Design, modeling and FEA analysis of internal symmetric and asymmetric involute spur gears

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Abstract. Recently, internal gears become widely used and highly required according to its ability to transmit large torque to weight ratio, high bending strength, smooth transmission, low noise, and low vibration. Therefore, internal gears are strongly recommended in particular “planetary gear transmission systems”. In this paper, internal involute spur gears are designed based on conventional approach of involute gears. Matlab is used to attain the points of internal gear and to get the main parameters of internal tooth profile, these points have been read by AutoCad to have two dimensional tooth profile. Finite element analysis using Ansys is achieved to analyze and compared the strength of three types of internal gears, Symmetric (20°-20°) that has been chose to be the standard case, Asymmetric (14.5°-35°) and Corrected Asymmetric (14.5°-35°). Tetrahedral type patch conforming method with element size1mm have been chose for fine meshing. The internal gear is fixed supported from the outer ring then, 6500 N.m torque is applied at the rotating axis of the pinion. The bending stresses are calculated based on Von-Mises theory using (FEA). Finally, this study is presented to demonstrate the superiority of internal asymmetric tooth and improve and optimize this significant type of gears.

1. Introduction
Gears are an extremely common component used in many applications. Therefore, gears and transmission designers face major challenges as a result of ever-increasing performance requirements on power density, compactness, reliability, efficiency, and noise. Some applications, the limitations of spacing, size, weight, and reduction ratio are required therefore, internal gears can meet easily these needs. Therefore, a number of researches have studied this type of gear to enhance and to get an optimal design of gear set. Ahmed A. Toman and Mohammad Q. Abdullah [1] introduced a developed analytical method to
calculate the stress concentration factor and assess the nominal and maximum tensile stress. The parametric equations for tooth fillets have also been rewritten to account for asymmetric fillet radiuses, asymmetric pressure angles, and profile shifting. Furthermore, using ANSYS, a numerical solution for evaluating the maximum fillet tensile stress and the combined tensile stress concentration factor was developed for the verification of the analytical method. Yang [2] An analysis of internal gear with asymmetric involute teeth was presented. The driven gear of an internal gear has been determined through the “double envelope concept”. Internal gears with helical teeth more preferable compared to internal gears with spur teeth. Muni and Muthuverappan [3] used direct and conventional method to designed asymmetric internal gears. They first obtained existence isograms of internal gears, then the synthesis of pinion cutters. They also defined the mathematical equations of internal gear teeth. Furthermore, FEA analysis and optimization were conducted to reach optimum design. Sekar and Ravivarman [4] provided a suggestion to improve the bending load capability of an asymmetric spur gear drive by having the same stresses in the asymmetric pinion and gear fillet regions, which may be accomplished by modifying the addendum height. For this updated addendum, the pinion and gear tooth proportion formulae have been calculated. The addendum adjustment factors required for a balanced maximum fillet stress condition were determined using FEM for various parameters. The bending load capacity of the simulated addendum adjusted asymmetric spur gear drives increased by 7% when compared to uncorrected asymmetric spur gear drives. Miryam B. Sánchez and Miguel Pleguezuelos [5] provided a new approach for calculating the strength of internal spur gear pairs. The meshing stiffness, which has been determined at any point along the path of contact, taking into account bending, shear, compressive, and Hertzian deflections, specifies the load sharing among pairs of teeth in simultaneous contact. C. Zaigang, et al. [6] investigates the effects of positive and negative profile shifting on the symmetric gear mesh stiffness. An analytical extended model was developed for the calculations of gears mesh stiffness taking into consideration the effect of profile shifting of gear tooth. A shift factor from (-0.33) to (0.99) were applied on external and internal involve symmetric gears. F. Karpat and S. Ekwaro-Osire [7] Internal gear design with various rim thicknesses and forms was examined. A static analysis of the gear bending stress and tooth displacement was also carried out. Maremuthu P. and Muthuverapan et al. [8] Using the finite element approach, the maximum fillet and contact stresses of asymmetric standard contact ratio spur gears were estimated (FEM). A.R. Rajesh, et al. [9] investigated the effects of the of profile shifting on standard spur gears bending stress, contact ratio, and the amount of load carrying. Results shows that it is possible to obtain a high contact ratio gear set by applying an optimum negative shift factors on both the pinion and the gear. Finally, some analysis and development of symmetric external and internal involute spur gear has been carried out in this study in order to improve the strength and durability of the gear when carrying loads.

2. Design and mathematical simulation
Internal gear teeth are placed in the inner ring and have a concave involute tooth profile, as is widely known. The addendum circle must be greater than the base circle because the involute lies outside the base circle of the internal gear; otherwise, the profile of top land will not be an involute. The design parameters for both external and internal gear are shown in table 1.
### Table 1. External and Internal gear parameters

| Case No. | Pinion | Internal gear |
|----------|--------|---------------|
|          | α_c   | α_d | h_a | h_d | r_{fc} | r_{fd} | x_c | x_d | m_o | N_1 | b  |
| 1        | 20°   | 20° | m_o | 1.157 m_o | 0.3 m_o | 0.3 m_o | 0   | 0   |      | 5   | 10 |
| 2        | 35°   | 14.5° | m_o | 1.157 m_o | 0.06 m_o | 0.209 m_o | 0   | 0   | 5    | 16  | 10 |
| 3        | 35°   | 14.5° | m_o | 1.157 m_o | 0.06 m_o | 0.209 m_o | 0   | +0.5|      |      |    |

#### 2.1. Generation of asymmetric external tooth profile

In a standard symmetric gear tooth, the pressure angle on both sides of involute profile is the same.

![Involute profile of symmetric spur gear tooth.](image)

From figure 1, it is clearly shown

\[
\text{inv } \phi - \text{inv } \alpha = \delta - \theta
\]
where $\delta = \frac{p_c}{r_p}$, $p_c = \pi \cdot m$ and $r_p = \frac{mN_1}{2}$, thus $\delta = \frac{\pi}{2N_1}$
then
$$\theta = \frac{\pi}{2N_1} + \text{inv } \alpha - \text{inv } \phi$$  \hspace{1cm} (2)

From figure 1, it can be deduced that
$$r_{bc} = r_p \cdot \cos \alpha \text{ and } r_{bc} = r \cdot \cos \phi$$
$$\phi = \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right)$$  \hspace{1cm} (3)
$$\theta = \frac{\pi}{2N_1} + \text{inv } \alpha - \text{inv } \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right)$$  \hspace{1cm} (4)

Involute angle by definition is $\text{inv } \alpha = \tan \alpha - \alpha$, therefore
$$\theta = \frac{\pi}{2N_1} + \tan \alpha - \alpha - \left( \tan \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right) \right) - \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right)$$  \hspace{1cm} (5)

Equation (5) shows angle $\theta$ at a given radius $r$, the radius $r$ is varies between base circle radius $r_b$ to addendum circle radius $r_a$. It defines location of points on the involute profile as shown in figure 1.
Finally, the coordinates $(x_i,y_i)$ of the involute profile are given by;
$$x_i = r_i \cdot \cos \theta_i$$
$$y_i = r_i \cdot \sin \theta_i$$  \hspace{1cm} (6)

Hence, $\theta_i$ is varied between $\theta_b \leq \theta \leq \theta_c$ for loaded side and, $-\theta_b \leq \theta \leq -\theta_c$ for unloaded side respectively.

2.2. Generation of asymmetric external tooth profile
The mathematical expression to the tooth profile of asymmetric external gear will be derived as it follows
Equation (10) gives the tip thickness on the drive side at any radius \( r \). Thickness at pitch circle radius, becomes equal to \( \alpha \) and so \((\text{inv } \phi - \text{inv } \alpha)\) becomes zero, and that means thickness at pitch circle radius does not depend on pressure angle, and it is constant at any pressure angle at the radius of pitch circle.

While \( r_{bc} = r_p \cdot \cos \alpha \) and \( r_{bc} = r \cdot \cos \phi \), then

\[
\phi = \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \tag{11}
\]

Thus

\[
\theta = \frac{\pi}{2 \cdot N_1} + \text{inv } \alpha - \text{inv} \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right) \tag{12}
\]

Since, by definition \((\text{inv } \alpha = \tan \alpha - \alpha)\), thus

\[
\theta = \frac{\pi}{2 \cdot N_1} + \tan \alpha - \alpha - \left[ \tan \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right) \right] - \left( \cos^{-1} \left( \frac{r_p}{r} \cdot \cos \alpha \right) \right) \tag{13}
\]
Equation (13) gives angle $\theta$ which is the angle with respect to $y$-axis at any radius of involute profile for given pressure angle.

Based on figure 2, the angle $\theta$ varied between $\theta_{bc}d \leq \theta_i \leq \theta_t d$ and $-\theta_{bc}c \leq \theta_i \leq -\theta_t c$, then the coordinates $(x_i, y_i)$ of the involute profile is given by;

$$x_i = r_i \cdot \cos \theta_i$$
$$y_i = r_i \cdot \sin \theta_i$$

Where $\theta_i$ is

$$\theta_i = \frac{\pi}{2x} + \tan \alpha - \alpha - \left(\tan \left(\cos^{-1}\left(\frac{r_p}{r_i} \cdot \cos \alpha\right)\right) - \left(\cos^{-1}\left(\frac{r_p}{r_i} \cdot \cos \alpha\right)\right)\right)$$

Where,

$\theta_{bc}d \leq \theta_i \leq \theta_t d$ and $\alpha = \alpha_d$ for drive side profile and, $-\theta_{bc}c \leq \theta_i \leq -\theta_t c$ and $\alpha = \alpha_c$ for coast side profile.

2.3. Generation of symmetric internal tooth profile

Assuming that point $M (x, y)$ is located on the involute tooth profile of a symmetric involute internal gear, the coordinates $(x, y)$ of point $M$ in the Cartesian coordinate system can be calculated as follows.

The trajectory of point $M$ on the line (AM) will be the involute of the circle when the line BM rolls along a circle. This circle is known as the base circle, and its radius is $r_b$, while the line BM is known as the generating line, and the angle $\theta_M$ is known as the evolving angle of point M. Assume that $r_M$ is the length of the line connecting point M on the involute to point O. The pressure angle $\alpha_M$ of involute at point M is formed when the involute engages with its conjugate tooth profile at point M, forming an angle between the force direction of point M and the velocity direction of point M, $\angle BOM$ is the symbol for it.

$$\cos \alpha_M = \frac{r_b}{r_M}$$

Due to

$$\tan \alpha_M = \frac{\overline{BM}}{r_b} = \frac{\overline{AB}}{r_b} = \frac{r_p(\alpha_M + \theta_M)}{r_b} = \alpha_M + \theta_M$$

$$\text{(16)}$$
The available

$$\theta_M = \tan \alpha_M - \alpha_M$$  \hspace{1cm} (17)

The involute function can then be expressed using \( inv\alpha_M \), as seen below.

$$inv\alpha_M = \theta_M = \tan \alpha_M - \alpha_M$$  \hspace{1cm} (18)

while the involute profile's polar coordinate equation is

$$r_M = r_b / \cos \alpha_M$$  \hspace{1cm} (19)

For ease of calculation and sketching, equation (19) can be transformed to a Cartesian coordinate equation, so let takes \( \angle AOB \) as a parameter, due to

$$\varphi_M = \alpha_M + \theta_M$$  \hspace{1cm} (20)

Simultaneous equations (19) and equation (20), then the available is

$$\varphi_M = \tan \alpha_M$$  \hspace{1cm} (21)

As a result, the Cartesian coordinate M \((x, y)\) in the xoy plane is:

$$x_M = r_M \cos \theta_M = \frac{r_b}{\cos \alpha_M} \cos (\varphi_M - \alpha_M)$$

$$= \frac{r_b}{\cos \alpha_M} (\cos \varphi_M \cos \alpha_M + \sin \varphi_M \sin \alpha_M)$$

$$= r_b (\cos \varphi_M + \varphi_M \sin \varphi_M)$$  \hspace{1cm} (22)

$$y_M = r_M \sin \theta_M = \frac{r_b}{\cos \alpha_M} \sin (\varphi_M - \alpha_M)$$

$$= \frac{r_b}{\cos \alpha_M} (\sin \varphi_M \cos \alpha_M - \cos \varphi_M \sin \alpha_M)$$

$$= r_b (\sin \varphi_M - \varphi_M \cos \varphi_M)$$  \hspace{1cm} (23)

The involute function's Cartesian coordinates will then be expressed as

$$x_M = r_b (\cos \varphi_M + \varphi_M \sin \varphi_M)$$

$$y_M = r_b (\sin \varphi_M - \varphi_M \cos \varphi_M)$$  \hspace{1cm} (24)

The above formula can be written in spatial coordinates as follows:

$$x_M = r_b (\cos \varphi_M + \varphi_M \sin \varphi_M)$$

$$y_M = r_b (\sin \varphi_M - \varphi_M \cos \varphi_M)$$

$$z_M = 0$$  \hspace{1cm} (25)

While this derivation related to symmetric internal gear then \( \alpha_d = \alpha_c = \alpha \), and \((r_b)\) In eq. 25 can be expressed as:

$$r_b = r \cos \alpha$$  \hspace{1cm} (26)
2.4. Asymmetric internal tooth profile generation
To obtain the coordinates of asymmetric internal tooth profile, that can be achieved by following equations of symmetric internal tooth profile starting from equation (15) to equation (25) supposing this derivation is related to the drive side of the internal gear then \( r_b \) at the working (loaded) side can be expressed as:

\[
r_b = r \cos \alpha_d
\]  

Similarly, on the non-working (unloaded) side of asymmetric internal gear, the Cartesian coordinate equation is

\[
x_m = r_b \left( \cos \varphi_m + \varphi_m \sin \varphi_m \right)
\]
\[
y_m = r_b \left( \sin \varphi_m - \varphi_m \cos \varphi_m \right)
\]
\[
z_m = 0
\]  

And \( r_b \) of the non-working (unloaded) side can be expressed as:

\[
r_b = r \cos \alpha_c
\]

To unify the coordinate system, the coordinate origin is the center of the base circle, the line between the center of the tooth thickness of the reference circle and the origin is the positive direction of the Y-axis, and the positive direction of the X-axis is horizontal to the right (the coordinate transformation of involute function). Create a set of converting coordinates. as shown in figure 4 then, the equation of point \( M \) is

\[
x_M = r_M \sin \varnothing
\]
\[
y_M = r_M \cos \varnothing
\]
\[
z_M = 0
\]

Where the angle \( \varnothing \) between the y-axis and the line (OM) is in equation (30).
Figure 4. Coordinate transformation.

Because the tooth thickness \( t_c \) at point Q and the y-axis is circular,

\[
t_c = r(\emptyset - (\text{inv} \alpha - \text{inv} \alpha_M))
\]

(31)

Then, the available in the expression of \( \emptyset \) is

\[
\emptyset = \frac{\pi}{2N_2} - \text{inv} \alpha_M + \text{inv} \alpha
\]

(32)

Simultaneous equations (30), (31), and (32), then coordinates of point \( M \) will be

\[
\begin{align*}
x &= \frac{r_b}{\cos \alpha_M} \sin \left( \frac{\pi}{2N_2} + \text{inv} \alpha - \text{inv} \alpha_M \right) \\
y &= \frac{r_b}{\cos \alpha_M} \cos \left( \frac{\pi}{2N_2} + \text{inv} \alpha - \text{inv} \alpha_M \right) \\
z &= 0
\end{align*}
\]

(33)

A computer program based on the above mathematical simulation has been built using a Matlab R2018b to generate the points of symmetric and asymmetric for both external and internal gear teeth, then these points are read in AutoCad 2020 program through SCR file to draw 2D then 3D models.

Figure 5. Flow chart of mathematical simulation.
Figure 6. Tooth profile of pinion $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ$ read by AutoCad.

Figure 7. Tooth profile coordinates of internal gear $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ$ read by AutoCad.

Figure 8. 2D draw of asymmetric internal gear $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ$.

Figure 9. 3D draw of asymmetric internal gear $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ$.

3. Design and mathematical simulation

3.1. Contact stress analysis

The contact stress analysis has done based on the modified Hertz equation below [10].

$$\sigma_c = \sqrt{\frac{2}{\pi} \frac{F_n}{u b m_o N_1 \tan \alpha_w \cos \alpha_d \left(1 - \frac{\nu_1^2}{E_2} + \left(1 - \frac{\nu_2^2}{E_1}\right)\right)}}$$  \hspace{1cm} (34)

This equation evaluates the gear tooth contact stress for both standard and non-standard spur gears in terms of the applied normal force, gear design parameters, and the properties of the design gear material.

3.2. Bending stress analysis

Bending stress analysis is calculated analytically depending on [11]. where for symmetric spur gear is

$$\sigma_{bending} = \frac{F_t}{b \cdot m_o Y}$$  \hspace{1cm} (35)

$$Y = \frac{1}{6m_o}\left(\frac{l^2}{h}\right)$$  \hspace{1cm} (36)
And for asymmetric spur gear, the bending stress is calculated based on [12].

\[ \sigma_{bending}' = \frac{F_t}{b \cdot m_o \cdot Y'} \]  
(37)

\[ Y' = \frac{1}{3m_o} \left[ \frac{(t_u+t_l)^2}{2h - \tan \beta_l (t_u-t_l)} \right] \]  
(38)

4. Finite element analysis (FEA)

FEA considers as the most accurate method to determining stresses and deflection information. Ansys package 18.2 is used as a computer aided stress analysis (contact & bending). After constructing the 3D models for the three study cases in AutoCad, it was transferred to Ansys with IGES file extension. In order to decrease time analysis only three teeth of pinion are used with three teeth of internal gear for contact stress investigation while the bending stress analysis used on one the tooth of pinion only since the ratio between the internal gear to the pinion is (2:1) according to the table 1, and that means the pinion will be the critical case of bending stress to be investigated and studied. A Polycarbonate material is used \( E = 2.8 \text{ GPa} \) and Poisson’s ratio \( \nu = 0.38 \) and the discretization of converting the whole of 3D model into very small elements size (1mm) chose to be Tetrahedral solid elements and enhanced using Patch Conforming Method. Frictionless support is used at the inner of the pinion, while internal gear is fixed supported at the outer surface and frictionless supported to the lateral surface. For contact stresses analyzing, frictional surface to surface has been used with 8226 number of elements for the target body (Internal gear) and 7365 for the contact body (Pinion) and 11573 total number of elements to analyze bending stresses while converging test has been done to select the best elements number. 6500 N.mm is used as an input torque applied to the pinion around its axis while the internal gear rotation is prevented. Figure 10 below shows the meshed finite element model of case (1)

Figure 10. FE model of symmetric internal gear with its pinion \( \alpha_d = 20^\circ, \alpha_c = 20^\circ \)

Figure 11. Boundary conditions for FEA of case (1)
5. Results

It is obvious from the two tables below that the maximum stress with different pressure angles and under different normal load shows the variation between the analytical and FEA for the three cases of study.

| Table 2. Analytical and numerical contact stress results |
|-----------------------------------------------|
| **Case** | **σ_c ans.** | **σ_c anl.** |
| 1        | 29.873       | 35.7053     |
| 2        | 39.577       | 41.0610     |
| 3        | 24.125       | 31.9611     |

| Table 3. Analytical and numerical bending stress results |
|-----------------------------------------------|
| **Case** | **σ_b ans.** | **σ_b anl.** |
| 1        | 13.711       | 14.8499     |
| 2        | 13.311       | 13.3895     |
| 3        | 10.824       | 11.6631     |

Both analysis results (analytically and numerically) show that the pressure angle for the three cases has a major effect on the results of bending stress, it is easily demonstrated that increasing the pressure angle will increase the gear's bending strength within a certain range. The maximum stress in the cross section of the gear tooth on the unloaded side decreases with increasing the pressure angle because the tooth thickness at the weakest part increases, according to the finite element analysis. When the pressure angle of the loaded side varies from 14.5 to 25, then the maximum stress at the unloaded side reduced about 37 % as in the 3rd case which is positively corrected by $\alpha = +0.5$ for the pinion and negatively corrected by $\alpha = -0.5$ for the internal gear. Furthermore, the contact stress results reveal that the pressure angle is varied, resulting in a difference in contact stress. It has been demonstrated that increasing the working pressure angle increases the contact of the gear's teeth at loaded side with a certain range. Therefore, the 2nd case of the study which has a pressure angle $\alpha_d = \alpha_w = 14.5^\circ$ gives the maximum value of contact stress 39.577 MPa while the 3rd case that is positively corrected which has a pressure angle at loaded side $\alpha_d = 14.5^\circ$ gives a minimum value 24.125 MPa since the working pressure angle become $\alpha_w = 24.32^\circ$ after correction thus the maximum contact stress enhance about 39 % compared with the 2nd case of the study.
Figure 12. Symmetric spur gear contact stress $\alpha_d = 20^\circ, \alpha_c = 20^\circ$.

Figure 13. Asymmetric spur gear contact stress $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ, x_d = 0$.

Figure 14. Contact stress of corrected asymmetric spur gear $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ, x_d = 0.5$.

Figure 15. Symmetric spur gear bending stress $\alpha_d = 20^\circ, \alpha_c = 20^\circ$. 
Figure 16. Asymmetric spur gear bending stress $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ, x_d = 0$.

Figure 17. Bending stress of corrected asymmetric spur gear $\alpha_d = 14.5^\circ, \alpha_c = 35^\circ, x_d = 0.5$.

Figure 18 show the values of contact stress with range $x = 0$ to $x = 0.5$, it should be noticed that $x = 0.5$ has been selected in this paper since the number of tooth of the pinion is 16 as it shown in table 1, while figure 20 clearly show that with increasing number of teeth, the contact stress for the three cases of study are also decreased. Also, In Fig.22 show a bending stress comparison between a standard case $\alpha_c = 14.5^\circ, \alpha_d = 14.5^\circ$ and asymmetric corrected case $\alpha_c = 35^\circ, \alpha_d = 14.5^\circ$, it can be easily noticed the high variation bending stress results between the two case since the bending stress decreased about 26%.

Figure 18. Contact stress under different correction factors.

Figure 19. Contact stress with respect to the working pressure angles.
In figure 22 show a bending stress comparison between a standard case $\alpha_c = 14.5^\circ, \alpha_d = 14.5^\circ$ and asymmetric corrected case $\alpha_c = 35^\circ, \alpha_d = 14.5^\circ$, it can be easily noticed the high variation bending stress results between the two case since the bending stress decreased about 26%, while the results of contact stress of FEA compared with the analytical results are shown in figure 24. the study is corresponding to the same data that have been presented in table 1. Figure 24 show a good agreement between analytical and FEA with acceptable discrepancies around both limits of the interval of a single tooth contact of pinion and internal gear. Also, when the whole load acting at the tip of a single pair teeth contacts a spur internal gear, the outcome is always bending stress. The theoretical results have been compared with the FEA as it shown in Fig.25. The results also show a good agreement since there is a slight difference between them. As a result, the critical stress calculation based on the modified ISO nominal stress calculation methodologies utilized in this study will yield accurate enough values for preliminary calculations or standardization reasons.
6. Conclusion

The research contents can be listed as it follows;

i. The research object is optimizing strength analysis of asymmetric internal spur gear $\alpha_c = 35^\circ, \alpha_d = 14.5^\circ$ through positively correcting the loaded side of the pinion by 0.5 and negatively with the same value for internal spur gear. Also, the principle of the tooth profile forming for both pinion and internal gear has discussed in detail.

ii. A 2D then a 3D model for the three cases of research is build up then analyzed the correlation between the maximum pressure, correction factor, face width, number of teeth with respect to contact and bending stress. The pressure angle limit range of asymmetric gear is significantly larger than that of conventional gear. The contact strength is modeled then simulated, the results showed that the maximum stress at loaded side decreased with increasing pressure angle, but this increasing is limited.

7. Nomenclature

- $b$: Face width of the tooth
- $E$: Modulus of elasticity
- $F_t$: Tangential force
- $h$: Bending moment arm
- $m$: Normal module
- $N_1$: Number of teeth of the pinion
- $N_2$: Number of teeth of the internal gear
- $r_p$: Pitch radius
- $r_b$: Base radius
- $t_d$: Tooth thickness of the drive side
- $t_c$: Tooth thickness of the coast side
- $u$: Speed ratio
- $\nu$: Poisson’s ratio
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