QCD axion dark matter and the cosmic dipole anomaly

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There is growing evidence that the cosmic dipole measured from the distant galaxy number-count is not consistent with that derived from the cosmic microwave background (CMB). We find that the QCD axion, a hypothetical particle originating from the spontaneous breaking of the Peccei-Quinn symmetry, could explain this dipole anomaly if it constitutes the dark matter of our universe.

I. INTRODUCTION

One of the mysteries is the existence of dark matter in our universe. It not only contributes to most of the matter component of our observed universe but also evolves with the initial stochastic density fluctuations and leads to the large-scale structure formation of our universe. However, the nature of dark matter is still unclear except for its gravitational interaction with normal matter. One of the most popular dark matter candidates is the weakly interacting massive particle (WIMP) where the dark matter relic originates from the freeze-out process during the early universe. Numerous experiments dedicate to looking for the particle nature of dark matter but there is no compelling evidence yet. Therefore it is intriguing to look for the other properties of dark matter which may provide clues about its origin.

On the other hand, it has been a long time since people try to test the cosmological principle by comparing the cosmic dipole measured from the distant galaxy number-count with that inferred from cosmic microwave background (CMB) [1–8]. It is known there is a dipole in the CMB measurement, which is believed due to the kinematic motion of the solar system relative to the CMB background. From the CMB dipole one can infer that the velocity of solar system relative to CMB is around 370 km/s [11]. Due to the presence of this velocity, we should observe similar dipole behavior from the distance galaxy-based measurements. Ellis and Baldwin [1] shows that for radio sources with identical flux density spectra $S \propto \nu^{-\alpha}$ (where $\nu$ is the frequency and $\alpha$ the spectral index) and integral source count above flux density threshold $S_\ast$, given by $dN(> S_\ast)/d\Omega \propto S_\ast^{-\beta}$, there exists a dipole for the galaxy number counts,

$$\Delta N/N = d_k \cdot \hat{n},$$

with $d_k = [2 + x(1 + \alpha)]\beta$ where $\beta = v/c$ and $v$ is relative velocity of the solar system to the ‘matter rest frame’. If we adopt the $v$ from the measurement of CMB in Eq. (1) as theoretical prediction for the galaxy number count dipole, then we can compare it with the measurement to check the consistence of the cosmological principle.

Recently one group analysis more than 1 million quasars from CatWISE catalogue [12] find that the amplitude of the number-count dipole is not consistent with that derived from the CMB and the deviation is at a confidence level around 4.9$\sigma$ [13, 14], bringing the suspicion on the cosmological principle (on large enough scale the universe should be homogenous and isotropic). A replacement of this fundamental principle has been considered recently [15]. However, before abandoning this fundamental assumption, it is necessary to examine whether this dipole anomaly can be explained under the standard framework of cosmic perturbation theory. It has been found that if there exists large isocurvature perturbation at the super horizon scale [16], the inconsistency of these two dipoles can be relieved.

If the isocurvature perturbation is from density fluctuation of the dark matter, it can not attribute to the WIMP dark matter because it would thermalize with the photon in the early universe and all the initial isocurvature perturbation would disappear [17, 18]. One of the interesting candidates might be the QCD axion [19–30], which is the hypothetical particle originating from the spontaneous breaking of the Peccei-Quinn symmetry to explain the strong CP problem [31, 32]. It has been known for a long time that the axion dark matter would generate an isocurvature perturbation around $H/\pi f_a$, where $H$ is the Hubble parameter during inflation and $f_a$ is the decay constant of the axion. Unfortunately, we do not observe the isocurvature perturbation from CMB [33], setting a strong limit on the ratio of the Hubble parameter and the axion decay constant $H/f_a \lesssim 10^{-5}$. In addition, for a moderate initial displacement angle of the axion $\theta \sim O(1)$, the axion decay constant should be around $10^{12}$ GeV to explain the dark matter relic abundance. Then the Hubble parameter should be less than $10^7$ GeV and a low scale inflation model is preferred.

On the other hand, if we want to solve the dipole anomaly, we need a pretty large isocurvature perturbation, but a large isocurvature perturbation at re-
combination is excluded by the current observation of CMB. However, explaining the dipole anomaly requires a large isocurvature perturbation at super horizon scale, if we can manage a large isocurvature perturbation at a large(super-horizon) scale but small enough at recombination, we could recoil this contradiction. In the following, we will present an axion model satisfying this requirement.

This letter is organized as follows. In Sec. II we will briefly overview the dipole anomaly and present the conditions for explaining it. In Sec. III we show an axion model which could explain the dipole anomaly and the numerical calculation is shown in Sec. IV. We draw our conclusion in V.

II. DIPOLE ANOMALY

Besides an average temperature of around 2.7 K, the anisotropy is also a typical feature of the CMB observation. The anisotropy can be decomposed into spherical harmonics,

\[ \frac{\Delta T}{T}(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}) , \]

where

\[ a_{lm} = \int d\Omega \frac{\Delta T}{T}(\hat{n}) Y_{lm}^*(\hat{n}) , \]

A rotationally-invariant angular power spectrum can be defined as [38]

\[ C_l = \frac{1}{2l+1} \sum_m (a_{lm}^* a_{lm}) . \]

Typically, \( l = 0, 1, 2 \) corresponds to the monopole, dipole, and quadrupole respectively. It is usually convenient to use another variable \( D_l \) defined as

\[ D_l = \frac{l(l+1)}{2\pi} C_l . \]

and then

\[ \frac{\Delta T_i}{T} = \sqrt{D_l} . \]

The dipole term has been measured by Planck to be [11]

\[ d^{\text{CMB}} = \left( 1.23357 \pm 0.00036 \right) \times 10^{-3} . \]

It is usually assumed that this dipole is from the Doppler effect due to the motion of the solar system with respect to the CMB rest frame. Then the relative velocity is [11]

\[ v_o = 369.82 \pm 0.11 \text{ km/s} , \]

with a direction \( l = 264^\circ, b = 48^\circ \).

\[ (\hat{v}) = 369.82 \pm 0.11 \text{ km/s} , \]

On the other hand, if the CMB dipole has a kinematic origin, there must be a kinematic dipole from the distant galaxy number-count according to the FLRW model. Recently there are many activities on the measurement of kinetic dipoles from distant radio galaxies or quasars and it has been found that these dipoles are 2-3 times larger than the expected value deduced from above \( v_o \) [13, 14]. If the dipole from galaxy number-count has the kinematic origin, it prefers a velocity [13]

\[ v'_o = 797 \pm 87 \text{ km/s} , \]

pointing in a similar direction. The inconsistency of these two velocities is around 4.9\( \sigma \). One might conclude that the comic principle should be discarded and the universe might have a preferred direction. However, before abandoning this fundamental assumption, we should examine whether this discrepancy can be explained in the perturbed FLRW model. Actually, there has been many studies showing that part of the CMB dipole could be intrinsic if there is large isocurvature perturbation at super horizon scale [40–42]. Therefore, assuming our velocity is \( v'_o \) relative to the cosmic rest frame and the intrinsic dipole cancels part of the kinematic dipole from CMB, the discrepancy of these two dipoles could be explained, i.e,

\[ d^{\text{CMB}} = d^{\text{kin}} + D^{\text{CMB}} = 1.23357 \times 10^{-3} , \]

where \( d^{\text{kin}} = v'_o/c \) is the kinematic dipole and the z-direction is chosen as the dipole direction. \( D^{\text{CMB}} \) is the intrinsic dipole of CMB from isocurvature perturbation in an opposite direction. Then we have

\[ D^{\text{CMB}} \approx -1.4 \times 10^{-3} - (v'_o - 797 \text{ km/s})/c . \]

To explain the anomaly to be within 2\( \sigma \), we need at least \( |D^{\text{CMB}}| > 8 \times 10^{-4} \).

Before getting this conclusion, one need also examine whether the super-horizon isocurvature affects the galaxy number-count dipole. The authors [16] find that the isocurvature perturbation at the super horizon scale only has an essential effect on the CMB dipole, not on the galaxy number-count dipole. Then the authors [16] interpret the dipole anomaly as a result of isocurvature perturbation at super horizon scale but with a single mode. However, in the realistic model, the power spectrum is usually a continuum. In this letter, we interpreted it due to the continuum isocurvature from the axion dark matter.

In this case, for the isocurvature perturbation, we have [16, 42]

\[ \frac{\Delta T}{T} = -\frac{1}{3} S , \]

3 Planck has also measured the relative velocity due to the aberration and modulation effects and finds that \( v_o = 384 \pm 78 \text{(stat)} \pm 115 \text{(syst) km/s} \) which has a large uncertainty [39].

4 Note that there are articles trying to use axion string to explain the CMB hemispherical anomaly which is total a different problem [43, 44].
here we ignore the matter contribution of the baryon. We can define the power spectrum of $\mathcal{P}_S(k)$ as,

$$\langle S_k S_{k'} \rangle = 2\pi^2 \frac{\mathcal{P}_S(k)}{k^3} \delta(k-k') \Theta(k-k_{\text{min}}) ,$$  \hfill (13)

where $k_{\text{min}}$ is the minimal comoving wave number where the isocurvature starts (or maybe taken as the start time for inflation). We have [42]

$$C_l = \frac{4\pi}{9} \int_{k_{\text{min}}}^{\infty} \frac{dk}{k^3} P_S(k) j_l^2(k r_{\text{dec}}) ,$$ \hfill (14)

where $k$ is comoving wave number and $r_{\text{dec}}$ is the comoving distance to the photon last scattering surface with $r_{\text{dec}} \approx 14.1$ Gpc. To explain the dipole anomaly, it requires

$$\sqrt{D_2} = \sqrt{2C_1} \lesssim 8 \times 10^{-4} \Rightarrow C_1 \gtrsim 3 \times 10^{-7} .$$ \hfill (15)

There are also constraints from multipole observations. For quadrupole [11], we have

$$D_2 \lesssim 2.5 \times 10^{-10} .$$ \hfill (16)

Thus we need

$$\frac{D_2}{D_1} \lesssim 4 \times 10^{-4} ,$$ \hfill (17)

or

$$\frac{C_2}{C_1} \lesssim 1.4 \times 10^{-4} .$$ \hfill (18)

Here we only consider the limit from quadrupole (for other multipoles, the constraint is much weaker).

Before presenting a concrete model, we can assume $P_S(k)$ just follows power law $P_S(k) = A(k/k_{\text{min}})^{n-1}, (n < 1)$. For $n > -1$ we have

$$C_l \approx \frac{4\pi A}{9} (k_{\text{min}} r_{\text{dec}})^{1-n} c(n,l) ,$$ \hfill (19)

where

$$c(n,l) = \int_{k_{\text{min}}}^{\infty} dk k^{n-2} j_l^2(k)$$

$$= 2^{n-4} \pi \frac{\Gamma(l+n/2-1/2)\Gamma(3-n)}{\Gamma(l+5/2-n/2)\Gamma^2(2-n/2)} .$$ \hfill (20)

We give some of the values of $c(n,l)$.

$$c(0,1) = 0.2, \quad c(0,2) = 0.03 ;$$ \hfill (21)

$$c(-0.9,2) = 1.17, \quad c(-0.9,2) = 0.015 .$$ \hfill (22)

Obviously the condition (18) can not be satisfied. Since the above formula is only valid for $n - 2 + 2l > -1$, for $n - 2 + 2l < -1$, the main part of the integration is around $k_{\text{min}}$. We can use the properties of the $j_l(x) \sim \frac{x^l}{(2l+1)!}$ for $x \ll 1$ and it is easy to find that

$$C_l \approx \frac{4\pi A}{9|n-1+2l|!(2l+1)!!} (k_{\text{min}} r_{\text{dec}})^{2l} .$$ \hfill (23)

The above formula is valid for $n - 2 + 2l < -1$.

Taking an example $n = -2$, $k_{\text{min}} r_{\text{dec}} = 0.01$, we have

$$C_2/C_1 \approx 9 \times 10^{-4} .$$ \hfill (24)

It is still too large to avoid the quadrupole limit. We could either make $k_{\text{min}} r_{\text{dec}}$ smaller or make $n$ smaller, for example, $k_{\text{min}} r_{\text{dec}} = 0.002$,

$$C_2/C_1 \approx 1.8 \times 10^{-4} .$$ \hfill (25)

In this case $A \sim 1$ is needed to explain the dipole anomaly within $2\sigma$.

For $n = -3$, $k_{\text{min}} r_{\text{dec}} = 0.01$ we have

$$C_2/C_1 \approx 2 \times 10^{-5} .$$ \hfill (26)

### III. LARGE ISOCURVATURE FROM THE AXION

The isocurvature perturbation of axion can be defined as,

$$S = \frac{\delta \rho_a}{\rho_a} - \frac{3}{4} \frac{\delta \rho_r}{\rho_r} \approx \frac{\delta \rho_a}{\rho_a} = \frac{2\delta \theta}{\theta} .$$ \hfill (27)

Since

$$P_{\delta \theta}^{1/2} = \frac{H}{2\pi \varphi(k)} ,$$ \hfill (28)

we have

$$P_{S}^{1/2} = \frac{H}{\pi \varphi(k) \theta} .$$ \hfill (29)

Note that $P_{S}^{1/2} \lesssim 1$ since for $\phi(k) \lesssim H, \delta \theta \sim \theta$. From the above equation, we see that a large isocurvature requires $\varphi \sim H$ at the super horizon scale, while $\varphi$ should be large enough at recombination. If the value of $\varphi$ rapidly increases into a very large value during inflation, the tension with the isocurvature limit from CMB observation could be relaxed, as illustrated in Fig. 1. In the following, we will describe a model of how to realize it.

The model contains a complex scalar fields $\Phi$ with Peccei-Quinn charge $+1$ and the potential can be written as

$$V(\Phi) = \lambda (\Phi \Phi^* - f^2/2)^2 .$$ \hfill (30)

Now we define the axion field as the angular mode of $\Phi$,

$$\Phi = \frac{1}{\sqrt{2}} \varphi \exp(i \frac{a}{f}) .$$ \hfill (31)

Assuming initially $\varphi$ is located close to the origin but with a displacement around $H$. Ignore the motion of angular direction, the equation of motion for $\varphi$ becomes

$$\ddot{\varphi} + 3H \dot{\varphi} + V'(\varphi) = 0 .$$ \hfill (32)
For the isocurvature perturbation at recombination, the value of $\varphi$ should be large enough, therefore we assume $H/f \lesssim 10^{-5}$. After a few e-fold the $\varphi$ already gets close to $f$ and the isocurvature limit can be easily avoided. To accommodate the dark matter relic for the axion, $f$ should be around $10^{12}$ GeV for $\theta \sim \mathcal{O}(1)$.

One may wonder why $\Phi$ is located around the origin of the potential. We can imagine in the beginning there is a coupling between the inflaton field $I$ and $\Phi$, 

$$V_I = g \frac{\Phi^4 I^n}{M_p^{n-2}} \quad (n \geq 2).$$  

During inflation, $I$ rolls from a large value to a smaller value, therefore this term contributes a large effective mass (larger than the Hubble parameter) for the $\Phi$ field, the minima of the potential is at the $\Phi = 0$ and the Peccei-Quinn symmetry is not broken. At a certain stage, when $I$ becomes small, the $V_I$ term becomes sub-dominant and the negative mass from potential in Eq. (30) dominates and then the potential develops a non-vanishing vacuum. Within a few e-fold, the field $\Phi$ goes random walking and is displaced from the origin with an amplitude around $H$. Later on, it follows the equation of motion through Eq. (32). If this happens at the super-horizon scale $k_{\text{min}}$, then the isocurvature is large at the super-horizon scale and smaller at the length scale of recombination.

### IV. NUMERICAL CALCULATION

For the numerical calculation, since 

$$P_s^{1/2} = \frac{H}{\pi \varphi(k)\theta},$$

we can define 

$$P_s(k) = P_s(k_{\text{min}}) \left( \frac{\varphi(k)}{\varphi(k_{\text{min}})} \right)^{-2},$$

where $P_s(k_{\text{min}}) \approx 1$. The $\varphi(k)$ follows the equation of motion of (Eq. 31) and $k = k_{\text{min}} \exp(Ht)$. Now we have 

$$C_l = \frac{4\pi}{9} \int_{k_{\text{min}}}^{\infty} dk \frac{dk}{k} P_s(k) J_l^2(k r_{\text{dec}}).$$

We define $\tau = Ht$, then 

$$C_l = \frac{4\pi}{9} \int_0^{\infty} d\tau P_s(k_{\text{min}}e^{\tau}) J_l^2(k_{\text{min}}e^{\tau} r_{\text{dec}}).$$

Here we show the evolution of $\varphi$ in Fig. 3 for $\lambda = 10^{-9}, 2 \times 10^{-9}, 4 \times 10^{-9}$ with the blue, red and green curves respectively. The initial condition is assumed that $\dot{\varphi} = 0$, $\varphi(k_{\text{min}}) = H/\pi$ and $f = 10^5 H$. As we see, for a smaller $\lambda$, the field will stay at the small value for a longer time. Nevertheless, it will go to the minima of the potential after a few e-folds. We note here that if the $\lambda$ is too small, it will stay at the small value for a too long time and the isocurvature becomes too large at the length scale of recombination. If it is too large, it will quickly go into the minima and the dipole anomaly can not be explained.
In the case of $\lambda = 4 \times 10^{-9}$, $k_{\text{min}} r_{\text{dec}} = 0.002$, $P_S(k_{\text{min}}) \approx 1$, we find that

$$C_1 = 1.4 \times 10^{-7} ;$$
$$C_2 = 1.2 \times 10^{-12} ;$$
$$C_2/C_1 = 8.4 \times 10^{-6} .$$

(38)

For $\lambda = 2 \times 10^{-9}$, $k_{\text{min}} r_{\text{dec}} = 0.002$ and $P_S(k_{\text{min}}) \approx 1$, we have

$$C_1 = 2.5 \times 10^{-7} ;$$
$$C_2 = 1.3 \times 10^{-12} ;$$
$$C_2/C_1 = 5.0 \times 10^{-6} .$$

(39)

For $\lambda = 10^{-9}$, $k_{\text{min}} r_{\text{dec}} = 0.002$ and $P_S(k_{\text{min}}) \approx 1$, we have

$$C_1 = 5.3 \times 10^{-7} ;$$
$$C_2 = 2.4 \times 10^{-12} ;$$
$$C_2/C_1 = 4.5 \times 10^{-6} .$$

(40)

To explain the dipole anomaly, we need $C_1 > 3 \times 10^{-7}$. We numerically find this condition requires $\lambda < 1.6 \times 10^{-9}$. On the other hand, to not give sizable isocurvature observation at the length scale of recombination($k = 0.002 \text{ Mpc}^{-1}$), we find $\lambda > 0.6 \times 10^{-9}$. Therefore for the parameter $k_{\text{min}} r_{\text{dec}} = 0.002$, $f = 10^5 H$, only a small region of $0.6 \times 10^{-9} < \lambda < 1.6 \times 10^{-9}$ is allowed.

In all the above calculations we ignore the baryon matter contribution to the matter component. There should be another factor $(\frac{\Omega_{cd}}{\Omega_{cd} + \Omega_{dm}})^2 = 0.7$ for all the $C_l$ calculation after considering baryon matter contribution.

V. CONCLUSION

In this letter, we present an axion model to explain the cosmic dipole anomaly. In the model, a large isocurvature is predicted at the super horizon scale to accommodate the dipole anomaly, while it remains small at recombination to evade the isocurvature limit from CMB observation. It can be realized if the radial mode of the axion evolves from a small value to a very large value in a very short time during inflation. However, to be consistent with the CMB isocurvature limit, we find for the parameter $k_{\text{min}} r_{\text{dec}} = 0.002$, $f = 10^5 H$, only a small region of $0.6 \times 10^{-9} < \lambda < 1.6 \times 10^{-9}$ is allowed. On the other hand, the observation of the dipole anomaly may provide the first evidence of the existence of the axion dark matter.

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