The Trojan Horse Method as indirect technique in nuclear astrophysics

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Abstract. The Trojan Horse Method is a successful indirect technique for nuclear astrophysics. It allows one to measure astrophysical rearrangement reactions down to the relevant energies, providing a successful alternative path to measure the astrophysical S(E) factor. The basic features will be discussed and some recent results will be presented.

1. Introduction
According to our knowledge, the source of energy that sustains burning stars for millions to billions of years is represented by nuclear reactions which are responsible also for the continual conversion of one element to another inside them [1, 2]. There are many different types of stars and each has its own unique set of nuclear reactions that change in character as the star evolves, in particular towards the end of its life. Over the past forty years nuclear physicists have been trying to measure the rates of the most relevant reactions, but there is still considerable uncertainty about their values. The problem lies in the fact that it is difficult to measure the rates at the thermal energies relevant to stellar interiors. Although these temperatures are high, on the order of hundred million degrees, the corresponding reaction rates are extremely small, making it difficult for them to be measured directly in the laboratory. Indeed, when studying charged particle reactions at sub-Coulomb energies, the Coulomb barrier causes a strong suppression of the cross-section, which drops exponentially with decreasing energy. This complicates a lot the experimental study. In addition, the electron screening effect due to the electrons surrounding the interacting ions prevents one to measure the bare nucleus cross-section. Up to date, the only way to get the ultra-low energy bare nucleus cross-section is by extrapolating the behaviour of the higher energy data. This is done by means of the definition of the astrophysical S(E) factor.
which represents essentially the cross-section free of Coulomb suppression:

$$S_b(E) = E \sigma_b(E) \exp(2\pi\eta)$$

(1)

where $\exp(2\pi\eta)$ is the inverse of the Gamow factor ($\eta$ is the Sommerfeld parameter), which removes the dominant energy dependence of $\sigma_b(E)$, due to the barrier penetrability. However the extrapolation may introduce additional uncertainties due for instance to the presence of unexpected resonances or to high energy tails of subthreshold resonances. Thus, a number of indirect methods (such as Asymptotic Coefficient Method, Coulomb Dissociation, Trojan Horse Method) [3, 4, 5, 6, 7] have been introduced as alternative approaches for determining the bare nucleus S(E) factors of astrophysical interest. They make use of direct reaction mechanisms, such as transfer processes (stripping and pick-up) and quasi-free reactions (knock-out reactions).

In particular the Trojan Horse Method (THM) [8, 9, 10, 11, 12, 13, 14] has been successfully applied several times to rearrangement reactions connected with fundamental astrophysical problems [15, 16]. Here we recall the basic ideas of the THM and we show some recent results.

2. The Trojan Horse Method

The THM selects the quasi-free (QF) contribution of an appropriate three-body reaction $A + a \rightarrow c + C + s$ performed at energies well above the Coulomb barrier to extract the cross section of a charged particle two-body process $A + x \rightarrow c + C$ in the Gamow energy window. This is done with the help of direct theory assuming that the nucleus $a$ is described in terms of the $x \oplus s$ cluster structure. In many applications [11, 10, 12, 13, 14], this assumption is trivially fulfilled: $a = \text{deuteron}$, $x = \text{proton}$, $s = \text{neutron}$. Thanks to the high energy in the $A + a$ entrance channel, the two body interaction can be considered as taking place inside the nuclear field, without experiencing either Coulomb suppression or electron screening effects. The $A + a$ relative motion is compensated for by the $x - s$ binding energy, determining the so called "quasi-free two-body energy" given by

$$E_{q.f.} = E_{Aa} - B_{x-s}$$

(2)

where $E_{Aa}$ represents the beam energy in the center-of-mass system and $B_{x-s}$ is the binding energy for the $x - s$ system. Then, a cutoff in the momentum distribution, which is related to the Fermi motion of $s$ inside the Trojan-horse $a$, fixes the range of energies around the "quasi-free two-body energy" accessible in the astrophysical relevant reaction. In the Impulse Approximation either in Plane Wave or in Distorted Wave (this does not change the energy dependence of the two-body cross section but only its absolute magnitude), the three body-cross section can be factorized as:

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto [KF |\varphi_a(p_{sx})|^2] \left( \frac{d\sigma}{d\Omega_{c.m.}} \right)^{\text{HOES}}$$

(3)

where KF is a kinematical factor containing the final state phase-space factor. It is a function of the masses, momenta and angles of the outgoing particles [11]; $\varphi_a(p_{sx})$ is proportional to the Fourier transform of the radial wave function $\chi(r)$ for the $x - s$ inter-cluster relative motion; $(d\sigma/d\Omega_{c.m.})^{\text{HOES}}$ is the half-off-energy-shell (HOES) differential cross section for the binary reaction at the center of mass energy $E_{c.m.}$ given in post-collision prescription by

$$E_{c.m.} = E_{cC} - Q_{2b}.$$ 

(4)

Here, $Q_{2b}$ is the $Q$-value of the binary reaction and $E_{cC}$ is the relative energy of the outgoing particles $c$ and $C$. 

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In a typical THM experiment the decay products (c and C) of the virtual two-body reaction of interest are detected and identified by means of telescopes (silicon detector or ionization chamber as ΔE step and position sensitive detector as E step) placed at the so called quasi free angles. After the selection of the reaction channel, the most critic point is to disentangle the quasi free mechanism from other reaction mechanisms feeding the same particles in the final state, e.g. sequential decay and direct break-up. An observable which turns out to be very sensitive to the reaction mechanism is the shape of the experimental momentum distribution of the spectator. In order to reconstruct the experimental momentum distribution, the energy sharing method \[17\] is applied for each pair of coincidence QF angles, selecting c-C relative energy windows of 50 to 100 keV. The extracted experimental momentum distribution is then compared with the theoretical one and further data analysis is limited to the data lying in the region where the agreement between the two distributions exists (usually within few tens of MeV/c).

Therefore, it is possible to derive the HOES \((dσ/dΩ)_{\text{HOES}}\) from the three-body coincidence yield by simply inverting eq.5. In a final step, the HOES cross section has to be related to the relevant on-energy-shell (OES) cross section by applying the corresponding corrections. In a heuristic approach this consists essentially in replacing the Coulomb suppression in the HOES cross section, by means of the penetrability factor:

\[
P_l(k_{Ax}R) = \frac{1}{G^2_l(k_{Ax}R) + F^2_l(k_{Ax}R)}
\]

with \(F_l\) and \(G_l\) regular and irregular Coulomb wave functions. It was demonstrated that there is no Coulomb barrier in the two-body amplitude extracted from the TH reaction \[18\] and this is due to the virtuality of particle \(x\). This seems to be the only consequence of off-energy-shell effects as suggested by the agreement between HOES and OES cross-sections for the \(^6\text{Li}(n,\alpha)^3\text{He}\) reaction \[19\].

This procedure does not allow us to extract the absolute value of the two-body cross section. However this is not a real problem since the absolute magnitude can be derived from a scaling to the direct data available at higher energies.

### 3. Results

A list of reactions studied by means of the THM is given in Table 1 together with the relevant references. The abundances of light elements have recently been readdressed as crucial observables for a diverse array of astrophysical topics. Their importance is indeed strongly related to cosmology as well as to stellar structure and evolution. In particular information on Be primordial abundance can provide a powerful test to discriminate between homogeneous and inhomogeneous primordial nucleosynthesis \[24\]. In addition Li, Be and B stellar abundances may help us to understand the stellar structure and the way the mixing processes take place in stellar envelopes \[25\]. THM data for the \(^6\text{Li}(p,\alpha)^3\text{He}\) reaction \[27\] are reported in fig.1, compared with the direct behaviour \[26\]. The disagreement at low energy is a clear evidence that THM data are not affected by the electron screening effect. The solid line in the figure represents a second order polynomial fit \[27\] to the THM data giving a bare nucleus \(S(0)\) value of \(S(0)=3.0\pm0.3\) MeVb.

Using the THM bare-nucleus \(S(E)\) factor, an estimate of the screening potential \(U_e\), was obtained by fitting the screened direct data with the function:

\[
S_s(E) = S_b(E)exp(\pi\eta U_e/E)
\]

where \(U_e\) is left as a free parameter. The fit yields \(U_e=(450\pm100)\) eV, which appear to be about a factor three larger than the adiabatic limit. The \(^3\text{He}(d,p)^4\text{He}\) and \(^2\text{H}(d,p)^3\text{He}\) reactions are involved in the primordial Big Bang nucleosynthesis, i.e. in the production of \(^2\text{H}, \(^3\text{He}, \(^4\text{He}\) and

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[17]:\[\]  
[18]:\[\]  
[19]:\[\]  
[20]:\[\]  
[21]:\[\]  
[22]:\[\]  
[23]:\[\]  
[24]:\[\]  
[25]:\[\]  
[26]:\[\]  
[27]:\[\]
The first time the whole Gamow window (94 THM down to 20 keV (to be compared to 73 keV in direct measurements), thus covering for 19 stars, whose chemical evolution is strongly influenced by

The THM bare nucleus S(E) factor for the limit [28].

cross-sections at low energy, significantly larger than could be accounted for from the adiabatic electron screening effect, this being responsible of the very pronounced enhancement in their baryon density of the universe. Moreover, they can give more hints in order to understand the

7 fluorine abundances in AGB stars are the adopted reaction rates [33, 34].

one, cannot be matched by the models. Among the primary sources of uncertainty in predicting observations provide evidence that fluorine is deeply produced within the interiors of these stars during helium flashes, and its abundance, enhanced by large factors with respect to the solar one, cannot be matched by the models. Among the primary sources of uncertainty in predicting fluorine abundances in AGB stars are the adopted reaction rates [33, 34].

The astrophysical S(E) factor for the

The THM bare nucleus S(E) factor for the limit [28].

complications due to the electron screening. The resulting S(E) factor is reported in fig.3 (red dots). Direct data [35] are also reported as open symbols. Good agreement is found in the energy range explored by direct data. The full line superimposed onto the data is the result of a non standard R-matrix calculation, which has been called Modified R-matrix approach [14]. It accounts for the virtuality of the proton in the entrance channel of the relevant two body reaction by replacing the entry resonance widths with the form factors. The calculations include the parameters associated with the first three resonances in the $^{15}$N + p system at

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Direct reaction & TH reaction & $E_{\text{inc}}$ (MeV) & Q (MeV) & TH$_{\text{nucl}}$ & ref \\
\hline
$^7\text{Li}(p,\alpha)^4\text{He}$ & $^7\text{Li}(d,\alpha \alpha)n$ & 19.22 & 15.122 & d = (p$n$) & [9] \\
$^6\text{Li}(d,\alpha\alpha)^4\text{He}$ & $^6\text{Li}(^3\text{Li},\alpha\alpha)^4\text{He}$ & 5 & 22.372 & $^6\text{Li} = (\alpha\alpha d)$ & [8] \\
$^6\text{Li}(p,\alpha)^3\text{He}$ & $^6\text{Li}(d,^3\text{He})n$ & 14, 25 & 1.795 & d = (p$n$) & [10] \\
$^{11}\text{B}(p,\alpha)^8\text{Be}$ & $^{11}\text{B}(d,^8\text{Be} \alpha)n$ & 27 & 6.36 & d = (p$n$) & [11] \\
$^{10}\text{B}(p,\alpha)^7\text{Be}$ & $^{10}\text{B}(d,^7\text{Be} \alpha)n$ & 27 & -1.079 & d = (p$n$) & [20] \\
$^9\text{Be}(p,\alpha)^6\text{Li}$ & $^9\text{Be}(d,^6\text{Li} \alpha)n$ & 22 & -0.099 & d = (p$n$) & [21] \\
$^2\text{H}(^3\text{He},p)^4\text{He}$ & $^6\text{Li}(^3\text{He},p \alpha)^4\text{He}$ & 5, 6 & 16.88 & $^3\text{He} = (p$n$) & [12] \\
$^2\text{H}(d,p)^3\text{H}$ & $^2\text{H}(^6\text{Li},t,p)^4\text{He}$ & 14 & 2.59 & $^6\text{Li} = (\alpha\alpha d)$ & [22] \\
$^{15}\text{N}(p,\alpha)^{12}\text{C}$ & $^{15}\text{N}(p,\alpha)^{12}\text{C}n$ & 60 & 2.74 & d = (p$n$) & [14] \\
$^{18}\text{O}(p,\alpha)^{15}\text{N}$ & $^{18}\text{O}(p,\alpha)^{15}\text{N}n$ & 54 & 1.76 & d = (p$n$) & [23] \\
$^1\text{H}(p,\alpha)^1\text{H}$ & $^2\text{H}(p,pp)n$ & 5, 6 & 0 & d = (p$n$) & [13] \\
\hline
\end{tabular}
\caption{Two-body reactions studied via the THM}
\end{table}

$^7\text{Li}$ nuclear ashes from the early universe, which are then used to extract information on the baryon density of the universe. Moreover, they can give more hints in order to understand the electron screening effect, this being responsible of the very pronounced enhancement in their cross-sections at low energy, significantly larger than could be accounted for from the adiabatic limit [28].

The THM bare nucleus S(E) factor for the $^3\text{He}(d,p)^4\text{He}$ is reported in fig.2. Direct data from [29, 30, 31, 32] are also reported. Again the disagreement at low energy is due to the electron screening affecting direct data. A polynomial fit to the THM data and an independent R-matrix calculation have been performed (see [12] for details). Importantly, the independent R-matrix calculation confirms the THM behaviour and provides a S(0) value of S(0)=(6.8±1.4)MeVb in agreement with the one derived from the fit S(0)=(6.4±1.3)MeVb. Following the procedure described before and using eq.8, the R-matrix parameterization provides a $U_e$ value of (126±29)eV, while the polynomial fit gives $U_e$=(155±34)eV. Both estimates are consistent and the first one appears to agree also with the theoretical adiabatic limit [28].

The $^{15}\text{N}(p,\alpha)^{12}\text{C}$, $^{18}\text{O}(p,\alpha)^{15}\text{N}$ reactions are involved in the production path of $^{19}\text{F}$ in AGB stars, whose chemical evolution is strongly influenced by $^{19}\text{F}$ abundance. Available data from observations provide evidence that fluorine is deeply produced within the interiors of these stars during helium flashes, and its abundance, enhanced by large factors with respect to the solar one, cannot be matched by the models. Among the primary sources of uncertainty in predicting fluorine abundances in AGB stars are the adopted reaction rates [33, 34].

The astrophysical S(E) factor for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ has been recently measured by means of the THM down to 20 keV (to be compared to 73 keV in direct measurements), thus covering for the first time the whole Gamow window (94 ± 66 keV for a temperature of 10^8 K) without the complications due to the electron screening. The resulting S(E) factor is reported in fig.3 (red dots). Direct data [35] are also reported as open symbols. Good agreement is found in the energy range explored by direct data. The full line superimposed onto the data is the result of a non standard R-matrix calculation, which has been called Modified R-matrix approach [14]. It accounts for the virtuality of the proton in the entrance channel of the relevant two body reaction by replacing the entry resonance widths with the form factors. The calculations include the parameters associated with the first three resonances in the $^{15}\text{N} + p$ system at
Figure 1. THM bare nucleus S(E) factor for the $^6\text{Li}(p,\alpha)^3\text{He}$ (black dots) [27] compared with direct data (open symbols) from [26]. The solid line is the result of a polynomial fit to the THM data.

Figure 2. THM atrophysical S(E) factor for the $^3\text{H}(d,p)^3\text{He}$ reaction (full dots) [12] compared with the direct behaviour (open symbols) [29, 30, 31, 32].

$E_{^5\text{N}_p} = 0.312, 0.9624$ and $1.0014$ MeV, and the subthreshold state at $E_{^5\text{N}_p} = -2.53$ MeV. For details concerning the formal expression of the reaction amplitude in terms of the form factors see [14]. The calculated S(0) value of the astrophysical factor ranges from 65.0 to 70.0 MeVb for a channel radius $r_0 = 5.0$ to $6.0$ fm, in agreement with the estimate derived from a Breit-Wigner extrapolation [35] on the direct data.

Recently we have extended the application of the THM to scattering processes. In particular, we have address the study of the $p + p$ elastic scattering through the $^2\text{H}(p,pp)n$ reaction [13].

Figure 3. THM S(E) factor for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction (red dots) [14] compared with the direct behaviour (open symbols) [35]. The full line is the result of a half-off-shell R-Matrix calculation.

Figure 4. THM two-body cross section (colored symbols) vs. pp relative energy [13]. Solid line represents the theoretical OES p-p cross section [36]. The dashed-dotted line is the calculated HOES cross section [13].
The aim of the experiment was to investigate the suppression of the Coulomb amplitude also for scattering. The extracted $p - p$ HOES cross section is presented in fig.5 as a function of $E_{pp}$ (black dots) and compared to the free $p-p$ cross section (solid line) [36] where the deep minimum due to the interference between Coulomb and nuclear amplitude dominates the excitation function. The dashed-dotted line represents the calculated HOES $p - p$ cross section [13] where the Coulomb amplitude is strongly suppressed compared to the nuclear one. We observe a striking disagreement between the THM (HOES) and the free $p-p$ (OES) cross sections throughout the region of the interference minimum, which is missing in the THM data. Instead, the calculated HOES $p - p$ nicely fits the THM data. In order to strengthen this result, the THM $p - p$ cross section was compared with the OES $n - n$, $p - n$ and pure nuclear $p - p$ ones [36] as reported in [13]. A good agreement shows up providing compelling evidence of the validity of the THM method for elastic scattering. Through a mechanism different from that of nuclear rearrangement reactions, these results strongly confirm the THM hypothesis, i.e. the suppression of Coulomb effects in the two-body cross section at sub-Coulomb energies. It turns out to be an universal effect whether we consider binary elastic or rearrangement processes.

References
[1] E.M. Burbige, G.R. Burbige, W.A. Fowler, F. Hoyle, Rev. Mod. Phys. 29, 547 (1957)
[2] W.A. Fowler, Rev. Mod. Phys. 56, 149 (1984)
[3] G. Baur and H. Rebel, J. Phys. G 20, 1 (1994) and references therein
[4] A.M. Mukhamedzhanov, R.E. Tribble et al., Phys. Rev. C59, 3418 (1999)
[5] A.M. Mukhamedzhanov, C.A. Gagliardi, R.E. Tribble et al., Phys. Rev. C63, 024612 (2001)
[6] S. Cherubini et al., Ap. J. 457, 855 (1996)
[7] C. Spitaleri, Problems of Fundamental Modern Physics II, World Scientific, p. 21-35, 1990
[8] C. Spitaleri et al., Phys. Rev. C 63, 005801 (2001)
[9] M. Lattuada et al., Ap. J. 562, 1076 (2001)
[10] A. Tumino et al., Phys. Rev. C 67, 065803 (2003)
[11] C. Spitaleri et al., Phys. Rev. C 63, 055806 (2004)
[12] M. La Cognata et al., Phys. Rev. C 72, 065802 (2005)
[13] A. Tumino et al., Phys. Rev. Lett., 98 252502 (2007)
[14] M. La Cognata et al., Phys. Rev. C (2007) to be published
[15] C.J. Copi, D.N. Schramm, M.S. Turner, Science 267, 192 (1995)
[16] L. Piau and S. Turck-Chieze, Ap.J. 566, 419 (2002)
[17] J. Kasagi et al., Nucl. Phys. A239, 233 (1975)
[18] A. M.Mukhamedzhanov et al., Eur. Phys. J. A27, 205 (2006)
[19] A. Tumino et al., Eur. Phys. J. A25, 649 (2005)
[20] L. Lamia et al. Nucl. Phys. A787, 309 (2007)
[21] S. Romano et al., Eur. Phys. J. A27, 221 (2006)
[22] A. Rinollo et al. Nucl. Phys. A 758, 146 (2005)
[23] M. La Cognata et al., J.Phys.G: Nucl. Part.Phys. 34, 1 (2007)
[24] R.N. Boyd and T. Kajino, Astr. Phys. J. 336, L55 (1989)
[25] A. Stephens and A.M. Boesgaard, Astr. Phys. J. 491, 339 (1997)
[26] S. Engstler et al., Z. Phys. A342, 471 (1992)
[27] A. Tumino et al., Nucl. Phys. A734, 639 (2004)
[28] G. Fiorentini, R.W. Kavanagh, C. Rolfs, Z. Phys. 350, 289 (1995)
[29] M. Aliotta et al., Nucl. Phys. A690, 790 (2001)
[30] W.H. Geist et al., Phys. Rev. C 60, 054003 (2001)
[31] A. Krauss et al., Nucl. Phys. A465, 150 (1987)
[32] T.W. Bonner, J.P. Conner, A.B. Lillie, Phys. Rev. 88, 473 (1952)
[33] M. Lugaro et al., Astrophys. Journal 615, 934 (2004)
[34] R.J. Stancliffe et al., arXiv :astro-ph/0504163v1
[35] A. Redder et al., Z. Phys. A305, 325 (1982)
[36] J.D. Jackson and J.M. Blatt, Rev. Mod. Phys. 22, 77 (1950)