ON THE CORRELATION BETWEEN SPIN PARAMETER AND HALO MASS

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\section{INTRODUCTION}

The physical mechanism by which galaxies acquire their angular momentum is an important problem that has been the subject of investigation for nearly sixty years (Hoyle 1945). This reflects the fundamental role played by angular momentum of galactic material in defining the size and shapes of galaxies (e.g., Fall & Efstathiou 1980). Yet despite its physical significance, a precise and accurate understanding of the origin of galactic angular momentum remains one of the missing pieces in the galaxy formation puzzle.

A fundamental assumption in current galaxy formation models is that galaxies form when gas cools and condenses within the potential wells of dark matter halos (White & Rees 1978). Consequently it is probable that the angular momentum of the galaxy will be linked to the angular momentum of its dark matter halo (e.g., Fall & Efstathiou 1980; Mo et al. 1998; Zavala et al. 2008). Within the context of hierarchical structure formation models, the angular momentum growth of a dark matter protohalo is driven by gravitational tidal torquing during the early stages (i.e., the linear regime) of its assembly. This “tidal torque theory” has been explored in detail; it is a well-developed analytic theory (e.g., Peebles 1969; Doroshkevich 1973; White 1984), and its predictions are in good agreement with the results of cosmological N-body simulations (e.g., Barnes & Efstathiou 1987; Warren et al. 1992; Sugerman et al. 2000; Porciani et al. 2002a, 2002b).

However, once the protohalo has passed through maximum expansion and the collapse has become nonlinear, tidal torquing no longer provides an adequate description of the evolution of the angular momentum (White 1984), which tends to decrease with time. During this phase it is likely that merger and accretion events play an increasingly important role in determining both the magnitude and direction of the angular momentum of a galaxy (e.g., Bailin & Steinmetz 2005). Indeed, a number of studies have argued that mergers and accretion events are the primary determinants of the angular momenta of galaxies at the present day (Gardner 2001; Maller et al. 2002; Vitvitska et al. 2002).

It is common practice to quantify the angular momentum of a dark matter halo by the dimensionless “classical” spin parameter (Peebles 1969),

\[ \lambda = \frac{J}{G M^{3/2}}, \]

where \(J\) is the magnitude of the angular momentum of material within the virial radius, \(M\) is the virial mass, and \(E\) is the total energy of the system. It has been shown that halos that have suffered a recent major merger will tend to have a higher spin parameter \(\lambda\) than the average (e.g., Hetznecker & Burkert 2006; Power et al. 2008). Therefore, one could argue that within the framework of hierarchical structure formation that higher mass halos should have larger spin parameters on average than less massive systems because they have assembled a larger fraction of their mass (by merging) more recently.

However, if we consider only halos in virial equilibrium, should we expect to see a correlation between halo mass and spin? One might naively expect that more massive systems will have had their maximum expansion more recently and so these systems will have been tidally torqued for longer than systems that had their maximum expansion at earlier times. This suggests that spin should increase with timing of maximum expansion and therefore halo mass. However, one finds at best a weak correlation between mass and spin for equilibrium halos at \(z = 0\) (e.g., Cole & Lacey 1996; Maccio et al. 2007; Bett et al. 2007, hereafter B07), and the correlation is for spin to decrease with increasing halo mass, contrary to our naive expectation.

In this paper, we report on a (weak) correlation between spin and mass for equilibrium halos at redshift \(z = 10\). The trend is for higher mass halos to have smaller spins, and is qualitatively similar to the one reported by B07 for the halo population at \(z = 0\). We present the main evidence in support of this correlation in \(\S 4\), and we consider its implications for galaxy formation in \(\S 6\).

\section{THE SIMULATIONS}

For the simulations presented in this paper we have adopted the cosmology as given by Spergel et al. (2003; \(\Omega_0 = 0.3, \Omega_{\Lambda} = 0.7, \sigma_8 = 0.9, \) and \(H_0 = 70 \) \(\text{km s}^{-1} \text{Mpc}^{-1}\)). Each run employed \(N = 256^3\) particles and differed in simulation box size \(L_{\text{box}}\), which leads to the particle mass \(m_p\) differing between runs—\(m_p = \rho_{\text{crit}}\Omega_0 L_{\text{box}}/N^3\), where \(\rho_{\text{crit}} = 3H_0^2/8\pi G\). This allows us to probe a range of halo masses at redshift \(z = 10\). The primary parameters of these simulations are summarized in Table 1.

Halos in all runs have been identified using the MPI parallelized version of the AHF halo finder (AMIGA’s Halo Finder), which is based on the MCF halo finder of Gill et al. (2004). For each halo we compute the virial radius \(R\), defined as the radius at

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which the mean interior density is \( \Delta_{\text{vir}} \) times the background density of the universe at that redshift. This leads to the following definition for the virial mass \( M \):

\[
M = \frac{4\pi}{3} \Delta_{\text{vir}} \Omega_{\text{crit}} R^3.
\]

Note that \( \Delta_{\text{vir}} \) is a function of redshift and amounts to \( \Delta_{\text{vir}} \approx 210 \) at redshift \( z = 10 \), \( \Delta_{\text{vir}} \approx 230 \) at \( z = 1 \), and the “usual” \( \Delta_{\text{vir}} \approx 340 \) at \( z = 0 \) (see Gross 1997). Table 1 summarizes the total number of halos (\( N_{\text{halos}} \)) recovered by AHF, while \( z_{\text{final}} \) gives the redshift to which the simulation has been evolved.

We add that we ran five realizations of B20 to redshift \( z = 10 \) in order to have a statistically significant sample of halos in that particular model. However, we also note that the fitting parameters presented in the following section are robust in the sense that they do not depend on whether we stack the halos from those five runs or use them individually.

3. THE HALO SAMPLE

We show in a companion paper (Power et al. 2008) that a substantial fraction of the halo population at high redshift is not in virial equilibrium. Because we wish to examine the spin distribution of equilibrium halos, it is important to account for unrelaxed systems when investigating correlations between spin and halo mass. For example, it has been shown that the spin can increase sharply in the aftermath of mergers with mass ratios as modest as \( 5:1 \) (e.g., Hetznecker & Burkert 2006), and that the degree to which a halo is in dynamical relaxation is as important as recent merging history in its influence on spin (D’Onghia & Navarro 2007). To ensure that halos in our sample are in virial equilibrium, we compute the virial ratio for each halo, which we define as

\[
Q = \frac{2T + S_p}{U} + 1.
\]

Here \( T \) represents the kinetic energy, \( U \) the potential energy, and \( S_p \) the surface pressure of a given halo of mass \( M \). By including \( S_p \), we can account for the effect of infalling material on the dynamical state of the halo. Each of these quantities are evaluated using all gravitationally bound particles, and we adopt the formula of Shaw et al. (2006) for the surface pressure term \( S_p \) (see eqs. [4]–[6] in their study).

In Figure 1 we show that the relation between halo mass and \( Q \) can vary with mass. This is apparent at redshift \( z = 1 \), where we find a trend for more massive halos to be less virialized. In contrast, high-redshift halos are less virialized on average (as indicated by the increased average \( \langle Q \rangle \approx -0.3 \)), but we find no apparent trend with mass.

Why is there a mass dependence at \( z = 1 \) but not at \( z = 10 \)? There are two factors. The first is that high-redshift halos “see” an effective slope of the initial power spectrum of \( n_{\text{eff}} \approx -3 \), and so the time at which a particular mass scale starts to collapse is relatively insensitive to mass. Therefore, we do not expect to find a strong correlation between virial state \( Q \) and mass. We have checked this with halo populations drawn from the simulations of scale-free cosmologies of Knollmann et al. (2008), and our

| Run     | \( L_{\text{box}} \) (h\(^{-1}\) Mpc) | \( m_p \) (h\(^{-1}\) M\(_{\odot}\)) | \( z_{\text{final}} \) | \( N_{\text{halos}}^{1+10} \) | \( N_{\text{halos}}^{10+100} \) | \( N_{\text{halos}}^{10+100} \) |
|---------|----------------------------------|----------------------------------|----------------|----------------|----------------|----------------|
| B01     | 1                                | \( 4.9 \times 10^3 \)            | 10             | \ldots          | 8780           | \ldots          |
| B02     | 2.5                              | \( 7.8 \times 10^4 \)            | 10             | \ldots          | 7991           | \ldots          |
| B05     | 5                                | \( 6.2 \times 10^5 \)            | 1              | 16917           | 6532           | 832            |
| B10     | 10                               | \( 4.9 \times 10^6 \)            | 1              | 18589           | 4360           | 949            |
| B20     | 20                               | \( 4.0 \times 10^7 \)            | 1              | 20514           | 10947          | 995            |
| B100    | 100                              | \( 4.9 \times 10^9 \)            | 1              | 24693           | \ldots         | 978            |

Fig. 1.—Relation between virial parameter \( Q \) (see eq. [3]) and halo mass for \( z = 1 \) (left) and \( z = 10 \) (right). The solid lines represent the adopted virialization criteria as given by eq. (4). Note that we already applied the mass cut of 600 particles per halo for this plot, and hence the number of halos appearing does not agree with the number given in Table 1. [See the electronic edition of the Journal for a color version of this figure.]
interpretation is consistent with the correlations we find in these runs. The second is that the typical collapsing mass \( M_\odot / C_3 \) at \( z = 10 \) is small—of order \( 10^3 h / C_0 \)—and because we resolve mass scales that have collapsed more or less simultaneously, we see a population that has yet to relax. At \( z = 1 \), the typical collapsing mass is much larger—of order \( 10^{11} h / C_0 \)—and so we resolve a population of halos whose mass accretion histories are more diverse. The most massive systems tend to be ones that have formed most recently, and are therefore the least dynamically relaxed.

We have used the following relation between \( Q \) and \( M_\odot / C_0 \) to classify dynamically relaxed and unrelaxed systems:

\[
Q_{\text{allowed}} / M_\odot / C_0 = 0.015 \text{ for } z = 1;
\]

\[
\text{const.} \text{ for } z = 10.
\]

(4)

We allow the \( Q \)-values of halos in our sample to deviate from these scaling relations by not more than

\[
Q_{\text{allowed}} - Q_{\text{lim}} \leq Q \leq Q_{\text{allowed}} + Q_{\text{lim}},
\]

(5)

with \( Q_{\text{lim}} = 0.15 \) (indicated by the dashed lines in Fig. 1). Furthermore, we consider only halos that contain at least \( N_{\text{min}} = 600 \) particles within their virial radius to ensure that we are not influenced by particle discreteness. Interestingly, when computing spin, the tightest restriction on particle number comes not from the calculation of angular momentum but from the calculation of the potential energy. By comparing analytic solutions with Monte Carlo realizations of Navarro et al. (1997) halos, we find that at least 600 particles are required if the energy is to be computed to better than 10%.

4. THE SPIN-MASS CORRELATION

Calculating the total energy \( E \) of a halo is computationally expensive, and so computing \( \lambda \) using equation (1) is also expensive. This prompted Bullock et al. (2001) to introduce a modified spin parameter

\[
\lambda' = \frac{J}{\sqrt{2MV_R}},
\]

(6)

where \( V = \sqrt{GM/R} \) measures the circular velocity at the virial radius \( R \) and \( J \) represents the absolute value of the angular momentum. We follow Bullock et al. and compute spin using equation (6).

![Fig. 2.—Correlation of spin parameter \( \lambda \) with mass \( M_\odot \) for \( z = 1 \) (left) and \( z = 10 \) (right). The binned data (histograms) have been fitted to a power law (dashed line, see eq. [9]). [See the electronic edition of the Journal for a color version of this figure.]

![Fig. 3.—Lognormal distributions of the spin parameter \( \lambda' \) for \( z = 1 \) (left) and \( z = 10 \) (right). [See the electronic edition of the Journal for a color version of this figure.]

In Figure 2, we investigate the correlation between halo spin \( \lambda' \) and mass \( M \). We show only halos that fulfill our selection criteria (individual points) and bin the data in five mass bins equally spaced in log-space between \( M_{\text{min}} \) and \( M_{\text{max}} \) of the considered halos at the respective redshift. The values plotted as histograms thereby represent the weighted mean of all spin parameters in the respective mass range, where the weight is inversely proportional to the error estimate

\[
\sigma_j = \frac{0.2}{\lambda'_j \sqrt{N}}
\]

for the spin parameter of a halo consisting of \( N \) particles as derived in Bullock et al. (2001, see eq. [7] in that study). The error bars indicate the standard deviation of the spin parameter values in the bin from the weighted mean.4

The best-fitting power laws to these histograms reveal that

\[
\lambda' \propto M^\alpha,
\]

with

\[
\alpha = \begin{cases} 
-0.002 \pm 0.149 & \text{for } z = 1, \\
-0.059 \pm 0.171 & \text{for } z = 10.
\end{cases}
\]

This indicates that there is a weak correlation at high redshifts for spin to decrease with increasing mass, albeit stronger than the one at \( z = 1 \). We compute Spearman rank correlation coefficients at \( z = 10 \) (1) and find \( R_s = -0.137 \) (0.06).

As an alternative approach, we fit a lognormal function to each of our halo samples at \( z = 10 \) and

\[
P(\lambda') = \frac{1}{\lambda' \sqrt{2\pi\sigma_0}} \exp\left[-\frac{\ln^2(\lambda'/\lambda'_0)}{2\sigma_0^2}\right].
\]

The resulting curves are presented in Figure 3, whereas the best-fit parameters, median values for \( \lambda'_\text{med} = \text{median}(\lambda' \text{, } i) \), and median halo masses \( M_{\text{med}} \) are given in Table 2. Inspection of the best-fit parameters confirms that the median spin declines as we move from less massive to more massive objects at high redshift.

### 5. Stability of Results

Because of the weak nature of the measured correlation it is vitally important to check its credibility by performing a statistical analysis. To this extent we investigate the sensitivity of the logarithmic slope \( \alpha \) with respect to a number of parameters that enter into its determination, namely, the number of bins \( N_{\text{bins}} \), used for the histograms, the virialization criterion parameterized via \( Q_{\text{lim}} \), and the minimum number of particles \( N_{\text{min}} \) within a halo’s virial radius. Note that we vary one parameter at a time, keeping the others at their fixed “standard” values. The results are presented in Tables 3, 4, and 5.

We find that bin number has practically no effect on the slope. We similarly find that varying the virialization criterion \( Q_{\text{lim}} \) has little effect on the slope of the relation between mass and spin, regardless of redshift (Table 4). In contrast, we find that the minimum number of particles within a halo’s virial radius has a strong and systematic effect on the result at \( z = 1 \)—as \( N_{\text{min}} \) increases, we find that the logarithmic slope becomes shallower. This does not appear to be true at high redshift, although as we go to earlier times we find that the number of massive halos becomes progressively smaller and our determination of \( \alpha \) becomes increasingly unreliable.

These tests lead us to believe that our results are both stable and reliable, and our main result holds: the correlation between spin parameter \( \lambda' \) and halo mass \( M \) is one order of magnitude larger at redshift \( z = 10 \) than at \( z = 1 \).

As a further test of the credibility of the correlation that we measure between halo mass and spin, we use the criteria of three other studies to select our halo sample. These are as follows.

1. Maccio et al. (2007) criteria:
   a) \( N_{\text{min}} = 250 \),
   b) \( x_{\text{eff}} < 0.04 \),
   c) \( P_{\text{rms}} < 0.4 \).

2. Bett et al. (2007) criteria:
   a) \( N_{\text{min}} = 300 \),
   b) \( Q_{\text{lim}} = 0.5 \).

### Tables

**Table 2**

| Run   | \( \lambda'_0 \) | \( \sigma \) | \( \lambda'_{\text{med}} \) | \( M_{\text{med}} \) |
|-------|------------------|-------------|--------------------------|------------------|
|       | \( h^{-1} \) M_\odot |             | \( h^{-1} \) M_\odot |                  |
| \( z = 1 \) |               |             |                         |                  |
| B01   | 0.042           | 0.538       | 0.041                    | 5.95E06          |
| B02   | 0.040           | 0.539       | 0.037                    | 8.19E07          |
| B05   | 0.036           | 0.516       | 0.035                    | 5.63E08          |
| B10   | 0.033           | 0.540       | 0.029                    | 4.14E09          |
| B20   | 0.030           | 0.251       | 0.027                    | 3.33E10          |
| B100  |                |             |                         |                  |

**Table 3**

| \( N_{\text{bins}} \) | \( \alpha_{z=1} \pm \sigma_\alpha \) | \( \alpha_{z=10} \pm \sigma_\alpha \) |
|------------------------|---------------------------------|---------------------------------|
| 4                      | -0.001 \pm 0.169               | -0.061 \pm 0.191               |
| 5                      | -0.002 \pm 0.149               | -0.059 \pm 0.171               |
| 6                      | -0.005 \pm 0.137               | -0.053 \pm 0.155               |
| 7                      | -0.006 \pm 0.128               | -0.056 \pm 0.144               |
| 8                      | -0.003 \pm 0.120               | -0.069 \pm 0.132               |

**Table 4**

| \( Q_{\text{lim}} \) | \( \alpha_{z=1} \pm \sigma_\alpha \) | \( \alpha_{z=10} \pm \sigma_\alpha \) |
|------------------------|---------------------------------|---------------------------------|
| 0.05                   | -0.007 \pm 0.148               | -0.058 \pm 0.153               |
| 0.10                   | -0.004 \pm 0.148               | -0.062 \pm 0.168               |
| 0.15                   | -0.002 \pm 0.149               | -0.059 \pm 0.171               |
| 0.20                   | -0.005 \pm 0.151               | -0.052 \pm 0.173               |
| 0.25                   | -0.002 \pm 0.153               | -0.052 \pm 0.173               |

*Note.—Here \( Q_{\text{lim}} = 0.15 \) and \( N_{\text{min}} = 600 \).*
We have performed a careful investigation of the relation between virial mass and dimensionless spin parameter for dark matter halos forming at high redshifts $z \geq 10$ in a LCDM cosmology. The result of our study, which is based on a series of cosmological $N$-body simulations in which box size was varied while keeping particle number fixed, indicates that there is a weak correlation between mass and spin at $z = 10$, such that the spin decreases with increasing mass. If there is a correlation at $z = 1$, we argue that it is significantly weaker than the one we find at $z = 10$; this is in qualitative agreement with the findings of previous studies that focused on lower redshifts (Maccio et al. 2007; Shaw et al. 2006; Lemson & Kauffmann 1999).

Interestingly, B07 find a weak correlation between median spin and halo mass at $z = 0$ in the Millennium Simulation (Springel et al. 2005), in the same spirit as the one presented here for $z = 10$: lower mass halos tend to have higher spins. However, as we show, the correlation between halo mass and spin is weaker at $z = 1$ than at $z = 10$, whereas the correlation reported in B07 for halos at $z = 0$ is much stronger than the one we find at $z = 1$.

This is not what one would expect, and so it is important to try and understand the source of the difference between our result at $z = 1$ and the B07 result at $z = 0$. B07 fitted a third-order polynomial to the median spins of halos in the mass range $3 \times 10^{11} \leq M/(h^{-1} M_{\odot}) \leq 3 \times 10^{14}$ at $z = 0$. The form of this polynomial is extremely sensitive to the precise values of the best-fit parameters (P. Bett 2007, private communication), and it is not straightforward to extrapolate its behavior outside of the given mass range and redshift. We derive our estimates of the power-law exponents from the spin distribution with respect to halo mass at $z = 1$. Our halos lie in the mass range $3 \times 10^{9} \leq M/(h^{-1} M_{\odot}) \leq 5 \times 10^{12}$. B07 base their median spins on $\sim 1.5$ million halos with correspondingly small errors, and note that the weak nature of the trend of spin with mass makes it hard to detect. This suggests to us that the correlation between mass and spin at $z = 10$ is remarkably strong rather than the correlation at $z = 1$ being too weak!

When studying correlations between halo mass and spin, great care must be taken in defining the halo sample. In particular, we find that mass resolution (i.e., the number of particles with which a halo is resolved) and the degree of virialization of a halo can have a significant effect on the strength of the correlation (at least at $z = 1$, see Table 5). This—along with other effects—is in agreement with the findings of B07.

We note that Power & Knebe (2006) demonstrated that the size of the simulation box can lead to a suppression of angular momentum in smaller boxes, due to the absence of longer wavelength perturbations in the initial conditions. This will lead to a bias in our estimate of $\lambda$ (approximately a $\sim 10\%$ effect), but we have verified that the spin distributions we obtain from a B20 run truncated on scales larger than the longest wavelength perturbation modeled in the B1 run produce results that are consistent. Indeed, we would expect the correlation to be strengthened if the B1 spins were corrected for box size effects.

It is interesting to speculate on the consequences of this correlation for galaxy formation at high redshifts and the galaxy population we observe today. In the standard picture of galaxy formation, gas cools onto dark matter halos and is shock heated to the virial temperature of the halo. The angular momentum of the gas and the dark matter should (initially) be similar because they are subject to the same tidal field. As the innermost densest parts of the gaseous halo cool, they will settle into a gaseous disk with a scale length determined by the specific angular momentum of the gas, which we would expect to be related to the angular momentum of the halo (e.g., Zavala et al. 2008).

If more massive halos at high redshifts show a tendency to have smaller spin parameters, the gas disks will have lower specific angular momenta and therefore will be more centrally concentrated. If star formation rate correlates with surface density, then we might expect the star formation rate to be enhanced in more massive halos. Because massive halos tend to form preferentially in high-density, highly clustered environments in which the merger rate also tends to be enhanced, then we might expect star formation to proceed more rapidly and at earlier times in these environments.

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### Table 5

| $N_{\min}$ | $\alpha_{z=1} \pm \sigma_{\alpha}$ | $\alpha_{z=10} \pm \sigma_{\alpha}$ | $N_{z=1}$ related halos | $N_{z=10}$ related halos |
|------|----------|-------------|----------------|----------------|
| 100... | 0.003 ± 0.129 | -0.037 ± 0.158 | 19707 | 5478 |
| 200... | 0.002 ± 0.137 | -0.038 ± 0.167 | 10809 | 2534 |
| 300... | 0.002 ± 0.142 | -0.035 ± 0.174 | 7336 | 1526 |
| 600... | -0.002 ± 0.149 | -0.059 ± 0.171 | 3811 | 660 |
| 1000... | -0.004 ± 0.155 | -0.058 ± 0.196 | 2303 | 343 |
| 2000... | -0.011 ± 0.164 | -0.048 ± 0.186 | 1154 | 120 |

**Note.**—Here $Q_{\min} = 0.15$ and $N_{\min} = 5$.  

### Table 6

| Criterion | $\alpha_{z=1}$ | $\alpha_{z=10}$ | $N_{z=1}$ related halos | $N_{z=10}$ related halos |
|----------|----------------|----------------|----------------|----------------|
| Neto et al. (2007) | 0.000 ± 0.149 | -0.041 ± 0.178 | 3486 | 429 |
| Maccio et al. (2007) | 0.001 ± 0.146 | -0.040 ± 0.184 | 4275 | 512 |
| Bett et al. (2007) | 0.003 ± 0.146 | -0.040 ± 0.175 | 8582 | 1955 |

**Note.**—Here $N_{\min} = 5$.  

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*References cited in the text are available in the original article.*
halos. Might this explain the effect of “downsizing” (e.g., Cowie et al. 1996), the successive shifting of star formation from high- to low-mass galaxies with decreasing redshift? We shall pursue this in a more quantitative manner in future work.

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