Orbital diamagnetism of weakly doped bilayer graphene in a magnetic field

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Abstract
We investigate the orbital diamagnetism of weakly doped bilayer graphene (BLG) in a spatially smoothly varying magnetic field and obtain the general analytic expression for the orbital susceptibility of BLG, with finite wavenumber and Fermi energy, at zero temperature. We find that the magnetic field screening factor of BLG is dependent on the wavenumber, which results in a more complicated screening behavior compared with that of monolayer graphene (MLG). We also study the induced magnetization and electric current in BLG, under a nonuniform magnetic field, and find that they are qualitatively different from those for MLG and the two-dimensional electron gas (2DEG). However, as for MLG, a magnetic object placed above BLG is repelled by a diamagnetic force from the BLG, which is approximately equivalent to the force produced by its mirror image on the other side of the BLG with a reduced amplitude dependent on the typical length of the systems. BLG shows crossover behaviors in the responses to the external magnetic field, intermediate between those of MLG and 2DEG.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Bilayer graphene (BLG), as a significant graphene-related material, has attracted much attention [1–3] due to its unusual electronic structure. Formed by stacking two monolayer graphene (MLG) layers in a Bernal stacking, bilayer graphene has four inequivalent sites in each unit cell, including A 1 and B1 atoms on the top layer and A2 and B2 atoms on the bottom layer. The distance \( a \) between A 1 and B 1 is about 0.142 nm and the vertical separation of the two layers \( d \) (which is also the distance between B 1 and A 2) is about 0.334 nm. There are three hopping parameters that characterize a BLG effective mass model: \( t \approx 3 \) eV represents the intralayer hopping energy (A i ↔ B i), \( \Delta \approx 0.35 \) eV the interlayer hopping energy (A 2 ↔ B 1), and \( \gamma_{3} \approx 0.3 \) eV another interlayer hopping energy (A 1 ↔ B 2). On its band structure side, the conduction and valence band touch at the two inequivalent corners (called K and K’ points) of the hexagonal Brillouin zone. Around the K and K’ points, BLG has a quadratic energy dispersion [1] similar to that of the regular two-dimensional electron gas (2DEG), but its low energy effective Hamiltonian is chiral without a band gap, similar to that of MLG. The quadratic dispersion of BLG can lead to broken-symmetry states that are induced by interactions among the charge carriers even in the absence of external magnetic and electric fields [4–11]. Another unique feature of BLG is that a widely tunable band gap can be conveniently realized by introducing an electrostatic potential bias between the top and bottom layers [1, 12–16].

The magnetic susceptibility of electronic systems comes from two contributions: one is Pauli paramagnetism which stems from spin polarization; the other is Landau diamagnetism which stems from the circulation of orbital currents. The orbital diamagnetism of carbon systems has attracted the interest of both experimental and theoretical physicists for a long time. It was first found by Krishnan [17] that the diamagnetic susceptibility of bulk graphite is large and anisotropic. McClure [18] showed that it arises from the Landau quantization of 2D massless Dirac fermions, which results in a delta function peak at zero energy in the orbital diamagnetism of graphite. Following the experimental fabrication of graphene, a great deal of work has been concerned with the orbital magnetism of graphene-related systems, such as nodal fermions [19], disordered graphene [20–22], few-layered graphene [23–25], and graphene in...
a nonuniform magnetic field [26, 27]. Recently, novel paramagnetic susceptibility has been found in MLG and BLG with a band gap [28] and doped graphene, to first order in the Coulomb interaction [29].

Safran [30] and Koshino and Ando [23] have derived an analytical expression for the orbital susceptibility $\chi(0)$ for bilayer graphene, but their results are limited to the case of zero wavenumber. In this paper we extend them to general cases and analytically study the orbital diamagnetism of weakly doped BLG (i.e., the Fermi energy $\epsilon_F$ is much smaller than the interlayer hopping energy) in nonuniform magnetic fields.

This paper is organized as follows. In section 2, the effective Hamiltonian of bilayer graphene and its corresponding eigenstates and eigenenergies are introduced. In section 3, the orbital susceptibility of bilayer graphene is studied. In section 4, we investigate and discuss the responses of bilayer graphene to several specific external magnetic fields. The conclusion is given in section 5.

2. The bilayer graphene effective model

In the low energy and long wavelength regime, the BLG Hamiltonian near a K point, in the absence of a magnetic field, can be written in an excellent approximate form [1] (we set $c = 1 = \hbar$ in this paper):

$$H_0 = \begin{pmatrix} 0 & \gamma k^- & 0 & 0 \\ \gamma k^+ & 0 & \Delta & 0 \\ 0 & \Delta & 0 & \gamma k^- \\ 0 & 0 & \gamma k^+ & 0 \end{pmatrix},$$

where $\gamma = 3ta^2/2 \approx 10^6$ m s$^{-1}$ is the monolayer graphene Fermi velocity, $t \approx 3$ eV is the in-plane hopping energy, $a \approx 0.142$ nm is the in-plane interatomic distance, $\Delta \approx 0.35$ eV is the interlayer hopping energy. $k = (k_x, k_y) = -i\mathbf{v}$ is a 2D wavevector operator, and $k_{\pm} = k_x \pm i k_y$. In order to solve the eigenvalue of the Hamiltonian (1), the wavefunction can be expressed as $(\psi_{A_1}, \psi_{B_1}, \psi_{A_2}, \psi_{B_2})$, where the four components represent the Bloch functions at $A_1$, $B_1$, $A_2$ and $B_2$ sites, respectively. We follow the stipulation of Ando [31], and define

$$\epsilon(k) = \sqrt{\left(\frac{\Delta}{2}\right)^2 + (\gamma k)^2},$$

$$\gamma k = \epsilon(k) \sin \psi,$$

$$\frac{\Delta}{2} = \epsilon(k) \cos \psi.$$  

Then the corresponding eigenstates of equation (1) are given as

$$\Psi_{sjk}(r) = \frac{1}{L^2} \exp(ik \cdot r) U[\theta_k] F_{sjk},$$

where $L^2$ is the area of the system, $s = \pm 1$ and $-1$ denote the conduction and valence bands, respectively, $j = 1, 2$ specifies two subbands within the conduction and valence bands respectively, $\theta_k = \arctan(k_y/k_x)$ is the polar angle of the momentum $k$,

$$U(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\theta} & 0 & 0 \\ 0 & 0 & e^{-i\theta} & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix},$$

and

$$F_{s1k} = \frac{1}{\sqrt{2}} \begin{pmatrix} s \cos(\psi/2) \\ -s \sin(\psi/2) \\ -c_2 \sin(\psi/2) \\ c_2 \cos(\psi/2) \end{pmatrix},$$

$$F_{s2k} = \frac{1}{\sqrt{2}} \begin{pmatrix} s \sin(\psi/2) \\ c_2 \cos(\psi/2) \\ -c_2 \sin(\psi/2) \\ s \cos(\psi/2) \end{pmatrix}.$$  

The corresponding eigenenergies of equation (1) are

$$\epsilon_{s1}(k) = 2s\epsilon(k) \sin^{2}(\psi/2),$$

$$\epsilon_{s2}(k) = 2s\epsilon(k) \cos^{2}(\psi/2).$$

Considering a magnetic field $\mathbf{B}(r) = [\nabla \times \mathbf{A}(r)]$, the Hamiltonian for the system is $H = H_0 + H_1$, with $H_1 = -\int d^2r \sum_{\mu \nu} j_{\mu}(r) A_{\nu}(r, t)$. The current operator at $r_0$ is given by

$$j_{\alpha}(r_0) = \frac{e}{2} [\hat{v}_a \delta(r - r_0) + \delta(r - r_0) \hat{v}_a], \quad (\alpha = x, y)$$

where $\hat{v}_a$ is the velocity operator

$$\hat{v}_a = \frac{\partial H_0}{\partial k_a} = \gamma \left( \sigma_a 0 \\ 0 \sigma_a \right), \quad (\alpha = x, y)$$

and $\sigma_x$, $\sigma_y$ are the Pauli matrices which act on the sublattice space within a layer.

3. Orbital susceptibility

The finite wavenumber susceptibility $\chi(q)$ can be obtained through the Kubo formula [32]. Within the linear response theory, the external vector potential $\mathbf{A}$ and the induced 2D electric current density $\mathbf{j}$ have a relation

$$j_{\mu}(q) = \sum_{v} K_{\mu v}(q) A_{\nu}(q),$$

and the orbital susceptibility $\chi(q)$ and the response tensor $K_{\mu v}(q)$ are related by

$$K_{\mu v}(q) = q^2 \chi(q) \left( \delta_{\mu v} - \frac{q_{\mu} q_{\nu}}{q^2} \right).$$

In the first-order perturbation, we have

$$K_{\mu v}(q) = -\frac{g}{L^2} \sum_{s' j' k} \frac{f[\epsilon_{s j}(k)] - f[\epsilon_{s j'}(k')]}{\epsilon_{s j}(k) - \epsilon_{s j'}(k')} I_{s' j'; s j},$$

where $g = g_s g_t = 4$ is the total degeneracy (here $g_s$ and $g_t$ denote the valley and spin degeneracy, respectively), $k' = k + q$, $\epsilon_{s j}(k)$ is the eigenenergy given by equations (8), $f(\epsilon)$ is...
the Fermi distribution function \( f(\varepsilon) = [1 + \exp(\beta(\varepsilon - \varepsilon_F))]^{-1} \)
where \( \varepsilon_F \) is the Fermi energy, and \( \beta = 1/(k_B T) \). \( I_{s',j'} \) is the current–current response matrix element expressed by
\[
I_{s',j'} = \left[ F_{s,k} U(\theta_k) v_{ij} U(\theta_k) F_{j',k} \right] \times \left[ F_{s',k} U(\theta_k) v_{ij} U(\theta_k) F_{j,k} \right],
\]
(14)
which determines the weight of the contribution of the transition from subband \( j \) to \( j' \), with \( ss' = +1 \) and \(-1\) denoting the intraband and interband transition, respectively.

Define the effective mass as \( m = \Delta/(2y^2) \approx 0.033m_e \) and the Fermi wavenumber as \( k_F = \sqrt{2m\varepsilon_F} \). For a low energy theory of bilayer graphene, there is a natural high energy cutoff wavenumber \( \Lambda \equiv \sqrt{2m\Delta} = \Delta/\gamma \). When \( k_F, q \ll \Lambda \), i.e., when bilayer graphene is weakly doped and the external field is smooth enough compared to the cutoff wavelength, the transitions inside the same \( j \) subband (including the intraband transitions from band \( (s, j) \) to band \( (s, j) \) and the interband transitions from band \( (s, j) \) to \((-s, j)\)) dominate the contribution to the response function. One of these transitions is the transition inside the lower two bands (i.e., the \( j = 1 \) subband):
\[
K_{\mu\nu,11}(q) = -\frac{g}{L^2} \sum_{s \in k} \frac{f[\varepsilon_{s1}(k)] - f[\varepsilon_{s2}(k')]}{\varepsilon_{s1}(k) - \varepsilon_{s2}(k')} I_{s',s},
\]
(15)
In the low energy limit, it can be approximately given as
\[
K_{\mu\nu,11}(q) \approx -\frac{g^2}{m^2L^2} \sum_{s \in k} \frac{f[\varepsilon_{s1}] - f[\varepsilon_{s2}]}{\varepsilon_{s1} - \varepsilon_{s2}} \times \left[ \frac{1}{2} \left( k + \frac{q}{2} \right)^2 \delta_{\mu\nu} + \frac{ss'}{8} F_{\mu\nu} \right],
\]
(16)
with \( \varepsilon_{s2} = k^2/(2m) \) and
\[
F_{\mu\nu} = (\delta_{\mu1}\delta_{\nu1} - \delta_{\mu2}\delta_{\nu2})[k^2 \cos 2\theta_k + k^2 \cos 2\theta_k + 2kk' \cos(\theta_k + \theta_{k'}) + \delta_{\mu2}\delta_{\nu2} \delta_{\nu1} - \delta_{\mu2}\delta_{\nu1} \delta_{\nu2}] \times \left[ k^2 \sin 2\theta_k + k^2 \sin 2\theta_k + 2kk' \sin(\theta_k + \theta_{k'}) \right].
\]
(17)
Then, we obtain
\[
K_{\mu\nu,11}(q) = \frac{g^2q^2}{8\pi m} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \log \frac{2k_F^2 + \sqrt{4k_F^4 + q^4}}{4\Lambda^2} + \frac{1}{3} \left( 1 - \frac{4k_F^2}{q^2} \right)^{3/2} \theta(q - 2k_F).
\]
(18)
The other is the transition inside the higher two bands (i.e., the \( j = 2 \) subband):
\[
K_{\mu\nu,22}(q) = \frac{g}{L^2} \sum_{s \in k} \frac{f[\varepsilon_{s2}(k)] - f[\varepsilon_{s2}(k')]}{\varepsilon_{s2}(k) - \varepsilon_{s2}(k')} I_{s',s},
\]
(19)
By using the fact that the subband \( j = 2 \) in the conduction band \( (s = +1) \) is empty in the weakly doped limit, it can be approximately given as
\[
K_{\mu\nu,22}(q) \approx \frac{2g^2}{m^2L^2} \sum_k \frac{1}{\varepsilon_k - \varepsilon_k} \times \left[ \frac{1}{2} \left( k + \frac{q}{2} \right)^2 \delta_{\mu\nu} + \frac{ss'}{8} G_{\mu\nu} \right],
\]
(20)
with
\[
G_{\mu\nu} = (\delta_{\mu1}\delta_{\nu1} - \delta_{\mu2}\delta_{\nu2})[k^2 \cos 2\theta_k + k^2 \cos 2\theta_k + 2kk' \cos(\theta_k + \theta_{k'}) + \delta_{\mu2}\delta_{\nu2} \delta_{\nu1} - \delta_{\mu2}\delta_{\nu1} \delta_{\nu2}] \times \left[ k^2 \sin 2\theta_k + k^2 \sin 2\theta_k + 2kk' \sin(\theta_k + \theta_{k'}) \right].
\]
(21)
Then, we have
\[
K_{\mu\nu,22}(q) = \frac{g^2q^2}{24\pi m} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right).
\]
(22)
From the above, we first obtain the analytic expression for the susceptibility of bilayer graphene at zero temperature as
\[
\chi(q; \varepsilon_F) = \frac{g^2}{8\pi m} \left[ \log \frac{2k_F^2 + \sqrt{4k_F^4 + 4q^4}}{4\Lambda^2} + \left( 1 - \frac{4k_F^2}{q^2} \right)^{3/2} \theta(q - 2k_F) \right].
\]
(23)
where \( \theta(x) \) is the step function. It is a central result of this paper. From equation (23), on the one hand, we can give the orbital susceptibility of BLG at zero Fermi energy as
\[
\chi(q = 0; \varepsilon_F) = \frac{g^2}{4\pi m} \left[ \log \frac{q}{2\Lambda} + \frac{1}{3} \right].
\]
(24)
This result is the same as the result given in [23, 30], which shows a logarithmically diverging behavior at \( \varepsilon_F = 0 \). On the other hand, we can give the orbital susceptibility of BLG at zero Fermi energy as
\[
\chi(q = 0; \varepsilon_F = 0) = \frac{g^2}{4\pi m} \left[ \log \frac{q}{2\Lambda} + \frac{1}{3} \right].
\]
(25)
This also shows a logarithmically diverging behavior at \( q = 0 \). It is easy to see that just by replacing \( \varepsilon_F \) with \( \gamma q/2 \) and increasing the coefficient to its double, we can transform the susceptibility \( \chi(q = 0; \varepsilon_F) \) into \( \chi(q; \varepsilon_F = 0) \).

The orbital magnetic susceptibility \( \chi(q) \) of bilayer graphene as a function of the wavenumber is shown in figure 1. For comparing the wavenumber dependent behaviors of the susceptibility of the MLG, BLG and 2DEG systems, we provide the finite wavenumber susceptibilities of MLG and 2DEG below, which were given in [26]:
\[
\chi(q; \varepsilon_F) = -\frac{g^2}{16} \frac{1}{q} \theta(q - 2k_F) \times \left[ 1 + \frac{2k_F}{q} \sqrt{1 - \left( \frac{2k_F}{q} \right)^2} - \frac{2}{\pi} \sin^{-1} \frac{2k_F}{q} \right]
\]
(26)
(for MLG),
\[
\chi(q; \varepsilon_F) = \frac{g^2}{24\pi m} \times \left[ 1 - \frac{4k_F^2}{q^2} \right]^{3/2} \theta(q - 2k_F) - 1]
\]
(27)
(for 2DEG).

At \( q = 0 \), the susceptibility of BLG \(-\chi(0, \varepsilon_F) \propto \log(A/\varepsilon_F)\) is rather different from those of MLG, where \(-\chi(0, \varepsilon_F) \propto \delta(\varepsilon_F)\), and 2DEG, where \(-\chi(0, \varepsilon_F) \propto 1\). By
comparing their diverging behaviors, it can be found that the BLG in some sense shows a behavior intermediate between those of the MLG and 2DEG. For small $q$, the $-\chi(q; \epsilon_F)$ of BLG deviates from the $-\chi(0; \epsilon_F)$ as $(q/2k_F)^3$, and falls more rapidly as $q$ increases. On the other hand, the susceptibility of MLG vanishes while that of 2DEG remains constant for the whole regime $q < 2k_F$ (see figure 1 of [26]). At $q = 2k_F$, the susceptibilities $\chi(2k_F; \epsilon_F)$ of MLG and 2DEG are both zero for MLG) which are independent of the Fermi wavenumber $k_F$, but for BLG we have

$$\chi(2k_F; \epsilon_F) = \frac{g e^2}{8\pi m} \left( \log \frac{\epsilon_F}{\Delta} + \frac{1}{3} + \frac{1 + \sqrt{5}}{2} \right), \quad (28)$$

which is dependent on $k_F$. In contrast to that of MLG but similarly to that of 2DEG, the susceptibility of BLG has no singular behavior at $q = 2k_F$, and it is continuous, as is its first derivative. For large $q$, and in particular for $q \gg 2k_F$, $-\chi(q)$ for BLG rapidly approaches the curve of equation (25) and falls as $\log(1/q)$, which is very different from the cases for MLG, where the susceptibility falls off more rapidly ($\sim 1/q$), and 2DEG, where it falls as $1/q^2$. Due to their having the same parabolic energy dispersion in the low energy limit, the susceptibilities of bilayer graphene and 2DEG share the same term $g e^2(1 - 4k_F^2/q^2)^{3/2}/(q - 2k_F/(24\pi m))$.

The generic response tensor $K_{\mu\nu}(q, \omega)$ can be decomposed into longitudinal and transverse components:

$$K_{\mu\nu}(q, \omega) = \chi_L(q, \omega) \frac{\rho_0 q \omega}{q^2} + \chi_T(q, \omega) \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \quad (29)$$

where $\chi_L$ and $\chi_T$ are the current–current response functions in the longitudinal and transverse channels, respectively. In the absence of long range order, there is a so-called diamagnetic sum rule for the longitudinal and transverse response functions [33]:

$$\lim_{q \to 0} \chi_L(q, 0) = \lim_{q \to 0} \chi_T(q, 0) = 0. \quad (30)$$

Because we are only concerned about the static susceptibility, $\chi_L(q, 0)$ vanishes for arbitrary $q$ [33], which satisfies the sum rule for $\chi_L$. As shown from equations (12) and (23), our calculation satisfies the diamagnetic sum rule for $\chi_T$. Since the orbital susceptibility $\chi(q) = -e^2\chi_T(q, 0)/q^2$, the existence of a finite orbital susceptibility means that $\chi_T(q, 0)$ vanishes as $q^2$ for $q \to 0$. It is easy to see that our calculation is consistent with this argument.

### 4. Responses to specific external magnetic fields

Now we study the responses of bilayer graphene to different types of magnetic field. First let us consider the case of neutral BLG (i.e., $\epsilon_F = 0$) under a sinusoidal magnetic field $B(r) = B_0 \cos qx\hat{e}_\gamma$. Defining $\chi(q) \equiv \chi(q, \epsilon_F = 0)$, we have the induced magnetization $m(r) = \chi(q)B(r)$, the induced current $j(r) = q\chi(q)B_0 \sin qx\hat{e}_\gamma$, and the $z$ component of the induced counter-magnetic field on BLG

$$B_{\text{ind}}(r) = -\alpha_z(q)B(r), \quad (31)$$

with the magnetic field screening factor

$$\alpha_z(q) = -\frac{ge^2}{2m} \left[ \log \frac{q}{2\Lambda} + \frac{1}{3} \right]. \quad (32)$$

When $q \to 0$, we have $\alpha_z(q) \to 0$; i.e., under a constant magnetic field there is no counter-magnetic field on BLG, which is different from the case for MLG. For MLG, the magnetic field screening factor is fixed and independent of $q$ and the specific form of the external field; the induced magnetic field above the graphene layer is simply equivalent to the field of a mirror image of the original object reflected with respect to the graphene layer but reduced by $\alpha_z$ [26]. However, this argument is not appropriate for the case of BLG, since the magnetic field screening factor of BLG is dependent on $q$ and complicated.

Next we consider the case of a line current $I$ flowing along the +$y$ direction above the BLG, and passing through the point $(0, 0, d)$ ($d > 0$). The $z$ component of the magnetic field on the BLG is $B(r) = -2I/x(x^2 + d^2)$. For $\epsilon_F = 0$, the induced magnetization can be given as

$$m(r) = -\frac{Ige^2}{2\pi m x^2 + d^2} \left[ \log(2\Lambda \sqrt{x^2 + d^2}) 
+ \frac{d}{x} \arctan \frac{x}{d} - yE + \frac{1}{3} \right], \quad (33)$$

where $yE \approx 0.577$ is the Euler constant. By using $j_{\text{ind}} = \nabla \times m(r)$, we obtain the induced electric current $j_{\text{ind}}(r) = j_x\hat{e}_x$, where

$$j_x = \frac{Ige^2}{2\pi m (x^2 + d^2)^2} \left[ (d^2 - x^2) \log(2\Lambda \sqrt{x^2 + d^2}) 
- yE + \frac{1}{3} \right] - 2d \frac{x}{d} \arctan \frac{x}{d} (x^2 + d^2). \quad (34)$$

The integral of $j_x$ over $x$ is exactly equal to 0, which means that the external electric current $I$ cannot induce an effective transport electric current on the BLG, in contrast to the case...
for MLG, where \( I \) induces an effective electric current \(-\alpha_s I\). The induced magnetic field on BLG can be given as \( B_{\text{ind}}(r) = B_z e_z \) with

\[
B_z = \frac{I g e^2}{m} \frac{1}{(x^2 + d^2)^2} \left[ 2 \pi \left( \log \frac{\sqrt{x^2 + d^2} + y \varepsilon_j - 1}{3} \right) + (d^2 - x^2) \arctan \frac{x}{d} \right].
\]

(35)

For MLG, the induced magnetic field \(-\alpha_s I x/(x^2 + d^2)\) is equivalent to the field created by a current \(-\alpha_j I\) flowing at \( z = -d \). This argument is not appropriate for the case of BLG as shown by equation (35). At large distance \((d \gg 1)\), the induced magnetic field has the proportionality \(\sim 1/x^2\), compared with \(\sim 1/x\) for MLG. However, the original current is repelled by a force of \(\approx \alpha_s (1/2d) I^2 / d\) per unit length, which can be approximately considered as a force created by a current \(\alpha_s(q) I\) at \( z = -d \) with \( q = 1/2d \).

As another typical example, we study the magnetization, electric current and magnetic field induced by a magnetic monopole \( q_m \) lying above the BLG. Suppose that \( q_m \) is located at the point \((0, 0, d)\) \((d > 0)\), and the BLG plane is \( z = 0 \). The magnetic field perpendicular to the BLG is given as \( B(r) = q_m I / r^3 \), with \( r = \sqrt{x^2 + y^2} \). For neutral BLG, the induced magnetization is given by

\[
m(r) = -q_m g e^2 \frac{1}{4\pi m d^2} \left\{ F \left( \frac{r}{d} \right) - (\log 2 \Lambda r - 1/3) \times \left[ 1 + \left( \frac{r}{d} \right)^{2/3} \right] \right\},
\]

(36)

where we define the function

\[
F(x) = \frac{1}{x^2} \int_0^\infty z J_0(z) \log z e^{-z/x} \, dz.
\]

(37)

At small distance \((r \ll d)\) and large distance \((r \gg d)\), the induced magnetization can be written as

\[
m(r) = q_m g e^2 \frac{1}{4\pi m} \times
\left\{ \begin{array}{ll}
\frac{\log 2 \Lambda d + y \varepsilon_j - 1}{3 - r/2} & (r \ll d) \\
\left( \frac{\log 2 \Lambda r - 1/3}{r^2} \right) / d^2 & (r \gg d)
\end{array} \right.
\]

(38)

It is interesting to see that at large distance the induced magnetization of BLG has the proportionality \( m(r) \propto 1/r^2 \), while that of MLG is \( \propto 1/r \) and that of 2DEG is \( \propto 1/r^3 \); i.e., the BLG shows a behavior crossing over from MLG to 2DEG. The integral of \( m(r) \) over the plane has a logarithmically diverging behavior \( \propto \log R \) when the distance \( R \to \infty \). The corresponding electric current \( j(r) = -(dq/dr)e_0 \) \( \equiv j_0 e_0 \), where \( j_0 \) is given at small and large distance by

\[
j_0 = \frac{q_m g e^2}{4\pi m} \times
\left\{ \begin{array}{ll}
3 r \log 2 \Lambda r / d^4 & (r \ll d) \\
2 / r^3 & (r \gg d)
\end{array} \right.
\]

(39)

Recall that our results are confined to the limit \( \Lambda r \gg 1 \), and therefore \( j_0 \) is positive throughout the realistic distance. The current \( j_0 \) in MLG is \( \propto r/(r^2 + d^2)^{3/2} \), and the asymptotic form is \( \propto r \) at small distance and \( \propto 1/r^3 \) at large distance. We can see that the induced currents \( j_0 \) of MLG and BLG are qualitatively different. However, like for the case of a line current, the force between the monopole and the BLG can be approximately written as \( \alpha_s(q) q_m^2 / (2d)^3 \), with \( q = 1/d \), which has the same form as that for MLG, \( \alpha_s q_m^2 / (2d)^3 \).

For doped bilayer graphene \((\varepsilon_r \neq 0)\), the corresponding response currents \( j_0(r) \) for different values of \( k_F \) are shown in Figure 2. We can see that the current changes slightly as the Fermi wavenumber increases from \( k_F d = 0 \) to 1; i.e., the induced current has a weak Fermi wavenumber dependence, which is rather different from that of MLG. This arises from the fact that the dominant term involving \( k_F \) in the susceptibility is logarithmic.

Unlike the traditional 2DEG, MLG has peculiar properties [26]: (1) the counter-field induced by the response current mimics a mirror image of the original object; (2) the object is repelled by a diamagnetic force from the MLG, as if there exists a mirror image with a reduced amplitude on the other side of the MLG. With the investigation above, we find that the argument of (1) cannot be extended to BLG. However, the argument of (2) can still be approximately correct for weakly doped BLG, and it is only necessary to replace the constant reduced amplitude with a reduced amplitude dependent on the typical length of the systems. The BLG, in some sense, still show a behavior intermediate between those of MLG and 2DEG.

It is of interest to compare the contribution of the Landau diamagnetism to the whole magnetism with that of the Pauli paramagnetism in BLG. At \( \varepsilon_r = 0 \), the Pauli spin susceptibility \( \chi_{\text{spin}}(q) \), which is given by the density–density response function [34], is equivalent to \( g_s m \mu_B^2 \log 4/2r \), where \( \mu_B \) is the Bohr magneton. With equation (25), we obtain the ratio \( \chi_{\text{spin}} / \chi_{\text{orb}} \sim (0.03)^2 / (\log 2 \Lambda / q) \), which is rather small in our theory \((\text{for } q \ll \Lambda)\).

The temperature also has an influence on the BLG diamagnetism. For \( q = 0 \), this has been discussed by...
5. Conclusions

In this paper, we study analytically the orbital magnetic susceptibility of weakly doped bilayer graphene (BLG) in spatially smoothly varying magnetic fields by means of the low energy Hamiltonian. The magnetization and electric current induced by nonuniform magnetic fields in BLG are studied; they are different from those for MLG and 2DEG, but the argument that a magnetic object placed above the BLG layer is repelled by a diamagnetic force which is equivalent to a force produced by a mirror image on the other side of the BLG can still approximately hold, just replacing the constant reduced amplitude with a reduced amplitude dependent on the typical length of the system. Logarithmically dependent behaviors are found to arise extensively in both the orbital magnetism and the physical quantities induced by a specific external field. As an intermediate between MLG and 2DEG, BLG shows crossover behaviors in the responses to external magnetic fields. Weak Fermi wavenumber dependent behaviors, as a distinctive electrical property of BLG, are found for the induced magnetization and electric current.

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