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Core crosstalk in ordered imaging fiber bundles

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Coherent fiber bundles are used widely for imaging. Commonly, disordered arrays of randomly sized fiber cores avoid proximity between like-cores, which would otherwise result in increased core crosstalk and a negative impact on imaging. Recently, stack-and-draw fiber manufacture techniques have been used to produce fibers with a controlled core layout to minimize core crosstalk. However, one must take manufacturing considerations into account during stack-and-draw fiber design in order to avoid impractical or unachievable fabrication. This comes with a set of practical compromises, such as using only a small number of different core sizes. Through characterization of core crosstalk patterns, this Letter aims to aid the understanding of crosstalk limitations imposed by such compromises in the core layout made for ease of fabrication.

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Coherent fiber bundles (CFBs) are a particularly powerful type of optical fiber that consist of thousands of cores that maintain their relative spatial orientation along the length of the fiber. In this manner, CFBs facilitate in vivo microscopy in regions of the body [1–5] including the respiratory system [6–8] or GI tract [9], where it would otherwise be challenging to observe biological processes in action. When using a CFB, image resolution can be increased by maximizing the packing density of the light-guiding cores until core crosstalk due to evanescent field coupling introduces blurring and reduces image contrast beyond an acceptable level [10–12].

Most commercially available CFBs are disordered in design: they consist of cores of a narrow range of diameters randomly distributed across the fiber to build the imaging field of view (FOV) [Figs. 1(a) and 1(b)]. Disordered CFBs rely on a high refractive index contrast between core and cladding to minimize core crosstalk. In a previous study of such disordered CFBs, we demonstrated that core proximity was the most dominant factor in core crosstalk [10,11]. We characterized a commercially available disordered multi-mode CFB (FIGH-30-650S, Fujikura) at two separate wavelengths (520 nm and 635 nm) and found that the mean light lost to crosstalk was 39% for 520 nm and 38.3% for 635 nm. We noted that, in line with coupled mode theory [13,14], the overall spread of crosstalk at 635 nm was broader than at 520 nm.

In contrast, ordered CFBs have a predetermined core placement implemented during the stacking process. In reality, this leads to a semi-regular repeating pattern due to the nature of the drawing process. The aim is to place similar sized cores maximally apart, while maintaining dense core packing. For example, we recently realized a fiber design consisting of 25-core unit arrays stacked and drawn to a total of 8100 cores across a 450 µm corner-to-corner (FOV) [15,16] [Figs. 1(c) and 1(d)]. The 25-core unit array is assembled from five core sizes, referred to herein as core classes [shown in Fig. 1(d) inset], which range from 2–3 µm in diameter and are few-moded [16]. We proposed this stack-and-draw technique as a method of fabrication using readily available OM1 telecommunications preforms (Draka). The benefit of the stack-and-draw method is that a high performance CFB can be achieved with commonly available preforms rather than relatively uncommon, heavily doped, and high numerical aperture (NA) preforms.

We observe comparable imaging resolution between our ordered CFB and commercial disordered CFBs evaluated by USAF targets. In other work, we observed comparable or improved performance compared to commercial imaging fibers when considering bulk transmission properties of fringe patterns [17]. This test is a valuable guide for fiber performance on average, across many cores. However, these evaluation methods describe fiber limitations due to macro scale core crosstalk properties. We note that with an ordered fiber, crosstalk artifacts from individual cores may be present and that they may not be well captured in fringe pattern quantization tests. CFBs are increasingly being employed in biomedical imaging scenarios where image features on the individual core scale are crucial,
enhanced to better visualize crosstalk spread. Thus, it should naturally be noted that these images do not represent the true intensity of light spread as a result of crosstalk. In each image, the illuminated core is highlighted by a magenta circle. It was determined that we could broadly categorize the crosstalk into two groups. The first group is marked by crosstalk dominated by proximal cores, shown in Fig. 3(a). The second is crosstalk dominated by core class, shown in Fig. 3(b).

The aims of this Letter are to (i) clearly demonstrate that attributes of the structure of ordered fibers are visible in the characterized core crosstalk, and (ii) as a result, suggest considerations that designers and users of these fibers should make.

For our investigation, we used a method of coupling laser light into individual cores of the CFB and acquiring images of the other end of the fiber. The method and analysis follow that of previous work, to which we direct the reader [10,11]. Only a minor modification was made by the use of a higher resolution camera (GS3-U3-32S4M-C, Point Grey). It can be summarized as: (i) light coupling to one core at the input of the fiber (confirmed to be on axis and with expected point spread function (PSF) [11]), (ii) image acquisition of the core crosstalk at the output of the fiber [Fig. 2(a)], (iii) repetition for a statistically relevant number of cores, and (iv) analysis of the spread of crosstalk as it pertains to neighboring layers of cores [Fig. 2(b)]. Flood illuminating a single core with an NA less than that of the various modes ensures simultaneous mode stimulation [11]. The fiber was held without a bend, and kept stationary throughout. The study here therefore gives a representation of core crosstalk in one fiber bend state. This will include coherence effects from laser illumination (commonly used in CFB imaging systems). As the fiber bends, coherent coupling between particular cores will vary; hence, a statistical representation of many core couplings is given in this Letter. This characterization process can be achieved by one operator in approximately 3 h per data set. Analysis of light intensities and linking this to core locations enables our results.

Figure 3 illustrates typical examples of core crosstalk patterns for 635 nm (left grid) and 520 nm (right grid), contrast enhanced to better visualize crosstalk spread. Thus, it should naturally be noted that these images do not represent the true intensity of light spread as a result of crosstalk. In each image, the illuminated core is highlighted by a magenta circle. It was determined that we could broadly categorize the crosstalk into two groups. The first group is marked by crosstalk dominated by proximal cores, shown in Fig. 3(a). The second is crosstalk dominated by core class, shown in Fig. 3(b).

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For each of the 300 scanned cores, the neighboring cores were classified and segmented to quantify the extent of the crosstalk. Figure 4 presents the proportion of overall light that is lost to crosstalk as a function of distance from the illuminated core, for 635 nm [Fig. 4(a)] and 520 nm [Fig. 4(b)]. Here, distance is defined as the Euclidean distance between the centers (center of mass) of each core. Each dot on the plot represents a core in the first, second, third, fourth, or fifth neighboring layer (green, blue, cyan, magenta, or orange, respectively). For 635 nm, it can be seen that a large proportion of light can be lost to a core that is at a great distance from the illuminated core. This is expected since the crosstalk one sees extending large distances (i.e., ≥ 2nd neighbor layer) is, in part, the result of coupling between cores of the same class within the same, and across separate, unit arrays within the fiber. In the case of 635 nm, one can see that sometimes a significant proportion of overall light occurs in the second and third neighbor layers. These are typically the neighbor layers that contain the closest cores of the same class to the illuminated core; see Fig. 2(b) for an example of neighbor classification. This effect is less apparent in the results for crosstalk at 520 nm, where the distribution shows that crosstalk is dominated by proximity to the illuminated core and is seen most significantly in the first neighbor layer.

Figure 5 shows these data as a box plot including the proportion of light that remains in the illuminated core. The median and mean light per core in each neighboring layer is indicated by the orange line and the purple line, respectively. The whiskers of the plot extend to the minimum and maximum value within the data. It can be seen that for the first, second, third, fourth, and fifth neighboring layers, the mean is significantly elevated from the median, showing that a small number of high crosstalk cores contributes to a skew in the distribution. Here, this effect is apparent at both 635 nm and 520 nm.

Figure 6 gives a comparison between the ordered fiber and the disordered fiber of the mean crosstalk per neighboring layer (combining all cores in that layer). The performance of the ordered fiber in this regard is higher than the disordered fiber for 520 nm, i.e., the proportion of light confined to the illuminated core is higher (80.71% versus 61%, respectively). However, there is a noticeable elevation of the mean crosstalk to the second and third layers for 635 nm in the ordered fiber when compared to the disordered fiber. We attribute this to crosstalk between cores of the same class. It should be noted that the ordered fiber was designed to be optimized for imaging in the green region of the spectrum, where tissue autofluorescence is strongest, and a number of well-established fluorophores reside. If such fibers are to be designed for use at longer wavelengths, these data show that consideration of crosstalk between cores of the same class becomes key for optimization of imaging performance.

Beyond the scope of this study, but of interest, would be to reclassify the cores in a system other than in neighboring layers. Due to the design of the ordered fiber, classifying cores into a grid-based geometry was considered. However, owing to fiber layout features, such as sheering between unit arrays, it was deemed to be non-trivial. We also considered a classification of core class rather than into neighboring layers. However, it was not possible during our study due to the resolution of our
optical endomicroscopy (OEM) system, which prohibited differentiation of cores based on size. A higher resolution OEM system may be able to provide the data required to complete such a study.

A key aim of this study was to use crosstalk characterization to consider the implications for future CFB design. Our results show that crosstalk in an ordered CFB is indeed noticeably affected by the repeated pattern of cores. Additionally, it is our experience that accurate theoretical modeling of such crosstalk effects is a challenge, and that experimental observation has granted us greater insights into fiber design. The experimental observation and analysis here provide a foundation for improving future fiber design.

There is growing interest in red and NIR imaging where tissue autofluorescence is lower and fluorescent labels are being developed. Thus to optimize imaging in the red, one should consider increasing the distance between cores of the same class beyond that shown here (the minimum in our ordered CFB was ∼10 μm). Similarly, attempts to reduce core spacing in a fiber for green imaging should maintain spacing between cores of the same class. A theoretically ideal ordered fiber would be an infinite arrangement of cores of markedly different sizes, which of course is practically unattainable. Thus, the stack-and-draw method makes a sensible trade-off between technical difficulty and outcome by utilizing a series of repeated arrays. Ultimately, one must consider: (i) the number of core classes, (ii) the range of sensible core class sizes, and (iii) the size of the unit array, which should be guided by the technical difficulty of stacking the design. For example, adding a core class within the same range of core sizes would achieve greater spacing between cores of the same class but would result in smaller variance between each class. Our results suggest that this may be a valuable trade-off to make, as crosstalk appears greatly dominated by cores that are identical rather than similar in size.

We note the observation of crosstalk over great distances (up to ∼25 μm). Much effort has been put into analysis and post processing techniques to improve image quality [19,20] as well as develop new imaging capabilities [21,22]. We propose that the data within this Letter are important for those who interpret or process the images. This is particularly key when weight is given to micro-scale features within an image (such as bacteria or cells), in which case knowledge of such crosstalk patterns may help to reduce false positives.

This study describes key results for the many existing users of commercial CFBs and future users of developmental imaging fibers exploiting ordered core patterns to help suppress crosstalk. The direct experimental observations presented here show that the core layout is clearly reflected in the observed crosstalk patterns. This is important for both fiber design and CFB image processing. We demonstrated that these specific crosstalk patterns are present in both green (520 nm) and red (635 nm), which relate to common fluorescence imaging bands, and showed that the effect is more significant at 635 nm. We believe these direct experimental observations and design considerations can contribute to imaging fiber development and analysis methods.

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