Set of dynamic restrictions imposed on robotic arm-based motion simulator phase coordinates

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Abstract. The paper discusses the motion cues simulation problem for aerospace applications. An algorithm was suggested for controlling a motion stand based on an industrial robotic manipulator. The paper provides simple mathematical model reflecting the movements of manipulator's endpoint in the vertical plane with the base performing rotational movements and presents a restriction set in the configuration space. We provide an algorithm for manipulator's stopping and returning to the start position of a motion simulation process active phase. A set of restrictions for the motion simulation active phase is also obtained.

1. Introduction

Mobile stands are actively used to provide motion simulation of aircraft movements when astronauts and pilots are trained. The purpose of motion simulation process is to simulate acceleration and angular motion characteristics on a mobile stand. The resulting forces and moments of forces should imitate real effects they have on the elements of the human bionavigation system (proof masses of astronaut's or pilot's vestibular system). Motion simulation is further described in [1]. Current mobile stands are most widely represented by various versions of platforms (in particular, the Stewart platform) and centrifuges. Motion simulation algorithms for such stands are described in the following papers: [2-4].

Modern industrial robotic manipulators can be used as simulator-stands combined with compact virtual reality (VR) technologies. The stand based on a robotic manipulator has higher dynamic capabilities than the above-mentioned configurations. Therefore, it is viewed as a new promising device that can be used in simulators, which is outlined in article [5].

This paper examines phase 2 of the motion simulation process: manipulator's slow return (with accelerations below the threshold of operator's vestibular system sensitivity) to a set of initial positions. Optimal control has been built for this phase, and a constraint zone within phase space has been found based on the above-mentioned control which allow for active motion simulation.

2. Motion Stand Structure.

Fig. 1 shows the simplest structure of the stand based on the robotic manipulator. Let O₃ denote manipulator's end effector, where a cockpit with a pilot is placed. In this case we assume that the size of the cockpit is insignificant and the pilot's proof mass is also concentrated at one point.
Let \( l_1 \) and \( l_2 \) denote the length of the first and second arms, \( \varphi_1 \) - first arm's angle with respect to the vertical, \( \varphi_2 \) - second arm's angle with respect to the vertical, \( \alpha \) - second arm's angle with respect to the first arm. Torques \( M_1 \) and \( M_2 \) are exerted in cylindrical joints \( O_1 \) and \( O_2 \) corresponding to manipulator's electric engine torques. We will assume that the manipulator has a moving base.

### 2.1. The set of restrictions for manipulator's end effector.

The coordinates of end effector \( O_3 \) satisfy the following relations:

\[
\begin{align*}
(x &= l_1 \sin \varphi + l_2 \sin(\phi + \alpha), \\
(y &= l_1 \cos \varphi + l_2 \cos(\phi + \alpha),
\end{align*}
\]

Note that \( \varphi_{\text{min}} \leq \varphi \leq \varphi_{\text{max}} \) and \( \alpha_{\text{min}} \leq \alpha \leq \alpha_{\text{max}} \). Fig. 2 shows a set of acceptable geometric positions \( P \) of a point \( O_3 \) in the vertical plane. The contour of a set \( P \) consists of arcs of several circles. The radius of the larger arc with centre \( O_1 \) is \( r = l_1 + l_2 \), and the radius of the smaller one \( \rho \) can be
calculated using the formula:

\[ \rho = \sqrt{l_1^2 + l_2^2 + 2l_1l_2 \cos \alpha_{\text{max}}} \]  

(2)

The set \( P \) has an unsymmetrical shape. To solve a motion simulation problem, it is necessary to have an area where it is possible to move with maximum acceleration equally in all directions from a current start position. A group of circles with radius \( R = l_1 + l_2 - \rho \) can be inscribed in a set \( P \), with their centres together making up an arc \( AB \). The circles themselves form an area \( D \) (shaded area in Fig. 2), which lies in \( P \). We will call this arc \( AB \) 'starting'.

2.2. The problem of stand's slow stop and return

Consider a certain model of an aircraft. Let us know all the forces acting on the aircraft, the parameters of the aircraft and the flight path. We need to simulate the vector of gravitational-inertial forces acting on the pilot.

Let the mass \( M \) of the aircraft be constant (fuel burns out fairly evenly and slowly). It is possible to consider a fairly small-time interval at which the fuel mass is constant. In this case:

\[ M(\vec{W} - \vec{g}) = \vec{T} + \vec{R}_{\text{aero}}, \]

where \( \vec{T} \) – engine forces, \( \vec{R}_{\text{aero}} \) – aerodynamic forces[1].

Let us introduce the vector of sensed acceleration:

\[ \vec{a} = \vec{W} - \vec{g}. \]

The overload vector (Fig. 3): \( \vec{n} = \frac{\vec{a}}{g_0} \), where \( g_0 \) is G-force absolute value on certain height.

![Fig. 3. Aircraft’s overload](image)

We will assume that the pilot is fastened in his seat and cannot make significant changes in his position \( N_p \) (Fig. 4). Therefore, we will perform calculations at the point N, which coincides with the center of mass of the pilot.

![Fig. 4. Pilot’s overload](image)

Since the pilot is not in the center of mass of the aircraft, additional accelerations appear:
\[ \vec{W}^N = \vec{W}^C + \vec{W}^\alpha, \] where \( \vec{W}^N \) is total pilot’s acceleration, \( \vec{W}^\alpha \) is additional accelerations caused by rotation.

Thus, the pilot overload vector is \( \vec{n}^p = \vec{n}^C + \frac{\vec{W}^\alpha}{g_0} \).

Motion simulation process consists of two stages:
1. active simulation phase: such dynamic control of the motion stand to ensure that pilot overload on the stand feels “similar” to the simulated \( \vec{n}^p \) (known from simulated aircraft dynamics),
2. stop-phase: stopping with acceleration considered to be subthreshold for the human vestibular system and, usually, returning to a start position.

2.3. The problem of stand's slow stop and return
We will set the optimal control task for the manipulator's fastest stopping and returning to the set of initial positions of the active simulation phase. We will choose an arc \( \overline{AB} \) as a set of initial positions. We assume that vestibular system sensitivity threshold for acceleration can be given separately on all axes and it is \( \delta \). We will introduce a fixed reference frame \( O_{x_1y_1z_1} \) and a moving one \( O_{xyz} \) (Fig. 1), which rotates with an angular velocity \( \vec{\omega}_\theta = (0, \omega, 0) \).

We will assume that the centrifugal and Coriolis forces of inertia are balanced out by the reaction forces of robotic manipulator's rigid structure, but they will be exerted on the human vestibular system, so we will present manipulator endpoint dynamics equations in a moving reference frame as follows:

\[
\begin{align*}
\dot{x} &= u_1 + (\omega^2 + \dot{\omega})x - 2\dot{z}\omega \\
\dot{y} &= u_2 \\
\dot{z} &= 2\dot{x}\omega
\end{align*}
\] (3)

Here \(-\delta \leq u_1 \leq \delta\) is \( x \)-axis control and \(-\delta \leq u_2 \leq \delta\) is \( y \)-axis control, which further help calculate engine torques \( M_1 \) and \( M_2 \) given the characteristics of the manipulator-stand. The problem of calculating the above-mentioned torques is beyond the scope of this article.

We will define the initial conditions that we get when the active simulation phase is over:
\[
x(0) = x_0, \quad \dot{x}(0) = v_1, \quad y(0) = y_0, \quad \dot{y}(0) = v_2.
\]

Final conditions: \( x(t_k) = x_k, \quad y(t_k) = y_k, \quad \dot{x}(t_k) = 0, \quad \dot{y}(t_k) = 0, (x_k, y_k) \in \overline{AB} \)

To obtain a stop-phase algorithm, we solve an extremum problem for (3):
\[
J = t_k \rightarrow \min_{u_1,u_2} \quad (4)
\]
t_k — total motion time to the endpoint on the arc \( \overline{AB} \) (Fig. 2). Using the Pontryagin's maximum principle we can find a solution of this problem [6].

2.4. Limited value of the speed of movement towards the frontier of the set of acceptable positions
Manipulator's end effector has a limited area of movement \( D \) (Fig. 5).
To stay within the geometric constraints, the speed of movement of the end effector at the boundary of the region $D$ must be zero. Let the manipulator be at a point $K$ at the first stage of motion simulation. The relevant nearest boundary point of the set $D$ is a point $K_B$. According to the control synthesis obtained for (4) if it moves from point $K$ to point $K_B$, it will stop with acceleration shown on $x$-axis that equals $-(\delta_1 + \omega^2x(t_k))$. This is how we can build a motion trajectory passing through the point $K_BX$ on the phase plane $xO\dot{x}$ and find velocity value $K_{VX}$ corresponding to the point $K_X$, as it is shown in Fig. 6.

![Fig. 6. Optimal movement to the frontier on the x-axis](image)

Likewise, it is possible to find a point $L_{BX}$, a trajectory point and find the corresponding velocity value $L_{VX}$ for the point $L_X$, as well as the velocity along the $y$-axis, with the only difference being that the trajectories on the phase plane $yO\dot{y}$ are not hyperbolas, but parabolas.

Thus, based on the optimal solution of (4) and the geometric constraints one can get a set $(x, v_x, y, v_y)$ where $x$ and $y$ are the coordinates of the points from the set $D$, and $v_x$ and $v_y$ show velocities along the $x$-axis and $y$-axis towards the boundary of the region $D$. This set is shown in Fig. 7 in such a way that the vertical axis shows velocity modulus $|v| = \sqrt{v_x^2 + v_y^2}$. To calculate the values, we have used the parameters of KR 470-2 PA robot and the sensitivity value of $\delta = 14$ mm/s in accordance with those presented in [7].

![Figure 7. Values of the modulus of the speed of movement towards the border](image)
3. Conclusion
We suggested an algorithm for controlling a motion stand based on an industrial robotic manipulator. The algorithm consists of two stages. Special attention has been given to stage 2, the task of end effector’s returning to the start position. The area where it can move during the motion simulation active phase has been found.

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