Numerical simulation of the hydrodynamic instabilities of Richtmyer-Meshkov and Rayleigh-Taylor

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Abstract. The paper presents the results of numerical simulation of the development of hydrodynamic instabilities of Richtmyer-Meshkov and Rayleigh-Taylor encountered in experiments [1-3]. For the numerical solution used the TPS software package (Turbulence Problem Solver) that implements a generalized approach to constructing computer programs for a wide range of problems of hydrodynamics, described by the system of equations of hyperbolic type. As numerical methods are used the method of large particles and ENO-scheme of the second order with Roe solver for the approximate solution of the Riemann problem.

1. Introduction.
An important section of modern computational fluid dynamics is the numerical simulation of the growth and development of hydrodynamic instabilities of the continuum. The complexity and specificity of the task and the possibility of comparison with experimental data allows not only to analyze the phenomenon, but also to use such tasks for the study of differential methods and verification of hydrodynamic codes. Examples of phenomena under investigation are wakes in the atmosphere and the ocean, the interaction of a blowing jet with the basic flow, the interaction of intense laser light with matter, many atmospheric phenomena, etc. A characteristic feature of these processes is the presence of high gradients of flow variables and high Reynolds numbers [4]. According to the Kolmogorov–Obukhov theory [5, 6], turbulent motion at high Re involves several ranges, namely, the energy, inertial, and dissipation ranges. For a broad class of phenomena at high Reynolds numbers, the effect of molecular viscosity on general flow characteristics in the energy and inertial ranges is overall insignificant. Accordingly, such phenomena are numerically studied as based on perfect fluid models, namely, on the Euler equations [4]. The work is devoted to numerical modeling of various natural experiments of hydrodynamic instabilities of Richtmyer-Meshkov [1,2] and Rayleigh-Taylor [3] using techniques of the TPS (Turbulence Problem Solver) software complex [7].

2. Experiments.

2.1. Richtmyer-Meshkov instability.
In [1] the nonlinear stage of growth of the RMI is experimentally investigated, immediately following the linear stage described by Richtmyer [2]. The experiments were carried out in a square tube with height of $h_0$ (figure 1), containing Zone 0 filled with xenon and Zones 1 and 2 filled with argon.
separated by a lavsan film. In the Zones 0 and 1 both gases are at rest, and the system is in equilibrium. Initial data was set as follows:

\[ \rho_0 = 5.894 \text{ kg} / \text{m}^3, \rho_1 = 1.784 \text{ kg} / \text{m}^3, \]
\[ u_{0,1} = u_{0,1} = 0, \]
\[ p_{0,1} = 50000 \text{ Pa}. \]

To the Zone 1 at speed of 600 m/s (Mach number equals 3.5) from the right side moves the front of Zone 2 which contains compressed argon: \( \rho_2 = \bar{\rho}, u_2 = \bar{u}, v_2 = \bar{v}, p_2 = \bar{p} \). For initial conditions to be correctly specified we need to calculate the state of the medium behind the shock wave. This state should specify the correct front speed. As it is known, the jumps of the values on the discontinuity are connected with the speed of the front by the Rankine-Hugoniot relations \([8-10]\). Thus we can obtain the initial conditions in the Zone 2 in figure 1:

\[ \tilde{u} - D = \frac{\sigma (-D)}{2 + \sigma} \frac{2(1+\sigma)p}{\rho(2+\sigma)(-D)} > 0, \]
\[ \tilde{\rho} = p + \rho(D-u)(\tilde{u}-u), \]
\[ \tilde{p} = \rho \frac{D-u}{D-\tilde{u}}. \]

Several experiments were carried out at different initial perturbation of the boundary. The length of the initial perturbation varied at a constant amplitude: 72 mm (one-mode perturbation), 36 mm (two modes), 24 mm (three modes), 12 mm (six modes) and 8 mm (nine modes). After 140 \( \mu \)s after the beginning of the interaction between the shock wave and films in the pipe the registration of the flow pattern on a photographic film was made. By this moment in the latter two cases the stage of fully developed turbulence practically occurred. For the first three variants of the experiment in the section 5.1 the results of the numerical calculation are presented.

2.2. Rayleigh-Taylor instability.
Consider one of the well-known experiments [3] to investigate the growth and development of this type of instability. The experimental installation is shown in figure 2. It represents a square-shaped tank which thickness is substantially less than width and height, i.e. the experiment can be considered as two-dimensional. Initially, by a thin partition the tank is separated into two parts, the upper being filled with ethanol, and the lower being filled with the air. The ratio of densities $\rho_1/\rho_2 = 600$. The bending of the partition specifies the initial disturbance of the boundary between the liquids $\zeta = a_0 \sin(2\pi x/\lambda)$, where $a_0 = 0.5 \text{ mm}$, $\lambda = 65 \text{ mm}$. Due to the special rocket engines, the tank is accelerated to significant accelerations in the range of $25 - 70 g_0$, where $g_0$ is the acceleration of gravity. The process is recorded on photographic film.

3. Mathematical formulation.

Traditionally, the classical laws of conservation of mass, momentum and energy of a continuous medium describe the considered problems. With their help, the emergence of vortices can be naturally observed which is discovered in the experiments [1-3]. The basis of the approach is the system of these laws in the approximation of the ideal medium of Euler, describing two-dimensional inviscid flow of compressible continuous medium:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = R.$$ 

Here $U$ is the vector of conservative variables, $F, G$ are flux vectors:

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(\rho E + p) \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(\rho E + p) \end{pmatrix}.$$ 

The right-hand term of our system is the vector of external forces, which is equal to the zero vector in the case of modeling Richtmyer-Meshkov instability. In the case of Rayleigh-Taylor instability, a flow occurs under a constant external acceleration $g$, and the vector of external forces becomes:

$$R = \begin{pmatrix} 0 \\ 0 \\ -\rho g \\ -\rho gv \end{pmatrix}.$$ 

Ideal gas equation of state $p = (\gamma - 1) \rho e$ was used.

4. Numerical simulations. Turbulence Problem Solver (TPS) software complex.

Numerical simulations were performed using the Turbulence Problem Solver (TPS) software package [6] developed by the authors. The package is implemented in C++ and provides a unified, object-oriented program interface for implementation of different numerical algorithms for solving hyperbolic problems. Using the software various problems of continuum mechanics has been numerically investigated: Rayleigh – Taylor instability problem, the shear layer problem, Kolmogorov problem of the formation of currents of the “parquet” type for an ideal medium, and others.
5. The results of numerical simulations.

5.1. Richtmyer-Meshkov instability.

For the numerical solution of the system of the Euler equations explicit finite volume method was used. To calculate intercell fluxes of conservative variables, ENO reconstruction of the second order was used. Primitive variables were reconstructed: density, velocity and pressure. Riemann problem was solved approximately using the Roe method. To avoid rarefaction shockwaves, entropy correction procedure was implemented. No artificial viscosity terms required because of sufficient scheme viscosity of the method itself, which allows to model flows even in the zones of large density and pressure gradients. The robustness of the method was verified on standard tests [11].

The system of equations was solved in the rectangular region covered with a uniform rectangular grid of 500x200 cells. The initial conditions were specified according to the experimental setting. The region is divided into three zones, the first two of which are resting the system xenon-argon with a curved boundary (see figure 1). In the third zone the state of argon behind the shock wave is calculated using the Rankine-Hugoniot relations.

![Figure 3](image)

Figure 3. The front of passed shock wave (SW), and the curved boundary of the contact discontinuity (CD) at the time moment of 80 µs.

To calculate flows through the border a single row of ghost cells is used. On the top and the bottom borders reflective boundary condition is used. In this case, the values of the density, pressure and tangential velocity components in the ghost cells are transferred from the inner region, and the values of normal velocity components change its signs to the opposite. On the left and right borders transmissive boundary condition is used, i.e. the values in fictitious cells are completely transferred from the inner region.
Figure 4. The development of Richtmyer-Meshkov instability at $t = 140 \mu s$: photographs [1] on the left, the numerical simulation results on the right.

For the three values of the wavelengths of the initial perturbation, the process of passing a shock wave through the contact border was simulated, accompanied by the development of instability. After reaching the film, the wave front begins to propagate into xenon, which leads to the simultaneous wave propagation in both media. Front velocities in these media differ: in xenon wave moves slower and it lags from the wave propagating through the argon. Figure 3 shows the shape of a shock wave front (left boundary) which is fully transmitted through the contact boundary for the case of $h = 36\ mm$. The perturbed contact boundary follows. By the time of $80 \mu s$, nonlinear stage of RMI is evolving.

Figure 4 shows a density distribution in the case of a two-mode perturbation of the boundary at the time moment of $140 \mu s$. The bright part corresponds to the middle region between the shock wave and the contact boundary (see figure 3). In the case of single-mode perturbations, the development of the instability occurs slower. The simulation of the nonlinear stage was turned into a transition stage, which lead to mushroom-shaped structure formation. Thus, due to reducing the length of the initial perturbation, we reduced the duration of each stage. Therefore, by the time moment of $140 \mu s$ in the case of $h = 9\ mm$, the pattern of developed turbulence in the mixing layer was recorded on the photographic film.

5.2. Rayleigh-Taylor instability.
Figure 5. The development of the Rayleigh-Taylor instability: experimental photographs are on the left, and numerical simulation results are on the right. Moments of time 0 ms (perturbed boundary), 8 ms and 16 ms are taken.

The system of the Euler equations in this numerical experiment was solved by the method of large particles of the second order accuracy on the grid of 600x520 cells. To avoid spurious oscillations artificial viscosity was used. The ability of the method of large particles to allow a sufficiently large ratio of the densities with high-quality coincidence pattern of instability tells about its high reliability, which also was tested on standard tests [11]. A growing perturbation on the boundary has a number of minor peculiarities: because of the smallness of the amplitude of perturbation of the perturbed boundary is sort of broken line. Every kink of the line develops into the individual bubble-like structure.

Figure 6 shows the trajectories of the mixing zone boundaries (left) and the evolution of the upper boundary velocity (right). The following procedure was used for determining the positions of boundaries. The density mesh function is averaged for each row of cells with the same vertical coordinate \( y \). Thus, we have a one-dimensional profile of the average density depending on the \( y \) coordinate. Further, by this profile, the upper (lower) boundary of the mixing zone are calculated.

They are the values of \( y \) coordinates in which the difference between average density and the density of alcohol (air) exceeds 5% of the densities difference. Boundary velocities are obtained by numerical differentiation of the coordinates by the time.

6. Conclusion.
Numerical experiments to investigate hydrodynamic instabilities of Richtmyer-Meshkov and Rayleigh-Taylor were made, using, respectively, the finite volume method based on the approximate solver of ROE and ENO-reconstruction and the method of large particles. For numerical simulation was used Turbulence Problem Solver (TPS) software package. The resulting patterns were obtained of growth and development of hydrodynamic instabilities, showing qualitative agreement with experimental data.

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Figure 6. Left: the trajectories of the upper and lower boundaries of the mixing zone; right: the evolution of the velocity of the upper boundary of the mixing zone at the development of Rayleigh-Taylor instability.

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