EFFECTS OF THE NUMBER OF RELAY ANTENNAS AND RELAY-POWER ON MIMO PRECODED TWO-WAY RELAYING

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Abstract
In this treatise a two-way Amplify and Forward (AF) relay-aided system is considered, which employs the so-called Arithmetic Sum of Average Bit Error Rate (ASABER) based MIMO precoding technique. The two-way AF relay system is comprised of the pair of transceiver nodes S₁ and S₂ and the Relay Node (RN) R, where each node is equipped with N₁, N₂ and N₃ antennas, respectively. We study the effects of varying N₃ for fixed values of N₁ and N₂ and as well as the effects of having a fixed transmission power at the RN on the achievable ASABER performance. Based on our intensive simulation campaign, we infer that the attainable diversity order is increased approximately by N₁ − min (N₁, N₂), whenever N₃ assumes a value higher than min (N₁, N₂) values. However, this observation is only valid for relay power p₂ ≥ (p₁, p₃), where p₁ and p₂ are the transmit power constraints imposed on the sources S₁ and S₂ respectively. We also observe that the ASABER MIMO precoder’s BER curve exhibits an error floor for p₁ ≤ (p₁, p₂).

Keywords:
Two-way Relay, Bidirectional Relay, Amplify and Forward, MIMO Precoding

1. INTRODUCTION

One of the challenging design objectives of next generation wireless communication systems is to support an increased data-rate right across the entire propagation cell. Relay-assisted wireless transmission schemes are capable of achieving this challenging goal and hence they have attracted substantial research efforts over the past decade. Diverse cooperative protocols have been proposed for exchanging information amongst the transmitter, receiver, and relay nodes. The basic two-way cooperative communication system consists of two transceiver nodes (SN), namely S₁, S₂ and a relay node R. All the nodes are assumed to be half-duplex, since practical transceivers cannot transmit and receive simultaneously¹. Since most practical communications sessions are bi-directional, they can invoke two-way relaying protocols. In [1] a detailed spectral efficiency study of diverse cooperative protocols was provided.

The family of cooperative systems may be classified as Amplify and Forward (AF), Decode and Forward (DF) as well as Compress and Forward (CF) regimes, based on the specific technique used for processing the received signal at the RN.

AF relaying is the least computationally complex technique amongst them, since no synchronization, demodulation or channel decoding is necessitated - the received signal is simply amplified by the RN. However, the signal and noise are jointly amplified, hence AF relaying fails to improve the Signal-to-Noise Ratio (SNR). In this contribution we consider an AF MIMO two-way relay-aided system.

The performance of every communication link is degraded by channel impairments. The attainable system performance may be improved by exploiting the knowledge of the channel’s unique, user-specific impulse response about to be experienced with the aid of transmit preprocessing techniques used at the transmitter. Naturally, the vital prerequisite of acquiring the required Channel Impulse Response (CIR) is its accurate estimation at the distant receiver, which then has to quantize and signal the CIR back to the transmitter. Linear precoding is one of the most appealing preprocessing techniques by virtue of its low implementational complexity.

In two-way relay-aided systems we have the freedom of designing precoders at S₁, S₂ and R. There are numerous methods available in the open literature, which discuss the design of linear precoders at the source and relay, which may invoke diverse optimization criteria, such as maximizing the achievable sum-rate, the Arithmetic Sum of Average Bit Error Rate (ASABER) criterion and the Arithmetic Sum of Average Mean Square Error (ASAMSE) criterion. In [2], [3], [4] and [5] optimal RN precoders were designed for maximizing the system’s sum-rate, while in [6] an optimal RN precoder was designed for minimizing the Sum of the Mean Squared Error (SMSE) of the two-way AF system, which was equipped with multiple antennas at all three nodes. These methods may be referred to as being ‘relay-only’ precoders (ROP), since they only specify the MIMO precoder at the RN.

In [7], [8], [9], [10] and [11]² the joint design of the SN and RN precoders was advocated for the sake of maximizing the sum-rate of the two-way relaying system. The joint design of SN and RN precoders was used for minimizing the ASAMSE and the ASABER criteria of the two-way relaying system in [8] and [12]. The method discussed in [12] and [11] was shown to outperform other existing methods.

Our main goal is to study the effect of varying the number of antennas at relay on the attainable ASABER performance of the MIMO-aided precoded two-way AF system. Here the precoders used at the SN and RN are jointly designed based on the ASABER criterion, as in [12].

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Let $N_1$, $N_2$ and $N_r$ denote the number of antennas at the two SNs and the RN node respectively and $L = \min (N_1, N_2)$ denote the number of parallel streams transmitted from $S_1$ to $S_r$ and vice versa. Based on our intensive simulation campaign, we will demonstrate that the approximate diversity order of the system is $(N_1 - L + 1) \times (N_r - L + 1) + (N_2 - L)$. The outline of the paper is as follows. In Section II we detail our system model, while in Section III we formulate expressions for characterizing the scenarios associated with the assignment of different transmit powers to the RN for different number of antennas employed at the relay. Finally, in Section III we discuss our simulation results, before concluding in Section V.

2. SYSTEM MODEL

We consider a two-way AF relay-aided communication system comprised of the transceiver nodes $S_1$, $S_2$ and RN $R$, as shown in Fig.1. The nodes $S_1$, $S_2$ and $R$ are equipped with $N_1$, $N_2$ and $N_r$ antennas, respectively. During the first communication phase the nodes $S_1$ and $S_2$ simultaneously transmit information to the RN $R$. In next phase of transmission the RN amplifies the received signal and broadcasts it to nodes $S_1$ and $S_2$. The following common assumptions are exploited in this treatise:

1. The channels are flat fading and reciprocal.
2. Perfect channel state information\(^\dagger\) (CSI) is assumed to be available for the channels spanning from $S_1 \rightarrow R$ and $S_2 \rightarrow R$ at all the three nodes.
3. The direct path between the SNs $S_1$ and $S_2$ is not exploited.

\(^\dagger\)In \cite{13}, \cite{14} and \cite{15} various channel estimation methods have been proposed for two-way relay channels.

Let $s_i \in \mathbb{C}^{L \times 1}$, $\forall i = 1, 2$ be the input information symbol vector at node $S_i$, where we have $L = \min (N_i, N_r)$. Let $p_r$ be the transmit power available at node $S_r$. The symbol vector transmitted from node $S_i$ is given by $x_i = F_i s_i$, $\forall i = 1, 2$. Here $F_i \in \mathbb{C}^{N_i \times L}$, $\forall i = 1, 2$ represents the linear precoder used at node $S_i$, where the design of $F_i$ satisfies the following power constraint,

$$tr(E[x_i x_i^H]) = tr[F_i F_i^H] \leq p_r.$$  \hfill (1)

During the first phase, the signal received at node $R$ is given by,

$$y_r = \sum_{i=1}^{2} H_i x_i + n_r,$$  \hfill (2)

where $H_i \in \mathbb{C}^{N_r \times N_i}$, $\forall i = 1, 2$ describes the channel spanning from node $S_i \rightarrow R$ and $n_r \in \mathbb{C}^{L \times 1}$ is the Additive White Gaussian Noise (AWGN) experienced at $R$, which is distributed as $CN(0, 1_{N_r \times N_r})$. In the second phase, node $R$ broadcasts the precoded signal $x_i$ given by

$$x_i = G y_r.$$  \hfill (3)

where, $G \in \mathbb{C}^{N \times N_r}$ is the linear precoder matrix of the RN. Let $p_r$ be the transmit power available at $R$, so that the design of $G$ satisfies the following relay-power constraint,

$$tr(E[x_i x_i^H]) = tr \left( G \sum_{i=1}^{2} H_i F_i F_i^H H_i^H + I \right) G^H \leq p_r.$$  \hfill (4)

The signals received at the nodes $S_1$ and $S_2$ are given by,

$$y_1 = H_1^T G H_1 x_1 + H_1^T G H_2 x_2 + H_1^T G n_r + n_1$$ \hfill (5)

$$y_2 = H_2^T G H_2 x_2 + H_2^T G H_1 x_1 + H_2^T G n_r + n_2,$$ \hfill (6)

where $n_i \in \mathbb{C}^{N_i \times 1}$, $\forall i = 1, 2$ is the AWGN vector satisfying $n_i \sim CN(0, 1_{N_i \times N_i})$. Let $\tilde{y}_1$ and $\tilde{y}_2$ be the signals obtained after canceling the self interference from the received signals $y_1$ and $y_2$ respectively, yielding,

$$\tilde{y}_1 = H_1^T G H_2 x_2 + \tilde{n}_1$$ \hfill (7)

$$\tilde{y}_2 = H_2^T G H_1 x_1 + \tilde{n}_2.$$ \hfill (8)

The effective additive noise at node $S_1$ is given as, $\tilde{n}_1 = H_1^T G n_r + n_1$. Let $R_{n, i}$ be the effective AWGN covariance matrix, which is given by,

$$R_{n, i} = E[\tilde{n}_i \tilde{n}_i^H] = H_i^T G G^H H_i^T + I_{N_i \times N_i}.$$  \hfill (9)

Following noise whitening, the signals received at nodes $S_1$ and $S_2$ are given by,

$$\tilde{y}_{1, \text{eff}} = \frac{1}{R_{n, 1}} \tilde{y}_1 = H_{1, \text{eff}} F_2 s_2 + \frac{1}{R_{n, 1}} \tilde{n}_1$$ \hfill (10)

$$\tilde{y}_{2, \text{eff}} = \frac{1}{R_{n, 2}} \tilde{y}_2 = H_{2, \text{eff}} F_1 s_1 + \frac{1}{R_{n, 2}} \tilde{n}_2,$$ \hfill (11)

where $H_{1, \text{eff}} = R_{n, 1}^{-\frac{1}{2}} H_1^T G H_2$ and $H_{2, \text{eff}} = R_{n, 2}^{-\frac{1}{2}} H_2^T G H_1$ are the effective channels spanning from $S_2 \rightarrow S_1$ and vice versa. The effective signals received at the nodes $S_1$ and $S_2$, namely $\tilde{y}_{1, \text{eff}}$ and $\tilde{y}_{2, \text{eff}}$ are then processed by linear MMSE equalizers, in order to generate the estimates of $s_2$ and $s_1$ respectively, namely,

$$\hat{s}_1 = D_2 \tilde{y}_{2, \text{eff}},$$ \hfill (12)
\[ \hat{s}_i = D_1 \hat{y}_{\text{eff}}, \]  
(13)

where \( \hat{s}_i \) is the estimated symbol vector of \( s_i \) and \( D_1 \in \mathbb{C}^{N_1 \times L} \), \( \forall i=1,2 \) is the equalizer’s weight matrix used at node \( S_i \).

\section*{3. EFFECT OF VARYING THE NUMBER OF ANTENNAS AT THE RELAY}

Our main goal is now to study the effects of varying the number antennas at the relay on the achievable ASABER performance of the precoded MIMO-aided two-way AF relaying system considered under the following scenarios.

\textit{Case 1:} Varying transmit power at the relay;
\textit{Case 2:} Fixed transmit power at the relay;

Before we commence our related study, let us first introduce the MSE matrices of \( E_1 \) and \( E_2 \) at nodes \( S_1 \) and \( S_2 \) respectively as,

\[ E_i(F_2,G,D_1) = E \left[ (s_i - \hat{s}_i)(s_i - \hat{s}_i)^H \right] \]

\[ = I_{LxL} - D_{eff}^H P_r - P_r^H D_1 + D_{eff}^H R_r D_1, \]  
(14)

\[ E_i(F_1,G,D_2) = E \left[ (s_i - \hat{s}_i)(s_i - \hat{s}_i)^H \right] \]

\[ = I_{LxL} - D_{eff}^H P_r - P_r^H D_2 + D_{eff}^H R_r D_2, \]  
(15)

where \( P_r, P_2, R_1 \) and \( R_2 \) are defined as follows:

\[ P_1 = H_1^H G H_2 F_2, \]  
(16)

\[ P_2 = H_2^H G H_1, \]  
(17)

\[ R_1 = P_1 F_1^H + H_1^H G H_1^H H_1^H + I_{N_1 \times N_1}, \]  
(18)

\[ R_2 = P_2 F_2^H + H_2^H G H_1 H_2^H + I_{N_2 \times N_2}. \]  
(19)

We note that for fixed values of \( F_1, F_2 \) and \( G \) the trace \( tr( E_i) \), \( \forall i=1,2 \) is a convex function with respect to \( D_r \). Therefore, the optimal equalizer weight matrix \( D_r \) is obtained by solving the equation \( \nabla E_i tr( E_i) = 0 \), which yields

\[ D_{\text{opt}} = R_{eff}^{-1} P_r, \quad \forall i=1,2. \]  
(20)

After substituting \( D_{\text{opt}} \), into Eq.(14) and Eq.(15), the MSE matrix \( E_1 \) and \( E_2 \) of node \( S_1 \) and \( S_2 \) may be written as,

\[ \hat{E}_1 = [I + F_2 R_{r,2} F_2^H]^{-1}, \]  
(21)

\[ \hat{E}_2 = [I + F_1 R_{r,1} F_1^H]^{-1}, \]  
(22)

respectively, where we have

\[ R_{r,1} = H_1^H G H_1 H_{n,1}^{-1} H_2^H G H_1, \]  
(23)

\[ R_{r,2} = H_2^H G H_1 H_{n,2}^{-1} H_2^H G H_2. \]  
(24)

Let us first define the relationship between the MSE, SINR and BER of the each individual stream as given in [16]. The MSE of the \( j^{th} \) node and \( j^{th} \) parallel stream is defined as,

\[ MSE_{i,j} = \left[ \hat{E}_{i,j} \right]^{-1}. \]  
(25)

Assuming that all the sub-streams use the same modulation constellation associated with Gray encoding, the corresponding BER and SINR are given by,

\[ BER_{i,j} = \frac{1}{m} \left( \alpha_j Q \left( \beta_j \sqrt{SINR_{i,j}} \right) \right), \]  
(26)

\[ SINR_{i,j} = \frac{1}{MSE_{i,j}} - 1 \quad \forall i = 1,2 \quad \forall j = 1,...,L. \]  
(27)

where \( m = \log_2 M \) is the number of bits per symbol and \( M \) is the size of the constellation used. The symbols \( \alpha \) and \( \beta \) depend on the transmitted signal constellation and \( Q(\cdot) \) is the Gaussian Q-function defined as

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt. \]  
(28)

The ASABER performance measure is defined as follows:

\[ f_x = \frac{1}{2L} \left( \frac{1}{m} \sum_{i=1}^{L} \sum_{j=1}^{L} \alpha_i Q \left( \beta_i \left( \frac{1}{\left[ \hat{E}_{i,j} \right]} - 1 \right) \right) \right). \]  
(29)

The effective channels \( H_{12}\text{eff} \) and \( H_{21}\text{eff} \) of the ASABER MIMO precoded system may be diagonalized after an appropriate rotation of the input data symbols. The effective channel matrices \( H_{12}\text{eff} \) and \( H_{21}\text{eff} \) have \((N_1 \times N_2) \) and \((N_2 \times N_1) \) elements respectively, and their size is independent of \( N_1 \). Hence as \( N_1 \) becomes higher than \( L \), the total number of independently fading paths spanning from \( S_1 \rightarrow R \rightarrow S_2 \) and \( S_2 \rightarrow R \rightarrow S_1 \) is increased, which increases the achievable diversity order of the system. Based on our simulation campaign, we are able to infer the approximate diversity-order in the form of

\[ d = (N_1 - L + 1) \times (N_2 - L + 1) + (N_1 - L). \]  
(29)

We note however that Eq.(29) is only valid for \( p_r \geq (p_1, p_2) \). When we have \( p_r < (p_1, p_2) \), instead of amplifying the received signal, the RN reduces the signal power and then broadcasts it. As a result, the BER curve of the ASABER technique exhibits an error floor in this reduced power region. Hence, the above-mentioned diversity-order given by Eq.(29) becomes invalid in this case.

\section*{4. SIMULATION RESULTS}

All the simulations we assumed having \( N_1 = N_2 = 2, L = 2 \) and the entries of the channel matrices \( H_1 \) and \( H_2 \) were independent and distributed as \( \mathcal{CN}(0,1) \). The entries of \( s_i \) and \( s_j \) assume one of the legitimate values from the QPSK constellation, where the bits are mapped to symbols using classic Gray coding. The BER was calculated using Eq.(27) for \( \alpha = 1 \) and \( \beta = 2 \), which again, corresponds to the QPSK constellation [16]. Table.1 lists the different precoding schemes used in this paper.

| Precoding Methods | Precoding Methods |
|-------------------|-------------------|
| UP | Uniform Power Allocation |
| ROP | Relay Only Precoding |
| JSRP | Joint Source Relay Precoding |
| ROPM | ROP with ASAMSE as cost func. |
| ROPB | ROP with ASABER as cost func. |
| JAM | JSRP with ASAMSE as cost func. |
| JABM | JSRP with ASABER and ASAMSE as cost func. |
| JAB | JSRP with ASABER as cost func. |

Table.1. List of different precoding schemes
Fig. 2. Performance comparison of different algorithms in terms of ASABER for $N_1 = 2$, $N_2 = 2$ and $N_r = 3$.

Fig. 3. Comparison of effect of varying $N_r$ from 2 to 5, when $N_1 = N_2 = 2$ in terms of ASABER measure.

Fig. 4. Comparison of effect of fixed power at RN i.e., $p_r = 15$dB, when varying $p_1 = p_2$ in terms of ASABER.

- As the number of antennas is increased, the JAM technique starts to perform more poorly than the ROPM and ROPB arrangements.
- The diversity order equation remains valid until we have $p_1 = p_2 \leq p_r$. However, once $p_1$ and $p_2$ are increased beyond $p_r$, the ASABER curve exhibits an “error floor”.
- The JAB and JABM techniques attain a similar performance and they outperform all the other methods.

The effect of varying $N_r$ from 2 to 5 in Case 1 and Case 2 may be summarized as follows:

- The attainable diversity order of the two-way AF relay system was increased in line with $N_r - L$ for each increment in $N_r$, when $N_1$ and $N_2$ were fixed.
- Case 1 outperformed Case 2 as $N_r$ was increased from 2 to 5.
- The JAB and JABM techniques outperformed all the other methods in all scenarios.

5. CONCLUSIONS

In this paper we have studied the quantitative effects of having a fixed relay power and a variable number of relay antennas on the achievable diversity gain and error rate performance of the system. As $N_r$ became higher than the number of parallel transmission streams $L$, the diversity order was increased by $(N_r - L)$ for each increment in $N_r$, when $N_1$ and $N_2$ were fixed.

Therefore maximum diversity gain for given $N_r$ at relay is a function of relative values of $p_1$, $p_2$ and $p_r$. This interplay between diversity gain and power constraint, especially for higher SNR, needs further study.

REFERENCES

[1] B. Rankov and A. Wittneben, “Spectral efficient protocols for half-duplex fading relay channels”, IEEE Journal on
Selected Areas in Communications, Vol. 25, No.2, pp. 379–389, 2007.

[2] N. Lee, H. Yang, and J. Chun, “Achievable sum-rate maximizing AF relay beamforming scheme in two-way relay channels”, in Proc. IEEE ICC 2008 Workshops, pp.300-305.

[3] R. Zhang, Y. Liang, C. Chai, and S. Cui, “Optimal beamforming for two-way multi-antenna relay channel with analogue network coding”, IEEE Journal on Selected Areas in Communications, Vol. 27, No. 5, pp. 699–712, 2009.

[4] R. Vaze and R. W. Heath Jr., “Optimal amplify and forward strategy for two-way relay channel with multiple relays”, in Proc. IEEE Information Theory Workshop, pp.181 – 185, 2009.

[5] K.-J. Lee, K.W. Lee, H. Sung, and I. Lee, “Sum-rate maximization for two-way MIMO amplify-and-forward relaying systems”, in IEEE 69th Vehicular Technology Conference, pp.1–5, 2009.

[6] N. Lee, H. Park, and J. Chun, “Linear precoder and decoder design for two-way AF MIMO relaying system”, in Proc. IEEE VTC, pp.1221–1225, 2008.

[7] T. Unger and A. Klein, “Maximum sum rate of non-regenerative two-way relaying in systems with different complexities”, in IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications, PIMRC 2008, pp.1-6, 2008.

[8] K. Lee, H. Sung, E. Park, and I. Lee, “Joint optimization for one and two-way MIMO AF multiple-relay systems”, IEEE Transactions on Wireless Communications, Vol. 9, No. 12, pp.3671–3681, 2010.

[9] S. Xu and Y. Hua, “Source-relay optimization for a two-way MIMO relay system”, in IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP), pp. 3038–3041, 2010.

[10] X. Wang and X.-D. Zhang, “Optimal Beamforming in MIMO Two-Way Relay Channels,” in IEEE - Global Telecommunications Conference, pp. 1–5, 2010.

[11] S.S. Rajeshwari, L. Hanzo, and K. Giridhar, “Low-Complexity Improved-Sum-Rate Two-Way Relays Relaying on Joint MIMO Precoding”, submitted to IEEE-Global Communications Conference (GLOBECOM), 2011.

[12] S.S. Rajeshwari and K. Giridhar, “New Approach To Joint MIMO Precoding For 2-way AF Relay Systems”, in proc. National Conference on Communications, pp.1 – 5, 2011.

[13] F. Roemer and M. Haardt, “Tensor-based channel estimation and iterative refinements for two-way relaying with multiple antennas and spatial reuse”, IEEE Transactions on Signal Processing, Vol. 58, No. 11, pp.5720–5735, 2010.

[14] B. Jiang, F. Gao, X. Gao, and A. Nallanathan, “Channel estimation and training design for two-way relay networks with power allocation”, IEEE Transactions on Wireless Communications, Vol. 9, No. 6, pp. 2022–2032, 2010.

[15] W. Gongpu, G. Feifei, Z. Xin, and C. Tellambura, “Superimposed Training-Based Joint CFO and Channel Estimation for CP-OFDM Modulated Two-Way Relay Networks”, EURASIP Journal on Wireless Communications and Networking, Vol. 2010, 2010.

[16] D. Palomar, M. Bengtsson, and B. Ottersten, “Minimum BER linear transceivers for MIMO channels via primal decomposition”, IEEE Transactions on Signal Processing, Vol. 53, No. 8, pp. 2866–2882, 2005.