Analysis of Geometric Characteristics of Cycloidal Transmission

Xiangmao Huang* and Jing Zhangb
Shuanghuan Driveline Co., Ltd, Zhejiang, China

E-mail: *xiangmaohuang@gearsnet.com, b zhangjing@gearsnet.com

Abstract. In order to improve the transmission performance of RV reducer, the equation of cycloid tooth profile is deduced according to the transmission principle of cycloid. The relationship between the curvature radius of cycloid tooth profile and the rotation angle is deduced. The rotation angle corresponding to the maximum curvature radius is solved and the value of the radius of needle tooth under the circumstance of avoiding top cutting is given. Pressure angle is an index to evaluate transmission performance. This paper deduces the relationship between pressure angle and rotation angle, simulates the influence of eccentricity, radius of pin teeth and radius of distribution circle of pin teeth on pressure angle. The simulation shows that the position with pressure angle of 90 degrees appears at the top and root of cycloidal gear, which is not conducive to the transmission performance of cycloidal gear, the position with the best transmission performance appears in the middle of cycloidal gear. The change speed of pressure angle near the root of tooth is faster than that near the top of tooth. The results show that increasing the eccentricity properly or reducing the radius of the center circle of the pin teeth can make the minimum curvature close to the root of tooth, which is beneficial to improving the load-carrying capacity of cycloidal gear.

1. Introduction

RV reducer is the key component of industrial joint robots. It is used in robot joints because of its high load-carrying capacity and impact resistance. RV reducer includes two stages of deceleration. The first stage is involute spur gear planetary transmission and the second stage is cycloidal transmission. Cycloid pinwheel transmission is driven by the meshing of cycloid gear and a number of pin teeth. Friction heat will be generated in the transmission process and then thermal deformation will occur, which will affect the overall transmission accuracy. Therefore, it is necessary to analyze the geometric meshing of cycloidal transmission for improving the overall performance of RV reducer.

Scholars have done a lot of research on cycloidal pinwheel transmission such as RV reducer. Among them, YANG et al. has studied the relationship between processing error, design parameters and transmission performance in cycloidal transmission and deduced the calculation formula of transmission backlash and torque fluctuation[1]. He Weidong et al. established the mechanical model of RV reducer, and put forward the mathematical model of cycloid gear optimization according to the clearance distribution of each tooth meshing[2-3]. In reference[4-6], the error analysis and experimental study of RV reducer are systematically carried out by using the equivalent model of geometry and mass spring. The main factors affecting the transmission accuracy of RV reducer are revealed and the conclusion beneficial to improving the transmission accuracy of
the system is drawn. In this paper, the geometric characteristics of cycloidal transmission are analyzed to provide guidance for the modification of cycloidal wheel.

2. Cycloid Gear Profile Equation
The principle of cycloidal transmission is shown in Figure 1. The track of the meshing point of the cycloidal pinwheel is a part of the circular arc of the pin teeth in the space coordinate system.

![Figure 1. Cycloid-pinion drive coordinate system](image)

Let \( S_0 \) be the reference coordinate system, \( S_1 \) is a pinwheel coordinate system, \( S_2 \) is a cycloid coordinate system. The origin of the cycloid coordinate system coincides with the origin of the reference coordinate system. The distance between the origin of the pinwheel coordinate system and the origin of the cycloid coordinate system is \( e \). \( r_{rp} \) is needle roller radius, \( r_p \) is the distribution circle radius of pin teeth. The position of point \( p \) on the pin tooth in the coordinate system of the pin wheel can be obtained.

\[
r_1(\alpha) = \begin{bmatrix}
-r_{rp}\sin\alpha \\
r_p + r_{rp}\cos\alpha \\
1
\end{bmatrix}
\]  

Let \( \theta_1 \) be the rotation angle of the needle wheel around the reference coordinate system, \( \theta_2 \) is the rotation angle of the cycloid wheel around the reference coordinate system. \( z_p \) is the number of pinwheel teeth. \( i^H \) is cycloidal pinwheel transmission ratio. The equation can be obtained.

\[
i^H = z_p/z_c = \theta_2/\theta_1
\]

The \( p \)-point trajectory on the pinwheel profile is transformed into the cycloid coordinate system by coordinates, the conjugate cycloid profile with the pinwheel can be obtained.

\[
r_2(\theta_1, \alpha) = M_{21}(\theta_1) r_1(\alpha)
\]

In the equation (3), coordinate transformation matrix \( M_{21}(\theta_1) \)

\[
M_{21}(\theta_1) = \begin{bmatrix}
\cos(\theta_1 - \theta_2) & -\sin(\theta_1 - \theta_2) & -\cos(\theta_1 - \theta_2) \\
\sin(\theta_1 - \theta_2) & \cos(\theta_1 - \theta_2) & -\sin(\theta_1 - \theta_2) \\
0 & 0 & 1
\end{bmatrix}
\]

By substituting the relevant parameters and formulas (1) (3) (4) into (2), the tooth profile equation of cycloid wheel in reference coordinate system \( S_0 \) can be obtained.

\[
\begin{align*}
x_0 &= -(r_p - r_{rp}S)\sin(1 - i^H)\theta_1 - (e - k_1r_{rp}S)\sin(i^H\theta_1) \\
y_0 &= (r_p - r_{rp}S)\cos(1 - i^H)\theta_1 - (e - k_1r_{rp}S)\cos(i^H\theta_1)
\end{align*}
\]

In the equation (5), \( S = (1 + k_1^2 - 2k_1\cos\theta)^{-1/2} \), \( k_1 = e z_p/r_p \).

3. Curvature radius of cycloid wheel
According to the basic definition of curvature radius of curve, the curvature radius of cycloid gear tooth profile at any point can be calculated as follows:
The radius of curvature of cycloid gear profile can be obtained by deriving formula (5) with respect to \( \theta_1 \) and substituting formula (6).

\[
\rho_c = \frac{(z^2+y^2)^{1/2}}{x^2+y^2}
\]  

(6)

Let \( \frac{d\rho}{d\theta_1} = 0 \), the rotation angle corresponding to the maximum curvature radius of cycloidal gear profile can be obtained as follows:

\[
\theta_1 = \arccos \frac{z_p(2k_p^2-1)-k_p^2+2}{k_p(z_p+1)}
\]

(7)

By substituting formula (8) into formula (7), the maximum curvature radius of the cycloid gear profile at the outer convex end is obtained as follows.

\[
\rho_{\text{max}} = r_p \sqrt{\frac{2(1-k_p^2)(z_p-1)}{(z_p+1)^3}}
\]

(9)

In the design of cycloid gear profile, it is necessary to avoid the phenomenon of tooth tip sharpening and tooth top cutting. For the concave profile, no matter what the radius of the pin tooth is, there will be no undercutting phenomenon. For outer convex profile, in order to avoid top cutting, the radius of needle teeth must be satisfied.

\[
r_{np} \leq r_p \sqrt{\frac{2(1-k_p^2)(z_p-1)}{(z_p+1)^3}}
\]

(10)

From the above relations, it can be concluded that reducing the eccentricity or increasing the radius of the distribution circle of needle teeth can avoid the top cutting phenomenon and improve the transmission performance. The curvature radius of the cycloid wheel varies with the rotation angle as shown in Figure 2 by simulating the parameters of the cycloid wheel in Table 1.

### Table 1. The parameters of the cycloid wheel

| Parameters                              | Value |
|----------------------------------------|-------|
| Eccentricity e/mm                      | 6     |
| needle roller radius \( r_{np} \)/mm   | 12    |
| distribution circle radius of pin teeth \( r_p \)/mm | 90    |
| the number of pinwheel teeth           | 7     |

Because of the infinite curvature radius of the cycloid gear profile, in order to describe the change rule conveniently, the relationship between curvature and rotation angle is taken as shown in Figure 3.
tooth side. The curvature of cycloid tooth profile changes quickly near the tooth root, and the curvature change near the tooth top is smaller than that near the tooth root. In the convex tooth profile section, the minimum curvature is not at the tooth top, but near the tooth top.

![Figure 3. The relation between curvature and rotation angle](image)

4. Pressure Angle of Cycloidal Tooth Profile
Pressure angle is an important index for judging transmission performance. From figure 4, in order to describe the transmission characteristics between cycloid gear and pin teeth, the angle between the relative speed of rotation of cycloid profile relative to geometric center and the force direction of cycloid profile at meshing point is defined as the pressure angle between cycloid profile and pin teeth.

![Figure 4. Pressure Angle of Cycloidal Tooth Profile](image)

By definition, the expression of pressure angle can be described as follows:

$$\alpha_k = \arccos\left(\frac{F_k \cdot v_k}{|F_k| \cdot |v_k|}\right) \quad k = 1, 2, 3 \ldots z_c$$

(11)

In the equation (11), $F_k$ is the force acting on the cycloid tooth profile at the meshing point K. The direction is the same as the normal vector $n_k$ of the cycloid profile at point K. $v_k$ is the velocity of the meshing point K relative to the center of cycloid wheel rotation. The expression of pressure angle can also be written as follows:

$$\alpha_k = \arccos\left(\frac{v_k}{|v_k|}\right) \quad k = 1, 2, 3 \ldots z_c$$

(12)

Bring the relevant values into equation (12), the equation can be obtained as follows:

$$\alpha_k = \arccos\left(\frac{-\sin(\theta_k(1+\lambda^2-2\lambda\cos\theta)\frac{r_p}{r_p})}{\lambda(1-2\lambda(\lambda\cos\theta_k(1+\lambda^2-2\lambda\cos\theta)\frac{r_p}{r_p})^2})} \quad k = 1, 2, 3 \ldots z_c$$

(13)

In the equation (13), $\lambda = \frac{r_p}{e z_c}$, $\theta_k = \theta + \frac{k-1}{z_p} 2\pi$

![Figure 5. The relationship between pressure angle of single tooth and rotation angle](image)
Figure 6. The relationship between pressure angle of full tooth and rotation angle

Figure 5 shows that the pressure angle equals 90 degrees when the rotation angle equals 0 degrees, 180 degrees and 360 degrees respectively. The three positions are at the root 1, the top 3 and the root 5 respectively, which are dead points. When the rotation angle is between 0 and 180 the pin tooth 1 has no transmission effect on the cycloid wheel. When the rotation angle is between 180 and 360 the pin tooth 1 has transmission effect on the cycloid wheel. When the rotation angle is equal to $2\pi - \arccos \frac{z_p (2k_r^2 - 1)}{k_s (z_p + 1)}$, the position 4 shown in Figure 5 has the smallest pressure angle, the best transmission performance and the smallest curvature. With this point as the demarcation point, along the root direction, the pressure angle increases rapidly, while along the top direction, the change rate is opposite. From the simulation results in Figure 6, the pressure angle of each pin tooth and cycloid gear profile has the same trend, but the phase angle exists in each curve. Of all the needle teeth, half of the pressure angle of the needle teeth is less than 90 degrees, only half of the needle teeth act as transmission.

5. Effect of Geometric Parameters on Pressure Angle

From the simulation results in figure 7 and figure 8, when the rotation angle is between 0 and 180 degrees, the pressure angle increases with the increase of eccentricity and decreases with the increase of the radius of the center circle of the pin teeth. When the rotation angle is between 180 and 360 degrees, the pressure angle is opposite to the eccentricity and the radius of the center circle of the pin teeth. Increasing the eccentricity or reducing the radius of the center circle of the pin teeth is beneficial to improving the transmission characteristics of cycloidal transmission. In addition, it can be seen from the simulation results that increasing the eccentricity or reducing the radius of the center circle of the pin teeth appropriately makes the rotation angle corresponding to the minimum curvature close to the root of the tooth, which is conducive to improving the load-carrying capacity of cycloidal gear. As can be seen from Figure 9, the radius of the pin tooth does not affect the pressure angle.
According to the principle of cycloidal transmission, there are relative sliding and rolling between cycloidal gear and pin teeth. The friction heat at the meshing point between cycloidal gear teeth and pin teeth is mainly caused by the relative sliding between them. Where the pressure angle is large is where the relative sliding occurs. From the above simulation results, it is known that the pressure angle of the top and the root of the tooth is the largest, the transmission performance is the worst, and the heat is the largest. This sub-region is not conducive to transmission. Therefore, the tooth top and root can be modified.

6. Conclusion
In view of the characteristics of cycloidal transmission, the equation of cycloidal tooth profile is established. The curvature radius of cycloidal tooth profile and the expression of pressure angle of cycloidal transmission are derived. The influence of geometric parameters on pressure angle is studied. From the above analysis, the following conclusions can be drawn:

1. Appropriate increase of eccentricity or reduction of the radius of the center circle of the pin teeth makes the rotation angle corresponding to the minimum curvature close to the root of the teeth, which is beneficial to improving the load-carrying capacity of cycloidal gear.
2. The dead-point position of cycloidal transmission occurs at the top and root of cycloidal gear, and the transmission performance is the worst. Therefore, the top and root of cycloidal gear can be modified.
3. The position with the best transmission performance appears in the middle of the cycloid profile, not in the top and root of the teeth.

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