Simulation and Analysis of Satellite Signature Effects on LAGEOS-1

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Abstract. In order to realize millimeter accuracy, a Retroreflector Uneven Distribution (RUD) model is proposed to investigate the satellite signature effect based on the actual distribution of retro-reflectors around the satellite. A satellite signature effect analysis platform is conducted to simulate the reflection characteristics of the satellite with the geometrical parameters and MATLAB programming language. The results indicate that the detected retro-reflectors are distributed dispersedly and asymmetrically, which lead to the distortion of retro-reflected pulse with a sharp leading edge. The value of CoM for LAGEOS-1 obtained is varied in a range from 246.6mm–249.8mm, which is coincide with the reference results offered by ILRS for Changchun Observation. This paper proves satellite signature effect is linked not only to the incidence angle but also the installation angle of each retro-reflectors on satellite.

1. Introduction
Since the first successful satellite observation in late 1960s, satellite laser ranging (SLR) has become one of the fundamental space-geodetic techniques widely applied in scientific geophysical research, such as measuring the gravity field of Earth, monitoring Earth rotation and determination of the Ocean and Earth tides[1–3]. With the rapid development of high-power lasers and single photon detectors, the precision of full rate residuals has improved from tens of meters to sub-centimetre level.

To achieve the full potential of satellite laser ranging for accurate geodesy, it is important to investigate all the systematic effects in the SLR measurement. As one of the major measurement errors, satellite signature effect caused by the multiple onboard retro-reflectors results in the temporal spread of echo pulse signals, especially for the large spherical satellite[4–5].

Based on the actual distribution and characteristics of retro-reflectors, a Retro-reflector Uneven Distribution (RUD) model is proposed and applied in the satellite signature effect analysis platform. The reflection characteristics of each retro-reflectors on satellite is simulated and a reasonable value of Center of Mass (CoM) for LAGEOS-1 is calculated in order to meet the needs about mm-precision of satellite laser ranging in the future.

2. Theory analysis
As a reference target for accurate laser station positioning, the Laser Geodynamics Satellite–1 (LAGEOS-1) was launched in 1976 by NASA. It has an orbit inclination of 110° over the Earth’s
equator, a semi-major axis of about 12,270 km and an eccentricity of about 0.004. The altitude of LAGEOS-1 is about 6000 km\cite{6}. Fig.1 is the photon of LAGEOS-1.

![Fig.1. External appearance of LAGEOS-1 satellite](image)

LAGEOS-1 is a sphere 60cm in diameter with 426 Cube Corner Reflectors (CCRs) covered over the surface to gather more return photons. 422 CCRs are made of fused silica glass for visible light and the others are made of germanium to reflect infrared laser pulse\cite{7}. In previous research\cite{8-9}, there always be a hypothesis of an homogeneous deposition of the retro-reflectors and the reflection of each one is treated as same for simplified calculation. However, the distribution of retro-reflectors on LAGEOS-1 is dense and uneven, as shown in Fig.2

![Fig.2. Latitude and longitude of each retro-reflector on LAGEOS-1](image)

In order to compute the optical response of satellite accurately, a Retro-reflector Uneven Distribution (RUD) model is proposed based on the analysis of distribution and characteristics of retro-reflectors on LAGEOS-1. The modeling steps are as follows: First, the echo signal intensity reflected by single retro-reflector on LAGEOS-1 is calculated. Next, the corresponding delay for each detected retro-reflector is simulated. And in the last step, the contribution of every retro-reflector involved in the reflection is added up to deduce the final satellite response histogram.

Let us denote the impulse intensity received by single retro-reflector as $I(t)$, and the corresponding impulse response intensity $R(t)$ is then\cite{8}:

$$R(t) = \tau \cdot I(t) = \tau \cdot \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{t - \mu}{\sigma} \right)^2}$$

(1)
For Changchun SLR system, the laser pulse is assumed to have a Gaussian profile with a FWHM (full width at half-maximum) of 50ps, \( \sigma \) is equal to 6.37mm, the time delay caused by satellite signature effect is \( \mu \) and the reflection coefficient is \( \tau \).

The reflection coefficient \( \tau \) is always considered as uniform and denoted as zero when the incidence angle exceeds 43 degrees. In this study, the reflection coefficient \( \tau \) is modeled to be proportional to the effective reflection area \( S(\phi) \) and reflection index \( n_{\text{ref}} \),

\[
\tau = n_{\text{ref}} \cdot S(\phi)
\]

(2)

For \( S(\phi) \), it is a result of the corresponding incident azimuth angular, which is expressed as \([9]\):

\[
S(\phi) = \frac{2}{\pi} \left\{ \sin^{-1} \left[ n \sin^{-1} \left( \frac{1}{n} \sin \phi \right) \right] \right. \\
- \sqrt{2} \left[ \sin^{-1} \left( \frac{1}{n} \sin \phi \right) \right] \left[ \frac{1}{n} \cos \phi \right] \\
-2 \tan^2 \left[ \sin^{-1} \left( \frac{1}{n} \sin \phi \right) \right] \cos \phi
\]

(3)

In previous analysis, the reflectance \( n_{\text{ref}} \) was uniformly treated as a constant \([6]\). However, the experiment results indicate \( n_{\text{ref}} \) is varied with the incident laser angular \([10]\):

\[
n_{\text{ref}} = y(\phi, \theta)
\]

(4)

Where \( \phi \) is the incident impulse azimuth angle and \( \theta \) is the incident impulse elevation angle. \( \phi \) and \( \theta \) is achieved by:

\[
\phi_i = \cos^{-1} \left( \frac{\vec{\mu}_i \cdot \vec{v}_i}{|\vec{\mu}_i| |\vec{v}_i|} \right)
\]

(5)

\[
\theta_i = \cos^{-1} \left( \frac{\vec{v}_i \times \vec{z}_i : \vec{\mu}_i \times \vec{z}_i}{|\vec{v}_i \times \vec{z}_i| |\vec{\mu}_i \times \vec{z}_i|} - \alpha_i \right)
\]

(6)

Where \( i \) is the number of retro-reflectors involved in SLR process, \( \vec{\mu}_i \) represents the impulse incident direction vector, \( \vec{v}_i \) and \( \alpha_i \) corresponds to the normal direction and installation angle of each retro-reflector respectively.

Based on above analysis, we simulate a two-dimensional map of the effective reflection coefficient \( \tau \) for an individual retro-reflector of LAGEOS-1. It is obviously seen that the value of \( \tau \) is variable, which is depended on the azimuth and elevation angles of incidence. The reflective intensity of central area on a retro-reflector is higher than the marginal zone.
Fig. 3. Effective reflection coefficient of a retro-reflector on LAGEOS-1 with respect to two-dimensional angles of incident laser pulse.

To derive the time delay of each retro-reflector, the geometrical parameters and material properties of satellite are analyzed, \( d_i \) is calculated to be \([13]\):

\[
d_i(\phi) = \frac{1}{c} [R \cdot \cos(\phi) - L \cdot \sqrt{n^2 - \sin^2(\phi)}]
\]  

(7)

Where \( R \) is defined as the radius of satellite, \( L \) corresponds to the distance of the vertex of the front face of the retro-reflector and \( c \) is the velocity of light.

Lastly, adding up the corresponding contribution of all the detected retro-reflectors, the satellite optical response intensity function denoted by \( P_s \) is then:

\[
P_s(t) = \sum_{i=1}^{k} P_i(t)
\]  

(8)

The internal and normalization of \( P_s \) is equal to reflection probability density function of satellite:

\[
p(t) = \int P_s(t)dL
\]  

\[
= \int \int P_s(t)dLdt
\]  

(9)

Where \( L \) is the optical path of laser propagation on the satellite. The value of CoM is numerically calculated using the function \( P(t) \):

\[
CoM = \int_{-\infty}^{\infty} c^2 t p(ct)dt
\]  

(10)

3. Numerical Simulation

Based on the Retro-reflector Uneven Distribution (RUD) and theory analysis above, a software platform for simulation of satellite signature effect is built in program language of MATLAB 8.0. The reflection intensity and distribution of detected retro-reflectors on satellite under the different conditions can be simulated. The program interface is shown in Fig.4.
In previous research, the number of detected retro-reflectors depends solely upon the incidence angle toward the satellite. The distribution of detected retro-reflectors is symmetrical, and the return intensity of these retro-reflectors are uniform within the limited incident angle, which amounts to 0.75 rad. However, the simulated results conducted by satellite signature effect analysis platform is obviously different. It can be seen that there are only several retro-reflectors involved in the echo signal reflection, and the number of detected retro-reflectors is decided both by the incident angle of laser impulse and the installation angle of each retro-reflectors on satellite. Fig.5 indicates that the distribution of these retro-reflectors under the different incident condition. The asymmetrical and dispersed distribution will distort the waveform of incident laser pulse.
Fig. 5. Reflected intensity distribution of LAGEOS-1 with different laser incident angle simulated by satellite signature effect analysis platform. (a) (0°, 67.5°); (b) (0°, 45°); (c) (0°, 22.5°); (d) (0°, 0°)

The distribution of reflected laser pulse is simulated shown in Fig.6. Compared with the incident beam, the intensity of the whole retro-reflected pulse is reduced and the full width at half-maximum is spread from 50ps to 72ps with a sharp leading edge. This is because the retro-reflectors closest to an observation respond strongly and ones located farther away respond weakly.

Fig.6. Distribution of the incident and retro-reflected laser pulse

The spreading temporal shape of the echo pulse caused by satellite signature effect will shift the value of CoM. In the previous research, the theoretical value of CoM is uniform and only depends on the geometric characteristics of satellite. In contrast, the simulated results in this paper reveal that the value of CoM is floated in a range from 246.6mm-249.8mm, and the average value is 248.1mm, which
is coincide with the reference results (247mm-257mm) offered by ILRS for Changchun Observation. It can be deduced that the value of CoM changes with respect to the optical path of incident laser travelled around the satellite. When the incident laser transmitted from the north pole to the equator of satellite, the variation of CoM for LAGEOS-1 is indicated in Fig.7.

![Fig.7. Influence of incidence angle on the value of CoM on LAGEOS-1](image)

### 4. Conclusion

In conclusion of this study, we propose a Retro-reflector Uneven Distribution (RUD) model and conduct a satellite signature effect analysis platform to simulate the satellite signature effect of LAGEOS-1 by practical considerations. Due to the uneven distribution of retro-reflectors, the retro-reflectors involved in reflection occupy a dispersed and asymmetrical range of satellite, which agree with the experimental results. The distribution of the retro-reflected laser pulse becomes much broader with a significant leading edge. And the value of CoM is also simulated. The calculated results vary dynamically with respect to the optical path of incident laser travelled around the satellite. For LAGEOS-1, the value of CoM is floated in a range from 246.6mm-249.8mm, which is coincide with the reference results (247mm-257mm) offered by ILRS for Changchun Observation.

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