A scalable system for primal-dual optimization

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Abstract

We present some of the most widely used architectures for Big Data, Hadoop and Spark, and develop several implementations exploiting the advantages of each. We implement a simplified version of the primal-dual optimization algorithm, described briefly in this paper, by choosing the smoothing functions to be \(|\|\cdot\|^2\) with a zero center point. Under the assumption that data is provided as a sparse matrix, we assess the scalability of the designed systems empirically by running them on sample tests.

1 Introduction

The following constrained convex minimization problem

\[
\min_{x} \{ f(x) : Ax = b, x \in X \}, \tag{1}
\]
captures a surprisingly broad set of problems in various disciplines, e.g., \([10, 12, 18, 21]\). \(f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}\) is a proper, closed and convex function, \(X \subseteq \mathbb{R}^n\) is a nonempty, closed and convex set, \(A \in \mathbb{R}^{m \times n}\) and \(b \in \mathbb{R}^m\). We assume that \(f\) and \(X\) are \(p\)-decomposable for \(p \geq 2\),

\[
f(x) := \sum_{i=1}^{p} f_i(x_i), \quad \text{and} \quad X := \prod_{i=1}^{p} X_i,
\]

where \(x_i \in \mathbb{R}^{n_i}\), \(X_i \in \mathbb{R}^{n_i}\), \(f_i : \mathbb{R}^{n_i} \to \mathbb{R} \cup \{+\infty\}\) is proper, closed and convex for \(i = 1, \ldots, p\), and \(\sum_{i=1}^{p} n_i = n\). This assumption is valid for many well-known concrete examples such as composite convex minimization, LASSO, groupd LASSO, basis pursuit, sparse logistic/multinomial logistic regression, signal/image recovery and and consensus optimization \([11, 13, 17, 19]\).

The Lagrange function associated with the linear constraint \(Ax - b\) is \(L(x, y) := f(x) + \langle y, Ax - b \rangle\) and the Lagrange dual problem of \((1)\) is

\[
g^* := \max \{ g(y) : y \in \mathbb{R}^m \} \quad \text{with} \quad g(y) := \inf \{ L(x, y) : f(x) + \langle y, Ax - b \rangle : x \in X \} \quad \text{\(2\)}
\]

\((1)\) and \((2)\) are referred as the primal-dual problems. By defining \(w := [x, y] \in \mathbb{R}^n \times \mathbb{R}^m\) to be the primal-dual variable, the function:

\[
G(\tilde{w}) := \max_{y \in \mathbb{R}^m} L(\tilde{x}, y) - \max_{x \in X} L(x, \tilde{y}) \equiv f(\tilde{x}) - g(\tilde{y}),
\]
is nonnegative and furthermore, because of strong duality (assuming Slater conditions holds, it is 0,i.e. \(G(w^*) = 0\), if and only if \(w^* = [x^*, y^*]\) is a primal-dual solution of \((1)\) and \((2)\). \(G\) is convex, but possibly non-smooth. For this we smooth the primal-dual functions using a Bregman distance (semi-Bregman distance) to

\[
g_\gamma(y) := \min_{x \in X} \{ L^\gamma(x, y) := f(x) + \langle Ax - b, y \rangle + \gamma d_\delta(x, x^c) \}\]

\[
f_\beta(\tilde{x}) := \max_{y \in \mathbb{R}^m} \{ L^\beta_\delta(\tilde{x}, y) := f(\tilde{x}) + \langle Ax - b, y \rangle - \beta b_\gamma(y) \}\].

and obtain an approximation of the gap function \(G_{\gamma, \beta}(\tilde{w}) \equiv f(\tilde{x}) - g_\gamma(y)\). For the choice of \(d_\delta(x, x^c) = \frac{1}{2} \sum_{i=1}^{p} \| x_i - x_i^c \|^2 \) and \(b_\gamma(y) = \frac{1}{2} \| y \|^2\), the smoothed primal has a Lipschitz constant of \(L_g := \sum_{i=1, p} \| A_i \|^2\) and the smoothed dual of 1, with \(A_i\) the matrix obtained by taking from \(A\) only the columns corresponding to \(x_i\). Furthermore we obtain a decomposable convex minimization program.

For any fixed choice of the center point \(x^c\) and using an accelerated scheme (heavy ball method), with a proper choice of the parameters \(\gamma_k\), \(\beta_k\) and \(\tau_k\) (accelerated scheme) in an ongoing work by Quoc Tran-Dinh, the sequence \(([x^k, y^k], \gamma_k, \beta_k)\) assures
an optimal convergence rate of $O(1/k^2)$ for $G_{γ_kβ_k}(x^k, y^k)$, and also a rate of convergence of the objective residual $|f(x^k) − f^*|$ and the primal feasibility gap $∥Ax^k − b∥_2$ depending on the rate of $\{γ_k, β_k\}$.

A system whose performance improves after adding more resources, proportionally to the added capacity, is said to be a scalable system. There are two broad categories of adding more resources horizontal scaling, add more nodes to a system, and vertical scaling, add resources to a single node in a system. Since the second option has a limit defined by current hardware advances, the first method is more attractive and will be the interest of this project. We can then measure scalability either by how the solution time varies with the number of nodes for a fixed total problem size, referred as strong scalability, or by how the solution time varies with the number of nodes for a fixed problem size per node, referred as weak scalability [7]. Scalability plays a more increasing role in the era of Big Data [16] and some common solutions approaches are widely accepted [8].

The goal of this project is to investigate Big Data architectures currently in use and to implement potentially scalable systems for the primal-dual optimization problem described above using this technologies and exploiting a particular assumption that $A$ is sparse. As a final result we will compare several implementations by analyzing the empirical performance on several test data sets.

The remainder of this project report is organized as follows. In the next section we provide the pseudocode of the primal-dual algorithm and how it is changed to exploit parallel distributed architectures. In section 3 we describe the technologies used and in section 4 we detail how the primal-dual algorithm was implemented in several choices. We provide the results of the tests performed in section 5 and analyze them. We end this report by drawing conclusions in section 6.

### 2 Primal-Dual Optimization

#### A1. Pseudocode for primal-dual algorithm

**Inputs:** $γ_0 > 0$

**Initialization:**

1. Set $L_g := \|A_i\|_2^2, 1 ≤ i ≤ p$
2. Set $L_g := \sum_{i=1}^{p} L_{g_i}$
3. Set $L_{\text{max}} := p \max\{L_{g_i} : 1 ≤ i ≤ p\}$
4. Set $c := 1, c := \max\{3, \bar{c}\}$
5. Set $\tau_0 := \frac{c}{c+2}$
6. Set $\beta_0 := \frac{3c^2L_g}{(c+2)^2\gamma_0}$
7. Compute $\bar{w}_0 := [\bar{x}_0, \gamma_0^0]$ by
   
   $$\bar{x}_0 := \arg\min_{x \in X} \{f_1(x) + (\bar{y}^k, A_1x_i - b_i) + γ_k+1d_s(x_i, \bar{x}^c_i)\}$$
   $$\gamma_0^0 := β_0^{-1}(Ax_0 - b)$$

For $k = 0$ to $k_{\text{max}}$

8. If stopping criterion, terminate.

9. $$\tau_k := \frac{c}{k+2}, \gamma_{k+1} := \frac{c}{k+4}$$

10. $$\bar{y}_k^h := (1 − τ_k)\bar{y}_k + τ_k\bar{y}_k^h(x_k^h).$$

($x_{k+1,i} = \arg\min_{x \in X_k} \{f_1(x) + (\bar{y}_k, A_1x_i - b_i) + γ_{k+1}d_s(x_i, \bar{x}^c_i)\}$)

11. **Do** $1 ≤ i ≤ p$ in parallel

12. **Do** $1 ≤ i ≤ p$ in parallel

13. **End For**

#### A2. Pseudocode for optimized parallel execution

**Inputs:** $γ_0 > 0$

**Initialization:**

1. Set $L_g := \|A_i\|_2^2, 1 ≤ i ≤ p$
2. Set $L_g := \sum_{i=1}^{p} L_{g_i}$
3. Set $L_{\text{max}} := p \max\{L_{g_i} : 1 ≤ i ≤ p\}$
4. Set $c := 1, c := \max\{3, \bar{c}\}$
5. Set $\tau_0 := \frac{c}{c+2}$
6. Set $\beta_0 := \frac{3c^2L_g}{(c+2)^2\gamma_0}$
7. Compute $\bar{w}_0 := [\bar{x}_0, \gamma_0^0]$ by

8. **Do** $1 ≤ i ≤ p$ in parallel

9. **End For**

For $k = 0$ to $k_{\text{max}}$

10. If stopping criterion, terminate.

11. $$\tau_k := \frac{c}{k+2}, \gamma_{k+1} := \frac{c}{k+4}$$

12. $$\bar{y}_k^h := (1 − τ_k)\bar{y}_k + τ_k\bar{y}_k^h(x_k^h).$$

($x_{k+1,i} = \arg\min_{x \in X_k} \{f_1(x) + (\bar{y}_k, A_1x_i - b_i) + γ_{k+1}d_s(x_i, \bar{x}^c_i)\}$)

13. **End For**

2
The pseudocode A1 is part of a paper draft in the LIONS group [6] and an early version is available at [20].

The \{ sign groups blocking operations and our goal is to reduce the number of synchronization points and take advantage of scheduling as much of the work in parallel non-blocking operations. By replacing $y^{k+1}_j$ with $y^k$ from $\{$ in the previous iteration, after we group terms we obtain [15]. We refer to $A^j$ to be $j^{th}$ partitioning block of $A$ by lines associated with $y_j$, i.e. if we partition into single elements, then $A^j$ refers to the $j^{th}$ row of $A$. Using linearity of the matrix multiplication, [15] is just one application of the forward matrix operator. The rest of the code is just a rearrangement of operations and grouping them conveniently. Because of the use of the previous iteration information, we must change the initialization procedure. We can use the same basics blocks of operation if we set the parameters as in A2.

The statement in [13] does not change the value of the input parameter $\gamma_0$ and only states that in [15] for $k = 0$, the value of $\gamma_0$ is not the input parameter.

The important aspect of the algorithm A2 is that it has the roughly the same complexity of one forward and a backward matrix application, plus the cost of the primal optimization problem, as the most basic primal-dual algorithms, namely the dual gradient mapping method (cite source), but the convergence guarantees are optimal. At the same time, we have 2 synchronization barriers, while the rest of the code is entirely parallelizable. We will assume that $f$ is $n$-decomposable, i.e. $p = n$, and thus $A_i$ is the $i^{th}$ column of $A$ and we choose $A^j$ to be the $j^{th}$ row.

3 Big Data Architectures/Paradigms

3.1 Hadoop Distributed Filesystem (HDFS)

HDFS is a file system optimized for storing large amounts of data, distributed across multiple machines (commodity hardware computers) grouped in a single entity, called cluster and each machine is referred as a node. It was inspired by Google distributed filesystem [15], but it is an open source implementation and because of its high availability and high throughput (at the expense of latency) it is a core component in most Big Data applications.

HDFS breaks files into small chunks/blocks, typical size is 64MB, which are stored as independent units. To be robust and fast performing (in the typical scenario of one write-multiple reads), the blocks are replicated on more than one node, typically 3, which can spread across the rack boundary. The files are delivered directly from the Datanodes that have the chunks, through the metadata kept in a NameNode. Further details of usage and detailed operations flows are available in [22].

3.2 Hadoop

Hadoop [5] is a Java open source implementation of Google MapReduce [14], but the concept of MapReduce has been generalized to a paradigm for parallel computation of data. Hadoop typically runs on top of a cluster with HDFS, which is our use case, but can also read/write from other sources than memory disk. Hadoop works with the concept of MapReduce Jobs, which can be seen as standalone applications, which are further broken into map and reduce tasks. To avoid going into unnecessary details, we can simplify by saying that each node is assigned a configurable number of map and reduce tasks, with the particularity that map tasks are assigned as close as possible to the node that has the chunk of data assigned to that particular map task. This is a concept of "bringing the computation to the data", and basically minimizes network transfer inside the cluster. If this transfer is some what controllable, the transfer of the map output to reduce tasks is hard to control, as with the exception of some functions, e.g. the key hashmap, it is much decided by Hadoop scheduling policies. In fact, this is also why Hadoop has become so popular, since the developer does not have to design this protocols and the system takes care of scheduling, transfer and assuring fault tolerance (rerunning failed tasks) or reducing job execution time (speculative execution).

Since its introduction in 2008, Hadoop has evolved to a mature version, but it is still subject to a lot of changes. The changes that had impact on this project most were the introduction of a totally new API, making a lot of resources absolute, including most published books, and the introduction of YARN as resource manager, that while still maintains compatibility with hadoop-1.x.x, the configuration files structure has changed. For development purposes I have used Hadoop 2.5.1, configured in Pseudo-Distributed Mode on a laptop with a 2.5GHz quad-core Intel Core i7 and 16GB, and the testing was supposed to be performed on a cluster with 10 nodes with 64GB and 24 cores running Hadoop 2.2.0 an using YARN. On the local version of the cluster, I have not used YARN.

An application can be designed in which Jobs are chained, such that the output of the first is used as the input of the second. The two major drawbacks are the fact that it requires a synchronization barrier, i.e. wait for the completion of the first job, and that the output of a Job is not persistent in memory and it is always written to disk (can produce significant overhead if the replication factor of HDFS is $> 1$). The final output is not the only one to be spilled to disk, as this can happen with map outputs, reducer inputs, during the shuffling phase, which to a certain extend can be controlled by setting job parameters [22] (page 181,182,277)

3.2.1 MapReduce

The MapReduce framework has a high level of abstraction, in which the programmer has to provide the implementation of two functions, the map and reduce. The input and output of this functions are pairs of user defined (key, value(s)) pairs, in which the value can be null, single or presented as a list. The key concept is that there is no communication between map processes and
neither between reduce processes (actually run as individual processes) which are executed in parallel across worker nodes in the cluster (depending on available resources and scheduling policies) and there is only one point of communication where map outputs are distributed as inputs to reduce processes. Because of this architecture, the same nodes can be scheduled to run reduce processes, after they have finished the assigned map work.

The map function basically defines the transformation $k_i, v_i \leftarrow \text{map}(s_j, p_j)$, where the function is applied subsequently to each individual input (key,value), referred as input split, from a chunk of a file, meaning we can not use any assumption on the particular order of input. The output can be stored to the local filesystem (not HDFS) or sent to the reducer as it is produced, to avoid traffic bursts. The output is grouped by key, with all the $k_i, v_i$ pairs that have the same key being sent to the same reducer. The allocation to worker nodes is done by applying normally a hashmap to the key that returns a number in the range of available nodes. This grouping and distribution process is called the shuffle phase. The reducer function basically defines the transformation $o_k, r_i \leftarrow \text{reducer}(k, \cdots, v_j, \cdots)$, where the function is applied once for each key in the output of map and all values associated with that key. This function usually is associative and commutative. To reduce communication overhead, the user can also define a combiner, which is very often the same function as the reducer, but acts on individual worker nodes (or containers in YARN), and thus processes the output of map from several chunks assigned to the same node. When using Hadoop, special care has to be taken since the combiner can be applied several times successive.

For clarity we have depicted in Fig. 1(a) a typical Job flow.

3.3 Spark

Spark [9] is a Scala open implementation, introduced recently [23], that is gaining more popularity in the realm of big data analytic since, unlike MapReduce, it has been designed to be used for iterative algorithms, that reuse the same working set of data across multiple parallel operations. Spark is a distributed in memory computational framework and for this introduces an abstract common data format that is used for efficient data sharing across parallel computation, called Resilient Distributed Datasets (RDD).

RDD is a read-only collection of objects partitioned across the nodes of the cluster that can be operated on in parallel and are initialized by either parallelizing a Scala collection or from distributed files in HDFS. There are two common types of operations on RDD: transformations, that create a new distributed dataset (or RDD) from the existing one, and actions, that return a value to the main application, running on a single node. Examples of transformation are map(func) that returns a new RDD formed by applying func to each record of the source RDD, union(otherRDD) that returns a new RDD containing the union with records from otherRDD, groupByKey() that returns the RDD of (K, Seq[V]) pairs from an RDD of (K, V) pairs, or join(otherRDD) which performs as the name implies a join on the key of the two RDDs. Typical actions are saveAsTextFile(path) that writes the elements of the RDD back into the HDFS filesystem, by calling toString on each record, collect() that return all the records of the RDD as an array at main application and also reduce(), that differs from reduceByKey() and returns the result to the main application.

By default are lazy, in the sense that partitions of the data set are materialized only when an action is required to be returned to the main application, and are not persistent, in the sense RDDs may be recomputed each time we run actions on them. This last behavior can be overwritten either by saving partitions to HDFS or by forcing caching of it, which still keeps the evaluation
lazy, but forces saving the results after first evaluation on each node. The level of caching can be a combination of memory, disk and serialization, although spilling to disk should be avoided unless partitioning is expensive, otherwise, recomputing a partition may be as fast as reading it from disk. Spark automatically persists intermediate data in shuffle operations, e.g. `reduceByKey()`, even without users calling persist. Since if a node fails during the shuffle, it is expensive to recompute. Spark is fault-tolerant, since the RDD have additional information, that allows recovery in the case of some node failures. RDDs can also be forced to be unpersisted and we can also define broadcast variables which give every node a read-only copy of the variable.

Spark works by constructing a Directed Acyclic Graph (DAG) from the transformation and actions chained by the user. Only when an action is invoked, this DAG is "evaluated" by splitting the work in stages and each stage into tasks. This tasks is referred to a lower level scheduler, in our case YARN, that schedules them based on locality or resubmits failed task. This is depicted in Figure [1(b)] where we can see that compared to `MapReduce`, which creates a DAG with two predefined stages, `map` and `reduce`, DAGs created by Spark can contain any number of stages. This allows some jobs to complete faster than they would in `MapReduce`, with simple jobs completing after just one stage, and more complex tasks completing in a single run of many stages, rather than having to be split into multiple jobs.

4 Implementation

4.1 Hadoop
4.1.1 One Job/iteration

If we refer to algorithm A2, we could prove that the dual variable \( \hat{y} \), can be eliminated, but this requires the algorithm to perform an \( A^T A x \) operation. As we assume we have access only to \( A \) and that the read access patterns are limited by the split into chunks and per input split application of the `map`, as briefly discussed in 3.2.1, it would require either to compute \( B = A^T A \) and use its entries as the method to derive the keys efficiently for the shuffle phase, or distribute the map output in a broadcast fashion (i.e. \( \{ \cdots, k_i, \cdots \}, v_i \leftarrow \text{map}(s_j, p_j) \)). In the first choice, we are not exploring the sparsity pattern of our matrix \( A \), as \( B \) has no guarantees to be sparse, and most of the times this is the case. In the second option, the bottleneck of the system will be the shuffle phase, more precisely the network transfer, making the parallel efficiency tend to 0. Without going into further details, we can conclude that one `job/iteration` is not efficient.

4.1.2 Implementation 1

For the remainder of this section we will concentrate on developing MapReduce implementations, which have two `job/iteration`. This `job` can be directly inferred from the listing of the pseudocode in A2, with `job1`, being described by [15] and `job2`, being described by [14]. This two `job` can also be seen as the application of linear algebra operations, forward operator (\( A x \)) and backward operator (\( A^T y \)), but different from the easy to code textbook material solutions from the web, we have one sensitive operation (solving the primal problem) and an increased number of variable types, that we have to track and make sure are available in full to the next `job`, within the same iteration, across iterations and during initialization. We stress that correct and efficient implementation of the forward and backward operator, and the proximal operator are crucial as this constitute the minimum complexity of any first order primal-dual method.

From the listing of pseudocode A2, it can be observed that the initialization steps have been designed such that we can use the same `jobs` as in the iterative block and appropriate parameters assure consistent results with A1.

This implementation used the idea to keep together the data (\( A, A^T, b \)) and the algorithm variables (\( x, x^+, \hat{y} \)), by constructing tuples (\( (i, j, a_{i,j}, x_j, x_j^+, y_i, b_i) \)) for \( a_{i,j} \neq 0 \). This approach exploits the sparsity of matrix \( A \) and having full information the shuffle phase can be made efficient, as we will detail below.

| MR1 - Job 1 |
|-------------|
| **Function:** `map` |
| **Inputs:** \( (i, (j, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i)) \) |
| 1: emit\( (i, (j, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i)) \) |
| **Function:** `reduce` |
| **Function:** `setup` |
| 1: Initialize from distributed cache \( \tau_k, L_g, \beta_k, \gamma_k \) |
| **Inputs:** \( (i, [\cdots, (j, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i), \cdots]) \) |
| 2: \( \hat{y}^{\text{new}} = (1 - \tau_k) \hat{y}_i + \frac{\tau_k}{\beta_k} a_{i,j} x_j \) |
| For \( j_i \) in input list |
| 3: \( \hat{y}^{\text{new}} = (1 - \tau_k) \hat{y}_i + \frac{\tau_k}{\beta_k} a_{i,j} x_j \) |
| For \( j_i \) in input list |
| 4: emit\( (i, (j_i, a_{i,j}, x_j, x_j^+, \hat{y}^{\text{new}}, b_i)) \) |

| MR1 - Job 2 |
|-------------|
| **Function:** `map` |
| **Inputs:** \( (j, (i, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i)) \) |
| 1: emit\( (i, (j, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i)) \) |
| **Function:** `reduce` |
| **Function:** `setup` |
| 1: Initialize from distributed cache \( \tau_k, L_g, \beta_k, \gamma_k \) |
| **Inputs:** \( (j, [\cdots, (i, a_{i,j}, x_j, x_j^+, \hat{y}_i, b_i), \cdots]) \) |

For \( i_t \) in input list
| 2: \( \hat{x}_j = a_{j_i,j} \hat{y}_i \) |
| 3: \( x_j^{\text{new}} = \arg \min_{x_j \in \mathbb{R}} \{ f_j(x_j) + (\hat{x}_j, x_j) + \gamma_{k+1} d_{S_j}(x_j, x_j^+) \} \) |
| 4: emit\( (i_t, (j, a_{i,j}, x_j^{\text{new}}, x_j^+, x_j^{*, \text{new}}, \hat{y}^{\text{new}}, b_i)) \) |
The pseudocode MR1, hides some of the difficulties in the actual implementation. Most important is that, non trivial at all, the values in the input list for the reduce may not all be in memory and Hadoop may be streaming them from disk. Even if in the new API we receive a Iterable and not an Iterator, we can not use cloning or iterate twice, as the list is void in the second pass, and the only acceptable solution is to cache the results in a ArrayList to reiterate later.

In an early solution presented, based on the move the computation to the data, the minimization was performed in the map, but this would result in many independent re-evaluations. Even if we constructed a custom FileInputFormat, to generate block of lines input splits, because we can not make any assumption on data ordering in the read, the worst case scenario remains the same. By moving the evaluation to the reduce, it is evaluated only once. The cost for this operation is that we need to pass $x$ in between jobs. $x^*$ is not needed to be passed, but for a reason detailed in Section 4.1.5, we have kept it.

### 4.1.3 Implementation 2

#### MR2 - Job1

**Function:** map  
**Function:** setup  
1: Initialize from distributed cache $n,p$  
**Inputs:** ($k_1, (k_2,\text{val},\text{tag})$) as: ($i, (j, a_{i,j}, \text{tag})$) | ($i, (0, b_i, \text{tag})$) | ($j, (0, x_j^*, \text{tag})$) | ($i, (0, y_i, \text{tag})$)  
2: Based on tag emit($k_1, (k_2,\text{val},\text{tag})$) $[a_{i,j}, b_i, y_i]$ or broadcast ($i, (k_1,\text{val},\text{tag})$) for $1 \leq i \leq n$ $[x_j, x_j^*]$  
**Function:** reduce  
**Function:** setup  
1: Initialize from distributed cache $\tau_k, L_g, \beta_k, \gamma_k, n,p$  
**Inputs:** ($i, \cdots (k_1,\text{val},\text{tag}), \cdots$)  
2: Sort and separate using tag  
3: $\hat{y}^{\text{new}} = (1 - \tau_k)\hat{y}_i + -(1 - \tau_k)\frac{\gamma_k}{L_g} + \frac{\tau_k}{\beta_k} b_i$  
For $k_1$ in input list  
4: $\hat{y}^{\text{new}}+ = (1 - \tau_k)\frac{\gamma_k}{L_g} a_{i,j} x_{j^*} + \frac{\tau_k}{\beta_k} a_{i,j} x_{j}$  
For $j_i$ in input list  
5: emit($i, (0, \hat{y}^{\text{new}}, \text{tag})$)

#### MR2 - Job2

**Function:** map  
**Function:** setup  
1: Initialize from distributed cache $n,p$  
**Inputs:** ($k_1, (k_2,\text{val},\text{tag})$) as: ($i, (j, a_{i,j}, \text{tag})$) | ($i, (0, y_i, \text{tag})$) | ($j, (0, x_j^*, \text{tag})$) | ($j, (0, x_j^*, \text{tag})$)  
2: Based on tag emit($k_1, (k_2,\text{val},\text{tag})$) $[x_j, x_j^*]$ or emit($k_2, (k_1,\text{val},\text{tag})$) $[a_{i,j}]$ or broadcast ($j, (k_1,\text{val},\text{tag})$) for $1 \leq i \leq p$ $[y_i]$  
**Function:** reduce  
**Function:** setup  
1: Initialize from distributed cache $\tau_k, L_g, \beta_k, \gamma_k, n,p$  
**Inputs:** ($j, \cdots (k_1,\text{val},\text{tag}), \cdots$)  
2: Sort and separate using tag  
For $k_1$ in input list  
3: $\hat{z}_j = a_{i,j} \hat{y}_i$  
For $k_1$ in input list  
4: emit($i, (j, a_{i,j}, \hat{x}_j, x_{j^*}^{\text{new}}, \hat{y}^{\text{new}}, b_i)$)  
5: $x_{j^*}^{\text{new}} := \arg \min_{x_{j^*}} \{ f_j(x_{j^*}) + (\hat{z}_j, x_j) + \gamma_{k+1} d_{S_i}(x_j, x_{j^*}) \}$  
6: $x_j^{\text{new}} := (1 - \tau_k) x_j + \tau_k x_{j^*}^{\text{new}}$  
For $i_j$ in input list  
6: emit($j, (0, x_{j^*}^{\text{new}}, \text{tag})$)  
7: emit($j, (0, x_j^{\text{new}}, \text{tag})$)

The advantage of the previous implementation is that by keeping data together with the optimization variables, in the shuffle phase the data is partitioned correctly and we do not end up with a broadcast behavior. At the same time it has the disadvantage that it requires writing the entire data and optimization variables to disk after each job. The data being considered "big", in this section we develop an algorithm that only requires reading from disk and writing to disk just the optimization variables. We cannot avoid shifting this data through the network, but if the network buffers do not spill to disk, then we are basically working "in memory". It is most probable not avoidable that the reduce does not spill to disk while waiting to receive the data from all maps, especially since a physical machine will collect data for several keys. It is important to underline that this is a operation done through the local filesystem and not the HDFS filesystem, which will perform replication across several machines.

In this version, the linear algebra operations are implemented as a "join", between the data and the optimization variables, stored in separate files, i.e., for $Ax$ the result is indexed by the row number from joining based on column the $A$ matrix, indexed by row and column number, with the $x$ vector, extended to be indexed by all row numbers and column number. The last extension is needed since we have separated the data and we need to assume that all rows may need a particular column of the optimization vector, resulting in the broadcast behavior. For the simple broadcast done in code MR2, we assume the number of columns and rows are known or can be determined in the initialization Job.

Again, for ease of readability, we did not provide the actual code design in MR2. The data and the optimization variables are kept in separate structures that allow easy retrieval, and also allow us to iterate twice (see Implementation 1 for details), with the distinction that since $A$ is sparse the most appropriate structure for the data is a HashMap, while for the optimization variables just simple vectors, if we know $n$ and $m$ (this variables are mostly dense). While we have solved one issue, we have traded network usage in the shuffle phase, since even if the data is appropriately partitioned we have to broadcast the optimization variables. We will try to address this problem in the next section.
4.1.4 Implementation 3 & 4

The need to broadcast the optimization variables comes from the fact that each record is read only by one map and the MapReduce framework does not allow communication between maps. For this we will use the DistributedCache option and distribute the optimization variables file. For details explained in the next section, we will use a Combiner (Implementation 4), which in our case is more or less a copy of the reducer and it is applied to the output of all map from several map tasks performed on the same worker node. Again, because only some chunks of a file are read by each node, we can not assume that we have the entire list of values that should normally be grouped to a single reducer and we will produce a partial result, i.e. emitting \((i,(0,y_{\text{new,partial}},tag_{\text{partial}}))\). The reducer (but also the combiner, since it can be recursively applied an indefinite number of times), if they encounter the tag_{\text{partial}} will simply need to add the value. Furthermore, once a partial result is obtained, all the values that produced it are no longer needed (i.e. we emit just one (key,value) pair containing just the partial result and discard all other (key,value) pairs received).

We have not included the code for this since it would be highly redundant, but we will refer to this implementations as MR3, using just the DistributedCache option, and MR4, when using also a combiner. The advantage over the previous versions is that network communication is decreased in the shuffle phase greatly, first by not broadcasting optimization variables and second because we no longer need to send also the data, just partial results. The trade-off is that we used DistributedCache and at first look resembles the broadcast of optimization variables. In fact, while the broadcast actually sends the same values over network for each key, using DistributedCache the files (containing the same set of values) are copied once to each worker node, which can host multiple map tasks, and are read only once in the setup procedure of the reducer (details in the next section), reducing greatly network usage for distributing optimization variables. The combiner has also the disadvantage that it blocks network transfer and each time the buffer is spilled to disk it is reapplied.

4.1.5 Implementation independent uses of Hadoop

The usual usage of MapReduce is to process text, such that the value is usually seen as "Text". In our case we would have needed to read the values as Text, convert it into String, split it by white space delimiters and then finally parse each split into its equivalent Int or Double value and inverse the process when writing back to disk. This is a costly operation and to avoid this I found two possible solutions, to either build a custom InputFormat and RecordReader or the second solution which proved more convenient, to use the already available SequenceFile classes. This classes add metadata to the files, which defines the structure of the binary data written, that the SequenceReader uses to make calls to the appropriate user-defined binary data read and write procedures (consistent) in the custom Writable classes. This use enforces that the types used for input and output match (between jobs and from map to reducer) and for this reason we need to keep the structure even if some of the values are irrelevant like \((i,(0,\hat{y}_i,\text{tag}))\) in MR2 or \(x_i^*\) in the input \((j,(i,a_{i,j},x_j,x_i^*,\hat{y}_i,b_i))\) to Job2 although it is not used.

In the new API, DistributedCache usage is marked as deprecated and is now part of the Job class. The threshold for total files size distributed is set to 10Gb, that in our case should be enough for the problems we wish to discuss (potentially limits \(n\) and \(p\)). Because the Jobs output Sequence files we needed to use a custom SequenceFile reader, when reading the distributed files in the setup procedure.

One particular discussion is that there exists a setup() procedure that is called once per map/reducer/combiner task, but the calls to the respective procedures are multiple in a single task. This is why it is a good practice is to read the DistributedCache files in this procedures.

Not detailed in this report are the Jobs that read raw data and transform it into the appropriate format for use in later Jobs and also the fact that we use the same Job for computing \(L_g\) to increase efficiency by a MapReduce counter and avoid running one more Job just to sum \(L_g\). However we trade precision, by finding an upper bound, since the counters can be only incremented by integer values in each reducer, we increment by ceil\((L_g, 10000)\) and divide the accumulated result by 10000.

4.2 Spark

I have used Scala for implementing the primal-dual optimization in Spark as this is the original language in which it was developed. The data, stored as tuples \((i,j,a_{i,j})\) or \((i,0,b_i)\) one per line, is read from HDFS into a distributed Collection, from which I construct to RDDs, one having as key the row index by mapping \((i,j,a_{i,j}) \rightarrow (i,(j,a_{i,j}))\) and the other in a similar way having as key the column index (filtering out \((i,0,b_i)\) tuples). By default, data is partitioned according to file origin, but repartitioning with a custom Partitioner based on key, we end up with partitions on each node that contain either entire values in several columns or entire values in several rows. Recomputing this partitions is costly due to the shuffle phase and we persist them with the option MEMORY_AND_DISK, which spills to disk RDD that do not fit entirely in memory. Taking as example computing \(y = Ax\), and referring to rows as the RDD partitioned by row indices, the vector \(x\) is broadcast to every node and by using the transformation mapPartitions, the list of available matrix elements in each partition is used, together with the local cached copy of \(x\), to implement the sparse multiplication. The output is the tuple \((i,y_i)\), with \(i\) being a row from the respective partition. Since this new value of \(y\) is needed in the next operation of \(A^T y\), they are collected to the main application and broadcast to be used in also the fact that we use the same

\[ x_i^{k+1} := (1 - \tau_k)x_i^k + \tau_kx_{y_{k+1}^i}(y_k^k) \]

As done in the main application after collecting the output of the RDD would block, we compute a new RDD \(x_i^{k+1}\) by reducing by key (with sum operator) the RDD computed in the previous iteration \(x_i^k\) mapped to \((1-\tau_k)x_i^k\) and the RDD \(x_{y_{k+1}^i}(y_k^k)\), obtained from
the mapPartitions applied to cols described earlier (also mapped by a scalar multiplication). This has the advantage that even this simple computation is performed in a distributed fashion. Furthermore, we set in the mapPartitions a flag to conserve partitions, which basically means that in principle there should be no shuffle in the reduceByKey() since $x^k_+$ and $x^k_{+,i,i}$ ($y^k$) should reside in the same partition (node). To collect the result in topological order (i.e. $t_1$, $t_2$, .. and not $t_{37}$, $t_{52}$, ...), we need to apply sortByKey() to the RDD before collect() and since this is done after we combine the two RDDs described earlier, it enforces our belief that there will be no shuffle.

The proximal operator is just a map transform of $(j, z_j) = \Rightarrow (i, \text{argmin}\{f_i(x_i) + \langle z_j, x_j \rangle + \gamma_{k+1} d_S(x_j, x^k_j)\})$ applied to the RDD cols.mapPartitions() which computes the $z_j$ with the meaning as in A2. Computing $L_y$ can now be seen symbolically as cols.map(\(e = \Rightarrow (e^2).\sum()\)). Unfortunately, there is also a downsides to using functional programming, and as a novice I found it really hard to use transformations efficiently that output the correct format and also allow chaining of "jobs". An unexpected behavior was that broadcast variables can not be updated by unpersisting followed by rewriting and apparently there is no workaround available for the moment (it is expected to be available in following releases of Spark).

5 Experiments

The two pseudocodes A1 and A2 were implemented in Matlab to verify that they are equivalent. The codes for MR1-4 and Spark were tested on small examples, first, to verify they provide the same result as A1. For the large test data, the output of all 5 was compared for correctness, as it is highly improbable to have all perform the same mistake because of parallelization that was not observable on the small example and because Matlab can not work with to large input on a single machine. To test the scalability, we can use a dummy prox function, still keeping the dependence on the dual variable and $\gamma$, argmin\(\{f_i(x_i) + \langle z_j, x_j \rangle + \gamma_{k+1} d_S(x_j, x^k_j)\} = z_j + \gamma_{k+1}$. The codes are run with the parameters id path $\gamma$ in n, where it is the number of iterations, id is a simulation identifier, path is the input HDFS folder that contains the files describing A and b. Because of a "bug" caused by running the new API 2.5.1 on a cluster with Hadoop 2.2.0 (described better here [3]), solved by [2], we added also for the Hadoop jobs two parameters corresponding to "mapreduce.job.maps" and "mapreduce.job.reducers" in the new API.

For testing we have generated sparse random matrices with the attributes summarized in Table 1. This are text files with each line containing one $(i,j,a_{ij})$ or $(0, b_i)$ tuple. We can observe that the nonzero elements are distributed uniformly in the matrix. Real data might be more skewed, but this would possibly lead us to the wrong conclusions about the scalability of our system, simply because the whole system would stall because of a task that processes more data. In this project we have not studied this particular case.

| Data set | \(m\) | \(n\) | min($A_{ij}$) | mean($A_{ij}$) | max($A_{ij}$) | min($A_{ij}^+$) | mean($A_{ij}^+$) | max($A_{ij}^+$) | nz$\text{nz}(A)$ | Total size |
|----------|--------|--------|--------------|----------------|------------|----------------|----------------|----------------|----------------|-------------|
| D1       | $10^6$ | $10^4$ | 1            | 10             | 29         | 876           | 1000           | 1119           | 10^7          | 232 MB      |
| D2       | $2 \times 10^6$ | $10^4$ | 1            | 10             | 29         | 1824          | 2000           | 2174           | $2 \times 10^7$ | 475.7 MB    |
| D3       | $10^6$ | $5 \times 10^4$ | 20           | 50             | 86         | 872           | 1000           | 1140           | $5 \times 10^7$ | 1.13 GB     |
| D4       | $2 \times 10^6$ | $5 \times 10^4$ | 21           | 50             | 86         | 1783          | 2000           | 2196           | $10^8$        | 2.32 GB     |
| D5       | $2 \times 10^6$ | $10^4$ | 55           | 100            | 152        | 1819          | 2000           | 2188           | $2^5$         | 4.62 GB     |
| D6       | $10^6$ | $5 \times 10^4$ | 18           | 59             | 92         | 9599          | 10000          | 10425          | $5^5$         | 11.8 GB     |

Table 1: Data sets attributes in order number of rows,columns, minimum, mean and maximum number of elements per row and column, number of nonzero elements, size on disk

To be able to visualize the results and analyze performance we have run the simulations with 2 iterations, such that we can define Stage 1 - reading the matrix and computing $L_y$, Stage 2 - initialize $x^0$ and $x^*_{n_0}$, Stage 3 - compute $y^0$, Stage 4 - compute $x^1$, $x^*_t$, Stage 5 - compute $\bar{y}^1$, Stage 6 - compute $\bar{x}^2$, $\bar{x}^*_{t_2}$ and output $\bar{x}^2$.

MR1 and MR2, both use a form of emitting pairs in a broadcast fashion (for MR1 when grouping data) which make them very impracticable to use. Furthermore MR1 will rewrite the entire data at each stage. We have defined the number of maps and reducers both to be 100, but this values are treated by Hadoop as guidelines, such that usually the number of map tasks equals the number of input splits of the input files. In this sense, Hadoop is a bit unpredictable when comparing it to Spark, that reserves a fixed number of resources.

Contrary to expectations the use of the Combiner does not help. By further inspecting run time statistics we observe that in stages 3 and 5, the output of the combiner is almost the same in number as the map output and this can be explained by the fact that in the input files describing A we have subsequent elements from the same column and thus the combiner simply performs one multiplication in place and outputs it as a partial value,i.e. does not aggregate several partial results. At the same time the cost of reading the optimization variable, even if it is from the local filesystem, adds a great overhead. However this partitioning of A is advantageous for stages 4 and 6. This can also be observed by comparing Table 2 with Table 3 where each stage 3 and 5 in MR4 lasts longer than each similar stage in MR3 and vice versa for stages 4 and 6. In a future implementation we would not use a combiner in stages 3 and 5.

While for MR3-4 the stages correspond to Jobs, for Spark the representation is symbolic since the actual organization of the stages is controlled by the DAG Scheduler. However, we can define stages by the points in which the RDD are collected to the main application. Table 5 measures strong scalability, but the increase is not linear, as expected for any program that is not
entirely parallel. The results is more easily seen in Figure 2(a). In Table 4, we can observe that the system is parallel efficient, and with the increase in data the useful work per data item also scales. The most costly stage is Stage 3, where the rows RDD is first evaluated and cached and since the data in files is grouped by columns, to repartition it by row we need to shuffle a lot of tuples, resulting in the high cost. It should not be attributed to the fact that $m > n$, since the data in both RDD is the same, as can be observed in Stage 5, where we work with the in memory values. Already in data set D5 we were occupying almost 90% of the cluster memory and for data set D6 Stage 3 fails, regardless of the solution we tried (changing the number of partitions, not caching or different levels of caching). Tracking the error we have discovered that in fact the OS has an upper limit on the number of files that can simultaneously be opened and during the shuffle phase, due to the high number of keys, Spark riches this limit. This can be changed, but at the same time makes us believe that adding more nodes (more memory) this problem can be avoided. Not caching would be a solution, but since we adopted a solution of iterating threw the list of values of each partition (not functional programming style) to implement the sparse multiplication, Spark can not reconstruct easily parts of the partition without recomputing it entirely.

In Figure 2(b) we aggregate the results from Table 2, 3 and 4 and we observe that all the solutions scale at a similar rate and while the Spark implementation is the fastest, it can not work with extremely large datasets. On the other hand the Hadoop implementations can process large sets, but we observe that the increase already tends to saturate.

6 Conclusions

In this project we have studied some of the most widely used architectures for Big Data, Hadoop and Spark, and developed several implementations exploiting, the advantages of each. We have implemented a simplified version of the primal-dual optimization algorithm, described briefly in this paper, by choosing the smoothing functions to be $\| \cdot \|_2$ with a zero center point. Under the assumption that data is provided as a sparse matrix, we have assessed the scalability of the designed systems empirically by running them on sample tests. While Spark is very well suited for iterative algorithms and performs best in times of running time, the "in memory" model has the disadvantage that it can not easily accommodate large sets, at least not in the implementation provided. On the other hand, the Hadoop implementations of MapReduce are more suited for processing large datasets at
Execution time as a number of worker nodes for a fixed dataset using Spark

Comparison of execution time as a function of data size for MR3, MR4, and Spark

the expense of running time. In terms of scalability, we can conclude that indeed both implementations are parallel efficient. MapReduce offers a higher level of abstraction, making it a simpler choice, but does not offer the flexibility of the functional programming model used in Spark.

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