Research in Area of Longevity of Sylphon Scraies

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Abstract. The proposed method of determining the sensitivity threshold for bellows expansion compensator by fitting a power polynomial of the empirical curves of distribution of durability for a large number of samples, followed by statistical analysis. For a reliable justification of resource products with regard to dispersion of their durability is of great importance the evaluation of the sensitivity threshold in cycles. Currently, the appointment of a guaranteed resource is made with high strength requirements. The proposed method allows to define sensitivity threshold value for the cycles between the curves of the distribution of longevity as having a distinct threshold, and not having it. Method is in good agreement with other known methods. Thus, by statistical processing of results of fatigue tests of joints obtained values of the sensitivity threshold in cycles and confidence interval for mathematical expectation of the magnitude of the threshold.

Introduction
Setting the reliable indicators is determined largely by a distribution function of operating time to failure. The value of operating time to failure in the increasing order reflects the empirical distribution function of endurance. For reliable study of items resource with a view of the dispersion of its endurance it is important to estimate the threshold of sensitivity to $N_0$ cycles. Currently, the appointment of the safe life is made without taking into account the values of $N_0$. Consequently a large endurance margin $n_N$ is necessary in order to account the dispersion of endurance. Safe life based on the average rated life is determined from the expression:

$$N_{\text{safe life}} = \frac{N_{\text{aver rated}}}{n_N}$$

(1)

As a rule such setting of safe life leads to increased strength properties of items and to increasing of laboriousness, metal consumption and excessive mass. Determination $N_0$ values allows us to justify the safe life of items. Researchers have found that the endurance has lower and upper boundaries under constant maximum voltage [8]. The lower boundary of endurance $N_0$ denotes the threshold of sensitivity to cycles, i.e. the largest possible number of cycles, $\sigma_{\text{max}}$ scarcely causes damage at a given voltage. The value $N_0$ separates insensitive zone of operating time to changing load.

It is difficult to estimate the $N_0$ empirically, as a considerable number of tests is required. Even more difficult it is to determine the threshold of sensitivity at relatively low voltages due to the large test duration.

Materials and methods for solving the problem
There are currently several methods for determining the threshold of sensitivity: graphic, symmetrical form bringing, least squares, maximum likelihood.

However, the effectiveness of these methods is not resolved until now. Application of these methods to the results of fatigue tests of syphons has not given results:
1. The method of reducing to the symmetrical form is applicable only to a comparatively large volumes of samples (more than 60), so it was not considered.

2. The method of least squares, as well as its graphic expression – a graphical method gives a solution only for the curve distribution with clearly defined lower boundary of endurance and not clearly upper limit. If endurance curves do not comply at least with one condition it means that the values $N_0$, calculated on these methods become unrealistic and even negative. Figure 1 shows the distribution curves of endurance [9]. The method of least squares does not give solutions for the curves 1, 2, 3, the solution for the curve 4 matches the value $N_0 = 6.1 \times 10^5$ cycles.

3. The maximum likelihood method does not give convergence solutions for samples less than 36, including curves (Figure 1). The need for solving this problem is recognized by the authors [1].

4. An approximate method [7] does not take into account the conditions of existence of the threshold of sensitivity; therefore it gives lower results (straight lines in Figure 1).

![Figure 1. Distribution curves of endurance](image)

In this research, the sensitivity threshold determination for compensator made of steel 12X18H10T, is performed by approximation of exponential polynomial of empirical curves distribution of endurance for a large number of samples, followed by statistical processing.

When the approximation is used under condition of sensitivity threshold display on the lower and upper boundaries of the empirical distribution curves.

Endurance test results of the products at a level of voltage $\sigma_{max}$ were located in the variation series in ascending order of endurance and were grouped by voltages with a deviation from one another not more than 10–15 MPa. The value of the probability of destruction $P_i$, estimated at the accumulated frequency, was calculated by the formula [10]

$$P_i = i - 0.5 \frac{n}{n}$$

(2)

$i$ – item number in the variational series, $n$ – the total number of tested items.
As noted in [2-4], sign of the sensitivity threshold $N_0$ is the curvature of the curve $P - \lg N$, built on probability paper, bump up. Analysis of the data presented in [6], shows that the distribution curves of endurance in the limit at probabilities of failure $P \to 0$ and $P \to 1$ form a right angle with $\lg N$ axis. Thus, the problem reduces to a dot approximating of the empirical distribution curves with zero derivative at the boundary points that are sought. To increase the accuracy, the approximation was made with the transition to a uniform probability scale. And the perpendicularity of distribution curve to the axis $\lg N$ in the limit is persists.

The solution of the problem can be represented in the form of approximating $m$–dimensional polynomial.

$$y(x) = \sum_{k=0}^{m} a_k x^k$$

$x$ represents the values of $P$. We consider at $P = 0, x = 0, N_a = a_0$.

Naturally, on the segment $N_1 < N_2 < \ldots < N_0$ empirical distribution function has a range of values, that means that the values of the desired distribution curve were set with an error. To solve the problem it is necessary to construct a function that gradually took place nearby the predetermined values. Polynomial is used in this function, at which the minimum of the functional is reached [5],

$$S[f(x_k)] = \alpha \int_{x_0}^{x_{n+1}} [f''(x_k)]^2 dx + \sum_{k=0}^{n+1} P_k[f(x_k) - y_k]^2$$

$\alpha$ – smoothing parameter; $P_k$ – mass coefficient ($P_k > 0$); $y_k$ corresponds to the values $N_k$.

Boundary condition for sought function will be:

$$f'(x_0) = \sum_{j=0}^{m} j a_j x_0^{j-1} = 0, \quad f'(x_{n+1}) = \sum_{j=0}^{m} j a_j x_{n+1}^{j-1} = 0$$

After differentiating the general expression for a system of equations has the following form:

$$\frac{\partial}{\partial a_i} S[f(x_k)] = \alpha \sum_{l=0}^{m} \sum_{j=0}^{m} \frac{2i(l - 1)j(j - 1)}{l + j - 3} a_j + 2 \sum_{k=0}^{n+1} P_k \left( \sum_{j=0}^{m} a_j x_k^j - y_k \right) x_k^l = 0_k$$

Thus, for determining $m + 3$ the unknowns $a_0, a_1, \ldots, a_m, y_0, y_{n+1}$, we have $m + 3$, equations. Taking into consideration that $x_0 = 0, x_{n+1} = 1$, the final system of equations for determining the coefficients in a general form can be written as:

$$\alpha \sum_{l=0}^{m} \sum_{j=0}^{m} \frac{i(l - 1)(j - 1)}{l + j - 3} a_j + \sum_{k=0}^{n+1} P_k \sum_{j=0}^{m} a_j x_k^{j+1} - y_0 x_0^l - y_{n+1} x_{n+1}^l = \sum_{k=1}^{n} y_k x_k^l$$

$$a_1 = 0$$

$$\sum_{j=0}^{m} j a_j = 0$$
Accuracy of approximation of a function to set values depends on the parameter $\alpha$ and mass coefficient $P_k$. The higher the mass coefficient $P_k$, the greater contribution the interpolation conditions make into the functional and the closer the smoothing function passes to the set values. Since the tests were conducted on identical equipment, with the same error, it is natural to take the values of the coefficients $P_k$ equal at all points of the empirical distribution curve.

Selecting $\alpha$ smoothing parameter is a problem. At a low $\alpha$ value the smoothing will be insignificant, at inflated $\alpha$ values the function will be extremely smooth. In this case, the choice of $\alpha$ parameter determines by the behavior of the first derivative of the approximating polynomial, which should not be less than zero.

As a result of the approximation of the empirical distribution curves of the fifth degree polynomial the corresponding values of sensitivity thresholds $N_0$ were obtained. Figure 2 shows the results of calculations $N_0$ values, depending on the voltage $\sigma_{\text{max}}$. $N_0$ values can be viewed as a random variable having a range of possible values.

In order to get the averaged curve $\lg N_0 = f (\sigma_{\text{max}})$, having a probability of $P = 0.5$, these presented results are approximated by a polynomial in the eighth power (dash-dot line).

![Figure 2](image)

**Figure 2.** The empirical regression line for the sensitivity threshold $N_0$ and its bounder of the confidence region

It is obvious that the resulting dependency without a large error can be replaced by a linear dependence, that is, the connection between the normally distributed random variable $y = \lg N_0$ and the value of $x = \sigma_{\text{max}}$ can be established by means of linear regression analysis. The empirical regression line has the form

$$Y = a + b (x-\bar{x})$$

(9)

where $Y$ - the evaluation of the conditional expectation value of $y = \lg N_0$ for a set value $x$; $\bar{x}$- sample average value of $x$.

Parameter estimation of the regression line was calculated from the formula:

$$\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i$$

(10)
\[ a = \bar{y} = \frac{1}{m} \sum_{i=1}^{m} y_i \]
\[ b = \frac{\sum_{i=1}^{m} (x_i - \bar{x})y_i}{\sum_{i=1}^{m} (x_i - \bar{x})^2} \]

\( m \) – the number of voltage levels.

Finally, for \( m = 29 \), we have:

\[ Y = 4.2754 - 0.0238 (x - 47.8896) \]

or:

\[ \text{lg}N_0 = 5.4152 - 0.0238 \sigma_{\text{max}} \]  \( \text{(11)} \)

The regression line corresponding to the resulting equation is shown in Figure 2 (solid line).

At probabilistic calculations of item strength to enhance their reliability it is not appropriate to use the average sampled values of the random variable, but the values of the boundaries of the confidence intervals, in particular, we should take the lower limit of the confidence interval instead of the sample average values of \( N_0 \). The boundaries of the confidence interval \( \Delta \) for the expectation \( N_0 \) are determined by the expression:

\[ Y - t_{\alpha,k} S_y < \Delta < Y + t_{\alpha,k} S_y \]  \( \text{(12)} \)

\( t_{\alpha,k} \) – student criterion for the significance level \( \alpha \) and the number of degrees of freedom \( K \); \( S_y \) – the variance of expected value estimation.

On Figure 2 the lower limit 99.9% of confidence region with a significance level \( \alpha = 0.001 \) is marked by the dashed line. The upper limit of endurance was not viewed as it had no practical interest.

**Results**

The proposed statistical processing method allows to determine the \( N_0 \) value for the distribution curves of endurance with a distinct sensitivity threshold, and without it. The method is consistent with other known methods, there is a pronounced tendency of the distribution curve to \( N_e \). For example, for distribution curve 4 (Figure 1) \( N_0 = 6.3 \ldots 10^5 \) cycles (least squares \( N_0 = 6.1 \ldots 10^5 \) cycles).

**Conclusion**

Thus, by means of statistical processing of the compensators fatigue test results we obtained the values of sensitivity threshold on cycles and the confidence interval for the expectation values of \( N_0 \).

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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