Research of influence of errors of cam manufacturing on the law of motion of output element

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Abstract. Insignificant error in the moment of manufacturing the mechanism can influence significantly on its' work. Research of influence of manufacturing error with manufactured mechanism usually is irrational, that's why necessity of evaluation at the stage of design occurs. Methods of evaluation of this influence are investigated in this paper. The research is implemented by developed algorithm, which also was realized in MATLAB and Python as well. Due this fact process of math simulation doesn’t occupy a lot of time and doesn’t need large calculating abilities. We research same data by independent ways by using 2 different programs and we compare results in order to check and improve model. Also, it’s interesting that technology of computer vision was applied in this work. So, presented paper will be interesting for everybody who practice design of cam mechanisms or research them.

1. Introduction

The cam mechanism is the highest kinematic pair, the input detail of which is a rotating cam and the output detail is a follower moving reciprocating (see figure 1). Currently, such mechanisms are widely used for automation purposes, a striking example is the wide range of cam mechanisms for internal combustion engines [1–3].

Usually, when we design a cam mechanism, it is necessary to solve the so-called profiling problem, which consists in finding the law of changing the radius of the cam from the angle of rotation. At the same time, the law of the follower motion (the dependence of acceleration on the angle of rotation of the cam) and the stroke of the follower, which are selected in accordance with the technological process, are set as the initial data.

Since there is a need to ensure the required law of motion in certain tolerances (in connection with ensuring the normal mode of operation of the mechanism and increasing its service life), there the question arises: what is the effect of the manufacturing error of the cam on the law of movement of the follower? Of course, the effect of cam profile distortion on the dynamic characteristics can be obtained by measurement, but this method is not suitable for use at the design stage. Thus, it becomes necessary to predict the effect of a random value of the manufacturing error on the output parameters of the mechanism. This article is dedicated to this issue.
In order to solve the problem, it is necessary to restore the law of movement of the follower, which is one of the main input parameters of the model used, using the existing cam shape. For this, an algorithm was developed that was implemented in the MATLAB and Python environments using the open source libraries NumPy, Matplotlib, and OpenCV.

2. **The main provisions of the model**

The model is based on relations (1–2) describing the geometry of the cam and the acceleration of its output link:

\[ S(\varphi) = \rho(\varphi) - r_0 \]  

(1)

\[ a(\varphi) = \frac{d^2 S(\varphi)}{d\varphi^2} \]  

(2)

where \( \varphi \) – cam angle; \( S(\varphi) \) – dependence of the follower movement on the angle of rotation of the cam; \( \rho(\varphi) \) – the law of changing the radius of the cam from the angle of rotation; \( r_0 \) – minimum cam radius corresponding to zero movement; \( a(\varphi) \) – dependence of the acceleration of the follower on the angle of rotation of the cam [5–8].

It should be noted that MATLAB and Python implement numerical differentiation, which greatly simplifies data processing, but at the same time leads to the appearance and accumulation of errors, which should be taken into account at the time of analyzing the results.

As suggested in [9-14, 18-21], a harmonic function was taken as a function describing the effect of the error on the cam profile (3):

\[ \chi = \chi_{max} \sin(k\varphi) + \chi_0, \]  

(3)

where \( \chi_{max} \) – tolerance value; \( k \) – parameter, characterizing the variation of the deviation function; \( \chi_0 \) – tolerance offset relative to nominal size.

Thus, the real cam profile \( \tilde{\rho}(\varphi) \) will be described as follows (4):

\[ \tilde{\rho}(\varphi) = \rho(\varphi) + \chi(\varphi) \]  

(4)

It should be noted that the described processing is carried out in a polar coordinate system. This approach allows you to remove the differences between the angle of rotation of the cam and the angular coordinate, which simplifies the implementation of the algorithm.
3. Obtaining a fist profile and the push movement law
The cam shown in the photography, which was taken as the object of study (figure 3).

![Figure 2. Testing cam.](image1)

![Figure 3. Cam shot profile.](image2)

Using a library of algorithms for computer vision and open-source image processing OpenCV v.4.1.2, a profile was obtained (figure 3) shown in figure 2 cams (212 points) [15, 16]. Points were drawn on the edge of the cam, a central point was set, and a segment of characteristic dimensions was indicated.

![Figure 4. The dependence of the movement of the follower on the angle of rotation of the cam.](image3)

Then, the dependence of the displacement of the follower on the angle of rotation of the cam (figure 4) was obtained using the above formula (2). The profile removal technique and processing will be described in more details in subsequent works.

4. Data processing
Since there are errors in profile removal associated with the applied technique, it was decided to approximate the law of motion.

Due to the fact that acceleration depends on displacement as a second derivative, relatively small fluctuations in the displacement function will introduce large distortions into the form of the acceleration
function. This distortion is especially pronounced in numerical differentiation. In this regard, approximation is necessary not so much from the point of view of increasing the accuracy of the results, but from the point of view of the need to obtain a qualitatively correct picture of the law of displacement adequate to the real one. Turn to the description of the approximation procedure.

Figure 4 shows that the graph has 2 inflection points, so the vector of values was divided into 3 sections and each of them was approximated by a second-order polynomial. The approximation obtained is shown in figure 5.

A smoothed cam profile was constructed using the approximated displacement vector, and further manipulations were carried out with it.

5. Getting a non-ideal profile
As noted above, in order to take into account the manufacturing error of the cam, it is necessary to add a harmonic function to the law of changing its radius.

In figure 6 were shown the simulated (with the production error) and ideal cam profiles, the amplitude of the deviation of the profile from the ideal was deliberately increased to display a quality picture by 30 times. Since 7 accuracy standards were chosen for modeling, the production error cannot have a maximum deviation from the nominal value of more than 35 microns. Therefore, without this additional increase introduced, the difference between the real and the ideal profile can be indistinguishable to the naked eye. Small differences in the geometry of the constructed profiles may seem deceptively insignificant for the operation of the mechanism.

Figure 5. Approximation of the obtained displacement.

Figure 6. Real and ideal cam profiles.
6. The real law of motion
After modeling the non-ideal cam profile, one can obtain the dependence of the accelerator follower on the angle of rotation of the cam and evaluate its deviation from the ideal (assumed during design). According to the above dependences (1-2), we have the picture shown in figure 7.

As can be seen from the figure, in the manufacture of the cam according to the 7th qualification [18], deviations of the law of acceleration of the follower are comparable with nominal accelerations. This can lead to unwanted beats, vibrations of the output link, etc. As a result of additional cyclic loads, the service life of the mechanism will decrease. Additional friction will lead to early wear of parts and improper operation of the mechanism, there will be a decrease in the efficiency of the machine as a whole.

The engineer will have to decide, based on the design goals and technical specifications for the design, whether such deviations are permissible, and adjust the tolerance for the manufacture of the cam and the method of its production.

Figure 8 shows the results of modeling the acceleration of the follower for a non-ideal cam profile with parameter k = 10, different from the previous example. The graph shows that despite the preservation of the previous tolerance on size, the deviation of the law of movement of the follower from the ideal increased. Thus, a separate task may be to study the influence of the parameter k on the dynamic characteristics of the mechanism.
7. Conclusion

Based on the results of this work, several conclusions can be drawn. Slight deviations of the cam shape from the ideal can lead to significant deviations of the movement’s law of the follower, which in turn reduces the service life of the mechanism, violates its operating modes, and in some cases leads to the inability to use the designed cam in the required process [22].

It is necessary to check the law of motion of the output link, according to the method described in the work. Based on this check, it is necessary to correct the manufacturing method of the cam and its tolerances.

The use of software packages greatly simplifies the task of data processing, but it is necessary to remember and take into account all the features of the calculations by the program. Since these features can affect the final result, its accuracy and general correctness [23].

In the future, it is planned to increase the accuracy of this mathematical model, for example, by using a different scheme for numerical differentiation, as well as combine and partially automate the procedures for removing the cam profile and its further processing, and consider other ways to obtain the cam profile using computer vision libraries. It is also planned to separately study the influence of the parameter k, to consider more complex laws of motion, for example, similar ones [6, 18–19].

It is also possible to expand the developed methodology to more complex mechanisms, which include a cam pair. Such an extension will require complication of the mathematical model, up to taking into account the influence of the rigidity of the links of the mechanism on its operation.

In addition, it is possible to add more advanced algorithms for analyzing the obtained image using a computer vision library and to take characteristics of cam mechanisms from the downloaded video file. Thus, it will be possible to get away from removing the cam shape, and immediately remove the movement of the follower directly.

In conclusion, we would also like to note that the described profile removal technique can be useful in studying the cam wear and the effect of this wear on the law of movement of the output link.

References

[1] Leonov I V 20015 Computer-aided design and analysis of cam gears Proceedings of Higher Educational Institutions. Machine Building 2 29–35
[2] Aleksandrov A A 2013 Internal combustion engines (Moscow: Machine Building)
[3] Vidal P 1985 Aide memoire d’automatique (Paris: Dunod)
[4] https://multiurok.ru/files/proekt-robototekhnika-kulachkovaia-peredacha
[5] Timofeev G A 2014 The theory of mechanisms and machines (Moscow: PublishingURAIT)
[6] Timofeev G A 2010 The theory of mechanisms and machines. Coursework design (Moscow: Publishing of Bauman Moscow State Technical University)
[7] Leonov I V 2009 The theory of mechanisms and machines (Moscow: High education)
[8] Levitskaya O N 1985 Course of the theory of mechanisms and machines (Moscow: High school)
[9] Telegin V V 2015 Research into impact of accuracy of cam manufacturing on mechanism dynamics of automatic machines. Modern science technologies 12 263–7
[10] Telegin V V 2006 Dynamic of mechanism of multipositional cold pressing machines (Lipetsk: Lipetsk State Technical University)
[11] Telegin V V 2013 Dynamic analysis of mechanism cutting for automaton cold pressing. Fundamental research 11 899–904
[12] Telegin V V 2012 Technology of digital prototypes in problems of researches the forge-press machines dynamics. Izvestia of Samara Scientific Center of the Russian Academy of Sciences 14 1306–9
[13] Telegin V V 2014 Construction of simulation models in tasks the research dynamics of mechanical systems. Fundamental research 12 2125–30
[14] Telegin I V 2013 Computer simulation of dynamic processes in hot-crane pres. Fundamental research 10 3414–8
[15] Barbashov N N 2012 Improving the characteristics of the flywheeled energy batteries of the drive
lifting-transporting machines (Moscow)

[16] OpenCV-Python Tutorials’s documentation Retrieved from https://opencv-python-tutorials.readthedocs.io/en/

[17] Baryshnikova O O, Leonov I V, Kuzenkov V V 2004 Using of MathCAD system for coursework design and homework on the theory of mechanisms and machines (Moscow: Publishing of Bauman Moscow State Technical University)

[18] Mezhennaya N M 2018 Testing of embedding with margin for discrete random sequences. Prikladnaya Diskretnaya Matematika 12–4

[19] Vlasova E A, Mezhennaya N M, Popov V S, Pugachev O V 2017 Methodological aspects of the discipline «Probability theory» in a technical university. Modern High Technologies 96–103

[20] Mezhennaya N M 2019 On the number of ones in the cycle of multicyclin sequence determined by boolean function. Siberian Electronic Mathematical Reports-sibirskie Elektronnye Matematicheskie Izvestiya 229–35

[21] Voronov S A, Veidun M A 2017 Mathematical modeling of the cylindrical grinding process. Journal of machinery manufacture and reliability 394–403

[22] Sadykhov G S, Babaev I A 2016 Computations of the least number of objects necessary for the cyclical reliability testing. Journal of Machinery Manufacture and Reliability 239–46

[23] Vlasova E A, Mezhennaya N M, Popov V S, Pugachev O V 2017 The use of mathematical packages in the framework of methodological support of probabilistic disciplines in a technical university. Bulletin of the Moscow State Regional University (Physics and Mathematics) 114–28