Experimental Verification of the Mathematical Model of an Opened Thin-Wall Shell Forced Vibrations with a Small Attached Mass and a Rigidly Clamped Edges

O E Sysoev², A Yu Dobryshkin¹, E O Sysoev³

¹Faculty of Cadastre and Civil Engineering, Komsomolsk-na-Amur State University, 27, Lenin Ave., Komsomolsk-na-Amur 681013, Russia
²Department of Civil Engineering and Architecture, Komsomolsk-na- Amur State University, 27, Lenin Ave., Komsomolsk-na-Amur 681013, Russia
³Department of Civil Engineering and Architecture, Komsomolsk-na Amur State University, 27, Lenin Ave., Komsomolsk-na-Amur 681013, Russia

E-mail: fks@knastu.ru, wwwartem21@gmail.com, sia@knastu.ru

Abstract. The article discusses the results of experimental studies on the verification of new mathematical models of an opened thin-wall shell forced vibrations with a small attached mass and a rigidly clamped edges. Nowadays, the opened thin-wall shells are widely used in aircraft manufacturing, buildings and structures construction, in various industries. Various small masses (fuel tanks, radio equipment, air conditioners, etc.) are attached to open shells. At the same time, these structures are affected by various loads (wind, snow and vibration), which causes forced oscillations, which are composed of a shell and natural oscillations lead to resonance phenomena, in some cases, structural destruction and man-made disasters. According to the results of research, an experimental verification of the mathematical model of an open thin-walled shell forced oscillations with a small added mass and rigidly clamped edges was carried out.

1. Introduction

Currently, the construction of thin-walled open shells with pinched supporting edges are becoming more widespread in the construction of buildings and structures will introduce architectural expressiveness, as well as in other industries due to economic efficiency. Attached masses are installed on these structures (balconies, air conditioners, canopies, and viewing platforms) that cause the launch of dynamic processes, such as the interaction of bending and radial vibrations. In addition, thin-walled open shells are affected by cyclic loads (wind and snow, tropical rain), which cause forced vibrations and have a significant impact on the oscillatory processes of the structure as a whole. The presence of all components of the oscillatory process leads to the imposition of wave processes [1]-[3] and an increase in the amplitudes of the oscillations and, as a general result, the destruction of the structure. Therefore, new refined mathematical models are required for calculating the vibrations of thin-walled open cylindrical shells, with initial shape irregularities, as well as their experimental verification.

This work reflects the study of forced vibrations of a thin rectangular in terms of an open shell, rigidly clamped on both sides. The equations of shell oscillations are obtained according to the generally accepted theory of shell oscillations, as well as experimental data reflecting the dependence...
of the effect of the added mass on the numerical characteristics of the natural oscillations of the shell oscillations. Oscillations with moderate amplitudes of forced oscillations were decomposed according to equations [5, 6]. The discrete nonlinear model of a thin shell oscillations, clamped at the edges, obtained in the course of research, was studied using the method of many scales.

The sample is an open shell, rectangular in plan, of aluminum alloy D19. Geometrical characteristics of the object: \( L = 890 \) mm, \( B = 370 \) mm, \( H = 0.4 \) mm. The scheme of the experiment is shown in Figure 1.

![Diagram of the action of forces.](image)

Figure 1. Diagram of the action of forces.

The experiment bench is rectangular. Test specimen using bolts M1.1 Ø6mm, with step \( S = 20 \) mm, and elongated rectangular plates of steel St3sp, screwed to the test bench, ideally creating conditions for the rigid clamping of the shell during its vibrations. Thus, the boundary conditions are as close to real as possible [7-10]. The stand is made of equal angles L45x3 of steel St3sp, made according to Interstate standard (GOST 8509-93). The attached mass is the accelerometer BC110, located on the sample according to Figure 3. The accelerometer BC110 measures the oscillation frequency with high accuracy in the ranges from 0.5 ... 10,000 Hz and has a sensitivity of 100 mV / g.

The oscillation measurement sensor BC 110 sends an electrical impulse to an analog signal amplifier, which, amplifying an electrical signal, and transmitting it along power lines further to an ADC (analog-to-digital converter). The digital signal through the circuit goes to a personal computer. To eliminate errors in determining the frequency characteristics and increase the accuracy of the study, an additional Zet701 probe is present in the experiment program. This sensor works on the principle of determining the magnetic component of the environment, located at a distance of no more than 2 mm from the shell. Z-lab software allows you to display and record vibrations in real time. The block diagram of the experimental setup for the experiment is shown in Figure 2.
2. Theoretical part

Consider the natural oscillations of a rectangular clamped along the contour of the shell \((-\frac{a}{2} \leq x \leq \frac{a}{2}, -\frac{b}{2} \leq y \leq \frac{b}{2})\). The original differential equation is:

\[ D \nabla^4 W + \rho W_u = 0, \]

where \(D = \frac{E_1 h^3}{12(1-\nu_1\nu_2)}\), \(\rho\) - mass per unit area of the shell; \(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\).

We introduce the notation: \(y = \frac{y}{b}; x = \frac{x}{a}; k = \frac{a}{b}\).

After the substitution of relations we get:

\(\nabla^4 W + \frac{\rho b^4}{D} W_u = 0\)

The solution of this equation will find the method of separation of variables: \((W=W(x,y)\cdot T(t))\)

After substituting the expression in the equation we get two equations:

\[ T'(t) + \theta^2 T = 0, \]
\[ \nabla^4 W - \lambda W = 0, \]

here \(\lambda = \rho b^4 D^{-1}\) - own number of tasks.

In order to obtain the solution of the forced oscillations problem, we indicate the boundary conditions of pinching in the differential equation:

\(W = 0, W_x = 0\) at \(x = \pm 0.5k,\)

\(W = 0, W_y = 0\) at \(y = \pm 0.5,\)

Enter the boundary conditions using the parameter \(\varepsilon\):

\(W = 0, (1-\varepsilon)W_{xx} \pm \varepsilon k W_x = 0;\) at \(x = \pm 0.5k,\)

\(W = 0, (1-\varepsilon)W_{yy} \pm \varepsilon k W_y = 0;\) at \(y = \pm 0.5,\)

When \(\varepsilon = 1\), the conditions of pinching along the contour are realized.

Next, we represent the eigenvalue of oscillations \(\lambda\) and the eigenform of \(W\) in the form of rows in \(\varepsilon\).

Substituting the series into a differential equation and realizing the boundary conditions, after splitting in powers of \(\varepsilon\), we obtain a recurrent sequence of boundary value problems:

\(\nabla^4 W - \lambda_0 W_j = 0,\)

\(W = 0, W_{0xx} = 0;\) at \(x = \pm 0.5k,\)
\[ W = 0, W_{0yy} = 0; \text{ at } y = \pm 0.5, \]
\[ \nabla^4 W - \lambda_0 W_j = \sum_{i=1}^{j} \lambda_i W_{j-i}; \]
\[ W_j = 0, W_{jxx} = \pm k \sum_{i=1}^{j} W_{ix}, \text{ at } x = \pm 0.5k, \]
\[ W_j = 0, W_{jyy} = \pm k \sum_{i=1}^{j} W_{iy}, \text{ at } y = \pm 0.5. \]

Let us describe in detail the construction of eigenvalues and forms for the case when eigen forms have direct symmetry in the \( x \) and \( y \) directions. In the zero approximation we have:
\[ \lambda_0 = \pi^2 \left( \frac{m^2}{k^2} + n^2 \right), n, m = 1,3,5, ..., \]
\[ W_0 = c \cos \frac{\pi m}{k} x \cos \frac{\pi n y}{k}. \]

The task of the first approximation is to write as follows:
\[ \nabla^4 W_1 - \lambda_0 W_1 = \lambda_1 \cos \frac{\pi m}{k} x \cos \frac{\pi n y}{k}; \]
\[ W_1 = 0, W_{1xx} = \pm \pi n m (-1)^{\frac{m-1}{2}} \cos \frac{\pi n y}{k} \text{ at } x = \pm 0.5k, \]
\[ W_1 = 0, W_{1yy} = \pm \pi n m (-1)^{\frac{n-1}{2}} \cos \frac{\pi m}{k} \text{ at } y = \pm 0.5. \]

The solution is sought by the method of separation of variables, presenting the function \( W_1 \) in the form:
\[ W_1 = Y_1 x \cos \frac{\pi m}{k} x + X_1 \cos \frac{\pi n y}{k}. \]

The eigenvalue \( \lambda_1 \) is also represented as the sum:
\[ \lambda_1 = \lambda_{1x} + \lambda_{1y}. \]

After substitution of expressions in the equation and accordance with the boundary conditions, we obtain two boundary value problems:
\[ Y_1^{iv} - 2 \pi^2 \frac{m^2}{k^2} Y_1^{ii} - \pi^4 n^2 \left[ 2 \frac{m^2}{k^2} + n^2 \right] Y_1 = \lambda_{1y} \cos \frac{\pi n y}{k}; \]
\[ Y_1 = 0, Y_1^{ii} = \pm \pi n m (-1)^{\frac{n-1}{2}}, \text{ at } y = \pm 0.5, \]
\[ X_1^{iv}(x) + 2 \pi^2 n^2 X_1^{ii}(x) - \pi^4 \frac{m^4}{k^4} + 2n^2 \left[ \frac{m^4}{k^4} + \frac{m^2}{k^2} \right] X_1(x) = \lambda_{1x} \cos \frac{\pi m}{k} x; \]
\[ X_1 = 0, X_1^{ii}(x) = \pm \pi n m (-1)^{\frac{m-1}{2}}, \text{ at } x = \pm 0.5k. \]

Integrating the equation in parts, we get:
\[ \int_{-0.5}^{0.5} u(y) \left[ Y_1^{iv} - 2 \frac{\pi^2 m^2}{k^2} Y_1^{ii} - \pi^4 n^2 \left[ 2 \frac{m^2}{k^2} + n^2 \right] Y_1 \right] dy \]
\[ + u(y) Y_1^{iii} \bigg|_{-0.5}^{0.5} + u^{ii}(y) Y_1 \bigg|_{-0.5}^{0.5} - u^{iii}(y) Y_1 \bigg|_{-0.5}^{0.5} - 2 \frac{\pi^2 m^2}{k^2} \left( u(y) Y_1 \right) \bigg|_{-0.5}^{0.5} \]
\[ - u^{i}(y) Y_1 \bigg|_{-0.5}^{0.5} = \lambda_{1y} \int_{-0.5}^{0.5} u(y) \cos \frac{\pi n y}{k} dy. \]

Equating the integrand to zero on the left side of the relation, we obtain an equation for the function \( u(y) \):
\[ u^{IV}(y) - 2 \frac{\pi^2 m^2}{k^2} u^{II}(y) - \pi^4 n^2 \left( 2 \frac{m^2}{k^2} + n^2 \right) u(y) = 0. \]

Require condition to be met:
\[ u(y) \left[ Y_1^{II} - \frac{\pi^2 m^2}{k^2} Y_1^I \right] \bigg|_{0,5\pi} - u^{II}(y) Y_1^I \bigg|_{0,5\pi} = 0. \]

The condition is satisfied if the coefficients at \( Y_1^I \) and \( Y_1^{II} \) are zero.

The general solution of the equation is:
\[ u(y) = c_1 \text{ch} \sqrt{\frac{m^2}{k^2} + n^2} y + c_2 \cos \pi ny. \]

Only the second term satisfies the boundary conditions, therefore:
\[ u(y) = c_2 \cos \pi ny. \]

From the solution we obtain the condition of solvability:
\[ \lambda_{1y} = 4\pi^2 n^2. \]

Now we define \( Y_1 \):
\[ Y_1 = \frac{n}{\pi \alpha} \left[ \frac{1}{2\pi^2 \beta_1} \text{ch} \pi \beta_1 y - y \sin \pi ny \right]. \]

There \( \alpha = n^2 + \frac{m^2}{k^2} \); \( \beta_1 = \sqrt{2 \frac{m^2}{k^2} + n^2}, n = 1,3,5, \ldots \)

We solve the boundary problem similarly, as a result we get:
\[ \lambda_{1x} = 4\pi^2 \frac{n^2}{k^2}; \]
\[ X_1 = \frac{m/k}{\pi \alpha} \left[ \frac{k(-1)^{m-1}}{2 \pi^2 \beta_2} \text{ch} \pi \beta_2 x - x \sin \frac{\pi m}{k} x \right]. \]

Here \( \beta_2 = \sqrt{\frac{m^2}{k^2} + 2n^2}, m = 1,3,5, \ldots \)

The first amendment to the eigenvalue \( \lambda_1 \) and the proper form \( W_1 \) for straight-symmetric forms is:
\[ \lambda_1 = 4\pi^2 \alpha; \]
\[ W_1 = \frac{1}{\pi \alpha} \left\{ n \left[ \frac{(-1)^{m-1}}{2 \pi^2 \beta_1} \text{ch} \pi \beta_1 y - y \sin \pi ny \right] \cos \frac{\pi m}{k} y \right\} + \frac{m}{\pi \alpha} \left\{ \frac{k(-1)^{m-1}}{2 \pi^2 \beta_2} \text{ch} \pi \beta_2 x - x \sin \frac{\pi m}{k} x \right\} \cos \pi ny, n, m = 1,3,5, \ldots \]

Similarly determined \( \lambda_2 \).

The recursive formulation of perturbation theory for an eigenvalue is:
\[ \lambda = \pi^4 \alpha^2 + 4\pi^2 \alpha \varepsilon \]
\[ + 4\pi \left\{ n \alpha + 2 \frac{n^2 m^2}{\pi \alpha^2} - 1 \frac{1}{2\pi} \left( k \frac{m^2}{k^2} \beta_1 \text{cth}(-1)^n \frac{\pi}{2k} \beta_1 + n^2 \beta_2 \text{cth}(-1)^n \frac{\pi}{2} \beta_2 \right) \varepsilon \right\} \]
\[ + \cdots, \]
\[ n,m = 1,3,5, \ldots \]
Results of experimental studies.

Figure 3. The first eigenvalue for a square in terms of a pinched open shell.

1 – Data from recursive perturbation theory;
2 - The data obtained using the Padé approximation;
3- Experimental data;

Figure 3 shows the dependence of the first eigenvalue of the \( \lambda (1) \) problem on \( \varepsilon \) for the recursive formulation of perturbation theory (curve 1) and the Padé approximation (curve 2), experimental data (curve 3). It can be seen that the limiting value of the parameter \( \varepsilon \), at which the difference in the results obtained using the recursive formulation of perturbation theory and the Padé approximation will be within 5%, is \( \varepsilon = 0.4 \). When \( \varepsilon = 1 \), the results obtained using the solution discussed above are very far from the numerical solution and can be used rather to estimate the eigenvalue from below. The results obtained using the Padé approximation are slightly higher than the numerical solution, but they can be used for all values of the parameter \( 0 \leq \varepsilon \leq 1 \). The results of experimental studies are consistent with the numerical results obtained using the recursive formulation of perturbation theory. When \( \varepsilon \leq 0.5 \), the results of experimental studies deviate from the results of the estimates obtained using the Padé approximation.

3. Conclusion
When calculating thin-walled open shells with rigid clamped edges, it is necessary to use the mathematical model presented in this paper, since this model is fully confirmed by experimental studies.

References
[1] Vlasov V Z 1949 The general theory of shells and its application in engineering (M.-L.: Gostekhizdat) 784 p
[2] Kubenko V D 1984 Nonlinear interaction of the forms of bending vibrations of cylindrical shells (Kiev: Naukova Dumka) 220 p
[3] Antufiev B A 2011 Oscillations of inhomogeneous thin-walled structures: monograph (M.: MAI Publishing House) 176 p
[4] Hempel K A 1965 Reference book on rare metals per. from English (M.: Mir) 946 p
[5] Sysoev O E, Dobryshkin A Yu, Naing N S 2017 Nonlinear Oscillations of Elastic Curved Plate Carried to the Associated Masses System International Conference on Construction Architecture and Technosphere Safety (ICCATS 2017) (Chelyabinsk, Russian Federation) IOP Conf. Series: Materials Science and Engineering 262 012055 doi:10.1088/1757-899X/262/1/012055

[6] Sysoev O E, Dobryshkin A Yu, Naing N S 2017 The influence of the added mass on the forced oscillations of open shells Scientific notes of KnAGTU 3 pp 110-116

[7] Sysoev O E, Dobryshkin A Yu, Nayn S N, Kokhorov K K 2017 Modern test benches for contactless studies of free oscillations of closed and open cylindrical shells Scientific Notes KnAGTU 1 pp 110-118

[8] Sysoev O E, Dobryshkin A Yu, Naing N S 2017 Effect of added mass and temperature shift on the natural oscillations of thin plates (membranes) Scientific notes KnAGTU 2 pp 105-111

[9] Sysoev O E, Dobryshkin A Yu, Naing S N 2017 The effect of the magnitude of the added mass on the forced oscillations of open shells of aluminum alloy d19 Scientific notes KnAGTU-2017 4 pp 100-106

[10] Sysoev O E, Dobryshkin A Yu, Nain Sit Naing 2018 Analytical and experimental study of free oscillations of open shells made of D19 alloy, carrying a system of attached masses Transactions of MAI 98

[11] Wang Q, Han D H, Nash P Liu 2017 Investigation on inconsistency of theoretical solution of thermal buckling critical temperature rise for cylindrical shell Thin-Walled Structures 119 pp 438-446

[12] Dobryshkin A Yu Research of the influence of the location of the joint mass on the forced vibration of a thin-contained extenced shell International Scientific Conference «Far East Con»

[13] Sysoev O E, Dobryshkin A Yu, Naing S N, Baenkhaev A 2019 Investigation to the location influence of the unified mass on the formed vibrations of a thin containing extended shell Materials Science Forum 945 pp 885-892 8 p

[14] Sysoev O E, Dobryshkin A Yu 2018 Natural vibration of a thin desing with an added mass as the vibrations of a cylindrical shell and curved batten ISSN 2095-7262 CODEN HKDXH2 Journal of Heilongjiang university of science and technology 28 1 pp 75-78

[15] Sysoev O E, Dobryshkin A Yu, Naing S N 2017 Curved plate carried to the associated masses system IOP Conf. Series: Materials Science and Engineering p 262

[16] Solovev D B 2019 Features of a Power Consumption of the Main Electro receivers of Coal Mine in the Conditions of the South of the Far East of the Russian Federation IOP Conference Series: Earth and Environmental Science 272 paper № 022001. [Online]. Available: https://doi.org/10.1088/1755-1315/272/2/022001