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The non-extensivity Parameter of a Thermodynamical Model of Hadronic Interactions at LHC energies.

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The LHC measurements above SPS and Tevatron energies give an opportunity to test predictions of the non-extensive thermodynamical picture of hadronic interaction to examine the measured transverse momentum distributions for new high interaction energy range. We determined the Tsallis model non-extensivity parameter for the hadronization process before short-lived particles decay and distort the initial $p_{\perp}$ distribution. We have shown that it follows exactly the smooth rise determined at lower energies below the present LHC record. The energy dependence of the $q$ parameter is consistent with expectations and the evidence of the asymptotic limit may be seen.

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The thermodynamical concept of the hadronization process following the fifty-year old Hagedorn idea was recently successfully used in [1] to describe multiplicities in $pp, e^+e^-$, and $NN$ high energy interactions. However, the main problem of such a treatment: the non-exponential tails of transverse momentum distributions remains. One explanation is proposed in [2] introducing the pre-hadronization dynamics of fireballs. A different, more general, solution is to extend the standard, Boltzmann statistics following the Tsallis idea [3]. The generalised, non-extensive statistics technique is often used to describe different kinds of phenomena: from interstellar dynamics to the cosmic ray source puzzle [4]. Among them high energy interaction phenomenology is one of the fields where the new statistics is very successful.

It was shown that such non-extensive statistics works well for $p_{\perp}$ spectra in $e^+e^-$ [5], and could be used for $pp$ and $pp$ interactions too [6, 7]. Here we show that the results of the Tsallis generalisation of the Becattini canonical thermodynamics works well when applied also to the LHC energies: 2.36 and 7 TeV.

In Refs. [9] and [10] the function of the form inspired by the Tsallis formula

$$E \frac{d^3N_{ch}}{dp^3} \sim \frac{dN_{ch}}{d\eta} \left( 1 + \frac{(q-1)}{T} \right)^{-\frac{1}{1-q}}$$

was used to fit the $p_{\perp}$ data. The shape of the invariant transverse momentum distribution is well described by the above formula with the values of the $q$ parameter of 1.13 and 1.15 (with a systematic uncertainty of order 0.005) and $T$: 130 and 145 MeV for energies 2.36 and 7 TeV, respectively.

We would like to use the same transverse momentum data, but not only to adjust the simple and convenient formula parameters. We try to say something about the mechanism of the hadronization process. The fits of the form of Eq.1 show the direction one can follow, but the physical conclusions should be based on rather more detailed analysis of the thermodynamical picture. We would like to remind that the word 'thermodynamical' here should be taken with care. As it was pointed out already by Hagedorn, (e.g., [8]) the meaning of the temperature $T$ in the "hadronic matter near the boiling point" is different than the ordinary temperature of the ordinary boiling soup.

The formalism we used in the present work is described in detail in [7]. The canonical partition function is generalised by replacing the exponential Boltzmann factor by the respective Tsallis formula:

$$\exp \left( -\frac{E}{T} \right) \rightarrow \left( 1 + \frac{(q-1)}{T} \sqrt{p^2 + m_j^2} \right)^{-\frac{1}{1-q}}$$

where $q$ is called the non-extensivity parameter. The transverse momentum distribution is given by the partition function $Z$

$$f \left( p_{\perp}^2 \right) dp_{\perp}^2 = \sum_j \sum_{n=1}^{\infty} (-1)^n \gamma_{\jmath, n} \left( 2J_j + 1 \right) V \frac{Z(Q^0 - nq_j)}{Z(Q^0)} \frac{1}{2\pi^2} \times \int dp_L x_j Z(Q^0 - nq_j) \, dp_{\perp}^2$$

where $x_j$ is the statistical weight (Eq.(2) of the particular state (with classical Boltzmann or modified Tsallis factor) $\gamma_{\jmath, n}$ is the strangeness suppression factor, $J_j$ is the spin of the $j$-th type hadron (quantum numbers $q_j$). $V$ denotes the hadronization volume.

Respective numerical integrations for the partition function determination have been performed. Then summation and integration of Eq.(3) have been performed taking into account almost one hundred species of daughter particles. All unstable particles were then decayed at the end and the final $p_{\perp}$ distribution was compared with the distributions measured by CMS at two energies: 2.36 [9] and 7 TeV [10]. The $T$ parameter is sensitive to small $p_{\perp}$'s while $q$ describes the non-exponential tails. No significant and systematic energy dependence of
FIG. 1: Transverse momentum distributions measured by CMS at 2.36 and 7 TeV and SPS at 900 GeV and respective non-extensive statistics fits (described in the text).

\( T \) was found, and eventually we found that this parameter can be fixed at 130 MeV as in our previous paper [7]. Results of the fits are shown in Fig. 1. Values of the \( q \)-parameter found for the \( \sqrt{s} \) of 2.36 and 7 TeV are 1.109±0.001 and 1.222±0.001, respectively. The quality of both fits (\( \chi^2/\text{ndf} \)) is quite good: 19/16 and 10/26, respectively.

The old data at 900 GeV from the SPS experiment [11] are shown together with the fit from [7]. As it is seen the \( p_{\perp}^{-4} \) steepness expectations at high \( p_{\perp} \) are still far and asymptotic possibility.

The non-extensive thermodynamics works well at lower energies, as it has been shown e.g. in [7], not only describing each individual energy transverse momentum spectrum, but the resulting parameter \( q \) behaviour exhibits the regular and expected smooth energy dependence. The low energy Boltzmann limit of \( q = 1 \) (observed for 100 GeV/c data) together with the upper (asymptotic, for infinite energy) limit of 1.25 [3] were combined in the functional form

\[
q(E) = \left[ 1.25 - A \, s^{-B} \right]_{>1}. \tag{4}
\]

Parameters \( A \) and \( B \) were adjusted to the data up to Tevatron energies in [7] and the result is shown in Fig.2. The values of \( A \) and \( B \) are 0.33 and 0.054, respectively.

In Fig. 2 new points at CMS energies are shown as solid circles. It is clearly seen that the non-extensive thermodynamical picture of hadron production applied to new LHC data on transverse momentum distributions measured at 2.3 and 7 TeV c.m.s. energies gives a good description of the experimental spectra. The \( T \) parameter could remain equal to 130 MeV and a smooth energy dependence of the non-extensivity parameter \( q \) found for lower energies is confirmed by the new data.

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