Assignments of Universal Texture Components
for Quark and Lepton Mass Matrices

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Abstract

We reanalyze the mass matrices model of quarks and leptons which gives a unified description of quark and lepton mass matrices with the same texture form. By investigating possible types of the assignment for the texture’s components of this mass matrix form, we find that a different assignment for up-quarks from one for down-quarks can lead to consistent values of CKM mixing matrix. This finding overcomes a weak point of the previous analysis of the model. We also obtain some relations among the CKM mixing matrix parameters, which are independent of evolution effects.
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Recent neutrino oscillation experiments [1] have highly suggested a nearly bimaximal lepton mixing ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \sim 1$) in contrast with the small quark mixing. In order to reproduce these lepton and quark mixings, mass matrices of various structures with texture zeros has been investigated in the literature. Symmetric or Hermitian six and five texture zero models were systematically discussed by Ramond et al. [2]. Before the work of Ramond et al., Fritzsch [3] proposed a six texture zero model, nonsymmetric or non-Hermitian six texture zero quark mass matrices model[nearest-neighbor-interaction(NNI) model] was proposed by Branco, Lavoura, and Mota [4]. Subsequently, Hermitian four texture model dealing with the quark and lepton mass matrices on the same footing are discussed by many authors [5]. Phenomenological quark mass matrices have been discussed from various points of view [6]. Recently a mass matrix model has been proposed [7] with the following universal structure for all quarks and leptons

$$
\begin{pmatrix}
0 & A & A \\
A & B & C \\
A & C & B
\end{pmatrix}
$$

(1)

This form has been originally used for leptons(neutrinos) in order to reproduce a nearly bimaximal lepton mixing [8]– [13]. Nevertheless it turns out that this model also leads to reasonable values of CKM quark mixing [7]. Unfortunately, however, the model in Ref [7] leads to $|V_{ub}| = \sqrt{m_u/m_c}$, and consequently to some what small predicted value for $|V_{ub}| = 0.0020 - 0.0029$ with respect to the present experimental value $|V_{ub}| = 0.0036 \pm 0.0007$ [14].

This is caused by the assignment used in ref [7] for texture’s components(A,B, and C) of the mass matrix in which $A = \pm \sqrt{m_i m_j}$, $B = \frac{1}{2} (m_3 + m_2 - m_1)$ and $C = -\frac{1}{2} (m_3 - m_2 + m_1)$ with the i-th generation mass $m_i$ are proposed both for up- and down-quarks. In the present paper, we concentrate ourselves on this type of mass matrix model and explore every new possible assignments for texture’s components(A, B, and C). We will point out that there exists another possible new assignment for A, B, and C and that a different type of the assignment for up-quarks from the one for down-quarks can lead to consistent values of CKM
mixing matrix. Namely we show the combination of the new assignment that $A = \pm \sqrt{\frac{m_2 m_3}{2}}$, $B = \frac{1}{2} (m_3 + m_2 - m_1)$ and $C = \frac{1}{2} (m_3 - m_2 - m_1)$ for up-quarks, and previously proposed one that $A = \pm \sqrt{\frac{m_2 m_1}{2}}$, $B = \frac{1}{2} (m_3 + m_2 - m_1)$ and $C = -\frac{1}{2} (m_3 - m_2 + m_1)$ for down-quarks is favorable to reproduce the consistent values of CKM quark mixing. This new finding of the another assignment overcomes the above weak point of the approach in Ref [7]. The new and positive feature of this work is presented in Table 1.

Our mass matrices $M_u$, $M_d$, $M_\nu$ and $M_e$ (mass matrices of up quarks $(u, c, t)$, down quarks $(d, s, b)$, neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and charged leptons $(e, \mu, \tau)$, respectively) are given as follows: [7]

$$M_f = P_f^\dagger \tilde{M}_f P_f^\dagger,$$  

(2)

with

$$\tilde{M}_f = \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} \quad (f = u, d, \nu, e),$$  

(3)

where $P_{Lf}$ is the diagonal phase matrices and $A_f$, $B_f$, and $C_f$ are real parameters. This structure of mass matrix was previously suggested and used for the neutrino mass matrix in Refs [8]– [13], using the basis where the charged-lepton mass matrix is diagonal, motivated by the experimental finding of maximal $\nu_\mu$–$\nu_\tau$ mixing [1].

Hereafter, for brevity, we will omit the flavour index. The eigen-masses of Eq. (3) are given by $\frac{1}{2} \left(B + C - \sqrt{8A^2 + (B + C)^2}\right)$, $\frac{1}{2} \left(B + C + \sqrt{8A^2 + (B + C)^2}\right)$, and $(B - C)$. Therefore, there are three types of assignments for the texture’s components of $\tilde{M}$ according to the assignments for the eigen-mass $m_i$ :

(i) Type A:

$$-m_1 = \frac{1}{2} \left(B + C - \sqrt{8A^2 + (B + C)^2}\right),$$  

(4)

$$m_2 = \frac{1}{2} \left(B + C + \sqrt{8A^2 + (B + C)^2}\right),$$  

(5)

$$m_3 = B - C.$$  

(6)
This is the case that \( B - C \) is the largest value. In this type, the texture’s components of \( \hat{M} \) are expressed in terms of \( m_i \) as

\[
A = \pm \sqrt{\frac{m_2 m_1}{2}},
\]
\[
B = \frac{1}{2} (m_3 + m_2 - m_1),
\]
\[
C = -\frac{1}{2} (m_3 - m_2 + m_1).
\] (7)

That is, \( \hat{M} \) is diagonalized by an orthogonal matrix \( O \) as

\[
O^T \hat{M} O = \begin{pmatrix}
-m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix},
\] (8)

with

\[
O \equiv \begin{pmatrix}
\pm c & \pm s & 0 \\
-s \sqrt{\frac{c}{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-s \sqrt{\frac{c}{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\] (9)

Here \( c \) and \( s \) are defined by \( c = \sqrt{\frac{m_2}{m_2 + m_1}} \) and \( s = \sqrt{\frac{m_1}{m_2 + m_1}} \). It should be noted that the elements of \( O \) are independent of \( m_3 \) because of the above structure of \( \hat{M} \). This type A is used in Ref [7].

(ii)Type B: This assignment is obtained by exchanging \( m_2 \) and \( m_3 \) in Type A.

\[
-m_1 = \frac{1}{2} \left( B + C - \sqrt{8A^2 + (B + C)^2} \right),
\] (10)
\[
m_2 = B - C,
\] (11)
\[
m_3 = \frac{1}{2} \left( B + C + \sqrt{8A^2 + (B + C)^2} \right).
\] (12)

In this type, the texture’s components of \( \hat{M} \) are expressed as

\[
A = \pm \sqrt{\frac{m_3 m_1}{2}},
\]
\[
B = \frac{1}{2} (m_3 + m_2 - m_1),
\]
\[
C = \frac{1}{2} (m_3 - m_2 - m_1).
\] (13)
The orthogonal matrix $O'$ which diagonalizes $\hat{M}$ ($(O'\hat{M}O' = \text{diag}(-m_1, m_2, m_3)$) is given by

$$O' \equiv \begin{pmatrix} \pm c' & 0 & \pm s' \\ -\frac{s'}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{c'}{\sqrt{2}} \\ -\frac{c'}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{s'}{\sqrt{2}} \end{pmatrix}. \quad (14)$$

Here $c'$ and $s'$ are defined by $c' = \sqrt{\frac{m_3}{m_3 + m_1}}$ and $s' = \sqrt{\frac{m_1}{m_3 + m_1}}$.

(iii) Type C: This assignment is obtained by exchanging $m_1$ for $m_2$ in Type B. However, this type is not useful in the discussion on the quark sector.

Taking the type A assignment of mass matrices both for up and down quarks, the authors in Ref [7] have discussed the quark mixing matrix and obtained the following prediction which is almost independent of the RGE effects,

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}} = 0.51 - 0.067. \quad (15)$$

By substituting the experimental values $|V_{cb}|_{exp} = 0.0412 \pm 0.020$ [14] into Eq.(15), one obtain

$$|V_{ub}| = \sqrt{\frac{m_u}{m_c}} |V_{cb}|_{exp} = 0.0020 - 0.0029. \quad (16)$$

However, this value is somewhat smaller than the present experimental value $|V_{ub}| = 0.0036 \pm 0.0007$ [14].

In this paper, taking the type B assignment for up quarks and the type A for down quarks, we reanalyze the quark mixing matrix of the model. In this assignment, $M_u$ and $M_d$ have the same zero texture with different assignments as follows:

$$M_u = P_u^\dagger \begin{pmatrix} 0 & \pm \sqrt{\frac{m_t^0 m_c^0}{2}} & \pm \sqrt{\frac{m_t^0 m_u^0}{2}} \\ \pm \sqrt{\frac{m_t^0 m_c^0}{2}} & \frac{1}{2} (m_t^0 + m_c^0 - m_u^0) & \frac{1}{2} (m_t^0 - m_c^0 - m_u^0) \\ \pm \sqrt{\frac{m_t^0 m_u^0}{2}} & \frac{1}{2} (m_t^0 - m_c^0 - m_u^0) & \frac{1}{2} (m_t^0 + m_c^0 - m_u^0) \end{pmatrix} P_u,$$

$$M_d = P_d^\dagger \begin{pmatrix} 0 & \pm \sqrt{\frac{m_b^0 m_s^0}{2}} & \pm \sqrt{\frac{m_b^0 m_d^0}{2}} \\ \pm \sqrt{\frac{m_b^0 m_s^0}{2}} & \frac{1}{2} (m_b^0 + m_s^0 - m_d^0) & -\frac{1}{2} (m_b^0 - m_s^0 + m_d^0) \\ \pm \sqrt{\frac{m_b^0 m_d^0}{2}} & -\frac{1}{2} (m_b^0 - m_s^0 + m_d^0) & \frac{1}{2} (m_b^0 + m_s^0 - m_d^0) \end{pmatrix} P_d. \quad (17)$$
where \( P_u \) and \( P_d \) are the \( CP \) violating phase factors. These quark mass matrices \( M_f = P_f^\dagger \tilde{M}_f P_f^\dagger \) \( (f = u, d) \) are diagonalized by the bi-unitary transformation

\[
D_f = U_{L_f}^\dagger M_f U_{R_f},
\]

(18)

where \( U_{L_u} \equiv P_u^\dagger O_u' \), \( U_{R_u} \equiv P_u O_u' \), \( U_{L_d} \equiv P_d^\dagger O_d \), and \( U_{R_d} \equiv P_d O_d \). Here \( O_u' \) and \( O_d \) are given by Eq. (14) and Eq. (9), respectively. Then, the Cabibbo–Kobayashi–Maskawa (CKM) \[15\] quark mixing matrix \( V \) is given by

\[
V = U_{L_u}^\dagger U_{L_d} = O_u'^T P_u P_d^\dagger O_d
\]

\[
= \begin{pmatrix}
    c_u' c_d + \rho s_u' s_d & c_u' s_d - \rho s_u' c_d & -\sigma s_u' \\
    -\sigma s_d & \sigma c_d & \rho \\
    s_u' c_d - \rho c_u' s_d & s_u' s_d + \rho c_u' c_d & \sigma c_u'
\end{pmatrix},
\]

(19)

where \( \rho \) and \( \sigma \) are defined by

\[
\rho = \frac{1}{2} (e^{i\delta_3} + e^{i\delta_2}) = \cos \frac{\delta_3 - \delta_2}{2} \exp i \left( \frac{\delta_3 + \delta_2}{2} \right),
\]

(20)

\[
\sigma = \frac{1}{2} (e^{i\delta_3} - e^{i\delta_2}) = \sin \frac{\delta_3 - \delta_2}{2} \exp i \left( \frac{\delta_3 + \delta_2}{2} + \frac{\pi}{2} \right).
\]

(21)

Here we have put \( P \equiv P_u P_d^\dagger \equiv \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}) \), and we have taken \( \delta_1 = 0 \) without loss of generality.

Then, the explicit magnitudes of the components of \( V \) are expressed as

\[
|V_{cb}^0| = |\rho| = \cos \frac{\delta_3 - \delta_2}{2}, \quad (22)
\]

\[
|V_{ub}^0| = |\sigma| s_u' = \sin \frac{\delta_3 - \delta_2}{2} \sqrt{\frac{m_u^0}{m_d^0}} \sqrt{m_u^0}, \quad (23)
\]

\[
|V_{cd}^0| = |\sigma| s_d = \sin \frac{\delta_3 - \delta_2}{2} \sqrt{\frac{m_d^0}{m_s^0}} \sqrt{m_d^0}, \quad (24)
\]

\[
|V_{us}^0| = c_u' s_d |1 + \rho \frac{c_u' c_d}{s_u' s_d}| = \sqrt{\frac{m_t^0}{m_t^0 + m_u^0}} \sqrt{\frac{m_s^0}{m_s^0 + m_d^0}} \times \left[ 1 + 2 \cos \frac{\delta_3 - \delta_2}{2} - \cos \frac{\delta_3 + \delta_2}{2} \sqrt{\frac{m_u^0 m_s^0}{m_t^0 m_d^0}} + \cos^2 \frac{\delta_3 - \delta_2}{2} \left( \frac{m_u^0 m_s^0}{m_t^0 m_d^0} \right)^{\frac{1}{2}} \right], \quad (25)
\]

\[
6
\]
\[ |V_{td}^{0}| = c_{ud}' s_{d} \rho \pm \frac{s_{u}' c_{ud}}{c_{u} s_{d}} = \sqrt{\frac{m_{t}^{0}}{m_{u}^{0} + m_{d}^{0}}} \sqrt{\frac{m_{d}^{0}}{m_{s}^{0} + m_{d}^{0}}} \times \left[ \cos^{2} \frac{\delta_{3} - \delta_{2}}{2} \pm 2 \cos \frac{\delta_{3} - \delta_{2}}{2} \cos \frac{\delta_{3} + \delta_{2}}{2} \sqrt{\frac{m_{u}^{0} m_{d}^{0}}{m_{l}^{0} m_{d}^{0}}} \left( \frac{m_{u}^{0}}{m_{l}^{0} m_{d}^{0}} \right) \right]^{\frac{1}{2}}. \quad (26) \]

\[ |V_{ts}^{0}| = c_{ud}' c_{d} \rho \pm \frac{s_{u}' c_{ud}}{s_{u} c_{d}} = \sqrt{\frac{m_{t}^{0}}{m_{u}^{0} + m_{d}^{0}}} \sqrt{\frac{m_{s}^{0}}{m_{s}^{0} + m_{d}^{0}}} \times \left[ \cos^{2} \frac{\delta_{3} - \delta_{2}}{2} \pm 2 \cos \frac{\delta_{3} - \delta_{2}}{2} \cos \frac{\delta_{3} + \delta_{2}}{2} \sqrt{\frac{m_{u}^{0} m_{d}^{0}}{m_{l}^{0} m_{d}^{0}}} \left( \frac{m_{u}^{0}}{m_{l}^{0} m_{d}^{0}} \right) \right]^{\frac{1}{2}}. \quad (27) \]

It should be noted that \(|V_{us}^{0}|, |V_{td}^{0}|, \text{ and } |V_{ts}^{0}|\) are almost independent of \((\delta_{3} + \delta_{2})\) and they are given from Eq. (25)-(27) as

\[ |V_{us}^{0}| \simeq \sqrt{\frac{m_{d}^{0}}{m_{s}^{0}}}, \quad (28) \]

\[ |V_{td}^{0}| \simeq \sqrt{\frac{m_{d}^{0}}{m_{s}^{0}}} \cos \frac{\delta_{3} - \delta_{2}}{2}, \quad (29) \]

\[ |V_{ts}^{0}| \simeq \cos \frac{\delta_{3} - \delta_{2}}{2}. \quad (30) \]

Therefore, the independent parameters in the expression \(|V_{ij}^{0}|\) are \(\theta_{u}' = \tan^{-1}(m_{u}^{0}/m_{l}^{0}), \theta_{d} = \tan^{-1}(m_{d}^{0}/m_{s}^{0})\), and \((\delta_{3} - \delta_{2})\). Among them, the two parameters \(\theta_{u}'\) and \(\theta_{d}\) are already fixed by the quark masses. Therefore, the present model has an adjustable parameter \((\delta_{3} - \delta_{2})\) in \(|V_{ij}^{0}|\), which is fixed to reproduce the observed CKM matrix parameters at \(\mu = m_{Z}\) [14]:

\[ |V_{us}|_{\exp} = 0.2196 \pm 0.0026, \quad |V_{cb}|_{\exp} = 0.0412 \pm 0.0020, \]

\[ |V_{ub}|_{\exp} = (3.6 \pm 0.7) \times 10^{-3}, \quad (31) \]

The relations in Eqs. (22)–(27) hold only at the unification scale \(\mu = M_{X}\). So we now consider evolution effects. As is well known [16], the evolution effects are approximately described as

\[ \frac{m_{u}^{0}}{m_{l}^{0}} \simeq \frac{m_{u}^{0}}{m_{d}^{0}} \simeq \frac{m_{d}^{0}}{m_{s}^{0}} \simeq \frac{m_{s}^{0}}{m_{d}^{0}}, \quad \frac{m_{u}^{0}}{m_{c}^{0}} \simeq \frac{m_{d}^{0}}{m_{s}^{0}} \simeq \frac{m_{s}^{0}}{m_{d}^{0}} \simeq 1 + \epsilon_{d}, \quad (32) \]

\[ \frac{m_{u}^{0}}{m_{c}^{0}} \simeq \frac{m_{d}^{0}}{m_{s}^{0}} \simeq \frac{m_{s}^{0}}{m_{d}^{0}} \simeq 1. \quad (33) \]
where \( m_q^0 \) and \( V_{ij}^0 \) \((q\) and \( V_{ij} \)) denote the values at \( \mu = M_X (\mu = m_Z) \). In the following numerical calculations, we use the running quark mass at \( \mu = m_Z \) and at \( \mu = M_X \) [17]:

\[
\begin{align*}
m_u(m_Z) &= 2.33^{+0.42}_{-0.45} \, \text{MeV}, & m_c(m_Z) &= 677^{+56}_{-61} \, \text{MeV}, & m_t(m_Z) &= 181 \pm 13 \, \text{GeV}, \\
m_d(m_Z) &= 4.69^{+0.60}_{-0.66} \, \text{MeV}, & m_s(m_Z) &= 93.4^{+11.8}_{-13.0} \, \text{MeV}, & m_b(m_Z) &= 3.00 \pm 0.11 \, \text{GeV}.
\end{align*}
\]

(34)

\[
\begin{align*}
m_u(M_X) &= 1.04^{+0.19}_{-0.20} \, \text{MeV}, & m_c(M_X) &= 302^{+25}_{-27} \, \text{MeV}, & m_t(M_X) &= 129^{+106}_{-40} \, \text{GeV}, \\
m_d(M_X) &= 1.33^{+0.17}_{-0.19} \, \text{MeV}, & m_s(M_X) &= 26.5^{+3.3}_{-3.7} \, \text{MeV}, & m_b(M_X) &= 1.00 \pm 0.04 \, \text{GeV}.
\end{align*}
\]

(35)

First we note that the predictions

\[
\begin{align*}
|V_{us}^0| &\simeq c_u s_d \simeq \sqrt{m_u^0 / m_d^0} \simeq 0.224, \\
\frac{|V_{cb}^0|}{|V_{td}^0|} &\sim \frac{s_d}{c_d} = \sqrt{m_d^0 / m_s^0} \simeq 0.224,
\end{align*}
\]

are almost independent of the RGE effects, because they do not contain the phase difference, \((\delta_3 - \delta_2)\), which is highly dependent on the energy scale as we discussed in the previous analysis and we know that the ratio \( m_d / m_s \) is almost independent of the RGE effects. We also obtain the relation

\[
\frac{|V_{ub}^0|}{|V_{cd}^0|} = \sqrt{(m_s^0 + m_d^0)/m_u^0(m_t^0 + m_u^0)m_d^0},
\]

(38)

which is independent of the phase difference.

Next let us fix the parameters \( \delta_3 - \delta_2 \) using the expression of Eq. (22) which holds at \( \mu = M_X \). From Eq. (22), with taking \( \epsilon_d = \frac{m_d^0 / m_u^0}{m_d^0 / m_b^0} - 1 = -0.149 \), we have

\[
\cos \frac{\delta_3 - \delta_2}{2} = |V_{cb}|_{\exp} (1 + \epsilon_d) = (0.0412 \pm 0.0020)(1 + \epsilon_d),
\]

(39)

\[
\delta_3 - \delta_2 = 175.98^\circ.
\]

(40)

Thus we obtain \( V_{ij} \) at \( \mu = m_Z \) as follows
\[ |V_{ub}| \approx |V_{ub}^0| \frac{1}{1 + \epsilon_d} = \sqrt{\frac{m_d^0}{m_s^0}} \sqrt{1 - |V_{cb}|_{\exp}^2 (1 + \epsilon_d)^2} \frac{1}{1 + \epsilon_d} \approx 0.0033, \]  \hspace{1cm} (41)

\[ |V_{cd}| \approx |V_{cd}^0| = \sqrt{\frac{m_d^0}{m_s^0}} \sqrt{1 - |V_{cb}|_{\exp}^2 (1 + \epsilon_d)^2} \approx 0.22, \]  \hspace{1cm} (42)

\[ |V_{ts}| \approx |V_{ts}^0| \frac{1}{1 + \epsilon_d} \approx |V_{cb}|_{\exp} \approx 0.041, \]  \hspace{1cm} (43)

\[ |V_{td}| \approx |V_{td}^0| \frac{1}{1 + \epsilon_d} \approx \sqrt{\frac{m_d^0}{m_s^0}} |V_{cb}|_{\exp} \approx 0.0092, \]  \hspace{1cm} (44)

which are consistent with the present experimental data. Therefore, the value of \((\delta_3 - \delta_2)\) in Eq. (40) is acceptable.

The remaining parameter \((\delta_3 + \delta_2)\) in this model remains a free parameter to be fixed by the observed CP-violating phase \(\delta\) in the standard representation of the CKM quark mixing matrix

\[
V_{\text{std}} = \text{diag}(e^{\alpha_1^u}, e^{\alpha_2^u}, e^{\alpha_3^u}) V \text{ diag}(e^{\alpha_1^d}, e^{\alpha_2^d}, e^{\alpha_3^d})
\]

\[
= \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} . \hspace{1cm} (45)
\]

Here, \(\alpha_i^q\) comes from the rephasing in the quark fields to make the choice of phase convention.

The \(\delta\) in Eq. (45) is expressed, in the present model, by

\[
\delta = -\frac{\delta_3 + \delta_2}{2} , \hspace{1cm} (46)
\]

so that from the observed value \(\delta_{\exp} = 59^\circ \pm 13^\circ\) we can fix \(\delta_3 + \delta_2 = -118^\circ \pm 26^\circ\).

Note that the mass matrix does not keep its texture form at \(\mu = m_Z\) because (1,2) and (1,3) components are not so small. Therefore, we consider the RGE effect more precisely.

In our model, the \(|V_{cb}|\) and the CP violating phase \(\delta\) are controllable parameters with the unobservable parameters \(\delta_2\) and \(\delta_3\), so we can adjust these parameters to the center values at \(\mu = m_Z\). On the other hand, \(|V_{us}|\) and \(|V_{ub}|\) are restricted in our model by mass regions in Eq.(34). Under these conditions, we estimate the numerical variation of the CKM matrix.
elements by using the two-loop RGE (MSSM (tan β = 10) case) for the Yukawa coupling constants [17]. Then, we obtain the following numerical results for the evolution effects:

\[
|V^{0}_{us}| = \left[ \frac{m^0_d}{m^0_d + m^0_s} \right] = 0.19 - 0.25 \quad \rightarrow \quad |V_{us}| = 0.19 - 0.25, \quad (47)
\]

\[
|V^{0}_{ub}| = \left[ \frac{m^0_u}{m^0_u + m^0_t} \right] = 0.0020 - 0.0034 \quad \rightarrow \quad |V_{ub}| = 0.0027 - 0.0038, \quad (48)
\]

\[
|V^{0}_{cb}| = 0.035 \quad \rightarrow \quad |V_{cb}| = 0.041, \quad (49)
\]

\[
\delta^0 = 59^\circ \quad \rightarrow \quad \delta = 59^\circ. \quad (50)
\]

These values are consistent with the approximations in Eqs.(36) - (44) and also the experimental data in Eq.(31).

In conclusion, we have reanalyzed the quark mixing matrix using the mass matrix model of Ref [7] with the universal texture form. We use different types of assignments for \(A_f, B_f,\) and \(C_f\) in \(\tilde{M}_f\) for (f = u and d). Namely, the type A for \(\tilde{M}_d,\) while the type B for \(\tilde{M}_u\) are considered in this paper. This is in contrast with the previous analysis in Ref [7] with use of the same type for both, which leads to somewhat small predicted value for \(|V_{ub}|\) compared with the experimental value. It is shown that the present model predicts consistent values of CKM mixing matrix and the above weak point of the previous model is overcome. We also have relations \(|V_{us}| \simeq \sqrt{m_d/m_s}\) and \(|V_{td}|/|V_{ts}| \simeq \sqrt{m_u/m_s}\) which are almost independent of RGE effects.

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TABLES

| down \ up | Type A | Type B | Type C |
|----------|--------|--------|--------|
|          | \frac{U_{ub}}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |
|          | \frac{|U_{ub}|}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |
|          | \frac{|U_{ub}|}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |
| Type A   | \frac{|U_{ub}|}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |
| Type B   | \frac{|U_{ub}|}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |
| Type C   | \frac{|U_{ub}|}{U_{cb}} = \sqrt{\frac{m_u^0}{m_d^0}} \lesssim \exp | \frac{|U_{us}|}{U_{cb}} = \sqrt{\frac{m_d^0}{m_u^0}} \simeq \exp | \frac{|U_{ud}|}{U_{cd}} = \sqrt{\frac{m_d^0}{m_u^0}} \gg \exp |

**TABLE I.** The results of quark mixing matrix element $U_{ij}$ are shown for three types of the assignment for up and down quarks mass matrices. These results are independent of the phases $\delta_2$ and $\delta_3$ and are useful for the consistency check between our model and the experiments because these values are hardly changed by the RGE effects from the GUT scale to the EW scale. The "exp" represents the corresponding experimental value of the left-hand side of inequality or equation. Here Type A is proposed in Ref [7]. Type B and Type C are new assignments proposed in the present paper. We find that only a combination of Type B for up and Type A for down quarks leads to the well consistent CKM mixing matrix. Other combinations fail to reproduce consistent quark mixing because of the results indicated in this table.