Performance investigation of NIM’s small force device based on electrostatic force principle

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Abstract. A small force device based on electrostatic force principle was developed by NIM. A flexure hinge mechanism with a balance lever is used as a force transmission unit. Inner and outer electrodes with thin cylinders configuration are assembled coaxially. The inner electrode is joined to the end of the flexure hinge mechanism. While a voltage is applied between inner and outer electrodes, an electrostatic force is generated. The force measurement range is from $10^{-8}$ N to $10^{-4}$ N. The main characteristics of the small force device such as creep, capacitance gradient and stiffness of flexure hinge mechanism were investigated. Force measurement uncertainty of the small force device is evaluated. The relative combined uncertainty of electrostatic force is less than $3.6 \times 10^{-4}$ in the force range of 10μN to 100μN.

1. Introduction
For fulfilling the demands of small force measurements in advanced materials, biology, microelectronics, traceable small force standards were developed by NMIs [1-3]. Generally, there are two main approaches for small force realization: mass-based and electrostatic force based principles [3]. NIM has developed small force standards based on two approaches in the range of micro-and nano-Newton [4-5]. In this paper, the performance investigation and uncertainty evaluation of the small force device based on electrostatic force principle are introduced.

As shown in Figure 1, the small force device based on electrostatic force principle consists of a flexure hinge mechanism, coaxial cylindrical capacitor (inner and outer electrodes), displacement measurement unit, coaxial measurement and adjustment system, electrostatic force measurement and control system. A flexure hinge mechanism with a balance lever is used as a force transmission unit. Differing from traditional hinge mechanism, the flexure hinge mechanism is equipped with a balance lever for balancing the weights of the inner electrode and itself. Inner and outer electrodes with thin cylinders configuration are assembled coaxially. The inner electrode is joined to the flexure hinge mechanism. The outer diameter of the inner electrode and the inner diameter of the outer electrode are 20mm and 21mm respectively. While a voltage is applied between inner and outer electrodes, an electrostatic force is generated. The electrostatic force is given by

$$ F_e = \frac{1}{2} \frac{dC}{dz} U^2 = \frac{1}{2} \cdot \frac{2\pi\varepsilon}{\ln(R_2/R_1)} U^2 $$(1)

Where $F_e$ is electrostatic force in nN, $dC/dz$ is capacitance gradient in pF/mm, $U$ is the applied voltage between two electrodes in V, $\varepsilon$ is dielectric constant between two electrodes, $R_1$ and $R_2$ are the outer radius of inner electrode and the inner radius of outer electrode in mm. From equation (1), capacitance gradient $dC/dz$ can be regarded as a constant. It is only related to dielectric and geometric dimensions of the electrodes.
Figure 1. NIM’s small force device based on electrostatic force principle

2. The performance investigation of the small force device

2.1. Creep measurements of flexure hinge mechanism
The creep is not only related to the internal stress of the flexure hinge mechanism, but also to environment factors. The creep of the flexure hinge mechanism was measured under the condition of unloading and loading with 5 mg standard weight respectively. The measurement results of creep are shown in Figure 2. Displacement changed significantly in the first 13 hours, while the temperature changed sharply. In the last 76 hours, the temperature fluctuation was within ±0.13°C. As a result, the displacement change was less than 1 μm. It is demonstrated that creep of the flexure hinge mechanism is less than 0.014 μm/h with stable temperature.

![Displacement vs. Time Graph](image1)

(a) under the condition of unloading  
(b) under the condition of loading

Figure 2. The measurement results of creep

2.2. Capacitance gradient measurements
Before capacitance gradient measurements were carried out, coaxiality of two electrodes should be adjusted by a coaxial measurement and adjustment system based on machine vision. In capacitance gradient measurement mode, outer electrode was driven by a translation stage and moved in vertical direction, while inner electrode was in a static state. At different positions, capacitance and displacement were measured simultaneously. The capacitance gradients were measured under different displacement steps and holding times. The measurement results are summarized in Table 1.

In addition, capacitance gradients were measured in a long period of time to investigate temperature influence on capacitance gradient. It was founded that capacitance gradient changes with the temperature, showing a general trend of decreasing with increasing temperature. According to a conservative estimation, the contribution of the capacitance gradient to the relative uncertainty of electrostatic force is less than 1 × 10⁻⁴, when the temperature fluctuates are within ±0.5°C.

According to the measurement results of capacitance gradient and equation (1), the realized electrostatic forces of the small force device are from 10⁻⁸ N to 10⁻₄ N, when the applied voltages from a few volts to several hundred volts.
2.3. Stiffness measurement of flexure hinge mechanism

The stiffness of flexure hinge mechanism is a vital parameter for the small force device. For investigating the stiffness, different voltages were applied between inner and outer electrodes. The applied voltages and relative displacements between electrodes were measured simultaneously. When voltages changed from 150 V to 400 V with 50 V step, displacements changed from -8 μm to -2 μm respectively. The stiffness of flexure hinge mechanism was derived from linear fit of electrostatic forces-displacements graph by least squares method. The slope of linear fit straight line is determined as the stiffness of flexure hinge mechanism, which is 11.679 N/m. With 1nm resolution of the displacement measurement unit, resolution of the force measurement is about 10^{-8} N.

3. The uncertainty evaluation of force measurement of the small force device

Based on equation (1), the relative combined standard uncertainty of electrostatic force realized by the small force device is derived as follows

$$u_r(F_e) = \sqrt{u_r^2(dC/dz) + 4u_r^2(U)}$$

Where $u_r(dC/dz)$ is the relative standard uncertainty due to capacitance gradient $dC/dz$, $u_r(U)$ is relative standard uncertainty due to applied voltage $U$. The standard uncertainty due to capacitance gradient $u_r(dC/dz)$ is combined by the standard uncertainties caused from measurement repeatability of capacitance gradient $u_r(Re)$, coaxial adjustment of two electrodes $u_r(\Delta a, \Delta \varphi)$, measurement results of capacitance bridge $u_r(C)$, resolution of the displacement measurement unit $u(\Delta d)$ and given by

$$u_r(dC/dz) = \sqrt{u_r^2(Re) + u_r^2(\Delta a, \Delta \varphi) + u_r^2(C) + u_r^2(\Delta d)}$$

From the measurement results of the capacitance gradient demonstrated in last section, the relative standard uncertainty caused by measurement repeatability of capacitance gradient is better than 1x10^{-4}.

Due to accuracy and resolution of coaxial measurement and adjustment system, axis offset $\Delta a$ and tilt angle $\Delta \varphi$ between two electrodes were adjusted within ±5 μm and ±0.1 ° respectively. The relative standard uncertainties due to axis offset and tilt angle are estimated less than 1.25x10^{-5}, 1.52x10^{-6} respectively. Conservatively, the uncertainty budget of capacitance gradient caused by coaxiality of two electrodes $u_r(\Delta a, \Delta \varphi)$ is less than 2x10^{-5}.

The capacitance value was measured by the AH-2700A capacitance bridge, which was calibrated by NIM’s electro-magnetic laboratory. The relative standard uncertainty of calibration result was $u_r(C)$ is 0.5x10^{-6} at 1 kHz, 10 pF.

The displacement of the inner electrode was measured by the RLE20 fibre optic laser encoder. Due to the limitation of resolution of the laser encoder, there is a measurement error of the balance position of the inner electrode. As a result, it results in an uncertainty of electrostatic force. Assuming that the stiffness of the flexure hinge mechanism is $k$, the standard uncertainty due to the resolution $\Delta d$ of the laser encoder $u(\Delta d)$ is calculated by

| Measurement points (μm) | Holding times (s) | Number of measurements | The mean values of capacitance gradient (pF/mm) | Standard deviation of capacitance gradient (pF/mm) |
|-------------------------|-------------------|------------------------|-----------------------------------------------|-----------------------------------------------|
| 0, ±50                  | 30                | 10                     | 1.019077                                       | 6.7x10^{-5}                                   |
| 0,10,20,30,40,50        | 30                | 35                     | 1.018901                                       | 1.8x10^{-5}                                   |
| 0,12.5,25,37.5,50,62.5, 75,87.5,100 | 30 | 10                     | 1.019012                                       | 6.7x10^{-5}                                   |
| 0, ±50                  | 5                 | 270                    | 1.018901                                       | 6.9x10^{-6}                                   |

Table 1. The measurement results of capacitance gradient
\[ u(\Delta d) = k \frac{\Delta d}{2\sqrt{3}} \] (4)

The resolution of RLE20 laser encoder \( \Delta d \) is 1nm, stiffness measurement result of flexible hinge mechanism \( k \) is 11.679 N/m. According to equation (4), \( u(\Delta d) = 3.37 \) nN. The relative standard uncertainty due to resolution of laser encoder \( u_r(\Delta d) \) is \( 3.37 / Fe \) (Fe in nN).

The voltage applied to inner and outer electrodes is measured by 2410 SourceMeter®. The digital voltmeter was calibrated by NIM’s electro-magnetic laboratory. The relative standard uncertainty of calibration result \( u_r(U) \) is \( 2.5 \times 10^{-5} \) in 200mV-1kV of voltage measurement range.

According to above analysis, the evaluation results in 10 \( \mu \)N to 100 \( \mu \)N are summarized in Table 2. It can be demonstrated that combined uncertainty of electrostatic force is less than \( 3.6 \times 10^{-4} \) in the force range of 10 \( \mu \)N to 100 \( \mu \)N.

**Table 2.** The standard uncertainty budgets and combined standard uncertainty of electrostatic force

| No. | Standard uncertainty budgets | Measured force (\( \mu \)N) |
|-----|------------------------------|-----------------------------|
|     |                              | 10  | 20  | 50  | 100 |
| 1   | Measurement repeatability of capacitance gradient \( u_r(\Re) \) | 1.0 \times 10^{-4} |
| 2   | Coaxiality of two electrodes \( u_r(\Delta a, \Delta \varphi) \) | 2.0 \times 10^{-5} |
| 3   | Capacitance bridge measurement \( u_r(C) \) | 0.5 \times 10^{-6} |
| 4   | Voltage measurement \( u_r(U) \) | 2.5 \times 10^{-5} |
| 5   | Resolution of laser encoder \( u_r(\Delta d) \) | \( 3.4 \times 10^{-4} \) | \( 1.7 \times 10^{-4} \) | \( 6.7 \times 10^{-5} \) | \( 3.4 \times 10^{-5} \) |
|     | Relative combined standard uncertainty | \( 3.6 \times 10^{-4} \) | \( 2.0 \times 10^{-4} \) | \( 1.3 \times 10^{-4} \) | \( 1.2 \times 10^{-4} \) |

4. CONCLUSION

The main characteristics of NIM’s small force device based on electrostatic force principle, such as creep, capacitance gradient and stiffness of flexure hinge mechanism were investigated. The realized electrostatic forces of the small force device are from \( 10^{-8} \) N to \( 10^{-4} \) N. The relative combined uncertainty of electrostatic force is evaluated less than \( 3.6 \times 10^{-4} \) in the force range of 10\( \mu \)N to 100\( \mu \)N.

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