Electro-optic entanglement source for microwave to telecom quantum state transfer

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We propose an efficient microwave-photonic modulator as a resource for stationary entangled microwave-optical fields and develop the theory for deterministic entanglement generation and quantum state transfer in multi-resonant electro-optic systems. The device is based on a single crystal whispering gallery mode resonator integrated into a 3D microwave cavity. The specific design relies on a new combination of thin-film technology and conventional machining that is optimized for the lowest dissipation rates in the microwave, optical and mechanical domains. We extract important device properties from finite element simulations and predict continuous variable entanglement generation rates on the order of a Mebit/s for optical pump powers of only a few tens of microwatt. We compare the quantum state transfer fidelities of coherent, squeezed and non-Gaussian cat-states for both teleportation and direct conversion protocols under realistic conditions. Combining the unique capabilities of circuit quantum electrodynamics with the resilience of fiber optic communication could facilitate long distance solid-state qubit networks, new methods for quantum signal synthesis, quantum key distribution, and quantum enhanced detection, as well as more power-efficient classical sensing and modulation.

I. INTRODUCTION

The development of superconducting quantum processors has seen remarkable progress in the last decade [1, 2], but long distance connectivity remains an unsolved problem. Coherent interconnects between superconducting qubits are currently restricted to an ultra-cold environment, which offers sufficient protection from thermal noise [3, 4]. A hybrid quantum network that combines the advanced control capabilities and the high speed offered by superconducting quantum circuits, with the robustness, range [5] and versatility [6] of more established quantum telecommunication systems appears as the natural solution. Entanglement between optical and microwave photons is the key ingredient for distributed quantum computing with such a hybrid quantum network and would pave the way to integrate advanced microwave quantum state synthesis capabilities [7–9] with existing optical quantum information protocols [10, 11] such as quantum state teleportation [12, 13] and secure remote quantum state preparation [14, 15].

Electro-optomechanical systems stand out as the most successful platforms to connect optical and microwave fields near losslessly and with minimal added noise [16, 17]. Very recently it has been shown that mechanical oscillators can also be used to deterministically generate entangled electromagnetic fields [18]. Mechanical generation of microwave-optical entanglement has been proposed [19–25] but an experimental realization remains challenging. Low frequency mechanical transducers typically suffer from added noise and low bandwidth, while high frequency piezoelectric devices require sophisticated wave matching and new materials, which so far results in low total interaction efficiencies [26–28], comparable to magnon-based interfaces [29].

Cavity electrooptic (EO) modulators are another proposed candidate [30–34] to coherently convert photons, or to effectively generate entanglement between microwave and optical fields, employing the Pockels effect and without the need for an intermediary oscillator. Here, a material with a large and broadband nonlinear polarizability $\chi^{(2)}$ is shared between an optical resonator and the capacitor of a microwave cavity [35–39], a platform that has recently been used for efficient photon conversion with bulk [40] and thin film crystals [41].

In this paper, we propose a multi-resonant whispering gallery mode (WGM) cavity electro-optic modulator whose free spectral range matches the microwave resonance frequency. It is optimized for optimal performance at ultra-low temperatures, in particular with respect to unwanted optical heating and thermal occupation of the microwave mode. We minimize the necessary optical pump power by maximizing the optical quality factor using a millimeter sized and mechanically polished bulk single crystal disk resonator [42]. Compared to nano- and micron-scale modulators its large size and surface area should facilitate a more efficient coupling to the cold bath and its large heat capacity is expected to result in slow heating rates in pulsed operation schemes. Compared to previous work [40] the disk is clamped in the center to avoid disk damage, air gaps and to minimize potential piezoelectric clamping losses. Importantly, finite-element modeling shows that a sufficient mode overlap and bandwidth at moderate pump powers can still be achieved using a combination of lithographically defined thin-film superconducting electrodes together with carefully shaped WGM disc cross-sections.

In the main part of the paper we develop the theory to analytically predict the entanglement properties under realistic conditions such finite temperature and asymmetric waveguide couplings. We show that it is feasible to deterministically generate MHz bandwidth continuous variable (CV) entanglement between the outputs of a pumped optical and a cold microwave resonator via spontaneous parametric downconversion (SPDC). We also present its performance for direct conversion-based and teleportation-based communication, as quantified by the quantum state transfer fidelities for a set of typical quantum states. Our results indicate that the pro-
posed entangler could serve as a repeater node to enable long distance hybrid quantum networks [43]. The developed theory results are applicable to any triply-resonant electro-optic transducer implementation.

II. SYSTEM

A. Hamiltonian of the system

As shown schematically in Fig. 1, we consider a WGM cavity electro-optic modulator containing a $\chi^{(2)}$ nonlinear medium that generates a nonlinear interaction between a single microwave cavity mode with frequency $\Omega$ and two modes of the WGM optical resonator corresponding to the central and the Stokes sideband modes with resonance frequencies $\omega_c$ and $\omega_s$, respectively. Such a single sideband situation can be achieved by making use of mode couplings of different polarization that lead to an asymmetry of the free spectral range of the WGM [40]. The total Hamiltonian describing the system is $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$ in which the free energy Hamiltonian is

$$\hat{H}_0 = h\omega_c a_c^\dagger a_c + h\omega_s a_s^\dagger a_s + h\Omega a_\Omega^\dagger a_\Omega,$$  

(1)

and the interaction Hamiltonian is

$$\hat{H}_{\text{int}} = g(a_\Omega a_s + a_s^\dagger a_\Omega^\dagger)(a_c + a_s),$$  

(2)

where $a_c$, $a_s$, and $a_\Omega$ are the annihilation operators of the central and Stokes sideband modes of the optical resonator, respectively, while $a_\Omega^\dagger$ is the annihilation operator of the microwave cavity and $g$ describes the coupling strength between the microwave and the two optical modes. Moving to the interaction picture with respect to $\hat{H}_0$ and setting $\Omega = \omega_c - \omega_s$, the system Hamiltonian reduces to

$$\hat{H} = g(a_c^\dagger \hat{a}_s \hat{a}_\Omega + a_s^\dagger \hat{a}_c \hat{a}_\Omega).$$  

(3)

The second part of this Hamiltonian describes a three-wave mixing process during which an optical photon with frequency $\omega_s$ and a microwave photon with frequency $\Omega$ are generated by annihilating an optical photon with frequency $\omega_c$. The coupling strength $g$ is determined by the spatial overlap of the electric fields $E_k = \sqrt{\hbar\omega_k/(2\epsilon_k V_k)}\psi_k(r, \theta, \phi)$ and the $\chi^{(2)}$ nonlinearity of the material [36, 40]:

$$g = 2\epsilon_0\chi^{(2)} \frac{\hbar\omega_c \omega_s \Omega}{8\epsilon_0 \epsilon_c \epsilon_\Omega C V_s V_\Omega} \int dV \psi_s^* \psi_c \psi_\Omega,$$  

(4)

where $\epsilon_0$ is the vacuum permittivity, $\psi_k(r, \theta, \phi)$ the field distribution functions, $\epsilon_k$ and $V_k$ are the relative permittivity and mode volume corresponding to mode $k$ with $k = s, c, \Omega$, respectively. The field distributions can be written in terms of the cross section $\Psi_k(r, \theta)$ and azimuthal distribution $e^{-im_\phi}$ as $\psi_k(r, \theta, \phi) = \Psi_k(r, \theta) e^{-im_\phi}$. The integral over the azimuthal variable $\phi$ is nonzero only if the relation $m_c = m_s + m_\Omega$ is fulfilled. This condition, known as phase matching or angular momentum conservation, returns a real value of the coupling constant $g$ presented in Eq (4).

We can linearize the Hamiltonian in Eq. (3) by limiting our analysis to the case where the center mode of the optical cavity is pumped resonantly by a strong coherent field at frequency $\omega_p = \omega_c$. In this condition the optical mode $\hat{a}_c$ can be treated as classical complex number $a_p = \langle \hat{a}_c \rangle$ and the linearized Hamiltonian becomes

$$\hat{H} = h\alpha_p (a_\Omega a_c + a_c^\dagger a_\Omega^\dagger).$$  

(5)

Here for simplicity we renamed the optical mode $\hat{a}_s \rightarrow \hat{a}_c$. The above Hamiltonian describes a parametric down-conversion process that is responsible for entangling the microwave mode $\Omega$ with the optical mode $\omega_o$. In a lossless system, this interaction could lead to an exponential growth of the energy stored in both modes and consequently lead to photon amplification of each mode.

B. Device implementation

The proposed system is based on a 3D-microwave cavity enclosing a mm-sized LiNbO$_3$ WGM-resonator with major radius $R$ operating at millikelvin temperature. At optical wavelengths, these mechanically polished resonators offer material-limited internal quality factors $Q_{i, \omega} \gtrsim 3.3 \times 10^8$ [44] and strong lateral confinement, presented by the optical mode cross-section $\Psi_k(r, \theta)$, on the order of tens of $\mu m^2$ [45]. In the microwave regime LiNbO$_3$ exhibits an internal quality factor $Q_{i, \Omega} \gtrsim 10^4$ in the X-band at millikelvin temperatures [46].
and a high electro-optic coefficient $r_{33} = 31 \text{ pm/V} \text{ at 9 GHz}$ [47, 48].

The large wavelength $\lambda_{\Omega} \gtrsim R$ of the microwave field causes considerable reduction of the spatial optical-microwave mode overlap, leading to a small microwave-optical mode coupling $g$. The proposed system tackles this problem by coupling the optical resonator to a metallic cavity. This hybrid device involves a monolithic LiNbO$_3$ resonator clamped at the center of a microwave cavity by two thin rods machined for example from aluminum or copper as depicted in Fig. 2(a). The LiNbO$_3$ resonator is coated with a thin film of superconductor such as Al or NbTiN forming the upper and lower electrodes of a capacitor for the microwave cavity. The thin film electrodes can be realized by evaporating metal on the full resonator’s surface followed by optical lithography on the resonator’s rim. The photoresist is developed and the unprotected thin metal band is etched. An interesting feature of this resonator fabrication process is the possibility to vary the gap size $d$ between the upper and lower electrodes independent of the disk thickness. Gaps from 1 mm down to 10 $\mu$m are feasible by adjusting the focus of the lithography laser. This results in a strong confinement of the microwave electric field at the resonator’s wedge-shaped rim, enhancing the mode overlap between the optical and microwave mode as shown in Fig. 2(b)-(d), and increasing the coupling constant $g$ (see Eq. (4)). In addition, the enclosing cavity offers a degree of freedom to control the microwave mode’s spatial distribution $\psi_{\Omega}(\vec{r})$, the microwave resonance frequency $\Omega$ and its coupling to a microwave coaxial waveguide $k_{c,\Omega}$.

To achieve optical-microwave mode interaction the energy and azimuthal momentum conservations must be fulfilled. For this system, we use and isolate two neighboring optical modes with angular number $m_x = m$ and $m_c = m + 1$, spectrally separated by a free spectral range (FSR) as experimentally shown in [40]. The energy conservation is fulfilled by matching the microwave mode frequency $\Omega$ to the optical FSR. Additionally, the microwave mode field distribution must have one oscillation around the resonator’s rim ($m = 1$) to fulfill the angular momentum conservation. We assume the center frequency of the WGM resonator with mode number $m_c = m + 1$ is coherently pumped via the evanescent coupling via a dielectric prism which also serves as the out-coupling port for the created Stokes-sideband with mode number $m_x = m$. On the microwave side, a pin coupler can be used to couple the microwave photons into a coaxial waveguide as depicted in Fig. 2(a). Here we work with a WGM resonator with an optical FSR of 9 GHz, corresponding to the typical frequency range of superconducting qubits and read-out resonators.

C. Numerical analysis of the system

Figure 2 (c) and (d) show the numerical simulation of the electric field distribution of the microwave and optical modes, respectively. The microwave electric field is constant in the region enclosing the optical field. We simulate a $z$-cut LiNbO$_3$ WGM-resonator with the major radius $R = 2.5 \text{ mm}$, height $H = 0.5 \text{ mm}$, and side curvature $R_c = 0.1 \text{ mm}$, enclosed by a cylindrical microwave cavity with diameter 5.5 mm and 1.3 mm hight. The optical WGM cross-section (FWHM) is analytically calculated to be $7.6 \times 17.8 \mu m^2$ [45]. For the electric fields along the $z$ direction, the integral term in Eq. (4) results in [40]:

$$g = \frac{1}{4\sqrt{2}} \cdot n_e^2 \cdot \omega_p \cdot r_{33} \cdot E_{\Omega,z}(\vec{r}_o),$$

(6)

where $n_e$ is the extraordinary optical refractive index of LiNbO$_3$ and $E_{\Omega,z}(\vec{r}_o)$ is the $z$-component of the single photon microwave electric field at the position $\vec{r}_o$ of the optical mode. The $1/\sqrt{2}$ correction term is due to the nature of the microwave stationary wave, which can be seen as two counter-propagating waves, one of which only propagates opposite to the optical mode and therefore does not interact with it.

Figure 3(a) shows the simulated microwave-optical coupling rate $g$ as a function of the electrodes gap size $d$. From a parametric fit to the simulated values, we find the dependency of coupling rate $g$ to the gap size $d$ scales with $g \sim d^{-0.8}$. Figure 3(b) shows the internal quality factor of the optical resonator $Q_{i,o}$ versus the gap size $d$. By decreasing the gap...
is maximized for a given pump power of the microwave (optical) cavity while $\kappa$ is the intra-cavity photon number due to the resonant optical cavity, respectively. Here we defined the critically coupled condition $\eta_{i,o} = 1/2$ the cooperativity is maximized for a given pump power

$$C = \frac{P_p g^2 Q^2_i,o}{\hbar \omega_p^2 Q_i,o}, \quad (7)$$

with the intrinsic quality factor $Q_{i,o} = \Omega(\omega_o)/\kappa_{i,o}(\omega_o)$ of the microwave (optical) mode. In Fig. 3(c) we plot the microwave-optical cooperativity $C$ as a function of the electrode gap size $d$ and for a fixed optical pump power of 10 $\mu$W.

This plot shows that the cooperativity increases by decreasing the gap size and it reaches its maximum value at $d \sim 50 \mu$m where $Q_{i,o}$ starts to saturate due to material absorption. To reach strong multi-photon microwave-optical interaction requires a cooperativity close to 1, which can be achieved by increasing the optical pump power to $P_p = 25.4 \mu$W.

It is important to note that this is lower than the cooling power of commercial cryostats at about 30 mK and that in practice only a small fraction of it would be dissipated into the cold stage of the dilution refrigerator, while the majority of the pump field is out-coupled together with the generated signal via an optical fiber e.g. by using a diamond prism with a basis angle of 63.5°. Nevertheless, in the following we will also consider the situation when the EO modulator is operated at the still plate at 800 mK and connected to a cold superconducting circuit at a few mK via a low-loss superconducting waveguide. The still stage of a modern dilution refrigerator offers cooling powers of at least 20 mW and the higher temperature offers higher thermal conductivities to connect the modulator more efficiently to the cold bath. Table I summarizes the full set of system parameters that will be used in the following unless otherwise stated.

| $\Omega/2\pi$ | $\omega_o/2\pi$ | $g/2\pi$ | $Q_{i,o}$ | $Q_{i,o}$ | $\eta_{i,o}$ | $\eta_{i,o}$ |
|---------------|----------------|---------|-----------|-----------|-------------|-------------|
| 9 GHz         | 193.5 THz      | 119 Hz  | $3 \times 10^3$ | $5 \times 10^8$ | 0.8         | 0.5         |

TABLE I. Reference values for the proposed system based on simulation ($\Omega$, $\omega_o$ and $g$) and characterization measurements of the system ($Q_{i,o}$ and $Q_{i,o}$). For generality we chose an asymmetric coupling situation $\kappa_o > \kappa_o$ and $\eta_o > \eta_o$. The necessary pump power to achieve $C = 1$ in this asymmetrically and over-coupled configuration is $P_{p,C=1} = 63.9 \mu$W.

III. SYSTEM DYNAMICS

In this section, we study the quantum dynamics of the proposed electro-optic modulator system presented in the previous section. We specifically focus on the conditions under which one can efficiently correlate and entangle optical and microwave fields using electro-optic interaction. The dynam-
ics of the system can be fully described using the quantum Langevin treatment in which we add the damping and noise terms to the Heisenberg equations for the system operators associated with Eq. (5). The resulting quantum Langevin equations for the intra-cavity optical and microwave modes are

\[
\dot{\hat{a}}_\Omega = -iG\hat{a}_\Omega - \frac{\kappa_\Omega}{2}\hat{a}_\Omega + \sqrt{\kappa_\epsilon,\Omega}\hat{a}_\epsilon,\Omega + \sqrt{\kappa_i,\Omega}\hat{a}_i,\Omega, \quad (8a)
\]

\[
\dot{\hat{a}}_\Omega = -iG\hat{a}_\Omega - \frac{\kappa_\Omega}{2}\hat{a}_\Omega + \sqrt{\kappa_\epsilon,\Omega}\hat{a}_\epsilon,\Omega + \sqrt{\kappa_i,\Omega}\hat{a}_i,\Omega, \quad (8b)
\]

where \( G = \sqrt{\gamma_p}ge^{i\phi_p} \) is the multi-photon interaction rate and \( \phi_p \) the phase of the pump. We also introduce the zero-mean microwave (optical) input noises given by \( \hat{a}_\epsilon,\Omega(t) \) and \( \hat{a}_i,\Omega(t) \), obeying the following correlation functions

\[
\langle \hat{a}_{k,\Omega}(t)\hat{a}_{k',\Omega}(t') \rangle = \bar{n}_{\Omega}(t-t') \delta(t-t'), \quad (9a)
\]

\[
\langle \hat{a}_{k,\Omega}(t)\hat{a}_{k',\Omega}(t') \rangle = (\bar{n}_{\Omega}(t)+1)\delta(t-t'), \quad (9b)
\]

\[
D(\omega) = M(\omega)^{-1}
\]

\[
M(\omega) = \left(\frac{\omega + \Delta\kappa}{2\Delta\kappa}\right)^2 + \frac{4\gamma_p^2}{\kappa_\Omega^2}\left(\kappa_\epsilon + \kappa_i\right)^2.
\]

using Eq. (10), and the bandwidth of the emitted radiation is

\[
\text{BW} = \sqrt{-C - \frac{\kappa_i^2 + \kappa_\epsilon^2}{2\kappa_\epsilon\kappa_i} + \sqrt{(1-C)^2 + \left(\frac{\kappa_i^2 + \kappa_\epsilon^2}{2\kappa_\epsilon\kappa_i}\right)^2}}
\]

with \( k = e, i \) where \( \bar{n}_{\Omega}(e) \) and \( \bar{n}_{\Omega}(i) \) are the equilibrium mean thermal photon numbers of the microwave (optical) fields.

The equations (8) describe the dynamics of the system and reveal the origin of the optical-microwave intra-cavity correlation, which arises from the cross dependency of microwave operator \( \hat{a}_\Omega \) on the optical mode operator \( \hat{a}_\epsilon \), and vice versa. However, in this paper we are interested in generating non-classical correlation and entanglement between itinerant electromagnetic modes, which can be calculated using the standard input-output theory [54]. We first solve the Eqs. (8) by moving to the Fourier domain to obtain the microwave and optical resonator variables. Then, substituting the solutions of Eqs. (8) into the corresponding input-output formula for the cavities’ variables, i.e., \( \hat{a}_{\Omega}^{\text{out}} = \sqrt{\kappa_\epsilon,\Omega}\hat{a}_{\Omega}(e) - \hat{a}_\epsilon,\Omega(e) \), we obtain

\[
\hat{S}_{\text{out}}(\omega) = \hat{S}_{\text{in}}(\omega) + \hat{S}_{\text{in}}^{\text{in}}(\omega)
\]

where \( \hat{S}_{\text{out}}(\omega) = [\hat{a}_{\Omega}^{\text{out}}(\omega), \hat{a}_{\Omega}^{\text{out}}(\omega)]^T \) is the output field matrix. The transformation matrix \( D(\omega) \) is given by \[32\]:

\[
D(\omega) = \left[\begin{array}{cc}
\left(\frac{\omega + \Delta\kappa}{2\Delta\kappa}\right)^2 + \frac{4\gamma_p^2}{\kappa_\Omega^2}\left(\kappa_\epsilon + \kappa_i\right)^2 & -iG\sqrt{\kappa_\epsilon\kappa_i}\kappa_\Omega

\hat{S}_{\text{in}}(\omega) &= \hat{S}_{\text{in}}^{\text{in}}(\omega) \end{array}\right]
\]

a cold waveguide \( \Delta\kappa \) which can be realized with superconducting cables connecting to the base temperature of the cryostat. As expected, the output of the microwave cavity increases considerably due to an increase of the modulator thermal noise \( \bar{n}_{\Omega}(e) \). While the photon occupation of the optical mode \( \bar{n}_{\Omega}(i) \) is negligible one can see that the thermal microwave noise leads to parametrically amplified optical noise at the resonator output at elevated temperatures.

Figure 4(b) shows the integrated optical and microwave output photon flux versus multi-photon cooperativity C. The photon numbers are increasing with C and diverge abruptly as the cooperativity approaches unity C → 1. In this limit the system reaches its instability and the linearization approximation used in the Hamiltonian Eq.(3) is not valid anymore. Therefore, for the remainder of the paper we study the generation of microwave-optics entanglement, conversion and quantum state transfer in the parameter range C < 1.

**IV. RESULTS**

In this section we verify the generation of microwave-optical two mode squeezing and deterministic entanglement of the output fields in the continuous variable domain.

**A. Two-mode squeezing**

First, we verify the generation of the two-mode squeezing at the outputs of the microwave cavity and optical resonator. For this reason it is convenient to define the field quadratures
the scattering matrix defined in Eq.(10) to calculate the sec-
Eq. (14).

Here we assume a cold waveguide 

These quadratures satisfy the bosonic commutator 

and we define the filtered output operators 

where we assume the filter function \( f_k(\omega, \sigma) \) with bandwidth \( \sigma_k (k = \alpha, \Omega) \) is acting on the output of each cavity. Note that the vacuum noise is 1/2 for the quadratures defined in Eq. (14).

In order to quantify entanglement, we first determine the covariance matrix (CM) of our system, which can be expressed as

\[
V_{jk} = \frac{1}{2} \langle \Delta \tilde{x}_j \Delta \tilde{x}_k + \Delta \tilde{x}_k \Delta \tilde{x}_j \rangle,
\]

where \( \Delta \tilde{x}_k = \tilde{x}_k - \langle \tilde{x}_k \rangle \) and \( \tilde{\mathbf{x}} = [\tilde{q}_o, \tilde{p}_o, \tilde{q}_\Omega, \tilde{p}_\Omega]^T \). Using the scattering matrix defined in Eq.(10) to calculate the second order moments of the output quadratures Eq. (15) at zero bandwidth \( \sigma = 0 \), we can compute the CM matrix of the system in the steady state

\[
V = \begin{bmatrix}
\begin{pmatrix} 0.5 + \frac{4C(1+\tilde{n}_o)\eta_0}{(1-C)^2} & \sqrt{4\tilde{n}_o\eta_0 C(1+2\tilde{n}_o)/(1-C)^2} \end{pmatrix} \mathbf{I} \\
\begin{pmatrix} \sqrt{\eta_0 \Omega(1+C+2\tilde{n}_o)/(1-C)^2} & 0.5 + \frac{4(C+\tilde{n}_o)\eta_0}{(1-C)^2} \end{pmatrix} \mathbf{Z}
\end{bmatrix}
\]

where \( \mathbf{I}_{2 \times 2} \) is the identity matrix, \( \mathbf{Z} = \text{diag}(1, -1) \), \( \tilde{n}_\Omega = \kappa_\Omega \tilde{n}_\Omega(T_b)/\kappa_\Omega \) is the microwave thermal mode occupancy. Here we assume a cold waveguide \( \tilde{n}_\Omega = 0 \) and \( \tilde{n}_o = 0 \).

For \( C = 0 \) the CM Eq.(18) takes on the values of the vacuum noise \( V = \mathbf{I}_{4 \times 4}/2 \) and the CM diverges at \( C = 1 \).

The existence of microwave-optical entanglement can be demonstrated using the quasi-probability Wigner function, which can be written in terms of the CM Eq. (18) and the optical and microwave quadratures \( \tilde{q}_k \) and \( \tilde{p}_k \)

\[
W(\mathbf{x}) = \frac{\exp(-\frac{1}{2}[\mathbf{x} \cdot \mathbf{V}^{-1} \cdot \mathbf{x}])}{\pi^2 \sqrt{\text{det}[\mathbf{V}]}},
\]

Figure 5(a) shows the Wigner function projected into the 4 different quadratures subspaces \( \{p_o, q_o\}, \{p_\Omega, q_\Omega\}, \{q_\Omega, p_o\} \), and \( \{p_\Omega, p_o\} \) where the complementary variables are integrated. As a reference, we also plot the Wigner function of the vacuum state \( \mathbf{V} = \mathbf{I}_{4 \times 4}/2 \) (red circle) corresponding to zero cooperativity \( C = 0 \). The single-mode projections \( \{p_o, q_o\} \) and \( \{p_\Omega, q_\Omega\} \) show an increase of the noise fluctuations, indicating the phase-independent amplification of the vacuum noise at the output of each cavity. The \( \{q_\Omega, p_o\} \) projections on the other hand, demonstrate the microwave-optical cross-correlation, originating from the electro-optic interaction, whose fluctuations in specific direction are squeezed below the quantum limit (blue line) and anti-squeezed in the opposite direction. In this plot the red (blue) line indicates a drop by \( e^{-1} \) of its maximum for the parameters \( C = 0.3 (C = 0) \) at \( T_b = 0 \). Unlike the ideal symmetric two-mode squeezer (\( V_{11} = V_{22} = V_{33} = V_{44} \)) whose quadrature squeezing appears along diagonal axes with squeezing angle \( \pm 45^\circ \) (black dashed lines), in general the electro-optic system is an asymmetric squeezer (\( V_{11} = V_{22} \neq V_{33} = V_{44} \)) when \( \eta_0 \neq \eta_\Omega \). The squeezing angle is then given by \( \tan(2\Theta) = \pm 2V_{13}/V_{33} - V_{11} \) and its value is 39.34° for system’s parameters in Figure 5(a).

In Fig. 5(b) we show the squeezed \( \Delta q_o^2 \) and anti-squeezing \( \Delta q_\Omega^2 \) quadrature variances as well as their product \( \Delta q_o \Delta q_\Omega \), which is related to the purity \( P = 1/(2\Delta q_\Omega \Delta q_o) \) of Gaussian states [55], as a function of the cooperativity \( C \). The variances are given as

\[
\Delta q_o^2 = \frac{\sqrt{(8C\eta_0 + \varepsilon)(8C\eta_\Omega + \varepsilon)/\varepsilon - T^2/\varepsilon}}{2[\varepsilon + 8C(\eta_\Omega(1) + \varepsilon)\sin^2(\Theta) + \eta_o(1) \cos^2(\Theta)] + T \sin(2\Theta)}.
\]
configuration. Due to the optical and microwave internal losses $\eta_k < 1$ ($k = o, \Omega$) the quadrature variances deviate from the uncertainty principle $\Delta q_- \Delta q_+ > 1/2$ in the proposed device as shown in Fig. 5(b).

### B. Microwave-Optical Entanglement

We are interested in the entanglement properties of the radiation leaving the system and we therefore study the bipartite microwave-optical entanglement, which can be quantified using the logarithmic negativity [56, 57]

$$E_N = \max \{0, -\log_2 (2\tilde{d}_-)\}. \quad (22)$$

where

$$\tilde{d}_- = 2^{-1/2}\sqrt{\Delta - \sqrt{\Delta^2 - 4\det(V)}}, \quad (23)$$

is the smallest symplectic eigenvalue of the partial transpose of the CM Eq. (18) with $\Delta = V_{23}^2 + V_{31}^2 + 2V_{32}^2$. In Fig. 6(a) we plot $E_N$ as a function of the cooperativity for two different temperatures 10 mK (solid line) and 800 mK (dashed line). One can see that a significant amount of microwave-optical entanglement is generated $E_N \sim 1$, even for moderate values of $C$, increasing with higher cooperativity and decreasing significantly at elevated bath temperatures $T_b$. In the low temperature limit $\eta_{l1} \approx 0$ and for the waveguide coupling matching $\eta := \eta_o = \eta_{l1}$, the logarithmic negativity (22) reduces to

$$E_N' = -\log_2 \left(1 - \frac{4\eta\sqrt{C}}{(1 + \sqrt{C})^2}\right). \quad (24)$$

We also calculate the distribution rate of the entangled fields emitted from the electro-optic system, which is given by

$$\#(\text{ebit/s}) = E_F \cdot \text{BW}/2\pi. \quad (25)$$

where we introduce the entanglement of formation

$$E_F(\rho) = (x_m + 0.5) \log_2(x_m + 0.5) - (x_m - 0.5) \log_2(x_m - 0.5) \quad (26)$$

with $x_m = (\tilde{d}_+^2 + 1/4)/(2\tilde{d}_-)$. From the obtained output operators in Eq. (16) with the rectangular filter $f_k(\omega, \sigma) = \Theta(\text{BW}/2 - |\omega|)$ we compute the average CM over the emission bandwidth, which is then used inside Eq. (26) returning the average entanglement of formation $E_F$.

Figure 6 (b) shows the total emission of entangled radiation as well as the bandwidth of the photon emission as a function of cooperativity $C$. For the considered system parameters given in Table I (blue solid line) a maximum value of 0.26 Mebit/s is reached at $C = 0.22$ with a photon emission bandwidth of 0.6 MHz at $P_o = 14 \mu W$. The most effective method to increase the BW and entanglement rate is to increase the optical waveguide coupling $\kappa_{e,o}$. The red lines in Fig. 6(b) show the situation for $\eta_o = 0.8$, yielding rates of $>1$ Mebit/s at $\sim 2$ MHz bandwidth at $C = 0.26$, which would

FIG. 5. Two-mode squeezing of the electro-optic output fields. (a) Normalized projections of the Wigner function of four output quadrature pairs for the same parameters as in Fig. 4(a). The solid red line (blue line) indicates a drop by $e^{-1}$ of its (the vacuum state’s) maximum. The black dashed-line marks the squeezing angle of 45° for an ideal squeezer. The squeezing angles for the asymmetric system in this representation are given by $\pm(90° - \Theta)$. (b) The squeezing $\Delta q_o^2$ (solid red line), anti-squeezing parameters $\Delta q_i^2$ (dashed red line), their product $\Delta q_- \Delta q_+$ (black line) and the variance of the vacuum (blue line) as a function of the cooperativity $C$ for the same parameters.

with $Y = 4\sqrt{\eta_o\eta_{l1}C}(1 + C)$ and $\varepsilon = (1 - C)^2$. Larger $C$ gives smaller $\Delta q_-$ (more squeezing) and larger $\Delta q_+$ (more amplification) at the outputs of the cavities. In the ideal case $\eta_o = \eta_{l1} = 1$ and for $0 < C < 1$ the above equation reduces to

$$\Delta q_-^2 = \frac{1}{2} \left(\frac{1 - \sqrt{C}}{1 + \sqrt{C}}\right)^2 < \frac{1}{2}; \quad (21a)$$

$$\Delta q_+^2 = \frac{1}{2} \left(\frac{1 + \sqrt{C}}{1 - \sqrt{C}}\right)^2 > \frac{1}{2}; \quad (21b)$$

which satisfies the minimum quadrature uncertainty $\Delta q_- \Delta q_+ = 1/2$. Moreover, we can define the electro-optic squeezing parameter as $\eta_{\text{EO}} = \ln \left(\frac{1 + \sqrt{C}}{1 - \sqrt{C}}\right)$ for this
now require a pump power of $P_p = 65 \, \mu W$, a value that is still feasible at the mixing chamber temperature stage of a dilution refrigerator. At significantly elevated bath temperatures of $T_b = 800 \, mK$ (dashed lines) the maximum entanglement rates drop by about a factor 5 in both coupling situations. It should be noted that further increasing the coupling to a cold waveguide on the microwave side $\eta_\Omega \simeq 1$ or alternatively by finding a way to lower the internal losses of the microwave mode, would result in a significantly smaller effective system temperature. Larger waveguide coupling strengths and higher available pump powers at the still stage of a dilution refrigerator together with higher thermal conductivities could result in significantly higher entanglement rates than discussed in this paper which focuses on currently accessible device parameters. In all cases the entanglement rate approaches zero at $C = 1$, following the decrease in photon emission bandwidth, see also Eq. (13).

V. QUANTUM STATE TRANSFER

An important feature of a hybrid quantum network is the ability to transfer quantum states between different nodes. The quality of the state transfer is characterized by the fidelity $F$ of an unknown arbitrary quantum state before and after the transduction, respectively. For Gaussian states the fidelity simplifies to [59]

$$F = \frac{\exp[-(x^{\text{out}} - x^{\text{in}})^T \cdot V_F^{-1} \cdot (x^{\text{out}} - x^{\text{in}})]}{\sqrt{\det(V_F/2)}}$$

with $x^{\text{in}} = (q^{\text{in}}_a, p^{\text{in}}_a)^T$, $x^{\text{out}} = (q^{\text{out}}_a, p^{\text{out}}_a)^T$ and $V_F = 2V_{in} + 2V_{out}$, where $V_{in(out)}$ are the input and output covariance matrices following the definition given in Eq. (17).

A. Teleportation

We propose the bidirectional microwave-to-optical quantum state transfer using the presented EO-device as an EPR source in an unconditional CV teleportation scheme. Assuming the standard Braunstein-Kimble set-up [58] with ideal Bell measurements and classical information transfer as depicted in Fig. 7(a), the state transfer fidelity for an unknown coherent squeezed state $|\psi_{\text{in}}\rangle = |\alpha, r\rangle$ is given by

$$F_{\text{TE}}^{\text{G}}(\alpha, r, C, \eta_o, \eta_\Omega) = (\Delta q_1^2 + 2\Delta q_2^2 \cosh(2r) + 1)^{-1/2} \, \hat{F}_{\text{TE}}^{\text{G}}(\alpha, r, C, \eta) = \det(2V_{in} + ZAZ + B - ZC - C^T Z^T)^{-1/2},$$

where $A = V_{11} I$, $B = V_{33} I$ and $C = V_{13} Z$. The fidelity Eq. (30) can be written in terms of the logarithmic negativity $E_N$ generated using the EO device

$$F_{\text{TE}}^{\text{G}}(\alpha, r, C, \eta, \Omega) = \left(1 + 2^{-1 - E_N(C)} \cosh(2r) + 2^{-2E_N(C)}\right)^{-1/2} \hat{F}_{\text{TE}}^{\text{G}}(\alpha, r, C, \eta, \Omega) = \left(1 + 2^{-1 - E_N(C)} \cosh(2r) + 2^{-2E_N(C)}\right)^{-1/2}$$

The fidelity in Eq. (31) is independent of the coherent state amplitude $\alpha$ due to the assumed ideal measurement of the quadratures $q_-$ and $p_+$, and a lossless classical information transfer in this protocol. The bandwidth of the teleportation is given by the photon emission bandwidth shown in the inset of Fig. 6(b).


\[
\Delta q^2 = 0.5 - \frac{\eta_o}{\eta_c + \eta_o} \\
\]

as shown in Fig. 7(b). An increased temperature leads to a significant reduction of the achievable state transfer fidelity. A fidelity of \( \sim 1 \) is achieved for a cooperativity close to 1 in the near lossless and perfectly over-coupled case. In this case the system thermalizes with the cold waveguide independent of its internal bath temperature.

Quantum state teleportation based on EO-entanglement can be used also with non-Gaussian states such as cat states that are readily available in superconducting circuits. Cat states are represented as a quantum superposition of two coherent states in the form

\[
|N\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + e^{i\phi}|\alpha\rangle)
\]

The state transfer fidelity using the proposed EO entanglement source is given by [58]

\[
F^\text{Sat}_{\text{TE}} = \frac{1}{1 + 2\Delta q^2 - e^{-2\Delta^2}}
\]

In Fig. 7(b) we show the teleportation fidelity of the cat state

\[
|\psi\rangle = |\alpha\rangle - |\alpha\rangle = |2\rangle - |\rangle
\]

as a function of the system parameters in Table I at zero temperature (red solid line), at 800 mK (red dashed line) as well as for a lossless system \( \eta_o = \eta_c = 1 \) (red dash-dotted line) where we consider \( \phi = \pi \). We find that the cat state transfer fidelities are lower compared to the coherent squeezed input state over the full range of parameters.

## B. Conversion

The EO system can also be used to directly convert the information between microwave and optical photons, schematically shown in Fig. 7(c). This is achieved by driving the lower frequency optical mode in the same scheme as given in [40, 41], changing the nonlinear interaction Hamiltonian into the so called beam splitter Hamiltonian, allowing coherent frequency conversion between the microwave and optical modes following the equations of motion:

\[
\hat{a}_o = -i\hat{g}\hat{a}_o - \frac{\Omega}{2}\hat{d}_o + \sqrt{\kappa_c}\hat{d}_{e,o} + \sqrt{\kappa_t}\hat{d}_{i,o}, \tag{33a}
\]

\[
\hat{a}_o = -i\hat{g}\hat{a}_o - \frac{\Omega}{2}\hat{d}_o + \sqrt{\kappa_c}\hat{d}_{e,o} + \sqrt{\kappa_t}\hat{d}_{i,o}. \tag{33b}
\]

Using the input-output theory to calculate the outputs of the optical and microwave modes, we can infer the photon conversion efficiency between the traveling microwave and opti-
ties.

The fidelity is strongly dependent on the field amplitude $\eta$. The fidelity in direct transduction shown in Figs. 7 (e) is significant fields [32].

\[
\frac{\langle a^\dagger_{\Omega,\Omega}(\omega) a_{\Omega,\Omega}(\omega) \rangle}{\langle a^\dagger_{\Omega,\Omega}(\omega) a_{\Omega,\Omega}(\omega) \rangle} = \frac{4C\eta_0 \eta_\Omega}{(1 + C - \frac{\kappa_\Omega}{2\eta_\Omega})^2 + \frac{\kappa_0^2}{4\eta_0^2} (\kappa_0 + \kappa_\Omega)^2}
\]

over the bandwidth [45]:

\[
BWC = \sqrt{C - \frac{\kappa_0^2 + \kappa_\Omega^2}{2\kappa_\Omega \kappa_0} + \sqrt{(1 + C)^2 + \left(C - \frac{\kappa_0^2 + \kappa_\Omega^2}{2\kappa_\Omega \kappa_0}\right)^2}} \times \sqrt{\kappa_\Omega \kappa_0}.
\]

The conversion bandwidth BWC increases with the cooperativity as shown in Fig. 7(d) and for the case of rate matching $\kappa_0 = \kappa_\Omega = \kappa$ achieves the maximum value of $\sqrt{2\kappa}$ for $C = 1$. For the coherent squeezed state $|\alpha, r\rangle$ the fidelity of the direct state transduction is given by

\[
F_{tr}^S(\alpha, r, C) = \exp\left(-2|\alpha|^2(\epsilon_3 - 1)^2 \left(\frac{\cos(\phi_\alpha)}{V_+} + \frac{\sin(\phi_\alpha)}{V_-}\right)\right),
\]

where

\[
V_\pm = (1 + \frac{\epsilon_2}{\epsilon_3} e^{-2r} - 1 + 2\bar{n}_\Omega/(\eta_0 C)),
\]

\[
\epsilon_2 = 1 + \cosh(2r) \quad \text{and} \quad \epsilon_3 = \sqrt{\frac{4\eta_\Omega \eta_0 C}{(1+C)^2}}.
\]

Figure 7(e) shows the fidelity of state transfer for the squeezed coherent input state $|\psi_{in}\rangle = |2\rangle$ as a function of $C$ for the system parameters in Table I at zero temperature (blue solid line), at 800 mK (blue dashed line) as well as for a lossless system $\eta_0 = \eta_\Omega = 1$ (red dash-dotted line). The lower bound of the fidelity ($C = 0$) is given by the overlap of the initial state and the vacuum state set by $\sqrt{\frac{1 - r - 2|\alpha|^2}{1 + r}}$. In comparison to the teleportation scheme shown in Fig. 7(b), the fidelity in direct transduction shown in Figs. 7(e) is significantly lower for this state. In general for direct conversion the fidelity is strongly dependent on the field amplitude $|\alpha|$, which can be seen from the numerator of Eq. (36) in the case $\eta_\Omega(\Omega) < 1$. However, it is important to note that many quantum communication protocols work with $|\alpha| \leq 1$ [12, 62, 63], a regime where both schemes offer more comparable fidelities.

The direct EO transducer can also be used to convert non-Gaussian states between microwave and optical fields. For a real $\alpha$ the fidelity of the conversion is

\[
F_{tr}^\text{cat}(\alpha, C) = \frac{1}{\epsilon_4(1 + \epsilon_5)} \left(\epsilon_3 e^{-2\alpha^2(1 + \epsilon_5)} \left(\epsilon_5 + 1\right) + 2 \cos(\phi) e^{-2\alpha^2(1 + \epsilon_5)} + e^{-2\alpha^2(1 + \epsilon_5)}\right)
\]

with $\epsilon_4 = (1 + \cos(\phi)) e^{-2\alpha^2(1 + \epsilon_5)}$ and $\epsilon_5 = 1 + \frac{8\eta_\Omega \eta_0 C}{(1+C)^2}$. The lower bound of this fidelity given by

\[
(1 + \cos(\phi)) / (\alpha^2 + e^{-\alpha^2} \cos(\phi)).
\]

VI. CONCLUSION

We have presented an efficient and bright microwave-optical entanglement source based on a triply resonant electro-optic interaction. We proposed a specific device geometry and material system, tested and simulated the most important system parameters and derived the theory describing the physics, entanglement generation and device performance for both teleportation and conversion type quantum state transfer.

The figures of merit for a quantum interface are efficiency and added noise, which both affect the achievable state transfer fidelity. For any realistic application with finite lifetime qubits, the transducer bandwidth determines if it is of practical use for quantum interconnects. On-chip integrated devices with small mode volume offer higher nonlinear coupling constants $g$ [41] compared to mm sized systems, but chip-level integration so far comes at the expense of a lower internal optical $Q_{\text{int}}$ [64], because surface qualities routinely achieved with mechanical polishing are difficult to realize in micro-fabrication processes. We have presented a new device geometry that offers the lowest losses without sacrificing coupling and as a result yields high predicted state transfer fidelities at practical bandwidth and realistic optical pump powers.

Our analysis shows that ultra-low losses, a prerequisite to achieve very strong waveguide over-coupling, turns out to be the most important aspect for any resonant quantum interface to approach the high efficiency and fidelity needed in realistic applications. In comparison, increasing waveguide coupling rates requires higher pump powers to achieve the same cooperativity and dissipates more optical energy in the over-
coupled regime, which leads to higher thermal bath occupations. Our analysis also pointed out the importance of low system temperatures, and mm sized devices not only offer lower optical absorption and scattering losses, which can easily break Cooper pairs in the superconducting microwave cavity, they also offer much larger volume, mass, heat capacity and surface area for effective thermalization to the cold bath in continuous and pulsed driving schemes.

The presented triply resonant modulator offers a very promising way forward in the field of hybrid quantum systems, both when used for entanglement swapping or for direct conversion of quantum states. Experimental tests will show if the proposed scheme can be implemented as expected and tell us more about the important LiNbO$_3$ material parameters and heating rates at millikelvin temperatures. In the context of classical and quantum communication applications, with the above given parameters, our system could also work as an ultra efficient electro-optic modulator with a $V_E$ as low as 12.4 mV that can be used for frequency comb generation [65].

Beyond that we also expect applications of microwave-optical entangled fields in the area of radio frequency sensing and low noise detection.

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VIII. AUTHOR CONTRIBUTIONS

A.R. W.H and J.M.F. conceived the project. Analytical analysis was done by A.R and S.B and FEM simulations by W.H. All authors contributed to the manuscript. S.B. and J.M.F. supervised the project.

IX. ADDITIONAL INFORMATION

The authors declare no competing interests.
[23] Wang, Y.-D. & Clerk, A. A. Reservoir-engineered entanglement in optomechanical systems. Phys. Rev. Lett. 110, 253601–253601−10 (2013).

[24] Tian, L. Robust photon entanglement via quantum interference in optomechanical interfaces. Phys. Rev. Lett. 110, 233602 (2013).

[25] Zhong, C. et al. Heralded Generation and Detection of Entangled Microwave–Optical Photon Pairs. arXiv e-prints arXiv:1901.08228 (2019). arXiv:1901.08228.

[26] Bochmann, J., Vainschenker, A., Awaschalom, D. D. & Cleland, A. N. Nanomechanical coupling between microwave and optical photons. Nature Physics 9, 712–716 (2013).

[27] Forsch, M. et al. Microwave-to-optics conversion using a mechanical oscillator in its quantum groundstate. arXiv:1812.07588v1 (2018).

[28] Shao, L. et al. Microwave-to-optical conversion using lithium niobate thin-film acoustic resonators. arXiv:1907.08593 (2019).

[29] Hisatomi, R. et al. Bidirectional conversion between microwave and light via ferromagnetic magnons. Phys. Rev. B 93, 174427 (2016).

[30] Matsko, A. B., Savchenkov, A. A., Ilchenko, V. S., Seidel, D. & Maleki, L. On fundamental quantum noises of whispering gallery mode electro-optic modulators. Opt. Express 15, 17401–17409 (2007).

[31] Tsang, M. Cavity quantum electro-optics. Physical Review A 81, 063837 (2010).

[32] Tsang, M. Cavity quantum electro-optics. II. Input-output relations between traveling optical and microwave fields. Physical Review A 84, 043845 (2011).

[33] Javerzac-Galy, C. et al. On-chip microwave-to-quantum coherent converter based on a superconducting resonator coupled to an electro-optic microresonator. Phys. Rev. A 94, 053815 (2016).

[34] Soltani, M. et al. Efficient quantum microwave-to-quantum conversion using electro-optic nanophotonic coupled resonators. Phys. Rev. A 96, 043808–043808–17 (2017).

[35] Cohen, D., Hossein-Zadeh, M. & Levi, A. Microphotonic modulator for microwave receiver. Electronics Letters 37, 300–301 (2001).

[36] Ilchenko, V. S., Savchenkov, A. A., Matsko, A. B. & Maleki, L. Whispering-gallery-mode electric-optic modulator and photonic microwave receiver. Journal of the Optical Society of America B 20, 333–342 (2003).

[37] Strekalov, D. V., Savchenkov, A. A., Matsko, A. B. & Yu, N. Efficient upconversion of subterahertz radiation in a high-Q whispering gallery resonator. Optics Letters 34, 713–715 (2009).

[38] Strekalov, D. V. et al. Microwave whispering-gallary resonator for efficient optical up-conversion. Physical Review A 80, 033810–5 (2009).

[39] Botello, G. S.-a. et al. Sensitivity limits of millimeter-wave photonic radiometers based on efficient electro-optic upconverters. Optica 5, 1210–1219 (2018).

[40] Rueda, A. et al. Efficient microwave to optical conversion: an electro-optical realization. Optica 3, 597–604 (2016).

[41] Fan, L. et al. Superconducting cavity electro-optics: A platform for coherent photon conversion between superconducting and photonic circuits. Science Advances 4 (2018).

[42] Strekalov, D. V., Marquardt, C., Matsko, A. B., Schwefel, H. G. L. & Leuchs, G. Nonlinear and quantum optics with whispering gallery resonators. Journal of Optics 18, 123002 (2016).

[43] Muralidharan, S. et al. Optimal architectures for long distance quantum communication. Scientific Reports 6, 20463–20463 (2016).

[44] Leidinger, M. et al. Comparative study on three highly sensitive absorption measurement techniques characterizing lithium niobate over its entire transparent spectral range. Opt. Express 23, 21690–21705 (2015).

[45] Sanchez, A. R. R. Resonant Electrooptics. doctoral thesis, Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU) (2018).

[46] Goryachev, M., Kostylev, N. & Tobar, M. E. Single-photon level study of microwave properties of lithium niobate at millikelvin temperatures. Physical Review B 92, 060406 (2015).

[47] Weis, R. S. & Gaylord, T. K. Lithium niobate: Summary of physical properties and crystal structure. Applied Physics A 37, 191–203 (1985).

[48] Wong, K., of Electrical Engineers, I. & service), I. I. Properties of Lithium Niobate. EMIS datareviews series (IN- SPEC/Institution of Electrical Engineers, 2002).

[49] McPeak, K. M. et al. Plasmonic films can easily be better: Rules and recipes. ACS Photonics 2, 326–333 (2015). PMID: 25950012, arXiv:https://doi.org/10.1021/ph5004237.

[50] Rakić, A. D. Algorithm for the determination of intrinsic optical constants of metal films: application to aluminum. Appl. Opt. 34, 4755–4767 (1995).

[51] Nguyen, D. T. et al. Ultrasubharmonic quantum detectors for both sub-terahertz and terahertz frequencies. J. Opt. Commun. Netw. 6, 458–466 (2014).

[52] Brecht, T. et al. Multilayer microwave integrated quantum circuits for scalable quantum computing. npj Quantum Information 2, 16002 EP – (2016).

[53] Wenner, J. et al. Surface loss simulations of superconducting coplanar waveguide resonators. Applied Physics Letters 99, 113513 (2011).

[54] Gardiner, C. W. & Zoller, P. Quantum Noise (Springer Series in Synergetics, 2004).

[55] Paris, M. G. A., Illuminati, F., Serafini, A. & De Siena, S. Purity of gaussian states: Measurement schemes and time evolution in noisy channels. Phys. Rev. A 68, 012314–012314 (2003).

[56] Vidal, G. & Werner, R. F. Computable measure of entanglement. Phys. Rev. A 65, 032314 (2002).

[57] Plenio, M. B. Logarithmic negativity: A full entanglement monotone that is not convex. Phys. Rev. Lett. 95, 090503 (2005).

[58] Braunstein, S. L. & Kiefer, H. J. Teleportation of continuous variable quantum states. Phys. Rev. Lett. 80, 869–872 (1998).

[59] Isar, A. Quantum fidelity for Gaussian states describing the evolution of open systems. The European Physical Journal Special Topics 160, 225–234 (2008).

[60] Fiurášek, J. Improving the fidelity of continuous-variable teleportation via local operations. Phys. Rev. A 66, 012304 (2002).

[61] Owari, M., Plenio, M. B., Polzik, E. S., Serafini, A. & Wolf, M. M. Squeezing the limit: quantum benchmarks for the teleportation and storage of squeezed states. New Journal of Physics 10, 113014 (2008).

[62] Wittmann, C. et al. Demonstration of near-optimal discrimination of optical coherent states. Phys. Rev. Lett. 101, 210501 (2008).

[63] Cook, R. L., Martin, P. J. & Geremia, J. M. Optical coherent state discrimination using a closed-loop quantum measurement. Nature 446, 774 EP – (2007).

[64] Zhang, M., Wang, C., Cheng, R., Shams-Ansari, A. & Lonar, M. Monolithic ultra-high-q lithium niobate microring resonator. Optica 4, 1536–1537 (2017).

[65] Rueda, A., Sedlmair, F., Kumari, M., Leuchs, G. & Schwefel, H. G. L. Resonant electro-optic frequency comb. Nature 568, 378–381 (2019).