Gravitational collapse due to dark matter and dark energy in the brane world scenario

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Gravitational collapse of FRW brane world embedded in a conformal flat bulk is considered for matter cloud consists of dark matter and dark energy with equation of state \( p = \epsilon \rho \) (\( \epsilon < -\frac{1}{3} \)). The effect of dark matter and dark energy is being considered first separately and then a combination of them both with and without interaction. In some cases the collapse leads to black hole in some other cases naked singularity appears.

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I. INTRODUCTION

In the last decade, two remarkable advancements has been done - one of them is the theoretical idea of the brane world scenario [1, 2] and the other one is the observational evidence that our universe is at present accelerating [3-5].

Randall and Sundrum [1] first proposed the bulk-brane model to explain the higher dimensional theory. According to them, we live in a four dimensional world (called 3-brane, a domain wall) which is embedded in a 5-dimensional space-time (called bulk). The extra fifth dimension may be finite or infinite in size. All matter fields are confined in the brane while gravity can only propagate in the bulk.

The observational fact of accelerating universe was first noticed from high red shift supernova Ia [3] and was confirmed from the cosmic microwave background radiation [4] and large scale structure [5]. In Einstein gravity present accelerating universe is possible with matter field (dark energy) having large -ve pressure (violating energy conditions). From observational point of view, our flat universe contains approximately 72% of dark energy, 21% of dark matter, 4.5% of visible matter (baryon matter) and 0.5% radiation. As the nature of dark energy [6-11] is unknown so several models have been proposed for it such as Chaplygin gas, phantoms, quintessence, a tiny +ve cosmological constant, dark energy in brane worlds.

Recently, Cai and Wang [12] have studied an important topic in gravitational physics namely black-holes and their formation in the universe considering gravitational collapse of dark matter in the background of dark energy. In the present work, formation of black-hole will be studied for brane world scenario with dark matter having dark energy in the background. The paper is organized as follows: in section II the basic equations for the gravitational collapse of a homogeneous and isotropic brane model having finite radius will be presented. Section III will deal with collapsing process for dark energy and dark matter separately to identify their individual role. In section IV, the collapse of dark matter in the presence of dark energy both without and with interaction between them (except the gravitational one) will be studied. The paper will end with conclusions in section V.

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II. GENERAL FORMULATION OF THE COLLAPSING PROCESS IN BRANE WORLD

Suppose in the 5D space-time (bulk) the brane-world is located at \( \chi(x^\mu) = 0 \) with \( x_\mu, \mu = 0, 1, 2, 3, 4 \) are the 5D co-ordinates. The effective action in 5D [13] is

\[
A = \int d^5x \sqrt{-\text{(5)}g} \left[ \frac{1}{2\kappa_5^2} \text{(5)}R - \text{(5)}\Lambda \right] + \int_{\chi=0} d^4x \sqrt{-\text{(4)}g} \left[ \frac{1}{\kappa_5^2} \kappa^\pm - \lambda + L_{\text{matter}} \right]
\]

(1)

Here \( \kappa_5^2 = 8\pi G^{(5)} \) is the 5D coupling constant, \( \kappa^\pm \) is the extrinsic curvature on either side of the brane, \( \lambda \) is the vacuum energy on the brane (brane tension), \( \text{(5)}\Lambda \) is the -ve vacume energy, the only source of the gravitational field in the bulk and \( L_{\text{matter}} \) is the Lagrangion for the matter field on the brane.

The variation of the action gives the Einstein equation on the bulk as

\[
G_{\mu\nu} = \kappa_5^2 \left[ -\text{(5)}\Lambda g_{\mu\nu} + \delta(\chi) \left( -\lambda g_{\mu\nu} + T_{\mu\nu}^{\text{matter}} \right) \right]
\]

(2)

Now, assuming \( z_2 \)-symmetry, the effective four dimensional gravitational equations on the brane take the form [2,14]

\[
G_{ij} = -\text{(4)}\Lambda g_{ij} + \kappa_4^2 T_{ij} + \kappa_5^2 \Pi_{ij} - E_{ij}
\]

(3)

where,

\[
\Pi_{ij} = \frac{1}{12} T_{ij} - \frac{1}{4} T_{ik} T_{jk} + \frac{1}{24} g_{ij} (3 T_{lm} T_{lm} - T^2)
\]

is the quadratic correction term, \( E_{ij} \) is the electric part of the 5D Weyl tensor,

\[
\text{(4)}\Lambda = \frac{\kappa_5^2}{2} \left( \text{(5)}\Lambda + \kappa_5^2 \lambda^2 / 6 \right)
\]

is the effective 4D cosmological constant and

\[
\kappa_4^2 = \kappa_5^2 \lambda / 6
\]

is the 4D gravitational coupling constant.

In the following assuming flat FRW space time

\[
\text{ds}^2 = dt^2 + a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

as our brane world embedded in a conformally flat bulk geometry with vanishing \( \text{(4)}\Lambda \), the effective Einstein equations become

\[
\frac{\dot{a}^2}{a^2} = \frac{\kappa}{3} [\rho_T (1 + \frac{\rho_T}{2\lambda})]
\]

(4)

\[
\frac{\ddot{a}}{a} = -\frac{\kappa}{6} [\rho_T (1 + \frac{2\rho_T}{\lambda}) + 3\varepsilon \rho (1 + \frac{\rho_T}{\lambda})]
\]

(5)

with \( \rho_T = \rho_M + \rho \). Here the energy momentum tensor \( T_{ij} \) is given by

\[
T_{ij} = (\rho_M + \rho + p) u_i u_j - p g_{ij}
\]

(6)

where \( \rho_M \) stands for the energy density of the dark matter, while \( \rho \) and \( p \) are respectively the energy density and pressure of the dark energy having equation of state

\[
p = \varepsilon \rho
\]
and $u_i$ is their four-velocity having expression

$$u_i = \delta_i^\lambda$$

in the comoving co-ordinate system. Using the energy-momentum conservation relation

$$T_{ij;k} g^{jk} = 0,$$

the interaction $Q(t)$ between dark matter and dark energy can be expressed as

$$\dot{\rho}_M + 3\frac{\dot{a}}{a}\rho_M = Q$$

(7)

$$\dot{\rho} + 3\frac{\dot{a}}{a}(1+\varepsilon)\rho = -Q$$

(8)

Now, if we consider gravitational collapse of a spherical cloud consists of above dark matter and dark energy distribution and is bounded by the surface $\Sigma : r = r_\Sigma$ then the metric on it can be written as

$$ds^2 = dT^2 - R^2(T) (d\theta^2 + \sin^2\theta d\phi^2))$$

(9)

Thus on $\Sigma : T = t$ and $R(T) = r_\Sigma a(T)$ where $R(r, t) \equiv ra(t)$ is the geometrical radius of the two spheres $t, r$ = constant. Also the total mass of the collapsing cloud is given by

$$M(T) = m(r, t) |_{r=r_\Sigma} = \frac{1}{2} r_\Sigma^3 \dot{a}^2 |_{\Sigma} = \frac{1}{2} R(T) \dot{R}^2(T)$$

(10)

The apparent horizon is defined as [15]

$$R_{\alpha\beta} R_{\alpha\beta} g^{\alpha\beta} = 0, \text{ i.e., } r^2 \dot{a}^2 = 1$$

(11)

So if $T = T_{AH}$ be the time when the whole cloud starts to be trapped then

$$\dot{R}^2(T_{AH}) |_{\Sigma} = r_{\Sigma}^2 \dot{a}^2(T_{AH}) = 1$$

(12)

As it is usually assumed that the collapsing process starts from regular initial data so initially at $t = t_i \ (< T_{AH})$, the cloud is not trapped i.e.,

$$r_{\Sigma}^2 \dot{a}^2(t_i) < 1, \quad (r_{\Sigma} \dot{a}(t_i) > -1)$$

(13)

Thus if equation (12) has any real solution for $T_{AH}$ satisfying (13) then black hole will form, otherwise the collapsing process leads to a naked singularity.

**III. ROLE OF DARK MATTER OR DARK ENERGY IN GRAVITATIONAL COLLAPSE**

This section will show the role of dark matter and dark energy separately during the collapse.

**CASE I : Collapse With Dark Matter: $\rho_M \neq 0, \rho = p = 0$**

Here from conservation equations (7) and (8), on integration one gets

$$\rho_M = \frac{\rho_0}{a^3}$$

(14)

Then using $\rho_M$ in equation (4) and integrating one obtains

$$a^3(t) = [a_0(t_0 - t) + A_0]^2 - A_0^2$$

(15)
Fig. 1 shows the behaviour of $\dot{R}$ of eq.(16) as the collapsing process approaches to singularity.

where $a_0 = \left(\frac{3\varrho_0}{2}\right)^{\frac{1}{2}}$, $A_0 = \left(\frac{\rho_0}{2\varrho}\right)^{\frac{1}{2}}$ and $t_0$ is an integration constant. The expression for relevant physical quantities are

$$
\begin{align*}
\dot{R}(T) &= -\frac{2}{3}R_0 (a^3 + A_0^2)^{\frac{1}{2}} \\
M(T) &= \frac{2}{9}R_0^3 \frac{(a^3 + A_0^2)}{a}\nonumber
\end{align*}
$$

(16)

with $R_0 = r_\Sigma a_0$.

The above solution shows that as $t \to \infty$, $a \to \infty$, $\rho_M \to 0$, $\dot{R} \to 0$ and $M(T) \to \frac{2}{9}R_0^3$. So the collapsing process (see fig.1) with dark matter starts at $t = t_i$ (say) when the condition (13) is satisfied and the time of formation of apparent horizon ($T_{AH}$) is given by the real root of the equation

$$
[a_0^2(T_0 - T)^2 + 2a_0A_0(T_0 - T)]^4 = \left(\frac{4R_0^2}{9}\right)^3 [a_0(T_0 - T) + A_0]^6
$$

(17)

Finally, the singularity will form at $T = T_0$. Note that during the collapse the dark matter energy density and the total mass of the cloud gradually increase and go to infinity at the singularity.

**CASE II : Collapse With Dark Energy:** $\rho_M = 0, \ p = \varepsilon \rho$

In this case the collapsing cloud is in the form of perfect fluid obeying barotropic equation of state: $p = \varepsilon \rho$, $\varepsilon$ is a constant. It is to be noted that when $\varepsilon < -\frac{1}{3}$, the strong energy condition is violated and the fluid is termed as dark energy. The solution for different values of $\varepsilon$ are given by
Fig. 2 and 3 show the variations of $\dot{R}$ with time in eq. (18) for $\varepsilon = -1/2$ and $\varepsilon = -3/4$ respectively.

\[
e^{3(1+\varepsilon)} = [a_1(t_0 - t) + A_1]^2 - A_1^2
\]

\[
\rho = \frac{\rho_1}{a_1(t_0 - t)}
\]

\[
\dot{R}(T) = -2R_1[a_1^{3(1+\varepsilon)} + \frac{A_1^2}{a_1^{3(1+\varepsilon)} - 1}]
\]

\[
M(T) = \frac{2R_1^2[a_1^{3(1+\varepsilon)} + A_1^2]}{9(1+\varepsilon)^2a_1^2a_1^{3(1+\varepsilon)} - 3}
\]

\[
a = a_2e^{\mu(t_0 - t)}
\]

\[
\rho = \rho_1
\]

\[
\dot{R}(T) = -R_2\mu e^{\mu(t_0 - t)}
\]

\[
M(T) = \frac{R_2^2\mu^2}{2}e^{3\mu(t_0 - t)}
\]

(18)

\[
\varepsilon > -1
\]

\[
\varepsilon = -1
\]

(19)

\[
\varepsilon < -1
\]

(20)

where the expressions for different constants are

\[
A_1 = \sqrt{\frac{\rho_1}{2\lambda}}, \quad a_1 = \sqrt{\frac{3\kappa \rho_1}{2}}(1+\varepsilon), \quad R_1 = r_\Sigma a_1, \quad \mu = \frac{\kappa \rho_1}{3}(1 + A_1^2)^{\frac{1}{2}}, \quad R_2 = r_\Sigma a_2,
\]

\[
a_3 = \sqrt{\frac{3\kappa \rho_1}{2}}(|\varepsilon| - 1), \quad a_0' = \sqrt{\frac{3\kappa \rho_1}{2}}, \quad R_0' = r_\Sigma a_0'
\]
From the above solution for \( \varepsilon > -1 \), (see fig.2 and fig.3) as before \( t = t_0 \) is the time of formation of singularity. For \( \varepsilon > -\frac{2}{3} \), the singularity is covered by the event horizon i.e., black hole will form, while for \(-1 < \varepsilon < -\frac{2}{3} \), the collapse will form a naked singularity with zero mass at \( t = t_0 \) i.e., the mass of a black hole decreases due to dark energy accretion and is consistent with the claim of Babichev et al [16].

As for equation of state \( p = -\rho \) (i.e., \( \varepsilon = -1 \)) the solution corresponds to de sitter space so it is not of much interest in the present context.

For \( \varepsilon < -1 \) i.e., when the matter in the brane is purely in the form of dark energy the behavior of both the physical and the geometrical quantities of interest at the time limits \( T = T_0 + \sqrt{\frac{3A_1}{a_3}} \) (\( = T_s \)) and \( T \to \infty \) are as follows:

\[
T \to T_s : \quad a \to \infty, \quad \rho \to \infty, \quad \dot{R}(T) \to -\infty, \quad M(T) \to \infty
\]

\[
T \to \infty : \quad a \to 0, \quad \rho \to 0, \quad \dot{R}(T) \to 0, \quad M(T) \to 0.
\]

Thus if the cloud is untrapped initially, it will remain so throughout the process and neither black hole nor naked singularity will form. However, if initially \( \dot{R}^2 > 1 \) (trapped) then the cloud will start untrapped at the instant given by the real root of the equation

\[
\left[ \frac{2}{3} R_0'(T - T_0)^{6(\varepsilon^{-1})} \right]^{6(\varepsilon^{-1})} = \left[ a_3(T - T_0)^2 - A_1 \right]^{6(\varepsilon^{-4})} \quad (21)
\]

Further, it is to be noted that with the collapsing process both the total mass and energy density of the cloud decrease and finally become zero in the limit \( T \to \infty \), though it \( (T = \infty) \) is not a singularity of space time.

**IV. EFFECT OF A COMBINATION OF DARK MATTER AND DARK ENERGY IN THE GRAVITATIONAL COLLAPSE**

The function \( Q(t) \) in the conservation equation (7) and (8) stands for the interaction between dark matter and energy.

**STEP-I : when \( Q = 0 \) i.e., no interaction:**

The integral of equations (7) and (8) give

\[
\rho_{DM} = \frac{\rho_0}{a^3}, \quad \rho = \frac{\rho_1}{a^3(1 + \varepsilon)} \quad \text{(22)}
\]

where \( \rho_0 \) and \( \rho_1 \) are positive integration constant. If \( \varepsilon < -1 \) then both \( a = 0 \) and \( a = \infty \) correspond to space-time singularity while for \( \varepsilon \geq -1 \), \( a = 0 \) is the only space-time singularity. These expressions for \( \rho_{DM} \) and \( \rho \) result an integral form of the field equation (4) as

\[
\int \frac{dy}{\sqrt{(\rho_0 + \frac{\rho_1}{y^{2\varepsilon}})(1 + \frac{1}{2\lambda y^2(\rho_0 + \frac{\rho_1}{y^{2\varepsilon}}))}} = \Lambda - \sqrt{\frac{3\kappa}{4}} t \quad \text{(23)}
\]

where \( y = a^{\frac{3}{2}} \) and \( \Lambda \) is an integration constant. This integration can not be evaluated in general. For \( \varepsilon = -1 \) the scale factor gives
Fig. 4 shows the variation of $\dot{R}$ with time in eq. (25).

$$a^3 = \frac{1}{\rho_1} \left[ \frac{2\lambda \rho_0}{\rho_1 + 2\lambda} \right] \cosh^2 \left[ \sqrt{\frac{\rho_1 (\rho_1 + 2\lambda)}{2\lambda}} \left( \frac{3\kappa}{4} (t_0 - t) + A \right) \right] - \rho_0 \right]$$  \hspace{1cm} (24)

with $A = \sqrt{\frac{2\lambda}{\rho_1 (\rho_1 + 2\lambda)}} \cosh^{-1} \sqrt{\frac{(\rho_1 + 2\lambda)}{2\lambda}}$.

The expressions for other relevant parameters are

$$\dot{R}(T) = -\sqrt{\frac{\kappa}{3a}} \sqrt{\rho_0 + \rho_1 a^3} \sqrt{1 + \frac{1}{2\lambda a^3} (\rho_0 + \rho_1 a^3)}$$ \hspace{1cm} (25)

$$M(T) = \frac{\kappa \rho_0}{6} (\rho_1 + \rho_0 a^3) \left[ 1 + \frac{1}{2\lambda a^3} (\rho_1 + \rho_0 a^3) \right]$$ \hspace{1cm} (26)

We note that at $T = T_0$ the space-time will be singular and the limiting value of different parameters are

$$t \to t_0: a \to 0, \rho_{DM} \to \infty, \rho = \rho_1, \dot{R}(T) \to -\infty, M(T) \to \infty$$

$$t \to \infty: a \to \infty, \rho_{DM} \to 0, \rho = \rho_1, \dot{R}(T) \to -\infty, M(T) \to \infty$$ \hspace{1cm} (27)

As the energy density for dark energy is constant so it behaves as cosmological constant. Also near the singularity ($a = 0$) the dark matter dominates over the dark energy. In fact, from the equation (22) we note that for general $\varepsilon < 0$, the dark matter approaches infinity faster than dark energy as the collapsing process approaches to singularity (see fig.4).

**STEP II :** $Q \neq 0$ i.e., Non Vanishing Interaction Between Dark Matter And Dark Energy:

Here according to Cai and Wang [17] let us assume

$$\frac{\rho}{\rho_M} = Ba^{3n}$$  \hspace{1cm} (28)
where $B(> 0)$ and $n$ are arbitrary constants. Defining, $\rho_T = \rho_M + \rho$, the total energy density, we have from conservation equations (7) and (8)

$$
\begin{align*}
\rho &= \frac{B \rho_2 a^{3n}}{a^3(1 + Ba^{3n})} \\
\rho_M &= \frac{\rho_2}{a^3(1 + Ba^{3n})} \\
\rho_t &= \frac{\rho_2}{[a^3(1 + Ba^{3n})^2]} 
\end{align*}
$$

(29)

with $\rho_2$, an arbitrary positive constant. Then the field equation (4) can be written in the integral form as

$$
\int \frac{(1 + Bz^n)\dot{z} dz}{\sqrt{z(1 + Bz^n)} + \frac{\rho_2}{2\lambda}} = -\sqrt{3\kappa \rho_2} t + \Lambda_1 
$$

(30)

where $z = a^3$ and $\Lambda_1$ is an integration constant. As before the integral can only be evaluated for $\varepsilon = -1$ and $n = 1$ and the solution is

$$
\begin{align*}
a^3 &= \frac{1}{12} [D^2 \cosh^2 \mu (t_0 - t) + F] - 1 \\
\rho_M &= \frac{\rho_2}{a^3} \\
\rho &= B \rho_2 \\
\dot{R}(T) &= -\frac{B \rho_2 a^{3n}}{a^3 a^3} \left[ \frac{\rho_2}{2\lambda} + \frac{a^3}{2D^2} \right] \\
M(T) &= \frac{M_0}{a^3} (1 + Ba^3) (\frac{\rho_2}{2\lambda} + \frac{a^3}{2D^2}) 
\end{align*}
$$

(31)

where

$$
D = \sqrt{\frac{2\lambda}{2\lambda + B \rho_2}}, F = \cosh^{-1}(\frac{1}{D}), R_2 = r_\Sigma \sqrt{\frac{\rho_2}{3}}, M_0 = \frac{1}{2} r_\Sigma R_2^2, \mu = \sqrt{\frac{3\kappa \rho_2 B(2\lambda + B \rho_2)}{8\lambda}}
$$

From equation (29), the limiting form of the energy densities are

$$
t \to t_0 : a \to 0, \rho \sim a^{3(n-1)}, \rho_M \sim a^{-3}, \rho_T \sim a^{-3} \\
t \to -\infty : a \to \infty, \rho \sim a^{-3(1+\varepsilon)}, \rho_M \sim a^{-3(1+\varepsilon+n)}, \rho_T \sim a^{-3(1+\varepsilon)}
$$

Thus we see that $a = 0$ is always a singularity of space-time while $a = \infty$ is also singular for $\varepsilon > -1$. Also for $n \geq 1$ near the singularity ($a = 0$) the dark energy becomes insignificant and the total matter is governed by dark matter. For $0 < n < 1$, though $\rho$ and $\rho_M$ both approaches to infinity near the singularity, but still the dark matter dominates over the dark energy.

Further, using this solution the expression for the interaction is given by (from equation (7))

$$
Q(t) = -3B(\varepsilon + n) \left( \frac{\dot{a}}{a} \right) \frac{a^{3n} \rho_t}{(1 + Ba^{3n})^2} 
$$

(32)

which approaches to $-\infty$ at the singularity.
V. DISCUSSION AND CONCLUDING REMARKS

The gravitational collapse of a homogeneous spherical cloud consists of dark matter and dark energy has been considered here in the background of brane world. To study the role of dark matter and dark energy in the final outcome of gravitational collapse, both of them have been considered separately as well as in combination with or without interaction.

When the collapsing fluid is in the form of dark energy with equation of state $p = \varepsilon \rho$, then the collapse will lead to a black hole for $\varepsilon > -\frac{4}{3}$ while the singularity will be naked for $-1 < \varepsilon < -\frac{2}{3}$. However for $\varepsilon = -\frac{4}{3}$ the end state of collapse will depend on the magnitude of $\frac{r_{\text{H}}}{2} \sqrt{\frac{2}{3}}$. (If this quantity is greater than unity then there will be black hole otherwise there will be a naked singularity). Then the time of formation of apparent horizon is given by

$$t_{\text{ah}} = t_0 + \frac{1}{a_1} \left( A_1 - \frac{1}{2R_1} \right)$$

This result is distinct from Einstein gravity where the critical role is played by $\varepsilon = -\frac{1}{3}$ and there is no restriction on the parameters involved.

Then the section IV a combination of dark matter and dark energy is considered as collapsing fluid. Due to the complicated form of the equations involved solutions are obtained only for particular value of the parameters. In step I solutions are obtained for $Q = 0$ (without interaction) for $\varepsilon = -1$ and solutions for $Q \neq 0$ (with interaction) are presented in step II for $\varepsilon = -1$ and $n = 1$. In both cases, the solutions are similar. At the time of singularity the collapsing cloud is clearly trapped. But for regular initial data, the initial instant of collapse should be chosen appropriately. As it is noted that $\dot{R} \to -\infty$ both at $t \to -\infty$ and $t = t_0$, so $\dot{R}$ has a finite maximum value (at the instant $t = t_{\text{max}}(< t_0)$). If this maximum value of $\dot{R}$ (which depends on the brane tension $\lambda$) is chosen to be less than unity (in magnitude) then any instant around $t_{\text{max}}$ may be taken as the starting epoch of the collapsing process.

Thus we conclude that the brane scenario has an effect on the collapsing process. Finally, if the brane tension approaches to infinity then one recovers Einstein gravity and the above results reduce to those of Cai and Wang.

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