ON THE MEANING OF LORENTZ COVARIANCE

László E. Szabó

Theoretical Physics Research Group of the Hungarian Academy of Sciences
Department of History and Philosophy of Science
Eötvös University, Budapest, Hungary
E-mail: leszabo@hps.elte.hu

In classical mechanics, Galilean covariance and the principle of relativity are completely equivalent and hold for all possible dynamical processes. In contrast, in relativistic physics the situation is much more complex. It will be shown that Lorentz covariance and the principle of relativity are not completely equivalent. The reason is that the principle of relativity actually only holds for the equilibrium quantities that characterize the equilibrium state of dissipative systems. In the light of this fact it will be argued that Lorentz covariance should not be regarded as a fundamental symmetry of the laws of physics.

Key words: special relativity, space-time, Lorentz covariance, relativity principle, equilibrium state, dissipative systems

1 INTRODUCTION

It is a widely accepted view that special relativity, beyond its claim about space and time (cf. [1]), is a theory providing a powerful method for the physics of objects moving at constant velocities. The basic idea is the following: Consider a physical object at rest in an arbitrary inertial frame $K$. Assume we know the relevant physical equations and know the solution of the equations describing the physical properties of the object in question when it is at rest. All these things are expressed in the terms of the space and time coordinates $x_1, x_2, x_3, t$ and some other quantities defined in $K$ on the basis of $x_1, x_2, x_3, t$. We now inquire as to the same physical properties of the same object when it is, as a whole, moving at a given constant velocity relative to $K$. In other words, the issue is how these physical properties are modified when
the object is in motion. The standard method for solving this problem is based on the
relativity principle/Lorentz covariance. It follows from the covariance of the laws of
nature relative to Lorentz transformations that the same equations hold for the primed
variables \(x'_1, x'_2, x'_3, t'\)… defined in the co-moving inertial frame \(K'\). Moreover, since
the moving object is at rest in the co-moving reference frame \(K'\), it follows from the
relativity principle that the same rest-solution holds for the primed variables. Finally,
we obtain the solution describing the system moving as a whole at constant velocity by
expressing the primed variables through the original \(x_1, x_2, x_3, t, \ldots \) of \(K\), applying
the Lorentz transformation.

This is the way we usually solve problems such as the electromagnetic field of a
moving point charge, the Lorentz deformation of a rigid body, the loss of phase suffered
by a moving clock, the dilatation of the mean life of a cosmic ray \(\mu\)-meson, etc.

In this paper I would like to show that this method, in general, is not correct; the
system described by the solution so obtained is not necessarily identical with the orig-
inal system set in collective motion. The reason is, as will be shown, that Lorentz
covariance in itself does not guarantee that the physical laws in question satisfy the
relativity principle in general. The principle of relativity actually only holds for the
equilibrium quantities characterizing the equilibrium state of dissipative systems.

2 THE RELATIVITY PRINCIPLE

Consider two inertial frames of reference \(K\) and \(K'\). Assume that \(K'\) is moving at
constant velocity \(v\) relative to \(K\) along the axis of \(x\). Assume that laws of physics are
know and empirically confirmed in inertial frame \(K\), including the laws describing the
behavior of physical objects in motion relative to \(K\). Denote \(x(A), y(A), z(A), t(A)\)
the space and time tags of an event \(A\), obtainable by means of measuring-rods and
clocks at rest relative to \(K\), and denote \(x'(A), y'(A), z'(A), t'(A)\) the similar data
of the same event, obtainable by means of measuring-rods and clocks co-moving
with \(K'\). In the approximation of classical physics \((v \ll c)\), the relationship be-
tween \(x'(A), y'(A), z'(A), t'(A)\) and \(x(A), y(A), z(A), t(A)\) can be described by the
Galilean transformation:

\[
\begin{align*}
  t'(A) & = t(A), \\
  x'(A) & = x(A) - v t(A), \\
  y'(A) & = y(A), \\
  z'(A) & = z(A).
\end{align*}
\]

Due to the relativistic deformations of measuring-rods and clocks, the exact relation-
ship is described by the Lorentz transformation:

\[
\begin{align*}
  t'(A) & = \frac{t(A) - \frac{v x(A)}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}, \\
  x'(A) & = \frac{x(A) - v t(A)}{\sqrt{1 - \frac{v^2}{c^2}}}.
\end{align*}
\]
\[
\begin{align*}
y'(A) &= y(A), \\
z'(A) &= z(A).
\end{align*}
\] (7) (8)

Since physical quantities are defined by the same operational procedure in all inertial frame of reference, the transformation rules of the space and time coordinates (usually) predetermine the transformations rules of the other physical variables. So, depending on the context, we will mean by Galilean/Lorentz transformation not only the transformation of the space and time tags, but also the corresponding transformation of the other variables in question.

Following Einstein’s 1905 paper, the Lorentz transformation rules are usually derived from the relativity principle—the general validity of which we are going to challenge in this paper. As we will see, this derivation is not in contradiction with our final conclusions. However, it is worth while to mention that Lorentz transformation can also be derived independently of the principle of relativity, directly from the facts that a clock slows down by factor \( \sqrt{1 - v^2/c^2} \) when it is gently accelerated from \( K \) to \( K' \) and a measuring-rod suffers a contraction by factor \( \sqrt{1 - v^2/c^2} \) when it is gently accelerated from \( K \) to \( K' \) (see [1]).

Now, relativity principle is the following assertion:

**Relativity Principle** The behavior of the system co-moving as a whole with \( K' \), expressed in terms of the results of measurements obtainable by means of measuring-rods, clocks, etc., co-moving with \( K' \) is the same as the behavior of the original system, expressed in terms of the measurements with the equipments at rest in \( K \).

Exactly as Galilei describes it:

... the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship’s motion is common to all the things contained in it [my italics], and to the air also. ([2], p. 187)

Or, in Einstein’s formulation:

If, relative to \( K \), \( K' \) is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to \( K' \) according to exactly the same general laws as with respect to \( K \). ([3], p. 16)

In classical physics, the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Galilean transformation. According to special relativity, the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames are connected through the Lorentz transformation. Consequently, the laws
of physics must preserve their forms with respect of the Galilean/Lorentz transformation. Thus, it must be emphasized, the Galilean/Lorentz covariance is a consequence of two physical facts: 1) the laws of physics satisfy the relativity principle and 2) the space and time tags in different inertial frames are connected through the Galilean/Lorentz transformation.

Let us try to unpack these verbal formulations in a more mathematical way. Let $\mathcal{E}$ be a set of differential equations describing the behavior of the system in question. Let us denote by $\psi$ a typical set of (usually initial) conditions determining a unique solution of $\mathcal{E}$. Let us denote this solution by $[\psi]$. Denote $\mathcal{E}'$ and $\psi'$ the equations and conditions obtained from $\mathcal{E}$ and $\psi$ by substituting every $x_i$ with $x'_i$, and $t$ with $t'$, etc. Denote $G_v(\mathcal{E}), G_v(\psi)$ and $\Lambda_v(\mathcal{E}), \Lambda_v(\psi)$ the set of equations and conditions expressed in the primed variables applying the Galilean and the Lorentz transformations, respectively (including, of course, the Galilean/Lorentz transformations of all other variables different from the space and time coordinates). Finally, in order to give a strict mathematical formulation of relativity principle, we have to fix two further concepts, the meaning of which are vague: Let a solution $[\psi_0]$ is stipulated to describe the behavior of the system when it is, as a whole, at rest relative to $K$. Denote $\psi_v$ the set of conditions and $[\psi_v]$ the corresponding solution of $\mathcal{E}$ that are stipulated to describe the similar behavior of the system as $[\psi_0]$ but, in addition, when the system was previously set, as a whole, into a collective translation at velocity $v$.

Now, what relativity principle states is equivalent to the following:

\begin{align*}
G_v(\mathcal{E}) &= \mathcal{E}', \quad (9) \\
G_v(\psi_v) &= \psi'_0, \quad (10)
\end{align*}

in the case of classical mechanics, and

\begin{align*}
\Lambda_v(\mathcal{E}) &= \mathcal{E}', \quad (11) \\
\Lambda_v(\psi_v) &= \psi'_0, \quad (12)
\end{align*}

in the case of special relativity.

Although relativity principle implies Galilean/Lorentz covariance, the relativity principle, as we can see, is not equivalent to the Galilean covariance (9) in itself or the Lorentz covariance (11) in itself. It is equivalent to the satisfaction of (9) in conjunction with condition (10) in classical physics, or (11) in conjunction with (12) in relativistic physics.

Note, that $\mathcal{E}$, $\psi_0$, and $\psi_v$ as well as the transformations $G_v$ and $\Lambda_v$ are given by contingent facts of nature. It is therefore a contingent fact of nature whether a certain law of physics is Galilean or Lorentz covariant, and, independently, whether it satisfies the principle of relativity. The relativity principle and its consequence the principle of Lorentz covariance are certainly normative principles in contemporary physics, providing a heuristic tool for constructing new theories. We must emphasize however that these normative principles, as any other fundamental law of physics, are based on empirical facts; they are based on the observation that the behavior of any moving physical object satisfies the principle of relativity. In the rest of this paper I will show, however, that the laws of relativistic physics, in general, do not satisfy this condition.
Before we begin analyzing our examples, it must be noted that the major source of confusion is the vagueness of the definitions of conditions $\psi_0$ and $\psi_v$. In principle any $[\psi_0]$ can be considered as a “solution describing the system’s behavior when it is, as a whole, at rest relative to $K$”. Given any one fixed $\psi_0$, it is far from obvious, however, what is the corresponding $\psi_v$. When can we say that $[\psi_0]$ describes the similar behavior of the same system when it was previously set into a collectives motion at velocity $v$? As we will see, there is an unambiguous answer to this question in the Galileo covariant classical physics. But $\psi_v$ is vaguely defined in relativity theory. Note that Einstein himself uses this concept in a vague way. Consider the following example:

Let there be given a stationary rigid rod; and let its length be $l$ as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of $x$ of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity $v$ along the axis of $x$ in the direction of increasing $x$ is then imparted to the rod. We now inquire as to the length of the moving rod, and imagine its length to be ascertained by the following two operations:

(a) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way as if all three were at rest.

(b) ...

In accordance with the principle of relativity the length to be discovered by the operation (a)–we will call it “the length of the rod in the moving system”–must be equal to the length $l$ of the stationary rod. [4]

But, what exactly does “a uniform motion of parallel translation with velocity $v$ ... imparted to the rod” mean? The following examples will illustrate that the vague nature of this concept complicates matters.

In all examples we will consider a set of interacting particles. We assume that the relevant equations describing the system are Galilean/Lorentz covariant, that is 9 and 11 are satisfied respectively. As it follows from the covariance of the corresponding equations, $G_v^{-1}(\psi_0')$ and, respectively, $\Lambda_v^{-1}(\psi_0')$ are conditions determining new solutions of $E$. The question is whether these new solutions are identical with the one determined by $\psi_v$. If so then the relativity principle is satisfied.

Let us start with an example illustrating how the relativity principle works in classical mechanics. Consider a system consisting of two point masses connected with a spring (Fig. 1). The equations of motion in $K$,

$$m \frac{d^2 x_1(t)}{dt^2} = k (x_2(t) - x_1(t) - L), \quad \text{(13)}$$

$$m \frac{d^2 x_2(t)}{dt^2} = -k (x_2(t) - x_1(t) - L), \quad \text{(14)}$$

5
Figure 1: Two point masses are connected with a spring of equilibrium length $L$ and of spring constant $k$

are indeed covariant with respect to the Galilean transformation, that is, expressing (13)–(14) in terms of variables $x', t'$ they have exactly the same form as before:

$$m \frac{d^2 x'_1 (t')}{dt'^2} = k (x'_2 (t') - x'_1 (t') - L), \quad (15)$$

$$m \frac{d^2 x'_2 (t')}{dt'^2} = -k (x'_2 (t') - x'_1 (t') - L). \quad (16)$$

Consider the solution of the (13)–(14) belonging to an arbitrary initial condition $\psi_0$:

$$x_1 (t = 0) = x_{10},$$

$$x_2 (t = 0) = x_{20},$$

$$\left. \frac{dx_1}{dt} \right|_{t=0} = v_{10},$$

$$\left. \frac{dx_2}{dt} \right|_{t=0} = v_{20}. \quad (17)$$

The corresponding “primed” initial condition $\psi'_0$ is

$$x'_1 (t' = 0) = x_{10},$$

$$x'_2 (t' = 0) = x_{20},$$

$$\left. \frac{dx'_1}{dt'} \right|_{t'=0} = v_{10},$$

$$\left. \frac{dx'_2}{dt'} \right|_{t'=0} = v_{20}. \quad (18)$$

Applying the inverse Galilean transformation we obtain a set of conditions $G^{-1}_{v} (\psi'_0)$ determining a new solution of the original equations:

$$x_1 (t = 0) = x_{10},$$

$$x_2 (t = 0) = x_{20},$$

$$\left. \frac{dx_1}{dt} \right|_{t=0} = v_{10} + v,$$

$$\left. \frac{dx_2}{dt} \right|_{t=0} = v_{20} + v. \quad (19)$$

One can recognize that this is nothing but $\psi_v$. It is the set of the original initial conditions in superposition with a uniform translation at velocity $v$. That is to say, the corresponding solution describes the behavior of the same system when it was (at $t = 0$) set into a collective translation at velocity $v$, in superposition with the original initial conditions.

In classical mechanics, as we have seen from this example, the equations of motion not only satisfy the Galilean covariance, but also satisfy the condition (10).
principle of relativity holds for all details of the dynamics of the system. There is no exception to this rule. In other words, if the world were governed by classical mechanics, relativity principle would be a universally valid principle. With respect to later questions, it is worth noting that the Galilean principle of relativity therefore also holds for the equilibrium characteristics of the system, if the system has dissipations. Imagine for example that the spring has dissipations during its distortion. Then the system has a stable equilibrium state in which the equilibrium distance between the particles is $L$. When we initiate the system in collective motion corresponding to (19), the system relaxes to another equilibrium state in which the distance between the particles is the same $L$.

Let us turn now to the relativistic examples. It is widely held that the new solution determined by $\Lambda_{\nu}^{-1} (\psi'_0)$, in analogy to the solution determined by $G_{-1}^{-1} (\psi'_0)$ in classical mechanics, describes a system identical with the original one, but co-moving with the frame $K'$, and that the behavior of the moving system, expressed in terms of the results of measurements obtainable by means of measuring-rods and clocks co-moving with $K'$ is, due to Lorentz covariance, the same as the behavior of the original system, expressed in terms of the measurements with the equipments at rest in $K$—in accordance with the principle of relativity. However, the situation is in fact far more complex, as I will now show.

Imagine a system consisting of interacting particles (for example, relativistic particles coupled to electromagnetic field). Consider the solution of the Lorentz covariant equations in question that belongs to the following general initial conditions:

$$r_i(t = 0) = R_i, \quad (20)$$
$$\frac{dr_i(t)}{dt} \bigg|_{t=0} = w_i. \quad (21)$$

(Sometimes the initial conditions for the particles unambiguously determine the initial conditions for the whole interacting system. Anyhow, we are omitting the initial conditions for other variables which are not interesting now.) It follows from the Lorentz covariance that there exists a solution of the “primed” equations, which satisfies the same conditions,

$$r'_i(t' = 0) = R_i, \quad (22)$$
$$\frac{dr'_i(t')}{dt'} \bigg|_{t'=0} = w_i. \quad (23)$$

Eliminating the primes by means of the Lorentz transformation we obtain

$$t^*_i = \frac{\gamma R_{xi}}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (24)$$

$$r'^{new}_i(t = t^*_i) = \left( \frac{R_{xi}}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{R_{yi}}{R_{zi}} \right). \quad (25)$$
and

\[
\frac{d \mathbf{r}_{1\text{ new}}(t)}{dt} \bigg|_{t^*_1} = \begin{pmatrix}
\frac{w \xi_1 + v}{1 + \frac{v^2}{c^2}} \\
\frac{w \xi_1}{c^2}
\end{pmatrix}.
\]

(26)

It is difficult to tell what the solution deriving from such a nondescript “initial” condition is like, but it is not likely to describe the original system in collective motion at velocity \(v\). The reason for this is not difficult to understand. Let me explain it by means of a well known old example [5–9]: Consider the above system consisting of two particles connected with a spring (two rockets connected with a thread in the original example). Let us first ignore the spring. Assume that the two particles are at rest relative to \(K\), one at the origin, the other at the point \(d\), where \(d = L\), the equilibrium length of the spring when it is at rest. It follows from (24)–(26) that the Lorentz boosted system corresponds to two particles moving at constant velocity \(v\), such that their motions satisfy the following conditions:

\[
\begin{align*}
 t^*_1 &= 0, \\
 t^*_2 &= \frac{\sqrt{v^2} d}{\sqrt{1 - \frac{v^2}{c^2}}}, \\
 r^{\text{new}}_{1\text{ new}}(0) &= 0, \\
 r^{\text{new}}_{2\text{ new}} \left( \frac{\sqrt{v^2} d}{\sqrt{1 - \frac{v^2}{c^2}}} \right) &= \frac{d}{\sqrt{1 - \frac{v^2}{c^2}}}. 
\end{align*}
\]

(27)

However, the corresponding new solution of the equations of motion does not “know” about how the system was set into motion and/or how the state of the system corresponding to the above conditions comes about. Consider the following possible scenarios:

**Example 1**  The two particles are at rest; the distance between them is \(d\) (see Fig. 2). Then, particle 1 starts its motion at constant velocity \(v\) at \(t = 0\) from the point of coordinate 0 (the last two dimensions are omitted); particle 2 start its motion at velocity \(v\) from the point of coordinate \(d\) with a delay at time \(t''\). Meanwhile particle 1 moves closer to particle 2 and the distance between them is \(d'' = d\sqrt{1 - \frac{v^2}{c^2}}\), in accordance with the Lorentz contraction. Now, one can say that the two particles are in collective motion at velocity \(v\) relative to the original system \(K\)—or, equivalently, they are collectively at rest relative to \(K'\)—for times \(t > t^*_2 = vd/ \left( c^2 \sqrt{1 - \frac{v^2}{c^2}} \right) \). In this particular case they have actually been moving in this way since \(t''\). Before that time, however, the particles moved relative to each other, in other words, the system underwent deformation.

**Example 2**  Both particles started at \(t = 0\), but particle 2 was previously moved to the point of coordinate \(d\sqrt{1 - \frac{v^2}{c^2}}\) and starts from there. (Fig. 3)
Figure 2: Both particles are at rest. Then particle 1 starts its motion at \( t = 0 \). The motion of particle 2 is such that it goes through the point \( (t^*_2, d') \), where \( d' = d/\sqrt{1 - v^2/c^2} \), consequently it started from the point of coordinate \( d \) at \( t'' = d \left( \frac{v}{c^2 \sqrt{1 - v^2/c^2}} - \left(1 - \sqrt{1 - v^2/c^2} \right) / \left(v \sqrt{1 - v^2/c^2} \right) \right) \). The distance between the particles at \( t'' \) is \( d'' = d \sqrt{1 - v^2/c^2} \), in accordance with the Lorentz contraction.

Example 3  Both particles started at \( t = 0 \) from their original places. The distance between them remains \( d \) (Fig. 4). They are in collective motion at velocity \( v \), although this motion is not described by the Lorentz boost.

Example 4  If, however, they are connected with the spring (Fig. 5), then the spring (when moving at velocity \( v \)) first finds itself in a non-equilibrium state of length \( d \), then it relaxes to its equilibrium state (when moving at velocity \( v \)) and—assuming that the equilibrium properties of the spring satisfy the relativity principle, which we will argue for later on—its length (the distance of the particles) would relax to \( d \sqrt{1 - v^2/c^2} \), according to the Lorentz boost.

We have seen from these examples that the relationship between the Lorentz boost—the motion determined by the conditions \( \Lambda_v^{-1}(\psi'_0) \)—and the systems being in collective motion—determined by \( \psi_v \)—is not so trivial. In Examples 1 and 2—although, at least for large \( t \), the system is identical with the one obtained through the Lorentz boost—it would be entirely counter intuitive to say that we simply set the system in collective motion at velocity \( v \), because we first distorted it: in Example 1 the particles were set into motion at different moments of time; in Example 2, before we set them in motion, one of the particles was relocated relative to the other. In contrast,
Figure 3: Both particles start at $t = 0$. Particle 2 is previously moved to the point of coordinate $d'' = d \sqrt{1 - v^2/c^2}$.

Figure 4: Both particles start at $t = 0$ from the original places. The distance between the particles does not change.
in Examples 3 and 4 we are entitled to say that the system was set into collective motion at velocity $v$. But, in Example 3 the system in collective motion is different from the Lorentz boosted system (for all $t$), while in Example 4 the moving system is indeed identical with the Lorentz boosted one, at least for large $t$, after the relaxation process. Thus, as Bell rightly pointed out:

Lorentz invariance alone shows that for any state of a system at rest there is a corresponding ‘primed’ state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'primed' of the original state, rather than into the 'prime' of some other state of the original system. ([9], p. 75)

However, neither Bell’s paper nor the preceding discussion of the “two rockets problem” provide a deeper explanation of this fact. For instance, after the above passage Bell continues:

In fact, it will generally do the latter. A system set brutally in motion may be bruised, or broken, or heated or burned. For the simple classical atom similar things could have happened if the nucleus, instead of being moved smoothly, had been jerked. The electron could be left behind completely. Moreover, a given acceleration is or is not sufficiently gentle depending on the orbit in question. An electron in a small, high frequency, tightly bound orbit, can follow closely a nucleus that an electron in a more remote orbit – or in another atom – would not follow at all. Thus we can only assume the Fitzgerald contraction, etc., for a coherent dynamical system whose
configuration is determined essentially by internal forces and only little
perturbed by gentle external forces accelerating the system as a whole.
([9], p. 75)

Of course, acceleration must be gradual so that the objects in question are not damaged.
(In our examples we omitted the acceleration period—symbolized by a black point on
the figures—for the sake of simplicity.) As the above examples show, this condition
in itself does not, however, guarantee that the Lorentz boosted solution describes the
original system gently accelerated from \( K \) to \( K' \). Before I proceed to formulate my
thesis about this question, let me give one more example.

**Example 5** Consider a rod at rest in \( K \). The length of the rod is \( l \). At a given moment
of time \( t_0 \) we take a record about the positions and velocities of all particles of the rod:

\[
\begin{align*}
  r_i(t = t_0) &= R_i, \\
dr_i(t) \bigg|_{t=t_0} &= w_i.
\end{align*}
\]

Then, forget this system, and imagine another one which is initiated at moment \( t = t_0 \)
with the initial condition \( 28 \)–\( 29 \). No doubt, the new system will be identical with a
rod of length \( l \), that continues to be at rest in \( K \).

Now, imagine that the new system is initiated at \( t = t_0 \) with the initial condition

\[
\begin{align*}
  r_i(t = t_0) &= R_i, \\
dr_i(t) \bigg|_{t=t_0} &= w_i + v,
\end{align*}
\]

instead of \( 28 \)–\( 29 \). No doubt, in a very short interval of time \((t_0, t_0 + \Delta t)\) this system
is a rod of length \( l \), moving at velocity \( v \); the motion of each particle is a superposition
of its original motion, according to \( 28 \)–\( 29 \), and the collective translation at velocity
\( v \). In other words, it is a rod co-moving with the reference frame \( K' \). Still, its length is
\( l \), contrary to the principle of relativity, according to which the rod should be of length
\( l \sqrt{1 - v^2/c^2} \)—as a consequence of \( l' = l \). The resolution of this “contradiction” is
that the system initiated in state \( 30 \)–\( 31 \) at time \( t_0 \) finds itself in a non-equilibrium state
and then, due to certain dissipations, it relaxes to the new equilibrium state. What
such a new equilibrium state is like, depends on the details of the dissipation/relaxation
process. It is, in fact, a thermodynamical question. The concept of “gentle acceleration” not only means that the system does not go irreversibly far apart from its equi-
librium state, but, more essentially, it incorporates the assumption that there is such a
dissipation/relaxation phenomenon.

Without entering into the quantum mechanics of solid state systems, a good way
to picture it is imagine that the system is radiating during the relaxation period. This
process can be followed in details by looking at one single point charge accelerated
from \( K \) to \( K' \) (see [10], pp. 208-210). Suppose the particle is at rest for \( t < 0 \), the
acceleration starts at \( t = 0 \) and the particle moves with constant velocity \( v \) for \( t \geq t_0 \).
Using the retarded potentials, we can calculate the field of the moving particle at some
time \( t > t_0 \). We find three zones in the field (see Fig. 6). In Region I, surrounding the particle, we find the “Lorentz-transformed Coulomb field” of the point charge moving at constant velocity. This is the solution we usually find in textbooks. In Region II, surrounding Region I, we find a radiation field traveling outwards which was emitted by the particle in the period \( 0 < t < t_0 \) of acceleration. Finally, outside Region II, the field is produced by the particle at times \( t < 0 \). The field in Region III is therefore the Coulomb field of the charge at rest. Thus, the principle of relativity never holds exactly. Although, the region where “the principle holds” (Region I) is blowing up at the speed of light. In this way the whole configuration relaxes to a solution which is identical with the one derived from the principle of relativity.

Thus, we must draw the conclusion that, in spite of the Lorentz covariance of the equations, whether or not the solution determined by the condition \( \Lambda_{\psi}^{-1}(\psi'_0) \) is identical with the solution belonging to the condition \( \psi_v \), in other words, whether or not the relativity principle holds, depends on the details of the dissipation/relaxation process in question, given that 1) there is dissipation in the system at all and, 2) the physical quantities in question, to which the relativity principle applies, are equilibrium quantities characterizing the equilibrium properties of the system. For instance, in Example 5, the relativity principle does not hold for all dynamical details of all particles of the rod. The reason is that many of these details are sensitive to the initial conditions. The principle holds only for some macroscopic equilibrium properties of the system, like the length of the rod. It is a typical feature of a dissipative system that it unlearns the initial conditions; some of the properties of the system in equilibrium state, after the relaxation, are independent from the initial conditions. The limiting (\( t \to \infty \)) electromagnetic field of the moving charge and the equilibrium length of a solid rod are good examples. These equilibrium properties are completely determined by the equations.
themselves independently of the initial conditions. If so, the Lorentz covariance of the
equations in itself guarantees the satisfaction of the principle of relativity with respect
to these properties: Let $X$ be the value of such a physical quantity—characterizing the
equilibrium state of the system in question, fully determined by the equations indepen-
dently of the initial conditions—ascertained by the measuring devices at rest in $K$. Let
$X'$ be the value of the same quantity of the same system when it is in equilibrium and
at rest relative to the moving reference frame $K'$, ascertained by the measuring devices
co-moving with $K'$. If the equations are Lorentz covariant, then $X = X'$. We must
recognize that whenever in relativistic physics we derive correct results by applying
the principle of relativity, we apply it for such particular equilibrium quantities. But
the relativity principle, in general, does not hold for the whole dynamics of the systems
in relativity theory, in contrast to classical mechanics.

When claiming that relativity principle, in general, does not hold for the whole
dynamics of the system, a lot depends on what we mean by the system set into uniform
motion. One has to admit that this concept is still vague. As we pointed out, it was
not clearly defined in Einstein’s formulation of the principle either. By leaving this
concept vague, Einstein tacitly assumes that these details are irrelevant. However, they
can be irrelevant only if the system has dissipations and the principle is meant to be
valid only for some equilibrium properties with respect to which the system unlearns
the initial conditions. So the best thing we can do is to keep the classical definition of
$\psi_v$: Consider a system of particles the motion of which satisfies the following (initial)
conditions:

$$
\begin{align*}
\mathbf{r}_i(t = t_0) &= \mathbf{R}_{i0}, \\
\frac{d\mathbf{r}_i}{dt}
\bigg|_{t = t_0} &= \mathbf{V}_{i0}.
\end{align*}
$$

(32)

The system is set in collective motion at velocity $\mathbf{v}$ at the moment of time $t_0$ if its
motion satisfies

$$
\begin{align*}
\mathbf{r}_i(t = t_0) &= \mathbf{R}_{i0}, \\
\frac{d\mathbf{r}_i}{dt}
\bigg|_{t = t_0} &= \mathbf{V}_{i0} + \mathbf{v}.
\end{align*}
$$

(33)

I have two arguments for such a choice: The usual Einsteinianderivation of Lorentz
transformation, simultaneity in $K'$, etc., starts with the declaration of the relativity
principle. Therefore, all these things must be logically preceded by the concept of a
physical object in a uniform motion relative to $K$. The second support comes from
what Bell calls “Lorentzian pedagogy”.

Its special merit is to drive home the lesson that the laws of physics in
any one reference frame account for all physical phenomena, including
the observations of moving observers. And it is often simpler to work in a
single frame, rather than to hurry after each moving objects in turn. ([9],
p. 77.)

3 CONCLUSIONS

We have seen that in classical mechanics the principle of relativity is, indeed, a univer-
sal principle. It holds, without any restriction, for all dynamical details of all possible
systems described by classical mechanics. In contrast, in relativistic physics this is not the case:

1. The principle of relativity is not a universal principle. It does not hold for the whole range of validity of the Lorentz covariant laws of relativistic physics, but only for the equilibrium quantities characterizing the equilibrium state of dissipative systems. Since dissipation, relaxation and equilibrium are thermodynamical conceptions *par excellence*, the special relativistic principle of relativity is actually a thermodynamical principle, rather than a general principle satisfied by all dynamical laws of physics describing all physical processes in details. One has to recognize that the special relativistic principle of relativity is experimentally confirmed only in such restricted sense.

2. The satisfaction of the principle of relativity in such restricted sense is indeed guaranteed by the Lorentz covariance of those physical equations that determine, independently of the initial conditions, the equilibrium quantities for which the principle of relativity holds. In general, however, Lorentz covariance of the laws of physics does not guarantee the satisfaction of the relativity principle.

3. It is an experimentally confirmed fact of nature that some laws of physics are *ab ovo* Lorentz covariant. However, since relativity principle is not a universal principle, it does not entitle us to infer that Lorentz covariance is a fundamental symmetry of physics.

4. The fact that the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Lorentz transformation is compatible with our general observation that the principle of relativity only holds for such equilibrium quantities as the length of a solid rod or the characteristic periods of a clock-like system.

The fact that relativity principle is not a universal principle throws new light upon the discussion of how far the Einsteinian special relativity can be regarded as a principle theory relative to the other (constructive) approaches (cf. [11], p. 57; [12–14]). It can also be interesting from the point of view of other reflections on possible violations of Lorentz covariance (see, for example, [15]).

It must be emphasized that the physical explanation of this more complex picture is rooted in the physical deformations of moving measuring-rods and moving clocks by which the space and time tags are defined in moving reference frames. In Einstein’s words:

*A Priori* it is quite clear that we must be able to learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, for the magnitudes $z, y, x, t$ are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. ([3], p. 35)

Since therefore Lorentz transformation itself is not merely a mathematical concept without contingent physical content, we must not forget the real physical content of Lorentz covariance and relativity principle.
Acknowledgements  The research was partly supported by the OTKA Foundation, No. T 037575 and No. T 032771. I am grateful to the Netherlands Institute for Advanced Study (NIAS) for providing me with the opportunity, as a Fellow-in-Residence, to complete this paper.

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