Whispering gallery mode enhanced optical force with resonant tunneling excitation in the Kretschmann geometry

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The boundary element method is applied to investigate the optical forces when whispering gallery modes (WGMs) are excited by a totally internally reflected wave. Such evanescent wave is particularly effective in exciting the high-$Q$ WGM, while the low angular or high radial order modes are suppressed relatively. This results in a large contrast between the forces on and off resonance, and thus allows for high size-selectivity. We fully incorporate the prism-particle interaction and found that the optical force behaves differently at different separations. Optimal separation is found which corresponds to a compromise between intensity and $Q$ factor.

Microcavities with whispering gallery mode (WGM) are important components in nanophotonics, nonlinear optics, quantum optics, and many other areas. As it is very difficult to accurately control the resonance frequency or the size of a micro-cavity during its fabrication, a convenient way to pick up the particles with the desired resonance frequencies or size is highly desirable. Recently, it was proposed in Ref. 2 that the optical force acting on a microsphere under an evanescent wave illumination is highly size-selective: at resonance the microsphere experiences a sizable force, whereas at off resonance the force is negligible. Utilizing this property and the high quality factor ($Q > 10^7$) of WGM, the size-selective force would allow for accurate size-sorting of microsphere cavities, with an accuracy of $\sim 1/Q$. However, the generation of evanescent wave requires a substrate. Ref. 2 considers only the regime where the microsphere is away from the near field of the substrate, and the interaction of the microsphere and substrate is only treated in a phenomenological manner.

In this article, using the boundary element method (BEM) and the Maxwell stress tensor, we intend to go beyond Ref. 2 by fully incorporating the influence of the substrate. We confine ourselves to 2D calculations with a transverse electric (TE) polarization, since 3D calculation is difficult due to the need of extremely fine meshes in the BEM calculation. We expect that our result should be qualitatively relevant for the 3D system. We consider the Kretschmann geometry as depicted in Fig. 1(f): a microcylinder (MC) of radius $a$ sits on top of a prism, both have a refractive index of $n_p = 1.5$. The combined system (in air) is then illuminated by an incident plane wave propagating at $\theta = 45^\circ$. As the critical angle of the prism is $\theta_c = 41.8^\circ$, the incident wave is totally internally reflected at the prism surface, and an evanescent wave is generated there. The transfer of photon momentum to the MC is allowed by the scattering or tunneling of the evanescent wave. We shall see that when the particle is not in the near field of the prism surface, the treatment of Ref. 2 gives a correct description of the situation. For the near field case, we observe a series of interesting phenomena, some are caused by the interplay between the increasing intensity and the decreasing $Q$ as the particle approaches the surface, and some are caused by the interaction between the particle and the substrate. The research on WGM-induced forces can be dated back to the early eighties when Ashkin reported the first observation of the force in air. Later, the WGM-induced forces were also observed by Fontes et al in water. Nevertheless, only a modest enhancement in the optical force has been achieved owing to the use of propagating waves in these works. Meanwhile, it is well-known that the Kretschmann geometry can efficiently excite the evanescent wave, and we observe that the force on the microsphere is large enough to form a resonance, where the optical force reaches a maximum.

![FIG. 1: (Color online) Optical forces at different height from the prism as functions of the wavelength. (Blue) dotted line: a microcylinder illuminated by a plane wave in the absence of the prism. (Black) solid line: the horizontal force $F_x$ acting on a microcylinder with the prism. (Red) dashed line: the vertical force $F_y$ with the prism. (a) $d = 1.1a$, (b) $d = 1.2a$, (c) $d = 1.3a$. Inset of (c): an enlarged section of (c). (d) $d = 1.1a$ and (e) $d = 1.2a$ for shorter wavelength. The $Q_{iso}$ and resonant wavelength for the typical modes in (d) and (e) are, respectively, 5399 and 0.31110a for TE$_{26}$, 7632 and 0.30047a for TE$_{27}$, 10817 and 0.29055a for TE$_{28}$, 15368 and 0.28128a for TE$_{29}$, 96 and 0.31044a for TE$_{32}$, 115 and 0.29092a for TE$_{33}$, 139 and 0.28892a for TE$_{34}$, and 170 and 0.27926a for TE$_{35}$. (f) Illustration of the Kretschmann geometry, where the evanescent wave is generated by total internal reflection on the top of the prism.]

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WGM of a high-\(Q\) microsphere. Kawata and Sugiura\(^3\) demonstrated versatile optical manipulation by evanescent waves subsequently.\(^{21}\) Although the Kretschmann geometry is well-known to be suitable for both optical manipulation and the excitation of WGM, the resonant forces associated with evanescent wave excited WGM have never been considered experimentally. Here, we hope to provide a more complete treatment for the WGM induced forces, and hope our work can stimulate experimental works in this area.

In Figs. 1(a)–1(c), we show the optical forces acting on a MC for \(\lambda = 0.32a - 0.4a\) (size parameter \(k_p a \approx 24 - 30\)). The (blue) dotted line represents both the horizontal and vertical forces (equal by symmetry) acting on a bare MC without the prism.\(^{32}\) The (black) solid line represents the horizontal force \(F_x\) and the (red) dashed line represents the vertical force \(F_y\), in the presence of the prism. For the case without the prism, the plane incident wave hits directly on the MC, and for the case with the prism, the plane wave is partially reflected by the prism surfaces and partially converted into an evanescent wave on the prism’s top surface. In the case of a bare MC, it can be seen from Fig. 1(a) that the force at resonance is comparable to the force at off resonance; thus size-selectivity is weak. On the other hand, the case with an incident evanescent wave (generated by the prism) is very size-selective, as we shall see below.

Let us first consider the horizontal force \(F_x\). As shown in Fig. 1(c) where the separation \(d = 1.3a\), it is quite clear that the force on first radial order resonance is significantly stronger than the force at off resonance, and force on the second order resonances can hardly be seen at all. This results in a high peak-to-baseline ratio, defined as the peak resonance force divided by the maximum of off resonance or lower-\(Q\) resonance force. This would imply the possibility of size-selective manipulation: upon illumination, only those particles whose sizes happen to be in resonance with the incident wave will experience a sizable optical force, whereas the off resonance particle will be left untouched. In the limit of large separation, the peak-to-baseline ratio can be several orders of magnitude higher than what is predicted here, as found out in Ref. 2. Indeed, the results presented in Fig. 1(c) already begin to resemble that of Ref. 2.

Although it is highly desirable to have a high peak-to-baseline ratio, it is also important to achieve large force. This case is illustrated in Figs. 1(a) and 1(b). As the separation decreases from \(d = 1.3a\) in Fig. 1(c) to \(d = 1.2a\) in Fig. 1(b), the force on first order resonance increases by a factor of \(\sim 2–3\) as the intensity becomes higher in the near field, while its linewidth is broadened slightly. At \(d = 1.2a\), the second order resonance can now be observed, due to the higher intensity. Although the peak-to-baseline ratio for \(d = 1.2a\) is smaller than that of \(d = 1.3a\), the size selectivity is still very strong. At an even closer separation \(d = 1.1a\) in Fig. 1(a), although the intensity of the evanescent wave increases, the strength of the first order resonance force remains at the same level as when \(d = 1.2a\). This is a consequence of the Lorentzian lineshape,\(^5\) which implies that the resonance force is inversely proportional to the linewidth, or directly proportional to its \(Q\). As \(d\) decreases, the stronger modal coupling between the prism and the MC broadens the linewidth (or decrease in \(Q\)), and causes the resonance force to decrease, compensating the increase in the local intensity \(I_{\text{loc}}\). The background force and the forces associated with the second order resonance have increased considerably, making the peak-to-baseline ratio small. We note that as the MC approaches the prism, the resonance frequencies do not shift and it is the \(Q\)’s that drops. This decrease in \(Q\) will significantly reduce the size-sensitivity of the resonance force, which scales like \(\sim 1/Q\).

A more explicit demonstration on the selectivity due to high-\(Q\) is shown in Fig. 1(d) and 1(e) which show the optical forces with the presence of the prism in higher size parameter regime \(\lambda = 0.28a - 0.32a\) (\(k_p a \approx 30 - 34\)). It is seen in Fig. 1(d) that with \(d = 1.1a\) several radially second order WGMs can be well excited, resulting in comparable optical forces of the radially first order WGMs as indicated by the downward arrows. We find that the \(Q\) value of the TE\(_{33}\) mode exceeds \(10^2\) (see the details in the caption of Fig. 1). The MC with such a value of \(Q\) can already collect sufficient incident field energy that yields a non-negligible amount of light scattering. Figure 1(e) shows that for increased separation to \(d = 1.2a\), these second order modes remain not well excited again, simply because of the decreased local field. At this separation, however, the radially first order WGMs with \(Q\) in the order of \(10^3\) and \(10^4\) can be well excited and are manifested as the several sharp peaks [as labeled in Fig. 1(e)].

Figure 2 clarifies the separation dependence of the optical force on and off resonance. Here we have chosen three different frequencies, one for TE\(_{22}\) (with quality factor of isolated particle \(Q_{\text{iso}} = 1395\))\(^{10}\) shown in Fig. 2(a), one for TE\(_{18}\) (\(Q_{\text{iso}} = 49\))\(^{10}\) shown in Fig. 2(c), and the last one for an off resonance frequency shown in Fig. 2(b). We remark that the strength of the resonance force depends on both the local intensity \(I_{\text{loc}}\) and the mode’s \(Q\), i.e. \(F \propto QI_{\text{loc}}\). The observed \(Q\) in the force spectrum is in fact related to the original quality factor for the isolated MC, \(Q_{\text{iso}}\), and the coupling \(Q\)-factor, \(Q_{\text{comp}}\), through the equation \(1/Q = 1/Q_{\text{iso}} + 1/Q_{\text{comp}}\). In Fig. 2(b) and 2(c), the optical force decreases monotonically as \(d\) increases. The off resonance case of Fig. 2(b) is straightforward to understand: as \(d\) increases, the local intensity decreases, so the force decreases as well. For the second order resonance TE\(_{18}\) case plotted in Fig. 2(c), the system’s \(Q\) is completely determined by its
increases, the total can draw a conclusion that a high value is the prerequisite for the resonant tunneling. Therefore in the scheme with the prism for optical sorting operated at a specific frequency, the high peak-to-baseline ratio, but if the MC is too far away, the force is too weak. Finally, we remark that the induced optical force can further be enhanced by coating the substrate with a metallic coating that supports surface plasmon, or by coating the prism with a dielectric cavity layer.

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