Reinforced Concrete Circular Deep Beams - Finite Element Parametric Study

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Abstract. This paper presents a parametric study using finite element analysis of twenty six reinforced concrete circular deep beams. The considered parameters are loading type (partial and full), diameter, height and width of the circular beam, in addition to number of supports. It is concluded that applying a central partial uniformly distributed loading by 33-100% of the span length increased the load capacity by about 4-39%, and at the same time, decreased the maximum deflection and torsional moments by about 10-30% and 4-26%, respectively as compared with equivalent concentrated central loading. Increasing beam diameter by 33-167% led to decrease the load capacity by about 7-35%, while the deflection and torsional moments increased by about 41-265% and 24-77%, respectively. The load capacity and torsional moments increased by about 14-68% and 14-86%, respectively whereas deflection decreased by about 7-23% when the height of beam increased by 11-56%. The load capacity, deflection and torsional moments increased by about 163-458%, 54-83% and 160-451%, respectively when the circular beam width increased by 50-150%. Finally, it is found that the load capacity increased by about 85-365%, while the deflection and torsional moments decreased by about 24-56% and 10-46%, respectively when number of support increased by 25-100%.

1. Introduction
Presently, circular beams are extensively used in the construction of circular foundation, dome, and base tank or to fulfill some aesthetic purposes in addition to their normal use in circular structures [1]. Such beams, when are loaded transverse to their planes, are subjected to torsion, bending, and shear. Therefore, a special feature of the analysis and design is the necessary to include combined effects. In a reinforced concrete structure, cracks form even at service load. Such cracking causes local reduction in torsional and flexural stiffness values resulting in internal forces redistribution.

Reinforced concrete circular deep beams are commonly supported over several supports. The ACI 318M-14 Code defines deep beams as [2]: "the structural members that are supported on a face and loaded on the opposite face such that shear stresses can develop between the supports and loads. That satisfies (a) or (b): (a) Single forces lie in a distance 2h from support face; and (b) Clear span is not more the four times the depth of the member h". However, most of the existing investigations to study the structural behaviour and strength of deep members have focused on single-span deep beams [3-8] and continuous deep beams [9-13], pile caps [14], corbels [15-19] but not focused on circular deep beams.

Finite element is a very useful tool for analyzing and designing reinforced concrete members [20-26]. The circular deep beams are very common in structural engineering, while lack of such studies on circular deep
beams is obvious. That is why this study offered analytical report of results for twenty-six reinforced normal concrete circular deep beams in order to investigate the effects of loading type, beam diameter, beam height, beam width and number of supports on the load capacity, deflection, torsional moments and failure location.

2. Finite element model
The methodology includes study of the parametric effect on circular deep beam using ETABS software. The whole work is divided into the following sequential steps.

2.1 Modeling
Three-dimensional (3D) finite element (FE) analysis has become a popular choice to predict the behavior of structural elements. The models are created using ETABS software. The beams are designed according to ACI 318-14 using the maximum load capacity. The vertical slice mesh is about 72-144 elements along the circular, chosen based on convergence studies carried out to determine the optimal mesh that gives a relatively accurate solution, taking low computational time. In the current study, steel was assumed to behave as an elastic perfectly plastic material in both tension and compression. After analysis, the results were obtained and evaluated.

2.2 Reinforced concrete circular deep beam study cases
A total of twenty-six reinforced concrete circular deep beams are designed. They are divided based on five parameters. The first parameter included five beams that had different types of loading, more specifically, vertical concentrated or uniform (partial or full) 33%, 50%, 67% and 100% of a span length. In addition to loading type, different parameters concerning circular deep beam dimensions are also studied. The second parameter included six beams that had various diameters; 3000 mm, 4000 mm, 5000 mm, 6000 mm, 7000 mm and 8000 mm. The third parameter included five beams that had various beam height values; 900 mm, 1000 mm, 1100 mm, 1200 mm, 1300 mm and 1400 mm. The fourth parameter included four beams that had various values of beam width values; 200 mm, 300 mm, 400 mm, and 500 mm. Finally, the fifth parameter included five beams with various number of equally spaced supports; 4, 5, 6, 7 and 8, Figure 1 shows the circular deep beam in 3D modeling.

2.3 Material properties
Circular deep beams are made up of two different materials including concrete and steel reinforcement. Each element type in this modeling had been used to represent a specified beam’s constituent. The real constants need the geometrical properties of the used elements such as cross-sectional area. Whereas the
needed material properties depend on mechanical tests such as compressive strength \( f'_c \), yield stress of main reinforcement \( f_y \), yield stress of web reinforcement \( f_{ys} \), Poisson’s ratio \( \nu \), modulus of elasticity of concrete \( E_c \) and modulus of elasticity of steel reinforcement \( E_s \). The material properties of circular deep beam are given Table 1.

| Table 1. Material Properties. |
|-------------------------------|
| \( E_c \) (MPa) | \( E_s \) (MPa) | \( f'_c \) (MPa) | \( \nu \) | \( f_y \) (MPa) | \( f_{ys} \) (MPa) |
| 24855.58 | 199947.98 | 27.58 | 0.2 | 1 | 413.69 | 413.69 |

3. Parametric study:
Using ETABS software, the results of load capacity, maximum deflection, maximum torsional moments and failure location of all beams are presented in the subsequent paragraphs.

3.1 The effect of loading type:
Circular beams are mostly act by concentrated and uniformly distributed load, for example, uniform load transferred from a dome or something else, concentrated or uniform (partial or full) 33%, 50%, 67% and 100% of the span length, Table 2.

| Table 2. Effect of loading type. |
|----------------------------------|
| Parameter | Diameter (mm) | Height (mm) | Width (mm) | No. of supports | Load capacity (kN) | Max mid-span Deflection (mm) | Max Torsional moments (kN-m) |
|----------|---------------|-------------|------------|-----------------|-------------------|----------------|-----------------------------|
| Concentrated load | 6000 | 1200 | 300 | 6 | 6608 | 0.518 | 58.969 |
| Partial uniform load (33%) | 6000 | 1200 | 300 | 6 | 6855 | 0.466 | 56.894 |
| Partial uniform load (50%) | 6000 | 1200 | 300 | 6 | 7134 | 0.436 | 54.313 |
| Partial uniform load (67%) | 6000 | 1200 | 300 | 6 | 7527 | 0.404 | 50.100 |
| Full uniform load (100%) | 6000 | 1200 | 300 | 6 | 9217 | 0.363 | 43.669 |

The wider the load distribution area, the greater the load capacity of the beam, the lower the deflection and torsional moments, because the concentration of the load causes a concentration in the stresses and thus an early failure occurs. More specifically:
1- Applying partial or full uniformly distributed load by about 33, 50, 67 and 100% of the span length increases the load capacity, in comparison with concentration load, by about 4, 8, 14 and 39%, respectively, Figure 2. Those differences in results can be attributed to the effect of stresses concentration
2- Applying partial or full uniformly distributed load by 33, 50, 67 and 100% of the span length led to deflection decrease by about 10, 16, 22 and 30% as compared with that occurred due to concentrated loading, Figure 3
3- Applying partial or full uniformly distributed load by 33, 50, 67 and 100% of the span length leads to torsional moments decrease by about 4, 8, 15 and 26% as compared with that occurred due to concentrated loading, Figure 4.

4- Under uniformly distributed load, beam failure occurs at the supporting points, while under the concentrated loading, the failure occurs in a zone that lies between loading and supporting points, Figure 5.

**Figure 2.** Effect of loading type on load capacity.

**Figure 3.** Effect of loading type on deflection.
3.2 The effect of dimensions variation:
A change in any of the beam dimensions leads to a major change in the way it carries the applied load. Dimensions are of the most important influencing parameters that must be taken into consideration, Table 3.

| Parameter | Diameter (mm) | Height (mm) | Width (mm) | No. of supports | Load capacity (kN) | Max mid-span deflection (mm) | Max torsional moments (kN-m) |
|-----------|---------------|-------------|------------|-----------------|--------------------|-----------------------------|-----------------------------|
| dimension diameter | 3000          | 1200        | 300        | 6               | 8485               | 0.217                       | 37.567                      |
|            | 4000          | 1200        | 300        | 6               | 7855               | 0.305                       | 46.473                      |
|            | 5000          | 1200        | 300        | 6               | 7216               | 0.405                       | 53.503                      |
|            | 6000          | 1200        | 300        | 6               | 6608               | 0.518                       | 58.969                      |
|            | 7000          | 1200        | 300        | 6               | 6050               | 0.647                       | 63.204                      |
|            | 8000          | 1200        | 300        | 6               | 5547               | 0.791                       | 66.496                      |

Figure 4. Effect of loading type on torsional moments.

Figure 5. Failure locations due to uniformly full load.
3.2.1 The effect of diameter:
This variable is the most influential. Six beams that had various circular diameter; 3000 mm, 4000 mm, 5000 mm, 6000 mm and 7000 mm under one single central concentrated load were investigated. Studying the effect of diameter shows that:
1- Increasing the circular diameter by 33-167% decreases the load capacity by about 7-35% as shown in Figure 6. The increase in diameter with keeping the same number of supports increases the distance between loading and supporting points, which increases the resulting moments.
2- The deflection increased by about 41-265% with increasing the circular diameter by about 33-167%, Figure 7 due to span length increase.
3- Torsional moments increased by about 24-77% with increasing the diameter by 33-167%, Figure 8, due to span length increase.
4 - The failure mode in all beams occurred between loading and supporting points, which indicates the shear and torsion failure. Figure 9 shows the failure of beam when diameter is 8000 mm.

| height  | 6000 | 900  | 300  | 6   | 4698 | 0.626 | 41.949 |
|---------|------|------|------|-----|------|-------|--------|
|         | 6000 | 1000 | 300  | 6   | 5334 | 0.580 | 47.621 |
|         | 6000 | 1100 | 300  | 6   | 5976 | 0.545 | 53.335 |
|         | 6000 | 1200 | 300  | 6   | 6608 | 0.518 | 58.969 |
|         | 6000 | 1300 | 300  | 6   | 7245 | 0.497 | 64.643 |
|         | 6000 | 1400 | 300  | 6   | 7882 | 0.480 | 70.317 |
| width   | 6000 | 1200 | 200  | 6   | 2513 | 0.336 | 22.642 |
|         | 6000 | 1200 | 300  | 6   | 6608 | 0.518 | 58.969 |
|         | 6000 | 1200 | 400  | 6   | 10461| 0.586 | 93.165 |
|         | 6000 | 1200 | 500  | 6   | 14022| 0.614 | 124.788|

Figure 6. Effect of diameter variation on load capacity.
Figure 7. Effect of diameter variation on deflection.
3.2.2 The effect of height:
To investigate the effect of height on the capacity of the 1-concentrated loaded beams, six beams that had various height values; 900 mm, 1000 mm, 1100 mm, 1200 mm, 1300 mm and 1400 mm were modelled. Studying the effect of height showed that:
1- Increasing the height by 11-56% increased the load capacity by about 14-68% as shown in Figure 10. Increasing height increases the concrete sectional area that resists the shear.
2- Deflection decreased by about 7-23% due to increasing the height by 11-56%, Figure 11. That took place because increasing height increases beam sectional area, i.e. increases resistance to the resulting moments.
3- Torsional moments increased by about 14-86% due to increasing the height by 11-56% as shown in Figure 12. Increasing the beam height increases torsional moments due to the increase in arm of the torsional forces.
4- The failure occurred in the zone lies between loading and supporting points, which indicates the occurrence of shear and torsion failure. Figure 13 shows how the failure of beam with a height of 1200 mm occurred between loading and supporting points.
3.2.3 The effect of beam width:

Four beams were modeled with various values of beam width; 200 mm, 300 mm, 400 mm and 500 mm:

1- Increasing beam width by 50-150% increased the load capacity by about 163-458% as shown in Figure 14. The increase in width increases the concrete sectional area that resists the applied shear.

2- The deflection increased by about 54-83% because of increasing the width by 50-150%, Figure 15. Increasing width leads to increasing load capacity, and hence, increasing deflection in a nonlinear manner.

3- The torsional moments increased by about 160-451% within reason increasing the width by about 50-150% as shown in Figure 16. Increasing width increases torsional moments due to increasing the arm of torsion forces.

4- The failure occurred in a zone lies between loading and supporting points, which indicates the occurrence of shear and torsion failure. Figure 17 shows how the failure of a beam with width of 500 mm occurred between loading and supporting points.
3.3 The effect of the number of supports:
To study the effect of this important parameter, five different circular deep beams by number of supports were analyzed, Table 4:
1- Increasing number of supports from 5 to 8 led to increase the load capacity by about 85-365%, Figure 18. The increase in the number of supports leads to a decrease in the shear span length and thus, an increase in the shear capacity happens.
2- Deflection decreased by about 24-56% because of increasing the number of supports from 5 to 8, Figure 19. The increase in the number of supports leads to a decrease in the span length and thus, a decrease in the deflection moments happens.
3- The torsional moments decreased by about 10-46% with increasing the number of supports from 5 to 8 as shown in Figure 20. The increase in the number of supports leads to a decrease in the span length and thus, a decrease in the torsional moments happens.
4- The failure occurred in a zone lies between loading and supporting points, which indicates the occurrence of shear and torsion failure. Figure 21 shows how the failure of a beam with four supports occurred between loading and supporting points.

| Parameter | Diameter (mm) | Height (mm) | Width (mm) | No. of supports | Load capacity (kN) | Max mid-span deflection (mm) | Max torsional moments (kN-m) |
|-----------|---------------|-------------|------------|-----------------|-------------------|-----------------------------|-----------------------------|
| support   | 6000          | 1200        | 300        | 4               | 2364              | 0.839                       | 76.382                      |
|           | 6000          | 1200        | 300        | 5               | 4365              | 0.634                       | 68.635                      |
|           | 6000          | 1200        | 300        | 6               | 6608              | 0.518                       | 58.969                      |
|           | 6000          | 1200        | 300        | 7               | 8470              | 0.473                       | 44.624                      |
|           | 6000          | 1200        | 300        | 8               | 11000             | 0.370                       | 41.194                      |
Figure 18. Effect of no. of support on ultimate capacity.

Figure 19. Effect of no. of support on deflection.
4. Conclusions
Using ETABS finite element software in order to study some circular deep beam influential parameters led to the following conclusions:
1- Applying partial or full uniformly distributed load by 33, 50, 67 and 100% of the span length increased the load capacity by about 4, 8, 14 and 39%, respectively, in addition to decreasing deflection and torsional moments by about 10, 16, 22 and 30% and 4, 8, 15 and 26%, respectively as compared with central concentrated loading. The difference in results took place due to the fact that increasing the load application area reduces the stress concentration and hence, reduces the resulting bending and torsional moments.
2- The load capacity decreased by about 7-35%, while the deflection and torsional moments increased by about 41-265% and 24-77% respectively, when diameter increased by 33-167%. Increasing diameter increases span length which leads to the happened differences.
3- The load capacity and torsional moments increased by about 14-68%, whereas the deflection decreased by about 7-23% when the beam height increases by 11-56%. Increasing height leads to increase the beam sectional area which increases strength.
4- The load capacity, deflection and torsional moments increased by about 163-458%, 54-83% and 160-451%, respectively when the beam width increased by 50-150%. Increasing width causes beam sectional area increase which increases strength.
5- Load capacity increased by about 85-365%, while the deflection and torsional moments decreased by about 24-56% and 10-46%, respectively when number of supports increased by 25-100%. Increasing the number of supports reduces the span length, therefore, increases the shear capacity and reduces the resulting bending and torsional moments.

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