Mathematical modeling and optimization of adaptive information system

Artem Obukhov*, Mikhail Krasnyanskiy and Maxim Nikolyukin
Tambov State Technical University, 392000 Tambov, Russian Federation

*Obukhov.art@gmail.com

Abstract. The paper considers mathematical modeling and optimization of adaptive information systems. The presented mathematical model is based on the apparatus of theories of sets and graphs and formalizes the main components and the relationships between them in adaptive information systems. Based on the developed model, a set of criteria is formulated for assessing the quality of systems. The multicriteria task of optimizing adaptive information systems is posed. An algorithm for its solution and the practical implementation of an adaptive information system for transmitting a video stream are considered. The stated approaches to formalization can be used for mathematical modeling and optimization of various adaptive information systems for solving digitalization problems in technical and industrial sectors.

1. Introduction
Digitalization of modern society creates the need to improve the functionality of information systems [1]. One of the directions in the development of systems is their adaptation to the capabilities and requirements of users. Adaptive information systems (AIS) mean systems that contain the possibility of modifying the algorithms of their functioning in response to user actions or changing the characteristics of the external environment [2]. Therefore, the urgent task is to develop and improve the mathematical support of AIS, the formulation and solution of problems of their optimization [3, 4].

In the framework of this work, we consider a mathematical model of AIS, presented using the apparatus of theories of sets and graphs. Based on the developed model, a set of criteria is formulated for assessing the quality of systems. The problem of optimizing AIS video stream transmission has been formulated and solved. The use of such a system is relevant for various industrial and technical systems that place high demands on the quality of the transmitted video signal while fulfilling the restrictions on the maximum load on the AIS hardware.

2. The mathematical model of AIS
At the first stage of research, an analysis of the structure of AIS was carried out. AIS components can be divided into 4 groups: Environment $E$, Control $C$, Model $M$, View $V$. We formalize the structure of AIS in the form of a graph (Fig. 1).
Based on this structure, a mathematical model of AIS has been developed, which has the following form in notation of set theory:

\[ MM_{AIS} = (STR, PRM, CRT), \]

where \( STR \) is the structure of the AIS, including many components and the relationships between them;

\( PRM \) is the AIS regulation parameters responsible for its functioning;

\( CRT \) is the set of evaluations of the effectiveness of AIS.

The structure of AIS in a formalized form is described by the following relationships and relationships between elements:

\[ S = (E, C, M, V). \]

The Environment includes many users \( U \), their access rights \( LAU \) and actions \( AU \), external factors \( F \), hardware \( HW \) and software \( SW \) parameters:

\[ E = (U, AU, LAU, HW, SW, F). \]

There are two types of impacts on AIS: user actions \( AP \) and external factors \( F \):

\[ AP = \{ap_i\}, F = \{f_i\}. \]

Each impact is mapped to set of events \( ev_i \):

\[ ap_i \rightarrow \{ev_i\}, f_i \rightarrow \{ev_i\}, \]

Control includes control blocks \( CB \) (software components of the interface with which the user interacts in the course of their actions), the event handler \( EV \), adaptation parameters \( AD \):

\[ C = (CB, EV, AD). \]

The event handler contains many functions \( f(ev_i) \) - software methods that implement response actions to events \( ev_i \):

\[ EV = \{f(ev_i)\}. \]
The handler $EV$ generates new parameters $AD = \{ad_i\}$ depending on the events $ev_i$ that occur according to the results of functions $\{f(ev_i)\}$:

$$\sum_i (g(ad_i) - g(ev_i)) \rightarrow 0.$$  

(8)

Functions $g(x)$ are used to evaluate the effect of an argument $x$ on the performance of AIS. Relation (8) reflects adaptability functions: parameter values $AD$ should level the effect of occurring events $\{ev_i\}$.

The Model consists of a model of information objects $MI$ (stored data and their structure), a model of functional blocks $MF$ (software modules, functions and procedures), an adaptation model $MA$ (intelligent modules that adapt the AIS to user equipment), an intermediary model $MP$ (communicates between other models):

$$M = \{MP, MI, MF, MI\}.$$  

(9)

The adaptation model provides a link between the firmware of users and a subset of adaptation parameters $AD^*$:

$$MA = SW + HW \rightarrow AD^* \in AD.$$  

(10)

The View contains the displayed data $ID$ and adaptive interface elements $IA$:

$$V = (ID, IA).$$  

(11)

The adaptive interface $IA$ is formed on the basis of adaptation parameters $AD$ obtained earlier:

$$AD \rightarrow IA.$$  

(12)

Consider the relationship between the components of the structural model of AIS:

$$D = (D_E, D_C, D_M, D_V, D_{CM}, D_{CV}).$$  

(13)

where $D$ is the set of edges of the graph of the AIS structure, which are the links between the components of the AIS structure.

Relationships between components AIS structure can be divided into intramodular ($D_E$, $D_C$, $D_M$, $D_V$) and intermodular ($D_{EC}$, $D_{CM}$, $D_{CV}$).

The subset of edges $D_E$ reflects the relationship between the user and his possible actions, access level and current equipment configuration:

$$D_E = (U \rightarrow AU, U \rightarrow LAU, U \rightarrow SW + HW).$$  

(14)

The relationships $D_C$ reflect the influence of external forces on the adaptation parameters $AD$ through the interaction of control units and an event handler:

$$D_C = (CB \leftrightarrow EV, EV \rightarrow AD, CB \rightarrow AD).$$  

(15)

The relationships $D_M$ implements the movement of information flows between different models. All models are linked through an intermediary model:

$$D_M = (MP \leftrightarrow MI, MP \leftrightarrow MF, MP \leftrightarrow MA).$$  

(16)

The relationships $D_V$ reflect the mutual interaction of data and adapted interface elements in which they are placed:

$$D_V = (ID \leftrightarrow IA).$$  

(17)

The relationships $D_{EC}$ show the influence of Environment objects (users and the external environment) on AIS on control units and an event handler:
The relationships $D_{CM}$ implements receiving data from a model in response to an event (for example, to display information in response to user actions):

$$D_{CM} = (EV \leftrightarrow MP).$$  \hspace{1cm} (19)

The relationships $D_{CV}$ fill and customize the interface:

$$D_{CV} = (AD \rightarrow IA, CB \rightarrow ID).$$  \hspace{1cm} (20)

AIS regulation parameters $PRM$ contains set of all parameters of AIS modules and components, including adaptation parameters $AD$:

$$PRM = \{prm\}, AD \subset PRM.$$  \hspace{1cm} (21)

Parameters $prm_i$ denote attributes of program methods and functions, component settings, are constraints and boundary conditions for processes and operations.

We will carry out optimization in several areas to assess the effectiveness of the AIS:

$$CRT = (CRT_e, CRT_p, CRT_c, CRT_A, CRT_Q)$$  \hspace{1cm} (22)

where $CRT_e$, $CRT_p$, $CRT_c$, $CRT_A$, $CRT_Q$ are evaluations of the economic costs, productivity, implementation complexity, adaptability and quality of AIS.

Evaluation of economic costs $CRT_e$ includes the following components:

$$CRT_e = E_{HW} + E_{SW} + E_{SPP} + E_{TR},$$  \hspace{1cm} (23)

$$E_{HW} = E_0 + E_w + E_{RE},$$  \hspace{1cm} (24)

where $E_{HW}$ are equipment costs: $E_0$ are initial costs for the purchase of server and client equipment for AIS; $E_w$ are costs of the operation of the AIS (electricity and overhead); $E_{RE}$ are costs of re-equipment and re-equipment of AIS (depreciation, replacement of components);

$$E_{SW} = E_L + E_{DEV},$$  \hspace{1cm} (25)

where $E_{SW}$ are software costs: $E_L$ are license fees for software products; $E_{DEV}$ is the cost of developing AIS, depending on the period of development;

$E_{SPP}$ are the costs of supporting AIS, depending on the complexity of the AIS and the reliability of its work;

$E_{TR}$ are the costs of developing AIS by users, which depend on the complexity of the AIS interface and the degree of its adaptability.

AIS performance assessment $CRT_p$ is determined by the number of operations performed over a certain period of time:

$$CRT_p = \frac{\sum_{i=1}^{k} on_i}{\sum_{i=1}^{k} t_i \cdot on_i},$$  \hspace{1cm} (26)

where $on_i$ is the number of operations of the $i$th type;

$t_i$ is the average time to complete operations of the $i$th type.

Assessing the complexity of implementing AIS $CRT_c$ is determined by combining several metrics:

$$CRT_c = \lambda_1 D_{COCOMPO} + \lambda_2 D_J + \lambda_3 D_R + \lambda_4 D_{CC} + \lambda_5 D_{CH} + \lambda_6 D_{ABC},$$  \hspace{1cm} (27)
where \( D_{COCOMO} \) is the estimate of the complexity of the project using the COCOMO method [5], \( D_J \) is the estimate of the complexity of the program code using the Jilb method [6], \( D_H \) is the estimate of the complexity of the program code using the Halstead method [6], \( D_{CH} \) is the cyclomatic complexity of the program code [6], \( D_{ABCD} \) is the metric of the ABC software [6], \( \lambda_k \) are normalizing weights, leading the values of \( CRT_C \) to the range \([0;1]\).

AIS adaptability assessment \( CRT_A \) is a linear convolution of a set of ergonomic criteria:

\[
R_A = \sum_{i=1}^{18} \lambda_i r_i, \tag{28}
\]

where \( \lambda_i \) is the weight coefficient (the sum of all coefficients is 1); \( r_i \) are the 18 ergonomic criteria presented in [7]. For their use in the mathematical model of AIS, an expert assessment will be used for each criterion. Values for the criteria range from 0 to 1.

The quality of the AIS \( CRT_Q \) is determined by expert judgment on the indicators ISO/IEC 25022 and 25023 [8]:

\[
CRT_Q = \sum_i \omega_i Q_i, \tag{29}
\]

where \( Q_i \) is the score for the \( i \)th indicator, \( \omega_i \) is the weight coefficient.

The presented mathematical model of AIS allows us to formalize the structure and relationships of the elements of AIS, to carry out its comprehensive assessment.

3. Problem statement of optimization of AIS

Based on the developed mathematical model of AIS, the following statement of the optimization problem is obtained: it is necessary to determine the structure \( STR \) and parameters \( PRM \) of the AIS at which the target function \( R \) of AIS optimization reaches an extremum:

\[
R(STR, PRM) = \begin{cases} 
CRT_E \to \min, \\
CRT_F \to \max, \\
CRT_C \to \min, \\
CRT_A \to \max, \\
CRT_Q \to \max,
\end{cases} \tag{30}
\]

in accordance with relationships of a mathematical model (1-29) and restrictions:

\[
CRT_E \leq E_{MAX}, \tag{31}
\]

\[
CRT_F \geq P_{MIN}. \tag{32}
\]

The optimization problem can be solved by various methods: Pareto, consecutive concessions, changes in restrictions, lexicographic optimization, linear convolution [9].

4. The algorithm for solving the statement of optimization of AIS

The solution of the optimization problem was carried out using the algorithm presented in Fig. 2. As an example of testing the mathematical model of AIS, we consider the development of a system for adapting the quality of video stream transmission. This system can be used in agriculture, when implementing security subsystems in home automation systems, industrial facilities, technical systems, etc.
1. Formalization of the structure of AIS
2. The definition of the range of values of the varied parameters AIS
3. Calculation of AIS metrics over the entire range of variable values
4. Search for the Pareto-set of optimal solutions
5. Transition to a single-criterion optimization problem
6. Determination of optimal values of AIS parameters
7. AIS Design

Figure 2. Algorithm for solving the AIS optimization problem

The structure of this AIS has the following form: \( M \) - database, \( V \) - client application, \( C \) - server (is engaged in video stream processing), \( E \) - environment includes users, their hardware and software.

Video quality parameters and the range of their values are defined in Table 1.

Table 1. Video quality options.

| Parameter         | Range of values          |
|-------------------|--------------------------|
| \( r \) (video resolution) | [640x480 ... 3840 x 2160] px |
| \( f \) (frames per second)   | [1 ... 60] fps           |
| \( b \) (bit rate)         | [192 ... 16000] kbps    |

Next, we will calculate the metrics (22). We accept the following assumption: as part of this task, optimization is carried out according to 2 criteria – quality \( \text{CRT}_Q \) (depends on the quality of the video frame, tends to the maximum) and productivity \( \text{CRT}_P \) (depends on the load on the system, tends to the minimum) AIS. The other criteria are not calculated, since the development of AIS is carried out on one version of existing equipment for research purposes, and the adaptability criterion is ensured by the performance of AIS at various loads. Then

\[
\text{CRT}_Q = r \cdot f \cdot b \to \text{max},
\]

\[
\text{CRT}_P = \frac{P_t(\text{CRT}_Q)}{C_t} \cdot 100\% \to \text{min},
\]

where \( C_t \) is the total processor time (for the selected hardware platform is 9454), \( P_t \) is the processor time spent on video processing. It is experimentally obtained that \( P_t(b) = 1.493b - 2378 \).

For the entire range of variable values, we calculate the metrics. Next, we define the Pareto-set of optimal solutions [9]. Within the framework of this task, these will be all solutions in which the load on the central processor of the server is less 70% (a large load can adversely affect the quality of the entire system as a whole, since the server processes other data besides processing the video stream) at the highest possible quality. Then we replace the performance criterion with a restriction of the following form: \( \text{CRT}_P < 70\% \). Then the task is converted to a single criterion. The optimization problem is solved by exhaustive search, since the values of the main parameters of the video stream vary in the range of small sets.

As a result, the following values of the criteria were obtained: for the worst parameters \( \text{CRT}_Q = 192 \), for optimal parameters \( \text{CRT}_Q = 6025 \).

Based on the solution, it is necessary to design an AIS. UML tools and deployment diagrams are used to design AIS (Fig. 3).
5. Conclusion
The paper considers the structure of the AIS, presented in the form of a graph. Based on this structure, a mathematical model of AIS has been developed. It includes the structure, parameters and evaluations and effectiveness of AIS. The structural components of AIS are divided into 4 main blocks: Environment, Control, Model and View. The main relationships between the components are considered.

The estimates of economic costs, work productivity, implementation complexity, adaptability and quality of AIS are formulated. The problem of optimizing AIS is based on the proposed mathematical model and criteria. An algorithm for its solution is formulated. The algorithm was tested in the implementation of the video stream quality adaptation system.

Further research will be related to the expansion of the mathematical model, its testing in solving practical problems of designing and optimizing AIS.

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