Distributed Joint Sensor Registration and Multitarget Tracking via Sensor Network

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This paper addresses distributed registration of a sensor network for multitarget tracking. Each sensor gets measurements of the target position in a local coordinate frame, having no knowledge about the relative positions (referred to as drift parameters) and azimuths (referred to as orientation parameters) of its neighboring nodes. The multitarget set is modeled as an independent and identically distributed cluster random finite set (RFS), and a consensus cardinality probability hypothesis density (CPHD) filter is run over the network to recursively compute in each node the posterior RFS density. Then, a suitable cost function, expressing the discrepancy between the local posteriors in terms of averaged Kullback–Leibler divergence, is minimized with respect to the drift and orientation parameters for sensor registration purposes. In this way, a computationally feasible optimization approach for joint sensor registration and multitarget tracking is devised. Finally, the effectiveness of the proposed approach is demonstrated through simulation experiments.

I. INTRODUCTION

Distributed multitarget tracking (DMT) on a sensor network made up of low cost and low energy consumption sensors has attracted great interest due to the rapid advances of wireless sensor technology and its wide potential application in both civil and defense fields. The use of such sensor networks can clearly enhance the performance while decreasing cost and facilitating deployment of surveillance systems. The goal of DMT is to achieve scalability and comparable performance with respect to centralized architectures. Exploiting random finite set (RFS) theory [1], [2], generalized covariance intersection (GCI), [3] and consensus [4]–[6], several effective DMT approaches have been proposed [7]–[14] assuming that all sensor nodes in the network had been correctly registered/aligned in a common global reference frame.

In many practical scenarios, however, the problem of sensor registration has to be tackled jointly with target tracking, since, in certain circumstances, it is hard to get accurate knowledge about the positions and/or orientations of the deployed sensor nodes. Most of the existing work on sensor registration is based on two approaches. In the first approach, called cooperative localization, each sensor is provided with direct measurements relative to positions of its neighbors [15]–[22]. Conversely, the second approach is based on exploiting some reference nodes of known positions (also called anchors) in the global coordinate system [23]–[27]. The locations of anchors are assumed known a priori or can be obtained by using global localization technology such as, e.g., Global Positioning System (GPS). Clearly, cooperative localization cannot be applied to situations wherein it is hard to obtain internode measurements, e.g., when it is difficult to directly provide measurements on relative orientations between sensor nodes. The anchor-based approach, instead, can only be used in some specific scenarios where either prior knowledge of the surveillance area is available or signals from the global localization equipment can be received. Conversely, in some specific applications, e.g., underwater or indoor environments, wherein the GPS signal cannot be received, this approach is not viable. It should also be mentioned that there have been interesting contributions to the problem of sensor registration of centralized multisensor systems [2, Ch. 12] exploiting possibly appeared targets as targets of opportunity. Specifically, in [31] and [32], the target state space is augmented to incorporate the sensor bias parameters and then the probability hypothesis density (PHD) filter is adopted to jointly estimate target and bias states. In [33], bias parameters and target states are jointly estimated by following a similar approach adopted for simultaneous localization and mapping (SLAM) [34]; specifically bias parameter particles (like robot trajectories in SLAM) are sampled with some prior distribution and each particle is associated to a bias-conditional PHD of the multitarget state (like map PHDs in SLAM). In [35], also based on the bias prior distribution, the bias and target states are mutually utilized to compensate the estimation process of each other,
and the \textit{generalized labeled multi-Bernoulli} filter is employed as multitarget tracker. In this paper, the interest is for a technique that neither needs sensing the positions of neighbors nor the presence of reference nodes and is able to work in a fully distributed fashion.

In this respect, some interesting techniques have been recently introduced [29], [30]. In particular, Kantas \textit{et al.} [29] exploited online distributed maximum likelihood and expectation maximization methods. The nodes iteratively exchange the local likelihoods based on the message passing (belief propagation) technique. The employed message passing method is well suited for networks with tree topology but cannot guarantee to avoid double counting in networks with cycles, and it is not robust with respect to changes of the network topology. Jiang \textit{et al.} [30] adopted the same strategy for sensor registration as in [29], while employing consensus instead of belief propagation for message passing, thus allowing to cope with networks having cycles and time-varying topology. Both contributions considered sensor registration for single-target tracking under the ideal condition wherein the target is assumed to always exist throughout the whole observation period, sensor nodes detect the target with unit probability, and the sensing process is not affected by false alarms (clutter).

In this paper, the aim is to solve sensor registration in the multitarget case, where phenomena like target existence/disappearance, missed/false detections of targets, and uncertain data associations must be accounted for. To the best of our knowledge, the only existing contribution in this context is the Bayesian approach of [37], wherein a Monte Carlo method is adopted to represent and compute in each node the posterior distribution of the relative positions (drift parameters) of its neighbors. In [37], distributed computation was accomplished by the message passing strategy that can suffer from the same problems of [29] with networks that change in time and/or contain loops. Conversely, the approach to this paper jointly solves the sensor registration and multitarget tracking problems over a sensor network in a distributed way by exploiting consensus. Furthermore, estimation of both relative positions and orientations is addressed.

In this paper, multiple targets are modeled as an independent and identically distributed (i.i.d.) cluster RFS [38], whose cardinality (the number of elements in the RFS) and the states of each set-member (target) are time-varying. A \textit{cardinality PHD} (CPHD) filter [39] can be run in each node of the network to update the local posterior density of the target set with the multitarget motion model and the available local measurements. When the sensor relative positions and orientations are known, the consensus method can be exploited as in [7] to fuse the local posterior into a global one in a fully distributed fashion. The resulting method, referred to as \textit{consensus CPHD} (CCPHD) filter in [7], provides an effective solution to DMT over a registered sensor network. The fusion rule adopted in [7] relies on the information-theoretic paradigm of the \textit{weighted Kullback–Leibler average} (WKLA) according to which the fused density is chosen as the one minimizing a special cost, defined as a \textit{weighted average Kullback–Leibler divergence} (WAKLD) from the local posteriors. In [40], it has been proved that the resulting WKLA multiant agent fusion, also known as GCI, turns out to be immune to double counting of information and is, therefore, resilient to the presence of loops in the sensor network. The minimum cost associated to the WKLA-fused density is known in the literature as \textit{GCI divergence} [13], [14]. The GCI divergence provides a sensible measure of the degree of dissimilarity among the set of local posteriors (see [13]), and can, therefore, be minimized with respect to the unknown drift and/or orientation parameters for sensor registration purposes.

Following the above-mentioned arguments, the GCI divergence is adopted in this paper as \textit{instantaneous cost} (IC) to quantify the amount of registration errors at each fusion step. Since the minimization of the IC would make the resulting estimates of the registration parameters sensitive to transient errors, a \textit{total cost} (TC), defined as the summation of ICs over fusion steps, would be a more appropriate candidate for sensor registration. It is shown that in the special case in which the orientation parameters are known \textit{a priori}, the TC can be recursively computed over time so that its direct optimization with respect to drift parameters can be a computationally feasible approach for estimating them. Conversely, in the case wherein both drift and orientation parameters are unknown, the recursive computation of the TC is no longer possible so that direct optimization of the TC would require excessive computational and memory loads not feasible for low cost sensor nodes. Hence, a sub-optimal approach is proposed by minimizing the IC at each fusion step and then combining the resulting instantaneous estimates of the registration parameters according to a suitable multihypothesis method in order to obtain estimates that are less sensitive to transient errors.

The remarkable features of the proposed sensor registration algorithm are that: 1) it requires no additional hardware devices, on-board and/or in the environment, for sensor localization; 2) it introduces no additional data exchanges, only slight extra computational load and memory space as compared to the original CCPHD filter. Furthermore, the proposed algorithm is insensitive to the type of strongly connected sensor network, a feature that is inherited from the properties of WKLA fusion and consensus [40]. Since i.i.d. cluster processes represent a quite general family of RFS processes [2], [41], the proposed sensor registration algorithm can be flexibly combined with several DMT algorithms.

It should be noted that, in our recent work [36], a similar approach has been successfully undertaken to perform sensor registration in the context of distributed detection and tracking (DDT) of a single-target on a sensor network wherein a consensus Bernoulli filter [9] is run at each sensor node. The novel contributions of this paper include: 1) the extension of sensor registration to the case of both unknown drift and orientation parameters, which is impossible to accomplish in the context of [36] (DDT of a single-target) due to the multiplicity of solutions for the orientation parameters; 2) sensor registration is performed simultaneously with
DMT, which represents a more general task for real-world applications. In this regard, if only the drift parameters are of interest and at most one target is present, this paper of [36] can be seen as a special case of this one.

II. PROBLEM FORMULATION AND BACKGROUND

A. Problem Formulation

The aim of this paper is to jointly perform sensor registration and DMT using a sensor network. Each node in the sensor network can get measurements of kinematic variables (e.g., angles, distances, Doppler shifts, etc.) relative to targets moving in the surrounding environment and can process local data as well as exchange data with neighbors. The network of interest has the following features: it has no central fusion node; sensor nodes are unaware of the network topology, i.e., the number of nodes and their connections; each node maintains its own local coordinate frame and has no knowledge about the locations as well as azimuths of its neighbors with respect to its local coordinates. In this paper, it is also assumed that the sensor network is time-synchronized. Please notice that time-synchronization can be achieved in different ways either before network deployment or during the network operation by means of some synchronization method [43]. Clearly, a further alternative could be to jointly perform time and space registration but this is out of the scope of this paper, and left for possible future work.

From a mathematical point of view, the sensor network can be described in terms of a directed graph $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of sensor nodes and $\mathcal{A} \subseteq \mathcal{N} \times \mathcal{N}$ the set of connections such that $(i, j) \in \mathcal{A}$ if node $j$ can receive data from node $i$. For each node $i \in \mathcal{N}$, $\mathcal{N}^i$ will denote the set of its in-neighbor nodes (including itself). The total number of nodes in the network will be denoted by $|\mathcal{N}|$, i.e., the cardinality of $\mathcal{N}$.

It is assumed in this paper that each node expresses positions and velocities with respect to a local Cartesian coordinate frame and that, without loss of generality, each node is located at the origin of its own frame. Let the pair $(\xi^i, \eta^i)$ denote the position of node $j$ in the local coordinates of node $i$ where $j \in \mathcal{N}^i$, and define the drift parameter vector from node $j$ to $i$ as $\vartheta^{i,j} = [\xi^{i,j}, \eta^{i,j}]^T$. Similarly, $\gamma^{i,j}$ is used to denote the orientation parameter from node $j$ to node $i$.

The single-target state expressed in the coordinates of node $i$ is denoted as $x^i = [\xi^i, \xi^{i,j}, \eta^{i,j}]^T$, where the pairs $(\xi^i, \eta^i)$ and $(\xi^{i,j}, \eta^{i,j})$ are the target position and, respectively, velocity in Cartesian coordinates. It is easy to check that the target states $x^i$ and $x^j$ are related by

\[
x^i = M^{i,j}x^j + T \vartheta^{i,j}
\]

where $T$ is the transition matrix defined as

\[
T = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]

Fig. 1. Drift and orientation between two sensor nodes.

$M(\gamma)$ is the rotation matrix defined as

\[
M(\gamma) = \begin{bmatrix}
cos(\gamma) & 0 & -\sin(\gamma) & 0 \\
0 & \cos(\gamma) & 0 & -\sin(\gamma) \\
\sin(\gamma) & 0 & \cos(\gamma) & 0 \\
0 & \sin(\gamma) & 0 & \cos(\gamma)
\end{bmatrix}
\]

and, for convenience, the shorthand notation $M^{i,j} = M(x^{i,j})$ is adopted. It is straightforward to check that the drift and orientation parameters satisfy the following properties:

\[
\dot{\gamma}^{i,j} = 0, \quad \gamma^{i,j} = 0, \quad \gamma^{j,i} = -\gamma^{i,j}, \quad M^{i,j} = I_4
\]

\[
\vartheta^{i,j} = -T^T M^{i,j} T \vartheta^{j,i}
\]

\[
M^{-1}(\gamma) = M^T(\gamma) = M(-\gamma)
\]

\[
\det(M(\gamma)) \equiv \det(M^T(\gamma)) \equiv 1, \text{ for any } \gamma.
\]

In order to keep dimension consistence between the drift parameter and the target state vector, we define $\vartheta^{i,j} = T \vartheta^{j,i}$, then the drift parameter $\vartheta^{i,j}$ can be easily recovered by computing $\vartheta^{i,j} = T^\top \vartheta^{j,i}$. The meaning of the drift and orientation (registration) parameters as well as the coordinate transformation (1) between two sensor nodes $i$ and $j$ are illustrated in Fig. 1.

The multitarget system in the surveillance area at time $t$ is modeled as an RFS $\mathcal{X}_t$, which consists of $|\mathcal{X}_t|$ targets. Let us denote by $\mathcal{X}_t^i$ the multitarget RFS expressed in the local coordinate frame of node $i$. The evolution of the target set $\mathcal{X}_t^i$ is supposed to be governed by the multitarget dynamics

\[
\mathcal{X}_{t+1} = \Phi_{t+1}(\mathcal{X}_t) \bigcup \mathcal{B}_{t+1}
\]

where $\mathcal{B}_{t+1}$ is the RFS of new-born targets at time $t + 1$ and

\[
\Phi_{t+1}(\mathcal{X}) = \bigcup_{x \in \mathcal{X}} \phi_t(x)
\]

\[
\phi_t(x) = \begin{cases}
\{x_+\}, & \text{with survival probability } P_{s,t} \\
\emptyset, & \text{otherwise}
\end{cases}
\]
with \( x_+ \) distributed according to the single-target Markov transition PDF \( \varphi_{t+1}(x_+|x) \). Notice that according to (8)–(10) each target in the set \( \mathcal{X}_{t+1} \) is either a new-born target from the set \( \mathcal{B}_{t+1} \) or a target survived from \( \mathcal{X}_t \), with probability \( P_{t,t} \), and whose state vector has evolved according to the single-target dynamics expressed by the transition PDF \( \varphi_{t+1}(\cdot|\cdot) \). In a similar way, observations are assumed to be generated, at each node \( i \in \mathcal{N} \), according to the measurement model

\[
\mathcal{Y}_i^t = \Psi_i^t(\mathcal{X}_i^t) \cup C_i^t
\]

where \( C_i^t \) is the clutter RFS (i.e., the set of measurements not due to targets) at time \( t \) and node \( i \), and

\[
\Psi_i^t(\mathcal{X}^i) = \bigcup_{x^i \in \mathcal{X}^i} \psi_i^t(x^i)
\]

\[
\psi_i^t(x^i) = \begin{cases} \{ y^i \}, & \text{with detection probability } P_{d,t}^i, \\ \emptyset, & \text{otherwise} \end{cases}
\]

with \( y^i \) distributed according to the single-sensor likelihood \( L_i^t(y^i|x^i) \) at node \( i \). Notice that according to (11)–(13) each measurement in the set \( \mathcal{Y}_i^t \) is either a false one from the clutter set \( C_i^t \) or is related to a target in \( \mathcal{X}^i \), with probability \( P_{d,t}^i \), according to the single-sensor likelihood.

The aim of this paper is, therefore, to estimate, at each node \( i \in \mathcal{N} \), the drift and orientation parameters \( \vartheta^{i,j} \) and \( \varphi^{i,j} \), only for \( j \in \mathcal{N} \setminus \{i\} \) (recall that each sensor can only communicate with neighbors) as well as the target set \( \mathcal{X}_i^t \) by collecting measurements and exchanging data with neighbors at each sampling interval. For convenience, let us also define \( \Theta^i = \text{col}(\vartheta^{i,j}, j \in \mathcal{N} \setminus \{i\}) \), \( \mathcal{M}^i = \text{block-diag}(\mathcal{M}^{i,j}, j \in \mathcal{N} \setminus \{i\}) \), \( \Gamma^i = \text{col}(\varphi^{i,j}, j \in \mathcal{N} \setminus \{i\}) \), and

\[
\mathcal{T}^i = \text{block-diag}(T, \ldots, T), \quad |\mathcal{X}^i| - 1 \text{ times}
\]

B. Single-Sensor CPHD Filtering

From a probabilistic viewpoint, an RFS \( \mathcal{X} \) is completely characterized by its multitarget density \( f(\mathcal{X}) \). It is worth pointing out that the multijobet density, while completely characterizing an RFS, involves a combinatorial complexity; hence simpler, though approximate, characterizations are usually adopted in order to keep the multitarget tracking problem computationally tractable. In this paper, it is supposed that the multitarget RFS \( \mathcal{X} \) is approximated as an i.i.d. cluster RFS whose density is given by

\[
f(\mathcal{X}) = |\mathcal{X}|! \cdot p(|\mathcal{X}|) \prod_{x \in \mathcal{X}} s(x)
\]

where \( p(n) \) is the cardinality distribution of the cardinality of \( \mathcal{X} \) and \( s(x) \) is the target spatial PDF. Clearly, an i.i.d. cluster process is completely characterized by the pair \((p, s)\).

The CPHD filter propagates in time the cardinality distribution \( p_t(n) \) as well as the target spatial PDF \( s_t(x) \) of \( \mathcal{X}_t \) assuming that the clutter RFS, the predicted and filtered RFSs are i.i.d. cluster processes. The resulting CPHD recursions (prediction and correction) can be found in [38].

Note that, in principle, the cardinality distribution \( p_t(n) \) is defined for a cardinality \( n \) of the multitarget set going from 0 to \( \infty \); this is, of course, computationally infeasible. For implementation purposes, it is enough to assume a sufficiently large maximum number of targets \( N_{\text{max}} \) in the scene. The spatial PDF \( s_t(x) \) can be represented with particles [26] or as a Gaussian mixture (GM) [39]. In this paper, we adopt the GM representation of the CPHD filter, referred to as GM-CPHD filter, also used in [7] for DMT.

C. Kullback–Leibler Paradigm for Multitarget Fusion

From an information-theoretic point of view, fusion of multiple RFS densities through the network can be regarded as finding the RFS density that minimizes the WAKLD among all the nodes of the sensor network [40]. To review the related concepts, let us first introduce the notion of Kullback–Leibler Divergence (KLD) from multitarget density \( g(\mathcal{X}) \) to \( f(\mathcal{X}) \) by

\[
D_{\text{KL}}(f \| g) \overset{\Delta}{=} \int f(\mathcal{X}) \log \frac{f(\mathcal{X})}{g(\mathcal{X})} d\mathcal{X}
\]

where the integral involved in (15) is the set integral defined in [1]. Then, the WKLA \( \overline{f}(\mathcal{X}) \) of the RFS densities \( f^i(\mathcal{X}) \) is defined as follows:

\[
\overline{f}(\mathcal{X}) \overset{\Delta}{=} \arg \inf_f \sum_i \omega^i D_{\text{KL}}(f \| f^i)
\]

where the cost \( J(f) \) to be minimized with respect to \( f(\cdot) \) is the WAKLD and the weights \( \omega^i \geq 0 \) must satisfy \( \sum_i \omega^i = 1 \). In particular, if \( \omega^i = 1/|\mathcal{X}| \) for \( i = 1, \ldots, |\mathcal{X}| \), (16) provides the (unweighted) KLA that averages the node densities giving to all of them the same level of confidence.

An interesting interpretation of such a notion can be given recalling that, in Bayesian statistics, the KLD (15) can be seen as the information gain achieved when moving from a prior \( g(\mathcal{X}) \) to a posterior \( f(\mathcal{X}) \). Thus, according to (16), the average density is the one that minimizes the weighted average of the information gains from the initial multitarget densities. Hence, this choice is consistent with the principle of minimum discrimination of information (PMDI).

**Theorem 1** (see [44]) The WKLA defined in (16) turns out to be given by

\[
\overline{f}(\mathcal{X}) = \frac{\prod_i [f^i(\mathcal{X})]^{\omega^i}}{\int \prod_i [f^i(\mathcal{X})]^{\omega^i} d\mathcal{X}}
\]

and the corresponding minimum WAKLD \( J(\overline{f}) \) is given by

\[
J(\overline{f}) \overset{\Delta}{=} \sum_i \omega^i D_{\text{KL}}(f \| f^i)
\]

\[
= - \log \left( \int \prod_i [f^i(\mathcal{X})]^{\omega^i} d\mathcal{X} \right).
\]
Notice that (17) corresponds to the normalized geometric mean of the densities \( f^j \). Hence, Theorem 1 shows that the WKLA actually coincides with the GCI fusion rule, originally proposed by Mahler [3] as a generalization of Covariance Intersection to arbitrary densities. The minimal cost \( \mathcal{J}(\mathcal{T}) \) in (18), which is always nonnegative and vanishes only when all the densities are coincident, is known in the literature as 

**GCI divergence** [13], [14]. As discussed in [13], the GCI divergence \( \mathcal{J}(\mathcal{T}) \) makes it possible to quantify, in a principled way, the degree of dissimilarity among a set of RFS densities within the context of GCI fusion.

When all the densities to be fused are i.i.d. cluster densities, the WKLA can be computed in closed form as follows.

**Theorem 2 (see [42])** Let all the \( f^j \) be i.i.d. cluster densities characterized by the pairs \((p^j, s^j)\). Then, the WKLA \( \mathcal{T} \) is again an i.i.d. cluster density characterized by the pair \((\mathcal{T}, \mathcal{S})\) with

\[
\mathcal{P}(n) = \frac{\prod_j \left[ s^j(x) \right]^{w_j} \left( f \prod_j \left[ s^j(x) \right]^{w_j} dx \right)^{\alpha}}{\sum_{n=0}^{\infty} \prod_j \left[ s^j(x) \right]^{w_j} \left( f \prod_j \left[ s^j(x) \right]^{w_j} dx \right)^{\alpha}}
\]

(19)

\[
\mathcal{S}'(x) = \frac{\prod_j \left[ s^j(x) \right]^{w_j}}{f \prod_j \left[ s^j(x) \right]^{w_j} dx}.
\]

(20)

In other words, (19) and (20) state that the fusion of i.i.d. cluster processes provides an i.i.d. cluster process whose spatial PDF is the weighted geometric mean of the node spatial PDFs, while the fused cardinality distribution is obtained by a more complicated expression (19) also involving the node location PDFs besides the cardinality distributions.

**D. Distributed Multitarget Tracking**

Let us preliminarily suppose that all the drift and orientation parameters \( \theta^{i,j}, \gamma^{i,j} \) \((i, j \in \mathcal{N})\) are known. Notice that this assumption is made here only for illustration purposes and will be relaxed later.

When the drift and orientation parameters are known, the ideas of Sections II-B and II-C can be combined so as to obtain an effective DMT algorithm. To see this, consider a generic time instant \( t \) and suppose that, in each network node \( i \), after the correction step of the GM-CPHD filter with local measurements, an RFS density \( f^i_0(\mathcal{X}^i) \) is available representing the information at node \( i \) on the i.i.d. cluster RFS \( \mathcal{X}^i \) (expressed in the local coordinates of node \( i \)). Clearly, \( f^i_0(\mathcal{X}^i) \) is characterized by the cardinality distribution \( p^i_0(|\mathcal{X}^i|) \) and the spatial PDF \( s^i_0(x^i) \).

If all the densities \( f^j(\mathcal{X}^j) \), \( j \in \mathcal{N} \), were available in node \( i \), fusion could be performed by: 1) expressing all the densities in the coordinates of node \( i \) by means of the change of coordinates (1) associated with \( \theta^{i,j}, \gamma^{i,j} \) \((i, j \in \mathcal{N})\); 2) computing the WKLA of such densities by means of the GCI fusion rule (19) and (20).

Clearly, in a distributed setting, it is not possible to directly compute the fused density \( f^i_0(\mathcal{X}^i) \) with (19) and (20) since not all the densities \( f^j(\mathcal{X}^j) \), \( j \in \mathcal{N} \), are available in node \( i \). However, it turns out that the collective average \( \mathcal{F} \) can be approximated to any desired degree of accuracy by means of distributed computation (i.e., exchanging only information with the neighbors). This is made possible by the **consensus method** that has emerged as a powerful tool for distributed computation over networks and has found widespread applications, e.g., in distributed parameter/state estimation. In essence, consensus aims at computing the collective average by iterating several times the computation of the local average over the subnetwork \( \mathcal{N}^i \) of in-neighbors of each node \( i \). In fact, it can be shown that, when the network is strongly connected, as the number \( L \) of consensus iterations increases, the density in each node converges to the collective average \( \mathcal{F} \), [7], [44].

Then, in practice, at each time \( t \) each node \( i \) of the network, after local GM-CPHD filtering, iterates for \( L \) times data-exchange with the neighbors and fusion of the received densities with the local one. More specifically, consider a generic node \( i \) at time \( t \) and suppose that \( \ell \) consensus iterations have been carried out. Then, the density at node \( i \) expressed in local coordinates is an i.i.d. cluster density characterized by the pair \((p^i_{\ell,t}, s^i_{\ell,t})\).

Notice that each \( s^i_{\ell,t} \) is expressed in the coordinates of node \( j \), that is, it is a function of \( x^j \). In order to compute the regional average over the subnetwork \( \mathcal{N}^i \), each node \( i \) applies the changes of coordinates (1) to the spatial PDFs of the neighbors so as to obtain the densities \( s^j_{\ell,t}(x^i; \theta^{i,j}, \gamma^{i,j}) \) for \( j \in \mathcal{N}^i \). Clearly, \( s^j_{\ell,t}(x^i; \theta^{i,j}, \gamma^{i,j}) = s^j_{\ell,t}(M^{i,j}x^i + \theta^{i,j}) \) and \( s^i_{\ell,t} = s^i_{\ell,t} \).

Then, the fused density at the next consensus step \( f^i_{\ell+1}(\mathcal{X}^i) \) is an i.i.d. cluster density characterized by the pair \((p^i_{\ell+1}, s^i_{\ell+1})\) where we have (21) as shown at the bottom of this page, and

\[
s^i_{\ell+1}(x^i) = \frac{\prod_{j \in \mathcal{N}^i} \left[ s^j_{\ell,t}(x^i; \theta^{i,j}, \gamma^{i,j}) \right]^{w^j}}{f \prod_{j \in \mathcal{N}^i} \left[ s^j_{\ell,t}(x; \theta^{i,j}, \gamma^{i,j}) \right]^{w^j} dx}. \]

(22)

The CCPHD filter is summarized in Table I. For a practical implementation of such an algorithm based on a GM approximation of the spatial PDFs \( s^i_{\ell,t} \), the interested reader is referred to [7].

\[
p^i_{\ell+1}(n) = \frac{\prod_{j \in \mathcal{N}^i} \left[ p^j_{\ell,t}(n) \right]^{w^j} \left( f \prod_{j \in \mathcal{N}^i} \left[ s^j_{\ell,t}(x; \theta^{i,j}, \gamma^{i,j}) \right]^{w^j} dx \right)^n}{\sum_{m=0}^{\infty} \prod_{j \in \mathcal{N}^i} \left[ p^j_{\ell,t}(m) \right]^{w^j} \left( f \prod_{j \in \mathcal{N}^i} \left[ s^j_{\ell,t}(x; \theta^{i,j}, \gamma^{i,j}) \right]^{w^j} dx \right)^m}. \]
III. DISTRIBUTED SENSOR SELF-LOCALIZATION

A. GCI Divergence for i.i.d. Cluster RFS Densities

The previous section has introduced the CCPHD filter, which assumes that the drift and orientation parameters between node $i$ and $j$, $(i, j) \in A$, are known a priori. However, in practice, registration parameters may not be known, or at least be known with insufficient accuracy. Then, $\theta^{i,j}$ and $\gamma^{i,j}$, $(i, j) \in A$, need to be estimated together or as a premise to the target RFS $\mathcal{X}$ without additional localization hardware in the sensor nodes and in the surrounding environment.

The sensor registration approach proposed in this section is suboptimal but has the twofold advantage of being scalable and not requiring any global information on the strongly connected network topology. As a further benefit, in order to keep the communication load as low as possible, the proposed approach will not require any additional data exchange with respect to the CCPHD filter of Table I.

The idea is to exploit the information-theoretic interpretation of the consensus step (21) and (22) in order to define a suitable cost function, which can be used for estimation of the registration parameters. As discussed in the previous section, each consensus step in node $i$ amounts to computing a regional average, according to the WKLA paradigm, over the subnetwork $\mathcal{N}^i$ of in-neighbors of node $i$. In fact, when the registration parameters are unknown, the fused density is a function of $\Theta^i$ and $\Gamma^i$, that is

$$f_{i,t,\ell}^{i,j} (\mathcal{X}^i; \Theta^i, \Gamma^i)$$

is expressed by the GCI divergence

$$J'_{i,\ell} (\Theta^i, \Gamma^i)$$

(24)

that represents a natural way of measuring the discrepancy among the multitarget densities to be fused. Notice, for instance, that the GCI divergence is equal to 0 if and only if all the densities to be fused are the same. Then, the idea is to apply once again the PMDI and estimate the drift and orientation parameters by minimizing the information gain, i.e., the GCI divergence. For this reason, hereafter, we refer to $J'_{i,\ell} (\Theta^i, \Gamma^i)$ as IC of node $i$ at time $t$ and consensus step $\ell$. The rationale for such a choice is that, since the RFS densities to be fused provide information on the same object set, then it is reasonable to choose the estimated registration parameters so that the densities overlap as much as possible, i.e., so that their discrepancy $J'_{i,\ell} (\Theta^i, \Gamma^i)$ is minimal.

Since in the considered setting all the densities to be fused are i.i.d. cluster densities, the IC $J'_{i,\ell} (\Theta^i, \Gamma^i)$ can be further specified as follows (see Appendix A for the proof).

**PROPOSITION 1** Let all the local multitarget densities $f_{i,t,\ell-1}^{i,j} \in \mathcal{N}^i$, be i.i.d. cluster densities characterized by the pair $(p_{i,t,\ell-1}^{i,j}, s_{i,t,\ell-1}^{i,j})$. Then, the IC can be computed as follows:

$$J'_{i,\ell} (\Theta^i, \Gamma^i) = -\log \left\{ \sum_{n=0}^{\infty} \psi_{i,t}^{n} \left[ \mathcal{W}_{i,t}^{n} (\Theta^i, \Gamma^i) \right]^{n} \right\}$$

(25)

where

$$\psi_{i,t}^{n} = \prod_{j \in \mathcal{N}^i} \left[ p_{i,t,\ell-1}^{i,j} (n) \right]^{\omega_{i,j}}$$

(26)

$$\mathcal{W}_{i,t}^{n} (\Theta^i, \Gamma^i) = \int \prod_{j \in \mathcal{N}^i} \left[ s_{i,t,\ell-1}^{i,j} (x; \theta^{i,j}, \gamma^{i,j}) \right]^{\omega_{i,j}} dx.$$  

(27)

It can be seen that $\psi_{i,t}^{n}$ is a constant independent of both the drift and orientation parameters. Conversely, $\mathcal{W}_{i,t}^{n} (\Theta^i, \Gamma^i)$, referred to hereafter as instantaneous reward factor (IRF) of node $i$ at time $t$ and consensus step $\ell$, is the only part of the IC related to the drift and orientation parameters. Please notice that minimizing the IC $J'_{i,\ell} (\Theta^i, \Gamma^i)$ is the same as maximizing the IRF $\mathcal{W}_{i,t}^{n} (\Theta^i, \Gamma^i)$, with respect to the registration parameters $\Theta^i$ and $\Gamma^i$. Notice that the IRF only depends on the discrepancy between the spatial PDFs of neighboring nodes. Furthermore, under the assumption that the local CPHD filters perform well, then all the local densities should provide a reasonably accurate estimate of the target set in local coordinates. In this case, the discrepancy between the spatial PDFs in two neighboring nodes $i$ and $j$ is mainly due to the registration error.

In order to better understand the above-mentioned argument, let us consider the following simple example. Assume that there is only one target in the surveillance area, and
the spatial PDFs of node $i$ and $j$ are both Gaussian, respectively, $s^i(x) = \mathcal{G}(x; \mu^i, P^i)$ and $s^j(x) = \mathcal{G}(x; \mu^j, P^j)$. After exponentiation of such posteriors, we have (recall [7, eq. (36)])

$$
[s^i(x)]^{\omega^i} = \chi(\omega^i, P^i) \mathcal{G}
\left(x; \mu^i, \frac{P^i}{\omega^i}\right)
$$

$$
[s^j(x)]^{\omega^j} = \chi(\omega^j, P^j) \mathcal{G}
\left(x; \mu^j, \frac{P^j}{\omega^j}\right)
$$

where

$$
\chi(\omega, P) = \sqrt{\frac{\det(2\pi P \omega^{-1})}{\det(2\pi P) \omega^{n}}}.
$$

Let the real target state be, respectively, expressed in the coordinates of nodes $i$ and $j$ as $\hat{\mu}^i$ and $\hat{\mu}^j$. Then, we have

$$
\tilde{\mu}^i = \mu^i + \epsilon^i, \quad \tilde{\mu}^j = \mu^j + \epsilon^j
$$

where $\epsilon^i$ and $\epsilon^j$ denote the estimation errors of nodes $i$ and $j$, respectively. Furthermore, we denote the real rotation matrix and drift parameters from node $j$ to $i$ as $M^j$ and $\bar{\omega}^j$, respectively. Then, we have the following relationship:

$$
\tilde{\mu}^i = (\bar{M}^j)^T \tilde{\mu}^j - (\bar{M}^j)^T \bar{\omega}^j.
$$

Exploiting the formula of Gaussian distribution multiplication [7, eqs. (37)–(40)], the GCI divergence evaluated at node $i$ is computed as

$$
\mathcal{J}^i = \mathcal{C} \cdot \mathcal{G}
\left(0; \tilde{\mu}^{i,j}, \frac{P^i}{\omega^i} + M^{i,j} \frac{P^j}{\omega^j} (M^{i,j})^T\right)
$$

where $\mathcal{C}$ is a constant and

$$
\tilde{\mu}^{i,j} = \mu^i - \mu^j
$$

$$
= \tilde{\mu}^i - \epsilon^i - (M^{i,j}) \mu^j + \theta^{i,j}
$$

$$
= \tilde{\mu}^i - \epsilon^i - (M^{i,j}) (\tilde{\mu}^j - \epsilon^j) + \theta^{i,j}
$$

$$
= \tilde{\mu}^i - \epsilon^i - M^{i,j} \left[\left(\bar{M}^j\right)^T \tilde{\mu}^j - \left(\bar{M}^j\right)^T \bar{\omega}^j - \epsilon^j\right] - \theta^{i,j}
$$

$$
= \left[I_k - M^{i,j} \left(\bar{M}^j\right)^T\right] \tilde{\mu}^j + M^{i,j} \left(\bar{M}^j\right)^T \bar{\omega}^j - \theta^{i,j} + M^{i,j} \epsilon^j - \epsilon^i.
$$

Under the precondition that all filters perform well, it is reasonable to assume that $\epsilon^i \approx 0$ and $\epsilon^j \approx 0$. Then, it is straightforward to see that the IRF, and hence the IC, is mainly affected by the discrepancy between the true and estimated registration parameters.

**B. GM Implementation**

In this section, we discuss how the IRF can be computed when a GM implementation of the CCPHD filter is adopted. Of course, this amounts to assuming that all the spatial densities $s_{i,t-1}^i$ are represented as GMs, i.e.,

$$
s_{i,t-1}^i(x^i) = \sum_{k=1}^{N_{i,t-1}^i} \alpha_{i,t-1}^{i,k} \mathcal{G}
\left(x^i; \mu_{i,t-1}^{i,k}, P_{i,t-1}^{i,k}\right)
$$

where $N_{i,t-1}^i$ is the number of Gaussian components, the weights $\alpha_{i,t-1}^{i,k}$ of the mixture are positive and such that $\sum_{k=1}^{N_{i,t-1}^i} \alpha_{i,t-1}^{i,k} = 1$, and $\mathcal{G}(x; \mu, P)$ denotes a Gaussian PDF with mean $\mu$ and covariance matrix $P$.

Preliminary operations for the computation of the IRF (27) in node $i$ are: the exponentiation by $\omega^{i,j}$ of each $s_{i,t-1}^j$; $j \in N^i$; and the application of the change of coordinates (1) for any $j \in N^i \setminus \{i\}$. In this paper, following [7], [45], the power of each $s_{i,t-1}^j$ is supposed to be approximated as a GM of the form

$$
\left[s_{i,t-1}^j(x^j)\right]^{\omega^j} \approx \sum_{k=1}^{N_{i,t-1}^j} \tilde{\alpha}_{i,t-1}^{j,k} \mathcal{G}
\left(x^j; \mu_{i,t-1}^{j,k}, P_{i,t-1}^{j,k}/\omega^j\right).
$$

There are several methods to determine the weight of each Gaussian component, such as the computationally cheap one adopted in [7] or the newly proposed method of [45]. However, the choice of a particular approximation method is immaterial for the subsequent developments as long as an approximation like (36) holds. Concerning the change of variables, it is immediate to check that each Gaussian in (36) can be rewritten as

$$
\mathcal{G}
\left(x^j; \mu_{i,t-1}^{j,k}, P_{i,t-1}^{j,k}/\omega^j\right) = \mathcal{G}
\left(x^i; \tilde{\mu}_{i,t-1}^{j,k}, \tilde{P}_{i,t-1}^{j,k}\right)
$$

where

$$
\tilde{\mu}_{i,t-1}^{j,k} = M_{i,t-1}^{j,k} \mu_{i,t-1}^{j,k} + \theta_{i,j}^{j,k}
$$

$$
\tilde{P}_{i,t-1}^{j,k} = M_{i,t-1}^{j,k} P_{i,t-1}^{j,k} (M_{i,t-1}^{j,k})^T / \omega^{j,k}.
$$

Hence, computation of the IRF (27) in node $i$ involves the product of $|N^i|$ GMs. With this respect, we observe that such a product is again a GM having a total of $\prod_{j \in N^i} N_{i,t-1}^j$ Gaussian components. Hereafter, for the sake of compactness, we will denote each of such components by means of a vector index, say $k$, taking value in the set

$$
\mathcal{K}_{i,t}^k = \times_{j \in N^i} \{1, \ldots, N_{i,t-1}^j\}
$$

where, $\times$ denotes Cartesian product. Accordingly, each element $k(j)$, $j \in N^i$, of the vector index $k \in \mathcal{K}_{i,t}^k$ expresses, which of the Gaussian components of node $j$ is used to form the $k$th component of the product.

Taking into account the above-mentioned considerations and notation, the following result can now be stated.

**Theorem 3** Let all the local spatial densities be represented by GMs as in (35) and let the approximation (36) be adopted. Then, the IRF turns out to be a GM given by

$$
\mathcal{W}_{i,t}^j(\Theta^j, \Gamma^j)
\approx \sum_{k \in \mathcal{K}_{i,t}^k} \tilde{\beta}_{i,t}^{j,k}(\Gamma^j) \mathcal{G}
\left(\Theta^j; \phi_{i,t}^{j,k}(\Gamma^j), \gamma_{i,t}^{j,k}(\Gamma^j)\right)
$$

where $\tilde{\beta}_{i,t}^{j,k}(\Gamma^j)$ is the number of Gaussian components, the weights $\alpha_{i,t}^{j,k}$ of the mixture are positive and such that $\sum_{k=1}^{N_{i,t}^j} \alpha_{i,t}^{j,k} = 1$, and $\mathcal{G}(x; \mu, P)$ denotes a Gaussian PDF with mean $\mu$ and covariance matrix $P$.

**Proof**
where

\[
\beta_{i,t,\ell}^{i,k}(\Gamma^i) = \det \left[ 2\pi \mathbf{P}_{i,t,\ell}^{i,k}(\Gamma^i) \right] \frac{1}{2} \det \left[ 2\pi \Psi_{i,t,\ell}^{i,k}(\Gamma^i) \right] \frac{1}{2} \times \prod_{j \in \mathcal{N}^i} \frac{1}{\sigma_{i,t,\ell}^{i,k(j)} \Gamma^i} \left( \frac{1}{\sigma_{i,t,\ell}^{i,k(j)} \Gamma^i} \right)^{1/2} \]
\]

(41)

\[
\phi_{i,t,\ell}^{i,k}(\Gamma^i) = E^i \mu_{i,t,\ell-1}^{i,k(j)} - M^i \mathbf{u}_{i,t,\ell-1}^{i,k(j)} - \mathbf{u}_{i,t,\ell-1}^{i,k(j)} \]
\]

(42)

\[
\gamma_{i,t,\ell}^{i,k}(\Gamma^i) = \Psi_{i,t,\ell}^{i,k} + E^i \mathbf{P}_{i,t,\ell-1}^{i,k(j)}(E^i)^\top \omega_{i,j}^{i,k(j)} \]
\]

(43)

and

\[
\Psi_{i,t,\ell}^{i,k} = \text{block-diag} \left[ \mathbf{P}_{i,t,\ell}^{i,k(j)}(j, j \in \mathcal{N}^i \setminus \{i\}) \right] \]
\]

(44)

The proof of Theorem 3 is given in Appendix A. Notice that the IC defined in (25) is minimized if and only if the sensor nodes are correctly registered, in which case the IRF \( \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i, \Gamma^i) \) is maximized. In order to take a further look at how the IRF can be used for sensor registration purposes, the following remarks are in order.

**Remark 1** Notice that (40) consists of \( |\mathcal{K}^{i,\ell}_{i,t,\ell}| = \prod_{j \in \mathcal{N}^i} N^{i}_{t,\ell-1} \) Gaussian components, where each component \( k \in \mathcal{K}^{i,\ell}_{i,t,\ell} \) represents a possible association among Gaussian components \( k(j), j \in \mathcal{N}^i, \) (which represent potential targets) of spatial PDFs from neighboring nodes. Generally speaking, correct associations among Gaussian components should approximately have the same drift and orientation parameters (i.e., the means \( \phi_{i,t,\ell}^{i,k}(\Gamma^i) \) of Gaussian components that represent correct target associations should be almost the same). By recognizing this, a convenient optimization approach to maximize \( \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i, \Gamma^i) \) can be adopted, as it will be discussed in the following sections.

**Remark 2** When the orientation parameters are known \textit{a priori} (e.g., the static sensor nodes are deployed to point toward the same direction), the weight, mean, and covariance of each Gaussian component in (40) turn out to be constant, i.e., independent of the orientation parameters \( \Gamma^i \), which means that the maximum of the IRF can be found by directly operating on GMs [47], [48].

**Remark 3** It should be noticed that the IRF \( \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i, \Gamma^i) \) cannot be directly computed when the spatial PDFs \( s_{i,t,\ell}(\cdot) \), \( i \in \mathcal{N}^i \), are represented by particles, i.e., \( s_{i,t,\ell}(\cdot) = \sum_{k=1}^{N^{i}_{t,\ell}} \alpha_{i,t,\ell}^{i,k} \delta_{i,t,\ell}^{i,k}(\cdot) \)

(45)

\[
\delta_{i,t,\ell}^{i,k}(\cdot) = \left\{ \begin{array}{ll} 1, & \mu = \mu_{i,t,\ell}^{i,k} \\ 0, & \text{otherwise} \end{array} \right. \]
\]

denotes the Dirac delta. This is due to the fact that each sensor node in the network would locally store and propagate its own set of particles, so that (27) would always be equal to zero. A viable approach is to use a GM to approximate the (delta-mixture) particle representation (e.g., by adopting the method of [28]), then fuse GMs and finally transform the fused GM back to particle representation.

C. Sensor Registration With Known Orientation Parameters

In this section, it is assumed that the orientation parameters between any pair of neighbor nodes are known \textit{a priori} and that, without loss of generality, \( \gamma_{i,j} = 0 \) for any \( (i, j) \in A \). Note that this case can occur in practice, e.g., when the nodes are equipped with magnetic sensors [49]. Then accordingly, we have \( \mathcal{M}^{i,j} = I_{4k} \), and (40) can be rewritten as

\[
\mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) = \sum_{k \in \mathcal{K}^{i,\ell}_{i,t,\ell}} \beta_{i,t,\ell}^{i,k}(\Theta^i, \phi_{i,t,\ell}^{i,k}, \gamma_{i,t,\ell}^{i,k}) \]
\]

(48)

where the weight \( \beta_{i,t,\ell}^{i,k}, \) mean \( \phi_{i,t,\ell}^{i,k}, \) and covariance \( \gamma_{i,t,\ell}^{i,k} \) of each Gaussian component can be computed through (41)–(47) by setting \( \mathcal{M}^i = I_{4k[|\mathcal{N}^i| - 1]} \). Hence, the IC (25) is independent of \( \Gamma^i \).

Since minimization of the IC \( \mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) \), or equivalently maximization of the IRF \( \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) \), would make the estimate too sensitive to transient errors in the local densities, in order to obtain a more reliable estimate of the drift parameters, we follow a strategy usually adopted in recursive parameter estimation and consider a TC up to the current time instant/consensus step

\[
\mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) = \sum_{t=1}^{L} \sum_{l=1}^{\ell} \mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) + \sum_{l=1}^{\ell} \mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i). \]
\]

(49)

In the sequel, an algorithm is provided for addressing minimization of the TC \( \mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) \) when a GM implementation of the CCPHD filter is adopted.

To this end, observe first that, in principle, computation of the IC \( \mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) \) as in (25) would involve the summation of an infinite number of GMs (all the powers of the IRF \( \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) \) for \( n \) going to infinity). However, in practice, for implementation purposes in the GM-CCPHD filter the cardinality distributions \( p^{i,\ell}_{i,t,\ell}(n) \), and hence the scalars \( c_{i,t,\ell}^{n} \), are all equal to zero for \( n \) greater than \( N_{\text{max}} \), the assumed maximum number of targets in the scene [39]. Hence, at most \( N_{\text{max}} \) GMs have to be taken into account in the summation so that the IC can be written as

\[
\mathcal{J}^{i,\ell}_{i,t,\ell}(\Theta^i) = - \log \left[ c_{i,t,\ell}^{0,0} + \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) \right] \]
\]

(50)

where \( c_{i,t,\ell}^{0,0} \) is given in (26), and

\[
\mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) = \sum_{n=1}^{N_{\text{max}}} c_{i,t,\ell}^{n} \left[ \mathcal{W}^{i,\ell}_{i,t,\ell}(\Theta^i) \right]^n \]
\]

(51)
is a GM. Then, it is an easy matter to check that also the TC can be written as
\[ J_{i,t}^l (\Theta^i) = -\log \left[ c_{i,t}^l + \tilde{W}_{i,t}^l (\Theta^j) \right] \] (52)
where \( c_{i,t}^l = \prod_{l=1}^L c_{i,t,l}^0 \prod_{l=1}^{L-1} c_{i,t,l}^0 \) and \( \tilde{W}_{i,t}^l (\Theta^j) \) is again a GM. Furthermore, \( c_{i,t}^l \) and \( \tilde{W}_{i,t}^l (\Theta^j) \) can be recursively computed as follows:
\[ c_{i,t}^l = c_{i,t}^{l-1} c_{i,t}^0 \] (53)
\[ \tilde{W}_{i,t}^l (\Theta^j) = c_{i,t}^0 \tilde{W}_{i,t}^{l-1} (\Theta^j) + c_{i,t}^l \tilde{W}_{i,t}^l (\Theta^j) + \tilde{W}_{i,t}^{l-1} (\Theta^j) \] (54)

Clearly, in the computation of \( \tilde{W}_{i,t}^l (\Theta^j) \) merging and pruning techniques can be adopted in order to keep the number of Gaussian components below a prespecified threshold, thus ensuring bounded complexity as \( t \) and \( l \) increase.

Hence, the estimated drift parameters at time \( t \) and consensus step \( \ell \) are obtained by solving the following optimization problem\(^1\)
\[ \hat{\Theta}_{i,t}^l = \arg \min_{\Theta^i} J_{i,t}^l (\Theta^i) = \arg \max_{\Theta^i} \tilde{W}_{i,t}^l (\Theta^j) \] (55)
which amounts to finding the global maximum of a GM. Candidate methods to solve (55) are, e.g., grid search, which first searches for a initial point and is then followed by a gradient-based algorithm, and multiple-initialization, which is run in parallel from multiple initial points [47], [48]. In this paper, the latter strategy is adopted. Note that, after several sampling times when the estimation of the multitarget states at sensor nodes becomes stationary, one can directly employ the estimated drift parameters at time \( t \), consensus step \( \ell - 1 \) (i.e., \( \hat{\Theta}_{i,t-1}^\ell \)) as initial point. The proposed sensor registration method with known orientation parameters is summarized in Table II.

### Table II

| Input: \( W_{i,t-1}^i (\Theta^i), C_{i,t-1}^1, \) and \( \hat{\Theta}_{i,t-1}^l (\Theta^j), j \in N^i \) | Output: \( \hat{\Theta}_{i,t}^l, W_{i,t}^i (\Theta^i), C_{i,t}^1 \) |
|---|---|
| 1 Compute the IRF \( W_{i,t}^i (\Theta^i) \) using (40)-(47) |  |
| 2 Compute \( W_{i,t}^i \) and \( c_{i,t}^0 \) using (51) and (26) |  |
| 3 Compute \( W_{i,t}^i (\Theta^j) \) and \( C_{i,t}^0 \) using (53)-(54) |  |
| 4 Perform pruning and merging on \( W_{i,t}^i (\Theta^j) \) |  |
| 5 Find \( \hat{\Theta}_{i,t}^l, W_{i,t}^i (\Theta^j) \) by maximizing \( W_{i,t}^i (\Theta^j) \) using the multiple initialization strategy |  |

---

\(^1\) Recalling that, by construction, \( \Theta^i = \text{col} (T^{\phi^i,j}, j \in N^i \setminus \{i\}) \), when writing (55), we intend that the optimization is performed with respect to the parameters \( \phi^{i,j} \) and \( \gamma^{i,j} \) with \( j \in N^i \setminus \{i\} \).

---

The dependence on \( \Gamma^i \) of the mean \( \phi_{i,t}^l (\Gamma^i) \) and covariance \( \gamma_{i,t}^{l,j} (\Gamma^i) \) of each Gaussian component of the IRF, merging and pruning strategies cannot be directly implemented, thus implying an exponential increase in the number of Gaussian components of the TC. In order to overcome such a drawback, in this section a different solution is proposed based on: 1) computation of instantaneous estimates obtained by maximizing the IRF; 2) combination of the instantaneous estimates computed at different time instances. Hereafter, we discuss in some detail these two steps.

1) **Computation of the Instantaneous Estimates:** The instantaneous estimates \( \hat{\Theta}_{i,t}^l, \hat{\Gamma}_{i,t}^l \) at time \( t \) and consensus step \( \ell \) are computed by solving the optimization problem
\[ (\hat{\Theta}_{i,t}^l, \hat{\Gamma}_{i,t}^l) = \arg \max_{\Theta^i, \Gamma^i} W_{i,t}^i (\Theta^j, \Gamma^i) \] (56)
To this end, a multiple initialization strategy has to be adopted. Because of the large dimension of the parameter space and of the nontrivial dependence of the IRF on the drift parameters \( \Gamma^i \), the choice of the initial points is a crucial issue. In what follows, we propose a procedure for the selection of the initial points that is based on some geometric insights on the sensor registration problem.

Generally speaking, in the context of multitarget tracking, sensor registration essentially amounts to matching the set of tracks of neighboring sensors by rotation and translation operations (see Fig. 2). With this respect, it is immediate to see that a necessary condition for sensor registration is that the number of tracks is at least equal to 3 (in fact, there is not a unique way for matching points, i.e., a single track, or segments, i.e., two tracks). Recall now that, in the context of the GM-CPHD filter, each Gaussian component of the spatial density \( s_{i,t}^l \) can be seen as a track for sensor \( i \). Hence, each component \( k \in K_{i,t}^l \) in \( W_{i,t}^i (\Theta^j, \Gamma^i) \) can be seen as an association among the tracks \( k(j), j \in N^i \), of the sensors in the neighborhood \( N^i \). Then, by selecting a triplet \( \{k_1, k_2, k_3\} \in K_{i,t}^l \) of components of \( W_{i,t}^i (\Theta^j, \Gamma^i) \), one has exactly three tracks \( \{k_1(j), k_2(j), k_3(j)\} \) for any
sensor \( j \in \mathcal{N}^i \) so that a tentative matching in terms of rotation \( \Gamma^i_{[k_1,k_2,k_3]} \) and translation \( \Theta^i_{[k_1,k_2,k_3]} \) can be easily computed (see Appendix B). Such tentative matchings can be used as initial points in the maximization (56) by applying the following steps: 1) list all the possible triplets of Gaussian components of the IRF; 2) for each triplet \( \{k_1,k_2,k_3\} \), find the corresponding initial point \( (\Theta^i_{[k_1,k_2,k_3]}, \Gamma^i_{[k_1,k_2,k_3]}) \) using the strategy described in Appendix B.

2) Combination of the Instantaneous Estimates: Combination of the instantaneous estimates \( (\hat{\Theta}^i_{t,\ell}, \hat{\Gamma}^i_{t,\ell}) \) computed at different \( t \) and \( \ell \) needs special care because some of such estimates can be unreliable under particular geometrical configurations of the targets present in the scenario. For instance, in Fig. 2(a), when the coordinates of node \( j \) are rotated of \( \gamma^{i,j} \) or \( \gamma^{i,j} + 120^\circ \) or \( \gamma^{i,j} + 240^\circ \), the triangles can be perfectly matched. This can also happen when the rotation degree is \( \gamma^{i,j} \) or \( \gamma^{i,j} + 180^\circ \) in Fig. 2(c). Hence, even when more than three targets are present, there may exist unsolvable ambiguities between two sensor nodes. Such ambiguities always occur when: 1) the number of targets is odd and the targets are uniformly located on a circle; 2) the number of targets is even and the targets are located on the vertices of any symmetrical polygon.

Generally speaking, when the moving targets are not organized as a group, conditions 1) and 2) occur only occasionally, which is enough for sensor registration. However, the unreliable instantaneous estimates generated in the presence of such ambiguities need to be singled out and eliminated. For this reason, a simple averaging of the instantaneous estimates (\( \hat{\Theta}^i_{t,\ell}, \hat{\Gamma}^i_{t,\ell} \)) is ruled out and, instead, a multihypotheses approach is adopted wherein a set of possible estimates and their respective weights (defined in terms of reward functions) are maintained, i.e., \( \Xi^i_{t,\ell} = \{\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}, \kappa^i_{t,\ell} \}_{k=1}^{N^i_{t,\ell}} \). At sampling time \( t \) and consensus step \( \ell \), the instantaneous estimates (\( \hat{\Theta}^i_{t,\ell}, \hat{\Gamma}^i_{t,\ell} \)) are computed first, then followed by association and estimation as summarized in Table III, where \( \delta_\theta \) and \( \delta_\gamma \) are two preset thresholds. Note that one can limit the memory space by imposing a maximum number of elements in \( \Xi^i_{t,\ell} \) so that, once \( N^i_{t,\ell} \) exceeds the limit, the element with minimal weight is deleted.

### Table III

**Sensor Registration With Both Unknown Drift and Orientation (Node \( i \), Time \( t \), Consensus Iteration \( \ell \))**

| Input: \( \Xi^i_{t-1,\ell} \) and \( f^i_{t-1} (X^i) \) | Output: \( f^i_t (X^i) \) and \( \Xi^i_{t,\ell} \) |
|---|---|
| 1 | Find the instantaneous estimates \( \hat{\Theta}^i_{t,\ell}, \hat{\Gamma}^i_{t,\ell} \) using a multi-initialization procedure |
| 2 | Let \( \mathcal{H} = \{ h : \|\hat{\Theta}^i_{t,\ell} - \tilde{\Theta}^i_{t,\ell} \|_2 \leq \delta_\theta \) \& \( \|\hat{\Gamma}^i_{t,\ell} - \tilde{\Gamma}^i_{t,\ell} \|_2 \leq \delta_\gamma \} \) |
| 3 | If \( \mathcal{H} = \emptyset \) |
| 4 | \( \Xi^i_{t,\ell} = \Xi^i_{t-1,\ell} \cup \{ (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}, \mathcal{W}^i_{t,\ell} (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}) ) \} \) |
| 5 | Else for \( h \in \mathcal{H} \) |
| 6 | Update the associated parameter set as |
| 7 | \( \hat{\Theta}^i_{t,\ell} = \kappa^i_{t,\ell} \hat{\Theta}^i_{t,\ell} + (1 - \kappa^i_{t,\ell}) \tilde{\Theta}^i_{t,\ell} \) |
| 8 | \( \hat{\Gamma}^i_{t,\ell} = \kappa^i_{t,\ell} \hat{\Gamma}^i_{t,\ell} + (1 - \kappa^i_{t,\ell}) \tilde{\Gamma}^i_{t,\ell} \) |
| 9 | where |
| 10 | \( \kappa^i_{t,\ell} = \frac{\mathcal{W}^i_{t,\ell} (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell})}{\mathcal{W}^i_{t,\ell} (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}) + \mathcal{W}^i_{t,\ell} (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell})} \) |
| 11 | Update the weight of the associated parameter as |
| 12 | \( \hat{\Theta}^i_{t,\ell} = \kappa^i_{t,\ell} \hat{\Theta}^i_{t,\ell} + 1 + \mathcal{W}^i_{t,\ell} (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}) \) |
| End |
| Registration parameter estimation |
| 14 | Let \( h^* = \max \{ h : \kappa^i_{t,\ell} \in \{1, \ldots, N^i_{t,\ell}\} \} \) |
| 15 | Then \( (\hat{\Theta}^i_{t,\ell}, \hat{\Gamma}^i_{t,\ell}) = (\tilde{\Theta}^i_{t,\ell}, \tilde{\Gamma}^i_{t,\ell}) \) |

### Table IV

**Joint Distributed Sensor Registration and Multitarget Tracking (Node \( i \), Time \( t \))**

| Input: \( f^i_{t-1} (X^i) \) and \( \Xi^i_{t-1,\ell} \) | Output: \( f^i_t (X^i) \) and \( \Xi^i_{t,\ell} \) |
|---|---|
| 1 | Carry out steps 1-2 in Table I |
| 2 | Set \( \Xi^i_{t,0} = \Xi^i_{t-1,\ell} \) |
| 3 | For \( \ell = 0, \ldots, L - 1 \) |
| 4 | Carry out steps 4-5 in Table I |
| 5 | If sensor registration has to be performed |
| 6 | Compute \( (\hat{\Theta}^i_{t,\ell+1}, \hat{\Gamma}^i_{t,\ell+1}) \) using the algorithm of Table III |
| 7 | End if |
| 8 | If consensus has to be performed |
| 9 | Set \( \Theta^i = \hat{\Theta}^i_{t,\ell+1}, \Gamma^i = \hat{\Gamma}^i_{t,\ell+1} \) |
| 10 | Carry out step 6 in Table I |
| 11 | Else |
| 12 | Set \( f^i_{t,\ell+1} (X^i) = f^i_{t-1} (X^i) \) |
| 13 | End |
| 14 | End for |
| 15 | Carry out steps 8-9 in Table I |

E. Distributed Joint Sensor Registration and Multitarget Tracking

Combining the proposed sensor registration algorithm with the CCPHD filter, the algorithm of Table IV for joint distributed sensor registration and multitarget tracking is obtained (for the sake of brevity, only the more general case of both unknown drift and orientation parameters is provided). Notice that, in practice, it may not be necessary/desirable to perform both sensor registration and consensus at all time instants. Specifically, the following practical suggestions can be given.

1) It is better not to carry out consensus steps at the beginning when sensor registration has not yet been achieved, since the information provided by the TC may not be sufficient to provide a reliable estimate of \( \Theta^i \) and \( \Gamma^i \), so that performing fusion with an imprecise sensor registration could lead to a performance deterioration as compared to the local CPHD filters. Hence, in practice, it is better to activate consensus only when a sufficient amount of data has been collected so that the sensor registration is reliable enough (e.g., after sensor registration has been performed a certain number of times).

2) At each sampling time, the sensor registration algorithm can be executed only once (for instance only when \( \ell = 1 \)), since sensor registration can be computationally demanding (as it involves an optimization routine), and the information about the drift and orientation parameters is maximal at the first consensus step. In the case
in which only the drift parameters are needed (since the orientation parameters are already known), one can further save computations by performing optimization of the TC only once every several time intervals.

3) Sensor registration can be performed only when sufficiently many targets are detected by the sensors (i.e., the cardinality estimation is above a certain threshold). For instance, a single target is enough for sensor registration with known orientation parameters and, otherwise, at least three targets are needed.

IV. SIMULATION EXPERIMENTS

In this section, the performance of the proposed algorithm is evaluated by carrying out simulations on a two-dimensional (planar) DMT scenario. The surveillance region is a square of $8000 \times 8000$ [m$^2$], which contains six targets. The target motion model used in the filter is a white noise acceleration model \cite{50} with standard deviation of the acceleration equal to 3 [m/s$^2$] and sampling interval equal to 1[s].

The algorithm is tested on a time-synchronized nonlinear sensor network consisting of 15 nodes. In the simulations, the orientation angle of each node with respect to the horizon is randomly chosen within the interval $(-90^\circ, 90^\circ]$. For each sensor $i$, the single-target likelihood has the form $L_i(x^i, y^i) = \mathcal{G}(\tilde{x}^i; h^i(x^i), R)$ where

$$h^i(x^i) = \left[ \sqrt{(\xi^i)^2 + (\eta^i)^2} \atan2(\xi^i, \eta^i) \right]$$

and $R^i = \text{diag}(\sigma^2_x, \sigma^2_y)$, with $\sigma_x = 20$ [m] and $\sigma_y = 0.1$ [\textdegree]. In this case, the GM representation of the spatial PDF (35) at each sensor node is propagated by employing the extended Kalman filtering recursion, see \cite[eqs. (46)–(51)]{39}.

The parameters of local CPHD filters are set as follows: $P_{s,i} = 0.9$, $P_{d,i} = 0.98$ for any $i \in \mathcal{N}$. The maximum number of targets that the CPHD filters can handle is set to $N_{\text{max}} = 10$. For the target birth, we assume six high-likelihood zones that are known \textit{a priori}. Accordingly, at sensor node $i$, a six-component GM has been hypothesized for the birth intensity in the local coordinates of node $i$, in which the following conditions hold:

1) the weight of each Gaussian component is set to 0.1;
2) the velocity components of each mean vector have been set to 0 [m/s] and the position components to the real target ones biased by 50 [m];
3) the covariance matrix of each Gaussian component is set to $\text{diag}(500$ [m$^2$], 10 [m$^2$/s$^2$], 500 [m$^2$], 10 [m$^2$/s$^2$]).

The clutter set at each sensor node is assumed to be a Poisson point process with expected number of clutter measurements per scan being 20 and uniform spatial distribution over the surveillance region. The simulation horizon is set to $T = 300$ [s] and the six targets appear/disappear as specified hereafter. Targets 1–4 appear at 1 [s], while targets 5 and 6 appear at 101 [s] and 121 [s], respectively. Then, targets 1 and 3 disappear at 151 [s] and 251 [s], respectively, while the remaining targets (2, 4, 5, 6) stay in the surveillance region until the end of the simulation. For each node $i$, the proposed algorithm starts only when the node itself and its neighbors detect more than three targets. Consensus is carried out starting from $t = 151$ [s] by using the estimated drift and orientation parameters to perform the changes of coordinates. The considered simulation scenario is depicted in Fig. 3.

Two different values of the number of consensus steps $L$ are adopted in order to perform DMT: 1) $L = 1$, which corresponds to the minimal information exchange among sensor nodes and clearly represents the best choice in terms of network efficiency; and 2) $L = 5$, which is equal to the diameter of the sensor network and, therefore, represents the best choice in terms of information diffusion across the network. In order to save computational resources, after consensus begins, the sensor registration algorithm is run by each sensor node only at the first consensus step, i.e., $\ell = 1$. Note that the information concerning the number of consensus steps at each time can either be preset in each node before deployment (e.g., embedded in the software code of each node) or be shared by routing operations \cite{52} before the sensor network starts to work. For the approximation of the GM power, we adopted the same strategy as in \cite{7}. The initial estimates of the drift parameters have been set to $\hat{\theta}_{i,0}^{j} = 0$ and $\gamma_{i,0}^{j} = 0$, for any $(i, j) \in \mathcal{A}$.

Fig. 4 analyzes the sensor registration performance by displaying the time evolution, averaged over 200 Monte Carlo runs, of the drift parameter estimation errors

$$\hat{\vartheta}_i \triangleq \frac{1}{|\mathcal{A}|} \sum_{(i,j) \in \mathcal{A}} \sqrt{(\vartheta_i^{j} - \hat{\vartheta}_i^{j})^T (\vartheta_i^{j} - \hat{\vartheta}_i^{j})}$$

and the orientation parameter estimation errors

$$\hat{\gamma}_i \triangleq \frac{1}{|\mathcal{A}|} \sum_{(i,j) \in \mathcal{A}} (\gamma_i^{j} - \hat{\gamma}_i^{j})^2.$$

It can be seen that the estimated errors of drift and orientation parameters exhibit a stable behavior with the satisfactory performance. Moreover, estimation errors of both offset $\hat{\vartheta}_i$ and orientation $\hat{\gamma}_i$ exhibit fast reduction with
the proposed sensor registration algorithm. More precisely, \( \hat{\theta}_i \) and \( \hat{\gamma}_i \) become smaller than the respective measurement noise standard deviations (i.e., \( \sigma_{\theta} \) and \( \sigma_{\gamma} \)) after less than 10 recursions, thus implying that after a short time DMT performance is not affected by registration errors.

Furthermore, the multitarget tracking performance of the proposed joint sensor registration and DMT (JSR-DMT) algorithm is evaluated with respect to the optimal subpattern assignment (OSPA) distance [51] (with order \( p = 2 \) and cutoff \( c = 15 \) [m]). In order to provide a performance baseline, the CPHD filter with perfect knowledge of the registration parameters (referred to as CCPHD-PK) is also involved in the simulations. Fig. 5 plots the time evolution of the network OSPA averaged over 200 Monte Carlo runs. It can be seen that, in the JSR-DMT algorithm, when sensor registration has been achieved and the consensus algorithm begins to work (\( t \geq 151 \) [s]), the performance of the DMT algorithm is much better as compared to the case without consensus (\( t < 151 \) [s]). Furthermore, it can also be seen that once consensus starts (\( t \geq 151 \) [s]) JSR-DMT exhibits almost the same accuracy as CCPHD-PK with both choices (\( L = 1 \) and \( L = 5 \)) of the number of consensus steps, thus demonstrating the effectiveness of the proposed sensor registration approach.

V. CONCLUSION

This paper has dealt with DMT over an unregistered sensor network. It has been shown how it is possible to jointly estimate, in a fully distributed and computationally feasible way, both the registration parameters (i.e., relative positions and orientations of the sensor nodes) as well as the number and kinematic states of the targets present in the surveillance region. The problem, referred to as distributed joint sensor registration and multitarget tracking, has been solved by using a CPHD filter in each sensor node of the network, for the update of a local posterior multitarget density and then minimizing an information-theoretic criterion that measures the discrepancy among such local posteriors for both sensor fusion and registration purposes. The effectiveness of the proposed approach has been successfully tested via simulation experiments.

APPENDIX A

Proof of Proposition 1: By recalling the definition of set integral, we have

\[
J_{r,t}^{i,j}(\Theta^i, \Gamma^i) = -\log \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j \in N^p} \left[ f_{t,1}^{j,i}(x_i^j; \theta_i^j, \gamma_i^j) \right]^{\omega^{j,i}} \, dx_i^j \cdots dx_n^j \right\}.
\]

Since each \( f_{t,1}^{j,i} \) is an i.i.d. cluster density, the above-mentioned identity implies

\[
J_{r,t}^{i,j}(\Theta^i, \Gamma^i) = -\log \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j \in N^p} \left[ n! p_{t,1}^{j,i}(n) \right]^{\omega^{j,i}} \right. \times \left\{ \prod_{k=1}^{n} s_{t,1}^{j,i}(x_k^j; \theta_i^j, \gamma_i^j) \right\} \, dx_i^j \cdots dx_n^j \right\}.
\]

Recalling now that \( \sum_{j \in N^p} \omega^{j,i} = 1 \) and, hence, \( \prod_{j \in N^p} (n!)^{\omega^{j,i}} = n! \), the above-mentioned equation yields

\[
J_{r,t}^{i,j}(\Theta^i, \Gamma^i) = -\log \left\{ \sum_{n=0}^{\infty} \prod_{j \in N^p} \left[ p_{t,1}^{j,i}(n) \right]^{\omega^{j,i}} \right. \times \left\{ \int \prod_{j \in N^p} s_{t,1}^{j,i}(x_i^j; \theta_i^j, \gamma_i^j) \, dx_i^j \right\}^n \right\}
\]

which can be rewritten as in (25).
Proof of Theorem 3: Observe preliminarily that the product of Gaussian distributions is again Gaussian and, specifically,
\[
\prod_j \mathcal{G}(x; \mu^j, P^j) = \det(2\pi P)^{1/2} \mathcal{G}(x; \mu, P) \prod_j \mathcal{G}(\mu^j, P^j)
\]
where \( P = \{\sum_j (P^j)^{-1}\}^{-1} \) and \( \mu = \mathcal{P}^* \sum_j (P^j)^{-1} \mu^j \).

Furthermore, it is an easy matter to check that the following identity holds
\[
\prod_j \mathcal{G}(\mu^j, P^j, P^j) = \left( \prod_j \frac{1}{\det(2\pi P^j)^{1/2}} \right) \\
\times \exp \left\{ -\frac{1}{2} \left( \sum_j (\mu^j)^\top (P^j)^{-1} \mu^j - \mu^\top \mathcal{P} \mu \right) \right\}.
\]

Consider now the IRF (27). Substituting (36) and (37) into (27), we have
\[
\mathcal{W}_{t,\ell}^{\psi}(\Theta^i, \Gamma^i) = \int \prod_{j \in \mathcal{N}^i} \left[ s_{t-1}^{j,j}(x; \theta^{i,j}, \gamma^{i,j}) \right]^{\omega_{i,j}} dx
\]
\[
= \int \prod_{j \in \mathcal{N}^i} \sum_k \delta_{i,j}^{i,j,k} \mathcal{G}(x; \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i), \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i)) dx
\]
\[
= \int \prod_{j \in \mathcal{N}^i} \sum_k \delta_{i,j}^{i,j,k} \mathcal{G}(x; \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i), \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i)) dx.
\]
Hence, by employing (58), we get
\[
\mathcal{W}_{t,\ell}^{\psi}(\Theta^i, \Gamma^i) = \sum_{k \in \mathcal{K}_{i,\ell}} \det(2\pi \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{1/2}
\times \int \mathcal{G}(x; \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i), \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i)) dx
\times \prod_{j \in \mathcal{N}^i} \delta_{i,j}^{i,j,k} \mathcal{G}(x; \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i), \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i)),
\]
where \( \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) = \{\sum_{j \in \mathcal{N}^i} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1}\}^{-1} \) and \( \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) = \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \sum_{j \in \mathcal{N}^i} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i) \). Noting that the integral in the above-mentioned expression is equal to 1 and exploiting the identity (59), we obtain
\[
\mathcal{W}_{t,\ell}^{\psi}(\Theta^i, \Gamma^i) = \sum_{k \in \mathcal{K}_{i,\ell}} \det(2\pi \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{1/2}
\times \exp \left\{ -\frac{1}{2} \left[ \sum_{j \in \mathcal{N}^i} (\tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \tilde{\mu}_{t,\ell}^{i,j,k}(\Gamma^i) \\
- (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \right] \right\}.
\]

Observe now that, since the matrices \( M^{i,j} \) are orthogonal, we have \( \det(2\pi \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i)) = \det(2\pi \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) / \omega^{i,j}) \). Furthermore, recalling the definitions of \( \Omega^i, \mathcal{M}_{t,\ell}^{i,j}, \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \), and \( \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \), it is immediate to check that \( \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \) can be rewritten as in the statement of the theorem and, moreover, we can also write
\[
\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) = \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \left[ (E^\top)^{-1} (\psi_{i,t,\ell}^{i,j,k}(\Gamma^i) \left[ (M^i \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) + \Omega^i) \\
+ (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \right]^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i) \right] \right] \sum_{j \in \mathcal{N}^i} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i). \]

Let us now define
\[
\xi_{i,t,\ell}^{i,j,k}(\Gamma^i) = (\psi_{i,t,\ell}^{i,j,k}(\Gamma^i) \left[ (M^i \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) + \Omega^i) \\
+ (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \right]^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i) \right] \right] \sum_{j \in \mathcal{N}^i} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i).
\]
Then, by using the matrix inversion lemma, we have
\[
\left[ \xi_{i,t,\ell}^{i,j,k}(\Gamma^i) \right] \left[ \xi_{i,t,\ell}^{i,j,k}(\Gamma^i) \right]^{-1}
= \psi_{i,t,\ell}^{i,j,k}(\Gamma^i) \left[ (\xi_{i,t,\ell}^{i,j,k}(\Gamma^i) \left[ (M^i \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) + \Omega^i) \\
+ (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} \tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i) \right]^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i) \right] \right] \sum_{j \in \mathcal{N}^i} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{\top} (\tilde{P}_{t,\ell}^{i,j,k}(\Gamma^i))^{-1} \mu_{t,\ell}^{i,j,k}(\Gamma^i).
\]

Considering (61)–(62) as well as the above-mentioned identities, the exponential argument in (60) can be rewritten as in (63). Finally, by substituting (63) into (60), the proof is
\[
\sum_{j \in N^t} (\hat{\mu}_{t,j,k}(j))_i \left( (\hat{\mu}_{t,j,k}(j))_i^{-1} \right) \]

\[
- (\tilde{\mu}_{t,k}(\Gamma^i))_i^T (\tilde{\mu}_{t,k}(\Gamma^i))_i^{-1} \tilde{\mu}_{t,k}(\Gamma^i)
\]

\[
= (M^t \mu_{t,i}^k + \Theta^i)^T \xi_{i,t}^k (\Gamma^i) \left( M^t \mu_{t,i}^k + \Theta^i \right) - 2 (M^t \mu_{t,i}^k + \Theta^i)^T \xi_{i,t}^k + \left[ s_{i,t}^k (\Gamma^i) \right]_i^T \left[ s_{i,t}^k (\Gamma^i) \right]_i^{-1} \xi_{i,t}^k
\]

\[
= (M^t \mu_{t,i}^k + \Theta^i)^T \xi_{i,t}^k (\Gamma^i) \left( M^t \mu_{t,i}^k + \Theta^i \right) - 2 (M^t \mu_{t,i}^k + \Theta^i)^T \xi_{i,t}^k + \left[ s_{i,t}^k (\Gamma^i) \right]_i^T \left[ s_{i,t}^k (\Gamma^i) \right]_i^{-1} \xi_{i,t}^k
\]

\[
= \Theta^i - (E^i \mu_{t,i}^k - M^t \mu_{t,i}^k)^T \xi_{i,t}^k (\Gamma^i) \left[ \Theta^i - (E^i \mu_{t,i}^k - M^t \mu_{t,i}^k)^T \xi_{i,t}^k (\Gamma^i) \right]_i^T \xi_{i,t}^k (\Gamma^i) \left[ \Theta^i - (E^i \mu_{t,i}^k - M^t \mu_{t,i}^k)^T \xi_{i,t}^k (\Gamma^i) \right]_i^T \xi_{i,t}^k (\Gamma^i).
\]

**APPENDIX B**

Recall that the Gaussian components are defined with respect to both position and velocity. However, by construction, \(\Theta^i = \text{col} (T \vartheta^{i,j}, j \in N^t \setminus \{i\})\). Hence, we are interested in matching only the positions of the tracks. For a given component \(k \in K^t_{i,t}\), the positions of the tracks \(j, k \in N^t\), can be matched by choosing \(\Theta^i\) and \(\Gamma^i\) such that

\[(T^i)^T \Theta^i - (T^i)^T \Phi^i_{i,t}(\Gamma^i) = 0\]

where \((T^i)^T \Theta^i = \text{col} (\vartheta^{i,j}, j \in N^t \setminus \{i\})\), and, in view of (42), we have

\[\Phi^i_{i,t}(\Gamma^i) = \text{col} \left( \mu_{t,j}^k(j) - M^t \mu_{t,j}^k(j), j \in N^t \setminus \{i\} \right)\].

By writing \(\mu_{t,j}^k(j) = [\xi_{j,t}^k(j), \eta_{j,t}^k(j), \dot{\xi}_{j,t}^k(j), \dot{\eta}_{j,t}^k(j)]^T, j \in N^t\), it is an easy matter to check that the following identity holds

\[
(T^i)^T \Phi^i_{i,t}(\Gamma^i) = \text{col} \left( b_{i,t}^k(j)^{-1} - A_{i,t}^k(j)^{-1} \omega^{i,j} , j \in N^t \setminus \{i\} \right)
\]

where

\[
\omega^{i,j} = \left[ \cos \gamma^{i,j} \sin \gamma^{i,j} \right]^T
\]

\[
b_{i,t}^k(j)^{-1} = \left[ \xi_{j,t}^k(j), \eta_{j,t}^k(j), \dot{\xi}_{j,t}^k(j), \dot{\eta}_{j,t}^k(j) \right]^T
\]

\[
A_{i,t}^k(j)^{-1} = \left[ \xi_{j,t}^k(j), \eta_{j,t}^k(j), \dot{\xi}_{j,t}^k(j), \dot{\eta}_{j,t}^k(j) \right]^T
\]

Consider now a triplet \(\{k_1, k_2, k_3\} \in K^t_{i,t}\). Given \(\omega^{i,j}\), the orientation parameter \(\gamma^{i,j}\) can be easily obtained. Then, the positions of three sets of tracks can be matched by choosing \(\Theta^i\) and \(\Gamma^i\) such that

\[
\begin{align*}
(T^i)^T \Theta^i - (T^i)^T \Phi^i_{i,t}(\Gamma^i) & \approx 0 \\
(T^i)^T \Theta^i - (T^i)^T \Phi^i_{i,t}(\Gamma^i) & \approx 0
\end{align*}
\]

where the approximation symbol accounts for the fact that, in general, the three conditions cannot be exactly satisfied together due to the uncertainties in the tracks. Then, the initial point associated to the triplet \(\{k_1, k_2, k_3\}\) can be obtained by finding a solution of (64), in the least-squares sense, by means of the simple procedure of Table V.

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