Recent development of quasiclassical operator method

To cite this article: V N Baier and V M Katkov 2009 J. Phys.: Conf. Ser. 198 012003

View the article online for updates and enhancements.

Related content

- Quasiclassical Describing of the Bloch Oscillation in a Superlattice by Fokker-Planck Equation
  Tian Qiang and Ma Benkun

- Quasiclassical Boundary Conditions for Superfluid Fermi Systems
  Jun'ichiro Hara and Katsuhiko Nagai

- YURI ALEKSANDROVICH SURINOV (on his sixtieth birthday)
  K Ya Kondrat'ev, V I Smirnov, L N Sretenskii et al.
Recent development of quasiclassical operator method

V. N. Baier and V. M. Katkov
Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia
E-mail: V.N.Baier@inp.nsk.su

Abstract. The basis of the quasiclassical operator (QO) method is described. Application of the method for series of problems are discussed. These are processes in the superposition of plane wave and constant field, radiation in linear colliders, radiation in inhomogeneous fields, the new approach to the pair creation by a photon in a magnetic field, the theory of radiation in oriented crystals, radiation spectra taking into account the energy loss.

1. Introduction. Basis of QO method

The method was developed by authors in series of paper (BKp67), (BK67), (BK68), (BK69). The first paper was devoted to study of the radiative polarization of electrons moving in a magnetic field. In following years the method was applied for study of many problems including photon emission and pair creation in media, in oriented crystals, in linear colliders, in inhomogeneous external fields, in very strong fields interesting for astrophysics, radiation in periodic structures.

List of the books and articles is given at the end of the paper.

There are two types of quantum effects at the radiation of high-energy particle in an external field (see Sec.2 in (BKS98)). The first one is associated with the quantization of particle motion in the field. For example, the commutator of the velocity components of relativistic particles in the magnetic field $H$ (where the energy levels is $\varepsilon = \sqrt{m^2 + 2eH\hbar n} \gg m$) is

$$[v_i, v_k] = \frac{ie\hbar}{\varepsilon^2} \varepsilon_{ikj} H_j,$$

and the uncertainty relation for the velocity components reads

$$\Delta v_i \Delta v_k \sim \frac{\hbar H}{\varepsilon^2} = \frac{H}{H_0 \gamma^2} = \frac{h\omega_0}{\varepsilon} \sim \frac{1}{2\hbar}, \quad H_0 = \frac{m^2}{e\hbar} = \left( \frac{m^2 c^3}{\hbar e} \right) = 4.41 \cdot 10^{13}\text{Oe}, \quad (2)$$

where we use units where $c = 1$, $\omega_0 = eH/\varepsilon$ is the Larmor frequency, $\gamma = \varepsilon/m$, so that with the energy rise the motion becomes increasingly classical.

The second type of quantum effects is associated with the recoil of particle at photon emission and is of the order $\hbar \omega/\varepsilon$. Already in the classical limit ($\hbar \omega \ll \varepsilon$) this type is principal, since $\omega \sim \omega_0 \gamma^3$, and grows linearly with the energy rise.

The first order matrix element of the photon emission by a charged particle in the external field may be represented in the form

$$U_{fi} = \frac{ie}{2\pi\sqrt{\hbar\omega}} \int dt \int d^3r F_{fs'}^+(r) \exp(i\varepsilon f t/\hbar)(e^s J) \exp[i(\omega t - kr)] \exp(-i\varepsilon t/\hbar) F_{is}(r), \quad (3)$$

$F$ is
where $F_{is}(r)$ is the solution of the wave equation in the given field with the energy $\varepsilon_i$ and in the spin state $s$, $e_\mu$ is the photon polarization vector, $k^\mu(\omega, k)$ is the photon 4-momentum, $J^\mu$ is the current vector.

For the states with $n \gg 1$ the following approximation may be made (see Eqs.(2.3)-(2.5) in (BKS98))

$$\exp(-i\varepsilon, t/\hbar)F_{is}(r) = \Psi_s(p)\exp(-i\mathcal{H}t/\hbar)|i>, \quad P^\mu = i\hbar \partial^\mu - eA^\mu;$$

where $\Psi_s(p)$ is the operator form of the particle wave function in the spin state $s$ in the given field. This form may be obtained from the free wave function via substitution of the variables by the operators: $p \rightarrow P$, $\varepsilon \rightarrow \mathcal{H} = \sqrt{P^2 + m^2}$. In the coordinate representation $|i>$ is the solution of the Klein-Gordon equation in the given field.

Substituting Eq.(4) into Eq.(3) and taking into account that the Schrödinger operators, standing between the exponential factors $\exp(\pm i\mathcal{H}t/\hbar)$, convert into the time-dependent Heisenberg operators, we obtain the following formula for the matrix element Eq.(3) (see Eqs.(2.6)-(2.11) in (BKS98)):

$$U_{fi} = |f><M|i|,$$

where

$$M = \frac{ie}{2\pi\sqrt{\hbar\omega}} \int dt \Psi_s^+(p)\{(e^*J), \exp[i(\omega t - kr(t))]|\Psi_s(p);$$

(5)

(6) here $p(t), J^\mu(t), r(t)$ are the Heisenberg operators of the particle momentum, current and coordinates respectively, the brackets {,} denote the symmetrized product of operators. Note, that $\Psi_s(p)\exp(-i\mathcal{H}t/\hbar)|i>$ is the operator solution of the wave equation for the arbitrary form of the initial state $|i>$.

We are interested in the transition probability with the photon emission summed over final particle states. Using the condition of the completeness of the states $\sum_f |f> = 1$ we obtain for the radiation probability

$$d\omega = <i|M+M|i> = d^3k.$$  

(7)

(8)

(9)

The next step of calculation is series of operator transformations ("disentanglement") in Eqs.(7), (6). As a result one obtains (see Eqs.(2.12)-(2.28) in (BKS98))

$$M = \frac{e}{2\pi\sqrt{\hbar\omega}} \int_{-\infty}^{\infty} R(t) \exp \left[ \frac{kx(t)}{\varepsilon - \hbar\omega} \right] dt = \frac{e}{2\pi\sqrt{\hbar\omega}} \int_{-\infty}^{\infty} R(t) \exp \left[ i \int_{0}^{t} \frac{kp(t')}{\varepsilon - \hbar\omega} dt' \right] dt,$$

(8)

here $R(t) = R(p(t))$, $R(p)$ is the matrix element for the free particles depending on the particle spin, $kp = \omega H - kp$, $|i>$ is the state vector of the initial particle at the time $t = 0$, and $p(t)$ is the operator of momentum in the Heisenberg picture.

The very important result of the method is that the recoil at radiation is incorporated into the theory in the universal form for any external field. There are two essentially different cases in application of the quasiclassical operator (QO) method. In the first case moving and scattering in a potential can be considered in classical terms: phase shifts are large, there is a correspondence between the impact parameter and the momentum transfer. Therefore it is possible to use the version of the QO method where one can substitute classical variables instead of operators in the expression Eq.(7) for the probability of process (see (BKp67),(BK68), (BKF72), (BLP82)). In the second case, the process of scattering can’t be described in classical terms. However, for the processes where large angular momentum contributes one can use the quasiclassical approximation for description of scattering including the situation where the phase shifts are small. For example, this formulation of the method must be applied for consideration of radiation from ultrarelativistic particles at scattering on atoms in a media.
1.1. Macroscopic case (BK67), (BK68)

For the particle with spin 1/2 the function $R(t)$ in Eq.(8) written in the two-component form is (see Eqs.(2.37)-(2.47) and Eqs.(4.1)-(4.4) in (BKS98), below we employ units such that $\hbar = 1$

$$R(t) = \varphi_s^\dagger (A + i\sigma B)\varphi_s, \quad A = \frac{1}{2} \left( 1 + \frac{\varepsilon}{\varepsilon'} \right) e v, \quad B = \frac{\omega}{2\varepsilon'} e \times b, \quad b = n - v + \frac{n}{\gamma}, \quad (9)$$

where $v = v(t)$ is the particle velocity, $\varepsilon' = \varepsilon - \omega, \gamma = \varepsilon/m$. Substituting Eq.(8) into Eq.(7) we get $(t = (t_1 + t_2)/2, \tau = t_2 - t_1)$

$$dW \equiv \frac{dw_n}{dt} = \frac{\alpha}{(2\pi)^2} \frac{d^3k}{\omega} \int R^* \left( t + \frac{\tau}{2} \right) R \left( t - \frac{\tau}{2} \right) \times \exp \left\{ -i \frac{\varepsilon}{\varepsilon'} \left[ k x \left( t + \frac{\tau}{2} \right) - k x \left( t - \frac{\tau}{2} \right) \right] \right\} d\tau. \quad (10)$$

In the magnetic field, as in the classical theory, the radiation takes place from a small part of the trajectory on which the particle turns on the angle $\sim 1/\gamma$. This means that one can expand the integrand in Eq.(10) on $\tau$ powers. For the spectral distribution of the radiation intensity, for example, one obtains (see Eq.(4.26) in (BKS98))

$$\frac{dI}{du} = \frac{\alpha m^2}{\pi \sqrt{3}} \frac{u}{(1 + u)} \left[ (1 + (1 + u)^2)K_{2/3}(\lambda) - (1 + u) \int_0^\infty K_{1/3}(z)dz \right], \quad (11)$$

where $K_{\nu}(z)$ is the MacDonald function,

$$u = \frac{\omega}{\varepsilon'}, \quad \lambda = \frac{2u}{3\chi}, \quad \chi = \frac{H_0}{H_0 m}, \quad (12)$$

here $\chi$ is the fundamental parameter. In the limit $\chi \ll 1$ one has the classical theory.

1.2. Microscopic case (BK69), Secs.16-18 (BKF73)

We consider the applicability of the QO method to the problem of radiation from a ultrarelativistic particle at potential scattering. In this process the emitted photon and the final electron are moving at small angles to the initial electron momentum and the large angular momenta $l \gg 1$ contribute. In this situation the quasiclassical scattering theory is applicable.

At very high energy of particle in a media, we can consider the case of complete screening, so that

$$a_s \ll \frac{1}{q_{\min}} = \frac{2\varepsilon(\varepsilon - \omega)}{\omega m^2} \equiv l_f, \quad (13)$$

where $a_s$ is the screening radius ($a_s \simeq 11Z^{-1/3} \lambda_c, \lambda_c = 1/m$), $Z$ is the charge of a nucleus, $q_{\min}$ is the minimal momentum transfer which is longitudinal (with respect to the momentum of the initial electron $p$), $l_f$ is the radiation formation length for the small angle scattering on an isolated atom. Note that in the frame of the QO method the radiation problem is solved for the case of arbitrary screening , see (BKF73). The impact parameters $p$, contributing into the scattering cross section, are small comparing with the formation length ($p \leq a_s \ll l_f$) in the screened Coulomb potential. This means that the scattering of ultrarelativistic particles (the virtual electron is close to the mass shell) takes place independently from the radiation process (see, (BK69), (BKF73), (BLP82)). Thus, we can present the cross section of radiation as a product of the probability of photon emission with the momentum $k$ at given momentum transfer $q_\perp (q_\perp p = 0)$, and the cross section of particle scattering $d\sigma(q_\perp)$ with the same momentum transfer $q_\perp$

$$d\sigma_\gamma = W_\gamma(q_\perp, k)d^4kd\sigma(q_\perp), \quad (14)$$
where the probability of photon emission is given in Eq.(7).

We show below that in the frame of the QO method the probability of radiation \( W_\gamma(q_\perp, k) \) is given by the trajectory of particle in "the form of an angle" in the momentum space

\[
p(t) = \vartheta(-t)p + \vartheta(t)(p + q_\perp),
\]

while the cross section \( d\sigma(q_\perp) \) should be taken in the eikonal form.

If the formation time of radiation is much longer than the characteristic time of scattering, one can present the dependence of the operator \( p_\perp(t) \) on time in Eq.(8) as

\[
p(t) = \vartheta(-t)p(-\infty) + \vartheta(t)p(\infty),
\]

\[
p_\perp(-\infty) \simeq p_\perp + \int_{-\infty}^{\infty} \nabla_\varrho V(\varrho, \varrho')dz', \quad p_\perp(\infty) \simeq p_\perp - \int_{\infty}^{\infty} \nabla_\varrho V(\varrho, \varrho')dz'.
\]

It should be pointed out that in the case when the scattering process is of nonclassical character the operators \( p(-\infty) \) and \( p(\infty) \) are noncommutative among themselves and, generally speaking, one can’t neglect their commutator. But these approximate expressions have the classical form.

Substituting the "trajectory" (16) in Eq.(8) we obtain

\[
M = \frac{ie(\varepsilon - \omega)}{2\pi\sqrt{\omega}} \left[ \frac{R(p(\infty))}{kp(\infty)} - \frac{R(p(-\infty))}{kp(-\infty)} \right].
\]

For the derivation of the differential cross section of bremsstrahlung in Eqs.(14), (7) it is necessary to insert the projection operator \( |f> < i| \) between the operators \( M^+ \) and \( M \), and to take \( |i> \) and \( |f> \) states so that the initial state is the eigenvector of the operator \( p(-\infty) \) and the final state is the eigenvector of the operator \( p(\infty) \):

\[
p(-\infty)|i> = p_i|i>, \quad |i> = \exp \left[ -i \int_{-\infty}^{\infty} V(\varrho, \varrho')dz' \right] |p_i>, \quad p(\infty)|f> = p_f|f>, \quad |f> = \exp \left[ i \int_{z}^{\infty} V(\varrho, \varrho')dz' \right] |p_f>.
\]

Using (18) we deduce for the matrix element of the operator \( M \) Eq.(17)

\[
M_{fi} = \frac{ie(\varepsilon - \omega)}{2\pi\sqrt{\omega}} \left[ \frac{R(p_f)}{kp_f} - \frac{R(p_i)}{kp_i} \right] < f|i>,
\]

\[
< f|i> = \int d^2\varrho \exp \left[ iq_\perp \varrho + i\chi(\varrho) \right] 2\pi\delta(p_f - p_i),
\]

where

\[
q_\perp = p_{i\perp} - p_{f\perp}, \quad \chi(\varrho) = -\int_{-\infty}^{\infty} V(\varrho, z)dz.
\]

In centrally symmetric potential we have

\[
\chi(\varrho) = -\int_{-\infty}^{\infty} V(\sqrt{\varrho^2 + z^2})dz.
\]

We introduce now the notations \( p_i = p, \quad p_f = p' + k \) where \( p' \) is the momentum of electron after photon emission. Then, neglecting the terms of the order of \( q_\parallel \) in the argument of the \( \delta \)-function in Eq.(19), we have in the region \( q_\perp \gg q_\parallel \), which contributes for the potential considered,

\[
\delta(p_f - p_i) \simeq \delta(\varepsilon' + \omega - \varepsilon), \quad \varepsilon' = \sqrt{p'^2 + m^2}.
\]
Using this relation one can express the propagator $kp_f$ through the propagator $kp'$ (up to terms of the order $1/\gamma^2$)

$$kp_f = \omega \sqrt{(p' + k')^2 + m^2} - k(p' + k)$$

$$= \omega \sqrt{\varepsilon^2 - 2kp' + kp' - \omega \varepsilon \approx \omega \varepsilon \left(1 - \frac{kp'}{\varepsilon^2}\right) + kp' - \omega \varepsilon = \frac{\varepsilon'}{\varepsilon} kp'.$$

Our final result for the differential probability of radiation in Eq.(14) is therefore

$$W_\gamma(q, k) = \frac{\alpha}{(2\pi)^2 \omega} \left[\frac{\varepsilon R(p' + k)}{kp'} - \frac{\varepsilon'R(p)}{kp}\right]^2. \quad (23)$$

Using the Moliere approximation of the atomic potential for the phase $\chi(\varrho)$ in Eq.(21) we have for the spectrum of bremsstrahlung in the case of complete screening

$$\frac{d\sigma}{d\omega} = \frac{4\pi^2 \alpha^3 \varepsilon}{m^2 \varepsilon \omega} \left[\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - \frac{2}{3}\right) \left(\ln(183Z^{-1/3}) - f(Z\alpha)\right) + \frac{1}{9}\right], \quad (25)$$

where

$$f(\xi) = \xi^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + \xi^2)}, \quad (26)$$

here the function $f(\xi)$ is the Coulomb correction. In the Coulomb field the term $1/9$ in the square brackets should be omitted and $\ln(183Z^{-1/3})$ should be substituted by $\ln(1/\delta) - 1/2$, where $\delta = q_{\text{min}}/m = \omega m/2\varepsilon\varepsilon'$.

2. Processes in plane wave and constant field (BKS91)

Substituting Eqs.(8),(9) into Eq.(7) and performing the integration over the photon emission angle and the summation over the polarization of final particles, we obtain the spectral distribution of radiation in the convenient form, where all cancelations in the leading terms have already been carried out (see also Eqs.(2.37)-(2.47) in (BKS98)):

$$\frac{d\omega}{d\omega} = \frac{i\alpha}{8\pi^2 \gamma^2} \int \int \frac{dt d\tau}{\tau - i0} \left[4 + \beta(\Delta_1 - \Delta_2)^2\right] \exp\left\{-\frac{i\tau}{\lambda_\omega} \left(1 + \int_{t_1}^{t_2} \Delta^2(t')dt'\right)\right\},$$

$$\beta = \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right), \quad \Delta_1 = \Delta(t_1), \quad \Delta = \Delta(t) = \frac{1}{m}(p(t) - \pi),$$

$$\pi = \frac{1}{\tau} \int_{t_1}^{t_2} p(t)dt, \quad \lambda_\omega = \frac{2\varepsilon\varepsilon'}{m^2 \omega}, \quad (27)$$

where $p = \varepsilon v$ is the momentum of the particle. Obviously, the quantity $\Delta(t)$ is unaffected by the substitution $p(t) \rightarrow p(t) + p_0$, where $p_0$ is the time-independent momentum. To pursue the analysis we need explicit expressions for the momentum $p(t)$ and the vector $\Delta(t)$ in the field under consideration. We will carry out calculations in the frame of reference in which the monochromatic plane wave, with the wave vector $q = q_0 \mathbf{q}$, is propagating in the direction $n = q_0/q$ opposite to the electron velocity. One can always find a relativistic frame of reference ($\gamma \gg 1$) in which the condition $q_0 \ll \varepsilon$ holds. This is necessary condition if we wish to treat the plane wave as classical. Solving the equation of particle motion in the electromagnetic field we get

$$\frac{p_\perp}{m} = \Omega t + \xi(t), \quad \xi(t) = \xi_2 \sin(\nu t + \varphi_0) + \xi_1 \cos(\nu t + \varphi_0),$$

$$\Omega = \frac{e}{m} F_\perp, \quad F_\perp = E - n(nE) + H \times n, \quad \nu = 2q_0, \quad \xi n = 0. \quad (28)$$
Here $E$ and $H$ are the electric and magnetic fields, both independent from time, and the orthogonal vectors $\xi_{1,2}$ characterize the intensity $\xi_0^2 = (\xi_1^2 + \xi_2^2)/2$ and the polarization of the wave. The corresponding Stokes parameters are

$$\lambda_3 = \frac{\xi_1^2 - \xi_2^2}{\xi_1^2 + \xi_2^2}, \quad \lambda_2 = \frac{(\xi_1 \times \xi_2) \cdot n}{\xi_0^2}. \quad (29)$$

Performing calculation we obtain the spectral probability of photon emission per unit time

$$\frac{d^2w_\gamma(t)}{dtdy} = \frac{dW_\gamma}{dy} = \frac{i\alpha m^2}{2\pi\varepsilon} \int_{-\infty}^{\infty} \frac{d\tau}{\tau - i0} \left[ 1 + \left( \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} \right) m_0 \right] \exp(-iu\tau\Phi(\varphi, \tau)), \quad (30)$$

where

$$y = \frac{\omega}{\varepsilon}, \quad u = \frac{y}{1+y}, \quad m_0 = \chi^2 = 2 + 2\chi\eta(\varphi)\tau \sin \frac{s\tau}{2} + \xi_0^2(1 - \lambda_3 \cos 2\varphi) \sin^2 \frac{s\tau}{2},$$

$$\Phi = \frac{1}{3} \lambda^2 + \frac{8}{s^2\tau} \chi\eta(\varphi) \left( \sin \frac{s\tau}{2} - \frac{s\tau}{2} \cos \frac{s\tau}{2} \right) +$$

$$\xi_0^2 \left[ 1 + \frac{2}{s^2\tau^2} (\cos s\tau - 1) \right] + \frac{\lambda_3}{s\tau} \cos 2\varphi \left( \sin s\tau + \frac{2}{s\tau} (\cos s\tau - 1) \right],$$

$$\eta(\varphi) = \xi_2 \cos \varphi - \xi_1 \sin \varphi, \quad \varphi = \nu t + \varphi_0, \quad \tau \rightarrow l_0\tau, \quad l_0 = \frac{\varepsilon}{m^2},$$

$$\Omega l_0 = \frac{e}{m} F_\perp l_0 = \chi, \quad 2\nu l_0 = \frac{4q_0\varepsilon}{m^2} \approx \frac{2q\varepsilon}{m^2} = s. \quad (31)$$

If the plane wave is absent ($\xi_1 = \xi_2 = 0$) Eq.(31) turns into the spectral probability in constant field in the quasiclassical approximation. In absence of constant field ($\chi = 0$) Eq.(31) turns into the exact spectral probability in the monochromatic plane wave. In the case $\xi_0 \ll 1$ one can expand the exponent in Eq.(31) over $\xi(\eta)$ and retain the terms $\propto \xi_0^3$:

$$dW_\gamma = dW_\gamma^F + \xi_0^2 dW_\gamma^\xi, \quad (32)$$

where $dW_\gamma^F$ is the spectral probability in constant field and $dW_\gamma^\xi$ is connected with the cross section of the Compton effect

$$d\sigma_c = \frac{8\pi\alpha\varepsilon}{m^2s} dW_\gamma^\xi. \quad (33)$$

The cross section $d\sigma_c$ describes the photon scattering on electron in the presence of an external field. A number of specific examples of the Compton effect in a static field is given in (BKS91).

### 3. Radiation in linear colliders (BKS90, BK04)

The particle interaction at beam-beam collision in linear colliders occurs in an electromagnetic field provided by the beams. The magnetic bremsstrahlung mechanism dominates and its characteristics are determined by the value of the quantum parameter $\chi(t)$ dependent on the strength of the incoming beam field at the moment $t$ (the constant field limit) $\chi = \gamma F/H_0$, where $F = |F|$, $F = E_\perp + \mathbf{v} \times \mathbf{H}$, $E$ and $H$ are the electric and magnetic fields in the laboratory frame, $E_\perp = E - \mathbf{v}(\mathbf{v} \times \mathbf{E})$.

The photon radiation length in an external field is

$$l_\gamma(\chi, u) = \lambda_\gamma \frac{H_0}{F} \left( 1 + \frac{\chi}{u} \right)^{1/3} = \frac{\lambda_\gamma\gamma}{\chi} \left( 1 + \frac{\chi}{u} \right)^{1/3}. \quad (34)$$
The field of the incoming beam changes very slightly along the formation length \( l_c \), if the condition \( l_c \ll \sigma_z \) if satisfied, providing a high accuracy of the magnetic bremsstrahlung approximation.

In the general case, when both polarization of electrons and photons is taken into account, the spectral probability of radiation per unit time has the form

\[
\frac{dw_\gamma}{dt} = dW_\gamma(t) = \frac{\alpha}{2\sqrt{3} \pi \gamma^2} \Phi_\gamma(1 + (\lambda \xi))d\omega; \quad \Phi_\gamma = \Phi_\gamma + \frac{\omega}{\varepsilon}(\zeta h)K_{1/3}(z),
\]

\[
\Phi_\gamma(t) = \beta K_{2/3}(z) - \int_z^\infty K_{1/3}(y)dy, \quad \beta = \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}, \tag{35}
\]

where \( z = 2u/3\chi(t) \), \( \lambda(\lambda_1, \lambda_2, \lambda_3) \) are the Stokes parameters of emitted photons for the following choice of axes: \( e_1 = (v \times h) \), \( h = F^*/F \), \( e_2 = h \), \( F^* = \epsilon/|\epsilon|[H_\perp + (E \times v)] \), \( \zeta \) is the spin vector of the initial electron in its rest frame. The vector \( \xi \) determines the mean photon polarization and its components are given by the following expressions:

\[
\xi_1 = \frac{\omega(\zeta \times h)}{e'\Phi_\gamma} K_{1/3}(z), \quad \xi_2 = \frac{(\zeta v)}{\Phi_\gamma} \left[ \left( \frac{\varepsilon}{\varepsilon'} - \frac{\varepsilon'}{\varepsilon} \right) K_{2/3}(z) - \frac{\omega}{\varepsilon} \int_z^\infty K_{1/3}(y)dy \right],
\]

\[
\xi_3 = \frac{1}{\Phi_\gamma} \left[ K_{2/3}(z) + \frac{\omega}{\varepsilon}(\zeta h)K_{1/3}(z) \right], \tag{36}
\]

here \( (\zeta \times h) = \zeta(v \times h) \).

Below we consider radiation from unpolarized electrons. The spectral probability of radiation is (35)

\[
\frac{dw_\gamma}{d\omega} = \frac{\alpha}{\pi \gamma^2 \sqrt{3}} \int_{-\infty}^\infty \Phi_\gamma(t)dt. \tag{37}
\]

For the Gaussian beams

\[
\chi(t) = \chi_0(x,y) \exp(-2t^2/\sigma_z^2), \tag{38}
\]

here the function \( \chi_0(x,y) \) depends on transverse coordinates.

It turns out that for the Gaussian beams the integration of the spectral probability over time can be carried out in a general form:

\[
\frac{dw_\gamma}{du} = \frac{\alpha m \sigma_z}{\pi \gamma \sqrt{6}} \left( \frac{1}{1 + u} \right)^2 \left[ \left( 1 + u + \frac{1}{1 + u} \right) \int_1^\infty K_{2/3}(ay) \frac{dy}{y\sqrt{\ln y}} \right. \\
-2a \int_1^\infty K_{1/3}(ay) \sqrt{\ln y} dy \left. \right], \tag{39}
\]

where \( a = 2u/3\chi_0 \). In the case when \( \chi_0 \ll 1 \) the main contribution into integral (39) gives the region \( y = 1 + \xi, \xi \ll 1 \). Taking the integrals over \( \xi \) we obtain

\[
\frac{dw_\gamma^{CF}}{du} \simeq \frac{\sqrt{3} \alpha m \sigma_z}{4\gamma} \frac{1 + u + u^2}{u(1 + u)^3} \chi_0 \exp \left( -\frac{2u}{3\chi_0} \right). \tag{40}
\]

For the round beams the integration over transverse coordinates is performed with the density

\[
n_\perp(q) = \frac{1}{2\pi \sigma_\perp} \exp \left( -\frac{q^2}{2\sigma_\perp^2} \right). \tag{41}
\]
The parameter $\chi_0(\varrho)$ we present in the form

$$\chi_0(\varrho) = \chi_m \frac{f(x)}{f_0}, \quad x = \frac{\varrho}{\sigma_\perp}, \quad f(x) = \frac{1}{x} \left(1 - \exp\left(-\frac{x^2}{2}\right)\right),$$

$$\chi_{rd} = 0.720 \alpha N \gamma \frac{\chi_m^2}{\sigma_z \sigma_\perp}, \quad f'(x_0) = 0, \quad f_0 = f(x_0) = 0.451256,$$

(42)

where $N$ is the number of electrons in the bunch. The result of calculation is shown in Fig.1.

![Figure 1](image_url)

**Figure 1.** The spectral intensity of radiation from the round beams in units $\alpha m^2 \sigma_z$ for $\chi_{rd}=0.13$ calculated according to Eqs.(39),(42).

The Laplace integration of Eq.(40) gives for the radiation intensity $dI/du = \varepsilon u/(1+u) dW/du$

$$\frac{dI_{as}}{du} \approx \alpha m^2 \sigma_z \frac{3}{4} \sqrt{\frac{\pi}{|f_0''|}} \left[ \frac{1 + u + u^2}{\sqrt{u}(1 + u)^2} f_0'' \chi_{rd} \exp \left( -\frac{2u}{3\chi_m} \right) \right],$$

(43)

where $f_0'' = f''(x_0) = -0.271678$.

For the flat beams ($\sigma_x \gg \sigma_y$) the parameter $\chi_0(\varrho)$ takes the form

$$\chi_0 = \chi_m \exp \left( -\frac{x^2}{2\sigma_x^2} \right) \left[ \text{erf} \left( \frac{y}{\sqrt{2}\sigma_y} \right) - i \text{erf} \left( \frac{x}{\sqrt{2}\sigma_x} \right) \right], \quad \chi_m = \frac{2N\alpha\gamma\chi^2_c}{\sigma_z \sigma_x},$$

(44)

here $\text{erf}(z) = 2/\sqrt{\pi} \int_0^z \exp(-t^2) dt$, $e_x$ and $e_y$ are the unit vectors along the corresponding axes.
To calculate the asymptotic of radiation intensity for the case $\chi_0 \ll 1$ one has to substitute

$$\chi_0 = |\chi_0| = \chi_m \exp \left( -\frac{x^2}{2\sigma_x^2} \right) \left[ \left( \text{erf} \left( \frac{y}{\sqrt{2}\sigma_y} \right) \right)^2 + \left( -i\text{erf} \left( \frac{-x}{\sqrt{2}\sigma_x} \right) \right)^2 \right]^{1/2} \tag{45}$$

into Eq.(40) and take integrals over the transverse coordinates $x, y$ with the weight

$$n_\perp(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right). \tag{46}$$

Integral over $x$ can be taken using the Laplace method, while at integration over $y$ it is convenient to introduce the variable

$$\eta = \frac{2}{\sqrt{\pi}} \int_w^\infty \exp(-t^2)dt, \quad w = \frac{y}{\sqrt{2}\sigma_y}. \tag{47}$$

As a result we obtain for the radiation intensity in the case of flat beams

$$\frac{dI_{f1}}{du} = \frac{9}{8\sqrt{2}(1-\frac{1}{2}\pi)} \alpha m^2 \sigma z^{5/2} \frac{1+u+u^2}{w^{3/2}(1+u)^4} \exp \left( -\frac{2u}{3\chi_m} \right). \tag{48}$$

4. Inhomogeneous fields (BKS89)

If the field varies slightly on the photon formation length, the vector $\Delta(t_2)$ in Eq.(27) as well as the exponential factor can be expanded in powers of $t_2 - t_1 = \tau$, with the required number of expansion terms. The first terms of the expansion which incorporates the particle acceleration, give the constant field limit, while remaining terms are the correction to this approximation. As the result the spectral intensity of radiation can be presented in the form

$$\frac{dI}{d\omega} = \frac{dI_0}{d\omega} + \frac{dI_c}{d\omega}, \quad \frac{dI_0}{d\omega} = \frac{\alpha m^2 \omega}{\sqrt{3\pi\varepsilon^2}} \left[ \beta K_{2/3}(z) - \int z K_{1/3}(y) dy \right],$$

$$\frac{dI_c}{d\omega} = \frac{\alpha m^2 \omega}{\sqrt{3\pi\varepsilon^2}} \left\{ -\frac{1}{36} \left[ (\mathbf{V} \nabla)^2 \mathbf{b} \right] \beta \left[ K_{2/3}(z) - \frac{2}{3z} K_{1/3}(z) \right] \right.$$

$$- \frac{1}{10} \left[ (\mathbf{V} \nabla)^2 \mathbf{b} + 3(\mathbf{V} \nabla)^2 \mathbf{b} \right] z K_{1/3}(z) - \frac{4}{3} K_{2/3}(z) + \beta \left[ 4 K_{2/3}(z) - \left( z + \frac{16}{9z} \right) K_{1/3}(z) \right] \left. \right\}, \tag{49}$$

where $\beta$ is defined in Eq.(35),

$$z = \frac{2m^2\omega}{3\varepsilon^2 |\mathbf{b}|}, \quad \mathbf{b} = \frac{e\mathbf{F}}{m}. \tag{50}$$

the vector $\mathbf{V}$ is the difference between the velocity of particle and the opposite beam (in the case when the beam shape is not changed during the collision). Here $dI_0/d\omega$ is the intensity spectrum in magnetic bremsstrahlung limit, $dI_c/d\omega$ is the gradient correction.

5. Pair creation by a photon in an external field (BKS98, BK07)

The probability of pair creation by a photon in the frame of QO method may be obtained from the probability of radiation from electron $dW = dI/\omega$ Eq.(11) using the QED crossing symmetry $p \rightarrow -p, k \rightarrow -k$, the final electron momentum $p' \rightarrow p', d^3k \rightarrow -d^3p$. This law is
exact for the differential probabilities. However, since in these processes all the final particles are moving within the angle $\sim 1/\gamma$ in the forward direction, the crossing symmetry relation are fulfilled within the relativistic accuracy also for the probabilities integrated over angles, i.e. for the spectral distributions. Substituting in Eq.(11) $\varepsilon \rightarrow -\varepsilon$, $\omega \rightarrow -\omega$, $\omega^2 d\omega \rightarrow -\xi^2 d\xi$ one obtains for the spectral distribution of created particle (see Eq.(3.50) in (BKS98))

$$dW = \frac{\alpha m^2}{\pi \varkappa} \frac{1}{\omega} \left[ \left( \frac{x}{1-x} + \frac{1-x}{x} \right) K_{2/3}(\xi) + \int_{\xi}^{\infty} K_{1/3}(z) dz \right] ,$$

where

$$x = \frac{\varepsilon}{\omega}, \quad \xi = \frac{2\omega}{3\kappa x (1-x)}, \quad \kappa = \frac{H\omega}{H_0 m} ,$$

$\omega$ is the energy of the initial photon, $\varepsilon$ is the energy of created positron and $\omega - \varepsilon$ is the energy of created electron.

Integrating over energy of created particle one obtains the total probability of pair creation (see Eq.(3.56) in (BKS98))

$$W = \frac{\alpha m^2}{3\sqrt{3}\pi \omega} \int_{-1}^{1} \frac{9 - v^2}{1 - v^2} K_{2/3}(\xi) dv ,$$

where $\xi = 8/[3\kappa(1 - v^2)]$ (the substitution was made $x(1 - x) = (1 - v^2)/4$).

In the limit $\kappa \ll 1$ the variable $\xi \gg 1$ and one can substitute into the integral Eq.(53) the asymptotic $K_\nu(z) = \sqrt{\pi/2z} \exp(-z)$. After integration one obtains in this limit

$$W^{(QO)} = \frac{3\sqrt{3} \alpha m^2 \kappa}{16\sqrt{2}\omega} \exp \left( -\frac{8}{3\kappa} \right) .$$

So at $\kappa \ll 1$ the probability of pair creation is exponentially small. Such situation is characteristic for all processes with the finite discontinuity of the square of system four momentum (the invariant mass of system): $\Delta_p^2 = (p + p')^2 - k^2 = (p + p')^2$, $\Delta_{p, \text{min}}^2 = 4m^2$.

In the case when the magnetic field is weak comparing with critical one and for high energy photons

$$\mu \equiv \frac{H}{H_0} \ll 1, \quad r \equiv \frac{k^2}{4m^2} \gg 1, \quad k_{\perp} H = 0, \quad \kappa = 2\sqrt{r} \mu \ll 1 ,$$

the standard QO method given above is applicable if $\kappa \gg 1/r$.

At $r \leq \mu^{-2/3}$ this method becomes inapplicable. In this region one can use the process probability found in Appendix B of (BK07), which is valid if $\mu \ll r - 1 \ll \mu^{-2}$. Using this result we get for the probability of pair creation by unpolarized photon

$$W^{(th)} = \frac{\alpha m^2 \mu}{4\omega} \frac{3r - 1}{\sqrt{r(r - 1)} l(r) \beta(r)} \exp \left( -\frac{\beta(r)}{\mu} \right) ,$$

where

$$l(r) = \ln \frac{\sqrt{r} + 1}{\sqrt{r} - 1}, \quad \beta(r) = 2\sqrt{r} - (r - 1) l(r) .$$

For large $r \gg 1$, limited by the condition $r \ll \mu^{-2}$ ($\kappa \ll 1$), we have

$$W^{(th)} \simeq \frac{3\alpha m^2 \mu}{8\omega} \sqrt{\frac{3r}{2}} \exp \left( -\frac{4}{3\mu \sqrt{r}} \right) = W^{(QO)} \exp \left( -\frac{4}{15\mu \sqrt{r}} \right) .$$
At $\kappa^3 \gg \mu^2$ Eq.(58) agrees with Eq.(54). The last expression has essentially wider region of applicability than pure $W^{(QO)}$. It is seen that the probability of pair creation depends separately on field (parameter $\mu$) and photon energy (parameter $r$), while in standard QO method this probability depends on combination $\sqrt{r}\mu$ only. The ratio $W^{(th)}/W^{(QO)}$ is shown in Fig.2. One can see that with $\mu$ decreasing the photon energy interval, in which Eq.(56) should be used, is significantly extended.

**Figure 2.** The ratio of the pair creation probability $W^{(th)}$ Eq.(56) to the standard $W^{(QO)}$ Eq.(54) vs the photon energy parameter $r$ for $\mu = 0.01$ (curve 1) and for $\mu = 0.001$ (curve 2).

At $\kappa \geq 1$ Eq.(56) becomes inapplicable. However in this case QO is valid, see Eq.(53).

6. General theory of radiation in oriented crystal (BK05, BK06)

Here we consider radiation from electron in an oriented single crystal for the case when the angle of incidence (angle between the electron momentum and axis) $\vartheta_0 \ll V_0/m$, $V_0$ is the scale of the axis potential (see below). In this case the distance of an electron from axis $\varrho$ as well as the transverse field of the axis can be considered as constant at the formation length. For an axial orientation of crystal the ratio of the atom density $n(\varrho)$ in the vicinity of an axis to the mean atom density $n_a$ is

$$\frac{n(x)}{n_a} = \xi(x) = \frac{x_0}{\eta_1} e^{-x/\eta_1}, \quad \varepsilon_0 = \frac{\varepsilon_0}{\xi(0)},$$

(59)

where

$$x_0 = \frac{1}{\pi d n_a a_s^2}, \quad \eta_1 = \frac{2u_1^2}{a_s^2}, \quad x = \frac{\varrho^2}{a_s^2}.$$  

(60)

Here $\varrho$ is the distance from axis, $u_1$ is the amplitude of thermal vibration, $d$ is the mean distance between atoms forming the axis, $a_s$ is the effective screening radius of the axis potential

$$U(x) = V_0 \left[ \ln \left( 1 + \frac{1}{x + \eta} \right) - \ln \left( 1 + \frac{1}{x_0 + \eta} \right) \right].$$

(61)
The local value of parameters $\chi(x)$, see Eq.(12), which determines the radiation probability in the field $\text{Eq.}(61)$ is

$$\chi(x) = -\frac{dU}{dq} \frac{\varepsilon}{m^3} = \chi_s f_a, \quad f_a = \frac{2\sqrt{x}}{(x + \eta)(x + \eta + 1)}, \quad \chi_s = \frac{V_0 \varepsilon}{m^3 \eta_s} \approx \frac{\varepsilon}{\varepsilon_s}. \quad (62)$$

The particular calculation below will be done for the tungsten and germanium crystals studied experimentally. The relevant parameters are given in Table 1.

**Table 1** Parameters of radiation (pair creation) process in the tungsten (the axis $< 111 >$) and germanium (the axis $< 110 >$) crystals for different temperatures $T$, the energies $\varepsilon$ and $\omega$ are in GeV

| Crystal | $T$(K) | $V_0$(eV) | $x_0$ | $\eta_1$ | $\eta$ | $\varepsilon_0$ | $\varepsilon_1$ | $\varepsilon_m(\omega_m)$ | $h$ |
|---------|--------|-----------|------|---------|-------|-----------|-----------|-----------------|------|
| W       | 293    | 413       | 39.7 | 0.108   | 0.115 | 7.43      | 0.76       | 34.8            | 14.35 | 0.348 |
| W       | 100    | 355       | 35.7 | 0.0401  | 0.0313 | 3.06      | 0.35       | 43.1            | 8.10  | 0.612 |
| Ge      | 100    | 114.5     | 19.8 | 0.064   | 0.0633 | 59        | 0.85       | 179             | 51    | 0.459 |

Here $\varepsilon_1$ is an estimate of electron energy when the intensity of coherent radiation is equal to the intensity of incoherent one. The maximal value of the parameter $\chi(x)$ is $\chi_m = \chi(x_m) = \varepsilon/\varepsilon_m$.

The spectral intensity of radiation can be presented in the form

$$dI(\varepsilon, y) = \frac{\alpha m^2 y dy}{2\pi} \left| 1 - \frac{y}{1 - y} \right|^{x_0} G(x, y), \quad G(x, y) = \int_0^\infty F(x, y, t) dt - r_3 \frac{\pi}{4},$$

$$F(x, y, t) = \text{Im} \left\{ e^{\varphi_1(t)} \left[ r_2 \nu_0^2 (1 + i\nu)(\varphi_2(t) + r_3 \varphi_3(t)) \right] \right\}, \quad b = \frac{4\chi^2(x)}{u^2 \nu_0^4},$$

$$y = \frac{\omega}{\varepsilon}, \quad u = \frac{y}{1 - y}, \quad \varphi_1(t) = (i - 1)t + b(1 + i)(f_2(t) - t),$$

$$\varphi_2(t) = \frac{\sqrt{2}}{\nu_0} \tanh \frac{\nu_0 t}{\sqrt{2}}, \quad \varphi_3(t) = \frac{\sqrt{2} \nu_0}{\sinh(\sqrt{2}\nu_0 t)}, \quad (63)$$

where

$$r_2 = 1 + (1 - y)^2, \quad r_3 = 2(1 - y), \quad \nu_0^2 = \frac{1 - y}{y} \frac{\varepsilon}{\varepsilon(x)}, \quad \varepsilon_\varepsilon(x) = \frac{\varepsilon_e(n_a)}{\xi(x) g} = \frac{\varepsilon_0}{g} e^{x/\eta},$$

$$\varepsilon_e = \frac{m}{16\pi Z^2 \alpha^2 \lambda^2 n_a L_0}, \quad L_0 = \ln(183Z^{-1/3}) - f(Za),$$

$$h(z) = -\frac{1}{2} [1 + (1 + z)e^2 \text{Ei}(-z)], \quad g = 1 + \frac{1}{L_0} \left[ \frac{1}{18} - h \left( \frac{u_1^2}{a^2} \right) \right]. \quad (64)$$

The found spectral intensity of radiation contains a very rich information. The intensity of coherent radiation $I(\varepsilon) = \int I^{\text{coh}}(\varepsilon, y) dy$ is the first term ($\nu_0^2 = 0$) of the decomposition of Eq.(63) over $\nu_0^2$

$$I^{\text{coh}}(\varepsilon) = \int_0^{x_0} I(\chi) dx \frac{dx}{x_0}. \quad (65)$$

Here $I(\chi)$ is the radiation intensity in constant field (magnetic bremsstrahlung limit). It is convenient to use the following representation for $I(\chi)$

$$I(\chi) = \int \frac{\alpha m^2 \chi^{\lambda+1+i\chi}}{(\lambda - i) \chi^{3/2}} \Gamma(1 - s) \Gamma(3s - 1) (2s - 1)(s^2 - s + 2) \frac{ds}{\cos \pi s}, \quad \frac{1}{3} < \lambda < 1. \quad (66)$$
Figure 3. The inverse radiation length in the tungsten crystal, axis <111> at different temperatures T vs the electron initial energy. The curves 1 and 4 are the total effect: $L^{cr}(\epsilon)^{-1} = I(\epsilon)/\epsilon$ Eq.(63) for $T=293$ K and $T=100$ K correspondingly, the curves 2 and 5 give the coherent contribution $I^{F}(\epsilon)/\epsilon$ Eq.(65), the curves 3 and 6 give the incoherent contribution $I^{inc}(\epsilon)/\epsilon$ Eq.(67) at corresponding temperatures T.

The second term of decomposition of Eq.(63) ($\propto \nu_0^2$) gives the intensity of incoherent radiation:

$$I^{inc}(\epsilon) = \frac{\alpha m^2}{60\pi e_0} \frac{\epsilon}{x_0} \int_0^{x_0} e^{-x/\eta} J(\chi) \frac{dx}{x_0}, \quad (67)$$

the new representation of $J(\chi)$ is

$$J(\chi) = \frac{i\pi}{2} \int_{\lambda-i\infty}^{\lambda+i\infty} \frac{\chi^{2s} \Gamma(1+3s)}{3^s \Gamma(s)} R(s) \frac{ds}{\sin^{3s/2} s}, \quad -\frac{1}{3} < \lambda < 0, \quad (68)$$

where

$$R(s) = 15 + 43s + 31s^2 + 28s^3 + 12s^4. \quad (69)$$

The inverse radiation length in tungsten crystal (axis <111>) $1/L^{cr}(\epsilon) = I(\epsilon)/\epsilon$ Eq.(63), as well as the coherent contribution $1/L^{F}(\epsilon) = I^{F}(\epsilon)/\epsilon$ Eq.(65) and the incoherent contribution $1/L^{inc}(\epsilon) = I^{inc}(\epsilon)/\epsilon$ Eq.(67) are shown in Fig.3 for two temperatures T=100 K and T=293 K as a function of incident electron energy $\epsilon$. One can see that at temperature T=293 K the...
intensity $I^{\text{coh}}(\varepsilon)$ is equal to $I^{\text{inc}}(\varepsilon)$ at $\varepsilon \simeq 0.4$ GeV and temperature $T=100$ K the intensity $I^{\text{coh}}(\varepsilon)$ is equal to $I^{\text{inc}}(\varepsilon)$ at $\varepsilon \simeq 0.7$ GeV. At higher energies the intensity $I^{\text{F}}(\varepsilon)$ dominates while the intensity $I^{\text{inc}}(\varepsilon)$ decreases monotonically.

Interrelation between the theory and experiment (very limited for the time being) is shown in Fig.4. One can see that there is quite satisfactory agreement of the theory and data.

**Figure 4.** Comparison of theory and experiment. (a) Enhancement of radiation intensity (the ratio $L^{BM}/L^{\text{F}}$) in the tungsten crystal, axis $<111>$, $T=293$ K. The curve 1 is for the target with thickness $l=200$ µm, where the energy loss was taken into account. The curve 2 is for a considerably thinner target, where one can neglect the energy loss ($L^{\text{F}} \rightarrow L^{\text{cr}}$). The data are from K.Kirshbom, et al, 2001 (see (BK06)).

(b) Enhancement of the probability of pair creation in the tungsten crystal for different temperatures, axis $<111>$. The data are from K.Kirshbom, et al, 1998 (see (BK05)).

7. Back-reaction: spectra of radiation taking into account energy loss in oriented crystals

The crystal radiation length $L(\varepsilon) = \varepsilon/I(\varepsilon)$, $I(\varepsilon)$ is the intensity of electron radiation, and the pair creation length $L_{\text{pr}}(\omega) = 1/W(\omega)$, $W(\omega)$ is the pair creation probability, are the function of energy in oriented crystals.

We consider the case when the target thickness $l$ is of the order $l \sim L(\varepsilon)$ in the intermediate energy region. Here we will neglect the energy dispersion. On this assumption the energy loss equation acquires the form

$$
dt = \frac{L(\varepsilon)}{\varepsilon} \, d\varepsilon, \quad t(\varepsilon, \varepsilon_0) = \int_{\varepsilon}^{\varepsilon_0} \frac{dx}{x} L(x), \quad \varepsilon = \varepsilon(\varepsilon_0, t).
$$

(70)
Now the photon spectral distribution taking into account the energy loss can be written in the form

\[
\omega \frac{dn_{\gamma}}{d\omega} = \int_0^l \left( \frac{dI(\varepsilon(\varepsilon_0, t), \omega)}{d\omega} \right) \vartheta(\varepsilon(\varepsilon_0, t) - \omega) dt
\]

\[
= \int_{\varepsilon_l}^{\varepsilon_0} \frac{L(\varepsilon)}{\varepsilon} \frac{dI(\varepsilon, \omega)}{d\omega} \vartheta(\varepsilon - \omega) d\varepsilon, \quad \varepsilon_l = \varepsilon(\varepsilon_0, l),
\]

where \(dI(\varepsilon, \omega)/d\omega\) is the radiation intensity spectral distribution (see Eq.(63)), \(\varepsilon_l = \varepsilon(\varepsilon_0, l)\) is the electron energy after traversing the thickness \(l\) by electron with the initial energy \(\varepsilon_0\). The result of calculation for the tungsten crystal is shown in Fig.5.

**Figure 5.** The spectral distribution of radiation at the initial electron energy \(\varepsilon_0 = 20\) GeV in the tungsten crystal, axis \(<111>\), \(T=100\) K in two targets with thickness \(l = 0.032\) cm= 0.77 \(L = 0.16 \ L_{pr}\) (curves 1 and 2) and \(l = 0.01\) cm= 0.24 \(L = 0.093 \ L_{pr}\) (curves 3 and 4) vs the photon energy \(\omega\). The curves 2 and 4 are calculated according to Eq.(63), while the curves 1 and 3 are calculated according to Eq.(71) which takes into account the electron energy loss.
QUASICLASSICAL OPERATOR METHOD: LIST OF REFERENCES

Books

(1) Baier V N Katkov V M and Fadin V M 1973 (BKF73), *Radiation from Relativistic Electrons* (in Russian) (Moscow: Atomizdat)

(2) Baier V N Katkov V M and Strakhovenko V M 1998 (BKS98), *Electromagnetic Processes at High Energies in Oriented Crystals* (Singapore: World Scientific)

(3) Berestetskii V B Lifshitz E M and Pitaevskii L P 1982 (BLP82) *Quantum Electrodynamics* 2nd ed (Oxford: Pergamon Press)

Articles

(1) Baier V N and Katkov V M 1967 (BKp67) Radiative polarization of electrons in a magnetic field *Soviet Phys. JETP* 25 944

(2) Baier V N and Katkov V M 1967 (BK67) Quantum effects in a magnetic bremsstrahlung *Phys. Lett.* A 25 492

(3) Baier V N and Katkov V M 1968 (BK68) Processes involved in the motion of high energy particles in a magnetic field *Soviet Phys. JETP* 26 854

(4) Baier V N and Katkov V M 1969 (BK69) Quasiclassical theory of bremsstrahlung by relativistic particles *Soviet Phys. JETP* 28 807

(5) Baier V N 1972 (B72) Radiative polarization of electrons in storage rings *Soviet Phys. Uspekhi* 14 695

(6) Baier V N, Katkov V M and Strakhovenko V M 1973 (BKS72) Radiation emitted by relativistic particles in periodic structures *Soviet Phys. JETP* 36 1120

(7) Baier V N and Katkov V M 1976 (BK76) Corrections to the eikonal approximation and the bremsstrahlung of electrons at high energies *Soviet Phys.-Doklady* 21 150

(8) Baier V N, Katkov V M and Strakhovenko V M 1981 (BKS81) Radiation of relativistic particles moving quasiperiodically *Soviet Phys. JETP* 53 688

(9) Baier V N, Fadin V S, Khoze V A and Kuraev E A 1981 (BFKK81) Inelastic processes in high energy quantum electrodynamics *Phys. Reports* 78 293

(10) Baier V N, Katkov V M and Strakhovenko V M 1988 (BKS88) Incoherent radiation and pair creation in crystals *Phys. Stat. Solidi (b)* 149 403

(11) Baier V N, Katkov V M and Strakhovenko V M 1988 (BKS88) Radiation from relativistic particles colliding in a medium in the presence of an external field *Soviet Phys. JETP* 67 70

(12) Baier V N, Katkov V M and Strakhovenko V M 1989 (BKS89) Quantum radiation theory in inhomogeneous external fields *Nucl.Phys.* B 328 387

(13) Baier V N, Katkov V M and Strakhovenko V M 1989 (BKSp89) Pair production by a photon in inhomogeneous external fields *Phys. Lett.* B 225 193

(14) Baier V N, Katkov V M and Strakhovenko V M 1990 (BKS90) Quantum beamstrahlung and electroproduction of the pairs in linear colliders *Particle Accelerators* 30 43

(15) Baier V N, Katkov V M and Strakhovenko V M 1991 (BKS91) Semiclassical theory of electromagnetic processes in a plane wave and a constant field *Soviet Phys. JETP* 73 945

(16) Katkov V M and Strakhovenko V M 2001 (KS01) Operator representation of the wave function for a charged particle in an external field and its application *JETP* 92 561

(17) Baier V N and Katkov V M 2004 (BK04) On coherent radiation in electron-positron colliders *Quantum aspects of Beam Physics* ed Chen P and Reil K (Singapore: World Scientific) p 3
(18) Baier V N and Katkov V M 2005 (BKPR05) Concept of formation length in radiation theory Phys. Reports 409 261

(19) Baier V N and Katkov V M 2005 (BK05) Coherent and incoherent pair creation by a photon in oriented single crystal Phys. Lett. A 346 359

(20) Baier V N and Katkov V M 2006 (BK06) Coherent and incoherent radiation from high-energy electron and the LPM effect in oriented single crystal Phys. Lett. A 353 91

(21) Baier V N and Katkov V M 2007 (BK07) Pair creation by a photon in a strong magnetic field Phys. Rev. D 75 073009

(22) Baier V N and Katkov V M 2009 (BK09) Spectra of radiation and created particles at intermediate energy in oriented crystal taking into account energy loss (Preprint hep-ph 0901.3951v1)