Asymmetric Nuclear Matter: A variational Approach

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We discuss here a self-consistent method to calculate the properties of the cold asymmetric nuclear matter. In this model, the nuclear matter is dressed with $s$-wave pion pairs and the nucleon-nucleon (N-N) interaction is mediated by these pion pairs, $\omega$ and $\rho$ mesons. The parameters of these interactions are calculated self-consistently to obtain the saturation properties like equilibrium binding energy, pressure, compressibility and symmetry energy. The computed equation of state is then used in the Tolman-Oppenheimer-Volkoff (TOV) equation to study the mass and radius of a neutron star in the pure neutron matter limit.

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I. INTRODUCTION

The search for an appropriate nuclear equation of state has been an area of considerable research interest because of its wide and far reaching relevance in heavy ion collision experiments and nuclear astrophysics. In particular, the studies in two obvious limits, namely, the symmetric nuclear matter (SNM) and the pure neutron matter (PNM) have helped constrain several properties of nuclear matter such as binding energy per nucleon, compressibility modulus, symmetry energy and its density dependence at nuclear saturation density $\rho_0$ [1, 2, 3] to varying degrees of success. Of late, the availability of flow data from heavy ion collision experiments and phenomenological data from observation of compact stars have renewed the efforts to further constrain these properties and to explore their density and isospin content (asymmetry) variation behaviours [4, 5, 6, 7].

One of the fundamental concerns in the construction of nuclear equation of state is the parametrization of the nucleon-nucleon (N-N) interaction. Different approaches have been developed to address this problem. These methods can be broadly classified into three general types [8], namely, the {	extit{ab initio}} methods, the effective field theory approaches and calculations based on phenomenological density functionals. The {	extit{ab initio}} methods include the Brueckner-Hartree-Fock (BHF) [9, 10, 11] approach, the (relativistic) Dirac-Brueckner-Hartree-Fock (DBHF) [12, 13, 14, 15, 16] calculations, the Green Function Monte-Carlo (GFMC) [17, 18, 19] method using the basic N-N interactions given by boson exchange potentials. The other approach of this type, also known as the variational approach, is pioneered by the Argonne Group [20, 21]. This method is also based on basic two-body (N-N) interactions in a non-relativistic formalism with relativistic effects introduced successively at later stages. The effective field theory (EFT) approaches are based on density functional theories like chiral perturbation theory [24, 25]. These calculations involve a few density dependent model parameters evaluated iteratively. The third type of approach, namely, the calculations based on phenomenological density functionals include models with effective density dependent interactions such as Gogny or Skyrme forces [26] and the relativistic mean field (RMF) models [27, 28, 29, 30]. The parameters of these models are evaluated by carefully fitting the bulk properties of nuclear matter and properties of closed shell nuclei to experimental values. Our work presented here belongs to this class of approaches in the non-relativistic approximation.

The RMF models represent the N-N interactions through the coupling of nucleons with isoscalar scalar $\sigma$ mesons, isoscalar vector $\omega$ mesons, isovector vector $\rho$ mesons and the photon quanta besides the self- and cross-interactions among these mesons [29]. Nuclear equations of state have also been constructed using the quark meson coupling model

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(QMC) \cite{31} where baryons are described as systems of non-overlapping MIT bags which interact through effective scalar and vector mean fields, very much in the same way as in the RMF model. The QMC model has also been applied to study the asymmetric nuclear matter at finite temperature \cite{32}.

It has been shown earlier \cite{33,31}, that the medium and long range attraction effect simulated by the $\sigma$ mesons in RMF theory can also be produced by the $s$-wave pion pairs. This “dressing” of nucleons by pion pairs has also been applied to study the properties of deuteron \cite{35} and $^4$He \cite{36}. On this basis, we start with a nonrelativistic Hamiltonian density with $\pi N$ interaction. The $\omega$—repulsion and the isospin asymmetry part of the NN interaction are parametrized by two additional terms representing the coupling of nucleons with the $\omega$ and the $\rho$ mesons respectively. The parameters of these interactions are then evaluated self-consistently by using the saturation properties like binding energy per nucleon, pressure, compressibility and the symmetry energy. The equation of state (EOS) of asymmetric nuclear matter is subsequently calculated and compared with the results of other independent approaches available in current literature. The EOS of pure neutron matter is also used to calculate the mass and radius of a neutron star. We organize the paper as follows: In Section II, we present the theoretical formalism of the asymmetric nuclear matter as outlined above. The results are presented and discussed in Section III. Finally, in the last section the concluding remarks are drawn indicating the future outlook of the model.

II. FORMALISM

We start with the effective pion nucleon Hamiltonian

$$\mathcal{H}(x) = \mathcal{H}_N(x) + \mathcal{H}_{int}(x) + \mathcal{H}_M(x),$$

where the free nucleon part $\mathcal{H}_N(x)$ is given by

$$\mathcal{H}_N(x) = \psi^\dagger(x) \varepsilon_x \psi(x),$$

the free meson part $\mathcal{H}_M(x)$ is defined as

$$\mathcal{H}_M(x) = \frac{1}{2} \left[ \partial^2_\mathbf{x} + (\nabla \varphi_i) \cdot (\nabla \varphi_i) + m^2 \varphi_i^2 \right],$$

and the $\pi N$ interaction \cite{33} is provided by

$$\mathcal{H}_{int}(x) = \psi^\dagger(x) \left[ - \frac{iG}{2\varepsilon_x} \sigma \cdot \mathbf{p} \varphi + \frac{G^2}{2\varepsilon_x} \varphi^2 \right] \psi(x).$$

In equations (2) and (4), $\psi$ represents the non-relativistic two component spin-isospin quartet nucleon field. The single particle nucleon energy operator $\varepsilon_x$ is given by $\varepsilon_x = (M^2 - \nabla^2_x)^{1/2}$ with nucleon mass $M$ and the pion-nucleon coupling constant $G$. The isospin triplet pion fields of mass $m$ are represented by $\varphi$.

We expand the pion field operator $\varphi_i(x)$ in terms of the creation and annihilation operators of off-mass shell pions satisfying equal time algebra as

$$\varphi_i(x) = \frac{1}{\sqrt{2\omega_x}} (a_i(x)^\dagger + a_i(x)), \quad \tilde{\varphi}_i(x) = i \sqrt{\frac{\omega_x}{2}} (a_i(x)^\dagger - a_i(x)),$$

with energy $\omega_x = (m^2 - \nabla^2_x)^{1/2}$ in the perturbative basis. We continue to use the perturbative basis, but note that since we take an arbitrary number of pions in the unitary transformation $U$ in equation (7) as given later, the results would be nonperturbative. The expectation value of the first term of $\mathcal{H}_{int}(x)$ in eq. (4) vanishes and the pion pair of the second term provides the isoscalar scalar interaction of nucleons thereby simulating the effects of $\sigma$-mesons. A pion-pair creation operator given as

$$B^\dagger = \frac{1}{2} \int f(\mathbf{k}) a_i(\mathbf{k})^\dagger a_i(-\mathbf{k})^\dagger d\mathbf{k},$$

is then constructed in momentum space with the ansatz function $f(\mathbf{k})$ to be determined later.

We then define the unitary transformation $U$ as

$$U = e^{(B^\dagger - B)},$$

where $\mathbf{k}$ is the momentum of the pion pair and $f(\mathbf{k})$ is the probability density for finding a pion pair with momentum $\mathbf{k}$.
and note that $U$, operating on vacuum, creates an arbitrarily large number of scalar isospin singlet pairs of pions. The “pion dressing” of nuclear matter is then introduced through the state

$$|f\rangle = U|\text{vac}\rangle = e^{(B^\dagger - B)}|\text{vac}\rangle,$$

where $U$ constitutes a Bogoliubov transformation given by

$$U^\dagger a_i(k)U = (\cosh f(k)) a_i(k) + (\sinh f(k)) a_i(-k)^\dagger,$$  \hspace{1cm} (8)

We then proceed to calculate the energy expectation values. We consider $N$ nucleons occupying a spherical volume of radius $R$ such that the density $\rho = N/(4\pi R^3)$ remains constant as $(N, R) \to \infty$ and we ignore the surface effects. We describe the system with a density operator $\hat{\rho}_N$ such that its matrix elements are given by

$$\rho_{\alpha\beta}(\mathbf{x}, \mathbf{y}) = Tr[\hat{\rho}_N \psi_\beta(\mathbf{y})^\dagger \psi_\alpha(\mathbf{x})],$$  \hspace{1cm} (9)

and

$$Tr[\hat{\rho}_N \hat{N}] = \int \rho_{\alpha\alpha}(\mathbf{x}, \mathbf{x})d\mathbf{x} = N = \rho V.$$  \hspace{1cm} (10)

We obtain the free nucleon energy density

$$h_f = \langle f|Tr[\hat{\rho}_N \hat{H}_N(\mathbf{x})]|f\rangle = \sum_{\tau=n, p} \frac{\gamma f^2}{6\pi^2} \left( M + \frac{3}{10} \frac{k_f^2}{M} \right).$$  \hspace{1cm} (11)

In the above equation, the spin degeneracy factor $\gamma = 2$, the index $\tau$ runs over the isospin degrees of freedom $n$ and $p$ and $k_f^2$ represents the Fermi momenta of the nucleons. For asymmetric nuclear matter, we define the neutron and proton densities $\rho_n$ and $\rho_p$ respectively over the same spherical volume such that the nucleon density $\rho = \rho_n + \rho_p$. The Fermi momenta $k_f^2$ are related to neutron and proton densities by the relation $k_f^2 = (6\pi^2 \rho_r/\gamma)^{1/3}$. We also define the asymmetry parameter $y = (\rho_n - \rho_p)/\rho$. It can be easily seen that $\rho_r = \frac{2}{3}(1 \pm y)$ for $\tau = n, p$ respectively.

Using the operator expansion of equation (9), the free pion part of the Hamiltonian as given in equation (3) can be written as

$$\hat{H}_M(\mathbf{x}) = a_i(\mathbf{x})^\dagger \omega_x a_i(\mathbf{x}).$$  \hspace{1cm} (12)

The free pion kinetic energy density is given by

$$h_k = \langle f|\hat{H}_M(\mathbf{x})|f\rangle = \frac{3}{(2\pi)^3} \int d\mathbf{k} \omega(\mathbf{k}) \sin^2 f(\mathbf{k}),$$  \hspace{1cm} (13)

where $\omega(\mathbf{k}) = \sqrt{k^2 + m^2}$. Using $\epsilon_x \simeq M$ in the nonrelativistic limit, the interaction energy density $h_{int}$ can be written from equation (12) as

$$h_{int} = \langle f|Tr[\hat{\rho}_N \hat{H}_{int}(\mathbf{x})]|f\rangle \simeq \frac{G^2 \rho}{2M} \langle f| : \varphi_i(\mathbf{x}) \varphi_i(\mathbf{x}) : |f\rangle.$$  \hspace{1cm} (14)

Using the equations (7), (8) and (9), we have from equation (15)

$$h_{int} = \frac{G^2 \rho}{2M} \left( \frac{3}{(2\pi)^3} \int d\mathbf{k} \frac{\sinh^2 f(\mathbf{k})}{2} + \sinh^2 f(\mathbf{k}) \right).$$  \hspace{1cm} (15)

The pion field dependent energy density terms add up to give $h_m(= h_k + h_{int})$ which is to be optimized with respect to the ansatz function $f(\mathbf{k})$ for its evaluation. However, this ansatz function yields a divergent value for $h_m$. This happens because we have taken the pions to be point like and have assumed that they can approach as near each other as they like, which is physically inaccurate. Therefore, we introduce a phenomenological repulsion energy between the pions of a pair given by

$$h_m^R = \frac{3\alpha}{(2\pi)^3} \int \sinh^2 f(\mathbf{k}) \ e^{R^2 k^2} d\mathbf{k},$$  \hspace{1cm} (16)
where the two parameters $a$ and $R_\pi$ correspond to the strength and length scale, respectively, of the repulsion and are to be determined self-consistently later. Thus the pion field dependent term of the total energy density becomes

$$h_m = h_k + h_{int} + h_m^R.$$ Then the optimization of $h_m$ with respect to $f(k)$ yields

$$\tanh 2f(k) = \frac{G^2 \rho}{2M} \cdot \frac{1}{\omega^2(k) + \frac{G^2 \rho}{2M} + a \omega(k) e^{R_\pi^2 k^2}}.$$ (18)

The expectation value of the pion field dependent parts of the total Hamiltonian density of eqn. (1) along with the modification introduced by the phenomenological term $h_m^R$ becomes

$$h_m = \frac{3}{2} \frac{1}{(2\pi)^3} \frac{G^2}{2M} \rho \left[ \rho_n I_n + \rho_p I_p \right]$$ (19)

and the integrals $I_\tau (\tau = n, p)$ given by

$$I_\tau = \int_0^{k_\tau} \frac{4\pi k^2 dk}{\omega^2} \left[ \frac{1}{(\omega + a e^{R_\pi^2 k^2})^{1/2}} + \frac{1}{(\omega + a e^{R_\pi^2 k^2})^{1/2} + G^2 \rho} \right]$$ (20)

and $\omega = \omega(k)$.

We now introduce the energy of $\omega$ repulsion by the simple form

$$h_\omega = \lambda_\omega \rho^2,$$ (21)

where the parameter $\lambda_\omega$ corresponds to the strength of the interaction at constant density and is to be evaluated later. We note that equation (21) can arise from a Hamiltonian density given in terms of a local potential $v_R(x)$ as

$$H_R(x) = \psi(x) \psi(x) \int v_R(x - y) \psi(y) \psi(y) dy,$$ (22)

where, when density is constant, we in fact have

$$\lambda_\omega = \int v_R(x) dx .$$

The isospin dependent interaction is mediated by the isovector vector $\rho$ mesons. We represent the contribution due to this interaction, in a manner similar to the $\omega$-meson energy, by the term

$$h_\rho = \lambda_\rho \rho_3^2$$ (23)

where $\rho_3 = (\rho_n - \rho_p)$ and the strength parameter $\lambda_\rho$ is to be determined as described later. Thus we finally write down the binding energy per nucleon $E_B$ of the cold asymmetric nuclear matter:

$$E_B = \frac{\varepsilon}{\rho} - M ,$$ (24)

where $\varepsilon = (h_f + h_m + h_\omega + h_\rho)$ is the energy density. The expression for $\varepsilon$ contains the four model parameters $a$, $R_\pi$, $\lambda_\omega$ and $\lambda_\rho$ as introduced above. These parameters are then determined self-consistently through the saturation properties of nuclear matter. The pressure $P$, compressibility modulus $K$ and the symmetry energy $E_{sym}$ are given by the standard relations:

$$P = \rho^2 \frac{\partial (\varepsilon/\rho)}{\partial \rho}$$ (25)

$$K = 9\rho^2 \frac{\partial^2 (\varepsilon/\rho)}{\partial \rho^2}$$ (26)

$$E_{sym} = \left( \frac{1}{2} \frac{\partial^2 (\varepsilon/\rho)}{\partial y^2} \right)_{y=0} .$$ (27)

The effective mass $M^*$ is given by $M^* = M + V_s$ with $V_s = (h_{int} + h_m^R)/\rho$. 

III. RESULTS AND DISCUSSION

We now discuss the results obtained in our calculations and compare with those available in literature. The four parameters of the model are fixed by self-consistently solving eqs. (24) through (27) for the respective properties of nuclear matter at saturation density $\rho_0 = 0.15$ fm$^{-3}$. While pressure $P$ vanishes at saturation density for symmetric nuclear matter (SNM), the values of binding energy per nucleon and symmetry energy are chosen to be $-16$ MeV and $31$ MeV respectively. In the numerical calculations, we have used the nucleon mass $M = 940$ MeV, the meson masses $m = 140$ MeV, $m_\omega = 783$ MeV and $m_\rho = 770$ MeV and the $\pi - N$ coupling constant $G^2/4\pi = 14.6$. In order to ascertain the dependence of compressibility modulus on the parameter values, we vary the $K$ value over a range 210 MeV to 280 MeV for the symmetric nuclear matter ($y = 0$) and evaluate the parameters. It may be noted that this is the range of the compressibility value which is under discussion in the current literature. For $K$ values in the range 210 MeV to 250 MeV, the program does not converge. The solutions begin to converge for compressibility modulus $K$ around 258 MeV. We choose the value $K = 260$ MeV for our calculations. In Table I we present the four free parameters of the model for ready reference.

| $a$ (MeV) | $R_\pi$ (fm) | $\lambda_\omega$ (fm$^2$) | $\lambda_\rho$ (fm$^2$) |
|-----------|-------------|----------------|----------------|
| 16.98     | 1.42        | 3.10           | 0.65           |

For this set of parameter values the effective mass of nucleons at saturation density is found to be $M^*/M = 0.81$. In the Fig. 1 we present the binding energy per nucleon $E_B$ calculated for different values of the asymmetry parameter $y$ as a function of the relative nuclear density $\rho/\rho_0$. The values $y = 0.0$ and 1.0 correspond to SNM and PNM respectively. As expected, the binding energy per nucleon $E_B$ of SNM initially decreases with increase in density, reaches a minimum at $\rho = \rho_0$ and then increases. In case of PNM, the binding energy increases monotonically with increasing density in consistency with its well known behaviour. In Fig. 2(a), we compare the $E_B$ of SNM as a function of the nucleon density with a few representative results in the literature, namely, the Walecka model [27] (long-short dashed curve), the DBHF calculations of Li et al. with Bonn A potential (short-dashed curve) (data for both the
models are taken from [13] and the variational A18 + δv + UIX* (corrected) model of Akmal at al. (APR) [21] (long-dashed curve). While the Walecka and Bonn A models are relativistic, the variational model is nonrelativistic with relativistic effects and three body correlations introduced successively. Our model produces an EOS softer than that of Walecka and Bonn A, but stiffer than the variational calculation results of the Argonne group. It is well-known that the Walecka model yields a very high compressibility $K$. However, its improvised versions developed later with self- and cross-couplings of the meson fields have been able to bring down the compressibility modulus in the ball park of $230 \pm 10$ MeV [21]. Our model yields nuclear matter saturation properties correctly alongwith the compressibility of $K = 260$ MeV which is resonsably close to the empirical data. In Fig. 2(b), we plot $E_B$ as a function of the relative nucleon density for PNM. Similar to the SNM case, our EOS is softer than that of Walecka and Bonn A models, but stiffer than the variational model. We use this EOS to calculate the mass and radius of a neutron star of PNM as discussed later.

The density dependence of pressure of SNM and PNM are calculated using the eqn. 25. These results are plotted (solid blue curves) in Figs. 3(a) and (b). Recently, Danielewicz et al. [4] have deduced the empirical bounds on the EOS in the density range of $2 < \rho/\rho_0 < 4.6$ by analysing the flow data of matter from the fireball of Au+Au heavy ion collision experiments both for SNM and PNM. These bounds are represented by the color-filled and shaded regions of the two figures. These bounds rule out both the “very stiff” and the “very soft” classes of EOSs produced, for example, by some variants of RMF calculations and Fermi motion of a pure neutron gas [4]. As shown in these figures, the EOS of SNM and PNM generated by our model are consistent with both the bounds.

![Graph showing the binding energy per nucleon as a function of relative nucleon density](image)

**FIG. 2:** (a) The binding energy per nucleon $E_B$ as a function of relative nucleon density $\rho/\rho_0$ for SNM. The results of present work (P.W.) are compared with the results of DBHF calculations with Bonn A potential [13], the variational calculations of the Argonne group [21] and the Walecka model [27]. The data for the Bonn A and Walecka model curves are taken from [13]. (b) Same as Fig.-(2a), but for PNM.

The potentials per nucleon in our model can be defined from the meson dependent energy terms of eqs. (19), (21) and (23). Contribution to potential from the scalar part of the meson interaction is due to the pion condensates and is given by $V_s = (h_{int} + k_m^R)/\rho$ as defined earlier. The contribution by vector mesons has two components, namely, due to the $\omega$ and the $\rho$ mesons and is given by $V_v = V_\omega + V_\rho = (h_\omega + h_\rho)/\rho$. In the Figs. 4(a) and (b), we plot $V_s$ and $V_v$ as functions of relative density $\rho/\rho_0$ calculated for PNM (Fig. 4(a)) and for SNM (Fig. 4(b)) respectively. The magnitudes of the potentials calculated by our model are weaker compared to those produced by DBHF calculations with Bonn A interaction [13] as shown in both the panels of Fig. 4. In Fig. 4(a), we show the contributions to the repulsive vector potential due to $\omega$ mesons (short-dashed curve), $\rho$ mesons (long-dashed curve) and their combined contribution (long-short-dashed curve). The contribution due to $\rho$ mesons rises linearly at a slow rate and has a low
Knowledge of density dependence of symmetry energy is expected to play a key role in understanding the structure and properties of neutron-rich nuclei and neutron stars at densities above and below the saturation density. Therefore this problem has been receiving considerable attention of late. Several theoretical and experimental investigations addressing this problem have been reported ([3, 8, 39] and references therein). While the results of independent studies show reasonable consistency at sub-saturation densities $\rho \leq \rho_0$, they are at wide variance with each other at supra-saturation densities $\rho > \rho_0$. This wide variation has given rise to the so-called classification of “soft” and “stiff” dependence of symmetry energy on density [38, 39].

Fig. 5 shows a representation of the spectrum of such results along with the results of the present work (solid blue curve). While the Gogny and Skyrme forces (dark rib-dotted and dotted curves respectively with data taken from [3, 39]) produce “soft” dependence on one end, the NL3 force (dot-dashed curve with data taken from [8]) produces a very “stiff” dependence on the other end. The analysis of experimental and simulation studies of intermediate energy heavy-ion reactions as reported by Shetty et al. [39] (red triangles and long-short-dashed red curve respectively), results of DBHF calculations of Li et al. and Huber et al. [13, 14, 40] (rib-dashed and magenta ribbed curve), variational model [3, 21] (short-dashed curve), RMF calculations with nonlinear Walecka model including $\rho$ mesons by Liu et al. [30] (long-dashed green curve) as shown in Fig. 5 suggest “stiff” dependence with various degrees of stiffness. The experimental results (represented by the red triangles with data taken from Shetty et al. [39]) are derived from the isoscaling parameter $\alpha$ which, in turn, is obtained from relative isotopic yields due to multifragmentation of excited nuclei produced by bombarding beams of $^{58}$Fe and $^{58}$Ni on $^{58}$Fe and $^{58}$Ni targets. Shetty et al. have shown that the results of multifragmentation simulation studies carried out with Antisymmetrized Molecular Dynamics (AMD) model using Gogny-AS interaction and Statistical Multifragmentation Model (SMM) are consistent with the above-mentioned experimental results and suggest (as shown by the red long-short-dashed curve) a moderately stiff dependence of the symmetry energy on density. Our results (represented by the solid blue curve) calculated using eqn. [27] are consistent with these results at subsaturation densities but are stiffer at supra-saturation densities. More
FIG. 4: (a) The potentials $V_s$, $V_\omega$ and $V_\rho$ (as defined in the text) in PNM as calculated by our model are compared with the Bonn A results of Li et al.\[13\]. The contributions made by the $\omega$-meson (short-dashed curve) and $\rho$-meson (long-dashed curve) mediated interactions are distinctly shown for comparison. (b) The potentials in SNM. Because of isospin symmetry, $V_\rho$ (see text for definition) vanishes. Both the scalar (solid curve) and vector (short-dashed curve) potentials produced by our calculations are weaker in magnitude compared to those of Bonn A calculations.

observational or experimental information is required to be built into our model to further constrain the symmetry energy at higher densities. In Fig. 5, the curve due to Huber et al.\[40\] (with data taken from\[29\]) correspond to their DBHF ‘HD’ model calculations which involves only the $\sigma$, $\omega$ and $\rho$ mesons. Similarly the long-dashed green curve due to Liu et al.\[30\] is from the basic non-linear Walecka model with $\sigma$, $\omega$ and $\rho$ mesons. Our formalism is the closest to these two models with the exception that in our model the effect of $\sigma$ mesons is simulated by the $\pi$ meson condensates. It is also noteworthy that our results are consistent with these results for densities up to $2\rho_0$.

The wide variation of density dependence of symmetry energy at supra-saturation densities has given rise to the need of constraining it. As discussed by Shetty et al.\[39\], a general functional form $E_{\text{sym}} = E_{\text{sym}}^0 (\rho/\rho_0)^\gamma$ has emerged. Studies by various groups have produced the fits with $E_{\text{sym}}^0 \sim 31 − 33$ MeV and $\gamma \sim 0.55 − 1.05$. A similar parametrization of the $E_{\text{sym}}$ produced by our EOS with $E_{\text{sym}}^0 = 31$ MeV yields the exponent parameter $\gamma = 0.85$.

We next use the equation of state for PNM derived by our model in the Tolman-Oppenheimer-Volkoff (TOV) equation to calculate the mass and radius of a PNM neutron star. The mass and radius of the star are found to be $2.25M_\odot$ and 11.7 km respectively.

IV. CONCLUSION

In this work we have presented a quantum mechanical nonperturbative formalism to study cold asymmetric nuclear matter using a variational method. The system is assumed to be a collection of nucleons interacting via exchange of $\pi$ pairs, $\omega$ and $\rho$ mesons. The equation of state (EOS) for different values of asymmetry parameter is derived from the dynamics of the interacting system in a self-consistent manner. This formalism yields results similar to those of the \textit{ab initio} DBHF models, variational models and the RMF models without invoking the $\sigma$ mesons. The compressibility modulus and effective mass are found to be $K = 260$ MeV and $M^*/M = 0.81$ respectively. The symmetry energy calculated from the EOS suggests a moderately “stiff” dependence at supra-saturation densities and corroborates the recent arguments of Shetty et al.\[39\]. A parametrization of the density dependence of symmetry energy of the form
\( E_{\text{sym}} = E_{\text{sym}}^0 \left( \frac{\rho}{\rho_0} \right)^\gamma \) with the symmetry energy \( E_{\text{sym}}^0 \) at saturation density being 31 MeV produces \( \gamma = 0.85 \). The EOS of pure neutron matter (PNM) derived by the formalism yields the mass and radius of a PNM neutron star to be \( 2.25 M_\odot \) and \( 11.7 \) km respectively.

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References

[1] M. Prakash, I. Bombaci, M. Prakash, P.J. Ellis, and J.M. Lattimer, Phys. Rep. 280, 1 (1997).
[2] J.M. Lattimer and M. Prakash, Phys. Rep. 333, 121 (2000); Astrophys. J. 550, 426 (2001); Science 304, 536 (2004).
[3] A. W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, Phys. Rep. 441, 325 (2005).
[4] P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002).
[5] D.J. Nice et al., Astrophys. J. 634, 1242 (2005).
[6] T. Klähn et al., Phys. Rev. C 74, 035802 (2006).
[7] J. Piekarewicz, Phys. Rev. C 76, 064310 (2007).
[8] C. Fuchs and H.H. Wolter, Eur. Phys. J. A 30, 5 (2006).
[9] M. Jaminon and C. Mahaux, Phys. Rev. C 40, 354 (1989).
[10] X.R. Zhou, G.F. Burgio, U. Lombardo, H.-J. Schulze, W. Zuo, Phys. Rev. C 69, 018801 (2004).
[11] M. Baldo and C. Maieron, J. Phys. G 34, R243 (2007).
[12] R. Brockmann and R. Machleidt, Phys. Rev. C 42, 1965 (1990).
[13] G.Q. Li, R. Machleidt and R. Brockmann, Phys. Rev. C 45, 2782 (1992).
[14] F. de Jong and H. Lenske, Phys. Rev. C 58, 890 (1998).
[15] T. Gross-Boelting, C. Fuchs and A. Faessler, Nucl. Phys. A 648, 890 (1999).
[16] E.N.E van Dalen, C. Fuchs and A. Faessler, Nucl. Phys. A 744, 227 (2004); Eur. Phys. J. A 31, 29 (2007).
[17] J. Carlson, J. Morales, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C 68, 025802 (2003).
[18] W.H. Dickhoff, C. Barbieri, Prog. Part. Nucl. Phys. 52, 377 (2004).
[19] A. Fabrocini, S. Fantoni, A.Y. Illarionov and K.E. Schmidt, Phys. Rev. Lett. 95, 192501 (2005).
[20] A. Akmal and V.R. Pandharipande, Phys. Rev. C 56, 2261 (1997).
[21] A. Akmal, V.R. Pandharipande and D.G. Ravenhall Phys. Rev. C 58, 1804 (1998).
[22] B.D. Serot, J.D. Walecka, Int. J. Mod. Phys E 6, 515 (1997).
[23] R.J. Furnstahl, Lect. Notes Phys. 641, 1 (2004).
[24] M. Lutz, B. Friman, Ch. Appel, Phys. Lett B 474, 7 (2000).
[25] P. Finelli, N. Kaiser, D. Vretenar, W. Weise, Eur. Phys. J A 17, 573, (2003); Nucl. Phys. A 735, 449 (2004).
[26] M. Bender, P.-H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 2003.
[27] J.D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974); B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[28] P. Ring, Prog. Part. Nucl. Phys. 73, 193 (1996); Y.K. Gambhir and P. Ring, Phys. Lett. B 202, 2 (1988).
[29] J.K. Bunta and S. Gmuca, Phys. Rev C 68, 054318 (2003); S. Gmuca, J. Phys. G 17, 1115 (1991).
[30] B. Liu, M. D Toro, V. Greco, E. Shen, G. Zhao and B.X. Sun, e-print Arxiv No. nucl-th/0702064.
[31] K. Saito and A.W. Thomas, Phys. Lett. B 327, 9 (1994); 335, 17 (1994); 363, 157 (1995); Phys. Rev. C 52, 2789 (1995); P.A.M. Guichon, K. Saito, E. Rodionov, and A.W. Thomas, Nucl. Phys. A 601 349 (1996); P.K. Panda, A. Mishra, J.M. Eisenberg, W. Greiner, Phys. Rev. C 56, 3134 (1997).
[32] P.K. Panda, G. Krein, D.P. Menezes and C. Providência, Phys. Rev. C 68, 015201 (2003).
[33] A. Mishra, H. Mishra and S.P. Misra, Int. J. Mod. Phys. A 7, 3391 (1990).
[34] H. Mishra, S.P. Misra, P.K. Panda, B.K. Parida, Int. J. Mod. Phys. E 2, 405 (1992).
[35] P.K. Panda, S.P. Misra, R. Sahu, Phys. Rev. C 45, 2079 (1992).
[36] P.K. Panda, S.K. Patra, S.P. Misra, R. Sahu, Int. J. Mod. Phys. E 5, 575 (1996).
[37] M. Prakash, T.L. Ainsworth, J.M. Lattimer, Phys. Rev. Lett. 61, 2518 (1988).
[38] J. R. Stone, J. C. Miller, R. Koncwiecz, F.D. Stevenson, and M. R. Strayer, Phys. Rev. C 68, 034324 (2003).
[39] D.V. Shetty, S.J. Yennello and G.A. Souliotis, Phys. Rev. C 76, 024606 (2007).
[40] H. Huber, F. Weber, and M. K. Weigel, Phys. Lett. B 317, 485 (1993); H. Huber, F. Weber, and M. K. Weigel, Phys. Rev. C 51, 1790 (1995).