Inflamagnetogenesis redux: Unzipping inflection-point inflation via various cosmoparticle probes

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In this paper I introduce a precise constraint on primordial magnetogenesis, for a generic class of single-field inflection point inflationary model followed by small field excursion below the Planck scale. I also establish a connection between the magnetic field at the present epoch and primordial gravity waves ($r$) via non-vanishing CP asymmetry parameter ($\epsilon_{\text{CP}}$), which triggers the leptogenesis scenario. Finally, I explore various hidden cosmophenomenological features of theoretical CMB B-mode polarization spectra, which can be treated as a significant probe to put further stringent constraint on low and high scale inflection point inflationary models after releasing the Planck B-mode polarization data.

In principle the CMB polarization field is conventionally decomposed into a sum of parity-even curl-free “E-mode” and parity-odd gradient free “B-mode” contributions [1]; since the B-mode polarization is absent from linear order scalar density perturbations which are the dominant source of CMB temperature and E-mode anisotropies, serves as a powerful test of various hidden aspects of cosmological physics, including primordial tensor and vector perturbations. The other complementary origin of the B-type polarization is Faraday rotation of the orientation of linear polarization due to the presence of a primordial magnetic field. In such a physical scenario if a purely “E-mode” polarization pattern is rotated by an angle, $\theta_F = \pi/4$ then it transforms into a purely “B-mode” polarization and for $\theta_F << \pi/4$, it will generate a component of the total B-type polarization. Besides the Faraday rotation effect also induces non-zero parity-odd cross correlations of type TB and EB, which are absent for standard cosmological scenarios. Primordial magnetic fields are interesting topics in the present day research in “particle cosmology” due to the three prime reasons:⇒ (1) they could serve as seed for the observed magnetic fields in galaxies and galaxy clusters, ⇒ (2) they can be treated as the clean probe of signal of primordial tensor perturbations, the detection of which would directly constrain the scale of inflation and ⇒ (3) also acts as a probe of gravitational lensing.

The earlier results obtained from the various famous CMB experiments by Planck, WMAP9 along with various combined constraints [2–4] have placed upper limits either on the B-mode polarization anisotropy or on the tensor-to-scalar ratio obtained from the inflationary picture. Very recently SPT confirms the first detection signatures of B modes sourced by gravitational lensing [5]. In CMB the B-modes is made up of primordial gravity waves [6], non-Gaussian fluctuations in the primordial fluctuations [7] and the contribution from the gravitational lensing. So the detection of B-modes via lensing also indirectly confirms the signature of the primordial gravity waves, provided we should need to separate the contribution of the primordial non-Gaussianity. However as the amount of non-Gaussian fluctuations are very small as predicted by the recent observation from Planck [8], I, henceforth, neglect such contribution from my theoretical analysis. In Fig (1) I have explicitly shown the schematic representation of the significant components of the CMB B mode polarization. Further in Fig (2) I have depicted the proposed cosmophenomenological technique to determine theoretical CMB B-mode polarization spectra from BB correlation using the models of inflection point inflationary setup below the Planck scale.

FIG. 1: Schematic diagram of the components of CMB B mode polarization.

The aim of this paper to present a precise constraint on primordial magnetogenesis, for a generic class of single-
field inflow point inflationary model followed by small field excursion characterized by two features \[9-12\], \(\phi_0 \leq M_p\) - vev of the inflaton must be bounded by the cut-off of the particle theory and \(\Phi \leq M_p\) - the inflaton potential has to be flat enough during which a successful inflation can occur. Using this sub-Planckian small field excursion criteria in this paper I first establish a connection between the magnetic field at the present epoch and primordial gravity waves (r) via non-vanishing CP asymmetry parameter (\(\epsilon_{CP}\)) from which I further explore various hidden cosmophenomenological features of CMB B-mode polarization spectra which can be useful to discriminate and to rule out various classes of inflationary models below the Planck scale.

A Gaussian random magnetic field for a statistically homogeneous and isotropic system is described by the equal time two-point correlation function in momentum space as \[13, 14\]:

\[
\langle B_i^\ast (\kappa, \eta)B_j (\kappa, \eta) \rangle = (2\pi)^3 \delta^{(3)}(\kappa - \hat{k}) P_{ij}(\hat{k}) P_B (k),
\]

where the plane projector onto the transverse plane is defined as \[13, 14\]:

\[
P_{ij}(\hat{k}) = \sum_{\lambda=\pm 1} e_i^\lambda (\hat{k}) e_j^{-\lambda}(\hat{k}) = (\delta_{ij} - \hat{k}_i \hat{k}_j)
\]

in which the divergence-free nature of the magnetic field is imposed via the orthogonality condition, \(\hat{k}_i \hat{e}_i^{\pm 1} = 0\). Here \(\hat{k}_i\) signifies the unit vector which can be expanded in terms of spin spherical harmonics. See \[13\] for details. Additionally, it is worthwhile to mention that in the present context, \(P_B (k)\) be the part of the power spectrum for the primordial magnetic field which will only contribute to the cosmological perturbations for the scalar modes and the Faraday Rotation at the phase of decoupling \[13, 15\].

The non-helical part of the primordial magnetic power spectrum is parameterized within the upper and lower cut-off momentum scale \((k_L \leq k \leq k_A)\) as:

\[
P_B (k) = A_B \left( \frac{k}{k_L} \right)^{n_B + \frac{2B}{3} \ln (\frac{k}{k_L}) + \frac{2B}{3} \ln^2 (\frac{k}{k_L}) + \cdots}
\]

where \(A_B\) represents amplitude of the magnetic power spectrum, \(n_B\) is the spectral tilt, \(\alpha_B\) is the running and \(\kappa_B\) be the running of the spectral tilt. Here the upper cut-off momentum scale \((k_A)\) corresponds to the Alfvén wave damping length-scale, representing the dissipation of magnetic energy due to the generation of magnetohydrodynamic (MHD) waves. Additionally, \(k_s\) being the pivot or normalization scale of momentum. It is important to note that the most recent observational constraint from CMB temperature anisotropies on the amplitude and the spectral index of a primordial magnetic field has been predicted by using Planck data as \(B_1^{\text{Mpc}} < 3.4 \text{mG}\) with \(n_B < 0\) \[3\]. If, in near future, Planck or any other observational probes can predict the signatures for \(\alpha_B\) and \(\kappa_B\) in the primordial magnetic power spectrum (as already predicted in case of primordial scalar power spectrum within \(1.5 - 2\sigma\) CL \[2\]), then it is possible to put further stringent constraint on the various classes of inflationary models.

I am now interested in the mean square amplitude of the primordial magnetic field on a given characteristic scale \(\xi\), on which I smooth the magnetic power spectrum using a Gaussian filter as given by \[14\]:

\[
B_\xi^2 = \langle B_i (x) B_i (x) \rangle_\xi = \frac{1}{2\pi^2} \int_0^{\infty} \frac{dk}{k_s} \left( \frac{k}{k_s} \right)^2 P_B (k) \exp \left( -k^2 \xi^2 \right).
\]

Here Eq (3) describes a more generic picture where the magnetic power spectrum deviates from its exact power law form in presence of logarithmic correction. Consequently, the resulting mean square primordial magnetic field is logarithmically divergent in both the limits of the integral as presented in Eq (4). To remove the divergent contribution from the mean square amplitude of the primordial magnetic field, I introduce here cut-off regularization technique in which I have re-parameterized the integral in terms of regulated UV (high) and IR (low) momentum scales. Most importantly the cut-offs \(k_A\) and \(k_L\) are momentum regulators to collect only the finite contributions from Eq (4). Finally I get:
where the regularized integral function $I_\xi(k_L; k_\lambda)$ is explicitly mentioned in the appendix. On the other hand, in absence of any Gaussian filter, the magnetic energy density can be expressed in terms of the mean square primordial magnetic field as [14]:

$$\rho_B = \frac{1}{8\pi} \langle B_i(x) B_i(x) \rangle = \frac{1}{8\pi^2} \int_0^\infty \frac{dk}{k_*} \left( \frac{k}{k_*} \right)^2 P_B(k)$$  \hspace{1cm} (6)$$

where I use Eq (5). Here the regularized integral function $J(k_L; k_\lambda)$ is explicitly written in the appendix.

Now to derive a phenomenological constraint here I further assume the fact that the primordial magnetic field is made up of relativistic species. In this physical prescription, the regularized magnetic energy density can be expressed as [16]:

$$\rho_B(k_L; k_\lambda) \sim \frac{\pi^2}{30} g_* T^4 \sim \mathcal{O}(10^{-13}) \times \frac{T^4}{\epsilon_{CP}}$$  \hspace{1cm} (8)$$

where the CP asymmetry parameter is defined as:

$$\epsilon_{CP} = \frac{\Gamma_L(N_R \rightarrow L_i \Phi^c) - \Gamma_L(N_R \rightarrow L_i^c \Phi)}{\Gamma_L(N_R \rightarrow L_i \Phi) + \Gamma_L(N_R \rightarrow L_i^c \Phi)} \lesssim \mathcal{O}(1|\lambda|^2)$$  \hspace{1cm} (9)$$

for the standard leptogenesis scenario [17, 18] where the Majorana neutrino ($N_R$) decays through Yukawa matrix interaction ($\lambda$) with the Higgs ($\Phi$) and lepton ($L$) doublets. Now combining Eq (7) and Eq (8) I derive the following simplified expression for the root mean square value of the primordial magnetic field at the present epoch in terms of the CP asymmetry parameter ($\epsilon_{CP}$) as:

$$B_0 \sim \mathcal{O}(10^{-14}) \times \frac{\sqrt{I_\xi(k_L = k_0; k_\lambda)}}{J(k_L = k_0; k_\lambda) \epsilon_{CP}}$$  \hspace{1cm} (10)$$

where I use the temperature at the present epoch $T_0 \sim 2 \times 10^{-4}$ eV and 1 Gauss $= 7 \times 10^{-20}$ GeV$^2$. In addition here I fix the IR cut-off scale at the present epoch. Consequently the momentum integrals satisfy the following constraint:

$$\sqrt{\frac{I_\xi(k_L = k_0; k_\lambda)}{J(k_L = k_0; k_\lambda)}} \sim 10^{-8}.$$  \hspace{1cm} (11)$$

The conformal symmetry of the quantized electromagnetic field breaks down in curved space-time which is able to generate a sizable amount of magnetic field during a phase of slow-roll inflation. Such primordial magnetism is characterized by the renormalized mean square amplitude of the primordial magnetic field at leading order in slow-roll approximation for comoving observers as [19]:

$$\rho_B(k_L; k_\lambda) = \frac{1}{8\pi} \langle B_i(x) B_i(x) \rangle \approx \frac{V^2 \epsilon_V}{2160 \pi^3 M_p^4}$$  \hspace{1cm} (12)$$

where the generic inflationary potential, $V(\phi)$ can be expanded around the vicinity of sub-Planckian VEV of inflation, $\phi_0(< M_p)$ by imposing the flatness condition, $V''(\phi_0) \approx 0$ as [9–11]:

$$V(\phi) = \alpha + \beta (\phi - \phi_0) + \gamma (\phi - \phi_0)^3 + \kappa (\phi - \phi_0)^4 + \cdots$$  \hspace{1cm} (13)$$

where $\alpha$ denotes the height of the potential which sets the scale of inflation, and the coefficients $\beta$, $\gamma$, $\kappa$ determine the shape of the potential in terms of the model parameters for which the model is fully embedded within a particle theory such as that of gauge invariant flat directions of Minimal Supersymmetric Standard Model (MSSM), or MSSM$\otimes U(1)_B-L$. Additionally in Eq (12), $\epsilon_V = (M_p V'/\sqrt{2}V)^2$ is the standard potential dependent slow-roll parameter and $M_p \sim 2.43 \times 10^{18}$ GeV be the reduced Planck mass. It is important to note that Eq (12)
is insensitive to the intrinsic ambiguities of renormalization in curved space-times. In general, one can expand $\epsilon_V$ at the momentum scale, $k_L \leq k \leq k_\Lambda$ around the pivot/normalization scale $k_* (\sim 0.002 \text{ Mpc}^{-1})$ as:

$$
\epsilon_V(k) = \epsilon_V(k_*) - \frac{\alpha_T(k_*)}{2} \ln \left(\frac{k}{k_*}\right) + \frac{\kappa_T(k_*)}{4 \ln^2 \left(\frac{k}{k_*}\right)} + \cdots
$$

(14)

where $\alpha_T(k_*)$ and $\kappa_T(k_*)$ are the running and running of the tensor spectral tilt evaluated at pivot $k = k_*$ scale and contributes in the next to leading order of the effective theory below UV cut-off. See the appendix where I have mentioned the inflationary consistency conditions which will contribute in Eq (14).

Let me fix the momentum scale at the pivot $k = k_*$ at which I can write,

$$
\epsilon_V(k_*) \approx \frac{r(k_*)}{16} \left[1 - 2C_E \left(\eta_V(k_*) + \frac{r(k_*)}{8}\right)\right] + \cdots,
$$

(15)

Finally using this constraint along with Eq (7) in Eq (12) I get the following simplified expression for the root mean square value of the primordial magnetic field in terms of the tensor-to-scalar ratio $r$ and slow-roll parameter $\eta_V$ (defined in appendix) as:

$$
V_s \leq (1.96 \times 10^{16} \text{GeV})^4 \left(\frac{r(k_*)}{0.12}\right),
$$

(16)

Finally my prime objective is to study how the derived relations in our paper further put a stringent constraint on the CMB BB polarization power spectra. To achieve this goal I start with a comoving radiation frequency $\nu_0$ in which we consider the Faraday rotation angle $\alpha(\hat{n})$ of the CMB linear polarization as a function of sky direction $\hat{n}$ along with the restriction that the full sky average of the rotation angle is zero. Now expanding the Faraday rotation angle in terms of spherical harmonics which leads to the definition of the angular power spectrum $C^\alpha_{\ell}$

which is pointing towards the following possibilities:

$\Rightarrow (1)$ For the large tensor-to-scalar ratio the significant features of CP asymmetry can be possible to detect in colliders experiments, $\Rightarrow (2)$ For very small tensor-to-scalar ratio the CP asymmetry is largely suppressed and can’t be possible to detect in the particle colliders. If, in near future, any direct/indirect observational probe detects the signatures of primordial gravitational waves by measuring large detectable amount of tensor-to-scalar ratio then it will follow the first possibility. In the first case it is highly possible to achieve the upper bound of CP asymmetry parameter, $\epsilon_{\text{CP}} \leq 10^{-6}$ by fixing the lower bound of the tensor-to-scalar ratio at very small value, $r(k_*) \sim 10^{-29}$ at the pivot scale $k_* \sim 0.002 \text{ Mpc}^{-1}$ to accommodate Majorana neutrino at the scale of $10^{10}$ GeV.
via the two-point correlation [21, 22]:

\[
\langle \alpha(\mathbf{n})\alpha(\mathbf{n}') \rangle = \sum_l \frac{2l+1}{4\pi} C^\alpha_l P_l(\mathbf{n} \cdot \mathbf{n}'),
\]

where \( P_l(\mathbf{n} \cdot \mathbf{n}') \) be the Legendre polynomial of order \( l \) and I have:

\[
C^\alpha_l \approx \frac{9l(l+1)A_B}{128\pi^3\alpha_{EM}\nu_0^2} \int_{k_L=\kappa_0}^{\kappa}\frac{dk}{k^2} \left( \frac{k}{\kappa} \right)^{n_B+2+\frac{\alpha_B}{2}\ln\left( \frac{\kappa}{\nu_0} \right)+\frac{\alpha_B}{4}\ln^2\left( \frac{\kappa}{\nu_0} \right)+\cdots} \left( \frac{\mathcal{J}_l(k\nu_0)}{k\nu_0} \right)^2 = \frac{9l(l+1)B_0^2\mathcal{M}(k_0, k_{\Lambda})}{64\pi^3\alpha_{EM}\nu_0^2 I_5(k_0; k_{\Lambda})} \text{ (20)}
\]

where \( \mathcal{M}(k_0, k_{\Lambda}) \) be the corresponding integral kernel, \( \alpha_{EM} \approx 1/137 \) be the fine structure constant and \( \nu_0 \) be the comoving frequency of the observed radiation. Finally, using Eq (20) the B-mode polarization angular power spectrum induced by the Faraday rotation field from the primordial E-mode polarization can be expressed as:

\[
C_{l}^{BB} = \frac{9B_0^2\mathcal{M}(k_0, k_{\Lambda})l(l+1)(l-2)!}{256\pi^4\alpha_{EM}\nu_0^2 I_5(k_0; k_{\Lambda})(2l+1)(l+2)!} \sum_{l_1,l_2} \left\{ \frac{l_1(l_1+1)(2l_1+1)(2l_2+1)(l_2-2)!}{(l_2+2)!} \right\} \left[ l_1^2(l_1+1)^2 - 2l_1^2(l_1+1)(l_2+1) - l_2^2(l_2+1)(l_1+1) + 2 \{ l_1(l_1+1) - l_2(l_2+1) - l(l+1) \} \right] C_{l_1}^{EE} \left( \Delta_{l_0; l_2} \right)^2 \text{ (21)}
\]

where the regularized angular power spectrum for the primordial E-mode polarization can be expressed as:

\[
C_{l_2}^{EE} = 16\pi^2 P_\mathcal{S}(k_\Lambda) \int_{k_L=\kappa_0}^{\kappa}\frac{dk}{k^2} \Theta_{E l_2}^2(k) \left( \frac{k}{\kappa} \right)^{n_S(k_\Lambda)+1+\frac{n_B(k_\Lambda)}{2}\ln\left( \frac{k}{\nu_0} \right)+\frac{n_B(k_\Lambda)}{4}\ln^2\left( \frac{k}{\nu_0} \right)+\cdots} = 16\pi^2 P_\mathcal{S}(k_\Lambda) \vartheta(k_0, k_{\Lambda}). \text{ (22)}
\]

where \( \vartheta(k_0, k_{\Lambda}) \) be the corresponding integral kernel. Here the subscript \( \mathcal{S} \) represents the scalar modes and the momentum dependent integral function [23],

\[
\Theta_{E l_2}(k) = \int_{0}^{\nu_0} d\eta \ K_E(k, \eta) \mathcal{P}_{E l_2}(k(\nu_0-\eta)), \text{ (23)}
\]

where \( K_E(k, \eta) \) characterizes the physical source kernel and \( \mathcal{P}_{E l_2}(k(\nu_0-\eta)) \) represents the geometric projection factor which can be expressed in terms of the spherical Bessel functions. Also \( \Delta_{l_0; l_2} \) are Clebsch-Gordan coefficients can be written in a closed form as [24]:

\[
\Delta_{l_0; l_2} = \frac{2(-1)^{l_1+l_2}l_1!}{(n-l_1)!l_1!l_2!(n-l_2)!(l_1+l_2)!} \text{ (24)}
\]

which is valid for \( l_1 + l_2 + l = 2n \), with \( n \) be a positive integer; \( l_1, l_2 \) and \( l \) must also satisfy the triangle inequalities. In Eq (21) the B-mode polarization angular power spectrum is constrained as the magnetic field at the present epoch \( B_0 \) satisfies all the previously derived constraints in terms of tensor-to-scalar ratio \( (r) \) and CP asymmetry parameter (\( \epsilon_{CP} \)). The modifications to the existing E-polarization power spectrum and the cross-correlation between E-polarization and temperature are negligible for small rotation angles, and consequently I ignore the contribution from the second-order effects as they are sufficiently small in the prescribed analysis presented in this paper.

In Fig (3) I have plotted the behaviour of the CMB B mode angular power spectra from BB correlation with respect to the multipole. Using Eq (21) I have obtained the blue and green dotted theoretical curves which correspond to the sub-Planckian inflection point models of inflation with low scale, \( V_*^{1/4} = 6.48 \times 10^8 \text{GeV} \) (with \( r_* \sim 10^{-29} \)) and high scale, \( V_*^{1/4} = 1.96 \times 10^{16} \text{GeV} \) (with \( r_* \sim 0.12 \)) at pivot scale of momentum, \( \kappa_* = 1 \).
FIG. 3: Variation of CMB B mode angular power spectra \((l(l+1)C^{BB}_l/2\pi)\) with respect to multipole \(l\). The blue and green dotted curves correspond to the sub-Planckian inflection point models of inflation with low scale, \(V_1^{1/4} = 6.48 \times 10^8\) GeV (with \(r_s \sim 10^{-29}\)) and high scale, \(V_2^{1/4} = 1.96 \times 10^{10}\) GeV (with \(r_s \sim 0.12\)) at momentum pivot scale \(k_s = 0.002\) Mpc\(^{-1}\). For both blue and green curves, the numerical value of the magnetic field at the present epoch is, \(B_0 \sim 10^{-9}\) Gauss. But for them the CP asymmetry parameter is different, \(\epsilon_{\text{CP}} \gtrsim 10^{-16}\) (for blue curve) and \(\epsilon_{\text{CP}} \gtrsim 10^{-27}\) (for green curve). The red dotted curve corresponds to the B mode caused due to the cosmic weak lensing. The bounded region between the blue and green dotted curves are the theoretically allowed region of generic inflection point inflationary models. For an example, within the framework of MSSM the inflection point technique holds good below the super-Planckian VEV, \(\phi_0\).

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To summarize, in the present article, I have established a generic connecting relationship between inflationary magnetogenesis and primordial gravitational waves via tensor-to-scalar ratio and CP asymmetry parameter for a generic sub-Planckian model of inflation with a flat potential, where inflation is driven near an inflection-point. For such a class of model it is also possible to predict amount of magnetic field at the present epoch by measuring CP asymmetry parameter or the tensor-to-scalar ratio. Most significantly, once the signature of primordial gravity waves will be predicted, it will be possible to comment on the associated CP asymmetry and vice versa. I have used important constraints arising from Planck on amplitude of power spectrum, spectral tilt and its running within \(1.5\sigma-2\sigma\) statistical CL. To this I have shown the behaviour of theoretical CMB B mode polarization power spectra for low and high scale inflection point models of inflation within \(10^{-29} \leq r_{0.002} \leq 0.12\), which is also consistent with the upper bound of tensor-to-scalar ratio obtained from Planck data. Further my aim is to carry forward this work in a more broader sense where I will extract the contribution from gravitational lensing and non-Gaussian contribution from the primordial magnetic field from the generic sub-Planckian inflationary setup expected to be reported shortly [33].

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Appendix

A. Momentum Integrals:
Additionally the momentum integrals used in

\[
I_\xi(k_L, k_\Lambda) := \left[ \frac{\sqrt{\pi} \, Erf \left( \frac{\xi}{k_L} \right)}{2 \xi k_s} \right] Q \ln \left( \frac{k}{k_s} \right) + \mathcal{F} \ln^2 \left( \frac{k}{k_s} \right) + \left( \frac{k}{k_s} \right) \left[ 2 \mathcal{F} \, \mathcal{P} F \left\{ \begin{array}{c} 1 \quad 1 \\ 2 \quad 2 \end{array} \right\} + \begin{array}{c} 3 \quad 3 \\ 2 \quad 2 \end{array} \right] \right] \bigg|_{k = k_\Lambda}^{k = k_L},
\]

\[
J(k_L, k_\Lambda) := \left[ (2 \mathcal{F} - \mathcal{P} + Q) \left( \frac{k}{k_s} \right) - (2 \mathcal{F} - \mathcal{P}) \right] \ln \left( \frac{k}{k_s} \right) + \mathcal{F} \ln^2 \left( \frac{k}{k_s} \right) \bigg|_{k = k_\Lambda}^{k = k_L}.
\]

where \(Q = n_B + 3\), \(\mathcal{P} = \alpha_B/2\) and \(\mathcal{F} = \kappa_B/6\).

Eq (5,7,10,17) can be written as:

\[ (25) \]

**B. Inflationary Consistency Conditions:**

In this paper we use the following slow-roll consistency relations at the pivot scale \(k_s\):

\[ (26) \]

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