Modeling and measuring the non-ideal characteristics of transmission lines

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We describe a simple method to experimentally determine the frequency dependencies of the per-unit-length resistance and conductance of transmission lines. The experiment is intended as a supplement to the classic measurement of the transient response of a transmission line to a voltage step or pulse. In the transient experiment, an ideal (lossless) model of the transmission line is used to determine the characteristic impedance and signal propagation speed. In our experiment, the insertion losses of various coaxial cables are measured as a function of frequency from 1 to 2000 MHz. A full distributed circuit model of the transmission line that includes both conductor and dielectric losses is needed to fit the frequency dependence of the measured insertion losses. Our model assumes physically-sensible frequency dependencies for the per-unit-length resistance and conductance that are determined by the geometry of the coaxial transmission lines used in the measurements.

I. INTRODUCTION

Lumped-element circuit analysis fails when the wavelengths of the signals of interest approach the size of the circuit elements and/or connecting wires. In this limit, the voltage and current along, for example, the length of a pair of wires are not uniform and a distributed circuit model of the wires must be used to properly analyze the circuit behavior. These so-called transmission line effects are rich in physics and, in many cases, can defy common intuition. Furthermore, due to the ever decreasing size of circuits and increasing data rates, transmission line effects are more prevalent than ever.

From a pedagogical standpoint, transmission lines are commonly used when deriving the expression for the thermal noise radiated by a resistor. In the derivation a transmission line, with both ends terminated by matched load resistors, is treated as a 1-D blackbody. The thermal power radiated by one resistor is completely absorbed by the other and the emitted radiation satisfies standing wave conditions (normal modes) set by the length of the transmission line. Transmission lines models have also been used to analyze problems in thermodynamics and mechanics. An RC transmission line circuit model has been used to understand diffusion of heat along the length of a conducting bar and an analogy has been made between a system of coupled pendula and coupled transmission lines. The physics of transmission lines has also been analyzed using a Lagrangian formalism and Fourier transforms.

It is also worth pointing out that there have been very clever uses of discrete transmission lines. In one example, reverse-biased variable capacitance diodes (varactor diodes) were used to construct a nonlinear transmission line that supports solitons. In a second example, the capacitance in some sections of the discrete transmission line was changed to mimic a change in refractive index. These structures were used to experimentally demonstrate the principles behind highly-reflective dielectric mirrors and confinement of electromagnetic (EM) waves by Bragg reflection. More recently, discrete left-handed transmission lines have been made from arrays of series capacitors and shunt inductors. Left-handed transmission lines have an unusual dispersion relation in which $\beta \propto -1/\omega$, where $\beta$ is the wavenumber and $\omega$ is angular frequency.

Many of the undergraduate transmission line experiments described in the literature focus on the propagation of short pulses along the length of a line that is assumed to be lossless. These measurements allow students to determine the propagation speed of the pulse and observe phase changes resulting from reflections at various load terminations. By tuning the load resistance to eliminate the reflections, students can also estimate the characteristic impedance of the transmission line. Another experiment in which dissipative effects are typically neglected is the transient response of a transmission line to an applied pulse that is many times longer than the time required to travel the length of the line. This measurement is particularly interesting and it is one that we demonstrate and discuss in this paper.

Our primary focus, however, will be on quantitatively measuring and analyzing transmission line dissipation due to conductor and dielectric losses. There are examples of undergraduate laboratories in which the dissipation due to a length of transmission line has been measured. However, in these cases, after making an attenuation measurement, there is little to no discussion about the origins of the dissipation and the relative importance of the various sources of loss. Our first objective in this paper is to make a simple, yet reasonably precise, measurement of the power dissipation due to a length of transmission line over a wide frequency range. The second, and more important, objective is to use the data and physical insights to quantitatively determine the relative magnitudes of conductor and dielectric losses as a function of frequency.

The outline of the paper is as follows: In Sec. I, the distributed circuit model of a transmission line and some of its important features are reviewed. Section II presents the transient response of transmission lines to a long-duration pulse. The experimental results are used to determine the per-unit-length capacitance and inductance of ideal (lossless) transmission lines. Dissipative...
FIG. 1. (a) A transmission line of length \( \ell \) connected to a signal source with output impedance \( Z_0 \) at \( x = -\ell \) and a load impedance \( Z_L \) at \( x = 0 \). (b) The distributed circuit model of a transmission line. The model accurately describes real transmission lines in the limit that \( \Delta x \to 0 \).

Effects are treated in Sec. IV. First, the expected frequency response of the transmission line insertion loss is calculated and then the experimental measurements are presented. In Sec. IV A, physically-motivated models for the frequency dependencies of the per-unit-length resistance and conductance of coaxial transmission lines are developed. These models are then used to fit the experimentally-measured insertion loss in Sec. IV B. Finally, the main results are summarized in Sec. V.

II. DISTRIBUTED CIRCUIT MODEL OF A TRANSMISSION LINE

Figure 1(a) shows a transmission line of length \( \ell \). One end, at \( x = -\ell \), is connected to a signal source that has output impedance \( Z_0 \) and the opposite end, at \( x = 0 \), is terminated by load impedance \( Z_L \). The equivalent distributed circuit model of the transmission line is shown in Fig. 1(b). It is made up of \( n = \ell/\Delta x \) daisy-chained segments, where \( \Delta x \) is the length of each segment. The segments have series inductance and resistance \( L \Delta x \) and \( R \Delta x \), respectively and shunt capacitance and conductance \( C \Delta x \) and \( G \Delta x \), respectively. The distributed circuit model describes the behavior of real transmission lines in the limit that the number of segments \( n \to \infty \) or, equivalently, \( \Delta x \to 0 \).

We now state some of the important results arising from an analysis of the distributed circuit model. Although we don’t derive these results, good treatments are given in Refs. 1 and 2 with additional insights provided in Ref. 20. For harmonic signals \( v_s = V_0 e^{j\omega t} \) of frequency \( \omega \), the voltage and current amplitudes at position \( x \) along the transmission line are given by

\[
V(x) = V_+ e^{-\gamma x} + V_- e^{\gamma x}
\]

(1)

\[
I(x) = \frac{1}{Z_c} [V_+ e^{-\gamma x} - V_- e^{\gamma x}].
\]

(2)

In these expressions, the propagation constant \( \gamma = \alpha + j \beta \) is complex and given by

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)},
\]

(3)

and

\[
Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}},
\]

(4)

is the characteristic impedance of the transmission line. The \( V_+ \) terms in Eqs. (1) and (2) represent signals propagating in the +x direction while \( V_- \) terms represent signals propagating in the -x direction. The \( V_+ \) and \( V_- \) coefficients are related via \( V_- = \Gamma V_+ \) where

\[
\Gamma = \frac{Z_L - Z_c}{Z_L + Z_c},
\]

(5)

is the reflection coefficient determined by the mismatch between the characteristic impedance of the line and the load termination.

A. Lossless transmission lines

In a lossless transmission line \( R \) and \( G \) are assumed to be negligible. In this limit, the propagation constant becomes completely imaginary such that there is no attenuation of the voltage and current amplitudes. Furthermore, the characteristic impedance becomes real and is typically designed to match the output impedance of
the signal source (usually 50 Ω). The results for a lossless transmission line are

\[ \gamma = j \beta = j \omega \sqrt{LC} = j \left( \frac{\omega}{v_0} \right) \quad (6) \]

\[ Z_c = \sqrt{L/C}, \quad (7) \]

where \( v_0 = 1/\sqrt{LC} \) is the signal propagation speed. Note that, if \( v_0 \) and \( Z_c \) are measured, then the inductance per unit length and capacitance per unit length of the transmission line can be determined via \( L = Z_0/v_0 \) and \( C = (v_0 Z_0)^{-1} \).

### III. Transient Response

We now consider a length of lossless transmission line with one end open \((Z_L \to \infty)\) and the opposite end connected to a resistance \( R_g \gg Z_c \). A square pulse of height \( V_0 \) is applied to the free end of \( R_g \). The width of pulse is chosen to be very long compared to the time \( \ell/v_0 \) that it takes signals to travel the length of the transmission line. After the pulse is applied, the time evolution of the voltage \( V_g \) at the junction between \( R_g \) and the transmission line is measured. The experimental setup is shown schematically in Fig. 2(a).

Although we do not provide a full analysis of this problem here, we refer the reader to the treatment given in Sec. 14.4 of Ref. 1. Figure 2(b) shows a measurement of \( V_g \) as a function to time using \( R_g = 1 \) kΩ and a 8.07-m length of semi-rigid UT-141 coaxial cable. The data were recorded using a Tektronix TBS 1104 digital oscilloscope (100 MHz bandwidth) and the pulse was generate using an HP 8011A pulse generator. The step-like pattern observed in \( V_g(t) \) is due to repeated reflections at the two ends of the transmission line. The reflection coefficient at the open end is \( \Gamma = 1 \) and at the source end it is \( \Gamma_g = (R_g - Z_c)/(R_g + Z_c) \).

The time between adjacent steps is \( \Delta t = 2 \ell/v_0 \) which corresponds to the time required for signals to travel twice the length of the line. A plot of the time of the \( N^{th} \) step versus \( N \) results in a straight line with slope \( m_1 = \Delta t \) which can then be used to determine the signal propagation speed.

For steps \( N \geq 2 \), the change in \( V_g \) is given by

\[ \left( \frac{\Delta V_g}{V_0} \right)_N = \frac{1 - \Gamma_g^2 \Gamma_g^N}{2\Gamma_g \Gamma_g^N}, \quad (8) \]

such that, because \( 0 < \Gamma_g < 1 \), the voltage steps decrease in size as \( N \) increases. A plot of \( \ln (\Delta V_g/V_0)_N \) versus \( N \) is linear with slope \( m_2 = \ln \Gamma_g \) which allows for a determination of the characteristic impedance \( Z_c \) of the transmission line.

Figures 3(a) and (b) show results of the analysis of the transient voltage steps for the semi-rigid coaxial cable. The slopes of the linear fits and the corresponding values of \( v_0 \) and \( Z_c \) are given in Table I. Although not shown, the transient responses of an RG-58 BNC coaxial cable and a high-quality (HQ) sma coaxial cable of unknown make and model were also measured. The results of those measurements are likewise summarized in Table II. The table also includes a determination of the dielectric constant \( \varepsilon'' = (c/v_0)^2 \) of the insulator separating the inner and outer conductors of the coaxial cables.

The RG58C/U coaxial cable specifications from Pasternack give a velocity of propagation that is 0.659c and a capacitance of 101.05 pF/m, both of which are in reasonably good agreement with our results. On the other hand, Pasternack specifies a characteristic
impedance of 50 Ω which is about 3.5 standard deviations away from our measurement. We will discuss a possible reason for this difference in Sec. IV B. The specifications for the UT-141-HA-M17 semi-rigid coaxial cable from Micro-Coax are away from our measurement. We will discuss a possible reason for this difference in Sec. IV B. The specifications for the UT-141-HA-M17 semi-rigid coaxial cable from Micro-Coax are ε₀ = 0.7c, C = 98.1 pF/m, and Z₀ = 50 ± 1Ω, all of which are in reasonable agreement with the results given in Table I.

### IV. DISSIPATION AND FREQUENCY RESPONSE

We now consider the simple scenario depicted in Fig. 1(a). The signal source outputs a sinusoidal wave of angular frequency ω and we wish to calculate the power delivered to the load impedance Z_L. We assume that both R and G are small but not negligible.

Assuming that the cross term RG in Eq. (3) is small and that, at all frequencies of interest, R/ωL + G/ωC ≪ 1, the propagation constant can be approximated as

\[ γ ≈ j \frac{ω}{v_0} + \frac{1}{2} \left( \frac{R}{Z_0} + GZ_0 \right) ≈ j \frac{ω}{v_0} + \frac{1}{2} α_+ , \tag{9} \]

where, as before, v₀ = 1/√LC and we denote the characteristic impedance of a lossless transmission as Z₀ = √L/C. As is usually the case, the output impedance of the signal generator is assumed to also be equal to Z₀. Using the same approximations, the characteristic impedance of a lossy transmission line given by Eq. (4) can be written as

\[ Z_c ≈ Z_0 \left[ 1 - \frac{jv_0}{2ω} \left( \frac{R}{Z_0} - GZ_0 \right) \right] \]

\[ ≡ Z_0 \left( 1 - \frac{jv_0}{2ω} α_- \right). \tag{10} \]

Next, using Eq. (1) and the fact that V_− = ΓV_+, V_+ can be expressed in terms of the voltage amplitude at \( x = -ℓ: V_+ = V_−, \) [e^{γℓ} + Γe^{−γℓ}]^{−1}. Substituting this result for V_+ back into Eq. (1) and using Eq. (5) for Γ allows one to solve for the voltage amplitude \( V_L = V(0) \) at the load impedance \( Z_L \)

\[ \frac{V_L}{V_−,} = 2 \left[ (e^{γℓ} + e^{−γℓ}) + \frac{Z_c}{Z_L} (e^{γℓ} − e^{−γℓ}) \right]^{-1}. \tag{11} \]

Substituting in the approximate forms of γ and Z_c from Eqs. (9) and (10) yields
\[
\left( \frac{V_L}{V_x} \right)^{-1} = \left[ \cos \frac{\omega x}{v_0} \cosh \frac{\alpha_+ x}{2} + \frac{Z_0}{Z_L} \left( \cos \frac{\omega x}{v_0} \sinh \frac{\alpha_+ x}{2} - \frac{v_0 \alpha_-}{2} \sin \frac{\omega x}{v_0} \cosh \frac{\alpha_+ x}{2} \right) \right] \\
+ j \left[ \sin \frac{\omega x}{v_0} \sinh \frac{\alpha_+ x}{2} + \frac{Z_0}{Z_L} \left( \sin \frac{\omega x}{v_0} \cosh \frac{\alpha_+ x}{2} + \frac{v_0 \alpha_-}{2} \cos \frac{\omega x}{v_0} \sinh \frac{\alpha_+ x}{2} \right) \right].
\]  
(12)

Equation (12) allows the frequency dependencies of both \(|V_L/V_x|\) and the phase difference between the voltages at \(x = -\ell\) and \(x = 0\) to be calculated. In the lossless limit, \(\alpha_+ = \alpha_- = 0\) such that

\[
\frac{V_L}{V_x} = \left[ \cos \frac{\omega \ell}{v_0} + j \frac{Z_0}{Z_L} \sin \frac{\omega \ell}{v_0} \right]^{-1}.
\]  
(13)

and, as expected, \(|V_L/V_x| = 1\) when \(Z_L = Z_0\).

Our objective was to measure the ratio of the signal power at \(x = 0\) to the power at \(x = -\ell\) as a function of frequency and then compare it to \(|V_L/V_x|^2\) calculated from Eq. (12). This comparison requires models for the frequency dependencies of the per-unit-length resistance and conductance of the coaxial transmission lines used in our measurements. These models are developed in the next section.

A. Models of resistance and conductance

First, we consider the resistance which is due to the usual Joule heating in conductors. Figure 4(a) shows a schematic diagram of a section of the center conductor from a coaxial cable. The current in the center conductor is restricted to a region that is within an EM skin depth \(\delta\) of the surface. For a good conductor

\[
\delta \approx \sqrt{\frac{2\rho}{\mu_0 \omega}},
\]  
(14)

where \(\rho\) is the resistivity of the conductor and \(\mu_0\) is the permeability of free space. Therefore, the cross-sectional area through which the current flows is \(A \approx 2\pi r_1 \delta\) where \(r_1\) is the radius of the center conductor and the per-unit-length resistance can be estimated as

\[
R \approx \frac{\rho}{A} = \frac{1}{2\pi r_1} \sqrt{\frac{\mu_0 \omega \rho}{2}}.
\]  
(15)

In this simple approximation, the contribution to \(R\) from the outer conductor, which provides a return path to the source for the current, has been neglected. Because the outer conductor’s inner radius \(r_2\) is about three times \(r_1\), its contribution to \(R\) is expected to be small compared to that of the center conductor. The important insight is that \(R \propto \omega^{1/2}\) with a constant of proportionality that is determined by \(\rho\) and geometrical factors.

Next, we turn our attention to \(G\). A schematic diagram of the coaxial cable cross-section is shown in Fig. 4(b).

FIG. 4. (a) Schematic drawing of the center conductor of a coaxial cable of radius \(r_1\). The shaded region depicts the cross-sectional area through which the current travels which is determined by the frequency-dependent EM skin depth \(\delta\). (b) Schematic drawing of a coaxial cable. The space between the center and outer conductors is filled with a dielectric material (shaded region) with complex relative permittivity \(\varepsilon_r = \varepsilon' - j\varepsilon''\). The dielectric has an inner radius \(r_1\) and an outer radius \(r_2\).

The per-unit-length capacitance of a coaxial cable is given by

\[
C = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln (r_2/r_1)},
\]  
(16)

where \(\varepsilon_r\) is the relative permittivity of the dielectric material filling the space between the center and outer conductors and \(\varepsilon_0\) is the permittivity of free space. For a lossy dielectric, the relative permittivity is \(\varepsilon_r = \varepsilon' - j\varepsilon''\).
such that the capacitive admittance becomes

$$Y_C = j\omega C = \frac{2\pi\omega\varepsilon_0 (j\varepsilon' + \varepsilon'\prime\prime)}{\ln(r_2/r_1)}.$$  (17)

The real term $G = 2\pi\omega\varepsilon_0\varepsilon''/\ln(r_2/r_1)$ is identified as the per-unit-length conductance.

In general, both the real and imaginary parts of $\varepsilon_r$ can have their own nontrivial frequency dependencies. The dielectrics in our coaxial cables are polytetrafluoroethylene (PTFE, Teflon) or polythene (PE). The loss tangent $\tan\delta_0 \approx \varepsilon''/\varepsilon'$ of Teflon have been measured precisely by a variety of techniques over a wide range of frequencies. From 100 MHz to 60 GHz, both $\varepsilon'$ and $\tan\delta_0$ have been shown to be independent of frequency, with $\varepsilon' \approx 2.05$ and $2 \times 10^{-4} < \tan\delta_0 < 3 \times 10^{-4}$.[27,31] In the analysis presented in Sec. [IV][II], which includes measurements that span 1 MHz to 2 GHz, we assume a frequency-independent $\varepsilon''$ such that $G \propto \omega$ with the constant of proportionality determined by $\varepsilon''$ and geometrical factors.

### B. Power ratio measurements

The simple experimental setup shown schematically in Fig. [II][III] was used to measure the rms power $P_L$ delivered to a load impedance at the end of a transmission line. The signal source used was a Rohde & Schwarz SMY 02 signal generator with a 50-Ω output impedance and the load impedance was a Boonton 41-4E power sensor with a 50-Ω input impedance coupled with a Boonton Model #42BD power meter. The dc recorder output of the power meter was monitored using a Keysight 34401A digital multimeter. A simple LabVIEW program was written to scan the frequency of the signal generator while writing the multimeter data to a file. The program made repeated measurements of the power at each frequency and the average of the values was recorded. Twenty averages were used in the measurements reported in this section.

The calibration of the Boonton power sensor is frequency dependent and there can be small variations in the power output by the signal generator when sweeping over a wide frequency range. To remove both of these effects from the measured data, we repeated the same frequency scan and measured $P_L$ with the power meter connected directly to the output of the signal generator. The ratio $P_L/P_{-L}$ is independent of both the calibration of the power sensor and small variations in the output power. Note that this power ratio is equivalent to the scattering parameter $S_{21}$ which can be measured quickly and precisely using a vector network analyzer (VNA). The results of the measurements for the RG-58 BNC, semi-rigid UT-141, and HQ sma cables are shown in Fig. [III][II]. The data are shown on a decibel scale and are a measure of the insertion loss resulting from the transmission lines. The rms power ratio $P_L/P_{-L}$ is also equivalent to $|V_L/V_{-L}|^2$ which can be calculated from Eq. (12). Table [III][II] shows that the measured insertion losses are in good agreement with manufacturer specifications.[23,24]

Fits to the insertion loss data, assuming $R = af^{1/2}$ and $G = bf$ are also shown in Fig. [III][II]. For all but the
The results of these estimates using \( r_1 = 0.460 \, \text{mm} \) and \( r_2 = 1.493 \, \text{mm} \) are given in Table III. For the semi-rigid UT-141, for which \( r_1 \) and \( r_2 \) are precisely known, we estimated the values of \( \rho \) and \( \tan \delta = \varepsilon''/\varepsilon' \) using

\[
\rho = \frac{(a r_1)^2}{\mu_0/(4\pi)} \quad (18)
\]

\[
\tan \delta_0 = \frac{b \ln (r_2/r_1)}{4\pi \varepsilon_0 \varepsilon'}. \quad (19)
\]

The results of these estimates using \( r_1 = 0.460 \, \text{mm} \) and \( r_2 = 1.493 \, \text{mm} \) are given in Table III.

The center conductor of the UT-141 cable is silver-plated copperweld (SPCW). Copperweld is a wire in which a copper cladding is bonded to a steel core. The conductivity of the cladding can be anywhere from 30 to 70% IACS (International Annealed Copper Standard, \( 58.2 \times 10^6 \, \Omega^{-1} \, \text{m}^{-1} \)). The silver plating will reduce the overall effective resistivity which, depending on the thickness of the plating, could be frequency dependent due to the changes in the EM skin depth. Given all of these considerations and the crudeness of our model of the per-unit-length resistance of the coaxial cable, the extracted estimate of \( \rho \), being about twice that of copper, is very reasonable.

The UT-141 coaxial cable has a Teflon dielectric. The extracted value of \( \tan \delta_0 \) falls directly within the range of values reported in the literature. This result is remarkable because typically precision resonator techniques are required to accurately measure dissipation in low-loss dielectrics.

When fitting the insertion loss data of the RG-58 cable, the characteristic impedance of 53.2 \( \Omega \) obtained from the transient analysis resulted in relatively large frequency oscillations in the fit function. A detailed view of the data and the oscillations are shown in Fig. 6. We speculate that dissipation in the RG-58 cable is non-negligible such that the lossless transmission line approximation used in the transient analysis breaks down. If we assume that \( V_g \) measured in the transient analysis is reduced by 0.2% each time the signal travels the length of the cable and back, then we find that the corrected data results in a characteristic impedance of 50 \( \Omega \). The black lines in Figs. 5(a) and 6 are fits to the RG-58 data in which \( Z_0 \) was included as an additional fit parameter. The best-fit value returned was \( Z_0 = 49.71 \, \Omega \) which results in oscillation amplitudes that are much closer to those observed in the measured data. If \( Z_0 \) is included as a parameter in the fits to the semi-rigid and HQ sma data, a fit value that is in experimental agreement with results from the transient analysis are returned. Finally, we note that the average period of the observed oscillations highlighted in Fig. 6 agrees very well with the expected value of \( (2\pi/v_0)^{-1} = 12.5 \, \text{MHz} \) based on the transient analysis.

Figure 5(b) shows plots of the frequency dependencies of \( R \) and \( G \) for all of the transmission lines measured using the values of \( a \) and \( b \) extracted from the
fits (see Table III). At the lowest frequencies, $R/Z_0$ is at least two orders of magnitude greater than $GZ_0$ such that $\alpha_+ \approx \alpha_- \approx R/Z_0$. However, because $G$ increases more quickly with frequency than $R$, its contribution to the overall insertion loss becomes more important as frequency is increased. In the case of the RG-58 cable, dielectric losses match conductor losses, and then surpass them, by 2 GHz. This crossover occurs by 25 GHz for the UT-141 and HQ sma cables. The dashed line in Fig. 5(a) shows a fit to the insertion loss of the RG-58 cable assuming that $G$ is negligible. Clearly, the dielectric losses need to be included to capture the full frequency dependence of the measured insertion loss.

Finally, we note that at high frequencies where dielectric losses dominate or in situations requiring very low attenuation, air-dielectric coaxial cables are available. In one example, a helical polyethylene spacer is used to keep the inner conductor concentric with the outer conductor, both of which are made from corrugated copper. The insertion loss of the HELIFLEX air-dielectric coaxial cable is specified to be 0.021 dB/m at 1 GHz which is ten times less than the value found for the "high-quality" sma cable characterized in this work.

V. SUMMARY

We have described two experiments that, together, can be used to fully characterize the properties of transmission lines. Both experiments are simple to set up and make use of equipment that is either commonly available in undergraduate labs or relatively inexpensive to acquire.

First, the transient response to a voltage step was used determine the transmission line signal propagation speed and characteristic impedance or, equivalently, the per-unit-length capacitance and inductance. In this well-known experiment, the data analysis assumes that the transmission lines are lossless. We found that this approximation worked well for the relatively low-loss semi-rigid UT-141 and high-quality sma coaxial cables. However, we found evidence suggesting that losses in the RG-58 coaxial cable were causing the characteristic impedance to be systematically overestimated.

Our primary objective was to use insertion loss measurements to determine the per-unit-length resistance and conductance of the same coaxial cables used in the first experiment. With one end of the transmission line driven by a sinusoidal voltage source, and assuming small but non-negligible losses, an expression for the power delivered to a load termination $Z_L$ was derived. Based on the geometry of the coaxial cables, the frequency dependence of the conductor losses was assumed to be determined by the EM skin depth such that $R \propto f^{1/2}$. An analysis of the dielectric losses, due to $\varepsilon''$, was used to deduce that $G \propto f$.

The insertion loss measurements, where possible, were compared to manufacturer specifications and found to be in good agreement. Fits to the data using the theoretical model developed were excellent. It was shown that both the $R$ and $G$ contributions were required to capture the full frequency dependence of the measured insertion losses. The parameters extracted from the semi-rigid UT-141 fit were used to make reasonable estimates the resistivity of the copper-clad center conductor and loss tangent of the Teflon dielectric.

These measurements also serve to highlight the importance of the non-ideal characteristics of transmission lines, which are often not emphasized in theoretical treatments at the undergraduate level. For example, despite impedance matching at the source and load, at 2 GHz only 20% of the incident power is delivered to the $Z_L$ termination at the end of a 7.61-m long RG-58 coaxial cable. For the highest-quality cable characterized in our measurements ($\ell = 8.04\, m$), although the power transfer efficiency increased, at 58% efficiency, substantial attenuation was still observed.

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