Supersymmetric thermalization and quasi-thermal Universe: consequences for gravitinos and leptogenesis

Rouzbeh Allahverdi\textsuperscript{1} and Anupam Mazumdar\textsuperscript{2}
\textsuperscript{1} Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada
\textsuperscript{2} NORDITA, Blegdamsvej-17, Copenhagen-2100, Denmark
E-mail: rallahverdi@perimeterinstitute.ca and anupamm@nordita.dk

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Abstract. Motivated by our earlier paper \cite{AllahverdiMazumdar}, we discuss how the infamous gravitino problem has a natural built-in solution within supersymmetry. Supersymmetry allows a large number of flat directions made up of gauge-invariant combinations of squarks and sleptons. Out of many, at least one generically obtains a large vacuum expectation value during inflation. Gauge bosons and gauginos then obtain large masses by virtue of the Higgs mechanism. This makes the rate of thermalization after the end of inflation very small and as a result the Universe enters a quasi-thermal phase after the inflaton has completely decayed. A full thermal equilibrium is generically established much later on when the flat direction expectation value has substantially decreased. This results in low reheat temperatures, i.e., \( T_R \sim O(\text{TeV}) \), which are compatible with the stringent bounds arising from the big bang nucleosynthesis. There are two very important implications: the production of gravitinos and generation of a baryonic asymmetry via leptogenesis during the quasi-thermal phase. In both the cases the abundances depend not only on an effective temperature of the quasi-thermal phase (which could be higher, i.e., \( T \gg T_R \)), but also on the state of equilibrium in the reheat plasma. We show that there is no ‘thermal gravitino problem’ at all within supersymmetry and we stress the need for a new paradigm based on a ‘quasi-thermal leptogenesis’, because in the bulk of the parameter space the old thermal leptogenesis cannot account for the observed baryon asymmetry.
Supersymmetric thermalization and quasi-thermal Universe

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1. Introduction

Primordial inflation [2] is the best paradigm for explaining the initial conditions for the structure formation and the cosmic microwave background (CMB) anisotropies [4]. The Universe is cold and empty after inflation and all the energy is stored in the inflaton condensate oscillating around the minimum of its potential. Reheating is an important stage for any inflationary model. It describes the transition from this frozen state to a hot thermal Universe. It involves various processes and eventually results in a thermal bath of elementary particles which contains the Standard Model (SM) degrees of freedom.

The inflaton decay is the most relevant part of the reheating. Only one-particle decay of the non-relativistic inflaton quanta were considered initially [5]. The treatment is valid if the energy transfer to the fields which are coupled to the inflaton takes place over many inflaton oscillations. This requires that the inflaton couplings to the SM fields are sufficiently small. Usually it is assumed that the plasma reaches complete kinetic and chemical equilibrium immediately after all the inflaton quanta have decayed. The reheat temperature of the Universe, $T_R$, is then quoted as [5]

$$T_R \sim 0.5 g_*^{-1/4} \sqrt{M_P \Gamma_d},$$

where $g_*$ is the total number of relativistic degrees of freedom, $\Gamma_d$ is the inflaton decay rate and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

It was also realized that the coherent oscillations of the inflaton can create particles non-perturbatively [7, 8]. This mechanism is called preheating and it is particularly efficient when the final products are bosonic degrees of freedom. It only takes about two dozen oscillations to transfer the energy from the homogeneous condensate to non-zero modes of the final state(s) [8]. However, despite efficiently transferring the energy, preheating does not result in a complete decay of the inflaton. An epoch of perturbative reheating is an essential ingredient of any potentially realistic cosmological model [9, 6]. In supersymmetry (SUSY) a first stage of preheating is naturally followed by the last stage of perturbative decay (see appendix A.1). For these reasons we concentrate on the perturbative inflaton decay, because of its relevance to create a thermal bath of SM particles.

Often cosmology is considered as a probe to the early Universe: a well known example is the Gravitino Problem in the context of a supersymmetric cosmology. SUSY introduces new degrees of freedom and new parameters. Most of them are rather poorly constrained from experiments. Cosmology however acts as a test bed where some of the SUSY particles...
can be tested. In this regard the reheat temperature plays an important role, as we shall explain below.

The gravitino is a spin-3/2 supersymmetric partner of the graviton, which is coupled to the SM particles with a gravitational strength. Gravitinos with both helicities can be produced from a thermal bath. There are many scattering channels which include fermion, sfermion, gauge and gaugino quanta, all of which have a cross-section $\propto 1/M_P^2$ \cite{14}, which results in a gravitino abundance (up to a logarithmic correction) as $\cite{15,16}$

$$\text{Helicity} \pm \frac{3}{2} \frac{n_{3/2}}{s} \simeq \left( \frac{T_R}{10^{10} \text{ GeV}} \right)^{10^{-12}},$$

(full equilibrium) \hspace{1cm} (1.2)

$$\text{Helicity} \pm \frac{1}{2} \frac{n_{3/2}}{s} \simeq \left( 1 + \frac{M_\tilde{g}^2}{12m_{3/2}^2} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right)^{10^{-12}},$$

where $M_\tilde{g}$ is the gluino mass. Note that for $M_\tilde{g} \leq m_{3/2}$ both the helicity states have essentially the same abundance, while for $M_\tilde{g} \gg m_{3/2}$ production of helicity $\pm 1/2$ states is enhanced due to their Goldstino nature. The linear dependence of the gravitino abundance on $T_R$ can be understood qualitatively. Since the cross-section for the gravitino production is $\propto M_P^{-2}$, the production rate at a temperature, $T$, and the abundance of the gravitinos produced within one Hubble time will be $\propto T^3$ and $\propto T$ respectively. This implies that the gravitino production is efficient at the highest temperature of the radiation-dominated phase of the Universe, i.e., $T_R$.

An unstable gravitino decays to particle–sparticle pairs, and its decay rate is given by $\Gamma_{3/2} \simeq m_{3/2}^3/4M_P^2$ \cite{14}. If $m_{3/2} < 50$ TeV, the gravitinos decay during or after big bang nucleosynthesis (BBN) \cite{17}, which can ruin its successful predictions for the primordial abundance of light elements \cite{18}. If the gravitinos decay radiatively, the most stringent bound, $(n_{3/2}/s) \leq 10^{-14}$–$10^{-12}$, arises for $m_{3/2} \simeq 100$ GeV–1 TeV \cite{19}.

On the other hand, much stronger bounds are derived if the gravitinos mainly decay through the hadronic modes. In particular, for a hadronic branching ratio $\simeq 1$, and in the same mass range, $(n_{3/2}/s) \leq 10^{-16}$–$10^{-15}$ will be required \cite{20}.

For a radiatively decaying gravitino the tightest bound $(n_{3/2}/s) \leq 10^{-14}$ arises when $m_{3/2} \simeq 100$ GeV \cite{19}. Following equation (1.2), the bound on reheat temperature becomes $T_R \leq 10^{10}$ GeV. This turns out to be a very stringent limit if the inflaton decay products immediately thermalize. In fact, for the inflaton mass $m_\phi = 10^{13}$ GeV, it is at best marginally satisfied even for a gravitationally decaying inflaton (see an example given in an appendix A.2). For a TeV gravitino which mainly decays into gluon–gluino pairs (allowed when $m_{3/2} > M_\tilde{g}$) a much tighter bound $(n_{3/2}/s) \leq 10^{-16}$ is obtained \cite{20}, which requires quite a low reheat temperature: $T_R \leq 10^6$ GeV.

The gravitino will be stable if it is the lightest supersymmetric particle (LSP), where $R$-parity is conserved. The gravitino abundance will in this case be constrained by the dark matter limit \cite{4}, $\Omega_{3/2} h^2 \leq 0.129$, leading to

$$\frac{n_{3/2}}{s} \leq 5 \times 10^{-10} \left( \frac{1 \text{ GeV}}{m_{3/2}} \right). \hspace{1cm} (1.3)$$

\textbf{8} From now on we take $g_*=228.75$ as in the case of minimal supersymmetric SM (MSSM).
Supersymmetry is introduced for the thermalization and quasi-thermal Universe. For $m_{3/2} < M_{\tilde{g}}$, the helicity $\pm 1/2$ states dominate the total gravitino abundance. As an example, consider the case with a light gravitino, $m_{3/2} = 100$ keV, which can arise very naturally in gauge-mediated models [21]. If $M_{\tilde{g}} \simeq 500$ GeV, see equation (1.2), a very severe constraint, $T_R \leq 10^4$ GeV, will be obtained on the reheat temperature.

As we have argued, gravitinos indirectly put constraints on the thermal history of the Universe; this is often known as the gravitino problem in models with weak scale SUSY. The reheat temperature, $T_R$, plays a very important role here as it sets the thermal (and possibly the largest) contribution to the gravitino abundance. However, besides the gravitino production, it has other implications for cosmology, for instance, thermal leptogenesis, thermal production of weakly interacting dark matter particles, etc. They are all directly or indirectly connected to the reheat temperature of the Universe.

The most important worry is the central assumption that the SM and or MSSM degrees of freedom thermalize instantly. In our opinion this is the key assumption which has not been questioned well enough in the literature. It is only very recently that various issues of thermalization have been considered in a non-SUSY case carefully [25]. We intend to cover this lapse and describe thermalization process within SUSY. The subject stands in its own right because SUSY introduces new ingredients into the game, i.e., flat directions. We will elaborate on the role of flat directions in thermalization, which was first pointed out in our earlier paper [1].

SUSY along with gauge symmetry introduces gauge-invariant flat directions. In any supersymmetric extension of the SM there are flat directions primarily made up of squarks, sleptons (SUSY partners of SM quarks and leptons) and Higgses. A flat direction in a cosmological context can take a large vacuum expectation value (VEV) by virtue of a shift symmetry. During inflation quantum fluctuations of any light field accumulate in a coherent state and its VEV makes a random walk with its variance growing linearly in time. The condensate becomes homogeneous on scales larger than the size of the Hubble radius with a growing VEV. However, in many cases (e.g., minimal supergravity) shift symmetry is not protected which truncates the VEV to be as large as the four-dimensional Planck scale, $M_P$. Moreover SUSY is broken and the flat directions obtain soft mass terms and $A$-terms. It is also possible that non-renormalizable terms arise in the superpotential after integrating out heavy degrees of freedom which are associated to a new physics at high scales. These contributions lift the flatness but still allow the VEV to be significantly large in a wide class of models; for a review see [26].

After inflation the VEVs of flat directions do not settle at the origin immediately. The cosmologically sliding VEV plays a crucial role in slowing down thermalization of the inflaton decay products as mentioned in [1]. The idea is very simple: if a flat direction develops a VEV, the SM gauge fields become massive, similar to the Higgs mechanism. The masses for the gauge bosons (and gauginos) are proportional to the VEV of the flat direction. This obviously breaks the charge and the colour in the early Universe, but the VEV of the flat direction finally vanishes and therefore there is no threat to the low-energy phenomenology.

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9 Similar results were obtained in connection to the brane worlds, see for instance [22], and for the discussions on reheating, see [23, 24].
Consider a situation where the inflaton has completely decayed into the MSSM degrees of freedom. The resulting plasma is initially far away from a full thermal equilibrium. The leading reactions which establish equilibrium are the $2 \to 2$ and $2 \to 3$ scatterings mediated by gauge fields [25, 29, 30]. The former lead to kinetic equilibrium, while the latter are required to change the total number of particles in order to give rise to a chemical equilibrium. However, these processes are suppressed by the VEV-dependent masses of the gauge bosons (and gauginos). As a result the Universe enters a period of a \textit{quasi-thermal} phase during which the reheat plasma evolves adiabatically. This lasts until the above-mentioned scatterings become efficient, at which time full equilibrium is finally attained, i.e., within one Hubble time.

Such a late thermalization also results in a low reheat temperature. For the intermediate values of the flat direction’s VEV, the reheat temperature within MSSM varies:

\[ 10^3 \text{ GeV} \leq T_R \leq 10^7 \text{ GeV}. \]

Interestingly such a low reheat temperature does not lead to a significant generation of the gravitinos. On the other hand, this is a \textit{bad news} for thermal leptogenesis which typically requires that $T_R \geq 10^9$ GeV [38].

One of the aims of this paper is to explore an alternative prescription for leptogenesis; here, we will introduce a new paradigm, quasi-thermal leptogenesis, which points towards a successful baryogenesis within SUSY.

The rest of this paper is organized as follows. In section 2 we briefly review thermalization after the perturbative inflaton decay (including relevant processes which lead to kinetic and chemical equilibrium), and then discuss the impact of supersymmetric flat directions on thermalization. Through numerical examples, in section 3, we will underline the dramatically altered picture which emerges. In section 4 we will consider thermalization in some of the other supersymmetric extensions of the SM, i.e., models with gauge mediation and split supersymmetry. After discussing particle production in the quasi-thermal phase in section 5, we will specialize to the gravitino production and the leptogenesis in sections 6 and 7 respectively. In section 8 we will make some remarks on thermalization after preheating. We have also included an appendix which helps the paper to be self-contained.

### 2. Thermalization in supersymmetric theories: part 1

#### 2.1. A brief recourse to thermalization

For a plasma which is in full thermal equilibrium, the energy density, $\rho$, and the number density, $n$, of a relativistic particles are given by

\[
\rho = \left( \frac{\pi^2}{30} \right) T^4, \quad n = \left( \frac{\zeta(3)}{\pi^2} \right) T^3, \quad \text{(Boson)},
\]

\[
\rho = \left( \frac{7}{8} \frac{\pi^2}{30} \right) T^4, \quad n = \left( \frac{3}{4} \frac{\zeta(3)}{\pi^2} \right) T^3, \quad \text{(Fermion)},
\]

The flat directions could also reheat the Universe and could also explain the fluctuations present in the cosmic microwave background radiation; see [27, 28]. However, there is a distinction: in this paper we demand that the inflaton decay is the major source for the entropy production.

Note that gravitinos are still produced during preheating [31]–[33] and the final stage of perturbative decay of the inflaton [34, 35] (see also [36, 37]). We will consider these cases separately in a different publication.
where $T$ is the temperature of the reheat plasma. Note that the $t$-channel singularity results in a cross-section $\alpha|t|^{-1}$. Here $t$ is related to the exchanged energy, $\Delta E$, and the momentum, $\vec{\Delta}p$, through $t = \Delta E^2 - |\vec{\Delta}p|^2$. The fine structure constant is denoted by $\alpha$ (note that $\alpha \geq 10^{-2}$ in the MSSM). This cross-section can be understood as follows: the gauge boson propagator introduces a factor of $|t|^{-2}$, while phase space integration results in an extra factor of $|t|$. Due to an infrared singularity, these scatterings are very efficient even in a dilute plasma.\footnote{\textsuperscript{12}There are also $2 \rightarrow 2$ scattering diagrams with a fermion or scalar exchange in the $t$-channel. Diagrams with a fermion (for example, gaugino) exchange have an amplitude $\propto |t|^{-1}$, which will be cancelled by the phase space factor $|t|$. This results in a much smaller cross-section $\propto s^{-1}$, where $s \approx 4E^2$ is the square of the centre-of-mass energy. Diagrams with a scalar (for example, Higgs) exchange are also suppressed by the following reason. A fermion–fermion-scalar vertex, which arises from a Yukawa coupling, flips the chirality of the scattered fermion. For relativistic fermions, as we consider here, the mass is $\ll E$ and a flip of chirality also implies a flip of helicity. This is forbidden by the conservation of angular momentum for forward scatterings, i.e., where $t \rightarrow 0$. As a result, the diagram in figure 2 has no $t$-channel singularity at all. Note that it is additionally suppressed by powers of Yukawa couplings compared to the diagram with gauge interactions in figure 1. For more details, see [20].}
Note that every degree of freedom of the MSSM has some gauge interactions with all other fields. Therefore any fermion in the plasma has $t$-channel scatterings off those fermions with the largest number density. For this reason the total number density, $n$, enters while estimating the scattering rate.

In addition one also needs to achieve chemical equilibrium by changing the number of particles in the reheat plasma. The relative chemical equilibrium among different degrees of freedom is built through $2 \to 2$ annihilation processes, occurring through $s$-channel diagrams. Hence they have a much smaller cross-section $\sim \alpha s^{-1}$. More importantly the total number of particles in the plasma must also change. It turns out from equation (2.1) that in order to reach full equilibrium, the total number of particles must increase by a factor of $n_{\text{eq}}/n$, where $n \approx \rho/m_\phi$ and the equilibrium value is $n_{\text{eq}} \sim \rho^{3/4}$. This can be a very large number; for examples given in the appendix, $n_{\text{eq}}/n \sim O(10^3)$. This requires that the number-violating reactions such as decays and inelastic scatterings must be efficient.

Decays (which have been considered in [39]) are helpful, but in general they cannot increase the number of particles to the required level. It was recognized in [25], see also [29,30], that the most relevant processes are $2 \to 3$ scatterings with gauge-boson exchange in the $t$-channel. Again the key issue is the infrared singularity of such diagrams shown in figure 2. The cross-section for emitting a gauge boson, whose energy is $|t|^{1/2} \ll E$, from the scattering of two fermions is $\sim \alpha^3 |t|^{-1}$. When these inelastic scatterings become efficient, i.e., their rate exceeds the Hubble expansion rate, the number of particles increases very rapidly [40], because the produced gauge bosons subsequently participate in similar $2 \to 3$ scatterings.

As a result the number of particles will reach its equilibrium value, $n_{\text{eq}}$, soon after the $2 \to 3$ scatterings become efficient. At that point the scatterings and inverse scatterings occur at the same rate and the number of particles will not increase further. Therefore full thermal equilibrium will be established shortly after the $2 \to 3$ scatterings become efficient. For this reason, to a very good approximation, one can use the rate for inelastic scatterings as a thermalization rate of the Universe, $\Gamma_{\text{thr}}$.

In order to estimate the thermalization rate, we need to choose an infrared cut-off on the parameter $t$. With a reasonable choice of a cut-off, it turns out that in realistic models the $2 \to 3$ scatterings have a rate higher than or the same as that of the inflaton decay rate (for more details we refer readers to [25,29,30]). Therefore, if the inflaton decay products have gauge interactions, the Universe reaches full thermal equilibrium immediately after the inflaton decay. The reason is that the $2 \to 3$ scatterings with gauge boson exchange
in the $t$-channel are very efficient. However this analysis crucially depends on having massless gauge bosons.

In general the reheat plasma induces a mass for all particles which it contains. This is however negligible in a dilute plasma as in the case after the perturbative inflaton decay. However, SUSY alters the situation quite dramatically, as we shall discuss below.

### 2.2. Flat directions in supersymmetry

The field space of supersymmetric theories contains many directions along which the $D$- and $F$-term contributions to the scalar potential identically vanish in the limit of unbroken SUSY. The most interesting such directions are those made up of SUSY partners of the SM fermions, namely the squarks and sleptons, and the Higgs fields. These directions have gauge and Yukawa interactions with matter fields, and hence decay before BBN. Therefore they do not lead to the cosmological moduli problem. As a matter of fact they can have very interesting cosmological consequences (for a review, see [26]).

The superpotential for the minimal supersymmetric standard model (MSSM) is given by (see for instance [41])

$$ W_{\text{MSSM}} = \lambda_u Q H_u u + \lambda_d Q H_d d + \lambda_e L H_d e + \mu H_u H_d, $$

where $H_u, H_d, Q, L, u, d, e$ in equation (2.2) are chiral superfields representing the two Higgs fields (and their Higgsino partners), LH (left-handed) (s)quark doublets, RH (right-handed) up- and down-type (s)quarks, LH(s)lepton doublets and RH(s)leptons respectively. The dimensionless Yukawa couplings $\lambda_u, \lambda_d, \lambda_e$ are $3 \times 3$ matrices in the flavour space, and we have omitted the gauge and flavour indices. The last term is the $\mu$ term, which is a supersymmetric version of the SM Higgs boson mass.

The SUSY scalar potential $V$ is the sum of the $F$- and $D$-terms, and reads

$$ V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 D^a D^a $$

where

$$ F_i \equiv \frac{\partial W_{\text{MSSM}}}{\partial \chi_i}, \quad D^a = \chi_i^* T^a_{ij} \chi_j. $$

Here the scalar fields, denoted by $\chi$, transform under a gauge group $G$, with the generators of the Lie algebra and gauge coupling denoted by $T^a$ and $g_a$ respectively.

For a general supersymmetric model with $N$ chiral superfields $X_i$, it is possible to find out the directions where the potential in equation (2.3) vanishes identically by solving simultaneously

$$ D^a X^\dagger T^a X = 0, \quad F_{X_i} = \frac{\partial W}{\partial X_i} = 0. $$

Field configurations obeying equation (2.5) are called respectively $D$-flat and $F$-flat.

$D$-flat directions are parameterized by gauge-invariant monomials of the chiral superfields. A powerful tool for finding the flat directions has been developed in [42]–[47], where the correspondence between gauge invariance and flat directions has been employed. There are nearly 300 flat directions within MSSM [47].
The flat directions are lifted by the soft SUSY-breaking mass term, $m_0$, 

$$V \sim m_0^2|\varphi|^2,$$

as well as higher-order terms arising from the superpotential \[^{13}\].

### 2.3. Flat directions during and after inflation

Because it does not cost anything in energy during inflation, where the Hubble expansion rate is $H_I \gg m_0$, quantum fluctuations are free to accumulate (in a coherent state) along a flat direction and form a condensate with a large VEV, $\varphi_0$. Because inflation smoothes out all gradients, only the homogeneous condensate mode survives. However, the zero-point fluctuations of the condensate impart a small, and in inflationary models a calculable, spectrum of perturbations on the condensate \[^{26}\]. In an abuse of language we will collectively call such condensates as flat directions.

After inflation, $H \propto t^{-1}$, the flat direction stays at a relatively larger VEV due to large Hubble friction term; note that the Hubble expansion rate gradually decreases but it is still large compared to $m_0$. When $H \simeq m_0$, the condensate along the flat direction starts oscillating around the origin with an initial amplitude $\sim \varphi_0$. From then on $|\varphi|$ is redshifted by the Hubble expansion $\propto H$ for a matter-dominated and $\sim H^{3/4}$ for a radiation-dominated Universe.

If higher-order superpotential terms are forbidden, due to an $R$-symmetry (or a set of $R$-symmetries) \[^{48}\], then we naturally have $\varphi_0 \sim M_P$ \[^{45}\]. On the other hand, $\varphi_0 \ll M_P$ will be possible if non-renormalizable superpotential terms are allowed. Let us first analyse the model-independent case where we treat $\varphi$ as a free parameter which can vary in a wide range with an upper limit $\varphi_0 \leq M_P$, and study various consequences. Note that a lower bound $H_I \leq \varphi_0$ is set by the uncertainty due to quantum fluctuations of $\varphi$ during inflation.

### 2.4. Flat directions and inflaton decay

If a flat direction which has a non-zero VEV has couplings to the inflaton decay product(s), then it will induce a mass, $y|\varphi|$, where $y$ is a gauge or Yukawa coupling. The inflaton decay at the leading order will be kinematically forbidden if $y|\varphi| \geq m_\phi/2$. One should then wait until the Hubble expansion has redshifted, $|\varphi|$, down to $(m_\phi/2y)$. The decay happens when (note that $|\varphi| \propto H$, after the flat direction starts oscillating and before the inflaton decays)

$$H_1 = \text{min} \left[ \left( \frac{m_\phi}{y\varphi_0} \right) m_0, \Gamma_d \right].$$

(2.7)

The inflaton also decays at higher orders of perturbation theory to particles which are not directly coupled to it \[^{50}\]. This mode is kinematically allowed at all times, but the rate is suppressed by a factor of $\sim (m_\phi/y|\varphi|)^2 \Gamma_d$. It will become efficient at

$$H_2 \sim \left( \frac{m_\phi m_0}{\varphi_0} \right)^{2/3} \Gamma_d^{1/3}.$$

(2.8)

\[^{13}\] Within supergravity there are corrections to the SUSY potential arising from the Kähler potential and also the mixing between superpotential and Kähler terms. We will consider these issues in section 4.
Therefore, if the decay products are coupled to a flat direction with a non-zero VEV, the inflaton will actually decay at a time when the expansion rate of the Universe is given by

$$H_d = \max [H_1, H_2].$$

(2.9)

In general it is possible to have $H_d \ll \Gamma$, particularly for large values of $\varphi_0$. Flat directions can therefore significantly delay inflaton decay on purely kinematical grounds.

### 2.5. Flat directions and thermalization

Flat directions can dramatically affect the thermal history of the Universe even if they do not delay the inflaton decay. The reason is that the flat direction VEV spontaneously breaks the SM gauge group. The gauge fields of the broken symmetries then acquire a supersymmetry conserving mass, $m_g \sim g|\varphi|$, from their coupling to the flat direction, where $g$ is a gauge-coupling constant.

The simplest example is the flat direction corresponding to the $H_d$ monomial. One can always rotate the field configuration to a basis where $H_{u,1} = H_{d,2} = 0$, with subscripts 1 and 2 denoting the upper and lower components of the Higgs doublets respectively. The complex scalar field, $\varphi = (H_{u,2} + H_{d,1})/\sqrt{2}$, represents a flat direction. It breaks the $SU(2)_W \times U(1)_Y$ down to $U(1)_{em}$ (exactly in a similar fashion as what happens in the electroweak vacuum). The $W^\pm$ and $Z$ gauge bosons then obtain a mass from their couplings to the Higgs fields via covariant derivatives.

The complex scalar field, $(H_{u,1} + H_{d,2})/\sqrt{2}$, and the real part of, $(H_{u,2} - H_{d,1})/\sqrt{2}$, also acquire the same mass as $W^\pm$ and $Z$, respectively, through the $D$-term part of the scalar potential. The Higgsino fields $H_{u,1}$ and $H_{d,2}$ are paired with the Winos, while $(H_{u,2} - H_{d,1})/\sqrt{2}$ is paired with the Zino, and acquire the same mass as $W^\pm$ and $Z$, respectively, through the gaugino–gauge–Higgsino interaction terms. In the supersymmetric limit, the flat direction and its fermionic partner $(\tilde{H}_{u,2} + \tilde{H}_{d,1})/\sqrt{2}$, as well as the photon and photino, remain massless.

Many of the flat directions break the entire SM gauge group. The prominent examples are flat directions corresponding to the $LLddd$ and $QuQu$ monomials. The whole SM gauge group can be also broken by multiple flat directions which individually break only part of the symmetry. For example, flat directions corresponding to the $LLe$ monomial break the $SU(2)_W \times U(1)_Y$ group, while directions corresponding to the $udd$ monomial break the $SU(3)_C$. Note that all independent flat directions can simultaneously acquire a large VEV during inflation.

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14 Flat directions can also affect inflaton decay in other ways [49].

15 Note that such a delayed decay can also naturally implement the modulated fluctuations mechanism for generating adiabatic density perturbations by converting their isocurvature perturbations [51]. Due to the dependence of $H_d$ on $\varphi_0$, fluctuations of the flat direction ($\sim H_1$) imply an inhomogeneous inflaton decay which can give rise to perturbations of the correct magnitude if $H_1 \sim 10^{-\varphi_0}$.

16 If the $LLe$ monomial develops a VEV during inflation, which breaks the electroweak symmetry, it generates a mass to the weak hypercharge gauge boson which mixes to the photon. It is then possible to excite that gauge boson from vacuum fluctuations, which would then be converted to the primordial magnetic field, see [52].

17 The only exceptional flat direction is the $H_u H_d$ direction. Since it has superpotential Yukawa couplings to all MSSM fields, it will induce a large mass to other scalar fields. As a result no other flat direction can develop a large VEV in the presence of $H_u H_d$. This is not the case for the $H_u L$ flat direction: $LLe$ has no Yukawa couplings to it, and hence can simultaneously obtain a large VEV. For a discussion on multiple flat directions, see [53].
Let us now imagine one of such flat directions which has obtained a large VEV during inflation. In which case a flat direction can crucially alter thermal history of the Universe by suppressing thermalization rate of the reheat plasma. Note that $m_g$ provides a physical infrared cut-off for scattering diagrams with gauge boson exchange in the $t$-channel shown in figures 1, 2. The thermalization rate will then be given by (up to a logarithmic ‘bremsstrahlung’ factor)

$$\Gamma_{\text{thr}} \sim \alpha^2 \frac{n}{|\varphi|^2}, \quad (2.10)$$

where we have used $m_g^2 \simeq \alpha |\varphi|^2$. After the flat direction starts its oscillations at $H \simeq m_0$, the Hubble expansion redshifts, $|\varphi|^2 \propto R^{-3}$, where $R$ is the scale factor of the FRW Universe. The interesting point is that $n \propto R^{-3}$, as well, and hence $\Gamma_{\text{thr}}$ remains constant while $H$ decreases for $H < m_0$. This implies that $\Gamma_{\text{thr}}$ eventually catches up with the expansion rate, even if it is initially much smaller, and shortly after that the full thermal equilibrium will be achieved.

Therefore the flat directions, even if they do not delay the inflaton decay, will modify the thermal history of the Universe. Depending on whether $m_0 > \Gamma_d$ or $m_0 < \Gamma_d$, different situations will arise which we discuss separately. If two or more flat directions with non-zero VEVs induce mass to the gauge bosons, then $|\varphi|$ denotes the largest VEV.

- $m_0 > \Gamma_d$: in this case the inflaton decays after the flat direction oscillations start. The inflaton oscillations, which give rise to the equations of state close to non-relativistic matter, dominate the energy density of the Universe for $H > \Gamma_d$. This implies that $R \propto H^{-2/3}$ and $|\varphi|$ is redshifted $\propto H$ in this period. We therefore have

$$\varphi_d \sim \left( \frac{\Gamma_d}{m_0} \right) \varphi_0, \quad (2.11)$$

where $\varphi_d$ denotes the amplitude of the flat direction oscillations at the time of the inflaton decay $H \simeq \Gamma_d$.

The number density of particles in the reheat plasma at this time is given by

$$n_d \sim \frac{10 \Gamma_d^2 M_P^2}{m_0}, \quad (2.12)$$

where we have used $\rho \approx 3 (\Gamma_d M_P)^2$. Note that $|\varphi|$ and $n$ are both redshifted $\propto R^{-3}$ for $H < \Gamma_d$. Thus, after using equation (2.10), we find that complete thermalization occurs when the Hubble expansion rate is

$$H_{\text{thr}} \sim 10\alpha^2 \left( \frac{M_P}{\varphi_0} \right) ^2 \frac{m_0^2}{m_\phi}. \quad (2.13)$$

- $m_0 < \Gamma_d$: in this case the flat direction starts oscillating after the completion of inflaton decay. The Universe is dominated by the relativistic inflaton decay products for $H < \Gamma_d$, implying that $R \propto H^{-1/2}$. The number density of particles in the plasma is redshifted $\propto H^{3/2}$ and, see equation (2.12), at $H = m_0$ it is given by

$$n_0 \sim \frac{10 \Gamma_d^2 M_P^2}{m_\phi} \left( \frac{m_0}{\Gamma_d} \right)^{3/2}. \quad (2.14)$$
Supersymmetry thermalization and quasi-thermal Universe

Note that \( n, |\varphi|^2 \propto R^{-3} \) and hence \( \Gamma_{\text{thr}} \) remain constant for \( H < m_0 \). The reheat plasma then fully thermalizes when the Hubble expansion rate is

\[
H_{\text{thr}} \sim 10\alpha^2 \left( \frac{\Gamma_d}{m_0} \right)^{1/2} \left( \frac{M_P}{\varphi_0} \right)^2 \frac{m_0^2}{m_\phi},
\]

(2.15)

Note that the kinetic equilibrium is built through \( 2 \to 2 \) scattering diagrams as in figure 1, which have one interaction vertex less than those in figure 2. This implies that the rate for establishment of kinetic equilibrium will be \( \Gamma_{\text{kin}} \sim \alpha^{-1} \Gamma_{\text{thr}} \). Therefore we have a typical relationship:

\[
\Gamma_d \gg \Gamma_{\text{kin}} > \Gamma_{\text{thr}},
\]

(2.16)
in SUSY\(^{18}\).

This implies that the Universe enters a long period of quasi-adiabatic evolution after the inflaton decay has completed. During this phase, the comoving number density and (average) energy of particles remain constant. It is only much later that the \( 2 \to 2 \) and \( 2 \to 3 \) scatterings become efficient and the plasma completely thermalizes. As we shall see, this has important cosmological consequences.

One comment is in order before closing this section. So far we have neglected the decay of the flat directions and their interactions with the reheat plasma. These effects are considered in appendix A.4, and shown to be negligible before the Universe fully thermalizes.

3. Reheat temperature of the Universe

The temperature of the Universe after it reaches full thermal equilibrium is referred to as the reheat temperature \( T_R \). In the case of SUSY, we therefore have

\[
T_R \simeq (H_{\text{thr}} M_P)^{1/2},
\]

(3.1)

where, depending on the details, \( H_{\text{thr}} \) is given by equations (2.13) and (2.15).

As mentioned earlier, we typically have \( H_{\text{thr}} \ll \Gamma_d \). Therefore in SUSY theories the reheat temperature is generically much smaller than the standard expression \( T_R \simeq (\Gamma_d M_P)^{1/2} \), which is often used in the literature and assumes immediate thermalization after the inflaton decay.

An interesting point to note that the reheat temperature depends very weakly on the inflaton decay rate; for instance, equation (2.15) implies that \( T_R \propto \Gamma_d^{1/4} \), while \( T_R \) is totally independent of \( \Gamma_d \) in equation (2.13). This is not difficult to understand: regardless of how fast the inflaton decays, the Universe will not thermalize until the \( 2 \to 3 \) scatterings become efficient. The rate for these scatterings essentially depends on the flat direction VEV and mass.

As expected, a larger \( \varphi_0 \) results in slower thermalization and a lower reheat temperature. On the other hand, larger values of \( m_0 \) lead to a higher \( T_R \). Since the

\(^{18}\) Relative chemical equilibrium among different degrees of freedom is built through \( 2 \to 2 \) annihilations in the \( s \)-channel with a rate \( \sim \alpha^2 n/E^2 \ll \Gamma_{\text{thr}} \). Hence the composition of the reheat plasma will not change until full thermal equilibrium is achieved.
flat direction oscillations start earlier in this case, its VEV (thus mass of gauge bosons) is redshifted faster and thermalization rate will be less suppressed.

The reheat temperature, $T_R$, depends on the inflation sector through the inflaton mass $m_\phi$. It is counterintuitive, but the fact is that, from the above equations (2.13), (2.15) and (3.1), a larger $m_\phi$ results in a lower reheat temperature. This is due to the conservation of energy which implies that the number density of inflaton decay products is inversely proportional to their energy, which is $\sim m_\phi$ initially. Since the scattering rate is proportional to the number density, a larger $m_\phi$ thus results in a smaller $\Gamma_{thr}$ and a lower $T_R$.

If $H_{thr}$ is very large or $\Gamma_d$ is very small, we may find $H_{thr} \geq \Gamma_d$. Obviously thermalization cannot occur before the inflaton decay has completed. This merely reflects the fact that the $2 \rightarrow 3$ scatterings are already efficient when $H \simeq \Gamma_d$. This will be the case if the flat direction VEV is sufficiently small at the time of the inflaton decay, and/or if the reheat plasma is not very dilute. The former happens for a small $\varphi_0$ or large $m_0$, while smaller values of $m_\phi$ lead to the latter case. The reheat temperature in such cases follows the standard expression: $T_R \simeq (\Gamma_d M_P)^{1/2}$.

3.1. Numerical examples

We now present some examples to demonstrate the impact of flat directions on the rate of thermalization. We choose the nominal value of the inflaton mass, $m_\phi = 10^{13}$ GeV, a typical value for the flat direction mass in models with a weak scale SUSY, $m_0 \simeq 100$ GeV–1 TeV, and four representative VEVs, $\varphi_0 = M_P, 10^{-2} M_P, 10^{-4} M_P$ and $\varphi_0 \leq 10^{-6} M_P$.

If the inflaton decays gravitationally, see the example in appendix A.2, we have $\Gamma_d \sim 10$ GeV. In this case $H_{thr}$ is given by equation (2.13). For larger inflaton couplings to matter, $\Gamma_d > m_0$ can be obtained, in which case $H_{thr}$ follows equation (2.15). As a sample case we have chosen $\Gamma_d = 10^{4}$ GeV.

It is evident that thermalization rate (hence $T_R$) becomes smaller as $\varphi_0$ increases. It is remarkable that in the extreme case where $\varphi_0 \sim M_P$, the reheat temperature can be as low as TeV. This is in stark contrast with the case if thermalization was instant (as expected in a non-SUSY case), which would result in hierarchically higher reheat temperatures $T_R > 10^9$ GeV (for the chosen values of $\Gamma_d$). Note that thermalization still remains quite slow for $\varphi_0 \sim 10^{-4} M_P$. It is only for $\varphi_0 \leq 10^{-6} M_P$ that the flat direction VEV is sufficiently small in order not to affect thermalization, thus leading to the standard expression $T_R \simeq (\Gamma_d M_P)^{1/2}$.

This clearly underlines the fact that complete thermalization can be substantially delayed in supersymmetry. Indeed, within the range determined by the uncertainty of the quantum fluctuations, $m_\phi \leq \varphi_0 \lesssim M_P$, flat directions considerably slow down thermalization. This has important cosmological consequences which will be considered in detail in the following sections.

4. Thermalization in supersymmetric theories: part 2

So far we have not made any specific assumption on the mediation of SUSY breaking to the observable sector, or the nature of higher-order terms which lift the flat directions. Instead we considered the typical case in models with soft masses at the TeV scale, and we treated the flat direction VEV as a free parameter which is bounded from above by $M_P$. In this section we shall study issues and some subtleties that may arise in more detail.
4.1. Higher-order superpotential terms and Kähler corrections

In models with gravity [41] and anomaly [54] mediation, SUSY breaking results in a usual soft term in the scalar potential, \( m_0^2 |\varphi|^2 \), where \( m_0 \simeq 100 \text{ GeV}–1 \text{ TeV} \). There is also a new contribution arising from integrating out heavy modes beyond the scale \( M \), which usually induces the non-renormalizable superpotential term:

\[
W \sim \lambda n \Phi^n M^{n-3},
\]

(4.1)

where \( \Phi \) denotes the superfield which comprises the flat direction \( \varphi \). In general \( M \) could be the string scale, below which we can trust the effective field theory, or \( M = M_P \) (in the case of supergravity). In addition, there are also inflaton-induced supergravity corrections to the flat direction potential. By inspecting the scalar potential in \( N=1 \) supergravity, one finds the following terms:

\[
( e^{K(\varphi^*, \varphi)/M_P^2} V(I) ), \quad \left( K_\varphi K^\varphi \varphi^* K^\varphi \left| W(I) \right|^2 / M_P^2 \right), \quad \left( K_\varphi K_\varphi^* D_I W^*(I) / M_P^2 + \text{h.c.} \right),
\]

(4.2)

where \( D_I \equiv \partial / \partial I + K_I W / M_P^2 \). The Kähler potential for the flat direction and the inflaton are given by \( K(\varphi^*, \varphi) \) and \( K(I^*, I) \), and \( W(I) \) denotes the superpotential for the inflaton sector.

All these terms provide a general contribution to the flat direction potential which is of the form [45]

\[
V(\varphi) = H^2 M_P^2 f \left( \frac{\varphi}{M_P} \right),
\]

(4.3)

where \( f \) is some function. Such a contribution usually gives rise to a Hubble-induced correction to the mass of the flat direction with an unknown coefficient, which depends on the nature of the Kähler potential. The relevant part of the scalar potential is then given by [45]

\[
V \supset \left( m_0^2 + c_H H^2 \right) |\varphi|^2 + \lambda n |\varphi|^{2(n-1)} / M^{2(n-3)},
\]

(4.4)

with \( n \geq 4 \). Note that \( c_H \) can have either sign. If \( c_H \gtrsim 1 \), the flat direction mass is \( \gtrsim H \). It therefore settles at the origin during inflation and remains there. Since \( |\varphi| = 0 \) at all times, the flat direction will have no interesting consequences in this case. The case with \( c_H < 0 \) will be more interesting. A negative \( c_H \) can arise at a tree-level [45], or as a result of radiative correction [56,57]. The flat direction VEV is in this case driven away from the origin and quickly settles at a large value which is determined by higher-order terms that stabilize the potential. A large VEV can also be obtained if \( 0 < c_H \ll 1 \).

19 The positive Hubble-induced mass to the flat direction has a common origin to the Hubble-induced mass correction to the inflaton in supergravity models. This is the well known \( \eta \)-problem [55], which arises because of the canonical form of the inflaton part of the Kähler potential. A large \( \eta \) generically spoils slow roll inflation. In order to have a successful slow roll inflation, one needs \( \eta \ll 1 \) (and hence \( c_H \ll 1 \)). Note that the origin of the Hubble-induced corrections to the mass of flat direction is again the cross terms between inflaton Kähler potential and superpotential terms and flat direction Kähler potential. There is no absolutely satisfactory solution to the \( \eta \) problem and therefore large coefficient, i.e., \( c_H \sim +\mathcal{O}(1) \). In this paper we assume that somehow the \( \eta \) problem has been addressed and therefore also \( c_H \ll +\mathcal{O}(1) \).
Table 1. The reheat temperature of the Universe for the inflaton mass, $m_\phi = 10^{13}$ GeV, and two values of the inflaton decay rate, $\Gamma_d = 10, 10^4$ GeV. The flat direction mass is $m_0 \sim 1$ TeV. The rows show the values of $T_R$ for flat direction VEVs varying in a wide range.

| VEV (in GeV) | $T_R(\Gamma_d = 10 \, \text{GeV})$ | $T_R(\Gamma_d = 10^4 \, \text{GeV})$ |
|-------------|----------------------------------|----------------------------------|
| $\varphi_0 \sim M_P$ | $3 \times 10^1$ | $7 \times 10^4$ |
| $\varphi_0 = 10^{-2}M_P$ | $3 \times 10^5$ | $7 \times 10^6$ |
| $\varphi_0 = 10^{-4}M_P$ | $3 \times 10^7$ | $7 \times 10^8$ |
| $\varphi_0 \leq 10^{-6}M_P$ | $3 \times 10^9$ | $7 \times 10^{10}$ |

It is possible to eliminate certain or all higher-order superpotential terms by assigning a suitable $R$-symmetry (or a set of $R$-symmetries) [48]. However, let us assume that all such terms which respect the SM gauge symmetry are indeed present. As shown in [47], all of the MSSM flat directions are lifted by higher-order terms with $n \leq 9$. If a flat direction is lifted at the superpotential level $n$, the VEV that it acquires during inflation will be $\varphi_1 \sim (H_I M^{n-3})^{1/n-2}$, where $H_I$ is the expansion rate of the Universe in the inflationary epoch. If $\epsilon_H < 0$, this will be the location of the minimum of the potential stabilized by the higher-order term.

After inflation, the flat direction VEV slides down to an instantaneous value: $(H(t)M^{n-3})^{1/n-2}$. Once $H(t) \simeq m_0$, the soft SUSY-breaking mass term in the potential takes over and the flat direction starts oscillating around its origin with an initial amplitude: $\varphi_0 \sim (m_0 M^{n-3})^{1/n-2}$. If $M = M_P$, we will have $\varphi_0 \sim 10^{10}$ GeV for $n = 4$ and $\varphi_0 \sim 10^{16}$ GeV for $n = 9$. In particular, $\varphi_0 > 10^{14}$ GeV for the flat directions lifted at $n \geq 6$ levels. These directions affect thermalization considerably and lower the reheat temperature of the Universe, as we discussed earlier; see table 1.

4.2. Models with gauge-mediated supersymmetry breaking

Our estimations of the thermalization timescale, equations (2.13) and (2.15), are valid if the flat direction potential is quadratic $m_0^2 |\varphi|^2$ for all field values (up to $M_P$). However, the situation is more subtle in models with gauge-mediated SUSY breaking [58].

In these models there exists a sector with gauge interactions that become strong at a scale $\Lambda_{\text{DSB}} \ll M_P$. This induces a non-perturbative superpotential and leads to a non-zero $F$-component, $\sim \Lambda_{\text{DSB}}^2$, for a chiral superfield which breaks supersymmetry. The gravitino mass will therefore be $m_{3/2} \sim \Lambda_{\text{DSB}}^2 / M_P$. At a next step, SUSY breaking is fed to a messenger sector with a mass scale $m_{\text{mess}}$. Finally, the soft SUSY-breaking parameters are induced in the observable sector by integrating out the messenger fields, which have some common gauge interactions with the observable sector, resulting in soft scalar masses $m_0$.

As a result, the flat direction potential has the following forms:

$$V = m_0^2 |\varphi|^2 \quad |\varphi| \leq m_{\text{mess}},$$

$$V \sim (m_0 m_{\text{mess}})^2 \ln \left( \frac{|\varphi|}{m_{\text{mess}}} \right) \quad m_{\text{mess}} \ll |\varphi| < \frac{m_0 m_{\text{mess}}}{m_{3/2}},$$

$$V = m_{3/2}^2 |\varphi|^2 \quad |\varphi| \geq \frac{m_0 m_{\text{mess}}}{m_{3/2}}.$$
This behaviour can be understood as follows. The flat direction obtains its soft SUSY-breaking mass by integrating out the messenger fields. For $|\varphi| > M_{\text{mess}}$ these fields acquire a large mass $\propto |\varphi|$ from the flat direction VEV; for details, see [21]. The flat direction mass, i.e., $\sqrt{V''}$, then decreases and the potential varies very slowly. At very large field values, however, dominance of the contribution from the gravity mediation recovers the conventional dependence on the potential, $|\varphi|^2$, although now the gravity-mediated soft SUSY-breaking contribution is $m_{3/2}$ instead of $m_0$.

Now let us find the thermalization timescale in models with gauge mediation. For small values of $m_{3/2}$, typically arising in these models, the flat direction starts its oscillations when $H = m_{3/2}$ and $V(\varphi_0) = m_{3/2}^2/2$. Also, the oscillations usually start after the inflaton has decayed, i.e., such that $\Gamma_d > m_{3/2}$.

Note that $|\varphi|^2$ and the number density, $n$, are redshifted alike $\propto R^{-3}$ for $H < m_{3/2}$ and hence $\Gamma_{\text{thr}}$ remains constant while $H$ decreases. This changes when $|\varphi| \lesssim m_0 m_{\text{mess}}/m_{3/2}$; after that, $|\varphi|$ is redshifted $\propto R^{-3}$ [21] and hence $\Gamma_{\text{thr}}$ increases $\propto R^3$. The transition occurs when $H \sim m_0 m_{\text{mess}}/\varphi_0$. Here we have assumed that the Universe is radiation dominated; thus $R \propto H^{-1/2}$, for $H < \Gamma_d$. Due to a rapid increase of the ratio $\Gamma_{\text{thr}}/H$, the reheat plasma reaches full thermal equilibrium when $|\varphi| > m_{\text{mess}}$. After using equations (2.10) and (2.14), we find that the Hubble expansion rate at the time of thermalization is then given by

$$H_{\text{thr}} \sim (10 \alpha^2)^{2/5} \left( \frac{\Gamma_d m_{3/2}}{m_0^2} \right)^{1/5} \left( \frac{M_P^4 m_{3/2}^3}{\varphi_0^3} \right)^{1/5}.$$ (4.6)

The maximum impact on thermalization happens in models with low-energy gauge mediation [21], which can give rise to very light gravitinos. In these models $\Lambda_{\text{DSB}}$, $m_{\text{mess}}$ and $m_0$ are all related to each other by one-loop factors: $m_{\text{mess}} \sim \Lambda_{\text{DSB}}/16\pi^2$ and $m_0 \sim m_{\text{mess}}/16\pi^2$ [21]. After using these relations and taking into account that $m_{3/2} \simeq \Lambda_{\text{DSB}}^3/M_P$, equation (4.6) reads

$$H_{\text{thr}} \sim (10^{-9} \alpha^2)^{2/5} \left( \frac{\Gamma_d m_{3/2}}{m_0^2} \right)^{1/5} \left( \frac{M_P^7 m_{3/2}^3}{\varphi_0^3} \right) m_{3/2}.$$ (4.7)

The reheat temperature then follows: $T_R \sim (H_{\text{thr}} M_P)^{1/2}$.

For example, consider a model where $\Lambda_{\text{DSB}} \sim 10^7$ GeV, $m_{\text{mess}} \sim 10^5$ GeV and $m_0 \sim 1$ TeV. The gravitino mass in this model is $m_{3/2} \simeq 100$ keV. Note that the potential is quadratic for $|\varphi| \leq 10^3$ GeV and $|\varphi| \geq 10^{12}$ GeV, while being logarithmic in the intermediate region. Table 2 summarizes the reheat temperature in this model for similar values of $m_\phi$, $\Gamma_d$ and $\varphi_0$ as in table 1. What we find is that $T_R$ is considerably lower in this case.

To conclude, in models with gauge mediation the Universe thermalizes even more slowly and the resulting reheat temperatures are even lower compared to that of the gravity mediation case. This is mainly due to the fact that the flat directions start their oscillations much later in this case.

4.3. Split supersymmetry

Now we consider the opposite situation where $m_0 \gg 1$ TeV. This happens in the recently proposed split SUSY scenario [59]. This scenario does not attempt to address the


Table 2. The reheat temperature of the Universe in a model with low-energy gauge mediation where the gravitino mass is $m_{3/2} = 100$ keV. The inflaton mass and its decay rate are the same as in table 1. The rows show the values of $T_R$ for flat direction VEVs chosen the same as in table 1.

| VEV (in GeV) | $T_R$ ($\Gamma_d = 10$ GeV) | $T_R$ ($\Gamma_d = 10^4$ GeV) |
|-------------|----------------------------|-----------------------------|
| $\varphi_0 \sim M_P$ | $8 \times 10^2$ | $3 \times 10^3$ |
| $\varphi_0 = 10^{-2} M_P$ | $2 \times 10^4$ | $6 \times 10^4$ |
| $\varphi_0 = 10^{-4} M_P$ | $5 \times 10^5$ | $2 \times 10^5$ |
| $\varphi_0 = 10^{-6} M_P$ | $10^7$ | $3 \times 10^7$ |

hierarchy problem. It allows the scalars (except the SM Higgs doublet) to be very heavy, while keeping the gaugino and Higgsino fields light. This removes problems with flavour changing and $CP$-violating effects induced by scalars at the one-loop level. On the other hand, it preserves attractive features like supersymmetric gauge coupling unification and a light LSP which can account for the dark matter.

Note that from equations (2.13) and (2.15), the thermalization timescale becomes shorter for larger values of $m_0$, because when $m_0$ is larger the flat directions start oscillating earlier. Their VEV and the induced masses for the gauge bosons will therefore be redshifted faster in this case. Let us consider the favoured range of the soft scalar masses in split SUSY: $10^8$ GeV $\leq m_0 \leq 10^{13}$ GeV. This removes the flavour changing and $CP$-violating effects and results in an acceptable gluino lifetime [60]. For the inflaton mass $m_\phi = 10^{13}$ GeV, we find $H_{\text{thr}} > 100$ GeV for all values of $\varphi_0 \leq M_P$.

Therefore in the case of split SUSY, we always have

$$T_R \geq 10^{10} \text{ GeV.} \quad (4.8)$$

This is fairly a robust prediction.

This might seem unacceptably high for thermal gravitino production. However, models of split SUSY can also accommodate gravitinos with a mass $m_{3/2} > 50$ TeV [60,61]. Such superheavy gravitinos decay before BBN [17] and hence are not subject to the tight bounds coming from BBN [19,20]. As a matter of fact, the gravitino overproduction can in this case turn into a virtue. The late decay of gravitinos, below the LSP freeze-out temperature, can produce the correct dark matter abundance in a non-thermal fashion [60,35,62].

Moreover, if produced abundantly, the gravitinos will dominate the Universe. Their decay in this case dilutes the baryon asymmetry. Consequently, larger parts of the parameter space will become available for thermal leptogenesis and flat direction baryogenesis [62].

We conclude that, due to larger scalar masses, thermalization is not affected by the flat directions in split SUSY. However, for $m_{3/2} > 50$ TeV, having high reheat temperatures and thermal overproduction of gravitinos can be a privilege rather than a handicap [35,62].

5. Particle production during thermalization
5.1. Quasi-adiabatic evolution of the Universe

Right after the inflaton decay has completed, the energy density of the Universe is given by $\rho \approx 3 (\Gamma_d M_P)^2$ and the average energy of particles is $\langle E \rangle \simeq m_\phi$.20 Deviation from full

20 For example, in a two-body decay of the inflaton, we have exactly $E = m_\phi/2$. 

equilibrium can be quantified by the parameter ‘$\mathcal{A}$’ [1], where
\[ \mathcal{A} \equiv \frac{3\rho}{T^4} \sim 10^4 \left( \frac{\Gamma_d M_P}{m_{\phi}^2} \right)^2. \] (5.1)

Here we define $T \approx \langle E \rangle/3$, in accordance with full equilibrium. Note that in full equilibrium, see equation (2.1), we have $\mathcal{A} \approx g_*(=228.75$ in the MSSM). On the other hand, see equation (A.3), after the inflaton decay we have $\Gamma_d \ll m_{\phi}^2/M_P$, which implies that $\mathcal{A} \ll 228.75$. One can also associate parameter $\mathcal{A}_i \equiv 3\rho_i/T^4$ to the $i$th degree of freedom with the energy density $\rho_i$ (all particles have the same energy $E$ and hence $T$, as they are produced in one-particle decay of the inflaton). Note that $\mathcal{A} = \sum_i \mathcal{A}_i$ and in full equilibrium we have $\mathcal{A}_i \approx 1$.

As we have discussed in section 2, thermalization is very slow in supersymmetry. As a result, the Universe enters a long phase of quasi-adiabatic evolution during which the comoving number density and comoving (average) energy of particles in the plasma remain constant. Since particles are produced in one-particle inflaton decay, the distribution is peaked around the average energy. The $2 \rightarrow 2$ scatterings (which become efficient shortly before complete thermalization) smooth out the distribution. This implies that the reheat plasma is in a quasi-thermal state which is not far from kinetic equilibrium, but grossly deviated from chemical equilibrium. In this period, which lasts until the $2 \rightarrow 3$ scatterings shown in figure 2 become efficient, the Hubble expansion redshifts $\rho_i \propto R^{-4}$, $n_i \propto R^{-3}$ and $T \propto R^{-1}$. Therefore $\mathcal{A}$ and $\mathcal{A}_i$ remain constant throughout this epoch. Note that $\mathcal{A}$ depends on the total decay rate of the inflaton $\Gamma_d$ and its mass $m_\phi$ through equation (5.1), while the $\mathcal{A}_i$ are determined by the branching ratio for the inflaton decay to the $i$th degree of freedom. The composition of the reheat plasma is therefore model-dependent before its complete thermalization. However, some general statements can be made based on symmetry arguments.

- For $CP$-conserving couplings, the inflaton decay produces the same number of particles and anti-particles associated to a given field.
- For a singlet inflaton (which is the case in almost all models)\(^{22}\) the gauge invariance implies that inflaton has equal couplings (thus branching ratios) to the fields which are in an irreducible representation of the gauge (sub)group. This holds in the presence of spontaneous symmetry breaking via flat direction VEVs, as long as the particle masses induced by coupling to the flat direction are (much) smaller than that of the inflaton mass.
- If the inflaton mass is (much) larger than the soft SUSY-breaking masses in the observable sector, the inflaton decay produces the same number of bosonic and fermionic components of matter superfields.\(^{23}\)

\(^{21}\) We would like to thank Antonio Masiero for highlighting this fact.

\(^{22}\) The only exceptional model is an example of assisted inflation [63] driven by $N$ supersymmetric flat directions [64] which are not gauge singlets.

\(^{23}\) This holds for a perturbative decay which is relevant for the last stage of reheating. Non-perturbative inflaton decay via preheating, which may happen at an initial stage, produces bosons much more abundantly than fermions as a result of strong SUSY breaking by large occupation numbers.
During the quasi-adiabatic evolution of the reheated plasma, i.e., for \( H_{\text{thr}} < H < \Gamma_d \), we have

\[
\rho_i = A_i \frac{3}{\pi^2} T^4, \quad n_i = A_i \frac{1}{\pi^2} T^4
\]

and the Hubble expansion rate follows

\[
H \simeq A^{1/2} \left( \frac{T^2}{3M_p} \right).
\]

In this epoch \( T \) varies in a range \( T_{\text{min}} \leq T \leq T_{\text{max}} \), where \( T_{\text{max}} \approx m_\phi/3 \) is reached right after the inflaton decay. Because of complete thermalization, \( T \) sharply drops from \( T_{\text{min}} \) to \( T_R \) at \( H_{\text{thr}} \), where the conservation of energy implies that

\[
T_R = \left( \frac{A}{228.75} \right)^{1/4} T_{\text{min}},
\]

The (final) entropy density is given by

\[
s = (2\pi^2/45) g_* T_R^3,
\]

where we take \( g_* = 228.75 \).

### 5.2. Quasi-thermal particle production

An important cosmological phenomenon is the production of (un)stable particles arising in theories beyond the SM in the early Universe. Let us consider such a particle, \( \chi \), with a mass, \( m_\chi \), which is weakly coupled to the (MS)SM fields. Through its couplings, \( \chi \) will be inevitably produced in a plasma consisting of only the (MS)SM particles. In full equilibrium, one can calculate the abundance of a given particle by taking a thermal average of the relevant processes (for example, see [65]). The only inputs required are the particle mass and its couplings to the (MS)SM fields, plus the reheat temperature \( T_R \). This leads to a lower bound on the relic abundance which is independent from the details of the reheating (except for \( T_R \)).

Here we are interested in particle production during the quasi-thermal phase of the Universe. Usually, scatterings are the most important processes for production of \( \chi \). The number density of \( \chi \), denoted by \( n_\chi \), then obeys

\[
\dot{n}_\chi + 3Hn_\chi = \sum_{i,j} \langle v_{\text{rel}} \sigma_{ij \rightarrow \chi} \rangle n_i n_j.
\]

Here \( n_i \) and \( n_j \) are the number densities of the \( i \)th and \( j \)th particles, \( \sigma_{ij \rightarrow \chi} \) is the cross-section for producing \( \chi \) from scatterings of \( i \) and \( j \), and the sum is taken over all fields which participate in \( \chi \) production. Also \( \langle \rangle \) denotes averaging over the distribution. Note that production will be Boltzmann suppressed if \( T < m_\chi/3 \). Therefore, to obtain the total number of \( \chi \) quanta produced from scatterings, it will be sufficient to integrate the RHS of (5.5) from the highest temperature down to \( m_\chi/3 \). The physical number density of \( \chi \) produced at a time \( t_1 \) will be redshifted by a factor of \( (R_2/R_1)^3 \) at any later time.

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24 One can express \( A_i \) in terms of a negative chemical potential \( \mu_i \), where \( A_i = \exp(\mu_i/T) \). Note that for a large negative chemical potential, i.e., in a dilute plasma, the Bose and Fermi distributions are reduced to the Maxwell–Boltzmann distribution and give essentially the same result. The assignment of a chemical potential merely reflects the fact that the number of particles remains constant until the number-violating reactions become efficient. It does not appear as a result of a conserved quantity (such as baryon number) which is due to some symmetry. Indeed, assuming that inflaton decay does not break such symmetries, here we assign the same chemical potential to particles and anti-particles.
If the Universe is filled by relativistic particles, \( H = (1/2t) \) and \( R \propto H^{-1/2} \). The relic abundance of \( \chi \), normalized by the entropy density, \( s \), will then be

\[
\frac{n_{\chi}}{s} \sim 10^{-5} \left( \frac{228.75}{A} \right)^{5/4} \sum_{i,j} \int_{T_{\text{min}}}^{T_{\text{max}}} A_i A_j \langle v_{\text{rel}} \sigma_{ij \rightarrow \chi} \rangle M_\text{P} dT,
\]

(5.6)

where we have used equation (5.4).

### 6. Gravitino production in a quasi-thermal case

As we discussed in section 3, slow thermalization in SUSY generically results in a low reheat temperature compatible with the BBN bounds on thermal gravitino production. However, gravitinos are also produced during the quasi-thermal phase prior to a complete thermalization of the reheat plasma. There are various channels for the gravitino production from scatterings of gauge, gaugino, fermion and sfermion quanta. The scattering cross-section for all such processes is \( \propto 1/M^2_\text{P} \), but the numerical coefficient depends on a specific channel. In full thermal equilibrium, all degrees of freedom (except for their bosonic or fermionic nature) have the same occupation number and hence obtaining the total production cross-section is rather easy. However, as mentioned earlier, the composition of the reheat plasma is model-dependent in a quasi-thermal phase. Therefore calculating the total cross-section is more involved in this case.

An important distinction is that we only need to consider scatterings of fermions and/or sfermions for the following reasons: the gauge and gaugino quanta have large masses \( \sim \alpha^{1/2} \varphi_d \) (induced by the flat direction VEV) at a time most relevant for the gravitino production, i.e., when \( H \simeq \Gamma_d \); therefore, they decay to lighter fermions and sfermions at a rate \( \sim \alpha^{3/2} \varphi_d^2/m_\phi \). Here \( \alpha^{3/2} \varphi_d \) is the decay width at the rest frame of gauge/gaugino quanta and \( \varphi_d/m_\phi \) is the time-dilation factor. The decay rate is \( \gg \Gamma_d \); thus gauge and gaugino quanta decay almost instantly upon production and they will not participate in the gravitino production\(^{25}\). This is indeed welcome as scatterings which include gauge and/or gaugino quanta in the initial state (particularly scattering of two gluons) have the largest production cross-section\(^{14,15}\).

As a consequence, production of the helicity \( \pm 1/2 \) states will not be enhanced in a quasi-thermal phase as scatterings with a gauge–gaugino–gravitino vertex will be absent.

Now let us discuss the relevant scattering processes and production cross-sections in the quasi-thermal phase\(^{26}\). The total cross-section, and cross-sections for multiplets comprising the LH (s)quarks \( Q \), RH up-type (s)quarks \( u \), RH down-type (s)quarks \( d \), LH (s)leptons \( L \), RH (s)leptons \( e \) and the two Higgs/Higgsino doublets \( H_u, H_d \) have been given separately for each channel\(^{27}\). Here \( 1 \leq i, j \leq 3, a, b = 1, 2 \) and \( 1 \leq \alpha, \beta \leq 3 \) are the flavour, weak-isospin and colour indices of scattering degrees of freedom respectively. Also \( \alpha_3, \alpha_2 \) and \( \alpha_1 \) are the gauge fine structure constants associated to the \( SU(3)_C, SU(2)_W \)

---

\(^{25}\) The inverse decay happens at a rate \( \sim \alpha n/E^2 \) and, as mentioned earlier, it is inefficient before complete thermalization of the plasma. This is just a reflection of the fact that the reheat plasma is out of chemical equilibrium for \( H > H_{\text{thr}} \).

\(^{26}\) One might wonder that scatterings of (s)fermions off the flat direction condensate would be the dominant source as the latter carries a much larger number of (zero-mode) quanta. However, these scatterings are suppressed for the same reasons as discussed in appendix A.4.

\(^{27}\) We also note that these cross-sections are free from infrared logarithmic divergences\(^{14,15}\).
• fermion + anti-sfermion → gravitino + gauge field, sfermion + anti-fermion → gravitino + gauge field. The total cross-section for this channel is \((1/32M_P^2) \times (48\alpha_3 + 21\alpha_2 + 11\alpha_1);\) see row E of the table on page 181 of the first reference in [15]. For different degrees of freedom we then find:

\[
\sigma_1 = \sigma_2 = \frac{1}{32M_P^2} \delta_{ij} \times \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} \right. + \left. \frac{2}{16} \alpha_2 \delta_{a\beta} + \frac{1}{216} \alpha_1 \delta_{a\beta} \delta_{ab} \right) Q, \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} \right. + \left. \frac{2}{7} \alpha_1 \delta_{a\beta} \right) u, \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} + \frac{1}{18} \alpha_1 \delta_{a\beta} \right) d, \\
\left( \frac{3}{16} \alpha_2 + \frac{1}{8} \alpha_1 \delta_{ab} \right) H, \\
\left( \frac{3}{16} \alpha_2 + \frac{1}{8} \alpha_1 \delta_{ab} \right) L, \\
\left( \frac{1}{2} \alpha_1 \right) \ e. \tag{6.1}
\]

• fermion + anti-fermion → gravitino + gaugino. The total cross-section for this channel is \((1/32M_P^2) \times (16\alpha_3 + 7\alpha_2 + 11\alpha_1/3);\) see row I of the table on page 181 of the first reference in [15]. For different degrees of freedom we then find:

\[
\sigma_3 = \frac{1}{32M_P^2} \delta_{ij} \times \\
\left( \frac{4}{27} \alpha_3 \delta_{ab} + \frac{1}{8} \alpha_2 \delta_{a\beta} + \frac{1}{108} \alpha_1 \delta_{a\beta} \delta_{ab} \right) Q, \\
\left( \frac{4}{27} \alpha_3 \delta_{ab} + \frac{1}{27} \alpha_1 \delta_{a\beta} \right) u, \\
\left( \frac{4}{27} \alpha_3 \delta_{ab} + \frac{1}{27} \alpha_1 \delta_{a\beta} \right) d, \\
\left( \frac{1}{8} \alpha_2 + \frac{1}{12} \alpha_1 \delta_{ab} \right) H, \\
\left( \frac{8}{16} \alpha_2 + \frac{1}{12} \alpha_1 \delta_{ab} \right) L, \\
\left( \frac{1}{4} \alpha_1 \right) \ e. \tag{6.2}
\]

• sfermion + anti-sfermion → gravitino + gaugino. The total cross-section for this channel is \((1/32M_P^2) \times (8\alpha_3 + 7\alpha_2/2 + 11\alpha_1/6);\) see row J of the table on page 181 of the first reference in [15]. For different degrees of freedom we then find:

\[
\sigma_4 = \frac{1}{32M_P^2} \delta_{ij} \times \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} + \frac{1}{16} \alpha_2 \delta_{a\beta} + \frac{1}{216} \alpha_1 \delta_{a\beta} \delta_{ab} \right) Q, \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} + \frac{2}{27} \alpha_1 \delta_{a\beta} \right) u, \\
\left( \frac{2}{27} \alpha_3 \delta_{ab} + \frac{1}{54} \alpha_1 \delta_{a\beta} \right) d, \\
\left( \frac{1}{16} \alpha_2 + \frac{1}{24} \alpha_1 \delta_{ab} \right) H, \\
\left( \frac{1}{16} \alpha_2 + \frac{1}{24} \alpha_1 \delta_{ab} \right) L, \\
\left( \frac{1}{8} \alpha_1 \right) \ e. \tag{6.3}
\]
The total cross-section for the gravitino production will then be given by

\[
\sigma_{\text{tot}} = \frac{1}{32M_P^2}\delta_{ij} \times
\]

\[
\left(\frac{2}{3}\alpha_3\delta_{ab} + \frac{9}{16}\alpha_2\delta_{\alpha\beta} + \frac{1}{24}\alpha_1\delta_{\alpha\beta}\delta_{ab}\right) Q,
\]

\[
\left(\frac{2}{3}\alpha_3\delta_{ab} + \frac{2}{3}\alpha_1\delta_{\alpha\beta}\right) u,
\]

\[
\left(\frac{2}{3}\alpha_3\delta_{ab} + \frac{1}{6}\alpha_1\delta_{\alpha\beta}\right) d,
\]

\[
\left(\frac{9}{16}\alpha_2 + \frac{3}{8}\alpha_1\delta_{ab}\right) H,
\]

\[
\left(\frac{9}{16}\alpha_2 + \frac{3}{8}\alpha_1\delta_{ab}\right) L,
\]

\[
\left(\frac{3}{8}\alpha_1\right) e.
\]

As we pointed out in the previous section, particles and anti-particles associated to the bosonic and fermionic components of the multiplets which belong to an irreducible representation of a gauge group have the same parameter \(A_i\). This implies that

\[
\Sigma_{\text{tot}} = \sum_{i,j=1}^{3} \sum_{a,b=1}^{2} \sum_{\alpha,\beta=1}^{3} A_{i,a,\alpha} A_{j,b,\beta} \langle \sigma_{\text{tot}} v_{\text{rel}} \rangle
\]

\[
= \frac{1}{32M_P^2} \sum [6\alpha_3\left(2A_Q^2 + A_u^2 + A_d^2\right) + \frac{9}{8}\alpha_2\left(3A_Q^2 + A_L^2 + A_H^2\right)]
\]

\[
+ \frac{1}{3}\alpha_1\left(A_Q^2 + 8A_u^2 + 2A_d^2 + 3A_L^2 + 6A_e^2 + 3A_H^2\right)].
\]

The sum is taken over the three flavours of \(Q, u, d, L, e\) and the two Higgs doublets. After replacing \(\Sigma_{\text{tot}}\) in equation (5.6) and recalling that \(T_{\text{max}} \approx m_\phi/3\), we obtain

\[
\frac{n_{3/2}}{s} \simeq \left(10^{-1} M_P^2 \Sigma_{\text{tot}}\right) \left(\frac{228.75}{A}\right)^{5/4} \left(\frac{T_{\text{max}}}{10^{10} \text{GeV}}\right) 10^{-12}.
\]

We recall that in full thermal equilibrium, \(\Sigma_{\text{tot}} = (4\pi/M_P^2) \times (16\alpha_3 + 6\alpha_2 + 2\alpha_1) \simeq (10^{-1}/M_P^2)\) (up to logarithmic corrections) [14,15]. It is evident that the exact abundance of the gravitinos produced during the quasi-thermal phase depends on the composition of the reheat plasma. One expects the number of gravitinos thus produced to be maximum if the inflaton mainly decays to one flavour of LH (s)quarks (which are charged under the whole SM group). In this case \(A_Q = 1/24\) for the relevant flavour, while \(A = 0\) for all other degrees of freedom. This results in a gravitino abundance:

\[
\frac{n_{3/2}}{s} \simeq \left(\frac{A}{228.75}\right)^{3/4} \left(\frac{T_{\text{max}}}{10^{10} \text{GeV}}\right) 10^{-12},
\]

where \(A\) is given by equation (5.1).

An important point to note is that \(T_{\text{max}}\) is accompanied by the factor \(A^{3/4}\) in equation (6.7). Therefore, despite the fact that \(T_{\text{max}} \approx m_\phi/3\) can be as large as \(10^{12}\) GeV, the gravitino abundance can be at a safe level. First consider the case for unstable gravitinos. For \(T_{\text{max}} \approx 10^{12}\) GeV, the tightest bound from BBN \((n_{3/2}/s) \leq 10^{-16}\) (arising for \(m_{3/2} \approx 1\) TeV and a hadronic branching ratio \(\approx 1\)) is satisfied if \(A \leq 10^{-6}\). Much

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28 The total number of degrees of freedom in one flavour of LH (s)quarks is \(2\) (particle – antiparticle) \(\times 2\) (fermion – boson) \(\times 2\) (weak – isospin) \(\times 3\) (colour).
weaker bounds on $\mathcal{A}$ are found for a radiative decay. For example, $\mathcal{A} \leq 10^{-3}$ (1) if $m_{3/2} \simeq 100$ GeV (1 TeV).

For stable gravitinos with $O(\text{keV})$ mass the dark matter limit, see equation (1.3), is satisfied if $\mathcal{A} \leq 10^{-6}$. A much more relaxed bound $\mathcal{A} < 1$ is obtained if $m_{3/2} \simeq 100$ MeV. Therefore, in general, the gravitino production during the quasi-thermal phase is safe\(^{29}\).

We conclude that late thermalization of the Universe due to SUSY flat directions eliminates the gravitino problem altogether in a natural way.

7. Leptogenesis in a quasi-thermal case

7.1. Basic concept

The baryon asymmetry of the Universe (BAU) parameterized as $\eta_B = (n_B - n_{\bar{B}})/s$ is determined to be $0.9 \times 10^{-10}$ by the recent analysis of WMAP data [4]. This number is also in good agreement with an independent determination from the primordial abundance of light elements produced during BBN [66]. Any mechanism for generating a baryon asymmetry must satisfy $B$- and/or $L$-violation, $C$- and $CP$-violation, and departure from thermal equilibrium $^{30}$.

Leptogenesis is an elegant mechanism which postulates the existence of RH neutrinos, which are SM singlets, with a lepton number-violating Majorana mass $M_N$. It can be naturally embedded in models which explain the light neutrino masses via the see-saw mechanism [70]. A lepton asymmetry can then be generated from the out-of-equilibrium decay of the RH neutrinos into Higgs bosons and light leptons, provided $CP$-violating phases exist in the neutrino Yukawa couplings [71]–[73]. The created lepton asymmetry will be converted into a baryonic asymmetry via sphaleron processes.

In thermal leptogenesis the on-shell RH neutrinos whose decay is responsible for the lepton asymmetry are produced via their Yukawa interactions with the SM fields in a thermal bath [74]. In SUSY there are RH sneutrinos which serve an additional source for leptogenesis [75]. This scenario works most comfortably if $T_R \gtrsim M_1 \geq 10^9$ GeV $^{31}$.

The decay of a RH (s)neutrino with mass $M_i$ results in a lepton asymmetry via one-loop self-energy and vertex corrections [73]. If the asymmetry is mainly produced from the decay of the lightest RH states, and assuming hierarchical RH (s)neutrinos $M_1 \ll M_2, M_3$, we will have $^{87}$

$$\eta_B \simeq 3 \times 10^{-10} \kappa \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right), \quad \text{(full equilibrium)}, \quad (7.1)$$

\(^{29}\) The above-mentioned constraints on $\mathcal{A}$ are comfortably satisfied in a generic inflationary model where the inflaton is a gauge singlet and couples to the MS(SM) gravitationally; see the discussion in appendices A.2 and A.3.

\(^{30}\) Since $B + L$-violating sphaleron transitions are active at temperatures $100$ GeV $\lesssim T \lesssim 10^{12}$ GeV [68], any mechanism for creating a baryon asymmetry at $T > 100$ GeV must create a $B - L$ asymmetry. The final asymmetry is then given by $B = a(B - L)$, where $a = 28/79$ in the case of SM and $a = 8/23$ for the MSSM [69].

\(^{31}\) There exist various scenarios of non-thermal leptogenesis [78]–[85] which can work for $T_R \leq M_N$. There are also leptogenesis models which implement soft SUSY-breaking terms $^{86}$. 

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\(^{86}\) The Journal of Cosmology and Astroparticle Physics 10 (2006) 008 (stacks.iop.org/JCAP/2006/i=10/a=008) 24
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for $O(1)$ $CP$-violating phases ($m_1 < m_2 < m_3$ are the masses of light mostly LH neutrinos). Here $\kappa$ is the efficiency factor accounting for the decay, inverse decay and scattering processes involving the RH states \[38,77\].

A decay parameter $K$ can be defined as

$$K \equiv \frac{\Gamma_1}{H(T = M_1)}, \quad (7.2)$$

where $\Gamma_1$ is the decay width of the lightest RH (s)neutrino. It can be related to an effective neutrino mass $\tilde{m}_1$ such that $\tilde{m}_1 = 10^{-3}$ eV, with the model-independent bound $m_1 < \tilde{m}_1$ [88].

If $K < 1$, corresponding to $\tilde{m}_1 < 10^{-3}$ eV, the decay of the RH states will be out of equilibrium at all times. In this case the RH states, which are mainly produced via scatterings of the LH (s)leptons off the top (s)quarks and electroweak gauge/gaugino fields [74], never reach thermal equilibrium. The cross-section for producing the RH (s)neutrinos is $\propto T^{-2} (M_1^2)$, when $T > M_1$ ($< M_1$) and hence most of them are produced when $T \sim M_1$. The efficiency factor reaches its maximum value for $\kappa \approx 0.1$ when $\tilde{m}_1 = 10^{-3}$ eV. For larger values of $\tilde{m}_1$ it drops again, because the inverse decays become important and suppress the generated asymmetry. Producing sufficient asymmetry then sets a lower bound, $M_1 \geq 10^9$ GeV [38]. Successful thermal leptogenesis therefore requires that $T_R \geq 10^9$ GeV. Note that this is at best marginally compatible with thermal gravitino production; see equation (1.2).

7.2. Quasi-thermal leptogenesis

The presence of flat directions slows down thermalization and lowers the reheat temperature, such that $T_R \ll 10^9$ GeV is naturally obtained (see tables 1 and 2). This is detrimental to thermal leptogenesis in the bulk of parameter space. Note that equation (7.1) implies that sufficient asymmetry will not be generated after the establishment of a full equilibrium\[32\]. Needless to mention, like gravitino production, leptogenesis can still occur during the quasi-thermal phase and this is the topic of our interest in this subsection.

In a quasi-thermal phase the reheat plasma is dilute, implying that the RH states are produced less abundantly than in full thermal equilibrium. Their abundance can be calculated from equation (5.6), where the production cross-section is $\sigma \propto 1/M_1^3$. Note that the abundance depends on $A_i$ and hence on the composition of the reheat plasma. Significant production of the RH (s)neutrinos requires that the LH (s)leptons and/or the top (s)quarks be present in the reheat plasma. The relevant channels for producing the lightest RH neutrino, $N_1$, and sneutrino, $\tilde{N}_1$, are scatterings of LH (s)leptons with the largest Yukawa coupling to $N_1$, $\tilde{N}_1$ off the LH top (s)quarks and anti-(s)quarks and annihilation of top (s)quark-anti(s)quark pairs\[33\]. Summing over all processes (including weak-isospin and colour indices) we find from equation (5.6) that

$$\frac{n_{N_1}}{s} \simeq 10^{-5} \Sigma_{N_1} (M_P M_1) \left( \frac{228.75}{A} \right)^{5/4} \quad (7.3)$$

\[32\] Thermal leptogenesis can work for $M_1 \ll 10^9$ GeV if the RH (s)neutrinos are degenerate [89], or for specific neutrino mass models [90].
\[33\] The electroweak gauge/gaugino fields have a large mass and decay almost instantly; therefore, unlike the case with full equilibrium they do not participate in $N_1$, $\tilde{N}_1$ production.
where
\[ \Sigma_{N_1} \propto \frac{1}{M_1^2} \times 18 (A_L A_{Q_3} + A_L A_t + A_{Q_3} A_t) \]  
(7.4)
is the production cross-section for \( N_1, \tilde{N}_1 \). Here \( A_L, A_{Q_3} \) and \( A_t \) denote the \( A \) parameter for the LH (s)leptons with the largest Yukawa coupling to \( N_1, \tilde{N}_1 \), the LH top (s)quarks and the RH top (s)quarks respectively. Note that in full equilibrium they are all \( \simeq 1 \).
The three terms inside the parentheses are the contributions from the above-mentioned processes respectively.
The final baryon asymmetry generated in the quasi-thermal phase will then be given by
\[ \eta_B \simeq 10^{-10} \left( \frac{228.75}{A} \right)^{5/4} (A_L A_{Q_3} + A_L A_t + A_{Q_3} A_t) \kappa \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right) \]  
(quasi-thermal).  
(7.5)
If the inflaton mainly decays to the top (s)quarks, we have \( A_{Q_3} = A_t = A/36 \),\(^{34}\) while \( A = 0 \) for all other degrees of freedom. This results in
\[ \eta_B \simeq 5 \times 10^{-9} \kappa \left( \frac{A}{228.75} \right)^{3/4} \left( \frac{M_1}{10^9 \text{ GeV}} \right). \]  
(7.6)
Since \( \eta_B \propto M_1 \), the maximum asymmetry is produced when \( M_1 \simeq 3T_{\text{max}} \approx 3 \times 10^{12} \text{ GeV} \). For the largest efficiency factor \( \kappa \simeq 0.1 \), generating the correct asymmetry requires that \( A \geq 10^{-3} \). This is compatible with the bound from the gravitino production for a radiatively decaying gravitino with \( m_{3/2} \simeq 100 \text{ GeV–1 TeV} \) (see the discussion in the previous section, equation (6.7)).

Note that from equations (6.7) and (7.5) the gravitino abundance and the baryon asymmetry produced during a quasi-thermal phase are both \( \propto A^{3/4} \). However, due to the dependence on the composition of the reheat plasma, the marginality between leptogenesis and gravitino production can be very different from that in the case of full equilibrium. The best-case scenario for leptogenesis occurs when the inflaton mainly decays to the top (s)quarks. In this case the marginality between leptogenesis, equation (7.6) and gravitino production, equation (6.7), is weakened by about one order of magnitude compared to the case of full equilibrium; see equations (1.2) and (7.1).

There are other interesting differences which arise in the case of a quasi-thermal leptogenesis. Because the plasma is dilute in this phase, \( N_1, \tilde{N}_1 \) will not be brought into equilibrium even if \( \tilde{m}_1 \gg 10^{-3} \text{ eV} \). On the other hand, see equation (5.3), the expansion rate of the Universe is (much) slower than the case with full thermal equilibrium when \( T \simeq M_1 \). This implies that out-of-equilibrium decay of \( N_1, \tilde{N}_1 \) requires that \( \tilde{m}_1 \ll 10^{-3} \text{ eV} \). Moreover, since \( \Delta L = 2 \) scatterings mediated by \( N_1, \tilde{N}_1 \) are much less efficient in a dilute plasma, the resulting bound on \( M_1 \) will be altered.

To conclude, late thermalization of the Universe implies that thermal leptogenesis cannot generate sufficient asymmetry in the bulk of the parameter space. The new paradigm is the quasi-thermal leptogenesis, but this depends on the composition of the reheat plasma. If the inflaton mainly decays to the top (s)quarks, the marginality between gravitino production and leptogenesis will be improved (compared to the case in full

^{34} Note that the total number of degrees of freedom in \( Q_3 \) and \( t \) is 36.
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As a result, the right amount of baryon asymmetry can be generated for sufficiently heavy RH (s)neutrinos. A more detailed and quantitative study is needed for a better understanding of the role of decays and inverse decays in a quasi-thermal leptogenesis.

8. Thermalization after preheating

So far we have focused on thermalization after the perturbative inflaton decay. Although for final reheating the perturbative decay is the most important, we make some comments on the situation after a non-perturbative decay of the inflaton also. For detailed studies on thermalization after preheating see [91]–[93].

Consider a simple chaotic inflation model as in appendix A.1 with the following potential:

\[ V \sim \frac{1}{2}m_\phi^2\phi^2 + h^2\phi^2\chi^2, \quad (8.1) \]

where \( \chi \) is another scalar field. Here we have considered only the real parts of \( \phi \) and \( \chi \). If \( h > 10^{-6} \), the inflaton oscillations decay to \( \chi \) quanta via broad parametric resonance [8]. If \( h > 10^{-4} \), resonant production results in an extremely efficient transfer of energy from the zero-mode condensate in a typical timescale \( \sim 100m_\phi^{-1} \) (which depends weakly on the coupling \( h \)) [8].

Resonant particle production and re-scatterings lead to the formation of a plasma consisting of \( \phi \) and \( \chi \) quanta with typical energies \( \sim 10^{-1}(hm_\phi M_P)^{1/2} \) [8]. This plasma is in kinetic equilibrium but full thermal equilibrium is established over a much longer timescale than preheating [91,92].

The occupation number of particles in the preheat plasma is \( \gg 1 \) (which is opposite to the situation after the perturbative decay). This implies that the number density of particles is larger than its value in full equilibrium, while the average energy of particles is smaller than the equilibrium value. It gives rise to large effective masses for particles which, right after preheating, are similar to their typical momenta [8]. Large occupation numbers also lead to important quantum effects due to identical particles and significant off-shell effects in the preheat plasma. Because of all these, a field theoretical study of thermalization is considerably more complicated in the case of preheating. Due to the large occupation numbers, one can consider the problem as thermalization of classical fields at early stages [91]–[93]. In the course of evolution towards full equilibrium, however, the occupation numbers decrease. Therefore a proper (non-equilibrium) quantum field theory treatment [94] will be inevitably required at late stages when occupation numbers are close to one.

Similar complications also arise when considering particle production during thermalization. However, let us make crude estimates based on equations (6.6) and (7.5). At the end of preheating \( \rho \sim 10^{-4}m_\phi^2M_P^2 \) and \( T \sim 10^{-1}(hm_\phi M_P)^{1/2} \). We therefore find from equation (5.1) that \( A \sim h^{-2} \gg 228.75 \). Equation (6.6) then implies overproduction of gravitinos in the preheat plasma unless \( T_{\text{max}} \ll 10^{10} \text{ GeV} \). This results in severe constraint on the models. For example, since \( T_{\text{max}} > m_\phi \), it requires that \( m_\phi \ll 10^{10} \text{ GeV} \), which is a disaster from the point of view of inflatons generating the density perturbations. On the other hand, equation (7.5) implies that successful leptogenesis is now possible for
$M \ll 10^9$ GeV. In both cases the situation is opposite to that after perturbative decay where $A \ll 1$.

However, we caution the reader that these should be only taken as crude estimates, since particle production from scatterings in the preheat plasma is more involved. In a dense plasma the scattering processes can be enhanced (for bosonic final states) or suppressed (for fermionic final states). Large effective masses can also kinematically suppress or shut-off some processes. We can nevertheless expect that our analysis captures the main qualitative aspects of particle production during thermalization after preheating.

9. Conclusion

In this paper we have presented a detailed account of the thermalization after inflation in SUSY and we discussed various implications. We have emphasized that the final stage of reheating is the perturbative inflaton decay, even if the inflaton condensate decays non-perturbatively. For a wide range of inflaton couplings perturbative decay happens when the inflaton quanta dominate the energy density and therefore generates entropy.

The most important result is that the rate of thermalization is extremely slow in SUSY. It is often wrongly assumed that the inflaton decay products immediately thermalize if they have gauge interactions. Our message is that this (although true in realistic models in the non-SUSY case) is not correct in SUSY.

In any SUSY extension of the SM there is a large number of flat directions which are made up of squark, slepton and Higgs fields. These flat directions acquire very large VEV during inflation which spontaneously break gauge symmetries in the early Universe. This induces very large masses to the gauge bosons (and gauginos) and suppresses main reactions which lead to kinetic and chemical equilibrium (i.e., the $2 \rightarrow 2$ and $2 \rightarrow 3$ scatterings with gauge boson exchange). As a result, the Universe enters a long period of a quasi-thermal phase during which the comoving number density and (average) energy of particles remain constant. This epoch lasts until the $2 \rightarrow 3$ scatterings become efficient, at which point the number of particles increases and full equilibrium is established. The main results are given in equations (2.13) and (2.15).

Slow thermalization substantially lowers the reheat temperature of the Universe. The reheat temperature is practically decoupled from the inflaton decay and can be as low as $O(\text{TeV})$, even for large inflaton decay rates. It varies in a typical range $10^3 \text{ GeV} \leq T_R \leq 10^7 \text{ GeV}$. This is underlined in equation (3.1) and demonstrated by the examples given in tables 1, 2.

We studied particle production in a quasi-thermal phase. The general results are given in equations (5.2), (5.3), (5.6). An important aspect of a quasi-thermal particle production is its dependence on the composition of the reheat plasma (before complete thermalization). The abundance of particles thus produced does not depend solely on the maximum temperature in the quasi-thermal phase. We then specialized to two cases of physical interest, namely gravitino production and leptogenesis, with the results given in equations (6.6) and (7.5) respectively.

The most important cosmological consequence of our study is the gravitino production. The Universe thermalizes at sufficiently low reheat temperatures which satisfy the tightest BBN bounds on thermal gravitino production. Our central message is that there is a natural resolution to the infamous Gravitino problem that lies within a consistent
treatment of thermal history of the Universe within SUSY. We emphasize that the built-in solution offered by SUSY renders any exotic modifications (such as late entropy release) unnecessary. Needless to say, this has very important implications for inflationary model building.

On the other hand, quasi-thermal leptogenesis is necessary to generate sufficient baryon asymmetry since slow thermalization results in $T_R \ll 10^9$ GeV, for which thermal leptogenesis does not work (unless in very special cases). As usual the question is the marginality between leptogenesis and gravitino production. Depending on the composition of the reheat plasma, this can be either relaxed or tightened compared to the case in full equilibrium. If the inflaton mainly decays into the top (s)quarks, it is possible to have successful quasi-thermal leptogenesis while keeping gravitino production under control. One can make further progress along these lines through more quantitative studies which takes into account of lepton-number violating interactions more carefully.

To conclude, supersymmetry dramatically modifies the thermal history of the Universe; most importantly, it provides a built-in solution in the form of flat directions which can naturally solve the gravitino problem. This, so far, neglected fact can remove one of the most serious obstacles for building consistent inflationary models in the framework of SUSY and in string-inspired theory.

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Appendix

A.1. Last stage of inflaton decay

Let us consider a simple model of chaotic inflation with a SUSY superpotential

$$W \supset \frac{1}{2} m_{\phi} \Phi \Phi + h \Phi \Psi \Psi, \quad (A.1)$$

A nice realization of chaotic inflation within supergravity is through implementing a shift symmetry [95]. If the inflaton Kähler potential has the form $K = (\phi + \phi^*)^2 / M_P^2$, instead of the minimal form $K = \phi^* \phi / M_P^2$, the scalar potential along the imaginary part of $\phi$ remains flat even for Transplanckian field values. Therefore it can play the role of an inflaton in a chaotic model. Note that a shift symmetry also ensures that the (positive) Hubble-induced corrections to the mass of flat directions vanish at the tree-level as the cross terms in equation (4.2) disappear. It is also possible to realize chaotic inflation for sub-Planckian field values in supergravity. For example, see the multi-axion-driven [96] assisted inflation [63].
where $\Phi$ is the inflaton superfield comprising the inflaton $\phi$ and the inflatino $\tilde{\phi}$. It is coupled to another superfield $\Psi$ whose bosonic and fermionic components are denoted by $\chi$ and $\psi$ respectively. Here we choose $m_\phi = 10^{13}$ GeV, so that inflaton fluctuations generate the right amount of density perturbations. Equation (A.1) results in the following interaction terms in the scalar potential:

$$V \supset h^2 \phi^2 \chi^2 + \frac{1}{\sqrt{2}} h m_\phi \phi \chi^2,$$

where we have considered the real parts of $\phi$ and $\chi$ fields. A nice feature is that SUSY relates the couplings of the cubic $\phi \chi^2$ and quartic $\phi^2 \chi^2$ interaction terms of the inflaton. Note that the cubic term is required for complete decay of the inflaton field.

At the end of inflation $|\phi| \sim O(M_P)$. Preheating occurs if $h > 10^{-6}$, in which case the $h^2 \phi^2 \chi^2$ term takes over and, for $h > 10^{-4}$, leads to an explosive transfer of energy from the homogeneous condensate to $\chi$ quanta [8]. Eventually, after re-scattering of $\chi$ quanta off the remaining condensate, a plasma is formed which consists of the same number of $\phi$ and $\chi$ quanta with typical energies $\gg m_\phi$ which is in kinetic equilibrium [91]–[93]. This stage completes over a rather short period of time $t \sim 100 m_\phi^{-1}$ [8]. Full thermal equilibrium takes much longer to establish, but the temperature of the resulting thermal bath will presumably be larger than $m_\phi$. This implies that the inflaton (and inflatino) quanta remain in thermal equilibrium as long as $T \gtrsim m_\phi$.

Once $T$ drops below $m_\phi$, due to Hubble expansion, the inflaton quanta become non-relativistic. The $h m_\phi \phi \chi^2 / \sqrt{2}$ term then takes over, leading to a perturbative decay of the inflaton to (real and imaginary parts of) $\chi$, plus the fermionic partner $\psi$, at a rate $\Gamma_d = (h^2/8\pi) m_\phi$. Note that, regardless of how large $h$ is, this stage of inflaton decay will be perturbative$^{36}$. The reason is that, unlike the initial condensate, there is no coherence among the decaying inflaton quanta at this stage.

The inflaton quanta dominate the energy density of the Universe at the time of decay and hence generate entropy, provided that

$$\Gamma_d \ll \frac{m_\phi^2}{M_P}.$$  

For $m_\phi = 10^{13}$ GeV, equations (A.2) and (A.3) result in $h < 10^{-2}$. This is much weaker than the condition $h \leq 10^{-6}$, which is required for the inflaton decay to be perturbative from the beginning. Therefore in SUSY a last stage of perturbative inflaton decay naturally follows preheating.

### A.2. Gravitationally decaying inflaton$^{37}$

As a first example, we consider a model of [97] in which the inflaton sector is gravitationally coupled to the MSSM sector. The scalar potential in supergravity is given by [41]

$$V = e^G \left( G_t G^t - \frac{3}{M_P^2} \right) M_P^6,$$  

$^{36}$ The situation is similar to that in the decay of SUSY partners of SM fields. These particles stay in thermal equilibrium at temperatures above their mass. Once $T$ drops below their mass, they decay very quickly, but perturbatively through gauge couplings of $O(1)$.

$^{37}$ This subsection is motivated by our discussion with Antonio Masiero.
where $G$ is the Kähler function and in minimal supergravity is defined as
\[ G = \frac{\chi_i \chi^*_i}{M_P^2} + \log \left( \frac{|W|^2}{M_P^6} \right). \] (A.5)

The $\chi_i$ denote the scalar fields in the theory and lower and upper indices on $G$ denote its derivative with respect to $\chi_i$ and $\chi^*_i$ respectively. The inflaton sector superpotential $W_\phi$ and the MSSM superpotential $W_{\text{MSSM}}$ are given by
\[ W_\phi = \frac{1}{2} m_\phi (\Phi - M_P)^2, \quad W_{\text{MSSM}} = y_{ijk} \Psi_i \Psi_j \Psi_k. \] (A.6)

Here $\Phi$ and $\Psi_i$ denote the inflaton and the MSSM chiral superfields, respectively, and $y_{ijk}$ are the MSSM Yukawa couplings. The minimum of inflaton potential is located at $\phi = M_P$, around which it takes the form $m^2_\phi (\phi - M_P)^2$. This model realizes new inflation in minimal supergravity. Obtaining density perturbations of the correct size from quantum fluctuations of the inflaton requires that $m_\phi = 10^{13}$ GeV \[97\]. Equation (A.4) leads to the following term
\[ y_{ijk} m_\phi \frac{M_P}{M_P} \phi^* \chi_i \chi_j \chi_k, \] (A.7)
in the scalar potential. If $m_\phi$ is much larger than the soft SUSY-breaking scalar masses, the partial width for inflaton decay to three scalars is $\sim y^2 m^3_\phi / M_P^2$. For $m_\phi = 10^{13}$ GeV this is always the case, particularly in models with weak-scale SUSY.

There is also a term
\[ e^{G/2} [G^{ij} - G^i G^j - G^k G^{ij}_k] \bar{\psi}_i \psi_j \] (A.8)
in the Lagrangian \[41\] which describes the couplings of fermionic partners of $\Psi$, denoted by $\chi$, in the two-component notation
\[ y_{ijk} \frac{1}{M_P} \phi \chi_i \bar{\psi}_j \psi_k. \] (A.9)

It results in a partial width for inflaton decay to two fermions and one scalar which is same as that for decay to three scalars $\sim y^2 m^3_\phi / M_P^2$. This implies that inflaton decay produces the same number of particles and sparticles. Because of the large top Yukawa coupling $y_t \approx 1$, the inflaton in this model mainly decays to the top (s)quarks, LH bottom (s)quarks, Higgs $H_u$ and Higgsino $\tilde{H}_u$. The total inflaton decay rate is therefore
\[ \Gamma_d \sim 10^{-2} \frac{m^3_\phi}{M_P^2}. \] (A.10)

Since the inflaton decays into three-body final states, the average energy of decay products is $\langle E \rangle \approx m_\phi / 3$ and hence $T_{\text{max}} \approx m_\phi / 9$. Equation (5.1) then implies that
\[ \mathcal{A} \sim \left( \frac{m_\phi}{M_P} \right)^2. \] (A.11)

For $m_\phi = 10^{13}$ GeV, equation (A.11) leads to $\mathcal{A} \sim 10^{-11}$. This implies that the reheat plasma is extremely dilute and hence substantially far from full equilibrium. For different degrees of freedom we have
\[ \mathcal{A}_i \sim y_i^2 \left( \frac{m_\phi}{M_P} \right)^2, \] (A.12)
where $y_i$ denotes the superpotential Yukawa coupling of the $i$th degree of freedom. Hence it is the largest for top (s)quarks, LH bottom (s)quarks, $H_u$ and $\tilde{H}_u$. 

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A.3. Right-handed sneutrino as the inflaton

It is also possible that the inflaton is directly coupled to some of the MSSM fields. This happens, for example, in the model of [98,99] where one of the RH sneutrinos $\tilde{N}$ plays the role of the inflaton. The relevant part of the superpotential in this case reads

$$W \supset \frac{1}{2} M_N N N + h_i H_u N L_i,$$  \hspace{1cm} (A.13)

where $N$ is the multiplet containing the RH sneutrino $\tilde{N}$ which plays the role of the inflaton (and its fermionic partner $N^c$) and $h_i$ are the Yukawa couplings governing the inflaton decay. With an appropriate choice of non-minimal Kähler function, the scalar potential remains flat at large field values $|\tilde{N}| > M_{P}$ and this model realizes chaotic inflation in supergravity [98,99] (see also [100]). Quantum fluctuations of the sneutrino result in density perturbations of the correct size, provided that $M_N = 10^{13}$ GeV.

The inflaton in this case mainly decays into the LH (s)leptons, $H_u$ and $\tilde{H}_u$. Note that the same number of particles and sparticles are produced in inflaton decay, so long as $M_N$ is much larger than soft SUSY-breaking masses (which is the case for weak-scale SUSY). The total inflaton decay rate is then given by

$$\Gamma_d = \frac{h^2}{4\pi} M_N; \quad h \equiv \sqrt{\sum_i |h_i|^2}.$$  \hspace{1cm} (A.14)

Since the inflaton decays into two-body final states, we have $\langle E \rangle = m_\phi/2$ right after the decay completes, implying that $T_{\text{max}} \simeq m_\phi/6$. Equation (5.1) then results in

$$A \sim 10^2 h^4 \left( \frac{M_P}{M_N} \right)^2.$$  \hspace{1cm} (A.15)

For $M_N = 10^{13}$ GeV and $10^{-6} \leq h \leq 10^{-3}$ we find $10^{-12} \leq A \leq 1$. For the $i$th (s)lepton doublet we have

$$A_i \sim 10^2 h_i^2 h^2 \left( \frac{M_P}{M_N} \right)^2.$$  \hspace{1cm} (A.16)

The (s)lepton singlets, (s)quarks, gauge fields and gauginos are not produced in two-body decays of the inflaton. However, they are inevitably produced at higher orders of perturbation theory [50]; therefore, they have much smaller but non-vanishing values of $A$.

A.4. Flat directions and thermalization: additional considerations

The main reason behind slow thermalization of the Universe in SUSY is that the flat direction VEV, and hence the mass of gauge bosons, remains large for a sufficiently long time. In section 2 we have assumed that the flat direction VEV is just redshifted by the Hubble expansion while oscillating. Here we consider further details of the flat direction dynamics and examine their importance.

- Flat direction decay: In our analysis we have assumed that flat directions do not decay until the Universe fully thermalizes. The flat directions have gauge and Yukawa couplings to other fields, generally denoted by $y$, which result in a decay
rate \( \Gamma_\varphi = (y^2/4\pi) m_0 \). Note, however, that the flat direction VEV induces a mass \( y|\varphi| \) for the decay products. The decay is therefore kinematically forbidden until \( y|\varphi| < m_0 \). On the other hand, the flat direction decay to particles to which it is not directly coupled is kinematically allowed at all times. However, such decays are mediated by the fields which are coupled to the flat direction, and hence suppressed by a factor \( \sim (m_0/y|\varphi|)^2 \) relative to the leading-order decay.

In both cases it turns out that the flat direction decays at a time when the expansion rate is given by

\[
H_\varphi < \frac{1}{4\pi} \frac{m_0^3}{|\varphi|^2}.
\]

(A.17)

The flat direction will not decay before the establishment of full thermal equilibrium if \( H_\varphi < H_{\text{thr}} \) where, depending on the case, \( H_{\text{thr}} \) is given by equations (2.13) and (2.15). This is generically the case, in particular for the examples of table 1.

One might also wonder whether the flat direction could promptly decay via preheating since \( y|\varphi| \gg m_0 \) at the onset of its oscillations [8]. However, SUSY-breaking soft mass terms generically lead to out-of-phase oscillations of the real and imaginary parts of the flat direction [45]. As shown in [101], a tiny effect of this type is sufficient to shut-off resonant decay of the flat direction. Similar effects will be present if two or more flat directions with non-zero VEV oscillate slightly out of phase.

- **Scatterings off the flat direction condensate:** Energetic particles in the reheat plasma scatter off the zero-mode quanta in the flat direction condensate. In order for the flat direction to affect thermalization, it should survive against evaporation by such scatterings (at least) until the Universe fully thermalizes. Scatterings which are mediated by gauge and gaugino fields, similar to diagrams in figure 1, play the main role here. However, the centre-of-mass energy in such scatterings is \( s^{1/2} \approx (4Em_0)^{1/2} \). It turns out that for \( H > H_{\text{thr}} \) we typically have \( 4Em_0 \ll \alpha|\varphi|^2 \). The evaporation rate is therefore given by

\[
\Gamma_{\text{eva}} \sim 10\alpha \frac{nEm_0}{|\varphi|^4},
\]

(A.18)

which is \( < \Gamma_{\text{thr}} \). This implies that the flat direction evaporates only after the establishment of full equilibrium. Note that the condensate contains zero-mode quanta with a number density \( n_\varphi = m_0|\varphi|^2 \). For sufficiently large values of \( \varphi_0 \) that affect thermalization, \( n_\varphi \) is much larger than the number density of particles in the reheat plasma \( n \). These zero-mode quanta can participate in the \( 2 \rightarrow 3 \) scattering similar to those shown in figure 2. Then one might wonder whether scatterings off the flat direction condensate would result in a thermalization rate larger than what we obtained in equation (2.10). However, emitting a gauge boson with a mass \( \simeq g|\varphi| \) requires that \( 4Em_0 > \alpha|\varphi|^2 \) (note that \( (4Em_0)^{1/2} \) is the centre-of-mass energy). As just mentioned, we have the opposite inequality for \( H > H_{\text{thr}} \). This implies that the \( 2 \rightarrow 3 \) scatterings off the condensate do not have enough energy to produce on-shell gauge bosons. Inelastic scatterings which happen at higher orders (mediated by off-shell gauge and gaugino fields) can increase the number of particles but their cross-section will be suppressed by (at least) a factor of \( 4Em_0/\alpha|\varphi|^2 \) relative to those
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in figure 2. This results in a rate

\[ \Gamma_{\varphi}^{\text{thr}} \sim 10^2 \alpha^2 m_0^2 \frac{E}{|\varphi|^2}, \]  

(A.19)

where we have used \( n_\varphi = m_0^2 |\varphi|^2 \). This turns out to be subdominant to \( \Gamma_{\text{thr}} \), see equation (2.10), whenever the flat direction VEV is large enough to affect thermalization.

• Early oscillations of the flat direction: The flat direction starts oscillating once the Hubble expansion rate drops below its mass, which we have considered to be the soft breaking mass \( m_0 \). The thermal effects can trigger early oscillations of the flat directions [102, 103]. This follows from a general consideration that the fields which are coupled to a flat direction (once kinematically accessible to a thermal bath) induce a plasma mass for it. If the plasma mass exceeds the expansion rate of the Universe at early times, it will trigger early oscillations of the flat direction. When the reheat plasma is in full thermal equilibrium, this indeed happens in many cases [102, 103]. Earlier oscillations of the flat direction also imply earlier thermalization of the reheat plasma, see equations (2.13) and (2.15), and hence higher reheat temperatures. Before the establishment of full equilibrium, however, the reheat plasma is dilute and the resulting plasma mass \( \sim (n/E)^{1/2} \) is smaller by a factor of \( (A/228.75)^{1/4} \). Moreover, particles which are coupled to the flat direction have a large mass, unless these couplings are very small, and hence decay via gauge interactions almost instantly. Note that the inverse decays are inefficient in a dilute plasma. Therefore the plasma masses are completely negligible and will not affect the flat direction dynamics.

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