State-of-charge estimation for second-life lithium-ion batteries based on cell difference model and adaptive fading unscented Kalman filter algorithm

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Abstract
Lithium-ion batteries retired from electric vehicles can provide considerable economic benefits when they are retired for secondary use. However, retired batteries after screening and restructuring still face the problem of inaccurate battery pack state-of-charge (SOC) estimation due to the existence of extreme inconsistency. To solve this problem, an adaptive fading unscented Kalman filtering (AFUKF) algorithm based on the cell difference model (CDM) is proposed in this paper for improving the accuracy of SOC estimation of retired lithium-ion battery packs. Firstly, an improved CDM based on a hypothetical Rint model is developed based on a second-order resistor/capacitor equivalent circuit model. Secondly, an AFUKF algorithm is developed to improve the adaptability and robustness of local state estimation against process modelling errors. Finally, characteristic data are obtained by conducting discharge tests on the screened retired lithium-ion batteries under specific operating conditions. The proposed method can improve the accuracy of SOC estimation of retired lithium-ion battery packs and provide a new idea for SOC estimation of retired lithium-ion battery packs, as shown by the simulated real experimental data.

Keywords: electric vehicles; echelon utilization; lithium-ion batteries; state-of-charge

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1. INTRODUCTION

In the past few years, growing concerns about energy and environmental pollution have led to a significant increase in the number of electric vehicles (EVs) and plug-in hybrid electric vehicles (HEVs). Lithium-ion batteries are widely used due to their high energy density, long service life and environmental friendliness [1–3]. However, after a few years of use in EVs, the capacity of lithium-ion batteries drops to 70–80% and are therefore determined to be unable to meet the energy and power requirements of EVs, and such batteries must face retirement in order to ensure safe operation as well as adequate power [4, 5]. In fact, lithium-ion batteries retired from EVs can be sorted and still be used in scenarios such as electric bicycles, smart grids and energy storage plants [6–8]. This not only reduces the pressure of entering the recycling phase, but more importantly, it can also effectively share the cost of using EVs through power battery recycling, which can make EVs widely popular and used in terms of economic efficiency [9–11]. Therefore, it is necessary to research on the effective and reliable secondary use of retired lithium-ion power batteries in order to achieve the promotion and sustainable development of EV in large numbers.

Retired lithium-ion power batteries exhibit significant inconsistencies, mainly due to inconsistent manufacturing of lithium batteries, and therefore cannot be directly used in second life cycle applications [12]. At the same time, lithium-ion batteries in EV are exposed to a variety of complex use environments, leading to various degradations in battery performance, which exacerbate the inconsistencies between batteries. Moreover, they are prone to overcharging or over-discharging, which leads to more serious ageing, thermal runaway and explosion hazards of retired lithium-ion power batteries [13, 14]. The literature [15, 16] therefore suggests the need to evaluate and screen retired lithium-ion power batteries before reassembling the battery pack.
to ensure their remaining capacity and ensure continued safety. However, even after evaluation and screening, retired lithium-ion power cells do not exhibit the same degree of consistency as newly manufactured lithium-ion batteries, with characteristic parameters such as open circuit voltage, terminal voltage, state-of-charge (SOC), internal resistance and charge/discharge current exhibiting strong inconsistencies [17, 18]. It is due to the inconsistencies that exist in these parameters that traditional methods for estimating the remaining capacity of lithium-ion battery packs are no longer adequate. Accurate estimation of the remaining capacity of retired batteries is beneficial for the power requirements, durability and safety of the battery pack. Therefore, the challenge of improving the accuracy of the SOC estimation of retired ion battery packs is fraught and a large number of researchers have conducted research to this end.

In terms of the different utilization background, a number of adaptive estimation methods have been proposed for different application contexts of lithium-ion batteries [19, 20]. These methods can be divided into three categories: simple computational, data-driven and battery-based modelling approaches [21, 22]. The simple calculation approach ignores the internal battery variations involved in different battery types and performs SOC prediction. It is implemented by means of simple correspondence models between external battery parameters, such as ampere-time integration, open-circuit voltage and impedance modelling methods [23–25]. The advantage of this typical estimation method is that it does not require complex modelling processing of the battery to accommodate different SOC levels, making it easy to apply at any time [26]. However, the disadvantage of this strategy is the low prediction accuracy and the need to restructure some other mathematical computational processes. Data-driven algorithms are similar to simple computational approaches, with the difference that the data are trained using exact modelling methods such as neural networks, fuzzy inference and support vector machines [27, 28]. The advantages of these types of algorithms are improved prediction accuracy and reduced computational effort. The disadvantage is that offline model adaptation training requires a large amount of data and takes a considerable amount of time. Also the training strategy has a significant impact on the prediction accuracy. In contrast, with the construction method of battery equivalent modelling, the battery will always be regarded as a dynamic power storage and supply system [29, 30] and then its state space description is completed by battery modelling processing and parameter identification. Considering its complex operating conditions, the estimated state quantities are improved by various filters or observers [31, 33] such as extended alman filter, unscented Kalman filter, adaptive Kalman filter, particle filter, sliding mode observer and other non-linear iterative computational methods. The advantages of these methods are high prediction accuracy and good convergence, while the disadvantages are high computational complexity of the battery model and more complex iterative computation. Therefore, these methods are more suitable for SOC estimation for batteries in battery management systems.

In addition, a lot of work has been done by researchers to improve the accuracy of SOC estimation for series-connected battery systems as well. In [34], the author stated that Rint model can work as cell difference model (CDM) to estimate SOC difference and identify the mean equivalent model through particle swarm optimization. However, this method only takes the model parameters difference between cells into consideration. The parameter difference is not equivalent to the remaining battery capacity difference, so the remaining capacity estimation is not accurate during the ladder utilization for retired batteries. In [35], the author built an equivalent circuit model (ECM) based on a model parameter adjuster to solve issues such as uncertain battery noise statistics and the divergence of estimation results, etc. Although this method raises the algorithm precision, it is not practicable to force transferring noise distribution of normal distribution because of the inconsistency between cells. In [36], the author provided one neural network algorithm that can take space and time features and train different discharge curves to capture the non-linear relationships between different parameters. However, those data cannot be gathered from real-time measurement, so it can only be used offline. In [37], the author stated a parameter-adaptive method with dead zone. This method can stop the real-time battery parameter update if the battery model error is unacceptable. Nevertheless, this method cannot solve the problem by setting dead zone if the error was too large initially. In [38], the author propose an average battery equivalent model that first identifies and quantifies the uncertainty in the parameters of the basic model and then uses a bias correction method to consider deviation effects and deviation functions, a neural network as an algorithm to build a deviation approximation model and, finally, the EKF algorithm to obtain the gradient of the deviation function required for the Jacob matrix calculation. Strictly speaking, the method is able to address the inaccuracy of the algorithm due to the inconsistency of the battery pack model parameters at the model level, but for retired batteries, their ageing characteristics are the main factor contributing to the imbalance generated by the battery pack.

Based on the above analysis, there are inconsistencies that are difficult to eliminate when estimating the SOC of a lithium-ion battery pack, such as inconsistencies in the initial SOC and inconsistencies in the battery model parameters. In particular, the capacity inconsistency is stronger for aged battery packs. However, considering the inconsistency of SOC and cell internal resistance, there is less research on this aspect of SOC estimation for retired lithium-ion battery packs. In order to address these inconsistencies in retired battery packs, the SOC, terminal voltage and cell internal resistance of each cell in the pack must be estimated jointly. To this end, a CDM with practical physical significance is proposed in this paper. A framework of Adaptive Fading Unscented Kalman Filtering (AFUKF) algorithm is developed that will reduce the divergence of the solution results for estimating SOC, model parameters and capacity in the CDM. And further experiments are conducted on a small battery in series under different operating conditions, setting the initial test SOC values of different individual cells and estimating the
terminal voltage and internal resistance of different retired cells to determine the SOC inconsistency. The rest of the paper is framed as follows: in Section 2, the single-cell ECM, i.e. the second-order resistor/capacitor (RC) ECM, is specifically analysed and a hypothetical Rint model is developed to simulate the difference between the single cell and the average cell. In Section 3, the adaptive fading traceless Kalman filtering algorithm is proposed to estimate the average SOC of the battery pack and the SOC inconsistency of each individual cell, and specific experimental test protocols and test results are given in Section 4. Section 5 specifically analyses the experimental results of the proposed model and algorithm.

2. BATTERY MODELLING

In practice, battery systems connected in series can provide higher voltages. However, due to the inconsistency of the battery cells, the performance of a series-connected battery system is largely dependent on the weakest cell, which can seriously reduce the accuracy of the battery system model when used. Therefore, in order to perform an accurate SOC estimation, the cell inconsistency must first be considered. Therefore a battery system model should first be developed. Three main classes of battery models have been used to describe the operation of batteries, electrochemical models, mathematical models and ECMs [39]. The electrochemical model, which uses complex non-linear differential equations to describe the battery chemistry, is the most accurate of the three models and is more suitable for electrolyte and electrode studies [40]. However, the computational complexity and time consumption of electrochemical models make it difficult to use them for real-time electrical simulations and engineering applications. Mathematical models that are useful to system designers are also not suitable for electrical simulation and control as they do not provide any information. ECMs not only accurately capture the dynamic operating characteristics of a cell through electrical components including capacitors and resistors, but also allow electrical simulation and are compatible with other circuits and systems. It is worthwhile to obtain a high average SOC estimation accuracy by sacrificing some computational effort, and several studies of second-order RC ECMs have been carried out in the literature [41, 42], which show that the model is plausible. The desired SOC estimation accuracy can be achieved.

2.1. Battery cell model (2RC ECM) description

Previous research has shown that although multi-order RC models have good accuracy, increasing the order of RC in ECM does not always improve the accuracy of the model due to the problem of overfitting. Then, finding the balance between calculation volume control and estimation error is essential. As shown in Figure 1a, a 2RC ECM for a lithium-ion series battery system is used in this paper, which provides an accurate account of the dynamic characteristics of the battery system. The circuit consists of a controlled voltage source ($U_{OC}$), an internal resistor ($R_0$), and two RC networks connected in series. $U_{OC}$ is the open voltage of the battery system, which will be formulated by SOC. While $R_0$ refers to the internal resistance of the battery system, the two RC network will show the time variance effect during the charging and discharging process in the battery system. $R_1$, $C_1$, $R_2$ and $C_2$ are battery parameters to represent the dynamic battery response and capacity. These parameters will vary with the SOC. $U_L$ and $I_L$ are terminal voltage and current respectively in the battery pack.

The equation for SOC is as follows:

$$\text{SOC}(t) = \text{SOC}_0 - \int \eta I(t) dt / C_0,$$  \hspace{1cm} (1)

where $\text{SOC}_0$ is the initial value of SOC and $\eta$ and $C_0$ are coulomb efficiency and nominal capacity. Based on Kirchhoff’s voltage law (KVL), the functional relationship between terminal voltage $U_L$ and current $I_L$ can be expressed as

$$U_L(t) = U_{OC}(t) - I_L(t) \left[ R_0(t) + \frac{R_1(t)}{1+R_1(t)C_1} + \frac{R_2(t)}{1+R_2(t)C_2} \right].$$ \hspace{1cm} (2)
Table 1. Calculation process for non-linear system.

Step 1: Initialization

\[ \dot{x}_0 = E[x_0], \quad P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]. \]

Step 2: Time update

Given the state estimate \( \hat{x}_{k-1} \) and its error covariance matrix \( P_{k-1} \), the sigma point can be selected in the following way:

\[
\begin{align*}
    x_{k-1} &= \hat{x}_{k-1}, i = 0, \\
    \tilde{x}_{k-1} &= \hat{x}_{k-1} + (\sqrt{n} \chi_{k-1})_i, i = 1, 2, \ldots, n, \\
    \tilde{x}_{k-1} &= \hat{x}_{k-1} - (\sqrt{n} \chi_{k-1})_n, i = n+1, n+2, \ldots, 2n,
\end{align*}
\]

Instantiate sigma points through the process model to generate a set of transformed samples:

\[
\begin{align*}
    x_{k/k-1} &= f(x_{k-1}), i = 0, 1, \ldots, 2n.
\end{align*}
\]

The predicted state mean and covariance are calculated as

\[
\begin{align*}
    \hat{x}_k &= \sum_{i=0}^{2n} \omega_i \tilde{x}_{k-1}, \\
    P_k &= \sum_{i=0}^{2n} \omega_i (x_{k/k-1} - \hat{x}_k)(x_{k/k-1} - \hat{x}_k)^T + Q_k,
\end{align*}
\]

Step 3: Sigma point update

Reselect a new set of sigma points with the mean \( \hat{x}_{k/k-1} \) and covariance \( P_{k/k-1} \):

\[
\begin{align*}
    x'_{k/k-1} &= \hat{x}_{k/k-1}, i = 0, \\
    \tilde{x}_{k/k-1} &= \hat{x}_{k/k-1} + (\sqrt{n} \chi_{k/k-1})_i, i = 1, 2, \ldots, n, \\
    \tilde{x}_{k/k-1} &= \hat{x}_{k/k-1} - (\sqrt{n} \chi_{k/k-1})_n, i = n+1, n+2, \ldots, 2n,
\end{align*}
\]

Step 4: Measurement update

The converted sigma point used for measurement is

\[
\gamma_{y/k-1} = h(x'_{k/k-1}).
\]

The predicted measured value passes:

\[
\hat{z}_{k/k-1} = \sum_{i=0}^{2n} \omega_i \gamma_{y/k-1} = \sum_{i=0}^{2n} \omega_i h(x'_{k/k-1}).
\]

The corresponding covariance matrix (innovation covariance matrix) is described as

\[
P_{k/k-1} = \sum_{i=0}^{2n} \omega_i (y_{k/k-1} - \hat{z}_{k/k-1})(y_{k/k-1} - \hat{z}_{k/k-1})^T + R_k.
\]

The covariance between the predicted state and the measured value is

\[
P_{z_{k/k-1}z_{k/k-1}} = \sum_{i=0}^{2n} \omega_i (x_{k/k-1} - \hat{x}_{k/k-1}) \times (y_{k/k-1} - \hat{z}_{k/k-1}).
\]

Kalman gain can be calculated as

\[
K_k = P_{z_{k/k-1}z_{k/k-1}} P_{k/k-1}^{-1}.
\]

The state estimate \( \hat{x}_k \) and the associated error covariance matrix \( P_k \) are updated to

\[
\hat{x}_k = \hat{x}_{k/k-1} + K_k (y_{k/k-1} - \hat{z}_{k/k-1}), \\
P_k = P_{k/k-1} - K_k P_{z_{k/k-1}z_{k/k-1}} K_k^T.
\]

Step 5: Repeat steps 2–4 for the next sample until all samples are processed

According to ECM in Figure 1a, the space-state representation is

\[
\begin{bmatrix}
    \text{SOC}_{\text{mean},k+1} \\
    U_{1,k+1} \\
    U_{2,k+1}
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & 0 \\
    0 & \exp\left(-\frac{\Delta t}{\tau_1}\right) & 0 \\
    0 & 0 & \exp\left(-\frac{\Delta t}{\tau_2}\right)
\end{bmatrix}
\times
\begin{bmatrix}
    \text{SOC}_{\text{mean},k} \\
    U_{1,k} \\
    U_{2,k}
\end{bmatrix}
+ \begin{bmatrix}
    -\eta \Delta t / C_0 \\
    R_1 \left[1 - \exp\left(-\frac{\Delta t}{\tau_1}\right)\right] \\
    R_2 \left[1 - \exp\left(-\frac{\Delta t}{\tau_2}\right)\right]
\end{bmatrix}
\times I_L,k
\]

where \( U_1 \) and \( U_2 \) are voltage values for capacitor \( C_1 \) and \( C_2 \), which \( \tau_1 \) and \( \tau_2 \) are both time parameters, which can obtained from

\[
\begin{align*}
    \tau_1 &= R_1 C_1, \\
    \tau_2 &= R_2 C_2.
\end{align*}
\]

Thus, the original battery system model can be obtained from equations (1)–(4). The original battery system ECM can simply be derived from the battery ECM and filtering methods can be used to improve the accuracy of the SOC estimation. However, since inter-cell variation in the battery system is largely ignored, the ECM will be constructed inaccurately and will reduce the SOC estimation of the battery system. Furthermore, assuming that \( I_L \),
and $U_L$ are the input and output variables of the system, respectively, the discrete-time measurement function can be written as

$$\mathbf{U}_{L,k+1} = \mathbf{U}_{oc,k+1} - R_{0,k+1}I_{L,k+1} - U_{1,k+1} - U_{2,k+1}$$ (5)

### 2.2. Battery pack model description

The accurate battery model can show the cell inconsistency between each other all-roundly. However, it is almost impossible to achieve modeling for hundreds of cells in the battery system because of the complexity. The battery inconsistency is measured in a longer time range, such as hourly, even daily, while the time variance between battery pack status is calculated in seconds, even in milliseconds. Hence, the inconsistency change in low frequency cells and the status change in high-frequency battery pack should be decoupled. From [43], several cell difference models are summarized. The CDM that consider SOC difference and internal resistance will be more reliable and accurate. Hence, this article will adopt Figure 1b as the cell difference model. This is a hypothetical Rint model with internal resistance difference and SOC difference into consideration. Therefore, this CDM can eliminate the SOC inconsistency caused by internal resistance. From Figure 1b, $\Delta R^i$ refers to the cell internal resistance difference. $\Delta U_{oc}^i$ is the i-th cell's OCV difference, which directly affects the SOC inconsistency. With SOC_{mean} known, the $\Delta SOC^i$ can be gathered from $\Delta U_{oc}^i$. $\Delta U_B^i$ is the i-th cell's difference between terminal voltage and average terminal voltage, which can be defined as

$$\Delta U_B^i = U_{B_i}^i - U_{mean}.$$ (6)

The $U_{B_i}^i$ is the battery pack voltage for terminal $I_L$, and $U_{mean}$ is the average terminal voltage for series-connected battery pack. For SOC_{mean}, the CDM can be presented as

$$\Delta U_B^i = \Delta U_{oc}^i \Delta SOC^i - I \Delta R^i,$$ (7)

where $\Delta U_B^i$ refers to the difference between terminal voltage and battery pack's average terminal voltage for battery i. The $\Delta U_{oc}^i$ is the difference with SOC_{mean} for difference battery i, which is under balanced condition. Comparing to SOC_{mean}, the lower SOC_i can increase while the higher SOC_i can decrease. As the I is the battery pack current, $\Delta R^i$ is the internal resistance difference. $\Delta R^i < 0$ refers to internal resistance lower than the battery i, and $\Delta R^i > 0$ refers to internal resistance higher the battery i.
Figure 3. AFUKF algorithm framework.

Figure 4. (a) Arbin constant temperature test chamber; (b) test platform for assembly and test between battery packs.

Relating to the cell average model, the output equation that explains the CDM should be linear. In this equation, $\Delta \text{SOC}$ is the only state variance. The input parameters include not only current $I$ observed by the sensor, but also the $\text{SOC}_{\text{mean}}$ derived from state-space representation. The output parameter is the terminal voltage difference $U_{oc}$. The state vector is defined as

$$x^i = \Delta \text{SOC}^i$$

(8)

and

$$f(x^i_m) = \Delta \text{SOC}^i_m,$$

(9)

where $\Delta \text{SOC}^i_m$ refers to the difference between SOC and $\text{SOC}_{\text{mean}}$ for battery $i$. To be notified, this equation is linear

$$g(x^i_m) = \Delta U^i_{oc,m} \left( \Delta \text{SOC}^i \right) - I_m \Delta R^i,$$

(10)

where $\Delta U^i_{oc,m}(\Delta \text{SOC}^i)$ is the difference between $\text{SOC}_{\text{mean}}$ and OCV under the balance point $m$ for battery $i$. The $\text{SOC}^i$ value can be dynamically adjusted. Similarly, by functioning $g(x^i_m)$ to $x^i_m$, with $x^i_m = \hat{x}^i_m$,

$$C^i_m = \frac{\partial g}{\partial x} \bigg|_{x^i_m=\hat{x}^i_m} = \frac{d \Delta U^i_{oc} \left( \Delta \text{SOC}^i \right)}{d \Delta \text{SOC}^i} \bigg|_{\Delta \text{SOC} = \Delta \hat{\text{SOC}}^i_m}$$

(11)

is calculated near $\text{SOC}_{\text{mean}}$, which can be presented as

$$C^i_m = \frac{\partial g}{\partial x} \bigg|_{x^i_m=\hat{x}^i_m} = \frac{d \Delta U^i_{oc} \left( \Delta \text{SOC}^i \right)}{d \Delta \text{SOC}^i} \bigg|_{\text{SOC}_{\text{mean}}=\hat{\text{SOC}}^i_m, \Delta \text{SOC} = \Delta \hat{\text{SOC}}^i_m}$$

(12)

In reality, $\text{SOC}_{\text{mean}}$, $\Delta U^i_{oc}$ and $\Delta \text{SOC}^i$ are correspondent. It is notable that

$$\Delta U^i_{oc} = \min \left( U^i_{oc} - U_{oc} \left( \text{SOC}_{\text{mean}} \right) \right)$$

(13)
when $SOC_{\text{mean}} + \Delta SOCi \leq 0$

$$\Delta U_{oc}^i = \max(U_{oc}) - U_{oc}(SOC_{\text{mean}})$$  \hspace{1cm} (14)

when $SOC_{\text{mean}} + \Delta SOCi \geq 1$, $\min(U_{oc})$ and $\max(U_{oc})$ are boundary voltages because $\Delta SOCi$ may go beyond the boundary. All the cells in the battery pack have the same SOC-OCV curve normally.

As for the identification of CDM, the input includes $SOC_{\text{mean}}$, battery pack current $I$ and cell difference information $\Delta SOCi$, $\Delta U_{oc}^i (i = 1, 2, \ldots, 8)$. Comparing to the average state of battery pack, the positive value means higher value and the negative value means lower value. Hence, there are two independent parameters to be estimated as

$$\theta^i = \left[ \Delta B_0^{i, \text{ch}} \text{ or } \Delta B_0^{i, \text{disch}} \right].$$  \hspace{1cm} (15)

The objective function adopted equation as

$$\min : f(\hat{\theta}^i) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \Delta U_{oc}^i - \Delta U_{ocj}^i \right)^2}$$  \hspace{1cm} (16)

where $n$ refers to the total collected data and $\Delta U_{oc}^j$ is the real difference of terminal voltage. The $\Delta U_{ocj}^i$ is the voltage difference between cell $I$ and cell $j$ derived from CDM.

For the estimation of SOC inconsistency, in addition to establishing an appropriate ECM, it is necessary to apply an effective algorithm to implement the model, which must converge and meet practical engineering applications without being too complex in addition to achieving accurate estimation of SOC. One of these algorithms, the EKF filtering algorithm, was used in the literature [44, 45] to estimate the difference between the average SOC and the SOC of a single cell. However, the battery pack is a strongly non-linear system and an EKF based on a first-order Taylor series expansion can produce large estimation errors or even divergence. The latter may be caused by severe disturbances or heavy load conditions. In order to circumvent the first-order approximation error of the EKF, some other nonlinear filters have been proposed and applied to the dynamic state estimation of the battery pack. For the problem that the retired lithium-ion battery pack system needs an accurate system model and accurate noise statistics, this paper proposes an AFUKF: AFUKF uses an adaptive fading factor to scale the process noise covariance or predicted state to compensate for the uncertainty of the process model. The specific implementation of this algorithm is described in detail in Section 3.

### 3. BATTERY SOC FILTER ESTIMATION METHOD

Unscented Kalman filter (UKF) has become a promising solution for SOC state estimation due to its simplicity of implementation, high estimation accuracy and fast convergence, as it does not require the computation of a Jacobi matrix. However, UKF requires an accurate system model and precise noise statistics. In practice it is difficult to meet these conditions. The system model usually involves uncertainties such as model parameter mismatch, system noise statistics errors and random drift of parameters, resulting in non-convergence or even divergence of the UKF solution results. For this reason, this paper will use the AFUKF algorithm to compensate for process model uncertainties by scaling the process noise covariance or predicted state covariance using adaptive fading factors.

#### 3.1. Classical UKF

Considering the non-linear dynamic system with additive noise as follows.

$$\begin{align*}
\dot{x}_k &= f(x_{k-1}) + w_k, \\
\dot{z}_k &= h(x_k) + v_k,
\end{align*}$$  \hspace{1cm} (17)

where $x_k \in R^n$ and $z_k \in R^n$ are the state vector and measurement vector for time $k$, $f(\cdot)$ and $h(\cdot)$ are the non-linear functions for describing the process and measuring the model. $w_k \in R^n$ and

### Table 2. Battery information.

| Material   | Nominal capacity | Nominal voltage | Lower cut-off voltage | Upper cut-off voltage | Diameter/height |
|------------|------------------|-----------------|-----------------------|-----------------------|-----------------|
| LiFePO₄    | 2600mAh          | 3.6 V           | 2.75 V                | 4.2 V                 | 18 mm/65 mm     |
$v_k \in \mathbb{R}^m$ are the process and model noises, which are assumed as white Gaussian noise with covariance with zero uncorrelated mean. In

$$
E\left[w_k w_j^T\right] = Q_k \delta_{kj},
E\left[v_k v_j^T\right] = R_k \delta_{kj},
$$

(18)

$Q_k$ is the non-negative definite matrix and $R_k$ is the positive definite matrix, while $\delta_{kj}$ is the Kronecker $-\delta$ function. The calculation process for the non-linear system described by Equation (12) can be concluded into Table 1.

Obviously, if the system model in Equation (17) has uncertainty, the gathered prediction state $\hat{x}_{k/k-1}$ and predictive measurement $\hat{z}_{k/k-1}$ will be inaccurate. This will cause further errors for Kalman gain $K_k$, making the state estimation worse. Hence, if no precise system models are provided, the filtering effect of classic UKF will be worse, or even dispersed.

### 3.2. Determination of adaptive fading factor

When less calculation is needed for AFUKF, the influence caused by process modelling error to filter solution will be greatly inhibited. When using this method to detect errors of process modelling, the adaptive fading factor will be structured based on innovation vector and its relevant statistics information. By using the adaptive fading factor to zoom the process noise covariance or predictive state covariance, the historical information weight is relatively small, while the current measurement information weight is relatively large. Thus, the UKF can be adjusted iteratively to ease the influence to estimated solution caused by process modelling error. Hence, after combining the AFKUF algorithm into the model created in the second section, it provides a solution to enhance the adaptability and robustness to the retired battery CDM.

It is assumed that the non-linear system process model to includes uncertainty in Equation (17). The core value for AFUKF is to add adaptive fading factor in the prediction of state covariance matrix to avoid the influence caused by former experiments.
Hence, the AFUKF with adaptive fading factor is presented in Figure 2, and detailed realization steps of AFUKF algorithm is in Figure 3.

4. EXPERIMENT

In order to verify the uncertainty in the cell modelling and SOC estimation, the ICR18650, a lithium-ion battery manufactured by LG Chem, was tested in a laboratory facility as shown in Figure 4. The basic parameters are shown in Table 2. Considering the batteries are retired, a set of experiments were designed to test the performance of different batteries after dividing them by capacity and SOC adjustments. In Figure 5, there are 16 cells with ~80% capacity, 8 of which are grouped into packs with an average capacity of 2080 mAh. The red dashed line is the boundary where the capacity difference is within ±1%. After selection, the cells could meet the capacity requirements of the experiment with SOC ranging from 100% to 65%. The SOC is considered constant during the experiment as all cells are tested in the same environment. The test bench consists of an Arbin BT2000 circulator, a temperature chamber for regulating the temperature and a computer for coding and storing the results of the experiments. The Arbin BT2000 circuit has a measurement error of less than 0.05% when measuring the system's internal current and voltage sensors, with a sampling frequency of 1HZ and a constant temperature of 25°C. Therefore, given an accurate initial SOC value, the SOC derived from the ampere-hour rating is accurate during the experiment. The battery will be tested under dynamic operating conditions (e.g. FUDS and DST). To clearly show the operating condition of the battery, the current profile and voltage in FUDS are shown in...
5. ANALYSIS AND DISCUSSION

In this section, the SOC estimation accuracy is verified by using current and cell voltage data collected using the AFUKF algorithm and estimating the SOC, SOC difference and $R_0$ for each cell based on the CDM under DST operating conditions. The estimated cell voltage is compared to the measured voltage to verify the accuracy of the internal resistance $R_0$. The results show the efficiency of the SOC estimation of the battery pack using AFUKF. Also, to further express the efficiency of AFUKF, the results are compared with UKF.

Figure 6a and b and the battery voltage and real time SOC values are shown in Figure 6c and d. The same information in the DST is shown in Figure 7a–d.

Figure 8a compares the estimated SOC with the actual SOC and shows the differences. With the CDM, the error between the estimated SOC and AFUKF converges to within 2%, representing its excellent performance. In Figure 8b, the estimated terminal voltages and the tested terminal voltages are also within reasonable intervals. Figure 8c shows the $\Delta$SOC for each cell’s difference between CDM and average SOC. The initial SOC values are consistent with the estimated SOC values and fluctuate very little. To ensure rapid convergence, the noise covariance was adjusted during the experiment and the SOC stabilized with iteration. In Figure 8d, the convergence of the estimation error for each image element is rapidly reduced to about 1%. Thus, the proposed method identifies the inconsistent SOC of each cell in the end-of-life battery pack, but provides fast convergence and stable variation in the results.

The first step of AFUKF algorithm using the CDM model is to select a specific cell. According to the analysis, theoretically,
any cell can be selected as the specific cell. It is only necessary to replace the specific cell with a virtual mean cell. The terminal voltage value of the mean cell is the average value of all the cell voltages. The SOC and $R_0$ values of the mean cell in the CDM model are obtained.

It can be seen from the results that the terminal voltage estimation error of this algorithm tends to stabilize with the increase of time, and the SOC estimation error is less than 2%, and the terminal voltage can be accurately estimated. The SOC estimation error of the traditional RC model is about 5%.

In Figure 9, a 3D coordinate system is given to clearly display the mapping relation between reflective internal resistance $R_0$, SOC and terminal voltage.

In addition, the error in the estimated terminal voltage of each cell is shown in Figure 10b, which increases at the end of the discharge process. Under the above conditions, the root mean square error is calculated to be 8.2 mV. Figure 10a shows the difference in the internal resistance of each cell. With both the CDM and AFUKF algorithms, the internal resistance uncertainty of each cell is reduced and the curve becomes stable with minimal fluctuations.

The same experiments were carried out under FUDS operating conditions with similar results. Based on these results, cell and battery SOC, $R_0$ and terminal voltage were considered as performance indicators in the CDM. Therefore, the CDM and AFUKF can reduce the uncertainty used for SOC estimation of retired cells. Therefore, we can conclude that the proposed method can identify SOC inconsistencies for batteries with good performance.

Furthermore, Figure 11 shows a comparison between UKF and AFUKF for SOC estimation. The AFUKF algorithm has a higher accuracy and lower absolute error, making it more suitable for use with this algorithm. In this work, AFUKF was used to estimate SOC inconsistently due to various arduous working conditions. For SOC estimation, AFUKF takes monitored currents and voltages as input and output, respectively, and then logically combines the method with the ECM-based method. Finally, an accurate SOC estimate is derived.

### 6. CONCLUSION

The SOC inconsistencies can affect the power, durability and safety of a battery pack for a second use. Therefore, a fundamental technology for retired battery packs is SOC inconsistency identification, which can be used as a reliable reference for battery condition assessment and battery balancing. In this paper, after analysing the characteristics of inconsistent battery model parameters, a CDM based on a second-order RC model is proposed to analyse the SOC inconsistency and internal resistance inconsistency of lithium-ion batteries using a hypothetical Rint model. Based on this model, AFUKF is proposed to estimate the SOC and $R_0$ of each cell in the battery pack. To verify the feasibility of the proposed method in this paper, a series of experiments are designed to identify the inconsistency of retired batteries and the estimation performance of the proposed method. Sample cells that meet the criteria for step-up utilization were selected through capacity testing of retired cells, and after conducting a series of discharge simulation experiments under FUDS and DST operating conditions, the experimental data under this operating condition were used to model and estimate the algorithm to verify the efficiency of the model and the estimated algorithm. Based on the results of the simulation experiments, the SOC error of the single-cell battery can be controlled to within $\pm$2% and can converge quickly. In particular, if the retired lithium-ion battery pack shows stronger inconsistencies in the charging and discharging phases, not only can the inconsistencies between the individual cells be identified by the model parameters, but also higher accuracy of SOC estimation for retired lithium-ion power batteries can be achieved. The method proposed in this paper is currently only implemented under offline operation, and more research needs to be done in the future before it can be applied online, while poor estimation performance at different time scales needs to be considered.

### 6. AUTHOR CONTRIBUTIONS

B.D. provided concepts, methods and models to verify the proposed SOC estimation based on a given algorithm and wrote the original manuscript. S.Y. helped with writing, editing and translating the manuscript and software support. Y.Z. provided key recommendations on the research scope and experiments. All authors have read and agreed on this version of manuscript.
CONFLICTS OF INTEREST

The authors declare that are no conflicts of interest regarding the publication of this paper.

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