GLUON CONDENSATE AND PARTON PROPAGATION IN A QUARK-GLUON PLASMA

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A calculation of the thermal quark propagator is presented taking the gluon condensate above the critical temperature into account. The quark dispersion relation following from this propagator is derived.

1 Introduction

Finite temperature QCD can be used to describe properties of a quark-gluon plasma (QGP) possibly created in the fireball of relativistic heavy ion collisions and in the early Universe. There are two different methods available, namely lattice QCD and perturbation theory.

Lattice QCD is a non-perturbative method, which can be used at all temperatures above and below the phase transition. However, it is very difficult to calculate dynamical quantities or to consider a system at finite baryon density or in non-equilibrium.

Using perturbative QCD, on the other hand, one is able to compute static as well as dynamical quantities at zero and finite baryon density and even out of equilibrium. However, the strong coupling constant $\alpha_s$ is small only for temperatures $T \gg T_c$. Furthermore, infrared singularities show up in perturbative calculations even within the hard thermal loop (HTL) resummation scheme.

Therefore it would be desirable to construct non-perturbative QCD Green functions. Here we suggest to include the gluon condensate into the parton propagators. Condensates of gluons and quarks describe the non-perturbative feature of the QCD ground state. They have been used in QCD sum rules to describe hadron properties at zero and finite temperature.

Whereas the quark condensate is related to chiral symmetry breaking, the gluon condensate is associated with the breaking of the scale invariance. Therefore, in contrast to the quark condensate, it does not vanish above the phase transition.
In the case of a pure gluon gas with energy density $\epsilon$ and pressure $p$ the gluon condensate can be related to the interaction measure $\Delta = (\epsilon - 3p)/T^4$ via

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 - \Delta T^4,$$

where $G^2 \equiv (11\alpha_s/8\pi) : G_a^{\mu\nu}G_a^{\mu\nu} :$ and $\langle G^2 \rangle_0 = (2.5 \pm 1.0) T_c^4$. Comparing $\langle G^2 \rangle_0$ with $\Delta T^4$, measured on the lattice between $T_c$ and $4T_c$, one observes that the thermal gluon condensate $\langle G^2 \rangle_T$ above $T_c$ is different from zero and increases like $T^\alpha$ with $\alpha$ between 3 and 4.

2 Gluon Condensate and Quark Propagator

As a first problem we want to study the influence of the gluon condensate on the quark and gluon propagation in the QGP. At zero temperature quark and gluon propagators containing condensates have been constructed already. Here we will extend these calculations to finite temperature, starting with the thermal quark propagator.

To lowest order in the gluon condensate the quark self energy is given by the diagram of Fig.1. The most general ansatz for the quark self energy in the rest frame of the QGP is given by

$$\Sigma(P) = -a(p_0, p) P - b(p_0, p) \gamma_0,$$

where we assumed a vanishing bare quark mass and used the notations $P = (p_0, \mathbf{p})$, $p = |\mathbf{p}|$.

The scalar functions $a$ and $b$ can be expressed as

$$a = \frac{1}{4p^2} \left[ tr \left( P\Sigma \right) - p_0 tr \left( \gamma_0\Sigma \right) \right],$$

$$b = \frac{1}{4p^2} \left[ P^2 tr \left( \gamma_0\Sigma \right) - p_0 tr \left( P\Sigma \right) \right].$$

The most general ansatz for the non-perturbative gluon propagator in Fig.1 at finite temperature is given by

$$\tilde{D}_{\mu\nu}(K) = \tilde{D}_t(k_0, k) P^t_{\mu\nu} + \tilde{D}_t(k_0, k) P^t_{\mu\nu},$$

where $\tilde{D}_t(k_0, k)$ is the thermal gluon propagator.
where we have subtracted the bare gluon propagator as we are not interested in perturbative corrections to the quark self energy. Consequently the gluon propagator is gauge independent. The projectors $P_{\mu}^{l,t}$ project onto the longitudinal and transverse degrees of freedom.

Using the imaginary time formalism and expanding the quark propagator in Fig.1 for small loop momenta, i.e. $k \ll p$ and $k_0 = 2\pi nT = 0$, we obtain

$$a = -\frac{4}{3} g^2 T \int \frac{d^3k}{(2\pi)^3} \left[ \left( \frac{1}{3} p^2 - \frac{5}{3} p_0^2 \right) k^2 \tilde{D}_l(0,k) + \left( \frac{2}{7} p^2 - 2p_0^2 \right) k^2 \tilde{D}_t(0,k) \right],$$

$$b = -\frac{4}{3} g^2 T \int \frac{d^3k}{(2\pi)^3} \left[ \frac{8}{3} p_0^2 k^2 \tilde{D}_l(0,k) + \frac{16}{13} p^2 k^2 \tilde{D}_t(0,k) \right]. \quad (5)$$

The moments of the longitudinal and transverse gluon propagators are related to the chromoelectric and chromomagnetic condensates via

$$\langle E^2 \rangle_T = \langle : G_0^a G_0^a : \rangle_T = 8T \int \frac{d^3k}{(2\pi)^3} k^2 \tilde{D}_l(0,k),$$

$$\langle B^2 \rangle_T = \frac{1}{2} \langle : G_{ij}^a G_{ij}^a : \rangle_T = -16T \int \frac{d^3k}{(2\pi)^3} k^2 \tilde{D}_t(0,k). \quad (6)$$

These condensates can be extracted from the expectation values of the space- and timelike plaquettes $\Delta_{\sigma,\tau}$ computed on the lattice, using

$$\frac{\alpha_s}{\pi} \langle E^2 \rangle_T = \frac{4}{11} \Delta_\tau T^4 - \frac{2}{11} \langle G^2 \rangle_0,$$

$$\frac{\alpha_s}{\pi} \langle B^2 \rangle_T = -\frac{4}{11} \Delta_\sigma T^4 + \frac{2}{11} \langle G^2 \rangle_0. \quad (7)$$

Combining (2), (5), (6), and (7), a gauge invariant expression for the quark self energy as a function of the measured plaquette expectation values and the zero temperature gluon condensate is found. The effective quark propagator can be written by decomposing it according to its helicity eigenstates

$$S(P) = \frac{1}{P - \Sigma(P)} = \frac{\gamma_0 - \hat{P} \cdot \vec{\gamma}}{2D_+(P)} + \frac{\gamma_0 + \hat{P} \cdot \vec{\gamma}}{2D_-(P)}, \quad (8)$$

where $D_{\pm}(P) = (p_0 \pm p)(1 + a) - b$.

### 3 Quark Dispersion Relations

The dispersion relations of quarks and gluons are one of the most important applications of thermal field theory. The quark dispersion relations follows
from $D_{\pm}(P) = 0$. Using the lattice results for the plaquette expectation values they have been determined numerically and are shown in Fig. 2 for various temperatures.

The dispersions exhibit two massive quark modes. The upper branch comes from the solution of $D_+ = 0$ and the lower one from $D_- = 0$. The lower branch, showing a minimum, corresponds to a so-called plasmino, possessing a negative ratio of helicity to chirality, and is absent in the vacuum. The plasmino branch disappears for large momenta rapidly indicating its collectivity.

At $p = 0$ both modes start from a common effective quark mass, which is given by

$$m_{\text{eff}} = \left[ \frac{2\pi^2}{3} \frac{\alpha_s}{\pi} \left( \langle E^2 \rangle_T + \langle B^2 \rangle_T \right) \right]^{1/4}. \quad (9)$$

In the temperature range $1.1T_c < T < 4T_c$ we found approximately $m_{\text{eff}} = 1.15T$.

The qualitative picture of this quark dispersion relation is very similar to the one found perturbatively in the HTL limit. The main difference is the different effective mass, which is given by $m_{\text{eff}} = gT/\sqrt{6}$ in the HTL approximation.

4 Conclusions and Outlook

The gluon condensate above the critical temperature has been measured on the lattice. This condensate describes non-perturbative effects in the QGP. Following the zero temperature calculation we have constructed an effective
quark propagator containing the gluon condensate. Furthermore we derived
the quark dispersion relations from this effective propagator. Similar as in
perturbation theory (HTL approximation) we found two branches correspond-
ing to a collective quark and a plasmino mode. Both branches start at the
same effective quark mass. In contrast to the HTL approximation, where the
effective quark mass is of order $gT$, we found a mass proportional to $T$.

As a possible application of this effective quark propagator we mention
the computation of the photon and dilepton production rates from the QGP. For
this purpose the photon self energy using effective quark propagators should
be evaluated.

Finally we discuss briefly the possibility to construct a thermal gluon prop-
agator for temperatures above $T_c$, containing the gluon condensate. This
would be of interest, because one could study the screening behavior of such a
non-perturbative propagator in the magnetic sector, where the absence of static
magnetic screening in the HTL resummed propagator leads to infrared singular-
ities in perturbative calculations.

As a consequence of the Slavnov-Taylor identities the gluon self energy
including the gluon condensate should be transverse. Already at zero tem-
perature this leads to the necessity to take ghost and higher order condensates
into account. Then one ends up with a complicated expression for the gluon
self energy containing unknown condensates. Furthermore the gluon self en-
ergy turns out to be gauge dependent, which leads to a gauge dependent gluon
dispersion within this approximation, rendering the physical meaning of this
dispersion unclear.

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