Investigation of Langdon effect on the nonlinear evolution of SRS from the early-stage inflation to the late-stage development of secondary instabilities

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In a laser-irradiated plasma, the Langdon effect can result in a super-Gaussian electron energy distribution function (EEDF), imposing significant influences on the stimulated backward Raman scattering (SRS). In this work, the influence of a super-Gaussian EEDF on the nonlinear evolution of SRS is investigated by three wave model simulation and Vlasov-Maxwell simulation for plasma parameters covering a wide range of $\kappa \lambda_{pe}$ from 0.19 to 0.48 at both high and low intensity laser drives. In the early-stage of SRS evolution, it is found that besides the kinetic effects due to electron trapping [Phys. Plasmas 25, 100702 (2018)], the Langdon effect can also significantly widen the parameter range for the absolute growth of SRS, and the time for the absolute SRS to reach saturation is greatly shortened by Langdon effect within certain parameter region. In the late-stage of SRS, when secondary instabilities such as decay of the electron plasma wave to beam acoustic modes, rescattering, and Langmuir decay instability become important, the Langdon effect can influence the reflectivity of SRS by affecting the secondary processes. The comprehension of Langdon effect on nonlinear evolution and saturation of SRS would contribute to a better understanding and prediction of SRS in inertial confinement fusion.

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I. INTRODUCTION

In laser-driven inertial confinement fusion (ICF), stimulated backward Raman scattering (SRS) is one important laser plasma instability (LPI), where an incident electromagnetic wave (EMW) resonantly decays into a backscattered EMW and a forward propagating electron plasma wave (EPW) [1]. SRS needs to be suppressed in ICF, since the backscattered light can take energy away from the incident laser, and the hot electrons generated by EPW of SRS can preheat the capsule [2].

Currently, the study of SRS usually assumes a Maxwellian electron energy distribution (EEDF) [3, 4]. However, for the laser-irradiated plasma in ICF, the dominant heating mechanism is inverse bremsstrahlung (IB) heating. The Langdon effect [5, 6], i.e., the EEDF tends towards a super-Gaussian form when the IB heating rate exceeds the electron thermalization rate, can become quite important under typical hohlraum conditions. It is found that in the linear convective regime, the Langdon effect can significantly enhance the gain of SRS and also leads to a shift in the scattered wavelength [7]. So it is necessary to take the Langdon effect into account in the investigation of SRS. In reality, for the widely spreading range of plasma parameters and laser intensity for typical ICF, there exists some zones where the SRS can grow absolutely with high reflectivity over a short distance [8–11], and the nonlinear effects become important to the evolution of SRS. Then, it is conceivable that both the absolute threshold and the nonlinear saturation of SRS can be affected by the Langdon effect. Nevertheless, the investigation of Langdon effect on the absolute SRS and its nonlinear behavior, which is quite important for the global understanding and proper modeling of SRS in ICF, is still lacking.

In this work, the influence of Langdon effect on the nonlinear evolution of SRS is investigated for a wide range of plasma parameters at both high and low-intensity laser drives. In the early growth stage of SRS, it is found that apart from the kinetic effect due to trapped electrons [12–14], Langdon effect can broaden the parameter range for absolute SRS modes, and significantly shorten the saturation time for the absolute SRS in certain parameter region. Over a long timescale evolution with the development of secondary instabilities such as the decay of the electron plasma wave to beam acoustic modes (BAM) [15–17], rescattering of the primary scattered wave [18, 19] and Langmuir decay instability (LDI) [20–22], the SRS typically demonstrates a change in dominant saturation mechanism accompanied by a change (usually drop) in the reflectivity. Under many circumstances, Langdon effect is found to be important to these secondary instabilities and thus to the nonlinear saturation mechanism and reflectivity of SRS in the late evolution stage.

This paper is organized as follows: In Section II, a three wave coupling model with consideration of the Langdon effect is given, as well as some theoretical analysis of SRS in the presence of Langdon effect. Besides, the physics model of a Vlasov-Maxwell code (VlaMaxW) is presented to account for both the Langdon effect and nonlinear kinetic effects. In Section III, the influences of super-Gaussian EEDFs on the nonlinear evolution of SRS are investigated for different stages. In Section IV, the conclusions and some discussions are given.

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II. MODELS WITH LANGDON EFFECT CONSIDERED

In SRS, the matching condition requires [1]
\[
\begin{align*}
\omega_0 &= \omega_s + \omega_t, \\
k_0 &= k_s + k_t,
\end{align*}
\]
where \(\omega_i\) and \(k_i\) are the frequencies and wavenumbers with the subscripts \(i = s, t\) for the pump wave, the scattered wave and the EPW, respectively. When the three waves of SRS are dominated by coherent single modes, the temporal-spatial evolution of SRS can be described by the envelope equations for their slowly varying envelopes. Writing the transverse electric fields of the pump wave and the scattered wave as \(\mathcal{E}_0 e^{-j\omega_0 t-jk_0 x}\), \(\mathcal{E}_s e^{-j\omega_s t-jk_s x}\), and the density perturbation of the EPW as \(\mathcal{D}\\delta n e^{-j\omega_t t-jk_t x}\), the envelope equations for the pump wave and the scattered wave can be written as [4, 23]
\[
\begin{align*}
(\partial_t + v_0 \partial_x) E_0 &= -j \frac{\omega_0^2}{2\omega_{pe}} \delta n_0 E_0, \\
(\partial_t - v_s \partial_x) E_s &= -j \frac{\omega_s^2}{2\omega_{pe}} \delta n_s E_0,
\end{align*}
\]
where \(\omega_{pe}\) is the plasma frequency, \(\delta n_0\) is the background electron density, \(\omega_{ic}\) and \(k_{ic}\) are the fundamental frequency and wavenumber of the pump wave \((i = 0)\) or the scattered wave \((i = s)\) respectively, and \(v_i = \epsilon^2 k_{ic}/\omega_{ic}\) is the group velocity. Here, \(\epsilon\) is the light speed. The fundamental modes satisfy the dispersion relation of EMW
\[
\omega_{ic}^2 = \omega_0^2 + k_{ic}^2 c^2. 
\]

To account for a non-Maxwellian EEDF, a kinetic description of EPW driven ponderomotively can be derived as [24],
\[
(1 + \chi_e) \frac{\delta n_1}{n_0} = -\frac{\chi_e e^2 k_{t0}^2 E_0^4}{2m_e^2 \omega_{pe}^2 \omega_{ic} \omega_{sc}},
\]
which can be rewritten as
\[
\mathcal{D} \frac{\delta n_1}{n_0} = \frac{\epsilon^2 k_{t0}^2 E_0^4}{2m_e^2 \omega_{pe}^2 \omega_{ic} \omega_{sc}},
\]
where \(\epsilon\) is the electron charge, \(m_e\) is the electron mass, and \(\mathcal{D}\) is a function of \(\omega_i\) and \(k_t\)
\[
\mathcal{D}(\omega_i, k_t) = -(1 + \frac{1}{\chi_e})
\]
describing the ponderomotive response. The electron susceptibility \(\chi_e\) is given by
\[
\chi_e(\omega, k) = \frac{\omega_{pe}^2}{k^2} \int f_{e0} dv \int \frac{\mathbf{k} \cdot \partial f_{e0}/\partial \mathbf{v} \mathbf{v} \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}} dv,
\]
where \(f_{e0}\) is the background EEDF. When Langdon effect is significant, the EEDF has a super-Gaussian form [6, 7],
\[
f_{e0}(v) = \frac{n_e m}{4\pi v_{the}^3 \beta_m^3 \Gamma(3/m)} \exp\left[-\frac{(v - v_{the})^m}{\beta_m v_{the}}\right],
\]
where \(m\) is the super-Gaussian exponent, \(\Gamma\) is the Gamma function, \(\beta_m = \sqrt{3(3/m)^m/(5/m)}\), and \(v_{the} = \sqrt{T_e/m_e}\) is the electron thermal velocity. Replacing \(\mathcal{D}(\omega_i, k_t)\) by the operator \(\mathcal{D}(\omega_{ic} + j\delta t, k_{ic} - j\delta k)\), and taking the slow envelope variation approximation [4]
\[
\mathcal{D}(\omega_{ic} + j\delta t, k_{ic} - j\delta k) = j(\partial\mathcal{D}/\partial \omega_i)(\delta t + v_i \delta k) + \mathcal{D}(\omega_{ic}, k_{ic}),
\]
the envelope equation of EPW can be derived as
\[
(\partial_t + v_i \partial_x + v_i + j\delta \omega) \frac{\delta n_1}{n_0} = \frac{j}{\omega_{ic} \omega_{sc}} \frac{e^2 k_{t0}^2 E_0^4}{2m_e^2 \omega_{pe}^2 \omega_{ic} \omega_{sc}}.
\]
Here only \(\partial \mathcal{D}/\partial \omega_i\) is retained in the derivatives for \(|\partial \mathcal{D}/\partial \omega_i| \ll |\partial \mathcal{D}/\partial \omega_t|\). The group velocity of the EPW
\[
\nu_i = \frac{\partial \mathcal{D}_r}{\partial \omega_t} / \frac{\partial \mathcal{D}_r}{\partial \omega_i},
\]
and the frequency mismatch
\[
\delta \omega_t = -\frac{\mathcal{D}_r}{\partial \omega_t} / \frac{\partial \mathcal{D}_r}{\partial \omega_i}.
\]

Notice that in this approach, the Langdon effect can be easily incorporated just by using Eq. (9) when calculating \(\chi_e\). Furthermore, the ponderomotive drive \(\mathcal{D}\) instead of the dielectric operator \(\mathcal{D}\) [4] is adopted to describe the response of the EPW field to the ponderomotive drive from the beating of the pump wave and the scattered wave. Especially for large \(k_t \lambda_{De}\) with \(\lambda_{De} = v_{the}/\omega_{pe}\) being the Debye length, this modification is necessary since \(3|\partial \mathcal{D}/\partial \omega_t| \ll |\partial \mathcal{D}/\partial \omega_i|\) can be satisfied for \(k_t \lambda_{De}\) ranging from 0 to 1 while the assumption \(3|\partial \mathcal{D} / \partial \omega_t| \ll |\partial \mathcal{D} / \partial \omega_i|\) is broken when \(k_t \lambda_{De}\) is near 0.4.

It is convenient to renormalize the wave amplitudes as
\[
\begin{align*}
a_0 &= \frac{E_0}{|E_{0L}|}, \\
a_s &= \frac{E_s}{|E_{0L}| \sqrt{\omega_{sc}/\omega_{pe}}}, \\
a_t &= \frac{E_t}{|E_{0L}| \sqrt{\omega_{sc}/\omega_{pe}}} \approx \frac{E_t}{|E_{0L}|} \sqrt{\omega_{sc}/\omega_{pe}},
\end{align*}
\]
where \(E_j = je \delta n_1/e_k k_t\) is the electrostatic field of EPW, and \(E_{0L}\) is the pump field amplitude incident at the left
boundary. Then, the three wave coupling (TWC) equations of SRS can be recast into the following simplified form

\begin{align}
(\partial_t + v_0 \partial_x) a_0 &= -\gamma_0 a_e a_t, \quad (16) \\
(\partial_t - v_0 \partial_x) a_s &= \gamma_0 a_0 a_t^*, \quad (17) \\
(\partial_t + v_0 \partial_x + \nu_l + j\delta\omega) a_t &= \gamma_0 \eta_0 a_s^*, \quad (18)
\end{align}

where \( \gamma_0 \) is the homogeneous growth rate of SRS when the Landau damping and frequency mismatch are ignored. By substituting the assumed solution forms \( E_s, \delta n_t/n_0 \propto e^{s^2} \) into Eq. (3) and Eq. (11)

\[ \gamma = \frac{k_f v_{os}}{2\sqrt{2\omega_s(\partial E_s/\partial \omega_s)}} \approx \frac{k_f v_{os}}{4} \frac{\omega_{pe}}{\sqrt{\omega_s}} \]  

(19)

can be obtained, where \( v_{os} = e|E_0|/m_e\omega_{pe} \) is the electron quiver velocity. Then, the instantaneous reflectivity \( R \) can be determined from the scattered wave amplitude emergent from the left boundary

\[ R \equiv \frac{|E_{sl}|^2 v_s/\omega_{pe}}{|E_{os}|^2 v_0/\omega_{pe}} = |a_{sl}|^2 v_s/v_0. \]  

(20)

In this definition, \( R < 1 \) limited by pump depletion can be guaranteed if \( R \) eventually reaches a constant value [8].

SRS can be in the convective and absolute instability regime or in the absolute instability regime [8, 25, 26]. The convective instability typically occurs under strong damping condition. The convective gain coefficient \( \kappa \) can be derived by assuming the solution form \( a_s, a_t \propto e^{-s^2} \), where the temporal and spatial derivatives in Eq. (18) can be ignored compared to the Landau damping, yielding

\[ \kappa = \frac{\gamma_0}{v_s} \frac{1}{\nu_l + j\delta\omega_l} = \frac{k_f^2 v_{os}^2}{8c^3 k_s} \frac{\gamma e}{1 + \chi e} \]  

(21)

which is just the classical formula for the kinetic convective gain coefficient of SRS [7, 24]. The strong damping condition \( \nu_l \gg \kappa R v_l \) such implies \( \nu_l \gg \gamma_0 \sqrt{v_l/v_s} \), under which the saturated reflectivity with pump depletion considered can be analytically determined from the TWC model, given by the Tang’s formula [8, 27],

\[ R(1 - R) = \varepsilon \left[ \exp \left[ G_R (1 - R) \right] - R \right], \]  

(22)

where \( \varepsilon = |a_{sl}|^2 |v_s/v_0| \) is determined by the seed light intensity at the right boundary, and \( G_R = 2\kappa R L \) is the energy gain of SRS for an amplification length \( L \).

The absolute instability usually occurs under strong laser drive or weak Landau damping. It is of important concern for the SRS control in ICFs [9], since SRS keeps growing until saturated by nonlinear effects, generally leading to a large reflectivity. When nonlinear effects can be ignored, by analyzing Eqs. (17-18) with Laplace transform, the absolute instability condition can be derived as [8, 28]

\[ 2\gamma_0 \sqrt{v_l/v_s} \geq \nu_l, \]  

(23)

and the absolute growth rate is

\[ \gamma_{abs} = 2\gamma_0 \sqrt{v_l/v_s} - \nu_l. \]  

(24)

This TWC model can describe the evolution of SRS both in the convective and absolute regimes with the consideration of Langdon effect and the the pump depletion, but excludes all the nonlinear kinetic effects such as reduction of Landau damping and frequency shift due to trapped electrons. So as simple as it is, the TWC model is helpful for understanding the initial growth of SRS and identify the onset of nonlinear kinetic effects.

As a super-Gaussian EEDF can significantly reduce \( \nu_l \) [7], it is expected that the absolute instability condition is easier to be met when Langdon effect is considered. Nevertheless, in most cases the linear condition fails to be a good criterion to judge whether absolute growth can occur, since the kinetic inflation [12, 14, 29], wherein the convective SRS is transformed into absolute SRS due to electron trapping effects, is found to be quite important for SRS. To take into account the nonlinear kinetic effects, an one spatial dimensional and one velocity dimensional (1D1V) Vlasov-Maxwell code (VlaMaxW) has been developed, which solves the Vlasov equations

\[ \partial f_{\alpha} / \partial t + \partial f_{\alpha} / \partial x = 0, \]  

(25)

\[ V_{px\alpha} = q_{\alpha} E_x - q_{\alpha}^2 \frac{\partial A_x}{\partial x}, \]  

(26)

together with Maxwell equations

\[ \partial B_z / \partial t = -\partial E_y / \partial x \]  

(27)

\[ \partial E_y / \partial t = -\partial (c^2 B_z) / \partial x - J_y / \epsilon_0 \]  

(28)

\[ \partial E_x / \partial t = -J_x / \epsilon_0 \]  

(29)

\[ E_y = -\partial A_y / \partial t \]  

(30)

\[ J_y = -\sum_{\alpha} q_{\alpha}^2 \frac{m_\alpha}{m_\alpha} \left[ f_{\alpha} \rho_{px\alpha} A_y \right] \]  

(31)

\[ J_x = q_{\alpha} \frac{m_\alpha}{m_\alpha} \left[ f_{\alpha} \rho_{px\alpha} p_{x\alpha} \right] \]  

(32)

by numerical methods similar to Ref. [10]. Here the cold plasma approximation for the transverse motion \( p_{y\alpha} = -q_{\alpha} A_y \) is assumed, and subscript \( \alpha = e, i \) for electron and ion, respectively. \( m_\alpha \) is the mass, \( q_\alpha \) is the charge, \( \epsilon_0 \) is the vacuum permittivity, \( p_{x\alpha} \) is the momentum along \( x \) direction, \( A_y \) is the vector potential, \( E_x \) is the electrostatic field, \( E_y \) and \( B_z \) are the transverse electric and magnetic fields, respectively.

In following, the Vlasov-Maxwell simulation is primarily used to investigate the influence of super-Gaussian EEDF on the nonlinear evolution (e.g., inflation and saturation) of SRS, while the TWC simulation is conducted
to provide a valuable reference to help comprehend the simulation results, where the fundamental mode of the EPW is chosen at the peak value of $\kappa R$. For small $k_l\lambda_{De}$, it is quite close to the natural mode of EPW which satisfies $1 + \chi_e(\omega_l, k_l) = 0$, and also $\omega_l \approx 0$. While for $k_l\lambda_{De} > 0.3$, this can deviate significantly from the natural mode, and also $\omega_l$ can be nonzero.

### III. LANGDON EFFECT ON THE NONLINEAR EVOLUTION OF SRS

In the Vlasov-Maxwell and TWC model simulations, a homogeneous He plasma with length $L_0 = 200 \lambda_0$ is assumed, where the laser vacuum length $\lambda_0 = 0.351 \mu m$, $L_0$ is on the speckle length scale beam with $f$ being the $U$ number [2], and the low-Z He plasma is the typical environment for significant SRS generation in ICFs [2, 30]. In the Vlasov-Maxwell simulation, also two additional collision layers with lengths $2 \times 20 \lambda_0$ and ramp electron density profiles are appended to both sides of the plasma to eliminate effects of the sheath field. One linearly polarized laser beam with intensity $I_0$ is incident from the left boundary at $t = 0$ with a rise time $20T_0$, while the seed light with frequency $\omega_{sc}$ and intensity $I_s = 10^{-4}I_0$ is incident from the right boundary at $t = 100T_0$ with a rise time $20T_0$, where $T_0 = 2\pi/\omega_0$ is the laser period. Since the typical range of super-Gaussian exponent $m$ for low-Z He plasma is between 2 and 3 as analyzed in Ref. [7], simulation cases with $m = 2$ and $m = 2.9$ are compared to demonstrate impacts of Langdon effect in this work. As known, $k_l\lambda_{De} \propto \sqrt{T_e/n_e}$ is a key parameter to determine the saturation mechanism of SRS [1, 31], so we widely scan $k_l\lambda_{De}$ from 0.48 to 0.19 in simulations, by changing $T_e$ from 5 keV to 0.8 keV with $n_e = 0.1 n_c$ ($n_c$ is the critical density). Laser intensity is typically chosen as $I_{15} = 2.82$ ($I_{15} = I_0/10^{15}$ Wcm$^{-2}$), which is achievable in small laser spectacles in ICF. Besides, a low laser intensity $I_{15} = 0.11$ is also used for the low $k_l\lambda_{De}$ cases.

In the Vlasov-Maxwell simulations, the temporal evolution of the electrostatic field in the $k_l$-space together with the instantaneous reflectivity are shown with different $T_e$ and $m$ in Fig. 1 and Fig. 2 for $I_{15} = 2.82$ and $I_{15} = 0.11$, respectively. Except the cases shown in Fig. 1(a) and Fig. 2(a), where the SRS reflectivity is saturated at a low level due to convective saturation [8, 28], the nonlinear effects are obvious in other cases, where the onset of early-stage saturation of the reflectivity generally occurs before significant broadening of the $k_l$-spectrum induced by the secondary instabilities and even the cascaded instabilities. Therefore, the SRS evolution can be approximately divided into the early-stage where the secondary instabilities play negligible roles, and the late-stage where the secondary instabilities dominate and result in non-stationary variation of the reflectivity. In the following Subsection III A, the influences of Langdon effect on the early growth and saturation of SRS are mainly studied. Then, the differences in the dominant saturation mechanism and reflectivity of SRS for different $m$ in the late-stage are discussed in Subsection III B.

#### A. The early-stage growth and saturation of SRS

As shown in Fig. 1 and Fig. 2, SRS grows convectively with a low saturation level at the larger $k_l\lambda_{De}$, but grows quickly to a high early-stage saturation level at the smaller $k_l\lambda_{De}$. This issue can be predicted by the linear criterion $\nu_0 \leq 2\gamma_0\sqrt{\nu_s/\nu_0}$ for absolute SRS, where $\nu_0$ is the initial Landau damping. Since $\nu_0$ is a rapidly decreasing function of $k_l\lambda_{De}$, there exists one critical $[k_l\lambda_{De}]_c$ at which the equality of the criterion can be satisfied. For $k_l\lambda_{De} > [k_l\lambda_{De}]_c$, the SRS growth is convective. The reflectivity can saturate at a lower level and the nonlinear effects are quite weak, as presented in Fig. 3(c-d). In such cases, the reflectivity calculated by the Tang’s formula (22) agrees well with the simulation results of the TWC model, as shown in Fig. 3(a-b). For $k_l\lambda_{De} < [k_l\lambda_{De}]_c$, the SRS growth is absolute, implying that SRS can keep growing until saturated by nonlinear effects, as shown by the time evolution of reflectivity in Fig. 3(c-d), which is calculated by the TWC model and hence the only nonlinear effect is the pump depletion. Since nonlinear effects that become important only at large reflectivity are necessary to prevent further growth of SRS, the reflectivity for absolute SRS growth can not be too small. Consequently, when $[G_R]_c = [k_l\lambda_{De}]_c$ is small and hence the convectively saturated level is low, there would be a sharp increase of reflectivity (especially, much sharper than the prediction by the Tang’s model) at the transition from convective to absolute SRS, as exemplified in Fig. 3(b). In comparison, the change of $R$ near $[k_l\lambda_{De}]_c$ is much more gradual in Fig. 3(a), since for $[G_R]_c > 15$ (cf. Table I), the reflectivity due to convective amplification is already sufficiently large to incur the nonlinear saturation effects.

In addition to decreasing $k_l\lambda_{De}$, increasing $m$ also leads to a decrease of $\nu_0$, and thus can result in the transition from convective to absolute SRS in the certain region of $k_l\lambda_{De}$. Comparing the lines with asterisks in Fig. 3(c) and (d), for the same laser and plasma parameters, SRS is in the convective regime when $m = 2$ but can grow absolutely and eventually saturated by nonlinear effects when $m = 2.9$. Consequently, $[k_l\lambda_{De}]_c$ becomes larger for a greater $m$, as listed in Table I, indicating that the Langdon effect can broaden the parameter range for absolute SRS instability as shown in Fig. 3. In Table I, it is found that the difference in $[k_l\lambda_{De}]_c$ for different $m$ results in close (slightly higher for increasing $m$) values of $|\nu_0|_c$ in the linear prediction. This is because in Eq. (23), $\gamma_0$ and $\nu_s$ are almost independent of $k_l\lambda_{De}$, while $\nu_0 \approx 3\nu_t(k_l/\omega_l \propto (k_l\lambda_{De})^2$ increases weakly with larger $[k_l\lambda_{De}]_c$ at greater $m$, in contrast to the strong rise of $\nu_0$ with decreasing $k_l\lambda_{De}$.

Considering the nonlinear kinetic effects cannot be in-
FIG. 1: The evolution of $k_l$-spectrum of the EPW field $[\log_{10}|E_i(k_l)|^2]$ for several cases with $T_e = 4.5, 3.2, 2.8$keV and $m = 2, 2.9$ when $I_{15} = 2.82$. The corresponding reflectivity versus time is displayed in the bottom panels, where dividing-line between the early stage and the later stage is marked by the red dotted vertical lines. The condition $n_e = 0.1$ $n_e$, $T_i = T_e/5$ and length of 200$\lambda_0$ is taken.

TABLE I: Summary of critical parameters for the transition from convective to absolute SRS growth from the linear TWC model and from the Vlasov-Maxwell simulation. The homogeneous He plasma with $n_e = 0.1$ $n_e$, $T_i = T_e/5$ and length of 200$\lambda_0$ ($\lambda_0 = 351$ nm) is taken.

| $I_{15}$ | $m$ | Linear TWC | Vlasov-Maxwell |
|----------|-----|-------------|----------------|
|          |     | $[k_l\lambda_{De}]_c$ | $\nu_{\text{NL}}_c$ | $\nu_{\text{NL}}_c$ | $G_R|_c$ | $[k_l\lambda_{De}]_c$ | $\nu_{\text{NL}}_c$ | $\nu_{\text{NL}}_c$ | $G_R|_c$ |
| 2.82     | 2   | 0.278       | 0.208          | 20.5         | 0.42     | 2.58       | 1.68          | 20.5         | 0.42     | 2.58       | 1.68          |
| 2.9      |     | 0.338       | 0.241          | 16.4         | 0.465    | 2.15       | 1.84          | 16.4         | 0.465    | 2.15       | 1.84          |
| 0.11     | 2   | 0.238       | 0.0358         | 4.99         | 0.269    | 0.154      | 1.12          | 4.99         | 0.269    | 0.154      | 1.12          |
| 2.9      |     | 0.295       | 0.0429         | 3.92         | 0.316    | 0.112      | 1.45          | 3.92         | 0.316    | 0.112      | 1.45          |

cluded by the TWC model, the Vlasov-Maxwell simulation results for some cases are shown in Fig. 4. Due to the nonlinear kinetic effects, the convective SRS in the linear prediction can change into absolute SRS. One example is presented in Fig. 4(c), where the initial growth of SRS agrees well with the TWC model before $t < 500T_0$, after which the TWC model predicts convective saturation while the Vlasov-Maxwell simulation exhibits the continual growth of SRS towards much higher reflectivity. A detailed investigation shows that with the growth of the EPW field, resonant electrons with $v_x \approx v_{\text{phi}}$ are trapped, resulting in flattening of the EEDF around $v_{\text{phi}}$, as shown in Fig. 4(d). Correspondingly, the Landau damping is reduced [32, 33] while the EPW frequency is downshifted causing a frequency mismatch [34–36]. This in turn induces a frequency upshift of the scattered wave that tends to restore the frequency matching resonance, as shown in the bottom panel of Fig. 4(c) for the adjustment period $500T_0 < t < 2700T_0$. The frequency mismatch induces a phase mismatch $\delta_{\text{mis}}$ and thus a reduction of growth rate by $\cos \delta_{\text{mis}}$, impairing the SRS growth, while the nonlinear reduction of $\nu_l$ favors the SRS growth. In the case shown in Fig. 4(c), the competition between these two factors results in oscillation of the reflectivity during $1500T_0 < t < 2500T_0$, wherein significant frequency mismatch exists in the plasma region further away from the left boundary (e.g. $x/\lambda_0 = 140$ in Fig. 4c). After a period of adjustment, ultimately at $t > 2700T_0$ over a large plasma region the frequency upshift of the scattered wave becomes sufficiently large to compensate the frequency downshift of the EPW, reducing the frequency mismatch to a low level. Consequently, the nonlinear absolute growth rate $\gamma_{\text{abs,NL}} \approx 2\gamma_0 \sqrt{\alpha/\nu_e} \cos \delta_{\text{mis}} - \nu_{\text{NL}}$ exceeds zero over a large plasma region, and SRS enters
into the absolute growth period. Until when $t > 3100T_0$, the EPW amplitude in the plasma region near the left boundary (e.g. $x/\lambda_0 = 60$ in Fig. 4c) is so large that the frequency downshift of the EPW cannot be completely compensated to maintain the SRS resonance, resulting in saturation of the absolute SRS growth.

To illustrate the Langdon effect on the early-stage saturation of SRS in Vlasov-Maxwell simulation, we need to measure the early-stage saturated reflectivity $R_{sat,e}$. However, as shown in Fig. 1(b-f) and Fig. 2(b-f), for the absolute SRS, the reflectivity versus time after the saturation is irregular and nonstationary, making it necessary to clearly define $R_{sat,e}$ in a reasonable way. Here we define $R_{sat,e}$ as the averaged reflectivity over the time period from the onset of early-stage saturation ($t_{sat}$) until the onset of significant secondary instabilities ($t_{early}$). Conveniently, $t_{sat}$ can be identified as the time when the growth of the reflectivity becomes flattened, and $t_{early}$ can be defined as the time until when 90% of the EPW field energy is contained within $[0.9k_{lc}, 1.02k_{lc}]$, as indicated in Fig. 4(a-b). For the convective SRS growth, on the other hand, the reflectivity would eventually become constant with time or in some cases oscillate in a regular way and hence have a constant mean value; correspondingly, it is natural to recognize the saturated reflectivity $R_{sat}$ as the steady (mean) value of the reflectivity.

In Vlasov-Maxwell simulation, $R_{sat,e}$ versus $k_l\lambda_{De}$ is shown in Fig. 5(a-b) for different $m$, whereas $t_{sat}$ versus $k_l\lambda_{De}$ is displayed in Fig. 5(c-d) for cases with absolute SRS growth. As a comparison, $R_{sat,e}$ with an alternative definition of $t_{early}$, i.e. the time until when 90% of the EPW energy is contained within $[0.95k_{lc}, 1.05k_{lc}]$, as well as the time-averaged reflectivity over the entire simulation time with $t \geq t_{sat}$, is also shown. As seen, $R_{sat,e}$ is insensitive to the individual choice in the definition of $t_{early}$. In fact, for relatively large $k_l\lambda_{De}$, kinetic electron trapping plays a key role. In such cases, the nonlinear evolution in the early-stage is predominately determined by (i) nonlinear reduction of Landau damping, (ii) nonlinear frequency downshift of EPW and the accompanied frequency upshift of the scattered wave, and (iii) pump depletion that becomes important when the reflectivity rises to a high level; while secondary instabilities that significantly broaden the $k_l$-spectrum of EPW and denote the end of the early-stage, mainly consist of trapped particle instability and generation of beam acoustic modes (cf. Section III B). As shown in Fig. 1(a-f) and Fig. 2(a-e), broadening of the $k_l$-spectrum of EPW is sudden and
FIG. 3: The saturated reflectivity versus $k_l\lambda_{De}$ from the TWC model (solid lines) for (a) $I_{15} = 2.82$ and (b) $I_{15} = 0.11$ at $m = 2$ (in red) and $m = 2.9$ (in blue). Correspondingly, the temporal evolution of the reflectivity for cases indicated by asterisks, crosses and triangles in (a) and (b) is presented in (c) and (d), respectively. In (a) and (b), the reflectivity versus $k_l\lambda_{De}$ calculated by the Tang’s formula is also plotted as dotted lines, while $[k_l\lambda_{De}]_c$ for absolute growth calculated from the linear criterion (23) is indicated for both $m = 2$ and $m = 2.9$.

abrupt. Consequently, $t_{\text{early}}$ can be delineated with a great accuracy, yielding a robust $R_{\text{sat, e}}$. Things are different for the three low $k_l\lambda_{De}$ cases indicated by open squares in Fig. 5(b,d), where the early-stage saturation is predominately caused by pump depletion, after which broadly featured LDI begins to become important, resulting in broadening of the $k_l$-spectrum. As shown in Fig. 2(f), in these cases broadening of the $k_l$ spectrum is much more gradual, making $t_{\text{early}}$ subject to individual choices. However, $R_{\text{sat, e}}$ remains insensitive to individual choices of $t_{\text{early}}$ since $R_{\text{sat, e}}$ is mainly contributed by the first reflectivity peak with large width and high amplitude, which is mainly saturated by the pump depletion and hence always contained within $t < t_{\text{early}}$.

In Fig. 5(a-b), with decreasing $k_l\lambda_{De}$, an abrupt rise in $R_{\text{sat, e}}$ appears at one critical $[k_l\lambda_{De}]_c$, where the transition from convective to absolute SRS growth occurs. $[k_l\lambda_{De}]_c$ as obtained from the Vlasov-Maxwell simula-
FIG. 4: The early-stage evolution of the instantaneous reflectivity for (a) $I_{15}$ = 2.82 and (b) $I_{15}$ = 0.11. The circles denote $t_{\text{early}}$ defined as the time until when 90% of the EPW energy is contained within $[0.98k_{lc}, 1.02k_{lc}]$. The diamonds denote $t_{\text{sat}}$ corresponding to the turning point of the reflectivity, which is determined by fitting $\log_{10} R$ versus $t$ with a piecewise linear function. The case $m = 2$ and $k_{l}\lambda_{De} = 0.415$ in (a) is replotted in (c), where the reflectivity versus time is compared to the prediction of TWC model in the upper panel, while the frequency shifts of the EPW and the scattered wave are displayed in the bottom panel for distances $x = 60\lambda_0$ and $x = 140\lambda_0$ from the left boundary. In (d), the flattened EEDF is shown for two cases in (a) at $m = 2$ and $m = 2.9$, where $v_{phl}$ is indicated by vertical dotted lines. The condition $n_e = 0.1 n_c$, $\lambda_0 = 351$ nm for a homogeneous He plasma with $T_i = T_e/5$ and length of 200$\lambda_0$ is taken.
FIG. 5: (a-b) $R_{\text{sat,}e}$ versus $k_l\lambda_{\text{De}}$ at different $m$ in Vlasov-Maxwell simulations for (a) $I_{15} = 2.82$ and (b) $I_{15} = 0.11$. Three types of cases are distinguished: (I) absolute growth when the $k_l$-spectrum broadening of EPW due to secondary instabilities is abrupt (open circles). (II) absolute growth when the $k_l$-spectrum broadening of EPW is gradual (open squares). (III) convective growth with rather weak nonlinear effects (solid circles). For comparison, $R_{\text{sat,}e}$ calculated using an alternative definition of $t_{\text{early}}$ (90% of the EPW field energy is contained within $[0.95k_l, 1.05k_l]$) is displayed by the plus symbols, while the time-averaged reflectivity over the entire simulation time with $t > t_{\text{sat}}$ is shown as open triangles. (c-d) $t_{\text{sat}}$ versus $k_l\lambda_{\text{De}}$ at different $m$ for cases with absolute growth. The condition $n_e = 0.1 \, n_c$, $\lambda_0 = 351$ nm for a homogeneous He plasma with $T_i = T_e / 5$ and length of $200\lambda_0$ is taken.

...ritions, as well as the initial Landau damping $\nu_{l0}$ and the convective gain $G_R$ corresponding to $[k_l\lambda_{\text{De}}]_c$, is listed in Table I for both $m = 2$ and $m = 2.9$ at $I_{15} = 2.82$ and $I_{15} = 0.11$. It can be seen that due to nonlinear kinetic effects, $[k_l\lambda_{\text{De}}]_c$ can far exceed that from the TWC model, leading to much smaller $[G_R]_c$ and hence a sharp rise in the reflectivity at the transition. Nevertheless, even in the presence of kinetic effects, $[k_l\lambda_{\text{De}}]_c$ is larger for a greater $m$. Also the values of $[\nu_{l0}]_c$ corresponding to $[k_l\lambda_{\text{De}}]_c$ are quite close for different $m$, as in the prediction of the TWC model. However, contrary to the TWC model, in the Vlasov-Maxwell simulation $[\nu_{l0}]_c$ is lower...
where the bouncing frequency \( \omega_B = \sqrt{\varepsilon E_l k_l/m_e} \). In this modified TWC model, when other parameters such as \( \gamma_0 \), \( v_l \), \( v_s \) and \( v_0 \) are kept nearly the same, the initial Landau damping \( \nu_0 \) and \( \delta \omega_l/\omega_B \) that depicts the strength of nonlinear frequency shift, determine the nonlinear evolution of SRS, and hence whether the absolute SRS growth can occur or not. Since the increase of either of the Landau damping or the frequency shift would impair the SRS growth, it can be expected that with increasing \( \delta \omega_l/\omega_B \), a lower \( \nu_0 \) is required for the onset of absolute SRS growth. This is indeed the effect of increasing \( m \), which leads to greater \( \delta \omega_l/\omega_B \) at the same \( \nu_0 \), as elucidated in Fig. 6. Consequently, \( [\nu_0] \) must be smaller for increasing \( m \) to overcome the effect of stronger nonlinear frequency shift at greater \( m \). It can be further understood that the different variation of \( \delta \omega_l \) and \( \nu_0 \) with \( m \) ultimately results from \( \delta \omega_l \propto \partial f_{e0}/\partial v_x \), in contrast to \( \nu_0 \propto \partial f_{e0}/\partial v_z \) [7].

This general understanding should hold even though the simplified model specified by Eqs. (33-34) is not precise, indicating that apart from the Landau damping, the influence of super-Gaussian EEDFs on the nonlinear frequency shift is also an important factor to affect the early-stage development of SRS.

Below \( [k_l \lambda_{De}] \), \( R_{sat,e} \) is quite insensitive to \( k_l \lambda_{De} \), except for an intermediate range of \( k_l \lambda_{De} \sim 0.33-0.4 \) for \( m = 2.9 \) and \( k_l \lambda_{De} \sim 0.28-0.35 \) for \( m = 2 \), where \( R_{sat,e} \) increases with decreasing \( k_l \lambda_{De} \) as shown in Fig. 5(a). For the high \( k_l \lambda_{De} \) range \( (k_l \lambda_{De} > 0.4 \) for \( m = 2.9 \) and \( k_l \lambda_{De} > 0.35 \) for \( m = 2 \)\), the phase velocity of the EPW is located in the bulk region of the EEDF, leading to strong Landau damping and also strong electron-trapping induced nonlinearity (e.g., \( \nu_0/\gamma_0 \gg 1 \) and \( \delta \omega_l/\gamma_0 \gg 1 \)). Despite the difference in the initial Landau damping, after the adjustment period, the absolute SRS growth becomes similar for different \( k_l \lambda_{De} \) and \( m \), until when the reflectivity reaches the level \( \sim 0.02 \), where secondary instabilities become important and the early-stage ends, as shown in Fig. 4(a). As a result, the dependence of \( R_{sat,e} \) on both \( k_l \lambda_{De} \) and \( m \) is much weak. In the intermediate range of \( k_l \lambda_{De} \) with weaker Landau damping and kinetic nonlinearity, the Landau damping and the kinetic nonlinear shift are comparable to \( \gamma_0 \) (e.g., \( \nu_0 \sim 0.5-2.9 \) and \( \delta \omega_l/\gamma_0 \sim 0.3-1.3 \)) for \( k_l \lambda_{De} \sim 0.28-0.35 \) at \( m = 2 \), \( I_{15} = 2.82 \) and \( \delta t_e/n_{e0} = 0.02 \). Thus, the nonlinear adjustment is insufficient to smear the effects of decreasing initial Landau damping and nonlinear frequency shift when \( k_l \lambda_{De} \) decreases or \( m \) increases. Consequently, \( R_{sat,e} \) increases with decreasing \( k_l \lambda_{De} \) or increasing \( m \).

For the low \( k_l \lambda_{De} \) range \( (k_l \lambda_{De} < 0.33 \) for \( m = 2.9 \) and \( k_l \lambda_{De} < 0.28 \) for \( m = 2 \)\), even with smaller initial Landau damping, the adjustment period, the Landau damping is nearly negligible. The absolute growth rate \( \gamma_{abs, NL} \sim \max[\gamma_{abs}] \sim 2\gamma_0 \sqrt{\nu_l/v_s} \), and also the evolution of SRS towards the early-stage saturation, becomes quite similar for different \( k_l \lambda_{De} \) and \( m \), as shown in Fig. 4(b). As a result, \( R_{sat,e} \) again becomes nearly independent of \( k_l \lambda_{De} \) and \( m \).

In Fig. 5(c-d), it can be seen that the saturation time \( t_{sat} \) is quite large near \( [k_l \lambda_{De}] \). As shown in Fig. 4(c), here an oscillating plateau of the reflectivity is formed before the absolute growth period, significantly lengthening the adjustment period. This is because the countervailing effects of nonlinear Landau damping reduction and the frequency shift induced phase mismatch, nearly balance during the adjustment period. A slight decrease of \( k_l \lambda_{De} \) or increase of \( m \), weakens both the Landau damping and the frequency shift, thus breaks the balance and significantly reduces the adjustment time, leading to a sharp drop of \( t_{sat} \) with decreasing \( k_l \lambda_{De} \) or increasing \( m \) near \( [k_l \lambda_{De}] \), as shown in Fig. 5(c-d). This decreasing trend of \( t_{sat} \) holds until for \( k_l \lambda_{De} \) far below \( [k_l \lambda_{De}] \), where \( t_{sat} \) tends to a value nearly independent of \( k_l \lambda_{De} \) and \( m \). Here, the SRS growth is absolute even in the absence of nonlinear kinetic effects, the adjustment period is negligible, and \( t_{sat} \) is primarily contributed by the absolute growth period. The absolute growth rate
Fig. 8(a-c). It can be seen that the BAMs developed upshifted relative to the linear resonance mode backscattered feature in the transverse electric field at $t > T_0$. This limits its further amplification, and also produces a beat pattern separated at $\tau = 2\pi/\Delta \omega$ [41]; correspondingly, many high frequency minor bursts appears during the quiescent period between 2500T_0 and 3000T_0. When the packet has almost convected out of the plasma ($t \sim 3200T_0$), a new major burst coming from new SRS growth in the plasma, now almost clear of the incoherent BAMs, occurs. Again, the pump depletion that takes effect instantaneously when the reflectivity is large, and the generation of BAMs whose effects last a long time, begin to suppress the reflectivity. The continual suppression and recovery of SRS lead to a sequence of major bursts with period about 1300T_0, intervened by many minor bursts. For $m = 2.9$, the decay to BAMs and the pump depletion play similar roles and act as the predominant saturation mechanism for $t_{early} < t < 4000T_0$. Nevertheless, as shown in Fig. 8(d-f), due to the smaller Landau damping and hence the greater growth rate and gain of SRS at larger $m$, significant SRS re-growth can occur behind the packet even when less than half the plasma is clear of the incoherent BAM field, permitting several packets to coexist and interact. For example, at $t \sim 2000T_0$, new packet II is formed near the left boundary when the previous packet I has just convected half through the plasma. The bursts of the reflectivity at $t \sim 2200T_0$ are generated deep inside packet I at $x \approx 150\lambda_0$, and further amplified across packet II on its path to the left boundary, similar to the high gain case in [41]. So, the bursts become overlapped, while the significant interaction between bursts leads to a less regular burst behaviour. Consequently, compared to $m = 2$, the quiescent period is significantly reduced, and the average reflectivity is enhanced. Besides, over a long timescale $t > 4000T_0$, in addition to decay to BAMs, resscattering also become important, featured by the remarkable $k_l$-features near $-0.54\omega_l/c$ [46]. The typical $\omega_s-k_s$ spectrum of the scattered wave, and the corresponding $\omega_s-k_l$ spectrum of the EPW, are shown in Fig. 7(c) and (d), respectively, where the feature with $\omega_s/k_l \approx (0.32\omega_0, -0.54\omega_0/c)$ and $(\omega_s, k_s) \approx (0.3\omega_0, 0)$ is due to rescattering of the primary upshifted backward scattered wave with $(\omega_s, k_s) \approx (0.64\omega_0, -0.54\omega_0/c)$, respectively. Consequently, the SRS becomes more chaotic, leading to more irregular variation of the reflectivity.

For $T_e = 2.8$ keV and $I_{15} = 2.82$ as shown in Fig. 1(e-f), the evolution of SRS after $t > t_{early}$ can be further divided into three periods: I. In the initial period with $t < 4000T_0$, the generation of BAMs plus the pump depletion remains the dominant saturation mechanism. The SRS reflectivity consists of overlapped bursts for both $m = 2$ and $m = 2.9$, and though less apparent, still the quiescent period is shorter for $m = 2.9$. II. In the intermediate period with $4000T_0 < t < 6000T_0$ for $m = 2$ and $4000T_0 < t < 7000T_0$ for $m = 2.9$, rescattering aids in limiting the SRS level. III. In the final period with $t > 6500T_0$ for $m = 2$ and $t > 8500T_0$ for $m = 2.9$, the decay to low-$k$ Langmuir branch (including modes 

$\gamma_{abs} \approx 2\gamma_0 \sqrt{v_l/v_t}$ is nearly independent of $k_l\lambda_{De}$ and $m$, and hence so is $t_{sat}$.

B. The late-stage saturation of SRS

Now we examine the impacts of Langdon effect on the nonlinear saturation of SRS in the late-stage when secondary instabilities are important. Since the nonlinear behaviour and dominant saturation mechanism in the late-stage are quite different between the high $k_l\lambda_{De}$ regime (0.25 $< k_l\lambda_{De} \leq 0.45$) which is investigated under a high intensity drive ($I_{15} = 2.82$), and the low $k_l\lambda_{De}$ regime (0.18 $< k_l\lambda_{De} \leq 0.3$) that is studied under a low intensity drive ($I_{15} = 0.11$), in the following we discuss them separately.

For $I_{15} = 2.82$ and $T_e = 3.2$ keV as shown in Fig. 1(c-d) for $m = 2$ and $m = 2.9$ respectively, a burst behaviour of the reflectivity is exhibited. In the active phase, the trapped particle induced nonlinearity ultimately results in a chaotic state, wherein a nearly continuous $k_l$-spectrum of the EPW field spreading from $k_0 \approx 1.5\omega_0/c$ to about $2\omega_l/c$ is generated. The downward broadening of the wavenumber is caused by vortex-merging processes [38, 39], giving rise to broadband incoherent EPW field consisting of beam acoustic modes [15–17], consistent with the nonlinear dispersion relation arising from the EEDF flattening near the phase velocity due to particle trapping. The decay of the resonant (and usually downshifted) EPW into BAMs breaks the three wave resonance condition, and can thus suppress the SRS and serve as an efficient saturation mechanism for SRS. The corresponding typical $k_l\omega_l$ spectrum is shown in Fig. 7(b), where the BAM feature is obvious. Note that the adjusted resonant point at $(\omega_s, k_l) \approx (0.37\omega_0, 1.5\lambda_{De}/c)$, which corresponds to the intersection of BAMs with the Stokes curve, is downshifted relative the linear resonance mode $(\omega_s, k_{sc}) \approx (0.39\omega_0, 1.47\lambda_{De}/c)$. Consequently, the backscattered feature in the transverse electric field at $(\omega_s, k_s) \approx (0.63\omega_0, -0.55\omega_0/c)$, as shown in Fig. 7(a), is upshifted relative the linear resonance mode $(\omega_s, k_{sc}) \approx (0.61\omega_0, -0.52\omega_0/c)$. For $m = 2$ where the bursts are well separated, the temporal-spatial evolution of the EPW, the scattered wave and the pump wave is shown in Fig. 8(a-c). It can be seen that the BAMs developed between 1400T_0 and 1800T_0 and their convection and damping lead to oscillation of the reflectivity during this period. Then at 2000T_0, a strong peak occurs. Accordingly, the laser pump is depleted, while the decay to BAMs is significantly enhanced by the strong EPW field. As a result, the reflectivity quickly drops to near zero at $t \sim 2200T_0$. Then, the pump intensity is restored. However, the BAM packet needs a much longer time ($\sim L/v_l$) to damp or convect out of the plasma [40]. Inside the packet, the rapid decay to BAMs caused by the large EPW field keeps the (upshifted) backscattered light at a low level. As this scattered light moves outside the packet into the unperturbed plasma region behind the packet, it is off-resonance with $\Delta \omega \approx \delta \omega_l$. This limits its further amplification, and also produces a beat pattern separated at $\tau = 2\pi/\Delta \omega$ [41]; correspondingly, many high frequency minor bursts appears during the quiescent period between 2500T_0 and 3000T_0.
due to rescattering and forward SRS) is significantly enhanced, leading to the $k_l$-spectrum continuously ranging from $k_{lc}$ down to zero. As shown in Fig. 7(f) for the typical $\omega_k$-k spectrum of the EPW, both BAMS and the low-$k$ Langmuir branch with features due to rescattering and forward SRS contained are nearly fully occupied. Correspondingly, the $\omega_k$-k spectrum of the scattered wave shown in Fig. 7(c) exhibits features of rescattering with $(\omega_k, k_e)$ around $(0.3\omega_0, 0)$ and forward SRS with $(\omega_k, k_e)$ around $(0.68\omega_0/c, 0.61\omega_0/c)$. Such fully developed incoherence results in the drop of SRS reflectivity in the final period, as demonstrated in Fig. 1(e-f).

The variation of the late-stage saturation mechanism with $T_e$, together with the corresponding average reflectivity, is summarized in Fig. 9(a) for $m = 2$ and $m = 2.9$ at $I_{15} = 2.82$. For all cases, initially the dominant saturation mechanism is decay to BAMS plus the pump depletion. In this period, the lower Landau damping and hence greater gain for increasing $m$ results in shorter quiescent period and hence greater average reflectivity. With decreasing $T_e$ or increasing $m$, rescattering and decay to low-$k_l$ Langmuir branch can become important in the later period, usually leading to a more chaotic state with reduced average reflectivity. Consequently, the dependence of the averaged reflectivity on $m$ becomes more uncertain in the later period, and the average reflectivity can be smaller for increasing $m$ in some cases.

For $T_e = 1.5$ keV and $I_{15} = 0.11$ as shown in Fig. 2(c-d), in the initial period after $t > t_{early}$ ($t < 12000T_0$ for $m = 2$ and $t < 7000T_0$ for $m = 2.9$), the dominant secondary process is trapped particle instability (TPI) [42-44], featured by the appearance of two sidelobes with $k_l = k_{lc} \pm \Delta k_{TPI}$ around the primary component $k_{lc}$. The sidelobe frequency is resonant with the electron bouncing frequency in the wave frame, thus satisfying $\Delta \omega_{TPI} - \Delta k_{TPI}v_{phl} = \pm \omega_B$ [43], where $\Delta \omega_{TPI} \equiv \omega_{TPI} - \omega_{lc}$, $\Delta k_{TPI} = k_{TPI} - k_{lc}$, and $v_{phl} = \omega_{lc}/k_{lc}$ is the EPW phase velocity. As shown in Fig. 10(a) for the typical $\omega_k$-k spectrum of the EPW when TPI dominates, the most significant mode occurs along the dispersion relation of EPW, thus $\Delta \omega_{TPI}/\Delta k_{TPI} = v_l$, giving $\Delta k_{TPI} = \pm \omega_B/(v_{phl} - v_l)$. For $T_e = 1.5$ keV, using $2\pi eE_l/m_e\omega_0c \approx 0.01$ estimated from the simulation data, it can be obtained $\Delta k_{TPI} \approx \pm 0.26\omega_0/c$, consistent with the sidelobe locations on the k1-spectrum as shown in Fig. 2(c-d).

As time evolves, Langmuir decay instability [20, 22], where a primary Langmuir wave (LW) decays into a secondary Langmuir wave and an ion acoustic wave (IAW), begins to become important. LDI satisfies the matching condition

$$\omega_{l,i} = \omega_{l,i+1} + \omega_a$$
$$k_{l,i} = k_{l,i+1} + k_a$$

where $i$ denotes the stage number of the Langmuir cascade with $i = 0$ corresponding to the primary LW, and $\omega_a$ and $k_a$ are the frequency and wave number of the IAW, respectively. The dispersion relation for the LW can be approximated by $\omega_{l,i}^2 = \omega_{pe}^2 + 3k_{l,i}^2v_{the}^2$, while the dispersion relation for the IAW is approximately $\omega_a = |k_a|c_a$, where $c_a = \sqrt{2T_e/M}$ is the acoustic velocity with $Z$ and $M$ being the charge and mass of the ion species. Substituting these dispersion relations into Eq. (35) yields $k_{l,i+1} \approx -k_{l,i} + \Delta k_{LDI}$ and $k_a \approx 2k_{l,i}$, where the wavenumber difference between two successive cascade step is

$$\Delta k_{LDI} \approx \frac{2\omega_a\omega_e}{3\omega_{the}^2}.$$  \hspace{1cm} (36)

The LDI threshold can be estimated by [45]

$$\epsilon_0E_{LDI}^2/n_0T_e = 16\frac{\nu_s}{\omega_e}$$
$$\approx \nu_s/\omega_a.$$  \hspace{1cm} (37)

where $\nu_s/\omega_a \approx 0.099$ for $T_e/T_i = 5$ in He plasma considered here. For $m = 2$ and $T_e = 1.5$ keV, when $t \approx 10000T_0$, $E_{i}/E_{LDI}$ \approx 1.14, LDI with one cascade step is excited and results in the reduction of reflectivity in the later period, as seen from Fig. 2(c). The corresponding $\omega_k$-k spectrum of the electrostatic field is shown in Fig. 10(b), where the features at $k_l \approx -1.51\omega_0/c + 0.07\omega_0/c$ due to the secondary LW and at $k_l \approx 3\omega_0/c$ due to IAW are obvious. For $m = 2.9$, due to decreasing $\nu_s$ with increasing $m$, $E_{LDI}$ is reduced while $E_i$ is generally greater, leading to $E_i/E_{LDI} \approx 15$ at $t \approx 7000T_0$. Consequently, at least five LDI cascade steps are apparent from Fig. 2(d) and Fig. 10(c) for $t > 10000T_0$, while the sidelobes of TPI becomes insignificant. This indicates that LDI cascade becomes the dominant saturation mechanism, limiting the reflectivity to a very low level about 5% for $t > 15000T_0$.

For $T_e = 1.2$ keV and $m = 2$ as shown in Fig. 2(e), in the initial period $t < 8000T_0$, TPI plus the pump depletion is still the dominant saturation mechanism. However, when $t \approx 8000T_0$, $E_i/E_{LDI} \approx 4$, so LDI cascade can be excited and the reflectivity in the later period is significantly reduced, as shown in Fig. 2(e) and Fig. 10(d). For $T_e = 1.2$ keV and $m = 2.9$ as shown in Fig. 2(f), $\nu_l/\omega_l \approx 10^{-6}$ is quite small. As a result, SRS is strongly driven and grows rapidly at the early time, leading to $E_l/E_{th\omega} \approx 100$ and $b_{ne}/n_e \approx 0.06$ at $t = 5000T_0$. Broad-featured LDI is developed, as shown in Fig. 10(e) for $6000T_0 < t < 8000T_0$. As more cascade steps are excited, the $k_l$-spectrum is gradually broadened (in contrast to the sudden broadening of the $k_l$-spectrum in other cases). When $t \approx 10000T_0$, multiple cascade steps with broad spectral features have been excited, as shown in Fig. 10(f). In this strongly-driven regime, SRS is quite turbulent, and the instantaneous reflectivity varies in a wide range. This leads to a greater average reflectivity in the later period compared to $m = 2$.

The variation of the late-stage saturation mechanism with $T_e$, together with the corresponding average reflectivity, is summarized in Fig. 9(b) for $m = 2$ and $m = 2.9$ at $I_{15} = 0.11$. Except for the three strongly driven cases with low $T_e$ ($T_e = 0.8$ keV and $m = 2$, $T_e = 0.8$ keV and
m = 2.9, $T_e = 1.2$ keV and $m = 2.9$), initially the dominant mechanism is TPI plus the pump depletion, while LDI and LDI cascade can develop over time for some $T_e$ and $m$, significantly reducing the reflectivity. Increasing $m$ is favorable for excitation of LDI or LDI cascade since the LDI threshold is reduced while the EPW amplitude is typically greater before the onset of LDI due to the lower Landau damping. This can in turn result in a much stronger drop of the reflectivity in the later period than $m = 2$, thus the reflectivity at $m = 2.9$ in the later period can be smaller than $m = 2$.

**IV. DISCUSSION AND SUMMARY**

In summary, the influence of Langdon effect on the nonlinear evolution of SRS over a long timescale is investigated for a wide range of plasma parameters. For the early-stage of SRS, it is found that the Langdon effect can significantly widen the parameter range for absolute SRS growth, and the kinetic nonlinear effect can widen this parameter range further. The time for SRS to reach the early-stage saturation is significantly reduced by the Langdon effect except when $k_i \lambda_{De}$ is far below $[k_i \lambda_{De}]$. For the late-stage of SRS, at high $k_i \lambda_{De}$, initially the dominant saturation mechanism is decay to BAMs plus the pump depletion, wherein the time-varying reflectivity is composed of a series of (possibly overlapped and irregular) bursts. The Langdon effect can shorten the quiescent period and hence increase the average reflectivity. Additional secondary instabilities such as rescattering of the primary scattered wave, and the generation of low-$k$ Langmuir branch can also develop over time, typically further reducing the reflectivity. The Langdon effect favors the development of additional secondary processes, though the effect is generally too weak to make a great difference in the reflectivity. At low $k_i \lambda_{De}$, the saturation mechanisms include TPI plus the pump depletion, which dominates in the initial period of the late-stage except for very low $k_i \lambda_{De}$, and LDI with single or multiple cascade steps, which typically becomes important in the later time period. The Langdon effect decreases the threshold for LDI, thus LDI or LDI cascade with more cascade step is easier to be excited. This can significantly
FIG. 8: The temporal and spatial evolution of (a,d) the electrostatic fields and the transverse electric fields of (b,e) the scattered wave and (c,f) the pump wave for \( m = 2 \) and \( m = 2.9 \). The root mean square over one wavelength is taken for each field. The reflectivity versus time is shown for comparison as black lines in the right region of panels (a,d). The condition \( n_e = 0.1 \), \( \lambda_0 = 351 \) nm for a homogeneous He plasma with \( T_i = T_e/5 \) and length of 200\( \lambda_0 \) is taken.

suppress the reflectivity at the later time, even leading to smaller reflectivity than \( m = 2 \). These findings are helpful to comprehend the evolution behavior of SRS in realistic ICF plasmas, where the prevalent existence of high intensity speckles and beam overlapping, in combination with the wide plasma parameter range, can make the Langdon effect quite important [7].

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FIG. 10: Some $\omega_l-k_l$ spectra of the electrostatic field $|\log_{10}[E_l(\omega_l, k_l)]|^2$ for cases in Fig. 2. In each panel, the upper part shows features due to the EPW, while the bottom part demonstrates features due to the LDI generated IAW. The dispersion relations for the EPW are shown as white dashed lines. The Stokes curves as the locus of the EPW modes that is phase matched for the electromagnetic decay of the pump, i.e., $(\omega_0 - \omega, k_0 - k_0)$ satisfies the dispersion relation of the EMW, is plotted as yellow lines. The condition $n_e = 0.1 \, n_i$, $\lambda_0 = 351$ nm for a homogeneous He plasma with $T_i = T_e/5$ and length of $200\lambda_0$ is taken.

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