The radiative corrections of the electroweak chiral Lagrangian
with renormalization group equations

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We show the computational procedure of the renormalization of the
electroweak chiral Lagrangian (the 4D Higgsless standard model), and pro-
vide one simplified version of its one-loop renormalization group equations,
which we demonstrate its simplicity and reliability. By analyzing the sol-
lutions of the one-loop renormalization group equations of the electroweak
chiral Lagrangian, we study the parameter space of the precision test pa-
rameters at ultraviolet cutoff with the current low energy experimental con-
straints. We find that the region of the permitted parameter space can be
greatly amplified (1 to 2 order) by the radiative corrections of those unde-
termined anomalous couplings.

05.10.Cc, 11.10.Hi, 12.15.Lk

The effective Lagrangian method is a bottom-to-up and model-independent approach
to understand experimental data [1–3]. The electroweak chiral Lagrangian (EWCL)
method (it was also called as the non-linear gauged sigma model or 4D Higgsless standard
model in some references) [4–6] is an effective field method which is expected to describe
the electroweak physics without a Higgs in the standard model, since till now there is no
direct evidence for the existence of such a scalar field. In principle, the EWCL with low
energy fermions can explain all the experiment data [7] below the energy scale $\mu = 200$
GeV or so. We can also use this theoretical framework to understand physics beyond
$\mu = 200$ GeV or so before we will find new particles and new resonances in the future
experiments. However, from the fact that the unitarity of the scattering amplitude of
the longitudinal vector bosons (according to the equivalent theorem [8], this corresponds
to the Goldstone scatterings) will be violated, we can deduce that new physics must be
below $4\pi v$, a few TeV [9].

Before a new resonance is found at experiments, with its anomalous couplings as pa-
rameters the EWCL can describe all possible effects of the new physics to the electroweak
bosonic sector, either the strong couplings and weak couplings origin of electroweak sym-

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metry breaking, or other possible scenarios (like extra dimensional Higgsless model [10]). There is a vast number of literature about the phenomenology of the EWCL [11] (For a review please refer [12]).

In this article, using the dimensional regularization and modified minimal subtraction scheme, we show the main steps of computational procedure of the renormalization of the EWCL and provide a simplified version of renormalization group equations (RGEs) which is easy to use. We also study the radiative corrections of anomalous couplings by solving the RGEs. We analyze the effects of anomalous couplings to the running of weak coupling constant \( g \), the \( U(1) \) coupling \( g' \), the vacuum expectation value \( v \), and the quadratic anomalous couplings.

Traditional wisdom might regard that the radiative corrections of anomalous couplings should be very small and there is no need to consider them. The argument is that in QCD (B physics, for instance), it is due to large \( g_s \) at low energy region that makes the QCD radiative corrections important. But in the electroweak case, from a few TeV down to 100 GeV or so, the weak coupling constant \( g_W \) is much smaller than \( g_s \). So the electroweak running effects should be small. Furthermore, the traditional power counting from the naive dimensional analysis [13] told us that the anomalous operators belong to \( O(p^4) \), so the radiative corrections of them should belong to \( O(p^6) \), therefore they should be of two loops contributions of \( O(p^2) \) operators and are supposed to be tiny.

However, we would like to point out several facts for the EWCL that this wisdom has neglected. 1) The naive dimensional analysis only counts the dimension of the operators and only qualitatively estimate the importance of the operators, but in a realistic case, the importance of a certain operator depends not only on its momentum dimension power but also on the magnitude of its effective couplings. For instance, although \( \mathcal{L}_0 \) in the EWCL has momentum power \( O(p^2) \), due to the smallness of its coupling \( \beta \), normally we know its effect to physics processes would be less important as \( \mathcal{L}_{WZ} \), of which the effective coupling is of \( O(1) \). 2) As matter of fact, some of \( \alpha_i \) might be of order \( O(1) \): 2.a) this phenomenological assumption is not contradicted to the current experiments [14–17]; 2.b) In principle, the strong electroweak theories [20] do not forbidden that some of anomalous couplings are \( O(1) \) (Of course, in the naive technicolor model [21], all anomalous couplings are of the same order and can not be of \( O(1) \)). Some of \( \alpha_i \) might reach \( O(1) \), as in non-standard Higgs model [19], for instance, since \( \alpha_i \) can receive large tree level contributions. Therefore, those operators in \( O(p^4) \) with large couplings should be more important than or at least equal to those operators in \( O(p^2) \) with tiny couplings. 3) In order to keep the generality and universality of the effective field theory method, before we really know the underlying theories, we treat all the relevant and marginal operators as of the same
importance. This phenomenological assumption is necessary as it is complimentary to the pure theoretical argument and estimate.

However, in order to control and estimate the contributions of the higher loop and higher order operators, we modify the power counting rule by setting the momentum power of $\alpha_i$, the couplings in the anomalous operators, to be $-2$. We will find that the radiative corrections of $\alpha_i$ always appear in the combination as $\alpha_i g_w^2$, $\alpha_i g^2$, and $\alpha_i G^2$, as shown in the RGEs in Eq. (29—42). These terms have the momentum power $O(p^0)$, which is the same as that of the contribution of Goldstones from the $O(p^2)$ operators. This modified power counting rule has been introduced when we considered the $SU(2)$ chiral Lagrangian [22] and works quite well.

Generally speaking, if the anomalous couplings are of $O(1)$, the unitarity of amplitudes of some processes of the theory might be violated, $W_L W_L \rightarrow W_L W_L$ for instance. This imposes the correlation between the magnitude of anomalous couplings and ultraviolet cutoff $\Lambda$: i.e. the higher the ultraviolet cutoff, the smaller the magnitude of the anomalous couplings, if the validity of the effective description should be preserved.

Loop calculations in the EWCL have been started quite a long time ago, and it is well known that the quartic, quadratic, and logarithmic divergences [23] are witnessed. Reference [24] introduces a Higgs field as a regulator. While in the reference [25], the authors used the method of higher covariant derivatives ($O(p^6)$) to regulate and parameterize the quartic and quadratic divergences, and the cutoffs of different types of vector bosons are set to be different. However, in the framework of the effective theory method [2], with the $\overline{MS}$ renormalization scheme and dimensional regularization, we can handle these divergences consistently and systematically. And after regularizing the quartic, quadratic and logarithmic divergence by using the dimensional regularization, anomalous couplings are logarithmically dependent on the universal cutoff.

Several works have analyzed the radiative correction to the precision test parameters [25–27] of anomalous operators by assuming the large anomalous couplings. The formula obtained there are quite complicated. Here we provide the RGE method, which is also not simple. However, for the physics permitted region, we can keep only the linear terms involving the anomalous couplings and the RGE method become very simple. Furthermore, we find that this simplified version of RGEs is quite reliable for the reasonable parameter space of the EWCL.

There are several technical difficulties in the actual calculation procedure: 1) There is a mixing between the photon and Z bosons mixing in the kinetic sector; 2) The number of relevant vertices is large, and how to construct the counter terms is a task; 3) Compared with the $SU(2)$ chiral Lagrangian, the calculation is complicated due to the non-degenerate
masses of A, Z and W bosons (and Goldstone bosons $\xi_Z$ and $\xi^\pm$). To overcome these technical difficulties, we use the path integral method [28], Stueckelberg transformation [29], background field method [30], heat kernel method [31] and short-distance expansion method [32] to extract the divergences. These methods work quite well, as we have demonstrated in the $SU(2)$ case [22]. Below we outline the main computational steps to the renormalization of EWCL and RGEs (Please refer to our long paper for details [33]).

**Step 1:** The EWCL [4–6] can be formulated as

\[
\mathcal{L}_{EW} = \mathcal{L}_{EW}^{p^2} + \mathcal{L}_{EW}^{p^4} + \cdots \tag{1}
\]

\[
\mathcal{L}_{EW}^{p^2} = \mathcal{L}_B, \tag{2}
\]

\[
\mathcal{L}_{EW}^{p^4} = \beta \bar{\mathcal{L}}_0 + \sum_{i=1}^{10} \alpha_i \bar{\mathcal{L}}_i \tag{3}
\]

where

\[
\mathcal{L}_B = -\bar{H}_1 - \bar{H}_2 + \bar{\mathcal{L}}_{WZ}, \tag{4}
\]

\[
\bar{H}_1 = \frac{1}{4g^2} W_{\mu\nu}^a W^{a\mu\nu}, \tag{5}
\]

\[
\bar{H}_2 = \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}, \tag{6}
\]

\[
\bar{\mathcal{L}}_{WZ} = \frac{v^2}{4} tr(V \cdot V), \tag{7}
\]

After using the relations of Lie algebra and the classic equation of motion to eliminate the redundant operators, the complete Lagrangian $\mathcal{L}_{EW}^{p^4}$ without violating discrete $C$, $P$, and $CP$ symmetries includes the following independent operators [4,6]:

\[
\bar{\mathcal{L}}_0 = \frac{v^2}{4} [tr(TV_\mu)]^2,
\]

\[
\bar{\mathcal{L}}_1 = i\frac{1}{2} B_{\mu\nu} tr(TW^{\mu\nu}),
\]

\[
\bar{\mathcal{L}}_2 = i\frac{1}{2} B_{\mu\nu} tr(T[V^\mu, V^\nu]),
\]

\[
\bar{\mathcal{L}}_3 = i tr(W_{\mu\nu}[V^\mu, V^\nu]),
\]

\[
\bar{\mathcal{L}}_4 = [tr(V_\mu V_\nu)]^2,
\]

\[
\bar{\mathcal{L}}_5 = [tr(V_\mu V^\nu)]^2,
\]

\[
\bar{\mathcal{L}}_6 = tr(V_\mu V_\nu) tr(TV^\mu) tr(TV^\nu),
\]

\[
\bar{\mathcal{L}}_7 = tr(V_\mu V^\mu) [tr(TV^\nu)]^2,
\]

\[
\bar{\mathcal{L}}_8 = \frac{1}{4} [tr(TW_{\mu\nu})]^2,
\]
\[ \mathcal{L}_9 = \frac{1}{2} tr(TW_{\mu\nu}) tr(T[V_{\mu}, V_{\nu}]), \]
\[ \mathcal{L}_{10} = [tr(TV_{\mu}) tr(TV_{\nu})]^2. \]  

where the auxiliary variable \( V_{\mu} \) and \( T \) is defined as
\[ V_{\mu} = U^\dagger D_{\mu} U = U^\dagger (\partial_{\mu} - i W^{a}_\mu T^a) U + i B_{\mu} \tau^3, \]  
\[ T = 2 U^\dagger T^3 U = U^\dagger \tau^3 U, \]

with \( \tau^3 \) is the third Pauli matrices. For the sake of convenience to refer, we can divide the parameters in the EWCL into four groups: 1) \( g, g' \) and \( v \), gauge couplings and vacuum expectation value respectively; 2) \( \alpha_1, \alpha_8 \), and \( \beta \), quadratic anomalous couplings; 3) \( \alpha_2, \alpha_3, \) and \( \alpha_9 \), triple anomalous couplings; 4) \( \alpha_4, \alpha_5, \alpha_6, \alpha_7, \) and \( \alpha_9 \), quartic anomalous couplings.

Here we would like to point out that due to the different definition in the covariant differential operator \( D_{\mu} U \) as given in Eq. (9), the signs of triple anomalous couplings is different than those given in [6], while the quadratic and quartic couplings have the same signs. The operators \( \bar{\mathcal{H}}_1, \bar{\mathcal{H}}_2, \) and \( \bar{\mathcal{L}}, i = 1, \cdots, 10 \) contribute the kinetic, trilinear, and quartic interactions. While operators \( \mathcal{L}_{WZ} \) and \( \mathcal{L}_0 \) contribute to the mass terms.

Compared with the canonical definition of the covariant differential operator \( D_{\mu} \), we have absorbed the gauge couplings in the definition of vector fields. The advantage of this definition is that it is much easier to define the counter terms. While the canonical definition (with the scaling transformation \( W(B) \rightarrow g W(g'B) \)) complicates the counter term structure.

**Step 2:** By using the background field method [30], we separate both the vector and Goldstone bosons into the classic and quantum parts. With the help of Stueckelberg transformation [29], the classic Goldstone can be absorbed by redefining the classic vector classic field, therefore does not appear in the computational procedure. At the same time, the gauge invariance is guaranteed at every computational step, due to the fact that the Stueckelberg field is an invariant combination with classic vector and Goldstone bosons under the background gauge transformation.

The Euler-Lagrange equation of motion of the classic fields in mass eigenstates can be deduced as
\[ C_1 \partial_{\nu} A^{\mu\nu} + C_2 \partial_{\nu} Z^{\mu\nu} - e (W^+_{\nu} W^{\mu\nu} - W^{-\nu} W^{+\mu}) - \frac{i}{2} C_5 \partial_{\nu} F^{\mu\nu} \]
\[ + \frac{e}{2} C_7 (Z \cdot W^+ W^{-\mu} + Z \cdot W^+ W^{+\mu} - 2 W^+ \cdot W^+ Z) = 0, \]
\[ C_3 \partial_{\nu} Z^{\mu\nu} + C_4 \partial_{\nu} A^{\mu\nu} - \frac{i}{2} C_6 \partial_{\nu} F^{\mu\nu}_z + \frac{1}{2} C_7 (W^+_{\nu} W^{\mu\nu} - W^{-\nu} W^{+\mu}) - 2 C_8 W^+ \cdot W^- Z^\mu = 0. \]
\[-C_9 Z_\nu (W^{\mu,+} W^{-\nu} + W^{\nu,+} W^{-\mu}) - 4C_{10} Z \cdot Z Z^\mu + \frac{G^2 \rho v^2}{4} Z^\mu = 0, \quad (12)\]

\[d_\nu W^{-\mu} - \frac{i}{2} C_5 W_\nu^- A^{\mu\nu} - \frac{i}{2} C_6 W_\nu^- Z^{\mu\nu} - \frac{i}{2} C_7 d_\nu F^{-\mu\nu} - \frac{i}{2} C_7 Z_\nu W^{-\mu\nu} - C_8 Z \cdot Z W^{-\mu}, \quad (13)\]

Where \( \rho = 1 + 2\beta \). The equation of motions are necessary to eliminate terms \( \partial \cdot Z \) and \( d \cdot W^\pm \) when we consider one-loop corrections.

The general renormalized effective generating functional can be expressed as

\[\Gamma_{\text{eff}}(W, B, U) = \Gamma_{\text{tree}}(W, B, U) + \delta \Gamma(W, B, U) + \Gamma_{\text{loop}}(W, B, U) \quad (14)\]

Where \( \Gamma_{\text{tree}}(W, B, U) = \mathcal{L}_{EW} \) given in Eqs. (1—8). Up to one-loop level, the counter term of the EWCL \( \delta \Gamma(W, B, U) \) can be simply formulated as

\[
\delta \Gamma(W, B, U) = 2 \frac{\delta g}{g} \bar{H}_1 + 2 \frac{\delta g'}{g'} \bar{H}_2 + \frac{v \delta v}{2} tr(V \cdot V) + \frac{\delta \beta v^2 + 2\beta v \delta v}{4} [tr(\mathcal{T} V_\mu)]^2 + \sum_{i=1}^{10} \delta \alpha_i \mathcal{L}^i \quad (15)
\]

Here we have used one of the advantages of the background field method that the renormalization constants of classic vector field can always be set as 1. The underlying reason for this advantage is that there are enough counter terms to absorb all divergence, the renormalization constant of the classic vector field is a redundancy. However, when we go beyond the one-loop level, the renormalization constant for vector field is necessary for the one-loop counter terms of quantum vector fields.

While \( \Gamma_{\text{loop}}(W, B, U) \) can be expressed as

\[
\exp\{\Gamma_{\text{loop}}(W, B, U)\} = \int_{\hat{W} \hat{B} \xi c} \exp\{\int_x \mathcal{L}(W, B, U; \hat{W}, \hat{B}, \xi, c)\}, \quad (16)
\]

which can be explicitly and systematically calculated to any loop level by using Feynman-Dayson expansion. However, up to one-loop level, the calculation becomes much simplified, since only the quadratic terms in \( \mathcal{L}(W, B, U; \hat{W}, \hat{B}, \xi, c) \) contribute. Formally, we can directly perform the Gaussian integral to get the \( \Gamma_{\text{loop}}(W, B, U) \).

**Step 3:** In order to perform the path integral in the mass eigenstates, we rescale the fields (both the classic and quantum fields) with their corresponding coupling constant

\[W \to g W, \quad B \to g' B, \quad (17)\]

and use the following relation between the mass and interaction eigenstates
\[ A = \sin \theta_W W^3, s + \cos \theta_W B^s, \quad Z = -\cos \theta_W W^3, s + \sin \theta_W B^s, \]

\[
W^+ = \frac{1}{\sqrt{2}} (W^{1, s} - iW^{2, s}), \quad W^- = \frac{1}{\sqrt{2}} (W^{1, s} + iW^{2, s}),
\]

where the angle \( \theta_w \) is the Weinberg angle and \( W^{i, s} \) and \( B^s \) are Stuckelberg fields which have combined both the background vector and Goldstone fields.

The rescaling and transformation from the mass and interaction eigenstates do not nontrivially change the measure of functional integral.

We find that to compute in this way the remnant \( U(1) \) gauge invariant is helpful and useful. Due to this \( U(1) \) symmetry, terms with \( A \) can only exist in \( A_{\mu \nu} \) and \( d_\mu = \partial - iQ_e e A_\mu \). Using this fact, we can simplify our treatment to \( A \) to a certain degree. We also use this explicit \( U(1) \) gauge symmetry to check our calculation in each step.

However, we can also perform the rescaling and transformation only for the quantum parts, the results for both treatments should yield the same results.

**Step 4:** After performing the functional Gaussian integral, we arrive at

\[
\exp \{ \Gamma_{\text{loop}}(W, B, U) \} = \exp \{ \Gamma_{\text{loop}}(W^\pm, Z, A) \},
\]

\[
= \int \exp \{- \int_x \mathcal{L}(W^\pm, Z, A; \tilde{W}^\pm, \tilde{Z}, \tilde{A}, \xi^\pm, \xi_Z, c^\pm, c_Z, c_A) \}. \tag{19}
\]

and

\[
\Gamma_{\text{loop}} = \frac{1}{2} \left\{ Tr \ln \Box_V + Tr \ln \Box'_\xi - 2Tr \ln \Box_c \\
+ Tr \ln \left( 1 - \tilde{X}^\mu \Box^{-1}_{V;\mu \nu} \tilde{X}^\nu \Box^{-1}_\xi \right) \right\}
\]

\[
= \frac{1}{2} \left\{ Tr \ln \Box_V + Tr \ln \Box'_\xi - 2Tr \ln \Box_c \\
+ Tr(X^{\alpha \beta} d_\alpha d_\beta \Box^{-1}_\xi) - \frac{1}{2} Tr(X^{\alpha \beta} d_\alpha d_\beta \Box^{-1}_\xi X^{\alpha' \beta'} d_{\alpha'} d_{\beta'} \Box^{-1}_\xi) \\
- Tr(\tilde{X}^\mu \Box^{-1}_{V;\mu \nu} \tilde{X}^\nu \Box^{-1}_\xi) + Tr(\tilde{X}^\mu \Box^{-1}_{V;\mu \nu} \tilde{X}^\nu \Box^{-1}_\xi X^{\alpha \beta} d_\alpha d_\beta \Box^{-1}_\xi) \\
- \frac{1}{2} Tr(\tilde{X}^\mu \Box^{-1}_{V;\mu \nu} \tilde{X}^\nu \Box^{-1}_\xi X^{\alpha \beta} d_\alpha d_\beta \Box^{-1}_\xi) + \cdots \right\}. \tag{20}
\]

Here \( Tr \) is over all Lorentz and group indices and over all coordinate points. Then the contributions of vector bosons, Goldstone bosons, and Ghost, have been compactly expressed. Each of terms in this expression can be expanded and corresponded to a set of gauge independent Feynman diagrams.

**Step 5:** After using the heat kernel and short distance expansion technique \([31, 32]\), we can extract all divergences in the \( \Gamma_{\text{loop}}(W^\pm, Z, A) \) and expressed them in the mass
eigenstate basis. The independent complete operators up to $O(p^4)$ in the mass eigenstate basis are listed as

$$L_{EW} = -\sum_{i=1}^{4} C_i O_i + \sum_{i=5}^{12} C_i O_i - O_{MW} - \rho O_{MZ},$$  \hspace{1cm} (21)$$

$$O_1 = \frac{1}{4} A_{\mu\nu} A^{\mu\nu},$$

$$O_2 = \frac{1}{2} A_{\mu\nu} Z^{\mu\nu},$$

$$O_3 = \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu},$$

$$O_4 = \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu},$$

$$O_5 = \frac{i}{2} A_{\mu\nu} W^{+\mu} W^{-\nu},$$

$$O_6 = \frac{i}{2} Z_{\mu\nu} W^{+\mu} W^{-\nu},$$

$$O_7 = \frac{i}{2} \left( W_{\mu\nu}^+ Z^{\mu} W^{-\nu} - W_{\mu\nu}^- Z^{\mu} W^{+\nu} \right),$$

$$O_8 = Z \cdot Z W^+ \cdot W^-,$$

$$O_9 = Z \cdot W^+ Z \cdot W^-,$$

$$O_{10} = Z \cdot Z Z \cdot Z,$$

$$O_{11} = W^+ \cdot W^- W^+ \cdot W^-,$$

$$O_{12} = W^+ \cdot W^+ W^- \cdot W^-,$$

$$O_{MW} = \frac{v^2}{4} W^+ \cdot W^-,$$

$$O_{MZ} = \frac{v^2}{8} Z \cdot Z,$$  \hspace{1cm} (22)$$

This set of operators is explicitly $U(1)$ gauge invariant, and is the complete operator set up to $O(p^4)$ in mass eigenstates.

Then the $\Gamma^{\text{loop}}(W^\pm, Z, A)$ is expressed as the combination of these independent operators, of which the coefficients are the functions of effective couplings $C_i$: $$\Gamma^{\text{loop}} = \sum_{i=1}^{12} \delta C_i (C_i) O^i.$$  \hspace{1cm} (23)$$

To construct the counter term, we need to restore from the operators in mass eigenstate basis back to the operators in electroweak interaction eigenstate basis. Therefore we use the relation between the operators in the mass and interaction eigenstates, which is given as
Here $H_i$ and $\mathcal{L}_i$ are operators defined in in the canonical vector boson fields $W$ and $B$, and are related with $\bar{H}_i$ and $\bar{\mathcal{L}}_i$ defined in Eqs. (5—8) by the scaling transformation of the fields: $gW \rightarrow W$ and $g'B \rightarrow B$.

These relations demonstrate how we can construct the effective Lagrangian in mass eigenstates and then by using the inverse Stueckelberg transformation to reach EWCL, as prescribed in reference [34].

Using this relation, we obtain $\Gamma^{\text{loop}}(W, B, U; C_i)$, which is expressed in the independent basis of weak interaction eigenstates.

$$O_{Mz} = -\frac{1}{2} \mathcal{L}_0,$$
$$O_{Mw} = -\mathcal{L}_{WZ} + \frac{1}{2} \mathcal{L}_0,$$
$$O_1 = \frac{1}{g^2 G^2} \left(g^4 H_2 + g^2 (\mathcal{L}_1 - \mathcal{L}_2) + g'^2 (-\mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6 - \mathcal{L}_7 + \mathcal{L}_8 - \mathcal{L}_9)\right),$$
$$O_2 = \frac{1}{g G^2 g'} \left\{ g^2 (2g^2 H_2 - \mathcal{L}_1 + \mathcal{L}_2) + g'^2 \left[ \mathcal{L}_1 - \mathcal{L}_2 + 2(\mathcal{L}_4 - \mathcal{L}_5 - \mathcal{L}_6 + \mathcal{L}_7 - \mathcal{L}_8 + \mathcal{L}_9) \right] \right\},$$
$$O_3 = \frac{1}{G^2} \left(g^2 H_2 - \mathcal{L}_1 + \mathcal{L}_2 - \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6 - \mathcal{L}_7 + \mathcal{L}_8 - \mathcal{L}_9\right),$$
$$O_4 = -\frac{1}{g^2 G^4} \left[g^4 (\mathcal{L}_6 - \mathcal{L}_7) + G^4 \mathcal{L}_8 - g^2 G^2 (G^2 H_1 - \mathcal{L}_3 + \mathcal{L}_9)\right],$$
$$O_5 = -\frac{1}{2g^4 Gg} \left[g^2 \mathcal{L}_2 + g'^2 (2\mathcal{L}_4 - 2\mathcal{L}_5 - 2\mathcal{L}_6 + 2\mathcal{L}_7 + \mathcal{L}_9)\right],$$
$$O_6 = \frac{1}{2g^2 G} (-\mathcal{L}_2 + 2\mathcal{L}_4 - 2\mathcal{L}_5 - 2\mathcal{L}_6 + 2\mathcal{L}_7 + \mathcal{L}_9),$$
$$O_7 = -\frac{1}{2g^2 G^3} \left[2g^2 (\mathcal{L}_6 - \mathcal{L}_7) + G^2 (\mathcal{L}_3 - \mathcal{L}_9)\right],$$
$$O_8 = \frac{1}{g^2 G^2} (\mathcal{L}_7 - \mathcal{L}_a),$$
$$O_9 = \frac{1}{g^2 G^2} (\mathcal{L}_6 - \mathcal{L}_a),$$
$$O_{10} = \frac{1}{G^4} (2\mathcal{L}_a),$$
$$O_{11} = \frac{1}{2g^4} (2\mathcal{L}_5 - 2\mathcal{L}_7 + \mathcal{L}_a),$$
$$O_{12} = \frac{1}{2g^4} (4\mathcal{L}_4 - 2\mathcal{L}_5 - 4\mathcal{L}_6 + 2\mathcal{L}_7 + \mathcal{L}_a).$$

(24)
\[ \Gamma^{\text{loop}} = \delta Z_{H_1}(C_i) \bar{H}_1 + \delta Z_{H_2}(C_i) \bar{H}_2 + \delta Z_{\mathcal{L}WZ}(C_i) \bar{\mathcal{L}}_{WZ} + \delta Z_{\mathcal{L}_0}(C_i) \bar{\mathcal{L}}_0 + \sum_{i=1}^{10} \delta Z_{\mathcal{E}_i}(C_i) \bar{\mathcal{E}}^i. \]  

(25)

And the relation between the effective coupling \( C_i \) and the anomalous couplings in the interaction eigenstate basis is given as

\[
C_1 = 1 - \frac{g^2 g'^2}{G^2} (2\alpha_1 + \alpha_8),
\]
\[
C_2 = \frac{gg'}{G^2} \left( \alpha_1 g^2 - \alpha_1 g'^2 + \alpha_8 g^2 \right),
\]
\[
C_3 = 1 - \frac{g^2}{G^2} \left( \alpha_8 g^2 - 2 \alpha_1 g^2 \right),
\]
\[
C_4 = 1,
\]
\[
C_5 = \frac{2gg'}{G} \left( 1 - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_8 + \alpha_9) g^2 \right),
\]
\[
C_6 = -\frac{2g^2}{G} \left( 1 - (\alpha_3 + \alpha_8 + \alpha_9) g^2 + (\alpha_1 + \alpha_2) g'^2 \right),
\]
\[
C_7 = \frac{2g^2}{G} \left( 1 - \alpha_3 G^2 \right),
\]
\[
C_8 = -\frac{g^4}{G^2} + 2\alpha_3 g^4 + (\alpha_5 + \alpha_7) g^2 G^2,
\]
\[
C_9 = \frac{g^4}{G^2} - 2\alpha_3 g^4 + (\alpha_4 + \alpha_6) g^2 G^2,
\]
\[
C_{10} = \frac{G^4}{4} (\alpha_4 + \alpha_5 + 2\alpha_6 + 2\alpha_7 + 2\alpha_8),
\]
\[
C_{11} = -\frac{g^2}{2} + \frac{g^4}{2} (2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_8 + 2\alpha_9),
\]
\[
C_{12} = \frac{g^2}{2} - \frac{g^4}{2} (2\alpha_3 - \alpha_4 + \alpha_8 + 2\alpha_9). \tag{26}
\]

Using this relation, the \( \Gamma^{\text{loop}}(W, B, U; C_i) \) is transformed to be \( \Gamma^{\text{loop}}(W, B, U; \alpha_i) \):

\[ \Gamma^{\text{loop}} = \delta Z_{H_1}(\alpha_i) \bar{H}_1 + \delta Z_{H_2}(\alpha_i) \bar{H}_2 + \delta Z_{\mathcal{L}WZ}(\alpha_i) \bar{\mathcal{L}}_{WZ} + \delta Z_{\mathcal{L}_0}(\alpha_i) \bar{\mathcal{L}}_0 + \sum_{i=1}^{10} \delta Z_{\mathcal{E}_i}(\alpha_i) \bar{\mathcal{E}}^i. \]  

(27)

**Step 6:** Using the \( \Gamma^{\text{loop}}(W, B, U; \alpha_i) \) given in Eq. (27) and the counter structure given in Eq. (15), it is straightforward to construct counter terms \( \delta \Gamma^{\text{loop}}(W, B, U; \alpha_i) \). With the constructed counter term, we arrive at the RGEs, which at one-loop level can be expressed as the general form:

\[ \frac{dc}{dt} = \frac{1}{8\pi^2} \beta_c. \]  

(28)
where the complete $\beta_c$ functions of each coupling of the EWCL are quite complicated (which will be provided in our full paper [33]).

In order to make the RGEs ease to use, we expand the effective couplings $C_i$ and keep only the linear terms of anomalous couplings $\alpha_i$ in the $\beta$ functions, then we get the following simplified version of $\beta$ functions of couplings in the EWCL

$$ \beta_g = g^2 \left\{ \frac{-29}{4} - \frac{\beta}{6} - \alpha_1 g^2 - 4\alpha_8 g^2 + \frac{5\alpha_2 g^2}{6} + \alpha_3 \left( \frac{50g^2}{3} - \frac{3g^2}{2} \right) + \frac{23\alpha_9 g^2}{6} \right\}, \quad (29) $$

$$ \beta_{g'} = g'^2 \left( \frac{1}{12} - \frac{\beta}{3} - 2\alpha_1 g^2 - 3\alpha_2 g^2 + \frac{5\alpha_3 g^2}{3} \right), \quad (30) $$

$$ \beta_{\alpha_1} = \frac{1}{12} + 4\alpha_1 g^2 - \alpha_8 g^2 + \frac{9\alpha_2 g^2}{2} - \frac{37\alpha_3 g^2}{6} - \frac{3\alpha_9 g^2}{2}, \quad (31) $$

$$ \beta_{\alpha_2} = \frac{-1}{24} - \frac{\beta}{6} - \frac{5\alpha_1 g^2}{2} + \alpha_2 \left( \frac{-e^2}{6} - 2g^2 + \frac{5g^2}{4} \right) + \alpha_3 \left( \frac{e^2}{3} - \frac{29g^2}{12} \right) + \alpha_9 \left( \frac{e^2}{6} - \frac{25g^2}{12} \right) $$

$$ + \alpha_4 \left( \frac{3g^2}{4} + \frac{g^2}{2} \right) + \alpha_5 \left( \frac{-3g^2}{2} + g^2 \right) + \frac{\alpha_6 g^2}{2} + \alpha_7 g'^2, \quad (32) $$

$$ \beta_{\alpha_3} = \frac{-1}{24} + \frac{\beta}{6} + \alpha_1 \left( \frac{e^2 - g^2}{4} \right) + \alpha_8 \left( -\frac{e^2}{4} + \frac{5g^2}{4} \right) $$

$$ + \alpha_2 \left( \frac{e^2 - 3g^2}{4} \right) + \alpha_3 \left( \frac{35e^2}{12} + 61g^2 \right) + \alpha_9 \left( \frac{-e^2}{6} - \frac{g^2}{6} \right) $$

$$ + \alpha_4 \left( \frac{-9g^2}{4} + \frac{5g^2}{8} \right) + \alpha_5 \left( \frac{9g^2}{4} - g^2 \right) + \alpha_6 \left( \frac{-9g^2}{4} + \frac{5g^2}{8} \right) + \alpha_7 \left( \frac{9g^2}{2} - \frac{g^2}{4} \right), \quad (33) $$

$$ \beta_{\alpha_4} = \frac{-1}{12} + \alpha_2 \left( \frac{e^2 - g^2}{6} \right) + \alpha_3 \left( \frac{-2e^2}{3} + \frac{43g^2}{6} \right) + \alpha_9 \left( \frac{-e^2}{3} + \frac{17g^2}{2} \right) $$

$$ + \alpha_4 \left( \frac{7g^2}{2} + 4g^2 \right) + \alpha_5 \left( 6g^2 - g^2 \right) + \alpha_6 \left( \frac{7g^2}{2} - \frac{3g^2}{2} \right) - 2\alpha_7 g'^2, \quad (34) $$

$$ \beta_{\alpha_5} = \frac{-1}{24} + \frac{\beta}{2} + \alpha_2 \left( \frac{-e^2}{3} + \frac{2g^2}{3} \right) + \alpha_3 \left( \frac{2e^2}{3} - \frac{37g^2}{6} \right) + \alpha_9 \left( \frac{e^2}{3} - 8g^2 \right) $$

$$ + \alpha_4 \left( \frac{3g^2}{2} - \frac{3g^2}{4} \right) + \alpha_5 \left( -g^2 + \frac{7g^2}{2} \right) + \alpha_6 \left( \frac{-g^2}{4} + \frac{5g^2}{4} \right) + \alpha_7 \left( \frac{7g^2}{2} + \frac{3g^2}{2} \right), \quad (35) $$

$$ \beta_{\alpha_6} = \alpha_2 \left( \frac{-7e^2}{6} + \frac{g^2}{2} - \frac{3e^2g^2}{2G^2} \right) + \alpha_3 \left( -9e^2 + \frac{3e^2g^2}{G^2} \right) + \alpha_9 \left( \frac{-4e^2}{3} - \frac{55g^2}{6} + \frac{3e^2g^2}{2G^2} \right) $$

$$ + \alpha_4 \left( \frac{3e^2}{2} - \frac{7g^2}{4} \right) + \alpha_5 \left( -7e^2 + \frac{7g^2}{2} \right) + \alpha_6 \left( \frac{3e^2}{2} - \frac{7g^2}{4} + \frac{13g^2}{2G^2} \right) $$

$$ + \alpha_7 \left( -7e^2 + 6g^2 + g^2 \right) + \alpha_{10} \left( 7g^2 - g^2 \right), \quad (36) $$

$$ \beta_{\alpha_7} = \frac{-3\beta}{4} + \alpha_2 \left( \frac{7e^2}{6} - \frac{3g^2}{4} + \frac{2e^2g^2}{G^2} \right) + \alpha_3 \left( 9e^2 - \frac{4e^2g^2}{G^2} \right) + \alpha_9 \left( \frac{4e^2}{3} + \frac{107g^2}{12} - \frac{2e^2g^2}{G^2} \right). \quad (37) $$
\[\begin{align*}
+\alpha_4\left(-\frac{9e^2}{2} + \frac{g^2}{2} + \frac{17g'^2}{8}\right) &+ \alpha_5\left(4e^2 - \frac{7g'^2}{4}\right) + \alpha_6\left(-\frac{9e^2}{2} + \frac{19g^2}{8} + \frac{g'^2}{8}\right) \\
+\alpha_7\left(4e^2 - \frac{25g'^2}{4} + \frac{g'^2}{4}\right) &+ 3\alpha_{10}g^2, \\
\beta_{\alpha_8} &= \frac{\beta}{2} + \alpha_1g^2 + 12\alpha_8g'^2 - \frac{5\alpha_2g^2}{6} + \frac{3\alpha_3g'^2}{2} + \frac{31\alpha_9g^2}{6}, \\
\beta_{\alpha_9} &= -\frac{\beta}{2} + \alpha_1\left(-e^2 + \frac{g^2}{4}\right) + \alpha_8\left(e^2 - \frac{15g^2}{4}\right) \\
&+ \frac{\alpha_2g'^2}{3} + \alpha_3\left(-\frac{13e^2}{4} - \frac{g'^2}{2}\right) + \alpha_9\left(\frac{67g^2}{12} + \frac{3g'^2}{2}\right) \\
&- \frac{9\alpha_4g^2}{8} - \frac{3\alpha_5g'^2}{4} + \alpha_6\left(\frac{9g^2}{4} - \frac{9g'^2}{8}\right) + \alpha_7\left(-\frac{9g^2}{2} - \frac{3g'^2}{4}\right), \\
\beta_{\alpha_{10}} &= -\frac{\alpha_2e^2g'^2}{2G^2} + \frac{\alpha_3e^2g'^2}{G^2} + \frac{\alpha_9e^2g'^2}{2G^2} \\
&+ \alpha_4\left(3e^2 - \frac{3g^2}{4} - 2g'^2\right) + \alpha_5\left(3e^2 - 3g'^2\right) + \alpha_6\left(3e^2 - g^2 - \frac{13g'^2}{4}\right) \\
&+ \alpha_7\left(3e^2 - 4g'^2\right) + \alpha_{10}\left(-12g^2 + g'^2\right), \\
\beta_v &= \frac{3g^2}{4} + \frac{3g'^2}{8} + \frac{5\beta g^2}{4} - \frac{11\alpha_{10}g'^2}{4} + \frac{8}{3}g^4 \\
&+ \alpha_2\left(-\frac{5g^2g'^2}{2} + \frac{3g'^4}{4}\right) + \alpha_3\left(g^4 - \frac{5g^2g'^2}{2}\right) + \alpha_9\left(\frac{g^4}{2} - \frac{3g^2g'^2}{4}\right) \\
&+ \alpha_4\left(-\frac{7g^4}{2} - \frac{3g^2g'^2}{4} - \frac{3g'^4}{8}\right) + \alpha_5\left(-\frac{27g^4}{4} - 4g^2g'^2 - 2g'^4\right) \\
&- \alpha_6\left(\frac{8G^4}{3} - 2\alpha_7G^4\right), \\
\beta_\beta &= \frac{-15\beta g^2}{4} - \frac{3g^2}{8} - \frac{3\beta g'^2}{4} + \frac{11\alpha_{10}g'^2}{4} - \frac{3\alpha_{10}g^4}{8} \\
&+ \alpha_2\left(\frac{5g^2g'^2}{2} - \frac{3g'^4}{4}\right) + \alpha_3\left(\frac{5g^2g'^2}{2} + \alpha_9\left(-\frac{g^4}{2} + \frac{3g^2g'^2}{4}\right)\right) \\
&+ \alpha_4\left(-\frac{19g^2g'^2}{4} - \frac{19g'^4}{8}\right) + \alpha_5\left(\frac{3g^2g'^2}{2} - \frac{3g'^4}{4}\right) + \alpha_6\left(-\frac{47g^4}{8} - \frac{41g^2g'^2}{4} - \frac{41g'^4}{8}\right) \\
&+ \alpha_7\left(-\frac{15g^4}{2} - \frac{7g^2g'^2}{4} - \frac{7g'^4}{2}\right) - \frac{11}{2}\alpha_{10}G^4. \\
\end{align*}\]

There are several comments on the $\beta$ functions in order: 1) The $\alpha_i$ always appear with $g^2$, $g'$, $e^2$, and $G^2$. If we count the couplings of operators with mass dimension 4 as $-2$, then the combination of $\alpha_3g^2$ is $O(1)$. 2) Although $\alpha_1$, $\alpha_8$ and $\beta$ belong to the quadratic anomalous, the $\beta$ function of $\beta$ parameter receives the radiative corrections from quartic anomalous couplings while those of $\alpha_1$ and $\alpha_8$ do not. The reason is easy
to understand in Feynman diagram as given in Figure 1. The $\alpha_1$ and $\alpha_8$ can only receive radiative corrections through the first diagram, Fig. 1(a), while $\beta$ can receive radiative corrections from both diagrams, Fig. 1(a) and 1(b). It is Fig. 1(b) that makes the quartic couplings contribute directly to $\beta$ parameter. 3) Due to its large numerical factor, $\alpha_3$ can affect much more than the rest to the running of $g$. Similarly, due to the large numerical factor in $\beta_{\alpha_1}$, $\alpha_2$ and $\alpha_3$ can affect the running of $\alpha_1$ significantly. 4) If we use the naive dimensional analysis from [13] and assume that all anomalous couplings are of the one-loop corrections of $O(p^2)$ operators, then all terms of $\alpha_i$ and $\beta$ in $\beta$ function should belong to two-loop order. At the one-loop level, we can neglect them, then we reach to the previous results obtained in [4, 5]:

\[
\begin{align*}
\beta_g &= \frac{g^3}{2} \left( -\frac{29}{4} \right), \\
\beta_{g'} &= \frac{g'^3}{2} \left( \frac{1}{12} \right), \\
\beta_{\alpha_1} &= \frac{1}{12}, \\
\beta_{\alpha_2} &= -\frac{1}{24}, \\
\beta_{\alpha_3} &= -\frac{1}{24}, \\
\beta_{\alpha_4} &= -\frac{1}{12}, \\
\beta_{\alpha_5} &= -\frac{1}{24}, \\
\beta_{\alpha_6} &= 0, \\
\beta_{\alpha_7} &= 0, \\
\beta_{\alpha_8} &= 0, \\
\beta_{\alpha_9} &= 0, \\
\beta_{\alpha_{10}} &= 0, \\
\beta_v &= v \left( \frac{3g^2}{4} + \frac{3g'^2}{8} \right), \\
\beta_\beta &= -\frac{3g'^2}{8}.
\end{align*}
\]

(43)

To reach this previous result is also one of the foolproof checks for our calculation procedure. This result comes from the contribution of pure Goldstone boson loops. 5) Compared with the simplified version of RGEs we have obtained previously in [35], this version is even simpler. 6) It might be helpful to express the simplified version of RGEs into the linear matrix form (higher order nonlinear terms have been dropped):
\[ \frac{d}{dt}\{\alpha_i\} = \frac{1}{8\pi^2}[\{C_{\alpha_i}\} + \beta_\gamma\{\alpha_i\}], \quad (44) \]

where the \(\{\alpha_i\}^T\) is defined as \(\{\alpha_1, \ldots, \alpha_{10}, \beta\}\), and the constant \(\{C_{\alpha_i}\}^T = \{-1/12, 1/24, 1/24, 1/12, 1/24, 0, 0, 0, 0, 3g'^2/8\}\). With the expression given in Eqs.(29—42), it is straightforward to determine the matrix \(\beta_\gamma\), which just indicates mixings between the anomalous operators.

Below we study some phenomenological applications of the RGEs. We take \(m_Z(m_Z) = 91.187 \text{ GeV}, g(m_Z) = 0.651, g'(m_Z) = 0.357, \text{ and } v(m_Z) = 246.708 \text{ GeV}\) as some of inputs.

We consider the constraints of the precision test parameters to the anomalous couplings (\(\alpha_1, \alpha_8, \text{ and } \beta\)) at the cutoff energy scale. The relations [6] between the quadratic anomalous couplings and the oblique precision test parameter \(S, T\) and \(U\) at \(\mu = m_Z\) are given as

\[
S = -16\pi\alpha_1, \quad (45)
\]
\[
T = \frac{\rho - 1}{\alpha_{em}} = \frac{2\beta}{\alpha_{em}}, \quad (46)
\]
\[
U = -16\alpha_8, \quad (47)
\]

According to the extant experiment results [14–16], we take the current constraints on the precision test parameters as

\[
S = -0.13 \pm 0.10, \quad (48)
\]
\[
T = -0.13 \pm 0.11, \quad (49)
\]
\[
U = 0.22 \pm 0.13. \quad (50)
\]

Therefore the lower energy boundary \(\alpha_1(m_Z), \alpha_8(m_Z), \text{ and } \beta(m_Z)\), is determined as our inputs. However, we would like emphasize that our theoretical framework assumes no Higgs. So these values of precision test parameters, which are obtained by fitting in the standard model with a Higgs, should not be regarded with too much seriousness. However, what we want to show below is the running behavior of \(\alpha_1, \alpha_8, \text{ and } \beta\) at the ultraviolet cutoff \(\Lambda\).

While for other anomalous couplings we take the following variation range as inputs

\[
(\alpha_2(m_Z) \alpha_3(m_Z) \alpha_9(m_Z)) \sim O(0.1),
\]
\[
(\alpha_4(m_Z) \alpha_5(m_Z) \alpha_6(m_Z) \alpha_7(m_Z) \alpha_a(m_Z)) \sim O(1). \quad (51)
\]

These are consistent with the current LEP measurement [14–17].
So with these low energy inputs at $\mu = m_Z$ as inputs, by solving the RGEs, we can study $\alpha_i(\Lambda)$ with the RGEs, which reflect the experimental constraints on the unknown possible underlying theories.

Before the actual numerical analysis, formally, we can express the solutions of $\alpha_i$ from the RGEs as

$$\alpha_i(m_Z) = \alpha_i(\Lambda) + \frac{1}{16\pi^2} \beta_{\alpha_i} \ln \left( \frac{\Lambda}{m_Z} \right)^2. \quad (52)$$

From this formal solution, we know that the low energy value of $\alpha_i(m_Z)$ is related with three factors: its initial value at the matching conditions $\alpha_i(\Lambda)$, which is determined by the underlying theory; $\beta_{\alpha_i}$ functions, which depends on the other anomalous couplings and reflects the contributions of the vector and Goldstone quantum; the ultraviolet cutoff $\Lambda$, which is energy scale where new physics should show up but is unknown to us. We would like to point out two obvious facts about this formal solutions: 1) Due to the fact that $\beta_{\alpha_i}$ is suppressed by loop factor $1/(8\pi^2)$, if $\alpha_i(m_Z)$ is small (say of order one-loop, like $\alpha_1$ and $\alpha_8$), then the radiative corrections are relatively important. 2) Generally speaking, for a specific set of $\alpha_i(m_Z)$, the larger the $\Lambda$, the larger the variation of $\alpha_i(\Lambda)$.

We consider three cases with different ultraviolet cutoffs: case 1, the ultraviolet cutoff is set as $\Lambda_{UV} = 600$ GeV; case 2, $\Lambda_{UV} = 1000$ GeV; and case 3, $\Lambda_{UV} = 3000$ GeV.

Figures (1—3) are devoted for the first group of parameters in EWCL, i.e. gauge couplings ($g$ and $g'$)and vacuum expectation value ($v$). In the Figs. 1(a—c), the correlations between $\alpha_3$ and $g$ is shown. Due to the large factor before $\alpha_3$ in the $\beta$ function of $g$, the $\alpha_3$ can affect the running of $g$. The phenomenological meaning of this fact is that it might provide an alternative way to determine or constrain the anomalous coupling $\alpha_3$ if the running of $g$ can be reliably measured in the future experiments to a precision $10^{-2}$.

While Figs. 2(a—c) show that the running of $g'$ is quite small and has a very weak correlation to $\alpha_3$. Such a difference between $g$ and $g'$ can be traced back to the $\beta$ functions of $g$ and $g'$ given in Eqs. (29—30). The direct physics reason includes that the degree of freedoms of the $U(1)$ representation in EWCL is much fewer than that of the $SU(2)$ one and the coupling of $U(1)$ is only half of that $SU(2)$.

Figs. 3(a—c) is devoted to reveal the correlation between the $\alpha_5$ and $v$, and due to the large numerical factor before $\alpha_5$ in the $\beta$ function of $v$, the running of $v$ can be affected by the value of $\alpha_5$. As the $\alpha_3$ in the case $g$, if the running of $v$ can be measured to a precision of $10^{-2}$, this will help to constrain quartic couplings, like $\alpha_5$.

From Figs. 1(d), 2(d) and 3(d), we know that the deviation of predictions of the simplified RGEs and the complete ones is small under the scanned region specified in Eq. (51).
Figures (4—6) are devoted to the quadratic group. Fig. 4(a—c) shows the correlation of $\alpha_1(m_Z)$ with its value at the UV cutoff. Due to the radiative effect from other anomalous couplings, $\alpha_1(\Lambda)$ can be either positive or negative. The direct reason is the contribution of $\alpha_2$ and $\alpha_3$ terms in the $\beta_{\alpha_1}$, which have relative large numerical factors. With the increase of $\Lambda$ the variation range of $\alpha_1(\Lambda)$ increases. In [15], the mechanism for the change of sign $S$ is solely due to the $\frac{1}{12}$ in the $\beta_{\alpha_1}$, the pure Goldstone contribution; while here we find the reason for the change of the sign is due to the combination of this $\frac{1}{12}$ and the interference of $\alpha_2$ and $\alpha_3$. If the interference is constructive, $\alpha_1(\Lambda)$ can have a quite large value with changed sign; if the interference is destructive, $\alpha_1(\Lambda)$ can keep its sign.

Fig. 5(a—c) shows the correlation of $\alpha_8(m_Z)$ with its value at the UV cutoff. Similar to the $\alpha_1$ case, due to the large contribution from triple couplings, $\alpha_8(\Lambda)$ can be either positive or negative.

The deviation range of $\alpha_1(\Lambda)$ and $\alpha_8(\Lambda)$ from the variation range of $\alpha_1(m_Z)$ and $\alpha_8(m_Z)$ can reach to one order. Fig. 6(a—c) shows the correlation of $\beta(m_Z)$ with its value at the UV cutoff. The value of $\beta(\Lambda)$ can deviate from $\beta(m_Z)$ significantly, and the deviation range from the variation range can reach to two orders.

Why is there such a big difference between $\beta$ and $\alpha_1$ (and $\alpha_8$)? The basic reason is that the $\beta$ can get the direct corrections from the unknown quartic couplings, which might reach $O(1)$; while $\alpha_1$ ($\alpha_8$) can only get the direct corrections from triple anomalous couplings. The correction of quartic couplings to $\alpha_1$ ($\alpha_8$) is indirect via triple anomalous couplings. This explains the difference. One interesting phenomenological consequence about the contributions of direct quartic anomalous couplings to $\beta$ parameter is that, if we can measure the running of $\beta$ to a certain precision, this will serve as an good way to constrain the magnitude of quartic anomalous couplings.

Figs. 4(d) and 5(d) show that the simplified version of RGEs is as good as the complete one. While for Fig. 6(d), when the magnitude of $\beta(\Lambda)$ is large ($|\beta(\Lambda)| > 0.1$), the deviation of the results of the complete RGEs and simplified RGEs becomes large.

In summary, we have shown the main steps and methods of the computational procedure for the renormalization of the EWCL. We arrived at the RGEs. And for the sake of easiness to use, we provide a simplified version, which, as has been demonstrated, is quite reliable for the parameter space we have considered. By using the one loop RGEs of EWCL, we have studied some region of the permitted parameter space of the EWCL at the ultraviolet cutoff by incorporating the current precision test constraints. We have found that due to the radiative corrections from triple anomalous couplings, $\alpha_1(\Lambda)$ and $\alpha_8(\Lambda)$ can have a considerable deviation from $\alpha_1(m_Z)$ and $\alpha_8(m_Z)$ (which can reach one order), and can be either positive or negative. While due to the contributions of quartic
anomalous couplings, $\beta(\Lambda)$ can have quite a large deviation from the $\beta(m_Z)$ (which can reach two order).

**ACKNOWLEDGMENTS**

The author would like to thank Dr. S. Dutta and Prof. K. Hagiwara for helpful discussions. The author is also indebted to Dr. H.J. He, Prof. Y. P. Kuang and Prof. Q. Wang, for some constructive suggestions. The work is supported in part by the Chinese Postdoctoral Science Foundation and the CAS K. C. Wong Postdoctoral Research Award Foundation, and in part by Grant-in-Aid Scientific Research from Ministry of Education, Culture, Science and Technology of Japan, and partially supported by the JSPS fellowship program.

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FIG. 1. Two types of Feynman diagrams for the radiative corrections to quadratic anomalous couplings
FIG. 2. The correlation between the running of gauge coupling constant $g$ and the anomalous coupling $\alpha_3$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1$ TeV; (c) is for case (3) where $\Lambda = 3$ TeV. For these three figures, x axes are the value of $g(600\text{GeV})$, $g(1\text{TeV})$, and $g(3\text{TeV})$, respectively; y axes are the value of $\alpha_3(600\text{GeV})$, $\alpha_3(1\text{TeV})$, and $\alpha_3(3\text{TeV})$, respectively. Fig. (d) is devoted to compare the value of $g(\Lambda)$ and $g^s(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.
FIG. 3. The correlation between the running of gauge coupling constant $g'$ and the anomalous coupling $\alpha_3$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1$ TeV; (c) is for case (3) where $\Lambda = 3$ TeV. For these three figures, x axes are the value of $g'(600\text{GeV})$, $g'(1\text{TeV})$, and $g'(3\text{TeV})$, respectively; y axes are the value of $\alpha_3(600\text{GeV})$, $\alpha_3(1\text{TeV})$, and $\alpha_3(3\text{TeV})$, respectively. Fig. (d) is devoted to compare the value of $g'(\Lambda)$ and $g'^s(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.
FIG. 4. The correlation between the running of vacuum expectation value $v$ and the anomalous coupling $\alpha_5$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1$ TeV; (c) is for case (3) where $\Lambda = 3$ TeV. For these three figures, x axes are the value of $v(600\text{GeV})$, $v(1\text{TeV})$, and $v(3\text{TeV})$, respectively; y axes are the value of $\alpha_5(600\text{GeV})$, $\alpha_5(1\text{TeV})$, and $\alpha_5(3\text{TeV})$, respectively. Fig. (d) is devoted to compare the value of $v(\Lambda)$ and $v^s(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.
FIG. 5. The running behavior of anomalous coupling $\alpha_1$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1 \text{ TeV}$; (c) is for case (3) where $\Lambda = 3 \text{ TeV}$. For these three figures, x axises are the value of $\alpha_1(m_Z)$; y axises are the value of $\alpha_1(600 GeV)$, $\alpha_1(1 \text{ TeV})$, and $\alpha_1(3 \text{ TeV})$, respectively. Fig. (d) is devoted to compare the value of $\alpha_1(\Lambda)$ and $\alpha_1^s(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.
FIG. 6. The running behavior of anomalous coupling $\alpha_8$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1$ TeV; (c) is for case (3) where $\Lambda = 3$ TeV. For these three figures, x axes are the value of $\alpha_8(m_Z)$; y axes are the value of $\alpha_8(600\text{GeV})$, $\alpha_8(1\text{TeV})$, and $\alpha_8(3\text{TeV})$, respectively. Fig. (d) is devoted to compare the value of $\alpha_8(\Lambda)$ and $\alpha_8^s(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.
FIG. 7. The running behavior of anomalous coupling $\beta$, (a) is devoted to case (1) where $\Lambda = 600$ GeV; (b) is devoted to case (2) where $\Lambda = 1$ TeV; (c) is for case (3) where $\Lambda = 3$ TeV. For these three figures, x axises are the value of $\beta(m_Z)$; y axises are the value of $\beta(600GeV)$, $\beta(1TeV)$, and $\beta(3TeV)$, respectively. Fig. (d) is devoted to compare the value of $\beta(\Lambda)$ and $\beta^*(\Lambda)$ by solving the complete version and simplified version of RGEs, respectively; the line $y = x$ is depicted for the sake of contrast.