SU(3) Breaking of Leading–Twist K and K* Distribution Amplitudes – a Reprise

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Abstract:

We review the status of the leptonic decay constants \( f_K \) and \( f_K^{\parallel,\perp} \) of the \( K \) and \( K^\ast \), respectively, and the SU(3) breaking quantities \( a_1(K) \) and \( a_1^{\parallel,\perp}(K^\ast) \), the first Gegenbauer-moments of the leading-twist distribution amplitudes of \( K \) and \( K^\ast \). We obtain new predictions from QCD sum rules which are relevant for the calculation of \( K \) and \( K^\ast \) form factors, for instance \( T_1^{B\rightarrow K^\ast} \), which determines the decay \( B \rightarrow K^\ast \gamma \), and for QCD factorisation calculations of nonleptonic \( B \) decays into strange mesons, for instance \( B \rightarrow K\pi \).

submitted to Physics Letters B

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1 Introduction

Hadronic light-cone distribution amplitudes (DAs) of leading twist play an essential rôle in the QCD description of hard exclusive processes. DAs enter the amplitudes of processes to which collinear factorisation theorems apply and were first discussed in the seminal papers by Brodsky, Lepage and others [1]. More recently, collinear factorisation has been shown to apply, to leading order in an expansion in $1/m_b$, also to a large class of nonleptonic $B$ decays [2], which has opened a new and exciting area of applications of meson DAs. These decays, and in particular their CP asymmetries, are currently being studied at the B factories BaBar and Belle and are expected to yield essential information about the pattern of CP violation and potential sources of flavour violation beyond the SM. The aim of this letter is to provide a reanalysis of SU(3) breaking effects in leading-twist $K$ and $K^*$ DAs, using QCD sum rules. Our letter is both a sequel to and an extension of previous work reported in Refs. [3, 4, 5, 6, 7]. The results are of immediate relevance for all predictions of $B \to (K, K^*)$ decay processes calculated in QCD factorisation.

We define two-particle DAs as matrix elements of quark-antiquark gauge-invariant non-local operators at light-like separations $z_\mu$ with $z^2 = 0$. For definiteness we consider distributions of mesons with an $s$ quark and a light antiquark $\bar{q}$. To leading-twist accuracy, the complete set of distributions comprises three DAs (we use the notation $\hat{z} = z^\mu \gamma_\mu$ for arbitrary four-vectors $z$):

$$
\langle 0 | \bar{q}(z) \hat{z} \gamma_5[0,0] | K(q) \rangle = i f_K(qz) \int_0^1 du e^{-i\bar{u}(qz)} \phi_K(u),
$$

$$
\langle 0 | \bar{q}(z) \hat{z}[0,0] | K^*(q, \lambda) \rangle = (e^{(\lambda)} z) f_{K^*}^{\parallel} m_{K^*} \int_0^1 du e^{-i\bar{u}(qz)} \phi_{K^*}^{\parallel}(u),
$$

$$
\langle 0 | \bar{q}(z) \sigma_{\mu\nu}[0,0] | K^*(q, \lambda) \rangle = i (e_\mu^{(\lambda)} q_\nu - e_\nu^{(\lambda)} q_\mu) f_{K^*}^{\perp} \int_0^1 du e^{-i\bar{u}(qz)} \phi_{K^*}^{\perp}(u),
$$

with the Wilson-line

$$
[z,0] = \text{Pexp} \left[ ig \int_0^1 d\alpha z^\mu A_\mu(\alpha z) \right]
$$

inserted between quark fields to render the matrix elements gauge-invariant. In the above definitions, $e^{(\lambda)}$ is the polarization vector of a vector meson with polarisation $\lambda$; there are two leading-twist DAs for vector mesons, $\phi_{K^*}^{\parallel}$ and $\phi_{K^*}^{\perp}$, corresponding to longitudinal and transverse polarisation, respectively. The integration variable $u$ is the (longitudinal) meson momentum fraction carried by the quark, $\bar{u} \equiv 1 - u$ the momentum fraction carried by the antiquark. The normalisation constants $f_{K^*}^{\parallel,\perp}$ are defined by the local limit of Eqs. (1) and chosen in such a way that

$$
\int_0^1 du \phi(u) = 1
$$

for all three distributions $\phi_{K}, \phi_{K^*}^{\parallel}, \phi_{K^*}^{\perp}$.

The most relevant parameters characterising SU(3) breaking are the decay constants $f_K$ and $f_{K^*}^{\perp,\parallel}$, and the first (Gegenbauer) moments of the corresponding leading-twist DAs, $a_1(K)$.
and \( a_1^\perp(K^*) \), defined as
\[
a_1(K) = \frac{5}{3} \int_0^1 du (2u - 1) \phi_K(u).
\]
and correspondingly for \( a_1^{\parallel}(K^*) \). \( a_1 \) describes the difference of the average longitudinal momenta of the quark and antiquark in the two-particle Fock-state component of the meson, a quantity that vanishes for particles with equal-mass quarks (particles with definite G-parity). The decay constants \( f_K \) and \( f_K^\parallel \) can be extracted from experiment; \( f_K^\perp \) has been calculated both from lattice [8] and from QCD sum rules, e.g. Ref. [9]. Although the results are in mutual agreement, the QCD sum rule calculations are still subject to improvement as we shall discuss in this letter. The situation is much less clear with \( a_1 \): no lattice calculation of this quantity has been attempted yet, so essentially all available information on \( a_1 \) comes from QCD sum rule calculations. To date, three different types of sum rules have been used to calculate \( a_1 \):

- sum rules based on the correlation function of two biquark currents of equal chirality, so-called diagonal sum rules [3, 4, 6];
- sum rules based on the correlation function of two biquark currents of different chirality, so-called nondiagonal sum rules [3, 4, 5];
- exact operator identities relating \( a_1(K) \) and \( a_1^\parallel(K^*) \) to quark-quark-gluon matrix elements of the meson, which in turn are calculated from QCD sum rules [7].

The basic argument in favour of nondiagonal sum rules is that they are of first order in SU(3)-breaking quantities: the perturbative contribution is \( \sim O(m_s) \), the leading nonperturbative contribution \( \sim \langle \bar{q}q \rangle - \langle \bar{s}s \rangle \), whereas for diagonal correlation functions the corresponding contributions are \( \sim O(m_s^2) \) and \( m_s \langle \bar{s}s \rangle - m_u \langle \bar{u}u \rangle \), respectively. The original calculation of Chernyak and Zhitnitsky using a nondiagonal sum rule yielded \( a_1 \sim 0.1 \) [3, 4], but unfortunately suffers from a sign-mistake in the perturbative contribution. This mistake was corrected in Ref. [5], which however entails that the two leading contributions come with different sign and cancel to a large extent. As pointed out in Ref. [6, 7], the resulting sum rules are sensitive to poorly constrained higher-order perturbative and nonperturbative corrections and hence numerically unreliable. Alternative sum rules for \( a_1 \) come from diagonal correlation functions, a route that was followed, for \( a_1(K) \), in Ref. [6]. Yet another possibility to pin down the elusive \( a_1 \) is offered by exploiting exact operator identities that relate \( a_1 \) to quark-quark-gluon matrix-elements which in turn are calculated by QCD sum rules; the corresponding results for \( a_1(K) \) and \( a_1^\parallel(K^*) \) can be found in Ref. [7]. In this letter we derive and analyse diagonal sum rules for the decay constants \( f_K \) and all three Gegenbauer moments \( a_1(K) \) and \( a_1^{\parallel,\perp}(K^*) \). We plan to come back to the analysis of operator identities and the corresponding sum rules for quark-quark-gluon matrix elements in a future publication.

Our letter is organised as follows: in Sec. 2 we calculate and analyse QCD sum rules for \( f_K, f_K^\parallel, f_K^\perp \) and \( a_1(K), a_1^{\parallel,\perp}(K^*) \). We summarise and conclude in Sec. 3. The appendix contains some remarks about the calculation of diagonal sum rules.
QCD Sum Rules for $f_K^{(\parallel, \perp)}$ and $a_1^{(\parallel, \perp)}(K^\ast)$

QCD sum rules are an established method for the calculation of hadronic matrix elements, see Refs. [10, 11] for the original papers and a recent review. The key feature of the method is the use of analyticity to relate the local short-distance operator product expansion (OPE) of a correlation function of two currents,

$$\Pi = i \int d^4 y e^{i q y} \langle 0 | T J_1(y) J_2(0) | 0 \rangle = \sum_n C_n(q^2) \langle O_n \rangle \equiv \Pi^{\text{OPE}}$$  \hspace{1cm} (4)

around $y = 0$ (as opposed to a light-cone expansion around $y^2 = 0$, which is appropriate for form factor calculations, cf. [12]) valid for $Q^2 \equiv -q^2 \ll 0$, to its dispersion relation in terms of hadronic contributions,

$$\Pi = \int_0^\infty ds \frac{\rho(s)}{s - q^2 - i0} \equiv \Pi^{\text{had}}$$  \hspace{1cm} (5)

where $\rho(s)$ is the spectral density of the correlation function along its physical cut. The OPE yields a series of local operators of increasing dimension whose expectation values $\langle O_n \rangle$ in the nonperturbative (physical) vacuum are the so-called condensates. In the sum rules analysed in this letter, we take into account the condensates and parameters listed in Tab. 1. As for the strange quark mass, we would like to recall that with present data there is a hint of a discrepancy between unquenched $n_f = 2$ and $n_f = 2 + 1$ results, the latter ones favouring smaller values $m_s(2 \text{ GeV}) = (78 \pm 10) \text{ MeV}$ [13]. Awaiting the clarification of this situation, we choose to stay with the result from $n_f = 2$ flavours given in Tab. 1.

The representation of the correlation function in terms of hadronic matrix elements can be written as

$$\rho(s) = f \delta(s - m_M^2) + \rho^{\text{cont}}(s),$$

where $m_M$ is the mass of the lowest-lying state coupling to the currents $J_{1,2}$ and $\rho^{\text{cont}}$ parametrises all contributions to the correlation function apart from the ground state. $f$, the residue of the ground state pole, is the quantity to be determined. A QCD sum rule that allows one to do so is obtained by equating the representations (4) and (5) and implementing the following (model) assumptions:

| $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3 \text{ GeV}^3$ | $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$ |
| $\langle \bar{q}\sigma g G q \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$ | $\langle \bar{s}\sigma g G s \rangle = (0.8 \pm 0.1) \langle \bar{q}\sigma g G q \rangle$ |
| $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = 0.012 \text{ GeV}^4$ | |
| $m_s(2 \text{ GeV}) = (100 \pm 20) \text{ MeV}$ | $\Lambda_{\text{QCD}}^{(3)\text{NLO}} = 384 \text{ MeV}$ |
| $m_s(1 \text{ GeV}) = (137 \pm 27) \text{ MeV}$ | $\alpha_s(1 \text{ GeV}) = 0.534$ |

Table 1: Input parameters for sum rules at the renormalisation scale $\mu = 1 \text{ GeV}$. The value of $m_s$ is obtained from unquenched lattice calculations with $n_f = 2$ flavours as summarised in [13].
\( \rho_{\text{cont}} \) is approximated by the spectral density obtained from the OPE above a certain threshold, i.e. \( \rho_{\text{cont}} \rightarrow \rho_{\text{OPE}}(s) \theta(s - s_0) \) with \( s_0 \approx (m_M + \Delta)^2 \) being the continuum threshold, where \( \Delta \sim O(\Lambda_{\text{QCD}}) \) is an excitation energy to be determined within the method. This assumption relies on the validity of semiglobal quark-hadron duality;

- instead of the weight-functions \( 1/(q^2)^n \) and \( 1/(s - q^2) \), one uses different weight-functions which are optimised to (exponentially) suppress effects of \( \rho(s) \) for large values of \( s \) and at the same time also suppress higher-dimensional condensates by factorials. This is achieved by Borel transforming the correlation function: \( B 1/(s - q^2) = 1/M^2 \exp(-s/M^2) \). A window of viable values of the Borel parameter \( M^2 \) and the continuum threshold \( s_0 \) has to be determined within the method itself by looking for a maximum region of minimum sensitivity (a plateau) in both \( M^2 \) and \( s_0 \);

- the OPE of \( \Pi \) can be truncated after a few terms. As is well known, this condition is fulfilled only for low moments, whereas for higher moments of the DAs in \((u - \bar{u})\) the nonperturbative terms become dominant.

After subtraction of the integral over \( \rho_{\text{OPE}} \) above \( s_0 \) from both sides, the final sum rule reads

\[
\mathcal{B}_{\text{sub}} \Pi^{\text{OPE}} \equiv \frac{1}{M^2} \int_0^{s_0} ds \, e^{-s/M^2} \rho_{\text{OPE}}(s) = \frac{f}{M^2} e^{-m^2_\Pi/M^2},
\]

which gives the hadronic quantity \( f \) as a function of the Borel parameter \( M^2 \) and the continuum threshold \( s_0 \) (and the condensates and short-distance parameters from the OPE).

We determine \( f_\parallel^{(\|,\perp)}(K^{(s)}) \) and \( a_1^{(\|,\perp)}(K^{(s)}) \) from the diagonal correlation function

\[
i \int d^4 y e^{i q y} \langle 0 | T \bar{q}(y) \Gamma s(y) \bar{s}(0) \Gamma [0, z] q(z) | 0 \rangle,
\]

where \( z_\mu \) is light-like and the Dirac structures \( \Gamma \) are given by

\[
K : \quad \Gamma = \hat{z} \gamma_5, \quad K^*_\parallel : \quad \Gamma = \hat{z}, \quad K^*_\perp : \quad \Gamma = \sigma_{\mu\nu} z^\nu.
\]

The calculation with nonlocal operators is very convenient, as it allows one to calculate all moments in one go. Specifying for instance to \( K^*_\parallel \), the sum rule reads

\[
\mathcal{B}_{\text{sub}} \Pi^{\text{OPE}} = (f_K^{\|})^2 e^{-m^2_{K^*}/M^2} \frac{1}{M^2} \int_0^1 du \, e^{i u(q z)} \phi_K^\parallel(u),
\]

where also \( \Pi^{\text{OPE}} \) is expressed as integral over \( u \), which naturally emerges as a Feynman parameter in the calculation, and comes with the same weight function \( \exp(i u(q z)) \). Sum rules for \( f_K \) are obtained as the lowest order in an expansion in \( q z \), those for \( a_1 \) by effectively replacing

\[
e^{i u(q z)} \rightarrow \frac{5}{3}(u - \bar{u}).
\]

For \( f_K^{\|} \) we find the following sum rules:

\[
f_K^2 e^{-m^2_{K^*}/M^2} = \text{SR}_+, \quad (f_K^{\|})^2 e^{-m^2_{K^*}/M^2} = \text{SR}_-,
\]
with \( SR_{\pm} = \frac{1}{4\pi^2} \int_{m_s^2}^{s_0} ds \, e^{-s/M^2} \left( \frac{s - m_s^2}{s^3} (s + 2m_s^2) + \frac{\alpha_s}{\pi} \frac{M^2}{4\pi^2} \left( 1 - e^{-s_0/M^2} \right) \right) \)

\[ + \frac{m_s \langle \bar{s}s \rangle}{M^2} \left( 1 + \frac{m_s^2}{3M^2} + \frac{13}{9} \frac{\alpha_s}{\pi} \right) + \frac{1}{12M^2} \left( \frac{\alpha_s}{\pi} G^2 \right) \left( 1 + \frac{1}{3} \frac{m_s^2}{M^2} \right) \]

\[ + \frac{4}{3} \frac{\alpha_s}{\pi} \frac{m_s \langle \bar{q}q \rangle}{M^2} \pm \frac{16\pi\alpha_s}{9M^4} \langle \bar{q}q \rangle \langle \bar{s}s \rangle + \frac{16\pi\alpha_s}{81M^4} \left( \langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2 \right). \] (9)

This sum rule was already given in Ref. [14], apart from the radiative and mass-corrections to the quark-condensate contribution which are new. The values of \( f_K \) and \( f_K^\parallel \) obtained from these sum rules are shown in Fig. 1, evaluating all scale-dependent quantities at the scale \( \mu = 1 \text{ GeV} \). The sum rule results agree very well with the experimental values [15]

\[ f_K = (0.160 \pm 0.002) \text{ GeV}, \quad f_K^\parallel = (0.217 \pm 0.005) \text{ GeV}. \]

For \( f_K \) the situation is slightly more subtle as the correlation function (7) contains contributions not only from the vector meson \( K^* \), but also from the axial-vector meson \( K_1 \). The same situation occurs for \( \rho \) and \( b_1(1235) \). In Ref. [16], where QCD sum rules for \( f_\rho^\perp \) were studied, it was argued that one can either explicitly include the contribution of \( b_1 \) in the hadronic parametrisation of this “mixed-parity” sum rule and use a suitably large value of the continuum threshold \( s_0 \approx 2.1 \text{ GeV}^2 \) or include it in the continuum and use a smaller value \( s_0 \approx 1.0 \text{ GeV}^2 \). Both procedures yield a stable sum rule and \( f_\rho^\perp(1 \text{ GeV}) \approx 160 \text{ MeV} \). For \( K^* \), however, the mixed-parity sum rule without \( K_1 \) does not display a stable plateau in \( M^2 \), which means one has to include the contribution of \( K_1 \) explicitly.\(^1\) There are actually two strange axial-vector mesons, \( K_1(1270) \) and \( K_1(1400) \), which are usually interpreted as mixture of a \( 3P_1 \) state, the \( K_a \), and a \( 1P_1 \) state, the \( K_b \) [17, 18]:

\[ K_1(1270) = K_a \cos \theta_K - K_b \sin \theta_K, \]

\[ K_1(1400) = K_a \sin \theta_K + K_b \cos \theta_K. \]

The results of Refs. [17, 18] indicate that the system is close to ideal mixing, i.e. \( \theta_K \approx 45^\circ \). To the accuracy needed in our sum rules it is then sufficient to replace the two resonances by one effective one with the mass \( m_{K_1} = 1.34 \text{ GeV} \) [18]. The mixed-parity sum rule obtained from the correlation function (7) now reads:

\[ (f_K^\perp)^2 e^{-m_{K^*}^2/M^2} + (f_{K_1}^\perp)^2 e^{-m_{K_1}^2/M^2} = \]

\[ \frac{1}{4\pi^2} \int_{m_s^2}^{s_0} ds \, e^{-s/M^2} \left( \frac{s - m_s^2}{s^3} (s + 2m_s^2) + \frac{\alpha_s}{\pi} \frac{M^2}{4\pi^2} \left( 1 - e^{-s_0/M^2} \right) \right) \]

\[ + \frac{m_s \langle \bar{s}s \rangle}{M^2} \left( 1 + \frac{m_s^2}{3M^2} + \frac{19}{9} \frac{\alpha_s}{\pi} \right) + \frac{2}{3} \left( 1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei}\left(-\frac{s_0}{M^2}\right)\right) \]

\(^1\)\( K_1 \) also contributes to the sum rule for \( f_K \), but can be safely absorbed into the continuum, as \( m_{K_1} \gg m_K \).
\[-\frac{1}{12M^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle \left\{ 1 - \frac{2m_s^2}{M^2} \left( \frac{7}{6} - \gamma_E + \text{Ei} \left( -\frac{s_0}{M^2} \right) - \ln \frac{m_s^2}{s_0} \left( 1 - \frac{M^2}{s_0} \right) e^{-s_0/M^2} \right) \right\}
\]
\[-\frac{1}{3M^4} m_s \langle \bar{s}\sigma g G s \rangle - \frac{32\pi\alpha_s}{81M^2} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2). \]  

(10)

The value of \( f_{\perp K_1}^+ \) itself can be obtained from a “pure-parity” sum rule which can be extracted from the correlation function (7) leaving one index uncontracted; explicit expressions are given in Ref. [14]. As is well-known, the diadvantage of pure-parity sum rules is that they come with a higher mass-dimension which increases the dependence of the result on the continuum model, which is however acceptable as long as we only need an estimate of \( f_{\perp K_1}^+ \) for use in Eq. (10). The pure-parity sum rule for \( f_{\perp K_1}^+ \), including SU(3)-breaking corrections, reads [14]:

\[
(f_{\perp K_1}^+)^2 m_{K_1}^2 e^{-m_{K_1}/M^2} = \frac{1}{8\pi^2} \int_0^{s_0} ds e^{-s/M^2} \left( \frac{s - m_s^2}{s^2} \right)^2 \left( s + 2m_s^2 \right) + \frac{1}{8\pi^2} \int_0^{s_0} ds e^{-s/M^2} \frac{\alpha_s}{\pi} \left( \frac{7}{9} + \frac{2}{3} \ln \frac{s}{\mu^2} \right) + \frac{32\pi\alpha_s}{81M^2} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2) - \frac{16\pi\alpha_s}{9M^2} \langle \bar{q}q \rangle \langle \bar{s}s \rangle
\]
\[+ m_s \langle \bar{q}q \rangle - \frac{m_s \langle \bar{s}s \rangle}{2} + \frac{1}{24M^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{1}{3M^2} m_s \langle \bar{s}\sigma g G s \rangle - \frac{1}{6M^2} m_s \langle \bar{q}\sigma g G q \rangle. \]  

(11)

Again all scale-dependent quantities are evaluated at \( \mu = 1 \text{ GeV} \). The results for \( f_{\perp K_1}^+ \) are shown in Fig. 2(a), from which we conclude

\[ f_{\perp K_1}^+ (1 \text{ GeV}) = (0.185 \pm 0.010) \text{ GeV}. \]  

(12)

This result is slightly smaller than, but still in agreement with, the one obtained in Ref. [19]. Using (12) as input in (10), we obtain the values for \( f_{\perp K}^+(1 \text{ GeV}) \) shown in Fig. 2(b), yielding

\[ f_{\perp K}^+ (1 \text{ GeV}) = (0.185 \pm 0.010) \text{ GeV}. \]  

(13)

The value of \( f_{\perp K}^+ \) at different scales can be obtained from the leading-order renormalisation-group improved relation

\[ f_{\perp K}^+(\mu) = f_{\perp K}^+(1 \text{ GeV}) \left( \frac{\alpha_s(\mu)}{\alpha_s(1 \text{ GeV})} \right)^{4/(3\beta_0)}. \]

The result (13) has to be compared with \((0.170 \pm 0.010) \text{ GeV}\) quoted in Ref. [9]. The main difference is that the sum rule (10) includes, in addition to the new terms in \( \alpha_s m_s \langle \bar{s}s \rangle \) and \( m_s^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \), in particular the contribution of \( f_{\perp K_1}^+ \), which allows one to obtain a stable plateau in \( M^2 \). Once we know the value of \( f_{\perp K}^+ \), we can now determine the continuum threshold to be used when \( K_1 \) is included in the continuum, which will be relevant for the determination of \( a_{\perp}^+(K^*) \). Fig. 3 shows that \( s_0 = (1.3 \pm 0.1) \text{ GeV}^2 \) is the appropriate value to use if the contribution of \( K_1 \) is not made explicit.
Figure 1: (a) $f_K$ as function of the Borel parameter $M^2$ for $s_0 = 1.1$ GeV$^2$. Solid line: central values of input parameters, dashed lines: variation of $f_K$ within the allowed range of input parameters. (b): the same for $f_K^\parallel$ with $s_0 = 1.7$ GeV$^2$.

Figure 2: (a) $f_K^\perp(1$ GeV) from Eq. (11) as function of the Borel parameter $M^2$ for $s_0 = (2.7, 2.9, 3.1)$ GeV$^2$ (solid lines from bottom to top). (b) $f_K^\perp(1$ GeV) from Eq. (10), using $f_{K_1}^\perp$ as input. Solid lines: $f_{K_1}^\perp = 0.180$ GeV and, from bottom to top, $s_0 = (2.3, 2.45, 2.6)$ GeV$^2$. Dashed lines: $f_{K_1}^\perp = 0.170$ GeV (top) and $f_{K_1}^\perp = 0.190$ GeV (bottom). Note that the optimum $s_0$ for $f_{K_1}^\perp$ is larger than for $f_K^\perp$, in agreement with the resonance structure in the $1^+$ and $1^-$ channels.

Figure 3: $f_K^\perp(1$ GeV) from Eq. (10), setting $f_{K_1}^\perp = 0$. From bottom to top: $s_0 = (1.2, 1.3, 1.4)$ GeV$^2$. 


The value for $a_1(K, 1 \text{ GeV})$ agrees with the one obtained in Ref. [6], although our uncertainty is slightly larger. $a_{1\|}(K^*)$ is smaller than $a_1(K)$, which follows from the fact that the right-hand sides of the sum rules (14) are essentially the same, except for the values of $s_0$, and one term which gives a positive contribution to $a_1(K)$, but a negative to $a_{1\|}(K^*)$. Since $f_{K\|} > f_K$, and the sensitivity of the sum rule on $s_0$ is small, one clearly expects $a_{1\|}(K^*) < a_1(K)$. Both results, however, markedly disagree with those obtained in Refs. [4, 5]. In Sec. 1 we have already mentioned the reasons for this discrepancy: in Ref. [5] a different set of sum rules, nondiagonal sum rules and a chirally odd correlation function, were used which exhibit large cancellations between the dominant terms. As discussed in Ref. [6, 7], these sum rules are numerically not reliable. Eq. (14) is free of such cancellations and hence expected to be more reliable. On the other hand, $a_1(K)$ and $a_{1\|}(K^*)$ in (15) are also smaller than the original results of Chernyak and Zhitnitsky [4]. This is due to the fact that in the corresponding nondiagonal sum rule Eq. (6.27) in Ref. [4], which contains no radiative corrections, the perturbative term has the wrong sign. Correcting the sign, and using the standard values of input parameters from Tab. 1, we obtain the results shown in Fig. 5, which are remarkably
Figure 4: (a) $a_1(K)$ as function of the Borel parameter $M^2$ for $s_0 = 1.1\text{ GeV}^2$ and $\mu = 1\text{ GeV}$. Solid line: central values of input parameters, dashed lines: variation of $a_1(K)$ within the allowed range of input parameters. (b): Same for $a_1^\parallel(K^*)$ with $s_0 = 1.7\text{ GeV}^2$.

Figure 5: $a_1(K)$ from the (tree-level) nondiagonal sum rule (6.27) in Ref. [4] after the correction of a sign-mistake in the perturbative contribution. $s_0 = (1.0, 1.2, 1.4)\text{ GeV}^2$ (from top to bottom).

Figure 6: $a_1^\perp(K^*, 1\text{ GeV})$ as function of the Borel parameter $M^2$ for $\mu = 1\text{ GeV}$. Solid line: central values of input parameters, dashed lines: variation of $a_1^\perp(K^*)$ within the allowed range of input parameters and $s_0 = (1.3 \pm 0.1)\text{ GeV}^2$. 
close to our result (15) from the diagonal sum rule. Unfortunately, once radiative corrections are included in the nondiagonal sum rule, the agreement with (15) is lost and one is back to the results obtained in Ref. [5] with negative \( a_1 \).

Let us finally turn to \( a_1^\perp(K^*) \). As with the decay constant, the sum rule obtained from the correlation function (7) contains contributions from the \( K_1 \), which in principle need to be subtracted. Based on the experience with the sum rule for \( f_K^\perp \), however, we decide to include these contributions in the continuum and use the continuum threshold \( s_0 = (1.3 \pm 0.1) \text{GeV}^2 \) determined from Fig. 3. The sum rule for \( a_1^\perp(K^*) \) reads

\[
a_1^\perp(K^*)(f_K^\perp)^2 e^{-m_{K^*}^2/M^2} = \frac{5}{4\pi^2} m_s^4 \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s - m_s^2)^2}{s^4} + \frac{10}{9} \frac{m_s\langle \bar{s}\sigma g G s \rangle}{M^4} \\
+ \frac{5m_s^2}{9M^4} \left( \frac{\alpha_s}{\pi} G^2 \right) \left( \frac{1}{4} + \gamma_E - Ei \left( -\frac{s_0}{M^2} \right) \right) + \ln \frac{m_s^2}{s_0} \left( \frac{M^2}{s_0} - 1 \right) e^{-s_0/M^2} \\
- \frac{5}{3} \frac{m_s\langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{m_s^2}{M^2} + \frac{\alpha_s}{\pi} \left[ -\frac{46}{9} + \frac{4}{3} \left( 1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + Ei \left( -\frac{s_0}{M^2} \right) \right) \right]\right\} \tag{16}
\]

and is plotted in Fig. 6. We find

\[
a_1^\perp(K^*, 1 \text{GeV}) = 0.04 \pm 0.03. \tag{17}
\]

3 Summary and Conclusions

In this letter we have calculated the decay constants \( f_K \) and \( f_K^{\parallel,\perp} \) of the \( K \) and \( K^* \) meson, respectively, and the first Gegenbauer moments of the leading-twist DAs, \( a_1(K) \) and \( a_1^{\parallel,\perp}(K^*) \). We find values for \( f_K \) and \( f_K^{\parallel} \) in agreement with experiment and

\[
f_K^\perp(1 \text{GeV}) = (0.185 \pm 0.010) \text{GeV}, \quad a_1(K, 1 \text{GeV}) = 0.050 \pm 0.025,
\]

\[
a_1^{\parallel}(K^*, 1 \text{GeV}) = 0.025 \pm 0.015, \quad a_1^{\perp}(K^*, 1 \text{GeV}) = 0.04 \pm 0.03.
\]

The value for \( f_K^\perp \) agrees, within uncertainties, with that found from lattice calculations [8]. The value of \( a_1(K) \) agrees with that found in Ref. [6], using the same method. \( a_1(K) \) and \( a_1^{\parallel,\perp}(K^*) \) disagree with the negative values found in Ref. [5] which is due to numerical instabilities of the nondiagonal sum rules used in that paper. \( a_1(K) \) and \( a_1^{\parallel}(K^*) \) are also smaller, by roughly a factor two, than the results obtained in Ref. [7] from exact operator relations between \( a_1(K) \) and \( a_1^{\parallel}(K^*) \) and quark-quark-gluon matrix elements. Whereas it seems rather unlikely that an increase in the accuracy of the sum rules for \( a_1 \), Eqs. (14), (16), by including corrections in \( m_s^2\alpha_s \) and \( m_s\alpha_s\langle \bar{q}\sigma g G q \rangle \) will change the results for \( a_1 \) by a factor two, the situation may be different with the sum rules for the quark-quark-gluon matrix elements employed in Ref. [7], which only contain terms in \( \sim m_s \). We plan to come back to this question in a separate publication.
Acknowledgements

R.Z. is supported in part by the EU-RTN Programme, Contract No. HPEN-CT-2002-00311, “EURIDICE”.

A Some Details of the Calculation

In calculating the contribution of the strange quark condensate, one has to include the first nonlocal term in the expansion of the quark condensate:

\[
\langle 0 | \bar{s}_i(x) s_j(y) | 0 \rangle = \frac{1}{12} \langle \bar{s}s \rangle \delta_{ij} \left( \delta_{\alpha\beta} + \frac{i}{D} m_s(x_\kappa - y_\kappa) (\gamma^\kappa)^{\beta\alpha} \right)
\]

\[
= \frac{1}{12} \langle \bar{s}s \rangle \delta_{ij} \left( \delta_{\alpha\beta} + \frac{1}{D} m_s \frac{\partial}{\partial Q_\kappa} (\gamma^\kappa)^{\beta\alpha} \right) e^{-iQ(y-x)} \bigg|_{Q=0}, \quad (A.1)
\]

where \(i, j\) are colour, \(\alpha, \beta\) spinor indices, and \(D\) is the number of dimensions; \(Q\) is an auxiliary momentum. It is the second term in (A.1) that causes a slight complication in the calculation of radiative corrections in the form of finite counter terms. Their origin is twofold: first, the factor \(1/D\) induces \(O(\epsilon)\) contributions at tree-level which cause finite counter terms upon renormalisation. Second, if the derivative in \(Q_\kappa\) yields a term \(\gamma_\kappa\) in the trace, the contraction over \(\kappa\) can also yield finite terms in the counter term, which indeed happens for the vertex correction diagrams. It appears that these finite counter terms have been missed by the authors of Ref. [6], for we reproduce the results in their appendix using (A.1) and dropping just the divergent terms.

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