Analysis of virtual inductances on the stability of the voltage control loops for LC-filtered voltage-controlled voltage-source inverters in microgrids

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Abstract
Voltage-controlled voltage-source inverters (VVSIs) have been widely used in microgrids. Typically, LC filters are adopted by the VVSIs to improve the quality of the output voltage. Resonances of the LC filters that may cause oscillations to the VVSIs can be dampened by well-designed voltage controllers. However, the virtual inductance (VI) that is widely applied to improve power sharing between VVSIs will reduce the stability margin of the voltage control loop and introduce new oscillations on the LC filters even if the well-designed voltage controllers have been used, which has not been reported in the literature. Furthermore, since the VI will introduce a cross-coupling into the system, the conventional methods for analyzing the resonances of the LC-filtered VVSIs cannot be used for analyzing the oscillations caused by the VI. Therefore, the stability of the LC-filtered VVSI with the VI is investigated based on a new model proposed in this paper. Besides, to enhance the stability of the VVSI, the application of a low-pass filter or a band-pass filter in the VI is studied. Moreover, when connecting the VVSIs, the effects of the line impedance on the stability of the VVSIs while using the VI are investigated. Finally, simulation and experimental results have been provided, which verifies the correctness of the analysis in this paper.

1 | INTRODUCTION

With the systematic integration of various distributed energy resources, microgrid becomes an increasingly competitive option for the distributed generation due to its flexibility and reliability [1–3]. Voltage-controlled voltage-source inverters (VVSIs) with LC filters are widely used in microgrids and multiple VVSIs can be paralleled to regulate the AC bus of the microgrid [4, 5]. Therefore, to retain the voltage stability of the AC bus of the microgrid, the voltage control loops of the VVSIs should be stable. The resonance of the LC filters brings a challenge to the voltage stabilities of the VVSIs [6, 7]. Conventionally, the resonance of the LC filters can be damped by the use of the voltage controllers [8, 9]. However, the virtual inductances (VIs) used for power sharing in the VVSIs will introduce a new oscillation to the system. It has been studied that the use of droop control [10–12] and VIs [1, 5, 13] can achieve average power sharing between the VVSIs without the use of communication. However, this important point has not been addressed in the previous literature; that is, the VI may interfere with the LC filter and reduce the stability margin of the voltage control loop, causing oscillations, even if well-designed voltage controllers have been used in the VVSI.

In the following, the applications of the VI methods will be introduced first. Then, the conventional voltage stability issues regarding how to damp the resonance of the LC filter by a well-designed voltage controller will be introduced. After that, the new stability issue introduced by the VI will be highlighted. Various VI methods have been used by the VVSIs due to their easy implementation [14–17]. In [14, 16], a full-order VI is...
adopted for improving the reactive power sharing and the soft-start operation in which the time differentiation of the output current is included. Since the time differentiation element of the full-order VI may cause the high-frequency noise amplification, a second-order general-integrator (SOGI) scheme for implementing the VI is proposed in [17], but its dynamic response is poor. To avoid the time differentiation element, a simplified VI method based on the fundamental components is proposed in [12], which is the most popular VI method so far [17–19]. Since the simplified VI will not cause high-frequency noises to the system, it has been directly applied to the VVSI without the use of low-pass filters (LPFs) or band-pass filters (BPFs) in many works of the literature [17–20]. Recently, the applications of adaptive VIs based on the simplified VI are widely investigated in [13, 20, 21]. However, the decrease in the stability margin of the system caused by the application of the VI is ignored [13, 20, 21]. In [22–24], the stability of the system when connecting multiple VVSIs with the use of the simplified VI is studied using the state-space model. However, the high-frequency characteristics of the system, for instance, the oscillation of the LC filter, are not considered. In [25], a single-input-single-output model of the VVSIs with the use of L filters and the simplified VI has been built in the continuous domain. However, when using LC filters in the VVSIs, the system becomes very complicated and the simplified model [22, 25] in the continuous domain is not precise. In [26], the high-frequency oscillation of an inverter-feed microgrid is studied; however, the stability of each inverter in the microgrid is not shown and each inverter should have some required margins when designing the controller for the inverter. Besides, the impacts of the VI are not considered in [26]. Therefore, when using the simplified VI, its detrimental effect on high-frequency stability is not investigated in the literature.

For the high-frequency stability of the VVSIs, many works have been focused on damping the resonance of the LC filters with well-designed voltage controllers, such as the single-loop controllers (SLCs) [8, 27] and multi-loop controllers (MLCs) [9]. The design method of the SLC for damping the resonance of the LC filter has been studied in [8]. However, for the VVSI, an inner current control loop is usually needed for overcurrent protection [28]. Therefore, the MLC with an inner current controller is adopted for the VVSI in this paper. In [9], the design method of the MLC for damping the resonance of the LC filter has been studied. It has been found that there is an optimized gain for the inner current controller which has the maximum damping capacity on the resonance of the LC filter. In [29], the impacts of the grid impedance on the stability of the voltage control loop are investigated. In [30], the optimal design methods for the voltage controller of the VVSI are studied. However, the use of the VI will reduce the stability margin of the voltage control loop and introduce a new high-frequency oscillation to the system, which should be prevented for the stability of the system. The mechanism of this oscillation introduced by the VI is different from the mechanism of the conventional oscillations. Unfortunately, the conventional models and methods in [7–9, 13, 10–12, 14] for analyzing the high-frequency stability of the VVSIs cannot be used for analyzing the stability issues caused by the VI.

As given in [7–9, 13, 10–12, 14, 31, 32], the conventional methods for analyzing the oscillations of the VVSIs are established in the stationary reference frame (αβ-frame). In these models, the α-axis components and the β-axis components are decoupled and there is no cross-coupling between the α-axis components and the β-axis components that can cause oscillations to the LC filters. Then, the models of the three-phase VVSIs can be equivalent to two single-phase ones with the same structures and either one of the two single-phase models can be used to analyze the stability of the VVSI. However, due to the use of the simplified VI, a cross-coupling is introduced between the α-axis components and the β-axis components [13, 18–20]. This cross-coupling makes the models of the VVSIs more complicated, and then the conventional models and methods for analyzing the stability of the system cannot be used.

Therefore, the method and the model for investigating the oscillations introduced by the VI on the LC filter are studied in this paper. Here, the main contributions are summarized. The mechanism that the VI can cause oscillations on the VVSI due to its interference with the LC filter has been revealed for the first time based on the model established in this paper. It is found that the use of the VI will reduce the stability margin of the system and may result in oscillations of the VVSI even a well-designed voltage controller is used. Besides, it is found that a low-pass filter or a band-pass filter can be used by the VI to prevent the oscillations. When using the low-pass filter or the band-pass filter in the VI, the stability of the VVSI is studied for the parameter range, which is the other contribution of this paper.

The use of the model and the method in this paper to analyze the stability of the VVSIs have the following merits. First, with the proposed models and analysis methods, the mechanism of the loss of stability is very clear and the stability margin of the VVSIs can be obtained when designing the voltage controllers. Second, the methods for designing the voltage controller in the literature can still be used, since the transfer function introduced by the VI is separated from the transfer function of the voltage controller and the issues with the VI introduced stability can be dealt with separately by the proposed filters. Finally, the model can be easily extended to study the impacts of the inductive line impedance and the impacts of other voltage controllers, for instance, the SLC, on high-frequency stability.

This paper is organized as follows: Section 2 briefly illustrates the model and the control of the VVSI with an LC filter and VI. Section 3 shows the derivation of the model when the VI is used. Then, Section 4 details the stability issues of the VVSI caused by the VI. Section 5 shows the stability enhancement of the system by using a filter in the VI. Section 6 shows the effects of the line impedance on the stability of the system. Section 7 shows the simulation and experimental results. Section 8 concludes the paper.
The model of the VI can be expressed as Equation (4) [12, 18], where \( \omega_0 L \alpha_1 \) is the value of the VI. \( v_{\text{VII}} \) and \( v_{\text{VIII}} \) are the voltage drop on the VI. The use of the VI is shown in Figure 2 [12, 18]. The voltage drop is subtracted from the voltage reference to mimic the voltage drop on a real inductor:

\[
G_{\text{V}} (s) = \frac{1 - e^{-s T_s}}{s}
\]

(2)

\[
G_{\text{C}} (s) = \frac{k_p + R \left( s \right)}{s^2 + \omega_0^2}
\]

(3)

Considering the control method is implemented in a digital controller, the discrete domain (\( \tau \)-domain) model of it is derived to precisely describe the model of the VVSI. The \( \tau \)-domain transfer function between the inductor current and the inverter output voltage \( G_{\text{IL}} (\tau) \) is given in Equation (5) and the \( \tau \)-domain transfer function between the capacitor voltage and the inverter output voltage \( G_{\text{LC}} (\tau) \) is given in Equation (6) [8]:

\[
G_{\text{IL}} (\tau) = \frac{i (\tau)}{v (\tau)} = \frac{1}{s L} \frac{s^2}{s^2 + \omega_r^2}
\]

(5)

\[
G_{\text{LC}} (\tau) = \frac{v (\tau)}{v (\tau)} = \frac{\omega_r^2}{s^2 + \omega_r^2}
\]

(6)

where \( \omega_r = 1/\sqrt{L C} = 2 \beta f_s \) and \( f_s \) is the resonant frequency of the LC filter.

According to Equations (5) and (6), the \( \tau \)-domain model of \( G_{\text{IL}} (\tau) \) and \( G_{\text{LC}} (\tau) \) can be derived based on the ZOH transformation [9], which are given in Equations (7) and (8), respectively. With the pre-warp discrete transformation, the discrete model of the resonant controller \( R(\tau) \) can be obtained as given in Equation (9):

\[
G_{\text{IL}} (\tau) = \frac{\sin (\omega_0 T)}{\omega_0 L} \frac{\tau - 1}{\tau^2 - 2 \tau \cos (\omega_0 T) + 1}
\]

(7)

\[
G_{\text{LC}} (\tau) = \frac{1 - \cos (\omega_0 T)}{\omega_0^2} \frac{\tau + 1}{\tau^2 - 2 \tau \cos (\omega_0 T) + 1}
\]

(8)

\[
R (\tau) = \frac{k_p \sin (\omega_0 T)}{2 \omega_0} \frac{\tau^2 - 1}{\tau^2 - 2 \tau \cos (\omega_0 T) + 1}
\]

(9)

Then, the \( \tau \)-domain control diagram of the VVSI using the VI can be established, which is shown in Figure 3. \( G_{\text{EIN}} (\tau) \) is the transfer function of the equivalent inner loop introduced by the current control loop. In the equivalent inner loop, the gain of the feedforward path is \( k_m \tau^{-1} \), and the gain of the feedback path is \( G_{\text{IL}} (\tau) \). The closed-loop transfer function of \( G_{\text{EIN}} (\tau) \) is
shown in Equation (10):

\[ G_{\text{EIN}}(z) = \frac{z^{-1} k_{\text{in}}}{1 + z^{-1} k_{\text{in}} G_{\text{IL}}(z)} \] (10)

Therefore, an additional loop that includes a cross-coupling between the \( \alpha \)-axis components and the \( \beta \)-axis components is introduced when using the VI. The stability of the system is affected by this additional loop, which will be analyzed in Section 3. The basic parameters of the system are given in Table 1.

### 3.1 STABILITY ANALYSIS OF THE VVSI USING THE VI

The \( z \)-domain control diagram of the VVSI has been established in Section 2 as shown in Figure 3, which includes the voltage controller and the cross-coupling introduced by the VI. The following section aims to derive the mathematical model of the VVSI when the VI is used according to the control diagram given in Figure 3.

In the following part, the model of the VVSI only using the voltage controller is derived first. Then, the model of the VVSI using the VI is further derived based on this model.

Without the VI, the additional loop in Figure 3 does not exist. The closed-loop transfer function between the voltage reference \( v_{\text{ref}}(z) \) and the capacitor voltage \( v_c(z) \) can be derived as Equation (11). In Equation (11), \( G_{\alpha,lc}(z) \) is the closed-loop transfer function of the voltage control loop. In Equation (12), \( G_{\alpha,je}(z) \) is the open-loop transfer function of the voltage control loop:

\[ G_{\alpha,lc}(z) = \frac{G_{\alpha,je}(z)}{1 + G_{\alpha,je}(z)} \] (11)

where

\[ G_{\alpha,je}(z) = G_{i}(z) \frac{G_{\text{LC}}(z) G_{\text{EIN}}(z)}{G_{\text{IL}}(z)} \] (12)

To investigate the stability of the system, the control diagram which includes the cross-coupling in Figure 3 can be simplified. The simplification of the control diagram in Figure 3 is explained based on the control diagrams in Figure 4. As seen in Figure 4, the control diagram in Figure 3 can be transformed into the one shown in Figure 4a.

In Figure 4, \( G_{\text{EIN}}(z) \) is the transfer function of the inner loop included in the additional loop. \( G_{\text{VIF}}(z) \) is the feed-forward transfer function introduced by the VI.
The control diagram in Figure 4b is simplified from the control diagram in Figure 4a. According to the control diagram in Figure 4b, the simplified diagram of the VVSI using the VI can be obtained as shown in Figure 4c. $G_{ad,VI}(z)$ shown in Figures 4b and c is the transfer function of the additional loop introduced by the VI.

To analyze the stability of the system when using the VI, the closed-loop transfer function of the VVSI has been derived based on the control diagram in Figure 4, which is $G_{c,VI}(z)$ as given in Equation (13). In Equation (13), $G_{c,lc}(z)$ is the transfer function of the voltage control loop which has been given in Equation (11). The derivation of Equation (13) has been given in the Appendix:

$$G_{c,VI}(z) = G_{c,lc}(z)G_{cp,VI}(z)G_{cn,VI}(z)$$

where

$$G_{cp,VI}(z) = \frac{1 + G_{o,lc}(z)}{1 + G_{o,lc}(z) - jG_{VIF}(z)}$$

$$G_{cn,VI}(z) = \frac{1 + G_{or,lc}(z)}{1 + G_{o,lc}(z) + jG_{VIF}(z)}$$

$$G_{VIF}(z) = \omega_0L_{VI}G_z(\omega_0)G_{EIN}(\omega_0)G_{IL}(\omega_0)$$

By comparing the closed-loop transfer function of the voltage control loop in Equation (11) with the one in Equation (13), it can be seen that another two transfer functions, that is, $G_{cp,VI}(z)$ and $G_{cn,VI}(z)$ are introduced into Equation (13) due to the use of the VI. Only if all these three transfer functions are stable, $G_{c,VI}(z)$ is stable and the VVSI is stable. The stability of $G_{c,lc}(z)$ is studied in the following section. The stabilities of $G_{cp,VI}(z)$ and $G_{cn,VI}(z)$ are explored in Section 4.

The method for designing the voltage control loop for a VVSI has been studied in [9]. It has been found that the proportional gain of the inner current controller has a value with which the VVSI can obtain an optimized damping capacity. According to the method in [9], the proportional gain of the inner current controller $k_{in}$ is set to 4.5. Besides, the proportional gain of the voltage controller should be smaller than an allowable maximum value to guarantee the stability of the system. With the use of various proportional gains by the voltage controller, the bode plots of the open-loop transfer function $G_{o,lc}(z)$ are shown in Figure 5. MATLAB software is used to obtain these bode plots according to the derived analytical models and are universal to the VVSI.

As seen in Figure 5, when the proportional gain of the voltage controller is 0.012, the gain margin (GM) of the system is around 10 dB, and when $k_p$ is 0.020, the GM of the system is around 6 dB. The voltage control loop of the VVSI is stable if the voltage controller uses these two gains. These two gains will be used in Section 4 when studying the stability of the VVSI. The gain of the resonant controller $k_c$ is set to 10 in this paper.

The stability of the voltage control loop has been studied above. The following part studies the output impedance of the VVSI at the fundamental frequency when using the VI. The output impedance of the VVSI can be calculated based on the control diagram shown in Figure 6 [18]. Since the main focus here is on the output impedance at the fundamental frequency, the effect of the control delay is very small. Then, the model of the VVSI in the s-domain is used here to simplify the analysis. $i_{o\alpha}$ is the output current of the VVSI, $v_{o\alpha}$ is the output voltage of the VVSI. The transfer function from $i_{o\alpha}$ to $v_{o\alpha}$ is the output impedance of the VVSI, which is shown in Equation (17):

$$Z_{\alpha}(s) = \frac{v_{o\alpha}}{i_{o\alpha}} = \frac{1 + N_1(s)}{1C_{s}^2 + N_1(s)C_s + D_1(s) + 1}$$

where

$$N_1(s) = k_m\left[1 + j\omega_0L_{VI}G_1(\omega_0)\right]G_{d}(s)G_{\gamma}(s)$$

$$D_1(s) = k_mG_{\eta}(s)G_{d}(s)G_{\gamma}(s)$$

Figure 7 shows the bode plots of the output impedance $Z_{\alpha}(s)$ with the use of various values by the VI. Therefore, without the VI, the output impedance at the fundamental frequency is resistance, and the value of the output impedance is very small. When using the VI, the output impedance of the VVSI is inductive, and the value of output impedance is dominated by the value of the VI. Therefore, the use of the VI is effective for the VVSI to modify the output impedance.
In this section, the model of the VVSI including the LC filter, the voltage control loop, and the cross-coupling introduced by the VI has been established. By studying the model, it is found that the cross-coupling will introduce closed loops that affect the stability of the VVSI. When using the VI, the VVSI is stable only if the voltage control loop \( G_{op} \) and the closed loops introduced by the cross-coupling of the VI \( G_{cp} \) and \( G_{cn} \) are all stable. The stability of \( G_{op} \) and \( G_{cn} \) can be guaranteed by the use of well-designed parameters in the controllers as has been analyzed in this section. The stabilities of \( G_{op} \) and \( G_{cn} \) will be analyzed in Section 4.

### 4 | STABILITY OF THE LOOPS INTRODUCED BY THE VI

To guarantee the stability of the VVSI, the closed loops introduced by the VI should be stable, of which the transfer functions are \( G_{op} \) and \( G_{cn} \) as shown in Equations (14) and (15). The stabilities of \( G_{op} \) and \( G_{cn} \) are analyzed in the following part of this section.

According to the NSC, the stabilities of \( G_{op} \) and \( G_{cn} \) can be checked by their open-loop transfer functions that are \( T_{op} \) and \( T_{cn} \) as shown in Equations (20) and (21), respectively. It should be noted that the NSC can still be used by \( G_{op} \) and \( G_{cn} \) even they include imaginary numbers as seen in Equations (14) and (15) [33]:

\[
T_{op} = G_{op} - jG_{vif}
\]

\[
T_{cn} = G_{cn} + jG_{vif}
\]

Combining the open-loop transfer function of the voltage control loop \( G_{op} \) in Equation (12) and the feedforward transfer function of the VI \( G_{vif} \) in Equation (16), the discrete transfer functions of \( T_{op} \) and \( T_{cn} \) can be derived as shown in Equations (22) and (23), respectively:

\[
T_{op} = \frac{G_z (z + k_1) - jk_2 (z - 1)}{Den_z (z)} \tag{22}
\]

\[
T_{cn} = \frac{G_z (z + k_1) + jk_2 (z - 1)}{Den_z (z)} \tag{23}
\]

where

\[
Den_z (z) = z (z^2 + 2 \cos (\omega_T) z + 1)
+ k_{in} \frac{\sin (\omega_T) (z - 1)}{\omega_L}
\tag{24}
\]

\[
k_1 = 1 - \cos (\omega_T) \tag{25}
\]

\[
k_2 = \omega_0 L \frac{\sin (\omega_T)}{\omega_L} \tag{26}
\]

Figure 8 shows the bode plots of \( T_{op} \) with various values of the VI and \( k_p = 0.020 \) shown in Equations (22) and (23), respectively.

\[
T_{op} = \frac{G_z (z + k_1) - jk_2 (z - 1)}{Den_z (z)} \tag{22}
\]

\[
T_{cn} = \frac{G_z (z + k_1) + jk_2 (z - 1)}{Den_z (z)} \tag{23}
\]

Figure 8 shows the bode plots of \( T_{op} \) with various values of the VI when the proportional gain of the voltage controller is 0.020. Therefore, the GM of \( T_{op} \) decreases with the increase of the VI. When \( \omega_0 L \) is greater than 5 \( \Omega \), the gain of \( T_{op} \) is greater than 10 dB at the frequency where the phase of \( T_{op} \) crosses –180°. According to the NSC, \( T_{op} \) is unstable.

Figure 9 shows the bode plots of \( T_{cn} \) with various values of the VI when the proportional gain of the voltage controller is 0.020. Therefore, the frequencies where the phases of \( T_{cn} \) cross –180°, the gain of \( T_{cn} \) is always smaller than 1. According to the NSC, \( T_{cn} \) is stable.

Therefore, when using the VI, the stability of \( T_{op} \) is affected. When using a large VI, \( T_{op} \) will be unstable, and the VVSI will be unstable.
The GM of $T_{op,VI}(z)$ is also related to the proportional gain of the voltage controller and the proportional gain of the inner current controller. Figure 10 shows the bode plots of $T_{op,VI}(z)$ with various values of the VI when the proportional gain of the voltage controller is 0.012. Therefore, when the VI is greater than 15, the magnitude of $T_{op,VI}(z)$ at the frequency where its phase crosses $-180^\circ$ is greater than 1. Then, the system is unstable.

As seen in Figure 8, when $k_p$ is 0.020, $T_{op,VI}(z)$ is unstable if the value of the VI is greater than 5. As seen in Figure 10, when $k_p$ is 0.012, $T_{op,VI}(z)$ is stable unless $\omega_0 L_{VI}$ is greater than 15. Therefore, the decrease of $k_p$ can enlarge the feasible value of the VI. However, this may deteriorate the transient response of the voltage controller, which is not encouraged.

The optimized value for the proportional gain of the inner current controller $k_{in}$ is 4.5, as has been studied in [9]. Figure 11 shows the bode plots of $T_{op,VI}(z)$ with various values of $k_{in}$ when $k_p$ is 0.020 and the VI is 10. As seen, when $k_{in}$ is larger than 4.5, the gain margins decrease, which deteriorates the stability of the system. Therefore, the recommended value of $k_{in}$ is 4.5.

As a conclusion of this section, the GM of the system decreases with the use of the VI. The system will be unstable with the increase of the VI. When the GM of the voltage control loop is 6 dB, a small VI can destabilize the system. In the following section, when using the VI in the VVSI, methods for the system to improve the stability will be studied.

### 5 STABILITY ENHANCEMENT OF THE VI

The use of the VI shown in this paper will not cause noise amplification [17–19]. However, it reduces the GM of the system and can cause the instability of the system, as has been analyzed in Section 4. Therefore, the VI cannot be used in the VVSI directly. A filter can be used in the VI to prevent the instability of the VVSI.

Figure 12 shows the structure of the control diagram with a filter used in the VI. This filter can be an LPF or a BPF. The transfer functions of the LPF and the BPF in the $s$-domain are shown in Equations (27) and (28), respectively. $\omega_{LPF}$ and $\omega_{BPF}$ are the bandwidth of the LPF and the BPF, respectively. The
stability of the VVSI using the filtered VI is studied in this section:

\[ G_{LPF}(s) = \frac{\omega_{LPF}}{s + \omega_{LPF}} \]  

(27)

\[ G_{BPF}(s) = \frac{2\omega_{BPF}s}{s^2 + 2\omega_{BPF}s + \omega_0^2} \]  

(28)

The discrete models of the LPF and the BPF should be used when implementing them in a digital controller, which are shown in Equations (29) and (30), respectively:

\[ G_{LPF}(z) = \frac{T\omega_{LPF}}{1 + T\omega_{LPF} + z^{-1}} \]  

(29)

\[ G_{BPF}(z) = \frac{\omega_{BPF} \sin(\omega_0 T) (z^2 - 1)}{\text{Den}_{BPF}} \]  

(30)

where

\[ \text{Den}_{BPF} = (\omega_0 + \omega_{BPF} \sin(\omega_0 T)) z^2 - 2\omega_0 \cos(\omega_0 T) s + \omega_0 - \omega_{BPF} \sin(\omega_0 T) \]  

(31)

The \( \omega_{LPF} \) is set to 5\( \omega_0 \), with which it will not affect the fundamental frequency current used by the VI. Besides, 5\( \omega_0 \) is far away from the oscillation frequency since the oscillation frequency of the LC filter is ten times larger than the fundamental frequency here.

When the bandwidth of the BPF is too small, the transient response of the BPF is slow. When the bandwidth of the BPF is too large, the effect of the VI on the system might not be completely eliminated. By trial and error, the bandwidth of the BPF \( \omega_{BPF} \) is set to 0.35\( \omega_0 \).

Figure 13 shows the bode plots of the output impedance of the VVSI with the LPF or the BPF used in the VI. According to the control diagram in Figure 6 and the transfer function in Equation (17), the output impedance of the VVSI can be obtained, which is not detailed here. Therefore, the use of the BPF and the LPF will not change the magnitude of the output impedance at the fundamental frequency, and the output impedance is inductive at the frequencies around the fundamental frequency.

Figure 14 shows the bode plots of \( T_{op,VI}(z) \) while using the LPF in the VI. Therefore, the system is stable when the VI is below 30. But the system will be unstable if the value of the VI exceeds 5 when the LPF is not used in the VI, as shown in Figure 8. In conclusion, the use of the LPF can extend the feasible value of the VI, but the GM of the system is still affected.

Figure 15 shows the bode plots of \( T_{op,VI}(z) \) while using the BPF in the VI. Therefore, the gain margin of \( T_{op,VI}(z) \) is almost not affected by the increase of the VI. But it should be noted that the transient response of the output power of the VVSI will be slow while using the BPF in the VI since the transient response of the BPF is much longer than the LPF, which will be verified in Section 6 by the simulation and experimental results.
In conclusion, both the LPF and the BPF can be used to enhance the stability of the VVSI. The BPF is more effective than the LPF in preventing the instability caused by the VI. If the VI is small, the use of the LPF is enough to retain the stability of the system. If the VI is large, the BPF has to be used to guarantee the stability of the system even though the BPF will cause a slow transient response to the output power of the VVSI.

6 EFFECTS OF THE LINE IMPEDANCE (LIM)

When connecting the VVSIs together, the LIM may affect the stability of the voltage control loop. To guarantee the stability of the VVSI with the use of MLC, the resonance frequency of the filter should be smaller than \( f_s/6 \) [9]. When the LIM is very small, it will make the resonance frequency of the filter greater than \( f_s/6 \) and the voltage control loop becomes unstable [29]. Therefore, an interface inductance is normally connected between the VVSI and the PCC as shown in Figure 16. \( L_{\text{ITI1}} \) and \( L_{\text{ITI2}} \) are the interface inductance. \( L_{\text{LIM1}}, \ L_{\text{LIM2}} \) and \( L_{\text{LIM3}} \) are the LIM. The total inductance is the sum of the LIM and the interface inductance. When the LIM exists, the total inductance increases, which will enhance the stability margin of the system. When each VVSI in the system with its interface inductance is stable and has some stability margins, the system with multiple VVSIs connected will be stable. The impacts of the LIM on the stability of the VVSI are shown in this section based on the model and the method above. In the following, because the line impedance and the interface inductance are series connected, the LIM refers to the sum of them.

Considering the impacts of the LIM, the transfer function between the output voltage of the inverter and the capacitor voltage and the transfer function between the output voltage of the inverter and the output current are altered, which should be \( G_{\text{LC,LIM}}(z) \) and \( G_{\text{IL,LIM}}(z) \) as shown in Equations (32) and (33). \( \omega_r \) is the resonant frequency of the filter considering the LIM:

\[
G_{\text{LC,LIM}}(z) = \frac{L_{\text{LIM}} (1 - \cos (\omega_r T))}{L + L_{\text{LIM}}} \frac{z + 1}{z^2 - 2 \cos (\omega_r T) z + 1}
\]  

\[
G_{\text{IL,LIM}}(z) = \frac{T_z}{(1 + L_{\text{LIM}})} (\chi - 1) + \frac{L_{\text{LIM}}}{1 + L_{\text{LIM}}} \sin (\omega_d T) \frac{\chi - 1}{\omega_d L_4} \frac{z - 1}{z^2 - 2z \cos (\omega_d T) + 1}
\]

With the use of \( G_{\text{LC,LIM}}(z) \) and \( G_{\text{IL,LIM}}(z) \), Figure 17 shows the bode plot of \( T_{\text{op,VI}}(z) \). As analyzed above, when using the VI, \( T_{\text{op,VI}}(z) \) can be used for investigating the stability of the VVSI. The LIM is 3.5 mH here. The proportional gain is 0.15, with which the gain margin of the voltage control loop without the VI is 6 dB. Therefore, the increase of the VI reduces the gain margin of the system and the system becomes unstable when the VI is around 6 Ω.

Figure 18 shows the bode plots of \( T_{\text{op,VI}}(z) \) with different LIM.

\[
\omega_d = \frac{\omega_r}{\omega_c}
\]

\[
\omega_c = \sqrt{\frac{L}{C}}
\]
the system with the VVSIs will be stable, since additional LIM will raise the stability margin.

Figure 19 shows the bode plots of $T_{op,VI}(\omega)$ with the LPF or the BPF. Therefore, with the use of the LPF and the BPF in the VI, the gain margin is almost not affected by the VI.

The simulation results of the system in Figure 16 will be given in Section 7 to prove the analysis in this section.

7 SIMULATION AND EXPERIMENTAL RESULTS

7.1 Simulation results

To verify the correctness of the analysis regarding the stability of the VVSI in this paper, a VVSI with the use of the VI and the voltage controller is established in the Matlab/Simulink, and the corresponding simulation results are given in this section.

The VVSI is simulated when the VI is not used. Figure 20a shows the simulation result of the output voltage. The transient response of the output voltage is adopted to show the state of the stability of the VVSI. The transient response of the output voltage is created by raising the magnitude of the voltage from 100 to 156 V. Therefore, the output voltage is very stable in the transient process.

The VVSI is simulated when the VI is 4 $\Omega$. Figure 20b shows the simulation result of the output voltage of the VVSI. Therefore, there is some oscillation on the output voltage in the transient process.

The VVSI is simulated when the VI is 6 $\Omega$. Figure 20c shows the simulation result of the output voltage. Therefore, the output voltage becomes unstable when the VI is 6 $\Omega$.

Hence, the simulation results are very consistent with the analysis in Section 4. The parameters of VVSI here are the same as parameters of the VVSI used for obtaining Figures 8 and 9.

In the following simulations, the LPF or the BPF is used in the VI. The parameters of the VVSI are that $L = 2.7$ mH, $C = 32$ uF, and $k_p = 0.020$, which are the same as the parameters used for obtaining the bode plots in Figures 14 and 15.

As seen in Figure 21a, the VVSI is stable when the VI reaches 15 $\Omega$ if the LPF is used in the VI. As seen in Figure 21b, the VVSI is stable when the VI reaches 30 $\Omega$ if the BPF is used in the VI. Compared with the results shown in Figure 20, it can be seen that the use of the LPF and the BPF can improve the stability of the system, which is consistent with the analysis in Section 5.

The transient response of the output power when using the LPF or the BPF in the VI is tested with two VVSIs paralleled as shown in Figure 16. The parameters of VVSIs here are the same as parameters of the VVSI used for obtaining the bode plots in Figure 19. Figure 22 shows the simulation results of the reactive power in the two VVSIs. Therefore, the reactive power of the VVSIs while using the BPF in the VI has a slow transient response, and the reactive power of the VVSIs while using the LPF in the VI has a quick transient response. While using the
TABLE 2 Parameters of the LIM

| Conditions | Parameters |
|------------|------------|
| Case I     | $I_{L \text{LIM1}} = 3.5 \text{ mH}, I_3 = 2.7 \text{ mH}, C_1 = 32 \text{ uF}, i_1 = 8 \text{ A}$ |
|            | $I_{L \text{LIM2}} = 5.5 \text{ mH}, I_2 = 2.7 \text{ mH}, C_2 = 32 \text{ uF}, i_1 = 8 \text{ A}$ |
| Case II    | $I_{L \text{LIM1}} = 3.5 \text{ mH}, I_3 = 3.7 \text{ mH}, C_1 = 23 \text{ uF}, i_1 = 5 \text{ A}$ |
|            | $I_{L \text{LIM2}} = 5.5 \text{ mH}, I_2 = 1.35 \text{ mH}, C_2 = 64 \text{ uF}, i_1 = 10 \text{ A}$ |

BPF or the LPF in the VI, the deviations between the steady-state reactive power are the same. The reason is that the magnitudes of the output impedance of the VVSIs at the fundamental frequency are the same.

7.2 Simulation results of system with two VVSIs

Figure 16 shows the system with two VVSIs. Two cases are simulated in this part. The parameters of the two cases are given in Table 2. In Case I, the LIM1 and the LIM2 are different. But the parameters of the VVSI are the same. In Case II, both the LIM and the parameters of the VVSIs are different. In both the cases, the LIM3 is 1.2 mH and the impedance of the load is $8 + j4 \Omega$.

Figure 23 shows the simulation results of Case I. The voltage control loop of VVSI is designed considering the impacts of the LIM. The gain margin of the voltage control loop without the VI is 6 dB. Therefore, the system is stable when the VI is 5 \Omega and the system is unstable when the VI changes to 6 \Omega. Therefore, when each VVSI is designed considering its interface inductance and has some stability margins, the system with the VVSIs will be stable. When the increase of the VI makes the VVSIs unstable, the system will be unstable. The parameters of VVSIs here are the same as parameters of the VVSI used for obtaining the bode plots in Figures 17 and 18.

7.3 Comparisons with existing methods

Figure 26 shows the simulation results of the system in Case I when using the full-order VI and the LPF as given in [14, 16]. The parameters of VVSIs here are the same as parameters of the VVSI used for obtaining the bode plots in Figures 8 and 9. Therefore, when the VI reaches 4 \Omega, a high-frequency oscillation occurs. In Figure 24, with the VI and the LPF, the system is stable when the VI reaches 12 \Omega.
Figure 27 shows the simulation results when using the SOGI-based VI as given in [17]. This VI method will introduce a negative resistance in the low-frequency range and then a low-frequency oscillation will be introduced when a large VI is used. Therefore, when the VI reaches $5 \Omega$, a low-frequency oscillation occurs. Therefore, the feasible range of the SOGI based VI is also smaller than the range of the VI with the use of the LPF.

7.4 | Experimental results

Figure 28 shows the 5 kW VVSI prototype used to verify the correctness of analysis regarding the stability of the VVSI in this paper. The control scheme of the VVSI is implemented in a DSP+FPGA digital controller where the DSP is TMS320F28335 from Texas Instruments, and the FPGA is Xilinx XC3S400. The parameters of the VVSI are shown in Table 1.

Figure 29 shows the output voltage of the VVSI when the VI is not used. The transient response is adopted to show the state of the voltage stability, which is created by raising the magnitude of the voltage from 100 to 150 V. Therefore, the output voltage is stable in the transient process.

Figure 30 shows the output voltage of the VVSI when the VI is $4 \Omega$. As seen, the output voltage has some oscillation in the transient process.

Figure 31 shows the output voltage of the VVSI when the VI is $6 \Omega$. As seen, the output voltage is unstable.
Hence, with the use of the well-designed voltage controller, the VVSI is stable. However, the use of the VI will reduce the stability of the system, and a large VI will cause the oscillation of the system. The parameters of VVSIs here are the same as parameters of the VVSI used for obtaining the bode plots in Figures 8 and 9.

Figure 32 shows the experimental result of the output voltage when using the LPF in the VI. As seen in Figure 32a, the VVSI is stable when the VI is 15 Ω. As seen in Figure 32b, there is a small oscillation in the transient process of the output voltage when the VI is 30 Ω. Compared with the results in Figure 31 when the LPF is not used, it can be concluded that the use of the LPF in the VI can improve the stability of the system. But the stability of the VVSI is still affected when a large VI is used in the VI.

Figure 33 shows the experimental result of the output voltage when using the BPF in the VI. Here, the VI is 30 Ω. As seen, even though the VI is very large, the system is stable while using the BPF in the VI.

The transient response of the output power of the VVSI is tested by connecting the VVSI to the grid. The reactive power of the VVSI is calculated in the DSP, and then it is exported through a DAC converter which is DAC7656 from Analog Devices. Figure 34a shows the experimental result of the reactive power while using the LPF in the VI. Figure 34b shows the experimental results of the reactive power while using the BPF in the VI. As seen, while using LPF in the VI, the transient response of the reactive power is quick. But the transient response of the reactive power is slow while using the BPF in the VI.

Figures 35 and 36 show the experimental results of the system given in Figure 16 in Case I. As seen in Figure 35, while
Using a small VI (1 \( \Omega \)), the system is stable, but the average reactive power sharing cannot be realized. While using a large VI (6 \( \Omega \)), the average reactive power-sharing can be realized, but the system becomes unstable, where a clear oscillation occurs. That is because the use of the large VI makes the voltage control loop lose the gain margin, which is consistent with analysis in Section 6.

As seen in Figure 36, while using the LPF in the VI, the system is stable even through the VI is 12 \( \Omega \). This is because the effect of the VI on the gain margin of the voltage control loop is reduced by the use of the LPF and both the voltage control loops of VVSI have some gain margins.

### 8 CONCLUSION

Considering the cross-coupling introduced by the VI, the model of the VVSI has been established in this paper. Based on the model presented in this paper, it is found that the cross-coupling will introduce an additional closed loop into the control system. By analyzing the additional closed loop, it is found that the use of the VI will reduce the GM of the system, which may cause the oscillation of the system. Both the LPF and the BPF can be used to enhance the stability of the system. When using the BPF in the VI, the oscillation caused by the VI can be completely prevented, but the transient response of the output power is slow. When using the LPF in the VI, the transient response of the output power is quick, but the GM of the system is still slightly affected by the VI if a large VI is used. Therefore, when a small VI is adopted, the use of the LPF is enough to prevent the oscillation caused by the VI. The model and method proposed in this paper can also be used for analyzing the impacts of the interface inductance and the LIM. When each VVSI in a microgrid with its interface inductance is stable and has some stability margin, the system with the VVSIs will be stable, since additional line impedance will raise the stability margin.

The VI is also an effective method for limiting the fault current of the VVSI, where a large VI is normally used. As shown in this paper, the VI may jeopardize the stability of the VVSI. Therefore, the stability of the VVSI using the VI as a fault current limiter requires further investigation.

### NOMENCLATURE

- \( f_r \): Resonant frequency of the LC filter
- \( G_{\text{EIN}}(\omega) \): Transfer function of the equivalent inner loop introduced by the current control loop


\( \omega_r(\bar{z}) \)  
\( G_{\alpha_{\text{le}}}(\bar{z}) \)  
\( G_{V_{\text{IF}}}(\bar{z}) \)  
\( G_{e_{\text{VI}}}(\bar{z}) \)  
\( G_i(\bar{z}) \)  
\( G_{c_{\text{le}}}(\bar{z}) \)  
\( G_{L_{\text{PF}}}(\bar{z}) \)  
\( G_{B_{\text{PF}}}(\bar{z}) \)  
\( G_{IL}(\bar{z}), G_{IL_{\text{lim}}}(\bar{z}) \)  
\( G_{IL_{\text{lim}}}(\bar{z}) \)  
\( G_{IL_{\text{lim}}}(\bar{z}) \)  
\( I_{c_{\text{VI}}}, I_{c_{\text{VI}}} \)  
\( \beta_k, \beta_e \)  
\( I_{c}, C \)  
\( \text{LIM} \)  
\( L_{\text{VI}} \)  
\( T_{op_{\text{VI}}}(\bar{z}), T_{op_{\text{VI}}}(\bar{z}) \)  
\( T_{s} \)  
\( T_{\text{ca}}(\bar{z}), T_{\text{cg}}(\bar{z}) \)  
\( \text{VI} \)  
\( \text{VVSIs} \)  
\( v_{\text{gs}}, v_{\text{gb}} \)  
\( v_{\text{ae}}, v_{\text{ag}} \)  
\( \omega_0 \)

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in the paper and combining the model of $G_{ad,VI}(z)$ in Equation (A2), the following expression can be obtained:

$$G_{o,VI}(z) = \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}}$$

(A4)

Considering the gain of the feedback path of the system is 1, combining the model of the open-loop transfer function $G_{o,VI}(z)$ in Equation (A4), the closed-loop transfer function of the VVSI can be obtained as shown in Equation (A5):

$$G_{c,VI}(z) = \frac{G_r(z)}{1 + G_{r}(z)}$$

$$= \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}} + \frac{G_{o,k}(z)}{1}$$

(A5)

By putting $(1+G_{o,k}(z))/(1+G_{o,k}(z))$ into Equation (A5), the expression can be manipulated to be:

$$G_{c,VI}(z) = \frac{(1 + G_{o,k}(z))}{(1 + G_{o,k}(z))}$$

$$\times \frac{G_{o,k}(z) (1 + G_{o,k}(z))}{(1 + G_{o,k}(z))^2 + G_{VIF}(z) G_{VIF}(z)}$$

(A6)

Then, the transfer function of the VVSI when using the VI can be derived according to the control diagram in Figure 4c. The feedforward gain of system can be derived as shown in Equation (A3):

$$G_{o,VI}(z) = G_r(z) G_{LC}(z) G_{ad,VI}(z) G_{EIN}(z)$$

(A3)

Considering that open-loop transfer function of the SLC, that is, $G_{o,le}(z) = G_r(z) G_{LC}(z) G_{EIN}(z)$ as shown in Equation (11) in the paper and combining the model of $G_{ad,VI}(z)$ in Equation (A2), the following expression can be obtained:

$$G_{o,VI}(z) = \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}}$$

(A4)

Considering the gain of the feedback path of the system is 1, combining the model of the open-loop transfer function $G_{o,VI}(z)$ in Equation (A4), the closed-loop transfer function of the VVSI can be obtained as shown in Equation (A5):

$$G_{c,VI}(z) = \frac{G_r(z)}{1 + G_{r}(z)}$$

$$= \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}} + \frac{G_{o,k}(z)}{1}$$

(A5)

By putting $(1+G_{o,k}(z))/(1+G_{o,k}(z))$ into Equation (A5), the expression can be manipulated to be:

$$G_{c,VI}(z) = \frac{1 + G_{o,k}(z)}{1 + G_{o,k}(z)}$$

$$\times \frac{G_{o,k}(z) (1 + G_{o,k}(z))}{(1 + G_{o,k}(z))^2 + G_{VIF}(z) G_{VIF}(z)}$$

(A6)

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(A3)

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$$G_{o,VI}(z) = \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}}$$

(A4)

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$$= \frac{G_{o,k}(z)}{1 + G_{VI}(z) G_{VIF}(z) \frac{1}{1 + G_{o,k}(z)}} + \frac{G_{o,k}(z)}{1}$$

(A5)

By putting $(1+G_{o,k}(z))/(1+G_{o,k}(z))$ into Equation (A5), the expression can be manipulated to be:

$$G_{c,VI}(z) = \frac{1 + G_{o,k}(z)}{1 + G_{o,k}(z)}$$

$$\times \frac{G_{o,k}(z) (1 + G_{o,k}(z))}{(1 + G_{o,k}(z))^2 + G_{VIF}(z) G_{VIF}(z)}$$

(A6)