A mean field approach to model levels of consciousness from EEG recordings

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Abstract

We introduce a mean-field model for analysing the dynamics of human consciousness. In particular, inspired by the Giulio Tononi’s Integrated Information Theory and by the Max Tegmark’s representation of consciousness, we study order-disorder phase transitions on Curie-Weiss models generated by processing EEG signals. The latter have been recorded on healthy individuals undergoing deep sedation. Then, we implement a machine learning tool for classifying mental states using, as input, the critical temperatures computed in the Curie-Weiss models. Results show that, by the proposed method, it is possible to discriminate between states of awareness and states of deep sedation. Besides, we identify a state space for representing the path between mental states, whose dimensions correspond to critical temperatures computed over different frequency bands of the EEG signal. Beyond possible theoretical implications in the study of human consciousness, resulting from our model, we deem relevant to emphasise that the proposed method could be exploited for clinical applications.

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I. INTRODUCTION

Consciousness is one of the most complex and fascinating phenomena in the brain, attracting the interest of a variety of scholars, spanning from neuroscientists to mathematicians, and from physicists to philosophers [1-5]. In addition, consciousness, as well as other complex systems as those we find in biology, social science, finance and artificial intelligence [6-12], has strongly benefited from the introduction of cross-disciplinary approaches. Despite a huge number of investigations, a lot of its aspects and mechanisms still require to be clarified. Given these observations, we focus on the challenge of quantifying consciousness, putting attention on the transition between mental states. So, we introduce a method for generating Curie-Weiss models [13, 14] from EEG signals, and then we analyse its outcomes by a machine learning classifier. As discussed later, although the spirit of this work is mostly theoretical, we conceive a framework that could support clinicians in some relevant tasks, e.g. in calibrating the optimal amount of anaesthetic for patients, and in assessing the cognitive conditions of unresponsive individuals. Nowadays, a number of devices allow studying the brain structure and its dynamics. Usually, the choice of a specific tool reflects both clinical needs and patient conditions. Here, we use recordings obtained by EEG analyses for two main reasons, i.e. its cheaper cost compared to other technologies and its non-invasive nature. At the same time, it is worth to report that the EEG is less advanced than other diagnostic tools, as fMRI, that generate images of higher quality (e.g. higher spatial resolution). Notwithstanding, stimulated by the above reasons, we aim to improve as much as possible the value of the information content of EEG signals, concerning the dynamics of human consciousness. Before moving to the proposed method, we very briefly introduce two seminal works on this topic, the Integrated Information Theory (IIT hereinafter) developed by Giulio Tononi [15] and the Max Tegmark’s manuscript on the physical representation of consciousness as a state of matter [3]. Both works contain ideas an observations that inspired us during our investigation. The IIT is based on the following core concept, that is the human consciousness results from integrated information generated by an ensemble of interacting elements. So, information emerges from collective action, and its content is extremely much richer than that one can obtain just by a simple summation of the individual contributions, like those provided by the elements belonging to the same ensemble. More in general the concept of collective effect pervades the field
of complex systems, as effectively explained by Anderson in ‘More is different’ [16]. Under that light, Tegmark proposed a computational description of consciousness [3], trying to address the IIT by the language of Physics, and developing both a classical and a quantum representation of this phenomenon. From his work [3], we take into account his ‘classical’ description of IIT. The latter is achieved via the Ising model, where a ‘conscious’ regime emerges only within a very restricted range of values. Such restricted range refers to the collective phenomena occurring when a spin system gets close to its critical temperature, and that assumption finds full agreement with the Damasio’s observations [17] on the conditions required for the reaching of homeostasis (i.e. some physical parameters, in the brain, have to be kept within a narrow range of values). Therefore, following ideas and insights of the above-mentioned authors, we propose a method for building a mean-field model from EEG data. Then, we analyse its behaviour by implementing Monte Carlo simulations, whose outcomes are expected to provide the information we need to quantify the transitions between mental states of individuals undergoing deep sedation. In particular, we consider the critical temperature computed in the various realisations of the model, e.g. those achieved on varying the frequency of the EEG signal. Details about the proposed model and the method for classifying mental states are provided in the next section. So now we take the opportunity for highlighting that, despite the increasing interest for modelling brain dynamics by networked approaches (e.g. [18, 20, 22, 29]), the present investigation is based on the modelling and the analysis of the distribution of electrical activity recorded in the scalp. Moreover, for each frequency band of the signal, as δ for 1 – 4 Hz and θ for 4 – 7 Hz, is defined a Curie-Weiss model whose interactions depend on the recorded phase differences across scalp locations. It is worth to note that previous studies showed that the α (8 – 12 Hz) band can be useful for quantifying transitions between mental states (e.g. [28]), as well as other signal components. At the same time, most of these works considered networks generated by avoiding to include ‘weak’ interactions. Notably, to remove weak interactions a threshold needs to be defined, and such practice has received some fair criticisms [30]. Remarkably, to the best of our knowledge, the definition of a suitable threshold is currently based only on rules of thumb. So, it is important to remark that the proposed model does not require to filter out or to cut off weak interactions. Eventually, let us observe that while from a neuroscience perspective the human consciousness might be investigated considering the full set of frequency bands of an EEG signal, those of major interest seem to be the δ, the α, and
the β band. However, due to the influence that has been reported between the propofol, i.e. the drug administered in our individuals during the examination, and the behaviour of the β band component in the EEG signal, we decided to take into account only the δ and α bands. Those interested in a more detailed list of features that can be used for analysing consciousness can see [33]. Summarising, our goal is to quantify consciousness, looking also at potential clinical applications. Notably, the EEG signal, as currently processed, provides some information about the state of consciousness of a patient, but it has some limitations. For instance, assessing the level of unconsciousness, or understanding why some individuals report having been fully aware (even if, obviously, unable to communicate) during surgery, is currently difficult by inspecting only the EEG signal. Therefore, under the assumption that the latter might contain more information than those currently extracted, we propose a method to improve its content (see also [34]). Notably, in mathematical terms, the proposed model can be thought of as a more rich representation of the EEG signal, being mapped to a novel vector space that we define state space of mental states. In relation to that, the Curie-Weiss model represents the tool to generate that space of states, by identifying the critical temperatures of each individual during an examination. Here, while critical temperatures are computed to generate the state space of mental states, they have no meaning with what is occurring in the brain of individuals. It is also worth to remark that our choice of using a model (i.e. the Curie-Weiss) usually adopted to describe collective phenomena, as phase transitions, aims to build a direct link with the IIT framework, where the concept of collective behaviour is central. The remainder of the paper is organised as follows: Section II introduces the proposed model. Section III shows results of numerical simulations. Eventually, Section IV ends the paper providing an overall discussion on this investigation, from its goal to the main outcomes, and on some possible future developments.

II. MODEL

In this section, we describe a framework for classifying mental states, whose variation is represented as a phase transition. In statistical mechanics, the most simple representation of phase transitions is achieved by the Ising model, and the latter has been used in [3] for showing how, in terms of information content, the set of states reachable by that model, at the critical temperature, contains suitable candidates for representing states of consciousness.
Therefore, we aim to evaluate with data (i.e. EEG recordings) whether that theoretical insight can be exploited for quantifying consciousness and performing classification tasks. It is worth to add that the Ising model has been used also by other authors for investigating different dynamics of the brain (see for instance [18, 20, 21, 35–37]). As above mentioned, the EEG signal can be decomposed into frequency bands and different measures can be adopted for its analysis, usually selected according to specific needs. For our purposes, a particularly useful parameter is the wPLI index [29] that quantifies the correlation between pairs of sensors in the scalp. Notably, for each frequency band, the interactions of the resulting Curie-Weiss are computed by scalar multiplication of the wPLI index with the relative power of the signal. It is worth to emphasise that while the wPLI and the power are strongly correlated, we found beneficial to combine them for realising useful mean-field models. In doing so, inspired by the Tegmark’s approach for studying the IIT by a simple physical system, and remaining in the land of the classical physics, we build a Curie-Weiss from EEG recordings assigning a spin to each sensor and computing interactions by the combination of the indexes above mentioned (i.e. wPLI and Power). Thus, starting with randomly assigned values of spins ($\sigma \pm 1$), and quenching interactions [14, 38], we study the dynamics of the system towards equilibrium. Following this method, it is possible to compute a critical temperature for each frequency band of the EEG signal, as $T_c^\alpha$ for the $\alpha$ band. However, since this signal varies over time, node interactions can vary as well. Here, actually, the variations of interactions are expected to be useful for detecting variations of mental states. For instance, as reported in [28], networks built using the wPLI index show variations as individuals undergo sedation and then recover to their original conscious state. To tackle this aspect, the EEG signal is sampled into four different points labelled as C, S, DS, R, representing consciousness, sedation, deep sedation, and recovery, respectively. Mental states C and DS are both classified as states of consciousness, in agreement with [39], while S and R are labelled as transition states. Also, C and DS can be viewed as two equilibrium states (although the deep sedation, i.e. DS, in our individuals has been induced by a drug). Let us now proceed to the formal definition of spin interactions. Recalling that the EEG signal is decomposed into 5 main frequency bands and that we extract 4 samples per recording, for each individual we can generate up to 20 different mean-field models. Since the wPLI quantifies the correlation between pairs of nodes, we indicate with $c_{i,j}^x$ the correlation between sensors $i$ and $j$ in the $x$th frequency band. Accordingly, the interaction
term $J$ reads

$$J^{x}_{i,j}(s) = P^{x}(s) \cdot c^{x}_{i,j}(s)$$

with $s$ sample (or mental state) and $P^{x}(s)$ power of the $x$th band for that specific sample. Thus, the Hamiltonian of the system is

$$H = -\frac{1}{N} \sum_{i,j=1\,\text{and} \, j \neq i}^{N} J_{i,j} \sigma_{i} \sigma_{j}$$

with $N$ number of sensors and $\sigma$ spin assigned to them. While the spin is a quantum property of particles, it finds large utilisation in non-physical models, typically for representing binary features. For instance, in social dynamics the spin can represent a binary opinion [40], in evolutionary game theory a strategy [41, 42], and in neural models it can indicate firing (+1) and resting (−1) states [43]. The proposed model does not define explicitly a connection between the value of the spin and the underlying neural activity since the spin dynamics is implemented only to obtain theoretical insights about the activity distribution recorded in the scalp. In particular, the studying of the order-disorder phase transitions occurring in each model realisation allows computing the set of critical temperatures $T_{c}$ associated to every mental state. Here, since our efforts are directed towards quantifying the human consciousness, we put the attention on the $\alpha$ and $\delta$ bands that, according to previous clinical studies (e.g. [44, 45]), seem to be quite relevant for investigating this complex phenomenon, as well as others as psychotic disorders [46]. The described method allows observing the motion that individuals take in the space of mental states. Such motion is defined along a path on a bidimensional plane, whose axes are $(T^{\alpha}_{c}, T^{\delta}_{c})$. Moreover, since recordings terminate once individuals recover their original cognitive state, the resulting path forms a closed cycle. Then, we implement a Machine Learning tool to exploit the resulting paths, occurring in the mental state space, for classification tasks, as for identifying the correct label of a point (e.g. $DS$). In particular, a classifier able to assign a label to each point has to take, as input, vectors with components $(T^{\alpha}_{c}, T^{\delta}_{c})$. As a huge literature suggests, classifiers can be realised by many algorithms, e.g. neural networks [47]. However, given the small size of our dataset, we implemented a Support Vector Machine (SVM hereinafter) [47]. Summarising, starting from EEG recordings, the proposed model generates bidimensional vectors, whose entries correspond to the critical temperatures computed in the $\alpha$ and $\delta$ frequency bands, state by state. These vectors constitute the input of an SVM implemented to discriminate between the two states of consciousness $C$ and $DS$. 

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III. RESULTS

The proposed model has been tested with a dataset of EEG signals obtained by recording 8 healthy volunteers undergoing sedation induced by propofol. Each recording started with individuals in the conscious state and terminated after their complete recovery. Four main mental states can be identified: awareness, sedation, deep sedation, and recovery, and for each of them, one sample is extracted from the recording. The resulting amount of samples, available for all 5 frequency bands, allows generating 20 Curie-Weiss configurations per individual. However, for the reasons reported above, only the $\alpha$ and $\delta$ bands are considered, therefore the number of configurations in this investigation is limited to 8. A model configuration has specific values of spin interactions. Let us remind that spins are the mathematical representation of the sensors in the scalp, however their value (i.e. $\pm 1$) is randomly assigned as below described. Instead, the interactions are computed by Eq. 1. Before proceeding showing results of the simulation, we compare the average value of interactions obtained by using the above equation (i.e. Eq. 1) with those (average) values that can be achieved by using the spectral power ($P$) and by the wPLI ($C$), individually. This comparison is performed over the 4 mental states and the results have been averaged over all individuals. Results are shown in Figure 1 which reports also the related standard error of the mean, i.e. $SEM = \frac{\sigma}{\sqrt{N}}$ with $\sigma$ standard deviation and $N$ number of individuals. A quick inspection of Figure 1 suggests that our method enhances in good extent the difference between the states $C$ and $DS$, embedding the contribution of the spectral power and that of the wPLI index. Let us highlight that our goal is now to extract information related to the mental states of individuals by processing the resulting Curie-Weiss models. So, the next step is focused on the identification of the critical temperature for the different configurations of the Curie-Weiss model. This process allows us to build a ‘state space’ of mental states, where we can quantify, and also visualise, the path followed by individuals across the clinical examination. Then, the paths of mental states are used to train a simple machine learning tool whose goal is to classify conscious states of our individuals.
FIG. 1. Average value of the interaction terms, over 4 different mental states, in Curie-Weiss built using three different approaches. In particular, the interactions have been defined by using (a) the power spectrum \( \langle P \rangle \), (b) the wPLI index, i.e. the correlation \( \langle C \rangle \) between pairs of sensors, and (c) the scalar product of the power with wPLI index (i.e. \( \langle P \cdot C \rangle \)). The legend indicates the considered frequency bands: red line for the \( \delta \) and the green line for the \( \alpha \), and the related diamonds and squares represent the average value for each label. The error bars have been calculated using the standard error of the mean (SEM). Then, semi-transparent points (i.e. red stars for \( \delta \) and green circles for \( \alpha \)) represent the single individuals.

A. Mental States I: Building the State Space

All numerical simulations have been performed on Curie-Weiss configuration related to every single individual, i.e. spin interactions have not been averaged as (instead) it has been done for the analysis shown in Figure 1. While spin interactions \( J \) are quenched, we study the dynamics of spins, that at the beginning of each simulation are randomly set to \( \pm 1 \). Notably, simulations are implemented for studying order-disorder phase transitions, and more specifically for identifying the critical temperature of each configuration. For that purpose, a useful parameter is the absolute value of the average magnetization of the system.
The latter strongly depends on the system temperature $T$, despite its definition does not include the temperature explicitly, and it reads

$$\langle M \rangle = \sum_{i}^{N} \frac{|\sigma_{i}|}{N}$$  \hspace{1cm} (3)

It is worth to observe that beyond identifying the critical temperature $T_{c}$ for each case, it is possible to assess if $T_{c} \approx \langle J \rangle$. The latter, as further explained later, can be a relevant feature for designing clinical applications based on the proposed model. Figure 2 reports the results obtained on one individual, randomly chosen among those available. To assess whether a temperature is 'critical', the variance of $M$ (indicated as $\sigma^{2}$) is analysed in function of the inverse temperature $\beta$ since the highest value of $\sigma^{2}$ is reached at $T = T_{c}$ —see inset of Figure 2. Then, once computed the critical temperature for all considered cases, we focus

FIG. 2. Average magnetization in function of the system temperature. A pictorial of a Curie-Weiss model is shown close to the main line, while the inset shows the variance of $M$ in function of the inverse temperature $\beta$ (on a semi-logarithmic scale).
on the numerical difference between these values and the average interaction term (i.e. $\langle J \rangle$) of each Curie-Weiss realisation. In doing so, we found that approximating the $T_c$ with the $\langle J \rangle$ gives small errors, limited to the 12.5% of $T_c$. Finally, results of the mean-field model are shown in Figure 3. Notably, plotting the values of the critical temperatures computed in the band $\delta$ ($T_c^\delta$) versus those computed in the band $\alpha$ ($T_c^\alpha$) allows us to visualise the path from the initial point $C$ to the point $DS$, and that of return. So, we have now a state space

FIG. 3. Diagram $\langle T_c^\alpha \rangle, \langle T_c^\delta \rangle$, with an inset showing the four points computed on a single individual (randomly chosen).

that contains 'mental paths', whose evolution (or motion) can be further analysed.

**B. Mental States II: Classification**

Each path obtained by the previous method represents an individual, then we aim to use this new representation for implementing a Machine Learning tool devised for classifying mental states. In our investigation, paths are composed of 4 points in bi-dimensional state space. Therefore, we can identify a vector, whose entries are the critical temperatures
computed in the bands $\alpha$ and $\delta$, for each mental state. The goal of this analysis is to assess whether these vectors are useful for generating a confident boundary that separates different mental states. In particular, for the sake of simplicity, we try to build a classifier able to discriminate between the state $C$ and state $DS$. Due to the small size of the dataset, we decided to use an SVM implemented using a kernel based radial basis function (as commonly adopted in classification tasks). The outcomes are shown in Figure 4, where the axes refer to the two critical temperatures of each individual (i.e. one per frequency band), not to their average values (as in the main plot of Figure 3). Let us briefly describe the procedure for training and testing the SVM. Notably, given a dataset of 8 individuals, the training has performed on 7 out of them, and the testing on the excluded one. This process was then repeated excluding each time one different individual. In doing so, we can evaluate 8 different testing phase results. Following the above procedure, now a further analysis compares the results that an SVM achieves when fed with vectors obtained by three different approaches. Notably, in all cases, vectors result from a mean-field model.

![Figure 4](image.png)

**FIG. 4.** Results of SVM applied to the EEG dataset on the plane $T_c^\alpha$ vs $T_c^\delta$. Different symbols refer to different individuals, while green indicates the conscious state ($C$) and red that of deep sedation ($DS$). The black line identifies the edge between the two classes, i.e. $C$ and $DS$ in the mental state space.
(with the method above described), but its interactions can be defined by Eq. 1 or by the power spectrum and the wPLI, individually. This comparison, shown in Figure 5, is quite relevant because we want to evaluate in which extent the utilisation of Eq. 1 provides an actual benefit. Remarkably, we found that our method produced the smallest error rate in

FIG. 5. Comparison between the results of SVM model build with data coming from three different methods: a) Using spectral power; b) Using wPLI; c) Critical temperatures obtained by using Eq. 1. Different symbols refer to different individuals. The green colour indicates the conscious state (C) and red one that of deep sedation (DS). The black line identifies the edge between the two classes, i.e. C and DS in the mental state space.

the task of classifying mental states —see Figure 6. In particular, the SVM trained and tested with our method did only one error, over 8 tests (i.e. 12%), while those trained with data coming from the two other methods did more errors (i.e. 33% and 25% by using power spectrum and wPLI, respectively). Before to conclude this section, we deem important to mention that actual measures might be used to compare outcomes of different models, as
FIG. 6. SVM error rate computed by data coming from three different methods: Critical temperatures ($T_c$) obtained by Eq. 1, Spectral power, and wPLI. Note: the lesser the better.

the AUC. At the same time in this investigation, considering the number of samples, we found beneficial to evaluate the error rate.

IV. DISCUSSION AND CONCLUSION

In this manuscript, we propose a method for quantifying human consciousness and classifying mental states using EEG signals (see also [33, 48]). In particular, we introduce a mean-field model of the distribution of electrical activity in the brain, whose outcomes are used for training a Machine Learning classifier. Inspired by the Tegmark’s work [3] about the IIT, we study order-disorder phase transitions occurring in Curie-Weiss models whose interactions depend on the phase differences across scalp locations. That analysis allows computing the critical temperature achieved for different configuration of the model, i.e. on varying the mental state and the considered frequency band of the signal. Here, the interactions between spins are computed by the scalar product of the power spectrum with the wPLI index, for each specific band. The benefits coming from this choice are reported in Figure 5 and Figure 6. Remarkably, the critical temperatures, computed by means of numerical simulations, allow defining a state space where we can observe the path of mental states.
It is worth to mention that, according to Giulio Tononi [39], consciousness emerges also during dreamlike phases of the sleep. Therefore, considering previous investigations stating that some individuals reported dreamlike activity, during an induced deep sedation [49], both awareness and deep sedation in principle should be considered as conscious states (see also [50]). The path between awareness and deep sedation shows, in the middle, the transition states (S and R). So, summarising, we have two conscious states and two transition states. It is worth to clarify that we are not using the formal meaning of ‘transition state’, as usually adopted in stochastic models (e.g. the voter model [51]), otherwise also the ‘deep sedation’ state would be defined a transition state since it lasts only for the duration of the drug effect. At the same time, these considerations could be extended further, since for instance states of coma could be properly classified as absorbing states, and so on. Hence, our definitions here have not that level of formality, despite we find interesting to investigate more on this. So, once defined mental state space, we use an SVM for tracing the boundary between states $C$ and $DS$. We remind that the dataset for performing the investigation has been obtained with 8 individuals wearing EEG sensors. Then, the EEG recordings have been used for building the mean-field model (i.e. Curie-Weiss) and generating a training dataset. The actual choice of the frequency bands, i.e. $\alpha$ and $\delta$, actually depends also on the clinical settings of examinations (e.g. the utilisation of propofol for inducing the sedation). Results suggest that the critical temperatures are useful to perform classification tasks, discriminating between the two conscious states. Therefore, thinking about the potential use of the proposed model, at clinical level, we conceive a framework composed of two elements: one devised for computing the average interaction term in different bands, that as we proven approximates the critical temperature in the related Curie-Weiss configuration, and the other based on an SVM, or on another Machine Learning algorithm. The fact that the average interaction can be approximated by the critical temperature, in principle, is not too surprising. Notably, the critical temperature of the 2D Ising model is $T_c = 1$ for $J = 1$. However, it has been worth, for the reasons above described, to confirm that hypothesis. Beyond the theoretical analysis, which requires further investigations to confirm our achievements, the framework that we conceive could support, after appropriate testing, clinicians in different scenarios. For instance, it could be useful for defining the optimal amount of drug for sedating a patient, or for classifying the level of unconsciousness of unresponsive patients. Also, it is interesting to observe that the mean-field model realised
by EEG recordings might be conceptually related to the ’classical’ Tegmark’s description of consciousness (i.e. that based on the Ising model). Notably, it would be useful to evaluate how to obtain only one model, across the different bands, and to study its behaviour at different temperatures. Furthermore, we highlight the possibility to extend further this work trying to improve the connection with the IIT framework, in order to develop mathematical tools able to analyse the human consciousness from a perspective supported by the Tononi’s insights. We then conclude emphasising that our results indicate that the EEG signal might be further exploited both for obtaining a deeper understanding of human consciousness, and for implementing novel tools to support clinicians in many complex and critical activities.

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