THEORY OF ELASTIC VECTOR MESON PRODUCTION: SOME RECENT DEVELOPMENTS

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Abstract

I review some recent developments in the QCD description of elastic vector meson production, focusing on issues like the meson wave function, the gluon density in the proton, factorisation, the use of parton-hadron duality and the nonperturbative vacuum.

1 Introduction

I will discuss some recent developments in the description of elastic vector meson production at high energy within QCD, restricting myself to the cases where either the photon virtuality $Q^2$ or the vector meson mass $M_V$ is large while the squared momentum transfer $t$ from the proton is small. I will highlight progress and problems in the theory of these processes, concerning issues such as the meson wave function, the gluon density in the proton and factorisation. I shall also spend some time on new approaches, namely the use of parton-hadron duality [1] and a nonperturbative description of the scattering [2]. My aim is not to attempt a detailed comparison of theory predictions with data but rather to indicate where and why the predictions of various authors are different. I will not have time to speak about phenomenological models based on Regge theory.

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Figure 1: One of the Feynman diagrams for $\gamma^* + p \to V + p$ with a vector meson $V = \rho, \rho', \omega, \phi, J/\psi$, etc. More diagrams are obtained by attaching the gluons to the quark loop in different ways.

2 Simple approach

The common picture of most QCD models for elastic vector meson production at high energy is shown in Fig. 1, where I also define the kinematics. At large $\gamma^* p$ c.m. energy $W$ the dominating exchange is the pomeron, which is described by the exchange of two gluons coupling on one side to the proton and on the other to a quark loop to which are attached the virtual photon and the vector meson. The vector meson couples to the quark line of the loop by its wave function $\psi(z, k_T)$. Here $z$ and $k_T$ are defined by a Sudakov decomposition for the quark momentum in the meson, $k = zq' + \zeta p + k_T$.

As a baseline let me present a very simple version of this model, recently used by Cudell and Royen [3]. For the meson it takes a nonrelativistic, constituent quark description, namely a wave function peaked at $z = \frac{1}{2}$ and $k_T = 0$, so that the quark and antiquark with mass $M_V/2$ share the four-momentum of the meson equally. It makes the approximation that the exchanged gluons do not interact and couple directly to the three constituent quarks of the proton, following the work of Low, Nussinov, Gunion and Soper [4].

With these assumptions Cudell and Royen find the $Q^2$- and $t$-dependence of the cross sections for $\rho, \phi$ and $J/\psi$ production in fair agreement with data from HERA, EMC and NMC, except for problems with the fixed tar-
get $J/\psi$ data. Let me remark that the cross sections for longitudinal and transverse photons respectively behave like $\sigma_L \sim 1/Q^6$ and $\sigma_T \sim 1/Q^8$ for $Q^2 \gg M_V^2$, while for nonasymptotic $Q^2$ the behaviour in this variable is more complicated, even in this simple model where a fixed strong coupling $\alpha_s$ was taken. One finds however a ratio $\sigma_L/\sigma_T = Q^2/M_V^2$ for all $Q^2 \geq 0$, which is in disagreement with the data—a point I will come back to in Sec. 4. As the approximation of noninteracting gluons does not allow to describe the energy dependence of the cross section the authors introduce an overall phenomenological factor in $\sigma_L$ and $\sigma_T$, and find consistency with the data if this factor is allowed to depend on $W$, but not on $Q^2$ or $M_V$. Little change in the results is obtained when the propagators of the exchanged gluons are taken as nonperturbative, as was done in earlier work of Donnachie, Landshoff and Cudell [5]. Finally, the authors observe that their model agrees with the data down to $Q^2 = 0$ even for the light mesons $\rho$ and $\phi$. Notice that in this case one has no large virtuality in the quark loop to justify the perturbative treatment of the quarks and their coupling to the photon.

3 Sophistication of the model

Having seen that even the simplest form of the model depicted in Fig. 1 does not fare badly in describing data we will now take a look at several of its refinements.

3.1 Meson wave function

The meson wave function $\psi(z,k_T)$ turns out to be a major source of uncertainty in the predictions of the model. It determines which virtualities dominate the integration over the quark loop and in particular influences the overall normalisation of the cross section, its $Q^2$-dependence and the production ratios for different mesons. Various choices have been made in the literature [2, 3, 4, 5, 6, 7, 8, 9, 10], sometimes leading to appreciable differences in numerical results. For $J/\psi$ photoproduction, as an example, Frankfurt et al. [11] find a suppression of the cross section due to finite $k_T$ in the wave function which is significantly stronger than the one estimated by Ryskin et al. [6]. For $\rho$ electroproduction a wide range of suppression factors has been obtained by Frankfurt et al. [8] according to the choice of $\psi(z,k_T)$, with a particular sensitivity to its large-$k_T$ tail.

On the theory side it is known that as the renormalisation scale for the wave function tends to infinity, corresponding to very large $Q^2$ in the process, the $k_T$-integrated wave function $\int dk_T^2 \psi(z,k_T)$ behaves like $z(1-z)$
due to QCD evolution, both for longitudinal and transverse meson polarisation. This is very different from the \( \delta(z - \frac{1}{2}) \) in the nonrelativistic wave function, but the difficult question is of course to assess how far one is from the asymptotic regime for the values of \( Q^2 \) one has in experiment. Investigations of the wave function using the operator product expansion and QCD sum rules have been made by Ball and Braun \[12\] and Halperin and Zhitnitski \[11\]. The latter find that at \( z \to 0 \) and \( z \to 1 \) the wave function should only depend on \( \frac{k_T^2}{z(1-z)} \), which in particular excludes a factorising ansatz \( \psi(z,k_T) = \phi(z) \cdot \chi(k_T) \). They also argue that from a conceptual point of view there should be no perturbative tail like \( 1/k_T^2 \) in the wave function and that the corresponding corrections to the meson-quark vertex should be explicitly treated as \( \alpha_s \)-corrections to the leading order result.

In the operator product expansion framework the effects of finite \( k_T \) in \( \psi(z,k_T) \) are of higher twist. It is important to realise that there are other higher twist contributions from diagrams where one or more gluons enter the wave function blob in Fig. 1. They correspond to higher fock states of the meson such as \( q\bar{q}g \) and \( q\bar{q}gg \), which have respectively been considered by Halperin \[13\] and Hoodbhoy \[10\]. Moreover, taking only into account a \( q\bar{q} \) wave function \( \psi(z,k_T) \) is not gauge invariant. To see this consider that a finite \( k_T \) involves the transverse component of the operator \( i\partial_\mu \) between quark fields in a matrix element. To recover gauge invariance one must pass to the covariant derivative, \( i\partial_\mu + A_\mu \), whose transverse component involves physical gluon degrees of freedom.

Finally let me mention a problem raised by Frankfurt et al. \[7\] concerning the \( J/\psi \) wave function. Calculating the transition amplitude of a \( J/\psi \) to a timelike photon via a charm quark loop with wave functions \( \psi(z,k_T) \) obtained in various nonrelativistic potential models they find that the region of \( k_T \) larger than the charm mass contributes up to 30% of the loop integral, thus calling into question the consistency of a nonrelativistic description. Note that the transition \( J/\psi \to \gamma^* \to e^+e^- \) is commonly used to fix the normalisation of the wave function from the semileptonic \( J/\psi \) decay width.

### 3.2 Quark mass

Which quark mass should be chosen in the loop of Fig. 1 is another source of uncertainty. Surprisingly this even holds for heavy mesons. Ryskin et al. \[6\] point out that the cross section scales like the eighth power of \( M_{J/\psi}/(2m_c) \), resulting in huge changes of its overall normalisation even if one allows for a modest change in the charm mass \( m_c \). Incidentally, Frankfurt et al. \[7\] advocate to use the running quark mass rather than the pole or constituent
mass.

In the case of light quarks both the choice of current and constituent masses has been made in the literature \cite{6, 8, 5, 9}. Considering the limit of zero quark mass \( m_q \) reveals that light meson production from transverse photons is a higher twist effect \cite{6}: the cross section \( \sigma_T \) is proportional to \( m_q^2 \) in the collinear approximation for the quarks, i.e. if one sets their transverse momentum \( k_T \) to zero in the calculation of the loop and uses the \( k_T \)-integrated meson wave function, corresponding to what one does when using collinear parton densities in DIS. At least one of the quantities \( m_q \) or \( k_T \) must be nonzero to obtain a nonvanishing \( \sigma_T \) from diagrams as shown in Fig. 1.

3.3 Off-diagonal gluon density

We now focus on the lower blob in Fig. 1, the coupling of the two gluons to the proton, including their interaction. If the outgoing proton had the same momentum as the incoming one this blob would be given by the gluon density in the proton. In our reaction this is however not the case: even if the proton has zero transverse momentum after the scattering it has lost a fraction \( x = (M_V^2 + Q^2)/(W^2 + Q^2) \) of its longitudinal momentum, which is necessary to make a timelike vector meson out of a spacelike photon. Thus the longitudinal momentum fractions \( x_1 \) and \( x_2 \) of the gluons with respect to \( p \) (cf. Fig. 1) are not equal, their difference being \( x_1 - x_2 = x \). One can argue that in the leading \( \ln Q^2 \cdot \ln(1/x) \) approximation it is indeed the gluon density \( g(x) \) that describes the blob \cite{14, 15}, as the typical values of \( x_1 \) and \( x_2 \) in the loop integration are of order \( x \) and to leading \( \ln(1/x) \) all such values are equivalent.

The most prominent phenomenological consequence of this description is the strong rise of the cross section in \( W \) when either of the scales \( Q^2 \) or \( M_V^2 \) is hard. At this point I should remark that only the imaginary part of the \( \gamma^* p \to V p \) amplitude is actually calculated from the Feynman diagrams of Fig. 1, cutting them in the \( s \)-channel. The contribution of the real part to the cross section is negligible if the \( W \)-dependence is weak as in the case of soft pomeron exchange, otherwise it can be accounted for in an approximate way \cite{14, 15}.

Beyond leading \( \ln(1/x) \) the quantity describing the blob in Fig. 1 is certainly different from the gluon distribution that contributes to inclusive DIS, and there has been much theoretical interest recently in such so-called “off-diagonal parton densities”. Strictly speaking they are not “densities” by the way, since they correspond to matrix elements of parton fields between different proton states and do not have a probability interpretation. Let me note that their evolution equations differ from the DGLAP equations by their
splitting functions [16, 17]. Some studies have been performed [10, 17] to estimate the difference between the off-diagonal and the usual, diagonal gluon densities, finding no big difference at small $x$, but much remains to be done in this field. An obvious remark is that the dependence of the off-diagonal gluon density on $t$ is specific of its asymmetric kinematics and cannot be predicted by approximating it with the diagonal density measured in other processes.

In the same way as in DIS at small $x$ one can also go beyond the leading $\ln Q^2$ approximation and instead of $g(x)$ consider the unintegrated gluon density $f(x, l_T)$ with a finite transverse momentum $l_T$ of the gluons [6].

The question of which factorisation scale is appropriate in the gluon density is numerically very important since we know from DIS that parton distributions at small $x$ change rapidly with this scale. Generically it is set by the typical virtualities in the quark loop, i.e. by $Q^2$ and the quark mass, but various concrete choices have been advocated in the literature: for $J/\psi$ production the scale $\frac{1}{4}(Q^2 + M_{J/\psi}^2)$ used by Ryskin et al. [6] is smaller than that of Frankfurt et al. [7], whereas Nemchik et al. [8] choose $Q^2 + M_{V}^2$ times a factor between 0.07 and 0.2 depending on the meson and its polarisation. Let me however emphasise that any choice of factorisation scale can only be an educated guess of which value will make higher order corrections small. As in other reactions such as for instance jet production experiment should feel free to try different scales $\text{const} \cdot (Q^2 + M_{V}^2)$, and also $\text{const} \cdot Q^2$ for light mesons.

### 3.4 Factorisation theorem

For the asymptotic limit where $Q^2$ is much larger than all masses in the process a factorisation theorem has been proven by Collins et al. [8] within perturbative QCD. By an analysis of Feynman diagrams and power counting arguments they find that to all orders in $\alpha_s$ the amplitude for $\gamma^* p \rightarrow V p$ factorises into a hard scattering part, a collinear, off-diagonal quark or gluon distribution in the proton, and a collinear $q\bar{q}$ wave function for the vector meson (see Fig. 2). The corresponding amplitude has a power behaviour like $1/Q$.

This power behaviour and factorisation property holds only for a longitudinal photon. If the $\gamma^*$ is transverse the authors find that the amplitude only behaves like $1/Q^2$, and that there is no factorisation like in Fig. 2: soft, “wee” partons can bypass the hard scattering and directly go from the proton blob $A$ to the meson blob $B$. The power behaviour in $Q$ confirms from a different point of view the statement that light meson production from transverse photons is a higher twist process.
Figure 2: Factorisation of the amplitude for $\gamma^* p \rightarrow V p$ with a longitudinal photon into a hard scattering part $H$, an off-diagonal quark or gluon density $A$ and a meson wave function $B$.

It is worth noting that these findings are not restricted to the small-$x$ region but valid for general $x$, where the quark distribution in the proton is relevant in addition to that of the gluons.

Finally it is clear that the direct phenomenological application of these results requires “sufficiently large” values of $Q^2$, and we have already seen that higher twist effects such as the transverse momentum in the meson wave function can be numerically important in the HERA regime. Also one should keep in mind that the $Q^2$-behaviour obtained by power counting can be strongly masked by logarithmic scaling violations which are strong at small $x$.

3.5 Rescattering effects

Let me just briefly mention that several papers \cite{6, 7, 9, 19} have investigated the effects of rescattering of the $q\bar{q}$-pair on the proton, in other words shadowing corrections or multiple-pomeron exchange, and found that they can be important in kinematic situations accessible at HERA. An observable that is naturally sensitive to such effects is the $t$-dependence of the cross section, which has been investigated in detail by Gotsman et al. \cite{19}.

In this context one may remark that an exponential parametrisation $d\sigma/dt \propto \exp(bt)$ at small and moderate values of $t$ is no more than a fit to a simple function: from theory one does not expect such a behaviour to be exact. Remember for instance that ordinary elastic form factors of hadrons
are not exponentials. One should be aware that just comparing an experimentally fitted slope parameter $b$ with the logarithmic slope of $d\sigma/dt$ at $t = 0$ calculated in theory can be misleading. To compare the full $t$-dependence is of course the most thorough way to proceed, but if one looks for convenient “handy” parameters other choices than $b$ might be useful, such as the mean value $\langle t \rangle$ proposed by Cudell and Royen [3].

4 The ratio $R = \sigma_L/\sigma_T$

As I have already emphasised the physics of light meson production is quite different with transverse and longitudinal photon polarisation. It turns out that $\sigma_T$ is more sensitive to small quark virtualities and thus to infrared physics than its counterpart for longitudinal photons [1, 8, 9]. This can be understood from the properties of the transition $\gamma^* \to q\bar{q}$: while for a longitudinal photon configurations are preferred where the longitudinal momenta of quark and antiquark are comparable, a transverse photon likes to split into a $q\bar{q}$-pair where the quark or antiquark carries only a small fraction of the photon momentum and is soft. This is the aligned jet configuration, which is of prime importance for the physics of diffraction. The interaction with the gluons hardly changes the longitudinal momenta of $q$ and $\bar{q}$, so that this configuration corresponds to small $z$ or $1 - z$ in the meson wave function. To which extent $\sigma_T$ is dominated by the soft region thus depends on how strongly small $z$ or $1 - z$ are suppressed in $\psi(z, k_T)$.

In the simple model presented in Sec. 2, where a meson wave function peaked at $z = \frac{1}{2}$ is used, the ratio $R$ comes out as $Q^2/M_V^2$, which is far too big to describe the data for $Q^2$ in the range of some GeV$^2$. In the model of Nemchik et al. [9] the greater infrared sensitivity of $\sigma_T$ is reflected in a lower factorisation scale of the gluon density than in $\sigma_L$, and the authors find that $R$ grows less fast than linearly in $Q^2$ and that its rate of growth in $Q^2$ depends on $x$.

4.1 Using parton-hadron duality

To circumvent the uncertainties associated with the meson wave function in this delicate context, Martin et al. [1] invoke parton-hadron duality: to obtain the cross section for $\rho$-production they calculate the diffractive production of an open $q\bar{q}$-pair by two-gluon exchange, project out the appropriate $q\bar{q}$ partial wave to make a vector meson of given helicity, and integrate the corresponding cross section over the $q\bar{q}$ invariant mass $M_{q\bar{q}}$ in an interval around $M_\rho$. The authors stress that in this mass range partonic final states
other than $q\bar{q}$ are heavily suppressed and that the dominating hadronic final state is a pair of pions. In the ratio $R$ several uncertainties are expected to cancel to a large extent, e.g. uncertainties about the appropriate interval of $M_{q\bar{q}}$ and about radiative corrections (the authors estimate that there should be a large $K$-factor). As in the calculation by Nemchik et al. \[9\] $\sigma_T$ is dominated by lower quark virtualities than $\sigma_L$, now in the context of open $q\bar{q}$-production, and through the different factorisation scales in the gluon density the authors obtain a ratio $R$ that grows less than linearly in $Q^2$ and is in fair agreement with HERA data for $R$ in the range $5 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$. Again this ratio is predicted to depend on $x$.

5 A nonperturbative model

To finish let me present an approach from a perspective of nonperturbative physics. The model of Dosch et al. \[2\] starts from the high-energy scattering of a quark or antiquark, which is described as the scattering in an external gluon field \[20\], like in the semiclassical model of diffraction of Buchmüller and Hebecker \[21\]. This gluon field is then averaged over in the sense of a path integral, according to the stochastic model of the nonperturbative QCD vacuum \[22\]. High-energy scattering thus becomes related with nonperturbative parameters such as the SVZ gluon condensate.

The nonabelian character of the gluon field is crucial in this model: not single quarks or antiquarks are scattered but rather colour singlet configurations of $q\bar{q}$ or $qqq$, their transverse separation having a strong influence on the strength of the scattering. The physical picture for vector meson production is then as shown in Fig. 3. For the three-quark wave function of the proton and the $q\bar{q}$ wave function of the vector meson an ansatz has to be made, whereas the splitting of the virtual photon into $q\bar{q}$ is calculated perturbatively. As in the simple two-gluon exchange model discussed in Sec. 2 the $W$-dependence of the cross section cannot be predicted in the present form of the model.

Its authors find fair agreement of their results with EMC and NMC data for $Q^2$ between 2 and 10 GeV$^2$ and $W$ between 10 and 20 GeV, looking at observables such as the $Q^2$- and $t$-dependence of the cross section and its normalisation, except that their cross section for the $\phi$ is about a factor 2 above the data. They also describe the ratio $R$ for $\rho$-production measured by NMC and the $t$-dependence of $J/\psi$ photoproduction. They conclude by emphasising the need to incorporate nonperturbative aspects of the photon wave function for small $Q^2$ and $M_V$, and perturbative gluon contributions which they expect to become important as $W$ increases from fixed target to
Figure 3: The process $\gamma^* p \rightarrow V p$ in the model of Dosch et al. [2]. Quarks and antiquarks are scattered in the gluon field of the nonperturbative QCD vacuum.

HERA energies.

6 Conclusions

The model depicted in Fig. 3 is a good candidate for the description of elastic vector meson production at large $Q^2$ or meson mass in QCD. Even a very simple version of this model as presented in Sec. 2 can describe quantitative features of the data.

Progress has been made in the theory of meson wave functions and of the off-diagonal gluon distribution in the proton. For the asymptotic regime of very large $Q^2$ a factorisation theorem has been worked out which further elucidates the different physics of $\sigma_L$ and $\sigma_T$.

On the phenomenological side various difficulties or uncertainties present themselves if one is to make quantitative predictions in the nonasymptotic regime, in particular for light meson production from transverse photons, or if in turn one aims to extract information from the data on the meson wave function or the gluon distribution. An important task is to identify observables which are sensitive to only a few theoretical effects.

Finally, as I showed in Sec. 4.1 and 5, there are promising alternative approaches, which highlight the connection of this reaction with open $q\bar{q}$-production, and with nonperturbative physics and the QCD vacuum.
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