Insurance valuation: A two-step generalised regression approach

IAA Webinar

Karim Barigou, joint work with Valeria Bignozzi and Andreas Tsanakas
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Introduction
Actuarial versus Financial valuation

Two types of valuations:

• **Traditional actuarial valuation:**
  \[ \rho[S] = e^{-r} \mathbb{E}^P[S] + \text{RM}[S] \]
  - Based on principle of **diversification** (LLN).
  - Risk margin to cover non-diversified risk.
  - The valuation is performed under the real-world measure \( \mathbb{P} \).

• **Financial valuation:**
  \[ \rho[S] = \mathbb{E}^Q[e^{-r}S] \]
  - Based on principle of **no-arbitrage** under a risk-neutral measure \( \mathbb{Q} \).
  - Set of feasible \( \mathbb{Q} \)'s follows from observed market prices.
  - In incomplete markets: **Infinite choice** of measures \( \mathbb{Q} \).
Fair valuation of insurance liabilities

Solvency II

The assets (liabilities) shall be valued at the amount for which they could be exchanged (transferred or settled), between knowledgeable and willing parties in a transaction under normal market conditions.

- Liabilities **traded** on the financial market:
  Fair value $\rightarrow$ Financial risk-neutral valuation

- Liabilities **independent** of the financial market:
  Fair value $\rightarrow$ Actuarial valuation

- Liabilities that are partly **dependent** on the financial market:
  
  What is the appropriate fair value?
Insurance liabilities are only partially hedgeable and therefore a two-step hybrid approach is usually considered:

- **Two-step hedge-based approach (Dhaene et al. 2017):**
  \[
  \rho[S] = \underbrace{V^\theta(0)}_{\text{Hedgeable part}} + \underbrace{\pi[S - V^\theta(T)]}_{\text{Actuarial principle on the residual part}}
  \]
  - First part is typically determined by e.g. quadratic hedging.
  - Second part is priced via a standard actuarial principle.

- **Two-step conditional approach (Pelsser & Stadje 2014):**
  \[
  \rho[S] = \mathbb{E}^Q \left[ \pi[S \mid Y] \right]
  \]
  - Inner step: Actuarial valuation conditional on traded asset prices.
  - Outer step: Financial valuation.
Growing research interest on fair/market-consistent valuation:

- The hedge-based valuation of Dhaene et al. (2017) in a one-period setting was further generalized in a discrete multi-period setting in Barigou et al. (2019) and in continuous-time in Delong et al. (2019).
- Assa & Gospodinov (2018) investigated market-consistent valuation in imperfect markets where the financial valuation is non-linear.
- The two-step conditional valuation was also investigated in Salahnejhad Ghalehjooghi & Pelsser (2021) for participating pension contracts.
- Deelstra et al. (2020) proposed a three-step valuation where the liability is decomposed into hedgeable, diversifiable and residual risk; with a generalization in Linders (2021).
- Fair valuation based on asymmetric hedging is considered in this paper: Barigou et al. (2022) and Chen et al. (2021).
The aim of this paper is twofold:

- We introduce a new valuation framework for insurance liabilities based on a two-step hedging procedure.
  - The resulting hedging strategies produce a residual which has a zero tail risk, as measured by a VaR or Expectile criterion. Hedging cost is then shared between policyholders and shareholders via appropriate cost-of-capital arguments.

- We propose a general backward iterations scheme to determine the valuation and hedging of liabilities in a multi-period framework.
Fair valuation in a one-period setting
Mathematical framework

- $\mathcal{C} \subseteq L^2(\Omega, \mathcal{F}, \mathbb{P})$ set of contingent claims
- $S \in \mathcal{C}$ insurance liability
- $y = (1, y_1, \ldots, y_n), \ n \in \mathbb{N}$ value of the financial market assets at time 0
- $Y = (e^r, Y_1, \ldots, Y_n), \ n \in \mathbb{N}, \ r \geq 0$, value of the financial market assets at time 1
- In this presentation, we assume $r = 0$
- $\beta = (\beta_0, \ldots, \beta_n) \in \mathbb{R}^{n+1}$ trading strategy

$$\beta \cdot y = \sum_{i=0}^{n} \beta_i \cdot y_i \quad \beta \cdot Y = \sum_{i=0}^{n} \beta_i \cdot Y_i$$
We can distinguish three types of claims:

- $C^h \subseteq C$ set of perfectly hedgeable claims:

\[
S^h = \beta \cdot Y = \sum_{i=0}^{n} \beta_i \cdot Y_i
\]

- $C^\perp \subseteq C$ set of claims independent of the financial market:

\[
S^\perp \text{ independent of } Y
\]

- Hybrid claims (our focus):

\[
S \in C\setminus(C^h \cup C^\perp)
\]
A valuation is a map $\rho : \mathcal{C} \to \mathbb{R}$, $S \mapsto \rho(S)$, that is

- Normalised: $\rho(0) = 0$
- Translation invariant: $\rho(S + m) = \rho(S) + m$, $\forall S \in \mathcal{C}$, $m \in \mathbb{R}$

$\rho$ is market-consistent if

\[
\rho(S + S^h) = \rho(S) + \beta \cdot y, \quad \text{for any } S \in \mathcal{C}, S^h = \beta \cdot Y
\]

$\rho$ is actuarial if

\[
\rho(S^\perp) = \mathbb{E}[S^\perp] + \text{RM}(S^\perp)
\]

for any claim $S^\perp \in \mathcal{C}^\perp$, where $\text{RM} : \mathcal{C}^\perp \to \mathbb{R}$ is a mapping that does not depend on current asset prices $y$

$\rho$ is a fair valuation (Dhaene et al. 2017, Barigou et al. 2019) if

it is market-consistent and actuarial
Two-step valuation with cost-of-capital approach

Fair valuation of hybrid claims is generally performed in two steps:

- **Quadratic hedging:**
  \[
  \theta = \arg \min_{\beta \in \mathbb{R}^{n+1}} \mathbb{E}[(S - \beta \cdot Y)^2] \quad \Rightarrow \quad \mathbb{E}[S] = \mathbb{E}[\theta \cdot Y]
  \]

- **Risk measure on the residual risk**
  \[
  \text{VaR}_\alpha(S - \theta \cdot Y) \quad \alpha \in (0, 1)
  \]

The fair value then is:

\[
\rho(S) = \theta \cdot y + i \cdot \text{VaR}_\alpha(S - \theta \cdot Y) \quad i \in (0, 1)
\]

Cost-of-capital approach

\[
\rho(S) = \mathbb{E}[S] + i \cdot \text{VaR}_\alpha(S - \mathbb{E}[S]) \quad i \in (0, 1)
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\]
Two-step valuation with a general loss function

**Error function**

\( \ell : \mathbb{R} \to [0, +\infty) \) such that \( \ell(x) = 0 \) iff \( x = 0 \)

\[ \xi = \arg\min_{\beta \in \mathbb{R}^{n+1}} \mathbb{E}[\ell(S - \beta \cdot Y)] \]

- \( \ell_\alpha(x) = \alpha x_+ + (1 - \alpha)x_- \) quantile regression

\[ \text{VaR}(S - \xi^{(\ell_\alpha)} \cdot Y) = 0 \]

- \( \ell_\tau(x) = \tau(x_+)^2 + (1 - \tau)(x_-)^2 \) expectile regression

We penalize (possibly asymmetrically) distance of the trading portfolio \( \beta \cdot Y \) from the liability \( S \)

- Föllmer & Leukert (2000) → Shortfall risk
- Rockafellar & Uryasev (2013) → Risk quadrangle
Figure 1: Quantile loss functions (solid lines) and expectile loss functions (dashed curves) for $\alpha \in \{0.4, 0.5, 0.6\}$. 
The main two steps:

- First step (quadratic hedging):
  \[ \theta = \arg \min_{\beta \in \mathbb{R}^{n+1}} \mathbb{E}[(S - \beta \cdot Y)^2] \]

- Second step (hedging residual with loss function \( \ell \)):
  \[ \eta = \arg \min_{\beta \in \mathbb{R}^{n+1}} \mathbb{E}[\ell(S - \theta \cdot Y - \beta \cdot Y)] \]

In total: \( \xi = \arg \min_{\beta \in \mathbb{R}^{n+1}} \mathbb{E}[\ell(S - \beta \cdot Y)] = \theta + \eta \)

**Two-step fair valuation**

\[ \rho(S) = \theta \cdot y + i \cdot \eta \cdot y, \quad i \in (0, 1) \]
Quantile hedging is a TVaR deviation minimiser

Assume that we want to hedge \( R(S, \theta) = S - \theta \cdot Y \) and the regulator imposes that \( \text{VaR}_\alpha(R(S, \theta) - \beta \cdot Y) = 0 \) for some trading strategy \( \beta \in \mathcal{B} \). Two possibilities:

- Invest \( \text{VaR}_\alpha(R(S, \theta)) \) in the risk-free asset. Call this strategy \( \nu \).
- Consider the quantile hedging strategy \( \eta \) such that \( \text{VaR}_\alpha(R(S, \theta) - \eta \cdot Y) = 0 \).

Lemma

The quantile hedging strategy satisfies:

\[
d\text{TVaR}_\alpha(R(S - \theta \cdot Y, \eta)) \leq d\text{TVaR}_\alpha(R(S - \theta \cdot Y, \beta)),
\]

for all hedging strategies \( \beta \) such that \( \text{VaR}_\alpha(R(S, \theta) - \beta \cdot Y) = 0 \). This is in particular the case for \( \beta = \nu \).

N.B: TVaR deviation: \( d\text{TVaR}_\alpha(X) = \text{TVaR}_\alpha(X) - \mathbb{E}(X) \).
Quantile hedging is a TVaR deviation minimiser

Assume that we want to hedge $R(S, \theta) = S - \theta \cdot Y$ and the regulator imposes that $\text{VaR}_\alpha(R(S, \theta) - \beta \cdot Y) = 0$ for some trading strategy $\beta \in \mathcal{B}$. Two possibilities:

- Invest $\text{VaR}_\alpha(R(S, \theta))$ in the risk-free asset. Call this strategy $\nu$.
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N.B: TVaR deviation: $d\text{TVaR}_\alpha(X) = \text{TVaR}_\alpha(X) - \mathbb{E}(X)$. 
Example

Equity-linked contracts sold to 1000 policyholders:

- Assets: $y = (1, 1)$, $Y = (1, Y_1)$, $Y_1 \sim LN(0.1, 0.2^2)$
- Mortality: $N$ is the number of survivors at time 1, $N \sim Bin(1000; 0.9)$
- $K$ is the guarantee level, $K = 1$
- Payoff: $S = N \times \max(Y_1, K)$

$S$ is highly but non-linearly correlated with a tradeable asset $Y_1$
Figure 2: Liability $S$ (left) and residual $S - \theta \cdot Y$ (right) against value of the risky asset $Y_1$. 
Table 1: Investment in risk-free and risky asset, from hedging strategies associated with the two-step valuation of $S$.

| Strategy               | risk-free asset | risky asset | cost of strategy |
|------------------------|-----------------|-------------|------------------|
| $\theta$               | 247             | 709         | 956              |
| $\xi$                  | 460             | 658         | 1118             |
| $\xi^{(l_T)}$          | 450             | 663         | 1113             |
| $(\text{VaR}_\alpha(R(S, \theta)), 0)$ | 163             | 0           | 163              |
| $\eta$                 | 213             | -52         | 161              |
| $\eta^{(l_T)}$         | 204             | -47         | 157              |

Our approach provides

- Higher investment in the risk-free asset (more stringent criterion)
- Higher sensitivity to adverse movement in the risky asset
Figure 3: Densities of residuals, for quadratic hedging of $S$, followed by investing the VaR of the residual in the risk-free asset (black); quantile hedging of $S$ (blue); and expectile hedging of $S$ (red).
The TVaR deviations are given by

\[
d\text{TVaR}_\alpha(R(S, \theta) - \text{VaR}_\alpha(R(S, \theta))) = 194.2 \\
d\text{TVaR}_\alpha(R(R(S, \theta), \eta)) = 182.5 \\
d\text{TVaR}_\alpha(R(R(S, \theta), \eta^{(l_\tau)})) = 182.6
\]

The fair value of \( S \) for a cost-of-capital rate \( i = 0.1 \):

\[
\phi(S) = \theta \cdot y + i \cdot \text{VaR}_\alpha(R(S, \theta)) = 972.6 \\
\rho(S) = \theta \cdot y + i \cdot \eta \cdot y = 972.4 \\
\rho^{(l_\tau)}(S) = \theta \cdot y + i \cdot \eta^{(l_\tau)} \cdot y = 972
\]

- In this example, the impact on the valuation of \( S \) is very limited, even though the hedging strategies and, importantly, the statistical behaviour of residuals, are different.
Fair valuation in a multi-period setting
Motivation

- The valuation problem is in general not a one-period static problem but a multi-period dynamic problem.

- In this dynamic context, the notion of time-consistency is important:

\[ \rho_t[S] = \rho_t[\rho_{t+1}[S]] \]
Objective: Determine a fair dynamic valuation for a claim $S$ with maturity $T$.

- The valuation should be time-consistent:

\[ \rho_t[S] = \text{Hedgeable part of } \rho_{t+1}[S] + \text{Risk margin} \]
**Procedure:** Backward iteration scheme:

\[
\rho_t(S) := \theta(t + 1) \cdot Y(t) + i \eta(t + 1) \cdot Y(t),
\]

where

\[
\theta(t + 1) = \arg \min_{\beta \in B(t)} \mathbb{E}_t \left[ (\rho_{t+1}(S) - \beta \cdot Y(t + 1))^2 \right]
\]

\[
\eta(t + 1) = \arg \min_{\beta \in B(t)} \mathbb{E}_t \left[ \ell_\alpha(\rho_{t+1}(S) - \theta(t + 1) \cdot Y(t + 1) - \beta \cdot Y(t + 1)) \right].
\]

**Notation:** \(Y(t) = (e^{rt}, Y_1(t), \ldots, Y_n(t))\) the vector of asset prices at time \(t \in \{1, 2, \ldots, T\}\), \(\mathcal{B}(t)\) the set of all real-valued \(\mathcal{F}_t\)-measurable trading strategies.
### Properties

- The valuation is **market-consistent, actuarial and time-consistent**.
- Yearly solvency constraints are satisfied:

\[
\text{VaR}_{\alpha, t} \left( \rho_{t+1}(S) - \theta(t+1) \cdot Y(t+1) - \eta(t+1) \cdot Y(t+1) \right) = 0,
\]

for \( t \in \{0, 1, \ldots, T - 1\} \).
Algorithm 1 Backward resolution of the dynamic fair valuation problem

1: $\rho_T \leftarrow S$
2: for $t = T - 1, T - 2, \ldots, 0$ do
3: \[ g_{t+1} = \arg \min_{g \in \mathcal{G}} \frac{1}{M} \sum_{i=1}^{M} \left( \rho_{t+1}(S) - g(Z^{(i)}(t)) \cdot Y^{(i)}(t+1) \right)^2 \]
4: \[ h_{t+1} = \arg \min_{g \in \mathcal{G}} \frac{1}{M} \sum_{i=1}^{M} \ell_{\alpha} \left( \rho_{t+1}(S) - g(Z^{(i)}(t)) \cdot Y^{(i)}(t+1) \right) \]
5: \[ \rho_t^{(i)}(S) = g_{t+1}(Z^{(i)}(t))Y^{(i)}(t) \]
\[ + i \left( h_{t+1}(Z^{(i)}(t)) - g_{t+1}(Z^{(i)}(t)) \right) Y^{(i)}(t) \]
6: end for

Assumptions:

- Risk drivers are markovian
- Hedging strategies are non-linear functions $g$ of risk drivers at time $t$. 
Numerical example: Equity-linked contracts
Application: Equity-linked life-insurance contracts

- Payoff at maturity $T$:

$$S = N(T) \times \max \left( Y^{(1)}(T), K \right),$$

with

- $N(T)$: Number of survivors at time $T$.
- $Y^{(1)}(T)$: Stock at time $T$.
- $K$: fixed guarantee level.

- Stock and force of mortality dynamics:

$$dY^{(1)}(t) = Y^{(1)}(t) \left( \mu dt + \sigma dW_1(t) \right)$$

$$d\lambda_x(t) = c\lambda_x(t)dt + \zeta dW_2(t)$$

with $W_1(t)$ and $W_2(t)$ correlated Brownian motions.

- Financial market: Risk-free asset $Y^{(0)}(t) = e^{rt}$ and risky asset $Y^{(1)}(t)$. 

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Application: Equity-linked life-insurance contracts

We have two neural networks:

\[ g_{t+1} : \mathbb{R}^2 \to \mathbb{R}^2, (Y_1(t), N(t)) \mapsto g_{t+1}(Y_1(t), N(t)) = \theta(t + 1), \]

\[ h_{t+1} : \mathbb{R}^2 \to \mathbb{R}^2, (Y_1(t), N(t)) \mapsto h_{t+1}(Y_1(t), N(t)) = \zeta(t + 1), \]

Specifications:

- \( N = 200.000 \) samples,
- Parameters for the financial market are \( r = 0.01, \mu = 0.02, \sigma = 0.1, K = 1, \delta = -0.5 \) and \( Y^{(1)}(0) = 1, \)
- The mortality parameters follow from Luciano et al. (2017) and correspond to UK male individuals who are aged 55 at time 0.
- \( l_x = 1000 \) initial contracts at time 0 with a maturity of \( T = 10 \) years.
Figure 4: Left: Evolution of the fair valuation from time 0 to maturity time $T = 10$. Right: Histogram of the final payoff $S = N(T) \times \max\left( Y^{(1)}(T), K \right)$. Shades in the fan represent prediction intervals at the 50%, 80% and 95% level.
Application: Equity-linked life-insurance contracts

\[ RB(t) = \xi(t + 1) \cdot Y(t) - \xi(t) \cdot Y(t), \quad \forall t \in \{1, \ldots, T - 1\} \]

**Figure 5:** Left: Rebalancing cost of the hedging portfolio at any rebalancing times \( t = 1, \ldots, T - 1 \). Right: total rebalancing cost. Shades in the fan represent prediction intervals at the 50%, 80% and 95% level.
Figure 6: Number of asset units bought at time $t = 5$ in the risk-free asset and risky asset under the quantile hedging strategy as function of the asset price $Y^{(1)}(5)$. This strategy corresponds to the expression $h_6(Y_1(5), N(t)) = \xi(6)$, with fixed mortality $N(t) = \mathbb{E}[N(5)]$. 
Conclusion
• There is no general agreement on the valuation of the “residual part”:
  – Esscher valuation, e.g. Deelstra et al. (2020)
  – Standard deviation principle, e.g. Delong et al. (2019), Barigou & Delong (2022)
  – Cost-of-capital principle, e.g. Pelsser (2011)

• We proposed to quantile-hedge the residual part.
  – Leads a VaR-neutral portfolio.
  – The hedging portfolio is a TVaR deviation risk minimiser.
  – The residual risk can still be hedged by moving from quadratic to quantile objective.

• We proposed a simulation-based algorithm that is valid for general loss functions $\ell$ and non-linear optimisers.
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Thank you for your attention! Any questions?

Reference: Barigou, K., Bignozzi, V., & Tsanakas, A. (2022). Insurance valuation: A two-step generalised regression approach. ASTIN Bulletin: The Journal of the IAA, 52(1), 211-245.

Contact: karim.barigou@univ-lyon1.fr
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