NUMERICAL STUDY OF STELLAR CORE COLLAPSE AND NEUTRINO EMISSION:
PROBING THE SPHERICALLY SYMMETRIC BLACK HOLE PROGENITORS WITH 3–30 $M_\odot$ IRON CORES

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ABSTRACT

The existence of various anomalous stars, such as the first stars in the universe or stars produced by stellar mergers, has been proposed recently. Some of these stars will result in black hole formation. In this study we investigate iron-core collapse and black hole formation systematically for the iron-core mass range of 3–30 $M_\odot$, which has not been studied well so far. Models used here are mostly isentropic iron cores that may be produced in merged stars in the present universe, but we also employ a model that is meant for a Population III star and is obtained by evolutionary calculation. We solve numerically the general relativistic hydrodynamics and neutrino transfer equations simultaneously, treating neutrino reactions in detail under spherical symmetry. As a result, we find that massive iron cores with $\sim 10 M_\odot$ unexpectedly produce a bounce, owing to the thermal pressure of nucleons before black hole formation. The features of neutrino signals emitted from such massive iron cores differ in time evolution and spectrum from those of ordinary supernovae. First, the neutronization burst is less remarkable or disappears completely for more massive models, because the density is lower at the bounce. Second, the spectra of neutrinos, except the electron type, are softer, owing to the electron-positron pair creation before the bounce. We also study the effects of the initial density profile, finding that the larger the initial density gradient is, the more steeply the neutronization burst declines. Furthermore, we suggest a way to probe into the black hole progenitors from the neutrino emission and estimate the event number for the currently operating neutrino detectors.

Subject headings: black hole physics — hydrodynamics — methods: numerical — neutrinos — radiative transfer — relativity

1. INTRODUCTION

Various anomalous stars, such as the first stars in the universe (so-called Population III stars) or stars produced by stellar mergers in stellar clusters, are being studied recently. As for the Population III stars, it is suggested theoretically that they are much more massive ($M \gtrsim 100 M_\odot$) than stars of later generations (e.g., Nakamura & Umemura 2001). On the other hand, N-body simulations show that the runaway mergers of massive stars occur and that very massive ($M \gtrsim 100 M_\odot$) stars are formed in a young compact stellar cluster (e.g., Portegies Zwart et al. 1999). Especially notable is a newly suggested formation scenario for supernovae by the formation of intermediate-mass black holes which requires the formation of a young supermassive black hole by the collapse of merged stars in a very compact stellar cluster (e.g., Ebisuzaki et al. 2001). These anomalous stars collapse to black holes without supernova explosions, it is supposed difficult to probe into their progenitors. One possible means of such a probe is, we think, to examine the neutrinos emitted during the black hole formation. For this purpose, systematic studies on black hole formation, including the effects of neutrinos, are needed.

So far, various numerical simulations of supernova explosions have been done by many authors. Similarly, numerical studies on black hole formation have also been produced recently (e.g., Fryer 1999; Linke et al. 2001; Fryer et al. 2001, hereafter FWH01; Sekiguchi & Shibata 2005, hereafter SS05; Nakazato et al. 2006, hereafter NSY06; Sumiyoshi et al. 2006, hereafter SYSC06). Fryer (1999) classified core collapse into three types: (a) Stars with $M \lesssim 25 M_\odot$ make explosions and produce neutron stars. (b) Stars with $25 M_\odot \lesssim M \lesssim 40 M_\odot$ also result in explosions, but produce black holes via fallback. (c) For $M \gtrsim 40 M_\odot$, the shock produced at the bounce can neither propagate out of the core nor make explosions. In any case, the core bounces once. SYSC06 computed fully general relativistic hydrodynamics under spherical symmetry, taking into account the reactions and transport of neutrinos in detail, and confirmed class c for the collapse of a progenitor with 40 $M_\odot$. On the other hand, much more massive stars result in black hole formation without bounce. SS05 studied the criterion for the collapse without bounce. Their computations are fully general relativistic, and they investigated the dynamics systematically, varying the initial mass and rotation. They concluded that nonrotating iron cores with a mass of $M_{\text{iron}} \gtrsim 2.2 M_\odot$ collapse to black holes without bounce. However, they employed the phenomenological equation of state and did not consider the effects of neutrinos.

There are also studies on the collapse of very massive stars in the context of the evolution of Population III stars. As mentioned above, Population III stars may be very massive, $M \gtrsim 100 M_\odot$. It is supposed that the pair creation of electrons and positrons makes a star unstable during the helium-burning phase if they do not lose much of their mass during the quasi-static evolution because of zero metallicity. Stars with $\lesssim 260 M_\odot$ reverse the collapse by rapid nuclear burnings and explode to pieces, which are called pair-instability supernovae, while more massive stars cannot halt the collapse and form black holes (e.g., Heger et al. 2003). Note that, however, these numbers are still uncertain at present (e.g., Ohkubo et al. 2006). Assuming that Population III stars with $M \gtrsim 300 M_\odot$ are formed and evolve without mass loss, FWH01 and NSY06...
showed that they collapse without bounce for spherically symmetric models under fully general relativistic computations while NSY06 treated the neutrino transport in more detail than FWH01. FWH01 also computed the collapse of a rotating star with 300 $M_\odot$ under Newtonian gravity and showed that it has a weak bounce and then recollapses to a black hole immediately. As for the collapse of supermassive stars with $M \gtrsim 5 \times 10^5 M_\odot$, Linke et al. (2001) found that they form black holes without bounce before becoming opaque to neutrinos.

The black hole formation of stars in the mass range between $\sim 100$ and $\sim 260 M_\odot$, which corresponds to an iron-core mass range between $\sim 3$ and $\sim 30 M_\odot$, has not been studied well so far, because they are supposed to explode as pair-instability supernovae during the quasi-static evolution if they are single stars. Recently, on the other hand, stars produced by stellar mergers in a young compact stellar cluster were studied in detail, and their evolutionary paths are beginning to be revealed (Suzuki et al. 2007). While they do not calculate the evolution of these stars up to the black hole formation, we speculate, as in § 3.5.1, that they may avoid the explosions as pair-instability supernovae and form a massive iron core in the above-mentioned range. Therefore, we investigate, in this study, the iron-core collapses systematically for iron-core masses of 3 and 30 $M_\odot$, although there is no evidence to show their existence so far.

To be more specific, we assume that the mass of an iron core is mainly determined by the entropy per baryon, and our investigation is done systematically for entropy. We solve the general relativistic hydrodynamics under spherical symmetry. We also solve the neutrino transfer equations simultaneously, treating neutrino reactions in detail. In addition to the isentropic iron-core models, we employ the realistic stellar model of 100 $M_\odot$ and zero metallicity, supposedly a Population III star, by Nomoto et al. (2005). We address the issues concerning the black hole formation of the merged stars in connection with our study and estimate the neutrino event number for the currently operating detectors. We also suggest a way to probe into the progenitors from the detection. We hope that this study will not only be a reference for future multidimensional computations but also provide a basis for neutrino astrophysics in black hole formation.

2. INITIAL MODELS AND NUMERICAL METHODS

At first, we construct the iron-core models, which we use as initial models for the dynamical simulation of the collapse. The progenitor with 40 $M_\odot$ in SYSC06 has an entropy per baryon $s \sim 1.5 k_B$ and an iron-core mass $M_{\text{iron}} = 1.98 M_\odot$, whereas the massive Population III star models in NSY06 have $s > 16 k_B$ and $M_{\text{iron}} > 50 M_\odot$. In this study we intend to bridge the gap of the black hole progenitors and discuss the neutrino emission systematically for this range. Unfortunately, realistic models of the progenitors for this range are rare, and “systematic” models for them are absent up to the present, since their astrophysical counterparts are not well known, as mentioned above. Therefore, we construct the initial models by ourselves.

We assume that the iron cores in equilibrium configurations collapse by photodisintegration, as is the case for the onset of ordinary core-collapse supernovae. We obtain the initial models, solving the Oppenheimer-Volkoff equation with the equation of state by Shen et al. (1998a, 1998b) assuming isentropy and that the electron fraction $Y_e = 0.5$ throughout the core. We define the mass of the iron core, $M_{\text{iron}}$, as the mass coordinate where the temperature is $5 \times 10^9$ K, whereas we set the outer boundary at a much larger radius so as not to affect the dynamics. For the systematic analysis, we set the initial central temperature to be $T_{\text{inh}} = 7.75 \times 10^9$ K, which is slightly higher than the critical temperature for photodisintegration (Fig. 1), and generate models 1a–6a with the values of entropy per baryon $s = 3 k_B – 13 k_B$, which have not been studied well so far, as mentioned above. In order to investigate the ambiguity in the onset of collapse, we also adopt a model (model 2b) with the same initial entropy per baryon as model 2a ($s = 4 k_B$) but having half the central density. The key parameters of these models are summarized in Table 1. In addition, we also employ the realistic stellar model of 100 $M_\odot$ with a vanishing metallicity by Nomoto et al. (2005) in order to validate the isentropic models. This model is supposedly a Population III star and resides in the range $s = 3 k_B – 13 k_B$.

As a next step, we compute the dynamics of spherically symmetric gravitational collapse with neutrino transport. As for our numerical methods, we follow NSY06 and use the general relativistic implicit Lagrangian hydrodynamics code, which simultaneously solves the neutrino Boltzmann equations (Yamada 1997; Yamada et al. 1999; Sumiyoshi et al. 2005). We consider four species of neutrino, $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, and $\bar{\nu}_\mu$, assuming that $\nu_e$ and $\bar{\nu}_e$ are the same as $\nu_\mu$ and $\bar{\nu}_\mu$, respectively, and take into account nine neutrino reactions listed in NSY06. We use 127 radial mesh points, while 12 and 4 mesh points are used for the energy and angular distribution of the neutrino, respectively. In order to assess the convergence of our results, we compute models with higher resolutions. They have the same initial conditions as model 2a. For model 2m, the number of radial mesh points is increased to 255. Model 2e uses 18 mesh points for the energy spectrum, while model 2g has six mesh points for the angular distribution.

It is noted that our method allows us to follow the dynamics with no difficulty up to the apparent horizon formation. The existence of the apparent horizon is the sufficient condition for the formation of a black hole (or, equivalently, of an event horizon). For the Misner-Sharp metric (Misner & Sharp 1964) adopted in our computations, the radius of the apparent horizon is written as

$$r = 2G\tilde{m}/c^2,$$

where $c$ and $G$ are the velocity of light and the gravitational constant, respectively (Van Riper 1979), $r$ is the circumference radius, and $\tilde{m}$ is the gravitational mass inside $r$. Since our models
are spherically symmetric, there is no difficulty in finding the horizon.

### 3. RESULTS AND DISCUSSIONS

In this section we show the results of our computations and discuss them. We study the dynamics of the collapse in § 3.1 and investigate the features of the neutrinos emitted during the collapse in § 3.2. In § 3.3 we investigate the role of the initial velocity and the deviation from equilibrium. We also make a comparison with the realistic progenitor models in § 3.4. Finally, we mention the astronomical counterparts of our models and the possibility of probing into the progenitors of the events in § 3.5.

To overview the characteristics of the models which we surveyed, we show in Figure 1 the evolution of the central density and temperature of our results together with those of other simulations of black hole formation. The trajectories of the current models shown by solid lines are between those of previous models reflecting the different values of entropy. From this figure we can recognize that our investigation bridges the gap between two previous studies, SYSC06 and NSY06.

#### 3.1. Dynamical Features

It is known that ordinary supernovae with $s \sim 1k_B$ bounce because their central density exceeds the nuclear density ($\sim 2.5 \times 10^{14} \text{ g cm}^{-3}$) and pressure drastically increases. From our computations, we find that models with $3k_B \leq s \leq 7.5k_B (M \leq 10.6 M_{\odot})$ have a bounce and that they recollapse to black holes. On the other hand, models with $s > 7.5k_B (M > 10.6 M_{\odot})$ collapse to black holes directly without bounce. We show the evolution of core collapse in Figure 2 for two representative cases.

In the case of $3k_B \leq s \leq 7.5k_B$, it is noted that the bounce mechanism of the core with $s \geq 3k_B$ is not the same as that of ordinary supernovae. The high-entropy cores bounce because of the thermal pressure of nucleons at subnuclear density. We can see this fact from the evolution of central density and temperature in the phase diagram of the nuclear matter at $Y_e = 0.4$ and

![Fig. 2.— Radial trajectories of mass elements. Left: Model 2a ($s = 4k_B$); time is measured from the bounce. Right: Model 5a ($s = 10k_B$); time is measured from the point at which the apparent horizon is formed.](image)
0.2 (Fig. 3). We note that for all models at the center $Y_e \sim 0.4$ and $\sim 0.2$ when $T \sim 1$ and $\sim 10$ MeV, respectively. These figures show that the models with higher entropies go from the nonuniform mixed phase of nuclei and free nucleons to the classical ideal gas phase of thermal nucleons and $\alpha$-particles, whereas that of an ordinary supernova goes into the uniform nuclear matter phase. In the ideal gas phase, the number of nonrelativistic nucleons and $\alpha$-particles is comparable to that of relativistic electrons. Since the adiabatic index of nonrelativistic gas is $\gamma = 5/3$ and that of relativistic gas is $\gamma = 4/3$, the collapse is halted and bounce occurs.

Because this bounce is weak and the shock is stalled, the inner core (or the proto–neutron star) grows beyond the maximum mass of the neutron star and recollapses to a black hole soon (left panel of Fig. 2). In Figure 4 we show the maximum mass of the neutron star assuming isentropy and the constant electron fraction ($Y_e = 0.1$) under the equation of state by Shen et al. (1998a, 1998b). It is noted that the maximum mass is larger than $3 M_\odot$ for the neutron star with high entropies, $s \gtrsim 4k_B$. Since the maximum mass of the neutron star depends on the equation of state, it should be noted that the time interval from the bounce to the recollapse also depends on it (SYSC06). We refer to this point again below.

In Table 1 we show the inner core mass, central density, temperature, and adiabatic index at the bounce together with the interval time from the bounce to the apparent horizon formation. We can recognize that the density and the adiabatic index at the bounce get lower for the models with higher initial entropies. These features indicate that the bounce is not due to the nuclear force but to the thermal pressure of nonrelativistic gas for high-entropy cores. Moreover, the interval time from the bounce to the apparent horizon formation is shorter for the higher entropy cores, because the initial mass of the iron core ($M_{\text{iron}}$) is larger than the maximum mass of the neutron star ($M_{\text{max}}$) for the models with high entropies and they can collapse to black holes quickly.

We also show the results for the models with higher resolutions in Table 1. The central density and the adiabatic index at bounce, which are key parameters in our analysis, are not very different for models 2a, 2g, and 2m. The central density at bounce of model 2e, which has a 1.5 times finer energy mesh, is different by 14% from that of model 2a, because neutrinos affect the entropy variations before the neutrino trapping. In fact, the central entropy at bounce of model 2a is 3.50$k_B$ while that of model 2e is 3.62$k_B$. However, qualitative features of their bounces are not changed. On the other hand, the interval times from the bounce to the apparent horizon formation are different by $\pm 15\%$ for models 2a, 2e, 2g, and 2m, because the start point of the recollapse is roughly determined by the maximum mass of the neutron star as mentioned above. Since the mass accretion rate is of the order of $10 M_\odot \text{ s}^{-1}$ during this phase in our models, the difference of $0.1 M_\odot$ in the maximum mass means the difference of 10 ms in the interval time, which is close to the discrepancies found here. Thus, the precise determination of the interval time is difficult in general. However, its dependence on the initial entropy is well established.

We compare our results with other studies. In SS05, the nonrotating models with an iron-core mass of $\gtrsim 2.28 M_\odot$ end up with black holes without bounce. In our models, on the other hand, it is shown that the iron core with $\lesssim 10.6 M_\odot$ (or the initial entropy $s \lesssim 7.5k_B$) has bounce before black hole formation. This discrepancy comes from the fact that their equation of state is parametric and does not properly take into account the effects of thermal nucleons in the collapsing phase. On the other hand, the rotating Population III star with $\sim 300 M_\odot$ has a weak bounce at $\rho_c \sim 10^{12} \text{ g cm}^{-3}$ in FWH01, and these authors adopt a realistic equation of state (Herant et al. 1994). Since the rotation tends to produce a bounce, we can predict that the bounce is inevitable...
for an iron core with the mass \(\leq 10 M_\odot\) irrespective of rotations, and the effects of thermal nucleons are crucial.

In the high-entropy case \(s > 7.5k_B\), more massive cores do not have a bounce, but form an accretion shock before the apparent horizon formation, because the outer region keeps collapsing supernovically while the central region becomes gravitationally stable by the thermal pressure of nonrelativistic gas. We can see this feature in the right panel of Figure 2. As the initial mass gets larger, the transition occurs smoothly from the collapse with bounce to the one without bounce. Incidentally, the features of direct collapse are almost the same as those for the Population III models in NSY06, where a detailed analysis can be found.

3.2. Neutrino Signals

In this section we discuss neutrino emission during core collapse. As mentioned above, we compute the collapse until the formation of the apparent horizon. However, the location of the event horizon is not known for our models, although it is proved mathematically that the event horizon is always located outside the apparent horizon. Moreover, the numerical difficulty prevents us from computing the dynamics until the apparent horizon swallows the shock surface entirely. Because of these facts, the total energy and number of emitted neutrinos have some ambiguities. In this study we estimate the upper and lower limits for the total energy and number of neutrinos, following NSY06. The upper limit is obtained with an assumption that all neutrinos in the region between the shock surface and the neutrino sphere flow out without being absorbed or scattered. For the lower limit, on the other hand, we assume that all neutrinos in this region are trapped and do not come out. Fortunately, for the models with bounce, these ambiguities are minor compared with the direct collapse models in NSY06, since the duration from the shock formation to the apparent horizon formation is longer and almost all neutrinos are emitted during this phase.

The calculated results of the neutrino emission are summarized in Table 2. It is noted that we assume that \(\nu_e\) and \(\bar{\nu}_e\) are the same as \(\nu_x\) and \(\bar{\nu}_x\) and that the luminosities of \(\nu_x\) and \(\bar{\nu}_x\) are almost identical, because they have the same reactions and because the difference between coupling constants is minor. In the following, ignoring this tiny difference, we denote these four species as \(\nu_x\) collectively. In Table 2 we can recognize that the total energy does not change monotonically with the initial entropy of the core. This is because the duration of the neutrino emission is longer for the lower entropy models, while the duration is shorter and the neutrino luminosity is larger for the higher entropy models.

In Figure 5 we show the time evolution of neutrino luminosity for several models under the assumption that the neutrinos outside the neutrino sphere flow freely after the apparent horizon formation. As mentioned above, the time interval from the bounce to the apparent horizon formation depends on the equation of state. In SYSC06, it is shown that the features of the neutrino emission, such as a neutronization burst, are not sensitive to the equation of state very much for the early phase. From Figure 5, we can see that the sign of a neutronization burst becomes less remarkable and disappears for the higher entropy models.

In order to analyze these features, we discuss the neutrino emission from model 1a, as a reference model. In the top left panel of Figure 6, we show snapshots of the luminosity of an electron-type neutrino as a function of the baryon mass coordinate. We can recognize that neutrinos are emitted mainly on the shock surface. The luminosity on the shock surface has a peak (e.g., at \(1.25 M_\odot\)) is the same irrespective of rotations, as can be evaluated roughly by the equilibrium value, and compare it with the results of our numerical simulations.

At first, the number density of neutrinos on the shock surface can be evaluated roughly by the equilibrium value,

\[
n_{eq}(\epsilon) d\epsilon \propto \frac{\epsilon^2}{\exp \left[\left(\epsilon - \mu_e\right)/(k_BT)\right] + 1} d\epsilon,
\]

where \(T\) and \(\mu_e\) are the temperature and the chemical potential of the electron-type neutrino in \(\beta\)-equilibrium at the shock surface, respectively, and \(k_B\) is the Boltzmann constant. Here the value \(\mu_e\) is defined as \(\mu_e \equiv \mu_e - (\mu_n - \mu_p)\), where \(\mu_e, \mu_n, \) and \(\mu_p\) are the chemical potentials of the electron, neutron, and proton, respectively, and they are given in the equation of state by Shen et al. (1998a, 1999b). The number flux is estimated as \(\chi_{eq} \cos(\theta)\), where \(\cos(\theta)\) is the mean value of the angular cosine over the neutrino angular distribution and \(\epsilon\) is the light velocity. In Figure 7 we compare the results of our numerical computation with the number flux estimated above. We can see that the equilibrium is not achieved completely, but the fraction is rather constant, ~0.6. Therefore, the luminosity is well estimated by

\[
L(\epsilon) d\epsilon = C \frac{16\pi^2r^2(\cos\theta)\epsilon^3}{h^3c^2\exp \left[\left(\epsilon - \mu_e\right)/(k_BT)\right] + 1} d\epsilon,
\]

where \(h\) is the Planck constant, \(r\) is the radius of the shock surface, and \(C \sim 0.6\).

From equation (3), we can see that the luminosity is determined by \(r, \mu_e, T,\) and \(\cos(\theta)\). According to our numerical computation,
$T \sim 1.5$ MeV and $(\cos \theta) \sim 0.5$ do not change very much on the timescale of the neutronization burst. Thus, the luminosity is dictated mainly by $r$ and $\mu_e$. Snapshots of the profiles of $\mu_e$ are shown in the bottom left panel of Figure 6, and we can see that $\mu_e$ has a peak on the shock surface for the following reason. When matter accretes onto the shock, the baryon mass density and the electron number density rise, leading to the increase of $\mu_e$ and, as a result, $\mu_n$. Immediately thereafter, neutronization occurs and the value of $(\mu_n - \mu_p)$ rises, which reduces $\mu_e$. We can recognize from Figure 6 that the peaks of $\mu_e$ and the luminosity are correlated. As for the time evolution, the luminosity on the shock surface is lower at the early phase, because the shock radius is small. On the other hand, it is also lower at the late phase, because $\mu_e$ is lower. This is the reason why the luminosity on the shock surface has a peak.

We now investigate model 4a, whose initial entropy is $s = 7.5k_B$. In the bottom right panel of Figure 6, snapshots of the profiles of $\mu_e$ for model 4a are shown. We can see that the value of $\mu_e$ at the shock surface is lower than that of model 1a at the early phase. This is because the baryon mass density on the shock surface of model 4a at the bounce is lower than that of model 1a, as mentioned above. Accordingly, the electron number density and $\mu_e$ are also lower for model 4a, and $\mu_e$ does not rise so high. This is the main reason why the neutronization burst is not remarkable. It is noted, moreover, that the electron fraction $Y_e$ on the shock surface of model 4a is lower than that of model 1a. This is because nuclei do not exist and the nucleons are already neutronized on the shock surface. The absence of nuclei is consistent with the fact that the higher the initial entropy is, the earlier nuclei dissolve into nucleons, as explained by Figure 3. In addition, we can see that the luminosity of $\nu_e$ rises monotonically, because the area of a shock surface increases, whereas $\mu_e$ is almost unchanging.

The results for the models with higher resolutions are shown in Figure 8. While the duration times of their neutrino emissions differ slightly among the models as mentioned above, the profiles of their neutronization bursts are not very different qualitatively. In fact, the luminosity declines a little after the peak and increases again for model 2a. This feature is well kept in other models with higher resolutions.

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**Fig. 5.**—Luminosities of $\nu_e$ (short-dashed lines), $\bar{\nu}_e$ (solid lines), and $\nu_x$ (long-dashed lines) as functions of $t$, where $\nu_x$ stands for $\mu$- and $\tau$-neutrinos and their antiparticles. Squares show the time when the apparent horizon is formed. Top left, top right, bottom left, and bottom right panels are for models 1a ($s = 3k_B$), 2a ($s = 4k_B$), 3a ($s = 5k_B$), and 4a ($s = 7.5k_B$), respectively.
We show the time-integrated neutrino spectra in Figure 9. We can see that the spectra become softer for higher entropy models, especially for $\bar{\nu}_e$ and $\nu_x$. In order to investigate this tendency, we show the time-integrated spectra of the neutrino emitted before and after the shock formation in Figure 10. We can see that, for higher entropy models, $\bar{\nu}_e$ and $\nu_x$ are also emitted before the shock formation. They are created by electron-positron pair annihilation, and their energy is relatively lower (≤ several MeV), because the temperature is low ($T$ ≤ 1 MeV). On the other hand, for lower entropy models, $\bar{\nu}_e$ and $\nu_x$ cannot be produced by the electron-positron pair process, because positrons are absent owing to Pauli blocking. As for the $\bar{\nu}_e$ and $\nu_x$ emitted after the shock formation, they are mainly created by bremsstrahlung. In this phase, the temperature near the neutrino sphere rises to $T$ several MeV, which makes the neutrino energies relatively high, $\sim$ 10 MeV. Since the low-energy (≤ several MeV) neutrinos are not emitted to any great extent and the spectra become harder for lower entropy models, the emission of low-energy $\bar{\nu}_e$ and $\nu_x$ is characteristic for the collapse of high-entropy cores.

### 3.3. Initial Velocity Dependence

We compare the results of models 2a and 2b, which are different in the initial values of central density and temperature, but have the same initial values of entropy per baryon ($s = 4k_B$). We can consider that models 2a and 2b are the same model but with different initial velocities, because the density profile of model 2b at the time when the central density reaches that of the initial model of 2a almost coincides with that of model 2a (Fig. 11). In reality, the onset of a collapse is determined not only by the core structure but also by the whole stellar structure. Thus, studying the initial velocity dependence of the core is meaningful. As a result of this comparison, we find that the initial velocity does not crucially affect the ensuing dynamics and the features of emitted neutrinos such as total number spectra or the time evolution
of the luminosity. This is because the velocity of model 2b at the
time in Figure 11 is several times lower than the sound speed at
each point. For instance, the fastest point of model 2b in Figure 11
has the velocity $v / C_{24} \approx 10^8 \text{ cm s}^{-1}$, while the sound speed is $\approx 7 \times 10^8 \text{ cm s}^{-1}$ there. If the supersonic region, where the infalling ve-
locity exceeds the sound speed, existed in the initial model, the
initial velocity profile could be important for the dynamics. How-
ever, since the temperature of our initial models is slightly higher

than the critical temperature for the photodisintegration instabil-
ity, they are unlikely to have a supersonic region.

3.4. Collapse of a Population III Star with 100 $M_\odot$

In this section, we consider yet another example of very mas-
sive stars, that is, a Population III star with 100 $M_\odot$. We use a model
constructed by Nomoto et al. (2005) with evolutionary calculations,
and we refer to it as model R. This model is very massive and its
entropy at the center is higher than that of ordinary supernova
progenitors when it starts to collapse, because the star does not
lose its mass at all in its evolution owing to its zero metallicity.
It should be emphasized that the isentropic models are meant
for the massive stars that may be produced in the present uni-
verse, for example, by stellar mergers in clusters, whereas model
R corresponds to a first-generation star in the past universe. Here
we are interested in the differences that these models may make.
In Figure 12 we show the comparison of the initial state of model
R and our isentropic models at the time when their central den-
sities become the same as that of model R. We can recognize that model R has an entropy $4k_B$ in the central region, which is
between those of model 1a and 2a, whereas the iron core of
model R is smaller than that of our models. In fact, the iron-core
mass of model R is $\approx 2.32 M_\odot$, which is close to that of model 1a.
We show some of the initial values at the center of model R in
Table 1. Incidentally, the initial velocity profile is taken into ac-
count for model R, although it is much lower than the sound
speed at each point.

As a result of collapse, model R has a bounce and recollapses
to a black hole. As shown in Table 1, the values of the central
density and the central adiabatic index of model R at the bounce
are between those of models 1a and 2a. This suggests that these values are determined by the initial central entropy as mentioned
in § 3.1. On the other hand, model R has a much longer time in-
terval from the bounce to the recollapse, compared with our

![Fig. 8.—Luminosities of $\nu_e$ as a function of $t$ for models 2a (solid line), 2m (short-dashed line), $2e$ (long-dashed line), and 2g (dot-dashed line). The meaning of the squares is the same as in Fig. 5.](image)

![Fig. 9.—Spectra of time-integrated emissions of $\nu_e$ (short-dashed lines), $\bar{\nu}_e$ (solid lines), and $\nu_x$ (long-dashed lines). Top left, top right, bottom left, and bottom right panels are for models 2a ($s = 4k_B$), 4a ($s = 7.5k_B$), 5a ($s = 10k_B$), and 6a ($s = 13k_B$), respectively.](image)

![Fig. 10.—Spectra of time-integrated emissions of $\nu_e$ (short-dashed lines), $\bar{\nu}_e$ (solid lines), and $\nu_x$ (long-dashed lines). The top left and top right panels give the time integrations of the emission before and after bounce, respectively, for model 2a ($s = 4k_B$). The bottom left and bottom right panels present the emission before and after shock formation, respectively, for model 6a ($s = 13k_B$).](image)
models, because the inner core mass of model R at the bounce \((M_{\text{bou}})\) is smaller and the lower density of the outer core (Fig. 12) gives lower accretion rates. For instance, at \(t = 0.06 \, \text{s}\), model R has a mass accretion rate \(\sim 4 \, M_{\odot} \, \text{s}^{-1}\) at the shock surface, whereas model 1a has \(\sim 11 \, M_{\odot} \, \text{s}^{-1}\). Thus, it takes much time until the inner core mass exceeds the maximum mass of the neutron star.

We show the total energy of neutrinos emitted during the collapse of model R in Table 2 and the time evolution of the emitted neutrino luminosity in Figure 13. We can see that the total energy of emitted neutrinos is larger than that of other isentropic models, despite the fact that the neutrino luminosity of model R is lower than those of our models. This is because model R neutrino emission lasts much longer. Moreover, the mean energy of the emitted neutrinos is larger for model R. This is also due to the longer duration time. The neutrino spectrum gets harder in the late phase, because the density of the accreting matter becomes lower and the temperature on the neutrino sphere gets higher. Thus, the longer the duration time of neutrino emission is, the larger the mean energy of the emitted neutrinos becomes. It is noted that the duration time is sensitive to the equation of state, which was mentioned above, and hence, the total and mean energy of emitted neutrinos is also sensitive to the equation of state.

In the following, we discuss the features of the emitted neutrinos from model R for the early phase, which is not sensitive to the equation of state as mentioned above. Comparing Figures 5 and 13, we can see that for model R, the peak luminosity of the electron-type neutrino by the neutronization burst is lower than those of our models. The reason why it is lower than that of model 1a \((s = 3k_{\text{B}})\) is because the chemical potential of an electron-type neutrino for model R is lower than that for model 1a, while the shock radii in both models are not so different from each other (right panels of Fig. 13). It is consistent with the fact that the density at the bounce of model R is lower than that of model 1a (Table 1). On the other hand, the shock radii of models with \(s \geq 4k_{\text{B}}\) (models 2a–4a) are larger than that of model R. This is the reason why the luminosity of the neutronization burst for
model R is lower than those of models 2a–4a. Furthermore, since the outer core density of model R is much lower than those of isentropic models (Fig. 12), $\mu_e$ drops quickly and the shock radius does not get much larger after the neutronization. From equation (3), these features lead to the fact that the luminosity of the electron-type neutrino after the neutronization burst drops more steeply for model R than for our models. It follows, then, that the decline of the neutronization burst depends not only on the initial entropy but also on the initial density profile. In particular, the larger the initial density gradient is, the more steeply the neutronization burst declines.

To sum up, the key parameters listed in Table 1 at bounce (e.g., central density, temperature) do not differ very much between the isentropic models and model R. This is not true for the time profile of the neutronization burst, because they depend not only on the central density at bounce but also on the initial density profile. However, since, in general, more massive iron cores have larger entropies, the following trend is generally true. The neutronization burst declines.

3.5. Astrophysical Implications

3.5.1. Progenitor of IMBH

For supermassive black holes (SMBHs) located at the center of many galaxies including ours, a new formation scenario via intermediate-mass black holes (IMBH) has recently been suggested (e.g., Ebisuzaki et al. 2001; Portegies Zwart et al. 2006). According to this scenario, very dense stellar clusters are initially formed in the vicinity of the galactic center (≤10 pc), and the massive stars with $\sim 20 M_\odot$ in them undergo runaway collisions to form IMBHs before they lose most of their mass by supernova explosions and/or pulsations. After that, these IMBHs merge together and finally form a SMBH. This scenario is supported by the discovery of the ultraluminous X-ray compact sources in the galaxy M82, which indicate the existence of IMBHs. It is conceivable that similar events occur in the Milky Way Galaxy as mentioned below. This scenario assumes that the supermassive stars formed by the runaway collisions would collapse to IMBHs when they are $\sim 1000 M_\odot$.

Recently, Suzuki et al. (2007) have studied the structures and evolution of these merged stars in the hydrogen-burning stage. According to them, the smaller star sits at the center of the larger star after the merger of two stars with different masses. It is also demonstrated that the merged stars become convectively unstable by the positive gradient of the mean molecular weight and that their evolution thereafter approach those of the single homogeneous star with the same mass and abundance. The central entropies of the merged stars will then be larger than those of the inhomogeneous single stars with the same mass. This suggests the possibility of forming the IMBH progenitors by the merger without experiencing the pair instability. Here we speculate on the entropy of these stars using previous studies on single Population III stars. Since the iron core of the Population III star with 100 $M_\odot$ has an entropy of $\sim 3.5 k_B$ (Nomoto et al. 2005), it is expected that these IMBH progenitors have entropies $\geq 3k_B$.

Even if the pair instability occurs, the massive stars corresponding to our models may still be formed. In fact, the positive entropy gradient and/or rotation may suppress the convection in the merged star, and the entropy at the center may remain low after the merge. Then the merged star has a massive envelope with a smaller core than the single stars with the same total mass. If the pair instability occurs for these objects, the nuclear burning may not produce total disruptions but lead to the eventual collapse. Again inferring from single Population III stars, we speculate that the central entropies of the merged stars will be smaller than $s \sim 16k_B$, which corresponds to 300 $M_\odot$ in NSY06. It is incidentally mentioned that the relations between the total mass and the iron-core mass of merged IMBH progenitors is highly uncertain at present.

In the preceding sections we have shown that the neutrino signals from the black hole formation are sensitive to the inner region of the progenitor. In this section, assuming our models correspond to the above-mentioned merged stars which collapse to IMBHs at the center of our Galaxy ($\sim 8.5$ kpc from the Sun), we estimate the neutrino event number for the currently operating detectors.

As for the event rate of the IMBH formation, based on the above-mentioned scenario and the fact that the SMBH residing in the center of our Galaxy (Sgr A*) is $\sim 3.5 \times 10^6 M_\odot$ and the
age of our Galaxy is \( \sim 10 \) Gyr, a very rough estimation for the formation rate of IMBHs with \( \sim 1000 M_\odot \) is \( \approx \) once per 1 Myr. It is, however, mentioned that this event rate may be underestimated, because star formation may not be continuous but triggered by some environmental effects (e.g., the merger of galaxies). Recent observations by Paumard et al. (2006) have revealed the existence of about 80 young massive stars within a distance of a parsec from Sgr A\(^*\), and some of them are identified as OB stars and their ages are about 6 \( \pm \) 2 Myr. These facts indicate that stars are actively formed in this region at present. Moreover, the IMBH candidate with \( \sim 1300 M_\odot \), IRS 13, is found in the same region (Maillard et al. 2004). Thus, Sgr A\(^*\) may be currently growing under this scenario.

In the following estimations for the neutrino event number, we do not take into account the neutrino mixing, although it should be. Because the mixing occurs mainly in the resonance regions and they are located outside the iron core of the progenitor, the neutrino oscillation does not affect the dynamics of the core. Unfortunately, the structures of the envelopes of merged stars, which are crucial for the neutrino mixing, are quite uncertain. There remain uncertainties as well on the mixing parameters, such as the mixing angle of \( \sin^2 \theta _{13} \) or the mass hierarchy. Thus, the precise evaluation of the neutrino flux including the neutrino mixing is deferred to future study.

### 3.5.2. Detection of Low-Energy \( \bar{\nu}_e \) by SuperKamiokande and KamLAND

As mentioned above, a good deal of low-energy \( \bar{\nu}_e \) are emitted from the collapse of the high-entropy cores, which softens the spectrum. We estimate the \( \bar{\nu}_e \) event number for SuperKamiokande III and KamLAND, currently operating neutrino detectors, under the assumption that the black hole formations considered in former sections occur at the center of our Galaxy. For both detectors, the dominant reaction is the inverse \( \beta \)-decay,

\[
\bar{\nu}_e + p \rightarrow e^+ + n, \tag{4}
\]

which is the only one we take into account. We adopt the cross section for this reaction from Vogel & Beacom (1999). For SuperKamiokande III, we assume that the fiducial volume is 22.5 kton and the trigger efficiency is 100\% at 4.5 MeV and 0\% at 2.9 MeV, which were the values at the end of SuperKamiokande I (Hosaka et al. 2006). For KamLAND, we assume 1 kton fiducial mass, which means that 8.48 \( \times \) 10\(^{31}\) free protons are contained (Eguchi et al. 2003). We also assume that the trigger efficiency is 100\% for all \( \bar{\nu}_e \) energy larger than the threshold energy of the reaction.

The results are given in Table 3. The total event number does not change monotonically with the initial entropy of the core, because the total number of neutrinos depends on both the core mass and the duration time of neutrino emission, as mentioned above. In order to investigate the hardness of the \( \bar{\nu}_e \) spectrum, we calculate the ratio of the event number of \( \bar{\nu}_e \) with \(< 10 \) MeV to that for all events. The ambiguity about the distance to the source is also canceled by this normalization. This ratio gets larger as the entropy of the core becomes higher. This suggests that we can probe the entropy of the black hole progenitor especially in higher regimes (\( s \geq 7.5 k_B \)), because the event numbers of \( \bar{\nu}_e \) with \(< 10 \) MeV are over 100 according to SuperKamiokande III.

| Model | \( N_{\bar{\nu}_e<10 \text{ MeV,SK}/N_{\bar{\nu}_e,SK} \) | \( N_{\bar{\nu}_e<10 \text{ MeV,Kam}/N_{\bar{\nu}_e,\text{Kam}} \) | \( N_{\bar{\nu}_e,\text{Kam}} \) |
|-------|---------------------------------|---------------------------------|-------------------|
| 1a     | 3.3                             | 5163                            | 3.3               | 174               |
| 2a     | 4.0                             | 4778                            | 4.0               | 135               |
| 1b     | 4.0                             | 4910                            | 4.0               | 139               |
| 3a     | 4.6                             | 4319                            | 4.6               | 122               |
| 4a     | 7.3                             | 4018                            | 7.3               | 114               |
| 5a     | 11.8                            | 5326                            | 12.0              | 151               |
| 6a     | 20.1                            | 9139                            | 20.5              | 259               |

Notes.—The subscript “\(< 10 \) MeV” means the event of \( \bar{\nu}_e \) with \(< 10 \) MeV, and the subscripts “SK” and “Kam” mean the prediction for SuperKamiokande III and KamLAND, respectively.

Since SNO can also detect the \( \bar{\nu}_e \) flux, we can estimate the intensity of the neutronization burst by comparing the event from the charged-current reaction of \( \nu_e \),

\[
\nu_e + d \rightarrow p + p + e^-, \tag{5}
\]

and that of \( \bar{\nu}_e \),

\[
\bar{\nu}_e + d \rightarrow n + n + e^+, \tag{6}
\]

using the SNO detector. SNO can also detect the neutral-current reaction,

\[
\nu + d \rightarrow n + p + \nu, \tag{7}
\]

for all species. It is noted that the neutral-current reaction contains \( \nu_e \) (\( \bar{\nu}_e, \nu_\mu, \nu_\tau \), and \( \bar{\nu}_\tau \)), and the neutrino sphere of \( \nu_e \) differs more from that of \( \nu_e \) than that of \( \bar{\nu}_e \) in general. Thus, for comparison with the reaction from equation (5), the reaction from equation (6) is more appropriate than the reaction from equation (7). On the other hand, we also use equation (7) for the comparison, because the event number of equation (7) is larger than that of equation (6). In our calculation, we use the cross sections from Ying et al. (1989) and assume that the trigger efficiency of these reactions is 100\%. In fact, it is \( \sim 92\% \) these days, which is the neutron (in the right-hand side of eq. [7]) capture efficiency on \(^{35}\)Cl and deuterons (Oser 2005).

In the following analysis, we regard the emission of neutrinos before \( t = 0.06 \) s as the neutronization burst, where the time \( t \) is measured from the bounce. The criterion \( t = 0.06 \) s is chosen empirically from our simulations as an expedient. The method for extracting the neutronization burst from detection should be reconsidered for more detailed studies. Here we calculate the event numbers for \( t < 0.06 \) s as well as those for the entire time duration of the neutrino emission, and the results are summarized in Table 4. We can recognize that the ratios of the \( \nu_e \) event number (\( N_{\nu_e<0.06 \text{ s}} \)) to the total event number of the charged-current reactions (\( N_{\nu_e<0.06 \text{ s}} + N_{\bar{\nu}_e<0.06 \text{ s}} \)) and that for the neutral-current reaction (\( N_{\nu<0.06 \text{ s}} \)) are larger for the models whose neutronization burst declines more steeply. Despite the fact that these neutronization burst numbers are of the order of 10, we can probe into the black hole progenitors in principle.

It is finally noted that the estimations in the current study are based on spherically symmetric models. If progenitors are rotating rapidly, the neutrino sphere will become nonspherical and
the neutrino emissions will be affected in general. This will be the subject of future investigations.

4. CONCLUSIONS

In this paper we have numerically studied gravitational collapse and black hole formation of massive iron cores systematically, taking into account the reactions and transports of neutrinos in detail. Massive iron cores with \( \sim 10 M_\odot \) have a bounce owing to thermal nucleons, following which they collapse to black holes when the maximum mass is reached. As for the emitted neutrinos, the spectra of \( \nu_e \) and \( \bar{\nu}_e = (\bar{\nu}_\mu, \bar{\nu}_\tau, \bar{\nu}_e) \) become softer for more massive models, or higher entropy models, because a high entropy generates a large number of electron-positron pairs, which create \( \bar{\nu}_e \) and \( \nu_e \). The neutronization burst from more massive iron cores becomes less remarkable or disappears completely, because the density at the bounce is lower and even the \( \nu_e \) number density in equilibrium becomes lower.

We have found that if the initial velocity is lower than the sound speed, it does not affect the collapse very much. We have also compared the collapse of our isentropic models with that of the realistic model, which is obtained by the detailed modeling of the evolution of Population III stars, and we have found that the steep decline of the neutronization burst depends not only on the initial entropy but also on the initial density profile. Moreover, assuming our models as the progenitors of IMBHs collapsing at the Galactic center, we have estimated the neutrino event numbers. As a result, for SuperKamiokande III, the ratio of the \( \bar{\nu}_e \) event number for \( < 10 \text{ MeV} \) to that for all events gets larger as the entropy of the core becomes higher, especially for \( s \geq 7.5k_B \).

We have suggested that we can use these features to probe into the progenitors. As for the lower entropy cores, despite the fact that the event number for the early phase of the emission is less than 100 by SNO, we have suggested that the steep decline of the neutronization burst can be distinguished in principle.

Concerning the prediction of neutrino event numbers, there is a room for further improvement. First, the effects of the neutrino oscillation should be taken into account. Second, multidimensional effects, such as rotation or magnetic field may be important, since they will affect the dynamics of collapse itself. This study will hopefully provide a first step toward a neutrino astrophysics for black holes.

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