Sparticle Masses, $\mu$ Problem and Anomaly Mediated Supersymmetry Breaking

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Abstract

Within the MSSM framework and with purely anomaly mediated supersymmetry breaking the slepton masses turn out to be tachyonic. We resolve this problem by introducing an anomaly free $U(1)$ gauge symmetry which provides positive $D$-term contributions to sparticle masses squared that are flavor conserving at one loop. Two realistic examples based on $SU(5)$ are presented. With $U(1)$ spontaneously broken at a scale $\sim 10^{16}$ GeV, the right handed neutrinos acquire masses $\lesssim 10^{14}$ GeV. This breaking scale of $U(1)$ also plays an important role in the proposed resolution of the MSSM $\mu$ problem.

1 Introduction

Despite its many attractive features the anomaly mediated supersymmetry breaking (AMSB) scenario [1] has one very serious shortcoming, namely within the MSSM framework the slepton masses turn out to be tachyonic. A number of attempts to resolve this problem have appeared in the [2]-[10]. Recent interest in this scenario is largely spurred by the fact that the gravitino mass in this approach can be considerably larger than a TeV, perhaps even as large as 100 TeV or so. This can be helpful as far as gravitino cosmology is concerned [3]. A gravitino with mass considerably greater than a TeV can decay before nucleosynthesis, thereby evading the severe constraints on the reheat temperature $T_r$ that follow from nucleosynthesis considerations [11]. Indeed, it has been argued that for $m_{3/2} > 60$ TeV, the upper bound on $T_r$ from nucleosynthesis effectively disappears [3]. A new bound on $T_r$ can arise from considerations of the LSP, especially

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if the latter happens to be the neutral wino. In this case $T_r$ is estimated to be $\lesssim 10^{11}$ GeV, which is still significantly higher than the bound $T_r \lesssim 10^5 - 10^9$ GeV for $m_{3/2} \sim$ TeV in gravity mediated supersymmetry breaking models. This makes inflationary model building and successful leptobaryogenesis considerably easier.

In this paper we propose to eliminate the tachyonic slepton masses by introducing an additional source of supersymmetry breaking for the MSSM fields via a superheavy messenger sector, taking care that the unification of the MSSM gauge couplings is preserved. An anomaly free $U(1)$ gauge symmetry is introduced under which both messengers and the MSSM fields transform non-trivially, and which gives rise to flavor universal 1-loop contributions to the sfermion squared masses. These new contributions can easily overcome the negative two loop AMSB contributions. Since the magnitude of the $U(1)$ gauge coupling is undetermined, the gravitino mass $m_{3/2}$ could be as high as $20 - 60$ TeV. We present two examples based on $SU(5)$. The first one is inspired by the decomposition $E_6 \rightarrow SO(10) \times U(1)$ and leads to a universal positive contribution for all sfermion squared masses, independent of flavor. In the second example the $\bar{5}$ matter fields receive twice the contributions as the $10$ fields. The $U(1)$ symmetry plays other essential roles. For instance, its breaking ensures generation of right handed neutrino masses of the desired magnitude. It also plays an important role in the resolution of the MSSM $\mu$ problem, and its spontaneous breaking may be linked to supersymmetric hybrid inflation.

## 2 Extended AMSB Scenario

In the minimal version of AMSB [1], the source of SUSY breaking is the non-zero $F_\phi$ component of a compensator superfield $\phi = 1 + \theta^2 F_\phi$, with $\langle F_\phi \rangle = m_{3/2}$. This causes the generation of soft SUSY breaking terms for the MSSM gauginos and sparticles through one and two loop contributions respectively. Namely, the gaugino masses at one loop level are [1]

$$M_{\lambda_a}(\mu) = \frac{\alpha_a(\mu)}{4\pi} b_a m_{3/2},$$

while the two loop contributions to sfermion squared masses are

$$m_i^2(\mu) = \frac{2m_{3/2}^2}{(4\pi)^2} \sum_{a=1}^3 c_i^a b_a \alpha_a^2(\mu),$$

where, for MSSM, $(b_1, b_2, b_3) = (-\frac{33}{3}, -1, 3), c_i^a > 0$, and the masses and couplings are evaluated at scale $\mu$. The mass of the gravitino $m_{3/2}$ can be in the $20 - 60$ TeV range because of suppression by loop factors $\sim \alpha_a^2/4\pi$. This insures that the sparticle masses can be in the TeV region as required by the gauge hierarchy problem. Although the scenario looks very attractive, its minimal version is ruled out because of the negative squared masses for the sleptons $(b_1, b_2 < 0)$. Thus, new contributions, especially for the sleptons, are needed in order to make the AMSB scenario realistic.
2.1 Generation of Soft Masses at One-Loop

In order to cure the problem of tachyonic sleptons we introduce $n$ pair of vector like messenger superfields: $(\Psi^+ \Psi)_i$, $i = 1, \cdots, n$. In addition, we introduce $U(1)$ gauge symmetry under which the MSSM states and messengers transform non trivially. From the multimessenger ($n > 1$) sector and $U(1)$ factor, the sfermion masses\(^2\) can obtain flavor universal 1-loop contributions [12, 13]. We will show that this gives an elegant possibility to avoid the shortcomings of the minimal AMSB scenario and to build realistic models.

The $D$-term corresponding to the $U(1)$ gauge factor is given by

$$D = Q_\Psi (\bar{\Psi}^+_i \Psi_i - \bar{\Psi}_i \Psi^+_i) + Q_f \bar{f}^+_i \bar{f}_i , \quad (3)$$

where $f$ denotes the MSSM states and $Q_{\Psi,f}$ stand for the $U(1)$ charges of the corresponding superfields. We assume that the messenger sector feels SUSY breaking through the non-zero $F_X$ component of a chiral superfield $X$, which then gets transferred to the visible sector. The relevant superpotential couplings for the messengers are

$$W(\Psi) = M_{ii} \bar{\Psi}^i \Psi_i + \lambda_{ij} X \bar{\Psi}^i \Psi^j , \quad (4)$$

where, without loss of generality, we have chosen a basis in which the matrix $M$ is diagonal with positive elements, and we also assume that the lowest component of $X$ has no VEV. The fermionic components from $\Psi_i, \bar{\Psi}_i$ have masses $M_{ii}$, while their scalar partners are split due to SUSY breaking. The scalar $2n \times 2n$ mass matrix is given by

$$\hat{M}_\Psi^2 = \frac{\bar{\Psi}^+_i \bar{\Psi}}{\Psi^i \Psi_i} \left( \begin{array}{cc} M^2 & F_X^\dagger \\ F_X & M^2 \end{array} \right) \lambda \quad \hat{M}_\Psi^2 \Psi$$

It is clear that even if the lowest component of $X$ has a VEV, it can be absorbed in $M$, and one can then choose the basis in which $M$ is diagonal. It is essential, though, that the matrix $\lambda$ which couples with $F_X$ in (4) is non-diagonal. Performing the transformation $U \hat{M}_\Psi^2 U^\dagger = \hat{M}_\Psi^2$ with

$$U = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1_{n \times n} & -1_{n \times n} \\ 1_{n \times n} & 1_{n \times n} \end{array} \right) \quad \text{and} \quad (\bar{\Phi}^+_i, \bar{\Phi}) = (\Phi^+_i, \Phi) U , \quad (6)$$

we find

$$\hat{M}_\Psi^2 = \frac{\bar{\Phi}^+_i \bar{\Phi}}{\Phi} \left( \begin{array}{cc} M^2 - (\mathcal{F} + \mathcal{F}^\dagger) & F_X^\dagger \mathcal{F} - \mathcal{F}^\dagger \\ \mathcal{F} - \mathcal{F}^\dagger & M^2 + \mathcal{F} + \mathcal{F}^\dagger \end{array} \right) \lambda , \quad (7)$$

where $\mathcal{F} = \frac{1}{2} F_X^\dagger \lambda$. In the basis (6) the $D$-term of eq. (3) becomes:

$$D = Q_\Psi (\bar{\Phi}^+_i \Phi_i + \Phi^+_i \bar{\Phi}_i) + Q_f \bar{f}^+_i \bar{f}_i . \quad (8)$$
From the $D^2$ term in the Lagrangian there exists an interaction term $Q_f \tilde{f}^i \bar{\Phi}_i \Phi_1 + h.c.$, and with a non-vanishing insertion between $\Phi$ and $\bar{\Phi}$ [$F - F^\dagger$ entry in matrix (7)], the sfermions $\tilde{f}$ acquire one loop masses [12, 13]:

$$\tilde{m}_f^2(1\text{-loop}) = Q_f \frac{\tilde{\alpha}}{4\pi} \Lambda^2,$$

where $\tilde{\alpha}$ is the $U(1)$ ‘fine structure’ coupling and the scale $\Lambda$ depends on $M_{ii}$ and the couplings $\lambda_{ij}$ ($F_{ij}$) [13]

$$\Lambda^2 = 2 Tr \left[ Q_f \sum_{i,j=1}^n \frac{|F_{ij}|^2 - |F_{ji}|^2}{(M_{ii})^2} f \left( \frac{M_{jj}^2}{M_{ii}^2} \right) \right], \quad \text{with} \quad f(x) = \frac{2}{1-x} + \frac{1+x}{(1-x^2)} \ln x. \quad (10)$$

Note that for this one loop contribution to be non-zero, it is essential that $Q_f \neq 0$ and the matrix $F(= \frac{1}{2} F_X \lambda)$ is neither hermitian nor symmetric.

The result in (9) can be interpreted as the generation of a non zero VEV of the $D$-term, $\langle D \rangle = \frac{\tilde{\alpha}}{4\pi} \Lambda^2$. Indeed, the messengers which couple to non-zero $F_X$, dynamically generate $\langle D \rangle$. This can be seen by performing the shift $D \rightarrow D + \langle D \rangle$ in (8) and squaring, and we see that the soft term $\tilde{m}_f^2 = Q_f \langle D \rangle$ emerges. In other words, since there are couplings $\int d^4 \theta f^i e^2 V f$ in the Kähler potential, the contribution $V = \theta^2 \bar{\theta}^2 \langle D \rangle + \cdots$ induces soft mass squared terms. Since each generation of particles with identical SM quantum numbers carry the same $U(1)$ charge, the universality of sparticle masses is ensured at 1-loop. This remains intact under renormalization since $\tilde{m}_f \propto \langle D \rangle$ belongs to the solution of RGE trajectory [6, 7]. Thus, the FCNC are naturally suppressed. Note that a potential mixing of the messengers with the MSSM matter fields which could lead to large flavor violation is readily avoided because of the MSSM ‘matter’ parity.

With $\lambda_{ij} \sim 1$ and $M_{ii}$ all of comparable magnitude, we have $\Lambda \sim \frac{F_X}{M_{ii}}$. With $F_X = m_{3/2} \bar{M}$ and $\tilde{\alpha} \sim 1/25$ (this is a reasonable value especially if $U(1)$ arises from some GUT unifying the MSSM interactions at scales~$10^{16}$ GeV) we should have $M_{ii} \sim (3 - 6) \cdot \bar{M}$ in order to get $\tilde{m}_f \sim 1$ TeV (with the gravitino mass $m_{3/2} \approx 20 - 60$ TeV). To estimate $\bar{M}$ note that the operators $\int d^4 \theta \frac{X_i}{M_{ii}} f^i f$ cause flavor violating contributions that are harmless if $\bar{M} \lesssim 3 \cdot 10^{14}$ GeV.

Thus, an order of magnitude estimate of the messenger masses is $M_{ii} \lesssim 10^{15}$ GeV. This bound can be modified if $\lambda_{ij}$ are not of order unity, in which case the $M_{ii}$'s can have different values. A nice feature of this scenario is that there is just one additional parameter $\tilde{\alpha} \Lambda^2$ which contributes to the sparticle masses (at one loop).

The messengers also contribute to gaugino masses so that the 1-loop expression for the latter is given by

$$M_{\lambda a} (\mu) = \frac{\alpha_a(\mu)}{4\pi} (b_a - n A \Delta b_a) m_{3/2}, \quad (11)$$

where $\Delta b_a$ comes from the messengers (for a pair $\bar{5} + 5$ of $SU(5)$, $\Delta b_a = -1$) and, for simplicity, it is assumed that all messengers have comparable masses and $F_X/M_{ii} = A m_{3/2}$. The 2-loop contribution to the sparticle masses also get modified and is given by

$$m_i^2(2\text{-loop}) = V \sum_{a=1}^3 \epsilon_i b_a \left[ \alpha_a^2(\mu) - A \alpha_a^2(\mu) \frac{n \Delta b_a}{b_a} + A^2 \left( \alpha_a^2(\mu) - \alpha_a^2(M_{ii}) \right) \frac{(n \Delta b_a)^2}{b_a^2} \right]. \quad (12)$$
\[-A \left( \alpha_a^2(\mu) + A \alpha_a^2(M_{ii}) \right) \frac{n \Delta b_a}{b_a} \], \quad (12)

(with this expression one can recover the result of [2] for concrete case with \(A = -1\) and \(n \Delta b_a \to -N\)).

It is natural to expect that the 1-loop contribution (9) dominates over the 2-loop AMSB contributions, so that the problem with tachyonic slepton masses can be easily avoided\(^3\). Therefore, we propose that the contributions (9) provide positive squared masses for all the sfermions. In particular, the \(U(1)\) charges of \(l\) and \(e^c\) should have the same sign (this is not case with \(U(1)_{B-L}\). (For attempts at resolving the problem with this abelian factor together with other contributions see refs. \([5, 7, 9]\)). We present below two new examples within the \(SU(5)\) framework which provide a successful realization of the mechanism presented in this section.

3 Models with an Additional \(U(1)\) Symmetry

3.1 Model A

Our first model is inspired by the grand unified group \(E_6\) \([14]\) with \(U(1)\) corresponding to the decomposition \(E_6 \to SO(10) \times U(1)\). In an \(SU(5)\) setting the anomaly free content per generation is

\[
\begin{align*}
10_1 &+ \mathbf{5}_1 &\text{(Chiral states of MSSM)} \\
\mathbf{5}_{-2} &+ \mathbf{5}_{-2} &\text{(Messengers)} \\
\nu^c_i &+ N_4 &\text{(MSSM singlets)},
\end{align*}
\]

(13)

where the subscripts label the \(U(1)\) charge. The three \((\mathbf{5} + \mathbf{5})_{-2}\) families which are crucial for anomaly cancellation can be used as messengers. That is, the number of messengers \(n = 3\) coincides with the number of MSSM quark-lepton generations. The messenger superpotential couplings are

\[
W_1 = (\delta_{ij} m_{ii} + \hat{\lambda}_{ij} \frac{X}{\phi M_\ast}) \chi_4 \bar{\mathbf{5}}_{-2} \mathbf{5}_{-2}^c,
\]

(14)

\((X\) and \(\phi\) have zero \(U(1)\) charge) where \(m_{ii}, \hat{\lambda}_{ij}\) are dimensionless couplings, \(M_\ast\) denotes the cut-off, and \(\chi_4\) is an \(SU(5)\) singlet superfield. The VEV of its scalar component breaks \(U(1)\) and also generates masses for the messengers. With \(M_{ii} = \langle \chi_4 \rangle m_{ii}, \lambda_{ij} = \hat{\lambda}_{ij} \langle \chi_4 \rangle / M_\ast\), the superpotential (14) has the ingredients needed for sfermion mass\(^2\) generation at 1-loop. With \(Q_{10} = Q_{\mathbf{5}}\), according to (9) we have the prediction:

\[
\tilde{m}_{\mathbf{5}_1}^2(1\text{-loop}) = \tilde{m}_{10}^2(1\text{-loop}) \equiv m^2.
\]

\(^3\)One might ask whether the additional two loop contributions are sufficient to overcome the negative 2-loop AMSB contributions, in which case the 1-loop contributions to the squared sparticle masses could be eliminated by invoking a suitable ‘messenger parity’ \([12]\). In the next section we present two examples based on \(SU(5)\) with three families of \(\mathbf{5} + \mathbf{\overline{5}}\) messengers and, as shown in [2], this is not sufficient to eliminate the tachyonic slepton masses.
Thus, we have a positive universal (1-loop) contribution for all the sfermions' squared masses. Radiative electroweak (EW) breaking and sparticle spectroscopy with this type of asymptotic relation has been discussed in the literature [3]. We see that it arises within a rather simple example of SUSY $SU(5)$ supplemented by a suitable anomaly free $U(1)$ gauge symmetry.

Note that since for each $SU(5)$ multiplet $\Psi$ we have $\text{Tr} Y\Psi = 0$, a $D$-term for $U(1)_Y$ is not generated at one loop (see discussion in [13]). Therefore, only the $D$-term of $U(1)$ plays a role in generating 1-loop sparticle masses.

For a more precise estimate of masses we should take into account the 2-loop contributions in (12). As an example we take $m_{3/2}/2 = 50$ TeV, messenger masses $M_i \approx 10^{15}$ GeV [i.e. $A \approx -1/6$ in (12)] and $\overline{m} = 500$ GeV in (15). We then obtain the following sparticle mass spectra at 1 TeV scale:

$$0.99 m_{\tilde{u}} \simeq 0.98 m_{\tilde{d}} \simeq 2.31 m_i \simeq 2.22 m_{\tilde{e}} \simeq m_{\tilde{q}} , \quad m_{\tilde{q}} \simeq 1.02 \text{ TeV} .$$

(16)

For this analysis we have ignored the Yukawa couplings which may be relevant for the third generation (small or moderate values of the MSSM parameter $\tan \beta$ are preferred by considering the Yukawa sector; see below). Note that the left and right handed sleptons of the first two generations are quasi-degenerate in mass to within 4%. The gaugino masses are determined according to (11),

$$M_{\lambda_1} \simeq 2.48 M_{\lambda_2} \simeq 0.5 M_{\lambda_3} , \quad M_{\lambda_1} \simeq 488 \text{ GeV} .$$

(17)

In the higgs sector, together with $H(\mathbf{5}_{-2})$, $\mathbf{10}(\mathbf{5}_2)$ which contain the MSSM doublets $h_u$, $h_d$ respectively, we have the singlets $\chi_{-4}$, $\chi_4$ whose VEVs break the $U(1)$ symmetry. The Yukawa superpotential couplings responsible for the generation of quark and lepton masses are

$$W_{1Y} = \mathbf{10}_1 \cdot \mathbf{10}_1 \cdot H(\mathbf{5}_{-2}) + \frac{\chi_{-4}}{M'} \mathbf{10}_1 \cdot \mathbf{5}_1 \cdot \mathbf{10}(\mathbf{5}_2) ,$$

(18)

where the generation indices are suppressed and the cut off $M'$ here (and below) is expected to be comparable to $M_{Pl}$. If $\langle \chi_{-4} \rangle / M' \ll 1$, the $b, \tau$ Yukawa couplings are sufficiently small and we expect that $\tan \beta$ is not too large.

The 1-loop soft mass squared for the MSSM higgs doublet $h_u$ is negative (because of its negative $U(1)$ charge). Including the $\mu$ and $B\mu$-terms the $h_u - h_d$ mass matrix (before loop corrections from the top and stop are included) is given by

$$h_u^\dagger \begin{pmatrix} \mu^2 - 2m_0^2 & m_0^2 \\ B\mu & \mu^2 + 2m_0^2 \end{pmatrix} h_d ,$$

(19)

where $m_0^2 \simeq (240)^2 \text{ GeV}^2$ denotes the (negative) 2-loop contribution. The generation of $\mu$ and $B\mu$-terms will be discussed in section 4.

Some comments about the $U(1)$ symmetry breaking are in order. The VEVs of the lowest components of $\chi_{-4}, \chi_4$ states are necessary for the generation of messenger masses. However, the scale of $U(1)$ breaking is not yet constrained, because the messenger masses also involve the Yukawa couplings $m_{ii}$. On the other hand, the MSSM singlet states $\nu_c^i$ in (13) can be used
as right handed neutrinos whose Majorana masses are related to the $U(1)$ breaking scale. The Dirac Yukawa couplings are $\delta_{ij} H_{-2} \bar{\nu}_i^c$. For the Majorana masses of $\nu^c_1$ we introduce the scalar singlets $\xi_{-1} + \xi_1$, and the masses arise through the coupling $(\nu^c_1 \xi_{-1})^2 / M_{Pl}$. To accommodate the atmospheric neutrino data one needs $\langle \xi \rangle \sim 10^{16}$ GeV, which gives $m_{\text{atm}} \sim \langle h_u^0 \rangle^2 M_{Pl} / \langle \xi \rangle^2 \sim 0.1$ eV. The states $N_4$ do not play any role here in generating the light neutrino masses. They decouple by acquiring superheavy masses via the couplings $(N_4 \chi_{-4})^2 / M_{Pl}$. The presence of right handed neutrinos will induce lepton flavor violating effects that are suppressed by an additional loop factor compared to the dominant flavor universal contributions proportional to $\langle D \rangle$. The soft SUSY breaking parameters, to leading order, still belong to the UV insensitive anomaly mediated trajectory [10].

We see that the $U(1)$ symmetry breaking plays an essential role for the generation of messenger masses as well as for realizing masses appropriate for the description of neutrino oscillations. In section 4, the importance of $U(1)$ in resolving the MSSM $\mu$ problem will be discussed.

3.2 Model B

The second $SU(5)$ scenario has the following ‘matter’ content:

$$
\begin{align*}
10_1 &+ \overline{5}_2 & \text{(Chiral states of MSSM)} \\
\bar{5}_{-2} &+ \overline{5}_{-3} & \text{(Messengers)} \\
N_5 & & \text{(MSSM singlet)} ,
\end{align*}
$$

with the messenger superpotential given by

$$
W_2 = (\delta_{ij} m_{ii} + \hat{\lambda}_{ij} X_{\phi M_s}) \chi_5 \overline{5}_{-3} \overline{5}'_{-2} ,
$$

where the singlet superfields $\chi_5, \chi_{-5}$ are needed to break $U(1)$. In 1-loop approximation the sparticle masses are predicted to be

$$
\tilde{m}^2_5 (1\text{-loop}) = 2 \cdot \tilde{m}^2_{10} (1\text{-loop}) \equiv \overline{m}^2 .
$$

The Yukawa superpotential involves the following couplings

$$
W_{2Y} = 10_1 \cdot 10_1 \cdot H(5_{-2}) + \overline{\chi}_{-5} \overline{5}_{-3} \overline{5}'_{-2} ,
$$

so that $\tan \beta$ is also not large for this model.

Taking into account the 2-loop contributions in (12), and with $\overline{m} = 500$ GeV, $m_{3/2} = 50$ TeV, $M_{ii} \approx 10^{15}$ GeV [i.e. $A \approx -1/6$ in (12)], the sparticle mass spectra is given by

$$
0.99 m_{\tilde{u}} \approx 0.88 m_{\tilde{d}} \approx 1.53 m_{\tilde{t}} \approx 2.22 m_{\tilde{e}} \approx m_{\tilde{q}} , \quad m_{\tilde{\nu}} \approx 1.02 \text{ TeV} .
$$

This is different from the one obtained for Model A due to the asymptotic relation (22) (there is no mass degeneracy any more between left and right handed sleptons). The gaugino masses are the same as for Model A (eq. (17)). The $h_u - h_d$ mass matrix for Model B coincides with (19).
4 $\mu$ Problem and $U(1)$ Symmetry

The presence of the $U(1)$ symmetry can be exploited to yield a solution of the $\mu$ problem in models with AMSB, along the lines proposed in [15] for gravity mediated SUSY breaking. To achieve the breaking of $U(1)$ with a renormalizable superpotential, a gauge singlet chiral superfield $S$ is introduced. The singlet $S$ triggers the $U(1)$ breaking, and its scalar and auxiliary components acquire VEVs after SUSY breaking which generate the $\mu$ and $B\mu$-terms of the MSSM. Last but not least, $S$ can play the role of the inflaton field in supersymmetric hybrid inflation [16].

The superpotential

$$W_S = \kappa S(\overline{\chi}\chi - \phi^2 M^2) ,$$

(25)

involves the pair $\overline{\chi}, \chi$ (needed for $U(1)$ breaking) and, in the unbroken SUSY limit (i.e. $\langle \phi \rangle = 1, F_\phi = 0$), $\langle S \rangle = 0$ (the dimensionless coupling $\kappa > 0$ without loss of generality). The presence of $\phi^2$ in (25) is required for the correct Weyl weight of $W_S$, with all superfields having the canonical Weyl weights (= 1). Note that we have also employed a $U(1) R$-symmetry such that $W_S$ and $S$ have a unit $R$-charge. With SUSY unbroken, the solution $\langle S \rangle = 0$, $|\langle \chi \rangle| = |\langle \overline{\chi} \rangle| = \phi M$ satisfies $F$ and $D$ flatness conditions. Substituting $\phi = 1 + \theta^2 m_3 / 2$ the scalar potential derived from (25) is given by

$$V = \kappa^2 |\overline{\chi}\chi - M^2|^2 + \kappa^2 (|\chi|^2 + |\overline{\chi}|^2) - (2\kappa M^2 m_3/2 S + h.c.)$$

(26)

The soft SUSY breaking term in (26) shifts the VEVs:

$$\langle S \rangle \simeq \frac{m_3/2}{\kappa} , \quad \langle \overline{\chi} \rangle = \langle \chi \rangle \simeq M(1 - \frac{m_3/2}{2\kappa^2 M^2}) ,$$

(27)

and also the corresponding $F$-terms

$$F_S \simeq - \frac{m_3/2}{\kappa} , \quad F_\chi = F_{\overline{\chi}} \simeq m_3/2 M .$$

(28)

Note that although SUSY is broken we still have $|\langle \overline{\chi} \rangle| = |\langle \chi \rangle|$ and therefore there are no additional contributions to $\langle D \rangle$. Also, $\overline{\chi}, \chi$ sit on the AMSB trajectory (i.e. $F_{\overline{\chi}} = F_\chi = m_3/2$) and therefore do not contribute, even not via loops, to the soft masses. With $\langle S \rangle = m_3/2/\kappa$, the superpotential coupling $\kappa_h S \overline{h_u} h_d$ gives $\mu \sim (\kappa_h/\kappa) m_3/2$, where $\kappa_h$ is a dimensionless coupling. For $m_3/2 \sim 50 - 100$ TeV, we need $\kappa_h \sim 10^{-2}\kappa$ to generate a $\mu$-term~ TeV.

Let us now recall that in sect. 3.1 we employed the $SU(5)$ singlets $\overline{\xi}, \xi$ to generate masses for the right handed neutrinos. If the gauge invariant combinations $h_u h_d$ and $\overline{\xi} \xi$ transform under some discrete symmetry such that $\xi \xi : h_u h_d$ is invariant, then the operator

$$W_\mu = \kappa_h \frac{\overline{\xi} \xi}{(\phi M_{Pl})^2} S \overline{h_u} h_d ,$$

(29)

with $\langle \overline{\xi} \rangle = \langle \xi \rangle \equiv \epsilon M_{Pl}$ gives

$$\mu \simeq \epsilon^2 \frac{\kappa_h}{\kappa} m_3/2 .$$

(30)
With $\kappa_h/\kappa \sim 1$, $\epsilon \sim 0.1$ is sufficient to guarantee $\mu \sim \text{TeV}$.

The $B\mu$ term turns out to be somewhat large ($\sim \mu m_{3/2}$) and some fine tuning may be necessary to implement radiative electroweak breaking. Namely, from (29), taking into account (27), (28), we have

$$B\mu = -\mu m_{3/2} \left( 3 - 2 \frac{F_\xi}{\langle \xi \rangle m_{3/2}} \right),$$

where $F_\xi$ denotes the VEV of the auxiliary component of $\xi$ ($\xi$) and its magnitude can be expected to be of order $F_\xi \sim \langle \xi \rangle m_{3/2}$. With $F_\xi/(\langle \xi \rangle m_{3/2}) \approx 3/2 + O(\mu m_{3/2})$ we can arrange that $B\mu \sim \mu^2$.

The superpotential (25) was extensively used for building inflationary scenarios within various realistic models [16]-[18]. Indeed, for successful inflation including leptogenesis, $\kappa$ near $10^{-2}$ is preferred [18], in which case $\epsilon \sim 10^{-2}$. This corresponds to $\langle \xi \rangle = \langle \xi \rangle \sim 10^{16}$ GeV, which is also preferred for realizing the observed neutrino masses (see sect. 3.1).

5 Conclusions

An anomaly free $U(1)$ gauge symmetry is employed to solve the problem of tachyonic slepton masses encountered in models with anomaly mediated supersymmetry breaking. Two simple examples based on $SU(5)$ are presented. With $U(1)$ spontaneously broken at a scale $M \sim 10^{16}$ GeV, the right handed neutrinos acquire masses $\lesssim 10^{14}$ GeV, which is suitable for realizing the correct light neutrino masses needed for neutrino oscillations and for implementing leptogenesis. A mechanism for resolving the MSSM $\mu$ problem is discussed in which the scale $M(\sim 10^{16}$ GeV) also plays an essential role. Finally, the breaking of $U(1)$ can be linked to hybrid inflation such that $\delta T/T$ is proportional to $(M/M_{Pl})^2$. Thus, $U(1)$ in our approach plays an essential role in the construction of realistic models utilizing anomaly mediated supersymmetry breaking.

Acknowledgments

Q.S. would like to acknowledge the hospitality provided by the Institut für Theoretische Physik in Heidelberg, especially Michael Schmidt and Christof Wetterich, and he also thanks the Alexander von Humboldt Stiftung. This work is supported by NATO Grant PST.CLG.977666 and by DOE under contract DE-FG02-91ER40626.

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