Learning Without a Global Clock: 
Asynchronous Learning in a Physics-Driven Learning Network

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In contrast, the elements of the brain (neurons and synapses) such that all elements change their resistance simultaneously.

Noise has been shown to improve memory retention in physical systems such as sheared suspensions [11,15]. Even when continually trained for several strain amplitudes, memories of the smaller amplitudes fade when the system is noiseless. With added noise, the system is able to retain memories of every trained amplitude. Noise prevents the system from reaching a fixed point, allowing it to remain out of equilibrium and retain ‘transient’ information of the smaller amplitudes indefinitely.

Learning is a special case of memory [4,5], where the goal is to encode targeted functional responses in a network [6–9]. Artificial Neural Networks (ANNs) are complex functions designed to achieve such targeted responses. These networks are trained by using gradient descent on a cost function, which evolves the system’s parameters until a local minimum is found [10,11]. Typically, this algorithm is modified such that subsections (batches) of data are used at each training step, effectively adding noise to the gradient calculation, known as Stochastic Gradient Descent (SGD) [12]. This algorithm produces more generalizable results [13–15], i.e. better retention of the underlying features of the data set, by allowing the system to escape non-optimal fixed points [16,17].

Recent work [18] has demonstrated the feasibility of entirely distributed, physics-driven learning in self-adjusting resistor networks. This system operates using Coupled Learning [19], a theoretical framework for training physical systems using local rules [20,22] and physical processes [23–25] in lieu of gradient descent and a central processor. Because of its distributed nature, this system scales in speed and efficiency far better than ANNs and is robust to damage, and may one day be a useful platform for machine learning applications, or robust smart sensors. However, just like computational machine learning algorithms, this system (as well as other proposed distributed machine learning systems e.g. [26,27]) relies on a global clock that synchronizes the learning rule, such that all elements change their resistance simultaneously. In contrast, the elements of the brain (neurons and synapses) evolve independently [28,29], suggesting that global synchronization is not required for effective learning. Desynchronizing the updates in machine learning is a largely unexplored topic, as doing so would be computationally inefficient. However in a distributed system, it is the less restrictive modality [30], removing the need for a global communication across the network.

Here we demonstrate that asynchronous implementation of coupled learning is effective in both simulation and experiment. Furthermore, we show that in the physical learning network where learning degrees of freedom are discretized, asynchronous learning actually improves performance by allowing the system to evolve indefinitely, escaping local minima. We draw a direct analogy between stochastic gradient descent and asynchronous learning, and show they have similar effects on the learning degrees of freedom in our system. Thus we are able to remove the final vestige of non-locality from our physics-driven learning network, moving it closer to biological implementations of learning. The ability to learn with entirely independent learning elements is expected to greatly improve the scalability of such physical learning systems.

INTRODUCTION

Coupled learning [19] is a theoretical framework specifying evolution equations that enable supervised, contrastive learning in physical networks. In the case of a resistor network, inputs and outputs are applied and measured voltages at nodes of the network, and the edges modify their resistance according to local rules. The learning algorithm is as follows. Input and output nodes are selected, and a set of inputs from the training set is applied as voltages on the input nodes, creating the ‘free’ response of the network. Using the measured outputs from this state $V^O_F$, the output nodes are clamped at voltages $V^C_O$ given by

$$\vec{V}^O_C = \eta \vec{V}^D + (1 - \eta) \vec{V}^O_F$$

(1)

where $V^D$ are the desired output voltages for this training example, and $0 < \eta \leq 1$ is a hyper parameter. Thus, the output
nodes are held at values closer to the desired outputs. When \( \eta \ll 1 \) this algorithm approaches gradient descent \([19]\). This generates the ‘clamped’ response of the network. The voltage drop across each edge in the free \( \Delta V_f \) and clamped \( \Delta V_c \) states then determine the change in resistance for that edge, given by

\[
\delta R_i = \frac{\gamma}{R_i} \left( [\Delta V_c]^2 - [\Delta V_f]^2 \right)
\]  

(2)

where \( R_i \) is the resistance of that edge and \( \gamma \) is a hyper parameter that determines the learning rate of the system. In effect, this learning rule lowers the power dissipation of the clamped state relative to the free state, nudging the entire system towards the (by definition) better clamped outputs. The system is then shown a new training example, and the process is repeated, iteratively improving the performance of the free state outputs. When a test set is given to the network to check its performance (by applying the input voltages appropriately) errors are calculated via the difference between the free state outputs and the desired outputs. A more detailed description of coupled learning is given in previous work \([19]\).

In this algorithm, it is implicitly assumed that all edges update at the same time. Here we relax this assumption, modifying the learning rule \([3]\) with a probabilistic element:

\[
\Delta R_i(p) = \begin{cases} 
\delta R_i & \text{with probability } p \\
0 & \text{otherwise} 
\end{cases}
\]

(3)

where \( 0 < p \leq 1 \) is the update probability and \( p = 1 \) recovers synchronized coupled learning. This change, especially for low \( p \), fundamentally changes how the system updates. Individual edges may spend long periods entirely static, while the system evolves around them, completely ignoring large changes along the way; that is, learning is desynchronized.

In simulation, we find that desynchronization does not hamper the learning process. In fact, the error as a function of training steps times \( p \) consistently collapses for all values of \( p \) for a variety of tasks and networks, as shown for a typical example in Fig. 1. This collapse occurs regardless of choice of hyper parameters \( \eta \) (nudge amplitude) and \( \gamma \) (learning rate).

This is a strikingly robust result, and signifies a substantive potential simplification for an experimental coupled learning system, namely removing the global clock.

**EXPERIMENTAL (DISCRETE) COUPLED LEARNING**

We next test desynchronous updates in an experimental realization of coupled learning. In recent work \([18]\), coupled learning was first implemented in a physical system. In this system, contrastive learning was performed in real time by using two identical networks to access the free and clamped states of the network simultaneously. The system was robust to real-world noise, and successfully trained itself to perform a variety of tasks using a simplified version of the update rule that allowed only discrete values of \( R \), specifically

\[
\delta R_i = \begin{cases} 
+r_0 & \text{if } |\Delta V_c| + |\sigma| > |\Delta V_f| \\
-r_0 & \text{otherwise} 
\end{cases}
\]

(4)

Note that we have explicitly added the measured bias of the comparators \( \sigma \), which we find manifests as a random, uniformly distributed variable from 0 to 0.05V. Each edge in the network performed this update individually, but did so all at once, synchronized by a global clock. Here, we implement this learning rule \([3]\) but incorporate a probabilistic element, such that with probability \( p \) each edge updates according to Eq. (4) on a given training step. Thus, we are able to tune the system from entirely synchronous (\( p = 1 \)) to entirely desynchronized (\( p \ll 1 \)). We implement this functionality via a separate circuit housed on each edge of the network, shown in Fig. 2A, that, when triggered, compares an oscillating voltage signal to a global ‘bias’ voltage, as shown in Fig. 2B. The components (comparators, capacitors, and resistors) used in each implementation of the oscillator vary slightly, changing the period, and thus the oscillating signals on each edge rapidly desynchronize. Thus, by changing the bias value, we can select \( p \) for our experimental system.

As with the continuous version of coupled learning, desynchronization does not prohibit the discrete, experimental system from learning. In fact, desynchronized learning performs better on average than synchronous learning for allosteric...

**FIG. 1. Coupled Learning is Successful Without a Global Clock**

(A) Simulated 143 edge coupled learning network. (B) Test set scaled error (error/error(\( t = 0 \)) curves averaged over 50 distinct 2-input 2-output regression tasks as a function of training steps times update probability \( p \). Colors denote differing values of \( p \) ranging from 0.1 to 1. Error bars at the terminus of each curve denote range of final error values for a given \( p \) when run for 20000 steps.

**FIG. 2. Circuitry for Realization of Desynchronous Coupled Learning.** (A) Image of the entire 16-edge network. Edges with LEDs on are active (updating) on this training step. (B) Diagram of the oscillator circuit in each edge in (A). A global bias voltage (red) determines \( p \). Each edge compares the bias against against a local oscillator signal (green) to determine if its resistance is updated.
Desynchronization Improves Discrete Network Solutions in Experiment and Simulation. (A) Scaled error (error/error(=0)) vs training steps scaled by update probability $p$ in experiment for an allosteric task. One typical raw (faded) and smoothed (color) curve is shown for each of the three values of $p$. (B) Three resistor values vs training steps scaled by update probability from the experiments shown in (A). (C) Scaled error at the end of training averaged over 25 allosteric tasks as a function of $p$. (D) Scaled error at the end of training averaged over 20 allosteric tasks as a function of number of targets. Each task has an equal number of sources, and half as many ground nodes as targets. Note the collapse of curves of varying $p$ as the task complexity grows. (E) Scaled test set error at the end of training in simulation averaged over 10 regression tasks. In (D) and (E) the same 143 edge simulated network from Fig. 1(A) is used with the discrete update rule (Eq. 5).

Thus we have, as before

$$\Delta R_i(p) = \begin{cases} \delta R_i & \text{with probability } p \\ 0 & \text{otherwise} \end{cases}$$

The addition of $\sigma$ leads to a tendency for the resistor values to drift upwards, just like in the experiment, finding lower power solutions, and putting the resistors in a regime where they can take smaller steps relative to their magnitude. From simulations of a 143-edge discrete network, we find that as allosteric task complexity (number of targets and sources) increases, the beneficial effects of desynchronous learning diminish, as shown in Fig. 3(D). More complex tasks require more desynchronous (lower $p$) learning to confer an advantage over synchronous learning. For tasks with enough targets, moderately desynchronous learning yields indistinguishable error from synchronous learning, as shown by the overlap of the blue and black curves on the right of Fig. 3(D).

Unlike the experimental 16 edge network, desynchronization does improve the error for our simulated 143-edge learning a two-source two-target regression task, as shown in Fig. 3(E). We believe that for such a task, our 16-edge experimental network is in the ‘too-complex’ regime, whereas our simulated 143-edge network is not, and therefore shows a monotonic trend in final error with $p$.

Linear tasks like allostery and linear regression do not have local minima when the resistors are free to change continuously. However, the discretization of resistor space creates many local minima, trapping the synchronous solution and preventing it from finding a global optimum. As $p$ decreases, solutions increasingly drift from those found for synchronous learning, as shown in Fig. 3. These behaviors suggest that desynchronization aids in exploring an under-constrained resistance space, and escaping local minima, much like stochastic gradient descent in machine learning.

**COMPARISON TO STOCHASTIC GRADIENT DESCENT**

In computational machine learning, artificial neural networks can be trained using batch gradient descent. In this algorithm, the entire set of training data is run through the network, and a global gradient is taken with respect to each weight in the network, averaged over the training set. The weights are then modified based on this gradient until a local minimum is found. In practice, this method is inefficient at best and intractable at worst [33]. A typical modification to this algorithm is known as stochastic gradient descent (SGD), where instead of the entire training set, a randomly selected subset of training examples (mini-batch) is used to calculate the gradient at each training step [12]. This effectively adds noise to the gradient calculation, speeds processing, and boosts overall performance by allowing the system to continually evolve, escaping from local minima in the global cost function. Stochastic gradient descent has been shown to improve learning performance in different settings, specifically in obtaining lower generalization (test) errors compared to full batch gradient descent. It is therefore argued that SGD
performs implicit regularization during training, finding local minima in the cost landscape that are more likely to generalize to unseen input examples [14].

This can be more clearly understood by describing training of a neural network as gradient descent dynamics of the learning degrees of freedom $w$ (edge weights in a neural network) with an additional diffusion term, following Chaudhari et al. [14]. We define $b$ as the fraction of training data points used in a mini-batch. Full-batch ($b = 1$) training simply minimizes the cost function $C(w)$, and thus the dynamics may be written as

$$\gamma^{-1}d\vec{w}(t) = -\nabla_w C(w) dt$$

(6)

which yields solutions $w_j$ that are minima of the cost function. When mini-batching, an additional diffusion term is added to the dynamics,

$$\gamma^{-1}d\vec{w}(t) = -\nabla_w C(w) dt + \sqrt{2\gamma b b^{-1} D(w)} d\vec{W}(t)$$

$$D(w) = [B^{-1} \sum_i \vec{\nabla}_w C_i \otimes \vec{\nabla}_w C_i] - \vec{\nabla}_w C \otimes \vec{\nabla}_w C$$

(7)

where the diffusion matrix $D(w)$ is defined by outer products of the individual training example gradients, $B$ is the total number of training examples, and $d\vec{W}$ is a Wiener process. These dynamics converge to critical points $w_j$ that are different from the minima of the cost function, $w_j$, by a factor that scales with the fraction of data points not included in each batch ($1 - b$). This difference is the hallmark of regularization, in this case performed implicitly by SGD.

In coupled learning, the desynchronization of edge updates is expected to yield a similar effect. Instead of having different training examples, learning stochastically uses the gradient at independent edges. Therefore we can define an effective diffusion matrix for desynchronized coupled learning by

$$\gamma^2 D_{eff}(R) = [N^{-1} \sum_i \Delta R_i \otimes \Delta R_i] - \Delta \vec{R} \otimes \Delta \vec{R}$$

(8)

where $N$ is the total number of edges. Note the similar form to the second line of Eq. (7). With this definition, the analogy of desynchronous coupled learning and SGD is clear, with the edge update probability $p$ playing the role of the batch fraction $b$, and thus we expect similar results for the two methods. We verify the analogy between desynchronous coupled learning and SGD in simulation.

For simulations with the continuous learning rule, we observe no change in final error when learning is desynchronized, consistent with this picture, as there are no local minima to escape. However, the analogy between SGD and desynchronization can still be explored by observing the solutions in resistor space. As a base case, we simulate a $N = 16$ edge network (the same structure as our experiment in Fig. 2(A)) using the original coupled learning rule (Eq. 2) with a full batch to solve a regression task with $B = 16$ training examples. That is for a given edge $i$,

$$\Delta R_i = \sum_{j=1}^B \Delta R_{ij} = \sum_{j=1}^B \gamma \left[\left|\Delta V_i^e\right|^2 - \left|\Delta V_i^f\right|^2\right]$$

(9)

where $j$ is the index of the training example, summed over all $B = 16$ elements of the training set. This is an entirely deterministic algorithm, given initial conditions of $R_i$, and thus a good basis for comparison. Then we compare two forms of stochasticity, randomly choosing edges (desynchronization) and randomly choosing training examples (SGD). With probability $p$ we update edges ($i$), and with probability $b$ we include each training example in the sum ($j$). For $b = 1$ we use a full batch, and for $p = 1$ we update every edge synchronously. Coupled learning as described in previous work [18] used $p = 1$ and $b \ll 1$ (a single training data point at a time). Decreasing $p$ (desynchronizing) and decreasing $b$ (stochastic mini-batching) do not meaningfully change the final error of the network’s solutions in continuous coupled learning, but do find different solutions than the full-batch synchronous case. In fact, we find they have the same relationships to the fully deterministic solutions,

$$b = 1 : \quad L_2(\bar{R}(p = 1), \bar{R}(p)) \sim (1 - p)^{2/3}$$

(10)

$$p = 1 : \quad L_2(\bar{R}(b = 1), \bar{R}(b)) \sim (1 - b)^{2/3}$$

(11)

Enforcing $p = b$ also gives the same power law, all seen in Fig. 3(A). We may also enforce a randomly selected but consistent fraction of edges ($\tilde{p}$) or of the training set ($\tilde{b}$) to be updated/included for each training step. This is the standard means of mini-batching in SGD, as mentioned previously. We find similar parallels between asynchronous and mini-batched learning in this condition, as seen in Fig. 3(B). The overall multiplicative factor separating the data can be explained by SGD and the desynchronous learning rule having a different effective learning rate. Matching these effective rates collapses all data in Fig. 3(A) and (B).
DISCUSSION

In this work we have demonstrated the feasibility of learning without a global clock in a physics-based learning network, both with a continuous state space of solutions and a discrete one, in simulation and experiment. In all cases desynchronizing the learning process does not hamper the ability of the system to learn, and in the discrete resistor space with many local minima, actually improves learning outcomes. We have shown that this improvement comes from a behavior analogous to stochastic gradient descent, namely that injecting noise into the learning process allows the system to escape local minima and find better overall solutions. Finally, we have strengthened this analogy by showing that mini-batching and desynchronization produce the same scaling of distance in solution space compared to a fully deterministic (full batch, synchronous) algorithm.

The freedom to avoid a global clock is an important step towards total decentralization of the learning process in a physical system; it is necessary to make a learning material. In this and previous [18] work, the experimental system is still run via a global clock, and thus requires a one bit communication with every edge to trigger resistor updates. However, the success of all values of $p$ demonstrates that edges with entirely self-triggered updates would function well. For a larger, less precise, tighter packed, or three dimensional learning system, removing this connection to each edge may greatly simplify construction. Furthermore, allowing desynchronization opens the door for learning with new types of systems, ones that cannot be synchronized such as elements updating out of equilibrium [34], via thermal noise [29] or other stochastic processes.

In discrete-valued coupled learning, mini-batching alone (the standard in Coupled Learning) gives inferior results to mini-batching plus asynchronous updates. This suggests that in other learning problems with many local minima, including in artificial neural networks, asynchronous updates could benefit the learning process. While we are not aware of this desynchronization algorithm used in such a way, similar methods such as dropout [45] have been shown to be beneficial in improving generalizability of solutions [56], similar to stochastic gradient descent. True desynchronization would be extremely inefficient in such a system, as then the entire gradient calculation is necessary for a single edge update. However, we have shown that benefits can be accrued by only moderate desynchronization, e.g. 80% update probability, which slows the learning process proportionately. The true test of the usefulness of this algorithm will be in larger, nonlinear networks solving complex problems with many minima. This is a subject for future work.

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