Estimating the quarantine failure rate for COVID-19

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Abstract
Quarantine is a crucial control measure in reducing imported COVID-19 cases and community transmissions. However, some quarantined COVID-19 patients may show symptoms after finishing quarantine due to a long median incubation period, potentially causing community transmissions. To assess the recommended 14-day quarantine policy, we develop a formula to estimate the quarantine failure rate from the incubation period distribution and the epidemic curve. We found that the quarantine failure rate increases with the exponential growth rate of the epidemic curve. We apply our formula to United States, Canada, and Hubei Province, China. Before the lockdown of Wuhan City, the quarantine failure rate in Hubei Province is about 4.1%. If the epidemic curve flattens or slowly decreases, the failure rate is less than 2.8%. The failure rate in US may be as high as 8.3%–11.5% due to a shorter 10-day quarantine period, while the failure rate in Canada may be between 2.5% and 3.9%. A 21-day quarantine period may reduce the failure rate to 0.3%–0.5%.

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1. Introduction

The current COVID-19 pandemic poses severe challenges to public health systems worldwide. Quarantine is a crucial control measure to reduce imported cases, and has been implemented in most countries, such as United States (Centers for Disease Control and Prevention, 2020), Canada (Government of Canada, 2020) and China (Kang et al., 2020).

One major challenge to quarantine is that COVID-19 patients may have a long incubation period. It is estimated that the median incubation period is approximately a week (Leung, 2020; Li et al., 2020; Ma et al., 2020; Tian et al., 2020; Tindale et al., 2020). As much as 7.3% patients could have an incubation period longer than the recommended 14-day quarantine period (Li et al., 2020). A patient may be quarantined in the middle of his/her incubation period. So not all of the patients with long incubation periods will fail the quarantine, i.e., showing symptoms after finishing quarantine. However, some patients may. After they finish quarantine, these patients may be mistakenly considered healthy by their close contacts, and are thus especially risky to causing community transmissions. The possibility of asymptomatic transmissions (Hellewell et al., 2020; Li et al., 2020; Park et al., 2020; Rothe et al., 2020; Ye et al., 2020) also increases this risk.

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Centers for Disease Control and Prevention (2020) suggests that travelers who are tested adhere to a 7-day quarantine period, and those who are not tested adhere to a 10-day period. This is shorter than the 14-day quarantine period recommended by WHO (World Health Organization, 2020). It has been suggested (Rolfes et al., 2021) that a shorter quarantine period may be easy to maintain, but it poses a risk of quarantine failure. To understand this risk, it is important to estimate the quarantine failure rate of a given quarantine period.

The quarantine failure rate not only depends on the fraction of patients with an incubation period longer than the quarantine period, it also depends on the probability distribution of the period from being infected to quarantine. This distribution depends on the epidemic curve. For example, during a fast exponential growth phase, more patients are infected in more recent days, and thus a randomly quarantined patient is more likely to be infected more recently. To assess the effectiveness of the 14-day quarantine policy, in this paper, we propose a formula to estimate the quarantine failure rate from the incubation period distribution and the epidemic curve.

We develop our formula in Section 2. In Section 3, we apply our estimations to United States, Canada, and Hubei Province, China. Concluding remarks are given in Section 4.

2. Quarantine failure rate

In this section, we aim to develop a formula to estimate the quarantine failure rate. This failure rate depends on both the incubation period distribution and the probability distribution of the period from being infected to quarantine. Assume that the incubation period has a cumulative distribution function (CDF) \( F(t) \). The quarantine period has a length \( T \). For a patient quarantined at time \( t \), the quarantine failure rate \( q(t) \) is the probability that the patient shows symptom after quarantine finishes at time \( t + T \). Given that the patient was infected at time \( s < t \), this probability is thus \( 1 - F(t + T - s) \). The failure rate is equal to the expected value of this probability with respect to the random time \( s \). Assume that the epidemic curve, i.e., the number of new infections at time \( t \) is \( X(t) \). Then a randomly chosen patient quarantined (and thus is asymptomatic) at time \( t \) was infected at time \( s \) with the following conditional probability density

\[
p(s|t) = \frac{[1 - F(t - s)]X(s)}{\int_{-\infty}^{t} [1 - F(t - s)]X(s)ds}.
\]

Thus, the failure rate is

\[
q(t) = \int_{-\infty}^{t} [1 - F(t + T - s)]p(s|t)ds = \frac{\int_{-\infty}^{t} [1 - F(t + T - s)][1 - F(t - s)]X(s)ds}{\int_{-\infty}^{t} [1 - F(t - s)]X(s)ds}.
\]

Let \( v = t - s \),

\[
q(t) = \frac{\int_{0}^{\infty} [1 - F(v + T)][1 - F(v)]X(t - v)dv}{\int_{0}^{\infty} [1 - F(v)]X(t - v)dv}.
\]

2.1. The discrete time formula with reported cases

The number of new infections \( X(t) \) is generally not available. We may infer \( X(t) \) from the number of reported cases \( C(t) \) on day \( t \) using the Richard-Lucy de-convolution method (Goldstein et al., 2009). Alternatively, we may simply assume that \( X(t) \) is proportional to \( C(t) \), i.e., \( X(t) = \rho C(t) \) where \( \rho > 0 \). Note that the constant \( \rho \) cancels in (1). Thus, after discretizing the time \( t \), (1) becomes

\[
q(t) = \frac{\sum_{\nu=0}^{\infty} [1 - F(\nu + T)][1 - F(\nu)]C(t - \nu)}{\sum_{\nu=0}^{\infty} [1 - F(\nu)]C(t - \nu)}.
\]

2.2. The exponentially growing phase

As a special case, if the number of new infections grows exponentially with a constant rate \( r \), i.e.,
\[ X(t) = e^{t - t_0}, \]

where \( t_0 \) is the time when the first case emerges. Then (1) becomes

\[
q = \frac{\int_0^\infty \left[ 1 - F(v + T) \right] [1 - F(v)] e^{-rT} dv}{\int_0^\infty [1 - F(v)] e^{-rv} dv},
\]

(3)

which does not depend on the time \( t \). Note that this formula is only applicable to the beginning exponential phase of an epidemic. If the exponential growth rate changes either due to control measures and the depletion of susceptibles, this formula is not applicable, because the probability for the length of infection before a patient is quarantined changes with the epidemic curve.

The failure rate \( q \) is a decreasing function of \( T \) and an increasing function of \( r \). This can be verified with straightforward calculations of the partial derivatives of \( q \) with respect to \( T \) or \( r \). Thus, for each fixed \( T \), the largest failure rate is achieved at \( r = \infty \), in which case \( q = 1 - F(T) \) (the fraction of patients with an incubation period longer than \( T \) days). Thus,

\[
q \leq 1 - F(T).
\]

This is because a patient may be quarantined late in his/her incubation period. Even if he/she has an incubation period longer than \( T \), the quarantine may not fail as the patient may show symptom before exiting quarantine.

2.3. Flat epidemics

If the epidemic curve is flat, i.e., \( r = 0 \), then (3) becomes

\[
q = \frac{\int_0^\infty \left[ 1 - F(v + T) \right] [1 - F(v)] dv}{\int_0^\infty [1 - F(v)] dv}.
\]

(4)

2.4. The incubation period distribution

Using Chinese case descriptions, Li et al. (2020) parametrized the incubation period distribution to be gamma distributed with a shape parameter 2.65 and a scale parameter 2.67. Here, we use this distribution to calculate the quarantine failure rate. Fig. 1 shows the quarantine failure rate of COVID-19 as a function of the quarantine period \( 14 \leq T \leq 22 \) and an exponential

![Graph showing the quarantine failure rate as a function of the exponential growth rate and the quarantine period.](image)

**Fig. 1.** The contour plot of the quarantine failure rate \( q \) as a function of the exponential growth rate \( r \) and the quarantine period \( T \), computed using (3). It is an increasing function of \( r \), and a decreasing function of \( T \).
growth rate $-0.2 \leq r \leq 0.8$. The failure rate is an increasing function of $r$, and a decreasing function of $T$. If the epidemic curve is constant or slowly decreases, i.e., the exponential growth rate $r \leq 0$, then the failure rate $q \leq 2.8\%$ from (4).

3. Applications

3.1. United States

We use (2) and the daily reported cases in US (Center for Systems Science and Engineering (CSSE) at Johns Hopkins University, 2020) for the period of March 1st, 2020 to January 2nd, 2021 to compute the quarantine failure rate in US. Fig. 2 compares the failure rates for the 14-day quarantine period recommended by WHO and the 10-day quarantine period recommended by US CDC for the travelers who are not tested. The failure rate of the 14-day quarantine period ranges from 2.7\% to 3.9\%, which is approximately 1/3 of the failure rate of the 10-day quarantine period (ranging from 8.3\% to 11.5\%). If the quarantine period is extended to 21 days, the failure rate ranges from 0.3\% to 0.5\%.

3.2. Canada

We use (2) and the daily reported cases in Canada (Center for Systems Science and Engineering (CSSE) at Johns Hopkins University, 2020) for the period of March 15th, 2020 to January 2nd, 2021 to compute the quarantine failure rate in Canada. The results are shown in Fig. 3. The failure rate ranges from 2.5\% to 3.9\%. If the quarantine period is extended to 21 days, the failure rate ranges from 0.3\% to 0.5\%.

3.3. Hubei Province, China

We use (3) to compute the failure rate of the 14-day recommended quarantine period in Hubei Province, China. Li et al. (2020) estimated that the exponential growth rate $r$ for Hubei before the lockdown of Wuhan City (the epicenter) on January 23 is 0.23, with a 95\% confidence interval (0.21, 0.26). The resulting failure rate for the 14-day quarantine policy is 4.1\% with a 95\% confidence interval (4.0\%, 4.2\%). For a 21-day quarantine period, the failure rate is 0.6\%.

4. Concluding remarks

The failure rate for the currently recommended 14-day quarantine period may range from 2.8\% (if the epidemic curve is flat) to 4.2\% (if the exponential growth rate was as high as that in Hubei Province, China before the lockdown of Wuhan City). For the period from March to December 2020, the failure rates in Canada are indeed approximately in this range. However, in

Fig. 2. The estimated failure rates for both a 10-day and a 14-day quarantine period in United State for the period of April 1st, 2020 to January 2nd, 2021. The upper panel shows the epidemic curve (from March 1st, 2020) used to calculate the failure rates, and the lower panel shows the estimated failure rates.
US, a shorter 10-day recommended quarantine period is recommended for travelers who are not tested, resulting in a threefold increase in the failure rate. If the quarantine period is increased to 21 days, the failure rates in US and Canada may be greatly reduced to 0.3%–0.5%. However, a proper quarantine period should always consider the tradeoff between practicality and a low quarantine failure rate. An excessively long quarantine period may result in less conformance to the quarantine rules, leading to more quarantine failure.

We did not consider reinfeict patients in this paper. Considering such patients may slightly affect the failure rate formula, because they may have a different incubation period distribution than the patients in primary infections. However, the effect of reinfection may be negligible because a) the estimated incubation period distribution used here is estimated from all patients, including reinfeict patients; and b) reinfeict patients may be only a small fraction of all patients, especially during the early exponential growth phase crucial for policy making. We also implicitly assumed that quarantine is perfect, i.e., individuals cannot be infected during quarantine. Infections during quarantine surely increase the quarantine failure rates.

The failure rate depends on the epidemic curve because this curve determines the distribution of the time of infection for a quarantined patient. For quarantine of international travelers, the epidemic curve of the source region of the travelers should be used. For local quarantine triggered by contact tracing, the local epidemic curve should be used. Social distancing and other control measures that lower the exponential growth rate also reduce the quarantine failure rate. Variants of concern typically spread with a larger transmission rate, so these variants may cause a slightly larger quarantine failure rate. In addition, patients infected by different strains may have a different incubation period distribution than the one used in this paper. This should be considered when studying the quarantine failure of variants of concern.

Declaration of competing interest

The authors have no conflict of interest.

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