Angular momentum conservation in measurements on spin Bose-Einstein condensates

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Abstract

We discuss a thought experiment where two operators, Alice and Bob, perform transverse spin measurements on a quantum system; this system is initially in a double Fock spin state, which extends over a large distance in space so that the two operators are far away from each other. Standard quantum mechanics predicts that, when Alice makes a few measurements, a large transverse component of the spin angular momentum may appear in Bob’s laboratory. A paradox then arises since local angular momentum conservation seems to be violated. It has been suggested that this angular momentum may be provided by the interaction with the measurement apparatuses. We show that this solution of the paradox is not appropriate, so that another explanation must be sought. The general question is the retroaction of a quantum system onto a measurement apparatus. For instance, when the measured system is entangled with another quantum system, can its reaction on a measurement apparatus be completely changed? Is angular momentum conserved only on average over several measurements, but not during one realization of the experiment?

Contents

1 The paradox
  1.1 Double Fock state and transverse spin measurements ........................................ 2
  1.2 Combined spin measurements .................................................................................. 4
  1.3 A non-local effect ...................................................................................................... 6
  1.4 But does angular momentum exist before it is measured? ...................................... 6

2 Role of the measurement apparatuses
  2.1 A preliminary experiment with a phase state ......................................................... 7
  2.2 Double condensate: angular momentum recoil of the apparatuses ......................... 8
  2.3 Role of entanglement ............................................................................................... 8

3 Discussion and conclusion

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The role of conservations laws in the process of quantum measurement, and in particular of the retroaction of the measured system on the measurement apparatus, has been discussed from the early days of quantum mechanics. A famous example is the Einstein-Bohr debate at the fifth Solvay Congress in Brussels, where Einstein invented a thought experiment with a moving double slit screen and a measurement of its momentum recoil during interaction with the test particle. Wigner also analyzed the relation between conservation laws and measurements, emphasizing that only observables commuting with the conserved quantities could
exactly be measured \[2\]. This line of thought was then continued by Araki and Yanase \[3\], Osawa \[4\], Loveridge and Bush \[5\], and others; Aharonov and Rohrlich have emphasized in their book \[6\] the fact that all measurements are relative (for instance a Stern-Gerlach apparatus does not measure the spin component along an absolute direction, but a direction that is fixed by the apparatus itself) and its impact on Wigner’s argument. Leggett and Sols \[7\] have discussed the spontaneous appearance of a relative phase between two large superconductors under the effect of quantum measurement. They have pointed out that the phase of a macroscopic current may in theory be determined by the interaction with a small measurement apparatus: “Can it be that by placing, let us say, a miniscule compass needle next to the system... we can force the system to ‘realize’ a definite macroscopic value of the current?”; of course, this seems paradoxical: how can a very small measurement apparatus make an arbitrarily large system to completely change state?

Bose-Einstein condensates in gases provide quantum systems offering interesting opportunities to test the laws in quantum mechanics \[8\], either in thought experiments as in the tradition created by the Einstein-Bohr debate, or in real experiments. A recent analysis \[9\] discusses a paradoxical thought experiment with spin condensates extending over long distances (Fig. 1). It assumes that two long condensates, in the + and − spin state respectively, overlap in two remote laboratories, where Alice and Bob perform spin measurements. If the populations of the two condensates are equal, the average of the spin angular momentum of the system vanishes. Nevertheless, if Alice performs transverse spin measurements in her laboratory and finds a polarization is some random direction \(\boldsymbol{u}_\phi\), even if she measures a small number of spins, a transverse angular momentum parallel to \(\boldsymbol{u}_\phi\) appears instantaneously in the entire system. This is in particular true in Bob’s laboratory, where all particles acquire a spin polarization that is parallel to \(\boldsymbol{u}_\phi\), even if Alice never interacted with them. Compatibility with relativity (no faster than light signalling) is maintained by the completely random direction of \(\boldsymbol{u}_\phi\), which ensures that no signal can be transmitted in this way at an arbitrary distance with no delay. Nevertheless, the spontaneous appearance of an angular momentum in Bob’s laboratory, without any local interaction, seems to violate angular momentum conservation. This is all the more true since Alice’s apparatus may have interacted with a microscopic number of spins only, while the angular momentum appearing in Bob’s laboratory is macroscopic; where does this angular momentum come from? We come back to this paradox in more detail in §1.1.

Paraoanu and Healey have studied this paradox \[10\] and concluded that, “while this Gedankenexperiment provides a striking illustration of several counter-intuitive features of quantum mechanics, it does not imply a non-local violation of the conservation of angular momentum”. They emphasize that, even if Bob’s system is projected by Alice’s measurement into an eigenstate of transverse angular momentum, “we cannot attribute physical reality prior to its measurement, even when the state of the system is an eigenstate of that observable... There is no angular momentum before it was measured”. The purpose of the present article is to complete this discussion: considering as suggested by Paraoanu and Healey that only measured quantities really exist, we study combined measurements performed by Alice and Bob and the angular momentum taken by their respective apparatuses. We conclude that this is not sufficient to ensure angular momentum conservation, so that the the paradox is not lifted; its solution should be sought in another direction.

1 The paradox

We first recall the double spin condensate paradox in more detail than in the introduction, and then how Paraoanu and Healey propose to solve it.

1.1 Double Fock state and transverse spin measurements

Figure 1 schematizes the thought experiment with two spin condensates. The first condensate has \(N_\alpha\) particles occupying a single particle state \(|\alpha\rangle = |u, +\rangle\) with orbital wave function \(u\) and spin state \(+\); the second has \(N_\beta\) particles occupying a single particle state \(|\beta\rangle = |v, −\rangle\) with orbital wave function \(v\) and spin state \(−\). The two wave functions overlap in regions of space A and B where Alice and Bob perform measurements.
The condensates are described by Fock states (we ignore thermal effects and condensate depletion due to interactions, assuming for instance that the systems are dilute gases; actually, in this article, for the sake of simplicity we take the word “condensate” as equivalent to “Fock state”). The initial state \(|\Psi_0\rangle\) of the whole quantum system is then:

\[
|\Psi_0\rangle = |N_\alpha, N_\beta\rangle = \frac{1}{\sqrt{N_\alpha! N_\beta!}} \left[ a_1^\dagger \right]^{N_\alpha} \left[ a_2^\dagger \right]^{N_\beta} |0\rangle
\]  

Here \(a_1^\dagger\) is the creation operator into the single particle state \(|u, +\rangle\), \(a_2^\dagger\) the creation operator into state \(|v, -\rangle\), and \(|0\rangle\) the vacuum state. We call \(Oz\) the quantization axis for spins, and will consider transverse spin measurements performed by Alice and Bob in any direction perpendicular to \(Oz\). This state has zero average spin angular momentum along these directions, and zero spin momentum along \(Oz\) as well if \(N_\alpha = N_\beta\).

![Figure 1: Two highly populated Fock states associated with opposite spin directions (+ and −) overlap in two remote regions of space, region A where Alice operates, and region B where Bob operates. A series of transverse spin measurements made by Alice triggers the appearance of a well-defined transverse orientation in her region, and also that of a parallel macroscopic transverse orientation in Bob’s region (quantum non-locality). Angular momentum seems to appear in region B from nothing, with no interaction at all.](image)

Now Alice makes \(P_A\) spin measurements in her laboratory, choosing transverse directions characterized by polar angles \(\varphi_1^A, \varphi_2^A, \ldots, \varphi_{P_A}^A\). These measurements are performed with different apparatuses situated in different regions of space of her laboratory, and they are independent; we assume that each apparatus detects one single particle (how to include the case where no particle is found in a particular region of space is discussed in [11]). Similarly, Bob performs \(P_B\) transverse spin measurements in his laboratory with polar angles \(\varphi_1^B, \varphi_2^B, \ldots, \varphi_{P_B}^B\). As in [9], we assume that the total number of measurements \(P = P_A + P_B\) is much smaller than the total number of particles \(N = N_A + N_B\). Under these conditions, the effects of the “quantum angle” [12] can be ignored and the relative phase behaves classically. The probability of an event where Alice obtains \(m_A^+\) results + and \(m_A^-\) = \(P_A - m_A^+\) results −, while Bob obtains \(m_B^+\) results + and \(m_B^- = P_B - m_B^+\) results −, is then given by:

\[
\mathcal{P}(m_A^+, m_A^-, m_B^+, m_B^- | \varphi_1^A, \ldots, \varphi_{P_B}^A, \varphi_1^B, \ldots, \varphi_{P_B}^B) = \\
\int \frac{2\pi}{2\pi} \prod_{j=1}^{P_A} \cos^2 \left( \frac{\lambda - \varphi_A^j}{2} \right) \prod_{j=m_A^+ + 1}^{m_A^-} \sin^2 \left( \frac{\lambda - \varphi_A^j}{2} \right) \prod_{k=1}^{m_B^+} \cos^2 \left( \frac{\lambda - \varphi_B^k}{2} \right) \prod_{k=m_B^++1}^{m_B^-} \sin^2 \left( \frac{\lambda - \varphi_B^k}{2} \right)
\]

The interpretation of this formula is straightforward: if the two condensates had a relative phase \(\lambda\), the probability of finding a result + with an apparatus oriented along direction \(\varphi\) is \(\cos^2 (\lambda - \varphi) / 2\), and that of finding the opposite result is \(\sin^2 (\lambda - \varphi) / 2\). The probability of finding the combined results is the product.
of the corresponding probabilities; in this formula, Alice’s results appear in the first two factors of the product, Bob’s results in the last two factors. But $\lambda$ is actually completely random between 0 and $2\pi$, so that an integral over $\lambda$ is necessary; although the integrand is a product, this integral introduces correlations between the successive results. This formula is general; for instance, it remains valid if Alice only makes measurements, not Bob; it is sufficient to put $m_B^+ = m_B^- = 0$, so that the last two products disappear from Eq. (2), leaving only a dependence on Alice’s measurement angles.

Remark: For clarity, in Fig. 1 we have assumed that the single particle state associated with the first condensate has an orbital wave function $u$ extending in space continuously from Alice’s to Bob’s laboratory; similarly, the second condensate extends continuously from one laboratory to the other. Nevertheless, we could also have assumed that any (or both) of the wave functions is the coherent superposition of two disconnected parts, one in region A and another in region B (as, for instance, in Bell experiments where photons propagate to opposite detectors without overlap of their wave packets). Provided both wave functions still overlap in each measurement region, nothing is changed in the calculations that will follow. The continuity of the condensates from region to the other plays no role in the physical effect we discuss, illustrating that it has nothing to do with some wave propagation along the condensates.

1.2 Combined spin measurements

The properties of formula (2) have been discussed in [9] and [12]. All results of measurements have exactly the same probability as for an ensemble of spins with well-defined initial orientations defined by angle $\lambda$, with a random distribution given by a function of this additional variable. Initially, the “additional” variable $\lambda$ has a completely uniform distribution between 0 and $2\pi$ and the first spin measurement provides a completely random result. Then, while measurements accumulate, the distribution of the “additional” variable becomes narrower and narrower. The Bayes theorem can be used to show that the $\lambda$ distribution after $Q$ measurements is nothing but the product of cosines and sines squared that appears under the integral of Eq. (2); so this distribution is known at any step of the experiment. At some point, this distribution becomes very narrow; Alice’s measurements have practically determined the relative phase between the condensates (with some uncertainty); if she makes further measurements in the direction in which the spins now point, she obtains results that are practically certain.

To illustrate the evolution of the $\lambda$ distribution, we have performed simple numerical simulations using the same method as in [13]. We rewrite Eq. (2), the probability of finding a $\pm$ spin along angle $\theta_m$ in the $m$th measurement, as

$$ P_m(\pm) \sim \int_0^{2\pi} \frac{d\lambda}{2\pi} g_m(\lambda) [1 \pm \cos(\lambda - \theta_m)] $$

In this equation, $g_m(\lambda)$ is the $\lambda$ distribution resulting from the $m-1$ previous measurements:

$$ g_m(\lambda) = \prod_{i=1}^{m-1} [1 + \eta_i \cos(\lambda - \theta_i)] $$

where $\eta_i = \pm 1$ is the result of the $i$th spin measurement result made along angle $\theta_i$. The initial distribution ($m = 1$) is perfectly flat and the first result completely random. Then Alice starts a series of 300 measurements along an arbitrary angle $\theta_A = 0$, with random results obtained from the $\lambda$ distribution at each step depending on all previous results according to Eq. (3). In our simulation she finds $n_+ = 269$ up spins and $n_- = 31$ down spins. But as the distribution of Fig. 2 shows, at this stage she is unable to tell whether the polarization that has developed during her measurements (or that was already there) is at a positive or negative angle with respect to her original zero angle. Note the sharpness of the distribution after this relatively small number of measurements. Alice then estimates that the polarization is at an angle

$$ \lambda_0 = 2 \cos^{-1} \left( \frac{269}{300} \right) = \pm 0.65 $$
Then, to establish the sign of the polarization Alice measures at $\theta_2 = \pi/2$ and finds $n_+ = 61, n_- = 270$; this means that the polarization angle is negative, with the estimate from this measurement given by:

$$2 \cos^{-1} \left( \sqrt{\frac{61}{300}} \right) = 2.20$$

relative to her new angle, or at $\lambda_0 = \pi/2 - 2.43 = -0.64$. The new single-peaked distribution is shown in Fig. 3.

The same calculations apply to Bob’s measurements. To make the argument easy to follow we suppose that Bob picks his first set of angles also to be $\theta_B = 0$; our simulation shows that he measures $n_+ = 269$, $n_- = 31$, implying an angle of $\pm 0.654$. If he does not know Alice’s results, he cannot determine its sign. He therefore makes a measurement at $\pi/2$ to determine it, to find $n_+ = 58, n_- = 242$ so that this estimate gives the polarization angle as 2.23 radians from his present angle, or at $\lambda_0 \approx \pi/2 - 2.23 = -0.66$. To check this result he measures 100 spins exactly at this angle and gets 99 spins up and only 1 down. The angle found by Bob is very close to what Alice found.
1.3 A non-local effect

According to Eq. (2), Alice and Bob share the same $\lambda$ distribution. This is the reason why, as we have just seen, if Bob starts making his experiments when Alice has finished hers, he will necessarily find a direction of $\lambda$ that falls inside the peak of the distribution produced by Alice. But if he is not aware of the results obtained by Alice, he will not notice anything special: to him, his results appear to be the same as if no previous measurement had been made. Nevertheless, when both compare their results at the end of the experiment, they can check that both have independently determined $\lambda$ distributions that are compatible: they then realize that Alice’s results have determined in advance the direction in which Bob has found the spins. In other words, Alice’s measurements fix the relative phase of the two condensates and therefore the direction of Bob’s spins, which is doubly paradoxical: first, if both laboratories are very remote, Alice cannot interact with Bob’s spins, which acquire a given direction by a non-local effect; second, Alice may perform her experiments on a small number of spins, 100 or 1000 for instance, while the spin orientation appearing in Bob’s laboratory may be macroscopic. How can a macroscopic physical quantity appear under the effect of arbitrarily remote measurements performed on a microscopic sample of atoms?

Another paradox arises when one asks the origin of the angular momentum associated with the transverse orientation of Bob’s spins, after Alice has made her measurements (but before Bob started to make his). Since these spins have interacted with no external physical system, one could expect that their angular momentum should remain unchanged. How can a measurement performed in Alice laboratory change the angular momentum contain in Bob’s, especially since this momentum is macroscopic? But quantum mechanics predicts the values that Bob can observe when measuring this angular momentum have indeed changed. The effect of Alice’s measurement is instantaneous, and has nothing to do with the propagation of any signal along the condensate (Bogolubov phonons for instance); it is a pure non-local effect arising from the quantum measurement postulate. The paradox is then that macroscopic angular momentum seems to appear from nothing in Bob’s laboratory.

1.4 But does angular momentum exist before it is measured?

Paraoanu and Healey [10] have studied the problem and propose a way to solve the paradox. They state that, after Alice has made her first measurements, "When Bob measures the spin-component on the large cloud of condensate in his region, far from Alice, he is almost certain to get a macroscopic result. But his does not mean that Alice’s measurement produces a ‘real’ macroscopic value of angular momentum in Bob’s region. Bob’s measurement does not simply reveal this macroscopic value of a pre-existing spin-component of or in the cloud produced by Alice’s projective measurement. Instead, the macroscopic spin-component ‘emerges’ during Bob’s measurement following an interaction with Bob’s measuring device.... Certainly nothing that happens near Alice creates a macroscopic angular momentum near Bob in violation of local conservation of angular momentum”. The central point of their argument is therefore that it is not possible to attribute physical reality to any physical quantity (here angular momentum) prior to its measurement, even if the state of the system is already an eigenstate of the corresponding observable before measurement.

If no “real” angular momentum exists in Bob’s laboratory before Bob measures it with his apparatus, and comes to existence only during his measurement, it becomes natural to assume that this apparatus takes the angular momentum recoil that is necessary to ensure a local conservation of angular momentum. The purpose of the present article is therefore to study the angular momentum recoil of both measurement apparatuses, and examine if Bob’s apparatus can take the recoil that is necessary to ensure momentum conservation; if so, the paradox can be lifted. Nevertheless, our analysis shows that this is not the case, so that the paradox remains.

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1 As we remarked at the end of §1.1, the wave functions do not necessarily extend continuously from one region to the other; the phenomenon cannot be seen as due to some kind of vibration propagating along the wave functions, even at infinite velocity.
2 Role of the measurement apparatuses

Before studying the paradoxical experiment, where the spin system is initially in a double Fock state (double condensate), we discuss a simpler case where the spin system starts from a phase state. In this case, a macroscopic transverse spin orientation exists from the beginning of the experiment.

2.1 A preliminary experiment with a phase state

Instead of (1), we now assume that the initial state of the measured system is a phase state, with the same total number of particles \( N = N_\alpha + N_\beta \). This state depends on an arbitrary phase parameter \( \lambda_0 \) and has the following expression:

\[
|\Psi\rangle = \frac{1}{\sqrt{N!}} \left[ e^{-i\lambda_0/2} a_0^+ + e^{i\lambda_0/2} a_0^- \right]^N |0\rangle
\]

(7)

It can be seen as a single condensate where all \( N \) particles are in the same individual state:

\[
|\phi(\lambda_0)\rangle = \frac{1}{\sqrt{2}} \left[ e^{-i\lambda_0/2} |u, +\rangle + e^{i\lambda_0/2} |v, -\rangle \right]
\]

(8)

where \(|+\rangle\) and \(|-\rangle\) are the single particle eigenstates of the \( Oz \) component of the spin and \(|u, v\rangle\) are the orbital states associated with the two condensates. Actually these orbital states play no particular role in the discussion \([11]\); we can assume that they coincide, or even ignore them. The spin of the single particle state \((8)\) is fully polarized in a transverse direction of the \( xOy \) plane making azimuthal angle \( \lambda_0 \) with axis \( Ox \). In this case, the probability \((8)\) is replaced by:

\[
\mathcal{P}(m_\alpha^+, m_\alpha^-, m_\beta^+, m_\beta^- | \varphi_A^1, \ldots, \varphi_A^m; \varphi_B^1, \ldots, \varphi_B^p) = \prod_{j=1}^{m_\alpha^-} \cos^2 \left( \frac{\lambda_0 - \varphi_A^j}{2} \right) \prod_{j=m_\alpha^+ + 1}^{m_\alpha^+} \sin^2 \left( \frac{\lambda_0 - \varphi_A^j}{2} \right) \prod_{k=1}^{m_\beta^-} \cos^2 \left( \frac{\lambda_0 - \varphi_B^k}{2} \right) \prod_{k=m_\beta^+ + 1}^{m_\beta^+} \sin^2 \left( \frac{\lambda_0 - \varphi_B^k}{2} \right)
\]

(9)

The integral over \( \lambda \) has now disappeared: all measurements are independent, which is not surprising since all spins are initially polarized in the same direction.

Consider a given spin measurement and the angular momentum transferred to the measurement apparatus. Initially, the average angular momentum projected along the measurement direction \( \varphi_A^j \) is equal to

\[
\left( \frac{\hbar}{2} \right) \cos \left( \frac{\lambda_0 - \varphi_A^j}{2} \right)
\]

after measurement, it takes the value \(+\hbar/2\) or \(-\hbar/2\) randomly, but on average it is:

\[
\left( \frac{\hbar}{2} \right) \cos \left( \frac{\lambda_0 - \varphi_A^j}{2} \right)
\]

(10)

The projected angular momentum may be larger or smaller, depending on measurement, but on average it keeps the same value as before measurement. The mean-square deviation \( \Delta \) of the angular momentum transfer during \( P \) identical measurements is given by:

\[
\Delta^2 = P \left( \frac{\hbar}{2} \right)^2 \left\{ \cos^2 \left( \frac{\lambda_0 - \varphi_A^j}{2} \right) \left[ 1 - \cos \left( \lambda_0 - \varphi_A^j \right) \right]^2 + \sin^2 \left( \frac{\lambda_0 - \varphi_A^j}{2} \right) \left[ 1 - \cos \left( \lambda_0 - \varphi_A^j \right) \right]^2 \right\}
\]

\[
= P \left( \frac{\hbar}{2} \right)^2 \sin^2 \left( \lambda_0 - \varphi_A^j \right)
\]

(11)

As a consequence, on average over many spin measurements, the measurement apparatuses takes zero angular momentum recoil. Nevertheless, this recoil has a fluctuation that is proportional to the square root of the number of realizations.
Without knowing the value of $\lambda_0$, Alice and Bob can determine it through their measurements. In a first step, Alice can choose any orientation $\varphi$ for her measurements, and obtain an approximate value of the cosine of the angle between this orientation and that of the spins. She can then choose a perpendicular direction to obtain the sine of this angle, removing the previous sign indeterminacy \[13\]. After a few measurements, the best way to narrow the distribution of possible values of $\lambda_0$ is to make spin measurements in a direction perpendicular to her first estimation of $\lambda_0$. In this way, after a reasonable number of measurements, 100 for instance, she has determined $\lambda_0$ with good accuracy. Finally, if she continues making spin measurements parallel to this direction, almost no fluctuation remains in the results: these measurements only confirm what she already knows, and perturb the spin system very little. On the whole, Alice can determine the value of $\lambda_0$ by transferring to her apparatus an angular momentum of the order of $10\hbar$, maybe ten times more if she really needs an excellent accuracy, but she certainly does not need to transfer a macroscopic number times of $\hbar$ to obtain the information.

Of course the same is true for Bob: to acquire information on the direction of the (pre-existing) spin angular momentum in his laboratory, he may need to transfer some angular momentum to his measurement apparatus, but again not more than 10 or 100 times $\hbar$.

### 2.2 Double condensate: angular momentum recoil of the apparatuses

We now come back to the original thought experiment with the initial state \[1\], where the initial angular momentum does not pre-exist measurements, but is created by them. We must then use Eq. \[2\] again, and we assume that $N_\alpha = N_\beta$. As we have recalled above (§1.1), when Alice starts performing measurements, the $\lambda$ distribution is completely flat between 0 and $2\pi$, and the probabilities of the two results are equal (in contrast with §2.1). But, as soon as measurements become available, the $\lambda$ distribution becomes a product of sines and cosines, which becomes a more and more peaked function. As above, we assume that Alice chooses appropriate measurement angles to avoid sign ambiguities on $\lambda$ and to optimize her definition of $\lambda$.

After 100 or 1000 measurements, the $\lambda$ distribution has become a narrow peak around some value $\lambda_0$, and one reaches a situation that is essentially the same as in §2.1.

Starting from this situation, Bob can obviously apply exactly the same strategy as Alice; he does not know the value of $\lambda_0$, but can measure it by transferring not more than 10 or 100 units of $\hbar$ to his measurement apparatus. After this preliminary operation, he can choose a direction of measurement that is parallel to $\lambda_0$ and make an arbitrary number of measurements, even macroscopic, without transferring appreciable momentum to his apparatus. Even if he continues to choose random directions of measurement, the angular momentum transfer increases only as the square root of the number of measurements, not linearly with it. So it is clear that the angular momentum transfer to his apparatus remains totally negligible with respect to the measured spin angular momentum.

We therefore see that, even if we consider that angular momentum does not exist before it is measured (including the case where the state before measurement is an eigenstate of the measured observable), the amount of angular momentum appearing in Bob’s laboratory cannot be compensated by an opposite angular momentum transferred to his apparatus. Again, angular momentum seems to have appeared from nowhere.

### 2.3 Role of entanglement

After Alice has performed at least one measurement, the two condensates are strongly entangled. This is because her measurements relate to a coherent superposition of the two spin states, so that they cannot distinguish between spins of each condensate. The unmeasured spins are therefore left in a coherent superposition of states where the population of each condensate fluctuates, with their sum constant.

This raises the general question of quantum measurements performed on entangled systems which, as we know, are not “separable” in quantum mechanics; they should be considered as a whole. Could it be that, when a measurement is performed on a small physical system that is entangled with another large system, the
reaction of the system on the measurement apparatus is completely changed by the entanglement, becoming macroscopic instead of microscopic?

The epitome of an entangled states is a so called GHZ state (“all or nothing state”), for which \(|\Psi\rangle\) is replaced by:

\[
|\Psi\rangle_0 = \frac{1}{\sqrt{N!}} \left[ \left( a_1^{\dagger} \right)^N \left( a_2^{\dagger} \right)^N \right] |0\rangle
\]

With this state, if Alice measures the \(Oz\) component of a single spin, all subsequent measurements (by her or by Bob) will find the same result as the first measurement: measuring the direction of one single spin instantaneously fixes a parallel direction for all other spins. Nevertheless, before the first measurement, the average value of the \(Oz\) total spin component vanished. Here again, angular momentum conservation during the first measurement would imply that the measuring apparatus must take the corresponding recoil. So one could conclude that, because Alice’s spins are part of a quantum system that is an indivisible whole, Alice’s apparatus takes a macroscopic angular momentum. We discuss this question further in the next section.

3 Discussion and conclusion

Our conclusion is therefore that, even if we consider only situations where angular momentum has been measured by Alice and Bob, the origin of the large angular momentum that appears in Bob’s laboratory cannot be found in his measurement apparatus, which actually absorbs very little angular momentum. If we wish to preserve local conservation of angular momentum during measurement processes, what are the possibilities to solve the paradox?

(i) A first possibility is to consider that angular momentum is not necessarily conserved in each realization of the experiment, but only on average. Indeed, the standard mathematical proof of angular momentum conservation when \(J\) and \(H\) commute shows that the average \(\langle J \rangle\) remains unchanged; the proof applies to any power of \(J\), in other words that the complete distribution of probability is constant. But the proof says nothing of individual events.

In this view, in quantum mechanics rotation invariance would not imply angular momentum conservation for single experiments, but only statistically for many realizations.

(ii) According to our analysis, it is not Bob’s apparatus that takes the angular momentum recoil, so could it be Alice’s that takes the macroscopic recoil? After all, it is her actions that apply state vector reduction and transform the initial state vector \(|\Psi\rangle\), with no transverse average angular momentum, into \(|\Phi\rangle\) where all spins are transversely polarized; in other words, it is her measurements that “fuse” the two condensates into one and create a single condensate described by a phase state with large angular momentum. In this case, one should consider that, even if he directly interacts with 100 spins only, since these spins are parts of (entangled with) a much larger quantum system, her measurement relates to the whole system.

The problem with such a view is that it seems to imply instantaneous signalling, creating a blatant contradiction with relativity. This is because Alice’s measurement could reveal if, for instance, Bob has destroyed his condensate, or rotated its spin direction by applying a magnetic field. The same reasoning applies to the GHZ state discussed above. So this explanation does not seem appropriate.

(iii) One could argue that some “super-selection rule” forbids the preparation of such double Fock initial states. Clearly, gaseous spin condensates extending over long distances are extremely fragile objects, even if repulsive interactions between the atoms tend to stabilize them. One can therefore question the accessibility of such double spin condensates. Nevertheless Bose-Einstein condensation seems to provide a mechanism to generate such double Fock states in dilute gases, since repulsive interactions favor the population of a single quantum state \(|14\rangle\), and this explanation sounds rather artificial.

(iv) Another explanation is to introduce additional variables, as suggested in [9]. In this case, quantum mechanics would not be considered complete. In this perspective, transverse angular momentum would exist from the beginning, and no paradox at all would occur. Nevertheless, as discussed in [12], these additional
variables should have a component having non-local evolution since they can give rise to Bell inequality violations.

In conclusion, the thought experiment we have discussed is not so remote from experimentally accessible situations; the spontaneous appearance of transverse spin polarization has already already been detected experimentally [15]. From a theoretical point of view, the compatibility between quantum mechanics and the no-signalling constraint of relativity has been the subject of many articles (for reviews, for instance see [16] or [17]). The usual conclusion [18] is that quantum mechanics remains compatible with relativity, even if parts of its formalism contains ingredients that are non-local and escape space-time description (“No story in spacetime can tell us how nonlocal correlations happen” [19]). Nevertheless, the present work focuses, not on the formalism, but on the results of measurements and their effects on measurement apparatuses, that is macroscopic events that can presumably be considered as space-time events. This makes the “tension” with relativity even more visible. It may be interesting to follow the line opened by Wigner [2] and examine in detail the general question of how quantum systems react on measurement apparatuses, in particular when they are entangled with another quantum system. This would be useful to better understand how quantum mechanics manages to remain fully compatible with relativity, and the reasons why a “peaceful coexistence between special relativity and quantum mechanics” [18] can be maintained. A conclusion of the present work is that, except if one prefers theories with non-local additional variables, the simplest way to preserve non-signalling is that proposed in (i) above: consider that angular momentum conservation applies only statistically, but not in each realization of an experiment.

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