Electromagnetic field emitted by core-shell semiconductor nanowires driven by an alternating current

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I. INTRODUCTION

Nanowires based on semiconductor materials have emerged as an advantageous platform for the realization of diverse applications in multiple areas such as photonic,[13] electronic,[14] photovoltaic,[15] or topological Majorana physics.[16] With diameters of tens to hundreds of nanometers, the microscopic crystal structure of the materials used becomes evident, resulting in polygonal cross sections. Nanowires based on III-V semiconductors fabricated by bottom-up growing techniques are commonly hexagonal,[17,18] but other geometries have been obtained, like triangular,[19,20] square,[21] and dodecagonal.[22] Nanowires of core-shell type can also have distinct core and shell polygonal shapes, like a triangular shell with a hexagonal core[23] or the other way around.[22]

By using various semiconductor materials, growing conditions, or controlling the thickness of the core or of the shell, the nanowire properties can be fine-tuned for desired or potential physical behavior.[23] In particular, with an insulating core surrounded by a conductive (doped) shell, the nanowire functions as a tubular conductor. Such a conductive nanotube may also be obtained by appropriate band alignment at the core-shell interface[23,24], or via the Fermi level pinning at the outer surface of the nanowire.[25]

In the case of a polygonal shell, a group of low energy electrons, including those in the ground-state, tend to localize at the corners of the shell.[26,27] These corner localization can be seen as the effect of particle accumulation at the bending of a two-dimensional quantum wire[28], representing a transverse polygonal cross section of the shell, or at the regions with the greatest surface curvature along the prismatic tubular shell.[29] For a polygon with N corners, and including the electron spin, there are 2N states localized in the corners. Depending on the shell thickness and sharpness of the corners, these states are separated by an energy interval varying from a few meV to tens of meV from the next 2N states, which are localized on the sides of the polygon.[30] This corner and side states correspond to low-energy transverse modes of a prismatic shell, where, due to the longitudinal motion, they generate states localized along edges or facets, respectively.

The controlled fabrication of core-shell nanowires and their internal structure motivated significant theoretical research on the electronic states in tubular prismatic geometry, i. e. in the shell, relevant for optical response,[31,32] electrical[33], and thermoelectrical conductivity[34], or topological physics.[35] At the same time, apart from the remarkable efforts for fabrication, relatively less experimental research has been devoted to the electrons localized in the shell. Notably, transport experiments showed oscillations of the longitudinal conductance in the presence of a magnetic field, related to flux periodicity.[36,37] More recently, the transverse conductance and Coulomb blocking effects have been investigated in triangular nanowires.[38] Also, optical emission excited by highly focused X rays indicated a nonuniform electron distribution in hexagonal multishell.[39] In addition, the localization of electrons on facets of different thicknesses a single hexagonal shell has also been probed via photoluminescence experiments.[40] However, more systematic experimental evidence of the quantum localization of electrons in the tubular prismatic geometry, and especially the presence of corner and side localized states with an energy gap separating them, has not been achieved yet. In this paper we explore theoretically the possibility to relate the quantum localization in the shell to the electromagnetic field radiated by the nanowire.
An interesting application of semiconductor based nanowires is as elements of optoelectronic circuits, where they could function as nanoantennas, with emitter and/or receiver function. It was already demonstrated that individual nanowires made of InAs were able to detect electromagnetic radiation in the terahertz domain. Later on, nanowires built from InP, have been studied by Grzela et al., in the near infrared domain (850 nm), and shown capable of directional emission and absorption via the internal Mie modes and More recently a multishell design of cylindrical nanowires has been proposed to achieve a superdirective emission in the optical domain.

This interest in nanoantennas based on nanowires also motivates us to consider the electromagnetic fields radiated by a polygonal core-shell nanowire due to an alternating current driven along the nanowire. We focus on core-shell nanowires made of semiconductors, where the charge transport occurs only within the shell, which acts as a tubular conductor, whereas the core is an insulator. This situation has been obtained by doping only the material of the shell, but not that of the core. If the radiated field of our nanowire, seen as an emitter nanoantenna, can be explored by a separate receiver nanoantenna, then the current and charge distribution within the nanowire could be obtained by solving an inverse problem. We emphasize again that the quantum localization of the electrons in a tubular prismatic geometry is not trivial, and offers a variety of application and manipulation possibilities (including contacts), which have not been investigated experimentally yet.

First we compute numerically the electronic quantum states for selected geometries, and then the current along an infinite prismatic shell driven by a time-dependent harmonic voltage bias. In this work we intend to emphasize the consequences of the internal geometry of such a nanowire, i.e. with a tubular prismatic shape, and of the associated localization properties, on the radiated electromagnetic field. In principle such effects can be observed in the neighborhood of the nanowire, i.e. close enough to the lateral surface, where the anisotropic distribution of currents should lead to complex field distribution. Whereas with increasing the distance between the observation point and the nanowire the internal current distribution becomes less and less important, until eventually the radiated field looks similar to that of a simple, unstructured wire of finite length. For these reasons we shall consider nanowires of an infinite length, and we shall calculate the radiated field in the proximity of the nanowire.

We describe our numerical methodology in Section II. Then, in Section III we qualitatively analyze the signature of the prismatic tubular geometry of the nanowire on the structure of the radiated electromagnetic field. We discuss the implications of the prism edge and facet localization, and in particular the anisotropy of the radiated field. Furthermore, we explore the variation of the radiated field when applying an external magnetic field perpendicular to the nanowire. The anisotropy of the radiated field is limited to the distances much smaller than the nanowire length, but depending on the geometry of the nanowire and on other parameters, that zone can be considerably large compared to the nanowire radius. Finally, in Section IV we collect our conclusions and comments on the tomography idea.

II. MODEL AND METHODS

We start by considering a system of non-interacting electrons confined in a polygonal ring. The model begins with a circular disk situated in the plane \((x, y)\) which is discretized in polar coordinates. On this grid, we superimpose polygonal constraints and we retain the points which lie inside the resulting boundaries, excluding the rest. With this method we define hexagonal, square, and triangular cross-sections. Further, we consider the nanowires to be infinite, assuming free particle propagation along their length, i.e., in the \(z\) direction. The Hamiltonian of the system is then expressed as follows:

\[
H = \frac{(-i\hbar \nabla + eA)^2}{2m_{\text{eff}}} - g_{\text{eff}} \mu_B \sigma B, \tag{1}
\]

where \(B = (B_x, B_y, 0)\) is an external magnetic field, transversal to the nanowire length, and \(A\) is the corresponding vector potential. The vector \(r = (x, y, z)\) defines the positions inside the shell, \(e\) is the electron charge, \(m_{\text{eff}}\) and \(g_{\text{eff}}\) are the effective electron mass and bulk g-factor of the material considered, \(\mu_B\) is the Bohr’s magneton and \(\sigma = (\sigma_x, \sigma_y, \sigma_z)\) are the spin Pauli matrices.

The Hilbert space associated with the polar grid is spanned by the position vectors \(|q\rangle = |r_q, \phi_q\rangle\), where \(r_q\) and \(\phi_q\) are the radial and angular coordinates of site \(q\), respectively, from which the spin is excluded. In order to obtain the eigenstates of the Hamiltonian, we solve the problem in two steps: we first obtain the transverse eigenstates \(|\alpha\rangle = \sum_q \psi(q, \alpha) |q\rangle\) and eigenvalues \(E_{\alpha} (\alpha = 1, 2, 3, \ldots)\), for \(B = 0\), i.e. the states of an electron in the polygonal ring representing the cross section of the nanowire. Then, we retain the transverse states with lowest energies, and together with the plane wave vectors in the \(z\) direction, \(|k\rangle = \exp(ikz)/\sqrt{L}\), where \(L\) is the length of the nanowire (considered infinite in our model), and with the spin states \(|s\rangle = \pm 1\), we form the basis \(|aks\rangle\). Finally, for \(B \neq 0\), we diagonalize numerically the total Hamiltonian in this basis, using a discretized series of \(k\) values, and obtain the eigenvalues \(E_{mks} (m = 1, 2, 3, \ldots)\) and eigenstates \(|mks\rangle\) of the Hamiltonian, expanded in the basis \(|aks\rangle\). The number of transverse states \(|\alpha\rangle\) needed to reach numerical convergence was typically 4\(N\) (excluding spin). The convergence was checked in several cases for an even larger basis set, and no change of the final results was observed. The numerical diagonals were performed with the Lapack library.
With this approach we obtain the first and the second lowest-energy groups of states for the infinite prismatic tubular nanowire, which are localized along the edges or along the facets of the prism, respectively. The charge density associated to these states is

$$\rho(r) = -e \sum_{mks} \mathcal{F}\left(\frac{E_{mks} - \mu}{k_B T}\right) \left(\langle |r|mks\rangle^2 - n_d\right), \quad (2)$$

where \(-e\) is the electron charge and \(n_d\) represents the density of the ionized donors. The next step is to compute the current density \(J\) inside the shell, carried by the edge- and facet-localized states:

$$J(r) = \sum_{mks} \mathcal{F}\left(\frac{E_{mks} - \mu}{k_B T}\right) \langle mks|j(r-r_0)|mks\rangle, \quad (3)$$

with \(\mathcal{F}(u) = 1/\left[\exp(u) + 1\right]\) being the Fermi function and \(u = (E_{mks} - \mu)/k_B T\), where \(\mu\) is the chemical potential, \(T\) the temperature, and \(k_B\) the Boltzmann’s constant. The operator \(j(r,r_0) = e\delta(r-r_0)v + e\delta(r-r_0)/2\) describes the contribution to the total current density at the position \(r\) from an electron situated at \(r_0\) and moving with a velocity \(v\). The velocity is defined by the operator \(v = i[H, r]/\hbar\).

If no longitudinal voltage bias is applied, the system is in equilibrium and the contributions to the total current of electrons moving with opposite velocities in the z direction compensate each other. When a voltage bias is considered, these contributions no longer cancel out and the nanowire carries a non-zero total current. In order to simulate the effect of a voltage bias, we create in our system an imbalance between electrons in states corresponding to positive and negative velocities, i.e., \(\partial E_{mks}/\partial k > 0\), and \(\partial E_{mks}/\partial k < 0\), respectively. Thus, we consider two different values, harmonically time-dependent, for the chemical potentials \(\mu_+\) and \(\mu_-\) for carriers moving in opposite directions, with \(\mu_\pm = \mu \pm V \sin(\omega t)\), where \(\omega\) is the frequency, \(2V\) the bias amplitude and \(\mu\) is the static chemical potential, which is determined by the carrier density at equilibrium.

Once the current density distribution is obtained, we calculate the time dependent scalar and vector potentials outside the nanowire,

$$V(r,t) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(r',t)}{|r-r'|} dr', \quad (4)$$

$$A(r,t) = \frac{\mu_0}{4\pi} \int \frac{J(r',t)}{|r-r'|} dr', \quad (5)$$

where \(\varepsilon_0\) and \(\mu_0\) are the vacuum electric permittivity and magnetic permeability, respectively, and the integration is carried out inside the shell.

To properly obtain the vector potential we consider the approximation \(r \ll L\). Having the vector potential it is straightforward to obtain the electromagnetic radiated field using the relations

$$E = -\nabla V - \frac{\partial A}{\partial t}, \quad \text{and} \quad B = \nabla \times A. \quad (6)$$

We thus neglect the retardation effects and displacement currents, since we will consider relatively low frequencies and quasi-stationary current. In all our calculations the contribution from the scalar potential was also very small. In fact, in Eq.(4) we neglected the polarization effects inside the nanowire, both due to the lattice dielectric response and due to the electron-electron Coulomb interactions. In principle such screening corrections lead to a dielectric constant \(\varepsilon > \varepsilon_0\) inside the nanowire and reduce the scalar potential everywhere, for the electron densities and frequencies considered in our work. Hence, with a more elaborated model replacing Eq.(4) the contribution of \(V\) to our results will remain negligible.

The next step is to estimate the frequency domain we can consider with our method. Typical nanowires made of semiconductor materials such as InAs, InP or GaAs have electron mobilities \(\mu_e\) in the range of 400-6000 \(\text{cm}^2/(\text{Vs})\), with scattering times in the range of tens of femtoseconds. Thus, for frequencies in the RF domain, from \(\text{kHz}\) to \(\text{GHz}\), achievable within an electric circuit connected to the nanowire, we can reduce the electron damping and losses in the semiconductor nanowire to the static resistivity, and ignore the dynamic corrections. For higher frequencies, towards the optical domain, the effects of the anisotropic and dynamic permittivity must be taken into account both inside and outside the nanowire, for example like in Ref. [37].

In our present RF regime the current can be injected into the nanowire through source-drain contacts covered with top-gate electrodes. This method has been used in a recent experiment to study the current output in core/shell GaSb/InAsSb NWs as a function of AC gate voltage and frequency, using a four-probe method. In our work we assume the contacts are far from each other, at several microns distance, possibly at the two ends of the nanowire, such that the length of the resulting antenna is much larger than the radius. As we mentioned in the Introduction, we are interested in the field distribution in the vicinity of the nanowire resulting from the prismatic geometry. At large enough distances from the nanowire its internal prismatic geometry becomes irrelevant, but instead, the geometry of the contacts may play a role. In addition, in practice the contacts contribute to the impedance of the nanowire, and therefore to the current strength, and also possibly to the current distribution in the contact region. Nevertheless, including these details in our model is beyond the scope of our present work, where we want to concentrate on the primary implications of current carrying tubular prismatic shell.

We consider InAs bulk parameters for the nanowire: \(m_{\text{eff}} = 0.023 m_e\) and \(g_{\text{eff}} = -14.9\). We use a voltage bias corresponding to \(V = 2.5 \text{ meV}\), and we consider a test frequency of 1 MHz. We are mostly interested in
the angular distribution of the radiated field around the nanowire, and for this purpose the frequency plays no qualitative role. It quantitatively affects though the conductivity, which drops down in the GHz domain due to damping, but also the power density, which - in principle - increases proportionally to the frequency squared. For our purpose we evaluate the power density of the radiated field with the time-averaged magnitude of the Poynting vector:

\[ \mathcal{S}(r) = \frac{1}{T} \int_0^T \frac{1}{\mu_0} (E(r, t) \times B(r, t)) \, dt. \]  (7)

In all the examples shown in this article the external radius of the shell (measured from the center of the polygon to one corner) and the thickness of the facets are \( R_{\text{ext}} = 30 \) and \( t = 6 \) nm, respectively.

III. RESULTS

For sufficiently thin shells with polygonal cross section, like ours, the electrons with the lowest energies are localized in the corners of the polygon and the electrons in the next layer of energy states are localized on the sides. The corner and side states are energetically separated by a gap interval that depends on the shape and on the aspect ratio of the polygon, i.e., the ratio between the thickness and external radius of the cross section. It increases with decreasing the shell thickness or the number of corners, and hence, in such a structure the subspace of corner states is potentially robust to many types of perturbation.

A. No external magnetic field

In Figs. 1(a-c) we show the energy spectra of nanowires with hexagonal, square, and triangular-hexagonal cross sections assuming no external field. The blue dashed horizontal lines indicate the chemical potential level. The energy intervals between corner and side states are \( \Delta_h = 21.2 \) meV for the hexagon, \( \Delta_s = 34.4 \) meV for the square and \( \Delta_t = 171.1 \) meV for the triangle. The chemical potential (blue-dashed line) corresponds to an electron density of \( n = 10^{17} \text{ cm}^{-3} \), a regime which can be achieved experimentally. For this carrier concentration the plasma frequency is of the order of \( 10^{15} \text{ Hz} \), far enough from our frequency domain, such that our quasi-static description of the electromagnetic field is reliable. Because of the spin and rotational symmetries of these geometries the states can be two- or four-fold degenerate, such that multiple energy levels overlap in each spectra. Each group of corner and side states consists of twelve levels for the hexagon, eight for the square, and six for the triangle, and the corresponding degeneracy patterns are 2442/2442, 242/242, and 24/42, respectively.

When the time dependent harmonic voltage is applied to the nanowire, the total current is zero at those instants of times when \( \mu_{\pm} = \mu \), i.e., when \( \sin(\omega t) = 0 \). However, at these instants the amplitude of the radiated electric field is maximal, as it depends on the time derivative of the current density, through the vector potential calculated with Eq. (7). On the contrary, when \( \sin(\omega t) = 1 \), and the applied voltage bias is maximum, i.e., \( \mu_{\pm} - \mu = 2V \), the current reaches its maximum, but its time derivative vanishes, and so does the radiated electric field. It is important to note that, when \( \mu_{\pm} \neq \mu \) or \( \sin(\omega t) \neq 0 \), depending on the amplitude of the harmonic voltage applied, electrons may move between different energy levels, which translates into changes of localization, and hence of the current distribution and radiated field, within one period of time.

In Figs. 1(d-f) we plot the time derivative of the current density, \( dJ/dt \), inside the nanowire cross-section. For simplicity we only show it in the case of \( \mu_{\pm} = \mu \), when the radiated field intensity is maximal. We use a carrier density sufficiently low in order to obtain a chemical potential at equilibrium (\( \mu \)) within the low energy bands.

In the absence of an external magnetic field the distributions shown for \( dJ/dt \) are qualitatively similar.
to the electron concentrations and current density at equilibrium\cite{15,62}. Obviously, the imposed voltage bias creates an imbalance between carriers moving in opposite directions along the length of the nanowire. In our case the total current driven along the nanowire ranges between a few nA to a few µA for voltage bias amplitudes of 1-10 meV\cite{43,63}.

In Figs. 1(g-i) we show the angular variation of the power density of the radiated electromagnetic field at the distance $d = 2R_{ext}$ from the center of the nanowires. The reference minimal values are $S_{min} = 3.01, 3.06, 1.76 \times 10^{-4}$ W/m$^2$ for the hexagonal, square and triangular nanowires, respectively. As expected from the current density distribution, the resulting field captures the internal geometry of the nanowires manifested by a number of peaks which is equal to the number of corners. The electrons localized in each corner form quasi-independent current channels which can be considered as individual nanowires.

In classical terms, one can attribute to the radiated field a singular behavior due to the quasi one or two dimensional current distributions within the prismatic shell with sharp corners and thin sides\cite{28}. However, here we take into account properly the transverse degree of freedom, consistently with the quantum localization of the electrons, along parallel current filaments with transverse geometry and different strengths. This special current distribution leads to multipole variations in the radiated field, of which the lower one will have the largest strength. We thus expect weakly imposed quadrupole, or hexapole, or higher order pattern in the radiation field, or more unusual patterns, as we will see in Subsection 11.13 due to the presence of a magnetic field.

Before that, in Fig. 2 we show how the differences between the maximal and the minimal value of the power density curves, shown in Fig. 1(g-i), decay with the distance from the center of the nanowire, within the near field zone, between $d = 2R_{ext}$ and $d = 10R_{ext}$. The fastest decay is observed for the hexagonal nanowires which has the least anisotropic geometry, closest to circular. As expected, the decay is less pronounced with decreasing the number of corners of the nanowire. The signals originating from the square and the triangular nanowires have similar intensity at the position $d = 2R_{ext}$, but they decay much slower around the triangular than around the square nanowire. The difference at $d = 10R_{ext}$ is around $10^5$, $10^3$ and $10^2$ times smaller for the hexagonal, square, and triangular nanowires, respectively.

**B. Effects of a transverse external magnetic field**

In this subsection we consider a uniform magnetic field perpendicular to the nanowire axis. Such a magnetic field may drastically perturb the current distribution within the tubular shell, mainly because the field component normal to the nanowire surface, which is responsible for the Lorentz force, varies with the angular coordinate. In the case of a circular cross section, and for strong enough fields, the electrons situated on the sides of the tubular shell, relatively to the direction of the magnetic field, experience nearly zero local magnetic field, and move on snaking trajectories along the wire, in opposite directions. On the contrary, the electrons situated on the top or bottom regions tend to perform local cyclotron loops\cite{28}. The situation becomes more complex for a polygonal shell, where the localization resulting from the geometry competes with the one induced by the magnetic field\cite{21,15,12}.

As we pointed out previously, when no longitudinal voltage is applied over the nanowire, the total current vanishes due to equal number of electrons moving to the left and right. This holds for both zero and non-zero transverse magnetic field. In the case of the current density $J$, which reflects the underlying charge and velocity distributions, the presence of a non-zero magnetic field leads to splitting due to the Lorentz-force. This can be seen in Fig. 3, where the equilibrium current density is split into channels consisting of electrons with positive and negative velocities, the former ones pushed to the right side of the sample, while the latter to the left side. The current density takes the form of loops along the z axis of the nanowire which close up at the ends of it (i.e. at infinity in our model). Then, even if the total integrated current is still zero, the current density is not compensated locally.

However, in equilibrium, the channels that form each loop are compensated, i.e., the same current flows in both directions, with each channel paired with the one on the opposite side of the geometric symmetry axis relative to the magnetic field direction. Depending on the localization strength, the pairing can happen within the same corner or side or on opposite ones. Thus, we observe that in the case of the hexagon (Fig. 3(a)), for which the localization is the weakest, there is one main loop formed on the two sides parallel to the magnetic field direction, where the snaking states are formed, and two additional loops originating from the split maxima in the corners situated at angles 0° and 180° with respect to the magnetic field. In the case of the square, Fig. 3(b), the situation...
FIG. 3. Current distribution in equilibrium inside the polygonal shells for $n = 10^{17}$ cm$^{-3}$ when a magnetic field of $B = 1$ T is applied perpendicular to one of the edges for the hexagonal (a), square (b) and triangular (c) shells. The units of the color scale are nA/nm$^2$.

is similar, with one main loop that captures most of the current and two additional ones on the corners perpendicular to the field direction. For the triangular case, Fig. 3(c), where the localization is the strongest, there are three loops and a pair of opposite channels in each corner.

If the magnetic field is strong enough, it can mix the two groups of corner and side states, leading to complex changes in the energy spectra and in the current distributions. The dispersion with respect to the wave vector $k$ when a magnetic field is applied perpendicular to one of the edges is shown in Figs. 4(a-c). We use the same carrier density $n = 10^{17}$ cm$^{-3}$ for the hexagon and the square cases, so that the position of the chemical potential would be at the level of the corner states if there was no magnetic field applied, as it was shown in Figs. 1(a-c), whereas for the triangular case we now use $n = 10^{18}$ cm$^{-3}$ in order to reach the side states.

The time derivative of the current density ($dJ/dt$), shown in Figs. 4(d-f), contrary to the current distribution, has only either positive or negative sign, as the electrons are oppositely accelerated or slowed down by the voltage bias, depending on its sign. For clarity, the time-derivative of the current distribution over the whole time period is shown in Fig. 2 of the Appendix.

In Figs. 4(g-i) we show how the magnetic field affects the relative differences of the power density. The minimum values of the power density are now $S_{\text{min}} = 1.74, 2.23, 15.45 \times 10^{-4}$ W/m$^2$ for the hexagonal, square and triangular nanowires, respectively. For the hexagonal nanowire we observe a clear difference with respect to the case without external magnetic field. Instead of the six corner-related peaks we now observe only two main ones with some shoulders. The reason is that now the Lorentz force pushes the current density to the sides where the snaking states are formed, which carry most of the current.

We note, however, that the time-average Poynting vector is an integration over a complete time period. When the chemical potentials are imbalanced, the nanowire quickly carries net current only in one direction and thus, over half of a period the contribution of one of the two sides is much smaller. On the contrary, the contribution of the two other regions where the additional loops are formed remains relevant during the complete period. Thus, on average, the two peaks of the maximum radiated power density appear on the angles corresponding to the two corners pierced by the field. For clarity, to illustrate this effect, we also show the current density distribution over a complete period in Fig. 8 of the Appendix.

The situation is, to some extent, similar in the case of the square, but the difference in this case lies in the fact that each corner hosts current in both directions. Thus, integrating over a period, we still observe four peaks in the power density but now with different intensities.

In the triangular case the corner localization is stronger and the current distribution is locked in each corner if the carrier concentration is too low. With $n = 10^{17}$ cm$^{-3}$ as before, the effects of a magnetic field of the order of 1 Tesla are negligible, and the radiated field profile is similar to the one in Fig. 4(h). For this reason we increase the carrier density to $n = 10^{18}$ cm$^{-3}$, so that the chemical potential crosses the next group of states, which are side localized, for which the localization is weaker. The Lorentz force then spreads the current loops laterally with respect to the magnetic field. For the field orientation shown in panels (a) and (b) while for the triangular nanowire in (c) $n = 10^{18}$ cm$^{-3}$. The color scale units are $\Lambda/(s$ mm$^2$).
is reflected in the power density curve where, we now observe only two clear peaks at angles corresponding to those corners, i.e., 120° and 240°.

Taking advantage of the localization mechanism induced by the external magnetic field, it is possible to tune the directivity of the nanowires. In Fig. 5, we consider the same parameters as in the previous examples, but we change the direction of the magnetic field. As can be seen in Figs. 3, a change in the direction of the magnetic field does not only shift the power density curves, as it happens for the hexagonal case, but also creates a different localization which leads to different current density distributions. Thus, this change has also implications on the power density curves, i.e., it mainly affects the overlapping of the contributions between the main loop and the weaker ones for the hexagon and the relative difference in their amplitude for the square. For the triangular nanowire the rotation of the field, now parallel to one of the sides, leads to the accumulation of current in the corner opposite to its direction, which translates in the formation of a dominant peak in the power density curve as seen in Fig. 5(i). The minimal values of the power density are in this later case $S_{\min} = 1.74, 2.37, 15.62 \times 10^{-4}$ W/m² for the hexagonal, square and triangular nanowires, respectively. Additionally, as can be seen in Fig. 5(c), because of the absence of an inversion center for the triangle, the energy curves are no longer symmetric with respect to the wave vector $k$ if the magnetic field points parallel to one

\[ \text{IV. CONCLUSIONS} \]

Polyagonal semiconductor core-shell nanowires exposed to a transverse external magnetic field incorporate complex electron localization mechanisms yielding a rich phenomenology. The direction and magnitude of the magnetic field allows for the tunability of the electron localization and creates current loops, or channels of current where electrons travel in opposite directions. The electromagnetic field emitted by these nanowires subjected to an alternate current along their length depends on the underlying electron and current distribution inside the nanowire.

In order to study (some) consequences of the electron localization inside a prismatic core-shell nanowire, we obtained the radiated power density as a function of the an-
ingle, in the neighborhood of it. We should mention that we on purpose avoided the term “near field”, which is more familiar to the electrical engineers, and implies a comparison between the wavelength of the radiated field and the distance from the antenna. Still, our nanowires should be considered electrically small antennas, i.e. with length much smaller than the wavelength. However, for our goal, the frequency, and thus the wavelength, are not essential parameters, the relevant domain for the field anisotropy being given by the geometry of the prismatic shell.

In a recent work a nonuniform optical field emitted by hexagonal core-shell structures with imperfect internal geometry, via space resolved photoluminescence, has been detected and associated with electron localization within the shell. In the present paper we considered instead the radio frequency domain. We showed that the radiated electromagnetic field can capture the electron and current density localization and we studied how it is affected by a magnetic field transverse to the nanowire, with different angular orientations.

The magnitude of the power emitted by the nanowires highly depends on the parameters used in the model, i.e., carrier density, amplitude of the voltage bias and, especially, frequency, and it can vary by orders of magnitudes. Rather than an in-depth study of the magnitude of the emitted power in different situations, our goal has been to demonstrate, more qualitatively than quantitatively, that the resulting field follows the anisotropy of the charge and current distribution within the shell of the nanowire, and has a tunable directivity, first via the geometry and internal structure of the nanowire, and second with an external magnetic field. Although we restricted our study to individual nanowires, the extension of the ideas to arrays of parallel nanowires, as nanowires are often grown, to achieve a combined effect and increased power, is straightforward.

If the electromagnetic field radiated by our core-shell nanowire could be measured with a receiver nanowire, at a sufficiently short distance, the data could be used for a tomography of the charge distribution in the nanowire. The receiver may be another nanowire, shorter than the emitter, made either of a semiconductor or a metal. Other solution could possibly follow the examples of the near-field optical spectroscopy of a Yagi-Uda nanoaenna, or using a nanoantenna array as receiver. Another interesting example is a tandem of receiving-transmitting nanoaennas situated 500 nm apart. Indeed, technical challenges are inevitable, like approaching the two emitter and the receiver at a distance within the emitter near field pattern, adjusting the relative distance and angular position of the two elements, etc. For example like in the atomic force or scanning tunneling microscopy. In addition, depending on the specific structure of the receiver, an interaction with the emitter nanowire is possible, and that should be included in the reconstruction of the current distribution. The design of a proper setup for such an experiment, where all these issues are taken into account, is nevertheless beyond the scope of our present work, that we hope will stimulate an experimental attempt.

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FIG. 7. Time-derivative of the current density distribution over a complete period when a magnetic field of $B = 1$ T is applied perpendicular to one of the edges of the nanowire. The carrier concentration is $n = 10^{16}$ cm$^{-3}$. 
FIG. 8. Current density distribution over a complete period when a magnetic field of $B = 1$ T is applied perpendicular to one of the edges of the nanowire. The carrier concentration is $n = 10^{16}$ cm$^{-3}$. 