Supporting Information for “The Fractal Nature of Clouds in Global Storm-Resolving Models”

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Introduction

In this Supporting Information we provide further details on the methodology used in our study. Text S1 and Figure S1 demonstrates the ability of the Area-Perimeter relationship to measure monofractal dimension by measuring the dimension of the Koch snowflake. We further use this toy example to assess the object size below which the method breaks down. Figure S2 shows a sample binary cloud field obtained by introducing a threshold in outgoing longwave radiation. Table S1 provides details of the DYAMOND simulations selected for this analysis.
Text S1. The Area-Perimeter fractal dimension

To demonstrate the utility of the Area-Perimeter method for estimating the fractal dimension, we use the approach to estimate the fractal dimension of the Koch snowflake. Copies of the Koch snowflake were produced at sizes ranging from two to 50 pixels across. All snowflakes had a depth of eight iterations. Figure S1 shows the area and perimeter of the resultant objects. Taking only the largest objects (black points), we measure the fractal dimension to be 1.2614, close to the analytic solution of ln 4 / ln 3 = 1.2619. The discrepancy is due to the finite depth of the snowflakes analysed, causing a systematic tendency towards a lower fractal dimension.

At a given resolution, the perimeter measurement fails for small objects: the estimated perimeter of the smallest Koch snowflakes in Figure S1 is systematically lower than expected given the known scaling law. We observed the deviation to become significant for objects smaller than 24 pixels. When applying the Area-Perimeter relationship to binary cloud fields, we therefore discard all cloud objects with area smaller than 24 pixels.
**Figure S1.** The area and perimeter of simulated Koch snowflakes. A linear fit to the largest objects (black data points) gives a measurement of 1.2614 for the fractal dimension.
**Figure S2.** (a) Outgoing longwave radiation (W m$^{-2}$) from the ICON 2.5km simulation at 04:00 UTC, 4th August 2016. (b) Binary cloud field obtained by applying a threshold of -157.8 W m$^{-2}$, equivalent to a cloud top temperature of 230 K. Before computing the fractal dimension, this binary field is cleaned by removing small objects, filling any holes, and removing clouds which touch the edge of the image.
| Name  | $\sqrt{A_{max}}$ (km) | $N_{lev}$ | $H_{top}$ (km) | CP | BL  |
|-------|---------------------|---------|---------------|----|-----|
| FV3   | 3.3                 | 79      | 39            | S  | K   |
| GEOS  | 3.3                 | 132     | 80            | D  | K   |
| ICON  | 2.5 or 5.0          | 90      | 75            | N  | T   |
| IFS   | 4.8                 | 137     | 80            | S  | K   |
| IFS   | 9.0                 | 137     | 80            | F  | K   |
| MPAS  | 3.8 or 7.5          | 75      | 40            | D  | T   |
| MPAS* | 3.8 or 7.5          | 75      | 40            | F  | T   |
| NICAM | 3.5 or 7.0          | 78      | 50            | N  | K   |
| SAM   | 4.3                 | 74      | 37            | N  | S   |
| UM    | 7.8                 | 85      | 85            | S  | K   |

Tabulated are the linear dimension corresponding to the largest tile, $\sqrt{A_{max}}$, the number of vertical levels, $N_{lev}$, and the height of the model top, $H_{top}$. CP denotes the cumulus parametrisation used: none (N); shallow (S); deep (D), where the scheme has been re-tuned or adjusted to account for the scales of motion being parametrised; or full (F) which is the un-tuned version of the scheme also used at coarser resolutions. BL denotes the boundary layer parametrisation used: a TKE-type scheme involving an additional prognostic equation (T), a diagnostic eddy diffusivity scheme (K), or a three-dimensional Smagorinsky-type scheme (S).