PERFORMANCE ANALYSIS OF INTEGRATED GPS/GLONASS CARRIER PHASE-BASED POSITIONING

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ABSTRACT  Due to the different signal frequencies for the GLONASS satellites, the commonly-used double-differencing procedure for carrier phase data processing cannot be implemented in its straightforward form, as in the case of GPS. In this paper a novel data processing strategy, involving a three-step procedure, for integrated GPS/GLONASS positioning is proposed. The first is pseudo-range-based positioning, that uses double-differenced (DD) GPS pseudo-range and single-differenced (SD) GLONASS pseudo-range measurements to derive the initial position and receiver clock bias. The second is forming DD measurements (expressed in cycles) in order to estimate the ambiguities, by using the receiver clock bias estimated in the above step. The third is to form DD measurements (expressed in metric units) with the unknown SD integer ambiguity for the GLONASS reference satellite as the only parameter (which is constant before a cycle slip occurs for this satellite). A real-time stochastic model estimated by residual series over previous epochs is proposed for integrated GPS/GLONASS carrier phase and pseudo-range data processing. Other associated issues, such as cycle slip detection, validation criteria and adaptive procedure(s) for ambiguity resolution, is also discussed. The performance of this data processing strategy will be demonstrated through case study examples of rapid static positioning and kinematic positioning. From four experiments carried out to date, the results indicate that rapid static positioning requires 1 minute of single frequency GPS/GLONASS data for 100% positioning success rate. The single epoch positioning solution for kinematic positioning can achieve 94.6% success rate over short baselines (<6 km).

1 Introduction

Real-time kinematic GPS precise positioning has been playing an increasing role in both surveying and navigation, and has become an essential tool for precise relative positioning. However, reliable and correct ambiguity resolution depends on observations upon a large number of GPS satellites. This constrains its performance, making it difficult to apply in areas where the number of visible satellites is limited.

The most obvious way to increase the number of tracking satellites is to integrate the GPS and GLONASS systems. Due to the different signal frequencies for the different GLONASS satellites, the commonly used DD procedure for carrier phase data processing cannot be implemented in its simplest form, as is done with GPS. To solve this problem several modelling methods have been proposed to date. Three general classes of integrated GPS and GLONASS functional models had been developed.
in the last decade. The first class is to introduce the known relative clock parameter, which is estimated by using pseudo-range measurements, into the GLONASS DD carrier phase observation equations (Pratt, et al., 1998; Leick, 1998). The second class is to estimate the clock parameters, baseline vectors and ambiguity parameters together in both the carrier phase and pseudo-range observation equations (Zhodzishsky, 1998; Koslov, 1997; Wang, 1998; Han, et al., 1999). However, for these two kind of techniques, the ambiguity resolution and positioning results are seriously affected by the remaining clock biases and the GLONASS pseudo-range inter-channel biases. The third method involves a two-level procedure or an iterative search approach to process DD observations (expressed in metres) without the receiver clock value (Wang, 2000; Habrich, et al., 1999). For this third class of techniques, if the ambiguity sets can be fixed correctly, the positioning results are not affected by the clock bias. Unfortunately, in a two-level search approach any accepted wrong SD GLONASS ambiguity value at the GLONASS reference satellite can cause systematic model errors that may affect DD ambiguity resolution. The iterative search approach is only suitable for a long session static positioning. A detailed review of a variety of mathematical modelling options for integrated GPS and GLONASS can be found in Wang, et al. (2001). In this paper three general classes of the integrated GPS and GLONASS functional models have been optimally integrated so that: 1) the ambiguity resolution is insensitive to the remaining clock biases and inter-channel biases, and 2) reliable and precise positioning results will not be affected by residual receiver clock biases.

High quality estimation results from the application of the Least Squares estimation technique requires specification of the optimal functional and associated stochastic model. The stochastic model is dependent on the choice of the functional model. Hence, for a different choice of functional model, the stochastic characteristics of unmodelled errors will be different, and the stochastic model must reflect this. According to the assumption that the accuracy of the one-way observations depends on signal-to-noise (Gianniou & Groten, 1996), or satellite elevation (Jin, 1995; Han, 1997), some approximate formulas to compute the variance-covariance matrix for DD observations have been proposed. However, constant coefficients for certain types of GPS receivers, which are empirically estimated from observations collected under specific observing conditions, are probably not well suited for other measurement environments. Due to the high temporal correlation of observations, the compensated method was proposed to estimate a scale factor in the stochastic model, using previous data collected over a certain period (Han, 1997). With this method we could derive a more realistic stochastic model, and hence increase the reliability of the ambiguity resolution and the positioning accuracy. However, it does not take account of the observations' spatial correlation, which would need refining of the variance-covariance matrix. The construction of the variance-covariance matrix can be carried out by using the residual series over the previous epochs, when the integer ambiguities are fixed correctly. On the basis of the estimated variance-covariance matrix of the residuals, and the relationship between the residuals and the observation errors, an improved variance-covariance matrix of the observations can be derived. Hence, the real-time stochastic model derived in this way will not only reflect the stochastic characteristics of the observation errors, but also the remaining biases due to multipath, atmospheric delay, inter-channel biases and orbital errors. This method could be used for different types (SD, DD, in metres or cycle units) of carrier-phase and pseudo-range observation combinations.

In this paper a novel data processing strategy for integrated GPS/GLONASS positioning, in combination with the real-time estimated stochastic model described above, is proposed. The performance of this data processing strategy will be demonstrated via examples of rapid static positioning and kinematic positioning.

2 Functional modeling strategy

The SD carrier phase observable between re-
receivers can be expressed as (e.g. Leick, 1998):
\[
\lambda_i^k \Phi_i^k = \rho_i^k + \lambda_i^k N_i^k - c \cdot d_{tk} - \frac{I_i^k}{(f_i^k)^2} + T_i^k + \epsilon_i^k
\]
(1)
where the subscripts \(k\) and \(l\) identify the ground receivers, and superscript \(p\) denotes the satellite. \(\Phi_i^k\) is the SD carrier phase observable expressed in units of cycles. \(\lambda_i^k\) and \(f_i^k\) are the wavelength and frequency of the \(L_1\) carrier wave. \(N_i^k\) is the SD integer ambiguity; \(d_{tk}\) is the difference between the two receiver clock biases in seconds; \(c\) is the speed of light; \(\frac{I_i^k}{(f_i^k)^2}\) is the SD ionospheric delay, where \(I\) is a function of the Total Electron Content; \(T_i^k\) is the SD tropospheric delay; and \(\epsilon_i^k\) is the carrier phase observation noise and remaining errors.

Eq. (1) is valid for GPS and GLONASS carrier phase measurements. However, \(L_1\) GPS signals have the same frequencies for all satellites, while GLONASS signals have different frequencies for different satellites.

2.1 Double-differenced observables

The DD observable in units of metres can be formed as:
\[
\lambda_i^k \Phi_i^k - \lambda_l^l \Phi_l^l = \rho_i^k + \lambda_i^k N_i^k - \lambda_l^l N_l^l - \frac{I_i^k}{(f_i^k)^2} - \frac{I_l^l}{(f_l^l)^2} + e_i^k
\]
(2)
It could be seen that data processing for integrated GPS and GLONASS system becomes more complicated because of the different frequencies for the GLONASS satellites. The GLONASS DD observables have more ionospheric delay than the GPS DD observables. However it could still be ignored if the distance between the two receivers is short enough.

Eq. (2) could then be rewritten as:
\[
\lambda_i^k \Phi_i^k - \lambda_l^l \Phi_l^l = \rho_i^k + \lambda_i^k N_i^k - \lambda_l^l N_l^l + e_i^k
\]
(3)
It is clear that for GPS carrier phase measurements the third term on the right side of Eq. (3) will disappear. For GLONASS carrier phase measurements the third term, or the SD integer ambiguity for the reference satellite, must be estimated before the DD integer ambiguities can be computed. The remaining errors from the third term could cause systematic model errors and may result in wrong DD ambiguity resolution, and hence degraded positioning accuracy.

An alternative approach is to form the DD observable after the SD observables are expressed in units of cycles:
\[
\Phi_i^k = \left(\frac{f_i^k}{c} \rho_i^k - \frac{f_i^k}{c} \epsilon_i^k\right) - \left(\frac{I_i^k}{(c f_i^k)^2} - \frac{I_i^k}{c T_i^k} - \frac{f_i^k}{c T_i^k}\right) - (f_i^k - f_l^l) \cdot d_{tk} + N_i^k + \epsilon_i^k
\]
(4)
The differenced receiver clock bias cannot be eliminated in Eq. (4). The second term (ionospheric delay) and the third term (tropospheric delay) will become slightly larger than in the case when both the frequencies are the same. By GPS and GLONASS pseudo-range measurements, the difference between the two receiver clock biases can be estimated, then it could be used to correct the fourth term for ambiguity resolution purposes. However, this receiver clock bias will significantly degrade the positioning accuracy.

2.2 An integrated three-step procedure

Due to the different frequencies for the different GLONASS satellites, the relative receiver clock bias \(\Delta dT\) cannot cancel in the GLONASS DD carrier phase (Eq. (4)). To solve this problem, the SD pseudo-range observations should be involved. Although the pseudo-range-derived receiver clock is good enough for ambiguity resolution purposes, it still affects the positioning results. Therefore Eq. (3) is used to determine the positioning results after the integer ambiguities are fixed. The third term on the right side of Eq. (3) will be considered as an additional unknown parameter before a cycle slip occurs on the GLONASS reference satellite. Hence the data processing procedure can be summarised as follows.

Step 1: The DD GPS pseudo-range observables and the SD GLONASS pseudo-range observables are used:
\[
P_{GPS}^G = \rho_i^k + \epsilon_i^k
\]
(5)
\[
P_{GLONASS}^G = \rho_i^k + c \cdot d_{tk} + \epsilon_i^k
\]
(6)
where \(P_{GPS}^G\) and \(P_{GLONASS}^G\) are the DD GPS pseudo-range observable and the SD GLONASS pseudo-range observable respectively. Why are DD GPS pseudo-range observables used rather than SD GPS pseudo-ranges? When the difference of the two receiver clocks is introduced into the GPS SD obser-
vation equation, the inter-channel bias between the GPS satellite and the GLONASS satellite may be introduced in order to derive equivalent results, making the data processing more complicated. This model was also identified as an optimal functional model by Rapoport (1997).

In this step, the difference in the two receiver clock biases, the initial coordinates and their variance-covariance matrix, can be derived for the ambiguity resolution process in the next step.

Step 2: DD GPS and GLONASS carrier phase observables in units of cycles, e.g. Eq. (4), will be used for ambiguity resolution. The fourth term on the right side of Eq. (4) will disappear for GPS measurements due to the fact that the same frequency is used for different GPS satellites. However, this term must be corrected by using the difference of the two receiver clock biases.

The frequency difference between GLONASS signals is smaller than 12.9 MHz for L1 observations, and less than 10.1 MHz for L2 observations. Hence the difference in the two receiver clock biases can be expected to be less than 10 ns (approximately 3 metres), and therefore this term can be corrected at the 0.1 cycle level. For ambiguity resolution purposes, the bias could be ignored without significant impact on the reliability. However, this error cannot be ignored when Eq. (4) is used to derive the positioning results. Furthermore, this term could not be considered as the same unknown parameter for different epochs.

Step 3: Although the integer ambiguity set could be determined in Step 2 by using Eq. (4), the positioning results will be affected by the receiver clock biases. However, the DD carrier phase observables in units of metres, e.g. Eq. (3), where the receiver clock biases are removed and the integer ambiguity set determined in Step 2, could be used. In this way the third term on the right side of Eq. (3) could be considered as an additional unknown parameter over different epochs before a cycle slip occurs on the GLONASS reference satellite.

This three-step procedure is an integrated way for processing combined GPS and GLONASS data, which takes advantage of features of the different DD combinations. It should be mentioned that: 1) the double-differencing operator is applied to the GPS measurements only or the GLONASS measurements only, rather than between GPS and GLONASS; and 2) cycle slip detection at the GLONASS reference satellite is required.

If a mixed double-differencing operator between GPS and GLONASS is used, the coefficient of the clock term will increase dramatically to a value between 26.6 MHz and 39.5 MHz for L1 observations, and from 18.4 MHz to 28.5 MHz for L2 observations. It is easily seen that the clock error effect on ambiguity resolution in the mixed formulation is more serious than in the separated formulation. The inter-channel bias must be accounted for in some way if the difference between GPS and GLONASS measurements is formed. Although inter-channel biases exist for measurements from different GLONASS satellites, they could be ignored for most applications. Hence the separated formulation of double-differences is much more reliable than the mixed formulation (Pratt, et al., 1998).

The slight disadvantage in the separated formulation is the reduced number of the double-differences (reduced by one). As with the case of the carrier phase, the performance of the separated pseudo-range combination is better than the mixed combination.

The second issue is cycle slip detection on the GLONASS reference satellite. It can be seen that the third term should be a constant when tracking at both receivers to the GLONASS satellite is maintained. However, when a cycle slip occurs on the SD carrier phase measurement involving the GLONASS reference satellite, this term will no longer be constant. A new unknown parameter must be introduced or this cycle slip must be repaired. One cycle slip will result in about 1.5 mm for the L1 observations, and 2.0 mm for the L2 observations in worst case. If the GLONASS reference satellite can be chosen from the middle of the GLONASS frequency range, it should be less than 1.0 mm. Cycle slips should be detected by using the SD carrier phase observables (Eq. (1)), which contains a strong effect from the difference of the two receiver clock biases. In practice, only significant cycle slips have been detected or recorded by
the receivers, and a new unknown parameter needed to be introduced.

3 Real-time stochastic model estimation

High quality results using Least Squares estimation techniques requires the correct selection of both the functional and stochastic models. The stochastic model is dependent on the choice of the functional model. Hence for a different choice of functional model, a different stochastic model may be necessary. GPS and GLONASS observations are affected by several kinds of errors and biases. When forming the double-differences, the main biases are caused by multipath effects, residual atmospheric errors, orbital errors, and inter-channel biases. Due to insufficient knowledge about these physical phenomena, the above biases cannot be rigorously accounted for through functional modelling. The stochastic model has to therefore model both the observation noise and the unmodelled residual biases.

3.1 Empirical stochastic model

The well-known elevation-dependent stochastic model is often used, which may be represented as an exponential function or an inverse of the sine of the satellite elevation angle (El-Rabbany, 1994; Jin, 1995). However, constant coefficients can only reflect error characteristics of the GPS receiver, rather than the unmodelled residual biases, which are probably related to the observing environment. In order to introduce this “environment information”, an adaptive stochastic model was proposed by Han (1997), in which a scale factor is introduced, and estimated in real-time:

$$\sigma = s \cdot (a_0 + a_1 \cdot \exp(-E/E_0))$$ (7)

where \(\sigma\) is the standard deviation of the carrier phase or pseudo-range observations; \(a_0, a_1\) and \(E_0\) are approximate constants; \(E\) is satellite elevation angle; and \(s\) is a scale factor, which can be estimated from moving 2-5 minutes windows of data. This model more or less accounts for the environmental impact on the stochastic model. However, the spatial correlation between the DD observables cannot be refined in this way. In other words, the diagonal elements of the a priori variance-covariance matrix cannot be accounted for by using a scale factor, in place of the appropriate non-diagonal elements. Therefore the construction of more rigorous stochastic model is still a challenge. An adaptive Kalman filter has been investigated for real-time stochastically modelling an integrated GPS/GLONASS/INS system (Wang, 1999).

3.2 Real-time stochastic modelling

On the basis of the fact that the residual series of Least Squares estimation contains sufficient information of the observation noise and biases, a more rigorous stochastic model is derived. The general Least Squares linearised observation equation and the criteria can be modelled as:

$$V_i = B_i X_i - L_i$$ (8)

$$V_i^T D_i^{-1} V_i = \min$$ (9)

where \(V_i\) and \(L_i\) are the vectors of all the measurements and residuals at epoch \(i\) respectively; \(B_i\) is the design matrix related to the vector of measurements \(L_i\); \(X_i\) is the estimated unknown parameter matrix; and \(D_i\) is the variance-covariance matrix of the measurements.

According to the minimum quadratic form of the residuals, the Least Squares estimated parameter \(\hat{X}_i\) are:

$$\hat{X}_i = (B_i^T D_i^{-1} B_i)^{-1} B_i^T D_i^{-1} L_i$$ (10)

Substituting Eq. (10) into Eq. (8), the estimated residuals are:

$$V_i = (B_i (B_i^T D_i^{-1} B_i)^{-1} B_i^T D_i^{-1} - E) L_i$$ (11)

The variance-covariance matrix can be derived from Eq. (11):

$$D_i = Q_{V_i} + B_i (B_i^T D_i^{-1} B_i)^{-1} B_i^T$$ (12)

where \(Q_{V_i}\) is the variance-covariance matrix of the residuals. Due to the similarity of the observation environments, the residuals of the observations show a high degree of temporal and spatial correlation in the short term. In other words, the residual series could be considered as a wide-sense stationary process. The actual variance-covariance matrix of the residuals can then be estimated from the previous residual series, whose ambiguity sets have already been fixed to the correct values, by using the following equation:

$$Q_{V_i} = \frac{1}{N} \sum_{k=1}^{N} (V_{i-k} V_{i-k}^T)$$ (13)
where $N$ is the width of the moving window. The minimum $N$ should not be less than the number of DD ambiguities. However if $N$ is too large, temporal and spatial decorrelation will occur, and the performance will decrease. Testing has shown that the optimal width of the moving window is in the range of 10-30 epochs with 1-second sampling rate. In practical applications, residuals from the ambiguities-fixed solutions should be used because the float ambiguity values may absorb some unmodelled errors.

In Eq. (12), the variance and covariance of the measurements can not be estimated directly. There is the need of an iterative procedure. The initial (or default) variance-covariance matrix is determined by using the previous variance-covariance matrix. On the basis of the previous measurement residuals, the variance-covariance matrix of the measurements can be rigorously estimated in real-time with Eqs. (12) and (13). Normally iterating twice is enough. The default stochastic model should be used at the beginning of the data processing, or for a new satellite, or after a long data gap.

The stochastic model in this paper reflects not only the stochastic characteristics of the observation noise, but also the residual biases due to multipath, atmospheric delays, the inter-channel biases and the orbital error remaining after double-differencing both the carrier phase and pseudo-range observations. With the help of the estimated variance-covariance matrix, the reliability of ambiguity resolution and the accuracy of the real-time kinematic positioning results can be significantly improved.

4 Ambiguity resolution, validation and adaptation

Eq. (4) with the clock bias correction can be used to estimate the real-valued ambiguities and their variance-covariance matrix. The associated stochastic model is derived from the residual series over the previous epochs. The LAMBDA procedure is then implemented to search the integer ambiguity set (Teunissen, 1994; Han & Rizos, 1995). The validation criteria test suggested by Han (1997), and the ratio test are implemented. If both tests are passed, the ambiguity resolution is assumed to be correct.

If the resolved integer ambiguities are incorrect, in general, the wrong integer ambiguities will refer to more than one satellite pair, and it is almost impossible to identify which ambiguities are incorrect. However, the fact that some significant biases are present in the observations can be confirmed. If there are enough satellites, eg. more than 5 GPS satellites, or more than 5 GLONASS satellites, or more than 7 GPS and GLONASS satellites, some of these observations could be eliminated. Following the outlier detection algorithm based on correlation analysis theory (Shi, 1998; Dai, et al., 1999), a procedure whereby one satellite is eliminated step-by-step has been implemented in the software (so that at least four DD carrier phase observables could still be formed). If ambiguity resolution has failed, the procedure is repeated until ambiguity resolution is successful, or less than four DD carrier phase observables can still be formed. If less than four DD carrier phase observables can be formed and the ambiguity test still fails, the ambiguity resolution procedure is considered as a failure. This adaptive procedure ensures the significant increase for ambiguity resolution success rate (Han, 1997).

5 Performance analysis of a novel data processing strategy

In order to test the performance of the proposed data processing strategy including the integrated three-step procedure to improve the functional model and the real-time stochastic modelling technique the following rapid static positioning and kinematic positioning experiments were carried out.

5.1 Rapid static positioning experiments

Rapid static positioning experiments have been carried out over different distances by using two GG24 integrated GPS/GLONASS single-frequency receivers. The reference receiver was set up on the Mather Pillar, on the roof of the Geography and Surveying building at the University of New South Wales. The rover receiver was set up at different sites which included the same roof nearby the refer-
ence receiver, at Coogee Beach, at Maroubra Beach, and at the La Perouse Beach. The baseline name, baseline length, number of satellites, observation span (total number of epochs) are given in Table 1. The positioning results can be easily checked from the repeatability of the baseline vectors for the different sessions. In all the data sets the cut-off elevation angle was set to 15 degrees during the processing.

| Name | Length/m | GPS/GLONASS satellites | Total epochs | Survey date |
|------|----------|------------------------|--------------|-------------|
| A1   | 12       | 8.5/7.3                | 14 362       | 1999-5-12   |
| A2   | 2 873    | 9.5/9.4                | 4 012        | 1999-5-11   |
| A3   | 4 053    | 9.5/5.5                | 6 868        | 1999-5-10   |
| A4   | 6 796    | 7.5/4.3                | 4 690        | 1999-5-10   |

The observations were divided into different sessions, 10 seconds in length for one set of sessions and 1 minute in length for another set of sessions. The data processing results are listed in Table 2 for the 10-second sessions and in Table 3 for the 1-minute sessions. It can be seen that rapid static positioning derives solutions with more than 98.3% success rate using 10 seconds of data, and with 100% success rate using 1 minute of data for each session.

| Name | Total sessions | Correct/% | Reject/% | Wrong/% |
|------|----------------|-----------|----------|---------|
| A1   | 1 406          | 99.3      | 0.7      | 0.0     |
| A2   | 401            | 100.0     | 0.0      | 0.0     |
| A3   | 631            | 99.7      | 0.3      | 0.0     |
| A4   | 418            | 98.3      | 1.7      | 0.0     |

| Name | Total sessions | Correct/% | Reject/% | Wrong/% |
|------|----------------|-----------|----------|---------|
| A1   | 233            | 100.0     | 0.0      | 0.0     |
| A2   | 66             | 100.0     | 0.0      | 0.0     |
| A3   | 109            | 100.0     | 0.0      | 0.0     |
| A4   | 75             | 100.0     | 0.0      | 0.0     |

The rapid static positioning results (1 minute for each session, Baseline A2) were also derived using Eq. (4), in which the clock biases are considered as unknown parameters, and plotted in Fig. 1 (gray colour, relative to the GPS-only solution using the whole data set). Compared with the positioning results (black colour, relative to the GPS-only solution using the whole data set) derived using Eq. (3), in which the third term of the SD integer ambiguity for the reference satellite is considered as an unknown parameter, it is clear that the suggested procedure derives positioning solutions with higher accuracy.

![Fig. 1](image-url)
ual series from previous epochs proposed in this paper. The results are listed in columns 3-5 in Table 4 and Table 5. The third column is the number (and percentage) of epochs for which ambiguity resolution is successful on an epoch-by-epoch basis. The fourth column is the number of epochs (and percentage) which do not pass the validation criteria test. The fifth column is the number of epochs (and percentage) which pass the validation criteria tests, but for which the result is incorrect. It can be seen that when using the elevation-dependent empirical stochastic model the success rates for single-epoch ambiguity resolution range from 80.1% to 83.3%. It also shows that quite a large percentage of epochs (0.2%) at Baseline A1 give the wrong ambiguity resolution results. If the real-time stochastic model estimated by using the residuals from the previous epochs (e.g. 10 epochs) is used, the success rates of ambiguity resolution range from 95.9% to 99.98%, and no wrong ambiguity resolution results are accepted. The results indicate that single-epoch ambiguity resolution can achieve very high success rates with redundant GPS and GLONASS satellite observations. The conclusion that can be drawn is that the estimated stochastic model from the residuals is in theory more rigorous, and in practice more powerful, than using other forms of arbitrary stochastic modelling.

Table 4  Single-epoch solution using the elevation

dependent empirical stochastic model

| Name | Total epochs | Correct/% | Reject/% | Wrong/% |
|------|--------------|-----------|----------|---------|
| A1   | 14 361       | 81.4      | 18.4     | 0.2     |
| A2   | 4 012        | 83.1      | 16.9     | 0.0     |
| A3   | 6 868        | 83.3      | 16.7     | 0.0     |
| A4   | 4 690        | 80.1      | 19.9     | 0.0     |

Table 5  Single-epoch solution using the real-time

stochastic model derived using residuals from

previous epochs, as proposed in this paper

| Name | Total epochs | Correct/% | Reject/% | Wrong/% |
|------|--------------|-----------|----------|---------|
| A1   | 14 361       | 97.60     | 2.40     | 0.0     |
| A2   | 4 012        | 99.98     | 0.02     | 0.0     |
| A3   | 6 868        | 95.90     | 4.10     | 0.0     |
| A4   | 4 690        | 96.10     | 3.90     | 0.0     |

The ratios of the validation criteria are plotted at each epoch in Fig. 2 for Baseline A2. The grey and black dots in Fig. 2 are ratios from the elevation-dependent empirical stochastic model and the estimated stochastic model from residuals, respectively. It can be seen that the ratios with the estimated stochastic model from residuals are much bigger than those with the elevation-dependent empirical stochastic model. It is believed that the larger the ratios are, the more reliable the ambiguity resolution is.

![Fig. 2 - Ratios of different stochastic models](image)

The positioning results are also derived by using two different stochastic models for Baseline A2. The differences of the three coordinate components, between the single-epoch solutions and the final baseline solution by using whole data set, are plotted in Fig. 3. The gray and black curves are po-

![Fig. 3 - Positioning results](image)
Positioning results by using the elevation-dependent empirical stochastic model (gray) and the estimated stochastic model from residuals (black), respectively. It can be seen that a realistic stochastic model can significantly improve the accuracy of the final positioning solutions.

A kinematic experiment was further carried out on 29 April, 1999 by using two GG24 GPS/GLONASS single-frequency receivers and three dual-frequency Leica SR399 GPS receivers. One GG24 receiver and one Leica SR399 were set up at the reference site. The other GG24 receiver and the other two Leica GPS receivers were mounted on a car. The trajectory of the rover receivers is shown in Fig. 4 (The purpose of using two Leica dual-frequency receivers is as a check on the derived GG24 positioning results.). The experiment started at the side of the M4 Motorway, Sydney, which is nearby the reference site. After the first 40 minutes in static mode, the car moved along the Motorway and the Great Western Highway, finishing the experiment in static mode for another 15 minutes. This is a single loop. A total of two loops were completed with 1 Hz data rate, and a total of 7571 epochs of data were collected. The number of observed satellites is plotted in Fig. 5.

In this kinematic experiment the data was divided into two groups according to the baseline length. The baseline lengths in one group were less than 6 km. The baseline lengths in the second group range from 6 km to 11 km. The results are summarised in Table 6. In order to check the kinematic results the Leica GPS receivers were used, as correct and reliable results from dual-frequency GPS receivers can be achieved by using current ambiguity resolution techniques. If the distance differences between the Leica rover receivers and the GG24 rover receiver exceeded some defined tolerance value (5 cm in this experiment), the ambiguities for the GG24 receivers were considered to have been fixed to the wrong values. From Table 6 it can be seen that with the elevation-dependent empirical stochastic model the success rate for ambiguity resolution is only 72.9% for the first short-range group, and a very poor 19.2% for distances ranging from 6 km to 11 km. The percentage of rejected epochs is 27.1% and 80.8%, respectively. If the real-time stochastic model estimated by using the residual series from the previous epochs is used, the success rate for ambiguity resolution is significantly improved to 94.6% for the short-range group and 56.4% for distances ranging from 6 km to 11 km. It can be seen that for the short-range baselines, the positioning results can achieve a reasonable success rate. However, when the distance between the two receivers is longer than about 6 km, the success rate with using single-frequency GG24 GPS/GLONASS receivers decreases quickly, if only a single epoch of data are used.

| Total epochs | Dist/km | Correct/% | Reject/% | Wrong/% |
|--------------|---------|-----------|----------|---------|
| Elevation dependent empirical model | 6 331 | <6 | 72.9 | 27.1 | 0.0 |
| | 1 240 | 6-11 | 19.2 | 80.8 | 0.0 |
| Estimated stochastic model | 6 331 | <6 | 94.6 | 5.4 | 0.0 |
| | 1 240 | 6-11 | 56.4 | 43.6 | 0.0 |
6 Conclusion

A new data processing strategy for integrated GPS/GLONASS positioning by using a three-step procedure, and associated real-time stochastic model estimated by using residuals from the previous epochs, has been introduced in this paper. The proposed functional model improves the performance because the ambiguity resolution process is insensitive to the residual clock biases and the inter-channel biases, and hence reliable and precise positioning results are obtained. The real-time stochastic model estimated from the residuals can significantly improve the ambiguity resolution success rates, as well as the accuracy of the final solutions.

The results indicate that rapid static positioning requires 10 seconds of data for success rates over 98%, or just 1 minute of data for a success rate of 100%, based on four experiments carried out over distances less than 6km. The single-epoch solution for kinematic positioning could achieve a 94.6% success rate over distances shorter than 6 km. However, when the range increases the success rate dramatically decreases. Investigations are underway and concerning the optimal strategy for dual-frequency GPS/GLONASS processing.

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