Faddeev calculation of $^6_{\Lambda\Lambda}$He using $SU_6$ quark-model baryon-baryon interactions

Y. Fujiwara,¹ M. Kohno,² K. Miyagawa,³ Y. Suzuki,⁴ and J.-M. Sparenberg⁵

¹Department of Physics, Kyoto University, Kyoto 606-8502, Japan
²Physics Division, Kyushu Dental College, Kitakyushu 803-8580, Japan
³Department of Applied Physics, Okayama Science University, Okayama 700-0005, Japan
⁴Department of Physics, Niigata University, Niigata 950-2181, Japan
⁵TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

(Dated: June 18, 2018)

Quark-model hyperon-nucleon and hyperon-hyperon interactions by the Kyoto-Niigata group are applied to the two-Λ plus α system in a new three-cluster Faddeev formalism using two-cluster resonating-group method kernels. The model fss2 gives a reasonable two-Λ separation energy $\Delta B_{AA} = 1.41$ MeV, which is consistent with the recent empirical value, $\Delta B_{exp} = 1.01 \pm 0.20$ MeV, deduced from the Nagara event. Some important effects that are not taken into account in the present calculation are discussed.

PACS numbers: 21.45.+v, 13.75.Ev, 21.80.+a, 12.39.Jh

A new discovery of the double Λ hypernuclei, $^6_{\Lambda\Lambda}$He, called the Nagara event [1] has provided an invaluable source of information for the strength of the ΛΛ interaction. Before this discovery, it had been believed that the two-Λ separation energy measured by $\Delta B_{AA}$ was $B_{AA}(^6_{\Lambda\Lambda}He) = B_{AA}(^4He) = 2E(He) - E(He)$ was fairly large, $\Delta B_{AA} \sim 4.3$ MeV, which implies that the ΛΛ interaction is more attractive than the corresponding $^1S_0 \Lambda N$ interaction. It was argued in Ref. [2] that the proper treatment of the ΛΛ coupling in the $^1S_0$ coupling model is important to reproduce this $\Delta B_{AA}$ value in the coupled-channel AGS formalism using the $^1S_0 \Lambda N$ interaction of the Nijmegen model D. It is now clear that the Nijmegen model D is not appropriate to describe the double Λ hypernuclei. Almost unique identification of the previous decay processes involved in the Nagara event enforced it necessary to reanalyze the previous three events of the double Λ hypernuclei $^6_{\Lambda\Lambda}$He [2, 3, 4] and led to the conclusion that the ΛΛ interaction is actually weakly attractive, under the assumption of possible involvement of excited states in the intermediate processes. The $\Delta B_{AA}$ value deduced from the Nagara event is $1.01 \pm 0.20$ MeV [5].

Based on this experimental information, several calculations have been carried out to determine the strength of the ΛΛ interaction precisely and to find an appropriate interaction model mainly among the meson-theoretical Nijmegen models. For example, Flikhaev, Gal, and Suslov [6] performed detailed Faddeev calculations using the $^1S_0$ cluster model with many phenomenological ΛΛ interactions and the so-called Isle Λ interaction with a repulsion core. They used the S-wave Λ interaction and $^1S_0$ potentials for all the allowed partial waves. Since $^6_{\Lambda\Lambda}$He is essentially an S-wave dominant system, their approximation is legitimate. Nevertheless, the Nijmegen soft-core model NSC97e [7] was found to have too weak ΛΛ interaction, corresponding to $\Delta B_{AA} \sim 0.66$ MeV [8].

We have discussed in Ref. [8] that the cluster model calculation with the Λ cluster needs a special care with an important rearrangement effect originating mainly from the starting energy dependence of the G-matrix interaction, when we consider composite-particle interactions starting from bare baryon-baryon interactions. For example, the energy loss of the interaction term in $^4He$ due to the added Λ particle is estimated to be $2.5 - 2.9$ MeV in the model-independent way. This effect plays a major role to explain the well-known overbinding phenomena of the $^4He$. This effect is renormalized in usual ΛN potentials by fitting the Δ separation energy $B_{Δ}(^4He) = 3.12 \pm 0.02$ MeV. In the ΛN system, however, there still remains an unrenormalizable effect mainly originating from the starting energy dependence of the ΔN interaction, which is found to be a repulsive effect of about 1 MeV [8]. As the result, the S-state matrix element of the ΔN interaction is not $-\Delta B_{ΔN} \sim -1$ MeV, but should be larger than $-2$ MeV. From this argument, we can conclude that the $^1S_0$ ΛΔ interaction of NSC97e is by far too weak, and there is no meson-theoretical models available to explain the Nagara event.

The purpose of this brief report is to show the extent how our quark-model baryon-baryon interaction fss2 [9, 10] can give a consistent description of the ΔN and ΛΔ interactions with the available experimental data of light single- and double-Λ hypernuclei. The model fss2 describes all the available nucleon-nucleon ($NN$) and hyperon-hyperon ($YN$) scattering data, by incorporating the effective meson-exchange potentials at the quark level. It is now extended to the arbitrary two-baryon systems of the octet baryons without introducing any extra parameters [10]. The strangeness $S = -2$ sector, in particular, involves several important aspects of the baryon-baryon interactions. First it contains the ΔN interaction, whose knowledge is essential to understand the binding mechanism of the double-Λ hypernuclei. The second is that the isospin $T = 0$ system corresponds to the so-called H-particle channel, in which a strong attraction is

*Electronic address: fujiwara@ruby.scphys.kyoto-u.ac.jp

[1] M. Kohno, [2] Y. Suzuki, [3] Y. Fujiwara, [4] T. Inoue, [5] M. Kohno, [6] Y. Fujita, [7] Y. Suzuki, [8] Y. Suzuki, [9] M. Kohno, [10] Y. Suzuki.
expected from the color-magnetic interaction of the quark model. The third is the existence of the Pauli-forbidden state at the quark level, with the $SU_3$ quantum number $(11)_3$. The existence of such Pauli-forbidden state usually implies a strong repulsion in some particular channels. It is therefore important to deal with the effect of the Pauli principle properly in the quark-model baryon-baryon interactions. Here we carry out Faddeev calculations of the $\Lambda\Lambda$ system, by directly using the quark-model baryon-baryon interactions in the strangeness $S = -2$ sector, and show that the $\Lambda\Lambda$ interaction of fss2 is consistent with the Nagara event after several corrections which are not easily incorporated in the present calculation.

The three-cluster Faddeev formalism used here is recently developed for general three-cluster systems interacting via two-cluster resonating-group method (RGM) kernels \[1\], \[2\]. A nice point of this formalism is that the underlying $NN$, $YN$, and hyperon-hyperon ($YY$) interactions are more directly related to the structure of the hypernuclei than the models assuming simple two-cluster potentials. The reliability of this formalism is already confirmed in several systems; i.e., the three-nucleon bound state \[3\], the hypertriton \[4\], the $3\alpha$ and $\Lambda\alpha$ systems \[5\]. The last application involves an effective range Gaussian potential generated from the phase-shift analysis of the $\Lambda\Lambda$ systems \[1\]. The three-nucleon ($NN$) component of the redundancy-free $T$-matrices is the $\Lambda\Lambda$ component of the redundancy-free $\Lambda\Lambda$-$\Sigma\Sigma$-$\Xi\Xi$ $T$-matrices in the specific channel with the strangeness $S = -2$ and the isospin $T = 0$. These $T$-matrices are generated from the RGM kernel of the $YY$ interaction, $V_{YY}^{RGM}(\epsilon_{YY})$, by solving the full coupled-channel Lippmann-Schwinger equation in the momentum space. The elimination of the Pauli-forbidden state with the $SU_3$ quantum number $(11)_3$ is automatically taken care of, simply by using the “RGM” $T$-matrix $T_{\Lambda\Lambda}(\epsilon_{\Lambda\Lambda})$ according to the prescription given in Ref. \[1\]. The total wave function $\Psi$ is orthogonal to this Pauli-forbidden state, if we formulate a full coupled-channel Faddeev equation for the $\Lambda\Lambda$-$\Xi\Xi$-$\Sigma\Sigma$ system. Such a calculation is not feasible for the time being, since we also need the $\Lambda\alpha$, $\Xi\alpha$, and $\Sigma\alpha$ interactions. Here we simply use the $\Lambda\alpha$ component of the redundancy-free $T$-matrix. The energy dependence involved in the RGM kernel and the $\tilde{T}$-matrix is treated self-consistently by calculating the matrix elements of the (quark-model) $\Lambda\Lambda$ Hamiltonian as

$$\epsilon_{\Lambda\Lambda} = \langle \Psi | h_{\Lambda\Lambda} + V_{\Lambda\Lambda}^{RGM}(\epsilon_{\Lambda\Lambda}) | \Psi \rangle .$$

The detailed prescription for the energy dependence of the RGM kernel and the Pauli-forbidden state in the quark-model baryon-baryon interaction is given in Ref. \[1\]. A Faddeev formalism involving two identical particles (or clusters) are spelled out in Ref. \[1\].

Since we are interested in the $J^p = 0^-$ ground state with the isospin $T = 0$, the channel specification scheme of the $\Lambda\alpha$ system is very simple. It becomes even simpler if we introduce no non-central forces since the $\Lambda\alpha$ interaction is known to involve a very weak spin-orbit force. In the ($\Lambda\Lambda$)-$\alpha$ channel, the exchange symmetry of the two $\Lambda$'s requires $(-)^{\lambda+S} = 1$, where $\lambda$ and $S$ are the relative orbital angular-momentum and spin values of the two-$\Lambda$ subsystem. The possible two-$\Lambda$ states are therefore $1^+_{\Lambda\Lambda}$ ($\lambda$=even) for $S = 0$ and $3^+_{\Lambda\Lambda}$ ($\lambda$=odd) for $S = 1$. If we neglect non-central forces, the spin value $S$ and the total orbital angular-momentum quantum number $L$ are good quantum numbers, and only $1^S_0$, $1^D_2$, $1^G_4$, · · · states of the $\Lambda\Lambda$ interaction contribute in the ground state with $L = S = 0$. Note that the orbital angular-momentum of the $\alpha$ particle, $\ell$, is equal to $\lambda$ since $J = 0$. Similarly, in the ($\Lambda\alpha$)-$\Lambda$ channel, the relative angular-momentum of the $\Lambda\alpha$ subsystem, $\ell_1$, is equal to the orbital angular momentum of the spectator $\Lambda$, $\ell_2$, because of the parity conservation and the possible spin value, $S = 0$ or $1$. These simplifications are of course the result of the channel truncation that we do not include the coupling to the possible $\Sigma\Xi\alpha$ and $\Sigma\Sigma\alpha$ configurations, in the present $\Lambda\Lambda\alpha$ model space. All the partial waves up to $\lambda_{\text{max}} = \ell_{\text{max}} = 6$-6 are included for $\lambda = \ell$ and $\ell_1 = \ell_2$. The momentum discretization points with $n_1 - n_2 - n_3 = 10$-10-5 in the previous notation \[1\] are used for solving the Faddeev equation. This ensures 1 keV accuracy.

Table \[0\] shows the $\Delta B_{\Lambda\Lambda}$ values in MeV, predicted by various combinations of the $\Delta N$ and $\Lambda\Lambda$ interactions.
TABLE I: Comparison of $\Delta B_{\Lambda\Lambda}$ values in MeV, predicted by various $\Lambda\Lambda$ interactions and $V_{\Lambda N}$ potentials. The $\Lambda\Lambda$ potential $V_{\Lambda\Lambda}$ (Hiyama) is the three-range Gaussian potential used in Ref. [17], and $V_{\Lambda N}(SB)$ the two-range Gaussian potential given in Eq. (3). FSS and fss2 use the $\Lambda\Lambda$ RGM $T$-matrix in the free space, with $\varepsilon_{\Lambda\Lambda}$ being the $\Lambda\Lambda$ expectation value determined self-consistently: $\Delta B_{\Lambda\Lambda}^{\text{exp}} = 1.01 \pm 0.20$ MeV [3].

| $V_{\Lambda\Lambda}$ | Hiyama | FSS | fss2 | SB |
|---------------------|--------|-----|------|----|
| $\Delta B_{\Lambda\Lambda}$ | $\Delta B_{\Lambda\Lambda}$ | $\varepsilon_{\Lambda\Lambda}$ | $\Delta B_{\Lambda\Lambda}$ | $\varepsilon_{\Lambda\Lambda}$ | $\Delta B_{\Lambda\Lambda}$ |
| SB | 3.618 | 3.657 | 5.124 | 1.413 | 5.938 | 1.910 |
| NS | 3.548 | 3.630 | 5.151 | 1.366 | 5.947 | 1.914 |
| ND | 3.181 | 3.237 | 4.479 | 1.288 | 5.229 | 1.645 |
| NF | 3.208 | 3.305 | 4.622 | 1.271 | 5.407 | 1.713 |
| JA | 3.370 | 3.473 | 4.901 | 1.307 | 5.702 | 1.824 |
| JB | 3.486 | 3.599 | 5.141 | 1.327 | 5.952 | 1.911 |

The results of a simple three-range Gaussian potential, $V_{\Lambda\Lambda}$ (Hiyama), used in Ref. [17] are also shown. We find that this $\Lambda\Lambda$ potential and the RGM $T$-matrix for the old version of our quark-model interaction FSS [13] yield very similar results with the large $\Delta B_{\Lambda\Lambda}$ values about 3.6 MeV, since the $\Lambda\Lambda$ phases shifts predicted by these interactions increase up to about 40°. The improved quark model fss2 yields $\Delta B_{\Lambda\Lambda} = 1.41$ MeV. The energy gain due to the expansion of the partial waves from the $S$-wave to the $I$-wave is $35 \sim -50$ keV, depending on the weakly attractive or repulsive nature of the $P$-wave $\Lambda N$ force. In Table I results are also shown for $V_{\Lambda\Lambda}(SB)$, which is a two-range Gaussian potential generated from the $^1S_0$ $\Lambda\Lambda$ phase shift of fss2, by using the supersymmetric inversion method [16]. This potential is given by

$$V_{\Lambda\Lambda}(SB) = -103.9 \exp(-1.176 \ r^2) + 658.2 \exp(-5.936 \ r^2),$$

where $r$ is the relative distance between two $\Lambda$’s in fm and the energy in MeV. This potential reproduces the low-energy behavior of the $\Lambda\Lambda$ phase shift of fss2 quite well, as seen in Fig. 1. We use this for all even partial waves and set the odd components zero by assuming the pure Serber type. [The odd components of the $\Lambda\Lambda$ interaction give no contribution to the present calculation in the $LS$ coupling scheme anyway.] We find that this $\Lambda\Lambda$ potential yields larger $\Delta B_{\Lambda\Lambda}$ values than the fss2 RGM $T$-matrix by 0.36 MeV - 0.58 MeV. We think that this difference of around 0.5 MeV between our fss2 result and the $V_{\Lambda\Lambda}(SB)$ result is probably because we neglected the full coupled-channel effects of the $\Lambda\Lambda$ channel to the $\Xi N\alpha$ and $\Sigma\Sigma\alpha$ channels. In our previous Faddeev calculation for $^3H$ [13], the energy gain due to the increase of the partial waves from the 2-channel ($S$-wave only) to 5-channel ($S + D$ waves) calculations is 0.36 - 0.38 MeV (see Table III). We should keep in mind that in all of these three-cluster calculations the Brueckner rearrangement effect of the $\alpha$-cluster with the magnitude of about 1 MeV (repulsive) is very important [8]. It is also reported in Ref. [13] that the quark Pauli effect between the $\alpha$ cluster and the $\Lambda$ hyperon yields a non-negligible repulsive contribution of 0.1 - 0.2 MeV for the $\Lambda$ separation energy of $^6\Lambda$He, even when a rather compact ($3\sigma$) size of $b \sim 0.6$ fm is assumed as in our quark-model interactions. Taking all of these effects into consideration, we can conclude that the present results by fss2 are in good agreement with the experimental value, $\Delta B_{\Lambda\Lambda}^{\text{exp}} = 1.01 \pm 0.20$ MeV, by the Nagara event [1].

Table II lists the energy decomposition to kinetic- and potential-energy contributions for the $\Lambda\Lambda\Xi N$-$\Sigma\Sigma$ coupled-channel system with the isospin $I = 0$. The single-channel phase shift of the $\Lambda\Lambda$ scattering, predicted by the SB potential, is also shown in circles.

![FIG. 1: $^1S_0$ phase shifts, predicted by fss2, in the $\Lambda\Lambda\Xi N$-$\Sigma\Sigma$ coupled-channel system with the isospin $I = 0$. The single-channel phase shift of the $\Lambda\Lambda$ scattering, predicted by the SB potential, is also shown in circles.](image-url)
TABLE II: Decomposition of the ground-state energy of $^{6}\text{He}$ ($E$), and the $\Lambda\Lambda$ ($\varepsilon_{\Lambda\Lambda}$) and $\Lambda\alpha$ ($\varepsilon_{\Lambda\alpha}$) expectation values to the kinetic- and potential-energy contributions. The SB $\Lambda\Lambda$ force is used. The unit is in MeV. The experimental value is $E^{\text{exp}} = -7.25 \pm 0.19$ MeV [1].

| $V_{\Lambda\Lambda}$ | kinetic + potential = total |
|---------------------|-------------------------------|
| Hiyama              | $E = 21.328 - 31.186 = -9.858$ |
|                     | $\varepsilon_{2\Lambda} = 11.869 - 6.779 = 5.089$ |
|                     | $\varepsilon_{\Lambda\alpha} = 10.388 - 12.203 = -1.815$ |
| FSS                 | $E = 20.179 - 30.076 = -9.897$ |
|                     | $\varepsilon_{2\Lambda} = 10.800 - 5.676 = 5.124$ |
|                     | $\varepsilon_{\Lambda\alpha} = 9.927 - 12.200 = -2.274$ |
| fss2                | $E = 17.111 - 24.764 = -7.653$ |
|                     | $\varepsilon_{2\Lambda} = 8.567 - 2.628 = 5.938$ |
|                     | $\varepsilon_{\Lambda\alpha} = 8.553 - 11.068 = -2.515$ |
| SB                  | $E = 17.439 - 25.589 = -8.150$ |
|                     | $\varepsilon_{2\Lambda} = 8.483 - 2.399 = 6.083$ |
|                     | $\varepsilon_{\Lambda\alpha} = 8.774 - 11.595 = -2.821$ |

in the $T$-matrix calculations. The results for the fss2 $\Lambda\Lambda$ and SB $\Lambda\Lambda$ model are $\Delta B_{\Lambda\Lambda} = 1.141$ MeV for the $\Lambda\Lambda$ single-channel calculation and 1.454 MeV for the $\Lambda\Lambda$-$\Sigma N$ double-channel calculation. The energy gain by the full coupled-channel $T$-matrix calculation is only 0.27 MeV. However, such truncation of channels spoils the exact treatment of the Pauli principle, and the RGM $T$-matrix does not satisfy the orthogonality condition to the Pauli forbidden (11)$_s$ state.

Summarizing this work, we have applied the quark-model $YN$ and $YY$ interactions, fss2 [8, 10] and FSS [18], to the Faddeev calculation of the $\Lambda\Lambda\alpha$ system for $^{6}\text{He}$, in the new three-cluster Faddeev formalism using two-cluster RGM kernels. The $\Lambda\alpha$ $T$-matrix is generated from the $\Lambda\alpha$ effective force, which is derived from the $^3\Sigma_0$ and $^3\Sigma_1$ $\Lambda\alpha$ phase shifts of fss2 by the supersymmetric inversion method [16]. With a single adjustable parameter, this $\Lambda\alpha$ force gives a realistic description of the $^3\Lambda\alpha\alpha$ and $^3\Lambda\alpha\alpha$ systems [8]. The $\Lambda\Lambda$ interaction of the quark-model baryon-baryon interactions is therefore reliably examined by solving the RGM $T$-matrix in the $\Lambda\Lambda$-$\Sigma N$-$\Sigma \Sigma$ coupled-channel formalism, and by using it in the coupled-channel Faddeev equation. Here we have used only $\Lambda\alpha$ configuration and obtained $\Delta B_{\Lambda\Lambda} = 1.41$ MeV for fss2, as a measure of the the two-$\Lambda$ separation energy. A simple Gaussian $\Lambda\alpha$ potential, reproducing the $^3\Sigma_0$ $\Lambda\alpha$ phase shift of fss2, yields $\Delta B_{\Lambda\Lambda} = 1.91$ MeV. Considering some repulsive effects from the Brueckner rearrangement of the $\alpha$-cluster ($\sim 1$ MeV) [8] and the quark Pauli principle between the $\alpha$ cluster and the $\Lambda$ hyperon ($\sim 0.1$ - 0.2 MeV) [19], we can conclude that the present results by fss2 are in good agreement with the experimental value, $\Delta B_{\exp} = 1.01 \pm 0.20$ MeV, deduced from the Nagara event [1]. Together with previous several Faddeev calculations, we have found that the model fss2 yields a realistic description of many three-body systems, including the three-nucleon bound state $^{12}\Lambda\alpha\alpha$, the hypertriton $^{14}F$, $^{13}\Lambda\alpha\alpha$, and $^{14}\Lambda\alpha\alpha$.

Acknowledgments

This work was supported by Grants-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (JSPS) (Nos. 15540270, 15540284, and 15540292).

[1] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).
[2] S. B. Carr, I. R. Afnan, and B. F. Gibson, Nucl. Phys. A625, 143 (1997).
[3] D. J. Prowse, Phys. Rev. Lett. 17, 782 (1966).
[4] M. Danysz et al., Phys. Rev. Lett. 11, 29 (1963).
[5] S. Aoki et al., Prog. Theor. Phys. 85, 1287 (1991).
[6] I. N. Filikhin, A. Gal, and V. M. Suslov, Phys. Rev. C 68, 024002 (2003).
[7] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
[8] M. Kohno, Y. Fujiwara and Y. Akaishi, Phys. Rev. C 68, 034302 (2003).
[9] Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto, and Y. Suzuki, Phys. Rev. C 65, 014002 (2002).
[10] Y. Fujiwara, M. Kohno, C. Nakamoto, and Y. Suzuki, Phys. Rev. C 64, 054001 (2001).
[11] Y. Fujiwara, H. Nemura, Y. Suzuki, K. Miyagawa, and M. Kohno, Prog. Theor. Phys. 107, 745 (2002).
[12] Y. Fujiwara, Y. Suzuki, K. Miyagawa, M. Kohno, and H. Nemura, Prog. Theor. Phys. 107, 993 (2002).
[13] Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, and H. Nemura, Phys. Rev. C 66, 021001(R) (2002).
[14] Y. Fujiwara, K. Miyagawa, M. Kohno, and Y. Suzuki, KUNS-1907, nucl-th/0404010 submitted to Phys. Rev. C.
[15] Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, D. Baye, and J.-M. Sparenberg, KUNS-1910, nucl-th/0404071 submitted to Phys. Rev. C.
[16] J.-M. Sparenberg and D. Baye, Phys. Rev. C 55, 2175 (1997).
[17] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Prog. Theor. Phys. 97, 881 (1997); Phys. Rev. C 66, 024007 (2002).
[18] Y. Fujiwara, C. Nakamoto, and Y. Suzuki, Phys. Rev. Lett. 76, 2242 (1996); Phys. Rev. C 54, 2180 (1996).
[19] Y. Suzuki and H. Nemura, Prog. Theor. Phys. 102, 203 (1999).
[20] K. S. Myint, S. Shimura, and Y. Akaishi, Eur. Phys. J. A 16, 21 (2003).