Influence of multiple in-medium scattering processes on the momentum imbalance of reconstructed di-jets

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Experimental data measured in $\sqrt{s} = 2.76$ TeV Pb + Pb collisions at the LHC show a significant enhancement of events with an unbalanced pair of reconstructed jet momenta in comparison with p + p collisions. This enhancement of momentum imbalance is caused by different momentum loss of the initial back-to-back di-partons by scatterings within the created dense medium. For investigating the underlying partonic momentum loss we extended the on-shell transport model BAMPS (Boltzmann Approach for Multi-Parton Scattering) for full heavy-ion collisions, which numerically solves the 3+1D Boltzmann equation based on $2 \rightarrow 2$ as well as $2 \leftrightarrow 3$ scattering processes, with the possibility of virtual initial partonic branching processes for jet particles. Due to the employed test-particle approach jet reconstruction within BAMPS events is not trivial. We introduce a method that nevertheless allows the microscopic simulation of the full evolution of both the shower particles and the underlying bulk medium in one common microscopic framework. With this method it is for the first time possible to use well-established experimental subtraction algorithms within a theoretical calculation. Furthermore, we investigate the influence of multiple in-medium scattering processes of the shower partons on the momentum imbalance $A_J$. Due to the available particle information in configuration as well as momentum space within BAMPS, it is possible to reproduce the entire evolution of the reconstructed jets within the medium. With this information we investigate the dependence of the jet momentum loss from the difference in the partonic in-medium path lengths.

I. INTRODUCTION

As already proposed in the 1980s by Bjorken [1], high transverse momentum ($p_t$) partons created in the initial hard partonic scattering processes of heavy-ion collisions are an excellent probe for investigating the properties of the created hot and dense medium. One of the first evidences for this energy loss of hard probes was the suppression of di-hadron correlations in heavy-ion collisions at RHIC [2]. This observation was further supported by measurements of the suppression of inclusive hadronic spectra compared to scaled p + p references at RHIC [3,4] as well as at LHC [5,6]. Due to the large collision energy of the LHC and thereby increased production cross section of high $p_t$ partons, the separation between these partons as well as their associated shower particles and the underlying background medium becomes even more distinct with the result that energy loss studies in terms of reconstructed jets become feasible.

By studying reconstructed jets within heavy-ion collisions, both the ATLAS [7] as well as the CMS experiment [8,9] reported the measurement of an enhanced number of events with an asymmetric pair of back-to-back reconstructed jets in comparison to vacuum p + p events. One observable for describing this asymmetry in reconstructed jet momenta is the momentum imbalance

$$A_J(p_{t;1}, p_{t;2}) = \frac{p_{t;1} - p_{t;2}}{p_{t;1} + p_{t;2}},$$

where $p_{t;1}$ ($p_{t;2}$) is the transverse momentum of the leading (subleading) jet—the reconstructed jet with the highest (second highest) transverse momentum per event.

Several theoretical models including analytical calculations based on perturbative quantumchromodynamics (pQCD) [10–12], Monte-Carlo codes employing jet energy loss within a hydrodynamic background medium [13,14] as well as partonic transport models utilizing a static [15,16] as well as an expanding partonic medium [17] aim to describe the measured momentum imbalance. What all of these models have in common is that they either do not simulate the full in-medium shower evolution by considering further multiple scattering processes of the shower and the medium and/or they do not employ a subtraction of background momentum from the reconstructed jets. However, it is not a priori clear if neglecting the further in-medium scattering processes by considering only the first shower-medium interaction is justified. Moreover, no present theoretical model uses a background subtraction that is similar to experimental background subtraction methods. This is however important to allow a reliable comparison with experimental data.

In the following we present our method for calculating the momentum imbalance $A_J$ of reconstructed jets in the framework of the microscopic transport model BAMPS [15], which takes the full 3+1-dimensional expansion of the partonic medium into account. Furthermore, BAMPS allows the simulation of the full evolution of both the shower—while considering multiple further in-medium scattering processes—and the bulk medium within the same approach. For the virtual partonic branching pro-
cesses within the on-shell transport model BAMPS we introduce an effective modeling of the initial vacuum splitting processes based on shower routines of the event generator PYTHIA [19]. The actual reconstruction of jets based on BAMPS particle spectra is done by the anti-ka algorithm as implemented in the package FASTJET [20]. For more information about the actual reconstruction of jets we refer to Ref. [21] [22].

We employ within BAMPS a test-particle approach to ensure that the Boltzmann equation is solved accurately. This approach provides the high statistics necessary for numerically solving the Boltzmann equation. However, this ansatz leads to issues when reconstructing jets: A jet reconstruction gives only sensible results when considering real particles instead of test-particles. Therefore we introduce within this paper a method for separating the evolution of the medium and the shower particles while still employing the same BAMPS scattering probabilities for both the shower and medium particles. This separation allows us to treat the jet shower particles as real particles. What is more, we will introduce a method to add to these shower particles an appropriate selection of medium particles to end up with full events now consisting of only real particles which permit a sensible jet reconstruction.

Furthermore, especially when reconstructing jets within heavy-ion events where a medium is present a consideration of background subtraction effects is important for studying the momentum loss of reconstructed jets. In contrast to most other theoretical calculations, where only the first scattering processes between shower particles and the medium are simulated, BAMPS propagates not only jet and associated shower particles, but also all associated particles from the bulk medium. Especially due to these further in-medium scattering processes an appropriate background subtraction becomes important. The mentioned full events consisting of only real particles allow us to use experimentally well-established subtraction algorithms for subtracting this non-neglectable background contribution.

The present paper is organized as follows. After a short review of the partonic transport model BAMPS in Sec. II we discuss in Sec. III how to model appropriately different effects on the momentum imbalance of reconstructed jets that emerge in the case of “vacuum” p + p events in which no medium creation is expected. In Sec. IV we present the method to calculate the momentum loss of reconstructed jets within BAMPS while considering further in-medium scattering processes and employing an experimental background subtraction. Finally, we show in Sec. V our results on the momentum imbalance $A_\gamma$ and study the influence of multiple in-medium scattering processes. Furthermore, we have a closer look on the jet momentum loss in comparison to the initial hard parton and its underlying path length dependence.

## II. PARTONIC TRANSPORT MODEL BAMPS

BAMPS is a full 3+1D transport model that aims to handle the collective propagation of medium particles as well as energy loss phenomena of high $p_t$ particles of heavy-ion collisions within a common framework. To this end it considers massless on-shell partons, whose evolution is described by the relativistic Boltzmann equation,

$$p^\mu \partial_\mu f(x,t) = C_{22} + C_{2,3}$$

which is solved numerically based on leading order pQCD matrix elements utilizing a stochastic collision algorithm and employing a test-particle ansatz. Included are both elastic $2 \leftrightarrow 2$ scattering processes in small angle approximation, like e.g. $gg \rightarrow gg$, and inelastic $2 \leftrightarrow 3$ interactions, like e.g. $gg \leftrightarrow gg g$, based on the (original) Gunion-Bertsch (GB) approximation

$$|M_{gg \rightarrow ggg}|^2 = \left( \frac{72\pi^2 \alpha_s^2 s^2}{(q_s^2 + m_D^2)^2} \right) \left( \frac{48\pi\alpha_s q_s^2}{k_s^2 ((k_s - q_s)^2 + m_D^2)} \right).$$

BAMPS studies have shown that especially the implemented inelastic $2 \leftrightarrow 3$ processes are responsible for achieving a thermalization time on the order of less than $\tau \approx 1$ fm [18]. As a remark, recent studies [23] show that the “original Gunion-Bertsch” matrix element is not truly valid in the forward and backward rapidity region of the emitted gluon. To this end an improved GB matrix element is proposed that shows a good agreement with the exact pQCD matrix element over the whole phase space. Although we expect a significant change in the partonic energy loss, the implementation of the “improved GB” matrix element into the BAMPS framework is still in progress and we will stick within this present paper to the “original GB” matrix element, also to allow to relate our results to previously published findings on the nuclear modification factor $R_{AA}$ obtained with the original GB approximation.

For modeling the Landau-Pomeranchuk-Migdal (LPM) effect, which is assumed to be an important quantum effect within a partonic QCD medium, an effective cutoff function $\theta (\lambda - \tau_g)$ in the radiative matrix elements is used where $\lambda$ is the mean free path of the jet particle and $\tau_g$ the gluon formation time. This effectively allows only independent inelastic scatterings. Any divergences within the matrix elements are screened by a Debye mass, which is dynamically computed on the basis of the current parton distribution [18].

Although the scattering processes of heavy quarks [24–26] are already implemented within BAMPS, throughout this work only scatterings of gluons, light quarks and anti-quarks are considered ($N_f = 3$). This is motivated by the rare production of initial heavy quarks and therefore minor influence of heavy quark jets in the analysis of the reconstructed jet momentum imbalance.

The parameters of BAMPS, which can be varied in reasonable ranges, are the freeze-out energy density $\epsilon$ and
the fixed-valued, strong coupling constant \( \alpha_s \) which alters the strength of the pQCD scattering processes. Based on studies investigating the elliptic flow \( v_2 \) \cite{27, 28}, the parameter set \( \epsilon = 0.6 \text{ GeV} \) and \( \alpha_s = 0.3 \) is chosen throughout this paper. However, the implementation of the running of the strong coupling constant \( \alpha_s \left( Q^2 \right) \), as it is already implemented for heavy quark interactions \cite{25}, is currently investigated and results of the implementation will be published in a separate publication soon.

Simulations by BAMPS with the chosen parameter set show a nuclear modification factor \( R_{AA} \) which overestimates the suppression of particles by a factor 4-5 \cite{28}. This is mainly caused by particular particle configurations which result from the complex interplay between the employed effective modeling of the LPM effect and the mentioned radiative matrix element in “original Gunion-Bertsch” approximation \cite{29}. However, as already pointed out we expect significant changes in the partonic energy loss due to the improvement of the GB matrix element.

Besides the study of the nuclear modification factor \( R_{AA} \) and the elliptic flow \( v_2 \) of light partons \cite{28, 30} as well as heavy quarks \cite{24, 28}, BAMPS also offers the possibility to study other observables concerning for example Mach cone structures \cite{31} or the viscosity over entropy ratio \( \frac{\nu}{s} \) \cite{32}. For more detailed information about BAMPS and different used parameter sets we refer to Refs. \cite{18, 29, 33}.

### III. MODELING OF VACUUM PARTON SHOWERS

#### A. Virtual splitting processes in an on-shell transport model

As it was observed in Ref. \cite{7, 9}, even in p + p collisions, where no medium creation is expected, a momentum asymmetry of the reconstructed jets with the two highest transverse momenta is found. This momentum imbalance is on one hand caused by the probabilistic nature of the independent vacuum splitting processes of the virtual high \( p_t \) partons, which are produced in the initial nucleon-nucleon collisions: The evolution in virtuality leads to different out-of-cone splittings of the two back-to-back partons and therefore to an imbalance in the reconstructed jets and jet momenta, respectively. On the other hand, also effects caused by the used jet reconstruction algorithm and its efficiency in reconstructing jets have an influence on the momentum imbalance. For example, by using a smaller cone radius less particles of the showers are reconstructed within the jets and therefore the momentum imbalance is enhanced.

In this work, partons are not only evolved within vacuum but also within a partonic medium. Therefore a simultaneous treatment of virtual splittings as well as medium-induced scattering processes is in principle necessary. However, because BAMPS is an on-shell transport model, the implementation of virtual splitting processes is not trivial. For that reason we assume a temporal separation between the initial virtual splittings within the vacuum and the subsequent in-medium scatterings. For modeling the virtual splittings we use the final-state shower routine PYSHOW of the event generator PYTHIA (version 6.4.25) \cite{19}.

The kinematics of the initial shower-initiating parton pair are sampled according to parton distribution functions (PDF) with a minimum transverse momentum of \( p_t,0 = 100 \text{ GeV} \) and leading order pQCD cross sections. Comparisons between different PDF show that the influence of the used parametrization on the momentum imbalance of reconstructed jets is modest. Throughout this paper, we use the PDF parametrization by GLÜCK, REYA and Vogt (GRV) \cite{34}. The generated back-to-back di-parton pairs are subsequently evolved within the PYSHOW routine. As a side remark, for ensuring overall momentum and energy conservation within one PYSHOW event it is important to consider both showering partons within one PYSHOW run instead of two single shower-initiating partons in two separate events. This method of sampling the initial partons with PDF and subsequently showering with PYSHOW is denoted in the following study as the “PDF+PYSHOW” method.

The shower evolution within PYSHOW is based on the differential splitting probability

\[
\frac{dP_n}{dz} = \sum_{b,c} \frac{\alpha_s}{2\pi} P_{n \rightarrow b,c}(z) dz
\]  

for parton \( a \) to split into partons \( b, c \), where parton \( b \) takes the fraction \( z \) of the parent four-momentum. As used in Q-PYTHIA \cite{35}, the maximum allowed virtuality of a radiating parton and therefore the starting point in virtuality of the evolution is chosen as \( Q_{\text{max}} = 4E_{\text{parton}} \). Due to the subsequent multiple splitting processes the parton virtuality decreases. In the standard version of PYSHOW this evolution stops when the single parton virtuality reaches \( Q_{\text{min,standard}} = 1 \text{ GeV} \) \cite{19}. The assumed separation between vacuum splittings and in-medium scatterings suggests a premature termination of the virtual splittings. Such a termination is more descriptive if it is done in time instead of parton virtuality. Therefore the shower evolution in virtuality is related to an evolution in shower time by the formation time \( \tau_f \) of the shower-initiating parton \cite{19}. Based on the uncertainty principle, the formation time of a parton in the lab frame is

\[
\tau_f = \frac{E}{Q^2},
\]

where \( E \) is the energy and \( Q \) the virtuality of the initial shower-initiating parton \cite{36}.

Defining a global termination time \( \tau_f \), one can fix a minimum virtuality under which the partons are forbidden to split further and thereby control the vacuum shower evolution. Depending on the used termi-
nation time, this limit in virtuality is above the standard $Q_{\text{min;standard}} = 1 \text{ GeV}$. In this work we use a termination time $\tau_f = 0.2 \text{ fm}$ which corresponds e.g. to a minimum virtuality $Q_{\text{min}} \approx 25 \text{ GeV}$ for a parton with $E = 100 \text{ GeV}$. Cross check studies within a static BAMPS simulation show that the influence of the used termination time on the momentum loss of reconstructed jets is rather modest for values of $\tau_f \approx 0.1 \text{ fm} - 2 \text{ fm}$.

Because BAMPS is an on-shell transport model the partons evolved within BAMPS have to fulfill the energy-mass relation $E^2 = p^2 + m^2$, which reads $|E| = |p|$ for massless partons. However, the finite virtuality of the shower particles violates this energy-mass relation. Curing this problem while simultaneously conserving energy and momentum is not possible. Nevertheless, in order to simulate the virtual partons within BAMPS, we modify the parton kinematics by setting them on the mass shell. To this end, we conserve the directions and values of the (three-)momenta but adjust the energies of the particles according to $|E| = |p|$ in order to fulfill the energy-mass relation.

B. Consideration of detector effects

Naive comparisons of the $A_J$ distribution of unterminated PDF+PYSHOW events without medium evolution or full, hadronic PYTHIA events with the $\sqrt{s} = 2.76 \text{ TeV}$ $p + p$ $A_J$ distribution measured by CMS [8] show that neither can describe the measured data out of the box. This is caused by the effect that the detectors themselves have only a finite resolution. This finite resolution leads to fluctuations in the reconstructed jets which results in an enhancement of momentum imbalance absent in the simulations and thereby to a difference between jets reconstructed on generator level and detector level.

As a first step for modeling the detector effects we reconstruct jets within this paper at “calorimeter level” and not at “particle level”. This is done by employing a “calorimeter”, which is modeled by a grid in rapidity $y$ and azimuthal angle $\phi$, in which the final particles of each simulation are sorted depending on their $y$ and $\phi$.

The size of the grid cells is based on the cell size of the CMS calorimeters. In doing so we employ no trigger condition for the individual particle transverse momentum like it should be done in principle when comparing with CMS data because of their strong magnetic field prohibiting particles with $p_t \lesssim 1 \text{ GeV}$ to reach the calorimeters. However, because of the employed “particle-flow” algorithm by CMS [37], we assume that also information of these particles with low momenta enter the finally reconstructed jets.

Moreover, motivated by theoretical calculations [10] and the detector response analysis by CMS [38], we use the following strategy for mimicking additional detector effects: We apply an independent smearing procedure on the leading and subleading jet momenta $p_{t;1}$ and $p_{t;2}$, which is based on a Gaussian function $N(p_{t;1}, c\sqrt{p_{t;1}})$ with width $\sigma = c\sqrt{p_{t;1}}$ and fit parameter $c$. This smearing procedure alters the two jet momenta independently, which leads to an additional amount of momentum asymmetry within one event. Assuming smooth $A_J$ distributions before smearing, the statistics of the simulations is enhanced by repeating the stochastic smearing $N_{\text{smeared}} = 1000$ times for each event.

Analytically this smearing procedure can be written as a folding between the initial jet momentum distribution $f(y_{ini}^{t;1}, y_{ini}^{t;2}, \phi_{ini}^{t;1}, \phi_{ini}^{t;2})$, where $y_{ini}^{t;1}$ is the initial transverse momentum, $y_{ini}^{t;2}$ the initial rapidity and $\phi_{ini}^{t;1}$ the initial azimuthal angle of the corresponding jet $i$, and Gaussian smearing functions $g(p_{t;1}^{\text{smeared}}, p_{t;1}^{\text{smeared}} - \Delta p_{t;1})$ with $p_{t;1}^{\text{smeared}}$ being the smeared leading/subleading jet momenta:

$$\frac{dN}{dA_J} = \int d\phi_1^{ini} \int d\phi_2^{ini} \int dy_1^{ini} \int dy_2^{ini} \int dp_t^{ini} \int d\tilde{p}_{t;1}^{\text{smeared}} \int d\tilde{p}_{t;2}^{\text{smeared}} \int \frac{dN}{dA_J} \bigg|_{A_J = a} f(p_t^{ini}, y_1^{ini}, y_2^{ini}, \phi_1^{ini}, \phi_2^{ini}) \delta \left(\phi_1^{ini} - \phi_2^{ini} - \pi\right) g(p_{t;1}^{\text{smeared}}, p_{t;1}^{\text{smeared}} - \Delta p_{t;1}) g(p_{t;2}^{\text{smeared}}, p_{t;2}^{\text{smeared}} - \Delta p_{t;2}) \Theta(p_{t;1}^{\text{smeared}} - p_{\text{Trigger;}1}) \Theta(p_{t;2}^{\text{smeared}} - p_{\text{Trigger;}2}) \Theta(y_{\text{Trigger;}j;ets} - y_{ini}^{t;1} - \Delta y_{ini}^{t;1}) \Theta(y_{\text{Trigger;}j;ets} - y_{ini}^{t;2} - \Delta y_{ini}^{t;2}) \Theta(|(\phi_1^{ini} - \Delta \phi_1) - (\phi_2^{ini} - \Delta \phi_2)| - \Delta \Theta_{\text{Trigger;}j;ets}) \delta(A_J (p_{t;1}^{\text{smeared}}, p_{t;2}^{\text{smeared}}) - a).$$

The medium modification of the jet momentum is described by $\Delta p_{t;i}$, while the functions $\Delta y_i$ and $\Delta \phi_i$ provide the deflection of the jets in rapidity and azimuthal angle due to the in-medium evolution. As a remark, the $\delta(\phi_1^{ini} - \phi_2^{ini} - \pi)$-function ensures an initial back-to-back parton direction in the transverse plane, while the $\theta$-functions provide the employed experimental trig-
For the shower events we create partonic show-
mer settings in which the energy loss is modeled by a
mon framework. This is different to most other energy
of a heavy-ion collision can be simulated within one com-
the jet energy loss as well as the medium bulk evolution
Sec. V are applied.

- The so fixed Gaussian smearing with the deter-
ained smearing factor is used in the following study as an
effective detector filter for the Pb + Pb results shown in
Sec. V. As a note, after applying the smearing procedure,
the unterminated PDF+Pyshow method also leads to a
leading jet distribution which is in agreement with the
experimental data [8].

IV. SIMULATION STRATEGY FOR PARTON
SHOWERS WITHIN BAMPS

A. Separation between shower and medium
evolution

In this section we introduce our method for simulating
the in-medium momentum loss of reconstructed jets
within a partonic transport model while employing a test-
particle ansatz. One of the advantages of BAMPS is that
the jet energy loss as well as the medium bulk evolution
of a heavy-ion collision can be simulated within one com-
mon framework. This is different to most other energy
loss calculations in which the energy loss is modeled by a
Monte-Carlo procedure embedded into a hydrodynamical
evolving background medium.

As described in Sec. I we simulate the medium evolu-
tion within BAMPS by numerically solving the 3+1D
Boltzmann equation for massless quarks and gluons. For
attaining this, it is necessary to employ a test-particle
ansatz [18]: the number of particles is scaled by a fac-
tor \( N_{\text{test}} \) (the number of test-particles per real particles),
while at the same time the cross sections and thereby
the scattering probabilities within BAMPS are decreased
by \( N_{\text{test}} \). Thus in total the interaction rate per particle
or the mean free path, respectively, is conserved but the
overall attainable statistics of the scattering processes is
significantly enhanced.

However, for studying reconstructed jets and high \( p_t \)
particles within BAMPS this scaling of particle numbers complicates the simulation: Jet reconstruction is obvi-
ously only reasonable if it is done event-by-event based
on real particles. This is not true when reconstructing
jets while test-particles are present. Due to this dif-
ficulties, we separate the simulation of the shower and
background regime. This allows us to simulate the back-
ground medium (\( N_{\text{test}} = 1 \)) and the shower particles
(\( N_{\text{test}} = 1 \)) with different test-particle numbers. Im-
portant to note is that we use for both regimes ex-
actly the same BAMPS interactions but only separate
the simulation into the actual energy loss and the evolu-
tion of the BAMPS medium. After the simulation both
regimes are appropriately combined again to form full
shower-medium events, which allows the reconstruction
of jets and even more the usage of experimental back-
ground subtraction methods. This combination of shower
and medium regime is described in Sec. IV B. As a re-
mark, besides the curing of the mentioned test-particle
issue, a separated simulation of the two regimes is ad-
ditionally useful for enhancing the statistics required for
rare processes like the production of high \( p_t \) partons.

Thus, our simulation strategy of parton showers within
BAMPS can be separated in the two following parts:

- For creating the underlying background event, we
  simulate heavy-ion collisions with fixed impact para-
  meter \( b \) within BAMPS with full 3+1D expansion
  and test-particle number \( N_{\text{test}} \) with no trigger for
  high \( p_t \) particles. As it is shown in Ref. [28], these
  events show a collective medium behavior in terms of
  the elliptic flow coefficient \( v_2 \) which is compatible
  with experimental data. At every timestep we
  keep track of every scattering process and the phase
  space information of every particle. With this in-
  formation it is afterwards possible to “offline recon-
  struct” this event and thereby the evolution of the
  expanding medium.

- For the shower events we create partonic show-
ers according to the introduced PDF+Pyshow
  method. These shower partons are real particles
  and thus have \( N_{\text{test}} = 1 \). As already noted, this is
  important for the subsequent reconstruction of jets
  where no test particles are supposed to be present.
  After showering and decreasing their virtuality we
  embed and subsequently evolve these “shower par-
  tons” within the recorded background event. The
  spatial insertion point of the initial shower parti-
  cles is chosen as the spatial creation point of the
  corresponding shower-initiating parton pair which is
  sampled by a Glauber modeling together with a
  Woods-Saxon density profile. During their evolu-
tion within the medium they are allowed to scatter
at every timestep with the recorded medium
particles by \( 2 \rightarrow 2 \) and \( 2 \leftrightarrow 3 \) processes with
the usual stochastic method employed within the

![Figure 1. \( A_J \) distribution for untermined PDF+Pyshow vacuum events with different smearing parameters in comparison with \( \sqrt{s} = 2.76 \) TeV p + p data [9]. All experimental trigger conditions as defined in Ref. [9] and mentioned in Sec. IV are applied.](image-url)
BAMPS framework. Similar to energy loss calculations within a hydrodynamical background the original evolution of the recorded bulk medium is not modified by the scattering processes between the shower and the medium particles.

In contrast to other models, we simulate within BAMPS not only the first scattering processes of the shower with the underlying medium but also the subsequent in-medium evolution of these shower partons as well as their scattering partners and gluons radiated in $2 \to 3$ processes. For considering these further in-medium scattering processes, the scattering products of shower-medium interactions—scattered medium particles and radiated gluons—become shower partons themselves. Thus scattered medium particles and radiated gluons are allowed to scatter again with other medium particles and evolve like the initial shower partons. This method is denoted in the following as the “multi-generation” scattering method.

However, these further scatterings can lead to a double counting of scattered medium particles: If the same medium particle is hit twice by the shower particles it would end up more than once as a shower parton. This is an effect caused by a finite number of test-particles: In the limit of infinite test-particle numbers this effect would be naturally cured since the probability of a repeated scattering with the same particle would vanish. To avoid this issue, while employing a finite number of test-particles, we assure that scattered medium particles become only shower partons when they scatter for the first time with a shower particle. Nevertheless, the actual scattering process takes place anyhow and changes the momenta of the outgoing shower parton. By doing so it is ensured that every scattered medium particle has only one trajectory within the medium and at the same time the effect of the medium on the momenta of the shower partons is preserved.

To compare our results with other theoretical approaches concerning the momentum loss of reconstructed jets, in which usually only the first scattering process with the medium is studied, we also define a “single-generation” scattering method. Within this method only the scattering of each initial shower parton with the medium is considered without simulating further interactions of the scattered medium particles. Additionally, also the gluons radiated by the shower partons are considered as shower particles as in the “multi-generation” method. Different to the “multi-generation” method where we simulated the whole shower evolution within the medium, we neglect within this method the evolution of scattered medium partons and its influence on the reconstructed jets. This is comparable to strategies pursued in e.g. MC energy loss simulations in a hydrodynamic background \cite{13} where also only the leading parton is tracked. As long as not noted otherwise, we use within this paper the “multi-generation” method.

As already mentioned, the scattering method within BAMPS employing stochastical probabilities is only meaningful while using the introduced test-particle approach. For that reason, scattering processes between shower particles, which are real particles and therefore have $N_{\text{test}} = 1$, are forbidden throughout this paper. In doing so one negects possible effects of collective behavior between the shower particles like e.g. Mach cone structures.

As an additional measure for separating between the shower and medium regime, the evolution of a shower parton is stopped when its transverse momentum reaches a minimum value $p_t;\text{cut}$. Added shower particles with less transverse momentum stream freely without any scattering process until the simulation ends. This cut in the shower evolution minimizes the numerical effort necessary for calculating many very low $p_t$ particles. These particles are supposed to be subtracted in experimental analyses anyhow since they are part of the background (cf. Sec. \ref{sec:results}).

As a side remark, by comparing the $A_J$ distribution of full hadronic PYTHIA $p+p$ events with partonic PYTHIA events one can show that the jetfinding procedure effectively removes any influence of the hadronization on the reconstructed jet momenta. This is an intended feature of modern jetfinding algorithms mainly facilitated by collinear and infrared safety. For that reason any effects of hadronization on the reconstructed jets are neglected throughout this paper.

The above described simulation strategy for parton showers is denoted within this paper as the “pure shower” simulation method.

In principle one could argue that the above described separation between shower and medium regime has an additional advantage for reconstructing jets: Jets can be reconstructed based on only the shower particles with no further need to consider any additional background subtraction method. However, we will see in Sec. \ref{sec:results} that despite the used momentum cut in the shower evolution—which is in principle arbitrarily selectable—the shower particles are able to decrease their energy and momentum by such an amount that there is no clean separation anymore between shower-medium and medium-medium interactions. This leads to an unassessable contamination of jets with background momenta that is not subtracted. This contamination with background momentum significantly contributes to the reconstructed jet momenta with the result that an appropriate background subtraction becomes crucial.

### B. Combination of shower and background medium

As mentioned in the previous subsection, one difficulty in reconstructing jets passing through partonic matter is the inevitable consideration of the underlying background medium and thereby a discrimination between jet and medium particles. This underlying background momentum is within BAMPS, as we will show in Appendix \ref{app:appA} of the order of $p_{t;\text{background}} \approx 240$ GeV within cones of
$\Delta R = 0.3$. In experimental studies, in which only information about the final particles is accessible, a suitable background subtraction method is therefore in any case needed and applied. Within theoretical simulations like BAMPS, information about all particles and their interactions is in principle available. Nevertheless, due to the equal treatment of medium and shower particles, also in BAMPS an appropriate subtraction of background momentum has to be discussed.

As we will show in Sec. V the introduced separation of the shower and medium simulations does not provide a clean separation between shower and medium partons for getting rid of the background contribution of reconstructed jets: When shower partons have lost a significant amount of energy/momentum, a clear distinction between shower-medium and medium-medium processes is no longer applicable. These processes contaminate the reconstructed jet momenta by an uncertain amount of background momenta. Therefore efficient and clean formulated methods for subtracting these background momenta become crucial in order to study the pure momentum loss of reconstructed jets. Although a microscopic handling of background subtraction seems to be possible we orientate us in the following on well-established background subtraction methods that are widely used in experimental data analysis. These subtraction methods are mainly motivated by estimating the average momentum density within a heavy-ion background medium and subtracting this density subsequently from the reconstructed jet momenta. Furthermore, the application of an experimentally used background subtraction scheme allows a consistent comparison to data.

Due to our chosen separation of shower and background event in the previous subsection, we first have to appropriately combine the shower and the medium part of our simulations to form full “shower+medium” events before employing experimental subtraction procedures. After the combination we are able to estimate correctly the underlying background momentum. To this end, we combine the previously simulated shower partons with the offline recorded background event, in which the scattered medium particles are removed. However, this embedding is not trivial: Due to the employed test-particle ansatz, the medium particles are test-particles ($N_{\text{test;medium}} \neq 1$) while the added shower particles are already real particles ($N_{\text{test}} = 1$) (cf. Sec. IV A). Thus, after simulating the event and before combining the two regimes we first have to scale both test-particle numbers to the same value. Because $N_{\text{test}} \neq 1$ identical jets within one event are not physical when subsequently reconstructing jets, we scale down our $N_{\text{test;medium}} \neq 1$ background medium to $N_{\text{test}} = 1$ while also the history of scattered medium particles is taken into account to prevent double counting. After this scaling, only real medium particles are left within the background event, which enables us to combine those with the shower partons to form full $N_{\text{test}} = 1$ “shower+medium” events. For more details about this scaling of test-particles and thereby generating appropriate background events see Appendix A where also the validity of this scaling is shown. This simulation strategy with considering the underlying background medium is denoted in the following as the “shower+medium” strategy.

Neglecting any hadronization effects, the so obtained events are similar to experimentally measured events. Thus one is now able to employ, for the first time, common experimental subtraction methods within a theoretical model. To subtract finally the background contamination from the reconstructed jets we employ the “CMS noise/pedestal subtraction method”. This algorithm is an iterative procedure consisting of estimating the background transverse momentum, reconstructing jets, excluding the reconstructed jets from the background estimation and again estimating the background. After some iterations the so obtained average background transverse momentum is subtracted from the finally reconstructed jets. For more information about this subtraction algorithm we refer to Ref. [40].

In summary, the simulation strategy introduced within this section can be sketched as shown in Fig. 2. Please note that this figure shows the complete workflow for simulating “shower+medium” events employing the “multi-generation” method.

V. SIMULATION RESULTS

After introducing our simulation strategy for parton showers within BAMPS in the previous section, we present our results for the momentum loss of reconstructed jets in terms of the momentum imbalance $A_J$. For this study we use full 3+1D, expanding BAMPS heavy-ion simulations of $\sqrt{s} = 2.76$ A TeV Pb + Pb collisions with test-particle number $N_{\text{test}} = 15$ and impact parameter $b_{\text{mean}} = 3.4$ fm. This mean impact parameter corresponds to an experimental centrality class of 0%-10% as determined in Monte-Carlo Glauber calculations [8]. The embedded parton showers are initialized by PDr+PYSHOW initial conditions ($\tau_f = 0.2$ fm) and afterwards evolved together with the medium particles by the “multi-generation” method as described in Sec. IV. Possible detector effects are considered by the methods described in Sec. III B. For the comparison with experimental data we employ for all results in this section the trigger conditions defined by CMS for their $A_J$ studies. These trigger conditions are shown in Tab. I.

A. Momentum asymmetry of reconstructed jets within BAMPS

As a cross-check of the employed sampling and subtraction procedure, we study the influence of the momentum cut $p_{\text{cut}}$ in the shower evolution (cf. Sec. IV) on the momentum imbalance $A_J$. To this end, Fig. 3 shows the $A_J$ distribution within BAMPS simulations for jets recon-
Figure 2. Flow chart showing the simulation strategy of parton showers within Bamps.

The momentum imbalance of the “pure shower” jets shows for $p_{t;\text{cut}} = 1\text{ GeV}$ a good agreement with the experimental data. However, this agreement is pure coincidence and is mainly caused by the contamination of background momentum within the reconstructed jets which is not accounted for by the missing background subtraction: A jet traversing the medium will lose energy and at the same time gather soft medium momenta within the cone by multiple further in-medium scatterings. Part of these momenta belong to the background medium, but are not subtracted in the end. This leads to a less pronounced momentum imbalance and thereby a better agreement with the data. This assumption is supported by the distribution for $p_{t;\text{cut}} \neq 1$. Depending on the momentum cut the momentum imbalance distribution can be arbitrarily shifted to lower or higher momentum imbalances: When using a higher momentum cut $p_{t;\text{cut}} = 1.5\text{ GeV} - 3\text{ GeV}$ the distribution is peaked at high $A_j$ values, while at low momentum cuts $p_{t;\text{cut}} = 1\text{ GeV}$ or $p_{t;\text{cut}} = 1.2\text{ GeV}$ the distribution is shifted to less momentum imbalance and closer to the experimental, background subtracted data. This shifting of the momentum imbalance shows again the higher contamination of jets by the underlying background momentum when using lower $p_{t;\text{cut}}$ value. Because this contamination is at this stage not subtracted the momentum imbalance gets less pronounced. Since the $p_t$ contamination is dependent on the number of scatterings of the jets it counteracts the actual momentum loss of the respective jet. Therefore parton showers that are allowed to scatter down to a lower transverse momentum regime show a less jet momentum asymmetry than shower evolutions with a larger cut.

Due to the dependence of the cone contamination on the number of in-medium scattering processes of the shower, a quantitative description of the background contamination is difficult. Therefore we employ the in Sec. [V] described method for scaling and combining the background event in order to employ experimentally established background subtraction methods. These events with “shower+medium” jets and a following background subtraction (Fig. 3b) show a stronger momentum imbalance after subtracting the background momenta. Furthermore, the momentum imbalance is now independent from the employed momentum cut in the shower evolution, which means that we are successful in efficiently subtracting the background contamination of the reconstructed jets for $p_{t;\text{cut}} \leq 3\text{ GeV}$. As a remark, the peak shapes of the imbalance distributions are caused and limited by the employed momentum trigger conditions and can be even more imbalanced when using different trigger conditions.

The efficient background subtraction leads to a momentum imbalance of the two leading jet momenta that is too strong compared to the experimental results. This momenta...
To compare our results with other common theoretical approaches concerning the momentum loss of reconstructed jets, we show in Fig. 4 the momentum imbalance of again “shower+medium” jets with background subtraction and “multi-generation” scatterings in comparison with the momentum imbalance resulting from jets reconstructed based on the “single-generation” method without any additional background subtraction. As a reminder, within the “single-generation” method we completely neglect the further evolution of scattered medium particles. This is comparable to the strategy pursued in MC energy loss studies in a hydrodynamic background where also only the leading parton is tracked. This method can be understood as the maximum possible (theoretical) microscopic background subtraction. However, as seen in Fig. 4 the agreement between the $A_J$ distributions of both scattering methods within BAMPS is good. This shows on one hand again that if one simulates the full in-medium shower evolution, as it is probably realized in nature, an appropriate treatment of the background momenta is crucial. On the other hand it demonstrates that while considering only the first scattering process of the shower partons an additional consideration of background subtraction is not essential for the momentum imbalance $A_J$. This means that the momentum imbalance distribution is dominated by the first shower-medium interaction and the influence of further in-medium scattering processes on the jet momentum loss is weakened by the subsequent background subtraction. However, as already mentioned before, the strong partonic energy loss together with the employed trigger conditions lead to the observed, peaked shape of the momentum imbalance distribution within BAMPS. After implementing the “improved GB” matrix element and thereby a more realistic jet momentum imbalance, a difference in the momentum imbalance of the “single-generation” and “multi-generation” method is possible and will be investigated in a future study. Moreover, since the background subtraction only affects the transverse momentum of the reconstructed jets within BAMPS, other observables like e.g. $\Delta \phi_{\text{jets}}$ distributions or jet shapes can provide a discrimination between the “single-generation” and “multi-generation” method.

B. Relation between initial partons and reconstructed jets

Simulations within transport models additionally benefit from the advantage of providing the full phase space information of every particle underlying the jet reconstruction. Despite the too strong momentum imbalance of the reconstructed jets within BAMPS employing the “original GB” matrix element, such studies can still provide a further understanding of the underlying mechanisms causing the momentum loss. For that reason we investigate the momentum loss of reconstructed jets by comparing the jet transverse momenta with the trans-
verse momenta of the initial hard scattered, unshowered partons of the nucleon-nucleon collisions. Fig. 4 shows this average momentum loss $\Delta p_t = p_{t,\text{jet}} - p_{t,\text{init. parton}}$ depending on the initial parton transverse momentum for the leading and subleading jets while employing again all experimental trigger conditions. Additionally, we show the momentum loss of the reconstructed jet with the third highest transverse momentum. This third jet can arise either by a hard gluon, which is radiated in initial vacuum splittings or in the subsequent in-medium evolution, or by shower particles resulting from the momentum loss of the two leading showers that are not reconstructed within the leading jets.

The momentum loss of the subleading jet and the third jet are constant over the whole considered $p_t$-range: While a subleading jet loses approx. 60% of its momentum, the third jet loses already approx. 80% of its momentum with respect to its initial shower-initiating parton. On the contrary, the leading jet shows a different momentum loss behavior: At lower parton momenta the leading jet has to gain up to 20 GeV to pass the employed trigger conditions which results in a gain of jet transverse momentum up to $p_{t,\text{initial parton}} \approx 120$ GeV.

The momentum loss of the subleading jet is a decreasing, almost linear function of $A_J$ with a momentum loss of $\approx 60\%$ at $A_J = 0.4$. In contrast, the dependence of the leading jet momentum loss from the initial parton momentum almost vanishes. This means that the observed momentum imbalance within BAMPs is mainly caused by the momentum loss of the subleading jet while the leading jet momenta consist of nearly the whole initial parton momentum. Again the reconstructed jet with the third-highest momentum suffers independently from the initial parton momentum a momentum loss of approx. 80%. This independence is mainly caused by the fact that the momentum imbalance $A_J$ only triggers on the two leading jet momenta and not on the third jet momentum.
Besides the momentum information of the initial particles, also their spatial positions are at hand. For this reason it is possible to investigate the underlying microscopic path length dependence responsible for the respective momentum imbalance. Together with the creation vertices of the initial partons and their initial direction in the transverse plane, one can geometrically define a distance between the creation point of the parton pair and the point where the partons leave the initial, almond-shaped collision zone. For more details about this definition of the transverse in-medium path length cf. Appendix C. By comparing the two distances \( L_i/L_s \), where \( L_i \) (\( L_s \)) is the longer (shorter) transverse in-medium path length of the two partons, one can define analogously to the momentum imbalance \( A_J \), the transverse length imbalance

\[
L_i = \frac{L_i - L_s}{L_i + L_s}.
\]  

(7)

Lower \( L_i \) values correspond to events in which the two initial partons have to traverse a similar transverse distance within the medium, while events with higher \( L_i \) are events in which one of the partons traverses more medium than the other in the transverse direction.

Fig. 7 shows the momentum imbalance for different length imbalance ranges. For di-jets with a similar transverse in-medium path lengths and thus a smaller \( L_i \), the momentum imbalance distribution emerges as flatter than the previously presented distributions. However, with increasing difference in the in-medium path lengths, the momentum imbalance again peaks at high \( A_J \) values, almost independent from the considered length imbalance region. This is again caused by the too large partonic energy loss within BAMPS, which leads to an insensitivity of the underlying path length dependence of the momentum loss. However, with a potentially lower partonic momentum loss resulting from the “improved GB” matrix element a stronger sensitivity of the momentum imbalance on the in-medium path length difference is expected.

VI. SUMMARY

Within this paper we presented the first results on reconstructed jets within the partonic transport model BAMPS. Heavy-ion simulations by BAMPS benefit from the possibility to calculate the full evolution of both the high \( p_t \) particles and the softer bulk medium in one common approach employing the same interactions. In contrast to other theoretical approaches, we simulate thereby not only the first scattering of jet shower particles with the medium but also subsequent further in-medium scatterings of both the shower and scattered medium particles. Because it is not a priori obvious that the first shower-medium interaction dominates the reconstructed jet momentum loss, the investigation of the full in-medium evolution of the parton shower is essential. While considering these “multi-generation” scattering processes, we show that it is important to appropriately subtract the contribution of the bulk medium to the reconstructed jets in order to compare with experimental data.

The employed test-particle approach complicates the simulation and reconstruction of jets within BAMPS. Nevertheless, we presented a method to simulate the momentum loss of reconstructed jets by dividing the simulation within BAMPS into the medium and the shower evolution. For reconstructing jets and afterwards subtracting any background momentum from the jets, we showed how to appropriately scale test-particle numbers and subsequently combine again the shower and medium regime in order to build full “shower+medium” events consisting of only real, physical particles.

Based on this procedure, we showed our results on the momentum imbalance \( A_J \) while ensuring a consistent comparison to data by facilitating for the first time the same background subtraction methods as used in the experimental analysis. For modeling the initial virtual splitting processes within an on-shell transport model like BAMPS, we used terminated di-jet showers generated by the PYSHOW routines of PYTHIA. Furthermore, to mimic detector effects we defined a filter based on comparisons to \( p + p \) data. The partonic energy loss of di-jet showers within expanding BAMPS media leads to a momentum imbalance which is too strongly enhanced with respect to data from central \( \sqrt{s} = 2.76 \text{A TeV} \) Pb + Pb collisions measured by CMS. This too strong momentum imbalance is consistent with studies of the single inclusive suppression of hadrons within BAMPS and is mainly caused by a strong partonic energy loss and gluon radiations by large angles resulting from the “original Gunion- Bertsch” matrix element. Due to corrections at forward and backward rapidity of the emitted gluon within the proposed “improved Gunion-Bertsch” matrix element we expect...
after its implementation significant changes not only for the energy loss per collision but also for the interaction rate of particles towards a less total momentum loss.

Furthermore, we investigated the influence of further in-medium scattering processes on the momentum imbalance $A_J$. To this end, we compared the “multi-generation” scattering method, where the full in-medium evolution of both the shower and medium particles is simulated, with a “single-generation” method that only considers the evolution of the leading parton together with its radiated gluons. This “single-generation” method is comparable with common other jet momentum loss calculations. We have shown that when reconstructing jets based on full evaluated showers in the “multi-generation” method a background subtraction becomes important. Moreover, we demonstrated that the momentum imbalance of reconstructed jets resulting from the “single-generation” method without an additional background subtraction is consistent with the “multi-generation” method accompanied by a background subtraction. This means that the first interaction between shower and background medium dominates the resulting momentum imbalance of reconstructed jets and the influence of further in-medium scattering processes is weakened by the subsequent background subtraction. However, we expect that future studies concerning other reconstructed jet observables, like e.g. $\Delta \phi_{\text{jet}}$ distributions or jet shapes, will lead to a discrimination between the “single-generation” and “multi-generation” method and thereby provide a further understanding of the role of the multiple in-medium scattering processes.

Besides the momentum imbalance $A_J$ we studied the relation between the reconstructed jets and their initial shower-initiating partons while employing the all experimental trigger conditions. Independent from the initial parton momentum, the reconstructed subleading jet loses $\approx 40$% of momentum with respect to its initial parton momentum in the given setting of the microscopic interactions. Investigations of the momentum loss causing the momentum imbalance showed that the subleading jet loses an amount of momentum proportional to the momentum imbalance $A_J$. Simultaneously, the leading jet loses almost no momentum apart from effects caused by the employed trigger conditions. The too strong energy loss within BAMPS with the “original GB” matrix element leads to only a modest sensitivity of the jet energy loss on the underlying in-medium path lengths.

In conclusion, we introduced a method for appropriately simulating reconstructed jets within the partonic transport model BAMPS. After implementing the “improved GB” matrix element and thereby an expected lower partonic energy loss, the presented method provides the possibility to study in a future publication further observables regarding reconstructed jets, for example, their azimuthal angular distribution or their shapes, which will offer additional insight in the microscopic interactions of high energy particles in the hot and dense QCD matter.

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Appendix A: Discussion of test-particle corrected background events

1. Sampling of test-particle corrected background events

As described in Sec. [IV] due to the employed test-particle ansatz one has to scale appropriately BAMPS background events ($N_{\text{test}} \neq 1$) before combining them with shower events ($N_{\text{real}} = 1$). Therefore it is necessary to determine which medium particle is real and which is a test-particle. This discrimination is achievable by stochastical methods: Whether a particle is real or a test-particle is decided by Monte-Carlo sampling.

Before going to the actual scenario of events with shower and medium interactions, we first discuss the sampling probabilities of a simpler scenario of BAMPS events without any additional shower partons. These events only consist of medium particles with test-particle number $N_{\text{test}} \neq 1$ and a combination of shower and medium part, which complicates the sampling procedure, is not needed. Within this simple scenario, the probability for a medium particle to be real is equally distributed among the medium particles. Therefore, this stochastical probability obviously reads

$$P_{\text{ref}} = \frac{N_{\text{real}}}{N} = \frac{1}{N_{\text{test}}},$$

where $N_{\text{real}}$ is the number of real particles, $N$ the total number of particles and $N_{\text{test}}$ as before the number of test-particles per real particles in this event. We will denote this simple sampling case in the following as the “reference sampling method”.

In contrast, when considering events in which the medium particles are scattered by added shower partons, the described reference sampling scenario and its probabilities cannot be applied anymore: Some of the medium particles have interacted with the shower during the simulation and became therefore shower partons themselves. If one simply uses the reference sampling
probabilities, some scattered medium particles could be sampled successfully as real medium particles and would end up therefore as a sampled medium particle as well as a shower particle in the same “shower+medium” event. To avoid this possible double counting, only medium particles that were not scattered by any shower parton are considered in the sampling process and may become real medium particles.

However, the exclusion of scattered medium particles from the sampling procedure leads to an overestimation in the sampling probabilities of the unscattered particles: Because there are already $N_{\text{scatt}}$ scattered, real medium particles, which were “chosen” during the shower simulation, only $N_{\text{real}} - N_{\text{scatt}}$ particles are still allowed to become real. Thus this exclusion has to change the sampling probabilities and becomes especially important in regions in which many scattered medium particles are present. To avoid the overestimation of medium particles, the sampling probabilities in events with added shower partons have to be modified depending on the number of scattered—and thereby already real—medium particles nearby in phase space. To account for this local dependence, the probabilities are calculated based on equally sized cells in rapidity $y$ and azimuthal angle $\phi$. This choice of cells is motivated by thinking of the experimental jet reconstruction based on calorimeter cells. The probability for an unscattered medium particle to be real within such a cell then reads

$$P_{\text{local}} = \frac{N_{\text{real;cell}} - N_{\text{scatt;cell}}}{N_{\text{cell}} - N_{\text{scatt;cell}}} = \frac{N_{\text{cell}}/N_{\text{test}} - N_{\text{scatt;cell}}}{N_{\text{cell}} - N_{\text{scatt;cell}}} = \left( \frac{A_2}{A_1} \right),$$

where $N_{\text{real;cell}} = N_{\text{cell}}/N_{\text{test}}$ is the number of real particles, $N_{\text{cell}}$ the total number of medium particles—including scattered and unscattered—and $N_{\text{scatt;cell}}$ the number of scattered medium particles within the respective cell. We will denote this sampling method in the following as the “local sampling method”. The size of the used cells is a crucial ingredient for the sampling procedure: As already visible in Eq. (A2) a too small cell size could lead to divergent or negative probabilities, while a too coarse cell size cannot correctly resolve the shower region in the $y$-$\phi$ space.

After sampling for every unscattered medium particle whether it is real or not, we end up with a pure $N_{\text{test}} = 1$ background event without any scattered medium particle. For obtaining the final “shower+medium” events we add these sampled medium particles to our previously simulated real shower partons, which consist of both the initial shower partons and scattered medium particles that became shower particles during the in-medium evolution. In this way we attain finally an event with only real particles left. Based on these events we are now able to calculate the average transverse momentum density and thereby employ common, well-established experimental subtraction methods.

### 2. Comparing different sampling methods

To check our proposed scaling techniques we want to compare in this appendix section events based on the local sampling method with events sampled by the reference sampling. Furthermore, the scaling of events to $N_{\text{test}} = 1$ will introduce uncertainties into the analysis of reconstructed jets. Before studying the subtracted jets it is therefore crucial to estimate these additional uncertainties of the used sampling algorithms.

For investigating the uncertainties of our sampling methods we study the summed transverse momentum within cones in $y - \phi$ that consist of only $N_{\text{test}} = 1$ medium particles. To create such test events we use offline recorded BAMS events that were traversed by a parton shower (cf. Sec. IV). Afterwards we sample for each unscattered medium particle whether it is real or not by the local sampling method. Instead of adding the sampled real medium particles to the shower partons to obtain a $N_{\text{test}} = 1$ “shower+medium” event, we subsequently combine in this section the sampled medium particles with the scattered medium particles before their respective interaction with the shower. Thus we should end up with events with only $N_{\text{test}} = 1$ medium particles while no original shower partons are present.

The so obtained events can be compared to the pure background events in which no shower was present and therefore the reference sampling method is applicable. As already pointed out in the previous section, the reference sampling is the correct method for scaling down the number of test-particles in events without any additional shower. Thus if we compare the locally sampled pure medium event to the sampled medium events with the reference method we get an estimate of the effects of the sampling procedure on the amount of transverse background momentum.

Because our main goal is to create an appropriate jet background we investigate the uncertainties in terms of background jet momenta. Since this sampling study should be understood as rather qualitative, it is assumed to be sufficient to calculate the transverse momentum within fixed jet cones instead of fully reconstructing jets. Therefore we neglect additional effects to the background momenta introduced by the full reconstruction algorithms. The energies and momenta of the jet cones are evaluated by summing up all four-momenta of sampled medium particles, which are located inside a cone with radius $\Delta R = \sqrt{\Delta \phi^2 + \Delta y^2}$ in the $y - \phi$ plane around a fixed axis. This choice of simple fixed cone “jet definition” is motivated by the first-generation jet cone algorithms, which were based on the idea of a conserved direction of energy flow.

Since we are interested in the different effects of the sampling procedures on the background cone momenta for the shower and the non-shower region, we investigate two different jet cone axes: an axis directed in the initial hard parton direction denoted as the “in-shower direction” and an axis with same azimuthal direction but dif-
shows the distribution of summed background momenta within a cone of $\Delta R = 0.3$ for the reference and the local sampling method. Shown are the distributions for the out-of-shower direction (Fig. 8a) and the in-shower direction (Fig. 8b).

Appendix B: Definition of in-medium path lengths

For studying the dependence of the momentum imbalance $A_J$ on the difference of the in-medium path lengths of the initial partons we defined in Sec. VIII the transverse length imbalance parameter $L_t$. To this end, a clear definition of the transverse in-medium path lengths is essential. Because BAMPS provides full phase space information including the spatial coordinates of every particle, one could in principle track the correct in-medium path length of each single parton. However, the definition of a path length of an object consisting of multiple particles, like e.g. a reconstructed jet, is difficult. Since the length imbalance $L_t$ measures only the relative difference between the path lengths, it is though possible to define the distances traveled by the partons not in terms of the actual in-medium path lengths but as the distance from the creation point of the initial parton pair to the edge of the collision zone $L_{i/s}$, in direction of the initial transverse parton momentum. Neglecting any transverse expansion, the difference between those distances estimates qualitatively the subsequent difference of the in-medium path lengths. Because we are interested in jets traversing the medium at mid-rapidity ($|y| < 2$), we neglect, as a first estimate, any longitudinal path length dependence and focus on the trajectories in the transversal plane.

For estimating the transverse spatial extension of the initial medium, we use an almond shape resulting from the collision of the two approximately circular nuclei approaching at impact parameter $b$. This picture is

Figure 8. Distribution of the background momentum $p_{t;cone}$ within a cone of $\Delta R = 0.3$ for the reference and the local sampling method. Shown are the distributions for the out-of-shower direction (Fig. 8a) and the in-shower direction (Fig. 8b).
schematically sketched in Fig. 9. The radii of these nuclei is estimated by the radius of the employed Woods-Saxon density profile and is parametrized by

$$R_A = 1.12 A^{1/3} - 0.86 A^{-1/3};$$  \hfill (B1)

which is e.g. $R_A \approx 6.49$ fm for $^{208}$Pb. With that radius and the impact parameter, the two nuclei shapes can be described by circles obeying

$$(x - b/2)^2 + y^2 = R_A^2.$$  \hfill (B2)

Assuming that the partons will fly on straight lines through the medium, which is a good approximation of high energy jets, their trajectory can be described in terms of simple geometry by

$$\left(\begin{array}{c} x \\ y \end{array}\right) = k \left(\begin{array}{c} p_x^0 \\ p_y^0 \end{array}\right) + \left(\begin{array}{c} x_p \\ y_p \end{array}\right),$$  \hfill (B3)

where $p_x^0 = \frac{p_x}{p_T}$ and $p_y^0 = \frac{p_y}{p_T}$ are the normalized transverse momentum components and $(x_p, y_p)$ the spatial creation point of the initial parton pair and $k$ is a proportionality factor.

With that information it is possible to calculate the intersection points between the two nuclei and the parton trajectory and thereby the exit points of the partons from the collision zone. Defining

$$\Delta x_{L/R} = x_p \pm b/2,$$  \hfill (B4)

one can show that the proportionality factor at which the line and the circles intersect, is

$$k_{L/R} = \pm \sqrt{(\Delta x_{L/R} p_x^0 + y_p p_y^0)^2 - \Delta x_{L/R}^2 - y_p^2 + R_A^2 - (\Delta x_{L/R} p_x^0 + y_p p_y^0)^2},$$  \hfill (B5)

where $k_{L/R}$ are the respective factors for the intersection with the left ($k_{L}$) and the right ($k_{R}$) circle. Because of the previous normalization of the parton momentum vector, the absolute values $|k_{L/R}|$ of the proportionality factors are the in-medium path lengths of the initial parton pair. As a remark, obviously every circle can have two intersection points. Depending on its corresponding $x$-value one has to decide which intersection point is used for determining the respective path length. Having done that, one is able to define the shorter path length as $L_s$ and the longer path length as $L_l$.

Because of the employed Woods-Saxon density distributions, there is a finite probability for events in which the parton pair is created outside the collision zone. In this case two scenarios are possible: One parton A will fly through the medium and parton B will fly through no medium at all. In this scenario, parton A has path length $L_l \neq 0$ fm and parton B has path length $L_s = 0$ fm. This will lead to a length imbalance of $L_i = 1$. The other possible scenario is that both partons will not pass the collision zone and thus get path lengths $L_{l/s} = 0$ fm. Obviously, this scenario leads to a length imbalance of $L_i = 0$.
