Compressibility in the Integer Quantum Hall Effect within Hartree-Fock Approximation

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Electron-electron interactions seem to play a surprisingly small role in the description of the integer quantum Hall effect, considering that for just slightly different filling factors the interactions are of utmost importance causing the interaction-mediated fractional quantum Hall effect. However, recent imaging experiments by Cobden et al. [1] and Ilani et al. [2] constitute strong evidence for the importance of electron-electron interactions even in the integer effect. The experiments report on measurements of the conductance and electronic compressibility of mesoscopic MOSFET devices that show disagreement with predictions from the single particle model. By diagonalising a random distribution of Gaussian scatterers and treating the interactions in Hartree-Fock approximation we investigate the role of electron-electron interactions for the integer quantum Hall effect and find good agreement with the experimental results.

1 Introduction

The integer quantum Hall effect (IQHE) — observed in two-dimensional electron systems (2DES) subject to a strong perpendicular magnetic field $B$ — has been explained in great detail based on single-particle arguments [4]. There exists one extended state in the centre of each disorder-broadened Landau level where the accompanying localisation-delocalisation transition is governed by the critical exponent $\nu = 2.34 \pm 0.04$ [5,6], the Hall conductivity $\sigma_{xy}$ jumps by $e^2/h$ and the longitudinal conductivity $\sigma_{xx}$ is finite.

However, recent experiments on mesoscopic MOSFET devices questioned that simple picture. Measurements of the Hall conductance as a function of magnetic field $B$ and gate voltage [1] exhibited regular patterns along integer filling factors. It was argued that, contrary to the single-particle picture, these patterns should be attributed to Coulomb blockade effects. Similar patterns have been found recently also in measurements of the electronic compressibility $\kappa$ as a function of $B$ and electron density $n$ [2]. From these measurements it turns out that the number of localised states is independent of $B$ which is inconsistent with the single-particle picture. The authors explain these results by the incomplete screening of the impurity charge density at the Landau level band edges.

In the present paper, we investigate the effects of Coulomb interactions on compressibility and Hall conductance within a Hartree-Fock (HF) approach. We show that the observed charging lines in the compressibility can be found within HF. Similar approaches [7,8] have recently found supporting results.

2 Hartree-Fock approximation in the Landau basis

We consider a 2DES confined to a square of size $L \times L$ with periodic boundary conditions. The Coulomb interaction is treated within the Hartree-Fock...
approximation \cite{9,10,11} and the resulting Hartree-Fock Hamiltonian is given as
\begin{equation}
H_{HF}^\sigma = H_0^\sigma + V_{HF}^\sigma = \frac{(p - eA)^2}{2m^*} + \frac{\sigma q^\ast \mu_B B}{2} + V_D(r) + V_{HF}^\sigma (r),
\end{equation}
where \(\sigma = \pm 1\) denotes the spin and \(V_D\) is a smooth disorder potential. The kinetic term in the Hamiltonian can be diagonalised by the Landau wave functions
\begin{equation}
\langle r | \phi_{m,k} \rangle = \sum_{m=-\infty}^{\infty} \frac{1}{\sqrt{2^m m! \pi l_c^2}} \exp \left[ iky - \frac{(x - k l_c^2 + u L)^2}{2l_c^2} \right] H_m \left( \frac{x - k l_c^2 + u L}{l_c} \right),
\end{equation}
where \(l_c = \sqrt{\hbar/eB}\) is the magnetic length and the Landau gauge \(A = B xe_y\) has been used. The Landau level index is denoted by \(m\) and the momentum \(k = 0, 1, \ldots, N_\phi - 1\), where the number of flux quanta is related to \(L\) and \(l_c\) by \(N_\phi = L^2/2\pi l_c^2\). The potential \(V_D\) is constructed as a sum of \(N_s\) Gaussian scatterers at random positions \(r_s\) with random strength \(V_s\) and fixed range \(d\) within the square,
\begin{equation}
V_D(r) = \sum_{s=-\infty}^{\infty} \sum_{s=1}^{N_s} \frac{V_s}{2\pi d^2} \exp \left[ \frac{(r - r_s + u L e_y)^2}{2d^2} \right].
\end{equation}
The electron-electron interactions are treated in Hartree-Fock approximation resulting in an effective Coulomb and exchange potential \(V_{HF}^\sigma\) which has to be calculated self-consistently. On solving the (self-consistent) eigenvalue equation \(H_{HF}^\sigma C_{\sigma}^\alpha = \epsilon_{\alpha}^\sigma C_{\sigma}^\alpha\), we obtain the single particle energies \(\epsilon_{\alpha}^\sigma\) and the expansion coefficients \(C_{\sigma}^\alpha\) of the HF wave functions \(\psi_\alpha\) = \(\sum_{\sigma,i} C_{\sigma}^\alpha |\phi_i\rangle\).

The total energy of the system is then calculated as
\begin{equation}
E_{N(eF)}^{\text{tot}} = \frac{1}{2} \sum_{\sigma,i,j} \epsilon_{ij}^\sigma \langle \phi_i | 2H_0^\sigma + V_{HF}^\sigma | \phi_j \rangle
\end{equation}
with the density matrix
\begin{equation}
\varrho_{ab}^\sigma = \sum_{\alpha} (C_{\alpha a}^\sigma)^* C_{\alpha b}^\sigma .
\end{equation}
Here \(i, j\) denote multi-indices counting both Landau level index \(m\) and momentum \(k\).

3 Electronic Compressibility and Hall Conductivity

The electronic compressibility \(\kappa\) is inversely proportional to the linear response of the chemical potential \(\mu\) to changes in the electron density \(n\),
\begin{equation}
\kappa \propto \left( \frac{dn}{d\mu} \right)^{-1} \quad \text{with} \quad \frac{dn}{d\mu} = L^2 \left( E_{N+1} - 2E_N + E_{N-1} \right),
\end{equation}
where \(E_N\) is the total energy for \(N = nL^2\) electrons. Calculations of the electronic compressibility are shown in Figs.\textsuperscript{11} and\textsuperscript{12} for non-interacting and interacting electron system in the same disorder configuration, respectively. The sample size in both cases is 300 nm which corresponds to \(N_\phi = 44\) at \(B = 2\ T\) and \(N_\phi = 131\) at \(B = 6\ T\). For the calculations we have used \(N_s = 200\) scatterers with range \(d = 20\ nm\) and strengths \(V_s/\text{nm}\) randomly distributed in \([-20, 20]\) meV. Only the lowest Landau level has been included into the calculation for the interacting system due to the large computational effort involved.

Without interactions the Hall conductivity can be calculated using the Kubo formula \cite{13}. In the strong field limit, the electron motion can be separated into guiding centre and cyclotron motion and the Kubo formula reads
\begin{equation}
\sigma_{xy} = -\frac{ne c^2}{B} \sum_{\alpha} \sum_{\beta} \left( \langle \psi_\alpha | \hat{X} | \psi_\beta \rangle \langle \psi_\beta | \hat{Y} | \psi_\alpha \rangle - \langle \psi_\alpha | \hat{Y} | \psi_\beta \rangle \langle \psi_\beta | \hat{X} | \psi_\alpha \rangle \right) \left( \epsilon_\alpha - \epsilon_\beta \right)^{-2}
\end{equation}
In the Landau basis, the velocity matrix elements can be written as

\[ \langle \phi_{n,i} | \hat{X} | \phi_{m,j} \rangle = \frac{i L^2}{\hbar} (k_i - k_j) \langle \phi_{n,i} | V_D | \phi_{m,j} \rangle \] (9)

and

\[ \langle \phi_{n,i} | \hat{Y} | \phi_{m,j} \rangle = \frac{L^2}{\hbar} \left( \sqrt{\frac{n}{2}} \langle \phi_{n-1,i} | V_D | \phi_{m,j} \rangle + \sqrt{\frac{m}{2}} \langle \phi_{n,i} | V_D | \phi_{m-1,j} \rangle \right. \\
- \left. \sqrt{\frac{n+1}{2}} \langle \phi_{n+1,i} | V_D | \phi_{m,j} \rangle - \sqrt{\frac{m+1}{2}} \langle \phi_{n,i} | V_D | \phi_{m+1,j} \rangle \right) \] (10)

In order to calculate \( \sigma_{xy} \) for the interacting HF system, we interpret the \( V_{HF} \) as an additional, mean-field potential and compute \( \sigma_{xy} \) using Eqs. (9) and (10) with \( V_D \rightarrow V_D + V_{HF} \). Results on the Hall conductivity in the non-interacting case are depicted in Figs. 3 and 4 using the same parameters as in the compressibility calculations.

4 Discussions and Conclusions

From our compressibility results for the non-interacting system, we find strongly incompressible lines along *even* — due to low spin-splitting — integer filling factors \( \nu = nh/eB \). These are accompanied by less pronounced lines away from integer \( \nu \), which are no longer perfectly parallel to \( \nu \). Their number increases with increasing magnetic field, i.e. the number of strongly localised states increase with \( B \) as expected in the single-particle picture.

In contrast, we find that in the HF-interacting system, the number of incompressible lines parallel to integer \( \nu \) remains independent of \( B \). They are exhibited only within a strip of constant density around integer filling factor. In this region of a nearly full (or empty) Landau level, the excess electron density is restricted by the band edges and becomes too low to provide a complete screening of the impurity density. The electron density is torn apart and the formation of charge or hole islands leads to the observed charging lines in the compressibility.
Fig. 3  Hall conductivity $\sigma_{xy}$ as a function of $B$ and $n$ for the lowest 2 Landau levels and the same disorder configuration as in Figs. 1 and 2. For clarity, we have subtracted the values of the plateau conductivities.

Fig. 4  $\sigma_{xy}$ as a function of electron density $n$ at $B = 5.9T$ for the lowest 3 Landau levels. Same sample as in Fig. 1 and 2.

This is in good agreement with the experimental results of Ref. [2]. In addition, we observe spin-splitting enhancement resulting in incompressible strips also along odd integer filling factors. We also observe lines of high contrast at $\nu \sim 1/2$ and 3/2. At present, these are unexplained and might be a numerical artifact.

In our conductivity results, we see maxima aligning with integer $\nu$ even for the non-interacting system. This is reminiscent of the results of Ref. [1]. However, the fidelity of extrema of $\sigma_{xy}$ appears to be lower. Results for the interacting system are currently in progress and will be presented in a following publication.

In conclusion, we have presented results from numerical calculations on Hall conductance and electronic compressibility as a function of both magnetic field and electron density and compared these to recent imaging experiments [1][2].

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