On the (Non) Perturbative Origin of Quark Masses in D-brane GUT Models

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Abstract

We examine the issue of generating the perturbatively absent $10 \cdot 10 \cdot 5^H$ SU(5)/flipped SU(5) Yukawa couplings in type II D-brane orientifold compactifications of string theory both at the perturbative (PER) and the non-perturbative (NP) level. We find at the PER level, higher order terms like $10 \cdot 10 \cdot 5^H \cdot 5^H \cdot 5^H \cdot 1^H \cdot 1^H$ in SU(5) may be responsible for the relevant quark mass generation in models with general intersecting D6-branes. Euclidean D2-brane instantons on the other hand can also generate at the NP via the term $10 \cdot 10 \cdot 5^H \cdot 5^H \cdot 5^H \cdot 5^H$ the relevant quark masses by the use of just the $U(1)_b$ brane, for SU(5) and flipped SU(5) GUTS classes of models. We provide local examples of rigid $O(1)$ instantons within the $T^6/Z_2 \times Z'_2$ toroidal orientifold with torsion, whose NP contribution to the masses gets minimal as it is induced by just a duplicated disk diagram.
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1 Introduction

Intersecting brane worlds (see [1–6] for reviews) provide us with a beautiful arena that has become in recent years the main playground for realistic model building (RMB) attempts. The main characteristics of these models against previous attempts in the context of RMB in string theory is the appearance of the right handed neutrino in the massless spectrum localized at the string scale. Often the baryon number B or B-L are gauged symmetries, hence they survive as global symmetries to low energies. This happens since the $B \wedge F_a$ couplings, that take part in a generalized Green-Schwarz mechanism, give masses to the associated U(1) gauge bosons $F_a$. Moreover, in orientifold compactifications of IIA (or IIB) in four dimensions, one finds that intersecting D-brane models possess perturbatively absent matter couplings (PAMC) [61] that has singled out all the SU(5)-like GUTS. In this case, the relevant Yukawa terms

$$\langle 10_{(2,0)} \cdot 10_{(2,0)} \cdot 5^H_{(1,1)} \rangle,$$

[see also section 2] are not allowed in SU(5)-like GUTS from intersecting branes due to charge non-conservation. Solutions to PAMC could be provided by the existence of instantons generating the relevant couplings. Because stringy instantons break global U(1) symmetries they can generate PAMC. Hence they have been used recently to generate various types of non-perturbative effects [7–59].
Instanton induced phenomenological couplings in the literature exist for the Majorana [12–14]/ Dirac [43] masses for neutrinos, $\mu$-terms [13], an instanton proposal [26] for the long standing problem of the perturbatively absent Yukawa coupling (1) of SU(5)-like GUTS [61] etc. In IIA compactifications with D-branes, the relevant non-perturbative effects are induced from Euclidean D2-brane instantons. The E2-instanton, wraps a three-cycle $\Pi_{E_2}$ and under a $U(1)_a$ transformation the instanton action $S_{E_2}$ transforms as

$$e^{-S_{E_2}} = e^\left[\frac{2\pi}{\ell_s}(-\frac{1}{gs} Vol_{E_2} + i f_{\mu E_2} C^3)\right] \rightarrow \Lambda_{\Pi_{E_2}},$$

$$\Lambda_{\Pi_{E_2}} = e^{Q_a(E_2)} e^{-S_{E_2}},$$

(2)

$$Q_a(E_2) = -N_a \Pi_{E_2} * [\Pi_a - \Pi_a']$$

(3)

where the D6-brane wraps the three-cycle $\Pi_a$ and its orientifold image $\Pi_a'$. Chiral fermions appear as open strings stretching between the two intersecting branes, while the number of chiral fermions between an instanton and a D6-brane $\alpha$ is described by the intersection number $I_{E_2\alpha}$. "Charged" fermionic zero modes appear at the intersection of E2 and a D6-brane $\alpha$. If $\Pi_i \Phi_i$ is the coupling, made of a product of $i$ chiral superfields, that is not invariant under a global symmetry then the instantons induce F-term couplings such that the superpotential term $W = \Pi_i \Phi_i e^{-S_{E_2}}$ is invariant under the global symmetries. For O(1) instantons wrapping a rigid, orientifold invariant cycle in the internal manifold the charge carried by $Q_a(E_2) = -N_a \Pi_{E_2} * [\Pi_a]$ is exactly the amount of $U(1)_\alpha$ zero modes carried by the fermionic zero modes between $E_2$ and the D6$_\alpha$. To establish notation a positive intersection number associates to $I_{E_2\alpha}$ the transformation representation behaviour ($\square_{E_2}, \square_\alpha$).

In this work, we investigate the generation of the SU(5) up-quark mass coupling (1)), that was known to be absent perturbatively in all D-brane SU(5) GUTS from intersecting brane worlds (IBW's), either in IIA(or in IIB), by finding a new type of SU(5) gauge group higher, than trilinear, invariant perturbative Yukawa coupling in the form

$$\frac{1}{M_s^{N+3}} \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5^H} \cdot \mathbf{\bar{5}^H} \cdot \mathbf{\bar{5}^H} \cdot \mathbf{\Phi_1^H} \cdots \mathbf{\Phi_N^H},$$

(4)

where $\Phi_i$ gauge singlet Higgs fields. In section 2 we present the general form of the higher dimensional perturbative coupling responsible for generating the up-quark mass based on eqn.(4). We exhibit the presence of this term in a non-supersymmetric background. Thus all SU(5) models from intersecting branes receive, beyond tree level, non-zero corrections to the mass of the up-quarks. In section 3, we exhibit the presence of this perturbative coupling in a N=1
supersymmetric $Z_2 \times Z_2$ orientifold. In section 4, we discuss a new form of instanton coupling which contributes to the mass of the up-quarks and originates from the perturbative coupling of sections 2 and 3. We also present a local realization based on the $T6/(Z_2 \times Z_2')$ orientifold with discrete torsion [73], in which the instanton contribution to the coupling (1) gets generated by a rigid O(1) instanton.

2 Perturbative induced quark masses for SU(5) D-brane models

2.1 The perturbative term in D-brane models

Model building based on SU(5)(or flipped SU(5)) GUT models from 4D type IIA compactifications, has been hampered as in these models the missing up(down) quark masses excluded the models phenomenologically. The first semirealistic SU(5) models historically were constructed in [60] (see also some recent work on [72], even though in a language T-dual to the one used in intersecting brane worlds(IBW)). In the context of IBW a two stack three generation non-susy SU(5) based on Z3 orientifolds appeared in [61], where the existing Yukawa couplings were listed. Also a toy four stack N=1 SU(5) model has been constructed on Z2 x Z2 orientifolds in [62]. Moreover, in [63] it was noticed that by a simple rescaling of the massless U(1) surviving the Green-Schwarz mechanism, the models of [61] were converted into flipped SU(5) ones, while it also appeared that the models were missing also the GUT Higgses. In [64] we proposed in which way one can identify GUT Higgses in a general non-supersymmetric flipped SU(5) model coming from intersecting branes and also we identified all the proton decay modes and presented a new doublet-triplet splitting mechanism. SU(5) models based on intersecting branes have been also constructed in different contexts (e.g. in the presence of fluxes; see also [66]) or in the context of other orientifold compactifications however, the basic spectrum structure is the one identified in [61], [63], [64].

We note that the matter field content (MFC) of an SU(5) GUT is

$$\overline{10} = (Q, u^c, e^c), \quad \bar{5} = (d^c, L), \quad 1 = \nu^c,$$

while the MFC of a flipped SU(5) GUT reads

$$\overline{10}_1 = (u, d, d^c, \nu^c), \quad \bar{5}_{-3} = (d^c, L), \quad 1_5 = e^c.$$  \hspace{1cm} (6)

In the minimal string version [61], [63], [64], of the SU(5) GUT models there are two stacks a, b, of intersecting D6-branes, associated to $U(5)_a$, $U(1)_b$ branes respectively. Its chiral spectrum can be seen in table (1). We note that the chiral spectrum of table (1) can be embedded also in a general string construction and thus our discussion in this section is quite generic. When embedded in a different
string construction, the D6-branes giving rise to the chiral spectrum of table (1) may be accompanied by a number of extra space filling D6-branes necessary to cancel tadpoles (RR tadpoles in a non-susy model; RR and NSNS tadpoles for N=1 models). To establish notation, we note that the multiplet $10_{(2,0)}$ also contains the GUT Higgs field which should appear as a vector-like pair as it has been firstly noted in [64]. The massless $U(1)$ surviving massless the Green-Schwarz mechanism is given by $U(1)_X$ seen in table (1) ($U(1)_Y$ in flipped SU(5)). In the $ab'$ sector where $I_{ab'} = 0$ there are present the non-chiral Higgs $5, \bar{5}$. If the SU(5) model is non-supersymmetric then the Higgs fields are coming from the lowest massive excitation spectrum of the fermions $5, \bar{5}$ with the same $U(1)$ charges (see [64] for details and also table (1)).

| sector | number | $U(5)_a \times U(1)_b$ reps. | $U(1)_X$ | $U(1)_Y$ |
|--------|--------|-------------------------------|----------|----------|
| $(a, a')$ | 3      | $10_{(2,0)}$                  | $\frac{5}{3}$ | 1       |
| $(a, b)$ | 3      | $\overline{5}_{(-1,1)}$      | $-\frac{6}{5}$ | -3      |
| $(b, b')$ | 3      | $1_{(0,-2)}$                 | 2        | 5        |
| $(a, a')$ | 1      | $10^H_{(2,0)} + \overline{10}^H_{(2,0)}$ | $\frac{1}{2}$ | $(1) + (-1)$ |
| $(a, b')$ | 1      | $5^H_{(1,1)} + \overline{5}^H_{(-1,-1)}$ | $(-1) + (1)$ | $(-2) + (2)$ |

Table 1: $SU(5)$ GUT via intersecting D6-branes: The massless $U(1)_a$, $U(1)_b = \frac{1}{5} U(1)_a - U(1)_b$ (in a flipped SU(5) parametrization $U(1)_Y = \frac{1}{2} U(1)_a - \frac{5}{2} U(1)_b$). The first three rows describe the chiral fermionic spectrum. The last two rows contains the matter Higgses: electroweak 5’s, $\overline{5}$’s and GUT 10, $\overline{10}$’s.

The Yukawa couplings giving masses to the down-quarks(up-quarks) for SU(5) (flipped SU(5)) respectively are given by the tree level expression $\overline{10} \cdot \overline{5} \cdot \overline{5}^H$. On the contrary the masses for up (down) quarks in SU(5)(flipped SU(5)), are given by the coupling (1), which is not allowed by charge conservation in orientifold compactifications of string theory. It violates the charge conservation by the amount $(U(1)_a, U(1)_b) = (5, 1)$ units. In this work, our first result is that the coupling (1) can be generated perturbatively. One can imagine that the simplest solution to the missing mass term (1) may be coming from the potential term

$$\frac{1}{M_s^5} \langle 10_{(2,0)} \cdot 10_{(2,0)} \cdot \overline{5}^H_{(1,1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{10}^H_{(0,2)} \cdot \overline{10}^H_{(0,2)} \rangle.$$

However a closer look reveals that a lowest order term exists, able to generate the relevant quark masses, via the Yukawa coupling

$$\frac{1}{M_s^5} \langle 10_{(2,0)} \cdot 10_{(2,0)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{5}^H_{(-1,-1)} \cdot \overline{10}^H_{(0,2)} \cdot \overline{10}^H_{(0,2)} \rangle.$$

(7)

(8)
The above term is gauge and charge invariant. Its gauge invariance is easily seen via the SU(5) tensor products

\[ 10 \cdot 10 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \sim 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \sim \bar{10} \cdot 5 \cdot \bar{5} \]

\[ \sim 10 \cdot \bar{5} \cdot \bar{5} \]  

(9)

The term of eqn.(8) cannot be used in the "re-scaled" two stack version of the present model that converts it to its flipped SU(5) version as the singlet in this case accommodates the right handed electron and cannot get a vev. Instead one might try to build a higher than two stack SU(5) GUT that has extra singlets to generate the required invariant in (8). The bosons associated to the \( I_{(0,2)} \) for the non-susy model are part of the massive spectrum that is localized in the \( bb' \) intersection, that allows as a Higgs field the vector-like pair \( I_{H}^{(0,2)} + \bar{1}_{H}^{(0,-2)} \) (see [64] and the 2nd ref. of [2] for details). After the associated Higgses receive vevs, the order of the quark masses is given by the perturbative evaluation of the above correlator which reads

\[ m_{U} = Y_{U} \frac{1}{M_{5}^{4}} \langle 5^{H} \rangle^{4} \langle 1^{H} \rangle^{2} e^{-\frac{1}{\pi \alpha} A}, \]

(10)

where \( A \) is the worldsheet area of the 8-point correlator in (8)(see also [71]).

By looking at the formulae (10) we see that the derived U-quark masses depend on three quantities, the string scale \( M_{s} \), the vev of 5-plet Higgses, and the area suppression factor \( A \). Before we give an estimate of the U-mass, let us make some remarks about these quantities.

With a High string scale:

The string scale in the present models and in general in models where orientifolded intersecting D6-branes wrap a six dimensional toroidal internal space [2], [60], [61], [64], [62] takes high values. On the other hand, the vev of the 5-plet Higgses can be taken to be (see N=1 examples in the next section) either at the electroweak scale \( v = 246 \text{ GeV} \) or high at its natural value that could be the string scale \( M_{s} \). In the previous case, namely Case I in eqn. (11), the mass of the U-quark is very suppressed and is excluded phenomenologically as a typical suppression factor can be of the order of \( 10^{-43} \times \text{Exp}[-M_{s}^{2}A] \) GeV with \( M_{s} \approx 10^{16} \) GeV and \( \langle 5 \rangle = v \). In the latter case, case II, the natural vev of the 5-plet higgses is of order \( M_{s} \) and hence the smallness of the up(down) quark masses is achieved from the exponential area suppression factor. The relevant estimates are summarized in eqn. (11). We assume that the areas of the second and third tori are close to zero and the eight term coupling can be approximated as \( M_{U} \sim e^{-R_{1}R_{2}M_{s}^{2}A^{(8)}} \equiv e^{-\tilde{A}} \) where the \( \tilde{A} \)-area may be of order one in string units.
The value of the up-quark as a current quark mass estimate (see p.479 of [65]) in the $\overline{\text{MS}}$, is between 1.5 - 4.0 MeV. Hence, in Case II for a typical value of the string scale $M_s = 10^{16}$ GeV, the mass of the up-quark e.g. 2 MeV is achieved for a suppression area factor \"area\" $\tilde{A} \approx 18.7$.

Summarizing, case I is excluded in models with a high string scale and the assumptions made in eq. (11). On the contrary, case II as it appears in eq.(11) is valid in D-brane models with a high and also a low string scale as it is shown next. In the following, we numerically examine under which conditions cases I, II are also valid in models with two extra dimensions, where we re-named them as Cases III, IV, respectively.

With a Low string scale; Models from Extra Dimensions

In type I compactifications if there are extra compact dimensions transverse to all stacks of branes then the string scale can be lowered to the TeV region and the gauge hierarchy problem is solved even without the presence of N=1 supersymmetry [76]. The only known realization of this scenario where the Standard model can be constructed in a string construction has appeared in the Standard-like models of [77], [78] where the D5-branes are filling M4 and wrapping two-cycles in the extra dimensional space $T^4 \times C/Z_N$. In [78] we have generalized the 4-stack constructions of [77] incorporating other four stack quiver SM’s and also accommodating the deformed Standard model configurations that have been used in the 5-and 6-stack SM-like models of [79], [80]. In those constructions, the D5-branes get localized at the orbifold singularity and only the massless chiral spectrum of Standard Model remains at low energy (in addition to some scalars where in cases could become tachyonic).

For the purposes of this paper we will silently accept, for the remaining of this section, that the relation (11) also holds and in SU(5)-like constructions from models with intersecting D5-branes, wrapping 2-cycles in $T^4 \times C/Z_N$ and having two extra dimensions transverse to the branes. String SU(5) models with two extra dimensions that localized the SM do not exist at present in the literature and may appear in [81]. In this way, if the $1 \, \text{TeV} \leq M_s < 10 \, \text{TeV}$ we see that
in case III, we get that the mass of the U-quark gets calculated in eqn.(12).

\[
M^4_u M_s^{-3} \cdot e^{-\bar{A}} \text{ GeV}; \quad \langle 5^H \rangle \sim v, \quad \langle 1 \rangle \sim M_s \sim 1 \text{ TeV}, \quad \text{Case III}
\]
\[
M_s \cdot e^{-\bar{A}} \text{ GeV}; \quad \langle 5^H \rangle \sim \langle 1^H \rangle \sim M_s, \quad \text{Case IV}
\]

(12)

| $M_s$ | 1 TeV | 5 TeV | 10 TeV | 1 TeV | 5 TeV | 9 TeV | 10 MeV |
|-------|-------|-------|--------|-------|-------|-------|--------|
| $M_u$ | 1.5 MeV | 1.5 MeV | 1.5 MeV | 4 MeV | 4 MeV | 4 MeV | 4 MeV |
| $A$   | 7.80  | 2.97  | 0.89   | 6.82  | 1.99  | 0.228 | −0.088 |

Table 2: Case III: Values of the String scale $M_s$ versus the area $A$, the mass of up(U)-quark $M_u$ and the vev of the singlet Higgses while keeping constant the values of the up-quark and the vev of the electroweak Higgs of the order of the electroweak scale $\nu = 246$ GeV.

**In Theories with Extra dimensions:** $M_s$ subject to $1 \text{ TeV} < M_s \leq 9 \text{ TeV}$

In particular the area factor expressed in terms of $M_s$, $M_u$, $M^4_w$ reads

\[
A = -\frac{M^3_s M_u}{M^4_W}
\]

(13)

From table (2), we observe that in Case III, when the string scale is low and the Higgs $5^H$ takes a vev of the order of the electroweak scale $\nu = 246$ GeV, the string scale is subject to an upper limit; its value can take any value from one (1) TeV to nine (9) TeV. For $M_s = 10$ TeV the area turns negative (we kept constant the value of the U-quark mass). On the other hand in Case IV, there is no constraint in the value of the string scale from first principles as the hierarchical mass of the up-quark 1.5-4.0 MeV is derived from the area suppression in the exponential.

### 3 Perturbative application to a 4D $\mathbb{Z}_2 \times \mathbb{Z}_2$ IIA N=1 SU(5) with D6-branes

An example of a N=1 supersymmetric SU(5) model exhibiting the relevant perturbative mass term for the up-quarks can be seen by choosing for example the model II.1.1 of [67]. The models come from an orientifold compactification of type IIA on a six-dimensional torus in a background of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold symmetry. The relevant rules for calculating the spectrum are summarized in appendix A. In this SU(5) model there are no filled branes to satisfy RR tadpoles. The wrappings are reproduced for convenience in table (3). The initial gauge group is a
$U(5) \times U(1) \times U(1)$.

There are two massless $U(1)$'s given by $6U(1)_a + 5U(1)_b, F_c$ and a massive $-5U(1)_a + 6U(1)_b$. The $F_c U(1)$ could be broken by the vev of the $1_{(0,0,-2)}$ multiplet.

In the present case the up-quark masses may receive perturbative contributions from the gauge invariant expression (4). For this particular model there are two different Yukawa contributing to the mass of the up-quarks. In the first mass term, see eqn.(14), we are using the Higgs 5-plets from the $N=2$ ab sector and the gauge singlets from the $bb'$ sector. In the second mass term contribution, see eqn. (15), we are using the Higgs 5-plets from the $N=2$ ac' sector and the gauge singlets from the cc' sector.

\[
\frac{1}{M_s^4} \mathbf{10}_{(2,0,0)} \cdot \mathbf{10}_{(2,0,0)} \cdot \bar{5}^H_{(-1,1,0)} \cdot \bar{5}^H_{(-1,1,0)} \cdot \bar{5}^H_{(-1,1,0)} \cdot \bar{5}^H_{(-1,1,0)} \cdot 1^H_{(0,-2,0)} \cdot 1^H_{(0,-2,0)} \quad (14)
\]

\[
\frac{1}{M_s^4} \mathbf{10}_{(2,0,0)} \cdot \mathbf{10}_{(2,0,0)} \cdot \bar{5}^H_{(-1,0,-1)} \cdot \bar{5}^H_{(-1,0,-1)} \cdot \bar{5}^H_{(-1,0,-1)} \cdot \bar{5}^H_{(-1,0,-1)} \cdot 1^H_{(0,0,2)} \cdot 1^H_{(0,0,2)} \quad (15)
\]

The mass of the up-quarks can be distinguished according to the order of the vacuum expectation value of the Higgs 5-plets and the gauge singlets. The up-quark mass estimate to both cases of eqn’s (14), (15) is identical to that of eqn. (11). See the subsection (4.2), for a comment on the associated Yukawa couplings of eqn’s (14), (15), that can generate additional contributions to the mass of the up-quarks through multi-instantons. In the present class of models, the perturbative mass of the up-quarks is a linear combination of the couplings (14) and (15).

### Table 3: $N=1 \mathbb{Z}_2 \times \mathbb{Z}_2$ $SU(5)$ chiral spectrum

| D6 | $n$ | $l_1$ | $l_2$ | $l_3$ | $n$ | $b$ | $c$ | $b'$ | $c'$ |
|----|-----|------|------|------|-----|-----|-----|------|------|
| a  | 10  | (0, -1)$\times$(1, 4)$\times$(1, 1) | -3  | 3    | 0   | 24  | 0   | 0    |
| b  | 2   | (-1, 3)$\times$(-1, 4)$\times$(1, 1) | -6  | 42   | -0  | 0   | -96 |
| c  | 2   | (-1, 0)$\times$(-1, 4)$\times$(7, 1) | 27  | -27  | -0  | -0  | -0  |

\[ x_B = 4x_A = 4x_C \]

4 Instantons and missing $SU(5)$ masses

#### 4.1 General $SU(5)$ u-mass instanton solutions

An instanton mechanism that can potentially generate the missing up(down) quark coupling for an $SU(5)/\text{flipped } SU(5)$ GUT of eqn. (1) was proposed [26] (without presenting details on a particular O(1) instanton solution), that uses three disk diagrams involving the $SU(5)_a$ and the $U(1)_b$ D6-branes to absorb the
relevant zero modes. In this case the violation of the U(1) charge that has to be absorbed by the instanton is the excess charge \((U(1)_a, U(1)_b) = (5, 1)\).

In this work, we find that the up(down) quark masses may also receive contributions to their masses via a different instanton mechanism. It can be generated by an E2-instantons carrying gauge group O(1), Sp(2) or U(1) and having the appropriate zero mode structure. For the case of the gauge group O(1) we present in the next section the explicit form of the O(1) stringy instanton. We find that stringy instantons give rise to the up(down) quark mass coupling via the coupling

\[
\frac{1}{M^3} \langle 10_{(2,0)} \cdot 10_{(2,0)} \cdot 5^H_{(-1,-1)} 5^H_{(-1,-1)} 5^H_{(-1,-1)} \rangle ,
\]

where we have used the general two stack D6-brane SU(5) Grand Unified model structure of section II. The excess charge which has to be absorbed by the instanton is \(U(1)_b = -4\). This coupling violates the \(U(1)_b\) symmetry and thus is perturbatively forbidden. Hence a non-perturbative contribution to the superpotential generated by a E2 instanton is a viable possibility. A general E2 instanton wrapping a three cycle and placed away from the orientifold fixed plane gives rise to four bosonic zero modes arising from the breakdown of the Poincare invariance and also four fermionic zero modes \(\theta_a, \bar{\tau}_a\) in the uncharged sector.

To generate a superpotential contribution we need instantons with \(O(1)\) Chan-Paton symmetry. These instantons are wrapping a rigid orientifold invariant three cycle, generate an \(O(1)\) gauge group on their volume and possess the two \(\theta_a\) uncharged instanton zero modes which saturate the \(d^2\theta 4d\) superspace integration. Hence each rigid \(O(1)\) instanton carries two neutral fermion zero modes, the goldstinos of the susys it breaks. However, as was noted in [23] instantons with \(Sp(2)\) or \(U(1)\) CP symmetries may also generate the relevant superpotentials if there is additional dynamics, e.g. fluxes, that can saturate the extra zero modes that could appear in those cases. Hence the induced instanton charge that cancels the \(U(1)_b\) violation has the following intersection numbers between the \(E_2\)-instanton and the D6 b-brane

- \(Sp(2)\) case : \(I_{E_2b} = -2\),
- \(O(1)\) case : \(I_{E_2b} = -4\),
- \(U(1)\) case : \(I_{E_2b} = -2\), \(I_{E_2b'} = 2\).

Thus in general we need four charged zero modes, a ”doublet”\(^1\) of \(\alpha_b\) zero modes coming from the intersection of the instanton M and the D6\(_b\) brane and another ”doublet” of \(\gamma^i\) zero modes coming the intersection of M with D6\(_{b'}\). Since the instanton lies in a \(\Omega\bar{\sigma}\) invariant position this guarantees that the uncharged part

\(^1\)The term ”doublet” is used in a broad sense; not connected to the group representation.
of the instanton measure contains two fermionic $\theta_a$ and four bosonic degrees of freedom $x^\mu$. The zero modes appearing in the intersections (18) gets saturated by two identical disk diagrams seen in figure 1.

![Diagram](image)

Figure 1: Absorption of charged zero modes by the instanton

These disk diagrams induce the non-perturbative contribution to the Yukawa coupling for the u-quarks which is based on the calculation of the path integral

$$\frac{1}{M_\phi^3} \int d^4x \ d^2\theta \ d^2\alpha \ d^2\gamma e^{-S_{E_2}^{\text{class}}} \ e^{Z'} \langle \alpha X \gamma \rangle \langle \alpha X \gamma \rangle,$$

$$X_i = X_j = 10 \cdot \bar{5}^I \cdot \bar{5}^I.$$

(19)

Also $S_{E_2}^{\text{class}}$ is the classical instanton action for the $E_2$ instanton, $e^{Z'}$ is the holomorphic part of the one-loop determinant arising from the annulus and Möbius diagrams ending on the instanton and the D6-branes or O-plane respectively; it can be interpreted as the one-loop Pfaffian. The existence of the path integral denotes the fact that all charged zero modes arising at the instanton intersections with D6-branes can be soaked up via disk diagrams $< \lambda_{-1/2}^a \Phi_{ab} \lambda_{-1/2}^b >$, where $\Phi_{ab} = \phi_{ab} + \theta \psi_{ab}$ denotes the chiral superfield arising at the intersection of branes a and b; $\lambda_i$ denote the charged zero modes (see for example [57]). The instanton suppression factor in (19) is

$$e^{-S_{E_2_1}^\text{vol}} = e^{-\frac{2\pi}{\alpha_s^2} \text{Vol}_{E_2_1}} = e^{-\frac{2\pi}{\alpha_a} \text{Vol}_{D6_a}},$$

(20)

where the $\frac{\text{Vol}_{E_2_1}}{\text{Vol}_{D6_a}}$ is given by

$$\frac{\text{Vol}_{E_2_1}}{\text{Vol}_{D6_a}} = \frac{1}{2} \left( \prod I \left[ \frac{(n_I^{E_2_1})^2 + (\tilde{n}_I^{E_2_1})^2 U_I^2}{(n_I^a)^2 + (\tilde{n}_I^a)^2 U_I^2} \right] \right)^{1/2}.$$

(21)
To generate the $O(1)$ MeV order of the up-quarks, or the $O(200)$ GeV mass of the top quark, assuming that there is no perturbative term which contributes to the quark masses, we need instanton suppression factors

$$e^{-S_{E21}^{u-quark}} \sim 10^{-21}, \quad e^{-S_{E21}^{t-quark}} \sim 10^{-16}$$

respectively, which are rather large estimates. We assume that the string scale $\approx 10^{18}$ GeV. Of course models with large numbers of exotics could achieve that goal, as we comment in the next section. Clearly such a model has to be presented for this scenario to be realized. See next section, where in a particular local $N=1 \mathbb{Z}_2 \times \mathbb{Z}_2'$ model, typical suppression factors are in the range $10^{-4}$ for the MSSM and can be as large as $10^{-21}$ when large number of exotics are present.

Thus in general the instanton eq. (19), constitutes a subleading correction to the perturbative mass term of the U-quarks, the latter given in eq.(8).

To be more concrete lets us comment on the 3 x 3 Yukawa coupling mass matrix for the up-quarks. The full matrix is a sum of a perturbative and a non-perturbative part. The perturbative generated Yukawa contribution to the masses, expressed by eq.(8), leads to Yukawa coupling matrices in the form

$$Y^{Per}_u = \begin{pmatrix} A_{u11} & A_{u12} & A_{u13} \\ A_{u21} & A_{u22} & A_{u23} \\ A_{u31} & A_{u32} & A_{u33} \end{pmatrix}$$ (23)

while the non-perturbatively generated instanton part is also given by

$$Y^{NP}_u = \begin{pmatrix} B_{u11} & B_{u12} & B_{u13} \\ B_{u21} & B_{u22} & B_{u23} \\ B_{u31} & B_{u32} & B_{u33} \end{pmatrix}.$$ (24)

We note that in general due to the instanton suppression factor $A_{ij} \gg B_{kl}$ may hold for arbitrary $i, j, k, l$. The precise form of the $A_{ij}$ also depends on the intersection numbers of the particular D-brane model. For the class of $O(1)$ instantons in eqn.(19) the mass matrix factorizes, giving a rank one matrix, thus the instanton contributes a correction to only one generation of U-quarks. As there is present the perturbative contribution to the U- mass, the instanton correction suggest to us that it may be contributing to the heaviest generation, the third (assuming that it is the one associated with the top quark). A different scenario could be realized in the following way. Let us suppose that the perturbative mass matrix has a texture that it leads to a rank one matrix. In this case, we can identify the non-zero perturbative mass eigenvalue with the mass of the third generation generating the mass of the top quark, while the rank one instanton

$^2$Additional worldsheet instanton suppression factors could also be created in the calculation of disk amplitudes associated with the intersection structure of the participating branes [19].
mass matrix can contribute to the mass e.g. of the second heaviest generation of the up-quarks, leaving massless the lightest generation, the first one.

After integrating out the charged zero modes the superpotential instanton contribution appears

\[ \int d^4 x d^2 \theta \ Y' \ U'_{(D)} \ 10 \ 10 \ 5 \ 5 \ 5 \ 5. \] (25)

The factor \( Y \) accommodates the classical suppression factor and \( e^{Z'} \), as well as the contribution of the disk amplitude \( \langle \alpha X \gamma \rangle \langle \alpha X \gamma \rangle \), the latter depending on the open string moduli in the sums of worldsheet instantons that connect the intersection points.

### 4.2 Instantons on the N=1 local model of section 3

Let us now make some observations concerning the N=1 \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) string model appearing in section 3. We have seen that for this model there are two different mass terms contributing to the mass of the up-quarks. Accordingly, we assume that the wrappings of this model constitute the ”observable part’ of a local model on a N=1 \( \mathbb{Z}_2 \times \mathbb{Z}_2' \), with no tadpole cancelation at this level. In this case, we find that there are two potential contributions to the up-quark mass through instantons. To saturate the instanton zero modes in this case, we will need a duplicate of the two diagrams of figure 1; the one pair identical to that of figure 1 and the second pair having replaced the b D6-brane with the c D6-brane. The instanton couplings for which instantons that may also additionally contribute corrections to the u-masses are

\[ \frac{1}{M_5^2} \ 10_{(2,0,0)} \cdot 10_{(2,0,0)} \cdot 5^H_{(-1,1,0)} \cdot 5^H_{(-1,1,0)} \cdot 5^H_{(-1,1,0)} \cdot 5^H_{(-1,1,0)}. \] (26)

with excess charge \( U(1)_b = 4 \) to be absorbed by one instanton and also

\[ \frac{1}{M_5^2} \ 10_{(2,0,0)} \cdot 10_{(2,0,0)} \cdot 5^H_{(-1,0,-1)} \cdot 5^H_{(-1,0,-1)} \cdot 5^H_{(-1,0,-1)} \cdot 5^H_{(-1,0,-1)}. \] (27)

with excess charge to be ”absorbed” by another instanton \( U(1)_c = -4 \). In this case, we will need at least two different O(1) instantons to absorb the relevant charges. An example of a single O(1) instanton compensating an excess charge -4 and generating the up-quark masses is described in the next section. In principle a set of two instantons typically has more neutral zero modes than is needed to lead to a superpotential. If you consider a set of two O(1) instantons, in principle you have eight charged fermion zero modes that have to be absorbed. For a multi-instanton process to contribute to the superpotential, one needs to make sure that all the extra fermion zero modes are lifted by some interaction. This has been shown only in few examples in [44]. More details will be presented elsewhere.
4.3 O(1) instantons for U-masses to a local $\mathbb{Z}_2 \times \mathbb{Z}_2'$ IIA

N=1 SU(5)

In this section, we will examine local setups of the stringy O(1) instanton, that can generate perturbatively the up-quark mass in SU(5) models, based on a background of an orientifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with Hodge numbers $(h_{11}, h_{22}) = (3, 51)$, the so called $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with discrete torsion. In these models, the orbifold quotient and the choice of discrete torsion is such that they contain rigid branes that freeze the position of the branes at the fixed points of the orbifold fixed points where the twisted cycles arise. Hence the corresponding adjoints associated to the transverse translations of the branes may not exist (models with completely rigid cycles have been considered in [75]. See also [74] for related constructions in type IIB).

Let us consider the chiral spectrum displayed in table (4). It corresponds to an initial $SU(5)_a \times U(1)_a \times U(1)_b$ gauge group at the string scale. The mass of the up-quarks is given by the coupling $10^{(2,0)} \cdot 10^{(2,0)} \cdot 5^H_{(1,1)}$ which is not allowed in perturbation theory due to the non-conservation of $U(1)_b$ charge. We will rather generate this coupling in this section in two ways, a) via non-perturbative effects from O(1) instantons and b) from perturbation theory at the end of the section.

The matter D6-branes are described by fractional branes that carry charge under one twisted sector. They wrap D6-branes along untwisted bulk and twisted three cycles. The bulk part of the wrappings is given in table (5). For the choice

| sector | $I_{ij}$ | $SU(5)_a \times U(1)_a \times U(1)_b$ |
|--------|---------|----------------------------------|
| $(a, a')$ | 3       | $10^{(2,0)}$                     |
| $(a, a')$ | 1       | $\overline{15}^{(-2,0)}$        |
| $(a, b)$  | 38      | $\overline{5}^{(-1,1)}$         |
| $(a, b')$| 0       | $5^H_{(1,1)} + \overline{5}^H_{(-1,-1)}$ |
| $(b, b')$| 26      | $1^{(0,2)}$                      |

Table 4: Chiral matter for local N=1 SU(5) GUT model with intersecting D6-branes on the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold.

\[ U^1 U^3 = \frac{3}{4}, \quad -12U^3 + 36U^1 - 13U^1 U^2 U^3 = 0, \quad (28) \]
N=1 supersymmetry is obeyed by the D6-branes, as they align with the orientifold planes. The complex structure moduli $U_i = (R^2/R_1)^{i(i)}$, where $i$ denotes the $i$-tori could be fixed according to the eqn’s (28). An example value, solving the complex structure constraints (28), is the choice

$$U^1 = 3, U^2 = \frac{140}{13}, U^3 = \frac{1}{4}$$

(29)

For definiteness we make the choice of crosscap orientifold charges

$$\eta_{\Omega R} = 1 \quad \eta_{\Omega \theta} = 1 \quad \eta_{\Omega R \theta} = -1 \quad \eta_{\Omega \theta R} = 1$$

(30)

The fractional matter D6-branes wrap fractional cycles, cycles are charged only under one twisted sector $g$ in the form

$$\pi^F = \frac{1}{2} \pi^B + \frac{1}{2} \left( \sum_{i,j \in S_g} \epsilon_{ij}^g \left[ \alpha_{ij}^g \times (n_a^I, \tilde{m}_a^J) \right] \right).$$

(31)

This class of cycles are only rigid in two tori and can move freely in the torus invariant under the action $g$. Then, for two fractional branes charged under a different twisted sector the intersection number is simply given by

$$\pi^F_a \circ \pi^F_b = \prod_{i=1}^3 (n_a^i \tilde{m}_b^i - n_b^i \tilde{m}_a^i)$$

(32)

The fractional 3 D6-branes generating the chiral matter spectrum of table (4) read

$$a : \frac{1}{2} [(3, 4)(0, -1)(1, 1)] + \frac{1}{2} \left( \sum_{i,j \in (2, 4) \times (3, 4)} [\alpha_{ij}^g \times (1, 1)] \right)$$

$$b : \frac{1}{2} [(1, 3)(-4, 1)(-1, 1)] + \frac{1}{2} \left( \sum_{i,j \in (1, 4) \times (3, 4)} [\alpha_{ij}^g \times (-1, 1)] \right)$$

(33)

Given our choice of fractional branes, if we were making another choice of crosscap charges e.g. as the one in appendix 8 of [73], our new choice may only have affected the numbers of antisymmetric and symmetric representations in the spectrum. In the latter case all K-theory constraints listed in [73] would have been satisfied. The next, beyond the tree level, general gauge invariant responsible for generating the mass of the up-quarks in the local SU(5) model via O(1) instantons is

$$\frac{1}{M^3_3} (\mathbf{10}_{(2, 0)} \cdot \mathbf{10}_{(2, 0)} \cdot \overline{\mathbf{5}}_{(-1, -1)}^H \cdot \overline{\mathbf{5}}_{(-1, -1)}^H \cdot \overline{\mathbf{5}}_{(-1, -1)}^H \cdot \overline{\mathbf{5}}_{(-1, -1)}^H)$$

(34)

\(^3\)Further details on the notation we are using can be seen in appendix B. We are using an identical notation to [59].
This mass coupling is not allowed in perturbation theory as there is a non-canceled $U(1)_b = -4$ excess charge. We have used the N=2 Higgses $\bar{5}$-plets from the ab' sector in order to generate it. The O(1) instanton able to and generating the dim=7 operator (in superpotential form)

$$W_{\text{non-pert}} = \frac{1}{M_s^4} 10_{(2,0)} \cdot 10_{(2,0)} \cdot \bar{5}^{H}_{(-1,-1)} \cdot \bar{5}^{H}_{(-1,-1)} \cdot \bar{5}^{H}_{(-1,-1)} e^{-S_{\text{ins}}}$$

thus absorbing the excess zero modes and giving a mass to the up-quarks is given by

$$E_{21} = \frac{1}{4} [(1,0)(0,1)(0,-1)] + \frac{1}{4} \left( \sum_{i,j\in(1,3)\times(3,4)} [a^b_{ij} \times (0,-1)] \right)$$

$$+ \frac{1}{4} \left( \sum_{i,j\in(\ast,\ast)\times(\ast,\ast)} [a^b_{ij} \times (1,0)] \right) + \frac{1}{4} \left( \sum_{i,j\in(\bullet,\bullet)\times(\bullet,\bullet)} [a^{b\prime b\prime}_{ij} \times (0,1)] \right).$$

The fractional instanton is charged under all the different twisted sectors of the orbifold. The meaning of $(\ast,\ast) \times (\ast,\ast)$ is that we can choose any combination of the following set of fixed points $\{(1,2)\times(1,2); (1,2)\times(3,4); (3,4)\times(3,4); (3,4)\times(1,2)\}$ the bulk wrappings of the fractional brane can go through, as there is no restriction from first principles. In a similar way, $(\bullet,\bullet) \times (\bullet,\bullet)$ means that we can choose any combination of the following sets $\{(1,3)\times(1,2); (1,3)\times(3,4); (2,4)\times(1,2); (2,4)\times(3,4)\}$. The instanton of eqn.(36) gives rise to the intersection number pattern

$$I_{E_{21}a} = 0, \quad I_{E_{21}b} = -4$$

thus canceling the $U(1)_b$ charge violation of (34). The relevant string diagrams generating the superpotential term are given in figure (1). All the states from the N=2 sector $E_{21} - a$ are massive. The bulk part of both $E_{21}, a$, D6-branes is parallel across the second torus, while simultaneously the twisted part of the branes runs through different fixed points. Thus all N=2 states gets massive and there are no vector-like states from this sector. The factor $\frac{1}{2}$ is due to the fact that the D6-brane wraps a fractional cycle while the instanton a rigid one.

Let us now derive some estimates on the instanton suppression factor for the local models of this section. Let us assume $\alpha_a = 1/24$, as in the MSSM, at string scale. If we choose the complex structure moduli, from eqn.(28), to be stabilized by the choice $U^1 = 1$, a N=1 consistent solution for the complex structure is $U^3 = 3/4, U^2 = 36/13$. In this way, we get the suppression factor

$$e^{-S_{E_{21}}^{\bar{b}}} \sim 10^{-4}.$$
Several values of $a_{GUT}$ against the suppression factor may be seen in table (6). Cases with the mark $\times$ are phenomenologically excluded as their contribution is bigger that the order of the mass of the quark.

| $a_{GUT}$ | $e^{-S_{E2}}$ | $M_u$ (MeV) |
|----------|---------------|-------------|
| $1/24$   | $10^{-4}$     | $10^{17}$   |
| $1/30$   | $10^{-5}$     | $10^{16}$   |
| $1/123$  | $4.43 \times 10^{-21}$ | $10^{16}$ |
| $1/124$  | $3.03 \times 10^{-21}$ | $4.43$ |
| $125$    | $2.07 \times 10^{-21}$ | $3.03$ |

Table 6: $a_{GUT}$ and instanton suppression factor values against the instanton contribution to the U-quark masses, for Georgi-Glashow SU(5)-type GUTS, from eqn.(39).

We observe that for D-brane models with a lot of extra matter $a_{GUT} \gg 1$, instanton suppression factors become large enough, so that they contribute significant corrections to the mass of the U-quarks for Georgi-Glashow GUTS (Equivalently for the instanton contribution to the down quarks for Flipped SU(5) GUTS).

Apart from string instantons which give rise to such a superpotential mass term in eqn. (35), a perturbative contribution to the up-quark mass exists for the present $\mathbb{Z}_2 \times \mathbb{Z}_2'$ models in the form

$$M_u^{\text{pert}} = \frac{10(2,0)10(2,0)}{M_s^5} \langle \overline{5}_{(1,-1)} \rangle \cdot \langle \overline{5}_{(1,-1)} \rangle \cdot \langle \overline{5}_{(1,-1)} \rangle \cdot \langle \overline{5}_{(1,-1)} \rangle \cdot \langle 1_{(0,2)} \rangle \cdot \langle 1_{(0,2)} \rangle e^{-A}$$

(39)

giving a mass of order $\approx M_u e^{-A}$.

We have shown that the long standing problem of missing up(down) quark masses that has singled out the global SU(5)/flipped SU(5) D-brane models in orientifold compactifications of type II theory can be resolved in terms of the previously unknown Grand Unified SU(5) GUT invariants

$$10 \cdot 10 \cdot \overline{5}^H \cdot \overline{5}^H \cdot \overline{5}^H \cdot \overline{5}^H$$

(40)

from higher orders of perturbation theory. Hence in all SU(5) models in IIA theory from intersecting branes, or in its dual theory in 4D SU(5)’s from IIB with or without fluxes, the relevant up-quark masses gets generated, minimally, from the dim=9 mass terms (8). As we have noted a perturbative mass term is not possible for flipped SU(5) GUTS.

We have also shown that Euclidean E2-instantons based on the new couplings can also contribute NP corrections to the mass of the relevant quarks. The presented instanton related mass corrections arising in eqn.(40) from SU(5) gauge invariants are also valid for the flipped SU(5) GUTS.

These corrections can be also extended to perturbatively generate missing up-quark and also its instanton associated u-quark mass contributing invariants, in higher GUTS. These are the ones with SU(6)/flipped SU(6) gauge groups where
the problem of missing up(down)-quark masses is more acute as also the lepton masses are not allowed in perturbation theory. The relevant gauge invariants now appear in the form of

\[ 15 \cdot 15 \cdot \overline{6}^H \cdot \overline{6}^H \cdot \overline{6}^H \cdot \overline{6}^H, \]

in terms of SU(6) representations and are discussed in [82].

5 Acknowledgements

We thank L. Ibanez, F. Marchesano, R. Richter and A. Uranga for useful discussions.

6 Appendix B - \( T^6/\mathbb{Z}_2 \times \mathbb{Z}_2 \) Orientifold

In this section we summarize the main points of the construction, and the details of our notation we are using in section 3.

We start with type IIA theory on \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2) \); with the orbifold group generators \( \theta, \omega \) acting on the complexified coordinates of a factorizable \( T^6 = T^2 \times T^2 \times T^2 \) tori as

\[ \theta: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3); \quad \theta': (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3). \]  

We implement an orientifold projection by \( \Omega R \), where \( \Omega \) is world-sheet parity, and \( R \) acts as

\[ R: (z_1, z_2, z_3) \rightarrow (\overline{z}_1, \overline{z}_2, \overline{z}_3). \]

To cancel the RR charge of the O6-planes, we introduce D6-branes wrapped on three-cycles that are products of one-cycles in each of the three two-tori. Each one-cycle is described by the wrapping numbers \((n_a^i, l_a^i)\). Under an \( \Omega R \) reflection a cycle \((n_a^i, l_a^i)\) is mapped to \((n_a^i, -l_a^i)\). That means that at the level of the spectrum, for a stack of \( N_a \) D6-branes along the cycle \((n_a^i, l_a^i)\) we also need to include its image with wrapping numbers \((n_a^i, -l_a^i)\). A torus can be tilted or untilted. For a tilted torus \( l - n \) is even. Avoiding multiply wrapped branes requires that \( m \) and \( n \) are relatively coprime, for tilted/untitled tori.

By convention to describe rectangular and tilted tori cycles we define

\[ l_a^i \equiv m_a^i, \text{ rectangular,} \quad l_a^i \equiv 2 m_a^i = 2 m_a^i + n_a^i, \text{ tilted.} \]
The intersection numbers of the homology cycles are computed using
\[ I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n_{i,a}^{1}l_{a}^{i} - n_{i,b}^{1}l_{b}^{i}), \quad I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n_{i,a}^{1}l_{b}^{i} + n_{i,b}^{1}l_{a}^{i}) \]
\[ I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{-3-k} \prod_{i=1}^{3} (n_{i,a}^{1}l_{a}^{i}), \quad I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{-3-k} (-l_{a,a}^{1}l_{a}^{3} + l_{a,a}^{2}n_{a}^{3} + n_{a,a}^{1}l_{a}^{2} + n_{a,a}^{1}n_{a}^{2}l_{a}^{3}) \]

where \( k = \beta_1 + \beta_2 + \beta_3 \) is the total number of tilted tori; by definition \( \beta = 0 \) for untitled tori; \( \beta = 1 \) for tilted tori.

The open string spectrum for branes intersecting at angles appears in [62] and is described in table (7).

| Sector | Representation |
|--------|----------------|
| \( aa \) | \( U(N_a/2) \) vector multiplet |
| \( ab + ba \) | \( I_{ab} (\square, \square) \) fermions |
| \( ab' + ba \) | \( I_{ab'} (\square, \square) \) fermions |
| \( aa' + a'a \) | \( \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,O6}) \) fermions |
| | \( \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,O6}) \) fermions |

Table 7: Spectrum of intersecting D6-branes

7 Appendix C - \( T^6/\mathbb{Z}_2 \times \mathbb{Z}'_2 \) Orientifold

In this appendix we briefly review the \( T^6/\mathbb{Z}_2 \times \mathbb{Z}'_2 \) orientifold with Hodge numbers \((h_{11}, h_{12}) = (3, 51)\) following [73], to which we refer the reader for further details.

The orbifold group acts as in eqn.(42) and where the combination \( \theta \theta' \) acts as a reflection in the first and third torus. Also present are the bulk cycles described in terms of the fundamental one-cycles \([a^I], [b^I]\) of the \( I \)-th \( T^2 \) and the associating wrapping numbers \( n^I_{a} \) and \( \tilde{m}^I_{a} = m^I_{a} + \beta^I n^I_{a} \). We note that \( \beta^I \) takes the value 0 and 1/2 for rectangular and tilted tori, respectively. We have also present the \( g \)-twisted cycles. The action of the group elements, \( \theta, \theta' \) and \( \theta \theta' \) possess 16 fixed points which after blowing up give rise to two-cycles of \( \mathbb{P}_1 \). Together with the fundamental one-cycle invariant under the respective group action the \( g \)-twisted cycles are constructed as

\[ \Pi^B_a = 4 \otimes_{I=1}^{3} (n^I_{a}[a^I] + \tilde{m}^I_{a}[b^I]), \]

\[ \Pi^g_{ij} = \left[ \alpha^g_{ij} \times (n^g_{j}, \tilde{m}^g_{j}) \right]. \]
In particular $i, j \in \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ labels one of the 16 blown-up fixed points of the orbifold element $g = \theta, \theta', \theta\theta' \in \mathbb{Z}_2 \times \mathbb{Z}_2'$; $I_g$ represents the $g$-invariant one-cycle with $I_g = 3, 1, 2$ for $g = \theta, \theta', \theta\theta'$.

Rigid cycles are charged under all three sectors $\theta$, $\theta'$ and $\theta\theta'$ and take the form

$$\Pi^F = \frac{1}{4} \Pi^B + \frac{1}{4} \left( \sum_{i,j \in S_g} \epsilon^\theta_{ij} \Pi^\theta_{ij} \right) + \frac{1}{4} \left( \sum_{j,k \in S_g} \epsilon^{\theta'}_{jk} \Pi^{\theta'}_{jk} \right) + \frac{1}{4} \left( \sum_{i,k \in S_g} \epsilon^{\theta\theta'}_{ik} \Pi^{\theta\theta'}_{ik} \right),$$

(48)

where $S_g$ is the set of fixed points in the $g$-twisted sector. The $\epsilon^g_{ij} = \pm 1$ correspond to the two different orientation the brane can wrap the blown up $\mathbb{P}_1$; they are subject to other consistency conditions which can be seen in detail in [73], including RR tadpoles and homological K-theory theory constraints. We do not list tadpole equations as for the purpose of this work, we use only local models on the $\mathbb{Z}_2 \times \mathbb{Z}_2'$.

The orientifold action $\Omega R$ for the bulk cycles appears as

$$\Omega R : [(n_1, \tilde{m}_1)(n_2, \tilde{m}_2)(n_3, \tilde{m}_3)] \rightarrow [(n_1, -\tilde{m}_1)(n_2, -\tilde{m}_2)(n_3, -\tilde{m}_3)].$$

(49)

For the $g$-twisted cycle the orientifold action is defined as

$$\Omega R : \left[ \alpha^g_{ij} \times (n^I, \tilde{m}^I) \right] \rightarrow -\eta_{KR} \eta_{KR} \eta_{KR} \left[ \alpha^g_{R(i)R(j)} \times (n^I, -\tilde{m}^I) \right],$$

(50)

where $\eta_{KR} = \pm 1$ denotes the orientifold charges of the different sectors and obeys

$$\eta_{KR} \eta_{KR} \eta_{KR} \eta_{KR} = -1.$$

(51)

The reflection $R$ leaves all fixed points of an untilted two-torus invariant and acts on the fixed points in a tilted two-torus as

$$R(1) = 1, \quad R(2) = 2, \quad R(3) = 4, \quad R(4) = 3.$$

(52)

Under the orientifold action (49) the orientifold appears as

$$\pi_{O6} = 2^3 \eta_{KR} [(1, 0)(0, 0)] + 2^{3-2\beta_1-2\beta_2} \eta_{KR} [(0, 1)(0, -1)(1, 0)]$$

$$+ 2^{3-2\beta_2-2\beta_3} \eta_{KR} [(1, 0)(0, 1)(0, -1)] + 2^{3-2\beta_1-2\beta_3} \eta_{KR} [(0, -1)(0, 0)]$$

The chiral matter spectrum is given by the intersection numbers of table (8). Throughout the paper we associate to positive intersection number $\pi_a \circ \pi_b$ to matter transforming as the bifundamentals of $(a, \overline{b})$. See also comments in the introduction. Given two branes $a$ and $b$ intersection numbers for the bulk part are defined as

$$\pi_a^B \circ \pi_b^B = 4 \prod_{i=1}^{3} (n^i_a \tilde{m}^i_b - n^i_b \tilde{m}^i_a)$$

(53)
| Representation | Multiplicity |
|----------------|-------------|
| \( \pi_a \)   | \( \frac{1}{2} (\pi_a \circ \pi_a + \pi_a \circ \pi_{O6}) \) |
| \( \pi_a \)   | \( \pi_a \circ \pi_a - \pi_a \circ \pi_{O6} \) |
| \( (a, b) \)   | \( \pi_a \circ \pi_b \) |
| \( (a, b) \)   | \( \pi_a \circ \pi'_b \) |

Table 8: Chiral spectrum for intersecting D6-branes.

and for the twisted sector they are defined as

\[
\left[ \alpha_{ij}^g \times (n_a^{I_g}, \tilde{m}_a^{I_g}) \right] \circ \left[ \alpha_{kl}^h \times (n_b^{I_h}, \tilde{m}_b^{I_h}) \right] = 4 \delta_{ik} \delta_{jl} \delta_{gh} (n_a^{I_g} \tilde{m}_b^{I_h} - n_b^{I_h} \tilde{m}_a^{I_g}) . \quad (54)
\]

Furthermore, \( N=1 \) supersymmetry is preserved by the branes as long as each brane satisfies

\[
\tilde{m}^1 \tilde{m}^2 \tilde{m}^3 - \sum_{I \neq J \neq K} \frac{n^I n^J \tilde{m}^K}{U^I U^J} = 0 \quad (55)
\]

and

\[
n^1 n^2 n^3 - \sum_{I \neq J \neq K} \tilde{m}^I \tilde{m}^J n^K U^I U^J > 0 , \quad (56)
\]

where \( U^I \) denotes the complex structure modulus \( U^I = R^I_X / R^I_Y \) of the \( I - th \) torus with radii \( R^I_X \) and \( R^I_Y \).
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