Coulomb Drag in Double Layers with Correlated Disorder

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Abstract

We study the effect of correlations between impurity potentials in different layers on the Coulomb drag in a double-layer electron system. It is found that for strongly correlated potentials the drag in the diffusive regime is considerably enhanced as compared to conventional predictions. The appropriate experimental conditions are discussed, and the new experiments are suggested.
Over the past decade the frictional drag in double-layer two-dimensional electron systems has been a subject of extensive experimental [1] and theoretical [2–5] studies. This phenomenon is manifested in the appearance of current $I_2$ or voltage $V_2$ in the “passive” layer 2 when the applied voltage $V_1$ causes the current $I_1$ to flow in the “active” layer 1. The strength of the drag is characterized by either transconductivity $\sigma_{21} = (I_2/V_1)_{V_2=0}$ or transresistivity $\rho_{21} = (V_2/I_1)_{I_2=0}$, which are related one to another as $\rho_{21} = -\sigma_{21}(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})^{-1} \approx -\sigma_{21}\sigma_{11}^2$, where $\sigma_{ii}$ are the intrinsic conductivities of the layers.

In the absence of tunneling (cf. Ref. [5]), the drag arises due to interlayer momentum transfer mediated by inelastic scattering (mainly, Coulombic) of carriers that belong to different layers. Hence, the drag provides a convenient tool for studying electron correlations in the coupled two-dimensional mesoscopic systems, including such important characteristics as the electron polarization function and the screened interlayer interaction.

In the conventional theory [2], the carriers in each layer are scattered by their own impurity potentials. As a result, the processes contributing to $\sigma_{21}$ can be understood in terms of coupling between independent thermal density fluctuations in different layers. The phase space available to the thermal excitations is small and limited by energies $\omega \lesssim T$. Therefore, the drag effect rapidly vanishes with decreasing temperature. For instance, $\rho_{21} \propto T^2 (T^2 \ln T)$ in a clean (dirty) normal metal [2] and $\rho_{21} \propto T^{4/3}$ for composite fermions in double-layers of electrons in the half-filled Landau levels [3].

The picture of independent impurity potentials used in Refs. [2,3] is well justified in the case of the standard experimental geometry [1], where two Si delta-doped layers (DDLs) are situated on the outer sides of the double quantum well. The DDLs not only serve as the reservoirs supplying carriers but also introduce disorder in the form of a smooth random potential (SRP) of the ionized donors. Moreover, due to the efficient screening the carriers in each quantum well experience only a SRP created by the nearest DDL.

Instead, one can consider an alternative geometry where a single DDL is located in the middle between the two electron layers, so that the SRPs in both layers are almost identical. This setup gives one an opportunity to study a new type of coherent effects in systems with spatially separated carriers.

In the present paper we investigate the influence of correlations between the impurity potentials in different layers on the transresistivity. We focus our attention onto the case of a long characteristic time, $\tau_g$, at which the carriers feel the difference between the SRPs in the two layers ($\tau_g \gg \tau_{tr}$, where $\tau_{tr}$ is the transport scattering time in each layer). We show that in this case the drag is strongly enhanced in comparison to the non-correlated situation. This enhancement is due to a possibility of a coherent motion of carriers propagating in different layers and feeling nearly the same random potential. As a result, the effective time of their interaction increases considerably. This gives rise to the new behavior of the transresistivity

$$\rho_{21}^{\text{corr}} \approx -\frac{\pi^4}{24} \frac{\hbar}{e^2} \frac{\ln(T\tau_g)}{(k_F d) \zeta \zeta}, \quad \tau_g^{-1} \ll T \ll \tau_{tr}^{-1}, \quad (1)$$

$$\rho_{21}^{\text{corr}} \approx -\frac{\pi^4}{6} \frac{\hbar}{e^2} \frac{(T\tau_g)^2}{(k_F d) \zeta \zeta}, \quad T \ll \tau_g^{-1}, \quad (2)$$

Here, $l = v_F \tau_{tr}$ is the electron mean free path, $k_F (v_F)$ is the Fermi momentum (velocity), $d$ is the interlayer distance (throughout this paper we assume $l \gg d$), and $\zeta$ is the
Thomas-Fermi momentum. This term yields the dominant contribution to $\rho_{21}$ within the entire experimentally accessible temperature range, provided that the system remains in the diffusive regime, $T \ll \tau_{tr}^{-1}$. Below we specify the experimental conditions necessary for the observations of the behavior described by Eqs. (3) and (4), and predict a suppression of these regimes by a weak magnetic field.

In general, a diffusive motion of carriers becomes coherent if there exist singularities in the particle-hole and particle-particle propagators ("Diffusons" and "Cooperons"). The equations for Diffusons in the double-layer system can be written as follows

$$
D_{kk'}^{ij}(q, \omega) = W_{kk'}^{ij} + \sum_{k_1} W_{kk_1}^{ij} G_{k_1+q}(\epsilon + \omega) G_{k_1}^{ij}(\epsilon) D_{k_1,k'}^{ij}(q, \omega). \tag{3}
$$

Here, the indices $i,j$ label the layers, $k$ and $k'$ are the momenta of the incoming and the outgoing electrons, $G_{k}^{R(A)i}(\epsilon) = [\epsilon - \xi_k \pm i/2\tau^i]^{-1}$ is the impurity averaged retarded (advanced) electron Green function, and $W_{kk'}^{ij} = \langle u'^i u^j \rangle_{imp}$ are the elastic electron scattering probabilities. The values of $W_{kk'}^{ij}$ at $i \neq j$ differ from zero due to correlations between the impurity potentials $u'$ in different layers. The total and the transport scattering times are defined, respectively, by the formulae

$$
1/\tau^{ij} = \langle W_{kk'}^{ij} \rangle_{k'}, \quad 1/\tau^{ij}_{tr} = \langle W_{kk'}^{ij} (1 - \hat{k}k') \rangle_{k'},
$$

where the symbol $\langle ... \rangle_k = \nu_F \int_0^{2\pi} ... d\varphi_k$ stands for the angle average over the Fermi surface, $\nu_F = m/2\pi$ is the single-spin density of states (assumed to be equal in both layers), $m$ is the effective electron mass, and $k = k/k$.

By solving the interlayer Diffusion equation with the use of the formalism developed in Ref. [3], we find that it is the characteristic time

$$
\tau_g^{-1} = \frac{\tau_{21}^{-1} - \tau}{\tau^2} \tag{4}
$$

(where $1/\tau = [1/\tau^{11} + 1/\tau^{22}]/2$) which determines a crossover between different regimes. Namely, for $\tau_g^{-1} \gtrsim \tau_g$, the interlayer Diffusons do not form, and the system remains in the ballistic regime at all temperatures, as far as the interlayer elastic scattering is concerned. It is only at $\tau_g \gg \tau_{tr}^{ii}$ that the solution of Eq. (3)

$$
D_{kk'}^{reg}(q, \omega) \simeq \frac{1}{2\pi \nu_F \tau^2} \frac{\gamma_k \gamma_{k'}}{D q^2 - i\omega + \tau^{-1}} + D_{kk'}^{reg} \tag{5}
$$

develops a quasi-Diffusion pole. Here $D = \nu_F^2 \tau_{tr}/2$ is the diffusion coefficient and $\gamma_k = 1 - i(\tau_{tr} - \tau)qk/m$. The regular part of the Diffusion $D_{kk'}^{reg}$ is given by the same expression as in the single-layer case [3]. We neglect the difference between the interlayer and intralayer scattering times by setting $\tau^{ij} = \tau$ and $\tau_{tr}^{ij} = \tau_{tr}$ everywhere except for the "gap" $\tau_g^{-1}$ in the first term of Eq. (3). For two identical SRPs one has $\tau_g = \infty$, and the interlayer Diffusion coincides with the intralayer one.

The same conclusions can be reached about the interlayer particle-particle propagator (Cooperon), which is given by Eq. (3) with the substitution of $\tau_g^{-1}$ by $\tau_g^{-1} + \tau^{-1}$, where $\tau^{-1}$ is the inelastic phase breaking time.
From Eq. (5) one can see that at $\tau_g \neq \infty$ there are no singularities in the interlayer Diffusons and Cooperons. Nonetheless, at frequencies $\tau_g^{-1} \lesssim \omega \lesssim \tau_{tr}^{-1}$ and momenta $(\tau_{tr}/\tau_g)^{1/2} \lesssim q l \lesssim 1$ the motion of carriers in the two layers is highly correlated.

The origin of the interlayer decoherence time $\tau_g$ can be explained as follows. Consider two coherent electron waves propagating in slightly different random potentials ($u + \delta u$ and $u - \delta u$). After passing through the distance of order the SRP correlation length $a$ they acquire a random phase difference $\Delta \phi \sim (2\delta u)v_F^{-1}a$. This leads to the electron’s phase diffusion with the diffusion coefficient $D_{ph} = \langle \Delta \phi \rangle^2 v_F a^{-1}$, and provides a complete loss of phase coherence over the time $\tau_g \sim D_{ph}^{-1}$.

The new correlation effects for the transconductivity are described by diagrams with two electron loops (one current vertex per each), connected not only by the interlayer Coulomb interaction lines but also by the impurity lines combining into the interlayer Diffusons and Cooperons. We find that, to any order in the interlayer interaction $V_{21}$, both the Diffuson and the Cooperon diagrams for which the electron energy does not change its sign at the current vertices give no contribution to $\sigma_{21}$ at zero external frequency. In order to prove this statement we extended the method proposed in Ref. [7] for treating the Hartree terms onto the double-layer case. This general method accounts for all the diagrams with an arbitrary number of impurity lines (also in the ballistic regime) and therefore remains valid in the presence of the gap $\tau_g^{-1}$ in Diffusons and Cooperons.

Perturbative calculations confirm the above statement. To illustrate this, consider the Diffuson diagrams which give rise to logarithmic conductivity corrections in the single-layer case [8]. Only two of such diagrams (see Fig. 1a) contain the current vertices inserted in different electron loops and hence are related to $\sigma_{21}$. We found that the logarithmic terms in these diagrams cancel against each other. To verify this in the case of SRPs one has to take into account the $q$-dependent residues of the diffusons as well as their regular parts (cf. Ref. [6]). Therefore, in the double-layer system only the intralayer conductivities $\sigma_{ii}$ acquire the logarithmic Hartree corrections due to the presence of the second layer.

Thus the only diagrams contributing to $\sigma_{21}$ are those with the energy sign changing at the current vertices. It can be checked that to any order in $V_{21}$ no Diffuson diagrams of this kind survive in the DC limit. On the contrary, the Cooperon diagrams do, and the first nontrivial contribution to $\sigma_{21}$ arises in the second order in the interaction $V_{21}$ and involves three Cooperons. It is, however, more instructive to sum up the entire interlayer Coulomb-Cooperon ladder (see Fig. 1b). Taking into account the smooth character of the donors’ potentials, one arrives at the two diagrams depicted in Fig. 1c. After the summation over electron frequencies and momenta, the contribution of these diagrams to the transconductivity takes the form

$$\sigma_{21}^{corr} = \frac{4e^2}{\pi hT} \int \frac{D(dq)}{Dq^2 + \tau_g^{-1} + \tau_{\varphi}^{-1}} \int_0^\infty \frac{d\omega}{\sinh^2 \frac{\omega}{2T}} \text{Im} \Psi_c(q, \omega) \text{Im} \Lambda_c(q, \omega),$$

(6)

The quantities $\Psi_c$ and $\Lambda_c$ are given by

$$\Psi_c(q, \omega) = \psi \left( \frac{Dq^2 - i\omega + \tau_g^{-1} + \tau_{\varphi}^{-1}}{4\pi T} + \frac{1}{2} \right),$$

$$\Lambda_c(q, \omega) = 2 \left[ \ln \frac{\varepsilon_0}{T} + \lambda_{21}^{-1} - \Psi_c(q, \omega) + \psi(1/2) \right]^{-1},$$
where $\psi$ is the digamma function, $\varepsilon_0 \propto \varepsilon_F$ is the upper energy cutoff, and $\lambda_{21}$ is the effective interaction constant. The above equations are valid with a logarithmic accuracy, since we omitted the terms originating from the regular parts of the Cooperons. The quantity $\lambda_{21}$ is defined as $\lambda_{21} = (4\pi^2\nu_F)^{-1} \langle V_{21}(\mathbf{p} - \mathbf{p}') \rangle_{\mathbf{p},\mathbf{p}'}$, with the screened interlayer Coulomb interaction being of the form

$$V_{21}(\mathbf{p}) = \frac{1}{2\nu_F} \left[ \frac{\kappa(1 + e^{-pd})}{p + \kappa(1 + e^{-pd})} - \frac{\kappa(1 - e^{-pd})}{p + \kappa(1 - e^{-pd})} \right].$$

Assuming that the screening is strong enough, $\kappa d \gg 1$, and that the interlayer Coulomb potential is sufficiently smooth, $k_Fd \gg 1$, one finds $\lambda_{21} \simeq \pi(4k_Fd\kappa d)^{-1}$.

Evaluation of the integrals in Eq. (3) yields

$$\rho_{21}^{corr} \simeq -\frac{2\pi^2}{3} \frac{\hbar}{e^2} \left( k_F \right)^2 \frac{1}{\left[ \lambda_{21}^{-1} + \ln(\varepsilon_0/T) \right]^2} \ln \frac{T\tau_{\phi}\tau_g}{\tau_{\phi} + \tau_g}$$

at $\tau_g^{-1} \ll T \ll \tau_{tr}^{-1}$ (when the domain of $\mathbf{q}$-integration is effectively limited by $Dq^2 \lesssim T$), and

$$\rho_{21}^{corr} \simeq -\frac{8\pi^2}{3} \frac{\hbar}{e^2} \left( k_F \right)^2 \frac{(T\tau_g)^2}{\left[ \lambda_{21}^{-1} + \ln(\varepsilon_0\tau_g) \right]^2}$$

at lower temperatures. These equations constitute our main result. Under realistic experimental conditions the value of $\lambda_{21}^{-1}$ is sufficiently large for one to neglect the logarithmic terms in the denominators of Eqs. (7) and (8). Also, since the interlayer decoherence time $\tau_g$ is temperature independent, the argument of the logarithmic function in the numerator of Eq. (7) is linear in temperature provided that $\tau_g \ll \tau_{\phi}$. Then Eqs. (7) and (8) reduce to Eqs. (1) and (2), respectively.

Now let us compare these equations with the results of the standard theory [2]:

$$\rho_{21}^{conv} = \frac{\hbar}{e^2} \frac{\pi^2\zeta(3)}{16} \frac{1}{(k_Fd)^2(\kappa d)^2} \left( \frac{T}{\varepsilon_F} \right)^2.$$

We see that at $T \ll \tau_{tr}^{-1}$ our result exceeds the conventional one: in the interval $\tau_g^{-1} \ll T \ll \tau_{tr}^{-1}$ the correlation effects lead to the smoother $T$-dependence, while at $T \ll \tau_g^{-1}$ the prefactor of the $T^2$-dependence is $(\tau_g/\tau_{tr})^2$ times greater in our case.

Two analogies should be mentioned. Firstly, Eq. (3) resembles the Maki-Thompson correction to the conductivity of a single-layer system [4]. However, in that case the corresponding processes yield a small correction to the Drude term while in the double-layer system they determine the leading contribution to $\sigma_{21}$. Also, in our situation there exists the temperature-independent quantity $\tau_g$ resulting in a new behavior at $\tau_g \ll \tau_{\phi}$ and $T \ll \tau_g^{-1}$. We note that this quantity plays a role similar to that of the magnetic field. On the other hand, a perpendicular magnetic field leads to a suppression of the transresistivity, since one has to replace $\tau_g^{-1}$ by $\tau_H^{-1} = 4DcH/(\hbar c)$ in Eq. (3) at $\tau_g^{-1} \ll \tau_H^{-1} \ll T$ and in Eq. (2) at $T, \tau_g^{-1} \ll \tau_H^{-1}$. This effect of the magnetic field could provide a test for the theory.

Secondly, it can be seen from Eq. (3) that the nontrivial contribution to $\sigma_{21}$ arises in the second order in $\lambda_{21}$ in which case one should replace $\text{Im}\Lambda_e$ by $2\lambda_{21}^2\text{Im}\Psi_e$. The resulting expression looks quite similar to the usual one obtained for the uncorrelated impurity...
potentials \cite{2}, since they both describe the second-order Coulomb interaction processes in terms of some effective bosonic modes. The difference is in the physical meaning of these bosonic modes, which are the (energy-integrated) interlayer Cooperons versus the interlayer plasmons in the case considered in this paper and in the conventional situation, respectively. Also, there is an extra $q^2$ factor in the latter case providing the rapid decay of $\rho_{21}^{\text{conv}}(T)$.

In the above consideration we neglected the Aslamazov-Larkin — type diagrams \cite{9} which contain two $\Lambda_c$ ladders, since being proportional to $T^2\lambda^2_{21}$, these terms appear to be of the order of the ordinary contribution given by Eq. (4).

Now we discuss the experimental conditions under which the above theory applies. For the standard geometry we have found that at $k_F a > 1$ and $k_F d > 1$ (here $a$ is the distance from DDL to the nearest quantum well) the condition $\tau_g \gg \tau_{tr}$ can never be satisfied as long as $\kappa d > 1$. At $\kappa d < 1$ and $\kappa a > 1$ it requires $2(\kappa d)^2(k_F a)^2 < 1$, which can only be possible at very small interlayer distances \cite{11}.

The situation is different for the suggested geometry with a single DDL located between the two quantum wells. Introducing a finite width of the DDL $\delta$ we find that at $k_F d > 1$ and $(2\delta/d)^2 < (\kappa d)^{-1}$ the condition $\tau_g \gg \tau_{tr}$ can be rewritten as

$$2(k_F \delta)^2 \ll \min[1, \kappa d] \quad (10)$$

For $\kappa \sim 0.02 \AA^{-1}$, $k_F \sim 0.015 \AA^{-1}$, $\delta \sim 10 \AA$, and $d \sim 400 \AA$ the above criteria are fulfilled, and there exists the regime of temperatures described by Eq. (11). Note that it might be easier to observe this regime in dirty samples (yet with $l \gg d$). For $l \sim 5000 \AA$ (which implies $\tau_{tr}^{-1} \sim 4 K$ and $\tau_g^{-1} \sim 0.2 K$) Eq. (11) yields $\rho_{21}$ of the order of a few m$\Omega$s within the entire temperature range $\tau_{tr}^{-1} \gtrsim T \gtrsim \tau_g^{-1}$, whereas the conventional theory would predict a rapid decay of the transresistivity from $\rho_{21} \sim 1 \text{m}\Omega$ at $T \sim \tau_{tr}^{-1}$ to $\rho_{21} \sim 3 \mu\Omega$ at $T \sim \tau_g^{-1}$.

In conclusion, we investigated the influence of correlations between impurity potentials in different layers of a double-layer electron system on the Coulomb drag effect. We found that for correlated potentials the low-temperature drag is substantially enhanced compared to the standard (uncorrelated) situation, whereas a weak magnetic field suppresses the effect.

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[11] We note, however, that unavoidable substrate roughnesses may lead to the correlated interface roughnesses of both quantum wells, thanks to the long-range character of the deformation field. Then in very clean samples where the interface roughness becomes the main scattering mechanism one can expect some coherence of the kind described in this paper to occur even in the conventional geometry.

Figure caption. Diffuson diagrams for transconductivity (a), interlayer Coulomb-Cooperon ladder (b), and two Cooperon diagrams which yield the main contribution to $\sigma_{21}$ in the considered case (c). Solid lines stand for electron Green functions, wavy lines, and single dashed lines denote the screened Coulomb interaction and the impurity potential, respectively. Double dashed lines represent the Diffusons and Cooperons, and the numbers label the layers.
