation (Kara et al., 2003), so its temperature varies with the changes of solar irradiance and divergence of heat flux. In Fig. 1b, the first minimum of semiannual component in tropical SAT occurs at day 45th in a year, consistent with the analytical solutions above, and the annual component is probably a result of energy transport by ocean currents or sensible and latent heat flux. There is a noticeable spatial feature of the amplitude ratio in Fig. 1d that, in both the tropical Pacific Ocean and Indian Ocean, the amplitude ratio of SAT is stronger in the west of them. Such phenomena is related to the ocean currents driven by trade winds. In the east boundary of the tropical Pacific, for example, the equatorward meridional ocean currents converge at the equator and flow westward under the driven of tropical easterly wind (Wyrtki, 1967; Fiedler et al., 1991). Those currents bring signals of strong annual component from subtropical oceans and weaken the amplitude ratio of SAT near the equator.

In the case of a very shallow oceanic mixed layer, the thermal radiation term contains strong variation which tends to directly balance the solar irradiance while the first term of Eq. (4) is small due to small heat capacity. In this case, SAT shares the same phase of the semiannual component and the same amplitude ratio with solar irradiance. The Antarctic region covered mostly by ice can be considered as an extreme example of such a case. The insulation of heat by ice makes the surface layer shallow and heat capacity small under solar radiation. In the oceanic portion of Antarctic, the ocean surface south of 70°S is partly covered by sea ice even in summer seasons of southern hemisphere (Parkinson and Cavalieri, 2012). The ice blocks local energy transport between air and ocean because of its poor thermal conduction as if there is a very shallow oceanic surface layer, i.e. the ice surface, when we focus on the temperature change of the interface between surface air and ocean water. The amplitude ratio of SAT is about 0.1 at 70°S, half of that in solar irradiance, but increases to 0.35 at 80°S which is very close to the value of solar irradiance (Fig. 2c).

The characteristics of SAT should lie between the results of two extreme cases for a medium oceanic heat capacity. However, the above analytical solutions in the case of constant oceanic mixed layer depth appears not to be capable of explaining the observed features of SAT over mid-latitude oceans. In the oceanic regions of latitudes between 30° and 60°, the amplitude ratio of SAT is significantly larger than that of solar irradiance. The amplitude ratio of SAT exceeds 0.2 over the North Atlantic Ocean and 0.3 over the North Pacific Ocean (Fig. 1d) while the amplitude ratio of solar irradiance is about 0.05 near latitude 40°. Another weakness of the above solutions is that the inferred
phase relation between the response (SAT) and forcing (solar irradiance) from
Eq. (5) is inconsistent with the observation in subpolar regions: the semiannual
component of SAT should lag that of forcing; in contrast, the phase of semi-
annual component of SAT leads by about 30 days in higher latitude regions
between 60° and 70°.

3.2 An oceanic mixed layer with a seasonally varying
dePTH

The weaknesses of the model solutions in the cases of a constant depth oceanic
mixed layer call for remediation. Since local heat capacity plays an impor-
tant role in the linearized model as discussed above, we suspect that these
weaknesses may be caused by assuming a constant mixed layer depth. Indeed,
the observed oceanic mixed layer has strong seasonal variation in middle and
high latitudes (Deser et al., 1996; Kara et al., 2003), and dynamics in mixed
layer is important in determining the seasonal cycle of sea surface tempera-
ture (Qu, 2001). With these in consideration, in this section, we will explore
whether including a seasonally varying mixed layer depth can overcome these
weaknesses and make the model solutions consistent with observations.

To examine the influence of annual component of heat capacity on local
SAT over oceans, in consistence with observation, the semiannual component
of solar irradiation is excluded, and, the solar irradiance $S$ is now expressed as

$$S \approx s_0 + s_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi_s \right).$$

The seasonally varying oceanic mixed layer depth $D$ is now approximated as

$$D = d_0 + d_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi_d \right),$$

where $d_0$ and $d_1$ are non-negative and $\phi_d$ ranges from 0 to $2\pi$.

With above approximations, the solution to Eq. (4) can be expressed as
(see Appendix A for more details)

$$T = a'_0 + a'_1 \sin\left(\frac{2\pi}{\tau_a} t + \phi'_1 \right) + a'_2 \sin\left(\frac{4\pi}{\tau_a} t + \phi'_2 \right),$$

(7)

where

$$B'_4 = \frac{4}{C_p d_0} (\epsilon - \beta) \sigma T_0^3,$$

$$a'_2 = \frac{B'_3}{\sqrt{B'_4^2 + \frac{16\pi^2}{\tau_a^2}}}, \phi'_2 = \phi_s + \phi_d + \arctan \frac{\tau_a B'_4}{4\pi}.$$
and, the amplitude ratio becomes

\[
\frac{a_1'}{a_1} = \frac{d_1}{2d_0} \sqrt{1 - \frac{3}{4 + m^2}}, \quad m = \frac{\tau_a B_4'}{2\pi}.
\]  

(8)

It is noted here that the analytical expression of \(a_1'\) is very complicated (see Appendix A) and we use \(a_1\) here instead to obtain a simplified expression. Our numerical calculation later provides more precise results.

The combination of seasonally varying heat capacity and solar irradiance of an annual component only does lead to a semiannual component in SAT. In most mid-latitude regions, the amplitude ratio of solar irradiance, \(s_2/s_1\), is always smaller than 0.1 and even close to 0 (near 50°N and 40°S). When the depth of oceanic mixed layer contains seasonality, e.g. \(d_1/d_0 = 0.4\), the amplitude ratio of SAT resulting from the annual variation of heat capacity can be significantly larger than 0.1. Besides, in the case of seasonally varying \(D\), the phase of semiannual component of SAT is modulated by annual component of solar irradiance and the seasonally varying depth of oceanic mixed layer. As the depth of mixed layer always reaches its maximum during February in the North Pacific Ocean (Kara et al., 2003), \(\phi_{s_1} + \phi_d\) in the SAT semiannual component is about \(-\pi/4\). The large oceanic mixed layer depth at that time infers that \(B_4'\to 0\) over ocean, indicating that the first maximum of semiannual component of SAT should appear around the 70th day of a year, very close to the observation (Fig. 1a). The above inference can also be applied to southern mid-latitudes where there is open ocean throughout the year and the obtained temporal location of the first peak of the semiannual component is also consistent with observation.

The solution of the seasonally varying depth case can also be used to explain the SAT variation over oceanic regions with seasonal ice, such as the ocean circling Antarctic (60° and 70°) (Gordon, 1981; Zwally, 1983; Zwally et al., 2002; Worby et al., 2008), the high latitude North Pacific Ocean (Parkinson et al., 1999), and Hudson Bay (Walsh and Chapman, 2001), which have open ocean in warm seasons and ice in cold seasons. When the ocean is open, local heat capacity depends on the depth of oceanic mixed layer as discussed above. When the ocean surface is covered by ice, the temperature variation of the mixed layer under the sea ice can no longer represent SAT and the SAT can be significantly colder than the mixed layer temperature. However, even in such a case, the effective heat capacity of the simplified ocean column water in our model can be approximated by a seasonal cycle, although its temporal shape may deviate from a simple sinusoidal curve. Our numerical results in the next section confirms this argument.
4 Model Results of SAT in Mid-latitude and Subpolar Oceans

In the previous section, to obtain analytical solutions, we have made various assumptions, some of them are too ideal and may not be applicable to particular regions of oceans. In this section, we solve Eq. (2) numerically, with model parameter selection guided by observations.

4.1 Model configuration

Since Eq. (2) is a forced one-dimensional (time) equation and has a damping term in longwave radiation, the numerical solution to it is not sensitive to numerical scheme as long as time step is sufficiently small. Here, we select the simplest forward finite difference scheme:

\[ C_p \frac{(D_m + \Delta D)T_{m+1} - (D_m T_m + \Delta DT'_m)}{\Delta t} = (1 - \alpha)\gamma S_m - (\epsilon - \beta)\sigma T_4^4 - F_m, \]

where \( \Delta D \) represents the additional part when oceanic mixed layer is deepening and \( T'_m \) is the corresponding temperature for this part (when mixed layer is shoaling, i.e. \( \Delta D < 0, T'_m = T_m \)), subscript \( m \) represents values at \( m \)th time step of a particular variables, and \( \Delta t \) is the length of temporal step for integral and is prescribed as 6 hours (21600 seconds). In this numeric model, the deepening and shoaling of oceanic mixed layer is prescribed instead of depending on ocean dynamic process, and the temperature in mixed layer is uniform at each step. Thus, \( T_{m+1} \) can be calculated by

\[ T_{m+1} = \frac{D_m T_m + \Delta DT'_m}{D_m + \Delta D} + \frac{\Delta t}{C_p(D_m + \Delta D)}[(1 - \alpha)\gamma S_m - (\epsilon - \beta)\sigma T_4^4 - F_m]. \] (9)

The initial temperature is set to be 290 K in most cases and in each case the model runs for 100 years to make sure that a stable seasonal cycle is achieved. The final year of each run is taken as the seasonal cycle to be further analyzed.

Instead of taking the original time series of the local solar irradiance \( S_m \), we decomposed it using Fourier Transform and retain only the first three terms: annual mean, annual component, and semiannual component, i.e.

\[ S_m \approx s_0 + s_1 \sin(\frac{2\pi}{\tau_a}t_m + \phi_{s1}) + s_2 \sin(\frac{4\pi}{\tau_a}t_m + \phi_{s2}). \]

When the seasonality of heat capacity is incorporated, \( D_m \) is given as

\[ D_m = d_0 + d_1 \sin(\frac{2\pi}{\tau_a}t_m + \phi_d). \]

Other parameters are assumed to be constant and will be specified in Section 4.2.
Fig. 3 Observed seasonal cycle of (a) solar irradiance at 40°N, (b) SAT over the North Pacific Ocean, and (c, d) modeled T anomaly. In each panel, the blue line is the seasonal cycle, and red and yellow lines are its annual and semiannual components, respectively.

4.2 Model results

We use this model to simulate the seasonal cycles over two different regions as examples, namely, the North Pacific Ocean, the Southern Ocean (the latitudinal sections marked with bold black line in Fig. 1d). For each location, we run the model in two parallel experiments, with one having fixed oceanic mixed layer depth only and the other additional seasonal cycle of the mixed layer depth.

The observed and simulated SAT over the North Pacific Ocean are compared in Fig. 3. Here, the seasonal cycle of solar irradiance is dominated by the annual component along 40°N while the amplitude ratio of observed SAT is about 0.2. With guidance of the diagnosed values from observations, the parameters are given by

\[
\alpha = 0.1, \gamma = 0.8, \epsilon = 0.95, \beta = 0.75, F = 160 \text{ W/m}^2, d_0 = 40 \text{ m}.
\]  

(10)

Fig. 3c shows the simulated SAT forced by solar irradiance containing annual and semiannual components in the constant mixed layer depth case. It is evident that, because the amplitude of semiannual component in the prescribed
solar irradiance is very small, the forced semiannual component in SAT is also very weak, in contrast to the strong semiannual component in observed SAT (Fig. 3b). In this case, the nonlinear term of thermal radiation \( (T^4) \) does not induce strong semiannual signals in SAT. From this model result, it is evident that the temperature tendency term is much larger than the thermal radiation term in Eq. (2) and the latter can be ignored so that the simulated response has a \( \pi/2 \) phase lag to the solar annual component of the solar irradiance. It is also evident that the model result is not consistent with the observed SAT seasonal cycle.

According to Kara et al. (2000a), the depth of oceanic mixed layer has remarkable seasonal variation in the North Pacific Ocean. Here, we use ARGO data to show this seasonal cycle of the mixed layer depth in the neighborhood of a particular location (40°N, 180°E) at the North Pacific. Fig. 4a shows the departure of sea temperature at different depths from the upper-most 10 meters. The strongest stratification often occurs in early autumn when the temperature of 20 m was cooler than surface by 2 to 4 K, implying a very shallow mixed layer. However, in early spring, the wind stress and the cooling of ocean surface deepen the layer of weak stratification in ocean, which favors vertical mixing, effectively leading to a deep mixed layer and large heat capacity.

With this in mind, a parallel experiment is carried out. In this experiment, the seasonality of heat capacity, based on the observation displayed by Fig. 4, is added and prescribed as

\[
\frac{d_1}{d_0} = 0.6, \phi_d = \frac{\pi}{6}.
\]

The above prescribed values of the phase \( \phi_d \) corresponds to the maximum of heat capacity appears on March 1.

The result of this experiment is shown in Fig. 3d. In this case, remarkable semiannual component of SAT, whose first maximum appears at March 11, emerges and it has similar phase to the observed. This result is also consistent with the theoretical result described by Eq. (7). The amplitude ratio of the resultant SAT is about 0.2, also consistent with the observation.

An alternative perspective of the oscillating SAT is not decomposing it in terms of Fourier components with constant frequency as done in Eq. (1), but consider it as an oscillation with changing frequency, \( i.e. \)

\[
T \propto e^{\int \omega(\tau) d\tau}.
\] (11)

From this perspective, the oscillating portion of the solution has a smaller instantaneous frequency, \( i.e. \omega(t) \) being small, for larger \( D \) and a larger instantaneous frequency for smaller \( D \). The characteristic of Eq. (11) is indeed what
we observed from the SAT variability presented in Fig. 1. Physically, when the heat capacity is large, the evolving seasonal cycle of the forcing can hardly force a fast phase change of the seasonal cycle of SAT, which makes the SAT trough lasting for longer time. The opposite can also be said when the heat capacity is small in seasonal progressing. This perspective appears to be very physically intuitive. This new perspective also raises the question on whether decomposing seasonal cycle into the annual component and its harmonics can help us understand the physics of seasonal cycle effectively.

The second region we pay attention to is the Southern Ocean. In this region, the progressing of seasonal ice leads to a strong seasonal cycle of the effective heat capacity for our model. In the seasons with ice cover, the effective heat capacity related to ice cover is very small. In contrast, in the seasons without

Fig. 4  (a) Observed sea temperature in different depth with respect to the mean temperature of upper most 10 meters and (b) vertical temperature profiles from ARGO during 2008-2013 (platform code: 2900741).
ice, the effective heat capacity can be large due to the deepen mixed layer. In accordance with that, we set model parameters for the constant mixed layer depth case as

\[ \alpha = 0.4, \gamma = 0.8, \epsilon = 0.95, \beta = 0.75, F = 50 \text{ W/m}^2, d_0 = 30 \text{ m}. \] (12)

There is strong semiannual component in solar irradiance near 62°S (Fig. 2 and 5a), and, as inferred by Eq. (5), solar irradiance forced semiannual component of SAT should show a lag of about 40 days to the corresponding component in solar irradiance as shown in Fig. 5c. However, the semiannual component of observed SAT leads the semiannual component of solar irradiance by about 30 days (Fig. 5b). Besides, the amplitude ratio of first two Fourier components in observed SAT is nearly 0.2, much larger than the prediction of Eq. (5), where the amplitude ratio of SAT is supposed to be less than 0.1, one-half of that ratio of solar irradiance. Thus, the model with this set of parameters is not capable of simulating well the observed SAT.

Physically, the semiannual component in observed SAT reflects the fast temperature change in southern winter. With the existence of sea ice, the surface temperature can suddenly fall or rise in response to even small net heating. Thus, we must take into account of the small effective heat capacity for our model in seasons with ice and large effective heat capacity in seasons without ice. With this in mind, we here select additional model parameters as

\[ \frac{d_1}{d_0} = 0.6, \phi_d = \frac{\pi}{3}. \]

The result of this model run is displayed in Fig. 5d. Clearly, the incorporation of a seasonal varying heat capacity leads to stronger SAT semiannual component which is consistent with observation.

5 Summary and Discussions

Outside the tropics, the largest variability of SAT is associated with its seasonal cycle. This seasonal cycle is closely tied to the seasonal cycle of solar irradiance associated with the tilted Earth revolving around the Sun. The seasonal cycle of SAT contains significant temporal asymmetry, such as widely trough and sharp ridge or taking different times marching from yearly minimum to maximum and from the maximum to minimum, in almost all regions of the Earth, including regions where the seasonal varying solar irradiance can be approximate well by a sinusoidal component with a yearly period. In another word, the temporal asymmetry of the seasonal cycle of SAT, when decomposed using Fourier Transform, can contain a large semiannual component even in the regions where solar irradiance only has an annual component. This raises the question of what being responsible for the temporal asymmetry of the SAT seasonal cycle.
In this study, we answer this question by both analyzing and numerically running a local energy budget model constructed based on first principles. The model is simple enough for us to obtain its analytical solutions in some special cases and thereby providing chance to reveal insights into physical causes of the temporal asymmetry of SAT seasonal cycle. These insights are further confirmed with the numerical solutions to the model. We also provide a more intuitive physical explanation of the asymmetry of SAT seasonal cycle.

We compared the modeled asymmetry of SAT with observations from two angles: amplitude and phase. We found that, in the regions where solar irradiance contains negligible semiannual component, such as mid-latitude oceans, the primary physical quantity that is responsible for the asymmetry of SAT seasonal cycle is the seasonal depth change of the oceanic mixed layer. The importance of the oceanic mixed layer depth change is also confirmed even in subpolar regions where the semiannual component of solar irradiance is not negligible, such as the Southern Ocean where has seasonal ice cover. We also found that, in the above mentioned regions, the semiannual component of solar irradiance plays very minor role in inducing the large semiannual component of SAT which is largely responsible the the temporal asymmetry of SAT seasonal cycle.

It is acknowledged that the energy budget model constructed here is extremely simple and does not explicitly include many nonlinear dynamical
processes of the local climate system. However, we believe that the model contains
essential important terms in understanding the seasonal cycle of SAT. For example, in our model analysis and model run, we ignored the net gain or
loss of heat related to heat flux term. This neglect has some physical basis. As
previous studies (Godfrey and Lindstrom, 1989; Trenberth et al, 2019) have
budgeted, the net heat flux caused by advection or mixing into or out of an
upper layer ocean column is of the order of a few tens W/m$^2$. This amount,
compared with the net downward radiation, which is of the order of a few
hundred W/m$^2$, is small and not likely to cause the significant change of the
solution and its physical interpretation. Since the parameters selected for the
model are within the observed range, the obtained solutions should reflect the
essential physics involved in inducing temporal asymmetry of SAT seasonal
cycle. Indeed, the high resembles in both phase and amplitude of SAT seasonal
cycle from model results to these of observations, to some degree, confirmed
the validity of model.

Declarations

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Appendix A Analytical solutions for seasonal heat capacity

With

$$S \approx s_0 + s_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi_{s1} \right), D = d_0 + d_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right),$$

and $F$ being assumed as a constant, Eq. (2) can be written as

$$C_p \left[ d_0 + d_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \right] \frac{dT}{dt} = \left( 1 - \alpha \right) \gamma [s_0 + s_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi_{s1} \right)] - (\epsilon - \beta) \sigma (4T_0^3 - 3T_0^4) - F.$$  \hspace{1cm} (A1)

By multiplying $1 - \frac{d_1}{d_0} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right)$ to both sides and taking advantage of

$$1 - \frac{d_1^2}{d_0^2} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \approx 1$$

by assuming $d_1 \ll d_0$, Eq. (A1) becomes

$$C_p d_0 \frac{dT}{dt} = \left[ 1 - \frac{d_1}{d_0} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \right] \left\{ (1 - \alpha) \gamma [s_0 + s_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi_{s1} \right)] - (\epsilon - \beta) \sigma (4T_0^3 - 3T_0^4) - F \right\}.$$  \hspace{1cm} (A2)
After reorganization, we obtain

\[
\frac{dT}{dt} + B_4 T = B_1 + B_{21} \sin \left( \frac{2\pi}{\tau_a} t + \phi_{s1} \right) + B_{22} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) + B_3 \cos \left( \frac{4\pi}{\tau_a} t + \phi_{s1} + \phi_d \right),
\]

(A3)

where

\[
B_1 = \frac{1}{C_p d_0} [(1 - \alpha) \gamma s_0 + 3(\epsilon - \beta) \sigma T_0^4 - F - \frac{1}{2} (1 - \alpha) \gamma s_1 \frac{d_1}{d_0} \cos(\phi_{s1} - \phi_d)]
\]

\[
B_{21} = \frac{1}{C_p d_0} (1 - \alpha) \gamma s_1
\]

\[
B_{22} = -\frac{d_1}{C_p d_0^2} [(1 - \alpha) \gamma s_0 + 3(\epsilon - \beta) \sigma T_0^4 - F]
\]

\[
B_3 = \frac{d_1}{2C_p d_0^2} (1 - \alpha) \gamma s_1
\]

\[
B_4 = \frac{4}{C_p d_0} (\epsilon - \beta) \sigma T_0^3 \left[ 1 - \frac{d_1}{d_0} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \right]
\]

Note that the semiannual part in Eq. (A3) (the forth term on the right-hand-side) does not come directly from solar irradiance.

The solution to this equation is hard to be explicitly expressed. To obtain an approximate solution, we ignore the parameter, \( \frac{d_1}{d_0} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \), in \( B_4 \), which results in the nonlinearity in thermal radiation term. Thus, in a stable solution, \( T \) can be expressed in a similar way of Eq. (5):

\[
T = a'_0 + a'_1 \sin \left( \frac{2\pi}{\tau_a} t + \phi'_1 \right) + a'_2 \sin \left( \frac{4\pi}{\tau_a} t + \phi'_2 \right),
\]

(A4)

where

\[
B'_4 = \frac{4}{C_p d_0} (\epsilon - \beta) \sigma T_0^3 \left[ 1 - \frac{d_1}{d_0} \sin \left( \frac{2\pi}{\tau_a} t + \phi_d \right) \right]
\]

\[
a'_2 = \frac{B_3}{\sqrt{B_4'^2 + \frac{16\pi^2}{\tau_a^2}}}, \quad \phi'_2 = \phi_{s1} + \phi_d + \arctan \frac{\tau_a B'_4}{4\pi}
\]

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