Density waves in the shearing sheet
III. Disc heating

B. Fuchs
Astronomisches Rechen-Institut, Mönchhofstr. 12–14, 69120 Heidelberg, Germany

Accepted Received ; in original form 2000 May

ABSTRACT
The problem of dynamical heating of galactic discs by spiral density waves is discussed using the shearing sheet model. The secular evolution of the disc is described quantitatively by a diffusion equation for the distribution function of stars in the space spanned by integrals of motion of the stars, in particular the radial action integral and an integral related to the angular momentum. Specifically, disc heating by a succession of transient, ‘swing amplified’ density waves is studied. It is shown that such density waves lead predominantly to diffusion of stars in radial action space. The stochastical changes of angular momenta of the stars and the corresponding stochastic changes of the guiding centre radii of the stellar orbits induced by this process are much smaller.

Key words: galaxies: kinematics and dynamics

1 INTRODUCTION

The shearing sheet (Goldreich & Lynden–Bell 1965, Julian & Toomre 1966) model has been developed as a tool to study the dynamics of galactic discs and is particularly well suited to describe theoretically the dynamical mechanisms responsible for the formation of spiral arms. For the sake of simplicity the model describes only the dynamics of a patch of a galactic disc. It is assumed to be infinitesimally thin and its radial size is assumed to be much smaller than the disc. Polar coordinates can be therefore rectified to pseudocartesian coordinates and the velocity field of the differential rotation of the disc can be approximated by a linear shear flow. These simplifications allow an analytical treatment of the problem, which helps to clarify the underlying physical processes operating in the disc.

In two previous papers (Fuchs 2001a, b, referred hereafter to as papers I and II) I considered the stellardynamical model of a shearing sheet and discussed the unbounded sheet and then the dynamical consequences, when inner boundary conditions are applied. The aim was to give a consistent theoretical description of pure swing amplification (Toomre 1981) as well as exponentially growing modes in the framework of the same model.

It is well known from numerous studies that, if a star disc is perturbed by a succession of spiral density waves, the stars are scattered randomly by the spiral arms and their velocity dispersion grows steadily so that the background states of the discs are evolving (Julian 1967, Carlberg & Sellwood 1985, Binney & Lacey 1988, Jenkins & Binney 1990). Dynamical disc heating has also been demonstrated in numerical simulations of the dynamical evolution of star discs such as by Sellwood & Carlberg (1984) or Toomre (1990).

In the present paper I discuss disc heating of the shearing sheet. I follow in particular the theory of diffusion of stars in the two–dimensional action integral space due to wave–star scattering developed by Dekker (1976). In section (2) I briefly describe the formal derivation of the diffusion equation in the framework of quasi–linear theory (Hall & Sturrock 1967) and in section (3) I calculate diffusion coefficients for scattering of stars by swing amplified density waves. This kind of spiral density waves has been shown to be relevant for disc heating by Toomre (1990), whereas mode–like density waves, on the other hand, do not heat effectively (Barbanis & Woltjer 1967, Lynden–Bell & Kalnajs 1972).

2 DERIVATION OF THE DIFFUSION EQUATION

When describing the disc heating effects of a succession of transient spiral density waves, one has to distinguish between the long time scales, on which the overall distribution function of stars in phase space is evolving, i.e. the dynamical heating time scale, from the shorter time scales, on which the individual density waves develop. As is well known from plasma physics (Hall & Sturrock 1967) this concept allows to derive from the Boltzmann equation in quasi–linear approximation a Fokker–Planck equation for the long term evolution of the distribution function. I follow here in particular the adaption of the formalism to the dynamics of stellar discs by Dekker (1976). The phase space distribution function can
be expressed with the aid of the action and angle variables, $J_1, J_2$ and $w_1, w_2$, respectively,

$$f(J_1, J_2, w_1, w_2; t).$$

(1)

The evolution of the distribution function with time is determined by the collisionless Boltzmann equation,

$$\frac{\partial f}{\partial t} + [f, H] = 0,$$

(2)

where $H$ is the Hamiltonian of the stellar orbits, and the square bracket denotes the usual Poisson bracket. Following Dekker (1976) I define a suitable mean of the distribution function by averaging it over the angle variables,

$$f = \frac{1}{4\pi^2} \int_0^{2\pi} dw_1 \int_0^{2\pi} dw f(J_1, J_2, w_1, w_2; t).$$

(3)

This averaged distribution function will evolve slowly on the long time scale. For the rapidly fluctuating part of the distribution function $\delta f = f - f$ one obtains from the general Boltzmann equation (2)

$$\frac{\partial \delta f}{\partial t} + \{ f, H \} = \{ \delta f, \delta H \} = 0,$$

(4)

where the Hamiltonian has been split up as $H = H_0 + \delta \Phi$ with $\delta \Phi$ denoting the fluctuations of the gravitational potential. The first Poisson bracket in equation (4) vanishes, because $\{ f, H \}$ depends only on the action integrals. The time derivative of the averaged distribution function is expected to be much smaller than that of the fluctuating part. Thus equation (4) can be cast into a Boltzmann equation for $\delta f$, and, if the quadratic term $\langle \delta f, \delta \Phi \rangle$ is neglected, into a linearized Boltzmann equation,

$$\frac{\partial \delta f}{\partial t} + [f, H_0] = - \{ f, \delta \Phi \},$$

(5)

which has been widely used to study the dynamics of galactic discs. $f$ has taken the role of the axisymmetric, stationary background distribution. Its slow evolution with time is neglected on short time scales, but taking the average of the Boltzmann equation (2) leads to

$$\frac{\partial \langle f \rangle}{\partial t} + \{ \langle f \rangle, H_0 \} = \{ \langle \delta f, \delta \Phi \rangle \} = 0.$$

(6)

The second and third terms of equation (6) vanish because $\langle \delta f \rangle = \langle \delta \Phi \rangle = 0$ by definition, so that equation (6) takes the form

$$\frac{\partial \langle f \rangle}{\partial t} = \langle \delta \Phi \delta \Phi \rangle = 0,$$

(7)

which describes the long-term evolution of the averaged distribution function in quasi-linear approximation. Equation (7) is also valid, if instead of action and angle variables other variables are used, as long as $J_1, J_2$ are integrals of motion and $w_1, w_2$ their canonical conjugates.

The potential perturbation $\delta \Phi$ can be Fourier transformed with respect to the angle variables,

$$\delta \Phi = \int dt_1 \int dt_2 \delta \Phi(t; t) e^{i(t_1 w_1 + t_2 w_2)},$$

(8)

and similarly

$$\delta f = \int dt_1 \int dt_2 \delta f(t; t) e^{i(t_1 w_1 + t_2 w_2)}.$$

(9)

On the other hand, the lhs of equation (5) represents the total derivative of $\delta f$ along stellar orbits in the axisymmetric part of the gravitational potential. Thus

$$\delta f = - \int_{t_0}^{t} dt' \left[ \langle f \rangle (J_1, t'), \delta \Phi(J_1, w_1, w_2; t') \right]$$

$$= \int_{t_0}^{t} dt' \sum_n \frac{\partial^2 \langle f \rangle}{\partial J_n^2} J_n \int dt_1 \int dt_2 \delta \Phi(t; t') e^{i(t_1 w_1 + t_2 w_2)},$$

(10)

where the integration is to be taken along ‘unperturbed’ orbits. The indices of the action and angle variables indicate that the variables, which are the independent variables of the distribution function and the gravitational potential, respectively, must be chosen according to the ‘unperturbed’ orbit starting at $J_{10}, w_{10}$, and terminating at $J_t, w_t$. In the next section I will apply equation (10) to a succession of uncorrelated swing amplification events of short duration. The typical integration interval $t - t_0$ will be then much smaller than the time scale, on which the averaged distribution function $f$ is evolving and equation (10) can be simplified to

$$\delta f = \sum_n \frac{\partial^2 \langle f \rangle}{\partial J_n^2} J_n \int_{t_0}^{t} dt' \delta \Phi(l, t') e^{i(t_1 w_1 + t_2 w_2)},$$

(11)

Comparison of equations (9) and (11) shows that

$$\delta f_1 = \sum_n \frac{\partial^2 \langle f \rangle}{\partial J_n^2} J_n \int_{t_0}^{t} dt' \delta \Phi(l, t') e^{i(t_1 w_1 + t_2 w_2)},$$

(12)

Upon inserting expressions (8) and (9) into equation (7) it is straightforward to evaluate the Poisson brackets and carry out the averaging with respect to the angle variables. After some algebra one obtains

$$\frac{\partial \langle f \rangle}{\partial t} = -i \int dt_1 \int dt_2 \sum_n l_n \frac{\partial}{\partial J_n} \langle \delta f \delta \Phi \rangle,$$

(13)

where use of the fact has been made that $\delta \Phi \delta \Phi = 0 \Phi \Phi$ so that $\delta \Phi$ is real. Since the potential perturbations are supposed to be a succession of short lived, uncorrelated fluctuations, it is customary to include into the averaging process an ensemble average over these fluctuations. Inserting finally equation (12) into (13) leads to a diffusion equation in action integral space,

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{1}{2} \sum_{m,n} \frac{\partial}{\partial J_m} D_{mn} \frac{\partial \langle f \rangle}{\partial J_n},$$

(14)

with diffusion coefficients

$$D_{mn} = 2 \int dt_1 \int dt_2 \int dt \int dt' \langle \Phi^* \Phi \rangle \langle \Phi(t) \Phi(t') \rangle e^{i(t_1 w_1 + t_2 w_2)}.$$
(1976) approach the duality of equations (5) and (7) is particularly instructive.

3 DISC HEATING BY SWING AMPLIFIED SPIRAL DENSITY WAVES

I apply the formalism of the previous section to calculate the diffusion coefficients for wave–star scattering by shearing, swing amplified spiral density waves. The dynamics of the density waves are modelled by a shearing sheet made of stars. This describes a patch of a thin galactic disc. Its centre orbits the galactic centre at galactocentric radius $r_0$ with an angular velocity $\Omega_0$. Pseudo-cartesian coordinates are defined with respect to the centre of the patch,

$$x = r - r_0, y = r_0(\theta - \Omega_0 t),$$

where $r$ and $\theta$ denote galactic polar coordinates, respectively. As explained in paper (I) the equations of motion of the stars are derived from the Hamiltonian

$$H_0 = \frac{1}{2}J_r^2 + \frac{1}{2}J_\theta^2(\theta - \Omega_0)^2 - 2A\Omega_0(r - r_0)^2,$$

or alternatively

$$H_0 = \kappa J_1 + \frac{A}{2B} J_2^2 - \frac{1}{2}\Omega_0^2 r_0^2,$$

where $A$ and $B$ denote Oort’s constants. The resulting orbits are simple epicyclic motions, which can be written as

$$x = \frac{J_2}{-2B} + \frac{2J_1}{\kappa} \sin w_1, y = w_2 - \frac{\sqrt{2}\kappa J_2}{B} \cos w_1,$$

with $\kappa = \sqrt{-4\Omega_0^2 B}$ the epicyclic frequency. $J_1$ is the radial action integral of an orbit. $J_2$ denotes the integral $J_2 = \dot{y} + 2\Omega_0 y$ of an epicyclic orbit and is related to the angular momentum of a star as $J_2 = (r^2 \dot{\theta} - r_0^2 \Omega_0)/r_0$. As can be seen from equation (19) the guiding centre radius of the orbit is given by

$$x_g = \frac{J_2}{-2B}.$$

$w_1 = \kappa t$ and $w_2 = \frac{A}{B} J_2 t$ are variables canonical conjugate to $J_1$ and $J_2$, respectively. The radial and circumferential velocities are given by

$$u = \sqrt{2\kappa J_1} \cos w_1, v = \frac{2B}{\kappa} \sqrt{2\kappa J_2} \sin w_1,$$

where $v$ is defined relative to mean shearing velocity $\dot{y} = -2Az$. The dynamics of a shearing sheet made of stars has been studied extensively by Julian & Toomre (1966) (cf. also Toomre 1981) and is discussed at length in paper (I) using strictly Eulerian coordinates. The principal result is that the wave crests of the density waves, which appear in the disc, swing around following the mean shearing motion of the stars. While the waves swing around their amplitudes are amplified transitorily and then die away. In paper (II) it is shown that also mode-like, quasi–stationary solutions can be constructed for the shearing sheet by introducing an inner reflecting boundary in the disc. Such kind of density waves is of no concern in the present context, however, since they hardly heat the disc at all.

The perturbations of the gravitational potential of the shearing sheet are customarily Fourier analyzed as

$$\delta \Phi = \int dk_x \int dk_y \delta \Phi_{k} e^{i(k_x x + k_y y)} = \int dk_x \int dk_y \delta \Phi_{k} \exp\left[k_x \frac{J_2}{-2B} + k_x \frac{2J_1}{\kappa} \sin w_1 + k_y w_2 - k_y \frac{\sqrt{2\kappa J_1}}{2B} \cos w_1\right].$$

The functional dependence on the variable $w_2$ is of the form as in equation (8) and I use in the following the wave number $J_2$ instead of $k_y$. The dependence on the angle variable $w_1$ can be easily adapted to the form of equation (8) by taking an inverse Fourier transform of equation (22) with respect to $w_1$. The resulting Fourier coefficients are inserted then into equation (15). There is, however, a difference of the meaning of the variable $w_2$ in this section from the meaning of the corresponding variable in the previous section. $w_2$ measures now the drift of the guiding centre of the orbit in the $y$–direction and is thus not an angle variable. Averaging the distribution function with respect to $w_2$ is impractical and I consider in the following the evolution of the distribution function in $(J_1, J_2)$–space, i.e. the distribution function integrated over $w_1$ and $w_2$, respectively, which is conceptually slightly different from the average (3). This leads to expressions for the diffusion coefficients of the form

$$D_{mn} = 2 \int_0^{2\pi} dw_1 \int_0^{2\pi} dw_1' \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_x' \int_{-\infty}^{\infty} dl_1 \int_{-\infty}^{\infty} dl_2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \left\{ -i \frac{1}{2} \frac{1}{2} \int \delta \Phi_{k_x} \frac{\delta \Phi_{k_x'}}{\delta J_2} \frac{\delta \Phi_{k_y}}{\delta J_1} \right\} 

\cdot \hat{e}^{-i\int \left[ \frac{1}{2} \left( J_2 t + J_2 t' \right) \right] - \frac{\kappa}{2} \left( J_2 t' \right) \frac{\delta \Phi_{k_x}}{\delta J_2} \frac{\delta \Phi_{k_y}}{\delta J_1} 

\cdot \hat{e}^{-i\int \left[ -\frac{1}{2} \frac{\kappa}{2} \left( J_2 t + J_2 t' \right) \right] - \frac{\kappa}{2} \left( J_2 t' \right) \frac{\delta \Phi_{k_x}}{\delta J_2} \frac{\delta \Phi_{k_y}}{\delta J_1} \right\}.$$

Note that the diffusion tensor is symmetric by construction.

In paper (I) it is illustrated how the shearing sheet responds to internal and external perturbations by developing density waves with shearing crests and amplitudes amplified while the waves swing around. Fig. 1 shows the distribution of amplitudes of the potential perturbations as function of wave numbers $k_x, k_y$, respectively, calculated using the formule of paper (I) (section 11.4) for the case when the swing amplification mechanism is fed by white noise. Toomre (1990) discusses the source of the noise and argues convincingly that swing amplified white noise explains exactly the behaviour of the star discs in his numerical simulations or that by Sellwood & Carlberg (1984). The distribution of amplitudes, which represents the superposition of many short lived shearing density waves, is quasi–stationary and can be modelled for positive wave numbers $k_y$ empirically as

$$\delta \Phi_{k} \equiv \hat{\Phi} e^{-\frac{(k_x - k_{\text{crit}})^2}{2k_{\text{crit}}^2} - \frac{(k_y - k_{\text{crit}})^2}{2k_{\text{crit}}^2}},$$

and continued at negative wave numbers $k_y$ as $\delta \Phi_{-k} = \delta \Phi_k$. The parameters are estimated as $k_{\text{crit}} = 1.5 k_{\text{crit}}$, $k_y = 0.5 k_{\text{crit}}$, $\sigma_{k_x} = 0.7 k_{\text{crit}}$, and $\sigma_{k_y} = 0.1 k_{\text{crit}}$ for the case $A = -B = \frac{1}{2}\Omega_0$. The critical wave number is defined as $k_{\text{crit}} = \kappa^2/(2\pi G \Sigma_0)$ with $G$ the constant of gravity and $\Sigma_0$
the surface density of the disc, respectively. Using this parametric model the quadratures in equation (23) can be carried out explicitly. Each spike in Fig. 1 represents a superposition of swing amplified density waves all travelling at a constant wave number $k_y$ along the $k_x$ abscissa at a speed of $k_{x, eff} = 2A_k y$. The wave numbers are given in units of $k_{x, crit}$, whereas the amplitudes are normalized arbitrarily. The parameters of the disc model are $A = -B = \frac{1}{2} D_0$ and $Q = 1.4$. See paper (I) for further details.

\[
\int_{t_0}^{t_1} dt' \delta(k_x' - k_x - 2A_k (t' - t)) e^{il_1(t' + t)} = \frac{1}{2A_k} e^{-il_1(w_1 - w_1')}.
\]  

I consider next in equation (23) the integration with respect to wave number $k_x$,

\[
\int_{k_{x, eff} = 2A_k y}^{k_{x, crit}} dk_x e^{-\frac{(k_x' - k_x)^2}{2\sigma_{k_x}^2}} \frac{1}{2A_k} e^{-il_1(w_1 - w_1')} e^{il_1(t' - t)}.
\]  

(28)

In equations (29) and (30) terms of the kind

\[
\sigma_{k_x}^2 \left( \frac{2l_1}{\kappa} \right)
\]  

are neglected as quadratically small. This is justified, because the majority of the stars have epicycle sizes smaller than the critical wave length $\lambda_{crit}$, the typical spacing between spiral arms (Julian & Toomre 1966, cf. also paper I). The epicycle size is determined by $\sqrt{2J_1/\kappa}$, whereas $\sigma_{k_x} \propto k_{x, crit} = 2\pi/\lambda_{crit}$ (cf. Fig. 1). Similarly terms of the kind

\[
\sigma_{k_x}^2 \left( \frac{\kappa}{2A_k} \right)^2
\]  

will be neglected, because

\[
\frac{\sigma_{k_x, \kappa}}{2A_k} \propto \frac{\sigma_{k_x}}{k_{x, eff}} \frac{1}{T_{orb}} \propto \frac{T_{acc}}{T_{acc}} \ll 1,
\]  

(33)

where $T_{acc}$ denotes the duration of disc heating by a single spike of the potential fluctuations, which is only effective close to the peak in Fig. 1, whereas $T_{orb}$ is the orbital period of the stars. In this aspect the disc heating mechanism described here is rather impulsive. Next I consider the integration in equation (23) with respect to $l_1$,

\[
\int_{-\infty}^{\infty} dl_1 \left\{ il_1^2 \right\} e^{-il_1(w_1 - w_1')}.
\]  

(34)

where the upper row refers to $D_{11}$, the middle row to $D_{12} = D_{21}$, and the lower row to $D_{22}$, respectively. The results are given by delta functions and derivatives thereof,
The basic differential equation involving the density waves is given by
\[ \frac{\partial \delta}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\sigma_x^2}{\sigma_y^2} \right) \] (41). I have chosen to ignore this and have replaced the integral by a saddle point approximation. The reason is that the model of disc heating used here (cf. equation 25) becomes unphysical for small circunferential wave numbers \( l_2 \), because the density waves approach then the WKB limit and become long lived, so that they do not heat the disc effectively. Due to the symmetry of the distribution of amplitudes (24) with respect to \( k \) the effect of density waves with negative wave numbers \( k_y \) can be taken into account by multiplying the diffusion coefficients by a factor of two. Assembling all results leads a diffusion tensor of the form.

\[ D_{\text{in}} = D_0 \begin{pmatrix} \left( \frac{\sigma_x^2}{\sigma_y^2} \right) & \frac{\sigma_x^2}{\sigma_y^2} J_1(0) \\ 0 & 1 \end{pmatrix}, \] (42)

with \( D_0 = 8\pi^2 |\Phi_0|^2 \sigma_x^2 \sigma_y^2 k_20 / A. \)

The diffusion equation takes the form
\[ \frac{\partial f}{\partial t} = \frac{1}{2} \frac{\partial}{\partial J_1} \left( D_{11} J_1 \frac{\partial f}{\partial J_1} \right) + \frac{1}{2} D_{22} \frac{\partial^2 f}{\partial J_2^2}, \] (43)

where the overhead tilde means the the \( J_1 \)-dependence of \( D_{11} \) has been written separately. For this kind of disc heating by transient spiral density waves no correlation between the diffusion in radial action and angular momentum space is found. The diffusion equation (43) is highly non-linear, because the diffusion coefficients depend on the distribution function \( f \) themselves. In particular, the effectiveness of swing amplification of spiral density waves depends critically on the value of the Toomre stability parameter \( Q = \sigma_y / (3.36 \sigma_0) \), where \( \sigma_y \) denotes the radial velocity dispersion of the stars. Thus, when the disc heats up, the amplitudes \( \Phi_0 \) of the density waves and the diffusion coefficients (42) will decrease and the disc heating rate slows down to zero. Numerical simulations of the dynamical evolution of galactic discs (Sellwood & Carlb erg 1984, Fuchs & v. Linden 1998) have shown that this can happen on comparatively short time scales, if the discs are left uncooled. Only if the discs are cooled dynamically by adding stars on low peculiar velocity orbits, the spiral density wave activity can be maintained at a constant level despite the rising velocity dispersions. In that case \( D_0 \approx \text{const.} \) and a simple solution of the diffusion equation (43) is found by separation of variables in the form
\[ f = \frac{1}{c_1 + \frac{J_1^2}{2} t} \sqrt{c_2 + \frac{J_2^2}{2} t} \exp \left\{ \frac{- \left( J_1^2 \sigma_1^2 / c_1 + \frac{J_2^2}{2} \right)}{c_1 + \frac{J_1^2}{2} t} \right\}, \] (44)

with arbitrary constants \( c_1 \) and \( c_2 \).

4 DISCUSSION AND CONCLUSIONS

The scattering of stars by shearing, short lived density waves leads to independent diffusion of stars in radial action – angular momentum space. This diffusion process has various implications. The radial action integral is related to the peculiar velocities of the stars as
\[ J_1 = \frac{1}{2 \kappa} \left( u^2 + \frac{k_y^2}{4B^2} v^2 \right). \] (45)

Thus a distribution function of the form
\[ \exp \pm J_1 / \left( c_1 + \frac{J_1^2}{2} t \right) \] (46)

implies an \( u^2 / 2\sigma_u^2 \) dependence of the velocity distribution with a predicted radial velocity dispersion \( \sigma_v(t) = \sqrt{\kappa(c_1 + \frac{J_1^2}{2} t)/2} \). Such a rise of the velocity dispersion with time fits ideally to the actual age–velocity dispersion relation...
observed in the solar neighbourhood (cf. Fuchs et al. 2001 for a recent review of the observational data). Unfortunately the diffusion coefficients cannot be estimated quantitatively, because the constant $|\Phi_0|^2$, which parameterizes the white noise, is not known a priori. But judging from the shape of the heating law (46) wave – star scattering of the kind discussed here might well have played an important role in the Milky Way disc. Whether this is the only disc heating mechanism is still a matter of debate (Fuchs et al. 2001).

The diffusion of the guiding centre radii of the stellar orbits can be estimated from the ratio of the diffusion coefficients,

$$D_{22} = \frac{\kappa}{4B^2} \frac{1}{2J_1^2} + \frac{\kappa^2}{2\Omega_0^2}. \tag{47}$$

As shown above $\kappa D_{11} t / (2J_1)$ is proportional to the square of the radial velocity dispersion, $\sigma_u^2(t)$. According to epicyclic theory (cf. equation 20)

$$\langle x^2_g \rangle = \frac{1}{4B^2} \langle J_2^2 \rangle, \tag{48}$$

so that the dispersion of the guiding centre radii of the stellar orbits is given by

$$\sqrt{\langle x^2_g \rangle} = \frac{1}{1 - 2B} \frac{\sigma_u}{2 \sqrt{2J_1^2 + \kappa^2/2\Omega_0^2}} = \frac{1}{2B} \sqrt{\frac{D_{22}}{4}}$$

Using the parameter values estimated above equation (49) implies $\sqrt{\langle x^2_g \rangle} = 0.2\sigma_u/\Omega_0$, if a flat rotation curve is assumed. A star with an age of 5 Gyrs like the Sun has typically in the solar neighbourhood a velocity dispersion of 50 km/s. Thus $\sqrt{\langle x^2_g \rangle} = 400$ pc $= 0.05 \, r_0$ in the solar neighbourhood, if a local angular velocity of $\Omega_0 = 26$ km/s/kpc is adopted. This confirms the conclusion of Binney & Lacey (1988) that the diffusion of guiding centre radii driven by a rapid succession of spiral density waves is rather small. This assumes that disc heating by transient spiral density waves is the only disc heating mechanism. However, if the Sun has indeed drifted from its birth place nearly 2 kpc radially outwards to its present galactocentric radius as suggested by Wielen, Fuchs, & Dettbarn (1996) and Wielen & Wilson (1997), this would mean that there must be other dynamical heating mechanisms of the galactic disc. It was shown by Fuchs, Dettbarn, & Wielen (1994) theoretically and by numerical simulations that Spitzer–Schwarzschild diffusion of stars due to gravitational encounters with massive molecular clouds, for instance, leads to a much more pronounced diffusion of the guiding centre radii of the stellar orbits, even if the mechanism does not heat the disc effectively.

ACKNOWLEDGMENTS

I thank A. Just and R. Wielen for helpful discussions. I am also grateful to the anonymous referee, whose comments lead to an improvement of the paper.

REFERENCES

Barbanis B., Woltjer L., 1967, ApJ, 150, 461