Bifurcation in kinetic equation for interacting Fermi systems

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The recently derived nonlocal quantum kinetic equation for dense interacting Fermi systems combines time derivatives with finite time stepping known from the logistic mapping. This continuous delay differential equation equation is a consequence of the microscopic delay time representing the dynamics of the deterministic chaotic system. The responsible delay time is explicitly calculated and discussed for short range correlations. As a novel feature oscillations in the time evolution of the distribution function itself appear and bifurcations up to chaotic behavior occur. The temperature and density conditions are presented where such oscillations and bifurcations arise indicating an onset of phase transition.

The relation between microscopic chaos and kinetic theory within the time evolution of averaged distribution functions is a matter of ongoing debate. If the coarse graining of phase space due to averaging is large enough, any microscopic signal of chaoticity like bifurcation is lost. The time evolution of the distribution function evolves continuously and smoothly in time. If the coarse graining is reduced by considering more microscopic fluctuations the time evolution of the distribution function itself can reveal signs of the underlying chaotic motion. To understand this relation different attempts have been made in the literature starting from standard mapping and deriving an appropriate kinetic description in terms of finite time step equations. Here the opposite route is proposed: Starting from a quantum statistical approach we employ the nonlocal kinetic theory for dense Fermi systems to show that indeed the resulting kinetic equation can exhibit a nonmonotonic time evolution in a specific temperature and density range. This is a consequence of the considered many-body correlations. Within a first approximation of relaxation times we show that a delay differential equation appears which interpolates between finite time stepping like logistic mapping and continuous time evolution by differential operators. It is argued that the occurring bifurcations in the time evolution are a signal of onsetting phase transition.

I. INTRODUCTION

The kinetic theory of interacting quantum or classical gases and the description of deterministic chaotic systems are mostly developed separately with very few overlapping. Almost exclusively for the case of Lorentz gas the interlink has been analyzed [1,2], and citation therein. The kinetic theory understands the increase of entropy as a result of many random collisions. The theory of deterministic chaos on the other hand represents the irreversibility by the characteristic measure of Lyapunov exponent. While the kinetic theory can be easily extended to quantum systems the quantum chaos is still a matter of debate about even the correct definition.

If both approaches describe some facet of irreversibility it should be possible to give relations between them. For instance, one can connect the transport coefficients with the Lyapunov exponent [3–8]. In [3,6] the fact, that the spreading of a small phase space volume is given by the sum of Lyapunov exponents, is used to give a relation between Lyapunov exponents and viscosity. In [4,5] the relation between transport coefficients and Lyapunov exponents was presented in terms of Helfand’s moments. The interlink was possible to establish by reinterpretation of the Helfand’s moments as stochastic quantities such that the mean variance of the time derivatives represents just the transport coefficients. In [7] the authors derive a density expansion of the largest Lyapunov exponent for hard sphere gases from a generalized Lorentz-Boltzmann equation. The intimate relation between transport coefficients and dynamical quantities like the Lyapunov exponent is even apparent in quantum Fermi gases in that an additional chaotic process behaves like an additional relaxation time [8].

Recently another more direct relation between kinetic theory and deterministic chaotic systems has been established [9]. It appears for the standard mapping that the phase-space averaged kinetic description can be formulated and solved in terms of finite time step equations instead of differential ones. We want to address here the opposite route starting from a continuous kinetic equation for strongly correlated quantum Fermi systems and will ask whether there is a reminiscence of the underlying chaotic motion in the time evolution of the distribution function itself. For a system of hard-sphere gases the appropriate kinetic description is the Enskog equation consisting of nonlocal off-sets in the collision integral. Here
we will consider the opposite extreme of particles interacting in such a way that they form short-living molecules as correlated states. We will show here that such collision duration leads to delay differential equations. In particular, it will be found that the time evolution of the distribution function itself undergoes oscillations up to chaotic behavior similar to Hopf bifurcations observed in mappings including memory [10]. This is due to a competition between attractive potential and repulsive Pauli-blocking. It can be considered as a dynamical signal of the onset of phase transition due to the formal similarity between mean-field phase transitions and bifurcations of one-dimensional discrete maps [11].

The understanding of the route to chaos has been paved with the logistic mapping [12]

\[ x_{k+1} = ax_k(1 - x_k) \] (1)

which starts to show successive bifurcations for increasing parameters \(3 \leq a\), the chaos occurring at \(a = 3.5699...\). An interesting extension of this logistic model has been studied by Berezowski [13] in that a finite inertia has been added to (1).

As done in [13] we perform a linear stability analysis of (2) according to \(f = f_0 + \delta f\) with the fixed point solution of \(-f_0 + a f_0 - a f_0^2 = 0\). Without loss of generality, we look only at the instable one \(f_0 = \frac{a - 1}{a}\). The Fourier-transformed disturbance \(\delta f(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} f(\omega)\) determines then the complex frequency \(\omega = \Omega + i\Gamma\) which gives for \(\Gamma > 0\) the instable and for \(\Gamma < 0\) the stable solutions. One gets from (2)

\[-i\omega \sigma + 1 - ae^{i\omega \tau_d}(1 - f_0) = 0.\] (3)

Separating real and imaginary part of (3) one has

\[ 1 + \Gamma \sigma = e^{-\Gamma \tau_d}(2 - a) \cos \Omega \tau_d \]

\[-\Omega \sigma = e^{-\Gamma \tau_d}(2 - a) \sin \Omega \tau_d.\] (4)

Dividing both sides of (4) gives the equation for the frequency \(\Omega\)

\[ \Omega \sigma + (1 + \Gamma \sigma) \tan \Omega \tau_d = 0 \] (5)

and adding both squared sides of (4) gives

\[ \sqrt{(1 + \Gamma \sigma)^2 + \Omega^2 \sigma^2} = e^{-\Gamma \tau_d}|2 - a|. \] (6)

Now we look for instable modes \(\Gamma > 0\) which leads to

\[ \sqrt{(1 + \Gamma \sigma)^2 + \Omega^2 \sigma^2} \geq |2 - a| \text{ for } \tau_d \leq 0. \] (7)

In the final form we assume \(\Gamma \sigma \ll 1\) and we see that the presence of the term \(\sigma\) shifts the onset of bifurcations and of chaos towards higher (lower) values of \(a\) if compared to the logistic map

\[ |a - 2| \geq \sqrt{1 + (\sigma \Omega)^2} \] (8)

for positive (negative) delay times \(\tau_d\) respectively. Furthermore an oscillation appears with a frequency \(\Omega\) determined by (5)

\[ \tau_d \Omega + \arctan(\sigma \Omega) = \pi. \] (9)

There exists a limiting value for the delay time \(\tau_d\) below which all oscillations disappear. These oscillations and the finite delay time \(\tau_d\) are the reasons why Hopf bifurcations can occur in the model (2) analogously to the model considered in [10] where an additional memory has been added to (1).

**II. NONLOCAL KINETIC THEORY**

It is now noteworthy to see that the model (2) can in fact be derived from microscopic quantum statistics and as such bears direct physical relevance. We will show that indeed for an interacting Fermi system at low temperatures the one-particle distribution function obeys a kinetic equation which can be written in the form of (2). In this way we will express the parameters \(a\) and \(\sigma\) by characteristic physical quantities of the system, the temperature \(T\), density \(n\), scattering length \(a_0\) and range of the potential \(r_0\).

First let us recall the quantum nonlocal kinetic equation for the quasiparticle distribution \(f\) [16,17] with quasiparticle energy \(\varepsilon\)

\[ \frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k} \frac{\partial f_1}{\partial r} - \frac{\partial \varepsilon_1}{\partial r} \frac{\partial f_1}{\partial k} = \int \frac{dp dq}{(2\pi)^3 \hbar^2} P \left[ \left(1-f_1\right)\left(1-f_2\right) f_3 f_4 - f_1 f_2 \left(1-f_3\right) \left(1-f_4\right) \right]. \] (10)

The superscripts denote the signs of non-local corrections: \(f_1 = f(k,r,t), f_2 = f(p,r-D_2,t), f_3 = f(k-q-D_3, r-D_3, t-D_3), f_4 = f(p-q-D_4, r-D_4, t-D_4)\). The first(second) part of the collision integral is called in(out)-scattering since it (de)populates the distribution \(f_1(k,r,t)\). The time-space symmetries and particle-hole symmetries of (10) are discussed in [18]. The form (10) used in the present paper, shows explicitly the particle-hole symmetry since the out-scattering is the particle-hole mirror of the in-scattering. Let us here remark only that it can be given an equivalent form with explicit time-space symmetry [18]. The scattering measure
where the equilibrium distribution $F$ is given by the modulus of the scattering T-matrix which is a complex quantity $T^\ast = |T^\ast| \exp i\phi$. The latter one can be found from scattering theory or experimental phase shift analysis. All corrections, the $\Delta$’s, describing the non-local and non-instant collision [16,17] are given by derivatives of the scattering phase shift $\phi = \Im \ln T^\ast (\omega, k, p, q, t, \tau)$

$$\Delta_t = \frac{\partial \phi}{\partial \omega} |_{\epsilon_1 + \epsilon_2} \Delta E = -\frac{1}{2} \frac{\partial \phi}{\partial t} |_{\epsilon_1 + \epsilon_2} \Delta_3 = -\frac{\partial \phi}{\partial k} |_{\epsilon_1 + \epsilon_2} \Delta_4 = -\left( \frac{\partial \phi}{\partial k} + \frac{\partial \phi}{\partial \epsilon} \right) |_{\epsilon_1 + \epsilon_2}.$$ (11)

The nonlocal kinetic equation (10) covers all quantum virial corrections on the binary level and conserves density, momentum and energy including the corresponding two-particle correlated parts [17]. The kinetic equation (10) allows for a simple classical interpretation of the complicated correlations covered by collisions. The collision integral can be understood as a collisional scenario that two particles are approaching until they reach their correlation distance $\Delta_2$ then they form a correlated pair with the collision duration of $\Delta_3$. During this correlated travel they can rotate so that they break up into single particles after the time $\Delta_t$ at the endpoints $\Delta_{3,4}$.

The classical kinetic theory of dense gases of Enskog-like equations as well as the Landau theory of interacting Fermi systems are limiting cases of this nonlocal kinetic equation. It requires no more technical problems than solving the Boltzmann equation [19,20]. From the formal point of view this kinetic equation has been derived taking into account all terms up to linear order in the quasiparticle damping.

We will now derive the equation (2) from (10) and will calculate the required parameters for a model T-matrix of separable potential. First we analyze the collision integral and assume quadratic dispersion relation for the quasiparticle energies. The $\delta$-function in $P$ representing the energy conservation in the collision integral reads

$$\delta\left( \frac{k^2}{2m_a} + \frac{p^2}{2m_b} - \frac{(k-q)^2}{2m_a} - \frac{(p+q)^2}{2m_b} \right) = \delta\left( |q|\left( \frac{k}{m_a} - \frac{p}{m_b} \right) \cdot |q| - \frac{q^2}{2} \left( \frac{1}{m_a} + \frac{1}{m_b} \right) \right) = \delta(|q|) \left( \frac{k}{m_a} - \frac{p}{m_b} \right) \cdot \frac{\partial}{\partial q} + \delta(q \neq 0)$$ (12)

with effective masses of particle $a,b$. The second part is the usual considered $\delta$-function excluding zero transferred momentum $q = 0$. This collision part will be treated in relaxation time approximation $\propto (F_0 - f)/\tau$ where the equilibrium distribution $F_0$ is specified later.

More interesting is to notice that the $q = 0$ channel leads to an additional part absent in usual local kinetic equations like the Boltzmann kinetic equation. To convince the reader of this novel observation let us rewrite the Pauli-blocking factors of (10) for the $q = 0$ channel where $f_3 \rightarrow f_1$ and $f_4 \rightarrow f_2$ according to (12)

$$f_3 f_4 (1 - f_2) (1 - f_1) |_{q=0} = f_3 (f_1 - f_1) f_4 (1 - f_2) |_{q=0} + f_3 (1 - f_1) f_4 (1 - f_2^\prime) |_{q=0} = f_1^\prime (f_1 - f_1) f_2 (1 - f_2) + f_1 (1 - f_1) f_2^\prime (1 - f_2)$$ (13)

The first part can be expanded

$$f_1 - f_1 = -\Delta_3 \frac{\partial f}{\partial t} - \Delta_4 \frac{\partial f}{\partial \epsilon} - \Delta_K \frac{\partial f}{\partial k} = -\Delta_3 \frac{\partial f}{\partial t} - \Delta K \frac{\partial f}{\partial k}$$ (15)

where in the last step we have replaced the time derivative of $f$ by the free drift motion $\frac{\partial f}{\partial t} = -\frac{\partial f}{\partial \epsilon} \frac{\partial \epsilon}{\partial t} + \frac{\partial f}{\partial t}$ leading to derivative of on-shell shifts (11). Therefore we can absorb the first part of (14) into the left (drift) side of the nonlocal kinetic equation (10). These gradients are absent for the now considered homogeneous case. Using thermal averaging of the occurring microscopic quantities $\tau, \tilde{\tau}$ and $\Delta_t$ denoted by $< ... >$ the second part of (14) leads to an additional term in the kinetic equation

$$\frac{df(k,t)}{dt} = -f(k,t - \Delta_t) [1 - f(k,t - \Delta_t)]$$ (16)

with

$$\frac{1}{\tau} = \frac{2\pi s}{\hbar} \int \frac{d\omega}{(2\hbar)^d} |T^\ast (k, p, q)|^2 \times \delta(\epsilon_k + \epsilon_p - \epsilon_{k+q} - \epsilon_{p-q}) f_p (1 - f_{k-q}) (1 - f_{p+q}) > \frac{1}{\tilde{\tau}} = \frac{2\pi s}{\hbar} \int \frac{d\omega}{(2\hbar)^d} |T^\ast (k, p, q)|^2 \times \delta((k-p) \cdot \frac{q}{m}) f_p (1 - f_p) > \Delta_t = -\frac{1}{\hbar} < \frac{\partial}{\partial \omega} \Im \ln T^\ast (k, p, q, \omega) |_{\omega = \epsilon_k + \epsilon_p} >.$$ (17)
The local equilibrium distribution $F_0 = \tilde{f}_0 - \tau/\tilde{f}_0(1 - \tilde{f}_0)$ is specified here in such a way that equilibrium $f_0$ is a solution of the stationary problem. Conservation laws are enforced in the usual way as for relaxation time approximation constructing the local equilibrium appropriately. We remark that for Fermi ground state $f_0 = (0, 1)$ we have $F_0 = (0, 1)$. For the reason of simplicity we will restrict to states above Fermi level $F_0 \approx 0$. All discussions can be performed with any given $F_0$.

One can see that the equation (2) appears from (16) if

$$a = \frac{\tau}{\pi} \quad \sigma = \tau \quad \tau_d = \Delta_t.$$  \hspace{1cm} (18)

Equations (16) and (17) are the main results of the paper. It shows that from the nonlocal kinetic equation for dense interacting Fermi systems one can derive an evolution equation which shows the onset of chaos in that an oscillation occurs and with increasing parameter $a$, see (8), bifurcations appear and deterministic chaos sets in. This is due to the fact that besides the stationary solution $f_0$ of (16) which corresponds to the stationary fixed point 0 of (2) we have a branching fixed point which corresponds to $1 - 1/a$. For the occupied states $F_0 \approx 1$ we would get the fixed points $(1, -1/a)$ of (2) which would result into a minus sign in front of the 2 in equation (8).

### III. MODEL CALCULATION

To be specific we will now calculate the parameter (18) for an interacting Fermi system. We use as an exploratory model the separable interaction [21,22] which is written in terms of the difference momenta of incoming $p_1$ and outgoing particles $p_2$

$$V(p_1, p_2) = \frac{(2\pi\hbar)^3\lambda}{m(\beta^2 + p_1^2)(\beta^2 + p_2^2)}$$  \hspace{1cm} (19)

in terms of two parameter, the coupling constant $\lambda$ and the inverse potential range $\beta$. With the help of this potential Bethe-Salpeter equation or retarded $T$-matrix equation $T^r = V + VG^rT^r$ can be solved

$$T^r(k, p, q, \omega) = \frac{(2\pi\hbar)^3\lambda/m}{(\beta^2 + (k-p)^2/4)(\beta^2 + (k-p + 2q)^2/4)} \times (1 + \frac{\pi^2\lambda}{\beta(\sqrt{m\hbar\omega + i\beta})})^{-1}.$$  \hspace{1cm} (20)

Calculating the scattering phase shift $\cot \delta = \text{Re} T^r/\text{Im} T^r$

$$p \cot \delta = -\frac{(p^2 + \beta^2/2)}{2\pi^2\lambda} + \frac{p^2 - \beta^2}{2\beta} = -\frac{\hbar}{a_0} + \frac{r_0}{2\hbar^2} + ..$$  \hspace{1cm} (21)

shows that the parameter $\beta$ and $\lambda$ are linked to the scattering length $a_0$ and the potential range $r_0$ in the following way

$$a_0 = \frac{2\hbar}{\beta}(1 - \frac{\beta^3}{\pi^2\lambda})^{-1}$$

$$r_0 = \frac{3\hbar}{\beta}(1 - \frac{4\hbar}{3a_0\beta}).$$  \hspace{1cm} (22)

Therefore we can describe with this simple rank-one separable potential the scattering length $a_0$ and the range $r_0$ of the interaction. Alternatively to the potential range one could fit to the bound state energy $\omega = E_B < 0$ which yields $\lambda = \beta(\beta - mE_B)^2/\pi^2$. We neglect here medium effects on the $T$-matrix which can be considered as well [22]. Now we can calculate the parameter (17) explicitly in the low temperature limit. The angular and energy integrals can be separated in the usual manner [23,24] and we obtain in lowest order temperature $T$

$$\frac{1}{\tau} = \frac{\pi T^2}{3\hbar e_f} a^2 c_1(\beta/p_f)s$$

$$\frac{1}{\tau} = \frac{4T}{\pi\hbar} a^2 \beta^2 \frac{c_2(\beta/p_f)s}{p_f^2}$$

$$\Delta_t = \frac{\hbar}{4\epsilon_f} c_3(a_s, \beta/p_f)$$  \hspace{1cm} (23)

with the Fermi energy $\epsilon_f = p_f^2/2m$ and the dimensionless scattering length $a_s = a_f p_f / \hbar$. The functions $c_1, c_2$ describing finite potential range effects and read

$$c_1(x) = \int_0^1 dy \frac{x^8}{(x^2 + 1 - y^2)^4} \approx 1 - \frac{8}{3x^4} + o(x^{-4})$$

$$c_2(x) = \int_0^1 dy \frac{2\sqrt{2x^6}}{(1 - y + 2x^2)^3(1 - y)} \approx 1 - \frac{1}{x^2} + o(x^{-4})$$

$$c_3(a, x) = \frac{a x^4 (-6 + 3 ax + 2 x^2)}{(1 + x^2)(4x^4 + (2 - ax)(2 - ax + 4x^2 - 4ax^3))} \approx \frac{a}{4(1 + a^2)} + o(x^{-2})$$  \hspace{1cm} (24)

where the first two integrals are elementary functions. The final result for the parameter of (2) are now represented in terms of the microscopic scattering length, $a_s = p_f a_0 / \hbar$, and the range of the potential (22) as

$$a = \frac{\epsilon_f}{T} \frac{12c_2 \beta^2}{\pi^2 c_1 p_f^2}$$

$$\sigma = \frac{\hbar e_f}{T^2} \frac{3}{\pi c_1 s} \frac{1}{a_s^2}$$

$$\tau_d = \frac{\hbar}{4\epsilon_f} c_3.$$  \hspace{1cm} (25)

From this we can now give the conditions for the onset of bifurcation and route to chaos according to (8).

First it is interesting to discuss the sign of $\tau_d$ required for (8). Since $c_3$ in (24) determine $\tau_d$ via (23) we obtain
\[ a_0 < 0 \quad \text{for } p_f \gg p_c \]
\[ 0 < a_0 < \frac{24}{3} : \quad \tau_d \geq 0 \quad \text{for } p_f \lesssim p_c \]
\[ a_0 > \frac{24}{3} : \quad \tau_d > 0 \quad (26) \]

with \( p_c = \beta/\sqrt{3 - \frac{4a_0}{3}} \). It shows nicely the interplay between the range of potential and the density. For lower densities than the critical one, \( p_f < p_c \), the delay time follows the sign of scattering length. For higher densities the delay time flips the sign if the scattering length is lower than \( 2h/\beta \).

Now we analyze (8) and (9) for the low temperature expansion compared to the Fermi energy. The final resulting condition for the occurrence of oscillations with the frequency \( \Omega \) reads

\[ \tau_d < 0: \quad T < \epsilon_f \frac{2\pi^2}{2c_s^3 c_f^3 \sqrt{1 - \zeta}} \left( \frac{h}{\beta a_0} \right)^2 \]
\[ \Omega = \epsilon_f \frac{2\pi^2}{2c_s^3 c_f^3 \sqrt{1 - \zeta}} \left( \frac{h}{\beta a_0} \right)^2 \left( \frac{T}{\epsilon_f} \right)^2 \]
\[ \tau_d > 0: \quad T > \epsilon_f \frac{2\pi^2}{2c_s^3 c_f^3 \sqrt{1 - \zeta}} \left( \frac{h}{\beta a_0} \right)^2 \]
\[ \Omega = \epsilon_f \frac{2\pi^2}{2c_s^3 c_f^3 \sqrt{1 - \zeta}} \left( \frac{h}{\beta a_0} \right)^2 \left( \frac{T}{\epsilon_f} \right)^2 \]

where \( \zeta = \frac{\pi^2}{6sc^3 c_f^2} \left( \frac{h p_f}{a_0 \beta^2} \right)^2 \ll 1 \) according to the expansion with respect to the range of the potential. These conditions for the temperature together with (26) allow to calculate the density and temperature range for which the oscillations appear and chaotic behavior is possible.

In the end we should discuss the validity range of the kinetic equation. The nonlocal kinetic equation (10) we have started from has been derived under the assumption that the delay time is smaller than the relaxation time. This translates into \( \tau_d < \sigma \) and from (25) we obtain

\[ T < \epsilon_f \frac{12}{\pi sc^3 c_f^3 a_s} \quad (28) \]

which has an overlapping region with (27) and is consistent with the low temperature expansion used here. To illustrate this we want to consider nuclear matter with realistic parameter. The triplet channel has a scattering length of \( a_0 = 5.379 \text{fm} \) and the deuteron bound state energy is reproduced if the parameter for the separable potential is \( \beta = 1.4488 \text{fm} \) [21]. Then the third case of (26) applies. The corresponding third case of (27) together with (28) gives then the criterion on the temperature \( 0.118 \epsilon_f > T > 0.0813 \epsilon_f / \text{fm} p_f \) which makes only sense if the density or the Fermi momentum \( p_f > 2.24 \text{fm} \) which corresponds to about four times nuclear matter density \( n_0 = 0.14 \text{fm}^{-3} \). The corresponding temperatures will be around 10MeV and the oscillation period \( 1/\Omega \approx 2 \text{fm}/c \). These parameter are reached in experimental heavy ion reactions. Therefore we will find a parameter range where bifurcation and the route to chaos can occur in the time evolution of the distribution function itself.

IV. CONCLUSION

Summarizing, dense interacting Fermi system are described by nonlocal kinetic equations which can be shown to be of the type of delay differential equations. They interpolate between finite time stepping of the logistic map and continuous time derivatives. The important novel observation is that the distribution function itself shows temporal oscillations around the equilibrium value with a basic frequency of about the Fermi energy. For certain density and temperature conditions, (27), this oscillation bifurcates and the route to chaotic time evolution sets in. It could be considered as a signal of onsetting phase transition in accordance with the analogy found in [11] between mean-field phase transitions and bifurcations of discrete maps.

In the end it should be commented on the entropy. It is clear that from the above described possibility of oscillations in the time evolution of the distribution function follows that also the entropy will oscillate. This has been seen already in the collisionless but selfconsistent Vlasov equation [25,26] which appears if we neglect collisions in (2) at all and calculated the quasiparticle energy on the drift side from the selfconsistent mean field. The inclusion of the collisions are expected to damp this oscillations. As we have shown above there might be a regime where the correlations are such that oscillations pertain. In such cases the entropy would oscillate. The complete balance equations [17] which follows from the nonlocal kinetic theory (2) show that the observables contain besides the Landau one–particle parts also correlated two-particle parts. Though the explicit forms are known for the density, current, energy and stress tensor and their corresponding conservation laws are proven [17], we have not been able yet to reconstruct the explicit form of the entropy beyond the single particle expression. Therefore we could not find an explicit H-theorem with two-particle correlations included. This will remain to future work.

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