Tolerance design of electromechanical products based on self-defined approximate model

J Deng¹, J M Lai¹* and G F Zhai¹

¹ School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin, Heilongjiang, P.R. China

E-mail:ljmllp1208@163.com

Abstract. Influenced by the uncertain factors in the manufacturing process, the output characteristics of electromechanical products inevitably deviate from the target value, resulting in the uneven quality of batch products. Generally, tolerance design based on a large number of input-output data is used to improve the quality consistency of batch products. As for electromechanical products, the output characteristics are usually obtained by the finite element method (FEM). However, FEM is so time-consuming that it is difficult to be applied to the tolerance design process. In this paper, a new tolerance design process based on approximate model is proposed. The period of tolerance design is greatly shortened. Firstly, a self-defined approximate model based on Taylor expansion principle is established. Then based on this model, the tolerance design with variable contribution rate is carried out. Finally, Monte Carlo method is used to compare the quality consistency of batch products before and after optimization. The accuracy and effectiveness of the method proposed in this paper are verified in the case of a certain type of electromagnetic relay (EMR).

1. Introduction
Electromechanical products are integrated with machine, electricity and magnetism, which have many components and complicated structure. In order to produce batch products with high quality consistency, each production process needs to be strictly controlled. However, due to the complexity of the structure and manufacturing process of electromechanical products and the influence of uncertain factors in the process of assembly and debugging, the output characteristics of a batch of products produced in the same process are decentralized [1], [2]. EMR is one of the most representative electromechanical products, which has become one of the electronic components with the worst reliability because the uncertainty of manufacturing process.

In general, EMR is inspected before delivery, but most manufacturers only pay attention to the qualified rate, and do not care about the quality distribution characteristics of batch products. In this way, the reliability of EMRs cannot be guaranteed. The best solution to this problem is to conduct quality analysis and robust design for EMRs (this paper focuses on tolerance design), which can greatly improve the quality consistency of products on the premise that the qualified rate of batch products is up to standard.

The process of quality analysis and tolerance design needs a lot of calculation to get enough data [3], [4]. The existing calculation methods for EMR include magnetic circuit method [5], finite element method (FEM) [6] and approximate modeling method. EMR has a very complex structure and a variety of materials, so it is difficult to use the common magnetic circuit method to model it. Moreover,
in the case of high nonlinearity, the accuracy of the model can not be guaranteed [7]. Although FEM can get high precision calculation results, its modeling process is tedious and the calculation time is long, which makes the application efficiency of this method very low. Fu et al. [8] proposed a method to reduce the calculation time of nonlinear time stepping finite element magnetic field. By combining Newton-Raphson method with incomplete Cholesky conjugate gradient method, the calculation time can be reduced by more than 50%. But this method makes the modeling process more complex, and the solution time is still very long for the complex nonlinear magnetic field. In recent years, many scholars have used mathematical methods to establish an approximate model between the input and output of EMR, which can make up for the shortcomings of the two methods mentioned above. The common approximate modeling methods are response surface method (RSM), radial basis function (RBF) method and Kriging method. Zeger bontinck et al. [9] established the mathematical model of rotor eccentricity effect of permanent magnet synchronous motor (PMSM) by RSM, which can simulate the multi-layer model of tilt eccentricity. However, this method is only suitable for two-dimensional system, which limits its application range. The advantage of RBF method is that it can approximate any nonlinear function. Lai et al. [10] proposed a new numerical method based on RBF to simulate the transient electromagnetic problems, which solved the one-dimensional solution problem under different boundary conditions well. However, the selection and establishment of the basis function are very complex, and the anti-interference ability of the established model is poor. Xia et al. [11] established the approximate model of electromagnetic device by Kriging method, which provides a way for the rapid calculation of electromagnetic device. However, the number of sample points used in the process of establishing the model is huge. Moreover, when the model parameters need to be changed, resampling is needed to train the new model. So that it is limited in the scope of application.

In addition, some scholars proposed using self-made function to establish the approximate model of electromagnetic system. For example, Ye et al. [12] used self-defined interpolation function to establish the dynamic output characteristic model of EMR, and used quantum particle swarm optimization algorithm to determine the coefficient of fitting function. Although the accuracy of the model obtained by this method is within a reasonable range, its modeling process ignores the interaction between input parameters, which makes the calculation results of the model near some points unsatisfactory.

For tolerance design, the robust tolerance design (RTD) method proposed by Taguchi [13, 14, 15] is the most classical one. In the method, the quality loss function is established to evaluate the economic loss caused by quality fluctuation. The tolerance of design parameters that have a large impact on output characteristics and low cost is minimized, and the tolerance of design parameters that have a small impact on output characteristics and high cost is appropriately expanded. In this process, the cost function is taken as the constraint objective, and the purpose of tolerance redistribution is to reduce the total cost of the product [16]. When the cost function reaches the standard, the new tolerance allocation scheme is determined. However, the empirical formula of tolerance allocation used in this method is subjective and random, which is easy to generate suboptimal solution [17]. Generally, only the expected quality loss is concerned in tolerance design based on the quality loss function method. In order to make the quality loss function more accurate, Ouyang et al. [18] proposed to establish the quality loss function by comprehensively considering the expectation and variance of quality loss. Among them, the expectation of quality loss represents location effect, and the variance of quality loss represents dispersion effect. Better quality performance can be obtained with this method. Youn et al. [19] proposed a general mathematical model to determine product tolerance, which contains the quality cost of quadratic loss function. Geometric decay function is used to represent manufacturing cost. It can be used to minimize the total cost. Then, in order to solve the error caused by the uncertainty in the product cost function and the quality loss function, Han et al. [20] proposed a new quality loss function based on the location effect and dispersion effect, considering the relationship between the model parameter uncertainty and the response. And the cost function is optimized to make the result of tolerance design closer to the
optimal solution. However, it is difficult to determine the quality loss function of electromechanical products, and more attention is paid to the improvement of product quality consistency, so the above methods are not suitable for the tolerance design of electromechanical products.

Considering the above factors, an automatic tolerance optimization method with variable contribution rate for electromechanical products based on self-defined approximate model is proposed in this paper. Firstly, an approximate model between the output characteristics and key design parameters of EMR is established. The modeling process is as follows: At first, the modeling process is decoupled into independent term modeling for single parameter change and interactive term modeling for multi parameter change; Then, the independent term model and the interactive term model are established based on the FEM simulation results. The interaction term model is used as the error compensation term. The accuracy of the model can be continuously improved through step-by-step interactive term modeling. Then the contribution rate of the key design parameters to the output characteristics is obtained based on the orthogonal experiment. Based on the tolerance step function and tolerance limit constraints, a new tolerance allocation scheme for design parameters is generated by contribution rate analysis. After that, Monte Carlo method was used to analyze the quality consistency. If the requirements are not met, the above tolerance design process shall be repeated.

The consistency of output characteristics before and after the optimization of tolerance scheme is compared, so as to verify the effectiveness of tolerance design results. The validity and correctness of the method proposed in this paper are verified by an example of tolerance design of a certain type of EMR.

2. Principle of approximate model

2.1. Process of model decoupling
When the input parameter \( X(x_1, x_2, ..., x_n) \) is based on the center value \( X_0(x_{10}, x_{20}, ..., x_{n0}) \) and fluctuates within the variable range \( TX(tx_1, tx_2, ..., tx_n) \), the output \( F \) also fluctuates accordingly based on the original value \( F_0(X_0) \). Therefore, if the output variation \( \Delta F(\Delta X) \) corresponding to the input parameter fluctuation \( \Delta X(\Delta x_1, \Delta x_2, ..., \Delta x_n) \) is established, the approximate model can be expressed as the following basic form:

\[
F = F_0 + \Delta F
\]

According to the Taylor expansion principle, the output variation \( \Delta F \) can be decoupled into a sum of \( \Delta F_d \) and \( \Delta F_i \), where \( \Delta F_d \) is the sum of output change amounts \( \Delta F_d(\Delta x_i) \) generated when each input parameter changes \( \Delta x_i \) individually, and \( \Delta F_i \) is the sum of the output variations \( \Delta F_i(\Delta x_{a}, \Delta x_{b}, ..., \Delta x_{m}) \) produced by the interactivity between the non-repetitive combinations \( (x_{a}, x_{b}, ..., x_{m}) \) of the input parameters. The details are as follows:

\[
\Delta F = \Delta F_d + \Delta F_i
\]

\[
\Delta F_d = \sum_{i=1}^{n} F_{di}(\Delta x_i)
\]

\[
\Delta F_i = \sum_{j=1}^{l} R_{ij}(\Delta x_a, \Delta x_b, ..., \Delta x_m)
\]

2.2. Modeling process of independent terms
Based on Taylor’s first order expansion, the output variation \( \Delta F(\Delta X) \) can be decoupled into the sum of the corresponding output change amounts when the input parameters change individually, as shown in equation (3), where \( R_i \) is the remainder of the expansion, which is regarded as the error term in this paper.
\[
\Delta F(\Delta x_1, \ldots, \Delta x_n) = \Delta F(0, \ldots, 0) + \sum_{i=1}^{n} \Delta x_i \frac{\partial F}{\partial x_i} + R_n = 0 + \sum_{i=1}^{n} \Delta x_i \frac{\partial F}{\partial x_i} + R_n
\] (3)

Based on equation (3), the independent term can be expressed as:

\[
\Delta F_{i}^n(\Delta x_i) = \Delta x_i \times \frac{\partial F}{\partial x_i}
\] (4)

In order to ensure the accuracy of the independent term model, appropriate sampling points can be selected between the upper limit value \((x_0+\Delta x_i, V)\) and the lower limit value \((x_0-\Delta x_i, L)\) of the variation range for each parameter \(x_i\), and the corresponding output \(S(x_i)\) is calculated. The number of sampling points should be determined by constant attempts to ensure enough accuracy with the minimum number. The sample size of each parameter is 7 in this paper. Then, the polynomial fitting method is adopted to establish a functional relationship between the input parameter \(x_i\) and the output \(F\). The modeling of independent terms is not affected by parameter nonlinearity.

Therefore, the mathematical model of the independent term can be further written in the following form \((F_0\) is the output when all input parameters are centered):

\[
\Delta F_{i}^n(\Delta x_i) = P_{i}(x_0 + \Delta x_i) - F_0
\] (5)

2.3. Modeling process of interaction terms

Based on equation (1) and equation (5), the error term in equation (3) can be obtained as follows:

\[
R_n = \Delta F(\Delta x_1, \Delta x_2, \ldots, \Delta x_n) - \Delta F_{i} = F - F_0 - \sum_{i=1}^{n} \Delta F_{i}^n(\Delta x_i)
\] (6)

If only the influence of individual parameters on the output is considered, it is difficult to ensure that the error term satisfies the required accuracy. Therefore, the amount of output change produced by the interactivity between parameters should also be considered. Suppose there are \(n\) input parameters, the number of two-factor interaction terms is \(C^2_n = n \times (n-1)/2\), and the number of \(q\)-factor interaction terms is \(C^n_q\). Obviously, it is impossible to model all interaction terms. Therefore, the significance analysis is used firstly to determine the parameter combinations \((x_{a}, x_{b}, \ldots, x_{m})\) that have the greatest impact on the output.

Weight \((W)\) is defined to describe the effect of parameter combination on the output. The larger the value of \(W\), the greater the effect on the output. \(\Delta F_{G}\) is assumed to be the output variation when the maximum variation of multiple parameters is taken at the same time. \(\Delta F_{i}\) is the calculated value of the independent term. Then equation (7) can be defined.

\[
W = \left| \frac{\Delta F_{G}(\Delta x_{a}, \Delta x_{b}, \ldots, \Delta x_{m}) - \sum_{i=a,b,\ldots,m} \Delta F_{i}}{\Delta F_{G}(\Delta x_{a}, \Delta x_{b}, \ldots, \Delta x_{m}) + F_0} \right| \times 100\%
\] (7)

For a parameter combination with significant influence, the function model between parameter variations \(\Delta x_{a}, \Delta x_{b}, \ldots, \Delta x_{m}\) and the output change amount \(\Delta F_{G}(\Delta x_{a}, \Delta x_{b}, \ldots, \Delta x_{m})\) is built and added to the established approximation model, in order to compensate the error. Based on Latin hypercube sampling method, sampling points are generated. The parameter variation \(\Delta x_{S}\) and the error \(\delta_{S}\) of each sampling point \(S\) is calculated, where \(\delta_S\) is the actual change amount minus the sum of the changes calculated by all independent term models, as shown in equation (8). Then, the functional relationship between the parameter variation \(\Delta x_{S}\) and the error \(\delta_{S}\) is established by polynomial fitting, that is, the interaction term model \(\Delta F_{i}(\Delta x_{a}, \Delta x_{b}, \ldots, \Delta x_{m})\) of the parameter combination.

\[
\delta_{S} = \left. \Delta F_{G} \right|_{S} - \sum_{i=a,b,\ldots,m} \left. \Delta F_{i} \right|_{\Delta x_{S}}
\] (8)
At this point, the error term of the model changes to:

\[ R_n = F - F_0 - \sum_{i=1}^{a} \Delta F_{di}(\Delta x_i) - \sum_{j=1}^{b} \Delta F_{dj}(\Delta x_j, \Delta x_j, \ldots, \Delta x_m) \]  \hspace{1cm} (9)

The error term of the above equation is rechecked. If the requirement is not satisfied, the combination parameter with the second largest weight is used to build the interaction term model, until the model accuracy is high enough.

3. Process of tolerance automatic optimization

The flow chart of adaptive tolerance variable step optimization algorithm proposed in this paper is shown in figure 1. Firstly, the central value \(x_0\) and the initial tolerance \(M_0\) of the key design parameters are determined as the input parameters, and the lower limit \(\zeta\) of the parameter tolerance and the qualified threshold range \(Q_r\) are determined as the constraints of the iterative model. Then the orthogonal experiment is designed according to the initial tolerance, and the output characteristic results are obtained by the fast calculation of each experimental scheme based on the approximate model proposed in this paper. After that, the weight coefficient \(\rho_i\) and acceleration coefficient \(\lambda\) are determined by contribution rate calculation and quality fluctuation range analysis respectively, so as to obtain tolerance step \(h_i\). Further, the new tolerance \(M_{i+1}\) at the end of this iteration process and the quality fluctuation range \(Q_{i+1}\) of output characteristics under the new tolerance range can be obtained. The results are analyzed. If the qualified threshold range \(Q_r\) is not met, the next iteration process will be carried out until the optimization target is reached.

![Figure 1. Tolerance optimization allocation process.](image-url)
4. Case study

In the magnetic holding electromechanical device, as shown in Figure 2, the armature magnetic holding torque F provided by the permanent magnet is an important property affecting the device's reliability.

In this case, seven key design parameters determined by simulation of the EMR are selected as the input, and the electromagnetic torque under 0V voltage is used as the output parameter to establish an approximate model, and on this basis, the tolerance optimization design is carried out. Information about the seven design parameters is shown in Table 1.

![Figure 2. Magnetic holding electromechanical device.](image)

Table 1. Design parameter center value and tolerance of electromagnetic system.

| Number | Parameters     | Center value and tolerance /mm |
|--------|----------------|--------------------------------|
| x₁     | A Yoke length  | 2.8±0.3                        |
| x₂     | B Armature length | 15.2±0.9              |
| x₃     | C Armature height | 0.57±0.45            |
| x₄     | D Armature width | 3.7±0.75               |
| x₅     | E Magnet length | 16.5±0.45             |
| x₆     | F Magnet width  | 4.1±0.3                    |
| x₇     | G Core radius  | 1.8±0.25                   |

4.1. Establishment of approximate model

4.1.1. Independent terms modeling

Yoke length x₁ is taken as an example, and nₙ=7 sampling points are uniformly selected in the variable range of 2.5~3.1mm. Then the calculation result of the armature magnetic retention force torque F is obtained through simulation modeling. In order to establish a functional relationship between the design parameter change amount Δx₁ and the output characteristic change amount ΔF₀(Δx₁), the sampling point calculation result is uniformly subtracted from the output characteristic F₀. By analogy, a complete independent terms model can be obtained, as shown in equation (10).

\[
\begin{align*}
F &= F₀ + \sum ΔFₙ₀ \\
F₀ &= 15.6 \\
ΔF_{d₁} &= -2.771Δx₁^² + 7.879Δx₁ \\
ΔF_{d₂} &= 0.865Δx₂^² + 2.211Δx₄ \\
ΔF_{d₃} &= -1.942Δx₃^² - 2.248Δx₃ \\
ΔF_{d₄} &= 2.211Δx₄ \\
ΔF_{d₅} &= 0.4756Δx₅^² - 0.7668Δx₅ \\
ΔF_{d₆} &= -6.633Δx₆ \\
ΔF_{d₇} &= 6.862Δx₇^² - 3.632Δx₇
\end{align*}
\]
4.1.2. Interactive terms modeling

Firstly, the interaction between the two factors is considered, and the interaction term for error compensation is established. There are 21 combinations between the seven design parameters of the electromechanical device, as shown in Table 2. For each interaction combination, the influence significance \( W \) is calculated using equation (7). Where \( \Delta F_i(\Delta x_{i1}, \Delta x_{i2}) \) and \( \Delta F_i \) needs to be obtained by simulation. The value of \( \Delta x_{i1}, \Delta x_{i2} \) is selected as the maximum value within the variable range. The calculation results of the significance \( W \) are shown in Table 2.

| No. | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) |
|-----|--------|--------|--------|--------|--------|--------|
| 1   | 25.02% | —      | —      | —      | —      | —      |
| 2   | 0.82%  | 12.9%  | —      | —      | —      | —      |
| 3   | 0.58%  | 0.34%  | 1.26%  | —      | —      | —      |
| 4   | 0.77%  | 0.89%  | 1.43%  | 0.98%  | —      | —      |
| 5   | 0.57%  | 0.33%  | 1.09%  | 0.73%  | 1%     | —      |
| 6   | 0.43%  | 0.38%  | 0.81%  | 0.56%  | 0.75%  | 0.57%  |

As can be seen from the above table, the interaction between parameters \( x_1 \) and \( x_2 \) is the strongest. The interaction term between these two parameters is established first. In the tolerance range of \( x_1 \) and \( x_2 \), 49 sample points are randomly selected (the rest parameters are central values). Then the actual torques \( F_i \) (\( i=1,2, \ldots, 49 \)) at the sample point is obtained by simulation, and the difference \( \delta_{x_i} \) between \( F_i \) and the established independent term model is the output of the interaction term model. After that, the polynomial fitting method is used to establish the functional relationship between the parameter variation \( \Delta x_{x_i} \) and the error \( \delta_{x_i} \), which is the interaction term model \( \Delta F_{i1}(\Delta x_{i1}, \Delta x_{i2}) \) of the parameter combination \((x_1, x_2)\), as shown in equation (11).

\[
\Delta F_{i1} = 3.2\Delta x_{i1}^4\Delta x_{i2} + 11.12\Delta x_{i1}^3\Delta x_{i2}^2 - 0.56\Delta x_{i1}^2\Delta x_{i2}^3 + 6.67\Delta x_{i1}\Delta x_{i2}^4 + 7.241\Delta x_{i1}\Delta x_{i2}^3 + 83.54\Delta x_{i1}^2\Delta x_{i2} + 6.211\Delta x_{i1}^2\Delta x_{i2} + 4.263\Delta x_{i1}\Delta x_{i2} \tag{11}
\]

As can be seen from Table 2, the interaction between parameters \( x_2 \) and \( x_3 \) cannot be ignored. To further reduce the error, the interaction term model of \( x_2 \) and \( x_3 \) can be established.

\[
\Delta F_{i2} = 3.264\Delta x_{i2}^3\Delta x_{i3} + 2.156\Delta x_{i2}^4\Delta x_{i3}^2 - 3.281\Delta x_{i2}^3\Delta x_{i3}^3 + 2.395\Delta x_{i2}^2\Delta x_{i3}^4 - 3.699\Delta x_{i2}\Delta x_{i3}^5 + 5.241\Delta x_{i2}\Delta x_{i3}^4 \tag{12}
\]

Then the approximation model of electromechanical device is improved to:

\[
F = F_0 + \sum_{i=1}^{7} \Delta F_{ai} + \sum_{i=1}^{2} \Delta F_{ni} \tag{13}
\]

To evaluate the model accuracy, 20 sample points are selected randomly, then the relative error value \( e \) of each point is calculated. The results are shown in Table 3.

| No. | \( e \) \( /\% \) | No. | \( e \) \( /\% \) | No. | \( e \) \( /\% \) | No. | \( e \) \( /\% \) |
|-----|-------|-----|-------|-----|-------|-----|-------|
| 1   | 0.6182 | 6   | 2.5988 | 11  | 1.1544 | 16  | 0.4727 |
| 2   | 0.8089 | 7   | 0.3320 | 12  | 0.4242 | 17  | 0.4622 |
| 3   | 0.7310 | 8   | 1.0440 | 13  | 0.1644 | 18  | 0.4915 |
| 4   | 0.6217 | 9   | 0.2625 | 14  | 0.9252 | 19  | 0.7164 |
| 5   | 1.1413 | 10  | 0.0060 | 15  | 0.3766 | 20  | 0.9954 |

It can be seen that the error between the model calculation results and simulation results is within 5\%, which meets the accuracy requirements. A total of 168 sampling points are used to establish the approximate model.
4.2. Tolerance optimization allocation process

According to the requirements of the electromagnetic torque index of the EMR, the qualified threshold range \( Q_r \) of its quality characteristic is determined as: electromagnetic torque 14.4 ~ 16.8 N\( \cdot \)mm.

Orthogonal experimental design. Taking the first iteration process as an example, the orthogonal experiment design and contribution rate analysis process are explained. First, the orthogonal table header is designed according to the total number of parameters and levels, as shown in Table 4. Then the orthogonal table \( L_{13}(3^7) \) is selected.

**Table 4. Test factor level table (first iteration process).**

| Level | A (mm) | B (mm) | C (mm) | D (mm) | E (mm) | F (mm) | G (mm) |
|-------|--------|--------|--------|--------|--------|--------|--------|
| 1     | 2.50   | 14.30  | 0.12   | 2.95   | 16.05  | 3.80   | 1.55   |
| 2     | 2.80   | 15.20  | 0.45   | 3.70   | 16.50  | 4.10   | 1.80   |
| 3     | 3.10   | 16.10  | 1.02   | 4.45   | 16.95  | 4.40   | 2.05   |

The approximate model is used to calculate the experimental schemes quickly, and the corresponding output characteristics are obtained. Then, the contribution rate is analyzed by using the calculation results, and the weight coefficient \( \rho_{i1} \) of the tolerance step required for the first iteration is obtained. The results of contribution rate analysis are shown in Table 5.

**Table 5. Contribution rate analysis results.**

| Parameters | Contribution rate* \( \rho_{i1} \) |
|------------|---------------------------------|
| A          | 34.88%                          |
| B          | 17.68%                          |
| C          | 25.90%                          |
| D          | 6.04%                           |
| E          | 0.38%                           |
| F          | 12.36%                          |
| G          | 2.76%                           |

*The sum of primary contribution rate and secondary contribution rate.

For the traditional tolerance design method, the contribution rate analysis results in the above table are used to allocate the tolerance manually according to the empirical formula. According to the quality consistency target range determined in this example, the consistency improvement target is determined to be 70%. The results of tolerance allocation determined by traditional methods are shown in Table 6.

**Table 6. Tolerance limit and optimal allocation scheme.**

| Parameters | Original tolerance | Tolerance limit \( \zeta \) | Traditional method | Improved method |
|------------|--------------------|-----------------------------|-------------------|-----------------|
| A (mm)     | ±0.3               | ±0.05                       | ±0.10             | ±0.05           |
| B (mm)     | ±0.9               | ±0.05                       | ±0.50             | ±0.30           |
| C (mm)     | ±0.45              | ±0.05                       | ±0.20             | ±0.17           |
| D (mm)     | ±0.75              | ±0.05                       | ±0.75             | ±0.31           |
| E (mm)     | ±0.45              | ±0.05                       | ±0.45             | ±0.40           |
| F (mm)     | ±0.3               | ±0.05                       | ±0.15             | ±0.05           |
| G (mm)     | ±0.25              | ±0.05                       | ±0.25             | ±0.10           |

Tolerance optimization allocation results. From the above contribution rate analysis, the weight coefficient of the initial tolerance step is obtained. In addition, the acceleration coefficient can be
obtained from the ratio $\tau_1$ of the current quality fluctuation characteristic $Q_1$ and the qualified threshold $Q_r$, and then the initial tolerance step $h_i$ of iteration can be determined. After one iteration, a new tolerance scheme is obtained as the initial condition of the next iteration. In this way, when the quality fluctuation range of output characteristics reaches the qualified threshold $Q_r$ or the tolerance reaches the lower limit of tolerance, the iteration process is stopped. In addition, the final tolerance obtained by the iterative algorithm is normalized to meet the requirements of tolerance annotation. The final result of tolerance optimization allocation is shown in Table 6.

$\tau_1$ is tracked and analyzed. After 32 iterations, the value is gradually reduced to 1, which shows that the new tolerance allocation scheme generated in each iteration makes the quality fluctuation range gradually approach the target range. The iterative algorithm has good convergence. In addition, it can be seen from the change trend that in the early stage of the iteration, the descent rate is fast, which makes the quality characteristics rapidly reduce from the position with large deviation from the target range to the edge of the target range. And the descent rate is slowing down continuously, so that the convergence accuracy at the end of the iteration is guaranteed. The curve of $\tau_i$ with the number of iterations is shown in Figure 3.

![Figure 3. The optimization process of tolerance distribution.](image)

The change processes of contribution rate and tolerance during optimization are shown in Figure 4 and Figure 5. As the iteration progresses, the contribution rate to the output characteristics is changed with the redistribution of the tolerance. With the optimal allocation of tolerance, the contribution rate of each parameter to the output characteristics is constantly adjusted, so that the optimal allocation result is achieved within the constraints. The advantage of variable contribution rate is that the tolerance reduction step size of different parameters can be adjusted in time. The parameter tolerance step size with large initial contribution rate is set to a larger value, and the parameter with small initial contribution rate is set to a smaller tolerance step size, so as to achieve the goal of adaptive control of tolerance step size. The variation trend of parameter tolerance is the same as $\tau_i$, which is from rapid to slow, which shows the efficiency of adaptive tolerance variable step optimization algorithm. It can be seen from the figure that the parameters A and F are not changed at some time before the end of iteration, indicating that the lower limit of allowable tolerance has been reached at that time.

Verification of tolerance allocation scheme. Monte Carlo method is used to analyze the tolerance allocation scheme and verify the accuracy of the optimization scheme. First of all, according to the parameters before and after optimization, 1000 virtual samples of relays are constructed respectively, and their mass distribution characteristics are compared, as shown in Figure 6. The range of acceptable quality thresholds is represented by the dotted red line in the figure. It can be seen from the figure that the quality consistency after optimization is significantly better than before. The change trend of probability density function of relay mass characteristic in the whole optimization process is shown in Figure 7. The improvement of mass characteristic brought by tolerance optimization is directly
reflected. In figure 7, different color curves from left to right represent different distribution characteristics of torque at different iteration times. It can be seen from the figure that the variance of mass distribution of relay electromagnetic torque of batch products is gradually reduced, and the expectation is gradually increased. Finally, the qualification threshold of quality consistency is reached.

![Figure 4. Contribution rate change process.](image1.png)  
![Figure 5. Tolerance change process.](image2.png)

The standard deviation and pass rate are taken as the indicators to evaluate the quality consistency, and the quality consistency of the output characteristics before and after optimization is compared. The results are shown in table 7.

**Table 7. Standard deviation and pass rate results before and after tolerance optimization.**

| Characterization parameters | Standard deviation | Qualified rate |
|----------------------------|--------------------|----------------|
| Original value             | 1.4239             | 80%            |
| Traditional method         | 0.8370             | 85%            |
| Consistency improvement rate | 41.22%            |                |
| Improved method            | 0.3983             | 99.6%          |
| Consistency improvement rate | 72.03%            |                |

![Figure 6. Mass distribution characteristics.](image3.png)  
![Figure 7. Change of probability density.](image4.png)

As can be seen from the table, the consistency of relay batch products has been increased by 72.03%, and the qualification rate has been increased from 80% to 99.6% with the method proposed in this paper. Compared with the traditional method, it has great advantages in both the range of consistency improvement and the qualified rate. The accuracy and superiority of this method are verified directly in theory.
5. Conclusions
The importance of product quality consistency is becoming increasingly significant. In this paper, a new approach of quality consistency design based on approximate modeling method is proposed.

- Based on the Taylor expansion principle, an approximate model between the key design parameters and the electromagnetic force of EMR is established. The accuracy of the model is greatly improved by using the interaction term as the error compensation term.

- The error between the EMR model established in this paper and the simulation results is less than 5%, which can meet the accuracy requirements. Using the finite element software to carry out tolerance design, a total of 32576 sample points are needed to be simulated. While only 168 sample points are needed with the approximate model established in this paper. The time-consuming problem of tolerance design with FEM is greatly improved.

- Robust tolerance design based on approximate model is carried out. Product consistency has been greatly improved. The effectiveness of the method is verified by a case study.

- The tolerance design method proposed in this paper can be applied to other mechanical and electrical products, and has certain value for improving the quality consistency and reliability of products.

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