The Investigation of a Ride Quality of Nonlinear Half-Car Model

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Abstract. The paper discusses the ride quality response of a half car model when a bilinear model and a power law model were implemented in the suspension of half car vehicle. The models were subjected to a step and double pothole disturbances. The nonlinearity of the model is contributed by the power law damper models which are located both at the front and at the rear of the vehicle. Passenger car dampers were initially tested on the damper test bench where the dynamic characteristics were obtained. From the experimental results it clearly showed that the automotive damper behaves nonlinearly. From the force-velocity graphs a backbone model was then developed and plot. Based on this graph two types of non-parametric model namely the bilinear model and power law model were then obtained through the Lavenberg-Marquardt curve fitting algorithm (LMA) method. From the multibody system simulations they clearly showed that the bounce and pitch responses of the bilinear damper model illustrates an overestimations of responses compared to the nonlinear damper model which may lead to a less accurate representation of a realistic damper.

1. Introduction
The damper or shock absorber is a part of the component in the suspension assembly that is responsible in determining the ride quality and stability of the vehicle. Dampers are tuned differently for diverse types of vehicles and the different tuning would depend on the vehicles’ capacity and utility.

While some researchers assumed the linear behavior of the shock absorber others have observed and concluded that the automotive damper is a nonlinear element and thus obtaining the nonlinear mathematical model is essential [1,2]. The representation of a single damping coefficient to a shock absorber may not be accurate since its physical characteristic would require it to have a varying damping coefficient at different level of disturbance frequencies [3,4]. When implemented onto a vehicle dynamics simulation the vehicle with a linear shock absorber model tends to overestimate its responses.

The purpose of this paper is to compare the variation of responses between a bilinear damper model with a power law damper model. The dampers were employed to a half car vehicle representation travelling on irregular road surfaces where it was subjected to bounce i.e. vertical motion and pitch i.e. rotational motion about its lateral axis.
2. Damper Models

The damper models implemented in this paper can be classified into two types i.e. bilinear model and power law model.

An experiment was conducted initially in order to obtain the characteristic of a typical passenger car damper. The data was analysed and the Lavenberg-Marquardt Algorithm (LMA) to attain the empirical models of both the bilinear and the power law models. Figure 1 shows the backbone data of the damper. In this case it shows the two regions of the damper i.e. rebound (moving up) and compression (moving down) [5]. To be more precise the rebound damping force is much higher compared to the compression region [6].

![Figure 1. Backbone data of a passenger car damper.](image)

2.1. Lavenberg-Marquardt Curve Fitting

The curve fitting process using the LMA is one of the steps in determining the accuracy of a realistic damper model. By using an opensource data analysis software the experimental results of the passenger car damper were plot as in figure 1. The LMA were implemented for two different models i.e. the bilinear model and the power law model with maximum iterations of 1000 and an error of 0.0001 for each respective model.

In statistics the adjusted $R^2$ can be used as an indicator of a good fit in which the value of the index can be between 0 and 1 where the value of closer to 1 indicating the better fit. The following table shows the results of the adjusted $R^2$ for both the bilinear model and the power law model after the implementation of the LMA. Table 1 clearly indicates that the power law model gives a better representation of a realistic damper. The accuracy of the model is critical since it will give the designer a good and correct sense of the ride quality of a vehicle.

| Model            | Rebound | Compression |
|------------------|---------|-------------|
| Bilinear Model   | 0.981   | 0.770       |
| Power Law Model  | 0.997   | 0.958       |

Table 1. Adjusted $R^2$ Values for the models.
2.2. Bilinear model
Since the rebound and compression damping forces are asymmetric thus the bilinear model was used. The term bilinear indicates that the damper would have the following characteristics:

\[
F_d(v) = \begin{cases} 
  d_1v & ; v \geq 0 \\
  d_2v & ; v < 0 
\end{cases}
\]  
(1)

Where \( F_d(v) \) is the damping force that changes with respect to velocity, \( d_1 \) and \( d_2 \) are the slopes of the model during rebound and compression respectively and \( v \) is the velocity of the piston rod of the damper. The bilinear model depicted in (1) can be illustrated as in figure 2. By implementing the LMA it was found that the values of \( d_1 \) and \( d_2 \) are 2287 N.s/m and 1220.5 N.s/m respectively for the front damper and for the rear damper the values of \( d_1 \) and \( d_2 \) are 1488.2 N.s/m and 943.55 N.s/m respectively.

2.3. Power Law model
The power law model can be defined in the following form:

\[
F_d(v) = \begin{cases} 
  c_1v^{e_1} & ; v > 0 \\
  c_2v^{e_2} & ; v < 0 
\end{cases}
\]  
(2)

Where \( c_1 \) and \( e_1 \) are the constants of the model for rebound and \( c_2 \) and \( e_2 \) are the constants for compression and can be illustrated as in figure 3. From the figure it clearly indicates that the damping forces during rebound and compression are not equal and this is in agreement with the experimental result shown in figure 1.
The values of the constants are obtained from the LMA and presented in table 2.

Table 2. Power Law Model Constant Values

| Symbol | Values | Front | Rear |
|--------|--------|-------|------|
| $c_1$  |        | 2302.1| 1573.12 |
| $c_2$  |        | 1217.8| 946.34 |
| $e_1$  |        | 0.74  | 1.03  |
| $e_2$  |        | 0.41  | 0.4   |

3. Theoretical Background

A half car model is a representation of a vehicle that consists of a body, a front wheel and a rear wheel. Between the wheels are the assembly of the suspension that includes the suspension coil spring and the damper. Figure 4 represents the schematic diagram of half car model.

Figure 4. Schematic diagram of a half car model.

The symbols denoted in figure 4 can be summarized and some of their values are presented in table 2.
Table 3. Half Car Model Parameters.

| Symbol | Connotation               | Value | Unit   |
|--------|---------------------------|-------|--------|
| $m_b$  | Mass of body              | 720   | kg     |
| $J$    | Body moment of inertia    | 737.4 | kg.m²  |
| $m_{wf}$ | Mass of front wheel     | 40    | kg     |
| $m_{wr}$ | Mass of rear wheel      | 40    | kg     |
| $a$    | Front to CG distance     | 1.08  | m      |
| $b$    | Rear to CG distance      | 1.48  | m      |
| $x_b$  | Body displacement at CG   |       | m      |
| $\theta$ | Body rotation (pitch angle) |     | rad    |
| $x_{wf}$ | Front wheel displacement |       | m      |
| $x_{wr}$ | Rear wheel displacement  |       | m      |
| $x_f$  | Road displacement at the front wheel |    | m      |
| $x_r$  | Road displacement at the rear wheel |   | m      |
| $F_{df}(\dot{x})$ | Front suspension damping force |   | N.s/m  |
| $F_{dr}(\dot{x})$ | Rear suspension damping force |   | N.s/m  |
| $k_{sf}$ | Front suspension spring constant | 1.996e+004 | N/m |
| $k_{sr}$ | Rear suspension spring constant | 1.996e+004 | N/m |
| $k_{wf}$ | Front wheel stiffness    | 1.755e+005 | N/m  |
| $k_{wr}$ | Rear wheel stiffness     | 1.755e+005 | N/m  |

The equations of motion of the half car model in the figure above can then be derived as in the following relationship:

$$
\begin{align*}
    m_s a_b + F_{df}(\dot{x}) + F_{dr}(\dot{x}) + k_{sf}(x_b - a \theta - x_{tf}) + k_{sr}(x_b + b \theta - x_{tr}) &= 0 \\
    J \alpha - F_{df}(\dot{x})a + F_{dr}(\dot{x})b - k_{sf}(x_b - a \theta - x_{tf})a + k_{sr}(x_b + b \theta - x_{tr})b &= 0 \\
    m_{wf} a_{wf} - F_{df}(\dot{x}) - k_{sf}(x_b - a \theta - x_{tf}) + k_{wf}(x_{wf} - x_{tf}) &= 0 \\
    m_{wr} a_{wr} - F_{dr}(\dot{x}) - k_{sr}(x_b - b \theta - x_{tr}) + k_{wr}(x_{wr} - x_{tr}) &= 0
\end{align*}
$$

In (3) the term $\dot{x}$ is the relative velocity between the body and the wheels $a_b$ is the linear acceleration of the body and $\alpha$ is the angular acceleration of the body. The relationship of (3) was solved by implementing the Runge-Kutta method through a multibody dynamic software.

4. Half-Car Model Simulation

The simulation of the half car model was done by using a multibody dynamics simulation software and the parameters in Table 3 are used. To compare the ride responses between the bilinear damper and the power law damper model the damping forces $F_{df}(\dot{x})$ in (3) were replaced with (1) and (2) respectively. The purpose of the simulation is to provide further proofs that the response of the half car model equipped with the bilinear damper model gives results which are different from the more accurate power law damper model.

As a disturbance to the vehicle two types of road profiles were implemented namely the step input and the double pothole disturbance input. The disturbances are illustrated as in the following figure.
As a measurement to ride responses the vertical and angular accelerations are used and from the data obtained the root mean square (rms) values will be calculated and the settling time of the body be taken.

5. Results
The results from the simulation can be divided into two groups based on the road disturbances. The main objective of this simulation is to compare the percentage variation between the bilinear model and the power law model. For each of the road disturbance the time history of the vertical position and acceleration and the pitch angle and angular acceleration of the body are presented. The vertical position and pitch angle indicate how fast the body move to its stable position after the disturbances have passed while the vertical acceleration and angular acceleration indicates the quality of the ride in which the smaller magnitudes of accelerations provide a better ride quality

Figure 6 below shows the response of the body’s vertical position and its pitch angle when subjected to a step road disturbance. From the figure it can be seen that there is a variation with respect to the overshoot response for both magnitudes. The most obvious being the response of the pitch angle where the overshoot of the power law damper model is almost doubled. Another obvious observation is the oscillatory response of the bilinear damper model in which the settling time of the aforementioned damper is larger i.e. at approximately 3 seconds compared to the power law damper which is approximately 1 second.

Figure 5. Road disturbances (a) step input (b) double pothole.
Figure 7. Vertical acceleration and angular acceleration response of the vehicle subjected to step disturbance.

Figure 7 shows the vertical acceleration measured at the centre of gravity of the body and its angular acceleration when the vehicle is subjected to the same step disturbance. From the vertical acceleration response it clear shows that a larger oscillation and larger settling time for the bilinear damper. However, an almost similar response can be seen for the angular acceleration result with some variation in the settling time of the vehicle.

Figure 8 is the vertical displacement and pitch angle response of the vehicle when it travelled to a double pothole at a speed of 10 m/s. The variations of results are again visible in the displacement response. The bilinear damper has shown an overestimation of displacement of approximately 20% more than the power law damper. However, a very close agreement of pitch angle response can be seen for both dampers with small magnitude of oscillations for the bilinear damper before reaching stable condition.

Figure 8. Vertical position and pitch angle response of the vehicle subjected to double pothole disturbance.

The following figure indicates the ride quality for both dampers when subjected to the double pothole road input. The most obvious observations from the figures are the larger oscillations of the power law damper and the overestimations of response for the bilinear damper especially in the
positive direction. The insert shows a close-up view of the vertical acceleration between 3 and 4 seconds.

![Figure 9. Vertical acceleration and angular acceleration response of the vehicle subjected to double pothole disturbance.](image)

To summarise the variations of response for all the quantities measured the following table presents the percentage difference of their root mean square (RMS) values. From the table it shows that in some responses the bilinear damper model could overestimate as much as 50% than the power law model damper. Since the power law model depicts a closer representation of a realistic damper therefore these percentage variations of the bilinear damper may not provide a good sense of simulated ride performances and stabilities.

| Road Disturbance | Quantity | RMS Value | %Difference |
|------------------|----------|-----------|-------------|
|                  | θ (pitch angle) | 0.003 | 0.004 | 36.1 |
|                  | α (angular acceleration) | 0.480 | 0.658 | 37.1 |
|                  | x (vertical displacement) | 0.069 | 0.094 | 37.1 |
|                  | a (vertical acceleration) | 0.491 | 0.744 | 51.5 |
| Step             | θ (pitch angle) | 0.007 | 0.008 | 3.6 |
|                  | α (angular acceleration) | 0.111 | 0.095 | -13.9 |
|                  | x (vertical displacement) | 0.011 | 0.014 | 23.8 |
|                  | a (vertical acceleration) | 0.127 | 0.163 | 27.8 |
| Double Pothole   | θ (pitch angle) | 0.007 | 0.008 | 3.6 |
|                  | α (angular acceleration) | 0.111 | 0.095 | -13.9 |
|                  | x (vertical displacement) | 0.011 | 0.014 | 23.8 |
|                  | a (vertical acceleration) | 0.127 | 0.163 | 27.8 |
6. Conclusions
An experiment was conducted to obtain the characteristic of a typical passenger car damper. From these data two types of damper models i.e. bilinear model and power law model were mathematically generated through the Lavenberg-Marquardt curve fitting algorithm. From the curve fitting algorithm it can be concluded that the power law model gives a better representation of a more realistic damper compared to the bilinear damper. By employing both dampers into a half car vehicle model the results of the simulations shows some degree of variations with respect to the body vertical position and pitch as well as the vehicle’s quality of ride. From the result of the simulations both dampers have shown some degrees of variations with respect to its overshoot, stability and settling time. The RMS values of the responses indicate that the bilinear model may not be an accurate damper representation to be implemented in the half car vehicle model.

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