New results for $t\bar{t}$ production at hadron colliders

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We present new theoretical predictions for the $t\bar{t}$ production cross section at NNLO at the Tevatron and the LHC. We discuss the scale uncertainty and the errors due to the parton distribution functions (PDFs). For the LHC, we present a fit formula for the pair production cross section as a function of the center of mass energy and we provide predictions for the pair production cross section of a hypothetical heavy fourth generation quark $t'$. 

1 $t\bar{t}$ - Production at Tevatron and LHC

The experimental measurements of the top mass $m_t$ and the $t\bar{t}$ production cross section have reached a relative accuracy of 0.75% [2] and 9% [3], respectively. Therefore it is necessary to provide improved theoretical predictions for the total cross section at the Tevatron and the LHC in perturbative QCD.

The total hadronic cross section for $t\bar{t}$ production depends on the top mass $m_t$, the center of mass energy $s = E^2$, the factorisation scale $\mu_f$, the renormalization scale $\mu_r$, and the PDF set and it is given by

$$\sigma(s, m_t, \mu_r, \mu_f) = \sum_{i,j=q,\bar{q}} f_{i/p}(\mu_f^2) \otimes f_{j/p}(\mu_f^2) \otimes \hat{\sigma}(m_t, \mu_r, \mu_f),$$

where $f_{i/p}$ are the proton PDFs. In the following, we discuss these dependencies.

We have updated the results from Refs. [4, 5] as follows. To obtain more reliable estimates of the scale uncertainty we have used the full dependence on $\mu_r$ and $\mu_f$. We have performed a consistent singlet - octet - decomposition when matching our NNLO threshold expansion at NLO. Further corrections (electroweak contributions [6, 7, 8], QCD bound state effects near threshold [9], and new parton channels $qq, \bar{q}\bar{q}$, and $q_i\bar{q}_j$ for unlike quarks opening at NNLO [10, 11]) are generally small and have been estimated.

As a new result we have studied the dependence of the $t\bar{t}$ production cross section on the definition of the mass parameter. We have used the $\overline{\text{MS}}$ mass scheme as an alternative mass description exploiting the conversion relation between the pole mass $m_t$ and the $\overline{\text{MS}}$ mass $\overline{m}(\mu_r)$ [12]. We find that the convergence of the perturbation expansion through NNLO is improved using the $\overline{\text{MS}}$ mass. This expansion has a considerably reduced scale dependence even at NLO. The NLO and NNLO corrections in the $\overline{\text{MS}}$ scheme are much smaller than the corresponding corrections in the pole mass scheme. Therefore, we find good properties of convergence of the perturbation series. From the measured $t\bar{t}$ cross section at the Tevatron [13] we derive a $\overline{\text{MS}}$ mass $\overline{m} = 160.0^{+3.3}_{-3.2}$ GeV, which corresponds to a pole mass of

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we arrive at the corresponding discussion. For the CTEQ6.6 PDF set and the LHC, the upper bound is shifted to larger values by a few percent. See also Fig. 1 and the LHC, the cross section is not a monotonically decreasing function if \( t \) for with \( x \) values of the total cross section as a function of \( m_t \) of the total cross section together with their error bands. This demonstrates the shrinking of the total error for the NNLO cross section. Having discussed scale uncertainty and PDF error, we present our prediction for the cross section at the Tevatron and the LHC. To obtain a more conservative error bound, we calculate the contribution of the scale uncertainty as

\[
\min_{\mu_r, \mu_f \in [m_t/2,2m_t]} \sigma(\mu_r, \mu_f) \leq \sigma(\mu_r, \mu_f) \leq \max_{\mu_r, \mu_f \in [m_t/2,2m_t]} \sigma(\mu_r, \mu_f). \tag{3}
\]

For \( t \bar{t} \) production at the Tevatron, this definition changes nothing, but for \( t \bar{t} \) production at the LHC, the upper bound is shifted to larger values by a few per cent. See also Fig. 1 and the corresponding discussion. For the CTEQ6.6 PDF set and \( m_t = 173 \text{ GeV} \) (pole mass), we arrive at

\[
\sigma(pp \rightarrow t \bar{t}) = 7.34^{+0.23}_{-0.38} \text{ pb @ Tevatron},
\]

\[
\sigma(pp \rightarrow t \bar{t}) = 874^{+14}_{-13} \text{ pb @ LHC 14 TeV}.
\]

For the LHC, we have calculated the total hadronic cross section as a function of the center of mass energy \( E \) for a value of \( m_t = 172.5 \text{ GeV} \) as used in ATLAS studies, see Fig. 6. We parametrize the result using the ansatz

\[
\sigma(E, \mu) = a + bx + cx^2 + dx \log \left( \frac{E}{\sqrt{s}} \right) + ex \log^2 \left( \frac{E}{\sqrt{s}} \right) + fx^2 \log \left( \frac{E}{\sqrt{s}} \right) + gx^2 \log^2 \left( \frac{E}{\sqrt{s}} \right) \tag{4}
\]

with \( x = E/\text{GeV} \) and \( \sqrt{s} = 14 \text{ TeV} \). The numerical values for the coefficients \( a, \ldots, g \) can be found in Tab. 4 for \( \mu = m_t, m_t/2, \) and \( 2m_t \). The fit is valid for \( 3 \text{ TeV} \leq E \leq 14 \text{ TeV} \) and has an accuracy of better than 0.05% within this range. This ansatz is justified by general limits for cross sections at high energies and is consistent with unitarity. Parametrisations of the total cross section as a function of \( m_t \) can be found for various scenarios in Ref. 14.

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Table 1: Numerical values of the coefficients (in pb) of Eq. 4 for $m_t = 172.5$ GeV and the PDF set CTEQ6.6.

| $a \times 10$ | $b \times 10^{-2}$ | $c \times 10^{-5}$ | $d \times 10^{-4}$ | $e \times 10^{-2}$ | $f \times 10^{-5}$ | $g \times 10^{-6}$ |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 3.42553       | -5.12699            | 4.09683             | -2.46892            | -3.93892            | -1.75175            | 2.02029             |
| 3.20912       | -4.85885            | 3.90541             | -2.33781            | -3.71957            | -1.65930            | 1.88132             |
| 3.31748       | -4.76706            | 3.82310             | -2.31392            | -3.73054            | -1.60468            | 1.74661             |

Figure 1: Contour lines of the total hadronic cross section from the independent variation of renormalization and factorization scale $\mu_r$ and $\mu_f$ for the Tevatron with $\sqrt{s} = 1.96$ TeV (left) and the LHC with $\sqrt{s} = 14$ TeV (right) with CTEQ6.6. The range of $\mu_r$ and $\mu_f$ corresponds to $\mu_r, \mu_f \in [m_t/2, 2m_t]$.

2 Predictions for $t'\bar{t}'$ Production at Tevatron and LHC

We briefly present theoretical predictions for the pair production cross section of a hypothetical heavy fourth generation quark $t'$ at the Tevatron and the LHC. In this calculation we have set the number of light flavours to $n_f = 6$. As one can see in Figs. [7] and [8] the cross section decreases very rapidly with increasing $t'$ mass. At the Tevatron, we predict for a 200 GeV $t'$ quark a cross section of $\sigma(pp \rightarrow t'\bar{t}') = 3.3 \pm 0.3$ pb and for $m_{t'} = 500$ GeV, we predict $\sigma(pp \rightarrow t'\bar{t}') = 1.3^{+0.2}_{-0.4}$ pb. Scale uncertainty and PDF error contribute roughly equal parts to the total error. At the LHC, we can test higher $m_{t'}$ masses. We predict for $m_{t'} = 500$ GeV a cross section of $\sigma(pp \rightarrow t'\bar{t}') = 4.0^{+0.5}_{-0.4}$ pb and for $m_{t'} = 2000$ GeV, we have $\sigma(pp \rightarrow t'\bar{t}') = 0.27^{+0.8}_{-0.9}$ pb. At the LHC, the PDF error is much larger than the scale uncertainty. Most $t'\bar{t}'$ pairs are produced via the $gg$ channel, the PDF error of the gluon PDF is large in the relevant kinematic region, i.e. at high $x$. The hadronic cross sections for $t'\bar{t}'$ production including the total error bands are presented in Fig. [7] for the Tevatron and in Fig. [8] for the LHC.

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Figure 2: NNLO $t\bar{t}$ production cross section at the Tevatron. The blue band indicates the scale uncertainty.

Figure 3: NNLO $t\bar{t}$ production cross section at the LHC. The blue band indicates the scale uncertainty.

Figure 4: Combined scale uncertainty and PDF error for $t\bar{t}$ production at NLO (blue band) and NNLO (red band) at the Tevatron.

Figure 5: Combined scale uncertainty and PDF error for $t\bar{t}$ production at NLO (blue band) and NNLO (red band) at the LHC.

Figure 6: $t\bar{t}$ production at the LHC as a function of the center of mass energy $E$ for $m_t = 172.5$ GeV and for three different scales $\mu = 1/2 m_t, m_t, 2 m_t$, see Eq. (4) and Tab. 1 for the parametrisation.
Figure 7: Pair production cross section for a hypothetical heavy fourth generation quark at the Tevatron. The violet band indicates the combined scale uncertainty and PDF error.

Figure 8: Pair production cross section for a hypothetical heavy fourth generation quark at the LHC. The violet band indicates the combined scale uncertainty and PDF error.

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