$B \to \pi$ Decay Form Factors from Covariant Confined Quark Model

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We evaluate $B \to \pi$ transition form factors in the full kinematical region within the covariant confined quark model. We compare the obtained results with other theoretical approaches.
1. Model

The covariant confined quark model [1, 2] is an effective quantum field approach to hadronic interactions based on an interaction Lagrangian of hadrons interacting with their constituent quarks. The value of the coupling constant follows form the compositeness condition \(Z_H = 0\), where \(Z_H\) is the wave function renormalization constant of the hadron. Matrix elements of the physical processes are generated by a set of quark loop diagrams according to the \(1/N_c\) expansion. The ultraviolet divergences of the quark loops are regularized by including vertex functions for the hadron-quark vertices. These functions also describe finite size effects related to the non-pointlike hadrons. The quark confinement [2] is built-in through an infrared cutoff on the upper limit of the scale integration to avoid the appearance of singularities in matrix elements. The infrared cutoff parameter \(\lambda\) is universal for all processes. The covariant confined quark model has limited number of parameters: the light and heavy constituent quark masses, the size parameters which describe the size of the distribution of the constituent quarks inside the hadron and the infrared cut-off parameter \(\lambda\). They are determined by a fit to available experimental data. We fix \(\Lambda\) parameters according to the experimental value of leptonic decay constants [3].

In calculations we used next values of the model parameters which are shown in Tab. 1.

**Table 1: CCQM model parameters: quark masses, meson size parameters and infra-red cut-off parameter (all in GeV)**

| \(m_u/d\) | \(m_s\) | \(m_c\) | \(m_b\) | \(\Lambda_B\) | \(\Lambda_\pi\) | \(\lambda\) |
|-----------|---------|---------|---------|-------------|-------------|---------|
| 0.241     | 0.428   | 1.67    | 5.05    | 1.963       | 0.871       | 0.181   |

More details concerning the model can be found in our previous papers [4–7]. Below, we list the definitions of the dimensionless invariant transition form factors together with the covariant quark model expressions that allow one to calculate them. We closely follow the notation used in previous papers [8, 9]

\[
\langle \pi(p_2) | \bar{d}O^\mu b | B(p_1) \rangle \\
= N_c g_{B\pi} \int \frac{d^4k}{(2\pi)^4} \bar{\phi}_B(-k + w_{13}p_1)^2)\phi_\pi(-(k + w_{23}p_2)^2) \\
\times \text{tr}[O^\mu S_1(k + p_1)\gamma^5 S_3(k)\gamma^5 S_2(k + p_2)] \\
= F_+(q^2)P^\mu + F_-(q^2)q^\mu, \\
\langle \pi(p_2) | \bar{d}\sigma^{\mu\nu}(1 - \gamma^5)b | B(p_1) \rangle \\
= N_c g_{B\pi} \int \frac{d^4k}{(2\pi)^4} \bar{\phi}_B(-k + w_{13}p_1)^2)\phi_\pi(-(k + w_{23}p_2)^2) \\
\times \text{tr}[i\sigma^{\mu\nu}(1 - \gamma^5)S_1(k + p_1)\gamma^5 S_3(k)\gamma^5 S_2(k + p_2)] \\
= \frac{iF_T(q^2)}{m_1 + m_2} (P^\mu q^\nu - P^\nu q^\mu + i\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma). \quad (1)
\]

In the above equations, \(P = p_1 + p_2\) and \(q = p_1 - p_2\) with \(p_1\) and \(p_2\) to be the momenta of \(B\) of mass \(m_1\) and daughter meson of mass \(m_2\), respectively. The on-shell condition also requires that \(p_1^2 = m_1^2 = m_B^2\) and \(p_2^2 = m_2^2 = m_\pi^2\).
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The form factors appearing in Eq. (1) and plotted in Fig. 1 are also represented in double pole approximation as

\[ F(q^2) = \frac{F(0)}{1 - a s + b s^2}, \quad s = \frac{q^2}{m_B^2} \]  \hspace{1cm} (2)

Note that this double pole parametrization is very precise and relative error for all the form factors with the exact results is less than 1\% for the entire momentum transferred square range.

2. Results and Discussion

The form factors for the \( B \to \pi \) transition are calculated in the full kinematical region of momentum transfer squared. We compare our form factors with other theoretical approaches the Tab. 2.

**Table 2:** \( B \to \pi \) form factors at maximum recoil in comparison with other theoretical works

| Models         | \( f_{+,0}(0) \)         | \( f_T(0) \)         |
|----------------|-------------------------|----------------------|
| Present        | 0.283 ± 0.019           | 0.268 ± 0.018        |
| LCSR [10]      | 0.280                   | 0.260                |
| LCSR [11]      | 0.285±0.016             | 0.267±0.015          |
| LCSR [12]      | 0.301 ± 0.023           | 0.273 ± 0.021        |
| LCSR [13]      | 0.21 ± 0.07             | 0.19 ± 0.06          |
| SUSY [14]      | 0.258                   | 0.253                |
| pQCD [15]      | 0.26±0.04 ± 0.03 ± 0.02 | 0.26±0.04 ± 0.03 ± 0.02 |
| SCET [16]      | 0.247                   | 0.253                |
| RQM [17]       | 0.217 ± 0.011           | 0.240 ± 0.012        |
| CQM [18]       | 0.29                    | 0.28                 |
| LFQM [19]      | 0.25                    | –                    |

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