Effect of magnetic field on the conducting fluids streaming in porous medium

P K Sharma
BUIT, Barkatullah University, Bhopal (M.P.)-462026 India

E-mail: pks_buit30@yahoo.com

Abstract. The Kelvin–Helmholtz instability of two superposed fluids of plasma streaming in opposite directions through porous medium has been investigated in a uniform two dimensional horizontal magnetic field. The medium is assumed to be conducting and incompressible. By applying the normal mode technique, the dispersion relation has been derived. From the dispersion relation the Kelvin Helmholtz instability condition is obtained. It is observed that the instability condition is modified due to the simultaneous presence of two dimensional horizontal magnetic field, porosity and suspended particles. It is found that the conditions for the Kelvin Helmholtz instability depends on the magnetic field, on the suspended dust particles and on the relaxation frequency of the particles. Numerical analysis is performed to show the effect of various parameters on the growth rate of Kelvin Helmholtz instability. It is also found that magnetic field, suspended dust particles and medium porosity has a stabilizing effect on the growth rate of unstable K-H mode.

1. Introduction
The Kelvin–Helmholtz (K-H) instability is a well-known instability that refers to the instability of plane interface between two superposed fluids flowing one over the other with relative velocity parallel to the interface. The hydromagnetic instabilities have wide applications in space plasma, astrophysical plasma and laboratory plasma. The flow problems of different fluids through porous medium have significant applications in petroleum production engineering, the chemical industry with regard to the recovery of crude oil from the pores of the reservoir rocks. Chandrasekhar [1] studied the classical K-H problems making use of single hydromagnetic equations.

The effect of suspended particle on the stability of superposed fluids might be of importance to industrial and chemical engineering. Saffman [2] had studied in detail, a dust gas in magnetohydrodynamics. Scanlon and Segel [3] have made a thorough study of the implication of suspended particles in hydromagnetics in the context of the Benard convection problem. Palaniswamy and Purushotum [4] have found the effect of fine dust to increase the region of instability in the shear flow of stratified fluids. Michael [5] has investigated the K-H instability of a dusty gas using Strokes drag force formula.

In the recent years, the wide applications of porosity in industrial, geophysical situation and chemical engineering has created considerable interest, particularly among geophysical fluid dynamics. Sharma and Spanos [6] have discussed the K-H and R-T instability of streaming fluids in a porous medium. Porosity is of use for the study of physical properties of comets, meteorites and interplanetary dust in astrophysical context McDonnel [7]. Prajapati et al. [8] have studied Kelvin Helmholtz and Rayleigh Taylor instability of two superposed magnetized fluids with suspended dust particles. Prajapati and Chhajlani [9] have
studied Kelvin Helmholtz and Rayleigh Taylor instability of streaming fluids with suspended dust particles flowing through porous media.

Keeping in mind the importance of suspended particles and porosity in modern technology and industries we have examined the problem of the Kelvin-Helmholtz instability of the plane interface separating two incompressible superposed conducting fluids in porous medium in the presence of two dimensional uniform magnetic field with suspended particles.

2. Equations of the problem
Consider two superposed perfectly conducting semi infinite homogeneous incompressible fluids flowing through porous medium of porosity $\varepsilon$ and medium permeability $k_1$. Suppose that a plane interface of discontinuity exist at $z=0$. Let the mixture of hydromagnetic fluid and suspended dust particles streaming together with unperturbed velocity $U(U,0,0)$ in the porous medium and two dimensional uniform horizontal magnetic field $H(H_x,H_y,0)$. Let $\rho$, $\mu$, $p$ and $u(u,v,w)$ denotes respectively the density, the viscosity, the pressure and velocity of the medium.

It is supposed that the suspended dust particles experience force $KN(V_U)$ in the fluid, where $K$ is a constant given by $K = 6 \pi a \mu$ (Stokes drag formula), ‘a’ being the particle radius. The linearized perturbation equations governing the fluids are written as follows:

$$\frac{\rho}{\varepsilon} \frac{\partial u}{\partial t} + \frac{1}{\varepsilon} (U.V)u = -\nabla \delta p + \frac{KN}{\varepsilon} (v - u) + \frac{\mu}{k_1} u + \frac{\mu}{4\pi} \left[ \nabla \times h \right] \times H + \left[ \nabla \times H \right] \times h, \tag{1}$$

$$\frac{\varepsilon}{\partial t} \delta p + (U.V)\delta p + (u.V)\rho = 0, \tag{2}$$

$$\varepsilon \frac{\partial h}{\partial t} = (H.V)u - (u.V)H \tag{3}$$

$$\nabla . u = 0, \tag{4}$$

$$\nabla . h = 0 \tag{5}$$

$$\left\{ \tau \left[ \frac{\partial}{\partial t} + \frac{1}{\varepsilon} (U.V) \right] + 1 \right\} v = u, \tag{6}$$

Where $\delta p$ and $\delta \rho$ are the perturbations in fluid pressure and density of fluid respectively and $\tau = m/6\pi a \mu$ denotes the relaxation time for the suspended dust particles.

Assuming the perturbation to be of the form $f(z) \exp(ik_x x + ik_y y + i\omega t)$, where $k_x$ and $k_y$ are the horizontal wave numbers $(k^2 = k_x^2 + k_y^2)$ and $n$ is the growth rate of the harmonic perturbations.

3. Dispersion relation
The following dispersion relation, governing the vertical component of velocity $w$ for the assumed configuration can be obtained by solving above equations and written as

$$\left[ i \left( n + \frac{k_x U}{\varepsilon} \right) + \frac{\omega}{k_1} + \frac{a_0}{\varepsilon} \frac{\left( in + i k_y \frac{U}{\varepsilon} \right)}{\tau \left( in + i k_y \frac{U}{\varepsilon} \right) + 1} \right] D(\rho Dw) - k^2 \rho w +$$

$$\left( \frac{(H_x k_y + k_y H_x)^2}{4\pi (\varepsilon n + i k_y U)} \right) \left( D^2 - k^2 \right) w = 0. \tag{7}$$

Where $a_0 = mN/\rho$ denotes the mass concentration of the particles and $D = d/dz$. Here we have assumed the same density of suspended particles in both the regions $z<0$ and $z>0$. 

2
Consider the configuration of two superposed fluids of densities $\rho_1$ (lower fluid) and $\rho_2$ (upper fluid) slipping past each other at horizontal interface $z=0$. In case of two regions of constant density, Eq.(7) reduces to

$$ (D^2 - k^2)w = 0. $$

After using the solutions of Eq.(4) for the upper and lower fluids and boundary conditions at the common interface of two fluids, we can obtain the following dispersion relation

$$ n^2 + \left[ \frac{2k_1}{\varepsilon} (\beta_1 U_1 + \beta_2 U_2) - i\frac{\varepsilon}{k_1} (\beta_1 \nu_1 + \beta_2 \nu_2) \right] + \left[ \frac{k_2^2}{\varepsilon^2} (\beta_1 U_1^2 + \beta_2 U_2^2) \right. $$

$$ - \left. \frac{i\varepsilon}{k_1} (\beta_1 U_1 + \beta_2 U_2) \right] + \frac{\alpha_1}{\varepsilon} (n + k U_1 / \varepsilon)^2 + \frac{\alpha_2}{\varepsilon} (n + k U_2 / \varepsilon)^2 + \frac{\alpha_3}{\varepsilon} (n + k U_2 / \varepsilon + 1) $$

$$ - 2(k \nu_2 + k \nu_2 / \varepsilon)^2 = 0, $$

Here $\alpha_1 = mN / \rho_1$, $\alpha_2 = mN / \rho_2$, $\beta_1 = \rho_1 / (\rho_1 + \rho_2)$, and $\beta_2 = \rho_2 / (\rho_1 + \rho_2)$.

For simplicity, we considered $V_{s2} = \frac{\mu_H}{4\pi(\rho_1 + \rho_2)}$

The above dispersion relation represents the influence of the suspended particles, dynamic viscosity and two directional horizontal magnetic field on Kelvin Helmholtz instability of two superposed fluids flowing through porous medium.

4. Discussion

In this problem we consider two superposed fluids of same densities and permeability in presence of suspended dust particles and flow velocities $U$ and $-U$ respectively with horizontal magnetic field.

For the simplicity of the problem we put $\alpha_1 = \alpha_2 = \alpha_0, \beta_1 = \beta_2 = 1/2, U_1 = U, U_2 = -U, \nu_1 = \nu_2 = \nu$ in dispersion relation (9) and get

$$ \sigma^2 + \sigma \left[ f_s(2 + \alpha_0) + \frac{\varepsilon \nu}{k_1} \right] + \sigma \left[ f_s^2(1 + \alpha_0) + \frac{2f_s \varepsilon \nu}{k_1} + 2(k \nu_1 + k \nu_2)^2 \right] $$

$$ + \sigma \left[ f_s^2 \frac{\varepsilon \nu}{k_1} + 4(k \nu_1 + k \nu_2)^2 f_s + \frac{k^2 U^2}{k_1} + \frac{f_s k^2 U^2}{\varepsilon^2} (\alpha_0 - 2) \right] $$

$$ - \left[ \frac{k^2 U^2}{\varepsilon^2} f_s^2(1 + \alpha_0) + \frac{k^2 U^2}{\varepsilon^2} \right] - 2(k \nu_1 + k \nu_2)^2 \left[ f_s^2 + \frac{k^2 U^2}{\varepsilon^2} \right] = 0, $$

The constant term in the dispersion relation (10) being negative means it will allow at least one real positive root leading to instability of the system.

The condition for the K-H instability of the system given by the constant term of Eq.(10) is

$$ \left. \frac{k^2 U^2}{\varepsilon^2} f_s^2(1 + \alpha_0) + \frac{k^2 U^2}{\varepsilon^2} \right] > 2(k \nu_1 + k \nu_2)^2 \left[ f_s^2 + \frac{k^2 U^2}{\varepsilon^2} \right] $$

11

The instability criterion (11), is therefore independent of permeability of the medium as well as the viscosity of the fluid. So the stability does not depend on either the permeability of the medium or the fluid viscosity, but depends on density and relaxation frequency of suspended dust particles. It is also clear that the inequality (11) depends on the Alfvén velocities and porosity of the medium. We also found that all these parameters have significant effect on the growth rate of unstable K-H mode.

In order to perform the numerical calculations on the growth rate of unstable K-H mode, we solve Eq.(10) numerically for positive roots in order to illustrate the influence of porosity, varying magnetic field and dust particle density. In fig.(1) and (2) we have plot the growth rate ($\sigma^*$) of unstable K-H mode vs relaxation frequency of dust particles ($f_s$) for the
Figure 1. The growth rate $\sigma^*$ (of unstable K-H mode) plotted against the relaxation frequency $f_s^*$ of suspended dust particles for the medium porosity $\varepsilon^*= 0.1, 0.2$ and $0.3$ with $\alpha=0.2$, $v^*=0.2$, $k_l=0.1$, $V_A=V_B=0.15$, $\theta=45^\circ$

Figure 2. The growth rate $\sigma^*$ (of unstable K-H mode) plotted against the relaxation frequency $f_s^*$ of suspended dust particles for the density of suspended dust particles $\alpha^*=0.2, 0.4$ and $0.6$ with $\varepsilon=0.2$, $v^*=0.2$, $k_l=0.1$, $V_A=V_B=0.15$, $\theta=45^\circ$

different values of medium porosity ($\varepsilon$) and dust particle density ($\alpha$). The numerical values of all the parameters are taken as arbitrary to study their effect on the growth rate. From the curves we find that growth rate of the system decreases as we increase the values of medium porosity and dust density, hence both the medium porosity and dust particle density have a stabilizing influence in presence of uniform two dimensional magnetic field.

5. Acknowledgement
The author is thankful to Prof. R.K Pandey, Director, BUIT and Prof. Nisha Dubey, Hon’ble V.C. Barkatullah University, Bhopal for their constant encouragement in this work. The author also expresses their sincere thanks to MPCST, Bhopal for providing Research Fellow and financial assistance in the research project.

6. References
[1] Chandrasekhar S 1991 *Hydrodynamic and Hydromagnetic Stabilit* (Clarendon Press Oxford)
[2] Saffman P G 1962 *J. Fluids Mech.* 13 120
[3] Scanlon J W and Segel L A 1973 *Phys. Fluids* 16 1573
[4] Palaniswamy V I and Purushotham G M 1991 *Phys. Fluids* 24 1224
[5] Michael D H 1965 *Proc. Comb. Phil. Soc.* 61 569
[6] Sharma R C and Spanos T J T 1982 *Can. J. Phys.* 60 1391
[7] McDonnel J A M 1978 *Cosmic Dust* (Toronto: John Wiley and Sons)
[8] Prajapati R P, Soni G D, Sanghvi R K and Chhajlani R K 2008 *Z. Naturforsch* 64a 455
[9] Prajapati R P and Chhajlani R K 2010 *J. Porous Media* 13(9) 765