Loschmidt echo with a non-equilibrium initial state: early time scaling and enhanced decoherence

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We study the Loschmidt echo (LE) in a central spin model in which a central spin is globally coupled to an environment (E) which is subjected to a small and sudden quench at \( t = 0^+ \), remains the same as the ground state of the initial environmental Hamiltonian before the quench; this leads to a non-equilibrium situation. This state now evolves with two Hamiltonians, the final Hamiltonian following the quench and its modified version which incorporates an additional term arising due to the coupling of the central spin to the environment. Using a generic short-time scaling of the decay rate, we establish that in the early time limit, the rate of decay of the LE (or the overlap between two states generated from the initial state evolving through two channels) close to the quantum critical point (QCP) of E is independent of the quenching. We do also study the temporal evolution of the LE and establish the presence of a crossover to a situation where the quenching becomes irrelevant. In the limit of large quench amplitude the non-equilibrium initial condition is found to result in a drastic increase in decoherence at large times, even far away from a QCP. These generic results are verified analytically as well as numerically, choosing E to be a transverse Ising chain where the transverse field is suddenly quenched.

PACS numbers: 03.65.Yz, 05.50.+q, 05.70.Jk, 64.70.qj, 64.70.Tg, 75.10.Jm

The emergence of the classical world from the quantum world, namely decoherence, or the quantum-classical transition through reduction of a pure state to a mixed state has been a subject of perpetual interest to the physics community. The concept of the LE has been proposed in connection to this quantum-classical transition in quantum chaos to describe the hypersensitivity of time evolution of a system to the perturbation experienced by its surrounding. The LE is defined as follows: if a quantum state \( |\psi\rangle \) evolves with two Hamiltonians \( H \) and \( H' \), respectively, the LE is the measure of the overlap given by

\[
\mathcal{L}(t) = |\langle \psi | \exp(iH't) \exp(-iHt)|\psi\rangle|^2.
\]

In recent years, the temporal evolution of the LE has been studied in the vicinity of a QCP. In this context, the central spin model (CSM) where a central spin (CS) is coupled globally to all the spins of an environment (E) which is chosen to be a transverse Ising chain has been introduced. The CS is assumed to be in a pure state initially while the spin chain is in the ground state. The interaction between the central spin and the environment effectively leads to two Hamiltonians which provide two channels of time evolution of the environmental ground state and lead to a decay in the LE. It has been reported that in the limit of weak coupling between the central spin and the environment, the LE shows a sharp decay close to the QCP of the spin chain and right at the QCP, it shows a collapse and revival as a function of time \( t \). This collapse and revival of the LE can be taken to be an indicator of the proximity to a QCP. It can also be shown that the CS makes a transition to a mixed state when the LE vanishes.

The CSM has been generalized to a more generalized environmental Hamiltonian, to the limit of strong coupling, to the case when the interaction between the CS and E is local (i.e., CS coupled to a single spin of the E) rather than global and also in the realization of a Schrödinger magnet. In the short-time limit, one finds a sharp decay of the LE given by the Gaussian form

\[
\mathcal{L}(t) \sim \exp(-\alpha t^2),
\]

where the decay rate \( \alpha \) is expected to capture the universality associated with the QCP of the E. In a recent experimental study with NMR quantum simulator, it has been observed that for a fixed short time the LE approaches a minima in the vicinity of the QCP of an antiferromagnetic Ising chain. The connection between the dynamic LE approach with the static fidelity approach has also been established.

In this paper, we study the temporal evolution of the LE following a sudden quench of the E by a perturbation at \( t = 0 \) so that the state of E at \( t = 0^+ \) is an eigenstate of the initial Hamiltonian \( (H_f) \) but not of the final Hamiltonian \( (H_F) \). Our aim here is to study how a sudden quench of the E influence the LE (or the decoherence of the CS) both in the early time limit and also in its time evolution. Denoting the excited (ground) state of CS by \( |e\rangle \) (\( |g\rangle \)) one can write down generic Hamiltonian of the system and environment in the form

\[
H(\lambda + \delta, g) = H_0 + (\lambda + \delta|e\rangle\langle e|) V_\lambda + g V_g,
\]

Here, \( H_0 + \lambda V_\lambda \) is the initial Hamiltonian \( H_f(\lambda) \), of the E where \( \lambda \) is the measure of the deviation from the QCP \( \lambda = 0 \) while \( g V_g \) denote the perturbing term responsible for the sudden quench so that the final Hamiltonian \( H_F(\lambda, g) = H_f(\lambda) + g V_g \); \( V_\lambda \) and \( V_g \) do not necessarily commute with each other. Unless otherwise stated \( g \) is always taken to be positive. The term \( \delta|e\rangle\langle e| V_\lambda \) is the global coupling between the CS and the E where we have assumed that the CS couples with the E only when it is

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in the excited state $|e\rangle$ and also the small $\delta$ limit throughout. Initially ($t = 0$) the CS is assumed to be in a pure state, $|\phi_g(t)\rangle = c_g|g\rangle + c_e|e\rangle$, with $|c_g|^2 + |c_e|^2 = 1$. On the other hand, since the E undergoes a sudden quench at $t = 0$, it is in the ground state of the Hamiltonian $H_I(\lambda)$ at $t = 0^+$ denoted by $|G(\lambda, g = 0)\rangle$.

As a consequence of the coupling between the environment and the CS, the state $|G(\lambda, g = 0)\rangle$ starts evolving following two separate hamiltonians $H_F(\lambda + \delta, g) = H_0 + (\lambda + \delta)V_\lambda + gV_g$ and $H_F(\lambda, g) = H_0 + \lambda V_\lambda + gV_g$ depending on the state of the CS. At an arbitrary time $t$, we can therefore write the state of the composite system as

$$|\psi(t)\rangle = c_g|g\rangle \otimes |\phi_g(t)\rangle + c_e|e\rangle \otimes |\phi_e(t)\rangle,$$

where $|\phi_g(t)\rangle = \exp(-iH_F(\lambda, g)t)|G(\lambda, g = 0)\rangle$ and $|\phi_e(t)\rangle = \exp(-iH_F(\lambda + \delta, g)t)|G(\lambda, g = 0)\rangle$. The LE at time $t$ is defined as

$$L_q(\lambda, g, t) = \langle |\phi_g(t)\rangle |\phi_e(t)\rangle^2$$

$$= \langle |G(\lambda, g = 0)\rangle |e^{iH_F(\lambda, g)t}e^{-iH_F(\lambda + \delta, g)t}G(\lambda, g = 0)\rangle.$$ (2)

To contrast with the equilibrium situation ($g = 0$), we note that in the present case $|G(\lambda, g = 0)\rangle$ is not an eigenstate of $H(\lambda, g)$. Throughout we shall denote the LE for the equilibrium case (no quenching) as $L$.

Having introduced our model, let us briefly summarize our results. In the following, we propose a general scaling for decay rate $\alpha$ of the LE in the short time limit close to the QCP and show that the scaling is unaffected by perturbation $V_g$. We however show that there is a crossover to the equilibrium behavior for a limit of the quenching amplitude $g$ when the correction in the LE due to the quenching becomes irrelevant. When the quenching is relevant, it deeply influences the temporal evolution of the LE and shows a faster decay as a function of time even when the E is quenched to the QCP and one observes a conspicuous absence of the proper revival when the environment is chosen to be a transverse Ising chain.

To arrive at the scaling relation valid in the short time limit, we assume the limit $\delta \rightarrow 0$ and truncate the exponential in Eq. (4) up to the order $t^2$. One can then express the LE in the form $L_q(\lambda, g, t) \approx 1 - \alpha t^2 \approx e^{-\alpha t^2}$, where $\alpha = \delta^2 [\langle V_\lambda^2 \rangle - \langle V_\lambda \rangle^2]$, where $\langle ... \rangle$ implies the expectation value in the state $|G(\lambda, g = 0)\rangle$. This implies that the small time limit behavior of the echo is independent of the quench amplitude $g$.

Let us note that the operator $\lambda V_\lambda$ is a relevant or marginal perturbation that drives the E away from the gapless QCP thereby generating a gap in the spectrum; it is then expected to scale in the same way as energy, implying $\lambda V_\lambda \sim \lambda^{\nu z}$ for $\lambda \gg L^{-1/\nu}$ where $\nu$ and $z$ are the correlation length and dynamical exponents, respectively, characterizing the QPT in the E driven by $\lambda$. We eventually find the general scaling form

$$\alpha \sim \delta^2 \lambda^{2\nu z - 2} (\lambda \gg L^{-1/\nu}); \quad \alpha \sim \delta^2 L^{2(1/\nu - 2z)} (\lambda \ll L^{-1/\nu}).$$

where $L$ is the linear dimension of the system. However, we note that the above scalings are valid only as long as $2/\nu - 2z > d$, where $d$ is the spatial dimension. Otherwise the contribution from the low energy modes become subleading and that from the higher energy modes dominate resulting in the scaling $\alpha \sim L^d$. In deriving the above relation, we have assumed that the CS is coupled to the E through the operator $V_\lambda$ which is chosen to be equal to $V_\lambda$ that drives the system away from the QCP. Otherwise, the critical exponent $\nu_3$, that determines the scaling of correlation length when the system is moved away from the QCP ($\lambda = 0$) due to the perturbation $V_\lambda$ (correlation length, $\xi \sim \delta^{-\nu_3}$), will replace $\nu$ in Eq. (4). We note that studies for the $g = 0$ case in the recent past have yielded similar results.$^{13}$

Let us now reconcile the scaling given in Eq. (4) with that presented in reference $^{13}$ when $\nu = z = 1$. In the short time limit, if one considers only low energy modes up to a cut off momentum $\tilde{k}$ (assuming that the critical mode $k_c = 0$), we find that the energy gap scales $\tilde{k}^2 \sim L^{-z}$ close to the QCP. With $\lambda V_\lambda \sim L^{-z}$, we find identical result $\alpha \sim \lambda^{-2} L^{-2z}$ to that given in $^{13}$ which has also been verified for situations with $z \neq 1$. Scaling relations presented in (Eq. 4) are also in congruence with the dimensional analysis noting that $\delta \sim \lambda \sim L^{-1/\nu}$ and $t \sim \lambda^{-\nu z} \sim L^{z}$.

**FIG. 1:** Variation of $L_q(t)$ as a function of $h_1$, for $L = 200$, $\delta = 0.025$, $g = 0.2$ and $t = 1$, as obtained numerically from Eq. (7). There is a sharp dip around the QCP.

Let us shift our attention beyond the short-time limit to explore the effect of quenching on the LE at an arbitrary time but for small values of $g$. Assuming that $|\partial G(\lambda, g)/\partial g|$ exists everywhere and using Eq. (4), one finds

$$L_q(\lambda, g, t) \approx L(\lambda, 0, t) + g \frac{\partial L_q(\lambda, g, t)}{\partial g}|_{g = 0} \cdots,$$ (5)

where $L(\lambda, 0, t)$ is the Loschmidt echo in the equilibrium situation $g = 0$ when the environment is initially in the state $|G(\lambda, 0)\rangle$ which is the ground state of $H_F(\lambda, g = 0) = H_F(\lambda)$. Hence, $\Delta_L = L_q(\lambda, g, t) - L(\lambda, 0, t)$ varies linearly in $g$ if $\partial L_q(\lambda, g, t)/\partial g|_{g = 0}$ exists and is non-zero. Demanding the Loschmidt echo to be dimensionless, we find that $\partial L_q(\lambda, g, t)/\partial g|_{g = 0}$ must have the dimension
and one recovers the results for the equilibrium case. The correction due to the quenching becomes irrelevant as given in Eq. (5). In the other limit of large $g$, $L \gg \lambda, g, t$ is in general finite far away from the QCP. It is worthwhile to note that recent studies have shown that non-equilibrium critical dynamics (achieved by a slow variation of a parameter) of the environment across its QCP also leads to dramatic enhancement in decoherence.

We elucidate the above generic theories using the exactly solvable example of a spin $1/2$ in an environment of a transverse Ising spin chain so that the composite Hamiltonian is

$$H_I(h + \delta) = -\frac{\delta}{2} \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \delta |e \rangle \langle e | \sigma_j^x).$$

The environmental spin chain undergoes a QPT at $h = h_\delta = \pm 1$. At time $t = 0$, the transverse field is suddenly changed from the initial value $h_t$ to the final value $h_f = h_t + g$ so that the initial state is the ground state $|G(h_0, 0)\rangle$ of the Hamiltonian $H_I(h_t)$. We are considering the case when the quenching along direction of the transverse field that drives the quantum transition (i.e., $V_0 = V_\lambda$).

The Loschmidt echo at time $t > 0$ can be calculated exactly in the form

$$\mathcal{L}_q(t) = |\langle G(h_0) | e^{-iH(t)} | G(h_0) \rangle|^2 = \prod_{k>0} F_k$$

where

$$F_k = |A_k e^{i\Delta_k t} + B_k e^{-iS_k t} - C_k e^{iS_k t} + D_k e^{-i\Delta_k t}|^2.$$  

And

$$A_k = \cos \alpha_k(h_t, h_f) \cos \alpha_k(h_t, h_f + \delta) \cos \alpha_k(h_f, h_f + \delta)$$

$$B_k = \cos \alpha_k(h_t, h_f) \sin \alpha_k(h_t, h_f + \delta) \sin \alpha_k(h_f, h_f + \delta),$$

$$C_k = \sin \alpha_k(h_t, h_f) \cos \alpha_k(h_t, h_f + \delta) \sin \alpha_k(h_f, h_f + \delta),$$

$$D_k = \sin \alpha_k(h_t, h_f) \sin \alpha_k(h_t, h_f + \delta) \cos \alpha_k(h_f, h_f + \delta),$$

$$\Delta_k = \varepsilon_k(h_f + \delta) - \varepsilon_k(h_f)$$

$$\delta_k = \varepsilon_k(h_f + \delta) + \varepsilon_k(h_f),$$

$$\alpha_k(m, n) = (\theta_m - \theta_n)/2, \theta_n = \tan^{-1} \sin \theta(k)/(m - \cos \theta),$$

$$\varepsilon_k(h) = 2 \sqrt{(h - \cos \theta)^2 + \sin^2 \theta}.$$

We note that the limit $g = 0$ (i.e., $h_t = h_f$), we recover the result of the reference [13] where we have used periodic boundary conditions such that the decoupled momentum modes $k$ are quantized as $2\pi p/L, p = 0, 1, 2, \ldots L$. 

\[ g^{-1} \sim L^{1/\nu_g} \] where $\nu_g$ is the correlation length exponent corresponding to the operator $V_\lambda$. We therefore propose a very interesting crossover scenario: in case the quenching parameter $g \gg L^{-1/\nu_g}$, the quenching influences the LE as given in Eq. 4. In the other limit $g \ll L^{-1/\nu_g}$, the correction due to the quenching becomes irrelevant and one recovers the results for the equilibrium case.

Eq. 4 also shows that $\Delta_L$ depends on the sign of $g$, i.e., the difference in the LE (compared to the $g = 0$ case) depends non-trivially on the direction of quench for small values of $|g|$. However, in the limit of large $|g| (|g| \gg L^{-1/\nu_g})$, $\mathcal{L}(\lambda, 0, t)$ is general finite far away from the QCP: this can be easily seen by expanding $\mathcal{L}(\lambda, 0, t)$ for small $\delta$ and noting that $\partial \langle G(\lambda, 0) \rangle / \partial \lambda$ is large only near the QCP and small otherwise.

On the other hand, in order to investigate the dependence of $\mathcal{L}_q(\lambda, g, t)$ on $g$, we expand $|G(\lambda, 0)\rangle$ in Eq. 4 as $|G(\lambda, 0)\rangle = \sum_n a_n |\Xi_n(\lambda, g)\rangle$ in terms of the eigenstates $|\Xi_n(\lambda, g)\rangle$ of $H_F(\lambda, g)$ (with corresponding probability amplitudes $a_n$ and eigenenergies $\epsilon_n(\lambda, g)$). Increase in $g$ increases the number of oscillatory terms contributing to the expansion of $\exp (iH_F(\lambda, g)t) |G(\lambda, 0)\rangle = \sum_n \exp (i\epsilon_n(\lambda, g)t) a_n |\Xi_n(\lambda, g)\rangle$, which finally leads to decrease in value of $\mathcal{L}_q(\lambda, g, t)$ at any finite time. Therefore in the limit of large $g$ we can expect $\Delta_L$ to be negative at large times irrespective of sign of $g$, or in other words, a sudden quench of an environment coupled to a qubit leads to enhancement in decoherence as compared to the case with equilibrium initial condition. We note that the dependence of $\mathcal{L}_q(\lambda, g, t)$ and $\Delta_L$ on $g$ as discussed above are not necessarily related to a QCP, and should be prominent even away from it. This is one of the central results of our work and it shows that equilibrium initial condition is absolutely necessary for a coherent time evolution in an open quantum system, even far away from the QCP. It is worthwhile to note that the above generic theories using the exactly solvable example of a spin $1/2$ in an environment of a transverse Ising spin chain so that the composite Hamiltonian is
We shall now analyse Eq. (7) numerically to study the proximity to the QCP. Fig. (1) shows that \( \mathcal{L}_q(t) \) plotted as a function of \( h_i \) shows a minimum near \( h_i = h_c - g = 1 - g \), thus detecting the presence of a quantum critical point even when the environment is suddenly quenched.

Let us now verify whether the scaling relations of \( \alpha \) in the early time limit given above holds true in this specific case. We present results in Fig. (2); the variation of \( \alpha \) is plotted as a function of \( \lambda = 1 - h \) close to the QCP with \( g = 0 \); we find the results are in perfect agreement with the proposed scaling. In inset of Fig. (2), we show that the short-time decay also depends on the symmetry of the phase on either side of the QCP. Moreover, \( \alpha \) is linear in \( L \) which is essentially a contribution from the high-energy modes as we show in Fig. (3) where we also show that the decay rate \( \alpha \) does not depend on \( g \) for \( g \neq 0 \).

In Fig. (4), we plot the behavior of \( \mathcal{L}_q(t) \) as a function of time, as obtained numerically using Eq. (7). Interestingly, for nonzero \( g \), \( \mathcal{L}_q \) initially decays for small time before rising again and showing small oscillations at large times. On the other hand the echo stays almost close to unity away from the QCP for \( g = 0 \). One therefore finds that quenching deeply influences the collapse and revival of the LE following a quench. In fact, absence of perfect revival (even when the E is quenched to the QCP with \( h_f + \delta = 1 \)) and very fast decay of the peak in \( \mathcal{L} \) establishes this conjecture. Moreover, we consider the limit \( \delta \ll g \) when \( \sin \alpha_k(h_f, h_f + \delta) \ll \cos \alpha_k(h_f, h_f + \delta) \) and one gets

\[
F_k \approx |A_k e^{i\Delta_k t} + D_k e^{-i\Delta_k t}|^2. \tag{8}
\]

Near the critical momentum mode \( k = 0 \), \( \Delta_k = \varepsilon_k(h_f + \delta) - \varepsilon_k(h_f) \sim 2(h_f + \delta - h_f) = 2\delta \) for \( g \gg L^{-1} \), eventually leading to

\[
F_k \approx (A_k + D_k)^2 \cos^2(\Delta_k t) + (A_k - D_k)^2 \sin^2(\Delta_k t). \tag{9}
\]

The time at which the maxima occurs can be determined by the condition \( \partial F_k / \partial t = 0 \) which yields \( t_{n,k} = n\tau / \Delta_k \sim \delta^{-1} \) (\( n = 1, 2, \ldots \)). However, the momentum dependent \( t_{n,k} \) results in the irregular oscillatory temporal behavior of \( \mathcal{L}_q(t) \), as shown in Fig. (4). In Fig. (5), we show the time \( t_0 \) at which the first maxima occurs indeed scale with \( \delta^{-1} \).

Let us now estimate characteristic value of \( g = g_0 \sim L^{-1/\nu_c} \); for \( g > g_0 \), we expect to see the influence of the non-equilibrium initial condition. Inspecting, Eq. (7), we expect a cross-over when the contributions from the terms \( C_k \) and \( D_k \) become negligible as compared to those coming from \( A_k \) and \( B_k \). This is ensured when \( \sin \alpha_k(h_i, h_f) \approx \cos \alpha_k(h_i, h_f) \). Using the initial and final parameters to be \( h_i = 1 - g/2, h_f = 1 + g/2 \), we find \( g_0 \sim L^{-1} \), as is expected from our previous analysis because \( \nu_c = 1 \), in the present case. In the limit of small \( g \) it is straightforward to verify numerically that the difference \( \Delta \) scales linearly with \( g \) as proposed in the generic case (see Fig. 4). Interestingly, as expected from our earlier discussions, even though the sign of \( \partial \Delta_L / \partial \vartheta \) is reversed for small \( |g| \) when direction of quench is reversed, in contrast in the limit of large \( |g| \) \((|g| \gg g_0) \) \( \Delta_L \) asymptotically decreases to \(-1 \) irrespective of the direction of quenching, signifying almost complete loss of coherence (see Fig. 4).

In conclusion, we have studied the LE both in the early time limit and as a function of time following a sudden quench of the environment. We have proposed a generic early time scaling which is independent of the quenching and determined entirely by the scaling dimension of the operator \( V_{\delta} \) that couples the CS to the E and the dynamical exponent \( z \) which is a characteristic of the associated QCP of the environment. Moreover, our study predicts an interesting crossover behavior. We also show that non-equilibrium initial condition can lead to drastic increase in decoherence in open quantum systems, even away from a QCP. For the specific example in which a transverse Ising chain is chosen as the E, we not only verify the generic scaling relations but also establish that
the perfect revival of LE does not occur. However, in this example we have chosen $V_0 = V_g = V_3$. It would be instructive to verify when the CS is coupled to the E through a longitudinal field term $\langle 0 | e_1 | \sum_i \sigma_i^x \rangle$ or the E is quenched by a term $g \sum_i \sigma_i^x$. Although the model then becomes non-integrable, the exponents $\nu_4$ and $\nu_3$ will be different from $\nu = 1$ (in fact, the critical exponent $\nu$ when the QCP is perturbed by a longitudinal field is $8/5$) and are expected to appear in the early time scaling and the crossover relation. Further, this work can also lead to additional studies on effects of different quenching schemes on decoherence and generalization to finite temperatures may also yield interesting results. Finally, recent experiments on Loschmidt echo using NMR quantum simulator and also other notable experiments on non-equilibrium quantum dynamics (for example see Ref. [27]) have created the possibility of experimental verification of our results in near future.

AD acknowledges CSIR, New Delhi, for support and SS for Junior Research Fellowship.