Exceptional Configurations with the Clover Action

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We study exceptional modes of both the Wilson and the clover action in order to understand why quenched clover spectroscopy suffers so severely from exceptional configurations. We show that a large clover coefficient can make the exceptional modes extremely localized and thus very sensitive to short distance fluctuations. We contrast this with the case of the Wilson action where exceptional modes correspond to large instantons. These modes are broadly extended and suffer much less from discretization errors.

1. Introduction

Exceptional configurations have recently been shown to be due to real eigenvalues of the Dirac operator occurring close to minus the bare quark masses used in spectroscopy. These real modes are the lattice counterparts of the continuum zero modes, shifted away from zero due to the additive mass renormalization. A chiral improvement of the lattice Dirac operator is expected to decrease the shift of zero-modes as well as their spread. It is thus surprising that this does not seem to be true for the simplest improvement on the Wilson action, the clover action. Although the clover term reduces the additive mass renormalization as compared to the Wilson action, the problem with exceptional configurations appears to be more severe for the clover than for the Wilson action; at least with the non-perturbatively determined value of the clover coefficient $c_{sw}$.

In the present paper we show why this happens by studying how the real modes of the Wilson Dirac operator change when the clover term is gradually turned on. We also make a first step towards establishing a connection between exceptional modes and instantons both for the Wilson and the clover case.

2. Wilson Eigenmodes and Instantons

Contrary to what happens in the continuum, the lattice Wilson Dirac operator does not have an exact zero-mode in the presence of an instanton. Instead, it has a definite chirality real eigen-mode appearing in the physical branch of the spectrum. On smooth one-instanton configurations the location of the real eigenvalue depends on the size of the corresponding instanton. Large instantons have modes closer to zero, small ones have modes farther away from zero, in the direction of the real doublet modes.

A similar connection between the instanton size and the location of the corresponding fermionic real mode can be found also on Monte Carlo generated gauge configurations. In Fig. we show the instanton size versus the corresponding fermionic eigenvalue both for smooth instantons and instantons identified on real Monte Carlo generated configurations at $\beta = 6.0$ on $12^4$ lattices. The instanton sizes were measured using the method of Ref. and the real modes were extracted exactly as in Ref. To find the corresponding instanton for each real mode, we compared the quark density of the given mode to the profile of the charge density. Fig contains results only from uniquely identifiable modes.

3. Eigenmodes with the Clover Action

3.1. Smooth instantons

We have seen that on fine enough lattices with the Wilson action there is a strong correlation between the instanton size and the eigenvalue of the corresponding fermionic real mode; (near) exceptional modes correspond to large instantons. We shall now explore what happens in the presence of the clover term. We start by looking at smooth
Figure 1. The instanton size (in units of the lattice spacing) vs. the eigenvalue of the corresponding fermionic real mode on smooth instantons (crosses) and on real Monte Carlo generated configurations at $\beta = 6.0$ (squares). The horizontal line indicates $-m_c$ for the $\beta = 6.0$ quenched ensemble.

In Fig. 1 we plot how the fermion eigenvalues associated to instantons of various sizes change as a function of the clover coefficient. For better legibility in the figure we include only the range of $c_{sw} \geq 1.0$. For the Wilson action ($c_{sw} = 0$) the eigenvalues corresponding to instantons of size $1.2 \leq \rho \leq 2.5$ are spread between 0.19 and 0.86.

As $c_{sw}$ increase from zero to the tree level value, $c_{sw} = 1$, all the modes move closer to zero and their spread decreases. If $c_{sw}$ is further increased, the modes start to pass through zero and also the trajectories corresponding to instantons of different sizes start to cross one another. By $c_{sw} = 1.4$ the order of all the eigenvalues for instanton sizes $\geq 1.5a$ has been reversed. As $c_{sw}$ is increased, modes of smaller and smaller instantons cross over to the other side of the distribution.

3.2. Monte Carlo Configurations

We studied an ensemble of $\beta = 5.7$ $6^3 \times 16$ lattices. We followed how the wave function of a few typical very exceptional modes changed with the clover coefficient starting from the Wilson action ($c_{sw} = 0.0$) up to the non-perturbatively determined value ($c_{sw} = 2.25$).

With the Wilson action we found that the quark density in modes close to $-m_c$ has an extended broad peak and for modes towards the doublers the peaks get narrower and sharper. This is in complete agreement with [7].

Let us now follow what happens to the wave function of a typical exceptional mode as the clover term is gradually turned on. Up to $c_{sw} = 1.0$ the main peak of the quark density remains at the same location. It only slowly gets narrower: at $c_{sw} = 1.0$ its width is 2.3. Increasing $c_{sw}$ further, another peak appears and its relative significance increases with the clover coefficient. By
$c_{sw} = 2.25$ the wave function is completely concentrated on a very sharp peak (of width $\approx 1$). The wave function of this $c_{sw} = 2.25$ exceptional mode is very similar to that of a Wilson mode lying halfway between the physical and the doubler branch.

We can now easily describe qualitatively how the exceptional modes change with the clover coefficient. The Wilson modes can be roughly thought of as being concentrated on single instantons. As the clover term is turned on, the zero modes corresponding to different instanton sizes get closer and the quark wave function spreads over several topological objects. We saw similar behavior on a subset of the Fermilab configurations given to us by H. Thacker. If $c_{sw}$ is further increased, the zero modes separate again, but this time the well localized modes (corresponding to smaller instantons) have smaller eigenvalues. The relative importance of the broader peaks decreases and the mode can become entirely concentrated in a very sharp peak. We want to emphasize however that on a given ensemble of gauge configurations at a given value of $c_{sw}$ different (near) exceptional modes can look qualitatively very different. Some of the modes are entirely concentrated in a sharp peak — typically these are the most exceptional ones — some have several peaks of various widths.

The sharply peaked modes look very much like (near) doubler modes of the Wilson action and the corresponding eigenvalues and the way they change with the clover term can be very sensitive to the fluctuations on the shortest distance scale. This is the reason why the clover coefficient cannot be optimized to minimize the spread of the real eigenvalues. On the other hand, this can be done if the shortest scale fluctuations are “tamed”. This is the idea behind our recently proposed new fermion action with fat gauge links and an optimized clover coefficient $[3]$. The role of the fattening is to make the fermion modes less sensitive to the short distance fluctuations. We checked the wave functions of the most exceptional modes of our fat link action and in our sample we never encountered any “anomalously” sharply localized mode.

We expect that minimizing the spread of real eigenvalues will also improve the situation with exceptional configurations. This is indeed what happens. On a set of 40 $6^3 \times 16$ Wilson $\beta = 5.7$ configurations we determined all the exceptional modes both for our optimized fat link action and the thin link clover action with $c_{sw} = 2.25$. For the fat link action all the modes occurred at pion masses $m_\pi < 0.3$ which corresponds to $m_\pi/m_\rho \leq 0.38$. whereas the clover action had five exceptional modes above $m_\pi = 0.3$ at $m_\pi = 0.42, 0.53, 0.39, 0.30, 0.62$ (in lattice units).

4. Conclusions

In the present paper we gave an explanation of why quenched clover spectroscopy on coarse lattices suffers so severely from exceptional configurations. Our main result is that increasing the clover coefficient can make exceptional modes extremely localized and thus very sensitive to short distance fluctuations. Therefore our results are quite worrisome for simulations done on coarse lattices with the clover action. It would be worth repeating this study on a finer lattice where instantons can be explicitly identified and correlated with peaks of the eigenmodes.

This work was supported by the US Department of Energy.

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