Phenomenological aspects of 10D SYM theory with magnetized extra dimensions

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Abstract
We present a particle physics model based on a ten-dimensional (10D) super Yang-Mills (SYM) theory compactified on magnetized tori preserving four-dimensional $\mathcal{N} = 1$ supersymmetry. The low-energy spectrum contains the minimal supersymmetric standard model with hierarchical Yukawa couplings caused by a wavefunction localization of the chiral matter fields due to the existence of magnetic fluxes, allowing a semi-realistic pattern of the quark and the lepton masses and mixings. We show supersymmetric flavor structures at low energies induced by a moduli-mediated and an anomaly-mediated supersymmetry breaking.

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1 Introduction

The standard model (SM) of elementary particles is a quite successful theory, consistent with all the experimental data obtained so far with a great accuracy. There are, however, many free parameters, which can not be determined theoretically, making the model less predictable. Among these parameters, especially, Yukawa coupling constants seem to be awfully hierarchical in order to explain the observed masses and mixing angles of the quarks and the leptons. It is argued that some flavor symmetries are helpful to understand such a hierarchical structure. (See, for a review, Ref. [1].) Another interesting possibility is a quasi-localization of matter fields in extra dimensions, where the hierarchical couplings are obtained from the overlap integral of their localized wavefunctions [2]. It is also suggested that the former flavor symmetries are realized geometrically as a consequence of the latter wavefunction localization in extra dimensions [3, 4, 1].

The SM does not describe gravitational interactions of elementary particles that could play an important role at the very beginning of our universe. Superstring theories in ten-dimensional (10D) spacetime are almost the only known candidates that can treat gravitational interactions at the quantum level. These theories possess few free parameters and potentially more predictive than the SM. Supersymmetric Yang-Mills (SYM) theories in various spacetime dimensions appear as low energy effective theories of superstring compactifications with or without D-branes. Thus, it is an interesting possibility that the SM is embedded in one of such SYM theories, that is, the SM is realized as a low energy effective theory of the superstrings. In such a string model building, how to break higher-dimensional supersymmetry and to obtain a chiral spectrum is the key issue. String compactifications on the Calabi-Yau (CY) space provide a general procedure for such a purpose. However, the metric of a generic CY space is hard to be determined analytically, that makes the phenomenological studies qualitative, but not quantitative.

It is quite interesting that even simple toroidal compactifications but with magnetic fluxes in extra dimensions induce chiral spectra [6, 7] in higher-dimensional SYM theories. The higher-dimensional supersymmetry such as $\mathcal{N} = 4$ in terms of supercharges in four-dimensional (4D) spacetime is broken by the magnetic fluxes down to $4D \mathcal{N} = 0, 1$ or 2 depending on the configuration of fluxes. The number of the chiral zero-modes is determined by the number of magnetic fluxes. A phenomenologically attractive feature is that these chiral zero modes localize toward different points in magnetized extra dimensions. The overlap integrals of localized wavefunctions yield hierarchical couplings in the 4D effective theory of these zero modes, that could explain, e.g., observed hierarchical masses and mixing angles of the quarks and the leptons [8]. Furthermore, higher-order couplings can also be computed as the overlap integrals of wavefunctions [9]. A theoretically attractive point here is that many peculiar properties of the SM, such as the 4D chirality, the number of generations, the flavor symmetries [3, 10, 11] and potentially hierarchical Yukawa couplings all could be determined by the magnetic fluxes.

Moreover if the 4D $\mathcal{N} = 1$ supersymmetry remains, a supersymmetric standard model could be realized below the compactification scale that has many attractive features beyond the SM.

$^1$Non-Abelian discrete flavor symmetries are also obtained within the framework of heterotic string theory on orbifolds [5].
such as the lightest supersymmetric particle as a candidate of dark matter and so on. In our previous work [12], we have presented 4D $\mathcal{N} = 1$ superfield description of 10D SYM theories compactified on magnetized tori which preserve the $\mathcal{N} = 1$ supersymmetry, and derived 4D effective action for massless zero-modes written in the $\mathcal{N} = 1$ superspace. We further identified moduli dependence of the effective action by promoting the Yang-Mills (YM) gauge coupling constant $g$ and geometric parameters $R_i$ and $\tau_i$ to a dilaton, Kähler and complex-structure moduli superfields, which allows an explicit estimation of soft supersymmetry breaking parameters in the supersymmetric SM caused by moduli-mediated supersymmetry breaking. The resulting effective supergravity action would be useful for building phenomenological models and for analyzing them systematically.

Motivated by the above arguments, in this paper, we construct a particle physics model based on 10D SYM theory compactified on three factorizable tori $T^2 \times T^2 \times T^2$ where magnetic fluxes are present in the YM sector. We search a phenomenologically viable flux configuration that induces a 4D chiral spectrum including the minimal supersymmetric standard model (MSSM), based on the effective action written in $\mathcal{N} = 1$ superspace. For such a flux configuration that realize a realistic pattern of the quark and the lepton masses and their mixing angles, we further estimate the sizes of supersymmetric flavor violations caused by the moduli-mediated supersymmetry breaking.

The sections are organized as follows. In Sec. 2 a superfield description of the 10D SYM theory is briefly reviewed based on Ref. [12], which allows the systematic introduction of magnetic fluxes in extra dimensions preserving the $\mathcal{N} = 1$ supersymmetry. Then, we construct a model that contains the spectrum of the MSSM, in which the most massless exotic modes are projected out due to the existence of the magnetic fluxes and a certain orbifold projection in Sec. 3. In Sec. 4, we numerically search a location in the moduli space of the model where a realistic pattern of the quark and the lepton masses and their mixing angles are obtained. Then, assuming the moduli-mediated supersymmetry breaking, we estimate the magnitude of the mass insertion parameters representing typical sizes of various flavor changing neutral currents (FCNC) in Sec. 5. Sec. 6 is devoted to conclusions and discussions. In Appendix A, the Kähler metrics and the holomorphic Yukawa couplings are exhibited for the MSSM matter fields in the 4D effective theory.

## 2 The 10D SYM theory in $\mathcal{N} = 1$ superspace

Based on Ref. [12], in this section, we review a compactification of 10D SYM theory on 4D flat Minkowski spacetime times a product of factorizable three tori $T^2 \times T^2 \times T^2$ and a superfield description suitable for such a compactification with magnetic fluxes in each torus preserving 4D $\mathcal{N} = 1$ supersymmetry. The geometric (torus) parameter dependence is explicitly shown in this procedure, which is important to determine couplings between YM and moduli superfields in the 4D effective action for chiral zero-modes.

The 10D SYM theory is described by the following action,

$$
S = \int d^{10}X \sqrt{-G} \frac{1}{g^2} \text{Tr} \left[ -\frac{1}{4} F^{MN} F_{MN} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda \right],
$$

(1)
where $g$ is a 10D YM gauge coupling constant and the trace is performed over the adjoint representation of the YM gauge group. The 10D spacetime coordinates are denoted by $X^M$, and the vector/tensor indices $M, N = 0, 1, \ldots, 9$ are lowered and raised by the 10D metric $G_{MN}$ and its inverse $G^{MN}$, respectively. The YM field strength $F_{MN}$ and the covariant derivative $D_M$ are given by $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$ and $D_M \lambda = \partial_M \lambda - i[A_M, \lambda]$ for a 10D vector (gauge) field $A_M$ and a 10D Majorana-Weyl spinor field $\lambda$, respectively. The spinor field $\lambda$ satisfies 10D Majorana and Weyl conditions, $\lambda^C = \lambda$ and $\Gamma \lambda = +\lambda$, respectively, where $\lambda^C$ denotes a 10D charge conjugation of $\lambda$, and $\Gamma$ is a 10D chirality operator.

The 10D spacetime (real) coordinates $X^M = (x^\mu, y^m)$ are decomposed into 4D Minkowski spacetime coordinates $x^\mu$ with $\mu = 0, 1, 2, 3$ and six dimensional (6D) extra space coordinates $y^m$ with $m = 4, \ldots, 9$. The zeroth component $\mu = 0$ describes the time component. The 10D vector field is similarly decomposed as $A_M = (A_\mu, A_m)$. The 10D background metric is given by

$$ds^2 = G_{MN} dX^M dX^N = \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n,$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$. Because we consider a torus compactification of internal 6D space $y^m$ by identifying $y^m \sim y^m + 2$ and the 6D torus is decomposed as a product of factorizable three tori, $T^2 \times T^2 \times T^2$, the extra 6D metric can be described as

$$g_{mn} = \begin{pmatrix} g^{(1)} & 0 & 0 \\ 0 & g^{(2)} & 0 \\ 0 & 0 & g^{(3)} \end{pmatrix},$$

where each entry is a $2 \times 2$ matrix and the diagonal submatrices are expressed as

$$g^{(i)} = (2\pi R_i)^2 \begin{pmatrix} 1 & \text{Re } \tau_i \\ \text{Re } \tau_i & |\tau_i|^2 \end{pmatrix},$$

for $i = 1, 2, 3$. The real and the complex parameters $R_i$ and $\tau_i$ determine the size and the shape of the $i$th torus $T^2$, respectively. The area $A^{(i)}$ of the $i$th torus is determined by these parameters as

$$A^{(i)} = (2\pi R_i)^2 \text{Im } \tau_i.$$

The complex coordinates $z^i$ for $i = 1, 2, 3$ defined by

$$z^i \equiv \frac{1}{2}(y^{2+2i} + \tau_i y^{3+2i}), \quad \bar{z}^i \equiv (z^i)^*,$$

are extremely useful for describing the action in 4D $\mathcal{N} = 1$ superspace, where the corresponding complex vector components $A_i$ are defined by

$$A_i \equiv -\frac{1}{\text{Im } \tau_i} (\tau_i^* A_{2+2i} - A_{3+2i}), \quad \bar{A}_i \equiv (A_i)^\dagger.$$

In the complex coordinate, the torus boundary conditions are expressed as $z^i \sim z^i + 1$ and $z^i \sim z^i + \tau^i$, and the metric is found as $h_{i\bar{j}} = 2 (2\pi R_i)^2 \delta_{i\bar{j}} = \delta_{i\bar{j}} e_i^i \bar{e}_j^j$ satisfying $2h_{i\bar{j}} dz^i d\bar{z}^j = -i \Gamma_{i\bar{j}} dz^i d\bar{z}^j.$
\[ g_{mn}dy^mdy^n = ds^2_{6D}, \] where \( e_i^i = \sqrt{2} (2\pi R_i) \delta_i^i \) is a vielbein, and the Roman indices represent local Lorentz space. The Italic (Roman) indices \( i, j \ldots (i, j, \ldots) \) are lowered and raised by the metric \( h_{ij} \) and its inverse \( h^{ij} (\delta_{ij} \text{ and its inverse } \delta^{ij}) \), respectively.

The 10D SYM theory possesses \( N = 4 \) supersymmetry counted by a 4D supercharge. The YM vector and spinor fields, \( A_M \) and \( \lambda \), are decomposed into (on-shell) 4D \( N = 1 \) single vector and triple chiral multiplets, \( V = \{ A_\mu, \lambda_0 \} \) and \( \phi_i = \{ A_i, \lambda_i \} \) \((i = 1, 2, 3)\), respectively, where the 10D Majorana-Weyl spinor \( \lambda \) is decomposed into four 4D Weyl (or equivalently Majorana) spinors \( \lambda_0 \) and \( \lambda_1 \). If we write the chirality associated with 6D spacetime coordinates \((x^\mu, z^i)\) in the \( i \)th subscript of \( \lambda \) like \( \lambda_{++} \), the decomposed spinor fields \( \lambda_0 \) and \( \lambda_i \) are identified with the chirality eigenstates \( \lambda_{\pm \pm} \) as \( \lambda_0 = \lambda_{+++}, \lambda_1 = \lambda_{+-+}, \lambda_2 = \lambda_{--+} \text{ and } \lambda_3 = \lambda_{--+} \) for the 4D chirality fixed, e.g. the positive chirality. Note that the components \( \lambda_{--}, \lambda_{++}, \lambda_{-+} \text{ and } \lambda_{++} \) do not exist in the 10D Majorana-Weyl spinor \( \lambda \) due to the condition \( \Gamma \lambda = +\lambda \).

The above \( N = 1 \) vector and chiral multiplets, \( V \) and \( \phi_i \), are expressed by vector and chiral superfields, \( V \) and \( \phi_i \), respectively as

\[
V \equiv -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta\bar{\theta}\lambda_0 - i\theta\bar{\theta}\bar{\lambda}_0 + \frac{1}{2}\theta\bar{\theta}\bar{\theta}D,
\]

\[
\phi_i \equiv \frac{1}{\sqrt{2}}A_i + \sqrt{2}\theta\lambda_i + \theta\phi_i,
\]

where \( \theta \) and \( \bar{\theta} \) are Grassmann coordinates of 4D \( N = 1 \) superspace. The 10D SYM action \((1)\) can be written in the \( N = 1 \) superspace as \([13]\)

\[
S = \int d^{10}X\sqrt{-G} \left[ \int d^4\theta K + \left\{ \int d^2\theta \left( \frac{1}{4g^2}W^\alpha W_\alpha + W \right) + \text{h.c.} \right\} \right].
\]

The functions of the superfields, \( K \), \( W \) and \( W_\alpha \), are given by

\[
K = \frac{2}{g^2}h^{ij}\text{Tr} \left[ (2\bar{\theta} + \bar{\phi}_i) e^{-V} (-2\bar{\theta} + \phi_j) e^V + \bar{\theta} e^{-V} \partial_j e^V \right] + K_{\text{WZW}},
\]

\[
W = \frac{1}{g^2}\epsilon^{ijk}e_i^i e_j^j e_k^k \text{Tr} \left[ \sqrt{2} \phi_i \left( \partial_j \phi_k - \frac{1}{3\sqrt{2}} [\phi_j, \phi_k] \right) \right],
\]

\[
W_\alpha = -\frac{1}{4}\bar{D}De^{-V}D_\alpha e^V,
\]

where \( \epsilon^{ijk} \) is a totally antisymmetric tensor satisfying \( \epsilon^{123} = 1 \), and \( D_\alpha (\bar{D}_\dot{\alpha}) \) is a supercovariant derivative (its conjugate) with a 4D spinor index \( \alpha (\dot{\alpha}) \). The term \( K_{\text{WZW}} \) represents a Wess-Zumino-Witten term which vanishes in the Wess-Zumino (WZ) gauge.

The equations of motion for auxiliary fields \( D \) and \( F_i \) lead to

\[
D = -h^{ij} \left( \bar{\partial}_i A_j + \partial_j \bar{A}_i + \frac{1}{2} [\bar{A}_i, A_j] \right),
\]

\[
F_i = -h^{ij}\epsilon^{kl}e_j^j e_k^k e_l^l \left( \partial_k A_l - \frac{1}{4} [A_k, A_l] \right).
\]
The condition $\langle D \rangle = \langle F_i \rangle = 0$ determines supersymmetric vacua. A trivial supersymmetric vacuum is given by $\langle A_i \rangle = 0$ where the full $\mathcal{N} = 4$ supersymmetry as well as the YM gauge symmetry is preserved. In the following, we select one of nontrivial supersymmetric vacua where magnetic fluxes exist in the YM sector, and construct a particle physics model with a semi-realistic flavor structure of (s)quarks and (s)leptons caused by a wavefunction localization of chiral matter fields in extra dimensions due to the effect of magnetic fluxes.

3 The model building

We consider the 10D $U(N)$ SYM theory on a supersymmetric magnetic background where the YM fields take the following 4D Lorentz invariant and at least $\mathcal{N} = 1$ supersymmetric VEVs,

$$\langle A_i \rangle = \frac{\pi}{\text{Im} \tau_i} (M^{(i)}\tilde{z}_i + \tilde{\zeta}_i),$$
$$\langle A_\mu \rangle = \langle \lambda_0 \rangle = \langle \lambda_i \rangle = \langle F_i \rangle = \langle D \rangle = 0. \quad (4)$$

Here $N \times N$ diagonal matrices of Abelian magnetic fluxes and those of Wilson-lines are denoted, respectively, as

$$M^{(i)} = \text{diag}(M^{(i)}_1, M^{(i)}_2, \ldots, M^{(i)}_N),$$
$$\zeta_i = \text{diag}(\zeta^{(i)}_1, \zeta^{(i)}_2, \ldots, \zeta^{(i)}_N). \quad (5)$$

The magnetic fluxes satisfying the Dirac’s quantization condition, $M^{(i)}_1, M^{(i)}_2, \ldots, M^{(i)}_N \in \mathbb{Z}$, are further constrained by the supersymmetry conditions $\langle D \rangle = 0$ and $\langle F_i \rangle = 0$ in Eq. (4), which are written as

$$^i \hat{R}^{ij} \left( \partial_i \langle A_j \rangle + \partial_j \langle A_i \rangle \right) = 0, \quad (6)$$
$$e^{ijkl}e^j_k e^l_i \partial_k \langle A_i \rangle = 0, \quad (7)$$

with $D$ and $F_i$ given by Eqs. (2) and (3), respectively.

One of the consequences of nonvanishing magnetic fluxes is the YM gauge symmetry breaking. If all the magnetic fluxes $M^{(i)}_1, M^{(i)}_2, \ldots, M^{(i)}_N$ take different values from each other, the gauge symmetry is broken down as $U(N) \rightarrow \hat{U}(1)^N$. The breaking pattern is changed as $U(N) \rightarrow \prod_a U(N_a)$ in the case with degenerate magnetic fluxes that are written, without loss of generality, as

$$M^{(i)}_1 = M^{(i)}_2 = \cdots = M^{(i)}_{N_1},$$
$$M^{(i)}_{N_1+1} = M^{(i)}_{N_1+2} = \cdots = M^{(i)}_{N_1+N_2},$$
$$\vdots$$
$$M^{(i)}_{N_1+N_2+\cdots+N_{N-1}+1} = M^{(i)}_{N_1+N_2+\cdots+N_{N-1}+2} = \cdots = M^{(i)}_{N_1+N_2+\cdots+N_{N-1}+N_N}, \quad (8)$$

with $\sum a N_a = N$ and all the fluxes $M^{(i)}_{N_1}, M^{(i)}_{N_1+N_2}, \ldots, M^{(i)}_{N_1+N_2+\cdots+N_{N-1}+N_N}$ take different values from each other. The same holds for Wilson-lines, $\zeta^{(i)}_1, \zeta^{(i)}_2, \ldots, \zeta^{(i)}_N$. In the following, indices

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2A similar study is possible by starting with other gauge groups \cite{14}. 

where \(a, b = 1, 2, \ldots, \tilde{N}\) label the unbroken YM subgroups on the flux and Wilson-line background \((4)\), and traces in expressions are performed within such subgroups.

On the \(\mathcal{N} = 1\) supersymmetric toroidal background \((4)\) with the magnetic fluxes \((5)\) as well as the Wilson-lines satisfying Eq. \((8)\), the zero-modes \((V^{n=0})_{aa}\) of the off-diagonal elements \((V)_{ab}\) \((a \neq b)\) of the 10D vector superfield \(V\) obtain mass terms, while the diagonal elements \((V^{n=0})_{aa}\) do not. Then, we express the zero-modes \((V^{n=0})_{aa}\), which contain 4D gauge fields for the unbroken gauge symmetry \(\prod_a U(N_a)\), as

\[
(V^{n=0})_{aa} = V^a.
\]

On the other hand, for \(\exists j \neq i\) with \(M_{ab}^{(j)} \equiv M_a^{(j)} - M_b^{(j)} < 0\), \(M_{ab}^{(i)} \equiv M_a^{(i)} - M_b^{(i)} > 0\) and \(a \neq b\), the zero-mode \((\phi^0_{j=0})_{ab}\) of the off-diagonal element \((\phi^j)_{ab}\) of the 10D chiral superfield \(\phi^j\) degenerates with the number of degeneracy \(N_{ab} = \prod_k M_{ab}^{(k)}\), while \((\phi^0_{j=0})_{ba}\) has no zero-mode solution, yielding a 4D supersymmetric chiral generation in the \(ab\)-sector \([12]\). The opposite is true for \(M_{ab}^{(j)} > 0\) and \(M_{ab}^{(i)} < 0\) yielding a 4D chiral generation in the \(ba\)-sector. Therefore, we denote the zero-mode \((\phi^0_{j=0})_{ab}\) with the degeneracy \(N_{ab}\) as

\[
(\phi^0_{j=0})_{ab} = g \phi^0_{ja}^g
\]

where \(I_{ab}\) labels the degeneracy, i.e. generations. We normalize \(\phi^0_{ja}\) by the 10D YM coupling constant \(g\). For more details, see Ref. \([12]\) and references therein.

### 3.1 Three generations induced by magnetic fluxes

We aim to realize a zero-mode spectrum in 10D SYM theory compactified on magnetized tori, that contains the MSSM with the gauge symmetry \(SU(3)_C \times SU(2)_L \times U(1)_Y\) and three generations of the quark and the lepton chiral multiplets, by identifying those three with degenerate zero-modes of the chiral superfields \(\phi^0_{ja}\).

For such a purpose, we start from the 10D \(U(N)\) SYM theory with \(N = 8\) and introduce the following magnetic fluxes

\[
F_{2+2r, 3+2r} = 2\pi \begin{pmatrix}
M_C^{(r)} 1_4 & M_L^{(r)} 1_2 \\
M_R^{(r)} 1_2
\end{pmatrix},
\]

where \(1_N\) is a \(N \times N\) unit matrix, and all the nonvanishing entries take different values from each other. These magnetic fluxes break YM symmetry as \(U(8) \rightarrow U(4)_C \times U(2)_L \times U(2)_R\). We consider the case that this is further broken down to \(U(3)_C \times U(1)_{C'} \times U(2)_L \times U(1)_{R'} \times U(1)_{R''}\) by the following Wilson lines

\[
\zeta_r = \begin{pmatrix}
\zeta_C^{(r)} 1_3 \\
\zeta_C^{(r)} 1_2 \\
\zeta_L^{(r)} 1_2 \\
\zeta_R^{(r)} 1_2 \\
\zeta_{R'}^{(r)} 1_2
\end{pmatrix},
\]

\((10)\)
where all the nonvanishing entries take different values from each other. The gauge symmetries $SU(3)_C$ and $SU(2)_L$ of the MSSM are embedded into the above unbroken gauge groups as $SU(3)_C \subset U(3)_C$ and $SU(2)_L \subset U(2)_L$.

A combination of the magnetic fluxes, that yield three generations from the zero-mode degeneracy and also the full-rank Yukawa matrices from the 10D gauge interaction as we will see later, are found as

\[
\begin{align*}
(M_C^{(1)}), M_L^{(1)}, M_R^{(1)} &= (0, +3, -3), \\
(M_C^{(2)}), M_L^{(2)}, M_R^{(2)} &= (0, -1, 0), \\
(M_C^{(3)}), M_L^{(3)}, M_R^{(3)} &= (0, 0, +1),
\end{align*}
\]

where the supersymmetry conditions (6) and (7) are satisfied by

\[
\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = \mathcal{A}^{(1)}/\mathcal{A}^{(3)} = 3.
\]

In this model, chiral superfields $Q$, $U$, $D$, $L$, $N$, $E$, $H_u$ and $H_d$ carrying the left-handed quark, the right-handed up-type quark, the right-handed down-type quark, the left-handed lepton, the right-handed neutrino, the right-handed electron, the up- and the down-type Higgs bosons, respectively, are found in $\phi_{ab}^{(4)}$ as

\[
\phi_{1ab} = \begin{pmatrix}
\Omega_C^{(1)} & \Xi_{CC}^{(1)} & 0 & \Xi_{CR}^{(1)} & \Xi_{CR'}^{(1)} \\
\Xi_{CC}^{(1)} & \Omega_C^{(1)} & 0 & \Xi_{CR}^{(1)} & \Xi_{CR'}^{(1)} \\
\Xi_{LC}^{(1)} & \Xi_{LC'}^{(1)} & H_u^K & H_d^K \\
0 & 0 & \Omega_L^{(1)} & \Xi_{RL}^{(1)} & \Xi_{RL'}^{(1)} \\
0 & 0 & \Omega_{RL}^{(1)} & \Xi_{RL}^{(1)} & \Omega_{RL'}^{(1)} \\
\end{pmatrix},
\]

\[
\phi_{2ab} = \begin{pmatrix}
\Omega_C^{(2)} & \Xi_{CC}^{(2)} & 0 & \Xi_{CR}^{(2)} & \Xi_{CR'}^{(2)} \\
\Xi_{CC}^{(2)} & \Omega_C^{(2)} & 0 & \Xi_{CR}^{(2)} & \Xi_{CR'}^{(2)} \\
\Omega_L^{(2)} & 0 & 0 & \Xi_{RL}^{(2)} & \Xi_{RL'}^{(2)} \\
0 & 0 & \Omega_L^{(2)} & 0 & 0 \\
0 & 0 & \Omega_{RL}^{(2)} & \Xi_{RL}^{(2)} & \Omega_{RL'}^{(2)} \\
\end{pmatrix},
\]

\[
\phi_{3ab} = \begin{pmatrix}
\Omega_C^{(3)} & \Xi_{CC}^{(3)} & 0 & 0 & 0 \\
\Xi_{CC}^{(3)} & \Omega_C^{(3)} & 0 & \Xi_{CR}^{(3)} & \Xi_{CR'}^{(3)} \\
\Omega_L^{(3)} & 0 & \Xi_{RL}^{(3)} & \Xi_{RL}^{(3)} & \Xi_{RL'}^{(3)} \\
0 & 0 & \Omega_L^{(3)} & 0 & 0 \\
\end{pmatrix},
\]

where the rows and the columns of matrices correspond to $a = 1, \ldots, 5 = C, C', L, R', R''$ and $b = 1, \ldots, 5 = C, C', L, R', R''$, respectively, and the indices $I, J = 1, 2, 3$ and $K = 1, \ldots, 6$ label the zero-mode degeneracy, i.e., generations.

Therefore, three generations of $Q$, $U$, $D$, $L$, $N$, $E$ and six generations of $H_u$ and $H_d$ are generated by the magnetic fluxes (11) that correspond to

\[
M_C^{(1)} - M_L^{(1)} = -3, \quad M_L^{(1)} - M_R^{(1)} = +6, \quad M_R^{(1)} - M_C^{(1)} = -3,
\]
\[ M_{C}^{(2)} - M_{L}^{(2)} = +1, \quad M_{L}^{(2)} - M_{R}^{(2)} = -1, \quad M_{R}^{(2)} - M_{C}^{(2)} = 0, \]
\[ M_{C}^{(3)} - M_{L}^{(3)} = 0, \quad M_{L}^{(3)} - M_{R}^{(3)} = -1, \quad M_{R}^{(3)} - M_{C}^{(3)} = +1. \]  

(14)

The zero entries of the matrices in Eq. (13) represent components eliminated due to the effect of chirality projection caused by magnetic fluxes. Because some vanishing fluxes are inevitable in Eq. (11) in order to realize three generations of quarks and leptons with the Yukawa coupling matrices of the full-rank, some of \( M_{ab}^{(i)} \) become zero in Eq. (14), that causes certain massless exotic modes \( \Xi_{ab}^{(r)} \) as well as massless diagonal components \( \Omega_{a}^{(r)} \), i.e., the so-called open string moduli, all of which feel zero fluxes. These exotics are severely constrained by many experimental data at low energies. In the following, we show that most of the massless exotic modes can be eliminated if we consider a certain orbifold projection on \( r = 2, 3 \) tori, that is, a sort of magnetized orbifolds [15].

### 3.2 Exotic modes and \( Z_2 \)-projection

Three generations of quarks and leptons are generated in the first torus \( r = 1 \) by the magnetic fluxes (11). The number of the degenerate zero-modes (generations) is changed by the orbifold projection [15]. We assume the \( T^6/Z_2 \) orbifold where the \( Z_2 \) acts on the second and the third tori \( r = 2, 3 \) in order to eliminate only the exotic modes without affecting the generation structure of the MSSM matter fields realized by the magnetic fluxes (11). Then, the \( Z_2 \) transformation of 10D superfields \( V \) and \( \phi_i \) is assigned for \( \forall m = 4, 5 \) and \( \forall n = 6, 7, 8, 9 \) as

\[
V(x, y_m, -y_n) = +PV(x, y_m, -y_n)P^{-1}, \\
\phi_1(x, y_m, -y_n) = +P\phi_1(x, y_m, -y_n)P^{-1}, \\
\phi_2(x, y_m, -y_n) = -P\phi_2(x, y_m, -y_n)P^{-1}, \\
\phi_3(x, y_m, -y_n) = -P\phi_3(x, y_m, -y_n)P^{-1},
\]  

(15)

where \( P \) is a projection operator acting on YM indices satisfying \( P^2 = 1_N \). The \( \phi_2 \) and \( \phi_3 \) fields have the minus sign under the \( Z_2 \) reflection, because those are the vector fields, \( A_i \) \( (i = 2, 3) \) on the \( Z_2 \) orbifold plane. Note that the orbifold projection (15) respects the \( \mathcal{N} = 1 \) supersymmetry preserved by the magnetic fluxes (11), because the \( Z_2 \)-parities are assigned to the \( \mathcal{N} = 1 \) superfields \( V \) and \( \phi_i \).

For the matter profile (13) caused by the magnetic fluxes (11), we find that the following \( Z_2 \)-projection operator,

\[
P_{ab} = \begin{pmatrix}
-1_4 & 0 & 0 \\
0 & +1_2 & 0 \\
0 & 0 & +1_2
\end{pmatrix},
\]

removes most of the massless exotic modes \( \Xi_{ab}^{(r)} \) and some of massless diagonal components \( \Omega_{a}^{(r)} \).
The matter contents on the orbifold $T^6/Z_2$ is found as

$$\phi_{I}^{ab} = \begin{pmatrix}
\Omega_C^{(1)} & \Xi_{CC'}^{(1)} & 0 & 0 & 0 \\
\Xi_{C'C}^{(1)} & \Omega_{C'}^{(1)} & 0 & 0 & 0 \\
0 & 0 & \Omega_L^{(1)} & \tilde{H}_u^{K} & \tilde{H}_d^{K} \\
0 & 0 & 0 & \Omega_{R'}^{(1)} & \tilde{\Xi}_{R'R''}^{(1)} \\
0 & 0 & 0 & \tilde{\Xi}_{R''R'}^{(1)} & \Omega_{R''}^{(1)} \\
\end{pmatrix},$$

$$\phi_{2}^{ab} = \begin{pmatrix}
0 & 0 & Q^j & 0 & 0 \\
0 & 0 & L^j & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}, \quad \phi_{3}^{ab} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
U^J & N^J & 0 & 0 & 0 \\
D^J & E^J & 0 & 0 & 0 \\
\end{pmatrix},$$

where $I, J = 1, 2, 3$ and $K = 1, \ldots, 6$ label the generations as before. There still remain massless exotic modes $\Xi_{ab}$ for $a, b = C, C'$ and $a, b = R', R''$ with $a \neq b$ as well as open string moduli $\Omega_{ab}^{(1)}$ for $a = C, C', L, R', R''$. That is one of the open problems in the $T^6/Z_2$ magnetized orbifold model. In the following phenomenological analyses, these exotic modes are assumed to become massive through some nonperturbative effects or higher-order corrections, so that they decouple from the low-energy physics.

Due to the orbifold projection (15), nonvanishing Wilson-line parameters in Eq. (10) are possible only in the first torus $r = 1$. We denote differences of these nonvanishing Wilson-line parameters as

$$\zeta_{C}^{(1)} - \zeta_{L}^{(1)} \equiv \zeta_Q, \quad \zeta_{R'}^{(1)} - \zeta_{C'}^{(1)} \equiv \zeta_U, \quad \zeta_{R''}^{(1)} - \zeta_{C'}^{(1)} \equiv \zeta_D,$$

$$\zeta_{C'}^{(1)} - \zeta_{L}^{(1)} \equiv \zeta_L, \quad \zeta_{R'}^{(1)} - \zeta_{C'}^{(1)} \equiv \zeta_N, \quad \zeta_{R''}^{(1)} - \zeta_{C'}^{(1)} \equiv \zeta_E,$$

whose numerical values are determined later phenomenologically.

### 3.3 Anomalous $U(1)$s and the hypercharge

Finally in this section, we discuss the $U(1)$ gauge fields and their charges in the low energy spectrum. As shown above, most of the exotic matter fields become massive (some of them are assumed) on the orbifold background, and then the low energy spectrum of this model is the MSSM-like matters with additional pairs of up and down type Higgs doublets. The gauge group is given by $SU(3)_C \times SU(2)_L \times U(1)^5$ and we denote each $U(1)_X$ charge as $Q_X$ for $X = a, b, c, d, e$. The particle contents and their gauge charges are summarized in Table 1.

As is well known, there are two non-anomalous local and global $U(1)$ symmetries in the MSSM that we denote $U(1)_Y$ and $U(1)_{B-L}$, respectively, where $Y$ represents the hypercharge and $B$ ($L$) is the baryon (lepton) charge. In our model the $U(1)_{B-L}$ is a local symmetry.

---

3Nonvanishing Wilson-line parameters would be possible also in the 2nd and the 3rd tori, if we allow non-zero VEVs of vector fields that are constants in the bulk but change their sign across the fixed points (planes) of the orbifold, that is beyond the scope of this paper. In this case localized magnetic fluxes at the fixed points might be induced which cause nontrivial effects on the wavefunction profile of the charged matter fields [16].
and there is an additional anomaly-free U(1) symmetry as a linear combination of all the U(1) groups denoted by $U(1)_D$ with the charge defined by $Q_D = Q_a + Q_b + Q_c + Q_d + Q_e$. Each $U(1)_X$ gauge symmetry has clear interpretation in terms of the global symmetries in the MSSM. For example, $Q_a$ is related to the baryon number and $Q_c$ is nothing but the lepton number. Thus one can obtain the $U(1)_{B-L}$ and the $U(1)_Y$ as linear combinations of the above five U(1) gauge symmetries. Here we take the $U(1)_{B-L}$ charge $Q_{B-L}$ as

$$Q_{B-L} = \frac{1}{3}Q_a - Q_b.$$ 

The U(1) hypercharge $Q_Y$ can be given by

$$Q_Y = \alpha Q_a + \left(\alpha - \frac{2}{3}\right) Q_b + \left(\alpha - \frac{1}{6}\right) Q_c + \left(\alpha - \frac{2}{3}\right) Q_d + \left(\alpha + \frac{1}{3}\right) Q_e,$$

where $\alpha$ is an arbitrary number. It is easy to check that these three U(1) symmetries, $U(1)_Y$, $U(1)_{B-L}$ and $U(1)_D$, are anomaly-free for both the mixed U(1) and non-Abelian gauge groups. As for $U(1)_D$ gauge group, there is no charged chiral matter under this gauge group, so the $U(1)_D$ gauge field can decouple.

In the following, we assume that the $U(1)_{B-L}$ is spontaneously broken at a high energy scale. There is another $U(1)$ gauge symmetry which has a property of Peccei-Quinn symmetry $U(1)_{PQ}$, whose charges for matter and Higgs fields are $-1/2$ and $+1$, respectively. This $U(1)_{PQ}$ symmetry prohibit the so-called $\mu$-term. However the $U(1)_{PQ}$ symmetry as well as the remaining fifth $U(1)$ symmetry are anomalous, and then we assume all the gauge fields of the anomalous $U(1)$s become massive via, e.g., the Green-Schwarz mechanism [17], and decouple from the low-energy physics. Then, it is interesting to survey the possibility of the dynamical generation of the $\mu$-term, although we just assume its existence in the following phenomenological analysis.

### 4 Flavor structures of (s)quarks and (s)leptons

In this section we show that a semi-realistic pattern of quark and lepton mass matrices are realized at a certain point of the (tree-level) moduli space in our model. The hierarchical
structure of the Yukawa couplings is achieved by the wavefunction localization of the matter fields in extra dimensions, whose localization profiles are completely determined by the magnetic fluxes \(\Phi_m\). More interestingly, if we embed the 10D SYM theory, which is the starting point of our model, into 10D supergravity, the flavor structure of the superparticles induced by the moduli-mediated supersymmetry breaking is also fully determined by the wavefunction profile. Therefore the supersymmetric flavor structure of the model can be analyzed based on the effective supergravity action derived through a systematic way proposed in Ref. [12].

The 4D effective action with the \(\mathcal{N} = 1\) local supersymmetry is generally written in terms of 4D \(\mathcal{N} = 1\) conformal supergravity [18] as

\[
S_{\mathcal{N}=1} = \int d^4x \sqrt{-g} C \left[ -3 \int d^4\theta \, \bar{C} C e^{-K/3} + \left\{ \int d^2\theta \left( \frac{1}{4} \sum_a f_a W^{a,\alpha} W^a_\alpha + C^3 W \right) + \text{h.c.} \right\} \right],
\]

where \(K\), \(W\) and \(f_a\) are the effective Kähler potential, the superpotential and the gauge kinetic functions, respectively, as functions of light modes as well as the moduli, and the chiral superfield \(C\) plays a role of superconformal compensator. Here and hereafter, we work in a unit that the 4D Planck scale is unity.

### 4.1 The MSSM sector in the 4D effective theory

The effective Kähler potential \(K\), superpotential \(W\) and gauge kinetic functions \(f_a\) \((a = 1, 2, 3)\) for the MSSM sector of our model on the \(T^6/Z_2\) magnetized orbifold at the leading order are found in the 4D effective action (17) as [12]

\[
K = K^{(0)}(\bar{\Phi}_m, \Phi^m) + Z_{\mathcal{I},\mathcal{J}}^{(Q)}(\bar{\Phi}_m, \Phi^m) \bar{Q}^\mathcal{I} \mathcal{Q}^\mathcal{J},
\]

\[
W = \lambda_{\mathcal{I},\mathcal{J},\mathcal{K}}^{(Q)}(\Phi^m) \mathcal{Q}^\mathcal{I} \mathcal{Q}^\mathcal{J} \mathcal{Q}^\mathcal{K},
\]

\[
f_a = S \quad (a = 1, 2, 3),
\]

where \(\mathcal{Q}^\mathcal{I}\) and \(\Phi^m\) symbolically represents the MSSM matter and the moduli chiral superfields,

\[
\mathcal{Q}^\mathcal{I} = \{Q^I, U^I, D^J, L^J, N^J, E^J, H_u^K, H_d^K\}, \quad \Phi^m = \{S, T_r, U_r\},
\]

respectively, the subscript \(r = 1, 2, 3\) labels the \(r\)th two-dimensional torus \(T^2\) among the factorizable three tori \(T^2 \times T^2 \times T^2\), and traces of the YM-indices are implicit. The explicit expressions of the moduli Kähler potential \(K^{(0)}(\bar{\Phi}_m, \Phi^m)\), the matter Kähler metrics \(Z_{\mathcal{I},\mathcal{J}}^{(Q)}(\bar{\Phi}_m, \Phi^m)\) and the holomorphic Yukawa couplings \(\lambda_{\mathcal{I},\mathcal{J},\mathcal{K}}^{(Q)}(\Phi^m)\) are exhibited in Appendix A.

We assume a certain mechanism of moduli stabilization and supersymmetry breaking that fixes VEVs of moduli superfields,

\[
\langle S \rangle \equiv s + \theta^2 F^S, \quad \langle T_r \rangle \equiv t_r + \theta^2 F^T_r, \quad \langle U_r \rangle \equiv u_r + \theta^2 F^U_r.
\]
Note that these VEVs determine 10D parameters $g$, $\mathcal{A}^{(i)}$ and $\tau_i$ as 
\[
\text{Re } s = g^{-2} \prod_{r=1}^{3} \mathcal{A}^{(r)}, \quad \text{Re } t_r = g^{-2} \mathcal{A}^{(r)}, \quad u_r = i\tau_r, \quad (20)
\]
and then $S$, $T_r$ and $U_r$ are called the dilaton, the Kähler moduli and the complex structure moduli, respectively, in the 4D effective theory. In the following analyses, numerical values of the moduli VEVs as well as the Wilson-lines are scanned phenomenologically.

### 4.2 Quark and lepton masses and mixings

First we analyze the flavor structure of the SM sector in our model. Canonically normalized Yukawa couplings between three generations of the quarks or the leptons and six generations of the Higgs doublets are calculated by

\[
y_{IJ}^{(U)} = \frac{\lambda_{IJK}^{(U)}}{\sqrt{Y_{IJ}^{(Q)} Y_{JK}^{(U)}}} H_a, \quad y_{IJ}^{(D)} = \frac{\lambda_{IJK}^{(D)}}{\sqrt{Y_{IJ}^{(Q)} Y_{JK}^{(D)}}} H_d,
\]

\[
y_{IJ}^{(N)} = \frac{\lambda_{IJK}^{(N)}}{\sqrt{Y_{IJ}^{(L)} Y_{JK}^{(N)}}} \bar{H}_a, \quad y_{IJ}^{(E)} = \frac{\lambda_{IJK}^{(E)}}{\sqrt{Y_{IJ}^{(L)} Y_{JK}^{(E)}}} \bar{H}_d
\]

where $Y_{IJ}^{(Q)}$ represents the superspace wavefunction coefficient of $\bar{Q}^I Q^J$ in the superspace action, which is related to the Kähler metric as

\[
Y_{IJ}^{(Q)} = e^{-K \Phi^m \Phi^m/3} Z_{IJ}^{(Q)}(\Phi^m, \Phi^m).
\]

The above Yukawa coupling (21) possesses the flavor symmetry $\Delta(27)$ due to the choice of the magnetic fluxes (11) selected for the three generations of quarks and leptons with the full-rank Yukawa matrices.

The up- and down-type quark masses ($m_u, m_c, m_t$) and ($m_d, m_s, m_b$), the neutrino Dirac masses ($m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}$) and the charged lepton masses ($m_e, m_\mu, m_\tau$) are the eigenvalues of the $3 \times 3$ mass matrices

\[
y_{IJ}^{(U)} (H_u^K) = y_{IJK}^{(U)} v_u, \quad y_{IJ}^{(D)} (H_d^K) = y_{IJK}^{(D)} v_d,
\]

\[
y_{IJ}^{(N)} (\bar{H}_a^K) = y_{IJK}^{(N)} v_u, \quad y_{IJ}^{(E)} (\bar{H}_d^K) = y_{IJK}^{(E)} v_d,
\]

respectively, that is,

\[
(V_L^{(U)})_I^J y_{IJ}^{(U)} (V_R^{(U)})_J^I = \text{diag} (m_u, m_c, m_t)_i^j / v_u,
\]

\[
(V_L^{(D)})_I^J y_{IJ}^{(D)} (V_R^{(D)})_J^I = \text{diag} (m_d, m_s, m_b)_i^j / v_d,
\]

\[
(V_L^{(N)})_I^J y_{IJ}^{(N)} (V_R^{(N)})_J^I = \text{diag} (m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau})_i^j / v_u,
\]

\[
(V_L^{(E)})_I^J y_{IJ}^{(E)} (V_R^{(E)})_J^I = \text{diag} (m_e, m_\mu, m_\tau)_i^j / v_d,
\]

(22)
where $V_{L,R}^{(Q_y)}$ for $Q_y = U, D, N, E$ are unitary matrices, and $\hat{I}, \hat{J} = 1, 2, 3$ label the (Dirac) mass eigenstates. The Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\text{CKM}} \equiv V_L^{(U)} V_L^{(D)\dagger}$ describes the flavor mixing in the quark sector, whose matrix elements are precisely measured by experiments.

For $v_u = v \sin \beta$, $v_d = v \cos \beta$ and $v = 174$ GeV, we numerically find that the following values of the tan $\beta$ and the VEVs of Higgs fields,

$$\tan \beta = 25$$

$$\langle H_u^K \rangle = (0.0, 0.0, 2.7, 1.3, 0.0, 0.0) v_u \times \mathcal{N}_{H_u},$$

$$\langle H_d^K \rangle = (0.0, 0.1, 5.8, 5.8, 0.0, 0.1) v_d \times \mathcal{N}_{H_d},$$

and those of the geometric moduli (20) as well as the Wilson-line parameters (16),

$$\pi s = 6.0,$$

$$(t_1, t_2, t_3) = (3.0, 1.0, 1.0) \times 2.8 \times 10^{-8},$$

$$(\tau_1, \tau_2, \tau_3) = (4.1i, 1.0i, 1.0i),$$

$$\left(\zeta_Q, \zeta_U, \zeta_D, \zeta_L, \zeta_N, \zeta_E\right) = (1.0i, 1.9i, 1.4i, 0.7i, 2.2i, 1.7i),$$

yield a semi-realistic pattern of the quark and the charged lepton masses as well as the CKM matrix at the electroweak (EW) scale shown in Table 2. The normalization factors $\mathcal{N}_{H_u} = 1/\sqrt{2.7^2 + 1.3^2}$ and $\mathcal{N}_{H_d} = 1/\sqrt{2(0.1^2 + 5.8^2)}$ in Eq. (24) are factorized just for convenience later in Eq. (27).

Here, we assume some nonperturbative effects [22] and/or higher-dimensional operators that effectively generate supersymmetric mass terms,

$$W_{\text{eff}} = \mu_{KL} H^K_u H^L_d.$$  

Because VEVs of these Higgs fields shown in Eq. (24) generate a semi-realistic pattern of the quark and the lepton masses and their mixing angles, here we consider the case that the supersymmetric mass parameters $\mu_{KL}$ are aligned in such a way that

$$\sum_{K,L} (U_{H_u})^K_{\hat{K}} \mu_{KL} (U^\dagger_{H_d})^L_{\hat{L}} = \delta_{\hat{K} \hat{L}} \mu_{\hat{K}},$$

$$\left|\mu_{\hat{K} = 1}\right| \ll M_{\text{GUT}} \lesssim |\mu_{\hat{K} \neq 1}|,$$

are satisfied with unitary matrices $U_{H_{u,d}}$, where $\hat{K}, \hat{L} = 1, 2, \ldots, 6$ label the supersymmetric mass eigenstates diagonalizing $\mu_{KL}$, and the VEVs $\langle H^K_{u,d} \rangle$ represent those shown in Eq. (24).

In this case, five of the six Higgs doublets other than $H^K_{u,d = 1}$ decouple from the light modes due to the heavy supersymmetric masses $\mu_{\hat{K} \neq 1}$. In the following, the numerical value of the $\mu$-parameter $\mu \equiv \mu_{\hat{K} = 1}$ is determined so that the EW symmetry is broken successfully yielding the observed masses of the $W$ and the $Z$ bosons, and then the masses and the mixing angles of quarks and leptons shown in Tables 2 and 3 are realized.

The VEV of dilaton $\pi s = 6.0$ yields the unified gauge couplings $4\pi/g_a^2 = 24$ at the GUT scale $M_{\text{GUT}} = 2.0 \times 10^{16}$ GeV, that is implemented in the MSSM with low energy data. We
Table 2: Numerical values of the quark masses \((m_u, m_c, m_t)\), \((m_d, m_s, m_b)\) and the charged lepton masses \((m_e, m_\mu, m_\tau)\) as well as the absolute values of the elements in the CKM matrix \(V_{\text{CKM}}\) at the EW scale, evaluated at a sample point in the moduli space of the 10D SYM theory identified by the magnetic fluxes \((11)\), the Wilson-lines and the VEVs of the moduli \((25)\) and the Higgs fields \((24)\). The experimental data \([21]\) are also shown. All the mass scales are measured in the unit of GeV.

| \(m_i, m_j, m_k\) | Sample values | Observed |
|-------------------|---------------|----------|
| \((m_u, m_c, m_t)\) | \(3.1 \times 10^{-3}, 1.01, 1.70 \times 10^{2}\) | \(2.3 \times 10^{-3}, 1.28, 1.74 \times 10^{2}\) |
| \((m_d, m_s, m_b)\) | \(2.8 \times 10^{-3}, 1.48 \times 10^{-1}, 6.46\) | \(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18\) |
| \((m_e, m_\mu, m_\tau)\) | \(4.68 \times 10^{-4}, 5.76 \times 10^{-2}, 3.31\) | \(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78\) |

\[
|V_{\text{CKM}}| = \begin{pmatrix}
0.98 & 0.21 & 0.0023 \\
0.21 & 0.98 & 0.041 \\
0.011 & 0.040 & 1.0 \\
\end{pmatrix}
\]

select the overall magnitudes of \(t_e\) so that the compactification scale, i.e., the mass scale of the lightest Kaluza-Klein mode becomes as high as \(M_{\text{GUT}}\), and their ratios are defined to preserve supersymmetry conditions \((12)\). Here, the running of the parameters from \(M_{\text{GUT}}\) to the EW scale is evaluated by the one-loop renormalization group (RG) equations of the MSSM. From Table 2 we find that the observed hierarchies among three generations of quarks and charged leptons are realized even with the above non-hierarchical VEVs of fields \((24)\) and \((25)\). It is quite interesting and suggestive that the complicated flavor structure of our real world could be realized at a certain point in the (tree-level) moduli space of the 10D SYM theory, whose action is simply given by Eq. \((1)\) at the leading order in a rigid limit.

In addition, if we assume some nonperturbative effects \([22]\) or higher-dimensional operators effectively generate Majorana masses\(^4\) for the right-handed neutrino \(N^J\) in the superpotential such as

\[
W_{\text{eff}} = M_{IJ}^{(N)} N^I N^J,
\]

a numerical value of the Majorana mass matrix \(M^{(N)}\) is found as

\[
M^{(N)} = \begin{pmatrix}
1.1 & 1.3 & 0 \\
1.3 & 0 & 3.2 \\
0 & 3.2 & 1.8 \\
\end{pmatrix} \times 10^{12} \text{ GeV},
\]

that yields a semi-realistic pattern of the neutrino masses and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix \([20]\) at the EW scale shown in Table 3.

\(^4\) Note that the Majorana mass term violates the flavor symmetry of the Yukawa couplings mentioned above. This fact can be a guiding principle for identifying an origin of the term.
Table 3: Numerical values of the neutrino masses \( (m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \) as well as the absolute values of the elements in the PMNS matrix \( V_{PMNS} \) at the EW scale, evaluated at the same sample point in the moduli space as in Table 2 but with the Majorana masses (29). The experimental data [21] are also shown. All the mass scales are measured in the unit of GeV.

### 4.3 Soft supersymmetry breaking terms

The low energy features of the superparticles in our model are governed by soft supersymmetry breaking parameters, namely, the gaugino masses \( M_a \), the scalar masses \( (m^2_{\tilde{\phi}})_{IJ} \) and the scalar trilinear couplings \( A^{(Q_y)}_{IJK} \) (normalized by the corresponding Yukawa couplings), those appear in the soft supersymmetry breaking terms as

\[
L_{soft} = -\frac{1}{2} M_a \chi^a \chi^a - (m^2_{\tilde{\phi}})_{IJ} \tilde{Q}^I \tilde{Q}^J - \frac{1}{6} \sum_{\{Q_y\}} y^{(Q_y)}_{IJK} A^{(Q_y)}_{IJK} \tilde{Q}^I \tilde{Q}^J \tilde{Q}^K + \text{h.c.,}
\]

where the superscript \( Q_y \) represents \( Q_y = U, D, N, E \) and \( (Q_L, Q_R, Q_H) = (Q, U, H_u), (Q, D, H_d), (L, N, H_u) \) and \( (L, E, H_d) \) for \( Q_y = U, D, N \) and \( E \), respectively. The tilded fields \( \tilde{Q}^I \) denote the scalar fields,

\[
\tilde{Q}^I = \{ \tilde{q}^I, \tilde{u}^I, \tilde{d}^I, \tilde{l}^I, \tilde{e}^I, \tilde{\nu}^I, \tilde{h}^K_u, \tilde{h}^K_d \},
\]

those are the lowest components of the chiral superfields \( \tilde{Q}^I \) in the \( \theta \) and the \( \tilde{\theta} \) expansion. Note that only the direction with \( \tilde{K} = 1 \) remains light in the Higgs sector of \( H_u^K \) and \( H_d^K \). The so-called \( B \)-term also appears as the soft supersymmetry breaking term. In the following, its value is determined numerically such that the EW symmetry is broken successfully.

The explicit moduli dependence of the Kähler and the superpotential (18) in the MSSM sector allows us to determine moduli-mediated contributions [23] to the soft supersymmetry breaking parameters (induced by nonvanishing \( F \)-components of \( S, T_r \) and \( U_r \)) as well as the anomaly-mediated one [24] (induced by a nonvanishing \( F \)-component of \( C \)). These contributions are summarized as [25]

\[
M_a = F^m \partial_m \ln(Re f_a) + \frac{b_a g_a^2 F^C}{8\pi^2 C_0},
\]
\( (m^2)^{IJ}_Q = -F^m \bar{F}^n \partial_m \partial_n \ln \gamma_Q^{(Q)} - \frac{\delta_{IJ}}{32\pi^2} \frac{d\gamma_Q^2}{d\ln \mu} \left| \frac{F^C}{C_0} \right|^2 + \frac{\delta_{IJ}}{16\pi^2} \left( \frac{F^m}{C_0} \right)^2 F^m \partial_m \gamma_Q^J + \text{h.c.} \),

\[ A^{(Q_J)}_{IJK} = -F^m \partial_m \ln \left( \frac{\lambda^{(Q)_I}_{JJK}}{\gamma^{(Q)_L}_{JL} \gamma^{(Q)_n}_{KK}} \right) - \frac{\gamma_Q^L + \gamma_Q^R}{16\pi^2} \frac{F^C}{C_0} \gamma_Q^J, \quad (30) \]

where \( \gamma_Q^J \) is the anomalous dimension of \( Q^J \), and \( F^m \) represents \( F \)-components of moduli superfields, that is,

\[ F^m = \{ F^S, F^T_r, F^U_r \}, \]

while \( C_0 \) and \( F^C \) are the lowest and the \( \theta^2 \) components of \( C \), respectively, in the \( \theta \) and the \( \bar{\theta} \) expansion. Here, we fix the dilatation symmetry by \( C_0 = \exp(K|_{\theta=\bar{\theta}=0}/6) \) that corresponds to the Einstein frame.

In the following, we study phenomenological aspects of our model at low energies, in the case that the above soft parameters are dominated by the moduli- and the anomaly-mediated contributions and the other contributions (such as the gauge-mediated one that is further model dependent) are negligible, by assuming a certain moduli stabilization and a supersymmetry breaking mechanism outside the MSSM sector that cause such a situation.

### 5 Phenomenological aspects at low energies

It has been found that the three generations of quarks and leptons are obtained from the degeneracy of chiral zero-modes due to the magnetic fluxes, yielding consequently the six-generations of up- and down-type Higgs doublets. Furthermore, a semi-realistic pattern of the quark and the charged lepton masses and the CKM mixings can be realized as shown in Tables at a certain point in the moduli space of the 10D SYM theory where the numerical values of the Higgs and the moduli VEVs as well as the Wilson-line parameters are given as shown in Eqs. (24) and (25).

The undetermined parameters so far are supersymmetry breaking order parameters \( F^m = \{ F^S, F^T_r, F^U_r \} \) and \( F^C \) mediated by moduli and compensator chiral multiplets \( \Phi^m = \{ S, T_r, U_r \} \) and \( C \), respectively. As a representative scale of the supersymmetry breaking \( M_{SB} \), we refer the \( F \)-component of the dilaton superfield \( S \),

\[ M_{SB} \equiv \sqrt{K_{SS}} F^S, \]

and define ratios,

\[ R^T_r = \frac{\sqrt{K_{TT_r}} F^T_r}{M_{SB}}, \quad R^U_r = \frac{\sqrt{K_{UU_r}} F^U_r}{M_{SB}}, \quad R^C = \frac{1}{4\pi^2} \frac{F^C/C_0}{M_{SB}}. \quad (31) \]

Here, we assume that CP phases of \( F^S, F^T_r, F^U_r \) and \( F^C \) are the same, and \( R^T_r, R^U_r \) and \( R^C \) are real. Then, there is no physical CP violation due to supersymmetry breaking terms. Otherwise, there would be a strong constraint on CP violation in the soft supersymmetry breaking terms.
As shown in Eq. (18), the gauge kinetic functions depend on only the dilaton superfield $S$ at the tree level in the (leading order) effective supergravity action. Then, the gaugino masses shown in Eq. (30) are determined by $F^S$ and $F^C$ at the compactification scale independently of $R^T_r$ and $R^U_r$. On the other hand, the lower bound on the gluino mass $M_3 \gtrsim 860$ GeV is found from the recent LHC data [27]. In the following we analyze phenomenological features of our model for $M_{SB} = 1$ TeV satisfying the above condition. By varying the ratios $R^T_r$, $R^U_r$ and $R^C$, we show phenomenological aspects of our model, especially, typical sizes of the flavor violations caused by superparticles.

Hereafter, we neglect all the Yukawa couplings except for those involving only the third generations, $y^{(U)}_{33}$, $y^{(D)}_{33}$, $y^{(N)}_{33}$ and $y^{(E)}_{33}$, for a numerical performance, when we evaluate soft parameters and their RG running. We also reevaluate accordingly the RG runnings of Yukawa couplings in this approximation that was not adopted in the analysis of quark and lepton masses and mixings.

### 5.1 Supersymmetric flavor violations

In models with a low-energy supersymmetry breaking, the flavor violations such as FCNCs caused by superparticles are severely constrained by the experiments. As measures of such supersymmetric flavor violations in our model, we adopt so-called mass insertion parameters [28], those we define as

$$
\delta_{LR}^{(Q_y)}_{IJ} \equiv \frac{v_f \left( V_L^{(Q_y)} a^{(Q_y)} \right)^\dagger V_R^{(Q_y)} \right)_{IJ}}{\sqrt{\left( V_L^{(Q_y)} m_{\tilde{Q}_L}^2 V_L^{(Q_y)} \right)^{II} \left( V_R^{(Q_y)} m_{\tilde{Q}_R}^2 V_R^{(Q_y)} \right)^{JJ}}} ,
$$

$$
\delta_{LL}^{(Q_y)}_{IJ} \equiv \frac{\left( V_L^{(Q_y)} m_{\tilde{Q}_L}^2 V_L^{(Q_y)} \right)^{IJ}}{\sqrt{\left( V_L^{(Q_y)} m_{\tilde{Q}_L}^2 V_L^{(Q_y)} \right)^{II} \left( V_L^{(Q_y)} m_{\tilde{Q}_L}^2 V_L^{(Q_y)} \right)^{JJ}}} ,
$$

$$
\delta_{RR}^{(Q_y)}_{IJ} \equiv \frac{\left( V_R^{(Q_y)} m_{\tilde{Q}_R}^2 V_R^{(Q_y)} \right)^{IJ}}{\sqrt{\left( V_R^{(Q_y)} m_{\tilde{Q}_R}^2 V_R^{(Q_y)} \right)^{II} \left( V_R^{(Q_y)} m_{\tilde{Q}_R}^2 V_R^{(Q_y)} \right)^{JJ}}} ,
$$

where $V_{L,R}^{(Q_y)}$ are given in Eq. (22), the matrices $(a^{(Q_y)})_{IJ} \equiv y^{(Q_y)}_{iJ,K=1} A_{ij,K=1}^{(Q_y)}$ originate from the scalar trilinear couplings, and $i$ labels the eigenstates defined by Eq. (27). The superscripts $Q_y$ represent $Q_y = U, D, N, E$ indicating the corresponding subscripts $\tilde{Q}_L = \tilde{q}, \tilde{\bar{q}}, \tilde{l}, \tilde{\bar{l}}$ and $\tilde{Q}_R = \tilde{u}, \tilde{\bar{u}}, \tilde{d}, \tilde{\bar{d}}$, respectively.

The $R^U_1$ dependence of the mass insertion parameters is shown in Figs. 1, 2, 3 and 4. Each parameter is constrained by various experiments, and most of them are free from these constraints in our model with $M_{SB} = 1$ TeV. However only one of those, the upper bound of

\footnote{A moduli mixing in the gauge kinetic functions could occur due to higher-order corrections if the SYM theory originates, e.g., from (magnetized) D-branes [25] that is beyond the scope of this paper.}
Figure 1: The mass insertion parameters \( (\delta_{LR,LL,RR})_{IJ} \) as a function of \( R^U \) evaluated at the same sample point in the moduli space as in Table 2 with the fixed values of \( R^U_{r \neq 1} = 0.9, R^T_r = 1 \) and \( M_{SB} = 1 \) TeV.

\[ \delta^U_{LR,LL,RR} (R^C=0) \]

\[ \delta^U_{LR,LL,RR} (R^C=1.5) \]

\[ |\delta^U_{LL,12}| \]

\[ |\delta^U_{RR,12}| \]

\[ |\delta^U_{LR,12}| \]

\[ -1.0 -0.5 \]

\[ 0.5 1.0 \]

\[ -1.0 -0.5 \]

\[ 0.5 1.0 \]

\[ 0.00005 \]

\[ 0.00010 \]

\[ 0.00015 \]

\[ \Delta U_{LL,RR,LR} / \left( R_C = 0 \right) \]

\[ \Delta U_{LL,RR,LR} / \left( R_C = 1.5 \right) \]

\[ \mu \rightarrow e\gamma \] transitions [28], is very severe as shown in Fig. 4. From this figure, we find that the value of \( F^U_1 \) is severely restricted, that is, the amount of supersymmetry breaking mediated by the complex-structure (shape) modulus of the first torus, \( U_1 \), must be extremely small. That is expected from the fact that only the \( U_1 \) distinguishes the flavors (the differences between the wavefunction profiles of chiral matter fields on the first torus) as can be seen in the expressions of Yukawa couplings (21). On the other hand, all the other moduli \( S, T_r, U_{r \neq 1} \) can mediate sizable supersymmetry breaking without conflicting with the experimental data concerning supersymmetric flavor violations. These flavor violations become smaller for larger values of \( M_{SB} \) due to the decoupling effect of the superparticles.

### 5.2 A typical superparticle spectrum

We show a typical superparticle spectrum at the EW scale by varying \( R^C \) with fixed values of \( M_{SB}, R^U, R^T \) and \( \tan \beta \) in Fig. 5. The supersymmetry breaking scale is again fixed as \( M_{SB} = 1 \) TeV. Because the value of \( R^U_{r=1} \) is severely constrained as shown in Fig. 4, an allowed small value \( R^U_{r=1} = -0.05 \) is chosen, while the ratio \( R^U_{r \neq 1} \) and \( R^T_r \) does not affect the spectrum so much and then \( R^U_{r \neq 1} = 0.9 \) and \( R^T_r = 1 \) \( (r = 1, 2, 3) \) are adopted here. As mentioned previously, the \( \mu \)-parameter is fixed in such a way that the EW symmetry is broken successfully yielding the observed masses of \( W \) and \( Z \) bosons. Curves describing some soft scalar masses in Fig. 5 are terminated at \( R^C \sim 1.6 \), because the EW symmetry is not broken successfully with \( R^C > 1.6 \).

A mediation mechanism of supersymmetry breaking, which is a sizable mixture of the modulus and the anomaly mediation, namely \( R^C \sim \mathcal{O}(1) \), is called the mirage mediation [29]. Especially, the mass spectrum with \( R^C \sim 1.6 \) in our model, where the gaugino masses and scalar masses respectively degenerate at the TeV scale, resembles that of the TeV scale mirage mediation model [30]. It is pointed out in this model that the notorious fine-tuning between supersymmetric and supersymmetry breaking parameters in the MSSM is dramatically alleviated.

As for the lightest superparticle in the above spectrum, we find that it is a neutralino. The eigenvalues of the neutralino and the chargino masses measured in the unit of GeV are listed
So far, we have considered the scenario with a low-energy supersymmetry breaking, and selected a small value $R_1^U = -0.05$ to be consistent with the experimental data concerning supersymmetric flavor violations. If we consider the case with larger values of $M_{SB}$, with which the flavor violations become smaller, the values of $R_1^U$ can reside in much wider region. However, there are other two factors restricting the values of $R_1^U$ besides those from FCNCs. One is related to the success of the EW symmetry breaking, and the other is related to obtaining non-tachyonic masses. We show the $R_1^U$ dependence of the masses of sfermions with $R^C = 1.5$ and $M_{SB} = 1$ TeV in Fig 6. In the figure, some curves are terminated at $R_1^U \sim \pm 0.2$ because the EW symmetry is not broken successfully for $|R_1^U| \gtrsim 0.2$, as the situation in Fig 5. We find that $R_1^U$ has to be in the range $|R_1^U| < 0.2$, where we obtain non-tachyonic masses. With other values of $R^C$ and $M_{SB}$, it is possible that the non-tachyon condition is more severe than the other. In some typical cases with $(R^C, M_{SB}/\text{TeV}) = (0, 1), (0, 10), (1.5, 1)$ and $(1.5, 10)$, we also find that the allowed region of the ratio $R_1^U$, where the EW symmetry is broken successfully and non-tachyonic masses are obtained, is roughly $|R_1^U| < 0.2$. That has to be in mind, especially when one considers larger values of $M_{SB}$.
Finally, we comment on the Higgs sector. In our model, there are some possibilities to obtain the mass of the lightest CP-even Higgs boson $m_h \sim 125$ GeV, which is indicated from the recent observations at the LHC [31]. First of all, as is well known, we can easily realize $m_h \sim 125$ GeV with $M_{SB} \sim 10$ TeV. The supersymmetric flavor violations are much smaller in this case than those we have studied above for $M_{SB} = 1$ TeV, and the bound on $R^U_1$ from the FCNCs disappears. Then, in this case $|R^U_1| < 0.2$ is suggested.

The second possibility is that, we can consider the next-to MSSM in some extensions of our model where $m_h \sim 125$ GeV could be realized with a low scale supersymmetry breaking $M_{SB} \sim 1$ TeV (see for review e.g. Ref. [32]). In this case, the supersymmetric flavor violations and the superparticle spectrum estimated above can be applied straightforwardly. Some analyses of such an extended Higgs sector in the TeV-scale mirage mediation models are performed in Ref. [33].

Besides these two, there is one more interesting possibility. Although we have worked on the 10D SYM theory in this paper, it would be straightforward to extend our model to SYM theories in a lower-than-ten dimensional spacetime, or even to the mixture of SYM theories with a different dimensionality. For example, in type IIB orientifolds, our model will be adopted not only to the magnetized D9 branes (a class of which is T-dual to intersecting D6 branes in IIA side), but also to the D5-D9 [34] and the D3-D7 brane configurations with magnetic fluxes in the extra dimensions. An interesting possibility is that the $SU(3)_C$ and the $SU(2)_L$ gauge groups of the MSSM originate from different branes with a different dimensionality, and then the moduli-dependence of the gauge kinetic functions are different by the gauge groups, that can cause nonuniversal gaugino masses at the tree level in the effective supergravity action. The situation may allow $m_h \sim 125$ GeV just within the MSSM with a low scale supersymmetry breaking without a severe fine-tuning [35].

Even in this case the same flavor structures in the MSSM sector would be realized as those in the 10D model presented in this paper, if these two branes share a single magnetized torus $T^2$ of the same structure as the first torus ($r = 1$) in our 10D model. Furthermore, the mixed brane configurations may allow an introduction of the supersymmetry-breaking branes sequestered from the visible sector, which coincide with the flavor structure derived in this paper. The model
Figure 4: The mass insertion parameters $(\delta^{E}_{LR})_{ij}$ as a function of $R^U_1$ evaluated at the same sample point in the moduli space as in Table 2 with the fixed values of $R^U_{r \neq 1} = 0.9$, $R^T_r = 1$ and $M_{SB} = 1 \text{ TeV}$. The horizontal dashed lines in the lower panels represent a typical value of the experimental upper bound restricting FCNCs that enhances $\mu \to e\gamma$ transitions [28].

building based on such mixed brane configurations will be reported in separate papers [36].

6 Conclusions and discussions

We have constructed a three-generation model of quark and lepton chiral superfields based on a toroidal compactification of the 10D SYM theory with certain magnetic fluxes in extra dimensions preserving a 4D $\mathcal{N} = 1$ supersymmetry. The low-energy effective theory contains the MSSM particle contents, where the numbers of chiral generations are determined by the numbers of the fluxes they feel, and the most massless exotics can be projected out by a combinatory effect of the magnetic fluxes and a certain orbifold projection.

We find that a semi-realistic pattern of the quark and the charged lepton masses and the CKM mixings is realized at a certain sample point in the (tree-level) moduli space of the 10D SYM theory, where the VEVs of the six Higgs doublets and of the geometric moduli as well as the Wilson-line parameters take reasonable numerical values without any hierarchies. In addition, it has been shown that a semi-realistic pattern of the neutrino masses and the PMNS mixings can be achieved at the same point of the moduli space, if we assume the existence of certain effective superpotential terms (28), those would be induced by nonperturbative effects and/or higher-order corrections. We have assumed the existence of such nonperturbative effects
Figure 5: The masses of the gauginos (the top panel) and of the sfermions (the bottom panel) at the EW scale as functions of $R_C$ evaluated at the same sample point in the moduli space as in Table 2 with fixed values of $R^U_1 = -0.05$, $R^U_{r\neq 1} = 0.9$, $R^T_r = 1$ and $M_{SB} = 1$ TeV.

and/or higher-order corrections making the remaining massless exotics heavy enough and also generating effectively the neutrino Majorana mass term (28) as well as the mu-term (26). Further studies are required to find the concrete origin of these effects.

Thanks to the systematic way of the dimensional reduction in a 4D $\mathcal{N} = 1$ superspace proposed in Ref. [12], the soft supersymmetry breaking parameters induced by the moduli-mediated supersymmetry breaking are calculated explicitly. Because the flavor structures of our model are essentially determined by the localized wavefunctions of the chiral zero-modes, the 4D effective theory possesses flavor dependent holomorphic Yukawa couplings and flavor independent K"ahler metrics for the MSSM matter fields. Under the assumption that the moduli-mediated low scale supersymmetry breaking dominates the soft supersymmetry breaking terms in the MSSM, we estimated the size of supersymmetric flavor violations by analyzing the mass insertion parameters governing various FCNCs, scanning supersymmetry breaking order parameters mediated by the dilaton, the geometric moduli and the compensator chiral superfields in the 4D $\mathcal{N} = 1$ effective supergravity.

The most stringent bound comes from the $\mu \to e\gamma$ on the size of the $F$-term in the chiral
Figure 6: The masses of the sfermions at the EW scale as functions of $R_U^1$ evaluated at the same sample point in the moduli space as in Table 2 with fixed values of $R_C^1 = 1.5$, $R_U^r = 0.9$, $R_T^r = 1$ and $M_{SB} = 1$ TeV.

multiplet of the complex structure modulus of the first torus where the SM flavor structure is generated via the wavefunction localization. The result provides a strong insight into the mechanism of moduli stabilization in our model. For instance, a mechanism of the moduli stabilization proposed by Ref. [37] would be suitable, that predicts vanishing $F$-terms of the complex structure moduli [25, 38] at the leading order. Therefore, it would be interesting to study a mechanism of the moduli stabilization and the supersymmetry breaking at a Minkowski minimum [37] by minimizing the moduli and the hidden-sector potential generated by some combinations [39] of nonperturbative effects and a dynamical supersymmetry breaking [40].

In our model, there are some possibilities to realize the mass of the lightest CP-even Higgs boson to be consistent with the recent observations at the LHC [31]. As mentioned at the end of Sec. 5 especially, it is very interesting to consider the D5-D9 [34] and the D3-D7 brane configurations with magnetic fluxes in the extra dimensions. With such brane configurations, we will be able to build more realistic models in which we can study concretely the Higgs sector as well as the supersymmetry-breaking sector, the mechanism of moduli stabilization and so on. Even in this case the same flavor structures would be realized as those in the 10D model presented in this paper, if these two branes share a single magnetized torus $T^2$ of the same structure as the first torus ($r = 1$) in our 10D model. The model building based on such mixed brane configurations will be reported in separate papers [36].

We have studied on the tree-level 4D effective theory of massless modes. Recently, massive modes were studied in Ref. [41]. They may have phenomenologically important effects on 4D effective theory. For example, the Kähler potential, superpotential and gauge kinetic functions would have threshold corrections due to massive modes and such corrections may affect the soft supersymmetry breaking terms. Thus, it is important to study such effects, although that is beyond our scope of this paper.
A Kähler metrics and holomorphic Yukawa couplings

The Kähler potential $K$, the superpotential $W$ and the gauge kinetic functions $f_a$ in the 4D effective supergravity action of our model are derived as \cite{12}

\begin{align*}
K &= K^{(0)}(\Bar{\Phi}^m, \Phi^m) + Z^{(Q)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{Q}^I \bar{Q}^J \\
&= K^{(0)}(\Bar{\Phi}^m, \Phi^m) + Z^{(Q)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{Q}^I \bar{Q}^J + Z^{(U)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{U}^I U^J + Z^{(D)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{D}^I D^J \\
&+ Z^{(L)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{L}^I L^J + Z^{(N)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{N}^I N^J + Z^{(E)}_{I,J}(\Bar{\Phi}^m, \Phi^m) \Bar{E}^I E^J \\
&+ Z^{(H_3)}_{K,L}(\Bar{\Phi}^m, \Phi^m) \Bar{H}_u^K H_u^L + Z^{(H_3)}_{K,L}(\Bar{\Phi}^m, \Phi^m) \Bar{H}_d^K H_d^L, \\
W &= \lambda_{IJK}^{(Q)}(\Phi^m)^{Q^I Q^J} Q^K \\
&= \lambda_{IJK}^{(U)}(\Phi^m) \bar{Q}^I U^J H_u^K + \lambda_{IJK}^{(D)}(\Phi^m) \bar{D}^I D^J H_d^K \\
&+ \lambda_{IJK}^{(N)}(\Phi^m) \bar{L}^I N^J H_u^K + \lambda_{IJK}^{(E)}(\Phi^m) \bar{E}^I E^J H_d^K, \\
f_a &= S \quad (a = 1, 2, 3),
\end{align*}

respectively, where $Q^I$ and $\Phi^m$ symbolically represents the MSSM matter and the moduli chiral superfields as shown in Eq. \cite{19}, $I, J = 1, 2, 3$ and $K, L = 1, 2, \ldots, 6$ label generations, and traces of the YM-indices are implicit.

In the Kähler potential, the YM-field independent part $K^{(0)}(\Bar{\Phi}^m, \Phi^m)$ is given by

\[K^{(0)}(\Bar{\Phi}^m, \Phi^m) = - \ln(S + \bar{S}) - \sum_r \ln(T_r + \bar{T}_r) - \sum_r \ln(U_r + \bar{U}_r),\]

and the Kähler metrics of chiral matters as functions of moduli are found as

\begin{align*}
Z^{(Q_L)}_{I,J}(\Bar{\Phi}^m, \Phi^m) &= \delta_{I,J} \frac{1}{\sqrt{3}}(T_2 + \bar{T}_2)^{-1}(U_1 + \bar{U}_1)^{-1/2}(U_2 + \bar{U}_2)^{-1/2} \exp \frac{4\pi}{3}(\Im \zeta_{Q_L})^2 \\
Z^{(Q_R)}_{I,J}(\Bar{\Phi}^m, \Phi^m) &= \delta_{I,J} \frac{1}{\sqrt{3}}(T_3 + \bar{T}_3)^{-1}(U_1 + \bar{U}_1)^{-1/2}(U_3 + \bar{U}_3)^{-1/2} \exp \frac{4\pi}{3}(\Im \zeta_{Q_R})^2 \\
Z^{(Q_H)}_{I,J}(\Bar{\Phi}^m, \Phi^m) &= \delta_{I,J} \sqrt{6}(T_1 + \bar{T}_1)^{-1} \left\{ \prod_{r=1}^{3}(U_r + \bar{U}_r)^{-1/2} \right\} \exp \frac{-4\pi}{6}(\Im \zeta_{Q_H})^2.
\end{align*}
where $Q_L = \{Q, L\}$, $Q_R = \{U, D, N, E\}$ and $Q_H = \{H_u, H_d\}$, and the Wilson-line parameters $\zeta_{Q_L}$, $\zeta_{Q_R}$ and $\zeta_{Q_H}$ are defined in Eq. (16).

On the other hand, in the superpotential, the holomorphic Yukawa couplings of chiral matters as functions of moduli are given by

$$
\lambda_{IJK}^{(Q_y)}(\Phi^m) = \sum_{m=1}^{6} \delta_{I+J+3(m-1), K} \vartheta \left[ \frac{3(I-J)+9(m-1)}{54} , 0 \right] \left( 3(\bar{\zeta}_{Q_L} - \bar{\zeta}_{Q_R}) , 54iU_1 \right),
$$

where the superscript $Q_y$ represents $Q_y = U, D, N, E$ and these also indicate corresponding subscripts $Q_L = Q, Q, L, L$ and $Q_R = U, D, N, E$, respectively, and $\vartheta$ represents the Jacobi theta-function:

$$
\vartheta \left[ \begin{array}{c} a \\ b \end{array} \right] (\nu, \tau) = \sum_{l \in \mathbb{Z}} e^{\pi i(a+l)^2 \tau} e^{2\pi i(a+l)(\nu+b)}.
$$

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