Low Scale String Unification and the Highest Energy Cosmic Rays

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Abstract. String unification at a scale of a few tens of TeV explains the existence of cosmic ray interactions beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff. Trans-GZK cosmic rays are neutrinos which can penetrate the cosmic microwave background. In interactions with atmospheric nuclei they have sufficient energy for exciting string modes. We present a model for the description of such interactions and discuss the properties of the resulting extensive air showers. Presently available data on trans-GZK cosmic rays suggest a string scale around 80 TeV.

It has been realized by Witten \cite{Witten} some time ago that in certain strongly coupled string models, the multidimensional (d=10 or 11) string scale and the four dimensional Planck scale are less rigidly coupled than it was previously believed. This insight gave rise to a flurry of papers: several authors pointed out that in such a scenario, the string scale could be of the order of a few TeV see \cite{2,3} for the original papers. Hence, even experiments at the LHC could provide some evidence for the existence of extra dimensions and string excitations. Along the same lines, we pointed out that in such a scenario the problem of the trans-GZK cosmic ray interactions may be resolved assuming that they are caused by high energy neutrinos \cite{4}. In fact, a neutrino penetrates the cosmic microwave background radiation (CMBR) essentially uninhibited. The CMS energy in a collision with a CMBR photon is of the order of 100 MeV: the interaction mfp in the collision is essentially infinite. By contrast, in an interaction with a nucleus in the atmosphere, the CMS energy is of the order of a few hundred TeV: hence, string modes are excited and the cross section grows to a hadronic size.

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Currently there is no string model known to be in agreement with experimental data. In particular, it is hard to calculate within the framework of a strongly coupled theory\(^2\) For this reason, we abstract features of current models which are likely to be present in future, phenomenologically successful theories.

The following basic ingredients are used:

- Unitarity of the $S$-matrix.
- A rapidly rising level density of resonances in dual models.
- Unification of interactions at around the string scale, hereafter denoted by $M$.
- Duality between resonances in a given channel and Regge exchanges in crossed channels.

Concerning the last item, it should be kept in mind that duality between resonances and Regge poles is exact only in the tree approximation to a string amplitude. It is unclear what the precise form of a generalization to world sheets of higher genus is: probably, resonances of finite width are dual to Regge cuts. Thus, our formulæ are likely to be valid to logarithmic accuracy. Using these ingredients and the optical theorem, one obtains that the total cross section in a neutrino-parton interaction is\(^3\)

\[
\hat{\sigma}(\hat{s}) = \frac{8\pi}{\hat{s}} \sum_j (2j + 1) (1 - \eta_j \cos(2\delta_j)), 
\]

where, as usual, $\eta$ and $\delta$ stand for the elasticity coefficient and phase shift of a given partial wave, respectively. The quantity $N(\hat{s})$ is the level of the resonance, equal to the maximal angular momentum.

For elastic resonances, $\eta = 1$ and $\delta \approx \pi/2$ within the width of the resonance. In that case, on resonance the total cross section is just proportional to the number of states at a given level. Due to the finite widths of resonances in any realistic model, it makes sense to average the cross section over an energy interval comparable to the widths of the resonances. In such an

\(^2\)By means of an explicit calculation, it was shown that weakly coupled string models cannot explain the trans-GZK cosmic ray interactions, cf. [5] and [6].

\(^3\)All energies are assumed to be large compared to the rest energies of the incoming particles.
approximation, one can introduce the density of states, \( d(\hat{s}) \) and regard \( N \) a continuous variable, such that \( N \approx \hat{s}/M^4 \). Using this, one gets from eq. (1):

\[
\hat{\sigma} \approx \frac{16\pi}{\hat{s}} d(\hat{s}) \tag{2}
\]

Clearly, as inelastic channels open up, the elasticity coefficients in eq. (1) become less than unity and eq. (2) is no longer valid. Without any detailed knowledge of the inelastic channels (world sheets of a higher genus in present day string models), one can estimate the behavior of the cross section as \( \hat{s} \to \infty \). Duality tells us that the leptoquark excitations should be dual to the exchange of the \( Z \)-trajectory in the \( t \)-channel. Hence, apart from logarithmic corrections,

\[
\hat{\sigma} \sim \hat{s}^{\alpha(0) - 1}, \tag{3}
\]

where \( \alpha(0) \) is the intercept (branch point, respectively) of the \( Z \) trajectory. Apart from corrections of the order of \( (M_Z/M)^2 \), one has \( \alpha(0) = 1 \), so that the neutrino-parton cross section tends to a constant. (We verify a posteriori that \( M_Z/M \ll 1 \), so that the power corrections to the cross section are insignificant at all energies of interest.)

The level density is a rapidly rising function of \( \hat{s} \). It is known that asymptotically it rises as \( \exp(a\sqrt{\hat{s}/M}) \), with \( a \) being some constant; see, for instance [7]. At the beginning of the spectrum, however, the rise is more rapid. The first few levels of the RNS model ([6] (loc.cit) can be well interpolated by the function

\[
d(N) \propto \exp 1.24N, \quad N \approx \hat{s}/M, \tag{4}
\]

see Figure 4. In this figure, the points have been calculated from the generating function of the level density, [4] eq. (4.3.64). Other string models exhibit a similar rapid rise of the level density. Due to one’s inability to carry out detailed calculations in a strongly coupled string model, we chose to interpolate between the low excitation regime, eq (3) and the asymptotic one, eq.(3). There are infinitely many functions, of course, interpolating between those limits: we were guided by a requirement of simplicity. Having experimented with a number of functional forms, we came to the conclusion that, after averaging over the parton distribution within the nucleon, the

\footnote{In the last formula, the Regge intercept has been neglected. However, we shall see shortly that the excitations begin to contribute significantly to the cross section for \( N \geq 10 \) or so; hence this approximation is justified.}
Figure 1: The level density of the RNS model. The points are from ref. [7]; the continuous curve is the best fit to the level density as explained in the text.

results are rather insensitive to the precise form of the $\nu$-quark cross section. For that reason, we chose a simple form satisfying the limits at low and high excitations:

\[
\hat{\sigma} = \Theta \left( \hat{s} - M^2 \right) \frac{16\pi}{M^2} 1 + \frac{\hat{s}}{M^2} \exp \left( 1.24 (N_0 - \hat{s}/M^2) \right)
\]  

In eq. (5), $M$ is the string scale and $N_0$ is a parameter measuring the onset of the “new physics”. In fact, one can convert that dimensionless parameter into an energy scale. Using our previous relations, one can write $N_0 \approx \hat{s}_0/M$, or in terms of a laboratory energy of the incoming neutrino, $N_0 \approx 2m\hat{E}_0/M$, $m$ being the mass of the nucleon. In all these equations, the “hat” over the energies involved serves as a reminder that the quantities have to be integrated over the parton distribution. As usual, the conversion is carried out by means of substitutions such as, $\hat{s} = xs x$ being the momentum fraction of a parton within the nucleon. The step function is inserted because the cross section of the “new physics” vanishes at CM energies below the mass of the first resonance.

Finally, the neutrino-nucleon cross section has to be constructed by integrating eq. (5) over the parton distribution in the atmosphere. In order to
do so, one takes into account the fact that the dominant nuclei in the atmosphere (N, O) contain an equal number of protons and neutrons. The parton distributions have been taken from CTEQ6, [8]. The dominant contribution comes from valence quarks; gluons do not contribute, since no presently known unification scheme contains “leptogluons”. Finally, the contribution of the sea is negligibly small, since the latter is concentrated around $x = 0$.

With the limited amount of information currently available on trans-GZK cosmic ray interactions, it is impossible to precisely determine the two parameters entering eq. (5). Nevertheless, the parameters can be bounded by the data. From a qualitative point of view, the limitations come from the facts that

- No deep showers have been observed by Fly’s Eye and HiRes.
- The trans-GZK showers reported by AGASA, Fly’s Eye and Hi Res appear to be “hadron-like”, i.e. they originate high in the atmosphere and appear to exhibit a development resembling proton induced showers.

Those constraints were analyzed by Sigl et al. and Weiler, [9, 10]. In essence, the absence of deep showers excludes a region of the neutrino cross section, approximately, $0.02 \text{mb} \leq \sigma \leq 1\text{mb}$. The cross section has to grow to roughly hadronic size around the “ankle” in the cosmic ray spectrum, approximately at $5 \times 10^{19} \text{eV}$ and stay of this size or grow slightly. Unless these conditions are satisfied, the neutrino model of trans-GZK cosmic rays fails.

A search of the parameter space yields reasonable values for $E_0$ and $M$: $E_0 \approx 5 \times 10^{10} \text{GeV}$ and $M \approx 80\text{TeV}$ gives a cross section which is rising sufficiently rapidly: thus it avoids the deep shower bound and at the same time, it gives sufficiently large cross sections in the trans-GZK energy region. These values of $E_0$ and $M$ give $N_0 \approx 15.6$ confirming the intuitive expectation. The neutrino-nucleon cross section with these values of the parameters is shown in Fig. (4). It is to be remarked that, due to the exponential dependence of eq. (5) on the parameters, one cannot vary their values over a broad range without getting a contradiction either with the bound on deep showers and/or with the required value of the cross section for trans-GZK showers.

Neutrino induced showers were simulated using the ALPS (Adaptive Longitudinal Profile Simulation) Monte Carlo package authored by Paul T. Mikulski, [11]. Similarly to earlier studies, see, e.g. [12] it was assumed that quarks and leptons are created in comparable numbers in an interaction as
Figure 2: The $\nu$-nucleon cross section calculated from eq. (5) and the CTEQ6 parton distribution. $E_0 = 5 \times 10^{19}$eV, $M = 80$TeV.

long as the CM energy of an interaction remains above $M$. Once the energy drops below $M$, the usual Standard Model cross sections govern the further development of the shower. A qualitative consequence of this feature is that, statistically, neutrino induced showers exhibit larger fluctuations than proton induced ones, see [12]. Detailed results of such a simulation are deferred to a forthcoming publication. Here we show an important characteristic of the model: the depth of the initial interaction and its rms deviation. It is clear from Fig. 5 that once the cross section becomes larger than about 20 mb or so, the shower starts high in the atmosphere. Hence, on an event by event basis, such showers are virtually indistinguishable from hadron induced showers. One will be able to test the validity of the scenario outlined here by a statistical study of the events observed in future detectors, such as EUSO, OWL and the Pierre Auger Observatory.

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5 This is a consequence of the fact that, once in the standard model regime, leptonic interactions have a lower average multiplicity than hadronic ones.
Figure 3: Depth of the initial interaction of a neutrino within the framework of the unified model. The string scale is kept fixed at 80 TeV; the cross section is varied by changing $E_0$.

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