Continuous Quantum Measurements via Random-Time Sampling

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Random-time sampling of quantum systems is studied as a general approach to measuring higher order quantum correlations with prospects for ultra-low-perturbation measurements. Our theory generalizes earlier works that were limited to two-level systems and second-order correlations. Random sampling naturally occurs, e.g., in an optical spin noise experiment when weak probe-laser light leads to random single-photon peaks in the detector time-trace. We show that a direct evaluation of these time-traces yields second and third-order spectra that are equivalent but not identical to quantum polyspectra of the usual continuous measurement regime. Surprisingly, broad-band system dynamics is fully revealed even for average sampling rates below the Nyquist frequency. Many applications are envisioned for high-resolution spectroscopy, circuit quantum electrodynamics, quantum sensing, and quantum measurements in general.

Probing of quantum systems is at the heart of quantum technologies for current and future applications in quantum sensing [1], quantum information [2], and quantum measurements in general [3]. Backaction of the measurement on the system is a major problem even of quantum-limited probing. Pfender et al. recently introduced a scheme of sequential instead of continuous weak measurements to dramatically reduce the influence of their probe system resulting in high-resolution spectra of the quantum system [4]. A similar scheme was studied by Liu et al. regarding third order correlations [5]. However, such undersampling schemes cause multiple replicas of original lines in the spectra as artifacts, which compromise applications where several distinct frequencies of the systems have to be probed. Here, we show that random-time sampling provides a way to access higher-order correlations of general quantum dynamics. Previously, Ruskov et al. had found in a two-level system the equivalence of the second-order spectrum of continuous measurements and random-time sampling [6, 7]. Similarly, Li et al. simulated a frequency-dependent higher-order cumulant of a randomly sampled two-level system but did not establish a connection with the corresponding continuous measurement [8]. Our theory and numerics show for general systems that second- and third-order spectra (quantum polyspectra) [9] directly calculated from traces of random-time quantum measurements share the same spectral features as those from usual continuous measurements [10–12]. Surprisingly, this holds true even for average sampling rates below the frequencies of the system dynamics. Apart from carefully devised random-time measurement schemes, random sampling can naturally occur if the probe itself shows quantum behavior. A weak probe laser beam in a spin noise experiment [13], e.g., will lead to random Poisson distributed click events in a photomultiplier tube. Our theory includes for the first time a quantum mechanical description of spin noise spectroscopy at low probe photon rates complementing earlier theories [14, 15].

Random-time sampling is inspired from classical signal processing [16–19]: Consider the autocorrelation of a stationary continuous random process $x(t)$

\begin{equation}
    g_x(\tau) = \langle x(t + \tau)x(t) \rangle,
\end{equation}

which yields the power spectrum via Fourier transformation according to the Wiener-Khinchin theorem. The angle brackets relate to the expectation value (here, the temporal average). Random-time sampling is described by a sum of Dirac-delta functions $y(t) = \sum_j \delta(t - t_j)$ where the times $t_j$ are Poisson distributed at an average rate $\gamma_p = \langle y(t) \rangle$. The random process $z(t) = x(t)y(t)$ represents the result of a random time-sampling of $x(t)$. Its autocorrelation

\begin{equation}
    g_z(\tau) = \langle x(t + \tau)y(t + \tau)x(t)y(t) \rangle \\
    = \langle x(t + \tau)x(t) \rangle \langle y(t + \tau)y(t) \rangle \\
    = g_x(\tau)\gamma_p\delta(\tau) + \gamma_p^2
\end{equation}

follows after noting that $x$ and $y$ are independent and

![Schematics of random-time sampling](image-url). The linearly polarized probe photons (red double arrows) arrive at random times in the interaction region where they interact and entangle with the quantum system (a) and may weakly change their orientation (blurred double arrow). After traversing the polarizing beamsplitter a photon with either horizontal or vertical polarization causes an event in the corresponding detector (b) giving rise to a positive or negative peak in the measurement trace (c).
therefore allow for the factorization in the second line. The result for \( g_x \) is equivalent to \( g_x \), except at \( \tau = 0 \). The discrepancy results only in an additional flat background contribution to the power spectrum of \( z(t) \) in comparison with the spectrum of \( x(t) \). A reconstruction of \( g_x \) from \( g_x \) does not depend on \( \gamma_p \) and is possible even for average sampling rates below the frequency range of \( x(t) \). Third-order moments and the corresponding bispectrum of \( z(t) \) have only scarcely been treated [18,19]. The bispectrum of \( z(t) \) contains the desired bispectrum of \( x(t) \) and four additional terms whose simple structure, however, allows for their subtraction [19]. It is not immediately clear if the success of classical random sampling can be repeated in the quantum case of sampling with, e.g., single photons. While the peaks in the classical \( z(t) \) are scaled with \( x(t) \), a single photon event in a detector contains no quantitative information on the quantum system. However, its probability to appear will depend on the actual state of the quantum system.

A quantum mechanical model of random sampling

Our theory of quantum measurements via random-sampling is based on continuous measurement theory [10,22]. The so-called stochastic master equation (SME) describes both system dynamics and the detector output \( z(t) \) with respect to a measurement operator \( A \) [20]. Only recently, analytical expressions for multi-time moments and quantum polynomials of the detector output \( z(t) \) [13,21] where directly calculated from the SME allowing for a direct comparison of experimental polynomials with analytical spectra [9]. In order to describe random-time sampling, we explicitly include the probe system (e.g. a photon with two polarization states or an electron spin) in the overall quantum system described by the Hamiltonian \( \hat{H} \).

The probe system enters incoherently an interaction region (a) with the quantum system at a certain rate \( \gamma_p \) (Fig. 1). The probe system will then slightly change its state due to interaction with the quantum system, thus gaining some knowledge about the quantum system. At the same time, the quantum system gets perturbed by the probe system (measurement backaction). The probe system leaves the interaction region at a rate \( \gamma_{\text{det}} \) towards the detector region (b) where a strong continuous measurement on the probe system (measurement backaction). The probe system leaves the interaction region at a rate \( \gamma_{\text{det}} \) towards the environment at a rate \( \gamma_{\text{det}} \). For \( \gamma_p \ll \gamma_{\text{out}} \) and \( \gamma_{\text{out}} \approx \gamma_{\text{det}} \), the time-dependent occupation of the detector region shows typical Poisson behavior (like clicks of a random radioactive decay).

The overall Hamiltonian \( \hat{H} \) consists of the quantum system \( \hat{s} \) and its interaction with the probe system

\[
\hat{H} = \hat{H}_s + \hat{H}_{\text{int}},
\]

where \( \hat{H}_s \) may be any system Hamiltonian. We employ a simple linear interaction

\[
\hat{H}_{\text{int}} = \hbar \gamma_p \hat{a}_z,
\]

where \( \beta \) corresponds to the interaction strength, \( s_z \) relates to some property of the system (e.g., the \( z \)-component of the spin of one of its electrons), and \( a_z \) relates to the \( z \)-component of the probe spin (analogous to photon angular momentum) in the interaction region (a). Note, that the probe system in the interaction region is described by three states: \( |a_{z,\uparrow}\rangle, |a_{z,\downarrow}\rangle \) and the empty state \( |\emptyset\rangle \). The operator \( a_z \) thus reads in that basis

\[
\hat{a}_z = 1_s \otimes |a_{z,\uparrow}\rangle \langle a_{z,\uparrow}| - \frac{1}{2} |a_{z,\downarrow}\rangle \langle a_{z,\downarrow}| \otimes 1_b, \tag{5}
\]

where we recognize the Pauli spin operator for the \( z \)-direction. The operators \( a_x \) and \( a_y \) follow analogously. The operators \( b_z, b_x, \) and \( b_y \) for the probe system in the detector region have the same representation in their basis as \( a_{x,\uparrow} \). The system dynamics given by the density matrix \( \rho(t) \) is governed by the SME (Ito-calculus) [20]

\[
d\rho = \frac{i}{\hbar} [\rho, \hat{H}] dt + \gamma_p D[\hat{a}_s](\rho) dt + \gamma_{\text{out}} D[\hat{a}_{ab}](\rho) dt \\
+ \gamma_{\text{det}} (D[\hat{a}_{b,\uparrow}](\rho) + D[\hat{a}_{b,\downarrow}](\rho)) dt \\
+ \beta^2 \mathcal{D}[A](\rho) dt + \beta \mathcal{S}[A](\rho) dW, \tag{6}
\]

with the damping terms

\[
\mathcal{D}[\rho](\rho) = \rho c^\dagger c - (c^\dagger c + c c^\dagger)/2 \tag{7}
\]

and backaction term

\[
\mathcal{S}[\rho](\rho) = c \rho c^\dagger - \text{Tr} [c + c^\dagger] \rho, \tag{8}
\]

where we used the notation of [21] and [9]. The differential \( dW \) relates to a stochastic Wiener-process, where \( \Gamma(t) = dW(t)/dt \) is white Gaussian noise, \( \langle \Gamma(t) \Gamma(t') \rangle = \delta(t - t') \). The resulting detector output [13]

\[
z(t) = \beta^2 \text{Tr}[\rho(t)(A + A^\dagger)/2] + \frac{1}{2} \frac{\beta^2}{\Gamma(t)} \tag{9}
\]

is a sum of the expectation value of measurement operator \( A = b_y \) and an omnipresent white background noise \( \Gamma(t) \) whose relative weight decreases with the measurement strength \( \beta \). The probe system entering the interaction region in state \( |a_{x,\uparrow}\rangle \) is modeled in the SME via the superoperator \( D[\hat{a}_s] \) with

\[
d_a = 1_s \otimes |a_{x,\uparrow}\rangle \langle a_{x,\uparrow}| \otimes 1_b. \tag{10}
\]

Similarly, the spin/polarization-conserving transition from region (a) to (b) is modeled via

\[
d_{ab} = (1_s \otimes |a_{x,\uparrow}\rangle \otimes 1_b) \cdot (1_s \otimes 1_s \otimes |b_{x,\downarrow}\rangle \langle b_{x,\downarrow}|) \\
+ (1_s \otimes |a_{x,\downarrow}\rangle \otimes 1_b) \cdot (1_s \otimes 1_s \otimes |b_{x,\downarrow}\rangle \langle b_{x,\downarrow}|). \tag{11}
\]

The probe system leaves the detector via

\[
d_{b,\uparrow/\downarrow} = 1_s \otimes 1_s \otimes |b_0\rangle \langle b_{z,\uparrow/\downarrow}|. \tag{12}
\]

After introducing the Liouvillian \( \mathcal{L}[\beta] \) the SME reduces to

\[
d\rho = \mathcal{L}[\beta](\rho) dt + \beta \mathcal{S}[A](\rho) dW. \tag{13}
\]
The z-direction of the electron spin is probed by a stream of single photons linearly polarized in xy-direction (Fig. 1). The two circular polarization states of the probe photon can be treated in full analogy to spin 1/2 systems altering in the z-direction via a strong continuous measurement (large $\beta$) which mimics a setup consisting of a polarizing beam splitter and a pair of photomultipliers. The detector output $z(t)$ will exhibit either a strong positive or a strong negative peak. Without interaction their probability of appearance is 50% each. The probabilities slightly change if the systems alters the photon’s polarization state in the interaction region.

Next, we give an illustrative example of a random-time measurement on a single electron spin that is precessing in an external magnetic field $B$ parallel to the $x$-direction,

$$H_s = \hbar \omega_L s_z,$$

where $\omega_L$ is the Larmor frequency.

The $z$-direction of the electron spin is probed by a stream of single photons linearly polarized in $xy$-direction (Fig. 1). The two circular polarization states of the probe photon can be treated in full analogy to spin 1/2 up and down states in $z$-direction. After interacting with the system ($s$), the photon polarization axis is slightly rotated in the $xy$-plane depending on the $z$-spin orientation (Faraday-effect). The photon polarization is then measured in $y$-direction and $x$-direction via a strong continuous measurement (large $\beta$) which mimics a setup consisting of a polarizing beam splitter and a pair of photomultipliers. The detector output $z(t)$ will exhibit either a strong positive or a strong negative peak. Without interaction their probability of appearance is 50% each. The probabilities slightly change if the systems alters the photon’s polarization state in the interaction region.

The measurement result, therefore, contains some (but not full) information about the $z$-spin orientation at the time of interaction. Figure 2(a) shows the detector output $z(t)$ found from numerical integration of Eq. (13) for $\omega_z/2\pi = 0.509$ GHz, $g = 100$ GHz, $\gamma_p = 0.5$ GHz, $\gamma_{out} = 100$ GHz, $\gamma_{det} = 100$ GHz, and $\beta^2 = 10^4$ GHz. In a spin noise experiment with a laser probe at 800 nm, a photon rate $\gamma_p = 0.5$ GHz corresponds to a light power of 0.12 nW. Peaks, both positive and negative, related to photon detection events are clearly visible in $z(t)$ on an otherwise Gaussian background noise $\beta \Gamma(t)/2$ [cmp. Eq. (9)] which would disappear only in the unrealistic limit of ultra-strong measurement of the probe system. The peaks vary in height and width closely resembling actual current traces of photomultiplier tubes. The expectation value $\langle z(t) \rangle$ of the spin $z$-component displays a coherent precession dynamics that is only weakly disturbed at the times of a photon detection event. At an increased average rate $\gamma_p = 5.0$ GHz of the incoming probe system, the precession dynamics is clearly distorted [Fig. 2(b)].

For the analysis of actual experiments, the power spectra $S_z^{(2)}(\omega)$ can be directly estimated from the experimental $z(t)$ (see App. B of [9]) without the need for introducing thresholds or counters for signal conditioning. Roughly speaking, the power spectrum $S_z^{(2)}(\omega)$ is via the Fourier transformation of $z(t)$ strongly related to the expectation value $\langle z(\omega) z^*(\omega) \rangle$. Here, we evaluate the general analytical expression for powerspectra $S_z^{(2)}(\omega)$ which is given in terms of the Liouvillean $L[\beta]$ and the measurement operator $A$, see Eq. (11) and Sec. XV.

The spectrum in Fig. 2(c) exhibits three contributions: (i) A constant white noise background due to Gaussian detector noise $\beta \Gamma(t)/2$ clearly visible at $\omega/2\pi \approx 100$ GHz, (ii) a broad Lorentzian peak centered at 0 Hz with a cut-off frequency at around 10 GHz which corresponds to the finite lifetime of the probe system in the interaction region, (iii) a narrow Lorentzian peak at the Larmor fre-
The power spectra for increasing measurement rates for oscillations with increased damping. Fig. 3 shows the spectra in the case of a continuous quantum measurement closely resemble that of case (a) except for the background and stripes. The color bar is scaled via the arsinh-function.

FIG. 4. (a) Power spectrum $S_2^{(2)}(\omega)$ and bispectrum $S_3^{(3)}(\omega_1,\omega_2)$ in GHz$^{-2}$ of the z-component of one spin in a coupled spin system measured via random sampling. An average background of $-4.04 \times 10^4$ GHz$^{-2}$ was subtracted. (b) The spectra in the case of a continuous quantum measurement closely resemble that of case (a) except for the background and stripes. The color bar is scaled via the arsinh-function.

Frequency $\omega_L$ originating from the system dynamics. For $\gamma_p = 5.0$ GHz the Larmor-peak exhibits a clear broadening and small shift to lower frequencies as expected for oscillations with increased damping. Fig. 3 shows the power spectra for increasing measurement rates $\gamma_p$ where background has been subtracted using power spectra for the case of no probe-interaction with the system, $H_{int} = 0$. At high rates the spectrum broadens and shifts to zero frequencies as the frequent measurements suppress all coherent dynamics. This behavior is known as quantum Zeno effect \cite{22}. Korotkov studied the Zeno transition using a continuous measurements approach. He found the same spectral features as in our case of random sampling \cite{21}.

Our numerics for $\gamma_p$ down to 0.05 GHz supports the width of the spectrum scales for lower rates linearly with $\gamma_p$ allowing for the detection of a fully coherent oscillation in the limit $\gamma_p \to 0$. This is in agreement with Ruskov’s early theory for a two-level system \cite{4}. Similarly, we found for general systems a sharpening of spectral features also in the third-order polyspectra (not shown). Nevertheless, an increase in interaction strength $g$ leads to a stronger disturbance of the system at any single probing event (see discrete distortion in $z(t)$ [Fig. 2(a)]). This changes the height of spectral features even for $\gamma_p \to 0$ revealing quantum backaction.

Bispectrum of a two-spin system Next, we demonstrate the generality of our random-sampling theory by calculating a third-order quantum polyspectrum (bispectrum) \cite{14} for two coupled spins precessing inside a tilted magnetic field. The bispectrum $S_3^{(3)}(\omega_1,\omega_2)$ is related to $\langle z(\omega_1) z(\omega_2) z^*(\omega_1 + \omega_2) \rangle$ (exact definition see \cite{23}). Bispectra of continuous quantum measurements have previously been used for evaluating transport experiments in nano-electronics \cite{9, 21} and for the analysis of non-Gaussian dephasing environments in circuit quantum electrodynamics (cQED) \cite{25}. The quantum system is defined by

$$H_s = \hbar \omega_L^{(1)} (\sin(\varphi) s_z^{(1)} + \cos(\varphi) s_z^{(1)}) + \hbar \omega_L^{(2)} (\sin(\varphi) s_z^{(2)} + \cos(\varphi) s_z^{(2)}) + g e^{i(\varphi(1) + \varphi(2))} s_z^{(1)} s_z^{(2)} + s_y^{(1)} s_y^{(2)} + s_z^{(1)} s_z^{(2)}],$$

where $\omega_L^{(1)} / 2\pi = 1.592$ GHz, $\omega_L^{(2)} / 2\pi = 0.0$ GHz, $g_e = 10$ GHz, and $\varphi = \pi/6$ is the tilt-angle of the magnetic field. $H_{int} = \hbar g s_z^{(1)} s_z^{(2)}$, where $g = 50$ GHz, $\gamma_p = 5$ GHz, $\gamma_{out} = 100$ GHz and $\beta^2 = 10^4$ GHz. An additional spin relaxation term $\gamma_{int} D[d_s(\rho)]$ with $d_s = [s_z^{(1)}(1) \otimes s_z^{(2)}] \otimes 1 \otimes 1_s \otimes 1_b$ is introduced which drives the first spin towards the $-z$ direction at a slow rate $\gamma_s = 0.1$ GHz. Consequently, $\langle s_z^{(1)} \rangle < 0$ and $\langle z(t) \rangle < 0$ will hold for the experimental setup. The spectrum $S_2^{(2)}(\omega)$ of the random sampling trace in Figure 3(a) shows several peaks corresponding to quantum beats between different energy eigenstates of the system. The third-order spectrum (Eq. (110) in \cite{15}) exhibits a negative background (subtracted in the Figure), straight lines that extend to higher frequencies, and several sharp peaks corresponding to the ones seen in the $S_3^{(3)}$ of the usual continuous measurement (Fig. 4(b)), where $\beta^2 = 0.5625$ GHz and $A = s_z^{(1)}$ calculated via standard theory \cite{15}. Benhenni and Rachdi show in their classical treatment of bispectra from random sampling that the asymptotic behavior of the additional contributions to $S_3^{(3)}$ makes it possible to separate them from the desired peak structure \cite{19}. After a first attempt we believe that separation of such contributions is also possible in the quantum case. A spectrum without background and stripes was found for $\langle z(t) \rangle = 0$ when $\gamma_s = 0$.

Conclusion Random-time sampling of quantum dynamics has been generalized to a spectroscopic tool that is capable of characterizing second- and third-order coherence even at very low average sampling rates. Random-time sampling therefore is a viable and more versatile alternative to recently introduced high resolution spectroscopy via sequential weak measurements \cite{4}. Random sampling by single-photon detectors also offers an alternative to amplification schemes via heterodyning in the very active field of spin noise spectroscopy \cite{26, 28}. Today’s experiments of cQED offer a great control of probe events with desired timing and interaction strength \cite{25, 30}. Especially the investigation of non-Gaussian environmental noise in cQED may benefit from measuring
the power spectrum and bispectrum of a detector qubit via random sampling with ultra-weak backaction [25].

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