Some considerations on the back door theorem and conditional randomization.

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Key words: causal inference, potential-outcome, identifiability, non-parametric structural equations, graphical methods

Abstract

In this work we propose a different \textit{surgical modified model} for the construction of counterfactual variables under non parametric structural equation models. This approach allows the simultaneous representation of counterfactual responses and observed treatment assignment, at least when the intervention is done in one node. Using the new proposal, the \textit{d}-separation criterion is used verify conditions related with ignorability or conditional ignorability and a new proof of the back door theorem is provided under this framework.

1 Introduction

In the potential outcome framework (Rubin, 1974), identifiability of the average treatment effect is guaranteed under the assumption of exchangeability (or ignorability). Let \( Y_t \) and \( Y_c \) denote the potential outcomes under treatment level \( t \) and \( c \), respectively. More precisely, \( Y_t \) is the outcome variable that would have been observed in a hypothetical world in which all individuals received treatment level \( t \), and \( Y_c \) is the outcome variable that would have been observed if all individuals were treated under level \( c \). If \( A \) denotes the observed treatment assignment, under exchangeability potential outcomes are independent of treatment assignment: \( Y_a \perp\!
\perp A \), for \( a = t, c \). If \( Y \) denotes the observed response, this assumption together with consistency (\( Y = Y_A \)) and positivity (\( 0 < P(A = t) < 1 \)), guarantees that \( E[Y_a] = E[Y|A = a] \), for \( a = c, t \) and so, the average treatment effect is identified by the distribution of observed data \( (A, Y) \), by the formula \( \text{ATE} = E[Y_t] - E[Y_c] = E[Y|A = t] - E[Y|A = c] \).

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When exchangeability is not a reasonable assumption, with the help of an observed vector $L$, conditional exchangeability still guarantees identifiability. Conditional exchangeability states that potential outcomes are independent of treatment assignment, given $L$: $Y_a \perp A \mid L$, for $a = c, t$.

This assumption together with consistency and positivity ($P(A = a \mid L = \ell) > 0$ if $P(L = \ell) > 0$), ensures that $E[Y_a] = E[E[Y \mid A = a, L]]$, for $a = c, t$, and so, the average treatment effect is identified by the distribution of observed data $(L, A, Y)$, by the formula $\text{ATE} = E[Y_t] - E[Y_c] = E[E[Y \mid A = t, L]] - E[E[Y \mid A = c, L]]$.

On the other hand, the $d$-separation criterion is a graphical tool designed to check independence and conditional independence between coordinates (or sub-vectors) of a random vector whose distribution satisfies the Markov factorization relative to a given directed acyclic graph (DAG). Throughout this work, we assume that the reader is familiarized with graphical models, DAG’s, and the results relating $d$-separation with conditional independences. Pearl’s book (Pearl, 2009a) contains all background information required to read this work.

Working with non parametric structural equation models (NPSEM), potential outcomes are defined replacing the equations related to the treatment nodes by the constants corresponding to the desired intervention: following Pearl’s work (Pearl (2009a)), if we are interested in $Y_t$, we use the modified model $M_t$, where the function associated to the node $A$ is fixed at $t$: $f_A = t$.

This construction does not allow $A$ (the observed treatment assignment) and $Y_t$ to be represented with the same set of structural equations. In this work we propose a new approach to construct the counterfactual variables using NPSEM in such a way that the potential outcomes and treatment variables are jointly represented by the same vector, at least when the intervention is done in one node. This allows the use of the $d$-separation rules to guarantee the identifiability of the distribution of counterfactual variables. Moreover, when the intervention addresses only one node, the assumptions of the back-door theorem imply conditional randomization.

The twin DAG’s presented by Balke and Pearl (1994) also allows to construct simultaneously observed and counterfactual variables. Richardson and Robins (2013) present a deep discussion about these models. However, considering that the back door theorem is one of the most popular criterion for identifying the distribution of counterfactual variables, we do care about constructing counterfactual variables and treatment assignments using the graph involved in the back door theorem, namely in the DAG where arrows emerging from nodes associated to the intervention are removed.

We started thinking about DAG’s and counterfactual variables a couple of years ago, inspired in the back door theorem. We felt that the $d$-separation criterion could be used to deduce randomization or conditional randomization, reconciling different approaches that give rise to the same formula for the distribution of the potential outcomes. We succeeded, at least, when the intervention is done in only one node and this case has been included in a Final Degree Thesis (Licenciatura en Matematica) (Cacheiro, 2011).

During the last years other authors have been concerned with this problem. In a working paper, Richardson and Robins (2013) present a graphical theory based on single world intervention graphs (SWIGs), unifying causal directed graphs and potential outcomes.

Although their graphs differ from the graphs in this paper, our perspectives are rather similar. One difference is that although they assume an underlying NPSEM, they do not assume independence of the disturbances (as we do) and prefer to work with the Finest Fully Randomized Causally Interpretable Structured Tree Graphs models (FFRCISTG).

In this work, we start assuming NPSEM with independent errors (NPSEM-IE). Although NPSEM-IE
are less general than FFRCISTG models, we decided to present our proposal under this setting, considering that these models are still adopted by a large portion of the community (like Pearl’s followers). However, in Section 4 we show that all the results presented in this work remain true if FFRCISTG models are assumed instead.

This work is organized as follows. In Section 2 we present a simple example with a three node DAG, explaining the main idea to construct jointly $A$ and $Y_a$. We check in this example that the relation with the back door theorem and conditional randomization holds. In Section 3 we generalize these results, first when the intervention is done in just one node and then generalized for many nodes. In Section 4 we discuss our results under FFRCISTG models.

To end this introduction we would like to establish a subtle difference of tenly omitted. Given a DAG $G$, we use $V = \{V_1, V_2, \cdots, V_n\}$ to denote the nodes of a graph, while random variables associated with a given node $V_i$ are denoted by $V_i$ or some perturbation of $V_i$, like $V_{i,t}$ or $V_i^t$, as it will be explained later on.

2 Toy Example - Main Idea

Assume that the causal diagram associated with the problem of interest is given by the DAG $G$

![Figure 1: the original DAG $G$.](image)

In terms of NPSEM’s (Pearl, 2009b) this means that there exists a set of functions $F = \{f_L, f_A, f_Y\}$ and jointly independent disturbances $U = \{U_L, U_A, U_Y\}$, that give rise to factual variables according to the following recursive system

$$
L = f_L(U_L), \quad A = f_A(L, U_A), \quad Y = f_Y(L, A, U_Y).
$$

We use $M = (F, U)$ to denote the model which defines factual variables. To emulate the intervention $do(a)$, Pearl (2009b) considers a model $M_a$ where the function $f_A$ is replaced by the constant $a$, while the disturbances remain unchanged: $M_a = (F_a, U)$, with $F_a = \{f_{a,L}, f_{a,A}, f_{a,Y}\}$, where

$$
f_{a,L} = f_L, \quad f_{a,A} = a, \quad f_{a,Y} = f_Y.
$$

Variables obtained iterating the functions in model $M_a$ using the same vector of disturbances $U = \{U_L, U_A, U_Y\}$ are denoted with subindex $a$: $L_a$, $A_a$ and $Y_a$. In this way, the counterfactual response of interest at level $a$ is given by $Y_a$.

Our proposal to represent counterfactual variables consist in the use of a new system of functions, in which the value $a$ is inserted in lieu of the variable corresponding to the node every time this one is required by the recursion. To do so we change the functions related to each node that have as parent. In the present example, $M^a = (F^a, U)$, with $F^a = \{f^a_L, f^a_A, f^a_Y\}$, where

$$
f^a_L = f_L, \quad f^a_A = f_A, \quad f^a_Y(\ell, u) = f_Y(\ell, a, u).
$$
Note that this new set of functions is compatible with the DAG $G_{\Delta}$, where arrows emerging from $A$ are removed:

![Diagram](image)

Figure 2: $G_{\Delta}$, constructed removing in $G$ arrows emerging from $A$.

Variables constructed iterating the functions in $F^a$ and using the same vector of disturbances $\{U_L, U_A, U_Y\}$ are denoted with supraindex $a$: $L^a$, $A^a$ and $Y^a$. Then, we get that the distribution of $(L^a, A^a, Y^a)$ is compatible with $G_{\Delta}$.

The following Lemma summarized the main results of this section.

**Lemma 1**

1. $L_a = L^a = L$, $A^a = A$ and $Y_a = Y^a$.

2. $A$ and $Y$ are $d$-separated by $L$ in $G_{\Delta}$ and so, since the distribution of $(L^a, A^a, Y^a)$ is compatible with $G_{\Delta}$, we get that $A^a$ is independent of $Y^a$ given $L^a$.

3. From the previous results, we conclude that $A$ is independent of $Y_a$ given $L$. Thus, conditional randomization holds.

### 3 Intervention with constant regimes

#### 3.1 First step: one node

Consider a causal DAG $G$ with nodes $V_1, \ldots, V_n$, labeled in a compatible way with $G$. Recall that in the graph terminology, we say that $V_i$ is a parent of $V_j$ if an arrow points from $V_i$ to $V_j$. We use $PA_G(V_j)$ to denote the set of parents of $V_j$ in $G$. If $V_i$ has a directed path to $V_k$ we say that $V_i$ is an ancestor of $V_k$, and use $An_G(V)$ to denote the set of ancestors of $V$ in $G$.

Consider a collection of independent random variables $U = \{U_1, \ldots, U_n\}$. Let $V_i$ denote the common support of any random variables associated with the node $V_i$ and let $\mathcal{U}_i$ denote the support of $U_i$. A set of functions $F = \{f_i : i \geq 1\}$ is said to be compatible with $G$ if for each $i = 1, \ldots, n$ we get that

$$f_i : \prod_{V_j \in PA_G(V_i)} V_j \times \mathcal{U}_i \longrightarrow V_i . \tag{1}$$

Given a set $F = \{f_i : i \geq 1\}$ of compatible functions with $G$, and independent $U = \{U_1, \ldots, U_n\}$, factual variables are defined by the recurrence

$$V_i = f_i(PA_i, U_i) ,$$

where $PA_i$ are the random variables (already defined by the recurrence) associated with the nodes in $PA_G(V_i)$. Note that, by construction, the distribution of $(V_1, \ldots, V_n)$ is compatible with $G$, meaning that
it satisfies the Markovian factorization induced by $G$. We use $M = (F, U)$ to denote the model that gives rise to factual variables.

In order to represent an intervention at level $a$ for a given node $A$, Pearl (2009b) defined the "Surgically modified model" $M_a = (F_a, U)$, considering $F_a = \{f_{a,i} : i \geq 1\}$, where $f_{a,i} = f_i$ if $V_i \neq A$ and for $V_j = A$, $f_{a,j} = a$. Counterfactual variables are defined by this new set of functions and the same disturbances $\{U_1, \cdots , U_n\}$, by the recurrence
\[
V_{a,i} = f_{a,i}(PA_{a,i}, U_i),
\]
where $PA_{a,i}$ are the random variables (already defined by the recurrence) associated with the nodes in $PA_G(V_i)$.

Before presenting our proposal for constructing counterfactual variables, recall that given a DAG $G$ and a node $A$ in $G$, $G_A$ is the graph obtained by removing from $G$ all arrows emerging from $A$. We will now introduce a new set of functions $F^a = \{f_i^a : i \geq 1\}$, compatible with $G_A$, that will allow the simultaneous definition of both the observed assignment random variable $A$ associated with the node $A$, and the counterfactual responses. To achieve this, if $A \notin PA_G(V_i)$ we get that $PA_G(V_i) = PA_G(V_i)$ and define $f_i^a$ being equal to $f_i$. When $A \in PA_G(V_i)$, $f_i^a$ is obtained fixing the value $a$ at the original function $f_i$. To be more precise, if $A \in PA_G(V_i)$, without loss of generality, we can assume that
\[
f_i : \prod_{V_j \in PA_G(V_i) \setminus A} V_j \times A \times U_i \rightarrow V_i,
\]
where $A$ denotes the set of possible values to be taken by variables associated to node $A$. Since $PA_G(V_i) = PA_G(V_i) \setminus A$ and $F^a$ should be compatible with $G_A$, we need $f_i^a$ to satisfy the following condition:
\[
f_i^a : \prod_{V_j \in PA_G(V_i) \setminus A} V_j \times U_i \rightarrow V_i.
\]
Then, for $\pi_i \in \prod_{V_j \in PA_G(V_i) \setminus A} V_j$, $u \in U_i$, we define
\[
f_i^a(\pi_i, u) = f_i(\pi_i, a, u).
\]
Let $V_i^a$ denote the variables obtained by the recurrence based on these new functions:
\[
V_i^a = f_i^a(PA_i^a, U_i),
\]
where $PA_i^a$ are the random variables (already defined by the recurrence) associated with the nodes in $PA_G(V_i)$. Note that the distribution of $(V_1^a, \ldots , V_n^a)$ is compatible with $G_A$. Let $M^a = (F^a, U)$.

The following Lemma explains how variables defined under models $M$, $M_a$ and $M^a$ are related.

**Lemma 2** The random variables associated with both modified models $M_a = (F_a, U)$ and $M^a = (F^a, U)$ are the same, with the exception of those associated with node $A$:
\[
V_i,a = V_i^a \quad \text{if} \quad V_i \neq A.
\]
Variables associated with the node $A$ defined by $M = (F, U)$ and $M^a = (F^a, U)$, respectively, are equal:
\[
A = A^a.
\]
Moreover, if $V_i$ is not a descendent of $A$, we get that
\[
V_i = V_{a,i} = V_i^a.
\]
To end this section, we state the back door Theorem (Pearl, 2009b), and provide a new proof of it.

**Theorem 3 The Back Door Criterion** Consider a set of nodes \( L \subset \{ V_1, \ldots, V_n \} \), such that \( L \cap A = \emptyset \). Assume that the following conditions hold:

1. No element of \( L \) is a descendent of \( A \) in \( G \),
2. \( L \) blocks all back door paths from \( A \) to \( Y \) in \( G \).

Then, \( Y_a \) in independent of \( A \) given \( L \) and so

\[
P(Y_a = y) = \sum_\ell P(Y = y|A = a, L = \ell)P(L = \ell).
\]

**Proof:** To prove that conditional ignorability holds, meaning that \( Y_a \) in independent of \( A \) given \( L \), we note that under the assumption of Theorem 3, considering the results presented in Lemma 2, we get that

1. If no element of \( L \) is a descendent of \( A \) in \( G \), then \( L = L_a = L^a \).
2. If \( L \) blocks all back door paths from \( A \) to \( Y \) in \( G \), then \( A \) and \( Y \) are \( d \)-separated by \( L \) in \( G_A \), and so \( A^a \) and \( Y^a \) are independent given \( L^a \).

Finally, applying again to the results stated in Lemma 2, we also know that \( A^a = A \) and \( Y^a = Y_a \). So, if \( L \) satisfies both conditions 1 and 2, we can conclude that \( Y_a \) is independent of \( A \) given \( L \). This means that conditional ignorability holds, as we wanted to prove. Thus, under positivity, the distribution of the counterfactual variables can be identified:

\[
P(Y_a = y) = \sum_\ell P(Y = y|A = a, L = \ell)P(L = \ell).
\]

\[\square\]

### 3.2 Intervention with constant regimes - many nodes

Assume now that we wish to intervene in a set of nodes \( A_{set} = \{ A_1, \ldots, A_k \} \). Consider \( a_i \in A_i \), where \( A_i \) denotes the support of variables associated with node \( A_i \), and let \( a = (a_1, \ldots, a_k) \). Following the new surgically modified model, we will change the functions related to those nodes whose parents have some \( A_j \).

As in the one node case, given a DAG \( G \), let \( M = (F, U) \) denote the model (compatible with \( G \)) for observed variables \( \{V_1, \ldots, V_n\} \). Let \( (V_{a,1}, \ldots, V_{a,n}) \) denote the vector of variables determined by the model \( M_a = (F_a, U) \) proposed by Pearl, with \( F_a = \{f_a,i : i \geq 1\} \), where \( f_a,i = f_i \) if \( V_i \) does not belong so the set \( A_{set} \), and when \( V_j = A_i \) for some \( i \), \( f_{a,j} = a_i \).

We will now generalize our construction presented for single node intervention in this new scenario. To do so, we consider \( M^a = (F^a, U) \), for \( F^a = \{f^a,i : i \geq 1\} \) compatible with \( G_{A_{set}} \), the graph obtained removing in \( G \) all arrows emerging from the set \( A_{set} \). Note that the set of parents of a given node \( V_i \) in \( G_{A_{set}} \) is obtained eliminating from the set of parents of \( V_i \) in the original DAG \( G \) all nodes in \( A_{set}^i = A_{set} \cap PA_G(V_i) \),
namely, we have that $\text{PA}_{G_{A_{\set}}} (\mathbf{V}_i) = \text{PA}_{G} (\mathbf{V}_i) \setminus \mathbf{A}_{\set}$ \textsuperscript{i}. Therefore, the definition of $f_i^a$ depends on whether the set $\mathbf{A}_{\set} = \mathbf{A}_{\set} \cap \text{PA}_{G} (\mathbf{V}_i)$ is empty or not. Now, if $\mathbf{A}_{\set} = \emptyset$, we get that $\text{PA}_{G_{A_{\set}}} (\mathbf{V}_i) = \text{PA}_{G} (\mathbf{V}_i)$ and we define $f_i^a = f_i$. When $\mathbf{A}_{\set} \neq \emptyset$, we can assume that

$$f_i : \prod_{v_j \in \text{PA}_{G} (\mathbf{V}_i) \setminus \mathbf{A}_{\set}} v_j \times \prod_{\mathbf{A}_j \in \mathbf{A}_{\set}} \mathbf{A}_j \times \mathbf{U}_i \rightarrow \mathbf{V}_i,$$

and consider

$$f_i^a : \prod_{v_j \in \text{PA}_{G} (\mathbf{V}_i) \setminus \mathbf{A}_{\set}} v_j \times \mathbf{U}_i \rightarrow \mathbf{V}_i,$$

where for $\mathbf{p}_i \in \prod_{v_j \in \text{PA}_{G} (\mathbf{V}_i) \setminus \mathbf{A}_{\set}} v_j$, $u \in \mathbf{U}_i$, we define

$$f_i^a (\mathbf{p}_i, u) = f_i (\mathbf{p}_i, a^i, u),$$

putting in $a^i$ all the coordinates of the vector $a = (a_1, \ldots, a_k)$ corresponding to the set $\mathbf{A}_{\set} = \mathbf{A}_j \in \mathbf{A}_{\set}$. In other words, when $\text{PA}_{G} (\mathbf{V}_i) \cap \mathbf{A}_{\set} \neq \emptyset$ we construct the function $f_i^a$ fixing at $f_i$ the value $a_j$, each time the value of the variable related to the node $\mathbf{A}_j$ is required by the original function $f_i$ (for each $j$ such that $\mathbf{A}_j \in \mathbf{A}_{\set}$).

Let $(\mathbf{V}_1^a, \ldots, \mathbf{V}_n^a)$ denote the vector of variables obtained by the recurrence based on these new functions $(F^a)$ and disturbances $U$. Once more, we get that the distribution of $(\mathbf{V}_1^a, \ldots, \mathbf{V}_n^a)$ is compatible with $G_{A_{\set}}$. The results are presented in what follows.

**Lemma 4** Let $A = (A_1, \ldots, A_k)$ and $A^a = (A_1^a, \ldots, A_k^a)$ denote the random variables related to the nodes $A_1, \ldots, A_k$, according to model $M$ and $M^a$, respectively. If $W \cap A_{\set} = \emptyset$, then the following version of the consistency assumption holds:

$$\{ A^a = a, \ W^a = w \} = \{ A = a, \ W = w \}.$$

The random variables associated with both modified models $M_a = (F_a, U)$ and $M^a = (F^a, U)$ are the same, with the exception of those associated with nodes in $A_{\set}$:

$$V_{i,a} = V_i^a \text{ \ if \ } V_i \notin A_{\set}. $$

Finally, we include a new proof of the Back Door Theorem, using the independences deduced from its assumptions and Lemma 4.

**Theorem 5** Back Door Criterion: Many Nodes Consider a set of nodes $L \subset \{ \mathbf{V}_1, \ldots, \mathbf{V}_n \}$, such that $L \cap A_{\set} = \emptyset$. Assume that the following conditions hold:

1. No element of $L$ is a descendent of $A_{\set}$.
2. $L$ blocks all back door paths from $A_{\set}$ to $Y$ in $G$.

Then,

$$P(Y_a = y) = \sum_{\ell} P(Y = y | A = a, L = \ell) P(L = \ell),$$
with $a = (a_1, \ldots, a_k)$.

**Proof:** Under the present assumptions we get that

1. If no element of $L$ is a descendent of $A_{\text{set}}$, then $L = L_a = L^a$.

2. If $L$ blocks all back door paths from $A_{\text{set}}$ to $Y$ in $G$, then $A_{\text{set}}$ and $Y$ are $d$-separated by $L$ in $G_{A_{\text{set}}}$, and so $A^a$ and $Y^a$ are independent given $L^a$.

Finally, if $L$ satisfies the previous conditions, by Lemma 4, we get that

$$\{A^a = a, L^a = \ell\} = \{A^a = a, L = \ell\}$$

and so, under positivity

$$P(Y^a = y) = P(Y^a = y) = \sum_{\ell} P(Y^a = y| L^a = \ell) P(L^a = \ell) = \sum_{\ell} P(Y^a = y| L^a = \ell, A^a = a) P(L^a = \ell) = \sum_{\ell} P(Y = y| A = a, L = \ell) P(L = \ell).$$

\[ \square \]

### 4 FFRCISTG models

In the previous results we have used the rules of $d$-separation to detect independence or conditional independence between variables of a random vector. To do so, given a graph $G$, all we required from the joint distribution of our vector was compatibility with $G$. When variables are constructed following a NPSEM-IE, the Markov factorization induced by $G$ holds automatically, and that is why our results are valid when the errors are independent.

However, the Markov factorization remains true under weaker conditions. For instance, let $v = (v_1, \ldots, v_n) \in \prod_{j=1}^n V_j$ and call $v_{pa_G(V_i)}$ the subvector of $v$ containing the coordinates related with nodes in the set $PA_G(V_i)$, namely $v_{pa_G(V_i)} = (v_j : V_j \in PA_G(V_i))$. If

$$\{f_i(v_{pa_G(V_i)}, U_i) : V_i \in G\}$$

is independent, for all $v \in \prod_{j=1}^n V_j$, (1)

then, the distribution of the vector whose variables are constructed with $M = (F, U)$ is compatible with the graph $G$. This condition, mainly defines the FFRCISTG models (Richardson and Robins 2013).

It is worthy to note that if $M = (F, U)$ satisfies condition (1) relative to $G$, the intervened model $M^a = (F^a, U)$ defined in Section 3.2 also satisfies condition (1) relative to $G_{A_{\text{set}}}$, since

$$\left\{f_i^a(v_{pa_G(V_i)}, U_i) : V_i \in G_{A_{\text{set}}} \right\} = \left\{f_i(v_{pa_G(V_i)}, U_i) : V_i \in G\right\}$$

where $v_{pa_G(V_i)}$ denotes the vector that results from replacing $v_j$ with $a_j$ for $\{j : A_j \in A_{\text{set}}\}$. Then, variables constructed by the model $M^a$ satisfy the factorization induced by the graph $G_{A_{\text{set}}}$, allowing the use of $d$-separation rules, and thus, extending our results for this new model.
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