Pulsar Kicks Induced by Spin Flavor Oscillations of Neutrinos in Gravitational Fields

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The origin of pulsar kicks is reviewed in the framework of the spin-flip conversion of neutrinos propagating in the gravitational field of a magnetized protoneutron star. We find that for a mass in rotation with angular velocity $\omega$, the spin connections entering in the Dirac equation give rise to the coupling term $\omega \cdot \mathbf{p}$, being $\mathbf{p}$ the neutrino momentum. Such a coupling can be responsible of pulsar kicks owing to the neutrino emission asymmetry generated by the relative orientation of $\mathbf{p}$ with respect to $\omega$. For our estimations, the large non standard neutrino magnetic momentum, $\mu_\nu \lesssim 10^{-11} \mu_B$, is considered.

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I. INTRODUCTION

The origin of the pulsar velocity represents till now an unsolved and still discussed issue of the modern astrophysics. Observations show that pulsars have a very high proper motion (velocity) with respect to the surrounding stars, with a three-dimensional galactic speed of $450 \pm 90\text{Km/sec}$ up to values greater than $1000\text{Km/sec}$ [1–6]. This suggests that nascent pulsars undergo to some kind of impulse (kick). After the supernova collapse of a massive star, neutrinos carry away almost all (99%) the gravitational binding energy ($3 \times 10^{53}\text{erg}$). The momentum taken by them is about $10^{43}\text{gr cm/sec}$. An anisotropy of $\sim 1\%$ of the momenta distribution of the outgoing neutrinos would suffice to account for the neutron star recoil of $300\text{Km/sec}$. Even though many mechanisms have been proposed, the origin of the pulsar kick remains an open issue.

An interesting and elegant mechanism to generate the pulsar velocity, which relies on the neutrino oscillation physics in presence of an intense magnetic field, has been proposed by Kusenko and Segré (KS) [7]. The basic idea is the following. Electron neutrinos $\nu_e$ are emitted from a surface placed at a distance from the center of a pulsar greater than the surfaces corresponding to muon neutrinos $\nu_\mu$ and tau neutrinos $\nu_\tau$. Such surfaces are called neutrinospheres. Under suitable conditions, a resonant oscillation $\nu_e \rightarrow \nu_\mu,\tau$ may occur between the $\nu_e$ and $\nu_\mu,\tau$ neutrinospheres. The neutrinos $\nu_e$ are trapped by the medium (due to neutral and charged interactions) but neutrinos $\nu_\mu,\tau$ generate via oscillations can escape from the protoneutron star being outside of their neutrinosphere. Thus, the “surface of the resonance” acts as an “effective muon/tau neutrinosphere”. In the presence of a magnetic field (or some other nonisotropic effect), the surface of resonance can be distorted and the energy flux turns out to be generated anisotropically. In [7], the anisotropy of the neutrino emission is driven by the polarization of the medium due to the magnetic field $\mathbf{B}$. The usual MSW resonance conditions turn out to be modified by the term [8–12]

$$\frac{eG_F}{\sqrt{2}} \sqrt{\frac{3n_e}{\pi^4}} \mathbf{B} \cdot \hat{\mathbf{p}},$$

where $\hat{\mathbf{p}} = \mathbf{p}/p$, $\mathbf{p}$ is the neutrino momentum, $e$ is the electric charge, $G_F$ is the Fermi constant, and $n_e$ is the electron density. Nevertheless, the neutrino masses required for the KS mechanism seem to be inconsistent with the present limits on the masses of standard electroweak neutrinos. These limits do not apply to sterile neutrinos (they may have only a small mixing angle with the ordinary neutrinos) [13,14]. Papers dealing with the origin of pulsar kicks can be found in [15–39].

The aim of this paper is to suggest a further mechanism to generate pulsar kicks. It is based on the spin flavor conversion of neutrinos propagating in a gravitational field generated by a rotating source. Even though the gravitational field per se cannot induce neutrino oscillations, unless a violation of the equivalence principle is invoked [40,41], it affects the resonance conditions (hence the probability that left-handed neutrinos convert into right handed neutrinos, the latter being sterile may escape from the protoneutron star). Such a modification is induced by spin

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connections entering in the Dirac equation in curved spacetimes. They give rise to the coupling term \( \omega \cdot p \), where \( \omega \) is the angular velocity of the gravitational source. The relative orientation of neutrino momenta with respect to the angular velocity determines an asymmetry of the neutrino emission, hence it may generate the pulsar kicks.

The paper is organized as follows. In Sect. 2 we shortly review the Dirac equation in curved space-times. Sect. 3 is devoted to the computation of the fractional asymmetry. The resonance and adiabatic conditions, as well as the spin flip probability are also discussed. Conclusions are drawn in Sect. 4.

II. DIRAC EQUATION IN THE LENSE–THIRRING GEOMETRY

The phase of neutrinos propagating in a curved background is generalized as [42–52]

\[
|\psi_f(\lambda)\rangle = \sum_j U_{fj} e^{i\int_\lambda^\lambda_0 p_{\mu\text{null}} d\lambda'} |\nu_j\rangle,
\]

where \( f \) is the flavor index and \( j \) the mass index. \( U_{fj} \) are the matrix elements transforming flavor and mass bases. \( p_{\mu\text{null}} = P_\mu p^\mu_{\text{null}} \), where \( P_\mu \) is the four–momentum operator generating space–time translation of the eigenstates and \( p^\mu_{\text{null}} = dx^\mu/d\lambda \) is the tangent vector to the neutrino world-line \( x^\mu \), parameterized by \( \lambda \). The covariant Dirac equation in curved space–time is (in natural units) [53]

\[
[\gamma^\mu(x)D_\mu - m(\lambda)]\psi = 0,
\]

where the matrices \( \gamma^\mu(x) \) are related to the usual Dirac matrices \( \gamma^a \) by means of the vierbein fields \( e^\mu_a(x) \), i.e.

\[
\gamma^\mu(x) = e^\mu_a(x) \gamma^a.
\]

The Greek (Latin with hat) indices refer to curved (flat) space–time. The connections \( \Gamma^\mu_\nu_\rho \) are the spin connections entering in the Dirac equation in curved spacetimes. They give rise to the coupling term \( \gamma^\mu \Gamma^\nu_\rho \equiv \gamma^\mu \Gamma^{\nu\rho}_\mu \gamma^5 \) acts differently on left- and right-handed neutrino states [42]. By writing \( \gamma^5 = \mathcal{P}_R - \mathcal{P}_L \), where \( \mathcal{P}_{L,R} = (1 \mp \gamma^5)/2 \), one immediately sees that left- and right-handed neutrinos acquire a gravitational potential which is opposite for the two helicities. The dispersion relation of neutrinos turns out to be modified by the term [42] \( p^\mu A_G^\mu \gamma^5 = -p \cdot A_G \gamma^5 \). In the case of neutrino oscillations, it is convenient to add, without physical consequences, a term proportional to the identity matrix, so that \( \gamma^5 \) can be replaced by the left-handed projection operator \( \mathcal{P}_L \). Spin-gravity coupling terms can be then pushed in the left-handed sector of the neutrinos effective Hamiltonian (see Eqs. (5)-(8)).
III. THE ASYMMETRIC EMISSION OF NEUTRINOS FROM PROTONeutRON STARS

Neutrinos inside their neutrinospheres are trapped by the weak interactions with the background matter. They therefore acquire the potential energy

\[ V_{\nu_f} \simeq 3.8 \times 10^{-14} \frac{\rho}{\text{gr cm}^{-3}} y_f(r, t) \text{eV}, \]

where \( f = e, \mu, \tau, \rho \) is the matter density, \( y_e = Y_e - 1/3 \) and \( y_{\mu, \tau} = Y_e - 1 \). In these expressions, \( Y_e \) is the electron fraction. In the present paper we shall consider the case in which matter induced effective potential is \( |V_{\nu_e}| \ll 1 \) (as shown in [54], \( V_{\nu_e} \) may cross the zero at \( r \sim 12 \text{km} \)). This occurs in the regions where the electron fraction \( Y_e \) assumes the value \( \approx 1/3 \) (\( y_e \ll 1 \)) [55–60,39,54].

Protoneutron stars possess large magnetic fields which may vary in a range of several order of magnitude. In such astrophysical systems, the interaction of neutrinos with (uniform) magnetic fields is described by the Lagrangian [61–63]

\[ \mathcal{L}_{\text{int}} = \bar{\psi} \mu \sigma^{ab} F_{ab} \psi, \]

where \( \mu \) is the magnetic momentum of the neutrino, \( F_{ab} \) is the electo-magnetic field tensor, and \( \sigma^{ab} = \frac{i}{2} [\gamma^a, \gamma^b] \). Spin flavor conversions of neutrinos propagating in magnetic fields are generated by the transition magnetic moments, which are non-diagonal terms in the effective Hamiltonian describing the neutrino evolution.

Taking into account the gravitational and magnetic interactions, the equation of evolution describing the conversion between two neutrino flavors \( f = e \) and \( f' = \mu, \tau \) reads (we refer to Dirac neutrinos, but the formalism also applies to Majorana neutrinos) [64]

\[ \frac{d}{d\lambda} \begin{pmatrix} \nu_{fL} \\ \nu_{f'L} \\ \nu_{fR} \\ \nu_{f'R} \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_{fL} \\ \nu_{f'L} \\ \nu_{fR} \\ \nu_{f'R} \end{pmatrix}, \]

(5)

where, in the chiral base, the matrix \( \mathcal{H} \) is the effective Hamiltonian

\[ \mathcal{H} = \begin{bmatrix} \mathcal{H}_L & \mathcal{H}_{ff'}^* \\ \mathcal{H}_{ff'} & \mathcal{H}_R \end{bmatrix}, \]

(6)

\[ \mathcal{H}_L = \begin{bmatrix} V_{\nu_e} + \Omega_G - \delta c_2 & \delta s_2 \\ \delta s_2 & V_{\nu_{f'}} + \Omega_G + \delta c_2 \end{bmatrix}, \]

(7)

\[ \mathcal{H}_R = \begin{bmatrix} -\delta c_2 & \delta s_2 \\ \delta s_2 & -\delta c_2 \end{bmatrix}, \quad \mathcal{H}_{ff'} = B_1 \begin{bmatrix} \mu_{ff} & \mu_{ff'} \\ \mu_{ff'} & \mu_{ff'} \end{bmatrix}. \]

(8)

\[ \Omega_G(r) = \frac{p_n A_{G}^\mu(r)}{E_l} = \frac{4GMR^2}{5r^3 E_l} \mathbf{p} \cdot \mathbf{\omega'}, \]

(9)

Here \( \delta = \frac{\Delta m^2}{2E_l} (\Delta m^2 = m_2^2 - m_1^2) \), \( c_2 = \cos 2\theta, s_2 = \sin 2\theta, \theta \) is the vacuum mixing angle, \( E_l \) is the energy measured in the local frame, \( B_1 = B \sin \alpha \) is the component of the magnetic field orthogonal to the neutrino momentum, and \( \beta \) is the angle between the neutrino momentum and the angular velocity.

\( \Omega_G \) is diagonal in spin space, so that it cannot induce spin-flips. Its relevance comes from the fact that it modifies the resonance conditions (resonances are governed by the \( 2 \times 2 \) submatrix in (6) for each pairs of states). For the transition \( \nu_{fL} \rightarrow \nu_{f'R} \) one in fact gets

\[ V_{\nu_e} + \Omega_G(\tilde{r}) - 2\delta c_2 = 0, \]

(10)

where \( \tilde{r} \) is the resonance point.
\( \Omega_c \) in (10) distorts the surface of resonance owing to the relative orientation of the neutrino momentum with respect to the angular velocity. As a consequence, the outgoing energy flux results modified. To estimate the anisotropy of the outgoing neutrinos, one needs to evaluate the energy flux \( \mathbf{F} \), emitted by nascent stars. According to the Barkovich, D’Olivo, Montemayor and Zanella paper [65], the neutrino momentum asymmetry is given by

\[
\frac{\Delta p}{p} = 1 \int_0^\pi \mathbf{F} \cdot \mathbf{\hat{n}} \, da \sim -\frac{1}{9} \hat{\rho},
\]  

(11)

The factor 1/3 comes from the fact that only one neutrinos species is responsible for the anisotropy (thus it carries out only 1/3 of the total energy). \( \mathbf{F} \) is the outgoing neutrino flux through the element area \( da \) of the emission surface. \( \omega' = \omega/|\omega| \) is the direction of the vector \( \omega \), whereas \( \mathbf{\hat{n}} \) is the unity vector orthogonal to \( da \) (notice that the area element in curved spacetime is \( \sqrt{-g} \, da \); in the weak field approximation one has \( \sqrt{-g} \, da \sim da \)). \( \hat{\rho} \) is the radial deformation of the effective surface of resonance. It shifts the resonance point \( \bar{r} \) to \( r(\phi) = \bar{r} + \hat{\rho} \cos \phi \), with \( \hat{\rho} \ll \bar{r} \) and \( \cos \phi = \omega' \cdot \mathbf{\hat{p}} \). To determine \( \bar{r} \), one uses the resonance condition

\[
2 \delta c_2 - V_{\nu_e}(\bar{r}) = 0.
\]  

(12)

The deformation \( \delta \) is evaluated by expanding (10) about to \( r(\phi) \), and using the shifts \( p \to p + \delta p \), \( V_{\nu_e} \to V_{\nu_e} + \delta V_{\nu_e} \) [65], where

\[
\begin{align}
\delta_p &= \frac{d \ln p}{dr} p \hat{\rho} \cos \phi = h_p^{-1} p \hat{\rho} \cos \phi, \\
\delta V_{\nu_e} &= \frac{d \ln V_{\nu_e}}{dr} V_{\nu_e} \hat{\rho} \cos \phi = h_{V_{\nu_e}}^{-1} V_{\nu_e} \hat{\rho} \cos \phi,
\end{align}
\]  

(13, 14)

Eqs. (12), (13), and (14), allow to write Eq. (11) as

\[
\frac{\Delta p}{p} = 4GMR^2 \omega' \left[ \int 1 \over 5r^3 \left( \frac{9V_{\nu_e} \hat{\rho} (h_p^{-1} + h_{V_{\nu_e}}^{-1})}{2} \right) \right].
\]  

(15)

To compute \( h_p^{-1} + h_{V_{\nu_e}}^{-1} \), a model for the protoneutron star has to be specified. For our purpose, we adopt the polytropic model. The inner core of a protoneutron star is consistently described by a polytropic gas of relativistic nucleon with adiabatic index \( \Gamma = 4/3 \) [66]. As in [65], we assume that such a model also holds for the rest of the star. The pressure \( P \) and matter density \( \rho \) are related by \( P = K \rho^\Gamma \) [66,65], where \( K = T_c/m_n \rho_c^{1/3} \simeq 5.6 \times 10^{-5} \text{MeV}^{-4/3}/r_c \), \( T_c = 40\text{MeV} \) and \( \rho_c \sim 10^{14} \text{g/cm}^3 \) are the temperature and the matter density of the core, respectively. The matter density profile \( \rho(r) \) can be chosen as [65]

\[
\rho^{-1}(x) = \rho_c^{-1} \left[ ax^2 + bx + c \right],
\]  

(16)

where \( x = r_c/r \),

\[
a = (1 - \mu) \lambda \Gamma, \quad b = (2\mu - 1) \lambda \Gamma, \quad c = 1 - \mu \lambda \Gamma,
\]

and \( \lambda \Gamma = GM_c(\Gamma - 1)/r_c \rho_c^{1-\Gamma} K \Gamma \simeq 0.87. \) \( r_c = 10\text{km} \) is the core radius and \( M_c \simeq M_\odot \) is the mass of the core (\( M_\odot \) is the solar mass). The parameter \( \mu \) is determined by setting \( \rho(R_s) = 0 \),

\[
\mu = \left[ \frac{R_s}{\lambda \Gamma (R_s - r_c)} - \frac{r_c}{R_s} \right] \frac{R_s}{R_s - r_c}.
\]  

(17)

The temperature profile \( T(r) \) is related to the matter density \( \rho(r) \) by the relation [66,65]

\[
\frac{dT^2}{dr} = -\frac{9\kappa L_c}{\pi r^2} \rho,
\]  

(18)

where \( L_c \sim 9.5 \times 10^{51} \text{erg/sec} \) is the core luminosity, and \( \kappa \sim 6.2 \times 10^{-56} \text{eV}^{-5} \). Eq. (18) can be integrate via (16)

\[
T(r) = T_c \sqrt{2\lambda \Gamma [\chi(x) - \chi(1)] + 1},
\]  

(19)

where \( \lambda \Gamma = 9\kappa L_c \rho_c/2\pi T_c^2 r_c \sim 1.95 \), and

\[
\chi(x) = c^3 x + \frac{3}{2} b c^2 x^2 + c(a c + b^2) x^3 +
\]  

(20)
b \frac{4}{3} (a c + b^2) x^4 + \frac{3 a}{5} (a c + b^2) x^5 + \frac{b a^2}{2} x^6 + \frac{a^3}{3} x^7.

Assuming the thermal equilibrium between neutrinos and the medium, so that \( p \sim T \) [65], and being \( V_{\nu_e} \sim \rho \), one can rewrite the inverse characteristic lengths \( h_{\nu}^{-1} \) and \( h_{V_{\nu_e}}^{-1} \) as \( h_{\nu}^{-1} \equiv h_T^{-1} \) and \( h_{V_{\nu_e}}^{-1} \equiv h_p^{-1} \). Eqs. (16) and (19) imply (at the resonance)

\[
\begin{align*}
\frac{d \ln T}{dr} &= -\lambda \frac{\rho (\bar{r})}{\rho_c} \left( \frac{T_c}{T (\bar{r})} \right)^2 \frac{\bar{x}}{\bar{r}}, \\
\frac{d \ln \rho}{dr} &= -3 \left( \frac{\rho_c}{\rho (\bar{r})} \right)^{1/3} (2 a \bar{x} + b) \frac{\bar{x}}{\bar{r}}. 
\end{align*}
\]

(21)

(22)

As before pointed out, \( V_{\nu_e} \ll 1 \) \( (y_e \ll 1) \) at \( r \sim 12 \text{km} \). For resonances occurring at \( \bar{r} \sim 12 \text{km} \), where \( \rho (\bar{r}) \sim 10^{11} \text{gr/cm}^3 \), Eqs. (16) and (17) give \( R_s \sim 12.5 \text{km} \). From Eqs. (19), (21), (22) and (15), one infers

\[
\frac{1}{10^{9.7}} \frac{\omega}{10^6 \text{Hz}} \sim 6, 
\]

(23)

where \( \frac{\Delta p}{p} \sim 0.01 \) and \( \omega' = 2 \omega \) (i.e. \( \omega \parallel \mathbf{x} \)) have been used. Typical angular velocities of pulsars \( \omega \sim (1 - 10^4) \text{Hz} \) lead to \( y_e \sim 10^{-14} - 10^{-10} \), as follows from Eq. (23). For neutrinos with momentum \( p \sim 10 \text{MeV} \), the resonance condition (12) can be recast in the form \( \Delta m^2 \cos 2 \theta \sim 7.6 \times 10^{14} y_e eV^2 \), i.e. \( 10^{-10} \lesssim \Delta m^2 \cos 2 \theta / eV^2 \lesssim 10^{-6} \). The latter is consistent with the best fit of solar neutrinos \( \Delta m_{\odot}^2 \sim (10^{-5} - 10^{-4}) eV^2 \) and \( 0.8 \lesssim \sin^2 2 \theta_{\odot} \lesssim 1 \) [67].

Some comments are in order:

- In addition to the level crossing (10), it must also occur that the crossing is adiabatic, i.e. the adiabatic parameter \( \gamma \), which quantifies the magnitude of the off-diagonal elements with respect to the diagonal ones of (6) in the instantaneous eigenstates, must be greater than one when valuated at the resonance \( \bar{r} \sim 12 \text{km} \). To show this, let us define 1) the precession length

\[
L = \frac{2\pi}{\sqrt{(2 \mu_{ff} B) + (V_{\nu_e} + \Omega_G - 2 \delta c_2)^2}},
\]

that at the resonance, it reduces to

\[
L_{\text{res}} = L (\bar{r}) = \frac{\pi}{\mu_{ff} B} \simeq 10^5 \frac{10^{-11} \mu_B}{\mu_{ff}} \frac{10^{13} \text{G}}{B},
\]

and 2) the width of the resonant spin flavor precession

\[
\Delta \nu = 2 \Lambda \Lambda,
\]

being \( \lambda = \pi / L_{\text{res}} \delta \) and

\[
\Lambda = \left( \frac{\rho (\bar{r})}{\rho (\bar{r})} \right)^{-1} \simeq \frac{V_{\nu_e} (\bar{r})}{V_{\nu_e} (\bar{r})}.
\]

\( \Lambda \) is derived assuming \( y'_{\nu_e} (r, t) = 0 (Y'_e \ll \rho' / \rho) \) [54]. The spin flavor conversion is adiabatic provided \( \Delta \nu \gg L_{\text{res}} \), or equivalently

\[
\frac{2(\mu_{ff} B)^2}{\delta \pi |\rho'/\rho|} \equiv \gamma \gg 1,
\]

where

\[
\mu_{ff} B \sim 5.6 \times 10^{-7} \frac{\mu_{ff}}{10^{-11} \mu_B} \frac{B}{10^{13} \text{G}} \text{ eV}.
\]

Since the magnetic fields inside to protoneutron stars are greater than \( B \gtrsim 10^{11} \text{G} \), whereas \( \mu_{ff} \lesssim 10^{-11} \mu_B \), as provided by astrophysical and cosmological constraints [68,69], one gets \( \gamma \gg 1 \), i.e. the adiabatic condition is fulfilled (such a result follows by using Eq. (16)).
• The conversion probability $P_{\nu f L \rightarrow \nu f' R}$ is given by [61]

$$P_{\nu f L \rightarrow \nu f' R} = 1/2 - (1/2 - P) \cos 2\tilde{\theta}_i \cos 2\tilde{\theta}_f,$$

where $P = e^{-\gamma \pi/2}$ is the Landau-Zener probability, and $\tilde{\theta}$ is defined as

$$\tan 2\tilde{\theta}(r) = 2\mu_{ff'} B_\perp / (\Omega_G(r) + V_{\nu e} - 2\delta c_2).$$

$\tilde{\theta}_i = \tilde{\theta}(r_i)$ is the initial mixing angle of neutrinos produced at $r_i$, and $\tilde{\theta}_f = \tilde{\theta}(r_f)$ is the mixing angle of neutrinos where the spin flip probability is evaluated. Since $\gamma \gg 1$, the Landau-Zener probability $P$ vanishes.

• The weak field approximation is fulfilled since $(4G M R^2/5r^3) \omega R_s \lesssim 10^{-2}$ as $\omega \lesssim 10^4$ Hz and $R_s \gtrsim 12$ km. This means that the geometry of the rotating sources is correctly described by (4) up to angular velocities $\omega \sim 10^4$ Hz. Since the typical angular velocity of the protoneutron star varies in the range $(10^2 - 10^3)$ Hz, the weak field approximation here used can be applied.

• The rotational effects are relevant during the time scale $t_0 \lesssim 10$ sec ($t_0$ is the time scale for the emission of the energy $\sim 0.5 \times 10^{53}$ erg by each neutrinos degree of freedom with $p \sim 10$ MeV) [69].

IV. CONCLUSION

In this paper it has been suggested a mechanism for the generation of pulsar kicks which accounts for the spin-gravity coupling of neutrinos propagating in a gravitational field of a rotating nascent star. Owing to the relative orientation of neutrino momenta with respect to the direction of the angular velocity, the energy flux turns out to be generated anisotropically. Results imply a correlation between the motion of pulsars and their angular velocities.

Spin-gravity coupling is strictly related to the so called gravito-magnetic effect, an effect predicted by General Relativity [72], as well as by many metric theories [73–75]. Its origin is due to the mass-energy currents (moving or rotating matter contributes to the gravitational field, in analogy to the magnetic field of moving charges or magnetic dipole). Experiments involving the technology of laser ranged satellites [76,77] are at the moment the favorite candidate to test gravito-magnetic effects. In connections with the mechanism proposed in this paper, a direct evidence of the gravito-magnetic effect might be provided by pulsar kicks. Future investigations on the velocity distribution of pulsars will certainly allow to clarify this still open issue.

Results of this paper (as well as the papers [19,22,36], in which pulsar kicks are discussed in relation to gravitational waves) have been obtained in semiclassical approximation, i.e. the gravitational field is described by the classical field equations of General Relativity. It will be of interest to investigate within the framework of quantum gravity theories.

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