Strong Lensing Constraints on Small-Scale Linear Power

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We place limits on the linear power spectrum on small scales ($k \gtrsim 50h$ Mpc$^{-1}$) using measurements of substructure in gravitational lens galaxies. We find excellent agreement with the simplest $\Lambda$CDM models, and in conjunction with other cosmological probes, place constraints on the neutrino mass $m_\nu$, tilt of the primordial power spectrum $n$, and mass of the dark matter particle $m$. We find $n > 0.94$, and for a Harrison-Zeldovich spectrum, $m_\nu < 0.74$ eV and $m > 5.2$ keV, at 95\% confidence.

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The amplitude of matter fluctuations on subgalactic scales can provide a wealth of information on a wide range of physics. For example, the inflaton potential\textsuperscript{[1]}, the neutrino mass\textsuperscript{[2]}, and the physics of the dark matter particles\textsuperscript{[3, 4]} can all cause measurable effects on the primordial power spectrum, especially on small scales. Hence, measurement of the small scale linear power spectrum is of great interest. Unfortunately, direct probes of the linear power on these scales has proven difficult, due to a host of astrophysical processes which complicate the interpretation of attempted measurements. The most promising avenue of attack has been study of the Lyman $\alpha$ forest absorption power spectrum\textsuperscript{[5]}, however it has been argued that inference of the linear power on these scales has proven difficult, due to a host of astrophysical processes which complicate the interpretation of attempted measurements. The most promising avenue of attack has been study of the Lyman $\alpha$ forest absorption power spectrum\textsuperscript{[5]}, however it has been argued that inference of the linear power spectrum from the Ly $\alpha$ forest will be difficult due to nonlinear effects\textsuperscript{[6]}. One relatively clean probe of the linear power spectrum is the number of collapsed halos as a function of mass. Assuming Gaussian density fluctuations, simple smoothing arguments may be applied to calculate the collapsed halo abundance given the linear power spectrum\textsuperscript{[7]}. This Press-Schechter formalism reproduces the mass function produced in cosmological simulations remarkably well, with only minor modifications\textsuperscript{[8]}. A consequence of Press-Schechter theory, subsequently confirmed by numerical simulations, is that simple inflationary cold dark matter (CDM) models predict halos with significant amounts of satellite substructure\textsuperscript{[9]}. The dearth of observed Local Group dwarf satellites, relative to these predictions, could in principle hint at suppression of fluctuations on small scales, implying new physics in inflation\textsuperscript{[10]} or in the dark matter sector\textsuperscript{[11, 12]}. Unfortunately, once again uncertainties in the astrophysics of such dwarfs, such as star formation or feedback processes\textsuperscript{[10]}, limit the utility of these objects in constraining the matter power spectrum. Instead, a direct probe of the satellite mass (as opposed to the light) is required. Gravitational lensing, as an effect sensitive to mass rather than light, can provide such a probe. Recently, the abundance of satellite substructure in a sample of elliptical galaxies has been measured using gravitational lensing\textsuperscript{[13]}. In this Letter, we quantify the constraints of this detection on the linear power spectrum.

Gravitational lens galaxies require a significant fraction, $0.006 < f_{\text{sat}} < 0.07$, of their mass to be in subhalos in order to be consistent with observations of lens image fluxes and relative positions\textsuperscript{[14]}. The most likely candidates for such satellites are the CDM subhalos predicted by Press-Schechter arguments and seen in N-body simulations; see Ref.\textsuperscript{[11]} for more details. Using the conditional mass function\textsuperscript{[12]}, which has been shown to match well the satellite mass function found in high resolution N-body simulations\textsuperscript{[11]}, this $f_{\text{sat}}$ directly translates into a measurement of the linear rms density fluctuations on such mass scales, $\sigma(M_{\text{sat}})$. For a Sheth-Tormen (ST) multiplicity function\textsuperscript{[8]}, we have

$$f_{\text{sat}}(M_{\text{sat}}) = A \left[ \text{erfc} \left( \frac{\alpha v}{2} \right) + \frac{\Gamma(\frac{1}{2} - p, \alpha v/2)}{2^p \sqrt{\pi}} \right]$$

$$\nu = \frac{\delta_c \left( D(z_{\text{sat}})^{-1} - D(z_{\text{gal}})^{-1} \right)}{\sigma^2(M_{\text{sat}}) - \sigma^2(M_{\text{gal}})}.$$  \hspace{1cm} (1)

(2)

Here, $\delta_c = 1.68$ is the critical linear overdensity for collapse\textsuperscript{[15]}, $\text{erfc}(x)$ is the complementary error function, $\Gamma(n, x)$ is the incomplete gamma function, $A^{-1} = 1 + 2^{-p} \pi^{-1/2} \Gamma(\frac{1}{2} - p)$ is a normalization constant, $D(z)$ is the linear growth factor normalized so that $D(z = 0) = 1$, and $z_{\text{gal}} \approx 1$ is a typical redshift at which the lens galaxy halo of mass $M_{\text{gal}} \approx 10^{12.5} M_{\odot}$ formed.

We estimate the typical formation redshift for satellites by assuming they are tidally truncated and setting their inferred density equal to 200 times the background matter density when they formed. Assuming a flat $\Lambda$CDM cosmology with $\Omega_M = 0.33$ and Hubble constant $h = H_0/(100 \text{ km/s/Mpc}) = 0.66$, and adopting the ST fit, the lens constraints on satellite Einstein radius and mass fraction translate into constraints on $\sigma(M)$. This is plotted in Figure\textsuperscript{[16]}.

Having determined the rms linear mass fluctuations on subgalactic scales, we now compare this result to the predictions of various cosmologies. A representative sample
The linear power spectrum constrains the neutrino mass, larger than not have features which break scale invariance on scales are tightly constrained; the inflaton potential can- arrive one-sided limits, we find Zeldovich spectrum of perturbations. If we instead de- confidence, consistent with a scale-invariant Harrison- currently possible. We find $0.04$ at 95.5% confidence. If we instead assume a logarithmic prior, which may be more appropriate, we find $m_{\nu} < 0.53$ eV at 95.5% confidence. Interestingly, the lensing upper limit on the sum of neutrino masses is quite close to the lower limit on neutrino mass measured by the LSND experiment [2], roughly $\sqrt{\Delta m^2} \sim 0.3$ eV.

Additionally, our measurement provides a lower limit on the energy spectrum of dark matter particles. We follow standard convention and characterize the energy spectrum by the mass of a neutrino with an equivalent free streaming scale. Warm dark matter models, with $m \lesssim 1$ keV, have been proposed as a modification to CDM, in order to account for the dearth of dwarf satellite galaxies and the lack of cusps in dark matter density in the central regions of galaxies [3]. The close agreement of lensing observations with CDM predictions indicates that the free streaming scale must be below the $\sim 20$ kpc scales probed by lensing. Assuming a uniform prior in the range $1$ keV $< m < 32$ keV, and assuming a Harrison-Zeldovich spectrum, we find that $m > 5.2$ keV is required at 95.5% confidence. Since the dark matter particle mass can be much larger than the range we have explored, our bound is conservative.

Certain effects which we have not considered will tend to suppress the effects of substructure, and hence bias our measurements to low $\sigma$. First, the conditional mass function ignores the effects of tidal stripping and disruption of satellite subhalos. Secondly, the finite angular sizes of radio QSO’s tend to wash out the lens effects of halo substructure. A proper treatment of these effects is beyond the scope of this Letter, however we note that both effects tend to increase the required abundance of satellite substructure and strengthen the constraints. Hence our limits should be regarded as conservative.

These constraints should all improve as the sample of suitable lens systems expands, and as the quality of the data improves. In general, multiply-imaged compact radio sources will be the preferred probe because of the high imaging resolutions available for radio sources using Very Long Baseline Interferometry (VLBI). For the current lenses, deep VLBI observations of extended, thin jet

![FIG. 1: Variance $\sigma$ on mass scale $M$. Contours depict the 68%, 90% and 95% confidence regions from lensing. The curves depict predictions for the cosmologies denoted which, unless labeled otherwise, are flat $\Lambda$CDM universes with $h = 0.66$, $\Omega_{\text{DM}}h^2 = 0.13$, $\Omega_{\text{B}}h^2 = 0.02$ and $h = 0.66 \pm 0.05$, and employ fitting functions for the transfer functions [3, 15]. The tilt of the primordial power spectrum, $n$, is related to the slow roll parameters of the inflaton potential [16]. Recent measurements [17] of the cosmic microwave background radiation constrain $n \approx 1 \pm 0.1$ (1 $\sigma$) on large scales ranging up to the horizon size. Lensing probes mass scales some 18 orders of magnitude smaller, so it provides the strongest constraints on the tilt that are currently possible. We find $0.93 < n < 1.04$ at 95.5% confidence, consistent with a scale-invariant Harrison-Zeldovich spectrum of perturbations. If we instead derive one-sided limits, we find $n > 0.94$ at 95.5% confidence. Inflationary models with broken scale invariance [2, 13] are tightly constrained; the inflaton potential cannot have features which break scale invariance on scales larger than $\sim 20$ kpc.

Experiments probing atmospheric [19] and solar neutrinos [20] have established that neutrinos have mass and thus constitute particle (hot) dark matter. However, these experiments do not provide a definite measure of the mass of neutrinos, but instead the squared mass difference $\Delta(m^2)$ between different neutrino flavors. The linear power spectrum constrains the neutrino mass,
structures in lenses [23] are a promising means of breaking the wide degeneracy between satellite mass and mass fraction. The Extended Very Large Array [24] and the proposed Square Kilometer Array radio array [25] can easily expand the sample of radio lenses by more than an order of magnitude, with a corresponding reduction in the uncertainties.

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[1] M. Kamionkowski and A. R. Liddle, Phys. Rev. Lett. 84, 4525 (2000).
[2] R. A. C. Croft, W. Hu and R. Davé, Phys. Rev. Lett. 83, 1092 (1999).
[3] P. Colin, V. Avila-Reese and Octavio Valenzuela, Astrophys. J. 542, 622 (2000); P. Bode, J. P. Ostriker and N. Turok, Astrophys. J. 556, 93 (2001).
[4] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. 84, 3760 (2000).
[5] L. Hernquist, N. Katz, D. H. Weinberg and J. Miralda-Escudé, Astrophys. J. Lett. 457, L51 (1996); R. A. C. Croft, D. H. Weinberg, N. Katz and L. Hernquist, Astrophys. J. 495, 44 (1998).
[6] M. White and R. A. C. Croft, Astrophys. J. 539, 497 (2000); M. Zaldarriaga, R. Scoccimarro and L. Hui, astro-ph/011123 (2001).
[7] W. H. Press and P. Schechter, Astrophys. J. 187, 425 (1974).

[8] R. K. Sheth and G. Tormen, Mon. Not. R. Astron. Soc. 308, 119 (1999).
[9] G. Kauffmann, S. D. M. White and B. Guiderdoni, Mon. Not. R. Astron. Soc. 264, 201 (1993); A. Klypin, A. V. Kravtsov, O. Valenzuela and F. Prada, Astrophys. J. 522, 82 (1999); B. Moore et al., Astrophys. J. Lett. 524, L19 (1999).
[10] J. S. Bullock, A. V. Kravtsov and D. H. Weinberg, Astrophys. J. 539, 517 (2000); E. Scannapieco, R. J. Thacker and M. Davis, Astrophys. J. 557, 605 (2001).
[11] N. Dalal and C. S. Kochanek, Astrophys. J. in press, astro-ph/0111459 (2001).
[12] R. G. Bower, Mon. Not. R. Astron. Soc. 248, 332 (1991).
[13] We ignore the slight dependence of δ_c on Omega_M, see V. R. Eke, S. Cole and C. S. Frenk, Mon. Not. R. Astron. Soc. 282, 263 (1996).
[14] E. F. Bunn and M. White, Astrophys. J. 480, 6 (1997). Note that we assume negligible contributions from tensor perturbations to the COBE normalization.
[15] D. Eisenstein and W. Hu, Astrophys. J. 511, 5 (1999).
[16] E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley (1994).
[17] C. Pryke et al., astro-ph/0104409 (2001); P. de Bernardis et al., astro-ph/0105258 (2001); M. Abroe et al., astro-ph/0111011 (2001).
[18] A. A. Starobinsky, JETP Lett. 55, 489 (1992).
[19] Super-Kamiokande collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998).
[20] SNO collaboration, Q. R. Ahmad et al., Phys. Rev. Lett. 87, 1301 (2001).
[21] A. D. Dolgov et al., hep-ph/0201287 (2002).
[22] C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995).
[23] C.S. Trotter, J.N. Winn, and J.N. Hewitt, Astrophys. J. 535, 671 (2000).
[24] EVLA web site http://www.aoc.nrao.edu/evla/
[25] SKA web site http://www.ras.ucalgary.ca/SKA