Massiveness of Glueballs as Origin of the OZI Rule

Wei-Shu Hou and Cheng-Ying Ko
Department of Physics, National Taiwan University
Taipei, Taiwan 10764, R.O.C.

Abstract

The heaviness of the glueball mass scale is suggested as the source of the OZI rule at low energy. The $J/\psi \rightarrow \rho \pi$ decay “anomaly” implies the vector glueball $O$ has mass $m_O \approx m_{J/\psi}$. Such a heavy mass is supported by other glueball studies. Glueball-meson matrix elements turn out to be not suppressed at all at the 1 GeV scale, and a simple and intuitive picture emerges which is consistent with the Gell-Mann–Okubo mass formula as well as the measured sign of $\phi$-$\omega$ mixing. The suppression of glueball mediated $\bar{q}_i q_i \leftrightarrow \bar{q}_j q_j$ transitions and the cancellation mechanism in two-step meson rescatterings are viewed as related by duality. Extensions to the $2^{++}$, $3^{--}$ meson sectors, and failure for $0^{\pm +}$ mesons are also discussed.
1. Introduction

When applying the Gell-Mann–Okubo mass formula to vector mesons, it was found that the physical $m_\omega$ differed significantly from $m_{\omega}$, which turned out to be near ideal. To explain why $\phi \to 3\pi$ is so suppressed as compared to $\phi \to K\bar{K}$ and $\omega \to 3\pi$, Okubo [2], Zweig [3], and Iizuka [4] independently suggested that strong processes in which the final states can only be reached through $q$ and $\bar{q}$ annihilations (disconnected quark lines) are suppressed.

The decay rate of $\phi \to 3\pi$ is not zero, hence the OZI rule is not exact, but since this decay rate is small, the violation of the OZI rule is soft. The purpose of this paper is to try to give a dynamical explanation of the OZI rule and its violation. The dynamical sources of OZI violation are usually classified as from two origins: (1) $q\bar{q}$ annihilations into multigluon intermediate states, and (2) unitarity correction effects involving (physical or virtual) rescattering of $q\bar{q}$ hadronic states. At $\omega$, $\phi$ energies, the relative importance of these two sources is still an open question.

At $J/\psi$ energy scale, the dynamical origin of OZI violation is understood as a manifestation [5] of asymptotic freedom (class (1)), which allows us to calculate OZI forbidden transition rates perturbatively. The question is how this picture can be extended [6, 7] down to the $\omega$ and $\phi$ scale. On the other hand, as championed by Lipkin [8], if one takes into account processes involving hadronic intermediate states (class (2)), e.g. $\phi \to K\bar{K} \to \rho\pi$ [9] or $\psi \to D\bar{D} \to \rho\pi$, OZI violation still needs to be understood. In these two-step transitions, each step is OZI allowed so the transition amplitude may not be small. It is suggested that cancellations [10] between different contributions may result from conditions imposed by SU(3) flavor symmetry, nonet symmetry and exchange degeneracy [8]. Following Lipkin’s discussions, Geiger and Isgur [11] have calculated these two-step transition amplitudes explicitly, using closure and spectator approximations and the $3P_0$ $q\bar{q}$ pair creation model. The results confirm that the cancellation mechanism indeed happens in detail.

The results after cancellation of two-step transitions could still dominate the transition amplitude. But this would not be predictive because of its complexity (some times up to $10^4$ terms [11]), nor does it constitute a fundamental explanation of the OZI rule. In this paper we assume that the cancellation is exact and consider gluonic intermediate states only, in particular emphasizing the role of gluonic bound states, i.e. glueballs. This will lead to a simpler and more intuitive picture for the dynamical origin of the OZI rule and its violation. We shall use vector mesons as the prime example, then extend to $2^{++}$ and $3^{--}$ nonets.
2. Vector Glueball Mass and Charmonium Decay

The discovery of \( J/\psi \) revived interests in the OZI rule in the 1970’s. In 1975, Freund and Nambu (FN) \(^{12}\) suggested that the breaking of the OZI rule in vector meson decays could be understood as due to the mixing of \( \omega, \phi, \) and \( J/\psi \) mesons with a new SU(4)-singlet meson \( O \), viewed as a “Pomeron daughter”. Denoting the dimension two \( O-V \) (\( V = \omega, \phi, J/\psi \)) transition amplitude as \( f_{OV} \), one has

\[
f_{O\psi} = f_{O\phi} = \frac{1}{\sqrt{2}} f_{O\omega} \equiv f. \tag{1}
\]

FN used dual dynamics to predict that \( m_O \sim 1.4-1.8 \text{ GeV} \). Taking \( f \) in Eq. (1) to be constant, their approach failed to predict \( J/\psi \rightarrow \rho \pi \) decay correctly.

By 1982, the so-called \( J/\psi \) vs. \( \psi' \) decay anomaly appeared \(^{13}\). Normally, one expects \( J/\psi, \psi' \rightarrow 3g \rightarrow X \) to differ only in the charmonium wave function at the origin, hence the ratio of branching ratios is expected to follow the so-called 15\% rule,

\[
\frac{\mathcal{B}(\psi' \rightarrow X)}{\mathcal{B}(J/\psi \rightarrow X)} \simeq \frac{\mathcal{B}(\psi' \rightarrow e^+e^-)}{\mathcal{B}(J/\psi \rightarrow e^+e^-)} \simeq 15\% \tag{2}
\]

which appears to hold for general \( X \). However, although \( \rho \pi \) and \( K^*\bar{K} \) decays are quite prominent (\( \sim 1\% \)) for \( J/\psi \), they were not seen in \( \psi' \) decay at all \(^{13}\). To explain this anomaly, Hou and Soni (HS) \(^{14}\) took \( f = f(q^2) \) and proposed a resonance enhancement model, viewing \( O \) as the lowest lying vector glueball. Assuming: i) \( J/\psi \rightarrow O \rightarrow \rho \pi \gg J/\psi \rightarrow ggg \rightarrow \rho \pi \), ii) \( J/\psi \rightarrow O \rightarrow \text{other} \ll J/\psi \rightarrow ggg \rightarrow \text{other} \), iii) \( \psi' \rightarrow O \rightarrow \text{ANY} \ll \psi' \rightarrow ggg \rightarrow \text{ANY} \), where \( ggg \) stands for three-gluon continuum states, one finds

\[
\frac{\Gamma(\psi' \rightarrow O \rightarrow \rho \pi)}{\Gamma(J/\psi \rightarrow O \rightarrow \rho \pi)} \simeq \left( \frac{m_{\psi'}^2 - m_O^2}{m_{\psi'}^2 - m_O^2} \right)^2 \frac{f_{O\psi}^2}{f_{O\psi'}^2}, \tag{3}
\]

where the energy denominator is the main enhancement factor for \( J/\psi \). Hence, as the anomaly deepened in 1986, the only way out was to have \( m_O \simeq m_{J/\psi} \), as pointed out by Brodsky, Lepage and Tuan (BLT) \(^{15}\). These authors also stressed that the \( VP \) modes should otherwise be suppressed by hadronic helicity conservation.

By 1996, the “15\% rule” has been confirmed for \(^{16}\) \( pp, p\bar{p} + n\pi, 5\pi, 7\pi, b_1\pi \) (AP) and \( \phi f_0 \) (VS) modes. However, the \( J/\psi \) anomaly persists for \( VP \) modes,

\[
\mathcal{B}(\psi' \rightarrow \rho \pi) < 2.9 \times 10^{-5}, \quad \mathcal{B}(\psi' \rightarrow K^{*+}K^-) < 3.2 \times 10^{-5}, \tag{4}
\]

while a similar situation starts to emerge for \( VT \) modes such as \( w f_2, \rho \omega_2 \) and \( K^*\bar{K}_2 \). Concurrently, however, the degeneracy of \( m_O \simeq m_{J/\psi} \) was challenged \(^{17}\) by a BES energy scan.
of $J/\psi \to \rho \pi$, which showed no sign of $O$ in the vicinity of $J/\psi$. It turned out, however, that there was an analysis fault in Ref. [17]. Upon closer scrutiny, it was found [18] that $m_O \simeq m_{J/\psi}$ is not ruled out by the energy scan so long that $\Gamma_O \gg \Gamma_{J/\psi}$, which should be the case. In fact, $O$ could hide even more easily in the radiative tail above the $J/\psi$ peak. Taking $m_O \simeq 3200$ MeV for example, one gets rather plausibly [18]

$$4\text{MeV} \lesssim \Gamma \lesssim 30\text{ MeV}, \quad \text{and} \quad \text{few} \% \lesssim B(O \to \rho \pi) \lesssim 25\%,$$

in contrast to the implicit need for $O \to \rho \pi$ dominance in the original HS model [14]. One also obtains the mixing parameters

$$f(m^2_{J/\psi}) \simeq 0.02\text{ GeV}^2, \quad \sin \theta_{O\psi} \simeq 0.03.$$  

The smallness of the $O-J/\psi$ transition strength $f(m^2_{J/\psi})$ conforms well with asymptotic freedom since it is $\propto \alpha_s^{3/2}$. Because $f(m^2_{J/\psi})$ is so small, one has a small mixing angle $\sin \theta_{O\psi}$ despite the proximity of $O$ and $J/\psi$ masses. This implies that the $J/\psi$ mass shift due to mixing with $O$ is negligible (at the sub-MeV level).

The above phenomenological arguments suggest an $m_O$ value which is much larger than the prediction of FN, and turns out to be fortuitously [18] close to $m_{J/\psi}$. Do we have other evidence to support this? After all, we have yet to establish any glueball state beyond doubt. We note, however, that recent experimental and lattice studies are converging [19] on $0^{++}$ and $2^{++}$ glueballs. In the $0^{++}$ case, there is an excess of isoscalar mesons: $f_0(1370)$, $f_0(1500)$ and $f_J(1710)$. Together with the $I = 1/2$ and 1 mesons $K^*_0(1430)$ and $a_0(1450)$, they cannot all fit into a $q\bar{q}$ nonet. Recent lattice calculations in the quenched approximation predict the $0^{++}$ glueball mass to be $1600 \pm 100$ MeV [19], which can fit either $f_0(1500)$ [20] or $f_J(1710)$ [21] as the $0^{++}$ scalar glueball $G$ while the other is dominantly $s\bar{s}$. It is likely that both states have large glueball admixtures. The situation seems cleaner though less established in the $2^{++}$ case. The even $^{++}$ state $\xi(2230)$ is the glueball candidate [22], which is very close in mass to the lattice expectation of $2400 \pm 120$ MeV [23]. All these states are seen in $J/\psi \to \gamma + X$ transitions but not seen in $\gamma\gamma$ production [24].

What does this have to do with the heaviness of $O$? Note that $0^{++}$ and $2^{++}$ quantum numbers can be constructed from two gluons but the $1^{--}$ quantum number demands three gluons. The $0^{++}$ quantum number is shared by the QCD vacuum hence is more complicated, but we could rather naively scale from the $2^{++}$ glueball to the 3-gluon case, which suggests $m_O$ to be in the ballpark of $m_{J/\psi}$. Indeed, the original HS model [14] was motivated by $m_O$ expectations from a constituent gluon picture [25]. The model predicts $m_{1^{--}}/m_{2^{++}} \simeq 1.5$. 

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and assuming the lattice/experimental result on $\xi(2230)$, it offers strong support for $m_O \sim m_{J/\psi}$. Similar result of $m_{3^-}/m_{2^+} \simeq 1.5$ is obtained for the 3-gluon 3$^-$ state. Lattice studies of the 1$^-$ glueball are unfortunately scarce and inconclusive, but it does turn out to be rather massive. In the following, we shall take the glueball masses to be $m_{0^+} = 1600$, 2230 MeV and $m_{1^-} \simeq m_{3^-} = 3200$ MeV respectively. Note that, compared to 15 years ago, these masses are 30–40% heavier.

3. Vector Glueball and OZI Dynamics

To understand the OZI rule in the glueball mediated picture, we need to have better understanding of the strength of the transition amplitude $f$ at scales much lower than the $m_{J/\psi}^2$ scale of Eq. (5). In the ideal mixing basis (limit of exact OZI rule), usually defined as

$$\phi_0 \equiv s\bar{s}, \quad \omega_0 \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \equiv n\bar{n},$$

the transition amplitude $f$ of Eq. (1) induces the mixing

$$\phi = \cos \delta \phi_0 + \sin \delta \omega_0$$
$$\omega = -\sin \delta \phi_0 + \cos \delta \omega_0,$$

where the angle $\delta$ is the (OZI violating) deviation from ideal mixing. To see how $\delta$ relates to $f$, note that Eq. (8) can be derived from the mass-squared matrix

$$M^2 = \begin{pmatrix} m_{\phi_0}^2 & T(\phi_0 \to \omega_0) \\ T(\omega_0 \to \phi_0) & m_{\omega_0}^2 \end{pmatrix}.$$

where the relation between $\delta$ and the off-diagonal transition amplitude $T(\phi_0 \to \omega_0)$ is

$$\tan \delta = \frac{T(\phi_0 \to \omega_0)}{m_{\phi_0}^2 - m_{\omega_0}^2}.$$  

The transition amplitude $T(\phi_0 \to \omega_0)$ is expanded by a complete set of gluonic states $|x\rangle$,

$$T(\phi_0 \to \omega_0) = \sum_x \frac{\langle \omega_0|H_{int}|x\rangle\langle x|H_{int}|\phi_0\rangle}{m_{\phi_0}^2 - m_{\omega_0}^2}.$$  

The matrix elements in the numerator, related to the $f_{OV}$ defined earlier, are in principle calculable. The $O$ dominance picture amounts to saturating the sum by $O$, namely $T(\phi_0 \to \omega_0) \simeq \sqrt{2}f^2(m_{\phi_0}^2)/(m_{\phi_0}^2 - m_{\omega_0}^2)$ where

$$f(m_{\phi_0}^2) = \langle O|H_{int}|q\bar{q}\rangle \bigg|_{q^2=m_{\phi_0}^2}. $$

4
O dominance can be partially understood by the fact that \( O, \omega \) and \( \phi \) are all lowest lying states, hence the matrix elements involving excited glue states are suppressed by both the numerator and the denominator. We finally get

\[
\tan \delta \simeq \frac{\sqrt{2} f^2(m_\phi^2)}{(m_\phi^2 - m_\omega^2)(m_\phi^2 - m_O^2)},
\]

where we have approximated \( m_{\phi_0}, m_{\omega_0} \) by \( m_\phi, m_\omega \).

From Eq. (13) and analogous relations for other mesons, we can determine \( f(m_{\text{had.}}^2) \) from meson and glueball masses and \( \delta_{\text{expt.}} \). The latter is calculated by using the quadratic Gell-Mann–Okubo mass formula, i.e. \( \delta_{\text{expt.}} = \delta_{\text{GMO}} \), which gives results in good agreement with those extracted from decay data [24, 29]. The results are given in Table 1. The distinction between \( \delta_{\text{GMO}} = \delta_{n\bar{n}} \) and \( \delta_{s\bar{s}} \) is explained in the next section. We have not presented the results for \( 0^-+ \) mesons since \( \eta-\eta' \) mixing is related to the famous \( U_A(1) \) anomaly, and does not follow the present formalism. We took \( s\bar{s} \) to be the \( f_J(1710) \) state for the \( 0^{++} \) entry as illustration, but the case is highly uncertain, and we do not pursue it further here.

| \( J^{PC} \) | \( n\bar{n} \) | \( s\bar{s} \) | Glueball | \( \delta_{n\bar{n}} \) | \( \delta_{s\bar{s}} \) | \( f(m_{\text{had.}}^2) \) (GeV^2) |
|---|---|---|---|---|---|---|
| 0^-+ | — | — | — | — | — | — |
| 0^{++} | \( f_0(1370) \) | \( f_J(1710) \) | \( G(1600) \) | 33° | — | \( \simeq 0.4 \) |
| 1^- | \( \omega(782) \) | \( \phi(1020) \) | \( O(3200) \) | 4° | —4° | \( \simeq 0.4 \) |
| 2^{++} | \( f_2(1270) \) | \( f_J'(1525) \) | \( \xi(2200) \) | —5° | 7° | \( \simeq 0.4 \) |
| 3^- | \( \omega_3(1670) \) | \( \phi_3(1850) \) | \( O_3(3200) \) | —3° | 3° | \( \simeq 0.4 \) |

Table 1: Values of mixing angle \( \delta \) from Gell-Mann–Okubo mass formula and glueball-quarkonium mixing strength \( f \) from Eq. (13). For \( 0^{++} \) and the distinction of \( \delta_{n\bar{n}} \) and \( \delta_{s\bar{s}} \) see discussion in text.

The \( \delta \) values for \( 1^-, 2^{++}, 3^- \) mesons are clearly different from \( 0^{++} \), all exhibiting near-ideal mixing. Quite remarkably, they all (including \( 0^{++} \)) give

\[
f(m_{\text{had.}}^2) \sim (0.6 \text{ GeV})^2, \tag{14}
\]

consistently, where \( m_{\text{had.}} \sim 0.8–1.7 \) GeV. Eq. (14) should not come as a surprise since, as seen from Eq. (12), \( f(m_{\text{had.}}^2) \) is a hadronic matrix element evaluated at normal hadronic
scales. Thus, in strong contrast to the smallness of \( f(m_{J/\psi}^2) \) in Eq. (3) at \( m_{J/\psi}^2 \) scale, \( f(m_{\text{had.}}^2) \) is not suppressed for the \( 1^- \), \( 2^{++} \), \( 3^{--} \) nonets. From this we infer that the smallness of OZI violation for these nonets is not due to the smallness of the glueball mixing strength, but because of the heaviness of the lowest glueball state that mediates the \( n\bar{n}-s\bar{s} \) mixing. Thus, it is the second factor in the denominator of Eq. (13) rather than the transition matrix elements in the numerator that controls the violation of the OZI rule. Even for the 2-gluon \( 2^{++} \) glueball, its mass scale is considerably above the \( f_2-f'_2 \) mixing scale. In contrast, because of the proximity of glueball and meson mass scales, as seen from Table 1, the OZI rule is badly broken in the \( 0^{++} \) sector. We illustrate our scenario pictorially in Fig. 1.

4. Sign of \( \phi-\omega \) Mixing

Although the glueball mediation scenario could intuitively and simply explain the OZI rule and its violation, there is one potential difficulty that has to be faced. Note that so far we have been cavalier with the sign of \( \delta \) in Eq. (13). Twenty years ago, Arafune, Fukugita and Oyanagi (AFO) [30] pointed out the importance of this sign, which seems to imply that the dominant contribution to OZI rule violation comes from \( SU_F(3) \) octet intermediate states (rescattering!) rather than singlet states such as \( O \). This threatens the foundation of the glueball mediation picture. Let us investigate this problem.

Defining the octet and singlet states in the usual way as [24]

\[
\begin{align*}
\omega_8 &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \\
\omega_1 &= \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}),
\end{align*}
\]  

(15)

then, in the convention of Eq. (7), we have

\[
\begin{align*}
\phi_0 &= \sqrt{\frac{1}{3}} \omega_1 - \sqrt{\frac{2}{3}} \omega_8, \\
\omega_0 &= \sqrt{\frac{2}{3}} \omega_1 + \sqrt{\frac{1}{3}} \omega_8.
\end{align*}
\]  

(16)

Since \( O \) couples only to the singlet component of \( \phi_0 \) and \( \omega_0 \), we have \( \langle \omega_0 | H_{\text{int}} | O \rangle \langle O | H_{\text{int}} | \phi_0 \rangle > 0 \). Together with \( m_O > m_{\phi_0} \), the glueball dominance model predicts that \( \tan \delta < 0 \) [30]. This contradicts the result from the Gell-Mann–Okubo mass formula listed under \( \delta_{n\bar{n}} \) in Table 1, as well as [31] the direct experimental probe of the sign of the mixing angle \( \delta \) via the interference between \( \phi \) and \( \omega \) in the \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) process. The observation of constructive interference in the energy domain of \( m_\omega < E_{cm} < m_\phi \) implies \( \tan \delta > 0 \).

A seemingly trivial way out is to change the convention from Eq. (7) to

\[
\begin{align*}
\phi_0 &\equiv -s\bar{s}, \\
\omega_0 &\equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),
\end{align*}
\]  

(17)
hence \( \langle \omega_0 | H_{\text{int}} | O \rangle \langle O | H_{\text{int}} | \phi_0 \rangle < 0 \) which naïvely leads to \( \tan \delta > 0 \). Things are not so simple, however, since instead of Eq. (8), one now has
\[
\phi = \cos \delta \phi_0 - \sin \delta \omega_0, \\
\omega = \sin \delta \phi_0 + \cos \delta \omega_0.
\] (18)

Following the same steps as Eqs. (7–11), one again runs into a sign problem. Since physics should be convention independent, we should view the implications of the \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \) experiment from this light. Treating \( \sin \delta \) as a perturbation, the \( \omega \) channel proceeds via \( \gamma^* \rightarrow \omega_0 \) and \( \omega_0 \rightarrow \pi^+ \pi^- \pi^0 \), whereas the \( \phi \) channel proceeds via \( \gamma^* \rightarrow \phi_0 \) and \( \omega_0 \rightarrow \pi^+ \pi^- \pi^0 \). Since the \( \gamma^- \omega_0 \) and \( \gamma^-\phi_0 \) couplings have opposite sign, and since the \( \omega, \phi \) propagators are also of opposite sign for \( m_\omega < E_{\text{cm}} < m_\phi \), constructive interference implies that, regardless of conventions for \( \phi_0 \) and \( \omega_0 \), the relative sign of the projection of \( \phi \) onto \( \phi_0 \) and \( \omega_0 \) is positive. This illustrates the problem one has with Eq. (18).

The solution to our problem lies in some interesting subtleties which are usually overlooked in casual applications of the GMO formula, namely, scale dependence and the value of \( m_{\pi 11}^2 \). To be consistent, we now adopt the conventions of the Particle Data Group (PDG). In terms of the octet and singlet states of Eq. (15), the physical states are defined as
\[
\omega = \cos \theta V_{\omega 1} \omega_1 + \sin \theta V_{\omega 8} \omega_8, \\
\phi = -\sin \theta V_{\omega 1} \omega_1 + \cos \theta V_{\omega 8} \omega_8.
\] (19)

Ideal mixing is defined as \( \theta_{\text{ideal}} = \sin^{-1}(1/\sqrt{3}) \approx 35.3^\circ \), that is
\[
\phi_0 = -\sqrt{\frac{1}{3}} \omega_1 + \sqrt{\frac{2}{3}} \omega_8, \quad \omega_0 = \sqrt{\frac{2}{3}} \omega_1 + \sqrt{\frac{1}{3}} \omega_8.
\] (20)

Thus, the PDG convention is in fact that of Eq. (17) rather than Eq. (7), i.e. \( \phi_0 \equiv -s\bar{s} \).

Eq. (19) supposedly diagonalizes the singlet-octet mass-squared matrix, or
\[
\begin{pmatrix}
  m_{11}^2 & m_{18}^2 \\
m_{81}^2 & m_{88}^2
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
m_\omega^2 & 0 \\
0 & m_\phi^2
\end{pmatrix}
\begin{pmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{pmatrix}
\]
\[
= \begin{pmatrix}
  \cos^2 \theta m_\omega^2 + \sin^2 \theta m_\phi^2 & \sin \theta \cos \theta (m_\omega^2 - m_\phi^2) \\
  \sin \theta \cos \theta (m_\omega^2 - m_\phi^2) & \sin^2 \theta m_\omega^2 + \cos^2 \theta m_\phi^2
\end{pmatrix}.
\] (21)

As is clear from Eq. (19), the physical \( \omega \) state descends from \( \omega_1 \) after mixing in \( \omega_8 \) at the \( \sin \theta_\omega \) level. Thus, the 88 element of Eq. (21) gives the standard formula
\[
\tan^2 \theta_\omega = \frac{m_\phi^2 - m_{88}^2}{m_{88}^2 - m_\omega^2}.
\] (22)
From the GMO formula $m^2_{88} = (4m^2_K - m^2_\rho)/3 \simeq 0.87 \text{ GeV}^2$, we find $\theta_\omega \simeq 39^\circ$, leading to $\delta_\omega \simeq +4^\circ$ listed in Table 1, hence $\tan \delta > 0$. But if one takes the convenient, usual assumption of a single rotation angle, one gets $\phi = \cos \delta \phi_0 - \sin \delta \omega_0$ from Eq. (18), which contradicts the $\omega$-$\phi$ interference experiment as mentioned earlier. We stress, however, that from the general point of view of QCD, Eq. (21) should be evaluated at a given scale. Since there are two different physical scales to the problem, namely $m_\omega$ and $m_\phi$, there is no reason why there should be just one mixing angle [32]. The two physical states $\omega$ and $\phi$ can have different “$\phi$-$\omega$ mixing angles” without violating orthogonality.

From the quark content of Eq. (15), the quark model gives (in the SU(2) limit)

$$m^2_{88} = m_0 + \frac{2}{3}(m_u + 2m_s), \quad m^2_{11} = m'_0 + \frac{2}{3}(2m_u + m_s).$$

(23)

Since they characterize different SU(3) multiplets, $m_0$ and $m'_0$ are usually treated as in principle different. However, since U(3) or nonet symmetry is more apparent for vector mesons, as reflected in $m_\omega \simeq m_\rho$, we take $m'_0 = m_0$. This conforms with flavor independence of QCD and is more reasonable than assuming the sum rule or trace relation $m^2_\phi + m^2_\omega = m^2_{88} + m^2_{11}$, i.e. equal mixing angles for $\phi$ and $\omega$. We therefore get a GMO-like formula

$$m^2_{11} = \frac{1}{3}(2m^2_K + m^2_\rho) \simeq 0.73 \text{ GeV}^2,$$

(24)

which is smaller than the $m^2_{11} \simeq 0.78 \text{ GeV}^2$ value from assuming equal mixing angles. Eq. (21) now gives a second rotation angle

$$\tan^2 \theta_\phi = \frac{m^2_{11} - m^2_\omega}{m^2_\phi - m^2_{11}},$$

(25)

leading to $\theta_\phi \simeq 31^\circ$, hence $\delta_\phi \simeq -4^\circ$, which is listed in Table 1 under $\delta_{ss}$. Thus, we see from Eq. (18) that the relative sign of the projection of $\phi$ onto $\phi_0$ and $\omega_0$ is now positive, which conforms with the interference experiment. Repeating the steps of Eqs. (11), one finds

$$\tan \delta_\phi = -T(\phi_0 \to \omega_0)/(m^2_{\phi_0} - m^2_{\omega_0}),$$

which is consistent with $\delta_\phi < 0$ for $\phi_0 = -s\bar{s}$ [33]. We have therefore constructed a completely consistent picture for OZI violation.

The physical states $\omega$ and $\phi$ as defined by Eqs. (22) and (25) are plotted in Fig. 2. Although they appear nonorthogonal, they are in fact orthogonal when evolved to the same scale [33]. Applying the same formalism to $2^{++}$ and $3^{--}$ mesons, we obtain the respective entries in Table 1, and the statevectors are also plotted in Fig. 2. Note that the $\delta$ angles for $2^{++}$ and $3^{--}$ are opposite those of the $1^{--}$ case. The result for $2^{++}$ is also supported by experimental data. For example, in the $\pi + N \to K^+K^- + N$ reaction which probes the
\[ \pi^+\pi^- \rightarrow f_2, \ f'_2, \ a_0^0 \rightarrow K^+K^- \] reactions, one observes destructive interference near the \( f'_2 \) mass over a broad \( f_2 \) and \( a_0^0 \) background \[34\]. Since the effect is seen at the \( m_{f_2}^2 \) scale, one should use \( \delta_{f_2} \simeq +7^\circ \) for both \( f'_2 \) and "\( f_2 \)" in an equation analogous to Eq. (18). Destructive interference follows naturally.

We conclude that the sign of \( \phi-\omega \) mixing, as pointed out by AFO \[30\], is indeed an important physical parameter. But contrary to the assertion of AFO, we find that our proposed glueball mediation scenario for OZI violation in 1\(^--\), 2\(^++\) and 3\(^--\) meson mixings is in harmony with both the GMO mass formulas and direct experimental probes.

5. Discussion and Conclusion

This work started with the observation in Ref. \[18\] that, while \( f(m_\phi^2) \sim 0.5 \) GeV\(^2\) as inferred from \( \phi \rightarrow \rho \pi \) decay is not suppressed at all, \( f(m_\phi^2)/(m_\phi^2-m_\omega^2) \) is very close to \( \sin \delta_{\omega\phi} \). This is now understood in terms of Eq. (13) since it turns out that \( f(m_\phi^2) \sim m_\phi^2 - m_\omega^2 \). We have extended the observation to 2\(^++\) and 3\(^--\) mesons. The empirical result of \( f(m_{\text{had.}}^2) \sim (600 \) MeV\(^2\)) is rather reasonable, since it is nothing but a strong interaction matrix element of mass-squared dimensions measured at normal hadronic scales (Eq. (12)).

We note from Fig. 2 that the angle between \( \phi \) and \( \omega \) is superficially less than 90\(^\circ\) while for 2\(^++\) and 3\(^--\) the angles are superficially larger than 90\(^\circ\). It is not clear what is the actual cause of this, but it is probably correlated with the fact that \( m_\omega > m_\rho \) while \( m_{f_2} < m_{a_2} \) and \( m_{\omega_3} < m_{\rho_3} \). The latter case of \( m_{n\bar{n}}(I = 0) < m_{n\bar{n}}(I = 1) \) seems more reasonable from the point of view of level repulsion induced by quantum mixing. Perhaps \( m_\omega > m_\rho \) is due to mesonic rescattering effects (class 2), but the results obtained by Geiger and Isgur with their central parameter values give also the wrong sign \[11\]. Related to this, note also that \( m_\phi^2 + m_\omega^2 > m_{\omega_8}^2 + m_{\omega_1}^2 \), while the situation is opposite for 2\(^++\) and 3\(^--\). This should be compared to the 0\(^--\) case where \( m_{\eta}^2 + m_{\eta'}^2 \) is considerably larger than \( m_{\eta_8}^2 + m_{\eta_1}^2 \), usually considered as a sign of the large glue content of \( \eta-\eta' \) mesons caused by the \( U_A(1) \) anomaly.

Since 1\(^--\) and 3\(^--\) are composed of three gluons while 2\(^++\) is made of two, one may ask why \( f(m_{f_2}^2) \) is not much larger than the other cases. We observe that the 2\(^++\) \( q\bar{q} \) mesons are \( P \)-wave, while the glueball is \( S \)-wave. Thus, besides the heaviness of the corresponding glueball as seen in Fig. 1, configurational mismatch may be part of the cause for near ideal mixing in the 2\(^++\) sector. Then why is the 3\(^--\) mesons not even closer to ideal mixing since the \( q\bar{q} \) mesons are \( D \)-wave? The answer is in part that they are indeed so. We further remark that the 3\(^--\) \( S \)-wave glueball can be viewed as composed of a gluon pair with total spin 2.
coupled to an additional spin 1 gluon. In terms of spin content, this matches onto the $q\bar{q}$ meson’s $L = 2$ and $S = 1$.

One implication of near-ideal mixing of $1^{--}$, $2^{++}$ and $3^{--}$ mesons is that the corresponding glueballs $O$, $\xi$ and $O_3$ are relatively clean, that is, they have only small admixtures of $q\bar{q}$ (including $c\bar{c}$) in them. Their decays therefore proceed differently than $q\bar{q}$ mesons, as reflected in the $\xi(2230)$. The indirectly inferred $O$ width of Eq. (3) is less than the already narrow $\xi$ width. This narrowness reflects the fact that the lowest lying glueball cannot decay via glueball channels, and their decay to $q\bar{q}$ final states are OZI (or $\sqrt{\text{OZI}}$?) suppressed. The “cleaness” of these glueballs would make their identification much easier, once they are seen. In comparison, the $0^{++}$ glueball $G$ mixes strongly with the neighboring $n\bar{n}$ and $s\bar{s}$ states, and will take much effort to establish its identity.

Finally, we think that the OZI suppression due to high glueball mass scale is at the root of the cancellation mechanism studied by Lipkin and by Geiger and Isgur. In short, the two results are related by duality. The annihilation via all possible gluonic states (class 1) and the rescattering via all possible (quark) hadronic states (class 2) are dual to each other, much like the equivalence between parton level inclusive cross sections and a complete set of allowed hadronic states. If in QCD OZI suppression comes about because of the heaviness of the glueball mass scale (single channel dominance is not necessary) as compared to the $q\bar{q}$ meson mass scale, such OZI suppression should then be automatically and strenuously maintained in terms of hadronic intermediate states. It is therefore both remarkable and understandable, then, that the cancellation mechanism fails to be operative precisely when the glueball mass scale is lowest, namely the $0^{++}$ sector.

In summary, we argue that the OZI rule for normal hadrons such as $\phi$ and $\omega$ is due to the heaviness of the mediating glueball mass scale rather than suppressed transition matrix elements, unlike the case for $J/\psi$ decay. Glueballs turn out to be rather heavy in QCD, but otherwise the effect is quantum mechanical. This seems to resolve a long standing riddle: why is the OZI rule operative at the 1 GeV scale? We find that it is necessary to treat the two physical nonet isoscalar mesons as having different mixing angles. If we treat the $m_q$-independent contributions to meson masses as $U(3)$ invariant rather than $SU(3)$ invariant, a simple, intuitive and consistent picture emerges for glueball dominance of OZI violation.

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Figure 1: Mass-squared spectrum. Solid lines denote mesons listed in the PDG 1996 booklet, dashed lines are glueball candidate states that needs further experimental confirmation, and dotdash line for $O$ stands for the $m_O$ value used in text. The shaded boxes are the expected glueball mass ranges from lattice or other estimates. The glueball states are considerably heavier than the isoscalar mesons, except for the $0^{++}$ case.
Figure 2: Physical isosinglet states vs. octet-singlet or ideally mixed states for (a) $1^{--}$, (b) $2^{++}$ and (c) $3^{--}$ mesons. The physical $\phi$ state would be on the wrong side of $\phi_0 = -s\bar{s}$ if one assumes equal rotation angles $\sin\theta_\phi = \sin\theta_\omega$. 