Membrane-capacitive transducer with compensation for the primary standard of pressure

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Abstract. The paper discusses improvements for the primary vacuum-meter transducer for measuring absolute pressure in a wide range from 0.1 to 10 Pa.

1. Introduction
The measurements of low absolute pressures (vacuum) are the basic measurements in a wide area of vacuum technologies. The value of these measurements is high for all branches of industry, related to the use of vacuum in technological processes in view of increasing demands of quality of products. Particularly significant are the vacuum measurements in atomic, aerospace and electronic industry, in metallurgy and other high-technology industries. Due to this, a number of instrumentations for low absolute pressure measurements is developing. So in recent years, a number of different types of precision instruments of a new generation for measuring low absolute pressures, both domestic and foreign, have appeared on in Russian Federation. The metrological support of these instruments requires the improvement of the reference base and the primary standard itself. Therefore, its modernization is an urgent task.

In the D I Mendeleev FSUE VNIIM, the unit of pressure in this range is carried out on the standard GET 49-2016 [1], which is improved and recertified in 2016. The standard of pressure GET 49-2016 is based on a membrane-capacitive transducer with compensation. The use of a membrane-capacitive transducer with compensation (MEPK) as a primary pressure standard was proposed in the thesis of Ryzhov V A in VNIIM in 1964 [2]. He proposed MEPK with the membrane and the electrode of equal diameters and numerous experimental studies of the prototype have been carried out.

2. Mathematical model of MEPK
The operation of the transducer MEPK is based on the effect of variation of the capacity of the capacitor formed with the membrane and the electrode (see figure 1) under the action on the membrane of the measured pressure $P$ and the electric force $F_e$ caused by applying the compensating voltage $U$ to the electrode. The action of this electric force can be considered as some “electric” pressure $P_e$. The electric field strength $E$ can be approximated by $U/d$, where $d$ is the distance between the membrane and the electrode. Then $P_e$ is given by the formula
\[ P_e = K_e U^2, \quad K_e = \frac{\varepsilon_0}{2 d^2}, \quad (1) \]

where \( \varepsilon_0 \) is the dielectric constant of vacuum.

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Figure 1. Construction of MEPK and the forces.

The membrane is bended under the action of pressures \( P \) and \( P_e \). Let \( z(r) \) be its transverse displacement, which we assume small compared to distance \( d \). Below the dimensionless variable \( w = z/d \) is used. The membrane of MEPK is supposed to have some initial tension \( T \) and its displacements are subject to the membrane equation [3]:

\[ \Delta w(r, \varphi) = -\frac{q}{Td} \quad , \quad (2) \]

where \( q \) is the total load per unit square of the membrane, and the boundary conditions \( w(R, \varphi) = 0 \) describing the fastening of the membrane on its boundary of radius \( R \). The load \( q \) is caused by pressure \( P \) and attractive electric force \( F_e \), represented by pressure \( P_e \). Dependence on the angle \( \varphi \) is excluded in view of cylindrical symmetry of the problem and equation (2) is reduced to an ordinary differential equation. As the first order approximation let the force applied to the membrane be piece-wise constant and \( F_e = 0 \) beyond the electrode, at \( r > R_e \). Then equation (2) can be rewritten as two equations:

\[ w''(r) + \frac{1}{r} w'(r) = -\frac{P - P_e}{T d} \quad , \quad r < R_e \quad (3) \]

\[ w''(r) + \frac{1}{r} w'(r) = -\frac{P}{T d} \quad , \quad r > R_e \]

which should be supplied with the conditions of continuity \( w(r) \) and \( w'(r) \) at \( r = R_e \).

When the membrane is bent, the distance between the electrode and the membrane changes. Accordingly, the electric field strength also changes. The operation of the MEPK is based on the fact that the feedback system maintains constant capacity of the capacitor \( C \) formed by the membrane and the electrode to which the compensating voltage \( U \) is applied. The formula for the capacitance is obtained by summing the elementary capacitances and has the form:

\[ C = \frac{\varepsilon_0}{d} 2\pi \int_0^{R_e} \frac{r \, dr}{1 + w(r)} \quad . \quad (4) \]

Thus the operation of MEPK is described by the system of equations in which the membrane equation (3) is complemented with the requirement of the capacitance (4) constancy, i.e. \( C[w(r)] = const \). Solving this system of equations [4] yields the relation, which expresses pressure \( P \) via the voltage \( U \) of the compensating electric field:
\[
P = K(U^2 - U_0^2), \quad K = K_eK_1, \quad K_1 = \frac{4\ln\left(\frac{R}{R_e}\right) + 1}{2\left(\frac{R^2}{R_e^2}\right) - 1}. \tag{5}
\]

Besides the force of pressure \(F\) and the force of electric field \(F_e\) the developed mathematical theory of MEPK [4] takes into account the "elastic" force \(F_m\) that acts on the edge of the membrane due to its fastening to the wall of the camera. Thus the force applied to the edge of the membrane is added to the electric force to provide the balance of forces acting on the membrane: \(F + F_e + F_m = 0\).

3. The comparison of MEPK and baratron

In the FSUE D I Mendeleev VNIIM, in 2016, an experimental work was performed to comparing the MEPK readings with the baratron readings, the calibration of which was carried out at the manufacturer. Both the MEPK and the baratron simultaneously measured the pressure in a volume filled with air at low pressure.

The membrane and the electrode radii are \(R_e = 20.73\) mm, \(R = 23.02\) mm and the distance between the electrode and the membrane \(d = 0.097\) mm are conditioned by the requirements of the construction of the transducer MEPK, constructed in 1980. The computed transformation coefficient \(K\) (5) for these parameters of MEPK is equal to

\[
K = 4.553 \cdot 10^{-4} \frac{[Pa/V^2]}{[Pa/V^2]} \quad \text{while} \quad K_1 = 0.9677. 
\]

The relative deviation \(w\) of the pressure readings \(P_{MPE}\) of MEPK with respect to the pressure \(P_b\) measured by baratron was determined in the three series of experiments:

\[
w = \left(\frac{P_{MPE} - P_b}{P_{MPE}}\right) \times 100. \tag{6}
\]

The results are shown in figure 2.

![Figure 2](image.png)

**Figure 2.** Relative deviation \(w\) in percent (6) of the pressure readings \(P_{MPE}\) of MEPK transducer and pressure readings \(P_b\) of the baratron dependently on the pressure in Pascals.

4. Results and discussion

Figure 2 shows the difference of readings of the two devices up to 4 % at low pressures which decreases at higher pressures, which is caused by temperature transpiration. The effect of temperature transpiration is observed in the case of temperature difference in different parts of the vacuum system and can be described by the formula [5]:

\[
\frac{P_{MPE}}{P_b} = \left(\frac{T_{MPE}}{T_b}\right)^{\frac{2L}{2(D+2L)}}, \tag{7}
\]

where \(T_{MPE}\) is the temperature readings of MEPK, equal to \(20\) °C, \(T_b\) is the temperature of baratron which measurement camera is heated to \(45\) °C, \(D\) is the effective diameter of the connecting pipe.
\[ D = 5 \text{ mm}, \] 
\[ L \text{ is the mean free path of gas molecules at mean pressure. For the air at the temperature of} \]
\[ T = 273 \text{ K it is} \]
\[ L_1 = 6.7 \cdot 10^{-3} \text{ m at the pressure of 1 Pa and at a different pressure} \]
\[ P \text{ it is:} \]
\[ L = \frac{L_1}{P}. \]

Theoretically computed curve of temperature transpiration practically coincides in its shape with the experimental data, which is shown in figure 3.

![Figure 3. Comparison of the theoretical and experimental transpiration curves.](image)

From the obtained experimental results it can be concluded that the accuracy of pressure measurements is on the order of 1 %. This estimate is based both on the magnitude of dispersion of the experimental data and on the magnitude of the deviation of the theoretical and experimental transpiration curves.

It is possible to consider the model of MEPK whose membrane has both the tension and the stiffness [4]. In this case coefficient \( K_1 \) does not depend on the mechanical characteristics of the membrane only for equal radii of the membrane \( R \) and the electrode \( R_e \) and is exactly equal to unity (see figure 4).

![Figure 4. The coefficient K as a function of R.](image)
To determine the parameters of MEPK it is necessary to perform only geometrical and electric measurements which can be presently performed with very high accuracy. Respectively, the standard of pressure is reduced to only length measurement and voltage measurement. This is especially important in the context of the transition to a new SI system [6], based on quantum effects, and in which electrical quantities become in some sense more primary.

Presently we work on further improvements of GET 49-2016. After this modernization the primary standard of pressure based on MEPK transducer will allow the unit of pressure to be reproduced with the accuracy of about 0.35 %.

References
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