Frame structures optimization based on evolutionary modeling in active overall stability constraints

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Abstract. A method of stochastic search for rational solutions for steel frame structures based on the genetic algorithm has been developed taking into account the presence of an active constraint on structure stability. To evaluate the stability, an internal iterative cycle is introduced into decision making process during which the finite element technique equation system is solved in the presence of a geometric matrix. An example of bar optimization taking into account the loss of stability under various loading options is considered. When implementing the genetic algorithm, an adjustable mutation operator is used along with penalty function, which regulates the degree of violation of constraints on the displacements of the frame system and critical stresses acting within the bar.

1. Introduction

The problems of bar system calculation and optimization taking into account the stability are currently of great interest in the scientific literature. A number of researchers consider this field as one of the foreground. This task becomes especially relevant when considering various impacts on structures under emergency actions [1-5]. Calculation of individual bars and bar systems is performed using two main approaches: according to the Euler scheme and the deformed scheme. At that, the mathematical model based on Bernoulli hypotheses is most commonly used. Kinematic relations based on Cauchy-Green finite deformation tensor determine geometrically nonlinear behavior of the bar. In works [6, 7], optimization of frame structures by mass minimization is performed taking into account the stability in the presence of initial imperfections and constraints to the probability of non-failure operation. Independent random failure values are considered distributed according to normal rule. Optimization is based on quadratic approximation of the target function and linear approximation of constraints. Article [8] presents an algorithm for calculating and optimizing the spatial frames taking into account the stability in elastoplastic structure work. Optimization is performed by minimum volume criterion. An iteration method of successive approximations is used to search for an extremum.

Genetic algorithms are one of the promising optimization methods [9-14]. In studies of this type, the structural stability assessment is made beyond the iteration process, which may distort the optimization results. In this paper, we suggest the genetic algorithm option using the stability condition in the form of an active constraint. This approach seems promising in assessing the survivability of structural systems in case of possible emergency situations.
2. Formulation of the optimization problem

Steel frame construction under consideration. Volume \( V_0 \) of its bar material is minimized:

\[
V_0(\Omega, Y) = \sum_{i=1}^{n} A_i l_i \rightarrow \min,
\]

where \( \Omega \) is the discrete set of possible configurations (options) of the load bearing structure geometry. These set is formed on the fixed topology of the deformed object by changing the coordinates of some basic and intermediate nodes; \( Y \) is a system of discrete sets of permissible values of variable sizes of the bar cross-sections; \( n \) - the number of bars; \( A_i, l_i \) – the cross-sectional area and length of bar.

The following active (used directly in the iterative scheme) constraints are taken into account:

A). The equilibrium of finite-element model nodes. Equilibrium equation [15] will be presented as follows:

\[
([K] + [K_G]) \{\delta\} = \{R\}
\]

where \([K]\) – general system stiffness matrix; \([K_G]\) – geometric matrix; \(\{\delta\}, \{R\}\) – nodal displacements and external nodal forces vector.

B). Strength and stability of bars per requirements of norms [16].

C). Structure stiffness. Each \( j \) nod of the discretized structure in each loading requires the inequality system:

\[
\begin{align*}
&f_{xj} \leq [f_x] \\
&f_{yj} \leq [f_y] \\
&f_{zj} \leq [f_z]
\end{align*}
\]

where \( f_{xj}, f_{yj}, f_{zj} \) are a displacements of \( j \) nod in the direction of the rectangular axis \( Oxz \); \([f_x]\) – \([f_x]\) is the permissible displacements values.

D). Overall stability. It is verified on the basis of introducing the geometric matrix of the finite element model into the calculation process.

E). Unification by structures and parameters.

F). Symmetry conditions.

Passive constraints are set, checked after the evolutionary search implementation:

G). Structure geometric stability condition.

H). Providing local strength and stability of structural elements.

I). Designing constraints (the possibility of nodal connections assembling, support and load diagram conditions, etc.).

3. Methods

Let us form discrete sets of valid parameters on which optimization is performed. To synthesize geometry variants on a given structural topology, we introduce \( \Omega \) set of coordinates of the finite-element model nodes:

\[
\Omega = \left\{ \{\Omega_x\}; \{\Omega_y\}; \{\Omega_z\} \right\},
\]

where \( \Omega_x = \{x_1, x_2, ..., x_j_0\} \) – range set of coordinate \( x_1 \) (\( j_1 = 1, 2, ..., j_0 \)); \( j_0 \) – overall number of nodes; \( \Omega_y = \{y_1, y_2, ..., y_j_0\} \) – range set of coordinate \( y_1 \); \( \Omega_z = \{z_1, z_2, ..., z_j_0\} \) – range set of coordinate \( z_j \).
Decompose the set $\Omega$ to disjoint subsets as follows:

$$\Omega = \Omega_A + \Omega_B + \Omega_D,$$

where $\Omega_A$ – set of independently variable coordinates; $\Omega_B$ – set of variable coordinates, which are linear functions of the elements of the subset $\Omega_{AI} \subset \Omega_A$; $\Omega_D$ – set of variable coordinates. The following relation is used

$$\{X_B\} = \{X_{BO}\} + [T]\{X_{AI}\},$$

where $\{X_B\}, \{X_{AI}\}$ – sets of member vectors $\Omega_B$ and $\Omega_{AI}$; $\{X_{BO}\}, [T]$ – vector and matrix of invariables, considering the relationship between the coordinates.

For each $j$ element of set $\Omega_{AI}$ the finite set $\tilde{\Omega}_j$ of the correspondent coordinate values is considered, allowed for selection during optimization.

A set $\Pi$, variable cross-sectional dimensions will be presented as follows:

$$\Pi = \{\Psi_1,...,\Psi_n\}, \Psi_i = \{(b_1;\delta_1),(b_2;\delta_2),\ldots,(b_i;\delta_i)\},$$

where $\Psi_i$ is a vector of dimensions describing the cross section of $I$ bar, modeled by the rectangles system; $b, \delta$ are the rectangle width and thickness, $js$ is the number of rectangles in $i$ bar cross section.

Individual $p$ is understood to be an object variant formed by selecting values from the sets of variable parameters. These sets are sorted in descending order. At each iteration of the genetic algorithm the following groups of individuals are considered called populations:

- current ($\Phi_1$). This population has the size $N$ and is used for storage at iteration of individuals randomly generated or modified by genetic operators. These individuals are tested by the survival criterion. The lower the objective function value, the higher the survival is (1);
- elite ($\Phi_2$). The population is used for saving during the search of the best solutions (individuals with the highest survival criteria). This population may have a size not exceeding $N$.

The population $\Phi_1$ is initially filled. $N$ of similar structure options is constructed. The parameter values for them are taken as maximum. Further, an iterative procedure is performed, each iteration of which includes such steps.

1. **Computation of individuals stress-strain state in the population $\Phi_1$.** Finite-element calculation is made in elastoplastic position using multiple-layer finite elements. The package of layers in such elements works in tension-compression; the hypothesis that there is no pressure of the layers on each other is considered valid; the cross-section deplanations are not taken into account.

At first iteration, an individual is calculated in linearly elastic position. Next, the decision on the nonlinear problem is made by successive approximations method. In each iteration $s > 1$ a system of equations is solved (2), at that, the finite-element model global stiffness matrix $[K^{(s)}]$ for iteration $s$ is build taking into account crosscutting material modules that are determined for each $i$ finite element according to the results of the iteration $s-1$. Stability matrix for finite element system $[K_{c}^{(s)}]$, obtained in iteration $s$, is expressed in axial forces in bars obtained in $s-1$ iteration.

2. **Accounting of the active constraints.**

Population $\Phi_1$ is divided into two equal groups $\Phi_{11}$ and $\Phi_{12}$. If for an individual in group $\Phi_{11}$ the set active constraints are not met, then this individual is substituted by any individual from the elite
population $\Phi_2$. If the population $\Phi_2$ does not include individuals yet, then a substitute individual is randomly generated. For individuals in group $\Phi_1$, strict performance of some active constraints is not necessary. If these restrictions are violated, a penalty is introduced for each of them $P$:

$$P = |k_p - 1| V(\Omega, Y),$$

$k_p =$ 

$$\xi_2 \chi \left( \frac{\sigma}{R} - 1 \right) \left[ \frac{\sigma}{R} - 1 \right] + \xi_2 \sum_{i=0}^{3} \chi \left( \frac{\delta_i}{\delta_i} - 1 \right) \left[ \frac{\delta_i}{\delta_i} - 1 \right] + \xi_2 \chi \left( 1 - n_y \right) \left( 1 - n_y \right)$

(8)

where $\xi_1 - \xi_4$ are the set positive numbers; $\sigma$ – maximal equivalent tension von Mises stress in a bar; $R$ – steel design resistance; $\chi(x)$ – Heaviside function: ($\chi(x) = 0$, если $x < 0$; $\chi(x) = 1$, если $x \geq 0$);

$$\delta_1 = f_y, \quad \delta_2 = f_y, \quad \delta_3 = f_y, \quad \delta_4 = \left[f_x\right], \quad \delta_5 = \left[f_x\right], \quad \delta_6 = \left[f_x\right]; \quad n_y$ – safety factor of stability.

1. **Elite population modification.** For individuals of population $\Phi_1$ the values of the target function are calculated and the following criteria are verified [17]:

$$\begin{cases}
\forall \left( p_{\theta_1} \in \Phi_1 \right) - \exists \left( p_{\theta_1} = p_{\theta_2} \right); \\
V(p_{\theta_1}) \leq \max \left\{ V(p_{\theta_2}) \right\}.
\end{cases}$$

(9)

If these criteria are met, an individual from $\Phi_1$ population is copied in $\Phi_2$ population. At the same time if there are $N$ individuals in the elite population already, then the individual with the biggest volume (1) is removed from it.

3. **Checking the condition for end of optimization.** The calculations demonstrate that the optimal synthesis of bar systems with failure to fulfill the criteria (9) during 200-300 iterations of the genetic algorithm attests to the fact that with the given initial system parameters, in accordance with the target function criterion, the best solution is obtained.

4. **Mutation** [18]. This operator is only used for individuals from the population $\Phi_1$. Let the parameter $i$ of some individual $p_{\theta_1}$ has the discrete set $T_i$ of allowable values. This set has $w_i$ elements. At interval (0; 1.0) taking into account the normal distribution, random values $m_p, m_{p1}, m_{p2}$ are generated that are compared with control numbers of mutation $m, m_1, m_2$. With $m_p \geq m$ the number of the value in the set $T_i$ is chosen randomly in the whole interval $[1; w_i]$. With $m_p < m$, cases presented in table 1 are considered:

| Number of the value in set $T_i$, prior to mutation | Actions at $m_{p1} < m_1$ | Actions at $m_{p1} \geq m_1$ | Actions at $m_{p2} < m_2$ | Actions at $m_{p2} \geq m_2$ |
|---|---|---|---|---|
| 1 | +1 | +2 | - | - |
| 2 | -1 | +1 | +2 | - |
| 3 | -2 | -1 | +1 | +2 |
| 4 | -2 | -1 | +1 | +2 |
| ... | ... | ... | ... | ... |
| $w_i - 2$ | -2 | -1 | +1 | +2 |
There, the values -2; -1; +1; +2 are whole numbers, to the value of which the number of the parameter value in the set $T_i$ will be changed.

5. Selection and crossover. The selection operator is applied to individuals from the population $\Phi_2$ by roulette-wheel selection method [19]. If there are no individuals in this population, then the choice is made from the population $\Phi_1$. Individual survival rate, which is estimated by the value of the target function is adopted as a criterion for the most probable selection of an individual for further changes. In this way, the improvement of object options from iteration to iteration is achieved. The crossover (exchange of parameter values) is implemented as a single-point operator. The conditions for selection of individuals for a purpose of crossover from groups $\Phi_1$ and $\Phi_2$ are adopted as the same.

To perform structural optimization under multivariate loading, the following steps are necessary to be performed:

1. Perform an evolutionary structural search at first loading $L_1$ and obtain optimal values vectors $\bar{\Omega}_1, \bar{Y}_1$, selected from the sets $\Omega_{t_1}, Y_{t_1}$.

2. Perform optimization for loading $L_2$, at that, the discrete set system $\Omega_{t_2}, Y_{t_2}$ is formed under the condition that the minimum allowable values of the vectors $\Omega_{2,\min}, \bar{Y}_{2,\min}$ will be respectively equal to the values of the vectors $\bar{\Omega}_1, \bar{Y}_1$:

$$\Omega_{2,\min} = \bar{\Omega}_1, \Omega_{2,\min} \in \Omega_{t_2}.$$  
$$\bar{Y}_{2,\min} = \bar{Y}_1, \bar{Y}_{2,\min} \in Y_{t_2}. \tag{10}$$

3. In accordance with the dependence (1-9), perform optimizations for all considered loadings. For optimization, taking into account the last loading $L_n$, it can be written as follows:

$$\Omega_{n,\min} = \bar{\Omega}_{n-1}, \Omega_{n,\min} \in \Omega_{t_n},$$  
$$\bar{Y}_{n,\min} = \bar{Y}_{n-1}, \bar{Y}_{n,\min} \in Y_{t_n}. \tag{11}$$

4. Results

The bar stability calculation. The considered bar is shown in Figure 1. Bar length $L=6 m$. The calculations were performed in a linear formulation. Two variants of calculations for the following loadings were considered: in the first, the action of force $P_1$ was taken into account $P_1$, in the second – of forces $P_1$ and $P_2$. Bar material – steel C235 with elastic modulus $E = 2.06 \cdot 10^5$ MPa. The cross section is taken constant by length in the form of a composite welded I-beam with the following dimensions: web – 16x1.5 cm, flanges – 20x2 cm.

The results are presented in table 2. The search for load peak values was carried out by the selection method. The transition of the system to a state close to the formation of a mechanism is taken as a criterion for stability loss (the determinant of the stiffness matrix is close to or equal to zero).
Analysis of the table shows that the results within the error margin correspond to those obtained in the finite element complex "NX Nastran".

| Loading | Calculation in software «Bgitafem» (developed algorithm), kN | Calculation in «NX Nastran» according to the buckling form, kN | Divergence |
|---------|---------------------------------------------------------------|---------------------------------------------------------------|------------|
| $P_1 \neq 0$ | $P_{1cr} = 1330$ | $P_{1cr} = 1450$ | 10.3% |
| $P_1 \neq 0$ | $P_{1cr} = 185$ | $P_{1cr} = 210$ | 11.9% |
| $P_2 = 0.5P_1$ | $P_{2cr} = 92.5$ | $P_{2cr} = 105$ |  |

The bar optimization. Two optimization cases were considered. The first (A) with force loading $P_1 = 1500$ kN, second (B) – at $P_1 = 1200$ kN, $P_2 = 500$ kN. Combinations of the sizes of I-section in the bar elements $l_1 - l_6$ shown in Figure 2 varied independently. Symmetry conditions were not taken into account.

The coordinates of node $u_3$ varied between allowable levels $a, b$. The coordinates of nodes $u_1, u_2, u_4$ were considered linearly independent from coordinate $u_3$ in accordance with the formula (6), at that $u_1 = u_5, u_2 = u_4$. For the node $u_3$ the set $\Omega_{u3} = \{-200; -175; -150; -125; -100; -75; -50; -40; -30; -20; -10; 0; 10; 20; 30; 40; 50; 75; 100; 125; 150; 175; 200\}, a = 200, b = -200, cm.

The sets of permissible combinations of dimensions of the bars cross sections are presented in table 3.

| Combination of sizes for a form | Sizes, sm |
|---------------------------------|-----------|
|                                 | $h_1$ | $\delta_1$ | $h_2$ | $\delta_2$ |
| 1                               | 10   | 1.0   | 8     | 0.6 |
| 2                               | 20   | 1.2   | 18    | 0.6 |
| 3                               | 25   | 1.6   | 23    | 0.8 |
In populations $\Phi_1$ и $\Phi_2$, during the evolutionary search, 20 individuals were examined. When assessing the efficiency of each individual, 10 internal iterations were used in which equation systems were solved (2). The optimization results are presented in Figure 3.

For each optimization run, an average of 600 calculations of structural variants was performed. A exhaustive search of options is associated with 359375 calculations. In order to assess the accuracy of the optimization technique for the structures shown in Figure 3, verification calculations were performed for stability according to the buckling mode in NX Nastran software package. The calculation results are presented in Table 4.

**Table 4. Obtained results verification**

| Optimization case | Stability ratio | Divergence |
|-------------------|-----------------|------------|
|                   | Software «Bgitaferm» | NX Nastran |         |
| (A)               | 0.92            | 0.81       | 11.82%   |
| (B)               | 0.87            | 0.77       | 12.56%   |

5. Discussion

Taking into account the stability of bar systems as an active constraint in the search for a solution significantly affects the optimization results. At the same time, an assessment of the overall stability for the bar structures only appears to be insufficient, verification of the local bar stability is also necessary. Such verification is typical for trusses, for example. When evaluating stability for a geometrically nonlinear calculation, 10 iterations were set. This value was adopted by us on the basis of examination of a work [15], where it is proved that the convergence of the nonlinear calculation is achieved in 3-5 iterations.

6. Conclusion

A methodology for optimizing steel bar structures based on evolutionary modeling, taking into account active constraints on strength, stiffness and stability has been developed. Adaptation of this technique to calculating the stability of reinforced concrete frame structures of buildings under emergency impacts seem promising.

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