K. Hagino · H. Sagawa

Are there good probes for the di-neutron correlation in light neutron-rich nuclei?

Received: date / Accepted: date

Abstract The di-neutron correlation is a spatial correlation with which two valence neutrons are located at a similar position inside a nucleus. We discuss possible experimental probes for the di-neutron correlation. This includes the Coulomb breakup and the pair transfer reactions of neutron-rich nuclei, and the direct two-neutron decays of nuclei beyond the neutron drip-line.

Keywords Neutron-rich nuclei · Di-neutron correlation · Coulomb breakup · pair transfer reactions · two-neutron decays

1 Introduction

One of the most important issues in many-body physics is to clarify the nature of correlations beyond the independent particle picture. In nuclear physics, the pairing correlation has been well recognized as a typical many-boy correlation[1, 2], which leads to a characteristic even-odd staggering of binding energy providing an extra binding for even-mass nuclei. A pairing interaction scatters nucleon pairs from a single-particle level below the Fermi surface to those above, and as a consequence, each single-particle level is occupied with a fractional occupation probability.

With the pairing correlation, one may naively expect that two nucleons forming a pair are located at a similar position inside a nucleus. A spatial structure of two valence neutrons has in fact attracted much attention in the past. One of the earliest publications on this problem is by Bertsch, Broglia, and Riedel, who solved a shell model for $^{210}$Pb and showed that the two valence neutrons are strongly clustered[3]. Subsequently, Migdal argued that two neutrons may be bound in a nucleus even though they are not bound in the vacuum[4].
The two-particle densities for $^{11}$Li (the left panel) and for $^6$He (the right panel) obtained with a three-body model calculation with a density dependent contact pairing interaction [11]. These are plotted as a function of neutron-core distance, $r_1 = r_2 \equiv r$, and the opening angle between the valence neutrons, $\theta_{12}$. The densities are weighted with a factor $8\pi^2 r^4 \sin \theta_{12}$.

The strong localization of two neutrons inside a nucleus has been referred to as the \textit{di-neutron correlation}. It has been nicely demonstrated in Ref. [5] that an admixture of configurations of single-particle orbits with opposite parity is essential to create the strong di-neutron correlation. This implies that the pairing correlation acting only on single-particle orbits with the same parity is not sufficient in order to develop the di-neutron correlation, and the pairing model space needs to be taken sufficiently largely so that both positive parity and negative parity states are included.

Although the di-neutron correlation exists even in stable nuclei, it is therefore more enhanced in weakly bound nuclei because the admixtures of single-particle orbits with different parities are easier due to the couplings to the continuum spectra [6; 7]. Three-body model calculations have revealed that a strong di-neutron correlation indeed exists in weakly-bound Borromean nuclei, such as $^{11}$Li and $^6$He [8; 9; 11; 12; 13]. For instance, Fig. 1 shows the two-particle density for the $^{11}$Li and $^6$He nuclei obtained with the three-body model calculation with a density dependent contact pairing interaction [11]. One can see that the densities are concentrated in the region with small opening angles, that is, nothing but the di-neutron correlation. It has been shown that the di-neutron correlation exists also in heavier neutron-rich nuclei [14; 15] as well as in infinite neutron matter [16]. The di-proton correlation, which is a counterpart of the di-neutron correlation, has also been shown to exist in the proton-rich Borromean nucleus, $^{17}$Ne [17].

From these studies, the di-neutron correlation seems to have been theoretically established. However, it is not straightforward to probe it experimentally. In this contribution, we discuss how one can probe the di-neutron correlation. To be more specific, we shall discuss the Coulomb breakup, the two-neutron transfer reactions, and the two-nucleon radioactivity as possible probes for the correlation.

### 2 Coulomb breakup of Borromean nuclei

Let us first discuss the Coulomb breakup reactions of Borromean nuclei, $^{11}$Li and $^6$He, for which the experimental data have been available in Refs. [18; 19]. Those experimental breakup cross sections, especially those for the $^{11}$Li nucleus, show a strong concentration in the low excitation region, reflecting the halo structure of these nuclei. Moreover, the experimental data for $^{11}$Li are consistent only with the theoretical calculation which takes into account the interaction between the valence neutrons, strongly suggesting the existence of the di-neutron correlation in this nucleus (see also Ref. [20]).

For the Coulomb breakup of Borromean nuclei, one can go one step further, given that the Coulomb breakup process takes place predominantly by the dipole excitation. The Coulomb breakup cross sections with the absorption of dipole photons are given by

$$
\frac{d\sigma}{dE_\gamma} = \frac{16\pi^3}{9hc} N_\gamma(E_\gamma) \cdot \frac{dB(E_1)}{dE_\gamma},
$$

where $N_\gamma$ is the number of virtual photons, and

$$
\frac{dB(E_1)}{dE_\gamma} = \frac{1}{2I_f + 1} |\langle \psi_f | |D| |\psi_i \rangle|^2 \delta(E_f - E_i - E_\gamma),
$$
is the reduced $E1$ transition probability. In this equation, $\psi_i$ and $\psi_f$ are the wave functions for the initial and the final states, respectively, $I_i$ is the spin of the initial state, and $D_\mu$ is the operator for the $E1$ transition. For the Borromean nuclei, assuming a three-body structure with an inert core, the $E1$ operator $D_\mu$ reads [21],

$$\hat{D}_\mu = e_{E1} (r_1 Y^{\pm \mu}(\hat{r}_1) + r_2 Y^{\pm \mu}(\hat{r}_2)),$$

where the $E1$ effective charge is given by

$$e_{E1} = \frac{2Z_c}{A_c + 2} e,$$

with $A_c$ and $Z_c$ being the mass and charge numbers for the core nucleus. $r_1$ and $r_2$ are the coordinates for the valence neutrons.

Using Eq. (2) and the closure relation for the final state, it is easy to derive that the total $E1$ strength (that is, the non-energy weighted sum rule) is proportional to the expectation value of the center of mass coordinate for the two valence neutrons, $\langle R^2 \rangle$, with respect to the ground state, that is,

$$B(E1) = \sum_f \frac{1}{2I_i + 1} |\langle \psi_f | D | \psi_i \rangle|^2 = \frac{3}{4\pi} e_{E1}^2 \langle R^2 \rangle,$$

with $R = (r_1 + r_2)/2$. Even though the $B(E1)$ strength distribution inevitably reflects both the correlation in the ground state and that in the final state, it is remarkable that one can extract the information which reflects solely the ground state properties after summing all the strength distribution. This implies that the average value of the opening angle between the valence neutrons can be directly extracted from the measured total $B(E1)$ value once the root-mean-square distance between the valence neutrons, $\langle r_{nn}^2 \rangle$, is available.

This quantity is related to the matter radius and $\langle R^2 \rangle$ in the three-body model as [8, 24, 22],

$$\langle r_{m}^2 \rangle = \frac{A_c}{A} \langle r_{m}^2 \rangle_c + \frac{2A_c}{A^2} \langle R^2 \rangle + \frac{1}{2A} \langle r_{nn}^2 \rangle,$$

where $A = A_c + 2$ is the mass number of the whole nucleus. The matter radii $\langle r_{m}^2 \rangle$ can be estimated from interaction cross sections. Employing the Glauber theory in the optical limit, Tanihata et al. have obtained $\sqrt{\langle r_{m}^2 \rangle} = 1.57 \pm 0.04, 2.48 \pm 0.03, 2.32 \pm 0.02, \text{and } 3.12 \pm 0.16 \text{ fm for } ^4\text{He}, ^6\text{He}, ^9\text{Li}, \text{and } ^{11}\text{Li}$, respectively [23, 24]. Using these values, we obtain the rms neutron-neutron distance of $\sqrt{\langle r_{nn}^2 \rangle} = 3.75 \pm 0.93 \text{ and } 5.50 \pm 2.24 \text{ fm for } ^6\text{He} \text{ and } ^{11}\text{Li}$, respectively. Combining these values with the rms core - di-neutron distance, $\sqrt{\langle R^2 \rangle}$, we obtain the mean opening angle of $\langle \theta_{nn} \rangle = 51.56_{-12.4}^{+12.2}$ and $56.2_{-21.3}^{+21.8}$ degrees for $^6\text{He}$ and $^{11}\text{Li}$, respectively [25]. These values are comparable to the result of the three-body model calculation, $\langle \theta_{nn} \rangle = 66.33$ and 65.29 degree for $^6\text{He}$ and $^{11}\text{Li}$, respectively [11], although the experimental values are somewhat smaller.

An alternative way to extract the value for $\sqrt{\langle r_{nn}^2 \rangle}$ has been proposed which uses the three-body correlation study in the dissociation of two neutrons in halo nuclei [24]. The two neutron correlation...
function provides the experimental values for $\sqrt{\left\langle \theta_{nn}^2 \right\rangle}$ to be $5.9 \pm 1.2$ and $6.6 \pm 1.5$ fm for $^6$He, $^{11}$Li, respectively [24]. Bertulani and Hussein used these values to estimate the mean opening angles and obtained $\left\langle \theta_{nn} \right\rangle = 83^{+20}_{-10}$ and $66^{+22}_{-18}$ degrees for $^6$He and $^{11}$Li, respectively [27]. After correcting the effect of Pauli forbidden transitions using the method presented in Ref. [21], these values are slightly altered to be $\left\langle \theta_{nn} \right\rangle = 74.5^{+11.2}_{-13.1}$ and $65.2^{+11.4}_{-13.0}$ degrees for $^6$He and $^{11}$Li, respectively [27]. Notice that these values are in a better agreement with the results of the three-body calculation [11], especially for the $^6$He nucleus.

In the absence of the correlations, the mean opening angle is exactly $\left\langle \theta_{nn} \right\rangle = 90$ degrees. The extracted values of $\left\langle \theta_{nn} \right\rangle$ are significantly smaller than this value both for $^{11}$Li and $^6$He, providing a direct proof of the existence of the di-neutron correlation in these nuclei. A small drawback is that this method provides only the average value of $\left\langle \theta_{nn} \right\rangle$ and a detailed distribution is inaccessible. In reality, the mean opening angle is most probably an average of a smaller and a larger correlation angles in the density distribution, as has been shown in Fig. 1.

3 Two-neutron transfer reactions

It has been recognized for a long time that two-neutron transfer reactions are sensitive to the pairing correlation [28, 29, 30]. The probability for the two-neutron transfer process is enhanced as compared to a naive expectation of sequential transfer process, that is, the square of one-neutron transfer probability [31, 32]. The enhancement of pair transfer probability has been attributed to the pairing effect, such as the surface localization of a Cooper pair [33, 34]. The pair transfer reaction is thus considered to provide a promising way to probe the di-neutron correlation. However, the reaction dynamics is rather complicated and has not yet been well established. For instance, it is only with a recent calculation that a theoretical calculation achieves a satisfactory agreement with the experimental data [34]. It would therefore be not surprising that the role of di-neutron in the pair transfer reaction has not yet been fully clarified.

One such example is a relative importance of the one-step (the direct pair transfer) process and the two-step (the sequential pair transfer) process. In heavy-ion pair transfer reactions of stable nuclei, both processes are known to play a role [35]. For weakly-bound nuclei, most of the intermediate states for the two-step process are likely in the continuum spectra. It is still an open question how this fact, together with the $Q$-value matching condition, alters the dynamics of the pair transfer reaction of neutron-rich nuclei [36].

On the other hand, the cross sections for the pair transfer reaction of the Borromean nuclei, $^{11}$Li and $^6$He, have been measured recently [37, 38, 39, 40, 41]. The data for the $^3$H$(^{11}$Li,$^3$Li)$^3$H reaction at $3$ MeV/nucleon indicate that the cross sections are indeed sensitive to the pair correlation in the ground state of $^{11}$Li [37]. That is, the experimental cross sections can be accounted for only when the $s$-wave component is mixed in the ground state of $^{11}$Li by 30-50%. Another important finding in this measurement is that significant cross sections were observed for the pair transfer process to the first excited state of $^9$Li [37], which has made a good support for the idea of phonon mediated pairing mechanism [42].

Further theoretical studies are apparently necessary in order to understand the connection between the two-neutron transfer process and the di-neutron correlation in Borromean nuclei. This has been left for future investigations.

4 Two-proton and two-neutron decays of nuclei beyond the drip lines

In the Coulomb breakup process discussed in Sec. 2, the ground state wave function of a two-neutron halo nucleus is firstly perturbed by the external electromagnetic field of the target nucleus. It may thus not be easy to disentangle the di-neutron correlation in the ground state from that in the excited states. The two-proton radioactivity, that is, a spontaneous emission of two valence protons, of proton-rich nuclei [43] is expected to provide a good tool to probe the di-proton correlation in the initial wave function. An attractive feature of this phenomenon is that the two valence protons are emitted directly from the ground state even without any external perturbation.

Very recently, the ground state two-neutron emissions have also been observed, e.g., in $^{10}$He [44, 45, 46, 47, 48], $^{16}$Be [49], $^{13}$Li [46, 50], and $^{26}$O [51, 52, 53, 54]. This is an analogous process of the
two-proton radioactivity, corresponding to a penetration of two neutrons over a centrifugal barrier. Since the long range Coulomb interaction is absent, one may hope that the ground state correlation can be better probed by studying the energy and the angular correlations of the emitted neutrons, as compared to the two-proton decays.

Figure 3 shows the calculated decay energy spectrum of $^{26}$O obtained with the $^{24}$O+$n$+$n$ three-body model for $^{26}$O. The calculations are carried out using the Green’s function method as explained in Refs. [7; 21; 55; 56], together with a density-dependent contact neutron-neutron interaction, $v$. In this formalism, the decay energy spectrum is given by,

$$\frac{dP}{dE} = \sum_k |\langle \Psi_k | \Phi_{\text{ref}} \rangle|^2 \delta(E - E_k) = \frac{1}{\pi} \Im \langle \Phi_{\text{ref}} | G(E) | \Phi_{\text{ref}} \rangle,$$

(7)

where $\Psi_k$ is a solution of the three-body model Hamiltonian with energy $E_k$ and $\Phi_{\text{ref}}$ is the wave function for a reference state. The reference state can be taken rather arbitrarily as long as it has an appreciable overlap with the resonance states of interest. Here, we employ the uncorrelated two-neutron state in $^{27}$F for it with the $[1d_{3/2} \otimes 1d_{3/2}]^{(I=0)}$ configuration, which is the dominant configuration in the initial state of the proton knockout reaction of $^{27}$F to produce $^{26}$O. In Eq. (7), $G$ is the correlated two-particle Green’s function calculated as

$$G(E) = (1 + G_0(E)v)^{-1}G_0(E) = G_0(E) - G_0(E)v(1 + G_0(E)v)^{-1}G_0(E),$$

(8)

with the uncorrelated Green’s function, $G_0$, given by

$$G_0(E) = \sum_{j_1,j_2} \frac{|\langle j_1j_2 | j_1j_2 \rangle|}{\epsilon_1 + \epsilon_2 - E - i\eta},$$

(9)

where $\eta$ is an infinitesimal number and the sum includes all independent two-particle states including both the bound and the continuum single-particle states. To this end, we use the Woods-Saxon potential for the neutron-$^{24}$O potential which reproduces the experimental single-particle energies of $\epsilon_{2s_{1/2}} = -4.09(13)$ MeV and $\epsilon_{1d_{3/2}} = 770^{+20}_{-10}$ keV for $^{25}$O. The parameters for the density-dependent zero-range pairing interaction are determined so as to yield the decay energy of 30 keV.

In the figure, we show the spectrum for the uncorrelated case by the dotted line. In this case, the spectrum has a peak at $E = 1.54$ MeV, that is twice the single-particle resonance energy, 0.77 MeV. With the pairing interaction between the valence neutrons, the peak energy is shifted towards lower energies. The decay spectrum obtained by including only the $[d_{3/2}]^2$ configurations is shown by the dashed line. In this case, the peak is shifted by ~0.5 MeV from the unperturbed peak at 1.54 MeV. The peak is further shifted downwards by the configuration mixing, and gets closer to the threshold energy. This comparison implies that the pairing correlation for the single configuration alone is not
enough to reproduce the empirical decay spectrum, and the di-neutron correlations between the two neutrons also play an essential role.

The angular distribution of the emitted neutrons can also be calculated with the two-particle Green’s function method \[21; 55\]. The amplitude for emitting two neutrons with spin components of \(s_1\) and \(s_2\) and momenta \(k_1\) and \(k_2\) reads \[55\],

\[
M_{j,l,k_1,k_2} = \langle (jj)^{(00)}|1 + vG_0 + vG_0vG_0 - \cdots |\psi_i\rangle = \langle (jj)^{(00)}|(1 + vG_0)^{-1}|\psi_i\rangle,
\]

where \(\psi_{jlm}\) is the spin-spherical harmonics, \(\chi_{\alpha}\) is the spin wave function, and \(\delta\) is the nuclear phase shift. Here, \(M\) is a decay amplitude calculated to a specific two-particle final state \[21\],

\[
P(\theta_{12}) = 4\pi \sum_{s_1, s_2} \int dk_1 dk_2 |f_{s_1 s_2}(k_1, \hat{k}_1 = 0, k_2, \hat{k}_2 = \theta_{12})|^2,
\]

where we have set z-axis to be parallel to \(k_1\) and evaluated the angular distribution as a function of the opening angle, \(\theta_{12}\), of the two emitted neutrons.

Fig. 4 shows the angular correlation so obtained. The dot-dashed line shows the distribution obtained without including the \(nn\) interaction, which is symmetric around \(\theta_{12} = \pi/2\). In the presence of the \(nn\) interaction, the angular distribution turns to be highly asymmetric, in which the emission of two neutrons in the opposite direction (that is, \(\theta_{12} = \pi\)) is enhanced, as is shown by the solid line. Grigorenko et al. have also obtained a similar result \[58\].

This behavior reflects properties of the resonance wave function of \(^{26}\text{O}\). That is, because of the continuum couplings, several configurations with opposite parity states mix coherently. Symbolically, let us write a two-particle wave function as,

\[
\Psi(r, r') = \alpha \Psi_{ee}(r, r') + \beta \Psi_{oo}(r, r'),
\]

where \(\Psi_{ee}\) and \(\Psi_{oo}\) are two-particle wave functions with even and odd angular momentum states, respectively. The coefficients \(\alpha\) and \(\beta\) are such that the interference term in the two-particle density, \(\alpha^* \beta \Psi_{oo}(r, r') + \text{c.c.}\), is positive for \(r' = r\) while it is negative for \(r' = -r\) so that the two-particle density is enhanced for the nearside configuration with \(r \sim r'\) as compared to the far side configuration with \(r \sim -r'\). This correlation appears in the opposite way in the momentum space. In the Fourier transform of \(\Psi(r, r')\),

\[
\tilde{\Psi}(k, k') = \int dr dr' e^{i \mathbf{k} \cdot \mathbf{r}} e^{i \mathbf{k'} \cdot \mathbf{r'}} \Psi(r, r'),
\]
there is a factor \(i^l\) in the multipole decomposition of \(e^{ik \cdot r}\). Since \((i^l)^2 = +1\) for even values of \(l\) and \(-1\) for odd values of \(l\), this leads to

\[ \tilde{\Psi}(k, k') = \alpha \tilde{\psi}_{\text{ee}}(k, k') - \beta \tilde{\psi}_{\text{oo}}(k, k'). \]

(15)

for the two particle wave function given by Eq. (13). If one constructs a two-particle density in the momentum space with this wave function, the interference term therefore acts in the opposite way to that in the coordinate space. That is, the two-particle density in the momentum space is hindered for \(k \sim k'\), while it is enhanced for \(k \sim -k'\).

From this argument, we can therefore conclude that, if an enhancement in the region of \(\theta \sim \pi\) in the angular distribution was observed experimentally, that would make a clear evidence for the di-neutron correlation in this nucleus, although such measurement will be experimentally challenging.

Incidentally, the tunneling decay of two fermionic ultracold atoms have been measured very recently [60] (see also Ref. [61] for an application of the Gamow shell model to this phenomenon). An attractive feature of this experiment is that several parameters are experimentally controllable, which include the sign and the strength of the interaction between the particles and the shape of a decaying potential. It may be useful to carry out in future detailed analyses of the tunneling decay of ultracold atoms in order to shed light on the two-proton and two-neutron decay problems in nuclear physics.

5 Summary

We have discussed possible experimental probes for the di-neutron correlation in neutron-rich nuclei, with which two valence nucleons are located at a similar position in the coordinate space. In particular, we have discussed the Coulomb dissociation of Borromean nuclei, the two-neutron transfer reactions, and the direct two-neutron decay of the unbound \(^{20}\)O nucleus.

For the Coulomb dissociation of Borromean nuclei, even though the detailed distribution is difficult to extract, one can use the cluster sum rule (that is, the non-energy weighted sum rule) to deduce the mean value of the opening angle between the valence neutrons. We have demonstrated that the mean opening angle is \(\langle \theta_{nn} \rangle = 74.5 \pm 11.1\) degrees for \(^6\)He and \(^{11}\)Li, respectively. These values are significantly smaller than the value for the uncorrelated distribution, that is, \(\langle \theta_{nn} \rangle = 90\) degrees, clearly indicating the existence of the di-neutron correlation in these Borromean nuclei.

For the two-neutron transfer reactions, the sensitivity of transfer cross sections to the pairing correlation has been well recognized for a long time. However, the reaction dynamics is rather complex, and the relation to the di-neutron correlation has not yet been clarified completely. Given the new experimental data for the two-neutron transfer reactions of the Borromean nuclei, it is now at a good time to discuss these reactions and clarify the reaction dynamics with the di-neutron correlation.

For the direct two-neutron decay, we have discussed the recent experimental data of the decay energy spectrum for the unbound \(^{20}\)O nucleus. We have shown that the decay energy spectrum can be accounted for only with the di-neutron correlation caused by a mixing of many configurations including the continuum. We have also discussed the angular correlations of the emitted two neutrons. We have argued that the di-neutron correlation enhances an emission of the two neutrons in the opposite direction (that is, the back-to-back emission), and indeed our three-body model calculation has revealed such feature. If the enhancement of the back-to-back emission will be observed experimentally, it will thus provide a direct evidence for the di-neutron correlation.

Even though we did not discuss them in this paper, there are other possible probes for the di-neutron correlation. Those include the nuclear breakup reaction [62], the \((p, d)\) scattering at backward angles [63, 64], and the knockout reactions of Borromean nuclei [65, 66, 67, 68]. It would be extremely intriguing if a clear and direct evidence for the di-neutron correlation could be experimentally obtained in near future using also these probes.

References

1. Ring, P., Schuck, P.: The Nuclear Many Body Problem. Springer-Verlag, New York (1980)
2. Brink, D.M., Broglia, R.A.: Nuclear Superfluidity: Pairing in Finite Systems. Cambridge University Press, Cambridge (2005)
3. Bertsch, G.F., Broglia, R.A., Riedel, C.: Qualitative description of nuclear collectivity. Nucl. Phys. A91, 123-132 (1967)
4. Migdal, A.E.: 2 interacting particles in a potential well. Soviet J. of Nucl. Phys. 16, 238-241 (1973)
5. Catara, F., Insolia, A., Maglione, E., Vitturi, A.: Relation between pairing correlations and two-particle space correlations. Phys. Rev. C28, 1091-1094 (1984)
6. Hagino, K., Tanihata, I., Sagawa, H.: Exotic nuclei far from the stability line. In: Henley, E., Ellis, S.D. (ed.) 100 Years of Subatomic Physics. World Scientific, Singapore, 231-272 (2013)
7. Sagawa, H., Hagino, K.: Theoretical models for exotic nuclei. Euro. Phys. J. A51, 102 (2015)
8. Bertsch, G.F., Esbensen, H.: Pair correlations near the neutron drip line. Ann. Phys. (N.Y.) 209, 327-363 (1991)
9. Zhukov, M.V., Danilin, B.V., Fedorov, D.V., Ban, J.M., Thompson, I.J., Vaagen, J.S.: Bound-state properties of Boreomean halo nuclei - ^6He and ^11Li. Phys. Rep. 231, 151-199 (1993)
10. Barranco, F., Bortignon, P.F., Broglia, R.A., Colo, G., Vigezzi, E.: The halo of the exotic nucleus ^11Li: a single Cooper pair. Eur. Phys. J. A11, 385-392 (2001)
11. Hagino, K., Sagawa, H.: Pairing correlations in nuclei on the neutron-drip line. Phys. Rev. C72, 044321 (2005)
12. Hagino, K., Sagawa, H., Carbonell, J., Schuck, P.: Coexistence of BCS- and BEC-like pair structures in halo nuclei. Phys. Rev. Lett. 99 022506 (2007)
13. Kikuchi, Y., Kato, K., Myo, T., Takashita, M., Ikeda, K.: Two-neutron correlations in ^6He in a Coulomb breakup reaction. Phys. Rev. C81 044308 (2010)
14. Matsuo, M., Mizuyama, K., Serizawa, Y.: Dis-neutron correlation and soft dipole excitations in medium mass neutron-rich nuclei near drip line. Phys. Rev. C71 064326 (2005)
15. Pillet, N., Sandulescu, N., Schuck, P.: Generic strong coupling behavior of Cooper pairs on the surface of superfluid nuclei. Phys. Rev. C76 024310 (2007)
16. Matsuo, M.: Spatial structure of neutron Cooper pair in low density uniform matter. Phys. Rev. C73 044309 (2006)
17. Oishi, T., Hagino, K., Sagawa, H.: Di-proton correlation in the proton-rich Borromean nucleus ^17Ne. Phys. Rev. C82 024315 (2010)
18. Nakamura, T., et al.: Observation of strong low-lying E1 strength in the two-neutron halo nucleus ^11Li. Phys. Rev. Lett. 96 252502 (2006)
19. Aumann, T., et al.: Continuum excitations in ^6He. Phys. Rev. C59 1252 (1999)
20. Esbensen, H., Hagino, K., Mueller, P., Sagawa, H.: Charge radius and dipole response of ^11Li. Phys. Rev. C76 024302 (2007)
21. Esbensen, H., Bertsch, G.F.: Soft dipole excitations in ^11Li. Nucl. Phys. A542 310-340 (1992)
22. Vinh Mau, N., Pacheco, J.C.: Structure of the ^11Li nucleus. Nucl. Phys. A607, 163-177 (1996)
23. Tanihata, I., et al.: Measurements of interaction cross sections and nuclear radii in the light p-shell region. Phys. Rev. Lett. 55 2676-2679 (1985)
24. Ozawa, A., et al.: Nuclear size and related topics. Nucl. Phys. A693, 32-62 (2001)
25. Hagino, K., Sagawa, H.: Dipole excitation and geometry of Borromean nuclei. Phys. Rev. C76 047302 (2007)
26. Marques, F.M., et al.: Two-neutron interferometry as a probe of the nuclear halo. Phys. Lett. B476, 219-225 (2000)
27. Bertulani, C.A., Hussein, M.S.: Geometrical Borromean halo nuclei. Phys. Rev. C76 051602(R) (2007)
28. Yoshida, S.: Note on the two-nucleon stripping reaction. Nucl. Phys. 33 685-692 (1962) 685
29. von Oertzen, W., Vitturi, A.: Pairing correlations of nucleons and multi-nucleon transfer between heavy nuclei. Rep. Prog. Phys. 64 1247-1337 (2001)
30. Potel, G., Idini, A., Barranco, F., Vigezzi, E., Broglia, R.A.: Cooper pair transfer in nuclei. Rep. Prog. Phys. 76 106301 (2013)
31. Corradi, L., et al.: Single and pair neutron transfers at sub-barrier energies. Phys. Rev. C84 034603 (2011)
32. Montanari, D., et al.: Neutron pair transfer in ^60Ni + ^118Sn far below the Coulomb barrier. Phys. Rev. Lett. 113 052501 (2014)
33. Insolia, A., Liotta, S., Maglione, E.: 2-particle surface correlations. J. of Phys. G15 1249-1263 (1989)
34. Potel, G., Barranco, F., Marini, F., Idini, A., Vigezzi, E., Broglia, R.A.: Calculation of the transition from pairing to pairing rotational regimes between magic nuclei ^100Sn and ^132Sn via two-nucleon transfer reactions. Phys. Rev. Lett. 107 092501 (2011)
35. Esbensen, H., Jiang, C.L., Rehm, K.E.: Coupled-channels analysis of ^58Ni + ^124Sn reactions Phys. Rev. C57 2401 (1998)
36. Vitturi, A., Sofia, H.M.: Two-particle transfer and pairing correlations: interplay of reaction mechanism and structure properties. Prog. Theo. Phys. Suppl. 196, 72-86 (2012)
37. Tanihata, T., et al.: Measurement of the two-halo neutron transfer reaction ^4H(^11Li,^6Li)^3H at 3A MeV. Phys. Rev. Lett. 100, 192502 (2008)
38. Oganesian, Y.T., Zagrebaev, V.I., Vaagen, J.S.: “Di-neutron” configuration of ^6Li. Phys. Rev. Lett. 82, 4996 (1999)
39. Raabe, R., et al.: Elastic Zn-transfer in the ^4He(^6He,^6He)^4He scattering. Phys. Lett. B458, 1 (1999)
40. Giot, L., et al.: Investigation of ^6He cluster structures. Phys. Rev. C71, 064311 (2005)
41. Chatterjee, A., et al.: \( \text{In} \) and \( \text{2n} \) transfer with the Borromean nucleus \(^{6}\text{He}\) near the Coulomb barrier. Phys. Rev. Lett. \textbf{101}, 032701 (2008)

42. Potel, G., Barranco, F., Vigezzi, E., Broglia, R.A.: Evidence for phonon mediated pairing interaction in the halo of the nucleus \(^{11}\text{Li}\). Phys. Rev. Lett. \textbf{105}, 172502 (2010)

43. Pützner, M., Karny, M., Grigorenko, L.V., Riisager, K.: Radioactive decays at limits of nuclear stability. Rev. Mod. Phys. \textbf{84}, 567-619 (2012)

44. Golovkov, M.S., et al.: The \(^{8}\text{He}\) and \(^{10}\text{He}\) spectra studied in the \((t,p)\) reaction. Phys. Lett. \textbf{B672}, 22-29 (2009)

45. Johansson, H.T., et al.: The unbound isotopes \(^{9,10}\text{He}\). Nucl Phys. \textbf{A842}, 15-32 (2010)

46. Johansson, H.T., et al.: Three-body correlations in the decay of \(^{10}\text{He}\) and \(^{13}\text{Li}\). Nucl Phys. \textbf{A847}, 66-88 (2010)

47. Sidordhuk, S.I., et al.: Structure of \(^{10}\text{He}\) low-lying states uncovered by correlations. Phys. Rev. Lett. \textbf{108}, 202502 (2012)

48. Kohley, Z., et al.: Unresolved question of the \(^{10}\text{He}\) ground state resonance. Phys. Rev. Lett. \textbf{109}, 232501 (2012)

49. Spyrou, A., et al.: First observation of ground state di-neutron decay: \(^{16}\text{Be}\). Phys. Rev. Lett. \textbf{108}, 102501 (2012)

50. Kohley, Z., et al.: First observation of the \(^{13}\text{Li}\) ground state. Phys. Rev. Lett. \textbf{87}, 011304(R) (2013)

51. Lunderberg, E., et al.: Evidence for the ground-state resonance of \(^{26}\text{O}\). Phys. Rev. Lett. \textbf{108}, 142503 (2012)

52. Caesar, C., et al.: Beyond the neutron drip line: the unbound oxygen isotopes \(^{25,26}\text{O}\). Phys. Rev. C\textbf{88}, 034313 (2013)

53. Kondo, Y., Nakamura, T., et al.: Invariant-mass spectroscopy of extremely neutron-rich nuclei with SAMURAI at RIBF. JPS Conf. Proc. \textbf{6}, 010006 (2015)

54. Kohley, Z., et al.: Study of two-neutron radioactivity in the decay of \(^{26}\text{O}\). Phys. Rev. Lett. \textbf{110}, 152501 (2013)

55. Hagino, K., Sagawa, H.: Correlated two-neutron emission in the decay of unbound nucleus \(^{26}\text{O}\). Phys. Rev. C\textbf{90}, 027303 (2014)

56. Hagino, K., Sagawa, H.: Three-body model calculation of the \(2^+\) state in \(^{26}\text{O}\). Phys. Rev. C\textbf{90}, 027303 (2014)

57. Hoffman, C.R., et al.: Determination of the \(N = 16\) shell closure at the Oxygen drip line. Phys. Rev. Lett. \textbf{100}, 152502 (2008)

58. Grigorenko, L.V., Mukha, I.G., Zhukov, M.V.: Lifetime and fragment correlations for the two-neutron decay of \(^{26}\text{O}\) ground state. Phys. Rev. Lett. \textbf{111}, 042501 (2013)

59. Kohley, Z., et al.: Three-body correlations in the ground-state decay of \(^{26}\text{O}\). Phys. Rev. C\textbf{91}, 034323 (2015)

60. Zürn, G., et al.: Pairing in few-fermion systems with attractive interactions. Phys. Rev. Lett. \textbf{111}, 175302 (2013)

61. Lundmark, R., Forssen, C., Rotureau, J.: Tunneling theory for tunable open quantum systems of ultracold atoms in one-dimensional traps. Phys. Rev. A\textbf{91}, 041601(R) (2015)

62. Assie, M., et al.: Neutron correlations in \(^{4}\text{He}\) viewed through nuclear break-up. Euro. Phys. J. A\textbf{42}, 441-446 (2009)

63. Horiuchi, W., Suzuki, Y.: Momentum distribution and correlation of two-nucleon relative motion in \(^{6}\text{He}\) and \(^{4}\text{Li}\). Phys. Rev. C\textbf{76}, 024331 (2007)

64. Suda, T.: private communications (2013)

65. Kondo, Y., et al.: Low-lying intruder state of the unbound nucleus \(^{13}\text{Be}\). Phys. Lett. B\textbf{690}, 245-249 (2010)

66. Kobayashi, N., et al.: One- and two-neutron removal reactions from the most neutron-rich carbon isotopes. Phys. Rev. C\textbf{86}, 054604 (2012)

67. Aksyutina, Yu., et al.: Momentum profile analysis in one-neutron knockout from Borromean nuclei. Phys. Lett. B\textbf{718}, 1309-1313 (2013)

68. Usaka, T., Sasano, M.: private communications (2015)
