On deformations of constrained Hamiltonian systems in BFV-formalism

I.L. Buchbinder\(^{(a,b,c)^1}\), P.M. Lavrov\(^{(a,c)^2}\),

\((a)\) Center of Theoretical Physics, Tomsk State Pedagogical University, Kievskaya St. 60, 634061 Tomsk, Russia
\((b)\) Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow region, Russia
\((c)\) National Research Tomsk State University, Lenin Av. 36, 634050 Tomsk, Russia

Abstract

We develop a general approach to constructing a deformation that describes the mapping of any dynamical system with irreducible first-class constraints in the phase space into another dynamical system with first-class constraints. It is shown that such a deformation problem can be efficiently explored in the framework of the Batalin-Fradkin-Vilkovisky (BFV) formalism. The basic objects of this formalism are the BRST-BFV charge and a generalized Hamiltonian that satisfy the defining equations in the extended phase space in terms of (super)Poisson brackets. General solution to the deformation problem is found in terms of a (super)canonical transformation with a special generating function which is explicitly established. It is proved that this generating function is determined by a single arbitrary function which depends only on coordinates of initial dynamical system. In principle, such a function may be non-local, but the deformed theory may nevertheless have a local sector. To illustrate the developed approach, we have constructed a non-local deformation of the Abelian gauge theory into a non-local non-Abelian gauge theory whose local sector coincides with the standard Yang-Mills theory.

Keywords: BFV-formalism, (super)canonical transformations, BRST-BFV charge, deformation procedure

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\(^1\)E-mail: joseph@tspu.edu.ru
\(^2\)E-mail: lavrov@tspu.edu.ru
1 Introduction

The gauge theories are the key elements of the Standard Model of Fundamental Interactions and its generalizations. Therefore, studying the structure of general gauge theories can in principle ensure the possibilities to formulate the new concrete gauge theories that can be useful for constructing the new models of fundamental interactions beyond the standard model. Recently, we have developed an approach to generate the new gauge theories on the base of known gauge theories [1], [2], [3] and describe their quantum aspects. The approach was formulated within the framework of the Batalin-Vilkovisky (BV) - formalism of covariant quantization of general gauge systems [4], [5].

In this paper we study a problem of generating the new gauge theories on the base of known gauge theories on the other hand using the Batalin-Fradkin-Vilkovisky (BFV) - formalism of canonical quantization of general gauge systems [6], [7], [8] (see also the reviews [9], [10], [11] and the book [12]). For some recent developments of this formalism see e.g. [13], [14], [15] and the references therein. The BFV-formalism, based on the fundamental principle of the BRST symmetry [16], [17], is the powerful quantization method for arbitrary dynamical systems with constraints in phase space. The central object of the BFV-formalism is a nilpotent BRST-BFV charge encoding the gauge structure in extended phase space involving the ghost coordinates and momenta. In these terms, a gauge invariant deformation of gauge theory means (super)canonical transformation of extended phase space preserving the nilpotency of the BRST-BFV charge. We prove that such a deformation can be described in general form in terms of generating function which is defined by one arbitrary function of coordinates of the initial dynamical system.

The paper is organized as follows. In section 2, we consider the classical aspects of the BFV-formalism for dynamical systems with irreducible first-class constraints. Section 3 is devoted to formulating the problem of gauge invariant deformation in extended phase space for gauge theories with first-class constraints and discussing a way of a possible solution to this problem. In section 4, we describe the exact solution of the deformation problem in closed form using the (super)canonical transformation in the initial phase space. The generating function of such a transformation is found in the explicit form. It should be noted that no special properties of the above (super)canonical transformation are assumed in advance. In particular, this transformation is not expected to be necessarily local, however it may have a local sector corresponding some new local gauge theory. In section 5, we apply the obtained results to Abelian gauge theory and construct the special canonical transformation such that a local part of the transformed gauge theory coincides with non-Abelian Yang-Mills theory. Section 6 is a summary of the results.

In this paper we systematically use the DeWitt’s condensed notations [18] and apply the symbols $\varepsilon(A)$ for the Grassmann parity and $\text{gh}(A)$ for the ghost number, respectively. The right and left functional derivatives are marked by special symbols ”←” and ”→” respectively.

2 Classical aspects of the BFV- formalism

We consider a dynamical system described by a Hamiltonian $H_0 = H_0(q, p)$ in the phase space of canonically conjugate variables $q^i$ and $p_i$

$$\{q^i, p_j\} = \delta^i_j, \quad \varepsilon(q^i) = \varepsilon(p_i) = \varepsilon_i, \quad \text{gh}(q^i) = \text{gh}(p_i) = 0, \quad i = 1, 2, ..., n, \tag{1}$$

and the set of irreducible first-class constraints

$$T_\alpha = T_\alpha(q, p), \quad \varepsilon(T_\alpha) = \varepsilon_\alpha, \quad \alpha = 1, 2, ..., m, \tag{2}$$

which satisfy the involution relations,

$$\{T_\alpha, T_\beta\} = T_\gamma U_{\alpha\beta}^\gamma, \quad \{H_0, T_\alpha\} = T_\beta V_{\alpha\beta}^\beta. \tag{3}$$
The structure coefficients $U_{\alpha\beta}^{\gamma}$ and $V_{\alpha}^{\beta}$ are in general functions of canonical variables $(q, p)$ with the properties $\varepsilon(U_{\alpha\beta}^{\gamma}) = \varepsilon_\alpha + \varepsilon_\beta + \varepsilon_\gamma$ and $\varepsilon(V_{\alpha}^{\beta}) = \varepsilon_\alpha + \varepsilon_\beta$. The symbol $\{ , \}$ means the Poisson superbracket defined for any set of variables $(Q^A, P_A)$, $\varepsilon(Q^A) = \varepsilon(P_A) = \varepsilon_A$ and any two quantities $F = F(Q, P)$ and $G = G(Q, P)$ by the rule

$$\{ F, G \} = F(\frac{\partial}{\partial Q^A} \to \frac{\partial}{\partial P_A} - \frac{\partial}{\partial P_A} \to \frac{\partial}{\partial Q^A}) G. \quad (4)$$

The Poisson superbracket obeys the following basic properties:

1) the Grassmann parity

$$\varepsilon(\{ F, G \}) = \varepsilon(F) + \varepsilon(G). \quad (5)$$

2) the generalized symmetry

$$\{ F, G \} = -\{ G, F \} (-1)^{\varepsilon(F)\varepsilon(G)}, \quad (6)$$

3) the Jacobi identity

$$\{ \{ F, G \}, H \} (-1)^{\varepsilon(F)\varepsilon(H)} + \text{cycle}(F, G, H) \equiv 0. \quad (7)$$

Description of a given dynamical system within the framework of BFV-formalism on classical level involves introduction of additional degrees of freedom. For theories with the irreducible first-class constraints (2) they include the set of variables

$$(C^\alpha, P_\alpha), \quad \varepsilon(C^\alpha) = \varepsilon(P_\alpha) = \varepsilon_\alpha + 1, \quad \text{gh}(C^\alpha) = -\text{gh}(P_\alpha) = 1, \quad \alpha = 1, 2, ..., m. \quad (8)$$

With the help of these variables one introduces the minimal extended phase space of the BFV-formalism with the coordinates

$$(Q^A, P_A) = (q^i, p_i, C^\alpha, P_\alpha). \quad (9)$$

On the minimal extended phase space (9), the BRST-BFV charge, $\Omega = \Omega(Q, P)$, $\varepsilon(\Omega) = 1$, and the generalized Hamiltonian, $\mathcal{H} = \mathcal{H}(Q, P)$, $\varepsilon(\mathcal{H}) = 0$, are defined as solutions to the equations

$$\{ \Omega, \Omega \} = 0, \quad \{ \mathcal{H}, \Omega \} = 0, \quad (10)$$

and the boundary conditions

$$\Omega\frac{\partial}{\partial C^\alpha}\big|_{C=0} = T_\alpha, \quad \mathcal{H}\big|_{C=0} = H_0. \quad (11)$$

Solutions to the basic equations (10) with boundary conditions (11) always exist in form of formal series with respect to ghost variables $C^\alpha$ [6], [7]. In the lowest order in the ghosts one gets

$$\Omega = T_\alpha C^\alpha + \frac{1}{2} P_\gamma U_{\alpha\beta}^{\gamma} C^\beta (-1)^{\varepsilon_\alpha} + \cdots, \quad \mathcal{H} = H_0 + P_\beta V_{\alpha}^{\beta} C^\alpha + \cdots. \quad (12)$$

In the case when structure coefficients are constants, the first two summands in (12) present exact solutions to the basic equations (10).

In terms of basic quantities $\Omega$ and $\mathcal{H}$ of the BFV-formalism, the quantization procedure is formulated. Here we do not concern the quantum aspects of the BFV-formalism and concentrate only on classical ones having in mind the deformation.
3 Deformation problem within the framework of canonical formalism

The deformation problem for theories with gauge freedom in Lagrangian formalism is formulated in Ref. [19] (see also [20], [21]). In this section, we are going to discuss aspects of this problem in the context of dynamical systems with constraints in phase space.

We consider the dynamical system described in the previous section. By deformation we will understand mapping characterized by the deformation parameter $g$, transforming the initial quantities $H_0, T_\alpha, \alpha = 1, 2, ..., m$ onto the deformed quantities $\tilde{H}_0, \tilde{T}_\alpha, \alpha = 1, 2, ..., m$ defined in the same phase space, so that the following equations hold

$$\{ \tilde{H}_0, \tilde{T}_\alpha \} = \tilde{T}_\beta \tilde{V}_\alpha^\beta, \quad \{ \tilde{T}_\alpha, \tilde{T}_\beta \} = \tilde{T}_\gamma \tilde{U}_{\alpha\beta}^{\gamma},$$

(13)

supported by the boundary conditions

$$\tilde{H}_0 \big|_{g=0} = H_0, \quad \tilde{T}_\alpha \big|_{g=0} = T_\alpha, \quad \tilde{U}_{\alpha\beta}^{\gamma} \big|_{g=0} = U_{\alpha\beta}^{\gamma}, \quad \tilde{V}_\alpha^\beta \big|_{g=0} = V_\alpha^\beta.$$ (14)

In principle, there exist two ways to solve the deformation problem under consideration. The first one can be based on the solution to the deformation procedure in Lagrangian formalism [19]. Following this way, we may propose to find solutions to the problem in Hamiltonian formalism in the form of series expansion in parameter $g$,

$$\tilde{H}_0 = H_0 + gH(1) + g^2H(2) + \cdots, \quad \tilde{T}_\alpha = T_\alpha + gT_\alpha^{(1)} + g^2T_\alpha^{(2)} + \cdots,$$

$$\tilde{U}_{\alpha\beta}^{\gamma} = U_{\alpha\beta}^{\gamma} + gU_{\alpha\beta}^{(1)\gamma} + g^2U_{\alpha\beta}^{(2)\gamma} + \cdots, \quad \tilde{V}_\alpha^\beta = V_\alpha^\beta + gV_\alpha^{(1)\beta} + g^2V_\alpha^{(2)\beta} + \cdots.$$ (16)

It yields the infinite system of equations,

$$\{ H_0, T_\alpha^{(1)} \} + \{ H_\beta, T_\alpha^{(1)} \} = T_\gamma V_{\alpha\beta}^{(1)\gamma}, \quad \{ H_0, T_\alpha^{(2)} \} + \{ H_\beta, T_\alpha^{(2)} \} = T_\gamma V_{\alpha\beta}^{(2)\gamma}, \quad \{ H_0, T_\alpha^{(n)} \} + \{ H_\beta, T_\alpha^{(n)} \} = T_\gamma V_{\alpha\beta}^{(n)\gamma},$$

(17)

(18)

(19)

and so on. Solutions to these equations seem to be quite a complicated task.

We also can attack the deformation problem considering the extended phase space and applying the BFV-formalism. In this case, the BRST-BFV charge $\tilde{\Omega}$ and the generalized Hamiltonian $\tilde{\mathcal{H}}$ satisfy the generating equations

$$\{ \tilde{\Omega}, \tilde{\mathcal{H}} \} = 0, \quad \{ \tilde{\mathcal{H}}, \tilde{\Omega} \} = 0,$$

(21)

and the boundary conditions

$$\tilde{\Omega} \frac{\partial C}{\partial C} \bigg|_{C=0} = \tilde{T}_\alpha, \quad \tilde{\mathcal{H}} = \tilde{H}_0 + \cdots,$$

(22)

where "..." in the last expression correspond to power-series expansions in the ghost variables. We can search for the solutions to the basic equations (21) in the form of series in deformation parameter $g$,

$$\tilde{\Omega} = \Omega + g\Omega(1) + g^2\Omega(2) + \cdots, \quad \tilde{\mathcal{H}} = \mathcal{H} + g\mathcal{H}(1) + g^2\mathcal{H}(2) + \cdots,$$

(23)

where $\Omega$ and $\mathcal{H}$ are the BRST-BFV charge and the Hamiltonian respectively for initial system. They satisfy the basic equations (10). As consequence, such a deformation procedure is described by the following infinite system of equations

$$2\{ \Omega, \Omega(1) \} = 0, \quad 2\{ \Omega, \Omega(2) \} + \{ \Omega(1), \Omega(1) \} = 0,$$

$$\{ \mathcal{H}, \Omega(1) \} + \{ \mathcal{H}(1), \Omega \} = 0, \quad \{ \mathcal{H}(1), \Omega(1) \} + \{ \mathcal{H}(2), \Omega \} = 0,$$

(24)

(25)
and so on. Note that solutions to the equations (24), (25) for some simple dynamical systems (in lower orders) are searched with the help of the cohomological methods (see e.g. [22], [23], [24], [25]). It is clear that it is difficult to expect to find a general solution to the infinite system of equations (21) in a compact and closed form. The above deformation procedure is described as an infinite series and therefore leads to the obvious problem of its convergence. Besides, it is unclear from the very beginning that the sum of the series really exists and whether it is local or no. In particular, it is not difficult to construct an example of non-local functional whose each term of expansion in power series will be local.

In our paper, we propose a new way for describing the consistent deformations of dynamical systems with constraints, which does not require expansion into series at all. This new way is based on the fundamental property of the basic equations (21) and (10) being invariant under (super)canonical transformations in the extended phase space.

4 Solutions to deformation problem

In this section, we describe a general approach to the solution of the deformation problem. Let \((Q^A, P_A) = (q^i, p_i; C^a, P_a)\) are the initial minimal extended phase space variables satisfying the commutation relations in terms of (super)Poisson brackets

\[ \{q^i, p_j\} = \delta^i_j, \quad \{C^a, P_b\} = \delta^a_b, \quad (26) \]

In this phase space, one considers the (super)canonical transformation of the form

\[ Q'^A = \overrightarrow{\partial}_{P_A} Y(Q, P'), \quad P_A = Y(Q, P') \overleftarrow{\partial}_{Q^A}, \quad (27) \]

where \(Y(Q, P')\), \((\varepsilon(Y(Q, P') = 0)\) is the corresponding generating function of the transformation. Then the canonically transformed BRST-BFV charge

\[ \Omega' = \Omega'(Q, P) = \Omega(Q'(Q, P), P'(Q, P)), \quad (28) \]

and the Hamiltonian

\[ H' = H'(Q, P) = H(Q'(Q, P), P'(Q, P)) \quad (29) \]

automatically satisfy the same basic equations

\[ \{\Omega', \Omega\} = 0, \quad \{H', \Omega\} = 0 \quad (30) \]

as well as \(\Omega\) and \(H\) (10).

We will search for the generating function of the canonical transformation in the form

\[ Y(Q, P') = P'_AQ^A + X(Q, P'), \quad (31) \]

where first term means the identical transformation and \(X(Q, P')\) is some unknown yet function. It yields

\[ Q'^A = Q^A + \overrightarrow{\partial}_{P'_A} X(Q, P'), \quad P_A = P'_A + X(Q, P') \overleftarrow{\partial}_{Q^A} \quad (32) \]

For further consideration, we choose the function \(X(Q, P')\) as follows

\[ X(Q, P') = p'_ih^i(q) \quad (33) \]

with some new function \(h^i(q)\). Then, it is easy to see that such a canonical transformation non-trivially affects only the variables \(q^i, p_i\) of the initial phase space. The corresponding transformation has the form

\[ q'^i = q^i + h^i(q), \quad p'_i = p_j(M^{-1}(q))_i^j, \quad (34) \]
where the matrix \( (M^{-1}(q))^{j}_i \) is inverse to the following one

\[
M^i_j(q) = \delta^i_j + h^i(q) \frac{\partial}{\partial q^j}.
\]  

(35)

As a result, one gets for the transformed BRST-BFV charge and Hamiltonian the following expressions

\[
\tilde{\Omega} = \tilde{\Omega}(Q, P) = \Omega(q + h(q), pM^{-1}(q), C, \mathcal{P}),
\]

\[
\tilde{H} = \tilde{H}(Q, P) = H(q + h(q), pM^{-1}(q), C, \mathcal{P}),
\]

(36)  

(37)

which satisfy the equations

\[
\{\tilde{\Omega}, \tilde{\Omega}\} = 0, \quad \{\tilde{H}, \tilde{\Omega}\} = 0
\]

(38)

and the boundary conditions

\[
\tilde{\Omega} \frac{\partial}{\partial C^\alpha} \bigg|_{C=0} = \tilde{T}_\alpha, \quad \tilde{H} = \tilde{H}_0 + \cdots,
\]

(39)

where \( \cdots \) means the corrected terms stipulated by the transformations (37). Here we have used the notations

\[
\tilde{H}_0 = \tilde{H}_0(q, p) = H_0(q + h(q), pM^{-1}(q)),
\]

\[
\tilde{T}_\alpha = \tilde{T}_\alpha(q, p) = T_\alpha(q + h(q), pM^{-1}(q)).
\]

(40)  

(41)

In its turn, the functions \( \tilde{H}_0 \) and \( \tilde{T}_\alpha \) satisfy the involution equations

\[
\{\tilde{H}_0, \tilde{T}_\alpha\} = \tilde{T}_\beta \tilde{V}_\alpha^\beta, \quad \{\tilde{T}_\alpha, \tilde{T}_\beta\} = \tilde{T}_\gamma \tilde{U}^\gamma_{\alpha\beta},
\]

(42)

where

\[
\tilde{V}_\alpha^\beta = \tilde{V}_\alpha^\beta(q, p) = V_\alpha^\beta(q + h(q), pM^{-1}(q)),
\]

\[
\tilde{U}^\gamma_{\alpha\beta} = \tilde{U}^\gamma_{\alpha\beta}(q, p) = U^\gamma_{\alpha\beta}(q + h(q), pM^{-1}(q)).
\]

(43)  

(44)

Thus, we see that the deformed theory is described by the first-class constraints \( \tilde{T}_\alpha \), Hamiltonian \( \tilde{H}_0 \) and structure functions \( \tilde{V}_\alpha^\beta \), \( \tilde{U}^\gamma_{\alpha\beta} \) which are constructed in the explicit form on the base of constraints \( T_\alpha \), Hamiltonian \( H_0 \) and the structure functions \( V_\alpha^\beta \), \( U^\gamma_{\alpha\beta} \) of the initial theory with help of an arbitrary function \( h(q) \). It is important to point out that no any a’priory restrictions of the function \( h(q) \) are not imposed 3.

It is also worth noting that the deformed BRST-BFV charge and Hamiltonian have the following structure

\[
\tilde{\Omega} = \tilde{T}_\alpha C^\alpha + \frac{1}{2} \mathcal{P}_\gamma \tilde{U}^\gamma_{\alpha\beta} C^\beta C^\alpha(-1)^{\epsilon^\alpha} + \cdots, \quad \tilde{H} = \tilde{H}_0 + \mathcal{P}_\beta \tilde{V}_\alpha^\beta C^\alpha + \cdots,
\]

(45)

that repeats the power-series of non-deformed quantities with replacement of all initial structure functions on the deformed ones.

Thus, we have constructed a special type of canonical transformation which maps an initial dynamical system with first-class constraints into the deformed dynamical system with first-class constraints. The new dynamical system fully satisfies all the properties of deformation procedure mentioned in the previous section. We have proved that the deformation procedure in the generalized canonical formalism can be described by only one generating function \( h(q) \), which depends on the coordinates \( q^i \) of the initial phase space. Moreover, it follows from the

3 Of course it is assumed that this function is differentiable and transforms as coordinates.
above relations that the function $h(q)$ indeed plays the role of the deformation parameter. We see that the description of the deformation procedure in the canonical formalism is similar to the description in the Lagrangian formalism [11, 2], where also only one arbitrary function is needed to encode the deformation. Compared to the cohomological approach to the problem of deformation, our method allows us to give general answers to questions concerning the structure of the deformation. First, all the arbitrariness in the deformation procedure is completely controlled by one generating function $h(q)$ of initial coordinates. Secondly, the result is completely obtained in a closed form and does not require expansion into series. In fact, this result can be interpreted as the sum of a series in the cohomological approach. Thirdly, in obtaining the general result, no assumptions are made about the locality or non-locality of the deformation structure. Therefore, in the general case, the deformation must be nonlocal.

5 Deformation of Abelian gauge theory into non-Abelian

In this section, we will show that within the framework of the canonical formalism one can construct such a deformation that transforms an Abelian gauge theory into some non-local non-Abelian gauge theory with a local sector corresponding to Yang-Mills theory. The structure of a gauge theory in the canonical formalism is defined by a system of first-class constraints and the Hamiltonian in involution with these constraints. Therefore, we will show that there exists a non-local deformation of the fields and conjugate momenta, which transforms, in local sector, the constraints and the Hamiltonian of the Abelian gauge theory into the constraints and the Hamiltonian of the Yang-Mills theory. The concrete choice of the function $h^i$ for appropriate deformation is based on the relations [34], [35].

Let us consider the $N$ copies of the Abelian gauge theory with fields $A^a_\mu$, $a = 1, 2, \ldots, N$; $\mu = (0, i)$; $i = 1, 2, 3$. In the canonical formalism, such a theory is described by the coordinates $A^a_0(\vec{x})$, $A^a_1(\vec{x})$ and the conjugate momenta $p^a_i(\vec{x})$, $p_i^a(\vec{x})$ obeying the first-class constraints

$$
p_i^a = 0, \quad \partial_i p_i^a = 0. \quad (46)
$$

The corresponding Hamiltonian has the form

$$
H_{\text{Abelian}} = \int d^3 x \left( \frac{1}{2} p_i^a p_i^a - A^a_0 \partial_i p_i^a + \frac{1}{4} F^a_{ij} F_i^a \right), \quad (47)
$$

where $F^a_{ij} = \partial_i A^a_j - \partial_j A^a_i$.

On the other hand, one considers the Yang-Mills theory with the fields $A^a_\mu$ and the structure constants $f^{abc}$. In the canonical formalism this theory is described by the coordinates $A^a_0(\vec{x})$, $A^a_i(\vec{x})$ and the conjugate momenta $p^a_0(\vec{x})$, $p_i^a(\vec{x})$ obeying the first-class constraints

$$
p_0^a = 0, \quad D_i^{ab} p_i^b = 0, \quad (48)
$$

where

$$
D_i^{ab} p_i^b = \partial_i p_i^a + g f^{abc} A^c_i p_i^b \quad (49)
$$

and $g$ is a coupling constant. The corresponding Hamiltonian has the form

$$
H_{\text{non-Abelian}} = \int d^3 x \left( \frac{1}{2} p_i^a p_i^a - A^a_0 D_i^{ab} p_i^b + \frac{1}{4} G^a_{ij} G_i^a \right), \quad (50)
$$

where $G^a_{ij} = \partial_i A^a_j - \partial_j A^a_i + g f^{abc} A^b_i A^c_j$. Derivation of the constraints and the Hamiltonians for Abelian and non-Abelian gauge theories see e.g. in [26], [27].

\[\text{In this section } i, j, k \text{ mean three-dimensional spatial indices.}\]
We will explore the deformation (34) of the form

\[ A_0^a = A_0^a, \quad p_0^a = p_0^a, \]
\[ A_i^a(x) = A_i^a(x) + h_i^a(x), \quad p_i^a(\vec{x}) = \int d^3 y \, p_j^b(\vec{y})(M^{-1})_{ji}^b(\vec{y}, \vec{x}), \]

where \( h_i^a \) depends on \( A_i^a(\vec{x}) \) and, accordingly to (35), the matrix \((M)^{ab}_{ij}(\vec{x}, \vec{y})\) looks like

\[ (M)^{ab}_{ij}(\vec{x}, \vec{y}) = \delta^{ab} \delta_{ij} \delta(\vec{x} - \vec{y}) + \frac{\delta h_i^a(\vec{x})}{\delta A_j^b(\vec{y})}. \]

with \( \vec{x} = (x^i; i = 1, 2, 3) \).

Let us consider the non-local function \( h_i^a(\vec{x}) \) in (52) as follows

\[ h_j^b = gf^{bcd} \frac{1}{\Delta} \left( \partial_k (A^c_k A^d_j) + \frac{1}{4} F^{cmn} A^d_k A^m_k A^n_j \right), \]

where \( \frac{1}{\Delta} \) is the inverse Laplacian. Here and further, the spatial indices \( \vec{x}, \vec{y} \) suppressed. It is evident that the obtained deformed theory should be non-local. Our goal is to explore the possibility that such a nonlocal theory has a local sector and to describe this local sector.

Taking into account the expression (53), we get for inverse matrix

\[ (M^{-1})_{ji}^{ba} = \delta^{ab} \delta_{ij} - \frac{\delta h_j^b}{\delta A_i^a} + \ldots, \]

where \( \ldots \) mean the terms of higher order in coupling constant. The relation (55) yields

\[ (M^{-1})_{ji}^{ba} = \delta^{ab} \delta_{ij} + gf^{bcd} \partial_i A^d_j + \]
\[ + \text{the non-local terms unessential for further consideration}. \]

The result (56) allows to write the deformation of the momenta \( p_i^{a}(\vec{x}) \) in (52) as follows

\[ p_i^{a'} = p_i^{a} + gf^{bad} p_j^b \partial_i \Delta^{-1} A_j^d + \]
\[ + \text{the non-local terms unessential for further consideration}. \]

Substituting the expression (57) into Abelian constraint \( \partial p_i^{a} \) one obtains

\[ \partial_i p_i^{a'} = D_i^{ab} p_i^b(\vec{x}) + \text{non-local terms}. \]

The local term in (58) corresponds to constraint in the non-Abelian theory (48). Another constraint \( p_0^a = 0 \) is fulfilled automatically since the \( p_0^a \) does not transform.

Now we turn to the Hamiltonian \( H_{\text{Abelian}} \) (17) and perform the transformations (52) with function \( h_i^a(\vec{x}) \) (54). It is evident that \( p_i^{a'} p_i^{a'} = p_i^{a} p_i^{a} + \text{non-local terms} \) and also \( A_i^a \partial_i p_i^{a} = A_i^a D_i^{ab} p_i^b + \text{non-local terms} \), where we have used the relation (59). The expression \( F_{ij}^a F_{ij}^a \) in the Hamiltonian \( H_{\text{Abelian}} \) (17) is transformed by the same way as the term \( F^{\mu \nu} F_{\mu \nu} \) in section 5 of ref. 11 with replacement \( \mu, \nu, \Box \) by \( i, j, \Delta \) respectively. That is, \( F_{ij}^a F_{ij}^a \) takes the form \( G_{ij}^a G_{ij}^a + \text{non-local terms} \). Therefore, the Hamiltonian \( H_{\text{Abelian}} \) (17) transforms into the Hamiltonian \( H_{\text{non-Abelian}} \) (50) up to non-local terms.

Thus, we see that the deformation (52) with non-local function \( h_i^a \) (54) maps Abelian gauge theory into some non-local non-Abelian gauge theory that contains a local sector coinciding with Yang-Mills theory.
6 Summary

In the present paper we have studied the deformation problem for an arbitrary dynamical system with irreducible first-class constraints in phase space and proposed a general method of solution to this problem. Our approach is based on the BFV-formalism, where the dynamical system is formulated in the extended phase space and its gauge structure is described by the BRST-BFV charge $\Omega$ and the generalized Hamiltonian $\mathcal{H}$ satisfying the equations (10) in terms of the Poisson superbrackets. Structure of the initial dynamical system is encoded by the boundary conditions (11). The generating equations (10) are invariant under (super)canonical transformations of the phase space variables (30).

We describe the deformation procedure in terms of special minimal (super)canonical transformation and construct this transformation in explicit and closed form. It is proved that such a transformation is defined by a single generating function $h(q)$ which depends only on coordinates of the initial phase space. Deformations of the initial Hamiltonian $H_0$ and the constraints $T_\alpha$ are described as change of variables in the arguments of these quantities, namely the shift of coordinates, $q^i + h^i(q)$, and the rotation of momenta, $p_j(M^{-1}(q))_j^i$, with the matrix inverse to matrix $M^i_j(q) = \delta^i_j + h^i_j(q)\partial_{q^j}$. The function $h(q)$ is completely arbitrary, there is no a’priory requirements on this function, in particular, it can be non-local. As a result, we find a solution to the deformation problem in closed and constructive form. The choice of the generating function just as $h(q)$ corresponds in fact to fixing the arbitrariness existing in the Hamiltonian gauge algebra for structure functions of involution equations [28].

To illustrate the possibilities of our method, we have applied the developed deformation procedure to system of several copies of an Abelian gauge theory formulated in the canonical formalism. We have constructed an appropriate function $h(q)$ mapping such a theory into a non-local non-Abelian gauge theory that has a local sector coinciding with the Yang-Mills theory in canonical formalism.

One can expect that the considered deformation procedure within the framework of BFV-formalism can be applied to construct the new gauge theories on the based of known gauge theories. As we have already mentioned, the deformation procedure is in general non-local. We believe that the most interesting aspect of applying our deformation procedure to concrete theories is the possibility of that a local sector exists in a non-local theory after deformation. It means that the non-local deformed action and non-local deformed gauge transformation may in principle have completely local sector which define a new local gauge theory obtained on the base of the given local gauge theory. These aspects deserve a special study in case of each concrete theory.

It is worth mentioning out that just the BFV-formalism is used in the modern higher spin theory to derive the interacting vertices for higher spin fields (see e.g. the recent papers [29], [30], [31] and the references therein). One can expect that the developed deformation procedure could allow to generate the interacting vertices by the deformation of free higher spin theories. Such a possibility in principle was demonstrated in [11] within the framework of the Lagrangian deformation procedure. We plan to consider the applications of our canonical deformation procedure in higher spin field theory in forthcoming papers.

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\[5\]See the new applications of this approach in higher spin field theory in [32], [33].
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