Cross sections of removal reactions populating weakly-bound residual nuclei

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In many instances, single nucleon removal reactions from neutron-proton asymmetric projectile nuclei populate final states in the residual nuclei that are very weakly bound. Familiar examples include neutron removal reactions from neutron-rich $^{11}$Be and $^{12}$Be, the latter populating the well-known 1/2+ halo ground-state and 1/2− excited-state of $^{11}$Be - both states less than 1 MeV from the first neutron-decay threshold. Numerous additional examples arise in reactions of asymmetric $p$- and $sd$-shell nuclei. The importance of this weak residue binding upon calculated single-nucleon removal reaction cross sections is quantified by means of model calculations that neglect or include the dissociation degree of freedom of the residual nuclei. The calculated removal-reaction cross sections for two representative $p$-shell projectiles indicate that an explicit treatment of these residue break-up effects is unnecessary and that the differences between the break-up and no break-up calculations are small provided a consistent description of the residue structure and density is used.

I. INTRODUCTION

The study of direct reactions in which a single nucleon is removed from a projectile nucleus continues to be instrumental in extracting spectroscopic information for exotic nuclei. Specifically, they contribute to the study of single-particle degrees of freedom and help identify the active valence single-particle orbitals near the Fermi-surfaces, their ordering and their occupancies. A primary motivation of such direct-reaction experiments and analyses is to infer quantitative spectroscopic information on rare, neutron-proton asymmetric nuclei. Extracting such information relies upon the ability of the combined direct-reaction and nuclear structure models used to calculate absolute cross sections with a fair degree of reliability. Whether the direct reactions used are intermediate-energy nucleon removal (knockout) reactions, the mechanism of interest here, or lower-energy single-nucleon transfer reactions, each projectile initial-state to residual nucleus final-state transition is quantified by comparing the measured final-state partial cross section with that computed with a direct-reaction model that: (a) includes sufficiently accurately (but usually approximately) the dominant reaction mechanisms, and (b) includes an initial- to final-state single-particle overlap function obtained from appropriate nuclear structure model calculations.

In the case of intermediate-energy single nucleon removal reactions, the eikonal direct-reaction dynamical model considered here, for details see for example Ref. [1] and references therein, has, typically, been combined with nuclear shell-model overlaps: that is the spectroscopic factors together with assumed radial form-factors of the removed nucleons [2]. Within this methodology, for reactions of a mass $A$ projectile with a target of light nuclei, usually Be or C, all details of the interaction of the residual, mass $A−1$ nucleus with the target are described by its eikonal-model elastic S-matrix, $S_A(b_r)$, computed as a function of their collision impact parameter $b_r$. It is this input to the reaction dynamical model that carries information on the size, structure and binding of the residual nucleus through the refractive and absorptive nature of the residue-target interaction.

In many cases, particularly when the projectile nuclei approach the drip-lines, the residual nuclei and/or their excited states populated by nucleon removal are only very weakly bound and so have spatially-extended wave functions. Examples include the neutron removal reactions from neutron-rich $^{11}$Be and $^{12}$Be, populating the 1− and 2− excited states of $^{10}$Be near 6 MeV [3], and the well-known 1/2+ halo ground-state and 1/2− excited-state of $^{11}$Be [4], respectively. Typically however, removal reaction analyses [2] have exploited conventional optical limit (OL) eikonal-model calculations for the residue-target S-matrix in which the residue structure is represented by parameterised neutron and proton one-body density functions - usually derived from Hartree-Fock calculations, empirical information on nuclear mass and/or charge radii, or from $(A,Z)$ size systematics. This density-based approach neglects the break-up degree of freedom of the populated residue final states.

An early mention of the possible effects of these break-up channels upon the residue-target elastic S-matrix [5] suggested an increased residue-target absorption - leading to a reduced residue survival probability and a reduced removal-reaction cross section. However, in that discussion, these derived cross section reductions were computed relative to no break-up, OL calculations that used only a simple (Gaussian) parameterised residue density and not the actual density computed consistently from the structure of the weakly-bound residue - information that was included in the comparison few-body calculations that included break-up degrees of freedom.

We present a more considered assessment of the importance of such break-up of weakly-bound residual nuclei on calculated single-nucleon removal reaction cross sections. Here, simplified but realistic model calculations that: (i) neglect, and (ii) include the break-up degree of freedom of
the residue, and that treat the residue structure in a consistent way, are compared. The $^9$C($-p$) and $^{10}$C($-n$) reactions, which involve removal of a weakly-bound proton, $S_p = 1.3$ MeV, and a strongly-bound neutron, $S_n = 21.3$ MeV, respectively, and that populate the weakly-bound $^8$B and $^9$C residues, with proton separation energies 0.137 MeV and 1.3 MeV, are used as representative test cases. Since only the ground states of these residues are particle bound, inelastic excitations other than break-up do not raise complications. We show that when the two sets of calculations are performed consistently, the computed nucleon removal cross sections when neglecting and including break-up degrees of freedom of the residues are essentially identical. An earlier, shorter version of this discussion was the subject of the supplementary material to a recent study of several $p$-shell mirror nuclei [6].

II. FEW-BODY CONSIDERATIONS

We consider the break-up degrees of freedom of weakly-bound residual nuclei within a few-body, eikonal-model approximation. In the calculations presented, the $^8$B and $^9$C reaction residues are treated as weakly-bound two-body (core + proton) systems and we neglect any explicit consideration of the internal degrees of freedom of these core nuclei, describing these using Gaussian densities with appropriate root mean squared (rms) radii. Effects of the structure of these residues on the eikonal-model $^9$C($-p$) and $^{10}$C($-n$) removal cross sections involve the residue-target elastic S-matrix. Here, these S-matrices are computed: (i) from the residue ground-state density, the optical limit, which neglects core+$p$ break-up channels, and (ii) taking account explicitly of the residue’s two-body cluster structure, its extended wave function, and its break-up. These two approaches were designated the static-density and few-body approaches, respectively, in the composite projectile reaction cross sections work of Refs. [7] and [8].

The inclusion of break-up, approach (ii), does not necessarily imply the residue-target S-matrix is more absorptive and the survival of the weakly-bound residue less likely – as the extended nature of the wave function of the residue affects the spatial overlap of its constituents with the target, see e.g. Refs. [7, 8] and Fig. 1. At beam energies of order 800 MeV/nucleon [7, 8], where the interactions with the target are overwhelming absorptive, including the effects of break-up of extended two- and three-body systems was actually shown to reduce their calculated reaction cross sections, $\sigma_R$, compared to calculations using the no break-up, density-based OL model [8]. Overall, the explicit treatment of the few-body nature of the collision found it to be more transparent and less absorptive. This result was shown by Johnson and Goebel [9] to be a very general result for strongly-absorbed systems, and was also discussed using a simplified, semi-analytic model in Ref. [10]. The reason for this reduced absorption, see Fig. 1, was that in many spatial configurations of the separated constituents they will not overlap with or interact strongly with the target. The additional transparency of the collision with the light target nucleus, due to the granular, cluster nature of the extended few-body system, was found to more than compensate for any additional absorption due to the removal of flux from the elastic channel into break-up channels.

Of course, the eikonal-model stripping and diffraction dissociation components of the nucleon removal cross-sections that produce the residues have a more complicated dependence on the residue-target S-matrix than simply its total absorptive content – as determined by its reaction cross section $\sigma_R$. These components are given explicitly in Eqs. (2) and (4) of Ref. [11]. In such removal reaction calculations, both the radial extent and geometry of the residue wave-function and of its density (principally its rms radius [12]) determine the absorption profile of the S-matrix as a function of impact parameter. This, in turn, determines which parts of the bound-state wave function of the removed nucleon are sampled and thus contribute to the cross section. Here, we quantify these residue structural effects upon the calculated nucleon removal cross-section at intermediate energies of order 100 MeV/nucleon.

III. DENSITY-BASED S-MATRICES

For the no break-up, density-based OL calculations, case (i) above, we require the neutron and proton single-particle densities of the residue. Here we compute these from the two-body, bound-state relative motion wave function, $\Phi_0$, of its core ($c$) and valence nucleon ($v$) and the internal densities of these constituents. The two-

![FIG. 1. Schematic of a sudden, nucleon (N) removal reaction at a projectile centre-of-mass (cm) impact parameter $b$, leaving a weakly-bound, two-body (core+$p$) residual nucleus with cm impact parameter $b_r$. In this few-body description, and for the spatial configuration shown, the core and $p$ constituents of the bound residue do not overlap and interact strongly with the target and will be transmitted. On the other hand, within a density-based description, with no reference to the orientation of the core+$p$ system, the one-body density of the residue (light-shaded circle) will overlap the target at this $b$, resulting in a degree of absorption and reduced transmission.](image)
body model residue density can be written
\[ \rho_\gamma(r) = \hat{\rho}_c(r) + \hat{\rho}_n(r), \tag{1} \]
where \( \hat{\rho}_c(r) \) and \( \hat{\rho}_n(r) \) are the contributions from the core and valence nucleon in the residue centre-of-mass (cm) frame \[5\]. Assuming that the free core has internal density \( \rho_c(r) \), then \( \hat{\rho}_c(r) \) is obtained by folding this intrinsic density with \( \rho_{cm}(r) \), the distribution of the motion of the cm of the core within the residue bound state. Thus,
\[ \hat{\rho}_c(r) = \int dx \rho_c(r - x) \rho_{cm}(x), \tag{2} \]
where the bound-state-generated core cm distribution is
\[ \rho_{cm}(r) = A_r^2|\Phi_0(A_r, r)|^2, \tag{3} \]
and \( A_r = A - 1 \) is the residue mass number. The valence nucleon density relative to the cm of the residue is
\[ \hat{\rho}_n(r) = \left[ \frac{A_r}{A_r - 1} \right]^3 |\Phi_0\left(\frac{A_r}{A_r - 1}, r\right)|^2. \tag{4} \]
In the calculations carried out here, the cores, of mass number \( A_c = A_r - 1 = N_c + Z_c \), are assumed to have a Gaussian density and the valence particle is a proton, in which case the proton and neutron densities of the residue are
\[ \rho_p(r) = \left[ \frac{Z_c}{A_c} \right] \hat{\rho}_c(r) + \hat{\rho}_n(r), \quad \rho_n(r) = \left[ \frac{N_c}{A_c} \right] \hat{\rho}_c(r). \]

When calculating the eikonal S-matrix for each particle \( x \) with the target from their densities: as is used here (a) in case (i) for the interaction of the residues, based on their proton and neutron densities, given above, and (b) in case (ii) for the interactions of the residue’s core and the proton with the target, these are computed in the optical limit (or \( tp \) and \( tpp \) folding) approximation to their optical potential, \( U_{xt} \). Given these complex optical potentials, each fragment \( x \)-target S-matrix, \( S_x \), is
\[ S_x(b) = \exp\left[i\mathcal{O}_{xt}(b)\right], \tag{5} \]
where the eikonal phase shift, \( \mathcal{O}_{xt}(b) \equiv 2\delta(b) \), as a function of impact parameter, is given by the \( R_3 \) integral (the \( z \)-component of \( R \) in the incident beam direction) through the optical interaction at each impact parameter \( b \equiv b_{xt} \). Specifically,
\[ \mathcal{O}_{xt}(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} dR_3 \ U_{xt}(\sqrt{b^2 + R_3^2}), \tag{6} \]
where \( v \) is the \( x \)-target relative velocity.

### IV. FEW-BODY MODEL S-MATRICES

For the few-body calculations, case (ii) above, that include break-up of the residue, the eikonal elastic S-matrix of the two-body, composite residue is constructed from the core and proton OL S-matrices, discussed above, as
\[ S_x(b_r) = \langle \Phi_0|S_c(b_c)S_p(b_p)|\Phi_0 \rangle_{\text{spin}}. \tag{7} \]
That is, the S-matrix product, \( S_c(b_c)S_p(b_p) \), that describes the combined core and proton system at a fixed vector separation, \( r \), and residue cm impact parameter, \( b_r \), must be integrated over all possible spatial configurations \( r \) weighted by the probability of each configuration in the core-proton relative motion wave function \( \Phi_0 \). This involves the residue ground-state position probability summed over the nucleon and core spin variables, denoted \( \langle \Phi_0 | \rho^2 | \Phi_0 \rangle_{\text{spin}} \), as in Ref. \[5\].

### V. CALCULATIONS AND RESULTS

We carry out model nucleon-removal calculations for the \( ^9\text{C}(\bar{p}) \) and \( ^{10}\text{C}(\bar{n}) \) reactions at 100 MeV/nucleon on a \(^{9}\text{Be} \) target, representative of reaction data of topical interest \[6\]. The parameters used are as follows.

The \(^7\text{Be} \) and \(^8\text{B} \) cores of the \(^8\text{B} \) and \(^9\text{Be} \) residual nuclei and the \(^9\text{Be} \) target are described by Gaussian densities with rms radii of 2.31 fm, 2.38 fm and 2.36 fm, respectively \[13\]. Thus, with \( g(\gamma, r) \) a normalised 3-dimensional Gaussian function
\[ g(\gamma, r) = (\sqrt{\pi}\gamma)^{-3} \exp(-r^2/\gamma^2) \tag{8} \]
the neutron and proton densities of these systems are taken to be \( \rho_p(r) = Z_r g(\gamma_x, r) \) and \( \rho_n(r) = N_r g(\gamma_x, r) \), where \( x = c, t \), and with \( \gamma \) values determined by the rms radii, given
\[ (r^2) = 3\gamma^2/2. \tag{9} \]

From the proton and neutron one-body densities of the target (\( t \)), of the residue (\( r \)), and of its constituent core (\( c \)) the residue, core and the proton interactions with the target are computed in the \( tpp \) and \( tpp \) double- and single-folding approximations. A Gaussian, finite-range nucleon-nucleon (NN) effective interaction is assumed,
\[ t_{jk}(r) = -\frac{\hbar v}{2} \sigma_{jk} (i + \alpha_{jk}) g(\beta_{jk}, r), \tag{10} \]
with \( j, k = n, p \) and where \( \sigma_{jk} \) and \( \alpha_{jk} \) are the NN total cross sections, taken from the parameterization of Ref. \[14\] and the ratios of the real to the imaginary parts of their forward-scattering amplitudes, taken from Ref. \[15\]. Here \( v \) is the particle-target relative velocity and the Gaussian range parameters \( \beta_{jk} \) are taken to be 0.5 fm.

The weakly-bound \(^8\text{B} \) and \(^9\text{C} \) residue two-body wave functions were calculated as \( p_{3/2} \) eigenstates with separation energies 0.137 MeV and 1.3 MeV in a Woods-Saxon plus Coulomb potential of standard geometry \( r_0 = 1.25 \) fm, \( a_0 = 0.7 \) fm and with a spin-orbit term of strength \( V_{so} = 6.0 \) MeV. The \( (p^8\text{B})^9\text{C} \) and \( (n^9\text{C})^{10}\text{C} \) radial overlaps were likewise described by normalized \( p_{3/2} \) Woods-Saxon eigenstates using the same geometry parameters with separation energies \( S_p = 1.3 \) MeV and \( S_n = 21.3 \) MeV, respectively. That is, we assume spectroscopic factors of 1.0 and calculate the cross sections for one unit of single-particle strength.
For $^9\text{C}(-p)$, the calculated reaction cross section $\sigma_R$ between the $^9\text{B}$ residue and the $^9\text{Be}$ target from the few-body and density-based S-matrices are 851.8 mb and 856.1 mb. The smaller few-body model reaction cross section is consistent with the higher-energy calculations in Refs. \[7, 8\] and the interpretations given in Refs. \[9, 11\]. The corresponding $^9\text{C}(-p)$ single-particle cross sections using the few-body (including break-up) and density-based (neglecting break-up) S-matrices are 61.25 mb and 61.47 mb, respectively.

For $^{10}\text{C}(-n)$, the corresponding few-body and density-based $^9\text{C}$ residue-target $\sigma_R$ are 866.2 mb and 868.0 mb. Again, the few-body S-matrix reduces the reaction cross section, the difference being somewhat smaller due to the increased proton separation energy from $^9\text{C}$ of 1.3 MeV. The $^{10}\text{C}(-n)$ single-particle cross sections when using the few-body and density-based residue-target S-matrices are 21.44 mb and 21.51 mb, respectively.

Thus, as found with earlier higher-energy analyses \[7, 8\], these consistent few-body model calculations indicate that the additional transparency of the residue-target collisions, due to their extended wave functions (Fig. 1) - that would tend to increase the residue survival probability and the removal cross section - is essentially balanced by the additional loss of flux from the elastic channel due to break-up, that drives a reduced core survival probability. These two competing aspects of the residue absorption in the collision are not identified separately within the model calculations discussed here. The overall effect upon the calculated nucleon-removal cross section, being 0.3–0.4%, is insignificant when compared to the precision of typical, available experimental measurements and with other parameter uncertainties and sensitivities in the model calculations presented.

VI. SUMMARY COMMENTS

We have compared model calculations of single-nucleon removal cross sections for the $^9\text{C}(-p)$ and $^{10}\text{C}(-n)$ reactions when: (i) neglecting, and (ii) including the break-up degrees of freedom of the weakly-bound $^9\text{B}$ and $^9\text{C}$ reaction residues. Importantly, the calculations of methods (i) and (ii) use the same parameters to describe these residues and their one-body density distributions are computed consistently from the inputs used in their few-body model description. The inclusion of break-up, through the eikonal elastic S-matrix of the two-body residues with the target, reduces the calculated reaction cross sections for the residue-target systems – the consequence of the granularity of the spatially-extended residues – as has been documented previously. We find that the inclusion of the break-up degree of freedom has a very small effect on the calculated single-particle nucleon-removal cross sections. For the representative $^9\text{C}(-p)$ and $^{10}\text{C}(-n)$ reactions considered, chosen to involve the removal of both strongly- and weakly-bound valence nucleons, the cross sections are found to be reduced by only 0.3–0.4%. We conclude that an explicit treatment of the break-up of weakly-bound residues is unnecessary provided a realistic description of the residue density is used in the no break-up, optical-limit calculations.

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