The meson spectra beyond a $q\bar{q}$ description

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Abstract. Despite the apparent simplicity of meson spectroscopy there are some states which cannot be accommodated in the usual $q\bar{q}$ structure. Among them there are either exotic states as the $X(1600)$, or the recently measured charmed states $D_s^*$ and some of the light scalar mesons. We present a description of the light scalar mesons in terms of $q\bar{q}$ and $qq\bar{q}\bar{q}$ components.

Nearly all known mesons made of $u, d,$ and $s$ quarks fit neatly into the multiplets expected in generic constituent quark models. The single, striking exception were the scalar mesons, i.e., $J^{PC} = 0^{++}$. Either they form an anomalously light nonet or a nonet and more scalar mesons appear in the energy region $1.2-1.5$ GeV [1]. Such a long-standing situation has been re-invigorated by the new data obtained at BaBar, CLEO, FOCUS and Belle, creating a challenging scenario. There have been reported a number of states compatible with meson quantum numbers, as the $D_s^*$’s [2], but either their masses do not fit into the quark model predictions in its many variations or they over-populate the expected number of states. For years multiquark states have been justified to coexist with $q\bar{q}$ states in the energy region around 1 GeV for the case of the light mesons [3]. This situation claims for a comprehensive study where two- and four-quark states are considered simultaneously.

A $qq\bar{q}\bar{q}$ state has the same quantum numbers and the same quark content as a $q\bar{q} - q\bar{q}$ meson scattering state. In a fairly precise way the $qq\bar{q}\bar{q}$ state can be considered as a piece of the meson-meson continuum that has been artificially confined by a confining boundary condition or potential that is inappropriate in the meson-meson channel [4]. If the multiquark state is unusually light or sequestered (by the spin, color and/or flavor structure of the wave function) from the scattering channel it may be prominent. If not, it would be just a silly way of enumerating the states in the continuum.

In this work we present a careful study of possible prominent four-quark states in the low-energy meson spectra. For this purpose, we will address the description of hadrons with zero baryon number described as clusters of quarks confined by a realistic interaction between them (it allows to describe the NN data and the baryon spectra). Our work tries to supersede other studies of four-quark systems devoted either to a particular set of states [5], or more general studies that in any case made a detailed comparison with $q\bar{q}$ predictions within the same model [6].

We have solved the two-body problem by means of the Schrödinger equation treating in an exact way the non-central terms of the interacting potential. The four-body problem has been solved by means of a variational method using as trial wave function the most general linear combinations of gaussians [7]. In particular, the non-quadratic terms that were neglected in previous works [8], have been considered. While they are known
to play a minor role in the description of light-heavy tetraquarks, they have a great influence in the light case. The mixing between the two- and four-body components will be parametrized as explained in the following.

The interacting potential and the method to solve the four-body problem have been tested in the case of a system whose quantum numbers can only be obtained by means of a multiquark description, the isospin two \( \langle q \bar{q} q \bar{q} | \rangle \). This state has been observed in the reaction \( \gamma \gamma \rightarrow \rho \rho \) near threshold, reported with a mass of \( 1600 \pm 100 \text{ MeV} \) and quantum numbers \( I^GJ^{PC} = 2^+ (2^+) \) [9]. It cannot be described as a \( q \bar{q} \) state, being therefore an exotic meson. It can be easily understood as a tetraquark made of four light quarks coupled to \( I = 2, S = 2 \) and \( L = 0 \). Our formalism predicts for this configuration an energy of \( 1500 \text{ MeV} \), in excellent agreement with the experimental data, giving confidence to the results obtained for the four-body configurations.

In non-relativistic quark models gluon degrees of freedom are frozen and therefore the total wave function for a \( B = 0 \) hadron may be written symbolically as

\[
|B = 0\rangle = \sum_n \Omega_n |(q \bar{q})^n \rangle = \Omega_1 |q \bar{q}\rangle + \Omega_2 |q \bar{q} q \bar{q}\rangle + \ldots. \tag{1}
\]

where \( q \) stands for quark degrees of freedom. As mentioned above, the energy of the \( q \bar{q} \) and \( q q \bar{q} \bar{q} \) configurations can be obtained from our hamiltonian, the results being resumed in table 1. One can see that neither the \( q \bar{q} \) nor the \( q q \bar{q} \bar{q} \) configurations match the experimental data. To consider the mixing between the two- and four-body configurations (the calculation of the coefficients \( \Omega_i \)), requires the knowledge of the operator annihilating a quark-antiquark pair into the vacuum. This could be done, for example, using a \( ^3P_0 \) model, but the result will always depend on the parameters used to describe the vertex. Therefore we have decided to parametrize these coefficients by looking to the quark pair that it is annihilated, and not on the spectator quarks that will form the final \( q \bar{q} \) state.

\[
\begin{align*}
\langle q \bar{q} q \bar{q} | O | q \bar{q} \rangle &= \langle q s \bar{s} s | O | s \bar{s} \rangle = \langle q q \bar{q} \bar{q} | O | q \bar{q} \rangle = C_q \\
\langle s \bar{s} s \bar{s} | O | s \bar{s} \rangle &= \langle q s \bar{s} q | O | q \bar{q} \rangle = \langle q s \bar{s} s | O | q \bar{q} \rangle = C_s.
\end{align*}
\]

The mixing parameters, \( C_s \) and \( C_q \), have been fixed using the best measured scalar states: the \( a_0(980) \) and the \( f_0(980) \).

The obtained mass and dominant flavor component for all the scalar mesons are given in table 2. The experimental data are taken from Ref. [9]. One should notice that such an interpretation of the light scalar mesons in terms of two- and four-quark components allows for a one-to-one correspondence between theoretical states and experiment. Our results assign a dominant tetraquark component to the \( f_0(980), a_0(1450) \) and the

| TABLE 1. Mass, in MeV, of the \( q \bar{q} \) and \( q q \bar{q} \bar{q} \) light isoscalar, isovector and \( I=1/2 \) systems. |
|-------------------------------------------------|
| \( f_0(600) \) | 648 | \( q \bar{q} \) | \( a_0(980) \) | 1079 | \( q \bar{q} \) | \( \kappa(800) \) | 1273 | \( q \bar{q} \) | \( \kappa(800) \) | 1273 |
| \( f_0(980) \) | 949 | \( q q \bar{q} \bar{q} \) | \( a_0(1450) \) | 1308 | \( q q \bar{q} \bar{q} \) | \( K^*_0(1430) \) | 1295 | \( q q \bar{q} \bar{q} \) | \( K^*_0(1430) \) | 1295 |
TABLE 2. Mass, in MeV, and flavor dominant component for the light isoscalar, isovector and \( I=1/2 \) mesons.

| \( I=0 \) | \( I=1 \) | \( I=1/2 \) |
|---|---|---|
| Mass | Exp. | Flavor | Mass | Exp. | Flavor | Mass | Exp. | Flavor |
| 568 | 400–1200 | \( q\bar{q} \) | 985 | 984.7±1.2 | \( q\bar{q} \) | 1113 | \( \approx 800 \) | \( q\bar{s} \) |
| 999 | 980±10 | \( qq\bar{q} \) | 1381 | 1474±19 | \( qq\bar{q} \) | 1440 | 1412±6 | \( qq\bar{s} \) |
| 1301 | 1200–1500 | \( s\bar{s} \) | 1530 | 1274±14 | \( s\bar{s} \) | 1784 | \( q\bar{s} \) |
| 1465 | 1507±5 | \( qs\bar{s} \) | 1640 | 1550±12 | \( q\bar{q} \) | 1831 | 1945±20 | \( qs\bar{s} \) |
| 1614 | 1713±6 | \( q\bar{q} \) | 1868 | \( q\bar{q} \) | 2060 | \( q\bar{s} \) |
| 1900 | 1992±16 | \( s\bar{s} \) | 1944 | \( s\bar{s} \) | 2224 | \( s\bar{s} \) |
| 2224 | 2197±17 | \( s\bar{s} \) | 2351 | \( s\bar{s} \) | |

\( K^*_0 (1430) \). The final physical picture arising from these results shows an involved structure for the flavor wave function of the light scalar mesons, in agreement with the complicated pattern observed for their decays. From our results one can also see how the predicted isoscalar state between 1.2 and 1.5 GeV may in fact correspond to two different resonances, as suggested by the PDG. We also observe that the description of the high energy states is worse than the low-energy data, what may be a consequence of not including in our calculation excited states of the four-quark configurations.

It seems therefore coherent to include simultaneously the two- and four-quark components of the wave function to describe the light scalar mesons. Preliminary studies of the charmed sector allows for a similar correspondence between reported states and theoretical numbers. The study of the pattern decays should definitively conclude the correctness of the present description.

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