Impact of New Physics on the EW vacuum stability in a curved spacetime background

E. Bentivegna\textsuperscript{a,b}, V. Branchina\textsuperscript{a,b}, F. Contino\textsuperscript{a,c} and D. Zappalà\textsuperscript{b}

\textsuperscript{a} Department of Physics and Astronomy, University of Catania, Via Santa Sofia 64, 95123 Catania, Italy

\textsuperscript{b} INFN, Sezione di Catania, Via Santa Sofia 64, 95123 Catania, Italy

\textsuperscript{c} Scuola Superiore di Catania, Via Valdisavoia 9, 95123 Catania, Italy

(Dated: August 4, 2017)

It has been recently shown that, contrary to an intuitive decoupling argument, the presence of new physics at very large energy scales (say around the Planck scale) can have a strong impact on the electroweak vacuum lifetime. In particular, the vacuum could be totally destabilized. This study was performed in a flat spacetime background, and it is important to extend the analysis to curved spacetime since these are Planckian-physics effects. It is generally expected that under these extreme conditions gravity should totally quench the formation of true vacuum bubbles, thus washing out the destabilizing effect of new physics. In this work we extend the analysis to curved spacetime and show that, although gravity pushes toward stabilization, the destabilizing effect of new physics is still (by far) the dominating one. In order to get model independent results, high energy new physics is parametrized in two different independent ways: as higher order operators in the Higgs field, or introducing new particles with very large masses. The destabilizing effect is observed in both cases, hinting at a general mechanism that does not depend on the parametrization details for new physics, thus maintaining the results obtained from the analysis performed in flat spacetime.

PACS numbers: 14.80.Bn, 11.27.+d, 04.62.+v
I. INTRODUCTION

One of the most important goals of present theoretical and experimental particle physics is the search for New Physics (NP) beyond the Standard Model (BSM), even though direct experimental searches up to now have not revealed any sign of it. When looking for patterns towards BSM theories, the stability analysis\cite{1-11} of the electroweak (EW) vacuum plays a crucial role. Earlier studies were mainly focused on establishing bounds for the Higgs boson mass, based either on the requirement that the Higgs effective potential $V(\phi)$ could not take values lower than the EW minimum $v$ (so that the latter is the stable vacuum of the theory) or on the possibility that our Universe sits on a metastable (false) vacuum state (i.e. $V(v)$ is not the absolute minimum of $V(\phi)$), with a lifetime larger than its own age\cite{12,13,2,10}.

The discovery of the Higgs boson boosted new interest on the stability problem. Clearly the goal is no longer to derive bounds on its mass, but rather to perform more refined analyses that should allow to discriminate between absolute stability or metastability for the EW vacuum\cite{14-18}, to study the cosmological impact of the vacuum stability condition during and after inflation\cite{19-29}, and to test the impact that different NP scenarios can have on the vacuum stability condition\cite{14,30-39}.

On the theoretical side, the stability analysis has its roots in a pioneering work of Bender and collaborators\cite{40}, where the tunneling for a quantum mechanical system with several degrees of freedom was studied with the help of saddle point techniques. This was later extended to quantum field theory by Coleman and Callan, who studied the decay of the false vacuum in a flat spacetime background\cite{41}, and then by Coleman and De Luccia\cite{42}, who included gravity in their analysis.

Physically the false vacuum decay is triggered by quantum fluctuations, that induce a finite probability for a bubble of true vacuum to materialize in a false vacuum sea. Both in flat and curved spacetime backgrounds, Coleman and collaborators considered a scalar theory where the potential $V(\phi)$ has a relative and an absolute minimum, at $\phi_{\text{false}}$ and $\phi_{\text{true}}$ respectively, such that the energy density difference $V(\phi_{\text{false}}) - V(\phi_{\text{true}})$ is much smaller than the height of the “potential barrier” $V(\phi_{\text{top}}) - V(\phi_{\text{false}})$, where $V(\phi_{\text{top}})$ is the maximum of the potential between the two minima. Under these conditions the true vacuum bubble is separated from the false vacuum sea by a “thin wall”, and this allows to treat the problem analytically, within the so called “thin wall” approximation.

Going back to the SM, it is known that due to the top loop corrections the Higgs potential $V(\phi)$ bends down for values of $\phi > v$, where $v \sim 246$ GeV is the location of the EW minimum, and for the present experimental values of $M_H$ and $M_t$, namely $M_H \sim 125.09$ GeV and $M_t \sim 173.34$ GeV\cite{13,44}, it develops a second minimum, much deeper than the EW one and at a much larger value of the field, $\phi_{\text{true}} \gg v = \phi_{\text{false}}$. The instability scale of the Higgs potential is then identified as the value $\phi_{\text{inst}}$ of the field such that $V(\phi_{\text{inst}}) = V(v)$ and $V(\phi) < V(v)$ for $\phi > \phi_{\text{inst}}$. For the values of the Higgs and top masses reported above, it turns out that $\phi_{\text{inst}} \sim 10^{11}\text{GeV}$. Clearly the conditions under which the thin wall approximation can be applied are not fulfilled in the SM case, so the results of\cite{11} and\cite{12} cannot be directly applied.

The EW vacuum stability condition was first studied in a flat spacetime background, and the interesting possibility that the SM is valid all the way up to the Planck scale $M_P$, or more generally to some very large scale $M_{\text{large}}$, meaning that NP shows up only at this scale, was investigated. In such a scenario, naturally prompted by the lack of direct observation of hints of new physics, the analysis was performed under the assumption that
the presence of NP at $M_{\text{large}}$ could be neglected for the computation of the tunneling time $\tau$ from the false to the true vacuum of the SM, so that $\tau$ was calculated by considering SM interactions only [10, 11, 14, 30, 32]. In fact it was argued that the relevant scale for tunneling is the instability scale $\phi_{\text{inst}} \sim 10^{11}$ GeV, and that the contribution to the tunneling rate coming from NP that lives at the scale $M_{\text{large}}$ should be suppressed. In other words, as $\phi_{\text{inst}} \sim 10^{11}$ GeV $\ll M_{\text{large}}$, a decoupling effect was expected [11].

It was later realized that the assumption that NP lives at $M_{\text{large}}$ ($\gg \phi_{\text{inst}}$) does not imply that it cannot affect the stability condition of the EW vacuum. On the contrary, the latter turns out to be very sensitive to unknown NP even if it lives at scales far away from $\phi_{\text{inst}}$, and the expected decoupling phenomenon does not take place [33–35, 38].

The reason why the decoupling theorem does not hold in this case is that tunneling is a non-perturbative phenomenon [38], while the former applies when calculating scattering amplitudes in perturbation theory at energies $E$ much lower than $M_{\text{large}}$. In this case the contributions to scattering amplitudes from physics that lives at $M_{\text{large}}$ is suppressed by factors of $E/M_{\text{large}}$ to the appropriate power, and this is how physics at the scale $M_{\text{large}}$ is decoupled from physics at the scale $E$.

For our tunneling phenomenon however, the bulk of the contribution to $\tau$ comes from the exponential of the (Euclidean) action calculated at the saddle point of the path integral for the tunneling rate, the so called bounce solution to the (Euclidean) Euler-Lagrange equation [41], and for this tree level contribution no suppression factors of the kind $(E/M_{\text{large}})^n$ can ever appear. If the Higgs potential is modified by the presence of NP at $M_{\text{large}}$, the new bounce is certainly different from the one obtained when these terms are neglected. The action calculated for this new bounce solution is also modified and (once exponentiated) it can give rise to a value of $\tau$ enormously different from the one obtained when the NP terms are neglected.

The inclusion of gravity in the vacuum stability analysis was pioneered in [42], where the case of the thin wall regime was studied. For the transition from a false Minkowski vacuum to a true Anti-de Sitter (AdS) vacuum, it was shown that, when the size of the Schwarzschild radius of the true vacuum bubble is much smaller than its size, i.e. when gravitational effects are weak, the probability of materialization of such a bubble is close to the flat spacetime result, while when the Schwarzschild radius becomes comparable to the bubble size, i.e. in a strong gravitational regime, the presence of gravity stabilizes the false vacuum, preventing the materialization of a true vacuum bubble. In other words, gravity tends to stabilize the false vacuum, and in a strong gravity regime the materialization of bubbles of true vacuum is quenched.

The above results are obtained in the thin wall regime, but they are commonly considered of more general validity. In particular it was recently claimed [45, 47] that in the presence of Planckian physics (strong gravity regime) the new bounce solutions [33, 35, 38] to the (Euclidean) equation of motion modified by the presence of new physics at $M_{\text{large}}$ should disappear, and that as a consequence these NP terms could not induce any modification to the tunneling rate.

As noted above however, the SM case is very far from the thin wall regime analyzed in [42], and before jumping to any conclusion, and also prior to studies that consider the inclusion of new physics, the stability analysis for the SM in the presence of gravity has to be performed. An early attempt to study the impact of gravity on the EW vacuum decay rate was done in [48], where a perturbative expansion of the bounce around the flat spacetime solution was considered. As later noted in [49] and then in [50] however, the boundary
conditions for the bounce solution, that are essential in the calculation of the decay rate, are not respected already at the first order of the expansion, and the results of this work are at least questionable.

In order to get close to the SM case, but still keeping a simple model as in [42], a scalar theory with a potential whose parameters can be adjusted to explore cases far from the thin wall regime was considered in [49], and a numerical analysis of the false vacuum stability condition was performed. The main result is that for the potential that well approximates the SM case, the stabilizing effect of gravity is hardly seen even in very strong gravity regimes. As suggested in [51–53], the total quenching of the vacuum decay rate can eventually be reached at some very high scale. As shown in [49] however, for the SM case such an effect takes place in a far transplanckian regime where the theory has already lost its validity (similarly to what happens for the Landau pole in QED, where the latter occurs at such a high energy scale that the theory has lost its significance several orders of magnitudes below that scale). The results obtained with the simple model considered in [49] were later confirmed in [50], where a bona fide SM Higgs effective potential was used.

The issue raised in [45–47] however, namely the possibility that strong gravity effects should make the presence of Planckian NP harmless in the calculation of the tunneling time, is a crucial open question. In order to complete the stability analysis of the EW vacuum it is of the greatest importance to understand if gravity really cancels the destabilizing effect induced by NP at high energies, as claimed in [45–47].

In the present work we address these issues, that are very important for current studies and for model building of BSM physics, where we are often confronted with the possibility of considering NP at Planckian and/or trans-Planckian scales. Anticipating on the results of the following sections, we will see that the tunneling time from the false to the true vacuum is still strongly dependent on NP even if it lives at very high ($\gg \phi_{\text{inst}}$) scales, thus confirming the results of the analysis performed in the flat spacetime background.

The rest of the paper is organized as follows. In Section II the general theoretical set-up for our work, mainly consisting of the equations that will be used in the subsequent numerical analysis, is presented. Moreover, in order to keep the present paper as self-contained as possible, and also to check our tools against known results, the EW vacuum stability analysis in the absence of new physics in both flat and curved spacetime backgrounds is briefly sketched, and the known results are recovered. In section III we study the impact of NP at the Planck scale on the stability condition of the EW vacuum when the presence of gravity is taken into account, parametrizing NP in terms of higher order operators. In Section IV a different parametrization for NP at high energy scales is used, namely we introduce a new boson and a new fermion (with very large masses) coupled to the Higgs boson. Section V is for our conclusions.

II. THEORETICAL BACKGROUND

In the present section we briefly review the theoretical background for the computation of the tunneling decay rate from a Minkowski false vacuum (minimum of the potential with vanishing energy density) to an Anti-De Sitter (AdS) true vacuum (minimum of the potential with negative energy density), considering both the flat and the curved spacetime background cases.
Flat spacetime. Let us begin by considering the flat spacetime Euclidean action for a single component real scalar field $\phi$:

$$S[\phi] = \int d^4x \left[ \frac{1}{2} (\partial_{\mu} \phi)^2 + V(\phi) \right],$$  

(1)

where $V(\phi)$ is a potential with a local minimum (false vacuum) at $\phi = \phi_{fv}$, and an absolute minimum (true vacuum) at $\phi = \phi_{tv}$.

In order to calculate the false vacuum lifetime we have to look for the so called bounce solution to the Euclidean Euler-Lagrange equation that have $O(4)$ symmetry and satisfy certain boundary conditions [11]. If $r$ is the radial coordinate, the equation takes the form:

$$\ddot{\phi}(r) + 3 \frac{\dot{\rho}(r)}{\rho(r)} = \frac{dV}{d\phi},$$  

(2)

where the dot indicates derivative with respect to $r$, and the boundary condition are:

$$\phi(\infty) = 0 \quad \dot{\phi}(0) = 0.$$  

(3)

Denoting with $\phi_b(r)$ the bounce solution, the action at $\phi_b$ is:

$$S[\phi_b] = 2\pi^2 \int_{0}^{\infty} dr \ r^3 \left[ \frac{1}{2} \dot{\phi}_b^2 + V(\phi_b) \right],$$  

(4)

and the decay rate $\Gamma$ of the false vacuum is given by:

$$\Gamma = D e^{-(S[\phi_b] - S[\phi_{fv}])} \equiv D e^{-B}$$  

(5)

where $B \equiv S[\phi_b] - S[\phi_{fv}]$ is the so called tunneling exponent and the exponential of $-B$ gives the “tree-level” contribution to the decay rate, while $D$ is the quantum fluctuation determinant. If $V(\phi_{fv}) = 0$, the action $S[\phi_{tv}]$ vanishes, and the tunneling exponent is simply $B = S[\phi_b]$. In order to determine the false vacuum decay rate in the flat spacetime case, in the following we integrate numerically Eq. (2) with boundary conditions (3), and use (4) to get the tunneling exponent $B$ of (5).

Curved spacetime. The next step is to study the impact of gravity on the vacuum decay rate, and to this end we consider the previous theory in a curved spacetime background. Including the Einstein-Hilbert term, the Euclidean action becomes:

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + V(\phi) \right]$$  

(6)

where $R$ is the Ricci scalar and $G$ is the Newton constant. Requiring again $O(4)$ symmetry, the (Euclidean) metric takes the form:

$$ds^2 = dr^2 + \rho^2(r) d\Omega_3^2$$  

(7)

where $d\Omega_3^2$ is the unit 3-sphere line element and $\rho(r)$ is the volume radius of the 3-sphere at fixed $r$ coordinate [12]. The bounce configuration needed to calculate the false vacuum transition rate is now given by the field and the metric solution, $\phi_b(r)$ and $\rho_b(r)$ respectively, of the coupled equations ($\kappa \equiv 8\pi G$):

$$\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} = \frac{dV}{d\phi}, \quad \dot{\rho}^2 = 1 + \frac{\kappa \rho^2}{3} \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) \right),$$  

(8)
where the first equation replaces (2), while the second is the only Einstein equation left by
the symmetry. For the decay of a Minkowski false vacuum to a true AdS vacuum, the case
of interest to us, the boundary conditions are:

\[ \phi_b(\infty) = 0 \quad \dot{\phi}_b(0) = 0 \quad \rho_b(0) = 0. \quad (9) \]

Asymptotically \( r \to \infty \) the bounce \( \phi_b(r) \) approaches the constant false vacuum solution
\( \phi_{fv} \), where \( V(\phi_{fv}) = 0 \) (Minkowski vacuum). In the same limit, from (8) we see that the
bounce solution metric \( \rho_b(r) \) approaches the flat spacetime metric:

\[ \rho_b(r) = r + c. \quad (10) \]

In the thin wall regime the constant \( c \) is obtained analytically \[42\], while in general it is
determined from the numerical integration of (8).

Differentiating (8) with respect to \( r \) we get:

\[ \ddot{\rho} = -\frac{\kappa}{3} \rho \left( \dot{\rho}^2 + V(\phi) \right), \quad (11) \]

that is a useful equation that we will use in our analysis as it is more robust than (8)
for numerical integration \[54\]. Finally the Ricci scalar \( R \) in terms of \( \rho \) is given by:

\[ R = -\frac{6}{\rho^2} \left( \rho \ddot{\rho} + \dot{\rho}^2 - 1 \right). \quad (12) \]

Inserting the above results in (6), the action for the bounce \( (\phi_b, \rho_b) \) becomes:

\[ S_b \equiv S[\phi_b, \rho_b] = 2\pi^2 \int_0^\infty dr \left[ \rho_b^3 \left( \frac{1}{2} \dot{\phi}_b^2 + V(\phi_b) \right) - \frac{3}{\kappa} \left( \rho_b^2 \dot{\rho}_b^2 + \rho_b \right) \right] + \frac{6\pi^2}{\kappa} \rho_b^3 \dot{\rho}_b \bigg|_0^\infty. \quad (13) \]

As the false vacuum action \( S[\phi_{fv}, \rho_{fv}] \) contains the same boundary term (the last one) of
(13), in the tunneling exponent \( B \) this term does not appear. Neglecting it, and using (8)
in (13) we finally get:

\[ S_b = 4\pi^2 \int_0^\infty dr \left[ \rho_b^3 V(\phi_b) - \frac{3}{\kappa} \rho_b \right]. \quad (14) \]

From (14), the false vacuum action \( S_{fv} \equiv S[\phi_{fv}, \rho_{fv}] \) can be easily derived. As already
said, we are interested in the decay of a Minkowski false vacuum (the state that corresponds
to the minimum of the potential with vanishing energy density, \( V(\phi_{fv}) = 0 \)) to an AdS true
vacuum. For the false vacuum case, \( \phi(r) = \phi_{fv} \), and \( \dot{\rho}^2 = 1 \), i.e. \( \rho_{fv}(r) = r + const. \) This
latter constant is fixed and is \( c \) of Eq. (10), as the bounce metric asymptotically tends to
the flat spacetime metric of the Minkowski false vacuum. In calculating the action \( S_{fv} \) we
must then integrate over the Minkowski spacetime from a real radius equal to zero up to
infinity. This corresponds to the integration range \([-c, \infty]\) in the \( r \) coordinate,

\[ S_{fv} = -4\pi^2 \int_{-c}^\infty dr \frac{3}{\kappa} (r + c) = -4\pi^2 \int_0^\infty dr \frac{3}{\kappa} (r + c) - \frac{6\pi^2}{\kappa} c^2, \quad (15) \]

so that for the tunneling exponent \( B = S_b - S_{fv} \) we have:

\[ B = 4\pi^2 \int_0^\infty dr \left[ \rho^3 V(\phi_b) - \frac{3}{\kappa} (\rho - r - c) \right] + \frac{6\pi^2}{\kappa} c^2. \quad (16) \]
In the following, the decay rate of the Minkowski false vacuum into the AdS vacuum will be calculated by integrating numerically Eqs. (8) and (11) with boundary conditions (9). Then with the help of (16) the tunneling exponent will be obtained.

As we are interested in the stability analysis of the EW vacuum, in our case the scalar field \( \phi \) is the Higgs field, and the potential \( V(\phi) \) is the Higgs effective potential. More specifically it is the renormalization group improved potential, that can be written as:

\[
V_{\text{SM}}(\phi) \sim \frac{1}{4} \lambda_{\text{SM}}(\phi) \phi^4, \tag{17}
\]

where \( \lambda_{\text{SM}}(\phi) \) is the quartic running coupling \( \lambda_{\text{SM}}(\mu) \) (\( \mu \) is the running scale) with \( \mu = \phi \) \cite{2, 55}.

In order to get \( \lambda_{\text{SM}}(\mu) \), the system of RG equation of the SM couplings has to be run. In \cite{50}, where a stability analysis of the SM in the presence of gravity was performed, an improved Higgs potential obtained with the help of three-loops beta functions was used, while in previous analyses two-loop beta functions were considered \cite{14}. In this respect, we note that the consistent counting of loops in the beta functions of different SM particles poses a problem in itself, as it was found that taking the same order for all the different components of the SM (Yukawa, gauge and quartic couplings) actually leads to inconsistencies \cite{56}.

The purpose of the present work however is to study the impact that NP at high energies can have on the stability condition of the EW vacuum when the SM coupling to gravity is taken into account. We are then not interested in precision measurements and/or refinements of previous analyses. For our illustrative scopes, the differences between these cases (the two loops counting, the three loops counting, and the “consistent” counting proposed in \cite{56}) are minimal, and they have no impact on the results and conclusions of our analysis.

We can then leave aside these questions and work in a simplified yet very robust framework, by using a good approximation of the SM effective potential that was obtained in \cite{57} by fitting the two-loops improved Higgs potential with the three parameter function \cite{57}:

\[
\lambda_{\text{SM}}(\phi) = \lambda_* + \alpha \left( \ln \frac{\phi}{M_P} \right)^2 + \beta \left( \ln \frac{\phi}{M_P} \right)^4, \tag{18}
\]

where \( M_P = 1/\sqrt{G} \) is the Planck mass. The fit gives:

\[
\lambda_* = -0.013 \quad \alpha = 1.4 \times 10^{-5} \quad \beta = 6.3 \times 10^{-8}. \tag{19}
\]

In the following we work with the Higgs potential (17) with \( \lambda_{\text{SM}}(\phi) \) given by (18) and (19).

Both in the flat and curved spacetime cases, an important parameter is the size \( R \) of the bounce, defined as the value of \( r \) such that

\[
\phi_b(R) = \frac{1}{2} \phi_b(0). \tag{20}
\]

Going back to (5) for the vacuum decay rate, we note that a good approximation to the prefactor for the case that we are considering is given in terms of the bounce size \( R \) and of \( T_U \), the age of the Universe, and the EW vacuum tunneling time \( \tau = \Gamma^{-1} \) turns out to be \cite{33}:

\[
\tau \simeq \left( \frac{R^4}{T_U^3} \right) e^B. \tag{21}
\]
In the following we use (21) to calculate the false vacuum lifetime.

Before ending this section and moving to the study of the impact of NP on the EW vacuum stability, we would like to test our tools starting with the known cases of the flat and curved spacetime backgrounds in the absence of NP (i.e. considering the SM alone), and briefly sketch the analysis for these cases.

Flat spacetime. In order to proceed with the numerical solution of the bounce equation (2), we begin by scaling the dimensionful field \( \phi \) and the radial coordinate \( r \) to dimensionless quantities, \( x \) and \( \varphi(x) \) respectively, by using Planck units:

\[
x \equiv M_P r \quad \varphi(x) \equiv \frac{\phi(r)}{M_P}
\]

(22)

Eq.(2), the boundary conditions (3) and the potential (17) then become:

\[
\varphi''(x) + 3 \frac{\varphi'(x)}{x} = \frac{dU}{d\varphi}
\]

(23)

\[
\varphi(\infty) = 0 \quad \varphi'(0) = 0
\]

(24)

\[
U(\varphi) = \frac{1}{4} \varphi^4 (\lambda_* + \alpha \ln^2 \varphi + \beta \ln^4 \varphi),
\]

(25)

where the prime indicates the derivative respect to \( x \). After the rescaling (22), the tunneling exponent (4) becomes:

\[
B = 2\pi^2 \int_0^{\infty} dx \ x^3 \left[ \frac{1}{2} \varphi_b'(x) + U(\varphi_b) \right].
\]

(26)

Solving numerically the bounce equation (23), with the Higgs potential given by (25) and (19), and inserting the result for \( \varphi_b(x) \) in (26), after using the values found for \( B \) and \( \mathcal{R} \), namely \( B = 2025.27 \) and \( \mathcal{R} = 10.7597 \), we finally get for the lifetime \( \tau \) of the EW vacuum:

\[
\tau_{\text{flat}} \sim 10^{639} T_U,
\]

(27)

in very good agreement with the results known in the literature. This is the first test of our numerical method, and also shows that we are considering a good approximation for the Higgs potential.

Curved spacetime. As in the case of flat spacetime, we move to dimensionless quantities. Defining the dimensionless curvature \( a(x) = M_P \rho(r) \), Eqs. (8), and (11) (that will be used in the following numerical integration) become:

\[
\varphi'' + 3 \frac{a'}{a} \varphi' = \frac{dU}{d\varphi} \quad a'' = -\frac{8\pi}{3} a \left( \varphi'^2 + U \right),
\]

(28)

where the potential \( U(\varphi) \) is the same as in (25). The corresponding boundary conditions are:

\[
\varphi(\infty) = 0 \quad \varphi'(0) = 0 \quad a(0) = 0 \quad a'(0) = 1.
\]

(29)

As we have already said, \( \rho(r) \sim r \) for \( r \to \infty \), and the asymptotic \( (x \to \infty) \) behavior of the bounce solution in the presence of gravity is the same as in the flat spacetime case.
FIG. 1: **Left panel:** Profile of the bounce solution $\varphi(x)$ in the presence of gravity. It is obtained for the potential (25) with the parameters $\lambda_*, \alpha, \beta$ given in (19). The center of the bounce is at $\varphi(0) = 0.0712$, its size is $R = 350.2996$ and the tunneling exponent is $B = 2062.5836$. **Right panel:** Difference between the curvature radius and its asymptotic value, $a(x) - x$, for the same parameters as in the left panel.

In terms of dimensionless quantities, from (16) we find for tunneling exponent:

$$B = 4\pi^2 \int_0^\infty dx \left[ a_b^3 U(\varphi_b) - \frac{3}{8\pi}(a_b - x - c) \right] + \frac{3\pi}{4} c^2$$

(30)

where $c$ (as already said above) is the constant that determines the asymptotic behavior of the metric $a(x)$ for $x \to \infty$, while $(\varphi_b, a_b)$ is the bounce solution to the system (28).

In the left panel of Fig. 1 the bounce profile $\varphi_b(x)$ is plotted. The right panel shows the difference $a_b(x) - x$: we clearly see how asymptotically $a_b(x)$ reaches the Minkowskian behavior $a(x) \sim x + c$, and we can read the value of the constant $c$. Finally, with the help of (21), we obtain the tunneling time in the presence of gravity:

$$\tau_{\text{grav}} \sim 10^{661} T_U.$$  

(31)

Once again we observe that the above result is in good agreement with known results [50]. Moreover, comparing (31) with the corresponding flat spacetime tunneling time (27), we see that gravity (as expected) tends to stabilize the EW vacuum.

**III. NEW PHYSICS: HIGHER ORDER OPERATORS**

The results briefly presented in Section II are known and concern the stability analysis under the assumption that, in the event that unknown NP lives at a scale $M_{\text{large}}$ much greater than the instability scale $\phi_{\text{inst}}$, it has no impact on the stability condition of the EW vacuum. In other words it is assumed that new physics at $M_{\text{large}}$ is decoupled from the physics that triggers the EW vacuum decay, and that it should be possible to calculate the tunneling rate ignoring these terms.

The analysis of the previous section is essential to set the proper framework where the effects of the presence of NP at $M_{\text{large}}$ can be properly investigated. To be specific, we consider the case where NP lives at the Planck scale $M_P$ (i.e. we choose $M_{\text{large}} = M_P$) and
parametrize it as in \cite{33,35} with the help of higher powers of $\phi$ added to the Higgs potential:

$$V_{NP}(\phi) = \frac{\lambda_6}{6} \phi^6 + \frac{\lambda_8}{8} \phi^8. \quad (32)$$

It was shown in \cite{33,35} for the flat spacetime case that when $\lambda_6 < 0$ and $\lambda_8 > 0$ the potential (32) destabilizes the EW vacuum. In other words, these NP terms favor the nucleation of true vacuum bubbles and, depending on the specific values of $\lambda_6$ and $\lambda_8$, this destabilization effect could dramatically reduce the EW vacuum lifetime $\tau$ in (27) and make it even shorter than the age of the Universe $T_U$. We now consider the same kind of analysis in the presence of gravity.

Adding the NP terms (32) to the SM Higgs potential (17), and moving again to dimensionless quantities, the new dimensionless potential $U(\varphi)$ becomes:

$$U(\varphi) = \frac{1}{4} \varphi^4 \left( \lambda_s + \alpha \ln^2 \varphi + \beta \ln^4 \varphi + \frac{2}{3} \lambda_6 \varphi^2 + \frac{1}{2} \lambda_8 \varphi^4 \right). \quad (33)$$

We are now ready to study the impact of high energy NP on the EW vacuum stability condition in the presence of gravity.

A first important result of our analysis is that for each value of the couple $(\lambda_6, \lambda_8)$ there is a different bounce solution to Eqs. (28), all of them being different from the solution obtained for the SM alone, i.e. the case $\lambda_0 = 0$, $\lambda_8 = 0$.

Contrary to the expectations of \cite{45,47} then, gravity does not induce the disappearance of the “new” bounce solutions related to the presence of new physics, here parametrized in...
TABLE I: Tunneling time for different values of $\lambda_6$ and $\lambda_8$, both for the flat and curved spacetime cases. We note that although gravity tends to stabilize the EW vacuum (the tunneling time $\tau_{\text{grav}}$ is always higher than the corresponding one in flat spacetime $\tau_{\text{flat}}$), new physics has always a strong impact.

| $\lambda_6$ | $\lambda_8$ | $\tau_{\text{flat}}/T_U$ | $\tau_{\text{grav}}/T_U$ |
|-----------|-----------|----------------|----------------|
| 0         | 0         | $10^{639}$    | $10^{661}$    |
| -0.05     | 0.1       | $10^{446}$    | $10^{453}$    |
| -0.1      | 0.2       | $10^{217}$    | $10^{598}$    |
| -0.15     | 0.25      | $10^{186}$    | $10^{512}$    |
| -0.3      | 0.3       | $10^{-52}$    | $10^{287}$    |
| -0.45     | 0.5       | $10^{-93}$    | $10^{173}$    |
| -0.7      | 0.6       | $10^{-162}$   | $10^{147}$    |
| -1.2      | 1.0       | $10^{-195}$   | $10^{-58}$    |
| -1.7      | 1.5       | $10^{-206}$   | $10^{-106}$   |
| -2.0      | 2.1       | $10^{-206}$   | $10^{-121}$   |

In order to illustrate these results, in Fig. 2 we show bounce solutions to Eqs. (28) for $\lambda_6 = -0.3$, $\lambda_8 = 0.3$ (yellow curve), $\lambda_6 = -0.4$, $\lambda_8 = 0.4$ (green curve) and compare them with the corresponding $\lambda_6 = 0$, $\lambda_8 = 0$ (blue curve) case. The profiles obtained are definitely new solutions to these equations related to the specific values of $\lambda_6$ and $\lambda_8$, clearly different from the bounce (blue curve) obtained for the SM alone ($\lambda_6 = 0$ and $\lambda_8 = 0$).

With the help of (21) we now calculate the EW vacuum lifetime for different values of the NP couplings $\lambda_6$ and $\lambda_8$. The fourth column of Table I contains different values of the tunneling time obtained for different couples ($\lambda_6$, $\lambda_8$). For comparison, the third column contains the corresponding values of $\tau$ for the flat spacetime analysis. First of all we note that the effect already seen in the previous section (also reported in the first line of the table, the case $\lambda_6 = 0$, $\lambda_8 = 0$), namely that the presence of gravity tends to stabilize the EW vacuum, is maintained even in the presence of new physics.

However, a simple inspection of this table shows that even though the presence of gravity tends to stabilize the EW vacuum as compared to the corresponding flat spacetime case, still for $O(1)$ values of the new physics couplings $\lambda_6$ and $\lambda_8$ the tunneling time can be made smaller than the age of the Universe $T_U$. Let us consider just a couple of examples. For $\lambda_6 = -0.3$ and $\lambda_8 = 0.3$ for instance, the EW vacuum in the flat spacetime background is unstable, being $\tau \sim 10^{-52}T_U$, but for the corresponding case with gravity included we observe a stabilization of the EW vacuum: $\tau \sim 10^{287}T_U$. There is a competition between the destabilizing effect of NP and the stabilizing effect of gravity. In this example, gravity takes over new physics and as a result the EW vacuum turns out to be stable. However for larger (absolute) values of the NP couplings, the destabilizing effect of NP takes over the stabilizing effect of gravity. For instance, for $\lambda_6 = -1.2$ and $\lambda_8 = 1.0$, despite the stabilizing effect of gravity ($\tau_{\text{grav}} \gg \tau_{\text{flat}}$), the EW vacuum turns out to be unstable: $\tau_{\text{grav}} \sim 10^{-58}T_U$.

The results discussed above with the help of Table I are better summarized in Fig. 3 where stability diagrams in the $(\lambda_6, \lambda_8)$ plane are presented for the range of values $-1.5 < \lambda_6 < 0.4$.
and $0.4 < \lambda_8 < 1.5$. In the left panel the flat spacetime case is considered, and we see that the stability region ($\tau > T_U$) is confined to the upper right corner of this figure. The right panel shows the stability diagram for the same range of values of the new physics couplings, and we see that the stability range here takes half of the diagram. These figures illustrate the features discussed above. On the one hand gravity tends to stabilize the EW vacuum. On the other hand, for natural values of the NP coupling constants $\lambda_6$ and $\lambda_8$, i.e. for $O(1)$ values of these couplings, the destabilizing effect of Planckian NP can take over the stabilizing effect of gravity.

Fig. 4 is a zoom of the stability diagram of Fig. 3 for the case where gravity is included in the restricted range of values $-0.6 < \lambda_6 < -0.4$ and $0.4 < \lambda_8 < 0.6$, where the dashed black lines are the level curves of the flat spacetime case (with the corresponding values of $\tau_{\text{flat}}$ reported). For the whole range of values of $\lambda_6$ and $\lambda_8$ considered in this figure, gravity wins over the destabilizing effect of NP ($\tau_{\text{grav}} > T_U$) and the EW vacuum turns out to be stable, while in the flat spacetime background it turns out to be unstable ($\tau_{\text{flat}} < T_U$).

Let us summarize the results of the present section. On the one hand we have shown that the inclusion of gravity in the stability analysis of the EW vacuum does not cause the disappearance of the “new” bounce solutions found in the flat spacetime case [33–35] that in turn cause the destabilization of the EW vacuum to the point that its lifetime can become smaller than the age of the Universe. On the other hand we have seen that the stability condition of the EW vacuum is the result of a competition between the destabilizing effect of NP and the stabilizing tendency of gravity. In particular we have also seen that outside a certain range of values of the NP couplings $\lambda_6$ and $\lambda_8$, the destabilizing effect of Planckian NP always takes over, thus making the EW vacuum unstable.
FIG. 4: Zoom of part of the right panel of Fig. 3 with the inclusion of the flat spacetime level curves (dashed lines). For the considered range of $\lambda_6$ and $\lambda_8$, in the flat spacetime case the EW vacuum is always unstable, while in the presence of gravity it is always stable.

IV. NEW PHYSICS: FERMIONS AND BOSONS WITH LARGE MASSES

In the present section the stability analysis of the EW vacuum will be performed by considering a different parametrization for NP at high energy scales. Actually in [33–35] and in the previous section the analysis was performed by parametrizing NP at the Planck scale in terms of few higher order (non-renormalizable) operators. This is just a convenient and efficient way of mimicking the presence of new physics, clearly not an (illegitimate) truncation of the UV completion of the SM. Some authors however expressed a certain skepticism on these results, suggesting that this effect should disappear when the infinite tower higher dimensional operators of the renormalizable UV completion of the SM is taken into account, so that the expected decoupling of very high energy physics from the mechanism that triggers the decay of the false vacuum should be recovered. It was actually suspected that this effect takes place above the physical cutoff, where the control of the theory is lost [58].

Although it is understandable that the parametrization of NP in terms of higher order operators could be the source of a certain confusion, the destabilizing effect has nothing to do with this parametrization. For the case of a flat spacetime background in [38] the stability analysis was performed by parametrizing NP in terms of renormalizable additional terms, with a fermion and a boson with very high masses that interact with the Higgs field, and it was shown that the destabilizing effect found in [33–35] is still present.

In this section we present the same kind of analysis of [38] taking into account the presence of gravity (i.e. considering the case of a curved spacetime background), and show that as for the case of the parametrization used in the previous section, gravity does not produce any washing out of the destabilizing effect of new physics, although it slightly mitigates it.

In order to illustrate the destabilization phenomenon we consider as in [38] a renormalizable model that is not a realistic high energy UV-completion of the SM but is very appropriate to the purposes of the present work. NP lives at very high energy scales and is parametrized by adding to the SM a scalar field $S$ and a fermion field $\psi$ that interact in
a simple way with the Higgs field $\phi$, with masses $M_S$ and $M_f$ of the scalar and fermion respectively well above the instability scale: $M_S, M_f \gg \phi_{\text{inst}}$.

Apart from the kinetic terms, the additional terms in the Lagrangian are:

$$\Delta L = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + g_S \phi^2 S^2 + M_f \bar{\psi} \psi + g_f \phi \bar{\psi} \psi.$$  \hspace{1cm} (34)

To understand how a NP Lagrangian of this kind can arise in a physical setup, we note that the large mass term $M_f$ can be thought as a sort of heavy right handed “neutrino” in the framework of a see-saw mechanism. While the corresponding light “neutrino” is totally harmless for the stability of the EW vacuum, the heavy “neutrino” can play an important role in destabilizing the vacuum. The scalar field $S$ counterbalances the destabilizing effect of $\psi$. Note that models with new scalar fields coupled to the Higgs (although admittedly unrealistic) have already been used to provide a stabilization mechanism for the Higgs effective potential [59, 60].

Before proceeding with our work, it is worth to mention a problem concerning the physical observables involved in the stability analysis, namely the gauge dependence of the effective potential away from the extrema [61–63]. In particular, absolute stability bounds on the Higgs mass (formally gauge independent) turn out to be gauge dependent at any order of perturbation theory. Only when a consistent resummation is considered the result can be made gauge independent, providing a slight improvement in mass bounds [63]. The gauge dependence of the instability scale has also been investigated and the range of uncertainty identified [62], but it is not known if it is possible to calculate this quantity in a gauge independent manner. Moreover, for the main quantity of interest to us, namely the false vacuum decay rate, again we know that it is a formally gauge-invariant quantity. In a truncated perturbative expansion, however, order-by-order gauge independence can possibly be achieved only after resumming the appropriate terms, as it was done for the energies at the minima of the effective potential [61]. Having these warnings in mind, and waiting for improvements on these gauge dependence issues, we proceed now with the analysis of our model following the usual pattern.

For the purposes of the present work, it is sufficient to consider the impact of these additional terms on the Higgs effective potential $V(\phi)$ at the one-loop level only. In fact we do not need a better level of precision as we are not interested in extracting numbers but we only want to illustrate the destabilization effect that arises from very high energy physics (see also the considerations developed below Eq. (17)). The one-loop contribution to $V(\phi)$ from these terms is:

$$V_1(\phi) = \frac{(M_S^2 + 2g_S \phi^2)^2}{64\pi^2} \left[ \ln \left( \frac{M_S^2 + 2g_S \phi^2}{M_S^2} \right) - \frac{3}{2} \right]$$

$$- \frac{(M_f^2 + g_f^2 \phi^2)^2}{16\pi^2} \left[ \ln \left( \frac{M_f^2 + g_f^2 \phi^2}{M_S^2} \right) - \frac{3}{2} \right],$$  \hspace{1cm} (35)

where the renormalization scale $\mu$ is taken as $\mu = M_S$. In this respect we note that at very high values of the running scale the SM quartic coupling reaches a plateau: $\lambda_{\text{SM}}(\mu)$ has practically the same value in the whole range $[M_f, M_S]$, and this is why we can use $\lambda_{\text{SM}}(M_S)$ as the threshold value for the coupling (even though strictly speaking we should use $\lambda_{\text{SM}}(M_f)$ (see below)), and can choose $\mu = M_S$ as the renormalization (threshold) scale.

The presence of the high energy NP of (34) is then taken into account by adding to the SM potential $V_{\text{SM}}(\phi)$ in (17) and (18) the contribution coming from $V_1(\phi)$. To this end
we have to implement the matching conditions described below. First of all we expand the potential $V_1(\phi)$ in powers of $\phi$ and isolate the constant, the $\phi^2$ and the $\phi^4$ terms. Then at the threshold scale $M_f$ we require that: (i) the renormalized cosmological constant $\Lambda$, given by the sum of all the constant terms (those coming from the SM potential and those coming from $V_1(\phi)$) vanishes, $\Lambda(\mu = M_f) \sim 0$; (ii) the renormalized mass term, given by the sum of all the coefficients of $\phi^2$, and identified with the SM mass parameter $m^2_{SM}(\mu = M_f)$ at the scale $M_f$, vanishes: $m^2_{SM}(\mu = M_f) \sim 0$ (more precisely we neglect this term to a very high degree of accuracy for the large values of $\phi$ considered); (iii) the renormalized quartic coupling, given by the sum of all the coefficients of $\phi^4$, is identified with the SM quartic coupling at the scale $M_f$, $\lambda_{SM}(\mu = M_f)$. In other words, at the scale $M_f$ this coefficient is matched with the value of the quartic coupling obtained by considering the running of the renormalization group equations for the SM couplings alone.

The above requirements for the renormalized cosmological constant and mass are well known features. For the renormalized $\Lambda$ (apart from the fine tuning problem) we can practically consider that $\Lambda(\mu = 0) \sim \Lambda(\mu = M_f) \sim 0$. The same is true for the renormalized mass, for which we take $m^2(\mu = 0) \sim m^2(\mu = M_f) \sim 0$, meaning that we neglect the $\phi^2$ term as compared to the $\phi^4$ and other terms for these large values of $\phi$, and that the running of the renormalized mass is totally harmless in this respect. For the quartic coupling we have a true matching condition. In fact we require that at the threshold scale $\mu = M_f$ the quartic coupling coincides with $\lambda_{SM}(\mu = M_f)$, that is obtained by running the renormalization group equations for the SM couplings only. Practically starting from the scale $M_f$, the potential is given by the SM contribution $V_{SM}(\phi)$ plus the contribution of $V_1(\phi)$ subtracted of its constant, quadratic and quartic powers of $\phi$, that we call $\nabla_1(\phi)$ from now on:

$$V_{tot}(\phi) = \frac{1}{4}\lambda_{SM}(\phi)\phi^4 + \nabla_1(\phi).$$

We are now ready to use our model of high energy NP to calculate the EW vacuum lifetime for different values of the masses $M_f$ and $M_S$ of $\psi$ and $S$, and for different values of the coupling constants. For our illustrative purposes we have chosen to consider the four following examples: (i) $M_S = 2.5 \times 10^{-1}$, $M_f = 3 \times 10^{-4}$, $g_S = 0.96$, $g_f^2 = 0.5$; (ii) $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-4}$, $g_S = 0.9$, $g_f^2 = 0.5$; (iii) $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-3}$, $g_S = 0.95$, $g_f^2 = 0.4$; (iv) $M_S = 1.5 \times 10^{-1}$, $M_f = 5 \times 10^{-3}$, $g_S = 0.92$, $g_f^2 = 0.4$.

First of all we have to solve the bounce equations (28) for $\varphi(x)$ and $a(x)$. In Fig. 5 the profiles of the bounce solutions $\varphi_b(x)$ for the four different cases (i), (ii), (iii) and (iv) and the corresponding plots of $a_b(x) - x$ are presented, with colors yellow, blue, green and red respectively.

These are the first relevant results of the present section. They show that, contrary to the claims in [15] [17], the presence of gravity does not cause the disappearance of the new bounce solutions due to the presence of high energy NP, that were already found in the case of the analysis carried in the flat spacetime background [38]. Therefore they confirm the results of the previous section, where high energy NP was parametrized in terms of higher order operators, and once again show that the appearance of new bounce solutions is not an artifact of the specific parametrization used in Section III.

Using (21) to calculate the vacuum lifetime, for the examples considered above we find in units of $\lambda_U$ (going from (i) to (iv)):

$$\tau = 10^{-65}, \tau = 10^{-93}, \tau = 10^{94}, \tau = 10^{307},$$

(37)
FIG. 5: Left panel: Profile of the bounce solutions $\varphi(x)$ for the potential (36) relative to the four cases considered in the text: $M_S = 2.5 \times 10^{-1}$, $M_f = 3 \times 10^{-4}$, $g_S = 0.96$, $g_f^2 = 0.5$ (yellow); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-4}$, $g_S = 0.9$, $g_f^2 = 0.5$ (blue); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-3}$, $g_S = 0.95$, $g_f^2 = 0.4$ (green); $M_S = 1.5 \times 10^{-1}$, $M_f = 5 \times 10^{-3}$, $g_S = 0.92$, $g_f^2 = 0.4$ (red). Right panel: the corresponding difference between the curvature radius and its asymptotic value, $a(x) - x$, for the same parameters as in the left panel.

to be compared with the corresponding results for the tunneling time obtained from the analysis performed in a flat spacetime background, where we have:

$$\tau = 10^{-80}, \tau = 10^{-103}, \tau = 10^{80}, \tau = 10^{293}. \quad (38)$$

Eqs. (37) and (38) together with Fig. 5 contain the main lesson of the present section. They definitely show that, even when gravity is included in the analysis, the presence of NP at high energy scales (even though much higher than the instability scale) can have an enormous impact on the vacuum lifetime. It is worth to remind here that when the calculation is performed in the curved spacetime background and the presence of high energy new physics is not considered, the tunneling time is given by (31) ($\tau \sim 10^{661} T_U$), while from (37) we see that $\tau$ strongly depends on the parameters of new physics, and can turn out to be even shorter than the age of the Universe.

Moreover, by comparing (37) and (38) we see that gravity slightly pushes toward stabilization (that is also what we observe for the case when new physics is not taken into account by comparing (27) with (31)), but this is a “tiny” effect, that (as we have just seen) does not generate the claimed [45–47] disappearance of the new bounce solutions.

Despite the fact that in our toy model NP lives at very high energy scales, much higher than the instability scale $\phi_{\text{inst}}$, the expectation that the tunneling time should be insensitive to it, in other words that the result shown in (31) should not be modified by the presence of NP at high energies, is not fulfilled. These results confirm the analysis of the previous section. Here, with the help of a fully renormalizable toy UV completion of the SM, we have shown that the EW vacuum lifetime strongly depends on NP even if the latter lives at very high energy scales.

These findings are at odds with a widely diffused expectation, based on a naive application of the decoupling argument, and show that the fact that the vacuum stability condition depends on physics that lives at very high energy scales is not due to an illegitimate extrapolation of the theory beyond its validity, as it was previously thought [64]. On the contrary, it is an illegitimate application of the decoupling argument to a phenomenon to which it cannot be applied, namely the (non-perturbative) tunneling phenomenon, that leads to the
expectation that physics at scales much higher than the instability scale \( \phi_{\text{inst}} \) should have no impact on the stability condition.

Before ending this section, we would like to stress once again that with respect to the previous section, where NP interactions were parametrized with the help of higher order non-renormalizable operators, here NP is given in terms of a fully renormalizable theory, thus showing that the effect that we present is a genuine physical effect and has nothing to do with the specific parametrization of NP.

V. SUMMARY, CONCLUSIONS, AND OUTLOOK

We studied the impact of very high energy NP (around the Planck scale \( M_P \)) on the stability condition of the EW vacuum by carrying the analysis in a curved spacetime background, i.e. by taking into account the presence of gravity. Despite the expectation [45–47] that strong gravity effects should act against the formation of new true vacuum bubbles, thus invalidating the results of previous analyses [33–35, 38] carried in a flat spacetime background, we found that these new solutions to the bounce equations persist even in the presence of gravity, and as in the case of the flat spacetime background analysis, they can have an enormous impact on the EW vacuum lifetime.

As in [33–35] we first performed the analysis by adding to the SM potential higher powers of the Higgs field, more precisely terms as \( \phi^6/M_P^2 \) and \( \phi^8/M_P^4 \) (Section III) that are certainly generated in a quantum gravity context [65]. Following [38] we then parametrized high energy new physics in a different manner, namely by adding to the SM potential a boson \( S \) and a fermion \( \psi \), with very large masses \( M_S \) and \( M_f \), coupled to the Higgs boson. As for the analysis carried in flat spacetime, in both cases we find that the presence of new physics can have an enormous impact on the EW vacuum stability condition.

These results definitely show that, irrespectively of the parametrization used to describe high energy new physics, it is not possible to ignore its presence when the stability analysis is performed. They are of the greatest importance for current studies and for model building of Beyond Standard Model physics, where we often have to take into account new physics at very high (Planckian and/or trans-Planckian) scales.

A question that is left open by the present analysis is the role that could be played by a non-minimal coupling of gravity to the Higgs boson. Work in this direction is in progress [66].

Acknowledgments

This work is carried out within the INFN project QFT-HEP and is supported in part by the HARMONIA project under contract UMO-2015/18/M/ST2/00518 (2016-2019). EB is also supported by the project “Digitizing the universe: precision modelling for precision cosmology”, funded by the Italian Ministry of Education, University and Research (MIUR).
Appendix A: Numerical computation of the bounce solution

The search for the bounce solution is a boundary-value problem specified by the values of $\varphi'(0)$ and $\varphi(\infty)$. This can be turned into an initial-value problem using the shooting method, whereby $\varphi(\infty)$ is replaced by $\varphi(0)$, and the appropriate value for the latter quantity is found iteratively, solving Eq. (23) (or its curved-space generalization) for different initial values until the desired $\varphi(\infty)$ is obtained.

As will be clear below, knowledge of the asymptotic behavior of $\varphi(x)$ for $x \to 0$ and $x \to \infty$ is a crucial ingredient for the efficiency of the shooting algorithm. To find the expected behavior of $\varphi(x)$ in the relevant regimes, we begin by expanding $\varphi(x)$ around $x = 0$:

$$\varphi(x) = B_0 + B_2 x^2 + B_3 x^3 + \ldots$$

(A1)

where the linear term is missing due to the condition $\varphi'(0) = 0$. Inserting this expansion in Eq. (23), with $U(\varphi)$ given by (25) we find that the coefficients of the odd-power terms vanish, $B_{2n+1} = 0$, while all the coefficients of the even-power terms $B_{2n}$ are functions of the first coefficient $B_0$ (called $B$ from now on). Truncating the expansion to the $x^2$ term:

$$\varphi(x) = B + \frac{B^3}{8} \left( \lambda_s + \frac{\alpha}{2} \ln B + \alpha \ln^2 B + \beta \ln^3 B + 3 \ln^4 B \right) x^2 + \ldots$$

(A2)

The coefficient of $x^2$ turns out to be negative, so near the origin the bounce profile behaves as an upside-down parabola.

As for the behavior of $\varphi(x)$ for $x \to \infty$, we note that $U(\varphi) \to 0$ for $x \to \infty$, and $\varphi(x) \to 0$ for $x \to \infty$. Asymptotically Eq. (23) and the corresponding solution are then:

$$\varphi''(x) + \frac{3}{x} \varphi'(x) = 0 \quad \Rightarrow \quad \varphi(x) = \frac{A}{x^2},$$

(A3)

where $A$ is one of the integration constants, while the second additive integration constant vanishes due to the condition $\varphi(\infty) = 0$. In other words, for the bounce solution $x^2 \varphi(x)$ has to reach a plateau for $x \to \infty$.

Numerically, we have implemented a fully adaptive algorithm designed to: (i) pick an initial guess ($\varphi(0) = B$, $\varphi'(0) = 0$), (ii) integrate Eq. (23) numerically while monitoring the behavior of $\varphi(x)$, and (iii) iteratively restart the procedure with a suitably corrected $B$ until the condition $\varphi(\infty) = 0$ is satisfied up to a prescribed tolerance. In practice, the numerical integration is carried out in the range $[x_{\min}, x_{\max}]$, where we have chosen $x_{\min} = 10^{-10}$ and $x_{\max} = 10^9$, so that the initial conditions for $\varphi$ and $\varphi'$ are given at $x = x_{\min}$ using Eq. (A2). With this boundary conditions, we find a class of solutions $\varphi_B(x)$, parametrized by $B$. Following the overshoot-undershoot argument of Coleman [41], we want to tune the parameter $B$ until we converge to the solution which reaches a plateau for $x \to \infty$.

We found that the characterization of the final state is of crucial importance to the effectiveness of the search. In particular, introducing the reference point $x_{\text{ref}} = x_{\max} - 10^3$ and a tolerance $\epsilon = 10^{-10}$, we found that the following three criteria are sufficient to lead the algorithm to the bounce solution in all cases (denoting $\tilde{\varphi}(x) = x^2 \varphi(x)$):

1. If $\tilde{\varphi}(x_{\max}) - \tilde{\varphi}(x_{\text{ref}}) > \epsilon$, the scalar field has reversed its direction before reaching $\varphi = 0$, and is returning towards its initial position. The initial guess for $\varphi(x_{\min})$ was therefore too low, and $B$ is correspondingly increased by a quantity $\delta$. 


FIG. 6: Convergence of $\varphi(x_{\text{min}})$ to its final value (left column), as well as of $\varphi'(x_{\text{max}}) + 2\varphi(x_{\text{max}})/x_{\text{max}}$ to zero (right column), in the four cases (flat and curved, with and without new physics) discussed in the appendix.

2. If $\varphi(x_{\text{max}}) - \varphi(x_{\text{ref}}) < -\epsilon$, the scalar field is overshooting the top of the hill. The initial guess for $\varphi(x_{\text{min}})$ was therefore too high, and $B$ is correspondingly decreased by a quantity $\delta$. 
3. If $|\tilde{\varphi}(x_{\text{max}}) - \tilde{\varphi}(x_{\text{ref}})| < \epsilon$, we consider that the plateau has been reached, i.e. that the bounce solution is found within the required precision.

Furthermore, the algorithm stores a state consisting of both $B$ and its value for the two preceding iterations. It is therefore possible to detect oscillations in $B$ and bisect (or otherwise decrease) $\delta$. We found that decreasing $\delta$ to $\delta/10$ each time $B$ changes trend leads to a particularly efficient search, that converges exponentially to the solution, as will be illustrated below.

The inclusion of gravity does not modify our algorithm, as we found that, for $x \to \infty$, the areal radius goes as $a \sim x + c$ (see text), i.e. the curvature tends asymptotically to zero (the value of the constant $c$ is given by $a(x) - x$ when the plateau is reached).

For this reason, the criteria for tuning the initial value of $\varphi$, introduced in the flat case, can also be used on a curved spacetime. Again, we implemented the boundary condition in $x_{\text{min}}$:

$$\varphi(x_{\text{min}}) = B \quad \varphi'(x_{\text{min}}) = 0 \quad a(x_{\text{min}}) = \epsilon' \quad a'(x_{\text{min}}) = 1,$$  \hspace{1cm} (A4)

with $\epsilon' \ll 1$ (clearly we cannot use $a(x_{\text{min}}) = 0$ due to the factor $a^{-1}$ in (28), $1$).

After solving the equations, the size $R$ of the bounce solution is obtained and we can compute the integral in Eq.(30).

We go now back to the flat spacetime case and include the NP terms of (32), thus obtaining the (dimensionless) potential (33). The bonus for our analysis is that this potential does not modify the asymptotic behavior of the bounce solution $\varphi_b(x)$ for $x \to \infty$, as in this limit we still have $U(\varphi(x)) \to 0$. Therefore the inclusion of NP does not lead to any substantial change in our numerical method. The only modification with respect to the flat spacetime case concerns the expansion of the bounce solution around the origin $x = 0$. In the flat case, by considering the integration range $[x_{\text{min}}, x_{\text{max}}]$, we found that the initial values for solving Eq. (2) with the shooting method are obtained once we take the expansion (A2) of $\varphi(x)$ and its first derivative at $x_{\text{min}}$. Repeating the same analysis for the potential (33), we find that NP simply leads to additional terms in these expressions. Thus, the new expansion for $\varphi(x)$ is given by (again up to $O(x^2)$):

$$\varphi(x) = B + \frac{B^3}{8} \left( \lambda_s + \lambda_6 B^2 + \lambda_8 B^4 + \frac{\alpha}{2} \ln B + \alpha \ln^2 B + \beta \ln^3 B + \beta \ln^4 B \right) x^2 + \cdots$$  \hspace{1cm} (A5)

which we use to set initial conditions for $\varphi(x_{\text{min}})$ and $\varphi'(x_{\text{min}})$. An analogous approach is followed when we consider the alternative parametrization of NP given in Eq. (36). Just like in the case without new physics, when we include gravity we use the initial values (A4), and the numerical integration of the equations of motion is reduced to the tuning of the parameter $B$.

Fig. 6 illustrates the exponential convergence of our algorithm in the four cases presented in this appendix.

[1] N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B158 (1979) 295.
[2] R.A. Flores, M. Sher, Phys. Rev. D27 (1983) 1679.
[3] M. Lindner, Z. Phys. 31 (1986) 295.
[4] M. Sher, Phys. Rep. 179 (1989) 273.
[5] M. Lindner, M. Sher, H. W. Zaglauer, Phys. Lett. B228 (1989) 139.
[6] C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl. Phys. B395 (1993) 17.
[7] M. Sher, Phys. Lett. B317 (1993) 159.
[8] G. Altarelli, G. Isidori, Phys. Lett. B337 (1994) 141.
[9] J.A. Casas, J.R. Espinosa, M. Quirós, Phys. Lett. B342 (1995) 171; Phys. Lett. B382 (1996) 374.
[10] G. Isidori, G. Ridolfi, A. Strumia, Nucl. Phys. B609 (2001) 387.
[11] J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP 0805 (2008) 002.
[12] M.S. Turner and F. Wilczek, Nature 298 (1982) 633.
[13] P. Hut and M-J. Rees, Nature 302 (1983) 508.
[14] G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia, JHEP 1208 (2012) 098.
[15] B. Garbrecht and P. Millington, Phys. Rev. D91 (2015) 105021.
[16] B. Grinstein and C.W. Murphy, JHEP 1512 (2015) 063.
[17] B. Garbrecht and P. Millington, J. Phys. Conf. Ser. 873 (2017) 012041.
[18] A. Andreassen, W. Frost, M.D. Schwartz, “Scale Invariant Instantons and the Complete Lifetime of the Standard Model”, arXiv:1707.08124.
[19] M. Herranen, T. Markkanen, S. Nurmí and A. Rajantie, Phys.Rev.Lett. 113 (2014) 211102.
[20] N. Khan and S. Rakshit, Phys. Rev. D90 (2014) 113008.
[21] M. Herranen, T. Markkanen, S. Nurmí and A. Rajantie, Phys.Rev.Lett. 115 (2015) 241301.
[22] J. Kearney, H. Yoo and K. M. Zurek, Phys. Rev. D91 (2015) 123537.
[23] L.A. Anchordoqui, V. Barger, H. Goldberg, X. Huang, D. Marfatia, L.H.M. da Silva and T. J. Weiler, Phys. Rev. D92 (2015) 063504.
[24] F. Kahlhoefer and John McDonald, JCAP 1511 (2015) 015.
[25] Y. Ema, K. Mukaida and K. Nakayama, JCAP 1610 (2016) 043
[26] Y. Ema, K. Mukaida and K. Nakayama, Phys. Lett. B761 (2016) 419.
[27] N. Okada and D. Raut, Eur. Phys. J. C77 (2017) 247.
[28] K. Urbanowski, Theor. Math. Phys. 190 (2017) 458.
[29] A. Stachowski, M. Szydlowski and K. Urbanowski, Eur. Phys. J. C77 (2017) 357.
[30] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312 (2013) 089.
[31] L.N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. Lett. 108 (2012) 151602; K. Chetyrkin and M. Zoller, JHEP 1206 (2012) 033; F. Bezrukov, M. Yu. Kalmykov, B. A. Kniehl, M. Shaposhnikov, JHEP 1210 (2012) 140.
[32] J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Riotto, A. Strumia, Phys. Lett. B709 (2012) 222.
[33] V. Branchina, E. Messina, Phys. Rev. Lett. 111 (2013) 241801.
[34] V. Branchina, E. Messina, A. Platania, JHEP 1409 (2014) 182.
[35] V. Branchina, E. Messina, M. Sher, Phys. Rev. D91 (2015) 013003.
[36] N. Haba, H. Ishida, R. Takahashi and Y. Yamaguchi, Nucl. Phys. B900 (2015) 244.
[37] P. M. Ferreira and B. Swiezewska, JHEP 1604 (2016) 099.
[38] V. Branchina, E. Messina, Europhys.Lett. 117 (2017) 61002.
[39] N. Chakrabarty and B. Mukhopadhyaya, Eur. Phys. J. C77 (2017) 153.
[40] T. Banks, C. M. Bender, T. T. Wu, Phys. Rev. D8 (1973) 3346; Phys. Rev. D8 (1973) 3366.
[41] S. Coleman, Phys. Rev. D15 (1977) 2929; C. G. Callan, S. Coleman, Phys. Rev. D16 (1977) 1762.
[42] S. Coleman, F. De Luccia, Phys. Rev. D21 (1980) 3305.
[43] G. Aad et al. (ATLAS Collaboration, CMS Collaboration), Phys. Rev. Lett. 114 (2015) 191803.
[44] ATLAS and CDF and CMS and D0 Collaborations, “First combination of Tevatron and LHC measurements of the top-quark mass”, [arXiv:1403.4427] [hep-ex].
[45] J. R. Espinosa, J.F. Fortin, M. Trépanier, Phys. Rev. D93 (2016) 124067.
[46] L. Di Luzio, G. Isidori, G. Ridolfi, Phys.Lett. B753 (2016) 150.
[47] J. R. Espinosa, PoS: TOP2015 (2016) 043.
[48] G. Isidori, V. S. Rychkov, A. Strumia, N. Tetradis, Phys. Rev. D77 (2008) 025034.
[49] V. Branchina, E. Messina, D. Zappalà, Europhys. Lett. 116 (2016) 21001.
[50] A. Rajantie, S. Stoppyra, Phys. Rev. D95 (2017) 025008.
[51] A. Aguirre, T. Banks, M. Johnson, JHEP 0608 (2006) 065.
[52] R. Bousso, B. Freivogel, and M. Lippert, Phys. Rev. D74 (2006) 046008.
[53] A. Masouni, S. Paban, E. J. Weinberg, Phys. Rev. D94 (2016) 025023.
[54] Y. Goto, K. Okuyama, Int. J. Mod. Phys. A31 (2016) 1650131.
[55] S. Coleman, E. Weinberg, Phys. Rev. D7 (1973) 1888.
[56] O. Antipin, M. Gillioz, J. Krog, E. Molgaard, F. Sannino, JHEP 1308 (2013) 034.
[57] P. Burda, R. Gregory, I. G. Moss, JHEP 1607 (2016) 025.
[58] J.R. Espinosa, G.F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia, N. Tetradis, JHEP 1509 (2015) 174.
[59] P.Q. Hung, M. Sher, Phys. Lett. B374 (1996) 138;
[60] J.A. Casas, V. Di Clemente, M. Quiros, Nucl. Phys. B581 (2000) 61.
[61] A. Andreassen, W. Frost, M. D. Schwartz, Phys. Rev. Lett. 113 (2014).
[62] L. Di Luzio, L. Mihaila, JHEP 1406 (2014) 079.
[63] A. Andreassen, W. Frost, M. D. Schwartz, Phys. Rev. D91 (2015), 016009.
[64] J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia and N. Tetradis, JHEP 09 (2015) 174.
[65] F. Loebbert, J. Plefka, Mod. Phys. Lett. A30 (2015) 1550189.
[66] E. Bentivegna, V. Branchina, F. Contino, D. Zappalà, work in progress.