ABOUT $s - \bar{s}$, $\Delta s - \Delta \bar{s}$ AND $D_d^{K^+ - K^-}$ IN $K^\pm$ PRODUCTION IN SIDIS

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Abstract

We consider semi-inclusive unpolarized DIS for the production of charged Kaons and the different possibilities, both in LO and NLO, to test the conventionally used assumptions $s - \bar{s} = 0$, $\Delta s - \Delta \bar{s} = 0$ and $D_d^{K^+ - K^-} = 0$. The considered tests have the advantage that they do not require any knowledge of the fragmentation functions.

1. Introduction

Inclusive deep inelastic scattering (DIS) gives information about the parton densities (PD) $q + \bar{q}$ and $\Delta q + \Delta \bar{q}$. Analogously, $e^+ e^- \to hX$ gives information about the fragmentation functions (FF) $D_q^{h + h}$. However, the new generation of semi-inclusive DIS (SIDIS) experiments performed with increasing precision and variety during the last years, present a new powerful instrument to reveal in more details the spin and flavour structure of the nucleon. However, as data is still not enough and not precise enough, it has become conventional to make certain reasonable sounding assumptions in analyzing the data. The usually made assumptions are:

$$s(x) = \bar{s}(x), \quad \Delta s(x) = \Delta \bar{s}(x), \quad D_d^{K^+}(z) = D_d^{K^-}(z).$$

In this paper we discuss to what extent these assumptions can be justified and tested experimentally, in both, LO and NLO in QCD. We suggest possible tests for the reliability of the leading order (LO) treatment of the considered processes. The considered tests do not require any knowledge of the (FFs). In more details these results are published in [1].

2. Positivity constraints

Here we discuss what we can learn about the strange quark densities from positivity conditions. If $s_+ (\bar{s}_+)$ and $s_- (\bar{s}_-)$ denote the $s(\bar{s})$-quarks with helicities along and opposite the helicity of the nucleon, the unpolarized and polarized parton densities are defined as follows:

$$s = s_+ + s_-, \quad \bar{s} = \bar{s}_+ + \bar{s}_-, \quad \Delta s = s_+ - s_-, \quad \bar{\Delta s} = \bar{s}_+ - \bar{s}_-. \quad (2)$$

Then from $s_+ \geq 0$ and $\bar{s}_- \geq 0$ the following positivity constraints follow:

$$|s - \bar{s}| \leq s + \bar{s}, \quad |\Delta s \pm \bar{\Delta s}| \leq s + \bar{s}. \quad (3)$$
i.e. all parton densities are constrained only by $s + \bar{s}$, our knowledge of the sum $\Delta s + \Delta \bar{s}$ does not put any additional limits. Note that $s - \bar{s} \leq 0$ and $\Delta s \pm \Delta \bar{s} \leq 0$.

From experiment, it is known with a good accuracy that $s + \bar{s}$ is different from zero only for small $x \lesssim 0.4$. Then (3) implies that only in this same interval, $x \lesssim 0.4$, the combinations $s - \bar{s}$ and $\Delta s \pm \Delta \bar{s}$ can be different from zero. Also, as $\int_0^1 dx (s - \bar{s}) = 0$, it follows that $(s - \bar{s})$ changes sign in $x = [0, 0.4]$.

3. SIDIS $e + N \to e + K^\pm + X$

Further we shall work with the difference cross sections in SIDIS. As shown in [2], the general expression for $K^\pm$ production in SIDIS is:

\[
\hat{\sigma}_p^{K^+ - K^-}(x, z) = \frac{1}{9} [4u_V \otimes D_u + d_V \otimes D_d + (s - \bar{s}) \otimes D_s]^{K^+ - K^-} \otimes \hat{\sigma}_{qq}(\gamma q \to qX) \tag{4}
\]

\[
\hat{\sigma}_n^{K^+ - K^-}(x, z) = \frac{1}{9} [4d_V \otimes D_u + u_V \otimes D_d + (s - \bar{s}) \otimes D_s]^{K^+ - K^-} \otimes \hat{\sigma}_{qq}(\gamma q \to qX). \tag{5}
\]

Here $D_q^{K^+ - K^-} \equiv D_q^{K^+} - D_q^{K^-}$, $\hat{\sigma}_{qq}$ is the perturbatively calculable, hard partonic cross section $q \gamma^* \to q + X$:

\[
\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}_{qq}^{(1)},
\]

normalized so that the LO contribution is $\hat{\sigma}_{qq}^{(0)} = 1$. For simplicity, we use $\hat{\sigma}_N^{K^\pm}$ and $\hat{\sigma}_N^{DIS}$ in which common kinematic factors have been removed [3].

As shown in [3], the advantage of the difference cross sections is that all terms in $\hat{\sigma}_N^{K^+ - K^-}$ are non-singlets both in PD and FF. This implies that 1) gluons do not enter – neither $g(x)$ nor $D_g^b(z)$ – and 2) their $Q^2$-evolution is rather simple.

As $D_s^{K^+ - K^-}$ is a favoured transition and thus expected to be big, eqs. (4) and (5) show that $\sigma_N^{K^+ - K^-}$ are sensitive to the combination $(s - \bar{s})$ which we are interested in. Up to now all analyses of data assume $s = \bar{s}$.

4. $s - \bar{s}$ and $D_d^{K^+ - K^-}$, LO

We consider $(\bar{\sigma}_p + \bar{\sigma}_n)^{K^+ - K^-}$ and $(\bar{\sigma}_p - \bar{\sigma}_n)^{K^+ - K^-}$. In LO we have:

\[
\bar{\sigma}_d^{K^+ - K^-} = (\bar{\sigma}_p + \bar{\sigma}_n)^{K^+ - K^-} = \frac{1}{9} [(u_V + d_V)(4D_u + D_d)^{K^+ - K^-} + 2(s - \bar{s})D_s^{K^+ - K^-}] \tag{7}
\]

\[
(\bar{\sigma}_p - \bar{\sigma}_n)^{K^+ - K^-} = \frac{1}{9} [(u_V - d_V)(4D_u - D_d)^{K^+ - K^-}] \tag{8}
\]

We define the following measurable quantities:

\[
R_+(x, z) = \frac{\sigma_d^{K^+ - K^-}}{u_V + d_V} = (4D_u + D_d)(z) \left[ 1 + \frac{(s - \bar{s})}{2(u_V + d_V)} \left( \frac{D_s}{D_u} \right)^{K^+ - K^-} \right] \tag{9}
\]

and

\[
R_-(x, z) = \frac{(\sigma_p - \sigma_n)^{K^+ - K^-}}{u_V - d_V} = (4D_u - D_d)^{K^+ - K^-}(z) \tag{10}
\]
Note that the $x$-dependence in (9) is induced solely by the difference $s - \bar{s}$, while in $R_-$ there is no $x$-dependence in LO. This result is independent of the FF. Then examining the $x$-dependence of $R_{\pm}(x, z_0)$ at some $z_0$, we can deduce the following:

1) if in some $x$-interval $R_+(x, z_0)$ is independent on $x$ then, we can conclude that $(s - \bar{s}) = 0$ in this $x$-interval. Recall that since $D_s^{K^+ - K^-}$ is a favoured transition ($D_s/D_u)^{K^+ - K^-} > 1$.

2) if $R_-(x, z_0)$ is also independent of $x$ , then this suggests that the LO approximation is reasonable.

3) if $R_+(x, z_0)$ and $R_-(x, z_0)$ are both independent of $x$, and if in addition, $R_+(x, z_0) = R_-(x, z_0), then both s - \bar{s} = 0$ in the considered $x$-interval and $D_s^{K^+ - K^-} (z_0) = 0$.

4) if $R_+(x, z_0)$ and $R_-(x, z_0)$ are both independent of $x$, but they are not equal, $R_+(x, z_0) \neq R_-(x, z_0), we conclude that s - \bar{s} = 0$ in the considered $x$-interval, but $D_s^{K^+ - K^-} (z_0) \neq 0$.

The above results 1) – 4) are independent of our knowledge of the FFs.

5) if $D_s^{K^+} +$ are known at some $z_0$, limits on $s - \bar{s}$ can be obtained. We have:

$$\frac{|(s - \bar{s})}{2(u_V + d_V)} (D_s/D_u)\big|^{K^+ - K^-} (z_0) \leq \frac{\delta r_+}{r_+} (11)$$

where $\delta r_+/r_+$ is the precision of the measurement: $R_+(x, z_0) = r_+(z_0) \pm \delta r_+(z_0)$.

6) if $R_+(x, z)$ is not a function of $z$ only, then NLO corrections are needed, which we consider below.

The above tests for $s - \bar{s} = 0$ and $D_s^{K^+ - K^-} = 0$ can be spoilt either by $s - \bar{s} \neq 0$ and/or $D_s^{K^+ - K^-} \neq 0$, or by NLO corrections, which are both complementary in size. That’s why it is important to formulate tests sensitive to $s - \bar{s} = 0$ and/or $D_s^{K^+ - K^-} = 0$ solely, i.e. to consider NLO.

5. $s - \bar{s}$ and $D_s^{K^+ - K^-}$, NLO

If an NLO treatment is necessary it is still possible to reach some conclusions, though less detailed than in the LO case. We now have:

$$\tilde{\sigma}_d^{K^+ - K^-} = \frac{1}{9} \left[ (u_V + d_V) \otimes (4D_u + D_d)\big|^{K^+ - K^-} + 2(s - \bar{s}) \otimes D_s^{K^+ - K^-} \right] \otimes (1 + \alpha_s C_{qq}) (12)$$

$$\tilde{\sigma}_d^{K^+ - K^-} = \frac{1}{9} (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes (4D_u - D_d)\big|^{K^+ - K^-} (13)$$

If instead of using (12) and (13), we succeed to obtain an acceptable fit for the $x$ and $z$-dependence of both $p - n$ and $p + n$ data with the same fragmentation function $D(z)$:

$$\tilde{\sigma}_d^{K^+ - K^-} \approx \frac{4}{9} (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z), (14)$$

$$\tilde{\sigma}_d^{K^+ - K^-} \approx \frac{4}{9} (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z). (15)$$

than we can conclude that both $s - \bar{s} \approx 0$ and $D_s^{K^+ - K^-} \approx 0$, and that $D(z) = D_s^{K^+ - K^-}$.

Note that for all above tests, both in LO and NLO approximation, we don’t require a knowledge of $D_s^{K^+ - K^-}$.
6. $\Delta s - \overline{\Delta \bar{s}}$ in $K^\pm$ production in SIDIS

Recently the COMPASS collaboration measured [4] the difference asymmetry in SIDIS with longitudinally polarized muons and protons:

$$A_{d}^{h-\bar{h}} = \frac{\Delta \bar{s}}{\bar{s}}. \quad (16)$$

and singled out the polarized valence quarks. Here we draw attention that if the same asymmetry is measured with final Kaons, information on $\Delta s - \overline{\Delta \bar{s}}$ can be obtained:

$$A_{d}^{K^+-K^-}(x, z) \simeq \frac{\Delta u_{V} + \Delta d_{V}}{u_{V} + d_{V}} \left\{ 1 + \frac{\Delta s - \overline{\Delta \bar{s}}}{\Delta u_{V} + \Delta d_{V}} - \frac{s - \bar{s}}{u_{V} + d_{V}} \right\} \left( \frac{D_{s}}{2D_{u}} \right)^{K^+-K^-} \right\} \quad (17)$$

The $z$-dependence of $A_{d}^{K^+-K^-}$ is present only if $\Delta s - \overline{\Delta \bar{s}}$ and/or $s - \bar{s}$ are non-zero. Thus, studying the $z$-dependence of $A_{d}^{K^+-K^-}$ one can obtain information about $\Delta s - \overline{\Delta \bar{s}} \simeq 0$, suppose we already have the information about $s - \bar{s} \simeq 0$, as discussed above.

At the end a few remarks on the measurability of the discussed asymmetries. In general, these are difference asymmetries and high precision measurements are required. In addition, the data should be presented in bins in both $x$ and $z$. Quite recently such binning was done in [5] for the very precise data of the HERMES collaboration on $K^\pm$-production in SIDIS on proton and deuterium. These results show that for $0.350 \leq z \leq 0.450$ and for $0.450 \leq z \leq 0.600$ in the $x$-interval $0.023 \leq x \leq 0.300$ the accuracy of the data allows to form the differences $(\sigma_{d})^{K^+-K^-}$ and $(\sigma_{p} - \sigma_{n})^{K^+-K^-}$ with errors not bigger than 7-13% and 10-15% respectively. Then one can form the ratios $R_+$ and $R_-$ with these precisions and perform the above tests.

Acknowledgements
When this work was finished we understood about [6], where similar questions were treated. This work was supported by a JINR(Dubna)-Bulgaria Collaborative Grant.

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