On the Proposals of Lorentz Invariance Violation Resulting from a Quantum-Gravitational Granularity of Space-time

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Abstract. We explore the possibility of the Lorentz Invariance violation that can come out from a quantum theory of gravity. We concentrate particularly in the combination of the known elementary particle interactions with a Planck-Scale preferred frame, the specific calculation lead to the conclusion that the effects of the Preferred Frame Granularity of Space-time would result in the very large effects. Finally we conclude that the enthusiasm for improved searches for Lorentz violation should be dampened, given the existing unsuccessful searches.

1. Introduction
Regardless of one’s preferences regarding the program to search for a quantum theory of gravitation, one of the main differences with other branches of physics has always been the clear theoretical expectation that no clues were to be expected to come from the empirical realm. Recently, motivated in part by speculations inspired on the different theoretical approaches to the subject, it has been suggested that quantum gravity could become manifest through slight deviations from Lorentz invariance [1]. The detailed form of these effects depends not only on the underlying approach (i.e. String Theory or Loop Quantum Gravity, to name the most popular two), but also on the particular form of the model. This has clearly serve to energize the experimental programs that have been ongoing, motivated on various grounds, to test Lorentz Invariance [2]. Quite remarkably the precision that has been achieved in many of the relevant phenomenological programs, ranging from very high energy particles to high precision laboratory experiments already call into question the specific form and magnitude of the sought for quantum gravity effects. In fact these studies have resulted in very tight bounds[3] on parameters that, although expected to be of order 1 in the absence of unexpected suppressions, can not been evaluated exactly. This drawback is a reflection of the fact that, despite their increasing sophistication, the current theoretical frameworks must still be regarded as heuristic constructs rather than detailed theoretical predictions. There is however, one common aspect of these ideas: the occurrence of a preferred reference frame associated with some sort of granularity of space-time resulting from the quantum gravity phenomena. The last feature could be seen to be associated, for instance, to the length the
strings of String theory or the nature of the fundamental geometric excitations in Loop quantum gravity.

The other feature, the preferential frame has been identified with the only such object that seems to be globally singled out in our Universe: The one selected by the Cosmic Microwave background. The search for such breakdown of Lorentz Invariance can thus be identified with the dependence of the laws of physics with the state of motion of the system in question with respect to that frame.

Therefore we might regard the existing program to search for the proposed L.I.V., as a modern day version of the eventually unsuccessful search for the Earth’s motion on the hypothetical Electromagnetic Ether. However in contrast to what was needed to search for the Ether, the current idea for experimental tests of these questions often calls for the study of particles with extremely high energies; cosmic rays, high energy gamma ray burst of cosmic origin, etc, because of the fact that the energy scale of ordinary physical phenomena is many orders of magnitude smaller than the Planck scale, the natural scale that would determine the situations in which those effects would presumably become large. In fact, many of the proponents of these ideas have argued that the effects one is looking for, would naturally be suppressed by powers of $E/M_{\text{Planck}}$. The simplest scenarios, and the ones that are already in deep conflict with experiments call for suppressions by $(E/M_{\text{Planck}})^{1}$, while more recent proposals, call for effects suppressed by $(E/M_{\text{Planck}})^{2}$. It is easy to see where this is leading.

In a recent work by J. Collins, A. Perez, L. Urrutia and H. Vucetich in collaboration with one of the authors [4], it was shown how the consideration of radiative corrections would in this context lead to the emergence of Lorentz violating effects that are quite large. The effects can be thought to be the result of the fact that although very high energy particles are in principle hard to come by, they do occur as virtual entities, contributing to all ordinary processes.

In fact quantum field theory (QFT), one of the best developed trusted and tested theories of the XXth century, teaches us that any process that is experimentally observed has contributions in which the intermediate, unobserved situations, involve particles of all energies and momenta, including energies arbitrarily higher than those of the particles present in initial and final stages. In particular it is noted that if ordinary theories are to be regarded as effective theories valid for particles that have energies and momenta, that in the preferred frame, are small compared with the Planck Scale, one needs, for the sake of consistency, to demand a cut-off of the four momenta relative to the preferential rest frame of the virtual particles appearing in any process. What we have found is that such frame specific cut-off would results in effects that can not be absorbed in the original terms of low energy theory. Moreover as could be expected these terms would be associated with a violation of Lorentz invariance, which as we shall see would have to be of an intensity that would have observable consequences which are ruled out even by low precision and quite old experiments.

Thus we will argue, that the ideas tied to space-time granularity, associated with a preferential frame, together with even the low precision tests of Lorentz invariance, (i.e. tests at the 1% precision or higher), are at odds with our current understanding of nature.

This article is organized as follows. In section 1 we will give a brief discussion of the expected phenomenology. In section 2 we will describe how the considerations of interactions, treated within the field theory theoretical scheme, lead to the conclusion that the effects of the Preferred Frame Granularity of Space-time would result in the very large effects mentioned above. The treatment provides an indication of the robustness of the analysis as we consider a slightly more general, and arguably more symmetric frame dependent cut-off than the one considered in [4].

In Section 3 we will discuss our results in light of other recent relevant works on the subject. References have been kept to a minimum due to space restrictions, and we appologize from the outset to all those colleagues whose work should have been cited here and has not. We refer the reader to the references in cited works for more complete account of the development of the
2. Quantum Gravity Effects on the Propagation of Free Matter Fields

The idea that quantum gravity would be associated with a granular structure with characteristic length scale $l_P$ has been used generically to argue that such structure would result in a breakdown of Lorentz invariance. This idea has been worked out in more detailed, for the most popular of the theoretical frameworks considered for constructing quantum gravity theories. Specific calculations in do in fact find preferred-frame effects associated with space-time granularity in the two most popular contenders for a theory of quantum gravity, which are string theory [5] and loop quantum gravity [6]. In these scenarios, the preferred frame and the consequent Lorentz violation occur even though the fundamental classical equations of both of the theories are locally Lorentz invariant.

In general the effect is to change the dispersion relation for particles propagating in vacuum is then given by

$$E^2 = |\vec{P}|^2 + m^2 + \xi E^3 / M_{\text{Planck}}$$

where $E$ and $\vec{P}$ are the energy and momentum of the particle in the preferential frame, $m$, is the particle’s mass and the $\xi$ is a dimensionless parameter that could depend on the particle species and or its helicity, and was expected apriori to be of order 1. We can write this modified dispersion relations in a covariant looking way by introducing the four velocity of the preferential frame $W^\mu$. The above equation then reads

$$P^\mu P_\mu = m^2 + \xi (W^\mu P_\mu)^3 / M_{\text{Planck}}$$

This has been the basis for the so called Quantum gravity Phenomenology, which has endeavored to set bounds on these parameters for various types of particles, and which has been a rather productive enterprise [7].

3. Effects of interactions

In all previous work on the field one consider as natural to assume that Lorentz violation at the Planck-scale would only cause extremely small macroscopic effects. This seems natural and in agreement with as our experience indicating that the details of physical phenomena on one distance scale do not directly manifest themselves in physics on much larger scales. However, in quantum field theories like the standard model, such decoupling of short-distance from long-distance phenomena is quite non-trivial [8]. The propagation of an isolated particle has contributions from Feynman diagrams containing “virtual particles”, and where all energies scales contribute entirely without suppression. When we work with the standard model by itself, i.e., ignoring gravity, as in almost all successful standard model phenomenology, there are indeed large contributions by high-energy virtual particles to low-energy processes. But [8] this does not give directly observable effects: The large effects of short-distance physics are equivalent to a “renormalization” of the parameters of the standard model. For example, the measured charge of the electron is not numerically equal to the parameter of the same name in the equations of the theory. When considering the type of situation we have been discussing, we must note that case we these virtual particles will explore all the possible energies and thus become affected by the space-time granularity (as well as by other possible quantum gravity effects) and will transfer these effects to the low energy particles that are the subject of direct observations. What would be the magnitude of these effects?

We addressed this issue by considering the simple case of Yukawa theory, the theory of a massive fermion field $\phi$ interacting with a scalar field $\Psi$ via the Yukawa term. This theory has the
advantage of simplicity together with the fact that represents one sector within the standard model. The lagrangian for this theory is

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \eta^{\mu \nu} - m_0^2 \phi^2 + \bar{\Psi} (i \gamma^\mu \partial_\mu - M_0) \Psi + \lambda_0 \phi \bar{\Psi} \Psi$$

(3)

where $m_0$ and $M_0$ are the bare masses of the scalar and fermion field respectively, and $\lambda_0$ is the bare coupling constant.

When considering the effects of space-time granularity on the propagation of free fields we need to take into account two facts: 1) The dispersion relations would be changed, and 2) the particles whose wave length, as measured in the frame associated the granular structure, (i.e. the frame where the granularity has length scale given by $l_{\text{Planck}}$) is shorter than the granularity scale, would not exist. These effects would be represented by the following changes in the propagator of the fermions

$$\frac{-i}{\gamma^\mu p_\mu - M_0 + i\epsilon} \rightarrow \frac{-i f \left( \frac{p^2}{\Lambda^2} \right) g \left( \frac{p_0^2}{\Lambda^2} \right)}{\gamma^\mu p_\mu - M_0 + \Delta \left( |p| \right) + i\epsilon}$$

(4)

where $\Delta$ again codifies the change in the dispersion relations and the cutoff functions $f$ and $g$ eliminate virtual as well as real particles with trans-plankian energy or momenta. The cutoff scale $\Lambda$ would normally be thought to be close to the Planck Mass scale, and the magnitude of the tree momenta, and energy is evaluated in the preferred frame, so $|p| = \sqrt{(\eta_{\mu\nu} - W_\mu W_\nu)p^\mu p^\nu}$ and $p_0 = p^\mu W_\mu$. The cut-off functions $f(x)$ and $g(x)$ are required to go to 1 as $x \rightarrow 0$ to recover low energy phenomenology, and to vanish as $x \rightarrow \infty$ to cut-off trans-plankian particles. There are similar changes in the bare scalar propagator.

The explicit analysis in [4] used a simple spatial momentum the cut-off, but the general analysis is expected to be very robust in the sense that one would expect other cut-off schemes to produce quite similar results. The reason for that is that there exist no compact region in momentum space which is invariant under the Lorentz group, and thus restriction of the virtual particles momentum to such compact region would be expected to yield the same type of Lorentz Violation effects we have encountered in the previous section. However, it is worthwhile to check this expectation more explicitly, and we do so here in a way motivated by a discussion with a colleague [10]. We consider a more symmetric form of the cut-off which treats the time component of the four momenta in a less unequal footing in comparison with the way the other components are treated. To do this we introduce a cutoff on the magnitude of the particle energy (as seen in the preferential frame), something that could be thought to be associates with a temporal discreteness in the space-time.

We are now interested in the effects of virtual processes, i.e. radiative corrections. We concentrate in the scalar self energy $\Pi(p)$. This quantity is the sum of all graphs with two external scalar lines and which can not be made disconnected by cutting a single internal line. The lowest order contribution corresponds to the virtual decay of the scalar into two fermions followed by their recombination into a scalar particle. We computed this quantity and following standard theorems showing that the would be divergences (in the absence of a cut-off) are confined to degree two polynomial in the momenta we expanded it about $p = 0$ to obtain

$$\Pi(p) = A + B p^2 + \Xi (p^\mu W_\mu)^2 + \Pi^{LI}(p) + O(p^4/\Lambda^2)$$

(5)

The interpretation of this result is the following: $A$ and $B$ are the standard mass and wave function renormalization constants, $\Pi^{LI}(p)$ is a standard cut off independent, and Lorentz invariant standard self energy term. The interesting term for us the term $\Xi (p^\mu W_\mu)^2$ which corresponds to the generated term in the lagrangian $(\Xi/2) \partial_\mu \phi \partial_\nu \phi W^\mu W^\nu$, and thus to a renormalization of the metric $\eta^{\mu\nu} \rightarrow g_{\text{Ren}}^{\mu\nu} = \eta^{\mu\nu} + \Xi W^\mu W^\nu$. 


In fact a direct calculation yields (after renormalization):

\[
\Xi = \frac{\lambda^2}{3\pi^3} \left[ \int drdy \frac{r^2 \left( 3f^2 (g/2) + g^2 (f^2) - 4y^2 f^2 (g')^2 \right)}{(y^2 - r^2 + i\beta)} + \int drdy \frac{2r^2 \left( r^2 f^2 (g/2) - (1/3)y^2 g^2 (f^2) \right)}{(y^2 - r^2 + i\beta)^2} \right]
\]  

(6)

where de arguments of \( g \) and \( f \) are \( y^2 \) and \( r^2 \), respectively. The noteworthy fact is that \( \Xi \) is independent of the cut-off scale \( \Lambda \). In the case where there is only a spatial momentum cut-off, corresponding to setting \( g \equiv 1 \) we obtain a result similar to the one of [4] and dependent on the coupling constant. Thus particles with different couplings would effectively propagate on different renormalised space-time metrics.

\[
\Xi = \frac{2\lambda^2}{9\pi^2} \left[ 1 + \int_0^\infty x f'(x)^2 dx \right]
\]  

(7)

We note that in this case the relevant quantity is positive definite, bounded from below by \( \frac{2\lambda^2}{9\pi^2} \). It is of course not our aim here to show that in the general case there exist no choice that would make the result vanish. Having the freedom to select two functions would now seem to open the possibility to make an extremely finned tuned choice. Our general aim is to explicitly show that no natural choice seems to emerge at this point, and that therefore it would be a small miracle if once such a specific choice is made, that the result would carry on to more complex diagrams contributing to the self energy, or to other irreducible n-point functions. However a rather natural case can be considered explicitly: The case in which the two cut-off functions \( f \) and \( g \) have the same functional form given by

\[
\begin{align*}
    f (r^2, a) &= 1 - \left[ 1 + \exp \left( -\frac{r^2 - 1}{a} \right) \right] \\
g (y^2, b) &= 1 - \left[ 1 + \exp \left( -\frac{y^2 - 1}{b} \right) \right]
\end{align*}
\]  

(8)

and where we leave the parameters \( a \) and \( b \) controlling the cut-off rate as independent. In this case we find, writing the quantity of interest \( \Xi = \Re (\Xi) + i\Im (\Xi) \)

\[
\Re (\Xi) = -\frac{\lambda^2}{9\pi^3} \int_0^\infty dr \int_0^\infty dy \left( f_a' \right)^2 g_b^2 \\
+ \frac{4\lambda^2}{3\pi^3} P \left( \int_0^\infty dr \int_0^\infty dy \frac{1}{(y^2 - r^2)^2} \left\{ f_a^2 \left[ 3r^2 (g_b^2)' - 2y^2 (g_b^2) \right] \\
+ r^4 (g_b^2)' (f_a^2)' + \frac{r^2 g_b^2}{3} \left[ (f_a^2)' - 2 (f_a^2) - y^2 (f_a^2)'' \right] \right\} \right)
\]  

(9)

and

\[
\Im (\Xi) = -\frac{\lambda^2}{3\pi^2} \int_0^\infty dx \left\{ f_a^2 \left[ 3 (f_b^2)' - 2 (f_b^2) \right] + x (f_b^2)' (f_a^2)' + \frac{f_b^2}{3} \left[ (f_a^2)' - 2 (f_a^2) - x (f_a^2)'' \right] \right\}
\]  

(10)

here the arguments of \( g_b \) and \( f_a \) are \( y^2 \) and \( r^2 \) respectively, the subscripts label the parameter of the function and finally, the argument for the \( f \) functions in the last equation is \( x \). We now
can naturally demand that the unitarity of evolution is not disrupted by the Quantum Gravity effects at hand. This would require at this order of the analysis the fixing of a specific value of \( b \) for every choice of \( a \) and this choice does in general lead to a specific value for the real part. We have investigated this issue numerically and find that in the result is a non-zero value of the real part given by:

\[
\Xi = Z^2
\]

(11)

Where \( Z \) is in the range \((-0.23, -0.30)\) for \( a \) in the range \((0.10, 0.30)\). Thus, in the more general scheme contemplated here, we find, as expected large unsuppressed Lorentz invariance violations as a result of the impact of interactions and the contribution of virtual particles to all ordinary processes as indicated by QFT. The main observable effects at low energies is thus to change the value, the maximum speed of propagation in the dispersion relation. The known widely different, couplings of different elementary fields imply that different fields have different values of \( c \), with fractional differences of the order of typical one-loop corrections in the standard model, around 0.1% to 10%. This is completely incompatible with the observed limits, which say that \( c \) is constant at a fractional level well below \( 10^{-20} \) [9, 3].

One could hope that other effects, arising also from the underlying quantum gravitational nature of space-time would cancel the effect we are considering, and in the absence of a precise theoretical description of such object it is impossible to prove that this might not happen, however barring a yet undiscovered principle, such cancellation would require a precise and unnatural fine tuning i.e. a seemingly miraculous and unexplained coincidence.

4. Discussion

The implications of our argument for both experiment and theory are quite profound. First, enthusiasm for improved searches for Lorentz violation should be dampened; given the stated motivations, the existing unsuccessful searches suffice by many orders of magnitude. Of course, it almost goes without saying that it is correct science to question and test accepted principles; on these grounds, tests of Lorentz invariance are worthwhile. But the enthusiasm has been engendered by arguments providing estimates of specific small orders of magnitude for Lorentz violation. The arguments have been shown here to have very serious problems in light of what is known about quantum field theory.

Some more comments are in order here: T. Jacobson likes to stress that given the fact that the group of Lorentz transformations is non compact is a good justification for exploring the possibility if its breakdown at some suitable large value of the boost factor.

Our analysis can be considered as the counterpart of this argument: As the set of four momenta of the virtual particles would be non-compact in any Lorentz invariant field theory, the introduction of any physical cut-off (such as one associated with space-time granularity), would result in a boundary in the momentum space of the virtual particles, and “such boundary would be observable from any point”, i.e. the effects would permeate to the observable processes at all energies. The latter results in a Lorentz violation that is independent of the value of the cut-off which in our context would be the Plank Mass or something close to it. Thus the effect is unsuppressed and would be so large as to become a blatant and not at all subtle violation of Lorentz Invariance. This latest point can be thought as the momentum space analog of Obler’s paradox regarding the brightness of the night’s sky (as stated within the Newtonian physical world view): A finite universe, no matter how large, would be different from an infinite universe no matter how sparsely populated by stars. In the alter the night would be as bright as the day, while in the former that would not be the case. The presence of the boundary makes a qualitative difference that could be perceived at any point no matter how distant it if from such boundary.

Regarding theoretical issues in this direction, if one is to keep the notion that quantum gravity is associated with discrete structures that define a preferential frame, the critical task would be
to find and implement a mechanism to give automatic local Lorentz invariance at low energies, despite a violation at the Planck scale. We assume here that the treatment involves real time, not an analytic continuation to imaginary time, as is common in treatments of quantum field theories in flat space-time.

We must of course concede that it is possible that some mechanism could be device to protect the Lorentz Invariance at ordinary energies, but such mechanisms in QFT are thought to be possible only when they are the result of a custodial symmetry. In fact in a recent paper [11] it has been argued that Supersymmetry could play precisely such custodial role. But the Supersymmetry group includes the Poincare group, so it seems very problematic to consider such extended symmetry in the context of Lorentz Violation. In fact, argues the author, one needs only the translation subgroup the Poincare group to provide the supersymmetric custodial mechanism. However this does not help much in the direction we would want, as it is precisely the translation symmetry, the one that is more obviously disrupted by any sort of fundamental granularity of space-time. So at this time no mechanism with the desired features appears to be known.

The breakdown of normal space-time, with the Planck length being a minimum measurable length, does indeed open the question as to whether Lorentz invariance is only an approximate symmetry valid only for relatively low-energy phenomena. But as indicated by the work [12, 13] have shown, the existence of a minimum measurable length does not of itself imply that local Lorentz invariance is violated. In particular the last article argues that the situation might be similar to the case rotational symmetry, where it is well known that the discreteness of the eigenvalues of the angular momentum operators does not at all imply a violation of rotational invariance in ordinary quantum mechanics. The point here is that the quantity that does transforms smoothly under a continuous rotation is the expectation value of the angular momentum, rather than the corresponding eigenvalues. There is however a problem with the attempt to carry this solution directly over to our case. It is clear that in order for the expectation value of linear momentum to transform in the satandard way under arbitrary boosts, the corresponding eigenvalues can not be bounded, as they would if there was indeed a spatial physical discreteness of a given length in the preferential frame.

Our conclusion is that the presumed fundamental discreteness, if it exists at all, must have a more subtle effect on low energy physics and cannot select a preferred local notion of Lorentz frame of the kind analyzed here.

On the other hand optimistic point of view must be stressed: a branch of theoretical physics that has been considered for a long time to suffer from detachment from experimental guidance now finds itself in the opposite situation. Because of mechanisms intimately tied to the known ultra-violet divergences in conventional quantum field theories, certain kinds of Planck-scale phenomena, like a preferred frame, manifest themselves suppressed by no powers of energy relative to the Planck energy, but only by two powers of standard model couplings. Lorentz invariance continues to play the powerful role in restricting the kinds of acceptable physical theories.

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[1] Amelino-Camelia, G., Ellis, J.R., Mavromatos, N.E., Nanopoulos, D.V. & Sarkar, S., “Test of quantum gravity from observations of γ-ray bursts”, (1998) Nature 393, 763.
[2] Colladay, D. & Kostelecký, V.A., “Lorentz-violating extension of the standard model”, (1998) Phys. Rev. D 58, 116002 (1998).
[3] R.J.Gleiser and C.N.Kozameh, “Astrophysical limits on quantum gravity motivated birefringence”, (2001) Phys. Rev. D 64, 083007. Jacobson, T., Liberati, S. & Mattingly, D., (2003) “A strong astrophysical constraint on the violation of special relativity by quantum gravity”, Nature 424, 1019. Sudarsky, D.,
Urrutia, L., & Vucetich, H., (2002) “New observational bounds to quantum gravity signals”, Phys. Rev. Lett. 89, 231301.

[4] “Lorentz Invariance in Quantum Gravity: A New finetuning problem? ” J. Collins, A. Perez, D. Sudarsky, L. Urrutia, y H. Vucetich, (1998) Physical Review Letters 93, 191301.

[5] Amelino-Camelia, G., Ellis, J.R., Mavromatos, N.E. & Nanopoulos, D.V., (1997) “Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity”, Int. J. Mod. Phys. A 12, 607. Ellis, J.R., Mavromatos, N.E. & Nanopoulos, D.V., (2000) “A microscopic recoil model for light-cone fluctuations in quantum gravity,” Phys. Rev. D 61, 027503.

[6] Gambini, R. & J. Pullin, J., (1999) “Nonstandard optics from quantum space-time”, Phys. Rev. D 59, 124021. Alfaro, J., Morales-Técotl, H.A. & Urrutia, L.F., (2002) “Quantum gravity and spin-1/2 particle effective dynamics”, Phys. Rev. D 66, 124006.

[7] For a recent and comprehensive review see D. Mattingly gr-qc/0502097.

[8] Low-energy effective theories Weinberg, S., “The Quantum Theory of Fields, Vol. II Modern Applications” (Cambridge Univ. Press, 1996).

[9] Coleman, S.R. & Glashow, S.L., (1999) Phys. Rev. D 59, 116008.

[10] Rafael Sorkin, private communication.

[11] S. G. Nibbelink & M. Pospelov “Lorentz Violation Supersymmetric Field Theories” hep-ph/0404271.

[12] F. Dowker et al., “Quantum Gravity Phenomenology, Lorentz Invariance and Discreteness”, arXiv:gr-qc/0311055.

[13] Rovelli, C. & Speziale, S., (2003) “Reconcile Planck-scale discreteness and the Lorentz-Fitzgerald contraction”, Phys. Rev. D 67, 064019.