Interacting hadron resonance gas model in magnetic field and the fluctuations of conserved charges

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In this paper we discuss the interacting hadron resonance gas model in presence of constant external magnetic field. The short range repulsive interaction between hadrons are accounted through van der Waals excluded volume correction to the ideal gas pressure. We assume uniform hardcore radius $r_h = 0.3$ fm for all the hadrons. We analyse the effect of uniform background magnetic field on the thermodynamic properties of interacting hadron gas. We especially discuss the effect of interactions on the behaviour of magnetization of low temperature hadronic matter. The vacuum terms have been regularized using magnetic field independent regularization scheme. We find that the magnetization of hadronic matter is positive which implies that the low temperature hadronic matter is diamagnetic. We further find that the repulsive interactions have very negligible effect on the overall magnetization of the hadronic matter and the diamagnetic property of the hadronic phase remains unchanged. We have also investigated the effects of short range repulsive interactions as well as the magnetic field on the baryon and charge number susceptibilities of hadronic matter within ambit of excluded volume hadron resonance gas model.

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I. INTRODUCTION

The phase diagram of strongly interacting matter described by quantum chromodynamics is one of the active topic of research today. At low temperature ($T$) and low baryon chemical potential ($\mu$) strongly interacting matter consist of colorless hadrons, while at high temperature and/or high baryon density the fundamental degrees of freedom are colored quarks and gluons. At low baryon density and high temperature this matter is expected to undergo phase transition from hadronic phase to quark gluon plasma phase (QGP). Lattice QCD simulation results show that this temperature driven phase transition is actually an analytic cross over\[1–6]\). At low temperature and high baryon density hadron-QGP phase transition is of first order as shown in many QCD effective model calculations\[7–15]\, although the first principle LQCD calculations are not available at high baryon density since the Euclidean formulation of the theory suffers from the sign problem\[16–18]\). If hadron-QGP phase transition is indeed first order at low temperature and high baryon density then we expect that the first order phase transition line should end at a critical point (CEP) as one moves towards high temperature and low density regime where the analytic crossover phase transition starts. Locating this QCD critical point is one of the hot topic of research in the experimental and theoretical high energy physics(see Ref.\[19\] for the recent review on the status of QCD critical point.).

Relativistic heavy-ion collision (HIC) experiments provides a unique opportunity to empirically study the QCD phase diagram over wide range of $T$ and $\mu$ values. The evolution of the matter created in HIC experiments can be simulated using equations of relativistic hydrodynamics\[20–23]\. In these hydrodynamic simulations the equation of state (EoS) as a function of control parameters $T$ and $\mu$ is the necessary input. In the off-central HICs huge magnetic field, $B \sim m_\pi^2 (\sim 10^{18}$G) are created due to relativistic motion of charged particles\[24–26]\. This additional parameter $B$ can affect the equation of state and hence it can have significant impact on the overall structure of the phase diagram. For instance, magnetic field may induce interesting phenomena on QCD matter, \textit{viz.} chiral magnetic effect\[27]\, magnetic catalysis\[28]\ and inverse magnetic catalysis\[29, 30]\ effect etc. Further, strong magnetic fields are also expected to be present in dense neutron stars\[31\] and they may be present during the electroweak transition in the early universe\[32, 33\]. Thus it is of utmost important to study the effect of magnetic field on the QCD EoS.

Since the first principle lattice QCD calculations has limited applicability at finite baryon density one has to resort to effective models, \textit{viz.} Nambu-Jona-Lasinio model\[72, 73\], quark-meson-coupling model\[36\], etc. The simplest effective model describing the hadronic phase of the strongly interacting matter is the hadron resonance gas model (HRG). This model is based on so called Dashen-Ma-Bernstein (DMB) theorem\[37\]. It can be shown that if the dynamics of thermodynamic system of hadrons is dominated by narrow-resonance formation then the resulting system essentially behaves like a noninteracting system of hadrons and resonances\[38–40]\. This ideal HRG model is successful in describing the hadron multiplicities produced in HICs\[41–49\]. On the other hand, it fails to account for the short range repulsive interactions between hadrons. In fact, it has been shown that the repulsive interactions modeled via excluded volume corrections to ideal HRG partition function can have significant effect on thermodynamic observables, especially higher order fluctuations\[50–53]\ as well as in the context of statistical hadronization\[54\]. Thus, it is necessary to include the repulsive interactions in the ideal HRG model if this model is to be used to understand the dynamics of hadronic matter in the context of HICs.
The study of correlations and fluctuations of conserved charges, *viz.* baryon number, strangeness and electric charge, has recently gained a lot of attention due to their reliability to understand the QCD phase transition. In fact, these quantities are very sensitive probes of deconfinement phase transition[55–63]. Moreover, near the critical end point (CEP) the fluctuations are supposed to diverge. The susceptibilities computed in QCD are related to the product of moments of the conserved charge distributions through the fluctuation-dissipation theorem. This theorem asserts that the measure of the intrinsic statistical fluctuations in a system close to thermodynamic equilibrium is provided by the corresponding susceptibilities. The importance point here is that moments of the conserved charges are experimentally measurable quantities. The first principle LQCD simulations has been performed to compute susceptibilities at zero chemical potential. The susceptibilities are found to rise rapidly around crossover region of the phase diagram. Recently the susceptibilities has been estimated within the ambit of hadron resonance gas model (HRG) and its extended version, namely excluded volume hadron resonance gas model (EHRG). While the Second order fluctuations and correlations estimated within ideal HRG model seem to agree reasonably well with the lattice data, higher order fluctuations show deviations close to the transition temperature, $T_c$. It has been argued that the breakdown of ideal HRG model near $T_c$ is the reflection of the fact that the hadrons melts quickly above $T_c$. However, recently it is shown that by including the van der Waals interactions between hadrons the higher order susceptibilities are also in agreement with lattice QCD data. This study has concluded that the van der waals interactions play very important role in describing the hadronic phase of QCD even near the phase transition region.

In this work our purpose is to analyse the effect of magnetic field on the EoS as well as conserved charge fluctuations of hot and dense hadronic matter using both HRG and EHRG models. The presence of magnetic field not only affect the EoS[64] but also the fluctuations and correlations[65]. In the present study we have investigated the effect of magnetic field in presence of repulsive interaction on the EoS as well as the susceptibilities of hadronic matter for the first time.

We organize the paper as follows. In Sec. II we recapitulate the thermodynamics of hadron resonance gas model in magnetic field. In Sec. III we derive the renormalized vacuum pressure using magnetic field independent regularization (MFIR) scheme. In Sec. IV we briefly discuss the extension of ideal HRG model to include repulsive interactions. In Sec. V we discuss the results and finally in Sec. VI we summarize and conclude.

II. HADRON RESONANCE GAS MODEL IN MAGNETIC FIELD

The free energy density in the presence of constant external magnetic field $B$ is written in terms of partition function as

$$F = -T \ln Z = F_{\text{vac}} + F_{\text{th}}$$

where $F_{\text{vac}}$ and $F_{\text{th}}$ are vacuum and thermal parts respectively. In the ideal HRG model the free energy of the system at low temperature and in the dilute gas approximation is approximated by the partition function of a gas of non-interacting hadrons and resonances with very narrow spectral width[38–40]. This result is based on the Dashen-Ma-Bernstein theorem.[37]. Thus, the free energy of non-interacting HRG model in presence of constant magnetic field is written as
\[
F_c = \pm \sum_i \sum_{s_z} \sum_{n=0}^{\infty} \frac{eB}{(2\pi)^2} \int dp_z \left( E_{i,c}(p_z, n, s_z) + T \ln(1 \pm e^{-(E_{i,c}-\mu_i)/T}) \right); \quad e_i \neq 0
\]  
\[
F_n = \pm \sum_i \sum_{s_z} \int \frac{d^3p}{(2\pi)^3} \left( E_{i,n} + T \log \left[ 1 \pm \exp \left( -\frac{(E_{i,n}-\mu)}{T} \right) \right] \right); \quad e_i = 0
\]  

Here \( e \) is the electric charge, \( E_{i,c/n} \) is the single-particle energy for charged/neutral particle, \( m_i \) is the mass, \( T \) is the temperature and \( \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q \) is the chemical potential. In the last expression, \( B_i, S_i, Q_i \), are respectively, the baryon number, strangeness and charge of the particle, \( \mu_i \)'s are corresponding chemical potentials. The upper and lower signs correspond to fermions and bosons respectively. We have incorporated the hadrons listed in the table 1.

For a constant magnetic field \( B \), the single particle energy levels for neutral and charged particles are respectively given by \([64]\)

\[
E_{i,n} = \sqrt{p_z^2 + m_i^2}
\]

\[
E_{i,c}(p_z, n, S_z) = \sqrt{p_z^2 + m_i^2 + 2e_iB(n + 1/2 - S_z)}
\]

where \( q_i \) is the charge of the particle, \( n \) is any positive integer corresponding to allowed Landau levels, \( s_z \) is the component of spin \( s \) in the direction of magnetic field. For a given \( s \), there are \( 2s+1 \) possible values of \( s_z \). The gyromagnetic ratios are taken as \( g_i = 2|e_i/e| \) \((e \) being elementary unit of electric charge) for all charged hadrons [67]. The pressure of ideal hadron resonance gas in magnetic field is \( P_{\text{id}}(B) = -F(B) \) and it can be written as

\[
P_{\text{id}}^c = \pm \sum_i \sum_{S_z} \sum_{n=0}^{\infty} \frac{eB}{(2\pi)^2} \int dp_z \left( E_{i,c}(p_z, n, S_z) + T \ln(1 \pm e^{-(E_{i,c}-\mu_i)/T}) \right); \quad e_i \neq 0
\]

\[
P_{\text{id}}^n = \pm \sum_i \sum_{S_z} \int \frac{d^3p}{(2\pi)^3} \left( E_{i,n} + T \log \left[ 1 \pm \exp \left( -\frac{(E_{i,n}-\mu)}{T} \right) \right] \right); \quad e_i = 0
\]

Note that while the thermal part of the pressure is naturally convergent in the UV limit the vacuum part is divergent and needs to be regularized.

### III. REGULARIZATION OF VACUUM PRESSURE

The vacuum pressure is divergent and it needs to be properly regularized first. It has been recently shown that the appropriate regularization scheme is necessary to avoid certain unphysical results. For instance, some studies have found the oscillations in the magnetization while others have found imaginary meson masses[68]. Both of these findings are unphysical and it can be attributed to an inappropriate regularization choices. These unphysical results arise especially in case of magnetic field dependent regularization schemes. Thus, it is utterly important to separate magnetic field dependent and independent parts are separated clearly through appropriate regularization scheme. Magnetic field independent regularization (MFIR) has recently been introduced to achieve this goal[69, 70]. We shall obtain the regularized vacuum pressure for spin \( 1/2 \) particles using MFIR method. Spin zero and spin one cases can be discussed in similar a manner(see Appendix B for renormalized vacuum pressure expressions).
The vacuum part of the pressure for a charged spin $\frac{1}{2}$ particle in magnetic field is

$$P_{\text{vac}}(S = 1/2, B) = \sum_{n=0}^{\infty} g_n \frac{eB}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} E_{p,n}(B)$$

where $g_n = 2 - \delta_{n0}$ is the degeneracy of $n$th Landau level. Now adding and subtracting lowest Landau level contribution (i.e. $n = 0$) from the above equation we get

$$P_{\text{vac}}(S = 1/2, B) = \sum_{n=0}^{\infty} 2 \frac{eB}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \left( E_{p,n}(B) - \frac{E_{p,0}(B)}{2} \right)$$

We regularize the divergence using dimensional regularization[71]. In $d - \epsilon$ dimension Eq. (9) can be written as

$$P_{\text{vac}}(S = 1/2, B) = \sum_{n=0}^{\infty} \frac{eB}{\pi} \mu^\epsilon \int_{-\infty}^{\infty} \frac{d^{1-\epsilon}p_z}{(2\pi)^{1-\epsilon}} \left( \sqrt{p_z^2 + m^2} - 2eBn - \sqrt{p_z^2 + m^2} \right)$$

where $\mu$ fixes the dimension of the above expression to one. The integration can be done using standard $d$–dimensional formula (see Appendix A). Integration of the first term in Eq.(10) gives

$$I_1 = \sum_{n=0}^{\infty} \frac{eB}{\pi} \mu^\epsilon \int_{-\infty}^{\infty} \frac{d^{1-\epsilon}p_z}{(2\pi)^{1-\epsilon}} (p_z^2 + m^2 - 2eBn) \frac{1}{\Gamma \left( -1 + \frac{\epsilon}{2} \right)} \Gamma \left( -1 + \frac{\epsilon}{2} \right) \zeta \left( -1 + \frac{\epsilon}{2}, x \right)$$

where $x = \frac{m^2}{2eB}$. The Landau infinite sum has been expressed in terms of Riemann-Hurwitz $\zeta$–function (see Eq. (A2)). Using the expansion of $\Gamma$-function (see Eq.(A4)) and the expansion of $\zeta$-function (see Eq.(A3)). Eq.(11) can be written as

$$I_1 = -\frac{(eB)^2}{2\pi^2} \left( -\frac{2}{\epsilon} + \gamma - 1 + \ln \left( \frac{2eB}{4\pi\mu^2} \right) \right) - \frac{1}{12} - \frac{x^2}{2} + \frac{x}{2} + \frac{\epsilon}{2} \zeta'(-1, x) + \mathcal{O}(\epsilon^2)$$

Integration of the second term in Eq.(10) can be simplified in similar manner. We obtain

$$I_2 = \sum_{n=0}^{\infty} \frac{eB}{\pi} \mu^\epsilon \int_{-\infty}^{\infty} \frac{d^{1-\epsilon}p_z}{(2\pi)^{1-\epsilon}} (p_z^2 + m^2)^{\frac{1}{2}}$$

$$= \frac{(eB)^2}{2\pi^2} \left( -\frac{x}{\epsilon} - \frac{1 - \gamma}{2} \right) + \frac{x^2}{2} \ln \left( \frac{2eB}{4\pi\mu^2} \right) + \frac{x}{2} \ln(x)$$

Thus the vacuum pressure in presence of magnetic field becomes

$$P_{\text{vac}}(S = 1/2, B) = \frac{(eB)^2}{2\pi^2} \left( \zeta'(-1, x) - \frac{2}{12\epsilon} - \frac{(1 - \gamma)}{12} - \frac{x^2}{\epsilon} - \frac{(1 - \gamma)}{2} \right) x^2$$

$$+ \frac{x}{2} \ln(x) + \frac{x^2}{2} \ln \left( \frac{2eB}{4\pi\mu^2} \right) + \frac{1}{12} \ln \left( \frac{2eB}{4\pi\mu^2} \right)$$

Above expression is still divergent. So we add and subtract $B = 0$ contribution from it. This zero field vacuum pressure in $d = 3 - \epsilon$ dimension is

$$P_{\text{vac}}(S = 1/2, B = 0) = 2\mu^\epsilon \int \frac{d^{3-\epsilon}p}{(2\pi)^{3-\epsilon}} \left( p^2 + m^2 \right)^{\frac{1}{2}}$$

$$= \frac{(eB)^2}{2\pi^2} \left( \frac{2eB}{4\pi\mu^2} \right)^{\frac{1}{2}} \Gamma \left( -2 + \frac{\epsilon}{2} \right) x^{2-\frac{\epsilon}{2}}$$

(15)
Above equation can be further simplified to

\[
P_{\text{vac}}(S = 1/2, B = 0) = -\frac{(eB)^2}{2\pi^2} x^2 \left( \frac{1}{\epsilon} + \frac{3}{4} - \frac{\gamma}{2} - \frac{1}{2} \ln \left( \frac{2eB}{4\pi\mu^2} \right) - \frac{1}{2} \ln(x) \right) \tag{16}
\]

where we have used the Γ-function expansion (see Eq. A5).

Adding and subtracting (16) from (14) we get the regularized pressure with vacuum part and magnetic field dependent part separated as

\[
P_{\text{vac}}(S = 1/2, B) = P_{\text{vac}}(1/2, B = 0) + \Delta P_{\text{vac}}(1/2, B) \tag{17}
\]

where

\[
\Delta P_{\text{vac}}(S = 1/2, B) = \frac{(eB)^2}{2\pi^2} \left( -\frac{2}{12\epsilon} + \frac{\gamma}{12} + \frac{1}{12} \ln \left( \frac{m^2}{4\pi\mu^2} \right) + \frac{x}{2} \ln(x) - \frac{x^2}{2} \ln(x) + \frac{x^2}{4} - \frac{1}{12} \ln(x) + \frac{1}{12} \right) \tag{18}
\]

The vacuum part can regularized using sharp cut-off \( \Lambda \) to obtain\cite{72, 73}

\[
P_{\text{vac}}(S = 1/2, B = 0) = -\frac{1}{8\pi^2} \left[ m^4 \ln \left( \frac{\Lambda + E_\Lambda}{m} \right) - E_\Lambda \Lambda (\Lambda^2 + E_\Lambda^2) \right] \tag{19}
\]

The field contribution given by (18) is still divergent due to presence of pure magnetic field dependent term \( \propto B^2 \). We cancel this divergence by redefining field dependent pressure contribution by including magnetic field contribution in it as

\[
\Delta P^r_{\text{vac}} = \Delta P_{\text{vac}}(B) - \frac{B^2}{2} \tag{20}
\]

The divergences are absorbed into the renormalization of the electric charge and magnetic field strength,

\[
B^2 = Z_e B^2_r; \quad e^2 = Z_e^{-1} e^2_r; \quad e_r B_r = e B \tag{21}
\]

where the electric charge renormalization constant is

\[
Z_e \left( S = \frac{1}{2} \right) = 1 + \frac{1}{2} e^2_r \left( -\frac{2}{12\epsilon} + \frac{\gamma}{12} + \frac{1}{12} \ln \left( \frac{M_*}{4\pi\mu^2} \right) \right) \tag{22}
\]

Here we fix \( M_* = m \), i.e to the physical mass of the particle. Thus the renormalized field dependent pressure (without pure magnetic field contribution) is

\[
\Delta P^r_{\text{vac}}(S = 1/2, B) = \frac{(eB)^2}{2\pi^2} \left( \zeta'(-1, x) + \frac{x^2}{2} \ln(x) - \frac{x^2}{2} \ln(x) + \frac{x^2}{4} - \frac{1}{12} \ln(x) + \frac{1}{12} \right) \tag{23}
\]

The renormalized \( B \) dependent pressure for spin zero and spin one can be obtain using similar method. These terms play very crucial role in determining the magnetization of the hadronic matter below \( T_c \).
IV. INTERACTING HADRON RESONANCE GAS MODEL IN MAGNETIC FIELD

The hadron resonance gas model defined by Eqs. (6) and (7) corresponds to non-interacting gas of hadrons. One can extend this ideal HRG model by taking into account the repulsive interactions between hadrons via Van der Waals (VDW) excluded-volume correction to the partition function. In the thermodynamically consistent excluded volume formulation one can obtain the transcendental equation for the pressure as [66]

\[ P^{EV}(T, \mu, B) = P^{id}(T, \tilde{\mu}, B), \]  

(24)

where \( \tilde{\mu} = \mu - vP^{EV}(T, \mu, B) \) is an effective chemical potential with \( v \) as the parameter corresponding to proper volume of the particle. For a particle of hard core radius \( r_h \), \( v = \frac{16}{3}\pi r_h^3 \). The excluded volume pressure can be obtained by solving Eq.(24) self consistently for given \( T, \mu \) and \( B \). The number density, energy density and entropy density, respectively can be written as [66]

\[ n^{EV}(T, \mu, B) = \frac{\sum_i n^{id}_i(T, \tilde{\mu}, B)}{1 + \sum_j v_j n^{id}_j(T, \tilde{\mu}, B)}, \]  

(25)

\[ \epsilon^{EV}(T, \mu, B) = \frac{\sum_i \epsilon^{id}_i(T, \tilde{\mu}, B)}{1 + \sum_j v_j n^{id}_j(T, \tilde{\mu}, B)}, \]  

(26)

\[ s^{EV}(T, \mu, B) = \frac{\sum_i s^{id}_i(T, \tilde{\mu}, B)}{1 + \sum_j v_j n^{id}_j(T, \tilde{\mu}, B)}. \]  

(27)

The factor \((1 + \sum_j v_j n_j(T, \tilde{\mu}, B))^{-1}\) suppresses the thermodynamical quantities at high temperature avoiding their singular behaviour which typically occurs in non-interacting HRG model. In this work we take uniform hardcore radius \( r_h \) and hence the proper volume factor \((v)\) for all the hadrons so that the suppression factor is \((1 + v \sum_j n_j(T, \tilde{\mu}, B))^{-1}\).

V. RESULTS AND DISCUSSION

Table I shows all the hadrons and resonances included in the HRG description. For the stability reasons as discussed in Ref.[64] we include only those hadrons having spin \( S < \frac{3}{2} \). We set hardcore radius of all the hadrons to \( r_h = 0.3 \) fm. This choice is based on the LQCD analysis of excluded volume HRG equation of state[77].

Fig.1(a) shows the variation of the vacuum pressure as a function of magnetic field. For spin-0, spin \( \frac{1}{2} \) and for spin-1 hadrons the vacuum pressure is positive for wide range of magnetic fields and hence we can safely assume that the HRG description is valid. Fig.1(b) shows the variation of pressure with temperature for HRG and EVHRG models both in presence and in absence of magnetic field. Note that the pressure in presence of magnetic field has magnetic field dependent vacuum contribution. Such contribution is zero if \( eB = 0 \). Hence the HRG pressure without magnetic field vanish at \( T = 0 \) GeV while it is non-zero at finite magnetic field. This is the reason we have plotted the pressure instead of scaled pressure \((\frac{P}{T^4})\). We further note that the pressure increases with temperature in all cases considered since the probability that a particular particle species populates is proportional to the Boltzmann factor \( e^{-m/T} \). It is necessary here to point out that, for HRG, the thermal
TABLE I. (Color Online) Hadrons and resonances included in the hadron resonance gas model. Particle data is taken from Ref.[78].

| hadron | m(GeV) | | | hadron | m(GeV) | | |
|--------|--------|---|---|--------|--------|---|---|
| \(\pi^+\) | 0.135  | 1 | 0 | 2 | \(p\) | 0.938  | 1 | 1/2 | 2 |
| \(\pi^0\) | 0.135  | 0 | 0 | 1 | \(n\) | 0.938  | 0 | 1/2 | 2 |
| \(K^\pm\) | 0.495  | 1 | 0 | 2 | \(\eta'\) | 0.958  | 0 | 0 | 1 |
| \(K^0\) | 0.495  | 0 | 0 | 2 | \(f_0\) | 0.980  | 0 | 0 | 1 |
| \(\eta\) | 0.548  | 0 | 0 | 1 | \(a_0\) | 0.980  | 0 | 1 | 1 |
| \(\rho^\pm\) | 0.776  | 1 | 1 | 2 | \(\phi\) | 1.020  | 0 | 1 | 1 |
| \(\rho\) | 0.776  | 0 | 1 | 1 | \(\Lambda\) | 1.116  | 0 | 1/2 | 1 |
| \(\omega\) | 0.782  | 0 | 1 | 1 | \(h_1\) | 1.170  | 0 | 1 | 1 |
| \(K^+_\pm\) | 0.892  | 1 | 1 | 2 | \(\Sigma^\pm\) | 1.189  | 1 | 1/2 | 2 |
| \(K^0\) | 0.892  | 0 | 1 | 2 | \(\Sigma^0\) | 1.189  | 0 | 1/2 | 1 |

FIG. 1. (Color Online) Left panel shows vacuum pressure of charged particles computed using MFIR scheme. Right panel shows the pressure as a function of temperature in presence magnetic field in HRG and EHRG.

part of the pressure in presence of magnetic field is smaller than that of \(eB = 0\) case although the pressure with vacuum part in former case appear to be greater than that of later. This can be accounted by comparing the dispersion relations in presence of magnetic field (Eq.(5)) and in the absence of magnetic field (Eq.(4)). The effective mass of the charged particle in presence of magnetic field is \(m_{eff}^2 = m^2 + 2eB(\frac{1}{2} - S)\). Thus the mass of spin-0 increases whereas those of spin-1 particles decrease if \(eB \neq 0\). For spin-\(\frac{1}{2}\) particles mass remains unchanged. Since the particles with spin-1 are heavier (lightest spin-1 particle \(\rho\) weighs 0.776 GeV which is \(\sim 6\) times the mass of pion), their contributions to pressure is much smaller compared to spin-0 particles and it only shows up at higher temperature. Since pions dominate the hadronic matter at low temperature, and since their effective mass is
higher in presence of magnetic field, the HRG pressure in presence of magnetic field is smaller than that of without magnetic field.

Fig.1(b) further shows the effect of repulsive interaction on the pressure. If we include the repulsive interactions through excluded volume correction to ideal gas partition function then the pressure is suppressed for both in $eB=0$ and $eB \neq 0$ cases. With finite size of hadrons the available free space for hadrons decreases with increasing temperature. This decreases the number density and hence the pressure of hadrons compared to free HRG case. From Fig.1 (b) it is seen that pressures for HRG and EVHRG models are almost identical up to $T \sim 0.1$ GeV for $eB=0$ case and up to $T \sim 0.12$ GeV for $eB=0.2$ GeV$^2$ case. Above these temperatures there is notable decrease in pressure for EVHRG case.

![Graph showing energy density and entropy density](image1)

**FIG. 2.** (Color Online) Energy density and entropy density in presence magnetic field estimated within ambit of HRG and EHRG.

Fig.2 shows effect of magnetic field on the energy density and entropy density estimated within ambit of HRG and EHRG. The effect of magnetic field as well as repulsive interactions on energy density is similar to that of pressure. The thermal part of the energy density in presence of magnetic field is smaller than that of $eB = 0$ case. The repulsive interactions further suppress the energy density. The effect of magnetic field as well as the repulsive interactions on the entropy density is quit interesting. In HRG, the entropy density rises rapidly as the temperature increases due to copious production of hadrons. As we have discussed above, the effective mass of spin-0 particles increases in presence of magnetic field. Thus their thermal production is accordingly suppressed by Boltzmann factor $e^{-m_{\text{eff}}/T}$ and the entropy production is suppressed in presence of magnetic field. The effect of repulsive interaction is also to suppress the entropy productions since the finite size of hadrons suppresses the number density at high temperature.

Fig.3(a) shows the behaviour of magnetization estimated within ambit of HRG and EHRG. The magnetization is positive indicating that the hadronic matter is paramagnetic. Fig.3(b) shows only thermal contribution to the magnetization. At very low temperature magnetization is practically zero due to absence charged hadrons. Probability that the particle species populates is proportional to the Boltzmann factor $e^{-m_{\text{eff}}/T}$. We note that the
pions, which are lightest hadronic species, are thermally excited at $T \sim 0.060$ GeV. Being scalar bosons their magnetization is negative (diamagnetic) and hence magnetization decreases with increase in temperature. It becomes positive only when lightest spin-1 particles $\rho$-mesons populates the hadronic matter and give positive contribution to hadronic matter. Thereafter, the magnetization rises rapidly as spin-$\frac{1}{2}$ also start to make positive contribution to the magnetization. It is interesting to note that even though the thermal part of magnetization becomes negative for certain temperature range the total magnetization including vacuum part is always positive. It turns out that the sign of magnetization of low temperature hadronic matter is a fundamental characteristic of thermal QCD vacuum.

The nth-order susceptibility is defined as
\[ \chi^n_x = \frac{1}{VT^3} \frac{\partial^n (\ln Z)}{\partial (\mu_T^n)^n} \]  

(28)

where \( \mu_x \) is the chemical potential for conserved charge \( x \). In this work we will take \( x \) to be baryon number \( B \) and electric charge \( Q \).

Fig. 4 shows baryon number susceptibilities estimated within HRG and EVHRG model in presence of magnetic field. In case of HRG model \( \chi^n_B \)s \( (n = 2, 4, 6) \) increases rapidly at high temperature. We note that the susceptibilities at a given temperature in presence of magnetic field is less than \( eB = 0 \) case. This is because the effect of magnetic field is to decrease the contribution to pressure from spin \( \frac{1}{2} \) charged particles as compared to the zero magnetic field case. (We haven’t considered any baryon with spin greater than \( \frac{1}{2} \)). At low \( T \), the dominating contribution to \( \chi_B \) comes from nucleons, namely protons and neutrons which carry baryon number \( 1 \). Note that the mesons, although they are predominant degrees of freedom at all temperatures, do not contribute to \( \chi_B \) since they do not carry baryon number. The probability that a baryon of mass \( m \) populate at given temperature \( T \) is proportional to the Boltzmann factor \( e^{-m/\sqrt{2}T} \). Thus as temperature increases other heavier baryons are thermally excited and start to contribute to the pressure and hence to \( \chi_B \). Since all the baryons have baryon number \( \pm 1 \), there is no difference in magnitude for higher order susceptibility for \( \chi_B \) in case of HRG model.

The effects of repulsive interactions accounted through excluded volume corrections can also be seen in Fig. 4. \( \chi^n_B \) with repulsive interactions is smaller as compared to ideal HRG model. The effect of repulsive interaction is not significant at low temperatures since population of various particles is low and hence there is enough room for particles in the system then. It is well known fact that the presence of repulsive interactions suppresses the pressure at high temperature as available space for additional number of hadrons and resonances decreases. This suppression is traded in to the \( \chi^n_B \). Thus, the presence of repulsive interactions affect the baryon number susceptibilities and, unlike the ideal HRG, this suppression effect becomes stronger for higher order \( \chi^n_B \)s i.e. \( \chi^n_B \)s of different orders are not of same magnitude when interaction is present. The \( \chi^6_B \) decreases with temperature for both \( eB = 0 \) and \( eB = 0.2 GeV^2 \) case.

![Figure 5](image)

**FIG. 5.** (Color Online) Electric charge susceptibilities of the hadronic matter estimated within ambit of HRG and EHRG.

Fig. 5 shows Electric charge susceptibilities estimated within HRG and EVHRG Model.
framework both in presence and absence of magnetic field. It is seen that for HRG model, charge susceptibilities of all orders are larger in magnitude than Baryon susceptibilities. This is expected as in our particle spectrum there are some electrically charged particles having very low mass for example charged pions and slight increase in chemical potential results in greater increment in pressure and hence greater increment in susceptibilities. It is also important to note that higher order electric charge susceptibilities increase at a faster rate than lower ones. This is also due to lower masses of electrically charged hadrons. On the other hand $\chi_{Q}^{n}$s in presence of magnetic field does not increase much due to suppression of contribution to pressure from electrically charged particles. At low temperature, dominant contribution to $\chi_{Q}^{n}$s comes from charged pions. As temperature increases, heavier hadrons start to contribute to pressure and hence to susceptibilities. Here we have also taken into account the effect of Van der Waals excluded volume repulsive interaction. The effect of hardcore repulsive interaction is to reduce pressure and hence $\chi_{Q}^{n}$ as compared to HRG Model. The effect of repulsive interaction is not significant at low temperatures since population of various particles is low and hence there is enough room for particles in the system then. The suppression effect is stronger for higher order susceptibilities. It is seen that unlike baryon susceptibilities, there is notable difference in magnitude between charge susceptibilities for eB=0 GeV$^{2}$ case and eB=0.2 GeV$^{2}$ case even at low temperatures. This is also due to lower mass of some electrically charged particles which are suppressed in presence of magnetic field as compared to eB=0 case.

VI. SUMMARY

To summarize, we analysed the effect of magnetic field on the hadronic matter within ambit of HRG and EVHRG models. The presence of magnetic field and the repulsive interactions accounted through excluded volume corrections significantly affect the static bulk thermodynamic quantities of hadronic matter. We observe that all the thermodynamical quantities are strongly suppressed due to non-zero background magnetic field and repulsive interactions. The magnetic field affects the effective masses of hadrons thereby affecting the thermal population probability. While effective mass of spin-0 particles increases in magnetic field, masses of spin-1 particles decreases. Upshot of this is to suppression of thermodynamic quantities. The repulsive interactions further enhance the suppression because of the finite size of hadrons which limits the available volume. The suppression of entropy density in presence of magnetic field and repulsive interaction is very important in the context of HIC experiments. We finally discussed the effect of magnetic field and repulsive interactions on the baryon number and electric charge susceptibility. We found that both of them are strongly suppressed at high temperature.

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Appendix A: Formulae

- d-dimensional integral

\[
\int_{-\infty}^{\infty} \frac{d^d p}{(2\pi)^d} \left(p^2 + m^2\right)^{-A} = \frac{\Gamma(A - \frac{d}{2})}{(4\pi)^{d/2} \Gamma(A) (M^2)^{(A - \frac{d}{2})}} 
\]

(A1)

- Riemann-Hurwitz ζ-function

\[
\zeta(z, x) = \sum_{n=0}^{\infty} \frac{1}{(x + n)^z}
\]

(A2)

with the expansion

\[
\zeta\left(-1 + \frac{\epsilon}{2}, x\right) \approx -\frac{1}{12} - \frac{x^2}{2} + \frac{x}{2} \epsilon + \zeta'(-1, x) + O(\epsilon^2)
\]

(A3)

and the asymptotic behavior of the derivative

- The expansion of Γ-function is

\[
\Gamma\left(-1 + \frac{\epsilon}{2}\right) = -\frac{2}{\epsilon} + \gamma - 1 + O(\epsilon)
\]

(A4)

and

\[
\Gamma\left(-2 + \frac{\epsilon}{2}\right) = \frac{1}{\epsilon} - \frac{\gamma}{2} + \frac{3}{4} + O(\epsilon)
\]

(A5)

where γ is the Euler constant.

- The limiting expression for natural logarithm

\[
\lim_{\epsilon \to 0} \epsilon^{-\epsilon/2} \approx 1 - \frac{\epsilon}{2} \ln(a)
\]

(A6)

Appendix B: Renormalized B dependent pressure for spin zero and spin one particles

- Spin-0 particle:

\[
\Delta P_{\text{vac}}^r (s = 0, B) = -\frac{(eB)^2}{4\pi^2} \left(\zeta'(-1, x + 1/2) - \frac{x^2}{2} \ln(x) + \frac{x^2}{4} + \frac{\ln(x) + 1}{24}\right)
\]

(B1)

- Spin-1 particle:

\[
\Delta P_{\text{vac}}^r (s = 1, B) = -\frac{3}{4\pi^2} (eB)^2 \left(\zeta'(-1, x - 1/2) + \frac{(x + 1/2)^2}{3} \ln(x + 1/2)
+ \frac{2}{3} (x - 1/2) \ln(x - 1/2) - \frac{x^2}{2} \ln(x) + \frac{x^2}{4} - \frac{7}{24} (\ln(x) + 1)\right)
\]

(B2)
[1] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B 643, 46 (2006) [hep-lat/0609068].
[2] S. Borsanyi et al. [Wuppertal-Budapest Collaboration], JHEP 1009, 073 (2010) [arXiv:1005.3508 [hep-lat]].
[3] S. Borsanyi et al., Phys. Rev. D 92, no. 1, 014505 (2015) [arXiv:1504.03676 [hep-lat]].
[4] P. Petreczky, AIP Conf. Proc. 1520, 103 (2013).
[5] H. T. Ding, F. Karsch and S. Mukherjee, Int. J. Mod. Phys. E 24, no. 10, 1530007 (2015) [arXiv:1504.05274 [hep-lat]].
[6] B. Friman, C. Hohne, J. Knoll, S. Leupold, J. Randrup, R. Rapp and P. Senger, Lect. Notes Phys. 814, pp.1 (2011).
[7] M. Asakawa and K. Yazaki, Nucl. Phys. A 504, 668 (1989).
[8] A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto and G. Pettini, Phys. Rev. D 41, 1610 (1990).
[9] A. Barducci, R. Casalbuoni, G. Pettini and R. Gatto, Phys. Rev. D 49, 426 (1994).
[10] J. Berges and K. Rajagopal, Nucl. Phys. B 538, 215 (1999) [hep-ph/9804233].
[11] A. M. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D 58, 096007 (1998) [hep-ph/9804290].
[12] O. Scavenius, A. Moisy, I. N. Mishustin and D. H. Rischke, Phys. Rev. C 64, 045202 (2001) [nucl-th/0007030].
[13] N. G. Antoniou and A. S. Kapoyannis, Phys. Lett. B 563, 165 (2003) [hep-ph/0211392].
[14] Y. Hatta and T. Ikeda, Phys. Rev. D 67, 014028 (2003) [hep-ph/0210284].
[15] A. Bhattacharyya, P. Deb, S. K. Ghosh, and R. Ray, Phys. Rev. D 82, 014021 (2010).
[16] S. Hands, Nucl. Phys. Proc. Suppl. 106, 142 (2002) [hep-lat/0109034].
[17] S. Aoki, Int. J. Mod. Phys. A 21, 682 (2006) [hep-lat/0509068].
[18] M. G. Alford, Nucl. Phys. Proc. Suppl. 117, 65 (2003) [hep-ph/0209287].
[19] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov and N. Xu, arXiv:1906.00936 [nucl-th].
[20] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001) [nucl-th/0011058].
[21] P. Romatschke, Int. J. Mod. Phys. E 19, 1 (2010) [arXiv:0902.3663 [hep-ph]].
[22] P. F. Kolb and U. W. Heinz, In *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 634-714 [nucl-th/0305084].
[23] P. Jacobs and X. N. Wang, Prog. Part. Nucl. Phys. 54, 443 (2005) [hep-ph/0405125].
[24] V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009) doi:10.1142/S0217751X09047570 [arXiv:0907.1396 [nucl-th]].
[25] A. Bzdak and V. Skokov, Phys. Lett. B 710, 171 (2012) doi:10.1016/j.physletb.2012.02.065 [arXiv:1111.1949 [hep-ph]].
[26] W. T. Deng and X. G. Huang, Phys. Rev. C 85, 044907 (2012) doi:10.1103/PhysRevC.85.044907 [arXiv:1201.5108 [nucl-th]].
[27] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008) doi:10.1103/PhysRevD.78.074033 [arXiv:0808.3382 [hep-ph]].
[28] I. A. Shovkovy, Lect. Notes Phys. 871, 13 (2013) [arXiv:1207.5081 [hep-ph]].
[29] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP 1202, 044 (2012) [arXiv:1111.4956 [hep-lat]].
[30] F. Preis, A. Rebhan and A. Schmitt, Lect. Notes Phys. 871, 51 (2013) [arXiv:1208.0536 [hep-ph]].
R. C. Duncan and C. Thompson, Astrophys. J. 392, L9 (1992). doi:10.1086/186413

T. Vachaspati, Phys. Lett. B 265, 258 (1991). doi:10.1016/0370-2693(91)90051-Q

J. R. Bhatt and A. K. Pandey, Phys. Rev. D 94, no. 4, 043536 (2016) doi:10.1103/PhysRevD.94.043536 [arXiv:1503.01878 [astro-ph.CO]].

S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992). doi:10.1103/RevModPhys.64.649

T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994) doi:10.1016/0370-1573(94)90022-1 [hep-ph/9401310].

B. J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D 76, 074023 (2007) doi:10.1103/PhysRevD.76.074023 [arXiv:0704.3234 [hep-ph]].

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187, 345 (1969). doi:10.1103/PhysRev.187.345

R. F. Dashen and R. Rajaraman, Phys. Rev. D 10, 708 (1974). doi:10.1103/PhysRevD.10.708

G. M. Welke, R. Venugopalan and M. Prakash, Phys. Lett. B 245, no. 2, 137 (1990). doi:10.1016/0370-2693(90)90123-N

R. Venugopalan and M. Prakash, Nucl. Phys. A 546, 718 (1992). doi:10.1016/0375-9474(92)90005-5

P. Braun-Munzinger, J. Stachel, J. P. Wessels and N. Xu, Phys. Lett. B 365, 1 (1996).

G. D. Yen and M. I. Gorenstein, Phys. Rev. C 59, 2788 (1999).

F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, Phys. Rev. C 64, 024901 (2001).

J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993) [hep-ph/9207204].

P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel, Phys. Lett. B 518, 41 (2001) [hep-ph/0105229].

J. Rafelski and J. Letessier, Nucl. Phys. A 715, 98 (2003) [nucl-th/0209084].

A. Andronic, P. Braun-Munzinger and J. Stachel, Nucl. Phys. A 772, 167 (2006) [nucl-th/0511071].

S. Chatterjee, R. M. Godbole and S. Gupta, Phys. Lett. B 727, 554 (2013) [arXiv:1306.2006 [nucl-th]].

S. Chatterjee, S. Das, L. Kumar, D. Mishra, B. Mohanty, R. Sahoo and N. Sharma, Adv. High Energy Phys. 2015, 349013 (2015).

M. Albright, J. Kapusta and C. Young, Phys. Rev. C 92, no. 4, 044904 (2015) doi:10.1103/PhysRevC.92.044904 [arXiv:1506.03408 [nucl-th]]

V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. 118, no. 18, 182301 (2017) [arXiv:1609.03975 [hep-ph]].

V. Vovchenko, A. Motornenko, P. Alba, M. I. Gorenstein, L. M. Satarov and H. Stoecker, Phys. Rev. C 96, no. 4, 045202 (2017) [arXiv:1707.09215 [nucl-th]].

A. Bhattacharyya, S. Das, S. K. Ghosh, R. Ray and S. Samanta, Phys. Rev. C 90, 034909 (2014).

P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465, 15 (1999)

A. Bhattacharyya, P. Deb, A. Lahiri, and R. Ray, Phys. Rev. D 82, 114028 (2010).

A. Bhattacharyya, P. Deb, A. Lahiri, and R. Ray, Phys. Rev. D 83, 041011 (2011).

P. Deb, A. Bhattacharyya, S. Datta, and S. K. Ghosh Phys. Rev. C 79, 055208 (2009).

A. Bhattacharyya, S. K. Ghosh, S. Majumder, and R. Ray, Phys. Rev. D 86, 096006 (2012).

A. Bhattacharyya, S. K. Ghosh, A. Lahiri, S. Majumder, S. Raha, and R. Ray, Phys. Rev. C 89, 064905 (2014).

A. Bhattacharyya, P. Deb, S. K. Ghosh, R. Ray, and S. Sur, Phys. Rev. D 87, 054009 (2013).
[61] A. Bhattacharyya, R. Ray, and S. Sur, Phys. Rev. D 91(R), 051501 (2015).
[62] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha and Sudipa Upadhaya, Phys. Rev. D 95, 054005 (2017).
[63] A. Bhattacharyya, R. Ray, S. Samanta and S. Sur, Phys. Rev. C 91(R), 041901 (2015).
[64] G. Endrödi, JHEP 1304, 023 (2013) [arXiv:1301.1307 [hep-ph]].
[65] A. Bhattacharyya, S. K. Ghosh, R. Ray and S. Samanta, EPL 115, no. 6, 62003 (2016) [arXiv:1504.04533 [hep-ph]].
[66] D. H. Rischke, M. I. Gorenstein, H. Stoecker and W. Greiner, Z. Phys. C 51, 485 (1991).
[67] S. Ferrara, M. Porrati and V. L. Telegdi, Phys. Rev. D 46, 3529 (1992).
[68] S. Fayazbakhsh and N. Sadooghi, Phys. Rev. D 88, no. 6, 065030 (2013) [arXiv:1306.2098 [hep-ph]].
[69] D. Ebert and K. G. Klimenko, Nucl. Phys. A 728, 203 (2003) [hep-ph/0305149].
[70] D. P. Menezes, M. Benghi Pinto, S. S. Avancini, A. Perez Martinez and C. Providencia, Phys. Rev. C 79, 035807 (2009) [arXiv:0811.3361 [nucl-th]].
[71] M. Peskin and D. Schroeder, An introduction to quantum field theory, Westview Press, U.S.A. (1995).
[72] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[73] T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994) [hep-ph/9401310].
[74] J. S. Schwinger, Phys. Rev. 82, 664 (1951).
[75] P. Elmfors, D. Persson and B. S. Skagerstam, Astropart. Phys. 2, 299 (1994) [hep-ph/9312226].
[76] J. O. Andersen and R. Khan, Phys. Rev. D 85, 065026 (2012) [arXiv:1105.1290 [hep-ph]].
[77] A. Andronic, P. Braun-Munzinger, J. Stachel and M. Winn, Phys. Lett. B 718, 80 (2012) [arXiv:1201.0693 [nucl-th]].
[78] J. Beringer et al. [Particle Data Group], Phys. Rev. D 86, 010001 (2012). doi:10.1103/PhysRevD.86.010001