The quark-gluon vertex in Landau gauge bound-state studies

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Received: date / Revised version: date

Abstract. We present a practical method for the solution of the quark-gluon vertex for use in Bethe–Salpeter and Dyson–Schwinger calculations. The efficient decomposition into the necessary covariants is detailed, with the numerical algorithm outlined for both real and complex Euclidean momenta. A model suitable for bound-state calculations is given together with results for the quark propagator and quark-gluon vertex for different quark flavours. The relative impact of the various components of the quark-gluon vertex is highlighted with the flavour dependence of the effective quark-gluon interaction obtained, thus providing insight for the construction of phenomenological models within Rainbow-Ladder. Finally, we solve the corresponding Green’s functions for complex Euclidean momenta as required for practical calculations.

PACS. 11.10.St – 11.15.-q – 11.30.Rd – 12.38.Aw – 12.38.Lg

1 Introduction

The phenomena of QCD such as confinement and dynamical chiral symmetry breaking (DCSB) are exemplified through the spectrum of hadrons, their decays, and transition form factors. Thus, a non-perturbative description of bound-states in terms of the component quarks and gluons is required. One such approach is the Dyson–Schwinger (DSE) and Bethe–Salpeter (BSE) equations which provide a framework where chiral properties – in particular the Goldstone nature of pions – can be taken into account through constraints such as the axial-vector Ward–Takahashi identity (axWTI) [1, 2] amongst others [3–5].

Since the DSEs constitute an infinite tower of coupled non-linear integral equations, truncations must be employed to make their solution tractable; for recent reviews, see Refs. [6–11]. These can be performed at varying levels of sophistication, with the simplest being that of Rainbow-Ladder (RL) wherein only the DSE for the quark is directly considered and the coupling between quarks and gluons is given by an effective interaction. This reduces the quark-(anti)quark interaction to a simple flavour independent coupling that is a capable model of DCSB. Such a truncation has yielded much phenomenological success for mesons [8, 12–20] as well as baryons [21–26].

However, the main detriment is that the structure remains that of a \( \gamma^\mu \otimes \gamma^\nu \) interaction with no variation in the projected interaction strength; to remedy this one must go beyond RL [2, 10, 17, 27–34]. This is fairly straightforward in principle since one knows the gluon propagator very well from both Lattice studies and functional methods. This leaves the quark-gluon vertex as one of the central objects to non-perturbative and bound state studies of hadrons, for it explicitly connects the matter sector (the quarks) to the gauge sector (the gluons).

Indeed, it is known that the enhancement that enables the effective quark-gluon interaction to trigger DCSB is provided by this vertex [35, 36]. Moreover, other non-perturbative effects such as pion-cloud corrections are contained within [31, 37–40], together with the dominant dependence on quark flavour necessary to make connection with the heavy quark limit [41–44].

In this paper, we present a solution strategy for a class of truncations of the quark-gluon vertex and quark propagator for both space-like [36, 45–47] Euclidean momenta and their analytic continuation to time-like momenta needed in bound-state equations [33, 48]. We provide a specific model, suitable for bound-state studies, and investigate the flavour dependence of the quark propagators and quark-gluon vertex. In the process, we comment upon the impact of different infrared properties of the Yang-Mills sector [49–57] – notably the gluon propagator and the three-gluon vertex [58–63].
2 Quark-gluon vertex

In general the vertex may be decomposed into twelve components

$$\Gamma^\mu(l, k) = \sum_{i=1}^{4} \sum_{a=1}^{3} c^i_a(l, k) L^\mu_{(a)} D^a_{(i)} , \quad (1)$$

where $c^i_a(l, k)$ are scalar coefficients that parametrise the Lorentz ($L^\mu_{(a)}$) and Dirac ($D^a_{(i)}$) parts of the basis (see the Appendix for details). For convenience, we collapse the indices $i, a$ and write

$$T_{1...4} = c^1_{(1)} , \quad T_{5...8} = c^1_{(2)} , \quad T_{9...12} = c^1_{(3)} , \quad (2)$$

where note that the indices $a$ may be referred to by symbols in the sequel.

Generically, the form of the DSE for the quark-gluon vertex is given by summation of self-energy contributions

$$\Gamma^\mu(l, k) = Z_{1f} \gamma^\mu + A_{NA}^\mu + A_{AB}^\mu + \ldots \quad (3)$$

where $A_{NA}^\mu$, $A_{AB}^\mu$ are representative corrections that we consider later and the ellipsis refers to other terms present in the DSE, see Fig. 2. Projection onto the coefficients is obtained via

$$c^i_{(a)}(l, k) = L^\mu_{(a)} \text{Tr} \left[ D^a_{(i)} \gamma^\mu \right] + L^\mu_{(a)} \text{Tr} \left[ D^a_{(i)} A_{NA}^\mu \right]$$
$$+ L^\mu_{(a)} \text{Tr} \left[ D^a_{(i)} A_{AB}^\mu \right] + \ldots \quad (4)$$

In an obvious notation, we write the contribution to the coefficient $c^i_{(a)}$ of each self-energy contribution as $c^i_{(a)}$, and $c^i_{(a)}$ respectively.

In Fig. 2 we give two forms of the DSE for the quark-gluon vertex, the difference being to which external leg the bare vertex is attached. Both equations are functions also of higher $n$-point functions which must be specified in some way, called the truncation, to enable solution.

In performing such a truncation we must find a balance between its numerical or algebraic complexity and the physics that it contains. In the case of the quark-gluon vertex, it is sensible to neglect the two-loop diagrams, and to eliminate non-primitively divergent $n$-point functions through use of a dressed skeleton expansion [36, 64] in combination with a non-perturbative ordering of diagrams provided by infrared power counting [55, 57]. The removal of such contributions can be mitigated in part by effectively dressing previously bare vertices through re-ordering of the diagrammatic resummation. We can then assume that a similar ordering also applies to the case where there is no scaling behaviour in the infrared by considering such solutions to be the limiting case of all decoupling solutions. These arguments typically apply in the absence of dynamically generated quark mass and must be considered carefully in a realistic calculation.

Typically, one considers the two contributions given in Fig. 3 with any combination of bare or dressed internal vertices, labelled 1 through 3 [35, 36]. For example, with all internal vertices dressed one would have a truncation reminiscent of the 3PI formalism [65], 2 and 3 dressed would match the second form of the DSE for the quark-gluon vertex.
Fig. 4: Quark self-energy contributions (exact) involving dressed $\Gamma_{qqg}^\mu$, $\Gamma_{ggg}^{\mu\rho}$ and $\Gamma_{ggg}^{\mu\nu}$ vertices.

Fig. 5: Bethe-Salpeter kernel from explicit cutting of quark lines (implicit cutting generates additional diagrams).

gluon vertex, and finally with 1 and 2 dressed the first form. Note, however, that in order to allow the diagrammatic construction of the quark-(anti)quark kernel consistent with the axial WTI [1, 2], the internal quark-gluon vertices must be of the form $\lambda(k^2)\gamma^\mu$ with $\lambda(k^2)$ a function of the squared gluon momentum $k^2$.

As an example, the self-energy contributions to the quark DSE are given in Fig. 4 using the second form of the DSE for the quark-gluon vertex. The corresponding symmetry preserving kernel, neglecting terms that require the cutting of internal vertices, is given in Fig. 5.

2.1 Method for use in the Quark DSE

We employ herein the colour reduced quark-gluon vertex $\Gamma_\mu(p_1, p_2) = l^\mu \Gamma_\mu(p_1, p_2)$ where $p_1$ and $p_2$ are the incoming and outgoing quark momenta, respectively, as given in Fig. 1. Throughout we will make use of transverse and/or orthonormal momenta [22, 66] which have proven very efficient in the covariant investigation of baryons [21, 24, 67–69]. The corresponding bases are given in the Appendix. In particular, the strategy is optimised such that the injection of a complex momentum (required for the analytic continuation of Euclidean momenta to the time-like region as probed in bound-state studies) can be easily considered. Numerical solutions are performed using the shell-method, as described in Section 2.4.

2.2 Non-Abelian diagram

Let us apply the transverse basis decomposition of Section A.1 to the case of the (a posteriori) dominant non-Abelian contribution to the quark-gluon vertex. From here on we focus only on the Dirac and Lorentz structure of the diagram, given in the left panel of Fig. 3, and implicitly contract all colour indices and suppress the renormalisation factors. The eventual choice of whether an internal vertex is taken dressed or not is left to the sophistication of the desired truncation, but note for consistency that each bare vertex must be associated with the corresponding multiplicative renormalisation factor.

Consider the following choice of four-momenta

$$k^\mu = p_2^\mu = |k| \left(0, 0, 0, 1\right),$$
$$l^\alpha = (p_1^\alpha + p_3^\alpha)/2 = |l| \left(0, 0, z', z\right),$$

where $z = \hat{k} \cdot \hat{l}$ and $z' = \sqrt{1 - z^2}$. The non-Abelian diagram of Fig. 3 would have the following form

$$A_{\text{NA}}^\alpha(l, k) = N_c \int \frac{d^4 q}{(2\pi)^4} \Gamma_\alpha(l_1, -q_1) S(q_3) \Gamma_\beta(l_2, -q_2) \times \Gamma_{qg}^{\alpha'\beta'}(q_1, q_2, p_3) D^{\alpha'\beta'}(q_1) D^{\beta'}(q_2),$$

where the relative quark momenta of the internal vertices are defined $l_1^\mu = (p_1 + q_3)^\mu/2$, $l_2^\mu = (p_2 + q_3)^\mu/2$, and $q$ is the Euclidean loop momentum of which $q_i$ are appropriate functions. The $N_c/2$ stems from the colour trace. This integrand can be separated into a Dirac and Lorentz part

$$M_{(a)}^{\alpha\beta} = \tilde{L}_(a) \Gamma_\alpha \Gamma_\beta \Gamma(q_1, q_2, p_3) D^{\alpha\beta'}(q_1) D^{\beta'}(q_2),$$
$$N_{(a)}^{i\alpha\beta} = \text{Tr} \left[ \tilde{D}^i_\alpha \Gamma_\alpha(l_1, -q_1) S(q_3) \Gamma_\beta(l_2, -q_2) \right],$$

where $a = (v, r, s)$, $i = 1, \ldots, 4$, and the contribution to the coefficients of the basis, Eq. (A.16), is obtained through projection

$$\hat{c}_{(a)}^{i, \text{NA}}(l, k) = N_c \int \frac{d^4 q}{(2\pi)^4} M_{(a)}^{\alpha\beta} N_{(a)}^{i\alpha\beta}.$$
of truncation schemes to be tackled more easily. For example, let us assume that tree-level quark-gluon vertices are employed under the loop integral, together with a tree-level three-gluon vertex. On replacing this with, say, a dressed three-gluon vertex \[ \text{[70]}, \] only \( M_{\alpha \beta}^{\mu \nu} \) would be modified. Similarly, any changes to the internal quark-gluon vertices affect only \( \mathcal{N}_{i,i}^{(a)} \). In addition, since the Lorentz part \( M_{\alpha \beta}^{\mu \nu} \) is typically independent of the quark propagator and quark-gluon vertices it can be pre-computed and stored. This is especially useful when employing traditional computer algebra methods to trace out the algebra, since the explicit contraction of the Dirac part with the Lorentz part tends to lead to large algebraic expressions.

2.3 Abelian diagram

The contribution often considered to be sub-leading, due to it being colour suppressed by \( N_{c}^{2} \), is the so-called Abelian diagram given in the right panel of Fig. 3. Considering the same choice of external momenta as before, this diagram with the Lorentz part tends to lead to large algebraic expressions.

\[
A_{AB}(l,k)^{\mu} = \frac{-1}{2N_{c}} \int \frac{d^{4}q}{(2\pi)^{4}} \Gamma_{\alpha}(l_{1},-q_{\alpha}) S(q_{1}) \Gamma_{\mu}(l_{2},p_{3}) \times S(q_{2}) \Gamma_{\beta}(l_{3},q_{3}) D^{\alpha \beta}(q_{3}), \tag{11}
\]

where \( l_{i}^{\mu} = (q_{1}+p_{1})^{\mu}/2 \), \( l_{2}^{\mu} = (q_{1}+q_{2})^{\mu}/2 \), \( l_{3}^{\mu} = (p_{2}+q_{2})^{\mu} \) are the relative momenta of the internal vertices. The Euclidean loop momentum is \( q \), of which \( q \) are appropriate functions. In the context of BSE studies, it has been investigated in Refs. \[ \text{[2, 27, 28, 30, 36, 48, 71]} \].

The Abelian contribution to the quark-gluon vertex does not benefit from the same separation of the Dirac part from the Lorentz part. Here we would find that

\[
e_{i,\alpha}^{\lambda,AB}(l,k) = \frac{-1}{2N_{c}} \int \frac{d^{4}q}{(2\pi)^{4}} D^{\alpha \beta}(q_{3}) \text{Tr} \left[ D_{\beta}(l_{3},q_{3}) \left( L_{\alpha}(l_{1},-q_{\alpha}) \right) \right] \Gamma_{\beta}(l_{3},q_{3}), \tag{12}
\]

which is the Lorentz contraction and Dirac trace applied to the single spin-line. Note that we used

\[
S(q_{1}) \Gamma_{\mu}(l_{2},p_{3}) S(q_{2}) := \chi_{\mu}(l_{2},p_{3}), \tag{13}
\]

with \( \Gamma \) now analogous to the Bethe-Salpeter amplitude and \( \chi \) its wavefunction. This provides significant simplification of the algebra in the case that the top-most vertex is dressed, and permits solution as an inhomogeneous BSE (wherein the inhomogeneous term would be the non-Abelian diagram plus the tree-level term).

Note that up to a colour factor and the additional inclusion of crossed-ladder contributions, the Abelian diagram with the 2+3 vertices dressed was solved inhomogeneously in Ref. \[ \text{[48]} \] in the context of the bound-state for a vector meson. Such methods can be similarly applied here also in the complex plane.

2.4 Analytic continuation

As indicated earlier, bound-state calculations require that the quark propagator must be analytically continued to a bounded parabolic region of the complex plane. We detail these steps here.

Starting with Eq. (14), one would ordinarily exploit \( O(4) \) symmetry and take the incoming momentum to be \( p^{\mu} = |p|(0,0,0,1) \). For its analytic continuation, we inject a complex momentum \( P^{\mu} = \text{i}M(0,0,0,1) \) parallel to \( p \), so that the incoming (viz. outgoing) quark momentum \( p^{\mu} \) is \( (p+P/2)^{\mu} \). It is obvious that the corresponding squared invariant \( p^{2} = (p+P/2)^{2} \) then maps out a bounded parabolic region of the complex plane with vertex \( -M^{2}/4 \). Thus the heavier the bound-state the larger the region in the complex plane that needs to be explored.

We work in Euclidean space where the integration measure is space-like and we are free to choose the momentum routing in the self-energy diagram of the quark DSE. For example, we can pass the momentum \( (p'-k) \) through the gluon and the loop momentum \( k \) through the internal quark propagator. Then, solution of the quark propagator in the complex plane requires the quark propagator on the positive real axis together with a prescription for the gluon and quark-gluon vertex (perhaps given by Ansatz in e.g. the RL truncation) valid for complex momenta. Such an approach requires no iteration of the quark DSE provided the quark propagator is known for \( p^{2} \in \mathbb{R}^{+} \). However, the obvious caveat is that typically we do not know the gluon or the quark-gluon vertex in the complex plane (exceptions are [33, 74]). Moreover, solution of the quark-gluon vertex may require the three-gluon vertex as input which has only been investigated for real momenta \[ \text{[60, 62, 63, 70, 75]} \].

The solution to this problem is to choose an alternative momentum routing so that the complex momentum \( p'-k \) passes through the internal quark propagator, and the real Euclidean loop momentum \( k \) through the gluon propagator. Since the parabolic regions are nested, we can expand out from the real-axis in parabolic shells, iterating for each until convergence is achieved and then proceeding outwards. Such a process is dubbed the shell-method \[ \text{[76]} \] although other similar techniques exist \[ \text{[77–79]} \]. Then, the momentum passing through the gluon is real and only
the relative quark-momentum of the quark-gluon vertex is continued to the complex plane. Through a judicious choice of the momenta we can again pass this complex momentum through the internal quark-line only. Thus, the quark-gluon vertex can also be solved for expanding parabolic shells. This necessitates a micro/macro iteration that is repeated until mutual convergence.

3 Ingredients

3.1 Quark propagator

The Green’s function fundamental to our discussion of mesons is that of the quark propagator, since without it we cannot describe the matter fields that form colourless bound-states. In its relatively simple structure are encoded such non-perturbative properties as the dynamical generation of mass and the realisation of a non-zero vacuum condensate through DCSB. Moreover, chiral symmetry as expressed through the axWTI connects the interaction part of the quark DSE to the quark-(anti)quark scattering kernel required in the covariant description of bound states.

This DSE for the (inverse) quark propagator, shown in Fig. 7, is

\[ S(p)^{-1} = Z_2 S_0^{-1}(p) - g^2 C_F Z_{1f} \int_\mathbb{Q} \gamma_\mu S(q) \Gamma_\nu(q, p) D_{\mu\nu}(k), \]

with \( k = q - p \) and \( \int_\mathbb{Q} = \int d^4q/(2\pi)^4 \). The fully-dressed inverse quark-propagator is diagonal in colour space

\[ S^{-1}(p) = -i\not{\psi} A(p^2) + i B(p^2), \]

and its bare counterpart given by \( S_0^{-1}(p) = -i\not{\psi} + Z_m m \). Here, \( Z_2, Z_m \) and \( Z_{1f} = Z_2 / Z_3 \) are respectively the renormalisation constants of the quark field, quark mass and quark-gluon vertex in Landau gauge using the miniMOM scheme [80]; \( Z_3 \) renormalises the ghost propagator. We deal with QCD where the number of colours is 3, and so the Casimir \( C_F = 4/3 \). The propagator is parametrised by two scalar functions \( A(p^2) = 1/Z_f(p^2) \) and \( B(p^2) = M(p^2)/Z_f(p^2) \) where \( Z_f \) is the quark wavefunction and \( M \) is the quark mass function. \( D_{\mu\nu}(k) \) is the gluon propagator in Landau gauge.

The solution of this system is therefore contingent upon two inputs. First is the aforementioned gluon propagator.

Second, to close the system, we need the quark-gluon vertex. For the latter, we may also require the three-gluon vertex.

3.2 Gluon propagator

The gluon propagator is given by

\[ D_{\mu\nu}(k) = T^{(k)}_{\mu\nu} Z(k^2) / k^2, \]

where \( Z(k^2) \) is the gluon dressing function and the transverse projector is \( T^{(k)}_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu / k^2 \). In Fig. 8 we show the dressing function of the gluon propagator for both scaling- and decoupling-type scenarios [81, 82]. As can be seen, they differ only below a hundred MeV. Due to screening in the infrared as a result of both explicit and dynamically generated masses, such deviations have little to no impact on the majority of hadronic properties [83].

The employed ghost and gluon propagators come with associated renormalisation constants \( Z_3 \) and \( Z_\alpha \), together with a particular choice of the strong coupling constant \( g_s^2 \) such that the running of the ghost-gluon vertex is given in physical units. For consistency, and to ensure that the reproduced anomalous dimensions are correct, all other renormalisation constants are appropriately related via their Slavnov-Taylor identities. This is necessary to obtain coincidence with the running coupling associated with each primitively divergent vertex at perturbative momenta.

A discussion of the propagators used here and the associated parameters is contained within Ref. [70].

3.3 Three-gluon vertex

The three-gluon vertex in pure Yang-Mills has been the focus of several investigations recently. Its tree-level structure can be directly read off from the QCD Lagrangian,
Eq. (8), is the non-Abelian contribution to the quark-gluon vertex, 

\[ \Gamma^{(0)}_{3g}(p_1, p_2, p_3) = \delta^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + \text{cyclic}, \]  

(17)

where the coupling constant and colour factor \( g f^{abc} \) are factored out. There are 10 longitudinal components and 4 transverse components, yielding 14 components in total. Examples of such a basis is given in Ref. [62, 84, 85]. In Landau gauge QCD it is sufficient retain only the transverse terms; a complete basis is presented in Ref. [70]. There, it was determined that the transversely projected tree-level component is dominant, with other structures contributing at the 10% level. Hence, here we keep only the tree-level component of the three-gluon vertex, as projected onto this transverse basis,

\[ \Gamma^{\mu_1 \mu_2 \mu_3}_{3g,TTT}(p_1, p_2, p_3) = F_1(p_1, p_2, p_3) \times T^{\mu_1 \rho_1}_{\mu_2 \rho_2}_{\mu_3 \rho_3} F^{(0) \mu_1 \mu_2 \mu_3}_{3g}, \]  

(18)

where \( F_1(p_1, p_2, p_3) \) is a scalar function parametrising the leading component of the three-gluon dressing; to good approximation this can described by the single combination \( S_0 = (p_1^2 + p_2^2 + p_3^2) / 6 \). Since the basis is transverse, the transverse projector contained within the gluon propagator, Eq. (16), is redundant. Thus, the Lorentz part of the non-Abelian contribution to the quark-gluon vertex, Eq. (8), is

\[ M^{\alpha \beta}_{\mu}(a) \equiv \left( L^\alpha_{(a)} \Gamma^{\alpha \beta}_{3g,TTT} \right) \frac{Z(q_1^2)}{q_1^2} \frac{Z(q_2^2)}{q_2^2}, \]  

(19)

with \( Z(p^2) \) the gluon dressing function.

### 3.4 Internal quark-gluon vertex

In principle one may choose to back-couple the full quark-gluon vertex internally and solve for the fully coupled system. This is a straightforward, but highly technical task when real space-like moments are considered. The calculation is significantly increased in complexity when the external quark momenta are continued into the complex plane as required in the solution of the quark DSE for bound-state studies. However, this complexity is increased further still when the corresponding quark-antiquark kernel is constructed. It is also not clear how feasible it is to then enforce consistency with the axial-vector Ward-Takahashi identity since the internal vertices must also be ’cut’. It has been shown in Ref. [86] that a quark-antiquark kernel inconsistent with chiral symmetry breaking can lead to a pion that is of mass 600 MeV in the chiral limit. This is a significant deviation that likely can’t be neglected even for systems consisting primarily of heavy quarks.

Hence, since the aim herein is to construct a quark-gluon vertex that is useful in practical bound-state calculations, we impose the following constraint on the internal quark-gluon vertices

\[ \Gamma^\mu(k, q) = L_1(q^2)^2 \gamma^\mu, \]  

(20)

where \( L_1(q^2) \) is a scalar function of the gluon momentum \( q \). The relative quark momentum \( k \) is neglected, and the asymptotic behaviour \( L_1 \to Z_1 \) at large momenta ensures multiplicative renormalisability. The function \( L_1(q^2) \) will be constructed to strongly resemble the calculated \( \gamma^\mu \) component including flavour dependence.

### 4 Truncating

There are three cases to consider each in the non-Abelian and Abelian diagrams, Fig. 3:

1. **(non-Abelian)** Both quark-gluon vertices (1+2) are dressed, the three-gluon vertex (3) is bare.
   - **(Abelian)** Two quark-gluon vertices (1+2) are dressed, third quark-gluon vertex (3) is bare.

2. **(non-Abelian)** One three-gluon vertex (3) and quark-gluon vertex is dressed, one quark-gluon vertex is bare.
   - **(Abelian)** Two quark-gluon vertices (2+3) are dressed, third quark-gluon vertex (1) is bare.

3. **(non-Abelian)** Both quark-gluon vertices (1+2) are dressed, the three-gluon vertex (3) is dressed.
   - **(Abelian)** All three quark-gluon vertices (1+2+3) are dressed.

Let us consider case 3 where all internal vertices are dressed and focus upon the non-Abelian diagram. To make explicit the possible types of solution that may result, we focus upon the scaling class of solutions which power-law like behaviour in the IR.

We considered several extreme scenario’s, with the following being the most interesting. First, assume that the three-gluon vertex features a zero crossing at some dynamically relevant scale (300 to 600 MeV), below which it becomes negative and induces an attractive as opposed to repulsive interaction. This can be inherited by the quark-gluon vertex together with an appropriate IR exponent. Indeed, such consistent solutions can be found but with a too low degree of dynamical mass generation. Upon increasing the strength of the internal vertices, the calculated vertex instead becomes positive and constant in the IR (scaling behaviour and consistency is lost). This suggests that the mass function screens the IR properties of the Yang-Mills sector, and we thus conclude that quark-gluon vertices that are finite in the IR are consistent for both scaling and decoupling type scenarios. Close to and in the symmetric phase, as well as for a (unphysically) strongly IR enhanced three-gluon vertex, scaling solutions would reappear.

Now, consider the differences between case 1 and case 2. For the non-Abelian diagram, we lose the enhancement present in one of the quark-gluon vertices, to be replaced by a dressed three-gluon vertex that provides screening at low to mid momenta. This again weakens the degree of dynamical mass generation provided. For the Abelian diagram, the number of enhanced vertices is the same but it remains \( N_c^2 \) suppressed. One could then conclude that the third diagram in the second from of the DSE for the quark-gluon vertex, featuring the \( \Gamma^{\mu \nu}_{\gamma g g g} \) vertex, provides a sizeable contribution.
On the other hand, the difference between case 1 and case 3 is a dressed three-gluon vertex and dressed quark-gluon vertex in the non-Abelian and Abelian diagrams, respectively. One can argue, since the origins of the effective re-summation are based around ghost dominance, that this dressed three-gluon vertex features only the ghost-loop. This entails that deviation from the tree-level form is shifted to be deeper in the IR, below 150 MeV. Since such features will be screened by the quark mass function, it seems reasonable to take the three-gluon vertex as bare to good approximation. Similarly, one would not expect the Abelian diagram to contribute significantly even with three internally dressed vertices.

Thus, we consider only the first form of the vertex DSE with internally dressed $1 + 2$ vertices and a bare three-gluon vertex. This is sufficient for the construction of phenomenological models. We checked other combinations of truncation discussed above, but report here only on the most relevant for bound-state studies. Moreover, we focus upon solutions of the decoupling variety since in practice we expect to find no differences.

### 4.1 Model for bound-states

The truncation employed is given in Fig. 9, wherein the presumed dominant non-Abelian contribution only is considered. All internal vertices are given by their tree-level structure and an associated dressing function provided by Ansatz. This is necessary since we must be able to provide the corresponding symmetry preserving BSE kernel, see [1, 2, 27, 33]. We already argued that the three-gluon vertex can be taken as bare to good approximation. Thus, we must specify the form of the internal dressing of the quark-gluon vertex. Extensive investigations lead to the following form that is itself consistent with the full numerically calculated vertex

$$
\lambda_1(p^2) = hZ_{1f}\left\{ \frac{A(M_0)}{1 + y} + \frac{z}{1 + z}\left[ \frac{4\pi}{\beta_0\alpha_\mu}\left( \frac{1}{\log x} - \frac{1}{x - 1} \right) \right]^{18/44} \right\}, \quad (21)
$$

with $h = 2.302$, $x = p^2/0.6$, $y = p^2/0.35$, $z = p^2/0.33$, $\beta_0 = 11N_c/3 - 2N_f/3$ with $N_f = 0$ and $\alpha_\mu = 0.7427$. The IR enhancement $A$ is a function of the quark mass at zero Euclidean momenta, $M_0$. Empirically, this is parametrised by the followed rational polynomial

$$
A(M_0) = \frac{a + bM_0 + cM_0^2}{M_0 + dM_0^3}, \quad (22)
$$

with $a \approx -0.79$, $b \approx 13.1$, $c \approx 5.74$, $d \approx 8.3$. These numbers have been determined recursively by iteration.

In Fig. 10 we display the vertex dressing for different quark masses. The important thing to note about this form of the vertex is the implicit flavour dependence. The combination of Eq. (21) and Eq. (22) is such that for heavy quarks, the vertex dressing reduces to $\lambda_1 \to Z_{1f}$ but features an enhancement sufficient for DCSB in light quarks.

### 5 Results

We solved the quark and quark-gluon vertex DSE for real space-like momenta according to the method outlined in Section 2.1, using the model discussed above. The calculation is quenched in the sense that quark-loop contributions to the gluon propagator are not considered. However, since the unquenching effects present in a dressed quark-gluon vertex are generally of more interest here, see Ref. [29] vs. Ref. [31, 39], this proves to be a good approximation. Unquenching effects in the Yang-Mills sector can be easily accommodated by modification of the ghost and gluon propagators.

With a bare three-gluon vertex, the overall renormalisation constant of the non-Abelian diagram is $Z_1$. Renormalisation of the quark-gluon vertex DSE itself is provided by the inhomogeneous term $Z_{1f}\gamma^\mu$. Note that these are not independent constants and are related to other renormalisation constants through Slavnov-Taylor identities,

$$
Z_1 = \frac{Z_3}{Z_5}, \quad Z_{1f} = \frac{Z_2}{Z_3}, \quad (23)
$$

which we discussed earlier.

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**Fig. 9:** Our truncation of the quark-gluon vertex DSE, featuring the dominant non-Abelian contribution. Internal vertices are provided by their tree-level structure and an associated dressing provided by Ansatz.

**Fig. 10:** The scalar dressing of the internal quark-gluon vertex for quarks of increasing mass. For heavy flavours, the vertex tends towards its bare form $Z_{1f}$.
We determined the chiral condensate from the trace of the quark propagator. Converting to $\overline{MS}$ at $\mu = 2$ GeV we found $-\langle \bar{\psi} \psi \rangle^{1/3} = 275$ MeV in good agreement with other phenomenological studies.

### 5.1 Flavour dependence

In Fig. 11 we show the quark mass function and quark wavefunction for chiral, $u/d$, $s$, $c$ and $b$ quarks. These are compared, in the case of light quarks, to a quenched lattice calculation in Fig. 12 wherein a similar qualitative behaviour is observed. The resulting quark mass function and quark wavefunction have the same behaviour as typically seen in other Dyson–Schwinger calculations, which reassuringly confirms that even simple models can perform adequately in this regard.

Since the calculation of the quark-gluon vertex depends both implicitly and explicitly on the quark flavour, we expect to see this reflected strongly in the vertex dressings. In Fig. 13 we give a representative momentum slice of the quark-gluon vertex for the transverse vertex dressings $T_1$ to $T_6$. As expected, the components $T_1$ and $T_6$ that reconstruct the tree-level $\gamma^\mu$ piece are dominant. For light quarks, the $T_4$ element contribution is next relevant, followed by $T_5$, $T_8$ and $T_3$. For heavy quarks, only $T_3$ and $T_8$ remain relevant. Moreover, there are obvious similarities between the coefficient functions which suggest that a minimal basis may be constructed that contains the dominant components. However, this is a model dependent statement and the hierarchy of structures could change for a different truncation.

To judge the actual relevance of each vertex component, we compare (for light and heavy quarks) its weighted contribution to the quark DSE under projection onto the vector and scalar parts of the propagator, Fig. 14. This is necessary, because in practice the kernel function can have a dramatic influence in terms of enhancement or suppression. One could judge the relevance by looking at some calculable quantity, such as the chiral condensate, but this would only consider the Dirac even parts of the vertex. We see the $T_1$ and $T_6$ pieces (components of the $\gamma^\mu$ term) dominating, particularly at large momenta. For the scalar part of the propagator, we see that $T_3$ and $T_8$ are relevant but are, in contrast, negligible for the vector part. Similarly, while $T_4$ is a strong contributor to the vector part of the propagator, it does not contribute to the scalar part.

Since one can see directly the relative importance of non tree-level terms in the quark DSE, one can surmise that they can have a similar impact in the interaction kernel appearing in Bethe–Salpeter studies of mesons and baryons. In particular, the flavour dependence is easily discernible with the result that for heavy-quarks, it essentially reduces to a single gluon exchange without vertex enhancement. In principle, one could use the results
Fig. 13: The eight transverse dressing functions of the quark-gluon vertex for incoming quark and gluon momentum \( p = k \), with \( p \cdot k = 0 \). We compare a light quark (left) to a heavy quark (right).

5.2 Quark for complex momenta

In Fig. 15 we give the real and imaginary parts of the quark mass function of light \( u/d \) quarks in a region of the complex plane centred on the origin; those for the quark wavefunction \( Z_f \), are similarly smooth. They are obtained via the shell-method as outlined in the introduction, with the quark-gluon vertex similarly analytically continued to the complex plane. Note that the vertex is similarly analytic in the region of the complex plane considered.

Each macro cycle in which the quark-gluon vertex is updated and the quark propagator solved, takes approximately three minutes on a modest single CPU core and is easily parallelised. Introducing a fully dressed three-gluon vertex does not impact upon performance appreciably. The introduction of fully dressed internal quark-gluon vertices will scale the algorithmic difficulty by approximately eight for each vertex, plus additional overhead for the interpolation of the dressing functions. Compared to this macro cycle, the micro cycle in which the quark-propagator is solved is essentially for free. The process is iterated until convergence is reached.

With the quark propagator thus obtained in the complex plane, we may proceed to construct the quark-gluon vertex as relevant for bound-state studies, using the enlarged basis given in Appendix B with a greater number of kinematic invariants. However, this procedure requires no further iteration and so results may be pre-calculated and tabulated for later use.

6 Conclusions and Outlook

We presented an adaptable approach to the decomposition and calculation of the quark-gluon vertex for both real and complex Euclidean momenta, as required for studies of hadronic bound-states. A suitable model was presented that reproduces QCD phenomenology whilst being compatible with lattice results. The quark-flavour dependence and the effective quark-gluon vertex were found to be sizeable, reducing to the expected single one-gluon exchange...
in the heavy quark limit. This highlights the inadequacy of simple RL studies of baryons and mesons which typically feature a flavour independent interaction.

There are several improvements that can be made. Firstly, whilst preserving the connection to a chiral symmetry preserving truncation of the quark-(anti)quark interaction, one could include an explicit solution of the three-gluon vertex in the non-Abelian contribution to the quark-gluon vertex. However, there one would need to include quark-loop effects and perhaps account for two-loop contributions in order for the result to be reliable. Secondly, one may include explicitly the Abelian contribution. This is a simple task for the quark-gluon vertex DSE (we explored this), but introduces crossed-ladder kernels in the BSE kernel without introducing sizeable corrections. Finally, complicating the preservation of the axWTI, one may choose to back-couple the quark-gluon vertex internally as considered in Refs. [45].

Acknowledgements

RW would like to thank R. Alkofer, G. Eichmann, C. S. Fischer, H. Sanchis-Alepuz and M. Vujinovic for useful discussions and a critical reading of the manuscript. This work was supported by the Helmholtz International Center for FAIR within the LOEWE program of the State of Hesse, by BMBF under contract 06GI7121, the Austrian Science Fund (FWF) under project number M1333-N16.

A Vertex basis for quark DSE

A.1 Transverse

In the case of real Euclidean momenta, the description of the quark-gluon vertex is straightforward. Given that we have two Dirac indices, one Lorentz index and two independent momenta \( p_1, p_2 \) the most naive basis decomposition would be

\[
\left( \gamma^\mu, \gamma^\mu_{1}, \gamma^\mu_2 \right) \times \left( 1, p_1, p_2, \gamma^\mu_1 p_2 \right),
\]

which features 12 components. This is of course not unique as we can construct different linear combinations of these basis elements, for instance such that the vertex is free of kinematic singularities [88, 89].

We specialise to the case of DSEs in Landau gauge, wherein every Lorentz index will ultimately be contracted by the transverse projector \( T^T_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu/k^2 \) contained within the gluon propagator. If we define the total incoming momentum of the gluon \( k^\mu = p^\mu_2 - p^\mu_1 \) and the relative quark momentum\(^1\) \( l^\mu = (p^\mu_2 + p^\mu_1)/2 \), we can construct the following orthonormal elements

\[
\begin{align*}
 t^\mu &= \hat{k}^\mu, \\
 s^\mu &= T^T_{\mu\nu} l^\nu, \\
 \gamma^\mu_T &:= T^T_{\mu\alpha} T^s_{\alpha\nu} \gamma^\nu = \gamma^\mu - \not{x} t^\mu - \not{s} s^\mu.
\end{align*}
\]

where the hat indicates normalisation. These then provide the orthogonal basis

\[
\left( \gamma^\mu_T, s^\mu, l^\mu \right) \times \left( 1, \not{s}, \not{x}, \not{x} \not{l} \right),
\]

where \( s \cdot t = 0 \), and \( \gamma^\mu_T s^\mu = \gamma^\mu_T l^\mu = 0 \). For later convenience we denote this Dirac part by

\[
D^{(1)} = \left( 1, \not{s}, \not{x}, \not{x} \not{l} \right),
\]

Due to the transversal nature of Landau gauge only those highlighted components contribute, thus requiring just 8 components to completely describe the quark-gluon vertex. Being orthogonal, the projectors for the scalar coefficients are easy to construct.

Let us focus on a particular choice of momentum frame. For squared momenta \( k^2, l^2 \) and their cosine \( z = \hat{k} \cdot \hat{l} \) we can write

\[
\begin{align*}
 k^\mu &= \left| k \right| \left( 0, 0, 0, 1 \right), \\
 l^\mu &= \left| l \right| \left( 0, 0, z', z \right),
\end{align*}
\]

\(^1\) Note that in practical calculations it is more prudent to employ unequal momentum partitioning in the quark-gluon vertex when defining the relative momentum \( l \).
where \( z' = \sqrt{1 - z^2} \). The orthonormal momenta, specifically for this frame, reduce to
\[
\begin{align*}
    t^\mu &= (0, 0, 0, 1), \\
    s^\mu &= (0, 0, 1, 0), \\
    r^\mu &= (0, 1, 0, 0), \\
    e^\mu &= (1, 0, 0, 0).
\end{align*}
\]  

We introduced the orthonormal vectors \( r^\mu, e^\mu \) to completely span the vector space. It is clear that we may then write
\[
\gamma_{TT}^\mu = \not{x}t^\mu + \not{r}r^\mu,
\]
which allows us to separate the Dirac from the Lorentz parts
\[
\begin{align*}
    L^\mu_{(H)} &= H^\mu, & \text{for } H &= \{v, r, s, t\}, \\
    D^\mu_{(H)} &= \left\{ \begin{array}{ll}
    D^\mu_{(1)} & \text{for } H = \{s, t\}, \\
    H D^\mu_{(1)} & \text{for } H = \{r, v\}.
    \end{array} \right.
\end{align*}
\]  

The basis is then given by
\[
L^\mu_v D^\mu_v + L^\mu_r D^\mu_r = L^\mu_{(1)} D^\mu_{(1)},
\]
where once again only those elements relevant in Landau gauge are highlighted. The reduced quark-gluon vertex, as a function of the relative and total momentum \( l \) and \( k \), is then written as
\[
\Gamma^\mu(l, k) = \sum_{i=1}^{4} \sum_{a=\{v,r,s\}} c^\mu_l(k) L^\mu_{(1)} D^\mu_{(1)},
\]
and the projectors, defined \( D^\mu_{(1)} \) and \( L^\mu_{(1)} \), satisfy
\[
\text{Tr} \left[ D^\mu_{(1)} D^\nu_{(1)} \right] = \delta_{\mu\nu}, \quad \text{for } \mu = v, r, s.
\]

No summation over \( a \) is implied on the right-hand side. In practical calculations, we reconstruct the \( \gamma_{TT}^\mu \) and \( c^\mu_l(k) \) by
\[
\begin{align*}
    c^\mu_l(k, l) &= \hat{L}^\mu_{(1)} \text{Tr} \left[ \hat{D}^\mu_{(1)} \Gamma^\mu(l, k) \right].
\end{align*}
\]

\section{Non-transverse}

Instead of the total/relative momentum basis above, which employs the transverse nature of Landau gauge to eliminate the \( L^\mu_v D^\mu_v \) basis elements defined in Eqs. (A.11–A.12), we can use a different set of orthogonal but non-transverse basis elements
\[
\begin{align*}
    t^\mu &= \hat{p}^\mu_2, \\
    s^\mu &= \hat{T}_{\mu\nu}^s \hat{p}^\nu, \\
    \gamma^\mu_{TT} &= \hat{T}_{\rho\sigma}^s \gamma^\nu = \gamma^\mu - \not{x}t^\mu - \not{s}s^\mu.
\end{align*}
\]

\section{Vertex basis for meson BSE}

Due to the kinematics of the meson BSE, the determination of the quark-gluon vertex requires further analytic continuation of the momentum variables.

Referring to Fig. 1, we define the external momenta \( p_i \) in terms of the total incoming momentum of the gluon, \( \Delta^\mu \), and two relative momenta \( \Sigma^\mu \) and \( \Omega^\mu \)
\[
\begin{align*}
    p^\mu_1 &= (\Sigma + \Omega)^\mu, \\
    p^\mu_2 &= (\Sigma + \Omega)^\mu + \Delta^\mu, \\
    p^\mu_3 &= \Delta^\mu.
\end{align*}
\]

Here, \( \Sigma \) and \( \Delta \) are real Euclidean momenta whereas \( \Omega \) contains the complex total momentum of the meson.

In bound-state calculations, a convenient explicit realisation of these momenta \( \Delta, \Sigma \) and \( \Omega \) is
\[
\begin{align*}
    \Delta^\mu &= |\Delta| (0, w'y', wy', y), \\
    \Sigma^\mu &= |\Sigma| (0, 0, z', z), \\
    \Omega^\mu &= |\Omega| (0, 0, 0, 1),
\end{align*}
\]

where \( w' = \sqrt{1 - w^2}, y' = \sqrt{1 - y^2}, z' = \sqrt{1 - z^2} \). That the vectors \( \Sigma \) and \( \Omega \) are not parallel, as in the case of the quark BSE, is the cause of the additional analytic continuation needed for the vertex in the BSE. To accommodate this additional angular dependence, it is also convenient to enlarge the basis for the quark-gluon vertex to include the total momentum of the meson.

\subsection{Transverse}

To avoid confusion, we will use different momentum labels here since our basic set of transverse orthogonal momenta are different to those in the previous section. To exploiting the transversality of Landau gauge, we introduce the following transverse projections of the momenta
\[
\begin{align*}
    \hat{\Omega}^\mu_T &= \hat{T}_{\mu\nu}^\alpha \hat{\Sigma}^\nu, \\
    \hat{\Sigma}^\mu_{TT} &= \hat{T}_{\rho\sigma}^s \hat{\Omega}^\nu_{TT} \hat{\Sigma}^\nu.
\end{align*}
\]

Given explicitly, their normalised form is
\[
\begin{align*}
    \hat{\Delta}^\mu &= (0, w'y', wy', y'), \\
    \hat{\Omega}^\mu_T &= (0, -w'y, -wy, y'), \\
    \hat{\Sigma}^\mu_{TT} &= (0, -w, w', 0).
\end{align*}
\]
with the angles $z$, $y$, and $w$ defined
\[
\Delta_{\perp,\Omega} \cdot \hat{\mathbf{S}}_{\perp,\Omega} = w, \\
\hat{\Delta} \cdot \hat{\Omega} = y, \\
\hat{\Sigma} \cdot \hat{\Omega} = z,
\]
where $\Delta_{\perp,\Omega} = T_{\mu}^{\Omega} \Delta^{\nu}$, $\Sigma_{\perp,\Omega} = T_{\mu}^{\Omega} \Sigma^{\nu}$ and the hat indicates normalisation. As before, $x' = \sqrt{1 - x^2}$. A suitable transverse and orthogonal basis is thus provided by
\[
\left( \frac{\mathbf{\mu}}{T_{TTT}, T_{TT}^{\perp}, \hat{\mathbf{D}}_{TT}}, \hat{\Delta}^{\mu} \right) \times D_{(1)}^{i},
\]
where we define the Dirac part, to be used later, as
\[
D_{(1)}^{i} = \left\{ 1, \hat{\Delta}, \hat{\mathbf{S}}_{TT}, \hat{\mathbf{D}}_{T}, \hat{\Delta} \hat{\mathbf{S}}_{TT}, \hat{\mathbf{D}}_{T}, \hat{\mathbf{S}}_{TT} \hat{\mathbf{D}}_{T}, \hat{\mathbf{S}}_{TT} \hat{\mathbf{D}}_{T} \right\}.
\]
Only those that remain under a transverse projection with respect to an external gluon of momentum $\Delta^{\mu}$ are highlighted. This constitutes 24 (reduced from 32) components that allow for a convenient representation of the quark-gluon vertex as needed in BSE calculations. It is here a function of three squared momenta and three angles, with one squared momentum ($\mathcal{Q}^2$) that corresponds to the mass of the bound-state considered as an external parameter in calculations.

Here, the triply transverse Dirac gamma matrix is defined
\[
\gamma_{TTT}^{\mu} = T_{\mu}^{\alpha} T_{\alpha}^{\beta} T_{\beta}^{\nu} \gamma^{\nu},
\]
where $\gamma^{\nu} = (\gamma^{1}, 0, 0, 0)$. As before, we may introduce the transverse momentum $v^{\mu} = (1, 0, 0, 0)$ as the orthogonal complement to $\Delta$, $\Omega$ and $\Sigma$ and thus define $\gamma_{TTT} = v^{\mu} \gamma^{\nu}$.

We separate the Lorenz from the Dirac parts through
\[
L_{(H)}^{\mu} = H^{\mu},
\]
for $H = \left\{ v, \hat{\mathbf{S}}_{TT}, \hat{\mathbf{D}}_{T}, \hat{\Delta} \right\}$,
\[
D_{(H)}^{i} = \left\{ D_{(1)}^{i}, \hat{\mathbf{D}}_{(1)} \right\},
\]
for $H = \left\{ v \right\}$.

As before, the reduced quark-gluon vertex, as a function of the relative momenta $\Omega$ and $\Sigma$ and the total momentum $\Delta$, can be written
\[
\Gamma^{\mu}(\Sigma, \Delta; \Omega) = \sum_{i=1}^{3} \sum_{a=\{v, \hat{\mathbf{S}}_{TT}, \hat{\mathbf{D}}_{T}, \hat{\Delta} \}} c_{a}^{i}(\Sigma, \Delta; \Omega) L_{(a)}^{\mu} D_{(a)}^{i},
\]
where the number of basis elements over which summation occurs is enlarged and we drop $a = \Delta$ due to transversality. The projectors, defined $D^{i}$ and $L_{(a)}^{\mu}$, satisfy
\[
\text{Tr} \left[ D_{(a)}^{i} D_{(a')}^{i'} \right] = \delta_{aa'} \delta_{ii'}, \quad L_{(a)}^{\mu} L_{(a')}^{\mu} = \delta_{aa'},
\]
such that
\[
\Gamma_{(a)}^{\mu}(\Sigma, \Delta; \Omega) = L_{(a)}^{\mu} \text{Tr} \left[ D_{(a)}^{i} L_{(a)}^{\mu} (\Sigma, \Delta; \Omega) \right] \quad \left( B.20 \right)
\]
where no summation on $a$ is implied on the right-hand side.

### B.2 Non-transverse

As an alternative to the basis above, we detail the obvious non-transverse basis that, despite requiring the full 32 elements of the extended basis to describe the vertex, provides for a simple means to tackle the calculation.

Consider the dressed quark-gluon vertex in the BSE, see Fig. 16. A typical choice for the momenta would be
\[
P^{\mu} = iM \left( 0, 0, 0, 1 \right),
\]
\[
p^{\mu} = |p| \left( 0, 0, z', z \right),
\]
\[
k^{\mu} = |k| \left( 0, w', w, y' \right).
\]
From these, it is convenient to construct the following orthonormal momenta
\[
t^{\mu} = \bar{p}^{\mu},
\]
\[
s^{\mu} = \bar{T}_{\mu}^{\alpha} p^{\alpha},
\]
\[
v^{\mu} = \bar{T}_{\mu}^{\alpha} T_{\alpha}^{\beta} T_{\beta}^{\gamma} v^{\gamma},
\]
\[
\gamma_{TTT}^{\mu} := \bar{T}_{\mu}^{\alpha} T_{\alpha}^{\beta} T_{\beta}^{\gamma} \gamma^{\nu} = \gamma^{\nu} - \xi t^{\nu} - \kappa s^{\nu} - \lambda v^{\nu}.
\]
We introduce the following orthonormal basis
\[
\left( \gamma_{TTT}^{\mu}, v^{\mu}, s^{\mu}, t^{\mu} \right) \times \left( 1, \xi, \kappa, \lambda, \mathcal{F}, \mathcal{F}, \mathcal{F}, \mathcal{F}, \mathcal{F}, \mathcal{F}, \mathcal{F}, \mathcal{F} \right).
\]
Here, we shaded the relevant basis elements in Landau gauge; this is all of them, since we do not exploit transversality due to contractions with the ubiquitous gluon propagator.

Introducing $v$, the orthogonal complement to $r$, $s$, $t$, whose values for this specific frame given in Eq. (A.9) we
\[
\begin{align*}
\text{Fig. 16: The BSE for a meson in a beyond-RL truncation.}
\end{align*}
\]
can write $\gamma^\mu_{\alpha\beta\gamma} = \gamma^\mu_{\alpha\beta} \gamma^\gamma$ in order to separate the Lorentz part from the Dirac part of the vertex. We define the Dirac part of Eq. (B.25)

$$D^{(1)}(H) = \{ 1, \gamma, \gamma^\mu, \gamma^\mu_{\alpha\beta\gamma}, \gamma^\mu_{\alpha\beta\gamma}, \gamma^\gamma \} ,$$  
(B.26)

and can then separate the Dirac from the Lorentz parts

$$L^\mu_{(H)} = H^\mu , \quad \text{for } H = \{ v, r, s, t \} \tag{B.27}$$

$$D^\mu_{(H)} = \left\{ \begin{array}{ll} D^{(1)}_{(H)} & \text{for } H = \{ r, s, t \} \\ H D^{(1)}_{(H)} & \text{for } H = \{ v \} . \end{array} \right. \tag{B.28}$$

As before, the reduced quark-gluon vertex is written

$$F^\mu(p_1, p_2) = \sum_{i=1}^{8} \sum_{a=\{v, r, s, t\}} c^i_a(p_1, p_2) L^\mu_a D^i_{(a)} , \tag{B.29}$$

where the number of basis elements over which summation occurs is enlarged. The projectors, defined $D^i$ and $L^\mu_a$, satisfy

$$\text{Tr} \left( D^i_{(a)} D^i_{(a')}^t \right) = \delta_{aa'} \delta_i^{i'} , \quad \delta^\mu_{(a)} L^\mu_{(a')} = \delta_{aa'} , \tag{B.30}$$

such that

$$c^i_a(p_1, p_2) = L^\mu_{(a)} \text{Tr} \left( D^i_{(a)} F^\mu(p_1, p_2) \right) . \tag{B.31}$$

with no summation over $a$ implied.

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