Optimal sub-parametric finite element approach for a Darcy-Brinkman fluid flow problem through a circular channel using curved triangular elements

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Abstract. In this paper a proposed sub-parametric finite element method is used to solve a Darcy Brinkman flow problem through a circular channel using curved triangular elements. The flow through an irregular geometry requires a very tedious computation. In order to solve the flow problem with a much effective computational process, a finite element approach with curved triangular elements and with less number of degrees of freedom is used. The usefulness of the method is the objective of the paper. Results are in good agreement with previous works done.

1. Introduction
The dynamics of porous medium is different from normal flows of streams and rivers. The pore matrix has its influence on the velocity distributions which is represented by its partial differential equation. Porosity depends on the pressure of the engineering system like a reservoir pressure. Porous medium effects various mechanical applications like thermal insulations, petroleum industry, heat exchangers and heat pipe technology. A lot of study has been carried out in the field of forced convection for a porous media by Haji-Shiekh and Vafai [1], Nield and Kuznetsov [2], Hooman [3], Narashimhan and Lage [4]. Work has also been done in steady flow through porous media by Greencorn [5], where the effects of different geometries of porous media is related to capillary pressure. Finite element method is found to generate a good result when compared to the experimental method in various works. In this work we have considered a flow through a circular channel with porous media and by the help of extensive computation using Mathematica 7.0 codes a finite element analysis is done. The study of a steady, uni-dimensional, linear flow is considered in the example. The use of curved triangular element is given in Ergatoudis et al. [6], the finite element procedure is given in Bhatti [7] and the matching of curved triangular elements with parabolic arcs , the description of the transformation is given in the works of Rathod et al. [8] and Nagaraja et al. [9]. The numerical integrations required in
calculating the finite element equations are carried using the numerical integration scheme given in the works of Rathod et. al. [10]. The numerical computation scheme adopted makes a faster calculation in comparison with the commercial software available. The scheme is adopted to solve the flow through the given circular porous channel as in (Figure 1).

2. Mathematical Formulation For Low Speed Flow

The momentum Darcy–Brinkman equation which governs the velocity \( u(x, y) \) profiles of the fluid flow in the \( z \)-direction for figure 1 is given by (see Keasavulu et. al. [11])

\[
\begin{align*}
\mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{u}{K} = \frac{d p}{d z}.
\end{align*}
\]

Due to the symmetric nature of the solution of the present problem, one-fourth of the geometry is considered as in figure 2. The boundary condition for the present geometry is given by

\[
u = 0 \text{ on } C1
\]

where \( C1 \) is the boundary of the cross-section; see Figure 3.

In order to solve the Partial Differential Equation Eq. (1) subject to the condition Eq. (2), the Galerkin weighted residual method is used; see Bhatti [9] and Keasavulu et. al. [11]. The region \( \Omega \) is divided into number of triangular elements \( \Omega_t \) and hence the boundary \( C1 \) is also divided as shown in the Fig. (4a&4b).

In the present example, experimentation is done with curved triangular elements with 6-noded quadratic and 10-noded cubic; see Kesavulu et. al. [11]. The equations of the shape functions are given in Nagaraja et. al. [9]. The Jacobian for the above transformation is found in Kesavulu et. al. [11]. The linear degree of the Jacobian reduces the computational time. By applying Galerkin finite element procedure and Gauss quadrature rule is applied for evaluating the above equations. After taking the effect of all the elements into account and then imposing the boundary conditions, we get a linear system of algebraic equations, on solving, we get the unknown nodal point velocity profiles (see Kesavulu et. al. [11]).

![Figure 1. The full circular channel flow domain.](image-url)
Figure 2. One fourth of the flow circular channel considered due to symmetry in the solution domain.

Figure 3. Cross-section of Figure 2.

3. Results and Discussions

In this 21st century the faster computation is the need of all the major analysis in mechanical industries. A faster computation can reduce the cost and saves time. The method used in the analysis of the circular flow shows a very effective and uniform results. The precise boundary effects are found in the classical finite element methods with a lot of computational effort. Hence the straight triangular elements are replaced in the edges with curved triangular elements to get the exact boundary effects with lesser elements. The coding for the designed algorithm is done in Mathematica7.0. It is observed that, as we increase the order and number of elements we get a better and much uniform flow with an increased accuracy. This shows the effectiveness of the method used. We can see the contour plots of the velocity distribution $U(X,Y)$ in a channel with porous materials which depicts the results in the present case.

In the Figs. 5a, 5b, 5c, 5d, 5e shows the effect of change in Darcy number $\sigma$ and Brinkman number $\Lambda$. In their respective contour plots it is observed that if we take the Darcy number as zero i.e. Fig. 5a, it means when there is no Darcy friction the flow velocity is the highest.
Secondly the Brinkman number is kept constant and the Darcy number is varied, the results shows as the Darcy number increase the flow velocity decreases. Comparison can be seen in the results of contour plots of Fig. 5b with Fig. 5c and Fig. 5d with Fig. 5e.

Thirdly the Brinkman number was varied and the Darcy is kept constant, as the Brinkman number is increased the flow velocity increases, comparison can be seen in Fig. 5b with Fig. 5d and Fig. 5c with Fig. 5e.

It may be concluded that the results depicted by the figures are in tune with the observation of Givler and Altobelli [12].

**Figure 4a.** Discretization of 8 elements of quadratic order.

**Figure 4b.** Discretization of 8 elements of cubic order.
**Figure 5a.** Contour plot for cubic 8 elements with \( \Lambda = 1, \sigma^2 = 0 \).

**Figure 5b.** Contour plot for cubic 8 elements with \( \Lambda = 1, \sigma^2 = 5 \).
Figure 5c. Contour plot for cubic with 8 elements with $\Lambda = 1$, $\sigma^2 = 10$.

Figure 5d. Contour plot for cubic with 8 elements with $\Lambda = 2$, $\sigma^2 = 5$. 
4. Conclusions.

- This method captures the effects for different irregular channels so it can be used for irregular geometries.
- As the friction increases the flow velocity reduces and as the porous nature increases the flow velocity increases, this phenomenon is precisely shown using the unique numerical scheme.
- A good accuracy is achieved using higher order curved triangular elements and with lesser number of elements.

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