Ultrahigh energy cosmic rays and supersymmetry

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Abstract

Recently, models proposing superheavy particles $X$ as source of ultrahigh energy cosmic rays have attracted some interest. The $X$-particles are either metastable relic particles from the early Universe or are released by topological defects. In these models, the detected air-showers are produced by primaries originating from the fragmentation of the $X$-particles. We present the fragmentation spectrum of superheavy particles calculated in SUSY-QCD. Then we discuss the status of the lightest supersymmetric particle as possible ultrahigh energy primary.

1 Introduction

Cosmic rays (CR) are observed in a wide energy range, starting from subGeV energies up to $3 \cdot 10^{20}$ eV. Apart from the highest energies, these particles are accelerated in our Galaxy, most probably by shocks produced by SNII explosions. There is no universal definition of Ultra High Energy Cosmic Rays (UHECR). We will use this term for energies $E \gtrsim 10^{19}$ eV, where a new, flatter component appears in the CR spectrum (Fig. 1). The highest energies detected so far are $2 - 3 \cdot 10^{20}$ eV.

It is natural to think that the UHE component has an extragalactic origin, since the galactic magnetic field cannot confine particles of these energies. Moreover, the acceleration of protons or nuclei up to $2 - 3 \cdot 10^{20}$ eV is difficult to explain with the known astrophysical galactic sources. The most prominent signature of extragalactic UHECR is the so called Greisen-Zatsepin-Kuzmin (GZK) cutoff. The energy losses of protons, nuclei and photons sharply increase at $E_{\text{GZK}} \sim 3 \cdot 10^{19}$ eV, reducing the mean free path length $l$ of these primaries to less than 50 Mpc or so. Thereby, the spectrum should become steeper above $E_{\text{GZK}}$ for any source with a distribution homogeneous on scales larger than $l$. There is another argument which disfavors the standard astrophysical sources: At energies $E \sim 10^{20}$ eV, the arrival direction of the primaries (which is known within several degrees) should point towards their site of origin. But no source of UHECR like, e.g., active galactic nuclei has been found within 50 Mpc in the direction of these events.

An elegant solution to the above problems are top-down models: In contrast to the standard sources, the primaries are not accelerated but are the fragmentation products of some decaying superheavy particle $X$. For $X$-particles with mass $m_X \gg E_{\text{GZK}}$, the acceleration problem is solved trivially. Moreover, these sources also evade detection by normal astronomical methods.

This contribution is organized as follows: In Section 2, the in our view two most promising top-down models are presented. In Section 3, the spectrum of hadrons produced in the decay of supermassive particles is calculated in supersymmetric QCD. As an application, the CR fluxes produced by cosmic necklaces are shown in Section 4. Finally, the status of the LSP as UHE primary is discussed in Section 5.

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Figure 1: Compilation of the CR spectrum from AGASA, Fly’s Eye, Haverah Park and Yakutsk. A two component fit to the spectrum is also shown.

2 Top-down models as new sources for UHECR

Three different possibilities for top-down models have been discussed in the literature:

1. Primordial black holes: During the final stage of their evaporation, they emit high energy particles. However, when the resulting spectra of cosmic rays are combined with various observational bounds on the mass fraction of the universe in black holes, one finds that the UHE CR flux from black holes is well below the observed flux.

2. Topological defects such as superconducting strings, monopoles, and monopoles connected by strings: Here we will concentrate on cosmic necklaces, since this model seems to provide the largest UHE particle flux for fixed density of electromagnetic cascade radiation.

Cosmic necklaces are hybrid defects consisting of monopoles connected by a string. These defects are produced by the symmetry breaking \( G \rightarrow H \times U(1) \rightarrow H \times Z_2 \). In the first phase transition at scale \( \eta_m \), monopoles are produced. At the second phase transition, at scale \( \eta_s < \eta_m \), each monopole gets attached to two strings. The basic parameter for the evolution of necklaces is the ratio \( r = m/(\mu d) \) of the monopole mass \( m \) and the mass of the string between two monopoles, \( \mu d \), where \( \mu \sim \eta_s^2 \) is the mass density of the string and \( d \) the distance between two monopoles. Strings loose their energy and can contract due to gravitational radiation. As a result, all monopoles annihilate in the end producing superheavy Higgs, gauge bosons and their supersymmetric partners which we call collectively \( X \)-particles. The rate of \( X \)-particle production can be estimated as

\[
\frac{dn_X}{dt} \sim \frac{r^2 \mu}{t^3 m_X}.
\] (1)

The flux of UHECR is determined mainly by two parameters, \( r^2 \mu \) and \( m_X \), which values must be of order \( 10^{27} \) GeV\(^2 \) and \( 10^{14} \) GeV, respectively, to have the flux close to the observed one. For a more complete discussion see Ref. [4].
3. Superheavy, metastable relic particles $X$: They constitute (part of) the cold dark matter (CDM) and, consequently, their abundance in the galactic halo is enhanced by a factor $\sim 5 \times 10^4$ above their extragalactic abundance. Therefore, the proton and photon flux is dominated by the halo component and the GZK-cutoff is avoided as was first pointed out in Ref. [5].

The necessary lifetimes, $\tau \gtrsim 10^{17}$ s, and mass ranges, $m_X \gtrsim 10^{13}$ GeV, of the $X$-particle arise quite naturally in several extensions of the standard model. The $X$-particle could be protected by some sort of $R$-parity which is extremely weakly broken by wormhole or instanton effects, or could belong to the hidden or messenger sector of SUSY models. The most promising production mechanism proposed so far is the enhancement of vacuum fluctuations of the $X$ field during inflation by gravitational interactions. For $10^{12}$ GeV $\lesssim m_X \lesssim 10^{13}$ GeV, this mechanism results in a $X$-particle abundance close to the critical one.

A common signature of all top-down models is the high photon/proton ratio $\gamma/p$. Since in the fragmentation process much more mesons than baryons are produced, the ratio $\gamma/p$ should be $\gtrsim 1$ at the highest energies. (The exact value depends on the fragmentation model and on the poorly known absorption length of UHE photons.) If the UHE primaries originate from the decay of some superheavy CDM particles, an additional signature is their anisotropy reflecting the non-homogeneous distribution of the CDM.

3 Fragmentation spectrum of hadrons in SUSY-QCD

The spectra of hadrons produced in deep-inelastic scattering and $e^+e^-$ annihilation are formed due to QCD cascading of the partons. In the Leading Logarithmic Approximation (LLA) which takes into account $\ln(Q^2)$ terms this cascade is described by the Gribov-Lipatov-Altarelli-Parisi-Dokshitzer (GLAPD) equation. This approximation is not valid for Bjorken $x \ll 1$, when colour coherence effects become important. A better approximation is the Modified LLA (MLLA), which takes into account both $\ln(Q^2)$ and $\ln(x)$ terms as well as angular ordering. In this section, we consider the limiting spectrum that has been obtained as an approximate solution to the MLLA evolution equations for small $x$. In fact, it describes well also the experimental data at $x \sim 1$.

At large energies $\sqrt{s} \gtrsim 1$ TeV the production of supersymmetric particles might substantially change the QCD spectra. Apart from future experiments at LHC, supersymmetry (SUSY) might strongly reveal itself in the decays of superheavy particles: As long as the virtuality $Q^2$ remains much larger than the SUSY scale $M^2_{\text{SUSY}}$, in the particle cascade initiated by the decay of a $X$-particle not only usual particles but also their supersymmetric partners participate. As we will see, the fragmentation spectrum of hadrons changes considerably going from QCD to SUSY-QCD. To obtain reliable results for UHECR fluxes, it is therefore necessary to calculate the spectra within SUSY-QCD.

We consider first the SUSY-QCD cascade in LLA. Neglecting terms proportional to $\alpha(Q^2)$ and keeping terms with $\alpha_s(Q^2)\ln(Q^2)$, the GLAPD equation can be written as:

$$\frac{\partial}{\partial \xi} D^B_A(x, \xi) = \sum_C \int_0^1 \frac{dz}{z} \Phi^C_A(z) D^B_C(x/z, \xi) - \sum_C \int_0^1 dz \, z \Phi^C_A(z) D^B_A(x, \xi),$$  \hspace{1cm} (2)

where $\Phi^B_A(z)$ is the splitting function characterizing the decay $A \rightarrow B + C$. Here $D^B_A$ is the distribution of partons $B$ inside the parton $A$ dressed by QCD interactions.
with coupling constant $\alpha(k_\perp^2)$, where $k_\perp$ is the transverse momentum and $x = k_\parallel/k_\parallel^{\text{max}}$ is the longitudinal momentum fraction of the parton $B$. The variable $\xi$ characterizes the maximum value of $k_\perp^2$ available in the considered process ($k_\perp^2 < Q^2$),

$$\xi(Q^2) = \frac{1}{\Lambda^2} \int_{\Lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_s(k_\perp^2)}{4\pi},$$

(3)

with $\Lambda \sim 0.25$ GeV as phenomenological parameter.

The supersymmetrization of Eq. (2) is simple: each parton $A$ should be substituted by the supermultiplet which contains $A$ and its superpartner $\tilde{A}$. We are mainly interested in the spectrum at small $x$, where gluons strongly dominate the other partons. Therefore, it is a reasonable approximation to consider only two partons, namely gluons $g$ and gluinos $\lambda$, in the tree diagrams.

After performing a Mellin transformation,

$$D_B^A(j, \xi) = \int_0^1 dx x^{-j-1} D_B^A(x, \xi),$$

(4)

Eq. (2) can be rewritten in matrix form,

$$\frac{\partial}{\partial \xi} D(j, \xi) = H(j) D(j, \xi),$$

(5)

where we have chosen as basis $(g, \lambda)$ and

$$H(j) = \begin{pmatrix} \nu_g(j) & \nu_\lambda(j) \\ \Phi_g^\lambda(j) & \nu_\lambda(j) \end{pmatrix}$$

(6)

$$\nu_g(j) = \int_0^1 dz \left[(z^{j-1} - z) \Phi_g^g(z) - \Phi_g^g(z)\right]$$

(7)

$$\nu_\lambda(j) = \int_0^1 dz \left(z^{j-1} - 1\right) \Phi_\lambda^\lambda(z).$$

(8)

After diagonalization of $H(j)$, the eigenvalues of $H$ are

$$\nu_\pm = \frac{1}{2} \left(\nu_g + \nu_\lambda \pm \left[(\nu_g - \nu_\lambda)^2 + 4\Phi_g^\lambda \Phi_\lambda^g\right]^{1/2}\right).$$

(9)

In the limit $\omega = j - 1 \to 0$, the leading term $\nu_+$ is given by

$$\nu_+ = \frac{4N_c}{\omega} - a + O(\omega)$$

(10)

with $a = \frac{11}{3} N_c = 11$. ($N_c = 3$ is the number of colours.)

Dokshitzer and Troyan [25] were able to express the limiting spectrum $D_{\text{lim}}$ which is a MLLA result as a function of the parameter $a$ calculated in LLA and $b$, the constant of evolution of $\alpha_s(k_\perp^2)$ in one-loop approximation, $k_\perp^2 d\alpha_s(k_\perp^2)/dk_\perp^2 = -b\alpha_s^2/(4\pi)$. Properly normalized, the function $D_{\text{lim}}(l, Y) = xD_{\text{lim}}(x, Y)$ gives $\sigma^{-1} d\sigma/dl$ in the case of $e^+e^-$ annihilation and the decay spectrum of $X$ particles, where $l = \ln(1/x)$, $Y = \ln[\sqrt{s}/(2\Lambda)]$ and $\sqrt{s}$ is the c.m. energy of an $e^+e^-$ pair or the mass $m_X$ of the superheavy decaying particle. It is given by

$$D_{\text{lim}}(l, Y) = K_{\text{lim}} \frac{4C_F}{b} \Gamma(B) \int_{-\pi/2}^{\pi/2} \frac{d\tau}{\pi} e^{-Ba} \left(\frac{b}{8N_c} \frac{\sin \alpha}{\alpha} \frac{Y}{y}\right)^B I_B(y)$$

(11)
with $a = 11$ and $b = b_{\text{SUSY}} = 9 - n_f = 3$ in SUSY-QCD. Furthermore, $\alpha = \alpha_0 + i \tau$, $\alpha_0 = \text{arctanh}(2\zeta - 1)$, $\zeta = 1 - l/Y$ and $C_F = (N_c^2 - 1)/(2N_c) = 4/3$. Finally, $I_B$ is the modified Bessel function of order $B = a/b$, and argument

$$y(\tau) = \left(\frac{16N_c}{b} \frac{\alpha}{\sinh \alpha} [\cosh \alpha + (1 - 2\zeta) \sinh \alpha] Y\right)^{1/2}.$$  \hspace{1cm} (12)

In Fig. 2, the SUSY-QCD limiting spectrum is shown in comparison with the QCD limiting spectrum. The maxima of the SUSY spectra are shifted to the right, and, since they are also narrower than the QCD spectra, the SUSY maxima are dramatically higher.

![Fig. 2 (left): Limiting spectrum $D_{\text{lim}}(l, Y)$ for SUSY-QCD (solid lines) and QCD (dashed lines). The QCD spectrum is scaled up by a factor 30. Both cases for $m_X = 10^{12}$ GeV (bottom), $m_X = 10^{13}$ GeV (middle) and $m_X = 10^{14}$ GeV (top).](image1)

![Fig. 3 (right): Predicted fluxes from cosmic necklaces with $r^2 \mu = 5 \cdot 10^{27}$ GeV for $m_X = 10^{14}$ GeV.](image2)

Fig. 2 (left): Limiting spectrum $D_{\text{lim}}(l, Y)$ for SUSY-QCD (solid lines) and QCD (dashed lines). The QCD spectrum is scaled up by a factor 30. Both cases for $m_X = 10^{12}$ GeV (bottom), $m_X = 10^{13}$ GeV (middle) and $m_X = 10^{14}$ GeV (top).

Fig. 3 (right): Predicted fluxes from cosmic necklaces with $r^2 \mu = 5 \cdot 10^{27}$ GeV for $m_X = 10^{14}$ GeV.

4 Application: UHECR fluxes from necklaces

We present now the application of the SUSY-QCD limiting spectrum to the calculation of UHECR spectra generated by the decay of superheavy particles\textsuperscript{[3]}. Let us discuss first the problem of the normalization of spectrum. The normalization constant $K_{\text{lim}}$ cannot be calculated theoretically and is normally found from comparison with experiment. Since the spectrum changes dramatically going from QCD to SUSY-QCD, we cannot use this value of $K_{\text{lim}}$. Instead we use as normalization condition

$$\int_0^1 dx xD_{i,\text{lim}}(x, Y) = 2f_i,$$  \hspace{1cm} (13)

where $i$ runs through $N$ (all nucleons) and $\pi^+$ and $\pi^0$ (charged and neutral pions), while $f_i$ is the fraction of energy carried by the hadron $i$. Note that the main contribution to the integral in Eq. (13) comes from large values $x \sim 1$, where the limiting spectrum might have large uncertainties. However, this is, in our opinion, the most physical way of normalization. The numerical values of $f_i$ are unknown at large $s$. One can assume that $f_\pi \approx 1 - f_{\text{LSP}}$, where $f_{\text{LSP}}$ is the energy fraction taken away by the lightest supersymmetric particle (LSP). According to a simplified Monte-Carlo simulation\textsuperscript{[4]}, $f_{\text{LSP}} \sim 0.4$. For the ratio $f_N/f_\pi$ we use $\sim 0.05$ inspired by $Z^0$ decay.
Let us assume that the decay rate of $X$-particles $\dot{n}_X$ in the extragalactic space does not depend on distance and time. Then taking into account the energy losses of UHE protons and the absorption of UHE photons due to pair production ($\gamma + \gamma \rightarrow e^+ + e^-$) on the radio and microwave background, the diffuse flux of UHE protons and antiprotons is

$$I_{p+\bar{p}}(E) = \frac{1}{2\pi} \frac{\dot{n}_X}{m_X} \int_0^\infty dt_g D_N(x_g, Y) \frac{dE_g(E, t_g)}{dE},$$

(14)

where $E_g(E, t_g)$ is the energy at generation time $t_g$ of a proton which has at present the energy $E$ and $x_g = 2E_g/m_X$. Denoting the proton energy losses on microwave radiation by $dE/dt = b(E, z)$, $dE_g/dE$ is given by

$$\frac{dE_g(E, z_g)}{dE} = (1 + z_g) \exp \left[ \int_0^{z_g} \frac{dz}{H_0} (1 + z)^{1/2} \left( \frac{\partial b(E, 0)}{\partial E} \right)_{E=E_g(z)} \right],$$

(15)

where $H_0$ is the Hubble constant and $z$ the redshift. The diffuse spectrum of UHE photons can be calculated as

$$I_\gamma(E) = \frac{\dot{n}_X}{\pi} \frac{1}{m_X} \int_{2E/m_X}^1 \frac{dx}{x} D_{\pi^0}(x, Y),$$

(16)

where $\lambda_\gamma$ is the absorption length of a photon. Finally, the diffuse neutrino flux is given by

$$I_\nu(E) = \frac{3\dot{n}_X(t_0)}{\pi H_0 m_X} \int_0^{z_{\text{max}}} dz (1 + z)^{3/2} \int_0^{E/(1+z)} \frac{dx}{x} D_{\pi^\pm}(x, Y),$$

(17)

where $1 + z_{\text{max}}(E) \approx m_X/(4E)$. The neutrino flux depends generally on the evolution of the sources,

$$\dot{n}_X(t) = \dot{n}_X(t_0) \left( \frac{t_0}{t} \right)^{3+p}.$$  

(18)

In Fig. 3, the spectra of UHE protons, photons, and neutrinos are shown together with experimental data for the model of cosmic necklaces with $r^2\mu = 5 \cdot 10^{27}$ GeV$^2$ and $m_X = 1 \cdot 10^{14}$ GeV. The proton flux is suppressed at the highest energies as compared with the calculations of Ref. [4], where the Gaussian SUSY-QCD spectrum was used.

5 The LSP as UHE primary

We have noted in Section 3 that usual particles as well as their supersymmetric partners participate in a particle cascade as long as their virtuality $Q^2$ remains larger than the SUSY scale $M_{\text{SUSY}}^2$. However, when $Q^2$ reaches $M_{\text{SUSY}}^2$, the supersymmetric particles stop branching and decay to the LSP. Thereby, UHE LSP will be produced in top-down models (assuming $R$-parity is unbroken). Since the LSP cannot be effectively accelerated in standard astrophysical sources, the detection of UHE LSP in extensive air shower (EAS) experiments would not only show that SUSY is realized by Nature but would be also a clear signature for top-down models.
5.1 Gluino as LSP

Hadronic bound states of the gluino $\tilde{g}$ (which we call generically $\tilde{g}$-hadrons $\tilde{G}$) were discussed already in the 80’s as primary of the UHECR\cite{13}. Recently, this idea was revived by Farrar et al.\cite{19} proposing as UHE primary the gluebarino $S = \tilde{g}uds$. A simple model that leads to the MSSM with gluinos as LSP was presented by S. Raby\cite{23}.

There are several arguments against a very light gluino with $\mathcal{O}(m_{\tilde{g}}) = 1 - 10$ GeV: First, the running of $\alpha_s$ gives\cite{2} the bound $m_{\tilde{g}} \geq 6.3$ GeV. Second, two searches for $\tilde{g}$-hadrons at Fermilab had negative results\cite{22}. Third, $\tilde{g}$-hadrons are produced by CR in the earth atmosphere. In the case that the lightest gluebarino is the state $\tilde{g}udd$ and stable, the light gluino as LSP is excluded by the search for heavy hydrogen\cite{2}. If the lightest gluebarino is neutral, as considered by Farrar et al., this argument\cite{2} still work if $S$ forms a bound state with the nuclei. Thus, in our view, a very light gluino is disfavoured. If the gluino is heavy enough (according Ref. [20] $m_{\tilde{g}} \gtrsim 50$ GeV), it will produce the standard missing energy signal in accelerator experiments. Therefore, the range $50$ GeV $\lesssim m_{\tilde{g}} \lesssim 154$ GeV is also excluded.

The interactions of UHE $\tilde{g}$-hadrons were already considered\cite{13} for the case of a glueballino. Two values determine the interaction of a UHE $\tilde{g}$-hadron with a nucleon. The first one is its total $\tilde{G}N$-cross-section which should be proportional to its size. However, different estimates have been made in the literature: $\sigma \sim \alpha_s(m_{\tilde{g}})\mu^2 \sim 1$ mb\cite{13}, where $\mu$ is the reduced mass of $\tilde{G}$, $\sigma \sim \Lambda_{QCD}^2 \sim 10$ mb\cite{2}, and $\sigma \sim (1 - 10)\sigma_{\pi N} \sim (3 - 30)$ mb\cite{2}. The second important value is the average energy transfer $\langle y \rangle$ per scattering. For the production of EAS in the atmosphere only interactions with large $y$ are effective: While a very light $\tilde{g}$-hadron would interact like a nucleon, a heavy one behaves in the atmosphere like a penetrating particle and should be distinguishable from a proton\cite{4,22}

5.2 Neutralino as LSP

In most SUSY models, the neutralino is the LSP and, in contrast to the gravitino and gluino, it is also a viable DM candidate. Let us consider the interactions of the neutralino $\chi$ relevant for their detection\cite{14}. Mainly two processes are important for its interaction with matter, namely the neutralino-nucleon scattering $\chi + N \rightarrow \text{all}$ and resonant production of selectron off electrons $\chi + e \rightarrow \tilde{e} \rightarrow \text{all}$. The first process is based on the resonant subprocess $\chi + q \rightarrow \tilde{q} \rightarrow \text{all}$ and on neutralino-gluon scattering. The latter subprocess is important, because for high energies, and consequently for small scaling variable $x$, the gluon content of the nucleon increases fast.

The cross-sections of all subprocesses start to grow at energies $s \gg m_{\tilde{g}}^2$. The rise with $s$ is caused by the decrease of $x_{\text{min}} = m_{\tilde{g}}^2/s$ and $x_{\text{min}} = (m_{\tilde{q}} + m_{\tilde{g}})^2/s$, and the corresponding increase of the number of partons with sufficient momentum in the nucleon. If squarks do not decay mainly into neutralino, i.e. $\Gamma_{\text{tot}} \gg \Gamma(\tilde{q}_{L,R} \rightarrow q + \chi)$, neutralino-gluon scattering gives the dominant contribution to the total cross-section. At energies $s \approx 10^{10}$ (GeV)$^2$ or $E_\chi \approx 5 \cdot 10^{18}$eV, the neutralino-nucleon cross-section is about $10^{-35} - 10^{-34}$ cm$^2$ (for $m_{\tilde{q}} \sim 900$ GeV), i.e. slightly lower than the neutrino-nucleon cross-section.

Let us consider now $\chi + e \rightarrow \tilde{e} \rightarrow \text{all}$ which is similar to the Glashow resonant scattering $\tilde{\nu}_e + e \rightarrow W \rightarrow \mu + \tilde{\nu}_\mu$. The resonant energy of the neutralino is

$$E_\chi = M_{\tilde{\nu}}^2/(2m_e) = 9.8 \cdot 10^8(M_{\tilde{\nu}}/10^3 \text{ GeV})^2 \text{ GeV}$$

(19)
and the cross-section is given by the usual Breit-Wigner formula. The resonant events are produced as a narrow peak at $E_\chi$ and give an unique signature for the neutralino.

6 Conclusions

Superheavy, metastable relic particle and cosmic necklaces are the two most promising sources for top-down models. Their signature is the high photon/proton ratio and the LSP as UHE primary. The UHECR flux produced by relic particles has additionally a small galactic anisotropy and, as most prominent signature, no GZK-cutoff. For a reliable calculation of UHECR fluxes the knowledge of the fragmentation spectrum of superheavy particles is necessary. We presented the limiting spectrum in SUSY-QCD that differs drastically from the QCD spectrum.

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