Relativistic slowing down shocks as sources of GRB lags

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Accepted . Received ;

ABSTRACT

We demonstrate which way slowing down ultrarelativistic shocks create GRB lags. Reflection process produces positive lags and Cracow acceleration produces negative lags. We describe the process of Cracow acceleration and present two ways in which the seed particles are injected into the upstream plasma. Strong decelerating shocks in the presence of the acceleration processes account for the observed hard energy spectra of accelerated electrons with spectral indices smaller than the value 2.2. We claim that during the strong deceleration stage the rise-fraction of seed particles is formed upstream of the shock. The rise-fraction feeds the Cracow surge and normal seed particles feed the reflection surge. We present the model of the microphysics of relativistic plasma that aims at explaining the required disturbances of the movement of particles upstream of the shock that allow for Cracow acceleration. We show that Cracow acceleration can produce UHECRs.

Key words. acceleration of particles — shock waves — cosmic rays — radiation mechanisms: non-thermal

1. Introduction

Fermi (1949) proposed a mechanism being supposed to explain cosmic rays acceleration. The idea is analogous to the acceleration of a tennis ball. Interstellar clouds are tennis rackets and a particle is the tennis ball. There is a small difference here, tennis rackets rarely decrease the energy of the ball. Cosmic rays are accelerated in this mechanism according to second order Fermi acceleration method. It should be called Fermi acceleration.

The concept that shock waves could accelerate particles appeared slowly. It was foreshadowed by Hoyle (1960) who postulated that shocks could efficiently accelerate particles but without specifying a mechanism. Parker (1958) and Hudson (1965, 1967) attempted to obtain such mechanism based on pairs of converging shocks and, most notably, Schatzman (1963) constructed a theory based on perpendicular shocks where particles keep bumping into a hydromagnetic shock front and increase their energy in the Fermi-like way.

The real acceleration mechanism was described in four seminal papers, Krymsky (1977), Axford et al. (1977), Bell (1978a,b) and Blandford & Ostriker (1978). Since many persons aspired to the name of the discoverer, therefore this mechanism was called diffusive shock acceleration. It is often called Fermi acceleration, but it is incorrect, since Fermi does not have anything to do with particle acceleration at shocks. The name stuck to this mechanism (see Hoshino 2008, page 940) and we use it in this paper.

One should, by the way, recall nomenclatures that mislead uninformed readers about a misunderstanding which persists in the literature. First order Fermi acceleration and second order Fermi acceleration are not a real mechanism. It is the method which produces the Fermi statistical dependence in which the proportional gain of energy is the same for all particles. There are two or more media in the method that are moving with the average speed equal to $V$. The speed of a particle is much larger that $V$ and is equal to $c$. The particle bounces between the media and gains energy. If the direction of $V$ is always opposite to the direction of $c$ in each collision then the average energy gain of the particle is proportional to $V/c$ and that is first order Fermi acceleration. If the directions are random then the average energy gain is proportional to $(V/c)^2$ and that is second order Fermi acceleration. The both methods appear in separate phenomena and there is no need of applying a shared name for them. It could sometimes lead to ambiguity. In rare cases when it is necessary one can use the phrase ‘all Fermi acceleration processes’ (Drury 1983, page 987). First order Fermi acceleration assumes that $V \ll c$. It implies that the distribution of accelerating particles in the medium becomes nearly isotropic so that the particles could reach the opposite medium. First order Fermi acceleration predicts that the energy of accelerating particles increases $\sim \gamma^2$ times in each cycle upstream-downstream-upstream if the mechanism is applied to relativistic shocks.

Some authors are trying to name the idea of the particle energy increase as a result of particles bouncing back and forth across the shock wave as Fermi acceleration, but it is incorrect. It is Hoyle-Schatzman idea.

In Fermi acceleration particles can cross the shock repeatedly thanks to magnetic field fluctuations downstream of the shock and subluminal shock geometry. Magnetic field fluctuations upstream of the shock do not play significant role here, since the shock will always catch up with the particle that is wandering there. This mechanism does not accelerate particles in superluminal shocks (Bell 1978a; Drury 1983) and therefore is limited to non-relativistic and poorly relativistic shocks, since relativistic shocks are generically superluminal.

For a lot of years an unresolved riddle existed in astronomy. Observations have shown that in areas where relativistic shocks are present particles are accelerating effectively, but nobody was able to find the mechanism.
Reflection process is connected with relativistic shocks. It was known that a particle increases its energy $\gamma^2$ times in the cycle upstream-downstream-upstream. Vietri (1995) proposed repeated crossings at an ultrarelativistic shock to occur and argued that it would lead to $\gamma^2$ energy gain per crossing cycle. Bednarz & Ostrowski (1998) found that because of strong anisotropy of cosmic rays upstream of the shock the gain equals to $\sim 2$, what was confirmed by Gallant & Achterberg (1999). However, flowing in particles have isotropic distribution and gain $\gamma^2$ times energy in the first cycle and it was called reflection (Bednarz & Ostrowski, 1999). In this paper we will show that reflection process is an important element of the cosmic ray acceleration.

After Bednarz & Ostrowski (1998) they have been trying to include the idea of cosmic ray anisotropy upstream of the shock, but in the Fermi acceleration context, Kirk et al. (2000); Achterberg et al. (2001); Vietri (2003); Lemoine & Pelletier (2003) - disordered shocks and Ellison & Double (2002) - parallel shocks. Fermi acceleration takes place at relativistic shocks in two cases not-appearing in the outer space, at practically parallel shocks (subluminal geometry) and at shocks with completely disordered magnetic field downstream of the shock (we named them disordered shocks). One would call these two theoretical cases Peacock acceleration (Peacock, 1981). The Weibel instability is preventing disordered and parallel shocks from being formed what observations of polarised emission are confirming (Steele et al., 2003; Götz et al., 2009).

Niemiec & Ostrowski (2006), Niemiec et al. (2006), Lemoine et al. (2006) were trying to examine the acceleration process and used ‘realistic’ magnetic field fluctuations upstream of the shock. They have failed to get effective acceleration at superluminal shocks. Pelletier et al. (2004) have been discussed it analytically and have been found to agree with the numerical results. They have concluded that the acceleration takes place if the fluctuations amplitude $\delta B$ is much larger than the homogeneous magnetic field $B_0$ upstream of the shock what produces disordered magnetic field downstream of the shock, but this is the case of Peacock acceleration. Their lack of effective acceleration results from this that they have applied normal conditions (types of fluctuation spectra) to the phenomenon which is completely unknown.

Cracow acceleration needs a type of the fluctuation spectrum where almost entire fluctuation energy is gathered in very intense waves about a small length. The length can be so small as the size of the atom. We do not belive in such fluctuations and propose an other solution to the problem in Section 2.

We expect that astronomical sites where particles are accelerated in Cracow acceleration are relativistic shocks with the Lorentz factor much larger than 1 and the helical (perpendicular to the shock normal) mean magnetic field produced by the Weibel instability (Medvedev & Loeb, 1999). In GRBs the process does occur not only in prompt gamma-ray pulses, but also in X-ray flares (Margutti et al., 2010) and in precursors (Burton et al., 2009) where very similar spectral properties are observed. Long and short GRBs are also showing the same acceleration process (Guiriec et al., 2010).

In pulsar wind nebulae (PWNe) where (contrary to GRBs) the plasma downstream of the shock is approximately at rest with respect to ISM, not only leptons but probably protons are being accelerated (Li et al., 2010). There not only the same mechanism of the acceleration is present, but probably similar physical conditions, since the assumed particle injection spectrum in the form of a broken-power law turns out to have spectral break at an similar energy for all sources (Ruccicatini et al., 2011).

We think that particles are accelerated in the mechanism of Cracow acceleration in gamma-ray binaries (Cerutti et al., 2000), in X-ray binaries where relativistic jets are formed and in jets thrown away by active galactic nuclei. An example of X-ray binaries is Cygnus X-3 where electrons gain energy at the place where the jet is recollimated by the stellar wind pressure and forms a shock (Dubus et al., 2010).

In the test particle limit the energy spectral index of accelerated particles is only dependent on the compression ratio of the plasma through the shock by $\beta = (R+2)/(R−1)$ (Bell, 1978; Drury, 1983). The formula is not valid for
relativistic shocks but it is good enough for the estimation how \( \beta \) increases when the compression falls down.

The magnetization parameter, \( \sigma \), is equal to the upstream Poynting flux relative to the total mass-energy flux (Appl & Camenzind 1988; Kennel & Coroniti 1984a). Kennel & Coroniti (1984a) presented the derivation of the Rankine-Hugoniot relations for perpendicular shocks. They found that \( R = 3(1 - 4\sigma) \) for \( \sigma \lesssim 0.01 \) and \( R = 1 + 1/(2\sigma) \) for \( \sigma \gtrsim 10 \). We estimate that Cracow acceleration needs \( \sigma < 0.1 - 0.01 \) to accelerate efficiently if one applies the usual plasma conditions, but we devise the model of loop plasma which could require large value of \( \sigma \) and standard compression \( R = 3 \).

Kirk & Skjæraasen (2003) have found that the Poynting flux can be dissipated before the pulsar wind reaches the inner edge of the Crab Nebula. This is in accordance with the value of \( \sigma = 0.003 \) near the termination shock of the nebula (Kennel & Coroniti 1984a) which is far more sufficient than Cracow acceleration requires.

The context of the paper is organized as follows. In Section 2 we present the model of the microphysics of relativistic plasma as well as the way the simulations of the relativistic plasma should be conducted. In Section 3 the mechanism of Cracow acceleration and in Section 4 the numerical simulations are described. In Section 5 we give thought to the influence of the deceleration of the shock on the energy spectra of accelerated particles. Results of the simulations are presented in Section 6. There are many tables and figures. Section 7 is devoted to the problem in what conditions Fermi acceleration is able to produce negative lags. In Section 8 we postulate the formation of the rise-fraction of particles upstream of the shock at the strong deceleration stage. The rise-fraction feeds the Cracow surge and normal seed particles feed the reflection surge. In Section 9 we show that Cracow acceleration can produce UHECRs. The seed particles problem is discussed in Section 10. We summarize and discuss our paper in Section 11.

2. The microphysics of relativistic flows

Relativistic plasma is an obscure phenomenon. Magnetic waves are too weak to disturb the particle movement so strong as Cracow acceleration requires. There must be some collective processes that build magnetic and electric fields on a micro-scale. Before we introduce the model of loop plasma let us present how small magnetic helical tubes could build the homogeneous magnetic field and the required disturbances. Let us consider two idealised cases.

Let us assume that the Weibel instability generates on a micro-scale small magnetic helical tubes. The axes of symmetry of these tubes are parallel to the global speed of the plasma. The tubes have different diameters and are scattered randomly across the space. Let us assume that the direction of the velocity of accelerated particles can diverge only a bit from the direction of axes of the tubes. This is the case of the accelerated particle wandering upstream of the relativistic shock. When the particle crosses the transverse section of a tube then the direction of its movement is changing slightly. The change is random and its value and direction depend on the location of the intersection point where the particle crosses the transverse section of the tube. These crossings correspond to scatterings in our simulations.

In addition, we assume that vectors of the magnetic field of small tubes are adding up and form one magnetic helical tube in the macro scale (the big tube). The particle wandering upstream of the big tube can see the big tube as the homogeneous magnetic field perpendicular to the shock normal. Now, we have two idealised cases, the random field and the homogeneous field, that are one physical reality. The value of the magnetic field of a small tube must be on average much larger than the value of the magnetic field of the big tube so that the addition of vectors is correct.

Now, our model of simulations fits into the theoretical model. In the theoretical model particles interact with the strong random magnetic field only and the homogeneous magnetic field is formed by the composition and dynamics of small tubes. Each scattering takes up a very short time span, equal to zero in the simulations, and corresponds to the change of the direction of the particle velocity caused by some group of small tubes. The wandering in the homogeneous magnetic field between scatterings takes up the entire amount of the time (the big tube). Our model of simulations enables any shortening of the time between scatterings and keeping the same pattern of the magnetic field fluctuations simultaneously.

One should remember that current numerical simulations and analytical models are useless in the topic of relativistic plasma since they do not reach the required accuracy. We will explain this opinion with the use of two examples of PIC simulations. Spitkovsky (2008) simulated relativistic shocks in 2D and used an artificial wall that had produced an artificial returning stream. Collective processes in plasma are sensitive to a flap of a bird’s wing and the stream is a hurricane which can do everything. The author has written that the shock moves to the right at 0.47c. We do not know how to transform the velocity to 3D, but if it has the same value in 3D then the compression is equal to 2. Such decline in the value of the compression from 3 to 2 means that the asymptotic value of the energy spectral index \( \beta_0 \) grows from 2.2 to \( \sim 4 \). The author has detected particle acceleration and got the index equal to 2.4. In our opinion the artificial returning stream accelerates particles and reduces the compression.

Nishikawa et al. (2006) performed simulations with \( \sim 3.8 \cdot 10^8 \) particles, but we have counted \( \sim 2.5 \cdot 10^9 \) what is important here. The important thing is that 1.25 \( \cdot 10^8 \) hydrogen atoms have mass \( 2.1 \cdot 10^{-16}g \), 1.25 \( \cdot 10^8 \) electron-positron pairs have mass \( 1.1 \cdot 10^{-19}(g, \text{ a flu virus has mass } 7 \cdot 10^{-13}g \) and typical plasma flows of GRBs have rest mass \( \sim 10^{27}g \) if we apply canonical energy release of \( \sim 10^{51} \text{erg} \), the Lorentz factor of the plasma equal to 1000 and neglect the Poynting flux.

The authors have scaled their simulations to density of the interstellar medium in order to get the length of the simulation box equal to 4 \( \cdot 10^5 \text{ cm} \) and have not scaled to density of the tail of a comet, for example, which is \( 10^5 \) times higher. The authors have compared the length of the simulation cell to the electron skin depth in order to show that the shock fits in the simulation box, but it is untrue. The thickness of the shock is comparable to the gyroradius of a thermal ion (Drury 1983) and it compares unfavourably with the size of their simulation box. The authors have not reached dynamic equilibrium so that they do not know if their results are stable. The authors have written, ‘‘the ‘flat’ (thick) jet fills the computational do-
main in the transverse directions (infinite width). Thus, we are simulating a small section of a relativistic shock infinite in the transverse direction and that is untrue again.

They have applied some boundary conditions and named them ‘thick jet’ or ‘infinite width’. All boundary conditions are artificial and can produce anything. Further below, we propose how to manage with the problem.

We have shown that the current PIC simulations of relativistic plasma are pointless. They have provided two intuitive hints only. First, relativistic plasma could produce seed particles (Nishikawa et al., 2003; Hededal et al., 2004) and second, the plasma could form current filaments (Medvedev & Loeb, 1999). We believe that the task of creation of relativistic plasma in PIC simulations from scratch is certainly not so perfect as presented in Figure 2. The system is not perfect, but the PIC simulations are required to prove it. The system must be conducted. We will demonstrate how to catch the asymptotic energy spectral index then the system accelerates particles in Cracow acceleration.

The proton-electron plasma is more difficult to analyse. We think that it consists of proton loops and electron loops. Diameters of the electron ovals are much smaller than proton ovals. The ovals also approach the geometry that minimizes the magnetic field opposite to the homogeneous magnetic field. Electron ovals tend to be close to the electric current of proton loops.

We name the described plasma loop plasma. The distribution of particle velocities in loop plasma is not the Maxwellian one so that the temperature is not defined in the usual meaning. It is hard to say what the downstream particles are. We assume that the plasma is Maxwellian and that the electric current is the same for all loops. Each loop contains identical electric charges, electrons or positrons, which are indicated in Figure 2 by $e^-$, $e^+$, respectively. The particles are arranged evenly along the loop in this way that the centrifugal force and the electric force are balanced by the Lorentz force. The isolated loop in the shape of a circle is in unstable equilibrium and the environment flattens it into the oval. We have no idea which oval is more stable, the one elongated parallel or perpendicular to the shock normal, but PIC simulations suggest that the ovals elongated parallel to the shock normal make relativistic plasma because current filaments tend to be parallel to the direction of the flow.

We think that the system presented in Figure 2 is stable and the Weibel instability rebuilds continuously the ovals, but the PIC simulations are required to prove it. The system is certainly not so perfect as presented in Figure 2. The ovals have different diameters, they oscillate, have global movements (the instantaneous velocity of a loop is $v_e$), and that is untrue again.

The ovals are being distorted in space. There is a fraction of particles that do not belong to any oval (free particles).

The ovals form the homogeneous magnetic field - $B$. They are divided into layers which are parallel to the shock normal. Each layer is divided into rows which are perpendicular to the shock normal. Ovals along a row are arranged alternately, the electron oval follows the positron oval and vice versa. We adopt the rule that the ovals approach the geometry that minimize the magnetic field opposite to $B$.

We have not carried out any calculations, but we think that two adjacent rows adhere to the shared adhesive line. We indicate lengths of semi-axes of the ellipse by $a$ and $b$, the distance between two consecutive ellipses along the row by $d$. Each layer is translated in regard to the symmetrical position of the adjacent layer for the length of $a$ parallel to the shock normal and $d/2$ perpendicular to the shock normal. The distance between two adjacent layers depends on values of $a$, $b$ and $d$.

In our model, not only the magnetic helical tubes (Figure 2) but also separation of electric charges disturb cosmic rays. It is easy to prove whether such disturbances enable cosmic rays to accelerate in Cracow acceleration. One should track the trajectory of high-energy particles upstream of the shock (Figure 2) with their initial momenta nearly parallel to the shock normal. The change of the direction of the momenta must be small while the tracking. The momenta must be all the closer to the shock normal the Lorentz factor of the shock is larger. Now, one derives the value of $Q_0$ from the tracking and puts it into the formula if the derived energy spectral index is close to the asymptotic energy spectral index then the system accelerates particles in Cracow acceleration.

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We name the described plasma loop plasma. The distribution of particle velocities in loop plasma is not the Maxwellian one so that the temperature is not defined in the usual meaning. It is hard to say what the downstream plasma will look like if loop plasma comes through the shock. However, it is not important for Cracow acceleration which will work anyway.

The PIC simulations are useless unless one is able to detect dynamic equilibrium of plasma. Creating relativistic plasma from scratch is outside the available computing power at the moment. The PIC simulations must hunt for dynamic equilibrium of plasma rather than try and create it from scratch. Below, we describe how the simulations should be conducted. We will demonstrate how to catch dynamic equilibrium of electron-positron loop plasma. At first, we take into account the upstream loop plasma only. The shock is far enough and does not affect the system.

One must use many processors at the same time (parallel programming). The plasma is divided into boxes. The boxes are within the space of a cuboid (the central cuboid).

Each electric current loop is within one loop box. Inside the loop box there are free particles also. Loop boxes must not overlap so that their linear dimensions are changing. If necessary, a few loops can be inside one box. The loop boxes can merge, split, disappear and be formed. Additionally,
the plasma is divided into free boxes that contain the remaining free particles inside empty spaces. All free boxes are the same and fill up the cuboid entirely, they do not change.

To save the computing power, one processor handles many loop boxes or free boxes during one computational step, but not at the same moment. The number of all boxes should exceed the number of processors ten times for example. The electromagnetic force inside a box is calculated exactly, the force from nearby boxes is also calculated exactly, but the force from any other box is taken from a pattern database.

We replace the boundary conditions with a pattern database of the electromagnetic field of pattern cuboids. The pattern cuboids surround the central cuboid up to a distance. There are no empty spaces inside the system. The distance must be large enough to not change results of simulations if it increases. If necessary, global electric and magnetic fields that represent the electromagnetic field of plasma outside the distance is added. Sizes of pattern cuboids can be different and the results can not depend on the sizes.

The biggest challenge is creating the pattern database and the efficient use of it. Each pattern in the database must have many items of accuracy. Let us say, we have 15 items of accuracy and number 0 represents the lowest accuracy. Results of simulations must be the same if we apply items numbered from a certain number to number 14.

The pattern database changes as simulations bring closer to the dynamic equilibrium target. The size of the central cuboid must be large enough to detect statistically independent pieces of it. Pattern cuboids will be formed by splitting and merging the central cuboid in order to get different sizes of them.

Plasma inside the central cuboid moves with a velocity in the observer’s frame. The plasma is supplied with boxes at one end of the central cuboid. The boxes disappear at the opposite end. The entering boxes are changing. They are replaced with boxes similar to those ones nearby the opposite end, but not too close to the boundary of the central cuboid. The simulations must run until all boxes inside the central cuboid are statistically homogeneous, pattern cuboids are similar to the central cuboid and the conditions do not change. It is the dynamic equilibrium of plasma.

The next stage is to catch equilibrium of the plasma around the shock. It is easy because we know what the entering boxes look like. The central cuboid must contain the upstream plasma, the shock and the downstream plasma. Initial boxes of the shock and downstream plasma can be any. Simulations are conducted in the shock rest frame.

The equation of motion of the shock must be known from astronomical observations in order to catch equilibrium of the slowing down shock. Simulations should be conducted in the shock rest frame. The entering boxes are supplied according to the equation of motion. The simulations must start with different starting points that are between the initial and the final Lorentz factor of the shock. At the end, the plasma must change in the same way independently of the choice of the starting point.

We have presented the sketch of simulations of relativistic plasma. We think that $10^5$ to $10^7$ processors are needed to perform the task. It requires much of intellectual efforts also, but it has to be done if we wish to figure out the microphysics of relativistic plasma.

The issue still remains to be explained whether the plasma that allow for Cracow acceleration exists. The answer is simple, observations resolve the problem. Cracow acceleration predicts a lot of peculiar facts as superluminal shocks acceleration, acceleration at shocks with extremely large Lorentz factors (pulsars), the negative GRB lag, the extremely short acceleration time and others.

Determining the acceleration time in pulsars would be the crucial proof. The plasma downstream of the shock in pulsars is at rest with respect to the interstellar medium (ISM) and the value of the homogeneous magnetic field can be detected there. The acceleration time in the mechanism of Cracow acceleration is equal to 1 or is shorter if measured in units of the homogeneous magnetic field downstream of the shock (Bednarz, 2000). Mechanisms in which the change of the direction of the movement of a particle enabling it the return to the shock takes place downstream of the shock can not achieve such short acceleration time.

3. The mechanism of Cracow acceleration

In this paragraph we follow simulations of Bednarz (2000) and Bednarz (2004). We will sometimes denote 'the plasma' upstream of the shock' as UP and 'the plasma' downstream of the shock' as DOWN. We have simulated relativistic shocks with Lorentz factors $\gamma > 1$ and the perpendicular homogeneous magnetic field. Downstream of the shock the pure homogeneous magnetic field is present, and upstream of the shock there are additional magnetic field fluctuations. The magnetic field fluctuations pattern UP is characterised by a value of the parameter $Q$ that fulfills the formula (Bednarz, 2004)

$$\frac{\kappa_\perp}{\kappa_\parallel} = \frac{1}{1 + Qp^2},$$

where $p$ is the particle momentum and $\kappa_\perp, \kappa_\parallel$ are perpendicular and parallel diffusion coefficients, respectively.

The numerical calculations involve particles with momenta systematically increasing over several orders of magnitude. In order to avoid any energy dependent systematic effect we consider the situation with all spatial and time scales—defined by the diffusion coefficient, the mean time between scatterings and the shock velocity—proportional to the particle gyroradius $r_p$, i.e. to its momentum. It means that the results are momentum independent and can be easily scaled to any momentum. In particular we will be using the parameter $Q_0 = Qp^2$ that determines magnetic field fluctuations UP independently of the value of the momentum.

We have applied the light velocity, $c$, equal to 1 and the time is measured in $r_p/c$ units. The plasma UP is hot and the compression is equal to $3U_0^2$, where $U_0$ is the shock velocity in the upstream plasma rest frame. The Lorentz factor of the plasma DOWN as seen UP, $\gamma_{ud}$, can be approximated through the formula $\gamma_{ud} = 0.71\gamma$.

Lengths of the magnetic field fluctuations required by Cracow acceleration must be much smaller than the mean time the particle spends UP between shock crossings, $\Delta t_u$, where $\Delta t_u$ is measured in the plasma rest frame. It means that the time period in which the particle and a fluctuation interact is much smaller than $\Delta t_u$. For better understanding we give $\Delta t_u$ in radians, for such an angle the particle
will turn in the wandering in the homogeneous field. The value is the same as in \( r_\gamma c \) units.

The mean change of the particle momentum direction between the moment when the particle is entering UP and the moment when it is leaving UP is equal to \( \Delta \alpha_u \) if measured in the plasma rest frame and \( \Delta \omega u \) if measured in the downstream plasma rest frame. Since upstream of the shock particles momenta are directed almost parallel to the shock velocity we assume that \( \Delta \alpha_u \approx \gamma \Delta \alpha_u \).

Let us consider the particle that crosses the shock from UP to DOWN (UP-DOWN) with parameters \( \phi \) and \( r \), where the particle phase \( \phi \in (0, 2\pi) \) is the angle between the direction DOWN-UP and the projection of the particle gyroradius onto a plane that is perpendicular to the homogeneous magnetic field, \( r \), see Bednarz & Ostrowski (1999).

The sense of the angle \( \phi \) is the same as the sense of rotation of the particle, and \( r, \phi \) are measured in the downstream plasma rest frame.

The particle will cross the shock again if there is a solution of the equation

\[
\gamma r \sin\left(\frac{\pi}{2} + \phi\right) \sin\left(\frac{\phi}{2}\right) + \frac{t}{3} = 0,
\]

at positive time \( t \).

We will therefore distinguish three ranges of particle parameters (see Figure 3), \( A_1 = \{ \phi : \phi \in (\phi_1, \phi_2) \} \) - the equation has not the positive solution and the particle does not return to the shock, \( A_2 = \{ \phi : \phi \in (\phi_2, \phi_3) \} \) - the equation has the positive solution and the particle returns to the shock, \( A_3 = \{ \phi : \phi \in (\phi_3, \phi_1) \} \) - the particle with parameters from this range crosses the shock DOWN-UP and can not cross the shock in the opposite direction, \( r \) sets the amount and determines \( \phi \) values.

Values of \( \phi_1 \) and \( \phi_2 \) are given by analytical formulae, \( \phi_1 = 2\pi - \arcsin(1/(3r)) \), \( \phi_2 = 3\pi/2 - \arccos(1/(3r)) \), and \( \phi_3 \) must be calculated numerically. A particle with \( r < 1/3 \) does not return to the shock. Chosen values of parameters are \( r = 1, \phi_1 = 5.94, \phi_2 = 1.96, \phi_3 = 3.48; r = 0.5, \phi_1 = 5.55, \phi_2 = 2.96, \phi_3 = 3.87; r = 1/3, \phi_1 = \phi_2 = \phi_3 = \frac{\pi}{2} \).

The change of the particle direction, \( \Delta \omega u \), consists of two parts, the change in the process of wandering in the homogeneous field, \( \Delta \omega_h \), and the change as the result of collisions with fluctuations, \( \Delta \omega f \), so we can write \( \Delta \alpha_u = \Delta \omega_h + \Delta \omega f \). We have neglected addition of vectors in this Section.

A particle is entering UP having parameters \((r, \phi)\) from the set \( A_2 \). If there are no fluctuations UP, the particle is returning DOWN having \((r, \phi)\) from the set \( A_1 \). That is the case of Fermi acceleration, where the acceleration is forbidden in superluminal shocks and \( \Delta \alpha_u = \Delta \omega f \).

In Cracow acceleration \( \Delta \omega f \) plays the key role. Fluctuations must be heavy so that \( \Delta \omega f \) is large enough. The angle change - \( \Delta \omega h \) is directing a particle this way so that it gets parameters from the set \( A_1 \) and \( \Delta \omega f \) is directing randomly, enabling the particle very much often changing \( \phi \) from \( A_3 \) to \( A_2 \) through \( \phi_3 \). When \( \Delta \omega f \) dominates then half of the particles change \( \phi \) to a value from the set \( A_2 \) and half from the set \( A_1 \). It implicates the short acceleration time equal to 1.

The change of the particle direction through \( \Delta \omega h \) is the linear process because \( \Delta \omega f \) is the diffusion process, and that is expressed by the formulæ \( (\Delta \omega f)^2 = C_f \Delta t_u \) and \( \Delta \omega_h = \Delta t_u \), where \( C_f \) is constant. It yields the relation \( \Delta \omega_d = 1.42(\sqrt{C_f} \Delta t_u + \Delta t_u) \). If \( \gamma \) grows then \( \Delta t_u \) decreases. One can see now that the relative \( \Delta \omega f \) contribution to \( \Delta \alpha_u \) grows with \( \gamma \), which means that the particle energy spectral index, \( \beta \), decreases. The index would increase but only if \( \Delta t_u \) decreased more quickly than \( 1/\gamma^2 \), but it does not occur as is shown below.

Let us return to the simulations. We have performed simulations for 14 \( \gamma \) values, \( \gamma = 2^4 \), where \( n = 2, 3, \ldots, 15 \) and 500 \( Q_0 \) values from 5 \cdot 10^{-1} to 5 \cdot 10^7 in logarithmic space. For larger \( \gamma \) we started the computations from larger \( Q_0 \) than 5 \cdot 10^{-1}, but being small enough in order to get the satisfying range of the asymptotic energy spectral index \( \beta_0 = 2.23 \) (Bednarz 2004).

The obtained \( \beta - \beta_0 \) values as a function of \( \ln(Q_0) \) are shown in Figure 4. Individual curves represent constant \( \gamma \) values and one can see that they are similar. We have converted the curves according to the formula \( W = \ln(Q_0) + 2(\ln(2.145) - \ln(\gamma)) \). After the transformation the curves have become identical, but the one for \( \gamma = 4 \), and the curve for \( \gamma = 8 \) has been standing out from this relation very slightly (see Figure 5). The relation \( \gamma^2/Q_0 = const \) for constant \( \beta \) is exact and there must be a simple theoretical explanation for it.

In order to get errors of fitting we have limited the range of the data to \( W \in (-4, 2.5) \), see Figure 5. We have excluded points for which \( \beta \leq \beta_0 \) and curves for which \( \gamma = 2^{14}, 2^{15} \) because their \( \beta \) values do not cover
the upper range of $W$. We have been fitting the relation

$$\beta - \beta_0 = A \exp(BW)$$

and have gotten $A = 1.007 \pm 0.004$, $B = 0.722 \pm 0.003$. Finally, we have received the important relation

$$(\beta - \beta_0)^{1.39} = Q_0 \left( \frac{2.145}{\gamma} \right)^2, \quad \gamma > 8. \tag{1}$$

The numerical precision prevented us from carrying out calculations for $\gamma$ larger than 32768, but we cannot see an obstacle that the above formula is not valid for all $\gamma > 8$.

We have repeated the above procedure for accurate values of $\gamma_{ad}$ instead of $\gamma$ and have gotten the same results but with the values of constants. Two first curves ($\gamma \in \{4, 8\}$) the same diverge from the relation as for $\gamma$. The separation of the curve for $\gamma = 4$ depictions where Cracow acceleration is starting weakening, for $\gamma = 2$ it turns off practically.

Further, we have checked how $\Delta t_u$ and $\Delta \alpha_u$ are changing with $\gamma$. The choice of $\ln(\gamma^2/Q_0)$ as independent variable enabled us to put curves in one line. In Figure 4 14 curves for constant $\gamma = 2^n, n = 2, \ldots, 15$ and in Figure 5 14 curves for constant $Q_0 \in \{1.4, 30, 297, \ldots, 8.3 \times 10^5\}$ are presented. The curves for $\gamma = 4$ (dashed lines) and $Q_0 = 1.4$ are diverging from general relations.

From Figure 4 it is seen that $\Delta \alpha_u \gamma$ is constant for large $\gamma$ and $\Delta t_u$ decreases with $\gamma$ faster than $1/\gamma$. Moreover, we have put lines that are indicating values of $\beta$ for fixed $\gamma^2/Q_0$ and now one can estimate how reducing the contribution of $\Delta \omega$, to $\Delta \omega_d$ is decreasing $\beta$ to $\beta_0$.

The aim of making Figure 5 was to show how $\Delta t_u$ was changing with $\gamma$. It decreases with $\gamma$ always slower than $1/\gamma^2$ and approaches the asymptotic value of $1/\gamma^2$ for large $\gamma$. It guarantees that Cracow acceleration does not fade with growing $\gamma$. For large $\gamma$, we have fitted the relation

$$\Delta t_u \gamma^2/\sqrt{Q_0} = 1.097 \pm 0.002. \tag{2}$$

Cracow acceleration could weaken with growing $\gamma$ because of the increasing energy of reflected particles, but it is not. Let $p_s$ be the momentum of a seed particle. Reflection process increases particle energy $\sim \gamma^2$ times, so that $Q_0 \sim Q(p_s \gamma)^2$ at the onset of Cracow acceleration process, so $1/(Qp_s^2) \sim \gamma^2/Q_0 = \text{const}$ if $\beta = \text{const}$. It means that the initial value of $\beta$ depends on plasma conditions upstream of the shock only ($Q$ and $p_s$). In real physical conditions this important result, similarly as previous results, can be altered by magnetic field fluctuations downstream of the shock.

Cracow acceleration could weaken with growing $\gamma$ as a result of reducing the effectiveness of reflection process, but it is a quite different problem for which plasma conditions should be examined.

4. Simulations

Below, the light velocity is used as the velocity unit, $c = 1$. As the considered particles are ultrarelativistic ones, $p = E$, we often put the particle momentum for its energy.

In the simulations we consider the mean magnetic field configuration perpendicular to the shock normal and magnetic field fluctuations. We restrict our consideration to the test-particle approximation in which it is assumed that par-
particles are scattered by scattering centres in the fluid but have no effect either on the fluid velocity or on the density of scattering centres. Between two successive scatterings the particle is assumed to proceed along the undisturbed path in the mean field. We model particle trajectory perturbations by introducing small-angle random momentum scattering along the mean field trajectory (Ostrowski [1991], Bednarz & Ostrowski [1996, 1998] Bednarz [2004]. The perturbed magnetic field represents the traditional picture based on the concept of magnetic scattering centres. It is simulated by the small amplitude particle momentum scattering within a cone with angular opening $\Delta \Omega$ less than the particle anisotropy $\sim 1/\gamma$, where $\gamma$ is the Lorentz factor of the shock. The particle momentum scattering distribution is uniform within the cone. For each $\Delta \Omega$ the adequate mean time between scatterings was chosen to keep the same magnetic field perturbations pattern.

The pattern is characterised by a value of the parameter $Q$ defined in Section 3. In this Section we use more convenient parameter to work with. We have defined $C = -\log_{10} Q$. If $C$ determines magnetic field fluctuations upstream of an ultrarelativistic shock moving with a constant velocity, then with growing $C$ particles are able to accelerate to higher energies in Cracow acceleration.

Particle momenta are measured in the unit of any moment $p_0$. We have applied seed particles momenta equal to 0.1 because of computational reasons, so that the value $\log_{10}(p) = 1$ means that the seed particle momentum was increased hundred times.

The value of the mean magnetic field taken in the upstream plasma rest frame, $B_u$, is the unit of the magnetic field. The particle charge, $e_0$, is the charge unit, so that the unit of time is $p_0/(B_u e_0 c)$.

Our simulations are intended to model decelerating ultrarelativistic shocks, so that we have chosen the equation of motion of the shock in the form

$$x_s(t) = -a \ln \frac{a + t}{a},$$

where $a > 0$ is a constant and $x_s, t$ are the distance and the time measured in the upstream plasma rest frame, respectively.

There is a lack of knowledge of the plasma conditions downstream of the decelerating relativistic shock, so that we have defined them. We have divided the motion of the shock into discrete instants. We have applied a constant shock velocity at each instant. The downstream plasma velocity and magnetic field are derived from the constant velocity relativistic shocks (Bednarz & Ostrowski, 1996) with the compression $R_h = 3U^2_s$ for the hot plasma, where $U_h$ is the shock speed in the upstream plasma rest frame. The shock is perpendicular, so that the instant formulae are

$$U = \frac{3U^2_h - 1}{2U_s}, \quad B = -B_u U_s \sqrt{\frac{9U^2_h - 1}{1 - U^2_s}}.$$

where $U$ is the downstream plasma velocity close to the shock measured in the upstream plasma rest frame and $B$ is the mean magnetic field close to the shock taken in the downstream plasma rest frame.

We had to accept a rule in order to get $U(x, t)$ and $B(x, t)$ fields downstream of the shock. The rule is that the decelerating shock ‘puts the instant fields back’ and goes on, but for simplicity its speed is always equal to the speed of light. The rule yields the fields

$$U(x, t) = \frac{3U^2_h (t - \Delta t) - 1}{2U_s (t - \Delta t)}, \quad (3)$$

$$B(x, t) = -U_s (t - \Delta t) \sqrt{\frac{9U^2_h (t - \Delta t) - 1}{1 - U^2_s (t - \Delta t)}}, \quad (4)$$

where $\Delta t$ is derived from $\Delta t = x_s (t - \Delta t) - x$ and $x_s$ are the distance and the time respectively, $x_s$ is the shock position and all quantities apart from $B(x, t)$ are measured in the upstream plasma rest frame.

We were injecting mono-energetic seed particles upstream of the shock with momenta distributed uniformly on the sphere. The seed particle density along the shock path was constant. Each particle trajectory was followed using numerical computations until the particle escaped through the free escape boundary placed far downstream from the shock or it reached the time larger than the assumed upper limit. When the particle was wandering downstream of the shock its momentum was Lorentz transformed to the respective plasma rest frame at each small step and its trajectory was computed in the frame.

The initial and final Lorentz factors of the shock were chosen. The factors and the equation of motion put the upper limit of time. We have divided the wandering time, measured in the upstream plasma rest frame, into ten equal periods. The successive time bins are labelled numbers from 0 to 9. The Lorentz factor range is labelled $\gamma_d = \gamma_\sim 3\gamma_f$, where $\gamma_i, \gamma_f$ are the initial and final Lorentz factors, respectively. For example, $\gamma_d = 40.5$ means that we started collecting data at the Lorentz factor of the shock equal to 40 and stopped at 5.

The particle crossings of the shock were divided into stages as described in Figure 8. The stages are labelled ‘su’, ‘rd’, ‘ru’, ‘p1’, ‘pu’, ‘pd’. The spectrum of the particles crossing the shock from DOWN to UP for the first time is labelled ‘rd’, for the second particle ‘p1’, for the third and following ‘pd’. The total spectrum of the particles crossing the shock from DOWN to UP is the sum of ‘rd’, ‘p1’, ‘pd’ spectra and is labelled ‘ad’. We introduced the token sp also, for instance sp=‘rd’.

Each time a particle crossed the shock the respective contribution was added to the given momentum bin for the appropriate time bin and stage. The momentum and time are measured in the upstream plasma rest frame.

We assume that particles emit synchrotron radiation. We neglect the self-absorption and take into account the total emission coefficient only (Pacholczyk [1970]).

$$\mathcal{E}_\nu = c_3 H \sin \vartheta \int_0^\infty N(E) F(x) dE,$$

where $c_3$ is a constant, $H$ magnetic field intensity, $\vartheta$ the angle between the mean magnetic field and the direction toward the observer, $N(E)$ - the particle energy distribution, $F(x) = x \int_{x'} K_{5/3}(x) dx$, $K_{5/3}(z)$ is the Bessel function of the second kind, $x = \nu / \nu_c$, $\nu$ - photon frequency, $\nu_c = c_1 H \sin \vartheta E^2$, $c_1$ a constant and $E$ is the particle energy.
We have calculated the spectral distribution of synchrotron radiation of accelerated particles using the formula

$$I(\nu) = \int_0^\infty \frac{N(p)}{p^2} \nu^2 dp,$$

where $I(\nu)$ is intensity of radiation and $N(p)$ the particle momentum distribution. Units of $I(\nu)$ and $\nu$ are unimportant here and we applied a constant equal to 1.

In order to represent each set of spectra concisely we have introduced the vector $(\gamma_d, a, C, sp)$. For instance, $(20, 5, 10^2, 3, pd)$ represents ten spectra of particles produced by a relativistic shock slowing down from Lorentz factor 20 to 5 with the parameter of the equation of motion equal to $10^2$, the upstream magnetic field fluctuations pattern measured by $C = 3$ and particles crossing the shock from DOWN to UP for the third and following times.

Fluctuations of the magnetic field downstream of the shock are unimportant now. They should be strong enough to allow effective reflections of seed particles. We have applied there $C_d = -\log_{10} 50$, where $C_d$ is the value of $C$ downstream of the shock. The parameter $C_d$ is a variable and $C$ a constant in Section 7.

5. Slowing down shocks

The velocity and magnetic fields (Equations 3, 4) downstream of the shock change the energy of wandering particles, so that they give some contribution to the total acceleration. We did not analyse the amount of the contribution, but it is of little significance to the paper. As a matter of fact our main results do not depend on a kind of acceleration but it has to be fast enough. Moreover, the presented figures suggest that the contribution does not influence reflection and Cracow acceleration processes significantly. We discuss the processes below, but one should remember that for strong deceleration they are affected by patterns of velocity and magnetic fields downstream of the shock.

Cracow acceleration is very sensitive to the physical conditions upstream of the shock. Its action in shocks moving with a constant velocity is very well known. The particles double their energy along every lap there (Figure 9) and cannot be accelerated if there is a lack of magnetic field fluctuations upstream of the shock.

In slowing down shocks other factors give their contribution, but it is still the same mechanism since the acceleration depends mainly on the fluctuations upstream of the shock.

Particles spend a bit more time upstream of the shock and receive larger momentum gains.

![Fig. 9.](image1) In constant velocity ultrarelativistic shocks particles double their energy along every lap.

![Fig. 10.](image2) In slowing down ultrarelativistic shocks particles spend more time upstream of the shock and receive larger momentum gains.
their momentum more and therefore the steeper and shifted spectra are seen at higher momenta.

Second, particles are staying longer upstream of the shock in an absolute unit of time (Figure 10). It means that particles that have higher energies return to the shock later. This entails that the delaying particles are stopping the high energy part of the spectrum from moving back to lower energies, what means that the part is moving back slower or standing or moving ahead. We call this phenomenon lingering.

There is still an issue to be addressed. Some particles enter UP with the directions of the momentum close to the limit of the transition. Their interaction with magnetic field fluctuations UP is weak and their increase of the momentum is small. If the shock has a constant velocity then a fraction of the particles can not reach the shock again, but when the shock is slowing down the particles get an additional time and change the trajectories in such a way that some particles from the fraction can reach the shock again. This leads to flatter spectra at lower energies.

In result, Cracow acceleration in slowing down shocks produces flatter spectra at lower energies and steeper at high energies.

The stronger deceleration of the shock the shorter duration of the episode if $\gamma_1$ and $\gamma_2$ are fixed. It means that the total time of the episode could be very small what yields two effects.

The first effect refers to reflection process. Reflecting particles gain higher energies if they are spending more time downstream of the shock. At first to the slowing down shock (from DOWN) arrive particles which increase the momenta less and then with better energy gains. These all particles have entered DOWN at the same Lorentz factor of the shock. This gives smaller range of energies (across all time bins) of particles reflecting from strong than from weak decelerating shocks.

The second effect refers to Cracow acceleration. Particles accelerating at strong decelerating shocks receive smaller energy gains because they have not enough time to receive larger.

### Table 1. $\gamma_d - 10\gamma_5$, spectrum - $pd$

| $a$ | $C$ | $n_p$ | $\nu_p$ | $n_n$ | $\nu_n$ | lag  |
|-----|-----|-------|---------|-------|---------|------|
| $10^2$ | 4  | ...  | ...  | ...  | ...  | R    |
| $10^3$ | 2  | ...  | ...  | 2  | 2.67 | N↑  |
| $10^3$ | 3  | ...  | ...  | 2  | 2.44 | N↑  |
| $10^4$ | 4  | ...  | ...  | 2  | 2.28 | N↑  |
| $10^4$ | 2  | ...  | ...  | 1  | 4.26 | N↑  |
| $10^4$ | 4  | ...  | ...  | 1  | 4.66 | N↑  |
| $10^5$ | 4  | ...  | ...  | 1  | 3.80 | N↑  |
| $10^5$ | 2  | ...  | ...  | ...  | ...  | N↑  |

### Table 2. $\gamma_d - 10\gamma_5$, spectrum - $p1$

| $a$ | $C$ | $n_p$ | $\nu_p$ | $n_n$ | $\nu_n$ | lag  |
|-----|-----|-------|---------|-------|---------|------|
| $10^2$ | 4  | ...  | ...  | ...  | ...  | R    |
| $10^3$ | 2  | ...  | ...  | 2  | 3.78 | N↑  |
| $10^3$ | 3  | ...  | ...  | 2  | 4.63 | N↑  |
| $10^4$ | 4  | ...  | ...  | 1  | 3.52 | N↑  |
| $10^4$ | 2  | ...  | ...  | ...  | ...  | D    |

### Table 3. $\gamma_d - 10\gamma_5$, spectrum - $ad$

| $a$ | $C$ | $n_p$ | $\nu_p$ | $n_n$ | $\nu_n$ | lag  |
|-----|-----|-------|---------|-------|---------|------|
| $10^2$ | 2  | ...  | ...  | 8  | 1.98 | N    |
| $10^3$ | 3  | ...  | ...  | 8  | 1.95 | N    |
| $10^4$ | 4  | ...  | ...  | 8  | 1.90 | N    |
| $10^5$ | 2  | ...  | ...  | 6  | 2.06 | N↑  |
| $10^2$ | 3  | ...  | ...  | 6  | 1.92 | N↑  |
| $10^2$ | 4  | ...  | ...  | 6  | 1.94 | N↑  |
| $10^3$ | 2  | ...  | ...  | 1  | 2.25 | N↑  |
| $10^3$ | 3  | ...  | ...  | 1  | 2.25 | N↑  |
| $10^3$ | 4  | ...  | ...  | 1  | 2.16 | N↑  |
| $10^4$ | 2  | ...  | ...  | 0  | 2.74 | N↑  |
| $10^4$ | 3  | ...  | ...  | 0  | 2.52 | N↑  |
| $10^4$ | 4  | ...  | ...  | 0  | 2.38 | N↑  |
| $10^5$ | 2  | ...  | ...  | ...  | ...  | D    |

### Table 4. $\gamma_d - 10\gamma_5$, spectrum - $rd$

| $a$ | $C$ | $n_p$ | $\nu_p$ | $n_n$ | $\nu_n$ | lag  |
|-----|-----|-------|---------|-------|---------|------|
| $10^2$ | 2  | 3  | 3.16 | ...  | ...  | P↑  |
| $10^3$ | 3  | 3  | 2.99 | ...  | ...  | P↑  |
| $10^4$ | 4  | 3  | 3.23 | ...  | ...  | P↑  |
| $10^5$ | 2  | 2  | 2.87 | ...  | ...  | P↑  |
| $10^2$ | 3  | 2  | 2.94 | ...  | ...  | P↑  |
| $10^2$ | 4  | 2  | 2.90 | ...  | ...  | P↑  |
| $10^3$ | 2  | 0  | 2.47 | ...  | ...  | P    |
| $10^3$ | 3  | 0  | 2.54 | ...  | ...  | P    |
| $10^3$ | 4  | 0  | 2.49 | ...  | ...  | P    |
| $10^4$ | 2  | ...  | ...  | ...  | ...  | D    |

### 6. Results

From graphs of particle spectra one can get the energy spectral index

$$\beta = 1 - \frac{\Delta \log_{10}(dN)/d\log_{10}(p)}{\Delta \log_{10}(p)} .$$

In each of photon spectra we distinguish the frequency $\nu_{max}$ where the maximum intensity $I(\nu_{max})$ is reached. In principle, we will limit ourselves to frequencies larger than $\nu_{max}$.

The first lap in the process of Cracow acceleration is represented by 'p1' spectra. They are similar to corresponding 'pd' spectra. We have not presented a 'p1' spectrum, but the observation is visible as one compares Tables 1 and 2. Our first result is that reflection process does not influence Cracow acceleration from the first lap, and therefore we can treat these processes as separate.

In Figures 11, 12, 13 and 14 are presented spectra that are produced by practically constant velocity shocks. The flat part of 'pd' spectra (Figure 14) has got $\beta$ in the range 2.5-2.7 what is larger than the limiting value of $\nu_{max}$.

In next three Figures (15, 16, 17) the particle spectra and Table 3 are compared with the three preceding figures then one can see to what extent some remarks presented in the previous Section are correct.

Our primary task is to present how lags are formed in photon spectra of slowing down ultrarelativistic shocks. We have examined the spectra and collected results in tables. The description of each table contains the values of $\gamma_d$, $sp$ and two first columns the values of $a$, $C$.  

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A lag will be visible as an intersection of two consecutive photon spectra (produced in two consecutive periods of time). Let us denote the frequency at which the intersection occurs as $\nu_i$, the number of the first spectrum as $n$ and the second as $n+1$. If the photon intensity of the $n+1$ spectrum is higher than the intensity of the $n$ spectrum just before $\nu_i$ (at lower frequency), then the positive lag appears. If the intensity of the $n+1$ spectrum is lower than the intensity of the previous spectrum just before $\nu_i$, then the negative lag appears.

The first occurrence of the positive lag in the set of photon spectra for consecutive periods of time was put into columns third and fourth, where $\nu_p$ is the number of the previous spectrum and $\nu_n$ is the frequency at which the intersection occurs. The first occurrence of the negative lag was collected in the fifth and sixth columns, where $\nu_n$ is the number of the previous spectrum and $\nu_p$ is the frequency at which the intersection occurs.

The seventh column is marked by 'lag' and contains the information how lags in the photon spectra are changing. The photon spectra that are only increasing (decreasing) their intensities at all frequencies are denoted as R (D). If the intensities at first are increasing and next they are decreasing at all frequencies, then they are denoted as RD.

If $R$ appears at some values of $a$ and $C$, then for all smaller values of $a$ and for the same value of $a$ and smaller values of $C$ $R$ appears also. In that case the subsequent items of $R$ are not included in the tables. If $D$ appears at some values of $a$ and $C$, then for all larger values of $a$ and for the same value of $a$ and larger values of $C D$ appears also, in that case the subsequent items of $D$ are not included in the tables.

If $n$ spectrum and $n+1$ spectrum build a positive lag at frequency $\nu_p$ and then $n+1$ spectrum and $n+2$ spectrum build a positive lag at frequency $\nu_{p2} < \nu_p$ and so on (or not), then the spectra are denoted as $P\uparrow$. If only one positive lag appears, then the spectra are denoted as $P$. Here it is necessary to add that we did not take into account positive lags that appeared at frequencies lower than $\nu_{max}$. In fact positive lags are always $P\uparrow$ and are moving very fast towards low frequencies passing through $\nu_{max}$.

If $n$ spectrum and $n+1$ spectrum build a negative lag at frequency $\nu_n$ and then $n+1$ spectrum and $n+2$ spectrum build a negative lag at frequency $\nu_{n2} < \nu_n$ and so on (or not), then the spectra are denoted as $N\downarrow$. If only one negative lag appears, then the spectra are denoted as $N$. The negative lag is moving slowly towards low frequencies and stops at high frequencies far before $\nu_{max}$.

If $n$ spectrum and $n+1$ spectrum build a negative lag at frequency $\nu_n$ and then $n+1$ spectrum and $n+2$ spectrum build a negative lag at frequency $\nu_{n2} > \nu_n$ and so on (or not), then the spectra are denoted as $N\uparrow$. If only one negative lag appears, then the spectra are denoted as $N$. These types of spectra are always $N\uparrow$. The case of $N$ arises due to the limitation of data and large errors at high frequencies.

An exception exists for small initial Lorentz factors of the shock and small $a$ (Figures 18, 19, 20). In Table 9 for the case of $(10,5,60, any, ad)$ the spectra number 8 and 9 build the negative lag N (Figure 18). First, we do not expect such small value of $a$ for small initial Lorentz factors. Second, $a$

| Table 5. $\gamma_d - 40_{-5}$, spectrum - $rd$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n2}$ | lag |
| 60 | 2 | 2 | 3.85 | ... | ... | $P\uparrow$ |
| 60 | 3 | 2 | 3.67 | ... | ... | $P\uparrow$ |
| 60 | 4 | 2 | 3.55 | ... | ... | $P\uparrow$ |
| $10^2$ | 2 | 1 | 4.08 | ... | ... | $P\uparrow$ |
| $10^2$ | 3 | 1 | 3.94 | ... | ... | $P\uparrow$ |
| $10^2$ | 4 | 1 | 4.17 | ... | ... | $P\uparrow$ |
| $10^3$ | 2 | ... | ... | ... | $D$ |

| Table 6. $\gamma_d - 40_{-5}$, spectrum - $pd$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n3}$ | lag |
| $10^2$ | 4 | ... | ... | ... | $R$ |
| $10^3$ | 2 | ... | ... | ... | $RD$ |
| $10^3$ | 3 | ... | ... | ... | $RD$ |
| $10^3$ | 4 | 1 | ... | 4.23 | $N\uparrow$ |
| $10^4$ | 2 | ... | ... | ... | $D$ |

| Table 7. $\gamma_d - 20_{-10}$, spectrum - $ad$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n3}$ | lag |
| 60 | 1 | 3 | 2.87 | ... | 6 | 3.45 | $P\uparrow N\downarrow$ |
| 60 | 2 | 3 | 2.89 | ... | 3 | 3.63 | $P\uparrow N\downarrow$ |
| 60 | 3 | 3 | 1.56 | ... | 5 | 2.78 | $P\uparrow N\downarrow$ |
| 60 | 4 | 5 | 1.45 | ... | 5 | 2.66 | $P\uparrow N\downarrow$ |
| 10^2 | 1 | 2 | 2.76 | ... | 3 | 4.01 | $P\uparrow N\downarrow$ |
| 10^2 | 2 | 2 | 2.99 | ... | 2 | 3.80 | $P\uparrow N\downarrow$ |
| 10^2 | 3 | 3 | 2.03 | ... | 3 | 3.06 | $P\uparrow N\downarrow$ |
| 10^2 | 4 | ... | ... | ... | 4 | 2.83 | $N\downarrow$ |
| 10^3 | 1 | ... | ... | ... | 1 | 3.87 | $N\uparrow$ |
| 10^3 | 2 | ... | ... | ... | 1 | 3.66 | $N\uparrow$ |
| 10^3 | 3 | ... | ... | ... | 1 | 3.56 | $N\uparrow$ |
| 10^3 | 4 | ... | ... | ... | 1 | 3.47 | $N\uparrow$ |
| 10^4 | 1 | ... | ... | ... | 0 | 4.52 | $N$ |
| 10^4 | 2 | ... | ... | ... | 0 | 4.47 | $N$ |
| 10^4 | 3 | ... | ... | ... | 0 | 4.31 | $N$ |
| 10^4 | 4 | ... | ... | ... | 0 | 4.14 | $N$ |
| 10^5 | 1 | ... | ... | ... | ... | $D$ |

| Table 8. $\gamma_d - 40_{-10}$, spectrum - $ad$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n4}$ | lag |
| 60 | 3 | 3 | 2.86 | ... | 3 | 3.64 | $P\uparrow N\downarrow$ |
| 60 | 4 | 4 | 2.10 | ... | 4 | 3.13 | $P\uparrow N\downarrow$ |
| 10^2 | 2 | 2 | 2.65 | ... | 3 | 4.13 | $P\uparrow N\downarrow$ |
| 10^2 | 3 | 2 | 2.80 | ... | 2 | 4.03 | $P\uparrow N\downarrow$ |
| 10^2 | 4 | 5 | 4.48 | ... | ... | $P\downarrow$ |
| 10^3 | 2 | 0 | 2.92 | ... | ... | $P$ |
| 10^3 | 3 | 0 | 3.50 | ... | 0 | 4.21 | $P\uparrow N\downarrow$ |
| 10^3 | 4 | ... | ... | ... | 1 | 4.42 | $N\uparrow$ |
| 10^4 | 2 | ... | ... | ... | ... | $D$ |

| Table 9. $\gamma_d - 20_{-10}$, spectrum - $ad$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n5}$ | lag |
| $10^3$ | 2 | ... | ... | ... | 4 | 3.49 | $N\uparrow$ |
| $10^3$ | 4 | ... | ... | ... | 5 | 3.26 | $N\uparrow$ |

| Table 10. $\gamma_d - 40_{-10}$, spectrum - $ad$ |
|---|---|---|---|---|---|
| $\alpha$ | $C$ | $\nu_p$ | $\nu_n$ | $\nu_{n6}$ | lag |
| $10^3$ | 2 | 2 | 2.81 | ... | 4 | 4.91 | $P\uparrow N\downarrow$ |
| $10^3$ | 4 | 2 | 3.54 | ... | 2 | 4.56 | $P\uparrow N\downarrow$ |
must grow from a moment. We expect that in real physical conditions such shocks form the spectra of the type $N^\uparrow$.

The structure $P^\downarrow$ often appears together with the structure $N^\downarrow$, in this case we will consider the structure $P^\downarrow N^\downarrow$. The untypical structure $PN^\uparrow$ in $(40_5, 10_3, 3, ad)$ arose from the small time resolution. An example of the structure $P^\downarrow N^\downarrow$ without details is presented in Figure 21 and with details in Figure 22.

Data presented in tables with 'rd' (Tables 4, 5) and 'pd' (Tables 1, 6) spectra give us the very important result that individual processes give their own contributions to the general spectrum. Reflection process generates the structure $P^\downarrow$ (the positive lag) and Cracow acceleration the structure $N^\uparrow$ (the negative lag). That is the only result we have obtained from the tables. Henceforth, the tables with 'ad' (Tables 3, 7, 8, 9, 10) spectra will be discussed.
When one is examining the tables, one can see that the structure $P_{\downarrow}$ occurs at large initial Lorentz factors of the shock and strong decelerations. If $\gamma_{i}$ is smaller and/or $a$ is larger then the spectra exhibit the structure $N_{\downarrow}$.

To obtain a better resolution of $(40, 5, 10^{3}, 2, \text{ad})$ and $(20, 5, 10^{3}, 4, \text{ad})$ spectra we have performed the same simulations as previously but $\gamma_{f}$ was equal to 10 instead of 5. The additional simulations were following $(40, 10, 10^{3}, 2, \text{any})$, $(40, 10, 10^{4}, 4, \text{any})$, $(20, 10, 10^{3}, 2, \text{any})$, $(20, 10, 10^{4}, 4, \text{any})$ and the outcome was put into Tables 9 and 10.

The spectra of $(40, 10, 10^{3}, 2, \text{ad})$ show the structure $P_{\downarrow}$ at low frequencies and the unimportant structure $N_{\downarrow}$ at higher frequencies and later times what is not visible for $\gamma_{d} = 40, 5$. The spectra represent almost pure $P_{\downarrow}$ that
is not suppressed by Cracow acceleration (Figure 23), we neglected results if \( \nu \) was high (here P for \( \nu = 4.75 \)).

The case of (40, 5, 10^3, 4, ad) is in fact P↓N↓, but N↓ is outside the range of detectable results (Table 8).

We have analysed the particle and photon spectra and reached the following conclusions

a) the structure P↓ is formed when the sudden fall in the spectra of reflected particles ‘rd’ is not suppressed by Cracow acceleration,
b) the structure P↓N↓ is formed when the growth of the high-energy tail of Cracow acceleration spectra ‘pd’ is accompanied by the sudden fall in the spectra of reflected particles ‘rd’,
c) the structure N↑ is formed due to lingering of the high-energy tail of Cracow acceleration spectra ‘pd’ in the presence of the weak fall in the spectra of reflected particles ‘rd’.

The fall in the spectra of reflected particles is best visible when one is looking at the high-energy tail of the spectra. Before doing so it is convenient to raise the particle spectra of initial times to the density level of the remaining spectra. Then, the high-energy tail will almost always move towards low energies.

We have done rough estimation of the fall and introduced the parameter: \( -\Delta p/\Delta t \), where \( \Delta p \) is the difference of the momenta measured at a constant particle density between the two particle spectra that differ by the time \( \Delta t \). The value of the parameter was measured at the high-energy tail. The results are collected in Table 11. In the table \( n_1 \), \( n_2 \) are numbers of spectra for times \( t_1 \), \( t_2 \), where momenta \( p_1 \), \( p_2 \) are measured at a selected density, respectively.
In spectra of \((20,10, 10^3, 2, \text{rd})\) we have observed a weak structure \(P_\downarrow\) at frequencies lower than \(\nu_{\text{max}}\). We have estimated that the case is close to the boundary between appearing and not-appearing of \(P_\downarrow\) and the boundary is at \(-\Delta p/\Delta t \simeq 8\). One should remember that both the parameter and the measurement are rough.

We denoted the spectra of \((20,10, 10^3, 2, \text{ad})\) as \(N\uparrow\) since \(N\) keeps still at first (Figure 24).

The lingering of the high-energy tail in Cracow acceleration spectra is presented in Figure 25 and related ‘rd’ spectra in Figure 26. The tail moves ahead if \(C\) is larger. Photon spectra with the structure \(N\uparrow\) are presented in Figure 27.

The phenomenon of the production of lags is simple. When one of the two acceleration processes dominates the other then appears \(P_\downarrow\) or \(N\uparrow\). When they compete then \(P_\downarrow\) is pulling the progenitor \(N\uparrow\) and changes it into \(N\downarrow\). This interaction is mutual and the progenitor \(N\downarrow\) is damping down \(P_\downarrow\) what makes it less distinctive.

Negative lags appear at frequencies higher than \(\nu_{\text{max}}\) only and the difference between the frequencies and \(\nu_{\text{max}}\) is not small then. Positive lags are found at frequencies both lower and higher than \(\nu_{\text{max}}\) but mainly around \(\nu_{\text{max}}\).

We normally expect that shocks with the small value of \(\gamma_i\) have large \(a\) and vice versa. If we adopt the above assumption then, according to the received results, there is a certain value of the initial Lorentz factor \(\gamma_i\) below which the positive lag is not visible and with growing \(\gamma_i\) positive lags become more and more harsh. If we assume additionally that the GRB luminosity is growing together with increasing \(\gamma_i\), then together with the increase in the distance to GRBs we will be observing the larger contribution of GRBs with stronger positive lags.

Our simulations show that the low energy electrons gain small \(\beta\) if the shock decelerates quickly and if \(\gamma\) decreases then \(\beta\) decreases too (see Figures 15 and 19). It is consistent with observations of the low energy electrons in plerions (Weiler & Panagia 1978, \(1.2 < \beta < 1.6\)), GRBs (Band et al. 1993, \(\beta \simeq 1.1\) typically), hotspots in radio galaxies (Stawarz et al. 2007, \(\beta < 2\)) and luminous blazar sources (Sikora et al. 2009, \(\beta \simeq 1.6\) typically). Moreover, the simulations show that the high energy electrons gain \(\beta\) larger than 2.23 if the shock decelerates slowly (see Figure 12, \(2.4 < \beta < 2.5\)) and it is consistent with observations of the sources with relativistic shocks where \(\beta \simeq 2.4\) is a typical value. Medium deceleration is presented in Figure 25 there are two flat parts in spectra. Our intuitive estimation is that the deceleration of a typical GRB is changing from \(a = 50\) to \(a = 9 \cdot 10^3\) during the phenomenon.

7. Fermi acceleration

In order to check whether Fermi acceleration is able to produce a GRB lag we have carried out additional simulations. We have switched Cracow acceleration off through the application of \(C = -2\) upstream of the shock for all simulations and we have switched Fermi acceleration on by choosing large values of \(C_d\) downstream of the shock. In this section we consider various values of \(C_d\) and we use the adapted vector \((\gamma_d, a, C_d, \nu_{p}\)) instead of the old one.

At fixed values \(a = 10^2\) and \(C_d = 5\) we carried out simulations for \(\gamma_d \in \{10.5, 20, 10, 40, 10\}\), and at fixed \(a = 10^3\) and two values of \(\gamma_d \in \{10.5, 20, 10\}\) we performed simulations for \(C_d \in \{1, 3, 4, 5, 6, 7\}\). It turned out that \(C_d = 1\) was a saturated value for the second set and increasing it did not change the results. The results have been collected in Tables 12-14.

We have checked spectra ‘rd’ and ‘pd’ and it turns out that \(P_\downarrow\) is a pure outcome of reflection process because spectra ‘pd’ do not show a lag in this case (Figure 25). The negative lag arises in full measure from Fermi acceleration what is visible in spectra ‘pd’. In this case spectra ‘rd’ are showing \(P_\downarrow\) which is suppressed by \(N\uparrow\) from spectra ‘pd’ and it gives spectra ‘ad’ with \(N\uparrow\) (see Figure 29).

Findings are simple. Fermi acceleration generates negative lags only in shocks that have small Lorentz factors and are slowing down slowly. In other conditions positive lags produced by reflection process are visible.

The first question is which way the negative lag is produced. We have not prepared an expert opinion on this issue, but it has emerged a simple interpretation after analysing particle spectra ‘pd’ (Figures 30-31). Spectra in Figure 30 are similar to the corresponding spectra with active Cracow acceleration. The spectra are falling at medium particle energies and moving forward at high-energy tail. In the process of Fermi acceleration particles are wandering UP over a distance shorter than in Cracow acceleration, but for small Lorentz factors of the shock it is enough to produce lingering of the high-energy tail in the particle spectra.

The second question is why there is no negative lag for \((10.5, 100, 5, \text{ad})\). It results probably from the fact that

### Table 11. Power of falling of ‘rd’ spectra

| \(\gamma_d\) | \(a\) | \(C\) | \(n_1\) | \(n_2\) | \(\Delta p/\Delta t\) |
|---|---|---|---|---|---|
| 40,5 | 60 | 3 | 3 | 6 | 19.8 |
| 10,5 | 60 | 2 | 4 | 8 | 4.59 |
| 40,5 | 10^3 | 3 | 0 | 1 | 0.412 |
| 20,10 | 10^3 | 3 | 5 | 0.012 |
| 20,10 | 10^3 | 3 | 6 | 7.83 |
| 40,10 | 10^3 | 2 | 5 | 26.93 |

### Table 12. \(a = 100\), spectrum - \(ad\)

| \(\gamma_d\) | \(C_d\) | \(n_p\) | \(\nu_p\) | \(n_o\) | \(\nu_o\) | lag |
|---|---|---|---|---|---|---|
| 10,5 | 5 | 2 | 1.88 | ... | ... | \(P_\downarrow\) |
| 20,10 | 5 | 7 | 3.70 | ... | ... | \(P_\downarrow\) |
| 40,10 | 5 | 5 | 4.04 | ... | ... | \(P_\downarrow\) |

### Table 13. \(\gamma_d - 10,5\), spectrum - \(ad\)

| \(a\) | \(C_d\) | \(n_p\) | \(\nu_p\) | \(n_o\) | \(\nu_o\) | lag |
|---|---|---|---|---|---|---|
| 10^3 | 1 | ... | ... | 2 | 1.97 | \(N\uparrow\) |
| 10^3 | 7 | ... | ... | 2 | 2.02 | \(N\uparrow\) |
| 10^3 | 5 | ... | ... | 2 | 2.02 | \(N\uparrow\) |
| 10^3 | 6 | ... | ... | 2 | 2.02 | \(N\uparrow\) |
| 10^3 | 7 | ... | ... | 2 | 2.02 | \(N\uparrow\) |

### Table 14. \(\gamma_d - 20,10\), spectrum - \(ad\)

| \(a\) | \(C_d\) | \(n_p\) | \(\nu_p\) | \(n_o\) | \(\nu_o\) | lag |
|---|---|---|---|---|---|---|
| 10^3 | 1 | 1 | 3.68 | ... | ... | \(P_\downarrow\) |
| 10^3 | 8 | 1 | 3.62 | ... | ... | \(P_\downarrow\) |
| 10^3 | 6 | 1 | 3.63 | ... | ... | \(P_\downarrow\) |
| 10^3 | 7 | 1 | 3.66 | ... | ... | \(P_\downarrow\) |
Fermi acceleration has longer acceleration time and produces steeper spectra than Cracow acceleration. It explains the lack of negative lags in photon spectra of rapidly slowing down shocks.

The third question is why one can not see negative lags for larger Lorentz factors of the shock. The answer is that the wandering time UP decreases more quickly with the growing Lorentz factor of the shock than the time in Cracow acceleration or Fermi acceleration fades out.

On the basis of the received results we put forward the hypothesis that the negative lag can be produced by any acceleration mechanism in which particles after reflection process are crossing the shock repeatedly from UP to DOWN and around DOWN to UP.

We have not examined Fermi acceleration in detail and we think that the Weibel instability and the fact that particles are accelerating in PWNe make this mechanism impossible. Our numerical code is still relatively straightforward and therefore it can not be ruled out on the basis of the current results that the negative lag arises due to Fermi acceleration. More detailed simulations and observations are needed in order to decide conclusively on the matter.

8. Two acceleration processes

Observations of GRBs with the Fermi Gamma Ray Space Telescope indicate that photons in the GeV range (the LAT) and photons in the MeV range (the GBM) arrive at different times. The GeV photons arrive sometimes later and sometimes before than the MeV photons [Abdo et al., 2009; Ghirlanda et al., 2010]. It seems that the observed spectra in the two ranges originate from different regions. However, it is the very same phenomenon that occurs in one region and we will describe it below.

In order to understand this phenomenon one should realise that Cracow acceleration does not have a general restriction to the maximum energy achieved by particles. The restrictions result from specific physical conditions only. Values of $C$ and $\gamma$ are two basic parameters that are limiting the energy of accelerated particles. Increasing any of them increases this limit.

The acceleration time is another important quantity and although it is constant downstream of the shock, it is $\gamma$ times smaller in the upstream plasma rest frame and grows together with reducing $\gamma$.

The Fermi Telescope observes bursts which have sufficiently large values of $C$ and $\gamma$ to allow for Cracow acceleration to take place at GeV energies detected on Earth. When the Fermi Telescope is detecting the onset of the GeV emission then the accelerated particles achieve energies enabling them to produce synchrotron emission in this range.
The two acceleration processes behave differently during the phenomenon. Reflection process is weakening almost all the time except for the short initial time when the number of particles entering the process is larger than the number of particles coming out. The weakening manifests itself by the moving of the maximum of the energy distribution of accelerated particles toward lower energies. We will call the maximum the reflection surge.

Cracow acceleration accelerates particles all the time but unevenly. The acceleration time decreases and this influences each particle the same. Reflection process provides but unevenly. The acceleration time decreases and this in-

maximum the reflection surge. accelerated particles toward lower energies. We will call the

fraction with lower energies and accelerates them. When the shock reaches the particles with higher energies, then the lower-energy particles were in time to accelerate to energies comparable to the higher-energy particles. As a result instead of the value of \( \beta \) typical for given \( \alpha \), e.g. \( \beta = 2.6 \), a flatter particle spectrum is arising, e.g. \( \beta = 1.4 \), which will approach \( \beta = 2.6 \) going to higher energies, but will enter the GeV photon spectrum range with \( \beta = 1.6 \) for example. Next, the normal seed particles, that produce \( \beta = 2.6 \), are falling within the GeV range and the photon spectrum starts falling.

The above description is compatible with the observation of GRB 090510. At the high-energy part of the photon spectrum the photon index is equal to \( \beta/2 + 1 \). In the GeV range the photon spectrum at first grows with the photon index equal to 1.8 (\( \beta = 1.6 \)), and then it is falling with the photon index equal to 2.3 (\( \beta = 2.6 \)).

9. UHECRs acceleration

We claim that the mechanism of Cracow acceleration produces the ultra-high energy cosmic rays (UHECRs). The restrictive requirement that rules out UHECRs acceleration in most astrophysical objects is that the particle gyroradius must be much smaller than the system size. The restriction does not exist if the acceleration takes place at standing or moving with a mildly relativistic velocity shocks (with respect to ISM) and UP has the Lorentz factor larger than \( \sim 1000 \) (with respect to ISM) and UHECRs come out from UP. Two kinds of sources fulfill the requirements, PWNe and relativistic flows in AGN jets. The PWN sources overcome the restriction of the Greisen-Zatsepin-Kuzmin cutoff also.

Below, we explain how Cracow acceleration is able to accelerate UHECRs. All quantities are given in the plasma rest frame. Let \( E_a \) be the particle energy UP, \( E_d \) the particle energy DOWN, \( B_u \) the homogeneous magnetic field UP and \( B_d \) the homogeneous magnetic field DOWN. When the particle crosses the shock from UP to DOWN then its energy changes according to the formula \( E_d = E_a / \gamma_{ud} \). When the particle crosses the shock from DOWN to UP then its energy changes according to the formula \( E_u = 2 \gamma_{ud} E_d \). This is a result of the particle anisotropy upstream of the shock. The relation between magnetic fields is \( B_d = 4 \gamma_{ud} B_u \). We assume that DOWN is at rest or moves with the velocity \( \lesssim 0.95c \) with respect to ISM.

Synchrotron losses are \( \sim E^2 B^2 \). It gives that the synchrotron losses of a particle that crossed the shock from 090510 was observed at MeV energies at first. This pulse could originate both from the reflection surge as well as from the Cracow surge. The pulse at GeV energies originates from the Cracow surge. The period of time between the pulses is considerable, therefore we can assume that during the GeV pulse the value of \( a \) is large and constant. The rise-fraction is present upstream of the shock during the pulse.

The rise-fraction consists of particles that are more close to the shock and have lower energies and larger anisotropy, and farther with higher energies and smaller anisotropy. The anisotropy does not play a significant role in the case. The acceleration time changes very slowly for large \( a \) and we can treat it as constant.

At first, the shock is getting the particles of the rise-

fraction with lower energies and accelerates them. When the shock reaches the particles with higher energies, then the lower-energy particles were in time to accelerate to energies comparable to the higher-energy particles. As a result instead of the value of \( \beta \) typical for given \( \alpha \), e.g. \( \beta = 2.6 \), a flatter particle spectrum is arising, e.g. \( \beta = 1.4 \), which will approach \( \beta = 2.6 \) going to higher energies, but will enter the GeV photon spectrum range with \( \beta = 1.6 \) for example. Next, the normal seed particles, that produce \( \beta = 2.6 \), are falling within the GeV range and the photon spectrum starts falling.

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field and electromagnetic disturbances are generated by synchrotron losses UP can be neglected. Synchrotron losses of protons and nuclei are neglected DOWN and their achieved maximum energies depend on the size of the system only.

The accelerated particles do not escape from DOWN if the perpendicular size of system is larger or equal to the particle gyroradius \( r_g \). It yields the formula for the maximum energy DOWN, \( E_{\max} \approx 300d_uB_3 \text{[eV]} \), where \( d_u \text{[cm]} \) is the size of DOWN perpendicular to the shock normal, \( B_3 \text{[G]} \). Protons and nuclei are able to reach energies equal to \( \sim 10^{-14} - 10^{-9}\text{eV} \) downstream of the shock and it is enough to produce UHECRs if the Lorentz factor of the shock \( \gamma \gtrsim 10^3 \) and the particles get out into ISM through UP.

We assume that a particle crossed the shock from DOWN to UP and its momentum was parallel to the shock normal at the beginning of the wandering UP. After a while the direction of the particle momentum changes and \( \alpha \) is the angle between the momentum and the shock normal. The angle \( \alpha \) is small therefore the distance the particle covered in the direction perpendicular to the shock normal is \( d_u \approx r_g\alpha^2/2 \), where \( r_g \) is the particle gyroradius UP. The value of \( d_u \) is the same both in the upstream and in the downstream plasma rest frame but the particle gyroradius grows \( \sim 8\gamma_{\text{up}}^2 \) times when the particle crosses the shock from DOWN to UP. The perpendicular (to the shock normal) length of the particle path DOWN is \( d_u \approx r_g \) and therefore \( d_u/d_d \approx 4\gamma_{\text{up}}^2\alpha^2 \).

We will estimate the value of \( \alpha \) in two ways. First, for \( \beta \) slightly larger than \( \beta_0 \) we apply \( \alpha \approx \Delta u \). We eliminate \( Q_0 \) from Equations 1 and 2 and get \( \Delta u \approx 0.5(\beta - 2.23)0.665/\gamma \). If we put the values \( \beta = 2.3, 2.4, 2.6 \) into the equation we get \( d_u/d_d \approx 0.013, 0.045, 0.13 \).

The electromagnetic forces that disturb the particle movement in the homogeneous magnetic field twist its path with different radii of curvature. If we exclude the contribution of the homogeneous magnetic field then we get the change of the direction of momentum and displacement of the particle caused by a scattering and, in consequence, the scattering radius. We will use the parameter \( \eta \) that is the ratio of the particle gyroradius to the mean scattering radius.

In our simulations \( \eta \) is equal to infinity. In real plasma \( \eta \) has a finite value. If \( \eta \) takes a finite value then the first estimation of \( d_u/d_d \) becomes larger for half of the particles. Because \( d_u/d_d \sim \alpha^2 \), the first estimation gives that the decrease of the shock must increase \( \alpha \) a few times to allow accelerated particles are come out from UP.

The second estimation is in the case of the neglected homogeneous magnetic field. The angle \( \alpha \) results from scatterings only and \( \beta = \beta_0 \). We apply \( \alpha \approx \Delta u \), where \( \Delta u \approx 1.3/\gamma \) (see Figure 4), but we divide the result by \( \eta \) because the particle turns in electromagnetic disturbances. It yields the second estimation \( d_u/d_d \approx 3.4/\eta \). We suggest that loop plasma has the value of \( \eta \) in the range \( 10 - 100 \). It gives that if \( \alpha \) increases \( \sim 2 \) times up to a few times because of the deceleration of the shock then the particles come out from UP.

In the model of loop plasma the homogeneous magnetic field and electromagnetic disturbances are generated by electric current loops. The loops could change if \( \gamma \) changes. We think that \( B_u \) could decrease with \( \gamma \). It would explain, at least partly, dissipation of the Poynting flux of the pulsar wind before it reaches the inner edge of the Crab Nebula.

A few times increase in the value of \( \alpha \) as the result of the shock deceleration is something expected. The slowing down velocity of the shock enables particles to spend longer time UP and it can not be neglected. If \( B_u \) decreases then \( r_g \) increases and, as a result, \( d_u \) increases and the particle spends longer time UP. It could even produce feedback that prevents the particle from returning to the shock. However, the reduction of the shock velocity can not be too large because the energy \( E_q \) increases \( \sim 2\gamma u_{\text{up}}\gamma_{\text{is}} \) times, where \( \gamma_{\text{is}} \) is the Lorentz factor of the plasma UP as seen in ISM at the moment when the particle comes out from UP.

The presented calculations are promising but further numerical simulations are needed to show that Cracow acceleration accelerates UHECRs. The first kind of simulations will be similar to presented in this paper but the numerical code will include an extended equation of motion and variable \( B_u \). The second kind of simulations should search for values of \( \eta \). One must choose parameters of loop plasma and follows particle trajectories in its electromagnetic field. The third kind of simulations will be similar to the first kind but UP will be replaced with loop plasma.

If Cracow acceleration accelerates UHECRs then astronomical observations will show that UHECRs arrive from systems where the shock move away from an observer and is slowing down from \( \gamma \gtrsim 10^3 \) to \( \gamma \gtrsim 10^2 \).

Fermi acceleration can not accelerate UHECRs because its change of the direction of the particle momentum that enables the return to the shock takes place DOWN.

10. Seed particles

In our model presented in Bednarsz (2004) we have proposed some acceleration mechanisms to be responsible for production of seed particles and that their radiation is detected as the precursor of GRBs. Actually, the mechanisms are ineffective in pre-acceleration of protons and nuclei and the analysis of spectra of GRB precursors is suggesting that they are produced in the same mechanism as GRBs. In our opinion, the mechanism described by Hoshino et al. (1992) is the only (among the proposed) effective mechanism that is able to produce seed leptons. This is the process of the downstream plasma acceleration of positrons to non-thermal distributions. Cracow acceleration needs seed particles upstream of the shock to accelerate particles. We propose two ways in which they could be injected into UP.

The first way occurs in the relativistic jets that are fired from a massive object close to the neighbourhoood. The jet production mechanism and indeed the jet composition on very small scales are not known at present. We propose that seed particles are present in jets from the beginning of producing them and are there all the time. We think that loop plasma generates seed particles both leptons and nuclei.

The second way is at work in a pulsar wind when it is streaming into the ambient medium and creating a standing shock. Seed particles can, similarly as in jets, be present from the moment of generating the wind, however it concerns leptons only. We propose that seed particles, particularly protons and nuclei, are entering the pulsar wind from its side boundary. These particles are coming out from the ambient medium being pushed by the gravitational force of the pulsar. Their Lorentz factor in the wind plasma rest.
frame is equal to $\sim \gamma_w$, where $\gamma_w$ is the Lorentz factor of the pulsar wind, so that they become seed particles automatically.

11. Summary and Discussion

Up to now, simulations of particle acceleration at relativistic shocks concerned the constant velocity shocks only. We are the first who have simulated particle acceleration at slowing down relativistic shocks and that is why we have solved the mysteries of photon and particle spectra of the astronomical objects that radiate the synchrotron radiation of particles accelerated at relativistic shocks. No one was trying to perform such simulations before because it was a very difficult task. It turns out that GRB lags and the hard low energy spectral indices arise due to the deceleration (Section 6).

We have presented the mechanism of Cracow acceleration (Section 9). The mechanism increases its strength when the Lorentz factor of the shock grows and initial $\beta$ does not depend on the Lorentz factor but on the conditions of the upstream plasma.

We have shown that the whole acceleration consists of two processes, reflection process and Cracow acceleration. We have assumed that the real deceleration is more dynamic than that in our simulations and the rise-fraction of particles upstream of the shock if formed at the strong deceleration stage. Acceptance of the assumption gives explanation to the question why two transitions are observed at one GRB and why the observation at high energy can follow the observation at low energy and vice versa (Section 8). It also explains why the photon spectrum is hard at the rise stage of the Cracow surge. Sometimes, two surges produce the double pulse, the first maximum in the pulse originates from the reflection surge and the second from the Cracow surge.

Cracow acceleration requires strong disturbances of the movement of particles upstream of the shock in order to accelerate the particles. We have shown that current PIC simulations of relativistic plasma are useless. We have presented the model of the microphysics of relativistic flows that would allow for acceleration in the mechanism of Cracow acceleration (Section 2). We have sketched the PIC simulations that are needed to solve the problem of the microphysics of relativistic plasma.

The current PIC simulations suggest that seed particles could be produced in the upstream plasma. We agree with the idea, but only proper PIC simulations can prove this. We proposed that particles of the ambient medium are injected into pulsar wind of PWNe and become seed particles (Section 10).

We claim that Cracow acceleration produces UHECRs which come out of UP (Section 9). Relativistic shocks are present in supernovae (SNe) and PWNe which are supposed to be the sources of cosmic rays in the Galaxy. They are also present in relativistic flows ejected from AGNs. The astronomical phenomena are widespread and show synchrotron radiation produced by energetic particles. The accelerated particles have power-law spectra similar to cosmic rays. The energy of these shocks is the main energy of the system. There are no other reasonable sources of cosmic rays but relativistic shocks. We claim that reflection process and Cracow acceleration produce almost all cosmic rays with energies higher than $\sim 50$ GeV.

Appendix A:

This Appendix will not be included in a review article.

In 1997 I was carrying out simulations of relativistic shocks. I inspected the results of the simulations and noticed that the part of $\Delta \theta_U$ caused by scatterings ($\Delta \theta_s$) diminishes with the time $\Delta t_s$ slower than the part of $\Delta \theta_U$ arising due to trajectory curvature in the uniform field component ($\Delta \theta_U$) when $\gamma$ grows and $\lambda$ is constant. I checked how quickly they are diminishing and got that $\Delta \theta_s \sim \sqrt{\Delta t_s}$ and $\Delta \theta_U \sim \Delta t_s$. I understood at once that it results from that the scattering is the diffusion process and the wandering in the uniform field is the linear process. I also knew that the scattering enables the particles the return to the shock and the wandering in the uniform field does not.

I wanted to write it down in Bednarz & Ostrowski 1998, but M. Ostrowski did not want. He made up the general conclusion 'The phenomenon of decreasing $\sigma$ to $\sigma_\infty$ at constant $\lambda$ and for growing $\gamma$ results from slower diminishing of the part of $\Delta \theta_U$ caused by scattering in comparison to $\Delta \theta_U$ arising due to trajectory curvature in the uniform field component.'

I agreed because M. Ostrowski was resolute and there was no free space in the article. I thought that physicists know at once that the scattering is the diffusion process and the wandering in the uniform field is the linear process.

During the 16th European Cosmic Ray Conference in Alcalá de Henares in 1998 Y. Gallant and other man came up to me. Gallant wanted to know how the particles are accelerated in relativistic shocks. They did not understand the general conclusion in Bednarz & Ostrowski 1998. My English was very poor but I wrote down some mathematical formulae and an example and explained about the diffusion process, the linear process and what enables particles the return to the shock. Gallant got the point. He wrote the paper Gallant & Achterberg 1999.

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Janusz Bednarz: Relativistic slowing down shocks as sources of GRB lags
