Dissimilar donuts in the sky? Effects of a pressure singularity on the circular photon orbits and shadow of a cosmological black hole

S. D. Odintsov\textsuperscript{1,2,3} and V. K. Oikonomou\textsuperscript{4(a)}

\textsuperscript{1} ICREA - Passeig Luis Companys, 23, 08010 Barcelona, Spain
\textsuperscript{2} Institute of Space Sciences (ICE, CSIC) - C. Can Magrans s/n, 08193 Barcelona, Spain
\textsuperscript{3} Institute of Space Sciences of Catalonia (IEEC) - Barcelona, Spain
\textsuperscript{4} Department of Physics, Aristotle University of Thessaloniki - Thessaloniki 54124, Greece

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Abstract – The black hole observations obtained so far indicate one thing: similar “donuts” exist in the sky. But what if some of the observed black hole shadows that will be obtained in the future are different from the others? In this work the aim is to show that a difference in the shadow of some observed black holes in the future might explain the H\textsubscript{0}-tension problem. In this letter we investigate the possible effects of a pressure cosmological singularity on the circular photon orbits and the shadow of galactic supermassive black holes at cosmological redshifts. Since the pressure singularity is a global event in the Universe, the effects of the pressure singularity will be imposed on supermassive black holes at a specific redshift. As we show, the pressure singularity affects the circular photon orbits around cosmological black holes described by the McVittie metric, and specifically, for some time before the time instance that the singularity occurs, the photon orbits do not exist. We discuss the possible effects of the absence of circular photon orbits on the shadow of these black holes. Our idea indicates that if a pressure singularity occurred in the near past, then this could have a direct imprint on the shadow of supermassive galactic black holes at the redshift corresponding to the time instance that the singularity occurred in the past. Thus, if a sample of shadows is observed in the future for redshifts $z \leq 0.01$, and for a specific redshift differences are found in the shadows, this could be an indication that a pressure singularity occurred, and this global event might resolve the H\textsubscript{0}-tension as discussed in previous work. However, the observation of several shadows at redshifts $z \leq 0.01$ is a rather far future task.

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Introduction. – Among other mysteries of contemporary research in astronomy and astrophysics, the H\textsubscript{0}-tension is the oldest and longstanding at least for three decades or more. The problem with the value of the Hubble rate at present day is that large redshift sources, like the Cosmic Microwave Background (CMB) radiation [1] indicate smaller values compared with small redshift sources like the Cepheids [2]. The tension can be explained theoretically by early dark energy [3–7]. Another groundbreaking explanation could be the abrupt change of physics before 70–150 Myrs ago [8–10] (see also [11]) which radically affected the Cepheid parameters, thus yielding a larger value for the value of the Hubble rate at present day. Although the H\textsubscript{0}-tension may be attributed to the Cepheid calibration [8,12], such an issue attracts the attention of cosmologist and astrophysicists, see for example refs. [13–33] and references therein.

In a recent work [11] we adopted the perspective of refs. [8–10] that an abrupt physics change before 70–150 Myrs ago might explain in a transparent way the H\textsubscript{0}-tension, and we theorized that such an abrupt and global in the Universe physics change might have occurred if the Universe experienced a pressure singularity (also known as Type II or sudden singularity). Such a singularity is not of the crushing type, and the only physical quantity that is divergent globally on the spacelike three dimensional hypersurface that is defined by the time instance at which the singularity occurs, is the pressure. The rest of the physical observables are finite, hence this can be viewed as a singularity through which the Universe passes relatively smoothly.

In this article we shall examine the qualitative effect of such a singularity on galactic supermassive black holes, and specifically on the circular photon orbits around the black hole and the corresponding shadow. Our proposal is based on the idea that if a pressure singularity occurred...
in the past, this could have potentially observable effects on supermassive black holes corresponding to redshifts around the time instance that the singularity occurred, so before 70–150 Myrs ago or for redshifts \( z \approx 0.01 \). The absence of the circular photon orbits could affect the ring of the shadow, so if scientists in the far future have available the shadows of a large sample of cosmological black holes for \( z \approx 0.01 \), if the shadows of some black holes at a specific redshift \( z_r \) are different than other black hole shadows at other redshifts, then this could indicate that a pressure singularity might have occurred at the specific redshift \( z_r \). The determination of the redshift might also determine the time at which the pressure singularity occurred.

In order to realize technically the above idea, we shall use the McVittie metric \([34–48]\). The motivation is rather simple, the expansion of the Universe at cosmological scales should somehow affect the cosmological black holes. So a more concrete approach for the shadow of the black hole of cosmological black holes should also take into account the expansion of the Universe, thus both the gravity of the black hole and the cosmic expansion at each part of the light trajectory. The shadow of a black hole is basically a dark spot in the direction of a black hole in the sky, which is viewable due to the background of other light sources nearby. The McVittie metric \([34–48]\) is the most refined description for describing a black hole embedded in an expanding Friedmann–Robertson–Walker (FRW) background. For a series of articles on the McVittie metric, see refs. \([34–48]\) and references therein. Although initially it was debatable whether the McVittie metric describes a black hole \([35]\), it is now widely accepted that indeed it describes a black hole in an expanding background \([36,44,45,47,48]\), although the accretion of matter and radiation is not allowed. The description of the McVittie solution as a black hole is further supported by the geodesically incompleteness of the McVittie metric, due to the existence of a null surface at a finite distance. Weakly gravitating systems the size of which is small compared to the comoving Hubble radius are not affected by the expansion of the Universe in a FRW Universe, however this is not the case for large scale structures. Indeed, in large scale structures and cosmological black holes, the effects of the expansion must be taken into account. Each galactic black hole, which is a supermassive one, tracks the orbit of the galaxy in the FRW spacetime, and thus the expansion of the Universe must be taken into account. The participation of even a strongly bound local object in the Universes expansion seems to be a general rule \([35]\). Hence by using the McVittie metric, we determine whether the condition which allows circular photon orbits is satisfied or not. As we demonstrate, for a spacetime with a pressure singularity, the circular photon orbits are not allowed for some time interval before the singularity, and we theorize how such a result could affect the shadow of the black holes being at a specific redshift. We discuss this perspective and we explain that technically it is hard to verify our proposal using present day’s technology, however in some decades from now, the resolution techniques will be refined and perhaps our proposal might be directly investigated experimentally.

**Photon orbits in McVittie’s black holes and pressure cosmological singularities.** – Before discussing the effects of a pressure singularity on the circular photon orbits in a McVittie black hole, let us recall the classification of finite-time spacetime cosmological singularities, following \([49]\). If the singularity occurs at the time instance \( t = t_s \), we have the following classification \([49]\):

- **Type I (“Big Rip”):** a typical crushing type singularity. As the finite-time singularity is approached at \( t \to t_s \), all the physical quantities that can be defined at the spacelike hypersurface defined by the time instance \( t = t_s \), such as the total effective pressure \( p_{\text{eff}} \) and energy density \( \rho_{\text{eff}} \), strongly diverge, including the scale factor \([50]\).
- **Type II (“sudden”):** this is known as the pressure singularity, firstly studied by Barrow in refs. \([51]\), see also \([52]\). This is the kind of singularity we shall be interested in this work, in which case as the singularity is approached, the scale factor and the energy density is finite, however the pressure diverges.
- **Type III:** for this case, as the singularity is approached, the scale factor is finite, however, both the pressure and the energy density diverge.
- **Type IV:** this is a mild singularity studied in detail in refs. \([49,53–58]\). In this case, as the singularity is approached, the scale factor, the energy density and the pressure are finite, and only the higher derivatives of the Hubble rate \( \frac{d^m H}{dt^m} \) diverge, for \( n \geq 2 \).

In a more transparent way, let us assume that the scale factor has the following simple form:

\[
a(t) \simeq c(t) + d(t)(t - t_s)^{n/2},
\]

where the functions \( c(t) \) and \( d(t) \) including their higher-order derivatives with respect to the cosmic time are finite at the cosmic time instance \( t = t_s \). Also we assume that \( \eta = \frac{2m}{n + 1} \) with \( m \) and \( n \) positive integers, in order to avoid having complex values for the scale factor. The values of \( \eta \) determine the singularity type that may occur at \( t = t_s \). The energy density is affected by the Hubble rate itself and the pressure by the energy density and the first derivative of the Hubble rate with respect to the cosmic time. The values of \( \eta \) affect the singularity type in the following way:

- For \( \eta < 0 \) a Type I singularity occurs, since the scale factor, the energy density and the pressure are divergent.
- For \( 0 < \eta < 1 \) a Type III singularity occurs.
For $1 < \eta < 2$ a Type II singularity, or pressure singularity, occurs, since only the pressure is divergent.

For $2 < \eta$ a Type IV singularity occurs.

Hence, for the pressure singularity one needs $1 < \eta < 2$, since in this case at $t = t_o$, only the derivative of the Hubble rate with respect to the cosmic time diverges. When the Universe goes through a pressure singularity, it remains geodesically complete, since the following integral takes finite values for all cosmic times [59]:

$$\int_0^\tau dt R_{0\nu0}(t).$$

(2)

However, the pressure diverges globally on the spacelike hypersurface defined by the time instances for which the singularity occurs.

Now we shall consider the effects of a pressure singularity on the circular photon orbits around black holes in an expanding spacetime. We shall discuss the impact of the absence of circular photon orbits around black holes on the shadow of a black hole, and also we shall discuss how to verify this observationally on the shadows of supermassive black holes but in the far future. Our main assumption will be that a pressure singularity occurred 70–150 Myrs ago, which corresponds to an abrupt physics change. In the spirit of ref. [8], this might explain the $H_0$-tension, since an abrupt physics change might affect directly the Cepheid parameters. So our assumption is that the pressure singularity indeed occurred before 70–150 Myrs ago, so at a redshift $z \leq 0.01$.

It is undoubtable at present day that the McVittie metric describes a black hole in a dynamically expanding FRW Universe [34–48]. The McVittie spacetime metric for a flat FRW background in geometrized units ($G = c = 1$), reads

$$ds^2 = -\left(1 + \frac{m(t)}{2r}\right)^2 dt^2 - 2\frac{m(t)}{2r} a(t)^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2\right),$$

(3)

where the function $m(t)$ is defined as follows:

$$m(t) = \frac{m_0}{a(t)},$$

(4)

where $m_0$ is the mass of the central body which is embedded in the expanding spacetime, so basically the mass of the black hole, and $a(t)$ is the scale factor of the FRW spacetime. When $a = 1$ the McVittie metric reduces to the Schwarzschild metric in isotropic coordinates, while in the limit $m_0 \rightarrow 0$, the FRW metric is recovered. Let us now consider the photon orbits in such a spacetime, and for geodesics paths on the plane $\theta = \frac{\pi}{2}$, due to the spherically symmetric spacetime, the conservation of the angular momentum yields

$$\dot{\phi} = \frac{L}{R^2}, \quad \dot{\theta} = 0,$$

(5)

where $R$ is the areal radius coordinate defined as follows:

$$R = a(t)r \left(1 + \frac{m_0}{2ra(t)}\right)^2.$$

(6)

The corresponding circular photon orbits geodesics equation reads [43]

$$\frac{L^2}{R^2} = (f^2 - g^2)^2,$$

(7)

where the functions $f$ and $g$ are defined as follows [43]:

$$f = \sqrt{1 - \frac{2m(t)}{R}}, \quad g = R \left(H + \frac{\dot{m}}{m} \left(f^{-1} - 1\right)\right),$$

(8)

with $H$ being the Hubble rate $H = \frac{\dot{a}}{a}$. The quantity $\chi(R, t) = f^2 - g^2 = g^{\mu\nu}\nabla_\mu R \nabla_\nu R$ basically defines the trapped and untrapped spacetime regions of the spherically symmetric spacetime. The condition for having circular photon orbits of radius $R_c$ for all cosmic times is [43]

$$\chi(R_c, t) = f^2 - g^2 = 1 - \frac{2m(t)}{R_c} - R_c^2 \times \left( H(t) + \frac{\dot{m}}{m(t)} \left(\frac{1}{\sqrt{1 - \frac{2m(t)}{R_c}}} - 1\right)\right)^2 > 0.$$

(9)

Obviously in the case that $\chi(t, R_c) < 0$, circular photon orbits cannot exist and as we shall now explain, this will be the case for cosmic times near a pressure cosmological singularity. In order to show this explicitly, let us assume that the scale factor of the Universe is approximately described by

$$a(t) = c + c|t|^n,$$

(10)

where $c$ is some arbitrary constant with units $[L]^{-1}$ in geometrized units and we shall assume that $\eta = \frac{2m}{2n+1}$, with $n$ and $m$ positive integers. Apparently, for values of $\eta$ satisfying the condition $1 < \eta < 2$, a pressure singularity occurs at the time instance $t = 0$. The time instance for which the singularity occurs is arbitrary, but we chose it to be $t = 0$ for convenience. This time instance $t = 0$ can be considered to be any time instance in the past of our Universe, so for example it can be before 70–150 Myrs ago. As we will now show, depending on the values of $c$ and $\eta$, the photon orbits with radii $2m < R_c \leq 3m$ in geometrized units around a black hole in an expanding Universe might not exist. For the scale factor (10) in fig. 1 we have plotted the quantity $\chi(R_c, t)$ for $R_c = 2.5 m_0$ and for $\eta = 5/2$ (left plot) and for $\eta = 2$ (right plot) taking $c = 1.5$ in units of length. The left plot describes the existence or not of circular photon orbits for the case that a pressure singularity occurs at $t = 0$. As is obvious, for a limited time interval before the singularity, the circular photon orbits do not exist, and now we shall discuss qualitatively what this result might mean for the
corresponding shadow of the black hole. The qualitative behavior presented in fig. 1 does not change for any radius values in the range $2m_0 < R_c \leq 3m_0$. What impact would the absence of circular photon orbits have on the shadow of a supermassive black hole at cosmological distances? Probably it would affect the inner part of the shadow, possibly it would affect the ring of the shadow. Thus the idea that we want to suggest with this work is simple: by observing a large sample of supermassive black holes shadows at cosmological distances with redshifts $z \leq 0.01$, is there any mentionable difference between the shadows and if yes, at which redshift are the shadows different? If such a scenario is verified, then this would be an indication that the Universe in the recent past has experienced a pressure singularity at the redshift where the singularity occurred. At the moment, the observation of a large sample of shadows at cosmological distances is rather technically limited. Indeed, the observation of the shadow of the M87 supermassive black hole [60,61] at $z = 0.004283$ is the best outcome that the current technology can achieve. Thus our proposal cannot be verified at the moment, since the technical requirements required exceed by far the current technology. Indeed, in order to capture the shadows of cosmological galactic supermassive black holes at redshifts $z \leq 0.01$, higher resolutions are required, so more refined VLBI techniques are required. Current VLBI techniques do not allow to reach higher resolutions at higher redshifts. Thus our proposal could be investigated experimentally in the far future, several decades from now. This includes also the related studies of the M87 and SgrA* which are too close to us, so their cosmological redshift is not of the order of $z \sim 0.01$, so not too large in magnitude. Moreover, considering the Centaurus A, which is an active galactic nucleus in the nearby galaxy NGC 5128 with redshift $z \sim 0.00183$, this supermassive black hole too is relatively chronologically close in the past, so the effects of a sudden past finite-time singularity would be more likely spotted on supermassive black holes corresponding to higher redshifts, of the order $z \sim 0.01$, thus these are future perspectives for our work, heavily relying on the refined VLBI techniques.

**Future perspectives.** – In this letter we considered the qualitative effects of a pressure cosmological singularity on the shadow of a galactic supermassive black hole at cosmological distances. Specifically we considered the McVittie metric and we investigated what effects a pressure singularity would have on the circular photon orbits around the supermassive black hole. As we showed, the circular photon orbits do not exist for a time interval before the singularity occurred. This feature can affect directly the shadow of the supermassive black hole, and specifically we theorized that the pressure singularity will affect all the supermassive galactic black holes at the redshift for which the circular photon orbits do not exist. Thus, if in the far future the shadow of a large sample of black holes is observed in detail for redshifts $z \leq 0.01$, it will be possible to verify experimentally our conjecture if the supermassive black holes at a specific redshift show similar characteristics, which are absent at different redshifts. This can be a direct indication that at a specific redshift in the past the Universe experienced a global physics change caused by a pressure singularity. Thus if such a scenario actually took place before 70–150 Myrs ago this could solve simultaneously the $H_0$-tension problem.

Let us now discuss the future perspectives of our theory, since for the moment it seems impossible to observe a large sample of shadows with high precision. The effect of expansion of the Universe on the nearby galactic black holes is tiny, only at cosmological distances the effect of expansion should be taken into account. Thus it is compelling to investigate cosmological black holes at cosmological distances of the order $z \leq 0.01$ in order to reveal differences between them, to pinpoint the absence of some characteristic relevant with the shadow of the black hole. This will reveal the possible effects of the pressure singularity in the past of our Universe. Precision is required though, to

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also take into account the effects of cosmic expansion, the shadow is a dynamical structure, not static [44,45,47,48], however with this paper we aimed to provide a qualitative description of the whole phenomenon. The whole procedure is a rather far future task because the current VLBI technique does not allow to reach different (higher) resolutions at higher redshifts and also at higher redshifts one must take into account the influence of the cosmic expansion on the angular diameter of the black hole shadow [42]. This might eventually cause differences between shadows at low and high redshifts, so one must also take this into account in order to pinpoint the effects of a pressure singularity. Moreover, a realistic approach should also take seriously into account the effects of the cosmic expansion on the surface brightness of light sources [42], and of course the rotation, but the main qualitative argument of this work does not change. If some difference is observed in a sample of different shadows at redshifts \( z \leq 0.01 \), this could be due to a pressure singularity occurring in the near past of our Universe at this specific redshift.

Although our proposal might be a far future proposal, the current scientific achievements are encouraging. Indeed, observations of high redshift supermassive black holes already exist in the literature [62]. Also supermassive black holes at large redshifts will have in general larger shadows [44]. Also the James Webb Space Telescope could reveal some properties of supermassive black holes at large distances. So perhaps in the next decades, scientists will be able to pinpoint differences in the shadow of supermassive black holes at redshifts \( z \leq 0.01 \). The techniques for obtaining the shadows of black holes are continuously refined [63,64], and also theoretical aspects are further studied [65].

Finally, it would be interesting to study the effects of a non-flat FRW expanding background on the McVittie metric in the case of a pressure singularity. The same analysis we performed in this paper should be repeated with non-zero spatial curvature. For astrophysical black holes, in which case the mass of the black hole or the radius of the black hole is smaller than the radius of the curvature, the effects of the curvature are negligible. However, for cosmological supermassive black holes, the effects of a non-flat expanding background should be significant. Thus it would be interesting to repeat the present study in the non-flat FRW case.

In conclusion, with this work we showed that the two shadows observed so far, the M87 [60] and the Sagittarius A* [66] seem like two identical “donuts”, the occurrence of a pressure singularity 70–150 Myrs in the past might affect the shadows of black holes at redshifts \( z \leq 0.1 \). Thus, although it is expected to see similar “donuts” in the sky, if some “donuts” are different, this might be due to a pressure singularity, an effect which solves simultaneously the \( H_0 \)-tension problem. It is furthermore interesting to note that one should include the perspective of pinpointing the modified gravity which may generate both the McVittie solutions and the past finite-time singularity, so the most generalized theory is the Horndeski theory, and recently McVittie’s solutions were considered in the context of Horndeski gravity in [67].

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