A dynamical probe of superfluidity in one-dimension:
The adiabatic quantum pump

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Abstract. Quantum pumping as a dynamical probe of the superfluid response of a one-dimensional (1D) quantum fluid is discussed. It is shown that a spatially periodic potential, oscillating adiabatically in time with frequency $\omega_0$, acts as a quantum pump inducing a continuous momentum current from broken spatiotemporal symmetries of the driven potential. The momentum current generated by the pump is strongly affected by the interactions. It has a power-law dependence on the frequency and the temperature with the exponent determined by the interaction. It depends on the phase difference between two umklapp terms of the drive, providing indication for the effect of quantum phase slips on the decay of superflow. Application of the results in understanding the superfluid properties of helium confined in nanometer-size pores or of laser-cooled atoms is briefly discussed.

1. Introduction
Dynamical effects and quantum transport phenomena are fundamental problems in physics. An intriguing example is the quantum pumping, where in the absence of a biased force a directed current of particles is produced along a periodic structure[1]. An adiabatic quantum pump is a device that generates a dc current by a periodic slow variation of some system characteristic, the variation being slow enough so that the system remains close to its ground state throughout the pumping cycle. From a fundamental physics standpoint, this mechanism represents a macroscopic quantum phenomenon reminiscent of superconductivity where current flows without dissipation. While quantum pumping has been mainly studied in solid state systems, as in quantum dots[2, 3], here we discuss quantum pumping in 1D quantum fluids as a dynamical probe of its superfluid response. Our pump is made of a spatially periodic potential, oscillating adiabatically in time with frequency $\omega_0$ on a one-dimensional quantum fluid described by a Tomonaga-Luttinger liquid. It induces a continuous momentum current from broken spatiotemporal symmetries of the driven potential which indicates the effect of quantum phase slips on the degree of dissipation of the superflow. The zero temperature analysis recently appeared[4] while the present study at finite temperature can be relevant in understanding the superfluid properties of helium confined in nanometer-size pores or of laser-cooled atoms in confined potentials.

2. The model and the pump
The proposed pump consists of a 1D quantum fluid exposed to a spatially periodic optical lattice potential oscillating wave like with frequency $\omega_0$ and momentum $q_0$. The low-energy properties
of the fluid are described by a Tomonaga-Luttinger liquid, and the pumping is carried out at small \( \omega_0 \), staying this way in the neighborhood of the Luttinger liquid fixed point. The low-temperature and frequency degrees of freedom of the system are described by two collective canonical conjugate fields, \( \theta(x, t) \) and \( \partial_x \phi(x, t)/\pi \), which describe the phase and the density fluctuations, respectively [5, 6] with Hamiltonian:

\[
H = \frac{\hbar}{2\pi} \int dx \left[ \frac{v_s}{K} (\nabla \phi(x))^2 + \frac{v_s K}{\hbar^2} (\pi \Pi(x))^2 \right].
\]

(\( \hbar = 1 \) hereafter). This Hamiltonian is a standard sound wave one in which the fluctuations of the phase \( \phi(x) \) represent the phonon modes of the density wave

\[
\rho(x) = \left[ \rho_0 - \frac{1}{\pi} \nabla \phi(x) \right] \sum_{p = -\infty}^{\infty} e^{2i(p\rho_0 x - \phi(x))},
\]

\( \rho_0 \) being the average particle density and \( v_s \) is the sound velocity. The Luttinger parameters \( v_s \) and \( K \) used in (1) are related by the compressibility of the fluid \( K/(\hbar^2 v_s \rho_0^2) \).

When the interaction goes to zero, \( K \) goes to infinity, while \( K = 1 \) for infinitely strong hard-core interactions (Tonks-Girardeau limit)[7]. The situation \( K < 1 \) can be instead realized in Bose-gases with long-range interactions, e.g. in dipolar gases.

An external potential \( V(x) \) which is periodic (with period \( l \) and period wavevector \( Q = \frac{2\pi}{l} \)) acting on a length \( L \) of the system is considered. Such potential couples to the density (2) and leading terms of such coupling are of the form: \( H_{n,m} \sim g_{n,m} \int dx e^{ik_{n,m} x} e^{i2n\phi(x)} \), the so-called umklapp terms, where \( k_{n,m} = n2\pi \rho_0 - mQ \) represents the momentum transfer. A commensurability between the boson density and the imposed periodicity implies \( k_{n,m} = 0 \) for some \( n, m \). Thus the smallest \( k_{n,m} \) provides us with a measure of the incommensurability between the 1D fluid density and the external potential. We then allow the external periodic potential to oscillate with frequency \( \omega_0 \) and propagate with characteristic momenta \( \{ q \} \) centered around a value \( q_0 \), \( V(x) \rightarrow V(x, t) = \sum_q A_q \cos(\omega_0 t - qx) V(x) \), i.e. it acts as a pump \( H^{ext}(t) = H^{pump} \). Close to the Luttinger fixed point, the umklapp terms, which are irrelevant operators in the renormalization group sense, will have time and phase dependent coupling constant of the form \( g_{n,m}(t) = g_{n,m} e^{i\omega_0 t - \varphi_{n,m}} \). The momenta \( \{ q \} \) in the driving potential break the reflection symmetry and this is reflected by the umklapp phases \( \varphi_{n,m} \) that for weak periodic potential are given by \( \varphi_{n,m} \sim nq_0/\omega_0 \). When reflection symmetry is present \( \varphi_{n,m} = 0 \) and no momentum current will be generated in the system.

The bosonized version of the pump Hamiltonian reads:

\[
H^{pump} = \sum_{n,m} H_{n,m}(t) = \sum_{n,m} g_{n,m} \frac{2n}{(2\pi \rho_0)^2} \int dx \left[ e^{i(\omega_0 t - \varphi_{n,m})} e^{ik_{n,m} x} e^{i2n\phi(x)} + h.c. \right],
\]

where the sum runs over \( n, m \) integers (\( m \) is the order of commensurability, e.g. \( m = 1 \) corresponds to one particle per site). Let us note that such irrelevant terms describe the effect of quantum phase slips responsible for the decay of the superflow. We wish to study the effect of the oscillating terms on the momentum current operator that in bosonized version is given by \( M_j = M \frac{\hbar}{2} \partial_t \phi = \Pi(x) \), where \( j(x, t) \) is the particle current operator. In the following, we shall consider the oscillating lattice as a perturbation around the Luttinger liquid fixed point and compute the current perturbatively, by the Keldysh technique[8]. Explicit calculations show that to get a continuous current out of the pumping potential at least two umklapp operators with a nonzero phase difference are required[9].

By using the bosonized expression of \( H^{pump} \), the pumped current to second order can be
calculated as:

\[ j^{\text{pump}}(\omega_0, T) = K v_s \sum_{n,n',m,m'} n_-(\rho_0 L)^{-n^2 \frac{K}{k_{n+,-m+}} I(\omega, k_{n+,-m+}, T) \frac{\sin(k_{n+,-m+} L)}{k_{n+,-m+}} \] (4)

where \( k_{n+,-m+} = \frac{(k_{n,m} \pm k_{n',m'})}{2} \), \( n_{\pm} = n \pm n' \) and similarly for \( m_{\pm} \), and,

\[ A_{n_{\pm},m_{\pm}} = \frac{g_{n,m}}{(2\pi \rho_0^{-1})^2 (2\pi \rho_0^{-1})^2} \sin \varphi_{n,m} \] (5)

is the area enclosed in a pumping cycle by the periodic parameters \( g_{n,m}(t) \) and \( g_{n',m'}(t) \), \( \varphi_{n,m} \) is the phase difference between two umklapp operators.

Using the finite temperature expression for the correlation functions of the boson operators[6] the expression for \( I(\omega_0, k_{n+,-m+}, T) \) reads:

\[ I(\omega_0, k_{n+,-m+}, T) = S\text{gn}(\omega_0) \sinh(\frac{\omega_0}{\pi T}) \] (6)

where \( s_{\pm} = (\frac{\omega_0 \pm v_s k_{n+,-m+}}{2T}) \); \( B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y) \) is the Euler beta function; \( K^{n',m'} = n'K \).

When we consider incommensurate fillings, \( k_{n+,-m+} \neq 0 \), two interesting regimes occur, \( T \ll \omega_0 \), or \( T \gg \omega_0 \). In the first case, we get the zero temperature result, i.e.

\[ I^{T=0}(\omega_0, k_{n+,-m+}) \simeq S\text{gn}(\omega_0 \sinh(\frac{\omega_0}{\pi T})) \Gamma^2(1 - K^{n',m'}) \left( \frac{\rho_0^{-1}}{2v_s} \right)^{2K^{n',m'} - 2} \frac{2K^{n',m'} - 2}{\omega_0} \frac{|\omega_0| - |k_{n+,-m+}|}{\omega_0} e^{-\frac{v_s k_{n+,-m+}}{2T}} \sinh(\frac{\omega_0}{\pi T}) \] (8)

For \( T \gg \omega_0 \) and \( \omega_0 \) not too small compared to \( v_s k_{n+,-m+} \) one has:

\[ I(\omega_0, k_{n+,-m+}, T) \simeq \sin^2(\pi K^{n',m'}) \Gamma^2(1 - K^{n',m'}) \left( \frac{\rho_0^{-1}}{2v_s} \right)^{2K^{n',m'} - 2} \frac{2K^{n',m'} - 2}{\omega_0} \frac{|\omega_0| - |k_{n+,-m+}|}{\omega_0} e^{-\frac{v_s k_{n+,-m+}}{2T}} \sinh(\frac{\omega_0}{\pi T}) \] (8)

where the exponential factor describes the suppression of processes involving momentum transfer \( k_{n+,-m+} \). When \( \omega_0 \to 0 \) at low-temperature the exponential factor in (8) prevails and the processes with the smallest \( k_{n+,-m+} \) are favored, i.e. the pumped current is suppressed and thus the system supports a superflow. At some commensurate point \( k_{n_0,m_0} \sim 0 \) and temperature not too low, we have to balance algebraic and exponential suppression in (8) and in the limit \( \omega_0 \ll T \), the dominant contribution to the d.c. current will be proportional to \( \left( \frac{2\pi \rho_0^{-1}}{v_s} \right)^{2K_{n_0,m_0}} \frac{2K_{n_0,m_0} - 1}{\pi} S\text{gn}(\omega_0) \).

By taking the first commensurate point and the Tonks- Girardeau limit (hard-core bosons) where \( K = 1 \), one recovers the same behavior as that of a non-interacting fermion gas \( I \simeq \omega_0^2 \). With interactions present, the current behaves as a power-law of the temperature with an exponent depending on the interactions, indicating a strong renormalization of the scattering process due to phase slips. The Fig. 1 (left panel) shows the low-frequency behavior of the pumped
Figure 1. (Color online) Low frequency behavior of the pumped current in arbitrary units having taken into account umklapp terms $g_{1,0}, g_{20}, g_{21}$. Left panel: The dashed line is for $T = 0.07$, the straight (black) line for $T=0.1$ and the straight (red) for $T=0.2$, the Luttinger parameter is $K = 1.5$, the system length (in units of $1/k_{20}$) is $L = 1.; \omega_0$ is in units of $v_s k_{20}$. Right panel: The straight line is for $K = 1.1$, the dashed (red) line for $K=1.4$ and the dashed (black) for $K=1.8$, the system length is $L = 1.$ and $\omega_0 = 0.2$, data in this panel have been magnified by a factor $10^2$.

momentum current (PMC) at finite temperature $T$, taking into account few umklapp terms. The pumped current is an increasing function of $T$ and has a characteristic power-law behavior (interaction dependent) at very small $\omega_0$. In the right panel, the PMC is plotted as a function of $T$ for different values of the Luttinger parameter. The figure clearly shows an onset temperature below which the pumped current is a constant, indicating the dissipative contribution from phase-slips. As $K$ increases the pumped current is decreases and this is in agreement with the expectation that decreasing the interaction tends to increase the superfluid response of the system. Let us note that the PMC is never zero, unless $\omega_0$ is zero, and the constant values goes to zero only for $L \to \infty$ at increasing $K$. Thus superfluidity could be observable in the dynamical sense in 1d for sufficiently long systems. A similar conclusion was drawn recently[10] by the analysis of the dynamical momentum response of a Tomonaga-Luttinger liquid subject to a periodic potential. Experiments of torsional oscillator in nanometer-sized pores filled with $^4$He[11] offer a way to detect the dynamical aspects of superfluidity. In conclusion, we have shown that a quantum pump made of a periodic potential oscillating with a finite frequency on a one dimensional quantum fluid could be used to probe the superfluid behavior at finite temperature.

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