Penta-Quark Anti-Decuplet in Anisotropic Lattice QCD

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The penta-quark(5Q) $\Theta^+(1540)$ is studied in anisotropic lattice QCD with renormalized anisotropy $a_s/a_t = 4$ for a high-precision measurement. Both the positive and the negative parity 5Q baryons are studied using a non-NK type interpolating field with $I = 0$ and $J = 1/2$. After the chiral extrapolation, the lowest positive parity state is found at $m_\Theta \simeq 2.25$ GeV, which is too heavy to be identified with $\Theta^+(1540)$. In the negative parity channel, the lowest energy state is found at $m_\Theta \simeq 1.75$ GeV. Although it is rather close to the empirical value, it is considered to be an NK scattering state rather than a localized resonance state.

LEPS group at SPring-8 discovered the first manifestly exotic hadron $\Theta^+$ at $1.54 \pm 0.01$ GeV with a width smaller than 25 MeV \cite{1}. The experimental discovery \cite{1} was motivated by a theoretical prediction \cite{2}. $\Theta^+$ is confirmed to have baryon number $B = 1$, charge $Q = +1$ and strangeness $S = +1$, which means that it is a baryon containing at least one $\bar{s}$. Hence, its simplest configuration is $uudd\bar{s}$, i.e., the penta-quark (5Q) state. There have been an enormous number of theoretical studies \cite{3,4} on $\Theta^+$ since its discovery. One of the most important topics in 5Q studies is its parity. Experimentally, the parity determination of $\Theta^+$ is quite challenging \cite{5,6}, while opinions are divided in the theoretical side \cite{8}.

There are several lattice QCD studies of $\Theta^+ \cite{7,8,9,10}$, which have not yet reached a consensus. Except for Ref. \cite{7}, all other calculations suggest that negative parity states are lighter than positive parity ones, and that the positive parity states are quite massive. Ref. \cite{9} has employed the NK-type interpolating field and found no signal on a 5Q resonance, whereas Refs. \cite{7,8} have employed non-NK type interpolating fields and claimed the existence of a 5Q resonance with negative parity. There is another type of lattice QCD studies of the static 5Q potential \cite{11} aiming at providing physical insights into the structure of penta-quark baryons.

In this paper, we study $\Theta^+$ for both parities with high-precision data generated by using the quenched anisotropic lattice QCD. We employ the standard Wilson gauge action at $\beta = 5.75$ on the $12^3 \times 96$ lattice with the renormalized anisotropy $a_s/a_t = 4$. The anisotropic lattice technique is known to work as a powerful tool for high-precision measurements \cite{12,13,14,15}. The lattice spacing is determined from the static quark potential adopting the Sommer parameter $r_0 = 0.5$ MeV leading to $a_s^{-1} = 1.100(6) \text{ GeV}$ ($a_s \simeq 0.18$ fm) \cite{13}. The lattice size $12^3 \times 96$ amounts to $(2.15 \text{ fm})^3 \times 4.30 \text{ fm}$ in the physical unit. For the quark part, we employ the $O(a)$-improved Wilson (clover) action \cite{13} with four values of hopping parameters as $\kappa = 0.1210(0.0010)0.1240$, which correspond to $m_q/m_\rho = 0.81, 0.77, 0.72$ and 0.65. By keeping $\kappa_s = 0.1240$ fixed for s quark, we change $\kappa = 0.1210 - 0.1240$ for u and d quarks for chiral extrapolation. Anti-periodic boundary condition (BC) is imposed on the temporal direction, whereas periodic BC is imposed on the spatial directions for quark fields. To enhance the low-lying spectra, we adopt a smeared source with the gaussian size $\rho \simeq 0.4$ fm. We use 504 gauge configurations to construct correlators of $\Theta^+$. For detail, see Ref. \cite{10}.

We consider a non-NK type interpolating field

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for $\Theta^+$ as

$$O \equiv \epsilon_{abc}\epsilon_{adg} \left( u_d^T C \gamma_5 d_e \right) \left( u_f^T C d_g \right) \left( Cs_f^T \right), \quad (1)$$

where $a-g$ denote color indices, and $C \equiv \gamma_4 \gamma_2$ denotes the charge conjugation matrix. The quantum number of $O$ is spin $J = 1/2$ and isospin $I = 0$. Under the spatial reflection of the quark fields, i.e., $q(t, \vec{x}) \rightarrow \gamma_4 q(t, -\vec{x})$, $O$ transforms exactly in the same way, i.e., $O(t, \vec{x}) \rightarrow +\gamma_4 O(t, -\vec{x})$, which means that the intrinsic parity of $O$ is positive. Although its intrinsic parity is positive, it couples to negative parity states as well [17].

We consider the asymptotic behavior of the zero-momentum projected correlator as

$$G_{\alpha\beta}(t) \equiv \frac{1}{V} \sum_{\vec{x}} \left\langle O_{\alpha}(t, \vec{x}) O_{\beta}(0, \vec{0}) \right\rangle, \quad (2)$$

where $V$ denotes the spatial volume. In the region of $0 \ll t \ll N_t$ with $N_t$ being the temporal lattice size, the correlator is decomposed into two parts as

$$G(t) = P_+ \left( C_+ e^{-m_+ t} - C_- e^{-m_- (N_t - t)} \right) + P_- \left( C_- e^{-m_- t} - C_+ e^{-m_+(N_t - t)} \right), \quad (3)$$

where $m_{\pm}$ refer to the energies of lowest-lying states in positive and negative parity channels, respectively. $P_{\pm} \equiv (1 \pm \gamma_4)/2$ serve as projection matrices onto the “upper” and “lower” Dirac subspaces, respectively, in the standard Dirac representation. Eq. (3) suggests that, in the region of $0 \ll t \ll N_t/2$, the backwardly propagating states can be neglected. Hence, “upper” Dirac subspace is dominated by the lowest-lying positive parity state, whereas “lower” Dirac subspace is dominated by the lowest-lying negative parity state. We utilize this property for parity projection.

In Fig. 1 we show the effective mass plots for both parity channels, which are obtained from a correlator with a smeared source and a point sink, adopting a typical set of the hopping parameters as $(\kappa_s, \kappa) = (0.1240, 0.1220)$. For both channels, we find plateaus in the region $25 \leq t \leq 35$. We simply neglect the data for $t > 35$, where backwardly propagating contributions are seen to become less negligible. The single-exponential fit is performed in the plateau region. The results are denoted by solid lines. The dotted lines indicate the p-wave (s-wave) NK thresholds for positive (negative) parity channels on the spatial lattice size $L \simeq 2.15$ fm. In Fig. 2 the masses of positive (triangle) and negative (circle) parity $\Theta^+$ are plotted against $m_n^2$. The open symbols denote direct lattice data. We find that the data behaves linearly in $m_n^2$. Such a linear behavior against $m_n^2$ is also observed for ordinary non-PS mesons and baryons [13,14]. We extrapolate the lattice data linearly to the physical quark mass region. The results are denoted by closed symbols. For convenience, we show p-wave (upper) and s-wave (lower) NK
threshold with dotted lines.

In the positive parity channel, the chiral extrapolation leads to $m_\Theta = 2.25$ GeV, which is much heavier than the experimentally observed $\Theta^+ (1540)$. In contrast, in the negative parity channel, the chiral extrapolation leads to $m_\Theta = 1.75$ GeV, which is rather close to the empirical value. However, from a recent progress using a new general method with a “hybrid boundary condition”, we have concluded that it is an NK scattering state. For detail, see Ref. [10].

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