Spin-polarized currents in corrugated graphene nanoribbons

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We investigate the production of spin-polarized currents in corrugated graphene nanoribbons. We model the corrugation as multiple regions with Rashba spin-orbit interactions. Concave and convex curvatures are modeled as Rashba regions with opposite signs. Numerical examples for different separated Rashba-zone geometries calculated within the tight-binding approximation are provided. Remarkably, the spin-polarized current in a system with several Rashba areas can be enhanced with respect to the case with a single Rashba part of the same total area. The enhancement is larger for configurations with multiple regions with the same Rashba sign. Additionally, we relate the appearance of the spin-polarized currents to novel symmetry relations between the spin-dependent conductances. These symmetries turn out to be a combination of different symmetry operations in real and spin spaces, as those occurring in non-planar systems like carbon nanotubes. Our results show that two-dimensional devices with Rashba spin-orbit interaction can be used as excellent spintronic devices in an all-electrical or mechanical setup.

I. INTRODUCTION

Corrugated graphene systems have been grown using different experimental techniques and theoretically explored within elastic deformation models, revealing interesting changes in the electronic and transport properties of such systems.1–6 Lattice deformations produce strain and may modify orbital hybridization; strain effects amount to the appearance of pseudomagnetic fields.4–10 Recently, such corrugations have been achieved by stacking graphene on a self-assembled periodic array of nanospheres; in such system it was possible to experimentally observe superlattice miniband effects.11 Another route to create periodic patterns was recently reported on buckled graphene deposited onto a NbSe2 substrate and also in other two-dimensional crystals.13,14 The large mismatch between the two materials forces a compressive strain in the graphene membrane, leading to periodic buckled structures whose geometry can be experimentally controlled. The formation of complex mosaic patterns in graphene flakes, by using in-situ uniaxial strain combined with atomic force microscopy, was recently reported.15 It was possible to estimate the minimum strain to apply to the substrate in order to create a mosaic morphology, and produce quantitatively controlled changes in the deformation applied to the sample, showing that a mechanically tuned device is thus feasible.

Folded graphene sheets have been described as origami-like structures with fascinating properties.16–18 In fact, curved graphene nanoribbons and nanotubes have been shown to present enhanced spin-orbit interactions.18–20 In general, curvature effects can produce spin-polarized currents, so corrugated graphene structures, as those depicted in Fig. 1(a), can be used efficiently in spintronic applications. From the theoretical viewpoint, a corrugated graphene nanoribbon (GNR) can be described as a superlattice composed of a series of regions with and without an applied electric field, with alternating direction. A structural inversion asymmetry is locally produced by the field, so a Rashba spin-orbit coupling is expected. Curved systems are known to induce an anisotropic charge distribution in the pz orbitals due to electronic repulsion, with the subsequent electric field, as schematically shown in Fig. 1(b). In that sense, different corrugation profiles may be explored within this framework, by turning on and off the Rashba spin-orbit interaction (SOI) with the purpose of obtaining spin-dependent currents. Other proposals for the enhancement of spin and valley polarizations have been reported, making use of magnetic barriers on strained graphene with Rashba spin-orbit.23 Our scheme does not require the use of magnetic elements, just mechanical deformations or electric fields.

On the other hand, periodically repeated regions with Rashba SOI interaction can be induced in planar systems, like graphene nanoribbons, by patterning multiple gates that produce external electric fields perpendicular to the plane of the nanoribbon.24,25 This is schematically shown in Fig. 1(c). These multiple Rashba SOI regions could also be induced in planar systems by proximity effects.26,27 Moreover, a recent theoretical work suggests the application of transverse electric fields in twisted ribbons, which is effectively subject to a periodic electrostatic potential along its length, with alternating signs.30 External gates can be used to tune the transport properties of graphene nanoribbons, as well as the value of the SOI coupling.
 implant the spin components. (b) Schematic of the right contact. Red and blue are used for the up and down spin-orbit interaction in the central part and a spin-polarized current is detected in the GNR. An unpolarized current from the left contact traverses to the central region and a corrugated graphene in a crest or hill profile. The unbalanced electronic density on the $p_z$ orbitals gives rise to an effective electric field perpendicular to the graphene sheet. The same as (a) but with the central region composed by a multiple-gated graphene nanoribbon.

Recently, we showed that depending on their symmetry and the chosen spin projection direction, graphene nanoribbons with a Rashba SOI region can yield spin-polarized currents. The fact that symmetry reasoning allows to elucidate whether the spin-conserved or spin-flip conductances are equal, can also be used to choose the most suitable ribbons and geometries for spintronic devices. We have also explored the production of spin-polarized currents in carbon nanotubes (CNTs) by the same physical mechanism, showing that the presence of periodic defects increases its value. Interestingly, for CNTs the symmetries are more general and it was necessary to consider separately symmetry operations in spin and real space to account for the relations between spin-resolved conductances. Since the periodic repetition of defects enhances the production of spin-polarized currents, it is natural to explore the optimal configurations for obtaining the maximum quantitative effect. We aim at elucidating the best configuration to maximize the realization of spin-polarized currents.

Our main findings are the following:

(i) Spin-polarized currents in these systems can be enhanced with respect to the case with one single Rashba region of the same size.

(ii) This effect is larger if the signs of the Rashba regions are the same.

(iii) For graphene systems with multiple Rashba regions different spatial and spin symmetry operations have to be considered in certain cases in order to explain the relations between spin-resolved conductances and the occurrence of spin polarized currents.

(iv) We have performed a symmetry analysis of the Hamiltonians that allows us to obtain relations between the spin-resolved conductances depending on the sequence of Rashba regions.

The paper is organized as follows. Section II describes the model employed and the geometries considered. Section III presents numerical calculations that demonstrate the optimal configurations for obtaining the maximum spin-polarized currents. Section IV discusses the symmetry issues raised by the numerical results. Finally, in Section V we summarize our conclusions.

II. MODELING SYSTEMS WITH MULTIPLE RASHBA REGIONS

The proposed device is composed of a corrugated graphene nanoribbon coupled to two leads as shown in Fig. 1(a). The corrugated central part plays the role of a conductor in the transport calculation. Alternatively, the conductor can be a graphene strip with multiple gates, as a truncated superlattice (see Fig. 1(c)). The corrugations or the gated area are described by a sequence of graphene regions perturbed by a Rashba-like SOI due to the electric field, separated by other areas without SOI, that we dub no-Rashba regions. We assume an unpolarized current coming from the left contact traverses the central part; due to the Rashba SOI a spin-polarized current can be detected in the right contact.

The whole system is described in the nearest-neighbor hopping tight-binding approximation. The Hamiltonian is given by the sum of a kinetic energy term $H_0 = t \sum_{<i,j>} c_{i\alpha}^\dagger c_{j\beta}$ and the Rashba spin-orbit interaction, given by

$$H_R = \frac{i \lambda_R}{a_{cc}} \sum_{<i,j>,\alpha,\beta} c_{j\beta}^\dagger \frac{1}{2} (\sigma \times d_{ij}) \cdot e_p c_{i\alpha} c_{j\beta} ,$$

where $t$ is the nearest-neighbor hopping parameter, $c_{i\alpha}^\dagger$, $c_{i\alpha}$ the destruction and creation operators, $\sigma$ the Pauli spin...
matrices, $d_{ij}$ the position vector between sites $i$ and $j$, $a_{cc}$ is the nearest-neighbor carbon-carbon distance in graphene, 1.42 Å, $\alpha, \beta$ are the spin projection indices, and $\lambda_R$ is the Rashba SOI strength due to curvature and that can be additionally tuned by an electric field.

If the corrugated graphene nanoribbon is shaped in a series of hills and crests, it is described by a sequence of positive and negative Rashba coupling regions to take into account the positive and negative concavities produced by the folding. For corrugations with the same curvature, as the bubbles reported in Ref. [12], the folds can be modeled as regions with the same sign of the Rashba SOI coupling. Any of these arrangements can be also achieved with multiple-gated systems, for which the sequence of the Rashba signs is determined by the transversal gates.

Some particular examples are shown in Fig. 2. In Fig. 2(a) the central conductor is chosen as a single region with positive SOI, denoted here as $+R$; this is the case studied in Ref. [32]. Figs. 2(b) and (c) depict two regions with the same SOI sign ($+R, +R$) and opposite signs ($+R, -R$), respectively, separated by a region without Rashba coupling. Finally, in Fig. 2(d) a positive-negative-positive sequence ($+R, -R, +R$) with spacers without SOI in between is depicted.

For those systems with the same sign of the Rashba coupling in all regions (Fig. 2(b)) or with an odd number of alternating Rashba regions (Fig. 2(d)), the sequence of signs of the Rashba term is the same starting from the left or from the right lead.

In fact, the symmetries of these systems are the same as for the single Rashba region (Fig. 2(a)) discussed previously [32]. Symmetries acting simultaneously in real and spin space are sufficient to explain the relations between spin-resolved conductances. We call these L-R symmetric systems. However, for systems with an even number of alternating Rashba regions (Fig. 2(c)), the sequence of signs of the Rashba regions starting from the right lead is reversed with respect to that obtained starting from the left lead. This should be considered in order to find the symmetries of the total Hamiltonian. We will later see that this situation calls for different symmetry operations in real and spin space, as in carbon nanotubes [34]. We denote these systems as L-R antisymmetric. These require additional symmetries with respect to the nanotube case, which involve the swapping of the Rashba regions.

The conductance is computed within the Landauer approach by using the Green function formalism [37, 39]. The spin-resolved conductance is given by

$$G_{\sigma\sigma'}^{LR} = \frac{e^2}{h} \text{Tr} \left[ \Gamma_{LR}^{\sigma} \Sigma_{LR}^{\sigma} \Gamma_{LR}^{\sigma'} G_{\sigma\sigma'}^{\text{av}} \right] ,$$  

where $G_{\sigma\sigma'}^{\text{av}}$ is the advanced (retarded) Green function of the conductor and $\Gamma_{LR}^{\sigma} = i(\Sigma_{L(R),\sigma} - \Sigma_{L(R),\sigma}^0)$ is written in terms of the $L$ ($R$) lead selfenergies $\Sigma_{L(R),\sigma}^0$. The spin polarization of the current in the $s$ direction is defined as $P_s = G^{\uparrow\uparrow}_{\uparrow\uparrow} + G^{\downarrow\downarrow}_{\downarrow\downarrow} - G^{\uparrow\downarrow}_{\downarrow\uparrow} - G^{\downarrow\uparrow}_{\uparrow\downarrow}$.

Seeing that the spin polarization of the current arises from the difference between the spin-conserved and/or the spin-flip conductances, in what follows we name spin-conserved polarization to the difference between the spin-conserved conductances $G^{\uparrow\uparrow}_{\uparrow\uparrow} + G^{\downarrow\downarrow}_{\downarrow\downarrow}$, and the difference $G^{\uparrow\downarrow}_{\uparrow\downarrow} + G^{\downarrow\uparrow}_{\downarrow\uparrow}$ is denoted as spin-flip polarization. As mentioned above, the spin projection direction that maximizes the effect is transversal, i.e., perpendicular to the current and electric field [32, 34]. Therefore, all the numerical calculations are done for this spin polarization direction. The Rashba coupling constant is taken as $\lambda_R = 0.1t$.

### III. RESULTS

In what follows, the widths of the armchair and zigzag graphene nanoribbons (AGNRs and ZGNRs) are given in dimers or chains, respectively [10]. For the lengths we use the translational unit cell (uc), equal to $3a_{cc}$ in AGNRs and $\sqrt{3}a_{cc}$ in ZGNRs (see, e.g., Ref. [32]).

Spin-dependent conductances projected on the $y$-direction for 11-AGNR systems are shown in Figs. 3(a) and (b) for $(+R,+R)$ and $(+R,-R)$ configurations, respectively. The systems are composed by 4 uc of Rashba region plus an intermediate region without SOI ($x=1$ uc). As explicitly depicted in the red square insets, the case $(+R,+R)$ exhibits different spin-conserved conductances ($G^{\uparrow\uparrow} \neq G^{\downarrow\downarrow}$). The situation is completely reversed for the

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**FIG. 2.** (Color online) Schematic view of different corrugated or multiple-gated systems: (a) homogeneous central part; (b) two Rashba regions with the same sign of the Rashba coupling separated by a no-Rashba region; (c) the same as (b) with opposite Rashba signs, and (d) three Rashba regions with alternating sign Rashba coupling, separated by no-Rashba regions.
FIG. 3. (Color online) Spin-dependent conductances projected on the $y$ direction for 11-AGNR systems with (a) (+R,+R) and (b) (+R,−R) composed by 4 unit cells of Rashba region plus an intermediated region without SOI of $x=1$ unit cell. The conductances $G_{\uparrow\uparrow}$, $G_{\downarrow\downarrow}$, $G_{\uparrow\downarrow}$, and $G_{\downarrow\uparrow}$ are plotted individually. The conductance for the pristine 11-AGNR (without Rashba SOI) is depicted in black dotted lines.

(+R,−R) case, where the conductance differences come from the spin-flip terms. For comparison, the conductance for an 11-AGNR without Rashba coupling is depicted in black dotted lines.

In Fig. 4(a) we present the polarization in the $y$ direction, $P_y$, of an armchair graphene nanoribbon with an 11-dimer width (11-AGNR), considering the (+R,+R) and (+R,−R) configurations with an intermediate region without SOI effects. The length of each region with SOI is 4 uc, whereas that of the spacer (no-Rashba region) is 1 uc. Given that the system presents electron-hole symmetry and therefore $P_E(E) = -P_E(-E)$ [32], the energy interval is represented from $-t$ to 0. $P_y$ is nonzero above/below ±0.4$t$, i.e., at energies for which the second channel in the leads is available for conduction. Since time-reversal symmetry holds, two channels at the outgoing lead are necessary to obtain spin polarization [41]. Obviously, it is possible to decrease the energy threshold for the spin polarization by increasing the width of the ribbon, but for the sake of clarity in the figures we stick to narrow ribbons.

It is important to notice that the spin-polarized current of the (+R,−R) device arises from the difference between spin-flip conductances, with a null spin-conserved polarization. On the contrary, the spin polarization of the (+R,+R) case is produced only by the inequality of spin-conserved conductances. This dissimilar behavior has its origin on the symmetries analyzed in section IV.

FIG. 4. (Color online) $P_y$ of 11-AGNR systems with a central region composed by parallel and antiparallel Rashba sequences: (a) (+R,+R) and (+R,−R); (b) (+R,+R,+R) and (+R,−R,+R), separated by no-Rashba 1-unit cell spacer.

Fig. 4(b) shows the polarization for the same 11-AGNR but with three Rashba regions: (+R,+R,+R) and (+R,−R,+R). Notice that, differently from the case of two Rashba regions with opposite Rashba signs, the $P_y$ corresponding to the (+R,−R,+R) configuration arises from the difference between spin-conserved conductances. We find that the spin-polarized current is larger for systems with the same sign of the Rashba coupling, either with even or odd number of Rashba regions.
FIG. 5. (Color online) Polarization $P_y$ of an 11-AGNR with a continuous central region (20 uc length) with positive Rashba SOI and a truncated superlattice with the same total area subject to SOI, divided in five periodically repeated regions of 4 uc separated by no-Rashba 1-uc spacers.

The role of scattering by multiple interfaces between Rashba and no-Rashba parts can be studied by comparing the polarization of a single Rashba area with that with multiple SOI regions with spacers, maintaining the total area and length affected by SOI constant. Since the spin of the carriers precesses when moving through a Rashba region, keeping the lengths equal amounts, in principle, to having the same SOI effects in both cases [42, 43].

We show in Fig. 5 the polarization corresponding to an 11-AGNR formed by five repetitions of the Rashba graphene region with and without spacers (20 uc in total); we name them superlattice (red curve) and continuous structures (black curve), respectively. We observe that, in general, the inclusion of more SOI/no-SOI interfaces enhances the production of spin-polarized currents for small system sizes. Although for some specific energies the maximum value can be reached for the continuous case, in overall, the superlattice yields larger absolute values for $P_y$, as it can be clearly seen in Fig. 5. This increasing of $P_y$ recalls the enhancement of spin polarization occurring when equidistant impurity defects appear in carbon nanotube with Rashba SOI [34]. Similarly, it indicates that scattering between Rashba and no-Rashba regions is a key ingredient in the increase of the spin-polarized conductance.

A. Spacer-dependent polarization

The spin-polarized current also depends on the size $x$ of the spacers. Results for a single spacer with different lengths ($x=1$ and 4 uc) are shown in Fig. 6(a) for both $(+R,+R)$ and $(+R,-R)$ configurations. Clearly, the best responses are obtained in the case with Rashba regions of the same sign (continuous lines), L-R symmetric but with multiple SOI areas. In Fig. 6(b) we compare the results for a system configuration with six repetitions of the same $+R$ region composed of 4 uc and with different spacer lengths ($x=1$, 2, and 4 uc). The results are compared to the continuous case (black curve). The maximum value is attained for the cases with larger spacers, but it also varies very rapidly; for some particular energies it can be even lower than for the no-spacer case.

As the number of spacer layers increases, many features appear in the spin polarization. To better quantify the amount of spin polarization over an energy interval we propose another magnitude given as an integrated polarization, i.e.,

$$\sum P_y = \int |P_y(E)|dE ,$$

(3)
in which the energy range for the integrated polarization was chosen to be from zero to $-1t$. As shown in Fig. 7(a) and (b), the integrated polarization increases dramatically if a small spacer is included ($x=1$ and 2 for the 11-AGNR and 11-ZGNR, respectively). It is important to remember that a single unit-cell spacer ($x=1$) corresponds to a Rashba region of different lengths for AGNR and ZGNR structures. However, for both configurations the polarization saturates already for $x \approx 2$ in the sense that the maximum integrated polarization value is already attained for that spacer size.

IV. SYMMETRY ANALYSIS

To understand all the features of the conductances and polarization of graphene nanoribbon superlattices we analyze the symmetries of the systems. In order to be able to explore these, we assume that all the Rashba regions in one device are of the same size, and the spacers are also identical. We would like to note that the spin-dependent conductance relations that we derive here are equally valid for other planar quasi-one-dimensional materials with multiple Rashba regions, so the interest of this analysis goes beyond graphene-based systems. We show that it is necessary to take into account different symmetries in real and spin spaces in order to keep the Rashba Hamiltonian invariant in the L-R asymmetric cases. The combination of spatial and spin symmetries provides relations between the spin-resolved conductances that allows for a full understanding of our results.

We start from the simplest situation. For L-R symmetric systems, as those labeled $a$, $b$, $d$ in Fig. 2, it is enough to consider the same symmetry operations in real and spin space, as shown for a nanoribbon with a single Rashba region in Ref. 32. If we take $x$ as the direction of the current and $y$ as the transversal direction, for a straight, two-terminal ribbon, these symmetries are $C_{2x}$, $M_x$, and $M_y$. $C_{2x}$ is a $\pi$ rotation over the $z$ axis, perpendicular to the plane of the nanoribbon. Let us recall that a rotation has the same effect in real and spin variables, but a mirror reflection acts differently in real ($r$) and spin space ($s$). We use superindices $r$, $s$ when necessary to distinguish between these. Thus, if $M_x$ changes the $x$ coordinate in real space from $x$ to $-x$, leaving the other two invariant, in spin space it leaves $\sigma_z$ invariant, and changes $\sigma_y \rightarrow -\sigma_y$ and $\sigma_x \rightarrow -\sigma_x$, due to the fact that spins transform as pseudovectors. In fact, note that a mirror reflection in spin space is equivalent to a $\pi$ rotation: $M_x^{(r)} = C_{2x}^{(s)}$, so we have that $M_x = M_x^{(r)} \otimes M_x^{(s)} = M_x^{(r)} \otimes C_{2x}^{(s)}$.

Besides these, we can also have other combinations of different real- and spin-space symmetries in L-R symmetric systems, but they give the same conductance relations as the former. Table II shows the conductances and the corresponding spin-polarization relations generated by each real and spin symmetries ($r \otimes s$) of the L-R symmetric cases ($a$, $b$, $d$ in Fig. 2). The symmetries are grouped in pairs that give exactly the same conductance relations. Notice that of each pair, one is trivial (at the bottom), in the sense that the same operation is performed in real and spin space, and the other combines two different symmetries in real and spin spaces. Therefore, for L-R symmetric systems, we could have derived the relations between spin-resolved conductances as in Ref. 32.

This is not the case for L-R antisymmetric devices. In Table II we display the symmetry relations of the truncated superlattice $c$, with an even number of Rashba regions with alternating signs. They are different from those present in the L-R symmetric systems.

We provide below an example for the derivation of the symmetries listed in Tables I and II. Suppose that our system has a certain spatial symmetry, neglecting the signs of the Rashba terms. If we apply this spatial symmetry operation to the Rashba Hamiltonian, we imme-
Pauli matrices associated to the $i = x, y, z$ direction. On the one hand, the mirror reflection on $x$ in real space operating on $H_R$ yields $M_x^{(r)} H_R \sim -\lambda(k_x \sigma_y + k_y \sigma_x) = H'_R$ since $(k_x, k_y, k_z) \rightarrow (-k_x, k_y, k_z)$. On the other hand, the spin rotation operation $C_{2x}^{(s)}$ transforms spins as $(\sigma_x, \sigma_y, \sigma_z) \rightarrow (\sigma_x, -\sigma_y, -\sigma_z)$, allowing us to recover the original Rashba Hamiltonian, i.e., $C_{2x}^{(s)} H_R = H_R$. Therefore, in the L-R invariant case $a$ the symmetry $M_x^{(r)} \otimes C_{2y}^{(s)}$ is present.

The same analysis can be done for the other cases depicted in Fig. 2. Without loss of generality, we focus on the cases with two Rashba regions, L-R symmetric (b) and L-R antisymmetric (c). For these, the Hamiltonian can be divided into parts, one for each Rashba region, namely $H_R = H_R^{(1)} + H_R^{(2)}$.

Continuing with the example of the spatial mirror symmetry $M_x^{(r)}$ for a L-R invariant system, when the two parts have the same Rashba sign, $(+R, +R)$ (case $b$), $M_x^{(r)} H_R = M_x^{(r)} H_R^{(1)} + M_x^{(r)} H_R^{(2)}$. We see that $M_x^{(r)}$ interchanges the two parts since the Rashba couplings are equal: $M_x^{(r)} H_R^{(1)} = \lambda \left(- k_x^2 \sigma_y - k_y^2 \sigma_x \right) = H_R^{(2)}$ and $M_x^{(r)} H_R^{(2)} = \lambda \left( k_x^2 \sigma_y + k_y^2 \sigma_x \right) = H_R^{(1)}$. The spin symmetry operation $C_{2x}^{(s)}$ is needed to leave the Hamiltonian invariant: $C_{2x}^{(s)} H_R^{(1)} = H_R^{(1)}$ and $C_{2x}^{(s)} H_R^{(2)} = H_R^{(2)}$, as $H_R^{(2)} = H_R^{(1)}$. Thus we have that $M_x^{(r)} \otimes C_{2x}^{(s)}$ guarantees the invariance of the Hamiltonian.

Now we show that for L-R antisymmetric systems as case $c$, a different spin symmetry is needed. Assume that the Rashba sign of $H_R^{(2)}$ is negative, then $H_R = H_R^{(1)} - H_R^{(2)}$. Applying $M_x^{(r)}$ to the Hamiltonian, $M_x^{(r)} H_R^{(1)} = \lambda \left( k_x^2 \sigma_y + k_y^2 \sigma_x \right) = H_R^{(2)}$ and $-M_x^{(r)} H_R^{(2)} = \lambda \left( -k_x^2 \sigma_y - k_y^2 \sigma_x \right) = -H_R^{(1)}$, leading to $M_x^{(r)} H_R = H_R^{(2)} - H_R^{(1)} = H_R$. Then, it is necessary a rotation $C_{2y}^{(s)}$ on $H_R^{(2)} - H_R^{(1)}$ to recover invariance, i.e., $(M_x^{(r)} \otimes C_{2y}^{(s)}) H_R = H_R$. With the same procedure, all the symmetries of the Hamiltonian can be found. They are collected in the first column of Tables II and III.

The second columns of these Tables show the conductance relations derived from the respective symmetries. Considering the same example, namely, $M_x^{(r)} \otimes C_{2y}^{(s)}$, it is possible to infer that the conductance from left (with spin $\sigma$) to right lead (with spin $\sigma'$), $G_{\sigma \sigma'}^{LR}$, depends on the spin polarization direction. In this way, $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{RL}$ for the $x$ and $z$ spin projection directions and $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{RL}$ for the $y$ spin direction. Note that $M_x^{(r)}$ changes the direction of the current; therefore, time-reversal symmetry is needed, $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{RL}$, to derive from the previous expressions the final equalities, namely, $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{RL}$ for the $x$ and $z$ spin directions and $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{RL}$ for the $y$ direction.

| Symmetries | Conductance | Spin polarization |
|------------|-------------|-------------------|
| $(r \otimes s)$ | $(x, y, z)$ | $G_{\uparrow \downarrow} - G_{\downarrow \uparrow}$ |
| $C_{2x}^{(r)} \otimes C_{2y}^{(s)}$ | $(x, z)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $= 0$ |
| $M_x^{(r)} \otimes C_{2y}^{(s)}$ | $(y)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $\neq 0$ |
| $I^{(r)} \otimes C_{2y}^{(s)}$ | $(x, y)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $\neq 0$ |
| $C_{2x}^{(r)} \otimes C_{2y}^{(s)}$ | $(z)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $= 0$ |
| $C_{2y}^{(r)} \otimes C_{2y}^{(s)}$ | $(y, z)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $\neq 0$ |
| $M_x^{(r)} \otimes C_{2y}^{(s)}$ | $(x)$ | $G_{\sigma \sigma'}^{LR} = G_{\sigma' \sigma}^{LR}$ | $= 0$, $\neq 0$ |

Immediately see that a spin symmetry operation has to be considered to restore the invariance of the Hamiltonian. Depending on whether we have an L-R symmetric or antisymmetric case, $(a, b, c, or d)$, this can be attained using different spin symmetry operations. Let us first consider L-R symmetric systems, such as cases $a$, $b$, and $d$. If $M_x^{(r)}$ holds, then $M_x^{(r)} \otimes C_{2y}^{(s)}$ leaves invariant the Rashba Hamiltonian. Differently, for the L-R antisymmetric systems, such as $c$, $(+R, -R)$, a different spin rotation symmetry $C_{2y}^{(s)}$ is required to guarantee its invariance, so the full symmetry operation is $M_x^{(r)} \otimes C_{2y}^{(s)}$.

This is best seen by taking the Rashba Hamiltonian in its continuum form [44], i.e., $H_R \sim \lambda(k_x \sigma_y - k_y \sigma_x)$, where $k_i$ is the momentum component and $\sigma_i$ the spin.
A summary of the predicted spin-resolved conductance relations for zigzag, anti-zigzag, armchair and anti-armchair based L-R antisymmetric systems are depicted in Fig. 8 separately for the three spin directions (x, y, z). The relevant spatial and spin symmetry operations are indicated for each lattice. Red and blue squares mark the cases with and without spin-polarized currents, respectively, and their origin, i.e., from spin-flip or spin-conserved conductance differences. We have checked all the cases with numerical calculations. If we pay attention to the transversal spin direction (y), in all the studied systems the polarizations \( P_y \) stem from spin-flip conductances, with the exception of the zigzag case that shows also a spin-conserved component, although this is small compared to spin-flip contribution. The polarizations in the other spin directions (x and z) are also small compared to y-direction due to the geometry of the Rashba spin-orbit interaction.

By varying the number of corrugations of the system (or modifying the voltage gates, depending on the particular setup) the quantitative response can be tuned, but also the system can change from L-R antisymmetric to L-R symmetric. This can change the symmetry properties, modifying the spin polarized current and its origin, the sign of the spin current and other characteristics, which can be exploited in the design of spintronic devices that harness this feature. For instance, a system with two gates can control the sign of the spin currents from negative (if the voltages are equal in the two gates) to positive (different voltages in the two gates); the type of polarization, spin-flip or spin-conserved can be selected, etc. Equivalently, a corrugated GNR can be mechanically modified to change the spin-polarized current, as a mechanical spin-flip switch, constituting a spin-straintronic device.

V. FINAL REMARKS

We have found that spin-polarized conductance can be enhanced in corrugated graphene nanoribbon systems, described successfully as multiple Rashba regions. The effect is larger for systems with the same sign of the Rashba SOI, that we have dubbed as L-R symmetric systems. The inclusion of SOI/no-SOI interfaces via small spacers (no SOI regions) enhances the effect, but the size dependence saturates soon.

Graphene systems with an even number of multiple Rashba regions with alternating signs call for separate spatial and spin symmetry operations in order to explain the relations between spin-resolved conductances. Although we have only presented here numerical examples with spin projected transversal to the current, more favorable for the obtention of the maximum spin polarization, we have performed a complete symmetry analysis including all the possible directions.

New spintronic devices can be designed by varying the induced Rashba SOI areas, i.e., changing the number of corrugations. This effect can be also produced and tuned by Rashba areas with external applied electric fields by multiple gates or by proximity phenomena with other materials. Importantly, the symmetry relations reported here are general and can be used to predict spin-polarized currents in other 2D materials.

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