Gain without inversion in a V-type system with low coherence decay rate for the upper levels.

Keywords: V-type three level system, Electromagnetically Induced Transparency, Lasing without inversion, Atomic Coherence, Autler Townes effect

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Abstract

Inversionless gain is observed in a V-type inhomogeneously broadened system without introducing any incoherent pumping and only by changing the collisional dephasing decay rate. In this system sub-Doppler linewidth is achieved with off-resonance pump detuning.
I. Introduction:

Over the last few years, atomic coherence effect such as coherent population trapping (CPT) [1,2], electromagnetically induced transparency (EIT) [3-5], gain without inversion (GWI) and its companion, lasing without inversion (LWI) [6-8], enhancement of refractive index [9,10], line narrowing [11] etc have been studied extensively both theoretically and experimentally due to their multiple applications in various fields. The area of application has a range from laser cooling to isotope separation and from ultrahigh-sensitive magnetometers to slowing down of light pulse [12,13]. One of the most potential applications of the atomic coherence is to extend the conventional laser sources to ultra-violet and possibly X-rays and even Gamma-ray spectral range, where the conventional methods based on population inversion are not available or are difficult to implement. This is performed through the conversion of incoherent energy into coherent light by a technique known as LWI [6]. In any commonly considered basis the amplification without inversion occurs due to the quantum interference between the two dressed-states created by the strong pump field [14]. The line shape of the probe absorption of a three level system is controlled by the four different contributions, among them two are absorptive and other two are dispersive contributions [15]. The absorptive contribution is always positive whereas the dispersive contribution is positive or negative depending upon the type of the scheme considered.

Most of the theoretical and experimental work on EIT and LWI are performed on three level systems (TLS) in the V, lambda (Λ) and cascade (Ξ) schemes. This is because these types of systems are easily available in various atomic samples such as Rb, Na, Cs etc. In recent years, EIT in Doppler broadened media has been observed experimentally using cw lasers as coupling lasers [16-18]. Recently Ahufinger et al [19] proposed an experiment for obtaining 0.4% inversionless gain in a cascade type TLS system applicable to cold free $^87$Rb atoms. Wu et al [20] have studied gain with or without inversion in a V-type system with two near degenerate excited levels driven by a strong pump and a weak probe field. They found that, due to quantum interference between two spontaneous decay channels, even in the absence of an incoherent pumping the probe gain is achieved. Boon et al [21] predicted an inversionless gain in a V-type mismatched Doppler-broadened medium and made a comparison on inversionless gain between matched and mismatched systems. In this article, we report gain without inversion in a Doppler-broadened V-type system without introducing any incoherent pumping. An analytical expression (first order in probe Rabi-frequency) of probe transition is obtained by solving the density matrix equations. We observed the gain without inversion via electromagnetically induced transparency in a Doppler broadened system. We observe that the significant EIT window is produced for a pump Rabi-frequency greater than the homogeneous but much less than the Doppler width. We also observe that EIT window crosses the zero line (Fig. 4(a)) with decreasing level dephasing rate ($\Gamma_{32}$). Off resonance pump detuning gives linewidth narrowing phenomenon for the V-type system.
II. Theory:

We consider a V-type three level system (T.L.S) having one common ground level \( |1\rangle \) and two upper levels \( |2\rangle \) and \( |3\rangle \) as shown in Fig.1 with energies \( \hbar \omega_1, \hbar \omega_2 \) and \( \hbar \omega_3 \) respectively. \( |1\rangle \rightarrow |2\rangle \) and \( |1\rangle \rightarrow |3\rangle \) are two dipole allowed transitions. The strong coherent pump or control field of frequency \( \Omega_1 \) with electric field amplitude \( \varepsilon_1 \) couples the \( |1\rangle \rightarrow |2\rangle \) transition. A weak probe field of frequency \( \Omega_2 \) with electric field amplitude \( \varepsilon_2 \) couples the \( |1\rangle \rightarrow |3\rangle \) transition whose dispersion and absorption signal we are interested in. The control and probe Rabi-frequencies are defined as \( \chi_1 = \frac{\varepsilon_1 \mu_{12}}{\hbar} \) and \( \chi_2 = \frac{\varepsilon_2 \mu_{13}}{\hbar} \), where \( \mu_{12} \) and \( \mu_{13} \) are the electric dipole moment of the two allowed transitions. The upper level(\( |3\rangle \)) decays to level \( |2\rangle \) (due to collision) and \( |1\rangle \) with decay rates \( \Gamma_{32} \) and \( \Gamma_{31} \). The level \( |2\rangle \) decays to \( |1\rangle \) with a decay rate \( \Gamma_{21} \). The interaction Hamiltonian of the atom and two fields can be written as

\[
H_I = -\frac{\hbar}{2}[\chi_1 e^{-i\Omega_1 t}|1\rangle\langle 2| + \chi_2 e^{-i\Omega_2 t}|1\rangle\langle 3|] + H.C. \tag{1}
\]

The density-matrix equations of motion may be obtained from Liouville equations under rotating wave approximation [22] we obtained the following density-matrix equations;

\[
\rho_{11} = i[\chi_1 (\rho_{21} - \rho_{12}) + \chi_2 (\rho_{31} - \rho_{13})] + \Gamma_{21}\rho_{22} + \Gamma_{31}\rho_{33} \tag{2}
\]

\[
\rho_{22} = i[\chi_1 (\rho_{12} - \rho_{21})] - \Gamma_{21}\rho_{22} + \Gamma_{32}\rho_{33} \tag{3}
\]

\[
\rho_{33} = i[\chi_2 (\rho_{13} - \rho_{31})] - \Gamma_{31}\rho_{33} - \Gamma_{32}\rho_{33} \tag{4}
\]

\[
\rho_{12} = i[\chi_1 (\rho_{22} - \rho_{11}) + \chi_2 \rho_{32}] - \rho_{12}(\gamma_{12} + i\Delta_1) \tag{5}
\]

\[
\rho_{13} = i[\chi_2 (\rho_{33} - \rho_{11}) + \chi_1 \rho_{23}] - \rho_{13}(\gamma_{13} + i\Delta_2) \tag{6}
\]

\[
\rho_{23} = i[\chi_1 \rho_{13} - \chi_2 \rho_{21}] - \rho_{23}(\gamma_{23} - i(\Delta_1 - \Delta_2)) \tag{7}
\]

and

\[
\rho_{11} + \rho_{22} + \rho_{33} = 1 \tag{8}
\]

where \( \gamma_{12}, \gamma_{13} \) and \( \gamma_{23} \) are decay rates for the off-diagonal elements between \( |1\rangle \) and \( |2\rangle \), \( |1\rangle \) and \( |3\rangle \) and \( |2\rangle \) and \( |3\rangle \) respectively. Off-diagonal or coherence decay rates(\( \gamma_{ij} \)) can be written in terms of the level or population decay rates(\( \Gamma_{ij} \))[23,24] as \( \gamma_{ij} = \frac{(\Gamma_{31} + \Gamma_{12})}{2}, \gamma_{12} = \frac{(\Gamma_{21} + \Gamma_{32})}{2} \).
where $\omega_{12}$ and $\omega_{13}$ are the transition frequencies corresponding to $|1>\rightarrow|2>$ and $|1>\rightarrow|3>$ transitions. The dispersion and absorption signal are determined from $\rho_{13}$ (first order in $\chi_2$) and keeping $\chi_1$ to all orders. Solving equations (2-8) in steady state we obtain

$$\rho_{13}^1 = -i\chi_2 \left[ \rho_{11}^0 (\gamma_{23} - i(\Delta_1 - \Delta_2)) + i\chi_1 \rho_{21}^0 \right] \frac{\chi_1^2}{[\chi_1^2 + (\gamma_{13} + i\Delta_2)(\gamma_{23} - i(\Delta_1 - \Delta_2))]^2}$$  \hspace{1cm} (9)

where $\rho_{11}^0$ and $\rho_{21}^0$ are the zeroth order contributions, written as

$$\rho_{11}^0 = \frac{2\chi_1^2\gamma_{12} + \Gamma_{21}(\gamma_{12}^2 + \Delta_1^2)}{\Gamma_{21}(\gamma_{12}^2 + \Delta_1^2) + 4\chi_1^2\gamma_{12}}$$ \hspace{1cm} (10)

and

$$\rho_{21}^0 = \frac{i\chi_1\Gamma_{21}(\gamma_{12} + i\Delta_1)}{\Gamma_{21}(\gamma_{12}^2 + \Delta_1^2) + 4\chi_1^2\gamma_{12}}$$ \hspace{1cm} (11)

Equ(9) gives us the absorption($\text{Im}(\rho_{13}^1)$) and dispersion($\text{Re}(\rho_{13}^1)$) signal of the probe transition. At low value of pump Rabi-frequency($\chi_1$) $\text{Im}(\rho_{13}^1)$ gives a Lorentzian lineshape corresponding to $|1>\rightarrow|3>$ transition. At higher value of $\chi_1$ it will split into two symmetric components commonly known as A-T doublet [25] when $\Delta_1 = 0$. To study the gain in the system we put $\Delta_1 = \Delta_2 = 0$ and obtain

$$\text{Im}(\rho_{13}^1) = \frac{\chi_1^4\Gamma_{21}\gamma_{12} - (2\chi_1^4\gamma_{21}\gamma_{23} + 2\chi_1^2\gamma_{12}\gamma_{13}\gamma_{23} + \gamma_{23}^2\gamma_{13}\Gamma_{21}\gamma_{12})}{(\Gamma_{21}\gamma_{12}^2 + 4\chi_1^2\gamma_{12})(\chi_1 + \gamma_{13}\gamma_{23})^2}$$ \hspace{1cm} (12)

So for $\Gamma_{32}(\gamma_{23} = \frac{\Gamma_{32}}{2}) \rightarrow 0$, $\text{Im}(\rho_{13})$ become positive for large value of $\chi_1$ leading to gain.

Non zero pump detuning gives an asymmetry in A-T doublet, the position of the A-T peaks can be found from the pole structure of the Equ(9). The real part of the pole structure gives the position of the resonance and the imaginary part represents the resonance width. The position of the resonances are

$$\Delta_2 = \frac{\Delta_1}{2} \pm \frac{1}{2} \sqrt{\Delta_1^2 + 4\chi_1^2}$$ \hspace{1cm} (13)

and the corresponding linewidths are

$$\Gamma = \frac{\Gamma_{31} + 2\Gamma_{32}}{4} \left[ 1 + \frac{\Delta_1\Gamma_{31}}{\sqrt{\Delta_1^2 + 4\chi_1^2}(\Gamma_{31} + 2\Gamma_{32})} \right]$$ \hspace{1cm} (14)

So for $\Delta_1 >> \chi_1$ the peak at $\Delta_2 = (\frac{\Delta_1}{2} + \frac{1}{2} \sqrt{\Delta_1^2 + 4\chi_1^2})$ has a linewidth smaller than the natural linewidth. Where as the other peak at $\Delta_2 = (\frac{\Delta_1}{2} - \frac{1}{2} \sqrt{\Delta_1^2 + 4\chi_1^2})$ is broadened more than the natural width so that they are equal to the unperturbed linewidth. This feature is also valid when the Doppler broadening is taken into account, in that case one peak has sub-Doppler width where the other peak is greater than the Doppler width. Similar result was obtained by Vemuri et al [11] in $\Lambda$ and cascede systems.
To obtain the probe absorption in a Doppler broadened system the Doppler broadening should be taken into account. For this purpose Eq.(9) should be integrated over the whole velocity range, the velocity distribution is conventionally taken as Maxwellian. To introduce the velocity of atom we change the pump and probe detuning as $\Delta_1(v) = (\Delta_1 \pm \frac{v}{c} \Omega_1)$ and $\Delta_2(v) = (\Delta_2 \pm \frac{v}{c} \Omega_2)$ where ± sign indicate the co and counter propagating pump and probe field propagation. We did our numerical calculation with co-propagating probe and pump beam. If the number of atoms with velocity $v$ per unit volume is $N(v)dv$, then the absorption of the probe laser is

$$\alpha = \int_{-\infty}^{+\infty} Im(\rho_{13}^{1}) \times \left( \frac{1}{u\sqrt{\pi}} \right) exp\left(-\frac{v^2}{u^2}\right) dv$$

(15)

Dispersion can be found similarly,

$$\beta = \int_{-\infty}^{+\infty} Re(\rho_{13}^{1}) \times \left( \frac{1}{u\sqrt{\pi}} \right) exp\left(-\frac{v^2}{u^2}\right) dv$$

(16)

where $u = \sqrt{\frac{2RT}{M}}$ denote the most probable velocity of the atom, and it is related to the Doppler width ($W_d$) of the probe spectrum as $W_d = (\omega_{13}\sqrt{ln2})c$.

III. Numerical calculation and results:

We study the absorption and dispersion signals of the probe transition in a Doppler broadened background for various values of (i) pump Rabi-frequencies($\chi_1$) (ii) Doppler width($W_d$ HWHM) and (iii) $\Gamma_{32}$. In Fig. 2 Electromagnetically Induced Transparency(EIT) is observed in a Doppler broadened back-ground with increasing pump Rabi-frequency. For numerical calculation in Fig. 2 we take the values of all population decay rates($\Gamma_{ij}$) as equal to 6 MHz and $W_d = 280$ MHz and in all the figures the probe Rabi-frequency($\chi_2$) is taken as 0.6 MHz. Here pump field is held on-resonance($\Delta_1 = 0$) with $|1\rangle \rightarrow |2\rangle$ transition. At zero value of pump field($\chi_1 = 0$) a Doppler broadened probe absorption signal is obtained(dotted curve in Fig. 2(a)). With increasing pump field intensity the absorption of the probe signal on the line center vanishes(solid curve of Fig. 2(a)) due to the quantum interference between the dressed states created by strong pump field[4]. As the probe absorption is cancelled in the line center the probe dispersion is increasing(solid curve of Fig. 2(b)) that means the system becomes more dispersive the phenomenon is known as EIT.

For non zero pump detuning($\Delta_1 \neq 0$) peak at $\Delta_2 = (\frac{\Delta_1}{2} + \frac{1}{2}\sqrt{\Delta_1^2 + 4\chi_1^2})$ is reduced in linewidth by a factor $[1 - \frac{\Delta_1\Gamma_{11}^{1}}{\sqrt{\Delta_1^2 + 4\chi_1^2}(\Gamma_{31} + 2\Gamma_{32})}]$ so it becomes narrow, while the other peak at $\Delta_2 = (\frac{\Delta_1}{2} - \frac{1}{2}\sqrt{\Delta_1^2 + 4\chi_1^2})$ is broadened by the same factor. We study this phenomena for various value of Doppler width($W_d$) starting from 20 MHz (dotted curve of Fig. 3(a)) to 280 MHz(solid curve of Fig. 3(a)) with a off-resonance pump detuning($\Delta_1$) equal to 100 MHz and $\chi_1 = 60$ MHz.

For a pump field Rabi-frequency($\chi_1$) = 60 MHz on-resonance with $|1\rangle \rightarrow |2\rangle$ transition ($\Delta_1$
To observe it we decrease the value of $\Gamma_{32}$ from 6 MHz (dotted curve of Fig. 4(a)) to zero (solid curve of Fig. 4(a)) at an interval of 2 MHz. It is seen from the figure that with reduction of $\Gamma_{32}$ EIT window increases and gain is achieved (equ. 12). It is clear that collisional dephasing changes the nature of the spectrum. A Rabi-like side band structure is observed in the gain profile equally spaced about the line center due to the split of ground state level. This type of splitting was previously observed in V-type system [26,27,21]. We study the effect of Doppler broadening on gain profile and observed that with increasing Doppler width ($W_d$) gain is decreases. The GWI predicted in this work may be observed for sodium D$_1$ transition ($S_{1/2} \rightarrow P_{3/2}$), $F = 1 \rightarrow F' = 1, 2$ or $F = 2 \rightarrow F' = 1, 2$ [28] where the decay rate $\Gamma_{32}$ will be negligible.

**IV. Conclusions:**

In this work we have presented the role of collisional dephasing and off-resonance pump detuning in a V-type Doppler broadened system in presence of a strong pump or coupling field. Recently Wu et al [24] observed how the coherent hole burning phenomena in a Λ-type Doppler broadened system change with atomic collision rate. Here we observed the probe transmission get amplified ($\text{Im}(\rho_{13}) > 0$) with decreasing $\Gamma_{32}$ and the maximum amplification is observed when $\Gamma_{32} = 0$. So it is seen that collisional dephasing act to reduce the effects of quantum coherence. In a V-type system if the upper levels are closely spaced then we can not ignore $\Gamma_{32}$ but if they are well spaced then we can ignore it.

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Figure Captions

**Figure 1**: Schematic diagram of a Doppler broadened three-level V-type atomic system driven by two coherent fields.

**Figure 2**: Probe absorption(a) and dispersion(b) signal in a Doppler broadened back ground with probe detuning($\Delta_2$) for various pump Rabi-frequency($\chi_1$). Other parameters are $W_d = 280$ MHz, $\Gamma_{ij} = 6$ MHz, $\chi_2 = 0.6$ MHz and $\Delta_1 = 0$.

**Figure 3**: Probe absorption(a) and dispersion(b) signal with probe detuning for various value of $W_d$. Other parameters are $\chi_1 = 60$ and $\chi_2 = 0.6$ MHz, $\Gamma_{ij} = 6$ MHz and $\Delta_1 = 100$ MHz.

**Figure 4**: Probe absorption(a) and dispersion(b) signal with probe detuning for various value of $\Gamma_{32}$. Other parameters are $W_d = 280$ MHz, $\chi_1 = 60$ and $\chi_2 = 0.6$ MHz, $\Gamma_{21} = \Gamma_{31} = 6$ MHz and $\Delta_1 = 0$. 
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