New $\Lambda_b(6072)^0$ state as a $2S$ bottom baryon

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As a result of continuous developments, the recent experimental searches lead to the observations of new particles that contain conventional and non-conventional hadrons. Among these hadrons are the excited states of the heavy baryons containing single bottom or charm quark in its valance quark content. The recently observed $\Lambda_b(6072)^0$ state is one of these baryons and possibly $2S$ radial excitation of the $\Lambda_b$ state. Considering this information from the experiment, we conduct a QCD sum rule analysis on this state and calculate its mass and current coupling constant considering it as a $2S$ radially excited $\Lambda_b$ resonance. For completeness, in the analyses, we also compute the masses and current coupling constant for the ground state $\Lambda_b^0$ and its first orbital excitation. We also consider the $\Lambda_c^+$ counterpart of these states and attain their masses as well. The obtained results are consistent with the experimental data and existing theoretical predictions.

I. INTRODUCTION

Due to the progress in experimental facilities and techniques, lately, we encountered observations of various new particles. Among these new states there exist not only excited states of the conventional baryons but also the exotic particles. These particles have been collected the attention since their investigations provide contributions to understand the dynamics of light quarks in the presence of one heavy quark. Scrutinizing these states contribute to the better understanding of the confinement mechanism and help us test the predictions of not only the quark model and the heavy quark symmetry but also that of other theoretical approaches used to investigate these states.

In the last few decades, we witnessed the observations of various excited baryons containing one heavy quark in their quark content. Among these states are the $\Omega_c^+(3000)^0$, $\Omega_c^+(3050)^0$, $\Omega_c^+(3066)^0$, $\Omega_c^+(3090)^0$, $\Omega_c^+(3119)^0$ states [1] observed from the investigation of the $\Xi^+K^-$ mass spectrum, $\Xi_c(6227)^+$ [2], $\Sigma_c(6097)^+$ [3], $\Xi_c^+(5935)^-$, $\Xi_c^+(5955)^-$ [4], $\Lambda_b^+(5912)^0$, $\Lambda_b^+(5920)^0$ [5], $\Lambda_b(6146)^0$, $\Lambda_b(6152)^0$ [6], $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$ and $\Omega_b(6350)^-$ [7]. These observations were followed by various theoretical investigations trying to explain their properties.

The mass spectrum and decay mechanisms of the heavy baryons containing single heavy quark were studied in various theoretical approaches. The properties of these baryons were extensively searched for by quark model [8–36], heavy hadron chiral perturbation theory [37–41], relativistic flux tube model [42], Bethe-Salpeter formalism [43], $3P_0$ model [44–51], lattice QCD [52–55], the bound state picture [56], light cone QCD sum rules [57–66] and QCD sum rules method [67–75], etc. For more related discussions about these states, we refer to the Refs. [76–81] and the references therein.

Nowadays the LHCb Collaboration announced the observation of another new beauty baryon state, which shows consistency with $2S$ radial excitation of $\Lambda_b^0$ baryon, in the $\Lambda_b\pi^+\pi^-$ invariant mass spectrum with a significance exceeding 14 standard deviations [82]. Its mass and width were reported, with an interpretation of its being $2S$ excited state, as $m_{\Lambda_b^{*\pi\pi}} = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV and $\Gamma = 72 \pm 11 \pm 2$ MeV, respectively. This observation is also consistent with the report of CMS collaboration [83] indicating a broad excess of events in the region of $6040 \rightarrow 6100$ MeV. In 2012, the LHCb Collaboration announced the observation of two narrow $\Lambda_b$ states decaying into $\Lambda_b^0\pi^+\pi^-$, which are $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$ and these states were interpreted as orbital excitations of $\Lambda_b^0$ baryon [5]. These baryons were studied using QCD sum rule approach in the heavy quark effective theory [69]. Later, in 2019, the LHCb collaboration reported the observation of another $\Lambda_b$ baryon doublet, namely $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, with an interpretation of their being 1D-wave state [6]. The mass predictions in the QCD sum rule method for these states were presented in Refs. [70, 75]. In the present work, we focus our attention to the newly observed state $\Lambda_b(6072)^0$ and perform an analysis on the mass of this particle considering its being first radial excitation, $2S$-state, with possible
quantum numbers \( J^P = \frac{1}{2}^+ \), as suggested by the LHCb Collaboration. To this end, we adopt the QCD sum rule method [84–86] with a proper interpolating current that couples the states with considered quantum numbers. This method is a non-perturbative method applied with success to calculate various properties such as spectroscopic and decay properties of hadrons giving consistent results with experimental observations. The interpolating current used in the calculation not only couples to the considered radially excited state but also to the ground and orbitally excited ones. Therefore in this work, we first calculate the mass and the current coupling constant of the ground state baryon, then we obtain the masses and current coupling constants of its first orbital and radial excitations. For completeness, we also include in our analyses the charmed counterpart of the considered states. The spectroscopic analysis of the considered state may shed light on the quantum numbers and structure of this state, improve our understanding of the strong interaction and help us test the predictions of the quark model and other theoretical models, as well.

The outline of this work is as follows: Sec. II provides the details of the QCD sum rules calculations for the masses and the current coupling constants of the considered states. In Section III the numerical analyses and the results are presented. The last section gives a summary of the results and conclusion.

II. QCD SUM RULE CALCULATIONS FOR THE \( \Lambda_b \) AND \( \Lambda_c \) STATES

The states considered in this work are analyzed through the following two-point correlation function applied for mass:

\[
\Pi(q) = i \int d^4xe^{iq\cdot x} \langle 0|\mathcal{T}\{\eta(x)\bar{\eta}(0)\}|0\rangle.
\]

(1)

The \( \eta(x) \) in Eq. (1) represents proper interpolating current formed from the related quark fields considering the valence quark content of the states and \( \mathcal{T} \) is used to represent the time ordering operator. The following interpolating current is used in the calculations:

\[
\eta = \frac{1}{\sqrt{6}} \epsilon^{abc} \left[ 2(u_a^T C d_b)\gamma_5 Q_c + 2\beta(u_a^T C\gamma_5 d_b)Q_c + (u_a^T CQ_b)\gamma_5 d_c + \beta(u_a^T C\gamma_5 Q_b)d_c + (Q_a^T Cd_b)\gamma_5 u_c + \beta(Q_a^T C\gamma_5 d_b)u_c \right],
\]

(2)

where \( Q \) represents \( b \) (\( c \)) quark field for bottom \( \Lambda_b \) (\( \Lambda_c \)) state; \( a, b \) and \( c \) are color indices, \( C \) is the charge conjugation operator and the \( \beta \) is an arbitrary parameter to be fixed later.

According to the standard calculation process of QCD sum rule method, the correlation function is calculated via two approaches. In the first one, it is calculated in terms of hadronic degrees of freedom and called the physical or hadronic side of the calculations. The result of this part contains the physical quantities such as mass and current coupling constant of the considered states. The second approach brings out the results in terms of QCD degrees of freedom such as quark-gluon condensates, QCD coupling constant, the mass of the quarks, etc. Therefore this side is named as the QCD side of the calculations. Matching the results of both sides, considering the coefficient of the same Lorentz structure, one gets the QCD sum rules for the physical quantities under question.

For the physical side of the calculations, the correlator, Eq. (1), is calculated by inserting a complete set of hadronic states into the correlator. This step turns the correlator into the following form

\[
\Pi^{\text{Phys}}(q) = \frac{\langle 0|\eta|\Lambda_Q(q, s)\rangle\langle \Lambda_Q(q, s)|\bar{\eta}|0\rangle}{m^2 - q^2} + \frac{\langle 0|\eta|\tilde{\Lambda}_Q(q, s)\rangle\langle \tilde{\Lambda}_Q(q, s)|\bar{\eta}|0\rangle}{m^2 - q^2} + \frac{\langle 0|\eta|\Lambda'_Q(q, s)\rangle\langle \Lambda'_Q(q, s)|\bar{\eta}|0\rangle}{m'^2 - q^2} + \cdots.
\]

(3)

The \( |\Lambda_Q(q, s)\rangle \), \( |\tilde{\Lambda}_Q(q, s)\rangle \) and \( |\Lambda'_Q(q, s)\rangle \) are used to represent the one-particle states of the ground, and its first orbital excitation \( 1P \) and first radial excitation \( 2S \) states, respectively, \( m, m' \) and \( m'' \) are their respective masses and \( \cdots \) represents higher states and continuum. The matrix elements present in the Eq. (3) are parameterized as follows:

\[
\langle 0|\eta|\Lambda_Q(q, s)\rangle = \lambda u(q, s),
\]

\[
\langle 0|\eta|\tilde{\Lambda}_Q(q, s)\rangle = \lambda'\gamma_5 u(q, s),
\]

\[
\langle 0|\eta|\Lambda'_Q(q, s)\rangle = \lambda' u(q, s),
\]

(4)

where \( \lambda, \lambda' \) and \( \lambda' \) are the corresponding current coupling constants and \( u(q, s) \) is the Dirac spinor. The matrix elements are placed inside the Eq. (3) and the summation over spins for Dirac spinors, which is given as

\[
\sum_s u(q, s)\bar{u}(q, s) = (\bar{q} + m),
\]

(5)
After the Borel transformation, the final result for the hadronic side becomes:

\[ \Pi^{\text{phys}}(q) = \frac{\lambda^2(q + m)}{m^2 - q^2} + \frac{\tilde{\lambda}^2(q - \tilde{m})}{\tilde{m}^2 - q^2} + \frac{\lambda'^2(q + \tilde{m}')}{m'^2 - q^2} + \cdots. \]  

(6)

After the Borel transformation, the final result for the hadronic side becomes:

\[ \Pi^{\text{phys}}(q) = \lambda^2(q + m)e^{-\frac{m^2}{2\sigma^2}} + \tilde{\lambda}^2(q - \tilde{m})e^{-\frac{\tilde{m}^2}{2\sigma^2}} + \lambda'^2(q + \tilde{m}')e^{-\frac{\tilde{m}'^2}{2\sigma^2}} + \cdots, \]

(7)

where \( \Pi^{\text{phys}}(q) \) denotes the correlation function after the Borel transformation.

For the QCD side, one computes the correlation function, Eq. (1), using the interpolating current given in Eq. (2) explicitly. To perform the calculations, first the possible contractions between the quark fields are carried out via Wick’s theorem. For the contracted quark fields the corresponding light and heavy quark propagators presented in coordinate space are used with the following explicit forms:

\[
S_{q,ab}(x) = i\delta_{ab}\frac{\not{k}}{2\pi^2x^4} - \delta_{ab}\frac{m_q}{4\pi^2x^2} - \delta_{ab}\frac{(\not{q}\gamma_q)}{12} + i\delta_{ab}\frac{x^2m_q}{48}(\not{q}\gamma_q) - \frac{i\gamma_{ab}G^{\alpha\beta}_{q}}{32\pi^2x^2}\left[\xi_{\alpha\beta} + \sigma_{\alpha\beta}\not{k}\right] - i\delta_{ab}\frac{x^2\not{q}G_{q}}{7776},
\]

(8)

and

\[
S_Q(x) = \frac{m_Q^2}{4\pi^2}K_1(m_Q\sqrt{-x^2}) - i\frac{m_Q^2}{4\pi^2K_2(m_Q\sqrt{-x^2})} - i\not{q}g_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 du \left[ \not{k} + m_Q \cdot 2(m_Q^2 - k^2)^2G^{\mu\nu}(ux)\sigma_{\mu\nu} + \frac{u}{m_Q^2 - k^2}G^{\mu\nu}(ux)\sigma_{\mu\nu} \right],
\]

(9)

where \( G_{\mu\nu} \) is the gluon field strength tensor, \( K_\nu \) is the Bessel functions of the second kind, \( G^{\alpha\beta}_{ab} = G^{\alpha\beta}_{A}t^{A}_{ab} \) with \( A = 1, 2, \ldots , 8 \) and \( t^A = \lambda^A/2 \). After the usage of the propagators, Fourier and Borel transformations are performed and this is followed by continuum subtraction. The result of the QCD side of the sum rule is as follows:

\[
\Pi^{\text{QCD}}(s_0, M^2) = \int_{(m_Q + m_u + m_d)^2}^{s_0} ds e^{-\frac{m^2}{\sigma^2}} \rho(s) + \Gamma,
\]

(10)

where, \( s_0 \) in Eq. (10) represents the continuum threshold, the \( \rho(s) \) is the spectral density that is obtained by taking the imaginary part of the results as \( \frac{1}{\pi} \text{Im}[\Pi^{\text{QCD}}(s_0, M^2)] \). As an example, for the structure \( \not{q} \), the results obtained for the \( \rho(s) \) and \( \Gamma \) are presented in the Appendix.

After the computation of both parts, the results are matched through the dispersion relations considering the coefficients of the same Lorentz structures, that is \( \not{q} \) or \( I \), and the QCD sum rules for the considered quantities are achieved from these matches as

\[
\lambda^2e^{-\frac{m^2}{\sigma^2}} + \tilde{\lambda}^2e^{-\frac{\tilde{m}^2}{\sigma^2}} + \lambda'^2e^{-\frac{\tilde{m}'^2}{\sigma^2}} = \Pi^{\text{QCD}}(s_0, M^2),
\]

(11)

from the coefficient of the \( \not{q} \) structure and

\[
\lambda^2me^{-\frac{m^2}{\sigma^2}} + \tilde{\lambda}^2\tilde{m}e^{-\frac{\tilde{m}^2}{\sigma^2}} + \lambda'^2m'e^{-\frac{\tilde{m}'^2}{\sigma^2}} = \Pi^{\text{QCD}}(s_0, M^2),
\]

(12)

from the \( I \) structure. The relation obtained using the \( \not{q} \) structure is used to derive the QCD sum rules for masses and coupling constants by following the ground state+continuum scheme in which we consider the second and third terms of the left-hand-side of Eqs. (11) as parts of the continuum. This results in the following equation for the masses of the considered ground states

\[
m^2 = \frac{d}{ds} \left[ \frac{\Pi^{\text{QCD}}_{\not{q}}(s_0, M^2)}{\Pi^{\text{QCD}}_{\not{q}}(s_0, M^2)} \right],
\]

(13)

and the current coupling constant is obtained as

\[
\lambda^2 = e^{\frac{m^2}{\sigma^2}} \left[ \frac{\Pi^{\text{QCD}}_{\not{q}}(s_0, M^2)}{\Pi^{\text{QCD}}_{\not{q}}(s_0, M^2)} \right],
\]

(14)
where \( m \) and \( \lambda \) represent the mass and corresponding current coupling constant for ground state \( \Lambda_Q \) baryon. Then we consider the first two terms on the left-hand side of Eq. (11) by increasing the threshold and the third one is taken in the continuum and using the results obtained from ground states as inputs we get the masses and current coupling constants for the first orbitally excited, \( 1P \), states. And finally, the results of the ground states and \( 1P \) states are used with a similar manner, namely ground-state+first orbitally+first radially excited states+continuum approach, to obtain the physical quantities of the radially excited, \( 2S \), states. These steps are explained also in the next section.

### III. NUMERICAL ANALYSES

The analytically obtained QCD sum rules results of the previous section are applied with the numerical input parameters and the two auxiliary parameters to attain the numerical values of the physical quantities, namely masses and the current coupling constants of the considered states. Some of the input parameters are presented in Table I. Though our main concern in the present work is the mass of newly observed \( 2S \) \( \Lambda_b(6072)^0 \) state, we also obtain the masses for the \( 1S \) and \( 1P \) excited states for both \( \Lambda_b^0 \) and \( \Lambda_b^+ \) states and their corresponding coupling constants. The auxiliary parameters \( s_0 \) and \( M^2 \) are determined from the analyses of the results following the standard criteria of QCD sum rules such as the convergence of the operator product expansion (OPE) and the dominance of the pole contribution. Moreover, in the working regions, a plateau of the results as a function of Borel parameters and threshold parameters is desired. Additionally, one more parameter, \( \beta \), enters the calculations in the QCD side from the explicit expression of the interpolating current. The working interval of the \( \beta \) is determined considering the flat regions of the results against the variation of the \( \beta \) parameter. That region is obtained from the analyses for both \( \Lambda_b \) and \( \Lambda_c \) cases as

\[
-1.0 < \cos \theta < -0.5 \quad \text{and} \quad 0.5 < \cos \theta < 1.0,
\]

where \( \beta = \tan \theta \).

The working intervals of Borel parameters, restricted by the convergence of OPE, pole dominance requirements, and the stability of the results in response to the variation of these parameters, are presented in Table II. For analyses, we take into account ground-state+first orbitally+first radially excited states+continuum and using Eq. (11) we move step by step as follows: First, we obtain the mass and current coupling constant for the ground state \( \Lambda_Q \) particles. To achieve these quantities we choose proper threshold parameters considering the ground-state+continuum scheme and the notion that the threshold parameter is related to the energy of the next excited state. Considering that we choose the proper interval for the \( s_0 \) as given in Table II. The masses and the current coupling constants obtained in this step are also given in Table II and these are used as inputs in the second step. Secondly, we consider the ground state+first orbitally excited state+continuum scheme and, with the same logic that is used for the determination of \( s_0 \) of the previous step, we determine a new \( s_0 \) working interval. The results obtained in these steps are presented

| Parameters | Values |
|------------|--------|
| \( m_{\Lambda_b} \) | \( 1.27 \pm 0.02 \) GeV \[87]\] |
| \( m_{\Lambda_b^0} \) | \( 4.18^{+0.04}_{-0.02} \) GeV \[87]\] |
| \( m_{\Lambda_b^+} \) | \( 2.16^{+0.02}_{-0.04} \) MeV \[87]\] |
| \( m_{\Lambda_c} \) | \( 4.67^{+0.35}_{-0.24} \) MeV \[87]\] |
| \( \langle \bar{q}q \rangle (1\text{GeV}) \) | \( (-0.24 \pm 0.01)^3 \) GeV\(^3\) \[88]\] |
| \( m_0^2 \) | \( (0.8 \pm 0.1) \) GeV\(^2\) \[88]\] |
| \( \langle \circ G^2 \rangle \) | \( (0.012 \pm 0.004) \) GeV\(^4\) \[89]\] |

TABLE I: Some input parameters used in the analyses.

| Particle | State | \( M^2 \) (GeV\(^2\)) | \( s_0 \) (GeV\(^2\)) | Mass (MeV) | \( \lambda \) (GeV\(^2\)) |
|----------|-------|----------------------|----------------------|------------|----------------------|
| \( \Lambda_b \) | \( \Lambda_b(\frac{1}{2}^-)(1S) \) | 6.0 \(-8.0 | 5.86^2 \- 5.90^2 | 5611.47 \pm 27.47 | 0.042 \pm 0.003 |
| \( \Lambda_b(\frac{1}{2}^-)(1P) \) | 6.0 \(-8.0 | 5.92^2 \- 5.96^2 | 5910.56 \pm 84.54 | 0.020 \pm 0.008 |
| \( \Lambda_b(\frac{1}{2}^-)(2S) \) | 6.0 \(-8.0 | 6.18^2 \- 6.22^2 | 6073.65 \pm 93.22 | 0.051 \pm 0.007 |
| \( \Lambda_c \) | \( \Lambda_c(\frac{1}{2}^-)(1S) \) | 3.0 \(-5.0 | 2.53^2 \- 2.57^2 | 2282.42 \pm 28.38 | 0.022 \pm 0.001 |
| \( \Lambda_c(\frac{1}{2}^-)(1P) \) | 3.0 \(-5.0 | 2.63^2 \- 2.67^2 | 2592.36 \pm 53.01 | 0.014 \pm 0.003 |
| \( \Lambda_c(\frac{1}{2}^-)(2S) \) | 3.0 \(-5.0 | 2.73^2 \- 2.77^2 | 2765.52 \pm 22.29 | 0.016 \pm 0.004 |

TABLE II: The auxiliary parameters and the results of masses and current coupling constants.
in Table II. And finally, we consider radially excited 2S state with ground-state+first orbitally+first radially excited states+continuum approach and attain the proper threshold parameter for this approach. The results obtained for 2S states are also depicted in Table II. The errors in the results arise from the errors of the input parameters and the uncertainties coming from the determinations of the working intervals for the auxiliary parameters.

IV. CONCLUSION

Focusing on the recently observed state Λ_b(6072), we studied the ground states 1S, first orbital 1P and first radial 2S excitations of the spin-1/2 Λ_c and Λ_b states. The experimentally observed values for the mass of Λ_b(6072) state is 

$m_{Λ_{b}^0} = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV with a width value $Γ = 72 \pm 11 \pm 2$ MeV [82]. In Ref. [82], it was underlined that this result is consistent with the predictions of the quark model for Λ_b(2S) state [10, 13, 16]. Motivated by this observation, we calculated the masses and current coupling constants for ground 1S, first orbitally excited 1P and first radially excited 2S states of Λ_b and Λ_c particles. For the analyses, we applied a powerful nonperturbative method, QCD sum rule with a suitable interpolating current form considering the quark content and quantum numbers of the considered states. The results presented in Table II for ground and first orbital excitations of Λ_b and Λ_c baryons are in good agreement with the present experimental findings given as: 

$m_{Λ_b} = 5619.60 \pm 0.17$ MeV [87], 
$m_{Λ_b(5812)} = 5912.20 \pm 0.13 \pm 0.17$ MeV [87], 
$m_{Λ_c^+} = 2286.46 \pm 0.14$ MeV [87], 
$m_{Λ_{b(2565)}} = 2592.25 \pm 0.28$ MeV [87].

As for the main focus of this work, the mass obtained for Λ_b(6072) as $m_{Λ_b(2S)} = 6073.65 \pm 93.22$ MeV is consistent with the experimental result, $m_{Λ_b} = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV [82]. The result is also consistent with the various theoretical predictions given for the radially excited Λ_b state with $J^P = 1\frac{1}{2}^+$ as $m = 6045$ MeV [10], $m = 6.107$ GeV [16], $m = 6089$ MeV [13], $m = 6106$ MeV [15], $m = 6153$ MeV[17]. In Ref. [19] the mass for this particle is calculated using hypercentral quark model with and without first order corrections to the confinement potential as $m = 6.026$ GeV and $m = 6.016$ GeV, respectively. The Ref. [91] presented the mass of the particle as $m = 5982 – 6127$ MeV obtained from the chiral quark model using five different sets of model parameters. As is seen from these results, the mass obtained in this work is in good consistency with the present theoretical predictions within the errors.

The mass for the 2S Λ_c state is also obtained for completeness and its value is attained as $m_{Λ_{c(2S)}} = 2765.52 \pm 22.29$ MeV. This result is also consistent with the mass value for Λ_c(2765)$^+$ given as $m_{Λ_{c(2765)}} = 2666.6 \pm 2.4$ MeV [87]. This particle is presented in PDG as Λ_c(2765)$^+$ or Σ(2765)$^+$ with unknown $I(J^P) = ?(?)$ quantum numbers. However in Ref. [92] its isospin was determined as zero and name for it was suggested to be Λ_c(2765)$^+$. In this work we obtained the mass for the first radial excitation of the Λ_c state with $J^P = 1\frac{1}{2}^+$ in consistency with the mass of the Λ_c(2765)$^+$ state. Our prediction is also consistent with the theoretical works with the following predictions for 2S wave Λ_c state: $m = 2775$ MeV [10], $m = 2772$ MeV [12], $m = 2769$ MeV [13], $m = 2769$ MeV [26], $m = 2.791$ GeV [16], $m = 2772$ MeV [30], $m = 2766$ MeV [42], $m = 2.758$ GeV [20], $m = 2857$ MeV [17], $m = 2785$ MeV [15], $m = 2749$ MeV [93] and $m = 2654 – 2825$ MeV [91] obtained with five different sets of model parameters. These results are in agreement with that of present work within the errors.

A comparison of the result of this work with the present theoretical and experimental findings indicates that the particle Λ_b(6072)$^0$ is the first radial excitation of the Λ_b baryon with the quantum numbers $J^P = 1\frac{1}{2}^+$. The consistency of the result for the first radial excitation of Λ_c with $J^P = 1\frac{1}{2}^+$ with other theoretical results and the present experimental value of Λ_c(2765)$^+$ is also considerable. Considering the findings of the Ref. [92] on its possibly being Λ_c(2765)$^+$ state, our result indicates that it may be first radial excitation of Λ_c state with quantum numbers $J^P = 1\frac{1}{2}^+$. Further studies on these states, including their masses and decay properties, and comparison with the result of the present work, may provide more clarification on the quantum numbers of these states. In this respect, the findings of the present work may also serve as inputs for future studies.

V. APPENDIX: THE RESULTS CORRESPONDING TO THE COEFFICIENT OF THE $\not{q}$ LORENTZ STRUCTURE

The results obtained in QCD side for the functions corresponding to the coefficient of the $\not{q}$ Lorentz structure including components of the results with different dimensions are presented in this section. We obtain,

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert.}}(s) + \sum_{N=3}^{13} \rho^{\dim N}(s); \quad \Gamma = \sum_{N=5}^{13} \Gamma^{\dim N},$$

(16)
where,

\[ \rho_{\text{pert.}}(s) = \frac{1}{3 \times 2^{11} \pi^4 s^2} \left\{ \left( m_b^2 - s \right) \left[ 20\beta \bar{m} m_b \bar{m} - 4\beta \bar{m} \bar{m} + 8\bar{m} \bar{m} + 3\beta \bar{m} - 21\beta m_b^4 (m_b^2 + s) \right] + 12 m_b^2 \beta^2 \beta \bar{m} - 3 \beta m_b^2 \right\}, \]  

(17)

\[ \rho_{\text{Dim}3}(s) = \frac{m_b^2 - s}{3 \times 2^{7} \pi^2 s^2} \left\{ \left( \bar{\pi} \bar{u} \right) \left[ 2\bar{\beta} m_b^2 + 2(1 - \beta(1 + 5\beta''')) m_b s - (m_b^2 + s) \right] \bar{\beta} \bar{m} (2 + 11\beta''') m_b^3 + 3 \beta m_b^2 \right\}, \]  

(18)

\[ \rho_{\text{Dim}4}(s) = \frac{g_s^2 G^2}{3 \times 2^{10} \pi^4 m_b s^2} \left\{ 54\beta m_b^2 \bar{m} s - 9\beta \bar{m}^2 m_b s - 3 \beta \bar{m}^2 s - 3 \beta''(4 + 13\beta''') m_b^4 \bar{m} + 24(1 + \beta \beta''') m_b - 3 \beta m_b s^2 + 18 \beta (m_b^2 - s) m_b^2 \bar{m} \log \left( \frac{s - m_b^2}{\Lambda^2 m_b^2} \right) \right\}, \]  

(19)

\[ \rho_{\text{Dim}5}(s) = -\frac{\beta''}{3 \times 2^{8} \pi^2 m_b^2 s^2 \Lambda^2} \left\{ m_0^2 \bar{\pi} \bar{u} \left[ M^2 (6\beta''' m_b s (4 m_d - m_b) - (4 - 11\beta''') m_b^5 - 48 \gamma_E \beta''' m_b s^2 \right] \right. \]  

\[ \left. + m_0^4 \beta'' (m_b^2 - (2 - 25\beta''') m_d) \right) - 48 \beta''' m_b^3 m_u^2 s^2 \log \left( \frac{s - m_b^2}{\Lambda} \right) \]  

\[ + m_0^2 \bar{\pi} \bar{u} \left[ M^2 (6\beta'' m_b^2 s (4 m_u - m_b) - (4 - 11\beta''') m_b^5 - 48 \gamma_E \beta''' m_u s^2 \right] \right. \]  

\[ \left. + m_0^4 \beta'' (m_u^2 - (2 - 25\beta''') m_u) \right) - 48 \beta''' m_b^3 m_u^2 s^2 \log \left( \frac{s - m_b^2}{\Lambda} \right) \}, \]  

(20)

\[ \rho_{\text{Dim}6}(s) = 0, \]  

(21)

\[ \rho_{\text{Dim}7}(s) = \frac{\beta''}{3 \times 2^8 \pi^2 M^6 m_b^2 s^2} \left\{ \langle g_s^2 G^2 \rangle \left( \bar{\pi} \bar{u} \right) \left[ 6 \beta''' M^6 m_b^2 (m_b^2 - 2 m_d) + m_d s^2 (m_b^4 (4 + 22 \beta''') + M^4 (2 + 23 \beta''') \right] \right. \]  

\[ \left. + M^2 m_b^2 (2 + 35 \beta''') \log \left( \frac{s - m_b^2}{\Lambda} \right) \right\}, \]  

(22)

\[ \rho_{\text{Dim}8}(s) = \frac{\langle g_s^2 G^2 \rangle}{3 \times 2^{12} \pi^4 M^6} m_b \bar{m} (-5 + \beta (4 + \beta)) \log \left( \frac{s - m_b^2}{\Lambda^2} \right), \]  

(23)

\[ \rho_{\text{Dim}9}(s) = \frac{\langle g_s^2 G^2 \rangle \bar{m} \left( \bar{\pi} \bar{u} \right)}{3 \times 2^{10} \pi^4 M^{10}} \left( m_u \bar{\pi} \bar{u} + m_d \bar{\pi} \bar{u} \right) \beta'' m_b^2 \left[ (4 + 11 \beta''') m_b^2 - M^2 (6 + 5 \beta''') \right], \]  

(24)

\[ \rho_{\text{Dim}10}(s) = 0, \]  

(25)

\[ \rho_{\text{Dim}11}(s) = -\frac{\langle g_s^2 G^2 \rangle^2 \left( m_u \bar{\pi} \bar{u} + m_d \bar{\pi} \bar{u} \right)}{3 \times 2^{11} \pi^2 M^{12}} \beta'' (2 + 11 \beta''') m_b^2 (2 m_b^2 - 5 M^2) \log \left( \frac{s - m_b^2}{\Lambda^2} \right), \]  

(26)
\[
\Gamma^{\text{Dim}13}(s) = \frac{(g^2 G^2)^2 m_0^4 (m_u \langle dd \rangle + m_d \langle uu \rangle)}{3^4 \times 2^{12} \pi^2 M^{16}} \beta''(2 + 11 \beta''') m_0^2 (m_b^4 - 7 M^2 m_b^2 + 9 M^4) \log\left[ \frac{s - m_b^2}{\Lambda^2} \right], \]

(27)

\[
\Gamma^{\text{Dim}5} = -\frac{1}{3 \times 2^8 \pi^2 m_b^2} \left\{ m_0^2 \langle uu \rangle \left[ e^{-\frac{m_b^2}{m_f^2}} \left[ 48 \gamma_E^2 \beta M^2 m_d + m_b^2 \left( 3 \beta'' m_m + 2 \beta'' (2 + \beta'' (11 + 24 \gamma_E) m_d) \right) \right] 
+ 2 \tilde{\gamma}_E m_0^2 m_d \log\left[ \frac{\Lambda^2}{m_b^2} \right] - e^{-\frac{m_b^2}{m_f^2}} \gamma_E \beta M^2 m_d \right] 
+ m_0^2 \langle dd \rangle \left[ e^{-\frac{m_b^2}{m_f^2}} \left[ 48 \gamma_E \beta^2 M^2 m_u + m_b^2 \left( 3 \beta'' m_d + 2 \beta'' (2 + \beta'' (11 + 24 \gamma_E) m_d) \right) \right] 
+ 2 \tilde{\gamma}_E m_0^2 m_u \log\left[ \frac{\Lambda^2}{m_b^2} \right] - e^{-\frac{m_b^2}{m_f^2}} \gamma_E \beta^2 M^2 m_u \right] \right\}, \]

(28)

\[
\Gamma^{\text{Dim}6} = \frac{\langle uu \rangle \langle dd \rangle}{3^2 \times 2^4 M^2} \beta'' \left[ m_b \tilde{m} (1 + 5 \beta) - 2 M^2 (2 + 11 \beta'') \right], \]

(29)

\[
\Gamma^{\text{Dim}7} = \frac{\langle uu \rangle (g^2 G^2)}{3^3 \times 2^8 \pi^2 M^4 m_b^2 (m_b^2 - s_0)^2} \left\{ e^{-\frac{m_b^2}{m_f^2}} \beta'' M^2 m_b^2 [4 + 22 \beta'''] m_b^4 (4 + 22 \beta'''') M^2 s_0 + m_b^2 M^2 (7 + 57 \beta''') 
- 2 m_0^2 \langle uu \rangle [(2 + 11 \beta''')] \left[ 2 \beta'' (2 + 11 \beta''') m_b^4 m_d + 8(-2 + \beta'''') M^4 m_b + 6 \beta'' (13 + 11 \beta + \gamma_E (2 + 23 \beta'''') M^4 m_u) + 3 M^2 m_b^2 (\beta'' (2 + 11 \beta''')(2 + \gamma_E) m_d - 2 (1 + \beta'''') m_u)] + \beta'' m_b^4 (m_b^2 - s_0)^2 \times \left[ 3 e^{-\frac{m_b^2}{m_f^2}} m_b \left[ (2 + 11 \beta''') m_b^4 + (2 + 23 \beta'''') M^2 m_b^2 + (2 + 23 \beta'''') M^4 \right] \log\left[ \frac{\Lambda^2}{M^2} \right] - 3 e^{-\frac{m_b^2}{m_f^2}} m_b^2 m_d 
\times \left[ (4 + 22 \beta'''') m_b^4 + (2 + 23 \beta'''') M^2 m_b^2 + (2 + 23 \beta'''') M^4 \right] \log\left[ \frac{s_0 - m_b^2}{\Lambda^2} \right] \right\} \right\}, \]

(30)

\[
\Gamma^{\text{Dim}8} = \frac{(g^2 G^2)^2}{3^4 \times 2^{13} \pi^2 M^4 m_b^2 (m_b^2 - s_0)^2} \left\{ 18 e^{-\frac{m_b^2}{m_f^2}} (5 + \beta) m_b^2 \tilde{m} M^2 (M^2 + m_b^2 - s_0) - e^{-\frac{m_b^2}{m_f^2}} (m_b^2 - s_0)^2 \times \left[ 16 (-2 + \beta'''') \tilde{m} m_b^2 - \beta'' (-30 + 11 \beta''') M^2 - 3 (13 + \beta (10 + 13 \beta)) M^4 m_b \right] 
- 9 \beta'' (5 + \beta) m_b^2 \tilde{m} m_b^2 (m_b^2 - s_0)^2 \left[ e^{-\frac{m_b^2}{m_f^2}} \log\left[ \frac{\Lambda^2}{m_b^2} \right] + 2 e^{-\frac{m_b^2}{m_f^2}} \log\left[ \frac{s_0 - m_b^2}{\Lambda^2} \right] \right] \right\} \right\}, \]

(31)
\[\Gamma_{\text{Dim} 9} = \frac{\langle g_4^2G^2 \rangle m_0^2 \langle \bar{\pi} u \rangle}{3^3 \times 2^{11} \pi^2 M^8} \left\{ 6 \beta'' M^2 m_0^2 m_d e^{-\frac{m_0^2}{M^2}} + m_0^6 \left( (15 + 17 \beta) M^2 - 6(2 + 11 \beta'') m_0^2 \right) + m_0^8 \left[ 6(2 + 11 \beta'') m_0^2 + (10 - 29 \beta'') m_0^2 \right] + \right. \]
\[+ m_0^2 [7 \beta'' M^2 s_0 - 2(2 + 11 \beta'') s_0^3 + 2(2 - 17 \beta'') M^4 s_0 + 2(6 + 61 \beta'') M^6] - \frac{1}{m_0^2} e^{-\frac{m_0^2}{M^2}} [4 \beta''(2 + 11 \beta'')] \]
\[\times m_0^2 m_d + \beta''(20 + 11 \beta'') M^2 m_0^2 + \beta''(2 + 39 \beta'') M^6 m_0^2 + 2(1 + \beta(4 - 5 \beta)) M^6 m_0^2 + 16b_6 M^4 m_0^2 m_d \]
\[+ M^2 m_0^2 [6 \beta''(6 + 37 \beta'') m_0^2 + (5 + \beta(2 + 5 \beta)) m_0^6] + 3e^{-\frac{m_0^2}{M^2}} \beta'' m_0^2 m_d [(4 + 22 \beta'') m_0^2 - (6 + 5 \beta'') M^2] \log \left( \frac{A^2}{m_0^2} \right) \]
\[+ 6e^{-\frac{m_0^2}{M^2}} \beta'' m_0^2 m_d [(6 + 5 \beta'') M^2 - 2(2 + 11 \beta'') m_0^2] \log \left( \frac{s_0 - m_0^2}{\Lambda^2} \right) \right\} \]
\[= \frac{\langle g_4^2G^2 \rangle m_0^2 \langle \bar{\pi} u \rangle}{3^3 \times 2^{11} \pi^2 M^8} \left\{ e^{-\frac{m_0^2}{M^2}} [3 M^6 + 9 M^4 m_0^2 - 2 m_0^6 + (3 \gamma_E - 8) M^2 m_0^2 + \right. \]
\[\times (m_0^2 - s_0^2) + 3e^{-\frac{m_0^2}{M^2}} M^2 m_0^2 \left( 2 m_0^4 - m_0^8 + 12 s_0^2 + 14 M^2 s_0 - M^4 \right) + m_0^6 (2 M^6 + \right. \]
\[\left. \left. + 7 M^4 s_0 - 24 M^2 s_0^2 - 8 s_0^3 + 5 M^2 s_0 (6 M^6 - 2 M^2 s_0 + M^2 s_0^2 - s_0^3) + m_0^2 (18 M^8 + 8 M^6 s_0 - 11 M^4 s_0^2 + 18 M^2 s_0^3 + 2 s_0^4) - 3 m_0^4 (m_0^2 - s_0^2) + \right] \right. \]
\[\left. - 3 m_0^2 (m_0^2 - s_0^2) \right\} \frac{\langle g_4^2G^2 \rangle m_0^2 \langle \bar{\pi} u \rangle}{3^{35} \times 2^{19} M^{12}} e^{-\frac{m_0^2}{M^2}} \beta'' m_b \left\{ 5(1 + 5 \beta) m_0^4 m_b - 3(1 + 5 \beta) M^2 m_0^2 m_b - (2 + 11 \beta'') M^2 m_b \right. \]
\[\left. + 12(2 + 11 \beta'') M^2 m_0^2 + (2 + 11 \beta'') M^4 m_b \right\}, \]
\[\Gamma_{\text{Dim} 11} = \frac{\langle g_4^2G^2 \rangle^2 \langle m_d \langle \bar{u} u \rangle + m_u \langle \bar{d} d \rangle \rangle}{3^5 \times 2^{11} \pi^2 M^{10} m_0^2 (m_0^2 - s_0^2)^4} \beta''(2 + 11 \beta'') \left\{ e^{-\frac{m_0^2}{M^2}} [3 M^6 + 9 M^4 m_0^2 - 2 m_0^6 + (3 \gamma_E - 8) M^2 m_0^2 \right. \]
\[\times (m_0^2 - s_0^2) + 3e^{-\frac{m_0^2}{M^2}} M^2 m_0^2 \left( 2 m_0^4 - m_0^8 + 12 s_0^2 + 14 M^2 s_0 - M^4 \right) + m_0^6 (2 M^6 + \right. \]
\[\left. \left. + 7 M^4 s_0 - 24 M^2 s_0^2 - 8 s_0^3 + 5 M^2 s_0 (6 M^6 - 2 M^2 s_0 + M^2 s_0^2 - s_0^3) + m_0^2 (18 M^8 + 8 M^6 s_0 - 11 M^4 s_0^2 + 18 M^2 s_0^3 + 2 s_0^4) - 3 m_0^4 (m_0^2 - s_0^2) + \right] \right. \]
\[\left. - 3 m_0^2 (m_0^2 - s_0^2) \right\} \frac{\langle g_4^2G^2 \rangle^2 \langle m_d \langle \bar{u} u \rangle + m_u \langle \bar{d} d \rangle \rangle}{3^5 \times 2^{19} M^{12}} e^{-\frac{m_0^2}{M^2}} \beta'' m_b \left\{ 5(1 + 5 \beta) m_0^4 m_b - 3(1 + 5 \beta) M^2 m_0^2 m_b - (2 + 11 \beta'') M^2 m_b \right. \]
\[\left. + 12(2 + 11 \beta'') M^2 m_0^2 + (2 + 11 \beta'') M^4 m_b \right\}. \]
\[ \bar{m} = m_u + m_d, \]
\[ \beta = (-1 + \beta)(1 + 5\beta), \]
\[ \tilde{\beta} = -1 + \beta^2, \]
\[ \beta' = 5 + \beta(2 + 5\beta), \]
\[ \beta'' = -1 + \beta, \]
\[ \beta''' = 1 + \beta. \]

(37)
