Comparative Analysis for Interval Modeling Algorithms of Wind Turbine Power Curve

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Abstract. Considering the stochastic uncertainty of wind power generation, joint estimation of wind turbine power curve (WTPC) in point and interval forms is carefully studied in this paper. Firstly, inducing factors of uncertain data and outliers types in measured data are analyzed to show necessity of uncertainty modeling of WTPC. Then, theoretic principles of different methods for WTPC modeling such as Gaussian Bin method, kernel density estimation (KDE), conditional kernel density estimation (CKDE), conditional probability via Copula, Gaussian process regression (GPR) and relevance vector machine (RVM), are seriously straightened up. Their theoretical essences are discussed and compared in depth. Subsequently, evaluation indexes for point and interval estimations of WTPC are defined to quantify modeling performance. Finally, using measured data from actual 1.5MW variable-speed variable-pitch (VSVP) wind turbines, simulations are executed to show diversities of these methods and their features are discussed from different views. Meanwhile, their potential application scenarios and usage strategies are carefully analyzed, including wind power prediction and operational performance evaluation of wind turbine in an infinite-time-horizon form or a sliding-window form. It is helpful to enhance development of WTPC modeling techniques.

1. Introduction
In recent decades, large-scale utilization of wind energy develops rapidly. Due to potential benefits in environment protection and reduction of carbon-dioxide emissions, it plays a more and more important role in the future energy structure [1]. However, as well-known, strong stochasticity and intermittence exist in the generated wind power, affecting safety of power-grid. To enhance controllability in utilization, wind power prediction is very critical in wind power dispatching and even in wind turbine control. How to obtain reliable and accurate prediction results is a research hotspot in both academia and industry.

From the view of time-scale, there are ultra-short term, short-term, medium-term and long-term predictions of wind power. About prediction procedure, there are direct and stepwise predictions [2]. For prediction form, point and interval predictions are mainly concerned. Point prediction is very common where prediction error is often used to evaluate its accuracy. If prediction values are optimized to fall into a specified interval with set confidence degree, interval prediction is realized.

Considering stochasticity of wind power, research on interval prediction attracts more and more attention. Using stepwise procedure, including wind speed prediction and WTPC modeling, influence of each step to the final predictions can be easily found and flexibly analyzed. Although interval prediction of wind speed or wind power has been studied, interval modeling of WTPC has not been
fully concerned. Except for wind power prediction, WTPC can also be used for theoretical power calculation, condition monitoring, operation performance evaluation and fault diagnosis of wind turbine. If interval model of WTPC is reasonably built, it is meaningful for above applications.

In [3], classical bin method is defined for point estimation of WTPC. In [4, 5], different parametric and non-parametric methods were reviewed where only point estimation of WTPC was discussed. It suggests that point estimation of WTPC has been popularly studied. In [6], Jin, et.al., adopted mean value and standard deviation to obtain uncertainty estimation of wind power in a simple form. In [7], power gradient formula was defined to represent uncertainty of wind power but accurate analytical formulas are difficult to be built in practice. In [8], Lin, et.al., used KDE to model uncertainty of WTPC in wind speed bins whereas point estimation based on KDE was not discussed. In [9, 10], joint probability distribution of wind speed and power was established via Gaussian distribution and Nadaraya-Watson estimator, respectively. However, point and interval estimation of WTPC and their relationship was not explicitly stated. In [11], interval modeling of WTPC via conditional Copula was studied while point estimation of WTPC wasn’t given, too. For above research, statistical methods are mainly used for interval estimation of WTPC. Yet, point estimation based on interval estimation and their relationship were not studied. In [12], interval prediction of wind power via GPR-Bagging was studied. In [13, 14], RVM was used for interval prediction of wind power. GPR [15] and RVM can be directly used for interval estimation of WTPC because they can give conditional probability of output against inputs. However, they haven’t been applied in interval estimation of WTPC.

Currently, point and interval estimation of WTPC and their relationship have not been studied adequately. Especially, different methods are dispersedly and simply used while their performances haven’t been carefully evaluated and compared. Besides, from a view of theory, how to understand essences of these interval estimation methods hasn’t been seriously explained. Considering great potential of uncertainty WTPC in many applications, comparison of different uncertainty modeling methods of WTPC are detailed studied in this paper. The common truth of these methods about point and interval estimation of wind power against wind speed will be revealed. All the discussions can be seen as an in-depth exploration to the existed research and a guide to future research for methodological improvement.

The rest of this paper is organized as follows. Section 2 analyzes uncertainty of WTPC and necessity of its interval estimation. Section 3 introduces the theoretical principles of different interval modeling methods. Section 4 lists the evaluation indexes for point and interval estimation. Simulation and analysis of these methods are compared in section 5. Section 6 concludes the paper.

2. Uncertainty analysis of WTPC

For modern wind turbine, WTPC is usually given by the original equipment manufacturer (OEM). A typical WTPC with variable-speed variable-pitch (VSV) capability is shown in Figure 1. It mainly consists of four operating regions. In region I, wind turbine un-connects power-grid and its generator idles with no-load until wind speed has overstepped the cut-in one. If wind speed can maintain rotor...
speed in the minimum level, generator connects power-grid via bi-directional pulse-width-modulation (PWM) converter. At this instant, wind turbine gets into region II whose task is maximum utilization of wind energy. Herein, rotor speed increases with wind speed until the rated value. Then, region III comes where rotor speed is kept at the rated value while output power increases with wind speed until the rated power, corresponding to the rated wind speed. Above rated wind speed, it is the region IV where wind turbine works until the cut-out wind speed.

For these regions, different operating strategies and controllers switch with the varying wind speed. In the switching areas, operational uncertainty may be brought in, leading to sparse outliers. Moreover, modern wind turbine has larger blade size, greater moment of inertia, stronger structural vibration and higher degrees of freedom, yielding bigger inertial time constant. Then, uncertainties of dynamic control process are increased. Besides, uncertainties may be induced due to random wind energy. Especially, turbulent wind speed caused by wake effect, rugged terrain, gust and wind direction, may draw into plenty of uncertainties. For a wind farm distributed in wide area, communication lines are long and many interferences may be injected. Meanwhile, accidental faults or performance deterioration of sensors and equipment can also add uncertainties.

Except above reasons, grid-connected wind farm should response to grid-power dispatching. Wind curtailments frequently happen in China, yielding limited output of wind power. A wind turbine may work at the limited power mode instead of the maximum power point tracking mode. Then, output power of wind turbine may stack below the OEM WTPC, producing the so-called stacked outliers.

![Figure 2. Raw data of wind turbine 1](image1)

![Figure 3. Raw data of wind turbine 2](image2)

Due to above sources of uncertainty, sparse and even stacked outliers exist in data records of SCADA system. Along WTPC of OEM, banding area of wind power against wind speed is often obtained, instead of a deterministic WTPC. Actual wind power banding areas are shown in Figure 2 and 3. To obtain reliable WTPC, outliers should be properly eliminated. However, stacked outliers are easily to be found while sparse outliers are mixed with valid data and difficult to be identified. Using effective algorithms, most and even all the stacked outliers can be cleaned while only partial sparse outliers can be cleaned [16]. As a result, although valid data have dominant proportion, the left data still distribute in banding shape along WTPC of OEM. Some uncertainties are inevitable and cannot be eliminated absolutely. Thus, interval modeling of WTPC becomes a very necessary representation.

### 3. Interval modeling algorithms

For point estimation methods of WTPC via parametric or non-parametric regression techniques, regression output is often the conditional expectation of input. Usually, statistical methods are used to establish the conditional probability distribution except some special regression algorithms such as GPR and RVM. Herein, the mainstream methods will be introduced in detail including their theoretical principles.

According to the IEC standard [3], bin method is a basic way to get point estimation of WTPC. Uniformly divide wind speed range into \( k \) bins with certain interval. Calculate mean values of wind speed and power by (1) and (2) in the \( i \)-th bin and \( k \) data pairs \((V_{i\text{mean}}, P_{i\text{mean}})\) \( (i=1,2,\ldots,k) \) can be
obtained. Then, use curve fitting methods to fit the data pairs and get WTPC. It is sensible to the quality of acquired data.

\[ V_{\text{mean}} = \frac{1}{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} V_j \quad (1) \]

\[ P_{\text{mean}} = \frac{1}{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} P_j \quad (2) \]

where \( i \) means the \( i \)-th bin; \( k \) is number of bins; \( V_{\text{mean}} \) is mean wind speed in the \( i \)-th bin; \( P_{\text{mean}} \) is mean power in the \( i \)-th bin; \( V_j \) and \( P_j \) are the \( j \)-th data points in the \( i \)-th bin, \( N_{\text{bin}} \) is number of points in the \( i \)-th bin. Generally speaking, although bin method is very simple, it is often a reference for the other methods of WTPC modeling.

3.1. Gaussian bin method

In [6], Gaussian distribution of wind power is assumed in each wind speed bin with the following form

\[ f(P_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(P_i - P_{\text{mean}})^2}{2\sigma_i^2}} \quad (3) \]

\[ F(P_i) = \int_{-\infty}^{P_i} f(x) \, dx = \Phi \left( \frac{P_i - P_{\text{mean}}}{\sigma_i} \right) \quad (4) \]

where \( P_i \) is power values in the \( i \)-th bin; \( \sigma_i \) is standard deviation; \( f(P_i) \) is probability density function; \( F(P_i) \) is cumulative distribution function. At each wind speed bin, mean value and standard deviation of wind power can be calculated, yielding its Gaussian distribution. Herein, Gaussian distribution of wind power can be seen as a conditional probability distribution versus wind speed in each bin. In [6], this technical route is easily executed while the rigorous argument is lacked. In [9], Villanueva. et.al validated this route in detail, where it was further used to perform long-term assessment of wind turbine combining the simulation based on Monte-Carlo method.

3.2. Kernel density estimation

Although Gaussian assumption is popular, it is not always coincident with factual data. If wind power data in each wind speed bin cannot always satisfy Gaussian assumption, affected by data size, faults and abnormalities, empirical probability modeling should be considered. Different from the parametric model via Gaussian distribution, empirical model is non-parametric, just depending on sampled data. It is applicable to the non-linear and non-homogeneous data with unknown distribution. It is suitable for WTPC modeling using measured data, whose probability distribution is unknown. Herein, KDE is used for probability modeling in each bin. A common KDE function is defined by [17]

\[ f(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{\| x - X_i \|}{h} \right) \quad (5) \]

where \( n \) is number of data points; \( h \) is bandwidth; \( X_i \) is the \( i \)-th actual data point; \( K(\cdot) \) is kernel function. Choosing of \( h \) and \( K(\cdot) \) determines its performance. The most-used kernel functions are exponential, uniform and Gauss kernel ones. If kernel function has been chosen, \( h \) becomes important to adjust the estimation accuracy. A bigger \( h \) may cover more details of distribution characteristics; a smaller \( h \) may bring in irregular probability density functions. Based on KDE, kernel regression can be used to get the point estimation. In [8], kernel regression was adopted to estimate point values of wind power while more comparison are needed about different data samples and methods.

3.3. Conditional kernel density estimation

In each wind speed bin, probability distribution of wind power is smoothed via KDE. Herein, CKDE can further smooth the probability distribution of wind power along the direction of wind speed. Then,
no wind speed bins exist. In section 3.2, KDE of single random variable is given by (5). For the case with bivariate random variables, the bivariate KDE is given as following [18]

\[
f(x_1, x_2) = \frac{1}{N_h h_2} \sum_{i,j} K_{ij}\left(\frac{\|x_i - x_j\|}{h_j}\right)
\]

(6)

where \(K_{ij}(\cdot)\) is a bivariate kernel function; \(h_1, h_2\) are bandwidths. Then, joint distribution can be given by the Nadaraya-Watson estimator [19], shown as following

\[
f(p|v) = \sum_{i=1}^{N} \frac{1}{h_P} K_P\left(\frac{\|p - P\|}{h_P}\right) \cdot \omega_k(V)
\]

\[
\omega_k(v) = \sum_{i=1}^{N} K_V\left(\frac{\|v - V_i\|}{h_V}\right)
\]

(7)

(8)

where bandwidths \(h_P\) and \(h_V\) control the smoothness of conditional probability densities along the \(P\) and \(V\) directions, respectively; \(K_P(\cdot)\) and \(K_V(\cdot)\) are kernel functions of \(P\) and \(V\). Compared with KDE of wind power in a wind speed bin, CKDE by the Nadaraya-Watson estimator is smoother along wind speed and power [10]. Based on CKDE, non-parametric regression can be calculated by the conditional expectation of wind power where

\[
\hat{R}_{P|V} = E(P|V = v)
\]

(9)

which can be used as point estimation of \(P\) when \(V=v\). In [20], point regression based on CKDE was adopted for wind power prediction. Point estimation of WTPC based on CKDE is very similar with point prediction.

3.4. Conditional probability based on Copula

Different from non-parametric method in section 3.3, conditional probability modeling based on Copula is usually a semi-parametric or parametric method [21]. For Copula theory, joint probability distribution of random variables \(x\) and \(y\) is \(F_{XY}\) which joins their marginal probability distributions \(F_X(x)\) and \(F_Y(y)\), shown as following

\[
F_{XY}(x, y) = C(F_X(x), F_Y(y))
\]

(10)

where \(u=F_X(x)\subseteq[0,1]\), \(v=F_Y(y)\subseteq[0,1]\). \(C(\cdot)\) is Copula function shown as

\[
C(u, v) = F_{XY}(F_X^{-1}(u), F_Y^{-1}(v))
\]

(11)

Then, modeling of \(F_{XY}\) is substituted by the separately modeling of \(F_X, F_Y\) and \(C(\cdot)\). Stochastic dependence between arbitrarily distributed variables \(x\) and \(y\) can be represented by that of uniform variables \(F_X\) and \(F_Y\). On this basis, conditional probability of \(F_Y(y)\) under \(F_X(x)\) is

\[
F(F_Y(y)|F_X(x)) = \frac{\partial(F_Y(y)|F_X(x))}{\partial F_X(x)}
\]

(12)

If \(X=V\) and \(Y=P\), conditional probability of \(F_Y(P)\) under \(F_Y(V)\) can be obtained. If \(F_Y(P)\) and \(F_Y(V)\) are modeled by a non-parametric method such as KDE, (12) becomes a semi-parametric form. If they are modeled by known probability function, (12) is a parametric form. No matter what method is adopted, mapping relation between random variables and their marginal distributions can be established. As a result, using (12), conditional probability distribution of \(P\) under \(V\) can be obtained. Conditional expectation of \(P\) under \(V\) is point estimation while the upper and lower boundaries with certain confidence degree can also be calculated. In [11], interval modeling of WTPC via conditional Copula has been studied whereas point estimation hasn’t been given.

3.5. Gauss process regression

As a probabilistic modeling method, GPR is based on the Bayesian framework. It is less parametric except the anterior hypothesis of multivariate Gaussian distribution. Meanwhile, the posterior
probabilities is derived strictly. To some extent, it strongly depends on measured data. For the data with noise, \(x \in \mathbb{R}^d\) and \(y \in \mathbb{R}\), \(y\) can be estimated as following
\[
\hat{y} = f(x) + \epsilon_x = f(x) + \mathcal{N}(0, \sigma^2_x)
\]
where \(f(x)\) is nonlinear mapping function, \(\epsilon_x\) is additional noise assumed to be independent Gaussian distribution with standard deviation \(\sigma^2_x\). Based on GPR theory, \(f(x)\) can be represented by [22]
\[
f(x) \sim \mathcal{GP}(m(x), K(\theta, x, x'))
\]
where \(m(x)\) is mean function, \(K(\theta, x, x')\) is covariance matrix between all possible pairs \((x, x') \in \mathbb{R}^d\). \(\theta\) is a given set of hyper-parameters, optimized to maximize the marginal likelihood \(p(f(x)|x)\). After training and testing, final \(\theta\) can be determined. For a new input \(x^*\), conditional probability under training and new input data are predicted by [23]
\[
p(y^*|x, x, y) = \mathcal{N}(y^*|m^*, \sigma^2_{y^*})
\]
where \(x\) and \(y\) are training data, \(m^*\) and \(\sigma^2_{y^*}\) are the predicted mean value and variance. Obviously, \(x\) and \(y\) are just used for modeling. (15) actually means the conditional probability of \(y^*\) under \(x^*\). As a result, not only point estimation but also the confidence boundary can be obtained. Thus, if GPR is adopted for WTPC modeling, both point estimation and confidence intervals can be calculated. In [15], interval modeling of WTPC was briefly discussed while detailed comparison was lacked. A known bottleneck of GPR is that its computational complexity is cubic in the number of input points, so it is unfeasible for large dataset.

3.6. Relevance vector machine
Compared with support vector machine, RVM has an identical functional form but provides probabilistic result. For the training data, input vector \(x \in \mathbb{R}^d\) and output vector \(y \in \mathbb{R}\), estimation of \(y\) can be represented by
\[
\hat{y}(x; \omega) = \sum_{i=1}^{N} \alpha_i \varphi(x, x_i) + \omega_b
\]
where \(\omega\) is weights which fulfills Gaussian distribution \(\omega \sim \mathcal{N}(0, \alpha^{-1}I)\). \(\varphi(\cdot)\) is kernel function. Then, \(y\) is shown by
\[
y = \hat{y}(x; \omega) + \epsilon_x
\]
where \(\epsilon_x\) is noise of output with independent Gaussian distribution \(N\sim(0, \sigma^2)\). It is actually equivalent to a GP model with following covariance function [24]
\[
K(x, x') = \sum_{i=1}^{N} \frac{1}{\alpha_i} \varphi(x, x_i) \varphi(x', x_i)
\]
where \(\alpha_i\) are variances of prior on weight vector. \(\alpha\) and \(\sigma^2\) are solved using the Maximum-Likelihood. Deduced from sparse Bayesian learning, conditional probability under training data and new inputs is also obtained. For a new input \(x^*\), predictive distribution is shown as following [29]
\[
p(y^*|x, x, y) = p(y^*|x, \alpha, \sigma^2) = \mathcal{N}(y^*|\hat{y}^*, \sigma^2_{y^*})
\]
where \(x\) and \(y\) are training data, \(\alpha\) and \(\sigma^2\) are solution of \(\alpha\) and \(\sigma^2\). \(\hat{y}\) and \(\sigma^2\) are the predictive mean value and variance. As a result, RVM can also be used for point or interval estimation of WTPC. In [13], RVM was used for short-term interval prediction of wind speed. In [14], improved RVM algorithm was used for direct interval prediction of wind power of wind farm. However, its application in interval modeling of WTPC has not been carefully studied.

4. Evaluation of point and interval estimation
For evaluation of point prediction of wind power, indexes based on output error such as root mean square error (RMSE) and mean absolute error (MAE) and their deformations are mostly used [5]. For evaluation of interval prediction, indexes such as prediction intervals coverage probability (PICP), prediction intervals normalized average width (PINAW) are commonly used [26]. Herein, similar
indexes are defined for evaluation of point and interval estimation of WTPC. In the $i$-th output interval versus a wind speed bin, the $i$-th output interval coverage probability (OICP) is defined as following

$$\text{OICP}^{(\tau)}(i) = \frac{1}{n_t(i)} \sum_{j=1}^{n_t(i)} \kappa_j^{(\tau)}$$

(20)

where $\tau$ is confidence degree for an output interval, $n_t(i)$ is size of testing data in the $i$-th interval, $\kappa_j^{(\tau)}$ is indicator of OICP under $\tau$. If output power falls into the interval, $\kappa_j^{(\tau)}=1$; otherwise, $\kappa_j^{(\tau)}=0$. OICP of each output interval should asymptotically approach output interval nominal confidence (OINC) as closely as possible. Then, the average coverage error (ACE) is used to evaluate interval estimation performance, shown as following

$$\text{ACE}^{(\tau)} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \text{OICP}^{(\tau)}(i) - \tau \right)$$

(21)

where $N_t$ is number of output intervals from testing data. Smaller the positive ACE$^{(\tau)}$ is, higher reliability the obtained output intervals possess. Except above evaluation, accuracy to interval modeling of WTPC is also important. Usually, sharpness index is used, called the output intervals average width (OIAW) in this paper, defined as following

$$\text{OIAW}^{(\tau)} = \frac{1}{q_t} \sum_{i=1}^{q_t} \left[ U^{(\tau)}(x_i) - L^{(\tau)}(x_i) \right]$$

(22)

where $q_t$ is number of testing data. $U^{(\tau)}(x_i)$ and $L^{(\tau)}(x_i)$ are upper and lower boundaries of output interval under input $x_i$ with confidence degree $\tau$. For the same $\tau$, smaller the OIAM is, better modeling performance the output intervals possess.

5. Simulation and comparative analysis

In order to compare interval modeling performance of above methods, measured data from the SCADA system of a wind farm in North China are adopted, which has the mainstream 1.5MW wind turbine with doubly-fed induction generator.

Wind speed and power data from January to October of 2018 are acquired with sampling period of 10 minutes. After data cleaning [16], 36000 data points are left, 32000 points for training and 4000 points for testing. From Figure 4 to 9 and Table 1 to 6, estimation effects of WTPC are different and conclusions from different views are discussed as following.

(a) Wind turbine 1

(b) Wind turbine 2

Figure 4. WTPC modeling via Gaussian bin method
Figure 5. WTPC modeling via KDE

Figure 6. WTPC modeling via CKDE

Figure 7. WTPC modeling via Copula
For conformity between curve shape and data distribution, statistical methods, such as Gaussian bin method, KDE, CKDE and Copula, perform better than GPR and RVM. Among the statistical methods, KDE, CKDE and Copula, perform better than Gaussian bin whose curve shape of transition operation region deviates from actual data distribution.

From view of modeling strategy, Gaussian bin and KDE belong to local modeling of WTPC while CKDE, Copula, GPR and RVM belong to global modeling of WTPC. Considering these statistical methods, for point estimation of WPTC, performance order is Gaussian bin method, KDE, Copula, CKDE. For interval estimation of WTPC, performance order is Copula, CKDE, KDE and Gaussian bin method. Then, comprehensive performance order is Copula, Gaussian bin, KDE and CKDE.

Meanwhile, compared with statistical methods, GPR and RVM have less interval estimation performance while their point estimation ability is similar with that of Gaussian bin method, KDE and Copula. Especially, RVM has better point estimation than GPR.

For modeling efficiency, GPR and RVM are more convenient based on Bayesian framework than that of statistical methods, so they are suitable for online estimation of WTPC and can be applied in super-short-term or short-term wind power prediction. The modeling processes of statistical methods are relatively complex. They are suitable for offline estimation of WTPC and can be applied in operational performance evaluation of wind turbine or wind farm.
Considering timeliness of models, all methods need to be updated in a finite-time-horizon way instead of an infinite-time-horizon way. Then, WTPC modeling in a sliding window form will be effective.

Table 1. Index calculation of Gaussian bin method

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9949 | 0.0449 | 566.745 |
| 0.90    | 146.0815 | 0.9079 | 0.0079 | 377.83  |
| 0.85    | 0.6192  | 0.2308 | 188.915 |

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9872 | 0.0372 | 663.7495 |
| 0.90    | 133.6074 | 0.9587 | 0.0587 | 442.4996 |
| 0.85    | 0.6967  | 0.1533 | 221.2498 |

Table 2. Index calculation of KDE in each bin

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9126 | 0.0374 | 389.8977 |
| 0.90    | 151.7917 | 0.8523 | 0.0477 | 320.5738 |
| 0.85    | 0.7875  | 0.0625 | 276.9662 |

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9493 | 0.0007 | 436.3148 |
| 0.90    | 140.4018 | 0.8925 | 0.0075 | 359.834 |
| 0.85    | 0.8855  | 0.0355 | 306.265 |

Table 3. Index calculation of CKDE in each bin

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9402 | 0.0098 | 315.6452 |
| 0.90    | 358.5084 | 0.8572 | 0.0428 | 255.8424 |
| 0.85    | 0.7401  | 0.1099 | 223.1967 |

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9523 | 0.0023 | 337.5849 |
| 0.90    | 219.8073 | 0.8559 | 0.0441 | 278.4118 |
| 0.85    | 0.7871  | 0.0629 | 237.4914 |

Table 4. Index calculation of conditional probability via Copula

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9467 | 0.0033 | 445.1377 |
| 0.90    | 141.586 | 0.8836 | 0.0164 | 367.3926 |
| 0.85    | 0.8318  | 0.0182 | 319.7442 |

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9374 | 0.0126 | 488.0534 |
| 0.90    | 152.6707 | 0.8716 | 0.0284 | 402.3225 |
| 0.85    | 0.8058  | 0.0442 | 347.4289 |

Table 5. Index calculation of conditional probability via GPR

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.8353 | 0.1147 | 429.7927 |
| 0.90    | 188.0964 | 0.8015 | 0.0985 | 368.2537 |
| 0.85    | 0.7675  | 0.0825 | 322.2857 |

| Turbine | RMSE | OICP   | ACE   | OIAW    |
|---------|------|--------|-------|---------|
| 0.95    | 0.9038 | 0.0462 | 414.1298 |
### Table 6. Index calculation of conditional probability via RVM

| Turbine 1 | RMSE  | OICP  | ACE   | OIAW  |
|----------|-------|-------|-------|-------|
| 0.95     | 133.0188 | 0.8683 | 0.3017 | 347.5487 |
| 0.85     | 0.8405 | 0.0095 | 304.1653 |
| Turbine 2 | RMSE  | OICP  | ACE   | OIAW  |
| 0.95     | 142.1473 | 0.8308 | 0.1192 | 346.4693 |
| 0.90     | 129.7741 | 0.8215 | 0.0785 | 287.0853 |
| 0.85     | 0.7653 | 0.0847 | 232.2211 |

### 6. Conclusion

Joint estimation of WTPC in point and interval forms is clearly stated in this paper. Considering potential applications and interference to SCADA data, necessity of WTPC modeling is deeply analyzed. Then, theoretical principles of different methods including point, interval estimation of WTPC and their relationship are carefully discussed for simultaneous point and interval estimation of WTPC. Meanwhile, special evaluation indexes including point and interval estimation of WTPC are defined. Using measured data, simulation results show the diverse modeling effects of different methods. In-depth explanation is given from several points. Besides, their features and applicable scenarios are explored in detail. The research in this paper provides good methodological support and theoretical guidance for WTPC modeling and its applications. On this basis, more improved interval modeling methods of WTPC can be explored. Besides, for evaluation of modeling performance using several indexes, multi-objective comprehensive evaluation should be concerned. They will be further studied in future.

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