Natural fermion mass hierarchy and mixings in family unification

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Abstract

We present an $SU(9)$ model of family unification with three light chiral families, and a natural hierarchy of charged fermion masses and mixings. The existence of singlet right handed neutrinos with masses about two orders of magnitude smaller than the GUT scale, as needed to understand the light neutrinos masses via the seesaw mechanism, is compelling in our model.

1 Introduction

The ideas of family symmetry and family unification has been with us for a while. Grand unified theories lend themselves to construction of such models, but most of the early models did not go as far as considering fermion masses and mixings. We know that there is a five orders of magnitude hierarchy among the charged fermions masses. There is also a two orders of magnitude hierarchy amongst the quark mixing angles. In addition, there are strong suppressions for the flavor changing neutral current processes. With the fairly accurate data on charged fermion masses and quark mixings, we are now in a position to attempt the construction of family symmetry models that include these parameters. The new data from the Tevatron and upcoming LHC will provide further constraints on such family unified models.

We have studied a class of $SU(N)$ family unification models, i.e., models where the families are not due to simple replication of the representation of the first fermion family. Grand unification requires at least an $SU(5)$ gauge group, but the only reasonable choice
(that avoids exotic fermions) of representations for the families are \((10 + \overline{5})_F\), and family unification is impossible. In \(SU(N)\) models, if the fermions all reside in totally antisymmetric irreducible representations (irrep), then there are guaranteed to be no exotic fermions. Such an idea was first proposed by H. Georgi [2], and subsequently used by many authors to build models with three chiral families [3] [4] [5][6]. We write the \(h^{th}\) totally antisymmetric irrep as \([1]^k\). Also in \(SU(N)\) there are two invariant tensors from which we can construct group singlets from the \([1]^k\)s. They are the Kroneker \(\delta^\alpha_\beta\) and the Levi-Civita tensor \(\varepsilon_{\alpha_1\alpha_2...\alpha_N}\) or it’s dual with all upper indices. The indices \(\alpha, \beta\) etc. runs from 1 through \(N\) of \(SU(N)\).

The number of totally antisymmetric irreps in the groups \(SU(6)\) and \(SU(7)\) are too small to arrange the realistic mass and mixing relations of the type to follow. An \(SU(8)\) model of family unification has been proposed by S. Barr [7][8], but we find it possible to arrange a more detailed phenomenology in \(SU(9)\), and this justifies our choice of gauge group.

2 Three families from \(SU(9)\)

In \(SU(9)\) our three family representation is \([1]\)

\[
126 + 84 + 2(36) + 14(\overline{9}).
\]

Note that this assignment is anomaly free.

We can write this in a shorthand notation as

\[
F^5 + F^3 + 2F^2 + 14F^8
\]

Where we have made the replacement \([1]^k\rightarrow F^k\). This can be written more completely as

\[
\mathcal{F} = F_{\alpha\beta\gamma\delta} + F_{\alpha\beta\gamma} + 2(F_{\alpha\beta}) + 14(F_{\alpha})
\]

where symbolically, \(F_{g-n} = \varepsilon_{g}F_{n}\).

Now let us consider the breaking of the \(SU(9)\) gauge symmetry to \(SU(5)\), which can be done most simply with vacuum expectation values (VEVs) for a set of Higgs fundamentals, \(H_i^\alpha, i = 1, 2, \ldots\). Four successive VEV is sufficient to break \(SU(9) \rightarrow SU(5)\). In this case the fermion irreps decompose as

\[
126 \rightarrow 5 + 4(10) + 6(\overline{10}) + 4(\overline{5}) + 1
\]

\[
84 \rightarrow (\overline{10}) + 4(10) + 6(5) + 4(1)\]

\[
36 \rightarrow 10 + 4(5) + 6(1)
\]

and

\[
\overline{9} \rightarrow \overline{5} + 4(1)
\]

Hence the complete set of fermions in the model is

\[
\mathcal{F} = 3(10 + \overline{5})_F + 15(5 + \overline{5})_F + 7(10 + \overline{10})_F + 73(1)_F.
\]

Note that the fermions in \(15(5 + \overline{5})_F + 7(10 + \overline{10})_F + 73(1)_F\) all acquire masses at the unification scale leaving three massless chiral families in \(3(10 + \overline{5})_F\).
3 Fermion Masses and Mixings

The assignment of the three light chiral families in the $SU(9)$ multiplets in our model are as follows.

3rd family: $(\overline{126}_3)_F \to t_L, t_R, b_L; \overline{9}_3 \to b_R$

2nd family: $84_2 \to c_L, c_R, s_L; \overline{9}_2 \to s_R$

1st family: $36_1 \to u_L, u_R, d_L; \overline{9}_1 \to d_R.$

In addition, the Higgs representations that we shall use are various $9_H$s, $36_H$s, and $126_H$s. The charged fermions will receive masses from the Yukawa interactions with the above Higgs multiplets which has electroweak VEVs. Since the top quark has a mass at the EW scale, its Yukawa coupling is of order one. So it is very reasonable to assume that only the top quark has dimension four Yukawa interaction, while the allowed interactions of the lighter quarks and charged leptons are of higher dimensions suppressed by a parameter $\varepsilon$. We will identify this parameter $\varepsilon$ with the ratio of the $SU(5)$ singlet Higgs VEV, $<1>$ and the unification scale, $M$.

3.1 Yukawa interaction for the up sector

Consistent with the $SU(9)$ symmetry, and the $Z_2$ symmetry, the allowed Yukawa interactions for the up sectors are as follows.

Dimension 4:

$$h^u_{33} \varepsilon \varepsilon F^5 F^5 H^8 = \varepsilon^{\alpha \beta \gamma \delta \lambda \zeta \eta \kappa} F_{\alpha \beta \gamma \delta} F_{\lambda \zeta \eta \kappa} H,$$

or somewhat more concisely

$$h^u_{33} (\overline{126}_3)_F (\overline{126}_3)_F \overline{9}_H.$$ 

Dimension 5:

$$\frac{1}{M} h^u_{32} (\overline{126}_3)_F (84_2)_F 36_H H;$$

$$\frac{1}{M} h^u_{22} (84_2)_F (84_2)_F 36_H (9_H).$$

Dimension 6:

$$\frac{1}{M^2} h^u_{31} (\overline{126}_3)_F (36_1)_F (36_H)^2 \overline{36}_H;$$

$$\frac{1}{M^2} h^u_{21} (84_2)_F (36_1)_F \overline{36}_H (9''_H)^2.$$
Dimension 7:

$$\frac{1}{M} h_{11}^u \left( 36_1 F (36_1 F) 36' (9''_H) \right)^3.$$ 

Note that the Yukawa interactions involving $c_L t_R$ has the same structure with $h_{32}^u$ replaced by $h_{23}^u$ above, and similarly for the $u_L c_R$ term. Also no lower dimensional Yukawa interactions are allowed for each terms. This is enforced by the $SU(9)$ invariance, as well as imposing the following $Z_2$ symmetry:

$\underline{126}_3 F, \underline{84}_2 F, \bar{9}_3 F, 9_H, 36_H, 9''_H, 126_H, \text{and } 126''_H$

have $Z_2 = +1$,

while

$\underline{36}_1 F, \bar{9}_1 F, \bar{9}_2 F, 9'_H, 9''_H, 36'_H$, and $126'_H$

have $Z_2 = -1$.

In each of the Higgs multiplets, there are $5_H, \bar{5}_H$, and $1_H$ under $SU(5)$. From each of the Yukawa interactions, we use one EW VEV arising from either $5_H$, or $\bar{5}_H$, and the rest from singlets. Thus a Yukawa interaction of dimension $4 + n$ above will give rise to the mass matrix elements of the form

$$(h_{ij}^u v) \bar{u}_i L u_j R (\varepsilon^n),$$

with

$$\varepsilon = \frac{<1>}{M},$$

where $<1>$ is the VEV of the $SU(5)$ singlet field contained in the above $SU(9)$ Higgs representations, and $M$ is the $SU(9)$ unification scale.

Collecting terms from the above Yukawa interactions, we obtain the following up quark mass matrix:

$$M_u = \begin{pmatrix} h_{11}^u \varepsilon^3 & h_{12}^u \varepsilon^2 & h_{13}^u \varepsilon^2 \\ h_{21}^u \varepsilon^2 & h_{22}^u \varepsilon^1 & h_{23}^u \varepsilon^1 \\ h_{31}^u \varepsilon^2 & h_{32}^u \varepsilon^1 & h_{33}^u \varepsilon^1 \end{pmatrix} v. \quad (1)$$

### 3.2 Yukawa interaction for the down sector

Since the bottom quark mass is very small compared to the EW scale, we do not allow any dimension 4 Yukawa coupling in the bottom sector. Again, consistent with the $SU(9)$ gauge symmetry and the $Z_2$ symmetry, the allowed Yukawa interactions in the down quark sector
are:

Dimension 5:
\[
\frac{1}{M} h^d_{33} \left( 126_3^c \right)_F (\bar{9}_3^c)_F 126_H 9_H;
\]

Dimension 6:
\[
\frac{1}{M} h^d_{32} \left( 126_3^c \right)_F (\bar{9}_2^c)_F (36)_H^2 9'_H;
\]
\[
\frac{1}{M} h^d_{23} \left( 84_2^c \right)_F (\bar{9}_3^c)_F 126_H (9'_H)^2;
\]
\[
\frac{1}{M} h^d_{22} \left( 84_2^c \right)_F (\bar{9}_2^c)_F 126'_H (9'_H)^2;
\]

Dimension 7:
\[
\frac{1}{M} h^d_{31} \left( 126_3^c \right)_F (\bar{9}_1^c)_F 36_H (9'_H)^3;
\]
\[
\frac{1}{M} h^d_{13} \left( 36_1^c \right)_F (\bar{9}_3^c)_F 126_H (9'_H)^3;
\]
\[
\frac{1}{M} h^d_{21} \left( 84_2^c \right)_F (\bar{9}_1^c)_F (36_H)^3 9'_H;
\]
\[
\frac{1}{M} h^d_{12} \left( 36_1^c \right)_F (\bar{9}_2^c)_F 126'_H (9'_H)^3;
\]
\[
\frac{1}{M} h^d_{11} \left( 36_1^c \right)_F (\bar{9}_1^c)_F (36_H)^4.
\]

From the above Yukawa interactions, we obtain the following down quark mass matrix.

\[
M_d = \begin{pmatrix}
  h^d_{11} \varepsilon^3 & h^d_{12} \varepsilon^3 & h^d_{13} \varepsilon^3 \\
  h^d_{21} \varepsilon^3 & h^d_{22} \varepsilon^2 & h^d_{23} \varepsilon^2 \\
  h^d_{31} \varepsilon^2 & h^d_{32} \varepsilon & h^d_{33} \varepsilon^4
\end{pmatrix} v .
\]

(2)

Note that all our Yukawa couplings, \( h^u_{ij} \) and \( h^d_{ij} \) are of order 1. The hierarchy in the fermion masses and mixings arises from the different degree of suppression coming from the ratio of the VEVs, i.e., \( \varepsilon \).

4 Phenomenology

**FCNC and Higgs Decays:** Note that the up quark and down quark mass matrices in our model are identical to those obtained in Lykken, Murdock and Nandi (LMN)\[9\][10]. So, as shown there, if we choose the parameter \( \varepsilon \) to be \( \sim 1/50 \), our model is in good agreement with all the quark masses and CKM mixings. However, the crucial difference is that in our present model the existence of three light chiral families, as well their mass and mixing
hierarchies has its origin in a gauge family symmetry, $SU(9)$. Another important difference is that our singlet Higgs fields have masses close to the GUT scale, not the EW scale. Thus the phenomenology of this model is very distinct from the LMN model. A further difference with the LMN model is in the flavor changing neutral current (FCNC) interactions. In our case, because the singlet Higgs fields are very heavy (close to the GUT scale), their VEVs do not contribute to the mass matrix elements, or to the Yukawa coupling matrix elements. Thus, in our model, the mass matrices and the Yukawa coupling matrices for the up and down quark sector are proportional, and hence there are no flavor changing neutral current interactions at the tree level. Thus the predictions of our model for the flavor changing neutral current processes are the same as in the SM. The same is true for Higgs boson decays.

**Neutrino Masses and Mixings:** Because we need $\tilde{9}_F$ of $SU(9)$ to obtain $\tilde{5}_F$ of $SU(5)$ for the chiral fermion families, $SU(5)$ singlet fermions are unavoidable. Thus, the existence of singlet right handed (RH) neutrinos are required in our model, similar to a $SO(10)$ GUT, and contrary to a $SU(5)$ GUT. These RH neutrinos get Majorana masses from the $SU(5)$ singlet Higgs whose VEVs are about 50 times smaller than the GUT scale, as needed to explain the hierarchy of quark masses and mixings. Thus our model naturally explain why the mass scale of the RH neutrinos are smaller than the $SU(9)$ GUT scale as needed to obtain the light neutrino masses at the observed level via the see-saw mechanism. Furthermore, the Dirac mass terms between the light neutrinos and the heavy RH neutrinos occur via dimension 4 operators at the tree level. Hence, in agreement with observation, there will not be large hierarchies among the light neutrino masses or among the neutrino mixing angles.

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