Critical temperature of a chain of long range interacting ferromagnets

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Abstract. The thermodynamic behavior of systems with long range interactions is anomalous, because there are some problems about defining the thermodynamic limit. A way to solve the problem is to use scaled thermodynamic quantities. In this work, we use a nonextensive scaling into Hamiltonian and characterize transitions between two different magnetic ordering phases. The critical temperature is estimated by Binder method. Ferromagnetic long range interactions are included in a special Hamiltonian through a power law that decays at large interparticle distance \( r \) as \( r^{-\alpha} \) for \( \alpha \geq 0 \). In addition, we improve the known nonextensive scaling and obtain the critical temperature for several values of \( \alpha \).

As the space dimension of a physical system decreases, magnetic ordering tends to vanish as fluctuations become relatively more important. This reason induced for a long time to specialists not to take interest in systems in one dimension. However, due to several theoretical and practical aspects, which appeared in physical properties of the systems in one dimension, carried to specialists to change such attitude [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11].

Microscopic anomalies, for instance anisotropy properties lead to some important modifications to the thermodynamic properties of systems. Then, ferromagnetism in one dimension has been reported and it is thought that anisotropy barriers are responsible in that novel effect [8].

Recently, the Gibbs potential of a one-dimensional metal at constant magnetization to second order is calculated, in the screened electron-electron interaction. At zero temperature, it is found a possible paramagnetic-ferromagnetic quantum phase transition in one-dimensional metals, that must be first order [9].

Moreover, transitions between two different magnetic ordering phases are obtained as a result of ferromagnetic long-range interactions, which have been included in a special Hamiltonian through a power law that decays at large interparticle distance. It is shown that if the range of interactions decreases, the trend of the critical temperature disappears, but if the range of interactions increases, the trend of the critical temperature approaches to the mean field approximation. The crossover between these two limiting situations is discussed. The critical
temperature has been estimated through Monte Carlo simulations by the Binder method [10].

The main aim in the present work is two fold:

(i) To obtain the critical temperature between two states of different magnetic ordering in a spin-$\frac{1}{2}$ Ising linear chain, where the range of interactions is, at least, comparable to the size of the chain.

(ii) To improve the according to a modofication in the known nonextensive scaling.

It is well known that there is a state of magnetic ordering in the mean field approximation. This approximation focuses on a single particle and assumes that the most important contribution to the interaction of such particle with all particles is determined by the mean field due to the other particles. The configuration with lowest energy is one in which the spins are totally aligned. Before, it was explained that no magnetic ordering is observed for finite range of interaction (e.g., first neighbors); now we illustrate the behavior of the system between infinite and finite range of interactions through a generic power law decaying as $1/r^\alpha$, where $r$ is the distance between two particles and $0 \leq \alpha < \infty$, ($\alpha = 0$ and $\alpha \to \infty$ close respectively to mean field and to independent spin approximations). Hence, the main question that we try to answer in the present work is “How does the critical temperature depend really on the range of the interaction?”

We enter the present discussion by revisiting some fundamental facts about the nonextensive thermodynamics that systems with long-range interactions show.

This way is useful to scale thermodynamic quantities, which depend strongly on the size of the system whose microscopic interactions are long-range. The explicit form of the Tsallis scaling appears by evaluating approximately the internal energy per particle. With this intention, we take into account interparticle potentials $v(r)$ with an attractive tail that decays as

$$v(r) = \frac{1}{r^\alpha}.$$  \hspace{1cm} (1)

Hence, if we take into account that the lattice is symmetric in the space, we must write

$$U_N \propto 2 \int_1^{N/2} dr g(r) v(r) \propto N^* \equiv 2 \left(\frac{N}{2}\right)^{1-\alpha} - 1 \frac{1}{1-\alpha},$$  \hspace{1cm} (2)

where $N$ is the size of systems and $g(r)$ (the pair distribution function) closes to 1 for $r \gg 1$. In the present topic we remember the Tsallis scaling is given the asymptotic value of the Eq. (2)

$$N^* = \begin{cases}
2^\alpha (N^{1-\alpha})/(1-\alpha) & \text{for } 0 \leq \alpha < 1 \\
2 \log(N) & \text{for } \alpha = 1 \\
2/(\alpha - 1) & \text{for } \alpha > 1.
\end{cases}$$  \hspace{1cm} (3)

In a previous work [10], the scaling proposed by Cannas and Tamarit (CT) [11] was used for estimating the critical temperature. However, in such proposal there is a miscalculation for $\alpha > 1$ and in Eq. (3) is shown the right asymptotic value for the scaling.

Due to the difficulty to define the thermodynamic limit, Tsallis has conjectured a possible way to solve the problem [12]. At this stage, we remark that it has been originally proposed as a nonextensive scaling for thermodynamic variables; this is, quantities like energy, as internal energy $U_N$, Gibbs free energy $G_N$ scale with $NN^*$, variables which include scalings with $N^*$ are
the intensive variables \((T, P, \text{etc})\). So, the fundamental equation of the thermodynamics given by

\[
\frac{G_N}{N^*N} = \frac{U_N}{N^*N} - \frac{T}{N^*N}S_N + \frac{P}{N^*N}V
\]

ensures the convergence in the thermodynamic limit \([12]\).

\[\text{Figure 1.}\] The trend of the critical temperature as a function of \(\alpha\), the range of interaction. For \(0 \leq \alpha \leq 1\) we see that the critical temperature coincides with the mean field value. The present result is compared with previous result.

An alternative model is studied here, the present scaling is not applied to normalize thermodynamic quantities, as it has been originally proposed, because some problems related to the interpretation have been appointed. For instance, the measured temperature is not \(T^*\), it is \(T\); the measured internal energy is not \(U^*_N\), it is \(U_N\), etc. In this way, now we put the scaling into the Hamiltonian. In addition, the system is described by

\[
H = -\sum_{i,j=1}^{n} J(i-j)s_is_j
\]

where \(s_i = \pm 1\) \(\forall i\) and \((i-j)\) is the distance between two sites and \(n\) is the number of particles in every cell.

\[
J(K) = \frac{1}{2} \sum_{l=-L}^{L} \frac{J_L^L(n)}{|nl+K|^{\alpha}}
\]

where

\[
(2L+1)n = N
\]

is the total number of particles in \(2L + 1\) repetition of a central configuration,

\[
J_L^L(n) = \frac{J}{N^*}
\]

where \(J\) is a positive parameter, \(J_L^L(n)\) measures the strength of the coupling that depends on the size \(n\) of the system, and we carry the scaling to the Hamiltonian. At this point, we remark that the coupling is \(J_L^L(n) \to (\alpha - 1)J/2\) for \(\alpha > 1\).

In Fig. 1, we depict the trend of the internal energy as a function of \(N\). We see that the mean field model is good for \(0 \leq \alpha \leq 1\), but a deviation from mean field is observed for \(\alpha > 1\). In a
previous work [10] we use the nonextensive CT scaling, which has been suggested in Ref.([11]) and we compare it to the result obtained from the present improvement.

In summary, a careful and simple evaluation gives the correct asymptotic value for the nonextensive scaling as it was shown in Eq.3, with this scaling we improve the trend of the critical temperature for $\alpha > 1$ and we verify that the mean field approximation is the best description for $\alpha \leq 1$. Finally, we can see that a suitable thermodynamics is derived from this form of Hamiltonian with nonextensive scaling.

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