Buckling analysis of laminated composite plate due to localized in plane loading

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Abstract. Laminated composite plates are inevitable parts of structure due to its superior properties than that of conventional material. Many times, these structural elements are subjected to the partial loadings during various applications which arises buckling in the elements. This necessitates proper analysis of these types of structural member for better safety and stability. In the present paper buckling analysis of laminated composite stiffened plate under in-plane localized edge loading is presented. Buckling load for various cases of partial edge loading are investigated for different fiber orientations and loading extent. Finite element method-based MATLAB program is developed for the present analysis. Validation study is done, which shows well agreement with the results already published.

Figure 1. Partial edge loading of Laminated composite plate at (a) centre (b) top of vertical sides (c) bottom of vertical sides (d) bottom to the left side and at top of right side.
1. Introduction
Application of composite panels in thin walled structures like automobiles, aerospace vehicles, submarines and others weight sensitive area has increased in last few decades. In the course of their services these structural members are usually subjected to in-plane uniform compressive loading. Therefore, analysis of these panels under localised uniform loading must be carried out for their better design and stability. Large number of research work are available for buckling behaviour of composite plates under uniform loading [1-3], while limited research is available for such plate under partial in-plane compressive loadings. Leissa [2] presented a review study of buckling of composite plate and discussed the post-buckling behaviour and influence of geometrical imperfections. Chia [3] used analytical method for analysis of post-buckling response of laminated elastic plates.

Analytical analysis of buckling of plate subjected to non-uniform in-plane edge loading is done by some researchers [4-5]. Brown [4] investigated elastic stability of thin plate under concentrated load. Kaldas and Dickinson [5] done calculation for stability of plates under in-plane stress using Rayleigh-Ritz integration. Liew and Chen [6] used a numerical method for buckling analysis of Mindlin plates under various locally distributed in-plane loadings and boundary conditions. Daripa and Singha [7] done nonlinear buckling analysis of isotropic and composite plates under localised in-plane compressive loading using four noded quadrilateral element and reveal the influence of location and variation of in-plane edge loading on critical buckling load. Using first order shear deformation theory with finite element buckling analysis of cross-ply laminated plate is done by Cagdas and Adali [8] for various boundary conditions when plate is subjected to varying edge loadings. Investigation is carried out for the optimal layer thickness of plate for maximum buckling load with various boundary restraints. A nonlinear bucking analysis using 3D cubic element of functionally graded plates under various non-uniform in-plane load patterns is done by Shariyat and Asemi [9] and buckling loads are determined. First order shear deformation theory using eight noded isoparametric shell element is used by Patel and Sheikh [10] for buckling response investigation of laminated composite plate under partial in-plane loadings. The influence of parameters like loading pattern, fiber orientation, and number of layers on buckling load is investigated. Kumar et al. [11] done nonlinear buckling analysis of cylindrical composite panels using higher order shear deformation theory (HSDT) under localized in-plane mechanical and thermal loading.

2. Mathematical formulation
A finite element based MATLAB program with HSDT using nine noded isoperimetric quadrilateral element is developed for buckling behaviour analysis of laminated composite plate subjected to partial edge in-plane loading. Material properties of composite plate considered for the present study are tabulated in Table 1.

| $E_1$ (GPa) | $E_2$ (GPa) | $v_{12} = v_{21}$ | $v_{23}$ | $G_{12}$ (GPa) |
|-------------|-------------|------------------|---------|----------------|
| 82.64       | 9.8034      | 0.3              | 0.0356  | 3.8545         |

Square plate having dimension 1 x 1 x 0.1 is taken for present analysis.

2.1. Displacement field
Displacement field for present paper is based on HSDT and can be given as,

\[
\begin{align*}
    u_1 &= u + z \theta_x + z^2 \phi_x, \\
    u_2 &= v + z \theta_y + z^2 \phi_y, \\
    u_3 &= w,
\end{align*}
\]

(1)
2.2. Strain-displacement relation

\[ \varepsilon_x = \frac{\partial u_x}{\partial x}, \quad \varepsilon_y = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \]

\[ \varepsilon_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}, \quad \varepsilon_{zx} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \]  

(2)

2.3. Stress-strain relations

Stress in x-, y- direction and in xy-plane can be evaluated using reduced constitutive matrix and Eq.1 as,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}
\]

(3)

Where \( \overline{Q} \) is reduced constitutive matrix given in appendix (A1).

Boundary condition applied in the present study is given as, 

\( u = 0 \) \text{ at } \( x = 0 \)

2.4. Finite element approach

Displacement vector can be given as, 

\[ U = [u, v_1, v_2, \ldots, v_n]^T \]

(4)

\[ U_i = \sum_{i=1}^{N_e} N_i^x u_i \quad \text{and} \quad U_i = \sum_{i=1}^{N_e} N_i^y v_i \]

(5)

Where, \( N_i^x \) is elemental interpolation function.

Strain vector can be given as

\[ \{\varepsilon\} = [B]\{U^r\} \]

(6)

The elastic stiffness matrix of laminated plate can be given as,

\[ [K_e] = \iint_{-1}^{1} [B]^T \overline{Q} [B] |J| d\xi d\eta, \]

(7)

Where, \([B]\) is kinematic matrix, \(|J|\) is determinant of Jacobian matrix and \( \overline{Q} \) is transformed constitutive matrix.

The geometric stiffness matrix of plate can be written as,

\[ [K_{ge}] = \iint_{-1}^{1} [B]^T [N_o] [B] |J| d\xi d\eta, \]

(8)

where, \([B]\) is geometric kinematic matrix and \([N_o]\) is load vector.

\[ [N_o] = [N_x, N_y, N_{xy}, N_{yx}], \]

Where, \([N_x]\) = 1/thickness, \([N_{xy}] = 0 \) and \([N_{yx}] = 0 \).

For partial edge loading \([N_x] = C_x\), where, \( C_x \) is x-coordinates of nodes of edge on which load is applied.

\[
[B_{ge}] = \begin{bmatrix}
0 & 0 & \frac{\partial N_x}{\partial \xi} & 0 & 0 & 0 \\
0 & 0 & \frac{\partial N_x}{\partial \eta} & 0 & 0 & 0
\end{bmatrix},
\]

(9)
2.5. Governing equation for buckling

The governing equation for buckling response of plate acted up on by in-plane partial loading may be given as,

\[ [(K)− \lambda^2 s] [v] = 0, \]

Non-dimensionalised critical buckling load for partial in-plane loading may be given as,

\[ \lambda_{cr} = \frac{\lambda_s c b^2}{E h^3}, \]

Where ‘c’ is percentage of width of plate on which partial load is acted upon.

3. Results and discussion

Table 2 Shows validation study for non-dimensional buckling load for a laminated plate with simply supported boundary conditions having stacking sequence (0/90/0/90/0) under in-plane partial compressive load for plate thickness ratio a/h=20 and a/h=100. It is revealed from the Table 2, that outcomes are in good agreement with the published result.

| c/a | a/h = 20  | a/h =100 |
|-----|-----------|----------|
|     | Present   | Ref. [12]| Present  | Ref. [12]|
| 0.2 | 15.4163   | 16.287   | 18.7168  | 19.563   |
| 0.4 | 17.9857   | 18.417   | 21.8363  | 21.590   |
| 0.6 | 20.5550   | 21.810   | 24.9558  | 25.110   |
| 0.8 | 23.1244   | 26.487   | 28.0752  | 30.205   |
| 1.0 | 30.3919   | 31.958   | 36.8987  | 35.962   |

Table 3. Influence of loading position on non-dimensional critical buckling load along with loading extent having stacking sequence (0/90/0/90/0) and a/h=100.

| Loading Positions | Extent of partial edge loading (in % of with of plate) | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|-------------------|-------------------------------------------------------|-----|-----|-----|-----|---|
| Case-I            | 18.7168                                               | 21.8363 | 24.9558 | 28.0752 | 36.8987 |
| Case-II           | 20.4184                                               | 23.8214 | 27.2245 | 30.6275 | 36.8987 |
| Case-III          | 20.4184                                               | 23.8214 | 27.2245 | 30.6275 | 36.8987 |
| Case-IV           | 25.5229                                               | 29.7768 | 34.0306 | 38.2844 | 36.8987 |

Table 3 Shows influence of non-dimensional buckling load with different loadings cases as shown in Figure 1 along with the percentage extent of loadings for laminated composite plate (0/90/0/90/0), plate span to thickness ratio a/h=100 when subjected in-plane partial uniformly distributed edge loadings. From Table 3 it is clear that, buckling load is maximum in Case-IV type of loadings which approx. 36 % more than that in case-I loading. With increase in extent of loading the critical buckling load increases in each case. For case-II and case-III the critical buckling is same due to symmetricity of loadings. For case-I critical buckling load is 97% less, when partial loading is applied on 20% of span than that when loading is on 100 % span of the plate.
Table 4. Influence of fiber orientation on buckling load for laminated plate with $a/h=100$, having loading condition as Case-I under 20% partial in-plane compressive edge loadings.

| Fiber orientation | Non-dimensionalised critical buckling load |
|-------------------|------------------------------------------|
| (0/90/0/90/0)     | 18.7168                                  |
| (60/-60/60/-60/60)| 24.3129                                  |
| (45/-45/45/-45/45)| 30.2476                                  |
| (30/-30/30/-30/30)| 26.9531                                  |

Table 4 shows influence of fiber orientations of critical buckling load for case-I i.e. when the loading is applied at 20 % of width of plate at mid for $a/h=100$. It is observed that laminated composite plate is more stable with $45^0$ fiber orientation. The plate with cross ply (0/90/0/90/0) shows minimum critical buckling load, because well-known fact that fibers have minimum strength along transverse direction.

4. Conclusions
Buckling responses of plate subjected to partial compressive in-plane loading to different extent (%) of width of plate is analysed in the present paper. Some conclusion based on results can be given as:

- Laminated composite plate with higher span to thickness ratio shows higher non-dimensional critical buckling load.
- With fiber orientation $45^0$ laminated composite plate is more stable.
- Laminated composite plate with cross ply shows minimum value of non-dimensional critical buckling load.
- When laminated plate is acted upon by symmetrical loading system then buckle easily.

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