Magnetohydrodynamic peristaltic motion of a Newtonian fluid through porous walls through suction and injection

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Abstract: In this paper, we investigate the peristaltic transport of a conducting Newtonian fluid bounded by permeable walls with suction and injection moving with constant velocity of the wave in the wave frame of reference under the consideration of long wavelength and low Reynolds number. The analytical solution for the velocity field, pressure gradient and the frictional force are obtained. The effect of suction/injection parameter, amplitude ratio and the permeability parameter including slip on the flow quantities are discussed graphically. It is found that the greater the suction/injection parameter, the smaller the pressure rise against the pump works. Further, the pressure rise increases with increasing Magnetic parameter.

1. Introduction
The centrality of peristaltic instrument has been thinking about in both physiological and mechanical conditions. The mechanism is naturally found in esophagus, gastrointestinal, urinary and reproductive tracts. The bio-medical apparatus such as blood pumping machine and dialysis machine are designed by the principle of peristalsis. In nuclear reactors, the mechanism is very helpful to pump the corrosive fluids.

The action of magnetic field has been found in the treatment of cancer tumor; in the controlling of bleeding during surgery, transport of drug to the target using magnetic particles etc. A few investigations on the influence of magnetic field with peristalsis have been reported in [1-5].

In general, the suction and injection mechanism has been found in the drug transport system. Ramesh Babu et al. [6] presented a theoretical on peristaltic motion of a Newtonian fluid in a porous channel through suction and injection. In another attempt, peristaltic pumping of Jeffrey fluid between the porous walls with suction and injection by Kavitha et al. [7]. They noticed that the flux increases with increasing suction/injection parameter in the free pumping region. Hari prabakaran et al. [8,9] and Hemadri Reddy et al.[10] have investigated the peristaltic pumping of a generalized Newtonian fluid through porous channel with suction/injection.

In the examination of some physiological situations, it is fascinating to study the peristaltic pumping of a conducting biofluid with suction and injection. The main objective of the present investigation is to introduce the magnetohydrodynamic peristaltic motion of a conducting Newtonian fluid with blowing and suction under lubrication approach. The analytical expressions for the velocity, pumping characteristics, frictional force of one wave length are obtained.
2. Mathematical formulation
We consider the flow of an incompressible conducting Newtonian fluid between two porous walls. The sinusoidal wave travelling occurs on the lower and upper porous walls of the channel. The fluid is blowing into the channel vertical to the lower porous wall with constant velocity $\bar{V}_0$ and is sucked out into the upper porous wall with same $\bar{V}_0$. Due to symmetric waves on the porous walls, it is enough to discuss for half width of the channel $a$.

![Figure 1. Physical Model](image)

The geometry of the wall deformation is drawn by the subsequent expression

$$H(\bar{X}, t) = a + b \sin \frac{2\pi}{\lambda} (\bar{X} - ct)$$  \hspace{1cm} (1)

In the above equation, $a$, $\lambda$ and $c$ designates the amplitude of the peristaltic wave, wavelength and the speed of the peristaltic wave.

The transformation from the fixed frame of reference $(X, Y)$ to the wave frame of reference $(x, y)$ is given by

$$\bar{x} = \bar{X} - ct, \bar{y} = \bar{Y}, \bar{u} = U, \bar{v}_0 = V_0 \quad \text{and} \quad p(\bar{\tau}) = \bar{P}(\bar{X}, \bar{Y}, \bar{\tau})$$  \hspace{1cm} (2)

where $\bar{u}, \bar{v}_0$ represents velocities in $\bar{x}, \bar{y}$ direction.

Let us introduce the subsequent non-dimensional variables:

$$x = \frac{2\pi \bar{x}}{\lambda}, y = \frac{\bar{y}}{a}, t = \frac{2\pi c t}{\lambda}, u = \frac{\bar{u}}{c}, v_0 = \frac{\bar{v}_0}{c}, h = \frac{H}{a}, R = \frac{\rho c a}{\mu}, p = \frac{2\pi a^3}{c \lambda} \bar{P}$$

$$\phi = \frac{b}{a}, \sigma = \frac{a}{\sqrt{K}}, \alpha = \left( \frac{a m}{\sqrt{K}} \right)^{-1}, \delta = \frac{2\pi a}{\lambda}, M = \sqrt{\frac{\sigma B_0 a}{\mu}}$$  \hspace{1cm} (3)

In the above non-dimensional variables, $M$ is the Hartmann number. The parameters $R, \delta, \sigma, \phi, m$ and $\alpha$ designates Reynolds number, dimensionless wave number, permeability parameter, amplitude ratio, slip parameter and permeability including slip parameter respectively.
The dimensionless form equations after dropping the bars under the assumptions of long wave length take the form.

\[
\frac{\partial^2 u}{\partial y^2} - k \frac{\partial u}{\partial y} - M^2 u = P
\]  
(4)

\[
\frac{\partial p}{\partial y} = 0
\]  
(5)

where \( k = Rv_0 \) and \( P = \frac{\partial p}{\partial x} \)

The relative boundary conditions in dimensionless form are

\[
\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0
\]  
(6)

\[
u = -1 - \alpha \frac{\partial u}{\partial y} \quad \text{at} \quad y = h
\]  
(7)

3. Solution

The solution of (4) subject to the boundary conditions are given by

\[
u = -1 + \left( \frac{P}{M^2} - 1 \right) \left[ \frac{e^{m_1 y}}{k_1} + \frac{e^{m_2 y}}{k_2} - 1 \right]
\]  
(8)

where \( m_1 = \frac{k + \sqrt{k^2 + 4M^2}}{2} \), \( m_2 = \frac{k - \sqrt{k^2 + 4M^2}}{2} \),

\[
k_1 = e^{m_1 h} (1 + \alpha m_1), \quad m_1 = \frac{m_2}{m_1} e^{m_1 h} (1 + \alpha m_1), \quad k_2 = e^{m_2 h} (1 + \alpha m_2), \quad m_2 = \frac{m_1}{m_2} e^{m_2 h} (1 + \alpha m_2)
\]

The volumetric rate of \( q \) is given by

\[
q = \int_0^h u dy = -h + \left( \frac{P}{M^2} - 1 \right) \left[ \frac{e^{m_1 h} - 1}{m_1 k_1} + \frac{e^{m_2 h} - 1}{m_2 k_2} - h \right]
\]  
(9)

The instantaneous flux at any axial station is given by

\[
Q(X,t) = \int_0^h U(X,Y,t) dY = \left( \frac{P}{M^2} - 1 \right) \left[ \frac{e^{m_1 h} - 1}{m_1 k_1} + \frac{e^{m_2 h} - 1}{m_2 k_2} - h \right]
\]  
(10)

Through the equation (9), we have
\[
\frac{dp}{dx} = M^2 \left[ \frac{q + e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} \right] \left[ \frac{e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} - h \right]^{-1}
\]

The averaging volume flow rate over one time period is

\[
\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1
\]

Through equation (11), we get

\[
\Delta p = \int_0^1 \left( \frac{dp}{dx} \right) dx = \int_0^1 M^2 \left[ \frac{q + e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} \right] \left[ \frac{e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} - h \right]^{-1} dx
\]

The dimensionless friction force \( F \) given by

\[
F = \int_0^1 h \left( -\frac{dp}{dx} \right) dx = \int_0^1 M^2 \left[ \frac{q + e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} \right] \left[ \frac{e^{m_1h} - 1}{m_1k_1} + \frac{e^{m_2h} - 1}{m_2k_2} - h \right]^{-1} dx
\]

4. Results and Discussion
The objective of this research is to study Magnetohydrodynamic Peristaltic Motion of a Newtonian fluid through porous walls. In order to find out numerical solutions, MATHEMATICA software is used. Figure 2 shows the impact of suction/Injection parameter \( k \) (\( k = 0.3, 0.6, 0.9, 1.2 \)) on pressure rise with flux being fixed other parameters. We perceive from this graph that the pressure rise reduces by an increase in \( k \) when \( \bar{Q} < 0.35 \) where as the pumping curves coincide in the first quadrant at \( \bar{Q} \approx 0.35 \). The behavior is contrary when \( \bar{Q} > 0.35 \). For free pumping the flux rises by increase in \( k \). Figure 3 reveals the dissimilar values of \( \phi \) (amplitude ratio) on \( \Delta P \) with fixed other parameters. It can be noticed that from this graph that the results rises in pumping region \( (\Delta P > 0) \) when \( \phi \) increased. Figure 4 depicts the influence of \( \alpha \) on \( \Delta P \) . It has been inferred that the pressure rise reduces in region \( (\Delta P > 0) \), the curves concurs in the free pumping region \( (\Delta P = 0) \) and we observe opposite behaviour in the co-pumping region \( (\Delta P < 0) \) by increase in \( \alpha \). Figure 5 shows the impact of Hartmann number \( (M = 0.25, 0.5, 0.75, 1) \) on \( \Delta P \) with fixed other parameters. It can be noticed that by rise in \( M \), the flux rises in pumping and free pumping regions. Figures 6-9 present the variations of friction force with different values of \( k, \alpha, \phi \) and \( M \). It can be noticed that from 6 and 8 graphs .the results in friction force rises and diminished by increase in \( k \) and \( \alpha \) . Figures 7 and 9 reveal the dissimilar values of \( \phi \) and \( M \) on \( F \) with fixed other parameters. It can be observed from these graphs that the friction force initially reduces and gradually rises by increase in \( \phi \) and \( M \). Therefore , we conclude from these figures that the friction force \( F \) has opposite behavior when we compared to \( \Delta P \)
Figure 2. The difference of pressure rise with flux for various value of $k$ with $\phi = 0.6, \alpha = 0.1, M = 0.5$.

Figure 3. The difference of pressure rise with flux for various values of $\phi$ with $k = 0.1, \alpha = 0.1, M = 0.5$. 
Figure 4. The difference of pressure rise with flux for various values of $\alpha$ with $k = 0.1, \phi = 0.6, M = 0.5$.

Figure 5. The difference of pressure rise with flux for various values of $M$ with $k = 0.1, \alpha = 0.1, \phi = 0.6$. 
Figure 6. The difference of frictional force with flux for various value of \( k \) with 
\[ \phi = 0.6, \alpha = 0.1, M = 0.5. \]

Figure 7. The difference of frictional force with flux for various values of \( \phi \) 
with \( k = 0.1, \alpha = 0.1, M = 0.5. \)
5. Conclusion
In the article, the theoretical analysis of MHD peristaltic flow of an incompressible Newtonian fluid between two permeable walls with suction and injection has been studied under the assumptions of long wave length. The analytical expressions for the velocity, pumping characteristics and frictional force of one wave length are obtained. The results are discussed with the help of graphs. The outcomes are highlighted as follows:

1. In free pumping region, the flux increases by increasing suction/injection parameter $k$.
2. For larger slip parameter $\alpha$, the pressure rise decreases in pumping region ($\Delta P > 0$), the curves coincides in the free pumping region ($\Delta P = 0$) and the trend reversed in the co-pumping region ($\Delta P < 0$).
3. For pumping and free pumping regions, the flux increases with increasing $M$.
4. The frictional force at the wall shows opposite behavior to that of $\Delta P$.
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