Radiative decay $Y(4260) \to X(3872) + \gamma$ involving hadronic molecular and charmonium components

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We apply a phenomenological Lagrangian approach to the radiative decay $Y(4260) \to X(3872)+\gamma$. The $Y(4260)$ and $X(3872)$ resonances are considered as composite states containing both molecular hadronic and charmonium components. Having a leading molecular component in the $X(3872)$ and a sole molecular configuration for the $Y(4260)$ results in a prediction compatible with present data.

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I. INTRODUCTION

The study of the radiative decay $Y(4260) \to X(3872) + \gamma$ among other production and decay modes can give additional information on the nature of the exotic states $Y(4260)$ and $X(3872)$.

In Refs. [1]-[9] we proposed and developed a phenomenological Lagrangian approach for the study of exotic mesons and baryons as hadronic molecules with a possible admixture of quark-antiquark (in case of mesons) and three-quark (in case of baryons) components. To set up the bound state structure of new exotic states we used the compositeness condition [10]-[12] which is the key ingredient of our approach. In Refs. [11, 12] and [1]-[7] it was proved that this condition is an important and successful quantum field theory tool for the study of hadrons and exotic states as bound states of their constituents. In particular, in Refs. [3, 8, 9] we considered strong and radiative decays of the $X(3872)$ and $Y(4260)$. A detailed analysis of the $X(3872)$ decay modes in the picture, where this state is considered as a superposition of the molecular $D^0D^0$, $D^±D^∗∓$, $J/ψω$, $J/ψρ$ and $c\bar{c}$ charmonium configurations, has been considered in Ref. [5]. It was shown that a compact charmonium component plays an important role in radiative transitions $X(3872) \to J/ψ(2s)\gamma$, which was in agreement with predictions obtained in potential models [13]-[15].

Here we present a quantitative study of the radiative decay mode $Y(4260) \to X(3872) + \gamma$ (see also the discussion in Refs. [16, 17]) by using the phenomenological Lagrangian approach developed previously in Refs. [1]-[6]. We assume that both $Y(4260)$ and $X(3872)$ states are composite states containing both extended hadronic molecular (with a typical size of $r > 1.5 - 2$ fm) and compact charmonium (with a typical size $r < 1$ fm) components. In Ref. [5] such a scenario was considered for the $X(3872)$ state, and here we extend this idea to the case of the $Y(4260)$ state. Following Refs. [9, 16, 18-20], we consider the molecular $DD_1(2420)$ assignment for the $Y(4260)$ state was proposed and studied, we assume that the $Y(4260)$ is a composite isosinglet state containing a molecular component made up of the pseudoscalar $D(1870)$ and the axial $D_1(2400)$ charm mesons and $c\bar{c}$ charmonium configuration

$$|Y(4260)⟩ = \frac{1}{2} \left| D_1D - \bar{D}D_1 \right⟩ \cos θ_Y + |c\bar{c}⟩ \sin θ_Y$$

with $J^P = 1^-$. $θ_Y$ is the mixing angle between the hadronic and the charmonium components. Here and in the following we will use the notation $D = (D^+, D^0)$, $D^* = (D^{*+}, D^{*0})$, $D_1 = (D^{*+}_1, D^{*0}_1)$ which stand for the doublets of pseudoscalar, vector and axial-vector $D$ mesons. In this paper we use the convention that the pseudoscalar and axial mesons do not change the sign under charge-parity transformation while the vector meson changes the sign. In previous papers we used another convention, but it does not affect the results.

The $X(3872)$ with $J^P = 1^+$ is set up as a superposition of the molecular $D^0D^∗− , D^+D^∗− − D^−D^∗+, J/ψω, J/ψρ,$ and the $c\bar{c}$ components as proposed in Ref. [21] and developed in Ref. [5]

$$|X(3872)⟩ = \cos θ_X \left[ \frac{Z_{J/ψω}^{1/2}}{\sqrt{2}} |D^0D^0⟩ - |D^+D^∗−⟩ + \frac{Z_{J/ψρ}^{1/2}}{\sqrt{2}} (|D^+D^∗−⟩ - |D^−D^∗+⟩) \right] + \sin θ_X |c\bar{c}⟩.$$

(2)
The mass of the $X(3872)$ state ($M_X$) is expressed in terms of the masses of the constituents $D^0$ and $D^{*0}$ and the binding energy $\epsilon_X$

$$M_X = M_{D^0} + M_{D^{*0}} - \epsilon_X,$$

where $\epsilon_X$ varies from 0 to 0.3 MeV. For these values of $\epsilon_X$ the charged combination $D^+D^{*-} - D^-D^{*+}$ and the other two components $J/\psi\rho$ and $J/\psi\omega$ are significantly suppressed [5, 21]. In particular, the probability for the leading molecular component $D^0D^{*0} - D^{*0}D^0$ with respect to the other hadronic components varies from 100% to 92%. The probabilities of the other hadronic components vary for the charged $D^+D^{*+} - D^-D^{*-}$ component from 0% to 3.3%, for the $J/\psi\omega$ component from 0% to 4.1% and for the $J/\psi\rho$ component from 0% to 0.6%. It should be clear that the $J/\psi\omega$ and $J/\psi\rho$ components do not contribute to the radiative decay $Y(4260) \rightarrow X(3872) + \gamma$, while the $D^+D^{*+} - D^-D^{*-}$ component could. However, the last contribution is further reduced due to the suppression of the coupling $g_{{D^*}D\gamma}$ by a factor 4 in comparison to $g_{D^*D\gamma}$ (see details in Sec.II). For the mass of the $Y(4260)$ state we use the central value $M_Y = 4250$ MeV.

As in the case of the $Y(4260)$ state we introduce a corresponding mixing angle $\theta_X$ encoding the mixing between the hadronic molecular and charmonium components in $X(3872)$. In this paper we use the convention that the pseudoscalar and axial mesons do not change the sign under charge-parity transformation while the vector meson changes the sign. In previous papers we used another convention, but it does not affect the results.

The paper is organized as follows. In Sec. II we briefly review the basic ideas of our approach and show the effective Lagrangians for our calculation. Then, in Sec. III, we proceed to derive the width of the radiative two-body decay mode $Y(4260) \rightarrow X(3872) + \gamma$. In our analysis we approximately take into account the mass distribution of the $Y(4260)$ state. Finally we present our numerical results and compare to recent limits set by experiment.

II. BASIC MODEL INGREDIENTS

Our approach to the possible composite structure of $X(3872)$ and $Y(4260)$ as mixed bound states containing respective hadronic molecular and charmonium components is based on interaction Lagrangians. They describe the coupling of the respective states to their constituents

$$\mathcal{L}_H(x) = H^\mu(x) \left( \cos \theta_H g_H^M J_{H;\mu}^M(x) + \sin \theta_H g_H^C J_{H;\mu}^C(x) \right)$$

where $H = X, Y$, $g_H^M$ and $g_H^C$ are the dimensionless couplings of the $H$ state to the molecular and charmonium components, respectively. Here, $J_{H;\mu}^M$ and $J_{H;\mu}^C$ are the respective interpolating hadronic and quark currents with quantum numbers of the $H$ state. In particular, these currents are written as

$$J_{X;\mu}^M(x) = \frac{iM_X}{\sqrt{2}} \int d^4y \Phi_X^M(y^2) \bar{D}_\mu^0(x + y/2)D^0(x - y/2) + \text{H.c.},$$

$$J_{Y;\mu}^M(x) = \frac{iM_Y}{2} M_Y \int d^4y \Phi_Y^M(y^2) \bar{D}_\mu(x + y/2)D(x - y/2) + \text{H.c.},$$

$$J_{X;\mu}^C(x) = \int d^4y \Phi_X^C(y^2) \bar{c}(x + y/2)\gamma_\mu\gamma_5c(x - y/2),$$

$$J_{Y;\mu}^C(x) = \int d^4y \Phi_Y^C(y^2) \bar{c}(x + y/2)\gamma_\mu c(x - y/2),$$

where $y$ is a relative Jacobi coordinate; $\Phi_H^M(y^2)$ and $\Phi_H^C(y^2)$ are correlation functions which describe the distribution of the constituent mesons and charm quarks in the bound states $X(3872)$ and $Y(4260)$, respectively.

A basic requirement for the choice of an explicit form of the correlation function $\Phi_H(y^2)$ is that its Fourier transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. For simplicity we adopt a Gaussian form for the correlation function. The Fourier transform of this vertex function is given by

$$\hat{\Phi}_H(p_E^2/(\Lambda_H^I)^2) = \exp(-p_E^2/(\Lambda_H^I)^2), \quad I = M, C$$

where $p_E$ is the Euclidean Jacobi momentum. $\Lambda_H^I$ stand for the size parameters characterizing the distribution of the two constituent mesons ($I = M$) or constituent charm quarks ($I = C$) in the $X(3872)$ and $Y(4260)$ systems.
that these scale parameters have been constrained before as \( \Lambda_X = 0.5 \) GeV \([5]\), \( \Lambda_Y = 0.75 \) GeV \([9]\) and \( \Lambda_H^C = 3.5 \) GeV \([5]\). The respective coupling constants \( g_{I}^{H} \) are determined by the compositeness condition \([1, 6, 10–12]\). It implies that the renormalization constant of the hadron state is set equal to zero with

\[
Z_{H}^{I} = 1 - \left( \frac{\Sigma_{H}^{I} (p^{2})}{p^{2}} \right)_{p^{2} = M_{H}^{I}} = 0 .
\]  

Here, \((\Sigma_{H}^{I})'\) is the derivative of the transverse part of the mass operator \((\Sigma_{H}^{I})^{\mu \nu}\) induced by the hadronic \((I = M)\) and the charm quark loop \((I = C)\), respectively. The mass operator including transverse and longitudinal parts is defined as

\[
\Sigma_{H}^{\mu \nu} (p) = g_{\perp}^{\mu \nu} \Sigma_{H} (p) + \frac{p^{\mu} p^{\nu}}{p^{2}} \Sigma_{H}^{L} (p) , \quad g_{\perp}^{\mu \nu} = g^{\mu \nu} - \frac{p^{\mu} p^{\nu}}{p^{2}} .
\]  

The diagrams corresponding to the contribution of the meson \((\Sigma_{H}^{M})^{\mu \nu}\) and charm quark loops \((\Sigma_{H}^{C})^{\mu \nu}\) are shown in Figs. 1 and 2. In the evaluation of meson and quark loop diagrams we use free propagators for the mesons and the charm quark with a mass \( m_c = 2.16 \) GeV \([12]\). In particular, the propagators for vector \( D^* \) and axial \( D \) mesons denoted by \( D_{\alpha \beta}^{*} \) and \( D_{1, \beta \beta} \) are given by

\[
D_{\alpha \beta} (k) = \frac{1}{M_{D}^{2} - k^{2}} \left[ -g_{\alpha \beta} + \frac{k_{\alpha} k_{\beta}}{k^{2}} \right] , \quad \mathcal{D} = D^*, D_1
\]  

and similarly for the pseudoscalar \( D \) meson with

\[
D (k) = \frac{1}{M_{D}^{2} - k^{2}} .
\]  

The propagator of the charm quark is written as

\[
S_{c} (k) = \frac{1}{m_c - k} .
\]  

The explicit expressions for the coupling constants \( g_{H}^{M} \) and \( g_{H}^{C} \) resulting from the compositeness condition are

\[
(g_{H}^{M})^{-2} = I(M_{X}^{2}, M_{D}^{2, o}, M_{D}^{2, o}), \quad (g_{H}^{C})^{-2} = \frac{1}{2} \left[ I(M_{Y}^{2}, M_{D}^{2, o}, M_{D}^{2, o}) + I(M_{Y}^{2}, M_{D}^{2, o}, M_{D}^{2, o}) \right]
\]  

FIG. 1: Hadronic molecular component contributions to the mass operators of \( X(3872) \) and \( Y(4260) \).

FIG. 2: Charmonium component contribution to the mass operators of \( H = X(3872), Y(4260) \).
on the binding energy $\epsilon_X$ when $\Lambda_Y$. Ref. [4]:

Once the masses of the composite states are fixed, the values for the corresponding couplings to their constituents can be extracted from the compositeness condition. In this work we only consider the neutral meson pair for the hadronic molecular generation of the coupling $g_y$. To calculate the radiative decay we need to specify how we couple the photons both for the hadronic and the charmion components.

For the hadronic case we need to specify the $D_1 \to D^*\gamma$ coupling. The relevant phenomenological Lagrangian generating the coupling $D_1^0 \to D^*\gamma$ is

$$
\mathcal{L}_{D^*D_1\gamma}(x) = \frac{e}{2} F_{\mu\nu}(x) \epsilon^{\mu\nu\alpha\beta} \left( g_{D^*\gamma} D_{1\alpha}^+(x) D_{1\beta}^-(x) + g_{D^*\rho_0} D_{1\alpha}^0(x) D_{1\beta}^0(x) \right) + \text{H.c.},
$$

where the couplings

$$
g_{D^*\gamma} = g_{D^*\rho_0} = \frac{1}{4} g_{D^*\rho_0},
$$

can be fixed using the estimate for the rate $\Gamma(D_1^0 \to D^*\gamma)$ given in Ref. [22]

$$
\Gamma(D_1^0 \to D^*\gamma) = \frac{\alpha^2}{24} g_{D^*\rho_0}^2 M_{D_1^0} (1 - r^2)^3 \left( 1 + \frac{1}{r^2} \right), \quad r = \frac{M_{D^*\gamma}}{M_{D_1^0}}.
$$

Table I. Results for the coupling constant $g_y$ depending on $\epsilon_X$.

| $\epsilon_X$ in MeV | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 2.06 | 2.07 | 2.08 | 2.09 | 2.10 | 2.11 |
|---------------------|------|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|-----|

The diagrams contributing to the two-body radiative decays $Y(4260) \to X(3872) + \gamma$ are summarized in Figs. 3 and 4. To calculate the radiative decay we need to specify how we couple the photons both for the hadronic molecular and the charmion components.
FIG. 3: Diagram describing the hadronic molecular component contribution to the radiative decay $Y(4260) \rightarrow X(3872) + \gamma$.

FIG. 4: Diagrams describing the charmonium component contribution to the radiative decay $Y(4260) \rightarrow X(3872) + \gamma$.

Using the numerical estimate of Ref. [22] with $\Gamma(D^0_1 \rightarrow D^{*0}_0 \gamma) = 245 \pm 18$ keV we get $g_{D^{*0}_0D^0_1\gamma} = 2.10 \pm 0.08$. Note that the relation between the couplings $g_{D^{*+}D^{0}_0\gamma}$ and $g_{D^{*0}_0D^0_1\gamma}$ has been deduced in the naive quark model [see Refs. [2, 3]]. In particular, the corresponding ratio $R = g_{D^{*0}_0D^0_1\gamma} / g_{D^{*+}D^{0}_0\gamma}$ is expressed in terms of the electric charges of the valence quarks of the neutral and charged $D$ mesons

$$R = \frac{e_c + e_u}{e_c + e_d} = 4.$$  \hspace{1cm} (20)

In case of charmonium we include the photons by direct coupling to the $c$-quarks as

$$\mathcal{L}_{\bar{c}c\gamma}(x) = \frac{2}{3} e A_\mu(x) \bar{c}(x) \gamma^\mu c(x)$$  \hspace{1cm} (21)

and in the interaction Lagrangian $\mathcal{L}_{H\bar{c}c}(x) = g_H^c H(x) J^c_{\mu}(x)$ in order to fulfill electromagnetic gauge invariance of these nonlocal Lagrangians. The gauging proceeds in a way suggested and extensively used in Refs. [12, 23, 24]. To guarantee local electromagnetic gauge invariance of the strong interaction Lagrangian one multiplies each quark and Dirac conjugated quark field in $\mathcal{L}_{H\bar{c}c}$ with a gauge field exponential $I(y,x) = \int_x^y dz_\mu A_\mu(z)$ as

$$c(y) \rightarrow e^{-i\frac{2}{3}c(x) I(y,x)} c(y), \quad \bar{c}(y) \rightarrow \bar{c}(y) e^{i\frac{2}{3}c(x) I(y,x)}.$$  \hspace{1cm} (22)

As a result the gauge-invariant (GI) nonlocal Lagrangians describing the coupling of the $X(3872)$ and $Y(4260)$ states to the charm quarks read

$$\mathcal{L}^{GI}_{H\bar{c}c}(x) = H(x)^\mu J_{H;\mu}^{\bar{c}c}(x),$$  \hspace{1cm} (23)
where \( J^C_{\mu}(x) \) is the GI interpolating charm-quark current with quantum numbers of the \( H = X(3872) \) or \( Y(4260) \) state:

\[
J^C_{\mu}(x) = \int d^4y \Phi^C_X(y^2) \bar{c}(x + y/2) \gamma_\mu \gamma_5 e^{i \frac{\phi}{2} (x + y/2, x - y/2)} c(x - y/2),
\]

\[
J^C_{\mu}(x) = \int d^4y \Phi^C_Y(y^2) \bar{c}(x + y/2) \gamma_\mu e^{i \frac{\phi}{2} (x + y/2, x - y/2)} c(x - y/2).
\]

The Lagrangian (21) generates the triangle diagram of Fig. 4 (see upper panel), while the two additional (bubble) diagrams in the lower panel of Fig. 4 are generated by terms of the Lagrangian (24) obtained after expansion of the gauge exponential up to first order in \( A^\mu \).

The diagram of Fig. 3 is separately gauge invariant because the \( D_1 D^* \gamma \) interaction Lagrangian (17) contains a contraction of the antisymmetric Levi-Civita tensor and the stress tensor of the electromagnetic field. The proof of gauge invariance of the set of diagrams in Fig. 4 is presented in Appendix A.

### III. DECAY MODES AND RESULTS

The two-body decay width for the transition \( Y(4260) \to X(3872) + \gamma \) is given by

\[
\Gamma(Y \to X\gamma) = \frac{P^*}{24\pi M_Y^2} \sum_{pol} |M_{inv}|^2
\]

where \( P^* = (M_Y^2 - M_H^2)/(2M_Y) \) is the three-momentum of the final states in the rest frame of the \( Y(4260) \) and \( M_{inv} \) is the corresponding invariant matrix element.

The contribution of the hadronic loop diagram to the invariant matrix element for the radiative transition is written as

\[
M_{inv} = e \epsilon_{\mu \nu \alpha \beta} \epsilon^\nu_{Y\alpha} (q) \epsilon^\nu_{X\nu} (p') \epsilon_{Y \beta} (p) q^\mu \Gamma^\alpha_{\mu \beta}^M
\]

where \( p, p', q, \) and \( \epsilon_{Y \beta} (p), \epsilon^\nu_{X \nu} (p'), \epsilon^\nu_{Y\alpha} (q) \) are the momenta and polarization vectors of \( Y(4260), X(3872) \) and the photon, respectively. The loop integral \( \Gamma^\alpha_{\mu \beta}^M \) contributing to the radiative decay of the \( Y(4260) \) state is given as

\[
\Gamma^\alpha_{\mu \beta}^M = g_{\text{eff}}^M \int \frac{d^4k}{\pi^2} \Phi_Y \left(- \left[ k + \frac{p}{2} \right]^2 \right) \Phi_X \left(- \left[ k + \frac{p}{2} + \frac{q}{2} \right]^2 \right) D_{\alpha \mu}^0 (k + q) D_{\beta \nu}^0 (k) D^0 (k + p),
\]

where

\[
g_{\text{eff}}^M = \frac{1}{16\pi^2} g_Y g_X g_{D^0 \bar{D}^0} M_Y M_X \cos \theta_X \cos \theta_Y
\]

is the effective coupling constant. Using the Feynman \( \alpha \) parametrization and performing the integration over the loop momentum we can expand the loop integral \( \Gamma^\alpha_{\mu \beta}^M \) in terms of three independent Lorentz structures for all involved particles on mass shell with \( p^2 = M_Y^2, p'^2 = M_X^2, q^2 = 0 \) [3]

\[
\Gamma^\alpha_{\mu \beta}^M = \frac{1}{M_Y^2} \left( g^{\alpha \alpha'} g^{\beta \beta'} pq F_1 + g^{\alpha \alpha'} p^\beta q^\beta F_2 + g^{\beta \beta'} p^\alpha q^\alpha F_3 \right).
\]

The \( F_i \) are relativistic form factors given by integrals over Feynman parameters. Note that our full phenomenological Lagrangian describing the radiative transition of the \( Y(4260) \) state has a simple structure reflected in the corresponding matrix element for the \( Y(4260) \to X(3872) + \gamma \) decay. It contains a smaller number of linearly independent Lorentz structures as was found for example in the tetraquark model [25]. In Ref. [25] it was also shown that in general the matrix element for the radiative transition between axial and vector meson states contains four independent Lorentz structures due to electromagnetic gauge invariance. However, as was also clearly demonstrated in Ref. [25] consideration of the additional Schouten identity, which forbids antisymmetric fifth-rank tensors in four dimensions [see details in Ref. [26]], leads to a further simplification of the Lorentz structure of the \( Y(4260) \to X(3872) + \gamma \) transition matrix element. The form factors \( F_1, F_2 \) and \( F_3 \) form two linearly independent combinations \( F_1 + F_2 \) and \( F_1 + F_3 \) corresponding to the \( E_1 \) and \( E_2 \) transitions [25].
Again in full consistency with the proof of Ref. [25], the combination of the four form factors $F_i$ are given by

$$F_i = g_{\text{eff}}^M R_i(M_{D^o}^2, M_{D}^2, \cdots) \ ,$$

where $R_i(M_{Y}^2, M_{X}^2)$ are the structure integrals with

$$R_i(M_{Y}^2, M_{X}^2) = \int_0^\infty \frac{d\alpha_1 d\alpha_2 d\alpha_3}{\Delta^2} f_i \exp[z(\alpha_1, \alpha_2, \alpha_3)] ,$$

$$f_1 = \left[ 1 + \frac{1}{2\Delta} \left( \frac{1}{M_{D_1}^2} + \frac{1}{M_{D_2}^2} \right) \right] \frac{2M_{Y}^2}{M_{Y}^2 - M_{X}^2} ,$$

$$f_2 = -x_1 x_2 \frac{M_{Y}^2}{M_{D_1}^2} ,$$

$$f_3 = -x_1 (x_1 + x_2 - 1) \frac{M_{Y}^2}{M_{D_2}^2} ,$$

and

$$z(\alpha_1, \alpha_2, \alpha_3) = -\alpha_1 M_{Y}^2 - \alpha_2 M_{X}^2 - \alpha_3 M_{D_1}^2 ,$$

$$+ M_{Y}^2 \left( \alpha_1 + \frac{s_Y + s_X}{4} - x_3^2 \Delta \right) + (M_{Y}^2 - M_{X}^2) \left( \frac{s_X}{4} - x_1 x_2 \Delta \right) ,$$

$$\Delta = \alpha_{123} + s_Y + s_X , \quad s_Y = \frac{1}{M_{Y}^2} , \quad \alpha_{123} = \alpha_1 + \alpha_2 + \alpha_3 ,$$

$$x_1 = \frac{1}{\Delta} \left( \alpha_1 + \frac{s_Y}{2} + \frac{s_X}{2} \right) , \quad x_2 = \frac{1}{\Delta} \left( \alpha_2 + \frac{s_X}{2} \right) .$$

Finally, the matrix element of the radiative transition $Y(4260) \rightarrow X(3872) + \gamma$ can be written in a manifestly gauge-invariant form

$$M_{YX\gamma}^M = \frac{e}{M_{Y}^2} \left( \epsilon_{p'q'r_1} \epsilon_1^* (\epsilon_2 \gamma p') (F_1 + F_2) + \epsilon_{p'q'r_2} \epsilon_2^* (\epsilon_3 \gamma q) (F_1 + F_3) \right) ,$$

where $\epsilon_{ABCD} = A^a B^b C^c D^\sigma \epsilon_{\alpha\beta\rho\sigma}$.

The contributions of the charm quark-loop diagrams of Fig. 4 are given in Appendix A. The corresponding matrix element is calculated using FORM [27] and is decomposed into four relativistic form factors factors $G_i (i = 1 \ldots 4)$ as

$$M_{YX\gamma}^C = \frac{e}{M_{Y}^2} \left( G_1 \left[ (p' \epsilon_1) \epsilon_{p'q'r_1} \epsilon_1^* - (p' \gamma) \epsilon_{p'q'} \epsilon_2^* \epsilon_1^* \right] + G_2 \left[ (q \epsilon_2^*) \epsilon_{p'q'r_1} \epsilon_1^* + G_4 (p' \gamma) \epsilon_{p'q'} \epsilon_2^* \epsilon_1^* \right] \right) .$$

Again in full consistency with the proof of Ref. [25], the combination of the four form factors $G_i$ is reduced to a matrix element with two Lorentz structures, where the form factors $G_i$ form two linearly independent combinations,

$$M_{YX\gamma}^C = \frac{e}{M_{Y}^2} \left( \epsilon_{p'q'r_1} \epsilon_1^* (\epsilon_2 \gamma p') \left( G_1 \frac{2M_{Y}^2}{M_{Y}^2 - M_{X}^2} + G_3 - G_4 \right) + \epsilon_{p'q'r_1} \epsilon_1^* (\epsilon_3 \gamma q) \left( G_1 \frac{M_{Y}^2 + M_{X}^2}{M_{Y}^2 - M_{X}^2} - G_2 - G_4 \right) \right) .$$

Combining the contributions of both the hadronic and charmonium components to the radiative transition $Y(4260) \rightarrow X(3872) + \gamma$ we form two linear-independent combinations of the corresponding form factors $F_1$ and $F_2$:

$$F_1 = F_1 + F_2 + G_1 \frac{2M_{Y}^2}{M_{Y}^2 - M_{X}^2} + G_3 - G_4 ,$$

$$F_2 = F_1 + F_3 + G_1 \frac{M_{Y}^2 + M_{X}^2}{M_{Y}^2 - M_{X}^2} - G_2 - G_4 .$$
Finally get the following estimate for the decay rate in the molecular picture

$$\Gamma(Y \rightarrow X + \gamma) = \frac{\alpha}{3} \left( \frac{P^+}{M_X^1} \right)^5 \left[ F_1^2 + M_X^2 F_2^2 \right].$$  \hspace{1cm} (37)

Numerical results for the radiative decay $\Gamma(Y(4260) \rightarrow X(3872) + \gamma)$ in a pure molecular scenario, which are about 50 keV, are summarized in Table II. In our calculations we use different values of the binding energy $\epsilon_X = 0.05, 0.1, 0.15, 0.2, 0.25$ and 0.3 MeV. When we also vary the cutoff parameters $\Lambda_X$ and $\Lambda_Y$ in the region $0.5 - 0.75$ GeV we finally get the following estimate for the decay rate in the molecular picture

$$\Gamma(Y \rightarrow X + \gamma) = 59 \pm 12.4 \text{keV}.$$  \hspace{1cm} (38)

Note that in Ref. [16] the result for $\Gamma(Y \rightarrow X + \gamma)$ is written in terms of the coupling constants $x$ and $c_0$ corresponding to the $XD^0D^{*0}$ and $D^0_dD^{*0}$ couplings, respectively [16]

$$\Gamma(Y \rightarrow X + \gamma) = 141^{+136}_{-91} (x^2 \text{GeV}) c_0^2 \text{keV},$$  \hspace{1cm} (39)

where in the numerical evaluation the coupling $c_0$ is taken as 0.4 and the coupling $x$ is fixed as

$$|x| = 0.97^{+0.40}_{-0.97} \pm 0.14 \text{GeV}^{-1/2}. $$  \hspace{1cm} (40)

**Table II.** Decay width for $Y(4260) \rightarrow X(3872) + \gamma$ in keV in a pure molecular scenario.

| $\epsilon_X$ in MeV |
|---------------------|
| 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 |
| 52.4 ± 4.0 | 52.5 ± 4.0 | 52.6 ± 4.1 | 52.7 ± 4.1 | 52.9 ± 4.1 | 53.0 ± 4.2 |

Inclusion of the charmonium components in the $X(3872)$ and $Y(4260)$ states adds the dependence on the mixing angles $\theta_X$ and $\theta_Y$. In this case we restrict to the binding energy $\epsilon_X = 0.1$ MeV and the central value of $g_{D^0D^{*0}Y}$ = 2.10.

The results for $\Gamma(Y \rightarrow H + \gamma)$ in terms of the mixing angles $\theta_X$ and $\theta_Y$ are given by

$$\Gamma(Y \rightarrow H + \gamma) = 52.5 \text{keV} \left[ 0.42(\cos \theta_X \cos \theta_Y + 1.51 \sin \theta_X \sin \theta_Y)^2 + 0.58(\cos \theta_X \cos \theta_Y + 1.23 \sin \theta_X \sin \theta_Y)^2 \right]. $$  \hspace{1cm} (41)

Here the first and second terms in square brackets represent the contributions of the $F_1$ and $F_2$ form factors, respectively. One can see that the pure charmonium scenario [when the molecular contributions in $X(3872)$ and $Y(4260)$ are neglected] gives the decay rate $\Gamma(Y \rightarrow H + \gamma) = 96.3 \text{keV}$. In Ref. [5] we considered three scenarios of the molecular-charmonium mixing in $X(3872)$, which give results for the $X(3872)$ decay rates compatible with data. In particular, for $\epsilon_X = 0.1$ MeV these scenarios correspond to the following choice of the mixing angle $\theta_X$: scenarios I ($\theta_X = 68.9^0$), II ($\theta_X = -12.6^0$) and III ($\theta_X = -20.2^0$). Keeping $\theta_Y$ as a free parameter and substituting the $\theta_X$ in the specific scenario we get

$$\Gamma(Y \rightarrow H + \gamma) = 6.8 \text{keV} \left[ 0.42(\cos \theta_Y + 3.91 \sin \theta_Y)^2 + 0.58(\cos \theta_Y + 3.19 \sin \theta_Y)^2 \right], $$  \hspace{1cm} (42)

for scenario I,

$$\Gamma(Y \rightarrow H + \gamma) = 50.0 \text{keV} \left[ 0.42(\cos \theta_Y - 0.34 \sin \theta_Y)^2 + 0.58(\cos \theta_Y - 0.27 \sin \theta_Y)^2 \right], $$  \hspace{1cm} (43)

for scenario II,

$$\Gamma(Y \rightarrow H + \gamma) = 46.2 \text{keV} \left[ 0.42(\cos \theta_Y - 0.56 \sin \theta_Y)^2 + 0.58(\cos \theta_Y - 0.45 \sin \theta_Y)^2 \right], $$  \hspace{1cm} (44)

for scenario III. Therefore the radiative decay $Y(4260) \rightarrow X(3872) + \gamma$ can also serve as a tool for testing a possible molecular-charmonium mixture of the $Y(4260)$ state. For example considering the mixing angle $\theta_Y$ as a free parameter in Fig. 5 we predict the rate $\Gamma(Y(4260) \rightarrow X(3872) + \gamma)$ as a function of $\sin \theta_Y$ for all three scenarios of the mixing in the $X(3872)$. The respective numerical values for $\Gamma(Y(4260) \rightarrow X(3872) + \gamma)$ are given in the range as

$$\Gamma(Y(4260) \rightarrow X(3872) + \gamma) = 48.6 \pm 41.8 \text{keV}.$$  \hspace{1cm} (45)
in scenario I,

$$\Gamma(Y(4260) \to X(3872) + \gamma) = 25.3 \pm 24.7 \text{ keV}$$

(46)

for scenario II and

$$\Gamma(Y(4260) \to X(3872) + \gamma) = 23.2 \pm 23.0 \text{ keV}$$

(47)

for scenario III. In the case of scenario I, where the charmonium components dominates in the $X(3872)$, an increase of $\sin \theta_Y$ leads to an enhancement of the radiative decay rate. In the case of scenarios II and III, which rely on a dominant molecular component in the $X(3872)$, we have the opposite behavior. In the following we assume the point of view that the $Y(4260)$ state has a very small charmonium component (since presently no direct evidence for a large admixture exists). Then we have the following estimate for the radiative decay rate of

$$\Gamma(Y(4260) \to X(3872) + \gamma) = 6.8 \text{ keV}$$

(scenario I), $50 \text{ keV}$ (scenario II) and $46.2 \text{ keV}$ (scenario III).

In the recent BESIII [28] measurement the product of the Born-order cross section for $e^+e^- \to X(3872)\gamma$ times the branching fraction of $X(3872) \to J/\psi \pi^+\pi^-$ relative to the cross section of $e^+e^- \to J/\psi \pi^+\pi^-$ is determined as

$$\frac{\sigma^B[e^+e^- \to X(3872)\gamma] \cdot B[X(3872) \to J/\psi \pi^+\pi^-]}{\sigma^B[e^+e^- \to J/\psi \pi^+\pi^-]} = (5.2 \pm 1.9) \times 10^{-3}.$$  

(48)

The ratio is determined at the center-of-mass energy $\sqrt{s} = 4.260$ GeV under the assumption that the $X(3872)$ is produced only in $Y(4260)$ radiative decay. They also assume that the $e^+e^- \to X(3872)\gamma$ cross section follows that of $e^+e^- \to J/\psi \pi^+\pi^-$. We further have the result of the Belle Collaboration [29] for the branching fraction with

$$2.3\% < B[X(3872) \to J/\psi \pi^+\pi^-] < 6.6\%.$$  

(49)

For this range of values we can deduce the ratio

$$R = \frac{\sigma^B[e^+e^- \to X(3872)\gamma]}{\sigma^B[e^+e^- \to J/\psi \pi^+\pi^-]} = 11.7 \pm 7.1\%.$$  

(50)

In Ref. [9] we have analyzed the strong two- and three-body decays of the $Y(4260)$ and found that

$$\Gamma(Y(4260) \to J/\psi \pi^+\pi^-) = 1 \pm 0.2 \text{ MeV}.$$  

(51)

Using this prediction we obtain ratios of decay rates with

$$R_Y^I = \frac{\Gamma(Y(4260) \to X(3872)\gamma)}{\Gamma(Y(4260) \to \pi^+\pi^-J/\Psi)} = 4.89 \pm 4.29\%$$  

(52)

for the scenario I,

$$R_Y^{II} = \frac{\Gamma(Y(4260) \to X(3872)\gamma)}{\Gamma(Y(4260) \to \pi^+\pi^-J/\Psi)} = 2.50 \pm 2.49\%$$  

(53)
for $\Pi$, and

$$R_{\gamma}^{III} = \frac{\Gamma(Y(4260) \rightarrow X(3872) + \gamma)}{\Gamma(Y(4260) \rightarrow \pi^+ \pi^- J/\Psi)} = 2.31_{-2.31}^{+2.34}\%$$

(54)

for the scheme III. All three values (within errors) are close to the lower range of the present experimental result for $R$.

In summary, using a phenomenological Lagrangian approach we give predictions for the radiative two-body decay rate $Y(4260) \rightarrow X(3872) + \gamma$. Both for the $X(3872)$ and the $Y(4260)$ we also include besides the molecular a charmonium component. In the case of the $X(3872)$ mixing schemes were previously fixed by the known strong and radiative decays. The full prediction for the radiative decay rate $Y(4260) \rightarrow X(3872) + \gamma$ is contained in Fig. 5 and spans the values from 0 to about 85 keV, depending on the charmonium admixture in the $Y(4260)$ and the dominant component in the $X(3872)$. For a $X(3872)$ which has leading molecular component and a $Y(4260)$ which has a vanishing charmonium content the prediction for the radiative rate is about 35 keV. This result together with the one for the strong three-body decay mode is able to explain the measured ratio $R$ within errors for this interpretation of the $Y(4260)$. It obviously would be helpful to also have a direct measurement of the radiative $Y(4260) \rightarrow X(3872) + \gamma$ mode to test the structure scenario given for the $Y(4260)$ and the $X(3872)$.

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Appendix A: Gauge-invariance of diagrams in Fig. 4.

Here, we prove the gauge invariance of the set of diagrams of Fig. 4. The sum of the contribution $M_{inv}^{C;\Delta}$ of the triangle diagram generated by the direct coupling of charm quarks to the electromagnetic field and of the two bubble diagrams $M_{inv}^{C;\text{bub}}$ generated by gauging the nonlocal strong Lagrangian describing the coupling of $X$ and $Y$ states to constituent charm quarks is denoted by the invariant matrix element $M_{inv}^{C}$. It is given by

$$M_{inv}^{C} = M_{inv}^{C;\Delta} + M_{inv}^{C;\text{bub}},$$

(A1)

where

$$M_{inv}^{C;i} = e \epsilon^*_{\gamma}(q) \epsilon^*_{X,\alpha}(p') \epsilon_{Y,\beta}(p) I_{\alpha\beta\nu}^{C;i}.$$  

(A2)

$I_{\alpha\beta\nu}^{C;i}$ are structure integrals generated by the triangle $i = \Delta$ and bubble $i = \text{bub}$ diagrams

$$I_{\alpha\beta\nu}^{C;\Delta} = -g_{\alpha\beta\nu}^{C} \int \frac{d^4k}{\pi^2} \Phi_Y^{C}(\cdot-k^2) \Phi_X^{C}(\cdot-(k+q/2)^2) \left( \gamma^\alpha \gamma^5 S_c(k+p/2) \gamma^\beta S_c(k-p/2) \gamma^\nu S_c(k+p/2-p') \right),$$

(A3)

$$I_{\alpha\beta\nu}^{C;\text{bub}} = -g_{\alpha\beta\nu}^{C} \int \frac{d^4k}{\pi^2} \left( \Phi_Y^{C}(\cdot-[k^2-kqt+q^2t/4]) \right)^\prime \Phi_X^{C}(\cdot-k^2) \left( \gamma^\alpha \gamma^5 S_c(k+p/2) \gamma^\beta S_c(k-p/2) \right)$$

$$- g_{\alpha\beta\nu}^{C} \int \frac{d^4k}{\pi^2} \Phi_X^{C}(\cdot-[k^2+kqt+q^2t/4]) \left( \gamma^\alpha \gamma^5 S_c(k+p/2) \right) \left( \gamma^\beta S_c(k-p/2) \right),$$

where

$$g_{\alpha\beta\nu}^{C} = \frac{g_{\alpha\beta,\nu}^{C}}{4\pi^2} \sin \theta_X \sin \theta_Y$$

is the effective coupling constant and

$$\left( \Phi_H^{C}(z) \right)^\prime = \frac{\Phi_H^{C}(z)}{dz}$$

(A5)
is the derivative of the correlation function.

Contraction of the structure integral \( I^{C;\Delta}_{\alpha\beta\nu} \) with the photon momentum \( q^\nu \) gives

\[
q^\nu I^{C;\Delta}_{\alpha\beta\nu} = -g_{\text{eff}}^C \int \frac{d^4k}{(2\pi)^4} \Phi_C^*(-k^2) \Phi_X^*(-(k + q/2)^2) \\
\times \left( \text{tr} \left( \gamma^\alpha \gamma^5 S_c(k + p/2) \gamma^\beta S_c(k + p/2 - p') \right) - \text{tr} \left( \gamma^\alpha \gamma^5 S_c(k + p/2) \gamma^\beta S_c(k - p/2) \right) \right).
\]

(A6)

Here we used the set of Ward-Takahashi identities on the quark level:

\[
g = S_c^{-1}(k - p/2) - S_c^{-1}(k + p/2 - p')
\]

\[
S_c(k - p/2) - S_c(k + p/2 - p') = S_c(k + p/2 - p') - S_c(k - p/2).
\]

(A7)

The contraction of the \( I^{C;\text{bub}}_{\alpha\beta\nu} \) with the photon momentum \( q^\nu \) gives

\[
q^\nu I^{C;\text{bub}}_{\alpha\beta\nu} \equiv -q^\nu I^{C;\Delta}_{\alpha\beta\nu} + \Delta I_{\alpha\beta}
\]

(A8)

where the term

\[
\Delta I_{\alpha\beta} = g_{\text{eff}}^C \int \frac{d^4k}{(2\pi)^4} \Phi_C^*(-k^2) \Phi_X^*(-(k^2) - \left[ \text{tr} \left( \gamma^\alpha \gamma^5 S_c(k + p/2) \gamma^\beta S_c(k - p/2) \right) \right] - \text{tr} \left( \gamma^\alpha \gamma^5 S_c(k + p/2) \gamma^\beta S_c(k - p/2) \right)) = 0
\]

(A9)

explicitly vanishes because of spinor traces.

Therefore, the sum of triangle and bubble diagrams is manifestly gauge-invariant

\[
q^\nu \left( I^{C;\Delta}_{\alpha\beta\nu} + I^{C;\text{bub}}_{\alpha\beta\nu} \right) = 0.
\]

(A10)
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