RG-invariant Sum Rule in a Generalization of Anomaly Mediated SUSY Breaking Models

Marcela Carena (a,b), Katri Huitu (c) and Tatsuo Kobayashi (d)

(a) Theory Division, CERN, CH-1211 Geneva 23, Switzerland
(b) Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
(c) Helsinki Institute of Physics, FIN-00014 University of Helsinki, Finland
(d) Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract

We study a generalization of anomaly-mediated supersymmetry breaking (AMSB) scenarios, under the assumption that the effects of the high-scale theory do not completely decouple and D-term type contributions can be therefore present. We investigate the effect of such possible D-term additional contributions to soft scalar masses by requiring that, for non-vanishing, renormalizable Yukawa couplings $Y^{ijk}$, the sum of squared soft supersymmetry breaking mass parameters, $M_{ijk}^2 \equiv m_i^2 + m_j^2 + m_k^2$, is RG-invariant, in the sense that it becomes independent of the specific ultraviolet boundary conditions as it occurs in the AMSB models. These type of models can avoid the problem of tachyonic solutions for the slepton mass spectrum present in AMSB scenarios. We implement the electroweak symmetry breaking condition and explore the sparticle spectrum associated with this framework. To show the possible diversity of the sparticle spectrum, we consider two examples, one in which the D-terms induce a common soft supersymmetry breaking mass term for all sfermion masses, and another one in which a light stop can be present in the spectrum.
1 Introduction

Supersymmetry (SUSY) provides a well-motivated extension of the Standard Model (SM) with an elegant solution to the so-called naturalness problem associated to the SM Higgs sector. In low energy supersymmetric models the electroweak scale is naturally of the order of the soft SUSY breaking parameters of the theory. Much work has been done in the search for an appropriate mechanism for SUSY breaking, but, at present, it remains unknown.

Two types of SUSY breaking mediation mechanisms, supergravity-mediated \cite{1} and gauge-mediated \cite{2}, have been studied extensively. Recently, another type of mediation mechanism, i.e. Anomaly–Mediated Supersymmetry Breaking (AMSB), has become under scrutiny \cite{3,4,5}. Each of these mechanisms has unique aspects which differ from each other. One of the important aspects of AMSB is that the soft SUSY breaking terms are Renormalization Group (RG) invariant, in the sense that they become independent of the specific ultraviolet boundary conditions. In fact, the magnitudes of the soft SUSY breaking terms at any scale are obtained in terms of values of the related gauge and Yukawa couplings at that scale, hence, they are determined as a function of the measured values of those couplings at the weak scale. Thus, the AMSB scenario yields definite phenomenological predictions. However, within the framework of the minimal supersymmetric standard model (MSSM), one of the predictions is quite problematic, since it implies the existence of negative values for the slepton squared masses. The simplest way to solve this problem is to add a universal contribution \( m_0 \) to all soft SUSY breaking scalar masses \cite{3,8}. There is, however, no dynamical explanation for the origin of this term and there is no obvious reason why extra contributions should not appear as well for the other soft SUSY breaking terms: gaugino masses and the trilinear Yukawa couplings \( A_f \) which couple Higgs and scalar fermion fields. Moreover, the addition of the universal value \( m_0 \) to all soft scalar masses, although provides a solution to the tachyonic spectra, it also violates the RG-invariance, which is one of the most attractive aspects in the AMSB scenario.

Independently of recent works on anomaly-mediated SUSY breaking, RG-invariant relations of soft SUSY breaking terms have been studied in the literature\cite{9,10}. The relation

\[ m_0 \]

\[ \text{See also Ref.}[6,7]. \text{In Ref.}[6] anomaly mediation has been discussed within the framework of supergravity. \]
to the case of anomaly mediated SUSY breaking has been clarified in Ref. [11]. In Ref. [10] the RG-invariant sum rule of soft scalar masses has been discussed and its importance has been emphasized. That is, the sum of three scalar masses squared, \( M_{i,j,k}^2 \equiv m_i^2 + m_j^2 + m_k^2 \), corresponding to chiral fields for which the Yukawa couplings \( Y^{ijk} \neq 0 \) are allowed, e.g. \((i,j,k) = (Q,u,H^2), (Q,d,H_1)\) and \((L,e,H_1)\) is more important to RG-invariance than any one of the scalar mass terms independently. This is the case since such a sum appears in the \( \beta \)-functions of the Yukawa couplings and soft SUSY breaking masses themselves. Hence, one could allow for additional contributions to each of the soft scalar masses, as long as the sum itself is not affected. For example, this situation can be realized by additional D-term contributions, which are proportional to a charge \( q_i \) of the field under a broken symmetry [12, 13] and such that in the allowed Yukawa couplings the charge must be conserved, \( q_i + q_j + q_k = 0 \). Then, in the sum of the soft SUSY breaking parameters, \( M_{i,j,k}^2 \), the D-term contributions cancel each other.

In the present work we shall investigate the possibility of having a similar behaviour as the one explained above, assuming a generalization of anomaly-mediated SUSY breaking models with residual, non-decoupling effects from extra \( U(1) \)'s at a high energy scale. In [3] it was concluded that all effects coming from a high scale theory decouple in pure anomaly mediation in the absence of light singlets. The authors of ref. [3] explored also extensions of the AMSB scenarios in which non-decoupling effects survive at low energies allowing, for example, for genuine D-term contributions. In a generic framework, D-term contributions have been proposed as a solution to the tachyonic slepton mass problem both in Refs. [3, 4]. Here we shall analyse the features of the particle spectrum depending on the specific charge assignments of the additional D-term contributions to each of the soft scalar mass parameters. We shall then explore the regions of the parameter space for which no tachyonic slepton masses appear. A novel point of the present work is to study the phenomenological aspects of these theories, making use of the RG-invariant sum of the soft SUSY breaking scalar squared masses. In this way, we preserve one of the most appealing features of the anomaly-mediated SUSY breaking scenarios, namely its high predictivity with induced soft

\[ ^2Q=(u,d) \] and \( L=(\nu_e, e) \) are the SU(2) left handed superfield doublets; \( H_1, H_2 \) are the two Higgs doublets and \( u, d, e \) are SU(2) right-handed superfield singlets.
masses which are independent of flavour physics, and we cure the main problem associated to them, namely the tachyonic solutions for the slepton sector.\footnote{There has been other attempts of constructing generalizations of AMSB models, curing the problem of tachyonic solutions in the slepton sector and still assuring a scale invariance of the solutions, like the anti-gauge mediated model of ref. \cite{5} or a complementary proposal in ref. \cite{14}.}

The paper is organized as follows. In Section 2 we define the framework: we assume additional D-term contributions to soft scalar masses $m_i^2$, requiring that the sum $M^2_{i,j,k} \equiv m_i^2 + m_j^2 + m_k^2$ remains RG-invariant, a property shared by AMSB scenarios. We implement the electroweak symmetry breaking condition and discuss generic class of models with suitable D-term contributions to investigate the properties of the mass spectrum. In Section 3 we investigate specific models in detail, imposing the present experimental bounds on supersymmetric particle masses. We present two examples with fixed charge assignments and study their mass spectra. Section 4 is devoted to our conclusions.

2 RG-invariant sum rule and D-term contributions

2.1 Sum rule

In the anomaly-mediated SUSY breaking scenario the soft SUSY breaking parameters, i.e. the gaugino masses $M_\alpha$, the soft scalar masses $m_i$ and the $A$-parameters are given by, \cite{3,4,5}

\begin{align}
M_\alpha &= \frac{\beta_{g_\alpha}}{g_\alpha} m_X, \\
m_i^2 &= -\frac{1}{4} \left( \sum_\alpha \frac{\partial \gamma_i}{\partial g_\alpha} \beta_{g_\alpha} + \sum_{Y_{ijk}} \frac{\partial \gamma_i}{\partial Y_{ijk}} \beta_{Y_{ijk}} \right) m_X^2, \\
A_{ijk} &= -\frac{\beta_{Y_{ijk}}}{Y_{ijk}} m_X,
\end{align}

where $g_\alpha$, with $\alpha = 1, 2, 3$, and $Y_{ijk}$, with $(i,j,k) = (Q,u,H_2), (Q,d,H_1), (L,e,H_1)$, are the gauge couplings and the Yukawa couplings, and $\beta_{g_\alpha}$ and $\beta_{Y_{ijk}}$ are their $\beta$-functions, respectively. Here $\gamma_i$ are the anomalous dimensions of the chiral superfields. For explicit calculations, it is convenient to define $m_F \equiv m_X/(16\pi^2)$, because $\beta$-functions include the loop-factor $16\pi^2$. \footnote{There has been other attempts of constructing generalizations of AMSB models, curing the problem of tachyonic solutions in the slepton sector and still assuring a scale invariance of the solutions, like the anti-gauge mediated model of ref. \cite{5} or a complementary proposal in ref. \cite{14}.}
Eqs. (1)-(3) are RG-invariant, that is, they are valid at any scale. Thus, the soft SUSY breaking terms are expressed as a function of gauge and Yukawa couplings at a given scale times the overall magnitude $m_F$. The consequent high predictability in the model leads to a problem in the MSSM, since the $\beta$-function coefficients for the weak gauge couplings, $b_1 = \frac{33}{5}$, $b_2 = 1$ and $b_3 = -3$ render the squared soft SUSY breaking parameters for the sleptons negative, yielding tachyonic slepton masses. Here we shall discuss phenomenological aspects of a solution to the tachyonic slepton mass spectrum, based on contributions from D-terms to those soft SUSY breaking parameters. We shall show that, depending on the specific D-term charge assignments, the mass spectrum can be very different from the one obtained in the framework of a universal $m_0^2$ contribution to all the soft SUSY breaking squared mass parameters.

At the present stage, it is a trivial statement that the sum,

$$\Sigma_{m^2_{AM}} \equiv (m_i^2)_{AM} + (m_j^2)_{AM} + (m_k^2)_{AM}, \quad (4)$$

is RG-invariant for non vanishing Yukawa couplings, $Y^{ijk} \neq 0$, because each of the soft scalar masses is RG-invariant. Now, let us assume additional contributions to soft scalar masses $m_i^2$, with the requirement that the sum $\Sigma_{m^2_{AM}}$ does not change. This can be realized by D-term contributions which are proportional to charges $q_i$ of the chiral superfields under a broken symmetry, that is, the total soft scalar mass is given by,

$$m_i^2 = (m_i^2)_{AM} + q_i m_D^2, \quad (5)$$

where $m_D$ is a universal parameter which defines the overall magnitude of the D-term contributions. We require for the allowed Yukawa couplings that the total charge should be conserved, i.e. $q_i + q_j + q_k = 0$. Hence, the sum does not change,

$$M_{i,j,k}^2 \equiv m_i^2 + m_j^2 + m_k^2 = \Sigma_{m^2_{AM}}, \quad (6)$$

and it is RG-invariant still after inclusion of the additional D-term contributions. The only effect of the D term is to modify the boundary conditions of the scalar masses at the scale where the U(1)’s get broken, but with no effects on the RG evolution of these masses. Eq. (4) is therefore valid at any scale. For a fixed charge assignment, the free parameters of the
theory are \( m_F \) and \( m_D \). Similarly, the sum, eq. (3), does not change if additional contributions to soft scalar masses are due to a certain type of supergravity theory, e.g. moduli-dominated SUSY breaking in perturbative heterotic string models \([13, 16]\). In this case, the charge \( q_i \) for the D-term contribution is replaced by \( q_i \propto 1 + n_i \), where \( n_i \) is the modular weight of the field \( \Phi_i \). Hereafter, we mean charge \( q_i \) as a coefficient of the deviation from the anomaly mediated soft SUSY breaking mass parameters squared, with the universal magnitude \( m^2_D \) in eq.(5), to include the realization by the moduli-dominant SUSY breaking. For the D-term contributions possible terms of \( O(g^4) \) may also appear \([5]\). Our assumption includes the fact that both the \( (m^2_i)_{AM} \) and the D-term contribution, which is of \( O(g^2) \), are the significant quantities and of comparable magnitudes. Thus, we neglect further contributions of \( O(g^4) \) compared with \( (m^2_i)_{AM} \). A similar statement has been done already in Ref.\([5]\).

2.2 Discussion on models

In order to proceed with our study, we need to assign the charges \( q_i \). For instance, in Ref. \([17]\) it is clarified that there are three U(1) symmetries, which are flavor-independent and allow the usual Yukawa couplings, i.e. \( R, A \) and \( L \) up to the baryon number symmetry \( B \). That is exactly consistent with the four degrees of freedom to deform each \( m_i \) keeping the sum of the scalar squared masses of the superfields, \( (Q, u, H_2), (Q, d, H_1) \) and \( (L, e, H_1) \) fixed. The generic U(1) symmetry \( X \) is a linear combination of them,

\[
X = mR + nA + pL + qB.
\]

We must then identify the charge \( q^X_i \) with \( q_i \) for each chiral superfield \( i = Q, u, d, L, e, H_1 \) and \( H_2 \). Charge assignments are shown in Table 1. Observe that the hypercharge is just a linear combination of the \( B, L \) and \( R \) symmetries, \( 3Y - 3R - 3L = -B \).

The extra U(1)’s appear naturally in GUT groups, like \( SO(10) \) and \( E_6 \). Indeed in the breaking \( E_6 \rightarrow SO(10) \times U(1) \) the extra U(1) charges have the same sign for both left- and right-handed lepton superfields\([18]\) providing a possibility to obtain positive slepton masses. We will discuss this kind of model in detail in Section 3.1.

It turns out that constructing a viable model starting from \( SO(10) \) is much more complicated. One finds immediately that when \( SO(10) \) breaks directly to the Standard Model
Table 1: Charge assignment

| Q | u | d | L | e | H1 | H2 |
|---|---|---|---|---|-----|-----|
| R | 0 | −1 | 1 | 0 | 1 | −1 | 1 |
| A | 0 | 0 | −1 | −1 | 0 | 1 | 0 |
| L | 0 | 0 | 0 | −1 | 1 | 0 | 0 |
| B | −1 | 1 | 1 | 0 | 0 | 0 | 0 |
| X | −q | −m + q | m − n + q | −n − p | m + p | −m + n | m |

gauge group or via $SU(5) \times U(1)$ with conventional assignments of the fermions in five and ten dimensional representations, one cannot get positive slepton masses squared. On the other hand, when the breaking is via $SU(2)_L \times SU(2)_R \times U(1)_{B−L}$, the charged slepton masses may become acceptable.

### 2.3 Generic aspects

Now let us discuss generic aspects of the sum rule. For concreteness, we write explicitly the three types of sum rules, valid for all three generations,

\[
m^2_{Q} + m^2_{u} + m^2_{H2} = (m^2_{Q} + m^2_{u} + m^2_{H2})_{AM},
\]

\[
m^2_{Q} + m^2_{d} + m^2_{H1} = (m^2_{Q} + m^2_{d} + m^2_{H1})_{AM},
\]

\[
m^2_{L} + m^2_{e} + m^2_{H1} = (m^2_{L} + m^2_{e} + m^2_{H1})_{AM}.
\]

Here we parameterize the deviations of the Higgs masses from the anomaly-mediated ones as

\[
m^2_{H_i} = (m^2_{H_i})_{AM} - d_{H_i}m^2_F.
\]

We fix the magnitudes of the $\mu$-term and the $B$-term by use of the minimization conditions for the Higgs effective potential, to assure proper radiative electroweak symmetry breaking,

\[
2\mu^2 + M_Z^2 = \frac{m^2_{H1} - m^2_{H2}}{-\cos 2\beta} - m^2_{H1} - m^2_{H2},
\]

7
\[ 2\mu B = \sin 2\beta \left( \frac{m_{H1}^2 - m_{H2}^2}{-\cos 2\beta} - M_2^2 \right). \] (13)

In the above, we have considered the minimization conditions derived from the tree level expression for the Higgs effective potential. The inclusion of the one-loop RG improved effective potential would modify the above equations in such a way that the quantitative behaviour of the solutions will be modified, but the qualitative features are expected to be similar. First of all, the condition \( m_L^2 + m_{\tilde e}^2 > 0 \) requires \( d_{H1} > 0.71 \). Another important condition for the successful electroweak symmetry breaking is the present experimental bound on \( m_A^2 \), where \( m_A^2 = 2\mu^2 + m_{H1}^2 + m_{H2}^2 \) is the squared mass of the CP-odd Higgs field.

For explicit models, which shall be discussed later, we require the fulfillment of the present experimental bound from LEP, \( m_A > m_A^{\exp} \simeq 88 \text{ GeV} \) \[23\]. However, if \( m_F^2 \) is large enough compared with \( M_Z^2 \), the condition \( m_A^2 > 0 \) is effectively equivalent to the experimental bound and corresponds to \( \Delta m^2 \equiv m_{H1}^2 - m_{H2}^2 > 0 \) (see Eq. \[12\]). In addition to the overall scale \( m_F^2 \), the difference \( \Delta m^2 \) depends on \( d_{H2} - d_{H1} \) and \( \tan \beta \). Thus, the condition \( \Delta m^2 > 0 \) leads to a minimum value of \( d_{H2} - d_{H1} \), which depends on \( \tan \beta \), and combined with \( d_{H1} > 0.71 \) leads to a minimum value for \( d_{H2} \). In the absence of D-terms, \( d_{H_i} = 0, i = 1, 2 \), we have \( \Delta m^2/m_F^2 = O(10) \) except around \( \tan \beta \sim 50 \). Such a large value of \( \Delta m^2 \) implies that a negative value of \( d_{H2} \) will be allowed, even after including a non-vanishing value of \( d_{H1} > 0.71 \). On the other hand, around \( \tan \beta = 50 \) we have \( \Delta m^2/m_F^2 = O(0.1) \) and then, for \( d_{H1} > 0.71 \), only positive values of \( d_{H2} \) are allowed. The smallness of \( \Delta m^2 \) close to \( \tan \beta = 50 \) is due to the fact that for such large value of \( \tan \beta \) the bottom Yukawa coupling becomes strong and very close in magnitude to the top Yukawa coupling. Hence, the evolution of the Higgs mass parameters is very similar, \( (m_{H1})_{AM} \simeq (m_{H2})_{AM} \), for \( \tan \beta \simeq 50 \). The solid line in Fig. 1 shows the minimum value of \( d_{H2} \) against \( \tan \beta \) under the condition \( \Delta m^2 > 0 \), that is, the condition \( m_A^2 > 0 \) for \( m_F^2 \gg M_Z^2 \). For \( \tan \beta > 50 \) there is a change in the slope for \( d_{H2} \). This is due to a change in the sign of the \( \beta \)-function of the bottom Yukawa coupling after a vanishing value is achieved due to a compensation between the effects associated with the strong gauge coupling and the bottom Yukawa coupling itself.

Now let us consider the minimum value of \( m_F \). The gaugino mass \( M_2 \) is obtained to be

\[ M_2 \simeq 0.43 \ m_F. \] (14)
As we shall show at the end of this section, in the whole \((d_{H1}, d_{H2})\) parameter space under consideration, we always have \(M_2 \ll |\mu|\). This implies that the lightest chargino is wino-like. Hence, the present experimental lower bound on the chargino mass, \(m_{\chi^\pm} < 90\,\text{GeV}\), implies that the mass parameter \(m_F\) is bounded to be

\[
m_F > 210\,\text{GeV.} \tag{15}
\]

For example, for \(m_F = 210\,\text{GeV}\), the condition \(m_A > m_A^{\text{exp.}} \simeq 88\,\text{GeV}\) lifts up the curve which defines the minimum value of \(d_{H2}\) in Fig. 1 by 0.3.

Next we calculate the stop mass, in particular the average stop mass \(m_{\tilde{t},\text{av}}\), which is defined as \(m_{\tilde{t},\text{av}}^2 \equiv (m_{\tilde{Q}}^2 + m_{\tilde{u}}^2)/2 + m_t^2\). In the limit under discussion in this section, \(m_{\tilde{t}}^2 \gg M_Z^2\), the contribution from the top quark mass is negligible and the average stop mass depends only on \(d_{H2}\). The dotted lines in Fig. 1 show contours of constant values of the ratio \(R_{\tilde{t}} = m_{\tilde{t},\text{av}}/M_2\) for \(R_{\tilde{t}} = 8, 10\) and 12. Obviously, as \(d_{H2}\) increases, \(R_{\tilde{t}}\) increases. The thick solid line in Fig. 2 shows the minimum value of \(R_{\tilde{t}}\) as a function of \(\tan\beta\). The minimum value of \(R_{\tilde{t}}\) is at \(\tan\beta \simeq 3\) and implies that \(m_{\tilde{t},\text{av}} \geq 6.6M_2\). The thin solid line gives a similar ratio for the average sbottom mass, \(R_b = m_{\tilde{b},\text{av}}/M_2\) with \(m_{\tilde{b},\text{av}}^2 \simeq (m_{\tilde{Q}}^2 + m_d^2)/2\). The dotted line corresponds to a similar ratio but for first and second generation squark masses \(R_{\tilde{u}} = m_{\tilde{u},\text{av}}/M_2\) with \(m_{\tilde{u},\text{av}}^2 \equiv (m_{\tilde{Q}_1}^2 + m_{\tilde{u}_1}^2)/2\) being the average squared mass in the up-squark sector. One can define the analogous quantity in the down sector \(R_{\tilde{d}} = m_{\tilde{d},\text{av}}/M_2\), with \(m_{\tilde{d},\text{av}}^2 \equiv (m_{\tilde{Q}_1}^2 + m_{\tilde{d}_1}^2)/2\). It turns out, however, that the down-squark sector is stable as a function of \(\tan\beta\), with \(R_{\tilde{d}}\) of order 10 for most \(\tan\beta\) regions. Such a behaviour is expected since the main dependence on \(\tan\beta\) in the first and second generation down-squark sector comes through \(d_{H1}\), which is fixed to its minimum value via the condition of positive slepton squared masses, \(d_{H1} > 0.71\). The up-squark sector instead, depends on \(d_{H2}\) and hence on \(\tan\beta\), as shown in Fig 2. From Fig. 2 it follows that, all squark masses are very heavy compared to gaugino mass parameters, as expected from the underlying structure of AMSB scenarios. This conclusion holds, unless there is a large hierarchy between the left and right handed soft SUSY breaking parameters in the squark sector, as we shall discuss below.
Fig. 1: The minimum value of $d_{H2}$ (solid line) and constant contours of $R_t$, the ratio of the average stop mass to the SU(2) gaugino mass parameter, $M_2$, which determines the chargino mass in this large $\mu$ scenario, (dotted lines).

Fig. 2: The minimum of the ratio of the average stop mass (thick solid line), of the average sbottom mass (thin solid line) and of the first and second generation up-sector squark masses (dotted line) to the SU(2) mass parameter $M_2$.

Now let us discuss the magnitude of $\mu$ in these models. In the case with $d_{Hi} = 0$, we
\[ \frac{\mu^2 + M_Z^2/2}{m_F^2} \sim 10, \]  
(16)

for any value of \(2 \leq \tan \beta \leq 60\). Moreover, from the expression for \(\mu^2\)

\[ \mu^2 = -m_{H1}^2 + [(m_A^2 + M_Z^2) \frac{\tan^2 \beta}{\tan^2 \beta + 1} - M_Z^2], \]  
(17)

in which the second term in RHS is always positive for \(\tan \beta > M_Z/m_A\), one can derive the following inequality,

\[ \mu^2 > -m_{H1}^2 > -(m_{H1}^2)_{AM}. \]  
(18)

The second inequality in eq. (18) is due to the constraint \(d_{H1} > 0\). We combine eqs. (14), (16) and (18), and find that \(|\mu|\) is larger than \(M_2\). Fig. 3 shows the minimum value of the ratio \(R_\mu \equiv \mu'/M_2\), with \(\mu' = \sqrt{\mu^2 + M_Z^2/2}\), as a function of \(\tan \beta\), in the parameter space \((d_{H1}, d_{H2})\) allowed by the conditions \(m_L^2 + m_e^2 > 0\) and \(\Delta m^2 > 0\). To realize the minimum value, an extreme value of the ratio \(r_d = d_{H2}/d_{H1}\) is sometimes required. For example, for \(\tan \beta = 3\) the minimum value of \(R_\mu \approx 2.5\) is obtained, and that is realized for \(r_d = -13\). Fig. 4 shows the minimum value \(R_\mu\) as a function of \(r_d\) for \(\tan \beta = 3, 20\) and 50. Note that as \(\tan \beta\) increases, the minimum values of \(d_{H2}\) and \(r_d\) increase. Hence, as a generic aspect of these models, the mass parameter \(|\mu|\) (as well as the the squark masses) is large compared with the gaugino mass parameters, \(M_1\) and \(M_2\).

Fig. 3: The minimum value of \(R_\mu = \sqrt{\mu^2 + M_Z^2/2}/M_2\) as a function of \(\tan \beta\).
3 Mass Spectrum

In order to show in detail possible mass spectra in generalized AMSB scenarios with extra D-term contributions to the sfermion mass parameters, we discuss two explicit examples in this section.

3.1 A simple example

First we consider a simple case, where additional contributions are degenerate for the sfermions and for the Higgs mass parameters, respectively,

\[ m_{f}^{2} = (m_{f}^{2})_{AM} + m_{D}^{2}, \quad m_{Hi}^{2} = (m_{Hi}^{2})_{AM} - 2m_{D}^{2}. \]  

(19)

This type of charge assignment can be realized through the breaking \( E_{6} \to SO(10) \times U(1) \).

In the following we shall calculate \( m_{A} \). Note that in this example \( m_{D} \) does not contribute to \( \Delta m^{2} \) or \( m_{A} \) and hence the mass \( m_{A} \) is determined by \( m_{F} \) and \( \tan \beta \). The result is shown in Fig. 5, where the region above the solid line defines the condition \( m_{A} > 88 \) GeV. As a result, \( m_{F} > 270 \) GeV is required around \( \tan \beta = 50 \). In most of the \( \tan \beta \) region, the requirement \( m_{A} > 88 \) GeV is less significant than the experimental lower bound on the chargino mass,
\( m_F > 210 \text{ GeV} \). The dotted lines in Fig. 5 correspond to \( m_A = 250, 500, 750 \) and 1000 GeV, respectively.

![Diagram showing lines of constant values of the CP-odd Higgs mass, \( m_A \), in the \( m_F-\tan \beta \) plane.](image)

Fig. 5: Lines of constant values of the CP-odd Higgs mass, \( m_A = 88, 250, 500, 750 \) and 1000 GeV, in the \( m_F-\tan \beta \) plane.

Now we discuss the slepton masses and the Higgsino mass parameter \( \mu \), in both the small and large \( \tan \beta \) scenarios. For example, we take \( \tan \beta = 3 \) and 50. Fig. 6 shows \( \mu \) and \( m_{\tilde{L}} \) for \( \tan \beta = 3 \). In this case the masses \( m_{\tilde{L}} \) and \( m_{\tilde{\epsilon}} \) are almost degenerate for all three generations, since the corresponding Yukawa couplings contributing to Eq. [2] are very small. The dotted lines correspond to \( |\mu| = 0.2, 0.5, 1.0, 1.5 \) and 2.0 TeV. The solid lines correspond to \( m_{\tilde{L}} = m_{\tilde{\epsilon}} = 85, 100, 200, 400 \) and 600 GeV. The region below \( m_{\tilde{L}} = m_{\tilde{\epsilon}} = 85 \) GeV is excluded by present bounds on the smuon mass from the combined results of the four LEP experiments\(^4\). The region to the left of the dot-solid line is excluded due to the present experimental bounds on the chargino mass.

\(^4\) The analogous lower bound on the selectron mass is about 90 GeV. The limits quoted here for slepton masses include data up to \( \sqrt{s} = 189 \) GeV. The inclusion of the latest data up to \( \sqrt{s} = 202 \) GeV will improve the bounds on smuons and selectrons by about 5 GeV, respectively. The precise value of the experimental bound is not crucial for the general analyses performed in this paper.
Fig. 6: Contours of constant values for the smuon masses for $m_{\tilde{L}} = m_{\tilde{e}} = 85, 200, 400, 600$ GeV (solid lines) and for $\mu = 0.2, 0.5, 1, 1.5$ TeV (dotted lines) in the $m_F - m_D$ plane, for $\tan \beta = 3$.

The first and second families of slepton masses for $\tan \beta = 50$ are almost the same as those for $\tan \beta = 3$. This is the case since, although the off-diagonal elements of the slepton mass matrices have a $\tan \beta$ enhancement factor, they are also proportional to the first and second generation lepton masses which are too small. Hence, ignoring the small $SU(2)_L \times U(1)_Y$ D-term contributions, the two mass eigenstates, for the selectrons and smuons are still approximately given by $m_{\tilde{L}}$ and $m_{\tilde{e}}$, respectively, independent of $\tan \beta$.

On the other hand, for large $\tan \beta$ the stau mass matrix has a sizable off-diagonal element, which reduces the eigenvalue of the lightest stau mass. For the same values of the parameters $m_F$ and $m_D$, the stau mass, $m_{\tilde{\tau}_1}$, is smaller than the first and second generation slepton masses and hence it excludes a wider region of that parameter space. In the following we consider the experimental lower bound on the lightest stau mass from LEP, $m_{\tilde{\tau}_1} > 70 GeV$ [24]. Fig. 7 shows the curves corresponding to the lightest stau mass, $m_{\tilde{\tau}_1} = 70, 200, 400$ and 600 GeV, as well as the curves for constant values of $|\mu|$ and the curve corresponding to the experimental lower bound, $m_{\tilde{L}} = m_{\tilde{e}} = 85$ GeV, for the smuons as a reference. The region to the left of the dot-solid line corresponds to values of the CP-odd mass $m_A$ which are experimentally excluded by LEP. The variation of the lightest stau mass, $m_{\tilde{\tau}_1}$ with the
sign of $\mu$ is negligible. In Fig. 7 the value of $|\mu|$ as a function of $m_F$ is slightly smaller than in the low $\tan \beta$ case of Fig. 6.

![Graph](image)

**Fig. 7:** Contours of constant values for the stau mass, $m_{\tilde{\tau}_1} = 70, 200, 400, 600$ GeV for $\tan \beta = 50$ (thick solid lines) and for $|\mu| = 0.2, 0.5, 1.0, 2.0$ TeV (dotted lines). The thin solid line shows the lower experimental bound on the smuon mass, $m_{\tilde{\mu}} \simeq m_{\tilde{e}} \simeq 85$ GeV.

Similarly we can discuss the predictions of this model for the stop sector. For the stop mixing angle $\theta_t$, $\sin 2\theta_t$ is always large. That is because $m_{\tilde{Q}}^2 - m_{\tilde{u}}^2$ for the stop is small compared with $|\mu|$ as well as $|A_t|$. For example, we have $\sin 2\theta_t > 0.8$ for $\tan \beta = 3$, $m_F < 500$ GeV and $\mu > 0$. The negative sign of $\mu$ leads to a slightly larger value of $\sin 2\theta_t$.

For $\tan \beta = 50$ we have $\sin 2\theta_t = 1$, with $m_F \leq O(1)$ TeV. Figs. 8 and 9 show the lightest stop mass for $\tan \beta = 3$ and 50 for $\mu < 0$. The solid lines correspond to different values for the lightest stop mass, $m_{\tilde{t}_1} = 0.09, 0.2, 0.5, 1.0$ and 1.5 TeV. Present experimental bounds exclude the region $m_{\tilde{t}_1} < 90$ GeV [24]. In Fig. 9 ($\tan \beta = 50$), the excluded experimental bound on the lighest stop mass is not shown, because it corresponds to a very narrow region already excluded by the stau mass constraint. The experimental bound of the chargino mass excludes the area to the left of the dot-solid curve and gives a stronger constraint, Eq. (15), which implies that the minimum of the $m_{\tilde{t}_1}$ is about 500 GeV for $\tan \beta = 3$. Furthermore, for $\tan \beta = 50$ we have the constraint due to $m_A$, i.e. $m_F > 270$ GeV, which excludes the region to the left of the dot-solid line in Fig. 9. Thus, in the case of large $\tan \beta$, with similar
values of the bottom and top Yukawa couplings, the minimum value of $m_{\tilde{t}_1}$ is about 800 GeV. The case with $\mu > 0$ leads to a slightly larger mass of the lightest stop.

In the case where a universal mass $m_0$ is added to solve the problem of tachyonic solutions in the slepton sector, the particle spectrum is such that the lightest stop is quite heavy and $|\mu|$ is large. Hence, the present example is phenomenologically not very different from the model with the universal $m_0$ addition. This simple example shows two features which are quite generic in this type of generalized AMSB models. One is that the lightest stop is in general quite heavy and the other is that for most $\tan \beta$ values (except around $\tan \beta = 50$), the chargino is predicted to be so light that present experimental bounds on chargino masses put constraints on the allowed values of $m_F$, which are stronger than those derived from demanding a successful electroweak symmetry breaking, with $m_A > 88$ GeV.

![Fig. 8: Contours of constant lightest stop mass, $m_{\tilde{t}_1} = 0.09, 0.2, 1, 1.5$ TeV, in the $m_D$-$m_F$ plane, for $\tan \beta = 3$ and $\mu < 0$. The regions below the dotted line and to the left of the dot-solid line are experimentally excluded.](image-url)
Fig. 9: Contours of constant lightest stop mass, $m_{\tilde{t}_1} = 0.2, 1, 1.5$ TeV, in the $m_D - m_F$ plane, for $\tan\beta = 50$ and $\mu < 0$ The same as in Fig. 8, the regions below the dotted line and to the left of the dot-solid line are experimentally excluded.

It is interesting to notice that, in most of the allowed regions of parameters, the lightest supersymmetric particle (LSP) is the wino-like neutralino and the next-to-LSP (NLSP) is the chargino. We have here the same spectrum for charginos as in minimal AMSB scenarios: LSP wino-like and almost degenerate with the chargino making detection difficult [8, 19, 20]. In addition, in the region close to the region where the lightest stau mass is close to its experimental limit, we may have the stau to be the LSP, although such region is narrow against $m_D$. A charged, stable LSP is cosmologically disfavoured. However, we can avoid this problem by demanding $m_{\tilde{\tau}_1} > M_2$ and none of the features discussed in this section will vary in any significant manner.

### 3.2 A model with a light stop

Here we consider a special case in which, by significantly shifting the degeneracy between the left and right handed stop/sbottom soft SUSY breaking parameters we obtain a light stop in the spectrum. Note that, in general, the supersymmetric spectrum is constrained by direct experimental searches and by the requirement that it provides a good description of the precision electroweak data. This requirement implies that, to avoid an unacceptable large
contribution from supersymmetric particle loops to the $\rho$ parameter, and unless unnatural cancellations take place, the soft SUSY breaking mass parameters for the left-handed top squark should be larger that 300 GeV \cite{21}. Quite generally, the heavier the supersymmetric spectrum, and in particular the heavier the left-handed sfermions, the better the agreement between the MSSM and the precision electroweak observables. Hence, in the following, we shall consider a case with a light stop, which is mainly right-handed, so that the model is not in any conflict with precision measurements.

First, let us discuss the charge assignment leading to a light stop. We take the sign assignment of charges, $q_i^X \equiv X_i$, such that $m^2_D > 0$ in eq.\((3)\).

Obviously, the minimal requirement that $m^2_L > 0$ and $m^2_e > 0$ implies $X_L = -n - p > 0$ and $X_e = m + p > 0$, which also yields the condition $X_{H1} = -m + n < 0$. In order to achieve proper electroweak symmetry breaking, a large $\Delta m^2$ is desirable, and then we need $X_{H1} - X_{H2} = -2m + n \geq 0$.

That also implies that $X_{H2} = m < 0$. Combining all the previous conditions we have

$$m > -p > n \geq 2m.$$  \hspace{1cm} (20)

The inequality $X_{H2} < 0$ requires $X_Q + X_u = -m > 0$. In order to obtain a light stop, either $X_Q$ or $X_u$ should be negative. Therefore, if $Q$ and $u$ belong to a multiplet of a larger gauge group like $SU(5)$, $(X_Q = X_u)$, the present scenario predicts heavy stops. If $X_Q < 0$, then the lightest stop would be left-handed and most probably in conflict with present constraints from precision measurements, unless its mass is sufficiently close to that of the left-handed lightest sbottom which would also appear in the spectrum. Alternatively, we have the possibility $X_Q > 0$ and $X_u < 0$, which may lead to a right-handed lightest stop. In any case, most sign assignment of charges are fixed in a light stop model.

As an example we take $(m, n, p, q) = (-2, -4, 3, -3)$, i.e.,

$$(X_Q, X_u, X_d, X_L, X_e, X_{H1}, X_{H2}) = (3, -1, -1, 1, -2, -2).$$  \hspace{1cm} (21)

In this case the CP-odd mass $m_A$ is independent of $m_D$ and the behaviour of the slepton masses is similar to the case in which all the squarks are quite heavy. Figs. 10 and 11 show the lightest stop mass for tan $\beta = 3$ and 50 and $\mu < 0$. For the small tan $\beta$ case, there is an allowed region in the $m_D - m_F$ plane where, even after imposing the chargino mass bounds
on $m_F$, $m_F \geq 210$ GeV, there are solutions which allow for a very light stop. In the case $\tan \beta \simeq 50$, for which $m_{H_1}^2 \simeq m_{H_2}^2$ and a more stringent bound on $m_F$ follows from the experimental bound on $m_A$, only heavy stops are allowed unless $m_D \geq 1$ TeV. The solid lines in Fig. 10 and 11 correspond to $m_{\tilde{t}_1} = 0.09, 0.2, 0.5, 1.0$ and 1.5 TeV. The region to the left of the curve $m_{\tilde{t}_1} = 90$ GeV corresponds to the experimentally excluded region of stop masses which excludes an important region of parameter space even for very large values of $m_D$. The region below the dotted line, $m_{\tilde{L}} < 85$ GeV and $m_{\tilde{\tau}} < 70$ GeV, Figs. 10 and 11 respectively, is experimentally excluded. For $\tan \beta \simeq 50$ and fixed values of $m_D$ and $m_F$, the lightest sbottom has a mass of similar magnitude to the lightest stop mass shown in Fig. 10.

![Fig. 10: Contours of the lightest stop mass, $m_{\tilde{t}_1} = 0.09, 0.2, 0.5, 1.0$ TeV, (solid lines) in the $m_D$-$m_F$ plane, for $\mu < 0$ and $\tan \beta = 3$. The regions below the dotted line, to the left of the dot-solid line and to the left of the line of $m_{\tilde{t}_1} = 90$ GeV are experimentally excluded.](image-url)
Fig. 11: Contours of the lightest stop mass, $m_{\tilde{t}_1} = 0.09, 0.2, 0.5, 1.0, 1.5$ TeV (solid lines) in the $m_D-m_F$ plane, for $\mu < 0$ and $\tan \beta = 50$. Analogous to Fig 10, the regions below the dotted line, to the left of the dot-solid line and to the left of the line of $m_{\tilde{t}_1} = 90$ GeV are experimentally excluded.

Similarly, we can discuss other cases leading to a light $\tilde{t}_1$. For example we can vary $q$ fixing $(m,n,p) = (-2, -4, 3)$ as in the previous case. Fig. 12 shows the lightest stop mass values as a function of $m_D$ for $m_F = 250$ GeV and $\tan \beta = 3$, for $q = -3, -4$ and $-5$.

Fig. 12 The lightest stop mass as a function of $m_D$, for $m_F = 250$ GeV, $\mu < 0$ and $\tan \beta = 3$ with $q = -3, -4$ and $-5$, respectively.
4 Conclusions

We have studied phenomenological aspects of a generalization of anomaly-mediated SUSY breaking scenarios with non-decoupling effects from a high scale theory, which allow for D-term contributions to be effective at the low energy scale. We have assumed that the additional D-term contributions to the soft supersymmetry breaking scalar mass parameters, $m_i$, are such that the sum $m_i^2 + m_j^2 + m_k^2$, corresponding to chiral fields in the allowed Yukawa couplings, $Y^{ijk} \neq 0$, remains RG-invariant as it occurs in the AMSB models. The extra D-term contributions, depending on the charge assignment of the extra U(1)’s involved in the model, can solve the problem of tachyonic solutions in the slepton sector, whereas preserving the flavour independence of the solutions. Most interesting, since the RG-invariant sums of non-vanishing Yukawa couplings $Y^{ijk} \neq 0$ appear directly in the renormalization group evolution of the Yukawa couplings and soft SUSY breaking masses themselves, one could allow for additional contributions to each of the soft scalar masses and, as long as the sum itself is not affected, the soft scalar masses will remain RG-invariant. Given a fixed charge assignment, the sparticle spectrum is uniquely determined as a function of the free parameters $m_F$ and $m_D$, $\tan \beta$ and the sign of $\mu$. In general, the average stop mass and the parameter $|\mu|$ are larger than the slepton masses and the gaugino mass parameters $M_1$ and $M_2$, as expected from the underlying AMSB structure. The mass spectrum in the most simple case is similar to the case in which a universal contribution $m_0^2$ is added, to all the soft SUSY breaking scalar mass parameters squared, to cure the tachyonic mass problem. However, as we have explicitly shown, it is possible to construct models in which a light stop, compatible with present electroweak precision measurements will naturally appear in the spectrum. Models with light third generation squarks demand a specific U(1) charge assignment and yield constraints for the model building.

Note added: After completion of this work, an article \cite{22} appeared, where $D$-term contributions and sum rules of soft scalar masses are also discussed.
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