STU/QCD Correspondence

J. Sadeghi* and B. Pourhassan†

*Email: pouriya@ipm.ir
†Email: b.pourhassan@umz.ac.ir

May 1, 2014

Abstract

In this review article we consider a special case of $D = 5, \mathcal{N} = 2$ supergravity called the STU model. We apply the gauge/gravity correspondence to the STU model to gain insight into properties of the quark-gluon plasma. Given that the quark-gluon plasma is in reality described by QCD, therefore we call our study STU/QCD correspondence. First, we investigate the thermodynamics and hydrodynamics of the STU background. Then we use dual picture of the theory, which is type IIB string theory, to obtain the drag force and jet-quenching parameter of an external probe quark.

Keywords: Gauge/Gravity duality; STU model; String theory; QGP; QCD.

Pacs: 04.65.+e, 11.25.Mj, 12.38.Mh.
# Contents

1 Introduction  \hspace{1cm} 3

2 STU Model
   2.1 Metric  \hspace{1cm} 6
   2.2 Equations  \hspace{1cm} 7
   2.3 Horizon structure  \hspace{1cm} 8
   2.4 Higher derivatives  \hspace{1cm} 10

3 Thermodynamics
   3.1 Quantities  \hspace{1cm} 11
   3.2 Dual picture  \hspace{1cm} 14
   3.3 Higher derivative correction  \hspace{1cm} 15

4 Hydrodynamics
   4.1 Ratio of shear viscosity to entropy  \hspace{1cm} 16
   4.2 Conductivity  \hspace{1cm} 20
   4.3 Higher derivative correction  \hspace{1cm} 21

5 Drag force
   5.1 Single quark solution  \hspace{1cm} 23
   5.2 Quasi-normal modes  \hspace{1cm} 28
   5.3 Effect of the constant electromagnetic field  \hspace{1cm} 30
   5.4 Higher derivative correction  \hspace{1cm} 32
   5.5 Quark-anti quark solution  \hspace{1cm} 33

6 Jet-quenching parameter
   6.1 One-charged black hole  \hspace{1cm} 41
   6.2 Two-charged black hole  \hspace{1cm} 42
   6.3 Three-charged black hole  \hspace{1cm} 43
   6.4 Effect of the constant electric field  \hspace{1cm} 44
   6.5 Higher derivative correction  \hspace{1cm} 45

7 Conclusion  \hspace{1cm} 46
1 Introduction

The relation between gauge theories and string theory has been the subject of many important studies in the last three decades. First, Maldacena [1] proposed the AdS/CFT correspondence, therefore the AdS/CFT correspondence sometimes called Maldacena duality. According to this conjecture there is a relation between a conformal field theory (CFT) in \( d \)-dimensional space and a supergravity theory in \((d + 1)\)-dimensional anti-de Sitter (AdS) space. Maldacena suggests that a quantum string in \((d + 1)\)-dimensional AdS space, mathematically is equivalent to the ordinary quantum field theory with conformal invariance in \( d \)-dimensional space-time which lives on the boundary of \( AdS_{d+1} \) space. The preliminary formulation of Maldacena are developed and completed by independent works of Witten [2] and Gubser, et al. [3]. The famous example of AdS/CFT correspondence is the relation between type IIB string theory in \( AdS_5 \times S^5 \) space and \( \mathcal{N} = 4 \) super Yang-Mills gauge theory on the 4-dimensional boundary of \( AdS_5 \) space. For more studying about the AdS/CFT correspondence and its applications see Ref. [4]. One of the most interesting application of the AdS/CFT correspondence is to study of quark-gluon plasma (QGP). A QGP or quark soup is a phase of quantum chromodynamics (QCD) which exists at extremely high temperature or density. This phase consists of free quarks and gluons, which are several of the basic building blocks of matter. The QGP created at CERN’s super proton synchrotron (SPS) firstly. Current experiments at Brookhaven national laboratory’s relativistic heavy ion collider (RHIC) are continuing this effort. Nowadays scientists at Brookhaven RHIC have tentatively claimed to have created a QGP with an approximate temperature of 4 trillion degrees Celsius. The study of the QGP is a testing ground for finite temperature field theory. Such studies are important to understand the early evolution of our universe. Already, there are many attempt to study QCD by using gauge/gravity duality which usually called AdS/QCD correspondence where the \( \mathcal{N} = 4 \) super Yang-Mills (SYM) plasma considered. The most important quantities of QGP are the shear viscosity, drag force and jet-quenching parameter. The shear viscosity is one of the important hydrodynamical quantities of QGP which relates to the important thermodynamical quantity so-called entropy, specially it is found that the ratio of shear viscosity \( \eta \) to the entropy density \( s \) had a universal value: \( \eta/s = 1/4\pi [5-18] \). However, for the several cases, this value may be enhanced or reduced [19-29]. For example, \( \alpha' \) corrections in string theory enhance the value of \( \eta/s \), but higher derivative corrections may be reduced it. In this paper we use diffusion constant [5] to obtain the ratio of shear viscosity to entropy density for the three-charged black hole in the STU model. Also we include higher derivative correction. The STU model admits a chemical potential for the \( U(1)^3 \) symmetry and this makes it more interesting. For instance, presence of a baryon number chemical potential for heavy quark in the context of AdS/CFT correspondence yields to introducing a macroscopic density of heavy quark baryons. Already the shear viscosity in the STU background computed [9,10] and higher derivative effects of the five-dimensional gauged supergravity [30] applied on the ratio of shear viscosity to entropy [31]. We should note that our paper is extension of the Refs. [10] and [31] because we are going to consider the STU model with three different charges [32], which corresponds to three different chemical potential, and arbitrary space curvature. The STU model is an
example of $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory which is dual to the $\mathcal{N} = 4$ SYM theory with finite chemical potential. The solutions of $\mathcal{N} = 2$ supergravity may be solutions of supergravity theory with more supersymmetry. Already the duality between gravity and $\mathcal{N} = 2$ gauged theory investigated and found that $\mathcal{N} = 2$ supergravity is an ideal laboratory [33-38]. Therefore, it may be to consider the STU model as a gravity dual of a strongly coupled plasma. In order to avoid naked singularity of the BPS black holes [37], non-extremal black holes of five dimensional $\mathcal{N} = 2$ AdS supergravity analyzed in the Ref. [38] and found a lower bound on the non-extremality parameter where the corresponding non-extremal black hole has regular horizon. On the other hand the $\mathcal{N} = 2$ supergravity theory in five dimensions can be obtained by compaction of the eleven dimensional supergravity in a three-fold Calabi-Yau [39]. The advantage of Kaluza-Klein reductional dimension and reduction of supersymmetry to obtain five-dimensional $\mathcal{N} = 2$ gauged supergravity is better understanding the nature, also some calculation such as quantum correction is very difficult in the theory with more supersymmetry. Moreover, the $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory is a natural way to explore gauge/gravity duality, and three-charge non-extremal black holes are important thermal background for this correspondence. Now, we called this duality as STU/QCD correspondence. The STU model describes a five-dimensional space-time which its four-dimensional boundary includes QCD. For these reasons we focused on the STU background and studied the problem of the drag force and jet-quenching parameter [40-43]. The calculation of energy loss of moving heavy charged particle through a thermal medium known as the drag force. One can consider a moving heavy quark (such as charm and bottom quarks) through the thermal plasma with the momentum $P$, mass $m$ and constant velocity $v$, which is influenced by an external force $F$. So, one can write the equation of motion as $\dot{P} = F - \zeta P$, where in the non-relativistic motion $P = mv$, and in the relativistic motion $P = mv/\sqrt{1 - v^2}$, also $\zeta$ is called friction coefficient. In order to obtain drag force, one can consider two special cases. The first case is the constant momentum ($\dot{P} = 0$). So, for the non-relativistic motion, one can obtain $F = (\zeta m)v$. In this case the drag force coefficient ($\zeta m$) will be obtained. In the second case, external force is zero, so one can find $P(t) = P(0)e^{\zeta t}$. In another word, by measuring the ratio $\dot{P}/P$ or $\dot{v}/v$ one can determine friction coefficient $\zeta$ without any dependence on mass $m$. These methods lead us to obtain the drag force for a moving heavy quark in the thermal plasma. The moving heavy quark in context of QCD has dual picture in the string theory where an open string attached to the D-brane and stretched to the horizon of the black hole. The existence of the black hole is necessary for considering the finite temperature field theory. Also the existence of the D-brane is necessary for considering the quark flavor. Moreover the existence of the rotating black holes in the five dimensional space is necessary for considering the finite chemical potential field theory. Already the issue of the drag force considered in the $\mathcal{N} = 4$ super Yang-Mills thermal plasma with several interesting backgrounds [44-50]. In the Ref. [45] the problem of the drag force for the arbitrary metric studied, and the R-charged black D3-brane background as an example considered. This is just STU model with three different charges after the special re-scaling which explain later in this paper. Therefore our work differs from the Ref. [45], so we don’t like to use any re-scaling on the original metric. Another important property of the QGP is called the jet-quenching parameter ($\hat{q}$). The knowledge about
this parameter increases our understanding about the QGP. In that case the jet-quenching parameter obtained by calculating the expectation value of a closed light-like Wilson loop and using the dipole approximation [50]. In order to calculate this parameter in QCD one needs to use perturbation theory. But, by using AdS/CFT correspondence the jet-quenching parameter calculated in non-perturbative quantum field theory. This calculations were already performed in the \( \mathcal{N} = 4 \) SYM thermal plasma with several interesting backgrounds [51-57]. Also the effect of higher derivative corrections such as Gauss-Bonnet on the drag force and the jet-quenching parameter has been studied [57, 58]. Hence, in the Ref. [53] we calculated the jet-quenching parameter in STU model include higher derivative correction and external electric field. We represent also our results in this paper. In the Ref. [40] we considered the moving quark at \( \mathcal{N} = 2 \) supergravity and obtained the drag force for the first time. In that paper we considered the non-extremal black hole with one charge and have shown that our results at near-extremal limit agree with the case of \( \mathcal{N} = 4 \) SYM theory. Then in the Ref. [41] we considered the non-extremal black hole with three equal charge and calculated the drag force for the three different spaces: three dimensional sphere, a pseudo-sphere and a flat space. These cases are just special case of STU model. So, in the Ref. [42] we extended our previous works to the general case of STU background, where the non-extremal black hole has three different charges. Also we studied the quark-anti quark \((q\bar{q})\) configuration and introduced rotating \(q\bar{q}\) pair in the STU background. Finally in the Ref. [43] we compute the jet quenching parameter for the case of the non-extremal black hole with three different charge. We generalize that work to the case of arbitrary curvature and obtain general expression of the jet-quenching parameter in this review article. There are also interesting hydrodynamical quantity such as thermal and electrical conductivity which can be calculated from gauge/gravity duality. In the recent work [59] the thermal and electrical conductivity calculated in presence of non-zero chemical potential and found that conductivities for gauge theories dual to R-charged black hole in \( d = 4 \) behaves in a universal manner. In the Ref. [59] R-charged black holes in arbitrary dimension considered and electrical conductivity computed. We use results of the Ref. [49] to write an expression for electrical conductivity as a hydrodynamical property of the QGP. In the Ref. [60] the STU model used to describe a relativistic fluid with multiple charges, and some transport coefficients relevant to the physics of the QGP calculated. Also in the Ref. [60] a time-dependent version of the STU model dual to a boost-invariant expanding plasma presented which may be useful for future studies based on this paper. In this paper we shall investigate some important properties of the QGP in the STU model with non-extremal black hole and three different charges. Indeed, we review some of the previous results and also add some new things, and collect all of them in this review article. Therefore, in section 2 we review basic properties of the STU model and obtain corresponding general relativity equations. In section 3 we extract thermodynamical quantities of the STU model, and in section 4 we compute the ratio of shear viscosity to entropy density. Then, in section 5 we consider the problem of the drag force for the several configurations. In section 6 we generalized computation of the jet-quenching parameter to the case of STU black hole with arbitrary curvature space. Finally in section 7 we summarized our results and give conclusion.
2 STU Model

2.1 Metric

The STU model is the special form of the $\mathcal{N} = 2$ supergravity in several dimensions. This model has generally 8-charged (4 electric and 4 magnetic) non-extremal black hole. However, there are many situations with less than charges such as four-charged and three-charged black holes. In that case there is great difference between the three-charged and four-charged black holes. For example if there are only 3 charges, then the entropy vanishes (except in the BPS case). So, one really needs four charges to get a regular black hole. In 5 dimensions the situation is different and actually much simpler, there is no distinction between BPS and non-BPS branch. So, in 5 dimensions the three-charged configurations are the most interesting ones [61]. Therefore, we begin with the three-charged non-extremal black hole solution in $\mathcal{N} = 2$ gauged supergravity which is called STU model and described by the following solution [62],

$$ds^2 = -\frac{f_k}{\mathcal{H}^2}dt^2 + \mathcal{H}^\frac{1}{3}(\frac{dr^2}{f_k} + \frac{r^2}{R^2}d\Omega_{3,k}^2),$$

where,

$$f_k = k - \frac{\mu}{r^2} + \frac{r^2}{R^2}\mathcal{H},$$

$$\mathcal{H} = \prod_{i=1}^{3} H_i,$$

$$H_i = 1 + \frac{q_i}{r^2}, \quad i = 1, 2, 3,$$

where $R$ is the constant AdS radius and relates to the coupling constant via $R = 1/g$ (also, coupling constant relates to the cosmological constant via $\Lambda = -6g^2$), and $r$ is the radial coordinate along the black hole, so the boundary of AdS space located at $r \to \infty$ (or $r = r_m$ on the D-brane). The black hole horizon specified by $r = r_h$ which is obtained from $f_k = 0$. In the STU model there are three real scalar fields as $X^i = H^\frac{1}{3}/H_i$, which satisfy the following condition. $\prod_{i=1}^{3} X^i = 1$. In another word, if we set $X^1 = S$, $X^2 = T$, and $X^3 = U$, then there is the $STU = 1$ condition. For the three R-charges $q_i$, in the equation (2), there is an overall factor such as $q_i = \mu \sinh^2 \beta_i$, where $\mu$ is called non-extremality parameter and $\beta_i$ are related to the three independent electrical charges of the black hole. Finally, the factor of $k$ indicates the space curvature, so the metric (1) includes a $S^3$ (three dimensional sphere) for $k = 1$, a pseudo-sphere for $k = -1$ and a flat space for $k = 0$. So, for $k = 1$, $k = 0$ and $k = -1$ one can write respectively,

$$d\Omega_{3,k}^2 \equiv \begin{cases} R^2(d\rho^2 + \sin^2 \rho d\theta^2 + \sin^2 \rho \sin^2 \theta d\phi^2) \\
 dx^2 + dy^2 + dz^2 \\
 R^2(d\rho^2 + \sinh^2 \rho d\theta^2 + \sinh^2 \rho \sin^2 \theta d\phi^2) \end{cases}$$
2.2 Equations

By introducing the new variable, 

\[ u = \frac{1}{6} \ln(H_1H_2H_3), \tag{4} \]

one can obtain the following independent Christoffel symbols,

\[
\begin{align*}
\Gamma^t_{rt} &= \frac{1}{2}(-4u' + \frac{f'_k}{f_k}), \\
\Gamma^i_{ri} &= u' + \frac{1}{r}, \\
\Gamma^r_{rr} &= u' - \frac{f'_k}{2f_k}, \\
\Gamma^r_{tt} &= \frac{1}{2}e^{-6u}f_k(f'_k - 4f_ku'), \\
\Gamma^r_{ii} &= -f_kr(1 + ru'). 
\end{align*} \tag{5}
\]

where the index \( i \) refers to the angular components. This yields us to the following non-zero components of Riemann tensor,

\[
\begin{align*}
R^t_{iti} &= \frac{r}{2}(4f_ku' + 4f_kru'^2 - f'_k - r f'_k u'), \\
R^t_{rrt} &= \frac{1}{2}(4u'' - \frac{f''_k}{f_k} - 12u'^2 + \frac{f'k u'}{f_k}), \\
R^t_{itt} &= \frac{1}{2}f_ke^{-6u}(f'_k u' - 4f_k u'^2 + \frac{f'_k}{r} - \frac{4f'_ku'}{r}), \\
R^i_{rir} &= -u'' - \frac{u'}{r} - \frac{f'_ku'}{2f_k} - \frac{f'k}{2f_k}, \\
R^r_{rrt} &= \frac{1}{2}f_ke^{-6u}(12f_k u'^2 + f''_k - 7f'_k u' - 4f_k u''), \\
R^r_{iri} &= -\frac{r}{2}(2f_ku' + f'_k + r f'_ku' + 2rf_k u''). 
\end{align*} \tag{6}
\]

Hence, we can extract the following components of the Ricci tensor,

\[
\begin{align*}
R^i_i &= e^{-2u}(\frac{f'_ku' - f'_k}{r} + 2f_ku'^2 - f'_ku - f_k u''), \\
R^t_t &= e^{-2u}(\frac{4f_k u' - f'_k}{2r} + 3f'_ku' - 4f_k u'^2 - \frac{f''_k}{2} + 2f_k u''), \\
R^r_r &= e^{-2u}(f_k u'' - \frac{f''_k}{2} - 6f_k u'^2 - \frac{f'_k u'}{r} + 3f'_ku' - \frac{f'_k}{2r}). \tag{7}
\end{align*}
\]

Finally one can find the Ricci scalar as the following,

\[
R = e^{-2u} \left[ \frac{2f_k u' - f'_k}{r} - 8f_k u'^2 + 5f'_ku' - 2f_k u'' - f''_k \right]. \tag{8}
\]
We can realize the relation (4) by computing the field strengths of the abelian gauge field which obtained by using components of the Ricci tensor as \( R_t^t - R_i^i \sim F_{rt}F^{rt} \) [10], and comparing it with the gauge field equation of motion [38].

### 2.3 Horizon structure

Now, we would like to discuss horizon structure of the metric (1). In the Ref. [38] the appropriate conditions for the existence of horizon in the STU model with \( k = 1 \) extracted. Here, we give similar discussion for arbitrary \( k \) and obtain exact relation for the black hole horizon. The \( f_k = 0 \) reduced to the following equation,

\[
r^6 + Ar^4 - B r^2 + q_1 q_2 q_3 = 0,
\]

where \( A \equiv q_1 + q_2 + q_3 + kR^2 \), and \( B \equiv \mu R^2 - q_1 q_2 - q_2 q_3 - q_3 q_1 \). A possible solutions of the equation (9) is given by,

\[
r_{\pm} = \pm \left( \frac{W^2 - 2AW + 4(3B + A^2)}{6W} \right)^{1/2},
\]

where we defined,

\[
W^3 = -36AB - 108 \prod_{i=1}^{3} q_i - 8A^3 \\
+ 12 \sqrt{-12B^3 - 3A^2B^2 + 54AB \prod_{i=1}^{3} q_i + 81(\prod_{i=1}^{3} q_i)^2 + 12A^3 \prod_{i=1}^{3} q_i}.
\]

![Figure 1: Typical horizon situation of STU black hole with \( k = -1 \) for small black hole charges.](image)
The $r_+$ denotes outer horizon, while the $r_-$ denotes inner horizon. Other solutions of the equation (9) are imaginary. In the Fig. 1 and Fig. 2 we draw horizon structure of STU black hole for different space curvatures.

However, with $r^2 \equiv x$ the function $x^2 R^2 f(x) = x^3 + A x^2 - B x + \prod_{i=1}^{3} q_i$ has two extremum at $x_{\pm} = \frac{A}{3}(-1 \pm y)$ where $y = 1 + z \equiv \sqrt{1 + \frac{3B}{A^2}} > 1$, so $x_- \leq 0$ is not acceptable region. Therefore, in order to have at least one horizon in the positive region it should be to have $x_+^2 f(x_+) \leq 0$ which implies that $-2z^3 - 3z^2 + c \leq 0$, where $c \equiv \left(\frac{A}{3}\right)^3 \prod_{i=1}^{3} q_i \leq 1$. In order to find $z$, we restrict ourself to the following cases, (i) $\frac{A}{R^2} \ll 1$ which implies $c \ll 1$, $A \approx kR^2$ and $B \approx \mu A$. (ii) $\frac{B}{R^2} \gg 1$ and $q_i \sim q$ which implies $c - 1 \ll 1$, $A \approx 3q$ and $B \approx \mu R^2 - 3q^2$. In the first case one can obtain $z = \sqrt{\frac{c}{3}}$ which yields to the following critical value for the non-extremality parameter,

$$\mu_c = 2\sqrt{\frac{k}{R^2} q_1 q_2 q_3 + \frac{q_1 q_2 + q_2 q_3 + q_1 q_3}{R^2}}. \quad (12)$$

It tells us that the first approximation is only valid for the cases of $k = 0$ and $k = 1$. We can see that the space-time including pseudo sphere ($k = -1$) yields to imaginary non-extremality parameter at critical point. In the second case one can obtain $z = 1/2$ which yields to the following critical value for the non-extremality parameter,

$$\mu_c = \frac{27}{4} \frac{q^2}{R^2} + \frac{5}{2} kq + \frac{5}{12} k^2 R^2. \quad (13)$$

Therefore, we success to obtain exact expression for horizon radius and calculate approximate values for the non-extremality parameter. In the next step we add higher derivative terms and give horizon radius, and try to obtain critical value of the non-extremality parameter.
2.4 Higher derivatives

The higher derivative corrections to R-charged $AdS_5$ black holes studied originally for the black hole with three equal charges [63] where four-derivative corrections to the bosonic sector of five-dimensional $\mathcal{N} = 2$ gauged supergravity considered. Then, the same problem in the STU model to linear order of the four derivative terms constructed [30]. Now, we would like to extend this work to the case of three different charges [42]. In that case the metric (1) remains unchange but,

$$f_k = k - \frac{\mu}{r^2} + \frac{r^2}{R^2} \prod_i (1 + \frac{q_i}{r^2}) + c_1 \left( \frac{\mu^2}{96r^6 \prod_i(1 + \frac{q_i}{r^2})} - \frac{\prod_i q_i (q_i + \mu)}{9R^2r^4} \right),$$

$$\mathcal{H} = \prod_{i=1}^{3} H_i,$n

$$H_i = H_i = 1 + \frac{q_i}{r^2} - \frac{c_1 q_i (q_i + \mu)}{72r^2(r^2 + q_i)^2}, \quad i = 1, 2, 3,$$  \hspace{1cm} (14)

where $c_1$ is the small constant horizon radius corresponding to the higher derivative terms. In that case the modified horizon radius for the case of $k = 1$ is given by the following expression,

$$r_h = r_{0h} + \frac{c_1 \prod_i (1 + \frac{q_i}{r_{0h}}) \left( \sum q_i^2 - \frac{26r_{0h}^2}{3} \sum q_i + 3r_{0h}^4 \right)}{576R^2 \left[ (\prod_i (1 + \frac{q_i}{r_{0h}}))^{\frac{2}{3}} (\frac{1}{3} \sum q_i - 2r_{0h}^2) - R^2 \right]} + \frac{2(\prod_i (1 + \frac{q_i}{r_{0h}}))^{\frac{2}{3}} (\frac{13}{3} \sum q_i - 3r_{0h}^2) + 3R^2}{576 (\prod_i (1 + \frac{q_i}{r_{0h}}))^{\frac{2}{3}} (\frac{1}{3} \sum q_i - 2r_{0h}^2) - R^2}.$$  \hspace{1cm} (15)

where $r_{0h}$ is the horizon radius without higher derivative corrections which is given by the equation (10). We should note that in order to obtain the expression (15) we removed $\mu$ by using $f_k = 0$. The $k = 1$ solutions is more appropriate to studies of the thermodynamic and hydrodynamic regimes of the theory, and also have interesting application in the horizon structure of the small black holes. Just as previous subsection it is interesting to find a critical value $\mu_c$. By using $\frac{\mu}{r^2} \ll 1$ approximation we find that all previous relations are valid just we find a difference in the $c$, so one can obtain,

$$c \equiv \left( \frac{3}{A} \right)^3 \prod_{i=1}^{3} q_i - \frac{c_1}{9} \prod_{i=1}^{3} q_i (q_i + \mu).$$  \hspace{1cm} (16)

In that case the critical value $\mu_c$ is root of the following equation,

$$4c_2 \mu^3 + R^2 \mu^2 + (4c_2 - 2)(q_1 q_2 + q_2 q_3 + q_1 q_3) \mu + 4(c_2 - 1)q_1 q_2 q_3 = 0,$$  \hspace{1cm} (17)

where we defined $c_2 \equiv \frac{2}{9} q_1 q_2 q_3$. It is clear that $c_1 = 0$ yields to the relation (12).
3 Thermodynamics

3.1 Quantities

In this section we are going to compute some thermodynamical quantities in the STU model with three different black hole charges for the arbitrary spaces. The thermodynamics of the STU model has been studied for special cases [10, 38, 64, 65]. So, the main goal of this section is to generalize previous studies and review the thermodynamics of the STU black hole solution generally. Also, we recall special re-scaling where the metric (1) changes to the dual picture namely $\mathcal{N} = 4$ SYM with finite chemical potential. According to the previous works the Hawking temperature of the black hole solution (1) will be as [62],

$$T = \frac{r_h}{2\pi R^2} \left( 2 + \frac{1}{r_h^2} \sum_{i=1}^{3} q_i - \frac{1}{r_h^2} \prod_{i=1}^{3} q_i \right)^{-1} \sqrt{\prod_{i=1}^{3} (1 + \frac{q_i}{r_h^2})}. \quad (18)$$

There is also a chemical potential which is given by the following relation,

$$\phi_i^2 = q_i(r_h^2 + q_i) \left( \frac{1}{R^2 r_h^2} \prod_{j\neq i} (r_h^2 + q_j) + k \right). \quad (19)$$

Also, the entropy density in $d = 4$ dimension is given by the following expressions for $k \neq 0$ and $k = 0$ respectively,

$$s = \frac{1}{4Gk^2 R^3} \left( r_h^3 \sqrt{\mathcal{H}(r_h)} \right),$$

$$s = \frac{1}{4G R^3} \left( r_h^3 \sqrt{\mathcal{H}(r_h)} \right), \quad (20)$$

where $G$ is Newton’s constant and relates to the AdS curvature as $G = \frac{4 R^3}{2 N^2}$, where $N$ is the number of colors. By combining relations (18) and (20) and relation $C_v = T \frac{\partial s}{\partial T}$ one can obtain the specific heat of the theory for $k \neq 0$ and $k = 0$ respectively,

$$C_v = \frac{r_h^2 \sqrt{\prod_{i=1}^{3} (r_h^2 + q_i)}}{4Gk^2 R^3} \frac{\bar{M}}{\bar{N}},$$

$$C_v = \frac{r_h^2 \sqrt{\prod_{i=1}^{3} (r_h^2 + q_i)}}{4G R^3} \frac{\bar{M}}{\bar{N}}. \quad (21)$$
where we defined,

\[
\tilde{M} = 6r_h^{10} + 7 \sum_i q_i r_h^8 + 2((\sum_i q_i)^2 + \sum_i q_i q_j)r_h^6 + (\sum_i q_i \sum_i q_i q_j - 3 \prod_i q_i) r_h^4
\]

\[
- 2 \sum_i q_i \prod_i q_i r_h^2 - \sum_i q_i q_j \prod_i q_i,
\]

\[
\tilde{N} = 2r_h^{12} + 3 \sum_i q_i r_h^6 + 6 \sum_i q_i q_j r_h^6 + (16 \sum_i q_i + \sum_j \sum_i q_i q_j^2) r_h^6
\]

\[
+ 6 \sum_i q_i \prod_i q_i r_h^4 + 3 \prod_i q_i \sum_i q_i q_j r_h^2 + 2 \prod_i q_i^2.
\]  

For the case of \( q = 0 \) one can obtain \( C_v = \frac{2\pi^2 N^2 T^3}{2k} \). Above relations show that the three cases of \( k = -1, 0, 1 \) yield to the same thermodynamical quantities. Therefore, in the following we can neglect constant curvature \( k \).

Another important thermodynamical quantity is the free energy of the theory,

\[
F = - \frac{4r_h^6 \sum_{i \neq j} (q_i q_j^2 - q_j q_i^2) + 3r_h^4 \sum_{i \neq j} (q_i q_j^3 - q_j q_i^3)}{12R^5 \prod_{i \neq j} (q_i - q_j)}
\]

\[
- \frac{6r_h^2 (\sum_{i \neq j} (q_i q_j^4 - q_j q_i^4) + 3 \sum_{i \neq j} (q_i^2 q_j^3 - q_j^2 q_i^3))}{12R^5 \prod_{i \neq j} (q_i - q_j)},
\]

up to \( O(\ln r_h) \). Importance of the free energy is its relation with the total energy and partition function, so by using the free energy (23) one can obtain,

\[
E \sim \frac{4r_h^8 + 3 \sum_i q_i - 6(\sum_i q_i^2 + 2 \sum_i q_i - 2 \sum_{i \neq j} q_i q_j) r_h^4 + 2 \sum_i q_i - 12 \prod_i q_i}{12R^5 r_h^2},
\]

and the partition function specifies by using the relation \( F = -T \ln Z \).

Now, we can discuss the above thermodynamical quantities for three different cases of one, two, and three-charged black hole.

(i) \( q_1 = q, q_2 = q_3 = 0 \): In that case the specific heat (21) reduced to the following expression,

\[
C_v = \frac{N^2 r_h^4 \sqrt{(r_h^2 + q)}}{2\pi R^6} \frac{6r_h^4 + 7q r_h^2 + 2q^2}{2r_h^6 + 3q r_h^4 + 16q}.
\]

where the horizon radius in terms of the temperature obtained from the relation (18) as the following,

\[
r_h^2 = \frac{1}{4}(-2q + 2\pi^2 R^4 T^2 + 2\sqrt{2q \pi^2 R^4 T^2 + \pi^4 R^8 T^4}).
\]

The free energy in this case vanishes, hence partition function derived as one \( (Z = 1) \), and the total energy becomes \( E = Ts = \frac{r_h^2}{R^6} (2r_h^2 + q) \). In the Fig. 3 we plot the specific heat in terms of the temperature (solid line of the Fig. 3).
(ii) $q_1 = q_2 = q, q_3 = 0$: In that case the specific heat (21) reduced to the following expression,

$$C_v = \frac{N^2(r_h^2 + q)}{2\pi R^6} \frac{2r_h^7 + 7qr_h^5 + 5q^2r_h^3 + q^3r_h}{r_h^6 + 3qr_h^4 + 3q^2r_h^2 + 16q + q^3},$$

(27)

where $r_h = \pi R^2 T$. The free energy, and hence partition function and total energy will be infinite in this case. It is important to note that the temperature of this situation is similar to the temperature of the zero-charge limit ($q = 0$), which is corresponding to the $N = 4$ SYM plasma. In the Fig. 3 we plot the specific heat in terms of the temperature (dotted line of the Fig. 3). We find that the specific heat for the case of two-charge black hole (the case of ii) is larger than the case of one-charge black hole (the case of i).

(iii) $q_1 = q_2 = q_3 = q$: In that case the specific heat (21) reduced to the following expression,

$$C_v = \frac{N^2r_h^2(r_h^2 + q)^3}{2\pi R^6} \frac{6r_h^{10} + 21qr_h^8 + 24q^2r_h^6 + 6q^3r_h^4 - 6q^4r_h^2 - 3q^5}{2r_h^{12} + 9qr_h^{10} + 18q^2r_h^8 + (48q + 3q^3)r_h^6 + 18q^4r_h^4 + 9q^5r_h^2 + 2q^6},$$

(28)

where,

$$r_h^2 = \frac{1}{6} \left[ \Psi + \frac{9q^2 + 4\pi^4 R^8 T^4}{\Psi} + 2\pi^2 R^4 T^2 \right],$$

(29)

with,

$$\Psi = 8\pi^6 R^{12} T^6 + 27q^2 \pi^2 R^4 T^2 - 27q^3 + 3q\pi R^2 T \sqrt{162q^3 + 27q^2 \pi^2 R^4 T^2} + 48q\pi^4 R^8 T^4.$$  (30)

In the Fig. 3 we give plot the specific heat in terms of the temperature (dashed line of the Fig. 3).

![Figure 3: Specific heat in terms of the temperature for $q = 1$. The solid line represents the case (i). The dotted line represents the case (ii). The dashed line represents the case (iii). We find that the value of the specific heat increases by number of the black hole charge.](image-url)
We find that the specific heat for the case of three-charge black hole (the case of iii) is larger than the case of one-charge black hole (the case of i) and two-charge black hole (the case ii). Before end of this subsection it is interesting to recall that the domain of thermodynamical stability is given by inequality \( \frac{q_1 + q_2 + q_3}{r_h^2} + \frac{q_1 q_2 q_3}{r_h^6} < 2 \). We will use this condition later.

### 3.2 Dual picture

As we mentioned already, the \( \mathcal{N} = 2 \) AdS\(_5\) supergravity solution (1) is dual to the \( \mathcal{N} = 4 \) SYM with finite chemical potential in Minkowski space. It can be shown by the following re-scaling [62],

\[
\begin{align*}
  r &\to \lambda^\frac{1}{4} r, \\
  t &\to \frac{t}{\lambda^\frac{1}{4}}, \\
  \mu &\to \lambda \mu, \\
  q_i &\to \lambda^\frac{1}{2} q_i, \\
\end{align*}
\]

and taking \( \lambda \to \infty \) limit while,

\[
d\Omega^2_{3,k} \to \frac{1}{R^2 \lambda^\frac{1}{2}} (dx^2 + dy^2 + dz^2),
\]

and also we set \( r_0^4 \equiv \mu R^2 \). Then, the solution (1) reduces to the following,

\[
ds^2 = e^{2A(r)} \left[ -\frac{f}{\mathcal{H}^2} dt^2 + \mathcal{H}^2 d\vec{X}^2 + \frac{\mathcal{H}}{f} dr^2 \right],
\]

\[
f = \mathcal{H} - \frac{r_0^4}{r^4},
\]

\[
\mathcal{H} = \prod_i (1 + \frac{q_i}{r^2}),
\]

where the geometric function \( A(r) \) defined as \( A(r) \equiv \ln \frac{r}{r_0} \), and \( r_0 \) is the horizon radius in the \( \mathcal{N} = 4 \) SYM theory. In that case the chemical potential conjugate to the physical charge for the \( U(1) \) R-charges is given by,

\[
\phi_i = \frac{r_h^2}{R^2 r_h^2} \frac{2q_i}{r_h^2} + q_i \sqrt{\prod_j (1 + q_j/r_h^2)}. 
\]

This is dual expression of the chemical potential which is given by the relation (19). For the special case of \( q_1 = q_2 = q_3 = q \) the Hawking temperature reads as,

\[
T_H = \frac{q + 2r_h^2}{2\pi R^2 \sqrt{q + r_h^2}},
\]

where the radius of the horizon (root of \( f = 0 \)) is given by,

\[
r_h^2 = \frac{1}{2} \left( \sqrt{4r_0^4 + q^2} - q \right). 
\]
In that case one can rewrite the chemical potential (34) in terms of the black hole charge and horizon radius,

$$\phi = \frac{r_h}{R^2} \sqrt{\frac{2q}{q + r_h^2}}. \quad (37)$$

Therefore the $q = 0$ limit is equal to the zero chemical potential limit. In that case the specific heat of the $AdS_5$ black hole is important parameter to find the phase transition which obtained as the following relation,

$$C_v \propto \frac{\tilde{T}^2}{\sqrt{\tilde{T}^2 - q}} \left[ (6\tilde{c} - 9q + \frac{2\tilde{c}\tilde{T}^2}{T^2 - q} + 12\tilde{T}^2)e^{-\frac{\tilde{c}}{T^2 - q}} - 6\tilde{T}^2 \right], \quad (38)$$

where $\tilde{c}$ plays role of a mass scale (relates to the dilaton field), and we defined $\tilde{T} \equiv \pi R^2 T_H$. It is clear that the case of $q = 0$ recovers results of the Ref. [66]. In the $\tilde{c} \to 0$ limit the sign of the specific heat is positive for $q = 0$, but in our case with $q \neq 0$ the sign of the specific heat is depends to the black hole charge. So, if $\tilde{T}^2 > 1.5q$ then the charged black hole is in stable phase. In the Ref. [66] it is found that the specific heat changes the sign at $\tilde{T}^2 \approx 0.75$ (for $\tilde{c} \neq 0$ and $q = 0$). In presence of the dilaton field ($\tilde{c} \neq 0$), and in unit of $\tilde{c}$, one can find that the phase transition temperature from unstable to stable black hole increases for the case of charged black hole. For example, in the case of $q = 1$ we find unstable/stable phase transition happen at $\tilde{T}^2 \approx 2.4$, so the charged black hole is in stable phase for $\tilde{T}^2 > 2.4$.

### 3.3 Higher derivative correction

The effect of the higher derivative corrections on the thermodynamical quantities such as Hawking temperature and entropy for the case of $k = 0$ and a black hole with three equal charges studied in the Ref. [63]. Here, we give extension to the case of arbitrary space curvature and three different charges. In that case the Hawking temperature obtained as,

\begin{equation}
T = \frac{r_h}{2\pi R^2} \left[ \frac{\mu R^2}{r_h^3} - \frac{1}{r_h^2} \sum_i (q_i \Pi_{j \neq i} (1 + \frac{q_j}{r_h^2})) + \Pi_i (1 + \frac{q_i}{r_h^2}) \right] \sqrt{\frac{1}{\Pi_i (1 + \frac{q_i}{r_h^2} - \frac{c_1 q_i (q_i + \mu)}{72r_h^2 (q_i + r_h^2)^2})} + \frac{4}{9\pi^2} \Pi_i q_i (q_i + \mu)} \right], \quad (39)
\end{equation}

where $r_h$ is given by the relation (15). Inserting $\mu$ from $f_k = 0$ into the relation (39) yields to the Hawking temperature in terms of horizon radius and charges of the black hole. In that case if we set $k = 0$ and $q_i = q$ then solution (39) agree with the result of the Ref. [63], ie,

$$T_{q_i=q,k=0} = \frac{(q_i + r_{0h}^2)^2}{2\pi L^2} \left[ \frac{2r_{0h}^2 - q}{r_{0h}^2} + \frac{c_1 (3q^3 + 4q^2 r_{0h}^2 + 59q r_{0h}^4 - 10r_{0h}^6)}{192 R^2 r_{0h}^4 (2r_{0h}^2 - q)} \right]. \quad (40)$$
Also \( c_1 = 0 \) limit of the relation (39) reduced to the relation (18). The modified entropy is obtained by using relations (14) and (20), so one can obtain,

\[
s = \frac{\sqrt{2}N^2}{1728\pi R^6} \prod_i \frac{72r_h^6 + 216q_i r_h^4 + 216q_i^2 r_h^2 + 72q_i^3 - c_1 q_i^2 - c_1 q_i \mu}{(q_i + r_h^2)^2},
\]

(41)

where \( r_h \) is given by the relation (15). Then, the specific heat can be obtained by using \( C_v = T \frac{\partial s}{\partial r_h} \left( \frac{\partial T}{\partial r_h} \right)^{-1} \). Numerically, we find that the specific heat enhanced due to the higher derivative terms.

In the next section we study some hydrodynamics aspects of the STU model and extract several interesting transport coefficients.

4 Hydrodynamics

4.1 Ratio of shear viscosity to entropy

In this subsection we are going to study universality of the ratio of shear viscosity to entropy density, \( \eta/s \). As we know the shear viscosity (\( \eta \)) is one of the important hydrodynamical quantities of QGP which relates to the thermodynamical quantity, so-called entropy. In the previous section we obtained the entropy of the theory. Let us now review some important studies about the shear viscosity. The ratio of shear viscosity to entropy density of the strongly coupled \( \mathcal{N} = 4 \) SYM thermal plasma investigated [5], and found that \( \eta = \frac{\pi}{16}N^2 T^3 \), where \( N \) is the number of coincident branes. Also \( s = \frac{\pi^2}{2}N^2 T^3 \), therefore \( \eta/s = 1/4\pi \) verified. Then, in the Ref. [6] argued that this value of \( \eta/s \) always saturated for gauge theories at large \( 't \) Hooft coupling. The Ref. [7] showed that this value is a lower bound for a wide class of systems, so \( \eta/s \geq 1/4\pi \). In that case in the Ref. [67] the leading correction to the shear viscosity in inverse powers of \( 't \) Hooft coupling using the \( \alpha' \)-corrected low-energy effective action of type IIB string theory computed. In the Ref. [10] by using Kubo formula [66, 67] the shear viscosity in the SYM theory dual to the STU model computed for the case of flat space (\( k = 0 \)). In the Refs. [9, 11] the viscosity of gauge theory plasma with a chemical potential obtained. They used the five-dimensional Reissner-Nordstrom AdS black hole, where the chemical potential is the one for the R-charges \( U(1)_3 \). In the Refs. [12, 14] the effect of curvature squared corrections, such as Gauss-Bonnet, on the \( \eta/s \) bound computed and found that the conjectured lower bound of \( 1/4\pi \) is violated for finite \( N \). These works generalized to the case of Gauss-Bonnet in arbitrary higher dimensions [24, 25], and showing that \( \eta/s \) reduced in these theories, but there is still a lower bound due to causality which may arise for the large Gauss-Bonnet coupling limit. Finite \( 't \) Hooft coupling corrections to the shear viscosity computed in the Ref. [13] and found that it disagrees with the equilibrium correlation function computations. This disagreement resolved in the Ref. [15].

There are several ways to compute the shear viscosity such as the Kubo formula, which relates the shear viscosity to the correlation function of the stress-energy tensor at zero spatial momentum. In this paper we would like to use diffusion constant to extract the ratio
of shear viscosity to entropy density \([5, 8]\). In that case for a general situation with given metric, \(ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{xx}dx^2\), the diffusion constant becomes,

\[
D = \frac{\sqrt{-g(r_h)}}{\sqrt{-g_{tt}(r_h)g_{rr}(r_h)}} \int_{r_h}^{\infty} dr \frac{-g_{tt} g_{rr}}{g_{xx} \sqrt{-g}}.
\]  \((42)\)

Then, we can use the following relations to investigate the universality of the ratio of shear viscosity to entropy density,

\[
\frac{\eta}{s} = TD, \quad \text{(43)}
\]

or

\[
\frac{\eta}{D} = sT + \mu \rho, \quad \text{(44)}
\]

where \(\rho\) is given by the following equation,

\[
\rho = \frac{\sqrt{2} \sum_i q_i N_i^2 r_h^2}{8 \pi^2 R^6} \sqrt{\prod_i (1 + q_i r_h^2)},
\]  \((45)\)

It is fact that both relations (43) and (44) yield to the same result. We should note that the relation (42) works in the flat space only, therefore we should set \(k = 0\) in our calculations. In that case one can obtain,

\[
\eta = \frac{1}{16 \pi GR^3} r_h^3 \sqrt{\mathcal{H}(r_h)},
\]  \((46)\)

which shows that \(\eta/s = 1/4\pi\) is valid only for the case of flat space. In order to discuss the shear viscosity we consider three different cases of one, two and three-charged black holes. In the first case we assume \(q_1 = q, q_2 = q_3 = 0\).

Figure 4: The graph of \(\eta\) for the case of one-charged black hole
In that case we have,

\[ \eta = \frac{1}{16\pi GR^3} r_h^3 \sqrt{1 + \frac{q}{r_h}}, \]  

(47)

where \( r_h = \frac{1}{2} \sqrt{-2q + 2\sqrt{q^2 + 8\mu}} \). In the Fig. 4 we draw shear viscosity in terms of the black hole charge. The Fig. 4 shows that the shear viscosity of the \( \mathcal{N} = 2 \) plasma, dual of one-charged black hole, decreased by increasing the black hole charge. In that case thermodynamical stability let us to choose \( q < 1.6 \). So, the shear viscosity never vanishes.

In the second case we assume \( q_1 = q_2 = q, q_3 = 0 \). In that case we have,

\[ \eta = \frac{1}{16\pi GR^3} r_h^3 (1 + \frac{q}{r_h}), \]  

(48)

where \( r_h = \sqrt{-q + \sqrt{2\mu}} \). We give plot of the shear viscosity as a function of black hole charge in the Fig. 5.

![Figure 5: The graph of \( \eta \) for the case of two-charged black hole.](image)

According to the Fig. 5 the shear viscosity for two-charged black hole increased by charge at the interval \( 0 < q < 0.8 \), and decreased by charge at the interval \( 0.8 < q < 1.4 \). But, thermodynamical stability tell us that allowed value of the black hole charge, in this case, is \( q < 0.4 \). Therefore the shear viscosity is completely increasing by \( q \) which is totally different with the previous case. It is interesting result that the value of a parameter is depend on the number of black hole charge. A black hole with odd number of black hole charge yields to decreasing function of \( q \) for shear viscosity, on the other hand, a black hole with even number of black hole charge yields to increasing function of \( q \) for shear viscosity. This assertion illustrated by studying three-charged black hole. We expect that three-charged black hole yields to decreasing function of \( q \) for shear viscosity.
In the last case we assume that black hole has three equal charges \((q_1 = q_2 = q_3 = q)\). In that case we have,

\[
\eta = \frac{1}{16\pi GR^3} r_h^3 \sqrt{(1 + \frac{q}{r_h})^3},
\]

(49)

where,

\[
r_h^2 = \frac{1}{6} \mathcal{M} - \frac{2b - \frac{2}{3}a^2}{\mathcal{M}} - \frac{1}{3}a,
\]

(50)

and we defined

\[
\begin{align*}
a &\equiv 3q, \\
b &\equiv 3q^2 - 2\mu, \\
\mathcal{M}^3 &\equiv 36ab - 108q^3 - 8a^3 \\
&\quad + 12 \sqrt{12b^3 - 3b^2a^2 - 54baq^3 + 81q^6 + 12q^3a^3},
\end{align*}
\]

(51)

Figure 6: The graph of \(\eta\) for the case of three-charged black hole.

We give plot of the shear viscosity (49) as a function of black hole charge in the Fig. 6. The Fig. 6 shows that the shear viscosity for the case of three-charged black hole decreased by \(q\). We see in the Fig. 6 that the shear viscosity goes to infinity for \(q \sim 0.8\), but thermodynamical stability tell us that \(q\) has lower value than 0.8, so the shear viscosity has finite value. We conclude that the shear viscosity is strongly depend on the black hole charges. The shear viscosity decreased by the black hole charge in the case of one-charged and three-charged black hole, but increased in the cases of two-charged black hole. However the ratio of the shear viscosity to entropy density has universal value.
4.2 Conductivity

Now, we would like to use results of the Ref. [59] to obtain the thermal and electrical conductivity. In the Ref. [59] it is found that the conductivities for gauge theories dual to R-charge black hole in 4, 5 and 7 dimensions behaves in a universal manner. According to [59] one can obtain,

\[ \sigma_H = r_h R^2 \left( \prod_i \left( 1 + \frac{q_i}{r_h^2} \right) \right)^{\frac{3}{2}}. \]  

(52)

And thermal conductivity is obtained as the following expression,

\[ \kappa_T = \left( \frac{\epsilon + P}{\rho} \right) \sigma_H T. \]  

(53)

where the energy density \( \epsilon \), pressure \( P \) and density of physical charge are defined as [68],

\[ \epsilon = \frac{3N^2r_h^4}{8\pi^2R^8} \prod_i \left( 1 + \frac{q_i}{r_h^2} \right), \]  

(54)

\[ P = \frac{N^2r_h^4}{8\pi^2R^8} \prod_i \left( 1 + \frac{q_i}{r_h^2} \right), \]  

(55)

where \( N^2 = 8\pi^2R^2 \) and we used \( 8\pi G = 1 \). In the Fig. 7 we draw graph of \( \kappa_T \) in terms of the temperature for the simplest case of \( q_1 = q, q_2 = q_3 = 0 \). It shows that the thermal conductivity vanishes at \( T \approx 28 \) MeV for the large black hole charge and \( T \approx 0.9 \) MeV for the small black hole charge. It means that the thermal conductivity decrease with the black hole charge.

Figure 7: The graph of \( \kappa_T \) for the one charged black hole with \( q = 10^6 \) and \( \mu = 0.5 \).
4.3 Higher derivative correction

If one include the higher derivative terms in STU model, then the value of $\eta/s$ increased which is agree with the results of Ref. [31]. In order to obtain effect of higher derivative terms exactly, we focus on the special case of one-charged black hole. To the first order of higher derivative correction the shear viscosity to entropy ratio takes the following form [31],

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + 4c_1 \left( \frac{q}{r_h^6} - 2 \right) \right),$$

where $r_h$ obtained by the relation (15) for the special case of $q_1 = q, q_2 = q_3 = 0$, and

$$r_{2h}^2 = \frac{q}{2} \left( -1 + \sqrt{1 + \frac{4\mu R^2}{q^2}} \right),$$

obtained by the equation (10). In the Fig. 8 we give plots of $\eta/s$ for special case of one-charged black holes. It shows that the higher derivative terms increases the value of $\eta/s$, so there is no condition for choosing small black hole charge. As expected the $c_1 = 0$ limit of the $\eta/s$ coincides with the results of subsection 4.1. The left side of Fig. 8 shows that the first order of the higher derivative terms increases the value of $\eta/s$ for $q^2/r_h^6 > 2$. On the other hand the right side of Fig. 8 tells that for the fixed $c_1$ the black hole charge increased the value of the $\eta/s$.

![Figure 8](image)

Figure 8: The graphs of $\eta/s$ for the case of one-charged black hole by choosing $\mu = 0.5$ and $R = 0.5$, in terms of (left) higher derivative parameter for $q = 1$ and (right) black hole charge for $c_1 = 0.0001$.

The extension of the relations (52) and (53) to include the higher derivative terms is more complicated. Therefore, just we draw graph of the thermal conductivity in terms of the higher
derivative parameter $c_1$ (the left plot of the Fig. 9), and in terms of the temperature (the right plot of the Fig. 9). These figures tell us that the large value of the higher derivative parameter yields to the negative thermal conductivity, which is not acceptable. For example in the case of $T = 250$ MeV one can obtain $c_1 \leq 0.6$. In that case for the case of $c_1 = 0.3$ the thermal conductivity become negative for $T > 450$ MeV.

![Figure 9: The graphs of $k_T$ for the higher derivative correction by choosing $\lambda = \sqrt{2}/2$, $\mu = 0.5$. Left: Thermal conductivity in terms of $c_1$ for $T = 250$ MeV. Right: Thermal conductivity in terms of $T$ for $c_1 = 0.3$.](image)

### 5 Drag force

Study of drag force on a moving heavy quark through a thermal plasma is interesting point to understand physics of charm and bottom quark at RHIC [70]. It is known that a moving quark in the $\mathcal{N} = 2$ thermal plasma corresponds to the stretched string from $r = r_m$ on the D-brane to the black hole horizon. So, calculating the energy loss of a heavy quark or drag force on the moving quark reduce to find components of momentum density along the string. The open string is described by the Nambu-Goto action,

$$S = -T_0 \int d\tau d\sigma \sqrt{-g},$$

where $T_0$ is the string tension. The coordinates $\tau$ and $\sigma$ are corresponding to the string world-sheet. Also, $g$ is determinant of the world-sheet metric $g_{\alpha\beta}$. We assume that the string moves along $x$ direction and use static gauge, where $\tau = t$ and $\sigma = r$. Therefore, the string world-sheet is described by $x(r, t)$, so in order to write lagrangian density we use the metric (1) and find,

$$g = \frac{1}{H^4} \left[ 1 - \frac{2H r^2}{f_k R^2} \frac{r^2}{r^2} + \frac{f_k r^2}{R^2} x'^2 \right],$$

where $H$ and $f_k$ are functions of $r$. The drag force is given by the change of the momentum density $\rho_{\alpha\beta}$ along the string, which is proportional to the gradient of the momentum density.

$$f_{\alpha\beta} = \frac{\partial}{\partial r} \rho_{\alpha\beta},$$
where dot and prime denote $t$ and $r$ derivatives respectively. By using Euler-Lagrange equation one can obtain the string equation of motion as the following expression,

$$\frac{\partial}{\partial r} \left( \frac{f_k r^2}{H^2 \sqrt{-g}} \right) = \frac{H^2 r^2}{f_k} \frac{\partial}{\partial t} \left( \frac{\dot{x}}{\sqrt{-g}} \right),$$  \hfill (60)

where $\sqrt{-g}$ is given by square of the relation (59). In order to obtain the total energy and momentum, drag force or energy loss of particle in the thermal plasma, we have to calculate the canonical momentum densities. In that case one can obtain the following expressions,

$$\left( \begin{array}{ccc} \pi^0_x & \pi^1_x & 0 \\ \pi^0_r & \pi^1_r & 0 \\ \pi^0_t & \pi^1_t & 0 \end{array} \right) = -\frac{T_0}{H^2 \sqrt{-g}} \left( \begin{array}{ccc} \frac{H r^2}{f_k R^2} \dot{x} & \frac{f_k r^2}{R^2} \dot{x}' & 1 - \frac{H r^2}{f_k R^2} x^2 \\ 1 + \frac{f_k r^2}{R^2} x^2 & -\frac{H r^2}{f_k R^2} x^2 & 0 \\ 0 & 0 & 0 \end{array} \right).$$  \hfill (61)

Corresponding to the single quark, in CFT side, we have an open string in $AdS$ space which stretched from $r = r_m$ on D-brane to $r = r_h$ at the horizon. In that case the total energy and momentum of string are obtained by the following integrals,

$$E = -\int_{r_m}^{r_h} \pi^0_t dr,$$

$$P = \int_{r_m}^{r_h} \pi^0_x dr.$$  \hfill (62)

In this section we would like to obtain drag force for single quark and also quark-anti quark configurations. Also we discuss quasinormal modes of the single quark solution. In that case we consider effects of adding B-field and higher derivative terms. Now we ready to obtain drag force for the single quark solution.

## 5.1 Single quark solution

There is the simplest solution for the equation of motion (60), namely $x = x_0$, where $x_0$ is a constant and the string stretched straightforwardly from D-brane at $r = r_m$ to the horizon at $r = r_h$. It means that in the dual picture there is a static quark in the thermal plasma. For such configuration one can obtain $-g = (H_1 H_2 H_3)^{\frac{1}{3}}$ and $\pi^0_x = \pi^0_r = \pi^1_t = \pi^1_x = 0$. It tells us that the drag force is zero, as it expected for the static quark. Only non-zero components of momentum densities are $\pi^1_r = \pi^0_t = -T_0 \left[ \prod_{i=1}^{3} (1 + \frac{q_i}{r_h}) \right]^{-\frac{1}{2}}$, so total energy of the string is obtained as,

$$E = T_0 \left[ r + \frac{1}{6r} \sum_i q_i + \frac{1}{36 r^3} \sum_{i \neq j} q_i q_j + \frac{1}{30 r^5} \prod_i q_i \right]_{r_m}^{r_h},$$  \hfill (63)

where we assume that the black hole charges $q_i$ are small. In zero temperature limit one can interpret $E$ as the rest mass of the quark, which is obtained by the following expression,

$$M_{\text{rest}} = T_0 \left[ r_m - r_h + \left( \frac{1}{r_m} - \frac{1}{r_h} \right) \sum_i q_i + \left( \frac{1}{r_m^3} - \frac{1}{r_h^3} \right) \sum_{i \neq j} q_i q_j + \left( \frac{1}{r_m^5} - \frac{1}{r_h^5} \right) \prod_i q_i \right].$$  \hfill (64)
In STU model there is non-extremal black hole which is described by non-extremality parameter $\mu$, but in the $\mathcal{N} = 4$ SYM theory there is near-extremal black hole. So, if we take $\mu \to 0$ ($q \to 0$) limit, we have near-extremal black hole, then the total energy of string obtained as $E = T_0 (r_m - r_h)$, so, the zero temperature is equal $r_h = 0$ and the physical mass of quark (rest mass) becomes $M_{rest} = T_0 r_m$.

Now, we are going to consider most physical time-dependent solution of moving heavy quark through the thermal $\mathcal{N} = 2$ plasma which is dual picture of a curved string described by $x(r,t) = x(r) + vt$, where $v$ is the constant velocity of the single quark. In that case by using equation of motion (60) one can find,

$$\frac{f_k r^2}{R^2 v \mathcal{H}^{\frac{1}{2}}} \sqrt{-g} x' = C, \quad (65)$$

where $C$ is an integration constant and $\sqrt{-g}$ is obtained in the following equation,

$$-g = \frac{1}{\mathcal{H}^{\frac{1}{2}}} \left[ 1 - \frac{\mathcal{H} r^2}{f_k R^2 v^2} + \frac{f_k r^2}{R^2} x'^2 \right]. \quad (66)$$

Solving the equation (65) for $x'$ yields,

$$x'^2 = \frac{C^2 v^2 R^2 \mathcal{H}^{\frac{1}{2}}}{f_k^2 v^2} \left( \frac{f_k R^2 - \mathcal{H} r^2 v^2}{f_k r^2 - C^2 v^2 R^2 \mathcal{H}^{\frac{1}{2}}} \right). \quad (67)$$

By using these results in the canonical momentum densities (61) we find,

$$\pi^1_x = -T_0 C v, \quad \pi^1_t = T_0 C v^2. \quad (68)$$

These expressions exactly coincide with those obtained in the $\mathcal{N} = 4$ SYM theory [44]. These expressions construct the rate of energy and momentum along the open string,

$$\frac{dP}{dt} = \pi^1_x |_{r=r_m} = -T_0 C v, \quad \frac{dE}{dt} = \pi^1_t |_{r=r_m} = T_0 C v^2. \quad (69)$$

Difference of our result with the $\mathcal{N} = 4$ SYM theory is the constant $C$. In order to find $C$ we use reality condition for $x'^2$ and $\sqrt{-g}$. This condition tell us that $x'^2$ and $\sqrt{-g}$ have real value along the length of the string. Therefore, we should find appropriate $r$, where nominator and denominator of the relation (67) become positive. For the small velocity we know that $f R^2 - \mathcal{H} r^2 v^2$ has a zero at $r = r_c > r_h$. We set this root in the denominator of the relation (67) and fix the constant $C$ as the following,

$$C = \left[ \prod_{i=1}^{3} \left( 1 + \frac{q_i}{r_c^2} \right) \right]^{\frac{1}{3}} \frac{r_c^2}{R^2}, \quad (70)$$
Figure 10: The graphs of the drag force for \( q_1 = q, q_2 = q_3 = 0 \) and the small velocity limit. We set \( \alpha' = 0.5, \lambda = 6\pi \) and \( \mu = 1 \). The solid, dotted, dashed and dash dotted lines correspond to \( v = 0.3, 0.5, 0.7 \) and 0.9 respectively. These show that by increasing velocity, the drag force increases. Left: drag force in terms of the temperature for \( q = 1 \). Right: drag force in terms of the black hole charge for \( T = 300 \text{ MeV} \). It tell us that the black hole charge increases the value of the drag force.

where,

\[
 r_c = r_h + \left( \frac{r^2v^2\mathcal{H}}{2R^2 \left[ \frac{\mu}{r^3} + \frac{r^2\mathcal{H}}{R^2} - \frac{q_1H_2H_3 + q_2H_1H_3 + q_3H_1H_2}{rR^2} \right]} \right)_{r=r_h} + \mathcal{O}(v^4) .
\]

(71)

It is important to note that this result is independent of curvature parameter \( k \), however we should set \( k = 1 \) in the relation (2) to have \( AdS_5 \times S^5 \) space. Combining the relations (69), (70) and (71) give us expression of the drag force which may be written as,

\[
 \frac{dP}{dt} = -T_0v \left[ \prod_{i=1}^{3} \left( 1 + \frac{q_i}{r_h^2} \right) \right]^{\frac{1}{2}} \frac{r_h^2}{R^2} (1 + \mathcal{O}(v^2)).
\]

(72)

Indeed the equation (72) is the momentum current into the horizon. Here, we have field theory interpretation of our system. One can image a single quark moving in a constant external field with strength \( \varepsilon = -\pi x^1 \). This external field keeps the curved string moving at the constant speed \( v \). We know that electromagnetic field lives on a D-brane on which this dragging string ends. The \( \varepsilon \) changes the boundary conditions for the string. Usually, the string should satisfy Dirichlet boundary conditions orthogonal to the D-brane and Neumann boundary conditions parallel with the D-brane. In the presence of \( \varepsilon \), the Neumann boundary conditions can be altered.
Figure 11: The graphs of the drag force for $q_1 = q_2 = q, q_3 = 0$ and the small velocity limit. We set $\alpha' = 0.5$, $\lambda = 6\pi$ and $\mu = 1$. The solid, dotted, dashed and dash dotted lines correspond to $v = 0.3, 0.5, 0.7$ and 0.9 respectively. These show that by increasing velocity, the drag force increases. Left: drag force in terms of the temperature for $q = 1$. Right: drag force in terms of the black hole charge for $T = 300$ MeV. It tell us that the black hole charge increases the value of the drag force.

Also by using the relations $\pi^1_x = -\zeta m v$ and,

$$D_q = \frac{T}{\zeta m},$$

one can obtain diffusion coefficient ($D_q$) of the quark. We try to discuss drag force and diffusion coefficient of the quark for three cases of one, two and three charged black holes. First, we assume $q_1 = q, q_2 = q_3 = 0$, so the horizon radius is given by the relation (26) and thermodynamical stability let us to choose $q \leq 6 \times 10^6$. In this case we draw plots of the drag force in terms of the temperature and the black hole charge in the Fig. 10. In that case diffusion coefficient of the quark obtained as the following expression,

$$D_q = \frac{2 r_h^2 + q}{2 \pi r_h^3 (1 + \frac{q}{r_h})^{\frac{3}{2}}}$$

So, for the $q \to 0$ limit we have $D_q = \frac{1}{\pi r_h}$, which means that diffusion coefficient of the quark found proportional to inverse of the temperature.

Second, we assume $q_1 = q_2 = q, q_3 = 0$, so the horizon radius is obtained from the relation (18) as $r_h = \pi R^2 T$ and thermodynamical stability let us to choose $q \leq 4 \times 10^6$. In this case we draw plots of the drag force in terms of the temperature and the black hole charge in the Fig. 11.
Figure 12: The graphs of the drag force for $q_1 = q_2 = q_3 = q$ and the small velocity limit. We set $\alpha' = 0.5$, $\lambda = 6\pi$ and $\mu = 1$. The solid, dotted, dashed and dash dotted lines correspond to $v = 0.3, 0.5, 0.7$ and $0.9$ respectively. These show that by increasing velocity, the drag force increases. Left: drag force in terms of the temperature for $q = 1$. Right: drag force in terms of the black hole charge for $T = 300$ MeV. It tells us that the black hole charge increases the value of the drag force.

In that case diffusion coefficient of the quark obtained as the following expression,

$$D_q = \frac{1}{\pi r_h (1 + \frac{q}{r_h})^\frac{3}{2}}.$$  \hfill (75)

So, for the $q \to 0$ limit we have $D_q = \frac{1}{\pi r_h}$. It means that diffusion coefficient of the quark found proportional of inverse of the temperature, which is expected.

Finally, we assume $q_1 = q_2 = q_3 = q$, so the horizon radius is given by the relation (29) and thermodynamical stability let us to choose $q \leq 15 \times 10^6$. In this case we draw plots of the drag force in terms of the temperature and the black hole charge in the Fig. 12. We found that the black hole charge increases the value of drag force. In that case diffusion coefficient of the quark obtained as the following expression,

$$D_q = \frac{2 + \frac{3q}{r_h} - \frac{q^3}{r_h^2}}{2\pi r_h (1 + \frac{q}{r_h})^\frac{3}{2}}.$$ \hfill (76)

Similar to the previous cases, for the $q \to 0$ limit, we have $D_q = \frac{1}{\pi r_h}$, which means that diffusion coefficient of the quark found proportional to inverse of the temperature.

As one can find from the Figs. 10-12 the behavior of the drag force for small black hole charge approximately are the same. So, in order to see difference of three cases we need to consider large black hole charges. In that case it is interesting to compare above three
different configurations with each other and also with the case of $q = 0$ limit. It is easy to check that $q \to 0$ limit of the relation (72) reduced to the drag force of $\mathcal{N} = 4$ SYM theory [44]. Therefore, in the Fig. 13 we draw graph of the drag force corresponding to four different situations. We found that the black hole charge increases the value of drag force.

5.2 Quasi-normal modes

In this subsection we consider small perturbations of a straight string which stretched from $r = r_m$ to $r = r_h$ in STU background with three non-zero charges. The quasi-normal modes give us information about the equilibrium state of the string after small perturbations. This allows us to obtain the friction coefficient $\zeta$ in the non-relativistic regime of the quark. In that case we consider the static quark in the $\mathcal{N} = 2$ supergravity thermal plasma without any external fields. Indeed, we want to study the behavior of the string at the $t \to \infty$ and low velocity limits. The small fluctuations around the straight string means that $\dot{x}^2$ and $x'^2$ are small, so one can neglect them in the expression (66). Then, under assumption of time-dependent solution of the form $x(r, t) = x(r)e^{-\zeta t}$, equation of motion reduces to the following relation,

$$\frac{f_k}{r^2 \mathcal{H}_\pi} \partial_r \frac{f_k r^2}{\mathcal{H}_\pi} x' = \zeta^2 x.$$  \hspace{1cm} (77)
In order to obtain friction coefficient, we assume that \( \zeta \) is small, so one can use the following expansion,
\[
x = x_0 + \zeta x_1 + \zeta^2 x_2 + \cdots.
\]  
(78)

Also, by applying Neumann boundary condition we find,
\[
x'(r_m) = \zeta x'_1(r_m) + \zeta^2 x'_2(r_m) = 0.
\]  
(79)

We should substitute the above relations to the equation (77) and compare appropriate coefficients, in that case the leading order yields to \( x_0 = A \), where \( A \) is a constant. Therefore, by using Neumann boundary condition and relation (79) one can obtain a quasinormal mode condition on \( \zeta \) as the following,
\[
\zeta = \frac{r_h^2}{r_m R^2} \left[ (1 + \frac{q_1}{r_h})(1 + \frac{q_2}{r_h})(1 + \frac{q_3}{r_h}) \right]^\frac{1}{3}.
\]  
(80)

Again, we may use this result to obtain drag force. In the large \( r_m \) limit which corresponds to the heavy quark, from the relation (64), one can obtain \( M_{rest} = T_0 r_m \). Also, we know that \( \dot{P} = -\zeta M_{rest} v \). Therefor we find,
\[
\frac{dP}{dt} \approx -T_0 v r_h^2 \left[ (1 + \frac{q_1}{r_h})(1 + \frac{q_2}{r_h})(1 + \frac{q_3}{r_h}) \right]^{\frac{1}{3}}.
\]  
(81)

We see that the relation (81) exactly coincide with the relation (72) which obtained for a slowly moving heavy quark.

Now, we can use these results to obtain the total energy and momentum of the string. By using the equation of motion (77), time dependent solution of the form \( \dot{x} = -\zeta x \), and momentum densities (61) one can obtain,
\[
\pi^0_x = -\frac{T_0}{\zeta R^2} f_k r_h^2 \partial_r \dot{x}'.
\]  
(82)

Using Neumann boundary condition together the total momentum integral (62) yields to the following result,
\[
P = \frac{T_0}{\zeta R^2} \frac{f_k(r_{min}) r_{min}^2}{(1 + q_1 r_{min}^2)(1 + q_2 r_{min}^2)(1 + q_3 r_{min}^2)} x'(r_{min}),
\]  
(83)

where we insert \( r_{min} > r_h \) as lower limit of integral to avoid divergency. The reason is that the quasi-normal modes diverge close to the horizon. In addition they are rapidly oscillating.

So, quantities like \( x(r_h) \) and \( \dot{x}'(r_h) \) are not well defined right at \( r = r_h \). In order to regulate these divergences we cut-off the integrals at a finite \( r_{min} \).

In order to obtain the total energy we keep second order of velocities and expand \( \sqrt{-g} \), then similar to the calculation of momentum one can find,
\[
\pi^0_t = -\frac{T_0}{\mathcal{H}^2} \left[ 1 + \frac{f_k r_h^2}{2 R^2} \dot{x}' + \frac{r_h^2}{2 R^2} \frac{\mathcal{H}}{f_k} \dot{x}'^2 \right].
\]  
(84)
Then, by using the equation of motion and Neumann boundary condition we get,

\[ E = T_0 \frac{dr}{H^\frac{3}{2}} - \frac{T_0}{2R^2} \frac{f_k(r_{\text{min}})r_{\text{min}}^2}{(1 + \frac{q_1}{r_{\text{min}}})(1 + \frac{q_2}{r_{\text{min}}})(1 + \frac{q_3}{r_{\text{min}}})} x(r_{\text{min}}) x'(r_{\text{min}}). \]  

(85)

According to the relation (64) one can interpret the integral of the right hand side of the equation (85) as rest mass of quark. So, combining the relations (83) and (85) yield to the relation between the energy and momentum as,

\[ E = M_{\text{rest}} + \frac{P^2}{2M_{\text{kin}}}, \]

where the kinetic mass defined as the following expression,

\[ M_{\text{kin}} \equiv \frac{T_0}{\zeta R^2} r_h^2 \left[ (1 + \frac{q_1}{r_h})(1 + \frac{q_2}{r_h})(1 + \frac{q_3}{r_h}) \right]^{\frac{1}{3}}. \]

(87)

It is interesting to note that the equation (86) is valid for every theories such as $\mathcal{N} = 4$ SYM theory and $\mathcal{N} = 2$ gauged supergravity.

### 5.3 Effect of the constant electromagnetic field

In the previous sections the moving heavy quark through the $\mathcal{N} = 2$ plasma considered without any external field. In this section we would like to introduce a constant electromagnetic field on the brane which affects on the motion of heavy quark. In the description of the AdS/CFT correspondence the endpoint of both fundamental and Dirichlet strings under influence of non-zero NS NS B-field background corresponds to the moving quark with a constant electromagnetic field. We assume that the constant electromagnetic field is along $x^1$ and $x^2$ directions. Therefore, we add a constant $B$-field in the form of $B = B_{01} dt \wedge dx_1 + B_{12} dx_1 \wedge dx_2$ to the line element (1), where $B_{01}$ is the constant electric field and $B_{12}$ is the constant magnetic field. Also $B_{01}$ and $B_{12}$ are antisymmetric fields and other components of the $B$-field are zero. We must note that the same work was done originally for $\mathcal{N}=4$ SYM theory [71].

Because of introducing $B_{01}$ and $B_{12}$, the curved string dual to the heavy quark may be described by the $x_1(r, t) = x_1(r) + v_1 t$, $x_2(r, t) = x_2(r) + v_2 t$ and $x_3(r, t) = 0$. Therefore, the square root of lagrangian density (59) takes the following form,

\[ -g = \frac{1}{H^\frac{3}{2}} \left[ 1 - \frac{\mathcal{H}r^2}{f_k R^2} \vec{\sigma}^2 + \frac{f_k r^2}{R^2} x'^2 - (B_{01} x_1' + B_{12} (\vec{v} \times \vec{x}'))^2 \right], \]

(88)

where $\vec{\sigma} = (v_1, v_2)$ is the vector of velocity and $\vec{x}' = (x_1', x_2')$ is the projected directions of string tail.

We consider three cases, first, we assume that only electric field is exist and $B_{12} = 0$, and second, we have non-zero magnetic field and there is $B_{01} = 0$, and finally we discuss about
the case where $\vec{v} \perp B_{01}$.

In order to study the effect of constant electric field we set $B_{12} = 0$ and choose $x_1$ as the moving direction of the quark, so we have $v_1 = v$, $v_2 = v_3 = 0$, $x_1(r, t) = x(r) + vt$ and $x_2(r, t) = x_3(r, t) = 0$. Therefore, the equation (88) reduced to the following relation,

$$
-g = \frac{1}{\mathcal{H}^2} \left[ 1 - \frac{\mathcal{H} r^2}{f_k r^2} v^2 + \left( \frac{f_k r^2}{R^2} - B_{01}^2 \right) x^2 \right].
$$

(89)

Therefore, comparing the relations (65) and (89) yields to the following expression,

$$
x'^2 = \frac{C^2 v^2 R^2 \mathcal{H}^2}{f_k r^2} \left( \frac{f_k r^2}{f_k r^2(1 - \frac{B_{01}^2}{f_k r^2})^2} - C^2 v^2 R^2 \mathcal{H}^2 (1 - \frac{B_{01}^2}{f_k r^2}) \right).
$$

(90)

It means that $r_c$ is given by the relation (71), but the constant $C$ modified as the following,

$$
C = \left[ \prod_{i=1}^3 \left( 1 + \frac{q_i}{r_c^2} \right) \right] ^{\frac{1}{3}} \frac{r_c^2}{R^2} \sqrt{1 - \frac{B_{01}^2 R^4}{\prod_{i=1}^3 (1 + \frac{q_i}{r_h^2}) r_c^4 v^2}}.
$$

(91)

It yields us to the expression of drag force,

$$
dP dt = -T_0 v \left[ \prod_{i=1}^3 \left( 1 + \frac{q_i}{r_h^2} \right) \right] ^{\frac{1}{3}} \frac{r_h^2}{R^2} \sqrt{1 - \frac{B_{01}^2 R^4}{\prod_{i=1}^3 (1 + \frac{q_i}{r_h^2}) r_h^4 v^2}} + \mathcal{O}(v^2).
$$

(92)

It is clear that $B_{01} \to 0$ limit of the equation (92) reduces to the equation (72). Also, we find that the effect of the constant electric field is decreasing the drag force, so this result agree with the result of the Ref. [71]. It is also interesting to write the linearized expression for small electric field of the relation (92). In that case the correction terms are of even powers of $B_{01}$, which is natural characteristic of the Nambu-Goto action.

In the second case we consider only constant magnetic field $B_{12}$. In that case one can choose $x_1(r, t) = x_1(r) + vt$, $x_2(r, t) = x_2(r)$ and $x_3(r, t) = 0$. Under these assumptions one can find,

$$
x'_1(r) = \pi_{x_1} \left[ \frac{\beta \left( \frac{1}{\mathcal{H}^2} - \frac{\mathcal{H} r^2 v^2}{f_k} \right)}{\frac{f_k r^2}{\mathcal{H}^2} \left( \frac{\pi_{x_1}^2}{\mathcal{H}^2} - \frac{f_k r^2}{f_k r^2} \right) \left( \frac{\pi_{x_2}^2}{\mathcal{H}^2} - \beta - \frac{\pi_{x_1}^2}{\pi_{x_2}} \right) + \beta} \right] ^{\frac{1}{2}},
$$

$$
x'_2(r) = \pi_{x_2} \left[ \frac{\beta \left( \frac{1}{\mathcal{H}^2} - \frac{\mathcal{H} r^2 v^2}{f_k} \right)}{\frac{f_k r^2}{\mathcal{H}^2} \left( \frac{\pi_{x_1}^2}{\mathcal{H}^2} - \frac{f_k r^2}{f_k r^2} \right) \left( \frac{\pi_{x_2}^2}{\mathcal{H}^2} - \beta - \frac{\pi_{x_1}^2}{\pi_{x_2}} \right) + \beta} \right] ^{\frac{1}{2}},
$$

(93)

where we defined, $\beta \equiv \frac{f_k r^2}{\mathcal{H}^2} - v^2 B_{12}^2$ and set $\pi_{x_i}^1 \equiv \pi_{x_i}$. Now, reality condition implies that $\pi_{x_2} = 0$, and therefore we yield to the expression (72). It tell us that there is no drag force in $x^2$ direction and therefore the constant magnetic field have no effect on the motion along
$x^1$ direction. Actually vanishing of $\pi_{x_2}$ is consequence of vanishing of $v_2$. According to these two cases, (electric and magnetic fields), we found that the constant magnetic field have no effect on the motion of string and it is appropriate electric field which keeps the string at constant speed $v$.

Finally, we consider the case of $\vec{v} \perp B_{01}$. It means that one may choose the solutions of equation of motion as, $x_1(r, t) = x_1(r)$, $x_2(r, t) = x_2(r) + vt$ and $x_3(r, t) = 0$. A possible drag force may be found as the following relation,

$$\frac{dP_2}{dt} = -T_0 \left[ \frac{f_k(r_h)^2}{\prod_i(1 + \frac{\mu}{r_h})^\frac{1}{2}} - v^2 B_{12}^2 + \mathcal{O}(v^2) \right],$$

(94)

and $\dot{P}_3 = 0$. In this case the constant electric field has no effect on drag force. It should be mentioned that this situation is not our interesting case which considered in this paper. The magnetic field on the brane has equivalent interpretation as the following. One can consider a moving heavy quark in the non-commutative plane. Both cases (ordinary theory with $B$-field and non-commutative theory without $B$-field) yield to similar result, which is decreasing the drag force or equivalently decreasing the effective viscosity of QGP. In the next step, without any external fields, we try to obtain effect of higher derivative terms on the drag force.

### 5.4 Higher derivative correction

Already we studied the effect of higher derivative correction on shear viscosity and conductivities of QGP. Now, we are ready to consider higher derivative effect on the drag force. By using the solution (14) one can obtain,

$$\frac{dP}{dt} = -T_0 v \left[ \prod_{i=1}^{3} \left( 1 + \frac{\mu}{r_h} - \frac{c_1 q_i (q_i + \mu)}{72 q_i^2 (r_h^2 + q_i)^2} \right) \right] \frac{r_h^2}{R^2} \left( 1 + \mathcal{O}(v^2) \right),$$

(95)

where $r_h$ is given by the relation (15), and we used,

$$r_c = r_h + \frac{r_h^2 v^2 \mathcal{H}(r_h)}{R^2 f'_k(r_h)} + \mathcal{O}(v^4),$$

(96)

with $f_k$ and $\mathcal{H}$ are given by the relation (14), and prime denotes derivative with respect to $r$. As expected, the $c_1 = 0$ limit of the $\frac{dP}{dt}$ coincides with the results of subsection 5.1.

In that case we draw figure of the drag force in terms of the higher derivative parameter (see Fig. 14) and find that for $c_1 < 1.5 \times 10^{-28}$ the effect of the higher derivative terms is to decrease the drag force, then for $c_1 > 1.5 \times 10^{-28}$ the value of the drag force increases. So, if we set $c_1 \approx 1.5 \times 10^{-28}$ drag force vanishes, also $c_1 = 0$ and $c_1 \approx 3 \times 10^{-28}$ yields to similar value of the drag force. Therefore, in order to obtain larger drag force than $c_1 = 0$ case we should set $c_1 > 3 \times 10^{-28}$. This situation is similar for the three cases of one, two and three charged black holes.
Figure 14: The graph of the drag force in terms of the higher derivative parameter for \( v = 0.5, \mu = 1, \alpha' = 0.5, \lambda = 6\pi, T = 300 \text{ MeV} \) and \( q = 1 \).

### 5.5 Quark-anti quark solution

Now, we consider a moving quark-antiquark pair which may be interpreted as a meson. Indeed there is a moving meson with the constant speed \( v \) in the \( \mathcal{N} = 2 \) supergravity thermal plasma. Already the energy of a moving quark-antiquark pair in \( \mathcal{N} = 4 \) SYM plasma calculated [72]. Now, we would like to repeat same calculations in the STU background. The quark-antiquark pair in the thermal QGP corresponds to an open string in \( AdS_5 \) space with two endpoints on the D-brane in the \((X,Y)\) plan. Two end points of string on the D-brane represent quark and antiquark which separated from each other by a constant \( l \). We assume that at the \( t = 0 \) string is straight and two endpoints of string move with the constant velocity \( v \) along the \( X \) direction. The dynamics of such configuration discussed in detail in the Ref. [72] for the \( \mathcal{N} = 4 \) SYM plasma. Here, for the \( \mathcal{N} = 2 \) supergravity thermal plasma one can obtain,

\[
-g = \frac{1}{\mathcal{H}^3} \left[ 1 + \frac{f_k r^2}{R^2} (x'^2 + y'^2) - \frac{\mathcal{H} r^2}{f_k R^2} (\dot{x}^2 + \dot{y}^2) - \frac{r^4}{R^4} \mathcal{H} (\dot{x}^2 y'^2 + y^2 x'^2 - 2\dot{x}\dot{y}x'y') \right],
\]

(97)

where \( f_k \) and \( \mathcal{H} \) are given by the relation (2). The equations of motion of \( x \) and \( y \) are given by the following equations, respectively,

\[
\frac{\partial}{\partial r} \left[ \frac{1}{\sqrt{-g}} \left( \frac{r^4}{R^2} \mathcal{H}^2 (\dot{y}^2 x' - \dot{x}\dot{y}y') + \frac{f_k r^2 x'}{\mathcal{H}^3} \right) \right] + r^2 \mathcal{H}^2 \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \frac{\dot{x}}{f_k} + \frac{r^2}{R^2} (y^2 \dot{x} - x' \dot{y}y') \right) \right] = 0,
\]

\[
\frac{\partial}{\partial r} \left[ \frac{1}{\sqrt{-g}} \left( \frac{r^4}{R^2} \mathcal{H}^2 (\dot{x}^2 y' - \dot{x}\dot{y}x') - \frac{f_k r^2 y'}{\mathcal{H}^3} \right) \right] + r^2 \mathcal{H}^2 \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( \frac{\dot{y}R^2}{f_k} + r^2 (x'^2 \dot{y} - x' \dot{x}y') \right) \right] = 0,
\]

(98)

33
then momentum densities obtained by the following equation,

\[
\begin{pmatrix}
\pi_x^0 & \pi_x^1 \\
\pi_y^0 & \pi_y^1 \\
\pi_r^0 & \pi_r^1 \\
\pi_l^0 & \pi_l^1
\end{pmatrix}
= -T_0 \frac{r^2 \mathcal{H}^2}{R^2 \sqrt{-g}} \times
\begin{pmatrix}
\frac{r^2}{R^2} \mathcal{H}^2 x'y' - \left( \frac{2 \mathcal{H}^2}{f_k} + \frac{r^2 \mathcal{H}^2 y'^2}{f_k} \right) \dot{x}' \quad \frac{r^2}{R^2} \mathcal{H}^2 y' \dot{y}' - \left( \frac{2 \mathcal{H}^2 r^2 y'^2}{f_k} - \frac{f_k}{f_k^2} \right) x' \\
\frac{r^2}{R^2} \mathcal{H}^2 y' \dot{x}' - \left( \frac{2 \mathcal{H}^2}{f_k} + \frac{r^2 \mathcal{H}^2 x'^2}{f_k} \right) y' \quad \frac{r^2}{R^2} \mathcal{H}^2 x' \dot{y}' - \left( \frac{2 \mathcal{H}^2 r^2 x'^2}{f_k} - \frac{f_k}{f_k^2} \right) y'
\end{pmatrix}
\]  

There are two interesting motion for the meson. The first one is the moving quark-antiquark pair with constant speed \(v\). The second case is rotational motion of the quark-antiquark pair.

The first system may be described by the \(x(r, t) = vt + x(r)\) and \(y(r, t) = y(r)\). These solutions satisfy boundary conditions as \(x(\infty, t) = vt\) and \(y(\infty) = \pm l/2\). In this case equation (99) reduces to the following expression,

\[
\begin{pmatrix}
\pi_x^0 & \pi_x^1 \\
\pi_y^0 & \pi_y^1 \\
\pi_r^0 & \pi_r^1 \\
\pi_l^0 & \pi_l^1
\end{pmatrix}
= -T_0 \frac{r^2 \mathcal{H}^2}{R^2 \sqrt{-g}} \begin{pmatrix}
-v \left( \frac{1}{f_k} + \frac{r^2 y'^2}{f_k^2} \right) & \frac{f_k}{f_k R^2} x' \\
\frac{f_k}{f_k^2} v x' & -\frac{f_k}{f_k} \left( \frac{r^2 v^2}{f_k^2} - \frac{f_k}{f_k^2} \right) y'
\end{pmatrix} \begin{pmatrix}
x' \\
y'
\end{pmatrix},
\]

where,

\[
-g = \frac{1}{\mathcal{H}^2} \left[ 1 + \frac{f_k R^2}{R^2} (x'^2 + y'^2) - \mathcal{H}^2 v^2 \frac{r^4}{f_k R^2} - \frac{r^4}{R^4} \mathcal{H}^2 v^2 y^2 \right].
\]

In order to obtain drag force, we calculate \(\pi_1^x\) and \(\pi_1^y\) components and solve them for \(x'\) and \(y'\) respectively and obtain,

\[
x'(r) = \pi_x^1 R \left( 1 - \mathcal{H} \frac{r^2 v^2}{f_k R^2} \right) \left( \frac{f_k}{\mathcal{H}} - \frac{r^2 v^2}{R^2} \left( T_0^2 \frac{r^2}{R^2} f_k \mathcal{H}^2 - \mathcal{H} \pi_{1x}^2 \right) - f_k \pi_{1y}^2 \right)^{-\frac{1}{2}},
\]

\[
y'(r) = \pi_y^1 R \left[ \left( \frac{f_k}{\mathcal{H}} - \frac{r^2 v^2}{R^2} \left( T_0^2 \frac{r^2}{R^2} f_k \mathcal{H}^2 - \mathcal{H} \pi_{1x}^2 \right) - f_k \pi_{1y}^2 \right)^{-\frac{1}{2}} \right].
\]

As before, by using reality condition one can obtain,

\[
\pi_{1y}^2 = \left[ \left( \frac{f_k}{\mathcal{H}} - \frac{r^2 v^2}{R^2} \left( T_0^2 \frac{r^2}{R^2} \mathcal{H}^2 - \mathcal{H} \pi_{1x}^2 \right) \right) \right]_{r=r_{min}},
\]

where \(r_{min}\) is turning point of string. One can check easily that \(r_{min} \geq r_c\) (\(r_c\) is critical radius which introduced in the subsection 5.1, but \(r_{min}\) differs from UV cut off which introduced
in the relation (83). If \( \pi^1_y = 0 \), then \( r_{\min} = r_c \) and above solutions are similar to single quark solution \( (l = 0) \). Here, in order the string have a chance of turning around smoothly, it requires that \( \frac{\partial y}{\partial x} = \frac{y'}{x'} = \infty \) at \( r_{\min} \) [72]. So, it is necessary to have \( \pi^1_x = 0 \). Therefore, one can find drag force as,

\[
\pi^1_y = \frac{T_0}{R} r_{\min} \mathcal{H}'(r_{\min}) \sqrt{\frac{f_k(r_{\min})}{\mathcal{H}(r_{\min})} - \frac{r_{\min} v^2}{R^2}}. \tag{104}
\]

In the second case we add a rotational motion with angular velocity \( \dot{\theta} \) to the motion of meson. Therefore, the string may be described by the \( x(r, t) = vt + x(r) \sin \theta \) and \( y(r, t) = y(r) \cos \theta \). So, Fig. 15 shows the configuration of rotating string. As we can see, \( \theta(t) \) is an angle with \( Y \) axis. These solutions satisfy boundary conditions \( x(\infty, t) = vt \pm \frac{l}{2} \sin \theta \) and \( y(\infty, t) = \pm \frac{l}{2} \cos \theta \), where for \( \theta = 0 \) reduce to the boundary condition without rotational motion. Also there is another condition due to our conjecture, \( \frac{y'}{x'} = \cot \theta \), which reduces to \( \frac{y'}{x'} \to \infty \) at the \( \theta \to 0 \) limit, which is agree with the first case.

These boundary conditions can also satisfy with two separated string which move at velocity \( v \) along \( X \) axis and simultaneously swing a circle with radius \( l/2 \). Specifying these boundary conditions doesn’t lead to a unique solution for equation of motion, so we should specify additional conditions for this motion. Here, we assume that the string is initially upright, move at velocity \( v \) and rotates around its center of mass.

Now, by using above solutions in the equation (99) and solving resulting equations with respect to \( x' \) and \( y' \) one can obtain following equations,

\[
A x'^2 + B y'^2 + C x' y' + D = 0,
A' x'^2 + B' y'^2 + C' x' y' + D' = 0, \tag{105}
\]
Figure 15: A rotating \( \cap \) - shape string dual to a \( q \bar{q} \) pair which can be interpreted as a meson. The points \( A \) and \( B \) represent quark and antiquark with separating length \( l \). The radial coordinate \( r \) varies from \( r_h \) (black hole horizon radius) to \( r = r_m \) on D-brane. \( r_c \) is a critical radius, obtained for single quark solution, which the string can't penetrate beyond it and \( r_{min} \geq r_c \). \( r_{min} = r_c \) is satisfied if points \( A \) and \( B \) located at origin \( (l = 0) \), in that case there is the straight string which is dual picture of the single static quark. \( \theta \) is assumed to be the angle with \( Y \) axis and the string center of mass moves along \( X \) axis with velocity \( v \).

where,

\[
A = R^2 \sin^2 \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 y^2 \theta^2 R^2 \sin^2 \theta}{f_k} \right) - T^2_0 H^2 R^2 \left( \frac{H^2 y^2 \theta^2 R^2 \sin^2 \theta}{f_k} \right)^2 \right],
\]

\[
B = R^2 \cos^2 \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 (v + x \theta \cos \theta)^2 R^2}{f_k} \right) - T^2_0 H^2 y^2 \theta^2 R^6 \sin^2 \theta (v + x \theta \cos \theta)^2 \right],
\]

\[
C = -2y \dot{\theta} R^4 \sin^2 \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 y^2 \theta^2 R^2 \sin^2 \theta}{f_k} \right) \right] (v + x \theta \cos \theta),
\]

\[
D = R^2 \frac{1}{2} \left( \frac{f_k}{H^2} - y^2 \dot{\theta}^2 \sin^2 \theta - (v + x \theta \cos \theta)^2 \right),
\]

\[
A' = R^2 \sin^2 \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 y^2 \theta^2 R^2 \sin^2 \theta}{f_k} \right) - T^2_0 H^2 y^2 \theta^2 R^6 \sin^2 \theta (v + x \theta \cos \theta)^2 \right],
\]

\[
B' = R^2 \cos^2 \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 (v + x \theta \cos \theta)^2 R^2}{f_k} \right) - T^2_0 H^2 y^2 \theta^2 R^6 \left( \frac{R^2 H^2 (v + x \theta \cos \theta)^2}{f_k} \right)^2 \right],
\]

\[
C' = -2y \dot{\theta} R^4 \sin^2 \theta \cos \theta \left[ \frac{1}{2} \left( \frac{f_k}{H^2} - \frac{H^2 y^2 \theta^2 R^2 \sin^2 \theta}{f_k} \right) \right] (v + x \theta \cos \theta),
\]
where we set \( \frac{r}{R} \equiv R \). We must note that the variable \( C \) in equations (105) and (106) is different with integration constant in equation (65) hence subsections 5.1 and 5.3. Therefore, from equations (105) one can obtain,

\[
x'(r) = 2 \left[ \frac{D(B - \frac{\pi_1^{12}}{\pi_2} B')}{C^2 - C'^2 - 4(BA - B'A')} \right]^{\frac{1}{2}},
\]

\[
y'(r) = 2 \left[ \frac{D(A - \frac{\pi_1^{12}}{\pi_2} A')}{C^2 - C'^2 - 4(BA - B'A')} \right]^{\frac{1}{2}}.
\]

(107)

Here, if the rotational motion vanishes (\( \dot{\theta} = 0 \)), from equations (104) one can see that coefficients of \( x'y' \) vanish \( (C = C' = 0) \) and our solutions recover the motion of quark-antiquark pair without rotation. In order to obtain drag force we use reality condition and find a relation between variable (106) as \( \frac{A}{A'} = \frac{B}{B'} = \frac{C}{C'} = \frac{D}{D'} = \frac{\pi_1^{12}}{\pi_1^{12}} \). Then one can find two equations as,

\[ C^2 - 4AB = 0 \]

\[ C'^2 - 4A'B' = 0. \]

These equations specify \( \pi_1^x \) and \( \pi_1^y \) respectively. After some calculations and simplifications we find,

\[
\pi_1^{12} = \frac{1}{2a} \left[ \pm \sqrt{b^2 - 4ac} - b \right],
\]

\[
\pi_1^{12} = \frac{1}{2a'} \left[ \pm \sqrt{b'^2 - 4a'c' - b'} \right].
\]

(108)

where

\[
a = \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} \cos^2 \theta(R_{\text{min}}^2 + \xi \chi),
\]

\[
b = T_0^2 \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} \chi \left(2R_{\text{min}}^4 \cos^2 \theta(R_{\text{min}}^2 \xi \chi - \zeta) \right),
\]

\[
c = T_0^4 \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right) R_{\text{min}}^6 \sin^2 \theta \xi \chi^2,
\]

\[
a' = \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} (R_{\text{min}}^2 \xi + \xi \chi),
\]

\[
b' = T_0^2 R_{\text{min}}^2 \chi \left(R_{\text{min}}^4 \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} - 2R_{\text{min}}^2 \zeta - \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} \xi \chi \right),
\]

\[
c' = T_0^2 \prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} R_{\text{min}}^6 \xi^2 \left(\prod_i \left(1 + \frac{q_i}{r_{\text{min}}^2} \right)^\frac{3}{2} - T_0^2 \right),
\]

(109)
with,

\[ \varsigma = \prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right) y^2 \dot{\theta}^2 R_{\text{min}}^2 \sin^2 \theta (v + x \dot{\theta} \cos \theta)^2, \]

\[ \xi = \frac{f_k(r_{\text{min}})}{\prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{3}{2}}} - \prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{1}{2}} (v + x \dot{\theta} \cos \theta)^2 R_{\text{min}}^2, \]

\[ \chi = \prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{1}{2}} y^2 \dot{\theta}^2 R_{\text{min}}^2 \sin^2 \theta - \frac{f_k(r_{\text{min}})}{\prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{3}{2}}}, \quad (110) \]

where \( R_{\text{min}} = \frac{r_{\text{min}}}{R} \) and \( r_{\text{min}} \) is the turning point. The direct consequence of rotational motion is that drag force is no longer constant. From equation (108) one can see that the momentum densities of string vary with respect to \( x(r) \) and \( y(r) \).

But, this result is not appropriate description of a meson. According to previous works [71, 72] the \( q\bar{q} \) pair should be close enough together and not moving too quickly. The presence of functions \( x(r) \) and \( y(r) \) in relations (110) is consequence of relativistic motion, which is not acceptable. On the other hand, because of non-vanishing drag forces, it is expected that the velocity of a \( q\bar{q} \) pair decreases. So, we consider a moving heavy \( q\bar{q} \) pair with non-relativistic speed, which rotates by angle \( \theta = \omega t \) around the center of mass. Indeed this situation is corresponding to motion of the heavy meson with large spin. Actually, in the very large angular momentum limit, a classical approximation is reliable. In this case, the angular velocity of the string is very small. Therefore, we are going to discuss the case of non-relativistic motion (\( \dot{\theta}^2 \to 0 \) and \( \dot{\theta} \nu \to 0 \)). In that case \( \varsigma = c = c' = 0 \) and we have,

\[ \pi_x^{12} = \frac{r_{\text{min}}^2 T_0^2}{R^2} f_k(r_{\text{min}}) \mathcal{H}^{-\frac{1}{2}}(r_{\text{min}}), \]

\[ \pi_y^{12} = \frac{r_{\text{min}}^2 T_0^2}{R^2} \left( f_k(r_{\text{min}}) - \mathcal{H}(r_{\text{min}}) \frac{r_{\text{min}}^2 v^2}{R^2} \right) \mathcal{H}^{-\frac{1}{2}}(r_{\text{min}}). \quad (111) \]

Now, we assume that \( v^2 \to 0 \) and angular velocity is infinitesimal constant (\( \dot{\theta} = \omega \ll 1 \)), and the quark-antiquark pair rotates around origin. In that case we neglect \( \omega^4 \) terms and obtain values of momentum densities as the following,

\[ \pi_x = \pi_y = T_0 \frac{r_{\text{min}}}{R} \frac{\left[ k - \frac{\mu}{r_{\text{min}}^2} + \frac{r_{\text{min}}^2}{R^2} \prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{3}{2}} \right]}{\prod_i \left( 1 + \frac{q_i}{r_{\text{min}}^2} \right)^{\frac{3}{2}}} \quad (112) \]

In order to obtain non-zero components of momentum densities (111) and (112) we should use negative sign in the relations (108). Therefore, correct sign in the equations (108) is minus sign, and we yield to constant drag forces as expected for the non-relativistic motion. In order to extend this work one may consider more quarks, such as four quarks in the baryon [63] through the thermal plasma.
6 Jet-quenching parameter

One of the interesting properties of the strongly-coupled plasma at RHIC is the jet quenching of partons produced with high transverse momentum. This parameter controls the description of relativistic partons and it is possible to employ the gauge/gravity duality and determine this quantity in the finite temperature gauge theories. In order to obtain the jet-quenching parameter one needs to rewrite the metric (1) in the light-cone coordinates. Therefore, one can introduce light-cone coordinates $x^\pm = t^\pm x_1/\sqrt{2}$, and rewrite the metric (1) in the following form,

$$ds^2 = \frac{1}{2} \left( \frac{H^2 r^2}{R^2} - \frac{f_k}{H^2} \right) \left( (dx^+)^2 + (dx^-)^2 \right) - \left( \frac{H^2 r^2}{R^2} + \frac{f_k}{H^2} \right) dx^+ dx^- + H^2 \left( \frac{r^2}{R^2} (dx_2^2 + dx_3^2) + \frac{dr^2}{f_k} \right).$$ \hspace{1cm} (113)

We begin with the general relation for the jet-quenching parameter [51],

$$\hat{q} \equiv 8\sqrt{2} \frac{S_I}{L^2},$$ \hspace{1cm} (114)

where $S_I = S - S_0$ ($S$ denotes $q\bar{q}$ pair action and $S_0$ denotes the action of isolated $q$ and $\bar{q}$). It means that the jet-quenching parameter is proportional to energy of the string, so we expect that this quantity will be opposite of the drag force which is indeed energy loss of the string. Therefore, calculation of the jet-quenching parameter reduces to obtain actions $S$ and $S_0$. One can image the situation with an open string whose endpoints lie on the brane. In the light-cone coordinates, the string may be described by $r(\tau, \sigma)$. We use the static gauge where $\tau = x^-$ and $\sigma = x^2 \equiv y$, and all other coordinates considered as constants. In that case $-\frac{L}{2} \leq y \leq \frac{L}{2}$, and $L^- \leq x^- \leq 0$, and because of $L^- \gg L$ one can assume that the world-sheet is invariant along the $x^-$ direction. Therefore, the string may described by the function $r(y)$, so the boundary condition is $r(\pm \frac{L}{2}) = \infty$. In this configuration, the induced metric on the string world-sheet obtained as the following,

$$2g = \left( \frac{H^2 r^2}{R^2} - \frac{f_k}{H^2} \right) \left( \frac{r^2}{R^2} + \frac{r'^2}{f_k} \right).$$ \hspace{1cm} (115)

Since equation (115) is dependent of coordinate $x^-$, one can integrate over $x^-$ and then the Nambu-Goto action is given by,

$$S = \frac{\sqrt{2} L^-}{2\pi \alpha'} \int_0^{\frac{L}{2}} dy \sqrt{\left( \frac{H^2 r^2}{R^2} - \frac{f_k}{H^2} \right) \left( \frac{r^2}{R^2} + \frac{1}{f_k} r'^2 \right)}. \hspace{1cm} (116)$$

One can remove the $r'$ by using the equation of motion. In that case, since the lagrangian density is time-dependent, one can write,

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial r'} - \mathcal{L} = \text{Const.} \equiv E. \hspace{1cm} (117)$$
Therefore, the following relation is obtained,
\[
r' = \frac{f_k r^2}{R^2 E^2} \left[ \frac{\mathcal{H}^2}{2 R^2} \left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} \right) r^2 - E^2 \right].
\] (118)

Equation (118) has two important poles where \( r' = 0 \). The main pole exist at the horizon. So, it is clear that the equation (118) has a zero at the horizon where \( f_k = 0 \). In this case the string comes from infinity \( r(L/2) = \infty \) and touches the horizon and returns to infinity \( r(-L/2) = \infty \). The second pole of equation (118) obtained by the following relation,
\[
\frac{f_k r^2}{\mathcal{H}^2 R^2} - \frac{\mathcal{H}^2 r^4}{R^4} + 2E^2 = 0.
\] (119)

In the Ref. [75] found that the string world sheet has one end at a Wilson line at the boundary with \( \operatorname{Im}[t] = 0 \), and the other end at a Wilson line the boundary with \( \operatorname{Im}[t] = -i\epsilon \). The only way that the string world-sheet linking these two Wilson lines can meet is if the string world-sheet hangs down to the horizon. Therefore, the only physical situation is the first case where the string touches the horizon. Also in our case, drawing the \( r'^2 \) in terms of \( r \) tells that the turning point of string should be \( r_h \).

By using equation (118) in (116), and also the new definition of \( B \equiv 1/E^2 \), one can rewrite the Nambu-Goto action in the following form,
\[
S = \frac{L^-}{2\pi \alpha'} \sqrt{B} \int_{r_h}^{\infty} dr \left( \frac{f_k r^2}{\mathcal{H}^2 R^2} - \frac{f_k}{\mathcal{H}^2} \right) \left[ 1 + \frac{R^2}{(\mathcal{H}^2 r^2 - f_k/\mathcal{H}^2) Br^2} \right].
\] (120)

For the low energy limit \( (E \to 0) \) we expand equation (120) to leading order in \( 1/B \). This is reasonable since the determination of \( \hat{q} \) demands the study of the small separation limit of \( L \). Then at the first order of \( 1/B \) one can obtain,
\[
S = \frac{L^-}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2\mathcal{H}^2}{f_k} \left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} \right)} \left[ 1 + \frac{R^2}{(\mathcal{H}^2 r^2 - f_k/\mathcal{H}^2) Br^2} \right].
\] (121)

Now, one can extract action \( S_0 \) which can be interpreted as the self-energy of the isolated quark and the isolated antiquark. In that case by using the result of the Ref. [56] one can obtain,
\[
S_0 = \frac{L^-}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2\mathcal{H}^2}{f_k} \left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} \right)}.
\] (122)

Therefore, we can extract \( S_I \) as the following,
\[
S_I = \frac{1}{\sqrt{B}} \frac{L^-}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2R^4}{(\mathcal{H}^2 r^2 - f_k/\mathcal{H}^2) Br^4}}.
\] (123)
On the other hand, one can integrate equation (118) and obtain the following relation for infinitesimal $\frac{1}{B}$:

$$\frac{L}{2} = R^2 \int_{r_h}^{\infty} dr \frac{1}{\sqrt{R^2 \left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} \right) f_k r^4}}.$$ (124)

Therefore, by using relations (115), (123) and (124) we can specify the jet-quenching as the following,

$$\hat{q} = (I(q))^{-1} \pi \alpha',$$ (125)

where,

$$I(q) = R^2 \int_{r_h}^{\infty} dr \frac{\sqrt{\left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} \right) f_k r^4}}.$$ (126)

In order to obtain the explicit expression of the jet-quenching parameter we set $k = 1$ and consider three special cases of one, two and three charged black hole.

### 6.1 One-charged black hole

One-charged black hole means that $q_1 = q, q_2 = q_3 = 0$. So, the integral (126) reduces to the following expression,

$$I(q_1) = R^4 \int_{r_h}^{\infty} \sqrt{\left( \frac{1}{r^2 - \mu} \right)} \frac{\left( \frac{q}{r^2} \right)^{\frac{1}{4}}}{\left( 1 + \frac{q}{r^2} \right)^{\frac{1}{4}}} dr,$$ (127)

where $r_h$ is given by the relation (26). In order to compare our result with $\mathcal{N} = 4$ SYM case we should use re-scaling (31), in that case it is easy to check that our results are agree with the case of $\mathcal{N} = 4$ SYM plasma. We show this point later for the case of three-charged black hole.

Numerically, we draw the curves of the jet-quenching parameter in terms of the black hole charge and the temperature in Fig. 16 and Fig. 17 respectively. These figures show that the jet-quenching parameter of the $\mathcal{N} = 2$ theory is larger than the jet-quenching parameter of the $\mathcal{N} = 4$ theory. For example by choosing $R^2 = \alpha' \sqrt{\lambda}$, $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$ and $T = 300$ MeV one can obtain, $\hat{q} = 42 \text{ GeV}^2/fm$. In that case the thermodynamical stability lets us choose $q \sim 10^6$ for $T = 300\text{MeV}$. On the other hand, for the small black hole charge, by taking $\alpha' = 0.5$ and $\lambda = 6\pi$ one can obtain $\hat{q} = 37.5 \text{ GeV}^2/fm$. It means that the black hole charge increases the jet-quenching parameter. In order to obtain $\hat{q} = 5 \text{ GeV}^2/fm$ the corresponding temperature of the QGP is $155\text{MeV}$, which is smaller than expected [76].
Figure 16: Plot of the jet-quenching parameter in terms of the black hole charge. We fixed our parameters as $\alpha' = 0.5$, $\lambda = 6\pi$, and $T = 300$ MeV. The solid line represents the case of $q_1 = q, q_2 = q_3 = 0$. The dotted line represents the case of $q_1 = q_2 = q, q_3 = 0$. The dashed line represents the case of $q_1 = q_2 = q_3 = q$. It shows that increasing the number of black hole charges increases the value of the jet-quenching parameter.

6.2 Two-charged black hole

Two-charged black hole means that $q_1 = q_2 = q, q_3 = 0$. So, the integral (126) reduces to the following expression,

$$I(q_{1,2}) = R^4 \int_{r_h}^{\infty} \sqrt{\rho(r^4 + (2q + R^2)r^2 - \mu R^2 + q^2)} \, dr,$$

(128)

where $r_h = \pi R^2 T$, and we defined,

$$\rho \equiv ((R^2 - 1)r^4 + (2qR^2 - R^2 - q)r^2 + R^2 q^2 + \mu R^2 - q^2).$$

(129)

Numerically, we draw curves of the jet-quenching parameter in terms of the black hole charge and the temperature in Fig. 16 and Fig. 17 respectively. These plots show that the jet-quenching parameter of the $\mathcal{N} = 2$ theory is larger than the jet-quenching parameter of the $\mathcal{N} = 4$ theory. Also, we find that the jet-quenching parameter of the two-charged black hole is larger than the jet-quenching parameter of the one-charge black hole. For example by choosing $R^2 = \alpha' \sqrt{\lambda}$, $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$ and $T = 300$ MeV one can obtain $\hat{q} = 49 \text{ GeV}^2/fm$. In that case the thermodynamical stability lets us choose $q \sim \times 10^6$ for $T = 300$ MeV. If we consider small values of the black hole charge then find the same value of the jet-quenching parameter as the previous case, and this point is illustrated in Fig. 17.
Therefore, in order to obtain $\hat{q} = 5 \text{ GeV}^2/fm$, the corresponding temperature of the QGP is $155\text{ MeV}$ for small black hole charge.

### 6.3 Three-charged black hole

Three-charged black hole means that $q_1 = q_2 = q_3 = q$. As we know, this configuration of STU model is identical to the Reissner-Nordstrom-AdS\textsubscript{5} black hole \cite{77}. In that case the integral (126) reduces to the following expression,

$$I(q_{1,2,3}) = R^4 \int_{r_h}^{\infty} \frac{r^2 (r^2 + q)}{\varrho (r^6 + (R^3 + 3q)r^4 + (3q^2 - \mu R^2)r^2 + q^3)} dr,$$

where we defined,

$$\varrho \equiv (R^2 - 1)r^6 + (3qR^2 - R^2 - 3q)r^4 + (3R^2q^2 + \mu R^2 - 3q^2)r^2 + (R^2 - 1)q^3),$$

and $r_h$ is given by the relation (29). Numerically, we give plots of the jet-quenching parameter in terms of the black hole charge and the temperature in Fig. 16 and Fig. 17 respectively. These plots show that the jet-quenching parameter of the $\mathcal{N} = 2$ theory is larger than the jet-quenching parameter of the $\mathcal{N} = 4$ theory. Also we find that the jet-quenching parameter of the three-charged black hole is larger than the jet-quenching parameter of the one-charge and two-charged black holes. For example by choosing $R^2 = \alpha'\sqrt{\lambda}$, $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$
and \( T = 300 \) MeV one can obtain \( \hat{q} = 58 \) GeV\(^2/fm\). In that case the thermodynamical stability lets us choose \( q \sim 10^6 \) for \( T = 300 \) MeV. If we consider small value of the black hole charge then find the same value of the jet-quenching parameter as the previous cases, and this point is illustrated in Fig. 17. Therefore, in order to obtain \( \hat{q} = 5 \) GeV\(^2/fm\) the corresponding temperature of the QGP is 155 MeV for a small black hole charge.

As we promised already in order to compare our results with the case of \( \mathcal{N} = 4 \) SYM we also perform the re-scaling (31) which yields us to obtain the following result,

\[
\hat{q} = \frac{r_0^2}{\pi \alpha' R^4} \left[ \int_{r_h}^{\infty} \frac{dr}{r^2 \sqrt{f/H}} \right]^{-1},
\]

(132)

where

\[
f = H^3 - \frac{r_0^4}{r_1^4},
\]

\[
H = 1 + \frac{q}{r_1^2},
\]

(133)

which agree with the results of the Refs. [52, 53], where the jet-quenching parameter calculated with the chemical potential. The horizon radius \( r_0 \) obtained for the case of zero-charge black hole. For the non-zero charge it is clear that the horizon radius decreases \((r_h < r_0)\). From the relation (19) we know that the \( q = 0 \) limit is equal to \( \phi = 0 \) limit and one can say that the jet-quenching parameter from the \( \mathcal{N} = 2 \) supergravity theory with zero chemical potential is equal to the jet-quenching parameter from the \( \mathcal{N} = 4 \) SYM theory.

### 6.4 Effect of the constant electric field

In this subsection similar to the subsection 5.3 we would like to add a two form \( F = B_{01} dt \wedge dx_1 \) as a constant electric field to the line element (1). Antisymmetric field \( B_{01} \equiv e \) is the constant electric field. Now, we are going to obtain the effect of the constant electric field on the jet-quenching parameter. In that case the Nambu-Goto action is given by the square root of the following equation,

\[
2g = \left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} + e \right) \left( \frac{r^2}{R^2} + \frac{r'^2}{f_k} \right).
\]

(134)

Therefore one can obtain the jet-quenching parameter as the following,

\[
\hat{q} = \frac{\left( I(q, e) \right)^{-1}}{\pi \alpha'},
\]

(135)

where,

\[
I(q, e) = R^2 \int_{r_h}^{\infty} \frac{dr}{\sqrt{\left( \frac{\mathcal{H}^2 r^2}{R^2} - \frac{f_k}{\mathcal{H}^2} + e \right)\mathcal{H}^2 f_k r^4}}.
\]

(136)
and $f$ and $\mathcal{H}$ is given by the relation (2). In order to find the effect of the constant electric field on the jet-quenching parameter we examine above integral for three different cases of one, two and three-charged black hole. Numerically, and under near boundary approximation, we draw graph of the jet-quenching parameter in terms of the constant electric field and find that the constant electric field increases the value of the jet-quenching parameter. In the Fig. 18 we draw the jet-quenching parameter in terms of the constant electric field for the large black hole charge.

![Figure 18: Plot of the jet-quenching parameter in terms of the constant electric field. We fixed our parameters as $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$ and $T = 300$ MeV. The solid line represents the case of $q_1 = q$, $q_2 = q_3 = 0$. The dotted line represents the case of $q_1 = q_2 = q$, $q_3 = 0$. The dashed line represents the case of $q_1 = q_2 = q_3 = q$. It show that the jet-quenching parameter increased by the constant electric field.](image)

6.5 Higher derivative correction

Finally, in absence of any external field, we would like to calculate the effect of higher derivative terms on the jet-quenching parameter. In that case the jet-quenching parameter obtained as the following expression,

$$\hat{q} = \frac{(I(q, c_1))^{-1}}{\pi \alpha'},$$

(137)

where,

$$I(q, c_1) = \int_{r_h}^{\infty} \frac{dr}{\sqrt{\left(\frac{H^2 r^2}{R^2} - \frac{L}{H}\right)f_k r^4}},$$

(138)
also, we used relations (14) and (15) for the case of \( q_1 = q_2 = q_3 = q \). One can study near boundary behavior of the jet-quenching parameter and find that the higher derivative terms include at \( \mathcal{O}(\frac{1}{\alpha'}) \). In that case we find that the higher derivative terms decrease the value of the jet-quenching parameter. So, for the fixed parameters such as \( \alpha' = 0.5, \lambda = 6\pi, T = 300 \text{ MeV} \) and small black hole charge, we obtain \( c_1 < 0.00021 \) to have positive jet-quenching parameter, which is agree with the result of the subsection 5.4. For example with the above fixed parameters and \( c_1 = 0.001 \) one can obtain \( \hat{q} = 4.6 \text{ GeV}^2/\text{fm} \) which is approximately value of the jet-quenching parameter of the \( \mathcal{N} = 4 \) SYM theory. In order to obtain \( \hat{q} = 5 \text{ GeV}^2/\text{fm} \) the corresponding higher derivative parameter should be \( c_1 \approx 97 \times 10^{-4} \) at \( T = 300 \text{MeV} \).

Again, we can use re-scaling (31) and obtain,

\[
\hat{q} = \frac{r_0^2}{\pi \alpha' R^4} \left[ \int_{r_h}^{\infty} \sqrt{\frac{H}{f}} \frac{dr}{r^2} \right]^{-1},
\]

(139)

where,

\[
f = (1 + \frac{q}{r^2})^3 - \frac{r_0^4}{r^4} + \frac{c_1 r_0^4}{24 R^2 r^4} \left[ \frac{r_0^4}{4 r^2 (r^2 + q)} - \frac{8q}{3} \right],
\]

\[H = 1 + \frac{q}{r^2} - \frac{c_1 q r_0^4}{24 R^2 r^4 (r^2 + q)}.
\]

(140)

and radius \( r_h \) is the root of the \( f = 0 \) from relation (140). The equation (139) may be solved numerically, and explicit expression of the jet-quenching parameter can be obtained. But it is clear that the effect of higher derivative correction is to decrease the jet-quenching parameter. One can check this statement by taking \( q = 0 \) limit. In this limit the jet-quenching parameter derived as,

\[
\hat{q}_0 = \frac{r_0^2}{\pi \alpha' R^4} \left[ \int_{r_h}^{\infty} \frac{6 R^2 r^4}{96 R^2 r^4 (r^4 - r_0^4) + c_1 r_0^8} dr \right]^{-1},
\]

(141)

where,

\[r_h^4 = \frac{r_0^4}{2} \left( 1 + \sqrt{1 - \frac{c_1}{24 R^2}} \right).
\]

(142)

In that case it is necessary that \( c_1 < 24 \alpha' \sqrt{\lambda} \). Comparing equation (141) with the jet-quenching parameter of the \( \mathcal{N} = 4 \) SYM theory tell us that the effect of \( c_1 \) is decreasing the jet-quenching parameter.

### 7 Conclusion

In this paper we studied some important problems to understand the nature of QGP more exactly. Indeed, we considered thermal QGP include a chemical potential. This chemical potential comes from \( \mathcal{N} = 2 \) supergravity in 5 dimensions. This theory contains a non-extremal black hole with three electrical charges and well known as STU model. First of
all we reviewed properties of STU model and extracted their equations. We studied thermodynamics of STU background and extracted the Hawking temperature, entropy density, specific heat and free energy of QGP. We found that the black hole charge increase the value of specific heat. In order to compare our results with the $\mathcal{N} = 4$ SYM plasma we used special re-scaling which actually was a transformation to the flat space.

We investigated the ratio of shear viscosity to entropy density and found that the universality of $\eta/s$ is valid also in STU model. Also, we found that the shear viscosity is decreasing for the cases of one-charged and three-charged black holes and is increasing for the case of two-charged black hole. We discussed briefly about thermal and electrical conductivities of QGP.

Then, we considered problem of the drag force and found energy loss of single quark and quark-antiquark pair. We showed that the value of the drag force enhanced due to the black hole charges. Also we calculated diffusion coefficient of the quark for the three special cases of one, two and three-charged black holse. We found that the effect of constant electric field is decreasing of the drag force but higher derivative terms may be increases or decreases the value of drag force. It depend to the value of higher derivative parameter. Here, we found interesting relation between drag force of the single quark (72) and entropy density (20) which is $s^2 \propto \dot{P}^3$. This relationship is valid also in the case of $\mathcal{N} = 4$ SYM plasma. We discuss about this relation and also other interesting relations end of this section.

Finally we studied the jet-quenching parameter and found that the jet-quenching parameter like the drag force enhanced due to the black hole charges. It means that the energy of the string in $\mathcal{N} = 2$ thermal plasma is larger than the string in $\mathcal{N} = 4$ thermal plasma, hence the string in $\mathcal{N} = 2$ thermal plasma lose more energy than the string in $\mathcal{N} = 4$ thermal plasma. In this case we found that the constant electric field enhances the jet-quenching parameter, but higher derivative terms decreases the value of jet-quenching parameter. We examine our solution for three special cases of one, two and three-charged black holes. All cases yield to the same value of the jet-quenching parameter for the small black hole charge. However, thermodynamical stability allow to choose the black hole charge of order $10^6$. In that case we found $\hat{q} = 42, 49$ and $58 GeV^2/fm$ for one, two and three-charged black hole respectively. These values of the jet-quenching parameter are far from experiments of RHIC (experimental data tell us that $5 < \hat{q} < 25$). There is no worry for this statement because the temperature of the $\mathcal{N} = 2$ supergravity theory should given smaller than the $\mathcal{N} = 4$ SYM theory. In that case with the temperature about $155 MeV$ we obtained the jet-quenching parameter in the experimental range.

Let us now compare some interesting ratios of QGP quantities. First, we summarize results of the $\mathcal{N} = 4$ SYM theory. The entropy density, drag force of moving heavy quark and jet-quenching parameter of $\mathcal{N} = 4$ SYM QGP are given by,

$$s = \frac{\pi^2}{2} N^2 T^3,$$
$$\dot{P} = \frac{\pi}{2} v \sqrt{\lambda} T^2,$$
$$\hat{q} = \frac{\pi^2}{a} \sqrt{\lambda} T^3,$$  \hspace{1cm} (143)
where $a = 1.311$ is a constant and $\lambda$ is 't Hooft coupling. Now, it is clear that,

$$\frac{s}{P} \propto T,$$

$$\frac{s}{q} \propto \text{Const.},$$

$$\frac{\dot{q}}{P} \propto T.$$  \hspace{1cm} (144)

It is interesting to study such relations in the $\mathcal{N} = 2$ supergravity thermal plasma. We obtained entropy density (20), drag force of moving heavy quark (72) and jet-quenching parameter (125) of $\mathcal{N} = 2$ QGP. We can draw graph of $s/\dot{P}$, $s/\dot{q}$ and $\dot{q}/\dot{P}$ to investigate behavior of these ratios. In the Fig. 19 we give $s/\dot{P}$ in terms of the temperature and find linear behavior of $T$. So, it is in agreement of $\mathcal{N} = 4$ case, therefore we can claim $s/\dot{P} \propto T$ is valid at the both $\mathcal{N} = 4$ and $\mathcal{N} = 2$ cases.

Also we draw $s/\dot{q}$ and $\dot{q}/\dot{P}$ in terms of the temperature in the Fig. 20, and find that $s/\dot{q}$ yields to a constant. Also $\dot{q}/\dot{P}$ has linear behavior of $T$ which are in agreement of $\mathcal{N} = 4$ case. In the recent works a general non-extremal rotating charged AdS black holes in five-dimensional $U(1)^3$ gauged supergravity [78] and also higher dimensional one studied [79, 80]. Now, it is interesting to generalized results of this paper to these cases.

Figure 19: Plot of the $s/\dot{P}$ in terms of the temperature $T$. We fixed our parameters as $\alpha' = 0.5$, $\lambda = 6\pi$ and $q = 1$. Three cases of one, two and three-charged black holes have similar manner for small black hole charge. It show that the $s/\dot{P}$ is linear in $T$.  

48
Figure 20: Plot of the $s/\dot{q}$ (dashed line) and $\dot{q}/\dot{P}$ (solid line) in terms of the temperature $T$. We fixed our parameters as $\alpha' = 0.5$, $\lambda = 6\pi$ and $q = 1$. Three cases of one, two and three-charged black holes have similar manner for small black hole charge. It show that the $s/\dot{q}$ is a constant and $\dot{q}/\dot{P}$ is linear in $T$.

Acknowledgments It is pleasure to thanks C. P. Herzog for reading manuscript and giving good suggestions. Also we would like to thanks Jose Edelstein and A. R. Amani for discussion about shear viscosity, and K. B. Fadafan for his collaboration about the jet-quenching parameter.

References

[1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity”, Adv. Theor. Math. Phys. 2 (1998) 231.

[2] E. Witten, ”Anti-de Sitter space and holography”, Adv. Theor. Math. Phys. 2 (1998) 253.

[3] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, ”Gauge theory correlators from noncritical string theory”, Phys. Lett. B428 (1998) 105.

[4] J. H. Schwart, ”Introduction to M Theory and AdS/CFT Duality”, Lecture Notes in Physics, Volume 525(1999), [arXiv:hep-th/9812037]. M. R. Douglas and S. Randjbar-Daemi, ”Two Lectures on AdS/CFT correspondence” [arXiv:hep-th/9902022]. J. L. Petersen, ”Introduction to the Maldacena Conjecture on AdS/CFT”, Int.J.Mod.Phys. A14 (1999) 3597. Horatiu Nastase, ”Introduction to AdS-CFT”, [arXiv:0712.0689 [hep-th]]. Igor R. Klebanov, ”TASI Lectures: Introduction to the AdS/CFT Correspondence”, [arXiv:hep-th/0009139]. Robi Peschanski, ”Introduction to String The-
ory and Gauge/Gravity duality for students in QCD and QGP phenomenology”, arXiv:0804.3210 [hep-ph]]

[5] G.Policastro, D.T.Son, A.O.Starinets, ”From AdS/CFT correspondence to hydrodynamics”, JHEP 0209 (2002) 043, arXiv:hep-th/0205052. Pavel Kovtun, Dam T. Son, Andrei O. Starinets, ”Holography and hydrodynamics: diffusion on stretched horizons”, JHEP 0310 (2003) 064, arXiv:hep-th/0309213.

[6] Alex Buchel, James T. Liu, ”Universality of the shear viscosity in supergravity”, Phys.Rev.Lett. 93 (2004) 090602, arXiv:hep-th/0311175. Alex Buchel, Robert C. Myers, Miguel F. Paulos, Aninda Sinha, ”Universal holographic hydrodynamics at finite coupling”, Phys.Lett.B669 (2008) 364, arXiv:0808.1837[hep-th]].

[7] P.Kovtun, D.T.Son, A.O.Starinets, ”Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics”, Phys.Rev.Lett. 94 (2005) 111601, arXiv:hep-th/0405231.

[8] Paolo Benincasa, Alex Buchel, ”Transport properties of N=4 supersymmetric Yang-Mills theory at finite coupling”, JHEP0601 (2006) 103, arXiv:hep-th/0510041.

[9] Omid Saremi, ”The Viscosity Bound Conjecture and Hydrodynamics of M2-Brane Theory at Finite Chemical Potential”, JHEP0610 (2006) 083, arXiv:hep-th/0601159.

[10] J. Mas, ”Shear viscosity from R-charged AdS black holes”, JHEP0603 (2006) 016, arXiv:hep-th/0601144.

[11] Kengo Maeda, Makoto Natsuume, Takashi Okamura, ”Viscosity of gauge theory plasma with a chemical potential from AdS/CFT correspondence”, Phys.Rev.D73 (2006) 066013, arXiv:hep-th/0602010.

[12] Yevgeny Kats, Pavel Petrov, ”Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory”, JHEP 0901 (2009) 044, arXiv:0712.0743 [hep-th]]

[13] Alex Buchel, ”Shear viscosity of boost invariant plasma at finite coupling”, Nucl.Phys.B802 (2008) 281, arXiv:0801.4421 [hep-th]]. Alex Buchel, Robert C. Myers, Aninda Sinha, ”Beyond $\eta/\sigma = 1/4\pi$”, JHEP 0903 (2009) 084, arXiv:0812.2521[hep-th]].

[14] Mauro Brigante, Hong Liu, Robert C. Myers, Stephen Shenker, Sho Yaida, ”Viscosity Bound and Causality Violation”, Phys.Rev.Lett.100 (2008) 191601, arXiv:0802.3318 [hep-th]].

[15] Alex Buchel, ”Resolving disagreement for $\eta/\sigma$ in a CFT plasma at finite coupling”, Nucl.Phys.B803 (2008) 166, arXiv:0805.2683 [hep-th]].

[16] Sayan K. Chakrabarti, Sachin Jain, Sudipta Mukherji, ”Viscosity to entropy ratio at extremality”, JHEP 1001 (2010) 068, arXiv:0910.5132 [hep-th]].
[17] Amos Yarom, ”Notes on the bulk viscosity of holographic gauge theory plasmas”, JHEP 1004 (2010) 024, [arXiv:0912.2100v1 [hep-th]].

[18] Sachin Jain, ”Universal properties of thermal and electrical conductivity of gauge theory plasmas from holography”, JHEP06(2010)023, [arXiv:0912.2719 [hep-th]]. Rong-Gen Cai, Zhang-Yu Nie, Ya-Wen Sun, ”Shear Viscosity from the Effective Coupling of Gravitons”, [arXiv:1006.0539 [hep-th]].

[19] Robert C. Myers, Miguel F. Paulos, Aninda Sinha, ”Quantum corrections to $\eta/s$”,Phys.Rev.D79 (2009) 041901, [arXiv:0806.2156 [hep-th]].

[20] Allan Adams, Alexander Maloney, Aninda Sinha, Samuel E. Vazquez, ”$1/N$ Effects in Non-Relativistic Gauge-Gravity Duality”, JHEP 0903 (2009) 097, [arXiv:0812.0166 [hep-th]].

[21] Xian-Hui Ge, Sang-Jin Sin, ”Shear viscosity, instability and the upper bound of the Gauss-Bonnet coupling constant”, JHEP 0905 (2009) 051, [arXiv:0903.2527 [hep-th]].

[22] Xian-Hui Ge, Sang-Jin Sin, Shao-Feng Wu, Guo-Hong Yang, ”Shear viscosity and instability from third order Lovelock gravity”, Phys. Rev. D80 (2009) 104019, [arXiv:0905.2675 [hep-th]].

[23] Xian-Hui Ge, Yoshinori Matsuo, Fu-Wen Shu, ”Viscosity Bound, Causality Violation and Instability with Stringy Correction and Charge”, JHEP 0810 (2008) 009, [arXiv:0808.2354 [hep-th]].

[24] X. O. Camanho, J. D. Edelstein, ”Causality constraints in AdS/CFT from conformal collider physics and Gauss-Bonnet gravity”, [arXiv:0911.3160 [hep-th]]. J. de Boer, M. Kulaxizi, and A. Parnachev, ”$AdS_7/CFT_6$, Gauss-Bonnet Gravity, and Viscosity Bound”, JHEP 1003 (2010) 097, [arXiv:0910.5347 [hep-th]].

[25] A. Buchel et al., ”Holographic GB gravity in arbitrary dimensions”, JHEP 1003 (2010) 111, [arXiv:0911.4257 [hep-th]].

[26] Fu-Wen Shu, ”The Quantum Viscosity Bound In Lovelock Gravity”, Phys.Lett.B685 (2010) 325, [arXiv:0910.0607 [hep-th]].

[27] X. O. Camanho, J. D. Edelstein, ”Causality in AdS/CFT and Lovelock theory”, [arXiv:0912.1944 [hep-th]].

[28] J. de Boer, M. Kulaxizi, and A. Parnachev, ”Holographic Lovelock Gravities and Black Holes”, JHEP 1006 (2010) 008, [arXiv:0912.1877 [hep-th]].

[29] X. O. Camanho, J. D. Edelstein, M. F. Paulos, ”Lovelock theories, holography and the fate of the viscosity bound”, JHEP 1105 (2011) 127, [arXiv:1010.1682 [hep-th]].
[30] Sera Cremonini, Kentaro Hanaki, James T. Liu, Phillip Szepietowski, "Black holes in five-dimensional gauged supergravity with higher derivatives", [arXiv:0812.3572 [hep-th]].

[31] Sera Cremonini, Kentaro Hanaki, James T. Liu, Phillip Szepietowski, "Higher derivative effects on eta/s at finite chemical potential", Phys.Rev.D80 (2009) 025002, [arXiv:0903.3244 [hep-th]]. Sera Cremonini, "The Shear Viscosity to Entropy Ratio: A Status Report", Mod. Phys. Lett.B 25 (2011) 1867-1888, [arXiv:1108.0677 [hep-th]].

[32] Robert C. Myers, Miguel F. Paulos, Aninda Sinha, "Holographic Hydrodynamics with a Chemical Potential", JHEP 0906 (2009) 006, [arXiv:0903.2834 [hep-th]].

[33] J. Sadeghi, B. Pourhassana, and A. R. Amani, "The effect of higher derivative correction on \( \eta/s \) and conductivities in STU model", [arXiv:1011.2291 [hep-th]].

[34] Carlos Hoyos-Badajoz, "Drag and jet quenching of heavy quarks in a strongly coupled N=2* plasma", JHEP 0909(2009) 068, [arXiv:0907.5036v3 [hep-th]]. D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, "Renormalization group flows from holography supersymmetry and a c-theorem", Adv. Theor. Math. Phys. 3 (1999) 363, [arXiv:hep-th/9904017].

[35] K. Pilch and N. P. Warner, "N = 2 supersymmetric RG flows and the IIB dilaton", Nucl. Phys. B 594 (2001) 209, [arXiv:hep-th/0004063].

[36] A. Buchel, A. W. Peet and J. Polchinski, "Gauge dual and noncommutative extension of an N = 2 supergravity solution", Phys. Rev. D 63 (2001) 044009, [arXiv:hep-th/0008076].

[37] K. Behrndt, A. H. Chamseddine and W. A. Sabra, "BPS black holes in \( \mathcal{N} = 2 \) five dimensional \( AdS \) supergravity", Phys. Lett. B442 (1998) 97.

[38] K. Behrndt, M. Cvetic and W. A. Sabra, "Non-extreme black holes of five dimensional \( \mathcal{N} = 2 \) AdS supergravity", Nucl. Phys. B553 (1999) 317.

[39] A. C. Cadavid, A. Ceresole, R. D’Auria, and S. Ferrara, "Eleven-dimensional supergravity compactified on Calabi-Yau three folds", Phys. Lett. B357 (1995) 76, [arXiv:hep-th/9506144].

[40] J. Sadeghi and B. Pourhassan, "Drag force of moving quark at the \( \mathcal{N} = 2 \) supergravity", JHEP12(2008)026, [arXiv:0809.2668 [hep-th]].

[41] J. Sadeghi, M. R. Setare, B. Pourhassan and S. Hashmatian, "Drag force of moving quark in STU background" Eur. Phys. J. C61 (2009) 527, [arXiv:0901.0217 [hep-th]].
[42] J. Sadeghi, M. R. Setare, and B. Pourhassan, ”Drag force with different charges in STU background and AdS/CFT”, J. Phys. G: Nucl. Part. Phys. 36 (2009) 115005. [arXiv:0905.1466 [hep-th]].

[43] K. Bitaghsir Fadafan, B. Pourhassan and J. Sadeghi, ”Calculating the jet-quenching parameter in STU background”, Eur. Phys. J. C (2011) 71:1785, [arXiv:1005.1368 [hep-th]].

[44] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, ”Energy loss of a heavy quark moving through $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma” JHEP 0607 (2006) 013, [arXiv: hep-th/0605158].

[45] C. P. Herzog, ”Energy loss of heavy quarks from asymptotically AdS geometries”, JHEP 0609 (2006) 032, [arXiv: hep-th/0605191].

[46] S. S. Gubser, ”Drag force in AdS/CFT”, Phys. Rev. D74 (2006) 126005.

[47] E. Nakano, S. Teraguchi and W. Y. Wen, ”Drag Force, Jet Quenching, and AdS/QCD”, Phys. Rev. D75 (2007) 085016.

[48] E. Caceres and A. Guijosa, ”Drag force in charged $\mathcal{N} = 4$ SYM plasma”. JHEP 0611 (2006) 077.

[49] J. F. Vazquez-Poritz, ”Drag force at finite ’t Hooft coupling from AdS/CFT”, [arXiv: hep-th/0803.2890]. A. Nata Atmaja and K.Schalm, ”Anisotropic Drag Force from 4D Kerr-AdS Black Holes”, [arXiv:1012.3800 [hep-th]].

[50] B. G. Zakharov, ”Radiative energy loss of high-energy quarks in finite-size nuclear matter and quark-gluon plasma”, JETP Lett. 65, (1997) 615, [arXiv:hep-ph/9704255]

[51] E. Caceres and A. Guijosa, ”On drag forces and jet quenching in strongly coupled plasmas”, JHEP0612 (2006) 068. Suphakorn Chunlen, Kasper Peeters and Marija Zamaklar, ”Finite-size effects for jet quenching”, [arxiv1012.4677 [hep-th]].

[52] F. L. Lin and T. Matsuo, ”Jet quenching parameter in medium with chemical potential from AdS/CFT”, Phys. Lett. B641 (2006) 45.

[53] Spyros D. Avramis, Konstadinos Sfetsos, ”Supergravity and the jet quenching parameter in the presence of R-charge densities”, JHEP0701(2007)065, [arXiv:hep-th/0606190].

[54] N. Armesto, J. D. Edelstein and J. Mas, ”Jet quenching at finite ’t Hooft coupling and chemical potential from AdS/CFT”, JHEP 0609 (2006) 039.

[55] J. D. Edelstein and C. A. Salgado, ”Jet quenching in heavy Ion collisions from AdS/CFT”, AIPConf.Proc.1031(2008)207-220, [hep-th/ 0805.4515]

[56] H. Liu, K. Rajagopal, U.A. Wiedemann, ”Calculating the jet quenching parameter from AdS/CFT”, Phys. Rev. Lett. 97 (2006) 182301.
[57] K. B. Fadafan, "Charge effect and finite 't Hooft coupling correction on drag force and Jet Quenching Parameter", \[arXiv:0809.1336\ [hep-th]].

[58] K. B. Fadafan, "$R^2$ curvature-squared corrections on drag force", JHEP 0812 (2008) 051, \[arXiv:0803.2777\ [hep-th]].

[59] Sachin Jain, "Universal thermal and electrical conductivity from holography", JHEP11(2010)092, \[arXiv:1008.2944\ [hep-th]].

[60] Tigran Kalaydzhyan and Ingo Kirsch, "Fluid-gravity model for the chiral magnetic effect", Phys.Rev.Lett. 106 (2011) 211601, \[arXiv:1102.4334\ [hep-th]].

[61] Vijay Balasubramanian, Finn Larsen, "On D-Branes and Black Holes in Four Dimensions", Nucl.Phys. B478 (1996) 199-208, [arXiv:hep-th/9604189].

[62] D. T. Son, A. O. Starinets, "Hydrodynamics of $R$-charged black holes", JHEP 0603 (2006) 052.

[63] James T. Liu, Phillip Szepietowski, "Higher derivative corrections to R-charged AdS$_5$ black holes and field redefinitions", Phys.Rev.D79 (2009) 084042, [arXiv:0806.1026 [hep-th]].

[64] M. Cvetic and S. S. Gubser, "Phases of R-charged black holes, spinning branes and strongly coupled gauge theories", JHEP 9904 (1999) 024, [arXiv:hep-th/9902195].

[65] M. Cvetic and S. S. Gubser, "Thermodynamic stability and phases of general spinning branes", JHEP 9907 (1999) 010, [arXiv:hep-th/9903132].

[66] Alex S. Miranda, C. A. Ballon Bayona, Henrique Boschi-Filho, and Nelson R. F. Braga, "Black-hole quasinormal modes and scalar glueballs in a finite-temperature AdS/QCD model", [arXiv:0909.1790 [hep-th]].

[67] A. Buchel, J. T. Liu and A. O. Starinets, "Coupling constant dependence of the shear viscosity in N=4 supersymmetric Yang-Mills theory", Nucl. Phys. B 707 (2005) 56, [arXiv:hep-th/0406264].

[68] Y. Kats and P. Petrov, "Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory", JHEP 0901 (2009) 044 [arXiv:0712.0743 [hep-th]].

[69] Sachin Jain, "Holographic electrical and thermal conductivity in strongly coupled gauge theory with multiple chemical potentials", JHEP 1003 (2010) 101, [arXiv:0912.2228 [hep-th]].

[70] PHENIX Collaboration, S. S. Adler et. al., "Nuclear modification of electron spectra and implications for heavy quark energy loss in Au+Au collisions at $s(NN)^{1/2} = 200$ GeV", Phys. Rev. Lett. 96 (2006) 032301. STAR Collaboration, M. Calderon de la Barca Sanchez et. al., "Open charm production from d+au collisions in star”, Eur. Phys.
J. C43 (2005) 187. **STAR** Collaboration, A. A. P. Suaide et. al., Charm production in the star experiment at rhic”, Eur. Phys. J. C43 (2005) 193.

[71] T. Matsuo, D. Tomino and W. Y. Wen, ”Drag force in SYM plasma with $B$ field from AdS/CFT”, JHEP0610 (2006) 055, [arXiv:hep-th/0607178].

[72] M. Chernicoff, J. A. Garcia and A. Guijosa, ”The Energy of a Moving Quark-Antiquark Pair in an $N=4$ SYM Plasma”, JHEP 0609 (2006) 068, [arXiv: hep-th/0607089].

[73] H. Liu, K. Rajagopal, U. A. Wiedemann, ”An AdS/CFT calculation of screening in a hot wind”, Phys.Rev.Lett.98(2007)182301, [arXiv: hep-ph/0607062].

[74] C. Krishnan, ”Baryon Dissociation in a Strongly Coupled Plasma”, JHEP 0812(2008)019, [arXiv:0809.5143 [hep-ph]].

[75] Francesco D’Eramo, Hong Liu, Krishna Rajagopal, ”Transverse Momentum Broadening and the Jet Quenching Parameter, Redux”, [arXiv:1006.1367 [hep-ph]].

[76] P. F. Kolb and U. Heinz, ”Hydrodynamic description of ultrarelativistic heavy-ion collisions”, [arXiv:nucl-th/0305084].

[77] Elena Caceres, Makoto Natsuume, Makoto Natsuume, ”Screening length in plasma winds”, JHEP 0610 (2006) 011, [arXiv: hep-ph/0607233].

[78] Shuang-Qing Wu, ”General Nonextremal Rotating Charged AdS Black Holes in Five-dimensional $U(1)^3$ Gauged Supergravity: A Simple Construction Method”, [arXiv:1108.4159 [hep-ph]].

[79] Shuang-Qing Wu, ”Two-charged non-extremal rotating black holes in seven-dimensional gauged supergravity: The single-rotation case”, [arXiv:1108.4158 [hep-ph]].

[80] Shuang-Qing Wu, ”General Rotating Charged Kaluza-Klein AdS Black Holes in Higher Dimensions”, Phys.Rev. D83 (2011) 121502(R), [arXiv:1108.4157 [hep-ph]].