Thermal correction to resistivity in dilute 2D systems

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We calculate the resistivity of 2D electron (hole) gas, taking into account the degeneracy and the thermal correction due to the combined Peltier and Seebeck effects. The resistivity is found to be universal function of temperature, expressed in units of \( \frac{k}{h} (kT)^{-1} \). The giant parallel magnetoresistivity found to result from the spin and, if exists, valley splitting of the energy spectrum. Our analysis of compressibility and thermopower points to thermodynamic nature of metal-insulator transition in 2D systems.

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Recently, a great deal of interest has been focussed on the anomalous transport behavior of a wide variety of low-density 2D electron\( ^{\text{[2, 3, 4, 5]}} \) and hole\( ^{\text{[2, 3, 4, 5]}} \) systems. It has been found that, below some critical density, the cooling causes an increase in resistivity, whereas in the opposite high density case the resistivity decreases. Another unusual property of dilute 2D systems is their enormous response to parallel magnetic field. At low temperatures the magnetic field found to suppress the metallic behavior of 2D electron (hole) gas and result in strong increasing of resistivity upon enhancement of spin polarization degree\( ^{\text{[6, 7]}} \). At high temperatures the parallel magnetoresistivity starts to be unaffected by magnetic field when the temperature exceeds a value being of the order of Zeeman energy. A strong perpendicular magnetic field, if applied simultaneously with the parallel one, results in suppression of parallel magnetoresistivity\( ^{\text{[8]}} \). Although numerous theories have been put forward to account for these effects, the origin of the above behavior is still the subject of a heated debate.

The ohmic measurements known to be carried out at low current \( I \to 0 \) in order to prevent heating. Usually, only the Joule heat is considered to be important. In contrast to the Joule heat, the Peltier and Thomson effects are linear in current. As shown in Refs.\( ^{\text{[9, 10, 11]}} \), the Peltier effect influences ohmic measurements and results in a correction to a measured resistance. When current is flowing, one of the sample contacts is heated, and the other cooled, because of the Peltier effect. The contact temperatures are different. The voltage drop across the circuit includes the Seebeck thermoelectromotive force, which is linear in current. Finally, there exists a thermal correction \( \Delta \rho \), to the ohmic resistivity, \( \rho \), of the sample. For degenerate electrons, \( \Delta \rho / \rho \sim (kT/\mu)^2 \), where \( \mu \) is the Fermi energy. Hence, the correction may be comparable with the ohmic resistance of a sample when \( kT \approx \mu \).

In the present paper, we report on a study of low-T transport in 2D electron (hole) gas, taking into account both the electron degeneracy and the Peltier-effect-induced correction to resistivity\( ^{\text{[2, 3, 4, 5]}} \). The parallel magnetoresistivity found to originate from the spin and, if exists, valley splitting of 2D energy spectrum.

Let us consider, for clarity, the (100) MOSFET 2DEG system. The electrons are assumed to occupy the first quantum-well subband with isotropic energy spectrum \( \varepsilon(k) = \frac{k^2}{2m} \), where \( m \) is the effective mass. The sample is connected (see Fig.1A, inset) by means of two identical leads to the current source. Both contacts are ohmic. The voltage is measured between the open ends( "e" and "d") kept at the temperature of the external thermal reservoir. The sample is placed in a sample chamber with mean temperature \( T_0 \).

According to our basic assumption, the contacts( "a" and "b") may have different respective temperatures \( T_a \) and \( T_b \). Including the temperature gradient term, the current density \( j \) and the energy flux density \( q \) yield

\[
    j = \sigma (E - \alpha \nabla T), \quad q = (\alpha T - \zeta/e)j - \kappa \nabla T, \tag{1}
\]

where \( E = \nabla \zeta/e \) is the electric field, and \( \zeta = \mu - e \varphi \) the electrochemical potential. Then, \( \sigma = Ne^2\tau/m \) is the conductivity, \( N \) the 2DEG density, \( \kappa \) the thermal conductivity, and \( \alpha \) the thermopower.

In general, one can unambiguously solve Eq.(1), and, then find a difference of contact temperatures \( \Delta T = T_a - T_b \) for an arbitrary circuit cooling. Since the electron-phonon coupling is weak below \( \approx 1K \), the heat conduction from 2DEG to mixing chamber could predominate occur through the contacts of the sample and the leads connected to them. However, the experimental observations\( ^{[12]} \) demonstrate that 2DEG alone is the dominant thermal resistance in this problem. Actually, the cooling is provided by electron thermal conductivity found to follow Wiedemann-Franz law \( \kappa = LT\sigma \), where \( L = \frac{e^2}{2\pi k} \) is the Lorentz number. Accordingly, we further consider adiabatic cooling, with the 2DEG thermally insulated from the environment. Then, we will omit the Joule heating for actual \( I \to 0 \) case. We emphasize that under the above conditions, the sample is not heated. Indeed, at small currents, \( T_a \approx T_b \approx T_0 \). Hence, the amount of the Peltier heat, \( Q_a = I \Delta \alpha T_0 \), evolved at contact "a" and that absorbed at contact "b" are equal. Here, \( \Delta \alpha \) is the difference of 2DEG and metal conductor thermopowers. If it is recalled that the energy flux is continuous at each contact, the difference of the contact temperatures yields \( \Delta T = \frac{\Delta \alpha I}{L\sigma w} \), where \( l_0 \) and \( w \) are respectively the sample length and width. Using Eq.(1), the voltage drop between ends "e" and "d"
is given by \( U = RI + \Delta \alpha \Delta T \), where \( R \) is the ohmic resistance of the circuit. The second term denotes conventional Seebeck thermoemotocore force. Since \( \Delta T \sim 1 \), we finally obtain the total 2DEG resistivity as follows

\[
\rho_{tot} = \rho(1 + \alpha^2/L),
\]

where \( \rho = 1/\sigma \) is the 2DEG ohmic resistivity. In Eq. (2) we ignore conductors resistances and take into account that for the actual case of the metal leads \( \Delta \alpha \sim -\alpha \).

Eq. (2) can also then be applied for 2D gas.

Using Gibbs statistics and the above energy spectrum, the 2DEG density \( N = -\left(\frac{\partial F}{\partial \epsilon}\right)_T \) yields

\[
N = N_0 \xi F_0(1/\xi),
\]

where \( \Omega = -kT \sum \ln(1 + \exp(\frac{\epsilon_n}{kT})) \) is the 2DEG thermodynamic potential, and \( \xi = kT/\mu \) the dimensionless temperature. Then, \( N_0 = D\mu \) is the density of strongly degenerate 2DEG, \( D = \frac{2m^*}{e^2} \) the 2DEG density of states(factor 2 accounts valley degeneracy) and \( F_0(\xi) \) the Fermi integral. In Fig. 1, we plot the temperature dependence of the dimensionless concentration \( n = N/N_0 \) given by Eq. (3). In the classical Maxwell-Boltzman limit \( (\xi < 0, |\xi| \ll 1) \), the 2D electron density is thermally activated, and, therefore, \( n = |\xi| \exp(-1/|\xi|) \). In the case of strongly degenerate electrons \( (\xi \ll 1) \), we obtain \( n = 1 + \xi \exp(-1/\xi) \). Then, at elevated temperatures \( \xi \geq 1 \), the dependence of the 2DEG density \( n = 1/2 + \xi \ln 2 \) becomes linear in temperature. It is to be noted that at finite temperature the 2DEG density always exceeds( see Fig. 3, inset b) the zero-temperature value, i.e \( N > N_0 \). As a consequence, the density, deduced from the period of Shubnikov-de-Haas quantum oscillations, \( N_{shdB} = N_0 \), is less than that, \( N_{Hall} = N \), obtained from low-field Hall resistivity data( see Ref. [13]) in consistent with experiments. We emphasize that the 2D electron density is a monotonic function of temperature (see Fig. 1, A). Therefore, one might expect that the ohmic resistivity \( \rho(T) \sim 1/N \) decreases with increasing temperature at constant carrier mobility. We now demonstrate that the total resistivity specified by Eq. (3) can, nevertheless, increase in a certain temperature range owing to thermal correction.

Following the conventional Boltzman equation formalism, the explicit formulae for the 2DEG thermopower (for the 3D case, see Pisarenko, 1940) can be written as

\[
\alpha = -\frac{k}{e} \left[ 2F_1(1/\xi) \left( F_0(1/\xi) - \frac{1}{\xi} \right) \right].
\]

Here, we assume, for simplicity, that the electron scattering is characterized by energy-independent momentum relaxation time. In the classical Maxwell-Boltzman limit \( (\xi < 0, |\xi| \ll 1) \) the thermopower is given by the conventional formulae \( \alpha = -\frac{k}{e}(2 - 1/\xi) \). Then, for strongly degenerated 2DEG \( (\xi \ll 1) \), we obtain the temperature dependence of the thermopower (Fig. 2B, inset) as \( \alpha = -\frac{k}{e}(\pi^2\xi/3 - (1 + 3\xi) \exp(-1/\xi)) \). At elevated temperatures \( (\xi > 1) \) the thermopower first grows with temperature, and then approaches the universal value \( \alpha_s = -\frac{k}{e} \left( 2\xi(1/\xi) \right) = -\frac{k}{e} \pi^2/3 \). Our support of the above behavior is confirmed by low-T thermopower measurements data[13], found to diverge at certain value \( \sim 0.6k/e \) being of the order of \( \alpha_s \) (see bold line in Fig. 2, inset a).

It’s worth noting that the thermopower can be of the order of \( k/e \). Accordingly, the thermal correction to resistivity may comparable with the ohmic resistivity of 2DEG. In Fig. 2B we plot the temperature dependence of the 2DEG resistivity given by Eq. (3) at different Fermi temperatures \( T_F = \mu/k \). At fixed temperature, the resistivity increases with decreasing of 2DEG degeneracy. At certain Fermi energy \( (T_F = 0.25K \ in \ Fig. 2B) \) the T-dependence of the resistivity exhibits metallic behavior at \( T < T_F \), and then becomes insulating ( i.e. \( \frac{d\rho}{dT} < 0 \) ) at \( T > T_F \). Within the low-temperature metallic region \( \xi \ll 1 \), the 2DEG resistivity can be approximated (see dashed line in Fig. 2B) with \( \rho_{tot} = \rho_0(1 + \pi^2\xi^2/3) \), where \( \rho_0 = \frac{h}{2\pi^2}(k_Fl)^{-2} \) is
FIG. 2: Zero-field T-dependence of resistivity and thermopower (inset (a)), given by Eqs. (2-4) for $T_F/K=2-0.25$ (step 0.25), 0.2-0.05 (step 0.05), 0.01, 0 (bold line). -0.1, -0.2. Asymptotes: $\xi \ll 1$ - dashed line, $\xi > 1$ - dotted line for fixed $T_F=0.25K$. Inset (b): 2DEG density vs Fermi energy at fixed temperature $T=0$ (piecewise bold line); 0, 0.25, 0.4K.

the resistivity at $T \to 0$, $k_F = \sqrt{2m\nu}/\hbar$ the Fermi vector, and $l = \hbar k_F\tau/m$ the mean free path. Then, for the high-temperature ($\xi > 1$) insulating region we obtain the asymptote $\rho^{\text{tot}} = \rho^0 \frac{1+\alpha^2/L}{2\ln 2+1/2}$, depicted in Fig.2 by dotted line. This results are confirmed by recent experiments showing that for temperatures well below the Fermi temperature the metallic region data obey a scaling, where the disordered parameter $k_F l$ and dimensionless temperature $T/T_F$ appear explicitly. These experimental observations therefore rule out interactions, the shape of the potential well, spin-orbit effects and quantum interference effects as possible origins of the metallic behavior mechanism. It is to be noted that the conventional theory used to explain 2D metallic behavior is, however, failed to account both $T \to 0$ and $T \geq T_F$ cases. It is of particular interest the 2DEG compressibility, $K = \frac{dN}{d\mu} = -\frac{d\rho}{d\mu}$, known to be a fundamental quantity generally more amenable to theoretical and experimental analysis.

For non-interacting 2DEG system Eq. (3) yields $K(\xi) = DF^0(1/\xi)$, where $F^0_n(z) = \frac{dF_n(z)}{dz}$ is the derivative of the Fermi function. At fixed temperature Fig. 3 represents the dependence of the inverse compressibility parameter, $d(\mu) = e/Ke^2$, vs Fermi temperature. For strongly degenerated electrons ($\xi < 1$) one obtains a constant value $d_0 = e/De^2$ in consistent with experimental findings. Then, upon decreasing of 2DEG density (i.e. $\mu \to 0$) the experimental data exhibit diminish and, furthermore, the negative inverse compressibility compared to $d_0$. Usually, this behavior is explained in terms of conventional Hartree-Fock exchange omitted in our simple approach. However, for extremely low 2D density the inverse compressibility data always exhibits an abrupt upturn which cannot be explained within Hartree-Fock scenario. We argue that the above feature has the natural explanation within our model (see dashed line in Fig. 3) since $d = d_0 exp(-1/|\xi|)$ at $\xi < 0$, $|\xi| \ll 1$ and, hence, exhibits T-activated behavior.

In general case of 2DEG placed in strong perpendicular magnetic field $B_\perp$ the compressibility yields
\[ K = \frac{D}{4\xi \nu} \sum_n \frac{1}{\cosh \left( \frac{\varepsilon_n - \mu}{2kT} \right)^2} \simeq D \left[ F'_0(1/\xi) + 4\pi^2 \xi \nu \sum_k (-1)^k k \cos(2\pi k \nu \xi) \right]. \quad (5) \]

Here, \( \Omega = -kT \sum \ln \left( 1 + \exp \left( \frac{\varepsilon_n - \mu}{kT} \right) \right) \) is the thermodynamic potential modified with respect to 2DEG spin-unresolved Landau level(LL) energy spectrum \( \varepsilon_n = \hbar \omega_c (n + 1/2) \), where \( n = 0, 1, \ldots \) is the LL number, \( \omega_c = eB/m^* \) the cyclotron frequency, and \( \Gamma \) the zero-width LL density of states. Then, \( \nu = \frac{\varepsilon_n}{\hbar \omega} \) is the filling factor. According to Eq.(5), at fixed magnetic field the dependence \( d(\mu) \) can be viewed( see Fig.1b) when \( \nu = \frac{\varepsilon_n}{\hbar \omega} \). Simultaneously, the thermopower exhibits a strong enhancement of thermopower and diminish of 2DEG density is responsible for the observed giant magnetoresistivity in 2DEG(Fig.4b). In the opposite spin-polarized case.

We now address the question of parallel field magnetoresistivity. The energy spectrum modified with respect to valley splitting \( \Delta_{\nu}[K] = \Delta_{\nu}^0 + 0.6B_0[K] \) and spin splitting \( \Delta_{\nu}(K) = g^*\mu_B B_0 = 2.6B_0[K] \) is represented in Fig.4a, inset. For simplicity, we neglect further the density dependence of \( g^* \)-factor. Then, addressing the unresolved problem to extract zero-field valley splitting within low-field Shubnikov de-Haas measurements, we assume that \( \Delta_{\nu}^0 = 0 \). Actually, these assumptions do not substantially affect our basic results. Note that the lowest spin-up states are always valley-split since the we have a ratio \( \beta = \Delta_{\nu}/\Delta_s = 0.23 < 1 \).

With the help of Gibbs approach, the explicit formulae for 2DEG density and thermopower yields

\[ N = \frac{N_0 \xi}{4} \sum_i F_0 \left( 1 - E_i / \xi \right), \quad (6) \]
\[ \alpha = -\frac{k}{e} \left[ \sum_i \left( \frac{F_1 \left( 1 - E_i / \xi \right) + E_i / \xi F_0 \left( 1 - E_i / \xi \right)}{\sum_i F_0 \left( 1 - E_i / \xi \right)} \right) - 1 \right], \]

where \( E_i = 0, B^*, \beta B^*, (1 + \beta)B^* \) is the energy deficit between the bottom of spin and(or) valley subbands and of the ground state( see Fig.4a, inset), \( B^* = \Delta_s / \mu \) the spin polarization factor. The system becomes completely spin-polarized when \( B^* = 1 \). Let us discuss qualitatively the 2DEG magnetoresistivity. At fixed temperature \( \xi < 1 \) the total number of electrons decreases(see Fig.4a) upon increasing of the parallel magnetic field as \( n = 1 - (1 + \beta)B^*/2 \) when \( B^* < 1 \). Simultaneously, the thermopower exhibits a strong enhancement( see Fig.4b) when \( \xi < B^* \). We argue, that both the enhancement of thermopower and diminish of 2DEG density is responsible for the observed giant magnetoresistivity in 2DEG(Fig.4b).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{(a)T-dependence of magnetoresistivity at fixed \( T_F = 0.25K \) for different degree of spin polarization( from top to bottom) \( B^* = 0.2; 0.15; 0.1 \) and \( 0.02 \). (b)Magnetoresistivity vs Zeeman splitting at fixed temperature \( T = 0.07K \) for Fermi energies denoted in Fig.2. Dotted line represents the condition \( B^* = 1 \) for spin-polarized 2DEG.}
\end{figure}

At fixed magnetic field the T-dependence of magnetoresistivity given by Eqs.(2),(6) is similar to that represented in Fig.4 for \( B=0 \). Then, for fixed Fermi temperature(see Fig.4b) the above dependence becomes unaffected by applied magnetic field above certain temperature being of the order of Zeeman energy. This find-
ing is consistent with experimental observations. It is worth to be noted that the conventional theory agrees with magnetoresistivity data only qualitatively.

Let us analyze in more detail the cooling conditions, which are known to influence the thermal correction to resistivity. It will be recalled that in the case of adiabatic cooling the electron temperature differs from the bath temperature $T_0$. We now consider the opposite situation of electron cooling due to, for example, finite strength of electron-phonon coupling. Following Ref. [20], below $\sim 0.6K$ in Si MOSFET’s the electron-to-phonon thermal exchange is given by $a(T^3 - T_0^3)$, where $a = 2.2 \times 10^{-8} W/K^3 cm^2$. When $T - T_0 \ll T_0$, the thermal correction to the resistivity $\Delta \rho$ is suppressed [10] by the factor $\gamma = \tanh \lambda l_0$, where $\lambda = l_0 T_0 \sqrt{3a/4k}$ is a dimensionless parameter. Actually, $\lambda$ is the ratio of outgoing and internal heat fluxes associated with phonon related thermal leakage and electron heat diffusion, respectively. When $\lambda \ll 1$, the local cooling due to phonons can be neglected and the adiabatic approach is well justified. In the opposite case of intensive cooling ($\lambda \gg 1$), the difference of the contact temperatures $\Delta T$ becomes smaller, and, therefore, the thermal correction to resistivity $\Delta \rho$ vanishes. For $l_0 = 1mm$, $T_0 = 50mK$, $\sigma = 2.1e^2/h = 8 \times 10^{-5} Ohm^{-1}$ (typical critical region conductance) we obtain $\lambda = 1.2$, and, therefore, $\gamma = 0.8$. It worthwhile to notice that Peltier correction to resistivity becomes greater at ultra-low temperatures for short samples, since $\lambda \sim l_0 T_0$.

We emphasize that both dc and ac ohmic measurements lead to a thermal correction. The correction is, however, strongly damped at high frequencies because of the thermal inertial effects. As demonstrated in Ref. [9], the above quasi-static approach is valid below some critical frequency $f_{cr} = \chi/l_0^2$. For example, for degenerate electrons the thermal diffusion coefficient $\chi$ is of the order of the diffusion coefficient $D = \frac{e^2}{h} \left( \frac{dN}{d\mu} \right)^{-1}$. Assuming $\sigma = e^2/h$, $l_0 = 1mm$, for GaAs-based structure we obtain $\chi \sim h/m$, hence $f_{cr} = 1.5kHz$. We suggest that the spectral dependence of the 2D resistivity can be used to estimate the thermal correction.

In conclusion, low-temperature ohmic measurements of a 2D electron (hole) gas involve a thermal correction caused by the Peltier effect. The resistivity of 2DEG with thermal correction included is found to be universal function of temperature, expressed in units $h/e^2(k_B l)^{-1}$. This universal behavior correlates with that found in experiments. Strong increasing, and, then subsequent saturation in parallel magnetoresistivity in Si-MOSFET found to results from both the spin and valley splitting of 2DEG energy spectrum compared with $B = 0$ case. We suggest ac measurements as a powerful tool illuminating the importance of thermal correction to resistivity. Our analysis of compressibility and thermopower points to thermodynamic nature of metal-insulator transition in 2D systems.

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