ON-SITE CALIBRATION OF A PHASE FRACTION METER BY AN INVERSE TECHNIQUE

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ABSTRACT. The formal calibration procedure of a phase fraction meter is based on registering the outputs resulting from imposed phase fractions at known flow regimes. This can be straightforwardly done in laboratory conditions, but is rarely the case in industrial conditions, and particularly for on-site applications. Thus, there is a clear need for less restrictive calibration methods regarding to the prior knowledge of the complete set of inlet conditions. A new procedure is proposed in this work for the on-site construction of the calibration curve from total flown mass values of the homogeneous dispersed phase. The problem is formulated as a set of integral equations, which could be formally solved by setting an appropriate approximation for the calibration curve. However, due to an intrinsically ill-conditioned characteristic, these equations cannot be solved in practical situations because of the severe influence of experimental errors. A solution to this problem is also proposed in this work. The method is based on minimizing an error functional constructed from a set of redundant measurements, which restores the lost information associated to the integration of the instantaneous mass flow rate in the one-dimensional one-velocity flow model. Numerical simulations performed for increasing errors demonstrate that acceptable calibration curves can be reconstructed, even from total mass measured within a precision of up to 4%. Thus, the method can readily be applied, especially in on-site calibration problems in which classical procedures fail due to the impossibility of having a strict control of all the input/output parameters.

Keywords: multiphase flow, instrumentation, phase meter, calibration
1 INTRODUCTION

The continuous measurement of physical parameters in multiphase flows is of great interest, not only for the monitoring and control of industrial equipment, but also to obtain phenomenological insight in research applications. In particular, the development of phase fraction sensors is a subject very frequently found in the related scientific literature, considering that this parameter is one of the most adequate when describing and analyzing two-phase flows.

A good illustration of this is the determination of composition and flow rates in the petroleum industry (Fischer, 1994). In the absence of an acceptable on-line measurement device, the most common procedure is to adopt a strategy based on the continuous separation of the fluid constituents and subsequent measurement by conventional single-phase techniques followed by recombination. Despite the extensive number of applications based on this approach, its efficiency and economic feasibility are not satisfactory. The large size of conventional separation equipment, such as hydro-cyclones and other centrifugal separators, is frequently restrictive and even prohibitive in offshore production systems for instance. In addition to this, in some situations, special thermal or chemical methods are required to deal with emulsion formation. Also, the flashing of dissolved gases from the liquid phase requires a more elaborate temperature and pressure control system implying in more complex operation conditions (Rajan et al., 1993). Thus, the development of simple, robust and inexpensive non-intrusive multiphase flow sensors suited for industrial applications is still an open problem.

In this context, and regarding the measurement of spatially averaged phase fractions, electrical sensing techniques are particularly attractive due to its capability of resolving fast changes in the flow structure, besides being simple to implement and not expensive as well. Its general principle of operation is based on differences or contrasts in the electrical properties of the phases of the mixture, and also on the assumption that the electromagnetic sensing field is instantaneously modulated by the flow (Chang and Watson, 1994). In general, problems associated with electrical sensing are related to electrochemical effects (Hemp, 1994), electrostatic stray charges (Green and Thorn, 1998), nonlinearity due to the influence of flow regimes (Andreussi et al., 1988), and electrode erosion or coating. One way of overcoming these problems is to design the sensor and the sensing strategy specifically for the desired application, therefore the need for on-site calibration methods (Duncan and Trabold, 1997).

The formal calibration procedure of a measurement device is based on the construction of the output/input relation by imposing known inputs and registering the corresponding outputs. In the case of a phase fraction probe this is not sufficient because, in addition to being correlated to the phase fraction, the probe's response will be also strongly correlated to the flow regime. In other words, the same phase fraction may result in different responses depending on the topological organization of the constituent phases within the sensing volume (see for instance the work of Andreussi et al., 1988). Attempts have been made aiming to minimize the influence of the flow regime, mostly relied on the optimization of the electrodes (Seleghim and Hervieu, 1998) or on the sensing strategy (Klug and Mayinger, 1994). A very promising approach is based on fuzzy and neural signal processing techniques and is implemented so to previously identify the flow regime and subsequently take the correct calibration curve (Tsoukalas et al. 1997; Mi et al. 1998; Crivelaro and Seleghim, 1999).

Thus, in a strict sense, the calibration of a phase fraction meter requires the ability to impose known phase fractions at known flow regimes, which can be straightforwardly done in laboratory conditions. In industrial conditions this is rarely the case. For instance, if it is
necessary to calibrate a fraction meter placed on a pneumatic conveying line, the control of the flow conditions would require the addition of auxiliary equipment that would produce significant disturbances in the operating conditions and, consequently, compromising the final result. Still, the majority of industrial scale pneumatic conveyors are designed to operate at nonpermanent flow conditions, which may take the form of discrete structures such as solid plugs and rolling dunes, or alternating flow regimes associated with varying inlet mass flow rates. This justifies the need for less restrictive calibration methods, in particular with regard to the prior knowledge of the complete set of inlet conditions.

The purpose of this paper is to contribute in this direction by proposing a new inverse procedure for the on-site construction of the calibration curve of a phase fraction meter from less restrictive data, i.e. not the instantaneous flow rate signal but its integral value. The problem will be precisely stated in section 2 and a numerical simulation will be presented in section 3 (in which the consequences and a solution for the problems associated with the inverse nature of the formulation will be shown). A final conclusion and the references are presented respectively in section 4 and 5.

2 STATEMENT OF THE PROBLEM

Consider the homogeneous flow of a two-phase mixture through a capacitive fraction meter installed on a light phase pneumatic transport system (Fig. 1). The volumetric solid fraction ($\alpha$) is defined as the ratio between the volumes occupied by the solids ($V_s$) and the total sensing volume ($V_s + V_a$), which, according to the one-dimensional one-velocity model (Bergels et al., 1981), can be written as

$$\alpha = \frac{V_s}{V_a + V_s} = \frac{Q_s}{Q_a + Q_s} = \frac{m_s}{\rho_s Q_a + m_s}$$

in which the volumetric flow rates ($Q_s$ and $Q_a$), solid density and mass flow rate is defined in Fig. 1.

![Figure 1: Homogeneous flow of a gas-solid mixture through a capacitive fraction meter](image)

The capacitance ($C$) measured between the sensing electrodes depends on how the electrical field traverses the sensing volume, which is related to the permittivity of the medium and also to the geometric organization of the different phases. Thus, the capacitance is strongly correlated to the solid fraction and to the flow regime (homogeneous by assumption), and the formal relation between these variables, i.e.

$$\alpha = \phi(C)$$
is known as the meter's calibration curve as mentioned before. Substituting equation (2) into (1) and isolating the solid mass flow rate yields:

\[ \dot{m}_s = \frac{\rho_s Q_a}{1 - \phi(C)} \]  

(3)

In an industrial pneumatic conveying system the instantaneous values of the air volumetric flow rate (the continuous phase) can be readily determined, for instance with an orifice plate or simply from the blower's performance curve by measuring its rotation and pressure rise. However, the measurement of the instantaneous values of the solid flow rate is quite complex without modifying the piping to install auxiliary equipment. In addition, the solid flow rate must be measured at the fraction meter's section since it can vary significantly along the transport line as well as in time (Ostrowski et al., 1999). A more convenient variable to measure would be the total solid mass \( M_s \) conveyed in a given time interval:

\[ M_s = \int \dot{m}_s \, dt \]  

(4)

This expression can be obtained from the integration of equation (3), with the additional assumption (for simplicity and without loss of generality) of a constant volumetric air flow rate. It will then result

\[ \frac{M_s}{\rho_s Q_a} = \int \frac{1}{1 - \phi(C)} \, dt \]  

(5)

To obtain the calibration curve \( \alpha = \phi(C) \) it is necessary to solve the integral equation (5) on the input data \( M_s, \rho_s, Q_a \) and \( C = C(t) \), the instantaneous capacitance values delivered by the fraction meter. Equation (5) can also be seen as a special case of an inhomogeneous Fredholm equation of the first kind, which is known to be ill conditioned. Consequently, as it will be shown on the sequel, if specific methods to deal with the ill-conditioned nature of the problem are not employed, the calibration curve will be extremely sensitive to small changes in the input parameters, which in fact are likely to happen due to intrinsic experimental errors. This is so because the integration of the unknown function in (5) causes an information loss. A proper method to deal with this must, in some way, restore the lost information from some prior knowledge or from redundant measurements.

3 NUMERICAL SIMULATION AND A SOLUTION TO THE ILL-POSED NATURE OF THE INVERSE PROBLEM

In order to demonstrate the statements above consider the following numerical simulations. First, suppose that the calibration curve is given by the following representative formula:

\[ \alpha = \phi(C) = 0.06 - \sqrt{0.036 - 0.12C} \]  

(6)
This equation is not known a priori and will have to be reconstructed from experimental data. Suppose now that, due to a specific operating condition, the instantaneous measured solids fraction values follow the equation

$$\alpha(t) = 0.025 \left[ 1 + \sin(\pi t) \right]$$  \hspace{1cm} (7)

This being, the substitution of (7) into (6) yields the instantaneous capacitance values delivered by the probe:

$$C(t) = \left[ 1 + \sin(\pi t) \right] \left[ 0.025 - 0.0052 \left[ 1 + \sin(\pi t) \right] \right]$$  \hspace{1cm} (8)

The resulting instantaneous solids mass flow rate for homogeneous flow can be determined by introducing (7) into expression (3), which yields

$$\dot{m}_s = \frac{\rho_s Q_a}{\left(0.025 \left[ 1 + \sin(\pi t) \right] \right)^{-1} - 1}$$  \hspace{1cm} (9)

The following figure illustrates the behavior of these curves for $\rho_s = 3000$ kg/m$^3$ and $Q_a = 0.01$ m$^3$/s

![Figure 3](image-url)

Figure 3: Capacitance in pF (equation (8)) and mass flow rate in kg/s (equation (9)) as a function of time in sec, resulting from (6) and (7) in an homogeneous gas-solid flow.

The inverse problem, such as formulated above, consists in reconstructing the calibration curve (2) (which was imposed to follow (6)) from measurements of the total mass $M_s$ and the instantaneous capacitance values (8). To do this we can start by expressing (2) according to

$$\alpha = \phi(C) = a_0 \phi_0(C) + a_1 \phi_1(C) + \ldots + a_n \phi_n(C) = \sum_{0}^{N} a_i \phi_i(C)$$  \hspace{1cm} (10)

where $\{\phi_i(\alpha)\}$ is a convenient set of known functions. Subsequently, substituting (8) into (10) and the result in (5) we obtain
\[
\frac{M_a}{\rho_s Q_a} \equiv \int_0^{\Delta t} \frac{1}{\sum_0^N a_i \phi_i \left( \left[ 1 + \sin(\pi t) \right] \left[ 0.025 - 0.0052 \left[ 1 + \sin(\pi t) \right] \right] \right)^{-1} - 1} \; dt
\]  \quad (11)

The difference between both sides of the expression above constitutes an error functional expressing how well is the approximation given by (10). The influence of experimental errors in the measurements can be introduced, for instance, by randomly perturbing \( M_s \) and \( C \). We thus define:

\[
e = \left( \frac{M_s(1 + \delta)}{\rho_s Q_a} - \int_0^{\Delta t} \frac{1}{\sum_0^N a_i \phi_i \left( \left[ 1 + \sin(\pi t) \right] \left[ 0.025 - 0.0052 \left[ 1 + \sin(\pi t) \right] \right] + \epsilon(t) \right)^{-1} - 1} \; dt \right)^2
\]

where \( \delta \) and \( \epsilon(t) \) are centered random variables.

The reconstruction of the calibration curve can now be achieved by searching the minimum of equation (12). The steepest descent method is employed in this work, with increments calculated according to the rule

\[
\Delta a_0 \left( \frac{\partial e}{\partial a_0} \right)^{-1} = \Delta a_1 \left( \frac{\partial e}{\partial a_1} \right)^{-1} = \cdots = \Delta a_n \left( \frac{\partial e}{\partial a_n} \right)^{-1}
\]  \quad (13)

Consider \( \delta = \epsilon(t) = 0 \) to illustrate the ill-posed nature of the problem. In this case, the minimum of (12) is associated with better-reconstructed curves as the order of the approximation in (10) is increased, as shown in the following figure.

Figure 4: Reconstruction of the calibration curve from simulated experimental data by searching the minimum of (12) for increasing approximation order in (10).

As it can be seen, the reconstruction works adequately under the assumption of no experimental errors. In a real situation however, these errors must be taken into account and, due to the inverse nature of the problem, this will produce an extremely negative affect. This can be illustrated by performing the same reconstruction as shown in Figure 4.b (third order approximation) and considering only a centered experimental error in the measurement of \( M_s \),
i.e. \( \epsilon(t) = 0 \) while \( \delta \) varies randomly between \( \pm \delta_{\text{max}} \). The following figure shows the influence of increasing values of \( \delta_{\text{max}} \) in the reconstructed calibration curve. Although neglecting the error in the measurement of the instantaneous capacitance values, even unrealistic experimental errors the order of \( \delta_{\text{max}} = 10^{-6} \) have a disastrous effect.

![Figure 5: The influence of experimental errors in the reconstruction of the calibration curve (third order polynomial) from the minimum of (12).](image)

As mentioned before, this problem is probably triggered by the integration of the unknown function \( \phi(C) \) in (5) rendering the problem extremely ill conditioned. To deal with this there are some mathematical methods, mostly based on the construction of regularizing operators from a priori information. Another way of overcoming the problem, which in fact is more natural in the case we are dealing with, consists in reintroducing the lost information by redundant measurements.

Consider then a set of measurements of the total mass \( M_s^{(k)} \) and the corresponding capacitance historic \( C^{(k)}(t) \), performed over different time intervals \( \Delta t^{(k)} \) \( (k = 1,2,\ldots,M) \). For the purposes of this numerical simulation, these data can be generated by randomly varying \( \delta \) and \( \epsilon(t) \), i.e.

\[
M_s^{(k)} = M_s + \delta^{(k)}
\]

\[
C^{(k)}(t) = [1 + \sin(\pi t)][0.025 - 0.0052(1 + \sin(\pi t))] + \epsilon^{(k)}(t)
\]

in which

\[
\delta^{(k)} \leq |\delta_{\text{max}}| \quad \text{and} \quad \epsilon^{(k)}(t) \leq |\epsilon_{\text{max}}|
\]

Within these definitions, the error associated with each measurement can be quantified by
\begin{equation}
\mathbf{e}^{(k)} = \left\{ \frac{M_s(1 + \delta^{(k)})}{p_s Q_{a}} - \frac{1}{\int \left( \sum_{i=0}^{N} a_i \phi_i ([1 + \sin(\pi t)][0.025 - 0.0052[1 + \sin(\pi t)] + e^{(k)}(t)])^{-1} - 1 \right) dt} \right\}^2 \tag{17}
\end{equation}

and a global error function can be defined by calculating the Euclidean norm of \{\mathbf{e}^{(k)}\}:

\begin{equation}
E = E(a_0, a_1, \ldots, a_n) = \sqrt{\sum_{k=1}^{M} e^{(k)}} \tag{18}
\end{equation}

The problem can now be solved by searching for the minimum of (18) instead of (12). The following figure shows the results for increasing error in the measurement of the total mass (\delta_{\text{max}}) and time intervals varying randomly in between 10 and 30s.

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**Figure 6:** The influence of experimental errors in the reconstruction of the calibration curve (third order polynomial) from the minimization of (18)
4 CONCLUSION

A new inverse procedure is proposed in this work for the on-site reconstruction of the calibration curve from total flown mass values of the homogeneous dispersed phase. The problem is formulated as an intrinsically ill conditioned integral equation solved by setting an appropriate approximation for the calibration curve (equation (10), figure 4). Under these circumstances, the method cannot be applied in practical situations due to the severe influence of very small experimental errors (illustrated in figure 5).

The proposed solution to this is based on minimizing an error functional constructed from a set of redundant measurements, which restores the lost information associated to the integration of the instantaneous mass flow rate in the one-dimensional one-velocity flow model in equation (5). Numerical simulations performed for increasing errors demonstrate that acceptable calibration curves can be reconstructed, even from total mass measurements within a precision of up to 2%. Thus, the method can readily be applied, especially in on-site calibration problems in which classical procedures fail due to the impossibility of having a strict control of all the input/output parameters.

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