Non-factorizable contributions to $\bar{B}_d^0 \to D_s^{(*)} D_s^{(*)}$ from chiral loops and tree level $1/N_c$ terms

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Abstract.
We point out that the amplitudes for the decays $B_0^d \to D_s^- D_s^-$ and $B_0^s \to D_s^+ D_s^-$ have no factorizable contributions. If one or two of the $D$-mesons in the final state are vectors (i.e. $D^*$'s) there are relatively small factorizable contributions through the annihilation mechanism. The dominant contributions to the decay amplitudes arise from chiral loop contributions and $1/N_c$ suppressed tree level. We predict that the branching ratios for the processes $\bar{B}_0^0 \to D_s^+ D_s^-$ and $\bar{B}_0^0 \to D_s^+ D_s^-$ are all of order $(2-3) \times 10^{-4}$, while $\bar{B}_d^0 \to D_s^+ D_s^-$ and $\bar{B}_s^0 \to D_s^+ D_s^-$ are of order $(4-7) \times 10^{-3}$. If both $D$-mesons in the final state are $D^*$'s, we obtain branching ratios of order two times bigger.

INTRODUCTION

Decay modes like $B \to \pi \pi, K \pi$ are intensively studied. From the theoretical side, for instance within QCD-factorization. The decays of a $B$- meson into two $D$-mesons are different because the energy release is only of order 1 GeV, and therefore (QCD-) factorization is not expected to hold. $B \to DD$ decays are also studied experimentally [1]. Here we discuss non-factorizable contributions to the decay modes $\bar{B}_d^0 \to D_s^{(*)} D_s^{(*)}$, where $D_s^{(*)}$ is a pseudoscalar or a vector meson. At quark level such decays occur through the annihilation mechanism $bq \to c\bar{c}$, where $q = d, s$ respectively. However, in the factorized limit the annihilation mechanism will give a vanishing amplitude due to current conservation (similar to $D^0 \to K^0 \bar{K}^0$ [2]), unless one or two of the $D$-mesons in the final state are vectors. The contributions due to the annihilation mechanism are proportional to a numerically non-favored Wilson coefficient. In contrast, the typical factorized decay modes which proceed through the spectator mechanism, $\bar{B}^0 \to D^+ D_s^-$ say, are proportional to the numerically favored Wilson coefficient.

In our approach [3, 4], the non-factorizable contributions are coming from chiral loops and from tree level amplitudes generated by soft gluon emission forming a gluon condensate. The gluon condensate contributions can be calculated within a recently

1 Presented by J.O. Eeg
developed Heavy Light Chiral Quark Model (HLχQM) \cite{5}. This model has been applied to processes involving $B$-mesons in \cite{6,7}. Both the chiral loop contributions and the gluon condensate contributions are proportional to the numerically favorable Wilson coefficient.

**FRAMEWORK**

**Effective Lagrangian at quark level**

The relevant effective Lagrangian at quark level reads:

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* \sum_i a_i(\mu) Q_i(\mu),$$  \hspace{1cm} (1)

where $q = d, s$ and $a_i(\mu)$ are Wilson coefficients that carry all information of the short distance physics above the renormalization scale $\mu$. The matrix elements of $Q_i(\mu)$ contain all non-perturbative, long distance physics below $\mu$. Within Heavy Quark Effective Theory (HQEFT) the effective non-leptonic Lagrangian $\mathcal{L}_W$ can be evolved down to the scale $\mu = \Lambda_\chi \simeq 1 \text{ GeV}$ \cite{8}.

The numerically relevant operators in our case are

$$Q_1 = 4(\bar{q}_L \gamma^\mu b_L) (\bar{c}_L \gamma_\mu c_L), \hspace{1cm} Q_2 = 4(\bar{c}_L \gamma^\mu b_L) (\bar{q}_L \gamma_\mu c_L),$$  \hspace{1cm} (2)

where $L$ denotes a left-handed particle. At $\mu = \Lambda_\chi$, which by construction is the matching scale within our approach \cite{3,5,6}, one finds $a_1 \simeq -0.35 - 0.07i$ and $a_2 \simeq 1.29 + 0.08i$. Note that the Wilson coefficients $a_i$ are complex below $\mu = m_c$ \cite{8}. In the next subsection we will show how the currents in the operators in (2) are bosonized.

In order to obtain all matrix elements of the Lagrangian (1) we need the Fierz transformed version of the operators in (2). To find these, we use the relation:

$$\delta_{ij} \delta_{ln} = \frac{1}{N_c} \delta_{in} \delta_{lj} + 2 t_{in}^a t_{lj}^a,$$  \hspace{1cm} (3)

where $i, j, l$ and $n$ are color indices running from 1 to 3 and $t^a$ denotes the color matrices, $a$ being the color octet index. One obtains

$$Q_1^F = \frac{1}{N_c} Q_2 + \bar{Q}_2, \hspace{1cm} Q_2^F = \frac{1}{N_c} Q_1 + \bar{Q}_1,$$  \hspace{1cm} (4)

where the superscript $F$ means “Fierzed”, and

$$\bar{Q}_1 = 4(\bar{q}_L \gamma^\mu t^a b_L) (\bar{c}_L \gamma_\mu t^a c_L), \hspace{1cm} \bar{Q}_2 = 4(\bar{c}_L \gamma^\mu t^a b_L) (\bar{q}_L \gamma_\mu t^a c_L).$$  \hspace{1cm} (5)

These operators generate contributions proportional to the gluon condensate.
Heavy Light Chiral Perturbation Theory

The Heavy Quark Effective Theory (HQEFT) Lagrangian is:

$$\mathcal{L}_{HQEFT} = \pm Q_v(\pm) iv \cdot DQ_v(\pm) + \mathcal{O}(m_Q^{-1})$$, \,(6)

where $Q_v(\pm)(x)$ is a (reduced) heavy quark field ($b$ or $c$ in our case) with velocity $v$, and $Q_v^-(x)$ is the field of a heavy anti-quark ($\bar{c}$ in our case). Furthermore, $m_Q$ is the heavy quark mass, and $D_\mu$ is the covariant derivative containing the gluon field.

After integrating out the heavy and light quarks, the effective Lagrangian up to $\mathcal{O}(m_Q^{-1})$ can be written as a kinetic term plus a term describing the chiral interaction between heavy and light mesons \([5, 9]\):

$$\mathcal{L}_\chi = -g_A \text{Tr} \left[ H_a(\pm) H_b(\pm) \gamma_\mu \gamma_5 A_\mu ba \right]$$, \,(7)

where $H_a(\pm)$ is the heavy meson field containing a spin zero and a spin one boson:

$$H_a(\pm) = P_{\pm}(P_{a\mu}^\pm \gamma_\mu - iP_{a5}^{\pm} \gamma_5)$$, \,(8)

Here $a, b$ are flavor indices and $P_{\pm} = (1 \pm \gamma \cdot v)/2$ are projecting operators. The axial vector field $A_\mu$ in (7) is defined as:

$$A_\mu = -\frac{1}{2} (\xi^+ \partial_\mu \xi - \xi \partial_\mu \xi^+)$$, \,(9)

where $\xi \equiv \text{exp}[i(\Pi/f)]$. Moreover, $f$ is the bare pion coupling and $\Pi$ is a 3 by 3 matrix which contains the Goldstone bosons $\pi, K, \eta$ in the standard way, and $g_A$ is the axial chiral coupling.

Based on the symmetry of HQEFT, we obtain the bosonized currents. For a decay of the $b\bar{q}$ system (see Fig. 2, left) we have \([5]\):

$$\bar{q}_L \gamma^\mu Q_b^+(\pm) v_b \rightarrow \alpha_H \text{Tr} \left[ H_c(\pm) \gamma_\alpha L H_b(\pm) \right]$$, \,(10)

where (up to QCD and $1/m_Q$ corrections\([5]\)) $\alpha_H = f_H \sqrt{m_H}$ for $H = B, D$. Further, $Q_{v_b}^+(\pm)$ is the heavy $b$-quark field, $v_b$ is its velocity, and $H_b^+(\pm)$ is the corresponding heavy meson field. For the $W$-boson materializing to a $D$, the bosonized current $\bar{q}_L \gamma^\mu Q_{v_c}^-(\pm)$ is also given by (10), but with $H_b^+(\pm)$ replaced by $H_c^+(\pm)$ representing the $D$ meson. $Q_{v_c}^-(\pm)$ is the field of the heavy $\bar{c}$ quark, and $v_c$ is its velocity (see Fig. 1 right).

The bosonized $b \rightarrow c$ transition current in Fig. 1 is given by

$$\bar{Q}_{v_b}^+(\pm) \gamma^\mu L Q_{v_c}^+(\pm) \rightarrow -\zeta(\omega) \text{Tr} \left[ H_c(\pm) \gamma_\alpha L H_b(\pm) \right]$$, \,(11)

where $\zeta(\omega)$ is the decay constant of the heavy meson $H_b$.
FIGURE 1. Factorized contribution for $\overline{B}^0 \to D^+ D_s^-$ through the spectator mechanism, which does not exist for decay mode $\overline{B}^0 \to D^+_s D_s^-$. There are similar diagrams with vector mesons.

FIGURE 2. Factorized contribution for $\overline{B}^0 \to D_s^+ D_s^-$ through the annihilation mechanism, which give zero contributions if both $D_s^+$ and $D_s^-$ are pseudoscalars.

where $\zeta(\omega)$ is the Isgur-Wise function for the $\bar{B} \to D^-$ transition, and $v_c$ is the velocity of the heavy $c$-quark. Furthermore, $\omega \equiv v_b \cdot v_c = v_b \cdot v_{\bar{c}} = M_B/(2M_D)$. For the weak current for $D\bar{D}$ production (corresponding to the factorizable annihilation mechanism in Fig. 2), the current $Q_{\bar{c}}^{(+)} \gamma^\mu LQ_{c}^{(-)}$ is given by (11) with $H_{\bar{b}}^{(+)}$ replaced by $H_{\bar{c}}^{(-)}$, and $\zeta(\omega)$ is replaced by $\zeta(-\lambda)$, where $\lambda = v_{\bar{c}} \cdot v_c = [M_B^2/(2M_D^2) - 1]$. Note that $\zeta(-\lambda)$ is a complex function which is less known than $\zeta(\omega)$.

The factorized contributions for the spectator and annihilation diagrams are shown in the Figs. 1 and 2. The first diagram does not give any (direct) contributions to the class of processes we consider, but is still important because it is the basis of our chiral loops.

The chiral loop amplitudes visualized in Fig. 3 are of order $(g_A m_K/4\pi f)^2$ compared to typical factorizable amplitudes in processes where these exist. For instance, the ratio between the chiral loop amplitude for $\overline{B}^0 \to D^+_s D_s^-$ and the factorized amplitude for $\overline{B}^0 \to D^+ D_s^-$ is $\simeq -0.20 + 0.26i$ (before the difference in KM structure is taken into account). For vectors (i.e. $D^+$ 's) in the final state there are similar diagrams with various combinations of pseudoscalars and vectors in the loop, but the diagrams in Fig. 3 constitute the two classes of diagrams [4].

In a complete analysis, counterterms to the chiral loops has to be included. These counterterms are not considered here (or in [3, 4]) and has to be considered together with the constant (non-logarithmic) chiral loop terms which we also have dropped in this analysis. The inclusion of counterterms and constant chiral loops terms will be discussed elsewhere.
FIGURE 3. Non-factorizable chiral loops for $B^0 \to D^+_s D^-_s$. There are similar diagrams for vector mesons in the final state. The two diagrams illustrate the two classes of diagrams.

One might also write down possible terms consistent with HQEFT and chiral symmetry, for instance the following three terms:

$$ Tr \left[ \xi^\dagger \sigma^{\mu \alpha} L H_{b}^{(+)} \right] \cdot Tr \left[ H_{c}^{(+)} \gamma_{\alpha} L H_{b}^{(-)} \gamma_{\mu} \right] , \quad Tr \left[ \xi^\dagger L H_{b}^{(+)} \right] \cdot Tr \left[ H_{c}^{(+)} \gamma^{\mu} L H_{b}^{(-)} \gamma_{\alpha} \right] , $$

$$ \varepsilon^{\mu \nu \alpha \lambda} (v_c + \bar{v})_{\nu} Tr \left[ \xi^\dagger \gamma^\mu L H_{b}^{(+)} \right] \cdot Tr \left[ H_{c}^{(+)} \gamma^{\mu} L H_{b}^{(-)} \gamma_{\alpha} \right] . $$

Such terms do not appear in the factorized limit, and will correspond to (at least) $1/N_c$ suppressed terms. Within pure chiral perturbation theory their coefficients are unknown, but they might be calculated within the HL$\chi$QM, as described in the next subsection.

The Heavy Light Chiral Quark Model

The HL$\chi$QM Lagrangian is

$$ \mathcal{L}_{\text{HL} \chi \text{QM}} = \mathcal{L}_{\text{HQEFT}} + \mathcal{L}_{\chi \text{QM}} + \mathcal{L}_{\text{Int}} . $$

The first term is given in (6) and the second term is described by the Chiral Quark Model of the light sector [10] involving interactions between quarks and (Goldstone) mesons:

$$ \mathcal{L}_{\chi \text{QM}} = \bar{\chi} \left[ \gamma^\mu (i \mathcal{D}_\mu + \gamma_5 \mathcal{A}_\mu) - m \right] \chi . $$

Here $m = (230 \pm 20)$MeV is the $SU(3)$ invariant constituent light quark mass, and $\chi$ is the flavor rotated quark field given by $\chi_L = \xi^\dagger q_L$ and $\chi_R = \xi q_R$, where $q^T = (u, d, s)$ is the light quark field. The covariant derivative $\mathcal{D}_\mu$ contains the soft gluon field forming the gluon condensates (besides some chiral interactions) [5, 6, 10].

The interaction between heavy meson fields and quarks is described by [5]:

$$ \mathcal{L}_{\text{Int}} = -G_H \left[ \chi_\alpha H_{a}^{(\pm)} Q_v^{(\pm)} + Q_v^{(\pm)} H_{a}^{(\pm)} \chi_\alpha \right] , $$

where the coupling constant $G_H = \sqrt{2m \rho / f}$, and $\rho$ is a hadronic parameter depending on $m$ (numerically $\rho$ is of order one). For further details, see ref. [5].

The gluon condensate amplitudes can be written, within the framework presented in the previous section, in a quasi-factorized way as a product of matrix elements of colored
![Figure 4](image)

**FIGURE 4.** Non-factorizable contribution for $\bar{B}^0 \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emission. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

The currents in (5), as visualized in Fig. 4, correspond to the bosonized colored current:

$$i\gamma^\mu Q_{vb}^{(+)} (\bar{Q}_v)_{1G} - \frac{G_H g_s}{64\pi} G_{\mu\nu}^a$$

$$\times \text{Tr} \left[ \xi^+ \gamma^\mu L H_b^{(+)} (\sigma_{\mu\nu} - F \{ \sigma_{\mu\nu}, \gamma \cdot v_b \}) \right], \quad (16)$$

where $G_{\mu\nu}^a$ is the octet gluon tensor, and $F \equiv 2\pi f^2/(m_c^2 N_c)$ is a dimensionless quantity of the order $1/3$. The symbol $\{ , \}$ denotes the anti-commutator. For the creation of a $D\bar{D}$ pair in the right part of Fig. 4, the colored current $\left( Q_{vb}^{(+)} i^a \gamma^\mu L Q_{\bar{v}}^{(-)} \right)_{1G}$ is bosonized similarly to (16), but involves $H_c^{(+)}$ and $H_c^{(-)}$.

Multiplying the two colored currents, and introducing gluon condensate contributions by the replacement:

$$g_s^2 G_{\mu\nu}^a G_{\alpha\beta}^a \to 4\pi^2 \left( \frac{\alpha_s}{\pi} G^2 \right) \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}), \quad (17)$$

we obtain a bosonized effective Lagrangian which is $1/N_c$ suppressed compared to the factorized contributions. This effective Lagrangian corresponds to a certain linear combination of a priori possible $1/N_c$ suppressed terms at tree level (in the chiral perturbation theory sense). Among these are the three terms in (12).

$1/m_b$ suppressed terms seem to be negligible. In order to include $1/m_c$ terms, one must for instance consider the $c\bar{c}$ production current to this order:

$$\Delta J_\mu (\bar{c}c) = \frac{1}{m_c} Q_{\bar{v}_c}^{(+)} \left( i\gamma^\mu D_{\perp}(v_c) \gamma^\mu L + \gamma^\mu Li\gamma \cdot D_{\perp}(v_c) \right) Q_{v_c}^{(-)}, \quad (18)$$

where $D_{\perp}^\mu (v) = (g^{\mu\nu} - v^\mu v^\nu) D_{\perp}$. Within the HLχQM one may estimate both factorizable and non-factorizable $1/m_c$ corrections due to this current There are also other operators. Compared to other contributions studied here, the relative size of $1/m_c$ suppressed contributions will be of order $\bar{m}/m_c$, where $\bar{m}$ is some hadronic parameter within the HLχQM with dimension mass, such as linear combinations of $m$ and $(\bar{q}q) / f^2$. The total contributions from such terms are significant and will be studied elsewhere.
RESULTS AND DISCUSSION

In our calculation we used the following input parameters: \(\alpha_B \simeq \alpha_D \simeq 0.33\ \text{GeV}^{-3/2}\), \(G_H = 7.5\ \text{GeV}^{-1/2}\) and \((\alpha_s \pi G^2) = [(315 \pm 20)\ \text{MeV}]^4\), \(g_{sf} = 0.6\), \(f_\pi = 93\ \text{MeV}\). We find the following branching ratios

\[
\begin{align*}
\text{Br}(\bar{B}_0 \rightarrow D^+_s D^-_s) &= 2.5 \times 10^{-4}, \\
\text{Br}(\bar{B}_s^0 \rightarrow D^+ D^-) &= 4.5 \times 10^{-3}, \\
\text{Br}(\bar{B}_0 \rightarrow D^+_s D^-_s) &= 3.3 \times 10^{-4}, \\
\text{Br}(\bar{B}_s^0 \rightarrow D^{++} D^-) &= 6.8 \times 10^{-3}, \\
\text{Br}(\bar{B}_0 \rightarrow D^+_s D^{*-}) &= 2.0 \times 10^{-4}, \\
\text{Br}(\bar{B}_s^0 \rightarrow D^+ D^{*-}) &= 4.3 \times 10^{-3}, \\
\text{Br}(\bar{B}_0 \rightarrow D^+_s D^{*-}) &= 5.4 \times 10^{-4}, \\
\text{Br}(\bar{B}_s^0 \rightarrow D^{++} D^{*-}) &= 9.1 \times 10^{-3}.
\end{align*}
\]

The contribution of the constant term and the corresponding counterterm can change the branching ratio for \(B\)-meson decaying into two pseudoscalars by about 10%, while in the case of decay into one pseudoscalar and one vector \(D\)-meson, this contribution is in the range of 20 – 40%. In the case of \(B\)-meson decaying into two vector mesons, the constant term is estimated to be 2-8 times larger than the logarithmic contribution, depending on the choice of the scheme in which the products of two Levi-Civita terms are considered. The uncertainty in input parameters can result in an additional error for the branching ratios. We estimate that it can be of the order of 20%. Within our approach the \(1/m_Q\) corrections, with \(Q = c, b\) have so far been omitted. At least the \(1/m_c\) corrections will be numerically significant.

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