Spin Dependence of Heavy Quark Fragmentation

Fernando Cornet
Departamento de Física Teórica y del Cosmos and Centro Anadalu de Física de Partículas,
Universidad de Granada, E-18071 Granada, Spain

Carlos A. García Canal
Departamento de Física, Universidad Nacional de La Plata, C.C.67, La Plata (1900), Argentina

We propose that the non-perturbative fragmentation functions describing the transition from a heavy quark to a heavy meson is proportional to the square of the produced meson wave function at the origin. We analyze the effects of this proposal on the number of pseudoscalar mesons compared to the number of vector mesons produced and find a good agreement with experimental data. Finally, we discuss further experimental checks for our hypothesis.

Keywords:

Heavy quark production in high energy collisions, either $e^+ e^-$, $ep$, $pp$ or $p\bar{p}$, provides a good laboratory to test QCD in both, its perturbative and non-perturbative sectors. In all the experiments both sectors contribute. In processes with a hadron in the initial state one has to consider the quark and gluon distribution functions in the initial hadron and the heavy quark fragmentation function describing the transition of the heavy quark into the measured final state hadronic system. In $e^+ e^-$ processes only this last non-perturbative piece contributes. It is, thus, clear that in order to get the maximum theoretical information from the increasingly precise experimental data on heavy quark production one must have a good description of all the pieces involved in the calculation, in particular of the fragmentation functions.

The heavy quark fragmentation functions receive two contributions. In high energy processes the heavy quark is produced far off mass-shell and it emits gluons until it becomes on mass-shell. This process can be calculated perturbatively via the usual DGLAP equations describing the evolution. In this way one obtains a better theoretical prediction for the partonic cross-section. At this point one still has to take into account the transition from the heavy quark into a hadron which is described by a purely non-perturbative fragmentation function. Since all the scaling violations are taken into account via the perturbative calculation, one expects the non-perturbative fragmentation functions to be scale independent [1].

The experimental situation on charm and bottom production in different collision events has been reviewed in [2]. These measurements are mainly devoted to test perturbative QCD and to extract fragmentation functions and fractions. The best description of the experimental data is obtained by using either a one free parameter non-perturbative phenomenological parametrization by Kartvelishvili, Likhodede and Petrov [3] or a two parameter one by Bowler [4]. The parametrizations by Peterson, Schlatter, Schmitt and Zerwas [5] is also widely used when analyzing experimental data. Other popular parametrizations are: Collins and Spiller [6], Colangelo and Nason [7] and Braaten, Cheung, Fleming and Yuan [8].

All these parametrizations provide different realizations of the original Bjorken [9] and Suzuki [10] proposal that the heavy quark fragmentation function, contrary to what happens with light quarks, should be very hard, i.e. the heavy quark should retain most of its momentum in the hadronization process. The exact shape of the dependence of the fragmentation function on the heavy quark momentum is controlled, in the parametrizations cited above, by some free parameters (the number of free parameters depends on the parametrization) that have to be fitted to the experimental data. In general these parameters have no absolute physical meaning (see for example [11] for a discussion).

In addition to the dependence of the fragmentation functions with the heavy quark momentum, the different experiments have also measured the relative number of charmed and bottom mesons produced in the pseudoscalar state ($D$ and $B$) and in the vector state ($D^*$, $B^*$). More specifically, they have measured the quantity $P_V$ defined by

$$P_V = \frac{V}{P+V}$$

where $V$ and $P$ stand for number of vector and pseudoscalar states produced, respectively. The present experimental
values for charmed mesons are:

\[
\begin{align*}
P^D_V &= 0.566 \pm 0.025^{+0.007+0.022}_{-0.022-0.023} \quad \text{ZEUS (photoproduction) [12]} \\
P^D_V &= 0.590 \pm 0.037^{+0.022}_{-0.018} \quad \text{ZEUS (DIS) [13]} \\
P^D_V &= 0.693 \pm 0.045 \pm 0.004 \pm 0.009 \quad \text{H1 [14]} \\
P^D_V &= 0.614 \pm 0.023 \
\end{align*}
\]

These values provide a world average

\[
P^D_V = 0.611 \pm 0.016.
\]

For \(B\) mesons there is only one measurement available [2]

\[
P^B_V = 0.75 \pm 0.04^{+0.023}_{-0.025}.
\]

The values for the charmed mesons are clearly smaller than the widely used naive spin counting prediction \(P_V = 0.75\), but this is not the case for the bottom mesons. So, whatever mechanism in the fragmentation process is claimed to be responsible for the decrease of \(P_V\) with respect to the naive prediction should, in a natural way, produce a much smaller effect for mesons containing a \(b\) quark than for mesons containing a \(c\) quark.

Inspired by the fact that positronium production cross-section (where the calculations are fully under control because the theory, QED, is perturbative) is proportional to the square of the wave function at the origin [16], in this paper we propose that the fragmentation function of a heavy quark, \(Q\), into a heavy meson \(M\) should be proportional to the square of the meson \(M\) wave function at the origin. At lowest order in Heavy Quark Effective Theory the wave functions at the origin for the pseudoscalar and vector states are the same and this would reproduce the naive spin counting result, but they differ at \(O(1/m_Q)\) and, obviously, the effects on the \(P_V\) predictions will be larger for mesons containing a \(c\) quark than for mesons containing a \(b\) quark. We, thus, propose to modify the naive \(P_V\) expression (1) and to consider instead

\[
P_V = \left( \frac{P}{V} + 1 \right)^{-1} = \left( \frac{1}{3} \frac{|\psi_P(0)|^2}{|\psi_V(0)|^2} + 1 \right)^{-1}.
\]

Notice that our predictions for the different rates \(P_V\) will be parameter free (once the wave functions at the origin are known), because we only modify the normalization of the fragmentation functions leaving their dependence on the heavy quark momentum unchanged.

It is important to notice that in the analysis of semileptonic meson decays a sensible improvement of the theoretical results when compared with experimental data, was obtained by breaking heavy quark symmetry and taking hyperfine interactions into account [17].

In the update [17] of the Isgur-Scora-Gronstein-Wise model [18] one finds approximate variational wave functions that consider separately each spin state. The distinction between spin states is mandatory in order to get agreement with the experimental data for the decays. The wave functions for the lowest lying pseudoscalar and vector states are written in terms of a single parameter \(\beta_S\) as

\[
\psi_{1S} = \frac{\beta_S^{3/2}}{\pi^{3/4}} \exp \left( -\frac{1}{2} \beta_S^2 r^2 \right)
\]

The value of \(\beta_S\) is fixed for each meson to properly describe its decay and the values obtained in Ref. [17] for different mesons are shown in Table I.

The results we obtain plugging the wave functions at the origin from [17] into our Eq. (5) are shown in the last column of Table I. We obtain a sensible reduction in the value of \(P^D_V\) with respect to the naive spin counting prediction and our result is in good agreement with the experimental data. For \(P^B_V\) the obtained reduction is much smaller and the result is within one standard deviation of the experimental data, Eq. (4). We should stress here once more that these numbers have been obtained without using any free parameter since the value of the \(\beta_S\) parameter has been fixed in an independent analysis.

We have also calculated our predictions for the relative number of \(D_s\) and \(D^*_s\) as well as \(B_s\) and \(B^*_s\) and shown the results in the last column of in Table I. The reduction with respect to the naive prediction is larger in the mesons containing a heavy and a strange quark than in the mesons containing a heavy and a \(u\) or \(d\) quark. This is a prediction of our assumption that can be experimentally checked. In particular, the low value of \(P^{D^*_s}_V\) looks promising for such a test.
TABLE I: Values of the parameter $\beta_S$ entering in the wave functions of the mesons, see Eq. (6) and predictions for heavy meson production rates

| Meson | Mass (MeV) | $\beta_S$ (GeV) | $P_V$ |
|-------|-----------|-----------------|-------|
| $D$   | 1864.1    | 0.45            | 0.64  |
| $D^*$ | 2006.7    | 0.38            |       |
| $D_s$ | 1969.0    | 0.56            |       |
| $D^*_s$ | 2112.0 | 0.44            |       |
| $B$   | 5279.3    | 0.43            | 0.71  |
| $B^*$ | 5325.0    | 0.40            |       |
| $B_s$ | 5369.6    | 0.54            | 0.69  |
| $B^*_s$ | 5412.8 | 0.49            |       |

The expression in Eq. (5) is only valid if both the pseudoscalar and vector mesons entering have the same quark content. If only one of them contains a strange quark (in addition to the heavy quark) one has to take into account also the strange suppression factor that is usually defined as the ratio of the number of charmed strange mesons with respect to the number of charmed non-strange mesons produced. However, under our hypothesis one should be careful because these ratios depend now on the wave functions at the origin. One would expect, however, that the strange suppression factor would only contain information about the relative probability of producing a $s\bar{s}$ from the vacuum compared to the probability of producing a pair of light quarks. This means, one would expect the strange suppression factor to be independent of the spin of the produced mesons. So, in order to define such a spin independent strange suppression factor one has to use:

$$\gamma_{SI} = \frac{|\psi_D(0)|^2}{|\psi_{D_s}(0)|^2}$$
$$\gamma_P = \frac{|\psi_{D^*}(0)|^2}{|\psi_{D^*_s}(0)|^2}$$

$$\gamma_V.$$

where $\gamma_P = \frac{D_s}{D}$ and $\gamma_V = \frac{D^*_s}{D^*}$ are the suppression factors measured in the pseudoscalar and vector channels, respectively. Using the $\beta_S$ parameters listed in Table I, it is clear that $\gamma_{SI} = 0.52 \gamma_P = 0.64 \gamma_V$.

Before ending we would like to briefly comment on the use of the model for fragmentation functions proposed in Ref. [8] in computing the ratio $P_V$. In this case one has spin dependent parametrizations depending on an extra parameter called $r$ related to the heavy quark and heavy meson masses. Clearly, in this case one can obtain agreement with experimental data by looking for the appropriate value of $r$, but also there is a dependence on the square of the wave functions at the origin (see Eqs. (31) and (32) in [8]) that, in order to be consistent with the results in Ref. [17], take different values for different mesons.

Final Remarks

The naive spin counting prediction for the ratio $P_V$ does not fit the experimental data, in particular in the case of charmed $D$ mesons. We propose that the fragmentation functions should be proportional to the square of the produced meson wave function at the origin. We have analyzed the effects of this proposal on the values of $P_V$ for charmed and bottom mesons and found very good agreement with the experimental data using values for the wave functions at the origin fixed from meson decays. As a way to check our proposal we estimate the values of $P_V$ for charmed-strange and bottom-strange mesons for which no experimental data are yet available. Further checks can be performed measuring our proposed spin dependence of the strange suppression factor.

Acknowledgements

We thank F. del Aguila, L. Labarga and M. Zambrana for useful discussions. F.C. acknowledges financial support from Junta de Andalucía (FQM-330) and Ministerio Español de Educación y Ciencia (FPA2006-05294). CAGC
warmly acknowledges the hospitality of Departamento de Física Teórica y del Cosmos of the Universidad de Granada that made possible this collaboration and to the Junta de Andalucía for financial support during his visit to Granada.

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