Effect of $H^\pm$ on $D_s^\pm \rightarrow \mu^\pm \nu_\mu$ and $D_s^\pm \rightarrow \tau^\pm \nu_\tau$

A.G. Akeroyd*

Korea Institute for Advanced Study, 207-43 Cheongryangri 2-dong, Dongdaemun-gu, Seoul 130-722, Republic of Korea

Abstract

We investigate the effect of a charged Higgs boson ($H^\pm$) on the decays $D_s^\pm \rightarrow \mu^\pm \nu_\mu$ and $D_s^\pm \rightarrow \tau^\pm \nu_\tau$, which will be measured with high precision at forthcoming CLEO-c. We show that a $H^\pm$ can suppress the branching ratios by $10\% \rightarrow 15\%$ from the Standard Model prediction, and we emphasize that such contributions should not be overlooked when comparing lattice calculations of $f_{D_s}$ to the values obtained from these decays.

Keywords: Rare D decay

*akeroyd@kias.re.kr
1 Introduction

Purely leptonic decays are the classic ways to measure the decay constants \( f_P \) of charged pseudoscalar mesons \( P^\pm \). For the light mesons, \( \pi^\pm \) and \( K^\pm \), the muonic decays \( \pi^\pm \rightarrow \mu^\pm \nu_\mu \), \( K^\pm \rightarrow \mu^\pm \nu_\mu \) have large branching ratios (BRs) and so their respective decay constants have been determined with high precision \([1]\) (< 1%). For the charmed pseudoscalar mesons \( D^\pm, D_s^\pm \) the BRs for the purely leptonic channels are much smaller than those for the above light mesons due to the dominance of weak decay mechanism \( c \rightarrow W^\pm q \) with a spectator quark. These smaller leptonic BRs together with the lack of a dedicated charm factory has resulted in vastly inferior experimental precision for the charmed meson decay constants compared to that for \( f_\pi \) and \( f_K \).

Current measurements of \( f_D \) and \( f_{D_s} \) have large errors of around 100% and 15% respectively \([1]\). With the imminent (summer 2003) commencement of the CLEO-c experiment \([2]\) this situation will improve dramatically in the next 2 → 3 years. Precise \( O(1 \rightarrow 2\%) \) measurements of \( f_D \) and \( f_{D_s} \) are expected and will constitute a vital test of lattice methods for the heavy quark systems, as well as providing crucial experimental input for calculations of the \( B \) meson decay constants \([2]\).

However, absent in the above discussion is the fact that the leptonic decays of \( D^\pm \) and \( D_s^\pm \) might be affected by physics beyond the Standard Model (SM). It is known that new charged particles which couple to the fermions would contribute at tree–level to these decays \([3]\). One such example is a charged Higgs boson \( H^\pm \), and in this paper we consider its effect on the decays \( D_s^\pm \rightarrow \mu^\pm \nu_\mu \), \( D_s^\pm \rightarrow \tau^\pm \nu_\tau \), and thus the measured value of \( f_{D_s} \). We point out that the possibility of such new physics contributions to these decays should not be overlooked when comparing the experimentally measured value of \( f_{D_s} \) to the lattice QCD predictions.

2 The decays \( D^\pm_{(s)} \rightarrow \mu^\pm \nu_\mu \) and \( D^\pm_{(s)} \rightarrow \tau^\pm \nu_\tau \)

Singly charged Higgs bosons, \( H^\pm \), arise in any extension of the SM which contains at least two SU(2) \times U(1) Higgs doublets, e.g. any Supersymmetric (SUSY) model. Together with \( W^\pm \) they mediate the leptonic decays \( D^\pm_{(s)} \rightarrow l^\pm \nu_l \) via the annihilation process shown below:

\[
\begin{align*}
D^\pm_{(s)} & \rightarrow W^* , H^\pm \\
& \rightarrow c , \nu \\
& \rightarrow d , s , \tau^\pm , \mu^\pm 
\end{align*}
\]

The tree–level partial width is given by \([3]\):

\[
\Gamma(D^\pm_{(s)} \rightarrow l^\pm \nu_l) = \left( \frac{G_F^2}{8\pi} m_{D_{(s)}} m_l^2 f_{D_{(s)}}^2 r_{(s)} |V_{cd_{(s)}}|^2 \right) \left( 1 - \frac{m_l^2}{m_{D_{(s)}}^2} \right)^2 \tag{1}
\]

where \( m_l \) is the mass of the lepton, \( m_{D_{(s)}} \) is the mass of the \( D^\pm_{(s)} \) meson, \( V_{cd_{(s)}} \) are CKM matrix elements, and

\[
r_{(s)} = \left[ 1 - \tan^2 \beta (m_{D_{(s)}}^2/m_{H^\pm}^2)(m_q/m_c) \right]^2 = \left[ 1 - R^2 m_{D_{(s)}}^2 (m_q/m_c) \right]^2 \tag{2}
\]
Table 1: SM predictions, current experimental BR, experimental error and CLEO-c expected errors for certain leptonic decays of $D^\pm$ and $D_s^\pm$

| Decay          | SM BR        | Current Exp BR   | Exp Error | CLEO-c Error |
|----------------|--------------|------------------|-----------|--------------|
| $D^\pm \to \mu^\pm \nu_\mu$ | $4.5 \pm 0.6 \times 10^{-4}$ | $8^{+16+5}_{-5-2} \times 10^{-4}$ | $\sim 100\%$ | $3.8\%$ |
| $D_s^\pm \to \mu^\pm \nu_\mu$ | $5.2 \pm 1.2 \times 10^{-3}$ | $5.3 \pm 0.9 \pm 1.2 \times 10^{-3}$ | $25\%$ | $3.2\%$ |
| $D_s^\pm \to \tau^\pm \nu_\tau$ | $5.1 \pm 1.2 \times 10^{-2}$ | $6.1 \pm 1.0 \pm 0.2 \times 10^{-2}$ | $25\%$ | $2.4\%$ |

where $r_{(s)} = 1$ in the SM, $R = \tan \beta/m_{H^\pm}$ and $\tan \beta = v_2/v_1$ (ratio of vacuum expectation values). The $H^\pm$ contribution interferes destructively with that of $W^\pm$, causing a suppression in the BR, with the largest deviations arising for large $R$. For $D^\pm$ this effect is essentially negligible ($r \approx 1$) due to the smallness of $m_d/m_c$, but for $D_s^\pm$ the scaling factor $r_s$ may differ from 1 due to the non–negligible $m_s/m_c$. This has been noted before [3], [4] but a numerical study was absent. In light of the high precision expected in the measurement of these leptonic decays at CLEO-c, we wish to quantify the previous qualitative analyses in order to see if the $H^\pm$ contribution can be significantly larger than the anticipated error in the measurement of BR($D_s^\pm \to \tau^\pm \nu_\tau, \mu^\pm \nu_\mu$).

The current experimental measurements and the SM predictions for the three leptonic decays which CLEO-c expects to measure are shown in Table 1. For the SM predictions we take the lattice results $f_D = 226 \pm 15$ MeV and $f_{D_s} = 250 \pm 30$ MeV [5], which induces an error of around 15% $\to$ 25% the BRs. The measurements of the $D_s^\pm$ decays are world averages taken from Ref. [6] and that for $D^\pm \to \mu^\pm \nu_\mu$ is taken from Ref. [7]. The expected errors from CLEO-c are shown in the final column.

3 Numerical Results

We now quantify the effect of the $H^\pm$ contribution on $r_s$ (eq.2). For the quark masses $m_s$ and $m_c$ we use the Particle Data Group values [1] and obtain $0.06 < m_s/m_c < 0.15$. The value of $R(= \tan \beta/m_{H^\pm})$ is best constrained from non–observation of the decay $B^\pm \to \tau^\pm \nu_\tau$, giving $R < 0.34 \pm 0.02 \pm 0.06 \text{GeV}^{-1}$, where the first error is from $f_B$ and the second is from possible large SUSY corrections [10]. Thus we take $R = 0.4 \text{ GeV}^{-1}$ as our largest value.

In Fig.1 we plot $r_s$ as a function of $R$ for various values of $m_s/m_c$. For $R = 0.4$ one has $r_s = 0.83(0.93)$ for the largest (smallest) values of $m_s/m_c$. This suppression is comfortably larger than the anticipated experimental error of 2% $\to$ 3% (shown by the horizontal line) in the measurement of BR($D_s^\pm \to \tau^\pm \nu_\tau, \mu^\pm \nu_\mu$). Thus the presence of $H^\pm$ would lead to a deceptive smaller measured value of the decay constant $f_{D_s}$. This effect was pointed out for the case of $f_K$ in Ref. [3], where BR($K^\pm \to \mu^\pm \nu_\mu$) can be suppressed by a factor comparable to that for the $m_s/m_c = 0.06$ curve. Although the effect of $H^\pm$ is less than the 25% error in BR($D_s^\pm \to \mu^\pm \nu_\mu, \tau^\pm \nu_\tau$) from the current lattice predictions of $f_{D_s}$ [5], there are already signs that the error in $f_{D_s}$ will be significantly improved in the near future. A recent paper [8] calculated $f_{D_s}$ with a precision of 4% ($252 \pm 9$ GeV) in the quenched approximation, while the techniques discussed in Ref. [9] promise comparable or smaller errors in the unquenched
Figure 1: $r_s$ as a function of $R(= \tan \beta / m_{H^\pm})$, for various values of $m_s/m_c$

approximation. With these anticipated reductions in the theoretical error of $f_{D_s}$, we suggest that the possible effects of any $H^\pm$ should not be overlooked when comparing the experimentally extracted $f_{D_s}$ to the prediction from lattice QCD.

An additional observable which will also be a test of lattice QCD is the ratio of the muonic decay rates $\mathcal{R}_\mu$ defined by

$$\mathcal{R}_\mu = \frac{BR(D^\pm_s \to \mu^\pm \nu_\mu)}{BR(D^\pm \to \mu^\pm \nu_\mu)} \sim \left(\frac{f_{D_s}}{f_D}\right)^2$$

(3)

The lattice prediction for $f_{D_s}/f_D$ is known with substantially greater precision than the individual values of the decay constants, and currently stands at 1.12(4) for unquenched calculations and 1.12(2) in the quenched approximation [5], i.e. an error < 4%. A similar ratio ($\mathcal{R}_\tau$) for the decays $D^{\pm}_{(s)} \to \tau^\pm \nu_\tau$ is also potentially an experimental observable, but is unlikely to be measured in the foreseeable future since CLEO-c has limited sensitivity to $D^\pm \to \tau^\pm \nu_\tau$ [2]. Hence we will only consider $\mathcal{R}_\mu$, whose current SM prediction is given by $\mathcal{R}_\mu = 12 \pm 0.8$, i.e. $\sim 7\%$ error. The current experimental measurement of $\mathcal{R}_\mu$ is based on 1 event for $BR(D^\pm \to \mu^\pm \nu_\mu)$ [4], whose central value is consistent with the old MARKIII limit of $BR(D^\pm \to \mu^\pm \nu_\mu) < 7.2 \times 10^{-4}$ [12]. Using the latter, a current lower bound would be $\mathcal{R}_\mu > 7 \pm 2$. The first accurate measurement of $\mathcal{R}_\mu$ is expected at CLEO-c with an error of around 7%, which is roughly the same as the error in the lattice prediction for $\mathcal{R}_\mu$. In contrast, in the case of the individual BRs the current theoretical error is substantially larger than the expected experimental error. The presence of $H^\pm$ would modify $\mathcal{R}_\mu$ by the factor $r_s$. Since the expected theoretical error in $\mathcal{R}_\mu$ should approach the percent level or less, $\mathcal{R}_\mu$ may also be a sensitive probe of physics beyond the SM. As an example, in SUSY models with $R$ Parity violating slepton interactions, $BR(D^\pm \to \mu^\pm \nu_\mu)$, which is essentially unaffected by $H^\pm$, can be significantly suppressed or enhanced by the cou-
pling combination $\lambda_{232}^2\lambda_{221}^1$, as discussed in Ref. [11]. Thus the presence of these couplings would give rise to a larger $R_\mu (\gg 12)$ or allow values close to the current experimental limit ($R_\mu \approx 7$) depending on the sign and magnitude of the product of $R$ Parity violating couplings $\lambda\lambda'$. Thus the first measurements of $R_\mu$ from CLEO-c are eagerly awaited.

Finally we note that any sizeable effects of $H^\pm$ on $\text{BR}(D^\pm_s \to \mu^\pm\nu_\mu, \tau^\pm\nu_\tau)$ and $R_\mu$ should manifest themselves in the purely leptonic $B^\pm$ decays, $B^\pm \to \tau^\pm\nu_\tau, \mu^\pm\nu_\mu$. This is because $r_s$ depends strongly on $R (= \tan\beta/m_{H^\pm})$, whose permitted value is constrained from the upper limits on the above $B^\pm$ decays. The $B$ factories will be sensitive to $R \sim 0.25$ with 400 fb$^{-1}$, and thus any significant suppression in $r_s$ from $H^\pm$ would be accompanied by a corresponding enhancement in $B^\pm \to \tau^\pm\nu_\tau$ (and $\mu^\pm\nu_\mu$).

4 Conclusions

We have studied the effect of a $H^\pm$ on the leptonic decays $D^\pm_s \to \mu^\pm\nu_\mu, \tau^\pm\nu_\tau$. We showed that $H^\pm$ can suppress the BRs by up to $10\% \to 15\%$, which is larger than the expected experimental error ($2\% \to 3\%$) from CLEO-c. We suggested that new physics effects like these should not be overlooked when comparing the experimental measurements of $f_{D_s}$ to the SM lattice QCD predictions.

References

[1] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66, 010001 (2002).
[2] I. Shipsey, arXiv:hep-ex/0207091
[3] W. S. Hou, Phys. Rev. D 48, 2342 (1993).
[4] J. L. Hewett, arXiv:hep-ph/9505246 J. L. Hewett, arXiv:hep-ph/9410314
[5] S. M. Ryan, Nucl. Phys. Proc. Suppl. 106, 86 (2002)
[6] S. Soldner-Rembold, arXiv:hep-ex/0109023
[7] J. Z. Bai et al. [BES Collaboration], Phys. Lett. B 429 (1998) 188.
[8] A. Juttner and J. Rolf [ALPHA Collaboration], Phys. Lett. B 560, 59 (2003)
[9] C. T. Davies et al. [HPQCD Collaboration], arXiv:hep-lat/0304004
[10] A. G. Akeroyd and S. Recksiegel, arXiv:hep-ph/0306037
[11] A. G. Akeroyd and S. Recksiegel, Phys. Lett. B 554, 38 (2003)
[12] J. Adler et al., Phys. Rev. Lett. 60, 1375 (1988) [Erratum-ibid. 63, 1658 (1989)].