Transfer of angular momentum from guided light to an atom near a vacuum-clad ultrathin optical fiber

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We calculate the axial orbital and spin torques of guided light on a two-level atom near a vacuum-clad ultrathin optical fiber. We show that the generation of these torques is governed by the angular momentum conservation law and is in agreement with the Minkowski angular momentum formulation. We find that, due to the interaction between the atom and guided light, photon angular momentum and atomic spin angular momentum can be converted into atomic orbital angular momentum.

It is well known that, like energy, linear and angular momenta can be transferred from light to atoms, molecules, and material particles and vice versa. For the field in a dielectric medium, several formulations for the linear and angular momentum densities remain contentious because the debate has not been settled by experiments. Depending on specific situations, one of the forms of momentum appears as the natural, experimentally observed momentum.

Angular momentum of guided light has been calculated in the Abraham \cite{6, 7} and Minkowski \cite{6, 8} formulations. It has been shown that the Abraham angular momentum of a photon in a quasicircularly polarized guided mode with an azimuthal mode order \( l \) increases with increasing \( l \) \cite{7} but is different from \( l \hbar \) \cite{6, 7}. Meanwhile, it has been pointed out that the Minkowski angular momentum per photon is quantized to be exactly equal to \( l \hbar \) \cite{6, 8}. The transfer of angular momentum of a scalar paraxial light beam to a particle in free space has been studied \cite{4}.

Recently, the force of guided light on an atom near a vacuum-clad ultrathin optical fiber has been investigated \cite{10}. The azimuthal component of the force leads to an axial torque and consequently to a transfer of angular momentum from guided light to the orbital motion of the atom.

In this Letter, we study the transfer of angular momentum from guided light to a two-level atom near a vacuum-clad ultrathin optical fiber.

We consider a two-level atom driven by a near-resonant classical field with optical frequency \( \omega_c \) and envelope \( \mathcal{E} \) near a vacuum-clad ultrathin fiber (see Fig. 1). The atom has an upper energy level \( |e \rangle \) and a lower energy level \( |g \rangle \), with energies \( \hbar \omega_c \) and \( \hbar \omega_g \). The atomic transition frequency is \( \omega_0 = \omega_c - \omega_g \). The fiber is a dielectric cylinder of radius \( a \) and refractive index \( n_1 > 1 \) and is surrounded by an infinite background vacuum or air medium of refractive index \( n_2 = 1 \). We use Cartesian coordinates \( \{x, y, z\} \), where \( z \) is the coordinate along the fiber axis, and also cylindrical coordinates \( \{r, \varphi, z\} \), where \( r \) and \( \varphi \) are the polar coordinates in the transverse plane \( xy \).

The atom interacts with the classical driving field \( \mathcal{E} \) and the quantum electromagnetic field. The quantum field in the presence of the fiber can be decomposed into the contributions from guided and radiation modes \cite{11}. In view of the very low losses of silica, we neglect material absorption. The Hamiltonian for the atom-field interaction in the dipole approximation is \cite{10}

\[
H_{\text{int}} = -\frac{\hbar}{2} \Omega \sigma_{eg} e^{-i(\omega_c - \omega_0)t} - i\hbar \sum_{\alpha} G_{\alpha} \sigma_{eg} a_\alpha e^{-i(\omega - \omega_0)t} \\
- i\hbar \sum_{\alpha} \tilde{G}_{\alpha} \sigma_{ge} a_\alpha e^{-i(\omega + \omega_0)t} + \text{H.c.,} \tag{1}
\]

where \( \sigma_{ij} = |i \rangle \langle j | \) with \( i, j = e, g \) are the atomic operators, \( a_\alpha \) and \( a_\alpha^\dagger \) are the photon operators, \( \Omega = \d_{eg} \mathcal{E}/\hbar \) is the Rabi frequency of the driving field, with \( \d = \d_{eg} = \langle e|\mathbf{D}|g \rangle \) being the matrix element of the atomic dipole operator \( \mathbf{D} \), and \( G_{\alpha} \) and \( \tilde{G}_{\alpha} \) are the coupling coefficients for the interaction between the atom and the quantum field \cite{10}. The notations \( \alpha = \mu, \nu \) and \( \sum_{\alpha} = \sum_\mu + \sum_\nu \) stand for the mode index and the mode summation. The index \( \mu = (\omega Nfp) \) labels guided modes, where \( \omega \) is the mode frequency, \( N = \text{HE}_{1m}, \text{EH}_{1m}, \text{TE}_{0m}, \) or \( \text{TM}_{0m} \) is the mode type, with \( l = 1, 2, \ldots \) and \( m = 1, 2, \ldots \) being the azimuthal and radial mode orders, \( f = \pm 1 \) denotes the forward or backward propagation direction along the fiber axis \( z \), and \( p = \pm 1 \) for HE and EH modes and 0 for TE and TM modes is the phase cir-
culation direction index [11]. The longitudinal propagation constant \( \beta \) of a guided mode is determined by the fiber eigenvalue equation. The index \( \nu = (\omega \beta p) \) labels radiation modes, where \( \beta \) is the longitudinal propagation constant, \( l = 0, \pm 1, \pm 2, \ldots \) is the mode order, and \( p = +, - \) is the mode polarization index. The notations \( \sum_{\mu} = \sum_{N_f p} \int_{0}^{\infty} d\omega \) and \( \sum_{\nu} = \sum_{l p} \int_{0}^{\infty} d\omega \int_{-\infty}^{\infty} d\beta \) denote the summations over guided and radiation modes.

In a semiclassical treatment, the center-of-mass motion of the atom is governed by the force \( F = -(\nabla H_{\text{int}}) \) [12, 13]. The azimuthal force component \( F_\varphi \) is responsible for the rotational motion of the atom around the fiber axis. The axial component of the orbital torque is \( T_z = r F_\varphi \). This torque component characterizes the rate of the change of the axial component of the orbital angular momentum of the atom. When we use the result of Ref. [10] for \( F_\varphi \), we find the following expression for \( T_z \):

\[
T_z = T_e^{(d_r)} + \rho_{ee} T_s^{(spon)} + T_{z}^{(vdW)e} + \rho_{g} T_{z}^{(vdW)g} \tag{2}
\]

Here, \( \rho_{ij} = \langle i | \rho | j \rangle \) with \( i, j = e, g \) are the matrix elements of the density operator \( \rho \) for the atomic internal state. The term \( T_{z}^{(d_r)} \) is the axial torque component resulting from the driving field and is given as

\[
T_{z}^{(d_r)} = \frac{\hbar}{2} \left( \rho_{ge} \frac{\partial \Omega}{\partial \varphi} + \rho_{eg} \frac{\partial \Omega^*}{\partial \varphi} \right) \tag{3}
\]

The term \( T_{z}^{(spon)} \) is the axial torque component produced by the recoil of spontaneous emission of the atom in the excited state \( |e\rangle \) and is given as

\[
T_{z}^{(spon)} = i \pi \hbar \sum_{\alpha_0} \left( G_{\alpha_0}^{e} \frac{\partial G_{\alpha_0}^{ee}}{\partial \varphi} + G_{\alpha_0}^{ee} \frac{\partial G_{\alpha_0}^{e}}{\partial \varphi} \right) \tag{4}
\]

where \( \alpha_0 \) denotes resonant guided modes and resonant radiation modes with the frequency \( \omega = \omega_0 \). Note that \( T_{z}^{(scat)} = \rho_{ee} T_{z}^{(spon)} \) is the axial torque component produced by the recoil of the photons that are scattered from the atom with the excited-state population \( \rho_{ee} \). The terms \( T_{z}^{(vdW)e} = -\partial U_e / \partial \varphi \) and \( T_{z}^{(vdW)g} = -\partial U_g / \partial \varphi \) are the axial torques resulting from the van der Waals potentials \( U_e \) and \( U_g \) for the excited and ground states. The potentials \( U_e \) and \( U_g \) are calculated by using the van der Waals potentials \( U_e \) and \( U_g \) for the excited and ground states. The potentials \( U_e \) and \( U_g \) are calculated by using the van der Waals potentials \( U_e \) and \( U_g \) for the excited and ground states.

It is clear that \( T_{z}^{(d_r)} \) is produced by the force \( F_{z}^{(d_r)} = T_{z}^{(d_r)} / r = (p_c l_c - q) \hbar (\Gamma_{\rho_{ee}} + \rho_{ee}) / r \), which is the azimuthal pressure force component.

Equation (4) describes the exchange of angular momentum between the guided driving field and the atom in the excitation process. According to [1, 2], the canonical angular momentum of a photon in the guided driving field in the Minkowski formulation is \( p_c l_c \hbar \). The change of the spin angular momentum of the atom due to an upward transition is \( q \hbar \). The scattering rate is \( \Gamma_{\rho_{ee}} \) and the upward-transition (photon-absorption) rate is \( \Gamma_{\rho_{ee}} + \rho_{ee} \). Then, it is clear from Eq. (7) that the angular momentum of absorbed guided photons is converted into the orbital and spin angular momenta of the atom. Thus, Eq. (7) is in agreement with the conservation of the total angular momentum of the atom-field system. Moreover, Eq. (7) is in agreement with the Minkowski formulation of angular momentum of light. Our result is consistent with the results of Refs. [14, 19].

The axial spin torque produced by the driving field is

\[
T_{z}^{(d_r)} = q \hbar (\Gamma_{\rho_{ee}} + \rho_{ee}) \tag{8}
\]
We have $T_z^{(drv)} + T_z^{(spon)} = p_{c} l_{c} \hbar (\rho_{ee} + \dot{\rho}_{ee})$. Note that Eq. (3) can be derived from the definition $\mathbf{T}^{(drv)} = (1/2)\text{Re}(\mathbf{D}^* \times \mathbf{E})$ for the spin torque on an oscillating electric dipole [21], where $\mathbf{D}$ is the envelope of the dipole positive-frequency component. Indeed, for the two-level atom considered here, we have $\mathbf{D} = 2d^* \rho_{eg}$. In the case where $d$ has a single nonzero spherical tensor component $d_q$, we have $T_z^{(drv)} = -q \hbar \text{Im}(\rho_{qe})$, which leads to Eq. (3). We find $T_z^{(drv)} = q/(p_{c} l_{c} - q)$. We can show that $T_z^{(drv)}/T_z^{(spon)} \neq j_z^{(\text{spin})}/j_z^{(\text{orb})}$, where $j_z^{(\text{spin})}$ and $j_z^{(\text{orb})}$ are the spin and orbital parts of the Minkowski angular momentum of guided light [8]. This means that the spin and orbital angular momenta of light are not transferred separately to the spin and orbital angular momenta of the two-level atom, unlike the case of small isotropic particles in free space [8].

We note that, for $l_c \geq 1$ and $q = p_c$, we have $(p_{c} l_{c} - q) \hbar = p_{c} (l_{c} - 1) \hbar$. In this case, Eq. (4) indicates that the photon angular momentum is converted into the atomic spin and orbital angular momenta. For $l_c \geq 1$ and $q = -p_c$, we have $(p_{c} l_{c} - q) \hbar = p_{c} (l_{c} + 1) \hbar$. In this case, Eq. (4) says that the orbital angular momentum and the change of the atomic spin angular momentum have the same sign and add up in generating the atomic orbital angular momentum. For $l_c \geq 1$ and $q = 0$, the total photon angular momentum is converted into the atomic orbital angular momentum.

Equation (4) can be used for not only hybrid modes ($l_c \geq 1$) but also TE and TM modes ($l_c = 0$). In the cases of TE and TM modes, we have $T_z^{(drv)} = -q \hbar (\rho_{ee} + \dot{\rho}_{ee})$, which indicates that the atomic orbital angular momentum can be generated from the atomic spin angular momentum through the interaction with a photon in a TE or TM mode having no angular momentum.

The conversion of atomic spin angular momentum into atomic orbital angular momentum via the interaction with a guided photon is possible because the guided mode of the atom at rest and in the steady-state regime. The dipole matrix-element vector $d$ has only one nonzero spherical tensor component $d_q$, where $q = 1, 0$, and $-1$. The power and detuning of the driving field are chosen to be $P = 1 \text{pW}$ and $\Delta = 0$. The fiber radius is $a = 350 \text{nm}$. The dipole magnitude $d$ corresponds to the natural linewidth $\gamma_0/2\pi = 6.065 \text{MHz}$ of the $D_2$ line of a $^{87}$Rb atom. The wavelength of the atomic transition is $\lambda_0 = 780 \text{nm}$. The refractive indices of the fiber and the vacuum cladding are $n_1 = 1.4537$ and $n_2 = 1$, respectively.

We now calculate the axial orbital torque $T_z^{(spon)}$ produced by the recoil of spontaneous emission. The expressions for the coupling coefficients $G_{\alpha \mu \nu}$ are given in Ref. [10]. In the case where a single spherical tensor component $d_q$ of the dipole matrix element vector $d$ is nonzero, we find $\partial G_{\alpha \mu}/\partial \varphi = i (pl - q) G_{\mu}$ and $\partial G_{\nu}/\partial \varphi = i (l - q) G_{\nu}$. In this case, the axial component of the orbital torque of spontaneous emission recoil is found from Eq. (4) to be

$$T_z^{(spon)} = q \hbar \Gamma - \hbar \sum_{\mu_0} p_l \gamma_{\mu_0} - \hbar \sum_{\nu_0} t_{\nu_0} \gamma_{\nu_0},$$

(9)

where $\gamma_{\mu_0} = 2\pi |G_{\mu_0}|^2$ and $\gamma_{\nu_0} = 2\pi |G_{\nu_0}|^2$ are the rates of spontaneous emission into guided and radiation modes [20].

Equation (9) describes the exchange of angular momentum between the quantum field and the atom in the spontaneous emission process. Indeed, the angular momentum of a photon emitted into a guided mode $\mu = (\omega N f p)$ or a radiation mode $\nu = (\omega 3\beta p)$ is $\hbar p_l$ or $\hbar t$, respectively, and the change of the spin angular momentum of the atom due to a downward transition is $-q \hbar$. Then, it is clear from Eq. (9) that the angular momentum of re-emitted photons is converted into the atomic spin and orbital angular momenta. Thus, we observe again the conservation of the total angular momentum of the

![FIG. 2. (Color online) Radial dependencies of the orbital and spin driving-field torques $T_z^{(drv)}$ (a) and $T_z^{(derv)}$ (b) on the atom being at rest and in the steady-state regime. The dipole matrix-element vector $d$ has only one nonzero spherical tensor component $d_q$, where $q = 1, 0$, and $-1$. The power and detuning of the driving field are chosen to be $P = 1 \text{pW}$ and $\Delta = 0$. The fiber radius is $a = 350 \text{nm}$. The dipole magnitude $d$ corresponds to the natural linewidth $\gamma_0/2\pi = 6.065 \text{MHz}$ of the $D_2$ line of a $^{87}$Rb atom. The wavelength of the atomic transition is $\lambda_0 = 780 \text{nm}$. The refractive indices of the fiber and the vacuum cladding are $n_1 = 1.4537$ and $n_2 = 1$, respectively.](image-url)

We have $T_z^{(drv)} + T_z^{(spon)} = p_{c} l_{c} \hbar (\rho_{ee} + \dot{\rho}_{ee})$. Note that Eq. (3) can be derived from the definition $\mathbf{T}^{(drv)} = (1/2)\text{Re}(\mathbf{D}^* \times \mathbf{E})$ for the spin torque on an oscillating electric dipole [21], where $\mathbf{D}$ is the envelope of the dipole positive-frequency component. Indeed, for the two-level atom considered here, we have $\mathbf{D} = 2d^* \rho_{eg}$. In the case where $d$ has a single nonzero spherical tensor component $d_q$, we have $T_z^{(drv)} = -q \hbar \text{Im}(\rho_{qe})$, which leads to Eq. (3). We find $T_z^{(drv)} = q/(p_{c} l_{c} - q)$. We can show that $T_z^{(drv)}/T_z^{(spon)} \neq j_z^{(\text{spin})}/j_z^{(\text{orb})}$, where $j_z^{(\text{spin})}$ and $j_z^{(\text{orb})}$ are the spin and orbital parts of the Minkowski angular momentum of guided light [8]. This means that the spin and orbital angular momenta of light are not transferred separately to the spin and orbital angular momenta of the two-level atom, unlike the case of small isotropic particles in free space [8].

We note that, for $l_c \geq 1$ and $q = p_c$, we have $(p_{c} l_{c} - q) \hbar = p_{c} (l_{c} - 1) \hbar$. In this case, Eq. (4) indicates that the photon angular momentum is converted into the atomic spin and orbital angular momenta. For $l_c \geq 1$ and $q = -p_c$, we have $(p_{c} l_{c} - q) \hbar = p_{c} (l_{c} + 1) \hbar$. In this case, Eq. (4) says that the orbital angular momentum and the change of the atomic spin angular momentum have the same sign and add up in generating the atomic orbital angular momentum. For $l_c \geq 1$ and $q = 0$, the total photon angular momentum is converted into the atomic orbital angular momentum.

Equation (4) can be used for not only hybrid modes ($l_c \geq 1$) but also TE and TM modes ($l_c = 0$). In the cases of TE and TM modes, we have $T_z^{(drv)} = -q \hbar (\rho_{ee} + \dot{\rho}_{ee})$, which indicates that the atomic orbital angular momentum can be generated from the atomic spin angular momentum through the interaction with a photon in a TE or TM mode having no angular momentum.

The conversion of atomic spin angular momentum into atomic orbital angular momentum via the interaction with a guided photon is possible because the guided mode is a structured field with a complex polarization profile. When an atom with a π, σ+, or σ− transition interacts with a plane-wave field in free space, in accordance with the selection rules, the atomic spin angular momentum is converted only to the photon spin angular momentum.

Note that, in the case where the atom is at rest and in the steady-state regime, we have $\dot{\rho}_{ee} = 0$. In this case, we obtain $T_z^{(drv)} = (p_{c} l_{c} - q) \hbar \Gamma \rho_{ee}$ and $T_z^{(spon)} = q \hbar \Gamma \rho_{ee}$, where $\rho_{ee}$ is given as $\rho_{ee} = \Omega^2 / (4\Delta^2 + \Gamma^2 + 2|\Omega|^2)$ [12]. We plot in Fig. 2 the torques $T_z^{(drv)}$ and $T_z^{(spon)}$ as functions of the radial position $r$ of the atom at rest and in the steady-state regime. The fact that the solid red curves of the figure have the same sign indicates that, for $q = p_c$, the angular momentum of guided light is converted into the orbital and spin angular momenta of the atom in the excitation process. The opposite signs of the dotted blue curves in Figs. 2(a) and 2(b) indicate that, for $q = -p_c$, the spin angular momentum is converted into the orbital angular momentum due to the excitation of the atom by guided light.
atom-field system and the agreement with the Minkowski formulation of angular momentum of light.

The axial spin torque produced by the spontaneous emission process is

\[ T_z^{(spon)} = -\hbar \rho_{ee}. \]  \hspace{1cm} (10)

We find \( T_z^{(spon)} + T_z^{(scatt)} = -\hbar \sum_{\mu_0} p_l \gamma_{\mu_0} - \hbar \sum_{\nu_0} l_\nu \gamma_{\nu_0}. \)

\[ \text{FIG. 3. (Color online) Radial dependencies of the orbital and spin scattering torques } T_z^{(scatt)} = \rho_{ee} T_z^{(spon)} \text{ and } T_z^{(scatt)} = \rho_{ee} T_z^{(spon)} \text{ for the parameters of Fig. 2.} \]

It is clear that the spontaneous-emission torques \( T_z^{(spon)} \) and \( T_z^{(spon)} \) do not depend on the driving field. However, the scattering torques \( T_z^{(scatt)} = \rho_{ee} T_z^{(spon)} \) and \( T_z^{(scatt)} = \rho_{ee} T_z^{(spon)} \) depend on the driving field through the excited-state population \( \rho_{ee}. \) We plot in Fig. 3 the torques \( T_z^{(scatt)} \) and \( T_z^{(scatt)} \) as functions of the radial position \( r \) of the atom being at rest and in the steady-state regime. The opposite signs of the curve for \( T_z^{(scatt)} \) for a given \( q \neq 0 \) and the corresponding curve for \( T_z^{(scatt)} \) indicate that the atomic spin angular momentum is converted into the atomic orbital angular momentum due to the scattering of light from the atom.

Finally, we calculate the axial torque components produced by the van der Waals potentials. In the case where a single spherical tensor component \( d_0 \) of the dipole matrix element vector \( \mathbf{d} \) is nonzero, the quantity \(|G_0|^2\) is independent of \( \varphi. \) In this case, the axial components of the torques of the van der Waals potentials vanish, that is, \( T_z^{(vdW)} = T_z^{(vdW)} = 0. \)

Combining the above results, we find that the total axial orbital torque is \( T_z = T_z^{(drv)} + \rho_{ee} T_z^{(spon)} \) and reads

\[ T_z = \hbar \rho_{ee} \left( p_c \Gamma - \sum_{\mu_0} p_l \gamma_{\mu_0} - \sum_{\nu_0} l_\nu \gamma_{\nu_0} \right) \pm \hbar \rho_{ee}. \]  \hspace{1cm} (11)

The total axial spin torque is \( T_z = T_z^{(drv)} + \rho_{ee} T_z^{(spon)} \) and reads

\[ T_z = \hbar \rho_{ee}. \]  \hspace{1cm} (12)

When the atom is at rest and in the steady-state regime, we have \( \rho_{ee} = 0. \) In this case, we obtain

\[ T_z = \hbar \rho_{ee} \left( p_c \Gamma - \sum_{\mu_0} p_l \gamma_{\mu_0} - \sum_{\nu_0} l_\nu \gamma_{\nu_0} \right) \]  \hspace{1cm} (13)

and \( T_z = 0. \)

\[ \text{FIG. 4. (Color online) Radial dependence of the total orbital torque } T_z. \] The driving field is in a quasicircularly polarized \( \text{HE}_{11} \) mode (solid red curves), a \( \text{TE}_{01} \) mode (dashed green curves), a \( \text{TM}_{01} \) mode (dotted blue curves), or a quasicircularly polarized \( \text{HE}_{21} \) mode (dashed-dotted magenta curves), with the power \( P = 1 \) pW. The polarization circulation index for the fields in the \( \text{HE}_{11} \) and \( \text{HE}_{21} \) modes is \( p_c = +1. \) Other parameters are as for Fig. 2.

We plot in Fig. 4 the total axial orbital torque \( T_z \) as a function of the radial position \( r \) of the atom at rest and in the steady-state regime. The results of calculations for different types of guided modes with a given power are shown. We observe from Fig. (a) that, for \( q = 1, \) the axial component \( T_z \) of the total orbital torque is larger for the \( \text{HE}_{11} \) and \( \text{HE}_{21} \) modes with \( p_c = q, 1 \) than for the \( \text{TE}_{01} \) and \( \text{TM}_{01} \) modes. However, Fig. (c) shows that, for \( q = -1, \) in the region \( r/a > 1.15, \) \( T_z \) is larger for the \( \text{TE}_{01} \) and \( \text{TM}_{01} \) modes than for the \( \text{HE}_{11} \) and \( \text{HE}_{21} \) modes with \( p_c = -q = 1. \) The occurrence of this feature is due to the fact that, for \( q = -1, \) the Rabi frequency \( \Omega \) is larger for the \( \text{TE}_{01} \) and \( \text{TM}_{01} \) modes than for the \( \text{HE}_{11} \) and \( \text{HE}_{21} \) modes with \( p_c = -q = 1. \)

In conclusion, we have calculated the axial orbital and spin torques of guided light on a two-level atom near a vacuum-clad ultrathin optical fiber. We have shown that the generation of these torques is governed by the angular momentum conservation law and is in agreement with the Minkowski angular momentum formulation. We have found that the photon angular momentum and the atomic spin angular momentum can be converted into the atomic orbital angular momentum.

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