1. Introduction

Recently, AUV is frequently used to carry out a variety of missions including exploration for the ocean floors or military purposes. But there are many difficulties to control AUV partly due to complex nonlinear dynamics and partly due to the sever and unpredictable environment in the ocean. Generally, the controlling scheme of an AUV are categorized into three categories such as heading control, dive plane control, and speed control. We only considered the dive plane dynamics for depth control in this thesis.

Many control techniques have been proposed for the depth control of AUV. Kadam et al.[3] linearized and approximated the overall depth control system of AUV as IPDT(Integral Plus Dead Time) system, and
tuned PD (Proportional-Derivative) controller with disturbance observer. But they did now show the disturbance rejection ability explicitly. Park et al.\cite{4} suggested PD controller control the vehicle pitch in an inner loop controller, and P controller control the depth of AUV as an outer loop controller. Vahid et al.\cite{6} proposed the same P+PD controller in order to promote the control ability of PD controller, but this method had the difficulty in tuning the controller and inability to disturbance rejection. Hong et al.\cite{8} used P+SMC(Sliding Mode Control) as feedback controller and adaptive feedforward controller to compensate the pitch angle, but this overall control system was too complicated to realize.

The FLC (Fuzzy Logic Control)\cite{12} and Neuro-fuzzy controller\cite{13} are adequate for the complex industrial process, but they may not give the excellent control performance if the controlled plant has uncertainty and high nonlinearity. Moreover, Traditional FLC showed the steady state error if the type of the controlled system is 0-type.

Ma et al.\cite{2} proposed SMC (Sliding Mode Control) and Kadar et al.\cite{9} suggested the DSMC (Discrete SMC) to control the depth of AUV. In spite of the SMC is robust in the sliding mode, the equivalent control input depends on the nominal parameters of AUV in the approaching mode. So the SMC may not guarantee the robust in case of severe parameter uncertainties.

In this thesis, we proposed robust tracking control of the depth control for AUV to tackle the tracking the desired input and the rejecting disturbance simultaneously. The technique is developed based an error dynamic equation using state feedback, the closed-loop control system will have the desired poles using the pole placement theory. So the asymptotic stability of the overall system is guaranteed by Lyapunov function, and it have the ability of tracking the desired input, disturbance rejection and robustness property. The simulation results showed that the proposed controller have better control performances than the results of P+PD in presence of environmental disturbances and uncertainties.

2. Dynamic equations of AUV

The motion of an AUV can be derived by six degrees of freedom differential equations of motion\cite{1,2} using two coordinate frames shown in Fig. 1 and the parameters listed in Table 1.

The positions vector $\eta_1 = [x \ y \ z]$ and Euler angles vector $\eta_2 = [\phi \ \theta \ \psi]$ are defined in the earth-fixed coordinate frame, the velocity vector $\nu_1 = [u \ v \ w]$ and the angular velocity vector $\nu_2 = [p \ q \ r]$ are defined in the body-fixed coordinate frame respectively.

The forces vector $\tau_1 = [X \ Y \ Z]$ and the moments vector $\tau_2 = [K \ M \ N]$ are defined in body-fixed frame.

![Fig. 1 Reference Frame of AUV\cite{2}](image)

| Table 1 Parameters of AUV\cite{2} |
|-----------------------------------|
| **DOF** | **Motion** | **Force & Moment** | **Velocity** | **Position & Euler angle** |
|---|---|---|---|---|
| 1 | surge | $X$ | $u$ | $x$ |
| 2 | sway | $Y$ | $v$ | $y$ |
| 3 | heave | $Z$ | $w$ | $z$ |
| 4 | roll | $K$ | $p$ | $\phi$ |
| 5 | pitch | $M$ | $q$ | $\theta$ |
| 6 | yaw | $N$ | $r$ | $\psi$ |
The linear velocity vector of AUV with respect to earth-fixed frame can be obtained by the time rate of the displacements as follows:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = J_1(\eta_2) \nu_1 = J_1(\eta_2) \begin{bmatrix}
u \\
w
\end{bmatrix}
\]  

where \( J_1(\eta_2) \) is the transformation matrix between two coordinate frame.

\[
J_1(\eta_2) = \begin{bmatrix}
c\psi & -s\psi & 0 \\
s\psi & c\psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The linearized equation of motion in depth plane is given as (5) and (6) from (2) and (3).

\[
\begin{bmatrix}
c\psi & -s\psi & 0 \\
s\psi & c\psi & 0 \\
\end{bmatrix}
\begin{bmatrix}
c\phi & c\psi & s\phi \\
-s\phi & c\phi & 0
\end{bmatrix}
\begin{bmatrix}
u \\
w
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & c\phi & -s\phi \\
0 & s\phi & c\phi
\end{bmatrix}
\begin{bmatrix}
u \\
w
\end{bmatrix}
\]

Therefore, the simplified equation of motion in depth plane can be written by assuming the origin of the body-fixed frame is center of mass as follows:

\[
m(\omega - u_0q) = \sum Z
\]

\[
= Z_x \dot{w} + Z_y \dot{q} + Z_w w + Z_q q + Z_\delta \delta_s
\]  

\[
I_y \dot{q} = \sum M
\]

Together the equations (7)-(10) can be conveniently written in a matrix form as:

\[
\begin{bmatrix}
m - Z_w & -Z_q & 0 & 0 & 1 & \dot{w} \\
-M_w & I_y - M_q & 0 & 0 & 0 & \dot{q} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
Z_\delta \\
M_\delta \\
\delta_s
\end{bmatrix}
\]

\[
\begin{bmatrix}
-Z_w & m u_0 & -Z_q & 0 & 0 \\
-M_w & -M_q & M_\theta & 0 & 0 \\
0 & -1 & 0 & 0 & 0 \\
-1 & 0 & u_0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{w} \\
\dot{q} \\
\dot{\theta} \\
\dot{\delta}_s
\end{bmatrix}
= \begin{bmatrix}
Z_\delta \\
M_\delta \\
\delta_s
\end{bmatrix}
\]

where \( \dot{w} \) is pitch moment due to \( q \), \( M_q \) is pitch moment due to \( q \), \( I_y \) is vehicle inertia around the pitch axes, \( M_\theta \) is the hydrostatic moment coefficient, \( u_0 \) is the desired reference velocity, and \( M_\delta \) is fin lift coefficient.

According to the parameter in Table 2[6], the linearized equation of motion in depth plane is given as:

\[
\dot{z} = -u_0 \theta + w
\]  

\[
\dot{\theta} = \cos \theta q
\]  

\[
\dot{\delta}_s = q
\]

According to the parameter in Table 2[6], the linearized equation of motion in depth plane is given as:

\[
\dot{z} = -u_0 \theta + w
\]  

\[
\dot{\theta} = \cos \theta q
\]  

\[
\dot{\delta}_s = q
\]
as (13).

\[
\begin{bmatrix}
\dot{q} \\
\dot{\theta} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
-10.6 & -14.8 & 0 \\
1 & 0 & 0 \\
0 & 0 & 20
\end{bmatrix}
\begin{bmatrix}
q \\
\theta \\
z
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \delta_s
\]

(13)

| \text{Parameter} | \text{Value} |
|------------------|-------------|
| \(M_y\)          | -458[kg m^2]\n| \(M_y\)          | 9826.2[kg m^2/s]\n| \(l_y\)          | 469[kg m/s]\n| \(M_z\)          | -1,575[\text{kg m/s}]\n| \(u_0\)          | 20          |

Table 2 Parameter Values of AUV

3. Robust Tracking Controller

In this section, we present the design of robust tracking controller to have the ability to track the step input and to reject the step disturbance.

Consider a dynamic equation given by (14).

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad y = Cx(t)
\]

where \(x\) is the state vector, \(u\) is the input, and \(y\) is the output.

Define the tracking error \(e(t)\) for step input as follows:

\[
e(t) \equiv y(t) - r(t)
\]

Taking the time derivative of equation (15) yields

\[
\dot{e}(t) = y'(t) = C \dot{x}(t)
\]

We define the two intermediate variables as follows:

\[
z = x \quad \text{and} \quad w = \dot{u}
\]

Then an augmented system is given as follows:

\[
\begin{bmatrix}
\dot{e} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
0 & C \\
A & 0
\end{bmatrix}
\begin{bmatrix}
e \\
z
\end{bmatrix} +
\begin{bmatrix}
0 \\
B
\end{bmatrix} w
\]

(18)

If the augmented system in equation (18) is completely controllable, we can find the closed loop system to be as equation (19) using the control feedback of the form as equation (20).

\[
\dot{v}(t) = \begin{bmatrix}
0 \\
-K_1B (A - BK_2)
\end{bmatrix}
\begin{bmatrix}
e(t) \\
z(t)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 \\
-K_1B (A - BK_2)
\end{bmatrix} v(t)
\]

(19)

\[
i = w = -K_1e - K_2z
\]

(20)

where \(K_1\) is scalar, \(K_2\) is \(n \times 1\) vector. \(n\) is the order of the system matrix \(A\).

The characteristic equation associated with equation (18) is as follows:

\[
\det \{\lambda I - (A - BK_2)\} + K_1B C \} = 0
\]

(21)

If all the roots of the characteristic equation place in the left half-plane using pole-placement theory, then the closed-loop system is asymptotically stable for any initial conditions \(v(t_0)\). \(v(t)\) is approaching to 0 as \(t\) is approaching to infinity.

As the augmented error equation have the tracking ability to step input, the steady-state error is zero. Integrating the equation (17) into the equation (21), and the corresponding the block diagram of the closed-loop system including the controller is shown in Figure 2.

In Figure 2, the controller includes one integrator because of the Laplace transformation of step input, so this method is also called the internal model design technique.

\[
u(t) = -K_1 \int_0^t e(\tau)d\tau - K_2 x(t)
\]

(22)

Fig. 2 Integral Controller for a step input
The design procedures of robust tracking controller for any other non-decaying input are the same as step input, the internal model of ramp input has two integrals.

4. Computer Simulation and Discussion

In this thesis, we simulate the depth control for AUV under the various condition using robust tracking control technique.

We first consider the step response of AUV, when the magnitude of step input is $-4$. If we assign the desired poles of the closed-loop characteristic equation as $-1 ± 0.6i$ and $-10$, then $K_1 = 8.0$ and $K_2 = [440.8 246.56 21.4]$. The parameters of P+PD are $k = 1.0$ and $k_p + k_d = 0.4 + 0.15s$ under the same desired poles. The simulation results using robust tracking controller and P+PD controller are shown in Fig. 3(a) and 3(b).

![Fig. 3(a) The Step Response of AUV](image)

![Fig. 3(b) The Detail Step Response in Partial Time](image)

In Fig. 3(b), the step response using robust tracking controller has maximum overshoot $-4.02$ at $5.46$[sec] and the step response using P+PD controller has the maximum overshoot $-4.04$ at $4.0$[sec]. The control performance such as rise time, settling time are almost same, but the response using P+PD controller is a little faster than the response using robust tracking controller.

In order to show the robustness for the step disturbance, we simulate the disturbance response when the magnitude of the disturbance is $+2$. The maximum value of disturbance response using robust tracking controller is about 0.12, and the disturbance is rejected at 3[sec] perfectly. Since the closed-loop transfer function from disturbance to output with P+PD controller is 0-type, so P+PD controller have no ability to handle the disturbance, in this case the final values of step disturbance is 5.

Now, we consider the robustness for the parameter uncertainty. The dynamic equation of the worst case\(^[13]\) in case of 50% variations from the nominal values are as follows:

$$
\begin{bmatrix}
\dot{q} \\
\dot{\theta} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
-5.3 & -7.6 & 0 \\
1 & 0 & 0 \\
0 & -30 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{\theta} \\
\dot{z}
\end{bmatrix} +
\begin{bmatrix}
-1.7 \\
0 \\
0
\end{bmatrix} \delta_s
$$

(23)

Fig. 5 is the step response with or without the parameter uncertainties when the proposed controller is used. In case of parameter uncertainties, the maximum overshoot is $-4.22$ at $3.18$[sec] and the
settling time is almost the same as without parameter uncertainty. This shows the proposed controller is robust in spite of severe parameter uncertainties.

Finally, we will show the simulation results when the disturbance and parameter uncertainties are exist simultaneously. Fig. 6 shows the disturbance response simulation results in the worst case. When the disturbance response simulation results when the parameter uncertainties. The maximum overshoot is $-5.45$ at $1.29$[sec], this value exceeds the sum of the disturbance response and the response when parameter uncertainties. And the output tracks the desired command signal precisely at $6.08$[sec] without steady-state error. This shows that the proposed robust tracking controller is robust amidst severe ocean conditions.

5. Conclusion

In this paper, we proposed the depth control of AUV using robust tracking control technique. Since the behavior of AUV is effected by the poorly known disturbance forces and moments, the depth control law is equipped to track the command signal and to reject disturbance at a time. Since the proposed control system contains the internal model of the Laplace transformation of the command signal and the state feedback controller, the overall closed-loop control system is stable and robust amidst severe ocean environment.

The computer simulation results showed the excellent control performance to track the command signal and the robustness under the severe parameter uncertainties.

Acknowledgment

“This research is supported by Kumoh National Institute of Technology (2018-104-076).”

References

1. Fossen, T. I., Marine Control Systems: Guidance, Navigation, and Control of Ships, Rigs and Unver Vehicles, Marine Cybernetics, Trondheim, Norway, 2002.
2. The Society of Naval Architects and Marine Engineers, Nomenclature for Treating the Motion of a Submerged Body Through a Fluid, Technical and Research Bulletin, No. 1-5, 1950.
3. Ma, S. J., Jun, B. H., Lee, P. M., and Kim, S. B., “Design on Yawing and Depth Controller and Analysis of Disturbance Characteristic about the AUV ISiMi”, Proceeding of the Korea Committee
for Ocean Resources and Engineering Conference, pp. 351–354, 2006.

4. Kadam, S. D. and Tiwari, K. N., “A Simplified approach to tune PD controller for the depth control of an autonomous underwater vehicle”, Communication, Computation and Networking Technologies, pp. 209-212, 2013.

5. Park, S. K., et al., “Design and Field Test of Heading and Depth Control Based on PD Control of Torpedo Type AUV, HW200”, Journal of Institute of Control, Robotics, and Systems, Vol. 21, No. 10, pp. 951-957, 2015.

6. Vahid S., and Javanmard K., “Modeling and Control of Autonomous Underwater Vehicle (AUV) In heading and Depth Attitude via PPD Controller with State Feedback”, International Journal of Coastal & Offshore Engineering, No. 4, pp. 11-18, 2016. ijcoe.org/article-1-81-en.html.

7. Ahn, J. K., Koh, G., and So, M. O., “Nonlinear PD Depth Control for Autonomous Underwater Vehicle”, Journal of Fisheries and Marine Sciences Education, Vol. 31, No. 4, pp. 949-959, 2019.

8. Hong, E. Y., Hong, G. S., and Chitre, M., “Depth Control of an Autonomous Underwater Vehicle, STARFISH,” Conference on Oceans 2010 IEEE, pp. 1-6, 2010.

9. Sarif, N. M., Ngadengon, R., Kadir, H. A., and Jalil, M. H. A., “Depth Control of Autonomous Underwater Vehicle Using Discrete Time Sliding Mode Controller,” Universal Journal of Electrical and Electronic Engineering 6(5B), pp. 96-102, 2019.

10. Reshmi, K. R. G., and Priya, P. S., “A Comparative Study on PID and Fuzzy Logic Controller for Depth Control of an Autonomous Underwater Vehicle,” International Advanced Research Journal in Science, Education and Technology, Vol. 3, No. 3, pp. 336-340, 2016.

11. Xu, B., Pandian, S. R, Sakagami, N., and Petry, F., “Neuro-fuzzy control of underwater vehicle-manipulator systems,” Journal of the Franklin Institute, Vol. 349, No. 3, pp. 1125-1138, 2012.

12. Dorf, R. C., and Bishop, R. H., Modern Control Systems, Pearson Edu., Inc., 13th Ed., 2017.

13. Lee, S., Kawk, J., and Chu, B., “Study of a Leveling Mobile Platform for Take-off and Landing of Unmanned Aerial Vehicles,” Journal of the Korean Society of Manufacturing Process Engineers, Vol. 19, No. 4, pp. 85-92, 2020.