Improving machining accuracy for a robotic arm with hybrid kinematic chains based on deformation characteristics

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Received: 10 December 2021 / Accepted: 20 June 2022 / Published online: 23 June 2022
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Abstract
Joint deformation greatly influences machining accuracy for a robotic arm. In this paper, the deformation characteristics of the robotic arm with hybrid kinematic chains are investigated to improve its machining accuracy. Firstly, the deformation model of the joints has been established based on the Strain energy method and Castigliano theorem according to the robot structure. Secondly, the deformation influence coefficient (DIC) is defined to investigate the deformation influence of main components on the end-effector, and the deformation characteristics are evaluated by the simulation. Finally, a small size robotic arm prototype is established and robotic drilling comparative experiments are conducted. The theoretical and experimental results show that the machining method can be selected according to the DIC, in which the force can be applied to the components with better stiffness. On the other hand, the deformation of driving components can also be reduced when the DIC cannot be adjusted to meet the accuracy requirement.

Keywords	Machining accuracy · Robotic arm · Hybrid kinematic chains · Deformation influence coefficient

1 Introduction
With the development of robotic technology, the requirements of industrial robots for fine working and high precision machining are being proposed, such as milling and drilling. However, low stiffness characteristics limit the application of industrial robots in the field of precision manufacturing [1]. Theoretical and experimental results show that the stiffness of a serial industrial robot is usually lower than 1 N/μm, while the stiffness of a CNC machine is usually greater than 50 N/μm [2]. In addition, the limitation of parallel-structured robots (such as Exechon and Tricept) is their restricted workspace, although they have better stiffness. Luo et al. [3] proposed a heavy-load and high-precision friction stir welding robot for welding large and complex thin-walled aluminum alloy 7075 surfaces. Xu et al. [4] designed a novel six-degrees-of-freedom (DOF) hybrid kinematic machine for polishing. Janez et al. [5] described an approach to improve the robot’s accuracy based on its structural properties, including manipulability, structural stiffness, structure inertial, damping ratios, and natural frequencies.

Stiffness analysis and optimization is an effective method for industrial robots to improve machining accuracy. Bu et al. [1] proposed a Cartesian compliance model to describe the robot stiffness in Cartesian space, and a quantitative index of the robotic processing performance is defined based on the compliance model. Xie et al. [6] built the mathematical models of machining error considering both kinematic errors and joint stiffness using speed and force adjoint transformation. Wei et al. [7] presented a finite element fitting method to analyze the stiffness distribution of the robot, and the low-, medium-and high-stiffness regions of the mechanism are shown through the stiffness distribution diagram. Zhao et al. [8] proposed a method of constructing the hybrid stiffness index to enhance the stiffness of the serial robot in completing friction stir welding tasks. The sigmoid function is used to process the dexterity or joint limit index and applies it to the stiffness index as a weight coefficient. The dimension synthesis of the serial robot is completed by maximizing the global soft stiffness index. Li et al. [9] divide the
positioning errors into the axial deformation error and the radial deformation error, and the positioning errors compensation method considering the end load and gravity based on the stiffness modeling is proposed. Zhang et al. [10] investigate the influence of robot stiffness properties on machining quality in drilling application, and the matching criterion of robotic drilling posture and thrust force is proposed, so as the optimized value of the pressing force can be predicted for stable machining. Lin et al. [11] presented an optimization method of the robot by taking the contour errors as the optimization index, and the optimal poses are obtained by minimizing contour error through a discrete searching algorithm. Stiffness optimization of industrial robots is easy to implement by adjusting the robot posture, however, the extent of improvement is limited by only using the stiffness optimization method. So, stiffness optimization is suitable for machining occasions where the stiffness requirement is not very high.

Error compensation is another way to decrease the positioning error for robotic machining. Zhu et al. [12] proposed a method that contains the two-dimensional manifold that reduces the dimensionality of the workspace, to improve the efficiency of error compensation. Yin et al. [13] proposed multi-iteration error compensation technology considering the coupling relationship between deformation and compensation, and a discrete control system for error compensation is constructed based on the milling force and errors predicted model. Ye et al. [14] focus on the contour errors of robotic machining, and a machining performance index is proposed to optimize the task-dependent workpiece placement. Li et al. [15] proposed a joint stiffness identification algorithm for the serial robot and a deformation compensation algorithm for the accuracy improvement. A drilling operation experiment is performed to identify the joint stiffness identification algorithm and the deformation compensation algorithm. Although the robot pose concerning the workpiece can be measured and modified through a multi-sensor measurement system, the inaccuracies of the test scenario and slippage on the workpiece impact the machining accuracy [16]. Error compensation method usually depends on the high precision multi-sensor measurement system which will increase the cost of the manufacturing system and the difficulty of compensation control. However, it is difficult to obtain satisfactory machining accuracy for robotic drilling even if the errors are compensated after the deformation is generated.

Precision Design is also an important approach to guarantee the accuracy of industrial robots. Luo et al. [17] propose a structural optimization method based on sensitivity analysis of the variable section of a slender robotic arm. The sensitivity analysis method and the sequential linear programming strategy are developed. The local sensitivity index is defined based on the projection theory of spatial vector to evaluate the contribution of an error component in a definite pose, and the results show that the pose accuracy can be increased greatly by improving very few error components [18]. Wang et al. [19] pay attention to the force transmission process of the robot, three indices are proposed which are forward force sensitivity, reverse force sensitivity and overall force sensitivity. Also, the structural optimization design schemes are presented based on the above indices. The ways of end-effector configuration affect the deformation of the robotic joint, the passive torque can be reduced to improve positioning accuracy due to selecting the proper configuration [20]. Luo et al. [21] used finite element analysis for designing the ram structure of friction stir welding robots, the lightweight design of the ram structure can be achieved through optimization, and the welding precision can be effectively improved. The precision design of the robot can fundamentally consider the influence of robotic stiffness on the machining accuracy; however, the open kinematic chain structure of traditional serial industrial robots makes it difficult to obtain better stiffness characteristics.

In this paper, a robotic arm with hybrid open- and closed-loop chains is introduced. To evaluate the machining accuracy of the hybrid robotic arm, the deformation characteristics of the robotic arm are investigated by proposing the deformation influence coefficient (DIC). Based on the deformation characteristics analysis, the machining accuracy improvement method is proposed. Then, a small size robotic arm prototype is established, and comparative drilling experiments are conducted with different poses and components. The results indicate that the proposed deformation analysis method is simple, easy to use, and effective.

The paper is structured as follows. In Sect. 2, the structure of the robotic arm with hybrid chains is described. In Sect. 3, the robotic joint deformation model is studied, and the effect of DIC on the machining accuracy is analyzed. In Sect. 4, the robotic drilling experiments are conducted to verify the proposed method. Finally, the paper is concluded in Sect. 5.

2 Robotic arm description

To improve machining accuracy and maintain a large workspace at the same time, a robotic arm with hybrid open- and closed-loop kinematic chains is proposed. The robotic arm has 5 DOF which compose of the waist, the big arm, the small arm, and the wrist joint, as shown in Fig. 1. The big arm and small arm are connected in series to form an open-loop chain, while the frame of the robotic arm itself forms a closed-loop chain. The big arm and small arm are composed of a multi-link parallelogram frame, and the electric cylinder is installed on the diagonal of the parallelogram which forms a truss structure [22]. The pitching action of the robotic arm
is realized by controlling the telescopic motion of the electric cylinders.

The long link of the parallelogram in the robotic arm is 1000 mm and the short link is 460 mm; the width of the long link is 220 mm and thickness is 60 mm in the small arm; the width of the long link is 300 mm and the thickness is 70 mm in the big arm. The structure of the robotic arm and its workspace in the $x$-$o$-$z$ plane is shown in Fig. 1.

3 Joint deformation modeling and analysis

3.1 Deformation modeling of robotic joint

Under the action of external load, the deformation of robotic components will eventually influence the end of the robot, which will affect the machining accuracy of the robot. The relationship between the deformation and the external force can be expressed as

$$\delta S_p = \begin{bmatrix} \delta S_{x} & \delta S_{y} & \delta S_{z} \end{bmatrix}^T = CF_e$$  \hspace{1cm} (1)

where, $C$ represents the compliance matrix; $F_e$ represents the external force, which can be expressed as

$$F_e = \begin{bmatrix} F_{ex} & F_{ey} & F_{ez} \end{bmatrix}^T$$  \hspace{1cm} (2)

The comprehensive displacement deformation of the robotic arm is superposed by the deformation of each joint, and the joint deformation is composed of the driving components’ deformation and the links’ deformation. To investigate the influence of each component deformation on the comprehensive deformation, the joint deformation model is established as follows. The robotic arm is established with hybrid kinematic chains, in which the links are connected with joints. Therefore, the robotic joint deformation model can be established based on the Strain energy method and Castigliano theorem. The deformation of electric cylinder AC and link AD, and the deformation of electric cylinder DE and link DF influence the displacement deformation of the end-effector. However, the deformations of EC and CB influence the perpendicularly of short links AB and CD rather than the displacement deformation of the end-effector. The load on link AD and electric cylinder AC causes the deformation of joint A along $x$-direction and $z$-direction, respectively. Similarly, the load on link FD and electric cylinder ED causes the deformation of joint D along $x$-direction and $z$-direction, respectively. The deformation of short and thick links AB, CD, and EF is ignored. So, the deformation of joint A along $x$-direction can be expressed as:
\[ \delta A_x = \frac{\partial U_A}{\partial F_{A_x}} = \frac{F_{AC} \partial F_{AC}}{K_{AC} \partial F_{A_x}} + \frac{F_{AD} \partial F_{AD}}{K_{AD} \partial F_{A_x}} \]  

where \( U_A \) represents the strain energy of joint A, \( F_{A_x} \) represents the force in the \( x \)-direction of joint A, \( K_{AC} \) represents the equivalent stiffness of electric cylinder AC, \( F_{AC} \) represents the force of electric cylinder AC, \( K_{AD} \) represents the equivalent stiffness of link AD, \( F_{AD} \) represents the force of link AD.

The deformation of joint A along \( z \)-direction can be expressed as:

\[ \delta A_z = \frac{\partial U_A}{\partial F_{A_z}} = \frac{F_{AC} \partial F_{AC}}{K_{AC} \partial F_{A_z}} + \frac{F_{AD} \partial F_{AD}}{K_{AD} \partial F_{A_z}} \]  

where \( F_{A_z} \) represents the force in the \( z \)-direction of joint A.

The deformation of joint D along \( x \)-direction can be expressed as:

\[ \delta D_x = \frac{\partial U_D}{\partial F_{D_x}} = \frac{F_{DE} \partial F_{DE}}{K_{DE} \partial F_{D_x}} + \frac{F_{DF} \partial F_{DF}}{K_{DF} \partial F_{D_x}} \]  

where \( U_D \) represents the strain energy of joint D, \( F_{D_x} \) represents the force in the \( x \)-direction of joint D, \( K_{DE} \) represents the equivalent stiffness of electric cylinder DE, \( F_{DE} \) represents the force of electric cylinder DE, \( K_{DF} \) represents the equivalent stiffness of link DF, \( F_{DF} \) represents the force of link DF.

The deformation of joint D along \( z \)-direction can be expressed as:

\[ \delta D_z = \frac{\partial U_D}{\partial F_{D_z}} = \frac{F_{DE} \partial F_{DE}}{K_{DE} \partial F_{D_z}} + \frac{F_{DF} \partial F_{DF}}{K_{DF} \partial F_{D_z}} \]  

where \( F_{D_z} \) represents the force in the \( z \)-direction of joint D.

To evaluate the deformation of each component separately and clearly, we define the deformation influence coefficient (DIC) \( \sigma \), which can be expressed as:

\[
\begin{align*}
\sigma_{AC_A} &= F_{AC} \frac{\partial F_{AC}}{\partial F_{A_x}} \\
\sigma_{AC_A} &= F_{AC} \frac{\partial F_{AC}}{\partial F_{A_z}} \\
\sigma_{AD_A} &= F_{AD} \frac{\partial F_{AD}}{\partial F_{A_x}} \\
\sigma_{AD_A} &= F_{AD} \frac{\partial F_{AD}}{\partial F_{A_z}} \\
\sigma_{DE_D} &= F_{DE} \frac{\partial F_{DE}}{\partial F_{D_x}} \\
\sigma_{DE_D} &= F_{DE} \frac{\partial F_{DE}}{\partial F_{D_z}} \\
\sigma_{DF_D} &= F_{DF} \frac{\partial F_{DF}}{\partial F_{D_x}} \\
\sigma_{DF_D} &= F_{DF} \frac{\partial F_{DF}}{\partial F_{D_z}}
\end{align*}
\]  

According to the deformation model of the robotic arm, the deformation influence coefficient (DIC) is defined according to Eqs. (7)–(10), which is affected by the robot’s structure, the external force and changes with the robot posture, and the deformation of the robot largely depends on the DIC. At the same time, the ratio of DIC can reflect the force distribution on the links and electric cylinders. Therefore, the ratio of DIC between the connecting link and the electric cylinder is considered to reduce the robot deformation. In addition, component stiffness is also the main factor affecting the deformation.

### 3.2 Simulation results and analysis

According to the structural parameters of the proposed hybrid robotic arm, the deformation model is established and the deformation characteristics are simulated by the MATLAB software. First, the effects of the link AD and the electric cylinder AC on the deformation of joint A along \( x \) and \( z \) directions are investigated.

Figure 2a shows the ratio of DIC \( \sigma_{AC_A} \) to \( \sigma_{AD_A} \), and Fig. 2b shows the DIC ratio of \( \sigma_{AC_A} \) to \( \sigma_{AD_A} \). The simulation results show that the link AD and the electric cylinder AC bear axial tension force or compression force at the same time in the workspace. In Fig. 2a, the ratio of \( \sigma_{AC_A} \) to \( \sigma_{AD_A} \) shows the relative influence of electric cylinder AC and link AD on the joint A in the \( x \)-direction. The variation range of \( \sigma_{AC_A}/\sigma_{AD_A} \) is 0.10–0.85. When the small arm is retracted, the DIC ratio of the electric cylinder AC relative to connecting link AD on the joint A along \( x \)-direction reaches the maximum value of 0.85, while, when the small arm is unfolded, the ratio of force influence coefficient reaches the minimum value of 0.10. Therefore, making the small arm work in a relatively unfolded state can make the force act on link AD that has better rigidity, which is conducive to reducing the deformation in the \( x \)-direction.

Similarly, the value of \( \sigma_{AC_A}/\sigma_{AD_A} \) shows the relative influence of electric cylinder AC and link AD on joint A along the \( z \)-direction. The variation range of \( \sigma_{AC_A}/\sigma_{AD_A} \) is 0.98–1.34, as shown in Fig. 2b. The simulation results show that when the small arm is unfolded, the ratio of DIC \( \sigma_{AC_A} \) is equivalent to \( \sigma_{AD_A} \). In other states, the DIC of electric cylinder AC on the deformation of joint A along \( z \)-direction is a little greater than that of the link AD.

Figure 3a shows the DIC of joint D along the \( x \)-direction. In some areas of the workspace, such as partially unfolded areas, the sign of DIC \( \sigma_{DF_D} \) changes while the sign of DIC \( \sigma_{DE_D} \) remains unchanged. Therefore, selecting the area with different DIC signs is a benefit to reduce the deformation of joint D along the \( x \)-direction.

The value of \( \sigma_{DE_D}/\sigma_{DF_D} \) shows the relative influence of electric cylinder DE and link DF on the joint D along the \( z \)-direction, as shown in Fig. 3b, and the variation range
of $\sigma_{DE_D}/\sigma_{DF_D}$ is 0.11–0.54. According to the simulation results, when the big arm is unfolded, the DIC $\sigma_{DE_D}$ is increased relative to the DIC $\sigma_{DF_D}$. Making the big arm work in the retracted state will reduce the deformation of the joint D along the $z$-direction. The equivalent stiffness of electric cylinders is usually lower than that of the links, therefore, making the DIC of the links larger than that of the electric cylinders as much as possible is conducive to reducing the deformation of robotic joints. Besides, the DIC of different areas in the workspace is different, and the directions of machining force need to be considered according to the accuracy requirement. In addition, because the equivalent stiffness of the electric cylinder is far less than the links, the effect of reducing the deformation of links is not obvious compared with that of electric cylinders, therefore, reducing the deformation of flexible parts which have relatively weak stiffness, that is, improving the equivalent stiffness $K$ in Eqs. (3)–(6) is a direct way to reduce the deformation of the robotic arm.

4 Robotic drilling experiments

To verify the deformation characteristics of the proposed robotic arm, a small size prototype is established that the length of the long link and short link in parallelogram frame is 360 mm and 240 mm, respectively, and the width of the links is 66 mm. The frame structure of the robot prototype is

Fig. 2 The ratio of deformation influence coefficient (DIC) for joint A

Fig. 3 The value and the ratio of DIC for joint D
The robotic drilling experimental system is established which includes a robotic arm, spindle feed mechanism, motorized spindle, drilling bit, workpiece, and fixture, as shown in Fig. 4. To ensure that the axes of the spindle and the drill bit are perpendicular to the plane of the worktable, the probe of the dial indicator is in contact with the moving end face of the spindle feeding mechanism along the X direction and Y direction respectively, and the normal angle between the spindle axis and the worktable is obtained by observing the reading change of the dial indicator during the spindle feeding process. After repeated adjustment, the included angle between the spindle axis and the normal of the horizontal plane of the worktable is less than 0.01°.

The parameters of the drilling experiment are shown in Table 1. An air-cooled spindle motor is selected as the motorized spindle for the machining. The power of the spindle motor is 1.5KW, and the maximum speed is 24000 rpm, the radial runout error is only 2 μm. The high-speed steel twist bit is selected in the experiment. BXTL150 series electric cylinders are selected for driving the robotic arm. The workpiece to be processed is made of 6061 aluminum alloy, with a length of 120 mm, a width of 60 mm, and a thickness of 5 mm. The maximum axial drilling force is about 49.2 N for drilling a 5 mm diameter hole. Fourteen holes are to be drilled on the workpiece which is marked as H1~H14, respectively.

Firstly, two groups of comparative drilling experiments are conducted with two different poses. Pose I has the angle $\theta_1$ 25° and angle $\theta_2$ 35°, which the ratio of $\sigma_{AC} / \sigma_{AD}$ is 0.4. Pose II has the angle $\theta_1$ 45° and angle $\theta_2$ 35°, which the ratio of $\sigma_{AC} / \sigma_{AD}$ is 0.6. The holes from H1 to H14 will be drilled in turn. The experiment results of drilling 5 mm holes by the robotic arm are shown in Fig. 5. From the overall experimental results, it can be seen that consistency is good for robotic drilling.

The results of the axial angle error of the holes obtained from the experimental measurement are shown in Table 2. In the experiment, the maximum angle error of the drilled holes along the $z$-direction at pose I is 0.93°, and the average angle error is 0.75°. Besides, the maximum angle error at pose II is 0.98°, and the average angle error is 0.92°.

The bending deformation of the bit causes the axial angle error for the drilled holes under the external force and causes the roundness error of the theoretical circular hole to be drilled at the same time. The diameter of the 5mm hole drilled by the robotic arm at the above two poses is measured at different circumference angles, as shown in Fig. 6. In addition, the actual diameters of the drilled holes (H1~H14) at pose I are shown in Table 3.

As shown in Table 3, the maximum tolerance of all the holes drilled on the 6061-aluminum alloy workpiece is 0.036mm, and the tolerance level of all holes can reach IT10, in which the most holes can reach IT9 at pose I. The same drilling process is conducted at pose II, and the actual diameters of the drilled holes are also measured using the same method, which is not listed in detail in the paper. Correspondingly, the tolerance level of the holes can only reach IT11 at pose II.

![Fig. 4 Robotic drilling experimental system](image-url)
Through the theoretical analysis results, the value of $\sigma_{\text{ACA}} / \sigma_{\text{AD A}}$ at pose II is larger than that at pose I, which makes the electric cylinder AC produce larger deformation and makes the end-effector produce larger deformation along the $x$-direction. The theoretical analysis results are also verified by the experiment results at the two poses. Therefore, the deformation characteristics analysis using DIC is effective to select the proper pose for improving machining accuracy.

According to Eqs. (3)–(6), in addition to analyzing the deformation influence coefficient, reducing the deformation of the components which has weak stiffness is another way to improve the machining accuracy. A comparative experiment is designed that Thomson PC25 series linear electric cylinder is selected to replace the original BXTL150 electric cylinders, as shown in Fig. 7. The equivalent axial stiffness of the Thomson electric cylinder is better than that of the BXTL150 series electric cylinder. The maximum axial load of Thomson PC25 is 1250N, the maximum stroke is 600mm, the maximum speed is 1.33m/s, the screw diameter is 10mm and the lead is 3mm. Panasonic A6 series low inertia AC servo motor is selected as the power source for the electric cylinder. The prototype for the comparative experiment is shown in Fig. 8.

The drilling experiments are also carried out under the drilling conditions shown in Table 1. The material of the workpiece used for drilling is still 6061 aluminum alloy with a thickness of 5 mm. The experiment results of angle errors at pose I and pose II are shown in Table 4. According to the experiment results, the average angle error is 0.064° at pose I, while the average angle error of the holes is 0.072° at pose II. According to experiment results, the selected Thomson electric cylinders greatly reduce the joint deformation, therefore, the drilling angle error of the prototype is reduced by one order of magnitude compared with the angle error using BXTL150 electric cylinders.

The roundness of the drilled holes can be evaluated through four indexes of the maximum aperture, minimum aperture, aperture difference, and average aperture of the holes. At the same time, the vertical accuracy of the holes can also be reflected by the roundness errors indirectly. In the experiment, the diameter of the hole drilled with different electric cylinders is measured every 30° on the circumference of the hole, as shown in Fig. 9. The experimental comparison results of maximum aperture, minimum aperture, aperture difference, and average aperture of the drilled holes are shown in Table 5. The maximum tolerance value is 0.006 mm, and the average tolerance value is 0.004 mm for drilling with Thomson series electric cylinders, while the tolerance value is less than the maximum tolerance value of 0.030 mm and the average tolerance value of 0.021 mm for drilling with BXTL150 electric cylinders. The experiment results show that the joint deformation is reduced significantly after replacing the electric cylinders of the robotic arm.

In further experiments, 7075 aluminum alloy workpieces with higher strength are selected, and the thickness of the workpiece increases to 8 mm. The drilling parameters are still selected as shown in Table 1. Under
Fig. 6 Diameter distribution of robotic drilling holes

(a) Drilled hole

(b) Diameter distribution of H1 holes at pose I

Fig. 7 Thomson series electric cylinders and servo drivers

Table 3 Actual diameters of the drilled holes at pose I (6061 aluminum alloy)

|   | -80° | -60° | -40° | -20° | 0°   | 20°   | 40°   | 60°   | 80°   |
|---|------|------|------|------|------|------|------|------|------|
| H1 | 5.019 | 5.021 | 5.011 | 5.008 | 5.005 | 5.012 | 5.019 | 5.020 | 5.022 |
| H2 | 5.020 | 5.019 | 5.026 | 5.013 | 5.009 | 5.015 | 5.012 | 5.010 | 5.029 |
| H3 | 5.018 | 5.009 | 5.012 | 5.016 | 5.026 | 5.015 | 5.024 | 5.012 | 5.011 |
| H4 | 5.007 | 5.018 | 5.031 | 5.025 | 5.021 | 5.019 | 5.021 | 5.013 | 5.007 |
| H5 | 5.001 | 5.002 | 5.008 | 5.009 | 5.001 | 5.013 | 5.010 | 5.009 | 5.009 |
| H6 | 5.029 | 5.027 | 5.038 | 5.029 | 5.011 | 5.015 | 5.012 | 5.041 | 5.020 |
| H7 | 5.020 | 5.030 | 5.048 | 5.049 | 5.041 | 5.020 | 5.021 | 5.029 | 5.021 |
| H8 | 5.028 | 5.029 | 5.022 | 5.026 | 5.015 | 5.031 | 5.040 | 5.029 | 5.026 |
| H9 | 5.029 | 5.018 | 5.019 | 5.023 | 5.032 | 5.029 | 5.015 | 5.006 | 5.005 |
| H10| 5.029 | 5.040 | 5.030 | 5.021 | 5.038 | 5.025 | 5.046 | 5.040 | 5.045 |
| H11| 5.041 | 5.048 | 5.048 | 5.048 | 5.046 | 5.029 | 5.018 | 5.012 | 5.021 |
| H12| 5.009 | 5.012 | 5.013 | 5.030 | 5.042 | 5.039 | 5.015 | 5.021 | 5.042 |
| H13| 5.025 | 5.020 | 5.021 | 5.035 | 5.038 | 5.012 | 5.020 | 5.029 | 5.028 |
| H14| 5.034 | 5.039 | 5.015 | 5.022 | 5.020 | 5.032 | 5.026 | 5.021 | 5.020 |
this condition, the axial drilling force for drilling a 5 mm hole increases to about 102.3 N, and the experimental effect of the robotic arm prototype drilling on 7075 aluminum alloy workpiece is shown in Fig. 10. The angle error results of the robot prototype for drilling 5 mm holes on 7075 aluminum alloy workpieces are shown in Table 6. As shown in Table 6, the average angle errors of the robot prototype drilling along the x-direction, y-direction, and z-direction are 0.104°, 0.281°, and 0.110°, respectively.

Table 7 shows the aperture distribution results of the robot prototype for drilling 5 mm holes on 7075 aluminum alloy workpieces along the z-direction, and the tolerance level of the drilled holes can be reached to IT6. Because of the high vertical accuracy of drilling, the burr of the holes is also significantly reduced, even if the feed force increases, awesome accuracy of drilling can be ensured.
Table 5 Diameter comparison of drilled holes using the robot prototype (6061 aluminum alloy)

| Electric cylinder | H1  | H2  | H3  | H4  | H5  | H6  | H7  |
|-------------------|-----|-----|-----|-----|-----|-----|-----|
| BXTL150           | 5.022 | 5.029 | 5.026 | 5.031 | 5.013 | 5.041 | 5.047 |
| Φ_{max} (mm)      | 5.005 | 5.009 | 5.009 | 5.007 | 5.001 | 5.011 | 5.020 |
| ΔΦ (mm)           | 0.017 | 0.020 | 0.017 | 0.024 | 0.012 | 0.030 | 0.027 |
| Φ_{avg} (mm)      | 5.015 | 5.017 | 5.016 | 5.018 | 5.007 | 5.025 | 5.031 |
| Thomson PC25      | 5.004 | 5.006 | 5.009 | 5.005 | 5.006 | 5.007 | 5.006 |
| Φ_{max} (mm)      | 5.002 | 5.002 | 5.003 | 5.001 | 5.004 | 5.001 | 5.002 |
| ΔΦ (mm)           | 0.002 | 0.004 | 0.006 | 0.004 | 0.002 | 0.006 | 0.004 |
| Φ_{avg} (mm)      | 5.003 | 5.004 | 5.006 | 5.005 | 5.004 | 5.004 |

Fig. 10 Drilling experiment effect using the robot prototype (7075 aluminum alloy)

Table 6 Angular errors of drilled holes using the robot prototype (7075 aluminum alloy)

| No. | Angle error Δ (°) | x- direction | y- direction | z- direction |
|-----|-------------------|--------------|--------------|--------------|
| H1  | 0.10              | 0.28         | 0.13         |              |
| H2  | 0.14              | 0.31         | 0.11         |              |
| H3  | 0.09              | 0.25         | 0.09         |              |
| H4  | 0.12              | 0.30         | 0.10         |              |
| H5  | 0.09              | 0.29         | 0.09         |              |
| H6  | 0.11              | 0.33         | 0.13         |              |
| H7  | 0.08              | 0.26         | 0.12         |              |
| H8  | 0.11              | 0.30         | 0.14         |              |
| H9  | 0.10              | 0.29         | 0.12         |              |
| H10 | 0.08              | 0.27         | 0.08         |              |
| H11 | 0.13              | 0.24         | 0.09         |              |
| H12 | 0.12              | 0.29         | 0.11         |              |
| H13 | 0.10              | 0.26         | 0.10         |              |
| H14 | 0.09              | 0.27         | 0.13         |              |
5 Conclusions

In this paper, the deformation influence coefficient (DIC) is proposed based on the Strain energy method and the Castigliano theorem to evaluate the deformation characteristics of the robotic arm with hybrid open- and closed-loop kinematic chains. By defining the deformation influence coefficient, the deformation influence of internal components in the workspace can be analyzed. According to the DIC and the component stiffness, the machining mode of the robotic arm can be reasonably planned to reduce the deformation in the machining process. To verify the influence of DIC on machining accuracy, comparative robotic drilling experiments are carried out. The theoretical and experimental results show that the pose can be selected according to the DIC which is beneficial to improve machining accuracy, as well as improve the equivalent stiffness of the key component. In addition, compared with the serial industrial robot, the advantage of the robotic arm with hybrid kinematic chains is that the electric cylinders, links, and other components can be selected according to the machining accuracy requirements. This feature makes the robotic arm more flexible to suit different tasks.

Funding This work was supported by the Liaoning Doctor Scientific Research Initial Fund (No.2021-BS-160), and the Research Support Fund for Introducing High-Level Talents to Shenyang Ligong University (No. 1010147000821).

Data availability Not applicable.

Code availability Not applicable.

Declarations

Ethics approval The work was original research that has not been published previously and is not under consideration for publication elsewhere, in whole or in part.

Consent to participate The authors all approved to participate.

Consent to publication It is approved by all authors for publication.

Conflict of interest The authors declare no competing interests.

References

1. Bu Y, Liao WH, Tian W, Zhang J, Zhang L (2017) Stiffness analysis and optimization in robotic drilling application. Precis Eng 53(49):388–400. https://doi.org/10.1016/j.precisioneng.2017.04.001
2. Pan ZX, Zhang H, Zhu ZQ, Wang JJ (2006) Chatter analysis of robotic machining process. J Mater Process Technol 173(3):301–309. https://doi.org/10.1016/j.jmatprotec.2005.11.033
3. Luo HT, Zhao FQ, Guo SW, Yu CS, Liu GM, Wu TK (2021) Mechanical performance research of friction stir welding robot for aerospace applications. Int J Adv Robot Syst 18(1):1–11. https://doi.org/10.1177/1729881421996543
4. Xu P, Cheung CF, Li B, Wang CJ, Zhao CY (2021) Dynamic analysis and experimental evaluation of a hybrid parallel-serial polishing machine with decoupled motions. J Mech Robot 13(6):1–13. https://doi.org/10.1115/1.4050829
5. Janez G, Timi K, Karl G, Miran B (2020) Accuracy improvement of robotic machining based on robot’s structural properties. Int J Adv Manuf Technol 108(5–6):1399–1399. https://doi.org/10.1007/s00170-020-05438-z
6. Xie H, Li WL, Zhu DH, Yin ZP, Ding H (2020) A systematic model of machining error reduction in robotic grinding. IEEE-ASME Trans Mechatron 25(6):2961–2972. https://doi.org/10.1109/TMECH.2020.2999928
7. Wei W, Cai GW, Gong JJ, Peng SX (2021) Modeling and analysis of the stiffness distribution of host-parasite robots. IEEE Access 9:86300–86320. https://doi.org/10.1109/ACCESS.2021.3063296
8. Zhao J, Duan YX, Xie BY, Zhang ZQ (2021) FSW robot system dimensional optimization and trajectory planning based on soft stiffness indices. J Manuf Process 63:88–97. https://doi.org/10.1016/j.jmapro.2020.05.004
9. Li YJ, Gao GB, Liu F (2020) Positioning error compensation for industrial robots based on stiffness modelling. Complexity 2020:1–13. https://doi.org/10.1155/2020/8850751
10. Zhang JL, Liao WH, Bu Y, Tian W, Hu JS (2020) Stiffness properties analysis and enhancement in robotic drilling application. Int J Adv Manuf Technol 106(11–12):5539–5558. https://doi.org/10.1007/s00170-020-05011-8
11. Lin JZ, Ye CC, Yang JX, Zhao H, Ding H, Luo M (2022) Contour error-based optimization of the end-effector pose of a 6 degree-of-freedom serial robot in milling operation. Robot Comput-Integr Manuf 73:1–11. https://doi.org/10.1016/j.rcim.2021.102257
12. Zhu WD, Li GH, Dong HY, Ke YL (2019) Positioning error compensation on two-dimensional manifold for robotic machining. Robot Comput-Integr Manuf 59:394–405. https://doi.org/10.1016/j.rcim.2019.05.013
13. Yin FC, Ji QZ, Wang CZ (2021) Research on machining error prediction and compensation technology for a stone-carving robotic

Table 7 Diameters of drilling holes in z-direction using the robot prototype (7075 aluminum alloy)

|          | H1  | H2  | H3  | H4  | H5  | H6  | H7  |
|----------|-----|-----|-----|-----|-----|-----|-----|
| φ_{max} (mm) | 5.011 | 5.012 | 5.011 | 5.010 | 5.011 | 5.010 | 5.009 |
| φ_{min} (mm) | 5.006 | 5.008 | 5.005 | 5.003 | 5.007 | 5.005 | 5.003 |
| Δφ (mm) | 0.005 | 0.004 | 0.006 | 0.007 | 0.004 | 0.005 | 0.006 |
| φ_{avg} (mm) | 5.009 | 5.010 | 5.008 | 5.007 | 5.009 | 5.008 | 5.006 |
| H8 | H9 | H10 | H11 | H12 | H13 | H14 |
| φ_{max} (mm) | 5.009 | 5.011 | 5.010 | 5.011 | 5.012 | 5.009 | 5.013 |
| φ_{min} (mm) | 5.003 | 5.006 | 5.002 | 5.005 | 5.005 | 5.004 | 5.006 |
| Δφ (mm) | 0.006 | 0.005 | 0.008 | 0.006 | 0.007 | 0.005 | 0.007 |
| φ_{avg} (mm) | 5.008 | 5.009 | 5.006 | 5.008 | 5.009 | 5.007 | 5.007 |
manipulator. Int J Adv Manuf Technol 115(5–6):1683–1700. https://doi.org/10.1007/s00170-021-07230-z
14. Ye CC, Yang JX, Zhao H, Ding H (2021) Task-dependent work-piece placement optimization for minimizing contour errors induced by the low posture-dependent stiffness of robotic milling. Int J Mech Sci 205:1–16. https://doi.org/10.1016/j.ijmecsci.2021.106601
15. Li GZ, Zhang FH, Fu YL, Wang SG (2019) Joint stiffness identification and deformation compensation of serial robots based on dual quaternion algebra. Appl Sci-Basel 9(1):1–18. https://doi.org/10.3390/app9010065
16. Frommknecht A, Kuehnle J, Effenberger I, Pidan S (2017) Multi-sensor measurement system for robotic drilling. Robot Comput-Integr Manuf 47:4–10. https://doi.org/10.1016/j.rcim.2017.01.002
17. Luo Z, Zhao XY, Liang L, Wang F (2012) Structure optimization of slender robot arm based on sensitivity analysis. Math Probl Eng 2012:1–17. https://doi.org/10.1155/2012/806815
18. Li J, Xie FG, Liu XJ, Mei B, Li HJ (2018) A spatial vector projection based error sensitivity analysis method for industrial robots. J Mech Sci Technol 32(6):2839–2850. https://doi.org/10.1007/s12206-018-0540-y
19. Wang YH, Tang XQ, Xiang CY, Hou SH (2021) Force sensitivity analysis and scale design of Stewart parallel manipulator. Adv Mech Eng 13(7):1–17. https://doi.org/10.1177/16878140211035996
20. Liang J (2015) A research on the mounted configuration of end-effector for robotic drilling. Robotica 33(10):2156–2165. https://doi.org/10.1017/S0263574714001313
21. Luo HT, Fu J, Wang P, Liu JG, Zhou WJ (2020) Design optimization of the ram structure of friction stir welding robot. Mech Adv Mater Struct 27(2):108–118. https://doi.org/10.1080/15376494.2018.1471758
22. Milutinovic DS, Sato R, Matsuura D, Ehmann K (2016) Mechanism for active pi-joint as an equivalent to the combination of revolute joint and proximal fixed-length link. Robot Comput-Integr Manuf 37:179–187. https://doi.org/10.1016/j.rcim.2015.05.005

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