Energy loss of charged particles in a two-dimensional Dirac plasma

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Abstract

The stopping power and energy loss rate of charged particles traversing a two-dimensional Dirac plasma is investigated. The Dirac plasma considered here models a solid state system, recently realized graphene monolayer, where the conduction electrons obey the Dirac-like equation and exhibit a linear in momentum dispersion relation. Theoretical work presented here is based on the dielectric response function and the dynamical structure function within the Random-Phase-Approximation (RPA).
I. INTRODUCTION

Recent fabrication of single layers of graphite (graphene monolayer)\cite{1} has posed the interesting question of charged particle energy loss in solid media where the conduction electrons behave distinctly different from those found in ordinary semiconductor heterostructures. In a two-dimensional electron gas (2DEG) realized in semiconductor heterostructures, the conduction electrons behave as ordinary electrons with a parabolic dispersion relation. In contrast, the dispersion relation obeyed by conduction electrons in graphene monolayer is linear in momentum. This occurs due to the unique crystal structure of graphene which is a two-dimensional (2D) honeycomb lattice of carbon atoms. Quantum-mechanical hopping between the sublattices of graphene leads to the formation of two energy bands, and their intersection near the edges of the Brillouin zone yields the conical energy spectrum. As a consequence, the dispersion relation of electrons and holes bands is linear near $K$, $K'$ points of the Brillouin zone which is given by $\epsilon_k = \hbar v_F k$. Hence, the conduction electrons, known as Dirac electrons, behave as massless particles with the effective speed of light $v_F \approx 10^6 m/s \approx c/300$\cite{2}. In a graphene monolayer, electron transport is essentially governed by Dirac’s (relativistic) equation, rather than the usual Schrodinger equation for nonrelativistic quantum particles. The relativistic behavior of graphene was first predicted by P. R. Wallace\cite{3}. It thus provides a unique opportunity to study relativistic quantum dynamics in condensed-matter systems. The Dirac-like gapless energy spectrum was also confirmed recently by cyclotron resonance measurements in graphene monolayer\cite{4}.

In this work, we address energy loss experienced by charged particles as they traverse a non-local dynamic quantum plasma of Dirac electrons, electrons that obey the Dirac equation. The energy loss calculation performed here requires that we determine the dynamic, non-local dielectric response function for the plasma of Dirac electrons. We determine the dielectric response function within the Random-Phase-Approximation(RPA) and employ it to calculate the dynamical structure function which yields the stopping power.

Since the work of Nozieres and Pines\cite{5}, conduction electrons in solids have been treated as a solid state plasma, a gas of electrons in a neutralizing positive background. Moreover, electron energy loss spectroscopy has been an important probe of the dielectric response properties of solid state systems\cite{6}. This was brought into clear focus by the pioneering work of Ritchie\cite{7} where the energy loss of a charged particle traversing a bounded solid...
state plasma was considered. Dynamical processes in a solid state medium have been investigated with a variety of spectroscopic tools which employ an almost monochromatic beam of electrons, photons, neutral atoms or ions which scatter inelastically from the medium under study [8]. Information concerning the electronic properties of the medium is obtained as a result of the interaction between the probes and the elementary excitations of the medium. Experimentally, this information is obtained by electron energy loss spectroscopy (EELS) and inelastic light scattering studies. Recently, energy loss spectroscopy of free-standing graphene films was performed [9].

The present paper is arranged as follows. In section II we give the formulation of the problem. The energy loss rate and stopping power of a 2D Dirac plasma is outlined. This theoretical investigation is based on the evaluation of dynamical structure function and energy loss function. In section III, we present the results and discussion. Concluding remarks are made in section IV.

II. FORMULATION

The system under consideration is a 2D quantum plasma of Dirac electrons i.e. Dirac electrons embedded in a uniform and rigid neutralizing background of positive charges. We begin by considering the low-energy electronic eigenstates for this system, given by

$$H_0 \mathbf{F}(\mathbf{r}) = \varepsilon \mathbf{F}(\mathbf{r}). \quad (1)$$

The Dirac-like Hamiltonian in eq(1) is [10]

$$H_0 = v_F \left( \begin{array}{cc} 0 & \hat{k}_x - i \hat{k}_y \\ \hat{k}_x + i \hat{k}_y & 0 \end{array} \right) = v_F (\sigma_x \hat{k}_x + \sigma_y \hat{k}_y) \quad (2)$$

where $\sigma_x, \sigma_y$ are Pauli matrices and $v_F$ is the two-dimensional Fermi velocity of Dirac electrons (with the characteristic velocity $v_F \simeq 10^6 m/s$) [10]. Conduction electrons in a Dirac plasma behave as massless Dirac particles with a linear dispersion relation $\epsilon_{sk} = skv_F$ [1], where $s = \pm 1$ indicate the conduction (+1) and valence (−1) bands, respectively. $\mathbf{k}$ is the two-dimensional wave vector and is given by $\mathbf{k} = k_x \hat{i} + k_y \hat{j}$, here $k_x = k \cos \theta_k$, $k_y = k \sin \theta_k$. Hence, electron dynamics is modelled by the following Dirac equation

$$v_F (\sigma_x \hat{k}_x + \sigma_y \hat{k}_y) \mathbf{F}_{sk}(\mathbf{r}) = \varepsilon \mathbf{F}_{sk}(\mathbf{r}). \quad (3)$$
From here onwards we take $\hbar = c = 1$ throughout this calculation. The wavefunctions appearing in eq. (3) are given by

$$F_{s,k}(r) = \frac{1}{L} F_{s,k} \exp(ik \cdot r),$$  

$$F_{s',k+q}(r) = \frac{1}{L} F_{s',k+q} \exp(i(k + q) \cdot r)$$  

where $L^2$ is the area of the plasma sheet and

$$F_{sk} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_k} \\ s \end{pmatrix}$$

and

$$F_{s',k+q} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_{k+q}} \\ s' \end{pmatrix}.$$  

The wavevector and frequency dependent longitudinal dielectric response function within the Random-Phase-Approximation (RPA) can be expressed as

$$\epsilon(q, \omega) = 1 - \nu_c \pi(q, \omega)$$

where $\nu_c = 2\pi e^2/\kappa q$ is the Fourier transform of two dimensional coulomb interaction, $\kappa$ is the background dielectric constant and $\pi(q, \omega)$ is the two-dimensional (2D) polarizability. The plasmon modes at finite wave vectors are given by the zeroes of the dielectric response function given by eq(8). Since, in this work, we are primarily interested in the energy loss due to plasmons, we consider the dielectric response function in the high frequency, long wavelength limit ($q \to 0$), which is given as

$$\epsilon(q, \omega) \approx 1 - \frac{\omega_p^2}{\omega^2} \left( 1 - \frac{\omega^2}{4E_F^2} \right).$$

Here $\omega_p = (g_sg_v e^2E_F/2\kappa)^{1/2}$ is the plasma frequency with $E_F$ the Fermi energy. Equivalently, in terms of dimensionless variables, we have

$$\epsilon(x, \nu) \approx 1 - \frac{2\pi r_s}{k_F} \left( \frac{g_sg_v n_s}{\pi} \right)^{1/2} \frac{x}{2\nu^2} \left( 1 - \frac{\nu^2}{2} \right).$$

In the above expression, we have used the dimensionless variables $x = q/k_F$, $\nu = \omega/E_F$, $r_s$ is the Wigner-Seitz radius, $\kappa$ is the background dielectric constant and $g_s = g_v = 2$ being the spin and valley degeneracies respectively. Energy loss function is the basic parameter which accounts for the energy lost by the incident charged particle. It is also the parameter which is of central importance in energy loss spectroscopy (EELS) experiments. Energy loss
function is defined as the imaginary part of the inverse dielectric response function. From
the dielectric response function given by eq(9) and introducing the infinitesimally small parameter $\eta$ we obtain
\[
\frac{1}{\epsilon(q, \omega)} = \frac{(\omega + i\eta)^2}{(\omega + i\eta)^2 - \omega_p^2 \left(1 - \frac{(\omega + i\eta)^2}{4E_F^2}\right)} = \frac{A}{\omega - \omega_p \sqrt{1 - \frac{\omega^2}{4E_F^2}} + i\eta} + \frac{B}{\omega + \omega_p \sqrt{1 - \frac{\omega^2}{4E_F^2}} + i\eta}
\]
(11)
Evaluating the coefficients $A$, $B$ and using Dirac’s prescription $\lim_{\eta \to 0} \frac{1}{\xi \pm i\eta} = \frac{1}{\xi} \mp i\pi \delta(\xi)$, in eq(11) and then expressing the variables in terms of dimensionless parameters $x$, $\nu$ and $r_s$ we obtain the energy loss function for graphene monolayer as
\[
\text{Im} \left(\frac{1}{\epsilon(x, \nu)}\right) = -\frac{\pi \left(\frac{g_s g_v r_s x}{E_F}\right)^{1/2}}{(1 - \frac{\nu^2}{4})} \left[ \delta \left(\frac{\nu}{1 - \frac{\nu^2}{4}}\right) - \sqrt{\frac{g_s g_v r_s x}{2}} \right] - \delta \left(\frac{\nu}{1 - \frac{\nu^2}{4}}\right) + \sqrt{\frac{g_s g_v r_s x}{2}} \right].
\]
(12)
The interaction of charged particles with condensed matter system can be studied by means of the system’s stopping power. The energy loss per unit path length is the stopping power $S \equiv -\frac{dE}{dx}$ and it accounts for the energy lost by an external charged projectile as it passes through matter. In the quantum mechanical framework, we consider an in-plane probe of charge $ze$, mass $m$ and velocity $v_p$, interacting with the many particle system under consideration by treating the incident particle state as a plane wave state. We calculate the probability that the point-like projectile loses an energy $\omega$ in a time interval $dt$ in interaction with the 2D Dirac plasma. During the interaction impulse transfer is $q$. Here $\omega$ and $q$ satisfy $\omega = q \cdot v_p + q^2/2m$. For a heavy incident particle, we neglect the recoil energy $q^2/2m$ to obtain $\omega = q \cdot v_p$. The stopping power is given by summing over energy difference weighted by transition rate $W$ times the inverse projectile velocity, mathematically
\[
\frac{dE}{dx} \equiv -\frac{1}{v_p} \sum_q (\epsilon_k - \epsilon_{k-q})W.
\]
(13)
We can expand energy loss in terms of $\Delta \omega \equiv q^2/2m$ to obtain
\[
\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_0 + \left(\frac{dE}{dx}\right)_1 + \ldots.
\]
(14)
Here we also make use of the sum rule $-\int_0^\infty \omega \text{Im} \{1/\epsilon(q, \omega)\} d\omega = \pi \omega_p^2/2$. To the lowest order, the stopping power for a 2D Dirac plasma given by eq(13) can be expressed in terms
of the dynamical structure functions $S(q, \omega)$ as

$$
\left( \frac{dE}{dx} \right)_0 = -\frac{1}{v_p} \int_0^\infty \int_0^{q v_p} d\omega d^2q \left( \frac{n_e^2 e^2 \omega^2}{\kappa q} \right)^2 S(q, \omega) \delta(\omega - q v_p)
$$

(15)

whereas the first order stopping power is given by

$$
\left( \frac{dE}{dx} \right)_1 = -\frac{1}{2m v_p} \int \int n_e q^2 d^2q \left( \frac{2\pi e^2}{\kappa q} \right)^2 \frac{\partial}{\partial \omega} [\omega S(q, \omega) \delta(\omega - q v_p)] d\omega.
$$

(16)

The dynamical structure function is the auto-correlation function of the Fourier components of the particle density and accounts for longitudinal charge oscillations of the electron density relative to the positive background. From the polarizability, we can obtain the dynamical structure factor at finite temperature for a 2D Dirac plasma as

$$
S(q, \omega) = \frac{\kappa^2 \omega_p^2}{2g_s g_v e^4 n_e E_F} \left( 1 - \frac{\omega^2}{4E_F^2} \right) (1 - \exp(-\beta \omega))^{-1} \left[ \delta(\omega - \omega_p) - \delta(\omega + \omega_p) \right]
$$

(17)

where $\omega_0 = \omega/\sqrt{1 - \omega^2/4E_F^2}$, $\beta = 1/k_B T$, $k_B$ is the Boltzmann constant, $T$ is the absolute temperature, $E_F = v_F k_F$, $k_F = (4\pi n_e/g_s g_v)^{1/2}$ is the Fermi wavevector and $n_e$ is the electron density. Dynamical structure factor provides direct physical information about longitudinal excitations in the system. It is the measure of the density-density correlations of the system. Calculations of dynamical structure function reveal that the electromagnetic response of a Dirac plasma is substantially different from a 2DEG system which is essentially due to the distinctly different dispersion relation of Dirac electrons in a 2D Dirac plasma as compared to ordinary electrons in a 2DEG system.

For a solid state Dirac plasma under consideration, we can classify the rate of energy loss and stopping power into two categories, one due to the particle-hole excitations and the other due to the plasma oscillations. So the total stopping power of the system is

$$
\frac{dE}{dx} = \frac{dE_{e-h}}{dx} + \frac{dE_{pl}}{dx}
$$

(18)

Employing the following relation in eq(15)

$$
\delta(\varepsilon - \varepsilon_n - V_n \cos(\frac{2\pi}{a} x_0)) = \frac{a}{2\pi} \left[ \delta(x_0 - a \frac{2\pi}{a} \arccos(\frac{\varepsilon - \varepsilon_n}{V_n})) + \delta(x_0 + a \frac{2\pi}{a} \arccos(\frac{\varepsilon - \varepsilon_n}{V_n})) \right]
$$

$$
\times \frac{1}{\sqrt{V_n^2 - (\varepsilon - \varepsilon_n)^2}} \Theta(|V_n| - |\varepsilon - \varepsilon_n|)
$$

(19)
For a single charged particle, $z = 1$, the lowest order stopping power, due to plasma oscillations, of a 2D degenerate Dirac plasma is given by

$$S \approx \frac{dE_{pl}}{dx} = -\frac{1}{v_p} \int_{0}^{q_{c}} \frac{e^2 \omega_p^2}{2\kappa \sqrt{1 - \frac{\omega_p^2}{4E_F^2}}} \ln \left( y + \sqrt{y^2 - 1} \right) \arccosh(y) dq$$

with $y = qv_p/\omega_p \sqrt{1 - \frac{\omega_p^2}{4E_F^2}}$.

### III. RESULTS AND DISCUSSION

The dynamic and static response properties of an electron system are all embodied in the structure of the dielectric response function. We have employed the Random-Phase-Approximation (RPA) based dielectric response function to determine the structure function and the energy loss function of a 2D Dirac plasma. The two types of excitations in the medium responsible for energy loss are the collective and single particle excitations. Collective excitations (plasmons) occur at a higher energy compared to single particle (electron-hole) excitations. At a frequency $\omega_0$ equal to $\omega_p$, collective excitations (plasma oscillations) dominate for $q^2 v_p^2 < \omega_p^2$ and these exist as long as the wave vector remains less than a critical value $q_c$. However, above this critical value plasmons undergo Landau damping generating electron-hole pair excitations. Therefore, energy loss rate and stopping power are affected by these two types of contributions, one due to the collective oscillations i.e. plasmons and the other due to the electron-hole excitations. The energy loss function taking into account both these mechanisms of energy loss is presented here. Collective modes with the onset of Landau damping change into electron-hole pairs. We have plotted the loss function given by eq(12) due to collective excitations in Fig.(1). Here we essentially restrict the energy loss to plasmon excitations by plotting the result at $x = 0.1, 0.5, 0.01, 0.05, 0.001$. The parameters used are: electron density $n_e = 3.16 \times 10^{15}/m^2$, $v_F = 10^6 m/sec$, $\kappa = 2.5$ (using SiO$_2$ as the substrate material), $r_s = 0.5$. Note that the critical value of the wave vector $q_c$ and the corresponding critical value of $x$ is 0.7 where the onset of Landau damping of plasmons occur for the parameters considered here. Dynamical structure function shows delta function peaks at the plasma frequency. The peaks in the figure are the manifestation of existence of plasmons in the system. In Fig.(2), in order to include the contribution of single particle excitations, we have plotted energy loss function against $\nu = \omega/E_F$ for following
values of $x \equiv q/k_F = 1.0, 1.5, 2.0$. The peaks in the energy loss function seen in the figure can be interpreted as occurring due to single particle excitations of the system. In Fig.(3), we have plotted energy loss function versus $x = q/k_F$ for various values of $\nu = \omega/E_F$. The relationship between energy loss rate $dE_{pl}/dt$ due to plasmons and velocity of the incident particle $v_p$ is shown graphically in figure(4). We have plotted the energy loss rate versus the dimensionless velocity $v_p/v_F$. Stopping power is shown in Fig.(5) where the energy lost by the incident particle is due to plasma oscillations. The stopping power decreases with the increase of the incident particle velocity. Furthermore, the lowest order energy loss rate and stopping power of the medium are found to be independent of the mass of the incident particle.

IV. CONCLUSIONS

An analysis of energy loss suffered by fast particles in their interaction with matter requires the calculation of energy loss rate and stopping power. In this work, we present a theoretical investigation of energy loss through the calculation of both the energy loss rate and the stopping power of a 2D solid state Dirac plasma. The motivation of this work is the recent realization of single layer of graphite, graphene monolayer. The quasiparticles in graphene monolayer are found to obey the Dirac equation with a linear in momentum dispersion relation. The work presented here is based on the the dielectric response function and the dynamical structure function within the Random-Phase-Approximation (RPA). The energy loss function taking into account the collective as well as single particle mechanisms of energy loss is presented here. In the collective excitation regime, energy loss function peaks at the plasma frequency, such peaks when detected by experiments, can also be used to identify energy at which plasmons occur in the medium. Furthermore, stopping power as a function of the incident particle velocity is also determined. Stopping power is found to decrease as the velocity of the incident particle is increased. Moreover, the lowest order energy loss rate and stopping power of the Dirac plasma are found to be independent of the mass of the incident particle.
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