Synchronization of Prefectural Business Cycles in Japan 1978–2018

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Abstract

Two decades of studies have found significant regional differences in the timing of transitions in national business cycles and their durations. Earlier studies detect regional synchronization during business cycle expansions and contractions in Europe (Grayer, 2007), the U.S. (Hamilton and Owyang, 2012; Chung, 2016), and Japan (Wall, 2007). We investigate those findings more comprehensively for Japan. We draw upon business cycle data spanning 1978-2018 for all 47 Japanese prefectures and measure synchronization between them using a method prominent in nonlinear sciences but infrequently applied in business cycle studies. Our findings confirm that synchronization in Japan’s prefectural business cycles increased during contractions and decreased during expansions throughout the period studied.

**Key words:** Regional business cycle, Synchronization, Hilbert transform, Fourier band-pass filter

**JEL Classifications:** C14, C65, E32, R19
1 Introduction

National economies consist of interlinked regional economies that react differently to changing macroeconomic forces, government policies, prices of imported materials, and technological innovation. Thus national business cycles are an admixture of regional cycles fluctuating diversely. Earlier studies of regional business cycles surveyed in Domazlicky (1980) generally examine how and why cycles differ. By contrast, the advent of the Economic and Monetary Union (EMU) in Europe has renewed research interest in similarities and synchronization among EU states’ business cycles because synchronization facilitates intra-EMU fiscal and monetary policies. Empirical studies, however, often reach divergent conclusions (Massmann and Mitchell, 2004; Gray, 2007; De Haan et al., 2008), likely because they use different raw data or methods of estimating cycles and gauging synchronization. Artis and Zhang (1999) found that synchronization intensified during the European Exchange Rate Mechanism period (1973–1995), but Massmann and Mitchell (2004) found periods of synchronization and desynchronization using identical but updated data. After conducting comprehensive study using six estimation methods and three measures of synchronization, Kappler and Sachs (2013) found little support for business cycle synchronization and degrees of synchronization fluctuate over time.

Euro-area studies of synchronization aroused interest in regional cycles within a country such as the United States and Japan. Clark and Wincoop (2001) found that business cycles of nine U.S. Census regions are significantly more synchronized than those of EU countries. Artis and Okubo
found that the degree of regional business cycle synchronization within Japan is strikingly higher than in the U.S. and the Euro area. These findings imply that national borders may dampen synchronization between regional business cycles.

Numerous studies regarding synchronization of regional business cycles, no matter whether they cover inter- or intra- national business cycles, found significant regional differences in the timing of transitions and duration of business cycles. Among them, several studies announced noteworthy results. Grayer (2007) found a recurring pattern of declining synchronization during expansions in Europe. Hamilton and Owyang (2012) and Chung (2016) noted that co-movement across states characterizes business cycle contractions in the U.S. Wall (2007) concluded that contractions tend to be experienced across most Japanese prefectures.

These results garnered via different data-sets and methods may suggest that the degree of synchronization between regional business cycles intensifies during contractions and diminishes during expansions. We examine that possibility using Japanese regional data and a method of identifying synchronization in time series that is prominent in nonlinear sciences (e.g., Pikovsky et al., 2001) but infrequently applied in business cycle studies. Before describing the method, we note how to extract business cycles from raw data. Following recent studies, we acknowledge that business cycles are relative to a trend and focus on their deviation from it.

Using monthly raw (not seasonally adjusted) observations of the index of industrial production (IIP), we employ a band-pass filter to extract time series that indicate business cycles in Japanese prefectures. The Hodrick–Prescott
filter (Hodrick and Prescott, 1997), a high-pass filter often used in economics, removes only trends with low frequencies. The Baxter–King (Baxter and King, 1997 (BK)) and Christiano–Fitzgerald (Christiano and Fitzgerald, 2003 (CF)) band-pass filters are also frequent in the literature, but we employ the Fourier band-pass filter that is mathematically and computationally simpler.

First, we convert fluctuations of time series into two-dimensional oscillations using the Hilbert transform. This enables us to identify “phases” of circular oscillations, which is defined as a position of a cyclically oscillating variable within one period. Converted oscillations include more information than the original one-dimensional time series and better assess synchronization. Second, we take the “phase difference” between two cycles as an indicator of their synchronization. Third, we use the phase difference to calculate a synchronization index that measures the constancy of the phase difference. If the phase difference of two cycles is nearly constant over time, this index indicates a value near 1, and we designate this situation as (phase) synchronization. In this sense, synchronization does not depend on the level of the phase difference but on its constancy. Our use of this method supports the hypothesis that synchronization between Japanese regional business cycles intensifies during contractions and diminishes during expansions.

An overview of other synchronization measures distinguishes our method from others. The most popular measure of synchronization, Pearson correlation coefficient, provides in one number the degree of similarity between series over a sampled period. It measures static relations between the series, \[ \text{correlation coefficient} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} \]

\[ \text{Ikeda et al. (2013)} \] also use this filter. \[ \text{The Hilbert transform is sometimes used in economics literature (see Ikeda et al. 2013).} \]
whereas synchronization is a dynamic phenomenon and its degree may vary over time. Moving window correlations and new time-varying indexes overcome that deficiency. As the European Commission (2006) mentions, however, correlation with a moving window is sensitive to the window’s length. Mink et al. (2007) proposed a multivariate, time-varying measure of synchronization based on an output gap. It gauges the percentage of regions over time whose output gap has the same sign as that of the reference region. However, this synchronization measure is nondifferentiable because it employs absolute values and graphs of calculated series exhibit numerous spikes.

By contrast, our method focuses on phase differences between two time series. Calculated using phase differences, the synchronization index of Rosenblum et al. (2001) captures the time-dependent degree of synchronization even if phase differences between two time series are large. If two time series exhibit identical behavior with a large phase difference, the correlation coefficient between them can be small.

The study proceeds as follows. We describe our data in Section 2 and our method in Section 3. Section 4 presents empirical results. Section 5 concludes.

2 Data

We employ monthly raw (not seasonally adjusted) IIP data (2010 average = 100) in Japan’s mining and manufacturing sectors from January 1978 to August 2018 (488 months) for 47 prefectures and Japan overall compiled by
the Ministry of Economy, Trade, and Industry (METI).\textsuperscript{3} Japan’s 47 prefectures roughly approximate Nomenclature of Units for Territorial Statistics (NUTS) Level 2 regions.

Figure 1 graphs time series of IIP data for selected prefectures.\textsuperscript{4} Figure 1a compares time series for Tokyo with those of Akita, Yamanashi, and Nara Prefectures, data for which exhibit the greatest synchronization with Tokyo. Figure 1b compares time series for Tokyo Miyagi, Wakayama, and Oita Prefectures, data for which exhibit least synchronization with Tokyo.

IIP data have long been endorsed for studying business cycles because data are monthly and correlate strongly with GDP. Although escalating value-added in the service sector might attenuate that correlation, Fulop and Gyomai (2012) concluded that IIP data proxy GDP for only eight (including Japan) of 38 OECD countries.\textsuperscript{5} Figure 2 supports our use of IIP data. It includes time series IIP data (Footnote 4) and monthly coincident composite indexes ((CI) 2015 average = 100) from January 1985 to August 2018 for Japan. It is compiled by the Economic and Social Research Institute (ESRI) of the Cabinet Office and accessed from the ESRI website. Figure 2 demonstrates that the timing of peaks and troughs in IIP data duplicate those of CI even though post-2000 deviations can be large. In short, IIP data still capture Japanese business cycles.

\textsuperscript{3}IIP data for Japan’s 47 prefectures are from NIKKEI NEEDS. Data for Japan are from the METI website.

\textsuperscript{4} We seasonally adjust all IIP series in this and the next figures are seasonally adjusted by purging high-frequency fluctuations on the basis of Fourier series representation. The lower cutoff frequency is $k_0 = 0$ and upper cutoff frequency is $k_1 = 40$. For $k_0$ and $k_1$, see the Appendix. Since the sample size of our IIP data is 488, the lower and upper frequencies correspond to $\infty$ (≈ 488/$k_0$) and 12 (≈ 488/$k_1$) months respectively.

\textsuperscript{5} The remaining seven countries are Brazil, Estonia, Germany, Switzerland, Sweden, Turkey, and the United Kingdom.
Figure 1: Seasonally-adjusted Time Series Comparison of IIP Data for Tokyo and Three Other Prefectures

Note: Figure 1a compares Tokyo with Akita, Yamanashi, and Nara Prefectures. Figure 1b compares Tokyo with Miyagi, Wakayama, and Oita Prefectures. Data for the former (latter) prefectures show the greatest (least) synchronization with Tokyo.
To measure synchronization of prefectural business cycles, we extract recurring patterns from the original time series. We employ the band-pass filter on the basis of Fourier series representation (details in the Appendix). Although other studies use BK and CF band-pass filters, we use the mathematically and computationally simpler Fourier filter. We reject the BK filter because it uses a moving average and would entail excluding a considerable number of data points at both ends to make it perform well. The time series filtered by Fourier and CF are nearly identical especially in timing of peaks and troughs, and qualitative results of synchronization analysis using these two series are identical.

3 Measuring Synchronization

We employ three procedures to measure synchronization between two scalar (one-dimensional) time series. First, we convert fluctuations in each scalar series into two-dimensional oscillations using the Hilbert transform to identify “phases”. Second, we take “phase differences” between two cycles as an indicator of their synchronization. Third, using the phase differences we calculate the Rosenblum et al. (2001) synchronization index.

3.1 Phase Synchronization

Synchronization is a phenomenon wherein multiple motions adjust their individual rhythms through mutual interactions to maintain a constant phase difference for a time. This phenomenon is also called phase locking or frequency entrainment. Phase synchronization differs from complete synchronization
in which multiple motions become completely identical.

Synchronization is exemplified with the aid of simple oscillators $s_1^t = \sin(2\pi t)$ and $s_2^t = \sin(2\pi t - \pi/2)$ in Figure 3. The phases of $s_1^t$ and $s_2^t$ are $2\pi t$ and $2\pi t - \pi/2$ respectively, rendering their phase difference as $\pi/2$. Time series $s_1^t$ and $s_2^t$ are synchronized on the grounds that their phase difference is constant over time.

### 3.2 Hilbert Transform and the Instantaneous Phase

Phases are crucial in synchronization analysis. However, the time-varying amplitudes and phases of business cycles make it impossible to extract their information from one-dimensional time series IIP data. Therefore, we construct a complex-valued time series $\hat{s}_t$ whose real part is actual data $s_t$ and imaginary part $s_t^H$ is generated from $s_t$ via the Hilbert transform:

$$\hat{s}_t = s_t + is_t^H.$$  

(1)

The Hilbert transform of $s_t$ is given by

$$s_t^H = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{s_\tau}{t - \tau} d\tau,$$

(2)

where $\text{P.V.}$ denotes Cauchy principal value integrals. Intuitively, the Hilbert transform provides a phase shift of $-\pi/2$ radian for every Fourier component of a function. For example, the Hilbert transform of $s_t = \cos(2\pi t)$ is $s_t^H = \cos(2\pi t - \pi/2) = \sin(2\pi t)$.

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6To be exact, the value of the phase is restricted to $[-\pi, \pi)$ by taking mod $2\pi$. “Phase” is defined in Section 3.2.
Figure 2: Time series IIP Data for Japan and Coincident CI spanning 1985:01 to 2018:08

![Graph](image)

Note: Timing of peaks and troughs in IIP data duplicate those of CI even though post-2000 deviations can be large.

Figure 3: Two Synchronized Time Series with a Temporally Constant Phase Difference

![Graph](image)

Note: The mark “•” (blue) represents time series $s_1(t) = \sin(2\pi t)$ and the mark “×” (orange) represents time series $s_2(t) = 2\sin(2\pi t - \pi/2)$. The phase difference is $\pi/2$ for all $t$. 
Now we can define a (instantaneous) phase at time $t$ using a point $P_t(s_t, s^H_t)$ on the complex plane as an angle $\phi_t$ that is formed between OP$_t$ and the horizontal axis in Figure 4:

$$\phi_t = \begin{cases} \tan^{-1} \left( \frac{s^H_t}{s_t} \right) & (s_t > 0) \\ \tan^{-1} \left( \frac{s^H_t}{s_t} \right) + \pi & (s_t < 0). \end{cases} \tag{3}$$

Since the value of phase ranges from $-\pi$ to $\pi$, it can be discontinuous over time. Using the phase $\phi_t$, we can rewrite Equation 1 as

$$\hat{s}_t = s_t + is^H_t = A_t \cos \phi_t + iA_t \sin \phi_t, \tag{4}$$

where the time-varying amplitude is represented as $A_t = \sqrt{s_t^2 + (s^H_t)^2}$. The above discussion assumes that $t$ is continuous, while we apply the procedure to discrete time series of IIP in Section 4.

### 3.3 Synchronization Index

To measure degrees of synchronization between two series for discrete time interval $1 \leq i \leq W$, we utilize synchronization index $\gamma^2 \in [0, 1]$ proposed by Rosenblum et al. (2001):

$$\gamma^2 = \left( \frac{1}{W} \sum_{i=1}^{W} \cos \psi_i \right)^2 + \left( \frac{1}{W} \sum_{i=1}^{W} \sin \psi_i \right)^2, \tag{5}$$
Figure 4: Constructing a Complex-valued Time Series $\hat{s}_t$ from One-dimensional Real Data $s_t$

Note: We construct $\hat{s}_t$ whose real part is actual data $s_t$ and imaginary part $s_t^H$ generated from $s_t$ via the Hilbert transform.
where \( \psi_i \) denotes the phase difference between two time series. When \( \psi_i \) is nearly constant over time, the value of \( \gamma^2 \) approaches 1, and we call that situation as (phase) synchronization. When \( \psi_i \) is chosen from the uniform distribution of \([-\pi, \pi)\), \( \gamma^2 \) approaches 0 as \( W \) increases (Muto and Saiki, 2020).

To examine the time evolution of \( \gamma^2 \), we presume \( W \) is an odd number of discrete time points and \( \gamma^2_t \) represents the strength of synchronization at time \( t \), which corresponds to the temporal center point of rolling window \( W \). Thus, instead of Equation 5, we calculate for each time \( t \)

\[
\gamma^2_t = \left( \frac{1}{W} \sum_{i=t-p}^{t+p} \cos \psi_i \right)^2 + \left( \frac{1}{W} \sum_{i=t-p}^{t+p} \sin \psi_i \right)^2, \tag{6}
\]

where \( p = (W - 1)/2 \) and \( 0 < p < t \). Throughout the analysis, we set \( W = 17 \), which equals approximately half of the shortest duration (36 months) of Japan’s past business cycles (Table 1). In Equation 6 the expected value of synchronization index \( \gamma^2_t \) with window size \( W = 17 \) is 0.059 when \( \psi_i \) is chosen randomly from the uniform distribution on [0,1] (Muto and Saiki, 2020).

### 4 Analysis

Before applying the Fourier band-pass filter to time series IIP data, we determine what frequency band corresponds to Japanese business cycles. Table 1 lists reference dates for cycles announced by ESRI, from which we identify a band spanning 36 to 86 months. We then extract time series with the frequency band of that range using lower cutoff frequency of \( k_0 = 6 \) and
upper cutoff frequency of \( k_1 = 14 \), which correspond to 81 (\( \approx 488/k_0 \)) and 35 (\( \approx 488/k_1 \)) months respectively. For \( k_0 \) and \( k_1 \) see the Appendix.

Figure 5 illustrates the band-pass-filtered time series of IIP for the sampled prefectures. Figure 5a compares time series for Tokyo and the three prefectures in Figure 1a. Although those prefectures are most closely synchronized with Tokyo, timing of peaks and troughs is sometimes disordered. Figure 5b compares time series for Tokyo with those for the three prefectures in Figure 1b. Since those prefectures are least synchronized with Tokyo, timing of peaks and troughs is considerably disordered. These two figures imply there are time points at which the degree of synchronization between prefectures is either high or low.

We next convert fluctuations in each band-pass-filtered scalar time series into two-dimensional oscillations using the Hilbert transform. Figure 6a depicts a two-dimensional trajectory of \( P_t(s_t, s_t^H) \) on the complex plane. The variable \( s_t \) denotes band-pass-filtered IIP data for Tokyo with the above-mentioned upper and lower cutoff frequencies and \( s_t^H \) denotes the Hilbert-transformed time series of \( s_t \). The trajectory of \( P_t(s_t, s_t^H) \) oscillates around the origin with evident frequencies and amplitudes. That finding implies business cycle fluctuations are adequately extracted using the band-pass filter with the lower and upper cutoff frequencies of \( k_0 = 6 \) and \( k_1 = 14 \).

Comparing Figure 6a with 6b and 6c uncovers the significance of the band-pass filter and selection of a frequency band. Figure 6b shows a trajectory of \( P_t(s_t, s_t^H) \) with the upper and lower cutoff frequencies of 6 (\( k_1 = 80 \)) and 81 (\( k_0 = 6 \)) months. Figure 6c shows a trajectory of \( P_t(s_t, s_t^H) \) with \( s_t \) detrended but not band-pass-filtered. Trajectories in these two figures consist of
Figure 5: Comparison of Band-pass-filtered Time Series of IIP Data for Tokyo and the Sampled Prefectures

Note: In Figure 5a Akita, Yamanashi, and Nara Prefectures are most closely synchronized with Tokyo. In Figure 5b Miyagi, Wakayama, and Oita Prefectures are least synchronized with Tokyo. The upper (lower) cutoff frequency of the band-pass filter is 35 (81) months.
numerous irregular oscillations, implying that time series $s_t$ contains higher-frequency fluctuations than Figure 6a and that business cycle fluctuations are not adequately extracted. Moreover, trajectories sometimes pass by the origin, which implies that phase movements exhibit abrupt jumps that may defeat synchronization analysis.

The converted trajectory on the complex plane via the Hilbert transform lets us identify the phase of circular oscillations and to calculate phase differences between two trajectories as an indicator of their synchronization. Thus we can compute the time evolution of synchronization index $\gamma_t^2$ between two trajectories by using Equation 6 to gauge the constancy of phase differences.

Figure 7 illustrates the time evolution of synchronization index $\gamma_t^2$ between Tokyo and the sampled prefectures. Prefectures in Figure 7a and 7b respectively correspond to those in Figure 1a and Figure 1b. Although prefectures in Figure 7a belong to the group most closely synchronized with Tokyo, the figure displays intervals when $\gamma_t^2$ bears low values. Figure 7b depicts prefectures that are least synchronized with Tokyo, so that the degree of synchronization is low versus Figure 7a. These figures imply that the degree of synchronization is basically high during most periods and declines concurrently for most prefectures. These phenomena nearly occur almost during business cycle expansions.

To scrutinize the degree of business cycle synchronization between prefectures, we calculate 1,081 ($=47 \times C_2$) series of $\gamma_t^2$ for all 2-tuples among Japan’s 47 prefectures. By $R(\gamma_t^2 \geq r)$ we denote the ratio of 2-tuples for which $\gamma_t^2$ bears value exceeding or equaling the threshold $r$ at each time $t$. By definition, $R(\gamma_t^2 \geq r) \in [0, 1]$. The larger the portion of prefectures synchronized,
Figure 6: Time Series of Trajectory $P_t(s_t, s_t^H)$ on the complex plane

Note: The variable $s_t$ denotes the band-pass-filtered IIP of Tokyo and $s_t^H$ its Hilbert-transformed time series. The upper (lower) cutoff frequencies of the band-pass filter are 35 (81) months in Figure 6a and 6 (81) months in Figure 6b. In Figure 6c, $s_t$ is de-trended but not band-pass-filtered.
Figure 7: Time Evolution of Synchronization Index $\gamma_t^2$ between Tokyo and the Sampled Prefectures

Note: In Figure 7(a) prefectures Akita, Yamanashi, and Nara are most closely synchronized with Tokyo. In Figure 7(b) prefectures Miyagi, Wakayama, and Oita are least synchronized with Tokyo.
the greater the value of $R(\gamma^2_t \geq r)$.

Figure 8 illustrates the time evolution of $R(\gamma^2_t \geq r)$ for $r = 0.7$, 0.8, and 0.9. It implies that $R(\gamma^2_t \geq r)$ is inclined to be low during expansions (1986:11 to 1991:02, 1993:10 to 1997:05, 2002:01 to 2008:02, 2009:03 to 2012:03, and 2012:11 to 2018:10), whereas $R(\gamma^2_t \geq r)$ is inclined to be high during contractions (1985:06 to 1986:11, 1991:02 to 1993:10, 1997:05 to 1999:01, 2000:11 to 2002:01, and 2008:02 to 2009:03). These observations support the hypothesis that the degree of synchronization between regional business cycles increases during contractions and decreases during expansions. That phenomenon was partly detected in the Euro area by Grayer (2007), in the U.S. by Hamilton and Owyang (2012) and Chung (2016), and in Japan by Wall (2007).

However, Figure 8 reveals a discrepancy between our results and our hypothesis, likely for two reasons. First, our band-pass filter does not extract an adequate trajectory if the duration of an expansion or contraction is too brief to accommodate its cutoff frequencies. The ratio $R(\gamma^2_t \geq r)$ shows relatively high values during the expansions from 1983:02 to 1985:06 (28 months) and 1999:01 to 2000:11 (22 months), and it shows relatively low values for the contraction spanning 2012:03 to 2012:11 (8 months). For those periods, we confirm that $R(\gamma^2_t \geq r)$ takes values that support our hypothesis if the frequency band is adequately chosen.

Second, observations may disparage our hypothesis during periods when expansions and contractions coexist—i.e., when Japan’s economy did not expand or contract unidirectionally. The contraction spanning 1980:02 to 1983:02 and expansions spanning 2009:03 to 2012:03 and 2012:11 to 2018:10
Table 1: Reference Dates for Japanese Business Cycles announced by ESRI

| Cycle  | Trough   | Peak     | Trough   | Duration |
|--------|----------|----------|----------|----------|
| 9th    | 1977:10  | 1980:02  | 1983:02  | 64 months|
| 10th   | 1983:02  | 1985:06  | 1986:11  | 45 months|
| 11th   | 1986:11  | 1991:02  | 1993:10  | 83 months|
| 12th   | 1993:10  | 1997:05  | 1999:01  | 63 months|
| 13th   | 1999:01  | 2000:11  | 2002:01  | 36 months|
| 14th   | 2002:01  | 2008:02  | 2009:03  | 86 months|
| 15th   | 2009:03  | 2012:03  | 2012:11  | 64 months|
| 16th   | 2012:11  | 2018:10  |          |          |

Note: Duration of cycles spans 36 to 86 months.

Figure 8: Time Evolution of the Ratio $R(\gamma_t^2 \geq r)$ for $r = 0.7$, 0.8, and 0.9

Note: By $R(\gamma_t^2 \geq r)$ we denote the ratio of 2-tuples among 47 Japan’s prefectures for which $\gamma_t^2$ shows a value greater than or equal to the threshold $r$ at each time $t$. 
correspond to such a jumble.

To illustrate, Figure 9 depicts the time evolution of the historical diffusion index (HDI) by Moore, 1961. HDI is the basis on which ESRI sets reference dates for business cycles. During the contraction spanning 1980:02 to 1983:02, HDI stays above the 50% line for 8 months from 1981:04 to 1981:11, which signals expansion during a contraction. Therefore, $R(\gamma_i^2 \geq r)$ in Figure 8 exhibits a sudden decline from 1981:07. During the expansion spanning 2009:03 to 2012:03, HDI declines sharply from 2010:04 to the bottom of 2011:03, indicating contraction during an expansion. Therefore, $R(\gamma_i^2 \geq r)$ in Figure 8 stays high until 2011:06. During the expansion from 2012:11 to 2018:10, HDI stays above the 50% line for 16 months, whereas HDI declines sharply and stays below the 50% line for 21 months. This finding implies, as before, contraction during a business cycle expansion. Therefore, $R(\gamma_i^2 \geq r)$ in Figure 8 increases during that period.

In a nutshell, our observations might deviate from our hypothesis because our method captures sensitively interims of expansion within contractions and vice versa. That does not constitute a defect in our method.

Finally, we offer a conjecture about why our hypothesis holds—i.e., why the degree of synchronization between regional business cycles increases during contractions and decreases during expansions. Under prospect theory by Kahneman and Tversky, 1979, people prefer avoiding losses over acquiring equivalent gains. That suggests industrial firms behave asymmetrically when business cycles enter contractions or expansions. When entering a contraction, firms trim production to avoid losses, and that behavior synchronizes...
well. When entering an expansion, some firms step up production and others do not, and their behavior tends not to synchronize well. Thus loss-averse behavior by firms engenders synchronization in production during contractions.

5 Conclusion

Using IIP data for all 47 prefectures in Japan and a method distinguished in nonlinear sciences to analyze synchronization between data series, we converted one-dimensional time series into two-dimensional circular oscillations via the Hilbert transform. Our quantitative results indicate synchronization of prefectural business cycles increases during business cycle contractions and decreases during expansions throughout the period studied.

One limitation of our results is that our method concentrates on a specific frequency band and will not extract an adequate trajectory if the duration of an expansion or contraction does not accommodate the cutoff frequencies
of the band-pass filter.

Future studies need to generalize our findings by applying our method to regional business cycles in other countries. It also would be useful to reexamine our hypothesis via different methods such as wavelet analysis, and to construct a macroeconomic dynamical model with loss-averse behavior of firms to explain our hypothesis.
Appendix: Fourier Filter

We briefly review the Fourier series of a function \( f \). For simplicity, let \( f \) be a real-valued continuous periodic function on \([0, L)\). The function \( f \) can be represented as a Fourier series:

\[
f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos \left( \frac{2\pi k x}{L} \right) + b_k \sin \left( \frac{2\pi k x}{L} \right) \right),
\]

where

\[
a_k = \frac{1}{L} \int_{0}^{L} f(x) \cos \left( \frac{2\pi k x}{L} \right) \, dx \quad (k = 0, 1, 2, 3, \ldots),
\]

\[
b_k = \frac{1}{L} \int_{0}^{L} f(x) \sin \left( \frac{2\pi k x}{L} \right) \, dx \quad (k = 1, 2, 3, \ldots).
\]

We can obtain a Fourier series for more general class of function \( f \) (see, e.g., Körner (1989)).

By taking a partial sum in Equation 7 we can create a band-pass-filtered periodic function \( \tilde{f} \) using a band \([k_0, k_1]\) with the lower and upper cutoff frequencies of \( k_0 \) and \( k_1 \) \((0 \leq k_0 \leq k \leq k_1)\) from a given function \( f \):

\[
\tilde{f}(x) = \sum_{k=k_0}^{k_1} \left( a_k \cos \left( \frac{2\pi k x}{L} \right) + b_k \sin \left( \frac{2\pi k x}{L} \right) \right) .
\]
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