Spatial Inhomogeneity of Kinetic and Magnetic Dissipations in Thermal Convection

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Abstract

We investigate the inhomogeneity of kinetic and magnetic dissipations in thermal convection using high-resolution calculations. In statistically steady turbulence, the injected and dissipated energies are balanced. This means that a large amount of energy is continuously converted into internal energy via dissipation. As in thermal convection, downflows are colder than upflows and the inhomogeneity of the dissipation potentially changes the convection structure. Our investigation of the inhomogeneity of the dissipation shows the following. (1) More dissipation is seen around the bottom of the calculation domain, and this tendency is promoted with the magnetic field. (2) The dissipation in the downflow is much larger than that in the upflow. The dissipation in the downflow is more than 80% of the total at maximum. This tendency is also promoted with the magnetic field. (3) Although 2D probability density functions of the kinetic and magnetic dissipations versus the vertical velocity are similar, the kinetic and magnetic dissipations are not well correlated. Our result suggests that the spatial inhomogeneity of the dissipation is significant and should be considered when modeling a small-scale strong magnetic field generated with an efficient small-scale dynamo for low-resolution calculations.

Key words: stars: interiors – Sun: interior – Sun: magnetic fields

1. Introduction

Thermal convection is one of the most important processes for understanding the differential rotation and the dynamo of the Sun. Because the rotation has a crucial role in introducing the anisotropy of the turbulence, it is important to estimate the convection velocity in the solar convection zone. When the convection velocity is fast (slow), the influence of the rotation is weak (strong). Recent observational and theoretical investigations suggest that the convection velocity in numerical calculations are significantly faster than in the real Sun (Hanasoge et al. 2012; Lord et al. 2014; Hotta et al. 2015b; Featherstone & Hindman 2016a, 2016b). This is an important unsolved problem in solar physics. A possible mechanism for suppressing the convection velocity is a magnetic field. Hotta et al. (2015a) found that the small-scale dynamo is very efficient in the solar convection zone, and the small-scale magnetic field can reach the equipartition level of the kinetic energy. This strong magnetic field acts like viscosity and reduces the convection velocity by a factor of 2 compared with the case without the magnetic field. To date, suppression by the magnetic field alone could not resolve the fast convection problem, but this could contribute to solving the problem. The viscosity-like behavior of the magnetic field also suggests that the real solar convection could be mimicked by an enhanced viscosity instead of a strong magnetic field. Because the strong small-scale magnetic field is only achievable with a high resolution—i.e., high Reynolds numbers, a calculation that requires a huge numerical resource—it would be useful to explore the possibility of mimicking the magnetic field by the viscosity to reduce the numerical cost. Such an approach has recently been taken by O’Mara et al. (2016). They found that an increase in the thermal Prandtl number \( Pr = \nu/\kappa \), where \( \nu \) and \( \kappa \) are viscosity and thermal conductivity, respectively, decreases the convection velocity. This result potentially indicates that a small-scale strong magnetic field contributes to suppressing the convective velocity, as the Lorentz force does not act as thermal conductivity (\( \kappa \)) but as viscosity (\( \nu \)).

In this paper, we explore the overlooked physical processes by only adding the viscosity to mimic the magnetic field. Of course, there is an important difference between the forms of the Maxwell and viscous stress tensors. We disregard this difference and focus on the dissipation. In a statistically steady turbulence, like the solar convection, the energy injection and the viscous and diffusive dissipations are balanced; i.e., all the injected energy must be dissipated in the end. Thus, a significant amount of the energy is continuously dissipated in small scales, where the viscosity and diffusivity are effective.

Because there is no special location in isotropic turbulence (e.g., Brandenburg 2014), the location of the dissipation is not important. In contrast, in thermal convection turbulence, the structures of the up and downflows are different. Typically, the downflow is colder than the upflow. When the kinetic and magnetic energies are dissipated more in the downflow region, this can change the thermal structure. In this study, we address this issue with high-resolution hydrodynamic and magneto-hydrodynamic (MHD) calculations. The effect of the magnetic field on the dissipation is also investigated. The strong magnetic field suggested in Hotta et al. (2015a) possibly changes the character of the dissipation, and we need to consider the difference in dissipation in low-resolution calculations when the strong magnetic field is mimicked by enhanced viscosity (O’Mara et al. 2016).

In addition, we investigate the effect of the magnetic Prandtl number (\( \text{Pm} = \nu/\eta \), where \( \eta \) is the magnetic diffusivity) on the inhomogeneity of dissipation. A small magnetic Prandtl number (\( \eta > \nu \)) makes the scale of the magnetic dissipation larger than that of the kinetic dissipation and vice versa. As a result, more (less) energy is dissipated through the magnetic dissipation with a small (large) magnetic Prandtl number. Brandenburg (2011, 2014) found that the ratio of the dissipation (\( \epsilon_\nu/\epsilon_\eta \)), where \( \epsilon_\nu \) and \( \epsilon_\eta \) are the kinetic and magnetic dissipation) decreases with decreasing the magnetic Prandtl number.

In the solar convection zone, the magnetic Prandtl number is very small (\( \text{Pm} \sim \text{10}^{5}-\text{10}^{-6} \)) (Christensen-Dalsgaard et al. 1996; Miesch 2005). If the magnetic Prandtl number...
changes the dissipation character, this should be considered even in high-resolution calculations when \( Pm \sim 1 \) is used.

2. Model

We solve the three-dimensional MHD equations in the Cartesian geometry \((x, y, z)\). Here, we define the \( x \)-direction as the gravity direction; the \( y \)- and \( z \)-directions are horizontal directions. The MHD equations with the gravity and radiative heating and cooling are expressed as

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}),
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \mathbf{p} + \left( \frac{\mathbf{B}^2}{8\pi} \right) \mathbf{v} - \rho \mathbf{g},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

\[
\frac{\partial E_{\text{total}}}{\partial t} = -\nabla \cdot \left[ \left( E_{\text{total}} + p + \frac{\mathbf{B}^2}{8\pi} \right) \mathbf{v} - (\mathbf{v} \cdot \mathbf{B}) \mathbf{B} \right] - \rho \mathbf{v} \cdot \mathbf{g} + \Gamma,
\]

where \( \rho, p, \mathbf{v}, \mathbf{B}, E_{\text{total}}, \) and \( \Gamma \) are the density, the pressure, the fluid velocity, the magnetic field, the total energy, and the time-independent heating and cooling, respectively. The ratio of the heat capacities is \( \gamma = 5/3 \). We adopt the fourth-order space-centered derivative and the Runge–Kutta method for time integration (Voglter et al. 2005). A stable calculation is achieved with the artificial diffusivity suggested by Rempel et al. (2009) and Rempel (2014). The details of the artificial diffusivity are explained in the next section. In this paper, we investigate the effect of a small-scale magnetic field on the dissipation. The strong magnetic field is generated by an efficient small-scale dynamo, which is only achievable by significantly reducing the viscosity and the magnetic diffusivity. Thus, we do not include explicit viscosity, magnetic diffusivity, and thermal conductivity. Only the artificial diffusivity is included. The initial conditions are the solution of the hydrostatic equilibrium \( dp/dx = -\rho g \) with a constant gravitational acceleration. The temperature gradient is expressed as \( \nabla = d \ln T/d \ln p \). Thus, the initial conditions are

\[
T_0 = T_b \left[ 1 - \frac{x}{H_b} \right],
\]

\[
p_0 = p_b \left[ 1 - \frac{x}{H_b} \right]^\gamma,
\]

\[
p_0 = p_b \left[ 1 - \frac{x}{H_b} \right]^\gamma - 1,
\]

where \( T_b, p_b, \rho_b, \) and \( H_b = -(d \ln p/dx)^{-1} |_{x = 0} = p_b / \rho_b g \) are the temperature, the pressure, the density, and the pressure scale height at \( x = 0 \), which is the bottom of the calculation domain. In the initial condition, the temperature gradient is set to the adiabatic value \( \nabla = (\gamma - 1)/\gamma \) and becomes superadiabatic after calculations start. The calculation domain is \((0, 0, 0) < (x, y, z)/H_b < (1.8, 3.6, 3.6)\). We have numbers of grid points of 512, 1024, and 1024 in the \( x \)-, \( y \)-, and \( z \)-directions, respectively. We adopt the impenetrable stress-free boundary condition for the velocity; i.e., \( v_x = \partial v_y / \partial x = \partial v_z / \partial x = 0 \) at both the top and bottom boundaries. At the top boundary, only the vertical magnetic field is allowed, \( \partial B_y / \partial x = B_z = B_y = 0 \), and the horizontal magnetic field is allowed at the bottom boundary, \( B_x = \partial B_y / \partial x = \partial B_z / \partial x = 0 \). The periodic boundary condition is adopted for all the variables in the horizontal direction. \( \Gamma \) is the time-independent cooling and heating around the top and bottom boundaries expressed as

\[
\Gamma = \frac{-d F_e}{dx},
\]

where \( x_{\text{max}} = 1.8 H_b \) and \( x_{\text{min}} = 0 \) are the locations of the top and bottom boundaries, respectively. We set \( d_{\text{max}} = 0.2 H_b \) and \( d_{\text{min}} = 0.4 H_b \) for the width of the heating and the cooling, respectively. \( F_o = 10^{-4} \rho_b c_b^3 \) is the energy flux through the calculation domain, where \( c_b = \sqrt{\rho_b / \rho_b} \) is the speed of sound at \( x = 0 \). The density contrast is \( \rho_b(x_{\text{min}}) / \rho_b(x_{\text{max}}) \sim 6.7 \) in this setting.

Although the calculations in this study are toy models, we implicitly assume that the computational domain extends from the base of the convection zone to somewhere in the upper convection zone. We note that the energy flux in this study \( (F_o = 10^{-4} \rho_b c_b^3) \) is much larger than the real solar value \( (F_o \sim 10^{-1} \rho_b c_b^3) \). Hotta (2017), however, shows that the convection property in the convection zone, is not influenced by the value of the energy flux when the convection velocity is normalized with a typical convection velocity \( v_c = (F_0 / \rho_b)^{1/3} \).

2.1. Artificial Diffusivity

We adopt the artificial diffusivity (hyperdiffusivity) developed by Rempel et al. (2009) and Rempel (2014). The diffusion-like equation is applied for all the variables \( (\rho, v, B, e_{\text{int}}) \), where \( e_{\text{int}} = p / (\gamma - 1) \) is the internal energy:

\[
\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{f_{i+1/2} - f_{i-1/2}}{\Delta x},
\]

where the diffusive flux is expressed as

\[
f_{i+1/2} = -\frac{1}{2} c_{i+1/2} \Delta x \Phi (u_{t+1} - u_{t-1})(u_{t+1} - u_{t-1}).
\]

The variables at the left and right sides of the cell surface are defined as

\[
u_i = u_i + 0.5 \Delta u_i,
\]

\[
u_t = u_{i+1} - 0.5 \Delta u_{i+1}.
\]

The difference in the variable in the cell is defined with the monotonized central minmod limiter as

\[
\Delta u_i = \text{minmod} \left[ \frac{u_{i+1} - u_{i-1}}{2}, 2(u_{i+1} - u_i), 2(u_i - u_{i-1}) \right]
\]
The limiter function is defined with
\[
\Phi = \max \left[ 0, 1 + h \left( \frac{u_{i+1} - u_i}{u_{i+1} - u_i} - 1 \right) \right],
\]
where \( h \) is a free parameter for the hyperdiffusivity. When we adopt \( h = 0 \), the diffusive flux is identical to the second-order local Lax–Friedrichs scheme. A less diffusive calculation is achieved with larger \( h \). In this work, \( h = 1 \) and \( h = 2 \) are adopted for the density \( \rho \) and the internal energy \( \epsilon_{\text{in}} \), respectively.

In this paper, the calculation includes three phases named Hydro, Large Pm, and Small Pm. We start the calculation without the magnetic field (phase Hydro). In this phase, \( h = 2 \) is adopted for the velocity. The calculation continues until the thermal convection reaches a statistically steady state. Then, a weak horizontal seed magnetic field is added (phase Large Pm). In this phase, \( h = 2 \) is adopted for both the velocity and the magnetic field. When we adopt the same values of \( h \) for the velocity and the magnetic field in this setting, the magnetic dissipation \( \epsilon_B \) is slightly smaller than the kinetic dissipation \( \epsilon_v \) (Figure 1). Thus, the second phase is named Large Pm. Here we assume \( \epsilon_v/\epsilon_B \sim \text{Pm} \) (Brandenburg 2014). When the small-scale dynamo reaches a statistically steady state, the \( h \) for the velocity increases to 500; i.e., the diffusivity for the velocity decreases (phase Small Pm).

The kinetic \( (\epsilon_v) \) and magnetic \( (\epsilon_B) \) dissipations at a grid point \( i \) are estimated with
\[
\epsilon_v = -\frac{1}{2} \sum_{m=1}^{3} \left( \rho_{i-1/2} f_{i-1/2}^m (v^m) \frac{v^m_i - v^m_{i-1}}{\Delta x} + \rho_{i+1/2} f_{i+1/2}^m (v^m) \frac{v^m_{i+1} - v^m_i}{\Delta x} \right),
\]
\[
\epsilon_B = -\frac{1}{8\pi} \sum_{m=1}^{3} \left( f_{i-1/2}^m (B^m) \frac{B^m_i - B^m_{i-1}}{\Delta x} + f_{i+1/2}^m (B^m) \frac{B^m_{i+1} - B^m_i}{\Delta x} \right),
\]
respectively, where \( m \) expresses three directions \( x, y, z \) and \( \rho_{i+1/2} = (\rho_{i+1} + \rho_i)/2 \). These dissipations are the loss of the kinetic \( (\epsilon_v) \) and magnetic \( (\epsilon_B) \) energies through the artificial viscosity. This is the mechanism to transform the kinetic and magnetic energies to the internal energy in the calculations. We note that because we use the total energy \( E_{\text{total}} \) for the energy Equation (5), \( \epsilon_v \) and \( \epsilon_B \) are not included in the equation. Any hyperdiffusivity becomes effective at a strong shear and current. Because we expect that the hyperdiffusivity possibly overestimates the spatial inhomogeneity of the dissipations, pseudo-dissipations \( D_v \) and \( D_B \) that are related to explicit viscosity and magnetic diffusivity are additionally estimated as
\[
D_v = \rho \sum_{i,k} \left( \frac{\partial v_i}{\partial x_k} \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \nabla \cdot \mathbf{v} \right),
\]
\[
D_B = \frac{1}{4\pi} |\nabla \times \mathbf{B}|^2.
\]
When the equation of motion has an explicit kinetic viscosity as

$$
\frac{\partial}{\partial t}(\rho v) = - \nabla \cdot \tau,
$$

(22)

where

$$
\tau_{ij} = -2\rho
\begin{bmatrix}
\epsilon_{ij} - \frac{1}{3}(\nabla \cdot \mathbf{v})\delta_{ij}
\end{bmatrix},
$$

(23)

$$
\epsilon_{ij} = \frac{\partial v_i}{\partial x_j},
$$

(24)

the kinetic energy loss rate is expressed with the kinetic pseudo-dissipation $D_v$ as $\nu D_v = \rho \epsilon_{ij} \partial v_i / \partial x_j$. With the explicit magnetic diffusivity as

$$
\frac{\partial B}{\partial t} = [...] - \nabla \times (\eta \nabla \times B),
$$

(25)

the magnetic energy loss is expressed with the magnetic pseudo-dissipation $D_B$ as $\eta D_B = \eta \|\nabla \times B\|^2 / (4\pi)$. Thus the pseudo-dissipations $D_v$ and $D_B$ could be the proxies of the explicit viscosity and magnetic diffusivity. The strong shear and current are selectively dissipated by the hyperdiffusivity. The pseudo-dissipations likely underestimate the inhomogeneity compared with reality, because we estimate these just after the artificial viscosity is exerted; i.e., the structure is smoothed. Thus, the realistic dissipation inhomogeneity could be somewhere between the dissipation and the pseudo-dissipation. In the following discussion, we show both the dissipations ($\epsilon_v$ and $\epsilon_B$) and the pseudo-dissipation ($D_v$ and $D_B$). We note that the ratios between the dissipations to the pseudo-dissipations can be the effective viscosity ($\nu_{eff} = \epsilon_v / D_v$) and magnetic diffusivity ($\eta_{eff} = \epsilon_B / D_B$).

The above issues are also discussed in Rempel (2017).

3. Result

Figure 2 shows the volume-rendered data of the vertical velocity $v_x / v_c$ in the phase Large Pm. The typical compressible thermal convection pattern—i.e., thin downflow surrounded by broad upflow—is observed. We achieve an efficient small-scale dynamo that suppresses the small-scale flows (Hotta et al. 2015a, 2016).

Figure 3 shows the fractions of the upflow (dashed) and downflow (solid) regions. The results in the phases Hydro (black), Large Pm (blue), and Small Pm (red) are shown. About 60% of the horizontal area is occupied by the broad upflow. This feature is similar to the calculation with higher density contrast ($\sim 600$: Hotta et al. 2014). The results with the magnetic field show a larger fraction of the upflow in the bottom half of the computational domain. Without the magnetic field, small-scale turbulent flow mixes the upflows and downflows, and the region of the upflows decreases smoothly toward the bottom of the calculation domain. The strong magnetic field generated by the small-scale dynamo suppresses the small-scale flow. Thus, the region of the upflow
increases because of the effect of stratification; i.e., the downflow becomes thinner with higher gas pressure around the bottom calculation domain. No significant difference is observed between results with different magnetic Prandtl numbers (blue and red lines).

Figure 4 shows the total dissipation (panel (a): $\epsilon_v + \epsilon_\eta$) and the total pseudo-dissipation (panel (b): $D_v + D_\eta$). For a comparison, panels (c) and (d) show quotients of the panels (a) and (b) divided by the background density ($\rho_0$), respectively. The results in the phases Hydro (black), Large Pm (blue), and Small Pm (red) are shown. The total dissipation density is estimated with vertical integration of the panel (a) as

$$\int_{x_{\min}}^{x_{\max}} (\epsilon_v + \epsilon_\eta) dx \sim \rho_0 \nu^3,$$

This is almost the same value as the energy flux imposed from the bottom boundary. We note that the dissipated kinetic and magnetic energies to the internal energy can return back to the kinetic energy through the pressure and buoyancy works again. Thus the total dissipation density does not mean the increase rate of the internal energy. As expected, a comparison between panels (a) and (b) shows that the inhomogeneity increased in the estimation of $\epsilon_v + \epsilon_\eta$; i.e., panel (a) shows a steeper increase of the dissipation at the bottom of the computational domain than panel (b). The increase in dissipation toward the bottom becomes larger when the magnetic field is included. Hotta et al. (2015a) shows that the downward Poynting flux efficiently transports magnetic energy to the bottom of the computational domain, and the small-scale dynamo is most effective around the bottom. Thus, a small-scale strong magnetic field is accumulated around the bottom. This magnetic field dissipates and contributes to the increase in dissipations around the bottom when the magnetic field is included. This increase is also seen even in the total pseudo-dissipation (panel (b): $D_v + D_\eta$), which likely reduces the space inhomogeneity; i.e., the increase in dissipation at the bottom is alleviated. In addition, this feature is seen even when it is divided by the background density (panels (c) and (d)). Hotta (2017) recently found that the small-scale dynamo with the artificial wall boundary at the bottom is not very different from that with the radiation zone using the realistic solar parameter, since the radiation zone is significantly stiff in terms of the thermal convection. In other word, the artificial wall at the bottom nicely mimics the realistic solar radiation zone. Thus we argue that the increase of the dissipation toward the bottom boundary is not just an artifact.

Figure 5 shows the dissipations at the upflow and downflow regions separately in the Large Pm phase. The results for the whole (black), downflow (blue), and upflow (red) regions are shown. In this paper, we show both the mean and integrated
dissipation for comparison. The integrated dissipation at the downflow region is calculated by integration of the dissipation over the downflow region. Then, the integrated dissipation is divided by the area of the downflow region, and the mean dissipation at the downflow region is calculated. The same routine is used for the mean and integrated dissipation at the upflow region. Panel (a) shows the horizontally averaged (mean) dissipation. The figure shows that the mean dissipations in the downflow region (blue line) are much larger than those in the upflow region (red line). As the downflow region is smaller than the upflow region, it is useful to see the integrated dissipations in these regions. Panel (b) shows the integrated dissipation over the corresponding area. Even after integration, much more dissipation is observed in the downflow region (Figure 5(b)). This feature is observed even for the pseudo-dissipations, where inhomogeneity is probably suppressed (Figures 5(c) and (d)). Thus, we conclude that the dissipation is dominantly located in the downflow region.

Figure 6 shows the ratio of the dissipation in the downflow region to that in the upflow region. Panels (a) and (b) show the mean and integrated dissipation ($\epsilon_v$ and $\epsilon_\eta$), and panel (c) and (d) show the mean and integrated pseudo-dissipations ($D_v$ and $D_\eta$), respectively. The kinetic (solid: $\epsilon_v$ and $D_v$) and magnetic (dashed: $\epsilon_\eta$ and $D_\eta$) dissipations are shown separately for the phases Hydro (black), Large Pm (blue), and Small Pm (red). The kinetic ($\epsilon_v$) and magnetic ($\epsilon_\eta$) dissipations show almost the same inhomogeneity between the upflow and downflow regions; i.e., the dashed and solid lines almost completely overlap. We note that this does not mean $\epsilon_v = \epsilon_\eta$ and merely indicates that the ratios between upflow and downflow are the same. All the results show that the largest and smallest inhomogeneities are shown in the Large Pm and Hydro phases, respectively. This means that the magnetic field increases the dissipation in the downflow region. The decrease in the magnetic Prandtl number reduces dissipation in the downflow region. As we change the kinetic viscosity to change the magnetic Prandtl number, this means that the magnetic diffusivity stays the same, and the pseudo-magnetic dissipation ($D_\eta$) is almost identical between the Small Pm and Large Pm phases.

Figure 7 shows 2D probability density functions (PDFs). The panels show the PDF of $\epsilon_v$ and $\nu$, (b) $\epsilon_\eta$ and $\nu$, (c) $\epsilon_v$ and $\nu_m$, (d) $D_v$ and $\nu$, (e) $D_\eta$ and $\nu$, and (f) $D_v$ and $D_\eta$. Panels (a), (b), (d), and (e) show similar behavior. The main dissipation occurs around $\nu \sim -2\nu$, and these distributions are almost symmetrical with respect to the $\nu = -2\nu$ axis. Although no significant difference can be seen between the distribution of the kinetic and magnetic dissipations, panels (c) and (f) show that the correlation of the kinetic and magnetic dissipation is not large. When there is strong kinetic dissipation ($\epsilon_v$ or $D_v$), strong magnetic dissipation ($\epsilon_\eta$ or $D_\eta$) is hardly seen, and vice versa.
4. Summary and Discussion

We perform high-resolution simulations to investigate the inhomogeneity of the kinetic and magnetic dissipations. Our main conclusions are as follows.

1. More dissipation is observed around the bottom of the calculation domain than at the middle. This tendency is promoted with a magnetic field with an efficient small-scale dynamo around the bottom of the calculation domain.

2. Dissipation in the downflow region is dominant. The ratio (downflow/upflow) is increased with the magnetic field, and a small magnetic Prandtl number produces a small ratio.

3. Although 2D probability density functions show a similar distribution between kinetic and magnetic dissipation, the precise location of the strong dissipation is different; i.e., the strong kinetic dissipation does not occur at the location where the strong magnetic dissipation occurs, and vice versa.

Figure 8 shows the root-mean-square (rms) entropy perturbation. The results in the Hydro (black), Large Pm (blue), and Small Pm (red) phases are shown. The values are typically $\Delta \rho$ in the middle of the calculation domain. The difference in dissipation between upflow and downflow is typically $\Delta \epsilon \sim \rho_0 v^2 / H_b$ (see panel (a) of Figure 5). This suggests that the difference in dissipation is large enough to diminish the thermal structure in the timescale of $\Delta t \sim 2H_b / v$, which is twice the turnover time of the convection cell. Although the thermal structure is not diminished because of continuous generation by the thermal convection itself, the difference in convection should not be ignored and should have a significant role in determining the thermal structure. The kinetic and magnetic energies are dissipated in the downflow with a magnetic field, indicating that the inhomogeneity of the dissipation tends to decrease the rms entropy and temperature perturbation. Figure 8, however, shows an increase in entropy perturbation with the magnetic field. Hotta et al. (2015a) show that a magnetic field suppresses the mixing between the downflow and upflow, and increases

Figure 7. Two-dimensional (2D) probability density functions (PDFs) of the dissipations ($\epsilon_\nu$, $\epsilon_\eta$, $D_\nu$, and $D_\eta$) and the vertical velocity ($v_x$). Each panel shows a PDF of (a) $\epsilon_\nu$ and $v_x$, (b) $\epsilon_\eta$ and $v_x$, (c) $\epsilon_\nu$ and $v_x$, (d) $D_\nu$ and $v_x$, (e) $D_\eta$ and $v_x$, and (f) $D_\nu$ and $D_\eta$ PDFs are calculated with the result in the Large Pm phase at $x = 0.9H_b$. A PDF of variables $a$ and $b$ ($F_{ab}$) are normalized to satisfy a relation $\int \int F_{ab} \mathrm{d}a \mathrm{d}b = 1$.

Figure 8. $\rho_0 T_0 s_{rms}$ shown for the Hydro (black), Large Pm (blue), and Small Pm (red) phases.
the entropy perturbation. As many mechanisms are involved in determining the thermal structure, the change in dissipation cannot directly change it. When we mimic a magnetic field with large viscosity in low-resolution calculations, we need to keep in mind the fact that the change in dissipation by a strong magnetic field may not be reproduced only with the enhanced viscosity.

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