Higgs mass sum rule in the light of searching for $Z'$ boson at the Tevatron

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Abstract

We discuss the Higgs boson mass sum rules in the Minimal Supersymmetric Standard Model in order to estimate the upper limits on the masses of stop quarks as well as the lower bounds on the masses of the scalar Higgs boson states. The investigation of the bounds on the scale of quark-lepton compositeness derived from the CDF Collaboration (Fermilab Tevatron) data and applied to new extra gauge bosons is taken into account. These extra gauge bosons are considered in the framework of the extended $SU(2)_h \times SU(2)_l$ interaction model.

1. In recent years, there are interesting discussions that the answer to the question of why the top quark is so heavy could be due to extra gauge interactions that single out the fermions of the third generation. In the simplest version of many extensions of the Standard Model (SM), for example, the extension of $SU(2)$ gauge group to $SU(2) \times SU(2)$ one [1-4], the massive $SU(2)$ extra gauge bosons (corresponding to the broken generators) could couple to fermions in different generations with different strengths.
Actually, in the model of extended weak interactions governed by a pair of $SU(2)$ gauge groups, i.e. $SU(2)_h \times SU(2)_l$ for heavy (third generation) and light fermions (labels $h$ and $l$ mean heavy and light, respectively), the gauge boson eigenstates are given as \cite{5}

$$A^\mu = \sin \theta (\cos \phi W_{3h}^\mu + \sin \phi W_{3l}^\mu) + \cos \theta X^\mu$$ \hspace{1cm} (1)

for a photon and

$$Z_1^\mu = \cos \theta (\cos \phi W_{3h}^\mu + \sin \phi W_{3l}^\mu) - \sin \theta X^\mu,$$ \hspace{1cm} (2)

$$Z_2^\mu = -\sin \phi W_{3h}^\mu + \cos \phi W_{3l}^\mu$$ \hspace{1cm} (3)

for neutral gauge bosons $Z_1$ and $Z_2$, respectively which give the neutral mass eigenstates $Z$ and $Z'$ at the leading order of $x$ \cite{6}

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} \simeq \begin{pmatrix} 1 & -\cos^3 \phi \sin \phi \\ \cos \phi \sin \phi & x \cos \theta \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix},$$ \hspace{1cm} (4)

where $\theta$ is the usual weak mixing angle and $\phi$ is an additional mixing angle due to the presence of two weak gauge groups $SU(2)_h \times SU(2)_l$, and the ratio $x$ is defined as $x = u^2/v^2$, where $u$ is the energy scale at which the extended weak gauge group $SU(2)_h \times SU(2)_l$ is broken to its diagonal subgroup $SU(2)_L$, while $v \approx 246$ GeV is the vacuum expectation value of the (composite) scalar field responsible for the symmetry breaking $SU(2)_L \times U(1)_{Y} \rightarrow U(1)_{em}$ in the model of extended weak interactions. The generator of the $U(1)_{em}$ group is the usual electric charge operator $Q = T_{3h} + T_{3l} + Y$.

At large values of $\sin \phi$, the $Z_2$-boson could have an enhanced coupling to the third generation fermions through the covariant derivative

$$D^\mu = \partial^\mu - ig \frac{\sin \phi}{\cos \phi} Z_{3h}^\mu \left( T_{3h} - \sin^2 \theta \cdot Q \right)$$

$$-ig Z_{3l}^\mu \left( \frac{\sin \phi}{\sin \phi} T_{3h} + \frac{\cos \phi}{\sin \phi} T_{3l} \right).$$ \hspace{1cm} (5)

2. The precision measurement of electroweak parameters narrowed the allowed region of extra gauge boson masses, keeping Higgs boson masses to be consistent with radiative corrections including the supersymmetric ones. In this work, we discuss another method for estimating the stop quark masses and upper limits on the CP-odd heavy and CP-even light Higgs boson masses.
in the Minimal Supersymmetric Standard Model (MSSM). We first show how
the existing Tevatron bounds on the scale of quark-lepton compositeness [6]
can be adopted to provide the upper limit of the quantity \( m_{\tilde{t}_1} \cdot m_{\tilde{t}_2} \), i.e. the
product of masses of stop eigenstates \( \tilde{t}_1 \) and \( \tilde{t}_2 \). We shall also intend to discuss
how the lower bound on the scalar Higgs bosons can be obtained from the
forthcoming Tevatron data. It should be pointed out that the Tevatron data [7]
for searching for the low energy effects of quark-lepton contact interactions
on dilepton production taken at \( \sqrt{s} = 1.8 \) TeV are translated into lower
bounds on the masses of extra neutral gauge bosons \( Z' \). Furthermore, one
can emphasize that the forthcoming experiments for trying to disc over an
evidence of supersymmetry in both Higgs and quark sectors could lead us to
estimation of the masses of neutral and charged extra gauge bosons \( Z' \) and
\( W^{\pm'} \), respectively. The models in which the precision electroweak data allow
these extra gauge bosons with their masses being of the order \( \mathcal{O}(0.5 \) TeV),
might be, e.g. the non-commuting extended technicolor models [2]. The \( Z' \)
and \( W^{\pm'} \) bosons with such masses are of interest, since they are within the
kinematic reach of the forthcoming Tevatron Run II experiments.

In the MSSM, the mass sum rule [8] at the tree-level

\[
m_h^2 + M_H^2 = M_A^2 + m_Z^2
\]

is transformed into the following one

\[
M_{Z'} = \frac{m_h^2 - M_A^2 + \delta_{ZZ'} - \Delta}{M_{Z'} + M_H} + M_H .
\]

In formulae (6) and (7), \( M_H \) is the mass of the CP-even heavy Higgs boson,
\( m_Z \) and \( M_{Z'} \) are the masses of \( Z \)- and \( Z' \)-bosons, respectively, and \( \delta_{ZZ'} = M_{Z'} - m_Z^2 \). The correction \( \Delta \) reflects the contribution from loop diagrams
involving all the particles that couple to the Higgs bosons [9,10]

\[
\Delta = \left( \frac{\sqrt{N_c} g m_t}{4 \pi m_W \sin \beta} \right)^2 \log \left( \frac{m_{\tilde{t}_1} \cdot m_{\tilde{t}_2}}{m_t^2} \right)^2 ,
\]

where \( N_c \) is the number of colors, \( m_t \) and \( m_W \) are the masses of top quark and
\( W \)-boson, respectively, \( \tan \beta \) defines the structure of the MSSM. The values
of \( \Delta \sim \mathcal{O}(0.01 \) TeV\(^2\) have been calculated [9] for any choice of parameters
in the space of the MSSM. We suggest that the measurement of \( M_{Z'} \) would
predict the masses of mass-eigenstates \( \tilde{t}_1 \) and \( \tilde{t}_2 \), since \( m_t \) and \( m_W \) are already
measured in experiment and \( m_h \) is restricted by the LEP 2 data \([11]\) as \( m_h < 130 \) GeV \([12]\); \( M_A \) and \( M_H \) are free parameters bounded by combined data coming from the MSSM parameter space and the experimental data \([13]\).

3. The Lagrangian density (LD) for an effective quark-lepton contact interaction looks like

\[
\mathcal{L} \supset \frac{1}{\Lambda_{LL}^2} \left[ g_2^2 (\bar{E}_L \gamma_\mu E_L) (\bar{Q}_L \gamma_\mu Q_L) + g_1^2 (\bar{E}_L \gamma_\mu \tau_a E_L) (\bar{Q}_L \gamma_\mu \tau_a Q_L) \right] + \frac{g_2^2}{\Lambda_{LR}^2} (\bar{e}_R \gamma_\mu e_R) (\bar{Q}_L \gamma_\mu Q_L) + \left[ \frac{1}{\Lambda_{LR}^2} (\bar{E}_L \gamma_\mu E_L) + \frac{1}{\Lambda_{RR}^2} (\bar{e}_R \gamma_\mu e_R) \right] \sum_{q=u,d} g_q^2 (\bar{q}_R \gamma_\mu q_R), \tag{9}
\]

where \( E_L = (\nu_e, e) \), \( Q_L = (u, d)_L \); \( g_i \) are the effective couplings and \( \Lambda_{ij} \) are the scales of new physics. The aim of the CDF collaboration analysis \([7]\) was to search for the deviation of the SM prediction in the dilepton production spectrum. If no such deviations have been found, the lower bound of the \( \Lambda \)-scale can be obtained. The embedding of the extra gauge bosons in the model beyond the SM gives rise to quark-lepton contact interactions in accordance to the following part of the LD (see \([6]\))

\[
\mathcal{L} \supset - \frac{g_2^2}{M_{Z'}^2} \left( \frac{\cot \phi}{2} \right)^2 \left( \sum_{l=e,\mu} \bar{l}_L \gamma_\mu l_L \right) \left( \sum_{q=u,d,s,c} \bar{q}_L \gamma_\mu q_L \right), \tag{10}
\]

where \( g = e / \sin \theta \).

We suppose that the couplings in the first two generations are same in strength. Comparing (9) and (10), one can get the following relation between \( M_{Z'} \) and \( \Lambda \) as

\[
M_{Z'} = \sqrt{\alpha_{em}} \Lambda \cot \phi / (2 \sin \theta), \tag{11}
\]

where the value of \( \Lambda \) was constrained from the CDF data at \( \sqrt{s} = 1.8 \) TeV as \( \Lambda > 3.7 \) TeV or 4.1 TeV, depending on the contact interactions for the left-handed electron or muon, respectively, at 95 % confidence level \([6,7]\).

In the decoupling regime of the MSSM Higgs sector where the couplings of the light CP-even Higgs boson \( h \) in the MSSM are identical to those of the SM Higgs bosons and thus, the CP-even mixing angle \( \alpha \) behaves as \( \tan \alpha \to - \cot \beta \) with the \( M_A \gg m_Z \) relation, one can get \( M_H^2 \simeq M_A^2 + m_Z^2 \sin^2(2\beta) + \mu^2 \) which leads to disappearance of the \( H \)-Higgs boson mass in (7). Here, \( \mu \) is
the positive massive parameter which can, in principle, be defined from the experiment searching for separation of two degenerate heavy Higgs bosons, \( A \) and \( H \). This behavior verified at the tree-level continues to hold even when radiative corrections are included. It has been checked that this decoupling regime is an effective one for all values of \( \tan \beta \) and that the pattern of most of the Higgs couplings results from this limit.

In studying the mass relation (7) from the extended electroweak gauge structure, we must be aware of the issues related to the structure of \( M_{Z'} \) in both sides of (7). We suppose that \( M_{Z'} \) in the l.h.s. of (7) is the mass in question to be determined using the Tevatron data (the CDF analysis [7,6]). Therefore, one can approximate the latter mass via the phenomenological relation (11) meanwhile the r.h.s. of (7) is model dependent where, to leading order, the mass \( M_{Z'} \) in the extended weak interaction model is \( M_{Z'} = m_W \sqrt{x}/\cos \phi \sin \phi \) [6] in the region where \( \cos \phi < \sin \phi \). With the help of the CDF restriction for \( \Lambda \) [7] entering into (11), one can easily find the upper limit on \( m_{\tilde{t}_1} \cdot m_{\tilde{t}_2} \) from the following relation

\[
\Delta < (B + M^*_{H}) (B - f C) + m_h^2 - m_{Z}^2 (1 - \sin^2 2\beta) + \mu^2, \tag{12}
\]

where \( M^*_{H} = (m_A^2 + m_Z^2 \sin^2 2\beta + \mu^2)^{1/2} \), \( f = f(\phi) = \cot \phi \sqrt{\alpha_{em}}/(2 \sin \theta) \), \( B = B(x, \phi) = m_W \sqrt{x}/(\cos \phi \sin \phi) \), and \( C \) is a minimal value of the \( \Lambda \) scale extracted from the CDF analysis [7]. The masses of \( Z \)- and \( W \)-bosons are currently known with errors of a few MeV each [14], whereas the mass of the top quark is known with errors of a few GeV [14]. The dominant error on the lower bound of \( m_{\tilde{t}_1} \cdot m_{\tilde{t}_2} \) comes from the errors on mass measurements of \( h \)- and \( A \)-Higgs bosons. In addition, the dependence of particle couplings via \( \tan \beta \) enters into the radiative correction \( \Delta \) in (8) and the redefinition of \( M_H \) because of the decoupling regime. Thus, the upper limit on \( m_{\tilde{t}_1} \cdot m_{\tilde{t}_2} \) can be accurately predicted by precision measurements of the lower bound of \( M_{Z'} \).

Fig. 1 shows the upper limit on \( \bar{L} \equiv \log \left( \frac{m_{\tilde{t}_1} \cdot m_{\tilde{t}_2}}{m_t^2} \right) \) for \( x = 2 \) and 3 as a function of \( \sin \phi \) at fixed values of \( \mu \) and \( M_A \). The regions of the parameter space lying below a given line are allowed by the present data. At present, the LEP bounds on the mass of \( A \)-Higgs boson are \( M_A > 88.4 \) GeV [11]. This result corresponds to the large \( \tan \beta \) region. We see that the function \( \bar{L} \) is rather sensitive within the changing of \( \sin \phi \), i.e. the ratio of gauge couplings \( g/g_t \). Here, \( g^{-2} = g_t^{-2} + g_h^{-2} \), where \( g_t \) is associated with the \( SU(2)_l \) group and defines the couplings to the first and second generation fermions, whose charges under subgroup \( SU(2)_l \) are as in the SM, while \( g_h \) has the origin
from the \( SU(2)_h \) group which governs the weak interactions for the third generation (heavy) fermions. In the range of \( \sin \phi \) presented in the Fig. 1, the width \( \Gamma_{Z'} \) of the \( Z' \) falls to a minimum in the neighborhood of \( \sin \phi = 0.8 \) [6], due to the decreasing couplings to two first generations of fermions. In the range \( \sin \phi > 0.8 \), \( \Gamma_{Z'} \) grows large, due to the rapid growth in the third generation coupling.

Fig.1 The upper limit on \( L \equiv \log \left( \frac{m_{\tilde{t}_1} \cdot m_{\tilde{t}_2}}{m_t^2} \right) \) as a function of \( \sin \phi \) for different values of \( x = 2 \) and \( 3; \mu = m_h = 120 \text{ GeV} \) (dashed line), \( \mu = 0 \) (solid line) for \( M_A = 0.8 \text{ TeV} \); \( \tan \beta = 30 \).

The CDF analysis of the contact interaction between left-handed muons and the up-type quarks is taken into account \( (C = 4.1 \text{ TeV}) \) in our calculations. The lower bounds of \( m_h^2 \) are illustrated in Fig.2. The constraints are given for different ratios of \( x = 2 \) and \( 3 \) as a function of \( \sin \phi \). In our calculations, the parameters of the model are chosen as \( M_A = 0.8 \text{ TeV}, m_h = 120 \text{ GeV}, \tan \beta = 30 \). Here, we did not use the mass difference between \( \tilde{t}_1 \) and \( \tilde{t}_2 \) mass eigenstates, and we set \( m_{\tilde{t}_1} = m_{\tilde{t}_2} = 1 \text{ TeV} \) (see Fig.2). The regions
The lower bound on $m^2_h$ as a function of $\sin \phi$ for different values of $\mu = 120$ GeV (dashed line), $\mu = 0$ (solid line) for $M_A = 0.8$ TeV; $\tan \beta = 30$. The regions of the parameter space lying above a given line are allowed by the present model.

of the parameter space lying above a given line are allowed by the present data. At the same time, a combined fit of the experimental data [11] gives $m_h = 90^{+55}_{-47}$ GeV. On the other hand, recent direct searches at LEP 2 give the lower bound on the Higgs boson mass which is 113.4 GeV [11].

In a more extended SUSY models, their mass sum rules can give some useful estimations with the help of the CDF data [7]. For example, in the minimal $E_6$ superstring theory, the particle spectrum consists of three scalar Higgs bosons $h$, $H_1$, $H_2$, a pseudoscalar Higgs $A$, a charged Higgs boson pair $H^\pm$, and two neutral gauge bosons $Z$ and $Z'$. There was obtained the mass sum rule on the tree-level in [15] in the form:

$$M_{Z'}^2 = m_h^2 + M_{H_1}^2 + M_{H_2}^2 - M_A^2 - m_Z^2.$$  \hspace{1cm} (13)

The analytical expressions for the loop corrections are unknown yet. By the way, the one-loop corrections can be summarized into the term logarithmically dependent on the SUSY sector mass scale [15]. Taking into account that $m_h$ can be identified with the lower bound on the Higgs boson mass
[11], we obtain the lower bound on the sum $M_{H_1}^2 + M_{H_2}^2$ at fixed $M_A$ as a function of $\sin \phi$:

$$\sum_{j=1}^{2} M_{H_j}^2 > M_A^2 + m_Z^2 - m_h^2 + \frac{\alpha_{em} C^2 \cot^2 \phi}{4 \sin^2 \theta}.$$  \(14\)

The results of the calculation of $M \equiv (\sum_{j=1}^{2} M_{H_j}^2)^{1/2}$ as the function of $\sin \phi$ is given in the Fig. 3

![Fig. 3](image)

Fig. 3 The lower bound on $M \equiv (\sum_{j=1}^{2} M_{H_j}^2)^{1/2}$ as the function of $\sin \phi$.

We have used the scales of new physics $\Lambda > C$ coming from the CDF analysis [7] at 95 % confidence level: $\Lambda > 4.1$ TeV and $\Lambda > 3.7$ TeV for left-handed muons and left-handed electrons, respectively, and up-type quarks.

4. To summarize, we have demonstrated that the study of the bounds on the scale of quark-lepton compositeness derived from the data taken at the Tevatron (CDF analysis [7,6]) and the ones applied to $Z'$ boson masses within the models of the extended $SU(2)_h \times SU(2)_l$ interactions can be combined with the measurement of the upper limits on the masses of mass-eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ and thus can sensitively probe radiative corrections to the MSSM Higgs
sector. Comparison of the experimentally measured radiative corrections combined into $\Delta$ with its calculations can give a precise estimation of the lower bounds of $h$ (as well as $A$)-Higgs boson masses. The analysis of the scale $\Lambda$ as well as the precise measurement of the lower bound on the $Z'$ boson mass at the Tevatron Run II can probe the CP-violating mixing between two heavy neutral CP-eigenstates $H$ and $A$, and as a consequence, the non-minimality of the MSSM Higgs sector.

It is expected that the Tevatron Run II experiments will be able to exclude $Z'$ bosons with masses up to 750 GeV. This leads to the restriction of the model scale parameter like $x$ which would grow. An important question is whether the forthcoming data at the Tevatron Run II at $\sqrt{s} = 2$ TeV will progress far enough to determine the lower bounds on the $\Lambda$-scale and the $Z'$ boson mass within the models considered in this work.

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$m_h^2 \ [\text{TeV}^2]$

$x = 8 \ 5 \ 3 \ 2$

$M_A = 0.8 \text{ TeV}$
$x = 8$  
$M_A = 0.8 \text{ TeV}$