Order and Creep in Flux Lattices and CDWs Pinned by Planar Defects

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The influence of randomly distributed point impurities and planar defects on the order and transport in type-II superconductors and related systems is considered theoretically. For random planar defects of identical orientation the flux line lattice exhibits a new glassy phase with diverging shear and tilt modulus, a transverse Meissner effect, large sample to sample fluctuations of the susceptibility and an exponential decay of translational long range order. The flux creep resistivity for currents \( J \) parallel to the defects is \( \rho(J) \sim \exp(-J_0/J)^\mu \) with \( \mu = 3/2 \). Strong disorder enforces an array of dislocations to relax shear strain.

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Introduction. Type-II superconductors can be penetrated by an external magnetic field in the form of quantized magnetic flux lines (FL). Under the influence of a transport current \( J \) FLs will move and hence give rise to dissipation. The resulting linear resistivity is proportional to the magnetic induction \( B \) [1]. To stabilize superconductivity it is therefore essential to pin FLs. One source of pinning is point disorder. In high-\( T_c \) materials point disorder is practically always existing because of the non-stoichiometric composition of most materials. Then the system regains superconductivity in the sense of the non-stoichiometric composition of most materials. After the system regains superconductivity in the sense of the non-stoichiometric composition of most materials.

An even more pronounced effect can be expected from planar defects like twin planes or grain boundaries, which will be considered in the present paper. Twins are ubiquitous in superconducting YBCO and La\(_2\)CuO\(_4\) where they are needed to accommodate strains arising from tetragonal to rhombic transformations. But also other causes are possible (see Fig.1). Planar defects occur frequently in families with the same orientation but random distances [1, 8] or in orthogonal families of lamella (“colonies”) [9]. The mean distance \( \ell_D \) of the defect planes is of the order of 10 nm [7] to \( \mu m \) [10]. Pinning of individual FLs by planar defects has been investigated in the past both for clean and disordered systems [2, 11, 12]. Recently it was shown that, depending on the mutual orientation of the FL lattice (FLL) and the defects, dilute planar defects are indeed a relevant perturbation even in the presence of point disorder [12], provided they are parallel to the main lattice planes of the FLL. In systems with parallel defect planes this is the generic situation since the FLL will rotate in such a position to reach maximum overlap with the defects (provided B is aligned with the defect planes). It turns out that the Bragg and the Bose

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barriers leading to a new creep law with $\mu = 3/2$, large sample to sample fluctuations of the magnetic susceptibility, an exponential suppression of translation order in the direction perpendicular to the defects, a resistance against shear deformations as well as the occurrence of a transverse Meissner effect. If only displacements perpendicular to the defects are considered, as in the main part of this paper, our results apply also to a wide class of systems which exhibit regular lattices of domain walls like magnetic slabs, charge density waves \cite{13} and incommensurate systems \cite{13}.

Model. We consider an Abrikosov FLL in the presence of randomly distributed point impurities and random defect planes, aligned with the magnetic field. Since in both types of imperfections superconductivity is suppressed they will attract FLLs. Then the Hamiltonian reads \cite{10}

$$
\mathcal{H} = \int d^d r \left\{ \frac{1}{2} \sum_{\alpha \beta \gamma \delta} c_{\alpha \beta \gamma \delta} (\partial_\alpha u_\beta) (\partial_\gamma u_\delta) + \sum_\alpha c_{44}^{(\alpha)} (\partial_z u_\alpha)^2 + 2 [V_p (r) + V_D (r)] \rho (u, r) \right\}
$$

where $\alpha, \beta, \gamma, \delta$ run over $x, y, u, \rho$ denotes the FL displacement. Only components of the elastic constants $c_{\alpha \beta \gamma \delta}$ with pairwise equal indices are non-zero \cite{17}. $\rho(u, r) = \rho_0 \left\{ -\nabla \cdot u + \sum G e^{G(r \cdot u)} \right\} \equiv \rho_s + \rho_p$ is the FLL density with $\rho_0 = B_0 / \phi_0$, $\phi_0$ is the flux quantum. $G$ is a reciprocal lattice vector of the FLL and $r_\perp = (x, y)$. $V_p (r)$ denotes the pinning potential resulting from randomly distributed point impurities. We will first consider the (realistic) case that all defect planes have the same orientation but random distances \cite{7}. Then the FLL will orient itself such that its main lattice planes will be parallel to the planar defects to allow for their maximal overlap \cite{13}. The defect pinning potentials then has the form $V_p (r) = -v_D \sum_\delta \delta (x - x_\delta)$ \cite{2} where we assumed that the defect planes are parallel to the $y$-$z$ plane. The $\delta$-functions are considered to have a finite width of the order of the superconductor coherence length $\xi_c$. A rough estimate for the defect strength is given by $v_D \approx H_i^2 \xi_c^2$, $H_i$ is the thermodynamic critical field. The statistical properties of the pinning energies are then encoded in their pair correlation functions $R_P (u)$ and $R_D (u)$ for point disorder and planar defects, respectively. Since the FLL density includes a slowly varying and a periodic part, $\rho_s$ and $\rho_p$, respectively, we decompose the pinning energy density accordingly. From the periodic part we get $R_D (u_x) = (v_D \rho_0)^2 / L^2 \sum_{n \neq 0} e^{i 2 \pi n u_x / \ell}$, $n$ is integer \cite{18}, $\ell \ll \ell_D$, where $\ell$ and $\ell_D$ are the mean spacing between the FLLs and the defect planes, respectively. The contributions from $\rho_p$ do not contribute to the glassy properties of the system since they can be eliminated by a simple transformation \cite{10}.

Since our main concern are the defect planes, it seems to be justified to start with a simplified model in which only the displacements $u_x \equiv u$ of the FLLs perpendicular to the defect planes are considered. Then only the elastic terms with the coefficients $c_{xxxx} \equiv c_{11}$, $c_{xyxy} \equiv c_{66}$ and $c_{44}^{(u)} \equiv c_{44}$ remain in the Hamiltonian. From a technical point of view it is convenient to consider a generalization of our model in $d$ dimensions by replacing $x$ by a $(d-2)$-dimensional vector $x$.

**Weak disorder.** In the absence of defect planes point impurities are relevant in less than 4 dimensions. The FLL exhibits a phase with quasi long range order: the Bragg glass \cite{3,4,5}, which exhibits a power law decay of $S_G (r) = \langle e^{i G (u_r - u_D)} \rangle \sim |r|^{-(d-4)}$. The Fourier transform of $S_G (r)$ is the structure factor which has Bragg peaks.

It was recently shown in \cite{13} that dilute planar defects can be a relevant perturbation also in the presence of point disorder. Indeed, distorting the initially ordered FLL in volume $L^{d-2} L_y L_z$, the energy gain is of the order $- (R''_{\mu} (0) L^{d-2} L_y L_z$ whereas the elastic energy loss is $c_{11} L_z L_y L^{d-4}$ since distortions are aligned parallel to the defects. For $L \gg L_D \sim c_{11}^2 / R''_{\mu} (0))^{(d-6)/4}$ the pinning energy gain wins and the FLL starts to disorder in the directions perpendicular to the defects. The critical dimension below which weak planar defects are relevant is $d = 6$.

For a more detailed study we use now a functional renormalization group approach in $d = 6 - \epsilon$ dimensions. We follow closely a related approach for columnar disorder \cite{21,22} but keep the unrescaled quantities which correspond to the effective parameters measured on scale $L$. To lowest order the flow equations for $\epsilon \ll 1$ read

$$
d \ln c_{1i} / d \ln L = 2 R''_{\mu} (0) L^2 / (4 \pi c_{11})^2, \quad i = 4, 6 \label{eq:1}
d \ln R_D (u) / d \ln L = R''_D (u) L^2 (R''_D (u) - 2 R''_D (0)) / (4 \pi c_{11})^2. \label{eq:2}
$$

Thermal fluctuations and point disorder are irrelevant for $\epsilon < 4$ and $\epsilon < 2$, respectively. There is no renormalization of $c_{11}$ because of a statistical tilt symmetry \cite{22}. For $L \rightarrow L_D$, many metastable states appear and $R''_D (0)$ develops a slope discontinuity at the origin which results in diverging elastic constants $c_{44}$ and $c_{66}$. The renormalization can however be continued to $L \gg L_D$ if one imposes a small but finite tilt of the FLL such that $R''(0)$ has to be replaced by $R''(0^+)$ in Eq. \cite{2}. In this case $c_{44}$ and $c_{66}$ remain finite but new terms of the form $\int_0^{2 \pi} \! d \phi \left\{ \sum_{\alpha} \cos \phi (\partial_\alpha u) + \sum_{\alpha} \sin \phi (\partial_\alpha u) / 4 \right\}$ are generated in the energy density which dominate the energy for small $u$. The fixed point function $R''_D (u, L) L^2 = (2 \pi c_{11})^2 \epsilon \left[ \frac{c_{11}}{3} - \frac{1}{4} (u - \ell)^2 \right]$ for $0 \leq u < \ell$ is periodic in $u$ with period $\ell$. The newly generated terms renormalize according to

$$
c_{66}^{-1/2} d \ln L \approx c_{44}^{-1/2} d \Sigma_z / d \ln L \approx \epsilon \sqrt{c_{11}^2 / 12} \ell. \label{eq:3}
$$

$\Sigma_z (y)$ has the meaning of a interface tension of a domain wall parallel to the $x$ and $y$ axes. $\Sigma_z$ can be measured by changing the external magnetic field by $H_x \delta \dot{u}$ which changes the Hamiltonian by $-(B_0 / 4 \pi) \int d^d r H_x \delta \dot{u}$. To
tilt the flux lines with respect to the $z$-axis, $H_x$ has to overcome the interface energy $\sim \Sigma_z^4$ which results in a threshold field $H_{x,c} = 8\pi\Sigma_2\ell/(\phi_0_\sqrt{3})$ below which FLs remain locked parallel to the planes. This is the transverse Meissner effect: a weak transverse magnetic field $H_x$ is screened from the sample. In this case $c_{44}$ is infinite! Only for $H_x > H_{x,c}$ the average tilt of the FLs becomes non-zero and $c_{44}$ stays finite. Moreover, there is a resistance against shear of the FLL: the shear deformation $\partial_\theta u_x$ is non zero (and $c_{66}$ finite) only if the shear stress $\sigma_{xy}$ is larger than a critical value $\Sigma_y/\ell$, otherwise $c_{66}$ is infinite. The divergence of $c_{66}$ is a new property which does not exist in Bose glass.

An infinitesimal change $\delta H, \delta x$ in the longitudinal field allows to measure the longitudinal susceptibility $\chi = B_n/\delta(\partial_\theta u_x)$ which is independent of the disorder as a result of the statistical tilt symmetry. The glassy properties of the systems can most easily be seen by the sample to sample fluctuations of the magnetic susceptibility $\chi^2 - \overline{\chi^2}$. Perturbation theory gives $\langle (\overline{\chi^2}) \rangle / \chi^2 = R_{uu}(0)\ell^*/(5c_{11}^2) \sim (L_\delta/L_D)^s$, i.e. the sample to sample fluctuations of the susceptibility grow with the the scale $L \lesssim L_D$, $d < 6$ which is a signature of a glassy phase.

The structural correlations in this phase are obtained in the standard way from $R_{uu}(u,L)$ which gives $S_G(x,y,z) \sim |x|^{-6-d}$. In $d \leq 4$ dimensions also the part of the pinning potential related to $\rho_\theta$ becomes relevant which gives the dominating contribution to the FL displacements. Both, a Flory argument and more detailed calculations for a related one dimensional problem give in $d = 3$ dimensions $S_G(x,y,z) \sim e^{-|x|/L_D}$. In the related study Villain and Fernandez found from a non-perturbative RG that for $d < 4$ the disorder renormalizes to strong coupling. We will show below that this case gives qualitatively the same results.

To get more information about a real 3-dimensional system we consider next the stability of this glassy phase with respect to point disorder by using an Imry-Ma argument. The energy gain from the point disorder in a system we consider next the stability of this glassy phase below that this case gives qualitatively the same results. Here we have taken into account that the elastic energy and the energy from the disorder scale in the same way. The saddle point $L_y/\Sigma_y = L_z/\Sigma_x \sim \ell^2/c_{11} \sim f^{-1}$ gives for the non-linear resistivity in $d = 3$ $(J \ll J_D$)

$$\rho(J) \sim e^{-(J/f)^{3/2}}, \quad J_D = C(\Sigma_1/\ell)^{3/2}c_{11}/BT^{1/3}.$$  

Thus the non-linear resistivity is reduced considerably with respect to the case of point impurities. A similar consideration for the Bose glass gives $\mu = 1$ which is, as far as we are aware, also a new result.

To summarize the results obtained so far we remark that the new phase described here is characterized by (i) diverging elastic constants $c_{44}$ and $c_{66}$ but a finite compressibility $c_{11}$, (ii) a transverse Meissner effect as well as a resistance against shear deformation, (iii) large sample to sample fluctuations of the susceptibility, (iv) an exponential decay of the structural correlations (in $d = 3$) and (v) a creep exponent $\mu = 3/2$. Since the totality of these properties is different from the Bragg glass or the Bose glass, we will call this new phase a planar glass. This phase is also different from that found for equally spaced defects which is incompressible.

### Strong disorder

If the disorder is strong, i.e. if $L_D \lesssim \ell_D$, (we ignore for the moment the point disorder) each defect will be completely overlapped by a FLL plane to gain its full energy. Integrating out the displacement field between two adjacent defect planes we get in $d = 3$

$$H/L_y = \sum_{i=1}^N \left\{ \frac{c_{11}}{2} \left( \frac{u_{i+1} - u_i}{\Delta x_{i+1}} - \rho_0 v_D \sum_n e^{iG_D n(x_i-u_i)} \right) \right\}$$

where $G_D = 2\pi/\ell, \Delta x_{i+1} = x_{i+1} - x_i$ and the sum over $i$ is over the defect planes. For $v_D \to \infty$ we have $x_i - u_i = \ell n_i$ with $n_i$ integer to minimize the pinning potential. Minimizing subsequently the elastic energy allows the exact determination of the ground state $u_i^0 = x_i - \ell \sum_{j=1}^i |\Delta x_j/\ell|$, where $\lfloor x \rfloor$ denotes the closest integer to $x$. For $\ell_D \gg \ell$, $S_G(\mathbf{r})$ is again decaying exponentially in the $x$-direction on scale $\ell_D$. Considering flux creep due to a driving force $\mathbf{f}$ perpendicular to the defect planes in $d = 3$ we obtain then the same form of the non-linear resistivity Eq. (5) as in the case of weak disorder. This formula applies for small currents where droplets cover many planar defects. Thus both weak and strong disorder give the same results for the correlations and the flux creep.

### Displacement parallel to the defects, dislocations

Next we include displacements $u_y$ parallel to the defects. In the case of strong disorder each defect is occupied by
a single FL layer and hence $u_x(x_i, y, z; n_i) = x_i - \ell n_i$, \forall y, z, to maximize the pinning energy gain. Even without point disorder we obtain then a non-zero displacement $u_y$. This can be seen most easily in the isotropic case where $\sigma \partial_x u_x = -\partial_y u_y$, here $\sigma = (c_{11} - c_{66})/(c_{11} + c_{66})$ is the Poisson number, $0 < \sigma < 1$ \cite{24}. The strain $\partial_x u_x$ in the segment between the defects at $x_{i+1}$ and $x_i$ is $\partial_x u_x \approx 1 - \ell \Delta n_{i+1}/\Delta x_{i+1}$ where $\Delta n_{i+1} = (n_{i+1} - n_i)$. The difference of the strain $\partial_y u_y$ in neighboring segments is then $\Delta \partial_y u_y \approx \sigma \ell (\Delta n_{i+1}/\Delta x_{i+1} - \Delta n_{i}/\Delta x_i)$ which is of the order $\pm \sigma \ell / L_D$. On the scale $L_y$ this implies $\Delta u_y \sim \pm \sigma \ell L_y / L_D$. To avoid a diverging shear energy one has to allow for dislocations with Burgers vector parallel to the $y$-direction sitting at the defects. Their distance in the $y$-direction is of the order $L_D / \sigma$. Comparing the energy of an edge dislocation piercing the crystal to the energy gain from the disorder we find that dislocations will be present if $\sigma \ell_6 \ell^2 \xi_c \ll \ell D / \sigma$. In general, the network of dislocations is likely to be valid for all current directions in the $xy$-plane. Since the Burgers vector of the dislocations is always parallel to the defects, creep in the $\pm y$-direction is of the order $\ell / \sigma P$. The creep law Eq. \ref{Eq:creep}. To describe creep parallel to the defects one has to take into account the interaction between the dislocation, a situation not considered so far \cite{30}. We leave this case for further studies. In the case of weak pinning qualitatively the same behavior can be expected on scales $L_x \gg L_D$, in particular if the flow is again to the strong coupling fixed point. If the samples exhibits orthogonal families of (non-intersecting) defects, long range order in the $xy$-plane is destroyed even without point disorder on scales larger than $L_D$. The creep is now limited by the slowest mechanism and hence Eq. \ref{Eq:creep} is likely to be valid for all current directions in the $xy$-plane.

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