Light-cone form of field dynamics in anti-de Sitter space-time and AdS/CFT correspondence

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Abstract

Light-cone form of field dynamics in anti-de Sitter space-time is developed. Using field theoretic and group theoretic approaches the light-cone representation for generators of anti-de Sitter algebra acting as differential operators on bulk fields is found. We also present light-cone reformulation of the boundary conformal field theory representations. Making use of these explicit representations of AdS algebra as isometry algebra in the bulk and the algebra of conformal transformations at the boundary a precise correspondence between the bulk fields and the boundary operators is established.

Keywords: Light-cone formalism, AdS/CFT correspondence.

PACS-99: 11.30-j; 11.25.Hf

Published in: Nucl.Phys. B563 (1999) 295-348

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1 Introduction

In spite of its Lorentz noncovariance, the light-cone formalism \[1\] offers conceptual and technical simplifications of approaches to various problems of modern quantum field theory. For example, one can mention the construction of light-cone string field theory \[2\]-\[3\] and superfield formulation for some versions of supersymmetric theories \[6\]-\[9\]. Sometimes, a theory formulated within this formalism turns out to be a good starting point for deriving a Lorentz covariant formulation \[10\],\[11\]. Another attractive application of the light-cone formalism is a construction of interaction vertices in the theory of higher spin massless fields \[12\]-\[16\]. Some interesting applications of light-cone formalism to field theory like QCD are reviewed in \[17\]. Discussion of super-p-branes and string bit models in the light-cone gauge is given in \[18\],\[19\] and \[20\] respectively.

Until now the light-cone formalism was explored in Minkowski space-time (for a review see \[10\]). The major goal of this paper is to develop light-cone form of field dynamics in anti-de Sitter space-time. A long term motivation comes from a number of the following potentially important applications.

One important application is to type IIB superstring in \(AdS_5 \times S^5\) background. Motivated by conjectured duality between the string theory and \(\mathcal{N} = 4, d = 4\) SYM theory \[21\],\[23\] the Green-Schwarz formulation of this string theory was suggested in \[24\] (for further developments see \[25\]-\[30\]). Despite considerable efforts these strings have not yet been quantized (some related interesting discussions are in \[31\]-\[33\]). As is well known, quantization of GS superstrings propagating in flat space is straightforward only in the light-cone gauge. It is the light-cone gauge that removes unphysical degrees of freedom explicitly and reduces the action to quadratical form in string coordinates. The light-cone gauge in string theory implies the corresponding light-cone formulation for target space fields. In the case of strings in AdS background this suggests that we should first study a light-cone form dynamics of target space fields propagating in AdS space-time. Understanding a light-cone description of AdS target space fields might help to solve problems of strings in AdS space-time.

The second application is to a theory of higher massless spin fields propagating in AdS space-time. Some time ago completely self-consistent interacting equations of motion for higher massless fields of all spins in four-dimensional AdS space-time have been discovered \[34\]. For generalization to higher space-time dimensions see \[35\]. Despite efforts the action that leads to these equations of motion has not yet been obtained. In order to quantize these theories and investigate their ultraviolet behavior it would be important find an appropriate action. Since the higher massless spin theories correspond quantum mechanically to non-local point particles in a space of certain auxiliary variables, it is conjectured that they may be ultraviolet finite (see \[36\],\[37\]). We believe that a light-cone formulation is what is required to understand these theories better. The situation here may be analogous to that in string theory; a covariant formulation of closed string field theories is non-polynomial and is not useful for practical calculations, while the light-cone formulation restricts the string action to cubic order in string fields.

Keeping in mind these extremely important applications, in this paper we develop and apply the light-cone formalism to the study of AdS/CFT correspondence at the level state/operators matching. As is well known in the case of the massless fields,
investigation of AdS/CFT correspondence requires analysis of some subtleties related to the fact that transformations of bulk massless fields are defined up to local gauge transformations. These complications are absent in the light-cone formulation because here we only deal with physical fields and this allows us to demonstrate AdS/CFT correspondence in a rather straightforward way.

In this paper we will focus on the light-cone formalism for integer (bosonic) arbitrary spin massless fields that can be formulated in any dimension. There is a number of reasons for this. First, the massless case is the simplest one to demonstrate all essential new features of light-cone formalism in AdS space-time. Second, higher spin massless field theory is an interesting and important subject which itself should be studied in detail. We develop also light-cone formulation for massive fields but in this case we do not discuss the AdS/CFT correspondence. Since the method we use is algebraic in nature, an extension of our results to the case of half integer spin fields (fermions) is straightforward and will be done elsewhere. A generalization to the case supersymmetric theories is relatively straightforward and will be studied in future.

The paper is organized as follows. In section 2 we describe various forms of the \( so(d - 1, 2) \) algebra and explain our notation.

In section 3 we develop light-cone formulation by using field theoretic approach. First we discuss the simplest case of spin one Maxwell field. Next we generalize our discussion to the case of totally symmetric and antisymmetric fields. We find an explicit representation for generators of anti-de Sitter algebra acting as differential operators on fields of arbitrary spin.

Based on these results in section 4 we establish defining equations for the AdS algebra generators for arbitrary symmetry type fields. We demonstrate that a field in \( d \) dimensional AdS space-time can be considered as a massive particle in \( (d - 1) \) flat space time with a continuous mass spectrum. We explicitly match the AdS algebra generators acting on bulk fields with conformal algebra generators acting on fields with continuous mass spectrum. This interrelation is behind AdS/CFT correspondence but it also suggests an idea of how massless higher spin fields in AdS space time and string theory at the boundary could be related. We shall briefly comment on this point.

In section 5 we use group theoretic approach to solve the defining equations. In this section we develop light-cone form of AdS generators for both massless and massive fields. We establish close correspondence between field theoretic and group theoretic approaches.

For comparison of boundary values of bulk light-cone fields with operators of boundary conformal theory we need to develop a light-cone formulation of conformal theories too. To our knowledge this formulation was not previously given in the literature. In section 6 we present light-cone formulation of conformal field theory for the case of arbitrary spin totally symmetric operators. We consider operators with canonical dimension as well as their conformal partners (which are sometimes referred to as shadow operators or sources). We investigate correspondence between solutions of equations of motion for totally symmetric bulk fields and operators of boundary conformal theory. We demonstrate that normalizable modes of bulk field are related to conformal operators while non-normalizable modes are related to conformal partners of these conformal operator.

Section 7 summarizes our conclusions and suggests directions for future research.
Appendices contain some mathematical details and useful formulae.

2 Various forms of $so(d - 1, 2)$ algebra and notation

First let us discuss the forms of AdS algebra, that is $so(d - 1, 2)$, we are going to use. AdS algebra of $d$ dimensional AdS space-time consists of translation generators $\hat{P}^A$ and rotation generators $\hat{J}^{AB}$ which span $so(d - 1, 1)$ Lorentz algebra. The commutation relations of AdS algebra are

\[
[\hat{P}^A, \hat{P}^B] = \lambda^2 \hat{J}^{AB}, \quad [\hat{J}^{AB}, \hat{J}^{CE}] = \eta^{BC} \hat{J}^{AE} + 3 \text{ terms},
\]

\[
[\hat{P}^A, \hat{J}^{BC}] = \eta^{AB} \hat{P}^C - \eta^{AC} \hat{P}^B,
\]

\[
\eta^{AB} = (-, +, \ldots, +), \quad A, B, C, E = 0, 1, \ldots, d - 1.
\]

The $\lambda$ is a cosmological constant of AdS space-time. Throughout this paper we use antihermitean form of generators: $G^\dagger = -G$. As $\lambda \to 0$ the AdS algebra becomes the Poincaré algebra

\[
\lim_{\lambda \to 0} \hat{P}^A = P^A_{\text{Poin}}, \quad \lim_{\lambda \to 0} \hat{J}^{AB} = J^{AB}_{\text{Poin}}.
\]

This form algebra is not convenient for our purposes. We prefer to use the form provided by nomenclature of conformal algebra. Namely we introduce new basis

\[
P^a \equiv \hat{P}^a + \lambda \hat{j}^{d-2a}
\]

new translation generators,

\[
K^a \equiv \frac{1}{2} (-\frac{1}{\lambda^2} \hat{P}^a + \frac{1}{\lambda} \hat{j}^{d-2a}),
\]

conformal boost generators,

\[
D \equiv -\frac{1}{\lambda} \hat{P}^{d-2}
\]

dilatation generator,

\[
J^{ab} \equiv \hat{j}^{ab}
\]

generators of $so(d - 2, 1)$ algebra.

Flat space limit in this notation is given by

\[
\lim_{\lambda \to 0} P^a = P^a_{\text{Poin}}, \quad \lim_{\lambda \to 0} (-\lambda D) = P^{d-2}_{\text{Poin}}, \quad \lim_{\lambda \to 0} \left( \frac{1}{2\lambda} P^a + \lambda K^a \right) = J^{d-2a}_{\text{Poin}}.
\]

In the conformal algebra basis one has the following well known commutation relations

\[
[D, P^a] = -P^a, \quad [D, K^a] = K^a, \quad [P^a, P^b] = 0, \quad [K^a, K^b] = 0,
\]

\[
[P^a, J^{bc}] = \eta^{ab} P^c - \eta^{ac} P^b, \quad [K^a, J^{bc}] = \eta^{ab} K^c - \eta^{ac} K^b,
\]

\[
[P^a, K^b] = \eta^{ab} D - J^{ab}, \quad [J^{ab}, J^{ce}] = \eta^{bc} J^{ae} + 3 \text{ terms},
\]

\[
\eta^{ab} = (-, +, \ldots, +), \quad a, b, c, e = 0, 1, \ldots, d - 3, d - 1.
\]

In this form the AdS algebra is known as the algebra of conformal transformations in $(d - 1)$-dimensional Minkowski space-time. In sections 3-5 we shall be interested in realization
of this algebra as the one of transformations of massless bulk fields propagating in $d$-dimensional AdS space-time while in section 6 we shall realize this algebra as the algebra of conformal transformations on appropriate operators.

Throughout this paper we shall use Poincaré parametrization of AdS space-time in which

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx_i^2 + dz^2 + dx_{d-1}^2), \quad z > 0.$$  

Here and below we set cosmological constant $\lambda$ equal to unity. The boundary at spatial infinity corresponds to $z = 0$. The Killing vectors in these coordinates are given by

$$\xi^{P^a,\mu} = \eta^{a\mu}, \quad \xi^{K^a,\mu} = -\frac{1}{2}x^2\eta^{a\mu} + x^a x^\mu, \quad \xi^{D,\mu} = x^\mu, \quad \xi^{J^{\alpha\beta},\mu} = x^\alpha \eta^{\beta\mu} - x^\beta \eta^{\alpha\mu},$$  

while the corresponding generators are defined as $G = \xi^{G,\mu} \partial_{\mu}$. To develop light-cone formulation we introduce light-cone variables $x^\pm, x^I$ where we use the convention

$$x^\pm \equiv \frac{1}{\sqrt{2}}(x^{d-1} \pm x^0), \quad x^I = x^i, z; \quad x^0 \equiv t, \quad x^{d-2} \equiv z,$$

$I, J, K, L = 1, \ldots, d-2$, \quad $i, j, k, l = 1, \ldots, d-3$.

In this notation scalar product of tangent space vectors is decomposed as

$$X^A Y^A = X^+ Y^- + X^- Y^+ + X^I Y^I, \quad X^I Y^I = X^i Y^i + X^z Y^z,$$

i.e. we use the convention $X^{d-2} = X^z$. The coordinate $x^+$ is considered as an evolution parameter. Here and below to simplify our expressions we will drop the metric tensors $\eta_{AB}, \eta_{ab}$ in scalar products.

Because we are going to describe the fields in the light-cone gauge let us discuss light-cone form of the above algebra. In light-cone formalism the AdS algebra splits into generators

$$P^+, P^i, J^{+i}, K^+, K^i, D, J^{+-}, J^{ij},$$  

which we refer to as kinematical generators and

$$P^-, J^{-i}, K^-,$$  

which we refer to as dynamical generators. For $x^+ = 0$ the kinematical generators are realised quadratically in physical fields while the dynamical generators receive corrections in interaction theory. In this paper we deal with free fields. The light-cone form of AdS algebra can be obtained from (2.3)-(2.5) with the light-cone metric having the following non vanishing elements $\eta^{++} = 1, \eta^{ij} = \delta^{ij}$.  

1Poincaré coordinates cover half of AdS space-time. Because we are interested in infinitesimal transformation laws of physical fields the global description of AdS space-time is not important for our study.

2The target space indices $\mu, \nu$ take the values $0, 1, \ldots, d - 1$.  

5
Instead of target space tensor fields we prefer to use tangent space tensor fields. To pass to tangent space tensor fields we should introduce local frame. One convenient choice is specified by the frame one-forms $e^A = e^A_\mu dx^\mu$ with

$$e^A_\mu = \frac{1}{z} \delta^A_\mu .$$

The connection one-forms, defined by $de^A + \omega^{AB} \wedge e^B = 0$, are then given by

$$\omega^{AB}_\mu = \frac{1}{z} (\delta^A_\mu \delta^B_\rho - \delta^B_\mu \delta^A_\rho ) .$$

Tangent space tensor fields are defined in terms of the target space ones as

$$\Phi^{A_1...A_s} = e^{A_1}_\mu ... e^{A_s}_\mu A^{\mu_1...\mu_s} .$$

To simplify our expressions we introduce creation and annihilation operators $\alpha^A, \bar{\alpha}^A$ and construct Fock space vector

$$| \Phi \rangle = \Phi^{A_1...A_s} \alpha^{A_1} ... \alpha^{A_s} | 0 \rangle , \quad \bar{\alpha}^A | 0 \rangle = 0 .$$

To describe totally antisymmetric fields we use anticommuting oscillators

$$\{ \bar{\alpha}^A, \alpha^B \} = \eta^{AB} , \quad \{ \alpha^A, \alpha^B \} = 0 , \quad \{ \bar{\alpha}^A, \bar{\alpha}^B \} = 0 ,$$

while for description of totally symmetric fields we use commuting oscillators

$$[ \bar{\alpha}^A, \alpha^B ] = \eta^{AB} , \quad [ \alpha^A, \alpha^B ] = 0 , \quad [ \bar{\alpha}^A, \bar{\alpha}^B ] = 0 .$$

The Lorentz covariant derivative for $| \Phi \rangle$ takes the form

$$D_\mu \equiv \partial_\mu + \frac{1}{2} \omega^{AB}_\mu M^{AB} , \quad M^{AB} = \alpha^A \bar{\alpha}^B - \alpha^B \bar{\alpha}^A .$$

where $M^{AB}$ is spin operator of Lorentz algebra $so(d - 1, 1)$. In the sequel we often use the notation

$$\alpha D \equiv \alpha^A D^A , \quad \alpha \bar{\partial} \equiv \alpha^A \bar{\partial}^A , \quad D_A \equiv e^\mu_A D_\mu , \quad \bar{\partial}_A \equiv e_\mu^A \partial_\mu , \quad \bar{\partial}^2 \equiv \bar{\partial}_A \bar{\partial}^A .$$

Also, we adopt the following conventions for derivatives $\partial^+ = \partial_-, \partial^- = \partial_+, \partial^I = \partial_I$, where $\partial_{\pm} \equiv \partial/\partial x^\pm, \partial_I \equiv \partial/\partial x^I$.

3 Light-cone formulation of field dynamics in AdS space-time. Field theoretical approach

In this section we shall develop light-cone formulation of field dynamics in AdS space-time by applying field theoretic approach. The basic strategy, which is well known, consists

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3 Throughout this paper we use a convention \{x, y\} = xy + yx.
of the following steps. First we start with gauge invariant equations of motion for free fields in AdS background. We shall impose light-cone gauge, solve the constraints, and derive equations of motion for physical degrees of freedom. Next we shall use the original global AdS group transformations of gauge fields supplemented by compensating gauge transformation to maintain the gauge. From these we shall get realization of AdS algebra on the space of physical degrees of freedom.

### 3.1 Maxwell field. Light-cone form of equations of motion

As a warm up let us consider spin one Maxwell field. Instead of $A^{\mu}$ with the equations of motion $D_{\mu}F^{\mu\nu} = 0$ we introduce tangent space field $\Phi^A$ defined by (2.10) and use the following form for equations of motion in tangent space

$$D_B F^{BA} = 0, \quad F_{AB} = D_A \Phi_B - D_B \Phi_A, \quad (3.1)$$

where $F^{AB}$ is the field strength in the tangent space while $D_A$ is covariant derivative

$$D_A \Phi_B = \hat{\partial}_A \Phi_B + \omega_{ABC} \Phi_C.$$ 

In Poincaré coordinates one has

$$D_A \Phi_B = \hat{\partial}_A \Phi_B + \delta^z_B \Phi_A - \eta_{AB} \Phi_z$$

and field strength takes the form

$$F_{AB} = \hat{\partial}_A \Phi_B - \hat{\partial}_B \Phi_A + \delta^z_B \Phi_A - \delta^z_A \Phi_B.$$ 

The equations (3.1) can be cast into the form

$$\hat{\partial}_B F^{BA} + (2 - d) F^{zA} = 0$$

and one has then the following second order equations of motion for the gauge field $\Phi^A$

$$(\hat{\partial}^2 + (1 - d)\hat{\partial}_z + d - 2) \Phi^A - \hat{\partial}^A (\hat{\partial} \Phi) + (d - 3) \hat{\partial}^A \Phi^z + (2 - d) \delta^z_A \Phi^z + 2 \delta^A_z (\hat{\partial} \Phi) = 0, \quad (3.2)$$

where $\hat{\partial} \Phi \equiv \hat{\partial}^A \Phi^A$. Since these equations are invariant with respect to the gauge transformation $\delta \Phi^A = \hat{\partial}^A \Lambda$ we can impose the light-cone gauge

$$\Phi^+ = 0. \quad (3.3)$$

Inserting this into equations (3.2) we get the following constraint

$$\hat{\partial}^A \Phi^A = (d - 3) \Phi^z. \quad (3.4)$$

---

\textsuperscript{4}Recall that in the Minkowski space-time the Maxwell equations in gauge $\Phi^+ = 0$ lead to the Lorentz constraint $\hat{\partial}^A \Phi^A = 0$. This is not the case in AdS space-time. Here, by virtue of the relation $D_A \Phi^A = \hat{\partial} \Phi + (1 - d) \Phi^z$, the constraint (3.4) does not coincide with the Lorentz constraint $D^A \Phi^A = 0$. 

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From (3.4) we express the $\Phi^-$ in terms of the physical field

$$\Phi^- = -\frac{\partial I}{\partial^+} \Phi^I + \frac{d - 3}{\partial^+} \Phi^z. \quad (3.5)$$

Note that the second term in r.h.s. of equation (3.5) is absent in flat space. It is this term that breaks $so(d - 2)$ manifest invariance and reduce it to $so(d - 3)$ one. By virtue of the constraint (3.4) the equations of motion (3.2) take the form

$$\left(\hat{\partial}^2 + (1 - d)\hat{\partial}_z + d - 2\right)\Phi^A + (d - 4)\delta^A_z \Phi^z = 0.$$  

From this we get the following equations for the physical fields $\Phi^i, \Phi^z$:

$$\left(\partial^2 - 1 + \frac{1}{4z^2}(d - 2)(d - 4)\right)\Phi^i = 0, \quad \left(\partial^2 - 1 + \frac{1}{4z^2}(d - 4)(d - 6)\right)\Phi^z = 0.$$  

Since this form of equations of motion is not convenient we introduce new physical field $\phi^I$ defined by

$$\Phi^I = z^{(d - 2)/2} \phi^I. \quad (3.6)$$

In terms of $\phi^I$ the equations of motion take the form

$$\left(\partial^2 - 1 + \frac{1}{4z^2}(d - 2)(d - 4)\right)\phi^i = 0, \quad \left(\partial^2 - 1 + \frac{1}{4z^2}(d - 4)(d - 6)\right)\phi^z = 0.$$  

Dividing by $\partial^+$ these equations can be rewritten in the Schrödinger form

$$\partial^- \phi^i = P^- \phi^i,$$

where the action of $P^-$ on physical fields is defined by

$$P^- \phi^i = \left(-\frac{\partial^2}{2\partial^+} + \frac{(d - 2)(d - 4)}{8z^2\partial^+}\right)\phi^i, \quad P^- \phi^z = \left(-\frac{\partial^2}{2\partial^+} + \frac{(d - 4)(d - 6)}{8z^2\partial^+}\right)\phi^z. \quad (3.9)$$

From equations of motion (3.7), (3.8) we see that in $d = 4$ the mass like terms cancel. This fact reflects the conformal invariance of spin one field in four dimensional AdS space-time. Gauge invariant action for spin one Maxwell field

$$S = -\frac{1}{4} \int d^4 x \sqrt{g} F^2_{AB}$$

takes the following form in terms of physical field $\phi^I$

$$S_{l.c.} = \int d^4 x \partial^+ \phi^I(-\partial^- + P^-)\phi^I. \quad (3.11)$$

---

5Here as well as while obtaining the constraint (3.4) we assume, as usual in light-cone formalism, that the operator $\partial^+$ has trivial kernel.

6Note that it is the field $\phi^I$ that has conventional canonical dimension $\Delta_0 = (d - 2)/2$. 

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3.2 Light-cone form of transformations for spin one physical degrees of freedom

Now let us consider transformation laws of physical field $\phi^I$. Toward this end we start, as usual, with original global AdS symmetries, supplemented by compensating gauge transformation required to maintain the gauge

$$\delta_{\text{tot}} A^\mu = \mathcal{L}_\xi A^\mu + \partial^\mu \Lambda, \quad (3.12)$$

where $\xi^\mu$ are the AdS target space Killing vectors and $\mathcal{L}$ is the Lie derivative given by

$$\mathcal{L}_\xi A^\mu = \xi^\nu \partial_\nu A^\mu - A^\nu \partial_\nu \xi^\mu.$$ 

In terms of tangent space field $\Phi^A$ (2.10) and tangent space Killing vectors\footnote{We use the notation $\xi^+, \xi^-, \xi^I$ and $\eta^+, \eta^-, \eta^I$ to indicate the Killing vectors in target space and tangent space respectively.} the transformations (3.12) take the form

$$\delta_{\text{tot}} \Phi^A = \mathcal{L}_\eta \Phi^A + \hat{\partial}^A \Lambda, \quad (3.14)$$

where $\mathcal{L}_\eta$ is given by

$$\mathcal{L}_\eta \Phi^A = \eta^B D_B \Phi^A - \Phi^B D_B \eta^A.$$ 

In Poincaré coordinates we use one has the representation

$$\mathcal{L}_\eta \Phi^A = (\eta \hat{\partial}) \Phi^A + \frac{1}{2} (\hat{\partial}^A \eta^B - \hat{\partial}^B \eta^A) \Phi^B + \frac{1}{2} \delta^A_\xi (\Phi \eta) - \frac{1}{2} \eta^A \Phi^z,$$

where $\eta \hat{\partial} \equiv \eta^A \hat{\partial}^A$, $\eta \Phi \equiv \eta^A \Phi^A$. As usual, the gauge parameter $\Lambda$ can be found from the requirement that the complete transformations (3.14) maintain the gauge (3.3), i.e. from the following equation

$$\delta_{\text{tot}} \Phi^+ = 0.$$ 

Solution to the equation for $\Lambda$ is found to be

$$\Lambda = -\frac{\partial^+ \Phi^I}{\partial^+ \Phi^I}.$$ 

Now plugging this $\Lambda$ and $\Phi^-$ given in (3.5) into $\delta_{\text{tot}} \Phi^I$ we find transformation laws for the physical field:

$$\delta_{\text{tot}} \Phi^I = (\eta \hat{\partial}) \Phi^I + \frac{1}{2} (\hat{\partial}^I \eta^J - \hat{\partial}^J \eta^I) \Phi^J + \frac{1}{2} \delta^I_\xi (\Phi \eta) - \frac{1}{2} \eta^I \Phi^z + \frac{1}{2} (\hat{\partial}^+ \eta^I \hat{\partial}^I - \hat{\partial}^+ \eta^I \hat{\partial}^I) \Phi^J + \frac{1}{2} \delta^I_\xi (\Phi \eta) - \frac{1}{2} \eta^I \Phi^z.$$
or in terms of $\phi^I$ (3.6) and $\xi^\mu$ (3.13)

$$
\begin{align*}
\delta_{\text{tot}} \phi^I &= (\xi \partial) \phi^I + \frac{d-2}{2z} \xi^z + \frac{1}{2} (\partial^I \xi^J - \partial^J \xi^I) \phi^J + \frac{1}{\partial^+} (\partial^+ \xi^I \partial^- J - \partial^+ \xi^J \partial^- I) \phi^J \\
&- \frac{d-4}{2z \partial^+} (\delta^I \partial^+ J + \delta^J \partial^+ I) \phi^J - \frac{1}{z \partial^+} \partial^+ \xi^z \phi^I.
\end{align*}
$$

Here and below we use the notation $\xi \partial \equiv \xi^\mu \partial_\mu$. To simplify our expressions we use creation and annihilation operators $\alpha^I$, $\bar{\alpha}^I$ (2.13) and introduce Fock vector for the physical field $\phi^I$

$$
|\phi\rangle \equiv \phi^I \alpha^I |0\rangle.
$$

The transformation laws of the physical field can be then cast into the following form

$$
\delta_{\text{tot}} |\phi\rangle = \left( \xi \partial + \frac{d-2}{2z} \xi^z + \frac{1}{2} \partial^I \xi^J M^{IJ} \right) + M^{IJ} \partial^+ \xi^I \partial^J - \frac{d-4}{2z \partial^+} \partial^+ \xi^I R_{IJ} - \frac{\partial^+ \xi^z}{z \partial^+} )|\phi\rangle \quad \text{(3.15)}
$$

where the spin operator $M^{AB}$ was defined by (2.14) while the operator $R^{AB}$ is given by

$$
R^{AB} \equiv \alpha^A \bar{\alpha}^B + \alpha^B \bar{\alpha}^A. \quad \text{(3.16)}
$$

The second and last two terms in r.h.s. of (3.15) are absent in Poincaré algebra transformation laws. It is these terms that break manifest $so(d-2)$ invariance to $so(d-3)$ one. Note that the operators $M^{IJ}$ and $R^{IJ}$ form $gl(d-2)$ algebra which has $so(d-2)$ subalgebra spanned by the spin operator $M^{IJ}$. Appearance of $R^{IJ}$ is not desirable. Fortunately, it turns out that the last two terms in r.h.s. of (3.15) can be expressed in terms of square of $M^{IJ}$. To avoid repetition we will demonstrate this explicitly when we shall consider arbitrary spin $s$ field whose particular case is the Maxwell spin one field.

### 3.3 Totally antisymmetric fields. Light-cone form of equations of motion

The next useful toy model is totally antisymmetric field $A^{\mu_1 \cdots \mu_s}$. As before we prefer to use tangent space field $\Phi^{A_1 \cdots A_s}$ defined by (2.10). Here from the very beginning we start with the generating function (2.11) and use the anticommuting oscillators (2.12). An appropriate generating function for the field strength is given by

$$
|F\rangle = \alpha D |\Phi\rangle,
$$

where the notation is given in (2.14), (2.15). The gauge transformation in terms of generating function $|\Phi\rangle$ take the form

$$
\delta_{gf} |\Phi\rangle = \alpha D |\Lambda\rangle. \quad \text{(3.17)}
$$
The fact that $|F\rangle$ is indeed invariant with respect of this transformation can be easily seen from the relation $(\alpha D)^2 = 0$. In terms of $|F\rangle$ the equations of motion take the form

$$\hat{\alpha} D |F\rangle = 0.$$  \hfill (3.18)

Making use of relations

$$\alpha D = \alpha \hat{\partial} - \alpha^z \alpha \bar{\alpha}, \quad \bar{\alpha} D = \bar{\alpha} \hat{\partial} - d \bar{\alpha}^z + \bar{\alpha}^z \alpha \bar{\alpha},$$

we transform the equations of motion (3.18) to the following form

$$(\hat{\alpha} \bar{\partial} + (s + 1 - d) \bar{\alpha}^z)(\alpha \hat{\partial} - s \alpha^z)|\Phi\rangle = 0.$$  \hfill (3.19)

The invariance with respect to the gauge transformation (3.17) allows us to impose light-cone gauge which in terms of generating function looks as

$$\hat{\alpha}^+ |\Phi\rangle = 0.$$  \hfill (3.20)

By applying $\hat{\alpha}^+$ to the equations of motion and taking into account this gauge we get a constraint which dividing by $\hat{\partial}^+$ can be cast into the form

$$(\hat{\alpha} \bar{\partial} + (s + 2 - d) \bar{\alpha}^z)|\Phi\rangle = 0.$$  \hfill (3.21)

Taking into account the gauge (3.20) we get from this the following constraint

$$(\hat{\alpha}^- \hat{\partial}^- + \bar{\alpha}^I \hat{\partial}^I + (s + 2 - d) \bar{\alpha}^z)|\Phi\rangle = 0.$$  \hfill (3.22)

Solution to this constraint is given by

$$|\Phi\rangle = \left(1 - \frac{\partial^I}{\partial^+ \alpha^+} \alpha^I + \frac{d - s - 2}{\partial^+} \alpha^+ \bar{\alpha}^z\right)|\Phi_{ph}\rangle,$$  \hfill (3.21)

where $|\Phi_{ph}\rangle$ is generating function of physical degrees of freedom, i.e. it depends only on $\alpha^I$:

$$|\Phi_{ph}\rangle = \Phi^{I_1 \ldots I_s} \alpha^{I_1} \ldots \alpha^{I_s} |0\rangle.$$  \hfill (3.22)

Now by inserting (3.21) into equations of motion (3.19) we get the following equations of motion for physical field $|\Phi_{ph}\rangle$

$$(z^2 \partial^2 + (2 - d) z \partial_z + (d - 2s - 2) \alpha^z \bar{\alpha}^z + s(d - s - 1))|\Phi_{ph}\rangle = 0.$$  \hfill (3.22)

We wish to express oscillator part of these equations in terms of spin operator $M^{IJ}$ alone. By using the representation (2.14) we get

$$\frac{1}{2} M^2_{ij} |\Phi_{ph}\rangle = (s(s + 3 - d) + (d - 2s - 2) \alpha^z \bar{\alpha}^z)|\Phi_{ph}\rangle,$$

$$M^2_{Ij} |\Phi_{ph}\rangle = -2s(d - 2 - s)|\Phi_{ph}\rangle,$$
where

\[ M^2_{ij} \equiv M^{ij}M^{ij}, \quad M^2_{IJ} \equiv M^{IJ}M^{IJ}. \]

With the help of these relationships we can cast the equations of motion (3.22) into the following form

\[ (z^2 \partial^2 + (2 - d)z\partial_z + \frac{1}{2}M^2_{ij} - M^2_{IJ})|\Phi_{ph}\rangle = 0. \]

Note that \( M^2_{ij} \) and \( M^2_{IJ} \) are nothing but the second order Casimir operator of the \( so(d-3) \) and \( so(d-2) \) algebras respectively. The fact that we can cast our equations into this form we consider as one of interesting results of this work. In order to cancel \( z\partial_z \) term we make the rescaling

\[ |\Phi_{ph}\rangle = z^{(d-2)/2}|\phi\rangle \]

and in such a way we get desirable light-cone form of equations of motion for physical field

\[ (z^2 \partial^2 + \frac{1}{2}M^2_{ij} - M^2_{IJ} - \frac{d(d-2)}{4})|\phi\rangle = 0. \]

By rewriting this equation in the Schrödinger form

\[ \partial^-|\phi\rangle = P^-|\phi\rangle \]

we can immediately get the hamiltonian

\[ P^- = -\frac{\partial^2}{2\partial^+} + \frac{1}{2z^2\partial^+} \left( -\frac{1}{2}M^2_{ij} + M^2_{IJ} + \frac{d(d-2)}{4} \right). \]

The action that leads to equations of motion (3.24) looks as

\[ S_{l.c.} = \int d^dx(\partial^+|\phi\rangle(-\partial^- + P^-)|\phi\rangle. \]

It is instructive to demonstrate how do the results of this section reproduce the ones for the spin one Maxwell field. Spin one vector field \( \phi^I \), which transforms in vector representation of \( so(d-2) \) algebra, is decomposed into vector representation \( \phi^i \) and scalar representation \( \phi \) of \( so(d-3) \) algebra. For both these components the Casimir operator of \( so(d-2) \) algebra \( M^2_{IJ} \) takes the same values

\[ M^2_{IJ}|\phi\rangle = -2(d-3)|\phi\rangle, \quad |\phi\rangle = |\phi_1\rangle + |\phi_0\rangle, \quad |\phi_1\rangle = \phi^i\alpha^i|0\rangle, \quad |\phi_0\rangle = \phi\alpha^z|0\rangle \]

while the Casimir operator of \( so(d-3) \) subalgebra \( M^2_{ij} \) gives

\[ M^2_{ij}|\phi_1\rangle = -2(d-4)|\phi_1\rangle, \quad M^2_{ij}|\phi_0\rangle = 0. \]

Now it is straightforward to see that the formulas for spin one (3.9),(3.10) are indeed reproduced.

---

8Here we use the fact that for totally antisymmetric spin \( s \) representation of \( so(N) \) algebra the Casimir operator \( M^2_{IJ} \) takes the value \( M^2_{IJ}|\phi\rangle = -2s(N-s)|\phi\rangle. \)
3.4 Light-cone form of transformations of totally antisymmetric field

Now we are studying transformation laws of physical degrees of freedom collected in $|\phi\rangle$. As in the case spin one field we start with original global AdS symmetries, supplemented by compensating gauge transformation required to maintain the gauge. The original global AdS transformations in terms of target space tensor field are given by

$$
\delta_{\text{isom}} A^{\mu_1 \ldots \mu_s} = \mathcal{L}_\xi A^{\mu_1 \ldots \mu_s},
$$

(3.27)

where the action of Lie derivative $\mathcal{L}_\xi$ is given by

$$
\mathcal{L}_\xi A^{\mu_1 \ldots \mu_s} = \xi^\nu \partial_\nu A^{\mu_1 \ldots \mu_s} - \sum_{k=1}^s A^{\mu_1 \ldots \mu_{k-1} \nu \mu_{k+1} \ldots \mu_s} \partial_\nu \xi^\mu_k.
$$

In terms of generating function for the tangent space field $|\Phi\rangle$ (2.11) and tangent space Killing vectors $\eta^A$ (3.13) these transformations take the form

$$
\delta_{\text{isom}} |\Phi\rangle = \mathcal{L}_\eta |\Phi\rangle, \quad \mathcal{L}_\eta = \eta^A D_A + \frac{1}{2} D^A \eta^B M^{AB},
$$

(3.28)

In Poincaré coordinates the Lie derivative takes the form

$$
\mathcal{L}_\eta = \hat{\eta} \partial + \frac{1}{2} \hat{\partial} A \eta^B M^{AB} + \frac{1}{2} M z B \eta^B.
$$

(3.29)

Now let us focus on the original AdS algebra transformations supplemented by compensating gauge transformation

$$
\delta_{\text{tot}} |\Phi\rangle = \mathcal{L}_\eta |\Phi\rangle + \alpha D |\Lambda\rangle.
$$

(3.30)

Our aim is to get transformation laws for the physical field $|\phi\rangle$ whose relationship to the gauge field $|\Phi\rangle$ is described by (3.21) (3.23). First we have to find appropriate $|\Lambda\rangle$. To this end we cast the transformations (3.30) into the following form

$$
\delta_{\text{tot}} |\Phi\rangle = (\eta \hat{\partial} + \frac{1}{2} \hat{\partial} A \eta^B M^{AB} + \frac{1}{2} M z B \eta^B) |\Phi\rangle + (\alpha \hat{\partial} - (s - 1) \alpha^z) |\Lambda\rangle.
$$

(3.31)

Then from the relation

$$
\bar{\alpha}^+ \delta_{\text{tot}} |\Phi\rangle = 0,
$$

(3.32)

which is amount to the requirement the complete transformations (3.30) maintain the gauge (3.20) we find the equation

$$
\hat{\partial} + \eta I \alpha^I |\Phi\rangle + (\hat{\partial} + \alpha^z \alpha^+ - \alpha D \alpha^+)|\Lambda\rangle = 0,
$$

which obviously has the following solution

$$
|\Lambda\rangle = -\frac{\partial^+ \eta^I \alpha^I}{\partial^+ \bar{\alpha}^I} |\Phi\rangle.
$$

(3.33)
Note that this solution is fixed by module the term \(\alpha D(f)\). Because this term does obviously not contribute to transformation of \(|\Phi\rangle\) we interested in we can ignore it.

Second let us note that in light-cone gauge the physical field \(|\Phi_{ph}\rangle\) is a coefficient \((\alpha^+)\) in \(|\Phi\rangle\). Due to that in order to find contribution to \(\delta_{tot}|\Phi_{ph}\rangle\) it is sufficient to analyse \((\alpha^+)\) terms in \((3.30)\). The light-cone gauge and the requirement \((3.32)\) which is already satisfied tell us that the terms proportional to \(\alpha^-\) are absent in both sides of \((3.30)\). Therefore we can restrict our attention to \((\alpha^+)\) terms in \((3.30)\). In this way we get the following extremely useful formula for Lie derivative of physical field

\[
\mathcal{L}_\eta|\Phi_{ph}\rangle = (\eta \hat{\partial} + \frac{1}{2} \hat{\partial}^{I} \eta^{I} M^{IJ} + \frac{1}{2} \eta^{J} M^{IJ} \hat{\partial}^{I} |\Phi_{ph}\rangle - \hat{\partial}^{+} \eta^{I} (\bar{\alpha} |\Phi\rangle) |\alpha^+ = 0, \quad (3.34)
\]

where equality \(\alpha^+ = 0\) indicates that we consider only \((\alpha^+)\) terms. The last term in \((3.34)\) can immediately be evaluated by using \((3.21)\)

\[
\bar{\alpha}^- |\Phi\rangle = (- \frac{\partial^{I}}{\partial^+} \bar{\alpha}^I + \frac{d - s - 2}{\partial^+} \bar{\alpha}^z) |\Phi_{ph}\rangle. \quad (3.35)
\]

Thus transformation laws of physical field are given by

\[
\delta_{tot}|\Phi_{ph}\rangle = \mathcal{L}_\eta|\Phi_{ph}\rangle + (\alpha^I \hat{\partial}^I - (s - 1) \alpha^z) |\Lambda\rangle |\alpha^+ = 0. \quad (3.36)
\]

From \((3.33)\) and \((3.21)\) we get immediately

\[
|\Lambda\rangle |\alpha^+ = 0 = - \frac{\partial^{+} \eta^{I}}{\partial^+} \bar{\alpha}^I |\Phi_{ph}\rangle. \quad (3.37)
\]

Inserting this into \((3.33)\) and taking into account \((3.34),(3.33)\) we get the following transformation laws for physical field

\[
\delta_{tot}|\Phi_{ph}\rangle = \left( \eta \hat{\partial} + \frac{1}{2} \hat{\partial}^{I} \eta^{I} M^{IJ} + \frac{1}{2} \eta^{J} M^{IJ} \hat{\partial}^{I} + \frac{d - 2}{2} M^{IJ} \hat{\partial}^{+} \eta^{I} \right) |\Phi_{ph}\rangle
\]

\[
+ \frac{2s + 2 - d}{2 \partial^+} \hat{\partial}^{+} \eta^{I} R^{I} - s \frac{\partial^{+} \xi^{z}}{\partial^+} |\Phi_{ph}\rangle. \quad (3.37)
\]

In terms of physical field \(|\phi\rangle\) \((3.23)\) and target space Killing vectors \(\xi^\mu\) \((3.13)\) this transforms to

\[
\delta_{tot}|\phi\rangle = \left( \xi \hat{\partial} + \frac{d - 2}{2z} \xi^{z} + \frac{1}{2} \hat{\partial}^{I} \xi^{J} M^{IJ} + M^{IJ} \hat{\partial}^{+} \xi^{I} \right) \hat{\partial}^{I} + \frac{2s + 2 - d}{2z \partial^+} \hat{\partial}^{+} \xi^{z} R^{I} - s \frac{\partial^{+} \xi^{z}}{\partial^+} |\phi\rangle. \quad (3.37)
\]

Note that for spin one this expression reduces to the one in \((3.15)\). As before one has term proportional to \(R^{I}\). Now let us demonstrate that this terms can be entirely expressed in terms of the spin operator \(M^{IJ}\). Indeed making of the following relationships

\[
\{ M^{zJ}, M^{IJ} \} = (d - 2 - 2 \alpha^J \bar{\alpha}^I) R^{zI} + 2 \delta_z^I \alpha^I \bar{\alpha}^J
\]
it is straightforward to see that the transformations (3.37) can be cast into the following desired form

\[
\delta_{\text{tot}} |\phi\rangle = \left( \xi \partial + \frac{d-2}{2z} \xi \sigma + \frac{1}{2} \partial^I \xi^J M^{IJ} + M^{IJ} \partial^+ \sigma^J \partial^+ - \frac{\partial^+ \xi^I}{2z \partial^+ \{M^{zj}, M^{jI}\}} \right) |\phi\rangle .
\]  

(3.38)

This remarkable formula demonstrates that the light-cone transformations of physical fields are indeed expressible in terms of the spin operator \(M^{IJ}\). Note that Poincaré algebra transformations involve only terms linear in \(M^{IJ}\). The presence of the term quadratic in \(M^{IJ}\) is an essential feature of the AdS algebra transformations. At the same time the AdS transformations do not involve higher than second order terms in \(M^{IJ}\). This implies, at least, that light-cone formalism in AdS space-time should not be much more complicated than the one in Minkowski space-time. With this optimistic conclusion let us proceed to the totally symmetric fields.

### 3.5 Totally symmetric fields in AdS space-time. Gauge invariant equations of motion

The case of totally symmetric fields is the most interesting one because of the following reasons. One is that the graviton falls into this representation. Another is that in four dimensional AdS space-time all massless physical fields are described by totally symmetric fields. Some time ago completely self consistent interacting equations of motion for such fields have been discovered \(34\). Therefore, before proceeding to the general light-cone case it is reasonable to study the totally symmetric fields in their own right. Note that we consider, as before, an arbitrary \(d\)-dimensional AdS space-time. The starting point in our derivation of light-cone equations of motion are the gauge invariant equations of motion.\(^9\) Some details of derivation of these equations of motion can be found in Appendix A. Let us present the result.

We formulate our equations of motion in terms of the generating function \(|\Phi\rangle\) defined by (2.11) where \(\Phi^{A_1...A_s}\) is totally symmetric tangent space tensor (2.10) while \(\alpha^A, \bar{\alpha}^A\) are commuting oscillators defined by (2.13). The fact that \(|\Phi\rangle\) is a spin \(s\) field is reflected by constraint

\[
\alpha \bar{\alpha} |\Phi\rangle = s |\Phi\rangle, \quad \alpha \bar{\alpha} \equiv \alpha^A \bar{\alpha}^A.
\]

In addition one imposes the double traceless condition

\(^9\)Gauge invariant action in \(d = 4\) AdS space-time for integer spin field has been established in \(38\) while for half integer spin in \(33\). We do not rely on these results because we need equations of motion for arbitrary space-time dimension. For arbitrary space-time dimension the gauge invariant action for massless field has been found in \(10,11\). There the action has been constructed in terms of linear field strength and the reason because we do not use these results is that it is quite difficult to transform the equations of motion for field strengths to the ones for gauge fields. The equations of motion for arbitrary space-time dimension in terms of gauge fields have been found in \(12-14\) but there these equations have been formulated in Lorentz gauge for gauge fields. However as we already demonstrated in light-cone gauge the Lorentz constraint does not follow from the AdS gauge invariant equations of motion. Thus for the purposes of this work we need to derive the equations in question from the very beginning.
\[(\tilde{\alpha}^2)^2 |\Phi\rangle = 0, \quad \tilde{\alpha}^2 \equiv \tilde{\alpha}^A \tilde{\alpha}^A.\]

The gauge invariant equations of motion look then as

\[\left( [\bar{\alpha} D, \alpha D] - \alpha D \tilde{\alpha} D + \frac{1}{2} (\alpha D)^2 \tilde{\alpha}^2 - 2 \alpha^2 \tilde{\alpha}^2 + 2(2s + d - 3) \right) |\Phi\rangle = 0, \tag{3.39}\]

where the corresponding gauge transformation is

\[\delta |\Phi\rangle = \alpha D |\Lambda\rangle \tag{3.40}\]

and the gauge parameter field \( \Lambda \) is subject to usual traceless condition

\[\tilde{\alpha}^2 |\Lambda\rangle = 0. \tag{3.41}\]

The above equations of motion can equivalently be rewritten in the form

\[\left( D_A^2 + \omega^{AB} D_B - \alpha D \tilde{\alpha} D + \frac{1}{2} (\alpha D)^2 \tilde{\alpha}^2 - \alpha^2 \tilde{\alpha}^2 - s^2 + (6 - d)s + 2d - 6 \right) |\Phi\rangle = 0, \tag{3.42}\]

which is more convenient in practical calculations. The formulation in terms of double traceless field is not convenient for our purposes. As is well known the double traceless spin \( s \) field can be decomposed into spin \( s \) traceless field the one of spin \( s - 2 \):

\[|\Phi\rangle = |\Phi_s\rangle + \alpha^2 |\Phi_{s-2}\rangle, \tag{3.43}\]

where the fields \( |\Phi_s\rangle \) and \( |\Phi_{s-2}\rangle \) satisfy the usual traceless condition

\[\tilde{\alpha}^2 |\Phi_s\rangle = 0, \quad \alpha^2 |\Phi_{s-2}\rangle = 0. \tag{3.44}\]

Inverse relations to the decomposition (3.43) are

\[|\Phi_s\rangle = (1 - \frac{\alpha^2 \tilde{\alpha}^2}{2(2s + d - 4)}) |\Phi\rangle, \quad 2(2s + d - 4) |\Phi_{s-2}\rangle = \tilde{\alpha}^2 |\Phi\rangle. \tag{3.45}\]

In terms of these new fields the equations of motion (3.42) take the form

\[\left( D_A^2 + \omega^{AB} D_B - \alpha D \tilde{\alpha} D - s^2 + (6 - d)s + 2d - 6 \right) |\Phi_s\rangle + \alpha^2 \left( D_A^2 + \omega^{AB} D_B - \alpha D \tilde{\alpha} D - s^2 + (2 - d)s + 2 \right) |\Phi_{s-2}\rangle + (2s + d - 6)(\alpha D)^2 |\Phi_{s-2}\rangle = 0. \tag{3.45}\]

This form turns out to be more convenient for deriving light-cone form of equations of motion. Gauge transformation for \( |\Phi_s\rangle \) looks as

\[\delta |\Phi_s\rangle = \left( \alpha D - \frac{\alpha^2}{2s + d - 4} \tilde{\alpha} D \right) |\Lambda\rangle. \tag{3.46}\]

Gauge transformation for the field \( |\Phi_{s-2}\rangle \) can be obtained straightforwardly from the formulas above but we do not need them because in light-cone gauge the constraints produced by equations of motion set the field \( |\Phi_{s-2}\rangle \) to be equal zero.
3.6 Totally symmetric fields. Light-cone form of equations of motion

Taking into account that the gauge field \(|\Phi_s\rangle\) and the gauge parameter field \(|\Lambda\rangle\) have the same number degrees of freedom and due to invariance with respect to gauge transformation (3.46) we can impose the light-cone gauge

\[ \bar{\alpha}^+|\Phi_s\rangle = 0. \] (3.47)

Note that in this gauge we get from (3.44) the constraint

\[ \bar{\alpha}_I^2|\Phi_s\rangle = 0, \] (3.48)

i.e. \(|\Phi_s\rangle\) becomes traceless field with respect to transverse indices. Acting with \(\bar{\alpha}^{+2}\) on the equations of motion (3.45) one proves that \(|\Phi_{s-2}\rangle = 0\). Then by acting \(\bar{\alpha}^+\) on the equations of motion we get the Lorentz like constraint

\[ \bar{\alpha}D|\Phi_s\rangle = -\frac{2(\hat{\alpha}^+ - \alpha^+\bar{\alpha}^z)}{\hat{\alpha}^+ - 2\alpha^+\bar{\alpha}^z} \bar{\alpha}^z|\Phi_s\rangle. \]

Dividing both the sides by \(\hat{\alpha}^+\) this constraint can be cast into form

\[ \bar{\alpha}^-|\Phi_s\rangle = \left(-\frac{\partial^I}{\partial^+} - \alpha^I\bar{\alpha}^+\right)\left(1 - \frac{1}{\hat{\alpha}^+\alpha^+\bar{\alpha}^z}\right)\left(1 - \frac{2}{\hat{\alpha}^+\alpha^+\bar{\alpha}^z}\right)^{2-d-s} \exp\left(-\frac{\partial^I}{\partial^+} - \alpha^I\bar{\alpha}^+\right)|\Phi_s\rangle. \] (3.49)

Solution to this equation is found to be

\[ |\Phi_s\rangle = P|\Phi_{ph}\rangle, \] (3.50)

where an operator \(P\) is defined by

\[ P \equiv \left(1 - \frac{2}{\partial^+} - \alpha^I\bar{\alpha}^+\right)\left(1 - \frac{1}{\hat{\alpha}^+\alpha^+\bar{\alpha}^z}\right)\left(1 - \frac{2}{\hat{\alpha}^+\alpha^+\bar{\alpha}^z}\right)^{2-d-s} \exp\left(-\frac{\partial^I}{\partial^+} - \alpha^I\bar{\alpha}^+\right). \]

The generating function \(|\Phi_{ph}\rangle\) consists of only physical degrees of freedom

\[ |\Phi_{ph}\rangle = \Phi_{I_1 \ldots I_s} \alpha^{I_1} \ldots \alpha^{I_s} |0\rangle \]

and due to (3.48) fulfills the traceless condition

\[ \bar{\alpha}_I^2|\Phi_{ph}\rangle = 0, \] (3.51)

which tells us that the field \(|\Phi_{ph}\rangle\) has the number of spin degrees of freedom equal to dimension of symmetric spin \(s\) irreducible representation of \(so(d-2)\) algebra. It is the field \(|\Phi_{ph}\rangle\) that describes physical degrees of freedom. Now we have to insert solution for \(|\Phi_s\rangle\) (3.50) into equations of motion (3.45). Taking into account that \(|\Phi_{s-2}\rangle = 0\) and rewriting the equations of motion (3.45) explicitly in Poincaré coordinates we get

\[ \left((\hat{\alpha}^A - \alpha^A\bar{\alpha}^z)^2 + (2\alpha^z - \alpha D)\bar{\alpha} D + (1 - d)(\hat{\alpha}^z - \alpha^z\bar{\alpha}^z) - s^2 + (5 - d)s + 2d - 6\right)|\Phi_s\rangle = 0. \]
Inserting the solution (3.50) in these equations we find after some tedious but straightforward calculations the following equations of motion for the physical field $|\Phi_{ph}\rangle$

\[
\left(\hat{\partial}^2 + (1 - d)\hat{\partial}^z + (2s + d - 6)\alpha^z\bar{\alpha}^z - \alpha^2\bar{\alpha}^2 - s^2 + (5 - d)s + 2d - 6\right)|\Phi_{ph}\rangle = 0.
\]

As before we wish to express the oscillator part of these equations in terms of spin operator $M^{ij}$ alone. Taking into account the relation

\[
\frac{1}{2} M^{ij}_2 = \alpha^2\bar{\alpha}^2 - (\alpha^j\bar{\alpha}^j)^2 + (5 - d)\alpha^j\bar{\alpha}^j
\]

and the traceless condition (3.51) one easily proves the following relationship

\[
\frac{1}{2} M^{ij}_2 |\Phi_{ph}\rangle = \left(-\alpha^2\bar{\alpha}^2 + (2s + d - 6)\alpha^z\bar{\alpha}^z - s^2 + (5 - d)s\right)|\Phi_{ph}\rangle.
\]

Making use of this relationship we get the following nice representation for the light-cone equations of motion

\[
\left(\hat{\partial}^2 + (1 - d)\hat{\partial}^z + \frac{1}{2} M^{ij}_2 + 2d - 6\right)|\Phi_{ph}\rangle = 0.
\]

Finally, in terms of the canonical normalized physical field $|\phi\rangle$ defined as in (3.23) one has the following form for equations of motion

\[
\left(z^2\partial^2 + \frac{1}{2} M^{ij}_2 - \frac{(d - 4)(d - 6)}{4}\right)|\phi\rangle = 0.
\] (3.52)

Transforming this equations into the Schrödinger form (3.24) we get the generator $P^-$ for physical totally symmetric field

\[
P^- = -\frac{\partial_i^2}{2\partial^+} + \frac{1}{2z^2\partial^+}\left(-\frac{1}{2} M^{ij}_2 + \frac{(d - 4)(d - 6)}{4}\right).
\] (3.53)

By inserting this $P^-$ and the field $|\phi\rangle$ into (3.20) we get immediately the action that leads to the above light-cone equations of motion.

### 3.7 Light-cone form of transformations of totally symmetric field

In order to find transformation laws of the physical field $|\phi\rangle$ we start as before with the global transformations for the gauge field $|\Phi_s\rangle$ supplemented by appropriate compensating gauge transformation

\[
\delta_{\text{tot}}|\Phi_s\rangle = \mathcal{L}_\eta|\Phi_s\rangle + \delta_{\text{gt}}|\Phi_s\rangle,
\] (3.54)

where the gauge transformation is given in (3.40). The Lie derivative is given in (3.29), where the anticommuting oscillators should be replaced by commuting ones. As above the gauge parameter field $|\Lambda\rangle$ is found from the equation

\[
\bar{\alpha}^+ \delta_{\text{tot}}|\Phi_s\rangle = 0.
\] (3.55)
Note that in order to find $\delta_{tot}|\Phi_{ph}\rangle$ it is sufficient to analyse $(\alpha^\pm)^0$ terms in (3.54). Before to solve the equation (3.55) we consider the contributions of global and gauge transformations in turn. Let us first consider original global AdS transformations. To this end we can use the formula (3.34). The last term in (3.34) can be obtained from (3.49) and is given by

$$\bar{\alpha}^-|\Phi_s\rangle|_{\alpha^+=0} = \left(-\frac{\partial^I}{\partial^+}\bar{\alpha}^I + \frac{s + d - 4}{\partial^+}\bar{\alpha}^z\right)|\Phi_{ph}\rangle. \quad (3.56)$$

Inserting this into (3.34) we get the following contribution of the original global AdS transformations

$$\mathcal{L}_\eta|\Phi_{ph}\rangle = \left(\eta\hat{\Theta} + \frac{1}{2}\hat{\Theta}^I\eta^J M^I J + \frac{1}{2}\eta^I M^2 I + \hat{\Theta}^+\eta^I\alpha^I\left(\frac{\partial^I}{\partial^+}\bar{\alpha}^J - \frac{s + d - 4}{\partial^+}\bar{\alpha}^z\right)\right)|\Phi_{ph}\rangle. \quad (3.57)$$

Now we should find a contribution of the gauge transformation. To this end we cast the gauge transformation (3.46) into the form

$$\delta_{gt}|\Phi_s\rangle = \left(\alpha\hat{\Theta} + (s - 1)\bar{\alpha}^z - \frac{\alpha^2}{2s + d - 4}(\bar{\alpha}\hat{\Theta} + (s - 1)\bar{\alpha}^z)\right)|\Lambda\rangle. \quad (3.58)$$

Since in r.h.s. of these expressions there is the annihilation oscillator $\bar{\alpha}^-$ it is clear that in order to find contribution of $|\Lambda\rangle$ into $\delta_{tot}|\Phi_{ph}\rangle$ we need only the first two leading terms in expansion of $|\Lambda\rangle$ in powers of $\alpha^+$:

$$|\Lambda\rangle = |\Lambda_0\rangle + \alpha^+|\Lambda_1\rangle + \ldots.$$

Plugging this into (3.58) and taking $(\alpha^\pm)^0$ terms we find

$$\delta_{gt}|\Phi_{ph}\rangle = (\alpha^I\hat{\Theta}^I + (s - 1)\bar{\alpha}^z)|\Lambda_0\rangle - \frac{\alpha^2}{2s + d - 4}\left((\bar{\alpha}\hat{\Theta}^I + (s - 1)\bar{\alpha}^z)|\Lambda_0\rangle + \hat{\Theta}^+|\Lambda_1\rangle\right). \quad (3.59)$$

All that remains to find explicit transformation is to find solution to $|\Lambda\rangle$. To this end we return to the equation (3.55). Using there an explicit form of the Lie derivative (3.29) and the gauge transformation (3.46) we get from (3.55) the following equation for $|\Lambda\rangle$

$$\hat{\Theta}^+\eta^I\alpha^I|\Phi_s\rangle + \left(\hat{\Theta}^+ - \frac{2\alpha^+}{2s + d - 4}(\bar{\alpha}\hat{\Theta} + (s - 1)\bar{\alpha}^z)\right)|\Lambda\rangle = 0. \quad (3.60)$$

Note that while deriving this equation we have used the fact that $\bar{\alpha}^+|\Lambda\rangle = 0$. This fact can be proved by analysis the requirement $\bar{\alpha}^+\delta_{tot}|\Phi_s\rangle = 0$ which is consequence of (3.55). From the equation (3.60) we get immediately

$$|\Lambda_0\rangle = -\frac{\partial^+\eta^I}{\partial^+}\alpha^I|\Phi_{ph}\rangle. \quad (3.61)$$

Then by acting on (3.60) with $\bar{\alpha}^-$ and taking $(\alpha^+)^0$ terms we obtain the relation

$$\hat{\Theta}^+\eta^I\alpha^I\bar{\alpha}^-|\Phi_s\rangle + \left(\bar{\alpha}^-\hat{\Theta}^+ - \frac{2}{2s + d - 4}(\bar{\alpha}^-\hat{\Theta}^+ + \alpha^I\hat{\Theta}^I + (s - 1)\bar{\alpha}^z)\right)|\Lambda\rangle|_{\alpha^+=0} = 0.$$
Taking into account (3.56) we get

\[
(\hat{\alpha}^I \hat{\partial}^I + (s - 1)\hat{\alpha}^z)|\Lambda_0\rangle + \hat{\partial}^+|\Lambda_1\rangle = -(2s + d - 4)\frac{\partial^+\eta^I}{\partial^+}\hat{\alpha}^I\hat{\alpha}^z|\Phi_{ph}\rangle. \tag{3.62}
\]

By plugging (3.61) and (3.62) into (3.59) and taking into account the relation

\[
(\alpha^I \hat{\partial}^I + (s - 1)\alpha^z)|\Lambda_0\rangle = \left(-\hat{\partial}^+\eta^I(\alpha^I \hat{\partial}^I + \frac{s - 2}{\partial^+}\alpha^z)\hat{\alpha}^I - s\frac{\partial^+\eta^z}{\partial^+}\right)|\Phi_{ph}\rangle.
\]

we get contribution of the gauge transformation

\[
\delta_{g\ell}|\Phi_{ph}\rangle = \left(-\hat{\partial}^+\eta^I(\alpha^I \hat{\partial}^I + \frac{s - 2}{\partial^+}\alpha^z)\hat{\alpha}^I + \frac{\partial^+\eta^I}{\partial^+}\alpha^2 J\hat{\alpha}^I\hat{\alpha}^z - s\frac{\partial^+\eta^z}{\partial^+}\right)|\Phi_{ph}\rangle.
\]

By summing this with contribution of the original global AdS transformations (3.57) we get complete transformation laws for the physical field |\Phi_{ph}\rangle:

\[
\delta_{\text{tot}}|\Phi_{ph}\rangle = \left(\eta^I \hat{\partial}^I + \frac{d}{2}\hat{\partial}^I \partial^+\frac{\partial^+\eta^I}{\partial^+} M^{I\ell} + \frac{1}{2}\eta^I M^{I\ell} + M^{I\ell}\hat{\partial}^+\eta^I \hat{\partial}^I + \frac{d - 2}{2\partial^+}\hat{\partial}^+\eta^I M^{I\ell} - \frac{2s + d - 6}{2\partial^+}\hat{\partial}^+\eta^I R^{I\ell} + \frac{\partial^+\eta^I}{\partial^+}\frac{\partial^+\partial^+\eta^I}{\partial^+}\alpha^2 J\hat{\alpha}^I\hat{\alpha}^z - s\frac{\partial^+\eta^z}{\partial^+}\right)|\Phi_{ph}\rangle.
\]

This can be rewritten in terms of the canonical normalized physical field |\phi\rangle (3.23) and target Killing vectors \(\xi^\mu\) (3.13) as follows

\[
\delta_{\text{tot}}|\phi\rangle = \left(\xi^I \hat{\partial}^I + \frac{d}{2}\hat{\partial}^I \hat{\partial}^+\frac{\partial^+\xi^I}{\partial^+} M^{I\ell} + \frac{1}{2}\xi^I M^{I\ell} + M^{I\ell}\hat{\partial}^+\xi^I \hat{\partial}^I + \frac{d - 2}{2\partial^+}\hat{\partial}^+\xi^I M^{I\ell} - \frac{2s + d - 6}{2\partial^+}\hat{\partial}^+\xi^I R^{I\ell} + \frac{\partial^+\xi^I}{\partial^+}\frac{\partial^+\partial^+\xi^I}{\partial^+}\alpha^2 J\hat{\alpha}^I\hat{\alpha}^z - s\frac{\partial^+\xi^z}{\partial^+}\right)|\phi\rangle. \tag{3.63}
\]

This form of transformations is quite different from the analogous expressions for antisymmetric field (3.37). Despite this fact, it turns out that the AdS transformations for totally symmetric physical field can also be entirely expressed in terms of the spin operator \(M^{I\ell}\). Indeed, making use of the relation

\[
\{M^{zj}, M^{ji}\} = (2\alpha^I \hat{\alpha}^J + d - 6)R^{I\ell} - 2\alpha^j \hat{\alpha}^I\hat{\alpha}^z - 2\alpha^I \alpha^z \hat{\alpha}^J + 2\delta^I_\ell \hat{\alpha}^J
\]

it is straightforward to see that (3.63) can be cast into the form

\[
\delta_{\text{tot}}|\phi\rangle = \left(\xi^I \hat{\partial}^I + \frac{d}{2}\hat{\partial}^I \hat{\partial}^+\frac{\partial^+\xi^I}{\partial^+} M^{I\ell} + \frac{1}{2}\xi^I M^{I\ell} + M^{I\ell}\hat{\partial}^+\xi^I \hat{\partial}^I + \frac{d - 2}{2\partial^+}\hat{\partial}^+\xi^I M^{I\ell} - \frac{2s + d - 6}{2\partial^+}\hat{\partial}^+\xi^I R^{I\ell} + \frac{\partial^+\xi^I}{\partial^+}\frac{\partial^+\partial^+\xi^I}{\partial^+}\alpha^2 J\hat{\alpha}^I\hat{\alpha}^z - s\frac{\partial^+\xi^z}{\partial^+}\right)|\phi\rangle. \tag{3.64}
\]

Thus, as before, light-cone transformations of physical field

(i) are expressible in terms of the spin operator \(M^{I\ell}\);
(ii) consist only of the terms linear and quadratic in \(M^{I\ell}\), i.e. do not involve higher powers in \(M^{I\ell}\).
Note that the form of transformations for totally symmetric field (3.64) coincides with the one for totally antisymmetric field (3.38). Unfortunately, this fact does not imply that the form of transformations given in (3.64) and supplied by appropriate spin operator $M^{IJ}$ is valid for fields of arbitrary symmetry. In this respect the situation differs from the one for Poincaré algebra transformations. The reason for this is that the AdS algebra transformations involve term quadratic in $M^{IJ}$. Light-cone form description of AdS algebra transformation laws for fields of arbitrary symmetry is given in section 5.

3.8 Light-cone form of generators of AdS algebra

Making use of the AdS transformations given in (3.38) and (3.64) we can represent them as differential operators acting on the physical massless field $|\phi\rangle$. Plugging the Killing vectors (2.7) in transformation laws (3.38) and (3.64) we get corresponding differential form of generators. Let us present the result.

Light-cone form of AdS algebra kinematical generators is given by

$$
P^i = \partial^i, \quad (3.65)$$

$$
P^+ = \partial^+, \quad (3.66)$$

$$
D = x^+ P^- + x^- \partial^+ + x^I \partial^I + \frac{d - 2}{2}, \quad (3.67)$$

$$
J^{+-} = x^+ P^- - x^- \partial^+, \quad (3.68)$$

$$
J^{+i} = x^+ \partial^i - x^i \partial^+, \quad (3.69)$$

$$
J^{ij} = x^i \partial^j - x^j \partial^i + M^{ij}, \quad (3.70)$$

$$
K^+ = -\frac{1}{2} (2x^+ x^- + x_1^2) \partial^+ + x^+ D, \quad (3.71)$$

$$
K^i = -\frac{1}{2} (2x^+ x^- + x_1^2) \partial^i + x^i D + M^{ij} x^j + M^{i-} x^+, \quad (3.72)$$

where

$$
M^{-i} \equiv M^{ij} \frac{\partial^j}{\partial^+} - \frac{1}{2z \partial^+} \{ M^{zj}, M^{zi} \}. \quad (3.73)
$$

Remaining generators which we refer to as dynamical generators are given by

$$
P^- = -\frac{\partial^2}{2 \partial^+} + \frac{1}{2z^2 \partial^+} A, \quad (3.73)$$

$$
J^{-i} = x^- \partial^i - x^i P^+ + M^{ij} \frac{\partial^j}{\partial^+} - \frac{1}{2z \partial^+} \{ M^{zj}, M^{ji} \}, \quad (3.74)$$

$$
K^- = -\frac{1}{2} (2x^+ x^- + x_1^2) P^- + x^- \partial^+ \left( \partial^I M^{IJ} - \frac{x^j}{2z \partial^+} \{ M^{zj}, M^{II} \} \right). \quad (3.75)
$$
The form of an operator $A$ depends on representation. For the case of totally antisymmetric massless field the operator $A$ takes the form (see (3.25))

$$A = -\frac{1}{2}M^2 + M_{ij} + \frac{d(d-2)}{4},$$

while for totally symmetric massless field one has (see (3.53))

$$A = -\frac{1}{2}M^2 + \frac{(d-4)(d-6)}{4}.$$

In what follows the operator $A$ is referred to as AdS mass operator.

A few comments are in order.

(i) the AdS mass operator $A$ for massless fields does not equal to zero in general. The operator $A$ is equal to zero only for massless representations which can be realized as irreducible representations of conformal algebra [45] which for the case of $d$-dimensional AdS space-time is the $so(d, 2)$ algebra.

(ii) Above representations have been derived by using the oscillator form realization of the spin operator $M^{IJ}$ (see (2.14)). However, having expressed generators in terms of the spin operator alone we can use arbitrary form of realization for the spin operator $M^{IJ}$. While exploiting such an arbitrary form of realization one should keep in mind however that spin operators $M^{IJ}$ for totally symmetric and antisymmetric representations satisfy the following defining constraints

$$(M^3)^{[I|J]} = \frac{1}{2}(M^2 + d^2 - 5d + 8)M^{IJ} \tag{3.76}$$

for case of totally antisymmetric field and

$$(M^3)^{[I|J]} = (-\frac{1}{2}M^2 + \frac{(d-4)(d-5)}{2})M^{IJ} \tag{3.77}$$

for totally symmetric field. These defining constraints can be derived by using oscillator form of realization for $M^{IJ}$ but they are valid, of course, for arbitrary form of realization of spin operator.

(iii) Spin operator for field of arbitrary representation, say mixed symmetry representation, does not satisfy the constraints (3.77), (3.76). In other words the constructed generators satisfy commutation relations of the AdS algebra provided the spin operator satisfy the defining constraints (3.77), (3.76). Therefore the above representations for generators are valid only for totally symmetric and antisymmetric fields.

(iv) As is well known, in the light-cone form the Poincaré algebra generators are realized nonlinearly with respect to $\partial^+$, namely, $\partial^+$ appears in denominators of some generators of Poincaré algebra. From the above expression it is seen that as compared to Poincaré algebra generators the AdS algebra generators take additional nonlinear dependence with respect to radial variable $z$. Thus the AdS generators are realized non linearly in two dimensional phase space $\partial^+$ (momentum), $z$ (coordinate). Note that at

---

10We use the notation $(M^3)^{[I|J]} = \frac{1}{2}M^{IK}M^{KL}M^{LJ} - (I \leftrightarrow J)$, $M^2 = M^{IJ}M^{IJ}$.

11The boost generator $J^{+-}$ scales the momentum $p^+$ which is representation of operator $\partial^+$ in momentum space. In string theory in Minkowski space-time this boost is realized as rescaling of string
the same time the generators are still local in coordinate $x^{-}$ and derivative with respect to $z$.

(v) If we restore the dependence on cosmological constant $\lambda$ then making use of (2.2) one can make sure that as cosmological constant tends to zero the light-cone generators of AdS algebra become the ones of the Poincaré algebra.

The above expressions give realization of AdS algebra generators as differential operators acting on physical fields. Now let us write down the realization of AdS algebra generators in terms of physical fields. As we mentioned above the kinematical generators $\hat{G}^{\text{kin}}$ are realized quadratically in the physical fields while the dynamical $\hat{G}^{\text{dyn}}$ are realized non-linearly. At a quadratical level both $\hat{G}^{\text{kin}}$ and $\hat{G}^{\text{dyn}}$ have the following representation

$$\hat{G} = \int dx^{-}d^{d-2}x \{\partial^{+}\phi \langle \phi \rangle \} ,$$

where $G$ are the differential operators given above. The field $\langle \phi \rangle$ satisfies the Poisson-Dirac commutation relation

$$[\langle \phi(x) \rangle, \langle \phi(x') \rangle]_{\text{equal } x^{+}} = -\frac{1}{2\partial^{+}}\delta(x^{-} - x'^{-})\delta^{d-3}(x - x')\delta(z, z') .$$

With these definitions one has the standard commutation relation

$$[[\phi], \hat{G}] = G\phi .$$

4 General light-cone formalism in AdS space-time

In the previous sections we have developed light-cone formulation starting with gauge invariant equations of motion. This strategy is difficult to realize in many cases because the gauge invariant formulations are not available in general. One of attractive features of light-cone formalism is that it allows to formulate field dynamics without knowledge of covariant formulation. Moreover, sometimes a theory formulated within this formalism turns out to be a good starting point for deriving a Lorentz covariant formulation. Derivation of covariant formulation of string field theories is one of the famous examples of exploiting this strategy (see [10, 11]). The practice we have got while deriving light-cone formulation for particular cases allows us to develop general light-cone formalism in AdS space-time. In this section we construct light-cone form of AdS algebra generators which are applicable to arbitrary symmetry representations, so called mixed symmetry representations. We show that these generators can be constructed in terms of spin operators and AdS mass operator. We find closed defining equations for the AdS mass operator.\[12\]

world sheet coordinate $\sigma$. The world sheet scale transformations associated with Virasoro algebra play defining role in string theory. In AdS space-time in addition to the scaling of $p^{+}$ there is the scaling generated by dilatation generator $D$. This $D$ scales, among others, the coordinate $z$. One can speculate that in string theory (and higher spin massless field theory) in AdS space-time there is an underlying invariance and thus an infinite dimensional algebra related to this scaling.

\[12\]A concrete representation for this operator will be found by applying group theoretic approach in the next section.
An attractive feature of the representation for generators we find is that they are valid (i) for massless and massive fields; (ii) for spin multiplets, i.e. these equations are in principle applicable to string theory; (iii) for supersymmetric theories. We also demonstrate how the light-cone description for totally symmetric and antisymmetric fields can be derived entirely within the light-cone formalism without using gauge invariant equations of motion.

Generators described in the previous section have been given for arbitrary value of evolution parameter $x^+$. As is well known, the generators for arbitrary $x^+$ can be obtained from the ones at $x^+ = 0$ by using the relation

$$G_{x^+} = e^{x^+ P^-} G_{x^+ = 0} e^{-x^+ P^-}.$$ 

From this and from commutation relations we learn that the following generators

$$P^i, \quad P^+, \quad P^-, \quad J^{ij}, \quad J^{-i}, \quad K^{-}$$

do not depend on $x^+$ while for the remaining generators one has

$$J^{+i} = J^{+i}_{|x^+ = 0} + x^+ P^i, \quad J^{+-} = J^{+-}_{|x^+ = 0} + x^+ P^-, \quad D = D_{|x^+ = 0} + x^+ P^-,$$

$$K^{i} = K^{i}_{|x^+ = 0} - x^+ J^{-i}, \quad K^{+} = K^{+}_{|x^+ = 0} + x^+ (D + J^{+-})_{|x^+ = 0} + x^2 P^-.$$ 

Thus without loss of generality we can put $x^+ = 0$. Below in this section all the generators are considered for $x^+ = 0$. To develop general light-cone we should make an assumption about form of generators. Based on our previous study we make the following assumptions.

(i) Taking into account that the kinematical generators take the same form for both symmetric and antisymmetric representations (with appropriate spin operator $M^{IJ}$) we suppose that they maintain this form for arbitrary representations

$$P^i = \partial^i, \quad (4.1)$$
$$P^+ = \partial^+, \quad (4.2)$$
$$J^{+i} = -x^i \partial^+, \quad (4.3)$$
$$J^{ij} = x^i \partial^j - x^j \partial^i + M^{ij}, \quad (4.4)$$
$$J^{+-} = -x^+ \partial^+, \quad (4.5)$$
$$D = x^- \partial^+ + x^i \partial^I + \frac{d-2}{2}, \quad (4.6)$$
$$K^{i} = -\frac{1}{2} x^2 \partial^i + x^i D + M^{iI} x^I, \quad (4.7)$$
$$K^{+} = -\frac{1}{2} x^2 \partial^+, \quad (4.8)$$

where the spin part $M^{IJ}$ should be taken in an appropriate representation.
(ii) The dynamical generator $P^-$ has the following form

$$P^- = -\frac{\partial_i^2}{2\partial^+} + \frac{1}{2z^2\partial^+} A. \quad (4.9)$$

Note that we do not make assumptions about the form of remaining generators $K^-$ and $J^{-i}$. Now the problem we are going to solve here is formulated as follows. Given spin operators $M^{ij}$ find the AdS mass operator $A$ and the remaining generators $J^{-i}$, $K^-$. Below by exploiting only the commutation relations of the AdS algebra we demonstrate that the above assumptions turn out to be sufficient to evaluate remaining generators and get closed defining equations for AdS mass operator $A$.

First of all from computation relations of $P^-$ with the kinematical generators given in (4.1)-(4.8) we conclude that the operator $A$ is independent of space-time coordinates $x^I$, $x^-$ and their derivatives $\partial^I$, $\partial^+$, and commutes with spin operator $M^{ij}$

$$[A, M^{ij}] = 0. \quad (4.10)$$

Second, from commutator

$$[P^-, K^i] = -J^{-i}$$

we find representation for $J^{-i}$

$$J^{-i} = x^- \partial^i - x^i P^- + M^{ij} \partial^j - \frac{1}{2z\partial^+} [M^z i, A] . \quad (4.11)$$

Using (4.9) and (4.11) we first find

$$[P^-, J^{-i}] = \frac{1}{4z^3\partial^+} \left( -2\{M^{zi}, A\} + [[M^z i, A], A] \right).$$

On the other hand because of AdS algebra commutation relation $[P^-, J^{-i}] = 0$ we conclude that the AdS mass operator $A$ should satisfy the following constraint

$$2\{M^{zi}, A\} - [[M^z i, A], A] = 0. \quad (4.12)$$

Making use of (4.7) and (4.11) we evaluate then the commutator

$$[K^i, J^{-j}] = -\delta^{ij} \left( -\frac{1}{2} x_k^2 P^- + x^- D \frac{1}{\partial^+} x^j \partial^j M^{jj} - \frac{x^j}{2z\partial^+} [M^{zi}, A] \right) + \frac{1}{2\partial^+} \left( \{M^{iL}, M^{Lj}\} + [M^{zi}, [M^z j, A]] \right)$$

From this and the AdS algebra commutation relation

$$[K^i, J^{-j}] = -\delta^{ij} K^-$$

we find the following representation for the generator $K^-$

$$K^- = -\frac{1}{2} x_k^2 P^- + x^- D \frac{1}{\partial^+} x^j \partial^j M^{jj} - \frac{x^j}{2z\partial^+} [M^{zi}, A] + \frac{1}{\partial^+} B, \quad (4.13)$$
provided the spin operators $M^{ij}$, AdS mass operator $A$ and new operator $B$ satisfy the constraint

$$[M^{z_i}, [M^{z_j}, A]] + \{M^{iL}, M^{Lj}\} = -2\delta^{ij}B.$$  

(4.14)

Note that this constraint gives definition of operator $B$ in terms of basic operators which are spin operator $M^{ij}$ and AdS mass operator $A$.

Thus we have derived representation for all generators of the AdS algebra. One can make sure that remaining commutation relations of the algebra are also satisfied. In fact, it is sufficient to check the commutation relation

$$[J^{-i}, J^{-j}] = 0.$$  

(4.15)

All others will then be satisfied because of Jacobi identities. Calculating the commutator in question we get

$$[J^{-i}, J^{-j}] = \frac{1}{2z^2\partial^2}(-M^{ij}A - M^{zj}AM^{zi} + M^{zi}AM^{zj} + \frac{1}{2}[[M^{z_i}, A], [M^{z_j}, A]]) .$$

Making use of Jacobi identities and defining equations (4.12), (4.14) one finds

$$[[M^{z_i}, A], [M^{z_j}, A]] = 2M^{ij}A - 2M^{zi}AM^{zj} + 2M^{zj}AM^{zi} ,$$

(4.16)

i.e. the commutation relation (4.15) is satisfied and there are no additional constraints on operator $A$.

Let us now summarize the results. Kinematical generators for arbitrary representations of AdS algebra are given in (4.1)–(4.8), while the dynamical generators are given by

$$P^- = -\frac{\partial_i^2}{2\partial^+} + \frac{1}{2z^2\partial^+}A ,$$

(4.17)

$$J^{-i} = x^-\partial^i - x^i\partial^- + M^{iL}\frac{\partial^j}{\partial^+} - \frac{1}{2z\partial^+}[M^{z_i}, A] ,$$

(4.18)

$$K^- = -\frac{1}{2}x_i^2P^- + x^-D + \frac{1}{\partial^+}x^i\partial^+M^{iL} - \frac{x_i}{2z\partial^+}[M^{z_i}, A] + \frac{1}{\partial^+}B ,$$

(4.19)

where AdS mass operator $A$ is (i) independent of $x^I, x^-\partial^I, \partial^+$, i.e. depends only on spin degree of freedom; (ii) invariant of spin part of the $so(d-3)$ algebra (4.10), and (iii) satisfies the defining equations

$$2\{M^{z_i}, A\} - [[M^{z_i}, A], A] = 0 ,$$

(4.20)

$$[M^{z_i}, [M^{z_j}, A]] + \{M^{iL}, M^{Lj}\} = -2\delta^{ij}B .$$

(4.21)

An attractive feature of the representation for generators in (4.17)–(4.19) as well as defining equations is that they are valid for massive fields, spin multiplets and supersymmetric theories.\footnote{Obviously, supersymmetry imposes additional constraints on the operator $A$.}
Thus the general strategy of finding light-cone form of generators consists of the following steps: (i) given spin, or spin multiplet, choose appropriate spin matrix $M^{ij}$; (ii) construct the most general operator $A$ which commutes with $M^{ij}$; (iii) find a solution to constraints (4.20), (4.21).

Making use of this representation we can then compute the second order Casimir operator of AdS algebra

$$Q = \frac{1}{2} J^2_{ab} + 2K^aP^a - D(D + 1 - d)$$

and find that all dependence on the space-time coordinates (orbital part) drops, i.e. the Casimir operator is entirely determined by the spin operator $M^{ij}$ and AdS mass operator $A$:

$$Q = -A + 2B + \frac{1}{2} M^2_{ij} + \frac{d(d - 2)}{4}.$$ (4.22)

Note that this fact is true for Poincaré algebra. Thus the equations of motion take the form

$$\left( -A + 2B + \frac{1}{2} M^2_{ij} + \frac{d(d - 2)}{4} - \langle Q \rangle \right) |\phi\rangle = 0 .$$ (4.23)

where $\langle Q \rangle$ is eigenvalue of the Casimir operator $Q$ in certain representation of the AdS algebra.

Before proceeding let to show in how manner these results can be used to demonstrate the fact that a field in the $d$ dimensional AdS space-time irrespective of its mass can be interpreted as a field in $(d - 1)$-dimensional Minkowski space-time with continuous mass spectrum. Toward this end let us rewrite the generators $P^-$ and $J^{-i}$ as follows

$$P^- = -\frac{\partial^2}{2\partial^+} + \frac{1}{2\partial^+} \hat{M},$$ (4.24)

$$J^{-i} = x^{-}\partial^i - x^i P^- + M^{ij} \frac{\partial^j}{\partial^+} - \frac{1}{\partial^+} \hat{M}^i ,$$ (4.25)

where we introduce the following operators

$$\hat{M} \equiv -\partial^2_z + \frac{1}{z^2} A , \quad \hat{M}^i \equiv M^{zi}z_i + \frac{1}{2z}[M^{zi}, A].$$ (4.26)

The operator $\hat{M}$ in (4.24) can be interpreted as mass operator for a field propagating in $(d - 1)$ dimensional Minkowski space-time while $\hat{M}^i$ is associated with spin operator of this field. We shall refer the $\hat{M}$ as Poincaré mass operator. To support this interpretation we should verify that these operators satisfy appropriate commutation relations between Poincaré mass operator and spin operator $\hat{M}^i$ [10]. Indeed, making use of the defining equations (4.20) and relationship (4.16) one finds that the operators $\hat{M}$ and $\hat{M}^i$ satisfy the following commutation relations

$$[\hat{M}, \hat{M}^i] = 0 , \quad [\hat{M}^i, \hat{M}^j] = M^{ij} \hat{M} .$$
These commutation relations coincide with the ones for mass and spin operators (see [10]). Therefore the generators $P^-, J^{-i}$ given in (4.24), (4.25) coincide exactly with ones for massive field in $(d - 1)$-dimensional Minkowski space-time. Thus we have demonstrated that field in the $d$-dimensional AdS space-time can indeed be interpreted as massive field in $(d - 1)$ dimensional space-time. In this matching the AdS algebra becomes the algebra of conformal transformation of boundary Minkowski space-time. The fact that in $(d - 1)$ dimensions the spectrum is continuous follows from the relation

$$e^{xD}P^2_a e^{-xD} = e^{-2x}P^2_a$$

(see [10]). Interesting point of our study is that it allows us to demonstrate explicitly how do the Poincaré mass and spin operators of massive field in Minkowski space-time relate with AdS mass and spin operator of field in AdS space-time (4.26).

As a illustration how does the general light-cone formalism work let us rederive the representation of AdS algebra for symmetric and antisymmetric fields. Note that in previous section we derived this representation by using gauge invariant equations of motion. Now we start directly with light-cone formalism and look for solution to defining equations (4.20), (4.21). Toward this end we look for the following form of operator $A$:

$$A = aM^2_{ij} + b,$$

where $a$ and $b$ arbitrary functions of spin Casimir operator $M^2_{ij}$. We are going to find these $a$ and $b$ by applying the defining equations. The above operator $A$ obviously commutes with $M^2_{ij}$. Next step is to analyse the defining constraint (4.20). This constraint takes the following form

$$2\{M^zi, A\} - [[M^zi, A], A] = -16a^2(M^3)^{[z][i]} + 2a(1 + 2a)\{M^zi, M^2_{kl}\} + (-8a^2M^2 + 4a^2(N - 2)^2 + 4b)M^zi = 0.$$  

At this point we should use defining equations for spin operators $M^{ij}$. By using the defining equations for spin operator of totally antisymmetric field (3.76) we express the first term in r.h.s. of equation (4.27) in terms of others. From this we get solution

$$a = -\frac{1}{2}, \quad b = M^2 + \frac{d(d - 2)}{4},$$

(4.28)

Exploiting defining equations for spin operator of symmetric field (3.77) in equation (4.27) leads to

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14 This fact was discussed in literature only for scalar field. Our formalism allows us to demonstrate this fact explicitly for arbitrary spin fields at the level of generators matching (4.26).

15 As a side a remark let us note that this fact might have interesting application to a problem of interrelation of higher spin massless spin fields living in $d = 11$ - dimensional AdS space-time, which is a bulk, and superstring theories living at $d = 10$ Minkowski space-time, which is a boundary. One can conjecture that string theory can be interpreted as resulting from some kind of a spontaneous breakdown of symmetries of higher spin massless fields theory. The problem of continuous mass spectrum could be solved then by spontaneous breakdown of original global AdS symmetries.
\[ a = \frac{1}{2}, \quad b = \frac{(d - 4)(d - 6)}{4}. \]  

(4.29)

From the second defining equations (4.21) we get the equations

\[ 4a\delta_{ij}M_{li}^2 + (1 + 2a)\{M^L_i, M^L_j\} = -2\delta_{ij}B \]

and taking into account (4.28) and (4.29) we get

\[ B = M_{zi}^2 \]

for both cases. These results coincide exactly with the those of the previous sections. Here we derived them in purely algebraic way without using covariant equations of motion. This derivation is much simpler and demonstrates efficiency of light-cone approach. Note that in this derivation we used only the defining equations for spin matrices \(M^{IJ}\) (3.77), (3.76). This is all what is required to get a light-cone description of a specific fields from general light-cone formalism.

5 Light-cone generators for arbitrary representations of AdS algebra. Group theoretic approach

In this section we develop light-cone form of AdS generators both for massless and massive fields. To do that we use method of induced representations [48], [49]. Let us briefly describe this method. Let \(g\) be a group and \(h\) its subgroup. The decomposition \(g = hg_x\) defines \(g_x\) as coset representative of \(g/h\). In the following the generators of \(h\) and \(g_x\) are denoted by \(H\) and \(K\) respectively. To define induced representation one introduces a space of functions \(\varphi\) subject to the following condition

\[ \varphi(hg_x) = \Delta(h)\varphi(g_x), \]

(5.1)

where \(\Delta\) is a representation of subgroup \(h\)

\[ \Delta(h_1h_2) = \Delta(h_1)\Delta(h_2). \]

(5.2)

Now the induced representation \(T_g\) is defined by

\[ (T_g\varphi)(g) = \varphi(gg_1). \]

(5.3)

The condition (5.1) tells that the functions \(\varphi\) are defined on coset space. It is convenient to introduce an unconstrained function \(\phi\) defined by \(\phi(x) = \varphi(g_x)\). In terms of \(\phi(x)\) the formula (5.3) takes then the form

\[ (T_g\phi)(x) = \Delta(h(x,g_1))\phi(xg_1), \]

(5.4)

where \(h(x,g)\) and \(xg\) are defined from a decomposition

\[ g_xg = h(x,g)g_{xg}. \]

(5.5)
Since we are interested in differential form for generators we consider the formula (5.4) for infinitesimal transformations

\[ g_1 = 1 + \epsilon_M G^M, \quad \epsilon \ll 1. \]

We use indices \( M, N, P \) to label all generators of algebra \( G^M \), while the indices \( \alpha, \beta, \gamma \) label generators of subalgebra \( H \). For infinitesimal transformations we have

\[ (xg_1)^{\dot{\mu}} = x^{\dot{\mu}} + \epsilon_M \xi^{M\dot{\mu}}, \quad h(x, g) = 1 + \epsilon_M \Omega^M_{\alpha} H^\alpha, \]

where \( \xi^{M\dot{\mu}} \) and \( \Omega^M_{\alpha} \) are Killing vectors and \( h \)-compensators respectively. They satisfy the relations

\[ ([\xi^M \partial, \xi^N \partial]) = f^{MN}_P \xi^P \partial, \quad (\xi^M \partial) \Omega_N - (\xi^N \partial) \Omega_M = f^{MN}_P \Omega^P_{\alpha} - f^{\alpha\beta}_\gamma \Omega^M_{\alpha} \Omega^N_{\beta}, \quad (5.6) \]

where \( (\xi^M \partial) \equiv \xi^{M\dot{\mu}} \partial_{\dot{\mu}} \). The \( x^{\dot{\mu}} \) are coordinates of coset space \( g \times x \). These coordinates should not be confused with the coordinates \( x^\mu \) used in previous sections to describe AdS space-time. Useful formula for calculation of Killing vectors and \( h \)-compensators is

\[ g_\alpha G^M g^{-1}_x = \xi^{M\dot{\mu}} \partial_{\dot{\mu}} g_x g^{-1}_x + \Omega^M_{\alpha} H^\alpha. \quad (5.7) \]

Using these relations and taking into account \( T_{g_1} = 1 + \epsilon_M G^M \) we get from (5.4) the following representation for generators \( G^M \) in terms differential operators acting on \( \phi(x) \)

\[ G^M = \xi^{M\dot{\mu}} \partial_{\dot{\mu}} + \Omega^M_{\alpha} \Delta^\alpha, \quad (5.8) \]

where spin operators \( \Delta^\alpha \) defined by \( \Delta^\alpha \equiv \partial^\alpha \Delta(h)|_{h=1} \) satisfy the commutation relations

\[ [\Delta^\alpha, \Delta^\beta] = f^{\alpha\beta}_\gamma \Delta^\gamma. \quad (5.9) \]

Using (5.8) it is easy to demonstrate that the generators \( G^M \) given in (5.8) satisfy the commutation relations

\[ [G^M, G^N] = f^{MN}_P G^P. \]

In next section we use the basic formulas (5.7) and (5.8) to obtain group theoretic representation of AdS algebra generators.

### 5.1 Light-cone form of generators of AdS algebra. Group theoretic representation

To apply the method of induced representations we should fix decomposition of AdS algebra into subalgebra \( H \) and coset space algebra \( K \). To do that we use Iwasawa decomposition \( G = H^m AN \), where \( H^m \) is a maximal compact algebra, \( A \) is a maximal abelian algebra in \( G/H^m \) and \( N \) nilpotent algebra in \( G/H^m \). The \( N \) transforms in certain representation of \( A \). Next, \( G \) can be decomposed as \( G = KANC \), where \( C \) are elements of \( H^m \) that commute with \( A \), while \( K \) is the remainder of \( H^m \): \( K = H^m/C \). For the case of AdS algebra we have
\[ A = D, J^+ - , \quad N = P^-, J^{-i}, K^-, K^i, \quad C = J^{ij} \]  
\[ K = P^i, P^+, J^{+i}, K^+ . \]  
\[(5.10)\]  
\[ (5.11)\]  
The subalgebra \( H \) is then \( G/K \), i.e. \( H = \text{ANC} \) and in terms of generators it looks as
\[ H = P^-, J^{-i}, K^-, K^i, D, J^+ - , J^{ij} . \]  
\[(5.12)\]
For AdS algebra the subalgebra \( H \) is analog of little group. Note that subalgebra \( H \) and coset space generators \( K \) can be represented as follows
\[ K = (P^+, J^{+i}, K^+) \supset P^i, \quad H = \left( (P^-, J^{-i}, K^-) \supset K^i \right) \supset (D, J^+ - , J^{ij}) . \]
\[(5.13)\]
Thus the coset space associated with \( K \) is used as carrier of representation, while \( H \) is used as inducing subgroup.

The coset representative \( g_x \) and coset coordinates we use are given by
\[ g_x = g_1 g_2 , \quad g_1 \equiv \exp (x^- P^+ + v^i J^{+i} + z^- K^+) , \quad g_2 \equiv \exp (x^i P^i) . \]  
\[(5.13)\]
The wavefunctions will depend on coordinates \( x^i, v^i, x^-, z^- \), which we refer as group theoretic coordinates. After certain redefinitions the coordinates \( x^i \) and \( x^- \) will be related with similar coordinates of field theoretical approach, while the coordinate \( z^- \) will be related with \( z \).

First we calculate the coset space generators (5.11) by applying the basic formula (5.8). For illustration purposes let us present the calculation of coset generators in details. To apply the formula (5.8) we need Killing vectors and \( h \)-compensators which can be obtained from (5.7). To this end we start with the following relation for right invariant form
\[ dg_x g_x^{-1} = (dv^i - z^- dx^i) J^{+i} + (dx^- + v^i dx^i) P^+ + dz^- K^+ + P^i dx^i . \]  
From this we get
\[ \partial_x g_x g_x^{-1} = P^+ , \quad \partial_{v^i} g_x g_x^{-1} = J^{+i} , \quad \partial_{x^-} g_x g_x^{-1} = K^+ , \quad \partial_{x^i} g_x g_x^{-1} = P^i + v^i P^+ - z^- J^{+i} . \]
Using then the relations
\[ g_x P^+ g_x^{-1} = P^+ , \quad g_x J^{+i} g_x^{-1} = J^{+i} - x^i P^+ , \]
\[ g_x K^+ g_x^{-1} = K^+ - \frac{1}{2} x^2_j P^+ + x^i J^{+i} , \quad g_x P^i g_x^{-1} = P^i + v^i P^+ - z^- J^{+i} , \]
and (5.7) we get the Killing vectors
\[ \xi^P^i, x^j = \delta^{ij} , \quad \xi^P^i, x^- = 1 , \quad \xi^{J^{+i}}, v^j = \delta^{ij} , \quad \xi^{J^{+i}}, x^- = -x^i , \]
\[ \xi^{K^+}, v^i = x^i , \quad \xi^{K^+}, x^- = -\frac{1}{2} x^2_j , \quad \xi^{K^+}, z^- = 1 , \]
while corresponding $h$-compensators are equal to zero. Note that these Killing vectors should not be confused with the ones in (2.7) which describe AdS space-time transformations. Making use of these expressions and formula (5.8) we get the following representation for coset generators $K$:

\[
P^i = \partial x^i, \quad (5.14)
\]

\[
P^+ = \partial x^-, \quad (5.15)
\]

\[
J^{+i} = -x^i \partial x^- + \partial \phi^i, \quad (5.16)
\]

\[
K^+ = -\frac{1}{2} x_j^2 \partial x^- + x^i \partial \phi^i + \partial z^- . \quad (5.17)
\]

Now let us derive the representation for generators of the subalgebra $\mathbf{H}$ (5.12). In accordance with general prescription of formula (5.8) we should introduce spin operators $\Delta^a$ for each generator of subalgebra $\mathbf{H}$ (5.12):

\[
\Delta P^-, \quad \Delta J^{-i}, \quad \Delta K^-, \quad \Delta K^i, \quad \Delta D, \quad \Delta J^+, \quad \Delta J^{ij} .
\]

These operators satisfy, by definition, the same commutation relations as generators of subalgebra $\mathbf{H}$ (5.12). Making use of the basic formula (5.8) we get then the following representation for the generators of the subalgebra $\mathbf{H}$:

\[
J^{+-} = -v^i \partial x^i - x^- \partial x^+ - z^- \partial z^+ + \Delta J^{+-} , \quad (5.18)
\]

\[
D = x^i \partial x^i + x^- \partial x^- - z^- \partial z^- + \Delta D , \quad (5.19)
\]

\[
J^{ij} = x^i \partial x^j - x^j \partial x^i + v^i \partial \phi^j - v^j \partial \phi^i + \Delta J^{ij} , \quad (5.20)
\]

\[
K^i = -\frac{1}{2} x_j^2 \partial x^j + x^i D + (v^j \partial \phi^i - v^i \partial \phi^j + \Delta J^{ij}) x^j - x^- \partial \phi^i + v^i \partial z^- + \Delta K^i , \quad (5.21)
\]

\[
P^- = -v^i \partial x^i + \frac{1}{2} v^2 \partial x^- + (z^-)^2 \partial z^- - (\Delta D + \Delta J^{+-}) z^- + \Delta P^- . \quad (5.22)
\]

Note that explicit expressions for the remaining generators $K^-$ and $J^{-i}$ can be found from above expressions for $P^-$, $K^i$ and commutation relations of the $so(d-1,2)$ algebra. The expressions for the generators given in (5.14)-(5.17) and (5.18)-(5.22) provide group theoretic representation for light-cone form of the AdS algebra generators.

### 5.2 Interrelation between group theoretic and field theoretic approaches. Solution to defining equations

Group theoretic representation for generators is constructed in a basis which differs from that used in field theoretic approach. Group theoretic representation is given in terms of coset space coordinates – group theoretic coordinates – defined by the relations (5.13), while in the field theoretic approach we used the usual space-time coordinates. Therefore in order to match these different forms we should transform them to the same basis.
We shall transform group theoretic representation to the one of field theoretic. There is a number of reasons for that. One reason is that we have formulated defining equations for generators written in space-time coordinates. Therefore to find the AdS mass operator and spin operator it is necessary to match orbital parts of generators. Another reason why we consider the field theoretic formulation as the preferable one, which is behind this work, is related to interaction vertices. To find interaction vertices one imposes locality condition, which implies that the vertices should be polynomial in derivatives with respect to transverse space-time coordinates. In other words, formulation in terms of space-time coordinates is preferable because in this form one can apply locality condition directly. So our aim here is to find transformation which match orbital part of group theoretic generators and the field theoretic ones. After that by comparing spin part we find immediately representation for AdS mass operator as well as spin operators $M^{IJ}$. This representation gives solution to defining equations of the previous section.

Before proceeding let us make some simplifications. We are interested in irreducible representation of AdS algebra. According to the general theory (see [48],[49]) in order to get such a representation it is sufficient to set spin operators corresponding the nilpotent algebra $N$ equal to zero

$$\Delta^P = \Delta^{J-} = \Delta^K = \Delta^K = 0. \quad (5.23)$$

For remaining spin operators we use the notation

$$\Delta^{J+} = -\lambda^+, \quad \Delta^K = -\lambda, \quad \Delta^{J+} = m^{ij}, \quad (5.24)$$

where the $\lambda^+$ and $\lambda$ are c-numbers, while spin operator $m^{ij}$ satisfies commutation relations of $so(d-3)$ algebra

$$[m^{ij}, m^{kl}] = \delta^{jk} m^{il} + 3 \text{ terms}.$$ 

$\lambda$ in (5.24) and below should not be confused with cosmological constant. Thus we shall use the generators (5.18)-(5.22) with spin operators given in (5.23), (5.24). Before proceeding it is important to understand which group theoretic variable is responsible for spin degrees of freedom. Below we show that the spin degrees of freedom are described by the variable $v^i$ together with spin operator $m^{ij}$. To be precise, let us make the following Fourier transform

$$f(x^i, x^-, z^-, v^i) = \int du \exp(-uv\partial^+) f_1(x^i, x^-, z^-, u^i),$$

where the $f_1$ is a new wavefunction. In basis of $f_1$ the coordinate $v^i$ takes the representation

$$v^i \to \frac{\partial v^i}{\partial +}, \quad \partial_u \to -u^i \partial^+.$$ 

It turns out that in group theoretic approach it is the new variable $u^i$ that is an analog of oscillators used in field theoretic approach to describe spin degrees of freedom. Remaining group theoretic variables $x^i, x^-, z^-$ are related to space-time coordinates of field theoretic approach. Let us now describe this interrelation in more detail.
First, we are trying to match coset generators \((5.11)\) given in \((4.1)-(4.8)\) and \((5.14)-(5.17)\). It is obvious that the generators \(P^i\) and \(P^+\) already coincide. Next step is to match the generators \(J^{+i}\). To do that we choose instead of the basis \(f_1\) the following new \(f_2\) basis defined by

\[
f_1(x^i, x^-, z^-, u^i) = e^{u\partial_z} f_2(x^i, x^-, z^-, u^i).
\]

Taking into account that in this new basis the coordinates take the representation

\[
x^i \to x^i - u^i, \quad v^i \to \frac{1}{\partial^+}(\partial_{u^i} + \partial_{x^i}), \quad \partial_{u^i} \to -u^i\partial^+
\]

one can make sure that the generator \(J^{+i}\) \(((5.16)\) takes the desired form given in \((4.3)\).

After that we should match the group theoretic \(K^+\) \(((5.17)\) and the one of field theoretic. To do that we choose instead of \(f_2\) the following new \(f_3\) basis

\[
f_2(x^i, x^-, z^-, u^i) = \int dz \exp(-\frac{1}{2}z^-\partial^+ (z^2 + u^2)) f_3(x^i, x^-, z, u^i).
\]

In this new basis the generator \(J^{+i}\) is not changed, while the generator \(K^+\) takes the desired form of field theoretic approach given in \((1.8)\). Thus at this stage group theoretic coset generators match with the ones of field theoretic.

Second step is to analyse the group theoretic dilatation generator \(D\) \(((5.19)\) which in basis of \(f_3\) takes the form

\[
D = x\partial_x + x^-\partial^+ + z\partial_z + u\partial_u + d - 1 - \lambda.
\]

This \(D\) consists of unwanted term \(u\partial_u\) which is absent in \(D\) of field theoretic approach. To remove this term we choose the basis \(f_4\) which is related to the previous basis \(f_3\) as follows

\[
f_3(x^i, x^-, z, u^i) = f_4(x^i, x^-, z, \zeta^i), \quad u^i \equiv \frac{1}{z}\zeta^i.
\]

It is straightforward to see that in the basis of \(f_4\) the unwanted term \(u\partial_u\) in \(D\) is cancelled and we get

\[
D = x^-\partial^+ + x^i\partial^i + d - 1 - \lambda.
\]

By now the orbital part of this operator coincides with the one of field theoretic approach \((4.6)\). Before we match spin part of \(D\) let us write down the expressions for the group theoretic generators \(J^{+-}\) \(((5.18)\) and \(P^-\) \(((5.22)\) in \(f_4\) basis

\[
J^{+-} = -x^-\partial^+ - \lambda^+, \quad P^- = -\frac{\partial^2}{2\partial^+} - \frac{c^\prime + 1}{2z\partial^+} \partial_z + \frac{1}{2z^2\partial^+}\left(\partial^2_{\zeta} + (\zeta \partial_{\zeta})^2 + c^\prime \zeta \partial_{\zeta}\right),
\]

where

\[
c^\prime \equiv d - 1 - 2\lambda - 2\lambda^+.
\]

All that now remains is to find the basis in which the generators \(J^{+-}\), \(D\), \(P^-\) given in \((5.25)\), \((5.26)\) take the form of the ones given in \((1.3)\), \((4.6)\) and \((1.3)\). This can be done by using the following \(f_5\) basis
In $f_5$ basis the generators $J^{+-}, D, P^-$ take precisely the desired form of the field theoretic approach given in (4.5), (4.6), (4.9) with the following representation for AdS mass operator $A$

\[ A = \partial^2_\zeta + (\zeta \partial_\zeta)^2 + c' \zeta \partial_\zeta + \frac{c'^2 - 1}{4}. \quad (5.28) \]

In order to complete our analysis we should evaluate the group theoretic generator $K^i$ (5.21) in the basis of $f_5$. The relatively straightforward calculations gives the desired form of field theoretic approach given in (4.7) with the following representation for spin operators

\[ M^{ij} = \zeta^i \partial_{\zeta^j} - \zeta^j \partial_{\zeta^i} + m^{ij}, \quad M^{zi} = \frac{1}{2}(1 - \zeta^2)\partial_{\zeta^i} + \zeta^i (\zeta \partial_\zeta - \lambda) + m^{ij} \zeta^j. \quad (5.29) \]

Note that in section 4 we have made assumption about form of kinematical generators (4.1)-(4.8) and dynamical generator $P^-$ (4.9). The derivation of this section demonstrates that this conjectured form is in fact a most general form. One can verify that the above expressions for AdS mass operator (5.28) and spin operator $M^{ij}$ (5.29) satisfy the defining equations (4.20), (4.21) and give the following representation for the operator $B$

\[ B = \frac{1}{2}(1 - \zeta^2)\partial^2_\zeta + (\zeta \partial_\zeta)^2 + \frac{c' + d - 5}{2} \zeta \partial_\zeta - m^{ij} \zeta^i \partial_{\zeta^j} - \lambda (\lambda + \frac{c' + d - 3}{2}). \quad (5.30) \]

Thus we have transformed the group theoretic form to the basis where the wavefunction depends on the space-time variables $x^r, x^I$ and spin variables $\zeta^i$. In addition wavefunction transforms in representation of little spin operator $m^{ij}$. In order to understand what kind of form of realization of spin degrees of freedom we have obtained we should inspect the spin operator $M^{IJ}$. It is straightforward to see that the representation we derived for $M^{IJ}$ is nothing but the stereographic form of realization of spin degrees of freedom. This fact is demonstrated in Appendix B. In other words, group theoretic approach naturally leads to the stereographic form of realization of spin degrees of freedom. At the same time, it turns out that the above representation for operators $A, B$ and $M^{IJ}$ can be put into the form which does not rely on stereographic form of realization of spin degrees of freedom. To this end we introduce the following operators

\[ p^i = \partial_{\zeta^i}, \quad k^i = -\frac{1}{2} \zeta^2 \partial_{\zeta^i} + \zeta^i (\zeta \partial_\zeta - \lambda) + m^{ij} \zeta^j, \quad (5.31) \]

\[ m^{ij} = \zeta^i \partial_{\zeta^j} - \zeta^j \partial_{\zeta^i} + m^{ij}, \quad d = \zeta \partial_\zeta - \lambda. \quad (5.32) \]

These operators satisfy commutation relations of $so(d - 2, 1)$ algebra

\[ [d, p^i] = -p^i, \quad [d, k^i] = k^i, \quad [p^i, p^j] = 0, \quad [k^i, k^j] = 0, \]
\[
[p^j, m^{jk}] = \delta^{ij} p^k - \delta^{ik} p^j, \quad [k^i, m^{jk}] = \delta^{ij} k^k - \delta^{ik} k^j,
\]
\[
[p^i, k^j] = \delta^{ij} d - m^{ij}, \quad [m^{ij}, m^{kl}] = \delta^{jk} m^{il} + 3 \text{ terms}
\]

The remarkable fact is that the operators \( A \) (5.28), \( B \) (5.30) and the spin operator \( M^{ij} \) (5.29) are expressible entirely in terms of above operators

\[
A = p^2 + d^2 + (c' + 2\lambda)d + \frac{(c' + 2\lambda)^2 - 1}{4}, \quad B = \frac{1}{2} p^2 + k p + (\lambda + \frac{c' + d - 3}{2})d, \quad (5.33)
\]
\[
M^{ij} = m^{ij}, \quad M^{zi} = \frac{1}{2} p^i + k^i. \quad (5.34)
\]

To summarize we have found two realization for spin operator \( M^{ij} \) and AdS mass operator. First realization given in (5.28), (5.29) is constructed in terms stereographic coordinates. The second realization given in (5.33), (5.34) is constructed in terms of generators of \( so(d-2, 1) \) and does not related to specific coordinates.

By inserting above representations for \( A \) and \( B \) into (4.22) we get for the Casimir operator

\[
-A + 2B + \frac{1}{2} M^{ij} + \frac{d(d-2)}{4} = -\lambda^+ (-\lambda^+ + 1 - d) - \lambda (\lambda + d - 3) + \frac{1}{2} m^{ij}_2. \quad (5.35)
\]

The positive energy lowest weight irreducible representations of \( so(d-1, 2) \) algebra denoted as \( D(E_0, \lambda, h) \), are defined by \( E_0 \), an lowest eigenvalue of the energy operator, and by \( (\lambda, h) \) which is the weight of the \( so(d-1) \) algebra representation in \( so(2) \oplus so(d-3) \) basis. In \( D(E_0, \lambda, h) \) the Casimir operator takes the value

\[
\langle Q \rangle = -E_0 (E_0 + 1 - d) - \lambda (\lambda + d - 3) + \frac{1}{2} \langle m^{ij}_2 \rangle, \quad (5.36)
\]

where \( \langle m^{ij}_2/2 \rangle \) is a eigenvalue Casimir operator of \( so(d-3) \) algebra representation. Inserting relations (5.35) and (5.36) into (4.23) suggests the following identification

\[
E_0 = \lambda^+. \quad (5.37)
\]

Below we demonstrate that the \( E_0 \) defined by this relation is indeed lowest energy value. The representation we obtained describes massive field in general. The basis for the spin states of massive field can be constructed as follows. Introduce the vector \( |h\rangle \) which is (i) the eigenvalue vector of operator \( d \), and (ii) a weight \( h \) representation of the \( so(d-3) \) algebra. This vector satisfies the following conditions

\[
p^i |h\rangle = 0, \quad d |h\rangle = -\lambda |h\rangle.
\]

The \( \lambda \) and \( h = (h_1, \ldots, h_{(d-3)/2}) \) as weights of the \( so(d-1) \) algebra representation satisfy the relation \( \lambda \geq h_1 \ldots \geq h_{[(d-3)/2]} \geq 0 \) for even \( d \) and the relation \( \lambda \geq h_1 \ldots \geq h_{[(d-5)/2]} \geq |h_{[(d-3)/2]}| \) for odd \( d \).

\[^{16}\text{The } \lambda \text{ and } h = (h_1, \ldots, h_{(d-3)/2}) \text{ as weights of the } so(d-1) \text{ algebra representation satisfy the relation } \lambda \geq h_1 \ldots \geq h_{[(d-3)/2]} \geq 0 \text{ for even } d \text{ and the relation } \lambda \geq h_1 \ldots \geq h_{[(d-5)/2]} \geq |h_{[(d-3)/2]}| \text{ for odd } d.\]
where we should exploit the following spin operator.

\[ \Lambda \equiv \sum_{\sigma=0}^{2\lambda} \oplus \Lambda^\sigma, \quad \Lambda^\sigma \equiv k^{i_1} \ldots k^{i_\sigma} |\mathbf{h}\rangle, \]  

(5.38)

where \( \Lambda^{2\lambda+1} = 0 \). Dimension of the space \( \Lambda \) coincides with a dimension of the \( so(d-1) \) algebra irreducible representation which is used to describe spin degrees of freedom of massive field. Therefore the \( \Lambda \) is appropriate to describe spin degrees of freedom of massive field. Obviously the space \( \Lambda \) is invariant under the action of spin operators \( M^{IJ} \) and AdS mass operator \( A \) (see (5.33), (5.34)). However for certain values of \( c' \) there is invariant subspace in \( \Lambda \). This invariant subspace describes spin degrees of freedom of massless field. In order to understand this fact better it is instructive to consider examples.

**Spin one Maxwell field.** Let us start with the simplest case of spin one Maxwell field. In this case we have \( \lambda = 1 \) and \( \mathbf{h} = 0 \). Wavefunction of spin massless field in stereographic coordinates is given by

\[ |\phi\rangle = |\phi_1\rangle + |\phi_0\rangle, \quad |\phi_1\rangle \equiv \zeta^i \phi^i, \quad |\phi_0\rangle \equiv (1-\zeta^2) \phi. \]

(5.39)

The \( |\phi_1\rangle \) and \( |\phi_0\rangle \) transform into one another under the action of spin operator \( M^{IJ} \), i.e. the vector \( |\phi\rangle \) transforms in representation of \( M^{IJ} \). This vector has \( d-2 \) degrees of freedom and therefore is appropriate to describe massless spin one field. For arbitrary \( c' \) the AdS mass operator \( A \) moves the \( |\phi\rangle \) out the form given in (5.39). Indeed straightforward calculation gives

\[ A|\phi\rangle = \kappa_1 |\phi_1\rangle + \kappa_2 |\phi_0\rangle - 2(d-1 + c')|V\rangle, \quad |V\rangle \equiv \phi, \]

where \( \kappa_1 \) and \( \kappa_2 \) are certain constants. Obviously the vector \( |V\rangle \) does not belong to the invariant subspace of \( |\phi\rangle \) given in (5.39). The contribution of \( |V\rangle \) can be cancelled whenever \( c' = 1-d \). Taking into account (5.27) and (5.37) we find \( E_0 = d-2 \) and this nothing but the lowest energy values for spin one massless field in \( d \)-dimensional AdS space-time ([24]). Thus the value \( c' \) is fixed from the requirement that the \( |\phi\rangle \) transforms into itself under the action of AdS mass operator \( A \).

**Totally antisymmetric spin s field.** In this case we have \( \lambda = 1 \) and the weights of vector \( |\mathbf{h}\rangle \) are given by \( \mathbf{h} = (1, \ldots ,1,0 \ldots ,0) \) (where the unity occurs \( s-1 \) times in this sequence). The invariant subspace in \( \Lambda \) appropriate to the description of massless field is given by the wavefunction.

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18 The vector \( |\mathbf{h}\rangle \), which is used to construct the space \( \Lambda \), is given by \( \alpha^{i_1} \ldots \alpha^{i_{s-1}} |0\rangle \). Then the space \( \Lambda \) is described by expression (5.38) and is given by \( \Lambda = (k^i k^j |\mathbf{h}\rangle, k^i |\mathbf{h}\rangle, |\mathbf{h}\rangle) \). The \( k^i \) is given in (5.31) where we should exploit the following spin operator \( m^{ij} = \alpha^i \tilde{\alpha}^j - \alpha^j \tilde{\alpha}^i \).
Then from (5.27), (5.37) we get value of \( E \).

In this case one has (5.43). The dots in (5.43) indicate the subleading terms which can be read from (B.6).

\[
|\phi_0\rangle \equiv \phi^{i_1 \ldots i_{s-1}} \left( (1 - \zeta^2) \alpha^{i_1} \ldots \alpha^{i_{s-1}} + 2 \sum_{k=1}^{s-1} \alpha^{i_1} \ldots \alpha^{i_{k-1}} \zeta \alpha \zeta \alpha \zeta \alpha^{i_{k+1}} \ldots \alpha^{i_{s-1}} \right) |0\rangle.
\]

As before the \(|\phi_1\rangle\) and \(|\phi_0\rangle\) transform into one another under the action of spin operator \(M^{IJ}\). The vector (5.40) has spin degrees of freedom appropriate to describe totally antisymmetric massless field. Decomposition (5.40) reflects the fact the antisymmetric rank \(s\) tensor of \(so(d-2)\) algebra can be decomposed into rank \(s\) antisymmetric tensor of \(so(d-3)\) algebra and the ones of rank \(s-1\). Let us inspect of action of AdS mass operator on \(|\phi\rangle\). Straightforward calculation gives

\[
A|\phi\rangle = \kappa_1 |\phi_1\rangle + \kappa_2 |\phi_0\rangle + 2(2s - 1 - d - c')|V\rangle,
\]

where

\[
|\phi_1\rangle \equiv \phi^{i_1 \ldots i_s} \zeta^{i_1} \alpha^{i_2} \ldots \alpha^{i_s} |0\rangle,
\]

\[
|\phi_0\rangle \equiv \phi^{i_1 \ldots i_{s-1}} \left( (1 - \zeta^2) \alpha^{i_1} \ldots \alpha^{i_{s-1}} + 2 \sum_{k=1}^{s-1} \alpha^{i_1} \ldots \alpha^{i_{k-1}} \zeta \alpha \zeta \alpha \zeta \alpha^{i_{k+1}} \ldots \alpha^{i_{s-1}} \right) |0\rangle.
\]

Again the vector \(|V\rangle\) does not belong to invariant subspace given in (5.40) and therefore we should cancel its contribution. To cancel the contribution of \(|V\rangle\) we set the coefficient in front of this vector equal to zero and this leads to the solution \(c' = 2s - d - 1\). Taking into account (5.27), (5.37) we get then \(E_0 = d - s - 1\). This is lowest energy value for massless totally antisymmetric field in AdS space-time (12, 13).

**Totally symmetric spin \(s\) field.** Above analysis can be immediately extended to arbitrary spin \(s\) totally symmetric field. In this case we have \(\lambda = s\) and \(h = 0\) and we start with wavefunction\(^{19}\)

\[
|\phi\rangle = |\phi_0\rangle + (1 - \zeta^2)|\phi_{s-1}\rangle + \ldots
\]

where

\[
|\phi_s\rangle \equiv \zeta^{i_1} \ldots \zeta^{i_s} \phi^{i_1 \ldots i_s}, \quad |\phi_{s-1}\rangle \equiv \zeta^{i_1} \ldots \zeta^{i_{s-1}} \phi^{i_1 \ldots i_{s-1}}.
\]

Complete expression of \(|\phi\rangle\) is given in Appendix B (see (B.6)). Here we exploit first two terms in expansion of (B.3), put there \(\rho = 1\) and use simplified normalization given in (5.43). The dots in (5.43) indicate the subleading terms which can be read from (B.6). In this case one has

\[
A|\phi\rangle = \kappa_1 |\phi_s\rangle + \kappa_2 (1 - \zeta^2)|\phi_{s-1}\rangle + 2(5 - 4s - d - c')|V\rangle + \ldots,
\]

where vector \(|V\rangle\) is given by

\[
|V\rangle = \zeta^{i_1} \ldots \zeta^{i_{s-1}} \phi^{i_1 \ldots i_{s-1}}.
\]

Because the vector \(|V\rangle\) again does not belong to the space of \(|\phi\rangle\) given in (5.43) we set factor in front of \(|V\rangle\) equal to zero. This can be achieved by choosing \(c' = 5 - d - 4s\). Then from (5.27), (5.37) we get value of \(E_0\)

\(^{19}\)In this case the space \(\Lambda\) is given by (5.38) where we set \(\lambda = s\), \(|h\rangle = 1\) and \(m^{ij} = 0\).
\[ E_0 = s + d - 3, \]

which is nothing but lowest energy value for totally symmetric massless fields in AdS space-time \[ [12]. \]

The lowest energy value for massless arbitrary type symmetry representation has been found in \[ [13] \] and is given by

\[ E_0 = \lambda - k - 2 + d, \]

where \( k \) is determined by the relation \( \lambda = h_1 = \ldots h_{k-1} > h_k \). It is for this value \( E_0 \) that there is invariant subspace in \( \Lambda \) appropriate to describe massless fields. Because detailed description of that invariant subspace is too involved, we hope to study it in future publications.

6 Light-cone form of conformal field theory

In this section we present light-cone reformulation of (free) conformal field theory. The reason for doing this is that we are going to establish AdS/CFT correspondence between bulk massless fields and conformal field theory operators. In the previous sections the bulk massless fields have been studied within the framework of the light-cone formalism. Therefore most adequate form for comparison is the light-cone form of conformal field theory.

6.1 Lorentz covariant form of conformal field theory

To keep our presentation as simple as possible we restrict our attention to the case of arbitrary spin totally symmetric operators that have canonical conformal dimension given below in \( (6.1) \). In this section we recall main facts of conformal field theory about these operators. The reason for this is that we use formulation in terms of generating functions and the resulting formulas look different from the ones that can be found in the standard literature on this subject (for instance see \[ [17] \] and reference therein).

In this section the \( \text{so}(d-1,2) \) algebra is considered as algebra of conformal transformations of \( (d-1) \)-dimensional Minkowski space-time. We are interested in spin \( s \) totally symmetric conformal operators

\[ O^{a_1 \ldots a_s}(x) \]

that have canonical conformal dimension \[ (6.1) \]

\[ \Delta = s + d - 3. \]

These operators, by definition, are traceless and divergence free.

---

\[ ^{20} \text{We do not discuss Lorentz covariant formulation for shadow operators which have conformal dimension } \Delta = 2 - s. \text{ In Appendix C we give directly light-cone formulation of such operators.} \]

\[ ^{21} \text{The fact that expression in r.h.s. of } (6.1) \text{ is nothing but the lowest energy value of spin } s \text{ massless fields propagating in } d \text{ dimensional AdS space-time has been demonstrated in } [12]. \]
\[ \mathcal{O}^{a_1 a_2 \ldots a_s} = 0, \quad \partial_{x^a} \mathcal{O}^{a_2 \ldots a_s} = 0. \]

As above to simplify our presentation we consider Fock space vector (generating function)

\[ |\mathcal{O}_{\text{cov}}\rangle \equiv \mathcal{O}^{a_1 \ldots a_s} \alpha^{a_1} \ldots \alpha^{a_s} |0\rangle. \quad (6.2) \]

In terms of generating function the traceless and divergence free conditions take the following form

\[ \bar{\alpha}^a \bar{\alpha}^a |\mathcal{O}_{\text{cov}}\rangle = 0, \quad (6.3) \]
\[ \bar{\alpha}^a \partial_{x^a} |\mathcal{O}_{\text{cov}}\rangle = 0. \quad (6.4) \]

Realization of conformal algebra generators on the space of operators \(|\mathcal{O}_{\text{cov}}\rangle\) is given by

\[ P^a = \partial^a, \quad (6.5) \]
\[ J^{ab} = x^a \partial^b - x^b \partial^a + M^{ab}, \quad (6.6) \]
\[ D = x^a \partial_{x^a} + \Delta, \quad (6.7) \]
\[ K^a = -\frac{1}{2} x_b^2 \partial^a + x^a (x^b \partial_{x^b} + \Delta) + M^{ab} x^b, \quad (6.8) \]

where \(\partial^a \equiv \eta^{ab} \partial_{x^b}\) and the so\((d-2,1)\) algebra spin operator \(M^{ab}\) is given by

\[ M^{ab} = \alpha^a \bar{\alpha}^b - \bar{\alpha}^b \alpha^a. \quad (6.9) \]

The condition that the conformal operator \(|\mathcal{O}_{\text{cov}}\rangle\) subject to the constraints (6.3), (6.4) should have the canonical conformal dimension (6.1) amounts to the requirement that this operator constitutes an invariant subspace in representation of conformal algebra. Indeed the operator \(\bar{\alpha}^a\) obviously commutes with all generators of conformal algebra. Operator \(\bar{\alpha}^a \partial_{x^a}\) commutes with the generators \(P^a, D\) and \(J^{ab}\) on the space of \(|\mathcal{O}_{\text{cov}}\rangle\). As to commutation relation of \(\bar{\alpha}^a \partial_{x^a}\) with \(K^a\) we get

\[ [\bar{\alpha}^b \partial_{x^b}, K^a] = \bar{\alpha}^a (\Delta - \alpha^b \bar{\alpha}^b - d + 3) + x^a \bar{\alpha}^b \partial_{x^b} + \alpha^a \bar{\alpha}^2. \]

Making use of this and the constraints (6.3), (6.4) we find

\[ [\bar{\alpha} \partial, K^a] |\mathcal{O}_{\text{cov}}\rangle = \bar{\alpha}^a (\Delta - s - d + 3) |\mathcal{O}_{\text{cov}}\rangle. \]

From this it clear that only for canonical conformal dimension (6.1) the operator \(|\mathcal{O}_{\text{cov}}\rangle\) subject to the constraints (6.3), (6.4) constitutes the invariant subspace in representation of conformal algebra.
6.2 Light-cone form of conformal field theory

Our derivation of light-cone form of generators of conformal algebra proceeds as follows. Recall that in the bulk the $so(d-1,2)$ algebra was realized on the space of unconstrained physical fields. On the CFT side our operators are subject to the divergence constraint \((6.4)\). It is reasonable to solve this constraint and formulate boundary conformal theory also in terms of unconstrained operators. Solution to the constraint \((6.4)\) is easily found to be

\[
|O_{\text{cov}}(\alpha^+, \alpha^-, \alpha^i)] = \exp \left( -\frac{\alpha^+}{\partial^+}(\bar{\alpha} \partial^- + \bar{\alpha}^i \partial^i) \right) |O(\alpha^-, \alpha^i)\rangle' \tag{6.10}
\]

A traceless operator $|\mathcal{O}\rangle'$ is an unconstrained operator. As compared to the original operator $|O_{\text{cov}}\rangle$ this $|\mathcal{O}\rangle'$ does not depend on oscillator $\alpha^+$. The second step in our derivation is to choose a new basis in which the generators

\[
P^a, \quad J^{+i}, \quad J^{+-}, \quad K^+, \tag{6.11}
\]

take form as simple as possible. Namely we choose a basis in which the generators given in \((6.11)\) do not depends on matrix $M^{ab}$. This can be done step by step. First we choose a basis which makes $J^{+i}$ independent of $M^{+i}$. Next we choose a basis which makes $J^{+-}$ independent of $M^{+-}$ and finally we choose a basis which makes $K^+$ independent of $M^{ab}$. Details can be found in Appendix C. Let us present the final light-cone form of the generators realized on conformal theory operators

\[
P^a = \partial^a, \tag{6.12}
\]
\[
J^{+i} = x^+ \partial^i - x^i \partial^+, \tag{6.13}
\]
\[
J^{+-} = x^+ \partial^- - x^- \partial^+, \tag{6.14}
\]
\[
J^{ij} = x^i \partial_{x^j} - x^j \partial_{x^i} + M^{ij}, \tag{6.15}
\]
\[
K^+ = -\frac{1}{2}(2x^+ x^- + x_i^2) \partial^+ + x^+ D, \tag{6.16}
\]
\[
D = x^+ \partial^- + x^- \partial^+ + x^i \partial_{x^i} + \hat{\Delta}, \tag{6.17}
\]
\[
J^{-i} = x^- \partial^i - x^i \partial^- + M^{ij} \frac{\partial^j}{\partial^+} - \frac{1}{\partial^+} M^i, \tag{6.18}
\]

where the spin operator $M^{ij}$ is the same as in \((6.9)\). The generator $M^i$ transforms in vector representation of the spin operator $M^{ij}$ and satisfies the commutation relations

\[
[M^i, M^{jk}] = \delta^{ij} M^k - \delta^{ik} M^j, \quad [M^i, M^i] = \square M^{ij},
\]

where $\square$ is the Dalamber operator in $(d-1)$ dimensional Minkowski space-time

\[
\square \equiv \partial_{x^a}^2.
\]

We use the following realization of unconstrained generating function $|\mathcal{O}\rangle$. We decompose $|\mathcal{O}\rangle$ into irreducible representations of $so(d-3)$ algebra
\[ |O\rangle = \begin{cases} \sum_{s'} \otimes |O_{s'}^{(1)}\rangle & s' = s, s - 2, s - 4, \ldots, s - 2\lfloor \frac{s}{2} \rfloor, \\ \sum_{s'} \otimes |O_{s'}^{(2)}\rangle & s' = s - 1, s - 3, s - 5, \ldots, s - 2\lfloor \frac{s-1}{2} \rfloor. \end{cases} \] (6.19)

Now a representation of spin part \( \hat{\Delta} \) of the dilatation operator \( D \) \((6.17)\) and the operator \( M^i \) on conformal operators \( |O_{s'}^{(1,2)}\rangle \) is determined to be

\[ \hat{\Delta} \equiv \alpha^i \bar{\alpha}^i + d - 3, \] (6.20)

\[ M^i |O_{s'}^{(1)}\rangle = \Box (\alpha^i - \frac{\alpha^2 \bar{\alpha}^i}{2s' + d - 7}) |O_{s' - 1}^{(2)}\rangle + \frac{\bar{\alpha}^i}{2s' + d - 3} |O_{s' + 1}^{(2)}\rangle \] (6.21)

\[ M^i |O_{s'}^{(2)}\rangle = \Box a(s, s') (\alpha^i - \frac{\alpha^2 \bar{\alpha}^i}{2s' + d - 7}) |O_{s' - 1}^{(1)}\rangle + \frac{a(s, s' + 1)}{2s' + d - 3} \bar{\alpha}^i |O_{s' + 1}^{(1)}\rangle \] (6.22)

where

\[ a(s, s') \equiv (s - s' + 1)(s + s' + d - 5). \]

Light-cone form of generators for shadow operator can be obtained from \((6.12)-(6.22)\) by making there the following substitutions: i) the operator \( \hat{\Delta} \) in \((6.16), (6.17)\) should be replaced by \( \tilde{\Delta} = 2 - \alpha^i \bar{\alpha}^i \); ii) in equations \((6.21), (6.22)\) the operator \( \Box \) should be deleted in front of \( |O_{s'}^{(1,2)}\rangle \) and should be placed in front of \( |O_{s' + 1}^{(1,2)}\rangle \).

### 7 Light-cone form of AdS/CFT correspondence

After we have derived the light-cone formulation for both the bulk fields and the boundary conformal theory operators we are ready to demonstrate explicitly AdS/CFT correspondence. Euclidean version of this correspondence for various particular cases has been studied in \([50]-[67]\). Intertwining operator realization of AdS/CFT correspondence was investigated in \([68]\). For review and complete list of references see \([69]\). Here we study correspondence for Lorentzian signature of AdS space-time. As far as we know, in this case the correspondence was discussed only for the case of the scalar field \([70]\). We establish correspondence for totally symmetric arbitrary spin massless fields. Namely, we match normalizable modes of solution to bulk equations of motion for massless fields and conformal operators with canonical conformal dimension. Also we make explicit map between non normalizable modes and shadow operators which are conformal partners of operators with canonical conformal dimensions.

What is required is to demonstrate that representation of the \( so(d - 1, 2) \) algebra on bulk physical fields coincides exactly with one for boundary unconstrained operators. Also, it is necessary to make sure that on both sides one has the same number of independent spin degrees of freedom.

As to spin degrees of freedom, the matching is straightforward. Indeed in \( d \)-dimensional AdS space-time the massless totally symmetric field has the following number of physical spin degrees of freedom

\[ 42 \]
\[ N_{d.o.f} = (2s + d - 4) \frac{(s + d - 5)!}{(d - 4)!} s! . \]

At the same time it is well known that this \( N_{d.o.f} \) is nothing but the number of independent components of traceless and divergence free operator in \((d - 1)\) dimensional space-time. Thus there is the same number of spin degrees of freedom on the both sides.

Now let us make a comparison of generators for bulk fields and boundary operators. Important technical simplification is that it is sufficient to make comparison only for part of algebra spanned by generators

\[ P^a, \ J^{+-}, \ J^{+i}, \ J^{ji}, \ J^{-i}, \ D, \ K^+ . \]

It is straightforward to see that the remaining generators \( K^- \) and \( K^i \) are obtainable from commutation relations of the \( so(d - 1, 2) \) algebra. We start with a comparison of the kinematical generators (2.8). As for the generators

\[ P^+, \ P^i, \ J^{+i} \]

they already coincide on both sides (see (3.65), (3.66), (3.69) and (6.12), (6.13)). Before matching generators \( K^+ \) let us consider the dilatation generators \( D \). Here we need explicit form of solution to bulk theory equations of motion. To this end we cast the equations of motion for totally symmetric field (3.52) into the following form

\[ \left( -\partial_z^2 + \frac{1}{z^2} \left( -\frac{1}{2} M_{ij}^2 + \frac{(d - 4)(d - 6)}{4} \right) \right) |\phi\rangle = \Box |\phi\rangle . \]

Then we decompose the field \(|\phi\rangle\), which transforms in representation of \( so(d - 2) \) algebra, into irreducible representations of \( so(d - 3) \) subalgebra \(|\phi_{s'}\rangle\)\(^{22}\)

\[ |\phi\rangle = \sum_{s'=0}^n \oplus |\phi_{s'}\rangle . \]

Because of the relation

\[ \frac{1}{2} M_{ij}^2 |\phi_{s'}\rangle = -s'(s' + d - 5) |\phi_{s'}\rangle \]

the equations of motion for \(|\phi_{s'}\rangle\) take the form

\[ \left( -\partial_z^2 + \frac{1}{z^2} \left( \nu^2 - \frac{1}{4} \right) \right) |\phi_{s'}\rangle = \Box |\phi_{s'}\rangle , \quad \nu \equiv s' + \frac{d - 5}{2} . \]

Normalizable solutions to these equations are

\[ |\phi_{s'}(x, z)\rangle = \sqrt{q z J_{s'+\frac{d-5}{4}}} (q z)^{-\left(s'+\frac{d-5}{4}\right)} |O_{s'}(x)\rangle , \quad q \equiv \sqrt{\Box} . \quad (7.1) \]

\(^{22}\)The \(|\phi_{s'}\rangle\) for \(s'\) given in (B.13) and (B.14) coincide with \(|\phi_{s'}^{(1)}\rangle\) and \(|\phi_{s'}^{(2)}\rangle\) respectively.
In the r.h.s. we use the notation $|O_{s'}\rangle$ since we are going to demonstrate that these operators are indeed the conformal operators. Namely we shall prove that AdS transformations for $|\phi_{s'}\rangle$ lead to conformal theory transformations for $|O_{s'}\rangle$. Asymptotic behavior of the solution above is given by

$$|\phi_{s'}(x,z)\rangle \xrightarrow{z \to 0} z^{s'+\frac{d-4}{2}}|O_{s'}(x)\rangle.$$ 

From this expression and (7.3) it is straightforward to see that

$$\lim_{z \to 0} z^{-s'+\frac{d-4}{2}}D_{\text{ads}}|\phi_{s'}\rangle = D_{\text{cft}}|O_{s'}\rangle. \quad (7.2)$$

Here and below we use the notation $G_{\text{ads}}$ and $G_{\text{cft}}$ to indicate realization of $\mathfrak{so}(d-1,2)$ algebra generators on the bulk fields (3.65)-(3.75) and conformal operators (6.12)-(6.18) respectively. Thus the operators $D_{\text{ads}}$ and $D_{\text{cft}}$ also match. Taking into account the expressions (3.71), (6.16) and (7.2) we get immediately

$$\lim_{z \to 0} z^{-s'+\frac{d-4}{2}}K^+_{\text{ads}}|\phi_{s'}\rangle = K^+_{\text{cft}}|O_{s'}\rangle.$$ 

The remaining generators of the algebra we need to match are $P^-$, $J^{+-}$, $J^{-i}$. Let us consider $P_{\text{ads}}$ and $P_{\text{cft}}^-$

$$P_{\text{ads}}^- = -\frac{\partial^2}{2\partial^+} + \frac{1}{2z^2\partial^+}(-\frac{1}{2}M^2_{ij} + \frac{(d-4)(d-6)}{4}), \quad P_{\text{cft}}^- = \partial^-.$$ 

In the bulk generator $P_{\text{ads}}^-$ given above we cannot send the coordinate $z$ to zero. The point is that the $P_{\text{ads}}^-$ consists of second derivative with respect to $z$. Therefore the operator $P_{\text{ads}}^-$ does not commute with $z$, $[P^- , z] \sim \partial_z$, and the r.h.s. is not equal on the representation space of bulk theory. Note that the bulk generators are defined on space of initial data, i.e., for $x^+ = \text{const}$ . Therefore the fact that we cannot send the coordinate $z$ to zero in the bulk $P^-$ implies that we cannot directly map the initial data of the bulk theory into the boundary conformal theory operators. The point is that in order to put $z$ equals to zero in bulk generators we have to replace the representation defined on initial data by the representation defined on the space of solutions. This implies simply that we should use the fact that on the space of solutions we have Schrödinger equations of motion $P_{\text{ads}}^-\phi = \partial^-\phi$. In other words, on the space of solutions we can simply replace $P_{\text{ads}}^-$ by $\partial^-$.

$$P_{\text{ads}}^- = \partial^-.$$ \quad (7.3)

So generators $P_{\text{ads}}^-$ and $P_{\text{cft}}^-$ also match. Taking this into account it is straightforward to see that the generators $J^{+-}_{\text{ads}}$ (3.68) and $J^{+-}_{\text{cft}}$ (6.14)

$$J^{+-}_{\text{ads}} = x^+P^- - x^-\partial^+, \quad J^{+-}_{\text{cft}} = x^+\partial^- - x^-\partial^+$$

also coincide. The last step is to match the generators $J^{-i}_{\text{ads}}$ and $J^{-i}_{\text{cft}}$. To this end we rewrite the $J^{-i}_{\text{ads}}$ given in (3.74) as follows

\[\text{[The $|O_{s'}\rangle$ for $s'$ given in the first row and the second row of (5.19) coincide with $|O_{s'}^{(1)}\rangle$ and $|O_{s'}^{(2)}\rangle$ respectively.]}\]
Using (7.3) and comparing the above expression for $J^{-i}_{ads}$ with $J^{-i}_{cft}$ given in (6.18) we conclude that all that remains to do is to match $M^i_{ads}$ given in (7.4) and $M^i_{cft}$ given in (6.21), (6.22). Technically, this is the most difficult point of matching. The fact of coincidence of the operators $M^i_{ads}$ and $M^i_{cft}$ we prove by direct calculation. The action of $M^i_{cft}$ is given in (6.21), (6.22). By acting with operator $M^i_{ads}$ on space of solutions given in (7.1) we obtain the representation of operator $M^i_{ads}$ in $|O_s'\rangle$. We should prove that this representation of operator $M^i_{ads}$ in $|O_s'\rangle$ coincides with (6.21) (6.22). This fact is proved in Appendix B.

Finally, let us write AdS/CFT correspondence for bulk symmetric spin $s$ massless field and corresponding boundary conformal theory operator. From (7.1) we can read the following relationship
\[
\lim_{z \to 0} z^{-\hat{\Delta} + \Delta_0} |\phi_{s'}(x, z)\rangle = |O_{s'}(x)\rangle, \quad s' = 0, 1, \ldots, s,
\] (7.5)

where $\hat{\Delta}$ is a spin part of dilatation generator $D_{cft}$ (6.20) while $\Delta_0$ is a canonical dimension of bulk massless field in $d$-dimensional AdS space-time
\[
\Delta_0 = \frac{d - 2}{2}.
\]

Above we matched normalizable solutions of bulk theory equations of motion and boundary conformal theory operators that have canonical conformal dimension.

The generalization of our analysis to the case non-normalizable modes is straightforward. In this case we are going to demonstrate that non-normalizable bulk modes correspond to the shadow operators of boundary conformal field theory. To this end let us write down explicitly the non-normalizable solutions to the bulk equations of motion
\[
|\phi_{s'}(x, z)\rangle_{\text{non-norm}} = \sqrt{qz} J_{-s' - \frac{d-6}{2}}(qz) q^{s' + \frac{d-6}{2}} |\tilde{O}_{s'}(x)\rangle.
\] (7.6)

They have the following asymptotic behavior
\[
|\phi_{s'}(x, z)\rangle_{\text{non-norm}} \overset{z \to 0}{\sim} z^{-s' - \frac{d-6}{2}} |\tilde{O}_{s'}(x)\rangle.
\]

Now the fact that non-normalizable solution indeed corresponds to shadow operator follows from the relation
\[
\lim_{z \to 0} z^{s' + \frac{d-6}{2}} D_{ads} |\phi_{s'}\rangle_{\text{non-norm}} = \tilde{D}_{cft} |\tilde{O}_{s'}\rangle,
\] (7.7)

where $\tilde{D}_{cft}$ is the dilatation operator for the shadow operator
\[
\tilde{D}_{cft} = x^a \partial_{x^a} + \tilde{\Delta}, \quad \tilde{\Delta} = 2 - \alpha^i \bar{\alpha}^i.
\] (7.8)

\footnote{To keep discussion from becoming unwieldy here we restrict our attention to even $d$. In this case the solutions given in (7.1) and (7.6) are independent.}
After this the remaining generators can be matched in the same manner as it was done for normalizable modes. Relation between non normalizable solutions and shadow operators is given by then by the formulas (7.5) where the spin operator $\hat{\Delta}$ should be replaced by the one for shadow field (7.8).

8 Conclusions

We have developed the light-cone formalism in AdS space-time. In this paper we applied this formalism to the study of AdS/CFT correspondence. Because the formalism we presented is algebraic in nature it allows us to treat fields with arbitrary spin on equal footing. Comparison of this formalism with other approaches available in the literature leads us to the conclusion that this is a very efficient formalism.

The results presented here should have a number of interesting applications and generalizations, some of which are:

(i) generalization to AdS/CFT correspondence between arbitrary spin massive fields and related operators at the boundary;

(ii) generalization to supersymmetry and applications to type IIB supergravity in $AdS_5 \times S^5$ background [71] and then to strings in this background;

(iii) extension of light-cone formulation of conformal field theory to the level of OPE’s and study of light-cone form of AdS/CFT correspondence at the level of correlation functions;

(iv) application of light-cone formalism to the study of the S-matrix along the lines of [72]–[76];

(v) applications to interaction vertices for higher massless spin fields in AdS space-time.

In this paper we have discussed AdS/CFT correspondence between massless arbitrary spin fields in AdS space-time and conformal arbitrary spin operators at boundary at the level of free equations of motion. By now it is known that to construct self-consistent interaction of massless higher spin fields it is necessary to introduce, among other things, a infinite chain of anti-de Sitter massless fields which consists of every spin just once [34]. This implies that to maintain AdS/CFT correspondence for such interaction equations of motion we should also introduce an infinite chain of conformal operators at the boundary. In this respect it would be interesting to extend the analysis of this paper to the case of that infinite chain of interacting massless fields and corresponding conformal operators.

We strongly believe that the light-cone formalism developed in this paper will be useful for better understanding of strings in AdS/RR-charge backgrounds.

Acknowledgments

I would like to thank A. Tseytlin for reading the manuscript and comments. This work was supported in part by INTAS grant No.96-538 and the Russian Foundation for Basic Research Grant No.99-02-17916.
Appendix A  Gauge invariant equations of motion for totally symmetric fields

In order to find gauge invariant equations of motion we use the algebra of commutation relations for operators that can be constructed by using the commuting oscillators $\alpha^A$ and Lorentz covariant derivative $D^A$. Starting with the commutator

$$[\hat{\partial}_A, \hat{\partial}_B] = \Omega^{ABC} \hat{\partial}_C,$$

$$\Omega^{ABC} \equiv -\omega^{ABC} + \omega^{BAC}, \quad \omega^{ABC} \equiv \epsilon^\mu_\nu \omega^{\nu BC},$$

where $\Omega^{ABC}$ is a contorsion tensor we get immediately the following basic commutation relation

$$[D_A, D_B] = \Omega^{ABC} D_C + \frac{1}{2} R^{CE}_AB M^{CE}. \quad (A.1)$$

Multiplying by oscillators both sides of (A.1) and taking into account the commutators

$$[\alpha^A, D_B] = \omega^{ABC} \alpha^C, \quad [\bar{\alpha}^A, D_B] = \omega^{ABC} \bar{\alpha}^C,$$

we get the following useful commutators

$$[\bar{\alpha} D, \alpha D] = D^2_A + \omega^{ABC} D_B - \frac{1}{4} R^{CD}_AB M^{CD} M^{AB},$$

where $R^{CD}_AB$ is a Rieman tensor in tangent space

$$R^{CD}_AB = \hat{\partial}^C \omega^{AB} D_B + \omega^{ACE} \omega^{EB} + \omega^{CD} E \omega^{EAB} - (C \leftrightarrow D).$$

Next important commutator is given by

$$[[\bar{\alpha} D, \alpha D], \alpha D] = 2 R^{A}_{AB} \alpha^A D^B - 2 R^{DE}_AB \alpha^A M^{DE} D_B + \frac{1}{4} D_C R^{DE}_AB \alpha^C M^{AB} M^{DE} + D_B R^{BC}_A \alpha^A M^{BC}, \quad (A.2)$$

where $R^A_B = R^C_{AB}$ is a Richi tensor in tangent space. Note that in this appendix and only in this appendix the $R^{AB}$ indicates the Richi tensor. In the remainder of this paper the $R^{AB}$ is used as it is defined in (3.16). To derive (A.2) one can use the following useful commutation relations

$$[D^2_A + \omega^{ABC} D_B, \alpha D] = R^{A}_{AB} \alpha^A D^B + R^{CD}_AB \alpha^B M^{CD} D^A + D_C R^{DE}_AB \alpha^B M^{CD},$$

$$[\omega^{ABC} D_B, \alpha D] = (\hat{\partial}^B \omega^{AAC} + \omega^{ABC} \omega^{BDF} \omega^{BDC}) \alpha^B D_C + \frac{1}{2} \omega^{ABC} R^{DE}_BC \alpha^C M^{DE}.$$ 

Note that all abovementioned commutation relations are valid for space-time of arbitrary geometry. Due to that these relations might be useful for a number of interesting applications to gauge invariant equations of motion for totally symmetric fields propagating in space-times of arbitrary geometry. For the case of AdS geometry the Rieman tensor satisfies the relationships
\[ R^{ABCD} = -\eta^{AC}\eta^{BD} + \eta^{AD}\eta^{BC}, \quad D_E R^A{}_C = 0. \]

With the above algebra of commutators at hand the derivation of the gauge invariant equations of motion is relatively straightforward. We look for equations of motion of the form \( L|\Phi\rangle = 0 \) where \( L \) is a polynomial of the second order in \( \alpha D \) and \( \bar{\alpha} D \) and impose the following conditions. i) Operator \( L \) commutes with oscillator number operator \( \alpha \bar{\alpha} \). ii) On the space of double traceless tensor the operator \( L \) commutes with operator \( (\bar{\alpha}^2)^2 \). This condition amount to the requirement that the equations of motion and double traceless condition respect each other; From this requirement we learn that the operator \( L \) does not consist of terms \( (\bar{\alpha}^2)^n, n > 1 \). iii) Operator \( L \) commutes with gauge transformation. This implies that equations of motion should be gauge invariant. Since we use gauge transformation of the form (3.40) we should impose the condition \( [L, \alpha D]|\Lambda\rangle = 0 \), where the gauge parameter transformation \( |\Lambda\rangle \) satisfies the traceless constraint (3.41). Making use of commutation relation above one can make sure that the unique operator \( L \) that satisfies these requirement is that given in (3.39).

In the rest of this appendix let us write down some important formulas we use in this paper. In Poincaré coordinates the Lorentz covariant derivative takes the form

\[ D^A = \hat{\partial}^A + \alpha^z \hat{\alpha}^A - \alpha^A \hat{\alpha}^z. \]

From this we get the following useful representations

\[ \alpha D = \alpha \hat{\partial} + \alpha^z \alpha \hat{\alpha} - \alpha^2 \hat{\alpha}^z, \quad \bar{\alpha} D = \bar{\alpha} \hat{\partial} + (2 - d) \bar{\alpha}^z - \alpha^z \bar{\alpha} \alpha + \alpha^z \bar{\alpha}^2, \]

\[ D_A^2 = (\hat{\partial}^A - \alpha^A \alpha^z)^2 + 2\alpha^z \hat{\alpha} D - \alpha^2 \bar{\alpha}^2 + (d - 1) \alpha^z \bar{\alpha}^z - \alpha \bar{\alpha}. \]

Some other commutation relations in Poincaré coordinates are

\[ [\bar{\alpha}^A, \alpha D] = \hat{\partial}^A + \alpha^z \hat{\alpha}^A + \delta_A^z \alpha \bar{\alpha} - 2\alpha^A \bar{\alpha}^z, \quad [\bar{\alpha}^A, \bar{\alpha} D] = \delta_A^z \bar{\alpha}^2 - \bar{\alpha}^z \hat{\alpha}^A, \]

\[ [\bar{\alpha}^A, D^2] = 2\delta_A^z \bar{\alpha} D - 2\bar{\alpha}^z D^A + (d - 2) \delta_A^z \hat{\alpha}^z - \hat{\alpha}^A. \]

These commutation relations are useful while studying the constraints that follow from equations of motion in the light-cone gauge.

**Appendix B** Various forms of realization of \( so(d - 2) \) algebra representations

**Stereographic form.** Here we wish to demonstrate that spin operator given in (3.29) comes from description of \( so(d - 2) \) algebra representation in stereographic coordinates. To this end we consider the simplest case of totally symmetric representations. Let \( \phi^{I_1 \ldots I_s} \) be a totally symmetric traceless tensor field which realizes irreducible representation of \( so(d - 2) \) algebra. Introduce vector \( a^I \) and consider a generating function

\[ |\phi\rangle = \phi^{I_1 \ldots I_s} a^{I_1} \ldots a^{I_s}. \]

The spin operator in this representation takes the form
By definition the $|\phi\rangle$ satisfies the constraints

$$a^I \partial_a^I |\phi\rangle = s|\phi\rangle, \quad \partial^2_a |\phi\rangle = 0. \quad (B.2)$$

The first constraint tells us that $|\phi\rangle$ is a monomial degree $s$ in $a^I$ while the second one is a traceless condition. Stereographic coordinates $\zeta^i$ are defined by the relations

$$a^i = 2\zeta^i \rho, \quad a^z = (1 - \zeta^2) \rho, \quad (B.3)$$

where $\rho$ is a scale parameter. Making use of chain rules

$$\partial a^i = \frac{1}{2\rho} \partial \zeta_i + \frac{\zeta_i}{1 + \zeta^2 (\partial \rho - \frac{1}{\rho} \zeta \partial \zeta)}, \quad \partial a^z = \frac{1}{1 + \zeta^2 (\partial \rho - \frac{1}{\rho} \zeta \partial \zeta)}, \quad \zeta^2 \equiv \zeta^i \zeta^i, \quad \zeta \partial \zeta \equiv \zeta^i \partial \zeta_i$$

one can verify that the constraints (B.2) and spin operator $M^{IJ}$ take the following form

$$\rho \partial \rho |\phi\rangle = s|\phi\rangle, \quad \left((1 + \zeta^2) \partial^2 \zeta - 4(s + \frac{d - 5}{2})(\zeta \partial \zeta - s)\right)|\phi\rangle = 0. \quad (B.4)$$

$$M^{ij} = \zeta^i \partial \zeta^j - \zeta^j \partial \zeta^i, \quad M^{zi} = \frac{1}{2}(1 - \zeta^2) \partial \zeta_i + \zeta^i (\zeta \partial \zeta - \rho \partial \rho). \quad (B.5)$$

The operator $\rho \partial \rho$ commutes with above spin operator $M^{IJ}$. Due to that in expression for $M^{zi}$ (B.5) we can replace the operator $\rho \partial \rho$ by its eigenvalue $s$. After this comparing the spin operators (B.5) with the ones of group theoretic approach (5.29) we conclude that the group theoretic approach indeed leads to the stereographic form of realization of spin degrees of freedom. Note that by inserting (B.3) into (B.1) the expression for $|\phi\rangle$ can be cast into the well known textbook form

$$|\phi(a^I)\rangle = \rho^s \sum_{s'=0}^{s} (1 + \zeta^2)^{s-s'} C_{s-s'}^{|a^I|s'}(t)|\phi_{s'}(\zeta^i)\rangle, \quad \zeta \partial \zeta |\phi_{s'}\rangle = s' |\phi_{s'}\rangle, \quad \partial^2 \zeta |\phi_{s'}\rangle = 0, \quad (B.6)$$

where $t \equiv (1 - \zeta^2)/(1 + \zeta^2)$ and $C_{\alpha}^{|a^I|s'}$ is a Gegenbauer polynom. The $|\phi_{s'}\rangle$ is totally symmetric traceless rank $s'$ tensor of $so(d-3)$ algebra. The formula (B.6) is a decomposition of irreducible representation of $so(d-2)$ algebra into ones of $so(d-3)$ algebra.

**Simple form.** Here we wish to describe an form of realization of totally symmetric $so(d-2)$ algebra representation in terms of $so(d-3)$ which as compared to stereographic form does not involve special functions like Gegenbauer polynoms and which we did not find in standard literature on this subject. This representation turns out to be more convenient for establishing AdS/CFT correspondence. As before we focus on symmetric field. Let $\phi^{I_1...I_s}$ be a totally symmetric traceless tensor field. Consider a generating function

$$|\phi\rangle \equiv \phi^{I_1...I_s} \alpha^{I_1} \ldots \alpha^{I_s} |0\rangle,$$
which satisfies the constraint
\[ \alpha^I \bar{\alpha}^I |\phi \rangle = s |\phi \rangle , \quad \bar{\alpha}^I \alpha^I |\phi \rangle = 0 . \] (B.7)

One can consider the second constraint in (B.7) as the second order differential equation with respect to oscillator variable \( \alpha^z \)

\[ (\bar{\alpha}^z + \omega^2) |\phi(\alpha^z, \alpha^i) \rangle = 0 , \quad \omega^2 = \bar{\alpha}^i \alpha^i . \]

Obvious solution to this equation is found to be
\[ |\phi(\alpha^z, \alpha^i) \rangle = \cos(\omega \alpha^z) |\phi_s \rangle + \sin(\omega \alpha^z) \omega |\phi_{s-1} \rangle , \] (B.8)

where \( |\phi_s \rangle \) and \( |\phi_{s-1} \rangle \) rank \( s \) and \( s - 1 \) traceful tensors, i.e. they are reducible representations of the \( so(d-3) \) algebra. The solution (B.8) reflects well know fact that symmetric traceless rank \( s \) tensor of \( so(d-2) \) algebra can be decomposed into symmetric rank \( s \) and \( s - 1 \) traceful tensors of \( so(d-3) \) algebra. These tensors satisfy the constraints
\[ \alpha^i \bar{\alpha}^i |\phi_s \rangle = s |\phi_s \rangle , \quad \bar{\alpha}^i \alpha^i |\phi_{s-1} \rangle = (s - 1) |\phi_{s-1} \rangle , \quad \bar{\alpha}^i |\phi_{s,s-1} \rangle = 0 . \]

Straightforward calculations gives
\[ M^i |\phi \rangle = \cos(\omega \alpha^z)(-|\phi_{s-1} \rangle) + \sin(\omega \alpha^z)\bar{\alpha}^i \alpha^i |\phi_s \rangle . \] (B.9)

Instead of the representation (B.8) we express \( |\phi \rangle \) in terms \( |\phi_{s,s-1} \rangle \) as follows
\[ |\phi \rangle = \begin{pmatrix} |\phi_s \rangle \\ |\phi_{s-1} \rangle \end{pmatrix} . \] (B.10)

Then from the (B.9) we get immediately the following representation for spin operator \( M^i \) on the generating function \( |\phi \rangle \)
\[ M^i = \sigma_-(\bar{\alpha}^i + \alpha^i \bar{\alpha}^2_j) - \sigma_+ \alpha^i , \quad \sigma_- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} , \quad \sigma_+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} . \] (B.11)

The representation of \( M^{ij} \) on \( |\phi \rangle \) has the usual form
\[ M^{ij} = \alpha^i \bar{\alpha}^j - \alpha^j \bar{\alpha}^i . \] (B.12)

For comparison with boundary theory it is convenient to exploit the following realization of \( |\phi_{s,s-1} \rangle \). Decompose the reducible generating functions \( |\phi_{s,s-1} \rangle \) into irreducible representations of \( so(d-3) \) algebra
\[ |\phi_s \rangle = \sum_{s'} (-\alpha_{s'}^2)^{[s'-1]} |\phi_{s'}^{(1)} \rangle , \quad s' = s, s - 2, s - 4, \ldots, s - 2[\frac{s}{2}] , \] (B.13)
\[ |\phi_{s-1} \rangle = \sum_{s'} (-\alpha_{s'}^2)^{[s'-1]} |\phi_{s'}^{(2)} \rangle , \quad s' = s - 1, s - 3, s - 5, \ldots, s - 2[\frac{s-1}{2}] , \] (B.14)
\( (\alpha^i \bar{\alpha}^j - s') |\phi^{(1,2)}_{s'}\rangle = 0 \). In this basis defining the action \( M^{zi} \) on \( |\phi^{(1,2)}_{s'}\rangle \) with the relation

\[
M^{zi}|\phi\rangle = \begin{pmatrix}
\sum \alpha^j \bar{\alpha}^i |\phi^{(1)}_{s'}\rangle + 2 \bar{\alpha}^i |\phi^{(2)}_{s' + 1}\rangle \\
\sum \alpha^j |\phi^{(1)}_{s'}\rangle + 2 \alpha^j \bar{\alpha}^i |\phi^{(2)}_{s' - 1}\rangle
\end{pmatrix}
\]

we get

\[
M^{zi}|\phi^{(1)}_{s'}\rangle = -\left(\alpha^i - \frac{\alpha^j \bar{\alpha}^i}{2s' + d - 7}\right) |\phi^{(2)}_{s' - 1}\rangle + \frac{\bar{\alpha}^i}{2s' + d - 3} |\phi^{(2)}_{s' + 1}\rangle , \tag{B.15}
\]

\[
M^{zi}|\phi^{(2)}_{s'}\rangle = -a(s, s')(\alpha^i - \frac{\alpha^j \bar{\alpha}^i}{2s' + d - 7}) |\phi^{(1)}_{s'}\rangle + a(s, s' + 1) \frac{\bar{\alpha}^i}{2s' + d - 3} |\phi^{(1)}_{s' + 1}\rangle . \tag{B.16}
\]

**Evaluation of action of operator \( M^i \).** Here by using above simple form of realization of \( so(d - 2) \) algebra representation we evaluate the action of operator

\[
M^i \equiv M^{zi} \partial_z + \frac{1}{2z} \{M^{zi}, M^{ji}\} \tag{B.17}
\]

on the space of solution given in (7.1). We use representation for spin operator \( M^{ij} \) given in (B.11), (B.12). For this representation we derive in a straightforward way

\[
\{M^{zi}, M^{ji}\} = \sigma_+ \left(\alpha^i (2\alpha_j \bar{\alpha}_j + d - 4) - 2\alpha_j^2 \bar{\alpha}^i\right) + \sigma_- \left(\alpha^i (M_{jk}^2 + 2(\alpha_j \bar{\alpha}_j)^2 + 2(d - 4)\alpha_j \bar{\alpha}_j + d - 6) - \alpha^i (d + 2 + 2\alpha_j \bar{\alpha}_j)\bar{\alpha}^j\right).
\]

By acting with this on \( |\phi\rangle \) given in (B.10) and using the decompositions (B.13), (B.14) we get the action of operator \( \{M^{zi}, M^{ji}\} \) on \( |\phi^{(1,2)}_{s'}\rangle \)

\[
\{M^{zi}, M^{ji}\}|\phi^{(1)}_{s'}\rangle = (2s' + d - 6) \left(\alpha^i - \frac{\alpha^j \bar{\alpha}^i}{2s' + d - 7}\right) |\phi^{(2)}_{s' - 1}\rangle + \frac{2s' + d - 4}{2s' + d - 3} \bar{\alpha}^i |\phi^{(2)}_{s' + 1}\rangle ,
\]

\[
\{M^{zi}, M^{ji}\}|\phi^{(2)}_{s'}\rangle = (2s' + d - 6) a(s, s') \left(\alpha^i - \frac{\alpha^j \bar{\alpha}^i}{2s' + d - 7}\right) |\phi^{(1)}_{s' - 1}\rangle + a(s, s' + 1) \frac{2s' + d - 4}{2s' + d - 3} \bar{\alpha}^i |\phi^{(1)}_{s' + 1}\rangle ,
\]

Now we (i) combine these relations with (B.15), (B.16) and find action of operator \( M^i \) (B.17) on \( |\phi^{(1,2)}_{s'}\rangle \); (ii) use solutions of equations of motion for \( |\phi_{s'}\rangle \) given in (7.1); (iii) exploit the following relationships for Bessel functions

\[
(\partial_z + \frac{2s' + d - 6}{2z}) \sqrt{z} J_{s' + \frac{d - 3}{2}}(z) = (\partial_z + \frac{2s' + d - 4}{2z}) \sqrt{z} J_{s' + \frac{d - 3}{2}}(z) = \sqrt{z} J_{s' + \frac{d - 3}{2}}(z)
\]

After this we get that the action of \( M^i \) on boundary values \( |\mathcal{O}^{(1,2)}_{s'}\rangle \) coincides with the action of \( M^i_{cf} \) given in (6.21), (6.22).
Appendix C Transformation of conformal theory generators to light-cone form

In this appendix we describe transformations that take the generators given in (6.5)-(6.8) to the those of light-cone form given in (6.12)-(6.18). Again we start our analysis with kinematical generators (2.8). First we consider the generators \( J^+ i \) and \( J^+ - \). The original \( J^+ i \) and \( J^+ - \) acting on \( |O_{cov}\rangle \) are defined by (6.6). Taking into account (6.10) it is straightforward to see that the generators \( J^+ i \) and \( J^+ - \) in \( |O\rangle' \) basis take the form

\[
J^+ i = x^+ \partial - x^i \partial^+ - \alpha^i \bar{\alpha}^+ - x^+ \partial - x^i \partial^+ - \alpha^i \bar{\alpha}^+.
\]

Our first step is to find the transformation that cancels out the oscillator terms in the generator \( J^+ i \). This is achieved by the following transformation

\[
|O\rangle' = \exp(\alpha^i \bar{\alpha}^+ \partial^i \partial) |O\rangle''
\]

In \( |O\rangle'' \) basis the generator \( J^+ i \) takes desired form given in (6.13) while the generator \( J^+ - \) is not changed (see (C.1)). Because of relations (6.10) and (C.2) the original \( |O_{cov}\rangle \) and \( |O\rangle'' \) are related as follows

\[
|O_{cov}\rangle = \exp(-\alpha^+ \bar{\alpha}^- + \alpha^- \bar{\alpha}^+) \exp(\alpha^i \bar{\alpha}^+ \partial^i \partial^+) |O\rangle''.
\]

Taking into account the formula

\[
e^{\alpha^i x^i} e^{\bar{\alpha}^i y^i} = e^{\alpha^i x^i + \bar{\alpha}^i y^i - \frac{1}{2} x^i y^i},
\]

this can be rewritten as

\[
|O_{cov}\rangle = e^{-\Gamma} |O\rangle'', \quad \Gamma = \frac{\alpha^+ \bar{\alpha}^+}{2 \partial^+} \Box + M^+ x^i \partial^i.
\]

In order to cancel oscillator term in expression for \( J^+ - \) (C.1) we make the transformation

\[
|O\rangle'' = (-\partial^+)^{\alpha^- \bar{\alpha}^+} |O\rangle'''.
\]

The original operator \( |O_{cov}\rangle \) (6.2) takes then the form

\[
|O_{cov}\rangle = e^{-\Gamma} (-\partial^+)^{\alpha^- \bar{\alpha}^+} |O\rangle'''.
\]

Thus in \( |O\rangle''' \) basis the generators \( J^+ i \) and \( J^+ - \) take the desired form given in (6.13) and (6.14). Let us now consider the remaining kinematical generator \( K^+ \). In \( |O\rangle''' \) basis this generator takes the following form

\[
K^+ = K^+_0 + (\Delta - \alpha^- \bar{\alpha}^+ + \frac{1}{2} \partial^+ \alpha^2 \bar{\alpha}^2 + \frac{1}{2} x^i \partial^i + x^+ (x^+ \partial - x^i \partial^i).
\]

Note that in \( |O\rangle''' \) basis the traceless condition (6.3) takes the following form

\[
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\]
\((\bar{\alpha}^2 - \Box \bar{\alpha}^2)|O\rangle'' = 0\).

Before to proceed we wish to transform this traceless condition to the form that does not consist of \(\Box\). To this end we introduce \(|O\rangle_{iv}\) basis

\[|O\rangle'' = \sqrt{\Box}(-\alpha - \bar{\alpha}^+)|O\rangle_{iv}.\]

In the \(|O\rangle_{iv}\) basis the traceless condition takes desired form

\[(\bar{\alpha}^2 - \bar{\alpha}^2)|O\rangle_{iv} = 0,\]

while the generators \(K^+, J^{-i}\) take the form

\[K^+ = K^+_0 + \Delta x^+ + \frac{\partial^+}{2}\Box (s(s + d - 5) + \frac{1}{2}I^2), \quad (C.3)\]

\[J^{-i} = x^- \partial^i - x^i \partial^+ + M^i \frac{\partial j}{\partial^+} - \frac{1}{\partial^+} M^i, \quad (C.4)\]

where the operator \(M^i\) is given in (B.12), while the operator \(M^i\) is given by

\[M^i \equiv \sqrt{\Box}(\alpha^- \bar{\alpha}^i + \alpha^i \bar{\alpha}^+).\]

In deriving of (C.3) the value of \(\Delta\) given in (6.1) should be used. Finally we are going to choose a basis in which the spin operator \(M^i\) and the generator \(K^+\) take the form given in (6.21), (6.22) and (6.16). To this end we use the following decomposition of operator \(|O\rangle_{iv}\)

\[|O\rangle_{iv} = \begin{pmatrix} |O_{s(1)}\rangle_{iv} \\ |O_{s-1}^{(2)}\rangle_{iv} \end{pmatrix}, \quad (C.5)\]

where \(|O_{s(1)}\rangle_{iv}\) and \(|O_{s-1}^{(2)}\rangle_{iv}\) are traceful rank \(s\) and \(s - 1\) tensors of \(so(d - 3)\) algebra. In such basis for \(|O\rangle_{iv}\) the operator \(M^i\) takes the form

\[M^i = \sqrt{\Box}(\alpha^i + \alpha^i \bar{\alpha}^2 + \alpha^i \bar{\alpha}^+ + \alpha^i \sigma_+). \quad (C.6)\]

Decomposing \(|O_{s,s-1}^{(1,2)}\rangle_{iv}\) into irreducible components we have

\[|O_{s(1)}^{(1)}\rangle_{iv} = \sum s' (\alpha^2)^{\frac{s'-1}{2}}|O_{s'}^{(1)}\rangle_{iv}, \quad |O_{s-1}^{(2)}\rangle_{iv} = \sum s' (\alpha^2)^{\frac{s'-1}{2}}|O_{s'}^{(2)}\rangle_{iv}, \quad (C.5)\]

where \(s'\) takes values given in (B.13) and (B.14) respectively. Inserting these expressions into (C.5) and by applying \(M^i\) (C.6) to (C.5) we get the following representation of \(M^i\) on \(|O_{s,s-1}^{(1,2)}\rangle_{iv}\)

\[\Box^{-1/2} M^i |O_{s,s-1}^{(1)}\rangle_{iv} = (\alpha^i - \frac{\alpha^2 \bar{\alpha}^i}{2s' + d - 7})|O_{s'-1}^{(2)}\rangle_{iv} + \frac{1}{2s' + d - 3} \bar{\alpha}^i |O_{s'+1}^{(2)}\rangle_{iv}, \quad (C.7)\]

\[\Box^{-1/2} M^i |O_{s,s-1}^{(2)}\rangle_{iv} = a(s, s') (\alpha^i + \frac{\alpha^2 \bar{\alpha}^i}{2s' + d - 7})|O_{s'-1}^{(1)}\rangle_{iv} + \frac{a(s, s' + 1)}{2s' + d - 3} \bar{\alpha}^i |O_{s'+1}^{(1)}\rangle_{iv}. \quad (C.8)\]
Then the final basis $|\mathcal{O}_{s'}^{(1,2)}\rangle_{iv}$ in which the operator $M^i$ and the generator $K^+$ take desired form given in (6.21), (6.22), (6.16) is found to be

$$|\mathcal{O}_{s'}^{(1,2)}\rangle_{iv} = \sqrt{-\Box^{(s-\alpha^i\bar{\alpha}^i)}}|\mathcal{O}_{s'}^{(1,2)}\rangle. \quad (C.9)$$

Now let us discuss the representation of conformal algebra generators in the space of shadow operators. To this end it turns out to be convenient to start with the form of generators given in $|\mathcal{O}\rangle_{iv}$ basis (see (C.3),(C.4)). First of all taking into account the relationship (6.1) the generator $K_0^+$ (C.3) can be rewritten as

$$K_0^+ = K_0^+ + \Delta x^+ + \frac{\partial^+}{2\Box}((\Delta - 2)(\Delta + 3 - d) + \frac{1}{2}M_{ij}^2). \quad (C.10)$$

Then because of relations

$$(\Delta - 2)(\Delta + 3 - d) = (\tilde{\Delta} - 2)(\tilde{\Delta} + 3 - d), \quad (K_0^+)^\dagger = -K_0^+ - (d - 1)x^+, \quad (K^+)^\dagger = -K^+ - (d - 1)x^+,$$

where $\tilde{\Delta}$ is a conformal dimension of shadow operator $\tilde{\Delta} = 2 - s$ we get

$$(K^+)^\dagger = -\tilde{K}^+, \quad \tilde{K}^+ \equiv K_0^+ + \Delta x^+ + \frac{\partial^+}{2\Box}((\tilde{\Delta} - 2)(\tilde{\Delta} + 3 - d) + \frac{1}{2}M_{ij}^2).$$

From these expressions it is seen that the new generator $\tilde{K}^+$ is obtainable from $K^+ \ (C.10)$ by making there the substitution $\Delta \to \tilde{\Delta}$. This suggests that $\tilde{K}^+$ gives rise representation in the space of shadow operator $|\mathcal{O}\rangle$. Additional support to this suggestion is that the following scalar product

$$\int d^{d-1}x \langle \tilde{\mathcal{O}}(x)||\mathcal{O}(x)\rangle$$

is invariant of conformal algebra transformations provided the $\tilde{\mathcal{O}}$ is transformed by $\tilde{K}^+$. By introducing new basis similar to (C.3), (C.9) one finds the representation of conformal algebra generators in space of shadow operator. These generators can be obtained from (6.12)-(6.16) making there the substitution $\Delta \to \tilde{\Delta}$. Note that for the case of shadow operator the expression $s - \alpha^i\bar{\alpha}^i$ in r.h.s. of (C.9) should be replaced by $\alpha^i\bar{\alpha}^i - s$. 

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