Second-order statistics of selection macro-diversity system operating over Gamma shadowed $\kappa$-$\mu$ fading channels

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Abstract
In this article, infinite-series expressions for the second-order statistical measures of a macro-diversity structure operating over the Gamma shadowed $\kappa$-$\mu$ fading channels are provided. We have focused on MRC (maximal ratio combining) combining at each base station (micro-diversity), and selection combining (SC), based on output signal power values, between base stations (macro-diversity). Some numerical results of the system's level crossing rate and average fading duration are presented, in order to examine the influence of various parameters such as shadowing and fading severity and number of the diversity branches at the micro-combiners on concerned quantities.

1 Introduction
Wireless channels are simultaneously affected by short-term fading and long-term fading (shadowing) [1]. Shadowing is the result of the topographical elements and other structures in the transmission path such as trees, tall buildings, etc. Short-term fading (multipath) is a propagation phenomenon caused by atmospheric ducting, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings. By considering important phenomena inherent to radio propagation, $\kappa$-$\mu$ short-term fading model was recently proposed in [2], as a model which describes the short-term signal variation in the presence of line-of-sight (LoS) components, and includes Rayleigh, Rician, and Nakagami-$m$ fading models as special cases [3]. An efficient method for reducing short-term fading effect at micro-level (single base station) with the usage of multiple receiver antennas is called space diversity. Upgrading transmission reliability without increasing transmission power and bandwidth while increasing channel capacity is the main goal of space diversity techniques. There are several principal types of space combining techniques that can be generally performed by considering the amount of channel state information available at the receiver [4-6].

While short-term fading is mitigated through the use of diversity techniques typically at a single base station (micro-diversity), the use of such micro-diversity approaches alone will not be sufficient to mitigate the overall channel degradation when shadowing is also concurrently present. Since they coexist in wireless systems, short- and long-term fading conditions must be simultaneously taken into account. Macro-diversity reception is used to alleviate the effects of shadowing, where multiple signals are received at widely located base stations, ensuring that different long-term fading is experienced by these signals [7,8]. At the macro-level, selection combining (SC) is used as a basically fast response handoff mechanism that instantaneously or, with minimal delay chooses the best base station to serve mobile based on the signal power received [9].

The performance analysis of diversity systems operating over $\kappa$-$\mu$ fading channels is rather scarce in the literature [10,11]. In [10], standard performance measures of maximal ratio combining (MRC) in the presence of $\kappa$-$\mu$ fading were discussed. Analytical expressions for the switching rate of a dual branch SC in $\kappa$-$\mu$ fading were derived in [11]. Macro-diversity over the shadowed fading channels was discussed by several researches [7-9,12]. Discussions
about the second-order statistics of various diversity systems can be easily found in the literature [13-15]. Second-order statistics analysis of macro-diversity system operating over Gamma shadowed Nakagami-m fading channels was recently proposed in [16]. Moreover, to the best knowledge of the authors, no analytical study investigating the second-order statistics of macro-diversity system operating over Gamma shadowed $\kappa$-$\mu$ fading channel has been reported in the literature.

This article delivers infinite-series expressions for level crossing rate (LCR) and average fading duration (AFD) at the output of SC macro-diversity operating over the Gamma shadowed $\kappa$-$\mu$ fading channels. Macro-diversity system of SC type consists of two micro-diversity systems and the selection (switching) is based on their output signal power values. Each micro-diversity system is of MRC type with an arbitrary number of branches in the presence of $\kappa$-$\mu$ fading. Received signal powers of the micro-diversity output signals are modelled by statistically independent Gamma distributions. Numerical results for these second-order statistical measures are also presented in order to show the influence of various parameters such as shadowing and fading severity and the number of the diversity branches at the micro-combiners on the system's statistics.

2 System model
The $\kappa$-$\mu$ distribution fading model corresponds to a signal composed of clusters of multipath waves, propagating in a nonhomogeneous environment. The phases of the scattered waves are random and have similar delay times, within a single cluster, while delay-time spreads of different clusters are relatively large. It is assumed that the clusters of multipath waves have scattered waves with identical powers, and that each cluster has a dominant component with arbitrary power. This distribution is well suited for LoS applications, since every cluster of multipath waves has a dominant component (with arbitrary power). The $\kappa$-$\mu$ distribution is a general physical fading model which includes Rician and Nakagami-m fading models as special cases (as the one-sided Gaussian and the Rayleigh distributions) since they also constitute special cases of Nakagami-m. $\kappa$ parameter represents the ratio between the total power of dominant components and the total power of scattered components. Parameter $\mu$ is related to multipath clustering. As $\mu$ decreases, fading severity increases. For the case of $\kappa = 0$, the $\kappa$-$\mu$ distribution is equivalent to the Nakagami-m distribution. When $\mu = 1$, the $\kappa$-$\mu$ distribution becomes the Rician distribution with $\kappa$ as the Rice factor. Moreover, the $\kappa$-$\mu$ distribution fully describes the characteristics of the fading signal in terms of measurable physical parameters [2].

Let us consider macro-diversity system of SC type which consists of two micro-diversity systems with switching between the base stations based on their output signal power values. Each micro-diversity system is of MRC type with an arbitrary number of branches in the presence of $\kappa$-$\mu$ fading. The optimal combining technique is MRC [4]. This combining technique involves co-phasing of the useful signal in all branches, multiplication of the received signal in each branch by a weight factor that is proportional to the estimated ratio of the envelope and the power of that particular signal and the summing of the received signals from all antennas. By co-phasing, all the random phase fluctuations of the signal that emerged during the transmission are eliminated. For this process, it is necessary to estimate the phase of the received signal, so this technique requires all the amount of the channel state information of the received signal, and separate receiver chain for each branch of the diversity system, which increases the complexity of the system. In [2,10], it is shown that the sum of $\kappa$-$\mu$ powers is $\kappa$-$\mu$ power distributed as well (but with different parameters), which is an ideal choice for MRC analysis. The expression for the pdf of the outputs of MRC micro-diversity systems is as follows [10]:

\[
\begin{align*}
    p(z|\Omega) & = \frac{I_{\mu}(1+\kappa)}{\kappa^\mu \mu^{\frac{\mu}{2}} \Gamma(\mu)} \exp\left(-\frac{\mu (1+\kappa) z}{\kappa\mu}\right) \\
    & \times \frac{1}{\mu^{\frac{\mu}{2}}} \exp\left(-\frac{\mu (1+\kappa) z}{\kappa\mu}\right) \\
    & \times (1 + \frac{\mu (1+\kappa) z}{\kappa\mu})^{\frac{\mu}{2} - 1} \\
    & \times \frac{I_{\mu,\mu-1}(1+\kappa)}{I_{\mu}(1+\kappa)}
\end{align*}
\]

(1)

In the previous equation, $I_{r}(\cdot)$ denotes the $r$-th-order modified Bessel function of first kind [17], eq. 8.445, $\mu_i$ and $\kappa_i$ are well-known $\kappa$-$\mu$ fading parameters of each micro-diversity system, while $L_i$ denotes the number of channels at each micro-level.

Since the outputs of a MRC system and their derivatives follow [9]:

\[
    z_i^2 = \sum_{k=1}^{L_i} z_{ik}^2 \quad \text{and} \quad \hat{z}_i = \sum_{k=1}^{L_i} \frac{z_{ik}}{z_i} \hat{z}_{ik}
\]

(2)

then $\hat{z}_i$ is a Gaussian random variable with a zero mean:

\[
    p(\hat{z}_i) = \frac{1}{\sqrt{2\pi}\sigma_{\hat{z}_i}} \exp\left(-\frac{\hat{z}_i^2}{2\sigma_{\hat{z}_i}^2}\right)
\]

(3)

and the variance given with [13]

\[
    \sigma_{\hat{z}_i}^2 = \frac{\sum_{k=1}^{N_i} z_{ik}^2 \sigma_{\hat{z}_{ik}}^2}{\sum_{k=1}^{N_i} z_{ik}^2}
\]

(4)
For the case of equivalently assumed channels
\[ \hat{\sigma}_z^2 = \hat{\sigma}_{z_1}^2 = \cdots = \hat{\sigma}_{z_N}^2, \quad k = 1, \ldots, N, \]
previous reduces into [18]
\[ \hat{\sigma}_z^2 = \hat{\sigma}_{z_k}^2 = 2\pi f_d^2 \Omega_i, \quad k = 1, \ldots, N, \]
where \( f_d \) is a Doppler shift frequency.
Conditioned on \( \Omega_i \), the joint PDF \( p_{z_1, z_2 \mid \Omega_i}(z_1, z_2 \mid \Omega_i) \) can be calculated as
\[ p_{z_1, z_2 \mid \Omega_i}(z_1, z_2 \mid \Omega_i) = \frac{I_{\mu_i}(1 + \kappa_i)\left(\frac{z_1}{\Omega_i}\right)}{\exp \left(\frac{z_1}{\Omega_i}\right) L_{\mu_i-1} \left(\frac{z_1}{\Omega_i}\right)} \times \exp \left(-\frac{\mu_i(1 + \kappa_i) z_1}{\Omega_i} - 2z_1 \mu_i \left(\frac{z_1}{\Omega_i}\right)\right) \times \frac{1}{2\pi \sigma_z} \exp \left(-\frac{z_1^2}{2\sigma_z^2}\right), \quad i = 1, 2. \]

It is already quoted that our macro-diversity system is of SC type and that the selection is based on the micro-combiners output signal power values. At the macro-level, this type of selection is used as handoff mechanism, that chooses the best base station to serve mobile, based on the signal power received. The joint probability density of the \( Z \) and \( Z \) conditioned on \( \Omega_1 \) and \( \Omega_2 \), equals the density of \( Z \) and \( Z \) at \( Z \) and \( Z \) for the case when \( \Omega_1 > \Omega_2 \), and equivalently the density of \( Z \) and \( Z \) at \( Z \) and \( Z \) for the case when \( \Omega_2 > \Omega_1 \). Now the unconditional joint probability density of the \( Z \) and \( Z \) is then obtained by averaging over the joint pdf \( p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) \) as
\[ p_{Z_1, Z_2}(z_1, z_2) = \int_{0}^{\infty} \int_{0}^{\infty} p_{Z_1, Z_2}(z_1, z_2 \mid \Omega_1, \Omega_2) p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) \, d\Omega_1 \, d\Omega_2 \]
\[ + \int_{0}^{\infty} \int_{0}^{\infty} p_{Z_1, Z_2}(z_1, z_2 \mid \Omega_1, \Omega_2) p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) \, d\Omega_2 \, d\Omega_1. \]

Similarly, this selection can be written through the cumulative distribution function (CDF) at the macro-diversity output in the form of
\[ F_z(z) = \int_{0}^{\infty} \int_{0}^{\infty} F_{z_1, z_2}(z_1, z_2) p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) \, d\Omega_1 \, d\Omega_2 \]
\[ + \int_{0}^{\infty} \int_{0}^{\infty} F_{z_1, z_2}(z_1, z_2) p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) \, d\Omega_2 \, d\Omega_1. \]
Here \( F(z_1, \Omega_i) \) defines the CDF of the SNR at the outputs of microdiversity systems given with
\[ F(z_1, \Omega_i) = \int_{0}^{\infty} p(t_1 | \Omega_i) dt_1. \]

Since base stations at the macro-diversity level are widely located, due to sufficient spacing between antennas, signal powers at the outputs of the base stations are modelled as statistically independent. Here long-term fading is as in [7] described with Gamma distributions, which are, as above mentioned, independent as
\[ p_{\Omega_1, \Omega_2}(\Omega_1, \Omega_2) = p_{\Omega_1}(\Omega_1) \times p_{\Omega_2}(\Omega_2) \]
\[ = \frac{1}{\Gamma(c_1) \Omega_1^{c_1-1}} \exp \left(-\frac{\Omega_1}{\Omega_0}\right) \times \frac{1}{\Gamma(c_2) \Omega_2^{c_2-1}} \exp \left(-\frac{\Omega_2}{\Omega_0}\right). \]

In the previous equation, \( c_1 \) and \( c_2 \) denote the order of Gamma distribution, the measure of the shadowing present in the channels. \( \Omega_0 \) and \( \Omega_0 \) are related to the average powers of the Gamma long-term fading distributions.

3 Second-order statistics
Second-order statistical quantities complement the static probabilistic description of the fading signal (the first-order statistics), and have found several applications in the modelling and design of wireless communication systems. Two most important second-order statistical measures are the LCR and the AFD. They are related to the criterion that can be used to determine parameters of equivalent channel, modelled by the Markov chain with the defined number of states and according to the criterion used to assess error probability of packets of distinct length [19].

Let \( z \) be the received signal envelope, and \( \dot{z} \) its derivative with respect to time, with joined probability density function (pdf) \( p_{z, \dot{z}}(z, \dot{z}) \). The LCR at the envelope \( z \) is defined as the rate at which fading signal envelope crosses level \( z \) in positive or negative direction and is mathematically defined by formula [9]
\[ N_z(z) = \int_{0}^{\infty} \dot{z} p_{z, \dot{z}}(z, \dot{z}) \, d\dot{z}. \]

The AFD is defined as the average time over which the signal envelope ratio remains below the specified level after crossing that level in a downward direction, and is determined as [9]
\[ T_z(z) = \frac{F_z(z \leq Z)}{N_z(z)}. \]

After substituting (10), (7), and (6) into (11), by using [15], Eq. 8 and following the similar procedure explained in [16, Appendix], we can easily derive the infinite-series expression for the system output LCR, in the form of
\[
\frac{N_0(\nu)}{J_0} = 2\sqrt{\pi} \sum_{p=0}^{\infty} \frac{\Gamma(p+1)}{\Gamma(p+\nu+1)} \sum_{m=0}^{\infty} \frac{\pi^m}{m!} \left(\frac{\nu}{\mu}\right)^{p+\nu+m} \exp\left(-\frac{\nu^2}{\mu^2}\right) \\
\sum_{j=0}^{\infty} \frac{1}{c_1(1+c_2)\Omega_0^2} (1+c_1) (1+c_2) \exp\left(-\frac{\nu^2}{\mu^2}\right) \quad (13)
\]

with \(K_n(.)\) denoting the \(n\)-th order modified Bessel function of first kind [17]. Similarly, from (8), we can obtain an infinite-series expression for the output AFD, in the form of

\[
\frac{z_0(\mu)}{J_0} = 2\sum_{p=0}^{\infty} \frac{\Gamma(p+1)}{\Gamma(p+\nu+1)} \sum_{m=0}^{\infty} \frac{\pi^m}{m!} \left(\frac{\nu}{\mu}\right)^{p+\nu+m} \exp\left(-\frac{\nu^2}{\mu^2}\right) \\
\sum_{j=0}^{\infty} \frac{1}{c_1(1+c_2)\Omega_0^2} (1+c_1) (1+c_2) \exp\left(-\frac{\nu^2}{\mu^2}\right) \quad (14)
\]

The infinite series from (13) and (14) rapidly converge for any value of the parameters \(c_i, \mu, \text{ and } \kappa_i, i = 1, 2\). In Table 1, the number of terms to be summed in (14), in order to achieve accuracy at the 5th significant digit, is presented for various values of system parameters.

### 4 Numerical results

Numerically obtained results are graphically presented in order to examine the influence of various parameters such as shadowing and fading severity and the number of the diversity branches at the micro-combiners on the concerned quantities. Normalized values of LCR, by maximal Doppler shift frequency \(f_d\), are presented in Figures 1 and 2.

We can observe from Figure 1 that lower levels are crossed with the higher number of diversity branches at each micro-combiner and larger values of shadowing severity parameters \(c_i\). From Figure 2, it is obvious that for higher values of \(\kappa\) fading severity parameter \(\mu\), and for higher values of dominant/scattered components power ratio \(\kappa_i\), LCR values decrease, since for smaller \(\kappa\) and \(\mu\) values, the dynamics in the channel is larger. Normalized AFD for various values of system parameters is presented in Figures 3 and 4. Similarly, with higher number of diversity branches, higher values of fading severity and higher values of shadowing severity, better performances of system are achieved (lower values of AFD).

### 5 Conclusion

In this article, the second-order statistic measures of SC macro-diversity system operating over Gamma shadowed \(\kappa-\mu\) fading channels with arbitrary parameters were analyzed. Useful incite-series expressions for LCR were presented.

| Table 1 Terms need to be summed in each sum of (14) to achieve accuracy at the 5th significant digit |
|---|---|---|---|
| \(c_1 = c_2 = 1\) | \(\Omega_{\theta_1} = \Omega_{\theta_2} = 1\) | \(\mu_1 = \mu_2 = 2\) | \(\mu_1 = \mu_2 = 3\) |
| \(z = -10\) dB | \(L = 2\) | \(c_1 = c_2 = 0.5\) | 9 | 12 |
| \(L = 2\) | \(c_1 = c_2 = 1\) | 12 | 15 |
| \(L = 3\) | \(c_1 = c_2 = 0.5\) | 12 | 14 |
| \(L = 3\) | \(c_1 = c_2 = 1\) | 16 | 21 |
| \(z = 0\) dB | \(L = 2\) | \(c_1 = c_2 = 0.5\) | 10 | 12 |
| \(L = 2\) | \(c_1 = c_2 = 1\) | 13 | 17 |
| \(L = 3\) | \(c_1 = c_2 = 0.5\) | 11 | 17 |
| \(L = 3\) | \(c_1 = c_2 = 1\) | 17 | 21 |
| \(z = 10\) dB | \(L = 2\) | \(c_1 = c_2 = 0.5\) | 12 | 13 |
| \(L = 2\) | \(c_1 = c_2 = 1\) | 15 | 18 |
| \(L = 3\) | \(c_1 = c_2 = 0.5\) | 12 | 15 |
| \(L = 3\) | \(c_1 = c_2 = 1\) | 16 | 23 |
Figure 1 Normalized average LCR of our macrodiversity structure for various values of shadowing severity levels and diversity order.

Figure 2 Normalized average LCR of our macrodiversity structure for various values of fading severity parameters $\kappa$ and $\mu$. 
Figure 3 Normalized average AFD of our macrodiversity structure for various values of shadowing severity levels and diversity order.

Figure 4 Normalized average AFD of our macrodiversity structure for various values of fading severity parameters $\kappa$ and $\mu$. 
and AFD at the output of this system were derived. The effects of the various parameters such as shadowing and fading severity and the number of the diversity branches at the micro-combiners on the system’s statistics were also presented.

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References
1. Gl. Stuber, Mobile Communication, 2nd edn. (Kluwer, Dordrecht, 2003)
2. MD Yacoub, The $\kappa\mu$ distribution and the $\nu\mu$ distribution. IEEE Antennas Propagation Mag. 49(1), 68–81 (2007)
3. JCS Filho, MD Yacoub, Highly accurate $\kappa\mu$ approximation to sum of $M$ independent non-identical Ricean variates. Electron Lett. 41(8), 338–339 (2005). doi:10.1049/el:20057727
4. WCY Lee, Mobile Communications Engineering (Mc-Graw-Hill, New York, 2001)
5. MK Simon, MS Alouini, Digital Communication over Fading Channels, 2nd edn. (Wiley, New York, 2005)
6. A Adinoyi, H Yanikomeroglu, S Loyka, Hybrid macro- and generalized selection combining microdiversity in lognormal shadowed rayleigh fading channels. In Proceedings of the IEEE International Conference on Communications. 1, 244–248 (2004)
7. PM Shankar, Analysis of microdiversity and dual channel macrodiversity in shadowed fading channels using a compound fading model. AEU Int J Electron Commun. 62(6), 445–449 (2008). doi:10.1016/j.aeue.2007.06.008
8. S Mukherjee, D Avizor, Effect of microdiversity and correlated macrodiversity on outages in a cellular system. IEEE Trans Wireless Technol. 2(1), 50–59 (2003). doi:10.1109/TWC.2002.806363
9. F Calmon, MD Yacoub, MRCS-selecting maximal ratio combining signals: a practical hybrid diversity combining scheme. IEEE Trans Wireless Commun. 8(7), 3435–3429 (2009)
10. M Miljic, M Hamza, M Hadzalic, BEP/SEP and outage performance analysis of L-branchmaximal-ratio combiner for $\kappa\mu$ fading. Int J Digital Multimedia Broadcasting. 2009, 1–8 (2009). Article ID 573404
11. X Wang, N Beaulieu, Switching rates of two-branch selection diversity in $\kappa\mu$ and $\nu\mu$ distributed fading. IEEE Trans Wireless Commun. 8(4), 1667–1671 (2009)
12. AA Abu-Dayya, CN Beaulieu, Micro- and macrodiversity MDPSK on shadowed frequency-selective channels. IEEE Trans Commun. 43(8), 2334–2342 (1995). doi:10.1109/26.403766
13. X Dong, NC Beaulieu, Average level crossing rate and average fade duration of selection diversity. IEEE Commun Lett. 10(5), 396–399 (2001)
14. NC Sagias, DA Zogas, GK Karagiannidis, GS Tombras, Channel capacity and second order statistics in Weibull fading. IEEE Commun Lett. 8(8), 377–379 (2004). doi:10.1109/LCOMM.2004.831319
15. Z Hadj-Welkov, N Zlatanov, GK Karagiannidis, On the second order statistics of the multihop Rayleigh fading channel. IEEE Trans Commun. 57(6), 1815–1823 (2009)
16. D Stefanovic, SP Panic, P Spalevic, Second-order statistics of SC macrodiversity system operating over Gamma shadowed Nakagami-m fading channels. AEU Int J Electron Commun. 65(3), 413–418 (2011). doi:10.1016/j.aeue.2010.05.001
17. I Gradshteyn, I Ryzhik, Tables of integrals, Series, and products, 1st edn. (Academic Press, New York, 1980)
18. SL Cotton, WG Scanlon, Higher-order statistics for kappa-mu distribution. Electron Lett. 43(22), 1215–1217 (2007). doi:10.1049/el:20072372
19. CD Iskander, PT Mathiopoulos, Analytical level crossing rate and average fade duration in Nakagami fading channels. IEEE Trans Commun. 50(8), 1301–1309 (2002). doi:10.1109/TCOMM.2002.801465