STABILITY OF THE DIRECTLY IMAGED MULTIPLANET SYSTEM HR 8799: RESONANCE AND MASSES

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ABSTRACT

A new era of directly imaged extrasolar planets has produced a three-planet system (Marois et al. 2008), where the masses of the planets have been estimated by untested cooling models. We point out that the nominal circular, face-on orbits of the planets lead to a dynamical instability in \( \sim 10^5 \) yr, a factor of at least 100 shorter than the estimated age of the star. Relaxing the face-on assumption, but still requiring circular orbits while fitting the observed positions, makes the problem even worse. Keeping the nominal orbits, but reducing the planetary masses, allows stability only for unreasonably small (\( \lesssim 2 M_{\text{Jup}} \)) planetary masses. A suite of numerical integrations shows the system can only survive until now if the inner two planets have a 2:1 commensurability between their periods, avoiding close encounters with each other through this resonance. This resonance implies the inner planet is eccentric (\( e > 0.04 \)) and that its current velocity is smaller than the nominal circular orbit, which can be confirmed with several more years of observations. That the resonance has lasted until now, in spite of the perturbations of the outer planet, leads to a limit \( \lesssim 10 M_{\text{Jup}} \) on the masses of the outer two planets. This constraint rules out certain versions of the core accretion hypothesis, and favors hot-start cooling models. If the outer two planets are also engaged in a 2:1 mean-motion resonance, which is consistent with the current data, the system could last until now even if the planets have masses of \( \sim 20 M_{\text{Jup}} \).

Subject headings: celestial mechanics — planetary systems — methods: numerical integration

1. INTRODUCTION

The method of directly imaging for the discovery of extrasolar planets has had spectacular first results in the last several years and months (Chauvin et al. 2004; Lafrenière et al. 2008; Marois et al. 2008; Kalas et al. 2008; Lagrange et al. 2008). These planets are young and massive and are thus still radiating their gravitational binding energy, which has been detected in the infrared.

Currently, masses of directly-imaged planets are estimated using untested models of this contraction and cooling process. Moreover, the models make a wide swath of predictions. In fact, in one particular model of the core-accretion hypothesis (Marley et al. 2007), when accretion halts the newly formed planets have smaller luminosities than those which have been recently detected. Conversely, at a certain mass and age, the “hot-start” models (which have initially extended envelopes and a large entropy per baryon) give the largest luminosity possible; even hotter models converge to a common track after a few Myr (Baraffe et al. 2002). Therefore, for a given age and luminosity, a lower limit on the mass seems to be a robust conclusion. Yet the newly-discovered three-planet system HR 8799 (= HD 218396) has masses that are seemingly beyond what can be accommodated in a stable system. These lower-limit masses are 5-11, 7-13, and 7-13 \( M_{\text{Jup}} \) for planets b, c, and d, respectively, based on a rather uncertain stellar age of 30-160 Myr (Marois et al. 2008), which is presumably also roughly the age of the planets. We integrated the Newtonian equations of motion\(^3\) of the proposed system, with circular, face-on orbits, and the nominal masses for all four bodies: 7, 10, 10 \( M_{\text{Jup}} \) for the planets and 1.5 \( M_{\odot} \) for the star. Figure 1 shows the results for the semi-major axis and maximum radial excursion of each planet as a function of time; we stop the integration soon after a close encounter at 0.18 Myr between the inner two planets (i.e., they enter within a Hill radius of one another, after which at least one of the three planets would be quickly ejected). A different integration with slightly different initial conditions resulted in a close encounter at 0.13 Myr between the outer two planets, confirming that the outcomes of such chaotic experiments are probabilistic. However, it is highly unlikely that any very nearby initial condition could last the star’s age of \( \gtrsim 30 \) Myr without instability setting in.

The goal of this paper is to determine system parameters that are consistent with the astrometric data, the inferred planetary masses, and with dynamical stability over the system’s age. In \( \S 3 \) we discuss astrometric constraints on the orbits from the discovery paper. In \( \S 4 \) we determine what planetary masses would be needed for the nominal orbits to be stable. In \( \S 5 \) we scan over non-circular orbits for the inner planet, which is currently unconstrained by the data. In \( \S 6 \) we discuss our preferred configuration for the planetary system: a mean motion resonance between the inner two planets. In \( \S 7 \) we discuss our conclusions.

2. ASTROMETRIC CONSTRAINTS

\(^3\) All integrations herein use the HYBRID algorithm of Mercury (Chambers 1999) with a timestep of 1000 days, which results in conservation of energy of 1 part in \( \sim 10^5 \) and angular momentum of 1 part in \( \sim 10^{12} \) before planets strongly scatter.
In Figure 2, we plot the sky-projected position and velocity vectors (Table 1) of the three planets, at the epoch 2008 Aug. 12, as determined by least-squares fit to the Keck astrometry of Marois et al. (2008). We use the nominal distance of the star 39.4 pc to convert observed angular separations to AU, and all the numbers herein will scale with this estimate. The impression given by Figure 2 is that we are seeing the planetary system face-on, with counter-clockwise, nearly circular orbits. This is what we call the “nominal model,” and we plot the implied orbits and velocity vectors for a 1.5 $M_\odot$ star, also in Figure 2. First, we point out some tension in the observed velocities. For both planets b and c, the observed velocity vector is $\sim 2\sigma$ away from perpendicular to the separation vector (from the star to the planet). Another problem is that, since all of the planets are bound mostly by the mass of the star, their circular orbits should follow the relation $v_{\text{orb}} = 2\pi \text{ AU yr}^{-1} (M_\ast/M_\odot)^{1/2}(a/\text{AU})^{-1/2}$. However, if all of the orbits are truly face-on and circular, their sky-projected separation $s = a$, and their sky-projected velocity $v_p = v_{\text{orb}}$. For the outer two planets, $s$ and $v_p$ are measured with high precision (Table 1), providing two independent measurements of the stellar mass. Given the nominal model, the stellar mass binding planet b is $M_\astb = 1.60 \pm 0.12 M_\odot$, yet the stellar mass binding planet c is $M_\astc = 1.38 \pm 0.08 M_\odot$. These values bracket the value of $M_\ast = 1.47 \pm 0.30 M_\odot$ preferred by combining parallax, magnitude, and spectroscopic information (Gray & Kavelaars 1999), but that they are in tension with each other: $\Delta M_\ast \equiv M_\astb - M_\astc = 0.22 \pm 0.15 M_\odot$, pointing to a failure of the face-on, circular model. The instability reported in the introduction is, however, the main failing of the nominal model. This leads us to search for another model in which the planets are still coplanar and circular, but the system plane is inclined by an angle $i$ to the plane of the sky, with an ascending node $\Omega$ measured East of North, and a to-be-determined consistent mass $M_\ast$. The sky-projection changes the magnitudes and directions of the velocity vectors and the inferred spacings of the planets, and taking it into account could lead us to infer a wider-spaced, more stable system. We focus only on circular and coplanar models in this section because, barring resonant effects, such systems are more amenable to long-term stability. The velocity field on the sky due to this model is:

$$
\begin{pmatrix}
  v_E \\
  v_N
\end{pmatrix} = n(x_E, x_N) \begin{pmatrix}
  -\alpha \sin \Omega \cos i - \beta \cos \Omega (\cos i)^{-1} \\
  \alpha \cos \Omega \cos i - \beta \sin \Omega (\cos i)^{-1}
\end{pmatrix},
$$

(1)

where

$$
\begin{pmatrix}
  \alpha \\
  \beta
\end{pmatrix} = \begin{pmatrix}
  \cos \Omega & \sin \Omega \\
  -\sin \Omega & \cos \Omega
\end{pmatrix} \begin{pmatrix}
  x_E \\
  x_N
\end{pmatrix},
$$

(2)

and

$$
n(x_E, x_N) = (G M_\ast)^{1/2} (\alpha^2 + \beta^2 (\cos i)^{-1})^{-3/4}
$$

(3)

is the mean motion as a function of position. We solve for these three parameters ($i$, $\Omega$, $M_\ast$), assuming the planets are on non-interacting Keplerian orbits, each of which only feels the mass of the central star. We neglect the errors on $x_E$ and $x_N$, but calculate $\chi^2$ values using $v_E$ and $v_N$, and their associated measurement errors, for all three planets (Table 1; 6 data points). Solutions are reported in Table 2. In model A, which is the nominal model, we fix the parameters to serve as a
baseline. In model B, we require face-on orbits, but let $M_\star$ float, the result being not far from the nominal stellar mass. In model C, we fix $M_\star = 1.5 M_\odot$, but let the orientation float. The orbits depart from face-on by $\sim 20^\circ$. In model D, we let all three parameters float, but penalized unreasonable stellar masses by including $[(M_\star - 1.5)/0.3]^2$ in $\chi^2$, as the stellar mass is independently measured. In model E, all three parameters float with no such mass penalty. The orientation-dependence of $\chi^2$ is shown in Figure 3 and the mass-dependence is shown in Figure 4. Interestingly, the best fits are for $M_\star$ much larger than the nominal value 1.5 $\pm$ 0.3 $M_\odot$. This is not surprising because we are introducing line-of-sight offsets and velocities, so a more massive star is needed to make such orbits circular. Figure 5 shows how the velocity vectors of model D falls into the 2-$\sigma$ error bands for each planet. However, the inner two orbits are closer spaced than the nominal model, and the instability is even more rapid: in an integration a close encounter occurred between c and d in only 2500 yr. Similarly, we fit the best orientation for $M_\star$ values between 1.1-3.0 $M_\odot$, spaced by 0.01 $M_\odot$, and integrated those orbits. We terminate each integration when any two planets pass within 1 Hill radius of each other, one is ejected (distance to the star $> 500$ AU with positive energy), or the system lasts 160 Myr. No three-planet systems generated in that way were stable for more than $1.5 \times 10^5$ yr. Therefore we find that more careful fits to the data, under the hypothesis of circular coplanar orbits, do not simply lead to a stable solution.

### 3. Much Lower Planetary Masses?

In three-body systems, conservation of total energy and angular momentum puts constraints on the possible motions [Marchal & Bozis 1982]. Applied to two-planet systems, we may define Hill stability as a constraint that the planet that is initially closer to the star stays closer to the star for all time. When the criterion for Hill stability is satisfied, a close encounter between the planets is prohibited (although escape of the outer planet to infinity, or the collision of the inner planet with the star, is not forbidden). Define $\Delta$ as the planets’ difference in semi-major axes in terms of mutual Hill radii: $R_H \equiv \frac{1}{2}(a_{in} + a_{out})\epsilon$, with $\epsilon \equiv \left\{m_{in} + m_{out}/(3M_\star)\right\}^{1/3}$. Gladman (1993) gave the Hill stability criterion as:

$$\Delta > \Delta_{crit} \equiv 2\sqrt{3}[1 + 3^{1/2}2 - \left(\frac{11m_{in} + 7m_{out}}{18M_\star}\right)3^{2/3}\epsilon^{-2} + ...].$$

By integrating systems near the boundary, Gladman found that if the planets initially have small eccentricity (radial excursions comparable or less than a Hill radius), then violating Hill’s stability criterion was necessary and sufficient for instability. This conclusion does not hold for planets in resonance with one another.

Evaluating these numbers using the nominal system with nominal masses 7, 10, 10 $M_{Jup}$, we have $\Delta_{cd} = 2.68$ and $\Delta_{crit,cd} = 4.03$ for the inner two, and $\Delta_{bc} = 3.69$ and $\Delta_{crit,bc} = 3.98$ for the outer two. Apparently both
sub-systems fail to satisfy the Hill stability criterion.

We perform numerical simulations to (1) confirm Hill stability when the criterion is satisfied and (2) find the timescale for instability when it is not satisfied. We are helped by the long orbital periods and short system age (only $\sim 10^9$ dynamical times), which allows suites of integrations to be rather inexpensive. First we survey the instability near the nominal orbits ("A"). Let us focus on the inner sub-system (c-d), as it is farther from stability, and ask the question: by what factor must we multiply the nominal masses for stability over 30 Myr? In Figure 6 we plot the instability timescale of the three-planet system, with each of the three planetary masses scaled by a common factor. Instability indeed occurs even if the sub-systems are initially Hill stable. In our case, the masses must be lower than about 1/5 of the nominal masses to remain stable 30 Myr. The implied upper limits of masses — 1.5, 2, and 2 $M_{\text{Jup}}$ — are then incompatible with any cooling model at ages greater than 30 Myr, even extreme hot-start models.

4. NON-CIRCULAR ORBITS?

In the previous section, we found that face-on, circular orbits could only be stable if the planetary masses were implausibly low. In this section, we choose the lowest planetary masses that are compatible with hot-start models, and we choose a non-circular orbit for planet d (its orbit is currently unconstrained by observations). We seek systems that remain stable until the lower limit on the stellar age of 30 Myr.

We first simulate the inner two planets, each of 7 $M_{\text{Jup}}$, in the absence of planet b. They are given coplanar orbits, with planet c on a circular orbit at $a_c = 37.97$ AU. The initial longitudinal separation is given by the observed positions, assuming face-on orbits ($\lambda_c - \lambda_d = 117^\circ$). We scan over a grid of semi-major axes for the inner planet. For $a_d < 24.44$ AU, $e_d$ is chosen so that aphelion is at 24.44 AU, and for $a_d > 24.44$ AU, $e_d$ is chosen so that perihelion is at 24.44 AU. These choices maximize the chance that the two planet system will be stable, while matching the constraint of the currently-observed separations from the star. We plot the instability times in Figure 7 as vertical lines. We repeat this calculation with planet b present with its nominal orbital elements — $a_b = 67.91$ AU, $e_b = 0$, $\lambda_b - \lambda_d = 105^\circ$ — and

![Figure 4](image1.png)

**Fig. 4.** — Model $\chi^2$ as a function of stellar mass, minimizing over system orientation ($i$ and $\Omega$) of circular, coplanar models. Masses above the nominal mass $1.5 M_{\odot}$ are preferred. The different crosses, each assigned a letter, are different solutions as given in Table 2.

![Figure 5](image2.png)

**Fig. 5.** — Observed sky-projected positions and velocities for the three planets, along with the velocity predictions of the D model of a circular, coplanar system.
that mensurate with planet c’s, and the observations require a requires that position. Currently, we observe than a coplanar and circular model would predict for planet d: the magnitude of its velocity should be smaller observed separation from the star. This may be verified planet b in which the resonant angle, between c and d, we search for a system near the center greater stability in the vicinity of the 2:1 resonance be-

\[ \lambda_c - \lambda_d \approx 0^\circ, \quad (5) \]

librates with small amplitude around 0\(^\circ\). Resonance requires that \( a_d \) is low enough for planet d’s period to com-

mensurate with planet c’s, and the observations require that \( e_d \) is high enough for planet d to reach its currently observed separation from the star. This may be verified in a few years’ time by measuring the displacement of planet d: the magnitude of its velocity should be smaller than a coplanar and circular model would predict for that position. Currently, we observe \( \lambda_c - \lambda_d \approx 116^\circ \), so

\[ \phi_d \approx 0^\circ \] implies \( \lambda_d - \omega_d \approx 128^\circ \). Thus we find small libra-

tion compatible with planet d being closer to apoapse at the current time, in which case the velocity should have a component away from the star. Integration with these initial conditions for planets c and d, in the absence of b, indeed shows libration and long-term stability (at least 160 Myr), for a wide range of initial \( a_d \) and \( e_d \) values.

In such solutions, the resonant angle for planet c does not librate. The resonance involves only the eccentricity of planet d. When planet b is added, planets b and c excite each other’s eccentricities and cause the amplitude of the resonant argument to fluctuate. Sometimes these exci-

ced eccentricities cause an encounter between b and c; sometimes the loss of libration in \( \phi_d \) allows an encounter between c and d. In Figure 7 we show an example of this instability, where planets c and b start in their nominal orbits, \( a_d = 23.32 \text{ AU}, e_d = 0.09, \phi_d = 0^\circ \), and all bodies have their nominal masses. Panel (a) shows the range of motion of each orbit versus time, panel (b) shows the resonant argument versus time, and the bottom panels show brief segments (~ 3 \times 10^4 \text{ yr}, labeled in panel b) of the motion of the resonant argument through phase space. Over such brief intervals, the libration amplitude holds rather steady, except at the very end of the integration. In this example, the instability causes an encounter between planets c and d at 7.5 Myr.

Compared to the no-resonance cases, this system showed considerable longevity. Given that the mecha-

nism for its eventual disruption was kicks between plan-

et b and c, systems with smaller masses should be more stable. In fact, Figure 7 shows four integrations in which the resonance allows planets of masses 5, 7, and 7 \( M_\text{Jup} \) to be stable for 30 Myr, which is consistent with the observed system. In Figure 10 we plot the time to in-

Fig. 7.— Positions of the planets, every \( \sim 3 \times 10^5 \text{ yr} \), in the numerical integration with \( a_d = 0.95(24.44 \text{AU}) = 23.22 \text{AU} \) from Fig. 7. The rotating coordinates are centered on the star with planet c on the positive horizontal axis. The circle is a distance from the star of 24.44 AU. When the inner planet lags the middle planet by \( \simeq 117^\circ \), its distance from the star is \( \simeq 24.44 \text{ AU} \), but when it reaches the middle planet’s longitude, it is always closer to the star. Planets are labeled near their currently-observed positions relative to one another. The 2:1 mean motion resonance protects the two planets from close encounters.

Fig. 8.— Instability time of coplanar systems. Lines: based on planets c and d (in the absence of planet b) from the nominal model, but choosing a non-zero eccentricity for planet d to satisfy its currently-observed distance from the star. Gray region: same as before, but in the presence of planet b with its nominal parameters. Planetary masses are taken to be 5, 7, and 7 \( M_\text{Jup} \) for b, c, and d respectively. A few three-planet systems last 30 Myr (dotted line), which is the minimum requirement, and these correspond to a 2:1 mean motion resonance between planets c and d. Current separations between the star and the planets are labeled \( s_d \) and \( s_c \).

with mass 5 \( M_\text{Jup} \), and plot those instability timescales in Figure 7 as a gray region.

We observe that a very narrow range of \( a_d \) is compatible with both the observed astrometry of the planets and with dynamical stability. The presence of the third planet narrows this range still further. The center of this range corresponds with the 2:1 mean motion resonance between planets c and d. (The position is offset from the location \( a_d = (1/2)^{2/3} a_c \) because the large mass ratios induce fast precession.) In Figure 8 we show how this resonance protects the planets from close encounters.

We ran identical simulations with planetary masses of 7, 10, and 10 \( M_\text{Jup} \), and found qualitatively similar results, except the most stable three-planet simulation lasted only 10 Myr. Thus it seems that even with resonan-

ce protection between the inner two planets, the nomi-

nal masses are slightly too high, and we must appeal to a younger age for the system. A younger system age sometimes the loss of libration in \( \phi_d \) allows an encounter between b and c; c, so they perturb each other less.

5. MEAN MOTION RESONANCE

Inspired by the fact that Figure 7 shows a region of greater stability in the vicinity of the 2:1 resonance between c and d, we search for a system near the center of the resonance. We find a solution in the absence of planet b in which the resonant angle,

\[ \phi_d = 2\lambda_c - \lambda_d - \omega_d, \]

allows an encounter between c and d. In Figure 7 we show an example of this instability, where planets c and b start in their nominal orbits, \( a_d = 23.32 \text{ AU}, e_d = 0.09, \phi_d = 0^\circ \), and all bodies have their nominal masses. Panel (a) shows the range of motion of each orbit versus time, panel (b) shows the resonant argument versus time, and the bottom panels show brief segments (~ 3 \times 10^4 \text{ yr}, labeled in panel b) of the motion of the resonant argument through phase space. Over such brief intervals, the libration amplitude holds rather steady, except at the very end of the integration. In this example, the instability causes an encounter between planets c and d at 7.5 Myr.

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et b and c, systems with smaller masses should be more stable. In fact, Figure 7 shows four integrations in which the resonance allows planets of masses 5, 7, and 7 \( M_\text{Jup} \) to be stable for 30 Myr, which is consistent with the observed system. In Figure 10 we plot the time to in-

\[ a_d / (24.44 \text{AU}) \]

\[ a_d / (24.44 \text{AU}) \]

\[ a_d / (24.44 \text{AU}) \]

\[ a_d / (24.44 \text{AU}) \]

\[ a_d / (24.44 \text{AU}) \]

\[ a_d / (24.44 \text{AU}) \]
stability of these resonant models versus a common mass scaling for all three planets. We find that both the resonance must be active and the planetary masses should be slightly lower than their nominal values.

At this point, we have found a way to calm the strongest interactions—those that cause instability on a few thousand orbits: a resonance between planets c and d that protects them from close encounters. This protects the system until the somewhat longer timescale interactions between b and c cause an instability. But those interactions can also be suppressed by postulating yet another resonance. We integrated the nominal masses with initial conditions as above except \( a_d = 23.42 \) AU instead of 23.32 AU. The resulting system showed resonance protection between planets b and c. The 2:1 resonance is active—this is possible far from its nominal location because the pericenters are precessing on nearly orbital timescales. In Figure 11, we show this system lasting for 160 Myr. In this example, the resonant argument \( \phi_c = 2\lambda_b - \lambda_c - \varpi_c \) is nearly librating, and \( \phi_b = 2\lambda_b - \lambda_c - \varpi_c \) is also affected. The angle \( \phi_d \) is librating with small amplitude the whole time (panels b and c). We verified that this solution, with resonances in both the inner and outer sub-systems, fits the astrometric data of Table 1.

In other test cases, we have observed three-body resonances, such as the Laplace resonance, with the angle \( \phi_L = \lambda_d - 3\lambda_c + 2\lambda_b \) librating temporarily. Such solutions are also consistent with the astrometric data. If verified, the planets of HR 8799 represent the first example of a three-body resonance in a planetary system, as opposed to a satellite system.
Our integrations show that a doubly-resonant system can be stable for 160 Myr, even for the nominal planetary masses. We are led to the question: how massive could the planets be in such a configuration, remaining stable for the estimated system age? We have not done an exhaustive search, but we have found an example in which the masses of the planets are each scaled up by a factor of 1.9 from the nominal model. That is, if this doubly-resonant configuration is correct, the planets could even have the masses of brown dwarfs. We find it remarkable that without resonance, the masses would be inferred to be no more than a few times Jupiter’s mass, but with the resonance, they might be even too massive to be called planets.

6. DISCUSSION

We have investigated the orbital stability of the newly-imaged planetary system HR 8799. The nominal orbital model and masses are not stable, so we tried the following alternatives.

• The system is not face-on, as in the nominal orbital model. No model with circular, coplanar orbits, that also fits the astrometry well, is stable.

• The planet’s masses are lower than calculated. This can happen if the cooling models underestimate the luminosity, though that is difficult to understand, as even hot-start models cannot produce the observed luminosity at such low masses ($\lesssim 2 M_{\text{Jup}}$). Such masses would be plausible if the system is younger than expected, but we would have to discard several astrophysical age estimates (Marois et al. 2008).

• The inner planets are in 2:1 resonance. This is our favored solution. It requires that the inner planet has a smaller semi-major axis than its current position. That is, it must have an eccentricity above 0.04, and be closer to apastron now. This configuration remains stable in the perturbing presence of planet b only if the masses of planets b and c are $\lesssim 10 M_{\text{Jup}}$. This solution predicts that the velocity of planet d, which should be measured in a few years, will prove to be smaller than the nominal face-on circular model predicts, and it should also have a component directed away from the star, if the resonant argument $\phi_d$ is librating with small amplitude about 0°.

• The outer planets are also in 2:1 resonance. This solution, which actually fits all the current data for the system, would be the first example of an exoplanetary system with multiple resonances active. If it is true, then the planetary masses could be at least 1.9 times bigger than their nominal values without violating stability constraints. It will be very interesting to find a test of this hypothesis.

This study brings up several issues for future observations of HR 8799, and directly-imaged multiplanet systems in general, as follows.

First, it serves as the first test of hot-start cooling models for exoplanets, which barely pass the test. We hope more detailed dynamical studies of this system will sharpen this test as more data are collected. If the doubly-resonant configuration is verified, it would considerably weaken this test. We have not yet used dust observations as a dynamical constraint. The spectral energy distribution reveals a massive debris disk surrounding the planetary system, with an orbital radius of $\gtrsim 66$ AU (Williams & Andrews 2006). Given that planet b is observed at $a_b \gtrsim 68$ AU, we expect that the inner edge of the debris disk must be $\gtrsim 90$ AU, and that future measurements and modeling will find that orbital radius to be plausible and even preferred. Such a model could in turn serve as a complementary test of planet b’s mass, in analogy to the test of the mass of Fomalhaut b (Chiang et al. 2008). Perhaps other directly-imaged systems will fortuitously arrange for complementary tests.

Second, we found evidence of a mean motion resonance at very large orbital separations, much further than those found by the radial velocity technique. (Correia et al. 2005) similarly proposed a 5:1 resonance in a system discovered by radial velocity, based on stability arguments. It has been suggested that the dissipation of the gas disk may trigger a rapid outward migration of planets formed closer in (Veras & Armitage 2004); if the inner planet catches up to a resonance of the outer planet, they would trap into resonance. We verified that if planets d and c were initially placed in circular orbits exterior to the 2:1 resonance, they are stable to collisions or ejections for $\gtrsim 30$ Myr, so getting into the resonance without first becoming unstable is not a problem in this case. The real difficulty with differential migration is that any additional migration, after the resonance is reached, efficiently increases the eccentricities. That this requires implausible fine-tuning has been discussed for the 2:1-resonant GJ876 system (Lee & Peale 2002), and the situation is even worse for this system, as any additional migration would move towards the dynamically near-by outer planet, and even moderate eccentricities would destabilize the three-planet system. If it is true that both sub-systems are in resonance, the fine-tuning problem may be even more problematic. We also showed how the perturbations by a third planet tend to disrupt a mean-motion resonance. This mechanism adds to a growing list of ways to disrupt resonances among planets, including turbulent fluctuations in a protoplanetary disk (Adams et al. 2008), tidal dissipation (Terquem & Papaloizou 2007), and scattering of planetesimals (Murray-Clay & Chiang 2006; Morbidelli et al. 2007).

Third, it is highly surprising that the first-reported detection of a directly-imaged multiplanet system would require dynamical stability arguments to correctly solve its orbits. This lends credence to the packed planetary system hypothesis (Barnes et al. 2008). However, planets that are discovered by direct imaging of their self-luminosity are biased to have high masses, and the stability of putative orbits of multiplanet systems discovered this way could frequently be in doubt. The bias towards large angular separation implies very long orbital periods will be common for such discoveries, and we foresee many stability analyses predicated on only the sky-projected positions and velocity vectors of those planets. We hope this paper proves to be a useful example of how to conduct such an analysis.
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