Abstract

Financial Reynolds number works as a proxy for volatility in stock markets. This piece of work helps to identify the predictability and herd behavior embedded in the financial Reynolds number (time series) series for both CNX Nifty Regular and CNX Nifty High Frequency Trading domains. Hurst exponent and fractal dimension have been used to carry out this work. Results confirm conclusive evidence of predictability and herd behavior for both the indices. However, it has been observed that CNX Nifty High Frequency Trading domain (represented by its corresponding financial Reynolds number) is more predictable and has traces of significant herd behavior. The pattern of the predictability has been found to follow a quadratic equation.

Keywords

econophysics, Hurst exponent, herding, High Frequency Trading

JEL Classification
A12, B4, D53, G1

INTRODUCTION

Econophysics gained momentum in the mid-1990s with trailblazing works of Rosario Mantegna and Eugene Stanley, continuing the legacy of Benoit Mandelbrot. Despite the plethora of knowledge being available (presented in literature review), a threaded study for finding a volatility proxy for India was found to be missing.

Many interesting works took place in the last decade or so. This current piece of study has been carefully cobbled from those works by eminent researchers. Jakimowicz and Juzwiszynb (2015) worked on Warsaw Stock Exchange using fluid mechanics analogy of Reynolds number to identify possible balance or equilibrium situation in a stock market. Cornelis Los (2004) used the same analogy, but for cash flow viscosity management. Two Chinese researchers (Zhang & Huang, 2010) have proved that stock market is nothing but a finite Hilbert space, where different variables (such as price, volume, circuit filter, etc.) could behave like eigenvectors in all possible directions. An eminent Romanian researcher (Racorean, 2015) echoed them as well while proving that bourses are nothing but quantum gates. Putting these thoughts together and stitching them carefully with specific stochastic oscillators generated financial Reynolds number. However, though the financial Reynolds number seemed to be novel, yet one daunting question was left to be answered. Could this very number be able to predict the explosions in the bourses and could it also spot herd behavior in the process. Every new study has to undergo critical test in the process of generating the critical mass around it. Hence, this work would be able to showcase the prowess and utility of the previously defined financial Reynolds number.
The previous work (Ghosh & Kozarević, 2018) rationally linked with valid logic and backed up by a strong measuring tool to identify and formulate an apt econophysics proxy (read as financial Reynolds number) for volatility number (explosive number) and further predicting the number for future as well. All the behavioral gaps were identified and validated as per behavioral theories. Quantum explosion in stochastic time series was predicted at the helm of the previous piece of work. However, some key questions left unanswered though. Since, the financial Reynolds number series is a stochastic time series; hence, Hurst exponent is an apt method of predicting the predictability. Moreover, any stochastic series is said to be having a pattern if the Hurst exponent is more than 0.5. Again, this study delves into the High Frequency Trading domain of Nifty as well. Hence, a comparison (between CNX Nifty Regular and CNX Nifty HFT) cannot be ruled out either. Nevertheless, the more is the Hurst exponent, the more is predictability and, thus, self-similar pattern in a recurring manner (confirming long memory process). Interesting to note that as the pattern intensifies, direction seems to be clear the indications are plausible enough for an evident herd behavior in the transaction levels of the underlying assets. Moreover, if the Hurst exponent is high, the fractal dimension is low confirming the relative smoothness of the surface and, thus, reaffirming the predictability quotient. One engaging puzzle remained unsolved even after solving all these puzzles. Econophysics often talks about power law connection, and quadratic equation connection. Hence, it can only be possible if financial Reynolds number series is connected rationally with such a generalized mathematical construct. Log periodicity has been long used by eminent researchers across the globe for finding econophysics links. This work is no exception either. Cumulative log periodic financial Reynolds numbers were arranged according to their observation point to check whether it follows any kind of pattern or not (see Figure 1). This work has only been carried out for the CNX Nifty HFT domain though.

This work has three clear objectives:

1) predictability of financial Reynolds number (introduced in the previous study) in both regular CNX Nifty and CNX Nifty HFT domain;

2) herding traces along with fractal dimensions in both regular CNX Nifty and CNX Nifty HFT domain;

3) finding an equation for cumulative log-periodic volatility of financial Reynolds number in CNX Nifty HFT domain.

Most studies have been aimed at herding and Hurst exponents of stock market closing prices or individual stock closing prices; however, this study aims to discover predictability and traces of herding using a volatility proxy. It’s a first study of this kind.

1. LITERATURE REVIEW

This innovative piece of work is a natural progression from the earlier work (Ghosh & Kozarević, 2018). The work took inspiration from a work where quantum field has been utilized as a financial field; the work used gauge transformation, or simply a change in coordinates (Ilinski, 2001). Another interesting piece of work proved the stocks to be behaving like tiny quanta, moving in linear path, where all the impurities of such a micro system can be absorbed by the macro system such as the bourse (Zhang & Huang, 2010). This work also showed a clear trace of wave particle duality in stocks. For a very short length of time, the movement is linear till the next move has been found, when it behaves like a wave. Stocks are found to be in a macro system (bourse), which interestingly stands as a proper analogue of a finite Hilbert space. However, purchase and sell happen on various tangible or intangible parameters directly or indirectly influencing the traders. Traders trade within the space of the circuit filters (defining the finite boundaries of the Hilbert
space). Their buying and selling behavior are completely probabilistic and, finally, leads to discovery of price (Ghosh & Kozarević, 2018). As this is related to information availability, hence, the direct intervention of information entropy cannot be ruled out either (namely Shannon entropy). Inside that finite Hilbert space (Schaden, 2002) (macro system), the boundaries are specific and defined clearly by circuit filters put into use by the respective regulatory authorities for the bourses. This, in turn, becomes a close proxy to a quantum well with finite boundaries (Zhang & Huang, 2010). On the other hand, the glaring warts of Black-Scholes option pricing model has been exposed many decades ago (Bouchaud, 1994). Geometric Brownian motion, Levy process and Weiner process over the years have showed the presence of residual risk and probabilities of both overestimation and underestimation on many instances. Nowadays, econophysicists are utilizing the idea of universality in social science (Sen Parongama, 2013). These agents-based complex systems (Sinha, 1996, 2010) enable the researchers to find the strength and stability of the economic systems as a whole from a completely diverse point of view.

Bachelier started this phenomena of econophysics amidst strong criticism more than a century ago (Bachelier, 1900). His seminal work of predicting cotton price in La Boursa in Paris caught Einstein who, in turn, produced one more work on similar lines (Einstein, 1906). In reality, Brownian motion used by Bachelier either overestimates or underestimates during crash or consolidation phases in economy due to the fundamental premise of Gaussian distribution. Mandelbrot introduced Levy flight, which is probability dependent along with geometry-based concept of “fractals”. A tiny part of an entire system will carry similar surface roughness and other traits of the entire setup. Ausloos, Belgian scientist (Ausloos, 1998), predicted 1987 US stock market crash with his econophysics model (though as hindcast). Quantum gate effect in stock markets (Racorean, 2015), Shannon entropy as a better proxy over GARCH (Bentes & Menezes, 2012) and cash-flow viscosity-based turbulence predictor (Los, 2004) were some of the breakthrough works in last two decades.

Jakimowicz and Juzwiszyn (2015) in their search of finding balance in volatile economic condition in Warsaw Stock Exchange (WIG) have used Osborne Reynolds equation and followed it with effective usage of Navier-Stokes equation.

A group of eminent researchers (Bellenzier, Vitting Andersen, & Rotundo, 2016) extended Nassim Nicholas Taleb’s work on effect of heuristics (Haug & Taleb, 2011). They found that stock market is a complex system, which is quite brittle due to mental accounting and anchoring. Inoua (2015) discussed about the substantial fluctuations of financial prices from the amplifying feedback of speculative demand and supply. Haven and Sozzo (2015) linked quantum mechanics with behavioral finance aided by heuristic decision-making process effortlessly.

Some innovative work has been done using Hurst exponent to identify nascent bubbles, traces of herd behavior, etc. In one such study, it has been found that self-similar process with high and very high Hurst exponent leads to outperformance against the benchmark return, whereas the low and very low value indicate significant non-performance (Fernández-Martínez, Sánchez-Granero, Muñoz Torrecillas, McKelvey, 2016). Another innovative piece of work from a South Korean group of researchers on different classification groups from rationality perspective showed that medium to high irrationality group has very high levels of Hurst exponent, proving it further that high Hurst exponent does indicate herd behavior (Kim & Kim, 2014).

Although the famous Benoit Mandelbrot started this revolution of long memory process for forecasting with little data but higher accuracy, yet it was described fluently by a group of researchers (Watkins & Franzke, 2017) who described the link that started with structural geometry and finished at long memory, self-similar process. Most of such econophysics studies for log-periodic cumulative representation link it to a probable power law condition (Didier Sornette, 2003; Feigenbaum, 2001; Gazola, Fernandes, Pizzinga, & Riera, 2008; Jhun, Palacios, & Weatherall, 2017; Sornette, 2009). Log-periodic cumulative volatility has been linked to financial crash conditions before in Hang Seng between 1970 and 2008 implementing bubble detection technique (Bree & Joseph, 2013; Johansen, Sornette, 2001).

Herding in bourses was unearthed by the famous LSV (Lakonishok-Shleifer-Vishny) way back in 1992. They have proved that herding germinates
from the “average tendency of a group of money managers to buy (sell) particular stocks at the same time” (Lakonishok, Shleifer, & Vishny, 1992). Though, it wasn’t explicitly termed as herding, but meant nothing but the same, where imitating noise traders were found to play pivotal role in the death of the bubble (Chen, Firth, & Rui, 2001; Johansen Anders, Ledoit Olivier, 2000). Momentum strategies of investment managers in developing nations were identified as a primary reason for herding (Bikhchandani & Sharma, 2000). Many seminal works has been found in this regard pinpointing herd behavior in various market conditions. Certain trailblazing work proved that Latam and North America has less herd behavior (Chang, Cheng, & Khorana, 2000; Chiang, & Zheng, 2010; Chiang, Li, & Tan, 2010; Lao, 2011), perhaps due to higher financial literacy or true sense of information cascading. Those studies also found that Asian peers, especially China, have herd behavior inbuilt that surfaces out during the crisis period.

This study is a natural amalgamation of four eminent works from 2001 till 2015. In a nutshell, it borrowed the concept of Reynolds number from both the Polish and the American researchers (Jakimowicz & Juzwiszyn, 2015; Los, 2004), defined its conditionality with the help of Romanian and Chinese works of eminent scientists (Racorean, 2015; Zhang & Huang, 2010) and constructed it in an independent manner by using two stochastic oscillators (namely RVI and EMV).

2. METHODOLOGY

In the previous work (Ghosh & Kozarevic, 2018) financial Reynolds number has been coined and calculated for CNX Nifty from February 15, 2000 to December 7, 2015. Financial Reynolds number was calculated and coined as a proxy for measuring volatility. Financial Reynolds number emerged as a ratio where Relative Volatility Index (RVI) (Dorsey, 1993) is the numerator and Ease of Movement (EMV) (Arms, 1996) is the denominator. Econometric tool GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) and feed forward, back propagating classical artificial neural network (ANN, machine learning tool) has been used to predict the financial Reynolds number for the future. The results show predictability in financial Reynolds number along with their explosive tendency and behavioral bias (Ghosh & Kozarevic, 2018).

Since both predictability and herd behavior can be indicated by the effective use of Hurst exponent, coupled with fractal dimension, thus, the same has been applied in this work. Once Hurst exponent confirms the pattern embedded in this time series, cumulative log-periodic version of the same series (consisting of financial Reynolds number) has been plotted to extract the equation hidden inside the same.

Two diverse domains of one stock exchange have been considered for this piece of empirical study for comparing their relative predictability. CNX Nifty Regular and CNX Nifty HFT (High Frequency Trading zone) have been considered in this current piece of work. The first one consists of 47,016 data points. CNX Nifty HFT data sets capture from February 1, 2013 to December 30, 2016. Hence, this consists of \(2.8 \times 10^7\) data points on a tick by tick basis. Hurst exponent (Hurst, 1951) has been checked periodically (checking window is adjusted at 200 observations) for both these data sets to determine the predictability. Hurst exponent uncovered the true sense of herd behavior as well in this work. Further delving in quest for a pattern in the newly developed volatility proxy log-periodic values were examined. Cumulative log periodicity has been implemented to reach an equation, depicting the overall pattern of cumulative volatility.

2.1. Hurst exponent

This is an asymptotic behavioral pattern of a rescaled time series.

\[
E \left[ \frac{R(n)}{S(n)} \right] = Cn^H, 
\]

where \(R(n)\) is the range of the values, \(S(n)\) is their standard deviation, \(E \left[ \frac{R(n)}{S(n)} \right] \) is the expected value, \(n\) is the number of data points in the specified time series, \(C\) is a constant, \(H\) is the Hurst exponent Relation of Hurst exponent with fractal dimension

\[
D = 2 - H. 
\]
However, some studies indicate that the relationship between fractal behavior and long memory dependence (Hurst exponent) is non-linear in nature (Gneiting & Schlather, 2001).

This piece of work is cobbled carefully between econophysics and behavioral finance.

3. RESULTS AND ANALYSIS

Table 1. Report of Hurst exponent in ReHFT domain (average Hurst exponent is 0.69812)

| Time period       | Obs range | Hurst exponent | Fractal dimension |
|-------------------|-----------|----------------|------------------|
| Feb 2013 to July 2013 | 1-105     | 0.59961        | 1.40039          |
| Aug 2013 to Feb 2014  | 106-305   | 0.88042        | 1.1958           |
| Mar 2014 to Oct 2014  | 306-505   | 0.64146        | 1.35854          |
| Nov 2014 to July 2015 | 506-705   | 0.63717        | 1.36283          |
| Aug 2015 to Mar 2016  | 706-905   | 0.61342        | 1.38658          |
| Apr 2016 to Dec 2016  | 906-1056  | 0.81664        | 1.18336          |

Note: Average on time stamped ReHFT data have been used to reduce complexity.

Analysis of Hurst exponent opens up three interesting discoveries related to volatility, herd behavior and fractal dimension. Hurst exponent ideally operates as described in Table 3.

Table 3. Zones of Hurst exponent

| Hurst exponent | Interpretation                              |
|----------------|---------------------------------------------|
| H < 0.5        | Non-persistent, no pattern, no herding, surface is rough (fractal) |
| H = 0.5        | Random walk, completely stochastic          |
| H > 0.5        | Persistent, clear pattern, trace of herding, surface is smooth (fractal) |

Overall observation suggests that Hurst exponent is significantly higher for ReHFT, or financial Reynolds number in HFT domain.

Table 4. Hurst exponent: interpretation

| ReHFT Nifty | 0.69812 | Predictable, self-similar, less-rough and traces of Herding |
|-------------|---------|-------------------------------------------------------------|
| Re Regular Nifty | 0.58676 | All the above but with a lower degree |

This proves that financial Reynolds number in High Frequency domain (ReHFT) predicting volatility is more predictable (see Tables 1 and 4) and could be termed as "self-similar “compared to its regular counterpart. It’s not stochastic at all; on the contrary, it has clear trace of herding. This means pattern, which is evident, and persistence, which is clearly been observed, were the results of a herd behavior. Investors are keen to ride trends and that causing Herding, (Linders, Dhaene, & Schoutens, 2015; Muñoz Torrecillas, Yalamova, & McKelvey, 2016) generating pattern.

Table 2. Report of Hurst exponent in Re regular domain (average Hurst exponent is 0.586762)

| Time period       | Obs number | Hurst exponent | Fractal dimension |
|-------------------|------------|----------------|------------------|
| Feb 2000 to Nov 2000 | 1-200     | 0.7212793      | 1.2787207        |
| Dec 2000 to Sept 2001 | 201-400   | 0.6041998      | 1.3958002        |
| Oct 2001 to July 2002  | 401-600   | 0.5963155      | 1.4036844        |
| Aug 2002 to April 2003  | 601-800   | 0.5148976      | 1.4851024        |
| May 2003 to Feb 2004  | 801-1000  | 0.7531655      | 1.2468345        |
| Mar 2004 to Dec 2004  | 1001-1200 | 0.6373432      | 1.3626568        |
| Jan 2005 to Sept 2005  | 1201-1400 | 0.6380365      | 1.369635         |
| Oct 2005 to July 2006  | 1401-1600 | 0.4496574      | 1.5503426        |
| Aug 2006 to May 2007  | 1601-1800 | 0.6648086      | 1.3351914        |
| Jun 2007 to Feb 2008  | 1801-2000 | 0.6886212      | 1.3123788        |
| Mar 2008 to Dec 2008  | 2001-2200 | 0.5331683      | 1.466816         |
| Jan 2009 to Oct 2009  | 2201-2400 | 0.5622112      | 1.4377889        |
| Nov 2009 to Aug 2010  | 2401-2600 | 0.5073273      | 1.4926727        |
| Sept 2010 to May 2011 | 2601-2800 | 0.7174276      | 1.2857254        |
| Jun 2011 to Mar 2012  | 2801-3000 | 0.5607086      | 1.4392914        |
| Apr 2012 to Dec 2012  | 3001-3200 | 0.5556666      | 1.4443314        |
| Jan 2013 to Oct 2013  | 3201-3400 | 0.5199249      | 1.4800752        |
| Nov 2013 to Aug 2014  | 3401-3600 | 0.5577099      | 1.4422901        |
| Sept 2014 to May 2015  | 3601-3800 | 0.5349345      | 1.4650655        |
| Jun 2015 to Dec 2015  | 3801-3919 | 0.4219888      | 1.5780113        |
For that matter, even the financial Reynolds number on Regular Nifty too is predictable and “self-similar” (see Tables 2 and 4). However, the extent is more so as the intensity for the HFT. Although fractals (Mandelbrot, 1963; Watkins & Franzke, 2017) are local properties and Hurst exponent is a global property (with long memory process), they seem to merge at a certain point. Roughness lowers for a self-similar process paving the way for plausible and possible prediction. Rougher surfaces with higher fractal dimensions occur for antipersistent processes with $0 < H < 0.5$ (see Table 3). The results obtained in this study match with the outcome of the South Korean study conducted back in 2014. The higher the Hurst exponent, the lower the fractal dimensions or roughness, and surely the self-similar process is highly predictable with a definite degree of herding.

Table 5. Certain periods (Re Regular NIFTY) with high Hurst exponent ($H > 0.65$) has clear event link as well

| Time period          | Event                                      |
|----------------------|--------------------------------------------|
| February 2000        | Ketan Parekh and UTI Scam                  |
| to September 2001    |                                            |
| May 2003             | Broad-based stock market rally              |
| to September 2005    |                                            |
| August 2006          | Global rally before the credit fiasco (US)  |
| to February 2008     |                                            |
| September 2010       | Recovery post Flash Crash in the US         |
| to May 2011          |                                            |

It has been found that whenever a particular condition occurred ($H > 0.65$), either local turmoil or global turmoil has been observed resulting in secular movements either upward or downward (see Table 5). Secular movements in stock markets do happen when herd behavior surfaces out. In fact, two periods (see Table 5) are found to be significantly higher from Hurst exponent point of view ($H > 0.8$). One seminal study report (Ormos & Timotity, 2016) indicates that heuristics and herding increase during crisis. Similar trace has been found below (April 2016 to December 2016, BREXIT effect, having $H = 0.81$).

Table 6. Certain periods (ReHFT NIFTY) with high Hurst exponent ($H > 0.65$) has clear event link as well

| Time period          | Event                                      |
|----------------------|--------------------------------------------|
| Aug 2013 to Feb 2014 | Indian currency crisis & NPA surge in banks|
| Apr 2016 to Dec 2016 | BREXIT                                     |

Situation based on evident herd behavior of the market participants intensified (see Table 6) in the CNX Nifty HFT domain. Hence, it can be confirmed that the HFT domain in the said index has been following a clear self-similar pattern and prominent traces of herd behavior is observed there.

Perhaps, one question remained unanswered here! Hence, the cumulative log-periodic financial Reynolds number in HFT domain (CLPREHFT) has been plotted as per the sequential observation (see Figure 1).

The expression for defining cumulative log-periodic volatility over a period of time emerged out as a quadratic equation (see Figure 1), where the dependent variable is the measure of cumulative volatility and the independent variable is observations (indicating time dependency):

$$ y = -0.0009x^2 + 0.9511x + 34.3201. \quad (3) $$

Inverse parabola is indicating an equation of common in most economic studies with profit or revenue function in question:

$$ y = -ax^2 + bx + c, \quad (4) $$

where $a$ and $c$ are constants, hence the square law comes out from cumulative log-periodic Reynolds number series generated out of Nifty data in HFT mode (amounting $2.8 \cdot 10^9$ data points). Log periodicity represents relative volatility between two trading days. That relative volatility piles up in a symmetric mode and further comes down in the exactly same way forming a perfect inverse parabola. The growth of volatility is symmetric and so as the decline. Thus, it can be observed that cumulative log-periodic financial Reynolds number is a direct variant of an inverse parabola, hence, prediction becomes relatively easy and accurate by the age old algebraic equation. The two coefficients in this case are “$a$” and “$c$” respectively. Hence, if $a=1$ and $c=0$, another simplified equation emerges:

$$ y = -x^2 + bx. \quad (5) $$

In all other cases, the researcher has to calculate both $a$ and $c$ to find out the perfect inverse parab-
Inverse parabola signifies the constant rate of acceleration and deceleration, which means cumulative volatility, is a function of inbuilt herd behavior. It accelerates when herd increases in the positive sense and decelerates when herd increases downwards. This reaffirms the periodic secular movement of volatility resulting from underlying herd behavior. Hence, the cumulative periodic volatility for a defined period (5 years in this case) can be obtained as a perfectly symmetric inverse parabola. This cumulative volatility is a function of the frequency as a quadratic equation. Cumulative volatility overshoots to a level of 280 and, then, experiences downward pool in the exact opposite direction (Sornette, 2003) with time and observation. Inertia of volatility brings it down to an expected lower level. Inertia in stock markets is often caused by strong presence of herd behavior. Some eminent works from eminent scientists across the globe have found similar trend of volatility over time. One such work calculated EU option prices using a stochastic differential equation with a quadratic volatility term (Andersen, 2011).

In simple terms, the cumulative log-periodic Re HFT (CLPREHFT) will increase with each observation, pick out around 500th observation before falling to zero around 1050th to 1090th observations. Such a method has been implied before for accurate prediction of currency futures (Bharadia, Christofides, & Salkin, 1996). Hence, this method could be beneficial in high frequency algorithmic domain with a higher degree of predictability (confirmed by the persistent value of Hurst exponent in Table 1) to predict the cumulative volatility and its movement as well. Moreover, financial Reynolds number can be calculated for any stochastic series having an underlying, such as stocks, commodities and currencies.
CONCLUSIVE NARRATION, CURRENT IMPLICATION AND LIMITATIONS

Characteristics and traits of financial Reynolds number were unearthed in this study in two different domains from the same stock exchange. Financial Reynolds number was introduced in a unique manner earlier (Ghosh & Kozarevic, 2018) to find an apt econophysics proxy for volatility. However, this study confirms the predictability aspect along with the strong footprints of herd behavior. The financial Reynolds number (depicting explosive elements in stock indices) in High Frequency Trading (HFT) segment of CNX NIFTY has been found to be more predictable (see Table 1), less rough in structure and more prone to herd behavior. “Crisis do trigger herd behavior in Asian markets” (Chiang & Zheng, 2010; Chiang, Li, J. & Tan, 2010) matched this study (see Tables 5, 6).

Whereas Reynolds Number generated from the regular segment of CNX NIFTY has been found to be less predictable (see Table 2), more rough in structure and less prone to herd behavior when compared with its HFT counterpart. The cumulative log-periodic Reynolds number in the high frequency segment (CLPREHFT) appears to be a perfectly symmetric inverse parabola, represented by a quadratic equation, making the cumulative volatility more predictable in CNX NIFTY HFT. These theoretically enhance the existing body of knowledge on financial Reynolds number. Perhaps, this helps to attain the critical mass, considering the infancy of this kind of work involving financial Reynolds number. However, the financial Reynolds number is yet to be tested on a global barometer encompassing most stock markets in the globe. Also, regular fractal dimension may be limited while searching for the holy grail of predictability. Hence, future work could well be considered in those lines. These findings could be useful for traders, foreign institutional investors and qualified institutional buyers operating in CNX NIFTY. Predictability, herding and cumulative volatility pattern could well be decoded for select indices used in this work. HFT domain being more predictable with fixed equation and more traces of herding seems to be the future interest for the foreign portfolio investors.

ECONOMIC INTERPRETATION

High frequency domain of a stock market is more predictable and contains an embedded pattern over its regular counterpart. However, this predictability comes with the problem of herd behavior, which could, in turn, form some kind of a positive rational bubble. Since, the regular domain is less predictable, thus, it carries less chances of forming a bubble (because of profound herd behavior) in future.

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