Seismic waveform inversion using a neural network-based forward

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Abstract. The purpose of seismic waveform inversion is to obtain a geological model that is optimally fitted to the predicted seismic record and the measured seismic record. Since the forward model is repeatedly called during the inversion process, in order to improve efficiency, an efficient forward calculation method must be employed. In this study, we take a 2D wave equation as an example and propose a deep learning method as a forward model to minimize the prediction error value of seismic records. And the velocity inversion test of the Marmousi model is carried out by conjugate gradient method. Numerical experiments show that compared with the traditional finite difference method, the method can greatly reduce the calculation amount and improve the calculation efficiency.

1. Introduction

Seismic wave forward modeling is an important basis for seismic exploration. It is widely used in seismic exploration and natural seismic research. By seismic forward modeling, we can check the rationality of design, and the reliability of the results of processing and interpretation, and the validity of inversion method and results. The regularity of the forward modeling in a given geological model can enhance people's understanding of the unknown model and thus contribute to the solution of geological problems. Generally, the forward modeling of seismic waves can be formulated as an operator equation

\[ d = g(m) \]  

where \( d \) is the seismic data vector, \( g \) is the forward operator, and \( m \) is the model parameter.

The main research methods of seismic wave field forward modeling are divided into ray tracing method and wave equation method [1, 2]. And the wave equation plays an important role in the propagation principle of seismic waves and the interpretation of complex formations [3, 4]. There are many ways to solve the wave equation [5, 6, 7], including finite difference method and finite element method [8-11]. However, the resolution of the finite difference operator is low in the time domain, and the stability and convergence of the algorithm are affected by the spatial sampling rate and the temporal sampling rate [12, 13]. The finite element method requires high memory and a large amount of computation of the computer, and different boundary conditions need to be set according to different situations. Therefore, we propose using neural network-based forward to replace the traditional finite difference method for seismic forward modeling.

Deep learning is a new field of machine learning that has drawn widespread interests by showing outstanding performances for recognition and classification in image and speech processing [14-17]. The forward problems in equation (1) describe a mapping between, which is usually, one set of continuous parameters to another set of continuous parameters [18]. We will refer to these methods in
general as regression networks, and it can be used to estimate the forward operator $g$ in equation (1). In the following, it is used to replace a computationally expensive forward problem with a regression network. The main advantage is that this will allow a very fast evaluation of the forward problem, making it possible to perform model inversion more efficiently. In addition, we use the adjoint-state method to compute the gradient of the objective function, hence the FWI workflow is simplified so we can focus mainly on the forward modeling and the model updating. Finally, the neural network-based forward model will be compared to using other widely used approximate forward models (based on finite difference approximations) in formulating. Numerical experiments performed on various data demonstrate the applicability of our method.

The paper is organized as follows. Section 2 gives a brief introduction to the basic inversion problem, and the concept of seismic forward modeling. The main methods of this paper are introduced in Section 3, including mathematical framework of using neural networks, preparation of training data sets, and forward modeling results based on a simple geological models. Section 4 shows the inversion results of numerical experiments on complex the Marmousi models using neural network forward modeling. Finally, the conclusion and future work are outlined in Section 5.

### 2. 2D acoustic-wave equation

For a 2D velocity-depth model, the acoustic-wave equation is expressed as

$$\frac{\partial^2 u}{\partial t^2} = v^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + s(t)$$

where $v(x,z)$ is the velocity of the longitudinal wave at the point $(x,z)$, $u$ is the wave field describing the velocity bit or pressure, and $s(t)$ is the source functions.

Indispensable conditions required for seismic forward modeling include seismic source functions, formation velocity (wave velocity), and boundary conditions. In order to highlight theoretical values in the model, this paper assumes that all noise, wave conversion, absorption, and diffraction are not considered. During the iterative forward modeling procedure, we use a perfectly matched layer (PML) absorbing boundary condition [19], the Ricker wavelet as the seismic source function

$$s(t) = e^{-2\pi r^2 t^2} \cos 2\pi ft$$

where $t$ is the time and $f$ is the center frequency, generally taken as 20-40Hz. In this paper, 20Hz is taken, $r$ is the parameter of the control band width, and it is choose to be 4.

### 3. Train data using neural networks

In this section, we first describe the mathematical framework of our novel method using neural network-based forward modeling. Then, we introduce the data preparation using a simple model and the forward implementation of the model.

#### 3.1. Mathematical framework based on neural network model

An expected result of forward modeling is to minimize the simulation data records generated by the network and the actual data records. We define the cost function as the sum of the squares of the seismic record errors

$$E = \frac{1}{2} \sum_{k=1}^{N} (x_k - d_k)^2$$

where $x_k$ is the corresponding data computed using the exact, but expensive, forward model based on finite-difference modeling, and $d_k$ is the simulated seismic data record.

As discussed, many types of machine-learning regression algorithms exist that can be used to obtain forward modeling data. In this study, we use a simple two layer feed-forward neural network,
that is, with one hidden layer, is considered to replace the accurate forward model [20]. Such a neural network that can be formulated as
\[
d_k = h_1(NM) \sum_j \alpha_{jk} h_2(NH) \sum_i \alpha_{ij} x_i)
\] (5)
where \(NM = 1000\) is the number of model parameters and \(NH\) is the number of hidden units. In this case, \(NH = 80\) hidden units are considered, \(h1\) and \(h2\) refer to two activation functions, in this case both chosen to be of tanh type. \(\alpha_{jk}^1\) and \(\alpha_{jk}^2\) refer to the weights of the units in the first and second layers. The main reason a two-layer feed-forward network is considered, is that the universal approximation theorem [21], which states that a feed-forward network with a single hidden layer containing a finite number of neurons can approximate any continuous function on compact subsets, under mild assumption on the activation function.

In actual training, we take the seismic wavelet as an input and repeatedly correct its weight such that the neural network will be able to estimate the data \(d\) given a specific model \(m\) in an optimal way. This is achieved by training the neural network based on the training data set, which we usually consider as an optimization problem, traditionally solved using the backpropagation algorithm.

The network learns by solving the optimization problem as
\[
\hat{\alpha} = \arg \min_{\alpha} \frac{1}{N} \sum_{i=1}^{N} (x_i - H(x_i, \alpha))^2
\] (6)
where \(H(x_i, \alpha)\) operator is implemented by using the neural network described above.

A commonly used minimization method is the gradient descent method to iteratively update the parameter vector as follows
\[
\alpha_{i+1} = \alpha_i - \delta \frac{\partial L_E}{\partial \alpha}
\] (7)
where \(\delta\) is the positive learning rate.

The neural network algorithm based on forward modeling is as follows:

1. Initialize the weight \(\alpha_{jk}^1\) and \(\alpha_{jk}^2\), select the learning rate \(\delta\), determine the error precision and the maximum number of iterations, and let \(i = 1\);
2. Input training sample pairs and calculate the output \(d_k\) according to equation (5);
3. Calculate the network output error according to equation (4). When the total network error is less than the error precision \(\epsilon\) within the maximum number of iterations, the algorithm converges to exit the training, otherwise it goes to step 4;
4. The backpropagation method. Adjust the weight of each layer according to equation (7), let \(i = i + 1\), turn to step 3.

3.2. Forward implementation

In order to reflect the application effect of neural network based forward model, we simulated the forward results of a simple geological model. In this paper, a three-layer horizontal layered homogeneous medium model is used. Based on the actual geological structure and the velocity data of various rocks, we determined the velocity and depth parameters of the three layers of horizontal layered uniformity. The seismic forward waveform records of the model are obtained under the condition that the seismic simulation records parameters are unchanged. Table 1 shows the structure and parameters of the geological model.
Table 1. Model construction and parameters.

| Velocity(m/s) | Depth(m) |
|--------------|----------|
| 1500         | 1000     |
| 2000         | 1200     |
| 2500         | 1400     |

Combined with the geological model, based on the MATLAB program, the seismic forward record using a neural network is obtained as shown in Figure 1. Figure 2 is an error curve of actual seismic records and simulated seismic records. Figure 2 shows use of the neural network-based forward model results in a modeling error bias is very close to zero, this is because the optimization of the neural network minimizes the mean square error. It shows that the neural network-based forward model has achieved good results.

![Figure 1. The seismic forward waveform record of the geological.](image1)

![Figure 2. Error curve of actual seismic records and simulated seismic records.](image2)

4. Inversion results based on Marmousi model

In order to verify the effectiveness of the proposed method, we applied the conjugate gradient method to the full-waveform inversion of the time domain acoustic wave and tested it using the Marmousi model (the Marmousi original model was created by the French Petroleum Institute in 1988 and is often used to test the correctness and rationality of advanced geophysical imaging methods). In time domain forward modeling, Ricker wavelet is used as source function, PML is used as absorbing boundary condition, and neural network-based forward is used to synthesize seismic data sets.

For each velocity model, we use horizontal and vertical spacing of 5 m, a total of 60 shots, shots are equally spaced on the interface of $z = 50 m$, 180 detectors are equally spaced surface. The velocity obtained by smoothing the real the Marmousi model at large scale is used as the initial guess. The inversion frequency is in uniform step from the minimum frequency of 5 Hz to the maximum frequency of 20Hz. A total of 12 frequencies participate in the inversion.

The inversion results of the Marmousi model are shown in Figure 3. Figures 3(a) and 3(b) are true and initial velocity models respectively. Figure 3(c) shows the results of the inversion velocity using the neural network for forward modeling. The results show that the inversion results reconstruct the general features of the model, and the boundaries part of the model is resolved reasonably well. It is shown that the neural network based on the forward model also has a good inversion result.

In order to test the effect of using neural network-based forward model in this paper, Table 2 lists the consumption time for forward modeling seismic records between neural network method and finite difference method, and shows the neural network-based forward has faster convergence velocity and less time consuming than the finite difference method.
5. Conclusions
In this paper, we starting from the time domain wave equation, the error squared sum of the wave field is taken as the objective function, and the minimum value is solved by using a neural network-based forward model. A new velocity model construction is demonstrated using neural network that allows the use of neural network forward modeling instead of computational complex forward modeling.
Under the premise of ensuring the same imaging results, the calculation amount is significantly reduced, and the calculation efficiency is improved. However, due to the limited number of training samples and oversimplified synthetic models and data, future work will focus on the improvements to the DL-based neural networks that will be applied for precise velocity model building with real seismic data.

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