Multicomponent force transducer calibration procedure using tilted plates

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Abstract. The calibration of a multicomponent force transducers (MFTs) represents a challenge in the meganewton range. In fact, the generation of transversal forces and moments is complex since a force standard machine (FSM) is only able to apply an uniaxial force. Furthermore since MFTs are composed of multi-transducers, each one dedicated to a particular component, correlations between force and moment components are possible. Therefore, a calibration system that could simultaneously generate all force/moment components and could be suitable in every FSM is needed. For this purpose, a couple of tilted plates was designed. Calibration measurements were performed on a 2 MN MFT at INRiM, LNE and PTB. Exploitation matrixes and performance indicators showed good results, unless small but not negligible correlations between MFT outputs. In particular some spurious values due to the uncertainty in the vertical force application point influenced the moment components.

1. Introduction
Calibration of MFTs strictly depends on the range of the applied transversal forces. From 0 N to an upper limit of 1 kN, it is possible to use two graduate rotating tables combined with a mass able to generate an accurate vertical force \cite{1}. To increase this range to the meganewton range, side loads are usually generated with systems based on pulleys, while moments are obtained with torque measurement structures \cite{2}. At the Physikalisch-Technische Bundesanstalt (PTB), a FSM able to apply and read all force and moment components was developed \cite{3}. In particular a different system for the application of side loads and torque moment was realised, each one with its advantages and disadvantages. Most of MFTs are composed of multiple force/moment outputs, each one dedicated to a single component. Nevertheless every output generally does not only depend on the relevant measured component. Therefore, a MFT calibration system should combine force and moment components in order to verify the exact correlation between them and, at the same time, should fit every type of FSM. In this work, a calibration method using tilted plates, which guarantees the simultaneous generation of force and moment components, is presented. Measurements performed at INRiM, PTB and LNE are compared in order to evaluate the correlation between the MFT outputs.

2. Calibration system
Before designing the calibration system, all the MFTs and FSMs located at INRiM, LNE and PTB were surveyed and screened. At the end of this work, a 2 MN MFT (with diameter of 170 mm and height of 300 mm) developed at INRiM [4] and able to simultaneously measure force and moment components \((F_x, F_y, F_z, M_x, M_y, M_z)\), was chosen and a system of tilted plates was designed. This system does not require any change on the structure of the FSM, unlike [2], is able to combine the desired force and moment vector components, unlike [1], and can be performed in any calibration laboratory, unlike [3]. The MFT has 6 independent output channels \((U_j)\), each dedicated to a single force and moment component. In our analysis, the torque moment \(M_z\) was excluded. Each channel has one or more strain gauges disposed anticlockwise along the MFT body. The MFT’s reference system inside the tilted plates is represented in figure 1.

![Reference system of the MFT inside a couple of tilted plates.](image)

Measurements were performed on the deadweight FSM with loads of 500 kN and 1000 kN at INRiM, on a deadweight FSM with a load of 500 kN at LNE, and on a hydraulic amplification FSM with loads of 500 kN, 1000 kN and 2000 kN at PTB. The main elements of the entire calibration system were the tilted plates. For this project three pairs of tilted plates were realised, with a tilt angle \(\alpha\) of 0°, 1° and 2°. The system allowed to tilt the MFT inside the FSMs and provided an orthogonal surface to apply the axial force. Furthermore, by rotating the MFT by an angle \(\omega\) inside the tilted plates, the vertical axial force \(F\) of the FSM was decomposed in the three components according to equation (1). The application of a moment was given by the eccentricity \(\varepsilon_x\) and \(\varepsilon_y\), along \(x\) and \(y\) axis respectively, with respect to the central axis of the FSM, as shown in equation (1).

\[
\begin{align*}
F_x &= F \cdot \sin \alpha \cdot \sin \omega \\
F_y &= F \cdot \sin \alpha \cdot \cos \omega \\
F_z &= F \cdot \cos \alpha \\
M_x &= F \cos \alpha \cdot (\varepsilon_x \cdot \sin \omega + \varepsilon_y \cdot \cos \omega) \\
M_y &= F \cos \alpha \cdot (\varepsilon_x \cdot \cos \omega + \varepsilon_y \cdot \sin \omega)
\end{align*}
\]

### 3. Calibration procedure

The calibration procedure of the MFT can be summarized as follows: the MFT was aligned and centred with the single vertical force load (reference point 0); the MFT was then positioned with an eccentricity of 3 mm along the \(y\)-axis (point A); the MFT was displaced with a further eccentricity of 3 mm still along the \(y\)-axis (point B); the MFT was displaced with a further eccentricity of 3 mm along the \(x\)-axis (point C); the eccentricity of the MFT along the \(y\)-axis was reduced of 3 mm (point D). For each step of the above procedure, the MFT was rotated by an angle \(\omega\) of 45° in order to complete a full turn. These steps were repeated for each pair of planes. From equations (1-2) it is possible to obtain all
the desired force and moment components, by combining the tilt angle $\alpha$, the angle of rotation $\omega$ and the eccentricity $\varepsilon$. Every combination of these three parameters leads to the creation of a different set of forces and moments, and this set of values can be represented in the experimental planes. The experimental planes are fundamental because they immediately show if the chosen points are sufficient to cover a large number of measurement conditions between two different components or if redundant points that can be avoided are present. Two examples for transverse forces $F_x$ and $F_y$, which are eccentricity-independent, and for moment components $M_x$ and $M_y$ are depicted in figure 2.

Figure 2. Experimental planes of $F_x$ vs $F_y$ (left) and $M_x$ vs $M_y$ (right).

4. Data analysis
In the ideal MFT, every output is only dependent on the relevant force or moment component. This condition can be easily analysed with the coefficient matrix. In fact, each force and moment component ($F_j$) can be seen as a linear combination of the MFT outputs, as shown in equation (2):

$$F_j = a_{1j}U_1 + a_{2j}U_2 + \ldots + a_{5j}U_5 = \sum_{i=1}^{5} a_{ij}U_i$$

(2)

Coefficients $a_{ij}$ are necessary to calculate the force/moment components $F_j$, and these coefficients can be obtained with $n$ linearly independent sets of values, i.e. the set of measurements coming from the experimental planes. Therefore, equation (2) can be solved as:

$$F = U \cdot A$$

$$U^T F = U^T U \cdot A$$

$$[U^T U]^{-1} U^T F = A$$

(3)

Matrix $A$ is also called exploitation matrix. For an ideal MFT, the exploitation matrix is a diagonal matrix, which means no correlation between force and moment components. The exploitation matrices obtained from the measurements performed at INRiM, LNE and PTB are shown in table 1.

| Table 1. The exploitation matrices from the measurements performed at INRiM, LNE and PTB |
|---------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $F_x$  | $F_y$  | $F_z$  | $M_x$  | $M_y$  | $F_x$  | $F_y$  | $F_z$  | $M_x$  | $M_y$  | $F_x$  | $F_y$  | $F_z$  | $M_x$  | $M_y$  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| INRiM |      |      |       |       | LNE   |       |       |       |       | PTB    |       |       |       |       |
| 497.1 | -10.9 | -6.4 | 63.0   | -8.7  | 307.2 | 55.4   | 20.7  | 35.1  | 10.1  | 275.2 | -137.6 | -10.2 | 18.1  | 3.7   |
| 2.5   | 491.0 | 3.5  | -13.2  | 55.0  | 54.5  | 474.5  | -3.2  | -2.5  | 54.9  | 115.9 | 406.1  | -16.6 | -27.4 | 38.8  |
| 5.6   | -8.0  | 1246.2 | 1.0   | -1.3  | 2.9   | -7.0   | 1245.6 | 0.6   | -1.0  | 1.4   | -9.7   | 1247.9 | 1.0   | -0.8  |
| 1.4   | 3.4   | 0.3  | 8.0   | -1.5  | 2.5   | 3.1    | -0.3  | 4.6   | 0.2   | -2.6  | 8.5    | -7.3  | 6.5   | -4.5  |
| 6.7   | 8.8   | 0.7  | -5.2  | 11.7  | 10.3  | 3.3    | -1.8  | -1.4  | 6.0   | 2.4   | 17.6   | 0.7   | -4.6  | 13.9  |

It is evident that the largest terms are located on the diagonals, however out-of-diagonal terms are not equal to zero. In order to evaluate the effects and the consistency of the force/moment components
interaction, performance indicators are introduced. The first parameter $I_1$ is an indicator of the "average deviation" from the ideal condition, which is given by equation (4):

$$I_1 = \frac{\left(\sum |a_{ij}|^n\right)}{\left(1+\sum |a_{ij}|^n\right)} \quad \text{with } i \neq j$$

Where $n$ is the normalised exploitation matrix order and $|a_{ij}|^n = |a_{ij}/a_{ii}|$ is the absolute value of the $ij$-th coefficient of the normalised exploitation matrix $A^*$. Nevertheless, the only parameter $I_1$ is not enough since it is not able to discriminate whether the total variation is due to a single value or if it is due to a general variation of the coefficients. For this reason, it is necessary to analyze the parameter $I_2$ which represents the “maximum deviation” of the out-of-diagonal coefficients of the matrix $A^*$:

$$I_2 = \max_{i \neq j} |a_{ij}|^n$$

A MFT has a low sensitivity to force/moment components interaction if such indicators are nearby equal to zero. Obtained experimental performance indicators are reported in table 2.

Table 2. Performance indicators in the different laboratories

| Institute | $I_1$ | $I_2$ |
|-----------|-------|-------|
| INRiM     | 0.37  | 0.75  |
| LNE       | 0.49  | 1.73  |
| PTB       | 0.56  | 1.30  |

5. Conclusions
A MFT calibration procedure using tilted plates was tested on a 2 MN MFT at INRiM, LNE and PTB. Experimental planes chosen for the calibration procedure covered all possible combinations of force and moment components. Exploitation matrices showed a good diagonalization for the force components, whereas moment components showed some spurious values, due to the uncertainty on the application point of the vertical force, that significantly influence the moment components. Differences between laboratories are due to the different FSMs used.

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7. References
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