Induced charges as probes
of low energy effective theories

V. A. Rubakov

Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary Prospect, 7a, Moscow 117312, Russia

Abstract

We suggest that the correspondence between gauge theories strongly coupled in
the infrared and their low energy effective theories may be probed by introducing
topologically non-trivial background scalar fields. We argue that one loop expressions
for the global charges induced in vacuum by these background fields are in some
cases exact in the fundamental theory, and hence should be matched in the effective
theory. These matching conditions are sometimes inequivalent to 't Hooft ones. A few
examples of induced charge matching are presented.

1. Low energy effective theories are used to describe low energy physics inherent in fun-
damental, “microscopic” theories which are strongly coupled in the infrared. An important
guide to infer the effective theories is provided by the ’t Hooft anomaly matching condi-
tions [1] (and their discrete analogs [2]). One way to understand the anomaly matching is
to introduce background gauge fields corresponding to (a subgroup of) the flavor symme-
try group; the anomalies in the flavor currents are then proportional to the topologicalcal
charge densities of the background gauge fields. This topological property is closely related
to the Adler–Bardeen theorem that guarantees that the anomalies are equal in the original,
“microscopic” theory and its low energy partner.

*E-mail: rubakov@ms2.inr.ac.ru
In this paper we suggest that the low energy theories may also be probed by introducing slowly varying scalar, rather than vector, background fields with topological properties. The global charges induced in vacuum by these fields are often (though not always) unambiguously calculable at one loop in the fundamental theory, and are proportional to the topological charges of the background. This topological property suggests, by analogy to the triangle anomalies, that the non-renormalization theorem similar to that of Adler and Bardeen should in many cases hold for induced charges, and that in these cases the induced charges, calculated at one loop in low energy and fundamental theories, should match.

Though induced charges and triangle anomalies are closely related to each other, they probe somewhat different aspects of the correspondence between fundamental and low energy theories. The background scalar fields provide masses to (some of) the fermions of the fundamental theory, so the induced charges probe the respond of the low energy theory to these masses. In some cases — in particular, in supersymmetric gauge theories — this respond is well understood, so the induced charge matching adds nothing to the analysis of the low energy theories. Indeed, we will see that there are very simple sufficient conditions that insure induced charge matching provided the ’t Hooft matching holds; these sufficient conditions are automatically satisfied in supersymmetric gauge theories. On the other hand, we will see that in non-supersymmetric models including those in which supersymmetry is slightly broken by small soft masses, induced charge matching places constraints on the low energy theories that are not equivalent to the ’t Hooft constraints.

In this paper we first present arguments supporting our conjecture of the absence of quantum corrections to induced charges, taking non-supersymmetric QCD as an example. We then give a few other examples of the induced charge matching and see that it sometimes occurs in a fairly non-trivial way. We conclude by pointing out that induced charges may suffer infrared problems in some theories, so the induced charge matching between low energy and fundamental theories may not occur.

2. We begin with conventional $SU(N_c)$ QCD with $N_f$ massless flavors. Let us introduce
background fields \( m^\tilde{p}_q(x) \) which are time independent and slowly vary in space. These are
\( N_0 \times N_0 \) matrices; hereafter the indices \( p, q, r; \tilde{p}, \tilde{q}, \tilde{r} \) run from 1 to \( N_0 \) with \( N_0 \leq N_f \). Let these fields couple to \( N_0 \) flavors of quarks and anti-quarks in the following way,

\[
m^\tilde{p}_q(x) \tilde{\psi}_{\tilde{p}} \psi^q + \text{h.c.}
\]

where \( \psi^i \) and \( \tilde{\psi}_j \) are left-handed quark and anti-quark fields, respectively. We assume for definiteness that the background fields have the following form

\[
m^\tilde{p}_q(x) = m_0 U^\tilde{p}_q(x)
\]

where \( m_0 \) is a constant and \( U \) is an \( SU(N_0) \) matrix at each point \( x \). We restrict the form of the background even further by requiring that \( U(x) \) is independent of coordinates at spatial infinity; by a global \( SU(N_f)_L \) rotation

\[
U(x) \rightarrow 1 \quad \text{as} \quad |x| \rightarrow \infty
\]

Under these conditions, the background fields are characterized by the topological charge

\[
N[U] = \frac{1}{24\pi^2} \int d^3x \, \epsilon^{ijk} \text{Tr} \left( U \partial_i U^\dagger U \partial_j U^\dagger U \partial_k U^\dagger \right)
\]

The background fields give \( x \)-dependent masses to \( N_0 \) quarks, while \( (N_f - N_0) \) flavors remain massless. These fields also explicitly break the flavor group down to

\( SU(N_f - N_0) \times SU(N_f - N_0) \times U(1)_B \times U(1)^f_8 \)

where \( U(1)_B \) is the baryon number (we assign baryon number 1 to each quark) and \( U(1)^f_8 \) is a vectorial subgroup of the original \( SU(N_f) \times SU(N_f) \) flavor group, whose (unnormalized) generator is

\[
T^f_8 = \text{diag} \left( 1, \ldots, 1, -\frac{N_0}{N_f - N_0}, \ldots, -\frac{N_0}{N_f - N_0} \right)
\]

( \( U(1)^f_8 \) is absent if \( N_0 = N_f \)).

The background fields \( m(x) \) induce global charges in vacuum. We are interested in the global symmetries which are unbroken by the background fields and under which the
massive quarks transform non-trivially. These are the baryon number and $T^f_8$. The one-loop calculation \[3\] gives for slowly varying $m(x)$

$$\langle B \rangle = N_c N[U] \tag{2}$$

$$\langle T^f_8 \rangle = N_c N[U] \tag{3}$$

As the right hand side of these relations equals the topological number, up to a color factor, we suggest that eqs. (2) and (3) are valid in full quantum theory.

To substantiate this conjecture, let us discuss the relation between induced charges and triangle anomalies; we consider induced baryon number as an example. The $x$-dependence of the background field $m(x)$ can be removed at the expense of modification of the gradient term in the quark Lagrangian. Namely, after the $SU(N_0)_L$ rotation of the left-handed quark fields $\psi^p$, $\psi(x) \to U^{-1}(x)\psi(x)$, $\bar{\psi}(x) \to \bar{\psi}(x)$, first $N_0$ quarks and antiquarks have $x$-independent masses $m_0$, and the gradient term of these quarks becomes

$$\bar{\psi} i\gamma \cdot \left( D + \frac{1-\gamma^5}{2} A^L \right) \psi$$

where $A^L_0 = 0$, $A^L_i = U \partial_i U^{-1}$, $D_\mu$ contains dynamical gluon fields, and we temporarily switched to four-component notations. This addition to the gradient term may be viewed as the interaction of massive quarks with the background pure gauge vector fields corresponding to $SU(N_0)_L$ subgroup of the flavor group; these background fields are small and slowly vary in space.

Now, consider an adiabatic process (either in Minkowskian or in Euclidean space-time) in which the background vector fields $A^L_i(x)$ (in the gauge $A_0 = 0$) change in time from $A^L_i = 0$ to $A^L_i = U \partial_i U^{-1}$ always varying slowly in space and vanishing at spatial infinity (an example of such a process is an instanton of large size). Suppose that this process begins with the system in the ground state which has zero induced charges because of the triviality of the background. As the background vector fields interact with massive degrees of freedom only, the system remains in its ground state in the entire process, at least order by order in perturbation theory. The induced baryon number in the final state — the quantity we are interested in — is equal to $< B > = \int d^4x \frac{\partial \mu j^B_\mu}{\partial \mu}$ which in turn is determined by the
anomaly in the gauge-invariant baryonic current $j^B_\mu$. Hence, we recover eq. (2):

$$< B >= \frac{N_c}{16\pi^2} \int d^4x \, F^L_{\mu\nu} \tilde{F}^L_{\mu\nu} = N_c N[U]$$

(4)

This observation relates induced charges and anomalies and strongly suggests that induced charges do not receive radiative corrections in the fundamental, “microscopic” theory.

This argument is still basically perturbative. One may wonder whether non-perturbative effects such as fermion level crossing might make the final state of the adiabatic process different from the ground state, i.e., whether the final state might actually contain excitations carrying non-zero net baryon number. In that case the baryon number induced in the ground state by the background field $A^L_i(x) = U\partial_i U^{-1}(x)$ would be different from eq. (4), as the anomaly determines the total change in the baryon number. To argue that this does not happen, we note that the appearance, in the final state, of excitations with non-zero net baryon number would show up as a non-vanishing index of the four-dimensional Euclidean Dirac operator $D[A^L] = \gamma \cdot \left( D + \frac{1-\gamma^5}{2} A^L(x) \right) + m_0$, so that the vacuum-to-vacuum amplitude would vanish while matrix elements of baryon number violating operators between the initial and final vacua would not. However, for arbitrary gluon fields, the eigenvalues $\omega$ of the operator $D[A^L = 0] = \gamma \cdot D + m_0$ obey $|\omega| > m_0$ (the Euclidean operator $\gamma \cdot D$ is anti-Hermitean) so the operator $D[A^L]$ has no zero modes when the background fields $A^L(x)$ are small ($A^L(x) \ll m_0$ at all $x$) and slowly vary in space-time. This argument implies that eq. (4) is valid in full quantum theory even at $m_0 < \Lambda_{QCD}$. Although the situation in theories with colored scalars is more complicated, it is likely that analogous arguments may be designed in those theories as well.

Let us see that the low energy effective theory of QCD — the non-linear sigma model — indeed reproduces eqs. (2) and (3). In the absence of the background fields, the non-linear sigma model action contains only derivative terms for the $SU(N_f) \times SU(N_f)$ matrix valued sigma-model field $V(x)$, including the usual kinetic term and the Wess–Zumino term. The background field $m(x)$ introduces a potential term into the low energy effective Lagrangian,

$$\Delta L_{eff} = \text{Tr} \left( m^\dagger V + V^\dagger m \right)$$
For slowly varying \( m \), the effective potential is minimized at

\[
V(\mathbf{x}) = \begin{pmatrix}
U(\mathbf{x}) & 0 \\
0 & 1
\end{pmatrix}
\]  

(5)

Hence, the induced baryonic charge appears at the classical level [4]; as the baryonic charge of \( V(\mathbf{x}) \) is equal to its topological number \( N[V] \) times \( N_c \), the induced baryonic charge is indeed given by eq.(2). Likewise, it follows from the structure of the Wess–Zumino term that the \( T^f_8 \) current of the configuration of the form (3) is (cf. [5])

\[
j^f_{8,\mu} = \frac{N_c}{24\pi^2} \epsilon_{\mu\nu\lambda\rho} \text{Tr} \left( U \partial_\nu U^\dagger U \partial_\lambda U^\dagger U \partial_\rho U^\dagger \right)
\]

so the \( T^f_8 \) charge of the configuration (3) is given by eq.(3).

We see that the induced charges in QCD and its low energy effective theory match rather trivially. The way the induced charges match becomes more interesting when low energy theories contain massless fermions.

3. Let us now consider supersymmetric QCD with \( N_c \) colors and \( N_f \) flavors. To be specific, we discuss the case \( 3N_c > N_f > N_c + 3 \). This theory exhibits the Seiberg duality [6]: the fundamental theory contains the superfields of quarks \( Q^i \), anti-quarks \( \bar{Q}^j \) and gluons, while its effective low energy counterpart at the origin of moduli space is an \( SU(N_f - N_c) \) magnetic gauge theory with magnetic quarks \( q_i \), magnetic anti-quarks \( \bar{q}^j \) and mesons \( M_j \) with the superpotential \( qM\bar{q} \).

Let us probe this theory by adding the scalar background fields \( m^i_p(\mathbf{x}) \) with the same properties as above, i.e., by introducing the term

\[
m^i_p(\mathbf{x})\bar{Q}^q Q^p
\]

(6)

into the superpotential of the fundamental theory. Let us take for definiteness

\( 2 \leq N_0 < N_f - N_c - 1 \). The calculation of the induced baryon and \( T^f_8 \) charges in the fundamental theory proceeds as above, and we again obtain eqs.(2) and (3).
Let us turn now to the effective low energy theory. For slowly varying \( m(x) \), the term \( \mathcal{B} \) translates into \( \text{Tr}(mM) \), so the total superpotential of the magnetic theory is

\[
qM\tilde{q} + \mu_0\text{Tr}(mM)
\]

(7)

where \( \mu_0 \) is the dimensionfull parameter inherent in the magnetic theory. The ground state near the origin of the moduli space has the following non-vanishing \( x \)-dependent expectation values\(^1\) of the magnetic quarks and anti-quarks,

\[
\langle q_p^q \rangle = \mu_p^q, \quad p = 1, \ldots, N_0, \quad q = 1, \ldots, N_0
\]

(8)

(here the upper and lower indices refer to magnetic color and flavor, respectively)

\[
\langle \tilde{q}_p^q \rangle = \tilde{\mu}_p^q, \quad p = 1, \ldots, N_0, \quad \tilde{q} = 1, \ldots, N_0
\]

(9)

(here the lower index refers to magnetic color). The expectation values obey

\[
\tilde{\mu}_p^r(x)\mu_q^p(x) = -\mu_0 m_q^r(x)
\]

They also satisfy the \( D \)-flatness condition at each point in space, \( \mu^{rq}_p \mu^r_q = \tilde{\mu}^q_p \tilde{\mu}^l_q \). With our choice of background fields, eq.(1), one has

\[
\mu = \pm \sqrt{\mu_0 m_0} W(x), \quad \tilde{\mu} = \mp \sqrt{\mu_0 m_0} \tilde{W}(x)
\]

where \( W \) and \( \tilde{W} \) are \( N_0 \times N_0 \) unitary matrices\(^2\) obeying

\[
\tilde{W}W(x) = U(x)
\]

(10)

Since the gradient energy has to vanish at spatial infinity, \( W(x) \) and \( \tilde{W}(x) \) are constant at \( |x| \rightarrow \infty \), so they can be characterized by their winding numbers \( N[W] \) and \( N[\tilde{W}] \). Because of eq. (10) one has

\[
N[W] + N[\tilde{W}] = N[U]
\]

\(^1\)Hereafter we use the same notations for superfields and their scalar components.

\(^2\)At \( m = m_0 \cdot 1 \), the matrices \( \mu \) and \( \tilde{\mu} \) are proportional to \( N_0 \times N_0 \) unit matrix, up to magnetic color rotation. At \( m = m_0 U(x) \) one has \( \mu = \pm \sqrt{\mu_0 m_0} U(x) \), \( \mu = \mp \sqrt{\mu_0 m_0} U_c(x) \) where \( U_c(x) \) is a slowly varying matrix belonging to \( SU(N_0) \) subgroup of the magnetic color group. The explicit form of \( U_c(x) \) is to be found from the minimization of the gradient energy, and it is not important for our purposes.
In this ground state, the magnetic color is broken down to $SU(N_f - N_c - N_0)$. At small $m_0$, the ground state (8), (9) is close to the origin of the moduli space, so the magnetic description is reliable.

Both the baryon number and $T_8^f$ are broken in this vacuum. However, there exist combinations of these generators and magnetic color generators that remain unbroken. Recalling [6] that the baryon number of magnetic quarks equals $N_c / (N_f - N_c)$ and that the magnetic quarks and anti-quarks transform as $(\bar{N}_f, 1)$ and $(1, N_f)$, respectively, under the global $SU(N_f) \times SU(N_f)$ group, the unbroken generators are

\begin{align*}
B' &= B - \frac{N_c}{N_f - N_c} T_{8mc}^m \\
T_8' &= T_8^f + T_{8mc}^m
\end{align*}

where $T_{8mc}^m$ is the following generator of the magnetic color

$$T_{8mc}^m = \text{diag} \left( 1, \ldots, 1, -\frac{N_0}{N_f - N_c - N_0}, \ldots, -\frac{N_0}{N_f - N_c - N_0} \right)$$

As the fundamental quarks and gluons are singlets under magnetic color, the induced charges $< B' >$ and $< T_8' >$ calculated in the magnetic theory should match eqs. (2) and (3). Let us check that this is indeed the case.\(^3\)

The induced charges appear in the magnetic theory through $x$-dependent mass terms of fermions. These are generated by the expectation values (8), (9). The mass terms coming from the superpotential (7) are

$$\tilde{\mu}_q^p(x) \Psi^i_p \psi^q_i + \mu_q^p(x) \tilde{\psi}^q_j \Psi^i_j$$

where $\Psi$, $\psi$ and $\tilde{\psi}$ are fermionic components of mesons, magnetic quarks and magnetic anti-quarks, respectively. The gauge interactions give rise to other mass terms,

$$\mu^p_q(x) \psi^a_p \lambda^q_a - \tilde{\mu}_q^p(x) \lambda^a_q \tilde{\psi}^q_a$$

\(^3\)This example explains why we prefer to deal with the induced charges rather than the induced currents. The currents corresponding to the generators (11), (12) may be quite complicated in the fundamental theory, so the calculation of their expectation values — induced currents — does not seem possible. On the other hand, mapping of charges in the fundamental and effective theories is dictated by symmetries alone.
where $\lambda_a^b$ is the gluino field, $a, b = 1, \ldots, (N_f - N_c)$ are magnetic color indices.

To calculate the induced baryon number $< B' >$ we observe that the only fermions carrying non-zero $B'$ are magnetic quarks $\psi_i^\alpha$ with $i = 1, \ldots, N_f$, $\alpha = (N_f - N_c - N_0 + 1), \ldots, (N_f - N_c)$, magnetic anti-quarks $\bar{\psi}_j^\alpha$ and gluinos $\lambda_p^\alpha, \lambda^p_\alpha$. Their $B'$-charges are

$$\psi_i^\alpha : \frac{N_c}{N_f - N_c} - \frac{N_c}{N_f - N_c} \left( - \frac{N_0}{N_f - N_c - N_0} \right) = \frac{N_c}{N_f - N_c - N_0}$$

$$\lambda_p^\alpha : \frac{N_c}{N_f - N_c - N_0}$$

$$\bar{\psi}_j^\alpha, \lambda^p_\alpha : - \frac{N_c}{N_f - N_c - N_0}$$

Hence, the induced $B'$ is due to the $x$-dependent mass term (14) and is equal to

$$< B' > = - \frac{N_c}{N_f - N_c - N_0} \cdot (N_f - N_c - N_0) \left( N[W^+] + N[\bar{W}^+] \right)$$

Due to eq. (10) it indeed coincides with $N_c N[U]$, the induced baryon number calculated in the fundamental theory.

The induced charge $< T_8' >$ is calculated in a similar way. The relevant $T_8'$ charges of magnetic quarks are

$$\psi_p^\alpha : - \frac{N_f - N_c}{N_f - N_c - N_0}$$

$$\psi_p^u : \frac{N_f}{N_f - N_0}, \quad u = (N_0 + 1), \ldots, N_f$$

and similarly for magnetic anti-quarks, gluinos and mesons. We find that both $x$-dependent mass terms, (13) and (14), contribute to $< T_8' >$, and obtain

$$< T_8' > = \frac{N_f}{N_f - N_0} \cdot (N_f - N_0) \left( N[W] + N[\bar{W}] \right) + \frac{N_f - N_c}{N_f - N_c - N_0} \cdot (N_f - N_c - N_0) \left( N[W^+] + N[\bar{W}^+] \right)$$

This is equal to $N_c N[U]$, so the induced $T_8'$ charges also match in the fundamental and low energy theories.

4. As our last example, let us consider supersymmetric QCD with small soft masses of scalar quarks, $m^2_Q$, that explicitly break supersymmetry [7]. We again probe this theory by
introducing the term (3) into the superpotential. The restrictions on $N_f$, $N_c$ and $N_0$ are the same as in the previous example.

The induced charges, as calculated in the fundamental theory, are still given by eqs. (2) and (3). The low energy theory near the origin is still the magnetic theory, but now with soft mass terms of scalar mesons and scalar magnetic quarks [7]. The scalar potential of the magnetic theory near the origin at small $m_0^2$ is determined both by the superpotential (7) and these soft terms,

$$V(M,q,\bar{q}) = |\bar{q}q + \mu_0 m_0|^2 + |qM|^2 + |M\bar{q}|^2 + m_M^2M^\dagger M + m_q^2(q\dagger q + \bar{q}\dagger \bar{q}) + D\text{-terms}$$

(15)

where $m_M^2$ and $m_q^2$ are proportional to $m_0^2Q$. Were the soft terms in eq. (15) positive, the ground state of this theory at $m_0^2 > \mu_0 m_0$ would be at the origin, $<q>=<\bar{q}>=<M>=0$. The masses of fermions in the magnetic theory would vanish, the induced charges $<B>$ and $<T_8^f>$ would be zero, so the induced charge matching would not occur. Hence, the induced charge matching requires that either $m_q^2$ and/or $m_M^2$ are negative, so that the ground state even at $m_0 = 0$ is away from the origin, or $m_q^2 = 0$, $m_M^2 \geq 0$ with the ground state being the same as in the previous example. This is in accord with explicit calculations: it has been found in ref. [8] (see also ref. [9]) that $m_q^2 < 0$ at $N_c + 1 < N_f < 3N_c/2$, i.e., when the magnetic theory is weakly coupled, while at $3N_c/2 \leq N_f < 3N_c$ one has $m_q^2 = m_M^2 = 0$ [10]. We conclude that the induced charge matching provides qualitative understanding of these results.\footnote{The same phenomenon occurs in softly broken supersymmetric theories with $SO(N_c)$ and $Sp(2k)$ gauge groups and fundamental quarks at $N_f$, $N_c$ and $k$ such that the Seiberg duality holds [11].}

It is worth noting that there exists an example [8] where soft masses of scalar quarks single out the vacuum at the origin of the moduli space (in the absence of the background fields $m(x)$). This is the theory with $Sp(2k)$ gauge group and $2k + 4 = 2N_f$ quarks $Q_i$, $i = 1,\ldots, 2N_f$, in the fundamental representation. The low energy effective theory [12] contains antisymmetric mesons $M_{ij}$ and has superpotential $\text{Pf} M$. One can probe this theory by adding $x$-dependent mass terms $m^{\bar{q}}(x)Q_{\bar{q}}Q_p$ where $p = 1,\ldots, N_f$, $\bar{q} = (N_f + 1),\ldots, 2N_f$.\footnote{The same phenomenon occurs in softly broken supersymmetric theories with $SO(N_c)$ and $Sp(2k)$ gauge groups and fundamental quarks at $N_f$, $N_c$ and $k$ such that the Seiberg duality holds [11].}
In the theory without soft supersymmetry breaking, the induced charges match much in the same way as in the previous example: scalar mesons obtain the expectation values
\[ \langle M_{\tilde{q}p}(x) \rangle \propto m_{\tilde{q}p}^\dagger(x) \] which give \( x \)-dependent masses to fermionic mesons. After the soft scalar quark masses are introduced, the scalar potential of the low energy theory contains soft meson masses,
\[ m_{MM}^2 M^\dagger M \] where \( m_{MM}^2 > 0 \) at \( k > 1 \). At first sight, this ruins the induced charge matching at small \( m_0 \), as the ground state appears to be at \( M = 0 \) and no \( x \)-dependent masses of fermionic mesons seem to be generated. However, the symmetries of the theory allow for a linear supersymmetry breaking term in the scalar potential, \( m_Q f(m_Q^2, m_0) m M \), which shifts the ground state to \( \langle M \rangle \propto m^\dagger \) and in this way restores induced charge matching. Hence, we argue that this linear term is indeed generated in the low energy theory.

5. To conclude this paper, let us make two remarks. Let us come back to the adiabatic process leading to eq. (4), and again discuss induced baryon number as an example. Our first remark is that the same adiabatic process may be considered within the low energy effective theory. The induced baryon number is now related to the anomaly in the effective theory, provided all low energy degrees of freedom interacting with \( SU(N_0) \) gauge fields become massive upon introducing the mass \( m_0 \) to \( N_0 \) flavors of fundamental quarks. As the \( U(1)_B \times SU(N_0)_L \times SU(N_0)_L \) anomalies are the same in the fundamental and low energy theories, the induced baryon numbers match automatically in that case. Hence, a sufficient condition for induced charge matching is that no low energy degrees of freedom transforming non-trivially under a subgroup of the flavor group remain massless when this subgroup is explicitly broken by masses of some fermions of the fundamental theory. This property is certainly valid in supersymmetric theories where no phase transition is expected to occur as the masses of some of the flavors are changed from small to large values, i.e., where massive flavors smoothly decouple. On the other hand, this property does not seem to be guaranteed in non-supersymmetric models, though it is intuitively appealing and may well

\[ \text{Note that a term linear in } q \text{ is not possible in the scalar potential of the magnetic theory in the previous example, as the magnetic quarks carry magnetic color.} \]
be quite generic.

Second, in more complicated models there may be fermions in the fundamental theory that interact with background fields $A^L(x)$ and remain massless even after the masses $m_0$ are introduced to some of the flavors. In that case the adiabatic process discussed above does not necessarily end up in the ground state (e.g., because some energy levels of massless fermions cross zero). The precise nature of the final state becomes a matter of complicated dynamics, so the induced charge matching need not necessarily occur.

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