Quantum Control with Measurements and Quantum Zeno Dynamics

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We introduce an efficient iterative method to prepare a target state in Hilbert spaces with high dimensionality using a combination of unitary evolution, measurements and quantum Zeno dynamics. The latter confines the evolution within Zeno subspaces of decreasing size. This gives an exponential speed up relative to the case of states evolving in the full Hilbert space between projective measurements. We demonstrate our approach on the control problem of rapidly transferring a superfluid into the Mott insulator in the Bose-Hubbard model. We also demonstrate the general applicability of the method by preparing arbitrary superpositions using random Hamiltonians.

I. INTRODUCTION

Preparation of specific target quantum states is a prerequisite for e.g. control of qubits, quantum computation, quantum metrology, and simulation of novel matter phases [1,2]. This type of control is typically achieved by manipulating the unitary dynamics using quantum optimal control theory [4,7].

Although quantum optimal control theory has been applied successfully in several systems, it is still challenging to control many-body systems such as ultracold atoms in an optical lattice [8]. Experimentally, these systems are typically created in the superfluid phase and must be transferred into the Mott insulator phase [6,10], which is the starting point for applications such as performing quantum logic gate operations [11,13], quantum simulations [10], and single atom transistors [17]. This transfer is difficult since the adiabatic time scales diverge close to the phase transition where the gap to the excited state closes in an infinite system [15]. There have been attempts to numerically optimize the transfer using optimal control theory and adiabatic ramp shapes [8,12,21], and the transition has been studied and optimized experimentally [8,20,21].

An alternative to unitary control is to steer the dynamics using the backaction associated with quantum measurements [22,24]. For instance, by measuring a sequence of observables in spin systems, it is possible to prepare desirable local properties as well as long range correlations [28]. Measurement based control of many-body systems requires inclusion of the quantum back-action in the modelling. Initial steps in this direction have been taken in Refs. [29,31], where it is shown that collective weak measurements of the on-site densities or coherences in an optical lattice can be used to engineer correlated tunneling and long-range entanglement. The weak measurements confine the system to distinct Zeno-subspaces defined by an effective non-Hermitian Hamiltonian, governing the evolution of the monitored system [31], and Raman-like transitions may be observed between these subspaces [32,33].

Here we discuss combinations of unitary and measurement-based dynamics that may offer better control strategies. Our focus will be on preparation of a target state. For this purpose, one previously proposed control strategy is Fixed Unitary Evolution and Measurements (FUMES) [34]. In FUMES it is the timing of the measurements rather than the unitary dynamics, which is optimized. This means that the unitary dynamics is given by a fixed static Hamiltonian while measurements, attempting to project the system into the target state, are performed at the times with highest success probability. In Ref. [34] it was shown that FUMES is competitive with other types of measurement-based control schemes such as Multiple Evenly Distributed Observables (MEDO) [35] and Mutually Unbiased Measurements (MUM) [36].

A projective measurement on a many-body system is typically realized by many individual (local) measurements. Even if the full projective measurement fails to produce the desired outcome, some of the individual measurements might still have succeeded. Despite exhibiting superior performance to MEDO and MUM, the FUMES strategy suffers from the drawback that it cannot maintain these partial successes. In this paper we propose to employ quantum Zeno dynamics to improve the FUMES strategy by freezing the state components prepared by each partial success [31]. This effectively confines the unitary dynamics to smaller Zeno subspaces similar to what was found in Refs. [32,33]. We demonstrate that this gives an exponential speed-up relative to FUMES.

In section II we introduce FUMES and Z-FUMES as methods for preparing a Mott insulator starting from the superfluid. In section III we analyze the performance of Z-FUMES in a more realistic setting using continuous homodyne measurements rather than projective measurements on the Bose-Hubbard model. In section IV we demonstrate the general applicability of Z-FUMES by simulating the preparation of arbitrary states using random Hamiltonians and measurements. Section V concludes the paper.
The quality of this transfer is quantified by the fidelity of the local atom-numbers, thus providing access to the set of observables \( \{ \hat{n}_i \} \) for both the projective measurement and the quantum Zeno dynamics. A simultaneous measurement of all observables collapses the state into a Fock-state \( |n_1, n_2, \ldots, n_L \rangle \) with \( \sum_j n_j = N \). This type of system can be controlled using the FUMES control strategy introduced in Ref. \( [37] \). In FUMES the system is projected into a Fock-state by simultaneous measurements of all the \( \hat{n}_i \) operators. This type of measurement will either succeed by projecting into the Mott state or fail by a projection into another Fock-state. For a non-zero \( J \) the Fock-states are not eigenstates of the model \( [11] \), which implies that if the measurement fails then the subsequent unitary dynamics drives the system out of the projected Fock-state. Hence, at later times there is again a non-zero probability of projecting into the target state. If the projections occur at arbitrary times then there is only a low probability of success \( [43] \). In FUMES this probability is improved by only measuring at peaks in the fidelity above some preset threshold. In an experimental setting, these peaks can be calculated prior to the experiment by solving the deterministic Schrödinger equation.

FUMES is problematic in the sense that for a large lattice it becomes exponentially improbable to project directly into the Mott state. In order to remedy this effect, we propose Z-FUMES, which is a combination of FUMES with quantum Zeno dynamics \( [37] \). Quantum Zeno dynamics in optical lattices have been reported experimentally in Refs. \( [41, 42] \). Although a projective measurement of all the sites may not have reached the Mott state some of the individual sites may still have the desired unit occupancy. In order to prevent these particles from tunneling away we propose to trap them using quantum Zeno dynamics, i.e. by performing rapidly re-
FIG. 2. (a) The expected time needed to reach a fidelity above 0.99 as a function of the lattice size for FUMES and Z-FUMES simulated under the Bose Hubbard model (1). The results are averaged over 1000 trajectories. (b) A toy model simulation of Z-FUMES with four measurements. Zeno locked sites are shown with white. The distributions of particles, \((n_1, n_2, \ldots, n_L)\) after a measurement is sampled from Eq. (2) for each sublattice with \(L = L_{\text{sub}}\) where \(L_{\text{sub}}\) is the sublattice length. (c) The probability distribution for the largest sublattice size, \(K\) after a measurement in a lattice of size \(L = 17\) calculated from Eq. (2). The white bar represents the median at \(K = 13\). The rightmost bar at \(K = L\) gives the probability for not locking any sites. The insert illustrates the average value of the relative size of the largest domain for different lattice sizes.

Zeno-locking the number of particles on a site does not only ensure the correct occupancy, it also prevents particles from tunneling across that site. This implies that locked sites effectively decouple the lattice into smaller parts. However, the Mott state can only be reached if each of these sublattices contain the correct number of particles. Hence, we should only Zeno-lock a given site whenever it contains a single particle and the right and left sublattices have matching numbers of sites and particles.

We compare FUMES and Z-FUMES for creating a Mott-state in this model for a system with size \(N = L = 7\). Between the measurements the evolution of the state is governed by the Hamiltonian (1) with \(U/J = 0\). The on-site density \(\langle \hat{n}_i \rangle\) during a Z-FUMES trajectory is shown in Fig. 1b. The state is initially in the superfluid phase and discrete changes in the density are introduced by projective measurements of the on-site density at specific times marked by dashed lines. At the time \(Jt = 4\), the two outer sites have been Zeno locked, creating a sublattice of length five. The edges of this sublattice are gradually Zeno locked in this trajectory, and after about \(Jt = 10\), the system has converged to the Mott insulator state with \(\langle \hat{n}_i \rangle = 1\) for \(i = 1, 2, \ldots, 7\). The gradual locking in the proper subspaces is the reason Z-FUMES converges faster than FUMES.

In Fig. 1c: the mean fidelity as a function of time is shown for both FUMES and Z-FUMES. The curves are obtained by averaging the results of 1000 simulated trajectories. The figure shows that Z-FUMES reaches an expected unit fidelity after about \(Jt = 20\) while after \(Jt = 70\) FUMES still only has a success rate of 60%. For comparison we also show the fidelity after a unitary linear ramp of the interatomic interaction strength from \(U/J = 0\) to \(U/J = 30\) during the same time-interval but in absence of any measurements. For each value of time the ramp is thus performed with different speeds. Figure 1c shows the final fidelity as a function of the total ramp time. Note, that this final fidelity is not the same as the success rates for FUMES and Z-FUMES shown in the same figure. For long ramp times the transfer is adiabatic and the fidelity approaches unity. The curve further exhibits characteristic rapid, coherent oscillations, which are due to the energy differences between populated eigenstates during the transfer. Contrary to FUMES, Z-FUMES greatly outperforms the linear ramp. We note, however, that the linear ramp never reaches a pure Mott state whereas FUMES leads to formation of the pure Mott state in 60% of the simulated runs.

B. Scaling with System Size

In this part, we discuss how FUMES and Z-FUMES scale with the lattice size in the Bose Hubbard model (1). For this purpose, we define \(T_{\text{conv}}\) as the time where the expected fidelity reaches \(\mathbb{E}[F] = 0.99\). The scaling of this...
quantity with the lattice size $L$ is illustrated in Fig. 2a where $T_{\text{conv}}$ is averaged over 1000 simulations. FUMES scales poorly with the lattice size as it becomes exponentially improbable to project the system to the Mott state. The improved scaling in Z-FUMES is due to the fact that each time a site is locked, the lattice is divided into smaller sublattices with a higher probability of measuring the desired outcome in subsequent measurements.

This intuitive picture can be substantiated with a toy model analysis of FUMES and Z-FUMES, where the backaction from the measurements is ignored by assuming a complete reshuffling after each measurement. In practice, this is achieved by assuming a Poisson number distribution on each site, corresponding to a superfluid state, with constraint $N = L$. Then the probability of measuring a particle distribution $n = (n_1, n_2, ..., n_L)$ is given by the multinomial expression,

$$P(n) = \frac{L!}{n_1!n_2! \cdots n_L!} \prod_{i=1}^L \left( \frac{1}{L} \right)^{n_i}.$$  \hspace{1cm} (2)

In FUMES the collective measurement either projects the state into the target state with $n = (1, 1, ..., 1)$ or not. I.e., it has the success probability,

$$P = \frac{L!}{L^L} \simeq \frac{\sqrt{2\pi L}}{e^L},$$ \hspace{1cm} (3)

where Stirling’s approximation, assuming large $L$, has been applied in the second identity. This confirms the discussion above concerning the exponential suppression of the success probability in FUMES as observed in Fig. 2a.

To gain further insight into the properties of Z-FUMES we perform a toy model simulation of the protocol in large lattice systems. In this toy model simulation, the outcome of a measurement is randomly sampled from the distribution (2). After each measurement we check if any site should be Zeno locked. If some sites have been Zeno locked then the system is divided into a number of smaller sublattices with length $L_{\text{sub}}$ and the outcome of a measurement on each sublattice is sampled from Eq. (2) with $L = L_{\text{sub}}$. One realization of this toy model simulation is shown in Fig. 2b where the bars represent the number of atoms on a given site and the white bars show the Zeno locked sites. This illustrates that after just four measurements most of the sites have been Zeno locked.

After a single measurement the convergence properties of Z-FUMES are dominated by the largest remaining sublattice. For a lattice of length $L$ we calculate the probability that the largest sublattice has length $K$ by summing the individual probabilities for all possible measurement outcomes. For an example with $L = 17$ the resulting probability distribution is shown in Fig. 2c. The median, 13 is indicated by a white bar. The insert depicts the average size of the largest domain divided by the total lattice size, $E[K]/L$, which appears to converge towards a value smaller than 0.75$L$. This implies that in Z-FUMES the largest domain size decreases exponentially with the number of measurements, resulting in the favourable scaling seen in Fig. 2a.

III. CONTINUOUS MEASUREMENTS

In the previous sections we have assumed that the measurements occur instantaneously. This is a valid assumption when the typical duration of a measurement is short compared to the dynamics of the measured system [44]. However, this is not generally true in practical applications with finite interaction strengths where the measurement record $I(t)$ is continuous. As a step towards experimental realizability we investigate the performance of Z-FUMES with weak continuous measurements.

Due to the backaction of the continuous measurements, the dynamics obeys a stochastic Schrödinger equation, which can be understood as the time evolution conditioned on the stream of measurement results in $I(t)$ [44]. At each instant in time, $I(t)$ is dominated by stochastic noise, resulting in a state $|\psi(t)\rangle$ which evolves in a random manner. This time evolution exhibits quantum jumps if measurement outcomes occur at discrete point in times as in photo detection while for example homodyne detection leads to a diffusive trajectory [45]. Quantum jump trajectories have previously been studied in the context of the Bose Hubbard model [29–31, 33, 40, 47]. Here we consider diffusion-type measurements which imply a stochastic Schrödinger equation of
where the $\hat{q}_j$ represent the current state of the system and $\hat{Q}_i$ is a Zeno subspace corresponding to a value of $q_i^{(i)}$ for a particular $Q_i$. Each Zeno-subspace contains a number of the basis states $\{\phi_j\}$. A simultaneous measurement of all the $Q_i$ yields a single state projection, which in this illustration is into the target state $|\psi_{\text{target}}\rangle$.

\[
|\psi(t)\rangle = d|t\rangle = dt \left[ -i\hat{H}(t) - \frac{1}{2} \sum_j \left( \gamma_j \hat{c}_j^\dagger \hat{c}_j + I_j(t) \hat{c}_j \right) \right] |\psi(t)\rangle ,
\]

where the $\hat{c}_j = \hat{n}_j$ are the measurement operators and the $\gamma_j$ corresponding measurement strengths, which determine the rate at which information is extracted and the $\gamma_j$ are uniform for all $j$. A measurement becomes projective in the limit $\gamma \to \infty$. Note that Eq. (4) does not preserve the normalization, which is instead imposed explicitly in each time step. The measurement record $I_j(t)$ for the $j$th detector reflects the current state of the system,

\[
I_j(t) = \gamma_j \langle \hat{c}_j^\dagger \hat{c}_j \rangle(t) + \sqrt{\gamma_j} \xi_j(t),
\]

where the $\xi_j(t) = dW_j(t)/dt$ are infinitesimal Wiener increments, representing white noise in the detection setup. Integration of the record allows one to determine the outcome of a measurement for large values of $\gamma$. In order to investigate the performance of FUMES and Z-FUMES at low values of $\gamma$ we assume that it is possible to quench the state such that the measurements are only performed with $J = 0$ in Eq. (1).

In Fig. 3 Z-FUMES is investigated for different values of the measurement strength in a system with $L = 5$ lattice sites. The figure also shows the expected fidelity for Z-FUMES simulated using discrete projective measurements as in Fig. 1. As expected, the fidelity of the continuous measurement scheme converges towards that corresponding to projective measurements as the measurement strength becomes large. At the time $Jt = 15$ more than 60% of the trajectories have converged with $\gamma/J < 0.5$, which signifies that Z-FUMES is effective for post-selected state preparation even with low values of the measurement strength, $\gamma/J < 1$.

### IV. GENERAL APPLICATION OF Z-FUMES

In this section we show that FUMES and Z-FUMES can also be applied to prepare an arbitrary target state of a system, almost independently of its Hamiltonian evolution. In order to demonstrate this in an unbiased way we assume random Hamiltonians drawn from the Gaussian Unitary Ensemble which ensures a uniform distribution in the space of Hamiltonians with a particular dynamical time-scale. This type of random Hamiltonians are used to model, e.g., chaotic systems with one and many particles.

We assume the target state $|\psi_{\text{target}}\rangle$ belongs to a set of orthonormal states $\{|\phi_j\rangle\}$. The system can be manipulated through the backaction from measuring a set of observables $\{Q_1, Q_2, ..., Q_n\}$. The measurement operators $\hat{Q}_i$ are linear combinations of the projection operators $\hat{Q}_i = \sum_j \gamma_j^{(i)} |\phi_j\rangle \langle \phi_j|$. The eigenvalues $\gamma_j^{(i)}$ should be constructed with a large degeneracy such that they define different subspaces, each with dimension around $\sqrt{d}$ where $d$ is the dimension of the full Hilbert space. A measurement of any $Q_i$ projects the state into one of these subspaces, which must satisfy that a simulta-
neous measurement of all the $\hat{Q}_i$ operators corresponds to a single state projection. A measurement of a $\hat{Q}_i$ is successful if it measures the same $\bar{q}_j^{(i)}$ as for the target state. Zeno-locking this value confines the time evolution to a smaller subspace containing the target state. The subsequent time evolution within this subspace is

$$\hat{U}_Z(\Delta t) = \hat{P} \exp\left(-i\hat{H}\hat{P}\Delta t\right), \quad (6)$$

where $\hat{P}$ is the projector on the locked Zeno-subspace \[51\]. A subspace should only be Zeno-locked if there is a sufficient coupling between the current state and the target state within that subspace.

These ideas constitute a direct generalization of the Bose-Hubbard control scheme discussed in the previous sections. There the orthonormal basis consists of the Fock-states and the individual on-site number operators can be written as linear combinations of projectors on these states. We have for instance, that the on-site density operator for the $i$th site is

$$\hat{n}_i = \sum_{n_{i,\ldots}} n_{i,\ldots}^{(i)} \hat{P}_i \hat{P}_i^\dagger \hat{P}_{i+1} \hat{P}_{i+1}^\dagger \cdots \hat{P}_{i+1} \hat{P}_{i+1}^\dagger \hat{P}_{n_{i,\ldots}}.$$

The eigenspectrum of the number operator on a single Fock-state is clearly degenerate. Measuring the $\hat{n}_i$ operators one-by-one gradually leads to a collapse onto a single Fock-state. The condition of only Zeno-locking in subspaces with sufficient coupling between the current state and the target state corresponds to exclusively locking sites with matching numbers of particles and sites on the left and right sublattices.

We have applied Z-FUMES to perform general state transfers using these random Hamiltonians. In order to show the average behavior, we have performed calculations for 1000 different random Hamiltonians and target states. Fig. \[5\] compares the behavior for small ($d = 16$) and large ($d = 100$) random Hamiltonians. As in the case of the Bose-Hubbard model, the Z-FUMES curves converge much faster than FUMES. Both Z-FUMES curves show a similar rate of convergence despite the system size differing by an order of magnitude. This shows that the favorable scaling of Z-FUMES with the system size, discussed in the previous section, carries over in the more general setting.

V. CONCLUSION

We have introduced a new protocol, denoted Z-FUMES, for multi-particle state preparation using unitary dynamics and measurement-based control. Our protocol steers the evolution towards a target state by measuring a set of observables. Each observable is Zeno-locked when an appropriate outcome is obtained, which confines the time evolution to gradually shrinking Zeno-subspaces. Z-FUMES gives an exponential speed up compared to other measurement-based control protocols.

We analyzed in detail the preparation of a Mott state using Z-FUMES. Here it is necessary to measure the density on each site, which can be realized experimentally using strong fluorescence imaging combined with Raman side-band cooling as shown in Refs. \[11\], \[42\]. We demonstrate furthermore that Z-FUMES can be applied for preparing a Mott state in more realistic settings relying on weak continuous rather than projective quantum measurements.

In this work we focused on local measurements in order to reach the Mott insulator state which is characterized by local properties. It is also possible to generate strongly correlated states using non-local measurements. This could be used to engineer exotic quantum state such as Schrödinger cats or other states with long-range correlations.

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