Ballistic collective group delay and its Goos–Hänchen component in graphene

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Abstract

We theoretically construct an experimental observable for the ballistic collective group delay (CGD) of all the particles on the Fermi surface in graphene. First, we reveal that lateral Goos–Hänchen (GH) shifts along barrier interfaces contribute an inherent component in the individual group delay (IGD). Then, by linking the complete IGD to spin precession through a dwell time, we suggest that the CGD and its GH component can be electrostatically measured by the conductance difference in a spin precession experiment under weak magnetic fields. Such an approach is feasible for almost any Fermi energy. We also indicate that it is a generally nonzero self-interference delay that relates the IGD to the dwell time in graphene.

1. Introduction

Among various proposed expressions [1, 2], group delay [3, 4] ($\tau_g$, also known as phase time in the literature) and dwell time [4, 5] ($\tau_d$) are two well-established time durations taken by a particle in the process of quantum tunneling through a barrier. The group delay is the duration between the appearance time of the reflection or transmission particle pulse and the arrival time of the incident pulse, while the dwell time is the time a particle lingers in the barrier region (see figure 1(b)). They describe the tunneling speed in different aspects, and are both of paramount importance for solid-state devices working at high frequencies [6]. Recently, as graphene has risen as a star material in condensed matter physics, extensive investigations [7–14] have concentrated on the group delay and/or dwell time in it. Unlike in single-electron devices, the current in bulk graphene devices is contributed by numerous electrons or holes on half of the Fermi surface. This fact implies that the collective group delay (CGD) of all these particles is more relevant than the individual group delay (IGD) of a single particle [7, 9–14] for evaluating the tunneling speed in graphene. The CGD can be defined as the summation of mode ($n$)-dependent IGDs weighted with the corresponding transmission probability ($T_n$), i.e., $\tau_g^C \equiv \sum_n T_n \tau_g^{(n)}$.

The CGD is not directly observable. Very recently, an approach using a Larmor clock [5, 15] was suggested to measure the transmission times $\tau^{(p)}$ ($p = 1, 2, \tau^{(1)} = \tau_g^C / \sum_n T_n$) at the specific Fermi energy that aligns with the charge neutrality point (or Dirac point) of the barrier [8]. However, tunability of the Fermi energy is necessary in graphene devices. The dynamics in graphene completely alters when the Fermi energy aligns with or diverges from the barrier Dirac point (i.e., pseudodiffusive [8] for the former and ballistic [16, 17] for the latter case). Therefore, how to measure the ballistic CGD at an arbitrary Fermi energy is still a basic problem. The solution of this problem is the primary motivation of this work.

On the other hand, the ballistic transport in graphene is essentially two-dimensional (2D). As a result, a lateral Goos–Hänchen (GH) shift [18] occurs for the reflected or transmitted wavepacket along the corresponding interface [19–22] (see figure 1(b)). Such a GH shift makes an intrinsic contribution to the IGD, which however has not been noticed in previous studies of graphene. Therefore, the complete expression of the 2D IGD, which is also the foundation of the measurement problem of CGD, must be...
clarified. The derivation of this expression is our secondary motivation.

In this work, through an analogy between ballistic electrons and photons, we obtain the complete expression of the 2D IGD and show that the inherent GH component adds an asymmetric feature to the IGD’s energy dependence. This can be found in section 2. Then, by linking the 2D IGD to spin precession through the 2D dwell time, we further suggest that, for any Fermi energy comparable with the barrier height, the CGD and its GH component can be probed through conductance measurements in a spin precession experiment under weak magnetic fields. This is presented in section 3, followed by a summary and remarks in section 4.

2. The inherent GH component in the 2D IGD

To investigate the 2D IGD, let us consider the 2D quantum tunneling through a potential barrier in graphene (see figures 1(a) and (b)). The size of the graphene sample is set to be smaller than the electron mean free path ($\ell \sim 0.5–1 \, \mu m$ [16, 17]) and the phase-relaxation length ($L_{\phi} \sim 3–5 \, \mu m$ [16]) to ensure that the system stays in the ballistic coherent regime. A real potential $V(x)$ occupies the region $0 < x < l$ and is translationally invariant in the y-direction; it can be induced by a top gate due to the electric field effect [23]. The Fermi energy is $E$ and can be controlled by a back gate [23]. The sample width $W$ in the y-direction is several times $l$ to ensure that the edge details are not important [24].

At the Fermi surface an electron with a central incident angle of $\alpha \in (-\pi/2, \pi/2)$ is incident from the left side of the barrier. In the stationary state description, it can be represented as a wavepacket of a weighted superposition of plane wave spinors [19, 22],

$$\Psi_i = \int \mathcal{dk} f(k) e^{i(k_0 x + k_0 y - \frac{x}{2})} e^{i\alpha/2}, \quad (1a)$$

$$\Psi_f = \int \mathcal{dk} f(k) |r| e^{i(\phi_0 - k_0 x + k_0 y - \frac{x}{2})} e^{i\beta/2}, \quad (1b)$$

$$\Psi_t = \int \mathcal{dk} f(k) |t| e^{i(\phi_0 + k_0 x + k_0 y - \frac{x}{2})} e^{i\alpha/2}. \quad (1c)$$

Here $i$, $r$, $t$ stand for incident, reflection, and transmission, respectively. $k = k_0 \hat{x} + k_0 \hat{y} = (\cos \alpha \hat{x} + \sin \alpha \hat{y})E/hv_F$ is the 2D wavevector ($v_F$ is the Fermi velocity in graphene), $\bar{k} = k(\bar{\alpha})$, and $f(k - \bar{k})$ is the spectral distribution, which can be assumed to be of Gaussian profile. $r = |r| e^{i\phi_0}$ and $t = |t| e^{i\phi_0}$ are the reflection and transmission coefficients, respectively. $\beta = \pi - \alpha$. The solution of the plane waves and the calculation of $r$ and $t$ can be found in appendix A. Ignoring the sublattice degree of freedom expressed by the column vectors, the total phases for the incident, reflection, and transmission $(\xi = i, r, t)$ packet
peaks at barrier interfaces read $\phi_\xi^T = \phi_\xi + k_\xi \sigma_\xi - E \tau_\xi^T / \hbar$, where $\phi_\xi = 0$, $\phi_\tau = \phi_\phi$, $\phi_\gamma = \phi_\phi + k_\phi l$, and $\sigma_\xi (\tau_\xi^T)$ are the centrally appearing loci (moments) of the electron rather than of its two sublattice components (see figure 1(b)). When the total phases are expanded as a Taylor series about $k$, the integrands in equation (1) oscillate rapidly except where the total phase is stationary, i.e., $(\partial \phi_\xi^T / \partial k)_k = 0$. Due to the relation between $k$ and $(E, \alpha)$, this condition means two independent conditions, $(\partial \phi_\xi / \partial \alpha)_E = 0$ and $(\partial \phi_\xi^T / \partial E)_\alpha = 0$. These two conditions respectively give the central loci and moments

$$\sigma_\xi = - \frac{(\partial \phi_\xi)}{\partial \xi} \text{ } E,$$

(2)

$$\tau_\xi^T = \hbar \left[ \frac{(\partial(\phi_\xi + k_\xi \sigma_\xi))}{\partial E} \right]_{\alpha}.$$  (3)

Since $\sigma_\xi = \tau_\xi^T = 0$, equations (2) and (3) also give the lateral GH shifts [25, 22] (see figure 1(b)) and 2D IGDs in reflection and transmission. These results are similar to the optical case [25] and can be calculated straightforwardly as we obtain $r$ and $t$. For incident, reflection, and transmission, the $A$ and $B$ sublattice components diverge from the central loci by distances of $\pm 1/2k_{\xi, \alpha}, \mp 1/2k_{\xi, \alpha}$, and $\pm 1/2k_{\xi, \alpha}$, respectively (see figure 1(b)). This comes from the column vectors in equation (1). As a result, the GH shift and hence the group delay of the reflection is sublattice dependent. In the following, we only concern ourselves with the central loci and moments.

For asymmetric barriers, there is a difference between the reflection and transmission in both $\phi_\xi$ and $\sigma_\xi$, so a bidirectional 2D IGD can be defined as $\tau_\xi = \sum_\xi |\xi|^2 \tau_\xi^T$. In the 1D case the bidirectional IGD has the expression $\tau_\xi^T = \sum_\xi |\xi|^2 \hbar (\partial \phi_\xi / \partial E)_\alpha$, which stems solely from the phase shifts introduced by scattering at the interfaces [26]. Comparing the two expressions, we can find that there are two aspects of differences between the 2D and 1D cases. Firstly, the differential in the 1D case becomes a partial differential in the 2D one, i.e., $d/E \rightarrow (\partial/\partial E)_{\alpha}$. Secondly, besides the scattering phase shifts in the 1D case, the GH shifts also contribute inherent phase shifts in the 2D case, i.e., $\phi_\xi \rightarrow \phi_\xi + k_\xi \sigma_\xi$.

To highlight the intrinsic contribution of the GH shifts, we rewrite the bidirectional 2D IGD as

$$\tau_\xi = \tau_\xi^T + \tau_\xi^{GH}.$$  (4)

The first component $\tau_\xi^T = \sum_\xi |\xi|^2 \hbar (\partial \phi_\xi / \partial E)_\alpha$ comes from the scattering phase shifts at the interfaces, which is the same as the 1D case, while the second component $\tau_\xi^{GH} = \sum_\xi |\xi|^2 \sigma_\xi \sin \alpha/\sqrt{V}$ results from the GH shifts, which is totally absent in the 1D case. Note that we have used the relation $h (\partial k_\xi / \partial E)_{\alpha} = \sin \alpha/\sqrt{V}$ to get the GH component, since the GH shifts are not explicit functions of the energy. We would like to stress that, although the GH component displays a partial derivative with respect to $k_\xi$ at fixed $E$ (i.e., $\tau_\xi^{GH} = \sum_\xi |\xi|^2 \sigma_\xi \sin \alpha/\sqrt{V}$ and $\sigma_\xi = -(\partial \phi_\xi / \partial k_\xi)_{E}$), this component essentially comes from the condition that the energy gradient of the GH phase shifts (i.e., $\partial/\partial E$) must vanish. We can understand the GH component as follows. At time $\hbar (\partial \phi_\xi / \partial E)_{\alpha}$, the wavefront of the transmitted beam reaches $(l, 0)$ and then propagates freely with a velocity of $\sqrt{V}$ to the final position $(l, \alpha)$ (see figure 1(b)). This step will have cost duration of $\sigma_\xi \sin \alpha/\sqrt{V}$ since the wavefront is perpendicular to the propagation direction. This picture also holds for the reflected beam. A weighted average of the transmission and reflection gives $\tau_\xi^{GH}$.

Figure 2 shows clearly the contribution of the GH component to the 2D IGD. One can see that $\tau_\xi^{GH}$ is symmetric about the center $(E/V = \cos^{-2}\alpha)$ of the transmission gap (TG) due to the symmetry of $\phi_\xi$ about it. In contrast, $\tau_\xi^{GH}$ stemming from the quantum GH shifts is asymmetric about the TG’s center. The quantum GH shift displays the same trend as the classical shift predicted by Snell’s law ($\sigma_\xi = \lambda \alpha / E \cos \gamma \cos \gamma$ (Equation 3)) being the refracted angle). The latter is obviously asymmetric as it is negative (positive) in the low (high) energy range (see figure 2). In total, the GH component not only quantitatively contributes a part of order $\sigma_\xi \sin \alpha/\sqrt{V}$ to the 2D IGD, but also qualitatively results in the remarkable asymmetric feature in the energy dependence of the 2D IGD. The numerical results differ from previous investigations on the 2D IGD [9, 10, 14] by a value of $\tau_\xi^{GH} \sim \sigma_\xi \sin \alpha/\sqrt{V}$, which becomes obvious when the barrier (no matter whether it is electric or magnetic) becomes longer and/or the effective refracted angle (determined by $E$, $\alpha$, and the barrier structure) becomes larger. The results for $\alpha = 0$ (normally incident) recover previous investigations on the 1D IGD [7, 11, 13], since no GH shift occurs in this case.

3. Measuring the CGD by spin precession induced conductance difference

Having obtained the complete expression of the IGD (equation (4)), we now seek a physical observable for the CGD at arbitrary Fermi energies. The Larmor precession of
the electron spin in a magnetic field provides a clock for studying the electron dynamics [5, 8, 27]. Here, we consider a configuration in which the magnetic field (B) is applied in the graphene plane along the x-axis (see figure 1(c)). In such a configuration, dynamical perturbation by the Lorentz force is avoided, and the only effect of the magnetic field is to cause spin precession round the x-axis. For electrons with spins initially directed along the y-axis, the duration of this precession leads to different transmission probabilities in the z- and ḟ-directions \((T_{zy} - T_{zy}) / (T_{zy} + T_{zy})\). The spin directions are fixed in the ferromagnetic electrodes 2 and 3 magnetized along the required axis (see figure 1(c)).

We calculate \(T_{zy} / T_{zy}\) in a pseudospin–spin direct product space (see appendix B for details; a similar method is used in [8]). An explicit equality between the spin polarization of the transmission probabilities \(P \equiv (T_{zy} - T_{zy}) / (T_{zy} + T_{zy})\) and the dwell time (calculated by equation (A.5))

\[
\tau_d = \frac{\int |\Psi(x)|^2 dx}{\omega y \cos \alpha}
\]

is found in the weak-field limit,

\[
\tau_d = (P / \omega_y) |B| \to 0.
\]

Here, \(\omega_y = g \mu_B B\) is the Larmor frequency, \(g\) is the gyromagnetic factor in graphene, \(\mu_B\) is the Bohr magneton, and \(B = \hbar / 2 E_0\) is the reduced magnetic field strength. This equality is clearly demonstrated in figure 3. Note that this equality is restricted to symmetric structures [28]. Under this restriction equation (6) holds for arbitrary incident energies and angles and can be interpreted physically as follows. Rewriting the equality as \(\omega \tau_d = P\) and multiplying both sides by \(\hbar / 2\), the right-hand side gives the expectation value of \(S_z\) for the transmitted electrons. This expectation value is determined by the product of the spin precession frequency (i.e., the Larmor frequency) and the time for which the precession persists (i.e., the dwell time). This is just what the left-hand side expresses. It should be noted that equation (6) also holds in systems with parabolic dispersion relations and scale envelope functions [5].

To obtain the sum of the dwell time on half of the Fermi surface, we multiply equation (6) by \((T_{zy} + T_{zy}) / 2\) (which tends to \(T\) when \(B \to 0\)). Then the right-hand side of equation (6) can be further related to an experimental observable, the conductance \((\tilde{G})\). This is because the conductance is defined as \(G(E) = G_0 \sum_n T_n\), where \(G_0 = 2e^2 / h\) is the quantum conductance considering the twofold valley degeneracy. Thus, equation (6) can be rewritten as

\[
\sum_n \tau_d(n)T_n = \frac{G_{zy}(E) - G_{zy}(E)}{2 \omega_y G_0} \lim_{B \to 0}.
\]

where \(G_{zy}\) and \(G_{zy}\) are the spin-resolved conductances through the barrier. To avoid the electrode contact induced potential [29], a four point probe [30] should be adopted in the proposed experimental setup (see figure 1(c)). The intrinsic conductance can be obtained by the current through electrodes 1 and 4 and the voltage between electrodes 2 and 3 at \(V_{23} \to 0\) (i.e., \(I_{14} / V_{23} \to 0\)).

We now try to link the CGD to the above conductance difference by the relation between the 2D IGD and the 2D dwell time. This relation can be obtained by making a variation of the time-independent Dirac equation with respect to the two independent variational parameters, \((k_x, k_y)\) or equivalently \((E, \alpha)\), at the barrier boundary. We make the variation with respect to \(E\) and \(\sin \alpha\), since the variational results about them can be expeditiously related to \(\tau_d^{2D}\) and \(\tau_d^{GH}\), respectively. In the variation, the 2D feature of the tunneling process and the spinor nature of envelope functions should be taken into account. The derivation details can be found in appendix C. The concise result for a rectangular barrier reads

\[
\tau_d = \tau_g + \tau_i,
\]

where \(\tau_i = \hbar [\text{Re}(r) \cos \alpha + \text{Im}(r) \sin \alpha] / (E \sigma / \langle E \rangle)\) is a self-interference delay stemming from the interference of the incident and reflection envelope functions in front of the barrier [1, 31]. The correctness of this relation can be verified by numerically calculating and comparing the explicit expressions of \(\tau_d\), \(\tau_g\), and \(\tau_i\).

Figure 4 shows the 2D IGD, the dwell time, and the self-interference delay in reduced form as a function of the Fermi energy at a fixed incident angle, where \(\epsilon \equiv E / E_0\) is the reduction factor. One can see that the reduced self-interference delay \((-\epsilon \tau_i)\) is symmetric about the TG’s center; it achieves a maximum at the TG’s center and oscillates around zero outside the TG. The self-interference delay itself is important only in the low energy range (diverging as \(E^{-1}\) when \(E \to 0\)). It disappears for (anti)resonant tunneling since there is no interference in front of the barrier. Accordingly, the IGD almost coincides with the dwell time except within the low energy range or around the TG’s center.

The collective self-interference delay \((\tau_i^C)\) can be calculated by using \(\sum_n \tau_n(E) \tau_i^{(n)}(E) = M \int_{-\pi/2}^{\pi/2} \tau_i(E, \tilde{\alpha}) T(E, \tilde{\alpha}) d(\sin \tilde{\alpha})\), where \(M = |E| W / h v_F = (|E| / E_0)(W / 2 \pi l_0) \equiv M_F M_W\) is the number of transverse modes (which should be rather large). As can be seen in the inset of figure 4, \(\tau_i^C\) oscillates with \(E\), with the amplitude disappearing exponentially at a relatively high Fermi energy. Therefore,
equations (7) and (8) imply that, for arbitrary Fermi energies comparable with $V$, the CGD can be directly measured by the spin precession induced conductance difference under weak fields

$$\tau^C_E \approx \frac{G_{zy} - G_{\bar{z}y}}{2\omega_B G_0} \bigg|_{B \to 0}. \quad (9)$$

This result, considering the measure of the CGD at almost arbitrary Fermi energies, covers the result considering the measure of the transmission time $\tau^{(1)}$ at the barrier Dirac point in [8], where a totally different analytical procedure was adopted.

Figure 5 shows the CGD, its scattering component, and the magnetic field dependent conductance difference as a function of the Fermi energy. To clearly show the oscillation details in the low energy range, all these qualities are plotted for a single mode (i.e., divided by $M$). As is seen, the spin precession induced conductance difference increases for weaker $B$. At $B = 10^{-3}$, it is already a rather good measurement of the CGD for $E > 0.2$ V. Thus, we provide an electrostatic approach to measure the CGD in graphene, which is feasible for almost arbitrary Fermi energies. This approach looks interesting when we notice that the CGD describes a dynamic process while the conductance is static.

Experimentally, the rectangular barrier can be induced by a top gate close to the graphene plane with a distance comparable with the Fermi length. To measure the CGD, one can apply a small magnetic field and measure $I$ at various back gates. Then one can switch the magnetization direction of electrodes 3 and 4 from along the $z$- to along the $\bar{z}$-axis, and make the measurement again. Thereby, one obtains $G_{zy}$ and $G_{\bar{z}y}$ curves similar to those shown in the inset in figure 5. The ratio of $(G_{zy} - G_{\bar{z}y})/(2\omega_B G_0)$ gives an energy-dependent curve for the applied magnetic field. One then decreases the magnetic field and obtains more such curves, which should be found to no longer change obviously at small enough magnetic fields. For Fermi energies comparable with the barrier height, the unchanged curve is just the CGD we want.

In figure 5, it is also noted that, due to the different signs of the GH shifts, the scattering component is always larger (smaller) than the expected value of the CGD when $E < V$ ($E > V$). The difference becomes obvious at Fermi energies away from the barrier Dirac point. Then, a comparison between the experimentally observed value of the CGD as above and the theoretical prediction of the CGD calculated by equation (4) away from the barrier Dirac point can be utilized to probe the inherent GH component of the CGD or equivalently the intrinsic effect of the GH shifts on the CGD in graphene.

4. Conclusions and remarks

In summary, we have obtained the complete expression of the 2D IGD in graphene with the intrinsic contribution of the GH shifts being contained. Moreover, we have proposed an approach to probe the ballistic CGD and its inherent GH component by conductance measurements in a weak-field spin precession experiment. This approach is feasible for almost arbitrary Fermi energies. We have also indicated that it is a generally nonzero self-interference delay that relates the 2D IGD to the dwell time in graphene. The inherent GH component of the 2D IGD should also be present in other 2D ballistic coherent electronic systems, since its derivation does not depend on the specific electronic excitation of graphene. The feasibility of the proposed approach to measure the CGD in these systems is an interesting question.

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Appendix A. Solution of envelope functions and calculation of reflection and transmission

Let us begin with the single-particle Dirac equation that governs the low energy excitation in graphene. It reads as

\[ [v \mathbf{p} \cdot \mathbf{\sigma} + V(x)] \psi(x, y) = E \psi(x, y), \]  

(A.1)

where the pseudospin matrix \( \mathbf{\sigma} \) has components given by the Pauli matrices and \( \mathbf{p} = (p_x, p_y) = \hbar (k_x, k_y) \) is the momentum operator. \( V(x) = V \) inside the barrier and 0 outside. The eigenstates \( \psi(x, y) \) are two-component spinors with each component being the envelope function at the sublattice site \( A/B \) of the graphene sheet. Due to the translational invariance along the \( y \)-axis, the envelope function can be separated as \( \psi = [\psi_A(x), \psi_B(x)]^T e^{ik_y y} \) with \( k_y = E \sin \alpha / \hbar V \). The \( A \) and \( B \) components along the \( x \)-direction are related by a pair of coupled first-order equations

\[ \frac{\partial}{\partial x} \psi_{A,B} = \pm k_y \psi_{A,B} - \frac{E - V}{i \hbar V} \psi_{B,A}, \]  

(A.2)

which imply a decoupled second-order equation for both the \( A \)- and \( B \)-components

\[ \left[ \frac{\partial^2}{\partial x^2} + \frac{(E - V)^2 - E^2 \sin^2 \alpha}{(\hbar V)^2} \right] \psi_{A,B} = 0. \]  

(A.3)

Outside the barrier the time-dependent right-going envelope function reads

\[ \left( \begin{array}{c} e^{-i\omega t/2} \\ e^{i\omega t/2} \end{array} \right) e^{ik_y y - iEt/\hbar}, \]  

(A.4)

while inside the barrier it becomes

\[ \left( \begin{array}{c} e^{-i\omega t/2} \\ e^{i\omega t/2} \end{array} \right) e^{ik_y y - iEt/\hbar}. \]  

(A.5)

Here, \( k_x^2 + k_y^2 = (E/\hbar V)^2 \), \( k_x^2 + k_y^2 = [(E - V)/\hbar V]^2 \), and \( \sin \alpha' = \hbar V k_x/(E - V) \). Note that a basis is used to ensure that the product of the two sublattice components is real. The left-going waves can be obtained from the right-going ones by the replacement \( k_x^{(l)} \mapsto -k_x^{(r)} \), \( \alpha^{(l)} \mapsto -\alpha^{(r)} \equiv \beta^{(l)} \). For \( 1 - |\sin \alpha'|/\cos^2 \alpha < E/\hbar V < 1 + |\sin \alpha'|/\cos^2 \alpha \), \( k_y \) becomes imaginary. This range is called the transmission gap (TG), within which the transport is evanescent.

Matching the modes at \( x = 0 \) and \( x = l \) we obtain

\[ A4^+ + rA4^- = pA5^+ + qA5^-, \]  

(A.6)

and matching at \( x = l \) we get

\[ pA5^+ + qA5^- = tA4^+, \]  

(A.7)

where \( \pm \) stand for the right- and left-going waves. \( r = |r|e^{i\phi} \) and \( t = |t|e^{i\phi} \) are the reflection and transmission coefficients, and \( p \) and \( q \) are complex coefficients. These four coefficients can be solved by the four independent equations contained in equations (A.6) and (A.7).

Appendix B. Calculation of the spin-resolved transmissions

In the pseudospin–spin direct product space, the Hamiltonian is given by

\[ H = I_x \otimes H_3 - \frac{1}{2} \hbar \omega_B \sigma_3 \otimes I_{ps}, \]  

(B.1)

where \( I_x \) and \( I_{ps} \) are identity matrices in real spin and pseudospin space, respectively, and \( H_3 = \mathbf{v} \cdot \mathbf{\sigma} + V(x) \). On the left side of the barrier, the envelope functions for spin along the \( y \)- and \( \bar{y} \)-directions respectively read

\[ \frac{1}{2} \left( \begin{array}{c} 1 \\ -i \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x + i k_y/E \hbar V \end{array} \right) e^{\pm i k_x x + i k_y y}, \]  

(B.2)

\[ \frac{1}{2} \left( \begin{array}{c} 1 \\ i \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x + i k_y/E \hbar V \end{array} \right) e^{\pm i k_x x + i k_y y}. \]  

(B.3)

Inside the barrier, the envelope functions for spin along the \( x \)- and \( \bar{x} \)-directions respectively are

\[ \frac{1}{2} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x^+ + i k_y/(E - V)/\hbar V + B/l_0 \end{array} \right) e^{\pm i k_x x + i k_y y}, \]  

(B.4)

\[ \frac{1}{2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x^- + i k_y/(E - V)/\hbar V - B/l_0 \end{array} \right) e^{\pm i k_x x + i k_y y}. \]  

(B.5)

where \( \kappa_x^\pm = \sqrt{[(E - V)/\hbar V \pm B/l_0]^2 - k_y^2} \). On the right side of the barrier, the expressions for spin along the \( z \)- and \( \bar{z} \)-directions become

\[ \frac{1}{0} \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x \end{array} \right) \right) e^{\pm i k_x x + i k_y y}, \]  

(B.6)

\[ \frac{0}{1} \otimes \left( \begin{array}{c} 1 \\ \pm \kappa_x \end{array} \right) \right) e^{\pm i k_x x + i k_y y}. \]  

(B.7)

Matching modes at \( x = 0 \) and \( l \) we arrive at the following linear equations:

\[ B_{24}^+ + r_{3y} B_{24}^- = A_1 B_{44}^+ + A_2 B_{44}^- + A_3 B_{54}^+ + A_4 B_{54}^-, \]  

(B.8)

\[ A_1 B_{44}^+ + A_2 B_{44}^- + A_3 B_{54}^+ + A_4 B_{54}^- = t_{3y} B_{64}^+ + t_{3y} B_{74}^+, \]  

(B.9)

where \( r_{3y}, r_{3y}, t_{3y}, \) and \( t_{3y} \) are the reflection and transmission coefficients for an electron incident from electrode 2 reflected to electrode 2, and transmitted to electrode 3, respectively. Together with the complex coefficients \( A_1, A_2, A_3, \) and \( A_4, \) we have eight unknowns, determined by the eight independent equations contained in equations (B.8) and (B.9).
Appendix C. Derivation of the relation between 2D group delay and dwell time in graphene

We carry out the energy-variational form and conjugate form of equation (A.3) and upon integration over the length of the barrier we get

$$\left( \frac{\partial \psi_{AB}}{\partial E} \frac{\partial \psi_{AB}}{\partial x} - \psi_{AB} \frac{\partial^2 \psi_{AB}}{\partial E \partial x} \right) \bigg|_{x=0}^{x=l},$$

$$= \int_0^l 2E \cos^2 \alpha - 2V(x) (\frac{\psi_{AB}}{\hbar \nu E})^2 \, dx.$$  \hspace{1cm} (C.1)

It is found that when it is evaluated by the envelope function outside (inside) the barrier, the left (right) part can be related to $\tau_S^\lambda (\tau_d)$. However, we should note that equation (C.1) is only valid inside the barrier as the spatial derivative of $\psi_{AB}$ is not continuous on the potential boundary. To overcome this dilemma, we express $\psi_{AB}/\partial x$ inside the barrier by equations (A.2) and their conjugate forms. Since $\psi_{AB}$ itself is continuous, envelope functions inside the barrier can be replaced by the corresponding ones outside the barrier. Then the left part of equation (C.1) can be evaluated. For the $A$-component it reads as

$$J + K + \cos \alpha(-ir + ir^*) / \hbar \nu E,$$  \hspace{1cm} (C.2a)

and for the $B$-component, the result becomes

$$J + K + \cos \alpha (ie^{-2i\alpha} r - ie^{2i\alpha} r^*) / \hbar \nu E,$$  \hspace{1cm} (C.2b)

where $J = \frac{\hbar \nu E}{\hbar} [-B(0) - A(0)] |r|^2 \phi'_i + [B(0) - A(0)] |r| \phi_i$ and $K = \frac{\hbar \nu E}{\hbar} [-B(0) - A(0)] |r|^2 \phi_i + [B(0) - A(0)] |r| \phi'_i$. Here, the relation of lossless barriers $|r|^2 + |r|^2 = 1$ has been used and the following notations are adopted: $O' \equiv (\partial O/\partial E)_{\psi N}$, $A(\lambda) = \sin \alpha + \dot{\lambda}(x) e^{i\alpha}$, $B(\lambda) = \sin \alpha - \dot{\lambda}(x) e^{-i\alpha}$, and $\dot{\lambda}(x) = 1 - V(x)/E$, the ratio of the kinetic energy inside and outside the barrier.

Since $\psi^* \psi = \psi_A^* \psi_A + \psi_B^* \psi_B$, the relation for each spinor component should be added to get the expression for the spinor, which at last reads

$$\int_0^l [\lambda(\lambda) - \sin^2 \alpha] |\psi(\psi)|^2 \, dx$$

$$\begin{align*}
\frac{\nu E}{\hbar} \cos \alpha & = \lambda(0)[r|^2 \phi'_i + \lambda(x)|r| \phi_i - i \dot{\lambda}(0) \hbar E |r|'| - i \dot{\lambda}(x) \hbar |r|' |r|^2 + \hbar \text{Re}(r) \cos \alpha + \text{Im}(r) \sin \alpha] \sin \alpha \frac{\partial k}{\partial E},
\end{align*}$$

where Re$(r)$ (Im$(r)$) is the real (imaginary) part of $r$. We can see that this equation is a general result that relates the integral of the weighted probability density inside the barrier (left part) and the weighted energy-variational behavior outside the barrier (right part). It is noted that the factor $\lambda(x)$ has a critical role in the expression.

To clearly relate the general result (C.3) with both $\tau_d$ and $\tau_G^\lambda$, we consider the restricted condition that $\lambda(x)$ is a constant under the barrier (i.e., a rectangular barrier). Note that this condition is not necessary for the common semiconductor case [26], a reflection of the spinor nature of graphene. Under such a condition, the third and fourth terms on the right-hand side of equation (C.3) disappear due to the lossless condition of the barrier, $|r| |r|^\prime + |r|^\prime |r| = 0$, and equation (C.3) can be rewritten in terms of $\tau_d$ and $\tau_G^\lambda$, i.e., as a sub-relation

$$\tau_d (\lambda - \sin^2 \alpha) = \tau_G^\lambda \lambda + \tau_G \cos^2 \alpha,$$  \hspace{1cm} (C.4)

where a self-interference delay is found from the last term of equation (C.3),

$$\tau_i = \frac{\hbar [\text{Re}(r) \cos \alpha + \text{Im}(r) \sin \alpha] \sin \alpha}{E \cos^2 \alpha}.$$  \hspace{1cm} (C.5)

Graphene is two-dimensional, which means that there are two independent variation parameters, $k_x$ and $k_y$ or $E$ and $\sin \alpha$. We have made the variations about $E$; the variation of equation (A.3) about $\sin \alpha$ reads

$$\left( \frac{\partial \psi_{AB}}{\partial \sin \alpha} - \psi_{AB} \frac{\partial^2 \psi_{AB}}{\partial \sin \alpha \partial x} \right) \bigg|_{x=0}^{x=l},$$

$$= \int_0^l -2E \sin \alpha - 2V(x) (\frac{\psi_{AB}}{\hbar \nu E})^2 \, dx.$$  \hspace{1cm} (C.6)

Following a similar method to that above, we straightforwardly obtain the sub-relation between $\tau_d$ and $\tau_G^\lambda$ under the same restricted condition of a constant $\lambda$. The sub-relation reads

$$\tau_d \sin^2 \alpha = \tau_G^\lambda \lambda + \tau_G (\lambda - \cos^2 \alpha).$$  \hspace{1cm} (C.7)

Making a simple addition of the two sub-relations in equations (C.4) and (C.7) and taking into account $\tau_g = \tau_S^\lambda + \tau_G^\lambda$ (see equation (4) in the main paper), we finally get

$$\tau_d = \tau_g + \tau_i.$$  \hspace{1cm} (C.8)

The correctness of this relation and the sub-relations can be verified by numerically calculating and comparing the explicit expressions of the five times. It may be valuable to indicate that many previous works [9–12] have also concerned such a relation and they all reached the same conclusion that $\tau_d = \tau_g$ (actually this means $\tau_d = \tau_S^\lambda$ since $\tau_G^\lambda$ is not included in their definition of the 2D IGD). The results obtained here clearly indicate that it is a self-interference delay that relates the group delay and dwell time in graphene, similarly to the common semiconductor case [26]. The numerical results for $\tau_d$ calculated by $\tau_S^\lambda$ in [9, 10] deviate from the original definition (5) due to the absence of the two terms $\tau_G^\lambda$ and $\tau_i$. The deviation is large in the low energy range where both $\tau_G^\lambda$ and $\tau_i$ become obvious.

For normal incidence or the 1D tunneling case ($\alpha = 0$), $\tau_i$ vanishes (see the factor $\sin \alpha$ in equation (C.5)), since there is no reflected portion and thus no interference happens in front of the barrier due to the Klein tunneling [32]. The relation and sub-relations revealed in equations (C.8), (C.4), and (C.7) and the expression for the self-interference delay in equation (C.5) also hold for the tunneling of massless Dirac particles in topological surface states [33], where the real electron spin rather than the sublattice structure in graphene provides the Dirac structure.
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