δT/T and Neutrino Masses in SU(5)

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Abstract

We implement inflation in a supersymmetric SU(5) model with U(1) R-symmetry such that the cosmic microwave anisotropy δT/T is proportional to \((M/M_{\text{Planck}})^2\), where \(M \sim M_{\text{GUT}} = 2 - 3 \times 10^{16}\) GeV, the SU(5) breaking scale, and \(M_{\text{Planck}} = 2.4 \times 10^{19}\) GeV. The presence of a global U(1)\(_X\) symmetry, spontaneously broken also at scale \(M_{\text{GUT}}\), provides an upper bound \(M_{\text{GUT}}^2/M_{\text{Planck}}^2 \sim 10^{14}\) GeV on the masses of SU(5) singlet right handed neutrinos, which explains the mass scale associated with atmospheric neutrino oscillations. The SU(5) monopoles and U(1)\(_X\) cosmic strings are inflated away. Although the doublet-triplet splitting requires fine-tuning, the MSSM \(\mu\) problem is resolved and dimension five proton decay is strongly suppressed.

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In a class of supersymmetric (SUSY) models of inflation associated with some symmetry breaking $G \rightarrow H$, the cosmic microwave anisotropy $\delta T/T$ turns out to be proportional to $(M/M_{\text{Planck}})^2$, where $M$ denotes the symmetry breaking scale of $G$, and $M_{\text{Planck}} = 1.2 \times 10^{19}$ GeV [1, 2]. Comparison with the determination of $\delta T/T$ by WMAP and several other experiments [3] leads to the conclusion [1, 4] that $M$ is comparable to $M_{\text{GUT}} \sim 2-3 \times 10^{16}$ GeV, the scale at which the three gauge couplings of the minimal supersymmetric standard model (MSSM) unify. In such models, the scalar spectral index of density fluctuations including supergravity (SUGRA) corrections is $n_s \approx 0.98 - 1.00$ [4, 5], in excellent agreement with observations [3]. An essential role in the discussion is played by a global $U(1)$ symmetry which helps determine the form of the tree level superpotential and has other important phenomenological implications. For instance, the $Z_2$ subgroup of $U(1)_R$ can be identified with the “matter parity” in the MSSM. This ensures the absence of rapid proton decay and stability of the LSP. Examples of $G$ include the groups $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$ [6], discussed in the context of inflation in Refs. [7] and [8], respectively.

Among supersymmetric grand unified theories (GUTs) $SU(5)$ [9] certainly is the simplest with the most direct connection to $M_{\text{GUT}}$ given above. It is therefore natural to try to realize this type of inflationary scenario in an $SU(5)$ framework [10]. The minimal $SU(5)$ model does not incorporate inflation and it also fails to provide an understanding of neutrino masses inferred from neutrino oscillations [11] and from cosmological considerations [3]. Our goal in this paper is to provide a suitable extension which can overcome both these shortcomings.

We have already indicated the important role to be played by the $U(1)_R$ symmetry. Next we consider another symmetry, namely $U(1)_X$, which also will be important for our analysis. This is a global symmetry of the minimal model [12] and its presence prevents the appearance of tree level superheavy masses ($\gtrsim M_{\text{GUT}}$) for the $SU(5)$ singlet right handed neutrinos. In our extended model, the breaking of $U(1)_X$ trig-
gers the $SU(5)$ breaking at $M_{GUT}$. Through non-renormalizable couplings, the right
handed neutrinos acquire masses of order $M_{GUT}^2/M_P \sim 10^{14} - 10^{15}$ GeV, where $M_X$
and $M_P$ ($\equiv M_{\text{Planck}}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV) denote the $U(1)_X$ breaking scale and the
reduced Planck mass. This can yield, via the seesaw mechanism \[13\], a light neutrino
mass of order $M_{W}^2/(10^{14}$ GeV) $\sim 10^{-1}$ eV, which is the appropriate mass scale for
atmospheric neutrino oscillations. With both the global $U(1)_X$ and $SU(5)$ spontaneously
broken during inflation, the $SU(5)$ monopoles and $U(1)_X$ cosmic strings are
inflated away. At the end of inflation the oscillating fields produce right handed neu-
trinos whose out of equilibrium decay lead via leptogenesis to the observed baryon
asymmetry of the universe \[14\]. The doublet-triplet splitting problem requires, as
usual, some fine-tuning. However, the mechanism which generates the MSSM $\mu$ term
after SUSY breaking also strongly suppresses dimension five proton decay mediated
by the superheavy color triplets higgsinos.

Let us recall the well known superpotential \[1, 15\],
\[
W = \kappa S(\phi\bar{\phi} - M^2) ,
\]
where $\kappa$ is a dimensionless coupling constant, and the superfields $\phi$ and $\bar{\phi}$ transform
non-trivially under $G$, with the gauge invariant combination $\phi\bar{\phi}$ carrying zero $U(1)_R$
charge. The singlet superfield $S$ and the superpotential $W$ carry unit $U(1)_R$ charges.
$S$ provides the scalar field that drives inflation. Note that the $U(1)_R$ symmetry ensures
the absence of terms proportional to $S^2$, $S^3$, etc in the superpotential, which otherwise
could spoil the slow-roll conditions needed for implementing successful inflation.\(^3\)
From $W$, it is straightforward to show that the SUSY minimum corresponds to non-
zero (and equal in magnitude) vacuum expectation values (VEVs) for $\phi$ and $\bar{\phi}$, while
$\langle S \rangle = 0$, and therefore $G$ is broken to a subgroup $H$.

An inflationary scenario is realized in the early universe with $\phi$, $\bar{\phi}$ and $S$ dis-

\(^3\)The issue of SUGRA corrections is more subtle. For $W$ given in Eq. \[1\], and the minimal Kähler
potential $K$, the flatness condition is not spoiled \[2\]. However, in some models such as the $SU(5)$
case discussed here, additional fields appear (from a hidden sector as well as the visible sector), and
special choices for the Kähler potential may be necessary to control SUGRA corrections \[10, 11\].
placed from their present day minima. Thus, for $S$ values in excess of the symmetry breaking scale $M$, the fields $\phi, \bar{\phi}$ both vanish, the gauge symmetry is restored, and a potential energy density proportional to $\kappa^2 M^4$ dominates the universe. With SUSY thus broken, there are radiative corrections from the $\phi-\bar{\phi}$ supermultiplets that provide logarithmic corrections to the potential which drives inflation \[1\]. After including the radiative corrections, the scalar potential from Eq. (1) turns out to give a scalar spectral index $n_s \approx 1 - 1/N$ (for $N \approx 50 - 60$ e-foldings) \[1\] and the quadrupole anisotropy $\delta T/T \equiv 8\pi \sqrt{N} (N_{l}^{45})^{1/2} x_l^{-1} y_l^{-1} f(x_l^{2})^{-1}$. \[2\]

where $x_l^2 = (|S|^2/M^2)_l$, $y_l \approx x_l(1 - 7/12x_l^2 + \cdots)$, $f(x_l^{2})^{-1} \approx 1/x_l^2$, for $S$ sufficiently larger than $M$. The subscript $l$ is there to emphasize the epoch of horizon crossing. $N$ indicates the dimensionality of the $\phi, \bar{\phi}$ representations, and $N_l \approx 45 - 60$ denotes the e-foldings needed to resolve the horizon and flatness problems. Comparison of Eq. \[2\] with the COBE and WMAP data \[3\] leads to the conclusion that the gauge symmetry breaking scale $M$ is very close to $10^{16}$ GeV \[4\], the SUSY GUT scale inferred from the evolution of the MSSM gauge couplings \[18\]. Thus, it is natural to try to realize the above inflationary scenario within a SUSY GUT framework. In this paper, we will attempt to provide a realistic scenario with $G = SU(5)$ and $H = SU(3)_c \times SU(2)_L \times U(1)_Y$.

The minimal $SU(5)$ model possesses a global $U(1)_X$ symmetry, where $X$ is given by the relation $B - L = X + \frac{2}{3}Y$, which holds for the MSSM fields, with $Y$ denoting the hypercharge. Note that $X$ coincides with $B - L$ for the $SU(5)$ singlet right handed neutrinos. By imposing a $U(1)_X$ symmetry one prevents the appearance of superheavy ($\gtrsim M_{GUT}$) masses for the right handed neutrinos $1_i$ ($i = 1, 2, 3$). The atmospheric neutrino data and leptogenesis scenario seem to require right handed neutrino of intermediate masses ($\lesssim 10^{14} - 10^{15}$ GeV). The masses for $1_i$ can arise from the spontaneous breaking of $U(1)_X$, for instance, via the non-renormalizable
couplings with an extra superfield $\mathcal{X}$:

$$\frac{y_{ij}}{M_P} \mathcal{X} \mathcal{X} 1_i 1_j,$$

(3)

where the $U(1)_X$ charge of $\mathcal{X}$ is listed in Table I, and the dimensionless coupling $y_{ij}$ are of order unity or smaller. A mass of order $10^{14} - 10^{15}$ GeV for the heaviest right handed neutrino can nicely explain via the seesaw mechanism the atmospheric neutrino mass scale ($\sqrt{\Delta m^2_{\text{ATM}}} \sim 5 \times 10^{-2}$ eV). This can be achieved with $\langle \mathcal{X} \rangle \sim M_{\text{GUT}}$. Hence, it would be desirable to construct a model such that the $U(1)_X$ and $SU(5)$ breakings are intimately linked.

If $\mathcal{X}$ is identified with the inflaton, as will be the case in our model, the coupling Eq. (3) can also lead to a successful leptogenesis [14]. The inflaton $\mathcal{X}$ exclusively decays into right handed neutrinos, and the reheat temperature $T_r$ turns out to be $\sim 10^{-1} M_{\nu^c}$ [8], where $M_{\nu^c}$ is the heaviest right handed neutrino mass allowed by the kinematics of the decay process. From the gravitino constraint $T_r \lesssim 10^9$ GeV [19], $M_{\nu^c} \sim 10^{10}$ GeV is required. Thus, the mass of $\mathcal{X}$ is constrained as follows:

$$m_{\mathcal{X}} \lesssim 10^{14} \text{ GeV},$$

(4)

and the mass(es) of the right handed neutrino(s) lighter than $\mathcal{X}$ should be $\lesssim 10^{10}$ GeV. In practice, $m_{\mathcal{X}}$ is taken to be of order $10^{12} - 10^{14}$ GeV, so that decay into the heaviest right handed neutrino is not allowed.

The $U(1)_R$ symmetry can forbid the bare mass term $M_{5_H 5_H}$. As a consequence, within this sector, the global symmetry is enhanced to $U(1)_X \times U(1)_{PQ}$, with $U(1)_X$ anomaly free. Motivated by this, we will impose $U(1)_{PQ}$ on the complete model. The $U(1)_R, U(1)_X$ and $U(1)_{PQ}$ charge assignments for the matter, 5-plet Higgs, and the extra superfield are shown in Table I.

|     | $10_i$ | $\mathbf{5}_i$ | $1_i$ | $\mathbf{5}_H$ | $\mathbf{5}_H$ | $\mathcal{X}$ |
|-----|-------|----------------|------|---------------|---------------|--------------|
| $U(1)_R$ | 1/2   | 1/2           | 1/2  | 0            | 0             | 0            |
| $U(1)_X$ | 1/5   | -3/5          | 1    | -2/5         | 2/5           | -1           |
| $U(1)_{PQ}$ | 1/5   | 2/5           | 0    | -2/5         | -3/5          | 0            |

**Table I**
Note that with the charge assignments shown in Table I, the operators leading to baryon and lepton number violations at low energies such as $\bar{5}, 5_H, 10, \bar{5}, 5_k, 10, 10, 10_k \bar{5}_l, 10, 10, 10_k 5_H, \bar{5}, 5, 5_H 5_H, \bar{5}, 5_H 5_H 5_H, \text{etc.}$ are forbidden in the absence of $U(1)_R, U(1)_X$, and $U(1)_{PQ}$ breakings.

To reiterate, the $SU(5)$ model we are after should have the following features.

• $U(1)_R$ and $U(1)_X$ are suitably utilized for the desired inflation and neutrino masses.
• Inflation is associated with the spontaneous breakings of $SU(5)$ and $U(1)_X$ at $M_{GUT}$.
• Monopoles and cosmic strings do not pose cosmological problems.
• The inflaton satisfying Eq. (4) decays only into right handed neutrinos via Eq. (3).
• The low energy spectrum should coincide with the MSSM field content.

Consider the following trial superpotentials

\[ W_1 = \kappa S \left[ \text{Tr} \left( \Phi^+ \Phi^- \right) - M^2 \right], \] (5)

\[ W_2 = S \left[ \kappa \text{Tr} \Phi^2 + \lambda S^+ S^- - \kappa M^2 \right], \] (6)

where $\Phi$’s and $S$’s denote the $24$-plets and singlet superfields, respectively. We normalize the $SU(5)$ generators $T^i$ such that $\text{Tr}(T^iT^j) = \delta^{ij}$. $\kappa, \lambda$ are dimensionless couplings, and the superscripts $\pm$ denote $U(1)_X$ charges of $\pm 1$ for the corresponding Higgs fields. We assign zero (unit) $U(1)_R$ charges to $\Phi^\pm$, $\Phi$, $S^\pm$ ($S$). From Eq. (1), both $W_1$ and $W_2$ appear to be viable candidates for inflation with both $SU(5)$ and $U(1)_X$ broken at the GUT scale. However after inflation ends, in either case, unwanted cosmological defects (monopoles and/or strings) would be produced. Furthermore, the octet and triplet states (and also a pair of $(3, \bar{3}), -5/6$, $(\bar{3}, 2), 5/6$ states) contained in $\Phi (\Phi^\pm)$ remain ‘massless’ after $SU(5)$ breaking. These states can acquire masses of order TeV when the $S$ VEV develops after SUSY breaking. This would seriously affect the running of the MSSM gauge couplings and therefore must be avoided.

Before proceeding to the model, let us consider the Higgs superpotential $W_3$ [20], which we will encounter as a part of the full construction:

\[ W_3 = M \text{Tr} \left( \Phi \Phi' \right) + \lambda \text{Tr} \left( \Phi \Phi' \Phi' \right), \] (7)
where $\Phi$ ($\Phi'$) is a $24$-plet superfield with unit (zero) $U(1)_R$ charge. Compared to minimal $SU(5)$, an additional $24$-plet Higgs $\Phi$ is introduced to realize the $U(1)_R$ symmetry. While $\langle \Phi \rangle = 0$ in the SUSY minimum, the scalar component of $\Phi'$ acquires a VEV ($\sim M/\lambda$) along the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet direction. The $(8,1)_0$, $(1,3)_0$, and $(1,1)_0$ components in $\Phi$ and $\Phi'$ pair up and become superheavy. While $\langle \Phi \rangle = 0$ in the SUSY minimum, the scalar component of $\Phi'$ acquires a VEV ($\sim M/\lambda$) along the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet direction. The $(3,2)_{5/6}$, $(1,3)_0$, and $(1,1)_0$ components in $\Phi$ and $\Phi'$ pair up and become superheavy. While $\text{Im}[(3, \overline{2})_{-5/6}]$ and $\text{Im}[(\overline{3}, 2)_{5/6}]$ obtain superheavy masses from the D-term potential, the goldstone modes $\text{Re}[(3, \overline{2})_{-5/6}]$, $\text{Re}[(\overline{3}, 2)_{5/6}]$ contained in $\Phi'$ are absorbed by the massive gauge bosons when $SU(5)$ is broken. Hence, the components $(3, \overline{2})_{-5/6}$ and $(\overline{3}, 2)_{5/6}$ from $\Phi$ remain ‘massless.’ To make them superheavy, a coupling such as $\langle \Sigma_{-1} \rangle \text{Tr}(\Phi \Phi)$ with a large VEV for $\Sigma_{-1}$ is needed, where $\Sigma_{-1}$ denotes a $SU(5)$ singlet field carrying a $U(1)_R$ charge of $-1$.

For a realization of inflation and $U(1)_X$ in $SU(5)$, we introduce some $SU(5)$ singlets and adjoint Higgs superfields with $U(1)_R$, $U(1)_X$ and $U(1)_{PQ}$ charges shown in Table II.

|       | $S$ | $S^+ | S^-$ | $Z$ | $P$ | $Q$ | $\Phi$ | $\Phi^+$ |
|-------|-----|--------|------|-----|-----|-----|--------|--------|
| $SU(5)$ | 1   | 1      | 1    | 1   | 1   | 1   | 24     | 24     |
| $U(1)_R$ | 1   | 0      | 0    | 2   | -1 | -1   | 1      | 0      |
| $U(1)_X$ | 0   | 1      | -1   | -3  | 2   | 4   | -2     | 1      |
| $U(1)_{PQ}$ | 0   | 0      | 0    | 0   | 0   | 0   | 0      | 0      |

Table II

We identify $S^-$ with $\overline{X}$ in Table I. To make $U(1)_X$ anomaly free, one could introduce, for instance, an additional singlet $(T)$ and an adjoint Higgs $(\Phi^+_1)$ with $U(1)_X$ charges $-3$ and $+1$, respectively. We assign a $U(1)_R$ charge of $1/2$ to $\Phi^+_1$, and a proper non-zero $U(1)_{PQ}$ charge to $T$. Their presence does not affect the inflationary scenario in any significant manner. The relevant renormalizable superpotential is given by

$$W_{\text{infl}} = \kappa_1 S [S^+ S^- - M^2] + Z [\kappa_2 P S^+ - \kappa_3 Q S^-] + \lambda Q \text{Tr}(\Phi \Phi) + \alpha S^+ \text{Tr}(\Phi \Phi^+) + \beta \text{Tr}(\Phi \Phi^+ \Phi^+) ,$$

where $\alpha$, $\beta$, $\lambda$, $\kappa$‘s are dimensionless coefficients. Note that through a suitable re-
definition of the superfields, the parameters in Eq. (8) can all be made real. Several comments are in order here. The $\kappa_1$ terms in Eq. (8) drives inflation accompanied by the spontaneous breaking of $U(1)_X$. Following Ref. (4), we assume that $\kappa_1$ is of order $10^{-3} - 10^{-2}$, which keeps the SUGRA corrections under control. The last two terms in Eq. (8) resemble $W_3$ discussed above, and therefore the $SU(5)$ breaking is triggered by a superlarge VEV of $S^+$. From the $\lambda$ term, the $(3, \Sigma)_{-5/6}$ and $(\Sigma, 2)_{5/6}$ components contained in $\Phi$ obtain superheavy masses with $Q$ acquiring a superlarge VEV via the $\kappa_2, \kappa_3$ terms in Eq. (8). [Through a non-renormalizable term $S^-S^-\Phi^{+}_{1/2}\Phi^{+}_{1/2}/M_P$, the component fields contained in $\Phi^{+}_{1/2}$ all acquire the same heavy mass. Thus, $\Phi^{+}_{1/2}$ leaves intact the unification of the MSSM gauge couplings. The VEV of $\Phi^{+}_{1/2}$ turns out to vanish at the SUSY minimum.] Consequently, without leaving any unwanted light fields, $SU(5) \times U(1)_X$ is broken to the MSSM gauge symmetry.

From Eq. (8), one derives the F-term scalar potential:

$$V_{\text{inf}} = \left| \kappa_1 S^+ S^- - \kappa_1 M^2 \right|^2 + \left| \kappa_2 P S^+ - \kappa_3 Q S^- \right|^2 + \left| \kappa_1 S S^+ - \kappa_3 Z Q \right|^2$$

$$+ \left| \kappa_1 S S^- + \kappa_2 Z P + \alpha \sum_i \Phi_i \Phi_i^+ \right|^2 + \left| \kappa_2 Z S^+ \right|^2 + \left| \kappa_3 Z S^- - \lambda \sum_i \Phi_i \Phi_i^+ \right|^2$$

$$+ \sum_i \left( \left| 2 \lambda Q \Phi_i + \alpha S^+ \Phi_i^+ + \beta \sum_{j,k} d^{ijk} \Phi_j^+ \Phi_k^+ \right|^2 + \left| \alpha S^+ \Phi_i + 2 \beta \sum_{j,k} d^{ijk} \Phi_j^+ \Phi_k^+ \right|^2 \right),$$

where $d^{ijk} \equiv \text{Tr}(T^iT^jT^k)$, and $\Phi_i^{(+)}$ ($i = 1, 2, 3, \cdots, 24$) denotes the component fields of the 24-plet Higgs. At the SUSY minimum of Eq. (9), the VEVs of $S, Z, \Phi$ vanish

$$\langle S \rangle = \langle Z \rangle = \langle \Phi_i \rangle = 0 , \quad i = 1, 2, 3, \cdots, 24 .$$

In the presence of soft SUSY breaking terms these fields can acquire VEVs of as much as a TeV. On the other hand, $S^\pm, P, Q, \Phi^+$ can develop VEVs such that $SU(5) \times U(1)_X$ is spontaneously broken to the MSSM gauge group,

$$\langle S^+ \rangle \langle S^- \rangle = M^2 , \quad \kappa_2 \langle P \rangle \langle S^+ \rangle = \kappa_3 \langle Q \rangle \langle S^- \rangle ,$$

$$|\langle \Phi^+_i \rangle| = \frac{\alpha \langle S^+ \rangle}{\beta d^{\text{GUT}}_{\text{SSS}}} \equiv M_{\text{GUT}} \sim M ,$$

$$\langle S \rangle = \langle Z \rangle = \langle \Phi_i \rangle = 0 , \quad i = 1, 2, 3, \cdots, 24 .$$
where \( \Phi^+_8 \) is the MSSM singlet contained in \( \Phi^+ \) (\( \langle \Phi^+_i \rangle = 0 \) for \( i \neq 8 \)), and \( d^{888} \) is given by \( \frac{1}{\sqrt{30}} \). By performing an appropriate \( U(1)_X \) transformation, we can rotate away the phase of \( \langle S^+ \rangle \). We identify \( \langle \Phi^+_8 \rangle \) with \( M_{\text{GUT}} \approx 2 - 3 \times 10^{16} \) GeV. We assume \( \langle S^+ \rangle \sim \langle S^- \rangle \sim M \sim M_{\text{GUT}} \), and \( \langle P \rangle \sim \langle Q \rangle \sim 10^{17} \) GeV with \( U(1)_R \) broken to \( Z_2 \) ("R-parity"). The VEVs of \( P \) and \( Q \) could be stabilized by including the soft terms and the SUGRA corrections to the scalar potential. We will see later that \( \kappa_{2,3} \) should be of order \( 10^{-4} - 10^{-3} \), while \( \lambda, \alpha, \beta \) are of order unity.

With \( U(1)_X \) broken at the \( M_{\text{GUT}} \) scale, we get an upper bound on the right handed neutrino Majorana mass by identifying \( S^- \) with \( X \) in Eq. (8):

\[
\frac{\langle S^- S^- \rangle}{M_P} \sim 10^{14} - 10^{15} \text{ GeV.}
\]  

Assuming the heaviest right handed neutrino of this mass, and with a Dirac neutrino mass for the third family of order the electroweak scale, we find a light neutrino mass
\[
M_W^2/(10^{14} \text{ GeV}) \sim \sqrt{\Delta m_{\text{ATM}}^2} \sim 5 \times 10^{-2} \text{ eV.}
\]

Next let us discuss how to implement doublet-triplet splitting. With some additional superfields, whose quantum numbers appear in Table III,

| \( SU(5) \) | \( H \) | \( \bar{H} \) | \( \Delta \) | \( C \) | \( A \) | \( B \) | \( N \) | \( \bar{N} \) |
|---|---|---|---|---|---|---|---|---|
| \( U(1)_R \) | 1 | 1 | 2 | -1 | -1 | 0 | 1/2 | 0 |
| \( U(1)_X \) | -7/5 | -3/5 | 0 | 0 | 2 | -2 | 0 | 0 |
| \( U(1)_{PQ} \) | 3/5 | 2/5 | 3/2 | -3/2 | -1 | -1/2 | 1/2 | -1/2 |

Table III

the relevant superpotential is

\[
W_H = y_a H \left[ S^+ + a \Phi^+ \right] \bar{5}_H + y_b \bar{H} \left[ S^+ + b \Phi^+ \right] 5_H + \Delta \left[ y_c MC - y_d AB \right] + y_e AH \bar{H} + \frac{y_\mu}{M_P} N N 5_H \bar{5}_H + \frac{y_n}{M_P} N N N N ,
\]

whose presence does not affect the conclusions based on Eq. (8). [The \( y \)'s and \( a, b \) denote dimensionless couplings.] From the \( y_c, y_d \) terms and the soft terms, \( C, \)
A, B can develop VEVs of order $M$, while $\langle \Delta \rangle$ is of order the gravitino mass scale $m_{3/2}$. Thus, $U(1)_{PQ}$ is also broken at $M_{GUT}$, but presumably this is acceptable within an inflationary cosmology \cite{21}. The $y_e$ term in Eq. (11) ensures that the additional 5-plets acquire superheavy masses. The VEVs $\langle S^+ \rangle, \langle \Phi^+ \rangle$ would make $5_H$ superheavy. Thus, two fine-tunings, $a = b = \sqrt{10/3} \langle S^- \rangle/\langle \Phi^- \rangle$ are necessary to obtain a pair of light Higgs doublets from $5_H, 5_H$. The non-renormalization theorem in SUSY ensures that such fine-tunings are stable against radiative corrections. We assume that $y$’s and $a (= b)$ are of order unity.

Due to the presence of SUSY breaking “$A$-terms,” the scalar components of $N, \bar{N}$ acquire intermediate scale VEVs, $\sim \sqrt{m_{3/2} M_P} \sim 10^{10}$ GeV. Consequently, a $\mu$ parameter ($\equiv y_\mu \langle NN \rangle/M_P$) of order $m_{3/2}$ is naturally generated \cite{22}. The presence of $U(1)_{PQ}$ resolves the strong CP problem \cite{23}.

The non-zero VEVs of $N, \bar{N}$ also break the $Z_2$ symmetry $\subset U(1)_R$, which can lead to a cosmological domain wall problem. We therefore assume that the VEVs $\langle N \rangle, \langle \bar{N} \rangle$ develop before or during inflation, so that the domain walls are inflated away. With $10,5_j \bar{5}_H, 5,5_H \langle N \bar{N} S^+ \rangle/M_P^2$, and the induced $\mu$ term in Eq. (14), the trilinear operator $10,5_j \bar{5}_k \langle S^- S^- A \rangle/M_P$ leading to “$R$-parity” violation in the MSSM is generated with a suppression factor $\mu^2/(M_P M_{GUT})$. If one requires an absolutely stable LSP, one could introduce a $Z_2$ “matter parity.”

Before discussing some aspects of inflation, let us point out an important consequence of the model related to proton decay. Because of the absence of a direct coupling $5_H \bar{5}_H$ in the $U(1)_R$ symmetric limit, the higgsino mediated dimension five operator relevant for proton decay, $10,10_j 10_k \bar{5}_i$, is suppressed by $\mu/M_{GUT}^2 \sim M_W/M_{GUT}^2$, which makes it harmless. The higher dimensional operator $10,10_j 10_k \bar{5}_i \times \langle S^- S^- A \rangle/M_P^4 \sim 10,10_j 10_k \bar{5}_i \times 10^{-7}/M_P$ is also suppressed. Thus, proton decay is expected to proceed via the superheavy gauge bosons, with life time $\sim 10^{34} - 10^{36}$ yrs.

In this model, inflation is implemented by assuming that in the early universe $\langle S \rangle \sim M$ with $\langle P \rangle \sim 10^{17}$ GeV. As a result, the VEVs of $S^+, Q$, and $\Phi^+$ vanish,
and a vacuum energy density proportional to $\kappa^2 M^4$ dominates the universe, as in inflation based on Eq. (1). Inflation is driven by the radiatively generated logarithmic inflaton potential. With $\langle \Phi \rangle$ as well as $\langle S^- \rangle$, $\langle Z \rangle$ non-zero during inflation \cite{17, 24}, $SU(5) \times U(1)_X$ is broken to $SU(3) \times SU(2) \times U(1)$, and the $SU(5)$ monopoles and $U(1)_X$ strings are inflated away. For simplicity, we assume that $\kappa_2 \langle P \rangle \sim \frac{1}{10} \kappa_1 M >> \langle \Phi \rangle, \langle S^- \rangle, \langle Z \rangle$ so that Eq. (2) with $N = 2$ still approximately holds. It turns out that with inclusion of soft SUSY breaking terms, $S^+, \Phi^+$ acquire VEVs during inflation that are proportional to $m_{3/2}$, the SUSY breaking scale. As inflation ends, $S^+$ and $\Phi^+$ develop GUT scale VEVs, while $\Phi$ is driven to zero, so that $SU(5)$ is broken to the MSSM gauge group.

With inflation over, the inflaton fields oscillate about their SUSY minima. With the masses of $S^+ (= |\kappa_1 \langle S^- \rangle| \sim |\kappa_2 \langle P \rangle| \lesssim 10^{14}$ GeV) and $\Phi^+_8 (= |\alpha \langle S^+ \rangle| \lesssim M_{\text{GUT}})$ smaller than the masses of the triplet Higgs contained in $5_H$, $\overline{5}_H$ ($\langle S^- \rangle \sim \langle \Phi^+_8 \rangle = M_{\text{GUT}}$), $S^+, \Phi^+$ can not decay through the terms in Eq. (14) into the triplet Higgs. A linear combination $S^+ - c \sqrt{\frac{2}{10}} \Phi^+_8 (\equiv \Psi)$, where $c \equiv a = b = \sqrt{10/3} \left[ \langle S^+ \rangle / \langle \Phi^+_8 \rangle \right]_{\text{min}}$, couples to the doublets in $5_H$, $\overline{5}_H$. We note that the VEV of $\Psi$ vanishes during and after inflation. Indeed, as $S$ (and $Z$) rolls down to the origin, $S^+$ and $\Phi^+_8$ also roll down, from the origin to their present values. If $\Psi$ remains zero throughout inflation,\footnote{We assume that this can be realized through a judicious choice of parameters in the potential in Eq. (2). This would be impossible had we assigned a zero $U(1)_X$ charge to $\Phi^+$, and the $5$-plet Higgs masses were given in terms of a mass parameter $M$ and $\langle \Phi^+ \rangle$.} the field $S^-$ exclusively decays into right handed neutrinos and sneutrinos via $S^- S^- 1_i 1_j / M_P$ and the superpotential couplings in Eq. (8). With $\kappa_1 \lesssim 10^{-2}$, $\kappa_3 \lesssim 10^{-3}$, $\langle S^+ \rangle \sim 10^{16}$ GeV, and $\langle Q \rangle \sim 10^{17}$ GeV, the inflaton $S^-$ fulfills Eq. (4). The subsequent out of equilibrium decay of the right-handed neutrinos yields the observed baryon asymmetry via leptogenesis \cite{14}.

Before concluding, one may inquire about implementing inflation in five dimensional (5D) $SU(5)$ models which have attracted much recent attention because of the relative ease with which the doublet-triplet problem is resolved and higgsino medi-
ated dimension five proton decay is eliminated [25]. Following Refs. [26, 24], inflation with $\delta T/T \propto (M_X/M_{\text{Planck}})^2$ can be realized in this case, where $M_X \sim 10^{16} \text{ GeV}$ denotes the $U(1)_X$ breaking scale. Thus, the desired atmospheric neutrino mass can be realized also in 5D $SU(5)$.

In conclusion, we have shown how inflation can be realized in SUSY $SU(5)$ with $U(1)_R$ symmetry playing an essential role. We have also discussed how neutrino masses can be understood in this setting by exploiting a global $U(1)_X$ symmetry also broken at a scale close to $M_{\text{GUT}}$. This inflationary model also possesses some interesting phenomenology. In particular, the troublesome dimension five proton decay mediated by higgsino exchange in minimal $SU(5)$ is strongly suppressed. It would be of some interest to extend the discussion to larger GUTs such as $SO(10)$ and $E_6$.

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