Lattice status of gluonia/glueballs

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I briefly review lattice QCD calculations that study the $0^{++}$ glueball and discuss implications for light flavour singlet $0^{++}$ mesons.

1. Introduction

The $0^{++}$ glueball in pure gauge theory is a well defined quantity that can be used to test our understanding and ability to solve theory. It is also interesting to understand how, or if, the $0^{++}$ glueball contributes to the physical light flavour singlet $0^{++}$ mesons.

Finding glueball degrees of freedom in the flavour singlet mesons is complicated, because other non-perturbative objects such as tetraquark, meson molecule or, even quark-antiquark degrees of freedom can also be building blocks of scalar mesons. There are reviews that discuss these broader issues in more detail [1–3].

1.1. Background to lattice QCD

Lattice QCD is based on a Monte Carlo process where a statistical sample of vacuum gauge fields is produced. On each sample of the QCD vacuum, an interpolating operator creates a hadron and after a specific time interval the hadron is destroyed. The choice of interpolating operator is particularly important for hadrons where it is not clear how the hadron is built out of quarks and gluons.

For example, to create a light flavour singlet $0^{++}$ hadron, possible interpolating operators are

\[ O_1 = \bar{q}q \]
\[ O_2 = \bar{q}_5 q \gamma_5 \]
\[ O_3 = U_{\text{plaq}} \]

where $U_{\text{plaq}}$ is a spatial plaquette of gauge fields with $0^{++}$ symmetry, and $q$ is a light quark operator.

The majority of recent lattice QCD calculations that include the dynamics of sea quarks have pion masses as low as 300 MeV [2], with a range of volumes and lattice spacings. These parameters have allowed lattice QCD to make contact with chiral perturbation theory for light pseudoscalar mesons [4]. For some quantities, there are now precision results from lattice QCD. For example the HPQCD collaboration obtained $m_c(m_c) = 1.268(9)$ GeV for the mass of the charm quark [5]. The LHPC collaboration [6] computed the nucleon axial charge to be $g_A = 1.212(84)$ from unquenched lattice QCD. The low lying glueball spectrum in pure $SU(3)$ gauge theory was accurately computed nearly ten years ago [7,8].

Unfortunately, lattice QCD calculations of scalar mesons are not as accurate as those of other quantities. The lattice QCD correlators for scalar mesons are more noisy than for $\rho$ and $\pi$ mesons, so much higher statistics are required. The light scalar mesons decay via S-wave decays, and current lattice QCD calculations are in the quark mass regime where some decay channels to two mesons are open. The results I will present for flavour singlet $0^{++}$ mesons largely use the last generation of lattice QCD calculations that are quenched, or dynamical QCD calculations with pion masses above 500 MeV [9,10].
1.3. Flavour singlet $0^{++}$ mesons

In nature and unquenched lattice QCD calculations glueball and $\bar{q}q$ operators will mix, so glueballs do not exist as separate particles. The lightest flavor singlet $0^{++}$ mesons listed in the PDG\cite{PDG} are: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, so it is expected that $0^{++}$ glueball degrees of freedom will contribute to some of these mesons. There are claims that the $f_0(980)$ and $f_0(600)$ may be molecules or tetraquark \cite{tetraquark}, so may not couple to $\bar{q}q$ interpolating operators in lattice QCD calculations. Morningstar and Peardon \cite{MorningstarPeardon} obtained $M_{0^{++}} = 1730(50)(80)$ MeV for the mass of the lightest $0^{++}$ glueball from quenched QCD. Chen et al. \cite{Chen} recently found $M_{0^{++}} = 1710(50)(80)$ MeV from quenched QCD. The quark model predicts that there should only be two $0^{++}$ mesons between 1300 and 1800 MeV, so if the mixing between the glueball and $\bar{q}q$ operators is weak, then the $0^{++}$ glueball is hidden inside the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ mesons. Klempt and Zaitev have recently argued that the experimental data don’t support that the $f_0(1370)$ is a separate resonance \cite{KlemptZaitev}.

Weingarten and Lee \cite{WeingartenLee} used quenched lattice QCD to estimate the mixing matrix between the glue and $\bar{q}q$ states. Weingarten and Lee \cite{WeingartenLee} predicted that the $f_0(1710)$ meson was $74(10)$\% $0^{++}$ glueball, and hence the mixing between the $0^{++}$ glueball and $\bar{q}q$ states is weak.

There are claims \cite{Wong} that continuum phenomenology is more consistent with a sizable contributions from the $0^{++}$ glueball to the $f_0(600)$ and $f_0(980)$ mesons, so it is important to study the effect of sea quarks on the masses of the state created by glueball interpolating operators.

The SESAM collaboration studied the glueball spectrum on unquenched lattices \cite{MacNeille}. McNeile and Michael studied the light $0^{++}$ spectrum with unquenched QCD \cite{MacNeilleMichael} at a coarse lattice spacing and found the mass of the lightest flavour singlet $0^{++}$ meson was very light. Using $0^{++}$ glueball operators, Hart and Teper \cite{HartTeper} found that

$$M_{0^{++}Unquenched} = 0.85(3)M_{0^{++}Quenched}$$

at a fixed lattice spacing of 0.1 fm. The UKQCD collaboration \cite{UKQCD} separately studied $0^{++}$ glue-
ball and $0^{++}$ $\overline{q}q$ operators on improved staggered gauge configurations, however higher statistics and an analysis similar to the one by Bernard et al. [25] is required.

Unfortunately, the existing unquenched lattice QCD calculations of the flavour singlet $0^{++}$ mesons don’t have the range of lattice spacings where a continuum extrapolation can be attempted. In quenched QCD it was found that the lattice spacing dependence of the mass of the $0^{++}$ glueball was strong. The use of a Symanzik improved gauge action by Chen et al. [8] and, Morningstar and Peardon [7], produced a smaller dependence on the lattice spacing of the scalar $0^{++}$ glueball mass, than for calculations that used the Wilson plaquette action. This is relevant to unquenched calculations, because any suppression of the mass of the flavour singlet $0^{++}$ mass may be due to lattice spacing effects.

The SCALAR collaboration [26], used unquenched lattice QCD, with Wilson fermions and the Wilson gauge action, to study the $0^{++}$ mesons. At a single lattice spacing $a \sim 0.2$ fm, with $\overline{q}q$ interpolating operators only, they obtain $m_{\overline{q}q} \sim m_\rho$. The lattice spacing dependence of this result needs to be quantified.

In unquenched QCD, both glue and $\overline{q}q$ states will couple to singlet $0^{++}$ mesons, so it is better to do a variational fit with both types of operators as basis interpolating operators. The variational technique analysis of the singlet $0^{++}$ mesons was done by Hart et al. [27]. A combined fit to $0^{++}$ glue and $\overline{q}q$ interpolating operators with two types of spatial smearing sources was done. The calculation used the non-perturbative improved clover action at a single lattice spacing [9]. Configurations from CP-PACS [10] with the Iwasaki gauge action and tadpole improved clover action were also used in the analysis, because this calculation should be less affected by lattice artifacts. A summary plot of the results, in units of $r_0$ ($1/r_0 \sim 400$ MeV) is in figure 1 (updated from [27]). The data with the bursts and squares (with the pion masses written near them) in figure 1 shows an additional reduction of the mass of the $0^{++}$ state over the pure glueball operators, as used by Hart and Teper [23].

Mathur at al. [28] recently claimed to get a result for the mass of the $f_0(600)$ ($\sigma$) from quenched lattice QCD with pion masses as low as 180 MeV. Using the interpolating operator in equation $2$ they obtain $m_{f_0(600)} \sim 550$ MeV. Mathur et al.’s [29] calculation is discussed in slightly more detail in [2]. The effect of sea quarks on this calculation needs to be quantified.

In [27] an attempt was made to compute the decay width for $f_0$ decay to two pions. Unfortunately much higher statistics will be required to obtain an accurate value for that width. Recent unquenched lattice QCD calculations have light enough quarks that the two meson decays of some scalar mesons are allowed and some preliminary evidence for $0^{++}$ decay has been presented [30].

Perhaps a more mundane issue with the improving unquenched lattice QCD calculations of flavour singlet quantities is just increasing the statistics in the Monte Carlo estimate. In table 2 I show the number of measurements done in some quenched and unquenched calculations of $0^{++}$ glueballs. This type of comparison between number of estimates, can be misleading because it depends on the autocorrelation times. The qualitative message from table 2 is that unquenched calculation of the flavour singlet $0^{++}$
Table 2
Comparison of statistics between quenched and unquenched glueball calculations.

| Group                  | Method     | statistics |
|------------------------|------------|------------|
| Morningstar et al. [7]  | quenched   | 6360       |
| Chen et al. [8]         | quenched   | 10000      |
| Hart et al. [27]        | unquenched | 500        |

mesons need at least 10 times as much statistics as currently used. Lattice QCD calculations with higher statistics are definitely required to study unquenching on the mass obtained from $2^{++}$ glueball interpolating operator [23]. There is a high statistics unquenched lattice QCD calculation in progress of light flavour singlet mesons, that has generated 6000 configurations at a single value of the lattice spacing [24, 31].

Chen et al. [8] and Meyer [32] have recently computed the matrix element of the energy momentum tensor with glueball states in quenched QCD calculations. Meyer [32] uses the computed matrix elements to study $J/\psi$ radiative decay.

2. Conclusions

There is “some” evidence that the flavour singlet $0^{++}$ $\eta$ and glueball interpolating operators, in unquenched lattice QCD calculations, are coupling to states around or below 1 GeV [27]. Although a continuum extrapolation is required for definite results. Unquenched lattice QCD calculations with $0^{++}$ tetraquark interpolating operators are required to clarify the composition of the lightest $0^{++}$ resonance.

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