Comparison of Methods for Calculating the Heating of Plastic Gears

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Abstract. Accurate determination of heating is significantly more important for plastic gears, even in simpler applications than for steel gears. These gears are often used without lubrication or with a single, lifetime lubrication. The load capacity of materials is significantly affected by temperature, even in ordinary cases. Even today, heating is often calculated for different heating cases according to the calculation method of VDI 2545 or the modified VDI 2545, although these methods have been proven to be obsolete. Since the main direction of our research is to explore the design features of drive units equipped with small plastic gears, we tried to assess in advance, using a series of calculations, how much difference there is between the results obtained with obsolete and modern calculation methods. In this article, we present the basic characteristics of the calculation methods and compare the results obtained for some m>1 mm and m<0.5 mm gears.

1. Introduction

Heating is a fundamentally bigger problem for plastic gears than for steel gears. The strength of plastics can vary significantly even at temperatures that occur in the everyday environment, such as in places exposed to direct sunlight. In gear drives, this can have a significant effect on the behavior of the drive unit without rapid failure of the components, with higher temperatures leading to greater deformation of the tooth surface, greater wear and consequently a reduction in the contact ratio. At the same time, drive units with plastic gears are often used without lubrication, and temperature also has a strong influence on the tribological properties of plastics. [1]

There are no thorough and detailed standards for the design of plastic gears, such as DIN 3990. A common method is to design such gears according to a common standard for metal gears, simply substituting the strength properties of plastics in the calculations. This method, however, does not address the problem that the strength of plastics can vary significantly with changes in operating ambient temperature. This is addressed by current standards and guidelines for the design of plastic gears, either by a computational method or by a combination of experiments and calculations. In Europe, the VDI 2545 guideline has long been the basis for the design of plastic gears. It was published in 1981 and withdrawn in 1996. Since no other similar guideline was available after its withdrawal, it was generally used in practice until the publication of VDI 2736 in 2016 and is still relevant today. Therefore, in this article we present the calculation method to calculate the heating of plastic gears according to VDI 2545. In addition, the calculation method of VDI 2736 is presented. AGMA and JIS standards for gear design recommend experimental methods for the determination of heating, of which we present here the method.
of JIS B 1759. Since there was no generally accepted method of measurement for 10 years, several material manufacturers have developed documents that are well suited to use within given limits. Here we present the method of Licharz company, which is a modified version of VDI 2545 based on empirical data.

2. Calculation methods of the VDI guidelines

2.1. VDI 2545 [2]

\[
\vartheta_{1,2} = \vartheta_k + 136 \cdot P \cdot \mu \cdot \frac{1 + u}{z_2} + \frac{17100}{(b \cdot z_1 z_2) (v \cdot m)^\kappa} + 6.3 \cdot k_3 \frac{k_2}{A}
\]  

(1)

Where:
- \( \vartheta_{1,2} \), the temperature of the gear, °C
- \( \vartheta_k \), the ambient temperature, °C
- \( P \), power, kW
- \( b \), width of the tooth face, mm
- \( \mu \), coefficient of friction
- \( z \), number of teeth
- \( m \), a module, mm
- \( v \), tangential velocity, m/s
- \( A \), surface of the gear casing, m²
- \( u \), gear ratio
- \( k_2 \), material-related factor
- \( k_3 \), gear-related factor, m²K/W

Table 1. The coefficient of friction and \( k_3 \) according to [2]

| Lubrication          | Oil mist lubrication | Assembly lubrication | Permanent lubrication (grease) | Dry | Oil lubrication |
|----------------------|----------------------|----------------------|--------------------------------|-----|-----------------|
| Coefficient \( \mu \) |                       |                      |                                |     |                 |
| PA-PA                | 0,07                 | 0,09                 | 0,04                           | 0,2 | 0,04            |
| PA-steel             |                       |                      |                                |     |                 |
| POM-POM, POM-steel   | 0,04                 |                       |                                |     |                 |
|                      | 0,28                 | 0,2                  | 0,2                            |     |                 |
|                       | 0,2                  |                      |                                |     |                 |
| \( k_3 \), m²K/W     | open gear            | 0                    |                                |     |                 |
|                      | partially open gear  | 0,04…0,13            |                                |     |                 |
|                      | closed gear          | 0,17                 |                                |     |                 |

Table 2. The value of \( k_2 \) and \( \kappa \) according to [2]

| Material          | PA-PA | PA-steel | POM-POM | POM-steel | for immersion and syringe lubrication: 0 |
|-------------------|-------|----------|---------|-----------|-----------------------------------------|
| \( k_2 \) flank temperature | 15    | 10       | 7       |           |                                         |
| \( k_2 \) root temperature | 2,4   | 1,0      | 2,5     |           |                                         |
| \( \kappa \)      | 0,75  | 0,75     | 0,4     | 0,4       |                                         |
2.2. Modified VDI 2545 [3]

\[
\vartheta_{1,2} = \vartheta_k + 136 \cdot P \cdot \mu \cdot \frac{1 + i}{z_2 + 5 \cdot i} \left( \frac{17100}{b \cdot z_{1,2}} \cdot \frac{k_2}{(v \cdot m)^3} + 7,33 \cdot \frac{k_3}{A} \right) \tag{2}
\]

Where:
- \( \vartheta_{1,2} \), the temperature of the gear, °C
- \( \vartheta_k \), the ambient temperature, °C
- \( P \), power, kW
- \( b \), width of the tooth face, mm
- \( \mu \), coefficient of friction
- \( z \), number of teeth
- \( m \), a module, mm
- \( v \), tangential velocity, m/s
- \( A \), surface of the gear casing, m²
- \( i \), transmission ratio
- \( k_2 \), material-related factor
- \( k_3 \), gear-related factor, m²K/W

| Table 3. Value of the factor \( k_2 \) according to [3] |
|-------------------------------------------------------|
| For the calculation of flank temperature:            |
| \( k_2=7 \), for mating components steel/plastic      |
| \( k_2=10 \), for mating components plastic/plastic   |
| \( k_2=0 \), in the case of oil lubrication            |
| \( k_2=0 \), at a velocity not bigger than 1 m/s      |

| Table 4. Value of the factor \( k_3 \) according to [3] |
|-------------------------------------------------------|
| \( k_3=0 \), for completely open gear                  |
| \( k_3=0,043-0,129 \) m²K/W, for partially open gear   |
| \( k_3=0,172 \) m²K/W, for closed gear                 |

| Table 5. The coefficient of friction according to [3] and [4] |
|-------------------------------------------------------------|
| \( \mu=0,04 \), for gears with permanent lubrication        |
| \( \mu=0,07 \), for gears with oil mist lubrication         |
| \( \mu=0,09 \), for gears with assembly lubrication         |
| \( \mu=0,2 \), PA/steel, dry                                |
| \( \mu=0,35 \), PA/PBT, dry                                 |

In the case of intermittent operation, the relative duty cycle \( ED \) is defined as the ratio between the load duration \( t \) and the overall cycle time \( T \) as a percentage [3]:

\[
ED = \frac{t}{T} \cdot 100 \tag{3}
\]

The value of the correction factor \( f \) as a function of the relative load cycle can be determined from a diagram. Thus, the equation modified by the correction factor \( f \) for intermittent operation [3]:

\[
\]
\[ \theta_{1,2} = \theta_k + 136 \cdot P \cdot \mu \cdot f \cdot \frac{1 + i}{z_2 + 5 \cdot i} \left( \frac{17100}{b \cdot z_{1,2} / (v \cdot m)^{3/4}} + 7.33 \cdot \frac{k_2}{A} \right) \]  

(4)

2.3. Calculation method of VDI 2736
The tooth root temperature [4]:

\[ \theta_{ft} = \theta_k + H_v \cdot P \cdot \mu \cdot \left( \frac{k_{\theta,Fuß}}{b \cdot z \cdot (v \cdot m_n)^{3/4}} + \frac{R_{\lambda,G}}{A_G} \right) \cdot ED^{0.64} \]  

(5)

The flank temperature [VDI 04]:

\[ \theta_{Fla} = \theta_k + H_v \cdot P \cdot \mu \cdot \left( \frac{k_{\theta,Fla}}{b \cdot z \cdot (v \cdot m_n)^{3/4}} + \frac{R_{\lambda,G}}{A_G} \right) \cdot ED^{0.64} \]  

(6)

Where [4]:
- \( \theta_k \), the ambient temperature, °C
- \( P \), nominal output, W
- \( \mu \), coefficient of friction
- \( H_v \), degree of tooth loss
- \( k_{\theta,Fuß} \), heat transfer coefficient of the plastic gear for the calculation of root temperature
- \( k_{\theta,Fla} \), heat transfer coefficient of the plastic gear for the calculation of flank temperature
- \( b \), face width, mm
- \( z \), number of teeth
- \( v \), tangential velocity, m/s
- \( m_n \), normal module, mm
- \( R_{\lambda,G} \), heat transfer resistance of the mechanism housing
- \( A_G \), heat-dissipating surface of the mechanism housing, m²
- \( ED \), relative tooth-engagement time

The degree of tooth loss [4]:

\[ H_v = \frac{\pi \cdot (u + 1)}{z \cdot \cos \beta_b} \cdot (1 - \varepsilon_1 - \varepsilon_2 + \varepsilon_1^2 + \varepsilon_2^2) \]  

(7)

Where [4]:
- \( u \), gear ratio
- \( z \), number of teeth
- \( \beta_b \), helix angle at the base circle
- \( \varepsilon_1 \) and \( \varepsilon_2 \), partial radial contact ratios of pinion and gear

\[ \varepsilon_i = \frac{z_i}{2 \cdot \pi} \cdot \left( \sqrt{\left( \frac{d_{ai}}{d_{bi}} \right)^2 - 1 - \tan \alpha_{wt}} \right) \]  

(8)

- \( d_{ai} \), addendum circle diameter, mm
- \( d_{bi} \), base circle diameter, mm
- \( \alpha_{wt} \), operating pressure angle in the transverse section

Table 6. Heat transfer coefficients in K(m/s)^0.75/mm^1.75/W [4]

| Lubrication | Pairing | \( k_{\theta,Fuß} \) | \( k_{\theta,Fla} \) |
|-------------|---------|----------------|----------------|

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Table 7. Heat transfer resistance of the mechanism housing in K m2/W [4]

| Mechanism                  | $R_{λ,G}$ |
|----------------------------|-----------|
| open housing               | 0         |
| partially open housing     | 0.015...0.045 |
| closed housing             | 0.06      |

3. Comparison of calculation methods in the VDI guidelines

All three VDI guidelines relate ambient temperature to the rest of the equation by addition. This means that none of the methods takes into account the fact that not only the strength but also other properties, such as tribological properties or elasticity, can change significantly with temperature in plastics.

Similarities in the VDI methods are that the power $P$, friction coefficient $μ$, width $b$, number of teeth $z$, tangential velocity $v$ and module $m$ are considered in a very similar way. Furthermore, all three methods take into account the size of the gear surface.

The VDI 2545 method can only be used for continuously running drive units. However, the modified VDI 2545 and VDI 2736 methods are not only applicable for continuous operation, but also for intermittently operating gear units. The former can be modified by a correction factor $f$, which is a function of the relative load cycle $ED$, the latter introduces this $ED$ value directly into the formula. This is advantageous, because plastic drive units often only perform setting tasks, i.e., they operate intermittently.

VDI 2545 and the modified VDI 2545 show significant similarities in structure and parameters. The differences are the empirical quantity of 6.3 in the former formula and 7.33 in the latter, and the possible values of the factors $k_2$ and $k_3$. VDI 2736 differs from the other two VDI methods in a number of respects, both in its structure and in the parameters it uses.

The material properties are taken into account by VDI 2545 and the modified VDI 2545 with the factor $k_2$. This factor can also take a different value when calculating the temperature of the tooth flank or the tooth root. For VDI 2736, there are two separate factors for calculating the temperature of the tooth root and the tooth flank. The factor $k_{ϑFuß}$ is the factor for the root and $k_{ϑFla}$ is the factor for the surface temperature. Both factors depend on the gear material.

The VDI 2545 method includes a factor that the other two VDI methods do not. This is the $κ$ factor, which depends on the gear material pairing. This is one of the reasons why this guideline is still often used today, because plastic gears are often paired with steel gears or plastic gears with different properties, for example to reduce noise or operating clearance. The modified VDI 2545 and VDI 2736 take this $κ$ factor as 0.75, although according to Miklós Antal [2] $κ$ is not 0.75 for all materials.

Various factors are also taken into account when determining the closure of the gearbox. VDI 2545 and modified VDI 2545 use the factor $k_3$ for this purpose, VDI 2736 uses $R_{λ,G}$. They have in common that three different values can be taken depending on whether the drive unit is open, partially closed or closed.

4. The JIS B 1759 standard

The Japanese standard JIS B 1759 presents a possible definition of the strength of plastic gears. For this purpose, the standard describes a test bench, which is used by taking measurements with the standard test parameters presented in the standard, as well as with different parameters, and then comparing them.

The JIS B 1759 standard considers the effect of temperature on the allowable dedendum bending stress of the gear material [5]:

$$σ_{FP} = σ_{Flim} \cdot Y_{NT} \cdot Y_Θ \cdot Y_{ΔΘ} \cdot Y_l \cdot Y_M$$  \hspace{1cm} (9)

Where [5]:
• \( \sigma_{\text{Flim}} \), the allowable bending stress of gear material, which can be determined by tests
• \( Y_{NT} \), the life coefficient
• \( Y_M \), mating gear coefficient
• \( Y_L \), lubrication coefficient
• \( Y_0 \), ambient temperature coefficient
• \( Y_{\Delta\theta} \), temperature increase coefficient

The ambient temperature coefficient for standard test parameters, i.e., for an ambient temperature of 23\(^\circ\)C is \( Y_0 = 1 \). The ambient temperature coefficient for a given temperature other than the standard test conditions is calculated by the following method [5]:

\[
Y_0 = \frac{\sigma_{\text{Flim}}(at \theta = \theta^*)}{\sigma_{\text{Flim}}} \tag{10}
\]

Where \( \sigma_{\text{Flim}} \) is the allowable bending stress of the gear material at the standard test parameters (\( \theta_K = 23\,^{\circ}\)C), \( \sigma_{\text{Flim}}(at \theta = \theta^*) \) is the allowable bending stress of the gear material at an ambient temperature different from the standard test parameters, the value of which can be determined by test bench tests. [5]

The temperature increase coefficient is a factor that takes into account the effect of the temperature increases due to friction and hysteresis on the allowable bending stress of the gear material. The temperature growth factor can be obtained as follows [5]:

- If the gear module or tooth width is smaller than the standard test gear: \( Y_{\Delta\theta} > 1 \)
- If the gear module or tooth width is greater than the standard test gear: \( Y_{\Delta\theta} < 1 \)
- If the rotation rate of the gear is less than the standard test gear: \( Y_{\Delta\theta} > 1 \)
- If the rotation rate of the gear is greater than for the standard tests: \( Y_{\Delta\theta} < 1 \)

Thus, the methodology of JIS B 1759 is very different from that used in the VDI guidelines. On the one hand, JIS B 1759 does not derive much of the factor from tables and diagrams of previous data but describes how to make measurements to obtain factors that affect strength.

In addition, JIS B 1759 does not specifically address the calculation of the gear temperature. Unlike the VDI guidelines, it incorporates the effect of temperature in the form of factors into the strength calculation formula.

Another important difference is that JIS B 1759 does not neglect the effect of ambient temperature on the gear material but presents a possibility to determine the effect of ambient temperature as a factor in the form of separate measurements.

In addition, however, it does not use a specific relationship to determine the additional temperature increase but presents a benchmark that can be obtained by comparing the parameters of the drive unit we use with the standard test parameters. This method carries some uncertainty, as the standard does not specify the scale at which a change in each parameter will change the factor, only whether it will be greater or less than one.

The temperature increase coefficient does not take into account the tribological properties of the gear material, nor the friction, nor the degree to which the gear is closed or open. It only takes into account the variation of certain geometrical dimensions of the gear and the speed. Furthermore, the JIS B 1759 standard cannot take into account intermittent operation.

5. Other calculation methods

Gear temperature according to [6]:

\[
\vartheta = \vartheta_K + \Delta \vartheta_b + \vartheta_f \tag{11}
\]

Where \( \vartheta_K \) is the ambient temperature, the increase in gear body temperature is [6]:

\[
Y = \frac{\sigma_{\text{Flim}}(at \theta = \theta^*)}{\sigma_{\text{Flim}}} \tag{10}
\]
\[ \Delta \theta_p = \frac{0.625 \cdot \mu \cdot T}{C_p \cdot \rho \cdot z \cdot L \cdot (r_0^2 - r_p^2)} \]

Where [6]:
- \( C_p \), the heat capacity of air
- \( \rho \), the air density
- \( T \), the transmitted torque
- \( L \), the gear face width
- \( r_0 \), the outer radius of the gears
- \( r_p \), the pitch radius

The flash temperature [6]:
\[ \theta_f = \frac{0.555 \cdot \delta H}{L \cdot (k \cdot \rho \cdot c \cdot w \cdot v_s)^{-1/2}} \quad (13) \]

Where [6]:
- the instantaneous energy loss due to friction
\[ \delta H = \frac{\pi \cdot P \cdot \mu}{z} \quad (14) \]
- \( k \), the thermal conductivity
- \( \rho \), the density
- \( c \), the specific heat of the surfaces
- \( v_s \), sliding velocity
- \( w \), the Hertzian contact width
- \( P \), the transmitted power
- \( z \) the number of teeth

The formula, similar to those in the VDI guidelines, does not take into account the effect of ambient temperature on the gear material. The difference is that the formula takes completely different parameters into account to calculate the temperature increase.

Compared to other temperature formulas, it breaks down the temperature change into two separate terms: one for the temperature inside the gear body and one for the temperature on the surface. The former depends on the torque and gear dimensions as well as on the air, while the latter depends on material properties, surface friction and Hertzian contact.

6. Calculations based on the methods presented

6.1. Definitions
A relevant temperature difference in this interpretation is a difference that exceeds 5 percent of the initial value.

Small gear: a gear with a maximum module of 0.5 mm and a maximum characteristic size of less than 30 mm.

6.2. Calculations
Using the methods presented here, calculations have been carried out for a number of different modules, ratios and powers, for a number of different materials. We were interested to see if there were any relevant differences between the results and, if so, to what extent the size of the gears affected the results.
For the three VDI procedures, the only relevant differences in the module range above 0.5 mm are in extreme cases where the applicability of a particular material or size range is questionable anyway. However, for small gears, relevant differences can be detected, especially for module of 0.3 mm and below. Table 8 is an extract of the calculations. These calculations are made here for POM material with 10/50 gear ratio and 0.3 mm module.

For this gear pair, the tooth surface load capacity at the critical starting temperature of 60 °C, which is not an exceptional value for example in a car dashboard in summer, at 10 watts of power and 10,000 rpm, the tooth surface safety factor is only 1.8, which is not a practical value for continuous operation, but is acceptable for intermittent operation.

Table 8. The results of the calculation

| VDI 2545 | Modified VDI 2545 | VDI 2736 |
|----------|-------------------|----------|
| Assembly lubrication | Dry | Assembly lubrication | Dry | Assembly lubrication | Dry |
| Flank Root Flank Root Flank Root Flank Root Flank Root Flank Root | | | | | |
| W | 62,247 61,18 66,99 63,66 63,06 61,255 66,81 62,79 60,041 60,016 60,09 60,04 |
| W | 61,123 60,59 63,49 61,83 61,53 60,628 63,40 61,39 60,020 60,008 60,05 60,02 |

Figure 1. Diagram of the calculated temperatures for assembly lubrication

Figure 2. Diagram of the calculated temperatures for dry operation
The values in the table show that VDI 2545 and VDI 2736 already give relevantly different values here. Calculations according to JIS require experiments to be carried out and are therefore not yet included in this calculation.

Summary
This article presents five methods that can be used to determine the in-service heating of plastic gears. Three of these methods have been used for comparative calculations to determine the extent to which the results obtained by the different methods differ from each other.

The calculations show that as the module is reduced, even ignoring all the other problems typical of small and micro-drives, the different methods give increasingly different results. This is no longer the case in the module range above 1 mm.

However, it is not possible to determine by calculation which method gives the correct result, experiments are needed. Therefore, the next step of our research is to carry out a series of experiments to show which calculation method is the most appropriate for small plastic gears and whether the application of known methods is even practical.

Literature
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