Disorder and Localization in the Lowest Landau Level

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We study the localization property of a two-dimensional noninteracting electron gas in the presence of randomly distributed short-range scatterers. We evaluate the participation number of the eigenstates obtained by exact diagonalization technique. At low impurity concentrations we obtain self-averaged values showing that all states, except those exactly at the Landau level, are localized with finite localization length. We conclude that there is no universal localization exponent and at least at low impurity concentrations localization length does not diverge.

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There has been a long lasting interest in understanding the localization problem in two-dimensional (2D) systems. According to scaling theory of localization [1, 2], all states in a 2D system are localized if a disordered potential is present. However, in the presence of a strong perpendicular magnetic field, where the time reversal symmetry is broken, extended states appear in the center of impurity-broadened Landau bands [3, 4]. If the scattering between Landau levels can be neglected, these extended states exist only at a single energy [5]. The width of the quantized plateau of the integer quantum Hall effect (QHE) depends on the ratio of number of localized to extended states [6].

In analogy with the quantum critical phenomena and other localization transitions, it has been proposed that localization length \( \xi(E) \) diverges as \( E \) approaches the critical energy \( E_c \), which is equal to Landau level energy, so that

\[
\xi(E) \propto |E - E_c|^{-\nu}
\]

(1)

where \( \nu \) is the localization critical exponent [1]. After the initial calculations of Aoki and Ando [10], several groups attempted to determine this critical exponent [13, 20]. Experimental results [22, 23] are generally in good agreement with the calculated values. Various techniques have allowed the computation of the exponent \( \nu \), and they strongly suggest a universal value close to \( \nu = 7/3 \) for the lowest Landau level (LLL). However, in spite of a great deal of experimental evidence and numerical simulations in its favor, there is no rigorous derivation of power law divergence in the localization length. Furthermore, even if the power law divergence is true, it is not clear whether the localization critical exponent is universal, independent of impurity concentration or parameters of the disordered potential [33].

Recently, we developed a method for a particle in the LLL moving in an arbitrary potential [34]. In this study we apply the method, which is basically an exact diagonalization technique, to a potential formed by randomly distributed short-range scatterers. We concentrate on low impurity concentrations where it is difficult to perform calculations by other methods due to the presence of zero eigenvalues associated with the extended states at the band center. At low enough concentrations, we obtain self-averaged values where energy spectrum or localization property of eigenstates do not change with increasing system size. Contrary to the widely accepted view, localization length does not diverge at low impurity concentrations but instead the maximum localization length grows exponentially with impurity density. Extrapolation to less pure systems suggests that localization length can become as large as the sizes of the samples used in QHE experiments which explains the observed divergence in measurements.

The Hamiltonian for a particle of mass \( m \) and charge \( q \), moving in 2D in the presence of magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) perpendicular to the plane and potential \( V \), is given by \( H = H_0 + V \) where

\[
H_0 = \frac{1}{2m}(\mathbf{p} - \frac{q}{c} \mathbf{A})^2
\]

(2)

Using the symmetric gauge \( \mathbf{A} = \frac{1}{2} \mathbf{B} \times \mathbf{r} \) and complex coordinates \( z = X + iY = \sqrt{qB/2\hbar c}(x + iy) \) where \( r = (x, y) \), the unperturbed Hamiltonian can be written as \( H_0 = \hbar \omega (a^\dagger a + 1/2) \) where \( a^\dagger = -\partial/\partial z + z^*/2 \). Since \( [a, a^\dagger] = 1 \), the energy eigenvalues are given by \( E_n = \hbar \omega (n + 1/2) \) where \( \omega = qB/2mc \) (\( q \) is assumed to be positive) and \( n = 0, 1, 2, \ldots \). When the magnetic field is very high the particle is confined into the LLL. This is a good approximation as long as the potential is small in comparison to Landau level splitting \( \hbar \omega \). We are going to measure energies from the LLL so that \( E_n = 0 \).

Now, let us consider the potential

\[
V(z, z^*) = V_0 \sum_i \delta(z - z_i)\delta(z^* - z^*_i)
\]

(3)

where \( z_i \) denotes the position of the \( i \)th impurity in complex coordinates defined above. According to our method [34], to find the nonzero eigenvalues, the matrix to be diagonalized is
\[ \langle i | \hat{V} | j \rangle = \frac{V_0}{\pi} \exp(z_i z_j^* - |z_i|^2/2 - |z_j|^2/2). \] (4)

Once $\hat{V}$ is diagonalized, the eigenfunctions $\psi(z, z^*)$ of $V$ can be constructed from $\psi_i$

\[ \psi(z, z^*) = \sqrt{\frac{V_0}{\pi^2|E|}} \sum_i \exp(z_i z_j^* - |z_i|^2/2 - |z_j|^2/2)\psi_i. \] (5)

We distinguish between the extended and the localized states via participation number $P$, which is the inverse of the mean fourth power of the amplitude. Therefore, given a wave function $\psi$, $P$ is defined as

\[ P = \frac{\int |\psi(r)|^2 dr}{\int |\psi(r)|^4 dr}. \] (6)

The participation number is a convenient quantity for distinguishing between localized and extended states since it takes a nonvanishing value for the former and becomes infinite for the latter. If a state is localized within a $d$–dimensional volume of average diameter $D$, $P$ behaves as $D^d$ irrespectively of the system size. For a plane wave it depends on the system size as $L^d$. In general, extended states lead to an effective dimensionality $d^*$, smaller than the real dimensionality $d$, which means that the states are not space-filling.

For $\psi$, participation number reduces to $P = 1/\sum_i |\psi_i|^4$ provided that $\psi$ is normalized. For a state localized on single impurity $P = 1$, while for uniform distribution over $N_i$ impurities $P = N_i$. Therefore, we can interpret $P$ as a measure of number of scatterers on which $\psi$ takes nonzero value. We note that, as can be seen from Eqn. 6, $\psi$ and $\psi_i$ have the same localization behavior, i.e. they are both extended or localized. Although we can evaluate corresponding $P$, for a state localized on single impurity $P = 1$, while for uniform distribution over $N_i$ impurities $P = N_i$. Therefore, $\psi$ is a measure of number of scatterers on which $\psi$ takes nonzero value. We note that, as can be seen from Eqn. 6, $\psi$ and $\psi_i$ have the same localization behavior, i.e. they are both extended or localized. Although we can evaluate corresponding $P$, for a state localized on single impurity $P = 1$, while for uniform distribution over $N_i$ impurities $P = N_i$. Therefore, $\psi$ is an extended state. On the other hand, if $P < \sqrt{N_i}$, then $\psi$ has no chance to be extended. In this way we obtain the number of extended states $N_e$, for a given system composed of $N_i$ impurities. Although $P > \sqrt{N_i}$, corresponding state can still be localized. Therefore, $N_e$ is only an upper bound for the number of extended states. Our definition of extendedness become exact if the sites at which $\psi$ is nonvanishing form straight lines at least in one direction.

Figure 2 shows variation of $N_e$ with $N_i$ for different impurity concentrations. Each point is obtained in such a way that number of different configurations times $N_i$ is $10^5$. Straight lines indicate that the two numbers are related by $N_e \propto N_i^{y}$ and the inset shows variation of $y$ with $f$. We note that $y$ depends upon impurity concentration. If we assume that the maximum localization length diverges as energy $E$ approaches the critical value $E_c$ (see Eqn. 4), then it is easy to show that $\nu = 1/(1 - y)$. The widely accepted value $\nu = 7/3$ corresponds to $y = 11/14$ which we obtain at $f = 0.97$. At lower $f$ values, $\nu$ becomes lower. However, we must be careful in comparing our results with the values in the literature since most, in fact to our knowledge all, of the existing calculations have been performed at impurity concentrations higher.
than unity. In addition to this, our definition of extendedness is somewhat arbitrary. In general, the number of extended states is less than \( N_e \). Finally, as we are going to see from the next figure, divergence of the localization length can merely be an artifact of finite size calculations.

As we go to more dilute systems and perform calculations in large enough systems a remarkable change occurs. As shown in Fig. 3, \( N_e \) no more increases with \( N_i \) but instead decreases and vanishes at the end. For \( N_i > N_i^* \) there are no extended states. We define \( N_i^* \) as the number of impurities for which \( N_e = 1 \). If we increase \( N_i \) further, we do not get any extended states. This result is independent of the arbitrariness of our method of distinguishing between extended and localized states.

At this stage, it is not possible to say whether there is no localization length divergence for larger concentrations but Fig. 4 gives some idea about the system sizes to be used in the experiment to observe the opposite is true, i.e. disorder potential is smoothly varying in comparison to the magnetic length. In that case we can use a correlated disordered potential where several \( \delta \)-functions are used to model the extended potential created by a single impurity atom. Another point that should be taken into account before quantitative comparison is that at low impurity concentrations electron-electron interactions become important.

\[
\begin{align*}
N_i^* & = \text{exp}(11.58f - 3.46) \\
N_i^* & = \text{exp}(13.04f - 1.52)
\end{align*}
\]

In conclusion, at low impurity concentrations we obtain self-averaged values showing that all states, except those exactly at the Landau level, are localized with finite localization length. We conclude that the localization length diverges.
exponent is not universal and at least at low impurity concentrations localization length does not diverge. Our results suggest that the same behavior can be observed for higher concentrations. If this is the case then there is no universal localization exponent, and in contrast to results of previous theoretical, numerical, and experimental studies, localization length does not diverge. On the other hand, if for higher concentrations there is a divergence, the transition between the two regimes is a very interesting problem to investigate.

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