Body Fixed Frame, Rigid Gauge Rotations and Large N Random Fields in QCD

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Abstract

The "body fixed frame" with respect to local gauge transformations is introduced. Rigid gauge "rotations" in QCD and their Schrödinger equation are studied for static and dynamic quarks. Possible choices of the rigid gauge field configuration corresponding to a nonvanishing static colormagnetic field in the "body fixed" frame are discussed. A gauge invariant variational equation is derived in this frame. For large number N of colors the rigid gauge field configuration is regarded as random with maximally random probability distribution under constraints on macroscopic-like quantities. For the uniform magnetic field the joint probability distribution of the field components is determined by maximizing the appropriate entropy under the area law constraint for the Wilson loop. In the quark sector the gauge invariance requires the rigid gauge field configuration to appear not only as a background but also as inducing an instantaneous quark-quark interaction. Both are random in the large N limit.

1 Introduction

Studies of non perturbative aspects of dynamics of non abelian gauge fields will continue to remain one of the focuses of theoretical activities. These fields appear at all
levels of the "elementary" interactions and even begin to enter at a more phenomenological macroscopic level in condensed matter systems. Quantum Chromodynamics represents a prime example of a strongly coupled theory with non abelian gauge fields. Despite many efforts, e.g. instantons [1], large N expansion [2, 3, 4, 5, 8], lattice gauge theory and strong coupling expansion [7, 9], topological considerations [10, 11], QCD sum rules [12], "spaghetti" vacuum [13], light cone approach [14], explicit color projection [15], and others [16, 17, 18], the quantitative understanding of the basic QCD features is still far from satisfactory. A sustained effort with different angles of attack is clearly in order with the hope that accumulated qualitative experience will finally lead to the development of quantitative calculational tools. This paper is a contribution to this effort.

Invariance under local gauge transformations is the most important feature of a non abelian gauge theory. In the framework of the Hamiltonian formulation of QCD I wish to explore the consequences of this invariance using some general methods common to molecular and nuclear physics. I wish to define an appropriate generalization of the body-fixed (intrinsic, rotating) frame formalism in the context of the local gauge transformations. After doing so one can attempt to separately investigate the dynamics of the gauge "rotations" of the frame and the intrinsic frame dynamics. These would be the analogs of rigid rotations and intrinsic vibrations in molecular and nuclear physics. Most of my interest in this paper will concentrate on the study of the "rigid gauge rotations". The study of the couplings between the "rotations" and the "vibrations" of the gauge field is deferred to future work. To avoid misunderstanding I wish to stress that by "gauge rotations" or "rotations of the gauge field" in this paper I will always mean Eq. (2.1) below, which includes the proper SU(N) rotation as well as the inhomogeneous "shift" term.

Perhaps the most important conceptual advantage of using the body-fixed frame associated with a given symmetry lies in the fact that one can freely approximate the dynamics in this frame without fears to violate the symmetry. In particular the use of this formalism appears to be fruitful provided there exist such a body-fixed, intrinsic frame in which the "rotational - vibrational" coupling can be considered as small. This in turn generically happens when the "rotational inertia" is much larger than the "inertia" associated with the intrinsic motion so that a variant of the Born-Oppenheimer approximation is valid. A typical situation is when the system’s ground state is strongly "deformed" away from a symmetric state. By deformation I mean absence of symmetry with respect to "intrinsic transformations", i.e. transformations in the body-fixed frame and not with respect to the transformations in "laboratory". Absence of symmetry in "laboratory" would correspond to the symmetry breakdown
which can not occur for a local gauge symmetry. Quantum mechanical examples of
deformed bodies are e.g. non spherical molecules, deformed nuclei, etc.

I do not have a priori arguments that the QCD vacuum is strongly "deformed"
in the above sense. Appearance of various QCD condensates, Ref. [12], suggests
that this may be true. The condensate wavefunction should then play a role of the
strongly deformed configuration. Another positive indication is the large N master
field concept, Ref. [1], according to which a special gauge field configuration should
exist which dominates the vacuum wave function or the corresponding functional
integral. It is expected, however, that the master field is not simply a fixed classical
configuration. It should rather be regarded as a statistical distribution allowing to
calculate quantities which are analogous to macroscopic thermodynamic quantities in
statistical physics, i.e. such that their fluctuations are suppressed in the large N limit,
Refs. [13, 20]. Glimpses of the meaning of these vague notions were found in various
matrix models, cf. Refs. [3, 8, 19, 20], and, e.g. in 1+1 dimensional QCD, Ref. [21].
If this viewpoint is correct then suitably chosen rigidly rotating "deformed" gauge
field could play a role of the master field provided one understands in which sense it
should also be statistical. The following formalism will clarify some of these issues
and provide a general framework in which they could be further discussed.

Works in the spirit of our study have already appeared in the past, cf. Refs.
[23, 24, 25, 26] and the analogy with various types of rotational motion is frequently
used in QCD. In this sense the present study is a continuation of these works.

This paper is organized as following. In Section 2 I introduce the transformation
to the body-fixed frame in the context of the simplest model of the gauge rotational
motion – the rigid gauge rotor. Giving a natural definition of the rigid gauge "rotations" I proceed to determine the appropriate generalization of the standard space
rigid rotor results – the moment of inertia tensor, the "body-fixed" frame, the generators of the "body-fixed" gauge group in terms of which the character of "deformation"
can be classified, etc. Despite severe limitation on the set of the allowed gauge field
configurations the model is gauge invariant. I work out in Section 3 the quantum
mechanics of the model. As with the space rotor the generators of the "laboratory"
and "body-fixed" gauge transformations provide a complete set of quantum numbers
for the wave functionals of the model. The vacuum has zero energy and is the most
disordered state. Higher states correspond to the presence of very heavy, i.e. static
quarks and antiquarks in the system. As an important example I consider the wave
function and the corresponding Schrödinger equation for a pair of static quark and
antiquark. This and other similar equations in the model are simple matrix equation
with the inverse "moment of inertia" determining the interaction between the color
sources and depending on the assumed rigid gauge configuration which plays the role of the "free parameter".

In Section 4 I discuss the meaning of the results obtained so far and possible choices of the rigid gauge field which physically represents a non vanishing colormagnetic field in the body-fixed frame. In the ground state this frame does not "rotate" but has random orientations in local color spaces at every space point. Introduction of static quarks forces the frame to "rotate" quantum mechanically at the points where the quarks are situated. The energy eigenvalues of these "rotations" are the energies of the quantum states of the colorelectric field generated by the quarks. The propagator of this field is the moment of inertia of the model and depends explicitly on the assumed configuration of the rigid static colormagnetic field. For zero field the propagator is a simple Coulomb while for a uniform field diagonal in color the propagator behaves asymptotically as a decaying Gaussian. The so called dual Meissner effect picture of the confinement, Ref. [27], could be implemented if a configuration of the rigid colormagnetic field is found which "channels" the colorelectric field and makes its propagator effectively one dimensional. It turns out that the creation of such a magnetic "wave-guide" is connected with existance of a zero eigenvalue of a certain operator in the model.

Since the quark color degrees of freedom are treated quantum mechanically the model allows for a possibility that confinement of fundamental representations does not automatically mean confinement of higher representations. I discuss this possibility and derive a variational equation for the rigid field. This equation is fully gauge invariant.

In Section 5 expecting that rigid gauge rotations should be relevant for the master field concept I study the model in the large N limit. Any candidate for the master field must be allowed to undergo free "gauge rotations" which can not be frozen by this limit and should induce an interaction between the quarks. Going to the body fixed frame of these rotations I regard the rigid gauge field configuration as random and introduce a natural requirement that it is least biased under constraints that it should reproduce gauge invariant quantities which can be regarded macroscopic-like in the large N limit. This means that it should be maximally random under these constraints. In order to make these ideas explicit I discuss in some detail the case of the uniform colormagnetic field. Such configuration in QCD was already discussed in the past, Refs. [28, 13] but it seems that its appearance in the interaction is a novel feature of the model. The detailed form of this interaction depends on the differences of the color components of the magnetic field. It is not confining for any finite number of colors. For $N \rightarrow \infty$ I assume that the form of the density of the color components of
the field is known. In 2+1 dimensions I choose it such that it gives area law for space oriented Wilson loops. I treat then the entire distribution of these components as a joint distribution of their probabilities and regard the adopted ”single component” density as an analog of a macroscopic quantity that must be reproduced by this joint distribution. I postulate that it must otherwise be maximally random, i.e. must have the maximum entropy (minimum information content) under suitable constraints. In this way I derive the maximally random distribution for this model. I discuss its relation to the large N limit of the Schrödinger equation for the static quark–antiquark system. I also give possible generalizations of this development to 3+1 dimensions.

In Section 6 I include dynamical quarks and show that the rigid gauge rotor limit corresponds exactly to the limit in QED in which only the instantaneous Coulomb interaction between the charges is retained. The major difference in QCD is that in this limit the quarks not only interact instantaneously via a more complicated interaction controlled by the rigid gauge field configuration, but at the same time are also found in a static colormagnetic field induced by this configuration. This dual appearance of the rigid field is a consequence of the gauge invariance and in the large N limit is apparently the way the master field should enter the quark sector of the theory. According to the ideology developed in Section 5 both the field in which the quarks move and their interaction should be considered as random in the large N limit. The random interaction between the quarks opens interesting possibilities to discuss the relationship between confinement and localization.

The body fixed Hamiltonian with dynamical quarks is gauge invariant. Its invariance with respect to global symmetries however is not guaranteed for an arbitrary choice of the rigid gauge configuration. I discuss possible variational approaches to determine this configuration and derive an analogue of the Hartree-Fock equations for the model.

In the rest of the Introduction I will establish my notations, cf., Ref.[29]. I consider the QCD Hamiltonian in d=3 space dimensions in the $A^0 = 0$ gauge,

$$H = \frac{1}{2} \int d^3x [(E^i_a(x))^2 + (B^i_a(x))^2] + \int d^3x q_\gamma^+(x) [\alpha^i \left( p^i - g A^i_a(x) \frac{\gamma^a}{2} \right) + \beta m] q_\delta(x).$$  \hspace{1cm} (1.1)

with

$$B^i_a(x) = \epsilon_{ijk} (\partial_j A^k_a + g f_{abc} A^j_b A^k_c),$$  \hspace{1cm} (1.2)

$i,j,k = 1,...,3$; $\gamma,\delta=1,...,N$; $a=1,...,N^2-1$ for SU(N) gauge group and $f_{abc}$ – the structure constants of the SU(N). Dirac and flavor indices are omitted and the summation convention for all repeated indices is employed here and in the following.
The gluon vector potential $A_i^a(x)$ and minus the electric field $-E_i^a(x)$ are canonically conjugate variables,

$$[E_i^a(x), A_j^b(y)] = i\delta_{ab}\delta_{ij}\delta(x - y), \quad (1.3)$$

and the quark fields obey the standard anticommutation relations.

The Hamiltonian (1.1) is invariant under the time independent gauge transformations. Using the matrix valued hermitian fields

$$A_{\alpha\beta}^i(x) = A_i^a(x)\lambda_{a\alpha\beta}\frac{\lambda}{2}, \quad E_{\alpha\beta}^i(x) = E_i^a(x)\lambda_{a\alpha\beta}\frac{\lambda}{2}, \quad (1.4)$$

where $\lambda^a$ are the SU(N) generators with the properties

$$[\lambda^a, \lambda^b] = 2if_{abc}\lambda^c; \quad \{\lambda^a, \lambda^b\} = \frac{4}{N}\delta_{ab} + 2d_{abc}\lambda^c;$$

$$Tr\lambda^a\lambda^b = 2\delta_{ab}; \quad \lambda_{a\beta}\lambda^a_{\gamma\delta} = 2[\delta_{a\beta}\delta_{\gamma\delta} - \frac{1}{N}\delta_{a\beta}\delta_{\gamma\delta}] \quad (1.5)$$

one can write the gauge transformation as

$$A^i \rightarrow SA^iS^+ + \frac{i}{g}S\partial^iS^+; \quad E^i \rightarrow SE^iS^+; \quad q \rightarrow Sq \quad (1.6)$$

where $S(x)$ are time independent but $x$ - dependent unitary $N \times N$ matrices, elements of the SU(N) group. The generators of this transformation

$$G_a(y) \equiv G^A_a(y) + G^q_a(y), \quad (1.7)$$

$$G^A_a(y) = \partial_iE_i^a(y) + gf_{abc}A^i_b(y)E^i_c(y), \quad G^q_a(y) = -gg^+(y)\lambda^a\frac{\lambda}{2}q(y) \quad (1.8)$$

are conserved,

$$\frac{\partial G_a(x)}{\partial t} = i[H, G_a(x)] = 0 \quad (1.9)$$

and it is consistent to impose the Gauss law constraints

$$G_a(x)|\Psi >= 0 \quad (1.10)$$

for all physical states. Although $G_a(x)$ do not commute, their commutators

$$[G_a(x), G_b(y)] = gf_{abc}\delta(x - y)G_c(x) \quad (1.11)$$

allow to set them all simultaneously zero.
2 Rigid Gauge Rotor.

In this section I will discuss the rigid gauge "rotations". Classically I define them as gauge field configurations of the type

\[ A^i(x,t) = U(x,t) a^i(x) + \frac{i}{g} U(x,t) \partial^i U^+(x,t) \] \hspace{1cm} (2.1)

where \( a^i(x) \) are t-independent, fixed as far as their x-dependence is concerned, "rigid" fields which I do not specify and leave them arbitrary for the moment. Eq.(2.1) is the simplest example of the transformation to the "body fixed" frame of the local gauge symmetry in which I have assumed that the dynamics of the field in this frame is very stiff so that the field can be approximately replaced by its static average. In general \( a^i \) is of course dynamical but should be viewed as constrained since \( U(x,t) \) already contains a third of the degrees of freedom. For non rigid \( a^i \) there is no obvious choice of the body-fixed frame and it can be constrained in a variety of ways, say, \( a^3 = 0 \) (axial gauge), \( \partial_i a^i = 0 \) (Coulomb gauge), etc. In our language these different gauge fixings correspond to different "rotating" frames. Since they are "non inertial" the dynamics will look very differently depending on the choice of the frame and different fictitious forces, the analogue of Coriolis and centrifugal forces, will be present. I am planning to discuss these issues elsewhere.

With the anzatz (2.1) the covariant derivatives are

\[ D^i = \partial^i - ig A^i = U(x,t) d^i(x) U^+(x,t), \] \hspace{1cm} (2.2)

with fixed, rigid

\[ d^i(x) = \partial^i - ig a^i(x). \] \hspace{1cm} (2.3)

Inserting (2.2) in the Hamiltonian (1.1) one finds that the gauge invariant potential term \( \sum_{i,a} (B^a_i(x))^2 \sim \sum_{i,j} Tr[D^i, D^j]^2 = \sum_{i,j} Tr[d^i, d^j]^2 \) is independent of the \( U \)'s, i.e. it is fixed, nondynamical in this model. The dynamics of the gauge field is governed by the kinetic energy, i.e. the term with the electric field in (1.1). Using \( \partial_0(U \partial_t U^+) = (i/2) U(\partial_0 \omega) U^+ \) with \( \omega = 2iU^+ \partial_0 U \) one finds

\[ -E^i = \partial_0 A^i = \frac{1}{2g} U[\omega, d^i] U^+, \] \hspace{1cm} (2.4)

and therefore the kinetic energy in (1.1) is

\[ \frac{1}{4} \int d^3 x Tr(\partial_0 A^i)^2 = -\frac{1}{16g^2} \int d^3 x Tr \left( \omega[d^i, [d^i, \omega]] \right) \] \hspace{1cm} (2.5)
where I have disregarded surface terms, ignoring for the moment possible non-vanishing fields at infinity, non-trivial topologies and other global issues (cf. below).

The double commutator in (2.5) with the summation over all indices, $x, i$ and the color is the straightforward generalization of the familiar double vector product summed over all particles indices in the moment of inertia tensor appearing in the kinetic energy of rigid space rotations of system of particles with fixed relative positions. Following this analogy the energy (2.5) of the rigid gauge rotations can be written

$$E_{rot} = \frac{1}{4} \int d^3 x Tr(\omega I \omega)$$

(2.6)

where the moment of inertia is defined as a differential matrix operator such that

$$I \omega = - \frac{1}{4g^2} \left[ d^i, [d^i, \omega] \right] =$$

$$= - \frac{1}{4g^2} \left( \partial_i - ig \partial_i a^i, \omega - 2i \Theta [a^i, \partial_i \omega] - g^2 [a^i, [a^i, \omega]] \right).$$

(2.7)

To obtain the corresponding Hamiltonian one can use the gauge field part $G^A$ of the generators (1.8). Using Eqs. (2.1), (2.4) and the definition (2.7) one finds

$$G^A = [\partial^i - igA^i, E^i] = \frac{1}{2g} U \left[ d^i, [d^i, \omega] \right] U^+ = -2gU(\omega)U^+. $$

(2.8)

Defining the gauge generators in the rotating frame $\hat{G} = U^+ G^A U$, expressing $\omega = -I^{-1} \hat{G}/2g$ from (2.8) and substituting in (2.6) one finds the Hamiltonian of the rigid gauge rotor

$$H_{rot}^A = \frac{1}{16g^2} \int d^3 x d^3 y Tr \hat{G}(x) I^{-1}(x, y) \hat{G}(y) = \frac{1}{4g^2} \int d^3 x d^3 y \hat{G}_a(x) I^{-1}_{ab}(x, y) \hat{G}_b(y).$$

(2.9)

where $I^{-1}_{ab}(x, y) = (1/4) Tr(\lambda^a I^{-1}(x, y) \lambda^b)$ is proportional to the inverse of the operator $-d^i_{ac} d^c_{cb}$ with $d^i_{ab} = \partial_i \delta_{ab} - g f_{abc} a^c$ and I assumed that this operator does not have zero eigenvalues. In a more careful way of handling fields at infinity one should avoid the integration by parts in (2.5). The inverse "moment of inertia" operator is then replaced by a less transparent

$$K_{aa'}(x, x') = \int d^3 y \left[ d^i_{ba'}(y) I^{-1}_{ca}(y, x) \right] \left[ d^i_{bc}(y) I^{-1}_{ca'}(y, x') \right].$$

(2.10)

Most of the following results remain valid for both forms of this operator.
The meaning of the preceding expressions is quite obvious. They are the field-theoretic generalization of the standard rigid rotor results. The unbroken local gauge symmetry of QCD means that there are free SU(N) color gauge "rotations" at every space point. Expression (2.9) shows that the "rotations" at different points as well as around different color axes are coupled via the non diagonal elements of the moment of inertia "tensor" $I_{ab}(x, y)$ in the manner similar to the coupling between rigid rotations around different space axes in systems of particles.

It does not seem to be useful to diagonalize the operator $I^{-1}$ in (2.9). The standard diagonal form of the rigid rotor Hamiltonian, i.e., $H = \frac{1}{2} \sum_a L_a^2 / I_a$ can be usefully achieved only in the case of rotations corresponding to a single SU(2) group to which the familiar rigid space rotations belong. Diagonalizing the moment of inertia in the case of higher groups will introduce combinations of the generators multiplied by matrices of orthogonal rotations. These in general will not have the group commutation relations. Already for a single SU(3) the group O(8) of rotations in the adjoint space needed in order to diagonalize the moment of inertia is much larger than SU(3).

The actual values of the moment of inertia depend on the rigid configuration $a^i(x)$ of the gauge field via the expression (2.7). This comprises the "free parameter" of the rigid gauge rotor model. For abelian theory or alternatively in the limit $g \to 0$ the inverse of $I(x, y)$ appearing in Eq.(2.9) is just the Coulomb propagator. In the opposite large $g$ or long wavelength limit $I(x, y)$ becomes a local tensor given by the last term in (2.7) which is obviously the SU(N) generalization of the moment of inertia expression.

An important feature of the Hamiltonian (2.9) is that despite the severe limitation of the allowed gauge field configurations imposed by (2.1) it remains gauge invariant. This is because (2.9) depends on $\hat{G}$ rather than $G^A$. Under a gauge transformation $U \to SU$, $G^A$ transforms as $SG^A S^+$ so that $\hat{G}(x)$ and therefore $H^A_{\text{rot}}$ stay invariant. The gauge invariance of (2.9) is the simplest illustration of the usefulness of the introduction of the body fixed frame. One can freely approximate the dynamics in this frame without fears of violating the symmetry with respect to which the frame has been defined, i.e. the local gauge symmetry in the present case.

Consider another transformation, $U \to US$. Referring to Eq.(2.1), one can interpret this transformation either as the change of $U$ i.e. the transformation of the intrinsic frame with respect to the rigid "shape" $a^i$ or as the change of $a_i$, $a^i \to S a^i S^+ + \frac{i}{g} S \partial^i S^+$, i.e. the transformation of the intrinsic "shape" with respect to the intrinsic frame. Such transformations obviously form a group of local SU(N) gauge transformations which I will call the intrinsic or "body fixed" gauge group to
distinguish it from the "laboratory" gauge group of the ordinary gauge transformations. According to two different interpretations of the intrinsic gauge transformations given above one has two options. One option is to regard \( \hat{G}'s \) as the generators of the intrinsic group. They act on the dynamical variables \( U \) but they have a disadvantage in that the "laboratory" group is not completely independent of such an intrinsic group, e.g. they both have identical Casimir operators. Another option is to formally introduce operators which gauge transform the intrinsic variables \( a^i \). They will have the same form as \( G^A \)'s but with \( a^i \) replacing \( A^i \). Defined in this way the intrinsic group will be completely independent of the "laboratory" gauge group but will act on nondynamical variables which do not appear in the wavefunctions. Convenience should dictate which one to use.

The above introduction of the intrinsic vs "laboratory" gauge groups is obviously quite general with e.g. the definition of \( \hat{G}(x) \) being independent of the rigid rotor restrictions set by fixing \( a^i \) to be nondynamical in \( (2.1) \). Unlike the local gauge symmetry in "laboratory", the symmetry in the "body fixed" frame can be broken. E.g., the gauge invariant Hamiltonian \( (2.9) \) is in general not invariant under the transformations of the intrinsic gauge group. This is a simple example of the situation to which I referred earlier as a possible existence of "deformation" vs impossibility of the symmetry breakdown in the context of non abelian local gauge theory. The character of the deformation can be classified using the intrinsic gauge group, e.g. in classification of possible "deformed shapes" of the rigid gauge rotor \( (2.9) \) by the transformation properties of the moment of inertia \( I_{ab}(x,y) \) under this group. Here I obviously adopt the second interpretation of the intrinsic group. The invariance of \( I_{ab}(x,y) \) under all intrinsic transformations would be analogous to the spherical rotor limit in the space rotation case. The invariance under a continuous subgroup of the intrinsic group is the analog of the axial symmetric rotor, etc. Discrete intrinsic subgroups should also be considered.

Consider a rigid gauge configuration \( a'' \) which is a gauge transform of \( a' \), \( a'' = S(a' + (i/g)\partial_t)S^+ \). The Hamiltonian \( (2.9) \) will have the same form with the same moment of inertia but with \( \hat{G} \) replaced by \( S^+ G S \). The eigenvalues of this transformed Hamiltonian will not change and will therefore depend only on gauge invariant combinations of the rigid \( a^i \), i.e. on the Wilson loop variables \( TrPexp(ig \oint a'dx_i) \).
3 Static Quarks

So far I have discussed the rigid gauge rotor limit of only the first term in (1.1). The resulting $H_{\text{rot}}^A$ is relevant for the discussion of very heavy quarks. They can be considered static as far as their translational motion is concerned. They still have a wavefunction describing the motion of their color degrees of freedom. Because of (1.10) this motion is coupled to the "rotations" of the gauge field which I will treat using (2.9).

In the limit of $m \to \infty$ the quark kinetic energy term $q^+ \vec{\alpha} \vec{p} q$ and the quark color current coupling $q^+ \vec{\alpha} \lambda^a q$ in Eq.(1.1) can be neglected and the resulting Hamiltonian decouples into a part containing the gauge field and another containing the quarks, $H = H_A + H_q$, where $H_A$ is the first term in (1.1) and $H_q = m \int d^3 x q^+(x) \beta q(x)$. The coupling appears only via the Gauss law constraint, Eq.(1.10). The wave function can not be taken as a product $\Psi = \Psi(q)\Psi(A)$ but should be a local color singlet. In the representation in which

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$q_\alpha(x) = a_\alpha(x) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_\alpha^+(x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with

$$\{a_\alpha(x), a_\beta^+(y)\} = \delta_{\alpha\beta} \delta(x-y)$$

$$\{b_\alpha(x), b_\beta^+(y)\} = \delta_{\alpha\beta} \delta(x-y), \text{etc...}$$

the eigenfunctions of $H_q$ are trivially written down. Consider, e.g.,

$$|\Psi(q) > \equiv |\text{vac}(q) > = |0 >, \quad (3.1)$$

$$|\Psi(q) > \equiv |x_0, \alpha > = a_\alpha^+(x_0)|0 > \quad (3.2)$$

$$|\Psi(q) > \equiv |x_0, \alpha; y_0, \beta > = a_\alpha^+(x_0)b_\beta^+(y_0)|0 > \quad (3.3)$$

These wave functions describe respectively zero quarks, one static quark at $x_0$ with color component $\alpha$ and a static quark - antiquark pair at $x_0$ and $y_0$. It is easy to form local color singlets with these wave functions. For e.g. the quark-antiquark pair it is

$$|\Psi > = \sum_{\alpha\beta} \Psi_{x_0,\alpha;y_0,\beta}(A)|x_0, \alpha; y_0, \beta >$$

(3.4)
with the wave functional $\Psi_{x_0,\alpha;0,\beta}(A)$ satisfying

$$G_a(x)\Psi_{x_0,\alpha;0,\beta}(A) = g\delta(x-x_0)\lambda_{\alpha\alpha}'2\Psi_{x_0,\alpha';0,\beta}(A) + g\delta(x-y_0)\bar{\lambda}_{\beta\beta}'2\Psi_{x_0,\alpha;0,\beta'}(A)$$

where $\bar{\lambda}_{\alpha\beta} = -\lambda^*_{\alpha\beta}$. The wave functional of the gauge field should be a singlet at every point in space except at the position of the quarks where it should transform as $N$ and $\bar{N}$ multiplets of $SU(N)$. This constraint together with the Schrödinger equation

$$H_A\Psi_{x_0,\alpha;0,\beta}(A) = E\Psi_{x_0,\alpha;0,\beta}(A)$$

completely defines the problem for the gauge field.

In the rigid gauge rotor limit $H_A$ is given by Eq. (2.9). The wavefunctions of this Hamiltonian are general functionals $\Psi[U(x)]$ of the $SU(N)$ matrices $U_{\alpha\beta}(x)$. Their scalar product is determined by functional integration over the $U$’s with the corresponding group invariant measure. The vacuum wave functional must obey $G_a^A(x)\Psi_{\text{vac}}[U] = 0$. This means that it is a constant independent of $U_{\alpha\beta}(x)$. Since also $\hat{G}_a(x)\Psi_{\text{vac}}[U] = 0$ the vacuum energy is zero according to (2.9). Regarding the parametrization of the $U$’s in terms of the appropriate Euler angles of the $SU(N)$ rotations at every space point, the constant $\Psi_{\text{vac}}[U(x)]$ means that all the ”orientations” of $U(x)$ at all points are equally probable, i.e. there are no correlations between the ”orientations” of the rigid gauge rotor at different points. This is as ”random” as the distribution of the $U$’s can get. The absence of correlations is the property only of the vacuum. For other states the ”orientations” of the gauge fields at different space points are correlated via the ”moment of inertia” operator.

In order to discuss the wave functions with non zero number of quarks it is sufficient to know some simple properties of the gauge generators $G_a^A(x)$ and $\hat{G}_a(x)$. Since $\hat{G}_a(x)$ are gauge scalars, they commute with $G_a^A(x)$,

$$[G_a^A(x), \hat{G}_b(y)] = 0$$

which means that together the generators of the ”laboratory” and the intrinsic gauge groups provide a complete set of commuting quantum numbers for the wave functionals $\Psi[U_{\alpha\beta}(x)]$. Indeed, since the Casimir operators for $G^A$’s and $\hat{G}$’s coincide one has e.g., for the SU(2) the $(G_a^A)^2$, $G_3^A(x)$ and $\hat{G}_3(x)$, i.e. three local commuting operator fields for the three fields of the Euler angles needed to specify the $U(x)$. In the SU(3) one has eight fields of the ”Euler angles” and eight local commuting generators made off $G^A$’s and $\hat{G}$’s – the two group Casimir operators, one Casimir
operator of an SU(2) subgroup for $G^A$'s, say $\sum_{a=1}^{3} (G^A_a)^2(x)$ and the corresponding one for the $\hat{G}$'s and respectively two pairs of the Cartan generators $-G^A_3(x), G^A_8(x)$ and $\hat{G}_3(x)$ and $\hat{G}_8(x)$. This counting continues correctly for any N, i.e. $N-1$ for the SU(N) Casimir operators, $2((N-1) + (N-2) + \ldots + 1)$ for the Casimir operators of pairs of SU(N-1)...SU(2) subgroups and 2(N-1) for the Cartan generators. Altogether there are $N^2 - 1$ local commuting operators as needed. An eigenfunction of this complete set of operators is the Wigner function $D_{L,K,K'}(U(x))$ of $U$ at a certain space point. $K$ and $K'$ are the quantum numbers of the "laboratory" and the intrinsic groups and $L$ determines the representation.

Since under an infinitesimal gauge transformations $U \rightarrow (1 + i\epsilon_a(x) A^a) U$ one can easily verify that

\[ \left[ G_a(x), U_{\alpha\beta}(y) \right] = g\delta(x-y) \frac{\lambda^a_{\gamma\beta}}{2} U_{\gamma\beta}(x) \]
\[ \left[ G_a(x), U^+_{\alpha\beta}(y) \right] = -g\delta(x-y) U^+_{\alpha\gamma}(x) \frac{\lambda^a_{\gamma\beta}}{2} \]
\[ \left[ \hat{G}_a(x), U_{\alpha\beta}(y) \right] = g\delta(x-y) U_{\alpha\gamma}(x) \frac{\lambda^a_{\gamma\beta}}{2} \]
\[ \left[ \hat{G}_a(x), U^+_{\alpha\beta}(y) \right] = -g\delta(x-y) \frac{\lambda^a_{\gamma\beta}}{2} U^+_{\gamma\beta}(x) \]

(3.8)

All the operators in the rigid gauge rotor model are functions of the G's and U's. E.g. consider the electric field operator. According to Eq. (2.4) it is

\[ E^i = -\frac{1}{4g^2} U[d^i, I^{-1}\hat{G}] U^+ \]

(3.9)

where I have expressed $\omega$ in terms of $\hat{G}$ using (2.8).

Using the relations (3.8) it is easy to write the general form of the wave functions for a single quark and for a quark–antiquark pair,

\[ \Psi_{x_0,a}(U) = U_{\alpha\gamma}(x_0)c_\gamma \]
\[ \Psi_{x_0,\alpha:y_0,\beta}(U) = U_{\alpha\gamma}(x_0)U^+_{\beta\delta}(y_0)c_{\gamma\delta} \]

(3.10)

They satisfy the conditions (3.5) following from the Gauss law with constant coefficients $c_\gamma$ and $c_{\gamma\delta}$ which give the probability amplitudes of the intrinsic quantum numbers $\gamma$ and $\delta$. They should be normalized, $\sum |c_\gamma|^2 = 1; \sum |c_{\gamma\delta}|^2 = 1$ to assure the normalization $\int d[U(x)] |\Psi[U]|^2 = 1$.
These amplitudes must be found by solving the corresponding Schrödinger equations but before describing this I wish to remark that the above form of the wave functions is valid also when the limitation of the rigid gauge rotations is relaxed and the most general gauge configurations are allowed. The parametrization (2.1) is still very useful but now with fully dynamical fields $a^i$ the variation of which should be limited only by a "gauge fixing" condition to avoid overcounting as described above. The dynamics will of course be that of the full QCD but the wave functions of the static quark and the quark–antiquark pair will have the same form (3.10). The difference will be that amplitudes $c_\gamma$ and $c_{\gamma\delta}$ will be functionals of $a^i(x)$ describing the space and color fluctuations of the "string" attached to the quark or between the quark and the antiquark. In the rigid gauge rotation case there are only color fluctuations described by constant amplitudes.

For quarks in higher representations the wave functions have the same form with $U$ replaced by the appropriate Wigner D-function. E.g. in the adjoint representation

$$
\Psi_{x_0, a}[U] = \text{Tr}(U(x_0)\lambda^a U^+(x_0)\lambda^b)c_b,
$$

etc.

I will now derive the Schrödinger equation for the string amplitudes $c_\gamma$ and $c_{\gamma\delta}$. Acting with the Hamiltonian (2.9) on (3.10), using (3.8) and the orthogonality of $U$'s with respect to the integration over the group, $\int dU U^*_{\alpha\beta} U_{\mu\nu} = \delta_{\alpha\mu} \delta_{\beta\nu}$, I find

$$
Q_{\alpha\gamma}(x_0)c_\gamma = E c_\alpha,
Q_{\alpha\gamma}(x_0)c_{\gamma\beta} + Q_{\beta\mu}^*(y_0)c_{\alpha\mu} - P_{\alpha\beta, \gamma\mu}(x_0, y_0)c_{\gamma\mu} = E c_{\alpha\beta},
$$

(3.11)

where I denoted

$$
Q_{\alpha\gamma}(x_0) = \frac{1}{4} I^{-1}_{ab}(x_0, x_0) (\lambda^a \lambda^b)_{\alpha\gamma},
$$

(3.12)

$$
P_{\alpha\beta, \gamma\mu}(x_0, y_0) = \frac{1}{2} I^{-1}_{ab}(x_0, y_0) \lambda^a_{\alpha\gamma} \lambda^b_{\mu\beta}.
$$

(3.13)

In SU(2) $Q_{\alpha\gamma}$ takes a particularly simple diagonal form, $Q_{\alpha\gamma} = \delta_{\alpha\gamma}(1/4) I^{-1}_{aa}(x_0, x_0)$. and is the eigenvalue for a single quark. For quarks in e.g. adjoint representation the lambda matrices in the expressions above are replaced by the corresponding group generators $if_{abc}$. The first two terms in the second line of (3.11) are the quark and the antiquark self energies whereas the last term is their interaction. In QCD one expects that terms like $Q$ are inflicted by the long and short distance divergences and should be properly regularized which I will assume for the rest of the paper. I will
further assume the translational invariance of $Q$, i.e. its independence of $x_0$. One can then rewrite Eq. (3.11) by transforming it to the basis in which $Q$ is diagonal. Defining its eigenvectors $Q^{(n)} = c_\alpha b^{(n)}$ and expanding $c_\gamma \beta = d_{mn} b^{(m)}_\gamma b^{(n)}_\beta^*$ one finds

$$<kl|P(x_0, y_0)|mn> d_{mn} = (E - \epsilon^k - \epsilon^l)d_{kl} \quad \text{(no sum over } k \text{ and } l) \quad (3.14)$$

where $<kl|P|mn> = P_{\alpha\beta, \gamma\mu} b^{(m)}_\gamma b^{(n)}_\mu b^{(k)}_\alpha b^{(l)}_\beta^*$. The Schrödinger equation (3.14) is $N^2 \times N^2$ matrix equation and the most interesting question of course concerns the dependence of its eigenvalues on the distance $|x_0 - y_0|$ for various possible choices of the rigid gauge field configuration $a^i(x)$ on which the matrix $P$ depends. I will address this question in the next section.

4 Choices of The Rigid Field. Mean Field Equations.

The rigid configuration if it exists in QCD must reflect the properties of the gluon condensate of the vacuum. One of the more accepted views of the QCD vacuum is that this is a condensate of non trivial topological configurations – the $Z(N)$ vortices, c.f.,[11]. Although such configurations are easily incorporated in the above formalism I was not able to overcome technical difficulties in working out a theory of their condensation.

On a heuristic level each $Z(N)$ vortex carries a unit of flux of the colormagnetic field. Condensation of the vortices presumably means that there is a non zero average of this field in the vacuum. Of course due to unbroken local gauge symmetry it must undergo free ”gauge rotations” at each space point. In the ground state this means that there are equal probabilities of all the ”orientations” yielding zero average value in the laboratory. The finite average value of the condensate field can only be ”seen” in the ”body fixed” frame and should appear in this picture in the manner similar to $a^i$ in the expression (2.1) for our rigid gauge rotations. The field strength

$$B^i(x) = U(x)b^i(x)U^+(x), \quad b^i = \frac{i}{g}\epsilon_{ijk}[d^j, d^k], \quad (4.1)$$

also averages to zero in the ground state but has a non zero value $b^i$ in the ”body fixed” frame.

Via the dynamics of $U(x)$ the anzatz (2.1) leads to colorelectric field (3.9) which propagates away from points where $\hat{G}(x)$ is non zero, i.e. from the location of static
quarks. The propagator of this field is controlled by the condensate field $a^i$ which enters the expressions for $I^{-1}$ and $d^i$. This propagator is a long range Coulomb potential for zero $a^i$ and is a Gaussian for $a^i$ corresponding to a uniform colormagnetic $b^i$ (cf., below). The screening of the propagation range of the colorelectric field in the presence of the colormagnetic "condensate" $b^i$ is reminiscent of the dual to the Meissner effect of screening of a magnetic field by the electric condensate of a superconductor. This possibility of the dual Meissner effect is of course a standard scenario for confinement in QCD. It is expected that tubes of flux of the colorelectric field are formed which connect quarks and make their energy depend linearly on the distance.

In the present formalism a way to attempt to model the formation of a confining string is to look for such a configuration of the rigid field $a^i(x)$ for which the propagator $I^{-1}$ behaves roughly speaking as one dimensional for large separations along some given line in space at the end of which quarks can be placed. This means that a sort of magnetic "wave guide" should be constructed so that the Green's function of the operator $-d^2_{ab} = -d^i_{ac}d^j_{cb} = -\left(\partial_i \delta_{ac} - gf_{acc'}a^j_{c'}\right)\left(\partial_j - gf_{cbb'}a^i_{b'}\right)$ is asymptotically $\propto |x - x'|$ along, say, one of the coordinate axes. In order to see the difficulties in finding such a configuration consider for simplicity 2 space dimensions and choose $a^1 = c(y)$ and $a^2 = 0$ with an arbitrary $c(y)$. This choice corresponds to the colormagnetic field $b(y) = \partial_y c(y)$ depending only on one coordinate $y$. The operator to invert is then

$$-(\partial_x - igc(y) \cdot F)^2 - \partial_y^2$$

where I denoted the color spin matrices $F^a_{bc} = if_{bac}$ and $c(y) \cdot F = c_a(y)F^a$. The propagator is then

$$\int_{-\infty}^{\infty} dk e^{i k(x-x')} \sum_n \frac{\chi_n(k,y)\chi_n(k,y')}{\epsilon_n(k)} = \epsilon_n(k)\chi_n(k,y).$$

In order to achieve the desired confining behaviour of the propagator the sum in (4.3) must be $\sim k^2$ for $k \to 0$. The simplest is to assume that the lowest eigenvalue of (4.4), $\epsilon_0(k)$ should vanish as $k^2$ for small $k$. However the operator in (4.4) is a sum of squares and does not have zero eigenvalues for non trivial regular $c(y)$. It is also not symmetric in $k$ for small values of $k$ but this seems to be less of a problem. The same conclusions seem to hold in 3 space dimensions. It is quite possible that perhaps a
singular configuration $a^i$ exists which leads to zero eigenvalue in (4.4) at zero $k$ but I was not able to find it.

The strong coupling limit of lattice QCD suggests that quarks in the fundamental representation are confined whereas they are only screened if put in the adjoint representation. This crucial difference comes from fairly simple quantum mechanics of color degrees of freedom related to matching of group representations in neighboring lattice points. In our rigid gauge rotor model a similar simple quantum mechanics of colors is retained. As a result the eigenenergies of a systems of static quarks will be determined by different combinations of the color components of $I^{-1}$ depending on the representation of the quarks. E.g., as already mentioned in Section 3 when the quarks are taken in the adjoint representation the $\lambda$ matrices in the expressions for $P$ and $Q$ in the Schrödinger equations (3.11) are replaced by their adjoint counterparts $F$.

In order to find the optimal $a^i$ in a systematic way one can follow a variational approach and minimize the ground state energy of the rigid gauge rotations for fixed positions of static quarks. This energy is given by a sum of the lowest eigenenergy of $H_{\text{rot}}^A$, Eq.(2.9) and the colormagnetic energy given by the second term in (1.1) with rigidly "rotating" $A(x)$, Eq.(2.1), i.e.

$$E[a^i] = E_{\text{rot}}[a^i] - \frac{1}{2g^2} \int d^3x Tr[d^i, d^j]^2$$

(4.5)

Variation of this expression gives

$$\partial_i f^{ij} - ig[a^i, f^{ij}] = \frac{1}{\delta a^i} \left( \frac{1}{2} \delta E_{\text{rot}} \right),$$

(4.6)

where $f^{ij} = (i/g)[d^i, d^j]$. Eq.(4.6) is obviously gauge invariant. In the vacuum $E_{\text{rot}}$ is zero and the minimization of the second term simply gives the classical equation for $a^i$ in the vacuum. For a quark-antiquark system $E_{\text{rot}}$ is non trivial and depends on the distance between the quarks. I plan to discuss the solutions of the equation (4.6) and their relation to confinement elsewhere.

In the rest of this Section as an illustration of a simple choice for the rigid field $a^i$ which allows to obtain some analytic results I consider it to be diagonal, $a^i_{\alpha\beta}(x) = \delta_{\alpha\beta} a^i_\alpha(x)$. The moment of inertia operator with such $a^i$ is

$$-\frac{1}{4g^2} [d^i, [d^i, \omega]]_{\alpha\beta} = -\frac{1}{4g^2} \left( \partial^i - ig(a^i_\alpha - a^i_\beta) \right)^2 \omega_{\alpha\beta}.$$

(4.7)

Using Green’s function satisfying

$$\left( \partial^i - ig(a^i_\alpha - a^i_\beta) \right)^2 J_{\alpha\beta}(x, y) = -\delta(x - y),$$

(4.8)
and following the procedure leading to Eq. (2.9) one finds the rigid gauge rotor Hamiltonian in this case

$$H = \frac{1}{4} \int d^2x \int d^2y \hat{G}_{\alpha \beta}(x) J_{\alpha \beta}(x, y) \hat{G}_{\beta \alpha}(y).$$  \hspace{1cm} (4.9)

The Schrödinger equation for the static quark–antiquark wave function (3.10) has the form (3.14) with

$$Q_{\alpha \gamma}(x_0) = \frac{1}{4} \delta_{\alpha \gamma} \left[ \sum_{\beta} J_{\alpha \beta}(x_0, x_0) - \frac{1}{N} \left(2J_{\alpha \alpha}(x_0, x_0) - \frac{1}{N} \sum_{\beta} J_{\beta \beta}(x_0, x_0) \right) \right]$$ \hspace{1cm} (4.10)

$$P_{\alpha \beta, \gamma \mu}(x_0, y_0) = -\frac{1}{2} \delta_{\gamma \mu} \delta_{\alpha \beta} J_{\alpha \gamma}(x_0, y_0) +$$ \hspace{1cm} (4.11)

$$+ \frac{1}{2N} \delta_{\alpha \gamma} \delta_{\mu \beta} \left[ J_{\beta \beta}(x_0, y_0) + J_{\gamma \gamma}(x_0, y_0) - \frac{1}{N} \sum_{\nu} J_{\nu \nu}(x_0, y_0) \right].$$

The diagonal components of $J$ are simple Coulomb propagators independent of the color so that the expressions for $Q$ and $P$ can be simplified further but I will not go into the details of this. Instead I will now consider the choice of $a^i$ which corresponds to a much discussed in the literature situation of a uniform colormagnetic field. I emphasize that in the present model this field is uniform in the intrinsic, "body fixed" frame. For simplicity I will first work in 2+1 dimensions and will try to extend to 3+1 in the next section. I set

$$a^i_\alpha(x) = \frac{1}{2} b_{\alpha} \epsilon_{ij} x^j$$ \hspace{1cm} (4.12)

where the space indices $i, j$ presently run over the values 1 and 2. In two space dimensions one can take $b$ diagonal in color since the transformation diagonalizing it is a part of $U$’s in (2.1). Explicit expression for $J$ is easily obtained in this case from the known Green’s function of a Schrödinger equation in a constant magnetic field, cf. Ref.[30],

$$J_{\alpha \beta}(x, y) = \frac{1}{4\pi} e^{i(g b_{\alpha \beta}/2) \epsilon_{ij} x^i y^j} \int_0^\infty \frac{ds}{\sinh s} e^{-[(gb_{\alpha \beta}/4)(x-y)^2 \coth s}} \hspace{1cm} (4.13)$$

where $b_{\alpha \beta} = b_\alpha - b_\beta$. For $x = y$ this expression is independent of color indices. It must be regulated to prevent the divergence, e.g.

$$\frac{g^2 N}{2\pi} \int_{s_0}^\infty \frac{ds}{\sinh s},$$ \hspace{1cm} (4.14)
where \( s_0 \) is a regularization cutoff. Although \( J(x, y) \) does not depend only on the distance \( |x - y| \) the Schrödinger equation (3.14) with \( P \) and \( Q \) based on such \( J \) is translationally invariant. Shifting the coordinates by say a vector \( h \) and simultaneously performing a gauge transformation of the wave function \( c_\alpha \rightarrow c_\alpha \exp \left[ i \frac{gb_\alpha}{2} \epsilon_{ij} h^i (x^j_0 - y^j_0) \right] \) leaves Eq.(3.14) invariant. The integral in the expression for \( J(x, y) \) can be expressed in terms of the Bessel function \( K_0(|g(b_\alpha - b_\beta)|z^2/4) \) with \( z = x - y \) and for \(|g(b_\alpha - b_\beta)|z^2 \rightarrow \infty \) it has the following asymptotic form

\[
\frac{1}{4\sqrt{\pi} \cdot (2|g(b_\alpha - b_\beta)||)^{-1/4}} \exp \left[ -|g(b_\alpha - b_\beta)|z^2/4 \right]. \tag{4.15}
\]

For finite values of \(|g|b_\alpha - b_\beta| \) it decreases as a Gaussian at large separations \( z \). This should lead to a similar decrease of the eigenvalues of (3.14) – an entirely unsatisfactory behavior as far as the confinement is concerned. In the next Section it will be seen that the situation may be different in the large \( N \) limit.

## 5 Large N Random Colormagnetic Fields.

As mentioned in the Introduction rigid gauge field rotations should be relevant for QCD in the large \( N \) limit where it is expected that a master field configuration dominates the vacuum, \[1,2\]. As in the case of a condensate such a configuration can not be just some fixed gauge field potential \( A^i_a(x) \). It must be allowed to undergo free gauge ”rotations” exactly as \( a^i \) in Eq.(2.1) since the gauge invariance is not expected to be broken in the large \( N \) limit. The dynamics of these rotations can not be ”frozen” and must be described by the gauge rotor Hamiltonian considered in Section 2. These ”rotations” induce an interaction between quarks as was shown in Section 3 for static quarks and will be demonstrated for dynamical quarks in Section 6 below where it will also be shown that in addition \( a^i \) appears as a background field in the Dirac operator.

Another important consideration is that for large \( N \) there is a large number of degrees of freedom operating at each space point which introduces statistical elements in the theory, cf. Refs. \[19, 20, 21\]. Experience with this limit for simple systems indicates that two types of gauge invariant physical operators should exist, analogues to macroscopic and microscopic observables in thermodynamics. The former depend on finite (relative to \( N \)) number of dynamical variables and involve sums over all labels of the degrees of freedom, i.e. the color indices. A simple example is \( a^i_{\alpha \beta} a^i_{\beta \alpha} \), etc. Operators without such summations, e.g., \( a^i_{\alpha \beta} \) with fixed \( \alpha \) and \( \beta \) belong to the
second type which must be regarded as microscopic observables like, e.g., a coordinate of a particle or a single spin variable in thermodynamic systems. The fluctuations of the macroscopic operators are suppressed and expectations of their products factorize at \( N = \infty \). This is not so for microscopic observables.

On the basis of these considerations one can adopt the following point of view. After allowing for free gauge rotations according to (2.1), i.e., after transformation to the body-fixed frame, one should consider \( a^i(x) \) as static random matrix functions described by a probability distribution \( P[a^i(x)] \). This distribution can be determined following the ideas of the random matrix theory, cf. Ref. [3]. To this end one should introduce the amount of information (negative entropy)

\[
I \{ P[a] \} = \int D\mu[a^i(x)] P[a^i(x)] \ln P[a^i(x)]
\]  

(5.1)

associated with the \( P[a^i(x)] \). Minimizing \( I\{P[a]\} \) subject to suitably chosen constraints on macroscopic-like variables should determine the least biased distribution \( P[a^i(x)] \). As in statistical mechanics the large \( N \) factorization should then simply appear as a consequence of the central limit theorem.

There are two crucial questions which need to be answered in following this procedure - what is the appropriate measure in the integral (5.1) and what are the variables which should be constrained. I hope to address the general answer to these questions in the future work. Presently I will illustrate how the procedure can be put to work for a uniform color magnetic field, Eq.(4.12).

In the limit of large \( N \) only the first terms in the expressions for \( P \) and \( Q \) above should be retained and the Schrödinger equation (3.14) for diagonal components of the string amplitude becomes

\[
-2g^2 \sum_{\beta} J_{\alpha\beta}(x_0, y_0) c_{\beta\beta} = (E - E_0^\alpha) c_{\alpha\alpha},
\]

(5.2)

where \( E_0^\alpha = 2g^2 \sum_{\mu} J_{\alpha\mu}(x_0, x_0) \). The non diagonal string amplitudes decouple and satisfy a trivial equation \((E_0^\mu + E_0^\nu)c_{\mu\nu} = 2E c_{\mu\nu} \) the eigenvalues of which are simply the sums of the selfenergies. Without careful treatment of long and short distance regularization in the large \( N \) limit one can not reliably discuss these eigenvalues taken separately and I will concentrate on Eq.(5.2). Using translational invariance and writing this equation for \( x_0 = 0 \) and \( y_0 = z \) one obtains

\[
\sum_{\beta=1}^{N} \int_0^\infty \frac{ds}{4\pi \sinh s} e^{-(g(b_\alpha - b_\beta))/4s^2} \coth s c_{\beta\beta} = \frac{E}{2g^2} c_{\alpha\alpha}.
\]

(5.3)
as the large N limit of the Schrödinger equation for a static quark–antiquark pair in the rigid gauge configuration corresponding to a uniform colormagnetic field in 2+1 dimensions. One must still specify the large N scaling of various quantities which enter this equation. Provided each term in the sum on the left hand side is of the same order of magnitude I get the standard scaling of the coupling constant requiring that $g^2 N$ is held fixed. In the exponential of the integrand one can then extract the finite combination $\bar{g} = g \sqrt{N}$. The problem is then to determine the scaling and in general the entire distribution of the field components $b_\alpha/\sqrt{N}$. Regarding the behavior at large separations $z$ one notes that if the limit of $N \to \infty$ is taken first in such a way that the differences $|b_\alpha - b_\beta|/\sqrt{N}$ decrease then the Gaussian decay can possibly be prevented.

I use this example to demonstrate how the ideas about the statistical nature of the large N limit can be used to determine the distribution of the components $b_\alpha/\sqrt{N}$. I consider what happens with the Wilson loop $W(C) = \frac{1}{N} < Tr P \exp (ig \oint_C A^i dx_i) >$ in the present theory. Choosing the loop perpendicular to the time axis, inserting (2.1) in $W(C)$, using its gauge invariance and the explicit form (4.12) of $a^i$ one finds

$$W(C) = \frac{1}{N} \sum_\alpha e^{igb_\alpha S} = \int_{-\infty}^{\infty} db \rho(b) e^{igbS} \tag{5.4}$$

where $S$ is the area of the loop and $\rho(b) = (1/N) \sum_{\alpha=1}^{N} \delta(b - b_\alpha)$ is the density of the field components. In the large N limit $\rho(b)$ can be approximated by a smooth function provided the range of variations of $b$ does not grow with N. Assuming this one easily finds simple expressions for $\rho(b)$, e.g. Lorenzian

$$\rho(b) = \frac{b_0 \sqrt{N}}{\pi (b^2 + Nb_0^2)} \tag{5.5}$$

which lead to the area law dependence of the Wilson loop, $W(C) = \exp(-\bar{g}b_0 S)$. The combination $\bar{g}b_0$ plays the role of the string constant. The placing of $N$’s in (5.3) was chosen in such a way as to have this constant finite for $N \to \infty$.

The choice (5.5) is the simplest possible. Any meromorphic function $\rho(b)$ with poles in the upper plane will give the area law with the string tension controlled by the position of the pole closest to the real axis. One can also take functions with other type of singularities in the upper complex plane, etc. The simple choice (5.5) gives area law for any $S$, missing entirely the asymptotic freedom behavior at small $S$. One can attempt to correct this by choosing more involved expressions for $\rho$. A much more serious problem is that the space oriented Wilson loop may not be a good measure...
of confining properties in the model where one has never worried about the Lorenz invariance.

Adopting any form of the ”single component” density $\rho(b)$ still leaves the distribution of the values of $b_\alpha$ needed in, e.g., Eq. (5.3) largely undetermined. Using statistical concepts described in the beginning of this Section one should view $b_\alpha$’s as random quantities and introduce their joint probability distribution $P(b_1, b_2, ..., b_N)$ which should be such that $\rho(b)$ is reproduced but is otherwise maximally random, i.e. contains least amount of information. The question immediately arises as to whether $\rho(b)$ is the only quantity which should constrain $P(b_1, ..., b_N)$ and what is the complete set of such constraints. In the absence of general answers I take $\rho(b)$ controlling $W(C)$ as an example and determine the distribution $P(b_1, ..., b_N)$ by minimizing the appropriate negative entropy (information) with this constraint.

The quantities $b_\alpha$’s are eigenvalues of a hermitian, in general complex matrix. The information content of a probability distribution $P(b_1, ..., b_N)$ of such eigenvalues is a well studied question, cf. Ref.[31]. It is

$$\int d\mu[b] P(b_1, ..., b_N) \ln P(b_1, ..., b_N)$$

(5.6)

where the measure is $d\mu[b] = \text{const} \prod_{\alpha>\beta} |b_\alpha - b_\beta|^2 db_1 db_2 ... db_N$, reflecting the repulsion of the eigenvalues. Minimizing (5.6) under the condition of a given $\rho(b) = (1/N) \sum_\alpha (b - b_\alpha)$ one finds

$$P(b_1, ..., b_N) = \text{const} \exp \left( \sum_{\alpha \neq \beta} \ln |b_\alpha - b_\beta| - 2N \sum_\alpha \int_{-\infty}^{\infty} \ln |b_\alpha - b'| \rho(b') db' \right).$$

(5.7)

Using, e.g. Eq. (5.3) for $\rho(b)$ this expression becomes explicitly

$$P(b_1, ..., b_N) = \text{const} \exp \left( \sum_{\alpha \neq \beta} \ln |b_\alpha - b_\beta| - N \sum_\alpha \ln (b_\alpha^2 + Nb_0^2) \right).$$

(5.8)

The constant in front of this expression must assure the normalization of $P$ and can be calculated by the methods described in Ref.[32]. Using the standard interpretation of $P(b_1, ..., b_N)$ as a partition function of a fictitious Coulomb gas one can say that the ”particles” $b_\alpha$ are ”repelled” from each other by the first term in its exponential but are kept within the interval $b_0$ by the second term representing the interaction with the background ”charge” distributed according to $\rho(b)$. The average distance $|b_\alpha - b_\beta| \sim b_0/N$ becomes very small in the large $N$ limit. The Schrödinger equation
(5.3) is now a random matrix equation with the probability distribution of its elements controlled by the $P(b_1, ..., b_N)$ above. The actual numerical solution of this equation is now in progress.

In a similar way one can consider rigid gauge configuration representing uniform colormagnetic field in $3 + 1$ dimensions. This field in the intrinsic frame corresponds to the choice

$$a^i(x) = \frac{1}{2} \epsilon_{ijk} b^j x^k$$

(5.9)

where now however the three color matrices $b^i$ in general cannot be assumed diagonal. For such non-diagonal colormagnetic field the inversion of the moment of inertia operator $-(1/4g^2)[d^i, [d^i, \omega]]$ requires a solution of a matrix differential equation. This equation simplifies considerably if $b^i$’s are nonetheless restricted to be diagonal, $b^i_{\alpha\beta} = \delta_{\alpha\beta} b^i_{\alpha}$. Then the equation (4.7) is still valid with the index $i$ now running from 1 to 3. In the following equation (4.8) for the Green’s function one should just replace $(b_\alpha - b_\beta) \epsilon_{ijkl} x^j$ by $\epsilon_{ijk} (b^i_{\alpha} - b^i_{\beta}) x^k$. The expression for this Green’s function is known and one can repeat all the steps leading to the static quark–antiquark Schrödinger equation which is the analog of Eq.(5.3) in 3+1 dimensions.

Turning again to the Wilson loop one finds in this case $\oint_C a^idx_i = b^j S_j$ with $S_j = (1/2) \epsilon_{jkl} \oint_C x^k dx_l$ so that $(\oint_C a^idx_i)_\alpha = b_\alpha S \cos \theta_\alpha$ (no sum over $\alpha$) where $S = \sqrt{\sum_i (S^i)^2}, b_\alpha = \sqrt{\sum_i (b^i_\alpha)^2}$ and $\theta_\alpha$ - the angle between the vectors $b^i_\alpha$ and $S^i$ at a given $\alpha$. S is the area of the loop when it is planar and is related to the minimal area in general. The Wilson loop is

$$W(C) = \frac{1}{N} \sum_\alpha e^{igSb_\alpha \cos \theta_\alpha} =$$

$$= \frac{1}{N} \sum_\alpha 2\pi \int_1^1 d(\cos \theta_\alpha) e^{igSb_\alpha \cos \theta_\alpha} =$$

$$= \frac{4\pi}{gs} \int_0^\infty \frac{db}{b} \rho(b) \sin(gbS),$$

(5.10)

where $\rho(b) = (1/N) \sum_\alpha \delta(b - b_\alpha)$ is the density of the positive lengths of the color components of the vector $b^i$. In (5.10) I have performed the angle averaging which must be present in the vacuum wavefunction. One can easily choose $\rho(b)$, e.g. the square of Lorentzian

$$\rho(b) = \frac{4b_0 b^2 \sqrt{N}}{\pi(b^2 + N b_0^2)^2}$$

(5.11)

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which gives the area law $W(C) = 4\pi \exp(-\bar{g}b_0 S)$. This choice is again not unique and gives the area law for any $S$. It has a powerlike tale as opposed to the perturbative Gaussian.

The statistical arguments for finding the entire distribution of $b'_\alpha$ can be used in $3 + 1$ dimensional case as well with the difference that in this case the density of the lengths of the vectors $b'_\alpha$ is fixed by, e.g. Eq.(5.11) and their directions are distributed isotropically.

6 Dynamic Quarks

Dynamic quarks can be easily included in the rigid gauge rotation model. For this I define quark fields in the "rotating frame", $q = U\hat{q}$, use Eq. (2.1) in the second term of the QCD Hamiltonian (1.1) and replace the first term in it by (2.9). Using moreover the Gauss law constraint (1.10) I can write the original QCD Hamiltonian (1.1) in the rigid gauge rotor limit as expressed in terms of the quark fields only,

$$H_{rot} = \frac{1}{2} \int d^3x d^3y \hat{\rho}_a(x) I_{ab}^{-1}(x, y, [a']) \hat{\rho}_b(y) + \int d^3x \hat{q}^+(x)[a^i(p^i - ga^i(x)) + \beta m]\hat{q}(x),$$

where $\hat{\rho}_a = \hat{q}^+ \gamma^a \hat{q}$ are the color quark densities in the rotating frame. The Hamiltonian $H_{rot}$ describes quarks with gauge strings attached to them, i.e. $q(x)$ are multiplied by $U^+(x)$. They move in an external colormagnetic field described by the vector potential $a^i(x)$ and interact via an instantaneous interaction $I_{ab}^{-1}(x, y, [a'])$ also depending on $a^i$ via Eq.(2.7). The simultaneous appearance of the rigid gauge field configuration both as a background and as "inducing" the quark-quark interaction is ultimately a consequence of the gauge invariance which requires that non dynamical rigid gauge fields appear only in the form (2.1).

$H_{rot}$ is gauge invariant since the operators $\hat{q}$ are. Moreover this Hamiltonian should only be used in the color singlet sector of the theory since I have used the Gauss law to derive it.

For the vanishing $a^i$ the Hamiltonian $H_{rot}$ describes free quarks interacting Coulombically. Also for a general non zero $a^i H_{rot}$ should be regarded as the QCD analogue of the QED Hamiltonian in which only the instantaneous Coulomb interaction between the charges has been retained. Indeed the analogue of the rigid gauge "rotations" (2.1) in QED is $A^i(x, t) = a^i(x) + \partial^i \chi(x, t)$ with abelian $U = \exp(ig\chi(x, t))$, fixed rigid $a^i(x)$ and dynamical $\chi(x, t)$. Repeating the steps leading to (6.1) one will derive in the QED case the Coulomb interaction between the charge densities.
Regarding the possible role of $a^i(x)$ as the master field in the large N limit one has in $H_{rot}$ a way in which this field should enter the quark sector of the theory, i.e. serving both as a background field and perhaps somewhat surprisingly also controlling the quark interaction. Following the developments of Section 5 this field should be regarded as random. The appearance of a random interaction between the quarks means that a possible mechanism for confinement of dynamical quarks in the large N limit could be related to the localization of their relative distances. The possible connection between confinement and localization has already been mentioned in the past but usually in the context of a random background field and not with random interactions as appear in the present model.

The Hamiltonian (6.1) takes exact account of the gauge symmetry. One must however also worry about global symmetries. For any N the Hamiltonian $H_{rot}$ may serve as a possible basis for various phenomenological developments. Both for this matter and conceptually one must face the issue that allowing for an arbitrary x-dependence of various color components of $a^i$ in (6.1) leads to breaking of important symmetries such as translational, rotational, Lorentz, time reversal, and various discrete symmetries. Of course the breaking of continuous space symmetries is not uncommon in phenomenology, e.g. the bag model, the quark potential model, the Skyrme model, etc. Symmetries can be restored by considering all configurations translated by the symmetry and integrating over them using collective coordinates. This of course applies to both continuous and discrete symmetries. In the absence of the guidance from the symmetries a more dynamical criterion for fixing $a^i$ seems to be the condition of lowest energy. This leads naturally to a generalization of the variational approach of Section 4 in which the variational energy should be replaced by the ground state energy $E_0[a^i(x)]$ of (the suitably regularized) $H_{rot}$ found for a given $a^i(x)$. Should the solution $a^i$ break a global symmetry, the symmetry ”images” of this $a^i$ will also be solutions and one should ”sum” over all of them in a standard way thereby restoring the symmetry. This variational approach may be combined with the Hartree-Fock method which should allow to calculate $E_0[a^i(x)]$ approximately. The Hartree-Fock approximation for fermions was shown to be consistent with the large N approximation, c.f., Ref.[22]. In a combined approach one should form for fixed $a^i$ an expectation value of $H_{rot}$ with respect to a trial state of a chosen color singlet configuration of quarks (e.g., vacuum, baryon, etc) which must be a product state, i.e. such that the expectation values with respect to this state have a non interacting factorized form, i.e.,

$$\langle \hat{\rho}_a(x)\hat{\rho}_b(y) \rangle = \lambda^a_{\alpha\beta} \lambda^b_{\gamma\delta} \langle \hat{q}^\dagger_\alpha(x)\hat{q}_\beta(x) \rangle \langle \hat{q}^\dagger_\gamma(y)\hat{q}_\delta(y) \rangle$$
\begin{equation}
-\langle \hat{q}^+_\alpha(x)\hat{q}_\beta(y) >\langle \hat{q}^+_\gamma(y)\hat{q}_\delta(x) >).
\end{equation}

For a global color singlet state $\langle \hat{q}^+_\alpha(x)\hat{q}_\beta(y) > = \delta_{\alpha\beta}\rho(y,x)$, with $\rho(x,y) = \frac{1}{N} \sum_\gamma < \hat{q}^+_\gamma(y)\hat{q}_\gamma(x) >$ and therefore

\begin{equation}
\langle \hat{\rho}_a(x)\hat{\rho}_b(y) > = -2\delta_{ab}\rho(x,y)\rho(y,x)
\end{equation}

Following the standard Hartree-Fock routine, cf., Ref.[33], the single quark density matrix can be expanded in terms of a complete set of functions, i.e., $\rho(x,y) = \sum_n f_n \psi_n(x)\psi^*_n(y)$. At this stage it is customary to add the so called Slater determinant condition which means that the single quark states $\psi_n$ have sharp occupations $f_n = 0$ or 1. This condition and its compatibility with the large N limit and Lorenz invariance were discussed in Ref.[21]. Adopting this condition, using (6.3) in forming the expectation $E_{HF}[\rho(x,y),a^i(x)] = \langle H_{rot} >$ and varying with the respect to $\psi_n(x)$’s with constraints on their normalization one obtains the Hartree-Fock equation

\begin{equation}
[a^i p^j + \beta m]\psi_n(x) - 2 \int d^3y I^{-1}_{aa}(x,y,\{a^j\})\rho(x,y)\psi_n(y) = \epsilon_n \psi_n(x),
\end{equation}

which appears here as a selfconsistent Dirac equation for the quark wave functions $\psi_n(x)$. Note that in this equation $a^i$ disappeared from the Dirac operator and enters only the interaction $I^{-1}_{aa}(x,y)$. Solutions of (6.4) determine the optimum $\rho$ and thus $E_{HF}$ for a given $a^i$.

Hartree-Fock equation similar to (6.4) has been investigated in the 1+1 dimensional QCD, [21] although there for obvious reasons the field $a^i$ was absent and $I^{-1}(x,y)$ was simply $|x - y|$. Both the t’Hooft meson spectrum and the baryon soliton solutions were found in this approach in 1+1 dimensions. For small quark masses the baryon was the realization of the skyrmion described in the quark language. If successful the approach based on Eq.(6.4) can possibly lead to similar results in 2+1 and 3+1 dimensions.

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