Cost optimisation of the design of reinforced concrete flat slab to BS8110

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Abstract. The optimum design of reinforced concrete flat slab could reduce its construction cost because it is usually employed in large floor area without any structural framing beams. The design consideration is based on the provision of BS8110 separating the slab into middle and column strips. The optimisation is to find the optimum slab thickness with least cost. The use of the in-built genetic algorithm function of MATLAB software was employed to minimise the design at various steel ratios. The constraints applied were non-linear and thus required a lengthy iteration cycles before convergence. The objective function for either strip is treated separately without any coupling in line with the design philosophy of BS8110. The optimum thicknesses for the middle and column strips are 130mm and 140mm respectively. The reinforcement ratios for the optimum design were also established to be 2% and 3.5% for the middle and column strips respectively.

1. Introduction
A flat slab is a reinforced concrete plate supported by columns only without any framing beams [1]. The columns can be provided with heads to improve punching shear resistance where there is occurrence of high shear [2]. Flat slab systems are quite popular and can be found around residential buildings as well as public buildings. With the absence of beams in flat slabs, lower storey heights are made possible. These lowered stories height ultimately leads to cost reduction. The most important aspects of any structural design are safety and cost. Generally, in the practical realm, economics is given little consideration. Suitable sections are designed as long as they satisfy the given conditions. Therefore, it is important to introduce the concept of optimisation to the design of structural members. These members are optimized within the actual constraints at both ultimate and service levels of the locally used design codes. To optimise any system, there are many approaches such as genetic algorithm, fuzzy logic, and neural network among many others. However, for structural members, optimisation, evolutionary genetic algorithms (EGA) presents a modern technique of optimisation. EGAs are subsets of evolutionary computation, a generic population-based meta-heuristic optimisation algorithm. This method leans on to the biological process which are easily programmed using any high level language.

Genetic Algorithm (GA) is a type of evolutionary algorithm and are global optimisation techniques. Various biological processes of reproduction and natural selection was imitated in order to solve for the ‘fittest’ solutions in [3]. Several of the genetic algorithms processes are random just like in evolution. However, with the genetic algorithm, there is an ability to control the algorithm technique and level of
randomization [4]. Genetic algorithm provides better solution than both exhaustive and random search algorithms without the need for extra information on the given problem. This makes them the perfect fit in finding solutions to different problems where other techniques and algorithms have failed. Thus, the current work studied the structural optimisation in reinforced concrete structures, with the focus on the cost optimisation of the design of reinforced concrete flat slabs. Efforts have been made to provide state of the art software that incorporates genetic algorithm demonstrated in MATLAB.

Reinforced concrete flat slabs are highly versatile elements widely used in building construction allowing flexible column grids. They generally transfer the loads directly to the supporting columns which are spaced suitably around the slab. The finishing of the slab obtained not only enhances the aesthetics of the building, but also helps to diffuse light better and reduce their vulnerability to fire. Furthermore, the absence of beams from flat slabs reduces the overall height of the entire building, thus saving some amount of cost. In addition to this, extra fittings to the building such as auto sprinkler become much easier to incorporate. There is however, limitation to the span and thickness of the slab in addition to their unsuitability for the support of brittle (masonry) partitions.

Galeb and Atiyah [5] worked on the optimum design of reinforced concrete waffle slabs using genetic algorithms. Two case studies were compared; the first is a waffle slab with solid heads, while the other is a waffle slab with band beams along the column centrelines. A MATLAB program was written for the optimisation and the result shows that the cost of formwork was reduced by 30%. It was also found that for a top slab of 62mm – 72mm was the most economic for slabs with solid heads, while most texts set their practical limit between 75mm and 125mm. Matthias et al. [6] worked on a multi-objective optimisation in which both cost and mass minimization of the structure is being done concurrently. Non-dominated Sorting Genetic Algorithm (NSGA – II) and meta-model was employed in getting the optimum solutions. The design variables are the cross-sectional dimensions of the structure to get optimum cost and weight. While there are several optimisation techniques to solve the constrained multi-objective problems, appropriate techniques depend on the type of problem and NSGA – II is a quite popular method especially for strength which was the reason for its choice. The entire procedure for the optimisation was implemented on MATLAB.

Ghandi et al. [7] presented a Cuckoo Optimisation Algorithm (COA) model for the cost optimisation of the one-way and two-way reinforced concrete (RC) slabs according to the ACI code. Its objective function is the cost of concrete and reinforcing steel. Constraints were also developed to conform to the requirements of ACI code. The model can be applied in practical designs so as to reduce project cost. This is also the first application of Cuckoo Optimisation Algorithm to the optimisation of reinforced concrete slabs. The result of the COA was compared with neural dynamics model, in which the COA achieved better results.

Sudarshana and Ramesh [8] demonstrated the design of short columns under biaxial bending making use of artificial neural network (ANN) and genetic algorithm (GA). It was demonstrated that the hybrid model does not perform significantly better than ordinary genetic algorithm. An optimisation model for the design of rectangular reinforced concrete beams subject to a specified set of constraints was developed by [3]. Genetic Algorithm was used to solve this, and the results were compared with a mathematical programming technique that deals with the non-linear equations of the existing model. The results showed that GA is quite efficient in optimisation. The current work is pinned on the use of evolutionary genetic algorithm (EGA) in MATLAB. The aim of the paper is to determine the most economical design variable for the construction of reinforced concrete flat slabs.

2. Methodology

The study was segmented into two stages. The first stage involved the determination of the design constraints and cost/objective functions, while the second stage involves the preparation and implementation of the suitable genetic algorithm to optimize the cost function for the construction of a reinforced concrete flat slab using MATLAB. The design variables are the various variables which are expected to be optimized for the objective function. For this project, a mono-variable of the thickness (h) of the flat slab was chosen. This was designated as \( x_1 \) to obtain the optimum range of the thickness.
using the genetic algorithm optimisation process. The aim of the research is to determine the most economical design variable for the construction of reinforced concrete flat slabs. This follows that the objective function has to be a function that will estimate the total cost for the design of reinforced concrete flat slabs. This function depends on the cost of concrete, formwork and reinforcement. For the flat slab, this was further divided into the cost of the middle strip and the cost of the column strip. The resulting moment on both strips of the slabs will be different due to the difference in the moment coefficients as expressed in the BS8110 part 1. This therefore results the problem to a multi-objective optimisation problem. The positions of the middle and column strips are shown in Figure 1. This was evaluated as follows.

\[ C_{col} = Q_c C_c' - W_s C_r' + A_f C_f' \]  

\[ C_{col} = x_1 l_1 \left( \frac{l_2}{2} \right) C_c' - \gamma(x_1) \left( \frac{l_2}{2} \right)(l_1)(\rho_s)C_r' + \left( \frac{l_2}{2} \right)(l_1)C_f' \]  

\[ C_{mid} = x_1 l_1 \left( \frac{l_2}{2} \right) C_c' - \gamma(x_1) \left( \frac{l_2}{2} \right)(l_1)(\rho_s)C_r' + \left( \frac{l_2}{2} \right)(l_1)C_f' \]  

2.1 Concrete

The popular concrete mix of 1:2:4 was taken into consideration which costs N35, 000 for a cubic metre of concrete in Nigeria.

Unit weight of concrete cost, \( C_c' = 35 \)

\[ \text{cost of concrete} = x_1 l_1 \left( \frac{l_2}{2} \right)C_c' \]

2.2 Formwork

The cost of the formwork is dependent on the total surface area covered by the formwork so as to support the concrete. The cost of a unit area of formwork was determined to be N9, 000.

Cost of unit area of formwork, \( C_f' = 9 \)

Cost of formwork = \( \left( \frac{l_2}{2} \right)(l_1)C_f' \)
2.3 Reinforcement

The cost of reinforcement depends on the weight of reinforcement required for the construction. The percentage of reinforcement for the slab was varied from 0% to the maximum percentage of 4%. The weight of the resulting reinforcement is calculated by first determining the area of the reinforcement. This is followed by estimating the volume by multiplying the area with the span. The weight was then established by multiplying the volume with the reinforcement density $\varphi$.

The cost of a tonnage of reinforcement was obtained to be N240, 000.

Cost of unit area of reinforcement, $C_r' = 240$

Cost of reinforcement $= p\varrho(x_1)(\frac{l_2}{2})(l_1)(\rho_s)C_r'$

2.4 Design constraints

The objective functions are subjected to the various constraints for the design of reinforced concrete flat slab according to BS8110 part 1 [9] as discussed below.

2.5 Minimum depth

The minimum thickness of a reinforced concrete flat slab should be taken as described in section 3.7.1 of BS8110 part 1 which states that in no case should the thickness of the slab be less than 125mm.

$g_1 = 0.125 - x_1 \leq 0$ \hspace{1cm} (4)

2.6 Serviceability constraints

The minimum reinforcement area for steel as specified by table 3.25 of BS 8110, which specifies 0.13% $bh$.

$\gamma \geq 0.13$ \hspace{1cm} (5)

$g_2 = 0.13 - \gamma \leq 0$ \hspace{1cm} (6)

The maximum reinforcement area for steel as specified by table 3.12.6 of the BS 8110, which specifies 4% $bh$.

$\gamma \leq 4.0$ \hspace{1cm} (7)

$g_3 = \gamma - 4 \leq 0$ \hspace{1cm} (8)

2.7 Flexural constraints

The design moments are distributed across each flat slab panel. All nominal flexural strength at the middle (positive), and two ends (negative) of the middle strips, $M_n$, should be greater than the ultimate design moment on the middle strips (mid) at each of these parts of the slab $M_u$ and $M_k$:

$M_u^- \leq M_n^- \leq 0$ \hspace{1cm} (9)

$g_4 = M_u^- - M_n^- \leq 0$ \hspace{1cm} (10)

$M_u^+ \leq M_n^+ \leq 0$ \hspace{1cm} (11)

$g_5 = M_u^+ - M_n^+ \leq 0$ \hspace{1cm} (12)

In the equations above, $M_u$ is calculated as described below using the simplified BS 8110 part 1 approach which requires that the conditions stated in section 3.7.2.7 of the code is satisfied.

$M_u = k_f \beta FL$ \hspace{1cm} (13)

Where $k_i$ and $\beta$ are coefficients obtained from tables 3.18 and 3.12 of BS 8110 part 1 respectively.

The nominal moments resistance of the slab can also be obtained as described below

$M_n = \gamma(x_1)(l_2)f_yz$ \hspace{1cm} (14)

Where $z$ is the lever arm for the strips of the slabs and is obtained as follows

$z = d\left(0.5 + \sqrt{0.25 - \frac{k}{0.9}}\right) \leq 0.95d$ \hspace{1cm} (15)
\[ k = \frac{M_u}{l_2(x_1)^2 f_{cu}} \leq k' \left(0.156\right) \]
\[ g_6 = k - 0.156 \leq 0 \]

2.8 Constant parameters

These are the final requirements for the optimisation problem. They are the given parameters that are not decision variables. For the purpose of this study, the various parameters and values were taken as described in the Table-1 below.

| Parameter        | Value     |
|------------------|-----------|
| \(l_1\)          | 4.0 m     |
| \(l_2\)          | 4.2 m     |
| Load on slab     | 14.0 kN/m² |
| \(f_{cu}\)       | 30.0 N/mm² |
| \(f_y\)          | 460.0 N/mm² |

3. Results and Discussion

The MATLAB built-in genetic algorithm comes with different plot functions to explore the fitness of the design variable. This plot functions were included to show the variations of the fitness of cost function at various reinforcement ratio. The value of the reinforcement ratio, \(\gamma\) was incremented by 0.5% between 0 and 4%. The software outputs for various reinforcement ratios are displayed for the middle and the column strips in the following sections.

3.1 Middle strip optimisation

Figure 2 shows the plots of various optimisation outputs such as the best fit of the objective function, the range of score of variable population and the optimal depth for the slab for the middle strip design with 0% reinforcement ratio. It is evident from the plots that the score of the variables is zero even at optimal depth of 130mm. No single member of the variable population shows any sign of optimisation because the plain concrete cannot survive under the loading in tension.

![Figure 2. Plots of optimisation statistics for 0% reinforcement](image)
Figure 3 shows the same optimisation statistical outputs for 1% reinforcement. It can be seen clearly that the score for the variable populations has jumped from zero to $1.1778 \times 10^5$. The various score range distribution is a fairly even with the highest number of individual having the same score value being 20. The optimisation statistical outputs for 1.5% is shown in Figure 4. At this reinforcement percentage, the score for the variable populations moved from the $1.1778 \times 10^5$ obtained from the 1% reinforcement percentage to $1.2192 \times 10^5$. Here, the range of the scores is well distributed among four values with the highest number of individuals being 33. The same optimisation statistical outputs for 2% reinforcement is shown in Figure 5 below. Here, the score range distribution shows very little variation with 48 individuals of the 50 population converging. The score range value was also recorded to be $1.2606 \times 10^5$.

**Figure 3.** Plots of optimisation statistics for 1% reinforcement

**Figure 4.** Plots of optimisation statistics for 1.5% reinforcement
Figure 6 shows the statistical outputs for the 2.5% reinforcement. Here, there was a $4.14 \times 10^3$ increase in the score value from the previous 2% reinforcement percentage to give a $1.3020 \times 10^5$ score. The score range value here was distributed between two score ranges with this highest number of individual at a particular score being 26. Figure 7 shows the optimisation statistical outputs for 3% reinforcement. The score for the variable population varies and reduces progressively, and the highest number of individuals with the same score being 29 and the least being 2. The score for the variable population also increased by the same value of $4.14 \times 10^3$ increase recorded for the change in percentage between 2% and 2.5% percentage reinforcement.

Figure 8 shows the optimisation statistical outputs for 3.5% reinforcement. There was an increase in the score to a value of $1.4263 \times 10^5$. The score range for the variable population were majorly dominated by two score values taking about 20 and 26 number of individuals. Figure 9 shows the optimisation statistical outputs for 4% reinforcement. There was an increase in the score to a value of $1.4263 \times 10^5$. The score range for the variable population converges to two scores with 26 and 20 number of individuals converging to similar scores.
Figure 7. Plots of optimisation statistics for 3% reinforcement

Figure 8. Plots of optimisation statistics for 3.5% reinforcement
3.2 Column strip optimisation

Figure 10 shows the plots of various optimisation outputs such as the best fit of the objective function, the range of score of variable population and the optimal depth for the slab for the column strip design with 0% reinforcement ratio. It is evident from the plots that the score of the variables is zero even at the optimal depth of 140mm. No single member of the variable population shows any sign of optimisation because the plain concrete cannot survive under the loading in tension. Figure 11 shows the same optimisation statistical outputs for 1% reinforcement. It can be seen clearly that the score for the variable populations has jumped from zero to $1.2908 \times 10^5$. The various score range distribution is a fairly reducing with the highest number of individual having the same score value being 27. The optimisation statistical outputs for 1.5% is shown in Figure 12. At this reinforcement ratio, the score for the variable populations moved from the $1.2908 \times 10^5$ obtained from the 1% reinforcement percentage to $1.3361 \times 10^5$ resulting in a $4.53 \times 10^3$ difference. Here, the range of the scores is concentrated between two scores with the number of individuals for each being 28 and 19.

The same optimisation statistical outputs for 2% reinforcement is as shown in Figure 13 below. Here, the score range distribution shows that the score for the variable population was centred around two score with the highest number of individual with the same score being 32. The score range value was also recorded to be $1.2606 \times 10^5$. Figure 14 shows the statistical outputs for the 2.5% reinforcement. Here, there was a $4.54 \times 10^3$ increase in the score value from the previous 2% reinforcement percentage to give a $1.4269 \times 10^5$ score. The score range value here was distributed between two score ranges with this highest number of individual at a particular score being 24 and 20 being the next. Figure 15 shows the optimisation statistical outputs for 3% reinforcement. The score for the variable population varies and reduces progressively, and the highest number of individuals with the same score being 25 and the least being 1. This is observed to be similar to that obtained for the same percentage in the middle strip. The score for the variable population also increased by the same value of $4.54 \times 10^3$ increase recorded for the change in percentage between 2% and 2.5% percentage reinforcement.
Figure 10. Plots of optimisation statistics for 0% reinforcement (column strip)

Figure 11. Plots of optimisation statistics for 1% reinforcement (column strip)
Figure 12. Plots of optimisation statistics for 1.5% reinforcement (column strip)

Figure 13. Plots of optimisation statistics for 2% reinforcement (column strip)
Figure 16 shows the optimisation statistical outputs for 3.5% reinforcement. There was an increase in the score to a value of $1.5177 \times 10^5$. The score range for the variable population was majorly dominated by a single score value with 38 number of individuals sharing the very same score. Figure 17 shows the optimisation statistical outputs for 4% reinforcement. There was an increase in the score to a value of $1.5631 \times 10^5$. The score range for the variable population varies, with the highest number of individuals sharing the same score being 31.
4. Conclusions

From the middle strip optimisation statistical outputs, it can be deduced that the optimum thickness of the slab is 130mm as generated by the software. From the various reinforcement ratio, the 2% ratio gives rise to almost all the variable population scoring the same value. This indicates that for the cost optimum design of the reinforced concrete flat slab, Engineers should endeavour to keep the thickness of the middle strip around 130mm and the reinforcement ratio should be kept below 2%.

The column strip optimisation as expected gives a slightly higher optimum thickness of 140mm. This can be linked to the higher percentage of moment shared with the middle strip. Furthermore, the reinforcement ratio at 3.5% gives the least variation with about 80% of the variable population converging to a single value. Hence, it can be further recommended that Engineers try to keep the thickness of the column strip around 140mm and the reinforcement ratio kept below 3.5%.

The current work is not exhaustive enough because the objective functions for both the middle and column strips are considered individually. It will be interesting to explore the interaction of these

Figure 16. Plots of optimisation statistics for 3.5% reinforcement (column strip)

Figure 17. Plots of optimisation statistics for 4% reinforcement (column strip)
objective functions as a multi-objective constrained optimisation whereby the Pareto plot could reveal the point of intersection of the two objective functions.

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