Black Holes, Interactions, and Strings

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ABSTRACT

We give some examples in which neglecting the interactions between particles or truncating the description of a black hole to the spherically symmetric mode leads to unphysical results. The restoration of the interactions and higher angular momentum modes resolves these problems. It is argued that mathematical consistency of the description of black holes in the Schwarzschild coordinate system requires that we neither truncate the theory nor ignore the interactions. We present two hypotheses on how matter must behave under large Lorentz boosts in order for black holes to be consistent with quantum mechanics. Finally, we argue that string theory exhibits these properties.

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1. Introduction

The vast majority of work that has been done on the subject of black hole evaporation in 3+1 dimensions, beginning with the seminal work of Hawking [1], has relied on the approximation of free fields propagating in the fixed background black hole geometry. There is a good reason for this, since solving an interacting quantum field theory, even in Minkowski space, is extremely complicated. Indeed, there are certain calculations for which the free field approximation gives a perfectly sensible answer. As an example, recall that the Euclidean continuation of the exterior Schwarzschild geometry for a black hole of mass $M$ is periodic in the Euclidean time variable $\Theta$ with period $8\pi MG$ ($G$ is the Newton constant). Therefore, the Euclidean Green functions of any quantum field theory on this background, interacting or not, will have this periodicity. This shows that the only static state of the system is a thermal state at the Hawking temperature $T_H = \frac{1}{8\pi MG}$. In particular, since this holds for free field Green functions, we see that free field theory is sufficient to get the basic thermodynamics of the system correct.

However, the free field approximation leads to puzzling conclusions for certain other questions. For instance, the free field approximation tells us that the black hole is in thermal equilibrium at the Hawking temperature. However, for ordinary systems one expects to achieve thermal equilibrium only if there are interactions present. One wonders how a black hole could circumvent this. A second example of the inadequacy of the free field approximation is that modes of arbitrarily high frequency, much higher than the Planck mass, appear in the calculation of the properties of the Hawking radiation. Since we have no knowledge of physics beyond the Planck scale, the calculation is suspect [2].

A further approximation is often invoked, in which the system is truncated to include only spherically symmetric modes. Since almost all of the escaping Hawking particles carry little or no angular momentum [3], and since, in the absence of interactions, the different angular momentum modes are decoupled, it is often argued that these higher angular momentum modes are irrelevant to the properties of the Hawking radiation. Indeed, the spherically symmetric description of a Schwarzschild black hole has been elevated from the status of an approximation to that of an independent 1+1 dimensional mathematical model [4]. As is well known, however, the resulting description of the details of the Hawking radiation leads to paradoxes and inconsistencies with quantum theory [5].
In the following we will give some examples of situations in which neglecting interactions and/or truncating the theory to only the spherically symmetric modes leads to unphysical results. We will then show how including the higher angular momentum modes and the interactions resolves these problems. We emphasize that the mathematical consistency of the description of the black hole in the Schwarzschild coordinate system requires that we not truncate the theory. Moreover, we will argue that for black holes to be consistent with quantum theory matter must have very specific properties under large Lorentz boosts. Finally, we will see that fundamental strings exhibit some of the necessary properties.

Let us begin by examining some consequences of truncating the theory to the $s$–wave sector. In the Schwarzschild coordinate chart $(t, r, \theta, \phi)$ the line element of the exterior Schwarzschild geometry has the form

$$ds^2 = -\left(1 - \frac{2MG}{r}\right)dt^2 + \left(1 - \frac{2MG}{r}\right)^{-1}dr^2 + r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right), \quad (1.1)$$

where the horizon is at $r = 2MG$. The entropy of the black hole is given by the Bekenstein-Hawking formula

$$S_{BH} = \frac{A}{4G}, \quad (1.2)$$

where $A = 16\pi M^2 G^2$ is the area of the horizon. For ordinary systems, the degrees of freedom that account for the entropy of a hot system are also those which thermalize, store, and eventually reemit any information which may have been absorbed by the system. Later we will discuss how superstring theory provides a description of the underlying degrees of freedom which give rise to this entropy. However, the specific nature of these degrees of freedom will not concern us here. For our purposes, a coarse grained description of these degrees of freedom, which we will call the stretched horizon [6], can be used.

A simple model of the stretched horizon can be constructed by considering a set of quantum fields $\phi^A$ propagating within a spherical shell of proper thickness $\varepsilon$ in the vicinity of the horizon [7]. The field theory is explicitly cut off by restricting to modes with momentum less than the Planck mass $m_{\text{Planck}} = G^{-1/2}$. The fields within this shell are coupled in some specific manner to the fields outside. The field operator can be written as

$$\phi^A = \sum_{\ell=0}^{\ell_{\text{MAX}}} \sum_{m=-\ell}^{\ell} \phi^{A}_{\ell,m}, \quad (1.3)$$

$$\phi^{A}_{\ell,m}(t,r,\theta,\phi) = f^{A}_{\ell,m}(t,r)\Omega^{A}_{\ell,m}(\theta,\phi),$$

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where \( \Omega_{\ell,m} \) is the appropriate spherical harmonic. Since we have restricted to momentum modes less than the Planck mass, the maximum allowed angular momentum is

\[
\ell_{\text{MAX}} \approx |\vec{L}| = |\vec{x} \times \vec{p}_{\text{MAX}}| = 2M \sqrt{G},
\]

and the total number of allowed angular momentum modes is

\[
N = \sum_{\ell=0}^{\ell_{\text{MAX}}} (2\ell + 1) = (\ell_{\text{MAX}} + 1)^2 \approx 4M^2 G \propto \frac{A}{4G}.
\]

If we now treat the fields outside the stretched horizon as a heat bath in thermal contact with the stretched horizon, the thermal entropy of the stretched horizon is proportional to \( N \), and thus is proportional to \( \frac{A}{4G} \). In other words, modes with angular momentum up to \( 2M \sqrt{G} \) are important in accounting for the Bekenstein-Hawking entropy (1.2). If one truncates the system down to the spherically symmetric modes (\( \ell = 0 \)), the above simple analysis shows that the entropy should no longer be proportional to the area of the black hole. It should not be surprising that when all of the degrees of freedom which could account for the entropy are truncated, information is lost.

2. Mirrors and the Origin of Hawking Radiation

Next, we shall consider the effects of neglecting interactions. To this end, we will examine the following gedanken experiment. Consider an evaporating Schwarzschild black hole of mass \( M \gg m_{\text{Planck}} \). Let us focus attention on a very unlikely event: suppose the Hawking radiation assembles itself into a spherical, perfectly reflecting mirror at a proper distance \( \varepsilon \) above the horizon. This mirror reflects the outgoing radiation back into the hole and any incoming radiation back out to infinity.* The region outside the mirror can then be studied as a system with a perfectly reflecting boundary condition at the mirror. The question we want to address is, for how long will the system continue to radiate as seen from the outside?

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* Now, we do not believe any more than you do that if \( \varepsilon = O(\ell_{\text{Planck}}) \), that any physical mirror could withstand the Planckian temperatures in this region. We are really considering a purely mathematical exercise involving a fixed, classical geometry describing the black hole, and quantum fields propagating on that geometry. Mathematically, the mirror is a reflecting boundary condition on the fields at the proper distance \( \varepsilon \) from the horizon. In the context of this mathematical model, we are interested in the consequences of nontrivial interactions between the various angular momentum modes.
We will first study the system using the approximation of free fields. Since the lifetime of the black hole is of order $G^2 M^3$, for times small compared to $G^2 M^3$ we can approximate the exterior geometry by the usual Schwarzschild geometry (1.1). Before the mirror appears, the state of the system is the Hartle-Hawking vacuum. An observer at fixed radial coordinate $r$ close to $2MG$ experiences an approximately thermal flux of Hawking particles with proper temperature given by

$$T = \frac{1}{8\pi MG \sqrt{1 - \frac{2MG}{r}}} ,$$  
(2.1)

which can be approximated by

$$T \approx \frac{1}{2\pi \rho}$$  
(2.2)

where $\rho$ is the proper distance from the event horizon. Thus an observer near the horizon basically sees the Unruh thermal state. An observer far from the black hole, however, would not describe the state as precisely thermal. Because of the angular momentum-dependent effective potential experienced by fields propagating in the fixed Schwarzschild spacetime, almost all of the Hawking radiation that reaches infinity is in low angular momentum modes.

We now consider the theory of a free massless scalar field $\phi$. The system is most easily analyzed if we change to the Regge-Wheeler tortoise coordinate

$$r_* = r + 2MG \log \left( \frac{r}{2MG} - 1 \right),$$  
(2.3)

for which the line element takes the conformal form

$$ds^2 = \left( 1 - \frac{2MG}{r} \right) [-dt^2 + dr_*^2] + r^2 d\Omega^2 .$$  
(2.4)

In these coordinates, the mirror surface is at $r_{\text{mirror}} = 2MG \left[ 2 \log \left( \frac{r}{4MG} \right) + 1 + \left( \frac{r}{4MG} \right)^2 \right]$. Writing the field as

$$\phi(t, r_*, \theta, \varphi) = \sum_{\ell,m} \int_{-\infty}^{\infty} dE \frac{e^{-iEt}}{(2\pi)} U_{E\ell m}(r_*) Y_{\ell m}^{\ast}(\theta, \varphi)$$  
(2.5)

the field equation for $U$ can be written as a time-independent Schrödinger equation

$$\left\{ -\frac{d^2}{dr_*^2} + V_{\text{eff}}(r_*; \ell) \right\} U_{E\ell m} = E^2 U_{E\ell m} ,$$  
(2.6)
where the effective radial potential is

\[ V_{\text{eff}} = \left[ r - \frac{2MG}{r^3} \right] \left[ \ell(\ell + 1) + \frac{2MG}{r} \right]. \]  

(2.7)

and \( E \) is the energy of the mode as seen by an observer at infinity. \( V_{\text{eff}} \) has a global maximum at \( r \approx \frac{8}{3}MG \), corresponding to the tortoise coordinate \( r_{\text{outer}}^* \). For the case \( \ell = 0 \), the barrier height is \( \frac{9}{1024(MG)^2} \), and it increases monotonically with \( \ell \). Since the great majority of the populated modes have energy \( E \approx T_H \), we see that only very few of the higher angular momentum modes can tunnel through the potential barrier and escape.

Now consider the effect of the appearance of the mirror. The higher angular momentum modes are effectively trapped between the mirror and the potential barrier at \( r_{\text{outer}}^* \). Only the lowest angular momentum modes can escape, and in the free field approximation they are decoupled from the higher angular momentum modes. For simplicity in this discussion, we will drop all but the \( s \)-wave. To calculate the lifetime of the radiation, we first calculate how long it takes an \( s \)-wave mode to propagate from the mirror surface to the outer reaches of the black hole, which we define to be \( r_{\text{outer}}^* \). We then multiply the result by \( 2e^{2A} \), where \( e^{-A} \) is the amplitude to tunnel through the barrier. The amount of time it takes for an \( s \)-wave to propagate from \( r_{\text{mirror}}^* \) to \( r_{\text{outer}}^* \) is simply

\[ \delta t = r_{\text{outer}}^* - r_{\text{mirror}}^* = MG \left( 4 \log \left( \frac{\sqrt{8}MG}{\varepsilon} \right) + 1 + O \left( \frac{\varepsilon^2}{M^2G^2} \right) \right) \sim MG \log \left( \frac{MG}{\varepsilon} \right) \]  

(2.8)

It turns out that the tunneling suppression is independent of \( M \) for particles with energy of order \( T_H \) and is \( O(1) \) for the \( s \)-wave particles [3].

Now, in the absence of a mirror, the black hole radiates for a time \( t_{\text{evap}} = O(G^2M^3) \). The above calculation tells us that the mirror will shut down the Hawking radiation after a time much less than \( t_{\text{evap}} \) unless \( \varepsilon \) is of order

\[ \varepsilon \sim MG \exp(-M^2G), \]  

(2.9)

which is an absurdly small distance and cannot be physically meaningful. If we restrict ourselves to distances larger than \( \ell_{\text{Planck}} \), the majority of the Hawking radiation responsible for the evaporation of the black hole is trapped behind the mirror. The conclusion is that the
Hawking radiation originates at distance scales of the order given in equation (2.9). Another consequence is that after a time of order $\delta t$, an external observer will be able to see his own reflection in the mirror surface. The photons he emits will propagate freely down to the mirror surface and reflect right back out.

The reasoning used above can be applied to determine where the Hawking radiation originates even without the mirror. For example, starting from a black hole of mass $M$ and Hawking temperature $T_H = \frac{1}{8\pi MG}$, suppose we let the black hole evaporate until its new temperature is given by $T'_H = T_H(1 + \delta)$. $\delta$ can be chosen arbitrarily small, say $10^{-6}$, so the approximation of the system by the static Schwarzschild metric is a good one. The amount of Schwarzschild time $t$ needed for this process is proportional to $\delta M^3 G^2$. A calculation identical to the above leads to the belief that the Hawking radiation which is responsible for this evaporation originates at distances of the absurdly small order given in equation (2.9).

Now, let us return to the real world, in which interactions exist between particles. We will again consider the case of a mirror appearing at proper distance $\varepsilon$ above the horizon. The region just outside the mirror is at proper temperature $T = \frac{1}{2\pi \varepsilon}$. The strength of the interactions in this region is governed by the values of running coupling constants, evaluated at momentum scales of order $T$. These interactions couple the different angular momentum modes, so that every now and then a higher angular momentum particle will get scattered into the $s$–wave, and may then escape. Since the higher angular momentum modes are essentially confined to the region between the mirror and $r_{*\text{outer}}$, the result is a slow replenishment of the $s$–wave, which allows the system to continue radiating much longer than one would expect from the naive free field calculation. Let us estimate how often an $s$–wave quantum is produced.

Consider a thin spherical shell at radial coordinate $r$. Let the proper distance from the shell to the horizon be $\rho(r)$, and let the proper thickness of the shell be $\Delta \rho(r)$. Most of the particles in the shell will have momentum of the order of the proper temperature of the shell,

$$T(r) = \frac{1}{8\pi MG \sqrt{1 - \frac{2MG}{r}}} \approx \frac{1}{2\pi \rho(r)}.$$  \hspace{1cm} (2.10)

Since most of the particles have momentum of order $T$, it makes no sense to choose $\Delta \rho(r)$ much smaller than $\frac{1}{T}$. Further, since the temperature is varying relatively rapidly, it also makes no sense to choose $\Delta \rho(r)$ much bigger than $\rho$. We will therefore choose $\Delta \rho(r) = \rho(r)$. 

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We can then treat the shell as an interacting neutral plasma at proper temperature $T(r)$. By dimensional analysis, the number of collisions per unit proper time per unit proper volume due to a particular interaction will have the form

$$\frac{dn}{dVd\tau} = \alpha^2(T)T^4,$$  \hspace{1cm} (2.11)

where $\alpha$ is an average dimensionless running coupling constant evaluated at the momentum scale $T$. The proper volume of the shell is $dV = 4\pi r^2 \Delta \rho$, so the number of collisions per unit proper time in this shell is

$$\frac{dn}{d\tau} = 4\pi \alpha^2 T^4 r^2 \Delta \rho.$$  \hspace{1cm} (2.12)

Since most of the particles have momentum of order $T$, only those modes with angular momentum less than

$$\ell_{\text{MAX}} \approx |\vec{L}| = |\vec{x} \times \vec{p}| = rT,$$  \hspace{1cm} (2.13)

can be relevant in either the initial or the final state of a collision. The total number of such modes is

$$N = \sum_{\ell=0}^{\ell_{\text{MAX}}} (2\ell + 1) = (\ell_{\text{MAX}} + 1)^2 \approx (rT)^2.$$  \hspace{1cm} (2.14)

When a typical pair of particles in the plasma collide, the probability that one of them is scattered into any given angular momentum mode is of order $\frac{1}{N}$. Thus, the number of $s$–wave particles produced per unit proper time in the shell is

$$\frac{dn_s}{d\tau} \approx \frac{1}{N} \frac{dn}{d\tau} = 4\pi \alpha^2 T^2 \Delta \rho.$$  \hspace{1cm} (2.15)

In order to compute the number of $s$–wave particles produced per unit Schwarzschild time, we simply need to multiply equation (2.15) by the redshift factor $\frac{d\tau}{dt}$. Inserting the expression (2.10) for the proper temperature, we obtain

$$\frac{dn_s}{dt} \approx \frac{\alpha^2(r) \Delta \rho(r)}{16\pi G^2 M^2 \sqrt{1 - \frac{2MG}{r}}}.$$  \hspace{1cm} (2.16)

This expression must be summed over shells, starting with the shell which begins at the mirror surface, at proper distance $\epsilon$ from the horizon. At temperatures below the mass of
the electron, all cross sections go rapidly to zero, so the last shell to be included in the sum should have temperature of order \( m_e \). The sum can be approximated by an integral

\[
\frac{dn_s}{dt} \approx \frac{1}{16\pi G^2 M^2} \int \frac{d\rho}{\varepsilon} \frac{\alpha^2}{\sqrt{1 - \frac{2MG}{r(\rho)}}},
\]

(2.17)

Since \( \alpha \) can vary no more rapidly than a logarithm, we replace \( \alpha(\rho) \) by an average \( \overline{\alpha} \), and the integral is easily evaluated. Dropping all but the leading behavior, we find

\[
\frac{dn_s}{dt} \sim \frac{\overline{\alpha}^2 \log\left(\frac{1}{m_e \varepsilon}\right)}{4MG}.
\]

(2.18)

This is to be compared to the ordinary flux of particles as seen by an observer at infinity,

\[
\frac{dn_{\text{Hawking}}}{dt} \sim \frac{1}{4MG}.
\]

(2.19)

Thus we see that the maximum replenishment rate is of the order of \( \overline{\alpha}^2 \log\left(\frac{1}{m_e \varepsilon}\right) \) times the Hawking rate. If the mirror appeared at a GUT distance above the horizon, the rate of replenishment would be insufficient to sustain the Hawking emission rate. However, the black hole would continue to radiate at a diminished rate until all of the particles in the thermal atmosphere above the mirror were depleted. Since the number of particles is approximately given by the entropy (1.2), we expect the black hole to radiate until a time of order

\[
t \sim \frac{M^3 G^2}{\overline{\alpha}^2 \log\left(\frac{1}{m_e \varepsilon}\right)}.
\]

(2.20)

On the other hand, the mirror need only be at a distance of order the Planck length to make gravitational interactions strong enough to sustain the Hawking radiation fully. In this case, no noticeable effect of the mirror could be discerned for a time of order \( G^2 M^3 \). This means that the true origin of the Hawking radiation is at distance of the order of the Planck length from the horizon. There is no need to invoke distances of order \( MG \exp(-M^2 G^2) \).

Regardless of where the mirror occurs, an external observer would not be able to see his reflection in the mirror until after a time of order \( G^2 M^3 \), if at all. As long as the thermal atmosphere of the black hole remained, the photons he emitted would be scattered and thermalized near the horizon. It would be expected that the information carried by these photons would not be radiated back out until a time of order \( G^2 M^3 \).
This example raises the interesting question of whether it is at all possible to detect the presence of a mathematical mirror located at a Planck distance from the horizon of an evaporating black hole. ‘t Hooft has speculated that such a mirror would, in fact, be undetectable [7].

3. Particles and Gauges

An argument which is often raised about calculations of the type presented above goes as follows. Consider an infalling observer and a stationary observer who stays permanently outside the black hole, both near the horizon. As mentioned previously, a stationary observer near the horizon sees a thermal bath of particles at proper temperature given in equation (2.1). For a sufficiently massive black hole, however, the infalling observer does not see the hot bath of particles near the horizon, since he can perform no local experiment to detect the presence of the horizon. Because the infalling observer does not see the thermal bath, it is claimed that it is not a physical phenomenon—only $s$–wave particles observed at distances greater than $r = 3MG$, the existence of which both a stationary and an infalling observer will agree upon, are physical. Therefore, it is claimed, the replenishment of the $s$–wave modes calculated above cannot be physical, and one is back to discussing absurdly short distances.

The fallacy of this argument comes from not being true to one’s choice of gauge, i.e., of one’s coordinate chart. The infalling and stationary observers describe physics in different gauges. It makes no sense to dismiss the description of a system made in one gauge as unphysical, while claiming that the description made in another is physical. Within a given gauge, the only criterion for the physicality of phenomena is that the theory be internally consistent. This means, in particular, that the stationary observer, who uses coordinates covering only the region outside the black hole, has no choice but to include all mathematical degrees of freedom that are required for a consistent description in his coordinate system. One is not allowed to throw away degrees of freedom in one gauge because they do not appear in another gauge.

As an example, consider quantum electrodynamics. If one chooses to perform calculations using the Coulomb gauge, one finds the existence of long range instantaneous interactions, but the Hilbert space of states is manifestly positive definite. If one instead chooses
to work in Lorentz gauge, one no longer finds long range instantaneous interactions, but
the Hilbert space one works in now contains unphysical states containing longitudinally and
timelike polarized photons.

Throwing away the long range instantaneous interaction in Coulomb gauge, because it
is not present in Lorentz gauge, is clearly a serious mistake. Quantum electrodynamics
in the Coulomb gauge requires the existence of the long range instantaneous interaction for
mathematical consistency. Likewise, throwing the longitudinal degrees of freedom in Lorentz
gauge because they are not present in the Coulomb gauge destroys the internal consistency
of the theory. It is an equally foolish argument to drop the effects of the high energy, high
angular momentum thermal atmosphere in the stationary coordinate system because it is
absent in the infalling coordinate system.

The description of the black hole in each coordinate system separately appears to be in-
ternally consistent. The infalling observer falls freely past the horizon, never seeing any high
energy thermal bath, but can never communicate this information to the observer outside.
The outside observer sees a hot thermal bath and sees the infalling observer disappear into
it. The confusion arises when one tries to relate the two descriptions. As long as the infalling
observer remains outside the black hole, however, there exists a gauge transformation which
will map his local description of physics into that given by the stationary observer. The
gauge transformation is simply the coordinate transformation between the two frames of
reference.

To be explicit, let us consider a region very near the horizon, which is approximated
by Rindler space. Rindler space is simply the section of Minkowski space as seen by a uni-
formly accelerated observer, and freely falling particles move on straight lines in Minkowski
space. The coordinate transformation between Rindler coordinates \((t, \rho, y, z)\) and Minkowski
coordinates \(\{x^\mu\}\) is given by

\[
\begin{align*}
x^0 &= \rho \sinh \left( \frac{t}{4MG} \right), \\
x^1 &= \rho \cosh \left( \frac{t}{4MG} \right),
\end{align*}
\]

The horizon is at \(t = \infty\). The effect of a time translation in the Rindler time is equivalent
to a boost in Minkowski space. Now consider a particle falling toward the horizon. The
relation between the rest frame of a freely falling particle and that of a stationary Rindler
observer is given by a time-dependent boost angle which increases to infinity as the particle approaches the horizon. This means that to relate the descriptions of physics in the two frames requires a knowledge of how physical states behave under extremely large Lorentz boosts. The boost angle becomes as large as $M^2 G$ during the lifetime of the black hole. It therefore follows that the Rindler momentum of the particle becomes as large as $e^{M^2 G}$. The black hole is the ultimate particle accelerator. In trying to formulate the relation between the physics in these two coordinate frames, we are driven to a range of relative momenta with which we have little experience.

This leads us to the following hypothesis. For a theory to be consistent with the existence of black holes, it must be such that the effect of a super-Planckian Lorentz boost on a system is equivalent to the accumulated effect of putting the system into contact with a thermal bath at Planckian temperature for a long period of time. We might therefore expect that under a sufficiently large boost, a particle appears to melt and diffuse over a large region of space.

When a freely falling observer passes the horizon, there is no longer any coordinate transformation which will map his local description of physics to that of the stationary observer. This is because the stationary observer can never measure what goes on behind the horizon. It is in this sense that measurements made by the two observers are complementary.

4. Superstrings and Black Holes

The hypotheses given above give us some idea of how matter should behave under large Lorentz boosts in a quantum theory of gravity containing black holes. Now we should look for ways to implement this idea. It will be seen that superstring theory exhibits the properties listed above. In the previous sections we assumed that the standard laws of physics hold down to the Planck scale. In string theory however, the new physics begins at the string scale which differs from the Planck scale by factors of the dimensionless string coupling constant $\kappa$. If $\ell_{\text{Planck}} = \sqrt{G}$ is the Planck length and $\ell$ is the string length then

$$\ell_{\text{Planck}} = \kappa \ell,$$

(4.1)

so that if $\kappa$ is very small the new physics begins at length scales appreciably larger than $\ell_{\text{Planck}}$. In what follows we will use units in which $\ell = 1$. From what we have argued, the
environment sufficiently near the horizon as described in Schwarzschild coordinates should resemble the phase of string theory above the Hagedorn temperature. We expect this to consist of a condensate or very dense hot soup of strings strongly interacting with each other [8, 9, 10]. Furthermore, it has been argued [9, 10, 11] that the Bekenstein-Hawking entropy per unit area of a horizon can be understood as the entropy of this soup of strings.

Let us proceed to consider the description of strings by a stationary Schwarzschild observer. The propagation of closed superstrings in a Schwarzschild background has not been completely analyzed. If we use the Rindler space approximation of the near horizon geometry, however, then we can use results from flat space string theory.

The points of Minkowski space occupied by a string are given by functions \( X^\mu(\tau, \sigma) \), where \( \tau \) and \( \sigma \) are coordinates on the string world sheet. In addition, the string has internal degrees of freedom implied by supersymmetry and compactification. In the light cone frame, \( X^+ = (X^0 + X^1)/\sqrt{2} = \tau \), and the dynamical degrees of freedom are the two transverse coordinates \( \{X^i\}_{i=1}^2 \), which are decoupled from the internal degrees of freedom. The normal mode decomposition of \( X^i \) for a noninteracting superstring is the same as for a free bosonic string

\[
X^i(\tau, \sigma) = x^i + p^i \tau + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \left[ \alpha_n e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n e^{-2in(\tau+\sigma)} \right], \tag{4.2}
\]

where \( x \) and \( p \) are the center of mass position and momentum, and the \( \alpha \) are the mode coefficients.

The transverse size of the string at light cone time \( \tau = 0 \) can be estimated by computing the expectation value of

\[
\bar{R}^2 = \frac{1}{\pi} \int d\sigma \left( \tilde{X}(\sigma) - \bar{x} \right)^2 \tag{4.3}
\]

in whatever state is under consideration. Now if an observer uses an apparatus with resolution time \( \varepsilon \) to measure the size of the string, he should only include in his description of the string modes with frequency less than \( \frac{1}{\varepsilon} \) in his frame of reference. The frequency of mode \( n \) is given by

\[
\nu_n = \frac{n}{P}, \tag{4.4}
\]

where \( P \) is the longitudinal momentum of the string as measured by the observer. This means that we need only include modes with \( |n| \leq N = P/\varepsilon \). For the ground state one
easily finds

\[ \langle 0 | \mathbf{R}^2 | 0 \rangle = \sum_{n=1}^{N} \frac{1}{n} \approx \log \left( \frac{P}{\varepsilon} \right). \]  \hspace{1cm} (4.5)

Here we see the first example of anomalous behavior of strings under Lorentz boosts. Instead of the transverse size being independent of momentum, one finds that it depends on the momentum logarithmically. A more complete analysis of the growth of the transverse size of free strings was made in [12], and indicates that in addition the length of string in the transverse plane is proportional to \( P \). As \( P \) increases, the string loops back over itself many times, in such a way that the total area occupied by the string only grows logarithmically. As the momentum goes to infinity, the string becomes dense over all of space. In Figures 1 (a) to 1 (f), we show a sequence of snapshots of a string falling toward a Rindler horizon, taken by a stationary Rindler observer at equal intervals of Rindler time. The figures were generated by imposing a smooth mode cutoff on a string wave function, a procedure very similar to that used in [12]. In figure 1 (a), only the very lowest modes appear, but as Rindler time progresses, more and more modes enter the description.

Even the modest logarithmic growth of a free string has surprising implications to a Schwarzschild observer. Recalling that the longitudinal momentum of a string falling toward the horizon as measured by the Schwarzschild observer grows exponentially with time, we see that the area of the region occupied by the string grows like \[ \langle \mathbf{R}^2 \rangle = t/4MG. \] This behavior has been successfully interpreted as an effective thermalization of the string, causing it to melt and diffuse over the horizon [13]. If no other effects take place the string would grow to a size comparable to the Schwarzschild radius in a time of order \( \kappa^2G^2M^3 \). If \( \kappa \) is small, this time is short in comparison with the evaporation time of the black hole.

The process is very similar to the stochastic evolution of a scalar field in an inflating universe. In both cases more and more modes enter the description with time. These modes enter with random phase and amplitude. In each case the growth and spreading can be described by stochastic interactions with a heat bath. In the string case the heat bath is provided by the Unruh effect. Thus we see that the second of our hypotheses is realized even at the level of free string theory.

Another suprise to the Schwarzschild observer is that the string fails to Lorentz contract as the momentum gets large. This is important for the finiteness of the entropy of the
horizon. The ordinary view is that particles Lorentz contract along the direction of their motion by a factor proportional to $\frac{1}{P}$. For this reason, an arbitrarily large number of particles can be stacked near the horizon [14]. This is in obvious contradiction to the finiteness of the Bekenstein-Hawking entropy (and the possibility of the existence of a mirror). The longitudinal behavior of strings is quite different. To compute the mean longitudinal spread $\Delta X^-$ at $\tau = 0$ we use the constraint equation [15]

$$\frac{\partial X^-}{\partial \sigma} = \frac{\partial \vec{X}}{\partial \sigma} \cdot \frac{\partial \vec{X}}{\partial \tau} + I,$$

where $I$ represents the contribution from compactified modes, fermionic modes, etc. We can rewrite equation (4.6) in terms of the transverse Virasoro generators, obtaining

$$\frac{\partial X^-}{\partial \sigma} = \sum_{n \neq 0} \left( L_n e^{in\sigma} - \tilde{L}_n e^{-in\sigma} \right),$$

which can be integrated to give

$$X^-(\sigma) = x^- + \sum_{n \neq 0} \frac{1}{in} \left[ L_n e^{in\sigma} + \tilde{L}_n e^{-in\sigma} \right].$$

Using the standard Virasoro algebra one finds

$$\langle 0 \mid (\Delta X^-)^2 \mid 0 \rangle \approx 4 \sum_{n=1}^{N} n \approx 2 \left( \frac{P}{\varepsilon} \right)^2,$$

Equation (4.9) indicates that no Lorentz contraction of the string distribution takes place. The spreading process begins to occur when the string reaches a distance of order the string scale from the horizon. The result is that in the Schwarzschild coordinates the bulk of the string never approaches closer than a distance of order $\ell_{\text{Planck}}$ to the horizon. This peculiar property of strings supports our first hypothesis that an external observer cannot detect the presence (or absence!) of a mirror at a distance of order $\ell_{\text{Planck}}$ from the horizon.

Eventually, the density of string will become so large that interactions can no longer be ignored, and more complicated phenomena occur, which probably cause the transverse growth to become even more rapid. As the string replicates, its transverse density increases.
At the center of the distribution the average number of strings $N$ passing through a region of area $A$ is of order $\exp(\bar{R}^2) \sim \exp \left( \frac{1}{\kappa^2} \right)$. However, this enormous density of string certainly leads to new effects once it becomes of order $\frac{1}{\kappa^2}$. At this time the probability for string interactions becomes unity and perturbation theory breaks down. One attractive possibility is that the growth of string density is cut off at this point. The result would be that the density grows until there is about one string per unit Planck area. This is also suggested by the fact that the entropy of a black hole is proportional to its area [16].

Perhaps the most remarkable aspect of the above description is that none of it is seen by an observer who falls through the horizon with the string. Such an observer sees the string with a fixed time resolution and therefore sees a constant transverse and longitudinal size as the horizon is crossed.

To conclude, we would like to point out some questions which need to be addressed. To begin, we must understand the behavior of the hot string soup at the horizon, both with and without a mirror present. Our speculation is that the presence of a mirror at a distance of about the Planck scale from the horizon will not affect the answer. Not unrelated to this, the behavior of string interactions when the density of string gets to be of order $\kappa^{-2}$ needs to be understood.

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FIGURE CAPTIONS

Figure 1 (a) - 1 (f): Snapshots of a string falling toward a Rindler horizon, taken by a stationary Rindler observer at equal increments of Rindler time. In Figure 1 (a), only the lowest modes contribute to the effective string wave functional, but as time progresses, more modes enter the description. Figure 1 (f) shows many modes have now entered the effective wave functional. We see the roughly linear growth of the area, and that the density of string near the center of the distribution is getting very large.
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