Research of the resistance of the rock mass when implementing a conical tool into it

BB Danilov1*, AI Chanyshev1,2, DO Cheshchin1
1Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
2Novosibirsk State University of Economics and Management, Novosibirsk, Russia
E-mail: *bbdanilov@mail.ru

Abstract. Among various types of resistance of the rock mass to the penetration of a solid body into it, those are determined, with the overcoming of which not rebound, but the depth of penetration (viscous-plastic resistance) is associated. It is shown how at a given initial speed of the tool movement, at the depth and time of penetration measured in experiments, for a given geometry of the tool, the desired resistance function consisting of a constant term and a function that depends on the speed of movement is restored. More acceptable is the geometry of the tool at which the resistance of the medium to penetration for a given depth is minimal.

1. Introduction
The study of the processes of static and dynamic interaction of two isotropic elastic bodies, the surfaces of which are approximated by two parabolas, was considered in [1–3]. The shock load at elastoplastic deformations was studied in [4–6]. Among many works in this direction, we will single out those that relate to the determination of the yield strength and strength of materials using the methods of Brinell and Vickers tests.

In the Brinell test method, indenters in the form of spheres are pressed, in the Vickers test method, regular tetrahedral diamond pyramids are pressed. The hardness of the materials is determined by the imprints of these figures.

Also, the problem of determining the resistance of a soil massif to deformation by various methods is considered in works [7–9].

In this work, similar problems are solved. The resistance of the medium to deformation is defined at a known value of the penetration depth of the tool, a known penetration time and the given values of the geometric parameters of the inserted tool. Thus, the task is reduced to the determination of such values of these parameters at which the resistance of the medium will be minimal, and the depth of penetration will be greatest.

2. Methods of the research
Consider the following situation. Let there be given a body with mass m, located at a height \( H \) above some obstacle - the surface of a rock mass (Figure 1).
We select the half-axis Ox, as shown in Figure 1: we will position the point O on the surface, the values of the x coordinates are positive if the body is penetrated in the rock mass.

To determine the motion of a body with mass m, we have Newton's equation of motion:

$$m \ddot{x} = mg$$

(1)

integrating which we find the velocity $V$ developed by the body when dropped from a height $H$:

$$V = \sqrt{2gH}.$$  

(2)

The body flies up to the obstacle with the speed $V$. Further, there are assumptions about the type of resistance of the medium located in the region $x > 0$. Let's consider various cases.

2.1. Case 1. Let the resistance of the medium be determined by the dependence

$$R = R_1 \cdot x,$$

(3)

where $R_1$ is an unknown constant.

Thus, the task is to determine $R_1$. To solve the problem, we have Newton's equation of motion of the form (1):

$$m \ddot{x} = mg - R_1 \cdot x.$$  

(4)

Integrating (4) with the initial conditions:

$$x|_{t=0} = 0, \quad \dot{x}|_{t=0} = V,$$

(5)

We get solution (4) in the form:

$$x = \frac{g}{\lambda^2} (1 - \cos \lambda t) + \frac{V}{\lambda} \sin \lambda t,$$

(6)

where $\lambda^2 = \frac{R_1}{m}$.

Relation (6) can be rewritten by converting it to one sinusoid:

$$x = \sqrt{\frac{g^2}{\lambda^2} + \frac{\lambda^2 V^2}{\lambda^2}} \sin \lambda \left(t - \frac{\gamma}{\lambda}\right) + \frac{g}{\lambda^2},$$

(7)

where $\tan \gamma = \frac{g}{\lambda V}$.

From this the dependencies for speed $\dot{x}$ and acceleration $\ddot{x}$ follow:

$$\dot{x} = \sqrt{\frac{g^2}{\lambda^2} + \frac{\lambda^2 V^2}{\lambda^2}} \cos \lambda \left(t - \frac{\gamma}{\lambda}\right),$$

(8)
\[
\ddot{x} = -\sqrt{g^2 + \lambda^2 V^2} \sin \left( t - \frac{\gamma}{\lambda} \right). 
\] (9)

From (8) we find that the velocity \( x \) will be equal to zero at:
\[
\lambda t_* - \gamma = \frac{\pi}{2}, \text{ t.e. } \lambda t_* = \frac{1}{\lambda} \left( \gamma + \frac{\pi}{2} \right). 
\] (10)

This means stopping the body in the rock mass.

Figure 2 shows graphs of functions \( x = x(t) \), \( x = x(t) \), \( x = x(t) \).

\[ \text{Figure 2. Graphs of dependences of speed } x \text{ on time } t \]

It can be seen that during the time interval \( t \in \left[ 0, \frac{\gamma}{\lambda} \right] \) the velocity increases from the \( V \) value to the value \( \sqrt{V^2 + \frac{g^2}{\lambda^2}} \). During the time interval, \( t \in \left[ \frac{\gamma}{\lambda}, \frac{1}{\lambda} \left( \gamma + \frac{\pi}{2} \right) \right] \) the speed drops from maximum \( \sqrt{V^2 + \frac{g^2}{\lambda^2}} \) to zero at \( t = \frac{\gamma}{\lambda} + \frac{\pi}{2\lambda} \).

In this case, the displacement \( x \) increases (Figure 2b) from the value \( x = 0 \) to the maximum value \( x = \frac{g}{\lambda^2} + \frac{\sqrt{g^2 + \lambda^2 V^2}}{\lambda^2} \).

Acceleration \( x \) at the moment of time \( t = 0 \) was equal to \( g \), at the moment of stopping the body \( (t = \frac{\gamma}{\lambda} + \frac{\pi}{2\lambda}) \). It changes sign and became equal to \( \ddot{x} = -\sqrt{g^2 + \lambda^2 V^2} \).
To determine the parameter $R_1$ in (3), it is possible to use the time at which the body was maximally immersed in the rock mass $t^*$ from (10). Knowing $t^*$ and $V$, we obtain the following equation for determining the parameter $\lambda$.

$$\arctg \frac{g}{\lambda V} - \lambda t^* + \frac{\pi}{2} = 0$$

From this equation we find $\lambda$, from (6) we find $R_1 = m \cdot \lambda$.

Let us now consider the rebound of the body at the moment of time $t \geq \frac{\gamma}{\lambda} + \frac{\pi}{2\lambda}$. In this case, as above, we have the equation of motion (4) and its solution (6). Let us postpone the origin of time at the moment of rebound. For $t = 0$, we have the following initial conditions:

$$\begin{align*}
x \big|_{t=0} &= x_{\text{max}} = \frac{g + \sqrt{g^2 + \lambda^2 V^2}}{\lambda^2}, \\
\dot{x} \big|_{t=0} &= 0.
\end{align*}$$

From these conditions we find:

$$\begin{align*}
x &= \frac{\sqrt{g^2 + \lambda^2 V^2}}{\lambda^2} \cos \lambda t - \frac{g}{\lambda^2}, \\
\dot{x} &= -\frac{\sqrt{g^2 + \lambda^2 V^2}}{\lambda} \sin \lambda t, \\
\ddot{x} &= -\sqrt{g^2 + \lambda^2 V^2} \cos \lambda t.
\end{align*}$$

Function graphs $x = x(t)$, $\dot{x} = x(t)$, $\ddot{x} = x(t)$ are shown in Figure 3.

![Figure 3. Velocity, displacement and acceleration plots at the rebound](image-url)
In this case, the speed $x$ after the rebound turns to zero at $t = \frac{\pi}{\lambda}$, the displacement $x$ at this time is equal to the value $x = \frac{g - \sqrt{g^2 + \lambda^2 V^2}}{\lambda^2}$, the acceleration from negative values goes into the region of positive values with the value $x = \sqrt{g^2 + \lambda^2 V^2} \geq g$.

Since the value $x$ is negative, it means that the body after rebounding is on the surface of the rock mass. The amount of lift above the surface $x = 0$ is $h = \left|x_\ast - x_{\text{max}}\right| = \frac{2 \sqrt{g^2 + \lambda^2 V^2}}{\lambda^2}$.

If this value is fixed, denoting by $h$, hence we obtain the equation for determining the parameter $\lambda^2$: $\lambda^2 h = 2 \sqrt{g^2 + \lambda^2 V^2}$, deciding which we find

$$\lambda^2 = \frac{R_1}{m} = \frac{2(V^2 + \sqrt{V^4 + h^2 g^2})}{h^2}$$ (14)

Further movement repeats what is stated in formulas (1)–(14). In this case, we obtain a new expression for the speed (2): $V = \sqrt{2hg}$ etc.

Let's make a few comments:
1. If the speed $V = 0$, then the maximum value of the displacement $x$ according to (7) will be equal to $x = \frac{2g}{\lambda^2} = \frac{2gm}{R_1}$. Hence, according to (3), the maximum resistance force is obtained twice as much as the weight of the load.
2. If $V = 0$, then $R_1$ according to (14) is also equal to $2gm$.

2.2. Case 2. Let the resistance of the medium is determined by a constant depending on the shape and geometry of the striker. In this case, Newton's equation of motion has the form:

$$m \ddot{x} = mg - R,$$ (15)

where $R$ – const.

Integrating (15) with the initial conditions (5), we obtain the dependence:

$$\begin{cases} 
\dot{x} = (g - \frac{R}{m}) t + V, \\
\ddot{x} = (g - \frac{R}{m}) t^2 + Vt, \\
\dddot{x} = (g - \frac{R}{m}).
\end{cases}$$ (16)

The speed $x$ turns to zero if $t = \frac{V}{(\frac{R}{m} - g)}$.

Since the time $t$ is positive, this implies the restriction:

$$\frac{R}{m} \geq g$$ (17)

Figure 4 shows the graphs of the functions (16).
From (16) and Fig. 4b follows that the depth of penetration of the initial body into the rock mass

\[ H = H^* \]

is the following value where

\[ H^* = \frac{2}{g(R - mg)} \]

whence

\[ R = m\left[g + \frac{V^2}{2H^*}\right] \]  

(18)

From (18) follows the formula

\[ RH^* = mgH^* + \frac{mV^2}{2} \]

expressing the law of conservation of energy.

It should be noted that in formula (18) \( H^* \) is the value depending on the shape and geometry of the working body of the tool at a given value of the speed \( V \).

2.3. Case 3. Let the resistance of the rock mass while the tool penetrating is determined by the expression

\[ R = R_0 + R_1x \]  

(19)

where \( R_0, R_1 \) – a priori unknown constants.

Substituting (19) into Newton's equation of motion, we obtain the following second-order differential equation:

\[ x + \frac{R_1}{m}x = g - \frac{R_0}{m} \]  

(20)

Solution of (20) is written as:

\[ x = C_1 \cos \lambda t + C_2 \sin \lambda t + \left(g - \frac{R_0}{m}\frac{t^2}{2}\right) \]
where \( C_1, C_2 \) – arbitrary constants, \( \lambda^2 = \frac{R_0}{m} \).

Under initial conditions \( x|_{t=0} = 0 \), \( \dot{x}|_{t=0} = V \), we get expressions:

\[
\begin{align*}
\dot{x} &= V \cos \lambda t + \left(g - \frac{R_0}{m}\right)t, \\
\ddot{x} &= \frac{V}{\lambda} \sin \lambda t + \left(g - \frac{R_0}{m}\right)\frac{t^2}{2} - \frac{R_0}{m}, \\
\dddot{x} &= -V \lambda \sin \lambda t + \left(g - \frac{R_0}{m}\right).
\end{align*}
\] (21)

Expressions (21) are the sums of two terms – the first terms are associated with vibrations (elastic vibrations) and the second terms – with plastic (inelastic) deformations. There is a moment in time at which the velocity \( \dot{x} \) turns to zero, after which the elastic displacements are restored.

2.4. Case 4. Let the resistance of the rock mass is determined by the expression

\[ R = R_0 + R_1 \cdot x, \] (22)

where \( R_0, R_1 \) – a priori unknown constants.

Substituting (22) into Newton’s equation of motion, we obtain the differential equation

\[ m \dddot{x} = mg - R_0 - R_1 \cdot x, \quad \text{or} \quad \dddot{x} + \frac{R_1}{m} \ddot{x} = g - \frac{R_0}{m}, \]

solution of which is the function:

\[ x = C_1 + C_2 e^{-\frac{R_1}{m} t} + \left(g - \frac{R_0}{m}\right)t. \]

Satisfying the initial conditions (5), we obtain

\[
\begin{align*}
\dot{x} &= (V + \frac{R_0}{m} - g)e^{-\frac{R_1}{m}} - \frac{R_0}{m} - g, \\
\ddot{x} &= \frac{m}{R_1} \left(V + \frac{R_0}{m} - g\right)(1 - e^{-\frac{R_1}{m}}) - \frac{R_0}{m} - g)t, \\
\dddot{x} &= -\frac{R_1}{m} \left(V - g + \frac{R_0}{m}\right)e^{-\frac{R_1}{m}}.
\end{align*}
\] (23)

From (23) directly follows that the acceleration \( \dddot{x} \) at \( t \geq 0 \) is always negative, i.e. function \( x = x(t) \) is decreasing from the value \( \dot{x} = V \) at \( t = 0 \) to the value \( \dddot{x} = -\left(\frac{R_0}{m} - g\right) \) at \( t \to \infty \).

In this case, there is a time \( t \) at which the speed \( \dot{x} \) turns to zero and the body stops. This time is determined by the expression

\[ t = t_* = \frac{m}{R_1} \ln \left(1 + \frac{V}{\frac{R_0}{m} - g}\right). \] (24)

Substituting this value of \( t \) into the expression for \( x \) from (23), we find the penetration depth
\[ H_* = \frac{mV}{R_1} - \left( \frac{R_0}{m} - g \right) I_* . \]  

(25)

Hence we find \( \frac{m}{R_1} = (H_* + (\frac{R_0}{m} - g)I_*)/V \), substituting this expression in (24). As a result, we obtain an equation for finding the unknown value \( R_0 \):

\[ Vt_* = \left[ H_* + (\frac{R_0}{m} - g)I_* \right] \ln \left( 1 + \frac{V}{\frac{R_0}{m} - g} \right) \]

In this equation, the values are assumed to be known (from experiment) \( V, t_*, H_*, m, g \). The unknown quantity is \( R_0 \). From where \( R_0 \) is founded and \( R_1 \) is founded from (25).

2.5. Case 5. Let the resistance of the medium is determined by the dependence

\[ R = R_0 + R_1 \left( x^2 \right) , \]  

(26)

Substituting (26) into Newton's equation of motion, we find the following differential equation

\[ \ddot{x} + \frac{R_2}{m} \left( \dot{x} \right)^2 + \frac{R_0}{m} - g = 0 . \]  

(27)

To find a solution to (27), the substitution is made [10] \( x = p(x) \), then \( \dot{x} = p'(x) \dot{p} x \) and equation (27) is rewritten in terms of the function \( p \):

\[ \frac{dp}{dx} = -\frac{R_2}{m} p^2 + \frac{R_0}{m} - mg , \]

Integrating this equation, we find that

\[ R_2 p^2 + R_0 - mg = Ce^{-\frac{2R_2}{m} p} , \]  

(28)

where \( C \) – arbitrary constant. Satisfying the initial condition \( x = p = V \) at \( t=0 \), we found that

\[ C = R_2 V^2 + R_0 - mg . \]  

(29)

Resolving (28) with respect to \( p = \frac{dx}{dt} \), find that

\[ p = \frac{dx}{dt} = \frac{C e^{-\frac{2R_2}{m} p} - R_0}{R_2} + mg \frac{R_2}{R_1} \]  

(30)

Integrating (30), we notice that

\[ \frac{R_2}{R_0 - mg} \arcsin \sqrt{\frac{R_0 - mg}{C} e^{-\frac{2R_2}{m} p}} = t + C_1 \]  

(31)

From the initial condition \( x|_{t=0} = 0 \), find \( C_1 \) in (31):

\[ C_1 = \frac{R_2}{R_0 - mg} \arcsin \sqrt{\frac{R_0 - mg}{C}} \]  

(32)

From (31) follows that
From (33) we find that

\[ \frac{R_x x_{\text{max}}}{m} = \ln \left( \frac{C}{\sqrt{R_0 - mg}} \right) \quad (34) \]

where \( x_{\text{max}} \) is the largest value of \( x \), which is reached when the argument of the sine in (33) is equal to \( \pi / 2 \). Taking into account (32), we obtain the time at which \( x \) reaches its maximum value:

\[ x_{\text{max}} = \frac{m}{2R_2} \ln \left( 1 + \frac{R_2 V^2}{R_0 - mg} \right) \quad (35) \]

at the appropriate time

\[ t = t_{\text{max}} = \frac{R_2}{\sqrt{R_0 - mg}} \left[ \frac{\pi}{2} - \arcsin \left( \frac{1}{\sqrt{1 + \frac{R_2 V^2}{R_0 - mg}}} \right) \right] \quad (36) \]

Knowing from experience the values of \( x_{\text{max}}, t_{\text{max}} \), we see that (35), (36) form a system of two equations to determine the parameters \( R_0, R_2 \) of the resistance of materials \( R \).

Note that formula (35), up to the gravitational acceleration \( g \), coincides with that given in [11, 12]. In [12], there is no formula (36) that forms with (35) a system of equations for finding \( R_0, R_2 \).

3. Conclusions
1. It has been shown that the most responsible for the penetration of solids into an obstacle is visco-plastic resistance. The visco-plastic resistance function is restored empirically by measuring the penetration depth and time.

2. To find the optimal tool geometry a series of experiments for different tool geometries is needed. The optimum is the geometry at which the resistance of the medium for a given penetration depth is minimal.

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