Binary collisions and the slingshot effect

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Abstract We derive the equations for the gravity assist manoeuvre in the general 2D case without the constraints of circular planetary orbits or widely different masses as assumed by Broucke (AIAA/AAS 1988) and obtain the slingshot conditions and maximum energy gain for arbitrary mass ratios of two colliding rigid bodies. Using the geometric view developed in an earlier paper by the authors (Rica da Silva, A., Lemos, J.P.S.: Am. J. Phys. 74, 584–590, 2006) the possible trajectories are computed for both attractive or repulsive interactions yielding a further insight on the slingshot mechanics and its parametrization. http://centra.ist.utl.pt/amaro/Collisions/Collisions.html. The general slingshot manoeuvre for arbitrary masses is explained as a particular case of the possible outcomes of attractive or repulsive binary collisions, and the correlation between asymptotic information and orbital parameters is obtained in general.

Keywords Binary collision · Gravity assist · Slingshot manoeuvre

1 Introduction

The slingshot or gravity assist manoeuvre (Broucke 1988; Longuski and Williams 1991; Johnson 2003; Dykla et al. 2004; Epstein 2005; Rica da Silva 2005; Rica da Silva and Lemos 2006) is often considered as part of a restricted three-body problem and its use has been associated in the literature mostly with spacecraft strategies (Racca 2003; Malyshev et al. 2003; Ocampo 2003) with some applications in astrophysics for the study of mass ejection from binary clusters (Saslaw et al. 1974) and the proposal of new General Relativity tests (Longuski et al. 2001). In reality, the design of spacecraft trajectories between two planets is a
many-body problem except for the slingshot part, which is in most designs well approximated by an elastic binary collision. This work focuses on the slingshot manoeuvre as a particular case of a general binary elastic collision between massive objects subjected to central interaction forces. In a previous work (Rica da Silva and Lemos 2006) the geometric determination of binary collisions was introduced, and the possible outcomes were in some cases surprising (Rica da Silva 2005). We have obtained a parametrization of all possible outcomes of a binary elastic collision in an arbitrary frame, and from the mass ratios and initial velocities as asymptotic initial conditions we obtain a picture not only of the final asymptotic velocities in terms of a single parameter \( \theta \) in the 2D case, but also the detailed description of the two-body motion that fits these asymptotic data in the case of the gravitational or Coulombian interaction. We are therefore in condition to determine which precise orbital parameters must be chosen to obtain a desired effect on a flyby of a satellite about a planet or star, be it a gravity assisted boost or capture. The conditions for a gravity-assisted manoeuvre of a satellite are often loosely associated to a flyby in front or behind the planet (Broucke 1988). Both in the case of attractive as well as repulsive collision forces, a harder look must be performed to really grasp what the critical ingredient is. In particular we show how the geometry and the timing of arrival at the point of closest approach (periapsis), to wit the relative position of the bodies with respect to the normal to the Center of Mass velocity \( \vec{V}_{cm} \) at that point, determines the outcome of the collision and refine the phenomenological rule-of-thumb that a flyby in front of the planet results in a slowing of the satellite whilst a flyby behind the planet’s trajectory would result in a boost (Labunsky et al. 1988). In fact, the asymptotic description of a collision is somewhat elusive in this respect. By scaling out the interaction in all space and time dimensions there is naturally a loss of information of precisely where and when are the two bodies for given initial velocities, so a determination must be made as to what corresponds in a real problem to these times and positions. It could be claimed that these initial velocities should be those of the bodies when their distance is equal to the sum of the radii of their respective spheres of influence (Barger and Olsson 1995). But since this too is a fuzzy concept this is not much of an improvement. In fact, that information is only present when enough conditions are specified to determine the collision outcome uniquely (Asada 2007). Thus, in the 2-dimensional case, the circumference of possibilities for the velocity outcomes of one of the bodies encodes the missing information about where the bodies initially are simultaneously when they have the given velocities. This can also be translated into an impact parameter in the non-inertial body-frame for one of the masses (or reduced-mass frame) but that begs the question of viewing the collision in the laboratory frame. In this paper the assumption will therefore be made that at \( t = 0 \) the Center of Mass (CM) will be at the origin of the laboratory frame (LF).

Diagrams like the one in Fig. 1, introduced in a previous work (Rica da Silva and Lemos 2006), are used to correlate the asymptotic information, provided by initial velocities far away from the periapsis, with the possible eccentricities, focal distances and other orbital parameters for open Keplerian orbits in case of gravitational attractive or Coulombian repulsive scattering. These diagrams depict the relation between incoming laboratory frame asymptotic velocities \( \vec{v}_o \) and \( \vec{u}_o \), of masses \( m_v \) and \( m_u \) respectively, and their final asymptotic velocities \( \vec{v}_1 \) and \( \vec{u}_1 \), through a computation involving the scattering angle \( \theta \) of the \( m_u \) mass, measured in its initial asymptotic \( u \)-body frame from the direction of the incoming relative velocity \( \vec{v}_o - \vec{u}_o \) of the \( m_v \) mass. In that \( u \)-body frame the circumferences of possible velocity outcomes are easily drawn and their image in the laboratory frame can easily be deduced, thus yielding information about the possible directions and magnitudes of asymptotic final velocities \( \vec{v}_1 \) and \( \vec{u}_1 \) for both masses. In this way the orbits can be viewed in the laboratory frame and a study can be made, for instance, for the optimal incidence angle on a planetary fly-by that delivers the maximum allowed velocity boost in a chosen direction. The energy