ABSTRACT

While low-rank matrix prior has been exploited in dynamic MR image reconstruction and has obtained satisfying performance, low-rank tensors models have recently emerged as powerful alternative representations for three-dimensional dynamic MR datasets. In this paper, we introduce a model-based deep learning network by learning the tensor low-rank prior of the cardiac dynamic MR images. Instead of representing the dynamic dataset as a low-rank tensor directly, we propose a learned transformation operator to exploit the tensor low-rank property in a transform domain. In particular, by generalizing the t-SVD tensor decomposition into a unitary transformed t-SVD, we define a transformed tensor nuclear norm (TTNN) to enforce the tensor low-rankness. The dynamic MRI reconstruction problem is thus formulated using a TTNN regularized optimization problem. An iterative algorithm based on ADMM used to minimize the cost is unrolled into a deep network, where the transform is learned using convolutional neural networks (CNNs) to promote the reconstruction quality in the feature domain. Experimental results on cardiac cine MRI reconstruction demonstrate that the proposed framework is able to provide improved recovery results compared with the state-of-the-art algorithms.

Index Terms— dynamic MR imaging, transform-based tensor low-rank, unrolling network

1. INTRODUCTION

Dynamic magnetic resonance imaging plays an important role in multiple clinical applications including cardiac, perfusion, and vocal tract imaging. Dynamic MRI collects more information than static MRI, which is helpful in detection of certain type of diseases (e.g., cardiovascular diseases). However, it is usually challenging to obtain dynamic MR images with high spatiotemporal resolution within clinically acceptable scan time. Low-rank Casorati matrix prior [1] has been utilized to reconstruct dynamic MR images, but it breaks the high-dimensional structure and may obtain suboptimal results.

Recently, the low-rank tensor-based models [2, 3] have been proposed, and had a wide range of applications, e.g., tensor completion, image denoising, and image reconstruction. In detail, the low-rank tensor priors are constructed via tensor decomposition, and there are two main tensor decompositions, i.e., the CP decomposition and the Tucker decomposition. For dynamic MRI reconstruction, the low-rank tensor prior under the Tucker decomposition [4] has been exploited, but the enhancement of reconstruction performance is limited.

Lately, the tensor singular value decomposition (t-SVD) [5] and the tensor nuclear norm (TNN) [6] have been proposed and obtained superior performance in the application of tensor completion. Furthermore, t-SVD is based on fast Fourier transform, which can be generalized into other transforms [7]. Song et al. [8] generalized the t-SVD based on FFT into the tt-SVD based on any one-dimensional unitary transform and obtained better tensor completion performance. They also found that different transform obtains significantly different performances. In that way, designing a proper transform becomes extremely important, but how to design is quite challenging. Meanwhile, the one-dimensional unitary transform is strict, which limits its application. For dMRI reconstruction, Ai et al. [9] has adopted the t-SVD based on FFT to reconstruct dMRI, but the reconstruction improvement is not obvious.

On the other hand, the unrolling deep neural network, which unrolls an iterative optimization method into a deep learning based network, attracts much attention and has been applied in dMRI reconstruction [10, 11, 12]. By integrating the model prior and the learning ability, the unrolling deep learning methods have obtained significant improvement in dMRI reconstruction.

In this paper, we propose a learned transform-based deep tensor low-Rank network (DTLR-Net) to reconstruct dMRI image, which adopts the tt-SVD based on a deep neural network learned transform. Specifically, we first generalize the t-SVD into a tt-SVD on any unitary transform and give the TTNN based on tt-SVD, then a TTNN minimization iterative algorithm is proposed to reconstruct dynamic MR image. Finally, A deep neural network unrolling the TTNN algorithm but learning the transform by CNN is proposed to exploit the transformed low-rank tensor prior in Dynamic MR
imaging. The proposed DTLR-Net brings a new perspective of exploiting the tensor low-rank prior by enforcing a low-
rank constraint on the feature domain learned by the neural network. Reconstruction results on a prospective Cine MR
dataset (real-time OCMR [13]) demonstrate the superior performance of the proposed TLR-Net over state-of-art methods.

2. THE TRANSFORMED TENSOR NUCLEAR NORM

In this paper, we denote tensors by Euler script letters, e.g., $\mathcal{X}$, matrices by bold capital letters, e.g., $\mathbf{X}$, vectors by bold lowercase letters, e.g., $x$, and scalars by lowercase letters, e.g., $x$. For a 3-way tensor $\mathcal{X} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$, we denote $\mathcal{X}^{(i)}$ as the $i$th frontal slice $\mathcal{X}(:,:,i)$, $i = 1, 2, ..., n_3$.

Let $\tilde{\mathcal{X}}_\mathcal{F}$ be the tensor obtained via applying a unitary transform $\mathcal{T}$ on $\mathcal{X}$, i.e., $\tilde{\mathcal{X}}_\mathcal{F} = \mathcal{T}(\mathcal{X})$. Due to the unitary property, $\mathcal{X}$ can also be obtained by applying the Hermitian transposed transform $\mathcal{T}^H$ on $\tilde{\mathcal{X}}_\mathcal{F}$, i.e., $\mathcal{X} = \mathcal{T}^H(\tilde{\mathcal{X}}_\mathcal{F})$. For the sake of illustration, we construct a block diagonal matrix based on the frontal slices of $\tilde{\mathcal{X}}_\mathcal{F}$ as follows,

$$bdiag(\tilde{\mathcal{X}}_\mathcal{F}) = \begin{pmatrix} \tilde{\mathcal{X}}^{(1)}_\mathcal{F} \\ \tilde{\mathcal{X}}^{(2)}_\mathcal{F} \\ \vdots \\ \tilde{\mathcal{X}}^{(n_3)}_\mathcal{F} \end{pmatrix},$$

(1)

and the block diagonal matrix can be converted back into a tensor by the following fold operator,

$$fold(bdiag(\tilde{\mathcal{X}}_\mathcal{F})) = \tilde{\mathcal{X}}_\mathcal{F}.$$  

(2)

The $\mathcal{T}$-product of $\mathcal{A} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$ and $\mathcal{B} \in \mathbb{C}^{n_2 \times n_3 \times n_3}$ based on a unitary transform $\mathcal{T}$ is a tensor $\mathcal{C} \in \mathbb{C}^{n_1 \times n_3 \times n_3}$, which can be expressed as

$$\mathcal{C} = \mathcal{A} \ast \mathcal{F} \ast \mathcal{B} = \mathcal{T}^H(fold(bdiag(\tilde{\mathcal{A}}_\mathcal{F}) \times bdiag(\tilde{\mathcal{B}}_\mathcal{F}))),$$

(3)

where ‘×’ denotes the standard matrix product. Then, the transformed tensor singular value decomposition of $\mathcal{X} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$ can be derived as follows

$$\mathcal{X} = \mathcal{U} \ast \mathcal{F} \ast \mathcal{S} \ast \mathcal{V}^H,$$

(4)

where $\mathcal{U} \in \mathbb{C}^{n_1 \times n_1 \times n_3}$ and $\mathcal{V} \in \mathbb{C}^{n_2 \times n_2 \times n_3}$ are unitary tensors with respect to $\mathcal{T}$-product, and $\mathcal{S}$ is a tubal diagonal tensor. Note that

$$\tilde{\mathcal{X}}^{(i)} = \tilde{\mathcal{U}}^{(i)} \tilde{\mathcal{S}}^{(i)} \tilde{\mathcal{V}}^{(i)H},$$

(5)

i.e., the transformed t-SVD is computed in the unitary $\mathcal{T}$ transformed domain via computing the matrix SVDs of every frontal slice of $\tilde{\mathcal{X}}_\mathcal{F}$. The transformed multirank of $\mathcal{X}$ is a vector $\mathbf{r} \in \mathbb{R}^{n_3}$ with its $i$th entry being the rank of $\tilde{\mathcal{X}}^{(i)}$. Based on the derivation in [8] and [6], we can obtain the transformed tensor nuclear norm (TTNN) of a tensor $\mathcal{X} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$, denoted as $\|\mathcal{X}\|_{TTNN}$, as the sum of the nuclear norms of all frontal slices of $\tilde{\mathcal{X}}_\mathcal{F}$, i.e.,

$$\|\mathcal{X}\|_{TTNN} = \sum_{i=1}^{n_3} \|\tilde{\mathcal{X}}^{(i)}\|_*,$$

(6)

which can also be considered as the convex envelope of the sum of the entries of the transformed multirank, similar as the original TNN [6].

The tensor completion problem based on TTNN can be formulated as follows:

$$\min_{\mathcal{X}} \tau \|\mathcal{X}\|_{TTNN} + \frac{1}{2} \|\mathcal{X} - \mathcal{Y}\|_F^2,$$

(7)

where $\mathcal{Y} = \mathcal{U} \ast \mathcal{F} \ast \mathcal{S} \ast \mathcal{V}^H \in \mathbb{C}^{n_1 \times n_2 \times n_3}$ is the collected data, $\tau > 0$ is the balancing parameter, and $\mathcal{X}$ is the data to be recovered. In order to solve the above problem, we propose the transformed tensor singular value thresholding algorithm ($\mathcal{T}$-TSVT) by defining the $\mathcal{T}$-TSVT operator $\mathcal{P}_{\tau,\mathcal{F}}$ as follows

$$\mathcal{P}_{\tau,\mathcal{F}}(\mathcal{Y}) = \mathcal{U} \ast \mathcal{F} \ast \mathcal{S} \ast \mathcal{V}^H,$$

(8)

where $\mathcal{S} = \mathcal{T}^H(\hat{\mathcal{S}}_{\mathcal{F},\tau})$ and $\hat{\mathcal{S}}_{\mathcal{F},\tau} = \max(\hat{\mathcal{S}}_{\mathcal{F},\tau} - \tau, 0)$. Specifically, the $\mathcal{T}$-TSVT operator first transforms $\mathcal{Y}$ into the unitary $\mathcal{F}$ transformed domain to obtain $\hat{\mathcal{Y}}_{\mathcal{F}}$. Then the matrix SVT is applied on every frontal slice of $\hat{\mathcal{Y}}_{\mathcal{F}} = \mathcal{U}_{\mathcal{F}} \mathcal{S}_{\mathcal{F}} \mathcal{V}_{\mathcal{F}}^H$, i.e., for the $i$th frontal slice, $\mathcal{P}_{\tau,\mathcal{F}}(\hat{\mathcal{Y}}_{\mathcal{F}}) = \mathcal{U}_{\mathcal{F}}^{(i)} \mathcal{S}_{\mathcal{F}}^{(i)} \mathcal{V}_{\mathcal{F}}^{(i)H}$. Finally, $\mathcal{F}$ is applied to convert the image back to the original image domain.

3. THE PROPOSED DTLR-Net

3.1. The TTNN-regularized minimization algorithm

We denote the distortion-free dynamic MR image as $\mathcal{X} \in \mathbb{C}^{n_x \times n_y \times n_t}$, where $n_x$ and $n_y$ denote the spatial coordinates, and $n_t$ is the temporal coordinate. The data acquisition of dMRI can be modeled as

$$\mathbf{b} = \mathcal{A}(\mathcal{X}) + \mathbf{n}$$

(9)

where $\mathbf{b} \in \mathbb{C}^m$ is the observed undersampled $k$-space data, $\mathcal{A} : \mathbb{C}^{n_x \times n_y \times n_t} \rightarrow \mathbb{C}^m$ is the Fourier undersampling operator, and $\mathbf{n} \in \mathbb{C}^m$ is the Gaussian distributed white noise.

The dMRI reconstruction problem can be formulated using the TTNN-regularized minimization algorithm as follows

$$\min_{\mathcal{A}} \frac{1}{2} \|\mathcal{A}(\mathcal{X}) - \mathbf{b}\|_F^2 + \lambda \|\mathcal{X}\|_{TTNN},$$

(10)

where $\lambda$ is the balancing parameter. The optimization problem (10) can be converted into the following constrained problem by the variable splitting strategy.

$$\min_{\mathcal{A}} \frac{1}{2} \|\mathcal{A}(\mathcal{X}) - \mathbf{b}\|_F^2 + \lambda \|\mathcal{Z}\|_{TTNN} \text{ s.t. } \mathcal{Z} = \mathcal{X}$$

(11)
The above problem can be rewritten as an unconstrained problem using the augmented Lagrangian function as follows:

\[
\mathcal{L}(\mathbf{X}, \mathbf{Z}, \mathbf{W}) = \frac{1}{2} \| \mathbf{A}(\mathbf{X}) - \mathbf{b} \|^2_F + \lambda \| \mathbf{Z} \|_{TTNN} + \left< \mathbf{W}, \mathbf{Z} - \mathbf{X} \right> + \frac{\mu}{2} \| \mathbf{Z} - \mathbf{X} \|^2_F
\]

where \( \mathbf{W} \) is the Lagrangian multiplier. The above problem can be efficiently solved with the alternating direction method of multipliers algorithm (ADMM), which yields to solving the following subproblems:

\[
\mathbf{Z}_n = \min_{\mathbf{Z}} \lambda \| \mathbf{Z} \|_{TTNN} + \frac{\mu}{2} \| \mathbf{Z} - \mathbf{X}_{n-1} + \frac{\mathbf{W}_{n-1}}{\mu} \|^2_F
\]

\[
\mathbf{X}_n = \min_{\mathbf{X}} \frac{1}{2} \| \mathbf{A}(\mathbf{X}) - \mathbf{b} \|^2_F + \frac{\mu}{2} \| \mathbf{Z}_n - \mathbf{X} + \frac{\mathbf{W}_{n-1}}{\mu} \|^2_F
\]

\[
\mathbf{W}_n = \mathbf{W}_{n-1} + \eta(\mathbf{Z}_n - \mathbf{X}_n)
\]

where the subscript \( n \) denotes the \( n \)th iteration. The subproblem (13) can be solved by the \( \mathcal{T} \)-TSVT algorithm in (8), and the \( \mathbf{X} \) subproblem (14) is a quadratic problem, which can be solved analytically. Thus, subproblems (13) to (15) can be efficiently solved using the following iterative steps:

\[
\begin{cases}
\mathbf{Z}_n = \mathcal{D}_{\lambda/\mu, \mathcal{T}}(\mathbf{X}_{n-1} - \frac{\mathbf{W}_{n-1}}{\mu}) \\
\mathbf{X}_n = (\mathbf{A}^H \mathbf{A} + \mu I)^{-1}(\mathbf{A}^H \mathbf{b} + \mu \mathbf{Z}_n + \mathbf{W}_{n-1}) \\
\mathbf{W}_n = \mathbf{W}_{n-1} + \eta(\mathbf{Z}_n - \mathbf{X}_n)
\end{cases}
\]

where 1 denotes an all-one tensor.

### 3.2. The DTLR-Net framework

Dynamic MR images reconstruction using the TTNN regularization algorithm illustrated above involves iteratively solving (16), which requires time-consuming parameters tuning. On the other hand, it is challenging to empirically select an appropriate unitary transform \( \mathcal{T} \) to obtain the optimal results. Moreover, the optimal \( \mathcal{T} \) may be different under different scenarios, which limits the application of the algorithm.

To address the abovementioned issues, we unroll the TTNN regularization algorithm into a deep neural network, which we term as deep tensor low rank network (DTLR-Net), where the hyperparameters and the unitary transform are learned by the proposed network. The framework of DTLR-Net is shown in Fig.1, where DTLR-Net unrolls the iterative steps of the TTNN regularization algorithm into a fixed number of iteration modules, each of which contains three blocks corresponding to the three subproblems in (16), i.e., the transformed low-rank tensor prior block \( \mathbf{Z}_n \), the reconstruction block \( \mathbf{X}_n \), and the multiplier update block \( \mathbf{W}_n \). In Fig.1, \( N \) denotes the number of iteration modules, and the subscript \( n \) denotes the \( n \)th iteration module. To be exact, the \( \mathbf{X}_n \) and \( \mathbf{W}_n \) blocks are conducted in the tensor operation manner which is same as the computation of iterative algorithm (16) except that the hyperparameters \( \lambda, \mu, \eta \) are learned. For the transformed low-rank tensor prior block \( \mathbf{Z}_n \), DTLR-Net conducts the \( \mathcal{T} \)-TSVT operation by learning the optimal transform \( \mathcal{T} \). Note that the transform \( \mathcal{T} \) and \( \mathcal{T}^H \) are learned by two separate non-interacting CNNs, which have the same structure containing three 3*3*3 convolutional blocks with 16, 16, and 2 channels, respectively. Moreover, we do not strictly constrain \( \mathcal{T} \) and \( \mathcal{T}^H \) to be unitary. Instead, a relaxed constraint that enforces the two transforms to be inverse, i.e., \( \mathcal{T}^H(\mathcal{T}(\mathbf{X})) = \mathbf{X} \), is incorporated into the loss function during the network training.

Specifically, according to the \( \mathcal{T} \)-TSVT operation mentioned in Section 2, the transformed low-rank tensor prior block first learns an optimized transform \( \mathcal{T} \) and convert \( \mathbf{X}_{n-1} - \frac{\mathbf{W}_{n-1}}{\mu} \) to the transformed domain, then applies the matrix SVTs on every frontal slice of the transformed image, and finally applies \( \mathcal{T}^H \) to convert the image back to the original image domain. The proposed DTLR-Net brings a new perspective of exploiting the tensor low-rank prior by enforcing a low-rank constraint on the feature domain learned by the neural network. The way that we exploit the tensor low-rank
property can be easily generalized to other problems, such as tensor completion [14] and robust PCA [8] problems.

The loss function of DTLR-Net is the combination of data fidelity and the invertibility of the transforms, i.e.,

\[
\text{Loss} = \sum_{(\tilde{X}, b) \in \Omega} ||\tilde{X} - f_{cnn}(b; \theta)||_F^2 + \zeta \sum_{n=1}^N \|\mathcal{F}(\mathcal{F}^{-1}(X_{n-1})) - X_{n-1}\|_F^2,
\]

where \(\Omega\) denotes the given training data, \(\tilde{X}\) is a fully sampled ground-truth data, \(b\) is the undersampled \(k\)-space data, \(f_{cnn}\) denotes the output of the network, and \(\zeta\) is a hyperparameter balancing the data fidelity and the invertibility of the transforms.

4. EXPERIMENTAL RESULTS

We evaluate the proposed DTLR-Net using the real-time OCMR dataset [12], which is a prospective cine MR dataset that contains 57 slices of fully sampled data collected on a 3T Siemens MAGNETOM Prisma machine and 126 slices of fully sampled data on a 1.5T Siemens Avanto and a 1.5T Siemens Sola machine. In order to investigate the robustness of the proposed network, we use mixed data under different settings, including 51 slices of 3T data for training, and 7 slices of 3T and 13 slices of 1.5T data for testing. In particular, we crop the training dynamic MR images into the size of \(144 \times 112 \times 16\), and the strides along the three dimensions are 15, 15, and 7. Finally, we obtain 1099 MR datasets with the size of \(144 \times 112 \times 16\) for training and 22 uncropped MR datasets for testing. All the multi-coil data are combined into single-coil images, and the coil sensitivity maps are computed by ESPIRiT [15]. For module configuration, we set DTLR-Net with 15 iteration modules. We adopt the exponential decay learning rate with an initial learning rate of 0.001 and a decay of 0.95. The Adam optimization is adopted in the DTLR-Net training and the hyperparameter \(\zeta\) is set to 0.001. In this paper, we train the DTLR-Net and the other deep learning based methods under comparisons with 50 epochs.

In order to evaluate the performance in dynamic MR images reconstruction, we compare the proposed DTLR-Net with two iterative algorithms, namely, k-t SLR [1] and LR (Casorati matrix low-rank regularization algorithm), and two unrolling deep learning methods, namely DC-CNN [11] and SLR-Net [12]. The reconstruction results are shown in Fig. 2.

![Fig. 2](image)

Table 1: The quantitative evaluations of five methods, where the SNR column shows the average SNR and the standard deviation on 22 test data.

| Method       | SNR (dB)       | Parameters | Time (s) |
|--------------|----------------|------------|----------|
| LR           | 16.86±3.00     | /          | 15.88 (CPU) |
| k-t SLR      | 18.54±1.91     | /          | 364.81 (CPU) |
| DC-CNN       | 18.53±1.64     | 954340     | 1.34 (GPU) |
| SLR-Net      | 19.24±1.76     | 293808     | 3.51 (GPU) |
| TLR-Net      | 21.40±2.19     | 259244     | 3.84 (GPU) |

The quantitative evaluations are reported in Table 1. It is shown that DTLR-Net requires the least parameters, and can provide the highest average SNR with an improvement of around 2dB over the SLR-Net. Note that the running time of DTLR-Net is slightly longer than SLR-Net, which may be due to the computation of the transformed t-SVD.

5. CONCLUSION AND DISCUSSION

We proposed a novel deep network that learns the transform-based tensor low-rank prior in dynamic MR imaging. Specifically, by generalizing the t-SVD decomposition to a unitary transformed t-SVD, we introduce the TTNN regularization problem to reconstruct the dynamic MR images. The transformed tensor singular value thresholding algorithm is used to efficiently solve the optimization problem using ADMM. By unrolling the iterative steps, we proposed the DTLR-Net. Instead of using a fixed unitary transformation, the proposed network is able to fully exploit the tensor low-rank property of the dataset in an optimized learned feature domain. Reconstruction results based on the OCMR dataset demonstrated the superior reconstruction performance of our network compared with the state-of-the-art algorithms.
6. REFERENCES

[1] Sajan Goud Lingala, Yue Hu, Edward DiBella, and Mathews Jacob, “Accelerated dynamic mri exploiting sparsity and low-rank structure: kt slr,” IEEE transactions on medical imaging, vol. 30, no. 5, pp. 1042–1054, 2011.

[2] Tamara G Kolda and Brett W Bader, “Tensor decompositions and applications,” SIAM review, vol. 51, no. 3, pp. 455–500, 2009.

[3] Bo Zhao, Kawin Setsompop, David Salat, and Lawrence L Wald, “Further development of subspace imaging to magnetic resonance fingerprinting: A low-rank tensor approach,” in 2020 42nd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC). IEEE, 2020, pp. 1662–1666.

[4] Kaiyan Cui, “Dynamic mri reconstruction via weighted tensor norm regularizer,” IEEE Journal of Biomedical and Health Informatics, vol. 25, no. 8, pp. 3052–3060, 2021.

[5] Misha E Kilmer and Carla D Martin, “Factorization strategies for third-order tensors,” Linear Algebra and its Applications, vol. 435, no. 3, pp. 641–658, 2011.

[6] Canyi Lu, Jiashi Feng, Yudong Chen, Wei Liu, Zhouchen Lin, and Shuicheng Yan, “Tensor robust principal component analysis with a new tensor nuclear norm,” IEEE transactions on pattern analysis and machine intelligence, vol. 42, no. 4, pp. 925–938, 2019.

[7] Eric Kernfeld, Misha Kilmer, and Shuchin Aeron, “Tensor–tensor products with invertible linear transforms,” Linear Algebra and its Applications, vol. 485, pp. 545–570, 2015.

[8] Guangjing Song, Michael K Ng, and Xiongjun Zhang, “Robust tensor completion using transformed tensor singular value decomposition,” Numerical Linear Algebra with Applications, vol. 27, no. 3, pp. e2299, 2020.

[9] Jianhang Ai, Shuli Ma, Huiqian Du, and Liping Fang, “Dynamic mri reconstruction using tensor-svd,” in 2018 14th IEEE International Conference on Signal Processing (ICSP). IEEE, 2018, pp. 1114–1118.

[10] Jian Zhang and Bernard Ghanem, “Ista-net: Interpretable optimization-inspired deep network for image compressive sensing,” in Proceedings of the IEEE conference on computer vision and pattern recognition, 2018, pp. 1828–1837.

[11] Jo Schlemmer, Jose Caballero, Joseph V Hajnal, Anthony N Price, and Daniel Rueckert, “A deep cascade of convolutional neural networks for dynamic mr image reconstruction,” IEEE transactions on Medical Imaging, vol. 37, no. 2, pp. 491–503, 2017.