Modern applications, such as social networking systems and e-commerce platforms are centered around using large-scale storage systems for storing and retrieving data. In the presence of concurrent accesses, these storage systems trade off isolation for performance. The weaker the isolation level, the more behaviors a storage system is allowed to exhibit and it is up to the developer to ensure that their application can tolerate those behaviors. However, these weak behaviors only occur rarely in practice, that too outside the control of the application, making it difficult for developers to test the robustness of their code against weak isolation levels.

This paper presents MonkeyDB, a mock storage system for testing storage-backed applications. MonkeyDB supports a Key-Value interface as well as SQL queries under multiple isolation levels. It uses a logical specification of the isolation level to compute, on a read operation, the set of all possible return values. MonkeyDB then returns a value randomly from this set. We show that MonkeyDB provides good coverage of weak behaviors, which is complete in the limit. We test a variety of applications for assertions that fail only under weak isolation. MonkeyDB is able to break each of those assertions in a small number of attempts.

CCS Concepts:

- Software and its engineering → Software testing and debugging;

Additional Key Words and Phrases: Weak Isolation Levels, Storage Systems, Causal Consistency, Read Committed, Randomized Testing

1 INTRODUCTION

Data storage is no longer about writing data to a single disk with a single point of access. Modern applications require not just data reliability, but also high-throughput concurrent accesses. Applications concerning supply chains, banking, etc. use traditional relational databases for storing and processing data, whereas applications such as social networking software and e-commerce platforms use cloud-based storage systems (such as Azure CosmosDb [Paz 2018], Amazon DynamoDb [DeCandia et al. 2007], Facebook TAO [Bronson et al. 2013], etc.). We use the term storage system in this paper to refer to any such database system/service.

Providing high-throughput processing, unfortunately, comes at an unavoidable cost of weakening the guarantees offered to users. Concurrently-connected clients may end up observing different views of the same data. These “anomalies” can be prevented by using a strong isolation level such as serializability, which essentially offers a single view of the data. However, serializability requires expensive synchronization and incurs a high performance cost. As a consequence, most storage systems use weaker isolation levels, such as Causal Consistency [Akkoorath and Bieniusa 2016; Lamport 1978; Lloyd et al. 2011], Snapshot Isolation [Berenson et al. 1995], Read Committed [Berenson et al. 1995], etc. for better performance. In a recent survey of database administrators [Pavlo 2017],...
86% of the participants responded that most or all of the transactions in their databases execute at read committed isolation level.

A weaker isolation level allows for more possible behaviors than stronger isolation levels. It is up to the developers then to ensure that their application can tolerate this larger set of behaviors. Unfortunately, weak isolation levels are hard to understand or reason about [Adya 1999; Brutschy et al. 2017] and resulting application bugs can cause loss of business [Warszawski and Bailis 2017]. Consider a simple shopping cart application that stores a per-client shopping cart in a key-value store (key is the client ID and value is a multi-set of items). Figure 1 shows procedures for adding an item to the cart (AddItem) and deleting all instances of an item from the cart (DeleteItem). Each procedure executes in a transaction, represented by the calls to Begin and Commit. Suppose that initially, a user \( u \) has a single instance of item \( I \) in their cart. Then the user connects to the application via two different sessions (for instance, via two browser windows), adds \( I \) in one session (AddItem(\( I \), \( u \))) and deletes \( I \) in the other session (DeleteItem(\( I \), \( u \))). With serializability, the cart can either be left in the state \( \{ I \} \) (delete happened first, followed by the add) or \( \emptyset \) (delete happened second). However, with causal consistency (or read committed), it is possible that with two sequential reads of the shopping cart, the cart is empty in the first read (signaling that the delete has succeeded), but there are two instances of \( I \) in the second read! Such anomalies, of deleted items reappearing, have been noted in previous work [DeCandia et al. 2007].

**Fig. 1.** A simple shopping cart service.

**Testing storage-based applications.** This paper addresses the problem of testing code for correctness against weak behaviors: a developer should be able to write a test that runs their application and then asserts for correct behavior. The main difficulty today is getting coverage of weak behaviors during the test. If one runs the test against the actual production storage system, it is very likely to only result in serializable behaviors because of their optimized implementation. For instance, only 0.0004% of all reads performed on Facebook’s TAO storage system were not serializable [Lu et al. 2015]. Emulators, offered by cloud providers for local development, on the other hand, do not support weaker isolation levels at all [cos 2020]. Another option, possible when the storage system is available open-source, is to set it up with a tool like Jepsen [jep 2020] to inject noise (bring down replicas or delay packets on the network). This approach is unable to provide good coverage at the level of client operations [Rahmani et al. 2019] (§8). Another line of work has focussed on finding anomalies by identifying non-serializable behavior (§9). Anomalies, however, do not always
correspond to bugs [Brutschy et al. 2018; Gan et al. 2020]; they may either not be important (e.g., gather statistics) or may already be handled in the application (e.g., checking and deleting duplicate items).

We present MonkeyDB, a mock in-memory storage system meant for testing correctness of storage-backed applications. MonkeyDB supports common APIs for accessing data (key-value updates, as well as SQL queries), making it an easy substitute for an actual storage system. MonkeyDB can be configured with one of several isolation levels. On a read operation, MonkeyDB computes the set of all possible return values allowed under the chosen isolation level, and randomly returns one of them. The developer can then simply execute their test multiple times to get coverage of possible weak behaviors. For the program in Figure 1, if we write a test asserting that two sequential reads cannot return empty-cart followed by $\{I, I\}$, then it takes only 20 runs of the test (on average) to fail the assert. In contrast, the test does not fail when using MySQL with read committed, even after 100k runs.

**Design of MonkeyDB.** MonkeyDB does not rely on stress generation, fault injection, or data replication. Rather, it works directly with a formalization of the given isolation level in order to compute allowed return values.

The theory behind MonkeyDB builds on the axiomatic definitions of isolation levels introduced by Biswas et al. [Biswas and Enea 2019]. These definitions use logical constraints (called axioms) to characterize the set of executions of a key-value store that conform to a particular isolation level (we discuss SQL queries later). These constraints refer to a specific set of relations between events/transactions in an execution that describe control-flow or data-flow dependencies: a program order $po$ between events in the same transaction, a session order $so$ between transactions in the same session\(^1\), and a write-read $wr$ (read-from) relation that associates each read event with a transaction that writes the value returned by the read. These relations along with the events (also called, operations) in an execution are called a history. The history corresponding to the shopping cart anomaly explained above is given on the bottom of Figure 1. Read operations include the read value, and boxes group events from the same transaction. A history describes only the interaction with the key-value store, omitting application side events (e.g., computing the value to be written to a key).

MonkeyDB implements a centralized operational semantics for key-value stores, which is based on these axiomatic definitions. Transactions are executed serially, one after another, the concurrency being simulated during the handling of read events. This semantics maintains a history that contains all the past events (from all transactions/sessions), and write events are simply added to the history. The value returned by a read event is established based on a non-deterministic choice of a write-read dependency (concerning this read event) that satisfies the axioms of the considered isolation level. Depending on the weakness of the isolation level, this makes it possible to return values written in arbitrarily “old” transactions, and simulate any concurrent behavior. For instance, the history in Figure 1 can be obtained by executing AddItem, DeleteItem, and then the two reads (serially). The read in DeleteItem can take its value from the initial state and “ignore” the previously executed AddItem, because the obtained history validates the axioms of causal consistency (or read committed). The same happens for the two later reads in the same session, the first one being able to read from DeleteItem and the second one from AddItem.

We formally prove that this semantics does indeed simulate any concurrent behavior, by showing that it is equivalent to a semantics where transactions are allowed to interleave. In comparison with concrete implementations, this semantics makes it possible to handle a wide range of isolation levels in a uniform way. It only has two sources of non-determinism: the order in which entire

\(^1\)A session is a sequential interface to the storage system. It corresponds to what is also called a connection.
transactions are submitted, and the choice of write-read dependencies in read events. This enables better coverage of possible behaviors, the penalty in performance not being an issue in safety testing workloads which are usually small (see our evaluation).

We also extend our semantics to cover SQL queries as well, by compiling SQL queries down to transactions with multiple key-value reads/writes. A table in a relational database is represented using a set of primary key values (identifying uniquely the set of rows) and a set of keys, one for each cell in the table. The set of primary key values is represented using a set of Boolean key-value pairs that simulate its characteristic function (adding or removing an element corresponds to updating one of these keys to true or false). Then, SQL queries are compiled to read or write accesses to the keys representing a table. For instance, a SELECT query that retrieves the set of rows in a table that satisfy a WHERE condition is compiled to (1) reading Boolean keys to identify the primary key values of the rows contained in the table, (2) reading keys that represent columns used in the WHERE condition, and (3) reading all the keys that represent cells in a row satisfying the WHERE condition. This rewriting contains the minimal set of accesses to the cells of a table that are needed to ensure the conventional specification of SQL. It makes it possible to “export” formalizations of key-value store isolation levels to SQL transactions.

Contributions. This paper makes the following contributions:

- We define an operational semantics for key-value stores under various isolation levels, which simulates all concurrent behaviors with executions where transactions execute serially (§4) and which is based on the axiomatic definitions in [Biswas and Enea 2019] (and outlined in §3),
- We broaden the scope of the key-value store semantics to SQL transactions using a compiler that rewrites SQL queries to key-value accesses (§5),
- The operational semantics and the SQL compiler are implemented in a tool called MonkeyDB (§6). It randomly resolves possible choices to provide coverage of weak behaviors. It supports both a key-value interface as well as SQL, making it readily compatible with any storage-backed application.
- We present an evaluation of MonkeyDB on several applications, showcasing its superior coverage of weak behaviors as well as bug-finding abilities (§7, §8).²

2 PROGRAMMING LANGUAGE

Figure 2 lists the definition of two simple programming languages that we use to represent applications running on top of Key-Value or SQL stores, respectively. A program is a set of sessions running in parallel, each session being composed of a sequence of transactions. Each transaction is delimited by begin and commit instructions³, and its body contains instructions that access the store, and manipulate a set of local variables Vars ranged over using x, y, . . .

In case of a program running on top of a Key-Value store, the instructions can be reading the value of a key and storing it to a local variable x (x := read(k)), writing the value of a local variable x to a key (write(k, x)), or an assignment to a local variable x. The set of values of keys or local variables is denoted by Vals. Assignments to local variables use expressions interpreted as values whose syntax is left unspecified. Each of these instructions can be guarded by a Boolean condition φ(Ʌ) over a set of local variables Ʌ (their syntax is not important). Other constructs like while loops can be defined in a similar way. Let P_kv denote the set of programs where a transaction body can contain only such instructions.

²Source code of our benchmarks is available as supplementary material.
³For simplicity, we assume that all the transactions in the program commit. Aborted transactions can be ignored when reasoning about safety because their effects should be invisible to other transactions.
We present the axiomatic framework introduced in [Biswas and Enea 2019] for defining isolation
along with a strict total order
\( r \)
:
\[ \text{read} \]

where
\( k \)
to an arbitrary set \( \text{OpId} \). We omit operation identifiers when they are not important.

\[ k \in \text{Keys} \quad x \in \text{Vars} \quad \text{tab} \in \mathbb{T} \quad \tilde{c}, \tilde{c}_1, \tilde{c}_2 \in \mathbb{C}^* \]

\[ \text{Prog} ::= \text{Sess} \mid \text{Sess} \mid \text{Prog} \]

\[ \text{Sess} ::= \text{Trans} \mid \text{Trans; Sess} \]

\[ \text{Trans} ::= \text{begin; Body; commit} \]

\[ \text{Body} ::= \text{Instr} \mid \text{Instr; Body} \]

\[ \text{Instr} ::= \text{InstrKV} \mid \text{InstrSQL} \mid x := e \mid \text{if}(\phi(\tilde{x}))\{\text{Instr}\} \]

\[ \text{InstrKV} ::= x := \text{read}(k) \mid \text{write}(k, x) \]

\[ \text{InstrSQL} ::= \text{SELECT} \; \tilde{c}_1 \; \text{AS} \; x \; \text{FROM} \; \text{tab} \; \text{WHERE} \; \phi(\tilde{c}_2) \mid \text{INSERT} \; \text{INTO} \; \text{tab} \; \text{VALUES} \; \tilde{x} \mid \text{DELETE} \; \text{FROM} \; \text{tab} \; \text{WHERE} \; \phi(\tilde{c}) \mid \text{UPDATE} \; \text{tab} \; \text{SET} \; \tilde{c}_1 = \tilde{x} \; \text{WHERE} \; \phi(\tilde{c}_2) \]

Fig. 2. Program syntax. The set of all keys is denoted by \( \text{Keys} \), \( \text{Vars} \) denotes the set of local variables, \( \mathbb{T} \) the set of table names, and \( \mathbb{C} \) the set of column names. We use \( \phi \) to denote Boolean expressions, and \( e \) to denote expressions interpreted as values. We use \( \mathbb{R}^* \) to denote vectors of elements.

For programs running on top of SQL stores, the instructions include simplified versions of standard SQL instructions and assignments to local variables. These programs run in the context of a database schema which is a (partial) function \( S : \mathbb{T} \to 2^\mathbb{C} \) mapping table names in \( \mathbb{T} \) to sets of column names in \( \mathbb{C} \). The SQL store is an instance of a database schema \( S \), i.e., a function \( D : \text{dom}(S) \to 2^\mathbb{R} \) mapping each table \( \text{tab} \) in the domain of \( S \) to a set of rows of \( \text{tab} \), i.e., functions \( r : S(\text{tab}) \to \text{Vals} \). We use \( \mathbb{R} \) to denote the set of all rows. The SELECT instruction retrieves the columns \( \tilde{c}_1 \) from the set of rows of \( \text{tab} \) that satisfy \( \phi(\tilde{c}_2) \) (\( \tilde{c}_2 \) denotes the set of columns used in this Boolean expression), and stores them into a variable \( x \). INSERT adds a new row to \( \text{tab} \) with values \( \tilde{x} \), and DELETE deletes all rows from \( \text{tab} \) that satisfy a condition \( \phi(\tilde{c}) \). The UPDATE instruction assigns the columns \( \tilde{c}_1 \) of all rows of \( \text{tab} \) that satisfy \( \phi(\tilde{c}_2) \) with values in \( \tilde{x} \). Let \( \mathcal{P}_\text{SQL} \) denote the set of programs where a transaction body can contain only such instructions.

3 ISOLATION LEVELS FOR KEY-VALUE STORES

We present the axiomatic framework introduced in [Biswas and Enea 2019] for defining isolation levels\(^4\) in Key-Value stores. Isolation levels are defined as logical constraints, called axioms, over histories, which are an abstract representation of the interaction between a program and the store in a concrete execution.

3.1 Histories

Programs interact with a Key-Value store by issuing transactions formed of read and write instructions. The effect of executing one such instruction is represented using an operation, which is an element of the set

\[ \text{Op} = \{ \text{read}_i(k, v), \text{write}_i(k, v) : i \in \text{OpId}, k \in \text{Keys}, v \in \text{Vals} \} \]

where \( \text{read}_i(k, v) \) (resp., \( \text{write}_i(k, v) \)) corresponds to reading a value \( v \) from a key \( k \) (resp., writing \( v \) to \( k \)). Each operation is associated with an identifier \( i \) from an arbitrary set \( \text{OpId} \). We omit operation identifiers when they are not important.

\[ \text{Definition 3.1. A transaction log } \langle t, O, \text{po} \rangle \text{ is a transaction identifier } t \text{ and a finite set of operations } O \text{ along with a strict total order } \text{po on } O, \text{ called program order.} \]

\(^4\)Isolation levels are called consistency models in [Biswas and Enea 2019].
The program order $\text{po}$ represents the order between instructions in the body of a transaction. We assume that each transaction log is well-formed in the sense that if a read of a key $k$ is preceded by a write to $k$ in $\text{po}$, then it should return the value written by the last write to $k$ before the read (w.r.t. $\text{po}$). This property is implicit in the definition of every isolation level that we are aware of. For simplicity, we may use the term transaction instead of transaction log. The set of all transaction logs is denoted by $\text{Tlogs}$.

The set of read operations $\text{read}(k, \_\_)$ in a transaction log $t$ that are not preceded by a write to $k$ in $\text{po}$ is denoted by $\text{reads}(t)$. As mentioned above, the other read operations take their values from writes in the same transaction and their behavior is independent of other transactions. Also, the set of write operations $\text{write}(k, \_\_)$ in $t$ that are not followed by other writes to $k$ in $\text{po}$ is denoted by $\text{writes}(t)$. If a transaction contains multiple writes to the same key, then only the last one (w.r.t. $\text{po}$) can be visible to other transactions (w.r.t. any isolation level that we are aware of). The extension to sets of transaction logs is defined as usual. Also, we say that a transaction log $t$ writes a key $k$, denoted by $t$ writes $k$, when $\text{write}(k, v) \in \text{writes}(t)$ for some $i$ and $v$.

A history contains a set of transaction logs (with distinct identifiers) ordered by a (partial) session order so that represents the order between transactions in the same session$^5$. It also includes a write-read relation (also called read-from) that “justifies” read values by associating each read to a transaction that wrote the value returned by the read.

Definition 3.2. A history $\langle T, \text{so}, \text{wr} \rangle$ is a set of transaction logs $T$ along with a strict partial session order so, and a write-read relation $\text{wr} \subseteq T \times \text{reads}(T)$ such that the inverse of $\text{wr}$ is a total function, and if $(t, \text{read}(k, v)) \in \text{wr}$, then $\text{write}(k, v) \in t$, and so $\cup \text{wr}$ is acyclic.

To simplify the technical exposition, we assume that every history includes a distinguished transaction log writing the initial values of all keys. This transaction log precedes all the other transaction logs in so. We use $h, h_1, h_2, \ldots$ to range over histories. The set of transaction logs $T$ in a history $h = \langle T, \text{so}, \text{wr} \rangle$ is denoted by $\text{Tlogs}(h)$.

For a key $k$, $\text{wr}_k$ denotes the restriction of $\text{wr}$ to reads of $k$, i.e., $\text{wr}_k = \text{wr} \cap (T \times \{ \text{read}(k, v) \mid v \in \text{Vals}\})$. Moreover, we extend the relations $\text{wr}$ and $\text{wr}_k$ to pairs of transactions by $\langle t_1, t_2 \rangle \in \text{wr}$, resp., $\langle t_1, t_2 \rangle \in \text{wr}_k$, iff there exists a read operation $\text{read}(k, v) \in \text{reads}(t_2)$ such that $\langle t_1, \text{read}(k, v) \rangle \in \text{wr}$, resp., $\langle t_1, \text{read}(k, v) \rangle \in \text{wr}_k$. We say that the transaction log $t_1$ is read by the transaction log $t_2$ when $\langle t_1, t_2 \rangle \in \text{wr}$.

3.2 Axiomatic Framework

A history is said to satisfy a certain isolation level if there exists a strict total order $\text{co}$ on its transaction logs, called commit order, which extends the write-read relation and the session order, and which satisfies certain properties. These properties, called axioms, relate the commit order with the session-order and the write-read relation in the history. They are defined as first-order formulas$^6$ of the following form:

$$\forall k, \forall t_1 \neq t_2, \forall \alpha.
\langle t_1, \alpha \rangle \in \text{wr}_k \land t_2 \text{ writes } k \land \phi(t_2, \alpha) \Rightarrow \langle t_2, t_1 \rangle \in \text{co}$$

(1)

where $\phi$ is a property relating $t_2$ and $\alpha$ (i.e., the read or the transaction reading from $t_1$) that varies from one axiom to another. Intuitively, this axiom schema states the following: in order for $\alpha$ to read specifically $t_1$’s write on $x$, it must be the case that every $t_2$ that also writes $x$ and satisfies $\phi(t_2, \alpha)$ was committed before $t_1$. The property $\phi$ relates $t_2$ and $\alpha$ using the relations in a history and the

$^5$In the context of our programming language, so would be a union of total orders. This constraint is not important for defining isolation levels.

$^6$These formulas are interpreted on tuples $\langle h, \text{co} \rangle$ of a history $h$ and a commit order $\text{co}$ on the transactions in $h$ as usual.
commit order. Figure 3 shows the axioms defining three isolation levels: Read Committed, Causal Consistency, and Serializability (see [Biswas and Enea 2019] for axioms defining Read Atomic, Prefix, and Snapshot Isolation).

For instance, Read Committed [Berenson et al. 1995] requires that every read returns a value written in a committed transaction, and also, that the reads in the same transaction are “monotonic”, i.e., they do not return values that are older, w.r.t. the commit order, than values read in the past. While the first condition holds for every history (because of the surjectivity of wr), the second condition is expressed by the axiom Read Committed in Figure 3a, which states that for any transaction \(t_1\) writing a key \(k\) that is read in a transaction \(t\), the set of transactions \(t_2\) writing \(k\) and read previously in the same transaction (these reads may concern other keys) must precede \(t_1\) in commit order. For instance, Figure 4a shows a history and a (partial) commit order that does not satisfy this axiom because \(\text{read}(k_1, 1)\) returns the value written in a transaction “older” than the transaction read in the previous read \((k_2, 2)\).

The axiom defining Causal Consistency [Lamport 1978] states that for any transaction \(t_1\) writing a key \(k\) that is read in a transaction \(t_s\), the set of \((\text{wr} \cup \text{so})^+\) predecessors of \(t_3\) writing \(k\) must precede \(t_1\) in commit order (\((\text{wr} \cup \text{so})^+\) is usually called the causal order). A violation of this axiom can be found in Figure 4b: the transaction \(t_2\) writing 2 to \(k_1\) is a \((\text{wr} \cup \text{so})^+\) predecessor of the transaction \(t_3\) reading 1 from \(k_1\) because the transaction \(t_4\), writing 1 to \(k_2\), reads \(k_1\) from \(t_2\) and \(t_3\) reads \(k_2\) from \(t_4\). This implies that \(t_2\) should precede in commit order the transaction \(t_1\) writing 1 to \(k_1\), which again, is inconsistent with the write-read relation \((t_2\text{ reads from } t_1)\).

Finally, Serializability [Papadimitriou 1979] requires that for any transaction \(t_1\) writing to a key \(k\) that is read in a transaction \(t_s\), the set of \(\text{co}\) predecessors of \(t_3\) writing \(k\) must precede \(t_1\) in commit order. This ensures that each transaction observes the effects of all the \(\text{co}\) predecessors.
begin;
write($k_1$, 1);
x2 = read($k_2$);
commit

write($k_2$, 1);
x1 = read($k_1$);
commit

write($k_1$, 0)
write($k_2$, 0)
write($k_1$, 1)
read($k_2$, 0)
read($k_1$, 0)

write($k_2$, 1)
read($k_1$, 0)
write($k_1$, 0)
write($k_2$, 0)

Fig. 5. The Causal semantics on the program in (a), assuming that the transaction on the left is scheduled first.

**Definition 3.3.** For an isolation level $I$ defined by a set\(^7\) of axioms $X$, a history $h = \langle T, so, wr \rangle$ satisfies $I$ iff there is a strict total order $co$ s.t. $wr \cup so \subseteq co$ and $\langle h, co \rangle$ satisfies $X$.

4 **OPERATIONAL SEMANTICS FOR $P_{KV}$**

We define a small-step operational semantics for Key-Value store programs, which is parametrized by an isolation level $I$. Transactions are executed *serially* one after another, and the values returned by read operations are decided using the axiomatic definition of $I$. The semantics maintains a history of previously executed operations, and the value returned by a read is chosen non-deterministically as long as extending the current history with the corresponding write-read dependency satisfies the axioms of $I$. We show that this semantics is sound and complete for any natural isolation level $I$, i.e., it generates precisely the same set of histories as a baseline semantics where transactions can interleave arbitrarily and the read operations can return arbitrary values as long as they can be proved to be correct at the end of the execution.

4.1 **Definition of the Operational Semantics**

We use the program in Figure 5a to give an overview of our semantics, assuming Causal Consistency. This program has two concurrent transactions whose reads can both return the initial value 0, which is not possible under Serializability.

Our semantics executes transactions in their entirety one after another (without interleaving them), maintaining a history that contains all the executed operations. We assume that the transaction on the left executes first. Initially, the history contains a fictitious transaction log that writes the initial value 0 to all keys, and that will precede all the transaction logs created during the execution in session order.

Executing a write instruction consists in simply appending the corresponding write operation to the log of the current transaction. For instance, executing the first write (and begin) in our example results in adding a transaction log that contains a write operation (see Figure 5b). The execution continues with the read instruction from the same transaction, and it cannot switch to the other transaction.

The execution of a read instruction consists in choosing non-deterministically a write-read dependency that validates Causal when added to the current history. In our example, executing read($k_2$) results in adding a write-read dependency from the transaction log writing initial values, which determines the return value of the read (see Figure 5c). This choice makes the obtained history satisfy Causal.

\(^7\)Isolation levels like Snapshot Isolation require more than one axiom.

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which are defined as tuples containing the following:

\[ (\text{if-true, if-false}) \]

if an if-true

the body of the transaction.

is no other live transaction (so and so to the last transaction log in so is appended to so contains a representation of so by associates session identifiers with sequences of transaction logs. Two transaction logs are ordered so Similarly, the session order so by fresh history so that remains to be executed from the current transaction, and

\[ \text{read-local} \]

write \( (k, v) \) is the last write on \( k \) in \( t \) w.r.t. \( \text{po} \) i fresh

\[ h, j, \gamma, x := \text{read}(k); B, P \Rightarrow_1 h \oplus_j \text{read}_i(k, v), j, \gamma[x \mapsto v], B, P \]

READ-EXTERN

\[ h = (T, \text{so}, \text{wr}) \]

\[ t \text{ is the id of the last transaction log in } \text{so}(j) \]

\[ \text{write}(k, v) \in \text{writes}(t') \text{ with } t' \in T \text{ and } t' \neq t \]

\[ \text{i fresh} \]

\[ h' = (h \oplus_j \text{read}_i(k, v)) \oplus \text{wr}(t', \text{read}_i(k, v)) \]

\[ h' \text{ satisfies } I \]

\[ h, j, \gamma, x := \text{read}(k); B, P \Rightarrow_1 h', j, \gamma[x \mapsto v], B, P \]

Fig. 6. Operational semantics for \( P_{\text{KV}} \) programs under isolation level \( I \). For a function \( f : A \rightarrow B, f[a \mapsto b] \) denotes the function \( f' : A \rightarrow B \) defined by \( f'(c) = f(c) \), for every \( c \neq a \) in the domain of \( f \), and \( f'(a) = b \).

The second transaction executes in a similar manner. When executing its read instruction, the chosen write-read dependency is again related to the transaction log writing initial values (see Figure 5d). This choice is valid under Causal. Since a read must not read from the preceding transaction, this semantics is able to simulate all the “anomalies” of a weak isolation level (this execution being an example).

Formally, the operational semantics is defined as a transition relation \( \Rightarrow_1 \) between \textit{configurations}, which are defined as tuples containing the following:

- history \( h \) storing the operations executed in the past,
- identifier \( j \) of the current session,
- local variable valuation \( \gamma \) for the current transaction,
- code \( B \) that remains to be executed from the current transaction, and
- sessions/transactions \( P \) that remain to be executed from the original program.

For readability, we define a program as a partial function \( P : \text{SessId} \rightarrow \text{Sess} \) that associates session identifiers in \text{SessId} with concrete code as defined in Figure 2 (i.e., sequences of transactions). Similarly, the session order \( \text{so} \) in a history is defined as a partial function \( \text{so} : \text{SessId} \rightarrow \text{Tlogs}^* \) that associates session identifiers with sequences of transaction logs. Two transaction logs are ordered by \( \text{so} \) if one occurs before the other in some sequence \( \text{so}(j) \) with \( j \in \text{SessId} \).

Before presenting the definition of \( \Rightarrow_1 \), we introduce some notation. Let \( h \) be a history that contains a representation of \( \text{so} \) as above. We use \( h \oplus_j (t, O, \text{po}) \) to denote a history where \( \langle t, O, \text{po} \rangle \) is appended to \( \text{so}(j) \). Also, for an operation \( o, h \oplus o \) is the history obtained from \( h \) by adding \( o \) to the last transaction log in \( \text{so}(j) \) and as a last operation in the program order of this log (i.e., if \( \text{so}(j) = \sigma; \langle t, O, \text{po} \rangle \), then the session order \( \text{so}' \) of \( h \oplus o \) is defined by \( \text{so}'(k) = \text{so}(k) \) for all \( k \neq j \) and \( \text{so}(j) = \sigma; \langle t, O \cup o, \text{po} \cup \{(o', o) : o' \in O\} \rangle \)). Finally, for a history \( h = \langle T, \text{so}, \text{wr} \rangle \), \( h \oplus \text{wr}(t, o) \) is the history obtained from \( h \) by adding \( (t, o) \) to the write-read relation.

Figure 6 lists the rules defining \( \Rightarrow_1 \). The \text{spawn} rule starts a new transaction, provided that there is no other live transaction (\( B = \epsilon \)). It adds an empty transaction log to the history and schedules the body of the transaction. \text{if-true} and \text{if-false} check the truth value of a Boolean condition of an \text{if} conditional. \text{write} corresponds to a write instruction and consists in simply adding a write


**SPAWN**
\[
\text{t fresh} \quad P(j) = \text{begin;}\text{commit;}S \quad \overline{B}(j) = \epsilon
\]
\[h, \gamma, \overline{B}, P \Rightarrow h \oplus_j \langle t, \emptyset, \emptyset \rangle, \gamma[j \mapsto \emptyset], \overline{B}[j \mapsto \text{Body}], P[j \mapsto S]\]

**READ-EXTERN**
\[
\overline{B}(j) = x := \text{read}(k); B \quad h = (T, \text{so}, \text{wr}) \quad t \text{ is the id of the last transaction log in } \text{so}(j)
\]
\[
\text{write}(k, v) \in \text{writes}(t') \text{ with } t' \in \text{compTrans}(h, \overline{B}) \text{ and } t \neq t'
\]
\[
i \text{fresh} \quad h' = (h \oplus_j \text{read}_i(k, v)) \oplus \text{wr}(t', \text{read}_i(k, v))
\]
\[h, \gamma, \overline{B}, P \Rightarrow h', \gamma[(j, x) \mapsto v], \overline{B}[j \mapsto B], P\]

Fig. 7. A baseline operational semantics for \(P_{KV}\) programs. Above, \(\text{compTrans}(h, \overline{B})\) denotes the set of transaction logs in \(h\) that excludes those corresponding to live transactions, i.e., transaction logs \(t'' \in T\) such that \(t''\) is the last transaction log in some \(\text{so}(j')\) and \(\overline{B}(j') \neq \epsilon\).

operation to the current history. READ-LOCAL and READ-EXTERN concern read instructions. READ-LOCAL handles the case where the read follows a write on the same key \(k\) in the same transaction: the read returns the value written by the last write on \(k\) in the current transaction. Otherwise, READ-EXTERN corresponds to reading a value written in another transaction \(t'\) (\(t\) is the id of the log of the current transaction). The transaction \(t'\) is chosen non-deterministically as long as extending the current history with the write-read dependency associated to this choice leads to a history that still satisfies \(I\).

An initial configuration for program \(P\) contains the program \(P\) along with a history \(h = (\{t_0\}, \emptyset, \emptyset)\), where \(t_0\) is a transaction log containing only writes that write the initial values of all keys, and empty current transaction code (\(B = \epsilon\)). An execution of a program \(P\) under an isolation level \(I\) is a sequence of configurations \(c_0c_1 \ldots c_n\) where \(c_0\) is an initial configuration for \(P\), and \(c_m \Rightarrow_I c_{m+1}\), for every \(0 \leq m < n\). We say that \(c_n\) is \(I\)-reachable from \(c_0\). The history of such an execution is the history \(h\) in the last configuration \(c_n\). A configuration is called final if it contains the empty program (\(P = \emptyset\)). Let \(\text{hist}_I(P)\) denote the set of all histories of an execution of \(P\) under \(I\) that ends in a final configuration.

### 4.2 Correctness of the Operational Semantics

We define the correctness of \(\Rightarrow_I\) in relation to a baseline semantics where transactions can interleave arbitrarily, and the values returned by read operations are only constrained to come from committed transactions. This semantics is represented by a transition relation \(\Rightarrow\), which is defined by a set of rules that are analogous to \(\Rightarrow_I\). Since it allows transactions to interleave, a configuration contains a history \(h\), the sessions/transactions \(P\) that remain to be executed, and:

- a valuation map \(\gamma\) that records local variable values in the current transaction of each session (\(\gamma\) associates identifiers of sessions that have live transactions with valuations of local variables),
- a map \(\overline{B}\) that stores the code of each live transaction (associating session identifiers with code).

Figure 7 lists some rules defining \(\Rightarrow\) (the others can be defined in a similar manner). SPANW\(^*\) starts a new transaction in a session \(j\) provided that this session has no live transaction (\(\overline{B}(j) = \epsilon\)). Compared to SPAN in Figure 6, this rule allows unfinished transactions in other sessions. READ-EXTERN\(^*\) does not check conformance to \(I\), but it allows a read to only return a value written in a completed (committed) transaction. In this work, we consider only isolation levels satisfying this constraint. Executions, initial and final configurations are defined as in the case of \(\Rightarrow_I\). The history of an execution is still defined as the history in the last configuration. Let \(\text{hist}_e(P)\) denote the set of all histories of an execution of \(P\) w.r.t. \(\Rightarrow\) that ends in a final configuration.

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We define an operational semantics for SQL programs (in $P$) such that the negation/complement of a relation). An axiom as in (1) is called monotonic, i.e., the set of models in the context of a pair $\langle h_2, co_2 \rangle$ is a superset of the set of models in the context of a prefix $\langle h_1, co_1 \rangle$ of $\langle h_2, co_2 \rangle$. For instance, the property $\phi$ in the axiom defining Causal is $(t_2, \alpha) \in (wr \cup so)^*$, which is clearly monotonic. In general, standard isolation levels are defined using a property $\alpha$ of the form $(t_2, \alpha) \in R$ where $R$ is an expression built from the relations $po$, $so$, $wr$, and $co$ using (reflexive and transitive closure and composition of relations [Biswas and Enea 2019]). Such properties are monotonic in general (they would not be if those expressions would use the negation/complement of a relation). An axiom as in (1) is called monotonic when the property $\phi$ is monotonic.

The following theorem shows that $hist_I(P)$ is precisely the set of histories under the baseline semantics, which satisfy $I$ (the validity of the reads is checked at the end of an execution), provided that the axioms of $I$ are monotonic.

**Theorem 4.1.** For any isolation level $I$ defined by a set of monotonic axioms, $hist_I(P) = \{ h \in hist_s(P) : h \text{ satisfies } I \}$.

The $\subseteq$ direction follows mostly from the fact that $\Rightarrow_I$ is more constrained than $\Rightarrow$. For the opposite direction, given a history $h$ that satisfies $I$, i.e., there exists a commit order $co$ such that $\langle h, co \rangle$ satisfies the axioms of $I$, we can show that there exists an execution under $\Rightarrow_I$ with history $h$, where transactions execute serially in the order defined by $co$. The prefix closure property is used to prove that $\text{read-extern}$ transitions are enabled (these transitions get executed with a prefix of $h$). See the supplementary material for more details.

It can also be shown that $\Rightarrow_I$ is deadlock-free for every natural isolation level (e.g., Read Committed, Causal Consistency, Snapshot Isolation, and Serializability), i.e., every read can return some value satisfying the axioms of $I$ at the time when it is executed (independently of previous choices).

## 5 COMPILING SQL TO KEY-VALUE API

We define an operational semantics for SQL programs (in $P_{SQL}$) based on a compiler that rewrites SQL queries to Key-Value $\text{read}$ and $\text{write}$ instructions. For presentation reasons, we use an intermediate representation where each table of a database instance is represented using a $\text{set}$ variable that stores values of the primary key$^8$ (identifying uniquely the rows in the table) and a set of key-value pairs, one for each cell in the table. In a second step, we define a rewriting of the API used to manipulate set variables into Key-Value $\text{read}$ and $\text{write}$ instructions.

**Intermediate Representation.** Let $S : T \rightarrow 2^C$ be a database schema (recall that $T$ and $C$ are the set of table names and column names, resp.). For each table $\text{tab}$, let $\text{tab.pkey}$ be the name of the primary key column. We represent an instance $D : \text{dom}(S) \rightarrow 2^R$ using:

- for each table $\text{tab}$, a set variable $\text{tab}$ (with the same name) that contains the primary key value $r(\text{tab.pkey})$ of every row $r \in D(\text{tab})$,
- for each row $r \in D(\text{tab})$ with primary key value $pkeyVal = r(\text{tab.pkey})$, and each column $c \in S(\text{tab})$, a key $\text{tab.pkeyVal.c}$ associated with the value $r(c)$.

$^8$For simplicity, we assume that primary keys correspond to a single column in the table.
Table:

|   |   |   |
|---|---|---|
| Id | Name  | City    |
| 1  | Alice | Paris   |
| 2  | Bob   | Bangalore |
| 3  | Charles | Bucharest |

Intermediate representation:

\[ A = \{1, 2, 3\} \]
\[ A.1.Id: 1, A.1.Name: Alice, A.1.City: Paris \]
\[ A.2.Id: 2, A.2.Name: Bob, A.2.City: Bangalore \]
\[ A.3.Id: 3, A.3.Name: Charles, A.3.City: Bucharest \]

**Fig. 8.** Representing tables with set variables and key-value pairs. We write a key-value pair as key:value.

**SELECT/DELETE/UPDATE**

\[
\text{rows := elements}(\text{tab})
\]
\[
\text{for } (\text{let pkeyVal of rows }) \{ \\
\text{\hspace{1cm} for } (\text{let c of } \vec{c}_1 ) \{ \\
\text{\hspace{2cm} val[c] := read}(\text{tab.pkeyVal.c}) \\
\text{\hspace{2cm} if ( } \phi[c] \mapsto \text{val[c]} : c \in \vec{c}_2 \text{ ) true } \\
\text{\hspace{2cm} // SELECT } \vec{c}_1 \text{ AS } x \text{ FROM } \text{tab WHERE } \phi(\vec{c}_2) \\
\text{\hspace{2cm} for } (\text{let c of } \vec{c}_1 ) \\
\text{\hspace{3cm} out[c] := read}(\text{tab.pkeyVal.c}) \\
\text{\hspace{3cm} x := x \cup out } \\
\text{\hspace{2cm} // DELETE FROM } \text{tab WHERE } \phi(\vec{c}_2) \\
\text{\hspace{2cm} remove}(\text{tab}, \text{pkeyVal}); \\
\text{\hspace{2cm} // UPDATE } \text{tab SET } \vec{c}_1 = \vec{x} \text{ WHERE } \phi(\vec{c}_2) \\
\text{\hspace{2cm} for } (\text{let c of } \vec{c}_1 ) \\
\text{\hspace{3cm} write}( \text{tab.pkeyVal.c}, \gamma(\vec{x}[c]) ) \\
\text{\hspace{1cm} } \}
\]
\[
\text{INSERT INTO } \text{tab VALUES } \vec{x} \\
\text{pkeyVal := } \gamma(\vec{x}[0]) \\
\text{if ( } \text{add}(\text{tab}, \text{pkeyVal}) \text{ ) } \{ \\
\text{\hspace{1cm} for } (\text{let c of } S(\text{tab}) ) \{ \\
\text{\hspace{2cm} write}( \text{tab.pkeyVal.c}, \gamma(\vec{x}[c]) ) \\
\text{\hspace{1cm} } \}
\]

**Fig. 9.** Compiling SQL queries to the intermediate representation. Above, \( \gamma \) is a valuation of local variables. Also, in the case of \text{INSERT}, we assume that the first element of \( \vec{x} \) represents the value of the primary key.

**Example 5.1.** The table \( A \) on the left of Figure 8, where the primary key is defined by the Id column, is represented using a set variable \( A \) storing the set of values in the column Id, and one key-value pair for each cell in the table.

Figure 9 lists our rewriting of SQL queries over a database instance \( D \) to programs that manipulate the set variables and key-value pairs described above. This rewriting contains the minimal set of accesses to the cells of a table that are needed to implement an SQL query according to its conventional specification. To manipulate set variables, we use \text{add} and \text{remove} for adding and removing elements, respectively (returning true or false when the element is already present or deleted from the set, respectively), and \text{elements} that returns all of the elements in the input set\(^9\).

SELECT, DELETE, and UPDATE start by reading the contents of the set variable storing primary key values and then, for every row, the columns in \( \vec{c}_2 \) needed to check the Boolean condition \( \phi \) (the keys corresponding to these columns). For every row satisfying this Boolean condition, SELECT continues by reading the keys associated to the columns that need to be returned, DELETE removes the primary key value associated to this row from the set \( \text{tab} \), and UPDATE writes to the keys corresponding to the columns that need to be updated. In the case of UPDATE, we assume that the values of the variables in \( \vec{x} \) are obtained from a valuation \( \gamma \) (this valuation would be maintained by the operational semantics of the underlying Key-Value store). \text{INSERT} adds a new primary key value to the set variable \( \text{tab} \) (the call to \text{add} checks whether this value is unique) and then writes to the keys representing columns of this new row.

\(^9\)\text{add}(s, e) and \text{remove}(s, e) add and remove the element \( e \) from \( s \), respectively. \text{elements}(s) returns the content of \( s \).
Manipulating Set Variables. Based on the standard representation of a set using its characteristic function, we implement each set variable `tab` using a set of keys `tab.has.pkeyVal`, one for each value `pkeyVal \in \text{Vals}`. These keys are associated with Boolean values, indicating whether `pkeyVal` is contained in `tab`. In a concrete implementation, this set of keys need not be fixed a-priori, but can grow during the execution with every new instance of an `INSERT`. Figure 10 lists the implementations of `add`/`elements`, which are self-explanatory (`remove` is analogous).

6 IMPLEMENTATION

We implemented MonkeyDB\textsuperscript{10} to support an interface common to most storage systems. Operations can be either key-value (KV) updates (to access data as a KV map) or SQL queries (to access data as a relational database). MonkeyDB supports transactions as well; a transaction can include multiple operations. Figure 11 shows the architecture of MonkeyDB. A client can connect to MonkeyDB over a TCP connection, as is standard for SQL databases\textsuperscript{11}. This offers a plug-and-play experience when using standard frameworks such as JDBC [Java Platform [n. d.]]. Client applications can also use MonkeyDB as a library in order to directly invoke the storage APIs, or interact with it via HTTP requests, with JSON payloads.

MonkeyDB contains a SQL-To-KV compiler that parses an input query\textsuperscript{12}, builds its Abstract Syntax Tree (AST) and then applies the rewriting steps described in Section 5 to produce an equivalent sequence of KV API calls (`read()` and `write()`). It uses a hashing routine (`hash`) to generate unique keys corresponding to each cell in a table. For instance, in order to insert a value `v` for a column `c` in a particular row with primary key value `pkeyVal`, of a table `tab`, we invoke `write(hash(tab, pkeyVal, c), v)`. We currently support only a subset of the standard SQL operators. For instance, nested queries or join operators are unsupported; these can be added in the future with more engineering effort.

MonkeyDB schedules transactions from different sessions one after the other using a single global lock. Internally, it maintains execution state as a history consisting of a set of transaction logs, write-read relations and a partial session order (as discussed in §3). On a `read()`, MonkeyDB first collects a set of possible writes present in transaction log that can potentially form write-read (read-from) relationships, and then invokes the consistency checker (Figure 11) to confirm validity under the chosen isolation level. Finally, it randomly returns one of the values associated with valid writes. A user can optionally instruct MonkeyDB to only select from the set of latest valid write per session. This option helps limit weak behaviors for certain reads.

The implementation of our consistency checker is based on prior work [Biswas and Enea 2019]. It maintains the write-read relation as a graph, and detects cycles (isolation-level violations) using DFS traversals on the graph. The consistency checker is an independent and pluggable module: we have one for Read Committed and one for Causal Consistency, and more can be added in the future.

\textsuperscript{10}We plan to make MonkeyDB available open-source soon.
\textsuperscript{11}We support the MySQL client-server protocol using https://github.com/jonhoo/msql-srv.
\textsuperscript{12}We use https://github.com/ballista-compute/sqlparser-rs

Fig. 10. Manipulating set variables using key-value pairs.
7 EVALUATION: MICROBENCHMARKS

We consider a set of micro-benchmarks inspired from real-world applications (§7.1) and evaluate the number of test iterations required to fail an invalid assertion (§7.2). We also measure the coverage of weak behaviors provided by MonkeyDB (§7.3). Each of these applications were implemented based on their specifications described in prior work; they all use MonkeyDB as a library, via its KV interface.

7.1 Applications

Twitter [Twissandra 2020]. This is based on a social-networking application that allows users to create a new account, follow, unfollow, tweet, browse the newsfeed (tweets from users you follow) and the timeline of any particular user. Figure 12 shows the pseudo code for two operations. A user can access twitter from multiple clients (sessions), which could lead to unexpected behavior under weak isolation levels. Consider the following scenario with two users, A and B where user A is accessing twitter from two different sessions, S₁ and S₂. User A views the timeline of user B from one session (S₁:Timeline(B)) and decides to follow B through another session (S₂:Follow(A, B)). Now when user A visits their timeline or newsfeed (S₂:NewsFeed(A)), they expect to see all the tweets of B that were visible via Timeline in session S₁. But under weak isolation levels, this does not always hold true and there could be missing tweets.

Shopping Cart [Sivaramakrishnan et al. 2015]. This application allows a user to add, remove and change quantity of items from different sessions. It also allows the user to view all items present in the shopping cart. The pseudo code and an unexpected behavior under weak isolation levels were discussed in §1, Figure 1.

Courseware [Nair et al. 2020]. This is an application for managing students and courses, allowing students to register, de-register and enroll for courses. Courses can also be created or deleted. Courseware maintains the current status of students (registered, de-registered), courses (active, deleted) as well as enrollments. Enrollment can contain only registered students and active courses, subject to the capacity of the course.

Under weak isolation, it is possible that two different students, when trying to enroll concurrently, will both succeed even though only one spot was left in the course. Another example that breaks the
// Get user's tweets
Timeline(user u) {
  Begin()
  key = "tweets:" + u.id
  T = read(key)
  Commit()
  return sortByTime(T)
}

// Get following users' tweets
NewsFeed(user u) {
  Begin()
  FW = read("following:"+ u.id)
  NF = {}
  foreach v ∈ FW:
    T = read("tweets:"+ v.id)
    NF = NF ∪ T
  Commit()
  return sortByTime(NF)
}

Fig. 12. Example operations of the Twitter app

| Application | Assertion                                      | Avg. time to fail (Iters) | (sec) |
|-------------|------------------------------------------------|---------------------------|-------|
| Stack       | Element popped more than once                  | 3.7                       | 0.02  |
| Courseware  | Course registration overflow                   | 10.6                      | 0.09  |
| Courseware  | Removed course registration                    | 57.5                      | 0.52  |
| Shopping    | Item reappears after deletion                   | 20.2                      | 0.14  |
| Twitter     | Missing tweets in feed                         | 6.3                       | 0.03  |

Table 1. Assertions checking results in microbenchmarks

application is when a student is trying to register for a course that is being concurrently removed: once the course is removed, no student should be seen as enrolled in that course.

Treiber Stack [Nagar et al. 2020]. Treiber stack is a concurrent stack data structure that uses compare-and-swap (CAS) instructions instead of locks for synchronization. This algorithm was ported to operate on a kv-store in prior work [Nagar et al. 2020] and we use that implementation. Essentially, the stack contents are placed in a kv-store, instead of using an in-memory linked data structure. Each row in the store contains a pair consisting of the stack element and the key of the next row down in the stack. A designated key “head” stores the key of the top of the stack. CAS is implemented as a transaction, but the pop and push operations do not use transactions, i.e., each read/write/CAS is its own transaction.

When two different clients try to pop from the stack concurrently, under serializability, each pop would return a unique value, assuming that each pushed value is unique. However, under causal consistency, concurrent pops can return the same value.

7.2 Assertion Checking
We ran the above applications with MonkeyDB to find out if assertions, capturing unexpected behavior, were violated under causal consistency. Table 1 summarizes the results. For each application, we used 3 client threads and 3 operations per thread. We ran each test with MonkeyDB for a total of 10,000 times; we refer to a run as an iteration. We report the average number of iterations (Iters) before an assertion failed, and the corresponding time taken (sec). All the assertions were violated within 58 iterations, in half a second or less. In contrast, running with an actual database almost never produces an assertion violation.

7.3 Coverage
The previous section only checked for a particular set of assertions. As an additional measure of test robustness, we count the number of distinct client-observable states generated by a test. A
client-observable state, for an execution, is the vector of values returned by read operations. For instance, a stack’s state is defined by return values of \texttt{pop} operations; a shopping cart’s state is defined by the return value of \texttt{GetCart} and so on.

For this experiment, we randomly generated test harnesses; each harness spawns multiple threads that each execute a sequence of operations. In order to compute the absolute maximum of possible states, we had to limit the size of the tests: either 2 or 3 threads, each choosing between 2 to 4 operations.

Note that any program that concurrently executes operations against a store has two main sources of non-determinism: the first is the interleaving of operations (i.e., the order in which operations are submitted to the store) and second is the choice of read-from (i.e., the value returned by the store under its configured isolation level). MonkeyDB only controls the latter; it is up to the application to control the former. There are many tools that systematically enumerate interleavings (such as \texttt{Chess} [Musuvathi and Qadeer 2008], \texttt{Coyote} [Microsoft Coyote 2019]), but we use a simple trick instead to avoid imposing any burden on the application: we included an option in MonkeyDB to deliberately add a small random delay (sleep between 0 to 4 ms) before each transaction begins. This option was sufficient in our experiments, as we show next.

We also implemented a special setup using the \texttt{Coyote} tool [Microsoft Coyote 2019] to enumerate all sources of non-determinism, interleavings as well as read-from, in order to explore the entire state space of a test. We use this to compute the total number of states. Figure 13 shows the number of distinct states observed under different isolation levels, averaged across multiple (50) test harnesses. For each of serializability and causal consistency, we show the max (as computed by \texttt{Coyote}) and versions with and without the delay option in MonkeyDB.

Each of these graphs show similar trends: the number of states with causal consistency are much higher than with serializability. Thus, testing with a store that is unable to generate weak behaviors will likely be ineffective. Furthermore, the “delay” versions of MonkeyDB are able to approach the maximum within a few thousand attempts, implying that MonkeyDB’s strategy of per-read randomness is effective for providing coverage to the application.

![Fig. 13. State coverage obtained with MonkeyDB for various microbenchmarks](image)

## 8 EVALUATION: OLTP WORKLOADS

OLTPBench [Difallah et al. 2013] is a benchmark suite of representative OLTP workloads for relational databases. We picked a subset of OLTPBench for which we had reasonable assertions. Table 2 lists basic information such as the number of database tables, the number of static transactions, how many of them are read-only, and the number of different assertions corresponding to system invariants for testing the benchmark. We modified OLTPBench by rewriting SQL join and aggregation operators into equivalent application-level loops, following a similar strategy as prior work [Rahmani et al. 2019]. Except for this change, we ran OLTPBench unmodified.

For TPC-C, we obtained a set of 12 invariants from its specification document [tpc 2020]. For all other benchmarks, we manually identified invariants that the application should satisfy. We asserted...
Table 2. OLTP benchmarks tested with MonkeyDB

| Benchmark   | #Tables | #Txns | #Read-only | #Assertions |
|-------------|---------|-------|------------|-------------|
| TPC-C       | 9       | 5     | 2          | 12          |
| SmallBank   | 3       | 6     | 1          | 1           |
| Voter       | 3       | 1     | 0          | 1           |
| Wikipedia   | 12      | 5     | 2          | 3           |

Fig. 14. Assertion checking: TPC-C

these invariants by issuing a read-only transaction to MonkeyDB at the end of the execution of the benchmark. None of the assertions fail under serializability; they are indeed invariants under serializability.\textsuperscript{13} When using weaker isolation, we configured MonkeyDB to use latest reads only (§6) for the assertion-checking transactions in order to isolate the weak behavior to only the application.

We ran each benchmark 100 times and report, for each assertion, the number of runs in which it was violated. Note that OLTPBench runs in two phases. The first is a loading phase that consists of a big initial transaction to populate tables with data, and then the execution phase issues multiple concurrent transactions. With the goal of testing correctness, we turn down the scale factor to generate a small load and limit the execution phase time to ten seconds with just two or three sessions. A smaller test setup has the advantage of making debugging easier. With MonkeyDB, there is no need to generate large workloads.

**TPC-C.** TPC-C emulates a wholesale supplier transactional system that delivers orders for a warehouse company. This benchmark deals with customers, payments, orders, warehouses, deliveries, etc. We configured OLTPBench to issue a higher proportion (> 85%) of update transactions, compared to read-only ones. Further, we considered a small input workload constituting of one warehouse, two districts per warehouse and three customers per district.

TPC-C has twelve assertions (A1 to A12) that check for consistency between the database tables. For example, A12 checks: for any customer, the sum of delivered order-line amounts must be equal to the sum of balance amount and YTD (Year-To-Date) payment amount of that customer.

Figure 14 shows the percentage of test runs in which an assertion failed. It shows that all the twelve assertions are violated under Read Committed isolation level. In fact, 9 out of the 12\textsuperscript{13}We initially observed two assertions failing under serializability. Upon analyzing the code, we identified that the behavior is due to a bug in OLTPBench that we have reported to the authors (link omitted).
assertions are violated in more than 60% of the test runs. In case of causal, all assertions are violated with three sessions, except for A4 and A11. We manually inspected TPC-C and we believe that both these assertions are valid under causal consistency. For instance, A4 checks for consistency between two tables, both of which are only updated within the same transaction, thus causal consistency is enough to preserve consistency between them.

These results demonstrate the effectiveness of MonkeyDB in breaking (invalid) assertions. Running with MySQL, under read committed, was unable to violate any assertion except for two (A10 and A12), even when increasing the number of sessions to 10. We used the same time limit of 10 seconds for the execution phase. We note that MySQL is much faster than MonkeyDB and ends up processing up to $50 \times$ more transactions in the same time limit, yet is unable to violate most assertions. Prior work [Rahmani et al. 2019] attempted a more sophisticated test setup where TPC-C was executed on a Cassandra cluster, while running Jepsen [jep 2020] for fault injection. This setup also was unable to violate all assertions, even when running without transactions, and on a weaker isolation level than read committed. Only six assertions were violated with 10 sessions, eight assertions with 50 sessions, and ten assertions with 100 sessions. With MonkeyDB, there is no need to set up a cluster, use fault injection or generate large workloads that can make debugging very difficult.

**SmallBank, Voter, and Wikipedia.** SmallBank is a standard financial banking system, dealing with customers, saving and checking accounts, money transfers, etc. Voter emulates the voting system of a television show and allows users to vote for their favorite contestants. Wikipedia is based on the popular online encyclopedia. It deals with a complex database schema involving page revisions, page views, user accounts, logging, etc. It allows users to edit its pages and maintains a history of page edits and user actions.

We identified a set of five assertions, A13 to A17, that should be satisfied by these systems. For SmallBank, we check if the total money in the bank remains the same while it is transferred from one account to another (A13). Voter requires that the number of votes by a user is limited to a fixed threshold (A14). For Wikipedia, we check if for a given user and for a given page, the number of edits recorded in the user information, history, and logging tables are consistent (A15-A17). As before, we consider small work loads: (1) five customers for SmallBank, (2) one user for Voter, and (3) two pages and two users for Wikipedia.

Figure 15 shows the results. MonkeyDB detected that all the assertions are invalid under the chosen isolation levels. Under causal, MonkeyDB could break an assertion in 26.7% (geo-mean)
runs given 2 sessions and in 37.2% (geo-mean) runs given 3 sessions. Under read committed, the corresponding numbers are 56.1% and 65.4% for 2 and 3 sessions, respectively.

9 RELATED WORK

There have been several directions of work addressing the correctness of database-backed applications. We directly build upon one line of work concerned with the logical formalization of isolation levels [Adya et al. 2000; Berenson et al. 1995; Biswas and Enea 2019; Cerone et al. 2015; X3 1992]. Our work relies on the axiomatic definitions of isolation levels, as given in [Biswas and Enea 2019], which also investigated the problem of checking whether a given history satisfies a certain isolation level. Our kv-store implementation relies on these algorithms to check the validity of the values returned by read operations. Working with a logical formalization allowed us to avoid implementing an actual database with replication or sophisticated synchronization.

Another line of work concentrates on the problem of finding "anomalies": behaviors that are not possible under serializability. This is typically done via a static analysis of the application code that builds a static dependency graph that over-approximates the data dependencies in all possible executions of the application [Bernardi and Gotsman 2016; Cerone and Gotsman 2018,?; Fekete et al. 2005; Gan et al. 2020; Jorwekar et al. 2007; Warszawski and Bailis 2017]. Anomalies with respect to a given isolation level then corresponds to a particular class of cycles in this graph. Static dependency graphs turn out to be highly imprecise in representing feasible executions, leading to false positives. Another source of false positives is that an anomaly might not be a bug because the application may already be designed to handle the non-serializable behavior [Brutschy et al. 2018; Gan et al. 2020]. Recent work has tried to address these issues by using more precise logical encodings of the application, e.g. [Brutschy et al. 2017, 2018] or by using user-guided heuristics [Gan et al. 2020].

Another approach consists of modeling the application logic and the isolation level in first-order logic and relying on SMT solvers to search for anomalies [Kaki et al. 2018; Nagar and Jagannathan 2018; Ozkan 2020], or defining specialized reductions to assertion checking [Beillahi et al. 2019a,b]. The CLOTHO tool [Rahmani et al. 2019], for instance, uses a static analysis of the application to generate test cases with plausible anomalies, which are deployed in a concrete testing environment for generating actual executions.

Our approach, based on testing with MonkeyDB, has several practical advantages. There is no need for analyzing application code; we can work with any application. There are no false positives because we directly run the application and check for user-defined assertions, instead of looking for application-agnostic anomalies. The limitation, of course, is the inherent incompleteness of testing.

Several works have looked at the problem of reasoning about the correctness of applications executing under weak isolation and introducing additional synchronization when necessary [Balegas et al. 2015; Gotsman et al. 2016; Li et al. 2014; Nair et al. 2020]. As in the previous case, our work based on testing has the advantage that it can scale to real sized applications (as opposed to these techniques which are based on static analysis or logical proof arguments), but it cannot prove that an application is correct. Moreover, the issue of repairing applications is orthogonal to our work.

From a technical perspective, our operational semantics based on recording past operations and certain data-flow and control-flow dependencies is similar to recent work on stateless model checking in the context of weak memory models, e.g. [Abdulla et al. 2015; Kokologiannakis et al. 2018]. This work, however, does not consider transactions. Furthermore, their focus is on avoiding enumerating equivalent executions, which is beyond the scope of our work (but an interesting direction for future work).
10 CONCLUSION

Our goal is to enable developers to test the correctness of their storage-backed applications under weak isolation levels. Such bugs are hard to catch because weak behaviors are rarely generated by real storage systems, but failure to address them can lead to loss of business [Warszawski and Bailis 2017]. We present MonkeyDB, an easy-to-use mock storage system for weeding out such bugs. MonkeyDB uses a logical understanding of isolation levels to provide (randomized) coverage of all possible weak behaviors. Our evaluation reveals that using MonkeyDB is very effective at breaking assertions that would otherwise hold under a strong isolation level.
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This section provides more details about the proof of correctness for our operational semantics.

\[ \text{SPAWN}^* \]
\[
\begin{array}{c}
t \text{ fresh } P(j) = \text{begin}; \text{Body}; \text{commit}; S \quad \tilde{B}(j) = \epsilon \\
h, \tilde{y}, \tilde{B}, P \Rightarrow h \uplus j \langle t, \emptyset, \emptyset \rangle, \tilde{y}[j \mapsto \emptyset], \tilde{B}[j \mapsto \text{Body}], P[j \mapsto S]
\end{array}
\]

\[ \text{IF-TRUE} \]
\[
\begin{array}{c}
\varphi(x)[x \mapsto \tilde{y}(j)(x) : x \in \tilde{x}] \text{ true } \quad \tilde{B}(j) = \text{if} (\varphi(\tilde{x})) \{ \text{Instr} \}; B \\
h, \tilde{y}, \tilde{B}, P \Rightarrow h, \tilde{y}, \tilde{B}[j \mapsto \text{Instr}; B], P
\end{array}
\]

\[ \text{IF-FALSE} \]
\[
\begin{array}{c}
\varphi(x)[x \mapsto \tilde{y}(j)(x) : x \in \tilde{x}] \text{ false } \quad \tilde{B}(j) = \text{if} (\varphi(\tilde{x})) \{ \text{Instr} \}; B \\
h, \tilde{y}, \tilde{B}, P \Rightarrow h, \tilde{y}, \tilde{B}[j \mapsto B], P
\end{array}
\]

\[ \text{WRITE} \]
\[
\begin{array}{c}
v = \tilde{y}(j)(x) \\ i \text{ fresh } \quad \tilde{B}(j) = \text{write}(k, x); B \\
h, \tilde{y}, \tilde{B}, P \Rightarrow h \uplus j \text{ write}_i(k, v), \tilde{y}[j, x \mapsto v], \tilde{B}[j \mapsto B], P
\end{array}
\]

\[ \text{READ-LOCAL} \]
\[
\begin{array}{c}
\text{write}(k, v) \text{ is the last write on } k \text{ in } t \\ i \text{ fresh } \quad \tilde{B}(j) = x := \text{read}(k); B \\
h, \tilde{y}, \tilde{B}, P \Rightarrow h \uplus j \text{ read}_i(k, v), \tilde{y}[j, x \mapsto v], \tilde{B}[j \mapsto B], P
\end{array}
\]

\[ \text{READ-EXTERN}^* \]
\[
\begin{array}{c}
\tilde{B}(j) = x := \text{read}(k); B \\ h = (T, so, wr) \\ t \text{ is the id of the last transaction log in } so(j) \\
\text{write}(k, v) \in \text{writes}(t') \text{ with } t' \in \text{compTrans}(h, \tilde{B}) \text{ and } t \neq t' \\ i \text{ fresh } \\ h' = (h \uplus j \text{ read}_i(k, v)) \oplus \text{wr}(t', \text{read}_i(k, v)) \\

\tilde{y}[j, x \mapsto v], \tilde{B}[j \mapsto B], P
\end{array}
\]

Fig. 16. A baseline operational semantics for \( P_{K\forall} \) programs. Above, \( \text{compTrans}(h, \tilde{B}) \) denotes the set of transaction logs in \( h \) that excludes those corresponding to live transactions, i.e., transaction logs \( t'' \in T \) such that \( t'' \) is the last transaction log in some \( so(j') \) and \( \tilde{B}(j') \neq \epsilon \).

### A CORRECTNESS OF THE OPERATIONAL SEMANTICS

This section provides more details about the proof of correctness for our operational semantics defined in Figure 6. The complete definition of the baseline semantics is given in Figure 16.

The notion of prefix of a tuple \( \langle h_2, co_2 \rangle \) is formally defined as follows. For a relation \( R \subseteq A \times B \), the restriction of \( R \) to \( A' \times B' \), denoted by \( R \downarrow A' \times B' \), is defined by \( \{(a, b) : (a, b) \in R, a \in A', b \in B'\} \). For \( h_1 = \langle T_1, so_1, wr_1 \rangle \) and \( h_2 = \langle T_2, so_2, wr_2 \rangle \), \( \langle h_1, co_1 \rangle \) is a prefix of \( \langle h_2, co_2 \rangle \), denoted by \( \langle h_1, co_1 \rangle \leq \langle h_2, co_2 \rangle \), iff \( T_1 = T_2 \cup \{t, O, po\} \), \( T_2 = T_2' \cup \{t, O, po'\} \), \( T_2 \subseteq T_2' \), \( O \subseteq O' \), \( po = po' \downarrow O \times O \), \( so_1 = so_2 \downarrow T_1 \times T_1 \), \( wr_1 = wr_2 \downarrow T_1 \times \text{reads}(T_1) \), and \( co_1 = co_2 \downarrow T_1 \times T_2 \).

Then, a property \( \varphi(t_2, \alpha) \) used to define an axiom like in (1), is called **monotonic** if for every \( \langle h_1, co_1 \rangle \leq \langle h_2, co_2 \rangle \),
\[
\forall t_2, \forall \alpha. \langle h_1, co_1 \rangle \models \varphi(t_2, \alpha) \Rightarrow \langle h_2, co_2 \rangle \models \varphi(t_2, \alpha).
\]

**Lemma A.1.** For any monotonic axiom \( X \), if \( \langle h_1, co_1 \rangle \leq \langle h_2, co_2 \rangle \), then \( \langle h_2, co_2 \rangle \) satisfies \( X \) if and only if \( \langle h_1, co_1 \rangle \) satisfies \( X \).

**Proof.** (Sketch) Given a monotonic axiom, the number of instantiations of \( \forall k, \forall t_1, \forall t_2, \) and \( \forall \alpha \) from (1) that satisfy the left-hand side of the entailment in the context of \( \langle h_1, co_1 \rangle \) is a subset of the same type of instantiations in the context of \( \langle h_2, co_2 \rangle \). Therefore, the co constraints imposed in

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the context of $\langle h_1, \text{co}_1 \rangle$ (by the right-hand side of the entailment) are a subset of the co constraints imposed in the context of $\langle h_2, \text{co}_2 \rangle$. Since the latter are satisfied (because $\langle h_2, \text{co}_2 \rangle$ satisfies $X$), the former are also satisfied and hence, $\langle h_1, \text{co}_1 \rangle$ satisfies $X$. □

Lemma A.1 extends obviously to isolation levels defined as conjunctions of axioms (which is the case for all the isolation levels that we are aware of [Biswas and Enea 2019]).

**Theorem A.2.** For any isolation level $I$ defined by a set of monotonic axioms,

$$\text{hist}_I(P) = \{ h \in \text{hist}_*(P) : h \text{ satisfies } I \}.$$ 

**Proof.** (Sketch) For the direction $\subseteq$, let $c_0 c_1 \ldots c_n$ be an execution under $\Rightarrow_I$, where $c_n$ is a final configuration. We need to show that the history $h_n$ contained in $c_n$ belongs to $\text{hist}_*(P)$ and that it satisfies $I$. The fact that $h_n \in \text{hist}_*(P)$ is a direct consequence of the fact that $\Rightarrow_I$ is more constrained than $\Rightarrow$. To prove that $h_n$ satisfies $I$, let $c_j$ be the latest configuration in the execution that is obtained from $c_{j-1}$ through an application of read-extern. By the definition of this rule, the history $h_j$ in $c_j$ satisfies $I$. Since the write-read relation of $h_j$ is identical to that of $h_n$, any axiom of the form (1) satisfied by $h_j$ is also satisfied by $h_n$ (the set of instantiations of $\forall t_1$ and $\forall \alpha$ in (1) that satisfy the left part of the entailment are the same in $h_j$ and $h_n$). Therefore, $h_n$ satisfies $I$, which concludes this part of the proof.

For the reverse, let $h = \langle T, \text{so}, \text{wr} \rangle \in \text{hist}_*(P)$ that satisfies $I$. Since $h$ satisfies $I$, there exists a commit order co such that $\text{wr} \cup \text{so} \subseteq \text{co}$ and $\langle h, \text{co} \rangle$ satisfies the axioms defining $I$. We show that there exists an execution $c_0 c_1 \ldots c_n$ under $\Rightarrow_I$ where transactions are executed serially in the order defined by co, such that $c_n$ is a final configuration that contains $h$. The only difficulty is showing that the read-extern transitions between two configurations $c_j$ and $c_{j+1}$ that add a write-read dependency $(t', \text{read}(k, v)) \in \text{wr}$ are enabled even though the transaction log $t$ containing read$(k, v)$ is “incomplete” in the history $h_j$ of $c_j$, and $h_j$ does not contain transactions committed after $t$. This relies on the prefix-closure property in Lemma A.1. Let co$_j$ be the order in which transactions have been executed until $c_j$. Then, $\langle h_j, \text{co}_j \rangle$ is a prefix of $\langle h, \text{co} \rangle$, and $\langle h_j, \text{co}_j \rangle \models I$ because $\langle h, \text{co} \rangle \models I$. □