Dilaton Stabilization and Supersymmetry Breaking by Dynamical Gaugino Condensation in the Linear Multiplet Formalism of String Effective Theory

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Abstract

We study dynamical gaugino condensation in superstring effective theories using the linear multiplet representation for the dilaton superfield. An interesting necessary condition for the dilaton to be stabilized, which was first derived in generic models of static gaugino condensation, is shown to hold for generic models of dynamical gaugino condensation. We also point out that it is stringy non-perturbative effects that stabilize the dilaton and allow dynamical supersymmetry breaking via the field-theoretical non-perturbative effect of gaugino condensation. As a typical example, a toy S-dual model of a dynamical $E_8$ condensate is constructed and the dilaton is explicitly shown to be stabilized with broken supersymmetry and (fine-tuned) vanishing cosmological constant.

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1 Introduction

Constructing a realistic scheme of supersymmetry breaking is one of the big challenges to supersymmetry phenomenology. However, in the context of superstring phenomenology, there are actually more challenges. As is well known, a very powerful feature of superstring phenomenology is that all the parameters of the model are in principle dynamically determined by the vev’s of certain fields. One of these important fields is the string dilaton whose vev determines the gauge coupling constants. On the other hand, how the dilaton is stabilized is outside the reach of perturbation theory since the dilaton's potential remains flat to all order in perturbation theory according to the non-renormalization theorem. Therefore, understanding how the dilaton is stabilized (i.e., how the gauge coupling constants are determined) is of no less significance than understanding how supersymmetry is broken.

Gaugino condensation has been playing a unique role in these issues: At low energy, the strong dilaton-Yang-Mills interaction leads to gaugino condensation which not only breaks supersymmetry spontaneously but also generates a non-perturbative dilaton potential which may eventually stabilize the dilaton. In the scheme of gaugino condensation the stabilization of the dilaton and the breaking of supersymmetry are therefore unified in the sense that they are two aspects of a single non-perturbative phenomenon. Furthermore, gaugino condensation has its own important phenomenological motivations: gaugino condensation occurs in the hidden sector of a generic string model [1, 2]; it can break supersymmetry at a sufficiently small scale and induce viable soft supersymmetry breaking effects in the observable sector through gravity and/or an anomalous U(1) gauge interaction [3].

Unfortunately, this beautiful scheme of gaugino condensation has been long plagued by the infamous dilaton runaway problem [2, 4]. That is, (assuming that the tree-level Kähler potential of the dilaton is a good approximation) one generally finds that the supersymmetric vacuum with vanishing gauge coupling constant and no gaugino condensation is the only stable mini-
mum in the weak-coupling regime. (The recent observation of string dualities further implies that the strong-coupling regime is plagued by a similar runaway problem [3].) Only a few solutions to the dilaton runaway problem have been proposed. Assuming the scenario of two or more gaugino condensates, the racetrack model stabilizes the dilaton and breaks supersymmetry with a more complicated dilaton superpotential generated by multiple gaugino condensation [4]. However, stabilization of the dilaton in the racetrack model requires a delicate cancellation between the contributions from different gaugino condensates, which is not very natural. Furthermore, it has a large and negative cosmological constant when supersymmetry is broken. The other solutions generically require the assistance of an additional source of supersymmetry breaking (e.g., a constant term in the superpotential) [2, 7]. It is therefore fair to say that there is no satisfactory solution so far.

Recently, several new developments and insights of superstring phenomenology are now known to play important roles in the above issues, and it is the purpose of this paper to show how these new ingredients can eventually lead to a promising solution. One of these new ingredients is the linear multiplet formalism of superstring effective theories [8, 9]: the dilaton superfield can be described either by a chiral superfield $S$ or by a linear multiplet $L$ [10], which is known as the chiral-linear duality. Since the precise field content of the linear multiplet $L$ appears in the massless string spectrum and $\langle L \rangle$ plays the role of string loop expansion parameter, stringy information is more naturally encoded in the linear multiplet formalism rather than in the chiral formalism. As will be pointed out later, stringy effects are believed to be important in the stabilization of the dilaton and supersymmetry breaking by gaugino condensation; therefore, it is more appropriate to study these issues in the linear multiplet formalism.

The other new ingredient concerns the effective description of gaugino condensation. In the known models of gaugino condensation using the chiral superfield representation for the dilaton, the gaugino condensate has always been described by an *unconstrained* chiral superfield $U$ which corresponds
to the bound state of $W^{\alpha}W_{\alpha}$ in the underlying theory. It was pointed out recently that $U$ should be a constrained chiral superfield \cite{1,2,3,4} due to the constrained superspace geometry of the underlying Yang-Mills theory:

\begin{align*}
U &= -(\mathcal{D}_{\alpha}^{\dot{\alpha}} - 8R)V, \\
\bar{U} &= -(\mathcal{D}^{\alpha}_{\dot{\alpha}} - 8R^\dagger)V, \tag{1.1}
\end{align*}

where $V$ is an unconstrained vector superfield. Furthermore, in the linear multiplet formalism the linear multiplet $L$ and the constrained $U, \bar{U}$ nicely merge into an unconstrained vector superfield $V$, and therefore the effective Lagrangian can elegantly be described by $V$ alone.

The third new ingredient is the stringy non-perturbative effect conjectured by S.H. Shenker \cite{5}. It is further argued in \cite{4} that the Kähler potential can in principle receive significant stringy non-perturbative corrections although the superpotential cannot generically. Significant stringy non-perturbative corrections to the Kähler potential imply that the usual dilaton runaway picture is valid only in the weak-coupling regime; as pointed out in \cite{4}, these corrections may naturally stabilize the dilaton.\footnote{Choosing a specific form for possible non-perturbative corrections to the Kähler potential, \cite{6} has discussed the possibility of stabilizing the dilaton in a model of gaugino condensation using chiral superfield representation for the dilaton. However, the issue of modular anomaly cancellation was not taken into account.} However, it may seem futile discussing whether the dilaton can be stabilized or not because we do not know what these non-perturbative corrections to the Kähler potential really are. On the other hand, in the spirit of effective Lagrangian approach it is always legitimate to ask the following interesting questions without having to specify the non-perturbative corrections to the Kähler potential: What is the generic condition for the dilaton to be stabilized? Is supersymmetry broken if the dilaton is stabilized? By studying generic models of static gaugino condensation with the above three new ingredients included, an interesting necessary condition for the dilaton to be stabilized has been derived in \cite{7}. It is also shown that supersymmetry is broken
as long as the dilaton is stabilized. Furthermore, explicit models with stabilized dilaton, broken supersymmetry and (fine-tuned) vanishing vacuum energy can be constructed.

As pointed out in [12], the kinetic terms for gaugino condensate naturally arise both from field-theoretical loop corrections and from classical string corrections [18]. Therefore, in this paper we would like to address the above questions in the context of dynamical gaugino condensation. In Sect. 2, the field component Lagrangian for the generic model of dynamical gaugino condensation is constructed, and its vacuum structure is analyzed. In Sect. 3, we review the study of static gaugino condensation [17] which is essential to the study of dynamical gaugino condensation later. The role of stringy non-perturbative effects in stabilizing the dilaton is also discussed. In Sect. 4, the S-dual models of dynamical gaugino condensation are studied. We discuss how the model of static gaugino condensation is related to the model of dynamical gaugino condensation and its implications. It is shown that the necessary condition of dilaton stabilization derived from static gaugino condensation also holds for generic models of dynamical gaugino condensation.

2 Generic Model of Dynamical Gaugino Condensation

It will be shown in this section how to construct the component field Lagrangian for the generic model of dynamical gaugino condensation using the Kähler superspace formalism of supergravity [19, 20]. We consider here orbifold models with gauge groups $E_8 \otimes E_6 \otimes U(1)^2$, three untwisted (1,1) moduli $T^I$ ($I = 1, 2, 3$) [24, 22, 23], and universal modular anomaly cancellation [21] (e.g., the $Z_3$ and $Z_7$ orbifolds). The confined $E_8$ hidden sector is described by the following generic model of a single dynamical gaugino condensate $U$ with Kähler potential $K$:

$$K = \ln V + g(V, \bar{U}U) + G,$$
\[ \mathcal{L}_{\text{eff}} = \int \! d^4 \theta \ E \left\{ \left( -2 + f(V, \bar{U}U) \right) + bV \right\} + \left\{ \int \! d^4 \theta \frac{E}{R} e^{K/2} W_{VY} + \text{h.c.} \right\}, \]
\[ G = -\sum_I \ln(T^I + \bar{T}^I), \] (2.1)

where \( U = -(\mathcal{D}_\alpha \mathcal{D}^\alpha - 8 R)V, \ \bar{U} = -(\mathcal{D}^{\alpha} \mathcal{D}_\alpha - 8 \bar{R}^I)V. \) We also write \( \ln V + g(V, \bar{U}U) \equiv k(V, \bar{U}U). \) The term \( \left( -2 + f(V, \bar{U}U) \right) \) of \( \mathcal{L}_{\text{eff}} \) is the superspace integral which yields the kinetic actions for the linear multiplet, supergravity, matter, and gaugino condensate. The term \( bVG \) is the Green-Schwarz counterterm \([21]\) which cancels the full modular anomaly here. \( b = C/8\pi^2 = 2b_0/3, \) and \( C = 30 \) is the Casimir operator in the adjoint representation of \( E_8. \) \( b_0 \) is the \( E_8 \) one-loop \( \beta \)-function coefficient. \( W_{VY} \) is the quantum superpotential whose form is dictated by the underlying anomaly structure \([25]\):

\[ \int \! d^4 \theta \frac{E}{R} e^{K/2} W_{VY} = \int \! d^4 \theta \frac{E}{R} \frac{1}{8} bU \ln(e^{-K/2U/\mu^2}), \] (2.2)

where \( \mu \) is a constant left undetermined by anomaly matching. \( g(V, \bar{U}U) \) and \( f(V, \bar{U}U) \) represent the quantum corrections to the tree-level Kähler potential. As illustrated in Sect. 1, \( g(V, \bar{U}U) \) and \( f(V, \bar{U}U) \) are taken to be arbitrary but bounded here. The dynamical model (2.1) is the straightforward generalization of the static model in \([17]\) by including the \( \bar{U}U \) dependence in the Kähler potential. We can also rewrite (2.1) as a single D term:

\[ K = \ln V + g(V, \bar{U}U) + G, \]
\[ \mathcal{L}_{\text{eff}} = \int \! d^4 \theta \ E \left\{ \left( -2 + f(V, \bar{U}U) \right) + bVG + bV \ln(e^{-K/2U/\mu^2}) \right\}. \] (2.3)

Throughout this paper only the bosonic and gravitino parts of the component field Lagrangian are presented. In the following, we enumerate the definitions of the bosonic component fields:

\[ \ell = V|_{\theta = \bar{\theta} = 0}, \]
\[ \sigma_{a\bar{a}}^m B_m = \frac{1}{2} [\mathcal{D}_\alpha, \mathcal{D}_{\bar{\alpha}}] V|_{\theta = \bar{\theta} = 0} + \frac{2}{3} \ell \sigma_{a\bar{a}}^a b_a, \]
\[ u = U|_{\theta = \bar{\theta} = 0} = -(\mathcal{D}^2 - 8R)V|_{\theta = \bar{\theta} = 0}, \]
\[ \bar{u} = \bar{U}|_{\theta = \bar{\theta} = 0} = -(\mathcal{D}^2 - 8R^\dagger)V|_{\theta = \bar{\theta} = 0}, \]
\[-4F_U = \mathcal{D}^2 U|_{\theta = \bar{\theta} = 0}, \quad -4\bar{F}_U = \bar{\mathcal{D}}^2 \bar{U}|_{\theta = \bar{\theta} = 0}, \]
\[ D = \frac{1}{8} \mathcal{D}_\beta (\mathcal{D}^2 - 8R) \mathcal{D}_\beta V|_{\theta = \bar{\theta} = 0} \]
\[ = \frac{1}{8} \mathcal{D}_\beta (\mathcal{D}^2 - 8R^\dagger) \mathcal{D}_\beta V|_{\theta = \bar{\theta} = 0}, \]
\[ t^I = T^I|_{\theta = \bar{\theta} = 0}, \quad -4F_T^I = \mathcal{D}^2 T^I|_{\theta = \bar{\theta} = 0}, \]
\[ \bar{t}^I = \bar{T}^I|_{\theta = \bar{\theta} = 0}, \quad -4\bar{F}_T^I = \bar{\mathcal{D}}^2 \bar{T}^I|_{\theta = \bar{\theta} = 0}, \] (2.4)

where \( b_a = -3G_a|_{\theta = \bar{\theta} = 0}, \ M = -6R|_{\theta = \bar{\theta} = 0}, \ \bar{M} = -6R^\dagger|_{\theta = \bar{\theta} = 0} \) are the auxiliary components of the supergravity multiplet. \((F_U - \bar{F}_U)\) can be expressed as follows:
\[ (F_U - \bar{F}_U) = 4i\nabla^m B_m + u\bar{M} - \bar{u}M, \] (2.5)
and \((F_U + \bar{F}_U)\) contains the auxiliary field \( D \). We also write \( Z \equiv \bar{U}U \), and its bosonic component \( z \equiv Z|_{\theta = \bar{\theta} = 0} = \bar{u}u \).

The construction of component field Lagrangian using chiral density multiplet method [19] has been detailed in [17], and therefore only the key steps are presented here. The chiral density multiplet \( r \) and its hermitian conjugate \( \bar{r} \) for the generic model (2.1) are:

\[ r = -\frac{1}{8}(\mathcal{D}^2 - 8R) \left\{ \left( -2 + f(V, \bar{U}U) \right) + bV G + bV \ln(e^{-K\bar{U}U}/\mu^6) \right\} , \]
\[ \bar{r} = -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger) \left\{ \left( -2 + f(V, \bar{U}U) \right) + bV G + bV \ln(e^{-K\bar{U}U}/\mu^6) \right\} , \] (2.6)

and the component field Lagrangian \( \mathcal{L}_{eff} \) is:
\[ \frac{1}{e} \mathcal{L}_{eff} = -\frac{1}{4} \mathcal{D}^2 r|_{\theta = \bar{\theta} = 0} + \frac{i}{2} (\bar{\psi}_m \sigma^m)^{\alpha} \mathcal{D}_\alpha r|_{\theta = \bar{\theta} = 0} \]
\[ - (\bar{\psi}_m \sigma^{mn} \bar{\psi}_n + \bar{M}) r|_{\theta = \bar{\theta} = 0} + \text{h.c.} \] (2.7)

We choose to write out explicitly the vectorial part of the Kähler connection \( A_m \) and keep only the Lorentz connection in the definition of the covariant
derivatives when we present component expressions. The $A_m|_{\theta=\bar{\theta}=0}$ for the
generic model (2.1) is:

$$A_m|_{\theta=\bar{\theta}=0} = -\frac{i}{4\ell} \left(1 + \ell g_\ell\right) B_m + \frac{i}{6} \left[ \frac{1 + \ell g_\ell}{1 - zg_\ell} - 3 \right] e_m a b a$$
$$+ \frac{1}{4(1 - zg_\ell)} \sum_I \left( \nabla_m \vec{t}^I - \nabla_m t^I \right)$$
$$- \frac{zg_\ell}{4(1 - zg_\ell)} \nabla_m \ln \left( \frac{\bar{u}}{u} \right).$$

(2.8)

The following are the simplified notations for partial derivatives of $g$:

$$g_\ell \equiv \frac{\partial g(\ell, z)}{\partial \ell}, \quad g_z \equiv \frac{\partial g(\ell, z)}{\partial z},$$

(2.9)

and similarly for other functions.

In the computation of (2.7), we need to decompose the lowest components of the following six superfields: $X_\alpha$, $\bar{X}^\dot{\alpha}$, $\mathcal{D}_\alpha R$, $\mathcal{D}^\dot{\alpha}R^\dagger$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_\dot{\alpha} \bar{X}^\dot{\alpha})$ and $(\mathcal{D}^2R + \bar{\mathcal{D}}^2R^\dagger)$ into component fields, where

$$X_\alpha = -\frac{1}{8}(\mathcal{D}_\alpha \mathcal{D}^\dot{\alpha} - 8R)\mathcal{D}_\alpha K,$$

$$\bar{X}^\dot{\alpha} = -\frac{1}{8}(\mathcal{D}^\alpha \mathcal{D}_\dot{\alpha} - 8R^\dagger)\mathcal{D}^\dot{\alpha} K,$$

$$(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_\dot{\alpha} \bar{X}^\dot{\alpha}) = -\frac{1}{8} \mathcal{D}^2 \mathcal{D}^2 K - \frac{1}{8} \mathcal{D}_\alpha \mathcal{D}^\alpha \mathcal{D}^2 K - \mathcal{D}^\alpha \mathcal{D}_\dot{\alpha} \mathcal{D}^\dot{\alpha} K$$
$$+ G^{\alpha\dot{\alpha}} [\mathcal{D}_\alpha, \mathcal{D}_\dot{\alpha}] K + 2R^\dagger \bar{\mathcal{D}}^2 K + 2R \mathcal{D}^2 K$$
$$+ (\mathcal{D}_\alpha \mathcal{D}^\alpha - 2\mathcal{D}_\alpha R^\dagger) \mathcal{D}^\dot{\alpha} K$$
$$+ (\mathcal{D}^\alpha \mathcal{D}_\dot{\alpha} - 2\mathcal{D}_\dot{\alpha} R) \mathcal{D}^\alpha K. \quad (2.10)$$

This is done by solving the following six algebraic equations:

$$\left( 1 + V \frac{\partial g}{\partial \ell} \right) \mathcal{D}_\alpha R + \left( 1 - Z \frac{\partial g}{\partial Z} \right) X_\alpha = \Xi_\alpha, \quad (2.11)$$
$$3\mathcal{D}_\alpha R + X_\alpha = -2(\sigma^{cb})_{\alpha\beta} T_{cb}. \quad (2.12)$$

$$\left( 1 + V \frac{\partial g}{\partial \ell} \right) \mathcal{D}^\dot{\alpha}R^\dagger + \left( 1 - Z \frac{\partial g}{\partial Z} \right) \bar{X}^\dot{\alpha} = \bar{\Xi}^\dot{\alpha}, \quad (2.13)$$
$$3\mathcal{D}^\dot{\alpha}R^\dagger + \bar{X}^\dot{\alpha} = -2(\bar{\sigma}^{cb})_{\dot{\alpha}\dot{\beta}} T_{cb}. \quad (2.14)$$

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\[
\left(1 + V \frac{\partial g}{\partial V}\right)(D^2R + \bar{D}^2R^\dagger) + \left(1 - Z \frac{\partial g}{\partial Z}\right)(D^\alpha X_\alpha + \bar{D}_\alpha \bar{X}^{\dot{\alpha}}) = \Delta, \quad (2.15)
\]
\[
3(D^2R + \bar{D}^2R^\dagger) + (D^\alpha X_\alpha + \bar{D}_\alpha \bar{X}^{\dot{\alpha}}) = -2R_{\beta a}^\beta + 12G^aG_a + 96RR^\dagger. \quad (2.16)
\]

The identities (2.12), (2.14) and (2.16) arise from the structure of Kähler superspace. The identities (2.11), (2.13) and (2.15) arise from the definitions of \(X_\alpha, \bar{X}^{\dot{\alpha}},\) and \((D^\alpha X_\alpha + \bar{D}_\alpha \bar{X}^{\dot{\alpha}}).\) The computation of (2.10) defines the contents of \(\Xi_\alpha, \bar{\Xi}^{\dot{\alpha}}\) and \(\Delta.\) Eqs.(2.8-16) describe the key steps in the computations of (2.7). In the following subsections, several important issues of this construction will be discussed.

### 2.1 Canonical Einstein Term

In order to have the correctly normalized Einstein term in \(L_{\text{eff}}\), an appropriate constraint should be imposed on the generic model (2.1). Therefore, it is shown below how to compute the Einstein term for (2.1). According to (2.7), the following are those terms in \(L_{\text{eff}}\) that will contribute to the Einstein term:

\[
\frac{1}{e}L_{\text{eff}} \ni \left[ 2 - f + \ell f_\ell - b \ell(1 + \ell g_\ell) \right] (D^2R + \bar{D}^2R^\dagger) \bigg|_{\theta = \bar{\theta} = 0} + \frac{1}{32} [zf_z + b \ell(1 - zg_z)] \left( \frac{1}{u}D^2\bar{U} + \frac{1}{\bar{u}}\bar{D}^2U \right) \bigg|_{\theta = \bar{\theta} = 0}. \quad (2.17)
\]

Note that the terms \(D^2\bar{U}\) and \(\bar{D}^2U\) are related to \(D^\alpha X_\alpha\) and \(D_\alpha \bar{X}^{\dot{\alpha}}\) through the following identities:

\[
D^2\bar{U} = 16D^\alpha D_a \bar{U} + 64iG^\alpha D_a \bar{U} - 48U G^a G_a + 48iU D^a G_a - 8\bar{U} D^\alpha X_\alpha + 16R^\dagger \bar{D}^2 \bar{U} + 8(D^\alpha G_{\alpha \ddot{\alpha}})(D^\ddot{\alpha} \bar{U}).
\]

\[
\bar{D}^2U = 16D^\alpha D_a U - 64iG^a D_a U - 48UG^\alpha G_a - 48iUD^\alpha G_a - 8U D_\dot{\dot{\alpha}} \bar{X}^{\dot{\alpha}} + 16R \bar{D}^2 U - 8(D^\dot{\dot{\alpha}} G_{\dot{\alpha} \ddot{\alpha}})(D^\ddot{\alpha} U). \quad (2.18)
\]
The contributions of \( (\mathcal{D}^2 R + \mathcal{D}^2 R^\dagger)|_{\theta=\bar{\theta}=0} \) and \( (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\bar{\alpha}} \bar{X}^{\bar{\alpha}})|_{\theta=\bar{\theta}=0} \) to the Einstein term are obtained by solving (2.15-16):

\[
(\mathcal{D}^2 R + \mathcal{D}^2 R^\dagger)|_{\theta=\bar{\theta}=0} \ni -\frac{2(1 - zg_\ell)}{(2 - \ell g_\ell - 3zg_s)} R_{ba}|_{\theta=\bar{\theta}=0}.
\]

\[
(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\bar{\alpha}} \bar{X}^{\bar{\alpha}})|_{\theta=\bar{\theta}=0} \ni +\frac{2(1 + \ell g_\ell)}{(2 - \ell g_\ell - 3zg_s)} R_{ba}|_{\theta=\bar{\theta}=0}. \tag{2.19}
\]

By combining (2.17-19), it is straightforward to show that the Einstein term in \( \mathcal{L}_{\text{eff}} \) is correctly normalized if and only if the following constraint is imposed:

\[
(1 + zf_\ell)(1 + \ell g_\ell) = (1 - zg_\ell)(1 - \ell f_\ell + f), \tag{2.20}
\]

which is a first-order partial differential equation. From now on, the study of the generic model (2.1) always assumes the constraint (2.20). (2.20) will be useful in simplifying the expression of \( \mathcal{L}_{\text{eff}} \), and it turns out to be convenient to define \( h \) as follows:

\[
h \equiv \frac{(1 + zf_\ell)}{(1 - zg_\ell)},
\]

\[
= \frac{(1 - \ell f_\ell + f)}{(1 + \ell g_\ell)}. \tag{2.21}
\]

Furthermore, the partial derivatives of \( h \) satisfy the following consistency condition:

\[
(h - \ell h_\ell)(zg_\ell - 1) + zh_\ell(1 + \ell g_\ell) + 1 = 0. \tag{2.22}
\]

Eqs. (2.21-22) will also be very useful in simplifying the expression of \( \mathcal{L}_{\text{eff}} \). Notice that \( h = 1 \) for generic models of static gaugino condensation, and (2.20) is reduced to an ordinary differential equation [17]. We will show in Sect. 4.2 how to construct physically interesting solutions for the partial differential equation (2.20).
2.2 Component Field Lagrangian with Auxiliary Fields

Once the issue of canonical Einstein term is settled, it is straightforward to compute $\mathcal{L}_{\text{eff}}$ according to (2.8-16). The rest of it is standard and will not be detailed here. In the following, we present the component field expression of $\mathcal{L}_{\text{eff}}$ as the sum of the bosonic Lagrangian $\mathcal{L}_B$ and the gravitino Lagrangian $\mathcal{L}_{\tilde{G}}$.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_B + \mathcal{L}_{\tilde{G}}. \quad (2.23)$$

\[
\frac{1}{e}\mathcal{L}_B = -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(h - \ell h_e)(1 + \ell g_s)\nabla^m \ell \nabla_m \ell \\
+ \frac{1}{2\ell}zh_z(1 + \ell g_s)\nabla^m \ln(\bar{u}u) \nabla_m \ell \\
+ \frac{u}{4\bar{u}}h_z\cdot g_z \frac{(2 - zg_s)}{(1 - zg_s)}\nabla^m \bar{u} \nabla_m \bar{u} \\
- \frac{1}{2}zh_z \left[ \frac{(2 - zg_s)}{(1 - zg_s)} - zg_s \right] \nabla^m \bar{u} \nabla_m u \\
+ \frac{\bar{u}}{4u}h_z\cdot g_z \frac{(2 - zg_s)}{(1 - zg_s)}\nabla^m u \nabla_m u \\
- \frac{zh_z}{2(1 - zg_s)} \sum_i \frac{1}{(t^i + \bar{t}^i)} \left( \nabla^m \bar{t}^i - \nabla^m t^i \right) \nabla_m \ln \left( \frac{\bar{u}}{u} \right) \\
+ \frac{zh_z}{4(1 - zg_s)} \sum_{i,j} \frac{1}{(t^i + \bar{t}^i)(t^j + \bar{t}^j)} \nabla^m \bar{t}^i \nabla_m \bar{t}^j \\
- \frac{1}{2} \sum_{i,j} \left[ 2(h + b\ell)\delta_{ij} + \frac{zh_z}{(1 - zg_s)} \right] \nabla^m \bar{t}^i \nabla_m t^j \\
+ \frac{zh_z}{4(1 - zg_s)} \sum_{i,j} \frac{1}{(t^i + \bar{t}^i)(t^j + \bar{t}^j)} \nabla^m t^i \nabla_m t^j \\
+ \frac{(2 - \ell g_e - 3zg_s)}{9(1 - zg_s)} b^a b_a \\
+ \frac{(1 + \ell g_s)}{4\ell^2(1 - zg_s)} B^m B_m
\]
\[
\frac{1}{e} \mathcal{L}_G = \frac{1}{2} e^{mnpq} (\tilde{\psi}_m \sigma_n \nabla_p \psi_q - \psi_m \sigma_n \nabla_p \tilde{\psi}_q )
\]
\[
- \frac{1}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u}/\mu^6) \right] \bar{u} (\psi_m \sigma^m \psi_n )
\]
\[
- \frac{1}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u}/\mu^6) \right] u (\tilde{\psi}_m \sigma^m \tilde{\psi}_n )
\]
\[
- \frac{1}{4} (h + b\ell) \sum_{I} \frac{1}{(t^I + \bar{t}^I)} e^{mnpq} (\tilde{\psi}_m \sigma_n \psi_p)(\nabla_q t^I - \nabla_q \bar{t}^I )
\]
\[
+ \frac{i}{4\ell} (h + b\ell)(1 + \ell g_{\ell}) (\eta^{mnpq}_w - \eta^{mqnp}_w) (\tilde{\psi}_m \sigma_n \psi_p) \nabla_q \ell
\]
\[-\frac{i}{4} \left[(1 - zg)(h + b \ell) - 1\right] \left(\eta^{mn} \eta^{pq} - \eta^{mq} \eta^{np}\right) (\bar{\psi}_m \sigma_n \psi_p) \nabla_q \ln(\bar{u}u)\]
\[+ \frac{1}{4} (h - 1 + b \ell) \epsilon^{mnpq} (\bar{\psi}_m \sigma_n \psi_p) \nabla_q \ln\left(\frac{\bar{u}}{u}\right).\]  

(2.25)

The bosonic Lagrangian $\mathcal{L}_B$ contains usual auxiliary fields and the vector field $B_m$ which is dual to an axion. The details of this duality and the structure of $\mathcal{L}_B$ will be discussed in the following subsections. The gravitino Lagrangian $\mathcal{L}_{\tilde{G}}$ is in its simplest form. An important physical quantity in $\mathcal{L}_{\tilde{G}}$ is the gravitino mass $m_{\tilde{G}}$ which is the natural order parameter measuring supersymmetry breaking. The expression of $m_{\tilde{G}}$ follows directly from $\mathcal{L}_{\tilde{G}}$:

\[m_{\tilde{G}} = \left\langle \frac{1}{8\ell} \left[ f + 1 + b \ell \ln(e^{-k} \bar{u}u/\mu^6) \right] u \right\rangle.\]  

(2.26)

Notice that $m_{\tilde{G}}$ contains no moduli $T^I$ dependence due to the Green-Schwarz cancellation mechanism in the linear multiplet formalism of string models with universal modular anomaly cancellation.

### 2.3 Duality Transformation of $B_m$

As pointed out in [11, 14], the constraint (1.1) allows us to interpret the degrees of freedom of $U$ as those of a 3-form supermultiplet, and the vector field $B_m$ is dual to a 3-form $\Gamma^{mpq}$. Since 3-form is dual to 0-form in four dimensions, $B_m$ is also dual to a pseudoscalar $a$. In this subsection, we show explicitly how to rewrite the $B_m$ part of $\mathcal{L}_B$ in terms of the dual description using $a$. According to (2.24), the $B_m$ terms in $\mathcal{L}_B$ are:

\[
\frac{1}{e} \mathcal{L}_B \ni + \frac{(1 + \ell g_s)}{4\ell^2 (1 - zg_z)} B_m B_m
\]
\[+ \frac{i}{2\ell} \left[h + b \ell - \frac{1}{(1 - zg_z)}\right] B_m \nabla_m \ln\left(\frac{\bar{u}}{u}\right)
\]
\[- \frac{i}{2\ell} \left[h + b \ell - \frac{1}{(1 - zg_z)}\right] \sum_l \left(\nabla^m \bar{t}^l - \nabla^m t^l\right) B_m
\]
\[- 2ih_z \left[1 - zg_z - \frac{1}{3}(1 + \ell g_z)\right] \left(u M - \bar{u} M\right) \nabla^m B_m
\]
They are described by the following generic Lagrangian of $B_m$:

$$\frac{1}{e} \mathcal{L}_{B_m} = \alpha B_m B_m + \beta \nabla B_m + \zeta B_m + \tau (\nabla B_m)^2. \quad (2.28)$$

To find the dual description of $\mathcal{L}_{B_m}$, consider the following Lagrangian $\mathcal{L}_{Dual}$.

$$\frac{1}{e} \mathcal{L}_{Dual} = \alpha B_m B_m + \beta \nabla B_m + \zeta B_m + a \nabla B_m - \frac{1}{4\tau} a^2. \quad (2.29)$$

In $\mathcal{L}_{Dual}$, the auxiliary field $a$ acts like a Lagrangian multiplier, and its equation of motion is:

$$a = 2\tau \nabla B_m. \quad (2.30)$$

Therefore, $\mathcal{L}_{B_m}$ follows directly from $\mathcal{L}_{Dual}$ using (2.30). On the other hand, we can treat the $B_m$ in $\mathcal{L}_{Dual}$ as auxiliary, and write down the equation of motion for $B_m$ as follows:

$$B_m = \frac{1}{2\alpha} \left( \nabla B_m + \nabla a - \zeta B_m \right). \quad (2.31)$$

Eliminating $B_m$ from $\mathcal{L}_{Dual}$ through (2.31) and then performing a field redefinition $a \Rightarrow a - \beta$, we obtain the Lagrangian $\mathcal{L}_a$ of $a$:

$$\frac{1}{e} \mathcal{L}_a = -\frac{1}{4\alpha} (\nabla a - \zeta) (\nabla a - \zeta) - \frac{1}{4\tau} (a - \beta)^2. \quad (2.32)$$

Therefore, $\mathcal{L}_a$ is the dual description of $\mathcal{L}_{B_m}$ in terms of $a$ which is interpreted as an axion. Notice that the generation of the axion mass in $\mathcal{L}_a$ corresponds to the appearance of $(\nabla B_m)^2$ in the dual description. In comparison with the model of static gaugino condensation [17], the model of dynamical gaugino condensation has one more axionic degree of freedom $a$ that is massive. As will be shown in Sect. 4.1, after integrating out the massive axion $a$, the axionic contents of the dynamical model are indeed identical to those of the static model. Therefore, this is consistent with the fact pointed out in [11, 13] that the $(\nabla B_m)^2$ term vanishes in models of static gaugino condensation.
(i.e., $h_z = 0$ in (2.27)), and therefore the corresponding axionic degree of freedom is massless.

According to (2.27-28) and (2.32), the $\mathcal{L}_{\text{eff}}$ defined by (2.23-25) is rewritten in the dual description as follows:

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\tilde{G}},
$$

where $\mathcal{L}_{\text{kin}}$ and $\mathcal{L}_{\text{pot}}$ refer to the kinetic part and the non-kinetic part of the bosonic Lagrangian respectively. $\mathcal{L}_{\tilde{G}}$ is defined by (2.25).

\[
\frac{1}{c} \mathcal{L}_{\text{kin}} = -\frac{1}{2} \zeta - \frac{1}{4\ell^2} (h - \ell h_c)(1 + \ell g_z) \nabla^m \nabla_m \ell
\]

\[
- \frac{(1 - z g_z)}{(1 + \ell g_z)} \ell^2 \nabla^m a \nabla_m \ell + \frac{1}{2\ell} z h_z (1 + \ell g_z) \nabla^m \ln(\bar{u} u) \nabla_m \ell
\]

\[
+ i \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right] \ell \nabla^m a \nabla_m \ln\left(\frac{\bar{u}}{u}\right)
\]

\[
- i \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right] \sum_I \frac{(\nabla^m I - \nabla^m \bar{I})}{(I + I)} \ell \nabla_m \ell
\]

\[
+ \frac{1}{4} \left\{ \frac{z h_z \cdot z g_z (2 - z g_z)}{(1 - z g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \frac{1}{\bar{u} u} \nabla^m \bar{u} \nabla_m u
\]

\[
- \frac{1}{2} \left\{ \frac{z h_z \cdot \left( \frac{2 - z g_z}{(1 - z g_z)} - z g_z \right)}{(1 + \ell g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \frac{1}{\bar{u} u} \nabla^m u \nabla_m u
\]

\[
+ \frac{1}{4} \left\{ \frac{z h_z \cdot z g_z (2 - z g_z)}{(1 - z g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \frac{1}{u^2} \nabla^m u \nabla_m u
\]

\[
- \frac{1}{2} \left\{ \frac{z h_z \cdot \left( \frac{2 - z g_z}{(1 - z g_z)} - z g_z \right)}{(1 + \ell g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \sum_I \frac{(\nabla^m I - \nabla^m \bar{I})}{(I + I)} \nabla_m \ln\left(\frac{\bar{u}}{u}\right)
\]

\[
+ \frac{1}{4} \left\{ \frac{z h_z \cdot \left( \frac{2 - z g_z}{(1 - z g_z)} - z g_z \right)}{(1 + \ell g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \sum_{I,J} \frac{\nabla^m I \nabla_m \bar{I}}{(I + I)(I + I)} \frac{\nabla^m \bar{I} \nabla_m I}{(I + I)(I + I)}
\]

\[
- \frac{1}{2} \sum_{I,J} \left\{ \frac{2(h + b \ell) \delta_{IJ} + \frac{z h_z}{(1 - z g_z)}}{(1 + \ell g_z)} + \frac{(1 - z g_z)}{(1 + \ell g_z)} \left[ h + b \ell - \frac{1}{(1 - z g_z)} \right]^2 \right\} \frac{\nabla^m I \nabla_m I}{(I + I)(I + I)} \frac{\nabla^m \bar{I} \nabla_m \bar{I}}{(I + I)(I + I)}
\]

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\[ + \frac{1}{4} \left\{ \frac{zh_s}{(1 - zg_s)} + \frac{zh_s}{(1 + zg_s)} \left[ h + b \ell - \frac{1}{(1 - zg_s)} \right]^2 \right\} \sum_{I,J} \frac{\nabla^m t^I \nabla^n t^J}{(t^I + t^J)(t^J + t^I)}. \] (2.34)

\[
\frac{1}{e} \mathcal{L}_{\text{pot}} = \frac{h_s(1 + \ell g_s)^2}{36(1 - zg_s)}(u\bar{M} - \bar{u}M)^2
- \frac{1}{9} \left[ 3 + (\ell h_\psi - h)(1 + g_\psi) \right] \bar{M}M
- \frac{1}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k}\bar{u}u/\mu^6) \right] \bar{M}M
- \frac{i}{4} \left[ 1 - \frac{(1 + \ell g_s)}{3(1 - zg_s)} \right] a(u\bar{M} - \bar{u}M)
+ \frac{1}{4} h_s(1 - zg_s)(F_U + \bar{F}_U)^2
+ \left\{ \frac{1}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k}\bar{u}u/\mu^6) \right]
+ \frac{1}{4}(\ell h_\psi + b\ell)(1 - zg_s)
- \frac{1}{6} h_s(1 + \ell g_s)(u\bar{M} + \bar{u}M) \right\} (F_U + \bar{F}_U)
+ (h + b\ell) \sum_I \frac{1}{(t^I + \bar{t}^I)^2} \bar{F}^I_I F^I_I
- \frac{1}{16\ell^2}(\ell h_\psi + h + 2b\ell)(1 + \ell g_s)\bar{u}u
- \frac{\bar{u}u}{16zh_s(1 - zg_s)} a^2. \] (2.35)

The $b^a b_a$ term has been eliminated by its equation of motion, $b^a = 0$, and $\mathcal{L}_{\text{kin}}$ is in its simplest form. Note that the kinetic terms of those axionic degrees of freedom $a$, $i \ln(\bar{u}/u)$ and $i(\bar{t}^I - t^I)$ are more complicated, which essentially reflects the non-trivial constraint (1.1) satisfied by $U$ and $\bar{U}$. An important issue is the structure of $\mathcal{L}_{\text{pot}}$, and it will be discussed in the next subsection.
2.4 Scalar Potential

It is straightforward to solve the equations of motion for the auxiliary fields \( b^a \), \( F^I_T \), \( \bar{F}^I_{\bar{T}} \), \( M \), \( \bar{M} \) and \((F_U + \bar{F}_{\bar{U}})\) respectively as follows:

\[
\begin{align*}
b^a &= 0, \\
F^I_T &= 0, \quad \bar{F}^I_{\bar{T}} = 0, \\
M &= -\frac{3}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right] u - \frac{3iu}{4} a, \\
\bar{M} &= -\frac{3}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right] \bar{u} + \frac{3i\bar{u}}{4} a, \\
(F_U + \bar{F}_{\bar{U}}) &= \frac{\ell h - h}{4zh_z} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right] \frac{\bar{u}u}{\ell} \\
&\quad - \frac{(\ell h + b\ell)}{2zh_z} \frac{\bar{u}u}{\ell}. \quad (2.36)
\end{align*}
\]

Note that \( \langle |M| \rangle = 3m_{\tilde{G}} \) because \( \langle a \rangle = 0 \) always. To obtain the scalar potential, the auxiliary fields are eliminated from \( \mathcal{L}_{\text{eff}} \) defined by (2.33), and \( \mathcal{L}_{\text{eff}} \) is then rewritten as follows:

\[
\frac{1}{e} \mathcal{L}_{\text{eff}} = \frac{1}{e} \mathcal{L}_{\text{kin}} - V_{\text{pot}} + \frac{1}{e} \mathcal{L}_{\tilde{G}}, \quad (2.37)
\]

where \( V_{\text{pot}} \) is the scalar potential. \( \mathcal{L}_{\text{kin}} \) and \( \mathcal{L}_{\tilde{G}} \) are defined by (2.34) and (2.25) respectively.

\[
V_{\text{pot}} = \frac{1}{16} (\ell h + h + 2b\ell)(1 + \ell g_z) \frac{\bar{u}u}{\ell^2} \\
+ \frac{1}{64zh_z(1 - zg_z)} \left\{ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right\}^2 \frac{\bar{u}u}{\ell^2} \\
- \frac{(2 - \ell g_z - 3zg_z)}{64(1 - zg_z)} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right]^2 \frac{\bar{u}u}{\ell^2} \\
+ \frac{(h - \ell h_z - 3zh_z)\bar{u}u}{16zh_z} a^2. \quad (2.38)
\]
Several interesting aspects of $V_{pot}$ can be uncovered. First, there is always a trivial vacuum with $\langle V_{pot} \rangle = 0$ in the specific weak-coupling limit defined as follows:

$$\ell \to 0, \quad z \to \frac{1}{\ell^2} e^{-1/b\ell} \to 0, \quad \text{and} \quad g(\ell, z), \ f(\ell, z) \to 0. \quad (2.39)$$

Note that quantum corrections to the Kähler potential, $g$ and $f$, should vanish in this limit. As expected, this is consistent with the well-known runaway behavior of the dilaton near the weak-coupling limit.

To proceed further, in the following of this subsection we only study $V_{pot}$ in the $z \ll 1$ regime. Since a physically interesting model of dynamical gaugino condensation should predict a small scale of condensation (i.e., $\langle z \rangle \ll 1$), there is no loss of generality in this choice. Note that in the $z \ll 1$ regime we have $h \approx 1, \ \ell h_z \approx 0, \ z h_z \approx 0 \text{ and } z g_z \approx 0$ up to small corrections that depend on $z$. The structure of $V_{pot}$ can be analyzed as follows: The only axion-dependent term in $V_{pot}$ is the effective axion mass term, the last term in $V_{pot}$. In order to avoid a tachyonic axion, the sign of the effective axion mass term must be positive. Therefore, the absence of a tachyonic axion requires $zh_z > 0$, which is the first piece of information about the $\bar{U}U$-dependence of the dynamical model. Furthermore, $\langle a \rangle = 0$ always, and therefore the last term in $V_{pot}$ is of no significance in discussing the vacuum structure. Because of $zh_z > 0$, the second term in $V_{pot}$ is always positive. The signs of the first term and the third term in $V_{pot}$ remain undetermined in general; however, near the weak-coupling limit the first term is positive and the third term is negative (which is expected because the third term is the contribution of auxiliary fields $M$ and $\bar{M}$). Notice that the second term in $V_{pot}$ contains a factor $1/zh_z$ ($1/zh_z \gg 1$), and therefore it is the dominant contribution to $V_{pot}$ except near the path $\gamma$ defined by

$$\{ f + 1 + b\ell \ln(e^{-k\bar{u}/\mu^6}) + 2(\ell h_z + b\ell)(1 - z g_z) \} = 0.$$ 

Hence, the vacuum always sits close to the path $\gamma$. This observation will be essential to the following discussion of vacuum structure.

The second piece of information about the $\bar{U}U$-dependence of the dy-
natical model can be obtained as follows. For \( 0 < \ell < \infty \), the first term and the third term in \( V_{pot} \) vanish in the limit \( z \to 0 \) generically. If \( h_z \) has a pole at \( z = 0 \), then the second term in \( V_{pot} \) also vanishes for \( z \to 0 \) and \( 0 < \ell < \infty \). Therefore, for those dynamical models whose \( h_z \) has a pole at \( z = 0 \), there exists a continuous family of degenerate vacua (parametrized by \( \langle \ell \rangle \)) with \( \langle z \rangle = 0 \) (no gaugino condensation), \( m_{\tilde{G}} = 0 \) (unbroken supersymmetry) and \( \langle V_{pot} \rangle = 0 \). In other words, in the vicinity of \( z = 0 \) those models always exhibit runaway of \( z \) toward the degenerate vacua at \( z = 0 \) which do not have the desired physical features; whether those models may possess other non-trivial vacuum or not is outside the scope of this simple analysis.

On the other hand, the dynamical models whose \( h_z \) has no pole at \( z = 0 \) are more interesting. If \( h_z \) has no pole at \( z = 0 \), then \( V_{pot} \to \infty \) for \( z \to 0 \) and \( 0 < \ell < \infty \). Therefore, for dynamical models whose \( h_z \) has no pole at \( z = 0 \), there is no runaway of \( z \) toward \( z = 0 \) except for the weak-coupling limit (2.39). Furthermore, it implies that gauginos condense \( (\langle z \rangle \neq 0) \) if the dilaton is stabilized \( (0 < \langle \ell \rangle < \infty) \). Based on the above observation, it can actually be shown that supersymmetry is broken \( (m_{\tilde{G}} \neq 0) \) and gauginos condense \( (\langle z \rangle \neq 0) \) if the dilaton is stabilized: As pointed out before, the second term in \( V_{pot} \) is generically the dominant contribution. In the following, the second term is rewritten in a more instructive form:

\[
V_{pot} \ni + \frac{1}{zh_z(1-zg_z)} \left\{ M_{\tilde{G}} + \frac{1}{4\ell}(\ell h_\ell + b\ell)(1-zg_z)|u| \right\}^2 , \quad (2.40)
\]

where

\[
M_{\tilde{G}} \equiv \frac{1}{8\ell} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) \right]|u| . \quad (2.41)
\]

The gravitino mass is related to \( M_{\tilde{G}} \) by \( m_{\tilde{G}} = \langle |M_{\tilde{G}}| \rangle \). If \( 0 < \langle \ell \rangle < \infty \), then \( \langle z \rangle \neq 0 \) and the second term should vanish at the vacuum (up to small corrections of order \( \langle z \rangle \)). That is, \( m_{\tilde{G}} = \langle |M_{\tilde{G}}| \rangle \approx \langle \frac{1}{4\ell}(\ell h_\ell + b\ell)(1-zg_z)|u| \rangle \approx \frac{1}{4}b(|u|) \neq 0 \) (up to small corrections of order \( \langle \bar{u}u \rangle \)). Therefore, for dynamical models whose \( h_z \) has no pole at \( z = 0 \), \textit{supersymmetry is broken and gauginos condense if the dilaton is stabilized}. The same conclusion has also
been established in the study of static gaugino condensation \cite{17}, which will be briefly reviewed in the next section.

As pointed out before, kinetic terms of the gaugino condensate $U$ naturally arise from field-theoretical loop corrections as well as from classical string corrections. As will be discussed in Sect. 4 these kinetic terms are S-duality invariant \cite{27} and correspond to corrections $\bar{U}U/V^2$, $(\bar{U}U/V^2)^2$, \ldots to the Kähler potential. This interesting class of S-dual dynamical models obviously belongs to the dynamical models whose $h_z$ has no pole at $z = 0$, and therefore it has the nice features established in the previous paragraph. In Sect. 4, S-dual dynamical models as well as the issue of dilaton stabilization will be studied in detail.

### 3 Review of Static Gaugino Condensation

Those features of static condensation \cite{14} which are essential to the study of S-dual dynamical models in Sect. 4 are briefly reviewed here. Considering the same string models as those in Sect. 2, we write the generic model of a static $E_8$ gaugino condensate as follows:

$$
K = \ln V + g(V) + G,
$$
$$
\mathcal{L}_{eff} = \int d^4\theta E \left\{ (-2 + f(V)) + bVG + bV \ln(e^{-K} \bar{U}U/\mu^6) \right\},
$$
$$
V \frac{dg(V)}{dV} = -V \frac{df(V)}{dV} + f,
$$
$$
g(V = 0) = 0 \ \text{and} \ \ f(V = 0) = 0.
$$

The Kähler potential depend only on $V$, and the condensate $U$ is therefore static. $g(V)$ and $f(V)$ represent quantum corrections to the Kähler potential. (3.2) guarantees the correct normalization of the Einstein term. The boundary condition (3.3) in the weak-coupling limit is fixed by the tree-level Kähler potential. Unlike the partial differential equation (2.20), (3.2) is an ordinary differential equation, and therefore $g(V)$ is unambiguously related to
Two important physical quantities of the static model are the gaugino condensate and the gravitino mass:

\[
\bar{u}u = \frac{1}{e^2} \ell \mu^6 e^{g-(f+1)/b\ell}. \tag{3.4}
\]

\[
m_{\tilde{g}} = \frac{1}{4} b \langle |u| \rangle. \tag{3.5}
\]

They imply that \textit{supersymmetry is broken and gauginos condense if the dilaton is stabilized}. These three issues are unified elegantly. Furthermore, supersymmetry is broken in the dilaton direction rather than in the direction of modulus $T^I$. The generic expression of scalar potential, which depends only on $\ell$, is:

\[
V_{\text{pot}} = \frac{1}{16 e^2 \ell} \left\{ (1 + f - \ell f) (1 + b \ell)^2 - 3 b^2 \ell^2 \right\} \mu^6 e^{g-(f+1)/b\ell}. \tag{3.6}
\]

In order to appreciate the significance of quantum corrections $g(V)$ and $f(V)$, a simple model with tree-level Kähler potential (i.e., (3.1) with $g(V) = f(V) = 0$) is considered, and its scalar potential $V_{\text{pot}}$ is shown in Fig. 1-A. Its $V_{\text{pot}}$ is unbounded from below in the strong-coupling limit $\ell \to \infty$, which is caused by a term of two-loop order, $-2 b^2 \ell^2$, in $V_{\text{pot}}$. This unboundedness simply reflects that (non-perturbative) quantum corrections, $g(V)$ and $f(V)$, to the Kähler potential should not be ignored, especially in the strong-coupling regime. It can be shown that the necessary and sufficient condition for $V_{\text{pot}}$ to be bounded from below is:

\[
f - \ell f_{\ell} \geq 2 \quad \text{for} \quad \ell \to \infty. \tag{3.7}
\]

(3.7) can also be interpreted as the necessary condition for the dilaton to be stabilized. Furthermore, it has been argued in detail [17] that non-perturbative quantum corrections to the Kähler potential may naturally stabilize the dilaton if (3.7) is satisfied. A nice realization of that argument is shown in Fig. 1-B, where the dilaton is stabilized and supersymmetry is broken with (fine-tuned) vanishing cosmological constant.
As the conclusion of this section, we comment on the meaning of the quantum corrections, $g(V)$ and $f(V)$, to the Kähler potential. Consider the unconfined string effective Lagrangian at the string scale $M_S$:

$$\begin{align*}
K &= \ln L + g(L) + G, \\
\mathcal{L}_{\text{eff}} &= \int d^4\theta E \left\{ (-2 + f(L)) + bLG \right\}, \\
\mathcal{W}^\alpha \mathcal{W}_\alpha &= -(\mathcal{D}_\alpha \mathcal{D}^{\dot{\alpha}} - 8R)L, \\
\mathcal{W}_\dot{\alpha} \mathcal{W}^{\dot{\alpha}} &= -(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^1)L,
\end{align*}$$

whose confined theory corresponds to (3.1). It is straightforward to compute the gauge coupling at the string scale, $g(M_S)$, defined by (3.8) as follows:

$$g^2(M_S) = \left\langle \frac{2\ell}{1 + f} \right\rangle. \tag{3.9}$$

According to (3.9), the $\hat{u}u$'s exponential dependence on $g^2(M_S)$ in (3.4) is consistent with the well-known analysis using renormalization group. (3.9) is also consistent with the interpretation of $g^2(M_S)$ in the chiral formalism of (3.8)\(^3\). In the chiral formalism, we always have $g^2(M_S) = \langle 2/(s + \bar{s}) \rangle$, where $S$ is the dilaton chiral superfield and $s = S|_{\theta = \bar{\theta} = 0}$. On the other hand, it has been shown\(^{[21]}\) that $1/(S + \bar{S})$ corresponds to $L/(1 + f)$ through a duality transformation from the linear multiplet formalism of (3.8) to the chiral formalism of (3.8). Therefore, the interpretations of $g^2(M_S)$ in both formalisms are consistent with each other. In the absence of $g(L)$ and $f(L)$, we have the usual relation $g^2(M_S) = 2\langle \ell \rangle$ at the string scale\(^{[21]}\). Therefore, the $1/(1 + f)$ factor in (3.9) is naturally interpreted as the renormalization of $g^2(M_S)$ by effects above the string scale; $g(L)$ and $f(L)$ are then interpreted as stringy corrections to the Kähler potential.

The above observation implies that the non-perturbative corrections, $g(V)$ and $f(V)$, to the Kähler potential of (3.1) should be interpreted as stringy.

\(^3\)The chiral formalism of (3.8) is obtained by performing a duality transformation\(^{[20]}\).\(^{[21]}\).
non-perturbative corrections. In this interpretation, it is actually stringy non-perturbative effects that stabilize the dilaton and allow dynamical supersymmetry breaking via the field-theoretical non-perturbative effect of gaugino condensation. Furthermore, (3.7) is now interpreted as the necessary condition for stringy non-perturbative effects to stabilize the dilaton. As we shall see in the next section, stringy non-perturbative effects also play the same crucial role in generic models of dynamical gaugino condensation.

4 S-Dual Model of Dynamical Gaugino Condensation

As has been discussed in Sect. 2.3, one of the motivations for studying models of dynamical gaugino condensation is the observation that kinetic terms of the gaugino condensate naturally arise from field-theoretical loop corrections as well as from classical string corrections. For example, the relevant field-theoretical one-loop correction has been computed using chiral formalism:

\[ \mathcal{L}_{\text{one-loop}} \ni \frac{N_G}{128\pi^2} \int d^4 \theta \left( S + \bar{S} \right)^2 \left( W^\alpha \bar{W}_{\dot{\alpha}} \right) \left( W_{\dot{\alpha}} \bar{W}^\alpha \right) \ln \Lambda^2, \quad (4.1) \]

where \( \Lambda \) is the effective cut-off and \( N_G \) is the number of gauge degrees of freedom. Therefore, the confined theory using linear multiplet formalism should contain a term which corresponds to (4.1):

\[ \mathcal{L}_{\text{eff}} \ni \int d^4 \theta \frac{\bar{U}}{V^2}, \quad (4.2) \]

as well as higher-order corrections \( \left( \frac{\bar{U}}{V^2} \right)^2, \left( \frac{\bar{U}}{V^2} \right)^3, \cdots \). These D terms are corrections to the Kähler potential, and will generate the kinetic

\[ ^4 \text{In the presence of significant stringy non-perturbative effects, (3.9) could have implications for gauge coupling unification. This is considered in the study of multi-gauginos and matter condensation.} \]
terms for gaugino condensate $U$. An interesting interpretation of these corrections is that they are S-duality invariant in the sense defined by Gaillard and Zumino \[27\]. This S-duality, which is an SL(2,R) symmetry among elementary fields, is a symmetry of the equations of motion only of the dilaton-gauge-gravity sector in the limit of vanishing gauge couplings. The implication of this S-duality for gaugino condensation has recently been studied in \[12\] using the chiral formalism.

Motivated by the above observation, we consider in this section models of dynamical gaugino condensation where the kinetic terms for gaugino condensate arise from the S-dual loop corrections defined by (4.2). More precisely, we consider the following dynamical model:

$$K = \ln V + g(V, X) + G,$$

$$\mathcal{L}_{\text{eff}} = \int d^4 \theta E \left\{ (-2 + f(V, X)) + bVG + bV \ln(e^{-K\bar{U}U/\mu^6}) \right\}, \quad (4.3)$$

$$\left(2 + X \frac{\partial f}{\partial X}\right) \left(1 - V \frac{\partial g}{\partial V}\right) = \left(2 - X \frac{\partial g}{\partial X}\right) \left(1 - f + V \frac{\partial f}{\partial V}\right). \quad (4.4)$$

For convenience, we have written the S-dual combination $(\bar{U}U)^{1/2}/V$ as a vector superfield $X$, and therefore its lowest component $x = X|_{\theta = \bar{\theta} = 0}$ is $x = (\bar{u}u)^{1/2}/\ell = \sqrt{z}/\ell$. Eq.(4.4) guarantees the correct normalization of the Einstein term. $g(V, X)$ and $f(V, X)$ satisfy the boundary condition in the weak-coupling limit defined by (2.39). We also assume that $g(V, X)$ and $f(V, X)$ have the following power-series representations\[5\] in terms of $X^2$:

$$g(V, X) \equiv g^{(0)}(V) + g^{(1)}(V) \cdot X^2 + g^{(2)}(V) \cdot X^4 + \cdots.$$  

$$f(V, X) \equiv f^{(0)}(V) + f^{(1)}(V) \cdot X^2 + f^{(2)}(V) \cdot X^4 + \cdots. \quad (4.5)$$

Furthermore, $g^{(n)}(V)$ and $f^{(n)}(V)$ ($n \geq 0$) are assumed to be arbitrary but bounded here. The interpretation of each term in (4.5) is obvious: As

\[5\] It should be noted that one can actually start with a more generic dynamical model by considering more generic $g(V, X)$ and $f(V, X)$, and the discussions of Sect. 4 remain valid.
has been discussed at the end of Sect. 3, \( g^{(0)}(V) \) and \( f^{(0)}(V) \) are stringy non-perturbative corrections to the Kähler potential. \( g^{(n)}(V) \cdot X^{2n} \) and \( f^{(n)}(V) \cdot X^{2n} \ (n \geq 1) \) are S-dual loop corrections to the Kähler potential in the presence of stringy non-perturbative effects.

It is also more convenient to use the coordinates \((\ell, x)\) instead of \((\ell, z)\) for the field configuration space. The component field expressions constructed in Sect. 2 can easily be rewritten in the new coordinates \((\ell, x)\) according to the following rules:

\[
\begin{align*}
\ell g_\ell & \to \ell g_\ell - xg_x, \\
zg_x & \to \frac{1}{2} xg_x,
\end{align*}
\]

(4.6)

where

\[
g_\ell \equiv \frac{\partial g(\ell, x)}{\partial \ell}, \quad g_x \equiv \frac{\partial g(\ell, x)}{\partial x}
\]

(4.7)

on the right-hand side of (4.6) are to be understood as partial derivatives in the coordinates \((\ell, x)\). The scalar potential of this generic model follows directly from (2.38):

\[
V_{\text{pot}} = \frac{1}{16} (1 + \ell g_\ell - xg_x) (h + \ell h_\ell - xh_x + 2b\ell) x^2 \\
+ \frac{1}{16xh_x(2 - xg_x)} \left\{ f + 1 + b\ell \ln(e^{-k\bar{u}u}/\mu^6) \right\}^2 x^2 \\
- \frac{(4 - 2\ell g_\ell - xg_x)}{64(2 - xg_x)} \left[ f + 1 + b\ell \ln(e^{-k\bar{u}u}/\mu^6) \right]^2 x^2 \\
+ \frac{(2h - 2\ell h_\ell - xh_x)\bar{u}u}{16xh_x} a^2.
\]

(4.8)

The kinetic terms also follow directly from (2.34). The absence of a tachyonic axion requires \(xh_x > 0\).

As discussed in Sect. 2.4, the S-dual dynamical model considered here belongs to the dynamical models whose \(h_z\) has no pole at \(z = 0\); part of its vacuum structure has already been analyzed in Sect. 2.4. It is concluded that supersymmetry is broken if the dilaton is stabilized. Therefore, we will focus on the issue of dilaton stabilization in the following subsection.
4.1 Low-Energy Limit of Dynamical Gaugino Condensation

Since a physically interesting model of dynamical gaugino condensation should predict a small scale of condensation (i.e., \( \langle x \rangle \ll 1 \)), it is clear from (4.8) that generally the condensate \( x \) and the axion \( a \) are much heavier than the other fields, and therefore should be integrated out. It is straightforward to integrate out \( a \) and \( x \) through their equations of motion: The equation of motion for \( a \) is

\[
a = 0.
\]

The equation of motion for \( x \) is

\[
f + 1 + b\ell \ln(e^{-k\bar{u}u/\mu^6}) + (2 - xg_x)(\ell h_x - xh_x + b\ell) = 0 + O(x^2). \tag{4.9}
\]

(4.9) can be re-written in a more instructive form:

\[
x^2 = \frac{h^6}{e^2\ell} e^{g_0(0) - (f_0 + 1)/b\ell} + O(x^4), \tag{4.10}
\]

where we have used the fact that \( g \approx g_0 \), \( f \approx f_0 \), \( h \approx 1 \), \( \ell g_i \approx \ell g_i(0) \), \( \ell f_i \approx \ell f_i(0) \), \( \ell h_i \approx 0 \), \( xg_x \approx 0 \), \( xf_x \approx 0 \) and \( xh_x \approx 0 \) up to corrections of order \( O(x^2) \). The (bosonic) effective Lagrangian, \( \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{kin}} - eV_{\text{pot}} \), of the dynamical model (4.3-5) after integrating out \( a \) and \( x \) is as follows:

\[
\frac{1}{e} \mathcal{L}_{\text{kin}} = -\frac{1}{2} \mathcal{R} - \frac{1}{4\ell^2} \left( 1 + \ell g_i(0) \right) \nabla^m \ell \nabla_m \ell
\]

\[
- (1 + b\ell) \sum_I \frac{1}{(t^I + \bar{t}^I)^2} \nabla^m \bar{t}^I \nabla_m t^I + \frac{1}{4\ell^2} \left( 1 + \ell g_i(0) \right) \tilde{B}^m \tilde{B}_m
\]

\[
+ O(x^2), \tag{4.11}
\]

where

\[
\tilde{B}_m \equiv -i \frac{b\ell^2}{(1 + \ell g_i(0))} \nabla_m \ln(\bar{u}/u)
\]

\[
+ i \frac{b\ell^2}{(1 + \ell g_i(0))} \sum_I \frac{1}{(t^I + \bar{t}^I)} (\nabla_m \bar{t}^I - \nabla_m t^I). \tag{4.12}
\]
\[ V_{\text{pot}} = \frac{1}{16e^2 \ell} \left\{ (1 + f^{(0)} - \ell f_{\ell}^{(0)}) (1 + b \ell)^2 - 3b^2 \ell^2 \right\} \mu^6 e^{g^{(0)} - (f^{(0)} + 1)/b \ell} + O(x^4). \] (4.13)

Furthermore, (4.4) leads to \( \ell g_{\ell}^{(0)} = f^{(0)} - \ell f_{\ell}^{(0)} \) to the lowest order in \( x^2 \).

In comparison with the static model of gaugino condensation \cite{17}, it is clear that the low-energy effective Lagrangian of the dynamical model are identical to the Lagrangian of the static model to the lowest order in \( x^2 \). Note that, in (4.13), the O \( (x^4) \) terms do not depend on the axionic degrees of freedom (i.e., \( i \ln(\tilde{u}/u) \) and \( i(\tilde{t}^I - t^I) \)), and therefore these axions remain massless, as expected. According to the equation of motion for \( x \), (4.10), \( x^2 \ll 1 \) actually holds for any value of \( \ell \). It implies that only the lowest-order terms (in \( x^2 \)) of (4.11) and (4.13) are important, and therefore the static model of gaugino condensation is indeed the appropriate low-energy effective description of the dynamical model. The above observation implies that the necessary and sufficient condition for \( V_{\text{pot}} \) of the dynamical model to be bounded from below is exactly the same as that of the static model:

\[ f^{(0)} - \ell f_{\ell}^{(0)} \geq 2 \quad \text{for} \quad \ell \to \infty, \] (4.14)

which depends only on stringy non-perturbative effects \( g^{(0)} \) and \( f^{(0)} \). (4.14) does not depend on the details of S-dual loop corrections, and therefore it holds for generic S-dual dynamical models. As discussed in Sect. 3, (4.14) can also be interpreted as the necessary condition for the dilaton to be stabilized. The above analysis shows that it is indeed stringy non-perturbative effects that stabilize the dilaton and allow supersymmetry breaking via gaugino condensation.

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On the other hand, these axionic degrees of freedom naturally acquire masses in scenarios of multiple gaugino condensation \cite{28}.

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4.2 Solving for Dynamical Gaugino Condensation

In the previous subsection, the dynamical model of gaugino condensation is analyzed through its low-energy effective Lagrangian. One can also analyze the dynamical model directly, and obtain the same conclusion. Here, we would like to present a typical example of dynamical gaugino condensation as a concrete supplement to the analysis of Sect. 4.1. Solving for dynamical gaugino condensation is generically difficult due to the partial differential equation, (2.20) or (4.4), which guarantees the correct normalization of the Einstein term. On the other hand, only those solutions of (2.20) which are of physical interest deserve study. Therefore, in the following we show explicitly how to construct the solution for the interesting S-dual model of dynamical gaugino condensation defined by (4.3-5). In order to simplify the presentation but leave the generality of our conclusion unaffected, we choose a specific form for \( f(V, X) \) in the following discussion: \( f(V, X) = f^{(0)}(V) + \varepsilon X^2 \), where \( \varepsilon \) is a constant and \( |\varepsilon| \) is in principle a small number because \( X \)-dependent terms arise from loop corrections. In this restricted solution space, (4.4) together with the boundary condition (2.39) can be re-expressed as an infinite number of ordinary differential equations with appropriate boundary conditions (evaluated at \( \theta = \bar{\theta} = 0 \)) as follows:

\[
\ell g^{(0)}_\ell = f^{(0)} - \ell f^{(0)}_\ell, \\
\ell g^{(1)}_\ell - \left( 1 - f^{(0)} + \ell f^{(0)}_\ell \right) g^{(1)} = -\varepsilon \cdot \ell g^{(0)}_\ell + 2\varepsilon, \\
\ell g^{(n)}_\ell - n \left( 1 - f^{(0)} + \ell f^{(0)}_\ell \right) g^{(n)} = -\varepsilon \cdot \ell g^{(n-1)}_\ell - \varepsilon(n - 1)g^{(n-1)}, \\
\text{for } n \geq 2.
\]

(4.15)

The associated boundary conditions in the weak-coupling limit are:

\[
g^{(0)}(\ell = 0) = 0, \quad f^{(0)}(\ell = 0) = 0, \\
g^{(1)}(\ell = 0) = -2\varepsilon, \\
g^{(n)}(\ell = 0) = -\frac{2}{n} \varepsilon^n \quad \text{for } n \geq 2.
\]

(4.16)
Therefore, $g(V,X)$ is unambiguously related to $f(V,X)$ in this interesting solution space.

First, notice that the boundedness of $g^{(n)}$ and $f^{(n)}$ can be guaranteed if (4.14) is satisfied. Therefore, the solution defined by (4.15-16) exists at least for viable dynamical models in the sense of (4.14). Secondly, $g^{(n)}$ is suppressed by a small factor $|\varepsilon|^n$, which is obvious from (4.15-16). Therefore, the solution defined by (4.15-16) converges for $x^2 < O(1/\varepsilon)$. Since a physically interesting model of gaugino condensation should predict a small scale of condensation (i.e., $\langle x^2 \rangle \ll 1$), this solution does cover the regime of physical interest.

(4.14) is the necessary condition for stringy non-perturbative effects to stabilize the dilaton. By looking into the details of the scalar potential, it can also be argued [17] that stringy non-perturbative corrections to the Kähler potential may naturally stabilize the dilaton if (4.14) is satisfied. In the following, the solution defined by (4.15-16) is used to construct a typical realization of this argument. Furthermore, as illustrated in Sect. 1, it is the typical feature of this example rather than the specific form of $g(V,X)$ and $f(V,X)$ assumed in this example that we want to emphasize.

In Fig. 2, the scalar potential $V_{\text{pot}}$ is plotted versus $\ell$ and $x$ for an example with $f(V,X) = f^{(0)}(V) + \varepsilon X^2$ and $f^{(0)}(V) = A \cdot e^{-B/V}$. There is a non-trivial vacuum with the dilaton stabilized at $\langle \ell \rangle = 0.52$, $x$ stabilized at $\langle x \rangle = \langle \sqrt{\bar{u}u}/\ell \rangle = 0.0024$, and (fine-tuned) vanishing vacuum energy $\langle V_{\text{pot}} \rangle = 0$. Supersymmetry is broken at the vacuum and the gravitino mass $m_{\tilde{G}} = 4 \times 10^{-4}$ in reduced Planck units. To uncover more details of dilaton stabilization in Fig. 2, a cross section of $V_{\text{pot}}$ is presented in Fig. 4. More

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5 In fact, there is one free parameter $\beta$ involved due to the fact that $g^{(n)}(\ell = 0)$ is not well-defined in (4.15); this ambiguity can be parametrized by $g^{(n)}(\ell = 0) = n\varepsilon^{n-1}\beta$. We take $\beta = 0$ here.

8 The generalization to generic $f(V,X)$ is straightforward.

9 This solution can in principle be extended into the $x^2 > O(1/\varepsilon)$ regime using the method of characteristics.
precisely, with the value of $\ell$ fixed, $V_{pot}$ is minimized only with respect to $x$; the location of this minimum is denoted as $(\ell, x_{\text{min}}(\ell))$. The path defined by $(\ell, x_{\text{min}}(\ell))$ is shown in Fig. 3. The cross section of $V_{pot}$ is obtained by making a cut along $(\ell, x_{\text{min}}(\ell))$; that is, the cross section of $V_{pot}$ is defined as $V'_{pot}(\ell) \equiv V_{pot}(\ell, x_{\text{min}}(\ell))$. Fig. 4 shows that the dilaton is indeed stabilized at $\langle \ell \rangle = 0.52$. Therefore, we have presented a concrete example with stabilized dilaton, broken supersymmetry, and (fine-tuned) vanishing cosmological constant. One can also consider the stringy non-perturbative effect conjectured by [15], and the generic feature is similar to that of Fig. 2. As pointed out in Sect. 1, this is in contrast with condensate models studied previously [2, 6, 7] which either need the assistance of an additional source of supersymmetry breaking or have a large and negative cosmological constant.

5 Concluding Remarks

This paper begins with two generic questions in the context of dynamical gaugino condensation: What is the generic condition for the dilaton to be stabilized? Is supersymmetry broken if the dilaton is stabilized? First, it is emphasized that the linear multiplet formalism of gaugino condensation is the framework in which these two questions can be defined and answered more appropriately. Secondly, the field component Lagrangian for the linear multiplet formalism of generic dynamical gaugino condensation is constructed as the grounds of this study; it may also be useful to future studies.

The second question can be answered in a very generic context: by analyzing the vacuum structure of generic dynamical models, it is found that, for dynamical models whose $h_z$ has no pole at $z = 0$, supersymmetry is broken if the dilaton is stabilized. In particular, a class of well-motivated models, the S-dual model of dynamical gaugino condensation, does belong to this category.

As for the first question, it is shown that the low-energy limit of dy-
namical gaugino condensation is appropriately described by static gaugino condensation. An interesting necessary condition (4.14) for the dilaton to be stabilized, which was first derived in the study of static gaugino condensation, is then shown to hold for generic S-dual models of dynamical gaugino condensation. Furthermore, the analysis of (4.14) shows that it is stringy non-perturbative effects that stabilize the dilaton and allow dynamical supersymmetry breaking via the field-theoretical non-perturbative effect of gaugino condensation. We also present a concrete example where the dilaton is stabilized and supersymmetry is broken.

For the string models considered here, supersymmetry is broken in the dilaton direction rather than in the direction of modulus $T_I$. However, the hierarchy between the Planck scale and the gravitino mass generated by the confined $E_8$ hidden sector is insufficient to account for the observed scale of electroweak symmetry breaking. This is simply due to the large gauge content of gauge group $E_8$. In realistic string models, the hidden-sector gauge group is in general a product group, and the gauge content of each non-abelian subgroup is smaller than that of $E_8$. Therefore, the hierarchy generated by realistic string models could be sufficient to explain the scale of electroweak symmetry breaking. On the other hand, the generalization of the current study to realistic string models is very non-trivial since multiple gaugino condensation as well as hidden matter condensation occurs in generic hidden sectors. Furthermore, the cancellation of modular anomaly is also a very important issue. These issues together with the stabilization of the moduli are considered in [28].

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FIGURE CAPTIONS

Fig. 1-A: The scalar potential $V_{pot}$ (in reduced Planck units) is plotted versus the dilaton $\ell$ without non-perturbative corrections to the Kähler potential.

Fig. 1-B: The scalar potential $V_{pot}$ (in reduced Planck units) is plotted versus the dilaton $\ell$ with appropriate non-perturbative corrections to the Kähler potential.

Fig. 2: The scalar potential $V_{pot}$ (in reduced Planck units) is plotted versus $\ell$ and $x$. $A = 6.8$, $B = 1$, $\varepsilon = -0.1$ and $\mu = 1$. (The rippled surface of $V_{pot}$ is simply due to discretization of $\ell$-axis.)

Fig. 3: $x_{min}(\ell)$ is plotted versus $\ell$ for Fig. 2.

Fig. 4: The cross section of the scalar potential, $V'_{pot}(\ell) \equiv V_{pot}(\ell, x_{min}(\ell))$ (in reduced Planck units), is plotted versus $\ell$ for Fig. 2.
Figure 1

(A) $V_{pot}$ vs. $l$

(B) $V_{pot}$ vs. $l$
Figure 2
Figure 3

\[ x_{\text{min}} \]

![Graph showing the function \( x_{\text{min}} \) as a function of \( l \). The graph starts at a minimum value around 0.0005 and increases to a peak near 0.003 at \( l \) around 0.4, before decreasing to a value close to 0 at \( l \) near 0.8.]
Figure 4

$V'_{pot}$ vs $l$