High AOA decoupling control for aircraft based on ADRC

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Abstract: In this paper, a practical decoupling control scheme for fighter aircraft is proposed to achieve high angle of attack (AOA) tracking and super maneuver action by utilizing the thrust vector technology. Firstly, a six degree-of-freedom (DOF) nonlinear model with 12 variables is given. Due to low sufficiency of the aerodynamic actuators at high AOA, a thrust vector model with rotatable engine nozzles is derived. Secondly, the active disturbance rejection control (ADRC) is used to realize a three-channel decoupling control such that a strong coupling between different channels can be treated as total disturbance, which is estimated by the designed extended state observer. The control surface allocation is implemented by the traditional daisy chain method. Finally, the effectiveness of the presented control strategy is demonstrated by some numerical simulation results.

Keywords: high angle of attack (AOA), decoupling control, linear extended state observer (LESO), active disturbance rejection control (ADRC), thrust vector technology, control allocation.

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1. Introduction

With larger flight envelope and higher agility, higher flight performance is urgently needed for advanced fighter aircraft [1]. Super maneuverability or higher agility at high angle of attack (AOA) can not only provide fighter aircraft with priority attack, but also increase its living opportunity. Nowadays, super maneuverability or higher agility has already been one of the important features for advanced fighter aircraft at high AOA [2]. However, the maneuver action at high AOA can make the fighter aircraft become unstable due to high coupling and complicated unsteady aerodynamics [3]. When maneuvering at high AOA, the air which flows through the aircraft will keep changing, that is, the attached flow becomes the vortex flow, and then changes into the separated flow. During this process, the unsteady aerodynamic force will make aircraft fighters have high nonlinearity, strong coupling and unstable character with uncertainty [4]. The control difficulty brings great challenges to flight controller designing. In past decades, many control methods were researched, such as the nonlinear dynamic inversion method (NDI) [5,6], variable structure control [7], and robust control [8]. However, these methods are almost sensitive to model uncertainty, which is usually called model based. Furthermore, these traditional control strategies usually need special dynamic decoupling controllers by utilizing the multivariable control idea to realize decoupling. However, for the moment, it is difficult to find a systematic or mature tool to analyze the robustness of the multivariable system, which is a controversial issue [9]. For example, the stability margin is often analyzed for a single-input-single-output (SISO) system. As a new control technology, active disturbance rejection control (ADRC) proposed by Han is not sensitive to model uncertainty [10]. To simplify parameters adjustment, the nonlinear part in ADRC is transformed to the linear part and becomes the linear ADRC (LADRC) [11]. On account of simplicity and effectiveness, the novel method ADRC has been studied and applied to a lot of practical problems in recent years [12 – 17].

Based upon LADRC, a decoupling control strategy for high AOA tracking is presented. The independent controllers for pitch, yaw and roll state variables are constructed respectively to reject the strong couplings between channels. The aerodynamic uncertainty and channels coupling are treated as total disturbance, which is estimated in real time by the linear extended state observer (LESO). Ignoring the estimation error in generalized disturbance, the plant can be reduced to a unit integrator. The proportional derivative (PD) control method can then be utilized to realize reference tracking. The corresponding numerical simulations results are carried out to validate the performance.

The remaining parts are arranged as follows. Section 2 presents the aircraft nonlinear model and the thrust vector model. Section 3 shows the proposed control strategy. Some simulation results are given in Section 4. Section 5 comes to a conclusion.
2. Aircraft model

The aircraft nonlinear dynamic model comes from a classical model [18]. Because of the singularity coming from high AOA or the high pitch angle, a mathematical model is derived from the body axis system to the track axis system. Moreover, the thrust vector with rotatable nozzles also needs to be utilized to cover the shortage of aerodynamic surfaces at high AOA. Thus, a nonlinear fighter aircraft model with the thrust vector model derived in the track axis system is given in this section.

2.1 Nonlinear dynamic model

The nonlinear model of the aircraft can be expressed as:

\[
\dot{\mathbf{V}} = \left( \frac{Y \sin \beta - D}{m} \right) - g \sin \gamma + \frac{1}{m} [T_x \cos \beta \cos \alpha + T_y \sin \beta + T_z \cos \alpha \sin \beta]
\]

\[
\dot{\alpha} = q - \tan \beta (p \cos \alpha + r \sin \alpha) + \frac{1}{m V \cos \beta} (-L + m g \cos \gamma \cos \mu) + \frac{1}{m V \cos \beta} (-T_x \sin \alpha + T_z \cos \alpha)
\]

\[
\dot{\beta} = -r \cos \alpha + p \sin \alpha + \frac{1}{m V \cos \beta} (Y \cos \beta + m g \cos \gamma \sin \mu) + \frac{1}{m V \cos \beta} (-T_x \sin \beta \cos \alpha + T_y \cos \beta - T_z \sin \beta \sin \alpha)
\]

\[
\dot{\gamma} = \frac{1}{m V}(L \cos \mu - Y \sin \mu \cos \beta) - \frac{1}{m V} \cos \beta T_y + \frac{1}{m V}(T_x \sin \beta \cos \alpha \sin \mu + T_z \cos \mu \sin \alpha) + \frac{1}{\sin \alpha} (T_z \sin \beta \sin \mu \sin \alpha - T_x \cos \alpha \cos \mu)
\]

\[
\dot{\chi} = \frac{1}{m V \cos \gamma} (L \sin \mu + Y \cos \mu \cos \beta) + \frac{T_y}{m V \cos \gamma} \cos \mu \cos \beta + \frac{T_x}{m V \cos \gamma} (-\cos \mu \cos \alpha \sin \beta + \sin \alpha \sin \mu) - \frac{1}{m V \cos \gamma} (T_z \sin \alpha \beta \cos \mu + T_x \sin \mu \cos \alpha)
\]

\[
\dot{\mu} = \frac{1}{\cos \beta} (p \cos \alpha + r \sin \alpha) + \frac{1}{m V \cos \gamma} (L \tan \beta + L \sin \mu \tan \gamma) + \frac{\cos \beta \tan \gamma \cos \mu}{m V} (T_y + Y) - \frac{1}{V \cos \mu} \cos \beta \tan \cos \gamma + \frac{\sin \alpha T_x - \cos \alpha T_z}{m V} \tan \beta \cos \gamma
\]

\[
\dot{\beta} = \frac{I_{zz} (l_a + l_T) + I_{xx} (n_a + n_T)}{I_{xx} I_{zz} - I_{zz}^2} + pq \frac{I_{xx} (I_{xx} - I_{yy}) + I_{zz}}{I_{xx} I_{zz} - I_{zz}^2} + qr I_{zz} (l_{yy} - I_{xx}) - I_{zz}^2 \frac{I_{zz}^2}{I_{xx} I_{zz} - I_{zz}^2}
\]

\[
\dot{\chi} = \frac{(m_a + m_T)}{I_{yy}} \frac{pr (I_{zz} - I_{zz}) - I_{zz}^2}{I_{yy} (p^2 - r^2)}
\]

\[
\dot{\mathbf{r}} = \left[ \begin{array}{c}
\mathbf{V} \cos \gamma \cos \chi \\
\mathbf{V} \cos \gamma \sin \chi \\
-\mathbf{V} \sin \gamma
\end{array} \right]
\]

along three-axis; \( D \) represents the aerodynamic drag, \( Y \) is the lateral force and \( L \) is the lift force. \( D, Y, L \) can be calculated as follows:

\[
\begin{bmatrix}
D \\
Y \\
L
\end{bmatrix}
= \begin{bmatrix}
-\cos \beta \cos \alpha & -\sin \beta & -\cos \beta \sin \alpha \\
-\sin \beta \cos \alpha & \cos \beta & -\sin \beta \sin \alpha \\
\sin \alpha & 0 & -\cos \alpha
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_{xtot} S_{\gamma} \\
C_{ytot} S_{\gamma} \\
C_{ztot} S_{\gamma}
\end{bmatrix}
\]

where \( \gamma \) is the dynamic pressure, \( S \) is the reference area of the aircraft, \( C_{itot} \) \((i = x, y, z)\) represents the total aerodynamic force coefficient. The aerodynamic torque can be obtained by

\[
\begin{bmatrix}
l_a \\
m_a \\
n_a
\end{bmatrix}
= \begin{bmatrix}
\mathbf{T}_{Sb} C_{xtot} S_{\gamma} \\
\mathbf{T}_{Sb} C_{ytot} S_{\gamma} \\
\mathbf{T}_{Sb} C_{ztot} S_{\gamma}
\end{bmatrix}
\]
where \( b \) denotes the wing span, and \( c \) represents the mean aerodynamic chord, \( C_{l_{tot}}, C_{m_{tot}}, C_{n_{tot}} \) represent the total aerodynamic torque coefficients.

### 2.2 Thrust vector model

At the end of the aircraft, there are two rotatable nozzles installed symmetrically. Every nozzle can deflect in the yaw and pitch directions. The deflection angles can be described by \( \delta_{yi} \) and \( \delta_{zi} \) (\( i = l, r \) denotes left and right), respectively. A pair of nozzles can produce required three axes torques through multiple combinations of deflections. Then, we can obtain the total deflections along the roll, yaw and pitch directions as

\[
\begin{align*}
\delta_x &= \frac{\delta_{xr} - \delta_{zl}}{2} \\
\delta_y &= \frac{\delta_{yr} + \delta_{yl}}{2} \\
\delta_z &= \frac{\delta_{xr} + \delta_{zl}}{2}.
\end{align*}
\]

Under the body coordinate system, the thrust along the three axes can be derived as

\[
\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \zeta_f T \begin{bmatrix} \cos \delta_{zi} \cos \delta_{yi} \\ \sin \delta_{yi} \\ -\cos \delta_{zi} \sin \delta_{yi} \end{bmatrix}
\]

(15)

where \( \zeta_f \) represents the loss coefficient of the thrust. Ignoring installation errors, we assume \( \zeta_f = \zeta_f, T_r = T_l, \delta_y = \delta_y \). Then, we can obtain

\[
\begin{align*}
T_x &= \zeta_f T \begin{bmatrix} \cos \delta_{zi} \cos \delta_{yi} \\ \sin \delta_{yi} \\ -\cos \delta_{zi} \sin \delta_{yi} \end{bmatrix} \\
T_y &= \zeta_f T \begin{bmatrix} \cos \delta_{zi} \cos \delta_{yi} \\ \sin \delta_{yi} \\ -\cos \delta_{zi} \sin \delta_{yi} \end{bmatrix} + T_l \zeta_f \begin{bmatrix} \cos \delta_{zi} \cos \delta_{yi} \\ \sin \delta_{yi} \\ -\cos \delta_{zi} \sin \delta_{yi} \end{bmatrix}
\end{align*}
\]

(16)

where \( T_x, T_y, T_z \) denote the thrust components about the three-axis, \( T \) represents the total thrust. When the nozzles deflection is within the deflection range (less than 20°), we can re-express (17) approximatively as follows:

\[
\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \zeta_f T \begin{bmatrix} \frac{1}{\delta_y} \\ \delta_y \\ -\delta_z \end{bmatrix}
\]

(18)

Let \( x_T, y_T, z_T \) represent the engine position coordinates respectively. The torque generated by the thrust vector can be expressed as follows:

\[
\begin{bmatrix} l_T \\ m_T \\ n_T \end{bmatrix} = \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} \otimes \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}
\]

(19)

### 3. Control strategy

To eliminate the strong coupling between different channels at high AOA, a decoupling rejection control strategy based on ADRC with the thrust vector technology is proposed. Considering the designing convenience, we choose the AOA \( \alpha \), the sideslip angle \( \beta \) and the roll angular rate \( p \) as controlled variables. In \( \alpha, \beta \) and \( p \) channels, we design independent SISO controllers respectively. Fig. 1 shows the whole control structure. The reference commands are defined as \( \alpha_d, \beta_d, p_d \) and the thrust command is represented by \( T_c. \delta_x, \delta_y, \delta_z \) represent the deflection angles of the aerodynamic control surfaces, respectively. \( \delta_x, \delta_y, \delta_z \) represent the deflection angles of the thrust vector along the three axes. The detailed controllers design process is shown in Fig. 1.

![Proposed decoupling control scheme](image)

3.1 AOA channel controller design

We reformulate (2) as

\[
\dot{\alpha} = q + f_\alpha
\]

(20)

\[
f_\alpha = -\tan \beta (p \cos \alpha + r \sin \alpha) + \frac{1}{\cos \beta m V} (mg \cos \mu \cos \gamma - L) + \frac{1}{\cos \beta m V} (-T_x \sin \alpha + T_z \cos \alpha).
\]

(21)

Differentiate (20), and substitute (8) and (19) into it, then

\[
\dot{\alpha} = \dot{f}_\alpha + \frac{I_{zz} - I_{xx}}{I_{yy}} p r - \frac{I_{zz}}{I_{yy}} (p^2 - r^2) + (\frac{1}{I_{yy}} - b_{0a}) (m_a + m_T) + b_{0a} v_1 = F_\alpha + b_{0a} v_1
\]

(22)

where \( v_1 \) is virtual control, which represents the expected pitch manipulation torque, and it depends on the pitch deflection of the aerodynamic elevator and the vector nozzles. According to (22), we can define
Then the original dynamics of the AOA channel will be described as
\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + b_{\alpha_0} v_1 \\
\dot{x}_3 &= H_\alpha \\
y &= [1, 0, 0] x
\end{aligned}
\]  
(24)

where \( b_{\alpha_0} \) is related to the system which is an adjustable parameter, \( H_\alpha \) is the derivative of the total disturbance. Then the original dynamics of the AOA channel will be simplified as a second order integral system with a generalized disturbance or total disturbance which causes deviation from the typical integrator system. The key of (24) is that \( x_3 \) is treated as a system state or a signal regardless of its original form. The state space expression for (24) can be described as
\[
\begin{aligned}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} &= 
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} v_1 + 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} H_\alpha \\
y &= [1, 0, 0] \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\end{aligned}
\]  
(25)

Then, we can establish the corresponding state observer as
\[
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 \\
\dot{\hat{x}}_2 &= \hat{x}_3 + b_{\alpha_0} \hat{v}_1 \\
\dot{\hat{x}}_3 &= \hat{H}_\alpha \\
\hat{y} &= [1, 0, 0] \hat{x}
\end{aligned}
\]  
(26)

where \( \hat{z}_j \) (\( j = 1, 2, 3 \)) represents the estimation results of \( x_j \) (\( j = 1, 2, 3 \)) respectively. If the observer gains, \( \beta_{\alpha_1}, \beta_{\alpha_2}, \beta_{\alpha_3} \) can be adjusted appropriately, \( z_j \) (\( j = 1, 2, 3 \)) can track the states \( x_j \) (\( j = 1, 2, 3 \)) approximately. In particular, the third state \( x_3 \) can be approximated by \( z_3 \). An observer with such capability is usually defined as the extended state observer (ESO) [10]. When the observer gains \( \beta_{\alpha_1} = 3\omega_o, \beta_{\alpha_2} = 3\omega_o^2, \beta_{\alpha_3} = \omega_o^3 \) are used, the observer is known as the bandwidth parameterized LESO [11,19]. \( \omega_o \) known as the bandwidth of the LESO is the only parameter that needs to be adjusted. When \( z_3 \approx x_3 \) is obtained by using the LESO, the disturbance compensation signal is used as follows:
\[
v_1 = \frac{-z_3 + u_0}{b_{\alpha_0}}
\]  
(27)

where \( u_0 \) represents the control law which needs to be designed. Finally, the original dynamic for AOA can be approximated to
\[
y \approx u_0.
\]  
(28)

Thus, the dynamic of \( \alpha \) becomes a second order integrator system. For such a typical system, the simple control law such as PD can be adopted.
\[
u_0 = k_{p\alpha}(\alpha_d - \alpha) - k_{d\alpha} \dot{\alpha}
\]  
(29)

Then we can obtain the final control law for the AOA channel
\[
v_1 = \frac{k_{p\alpha}(\alpha_d - \alpha) - k_{d\alpha} z_2 - z_3}{b_{\alpha_0}}
\]  
(30)

where \( \alpha_d \) is the command signal for the AOA channel. To generate the proper reference signal [20], a fastest tracking differentiator (TD) is employed, and its specific form can be described as
\[
\begin{cases}
\dot{s}_{\alpha_1} = s_{\alpha_2} \\
\dot{s}_{\alpha_2} = fhan(s_{\alpha_1} - \alpha_d, s_{\alpha_2}, \varepsilon, \eta)
\end{cases}
\]  
(31)

where the output signal \( s_{\alpha_1} \) can track \( \alpha_d \); \( s_{\alpha_2} \) represents the differential signal of \( s_{\alpha_1} \); \( \varepsilon \) determines the TD convergence speed and is usually called the speed factor, and \( \eta \) is called the filtering factor; \( fhan(\cdot) \) is an optimal synthetic function and its specific form is expressed as
\[
\begin{aligned}
d &= \varepsilon \eta^2 \\
c_0 &= \eta s_{\alpha_2} \\
y &= (s_{\alpha_1} - \alpha_d) + c_0 \\
c_1 &= \sqrt{(d^2 + 8|y|d)} \\
c_2 &= c_0 + 0.5(c_1 - d) \text{signy} \\
c &= f\,\text{s}(y, d)(y + c_0) - (-1 + f\,\text{s}(y, d))c_2 \\
f\,\text{s}(x, d) &= 0.5[\text{sign}(d + x) - \text{sign}(-d + x)] \\
f\,\text{han} &= -(\frac{ce}{d})f\,\text{s}(c, d) + \text{signc} \\
r(-1 + f\,\text{s}(c, d))
\end{aligned}
\]  
(32)

For the convergence of the ESO, we consider the AOA channel and let \( e_j(t) = x_j(t) - z_j(t) \) (\( j = 1, 2, 3 \)). Combining (25) and (26), we describe the observation error dynamics as
\[
\begin{cases}
\dot{\hat{e}}_1(t) = e_2(t) - 3\omega_o^2 e_1(t) \\
\dot{\hat{e}}_2(t) = e_3(t) - 3\omega_o^2 e_1(t) \\
\dot{\hat{e}}_3(t) = H_\alpha(t) - \omega_o^3 e_1(t)
\end{cases}
\]  
(33)
where $H_\alpha(t)$ indicates the total disturbance in the AOA channel. Define $\Theta_i(t) = e_i(t)/\omega_o^{-1}$ ($i = 1, 2, 3$). Equation (33) can be reformulated as

$$\dot{\Theta}(t) = \omega_o A_{\Theta} \Theta(t) + B_{\Theta} \frac{H_\alpha(t)}{\omega_o}$$

(34)

where $A_{\Theta} = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$, $B_{\Theta} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

**Theorem 1** Assuming $H_\alpha(t)$ is bounded, there exists a positive constant $\sigma_i > 0, \omega_o > 0$, and finite time $T_{\Theta} > 0$, such that $|\Theta_i(t)| \leq \sigma_i$ ($i = 1, 2, 3$) for all $t \geq T_{\Theta}$.

**Proof** The solution of (34) can be expressed as

$$\Theta(t) = e^{\omega_o A_{\Theta} t} \Theta(0) + \int_0^t e^{\omega_o A_{\Theta} (t - \tau)} B_{\Theta} \frac{H_\alpha(t)}{\omega_o} \, d\tau.$$  (35)

Let

$$p(t) = \int_0^t e^{\omega_o A_{\Theta} (t - \tau)} B_{\Theta} \frac{H_\alpha(t)}{\omega_o} \, d\tau.$$  (36)

Assume $H_\alpha \leq K, K > 0$, then

$$|p_i(t)| \leq \frac{K}{\omega_o} \left( |(A_{\Theta}^{-1} B_{\Theta})_i| + |(A_{\Theta}^{-1} e^{\omega_o A_{\Theta} t}) B_{\Theta} | \right),$$

$$i = 1, 2, 3.$$  (37)

From $A_{\Theta}^{-1}$ and $B_{\Theta}$, we can obtain

$$|(A_{\Theta}^{-1} B_{\Theta})| = \begin{cases} 1, & i = 1 \\ 3, & i = 2, 3 \end{cases}$$

(38)

Since $A_{\Theta}$ is Hurwitz, there exists a setting time $T_{\Theta} > 0$, and for all $t \geq T_{\Theta}$ ($i, j = 1, 2, 3$), such that

$$|e^{\omega_o A_{\Theta} t}| \leq \frac{1}{\omega_o}.$$  (39)

$T_{\Theta}$ depends on $\omega_o A_{\Theta}$. Let $A_{\Theta}^{-1} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ and

$$e^{\omega_o A_{\Theta} t} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix},$$

it follows

$$|(A_{\Theta}^{-1} e^{\omega_o A_{\Theta} t} B_{\Theta})| = |c_{11}d_{13} + c_{12}d_{23} + c_{33}d_{33}| \leq \frac{1}{\omega_o},$$

$$i = 1$$

$$\frac{1}{\omega_o},$$

$$i = 2, 3.$$  (40)

Thus, from (37), (38) and (40), we can obtain

$$|p_i(t)| \leq \frac{3K}{\omega_o^3} + \frac{4K}{\omega_o^3}.$$  (41)

Let $\Theta_{sum}(0) = \sum_{i=1}^{3} |\Theta_i(0)|$, we have

$$|e^{\omega_o A_{\Theta} t} \Theta(0)| \leq \frac{\Theta_{sum}(0)}{\omega_o^3}. $$

(42)

Then, according to (35) and $\Theta_i(t) = e_i(t)/\omega_o^{-1}, i = 1, 2, 3$, for all $t \geq T_{\Theta}$, we have

$$|e_i(t)| \leq \frac{\Theta_{sum}(0)}{\omega_o^3} + \frac{3K}{\omega_o^3} + \frac{4K}{\omega_o^3} = \sigma_i.$$  (43)

In summary, from (43), the estimation error of the ESO is bounded and its upper bound decreases when the observer bandwidth increases.

### 3.2 Sideslip angle controller designing

We can reformulate (3) as

$$\dot{\beta} = -r \cos \alpha + f_\beta$$

(44)

$$f_\beta = p \sin \alpha + \frac{1}{mV}(Y \cos \beta + mg \cos \gamma \sin \mu) + \frac{1}{mV}(-T_\epsilon \sin \beta \cos \alpha + T_y \cos \beta - T_z \sin \beta \sin \alpha).$$  (45)

Considering the symmetry of the AOA and the sideslip angle, we can directly give the virtual control law for the $\beta$ channel as

$$v_2 = \frac{k_{p_3}(s_{31} - \beta) - k_{d_3}r - z_{33}}{b_{\beta}}$$

(46)

where $z_{33}$ can be obtained by

$$\begin{bmatrix} \dot{z}_{13} \\ \dot{z}_{23} \\ \dot{z}_{33} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{13} \\ z_{23} \\ z_{33} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{\beta} \end{bmatrix} v_2 + \begin{bmatrix} 3\omega_o \\ 3\omega_o^2 \\ \omega_o^3 \end{bmatrix} \begin{bmatrix} \beta \\ 0 \end{bmatrix} - [1, 0, 0] \begin{bmatrix} z_{13} \\ z_{23} \\ z_{33} \end{bmatrix}.$$  (47)

$s_{31}$ is produced by TD as

$$\begin{bmatrix} s_{31} \\ s_{32} \end{bmatrix} = f_{han}(s_{31} - \beta_d, s_{32}, \varepsilon, \eta).$$  (48)

### 3.3 Roll angular rate controller designing

In practice, it is difficult to measure the bank angle $\mu$ accurately. Therefore, we select roll angular rate $p$ as the controlled state to realize the proper curve of $\mu$. We rewrite (7) as

$$\dot{p} = F_p + b_{p_3}v_3$$

(49)

$$F_p = \frac{I_{zz}(n_{ia} - n_\tau I_{zz})}{I_{xx}I_{zz} - I_{zz}^2} + \frac{I_{zz}(I_{xx} - I_{yy} + I_{zz})}{I_{xx}I_{zz} - I_{zz}^2}pq + \frac{I_{zz}(I_{yy} - I_{zz} - I_{zz}^2)}{I_{xx}I_{zz} - I_{zz}^2}qr + \left( \frac{I_{xx}I_{zz} - I_{zz}^2}{b_{p_3}} \right) (l + \tau).$$  (50)
Define $x_1 = p$, $x_2 = F_p$, $\dot{F}_p = H_p$, then we can get the state-space expression

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 & x_1 \\ 0 & 0 & x_2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} b_{p0} \\ 0 \end{bmatrix} v_3 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} H_p \ . \tag{51}$$

Thus, the corresponding LESO can be designed as

$$\begin{bmatrix} \dot{z}_{1p} \\ \dot{z}_{2p} \end{bmatrix} = \begin{bmatrix} 0 & 1 & z_{1p} \\ 0 & 0 & z_{2p} \end{bmatrix} + \begin{bmatrix} b_{p0} \\ 0 \end{bmatrix} v_3 + \begin{bmatrix} 2\omega_0 \\ \omega_0 \end{bmatrix} \left( y - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right). \tag{52}$$

Finally, the whole control law is

$$v_3 = \frac{k_p(s_{p1} - p) - z_{2p}}{b_{p0}} \tag{53}$$

where $s_{p1}$ is generated by the TD

$$\begin{cases} \dot{s}_{p1} = s_{p2} \\ \dot{s}_{p2} = \text{fan}(s_{p1} - p_d, s_{\beta2}, \varepsilon, \eta) \ . \tag{54} \end{cases}$$

4. Control surfaces allocation

The controller outputs of three channels characterize the corresponding need of the expected moment of the force about the three axes. The goal of control surfaces allocation is to finish calculation for the deflection angle values of aerodynamic surfaces and the thrust vector. Thus, we define $x_1 = [p, q, r]^T$, $u = [\delta_e, \delta_a, \delta_r, \delta_y, \delta_z]^T$, $\varpi = [V, \alpha, \beta, p, q, r]^T$, $F(\varpi) = [f_p(\varpi), f_q(\varpi), f_r(\varpi)]^T$. Then, we can reformulate (7), (8) and (9), that is

$$\begin{cases} \dot{x}_1 = G(\varpi)u + F(\varpi) \\ y_1 = x_1 \ . \tag{55} \end{cases}$$

The expressions of $F(\varpi)$ and $G(\varpi)$ can be described as

$$F(\varpi) = \frac{I_{xx}I_{xx}pq - I_{xx}I_{yy}pq + I_{xx}I_{zz}pq}{-I_{zz}^2 + I_{xx}I_{zz}} + \frac{(I_{yy}I_{zz}^2 - I_{zz}^2)^2 + (I_{zz}pr - I_{xx}pr) - (I_{xx}p^2 - I_{zz}r^2) - I_{yy}g}{I_{yy}pr - I_{xx}pr} - \frac{I_{xx}I_{zz}^2}{I_{zz}^2 + I_{xx}I_{zz}}$$

$$- \frac{I_{xx}I_{zz}r^2 - I_{yy}I_{zz}r^2 - I_{zz}I_{xx}r^2}{-I_{zz}^2 + I_{xx}I_{zz}}$$

$$G(\varpi) = \begin{bmatrix} g_{\delta_e}(\varpi) \\ g_{\delta_a}(\varpi) \\ g_{\delta_r}(\varpi) \\ g_{\delta_y}(\varpi) \\ g_{\delta_z}(\varpi) \\ g_{\beta}(\varpi) \\ g_{\alpha}(\varpi) \end{bmatrix} \tag{57}$$

where $g(\varpi)$ denotes control derivatives with respect to aerodynamic deflections and thrust vector deflections, respectively. In addition, we can define $v = [v_1, v_2, v_3]^T$, then the control surfaces allocation can be explained as follows. We let $v(t) \in \mathbb{R}^3$ as the expected virtual control command and $u(t) \in \mathbb{R}^6$ as the control surface deflection. In a word, the goal of control surfaces allocation is to obtain the solution of the equation under the following mapping relation $G(\varpi) : \mathbb{R}^6 \rightarrow \mathbb{R}^3$,

$$\begin{cases} \dot{x}_1 = G(\varpi)u + F(\varpi)u \\ y_1 = x_1 \ . \tag{58} \end{cases}$$

with the control input constraint $\Omega = \{u(t) \in \mathbb{R}^6 | u_{\min} \leq u \leq u_{\max}, \Gamma_{\min} \leq \Gamma \leq \Gamma_{\max} \}$, where $u_{\min(max)}$ and $\Gamma_{\min(max)}$ denote the lower bound (upper bound) of the control surface deflection and its rate of change. In order to extend the lifespan of the engine nozzles, the principle of control surfaces allocation is that the deflection of the thrust vector can be reduced as far as possible, that is, aerodynamic control has a higher priority than the thrust vector. Based on the above principle, we divide the control input into two parts

$$\begin{cases} u = [u_{\text{aero}}, u_{\text{tv}}]^T \\ G(\varpi) = [G_{\text{aero}}, G_{\text{tv}}]^T \ . \tag{59} \end{cases}$$

where $u_{\text{aero}} = [\delta_e, \delta_a, \delta_r]^T$, $u_{\text{tv}} = [\delta_y, \delta_y, \delta_z]^T$, $G_{\text{aero}}$ represents the left three columns of $G(\varpi)$, and $G_{\text{tv}}$ denotes the right three columns. Moreover, we let $P_{\text{aero}} = G_{\text{aero}}^{-1}$, $P_{\text{tv}} = G_{\text{tv}}^{-1}$ represent the inverse matrices of $G_{\text{aero}}$ and $G_{\text{tv}}$, respectively. Table 1 shows the limitations of the position and the rate of change. Fig. 2 gives the overall description of the control allocation process. The aerodynamic control deflections should try to satisfy

$$u_{\text{aero}} = P_{\text{aero}}v. \tag{60}$$

| Control surface | Bandwidth (rad/s) | Rate (°/s) | Position (°) |
|-----------------|------------------|------------|--------------|
| Aileron         | 20.2             | ±80        | ±21.5        |
| Elevator        | 20.2             | ±80        | ±25          |
| Rudder          | 20.2             | ±120       | ±30          |
| Roll            | 30.2             | ±150       | ±20          |
| Yaw             | 30.2             | ±150       | ±20          |
| Pitch           | 30.2             | ±150       | ±20          |
Fig. 2 Control allocation diagram

If the aerodynamic control deflections calculated by (60) are all in the limitation, then the control allocation is over. Or else, when $u_{aero}$ exceeds the limitation, the thrust vector $u_{tv}$ is required to eliminate the control allocation error $E$.

$$
\begin{align*}
  u_{aero} &= \text{Sat}_{aero}(P_{aero}v) \\
  E &= v - G_{aero}u_{aero} \\
  u_{tv} &= \text{Sat}_{tv}(P_{tv}E)
\end{align*}
$$

(61)

The expression of $\text{Sat}(x, \bar{k})$ is

$$
y = \text{Sat}(x, \bar{k}) = \begin{cases} 
  k, & x > \bar{k} \\
  x, & -\bar{k} \leq x \leq \bar{k} \\
  -k, & x < -\bar{k}
\end{cases}
$$

(62)

where $\bar{k}$ denotes the upper bound. To sum up, such a control allocation process can realize the maximum usage of the aerodynamic control and minimize the usage of nozzles deflections.

5. Simulation verification

Some numerical simulations are conducted to demonstrate the effectiveness and robustness of the presented control strategy. Herbst maneuver is a typical super maneuvering action. Thus, the controller performance is validated by selecting a Herbst-type maneuver. Moreover, we select the initial flight speed, flight height and AOA as $V = 90$ m/s, $h_0 = -z_{E0} = -1200$ m and $\alpha_0 = 10^\circ$, respectively. The controllers parameters are adjusted as $\omega_0 = 10$, $k_{pa} = 50$, $k_{da} = 0.02$, $k_{p\beta} = 20$, $k_{d\beta} = 0.02$, $k_p = 30$, $b_{\alpha} = 8$, $b_{\beta} = -2.5$, $b_{\dot{\alpha}} = -2$. During the maneuver, the maximum thrust value is utilized.

5.1 Performance verification

The Herbst maneuver is to realize the rapid turn with a small radius. Thus, we consider a Herbst-type maneuver, and its end condition is that the flight direction of the fighter aircraft changes $180^\circ$. Figs. 3–6 present the simulation results. From these figures, the AOA and roll angular rate $\dot{p}$ track the desired value well, and the actual value of sideslip angle $\beta$ keeps almost zero. For $\alpha$, $\beta$ and $p$, an appropriate reference is used to go through the fastest TD, and the outputs of TD are chosen as the final reference signals which are shown in Fig. 3(a), Fig. 3(b) and Fig. 3(c), respectively. From Fig. 3(a), at $t = 1.5$ s, AOA begins to change and then reaches $62.5^\circ$ after about 2 s. AOA keeps $62.5^\circ$ for a little while, and meanwhile the roll angular rate changes as Fig. 3(c) such that the aircraft can roll around the velocity vector. After rolling, the aircraft swoops for a while and then the Herbst-type maneuver is over. During this maneuver action, the velocity changes as shown in Fig. 3(d), and it decreases at first and then increases. At high AOA, when $p$ changes properly, $\mu$ can change as the expected curve shown in Fig. 3(f). From Fig. 3(g), it can be seen that the heading angle of the flight speed changes $180^\circ$. The aerodynamic deflections are shown in Fig. 4. From Fig. 4, we can see that the elevator reaches the deflection limitation, indicating that the aerodynamic surface is insufficient. The thrust vector deflections are illustrated in Fig. 5. Moreover, all the thrust vector controls do not exceed the saturation limitation. From Fig. 6, the turning diameter is about 150 m.
5.2 Monte Carlo test
Monte Carlo simulation is conducted to realize the robustness verification. We select 43 aerodynamic parameters
and perturb them randomly within ±30% limits. Five hundred simulations are carried out totally. The simulation test results are shown in Fig. 7. It can be easily known that the response indicates good robustness.

![Fig. 7 Monte Carlo simulation test](image)

6. Conclusions

In this paper, a decoupling control scheme is proposed to implement the super maneuver with the thrust vector at high AOA. Independent controllers for three channels are designed based upon the linear active disturbance rejection control instead of the traditional multivariate control idea. LESOs are constructed to get the estimation of the total disturbance including strong coupling from channels and aerodynamic uncertainties, which are compensated in real time. The traditional daisy chain method is utilized to complete the control surface allocation. Simulation results are carried out to validate the effectiveness and robustness of the proposed coupling rejection scheme.

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