Remarks on Cosmic String Formation during Preheating on Lattice Simulations

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We reconsider the formation of (global) cosmic strings during and after preheating by calculating the dynamics of a scalar field on both two- and three-dimensional lattices. We have found that there is little differences between the results in two and three dimensions about the dynamics of fluctuations, at least, during preheating. Practically, it is difficult to determine whether long cosmic strings which may affect the later evolution of the universe could ever be produced from the results of simulations on three-dimensional lattices with smaller box sizes than the horizon. Therefore, using two-dimensional lattices with large box size, we have found that cosmic strings with the breaking scale \(\eta \sim 10^{16}\text{GeV}\) are produced for broad range of parameter space in \(\eta\), while for higher breaking scales \((\eta \sim 3 \times 10^{16}\text{GeV})\), their production depends crucially on the value of the breaking scale \(\eta\) in our simulations.

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One of the hot topics at the reheating stage after inflation is the possibility of the formation of topological defects \([4,5]\). This phenomenon is due to large nonthermal fluctuations during preheating \([1,2]\) and efficient rescattering \([4,5]\) both of which are caused by the Bose enhancement effects.

At the preheating stage, very large nonthermal fluctuations are produced, \(\langle \delta \phi^2 \rangle \sim c^2 M_p^2\), where \(M_p\) is the Planck mass and \(c = 10^{-2} - 10^{-3}\) \([6,7]\). These fluctuations make the shape of the effective potential of the field \(\phi\) change to the type of the potential which has a minimum at the origin if the potential \(V(\phi)\) is of spontaneous symmetry-breaking type. It may be regarded that the symmetry is restored \([5,7]\). Later, when the amplitude of these fluctuations is redshifted away by the cosmic expansion, the symmetry is spontaneously broken and topological defects may be created. Thus, the mechanism for producing the topological defects seems to be somewhat similar to the Kibble mechanism in high temperature theory.

For the order estimation of the critical value of the breaking scale where cosmic strings are not formed above that value, it is sufficient to see the amplitude of the nonthermal fluctuations produced during preheating: \(\langle \delta \phi^2 \rangle^{1/2} \sim 10^{16}\text{GeV}\) \([8,9]\). This order estimation is in good agreement with numerical estimations done in Refs. \([1,2]\). But numerical simulations on the lattices which follow the dynamics of the scalar fields reveals several new results, such as the effects of the rescattering \([1,2]\) or the number of defects estimation \([4,5]\).

In our previous paper \([2]\) we investigated the dynamics of a complex scalar field using two-dimensional lattice simulations, taking into account that the box size is large enough to cover the horizon size and that the lattice size is small enough to identify cosmic strings safely, and concluded that (long) cosmic strings would not be produced if the breaking scale \(\eta\) was larger than \(\eta \sim 3 \times 10^{16}\text{GeV}\). On the other hand, the authors of Ref. \([5]\) showed the possibility that cosmic strings could be formed even if \(\eta = 6 \times 10^{16}\text{GeV}\) using the three-dimensional lattice where the box size is smaller than the horizon. In this paper, we will discuss both two- and three-dimensional lattice simulations, and make certain that these results are consistent, commenting on the limitations of both simulations.

First we show that results from lattice simulations in two and three dimensions are not different from each other when we study the parametric resonance during preheating. To be concrete, let us consider a complex scalar field with the effective potential:

\[
V(\Phi) = \frac{\lambda}{2}(|\phi|^2 - \eta^2)^2, \tag{1}
\]

where \(\lambda\) is a small coupling constant. This model has a global U(1) symmetry, and cosmic strings are formed when the symmetry is spontaneously broken. What we have to do is to integrate the equation of motion:

\[
\ddot{\varphi} + 3H \dot{\varphi} - \frac{1}{a^2} \nabla^2 \varphi + (|\varphi|^2 - \eta^2) \varphi = 0. \tag{2}
\]

For numerical simulations, it is convenient to use rescaled variables:

\[
a(\tau) d\tau = \sqrt{\lambda} \Phi_0 a(0) dt, \tag{3}
\]

\[
\varphi = \Phi_0 a(0) \varphi, \tag{4}
\]

\[
\xi = \sqrt{\lambda} \Phi_0 a(0) \xi, \tag{5}
\]

where \(\Phi_0 \equiv |\Phi(0)|\). Setting \(a(0) = 1\), we obtain

\[
\varphi'' - \frac{a''}{a} \varphi - \nabla^2 \varphi + (|\varphi|^2 - \eta^2 a^2) \varphi = 0, \tag{6}
\]
where \( \tilde{\eta} \equiv \eta / \Phi_0 \) and the prime denotes differentiation with respect to \( \tau \). The second term of the left-handed side can be omitted since the energy density of the universe behaves like radiation at the early times and also the scale factor becomes very large later. In this case, the rescaled Hubble parameter \( h(\tau) \equiv H(\tau) / \sqrt{\lambda} \Phi_0 \), and the scale factor \( a(\tau) \) become

\[
h(\tau) = \frac{\sqrt{2}}{3} a^{-2}(\tau), \tag{7}
\]

and

\[
a(\tau) = \frac{\sqrt{2}}{3} \tau + 1, \tag{8}
\]

respectively, when \( \Phi \) is assumed to be an inflaton (even if \( \Phi \) is not an inflaton, results are the same as in the case of rescaling the breaking scale in an appropriate way, see Ref. [4]) and we set \( a(0) = 1 \). For the initial conditions we take

\[
x \equiv \text{Re} \varphi(0) = 1 + \delta x(x), \tag{9}
\]

\[
y \equiv \text{Im} \varphi(0) = \delta y(x), \tag{10}
\]

where the homogeneous part comes from its definition (we call \( x \) direction for real direction and \( y \) for imaginary), and \( \delta x, y(x) \) is a small random variable of \( O(10^{-7}) \) representing the fluctuations. We also set small random values for velocities.

Since the physical length and the horizon grow proportional to \( a \) and \( a^2 \), respectively, the rescaled horizon grows proportional to \( a \). The initial length of the horizon is \( \ell_h(0) = 3 / \sqrt{2} \approx 2.12 \). Therefore, the rescaled horizon size grows as \( \ell_h(\tau) = \ell_h(0) a(\tau) \). It is thus better to take the box size larger than \( \ell(\tau_{\text{end}}) \) where \( \tau_{\text{end}} \) is the time at the end of the simulation. This is because those strings whose lengths are shorter than the horizon scale will be in the form of loops, which will shrink and disappear very soon. Only longer strings than the horizon will survive to affect the later evolution of the universe. On the other hand, the width of the topological defect is \( (\sqrt{2} \eta a(\tau))^{-1} \) which corresponds to \( (\tilde{\eta} a(\tau))^{-1} \) in the rescaled units. Since it decreases with time, one lattice length should be at least comparable with the defect width at the end of the calculation.

Leaving these facts aside, let us just compare the evolution of fluctuations on two-dimensional lattices with three-dimensional ones. Here we take \( 128^3 \) three-dimensional lattices and \( 4096^2 \) two-dimensional lattices with the lattice size \( \Delta \xi = 0.3 \) for both cases. We find no difference between both growth exponents (see Fig. 1). Notice that the data used on the two-dimensional lattice (top panel) is the same used in Fig. 4 of Ref. [4], where it was plotted linearly instead of logarithmically as in Fig. 1. Moreover, we obtain very similar results

\[
\text{in two- and three-dimensional lattices, at least, on the effects of parametric resonance during preheating.}
\]

Since small loop strings will shrink and disappear very soon, and cause no influence on the universe, we are interested only in infinitely long strings. In three-dimensional lattices, they can be considered as those strings that penetrate through the box of the lattice (come into the box from one side and go out to the other side) and survive until the later time. Actually, we have found tempo-

\[\text{\[4\]}\]

\[\text{\[5\]}\]

\[\text{\[6\]}\]

\[\text{\[7\]}\]

\[\text{\[8\]}\]

\[\text{\[9\]}\]

\[\text{\[10\]}\]
rary formation of cosmic strings at any breaking scale, which confirms the result of Ref. [5]. In most values of the breaking scale, however, these strings are in the form of small loops, and disappear very quickly. We have also found strings longer than the box size. In Fig. 2, we take the breaking scale $\eta = 3.08 \times 10^{16}$ GeV, the lattice size $\Delta \xi = 0.3$, and integrate the equation of motion until $\tau = 280$. If we integrated until the time before $\tau = 265$, we would conclude that cosmic strings were formed for this breaking scale. However, as we can see in Fig. 2 these long strings feel the attractive force from each other, and they form into loops beyond the box size, which is much smaller than the horizon scale: $N\Delta \xi = 38.4 \ll \ell_h(\tau = 280) \approx 282$, where $N$ is the number of the lattice.

We agree with the authors of Ref. [5] that longer strings tend to be created when the velocity of the real part of the oscillating homogeneous mode $x$ becomes almost zero at the moment when it passes through the origin of the effective potential (in their words, when the moment of the symmetry breaking nearly coincides with the moment when $\langle \phi_1 \rangle (= x)$ passes through zero [5]). They argued that it leads to the fact that the long string formation is a non-monotonic function of the breaking scale [5], which we confirm to some extent. However, in more than one hundred runs of our simulations, for almost all the breaking scales higher than $3 \times 10^{16}$ GeV, we have not found long strings stretched out beyond the box size except for two breaking scales: $\eta \simeq 3.08 \times 10^{16}$ GeV and $\eta \simeq 3.16 \times 10^{16}$ GeV on the $128^3$ lattices with $\Delta \xi = 0.3$ (Actually, long strings cannot be found at these scale, if another initial configurations of initial fluctuations are used). Even in these scales, long strings deform into loops and shrink and disappear very soon.

Therefore, we cannot make any definite conclusion on the formation of cosmic strings in the sense that we cannot tell whether or not they may affect the evolution of the universe. This is because the box size cannot be taken to be larger than the horizon scale in three-dimensional lattices. We have thus studied the formation of cosmic strings in two-dimensional lattices in the previous paper [4], abandoning one dimension in space because of the lack of memory capacity of computers.

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$^1$We have recently found long strings formed at $3.01 \times 10^{16}$ GeV, but they are very unstable in the sense that strings in the form of loops develop into the form of long strings, and later they again deform into loops and disappear. Notice that the range of the breaking scale where long strings are formed is also very narrow at this scale as in the above two scales. Actually, these scales correspond to the circumstances that the velocity of the real part of the oscillating homogeneous mode $x$ becomes almost (but not exactly) zero at the moment when it passes through the origin of the effective potential, so there may be more breaking scales where long cosmic strings are formed.

FIG. 2. Evolution of cosmic strings for $\eta = 3.08 \times 10^{16}$ GeV. We plot from $\tau = 264$ (top) to $\tau = 270$ (bottom).
We can see that the size of the narrow regions are both of the breaking scale for the fixed initial conditions: \( \Delta \eta \) are in the only very narrow region of the parameter space. In this case, therefore, the cases when those strings stretched across the box size have been once formed, for the sake of the conservative discussion, we have found that the corresponding cases are in the only very narrow region of the parameter space of the breaking scale for the fixed initial conditions: \( \Delta \eta / \eta \simeq 3 \times 10^{-3} \). See Figs. 3 and 4. Figures 3 and 4 show the lifetime \( (\tau_d - \tau_f) \) with respect to the breaking scale \( \eta \) for \( \sim 3.08 \times 10^{16} \text{GeV} \) and \( \sim 3.16 \times 10^{16} \text{GeV} \), respectively. We can see that the size of the narrow regions are both \( \Delta \eta \simeq 0.009 \times 10^{16} \text{GeV} \). Here \( \tau_d \) is the time when the long strings destruct into loops, and \( \tau_f \) is the formation time for those strings.

However, this criterion may not be so sufficient, since those strings stretched across the lattice box will deform into loops and disappear very soon. We can take other criterion, which may reflect the idea that it is important to consider a long cosmic string. When we observe the lifetime \( (\tau_d - \tau_f) \), we find that long strings survive a few times longer at a certain range of the breaking scale, as shown in Figs. 3 and 4. We can expect that long strings that stretch beyond the horizon size will be formed only within such very narrow regions. In this case, therefore, the breaking scale should be in the very narrow ranges \( (\Delta \eta / \eta \simeq 10^{-4}) \) in order for (infinitely) long strings to be formed for the fixed initial conditions in our simulations (These features can be also seen on the lattices with larger box size: \( N = 200 \)). In other words, the long string formation is very sensitive to the breaking scale.

These results can be understood as follows. If there is no gradient force, the dynamics of the field is determined only by the homogeneous mode. But the initial values of the field at each point on the lattice differs from each other by \( O(10^{-7}) \), we naively expect that cosmic strings are formed only in the very narrow ranges of the breaking scale of \( O(10^{-7}) \). Owing to the preheating stage, fluctuations become large, so that the degree of the narrow ranges is somewhat relaxed. However, it seems that the full development of rescattering does not occur yet for \( \eta \sim 3 \times 10^{16} \text{GeV} \), as we mentioned in Ref. 4. We will see later that the fundamentally different feature appears for the lower breaking scales.

As mentioned above, it is difficult to tell whether long cosmic strings are formed on three-dimensional lattices with a smaller box size than the horizon volume, and this is why we have calculated in two dimensions, in order to make definite conclusions using both two and three-dimensional simulations complementarily. Similar results are found in two-dimensional lattices. In the two-dimensional case, we cannot distinguish long strings from loops, because all the strings are assumed to be infinitely different scales, and they are also very narrow.

\[ \frac{\eta}{10^{16} \text{GeV}} \]

FIG. 3. Dependence of the lifetime \( (\tau_d - \tau_f) \) when long strings penetrate the box of the lattice on the breaking scale \( \eta \) around \( \eta = 3.078 \times 10^{16} \text{GeV} \). Here \( \tau_d \) is the destruction time when a long string deforms into a loop, and \( \tau_f \approx 250 \) is the formation time when a long string is first formed. Notice that only loop strings can be formed outside the narrow strip of the breaking scale \( (\eta < 3.075 \times 10^{16} \text{GeV}, 3.082 \times 10^{16} \text{GeV} < \eta) \).

\[ \frac{\eta}{10^{16} \text{GeV}} \]

FIG. 4. Dependence of the lifetime \( (\tau_d - \tau_f) \) when long strings penetrate the box of the lattice on the breaking scale \( \eta \) around \( \eta = 3.158 \times 10^{16} \text{GeV} \). Notice that the formation time is \( \tau_f \approx 250 \), and only loop strings can be formed outside the narrow strip of the breaking scale \( (\eta < 3.156 \times 10^{16} \text{GeV}, 3.164 \times 10^{16} \text{GeV} < \eta) \).

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\(^8\)For other initial conditions, narrow ranges of the breaking scale where long cosmic strings are produced appear in the different scales, and they are also very narrow.
stretched along the z direction. Instead, we observe the number of cosmic strings in the horizon size at each time. The physically meaningful criterion for the formation of long cosmic strings is whether the number of strings per horizon remain (almost) constant or not as time goes on. Even if we cannot distinguish very long strings from loop ones, we can regard very nearly located string-antistring pair in two dimensions as a small loop string in three dimensions. Actually, they annihilate very soon, similar to small loop strings which will shrink and disappear very quickly. Thus, when we observe the number of strings as time goes on, it will remain at a certain value if long strings are formed, since loop strings (string-antistring pairs in terms of two dimensions) will disappear and very long strings (isolated strings in terms of two dimensions) will remain. Notice that we include both strings and antistrings in the numbers. As a result, we find that it depends on the breaking scale very crucially at the scale $\eta \sim 3 \times 10^{16}$ GeV, as is seen in Fig. 5. At the scales $\eta \sim 3.02 \times 10^{16}$ GeV and $\eta \sim 3.09 \times 10^{16}$ GeV, a dozen cosmic strings are formed, and the numbers do not decrease so much. On the other hand, at the other scales, the numbers of strings decreases, and finally, we find no strings at all. Thus, the value of the breaking scale has to be in the very narrow ranges such as the degree of $\Delta \eta / \eta \simeq 10^{-2}$, where $\Delta \eta$ is defined as the breaking scales at which the number of strings per horizon remains at a certain value as time goes on. This implies, together with the results in three dimensions, that long string formation is very sensitive to the breaking scale at these scales.

Moreover, we have simulated 30 realizations of initial conditions for fluctuations for each breaking scale, and find that the dependence of the average numbers of strings per horizon on the breaking scales, shown in Fig. 6, coincides with the above particular one of Fig. 5. This not only confirms the crucial sensitiveness to the breaking scale, but also implies that the main factor which determines whether long strings are produced is the value of the breaking scale. Initial fluctuations do not affect the dynamics of the scalar field so much at $\eta \sim 3 \times 10^{16}$ GeV.

On the contrary to the cases with somewhat higher breaking scale such as $\eta \sim 3 \times 10^{16}$ GeV, many cosmic strings are formed at $\eta \sim 10^{16}$ GeV as is seen in Fig. 7. We have found more than a dozen strings per horizon size at any value of the breaking scales near $\eta \sim 10^{16}$ GeV, and the numbers do not decrease so much. We thus conclude that the formation of cosmic strings occurs in the very broad region of the breaking scale at $\eta \sim 10^{16}$ GeV.

Finally, we should comment on the relation between the symmetry restoration and the topological defect (cosmic string) formation. To know whether the symmetry is restored or not is a difficult task to do, and its methods are somewhat uncertain. The authors of Ref. [5] argued it in terms of the shape of the effective potential for the scalar field. If the potential has a minimum at the origin, the symmetry is restored. We may make sure that the potential has a minimum at the origin in the following way, as done in Ref. [5]. The field $\phi$ (radial direction of $\Phi$) will oscillate around the origin in the potential of the form $(\phi^2 - \eta^2)^2$ when its amplitude is larger than $\sqrt{2} \eta$. But, as we can see in Fig. 8, $\phi$ is oscillating around the origin even when its amplitude becomes smaller than the breaking scale $\eta$. It is thus very clear that the effective
FIG. 7. Dependence of the number of strings per horizon on the breaking scale $\eta$ around $\eta = 10^{16}\text{GeV}$ at $\tau = 1420$ (dotted), and $\tau = 1800$ (solid) in two-dimensional simulations.

potential should have a minimum at the origin. We can see this phenomena in all cases in Fig. 8. However, in the above, we actually see that topological defects are produced only when the breaking scale is $\eta = 10^{16}\text{GeV}$, not in the cases of $\eta = 3 \times 10^{16}\text{GeV}$ and $\eta = 6 \times 10^{16}\text{GeV}$. Therefore, the symmetry must be fully restored only in the case of $\eta = 10^{16}\text{GeV}$, where rescatterings play a crucial role for that.

Then what is the relation between the symmetry restoration and defect production? We conclude that defect formation is the signal of the full restoration of symmetry. We thus discriminate between symmetry restoration and the shape of the effective potential which has a minimum at the origin. In other words, you cannot tell if the symmetry is restored only by observing the shape of the potential.

In conclusion, we have reconsidered the formation of (global) cosmic strings during and after preheating by calculating the dynamics of the scalar field on both two- and three-dimensional lattices, and confirmed the results of both Ref. [5] and our previous ones [4]. We have found that there are little differences between the results in two and three dimensions, at least, at the preheating stage. It is obvious that phase-space volume is larger in the three dimensions than two, and this effect might affect somehow in the rescattering stage, but we expect that it seems subdominant when we observe our numerical simulations.

Practically, it is difficult to determine whether long cosmic strings which may affect the later evolution of the universe could ever be produced from the results of simulations on three-dimensional lattices, since they will deform into large loops and disappear very soon because

FIG. 8. Evolution of the $\Phi$-field for various breaking scales. $\eta = 6 \times 10^{16}\text{GeV}$, $\eta = 3 \times 10^{16}\text{GeV}$, and $\eta = 10^{16}\text{GeV}$ from the top to the bottom. Here the amplitudes are plotted in the physical unit, not in the rescaled one.
of the small box size of the lattices. Moreover, we have found that cosmic strings with a higher breaking scale than $3 \times 10^{16}$ GeV could only be produced in the very narrow ranges of the breaking scale in our simulations. In Ref. [5] they referred to this formation of cosmic string as the formation of a nonmonotonic function of the breaking scale. We confirm this result and, in addition, find that the formation of long cosmic strings occurs in very small parameter space of the breaking scale $\eta$. In other words, it is very sensitive to the value of the breaking scale. In two-dimensional simulations, long strings and loops can be distinguished to some extent, even though it is difficult, by observing the evolution of the number of strings per horizon, and we have found similar phenomenon that long strings are produced only within very narrow range of the breaking scale around $\eta \sim 3 \times 10^{16}$ GeV for fixed initial conditions. On the contrary, they are produced for a wide range of the breaking scale when $\eta \sim 10^{16}$ GeV.

Cosmologically, we are interested only in (infinitely) long cosmic strings stretched beyond the horizon. Those strings with breaking scale $\eta \lesssim 10^{16}$ GeV are naturally formed after preheating, since they are produced independent to the actual values of the breaking scale. In other words, they are produced and their numbers remain constant in every horizon volume.

On the other hand, when the breaking scale is larger ($\eta \sim 3 \times 10^{16}$ GeV), it is very difficult to connect our results directly to the actual probability of the string formation. What we have found is that the formation of a cosmic string with $\eta \sim 3 \times 10^{16}$ GeV depends crucially on the breaking scale. As mentioned, since initial fluctuations do not affect whether long strings are formed so much, we can tell how many long strings are produced, if the value of the breaking scale and the initial condition for the homogeneous mode are fixed, which is, in general, determined if the inflation model is specified.

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