Possibility of Microturbulence Diagnostics in a Magnetically Confined Plasma Using Multiple Scattering Effects

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Abstract

The idea of new diagnostics method for the small-scale irregular structures of magnetically confined plasma is suggested in the present paper. The method can be based on measurements of intensity attenuation of the normal sounding waves. Anomalous attenuation arises due to multiple scattering effects investigated earlier for ionospheric radio propagation. It has been shown that multiple scattering regime can realize in a tokamak plasma. Calculations of normal sounding wave anomalous attenuation in a tokamak plasma have been carried out. This quantity is large enough to be registered experimentally.

1 Introduction

Anomalously large level of energy and particle transport is one of the main problems in the magnetic confinement fusion research. The transport is thought to be enhanced by small scale plasma turbulence [1-3]. Therefore, the determination of the microturbulent fluctuations properties is necessary for understanding and improvement of plasma confinement.

At present, there are several basic methods of microturbulence diagnostics: Langmuir probes, heavy ion beam probes, scattering, beam emission spectroscopy, electron cyclotron emission, reflectometry and some others. Each of these techniques has limited field of application, merits and demerits [4]. Joint usage of different methods allows to obtain more valuable and more accurate information about microturbulence properties. The application of the existing methods at large devices of the future (such as ITER) requires additional studies and probably some of them will not be possible or will become more difficult there. Therefore, the development of new diagnostic methods is an actual problem.

In this paper it is suggested the idea of using of electromagnetic wave multiple scattering effects for diagnostics of spatial spectrum of small scale electron density fluctuations in a magnetically confined plasma. Unlike existing scattering techniques, instead of the scattered field registration, it is suggested to measure the power of signal reflected from plasma. The theoretical basis of the idea has been taken from the works dealing with the application of the multiple scattering theory to the ionosphere radio wave propagation. The attenuation of the vertical sounding signal is one of the consequences of the theory. This phenomenon is well known and it has been observed in a number of ionospheric experiments [5-7]. However, ionosphere parameters, properties of random irregularities and sounding frequencies strongly differ from those in laboratory plasma. For instance, maximum average electron density in the ionosphere is $10^6 \text{ cm}^{-3}$ and in the magnetically confined plasma is $10^{13} - 10^{14} \text{ cm}^{-3}$. Typical sounding signal frequency for reflection case is about $10\,\text{MHz}$ in the ionosphere and $10 - 100\,\text{GHz}$ in tokamak plasma. Small scale ionospheric irregularities of importance for the scattering process have size of
1 – 10 km across the magnetic field and in the tokamak plasma the irregularities are of 1 – 5 cm in diameter. This strong difference of parameters demands separate study for the case of high-temperature magnetically confined plasma. Such study is presented in this paper.

The analysis and numerical estimates are carried out for the large tokamak (with minor radius $a \geq 1 \, \text{m}$), however, the suggested idea can also be realized for the next generation devices, such as NSTX [8].

We are starting the paper with the description of the suggested experiment scheme, presentation of the electron density irregularity spatial spectrum model and estimation of plasma optical depth. Then, the radiative transfer equation in a randomly irregular magnetized plasma and its approximate analytical solution are presented. The next section is devoted to the analysis of the applicability of this theory to the tokamak plasma. Then, the results of numerical estimations of the anomalous attenuation effect are presented. Finally, the obtained results are discussed and the conclusions are presented.

2 Multiple scattering regime in a tokamak plasma

Scheme of the experiment for density fluctuation study is shown in Fig. 1. It is suggested to measure the intensity of normally reflected signal. If the scattering is multiple, than, according to the existing theory [9], in the case of the normal sounding this can cause considerable attenuation of the reflected signal. The attenuation value is proposed to be used for irregularities study. To find out the conditions, in which the scattering is multiple, it is necessary to carry out the estimation of plasma slab optical thickness. The optical thickness $L$ for scattering process is determined by the expression

$$L = \int \sigma_0 dS,$$

where $\sigma_0$ is full scattering cross-section of a unit volume, $dS$ is the element of nonperturbed ray trajectory. The value of $\sigma_0$ is determined by integration of the differential scattering cross-section $\sigma$ over full solid angle

$$\sigma_0 = \int \frac{\sigma d\Omega}{4\pi}. \quad (2)$$

The calculation will be carried out in the isotropic plasma approximation. It is implied using the expression of the differential scattering cross-section for isotropic plasma. Unlike of the isotropic plasma case, the expression for differential cross-section for magnetized plasma contains dimensionless multiplier (so called "geometrical factor"), depending on the polarization of incident and scattered waves [10]. If the wave frequency is not close to some plasma resonance, then the geometrical factor is about a unit. We will neglect of this cross-section dependence on the polarization and set the geometrical factor equal to unit. Also we will use the refractive index for isotropic plasma.

Utilization of this simplifications is justified by calculation results for the ionosphere. Calculation of the ionosphere optical depth does not lead to considerable quantitative difference from the same calculation in the isotropic plasma approximation [9].

Although tokamak plasma temperature is high ($T \sim 10^8 \, \text{K}$), the electron thermal motion in our problem does not have considerable influence on radiation propagation. It is bound up with the fact that in the case of normal sounding we are interested in waves propagating nearly perpendicularly to the magnetic field. In this case temperature correction for the refractive index is exponentially small [11].
and one can use the refractive index for a cold plasma \((n = 1 - v, \nu = \omega^2/\omega^2_e, \omega_e - \text{plasma frequency}, \omega = 2\pi f, f - \text{wave frequency})\).

The differential scattering cross-section in isotropic plasma takes the form [12]

\[
\sigma (\alpha_0, \beta_0, \alpha, \beta) = \frac{\pi}{2} k_0^4 v^2 F \left[ \bar{k}' (\alpha, \beta) - \bar{k} (\alpha_0, \beta_0) \right],
\]

where \(k_0 = \omega/c, F (\bar{k})\) is density fluctuation spatial spectrum, \(\alpha_0, \beta_0, \alpha, \beta\) - polar and azimuthal angles of wave vectors \(\bar{k}\) and \(\bar{k}'\) of incident and scattered waves, respectively.

For numerical calculation of optical depth it is necessary to concretize the model of irregularity spectrum being based on existing experiment information. According to experimental data, the irregularities in tokamak plasma are strongly stretched along the magnetic field: \(l_{||} \sim 10^2 - 10^3 \, \text{cm}, l_{\perp} \sim 1 - 5 \, \text{cm},\) where \(l_{||}\) and \(l_{\perp}\) are typical irregularity sizes in the parallel and perpendicular to the magnetic field directions. Since longitudinal sizes exceed transverse ones by \(100^{-2}\) to \(1000\) times, we can use for our estimates the approximation of infinitely stretched irregularities with spatial spectrum

\[
F (\bar{k}) = C_A \left( 1 + \frac{\kappa_\perp^2}{\kappa_{0,\perp}} \right)^{-\nu/2} \delta \left( \kappa_\parallel \right),
\]

where \(C_A\) is normalizing constant, \(\kappa_{0,\perp} = 2\pi/l_{0,\perp}\), \(l_{0,\perp}\) is external irregularity scale length, \(\kappa_{\perp}\) and \(\kappa_{\parallel}\) are transverse and longitudinal to the magnetic field components of the irregularity spatial harmonic, \(\nu\) is spectrum index and \(\delta (x)\) is delta-function. Spectrum \(\bar{D}\) dependence on \(\kappa_{\perp}\) for \(\kappa_{\perp} >> \kappa_{0,\perp}\) takes form \(F \sim \kappa_{\perp}^\nu\) what is consistent with the existing experimental data for \(\nu\) from 2 to 3.5 [4,13].

For the spectrum normalization a certain value of the relative irregularities level in a some scale \(R\) can be used. The most natural analog of this physical value in the locally homogeneous random field theory is the structural function [12]

\[
D \left( \bar{R} \right) = \left\langle \left[ \frac{\delta n_e}{n_e} (\bar{r} + \bar{R}) - \frac{\delta n_e}{n_e} (\bar{r}) \right]^2 \right\rangle,
\]

where \(\delta n_e/n_e\) is relative electron density perturbation, \(\langle \rangle\) means ensemble average. To determine the normalizing constant \(C_A\) we will normalize the spectrum \(\bar{D}\), following to the method of [9], by the value of structural function \(\bar{D}\), choosing irregularity scale length \(\bar{R}\) in transverse to the magnetic field direction being corresponded to the interested spectrum interval. An important property of the structural function is that perturbations \(\frac{\delta n_e}{n_e}\) of large spatial scale lengths (with typical size \(l >> R\)) do not have influence on it. The structural function is connected with spatial spectrum by the following expression [12]

\[
D \left( \bar{R} \right) = 2 \int F (\bar{k}) \left( 1 - \cos \bar{k} \bar{R} \right) d^3 \kappa.
\]

Thus, setting relative density perturbation \(\frac{\delta n_e}{n_e} = \delta_R\) in a certain scale \(R\), assuming \(D \left( \bar{R} \right) = \delta_R^2\) and using then formula \(\bar{D}\) we determine the normalizing constant:

\[
C_A = \delta_R^2 \frac{\Gamma (\nu/2)}{2 \pi \kappa_{0,\perp}^2} \left[ \Gamma \left( \frac{\nu - 2}{2} \right) - 2 \left( \frac{R \kappa_{0,\perp}}{2} \right)^{\nu/2} K_{\nu/2} (R \kappa_{0,\perp}) \right]^{-1},
\]

where \(\Gamma (x)\) is gamma-function, \(K (z)\) is McDonald function [14].
Numerical calculation of the optical depth for the ray trajectory with coinciding incident and reflected ray paths (see Fig. 1, where, however, the incident and reflected rays are drawn separately for clearness) was carried out for the linear regular density profile with \( n_e = 10^{14} \text{ cm}^{-3} \) at distance 100 cm from the slab boundary (see Fig. 2). The following values of spectrum parameters were chosen: external irregularity scale length \( l_0 = 5 \text{ cm} \), spectrum index \( \nu = 2.5 \), irregularity level \( \delta_R = 1; 1.5; 2\% \), normalization scale \( R = 1 \text{ cm} \). The magnetic field direction was chosen perpendicular to the ray path. The value of magnetic field is of no importance in used approximation, but its direction determines the irregularity orientation. The calculation results are shown in Fig. 3 in the form of dependence of the optical depth \( L \) on the sounding wave frequency \( f \). Chosen frequency interval corresponds to wave penetration depth from \( z = 50 \text{ cm} \) to \( z = 100 \text{ cm} \). The obtained results show that in the chosen frequency (or reflection level) band the optical depth is considerably more than a unit (unit optical depth corresponds to \( L = 4.3 \text{ dB} \)) for relative irregularity level \( \delta n_e \geq 1\% \).

Thus, for parameters characterised of tokamak irregularities and plasma, the realization of the multiple scattering mode is possible.

### 3 Radiation transfer in a randomly irregular magnetized plasma

In the considered case of normal sounding, the rays situated near the normal ray trajectory give the basic contribution to the reflected signal power. That is why we will assume the plasma layer to be plane stratified. As it has been shown in [15,16], radiation energy transfer with multiple scattering effects accounting for the case of total internal reflection from a plane stratified layer of randomly irregular plasma can be described by the equation of radiation energy balance (REB) in ray tubes. This equation is written in terms of the invariant ray variables (coordinates). The latter ones permit to take into account naturally of regular refraction and give the most simple form to the equation.

The invariant ray variables are introduced by setting the basic plane out of the layer and parallel to it. Let us introduce Cartesian orthogonal coordinates \((x, y, z)\) with \( z \)-axis being directed along the plasma density gradient. Then \( XOY \) plane can be considered as the basic plane. The plasma occupies the region \( z > h_0 \) (see Fig. 4). The coordinates \( \vec{\rho} = (x, y) \) of intersection point of a ray trajectory going out of the layer with the basic plane as well as ray polar \( \theta \) and azimuthal \( \varphi \) arrival angles in this point completely determine ray trajectory within the plasma layer and outside of it. In this meaning they are called "invariant". The equation has the following form

\[
\frac{d}{dz} P(z, \vec{\rho}, \omega) = \int Q(z, \omega, \omega') \left\{ P(z, \vec{\rho} - \vec{\phi}(z, \omega', \omega), \omega') - P(z, \vec{\rho}, \omega) \right\} d\omega ,
\]

where \( \omega = \{\theta, \varphi\} \); \( d\omega = d\theta d\varphi \); \( P \) - radiation energy flux density in a unit solid angle in direction determined by angles \( \theta, \varphi \), at the point \( \vec{\rho} \) of basic plane;

\[
Q(z, \omega, \omega') = \sigma(\omega, \omega') C^{-1}(z, \omega) \sin \theta \left| \frac{d\Omega}{d\Omega'} \right| ,
\]

\( \sigma(\omega, \omega') \) is scattering differential cross-section, \( C^{-1}(z, \omega) \) is cosine of angle between ray trajectory and \( z \)-axis at level \( z \), \( \left| \frac{d\Omega}{d\Omega'} \right| \) is Jacobian of transition from current wave vector angles to invariant ones, \( \vec{\phi}(z, \omega', \omega) \) is vector connecting points of intersection with basic plane of two ray trajectories determined by invariant angles \( \omega \) and \( \omega' \) under the condition that trajectories intersect each other at level \( z \). Using
of the invariant ray coordinates allows one to introduce the small angle scattering approximation in the invariant ray coordinates [9]. This approximation is valid if the most probable difference of invariant angles in each scattering act is small. It must be noticed that the applicability field of this approximation is somewhat more wide than that of the ordinary small angle scattering approximation. In particular, when the scattering occurs near reflection level, small difference of the invariant angles can correspond to considerable difference of wave vector orientation angles. This approximation allows one to obtain an analytical solution of the equation (8). The solution consists of two terms. The first term gives the basic radiation energy flux

\[ \tilde{P}(z, \rho, \omega) = \frac{1}{(2\pi)^2} \int d^2 q P_0(q, \omega) \cdot \exp \left\{ i\tilde{q}\tilde{\rho} + \int_0^z dz' \int d\omega' Q(z', \omega, \omega') \left[ e^{-i\tilde{q}\tilde{\Phi}(z'; \omega, \omega')} - 1 \right] \right\}, \tag{9} \]

where \( P_0(q, \omega) \) is the Fourier transform of the energy flux spatial-angular distribution \( P_0(\rho, \omega) \) of the radiation reflected from the layer in absence of irregularities. This undisturbed flux is determined by the source directivity diagram and the regular layer parameters. The second term (not shown in (9)) has the sense of difference between approximate and exact solutions of the equation (8). It may be shown using asymptotic estimates that under considered approximation the second term is small.

4 Applicability of the radiation transfer theory for tokamak plasma

In the next section the outlined theory will be applied to calculation of the normal sounding signal attenuation in a tokamak plasma layer. But before that, the applicability analysis of used approximations has to be carried out. First of all, for the transfer theory utilizing it is necessary to clarify the applicability of the geometrical optics approximation for the average field. The radiation wavelength (\( \lambda \sim 0.3 \, cm \) for \( f \sim 100 \, GHz \)) must be much less than the average density regular distribution scale length. If the density profile is sufficiently smooth, then this scale length is about the tokamak minor radius (\( a \sim 1 \, m \) for large devices). So, the geometrical optics approximation is valid in this case.

The next assumption to be verified is the validity of the small angle scattering approximation in the invariant ray coordinates. The frequency band of interest is 60 – 90 \( GHz \), what corresponds to wave length band of 0.3 – 0.5 \( cm \), but the minimum irregularity scale length is 1 \( cm \), and it is at least two times larger than the wave length. It means that in the entry region of the plasma layer the usual small angle scattering approximation is valid. In the plasma layer depth, near the wave reflection level, as it was mentioned in the previous section, there exist additional reasons for using the small angle scattering approximation in the invariant ray coordinates.

5 Numerical calculation of the reflected wave attenuation under normal sounding of tokamak plasma layer

For the calculation of the signal attenuation due to multiple scattering we use the solution (9) of the equation (8) in the small angle scattering in the invariant ray coordinates approximation. We assume the antenna to have small sizes and wide...
Let the antenna be situated in the coordinate center, point $O$ (see Fig. 4), at the distance $h_0 = 10 \, \text{cm}$ from the layer boundary.

Then, we take the function $P_0$ in the form

$$P_0(\hat{\rho}, \theta, \varphi) = \tilde{P}_0(\hat{\rho}) \delta \left( -\cos \theta + \cos \theta_0(\hat{\rho}) \right) \delta \left( \varphi - \varphi_0(\hat{\rho}) \right), \quad (10)$$

where $\theta_0(\hat{\rho})$ and $\varphi_0(\hat{\rho})$ are angle coordinates of the ray coming to the point $\hat{\rho}$ when neglecting the scattering.

The calculation is carried out for the same linear density layer (Fig. 2) of a cold isotropic plasma and for the same frequency interval what have been used in section 2 for the optical depth estimates. The function $\Phi$ for the plane isotropic plasma layer can be obtained in analytical form

$$\Phi_x(v, \theta, \varphi, \theta', \varphi') = f(\theta') \cos \varphi' - f(\theta) \cos \varphi, \quad \Phi_y(v, \theta, \varphi, \theta', \varphi') = f(\theta') \sin \varphi' - f(\theta) \sin \varphi, \quad (11)$$

\[
f(\theta) = 2H \sin \theta \left( \cos \theta + \sqrt{n^2 - \sin^2 \theta} \right) + h_0 \tan \theta,
\]

where \( H = \frac{dv}{dz} \).

The intensity of normally reflected signal is obtained using formula (9), after substituting of $\hat{\rho} = 0$ and integration over angle variables. In view of the integrand complicity, the calculation is carried out numerically. The numerical results for various irregularity spectrum parameters are shown in Fig. 5 in a form of dependence of the signal attenuation on frequency.

The attenuation for three different irregularity levels $\delta_R = 1; 1.5; 2\%$ and $l_0 = 10 \, \text{cm}$, $\nu = 2.5$ is illustrated in Fig. 5(a). The first, quite natural conclusion, is that the attenuation increases with the fluctuation amplitude. The results obtained for $l_0 = 3, 5, 10 \, \text{cm}$, $\nu = 2.5$ and $\delta_R = 1.5\%$ are presented in Fig. 5(b). The attenuation slightly increases with the external irregularity scale length. Figure 5(c) shows the results obtained for $\nu = 2.5, 2.75, 3$, $l_0 = 10 \, \text{cm}$ and $\delta_R = 1.5\%$. One can see that the attenuation also grows with increase of the spectrum index $\nu$. Finally, all Figs. 5 (a)-(c) show the attenuation growth with frequency increase. In the chosen frequency band ($60 \, 90 \, \text{GHz}$) total attenuation variation is $2 \, 4 \, \text{dB}$.

The main feature of the presented results, of importance for the present paper basic topic, is that the signal attenuation caused by scattering amounts of $3 \, 7 \, \text{dB}$ and can be measured in experiment.

6 Conclusion

The paper considered the problem of the sounding electromagnetic wave propagation in a magnetically confined plasma with accounting of multiple scattering effects. It was shown that, for typical tokamak plasma and irregularity parameters, the multiple scattering regime can take place. The anomalous attenuation of the normal sounding signal is one of the consequences of this fact. The numerical calculations of the anomalous attenuation were carried out. It was shown that the attenuation value is sufficiently large to be registered by experimental facilities. The attenuation dependences on signal frequency and irregularity spatial spectrum parameters were obtained. Since the anomalous attenuation depends on the spectrum parameters, its measuring can be used for stating and solving of the inverse problem. Thus the aim of the irregularity characteristics determination using observations of the attenuation can be reached. Utilization of this method can broaden the possibilities of the existing microturbulence diagnostic methods.
7 References

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