Universal behavior of magnetoresistance in quantum dot arrays with different degrees of disorder

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Abstract

The magnetoresistance in a two-dimensional array of Ge/Si quantum dots was studied in a wide range of zero magnetic field conductances, where the transport regime changes from a hopping to a diffusive one. The behavior of the magnetoresistance is found to be similar for all samples—it is negative in weak fields and becomes positive with increasing magnetic field. The result apparently contradicts existing theories. To explain experimental data we suggest that clusters of overlapping quantum dots are formed. These clusters are assumed to have metal-like conductance, the charge transfer taking place via hopping between the clusters. Relatively strong magnetic field shrinks electron wavefunctions, decreasing inter-cluster hopping and, therefore, leading to a positive magnetoresistance. Weak magnetic field acts on ‘metallic’ clusters, destroying the interference of the electron wavefunctions corresponding to different paths (weak localization) inside clusters. The interference may be restricted either by inelastic processes, or by the cluster size. Taking into account weak localization inside clusters and hopping between them within the effective medium approximation, we extract effective parameters characterizing charge (magneto-) transport.

1. Introduction

This work is aimed at investigation of the (magneto-) transport in dense two-dimensional (2D) arrays of self-assembled Ge-in-Si quantum dots (QDs) grown by molecular-beam epitaxy. Depending on the QD parameters, such arrays demonstrate a variety of transport regimes in zero-\(B\) field. It has been previously shown [1] that the variation of the (hole) filling factor of QDs and their structural parameters, density, size and composition leads to an essential variation in conductance (from \(10^{-12}\) to \(10^{-5}\ \Omega^{-1}\)) and stimulates a crossover from variable range hopping (VRH) [2] to diffusive transport.

The present work shows that transport properties of a given sample can be governed by interplay between diffusive and hopping contributions. Both mechanisms can simultaneously determine the transport behavior of the QD array in a given sample. Interplay of these types of transport reveals itself in unusual dependences of the conductance on temperature, \(T\), and magnetic field, \(B\). In particular, the temperature dependence of zero-\(B\) conductance and its magnetic field dependence at large \(B\) can be compatible with VRH, while weak-field magnetoresistance (MR) can be rather described by diffusive conductance along the weak localization (WL) concept. To the best of our knowledge, such a combination has never been observed—usually the observed dependences can be interpreted assuming a single dominant conduction mechanism.

In particular, for hopping conductance, theory predicts an exponential increase of the conductance with temperature and exponentially large positive MR in high magnetic fields due to the shrinkage of the localized wavefunctions in the magnetic
field. Hence, the overlap between localized states decreases, reducing the hopping probability [3]. In relatively weak magnetic fields and in the VRH regime the theory predicts a negative contribution to MR associated with suppression by the magnetic field of destructive interference of the wavefunctions corresponding to electron tunneling between the localized states along different paths [4]. According to the theory, the interference within cigar-shaped domains of impurity states with the size of the hopping length, \( r_h \), may considerably change the hopping probability, leading to a decrease of \( \ln[R(B)/R_0] \) linear in magnetic field \( B \). Here \( R(B) \) is the resistance of the sample in magnetic field \( B \).

For diffusive transport in the WL regime, the main mechanism of MR is the interference between different closed-loop trajectories of delocalized electrons experiencing elastic scattering leading to enhanced backscattering [5]. The decrease of resistance is due to a magnetic-field-induced phase shift between the wavefunctions characterizing clockwise and counter-clockwise trajectories, which suppresses the coherent backscattering and leads to negative MR. The standard theory of WL is applicable at large conductance, \( G \gg G_0 = e^2/2\pi^2\hbar \), where \( e \) is the electron charge while \( \hbar \) is the Planck constant. In the applicability domain of the WL theory, the quantum contribution should be less than the Drude conductance,

\[
G_D = e^2 n \tau / m = \pi k_F G_0 .
\]

Here \( n \) and \( m \) are the electron density and mass, respectively, \( \tau \) is the elastic transport mean free time, \( k_F \) and \( l \) are the Fermi quasimomentum and the classical mean free path, respectively. Therefore, the WL theory works well for high values of \( k_F l \). An attempt to extend the WL theory up to \( k_F l \sim 1 \) by including higher corrections in \( (k_F l)^{-1} \) was made by the authors of [6]. They reported good agreement between the experiment and the theory down to conductance values as low as \( 10^{-2} G_0 \).

Thus, if a particular mechanism dominates the conductance, then the behavior of MR should be fully determined. The dominant mechanism can be, in turn, found from analysis of the temperature dependence of zero-\( B \) conductance.

We will show that the MR behavior is fairly similar for all our samples, independently of the magnitudes and temperature dependences of their zero-\( B \) resistance. Namely, for all samples MR is negative in a weak magnetic field and becomes positive as magnetic field increases. Moreover, even for the samples with large resistance, the observed negative MR contradicts the theory [4] based on account of interference in hopping transport. At the same time, in high magnetic field MR is positive even in highly conductive samples, and its \( B \)-dependence is similar to that for the hopping mechanism.

To interpret the observed unusual behavior of MR, we suggest that the arrays contain clusters of close QDs behaving as pieces of diffusive conductor embedded in a hopping medium. The resistance of the hopping medium depends on areal density, structural parameters and (hole) filling factors of QDs, which determine the localization length, \( \xi \), of the hopping problem. Diffusive conductance inside the clusters is described by the WL approach assuming that the interference is limited by the shortest of the two lengths—the phase breaking length, \( L_\phi \), and the typical cluster size, \( \xi \). The negative MR in weak magnetic fields is explained by the magnetic-field-induced suppression of the quantum interference inside the clusters, while the inter-cluster hopping is only weakly sensitive to the magnetic field. However, in high magnetic fields it provides a dominant positive contribution to the MR. We will develop a semi-quantitative model based on the effective medium approximation (EMA), which allows us to evaluate both the phase breaking length and the characteristic size of the clusters for different samples and follow the dynamics between these two lengths when changing the structural parameters of QDs and temperature. This picture qualitatively agrees with the observed dependences of resistance on both magnetic field and temperature.

The physical reason for clustering in QD arrays is a very interesting issue, which we are not able to approach in this paper. Specific clustering in strongly correlated systems of electrons in magnetic fields was predicted in [7–9]. The main properties of non-uniform many electron states essentially depend on interaction potentials and properties of electron wavefunctions in magnetic field. In our case, the shapes of quantum dots change as a result of heat treatment of the samples, and one can expect that such changes influence the details of interaction potentials. In principle, they can even be non-monotonic, bringing the system closer to the model [9]. However, this is just a speculation. The external magnetic field within all the domains studied does not reconstruct the arrays. Therefore, though analogy is very interesting, it is hard to expect that detailed mechanisms of clustering are similar.

The paper is organized as follows. The samples and the experimental conditions are described in section 2. The analysis of the conductance versus temperature in the samples under study is given in section 3. The study of MR is reported in section 4: positive MR in section 4.1, negative MR in section 4.2. Our model and interpretation of the observed behaviors of resistance and MR are discussed in section 5.

2. Samples and experiment

The samples were grown on a (001) p-Si substrate with resistivity of 20 Ω cm by molecular-beam epitaxy of Ge in the Stranski–Krastanov growth mode. There were two regimes of growth allowing us to obtain QD arrays with different areal densities. In the first case, the growth temperature for a 10 monolayer (ML) Ge layer was 300 °C and the growth rate was 0.2 ML s\(^{-1}\). As a result, the areal density of the dots was shown to be \( \sim 4 \times 10^{11} \) cm\(^{-2}\). In the second case, decrease of the Ge growth temperature to 275 °C with simultaneous increase of the growth rate allows us to reach twice the QD areal density (\( \sim 8 \times 10^{11} \) cm\(^{-2}\)). To supply holes to the dots, a boron δ-doped Si layer was inserted 5 nm below the Ge QD layer. Because the ionization energy of boron impurities in Si is 45 meV and the energies of the first ten hole levels in Ge QDs grown at 300 °C are 200–400 meV [10], at low temperatures holes leave impurities and fill levels in QDs. To get significant changes of the localization radius \( \xi \), the
number of holes per dot was chosen to be 2 and 2.4 for the samples with smaller density. These samples were exposed to additional annealing in Ar atmosphere for 30 min at 550, 575, 600 and 625 °C. It was proposed that annealing will lead to the smearing of the QD shape and Ge–Si intermixing, increasing overlap between the hole wavefunctions belonging to adjacent QDs. The silicon cap layer had a thickness of 40 nm. A metal (Al) source and drain electrodes were deposited on the top of the structure and then heated to 480 °C to form reproducible Ohmic contacts. The temperature stability was controlled using a Ge thermometer. The magneto-transport measurements were carried out at 1.3–4.2 K in magnetic fields of 0–10 T.

### 3. Temperature dependence of conductivity

Figure 1 demonstrates the temperature dependences of sheet conductance as Arrhenius plots at zero magnetic field for the low-density samples 2–6 with filling factor ν ∼ 2.4 annealed at different temperatures, for sample 7 with ν ∼ 2 annealed at 550 °C and for sample 1 with double density of dots (ν ∼ 3). One can see that the conductance strongly depends on the sample parameters and differs by several orders of magnitude. Within the domain 480–625 °C of annealing temperatures the conductance increases with annealing temperature. It is seen that the conductance of highly conductive samples (1–3) weakly depends on temperature, that is typical for diffusive conduction. At the same time, the temperature dependence of the conductance of more resistive samples (4–7) is strong, indicating a VRH mechanism. Consequently, for data analysis we used both approaches.

The temperature dependence of the VRH conductance was specified as

$$G(T) = \gamma T^m \exp[-(T_0/T)^x]$$

where $\gamma$, $m$ and $T_0$ are material-dependent constants, $x = 1/3$ corresponds to the 2D Mott law while $x = 1/2$ corresponds to the Efros–Shklovskii (ES) law. We analyzed the temperature dependence of the reduced activation energy,

$$w(T) = \partial \ln G(T)/\partial \ln T = m + x(T_0/T)^x,$$

using the method proposed in [11]. We have found that, for samples 4–7, the temperature dependences of the conductance are well described by the ES law with $x \approx 0.5$. From the fitted value of $T_0$ we have extracted the localization length as

$$\xi = Ce^{\gamma/\hbar k_B T_0}$$

where the theoretical value of the constant $C$ for 2D single-electron hopping is 6.2 [12], $\kappa$ is the static dielectric constant, and $k_B$ is the Boltzmann constant. The values of $\xi$ for each sample, calculated using the above mentioned formula, are collected in Table 1. One can see that the smaller the conductance is the more localized is the system, as it should be in accordance with the conventional scaling theory [13].

More conductive samples (1–3) with conductance $\sim e^2/h$ and weakly dependent on temperature cannot be accounted for by the VRH mechanism. Therefore, we assume that the mechanism of conductance in those samples has a pronounced diffusive contribution.

In the diffusive regime, the classical contribution to the conductance is temperature independent and is described by the Drude formula (1). Quantum corrections can be reliably evaluated when they are small compared with the Drude conductance. They can be specified as WL corrections, which are due to interference of elastically scattered electrons and corrections induced by electron–electron interaction. The WL corrections lead to a logarithmic decrease of the conductance with decreasing temperature. For the 2D case this correction gives a negative contribution

$$\Delta G_{WL} \approx -G_0 \ln(L_0/l)$$

where

| $N$ | $\xi$ | $\alpha$ | $L_0^*$ | $G_0(0)$ | $G_c(0)$ | $L_\phi$ | $\delta$ |
|-----|-------|---------|-------|---------|---------|---------|-------|
| 1   | 1     | 41      | 1.7 $\times 10^{-5}$ | 1.8 $\times 10^{-5}$ | 46       | 0.03    |
| 2   | 1     | 32      | 1.6 $\times 10^{-5}$ | 2.0 $\times 10^{-5}$ | 37       | 0.15    |
| 3   | 1     | 41      | 5.8 $\times 10^{-7}$ | 2.0 $\times 10^{-5}$ | 56       | 0.31    |
| 4   | 91    | 1       | 4.4 $\times 10^{-9}$ | 9.4 $\times 10^{-6}$ | 58       | 0.46    |
| 5   | 62    | 0.017   | 8.7 $\times 10^{-9}$ | 6.2 $\times 10^{-7}$ | 270      | 0.37    |
| 6   | 47    | 0.0014  | 2.8 $\times 10^{-9}$ | 4.6 $\times 10^{-8}$ | 1000     | 0.31    |
| 7   | 21    | 0.00014 | 1.6 $\times 10^{-9}$ | 2.1 $\times 10^{-9}$ | 5000     | 0.50    |

Figure 1. Temperature dependence of the sheet conductance, $G(T^{-1})$, for the samples with different structural parameters. 1, high-density sample with $\nu \sim 3$; 2–6, low-density samples with $\nu \sim 2.4$ and different annealing temperatures (°C: 2, 625; 3, 600; 4, 575; 5, 550; 6, 480); 7, low-density sample with $\nu \sim 2$, 550 °C.
where \( L_\phi \) is the phase breaking length, \( L_\phi \sim T^\alpha, \alpha < 0 \). As a result, \( \Delta G_{WL} \propto \ln T \). The quantum correction due to electron–electron interaction is also proportional to \( \ln T \) [16]. Estimates show that for our samples the interaction-induced contribution is less important than the WL one.

Figure 2 demonstrates fitting the conductances of samples 1–3 from figure 1 by the \( G \propto \ln T \) law (gray lines). One can see that fitting by this law is really good only for sample 1; for samples 2 and 3 some deviations can be observed. Previously, we have shown [1] that at \( G \leq 10^{-2}c^2/h \) the temperature dependence of the conductance is well described by the ES law, whereas at \( G \geq 0.4c^2/h \) the mechanism of the conductance is close to metallic. Corresponding boundaries are shown in figure 1 as dashed lines. Based on this classification, we conclude that only sample 1 shows diffusive behavior of the conductance; samples 2 and 3 belong to the intermediate region, while samples 4–7 show VRH.

4. Magnetoresistance of QD array

4.1. Positive magnetoresistance

Typical dependences of the relative resistance, \( R(B)/R_0 \), where \( R_0(T) = R(B = 0, T) \), on transverse magnetic field \( B \) at \( T = 4.2 \) K are shown in figure 3. One can see that all samples demonstrate non-monotonic magnetic field dependences of the resistance, \( R \). In low fields \( R \) decreases with magnetic field, and then crosses over to an increase. This increase is especially pronounced for highly resistive samples. We attribute this increase leading to positive MR to a shrinkage of the wavefunctions of localized holes by a transverse magnetic field.

It worth noting that pronounced positive MR was observed in several crystalline materials with 2DEG having different degrees of disorder. This MR has a clearly observed saturation in strong magnetic field and is only weakly dependent on the angle between the direction of the magnetic field and the 2DEG plane. Based on such behavior, this MR was conventionally interpreted as a manifestation of magnetic-field-induced spin polarization in combination with on-site and/or inter-site electron–electron correlations (see, e.g., [17, 18] and references therein), as well as temperature- and magnetic-field-dependent disorder (see, e.g., [19, 20] and references therein).

The MR observed in Ge-in-Si QD arrays does not show a clear trend to saturation. Furthermore, our preliminary experiments have shown that positive MR essentially depends on the angle between applied magnetic field and the array plane. Therefore, we assume that the observed MR is due to orbital effects and ascribe it to shrinkage of the hole wavefunctions due to magnetic field [21].

According to [21], in the case of VRH and relatively weak magnetic fields this shrinkage results in a quadratic dependence of \( \ln R \) on \( B \):

\[
\ln[R(B)/R(0)] = (B/B_0)^2.
\]

Here \( B_0 = c\hbar\xi_0^{3/2}/(s^{1/2}e^2) \), \( \xi_0 = (T_0/T)^{1/2} \), while \( s \) is a numerical coefficient \( \approx 10^{-3} \). According to this expression, \( B_0 \propto T^{-3/4} \).

Shown in figure 4 are the dependences \( \ln[R(B)/R_0] \) versus \( B^2 \) for sample 5, where conductance in zero magnetic field is well described by VRH. Despite the fact that the experimentally measured positive MR is not too large and log-log plots are not fully reliable, one can notice that the data collapse if one assumes that \( B_0 \) is approximately proportional to \( T \). In the most conductive sample 1, because of the small value of positive MR, \( \lesssim 10\% \), we can approximate \( \ln[R(B)/R_0] \) as \( \Delta R(B)/R_0 = [R(B) - R_0]/R_0 \). The inset to figure 4 demonstrates the temperature dependence of \( B_0 \) for sample 1 and its approximation by the \( T^{-3/4} \) law. This dependence is typical for the VRH regime, though at \( B = 0 \) the conductance of sample 1 is typically diffusive. We think that in the absence of magnetic field sample 1 is close to the metal-to-insulator transition and external magnetic field drives it to the insulating state. This is, in our view, the reason why the sample having typically diffusive conductance in the...
absence of magnetic field demonstrates VRH behavior in high fields.

Unfortunately, at the present time there is no quantitative theory of the magnetic-field-induced metal-to-insulator transition. Nevertheless, we can see that equation (4) reasonably represents the observed dependences of $\ln[R(B)/R(0)]$ on both magnetic field and temperature. Obviously, for more resistive samples, the positive MR is larger, showing that their behaviors in magnetic field are closer to those typical for VRH.

### 4.2. Negative magnetoresistance

Negative MR in weak magnetic fields was observed in all investigated samples. Insets in figure 3 show enlarged low-field plots. The left inset shows $\Delta G(B) ≡ G(B) − G(0)$ for high-conductivity samples while the right one shows $R(B)/R(0)$ for low-conductivity ones. Assuming that conductance of the low-conductivity samples is described by the VRH model, one is tempted to use the relevant theory of negative hopping magnetoresistance, see [22] for a review. According to this theory, the logarithm of the ratio $R(H)/R_0$ in relatively weak magnetic fields decreases proportionally to $B$ [4]. The experimental magnetic field dependences of $\ln[R(R_0)]$ versus $B$ for samples 4–7 are shown in figure 5. Numbers on the curves show the values of the localization length obtained from equation (3). The inset shows the absolute value of the slope, $|\ln[R(R_0)]/R|$, as a function of $\xi$.

### 5. Model and interpretation

To explain the negative weak-field MR observed in all our samples, as well as the crossover from a negative to a positive MR, we propose a model, based on the well known fact of inhomogeneous distribution of self-assembled QDs in the growth plane.

We assume that our samples with high density of Ge nanoislands contain clusters of closely located QGs with diffusive conductance (‘metallic droplets’) embedded in a host VRH material. Then the transport through the QD array under study can be represented as a combination of the ‘intra-cluster’ diffusive conductance and ‘inter-cluster’ hopping one.

If the metallic clusters occupy small areal fraction $\delta$ of the sample, $\delta < 1/2$, i.e. the system is far enough from the percolation threshold, the effective conductivity, $\sigma_e$, of a mixture can be found using the effective medium approximation, see [25] for a review. For a 2D array of circular inclusions with conductivity $G_c$ in the matrix with conductivity $G_h$ the effective medium equation for $G_c$ reads as

$$\delta \left( \frac{G_c - G_e}{G_c + G_e} \right) + (1 - \delta) \left( \frac{G_h - G_e}{G_h + G_e} \right) = 0.$$  

The solution of equation (5) is

$$G_c = \frac{1}{2} (1 - 2\delta)(G_h - G_e) + \frac{1}{2} \sqrt{(G_c + G_h)^2 + 4\delta(\delta - 1)(G_c - G_h)^2}.$$  

We specify the hopping conductance in magnetic field as $G_h(B, T) = G_h(0, T) \exp(-B/B_0)$ and the conductance of the clusters as $G_c = G_c(0, T) + \Delta G_{WL}$. The weak localization
contribution, $\Delta G_{\text{WL}}$, is determined as

$$\frac{\Delta G_{\text{WL}}}{G_0} = \alpha G_c,$$

$$G(B, L_\phi^*, l) = \left[ \psi \left( \frac{1}{2} + \frac{\hbar}{2Be\ell^2} \right) - \psi \left( \frac{1}{2} + \frac{\hbar}{2Be\ell^2} \right) + 2 \ln \left( \frac{L_\phi^*}{l} \right) \right].$$  \hspace{1cm} (7)

Here $\alpha$ is a constant of the order of one, $\psi(x)$ is the digamma function,

$$\psi(x) = \frac{d}{dx} \ln \Gamma(x), \quad \Gamma(x) = \int_0^\infty dt \, t^{-1} e^{-xt},$$

and $L_\phi^*$ is an effective phase breaking length, which will be used as an adjustable parameter. This expression is similar to that obtained by Wittmann and Schmid [26] as a modification of the Hikami–Larkin–Nagaoka theory [27]. In the original expression, $\alpha = 1$ and $L_\phi^* = L_\phi \equiv \sqrt{2D\tau_\phi}$, where $\tau_\phi$ is the phase breaking time and $D$ is the diffusion coefficient. It was obtained using the diffusion approximation valid for relatively long electron trajectories subjected to interference. Recently it was shown [6, 28] that a variety of experimental results can be described by equation (7), but with renormalized $\alpha$ and $L_\phi^*$, in a much broader domain of conductances than is required for validity of the diffusion approximation.

Therefore, we employ equations (6) and (7) and consider $G_{\text{h.c}}(0), B_0, \delta$, as well as $\alpha$, $L_\phi^*$ and the elastic mean free path $l$, as adjustable parameters. It was found that in the highly conducting samples (2–3) with QD areal density $4 \times 10^{11}$ cm$^{-2}$ $l \approx 1$ nm, while in sample 1 with double density of QDs $l \approx 7$ nm. These values correspond to typical distances between QD centers in these samples. Therefore, we suggest that the main scattering centers for the delocalized electrons are the QDs. To decrease the number of adjusted parameters we fixed the mean free path $l$ for all the samples with the same areal density as the typical distance between the QDs. At the beginning of the approximation, we put $\alpha = 1$ and estimate the quantity $B_0$ from positive MR in intermediate magnetic fields where the transport is dominated by VRH. Then we find $G_c$, h and $\delta$ using the effective medium approximation (6). The parameters relevant to WL are found putting $G_c = G_c(0, T) + \Delta G_{\text{WL}}$ and then using equation (7). After that we modify $\alpha$ and then repeat the procedure to get the best fit. It turns out that the best fit for the more conductive samples (1–4) is achieved for $\alpha = 1$.

Shown in figure 6 are examples of fitting of the experimental data for samples 2, 4 and 6 by equations (6) and (7). The extracted parameters for all the samples are collected in table 1.

We conclude that the proposed model provides a reasonable interpretation of the experimental findings for samples 3–5, where the partial conductances $G_h$ and $G_c$ are essentially different, making EMA efficient. For these samples we are able to (quantitatively) reproduce the non-monotonic dependence of resistance on magnetic field. The extracted adjustable parameters $\delta, G_{\text{h.c}}, \alpha$ have reasonable magnitudes, compatible with the initial assumptions.

For highly conductive samples (1, 2), our fitting provides low values of the fraction $\delta$ of metallic clusters. In addition, fitted values of $G_c$ and $G_h$ in zero magnetic field are close to each other. These results apparently contradict the concept of hopping between relatively rare metallic clusters. However, in high magnetic fields these samples show noticeable positive MR, which cannot be interpreted by the classical theory of metallic conductance. We think that magnetic-field-induced shrinking of holes’ wavefunctions leads to a decrease of their overlap between different clusters. As a result, the magnetic field drives the sample into insulating regime—the regions of diffusive hole transport become connected by hopping regions, which are responsible for observed positive MR. Unfortunately, the crossover region cannot be quantitatively accounted for by our simple model. We are not aware of any quantitative theory of such a crossover. Therefore, behaviors of the resistance of samples 1 and 2 are interpreted only qualitatively.

For highly resistive samples (5–7) our procedure leads to low values of the adjustable parameter $\alpha$ for highly resistive samples. The fitted value of $\alpha$ decreases with decrease of conductance. A similar trend was observed in [6], where this behavior has been attributed to a crossover between WL and the so-called weak insulator (WI) regime. The WI regime is the case close to the metal-to-insulator transition where $k_F l \gtrsim 1$ and $G(0) \lesssim G_0$. A special name for this regime was introduced because the MR is still well described by the WL expression (7) with reduced prefactor $\alpha$ and fitted parameter $L_\phi^* \lesssim \phi$ [29]. In particular, the authors of [6] have shown that this approach is compatible with experimental results for disordered 2D GaAs/InGaAs quantum well structures down to $G \gtrsim 10^{-2} G_0$.

The theory [6, 30] also predicts decrease of $\alpha$ with decrease of conductance. However, the theory is not valid for very small values of $\alpha$, such as those extracted from fitting of the experimental data for samples 6 and 7. Therefore, the above method of extracting parameters for these samples with high resistance should be considered as an empirical one.

When fitting their result, the authors of [29] assumed the cut-off length $L_\phi^*$ is given by the empirical expression

$$\frac{1}{L_\phi^*} = \frac{1}{L_\phi^2} + \frac{1}{\xi^2}$$

\hspace{1cm} (8)
where \( L_\phi \) is the dephasing length and \( \xi^* \) is the 2D localization length [14].

\[
\xi^* \approx l \exp(\pi k_F l/2).
\]

(9)

We also use the interpolation (8) assuming that there exists a characteristic cut-off length \( \xi_c \), which limits the size of interfering trajectories. This length can be ascribed to characteristic size of clusters with diffusive conductance size of interfering trajectories. Having this mechanism in mind we replace \( \xi^* \rightarrow \xi_c \) in equation (8).

At low temperatures the dephasing length increases, and the fitted value \( L_\phi^\ast \) should saturate at the typical cluster size, \( \xi_c \). Interestingly, the values of \( \xi_c \sim 60–80 \) nm, compatible with our experimental results for all the samples, have the same order of magnitude as the localization length \( \xi^* \), which can be roughly estimated from the density of QDs using equation (9). The ‘hopping’ localization length, \( \xi \), given by equation (3) also has the same order of magnitude. At \( L_\phi \lesssim \xi_c \), temperature dependence of \( L_\phi \) should manifest itself through temperature dependence of the negative MR. We will address this issue in section 5.1.

5.1. Temperature dependence of the phase breaking length

As the conductance of the samples is determined by a competition between ‘intra-cluster’ and ‘inter-cluster’ contributions, its temperature dependence depends on their interplay. For most resistive samples (4–7) the temperature dependence of zero-field conductance is dominated by the ‘inter-cluster’ hopping, resulting in the Efros–Shklovskii law. For more conductive samples the zero-B \( G(T) \) dependence can be approximated by a logarithmic law that is compatible with the WL mechanism. Assuming this mechanism one can ascribe the \( G(T) \)-dependence to the temperature dependence of the dephasing length \( L_\phi \). This dependence can be extracted in two ways: (i) from the temperature dependence of the conductance, and (ii) from the MR data measured at different temperatures. In both procedures we actually extract the value of \( L_\phi^\ast \) rather than the ‘true’ \( L_\phi \), which is determined by inelastic processes. As follows from equation (8), at \( L_\phi \ll \xi_c \) the effective length \( L_\phi^\ast \) coincides with the dephasing length \( L_\phi \). In the opposite case \( L_\phi^\ast \approx \xi_c \) and it is temperature independent since the interference area is limited by the cluster size. Even if \( L_\phi \ll \xi_c \) at relatively high temperature, this relationship can be reversed at low temperatures since \( L_\phi \) is a decreasing function of temperature. Therefore, the temperature dependence of \( L_\phi^\ast \) saturates at low temperatures. Temperature-independent \( L_\phi^\ast \) is also characteristic for the samples with large structure disorder where \( L_\phi \gtrsim \xi_c \) at any accessible temperature.

Shown in figure 7 is the \( L_\phi(T) \) dependence for the most conductive sample 1, restored from the \( G(T) \) data (closed symbols) and from the MR data (open symbols) using equations (8) and (9). The gray line is the approximation of the experimental data by \( L_\phi \approx T^{-0.56} \); the exponent 0.5 is typical for the dephasing due to electron–electron scattering in 2D systems [31]. This behavior agrees with the results of MR measurements, according to which for sample 1 \( L_\phi \ll \xi_c \) at 4.2 K. Obviously, the \( L_\phi \) values obtained from the MR measurements at low temperature (open symbols) roughly extend the approximating line. The \( L_\phi^\ast(T) \) dependences for samples 1 and 3 extracted from the temperature dependence of conductance (closed symbols) and MR data (open symbols) are shown in the inset. One can see a clear tendency to saturation at low temperatures for sample 3. Although for this sample the inequality \( L_\phi \ll \xi_c \) holds, it is not very strong and the temperature dependence of \( L_\phi^\ast \) extracted from fitting experimental results tends to saturation. Therefore, only sample 1 remains metallic in the whole investigated range of temperatures, where its low-field MR can be interpreted through the WL theory with fitted \( L_\phi(T) \).

6. Conclusion

We have studied the conductance of dense 2D arrays of Ge-in-Si quantum dots versus temperature and magnetic field. The set of samples was obtained by different annealing procedures and growth regimes resulting in very different values of resistance and quantitatively different temperature dependences. However, we found that magneto-transport was rather universal: all the samples showed negative MR in weak magnetic fields, which then crossed over to a positive MR in high magnetic fields. To interpret the results we have proposed a model describing dependences of resistance of dense QD arrays on both temperature and magnetic field. The model assumes that the samples contain clusters of closely located QDs with diffusive intra-cluster conductance. These clusters are embedded in a host VRH material responsible for the inter-cluster transport strongly dependent on the QDs structural parameters and their filling with carriers. Annealing of the samples provokes a reconstruction of the QDs leading to increase of the wavefunction overlapping and increase of the conduction inside clusters as well between them. The resulting conductance of the structure is
determined by a competition between intra- and inter-cluster contributions, which we describe in terms of the effective medium approximation. Regarding magneto-transport, we conclude that positive magnetoresistance in high magnetic field is a result of suppression of the conductance of the hopping matrix due to shrinkage of the holes’ wavefunctions, whereas negative weak-field MR is due to weak localization contributions to the conductance of the clusters. We were able to extract relevant parameters for the model, which facilitates description of the crossover from a definitely insulating (VRH) behavior to a quasi-metallic one. The model provides a reasonable interpretation of our experimental results.

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