Hydrodynamic Effects on the QPO Frequencies of Accreting Compact Objects

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ABSTRACT

The variability properties of accreting compact objects predicted by dynamical models are characterized by a number of distinct frequencies that are specific to each model. Because the accretion disks around neutron stars and black holes are hydrodynamic flows, these modulation frequencies cannot be strictly equal to test particle frequencies. I discuss several implications of hydrodynamic corrections to the modulation frequencies predicted by dynamical models. Finally, I show that the recent detection of yet a third kHz QPO in three neutron-star systems favors a previously developed model that attributes the various QPO frequencies to fundamental general relativistic frequencies in the accretion flow.

Subject headings: accretion, accretion disks — stars: neutron — stars: rotation — X-rays: stars

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1. INTRODUCTION

The accretion flows around compact objects are turbulent and are, therefore, expected to be highly variable. The frequencies that correspond to the dynamical timescales can be as high as \( \simeq 1 \) kHz and depend strongly on the distance from the central object. For example, for a Kerr spacetime and in the limit of slow rotation, the Keplerian orbital frequency at a radius \( r \) scales as \( \sim r^{-3/2} \), the periastron precession frequency as \( \sim r^{-5/2} \), whereas the nodal precession frequency (if it is caused by frame dragging) scales as \( \sim r^{-3} \) (see, e.g., Perez et al. 1997; Stella, Vietri, & Morsink 1999). Because of such strong dependences, it is surprising that the observed power-density spectra of accreting systems are characterized by a distinct number of frequencies in the form of relatively coherent \( (\Delta \nu/\nu \gtrsim 10^{-2}) \) quasi-periodic oscillation (QPO) peaks.

Modeling theoretically the presence of multiple QPO peaks is challenging. Current models seek to identify characteristic radii in the accretion flow that pick only a small range of frequencies and hence produce narrow QPO peaks in the power spectra. Such radii include the radius of the
innermost stable circular orbit (which is important in the diskoseismic models of, e.g., Nowak & Wagoner 1991; Perez et al. 1997; Silbergleit & Wagoner 2000), the magnetospheric radius (Alpar & Shaham 1985, Strohmayer et al. 1996), or the sonic-point radius (Miller, Lamb, & Psaltis 1998). The simultaneous presence of low- and high-frequency QPOs is then attributed either to multiple characteristic radii in the accretion flow (as discussed, e.g., in Miller et al. 1998; Psaltis et al. 1999b) or to different characteristic frequencies occurring at the same region in the flow, as in the case of the diskoseismic models (see Wagoner 1999) or the relativistic models of Stella et al. (1999) and Psaltis & Norman (2000). In the first class of models, the observed tight correlations between the different QPO frequencies (see, e.g., Psaltis et al. 1999b; Psaltis, Belloni, & van der Klis 1999a) may be accounted for by combining the dependence of each QPO frequency on the mass accretion rate (see, e.g., Psaltis et al. 1999b). In the latter case, frequency correlations can be easily reproduced since all QPO frequencies are determined almost entirely by a single parameter (e.g., the characteristic transition radius; Psaltis & Norman 2000).

A characteristic signature of each QPO model is the spectrum of QPO frequencies that it predicts beyond the ones currently detected (see, e.g., Miller 2000). For this reason, the inadequacy of current data to distinguish between different models can be overcome with the detection of additional QPO peaks. Even though some attempts for detecting expected QPO peaks at different frequencies have given negative results (Méndez & van der Klis 2000), a third kHz QPO has been recently discovered in three atoll sources (Jonker, Méndez, & van der Klis 2000). This new QPO is significantly weaker than the other kHz QPOs and has a frequency that is \( \simeq 60 \) Hz higher than that of the previously known lower kHz QPO.

Motivated by this detection, I discuss in this Letter the importance of the hydrodynamic corrections to the predicted QPO frequencies that are introduced by the previously developed dynamical model that identifies the various QPO frequencies with general relativistic frequencies in the accretion disk (Psaltis & Norman 2000). I then use this property to show that the model can account for the observed properties of the newly discovered QPO.

2. HYDRODYNAMIC EFFECTS ON THE PREDICTED QPO FREQUENCIES

The high frequencies that characterize the observed QPOs in neutron star sources imply that the accretion flows around them must be modulated at the dynamical timescale. A simple estimate of the three characteristic dynamical timescales at distance \( r \) away from a compact object is given by the three test-particle frequencies, i.e., the orbital frequency in the azimuthal direction, the epicyclic frequency in the radial direction, and the oscillation frequency in the vertical direction. For a slowly rotating compact object these frequencies are (see, e.g., Perez et al. 1997)

\[
\Omega^2 = \frac{M}{r^3[1 + \alpha_*(M/r)^{3/2}]},
\]

\[
\kappa^2 = \Omega^2(1 - 6M/r),
\]
and

$$\Omega_\perp^2 = \Omega^2[1 - 4\alpha_s(M/r)^{3/2}]$$, \hspace{1cm} (3)

respectively, where $M$ is the mass and $\alpha_s$ the specific angular momentum per unit mass of the compact object, and I have set the fundamental constants to $c = G = 1$. The resulting orbital frequency ($\Omega/2\pi$), periastron precession frequency $[(\Omega - \kappa)/2\pi]$, and nodal precession frequency $[(\Omega - \Omega_\perp)/2\pi]$ were shown to be comparable to and follow similar correlations with the three observed QPO frequencies (Stella & Vietri 1998, 1999; Stella et al. 1999).

An accretion disk, however, is a hydrodynamic flow and hence the test-particle frequencies (1)–(3) cannot be strictly equal to the frequencies at which the flow is modulated. For example, the frequencies of the trapped $g$– and $c$–modes in the model discussed in Wagoner (1999) are determined mostly by the maximum of the radial epicyclic frequency (eq. [2]) and the nodal-precession frequency ($\Omega - \Omega_\perp$) but also depend weakly on the local sound speed. In the case of the model discussed in Psaltis & Norman (2000), the inner accretion disk is modulated mainly at three characteristic frequencies,

$$2\pi f_1 = \Omega - \Omega_\perp \left[1 + \frac{\Omega_c^2(\Omega_\perp^2 - \kappa^2)}{2\Omega_d^2\Omega_\perp^2 + 2(\kappa^2 - \Omega_\perp^2)^2}\right]$$, \hspace{1cm} (4)

$$2\pi f_2 = \Omega - \kappa \left(1 + \frac{\Omega_c^2}{2\kappa^2}\right)$$, \hspace{1cm} (5)

and

$$2\pi f_3 = \Omega$$, \hspace{1cm} (6)

where $\Omega_c \equiv c_s/r$, $c_s$ is the speed of sound, and $\Omega_d \sim u_r/r$ is the inverse radial drift timescale at the transition radius $r$. In the notation of Psaltis & Norman (2000), the mode ($m = 1, n = 0$) provides the biggest contribution to the peak at frequencies $f_2$ and $f_3$, whereas the mode ($m = 1, n = 1$) provides the biggest contribution to the peak at frequency $f_1$.

It is important to note here that the corrections to the three dynamical frequencies introduced by the model are specific to the fact that they describe the modulation frequencies of the density in the inner disk and depend on the details of the calculation. For example, because of the approximations used, the effects of radial pressure forces, which will affect the hydrodynamic corrections, have not been taken explicitly into account. The corrections introduced to the modulation of the X-ray luminosity will be different in detail but similar in magnitude, if such effects are taken into account. However, hydrodynamic corrections are generic to a dynamical model and have three important implications.

First, the observed QPO frequencies will not be strictly equal to the general relativistic frequencies; the corrections can be as large as $10 - 30\%$, depending on the local sound speed and the viscosity in the accretion flow (see, e.g., Psaltis & Norman 2000). This property resolves entirely the small but statistically significant discrepancy in the fits of the uncorrected general relativistic frequencies to observations (Psaltis et al. 1999b; Marković & Lamb 2000). An example
Fig. 1.— The effect of hydrodynamic corrections on the frequency difference $f_3 - f_2$, calculated for a non-rotating compact object of mass $2.05M_\odot$, a viscosity parameter $\alpha = 0.2$, a fractional width of the transition region of $\delta r/r = 0.01$, and different values of the disk scale-height $h/r$ (see Psaltis & Norman 2000 for definitions). The data points are for the neutron-star source Sco X-1 and correspond to the peak separation of the kHz QPOs and the frequency of the upper QPO peak (after van der Klis et al. 1997).

is shown in Figure 1, where the predicted frequency difference $f_3 - f_2$ is compared to the changing peak separation of the kHz QPOs in the neutron-star source Sco X-1 (van der Klis et al. 1997), for standard values of the model parameters. The observed decrease of the kHz QPO peak separation with increasing frequency is slower than predicted by the model but can be accounted for with a simultaneous small increase of the disk scale height. The resulting frequencies are in good agreement with the data without the need for introducing characteristic frequencies of elliptical orbits (as originally suggested by Stella & Vietri 1998), which are unlikely to be sustained in an accretion disk. Note here that the results plotted in Figure 1 are for a non-rotating compact object and were not obtained from formally fitting the data, but are shown to illustrate the effect of hydrodynamic corrections. However, detailed comparisons with data of the dynamical frequencies calculated for realistic neutron-star spacetimes were shown to affect only slightly the conclusion (Marković & Lamb 2000).
Second, the centroid frequencies of QPO peaks produced by resonances that correspond to harmonics of general relativistic frequencies are not exactly harmonically related. This is true, because the hydrodynamic corrections are different for the different modes and hence the ratio of the frequencies of the modes is not an integer number but depends on the properties of the flow. Note, however, that if the presence of additional harmonics besides the fundamental is caused by the non-sinusoidal profile of the modulation of the X-ray flux and not by the presence of higher-order modes, then the frequency ratio will be an integer. This might lead to the presence of multiple peaks at nearby frequencies with possibly different FWHMs, as for example observed in the case of Cyg X-1 by Nowak (2000).

Finally, different modes may contribute power to the same QPO peak but at slightly different centroid frequencies, again because of the different hydrodynamic corrections. For example, all the \((m = 1, n \geq 0)\) modes in the analysis of Psaltis & Norman (2000) contribute to the variability power at frequencies comparable to \(\sim f_2\), i.e., to the lower kHz QPO (the contribution of the modes with \(n > 0\) is small and was not explicitly pointed out in the original analysis, even though it was present). Indeed, expanding the general expression for the response function (eq. [27] in Psaltis & Norman 2000) for the \((m = 1, n = 1)\) mode and at frequencies \(2\pi f \sim \Omega - \kappa\), it simplifies to

\[
A_{11} \simeq \left\{ \left[ 1 + \frac{\Omega_c^2}{4\omega_b}\frac{\Omega_d}{(2\pi f - \Omega + \kappa)^2} \right]^2 + \left[ -\frac{\Omega^2_{1} - \kappa^2}{\kappa \omega_b} + \left( \frac{\Omega_c}{2\omega_b} \right)^2 \frac{\Omega_c}{2\pi f - (\Omega - \kappa)} \right]^2 \right\}^{-1/2},
\]

when \((\delta r/r)(\kappa/\Omega_c)^2 \ll 1\). Equation (7) describes a narrow resonant mode centered at a frequency

\[
2\pi f_{2b} \simeq \Omega - \kappa \left[ 1 - \frac{\Omega_c^2}{2(\Omega^2_{1} - \kappa^2)} \right],
\]

and with a FWHM of

\[
\delta f_{2b} \simeq \frac{\sqrt{3}}{2\pi} \omega_b \left( \frac{\Omega_c^2}{\Omega^2_{1} - \kappa^2} \right),
\]

where \(\omega_b = (r/\delta r)\Omega_d\) and \(\delta r/r\) is the fractional width of the transition region. This resonance is to be compared to the corresponding resonance of the \((m = 1, n = 0)\) mode, which is described by (see, eq. [35] of Psaltis & Norman 2000)

\[
A_{10} \simeq \left\{ 1 + \left[ \frac{\kappa}{\omega_b} + \left( \frac{\Omega_c}{2\omega_b} \right)^2 \frac{\Omega_c}{2\pi f - (\Omega - \kappa)} \right]^2 \right\}^{-1/2},
\]

has a peak at frequency \(f_2\) and a FWHM of

\[
\delta f_2 \simeq \frac{\sqrt{3}}{2\pi} \omega_b \left( \frac{\Omega_c^2}{\kappa^2} \right),
\]

when \((\delta r/r)(\kappa/\Omega_{r\text{mc}})^2 \ll 1\).
The response function for modes \((m = 1, n = 0)\) and \((m = 1, n = 1)\) at high frequencies, calculated for \(\Omega/2\pi = 1138\) kHz, \(\kappa/2\pi = 280\) Hz, \((\Omega - \Omega_\perp)/2\pi = 50\) Hz, \(\Omega_c/2\pi = 200\) Hz, and \(\omega_b/2\pi = 50\) Hz. The parameters correspond to typical values of the hydrodynamic corrections in an accretion disk (see eq. [14]) and were chosen to reproduce the observed power spectrum of the source 4U 1728–34 (Jonker et al. 2000).

Figure 2 shows the resulting response of these lowest-order modes at high frequencies. As it is also apparent from equations (5) and (8), the two peaks at frequencies comparable to \(\simeq f_2\) are displaced relative to each other and, in the particular model considered here, their separation is equal to

\[
\frac{\Omega^2_c}{4\pi\kappa} \left(1 + \frac{\kappa^2}{\Omega^2_\perp - \kappa^2}\right).
\]

As a result, two QPOs at frequencies comparable to that of the lower kHz QPO may appear in the power spectra of accreting compact objects. Note here that the two frequencies are identical in the limit \(\Omega_c \to 0\), i.e., for a purely kinematic model.

The amplitudes of the two QPOs will depend on the power-spectrum of the driving perturbations, which might be different in the two modes that produce the two QPOs, even though their frequencies are very similar. (The maximum of the response function does \textit{not} provide an estimate of the amplitudes of the QPOs.) However, even for a comparable amplitude
of perturbations in both modes, the resonance described by $A_{10}$ will produce the dominant QPO peak (see also Psaltis & Norman 2000), whereas the resonance described by $A_{11}$ will produce a weaker sideband, as Figure 2 suggests. In the absence of any additional broadening mechanisms, the FWHM of the two lower kHz QPOs (at frequencies $f_2$ and $f_{2b}$) will be significantly smaller than the FWHM of the upper kHz QPO (at frequency $f_3$). However, the two lower kHz QPOs may have a difference of up to $\sim (\Omega_\perp/\kappa)^2 \lesssim 15$ in their relative FWHM.

3. DISCUSSION

In the previous section, I discussed a number of implications of the hydrodynamic corrections introduced to the predicted QPO frequencies by dynamical models. Moreover, I showed that the previously developed model that attributes the observed QPOs to general relativistic frequencies in the accretion flow can account for the presence of multiple kHz QPOs, as recently observed by Jonker et al. (2000). In this section I will concentrate to the case of the well studied source 4U 1728−34 (Jonker et al. 2000) and discuss the implications of the newly observed QPOs for various models. In this source, four QPOs are observed simultaneously at the following frequencies: $\nu_1 = 41.5 \pm 0.2$ Hz (the low-frequency QPO), $\nu_2 = 795$ Hz (the lower kHz QPO), $\nu_{2b} = 859 \pm 2$ Hz (the newly discovered QPO), and $\nu_3 = 1138 \pm 2$ Hz (the upper kHz QPO).

According to beat-frequency models (see, e.g., Strohmayer et al. 1996; Miller et al. 1998), the highest QPO frequency $\nu_3$ is that of the Keplerian frequency of a characteristic radius in the accretion disk and the difference $\nu_3 - \nu_2$ is comparable to the spin frequency of the neutron star. In fact, oscillations during Type I X-ray bursts have been detected from 4U 1728−34 at a frequency of $\simeq 363$ Hz, which are also thought to correspond to the spin frequency of the neutron star (Strohmayer et al. 1996). In this picture, the presence of the newly discovered QPO at a frequency that is $\simeq 64$ Hz higher than $\nu_2$ might be accounted for if the radiation that emerges from the neutron star is modulated not only at the stellar spin frequency, but also at a frequency equal to $\simeq 64$ Hz, introducing sidebands to the stellar spin (see, however, Alpar 1986 for an alternative way of generating sidebands in a beat-frequency model, because of the radial drift of the accreting material). As pointed out by Jonker et al. (2000), this latter frequency is not equal to the frequency of the observed QPO at $\nu_1$ and therefore the accretion flow needs to produce modulations at two distinct, unrelated, low frequencies. One such low frequency is the nodal precession frequency $(\Omega - \Omega_\perp)/2\pi$. Note, however, that, as also discussed by Psaltis et al. (1999b) for other similar cases, the QPO at frequency $\nu_1$ cannot be caused by frame-dragging effects if the spin-frequency of the neutron star is equal to $\simeq 363$ Hz. (In the relativistic model described in §2 the frequency $\nu_1$ is related to the frame-dragging frequency, but the peak separation of the kHz QPOs is not equal to the spin frequency of the neutron star.) This can also be extended to the case of the $\simeq 64$ Hz frequency, which is even higher than $\nu_1$. Indeed, such a high nodal precession frequency would require an unrealistically high neutron-star moment of inertia of (see, e.g., Psaltis
et al. 1999b)

\[
\frac{I_{45}}{M/M_\odot} \gtrsim 3.1 \left( \frac{\nu}{64 \text{ Hz}} \right) \left( \frac{\nu_{\text{spin}}}{363 \text{ Hz}} \right)^{-1} \left( \frac{\nu_3}{1138 \text{ Hz}} \right)^{-2},
\]

(13)

where \( I_{45} \equiv I/10^{45} g \text{ cm}^3 \) is the moment of inertia, \( M/M_\odot \) the mass in solar units, and \( \nu_{\text{spin}} \) the spin frequency of the compact object. Even if a frequency of \( \simeq 64 \text{ Hz} \) corresponded to the first overtone of the nodal precession frequency, the required moment of inertia \( [I_{45}/(M/M_\odot) \gtrsim 1.55] \) would be higher than allowed by general relativity for a slowly rotating star, if any of the current equations of state for neutron-star matter were valid up to \( \simeq 1.5 \) times the nuclear saturation density. As a result, none of the two required low frequencies can be equal to the frame-dragging frequency, if the spin frequency of the neutron star in 4U 1728–34 is equal to the burst oscillation frequency.

In the case of the dynamical model described in Psaltis & Norman (2000), the presence of multiple QPO peaks in the power-density spectra is a direct consequence of the fact that in strong-field general relativity the three dynamical frequencies (eq. 13) are significantly different from each other. In particular, all the QPO peaks observed so far can be accounted for by only the two lowest-order modes \((m = 1, n = 0)\) and \((m = 1, n = 1)\).

In this interpretation, the difference \( f_{2b} - f_2 \) depends mostly on the radial epicyclic frequency and the inverse sound-crossing time (see, eq. [12]). For a standard \( \alpha \)-disk, the latter is equal to \( \Omega_c \simeq (h/r)\Omega_\perp \), where \((h/r)\) is the scale-height of the disk (see, e.g., Psaltis & Norman 2000). As a result, the predicted frequency separation between the two lower-kHz QPO peaks becomes

\[
f_{2b} - f_2 \simeq 66 \left( \frac{h/r}{0.12} \right)^2 \left( \frac{\Omega/2\pi}{1138 \text{ Hz}} \right)^2 \left( \frac{\kappa/2\pi}{280 \text{ Hz}} \right)^{-1} \text{ Hz}
\]

and hence the observed frequencies can be easily accounted for, for typical values of the model parameters (see also Fig. 2).

Higher-order modes will also contribute to the variability power at frequencies comparable to \( f_2 \) and may produce additional structure to the lower kHz QPO peak. The overall spectrum of observed modes will depend mostly on the amplitude of the perturbations that drive the variability outside the characteristic transition radius. A detailed analysis of the various predicted resonant frequencies and their amplitudes is beyond the scope of this paper and will be presented elsewhere.

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