Non-extensivity Effects and the Highest Energy Cosmic Ray Affair

Luis A. Anchordoqui\textsuperscript{a} and Diego F. Torres\textsuperscript{b}

\textsuperscript{a}Department of Physics, Northeastern University, Boston, Massachusetts 02115
\textsuperscript{b}Instituto Argentino de Radioastronomía, C.C.5, 1894 Villa Elisa, Buenos Aires, Argentina

Abstract

Recent measurements of the cosmic microwave background confirm that it is described by a Planckian distribution with high precision. It is non-extensivity bounded to be less than some parts in $10^5$, or to some parts in $10^4$ at most. This deviation may appear minuscule, but may have a non-negligible effect on a particle propagating through this background over the course of millions of years. In this paper we analyze the possible influence of such a slight deviation upon the propagation of nuclei and protons of ultra-high energy. These particles interact via photopion and photodisintegration processes which we examine taking into account a slight non-extensive background. We show that such a deviation does not exhibit a significant difference in the energy attenuation length of extremely high energy cosmic rays.

\textit{PACS number(s):} 0.5.20.-y, 98.70.Sa

\textit{Key words:} non-extensive statistics, cosmic rays
The influence of non-extensivity in cosmology, astrophysics, and cosmic ray physics has been a subject of recent interest (see, for instance, Refs. [1]). In particular, the effect of a possible and very slight non-extensivity upon the cosmic microwave background (CMB) has been thoroughly investigated (see Refs. [2–5] among others). The CMB has proven to be described by a Planckian distribution, with a precision ranging from $\sim 5$ parts in $10^4$ [2] to $\sim 4$ parts in $10^5$ [3]. These bounds are given in the form of constraints in the parameter space, here represented by $|q-1|$, where $q$ is the non-extensivity parameter that appear in Tsallis’ extension of statistical mechanics [6]. Within this framework, the generalized photon energy distribution of Planck’s radiation law is [2]

\[
D_q(\nu) = \frac{8\pi k^3 T^3}{c^3 h^2} \frac{x^3}{e^x - 1} (1 - e^{-x})^{q-1} \\
\times \left[ 1 + (1 - q)x \left( \frac{1 + e^x}{1 - e^{-x}} - \frac{x}{2 (1 - e^{-x})^2} \right) \right],
\]

(1)

where $x = h\nu/kT$. Therefore, the expression for $D_q(\nu)c^3h^2/8\pi k^3T^3$ generalizes the Planck’s law, and in the limit of $x \ll 1$, the Rayleigh-Jeans’ law. For $h\nu|1-q| \gg kT$, the $(1-q)$ correction diverges, which renders the expansion invalid. For $q \sim 1$, because of the suppression of the exponential factor, the divergent region has no practical importance. Consequently, the total emitted power per unit surface is given by

\[
P_q \propto \int_0^\infty D_q d\nu \sim \sigma_q T^4,
\]

(2)

where a $q$-dependent Stefan-Boltzmann constant has appeared. The thermal spectrum given in Eq. (1) was applied to the CMB in order to test for deviations from Planck’s law [2–5]. However, the influence of such a possible deviation has not been yet fully explored. In this letter, we aim to look for any possible non-extensivity effects in the propagation of ultra-high energy cosmic rays (CRs).

CRs do not travel unhindered through intergalactic space, as there are several processes that can degrade the particles’ energy. In particular, the thermal photon background becomes highly blueshifted for ultra-relativistic protons. The reaction sequence $p\gamma \rightarrow \Delta^+ \rightarrow \pi^0 p$ effectively degrades the primary proton energy. This provides a strong constraint on the proximity of CR-sources, a phenomenon known as the Greisen-Zatsepin-Kuz’m‘min (GZK) cutoff [7]. Specifically, less than 20% of $3 \times 10^{20}$ eV ($1 \times 10^{20}$ eV) protons can survive a trip of 18 (60) Mpc. For nuclei and photons the situation is, in general, more drastic [8].

In recent years, a handful of extensive air showers have

\footnote{It should be stressed that super-heavy nuclei can break the GZK barrier [9].}
been observed with nominal energies at or above $10^{20} \pm 30\%$ eV [10]. The arrival directions of the primary particles are distributed widely over the sky, with no plausible astrophysical sources within the GZK distance ($\approx 50$ Mpc) [12]. Furthermore, although the first five events with $E > 8 \times 10^{19}$ eV (at 1-standard deviation) did in fact point toward high redshift compact radio-loud quasars (astrophysical environments that could accelerate CRs above the GZK energies via shock mechanisms) [13], with the inclusion of subsequent data this association is controversial [14]. Certainly the energy window of “super-GZK” CRs greatly exceeds that of man-made accelerators, and so speculation on new high-energy physics is open [11].

The interaction length of a nucleon to undergo photopion production is $\approx 6$ Mpc, whereas a typical nucleus suffers disintegration (against photons of the microwave and infrared radiation backgrounds), losing about 3-4 nucleons per traveled Mpc [8]. Consequently, if taking $q \neq 1$ leads to unobservable effects in the nucleus photodisintegration rate, this also implies unobservable effects in the photopion production process. For iron nuclei with a Lorentz factor $\log \Gamma \geq 9.3$, the CMB provides the target photon field, hence they are excellent candidates to test whether the inclusion of non-extensivity effects may alter the “average particle kinematics”. In the universal rest frame (in which the microwave background radiation is at 2.73 K), the disintegration rate $R$, of an extremely high energy nucleus of iron ($^{56}$Fe), propagating through an isotropic soft photon background of density $n$, is given by [15],

$$R = \frac{1}{2\Gamma^2} \int_{0}^{\infty} d\epsilon \frac{n(\epsilon)}{\epsilon^2} \int_{0}^{2\Gamma \epsilon} d\epsilon' \sigma(\epsilon'),$$

(3)

where $\sigma$ stands for the total photodisintegration cross section. According to the photon energy in the nucleus rest frame ($\epsilon'$) one can distinguish different regimes. From $\epsilon' \approx 2$ MeV up to $\epsilon' \sim 30$ MeV, the total photon absorption cross section is characterized by a broad maximum, designated as the giant resonance, in which a collective nuclear mode is excited with the subsequent emission of one (or possible two) nucleons. Beyond the giant resonance and up to the photopion production threshold, i.e. $\epsilon' \approx 150$ MeV, the excited nucleus decays dominantly by two nucleon (quasi-deuteron effect) and multi-nucleon emission. The giant dipole resonance can be well represented by a Lorentzian curve of the form [16],

$$\sigma(\epsilon') = \sigma_0 \frac{\epsilon'^2 \Gamma_0^2}{(\epsilon_0^2 - \epsilon'^2)^2 + \epsilon'^2 \Gamma_0^4},$$

(4)

However, the abundance of nuclei heavier than iron (in the cosmic radiation) is expected to be smaller by 3 or 5 orders of magnitude relative to the lighter ones.
where $\sigma_0 = 1.45A$ mb, and $\Gamma_0 = 8$ MeV is the energy bandwidth. The peak energy depends on the mass number of the nucleus: $\epsilon_0 = 42.65A^{-0.21}$ MeV for $A > 4$, and $\epsilon_0 = 0.925A^{2.433}$ MeV for $A \leq 4$. Above 30 MeV the cross section is roughly a constant, $\sigma = A/8$ mb. In Fig. 1 we show the variation of $dR/d\epsilon$ as a function of the background photon energy for different values of the non-extensivity parameter in $n(\epsilon) \equiv D_q(\nu)/\hbar\nu$. It is important to stress that $1 - q = 10^{-5}$ is admitted for the most restrictive cosmological bounds existing today: those obtained from the CMB. The effects of $q \neq 1$ are a small decrease in the peak value and a shorter tail.

At this stage, it is noteworthy that the nucleus emission is isotropic in the rest system of the nucleus. Moreover, for photodisintegration the Lorentz factor is conserved during propagation. Thus, the photodisintegration average loss rate reads,

$$\frac{1}{E} \frac{dE}{dt} = \frac{1}{A} \frac{dA}{dt}, \quad (5)$$

and the total energy loss time $\tau$ is given by

$$\tau^{-1} \equiv \frac{1}{E} \frac{dE}{dt} = \frac{1}{\Gamma} \frac{d\Gamma}{dt} + \frac{R}{A}. \quad (6)$$

Above $2 \times 10^{20}$ eV the reduction in $\Gamma$ (due to $e^+e^-$ production) can be safely neglected [8], so using the scaling for the cross section proposed in [8],

$$\left. \frac{dA}{dt} \right|_A \approx \left. \frac{dA}{dt} \right|_{Fe} \frac{A}{56} = \left. \frac{R(\Gamma)}{Fe} \frac{A}{56} \right|_t \quad (7)$$

it is straightforward to obtain the photodisintegration history in terms of the nucleus time flight:

$$A(t) = 56 \exp[-R|_{Fe} t/56]. \quad (8)$$

It is easily seen that although non-extensivity effects are observable in the derivative of $R$, they disappear when one integrates over the energy. Even if we artificially enhance the difference in the value of $|q - 1|$, and take for instance $q = 0.95$ (see Fig. 2), the mean value of the nucleon loss is within the errors of a Monte Carlo simulation [17].

---

2 Note that processes which involve the creation of new particles which carry off energy (pair production, photopion production) do not conserve $\Gamma$.

3 The case could be made for such high values of $|q - 1|$ in the early universe (before the time of primordial nucleosynthesis) but they are certainly not in agreement with CMB measurements.
In summary, photopion and photodisintegration in a non-extensivity framework do not differ from the standard case (when non-extensivity is bounded as explained) even when nuclei or protons are compelled to interact with the background through a significant part of the size of the entire universe.

In closing, we would like to point out that the most efficient target for extremely high energy gamma rays to produce $e^+e^-$ pairs are background photons of energy $\sim m_e^2/E \lesssim 10^6$ eV $\approx$ 100 MHz (radio-waves), with a mean free path of 20 - 40 Mpc at $3 \times 10^{20}$ eV (see Protheroe-Johnson in Ref. [8]). However, at $10^{15}$ eV the energy loss rate is dominated by collisions with the CMB. The relic photons drop the energy attenuation length down to $10^{-2}$ Mpc. Then, from the phenomenological point of view, it would perhaps be interesting to analyze whether non-extensivity effects yield any observable deviation on the energy loss time of photon-photon interaction.

Acknowledgments

We would like to thank Tom McCauley for a critical reading of the manuscript. Remarks by an anonymous referee are gratefully acknowledged. This work was supported by CONICET, and Fundación Antorchas through a grant to D.F.T.

References

[1] V. H. Hamity and D. E. Barraco, Phys. Rev. Lett. 76, 25 (1996); D. F. Torres, H. Vucetich and A. Plastino, Phys. Rev. Lett. 79, 1588 (1997); E80, 3889 (1998); G. Kaniadakis, A. Lavagno and P. Quarati, Phys. Lett. B369, 308 (1996); D. F. Torres and H. Vucetich, Physica A259, 397 (1998); U. Tirnakli and D. F. Torres, Physica A268, 225 (1999); G. Wilk and Z. Wlodarczyk, Nuc. Phys. B. (Proc. Suppl.) 75A, 191 (1999); M. Rybczynski, Z. Wlodarczyk, G. Wilk, Nucl. Phys. B (Proc. Suppl.) 97, 85 (2001); V. H. Hamity and D. E. Barraco, Physica A282, 203 (2000); H. P. de Oliveira, S. L. Sautu, I. D. Soares and E. V. Tonini, Phys. Rev. D60, 121301 (1999); A. Lavagno, G. Kaniadakis, M. Rego-Monteiro, P. Quarati and C. Tsallis, Astrophysical Letters and Communications 35, 449 (1998).

[2] C. Tsallis, F. Sa Barreto and E. D. Loh, Phys. Rev. E52, 1447 (1995).

[3] A. R. Plastino, A. Plastino and H. Vucetich, Phys. Lett. A207, 42 (1995).

[4] A. B. Pinheiro and I. Roditi, Phys. Lett. A 242, 296 (1998).

[5] D. F. Torres, Physica A 261, 512 (1998).
[6] C. Tsallis, J. Stat. Phys. 52, 479 (1988); see also, C. Tsallis, Fractals 6, 539 (1995); C. Tsallis, Physica A221, 227 (1995); E. M. F. Curado and C. Tsallis, J. Phys. A24, L69 (1991); Corrigenda: A24, 3187 (1991) and A25, 1019 (1992).

[7] K. Greisen, Phys. Rev. Lett. 16 (1966) 748; G. T. Zatsepin, and V. A. Kuz’min, Pis’ma Zh. Éksp. Teor. Fiz. 4 (1966) 114 [JETP Lett. 4, 78 (1966)].

[8] J. L. Puget, F. W. Stecker and J. H. Bredekamp, Astrophys. J. 205, 638 (1976); R. J. Protheroe and P. Johnson, Astropart. Phys. 4, 253 (1996).

[9] L. A. Anchordoqui, M. Kirasirova, T. P. McCauley, S. Reucroft and J. D. Swain, Phys. Lett. B. 492, 237 (2000) [astro-ph/0007408].

[10] For a recent survey see for instance, S. Yoshida and H. Dai, J. Phys. G 24, 905 (1998).

[11] For a comprehensive review on the origin and nature of the highest energy CRs the reader is referred to P. Bhattacharjee, and G. Sigl, Phys. Rep. 327, 109 (2000). See also recent updates: G. Sigl, [astro-ph/0008364]; T. J. Weiler, [astro-ph/0103023].

[12] M. Hillas, Nature 395, 15 (1998).

[13] G. R. Farrar, and P. L. Biermann, Phys. Rev. Lett. 81, 3579 (1998).

[14] G. Sigl, D. F. Torres, L. A. Anchordoqui and G. E. Romero, Phys. Rev. D 63, 081302(R) (2001) [astro-ph/0008363]; A. Virmani, S. Bhattacharya, P. Jain, S. Razzaque, J. P. Ralston, and D. W. McKay, [astro-ph/0010235].

[15] F. W. Stecker, Phys. Rev. D 180, 1264 (1969).

[16] S. Karakula and W. Tkaczyk, Astropart. Phys. 1, 229 (1993).

[17] L. A. Anchordoqui, Ph.D. thesis, [astro-ph/9812445]. See in particular Fig. 2.
Fig. 1. Variation of $\frac{dR}{d\epsilon}$ with respect to the background photon energy for different values of $q$ and $\log \Gamma = 10.3$. The curve with the higher peak stands for the typical Planckian spectrum, whereas the other curve owns a deviation from Planck spectrum of $1 - q \equiv 10^{-5}$.

Fig. 2. Evolution of the mass number $A$ of the surviving fragment vs. travel time for injection energy $\log \Gamma = 10.3$. It is important to keep in mind that 1 Mpc $\equiv 1.03 \times 10^{14}$ s $\equiv 3.26 \times 10^{6}$ light years $\approx 3 \times 10^{19}$ km.