Wake asymmetry weakening in viscoelastic fluids

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Abstract

Viscoelasticity weakens the asymmetry of laminar shedding flow behind a blunt body in a free domain. In the present study, this finding is confirmed by three unsteady viscoelastic flows with asymmetric configuration against each blunt body, i.e., flow over a hydrofoil with various angles of attack, flow over a rotating circular cylinder, and flow over two side-by-side circular cylinders placed closely. At high Weissenberg number (We), an arc shape region with high elastic stress, which is similar to shock wave, forms in the frontal area of each blunt body. This region acts as a stationary shield to divide the flow into different regimes. Thus, the free stream resembles to pass this shield instead of the original blunt body. As this shield has symmetric feature, the wake flow restores symmetry.

Keywords: Viscoelastic flow, shedding flow asymmetry, hydrofoil, rotating circular cylinder, side-by-side cylinders.

I. Introduction

When a long cylindrical structure is immersed in a cross-flow, Kármán vortices are periodically shed downstream of the structure at sufficiently large Reynolds number (Re).1 These vortex shedding phenomena from blunt cylinders have received great attention since they are associated with numerous cases of flow-induced structural and acoustic vibrations.2,3 For the simplest case of unconfined viscous flow past a circular cylinder, depending upon Re, the flow undergoes several transitions from one flow regime to another. At very low Re, since fluid inertia is negligible, fluid parcels are able to adjust the shape of the submerged blunt body and thus closely follow its contours, i.e., the flow remains attached to the surface. The flow behaves symmetrically at front and back, up and down area of rigid body. As Re is increased gradually (Re > ~5), fluid inertia increases and the adverse pressure gradient along the surface of the blunt body leads to the appearance of a separation bubble. The flow remains symmetric at up and down areas of cylinder. But the flow in front and rear of the cylinder is asymmetric. With a further increment in Re, the separation bubble grows in size until the wake becomes asymmetric (about the mid-plane) and unsteady at Re ≈ 48. Beyond this Re, the vortices are shed downstream of the

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cylinder and the wake becomes periodic in time, albeit the flow is still two-dimensional and laminar. The transient flow shows the up-down asymmetry. However, previous numerical simulation and experimental results indicated that the time average flow field around a cylinder still maintains the spatial symmetry.\textsuperscript{4,5} Statistically, the time average lift acted on cylinder is about zero.

Flow over blunt bodies is also affected by the shape,\textsuperscript{6} orientation,\textsuperscript{7} number and arrangement,\textsuperscript{8} and motion (such as rotation)\textsuperscript{9} of the blunt bodies, as well as the blockage ratio of flow domain,\textsuperscript{10} etc. In Newtonian fluid, when the flow configuration (i.e., geometry or boundary condition) is asymmetric, the pressure distribution on the blunt body and the vortex shedding in the wake show up-down asymmetry. For example, shear layer attachment on a hydrofoil could lead to an effective change of the flow pattern based on the inclination of the hydrofoil,\textsuperscript{11} which may lead to asymmetric flow beyond a certain angle of attack ($\alpha$). With an increase in $\alpha$, the lift coefficient of hydrofoil gradually increases. This results from the asymmetric pressure distribution on the upper and lower walls of the hydrofoil, while the vortex shedding trajectory in the wake deviates from the center line of the flow field. Over a rotating circular cylinder, the cross flow would also generate lateral lift due to asymmetry. As the cylinder rotation drives additional motion of the surrounding fluid, the fluid velocity increases at one side of the cylinder and decreases at the other side. This difference causes the pressure on the upper and lower sides of the cylinder to be inconsistent, resulting in the force perpendicular to the incoming flow direction, which is the so-called Magnus effect.\textsuperscript{12} Correspondingly, the wake flow at $Re = 100$ could be divided into four categories, including two types of vortex shedding modes and two types of steady states.\textsuperscript{9} For unsteady flow over two side-by-side circular cylinders in a free domain, a strong repulsive force exists between the two cylinders if they are placed closely.\textsuperscript{13} The two cylinders have the same diameter $D$ and the minimum spacing between two cylinders is $L_D$. The flow behaviors at the upper and lower sides of each cylinder show different features. For example, when the fluid flows past the upper side of the top cylinder, the velocity gradually recovers the incoming flow velocity along the streamwise direction. However, the flow velocity past the lower side of the top cylinder is much smaller, due to the blockage effect of the narrow gap between the two cylinders. Thus, the flow is asymmetric with respect to the horizontal center line of each cylinder, which results in the nonzero time average lift force acted on each cylinder. Moreover, the shear strain rates of the upper and lower sides of each cylinder are different, which affect the vortex shedding pattern from the cylinder. Special to $Re = 100$ (based on $D$), the flow behaviors can be classified into four different modes, depending on $L_D/D$, i.e., the single vortex street mode when $L_D/D < 0.4$, the flip-flopping mode when $0.4 \leq L_D/D < 1.5$, the in-phase-synchronized flow mode when $1.5 \leq L_D/D < 2.0$, and the antiphase-synchronized flow mode when $L_D/D \geq 2.0$.\textsuperscript{8}

Adding dilute concentration of polymers in a fluid could significantly change the flow characteristics. For the internal and external flows in the turbulent flow regime, when a small amount of polymer is added into water, the friction factor and the drag coefficient are known to dramatically decrease, which implies many applications ranging from fluid transportation to flow control.\textsuperscript{12,13} For example, Xiong et al.\textsuperscript{14} proposed a strategy to suppress vortex-induced vibration by introducing a small amount of soluble long-chain polymer in water. Adding soluble polymer into water could also suppress cavitation.\textsuperscript{15-18} Dissolving polymer into a working fluid may either inhibit heat dissipation\textsuperscript{19,21} or enhance heat transfer.\textsuperscript{22-24} Solutions containing polymer additives may exhibit more complex rheological behavior than Newtonian fluids, owing to the
non-Newtonian effects such as shear-thinning, anisotropy, and viscoelasticity. Therefore, the underlying mechanisms behind these applications are very complicated. When the geometry and the boundary conditions are symmetric, flow asymmetry is more likely to occur in viscoelastic fluid, compared with pure shear-thinning fluid or Newtonian fluid flow.\(^{25-27}\) Haward \textit{et al.}\(^{25}\) experimentally studied the flow of a dilute polymer solution (low polydispersity sample of tactic polystyrene dissolved in a viscous organic solvent dioctyl phthalate) over a confined cylinder. This fluid is essentially non-shear-thinning over 3 decades in shear rate. They found that at high Weissenberg number \((We)\), a flow asymmetry appeared upstream of the cylinder, due to a high local tensile Weissenberg number. Later, Haward \textit{et al.}\(^{26}\) investigated a series of shear-banding viscoelastic wormlike micellar solutions (hydrolyzed polyacrylamide dissolved at different concentrations in deionized water) and found that strong flow asymmetry appears not only in front of the cylinder but also at both sides of the cylinder. The asymmetry was also found to develop from an initially random flow fluctuation of the highly-stressed downstream birefringent wake when \(We\) is beyond a critical value \(We_c\). Besides, the asymmetry also widely appears in other flows, such as cross-flow\(^{28,29}\), planar expansion,\(^{30}\) etc.

Xiong \textit{et al.}\(^{31}\) simulated viscoelastic flow over a hydrofoil with a large \(a\) and found that the wake field gradually retains symmetry (associated with a decreasing lift) when increasing \(We\). For viscoelastic flow over two side-by-side circular cylinders, the lift force between the two cylinders with a low \(L_D\) become lower when increasing \(We\).\(^{32}\) In these two numerical simulations, it is observed that the lift force acted on every blunt body is weakened in viscoelastic fluid, comparing with that in Newtonian fluid. This raises questions as to whether and why the flow asymmetry against each blunt body caused by the asymmetric geometry or boundary condition could be weakened in viscoelastic fluid.

To shed light on this matter, we propose three numerical examples with asymmetric flow configuration against each blunt body, i.e., flow over a hydrofoil under various \(a\), flow over a rotating circular cylinder, and flow over two side-by-side circular cylinders placed closely. Flow over a NACA0012 with angle of attack is the representative of geometric asymmetry. Other geometric asymmetry examples include an inclined flat plate,\(^{33}\) an inclined square cylinder,\(^{34}\) etc. Flow over a rotating circular cylinder is the representative of asymmetry in boundary condition. The asymmetric slip distribution on the cylinder surface also belongs to this category.\(^{35}\) Flow over two side-by-side circular cylinders placed closely, which could be regarded as one blunt body, is the representative of multibody flow. The flow asymmetry comes from the asymmetry behavior of the surrounding flow as discussed above. Other blunt body flows, such as flows over three side-by-side circular cylinders,\(^{36}\) side-by-side spheres or disks,\(^{37}\) and three circular cylinders in equilateral-triangular arrangements,\(^{38}\) can be classified in this category. For these three types of flows, due to geometric asymmetry, boundary asymmetry or asymmetry behavior of surrounding flow, the lift force acted on each blunt body is not zero, while the wake field becomes asymmetric.

In each example, we carefully explore the influence of viscoelasticity on the laminar unsteady wake behavior, particularly the flow asymmetry. This paper is organized as follows. The governing equations and the numerical methods are presented in Sec. II. In Sec. III, the results of lift force and flow characteristics of the three flow cases are presented and the corresponding underlying mechanisms on flow asymmetry weakening are discussed. Finally, the main conclusions and the future outlook are provided in Sec. IV.
II. Mathematical formulation

While a small amount of polymers are added into water, the incompressible Navier-Stokes (N-S) equations are slightly modified and have the following form on a differential fluid element:\textsuperscript{39,40}

\begin{equation}
\frac{\partial u_j}{\partial x_j} = 0,
\end{equation}

\begin{equation}
\rho \frac{\partial u_j}{\partial t} + \rho u_j \frac{\partial u_j}{\partial x_j} = - \frac{\partial p}{\partial x_j} + \mu_x \frac{\partial^2 u_j}{\partial x_i^2} + \frac{\mu_p}{\lambda} \frac{\partial \tau_{ij}^p}{\partial x_j},
\end{equation}

where the summation convention is used, with \(i\) and \(j\) being the summation indices, \(x\) is the coordinate, \(t\) is time, \(u\) is the velocity, \(p\) is the pressure, \(\rho\) is the density of fluid, \(\mu_x\) and \(\mu_p\) are the viscosity contribution from the solvent and the polymer, respectively, \(\lambda\) is the relaxation time of polymer, \(\tau_{ij}^p\) is the additional polymer stress. The last term in Eq. (2) represents the influence of \(\tau_{ij}^p\) due to the elasticity of polymers in the flow. The total viscosity of the solution is defined as \(\mu = \mu_p + \mu_x\). The polymer viscosity ratio at vanishing shear rate is defined as \(\beta = \mu_p/\mu_x\), which is a measurement of polymer concentration and molecular characteristics. Eq. (2) restores to the original N-S equations when \(\beta = 0\). The polymer stress can be modeled by a molecular-based Peterlin approximation of the finitely extensible nonlinear elastic (FENE-P) model, which describes an individual member of polymers in a dilute concentration as a dumbbell connected with a finitely extensible nonlinear elastic spring by the way of a balance of forces acting on each bead. In this model, the polymeric stress \(\tau_{ij}^p\) can be determined using kinetic theory:\textsuperscript{41,42}

\begin{equation}
\tau_{ij}^p = \frac{c_{ij}}{1 - \frac{c_{ik} c_{kj}}{L^2}} \delta_{ij},
\end{equation}

where \(c_{ij}\) represents the polymer conformation tensor, which is defined as the pre-averaged dyadic product of the polymer end-to-end vector, \(\delta_{ij}\) is the Kroenecker delta function, and \(L\) is the maximum polymer extensibility, which is normalized by the equilibrium length of a linear spring \((\kappa T/H)^{1/2}\) with \(T\) the absolute temperature, \(\kappa\) the Boltzmann constant, and \(H\) the Hookean spring constant for an entropic spring. The polymer conformation tensor is governed by the following hyperbolic transport equation:

\begin{equation}
\frac{\partial c_{ij}}{\partial t} + u_k \frac{\partial c_{ij}}{\partial x_k} - c_{ik} \frac{\partial u_j}{\partial x_k} - c_{kj} \frac{\partial u_i}{\partial x_k} = - \frac{\tau_{ij}^p}{\lambda}.
\end{equation}

It is noted that the FENE-P model is employed in this work based on its ability to properly represent the finite extensibility of the polymer. In order to obtain bound solutions for problems with high \(We\) and large strain rate, the finite extensibility is necessary. Those linear spring models such as the Oldroyd-B model cannot be faithfully used in real engineering problems. Furthermore, the FENE-P model has been widely used in previous studies involving viscoelastic flows at high \(Re\) and can successfully produce accurate physical results in these problems.
In this paper, $\beta$ is set to be very small ($\beta = 0.1$) to minimize the shear-thinning effect and $L$ is set to 100. Under these parameters, the FENE-P model could describe the rheological behavior of a dilute polymer solution more explicitly. Other values of $L$ are also investigated to provide a basic evaluation of the influence of the maximum polymer extensibility. As $L$ tends to infinite, the present FENE-P fluid approaches to the Oldroyd-B fluid.

To solve the above equations numerically, a finite volume commercial solver ANSYS FLUENT associated with a self-developed user defined function (UDF) for the elastic stress transport equation, i.e., Eq. (4), is applied. The QUICK scheme and the second order implicit scheme are adopted to discretize the space and temporal domains, respectively. An artificial term $\kappa \Delta c$ is added to the right hand side of Eq. (4). This implementation has been proved to be effective to ensure numerical stability.\(^{43,45}\) In this study, $\kappa = 0.1\mu$ is adopted by considering both the numerical stability and accuracy of numerical approximation.\(^{39,40}\) Furthermore, an under-relaxation scheme is utilized to avoid the rapid increase of the numerical error.

### III. Results and Discussion

#### A. Viscoelastic flow over a hydrofoil

First, we consider viscoelastic flow over a standard NACA0012 hydrofoil in a free domain, as shown in figure 1. The chord length of the hydrofoil is set as $a$. The attack angle between the hydrofoil and the incoming flow is recorded as $\alpha$. More information about this hydrofoil can be found in Kouser et al.\(^7\) The entrance of the calculation domain is semicircular. The distance between the foremost end of the semicircle and the rearmost end of the hydrofoil is set as 6$a$. The distance between the outlet and the rearmost end of the hydrofoil is set as 18$a$. The distance between the upper and lower boundaries is set as 12$a$. The rearmost end of the hydrofoil is set at $(x, y) = (0, 0)$.

![Fig. 1](image_url). The schematic of the unconfined flow around a NACA0012 hydrofoil. Reprinted from Phys. Fluids, 30, 013104 (2018), with the permission of AIP publishing.
Fig. 2. The time average lift coefficients in (a) Newtonian fluid and (b) viscoelastic fluid. Reprinted from Phys. Fluids, 30, 013104 (2018), with the permission of AIP publishing.

Fig. 3. The time average streamlines. The left and right columns represent $\alpha = 15^0$ and $20^0$, respectively. The rows from top to bottom denote $We = 0$ (Newtonian fluid), $We = 0.2$, $We = 0.5$, $We = 1.0$, and $We = 1.5$, respectively. The red arrow and circle in the right top panel highlight the small anti-clockwise rotating recirculation bubble between the clockwise recirculation and the foil, which gradually disappears with increasing $We$ as shown in the below panels. And this small recirculation disappears as the increase in the Weissenberg number. Reprinted from Phys. Fluids, 30, 013104 (2018), with the permission of AIP publishing.
The no-slip boundary condition is imposed at the surface of the hydrofoil. The inlet velocity is uniform and set to $\bar{u} = (U_\infty, 0)$. The symmetry boundary condition is imposed at the upper and lower boundaries. The pressure of the outlet boundary is set as $p = 0$. For the conformation tensor, the no-flux condition is approximated at all boundaries. For this numerical example, the Reynolds number is defined as $Re = U_\infty a / \mu$ and fixed at 1000 and the Weissenberg number is defined as $We = \lambda U_\infty / a$, which ranges from 0 to 2. The computational grid is generated by the commercial software ICEM, and the detailed information can be found in our previous studies.\(^\text{7,31}\)

The time average lift coefficient ($\bar{C}_l$) of the hydrofoil at various $\alpha$ and $We$ is plotted in figure 2. In Newtonian fluid, $\bar{C}_l$ increases with $\alpha$, as shown in figure 2(a). Our results agree well with those reported in Kutulus,\(^\text{46}\) Liu et al.,\(^\text{47}\) and Khalid et al.\(^\text{48}\) In viscoelastic fluid, regardless of $\alpha$, $\bar{C}_l$ gradually decreases with an increase in $We$. Besides $L = 100$, $L = 10$ and 200 are also considered. An increase in $L$ strengthens viscoelasticity, which further suppresses $\bar{C}_l$. The magnitude of lift
could reflect the strength of flow asymmetry. The decreasing $C_l$ indicates that the flow asymmetry is weakened.

The time average streamlines for $\alpha = 15^0$ and $20^0$ are shown in figure 3. The recirculation region behind the hydrofoil gradually elongates with increasing $We$. The instantaneous vorticity contours are shown in figure 4. In Newtonian fluid, at high $\alpha$ (e.g. $\alpha = 20^0$), the trail of wake vortex leans to one side of the hydrofoil. However, as $We$ increases, the vortex trajectory gradually approaches to the center line of the flow field. The typical instantaneous positions of vortex cores for Newtonian and viscoelastic fluids at $\alpha = 10^0$ and $20^0$ are extracted and plotted in figure 5. Obviously, with an increase in $We$, the vortices on both sides of the hydrofoil tend to be symmetric against the center line of the flow field.

B. Viscoelastic flow over a rotating circular cylinder

Second, we consider viscoelastic flow over a rotating circular cylinder in a free domain, as shown in figure 6. The whole computational domain has a rectangular shape, with the length $L_u+L_d$ and the width $H$. The center of the cylinder is set at $(x, y) = (0, 0)$. The diameter of the cylinder is $D$. The distance between the inlet and the cylinder center is set as $L_u = 25D$. The distance between the outlet and the cylinder center is set as $L_d = 75D$. $H$ is set as $50D$. The free stream boundary condition with the uniform velocity $u = (U_\infty, 0)$ is imposed at the inlet. The no-slip boundary condition is adopted on the cylinder surface. The angular velocity of rotating cylinder is $\omega$ and the dimensionless rotation velocity rate is defined as $\alpha_r = \omega D/(2U_\infty)$. In this study, $\alpha_r$ ranges from 0 to 6. The symmetry boundary condition is imposed at the upper and lower boundaries of the computational domain. The pressure at the outlet is set as $p = 0$. For the conformation tensor, the no-flux condition is approximated at all boundaries. The Reynolds number is defined as $Re = U_\infty D \rho/\mu$ and fixed at $Re = 100$ and the Weissenberg number is defined as $We = \lambda U_\infty D$, which ranges from 0 to 10.

![Fig. 6. The schematic of flow over a rotating circular cylinder.](image)
Fig. 7. The absolute time average lift coefficients of the rotating cylinder in (a) Newtonian fluid and (b) viscoelastic fluid.

Fig. 8. The instantaneous vorticity contours (left column) and the time average streamlines (right column) at \((Re, We, L) = (100, 10, 100)\). The rows from top to bottom represent \(\alpha_r = 0, 1, 3\) and 6, respectively.

Fig. 9. A simple model to illustrate the feature of the viscoelastic flow over a circular cylinder rotating at a high \(We\).
The absolute time average lift coefficients ($|\tilde{C}_l|$) of the cylinder is calculated and plotted in figure 7. In Newtonian fluid, $|\tilde{C}_l|$ increases with $\alpha$, as shown in figure 7(a). The comparison between the present results and those of Bourguet & Jacono\textsuperscript{39} and Stojković \textit{et al.}\textsuperscript{9} shows excellent agreement. In viscoelastic fluid, $|\tilde{C}_l|$ gradually decreases with an increase in $We$ for all $\alpha$. As mentioned above, the magnitude of lift could reflect the strength of flow asymmetry. The decreasing $|\tilde{C}_l|$ indicates that the flow asymmetry is weakened.

The instantaneous vorticity contours (left column) and the time average streamlines (right column) at ($Re, We, L$) = (100, 10, 100) for different $\alpha$ are shown in figure 8. For different $\alpha$, the flow fields behave quite similarly. We mainly concern the flow behaviors in two regions, i.e., a thick rotating boundary layer flow around the rotating circular cylinder and vortex shedding in the wake. A simple model, as shown in figure 9, could be used to describe the flow behavior of the viscoelastic fluid past a rotating circular cylinder at a high $We$ (such as $We = 10$). $Da$ denotes the thickness of the rotating boundary layer ($Da > D$), which increases with $\alpha$. We notice that the profile of the circumferential velocity $u_\theta$ along the radial direction $r$ in the rotating boundary layer is almost completely consistent, as shown in figure 10. $u_\theta$ gradually decays with $r$ and tends to zero at the outer edge of the rotating boundary layer $r = Da / 2$. Thus, this outer edge $r = Da / 2$ acts as a dummy stationary solid wall and divides the flow into two regimes, as shown in figure 9. The flow induced by the rotating cylinder is shielded by this dummy wall and does not communicate with the free stream outside. On the other hand, the free stream outside resembles to pass a large stationary cylinder with $r = Da / 2$ and the wake becomes symmetric as shown in figure 8.

![Fig. 10. Circumferential velocity ($u_\theta$) profiles along the front, back, upper and lower lines near the cylinder for ($Re, We, L, \alpha$) = (100, 10, 100, 6). $r = 0.5$ denotes the cylinder surface.](image)

**C. Viscoelastic flow over two side-by-side circular cylinders placed closely**

Third, we consider viscoelastic flow over two equal-size side-by-side circular cylinders in a free domain, as shown in figure 11. The diameter of each cylinder is $D$ and the nearest distance between the two cylinders is $L_D$. The detailed flow features has been reported in our previous study.\textsuperscript{32} Our previous results indicated that if the cylinders placed closely, the flow behaves like flow over a large cylinder (the cross-sectional area facing the incoming flow is about $2D + L_D D$).\textsuperscript{32} However, if $L_D$ is increased, such as $L_D = 3D$, the two cylinders cannot be considered as one body. Note that the present study only concerns the wake asymmetry weakening of one blunt
body or several closely placed bodies which can be regarded as one body. Thus, only the results for \( L_D = 0.1D \) is presented here.

The whole computational domain has a rectangular shape, with the length \( L_u + L_v \), and the width \( L_u \). The distance between the inlet and the cylinder center is set as \( L_u = 50D \). The distance between the outlet and the cylinder center is set as \( L_v = 120D \). \( L_v \) is set as \( 50D \). The middle point of the two cylinders is set at \((x, y) = (0, 0)\). The free stream boundary condition with the uniform velocity \( \vec{u} = (U_{\infty}, 0) \) is imposed at the inlet. The no-slip boundary condition is adopted on the two cylinder surfaces. The symmetry boundary condition is imposed at the two lateral boundaries of the computational domain. The pressure at the outlet is set as \( p = 0 \). For the conformation tensor, the no-flux condition is approximated at all boundaries. The Reynolds number is defined as \( Re = U_{\infty}D\rho/\mu \) and fixed at \( Re = 100 \). The Weissenberg number is defined as \( We = \lambda U_{\infty}/D \), which ranges from 0 to 8. The computational grid is generated by the commercial software ICEM, and the corresponding details could refer to our previous study.\(^{32}\)

We first consider the repulsive force between the two cylinders. In Newtonian fluid, the absolute time average lift coefficient of each cylinder is about 1.5. This means that if unfixed, the two cylinders would be separated due to the strong repulsion between them. However, in viscoelastic, the repulsive force becomes weak when \( We \) increases, as shown in figure 12(a). The reduction in the repulsive force is mainly due to the redistribution of the cylinder surface pressure, as shown in figure 12(b).

The instantaneous vorticity contours (left column) and the time average streamlines (right column) are shown in figure 13. For both Newtonian and viscoelastic fluids, the flow resembles the flow around a single cylinder. The transient vortex shedding trajectory remains symmetric with the center line of the flow field, but is asymmetry against the horizontal center line of each cylinder. With an increase of \( We \), the time average main recirculation region is obviously elongated. Moreover, the small recirculation bubbles on the cylinder surface gradually disappear with increasing \( We \). These mean that the time average wake field behind each cylinder tends to be symmetric against its own horizontal center line.

**Fig. 11.** The schematic of unconfined flow over two side-by-side circular cylinders. Reprinted from Phys. Fluids, 32, 083106 (2020), with the permission of AIP publishing.
Fig. 12. (a) The variation of the absolute time average lift coefficient with $We$. (b) The time average pressure profiles along the surface of the upper cylinder in Newtonian and viscoelastic fluids. The symbol $X$ is equivalent to $x/D$. Adapted from Phys. Fluids, 32, 083106 (2020), with the permission of AIP publishing.

Fig. 13. The instantaneous vorticity contours (left column) and the time average streamlines (right column) at $LD = 0.1D$; (i) $We = 1$, (ii) $We = 2$, and (iii) $We = 4$. Adapted from Phys. Fluids, 32, 083106 (2020), with the permission of AIP publishing.

D. Underlying mechanism on flow asymmetry weakening

In order to understand the physical mechanism on flow asymmetry weakening, we investigate the distributions of elastic stress for the above three examples.

For the viscoelastic flow over the NACA0012 hydrofoil, a region with high elastic stress appears around the leading edge and then extends to the wake region (combine the regions of high conformation tensor components $c_{xx}$ and $c_{yy}$ in figure 14(a) and (b)). $\alpha$ does not fundamentally affect the distribution of the elastic stress (data not shown). Ritcher et al.\textsuperscript{40} pointed out that this
elastic stress concentrated area is similar to shock wave and the fluid in this area behaves like a solid, experiencing low velocity and high pressure. The corresponding velocity and pressure coefficient distributions are shown in figure 15. The velocity becomes low near the hydrofoil. The velocity distributions near the upper and lower surfaces of the hydrofoil tend to be symmetric, which results in the lower pressure difference between the upper and lower surfaces.

Fig. 14. The instantaneous distributions of (a) $c_{xx}$ and (b) $c_{yy}$ for the viscoelastic flow over the NACA0012 hydrofoil at $\alpha = 20^\circ$ and $We = 1.5$.

Fig. 15. The time average distributions of (a) the $x$ velocity component $\bar{u}$ and (b) the pressure coefficient $\overline{C_p}$ for the viscoelastic flow over the NACA0012 hydrofoil at $\alpha = 20^\circ$ and $We = 1.5$.

The elastic stress distribution for the viscoelastic flow over a rotating circular cylinder is shown in figure 16. Two elastic stress boundary layers can be identified by combining the regions of high conformation tensor components $c_{xx}$ and $c_{yy}$ in figure 16(a) and (b), with the help of the simple model sketched in figure 9. Within the annulus formed between the original cylinder surface and the dummy stationary wall, a thick rotating elastic stress boundary layer forms, which fully fills the annulus. The velocity at the dummy stationary wall is close to zero, as shown in figure 10. In addition, similar to viscoelastic flow over a hydrofoil, another elastic stress boundary layer similar to shock wave appears in front of the dummy stationary cylinder, and then extends to the wake. Now the free stream resembles to past a large stationary cylinder and the wake field restores its symmetry with increasing $Wi$ as shown in figure 8.
Fig. 16. The instantaneous distributions of (a) $c_{x\alpha}$ and (b) $c_{yy}$ for the viscoelastic flow over a rotating circular cylinder at $\alpha_r = 6$ and $We = 10$. The dash lines denote the position of the dummy stationary solid wall.

Fig. 17. The instantaneous distributions of (a) the circumferential velocity $u_\theta$ and (b) the pressure coefficient $C_p$ for the viscoelastic flow over a rotating circular cylinder at $\alpha_r = 6$ and $We = 10$.

For the viscoelastic flow over two side-by-side circular cylinders placed closely, the elastic stress is concentrated near the front edge of each cylinder, connects into one piece, and extends along the outer side of each cylinder towards the downstream of the cylinders, as shown in figure 18. The velocity becomes low near the cylinders, which is similar to the behavior to flow over the NACA0012 foil. The velocity difference between the upper and lower sides of each cylinder becomes not obvious, as shown in figure 19(a). Then, the pressure distribution tends to be symmetric against its own horizontal center line, as shown in figure 19(b). In the wake, the time average flow field also becomes symmetry against its own horizontal center line, as shown in figure 13(b).

However, if the two cylinders placed far away from each other (such as $L_D = 3D$), the elastic stress distribution in front of each cylinder cannot connect together, but surrounds the cylinders separately, which strengthens the flow asymmetry, as reported in our previous study.\textsuperscript{32}

A common feature can be summarized from the above stress distributions for the three
examples, i.e., an region with high elastic stress similar to shock wave appears near the frontal edge of the blunt body and extends downstream, as sketched by the red solid lines in figure 20. This region acts as a stationary shield to divide the flow into different regimes. Thus, the free stream resembles to pass this shield instead of the original blunt body. As this region is nearly symmetric against the horizontal center line of the corresponding flow configuration, the wake flow recovers symmetry. Specially, the high elastic stress regions in front of each cylinder of the side-by-side configuration connect into one piece as shown in figure 18(b) and 20(c), which blocks the flow through the gap between the two cylinders. In this sense, the horizontal center line of this flow configuration can be regarded as an imaginary shield indicated by the dash line in figure 20(c). Therefore, the wake flow behind each cylinder does not interact with each other and recovers symmetry against its own horizontal center line.

Fig. 18. The instantaneous distributions of (a) $c_{xx}$ and (b) $c_{yy}$ for viscoelastic flow over two side-by-side circular cylinders at $L_D = 0.1D$ and $We = 8$.

Fig. 19. The distributions of (a) the time average $x$ velocity ($\overline{u}$) and (b) the time average pressure coefficient ($\overline{C_p}$) for viscoelastic flow over two side-by-side circular cylinders at $L_D = 0.1D$ and $We = 8$.

Fig. 20. Schematic diagram of regions with high elastic stress (the red solid lines): (a) flow over a hydrofoil, (b) flow over a rotating circular cylinder, and (c) flow over two side-by-side circular cylinders placed closely.
IV. Conclusion Remarks

In this paper, we report that viscoelastic wake flow asymmetry in a free domain is no longer sensitive to asymmetry in geometry, boundary condition, or surrounding flow behavior, through numerical simulation. Two-dimensional direct numerical simulations based on the FENE-P model (the finite-extensible nonlinear elastic model with the Peterlin closure) are conducted. Three asymmetric flows are considered in this study, i.e., flow over a hydrofoil with different angles of attack at $Re = 1000$, flow over a rotating circular cylinder at $Re = 100$, and flow over two side-by-side circular cylinders placed closely at $Re = 100$. The three flows are the representative of geometric asymmetry, boundary condition asymmetry, and asymmetry behavior of surrounding flow, respectively.

The lift coefficient can be used to describe the asymmetry degree of the flow field near the blunt body. The lift coefficient acted on blunt body becomes lower in viscoelastic fluid compared that in Newtonian fluid. Accordingly, the viscoelastic wake flow tends to be symmetry with respect to the horizontal center line of the flow configuration for the hydrofoil with an angle of attack or the rotating circular cylinder. For viscoelastic flow over two side-by-side circular cylinders placed closely, the time average wake field behind each cylinder tends to be symmetry against its own horizontal center line.

A careful examination of the elastic stress distributions of the three examples reveals a common feature, i.e., an region with high elastic stress similar to shock wave appears near the frontal edge of the blunt body and extends all the way downstream. The blunt body seems to be wrapped by this region, which retains a symmetric shape with respect to the horizontal center line of the flow configuration. The free stream cannot directly interact with the blunt body but flows around this region, as if the free stream passes an imaginary symmetric blunt body with larger characteristic length. Thus, the wake flow recovers symmetry. Specially, the high elastic stress region in front of the two side-by-side cylinder placed closely blocks flow through the gap between the two cylinders. In this sense, the horizontal center line of this flow configuration can be regarded as an imaginary solid wall, which inhibits the interaction of the wake flow behind each cylinder. Thus, the wake behind each cylinder recovers symmetry against its own horizontal center line.

The influence of elastic stress distribution on lift and flow symmetry discovered in this paper could be used for precise flow control. It would be interesting to investigate how to obtain the desired lift and vortex shedding trajectory by controlling the polymer concentration distribution around a hydrofoil to redistribute elastic stress in future. The flow asymmetry could cause higher flow-induced vibration response for a rotating circular cylinder. Xiong et al. proposed that polymer addition could restrain vortex-induced vibration of a non-rotating cylinder. It is reasonable to speculate that polymer addition could also effectively suppress vortex-induced vibration of a rotating circular cylinder, due to the flow asymmetry weakening discussed above, which entails future investigations.

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