Dissipative pulsar magnetospheres

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Abstract. A dissipative axisymmetric pulsar magnetosphere is calculated by a direct numerical simulation of the strong-field electrodynamics equations. The magnetic separatrix disappears; it is replaced by a region of enhanced dissipation. With a better numerical scheme, one should be able to calculate the bolometric light curves for a given conductivity.

Keywords: neutron stars, magnetic fields

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1. Introduction

Recent progress in calculating pulsar magnetospheres ([2] and references therein) is based on force-free electrodynamics (FFE). FFE describes electromagnetic fields of special geometry—for each event, the electric field is smaller than and perpendicular to the magnetic field. When applied to pulsars, FFE predicts a singular current layer along the magnetic separatrix surface. Within the light cylinder, the singularity is of an admissible type—although the current is singular, the electric field remains smaller than and perpendicular to the magnetic field. Outside the light cylinder, the singularity actually violates the FFE equations. For this reason, and also in order to understand dissipation processes leading to the observed radiation, one wants to use a dissipative generalization of FFE. Such a theory—strong-field electrodynamics (SFE)—has recently been proposed [1].

SFE describes electromagnetic fields of arbitrary geometry. SFE is Maxwell theory with the semi-dissipative Lorentz-covariant Ohm’s law

\[ j = \frac{\rho E \times B + (\rho^2 + \gamma^2 \sigma^2 E_0^2)^{1/2}(B_0 E + E_0 B)}{B^2 + E_0^2}, \]

where \( \rho \equiv \nabla \cdot E \) is the charge density, \( B_0^2 - E_0^2 \equiv B^2 - E^2, B_0 E_0 \equiv E \cdot B, E_0 \geq 0 \) represent the field invariants, and \( \gamma^2 \equiv (B^2 + E_0^2)/(B_0^2 + E_0^2) \). The conductivity scalar \( \sigma = \sigma(B_0, E_0) \) is an arbitrary function of the field invariants. SFE comes from the following simple postulate: for each event, in the frame where the electric field is parallel to the magnetic field and the charge density vanishes, the current \( \sigma E \) flows along the fields. The covariant formulation of SFE is

\[ j^2 = -\sigma^2 E_0^2, B_0 F_j = E_0 \tilde{F} j \] (compare to FFE: \( E_0 = 0, F_j = 0 \)).

2. Axisymmetric pulsar in SFE

We simulated time-dependent axisymmetric Maxwell equations with the SFE Ohm’s law. We used the regularization described in [1], and checked that the results are independent of the regularization to the specified accuracy.

The initial field was vacuum around the rotating magnetized neutron star. In spherical coordinates, in pulsar units (\( \mu = \Omega = c = 1 \)), the vacuum field is given by the magnetic field invariants.
Figure 1. $\sigma = 10$, $r_s = 0.5$. Thick: magnetic stream function $\psi$, intervals of $\psi(r = 1, \theta = \pi/2)/5$. Thin: electric potential $\phi$, same intervals. Dotted: poloidal current $I$; see figure 3 for the current distribution on the surface of the star.

Figure 2. Left: invariant $E_0$, intervals $E_0\max/5$. Right: invariant $E^2 - B^2$, intervals $(E^2 - B^2)\max/5$, showing only positive isolines.

stream function $\psi = \sin^2 \theta/r$ and the electrostatic potential $\phi = (1/3 - \cos^2 \theta)r_s^2/r^3$, where $r_s$ is the radius of the star. We then evolved the three components of the electric field ($E_r, E_\theta, E_\phi$), the magnetic stream function $\psi$, and the toroidal magnetic field $B_\phi$. The boundary conditions on the surface of the star are $\psi = \sin^2 \theta/r_s$ and $E_\theta = -\sin(2\theta)/r_s^2$. 

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Figure 3. Left: poloidal current distribution over the surface of the star. Right: Poynting luminosity at different radii.

Figure 4. Same as figure 1; $\sigma = 50$, $r_s = 0.2$.

We show the magnetosphere at $t = 6$, when the field configuration within $r = 2$ (two light cylinders) saturates. The outer boundary was at $r = 5$—out of causal contact with the $r < 2$ zone. The saturated electric field is potential. The saturated toroidal field is given by the poloidal current $B_\phi \equiv 2I/(r \sin \theta)$.

We show two cases. The $r_s = 0.5$, $\sigma = 10$ results (for the Poynting power) are accurate to about 2%. Figure 1 shows the field lines, figure 2, the field invariants, figure 3, the
integrated poloidal current as a function of the latitude and the Poynting luminosity (in pulsar units) as a function of radius. The luminosity decreases due to dissipation. Figures 4–6 show the $r_s = 0.2$, $\sigma = 50$ results; these are accurate to about 10%.

3. Conclusion

Our code is amateurish—a straightforward discretization with diffusive stabilization. A professional numerical scheme might be of interest for two reasons:

(i) Our pulsar, even at $\sigma = 50$, is too dissipative. It would be interesting to increase $\sigma$ to the values for which the return current becomes space-like.

2 Available upon request from ag92@nyu.edu.
(ii) With a good 3D code, it should be possible to calculate the pulsar bolometric light curves for a given surface field and conductivity\(^3\). To calculate emission, one assumes efficient heat to light conversion and strong beaming along the field lines (in the frame where the fields are parallel).

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References

[1] Gruzinov A, 2008 arXiv:0802.1716
[2] Spitkovsky A, 2006 Astrophys. J. 648 L51 [SPIRES]

\(^3\) In SFE, \(\sigma\) is an arbitrary function of the field invariants. In the real world, the star breaks Lorentz invariance.