Black Hole Skyrmions with Negative Cosmological Constant

Noriko Shiiki\(^\ast\) and Nobuyuki Sawado\(^\dagger\)

Department of Physics, Tokyo University of Science, Noda, Chiba 278-8510, Japan

(Dated: March 24, 2022)

We study spherically symmetric black hole solutions with Skyrme hair in the Einstein-Skyrme theory with a negative cosmological constant. The dependence of the skyrmion field configuration on the cosmological constant is examined. The stability is investigated in detail by solving the linearly perturbed equation numerically. It is shown that there exist linearly stable solutions in the branch which represents unstable configuration in the asymptotically flat spacetime.

1. INTRODUCTION

It has been known that the Einstein-Skyrme system possesses black hole solutions as well as regular solutions. The black hole solution with \(B = 1\) Skyrme hair was found in Refs. [2, 3, 4, 5]. It was shown that there exist two fundamental branches of the solutions and remarkably one of the branches represents stable configuration under linear perturbations [2, 3, 4]. This is a counter example to the no-hair conjecture for black holes. The no-hair conjecture states that the only allowed degrees of freedom of stationary black holes are global charges such as mass, angular momentum and electric and magnetic charges [5]. However, this conjecture concerns only linear field sources and asymptotically flat spacetime. In fact, if we consider nonlinear field sources such as non-abelian gauge fields [8] and Skyrme fields or asymptotically anti-de Sitter spacetime [9], black hole solutions with hair can exist. Following the discovery of black holes \(B = 1\) Skyrme hair, the \(B = 2\) Skyrme black hole solution with axisymmetry was constructed by us in Ref. [10]. The review of the black hole solutions with Skyrme hair is given in Ref. [11]. All of these solutions are, however, obtained in the asymptotically flat spacetime. Thus we consider spherically symmetric black hole solutions with Skyrme hair in the asymptotically anti-de Sitter (AdS) spacetime with a negative cosmological constant in this paper.

There has been an increasing interest in the AdS spacetime. Especially the AdS black hole is an interesting object from the holographic point of view in the form of AdS/CFT correspondence [12, 13]. Maldacena proposed the correspondence between the large \(N\) limit of conformally invariant \(SU(N)\) gauge theory and supergravity (string theory) on the product of AdS space with a compact manifold in a low energy limit where the dynamics on the brane decouples from the bulk. Witten elaborated on the idea of the correspondence and related the thermodynamics of \(N = 4\) super Yang-Mills theory in four dimensions to the thermodynamics of Schwarzschild black holes in AdS space [13]. The study of the AdS black hole in terms of conformal field theory showed that the Hawking-Page phase transition between AdS space and a AdS black hole corresponds to the confining-deconfining transition in the dual gauge theory [14]. It may be thus useful to consider the Skyrme model which is interpreted as QCD in the large \(N\) limit in the asymptotically AdS spacetime to understand the AdS/CFT correspondence.

Brane world cosmology also indicates that there was a period when spacetime was AdS with a negative cosmological constant in the early universe (for example, see [15, 16, 17, 18]). If this scenario is true, AdS black holes should have been produced as the primordial black holes. The solutions we obtain in this paper provide a semiclassical framework to study the interaction of baryons with the AdS primordial black hole.

In the context of the Einstein-Yang-Mills (EYM) theory, it was shown that while all black hole solutions are unstable for \(\Lambda \geq 0\) [19, 20], there exist stable black hole solutions for \(\Lambda < 0\) [21]. We investigate the stability of the AdS Skyrme black hole and discuss in detail to see if the cosmological constant change the stability of the AdS black hole skyrmion.

2. THE EINSTEIN-SKYRME MODEL

The Einstein-Skyrme system with the cosmological constant \(\Lambda\) is defined by the action

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G} (R - 2\Lambda) + \frac{F^2}{16} g^{\mu\nu} \text{tr} (L_\mu L_\nu) + \frac{1}{32\pi^2} g^{\mu\nu} g^{\rho\sigma} \text{tr} ([L_\mu, L_\rho][L_\nu, L_\sigma]) \right\}
\]

where \(L_\mu = U^\dagger \partial_\mu U\) and \(U\) is an \(SU(2)\) chiral field.

We require that the spacetime recovers the AdS solution at infinity and parameterize the metric as

\[
d^2 = -e^{2\xi(r)} C(r) dt^2 + C(r)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)
\]
where
\[ C(r) = 1 - \frac{2Gm(r)}{r} - \frac{\Lambda r^2}{3}. \] (3)

The topology of AdS spacetime is \( S^1 \times \mathbb{R}^3 \) and hence the timelike curves are closed. This can be, however, unwound if we consider the covering spacetime with topology \( \mathbb{R}^4 \). For the spacetime to recover the anti-de Sitter spacetime asymptotically, solutions satisfy
\[ m(r) \to \text{const.}, \quad \delta(r) \to 0 \quad \text{as} \quad r \to \infty. \] (4)

At the horizon \( r = r_h \), we have \( C(r_h) = 0 \) which reads
\[ m(r_h) = \frac{r_h}{2G} - \frac{\Lambda r_h^3}{6G}, \quad \delta(r_h) = \delta_h \] (5)
where \( \delta_h \) is provided by hand as a shooting parameter so as to satisfy the asymptotic boundary condition in (4).

The \( B = 1 \) skyrmion can be obtained by imposing the hedgehog ansatz on the chiral field
\[ U(\vec{r}) = \cos f(r) + i\vec{n} \cdot \vec{\tau} \sin f(r). \] (6)

Introducing the dimensionless variables
\[ x = eF_\pi r, \quad \mu(x) = eF_\pi Gm(r), \quad \tilde{\Lambda} = \Lambda/e^2 F_\pi^2 \] (7)
with
\[ C(x) = 1 - \frac{2\mu(x)}{x} - \frac{\tilde{\Lambda} x^2}{3}, \] (8)
one gets the energy
\[ E_S = 4\pi \frac{F_\pi}{e} \int \left\{ \frac{1}{8} \left( Cf'^2 + \frac{2\sin^2 f}{x^2} \right) + \frac{\sin^2 f}{2x^2} \left( 2Cf'^2 + \frac{\sin^2 f}{x^2} \right) \right\} e^x x^2 dx \] (9)
\[ = 4\pi \frac{F_\pi}{e} \int \left( \frac{1}{8} Cuf'^2 + \frac{1}{4x^2} v^2 \right) e^x dx \] (10)
where we have defined \( u = x^2 + 8\sin^2 f \) and \( v = \sin^2 f(x^2 + 2\sin^2 f) \).

The covariant topological current is defined by
\[ B^\mu = -\frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2} \sqrt{-g} \text{tr} \left( U^{-1} \partial_\nu U U^{-1} \partial_\rho U U^{-1} \partial_\sigma U \right) \] (11)
whose zeroth component corresponds to the baryon number density
\[ B^0 = -\frac{1}{2\pi^2} e^{-\delta} \frac{f'^2 \sin^2 f}{r^2}. \] (12)

We impose the boundary condition on the profile function as
\[ f(x) \to 0 \quad \text{as} \quad x \to \infty, \] (13)
and at the horizon \( x = x_h \), \( f(x_h) = f_h \) is a shooting parameter determined so as to satisfy (13). Then the baryon number becomes
\[ B = \int \sqrt{-g} B^0 d^3x = -\frac{2}{\pi} \int_{f_h}^0 \sin^2 f df = \frac{1}{2\pi}(2f_h - \sin 2f_h). \] (14)

If the solution is regular and there is no horizon, the \( B = 1 \) skyrmion is recovered with \( f_h = \pi \). Indeed, as shown later numerically, the value of \( f_h \) approaches to \( \pi \) as \( x_h \) goes to zero. However, if the solution is a black hole, \( f_h \) takes the value less than \( \pi \), which means that the solution possesses fractional baryonic charge. Topologically, the presence of a
The profile function numerically computed for several values of gravitational fields. The variation of the static energy (10) with respect to the profile. Thus the coupled field equations to be solved are given by

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \] (15)

which reads

\[ G_{00} = 8\pi G T_{00} - \Lambda g_{00} \rightarrow 1 - C - C' x = \frac{\alpha}{4} \left(C u f'^2 + \frac{2v}{x^2}\right) + \tilde{\Lambda} x^2 \] (16)
\[ G_{11} = 8\pi G T_{11} - \Lambda g_{11} \rightarrow -1 + C + \left(\frac{e^{2\Phi} C'}{e^{2\Phi}}\right) x = \frac{\alpha}{4} \left(C u f'^2 - \frac{2v}{x^2}\right) - \tilde{\Lambda} x^2. \] (17)

where we have defined the coupling constant \( \alpha = 4\pi GF^2_\pi \). Consequently following two equations are obtained for the gravitational fields

\[ \delta' = \frac{\alpha}{4x} u f'^2, \quad -(C x)' + 1 = \frac{\alpha}{4} \left(C u f'^2 + \frac{2v}{x^2}\right) + \tilde{\Lambda} x^2. \] (18)

The variation of the static energy (10) with respect to the profile \( f(x) \) leads to the field equation for matter

\[ f'' = \frac{1}{e^\delta C u} \left[-(e^\delta C u)' f' + \left(4C f'^2 + 1 + \frac{4\sin^2 f}{x^2}\right) e^\delta \sin 2f\right]. \] (19)

Thus the coupled field equations to be solved are given by

\[ \delta' = \frac{\alpha}{4x} u f'^2 \] (20)
\[ \mu' = \frac{\alpha}{8} \left(C u f'^2 + \frac{2v}{x^2}\right) \] (21)
\[ f'' = \frac{1}{e^\delta C u} \left[-(e^\delta C u)' f' + \left(4C f'^2 + 1 + \frac{4\sin^2 f}{x^2}\right) e^\delta \sin 2f\right]. \] (22)

In order to determine the boundary conditions on the regular event horizon, let us expand the fields around the horizon \( x_h \)

\[ \mu = \frac{x_h}{2} - \frac{\tilde{\Lambda} x_h^3}{6} + \mu_1(x - x_h) + O((x - x_h)^2) \] (23)
\[ f = f_h + f_1(x - x_h) + O((x - x_h)^2) \] (24)
\[ \delta = \delta_h + \delta_1(x - x_h) + O((x - x_h)^2). \] (25)

Inserting them into the field equations (20)-(22), one obtains

\[ \mu_1 = \frac{\alpha}{4} \left(1 + \frac{2\sin^2 f_h}{x_h^2}\right) \sin^2 f_h \] (26)
\[ f_1 = \frac{x_h^2 + 4\sin^2 f_h}{x_h(x_h^2 + 8\sin^2 f_h)(1 - 2\mu_1 - \tilde{\Lambda} x_h^2)} \sin 2f_h \] (27)
\[ \delta_1 = \frac{\alpha}{4x_h} (x_h^2 + 8\sin^2 f_h) f_1^2. \] (28)

The profile function numerically computed for several values of \( |\tilde{\Lambda}| \) are shown in Fig. 1. The horizon radius and the coupling constant are fixed with \( x_h = 0.1 \) and \( \alpha = 0.02 \). There are two branches of solutions for each value of the cosmological constant as well as the coupling constant as was seen in the asymptotically flat spacetime case. We define the solution with larger (smaller) values of \( f_1 \) as upper (lower)-branch. It is shown that the skyrmion shrinks as \( |\tilde{\Lambda}| \) becomes large in the upper branch. In Refs. 2, 6, one can see that the skyrmion shrinks as the coupling constant becomes large in the upper branch. Therefore increasing the value of \( |\tilde{\Lambda}| \) gives similar effects on the skyrmion as increasing the value of the gravitational constant. On the other hand, in the lower branch, the change in size is much
smaller and is almost unrecognizable. In the asymptotically flat spacetime, the solution in the lower branch expands in size as the coupling constant increases. Fig. 2 shows the value of \( f_h \) as a function of \( x_h \). It is observed that \( f_h \) continuously approaches to \( \pi \) as \( x_h \) goes to zero, recovering the \( B = 1 \) regular skyrmion solution at \( x_h = 0 \).

The black hole mass-horizon radius relation is shown in Fig. 3. It has been shown that the area of an AdS black hole with scalar hair proportional to the entropy [22, 23]. Thus we infer that this relation still holds for AdS black holes with Skyrme hair as

\[
S = \frac{\pi r^2}{4\hbar G} = \frac{\pi^2}{\hbar e^2} \left( \frac{x_h^2}{\alpha} \right).
\]

One can see that the upper and lower branch correspond to the high- and low-entropy branch respectively. The cosmological constant reduces the entropy of the black hole. The reduction of the entropy is also seen when the coupling constant increases.

Fig. 4 shows the parameter \( f_h \) as a function of \( |\tilde{\Lambda}| \) for \( \alpha = 0.0, 0.02, 0.04 \) with \( x_h = 0.1 \) fixed. The value of \( f_h \) is directly related to the baryon number as can be seen from Eq. (14). Thus, in the upper branch, the baryon number becomes smaller as \( |\tilde{\Lambda}| \) becomes larger, which represents the baryon more absorbed by the black hole. In the lower branch, the baryon number slightly increases as \( |\tilde{\Lambda}| \) becomes large. This result also shows that the cosmological constant gives a similar effect on the skyrmion as the coupling constant.

We found the maximum value of \( |\tilde{\Lambda}| \) above which there exists no solution for each value of the coupling constant. In Fig. 5 the maximum value of \( |\tilde{\Lambda}| \) is shown as a function of \( \alpha \). The maximum value decreases as \( \alpha \) increases, and at \( \alpha = 0.126 \) it becomes zero. Thus at \( \alpha = 0.126 \), the asymptotically AdS solution does not exist and only the asymptotically flat solution exists.

### 3. LINEAR STABILITY ANALYSIS

In this section we shall examine the linear stability of the black hole solutions described in the previous section. Let us consider the time-dependent small fluctuation around the static classical solutions \( f_0, \delta_0 \) and \( \mu_0 \)

\[
f(r,t) = f_0(r) + f_1(r,t) \quad \delta(r,t) = \delta_0(r) + \delta_1(r,t) \quad \mu(r,t) = \mu_0(r) + \mu_1(r,t).
\]

From the time-dependent Einstein-Skyrme action

\[
S = -\frac{\pi e^2 F_4^4}{2} \int \left[ \left( -\frac{1}{e^4 C} \dot{f}^2 + C f'^2 \right) u + v \right] e^\delta dx,
\]

one obtains the time-dependent field equation as

\[
(e^\delta C u f')' + \frac{1}{2} \left( \frac{1}{e^4 C} \dot{f}^2 - e^\delta C f'^2 \right) u_f - \frac{e^\delta v_f}{x^2} = \frac{1}{e^4 C} u \dot{f}
\]

where we have defined \( u_f = \delta u/\delta f \) and \( v_f = \delta v/\delta f \).

The time-dependent Einstein equations are derived as

\[
G_{00} = 8\pi G T_{00} \rightarrow 1 - C - C' x = \frac{\alpha}{4} \left[ \left( \frac{1}{e^{25} C} \dot{f}^2 + C f'^2 \right) u + \frac{2v}{x^2} \right]
\]

\[
G_{11} = 8\pi G T_{11} \rightarrow -1 + C + \frac{(e^{25} C)'}{e^{25} C} x = \frac{\alpha}{4} \left[ \left( \frac{1}{e^{25} C} \dot{f}^2 + C f'^2 \right) u - \frac{2v}{x^2} \right]
\]

which reads the following two equations

\[
\delta' = \frac{\alpha}{4x} \left( \frac{1}{e^{25} C^2} \dot{f}^2 + f'^2 \right) u
\]

\[
-(C x)' + 1 = \frac{\alpha}{2x^2} v + C \delta' x.
\]
Substituting Eqs. (30), (32) into Eqs. (37) and (38) gives the linearized equations
\[ \delta_1' = \frac{\alpha}{2x} (2u_0 f_0' f_1 + u f_0 f_1'^2) \] (39)
\[ -(e^{\delta_0} C_1 x)' = \frac{\alpha}{2x^2} e^{\delta_0} v f_0 + e^{\delta_0} C_0 \delta_1 x. \] (40)

Eq. (39) and the classical field equation derived from Eq. (34) which is
\[ \frac{e^{\delta_0} v f_0}{x^2} = (e^{\delta_0} C_0 u_0 f_0')' - \frac{1}{2} e^{\delta_0} C_0 u f_0 f_0'^2, \] (41)
are inserted into Eq. (40) and resultantly one gets
\[ -(e^{\delta_0} C_1 x)' = \frac{\alpha}{2} (e^{\delta_0} C_0 u_0 f_0' f_1)' \] (42)
which can be integrated immediately to obtain
\[ C_1 = -\frac{\alpha}{2x} C_0 u_0 f_0' f_1. \] (43)

Similarly let us linearize the field equation (34). Using Eqs. (30), (31) and (32), one arrives at
\[ (e^{\delta_0} C_0 u_0 f_0')' - U_0 f_1 = \frac{1}{e^{\delta_0} C_0} u_0 \tilde{f}_1 \] (44)
where
\[ U_0 = -(e^{\delta_0} C_0 u_0 f_0')' + \left( \frac{\alpha}{2x} e^{\delta_0} C_0 u_0^2 f_0'^3 \right)' - \frac{\alpha}{2x} e^{\delta_0} C_0 u f_0 f_0'^2 \]
\[ + \frac{1}{2} e^{\delta_0} C_0 u f_0 f_0'^2 + \frac{e^{\delta_0} v f_0}{x^2}. \] (45)

Setting \( f_1 = \xi(x)e^{\omega t}/\sqrt{u_0}, \) we derive from Eq. (44) as
\[ - (e^{\delta_0} C_0 \xi')' + \left[ \frac{1}{2\sqrt{u_0}} \left( e^{\delta_0} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right] \xi = \omega^2 \frac{1}{e^{\delta_0} C_0} \xi. \] (46)

Let us introduce the tortoise coordinate \( x^* \) such that
\[ \frac{dx^*}{dx} = \frac{1}{e^{\delta_0} C_0} \] (47)
with \(-\infty < x^* < +\infty\). Eq. (46) is then reduced to the Strum-Liouville equation
\[ - \frac{d^2 \xi}{dx^{*2}} + \hat{U}_0 \xi = \omega^2 \xi \] (48)
where
\[ \hat{U}_0 = e^{\delta_0} C_0 \left[ \frac{1}{2\sqrt{u_0}} \left( e^{\delta_0} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right]. \] (49)

The solution is linearly stable if there exist no negative eigenvalues since imaginary \( \omega \) represent exponentially growing modes. Unfortunately the potential \( \hat{U}_0 \) has a complicated form and we are unable to discuss the stability analytically.

Thus we solve the wave equation (48) numerically under the boundary conditions \( \xi \to 0 \) as \( x \to x_h \) and \( x \to \infty \) which ensure the norm of the wave function to be finite. We show the wave function in Fig. 6 for \( |\tilde{\Lambda}| = 1 \) with \( \alpha = 0.02 \) and \( x_h = 1 \) fixed. The ground state corresponds to the wave function with no node. The first excited state corresponds to the wave function with one node. Remarkably both the lower and upper branch solution have ground states with \( \omega^2 \) positive and hence they are stable. Fig. 7 shows the wave function of the lower branch for \( |\tilde{\Lambda}| = 0.5 \). The ground state has a negative eigenvalue \( (\omega^2 = -0.273) \) and thus the solution is unstable. We have found the critical value of \( |\tilde{\Lambda}| \) at which the stability of the lower branch solution changes and shown in Fig. 5 as a dotted line. For \( |\tilde{\Lambda}| = 0, \) the
lower branch solution has one negative mode and for $\omega^2 \geq 0$, the mode becomes continuous (see Fig. 8). Hence it is unstable for all values of the parameters $\alpha$ and $x_h$, which confirms the previous results in Ref. [5, 6].

The results we have obtained indicate that the cosmological constant stabilizes the black hole skyrmion. This is consistent with the result of the EYM black hole where it was shown that the EYM black hole is unstable for $\Lambda \geq 0$ [13, 20], but the stable EYM black hole exists for $\Lambda < 0$ [21].

Another important feature for the black hole skyrmion with $\Lambda < 0$ is that only discrete modes exist in both of the branches. It is because the potential behaves asymptotically as $x^2$, meaning all the eigenvalues are discretized analogous to the harmonic oscillator eigenvalues.

4. CONCLUSIONS

We have studied the black hole skyrmion with a negative cosmological constant. There exists two fundamental branches of the solutions; the upper- and lower-branch corresponding to the high- and low-entropy branch respectively. The increase in the absolute value of the cosmological constant $|\Lambda|$ gives similar effects on the skyrmion as increasing effectively the value of the gravitational constant fixing the pion decay constant to be the experimental value. The skyrmion shrinks and the baryon number is more absorbed by the black hole as $|\Lambda|$ increases. There is the maximum value of $|\Lambda|$ above which no solution exists for each value of the coupling constant. In particular, we observed that at $\alpha = 0.126$, no asymptotically AdS solution exists and only the asymptotically flat solution exists.

The linear stability was examined in detail by solving the linear perturbed wave equation numerically. In the asymptotically flat case, the upper branch is stable and the lower branch is unstable. However, in the AdS case, surprisingly there exist stable solutions even in the lower branch depending on the value of $|\Lambda|$. The observation that the negative cosmological constant stabilizes the black hole skyrmion solution is consistent with the result for the EYM black hole where there exist stable solutions only when $\Lambda < 0$ [19, 20, 21]. This implies that the catastrophe theory for the stability analysis of hairy black holes [24] may not be applicable to the black hole solution with a negative cosmological constant.

The solutions we obtained in this paper provide a semiclassical framework to study the interaction of baryons with the AdS primordial black hole. If the AdS primordial black holes were created in the early universe as indicated in Refs. [15, 16, 17, 18], it would have induced baryon decay. The Einstein-Skyrme theory should be useful as a simple framework to study such decay process.

Since the Skyrme model corresponds to QCD in the large $N$ limit, it may be worth understanding the Skyrme model in the context of the string theory. With the assumption that the brane is AdS, the extension of recently discovered brane-skyrmions to AdS brane-skyrmions would be one of the possibilities [25].

* Electronic address: norikoshiki@mail.goo.ne.jp
† Electronic address: sawado@ph.noda.tus.ac.jp

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FIG. 1: The profile function $f$ as a function of the radial coordinate $x$ for $|\tilde{\Lambda}| = 0.0, 0.5, 1.0$ with $x_h = 0.1$ and $\alpha = 0.02$ fixed.

FIG. 2: The value of the profile at the horizon $f_h$ as a function of $x_h$ with $\alpha = 0.02$ and $|\tilde{\Lambda}| = 1.0$ fixed.

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FIG. 3: The horizon radius $x_h$ as a function of the black hole mass $M_{bh}$ for $|\tilde{\Lambda}| = 0.0, 1.0, 2.0$.

FIG. 4: Parameter $f_h$ as a function of $|\tilde{\Lambda}|$ for $\alpha = 0.0, 0.02, 0.04$.

FIG. 5: The solid line shows the $\alpha$ dependence of the maximum value of $|\tilde{\Lambda}|$ above which there exists no black hole solution. The dotted line shows the $\alpha$ dependence of the value of $|\tilde{\Lambda}|$ above which the lower branch solution changes its stability to become stable.
FIG. 6: Wave function $\xi$ for the upper and lower branch with $|\tilde{\Lambda}| = 1$ and $\alpha = 0.02$.

FIG. 7: Wave function $\xi$ for the lower branch with $|\tilde{\Lambda}| = 0.5$ and $\alpha = 0.02$.

FIG. 8: Wave function $\xi$ for the lower branch with $|\tilde{\Lambda}| = 0.0$ and $\alpha = 0.02$. 