Instanton effects and chiral symmetry breaking in QCD

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In this paper we discuss the spontaneous breakdown of chiral symmetry in Quantum Chromodynamics by considering gluonic instanton configurations in the partition function. It is shown that in order to obtain nontrivial fermionic correlators in a two dimensional gauge theory for the strong interactions among quarks, a regular instanton background has to be taken into account. We work over massless quarks in the fundamental representation of SU(\(N_c\)). For large \(N_c\), massive quarks are also considered.

1. INTRODUCTION

In the massless limit, Quantum Chromodynamics is not only invariant under right (\(R\)) and left (\(L\)) chiral symmetry transformations but also under two independent Abelian symmetry groups U(1). The massless theory is thus symmetric under a large unitary group U(\(N_f\)) \times U(\(N_f\)). The vector subgroup is realized in the normal mode, i.e. the QCD vacuum is also invariant under U(\(L\)) \times U(\(R\)) transformations. The hadrons form degenerate SU(\(N_f\)) multiplets for small \(N_f\) and baryon number is conserved. The remaining symmetries -axial SU(\(N_f\)) and axial U(1)- corresponding to U(\(L\)) \times U(\(R\)), are not manifest in particle degeneracies as we do not observe parity doubling of baryon states. This phenomenon is ascribed to the vacuum of the theory which is said to spontaneously breakdown the axial symmetry (SSB). In a three flavor world it has been experimentally verified that SSB is true for the special part of the axial group. Actually, eight approximately massless pseudoscalar mesons are observed in the hadronic spectra. However, there is one missing particle which would be associated with U(1) according to the Goldstone theorem, since the transformation \(\psi_f \rightarrow e^{i\gamma_5 \theta} \psi_f\) is neither realized as a symmetry of the vacuum nor a la Goldstone. Although we do not exactly know the essential physical mechanism underlying chiral symmetry breaking, the answer to this famous problem lays on the so-called Adler-Bell-Jackiw anomaly. This explicit symmetry breaking at the quantum level has a close connection to a, seemingly, topological nature of the QCD ground state.

In two dimensions, it turns out that there is no possible Goldstone mode associated with any continuous (global) symmetry of the Lagrangian \([1]\). If chiral symmetry cannot be broken by the vacuum state and parity is a good symmetry of the Lagrangian, the 2D spectrum should contain parity doublets. However, they do not appear, in direct correspondence with the 4D situation. The mechanism which eliminates the pairing among meson states will be clarified in a path-integral calculation. We will see that a non-Abelian chiral anomaly takes place provided a topological structure is taken into account. In this way, not only the Abelian part of the axial symmetry but also the non-Abelian counterpart are dynamically broken due to quantum fluctuations. It shall be made manifest by performing a fermion chiral decoupling.

The non-perturbative vacuum structure of QCD can be expressed in terms of nonzero v.e.v. of various composite operators. These condensates have been introduced as phenomenological parameters in a non-perturbative generalization of the operator product expansion, which can be related to observable hadronic properties by the sum rule method. The lowest dimensional condensates are well-known phenomenologically, but considerable uncertainty prevails about the value of higher dimensional correlators. Conden-
states can be understood as being generated by certain nonperturbative fluctuations of the fields. In particular, instantons would be responsible for the SSB of chiral symmetry. A quark condensate arise due to the delocalization of fermion zero modes associated with the instantons in the medium.

Therefore, the study of fermionic correlation functions is a key issue of Quantum Chromodynamics as these are suitable quantities to shed light on QCD non-perturbative aspects and hadron physics. Actually, the correlation among quark fields can be written in terms of dispersion relations involving matrix elements among the vacuum and physical hadronic Fock states. Consequently, fermionic correlators of fundamental degrees of freedom are directly connected to the analysis of multipoint fermionic correlators. The QCD vacuum has non-perturbative condensation effects, seemingly arising due to an underlying symmetry breaking coming from non-perturbative fluctuations of the fields.

In particular, instantons would be responsible for certain nonperturbative fluctuations of the fields. These highly complicated problems can be faced only by assuming sensible simplifications in the theory itself, or for instance, on the number of space-time dimensions. Quantum chromodynamics in two dimensions is a convenient framework for several reasons: the use of fundamental degrees of freedom, its non-Abelian character, the chirality properties of the theory, the existence of analytical results, etc. We will then work with the following Lagrangian of SU($N_c$) gauge fields coupled to Dirac fermions in an Euclidean space:

$$L = \bar{\psi} \gamma^\mu \delta_{\mu\nu} \psi + A_{\mu a} \bar{\psi} \gamma^\mu \gamma^\nu \psi + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a$$

where $q$ runs from 1 to $N_c$ and $a$, the label of the generators, goes from 1 to $N_c^2 - 1$. For short, we will discuss in detail the issues for only one flavor, but the procedure can be readily extended to arbitrary $N_f$. From now on we drop the $c$ subindex from $N_c$.

In order to compute fermionic correlators, a path-integral approach is very appropriate, especially to handle different topological sectors. A necessary first point in the procedure is to separate the gauge field as a sum of a fixed classical background carrying a topological charge $n$, and a quantum fluctuation belonging to the trivial sector: $A_{\mu}^a(x) = A_{\mu}^{a(n)} + a_{\mu}^a$. In this way one is able to decouple gauge fields from fermionic fields within the $n=0$ sector, yielding a non-Abelian jacobian of the Fujikawa type. In order to do this in a gauge independent way we have to use a group valued representation for all the fields and perform a chiral rotation of the spinors. It is worthwhile to show the jacobian resulting from the decoupling, since it first makes apparent the axial anomaly, responsible for the full chiral symmetry breaking in 2D. In terms of group fields $u$, $v$ and $d$ which represent $a_+$, $a_-$ and $A_{\nu}^{(n)}$ respectively ($A_{\nu}^{(n)} = 0$), the fermion determinant can be suitably factorized, in an arbitrary gauge, by repeated use of the Polyakov-Wiegmann identity, resulting in the following formal identity for the jacobian

$$J = \frac{\det \mathcal{D}[A^{(n)} + a]}{\det \mathcal{D}[A^{(n)}]} = \mathcal{N} \exp -S_{eff}(a, A^{(n)})$$

2. MODEL

For instance, a non-perturbative structure of the QCD vacuum comes about from the Gell Mann-Oakes-Renner relation,

$$\langle \bar{u}u + \bar{d}d \rangle = -2f_0^2 m_0^2/(m_u + m_d),$$

since such a nonzero result indicates the existence of a dynamical mass in the massless quark propagator which would vanish in the perturbative calculation. The strong attractive force in the $J^P = 0^+$ channel induces the instability of the Fock vacuum of massless quarks to realize the nonperturbative ground state with quark antiquark condensation. This is quite similar to the BCS mechanism of superconductivity. The pair condensation breaks the full symmetry down to the vector subgroup and the dynamical quark mass is generated.

Therefore, the study of fermionic condensates is especially useful in connection with chiral symmetry breaking coming from non-perturbative effects, seemingly arising due to an underlying topological structure.
where

\[ S_{eff}(a,A^{(n)}) = W[u] + W[v] + \frac{1}{4\pi} \text{tr}_c \int d^2 x (u^{-1} \partial_+ u) (d \partial_- d^{-1}) + \frac{1}{4\pi} \text{tr}_c \int d^2 x \left( d^{-1} \partial_+ d \right) (v \partial_- v^{-1}) + \frac{1}{4\pi} \text{tr}_c \int d^2 x \left( u^{-1} \partial_+ u \right) d \left( v \partial_- v^{-1} \right) d^{-1}, \]

\[ W[u] \text{ being the usual Wess-Zumino-Witten action. } \]

A lengthy calculation leads to the exact expressions for the correlation functions of an arbitrary number of fermionic bilinears in the SU(N) 2D gauge theory. As it is a very large expression we shall not quote it in this note but we can state here that our results should show that the simple product of fermionic and bosonic path-integrals one finds in the Abelian case becomes here an involved sum due to color couplings, bringing about the extended Wess-Zumino-Witten action coming from the decoupling. Nevertheless, we can say that expressions show in a clear way both sectors completely decoupled. The fermionic path-integral can be easily performed, amounting to a sum of products of zero modes of the Dirac operator in the multistanton background. In the bosonic sector, the presence of the Maxwell term crucially changes the effective dynamics with respect to that of a pure Wess-Zumino model. One then has to perform approximate calculations to compute the bosonic factor, for example, by linearizing the group transformations; nevertheless, the point relevant to our discussion of obtaining an nonzero fermion correlators is manifest in our result.

We should note however that an immediate by-product of our approach gives a null elementary (lowest dimensional) condensate, \( \langle \bar{\psi} \psi \rangle = 0 \), for any number of colors. For finite \( N \), it is consistent with alternative approaches, but, on the other hand, independent analytical calculations coming from dispersion relations and canonical quantization have found that it is possible to obtain a nonzero condensate when the large \( N \) limit is considered in massive QCD2. These alternative approaches, which consider just trivial topology, would in principle be in constrast with Coleman’s theorem as we have discussed in the introduction. However, it has been shown that in the weak coupling regime, or ‘t Hooft phase, there is in fact a Berezinskii-Kosterlitz-Thoules (BKT) realization of the chiral symmetry, defining a phase transition between a chiral symmetric and a chirally broken phase. What we want to point out now is that by considering the massive version of our model for large \( N \), we may also obtain a nonzero outcome for \( \langle \bar{\psi} \psi \rangle \) in the chiral limit.

Regarding the topological aspects of our approach, here we are considering fundamental quarks in a background given by the gluonic component of \( Z_N \) vortices which give a realization in an Euclidean 2D theory of regular instanton configurations. This picture allows an interesting connection between an underlying topological structure in the theory and the existence of nonzero fermion condensates. This is a very suitable feature for a model approach to the strong interactions among quarks. The relevance of instantons has been amply demonstrated by phenomenology as well as by lattice calculations. In particular, instantons allow a good description of chiral symmetry breaking and shed light on the nonperturbative phenomena determining the structure of hadrons. An instanton vacuum certainly provides a convenient tool for computing correlation functions and a microscopic picture of the nonperturbative configurations of the gluon field.

The minimal correlation function, for the massive partition function, can be readily written as

\[ \langle \bar{\psi} \psi(\omega) \rangle_M = \sum_n \left( \langle \bar{\psi} \psi(\omega) \rangle^{(n)}_{M=0} + M \int d^2 x (\bar{\psi}(\omega) \bar{\psi}(x))^{(n)}_{M=0} + \frac{1}{2} M^2 \int d^2 x d^2 y (\bar{\psi}(\omega) \bar{\psi}(x) \bar{\psi}(y))^{(n)}_{M=0} + \ldots \right) \]

Written in this fashion, it is apparent that for \( M \neq 0 \) the elementary condensate receives contributions from every higher order correlator coming from the massless theory. As we have mentioned, these have precisely been calculated in a general way. It follows from the discussion above that
these correlators are given by the fermion zero modes determined by the topological background.

Since the fermionic sector is completely decoupled, the counting of the nonzero terms simply follows from that of the Abelian case because the topological structure here can be read out from the torus of the gauge group. To be more specific, in a compactified space, there exist $nN$ normalizable zero modes in topological sector $n$ \[10\]; this implies the vanishing of the first summatory in eq.(1) in every topological sector. However, for higher powers of $M$ it is clear that certain nonzero contributions come into play. The zero modes, with a definite chirality, set the integration rules for computing v.e.v’s. By using the chiral decomposition \[\langle \bar{\psi} \psi(x) \rangle = \langle \bar{\psi}_R \psi_R(x) \rangle + \langle \bar{\psi}_L \psi_L(x) \rangle\] one can see that the first $N$ powers of $M$ ($j = 1 \ldots N$) receive an input from the trivial topological sector alone. For $j \geq N$, the $n = 1$ sector starts contributing together with $n = 0$. For higher powers, $j \geq 2N$, the contribution of topological sector $n = 2$ starts on, etc. Now, it can be easily seen that the number of contributions grows together with the number of colors. As we let $N$ go to infinity the elementary condensate in the massive theory does so. On the other hand, since within each nontrivial topological sector the number of zero modes grows also to infinity, one has divergent v.e.v. everywhere in the series expansion of eq.(1). Accordingly, high order terms can also produce a nonzero outcome in the chiral limit. This is a pleasant result of our model, in order to mimic real four dimensional QCD where an instanton vacuum seems to be closely connected to this phenomenon.

Therefore, it is clear that the limit $M \to 0$ becomes matter of a careful analysis; namely, combined with a large number of colors, eq.(1) leaves place enough for a nontrivial elementary condensate even in the chiral limit. This result is of phenomenological interest as it puts forward possible roots to the BKT phenomenon, arising from topological considerations in a path-integral approach.

3. CONCLUSION

We have discussed chiral symmetry breaking in a two dimensional model for the gluon interactions among quarks. Our approach allows a systematic procedure for computing arbitrary correlation functions of fermionic operators. We have shown that a topological background is crucial for obtaining a whole class of nonzero condensates of the fundamental fields which signal the dynamical breakdown of the full chiral symmetry group.

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