Statistical Analysis of the Fatigue Failure Phenomenon of Powder Bed by Loading with Dynamic Repeated Tensile Stress

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Abstract

To study more quantitatively the fatigue failure phenomenon of powder beds, which was earlier discovered and reported by a part of the authors, this study was an attempt to obtain the relation between repeated tensile stress and fatigue life, which is usually represented by S-N relation, in a wide range of the repeating number N. Experiments were carried out by using vibration to exert dynamic repeated tensile load on the powder bed, with a vertically vibrating cell and a horizontally splitting cell. The frequency of vibration used was in the region of 5–300 Hz, and Kanto loam powder (JIS-11), fused alumina powder and lactose powder were used as samples.

The results obtained are analyzed statistically and quantitatively concerning the distribution of fatigue failure life.

It is found that within the limited range of the number of repeated stress less than $10^6$, the existence of a lower limit of tensile stress (endurance limit) where the powder bed fails is confirmed at a stress ratio of the order of 0.7. The distribution of fatigue life is expressed by a Weibull distribution, and then it is shown quantitatively that the distribution of fatigue life of the powder bed is wide compared with other kinds of material, and it is suggested that the S-N relation of the powder bed obtained previously can be divided into two ranges, a sloping part and a horizontal part.

1. Introduction

Mechanical properties of fine-powder bed have been investigated primarily by means of static failure tests for tensile and shear. In practical operations, however, there are specific phenomena which are difficult to be verified by use of such knowledge.

Consider, for instance, repeated application of impact stress perpendicularly to a dust layer deposited on a fabric filter. A part of the authors observed that the dust layer was allowed to be dislodged by the repeated application of relatively small impact stress that was insufficient enough to cause the dislodgement by only one trial. This fact could not be explained by an existing static model only based on a cohesion and separation force.

To solve the above problem, the authors introduced a concept of “fatigue failure” as a fundamental mechanical property of powder bed into a study on the dislodgement caused by a shear-directional repeated stress. This study provided the fact that the shape of S-N relation curves, which were used to express fatigue failure phenomenon, would be depend-
ent on stress direction: vertical or shear direction.

Based on such an observation, a part of the authors started a preliminary study on the application of quasi-static repeated tensile stress to powder bed. It was ascertained from this study using a simplified test apparatus that a fatigue failure could take place in powder bed.

The object of the present report, continued from the last report, is to relate the stress with the repeat number of applied stress required to cause split of powder bed so as to analyze more quantitatively fatigue failure phenomenon. In the present work, vibration techniques were used to obtain larger repeated number of stress application during a unit time. Such a test implies the extension to the treatment of fatigue failure under dynamic conditions. The obtained result will be discussed in such a way as that of material science and also a statistical analysis of fatigue life distribution, and this phenomenon will be considered quantitatively.

2. Experimental apparatus and methods

Two kinds of vibration (high and low frequency) were adopted to apply repeated tensile stress to powder bed. High frequency vibration was used for the case where the split of powder bed needed many repeated times of stress application; while low frequency vibration was done for few repeated times.

2.1 Repeated application of high frequency vibration (Experiment I)

This experiment was carried out to break off an exposed part of sample material in a cell by applying repeatedly tensile stress caused by a vertical vibration. The apparatus and the cell used in this experiment are schematically illustrated in Fig. 1 (a). The cell, made of stainless steel with diameters of 25.2 mm (inside) and 30 mm (outside), and a depth of 32 mm, was mounted on an electromagnetic vibrator (Node, VT-10) with the continuously controlled range of 5 to 50000 Hz.

The test cell 1 and a split-type frame 2 were fixed by using a ring 3 as shown in Fig. 1 (b). A sample powder was fed into this test cell with a spatular and was consolidated with a weight as shown in Fig. 1 (c). In this case, each height of a powder bed was adjusted to be by 1 mm higher than the frame depth after pre-consolidation. After this consolidation was made for 10 minutes, the weight, the piston 4, and the ring 3 were removed. Then the powder bed was cut off at the top of the frame 2, and finally the frame 2 was removed to complete forming of the test sample.

The experiment was carried out in the following procedure. First, the high-frequency vibration was applied increasingly until the set value of acceleration was achieved. Then this vibration was applied continuously in a constant magnitude until the exposing part of the powder bed was fallen on the tray 6. At that time, the period from the start of the constant vibration to fall of the powder bed was counted as an effectively operated time, and its mass in the tray 6 was weighed. The vibratory acceleration was measured by a pick-up (EMIC
Piezoelectric type, Model 541-AT) mounted perpendicularly on the cell.

In this experiment, pre-consolidation stress and vibration frequency were varied. The former was limited within 11.4 and 41.3 kPa because too little pre-consolidation stress could not keep the powder bed from dropping off out of the cell. For the latter, 70, 100, and 300 Hz were adopted because the acceleration ratio of vertical direction to horizontal one was less than 5 percent that was measured by the pick-up mounted on the upper part of the cell side. The time required to reach the set value was a few seconds by adjusting the rise-up time to be constant. Since this method made use of high-frequency vibration, the time required was very short when the failure took place in a few repeated times. This does not tend to give exact time which would be required to cause failure. Therefore the iterative loading by low-frequency vibration is required. As this method, however, acceleration large enough to cause failure was not able to be obtained due to the limited amplitude of vibrator, the authors used the following low-frequency vibratory tester to achieve an expected object.

2. 2 Application of iterative loading by low-frequency vibration (Experiment II)

A hunger-type cohesion tester was used as a test equipment for this experiment. As shown in Fig. 2, the movable cell of this tester was connected with a vibrator to exert iterative stress on a powder bed in the cell by a low-frequency vibration.

The sample powder was packed by a spatula into the split cell with a inside diameter of 50 mm an a depth of 20 mm. The powder bed was formed by cutting off the over-flowing material after 10 minutes pre-consolidation. The experiment was made in the following procedure. First, the vibration was started to apply on the movable cell which was kept fixed by a clamp. Then, when the magnitude of vibration was increased gradually and reached the set value, the fix was released. After the stress was applied repeatedly, failure of the powder bed was observed to take place. At that time, the period from release of the hook to failure of the powder bed was measured.

Table 1 shows the average particle diameter, true density, and porosity of the fracture surface of the three kinds of sample materials used. In this experiment, the pre-consolidation pressure varied from 4.0 to 21.5 kPa and the vibration frequency was 5 and 20 Hz. Although displacement of the powder bed could not be obtained in the experiment I. This value was obtainable in this experiment, by using a differential transducer.

3. Experimental result

As is well known in the field of material science, fatigue failure phenomenon of solid materials have been investigated by using S-N relations; $S$ and $N$ represent an applied stress and its application frequency (hereinafter referred to as “life”), respectively.

In the present work, this $S$-N relation was introduced by taking the maximum value of stress $\sigma_s$ as a representative one in the experiment I and II. The value of $\sigma_s$ in the experiment I is given by

$$\sigma_s = \frac{m_T(a_0 \omega^2 + g)}{A}$$  \(1)
Where \( m \) is the mass of a sample material falling on the tray, \( a_0 \) the amplitude of vibration, \( \omega \) the angular frequency, and \( A \) the cross-sectional area of fracture surface. Also, \( N_b \) is the repeat time of applied vibration.

Figure 3 shows a typical result of the \( S-N \) relations of Kanto Loam (JIS No.11) of which data were abundantly obtainable. The values at \( N = 10^6 \) denote average ones of static tensile strength at various porosities determined by a hunger-type cohesion tester. As indicated clearly in this figure, the fatigue strength increased as the porosity decreased, that is, the static strength increased. The effects due to the difference in the type of apparatus, the cell shape, and the vibration frequency in the experiment I and II could not been observed obviously. These tendencies were similar in other sample materials used. Thus, the following discussion will take into no account of these factors.

In Fig. 4, all the results obtained in the present experiments are indicated in the term of dimensionless value that represents the ratio of actual stress to average static tensile strength \( \sigma_0 \). The use of this stress ratio, independent of the magnitude of static strength, would permit the \( S-N \) relation to be more distinguished. In this figure, the data where no failure was observed (represented by the symbol \( \circ \)) are found to exist at \( N_b \) of more than \( 3 \times 10^6 \). Also, when the stress ratio was less than 0.7, there was no failure at \( N_b \) of less than \( 10^6 \). This implies that there would be a minimum stress, so-called endurance limit, below which there is no possibility to cause a fatigue failure. When the stress ratio was more than a unit, on the other hand, a fatigue failure took place. This phenomenon should be noticed as one of the dynamic characteristics because it could not be seen in a static test.

4. Statistical analysis on phenomenon of fatigue failure

Wide variation of data shown in Fig. 4 implies that there could be fatigue failure life distribution according to certain stress ratios. To bring out more distinct relations, the authors tried to analyze statistically the stress ratio distribution of Fig. 4, as follows.

4. 1 Procedure of statistical analysis

The statistical analysis was carried out, based on the following method: Standard Method of Statistical Fatigue Testing (JSME-S-002-1981)\(^4\):

![Fig. 3 An example of S-N relation](image-url)
1) Data of the stress ratio were classified into 2 to 7 segments containing same numbers of data in each experimental condition.
2) Order of small to large number was adjusted for each segment to arrange as a serial statistical value.
3) Median rank was used to obtain a mortal probability corresponding to each serial number, that is, the portion of powder bed broken under its own life, because the distribution shape of population was unknown.

In the present work, analysis was carried out by including data on survivalship. Fatigue life distributions obtained by the above method could be the most successfully arranged by the cumulative Weibull-distribution function of two population parameters expressed as

\[ P(N) = 1 - \exp \left( -\left( \frac{N}{N_c} \right)^m \right) \]  

or written by the probability-density function:

\[ p(N) = \left( \frac{1}{N_c} \right)^m N^{m-1} \exp \left( -\left( \frac{N}{N_c} \right)^m \right) \]  

where \( N_c \) is the characteristic fatigue life in Weibull-distribution function at the failure probability of 63.2 percent. The value of \( m \), so-called sharp parameter, characterizes the variation of fatigue lives and is expressed as the slope on the Weibull-probability graph.

4.2 Result of statistical analysis

The data on Kanto Loam (JIS No.11) with a porosity of 0.740 were most abundantly obtainable in the present work. The result of analyzing statistical fatigue failure life distribution of these data is expressed by Weibull distribution in Fig. 5, where the range of the stress ratio is also attached. The solid line represents a least square approximation of mortal probability obtained from a failure life and a median rank, while the one-point-dashed and broken lines denotes the limits of 90 percent in confidence.

Figure 6 shows the measured result of fused almina (W.A. #8000) with a porosity of 0.675 whose data were also much obtained. As seen in this figure, the characteristic fatigue life in Weibull-distribution function, \( N_c \), would be much dependent on the region of the stress ratio and also the shape parameter in Weibull-
distribution function, $m$, which characterizes the spread of the distributions. Similar results were obtainable on lactose powder, through they are not indicated in the present paper.

In Figs. 7 and 8, the mean values of the characteristic life $N_c$ and the shape parameter $m$ are indicated as a function of the mean value of stress ratio $\overline{\sigma}/\overline{\sigma}_2$. These data are all that were gained in the present work within the confidence zone of 90 percent. The value of $N_c$ is found to decrease with increasing the stress ratio in Fig. 7. In particular, the data of Kanto Loam (JIS No.11) with a porosity of 0.72 and 0.74, which were abundantly obtainable in the present work, can be devided into two regions: one has a steep slope within the stress ratio of 0.7 to 0.85 and the other has a gentle slope at the ratio of larger than 0.85. Such a tendency is supported by the 90 percent confidence zone. Thus, this implies that the present analysis used the number of data enough to estimate qualitative variations in $N_c$.

It is also considered that the position and
sharpness of $S$-$N$ curves depend on experimental conditions, because estimates of other data which were scantily obtainable distant from the hard line in Fig. 7.

On the other hand, the shape parameter $m$ was almost constant, independently of the stress ratio, as shown in Fig. 8. Therefore, the shape of fatigue-life distribution would be almost unchangeable within the conditions adopted in the present work.

The above statistical analysis could lead to a significant result: two slopes representing different tendencies of decreasing $N_c$. This suggests that fatigue failure phenomenon could be classified into two regions.

5. Discussion

As illustrated in Fig. 7, it is shown that the characteristic life $N_c$ decreased rapidly below a certain value as the stress ratio was reduced. This is probably because of not only increase of the failure life but also increase of the survival possibility without failure of powder bed, accompanied by a decrease of the stress under the present test condition.

To demonstrate the relationship between such a survivalship phenomenon and the difference of the slopes indicated in Fig. 7, consider Fig. 9 that shows a mortal probability obtained from the present work. The value of $P_{3 \times 10^4}$ was defined here as a mortal probability where failures of powder bed took place at the repeated times of less than $3 \times 10^4$; for there were no data of failure obtained at $N$ of more than $3 \times 10^4$.

The failure probability at $N \leq 3 \times 10^4$ was obtained as follows: After the stress ratio was classified into some segments containing the same numbers of data, that probability was represented by the ratio of the failure data to the total data containing unbroken data in each segment.

In Fig. 9 the mortal probability $P_{3 \times 10^4}$ approached to almost a unit at the stress ratio of 0.85 where the slope of the line changed abruptly in Fig. 8. When $\bar{\sigma}_f/\bar{\sigma}_2 < 0.85$, there would exist a limiting region including survivalship under the finite repetition of stress loading. The characteristic life $N_c$ would increase infinitely, when $P_{3 \times 10^4} = 0$ or $\bar{\sigma}_f/\bar{\sigma}_2 \approx 0.68$. This implies that the $S$-$N$ relationship could be expressed as a horizontal line parallel to the $N_b$ axis.

When $\bar{\sigma}_f/\bar{\sigma}_2 > 0.85$, on the other hand, failure of powder bed never failed to take place at $N_b$ of less than $3 \times 10^4$. Fig. 7 shows that the $S$-$N$ relation in this mortal region was declined against the $N_b$ axis, compared with the horizontal region.

Figure 10 shows $S$-$N$ curve based on assuming the presence of inclined and horizontal part for date on the half survivalship in number of Kanto Laom (JIS No.11) indicated in Fig. 7. It is found from this figure that the line would become horizontal at $N_b$ of more than about $10^6$. However, whether the $S$-$N$ curve is parallel to the $N_b$ axis seems to be dependent on the data which can be given at $N$ of more than $10^6$. The stress ratio at $N = 10^6$ was over a unit. This is probably because the tensile stress $\bar{\sigma}_2$ would vary at the extent of 5 percent and also dynamic, but very short loading, repetition could provide larger stress required to cause failure by each loading than static action. In practice, it has been ascertained in the field
of soil mechanics that a tri-axial compression test permits a failure strength to increase when the loading time is limited within a second 1). Thus, the $S$-$N$ curve obtained in the present work is considered to include an effect of increasing strength resulting from the dynamic characteristics of powder bed.

The shape parameter $m$ can be estimated to be constant at both inclined and horizontal part of $S$-$N$ relation, judging from the result of Fig. 8. Other materials, like iron steel, allow $m$ to increase in the inclined part due to relatively narrow distribution and to decrease in the horizontal part due to broad distribution. Therefore, the $S$-$N$ curve for these materials can be classified easily into inclined and horizontal parts. As for powder bed, on the other hand, there are wide distributions of fatigue life even in a inclined part. This implies that data of powder bed tend to vary considerably in the view of fatigue failure phenomenon as well as other specific characteristics. It should be also noticed that the probability-density distribution function $p(N)$ decreased monotonously with an increase of $N$ because the values of $m$ were almost below a unit.

6. Conclusion

In the present work, experiments were conducted by using vibratory motions to load repeatedly powder bed with tensile stress less than static strength. The conclusions obtained are:

1) There existed a lower limit stress (endurance limit) in fatigue failure of powder bed suffering from dynamic tensile stress load under the repetition of $10^8$ times.

2) Application of the tensile stress repeated over the lower limit permitted the mortal life due to fatigue failure to be expressed as a Weibull distribution.

3) The $S$-$N$ relation of tensile fatigue failure was shifted from inclined part to horizontal one as the stress ratio decreased.

4) The shape parameter was kept constant $(m \approx 0.5)$, in both the inclined and the horizontal part. The fatigue failure test of powder bed was found to provide the wide range of data.

Nomenclature

- $A$: sectional area of vertical cell [cm²]
- $d_p$: mean particle diameter [µm]
- $f$: frequency of vibration [Hz]
- $g$: gravitational acceleration [m/s²]
- $m$: shape parameter in Weibull distribution function
- $\bar{m}$: estimate of shape parameter in Weibull distribution function
- $m_f$: mass of separated powder bed [kg]
- $N$: number of repeated stress
- $N_b$: number of repeated stress to failure, fatigue life
- $N_c$: characteristic fatigue life in Weibull distribution function
- $\bar{N}_c$: estimate of characteristic fatigue life in Weibull distribution function
- $N_{50}$: fatigue life that failure probability is 50 percent
- $P_{3 \times 10^4}$: failure probability at $N \leq 3 \times 10^4$
- $P$: cumulative distribution of fatigue life
- $p$: probability density of fatigue life
- $a_0$: amplitude of vibration [m]
- $e$: porosity rate [-]
- $\rho_p$: particle density [kg/m³]
- $\sigma_s$: repeated tensile stress [kPa]
- $\bar{\sigma}_s$: mean repeated tensile stress [kPa]
- $\bar{\sigma}_s$: mean static tensile strength of powder bed [kPa]
- $\omega$: angular frequency [rad/s]

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