Probing Primordial Symmetry Breaking with Cosmic Microwave Background Anisotropy

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There have been vigorous research attempts to test various modified gravity theories by using physics of the cosmic microwave background (CMB) anisotropy. In this article, with the CMB we try to distinguish two different scenarios of spontaneous symmetry breaking (SSB) in primordial era of the universe. The first scenario is a broken symmetric theory of gravity, which was suggested by A. Zee in 1979. The second scenario is an application of Palatini formalism to Zee’s model and it combines the idea of SSB with Weyl scalar-tensor geometry.

Since our new model based on Palatini formalism has geometrical feature, the scalar field coupled with gravity affects an evolution of Hubble parameter whereas Zee’s model gives additional contributions to the energy-momentum tensor. Furthermore, it turns out that our model has different (from Zee’s) sensitivity of CMB anisotropy power spectra with the scalar field mass. This fact enables us to verify distinct kinds of primordial symmetry breaking.

I. INTRODUCTION

After the discovery of Cosmic Microwave Background (CMB), there have been many studies to understand its physical implications. Especially, its anisotropy has been vigorously studied to verify and give restriction on free parameters of modified gravity theories such as Brans-Dicke theory [1], Horndeski theory [2], and $f(R)$ gravity [3]. They have been developed to alleviate or give a clue for cosmological problems such as the cosmic constant problem.

Among the modified theories, there also have been attempts to apply an idea of symmetry breaking to cosmology. The study on symmetries and their breaking phenomena is one of the most important development in modern physics. The idea of spontaneous symmetry breaking, especially after the appearance of Higgs field theory [4-6], became one of the main subject area in particle physics. This trend also have affected studies in cosmology such as cosmic acceleration, the theory of inflation and the cosmic constant problem [7]. There are other tries for doing it in the primordial era. Broken-symmetric theory of gravity proposed by Zee [8] is a remarkable one among these attempts, in which the following action is considered:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \phi^2 R + \frac{1}{2} \phi^{\alpha\beta} \nabla_{\alpha} \phi \nabla_{\beta} \phi - V(\phi) \right] + S_M,$$

(1)

with the Higgs-type potential

$$V(\phi) = \frac{1}{2} V_0 (\phi^2 - v^2)^2,$$

(2)

which invokes symmetry breaking in primordial era with the value $v = M_P$ where $M_P$ is Planck mass. This model gives us deep insights. One of main features of the model is it cannot be distinguished from general relativity (GR) by current observation, if we consider only large scale and recent era of our universe. Its properties on perturbative scale, however, were not studied. Hence, we should consider also the perturbative scale to verify the model.

In this paper, we revisit Zee’s model by studying how it affects on CMB anisotropy spectra. Furthermore, we modify the model by Palatini formalism.

In GR, for an uniqueness of a connection we assume metric compatibility, which says that a covariant derivative of the metric tensor is zero. From this assumption we can compute Affine connection from the metric tensor. In contrast, in the Palatini formalism it is assumed that the connection is independent from the metric tensor. Especially, when we apply it to the modified gravity theories, metric compatibility may also not be true anymore. There are two main application of Palatini formalism to modified gravity theories. They are known as Weyl scalar-tensor geometry (or Weyl geometry) [9] and Palatini $f(R)$ gravity [10], which are their adoptions to Brans-Dicke theory and $f(R)$ gravity. In this paper we try to give a possibility of observational verification of primordial symmetry breaking by using an idea of Weyl geometry, and we propose a slightly different model for broken-symmetric theory of gravity.

Throughout the paper, to construct perturbation theory we use covariant and gauge-invariant (CGI) formalism, which developed by [11-13]. The reason why we choose CGI formalism is that we choose CAMB [14] to execute numerical analysis. CAMB takes CGI formalism to be its fundamental numerical strategy, due to the well-known fact that there are a gauge choosing problem in cosmological perturbation theory (See [15] for detailed explanation, for example.) and the numerical stability issues in Newtonian gauge. Furthermore, in non-minimally coupled gravity theories different gauge could give different results due to prescence of the scalar field. However, CGI formalism gives a simple remedy by using 1+3 decomposition of Einstein field equation and using a fundamental 4-velocity of fluid as a source for basic kinematical
quantities which are always gauge-independent. It has a clear physical meaning and its equations are equivalent to synchronous gauge if we take the observer 4-velocity to correspond to cold dark matter (CDM) velocity.

This paper is organized as follows: In section 2 we review Zee’s model and Weyl geometry, then we construct the SSB theory under Palatini formalism. Furthermore, we explain why we have to consider perturbation and verify our model. In section 3 we construct linearized perturbation theory based on CGI formalism. In section 4 we perform numerical analysis via numerical cosmology code CAMB. In section 5 we summerize our results and discuss their physical meanings.

II. BROKEN-SYMMETRIC THEORY OF GRAVITY AND PALATINI FORMALISM

Our first goal is applying Palatini formalism to Zee’s model. We first briefly review broken-symmetric theory of gravity (we abbreviate it as the Zee model). Then we review Weyl geometry and its main properties.

The equations of motion obtained from the action (1) are

\[ G_{\mu\nu} = \frac{2}{\phi^2} T_{\mu\nu} - T^{(\phi)}_{\mu\nu}, \tag{3} \]

\[ \Box \phi + \partial_\mu V(\phi) - \phi R = 0, \tag{4} \]

where we define energy-momentum(EM) tensor for matter and \( \phi \):

\[ T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} (\int d^4x \sqrt{-g} \mathcal{L}_M), \tag{5} \]

\[ T^{(\phi)}_{\mu\nu} \equiv \frac{2}{\phi^2} \partial_\mu \phi \partial_\nu \phi - \frac{2}{\phi^2} g_{\mu\nu} \left[ \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - V(\phi) \right] - \frac{1}{\phi^2} (\nabla_\mu \nabla_\nu \phi^2 - g_{\mu\nu} \Box \phi^2). \tag{6} \]

From equation (4), we see that the potential given by (2) invokes symmetry breaking and make the field \( \phi \) settle down at the minimum \( \phi = \nu = M_P \). After the symmetry breaking, the theory recovers GR in large scale so that

\[ G_{\mu\nu} = \frac{2}{\nu^2} T_{\mu\nu}. \tag{7} \]

However, on the perturbative scale differences arise. Let us expand the scalar field as \( \phi \sim \nu + \varphi \) where \( |\varphi| \ll 1 \) is first order perturbation variable. Clearly, one can see that the extra term \( \nabla_\mu \nabla_\nu \varphi^2 - g_{\mu\nu} \Box \varphi^2 \) in EM tensor of \( \phi \) (6) due to non-minimal coupling does not vanish in first order. However, we show that under Palatini formalism this term vanishes whereas the scalar field affects CMB anisotropy in other ways. To show this, let us review Weyl geometry, which is the simplest application of Palatini formalism to the theory with non-minimal coupling.

In Palatini formalism, we think of the connection and the metric as independent quantities. For an illustration, let us apply it to the action (1). For convenience we redefine the variables \( \psi = -2 \ln \phi \) so that

\[ S = \int d^4x \sqrt{-g} e^{-\psi} \left[ \frac{1}{2} R + \frac{1}{8} g^{\alpha\beta} \partial_\alpha \psi \partial_\beta \psi - V(\psi) \right] + S_M. \tag{8} \]

From the variation with respect to the connection \( \Gamma_{\mu\nu}^{\alpha} \), one directly obtains a new kind of covariant derivative, \( \psi \nabla \), namely, with no metric compatibility. [16]

\[ \psi \nabla_\alpha g_{\mu\nu} = g_{\mu\nu} \partial_\alpha \psi. \tag{9} \]

Now we set the derivative operator \( \psi \nabla_\alpha \) satisfying (9) to be fundamental derivative operator, instead of original covariant derivative \( \nabla_\alpha \) which satisfies metric compatibility condition. We call this kinds of theories with the covariant derivative given by (9) as Weyl geometry. With this setting, we call new connection \( \psi \nabla_\alpha \) satisfying (9) Weyl connection. It is related with the original affine connection \( \Gamma_{\mu\nu}^{\alpha} \) as:

\[ \psi \Gamma_{\mu\nu}^{\alpha} = \Gamma_{\mu\nu}^{\alpha} + \Delta_{\mu\nu}^{\alpha}, \tag{10} \]

where

\[ \Delta_{\mu\nu}^{\alpha} = -\frac{1}{2} (g^{\alpha\rho} \psi_{\rho\nu} + g^{\alpha\rho} \psi_{\rho\mu} - g_{\mu\nu} \psi^{\alpha}). \]

One can easily see that Weyl geometry has new kind of symmetry, because (9) and (10) is invariant under the transformation

\[ (g_{\mu\nu}, \psi) \rightarrow (e^f g_{\mu\nu}, \psi + f), \tag{11} \]

where \( f \) can be any function on spacetime. This kinds of transformation consist a group, namely \( G_{\text{weyl}}(\psi) \). Any operation in \( G_{\text{weyl}} \) is called a frame transformation and pair \( (M, g_{\mu\nu}, \psi) \) is called a frame where \( M \) is spacetime manifold. Especially, if we take \( f = -\psi \) and define \( \gamma_{\mu\nu} = e^{-\psi} g_{\mu\nu} \), namely effective metric, then from (9) it is clear that \( \psi \nabla_\alpha \gamma_{\mu\nu} = 0 \). So for the covariant derivative operators the metric \( \gamma_{\mu\nu} \) behave like original metric \( g_{\mu\nu} \) for derivative operator in GR. Hence, we can take similar step like GR for computation involving derivation if we use the rescaled metric \( \gamma_{\mu\nu} \) in the frame. Hence we call the frame \( (M, \gamma_{\mu\nu}, 0) \) a Riemann frame, whereas the frame with \( g_{\mu\nu} \) is called a Weyl frame. From now on, we will only use Riemann frame wherever we discuss our new theory. In this frame the field \( \psi \) appear to be minimally coupled, hence one can expect that the additional term \( \nabla_\mu \nabla_\nu \varphi^2 - g_{\mu\nu} \Box \varphi^2 \) in the EM tensor (6) does not appear anymore.

To show this, we apply Palatini formalism to the Zee model to construct Weyl geometrical version of broken-symmetric theory of gravity. First, we adopt our action to be

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \frac{1}{8} \gamma^{\alpha\beta\mu} \nabla_\alpha \psi^\mu \nabla_\beta \psi - V(\psi) \right] + S_M. \tag{12} \]
The equations of motion are
\[ \frac{M^2_e}{2} G_{\mu \nu} = w T_{\mu \nu} - w T^{(w)}_{\mu \nu}, \] (13)
\[ w \Box \psi + 4 \partial_\phi V_{eff}(\psi) = 0, \] (14)
where we define Weyl-type energy-momentum(EM) tensor for matter and \( \psi \):
\[ w T_{\mu \nu} = -\frac{2}{\sqrt{-\gamma}} \delta S_M, \] (15)
and the effective potential
\[ \partial_\phi V_{eff}(\psi) = \left[ 2V(\psi) + \partial_\phi V(\psi) \right] e^{2\psi}. \] (17)

Since \( V_{eff}(\phi = 0) \) and \( V_{eff}(\phi > 0) \) differ, the potential can invoke symmetry breaking at \( \phi = \phi_c \). After the symmetry breaking, we may expand \( \psi = -\ln \psi + \zeta \) where \( |\zeta| \ll 1 \). Before we discuss effects of scalar field \( \zeta \) in CMB anisotropy, we explain a geometric role of the symmetry breaking for deeper understanding. As we have shown before, the action (12) is invariant under the operation of the transformation group \( G_{weyl}(\psi) \). The invariant quantity corresponds to this group is of course Weyl connection \( \Gamma_{\mu \nu}^{\alpha} \). After the symmetry breaking, on the large scale, the field \( \psi \) to be fixed at \( \psi = -\ln v + \zeta \) where \( v \) and \( \zeta \) are constant. Therefore after the symmetry breaking we can see the effects of \( \psi \) only through the perturbation scale, where the group \( G_{weyl}(\zeta) \) has nontrivial elements.

Let us try to be more quantitative. We briefly explain the behavior of perturbation of some quantity, namely \( X_a \). The first order term of the derivative of \( X_a \) in GR is given by \( \delta(\nabla_b X_a) = \nabla_b (X_a + \delta X_a) - \nabla_b X_a \). In our new model, however, this is not correct since \( w \nabla_b X_a \) contain perturbative term in itself. To be explicit, by expanding \( w \nabla_b (X_a + \delta X_a) \)
\[ w \nabla_b (X_a + \delta X_a) = \partial_b X_a - w \Gamma_{ba}^c X_c + \partial_b \delta X_a - w \Gamma_{ba}^c \delta X_c \]
\[ = \partial_b X_a - \Gamma_{ba}^c X_c + \partial_b \delta X_a - \Gamma_{ba}^c \delta X_c \]
\[ - \Delta_{ba}^c X_c, \] (19)
and we can see that the first order perturbation of \( \delta (w \nabla_b X_a) = \partial_b \delta X_a - \Gamma_{ba}^c \delta X_c - \Delta_{ba}^c X_c \) differs from the one in GR. The new factor \( \Delta_{\mu \nu}^c X_c \) appears to satisfy the invariance under the group \( G_{weyl}(\zeta) \). Therefore, in the first order there are differences from GR in our model even if the contribution to the EM tensor from the field \( \psi \) in our model vanishes. In next section we investigate the more concrete feature with the covariant formulation of kinematics and dynamics of perturbations.

III. CGI FORMALISM

A. 4-velocity and kinematical quantities

Here we use CGI formalism whose equations are free from gauge choice problem. To construct the formalism, we use 1+3 decomposition. For a given coordinates \( x_a \), we define a 4-velocity in each model
\[ u_a = \frac{dx_a}{d\tau}, \] (20)
\[ w u_a = \frac{dx_a}{d\tau}, \] (21)
with a comoving time \( \tau \) which satisfies in the Zee model \( \tau = \int d\tau \sqrt{-g_{ab} u^a u^b} \) whereas in our model \( \tau_w = \int d\tau \sqrt{-\gamma_{ab} u^a u^b} \). Notice that in our model we must use \( \gamma \) and not \( g \) since any geometrical quantities should be invariant under the group \( G_{weyl}(\zeta) \). The other procedures are similar in both models. Given fundamental 4-velocity \( u_a \) satisfying \( u_a u^a = -1 \), we define orthogonal spatial derivative as
\[ D_{c} S_{a...b...c...} = h^{f}_{[c} h^{a}_{d} \cdots h^{e}_{b]} \cdots \nabla_f S^{d...e...}, \] (22)
\[ w D_{c} S_{a...b...c...} = w h^{f}_{[c} w^{a}_{d} \cdots w^{e}_{b]} \cdots \nabla_f S^{d...e...}, \] (23)
where \( h_{ab} = g_{ab} + u_a u_b \) and \( w h_{ab} = \gamma_{ab} + w u_a u_b \) are the 3-dimensional projection tensor.

The covariant derivative of \( u_a \) is expressed by
\[ \nabla_h u_a = \frac{1}{3} \Theta h_{ab} + \sigma_{ab} + \omega_{ab} - \dot{u}_a u_b, \] (24)
where we define the kinematic quantities: volume expansion \( \Theta = D^a u_a \), shear \( \sigma_{ab} = (h^{[a}_{c} h^{b]}_{d} - \frac{1}{3} h^{cde} h_{abc}) \nabla_c u_d \) (that is, the projected symmetric trace-free (PSTF) part of spatial derivative of 4-velocity), vorticity \( \omega_{ab} = D_{(b} u_{a)}, \) and acceleration \( \dot{u}_a = w \nabla_a u_a \) (the time derivative of 4-velocity) [17]. The quantities in our model just can be given with the superscript \( w \). In general, the PSTF part of any vector and tensor are given by
\[ V_{(a)} = h_{ab} V^b, \] (25)
\[ S_{(ab...z)} = (h_d h_b \cdots h_z^w - \frac{1}{3} h^{cd...w} h_{ab...z}) S_{cd...w}, \]  
\[ \dot{S}_{ab...} = u^c \nabla_c S_{ab...}. \]  

Finally, let us define some important quantities. We define Riemann tensor by
\[ R_{abcd} = \nabla_a \nabla_d u_b - \nabla_b \nabla_d u_a, \]  
and 3-Riemann tensor
\[ [D_a, D_b] u_c = \frac{3}{2} R_{abcd} u^d. \]  
The another important quantity is Weyl tensor, which is defined by
\[ C^{ab}_{\phantom{ab}cd} = R^{ab}_{\phantom{ab}cd} + \frac{1}{2} (\delta^a_b R^c_d - \delta^a_c R^b_d + \delta^b_c R^a_d - \delta^b_d R^a_c) \]  
\[ + \frac{1}{6} R (\delta^a_b \delta^c_d - \delta^a_d \delta^c_b). \]  

B. Dynamics at first order approximation

Now we perform linearized theory in covariant formalism of two models, which is constructed in Friedmann–Lemaître–Robertson–Walker (FLRW) geometry in large scale. In FLRW geometry, all the kinematical quantities are zero except for volume expansion \( \Theta = 3H \), where \( H \) is defined as Hubble parameter. In this geometry, EM tensor for fluid generally given by
\[ T_{ab} = \rho u_a u_b + 2\eta_{ab} + P_{ab} + \pi_{ab}, \]  
becomes
\[ T_{ab} = \rho u_a u_b + \pi_{ab}, \]  
and it means that it should be perfect fluid so \( \eta = 0 \) and \( \pi_{ab} = 0 \). Moreover, the spatial derivative of all the nonzero quantities must vanish at zeroth order scale.  
Here we display first order dynamical equations. At first order scale, the terms which are not at zero-th order may appear unless there is higher order terms like quadratic or cubic terms. Using definitions above and from the relation of Riemann tensor and 4-velocity (28) we obtain 1+3 decomposition of Einstein equation
\[ - R_{bc} u^b u^c = \dot{\Theta} + \frac{1}{3} \Theta^2 - D_\alpha u^\alpha, \]  
\[ R_{(\alpha)b} u^b = D^b \sigma_{ab} - \text{curl} \omega_a = - \frac{2}{3} D_\alpha \Theta, \]  
where the curl operator and vorticity vector are defined
\[ \text{curl} V_a = \eta_{abc} D^b V^c, \]  
and it means that it should be perfect fluid so \( \eta_{abc} = 0 \) and \( \pi_{abc} = 0 \). Moreover, the spatial derivative of all the nonzero quantities must vanish at zeroth order scale.  

For our model, however, in the view of effective metric the field \( \psi \) behave as like minimally coupled field.

\[ \omega_a = \frac{1}{2} \eta_{abc} w^{bc}, \]  
where \( \eta_{abc} = \eta_{abcd} u^d \) and \( \eta_{abc} = \eta_{[abc]} \) satisfies \( \eta_{0123} = -\sqrt{-g} \). The first one is Raychaudhuri equation, which is generalization of Friedmann equation and the second one is spatial projection of Einstein equation. Moreover, from the 'Electric' and 'magnetic' part of Weyl tensor which are defined by
\[ E_{ab} \equiv \eta_{abcd} u^a u^b, \]  
\[ H_{ab} = \frac{1}{2} \eta_{cd} C^{cd}_{\phantom{cd}be} u^e, \]  
one obtains
\[ E_{ab} = \frac{1}{2} R_{(ab)} - \dot{\sigma}_{(ab)} - \frac{2}{3} \Theta \sigma_{ab} + D_\alpha (\dot{u}_b), \]  
\[ H_{ab} = \text{curl} \sigma_{ab} + D_\alpha (\omega_b). \]  

For 3-curvature whose definition is given by (29), one obtains
\[ 3R = 2(G_{ab} u^a u^b + \frac{1}{3} \Theta^2). \]  

For the matters, from the conservation laws one obtains
\[ \dot{\rho} + (\rho + p) \Theta + D_a q^a = 0, \]  
\[ \dot{q}_a + 4H q_a + (\rho + p) u_a + D_a p + D^b \pi_{ab} = 0. \]  

One may decompose EM tensor for the scalar field to the form (31) and apply the equations above. At the large scale, Weyl manifold recovers its metric compatibility and all the quantities coincide with GR’s results so that \( u^a \Theta = \Theta \). However, when it comes to the first order scale there arises difference. First, in our model the perturbation of kinematical quantities differs from approaches based on metric compatibility. As explained in the last paragraph of the section II, one need to consider weyl field part when computing any kind of perturbative variables. Let us compute \( u^a D_a u^\Theta \) directly:
\[ u^a D_a u^\Theta = D_a (\Theta + \psi) \]  
\[ = D_a \Theta + \Theta D_a \psi. \]  

All the other kinematical quantities comes out to be same with GR at the first order scale. This clearly shows geometrical effects in Weyl manifold when we consider the effective metric \( \gamma = e^{-\psi} g \), the spatial growth of \( \psi \) (as time goes) makes the 4-velocity \( u^a \) to be disorted.

For the matter terms, however, the Zee model differs from GR whereas our model is as same as GR without additional contribution from the scalar field. This fact comes from different definitions of EM tensor for the scalar field. In the Zee model, the effect of nonminimal coupling appear as additional term \( \nabla_a \nabla_b \phi^2 - g_{ab} \Box \phi^2 \) in EM tensor (6) and at the first order EM tensor become
\[ T^{(\psi)}_{\mu\nu} = - \frac{1}{2} (\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi). \]
C. Scalar perturbation

For the numerical computation, we compute the full set of the perturbation evolution equations and their constraints. First, let us define some perturbation variables. For the comoving spatial gradient of the matter density and expansion we define

\[ \Delta_a = \frac{S}{\rho} D_a \rho, \]

\[ Z_a = SD_a \Theta, \]

and for the 3-curvature

\[ \eta_a = \frac{1}{2} SD_a^3 R. \]

Futhermore, we also define variables for scalar field perturbation:

\[ \phi_a = \kappa SD_a \phi, \]

\[ \psi_a = SD_a \psi. \]

(\( \kappa = v^{-1} \) and \( S \) is scale factor) Physically, they correspond to the difference of observables between one observer A and the other who is apart from A with the direction of spatial unit vector \( e^a \).

After manipulating the equations, we expand them as series with harmonic coefficients. First, we define scalar eigenfunction \( Q^{(k)} \) which satisfies generalized Helmholtz equation

\[ S^2 D_a D^a Q^{(k)} = k^2 Q^{(k)}, \]

at zeroth order. We also define its multipole term \( Q^{(k)}_{a_1 a_2 \ldots a_l} \) by

\[ Q^{(k)}_{a_1 a_2 \ldots a_l} = (\frac{S}{k})^l D_{a_1} D_{a_2} \ldots D_{a_l} Q^{(k)}. \]

Now we expand variables following the notation of \[18,19\]. For the kinematical quantities,

\[ Z_a = \sum_k \frac{k^2}{S} Z_k Q^{(k)}_a, \]

\[ \sigma_{ab} = \sum_k \frac{k}{S} \sigma_{k ab} Q^{(k)}_a, \]

\[ \eta_a = \sum_k \frac{k^3}{S^2} \eta_k Q^{(k)}_a, \]

and for the matters,

\[ \Delta_a = \sum_k k \Delta_k Q^{(k)}_a, \]

\[ \phi_a = \sum_k k \phi_k Q^{(k)}_a, \]

\[ \psi_a = \sum_k k \psi_k Q^{(k)}_a. \]

For the briefness, we only display and discuss equations which differ from GR. We will only consider adiabatic perturbation and CDM frame whose 4-velocity corresponds to CDM velocity and hence \( a_a = 0 \). Futhermore, we may set vorticity to be zero. From now on we only use conformal time given by \( Sd\tau = dt \) and conformal time derivative using prime(\( t \)) notation.

First we compute perturbations of scalar quantities in the Zee model. The evolution of expansion perturbation is given by

\[ Z'_k = -H Z_k - \frac{\kappa^2 S^2}{2k} \sum_i [1 + 3 \frac{dp^{(i)}}{d\rho^{(i)}}] \rho^{(i)} \Delta^{(i)}_a - \frac{S}{k} (3\phi'_k + 6H \phi'_k + 5 \frac{k^2}{S^2} \phi_k), \]

where \( H = da/d\tau \) is conformal Hubble factor and the index \( i \) denotes each components of matter fluids. It corresponds to \((00)\) components of Einsteins equation. \((0i)\) components corresponds to

\[ \sigma_k = Z_k + \frac{3\kappa^2 S^2}{2k^2} \sum_i \rho^{(i)} q^{(i)}_k - \frac{3S}{k} (\phi'_k - H \phi_k). \]

The 3-curvature formula (41) leads to

\[ k^2 \eta_k = -2kH \Delta_k + \kappa^2 S^2 \sum_i \rho^{(i)} \Delta^{(i)}_k - 2k^2 \phi_k. \]

The scalar field \( \phi_k \) has the following evolution equation:

\[ \phi''_k + 2H \phi'_k + (k^2 + v^2 V_0 S^2) \phi_k = \frac{\kappa^2 S^2}{4} \sum_i [1 - 3 \frac{dp^{(i)}}{d\rho^{(i)}}] \rho^{(i)} \Delta^{(i)}_a. \]

Now we compute perturbations in our model. The perturbation evolution of expansion and 3-curvature are given by

\[ Z'_k = -H Z_k - \frac{\kappa^2 S^2}{2k} \sum_i [1 + 3 \frac{dp^{(i)}}{d\rho^{(i)}}] \rho^{(i)} \Delta^{(i)}_a - \frac{3S}{k} [(H' + H^2) \psi_k + H \psi'_k], \]
\[ k^2 \eta_k = - 2k \mathcal{H}_k Z_k + \kappa^2 S^2 \sum_i \rho^{(i)} \Delta^{(i)}_k - 6k \mathcal{H} \psi_k. \] (66)

In our model the matter evolution equation also changes:
\[
(\Delta^{(i)}_k)' = -3 \mathcal{H}[\left( c_s^{(i)} \right)^2 - \rho^{(i)}_k |\Delta^{(i)}_k| - k[(1 + \rho^{(i)}_k) Z_k + q_k]]
- 3(1 + \rho^{(i)}_k) \mathcal{H} \psi_k.
\] (67)

The evolution equation for \( \psi_k \) is
\[ \psi_k'' + 2 \mathcal{H} \psi_k' + (k^2 + 8V_0 S^2) \psi_k = 0. \] (68)

### IV. NUMERICAL ANALYSIS

In this section, we present our numerical results obtained by modification of CAMB. CAMB is written by Antony Lewis and Anthony Challinor. It is in fortran 90, fast and accurate enough for the statistical analysis of many data such as Planck 2018 data, and hence it is used generally to analyze new CMB physics. To obtain numerical results, we modified the set of equations in equations.f90 file in folder fortran. Furthermore, we added new scalar field whose evolution is given by (64) and (68).

Although CAMB can solve these two equations exactly, as the potential (or the mass of the scalar field) get higher value the field oscillates more rapidly in its restrictive region (the field should be diluted fast in radiation epoch, so the evolution itself recovers GR) so to solve other coupled equations (such as matter perturbation evolution) exactly CAMB needs to split the region with extremely small time spacing to reflect the rapid changes of the field, which results the code to be much slower than the usual cases [20]. As the field mass become more heavier the theory approaches closely GR. Unfortunately, we could not avoid this numerical issue in the both model and hence we had to develop approximation scheme for the rapidly oscillating field. Our scheme is not to be quantitively accurate but to cut down the time-consuming computation of the code and to be qualitative to observe exotic features of the physics.

The approximation scheme came from the fact that the field asymptotically decrease with the factor \( 1/S \) as time goes by. Therefore, we may introduce new variable \( \varphi_k = S \phi_k \). The equation for \( \varphi_k \) shows the oscillating property more explicitly:
\[ \varphi_k'' = -(k^2 + m^2 S^2 - \frac{S''}{S}) \varphi_k, \] (69)

where \( m \) is given by in our model \( m^2 = 8V_0 \) and in the Zee model \( m^2 = V_0 v^2 \). Here we drop out the oscillation part and only extract effects of the mass on an asymptotic line of the solution of (70). Hence we decompose \( \varphi_k \) as the product of two variables \( g \) and \( h \):
\[ \varphi_k^\pm = e^{g \pm ih}, \] (70)

where \( \pm \) denotes two solution of (69). The equation for \( g \) and \( h \) are
\[ g'' = -(k^2 + m^2 S^2 - \frac{S''}{S}) - (g')^2 + (h')^2, \] (71)
\[ h' = e^{-2g}. \] (72)

Further, We assume that \( |g''| \gg (g')^2 \) (\( g \) itself changes slowly but \( g'' \) has much more bigger value when the field mass is heavier) and neglect the oscillation term \( h \) and obtain
\[ g'' \approx -(k^2 + m^2 S^2 - \frac{S''}{S}), \] (73)

which makes possible us to approximate the field as \( \phi_k \approx e^g/S \). For the equation (65), we may neglect the right-hand side of the equation (64) since the difference between \( \psi_k \) and \( \phi_k \) is much less than the difference between our approximation and the original solution.

For the initial condition, with the analogy with gravitational waves, we can set \( \psi_k = 1 \) and \( \psi_k' = 0 \) at the super-horizon scale. We also set \( \phi_k = 1 \) and \( \phi_k' = 0 \). Note that the initial values of \( g \) are
\[ g = \log S, \] (74)
\[ g' = \mathcal{H}. \] (75)

Next we plot the CMB temperature anisotropy multipole in our model and the Zee model. We take an unit of the mass to be Megaparsecs, since it is the default unit of CAMB. For convenience, we adopt new variable:
\[ \zeta = \log(1 + \frac{1}{m}). \] (76)

For cosmological parameters we use Planck 2018 results [21]: Hubble factor \( h = H_0/100 = 0.6732117 \), baryon
density $\Omega_b h^2 = 0.0223828$, cold dark matter density $\Omega_c h^2 = 0.1201075$, neutrino density $\Omega_{\nu} h^2 = 0.6451439 \times 10^{-3}$, scalar power spectra amplitude $A_s = 2.100549 \times 10^{-9}$, and scalar spectral index $n_s = 0.9660499$, and we only consider flat universe. As the figure 1 and 2 shows, the field mass mainly affects to high-$l$ part of the multipole. The first peak of spectra and the damping with oscillation tend to get lower value as field mass have lower value. Basically, as the mass gets bigger the field become more oscillating and hence dissipate more fast, this is reasonable results. In here the effect of the model on CMB spectra dominates small scale. We can understand it through Weyl potential, which is given by

$$E_{ab} = \sum_k \frac{k^2}{S^2} \tilde{\Phi} E(k) G_{ab}^k.$$  

We plot Weyl potential in figure 3 to show the effects of new scalar field on the potential. As one can see, the effect is almost restricted to large $k$, which means small wavelength. As the momentum is bigger the field tend to decrease more quickly, this result is physically acceptable. The connection between Weyl potential and damped oscillation in CMB spectra is shown in [17]:

$$\Delta'' + \frac{HR}{1 + R} \Delta' + \frac{k^2}{3(1 + R)} \Delta = -8(\Phi''_E + \frac{HR}{1 + R} \Phi'_E),$$  

where $R = 3\rho_b / 4\rho_\gamma$ and $\Delta = S D^a \Delta_a$. The right-hand side of (77) expresses an oscillator equation. Since the left-hand side which corresponds to external force term reflects rapid change of Weyl potential at small scale, which is increased by smaller field mass, disturbs oscillation, we conclude that the small mass of the field slows down the oscillation more quickly. However, as the acoustic peaks subside the effects of additional field also get smaller.

Another noticeable result is the extreme shape changes in our model. As figure 1 shows (red line), when $\zeta$ is much more lighter than $10^{-9}$ the first peak of the spectra get much more diminished and oscillation at high $l$ is increased. This is also same effect due to the deviation of Hubble parameter, which comes from the geometrical property of the scalar field. Hence the scalar field appears in all the density perturbation equation and its effects should be more strong than the Zee model. In the Zee model, however, we could not find numerical noticeable differences when $\zeta > 1$. Since in the Zee model the only additional effects on CMB power spectra comes from the additional EM tensor, the effects of scalar field is much more restrictive.

V. CONCLUSION

In this paper, we suggested a new kind of primordial symmetry breaking which has geometrical feature, achieved by application of Palatini formalism to non-minimally coupled gravity. Furthermore, we computed perturbation to verify the models with observation. Our method is based on CGI formalism, and clearly it has no gauge ambiguity to be suitable for our approach.

Although they are under the same symmetry breaking mechanism, two models-the Zee model and our model have different sensitivity to the potential scale (or the field mass). In the Zee model the additional term appearing in the scalar field part of the EM tensor due to non-minimal coupling is only thing which can affect perturbation. However, in our model based on Palatini formalism in the effective metric (or Riemann frame) the field should be regarded as minimally coupled field. The adoption of Riemann frame is inevitable because to apply CGI formalism we had to choose proper comoving time that should be invariant under the transformation (18). Nevertheless, on the perturbative scale we could find deviation term in the expansion factor so the the perturbation evolves in the different way.
To plot the computed results we used the fortran code CAMB. However, for the numerical reason to speed up the code we had to apply specific approximation scheme for the large field mass. Under this approach, we concluded that our model is more sensitive to the field mass than the Zee model. The additional scalar field affects Weyl potential at short wavelength region and it makes peak and the oscillation part on the small scale of CMB spectra to be lower than GR. The geometrical impact on Hubble parameter in our model is much more greater than additional terms in energy-momentum tensor given by the Zee model.

Our studies have some noticable meanings. Although Zee said that there is no observational test that can verify the theory in his article, our approach based on CGI formalism opens way to probe primordial symmetry breaking through cosmic observation. Both Zee’s approach and our approach based on Palatini formalism does not concern other forces nor unify gravity and other forces, but it might give some clue for the other way of test symmetry breaking phenomena in cosmology.

However, our results are very restrictive to some simple cases. We only considered simplest scalar-tensor theory and does not considered torsion. Weyl geometry, which motivated our studies, is actually a simple case without torsion of Lyra’s geometry. Although it is hard to apply CGI formalism to theories with torsion (since one may cannot choose proper foliation in this case and hence cannot define 4-velocity and kinematical quantities), many high-energy gravity theories in these days concerns torsion so we need to develop appropriate mathematical formulations for them. Of course, comparing our theory with observational data such as Planck 2018 data or more detailed study on differences between the Zee model and our model can be also one of the most important further studies. The more careful and concentrated researches are needed.

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