Inverse Modeling of Gravity Data with Two Layers Density in Sedimentary Basin Structure

Eko Januari Wahyudi¹ and Wawan Gunawan A. Kadir¹
¹Institut Teknologi Bandung

Abstract. Study of sedimentary basin structure and basement variation in the subsurface using gravity methods are very common in early stage of hydrocarbon exploration. Most of inverse modeling are designed with several series of juxtaposed vertical prism in the subsurface. In this study, we utilize two layers of vertical prism below each gravity stations to represent homogeneous density of sedimentary layer and basement in the subsurface. In order to reach best miss-fit data we used iterative calculation utilizing differences between observed and calculated data. Performance of iterative calculation will attempt to find solution of interface density in the subsurface. In this paper, we use two synthetic data of symmetrical sedimentary basin model for inversion performance test. The performance test will evaluate the effect of initial model, exploration well data, topographic data, and noisy data effect. Based on synthetic data test, the performance of iterative method provide small order of miss-fit data with residual solution model of noisy data in the range -44 to 35 meters. Real data application identified three shallow sedimentary layer and two deeper sedimentary layer in the study area.

1. Introduction
Study of sedimentary basin structure is very common conducted using gravity data. Density contrast between interface layer (sedimentary and basement) provide the source of gravity anomalies that we can observe in the surface. Inverse modeling of gravity data observation can be targeted to provide subsurface structure as sedimentary thickness variation. There are several publication of sedimentary inverse modeling study since 70’s that we can use as references ([1]-[14]). In this paper, we will use iterative method to find the solution of subsurface interface using two layers density (sedimentary layer in the top and basement in the bottom layer). Design of the iterative calculation will be described in the next section and we will evaluate the performance using two synthetic model of symmetrical sedimentary basin. Real data application also conducted using S-N section of gravity data that observed in small area in East Java Basin [15].

2. Methodology
We design the iterative calculation start from subsurface model and simplified as illustrated in the figure 1. The subsurface model consists of M-series juxtaposed vertical prism. Vertical series of subsurface model placed below each gravity stations (N-series data). Two layers of vertical prism below each gravity stations represent two layers in the subsurface (homogeneous density of sedimentary layer ($\rho_s$) and basement ($\rho_b$)). Each vertical prisms (of sedimentary layer and basement) has length in 3D cartesian coordinate system (dx, dy, and dz). The lower bound of basement vertical prisms in the subsurface model are limited with certain depth, while the upper bound of sedimentary vertical prisms in the subsurface model are limited with topographic. Interface for each vertical series (between basement and sedimentary layer) are representation of our subsurface structure that will be updated as our solution.
In order to minimize data miss-fit we used iterative calculation of differences between observed \((g_{\text{obs}})\) and calculated \((g_{\text{cal}})\) data. Iterative calculation attempt to modify model solution \((z(j))\) and updated using slight modification from Chakravarthi [10], as follow:

\[
z^{(k+1)}(j) = z^{(k)}(j) + \frac{\text{Rand}(g_{\text{obs}}(i)-g_{\text{cal}}(j))}{G\rho_s}.
\] (1)

Where \(i, j\) and \(k\) respectively denotes index number of data observation, vertical series in subsurface model, and iteration. Random number \((\text{Rand})\), universal gravitational constant \((G)\), and sedimentary density \((\rho_s)\) used as nominator and denominator in the equation (1) in order to provide rough estimation of updated solution. Flow-chart of the program will be described in several steps. Start from the input, initialization, initial model, iterative process (update model, well constraint, miss-fit evaluation), stopping criteria, and output. The procedure of iterative calculation described with program flow-chart as seen in figure 2.

**Input** of the program consist of three categories:
1. Gravity data observation and station coordinates. In this category, program will accomodate information related with station’s interval, target anomalies (residual Bouguer anomaly), N-obs data, and instrument height from the surface.
2. Subsurface model parameters. In this category, we can accomodate following information: well data, well location/coordinate, depth estimation from spectral analysis, M-vertical series of prism, length of body strike in axis-Y direction, coordinat prisms, maximum depth for subsurface model, sedimentary layer density, and basement density.
3. Iterative parameters for maximum iteration number and realization number for statistical purpose.

**Initialization** of the program used to introduce:
1. Default constants (universal gravitational constant, unit conversion, prism’s index).
2. Empty matrix as storage space of objective function (data miss-fit), calculated \((g_{\text{cal}})\), and model solution.
Figure 2. Flow-chart of the program to seek model solution of sedimentary basin structure.

After initialization, next step from the program is generating initial model and several calculation (Kernel matrix, $g_{calc}$, and miss-fit). Initial model can be generated as follow:

1. Random interface between sedimentary and basement based on random number.
2. Flat interface between sedimentary and basement based on depth estimation from spectral analysis.
3. Bowl shape interface between sedimentary and basement based on mimicking the input of gravity anomalies.
4. Subjective initial model based on user interpretation.

Iterative process to update model solution for each vertical series is following mathematic formula of equation (1). Well data also accomodate in iterative process (as well-constraint) in order to avoid updated solution in this well location. After model solution in well location is tied-up, program calculate Kernel matrix, $g_{calc}$, and miss-fit. Miss-fit evaluation will compare current and previous model. Miss-fit evaluation only accept down-hill changes of the objective function (miss-fit). Miss-fit ($\phi$) in this study calculated with this equation:

$$\phi = \sum_{i=1}^{N} (g_{obs}(i) - g_{calc}(i))^2$$  \hspace{1cm} (2)

Stopping criteria the iterative process is sets-up using maximum iteration number. If maximum iteration number is met, we get model solution as our output. The program working with random number, so it is possible we will get different solution as realization. We can evaluate deviation of solution based on statistical point of view from several realization.

3. Synthetic Data
Forward calculation for synthetic data uses equation of gravitation caused by 3-D polygonal prism body. Vertical component of gravitation ($g_z$) in the coordinate O (0, 0, 0) caused the body, mathematically expressed with this following equation:

$$g_z = G\rho \int_{\gamma=z_1}^{\gamma=z_2} \int_{\beta=y_1}^{\beta=y_2} \int_{\alpha=x_1}^{\alpha=x_2} \frac{z}{r^3} d\alpha d\beta dy.$$  \hspace{1cm} (3)
\[ g_z = G \rho \left[ \alpha \ln(\beta + r) + \beta \ln(\alpha + r) - \gamma \tan^{-1} \left( \frac{\alpha \beta}{y r} \right) \right] \cdot \]  

The formula in equation (3) accommodate universal gravitational constant \((G)\), density of prism \((\rho)\), resultant distance between 3-D body and gravity station \((r)\), vertical distance between 3-D body and gravity station \((z)\) and prism edges coordinate \((x_1, x_2, y_1, y_2, z_1, z_2)\). Next, by integral the equation (3) we get equation (4). Equation (4) is gravitation effect from one prism body, while for the overall model that consists of several prism bodies, the calculation conducted cumulatively as described by Pluoff [16].

Synthetic model A with flat topographic shown in **figure 3a**. The model A is calculated over 11 stations with 500 m of station’s interval in flat topographic. Synthetic model consists of 11 vertical series (of sedimentary and basement) with 2700 m in maximum depth. Length of each vertical prisms in x-direction \((dx)\) and y-direction \((dy)\) respectively are 500 m and 10000 m, while the length of prisms in z-direction \((dz)\) are varied symmetrically as shown in numbers below the surface in **figure 3a** (900, 1020, 1140, 1260, 1380, 1500, 1380, 1260, 1140, 1020, 900). Density contrast for numerical example used in the sedimentary and basement model respectively are -0.10 g/cc and +0.10 g/cc.

![Figure 3. Model illustration: (a) model A with flat topography and (b) model B with topographic variation.](image)

Synthetic model B with topographic variation for algorithm performance test shown in **figure 3b**. Model B is similar as the previous one, but topographic variation above sea level shown in numbers above the surface (figure 3b). The sedimentary thickness are varied symmetrically from mean sea level (same with model A). In this study, algorithm performance test for model B will be conducted in two types synthetic data: (1) free-noise and (2) with noise. We use random noise within the range of \pm 10 \mu Gal.

### 4. Performance Test

Performance test using synthetic data A will be evaluate the effect of initial model and well data. Synthetic data A is tested with three types of set-up, as follow: random initial model, flat initial model, and flat initial model with well constraint. We conducted inversion with 1000 iteration for 100 realization in order to provide statistical sample.

Random initial model introduce to the iterative calculation as simulation of no subsurface data consideration. This set-up try to simulate data driven in the iterative process to seek model solution. Inversion performance shown as the average changes of data miss-fit over iteration number. Miss-fit is average value from 100 realization (**figure 4**). As we can see from the **figure 4**, miss-fit (of random initial model) curve changes significantly at the first 50 iteration and then the data miss-fit changes looks like as linear downward (up to \(10^{-8}\) order).

We also can use depth estimation from spectral analysis results as flat initial model to the iterative calculation. In this example (synthetic data A), we get rough result as depth estimation from spectral
analysis is 857.84 m. Average miss-fit is calculate from 100 realization (figure 4). As we can see from the figure 5, data miss-fit (of flat initial model) changes significantly at the begining of 50 iteration and then the data miss-fit continously curve down to $10^{-9}$ order.

Well constraint introduce to the iterative calculation can be very helpfull to reduce ambiguity of inverse model solution in the subsurface. Average miss-fit is calculate from 100 realization (figure 4). As we can see from the figure 4, data miss-fit (of flat initial model with well constraint) changes significantly at the begining of 50 iteration and then the data miss-fit continously curve down to $10^{-11}$ order.

Average model (solution) from inverse modeling is shown in figure 5. Maximum differences between synthetic and inverse model in the range of -6 to 8 meters. Using flat initial model provide smaller standard deviation of solution model compare to random initial model. Big difference between solution model and synthetic model shows in the centre-part of the section. Figure 5a shows the comparison of 11 solution model within depth range of 800-1500 m, while figure 5b shows the zoom in of comparison 3 solution model in the centre-part of the section (within depth range of 1350-1550 m). Table 1 shows the summary of comparison from statistical point of view. This performance test of inversion with free-noise data and simple synthetic model is completely recover the true solution, because almost to zero difference between synthetic and inverse model (flat initial model with well constraint).

Performance test using synthetic data B will be evaluate the effect of topographic and noisy data. Flat initial model and well data are introduce into the calculation of 1000 iteration for 100 realization. Average and median of inverse model are shown in table 2 to provide sedimentary thickness comparison from synthetic and inverse model. With small topographic variation in the syntethic model B (0 to 30 m above the sea level), the iterative calculation of inverse model possible to recover the true solution. The difference solutions for each vertical series between synthetic and inverse model as can be seen in table 2 in the range of -4 to 2 meters. More realistic input data are simulated with noisy data in the synthetic data model B. Maximum difference between synthetic and average inverse model in the range of -44 to 35 meters. Comparison of solution model using synthetic data B is shown in figure 6.

![Figure 4](image)

**Figure 4.** Comparison of inversion performance using synthetic data A describe as down-hill curves of average miss-fit over iteration number.
Figure 5. Comparison of solution model using synthetic data A: (a) depth interval 800-1500 m, (b) depth interval 1350-1550 m.

Table 1. Comparison of sedimentary thickness (m) between synthetic model A and inverse models.

| no | synt. | random initial model | flat initial model | flat initial model w/ well const. |
|----|-------|----------------------|--------------------|-----------------------------------|
|    |       | avg. | med.  | min. | max. | avg. | med.  | min. | max. | avg. | med.  | min. | max. |
| 1  | 900.00| 899.94| 900.06| 898.37| 900.42| 900.08| 900.08| 900.07| 900.08| 900.00| 900.00| 899.95| 900.00|
| 2  | 1020.00| 1019.74| 1019.80| 1016.28| 1022.95| 1019.75| 1019.75| 1019.74| 1019.77| 1020.00| 1020.00| 1020.00| 1020.23|
| 3  | 1140.00| 1140.61| 1140.70| 1128.16| 1151.20| 1140.87| 1140.87| 1140.82| 1140.91| 1139.99| 1140.00| 1139.49| 1140.00|
| 4  | 1260.00| 1257.48| 1258.17| 1224.78| 1285.98| 1257.76| 1257.76| 1257.66| 1257.89| 1260.01| 1260.00| 1260.00| 1260.68|
| 5  | 1380.00| 1385.67| 1385.11| 1316.15| 1462.74| 1385.26| 1385.27| 1384.96| 1385.51| 1380.00| 1380.00| 1379.65| 1380.00|
| 6  | 1500.00| 1493.66| 1492.38| 1390.18| 1602.27| 1492.42| 1492.41| 1492.07| 1492.85| 1500.00| 1500.00| 1500.00| 1500.00|
| 7  | 1380.00| 1385.67| 1385.58| 1315.10| 1465.45| 1385.26| 1385.27| 1384.96| 1385.51| 1380.00| 1380.00| 1379.65| 1380.00|
| 8  | 1260.00| 1257.48| 1257.57| 1222.63| 1287.27| 1257.76| 1257.76| 1257.66| 1257.89| 1260.01| 1260.00| 1260.00| 1260.68|
| 9  | 1140.00| 1140.61| 1140.91| 1127.42| 1152.36| 1140.87| 1140.87| 1140.82| 1140.91| 1139.99| 1140.00| 1139.49| 1140.00|
| 10 | 1020.00| 1019.74| 1019.73| 1015.79| 1023.23| 1019.75| 1019.75| 1019.74| 1019.77| 1020.00| 1020.00| 1020.00| 1020.23|
| 11 | 900.00| 899.94| 900.08| 898.29| 900.42| 900.08| 900.08| 900.07| 900.08| 900.00| 900.00| 899.95| 900.00|
Table 2. Comparison of sedimentary thickness (m) from msl between synthetic model B and inverse models.

| no | synt. | free-noise data | noise data added |
|----|-------|-----------------|------------------|
|    |       | avg. Δ solution | avg. Δ solution  |
| 1  | 900.00| 900.49 -0.49    | 901.00 -1.00     |
| 2  | 1020.00| 1019.99 0.01  | 1016.84 3.16     |
| 3  | 1140.00| 1139.55 0.45  | 1127.22 12.78    |
| 4  | 1260.00| 1258.99 1.01  | 1303.10 -43.10   |
| 5  | 1380.00| 1383.11 -3.11 | 1345.48 34.52    |
| 6  | 1500.00| 1500.00 0.00  | 1500.00 0.00     |
| 7  | 1380.00| 1378.20 1.80  | 1381.30 -1.30    |
| 8  | 1260.00| 1261.22 -1.22 | 1278.71 -18.71   |
| 9  | 1140.00| 1140.18 -0.18 | 1129.43 10.57    |
| 10 | 1020.00| 1020.27 -0.27 | 1017.90 2.10     |
| 11 | 900.00 | 900.26 -0.26  | 902.38 -2.38     |

Figure 6. Comparison of solution model using synthetic data B (depth interval 850-1550 m).

5. Real Data Application

Grid survey of gravity data observation conducted in study area that located close to Blora in Central Java, Indonesia. About 250 m interval grid station over 5x5 km square are acquired in 2014 using two Scintrex CG-5 (blue and red team). Summary of the survey is shown in table 3. Relative gravity data from 402 stations are processed to get Complete Bouguer Anomaly, then regional-residual anomaly are separated using moving average filter. Residual anomaly and topographic map are sliced diagonaly to provide SW-NE section as the input data of inverse modeling.

The geological formations imaged on seismic profile indicate that the center part of study area is an anticline structure whose axis is continued for E-W direction [17]. Simplified surface geology map [18] (figure 7) also confirm anticline structure in the center part of study area. High residual anomaly in the center-part of study area expected as an anticline structure in the inverse modeling. Density contrast as an input of first (upper) layer in this study area are test in the range of 0.00 to -0.010 g/cc, while density contrast as an input of second (lower) layer in the range of 0.00 to +0.10 g/cc. Combination of first and second layer provide 121 pairs as an input to be calculated in the scheme of 100 iteration number. Strike length (in the y axis of subsurface model) as an input of first and second layer are 1 km and 1.5 km.
respectively, while length interval in the x axis of subsurface model is 50 m. Maximum depth (in the z axis of subsurface model) are set-up in 5 km. Total 137 vertical series of subsurface model calculated below 137 data input. Data input are topographic and residual anomaly with 50 m interval data in the direction of SW-NE azimuth.

Table 3. Survey summary.

|                  | Team       | 2014       |
|------------------|------------|------------|
| **Survey**       | Blue       | Red        |
| **Date**         | 19-29 Sep  | 18-29 Sep  |
| **Gravimeter**   | CG-5 822   | CG-5 1092  |
| **Tot. Time Duration** | 37560 min. | 43020 min. |
| **Good Leveling**| 98.02%     | 98.26%     |
| **Auto Rej. > 5%**| 19.59%     | 17.91%     |
| **Average daily drift** | 42 µGal | 46 µGal |
| #Day(s)          | 10         | 11         |
| #Reading(s)      | 1113       | 1379       |
| #Observation(s)  | 243        | 239        |
| #Station(s)      | 206        | 202        |
| #Repeat Obsv.(s) | 37         | 37         |
| #VG Station(s)   | 3          | 2          |

![Simplified surface geology map](image)

**Figure 7.** Simplified surface geology map [18] in study area consist of Lidah Formation (QTI), Mundu Formation (Tpm), Ledok Formation (Tml), and Wonocolo Formation (Tmw).

As the input of iterative calculation described in the previous paragraph, we will get 121 model as possible solution for our study. Miss-fit data for each model solution had been minimized over 100 iteration. The example of miss-fit data changes over the iteration number shown in **figure 8**. There are 27 model solution that have small missfit data (less then 1 mGal). We consider the 27 possible model solution to be the best solution in average value as shown in **figure 9**. The density of first and second layer based on 27 best model respectively in the range of -0.10 to -0.07 g/cc and 0.04 to 0.10 g/cc. Based on the results in **figure 9**, we get average density for the first and second layer are -0.09 and 0.07 g/cc. Simplified subsurface structure and the depth variation of sedimentary layer (first layer) described in **figure 10**. Shallow sedimentary layer (less then 1 km depth) shown in **figure 10** as 1A, 1B, and 1E, while deeper sedimentary layer (1C and 1D) located beside the center anticline structure (2B).
Figure 8. Example of miss-fit data changes (in the log scale) during iterative process.

Figure 9. Average model from 27 best miss-fit data (depth interval 0-5 km).

Figure 10. Simplified subsurface structure.

6. Conclusion
Iterative calculation using gravity data can be used to determine the interface density contrast in the subsurface between sedimentary layer and basement layer. Based on synthetic data test, the performance of iterative method using synthetic data of model A provide small order of miss-fit data ($10^8$ to $10^{11}$).
The performance test using synthetic data of model B (topographic and noisy data added) provide residual solution of model in the range -44 to 35 meters. Real data application identified three shallow sedimentary layer and two deeper sedimentary layer in the study area.

7. Acknowledgments
Authors acknowledge support of the P3MI-ITB, SATREPS project by JICA-JST, and other Gundih CCS pilot project team members that help data acquisition.

8. References
[1] Fournier K P and Krupicka S F 1975 A new approximate method for directly interpreting gravity anomaly profiles caused by surface geologic structures Geophysical Prospecting 23 80
[2] Overmeeren R A V 1975 A combination of gravity and seismic refraction measurements, applied to groundwater near Taltal, Province of Antofagasta, Chile Geophysical Prospecting 23 248
[3] Verma R K, Majumdar R, Ghosh D, Ghosh A and Gupta N C 1976 Results of gravity survey over Raniganj coalfield, India Geophysical Prospecting 24 19
[4] Pedersen L B 1977 Interpretation of potential field data a generalized inverse approach Geophysical Prospecting 25 199
[5] Baldi P and Unguendoli M 1978 Inversion of gravity profiles by polynomial method Geophysical Prospecting 26 247
[6] Rao D B, Prakash M J and Babu N R 1990 3D and 2½D modelling of gravity anomalies with variable density contrast Geophysical Prospecting 38 411
[7] Lee T C and Biehler S 1991 Inversion modeling of gravity with prismatic mass bodies Geophysics 56 1365
[8] Mickus K L and Peeples W J 1992 Inversion of gravity and magnetic data for the lower surface of a 2.5 dimensional sedimentary basin Geophysical Prospecting 40 171
[9] Rao V C, Pramanik A G, Kumar G V R K and Raju M L 1994 Gravity interpretation of sedimentary basins with hyperbolic density contrast Geophysical Prospecting 42 825
[10] Chakravarthi V 1995 Modelling of density interface with binomial density variation Journal of Applied Geophysics 34 69
[11] Abdeslem J G 2000 2-D inversion of gravity data using sources laterally bounded by continuous surfaces and depth-dependent density Geophysics 65 1128
[12] Chappell A and Kusznir N 2008 An algorithm to calculate the gravity anomaly of sedimentary basins with exponential density-depth relationships Geophysical Prospecting 56 249
[13] Silva J B C, Oliviera A S and Barbosa V C F 2010 Gravity inversion of 2D basement relief using entropic regularization Geophysics 75 129
[14] Zhou X 2010 Analytic solution of the gravity anomaly of irregular 2D masses with density contrast varying as a 2D polynomial function Geophysics 75 111
[15] Wahyudi E J, Marthen R, Fukuda Y, Nurali Y 2017 Time-lapse microgravity data acquisition in baseline stage of CO2 injection Gundih pilot project IOP Conf. Series: Earth and Environmental Science 62 012047
[16] Pluoff D 1976 Gravity and magnetic fields of polygonal prisms and application to magnetic terrain corrections Geophysics 41 727
[17] Tsuji T, Matsuoka T, Kadir W G A, Hato M, Takahashi T, Sule M R, Kitamura K, Yamada Y, Onishi K, Widarto D S, Sebayang R I, Prasetyo A, Priyono A, Ariadji T, Sapie B, Widianto E, Asikin A R and Gundih CCS project team 2014 Reservoir characterization for site selection in the Gundih CCS project, Indonesia Energy Procedia 63 6335
[18] Pringgoprawiro H and Sukido 1992 Geologic map of the bojonegoro quadrangle, East Jawa Geological Research and Development Centre