Experimental and Numerical Analysis on Pinned-fixed Beam-columns

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**Abstract.** Slender steel beam-column subjected to axial force and bending moments exhibits complex behaviour, which is necessary to include into design calculation. The design procedure for verification of beam-columns loaded in this way can be found in European Standards for Steel structures. Another approach for verification of beam-column can be also found in European Standards for Aluminium structures. The aim of this paper is to present the experimental and numerical analysis on pinned-fixed beam-columns, the results of which will be applied for comparison of these two standards.

1. **Introduction**

It is not correct to simply superpose the effects of axial forces and bending moments on slender steel members. The deflections are influencing internal forces, bending moments mostly, so they are not only dependent on loading, but also on deflections along the member. The geometrical and material imperfections are an additional phenomenon, which has significant impact to resistance of members subjected to combined stresses, so-called beam-columns.

Let’s consider model of imperfect beam-column in Figure 1 loaded by transverse uniformly distributed load, labelled as \( q \), which is generating bending moment and also loaded by axial force \( N \) as in [1]. Let us assume that the lateral-torsional buckling is prevented.

**Figure 1.** Beam-column loaded by bending moment and axial force
The first order bending moment $M_0$ generated by transverse load is than increased due to the action of axial force $N$, initial deflection $w_0$ and incremental deflection $w$. This results in the total bending moment given by equation:

$$M = M_0 + N(w(x) + w_0(x)). \quad (1)$$

The effect of all imperfections is included in by initial equivalent sinusoidal curvature with amplitude $e_{0d}$ in the middle of the span, given by:

$$w_0(x) = e_{0d} \sin\left(\frac{\pi x}{L}\right). \quad (2)$$

The equilibrium of bending moments for cross-section at the distance of $x$ along the member results in the differential equation:

$$EIw'''' + Nw(x) = -Nw_0(x) - M_0(x), \quad (3)$$

which can be modified on the assumption of constant cross-section along the member into equation:

$$w'''' + \frac{N}{EI}w(x) = -\frac{N}{EI}w_0(x) - \frac{M_0(x)}{EI}, \quad (4)$$

$$w'''' + a^2w(x) = -a^2w_0(x) - \frac{M_0(x)}{EI}, \quad (5)$$

where $M_0(x) = qx(L - x)/2$.

General solution of equation (5) can be written in the following form:

$$w(x) = C_1 \sin ax + C_2 \cos ax + w_{w0} + w_q, \quad (6)$$

where $w_{w0}$ is particle integral for effect of initial curvature of the beam-column and $w_q$ is particle integral for effect of transverse load $q$. Equation (5) can be than modified into following form:

$$w(x) = \frac{Ne_{0d}}{N_{cr} - N} \sin\left(\frac{\pi x}{L}\right) + \frac{q}{a^2N} \left[\cos ax - 1 + \frac{(1 - \cos \alpha L) \sin ax}{\sin \alpha L} - \frac{a^2Lx}{2} + \frac{a^2x^2}{2}\right]. \quad (7)$$

The final equation for combination of loads according to Figure 1 can be obtained considering bending moments in general differential form $M = -EIw''''$:

$$M = Ne_{0d}\left(\frac{N_{cr}}{N_{cr} - N}\right) + \frac{q}{a^2}\left(\frac{1}{\cos(\alpha L/2)} - 1\right), \quad (8)$$

where $N_{cr}$ is critical flexural buckling load.

The presented method of calculation of internal forces of beam-column subjected to axial force and transverse loads is for practical application unacceptable, because of its complexity. If the initial curvature of deflection is in the sinusoidal shape, the second-order bending moment $M''''$ for beam-column subjected to axial load and bending moments and axial load can be simply defined in following form:
which would be accurate, if the transverse load has sinusoidal distribution along the member. The second order bending moment can be defined more accurately according to Dischinger coefficient:

\[ M'' = \frac{M_0}{\left(1 - \frac{N}{N_{cr}}\right)} \]  

(9)

The coefficient \( \mu \) is taking to account the shape of bending moment distribution along the member. It has the value \( \mu = 0.27 \) in case of constant bending moment, value \( \mu = -0.19 \) in case of triangle, value \( \mu = 0.028 \) in case of parabolic shape of the bending moment and \( \mu = 0 \) if the bending moment has sinusoidal shape. Because the Dischinger coefficient can be acquired only for elementary cases of transverse load, this also cannot be assumed as a general approach.

The presented simplified calculation methods are accurate only for beam-columns subjected to axial load and simple distribution of transverse loads. However, the beam-columns are subjected to different combination of transverse loads in design practice. Therefore, it is necessary to assume another simplifications for design practice, which can be found in [2,3].

2. Verification of beam-columns according to European standards

2.1 Verification according to EN 1993-1-1

The approaches according to this standard are based on the equivalent column method. The concept of this method is substituting a member with arbitrary boundary conditions by an equivalent one, which has simply supported end conditions in order to further check its resistance by the given standard interaction formulae. For members under axial compression plus bending, the equivalent member is defined by its specific buckling length and the bending moment diagram along this equivalent length. In this view, the equivalent member is a full representation of the real member and the standard design formulae for simply supported members can be applied to it.

2.1.1 EN 1993-1-1 Annex A

This design method is aimed at proposing general, transparent, consistent and accurate interaction criteria. The real bending moment is substituted by equivalent sinusoidal bending moment by means of \( C_m \) factor. The proposal has been derived as far as possible on theoretical aspects. In this method, each coefficient in formulae represents a single physical effect. It can be effective to identify the governing phenomenon and to propose an adequate design. As this method is derived to be as general as possible, it is covering a wide range of configurations, including unusual ones.

2.1.2 EN 1993-1-1 Annex B

The objective of this method is more user-friendly proposal reducing the amount of design work. The aim is reached by means of providing design formulae in the basic format of the theoretical buckling equations using reduced number of coefficients for the calculation of the resistance against buckling. The actual distribution of bending moment is substituted by means of equivalent uniform bending moment for non-uniform moment diagrams.

Both methods differentiate between members susceptible and not susceptible to torsional deformations. Members not susceptible to torsional deformations are hollow sections and open sections with appropriate torsional restraints. Accordingly, it provides two design equations for in-plane and out-of-plane buckling for two cases as follows:

\[ \frac{N_{Ed}}{\chi_{y,RK}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \tau_{fM_1}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\tau_{fM_1}} \leq 1, \]  

(11)
\[
\frac{N_{Ed}}{\frac{\gamma_{M1}}{2\gamma N_{Rk}}} + k_{zy} \frac{M_{y,Ed}+\Delta M_{y,Ed}}{\frac{\gamma_{M1}}{XLT}} + k_{zz} \frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{\gamma_{M1}}{Y_{M1}}} \leq 1,
\]

where:

- \( N_{Ed}, M_{y,Ed}, \) and \( M_{z,Ed} \) are the design values of the compression force and the maximum bending moments about the y-y and z-z axis along the member,
- \( \Delta M_{y,Ed} \) and \( \Delta M_{z,Ed} \) are the bending moments due to the shift of the centroid axis for class 4 of cross-sections,
- \( \chi_y \) and \( \chi_z \) are the reduction factors due to flexural buckling,
- \( \chi_{LT} \) is the reduction factor due to lateral torsional buckling,
- \( k_{zy}, k_{yz}, k_{zy}, k_{zz} \) are the interaction factors; they are relative to either method Annex A or Annex B of standard EN 1993 – 1 - 1 ,
- \( N_{Rk}, M_{y,Rk}, \) and \( M_{z,Rk} \) are the characteristic values of resistances to normal force and bending moments, y-y and z-z axis,
- \( \gamma_{M1} \) is the partial factor for resistance of members to instability assessed by member checks.

The interaction factors are derived differently for class 1 or 2 cross-sections and for 3 or 4 ones respectively. Initially, it is necessary to classify the cross-section in accordance with standard. Although, the classification may be done for compression and bending moment separately, however, for combination of compression and bending moment it should be accomplished too.

### 2.2 Verification according to EN 1999-1-1

Classification of cross-sections for members with combined bending and axial forces is performed for the loading components separately. No classification is recommended for the combined state of stresses. The combined state of stresses is accounted for in the interaction formulae. Those formulae are the same for all the cross-section classes. The influence of yielding and local buckling is taken into account in the denominators and the exponents, which are functions of the member slenderness. Cross-section check is included in the assessment of flexural and lateral-torsional buckling, so there is not necessary to verify the cross-sectional resistance. Nevertheless, more cross-sections along the member length are needed to be checked.

Beam-column with open double-symmetric cross-section has to be verified for flexural buckling according to these two expressions:

\[
\left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right) \xi_{yc} + \frac{M_{y,Ed}}{\eta_0 M_{y,Rd}} \leq 1,
\]

\[
\left( \frac{N_{Ed}}{\chi_y \omega_x N_{Rd}} \right) \eta_c + \frac{M_{z,Ed}}{\eta_0 M_{z,Rd}} \xi_{zc} \leq 1,
\]

where:

- \( \xi_{yc} = 0.8 \) or alternatively \( \xi_{yc} = \xi_0 \gamma_y \), however \( \xi_{yc} \geq 0.8 \),
- \( \eta_c = 0.8 \) or alternatively \( \eta_c = \eta_0 \gamma_z \), however, \( \eta_c \geq 0.8 \),
- \( \xi_0 \) is defined in the section 6.2.9.1 of standard,
- \( \omega_0 \) is the coefficient taking into account the effect of cross welds, for cross-section with no cross welds \( \omega_0 = 1 \),
- \( N_{Ed}, M_{y,Ed}, \) and \( M_{z,Ed} \) are the design values of the compression force and the bending moments about the y-y and z-z axis in the verified cross-section,
$N_{Rd}$, $M_{y,Rd}$, and $M_{z,Rd}$ are the design values of resistances to normal force and bending moments about the $y$-$y$ and $z$-$z$ axis in the verified cross-section.

Beam-columns with open double-symmetric or mono-symmetric cross-sections have to be verified for lateral torsional buckling about the weak axis of cross-section according to following expression:

$$
\left(\frac{N_{Ed}}{\chi_{x} \omega_{x} N_{Rd}}\right)^{\frac{\eta_{c}}{\chi_{c}}} + \left(\frac{M_{y,Ed}}{\chi_{yLT} \omega_{yLT} M_{y,Rd}}\right)^{\frac{\gamma_{c}}{\gamma_{c}}} + \left(\frac{M_{z,Ed}}{\omega_{0} M_{z,Rd}}\right)^{\frac{\xi_{c}}{\chi_{c}}} \leq 1, \quad (15)
$$

$$
\omega_{x} = \frac{\omega_{0}}{\chi + (1-\chi) \frac{x}{l_{c}} \sin \frac{\pi x}{l_{c}}}, \quad (16)
$$

$$
\omega_{yLT} = \frac{\omega_{0}}{\chi_{yLT} + (1-\chi_{yLT}) \frac{x}{l_{c}} \sin \frac{\pi x}{l_{c}}}, \quad (17)
$$

where:

- $\omega_{x}$ and $\omega_{yLT}$ are the coefficients taking into account the distribution of the secondary bending moment along the member,
- $x$ is the distance between support or point of inflection in the case of elastic flexural buckling and the point of verification,
- $\omega_{0}$ is the coefficient taking into account the effect of cross welds, for cross-section with no cross welds $\omega_{0} = 1$,
- $\gamma_{c} = \gamma_{0}, \chi = \chi_{0}$, or $\chi_{c}$ are the reduction factors due to flexural buckling depending on the direction of buckling,
- $l_{c}$ is the flexural buckling length,
- $N_{Ed}$, $M_{y,Ed}$, and $M_{z,Ed}$ are the design values of the compression force and the bending moments about the $y$-$y$ and $z$-$z$ axis in the verified cross-section,
- $N_{Rd}$, $M_{y,Rd}$, and $M_{z,Rd}$ are the design values of resistances to normal force and bending moments about the $y$-$y$ and $z$-$z$ axis in the verified cross-section.

3. Experimental and numerical analysis

The comparison for some load cases between design approaches according to EN 1993-1-1 [2] and EN 1999-1-1 [3] for beam-columns subjected to axial load and bending moments is presented in [4]. However, the comparison is only between these two approaches. The comparison with experimental analysis can be seen in [5] but it is only done for one load case and one relative slenderness. Therefore, the experimental and numerical analysis is undertaken by Department of Structures and Bridges at University of Žilina on pinned-fixed beam-columns subjected to axial force and bending moment induced by eccentric axial forces at the top sides of the members.

For experimental investigation, the hot-rolled section of IPE 100 was chosen belonging to the first class from the viewpoint of cross-sectional classification. The specimens total lengths are 1800 mm and 2370 mm, so that the appropriate relative slenderness are $\bar{l}_{x} = 1.28$ and $\bar{l}_{z} = 1.69$ for buckling about the $z$-axis.

Both beam-columns ends are equipped with the 30 mm (at the bottom) or 20 mm (at the top) thick end – plates ensuring the zero warping deformations of beam-column edges. The fixed edges are situated at the bottom ends of the vertically tested specimens while the hinged boundary conditions were simulated at the tops of samples using the specially adjusted device.
The set of 3 types of members are prepared for testing, because of 3 combinations of loading. There are going to be 3 specimens investigated for every combination of loading for both relative slenderness, thus 18 specimens in all will be observed.

Members of type A are centrically loaded by axial force at the top side, thus without eccentricities in both main axis of cross-section. Consequently, members of type B has eccentricity at the direction of y-axis (axial force will be generating bending moment about the weaker axis) with value $e_y = 10$ mm and members of type C has eccentricity at the direction of z-axis (bending moment about the stronger axis) with value $e_z = 30$ mm.

The strains and lateral deflections will be measured in chosen locations as it can be seen from figure 3. The strains and lateral deflections will be monitored using gauges 6/120 LY11 (HBM) and potentiometer sensors of deformations TR50 recorded by means of Spider 8, also enabling recording of the actual value of compressive force monitored by dynamometer situated at the top end of the tested beam-column.
Figure 3. Locations for measuring strains and deflections for beam-columns with length 2370 mm

Figure 4. Prepared specimen for test and numerical model in Ansys Workbench

The actual geometrical and material characteristics of the beam-columns were determined and evaluated statistically. The initial imperfections along the member span in direction of both axis were measured by means of geodetic method in 15 places along the member and for the members with
length $L = 1800 \text{ mm}$ their maximal amplitude is $e_z = L/1881$ and $e_y = L/1210$. For the members with length $L = 2370 \text{ mm}$ their maximal amplitude is $e_z = L/3095$ and $e_y = L/1115$. The material characteristics were determined by means of the tensile tests on 6 specimens, 3 were from the web and 3 from the flanges of beam-columns. The average yield stress for the flange is $f_y = 300 \text{ MPa}$ and for the web is $f_y = 323 \text{ MPa}$, so the steel with such a yield strength is in accordance with yield strength of grade S235, as was also observed in [6].

The geometric characteristics of the cross-section was also measured. The thicknesses of the web and flanges were measured by ultrasonic coating a thickness gage Sonatest CT-GAGE at 5 places along every member. The total height and width of the cross section were measured with slide gauge. The average actual and tabular geometric characteristics can be seen in table 1 and table 2.

Table 1. Actual and basic cross-sectional dimensions of specimens

| Actual cross-sectional dimensions of IPE 100 | Basic cross-sectional dimensions of IPE 100 |
|-------------------------------------------|------------------------------------------|
| H = 100.65 mm | H = 100 mm |
| B = 56.25 mm | B = 55 mm |
| $t_w$ = 4.4 mm | $t_w$ = 4.1 mm |
| $t_f$ = 5.92 mm | $t_f$ = 5.7 mm |
| $r$ = 7 mm | $r$ = 7 mm |

Table 2. Actual and basic cross-sectional properties of specimens

| Actual cross-sectional properties of IPE 100 | Basic cross-sectional properties of IPE 100 |
|-------------------------------------------|------------------------------------------|
| A = 1099 mm$^2$ | A = 1030 mm$^2$ |
| $I_y$ = 1831 600 mm$^4$ | $I_y$ = 1710 000 mm$^4$ |
| $I_x$ = 176 930 mm$^4$ | $I_x$ = 159 000 mm$^4$ |
| $W_{el,y}$ = 36 395 mm$^3$ | $W_{el,y}$ = 34 200 mm$^3$ |
| $W_{el,x}$ = 6 291 mm$^3$ | $W_{el,x}$ = 5 790 mm$^3$ |
| $W_{pl,y}$ = 42 056 mm$^3$ | $W_{pl,y}$ = 39 400 mm$^3$ |
| $W_{pl,x}$ = 9 958 mm$^3$ | $W_{pl,x}$ = 9 146 mm$^3$ |

Before the tests is also important to know estimated ultimate forces due to the loading organization and safety reasons. Therefore, the ultimate forces were calculated according to already mentioned design approaches presented in European standards [2,3] and by means of numerical simulations in Ansys Software [7].

The numerical model was created on basis of GMNIA simulation in the commercial software ANSYS Workbench. The actual averaged cross-section dimensions were used, but the fillets at the web-flange joint were not modelled, as their influence on load-carrying capacity was found as negligible [8]. The beam-columns were modelled using the homogeneous structural solid element SOLID 185. It is an 8-node solid element that is suitable for 3D modelling of solid structures having three degrees of freedom at each node. It has large deflection and large strain capabilities, plasticity, hyperelasticity, stress stiffening and creep. The enhanced strain formulation was considered, as it is recommended in [9,10]. This formulation prevents shear locking in bending dominated problems and volumetric locking in nearly incompressible cases. It is a well-known assumption that in case of the numerical analysis on FEM basis, the smoother the mesh is, the more accurate the results are.

However, the calculation time is also increasing with increasing number of finite elements included in the model. So the mesh was defined by 10 elements for the flange width, 20 elements for the web height and by 2 for the flange thickness. It was proven, that the smoother mesh with doubled amount of finite elements per width, height and thickness of the parts of cross-sections provide almost the
same results [11]. The number of finite elements for the longitudinal direction was chosen in such a way, that the maximal aspect ratio of the longest and shortest edges of an element does not exceed the maximal acceptable aspect ratio for quadrilaterals [7]. According to Annex C of Eurocode 3 part 1-5 [12], it is recommended to use one of four following material models when performing plated finite element analysis:

- elastic-plastic without strain hardening,
- elastic-plastic with a nominal plateau slope of 1 MPa,
- elastic-plastic with a strain hardening slope of E/100,
- true stress-strain curve modified from test results.

According to [13] in buckling analysis, the strains at the point of maximum loads are often limited in magnitude allowing a simple material model without considering the difference between true stresses-true strains and the engineering stress-strains. So in order to avoid convergence problems, the bi-linear model with hardening slope of E/1000 was used.

To calculate the maximal axial force for testing, the shape of geometric imperfection was extracted from the previous linear buckling analysis and the amplitude was derived according to 5.3.2 (11) in Eurocode 3, part 1-1 [2]. The fixed support was modelled by fixing the displacements in all directions for bottom area of the bottom end plate and the hinged support was modelled by fixing the displacements in both horizontal directions for one node at the centre of the top-end plate. Thus the vertical and rotational displacements are allowed for this node. The impact of residual stresses was not considered in this analysis.

The comparison of ultimate load forces determined according to design calculations and numerical simulations can be seen in table 3 and table 4.

| Type of the specimen | Calculation method | A | B | C |
|----------------------|--------------------|---|---|---|
| EN 1993- 1 -1 Method A | 95 | 74 | 84 |
| EN 1993- 1 -1 Method B | 95 | 77 | 78 |
| EN 1999- 1 -1 | 95 | 73 | 90 |
| Numerical analysis | 103 | 82 | 99 |

**Table 4.** Calculated ultimate load forces for specimens with length L = 1800 mm

| Type of the specimen | Calculation method | A | B | C |
|----------------------|--------------------|---|---|---|
| EN 1993- 1 -1 Method A | 144 | 102 | 122 |
| EN 1993- 1 -1 Method B | 144 | 103 | 108 |
| EN 1999- 1 -1 | 144 | 103 | 132 |
| Numerical analysis | 154 | 106 | 147 |

**4. Results and discussions**

There is presented a preparation for experimental test on pinned-fixed beam-columns subjected to combinations of axial force and bending moments generated by eccentricities in the hinged end (at the top plate). There is also a comparison of ultimate load forces for every type of specimen gathered by
verification methods provided by European Standards and numerical GMNIA analysis, indicating differences in comparison to those standards mostly up to 10%.

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