THE YELLOW AND RED SUPERGIANTS OF M33

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ABSTRACT

Yellow and red supergiants are evolved massive stars whose numbers and locations on the Hertzsprung–Russell (H–R) diagram can provide a stringent test for models of massive star evolution. Previous studies have found large discrepancies between the relative number of yellow supergiants (YSGs) observed as a function of mass and those predicted by evolutionary models, while a disagreement between the predicted and observed locations of red supergiants (RSGs) on the H–R diagram was only recently resolved. Here, we extend these studies by examining the YSG and RSG populations of M33. Unfortunately, identifying these stars is difficult as this portion of the color–magnitude diagram is heavily contaminated by foreground dwarfs. We identify the RSGs through a combination of radial velocities and a two-color surface gravity discriminant, and after re-characterizing the rotation curve of M33 with our newly selected RSGs, we identify the YSGs through a combination of radial velocities and the strength of the O i \( \lambda 7774 \) triplet. We examine \( \sim 1300 \) spectra in total and identify 121 YSGs (a sample that is unbiased in luminosity above \( \log(L/L_\odot) \sim 4.8 \)) and 189 RSGs. After placing these objects on the H–R diagram, we find that the latest generation of Geneva evolutionary tracks shows excellent agreement with the observed locations of our RSGs and YSGs, the observed relative number of YSGs with mass, and the observed RSG upper mass limit. These models therefore represent a drastic improvement over previous generations.

Key words: galaxies: individual (M33) – galaxies: stellar content – stars: evolution – supergiants

Online-only material: color figures, machine-readable tables

1. INTRODUCTION

Yellow and red supergiants (F- to G-type supergiants and K- to M-type supergiants, respectively) represent evolved phases in the lives of massive stars whose populations can provide stringent tests of current models of stellar evolution. The yellow supergiant (YSG) phase (defined as the region where 7500 K > \( T_{\text{eff}} > 4800 \) K and \( \log(L/L_\odot) > 4 \)) is a short-lived, transitional phase. Stars with initial masses between approximately 9 \( M_\odot \) and 40 \( M_\odot \) briefly pass through this region of the Hertzsprung–Russell (H–R) diagram while transitioning either from the main sequence to the red supergiant (RSG) phase or, in some cases, from the red back to the blue. The lifetime of the YSG phase is only on the order of tens of thousands of years and, as such, these stars are very rare. The RSG phase, on the other hand, is longer lived, occupying most of the He-burning phase, and representing the final stage in the evolutionary process for stars of an initial mass between approximately 9 \( M_\odot \) and 25 \( M_\odot \).

Details of the evolutionary process for stars in this portion of the H–R diagram are far from straightforward. To emphasize this point, in Figure 1 we present a set of \( Z = 0.014 \) (solid lines) and \( Z = 0.006 \) (dashed lines) evolutionary tracks from the Geneva group (Ekström et al. 2012; V. Chomienne et al. 2012, in preparation) zoomed in on this region. The left panel presents models computed with realistic initial rotation speeds, while the right panel presents models with no initial rotation for comparison. The number of loops in these models is striking, with the 9 \( M_\odot \) tracks going through a characteristic “blue loop” before terminating in the RSG region, while many of the higher mass tracks only briefly pass through this portion of the H–R diagram before looping back to end their lives as Wolf–Rayet (W–R) stars or luminous blue variables. The location at which these upper loops occur therefore dictates the divide between massive stars that end their lives as RSGs and those that do not (i.e., they define an upper mass limit for RSGs). This, in turn, has a significant effect on the predicted ratios of both supergiant and supernova types (for a discussion see Meynet et al. 2011).

The precise locations at which both these loops and the main RSG branches occur provide one of the key observational tests that can be carried out in this portion of the H–R diagram. Not only are their locations incredibly sensitive to both metallicity and to the treatment of uncertain parameters such as mass loss, overshooting, and rotationally induced mixing, but they also define forbidden regions on the H–R diagram—a prediction easily tested by obtaining effective temperatures and luminosities for observed RSG and YSG populations.

Indeed, until recently, there was a discrepancy between the locations of observed RSGs on the H–R diagram and those predicted by stellar evolution theory, with the models failing to produce RSGs as cool or luminous as those observed (Massey 2003). Although a number of assumptions made in the modeling process could cause this discrepancy (e.g., uncertain RSG opacities and simplifications invoked by mixing-length theory), the fault actually lay with the derived locations for

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4 This can easily be seen from Figure 1, where the higher metallicity tracks both possess cooler RSG branches and loop back to the blue portion of the H–R diagram at a lower initial mass than the low-metallicity tracks, and the non-rotating tracks systematically extend to cooler temperatures before looping back to the blue.

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the observed samples. Using state-of-the-art MARCS model atmospheres (Gustafsson et al. 2003, 2008; Plez et al. 1992; Plez 2003), Levesque et al. (2005) derived new temperatures and luminosities for Galactic RSGs and found excellent agreement with the solar metallicity Geneva evolutionary models (Meynet & Maeder 2003). Since then, agreement has been found across a wide range of metallicities from the Magellanic Clouds (Levesque et al. 2006) to M31 (Massey et al. 2009).

However, the main reason this portion of the H-R diagram is so ideal for observational testing is the incredibly short lifetime of the YSG phase. With assumptions about the star formation rate and initial mass function (IMF) the YSG lifetimes predicted by the models for stars of various masses can be used to predict the relative number of stars that should appear in various luminosity (mass) bins. However, these predicted lifetimes are also highly sensitive to the uncertain parameters mentioned above (mass loss, overshooting, rotationally induced mixing), and because of their short duration, even small variations can dramatically change the predicted relative number of stars. As Kippenhahn & Weigert (1990, p. 468) put it, “[The yellow supergiant] phase is a sort of magnifying glass, revealing relentlessly the faults of calculations of earlier phases.”

In previous studies, Drout et al. (2009) and Neugent et al. (2010) examined the YSG populations of M31 and the Small Magellanic Cloud (SMC), respectively. They found that the relative number of observed YSGs as a function of luminosity differs dramatically from those predicted by the lifetimes of the evolutionary models. In both cases, the models overestimate the number of high-luminosity stars with respect to low-luminosity stars by factors of 10 or more. We note that since this problem has manifested itself in both M31 and the SMC (whose metallicities differ by a factor of 10), the cause cannot simply be the prescriptions used for the metal-line-driven (and, hence, metallicity dependent) main-sequence mass loss.

In this paper we examine the case of M33, whose metallicity, although debated (see Section 4.1), likely lies intermediate between the SMC and solar (Z = 0.006 and Z = 0.014, respectively). This and a companion paper examining the RSG and YSG contents of the Large Magellanic Cloud (LMC; Neugent et al. 2012) are part of a continued effort to characterize the YSG and RSG populations in local group galaxies of various metallicities in order to more fully assess the problems that may be present in the current evolutionary models. These tests are vital not only for our understanding of stellar physics but also because spectral synthesis codes (such as Starburst99; Leitherer et al. 1999) are only as good as the evolutionary models that they utilize. At the same time, we are employing a newer generation of the Geneva evolutionary models.

One large hurdle that must be overcome in order to actually carry out these tests is, ironically, identifying the supergiants. When one looks toward a local galaxy at the colors and magnitudes of YSGs, the bona fide supergiants are masked by a veritable sea of foreground dwarfs. This is clearly demonstrated in Figure 2, where the upper panel is a color–magnitude diagram (CMD) of M33 constructed from the photometry of the Local Group Galaxies Survey (LGGS; Massey et al. 2006) and the lower panel is the predicted locations for foreground dwarfs constructed from the Besançon Model for the Milky Way (Robin et al. 2003). Although the red and blue portions of the CMD are relatively uncontaminated, the overlap in the YSG regime is striking. Drout et al. (2009) and Neugent et al. (2010) overcome this problem by using the relatively large systematic velocities of M31 and the SMC to distinguish extragalactic supergiants based on their radial velocities. In M33, modest systematic (−180 km s\(^{-1}\)) and line-of-sight rotational (∼80 km s\(^{-1}\))

### Figure 1.

New Geneva Group Z = 0.014 (solid lines) and Z = 0.006 (dashed lines) evolutionary models (Ekström et al. 2012; V. Chomienne et al. 2012, in preparation). The left panel displays models with an initial rotation of 0.4\(v_{\text{crit}}\), where \(v_{\text{crit}}\) is critical velocity at which the centrifugal force exactly balances the gravitational acceleration (see Section 4.2 for further discussion). The right panel presents models with no initial rotation and is shown for comparison. Vertical lines designate the yellow supergiant region (4800 K < \(T_{\text{eff}}\) < 7500 K).

(A color version of this figure is available in the online journal.)

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5 We note that a similar test is not possible for the RSGs because the near-vertical nature of the tracks in the region makes it very difficult to distinguish stars in different mass bins.
Figure 2. Top: a color–magnitude diagram for M33 constructed from the LGGS photometry. Bottom: location of foreground contamination as predicted by the Besançon model for the Milky Way (Robin et al. 2003).

Figure 3. Photometry of isolated RSG candidates in M33. Two separate bands are obvious, with the upper band representing likely supergiants and the lower band representing likely dwarfs.

velocities (Kerr & Lynden-Bell 1986; Zaritsky et al. 1989) should still allow some separation of supergiant candidates via this method.

For the case of the RSGs, the situation is not nearly as dire. Massey (1998) demonstrates that local group RSGs may be separated from foreground dwarfs by means of a $B - V$, $V - R$ two-color diagram. At cool effective temperatures $B - V$ is primarily a surface gravity indicator (due to line-blanketing in the blue), while $V - R$ remains an effective temperature indicator. Using the improved photometry of the LGGS, Massey et al. (2009) examine M31 in more detail and demonstrate that stars selected in this manner show excellent agreement with stars whose radial velocities are consistent with their location in M31. In Figure 3, we plot the $B - V$ versus $V - R$ colors from the LGGS for isolated stars toward M33 with $V < 20$ and $V - R > 0.6$. Two distinct bands are evident in this plot, with the upper band representing likely supergiants and the lower band likely dwarfs. However, the exact selection criterion that should be utilized has yet to be thoroughly tested at sub-solar metallicities.

Thus, our aims in this paper are twofold: (1) to characterize the RSG and YSG contents of M33 and (2) to further assess any contradictions that may be present between the current evolutionary tracks and our supergiant populations. In Section 2, we describe our data and reductions. In Section 3, we describe the process by which we distinguish M33’s RSGs and YSGs from the population of foreground dwarfs, and in Section 4 we present a comparison of our results to the current set of evolutionary tracks.

2. OBSERVATIONS AND REDUCTIONS

In order to distinguish genuine RSGs and YSGs in M33 from foreground Milky Way dwarfs, we obtained radial velocities for $\sim$1300 red and yellow stars toward M33 using the Hectospec multi-fiber spectrograph on the 6.5 m MMT telescope. The observations were obtained over the course of two years, with our yellow sample observed in the fall of 2009 and our red sample (along with repeat observations for a subset of the yellow sample) in the fall of 2010. In this section, we describe our sample selection, observations, and initial data reduction.

2.1. Selection Criteria

Our samples were initially selected photometrically from the contents of the LGGS of Massey et al. (2006). The YSG candidates were selected to have $V < 18.7$, $U - B > -0.4$, and $0.0 \leq B - V \leq 1.4$. We note that these selection criteria are very similar to those used in Drout et al. (2009). The magnitude cutoff (roughly corresponding to log $L/L_\odot \sim 4.5$) is selected to provide adequate signal to noise in our spectroscopy, and the color range roughly corresponds to that over which F- and G-type supergiants cannot be photometrically distinguished from foreground dwarfs. With these definitions we constructed a catalog of 1438 YSG candidates.
The sample of red stars was initially selected to have $V < 20$ and $V - R > 0.6$. These criteria were selected to provide adequate signal to noise for spectroscopy, to help avoid confusion with low-mass asymptotic giant branch stars, and to restrict the sample to K-type stars and later (see discussion in Massey et al. 2009). We then applied the additional criterion that our targets be "isolated" to ensure, to the best of our ability, that our spectroscopy probes a single star. Using the $V$-band photometry of the LGGS, we estimated the crowding of each red star that falls within the above criteria. We restricted our sample to stars for which we can expect the contamination from objects other than the star itself in a Hectospec fiber (diameter $\sim 1.5''$) to be $<20\%$. With this additional requirement, we construct a catalog of 1098 red stars. This represents $\sim 42\%$ of the 2602 stars in the LGGS that matched the initial photometric criteria. We then applied the photometric criteria described in Section 1 to assess the LGGS that matched the initial photometric criteria. We then restricted our sample to stars whose $BVR$ photometry places them in the supergiant candidate regime and who also have $K$-band data from the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006). The 2MASS criteria were imposed because we would like to utilize $K$-band photometry and $V - K$ colors to determine the effective temperatures and bolometric luminosities of our RSG candidates. After these stars, the order of priority was our repeat yellow targets, the remaining RSG candidates, and finally the red catalog stars whose $BVR$ photometry indicates they are likely dwarfs. This final category is included to act as a test of our photometric selection criteria. During this process, the RSGs were given some short shrift as our higher priority was obtaining an unbiased selection of YSGs. In the end, we were able to assign fibers to $\sim 68\%$ of our yellow catalog and $\sim 15\%$ of our red catalog ($\sim 37\%$ of the isolated stars).

2.2. Observations

Hectospec is a 300 optical fiber fed spectrograph (Fabricant et al. 2005) with a $1^\circ$ field of view deployed on the MMT 6.5 m telescope. Hectospec observations are carried out in a queue mode in order to spread the effects of bad weather over all of the selected programs. As a result, our observations were obtained over three nights in 2009 October, one night in 2009 November, one night in 2009 December, and two nights in 2010 November (NOAO proposals 2009B-0149 and 2010B-0260). All observations were obtained using the 600 line mm$^{-1}$ grating, yielding a dispersion of 0.55 Å pixel$^{-1}$ and a (5 pixel) spectral resolution of 2.8 Å. The grating was centered on 7800 Å such that the wavelength coverage (6500–9000 Å) included both the Ca II triplet ($\lambda\lambda 8498, 8543, 8662$) for cross-correlation and the O I $\lambda 7774$ line (which has been shown to be a luminosity indicator in galactic F-type supergiants; see Section 3.3).

| Configuration | Exp Time (minutes) | UT Date (s) | N Stars Observed |
|---------------|--------------------|-------------|------------------|
| 2009 Observations |  |  |  |
| Brt-1 | 15 | 2009 Oct 16 | 124 |
| Fnt-1 | 60 | 2009 Oct 15 | 217 |
| Fnt-2 | 60 | 2009 Oct 27 | 210 |
| Fnt-3 | 60 | 2009 Nov 25 | 156 |
| Fnt-4 | 60 | 2009 Dec 15 | 207 |
| 2010 Observations |  |  |  |
| Brt-1 | 36 | 2010 Nov 26 | 56 |
| Fnt-1 | 75 | 2010 Nov 26, 27 | 211 |
| Fnt-2 | 80 | 2010 Nov 27 | 190 |

The observations obtained in 2009 consisted of one field containing bright catalog stars ($V \sim 12.7–15.5$) and four fields containing faint catalog stars ($V \sim 15.0–18.7$), with overlap to check on the consistency of radial velocity measurements. Each field was observed in three consecutive exposures to help eliminate cosmic rays; the total integration times for the bright and faint fields were 15 minutes and 60 minutes, respectively.

The observations in 2010 were again broken into a bright field ($V \sim 15.3–17.7$ for the yellow stars and $V \sim 14.2–17.0$ for the red stars) and two fields of fainter stars ($V \sim 17.0–20.0$ for the YSG candidates and $\sim 17.0–18.7$ for the RSG candidates). The bright field was observed through inclement weather for 36 minutes. One of the two faint fields was also observed that same night for 75 minutes but was repeated the next evening under clear conditions for 80 minutes, along with the second faint field. Information on our observed fields is summarized in Table 1.

The observations obtained in 2009 consisted of one field containing bright catalog stars ($V \sim 12.7–15.5$) and four fields containing faint catalog stars ($V \sim 15.0–18.7$), with overlap to check on the consistency of radial velocity measurements.

For our radial velocity standards we utilize 10 yellow stars in the direction of M31 whose radial velocities were determined in Drout et al. (2009). The observations were taken in the fall of 2009 with the same grating and over the same wavelength range as the M33 data described here.

In observing our fields, we relied on the third US Naval Observatory (USNO) CCD Astrobot Graph Catalog (UCAC3; Zacharias et al. 2010) to provide guide stars with negligible proper motions. The LGGS coordinates were tied to an earlier version of the USNO catalogs. In order to transform the LGGS coordinates to the UCAC3 “system” (presumably identical to the Interna tional Celestial Reference System, ICRS), we used stars in common to the two surveys to determine $\alpha_{UCAC3} = \alpha_{LGGS} - 0^\circ 03$ and $\delta_{UCAC3} = \delta_{LGGS} - 0^\circ 14$. The names of all our targets contain their LGGS coordinates, and these relations may therefore be used to determine their ICRS coordinates.

The reductions followed the same procedure as described in Drout et al. (2009) for Hectospec data, except that the zero point of the wavelength scale was set by the OH $\lambda 8399$ line. We are indebted to Susan Tokarz of the CfA’s OIR group for performing these reductions through their standard pipeline.

3. ANALYSIS

3.1. Observed Radial Velocities

In order to begin assigning membership to our supergiant candidates, we first measured radial velocities for our $\sim 1300$ observed spectra via Fourier cross-correlation with a set of radial velocity standards. Before cross-correlation, each spectrum was normalized with a 9th order cubic spline fit and then had 1.0
subtracted to remove the continuum. All cross-correlations were carried out in the IRAF routine “fxcor.”

As mentioned in Section 2.2, for radial velocity standards we used spectra of stars in the direction of M31 whose radial velocities had been measured by Drout et al. (2009). We initially intended to carry out all of our cross-correlations in the region of the Ca II triplet and thus choose five stars, J004535.23+413600.5, J004251.90+413745.9, J004532.62+412227.8, J004618.59+414410.9, and Mag-253496, whose spectra contained particularly strong Ca II lines. When the velocities from Drout et al. (2009) were applied to the spectra and the standards were cross-correlated against each other, the results were consistent to within $\sim 1.5$ km s$^{-1}$ with standard deviations of the mean ranging from 0.8 to 1.3 km s$^{-1}$. Our $\sim 1300$ program spectra were then cross-correlated against all five standards. All cross-correlations involving these standards were carried out over the range 8465–8680 Å to include the Ca II triplet and to exclude featureless continuum and strong atmospheric bands.

As described in detail by Cenarro et al. (2001) at the temperatures of A- to F-type stars, the lines of the Ca II triplet ($\lambda\lambda$8498, 8543, 8662) can become contaminated by the Paschen lines, P13, P15, and P16 ($\lambda\lambda$8502, 8545, 8665). Suspecting that this could affect the accuracy of the radial velocity measurements for our warmer stars, we examine our spectra for evidence of the other lines in the Paschen series. Indeed, we found $\sim 120$ spectra in our sample ($\sim 9\%$ of the total sample) that showed obvious Paschen lines. As expected, within these stars we observe a general trend such that as $B - V$ increases (and therefore effective temperature decreases), the strength of the three lines near the Ca II triplet increases relative to the other Paschen lines. We find that at a $B - V$ of $\sim 0.55$ (in the middle of the YSG range) this relative increase is apparent, but not overwhelming, indicating that the lines in the spectra are likely a blend of the Ca II triplet lines and Paschen lines. Conversely, by a $B - V$ of $\sim 1.4$ (approximately the edge of our defined YSG range), either no additional Paschen lines are evident above the noise or they are so dwarfed by the main three lines in the spectra that it is evident the Ca II triplet completely dominates. See Figure 4 for an illustration of this effect.

For the spectra with Paschen lines we sought new radial velocity standards. From the spectra of yellow stars in M31 we identify five stars with obvious Paschen lines: J003930.55+403135.2, J003948.85+403844.8, J004009.13+403142.9, J003949.86+405305.8, and J004120.56+403515.4. When these were cross-correlated against one another, the results were consistent with the “known” values from Drout et al. (2009) to within $\sim 1.5$ km s$^{-1}$ and the standard deviations from the mean varied between 0.5 and 1.0 km s$^{-1}$. The $\sim 120$ spectra with obvious Paschen lines were then cross-correlated against these standards over the range 8400–8900 Å, to include the P11 to P19 lines of the Paschen series.

These new radial velocity standards all possess intermediate temperatures within the YSG range. We found that use of these standards improved our cross-correlations for all but our hottest yellow stars (which represent spectra most strongly dominated by the Paschen lines). For these $\sim 30$ spectra we found no suitable standards within our M31 sample. Instead, we chose six of the highest quality spectra (J013254.33+303603.8, J013244.40+301547.7, J013252.56+303419.6, J013327.81+302106.0, J013350.71+304254.3, J013327.65+303751.2), measured their radial velocities by hand, and used them as standards for the rest of the sample. Given our means of measuring the standards’ velocities, we expect our radial velocities for these $\sim 30$ spectra to process higher errors than the rest of our sample. However, we do not expect this fact to alter which stars we select as M33 members (see Section 3.3). In addition, all these stars actually fall outside our defined YSG range (4800–7500 K) and, thus, will not affect our comparison to the current set of evolutionary tracks.

The error associated with our measured radial velocities was quantified in several independent ways. First, each measured velocity, $V_{\text{obs}}$, is produced by taking the average result of...
cross-correlating the spectra against five independent standards. The standard deviation of the mean for this averaging process was typically on the order of 1.5 km s\(^{-1}\), although this number may underestimate our true error. For the \(\sim 30\) hottest stars in our sample, the average standard deviation was slightly larger: on order of 4 km s\(^{-1}\). This is a result of the larger uncertainty in the standards’ velocities. Additionally, the IRAF package “fxcor” outputs both an internal error, obtained from the uncertainty of the Gaussian fit to the center of the correlation function, and the Tonry & Davis (1979) \(r\)-parameter, the ratio of the peak of the correlation function to its noise. The average internal error and \(r\)-parameter for our sample are 2.4 km s\(^{-1}\) and 38.6, respectively. Further examination reveals that stars with low \((r < 25)\), medium \((25 < r < 40)\), and high \((r > 40)\) \(r\)-parameters have typical errors of 3.5–5.5 km s\(^{-1}\), 2.0–3.5 km s\(^{-1}\), and 1.5–2.0 km s\(^{-1}\), respectively.

Over the course of our program, we also obtained multiple observations of a number of our targets. Notably, in the 2010 set of observations, the fnt-1 field was repeated on two consecutive nights. In Figure 5 (left), we compare the velocities measured each night. We see that, in general, the agreement is good. The average difference between the observations is \(-1.3\) km s\(^{-1}\) with a standard deviation of 8.0 km s\(^{-1}\). Additionally, 25 and 42 stars were observed in common between the “faint” and “bright” fields in 2010 and 2009, respectively. A comparison of the two measured velocities for the 25 stars from 2010 yields similar results to the fnt-1 stars. However, the 42 stars observed twice in 2009 had an average difference of \(-1.0\) km s\(^{-1}\) with a standard deviation of 3.1 km s\(^{-1}\). Thus, we suspect that the poor weather during our second observing run has produced higher uncertainties in the 2010 data. Finally, 79 of the stars observed in 2010 were repeat observations of yellow stars observed in 2009. In Figure 5 (right) we plot the velocities measured each year. The average difference between the observations is \(-0.7\) km s\(^{-1}\) with a standard deviation of 7.0 km s\(^{-1}\).

We do note that, although our sample contained both red and yellow stars, all of our radial velocity standards were yellow stars. Thus, it is likely that the velocities we measure for our red sample will have a higher associated error. In Figure 6 we plot Tonry & Davis (1979) \(r\)-parameter versus \(V\)-band magnitude for both our red and yellow samples. Indeed, it can be seen that, for a given magnitude, the red sample has a lower average \(r\)-parameter. However, we do not believe that this effect influences our results.

Recall that our purpose in measuring radial velocities was to attempt to discern membership in M33. As mentioned in Section 1, M33 has a systematic velocity of approximately \(-180\) km s\(^{-1}\) and has a line-of-sight rotation of approximately
80 km s\(^{-1}\). We therefore expect M33 members to have velocities roughly ranging from \(-100\) km s\(^{-1}\) to \(-260\) km s\(^{-1}\), while the foreground dwarfs should be scattered about 0 km s\(^{-1}\). Thus, we find that, for essentially all of our targets, we obtained radial velocities to suitable precision for our purposes. We did choose to reject a handful of stars whose errors were above \(\sim 10\) km s\(^{-1}\), and examination revealed no obvious spectral features above the noise. In the end, we obtained radial velocities for 408 individual red stars and 916 individual yellow stars in the direction of M33. Their velocities, errors, and other identifying information are summarized in Tables 2 and 3. For stars observed in multiple fields we adopt the average values. If the velocities obtained from two different observations of the same star varied by more than \(10\) km s\(^{-1}\), we note this fact in Tables 2 and 3.

### 3.2. Membership Determination: Red Supergiants

In order to assess the accuracy of our two-color discriminant for RSGs, we compare the stars we select photometrically as supergiants to those we expect to be M33 members based on their radial velocities. In general, the radial velocity of a disk galaxy can be approximated as \(V_r = V_0 + V(R)\sin\xi\cos\theta\), where \(V_0\) is the systemic radial velocity, \(\xi\) is the angle between the line of sight and the perpendicular to the plane of the galaxy, and \(V(R)\) is the rotational velocity within the plane at a radial distance \(R\), and \(\cos\theta = X/R\), where \(X\) is the position along the major axis (Rubin & Ford 1970). If the rotation curve were flat \((V(R) = \text{constant})\), this would be a linear relationship between \(V_r\) and \(X/R\).

To assess whether such a linear relationship is a valid approximation for M33, we examine the radial velocities obtained for 55 H\(\alpha\) regions in M33 by Zaritsky et al. (1989). We first calculate the parameter \(X/R\) for each H\(\alpha\) region, assuming a position angle of 23° and an inclination angle of 56° (following Kwitter & Aller 1981 and Zaritsky et al. 1989), and plot the results versus radial velocity in Figure 7. This shows a clear linear trend, indicating that the kinematics of M33 are dominated by dark matter. We note that Zaritsky et al. (1989) provide velocities relative to one of the H\(\alpha\) regions in the central portion of...
Figure 7. Radial velocity vs. $X/R$ for 55 H\textsc{ii} regions in M33 from Zaritsky et al. (1989). Radial velocities were measured with respect to one of the H\textsc{ii} regions near the center of the galaxy. The relation between the two variables is well approximated by a linear fit (solid black line), indicating that the rotation curve of M33 is nearly flat. Using this fit, we are able to define an expected radial velocity for a bona fide M33 member as a function of position in the galaxy ($X/R$).

Figure 8. Observed velocity minus expected M33 velocity vs. $X/R$ for our sample of red stars. Genuine M33 members should be centered about zero difference, while the foreground dwarfs make up the diagonal band. Stars photometrically classified as supergiants are plotted in red. One can see there excellent agreement between these two selection criteria. (A color version of this figure is available in the online journal.)

Figure 9. Two-color diagram for our observed sample of red stars (filled points) and the stars from Massey (1998), whose classifications are at odds with those predicted by the LGGS photometry (stars). The dashed line represents our adopted photometric cutoff between supergiant candidates (above line) and likely foreground dwarfs (below line). Stars whose radial velocities are consistent with the kinematics of M33 are plotted in red. (A color version of this figure is available in the online journal.)
From this we see that of the five stars classified photometrically as supergiants (but with radial velocities inconsistent with M33), three lie very close to our photometric cutoff and are more naturally associated with the band of likely dwarfs. An examination of the two remaining stars reveals that they are highly contaminated by light from other objects in the B band. Had our initial criteria that the stars be “isolated” been based on B-band photometry (rather than V band), these stars would not have been included in our sample.

To explain the stars that appear photometrically as dwarfs but kinematically as supergiants, we note that falling near the zero point of the y-axis in Figure 8 does not necessarily prove membership in M33. A portion of M33’s $-180$ km s$^{-1}$ systematic velocity is actually reflex motion of the Sun (Courteau & van den Bergh 1999), and we thus expect some halo dwarfs and giants to possess negative radial velocities. An examination of the Atlas of Galactic Neutral Hydrogen by Hartmann & Burton (1997) reveals that M33 only stands out from the Galactic clutter at radial velocities less (more negative) than $-150$ km s$^{-1}$. This suggests that a star with $V_{\text{hel}} > -150$ km s$^{-1}$ should not be selected as a member of M33 based on radial velocity alone.

We additionally examine the velocities of red foreground stars (covering the same area as our observations) predicted by the Besançon model for Galactic dwarfs (Robin et al. 2003). In Figure 10, we plot a histogram of the velocities predicted by the model (solid, black) and the velocities of our observed red sample (dashed, red). From this we see an excess of stars at high radial velocities in our observed sample, representing the genuine M33 RSGs. We note that the peak of stars centered about 0 km s$^{-1}$ (the foreground dwarfs) is much smaller in our observed sample because we only observed a fraction of the red stars in our initial catalog. We choose not to scale the output of the Besançon model by this factor because during our sample selection we favored stars we believed to be supergiants based on our photometric criteria. Thus, the fraction of total stars we observe is likely not representative of the fraction of foreground dwarfs we observe. From this histogram we expect to find a handful of foreground dwarfs scattering out to approximately $-260$ km s$^{-1}$. A foreground star with such a velocity would fall within 20 km s$^{-1}$ or below its expected M33 velocity over the entire range of $X/R$ in Figure 8.

As a final check, we re-examine the 65 red stars toward M33 that were classified as RSGs by Massey (1998). Sixteen of these stars were also observed as part of our sample, and we find complete agreement between our classifications and those of Massey (1998). In addition, 58 of the stars now have improved photometry as a result of the LGGS. When we place these stars on the $B - V$ versus $V - R$ diagram (with the improved LGGS photometry), we find that for eight of the stars the classifications given by Massey (1998) would not be supported by our photometric cut. These are plotted as stars in Figure 9.

An examination reveals that all eight stars are located in very crowded regions of M33, thus validating our decision to include only isolated stars in our sample.

Thus, we conclude that it is likely possible for (1) some foreground star to possess radial velocities as (or more) negative than genuine M33 members over the entire surface of the galaxy and (2) stars that are sufficiently crowded to appear to possess colors in disagreement with our photometric selection criteria. Despite these facts, when considering only ($V$-band) isolated stars, our two selection criteria agree for 96% of our sample. We therefore believe that the two-color discriminant described in Massey (1998) does, in fact, act well at discerning RSGs at M33 metallicities. For the purposes of comparing our results to the current evolutionary models, we divide our stars into three ranks: rank 1 stars were selected as supergiants both photometrically and kinematically, rank 2 stars were selected by one method, but not the other, and rank 3 stars were selected by neither method and are likely foreground dwarfs. After checking our rank 1 and rank 2 samples for known stellar clusters (finding none) and assigning the three stars initially selected photometrically as supergiants, but which fall much closer to the band of foreground dwarfs as rank 3 stars, we are left with 189 rank 1 stars, 12 rank 2 stars, and 207 rank 3 stars.

Prompted by the two stars described above that were included in our sample, despite being crowded in the B band, we also examine the level of crowding of our rank 1 stars in the B and R bands. Only two stars showed moderate crowding in the R band, but 67 stars ($\sim$35% of our rank 1 stars) showed moderate to high crowding in the B band. Given that all of these objects have observed minus expected radial velocities less than 20 km s$^{-1}$, we do not believe that this crowding invalidates our classification of these stars as very probable M33 RSGs. However, we do indicate rank 1 and rank 2 stars that possess moderate levels of crowding in the B band in Table 2.

3.3. Membership Determination: Yellow Supergiants
3.3.1. Radial Velocities

We now turn to our yellow sample and begin by examining their radial velocities. In Section 3.2, we characterized the rotation curve of M33 based on the radial velocities of 55 H ii regions from Zaritsky et al. (1989). However, these radial velocities were obtained in a relative, as opposed to absolute (heliocentric), sense and often had high associated errors (10–15 km s$^{-1}$). Thus, to calculate the expected radial velocities for our yellow sample, we opt to use a characterization of M33’s rotation curve based on our probable RSGs.
In Figure 11, we plot $X/R$ versus observed radial velocity for our rank 1 RSGs. A linear least-squares fit to the sample yields a relation (in km s$^{-1}$) of

$$V_{\text{expect}} = -182 - 81.9(X/R)$$

with a dispersion of 14.3 km s$^{-1}$. We see that the slope produced by this relation varies slightly from that produced by the H ii regions of Zaritsky et al. (1989). We also note that this relation accurately reproduces the systemic velocity of M33 ($-180 \pm 3$ km s$^{-1}$; Kerr & Lynden-Bell 1986).

Using this relation, we calculate the expected radial velocity for each of our yellow stars, were it a genuine M33 member. In Figure 12, we plot the observed minus expected radial velocities versus $X/R$ for our yellow sample. As in Figure 8 for our red sample, we expect the genuine M33 members to fall near the zero point of the $y$-axis. We note that although two bands of stars are evident in this figure, the separation is not nearly as distinct as seen in Figure 9 of Drout et al. (2009) for the case of M31 (where both the systematic and rotational velocities are larger than the modest values of M33). It is not until the value of $X/R$ approaches zero that the two bands are noticeably distinct. Thus, if we were to restrict our sample of potential supergiants to those that Figure 12 indicates possess a modest separation from the bulk of Milky Way dwarfs, we would be left only with stars in the NW portion of M33.

Additionally, the Besançon model for galactic dwarfs applied to the color and magnitude range of our yellow sample reveals that we can expect modest contamination from high-velocity foreground dwarfs across the entire face of M33. In Figure 13, we plot a histogram of the velocities predicted by the Besançon model (solid, black) and the observed velocities of our yellow sample (dashed, red). In this figure, the output of the foreground model has been scaled by the fraction of our initial yellow catalog we actually observed. We see that at a radial velocity of approximately $-200$ km s$^{-1}$ (corresponding to $X/R = 0.22$ for a typical M33 member in our formulation of M33’s rotation curve) a majority of the stars in our sample should be genuine M33 members. However, at a velocity of $-170$ km s$^{-1}$ ($X/R = -0.15$ for a typical M33 member) a significant portion of our sample is likely composed of foreground dwarfs. Based on this model, we can predict 2 foreground stars with velocities beyond $-300$ km s$^{-1}$, 9 foreground stars beyond $-200$ km s$^{-1}$, and 17 foreground stars beyond $-150$ km s$^{-1}$ (the region in which
we also expect to find 65% of our genuine supergiants). Thus, we conclude that even in M33, which possesses a modest radial velocity separation, it would be prudent to obtain an independent means of membership determination to complement the radial velocity data.

3.3.2. O\textsc{i} $\lambda7774$ Triplet

To address this issue, we now examine the possibility of using the O\textsc{i} $\lambda7774$ triplet as a luminosity indicator at M33 metallicities. Osmer (1972) demonstrated that O\textsc{i} $\lambda7774$ is one of the strongest spectral features in high-luminosity A- to F-type supergiants, yet it is relatively weak in dwarfs at galactic metallicities. Przybilla et al. (2000) later showed that this feature is a result of non-LTE atmospheric effects, exacerbated by sphericity. Although promising, it is not yet understood whether this luminosity dependence extends to the cooler G-type supergiants and what effect, if any, metallicity creates.

As an initial test we measured the equivalent width of the deepest spectral feature in the wavelength range 7760–7785 Å. The continuum level was determined by averaging the intensity over a length of the normalized spectra at both longer and shorter wavelengths for these stellar parameters. Most notably for our purposes, this includes a CN band head at $\sim7774$ Å along with companion features at $\sim7865$ Å and $\sim7938$ Å.\(^6\)

Thus, in order to determine whether the population of stars with high equivalent widths represents a sample of M33 supergiants with a strong O\textsc{i} $\lambda7774$ feature, we examine all stars with an initial equivalent width greater than 0.5 Å, all stars with a velocity difference less than 100 km s$^{-1}$ in Figure 12, and all the stars observed in the 2009 fnt-4 field. We distinguish a CN band head near $\lambda7774$ from a strong O\textsc{i} $\lambda7774$ feature by (1) the presence or absence of the other two band heads and (2) the width of the feature. We find that our spectra fall roughly into four categories: spectra with no obvious features around $\lambda7774$; spectra with three broad CN features that possess a natural progression of line depths; spectra with features at all three band head wavelengths, but whose feature at $\lambda7774$ is deeper and narrower than the other two; and spectra with a single, narrow feature at $\lambda7774$. Examples of the latter three are shown in Figure 15.

We believe that spectra falling in the latter two categories possess real O\textsc{i} $\lambda7774$ features. The red histogram in Figure 14 (right) represents the equivalent widths from these stars. We see that in almost every case the strongest equivalent widths represent real O\textsc{i} $\lambda7774$ features. We also note that with the exception of a handful of spectra (all of which possessed particularly high noise levels) the spectra with strong O\textsc{i} $\lambda7774$ are the same spectra that possess Paschen lines (see Section 3.1). This is promising, as the Paschen lines are expected to remain stronger to later spectral types for supergiants than for giants and dwarfs (Andrillat et al. 1995).

In Table 3 we list our measured equivalent widths for only the stars we believe have true O\textsc{i} $\lambda7774$ features. Propagating the errors in our calculation of the mean continuum level leads to errors in our measured equivalent widths on the order of 0.02 Å, while a comparison of the two equivalent widths measured for stars observed in both 2009 and 2010 reveals an average difference of 0.02 Å with a standard deviation of 0.36 Å.

3.3.3. Radial Velocities and the O\textsc{i} $\lambda7774$ Triplet

In Figure 16 we again plot the observed minus expected velocities versus $X/R$ for our yellow sample. The green points now represent stars with significant amounts of O\textsc{i} $\lambda7774$. We see that there is excellent agreement between stars with a distinguishable O\textsc{i} $\lambda7774$ feature and stars that have radial velocities consistent with the kinematics of M33. This indicates that the O\textsc{i} $\lambda7774$ triplet can, indeed, be used as a luminosity indicator for F- and G-type stars at the metallicity of M33.

We do note that there are (1) two stars that seem to possess significant amounts of O\textsc{i} $\lambda7774$ but whose velocities place them in the middle of the strong band of foreground dwarfs and (2) a handful of stars that possess velocities consistent with, or more negative than, their expected M33 metallicities, but which do not possess significant amounts of O\textsc{i} $\lambda7774$.

To explain the latter, we first checked our sample for possible non-stellar objects. Using the $R$-band images of the LGGS, we measured the FWHM of the point-spread function for objects in our sample with either an observed minus expected

\(^6\) We are indebted to Nelson Caldwell for first noting to us the possible confusion of the O\textsc{i} $\lambda7774$ feature with the CN band head.
Figure 15. Sample yellow spectra in the wavelength range 7700–8000 Å. Top right: three CN band heads with a progression of lines depths. Top left: two CN band heads are still evident, but the feature at ∼7774 Å has a much larger relative depth, representing the presence of a strong O i λ7774 feature. Bottom center: an isolated O i λ7774 feature.

velocity less than 100 km s⁻¹ or significant amounts of O i λ7774. This process revealed seven previously known clusters in our sample (J013350.92+303936.9, J013419.85+303613.1, J013456.51+304100.4, J013400.19+303747.3, J013403.30+302756.1, J013256.08+303826.0, J013418.67+303137.7) and one additional extended object on the outskirts of the LGGS images (J013224.07+301242.9). Despite this fact, our sample still contains eight stars with −200 > V_{obs} > −300 km s⁻¹ and one star with V_{obs} < −300 km s⁻¹ that do not possess noticeable amounts of O i λ7774. As described above, however, this is consistent with the numbers predicted by the Besançon model for foreground stars. Thus, although it is not impossible for our sample to contain some bona fide M33 members without obvious O i λ7774, we find that our data are not inconsistent with all F- and G-type supergiants in M33 possessing significant amounts. A fuller examination of the temperature and metallicity dependence of this line will be addressed in a future paper.

For the purpose of comparing our sample to the current evolutionary models we assign the following ranks to our yellow sample: rank 1 stars possess both velocities consistent with M33 and significant amounts of O i λ7774, rank 2 stars have either a velocity difference less than 60 km s⁻¹ and a V_{expect} less than −150 km s⁻¹ or significant amounts of O i λ7774 and a velocity inconsistent with M33, rank 3 stars have a velocity difference less than 60 km s⁻¹ and V_{expect} greater than −150 km s⁻¹, and the rest of the sample is designated as rank 4. Even if it is possible for a genuine supergiant in our sample to lack a noticeable O i λ7774 feature, we expect our rank 3 sample to be heavily contaminated by foreground dwarfs (recall from Section 3.2 that
M33 only stands out from the Galactic clutter at $-150\rm\ km\ s^{-1}$). With these definitions our sample contains 121 rank 1 stars, 14 rank 2 stars, 68 rank 3 stars, 705 rank 4 stars, and 8 clusters. This corresponds to a foreground contamination of $\sim83\%$.

In Figure 17, we present the location of our newly determined RSGs and YSGs on the CMD of M33. Black dots represent the LGGS sample, red $\times$'s our rank 1 RSGs, green $\times$'s our rank 1 YSGs, and blue $\times$'s the M33 W-R population, determined by Neugent & Massey (2011). In Figure 18, we also show the spatial distribution of all three populations in the disk of M33. We note that the lack of RSGs in the central region of the galaxy is an observational bias, created by our restriction to isolated red stars as described in Section 2.2.

### 3.4. Variability

Although relatively stable in the $K$ band, some RSGs are known to possess variations in the $R$ band from 0.2 to 1 mag across a range of metallicities (Josselin et al. 2000; Levesque et al. 2007; Massey et al. 2009). In some cases this effect is at least partially due to variable dust extinction (Massey et al. 2005, 2007), although Levesque et al. (2007) also examine a handful of late-type RSGs in the Magellanic Clouds whose variability seems to be due to physical changes in the stars themselves. These unusual stars may inhabit the Hayashi forbidden region and, hence, be hydrostatically unstable. Given the potential insight that variability can provide, we wish to examine which, if any, of our stars are known to be variable.

Macri et al. (2001), Hartman et al. (2006), and Javadi et al. (2011) present the most recent catalogs of variable point sources in M33. Macri et al. (2001) identify 544 BVI variables in two fields in the central region of M33 as part of the DIRECT project; Hartman et al. (2006) examine a field approximately 1 deg$^2$ (well matched to the LGGS photometry) and identify 36,000 variable objects in $g', r', or i'$; and Javadi et al. (2011) examine the central square kpc for JHK variable objects.
Table 4
Derived Properties of Probable Yellow Supergiants

| Star Name          | Rank | $\log T_{\text{eff}}$ | $\log L/L_\odot$ | Known Variable |
|--------------------|------|-----------------------|-------------------|----------------|
| J013226.98+303659.7 | 2    | 3.777                 | 4.95              | 1              |
| J013220.20+303430.2 | 1    | 3.849                 | 5.48              | 1              |
| J013231.71+302940.6 | 1    | 3.913                 | 4.61              | 1              |
| J013231.94+303516.7 | 1    | 3.753                 | 5.03              | 1              |
| J013233.85+302728.9 | 1    | 3.804                 | 5.37              | 1              |
| J013235.17+303331.6 | 1    | 3.946                 | 4.53              | 1              |
| J013240.21+304149.6 | 2    | 3.789                 | 4.58              | 1              |
| J013241.21+304446.4 | 2    | 3.769                 | 4.86              | 1              |
| J013243.77+304556.3 | 2    | 3.751                 | 4.78              | 1              |
| J013244.40+301547.7 | 1    | 3.919                 | 5.37              | 1              |

Notes.

- (1) Hartman et al. 2006; (2) Macri et al. 2001; (3) Javadi et al. 2011.

(This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)

Table 5
Derived Properties of Probable Red Supergiants

| Star Name          | Rank | $\log T_{\text{eff}}$ | $\log L/L_\odot$ | Known Variable |
|--------------------|------|-----------------------|-------------------|----------------|
| J013220.36+302938.0 | 1    | 3.627                 | 4.29              | 1              |
| J013222.59+302741.8 | 1    | 3.613                 | 4.38              | 1              |
| J013224.33+303155.5 | 1    | 3.611                 | 4.40              | 1              |
| J013229.64+302802.7 | 1    | 3.603                 | 4.33              | 1              |
| J013232.49+303521.8 | 1    | 3.564                 | 4.83              | 1              |
| J013233.49+301511.7 | 1    | 3.572                 | 4.60              | 1              |
| J013233.55+302543.1 | 1    | 3.600                 | 4.50              | 1              |
| J013233.77+302718.8 | 1    | 3.545                 | 5.24              | 1              |
| J013233.93+302731.4 | 1    | 3.593                 | 4.94              | 1              |
| J013234.03+302903.6 | 1    | 3.624                 | 4.32              | 1              |

Notes.

- (1) Hartman et al. 2006; (2) Macri et al. 2001; (3) Javadi et al. 2011.

(This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)

A comparison of our RSG sample with these catalogs reveals that, indeed, 127 of our 189 rank 1 stars (67%) and 6 of our 12 rank 2 stars are variable in the visible bands. Thus, although our rank 2 stars are less likely to be genuine M33 members than our rank 1 stars, these data are not inconsistent with many of our rank 2 stars being genuine members. In contrast, only 5 of our 207 rank 3 red stars were flagged as visible variables. Additionally, one RSG was flagged as JHK variable in the catalog of Javadi et al. (2011). However, due to the effects of crowding, only two of our rank 1 RSGs were located in the central square kpc (the region covered by the Javadi et al. 2011 survey).

A similar comparison with our YSG sample revealed only 25 known variables in our 121 rank 1 stars (20%) and 0 of our 14 rank 2 stars. These variables are split between optical (15 stars over the entire disk of M33) and infrared (10 stars in the central square kpc). Interestingly, our YSG sample contains only 12 stars in the central square kpc, 10 of which are variable in the IR bands, indicating that the IR variability of YSGs warrants more detailed future investigation. Stars found to be known variables are marked in Tables 4 and 5.

3.5. Completeness

An accurate and meaningful comparison of the relative number of observed YSGs at various luminosities to those predicted by the evolutionary tracks requires that our observed sample be unbiased in luminosity. To assess what, if any, such biases may be present, we examine the completeness of our observed sample. In Figure 19, we present the distribution of V-band magnitudes for the 1438 yellow LGGS stars that met our original selection criteria (black) and the 916 yellow stars for which we actually obtained radial velocities (red). We see that there is excellent agreement between the two distributions, with our observed sample falling off only at the highest magnitudes. If we conservatively set our limiting magnitude to 17.5 (which very roughly corresponds to $\log L/L_\odot = 4.8$), we find that our sample should be complete down to $\sim 15 M_\odot$ (see Figure 1).

As described in Section 1, a similar comparison of the relative numbers of our RSG sample at various luminosities to those predicted by the models is made incredibly difficult by the near-vertical nature of the evolutionary tracks. Thus, we will merely be examining the locations of our RSG populations on the H-R diagram, a task that does not depend quite so stringently on the completeness of our sample.

4. THE H-R DIAGRAM

4.1. Making the H-R Diagram: Effective Temperatures and Luminosities

In order to compare our populations of M33 RSGs and YSGs to the current evolutionary tracks, we must first transform their magnitudes and colors to effective temperatures and luminosities.

4.1.1. Yellow Supergiants

For the YSGs, we follow the same procedure as Drout et al. (2009) and Neugent et al. (2010), using the Kurucz (1992) Atlas 9 model atmospheres to transform $B - V$ colors into effective temperatures and luminosities.
temperatures and luminosities. The $B$- and $V$-band photometries for all of our stars were taken from the LGGS, and we apply a constant reddening correction of $E(B - V) = 0.12$, in keeping with the median value found by Massey et al. (2007).

Before choosing which Atlas 9 model to use for the transformations, we first note that the metallicity of M33 has long been a topic of debate. As summarized in detail in Neugent & Massey (2011), there is strong evidence for a metallicity gradient within the plane of M33. Through the years strong gradients (e.g., Kwitter & Aller 1981; Zaritsky et al. 1989; Garnett et al. 1997), weak gradients (e.g., Crockett et al. 2006; Rosolowsky & Simon 2008; Bresolin 2011), and two-component models (Magrini et al. 2007) have all been suggested. By examining the ratio of WC to WN W-R stars in M33, Neugent & Massey (2011) suggest that the central regions may lie closer to solar metallicity while the outer regions are closer to that of the LMC ($\sim$0.5 solar).

Based on this fact, we initially compute a transformation from $B - V$ to effective temperature using the $Z = 0.6 Z_\odot$ Atlas 9 models. A least-squares fit to the low surface gravity models over the temperature range 4000–10,000 K yields the relation

$$
\log T_{\text{eff}} = 3.936 - 0.5272(B - V)_0
+ 0.4752(B - V)_0^2 - 0.1749(B - V)_0^3.
$$

In order to assess any errors that may result from the metallicity gradient present in M33, we also compute an analogous relation using the $Z = Z_\odot$ Atlas 9 models. We find that the difference between the two transformations is negligible. For a star with $(B - V)_0 = 0.6$ the $Z = 0.6 Z_\odot$ relation gives $\log T_{\text{eff}} = 3.753$ while the $Z = Z_\odot$ relation gives $\log T_{\text{eff}} = 3.759$. As there is only a slight (0.06 dex) difference, we conclude that our original transformation is acceptable over the entire area of M33. Since we restricted our initial fit to the temperature range 4000–10,000 K, the relation for $\log T_{\text{eff}}$ given above is only valid for the range $-0.11 \leq (B - V)_0 \leq 1.57$ or $0.01 \leq (B - V) \leq 1.69$. We find, however, that this range encompasses our entire yellow sample.

The bolometric corrections for our yellow sample are modest (a few tenths of a magnitude) and were also computed using the Atlas 9 model atmospheres. The models yield a relation

$$
BC = -260.05 + 135.205 \times \log T_{\text{eff}} - 17.5746 \times (\log T_{\text{eff}})^2,
$$

which is also valid over the temperature range 4000–10,000 K. Adopting a distance modulus of 24.60 (van den Bergh 2000), we compute effective temperatures and bolometric luminosities for our rank 1 and rank 2 YSGs. These derived properties are listed in Table 4.

### 4.1.2. Red Supergiants

Deriving a place on the H-R diagram for RSGs based on photometry is a more uncertain process than that described above for YSGs.

First, the derivation of the effective temperature is complicated by the fact that RSGs have so many atomic and molecular lines that one is not observing the true continuum, but rather regions of relative transparency between overlapping absorption bands. Thus, $B - V$ is no longer a color indicator but is instead primarily sensitive to surface gravity, due to line blanketing in the $B$ band (see discussion in Massey 1998, and references therein). Furthermore, their surfaces appear to be covered in large spots (Freytag et al. 2002), resulting in slight differences in the effective temperatures and radii one derives from different bandpasses (Young et al. 2000). Levesque et al. (2006) found, using the state-of-the-art MARCS models (Gustafsson et al. 2003, 2008), that the effective temperatures derived from $V - K$ were hotter than those derived by fitting spectral features (i.e., the TiO bands) by 105 K for the LMC and by 170 K for the SMC. They found, however, good agreement between the temperatures derived from $V - R$ and the spectral fitting. Massey et al. (2009) attribute the problem with $V - K$ to the inherent limitations of the one-dimensional models, given the presence of cooler and warmer regions on the surface.

Second, the bolometric corrections are both a large and a strong function of the assumed effective temperature, while for the YSGs the bolometric corrections were small and fairly insensitive to the effective temperature. This problem can be partially alleviated by using the $K$-band photometry and applying a bolometric correction to $K$, as discussed by Levesque et al. (2006). There is an additional advantage to using $K$ to derive the luminosity, namely, that it is less affected by uncertainties in the interstellar (and circumstellar) reddening. Massey et al. (2009) rederived the bolometric luminosities of RSGs in the SMC, LMC, Milky Way, and M31 and found good agreement on average with the values derived from $V$ and $K$ when the same effective temperatures were used, although the individual scatter was large.

All of the RSGs in our sample have $BVR$ data from the LGGS, and hence we can use the $V - R$ photometry to derive effective temperatures. We adopt the conversions given in Section 3.3 of Levesque et al. (2006) for the LMC, since the metallicity of M33 is similar. Some of the RSGs in our sample also have 2MASS photometry, and for those stars we compute the bolometric luminosity from $K$, after converting the 2MASS $K_s$ to standard $K$ following Levesque et al. (2006). For stars without 2MASS, we compute the bolometric correction to the LGGS $V$ following the transformations in Levesque et al. (2006) and adopting a reddening of $E(B - V) = 0.12$, typical for OB stars in M33 (Massey et al. 2007).

For the stars with 2MASS data, the bolometric luminosity we derive from the $K$-band is, on average (median), 0.2 mag fainter than that from $V$. Increasing the assumed reddening to $E(B - V) = 0.5$ would remove most of the discrepancy. This is consistent with our earlier findings that RSGs invariably have circumstellar reddening, attributable to dust formation (Massey et al. 2005). Of course, changing the assumed reddening would also change the derived effective temperatures. For now we have taken the conservative approach of adopting the lower reddening ($E(B - V) = 0.12$) but note that this probably underestimates the effective temperature for some stars. It also means that the $V$-band bolometric correction is more positive for $V$ but more negative for $K$.

---

8 Recall that in our previous comparisons between the luminosities derived from $V$ and $K$ (Massey et al. 2009) we explicitly determined the reddening in a star-by-star basis using spectral fitting.

9 We can see this numerically as follows. Consider a star with a $V - R$ value of $\sim 1.0$. If $E(B - V) = 0.12$, this star will have an effective temperature of 3800 K (corresponding to $(V - R)_0 = 0.9$), as $E(V - R) = 0.642E(B - V)$ according to Schlegel et al. (1998). The bolometric correction will be $-1.2$ mag at $V$ and $+2.7$ mag at $K$, while the correction for interstellar reddening will be $-0.37$ mag at $V$ and $-0.04$ mag at $K$. Thus, the apparent magnitude will have to be corrected by $-1.6$ mag at $V$ and $+2.7$ mag at $K$, as well as by the true distance modulus. If, however, the reddening had been underestimated by 0.5 mag in $E(B - V)$, then the star is actually 0.3 less red in $V - R$ and the effective temperature is more like 4300 K rather than 3800 K. The bolometric correction will be only $-0.6$ mag in $V$ and $+2.3$ mag at $K$, while the interstellar extinction will be $-1.9$ mag at $V$ and $-0.2$ mag at $K$. Thus, the total corrections will be $-2.5$ mag at $V$ and 2.1 mag at $K$. By assuming too low a
Thus, again adopting a distance modulus of 24.60, we calculate effective temperatures and bolometric luminosities for our rank 1 and rank 2 RSGs. These derived properties are listed in Table 5.

4.2. Testing the H–R Diagram: Models and Observations

Having selected our supergiant populations and transformed their photometry into effective temperatures and luminosities, we now compare our results to the Geneva evolutionary tracks. As mentioned in Section 4.1, the metallicity of M33 is a complex issue and we therefore opt to compare our supergiant populations to evolutionary tracks at multiple metallicities. Additionally, in the time since our initial studies examining the YSG content of M31 and the SMC, the Geneva group has begun the release of a new generation of models (Ekström et al. 2012; V. Chomienne et al. 2012, in preparation). Throughout this section we compare our observed populations to both this new set of models and the previous generation (Meynet & Maeder 2003, 2005; Schaerer et al. 1993). This will allow us to properly assess what effect the modified physics in the new models creates. Recall that Drout et al. (2009) and Neugent et al. (2010) identified a discrepancy between the observed number of YSGs with luminosity and those predicted by the previous generation of models.

A thorough description of the differences between the two sets of models is given in Ekström et al. (2012); however, they can be summarized briefly as follows: (1) the initial composition differs, with the new models based on the more recent analysis of solar abundances by Asplund et al. (2005). As a result, the new models adopt Z⊙ = 0.014, while the historic value is Z⊙ = 0.02. (2) The opacity tables differ as a result of the changed initial composition. (3) The reaction rates have been updated. (4) A new prescription for RSG mass loss is adopted. An effect of this new prescription is to increase the time-averaged RSG mass loss by a factor of ∼10 for stars above ∼15 M⊙. (5) A new prescription for the shear diffusion coefficient, which describes rotationally induced mixing, is utilized. (6) Different initial rotational velocities are considered. The previous generation of rotating models used an initial velocity of 300 km s\(^{-1}\) for all masses. In contrast, the new models are computed with an initial rotation velocity \(v_{\text{rot},i} = 0.4v_{\text{crit}}\), where \(v_{\text{crit}}\) is the critical velocity at which the centripetal force balances the gravitational acceleration. The value 0.4 \(v_{\text{crit}}\) lies between 200 and 300 km s\(^{-1}\) for the mass range examined below.\(^{10}\) We note that both generations of models were also computed with zero initial rotation for comparison. Throughout this section we will refer to non-rotating tracks as S0, the old generation rotating tracks as S3, and the new generation rotating tracks as S4.

In Figures 20 and 21, we plot the location of the 121 rank 1 YSGs and 189 rank 1 RSGs, selected in Sections 3.2 and 3.3, along with the \(Z = 0.014\) and \(Z = 0.006\) (Figure 20) new generation models and \(Z = 0.02\) and \(Z = 0.008\) old generation models (Figure 21). These models should represent roughly solar and LMC-like metallicities, and the values for \(Z\) vary between the two generations as a result of the updated compositions described above. In both panels of Figure 20 the \(Z = 0.014\) models are represented by solid lines and the \(Z = 0.006\) models are represented by dashed lines. Similarly, in Figure 21, the \(Z = 0.02\) models are represented by solid lines and the \(Z = 0.008\) models are represented by dashed lines. In both figures, the top panel presents models that include the effects of rotation (described above) while the bottom panel presents models that neglect the effects of rotation (shown for comparison). The YSG region (7500 K > \(T_{\text{eff}}\) > 4800 K) is designated by vertical lines.

4.2.1. Locations

In the recent literature, single-star evolutionary models have been found to be fairly effective at reproducing the locations

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\(^{10}\) At Galactic metallicities an initial rotation of 300 km s\(^{-1}\) would correspond to an average main-sequence rotational velocity of 180–240 km s\(^{-1}\). This value is in agreement with observations of Galactic O-type stars (Conti & Ebbets 1977; Penny 1996), but the appropriate value to use at the metallicities of M33 has yet to be established (e.g., Penny & Gies 2009).
The presence of an RSG branch greatly increases the amount of time spent and, hence, the number of stars observed in a given region of the H-R diagram. Thus, our distribution of RSGs is consistent with the results of the Geneva Z = 0.014 models (Figure 20, solid lines), which predict an RSG branch up to log $L/L_\odot \sim 5.2$, while tracks above this point only briefly loop though the RSG portion of the H-R diagram. By contrast, the previous generation of Geneva models predicts RSG branches above log $L/L_\odot \sim 5.2$.

We next draw attention to the five most luminous stars in our sample: two YSGs and three RSGs. These stars appear to be inconsistent with the new generation S4 models and both the S0 and S3 old generation models, all of which loop back to the blue portion of the H-R diagram without extending to such cool temperatures. Further examination reveals that two of the three RSGs (J013418.56+303808.6 and J013339.28+303118.8) are moderately crowded in the B band (as described in Section 3.2), but that the most luminous star (J013312.26+310053.3) is particularly isolated. Additionally, the two most luminous YSGs (J01358.05+304539.9 and J013349.86+303246.1) both possess particularly strong and isolated O $\lambda 7774$ features. Thus, for at least three of these five stars, we see no reason to suspect that they were falsely identified as rank 1 supergiants.

As described in detail by Meynet et al. (2011), two main factors that affect if, and where, a single star will loop back toward the blue are mass loss during the RSG phase and rotationally induced mixing (both of which have been modified in the new generation models; see above). Higher values of either parameter tend to favor blueward evolution. Indeed, we see that the new generation S0 tracks (Figure 20, bottom panel) proceed to cooler temperatures than the S4 tracks (Figure 20, top panel) before looping back to the blue. Although the distribution of stellar rotational velocities has yet to be fully decoupled from the effect of inclination (since one can only measure $v \sin i$), there is increasing evidence that some massive stars are born as genuine slow rotations (e.g., Huang et al. 2010). Thus, although the location of our five most luminous supergiants could likely be explained by a change in the mass-loss prescriptions used by the Geneva models, it is possible that they are simply slow rotators. If this were the case, their locations would be fully consistent with the new generation models presented here.

Finally, we call attention to the effective temperatures of a majority of our RSG population. As can be seen in Figure 20, the predicted location of the RSG branch moves to warmer temperatures as metallicity decreases. Recalling that M33 may possess a strong metallicity gradient, in Figure 22 we again plot the locations of our RSGs on the H-R diagram, but now separated into bins based on the value of $\rho$ (the distance from the center within the plane of M33, normalized by the D25 isophotal radius of 30′). Moving from left to right, the panels are arranged innermost to outermost. The evolutionary models present in this figure are the S4 models from the top panel of Figure 20. From this we see that our RSG sample is consistent with a metallicity gradient within the disk of M33. The supergiants in the outermost region of M33 are systematically warmer than those in the innermost region, and the two populations are consistent with the $Z = 0.006$ and $Z = 0.014$ Geneva models, respectively. The inner two panels appear to show a progression (albeit with some scatter) between the two.

We do note that a handful of our RSGs appear to be inconsistently cool compared to both the $Z = 0.014$ and

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11 At least once the RSG effective temperature problem was resolved by Levesque et al. (2005) and subsequent papers.

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12 Computed for each star assuming $\delta_{2000} = 0^h 33^m 50.89$, $\alpha_{2000} = 30^h 39' 36.8''$, an inclination angle of 56°, and a position angle of the major axis of 23°, following Kwitter & Aller (1981) and Zaritsky et al. (1989).
Figure 22. Location of our rank 1 RSGs binned by radius within the disk of M33. Moving from left to right, the panels are arranged innermost to outermost. One can see that RSGs in the outer regions of M33 ($\rho > 0.8$, right panel) are systematically warmer than RSGs in the inner region ($\rho < 0.4$, left panel), as expected if the outer regions lie at lower metallicity.

(A color version of this figure is available in the online journal.)

$Z = 0.006$ Geneva evolutionary models. Most notable is the group at $\log T_{\text{eff}} \sim 3.53$ with $4.9 < \log L/L_\odot < 5.2$. Further examination of these stars reveals that they are all known variables, as discussed in Section 3.4. Thus, it is possible that our sample contains several of the unusually cool, and potentially unstable, RSGs described by Levesque et al. (2007). Or, it could be that these stars are like the “too cool” LMC RSGs discussed by Neugent et al. (2012), with high circumstellar reddening.

4.2.2. Relative Numbers

Having examined the locations of our RSGs and YSGs on the H-R diagram, we now assess the relative numbers of our YSGs at various luminosities. If we assume that the star formation rate averaged over the entire disk of M33 has been relatively constant over the past 20 Myr (the lifetime of a $12 M_\odot$ star), then the number of stars expected between masses $m_1$ and $m_2$ will be proportional to

$$N^{m_2}_{m_1} \propto [m_1^{\Gamma}]^{m_2} \times \bar{\tau},$$

where $\Gamma$ is the slope of the IMF, taken here to be $-1.35$ (Salpeter 1955), and $\bar{\tau}$ is the average duration of the evolutionary phase for masses $m_1$ and $m_2$.

In Table 6, we list the theoretical YSG lifetimes obtained from the new and old generation Geneva models. Note that (1) we include the lifetime of the $32 M_\odot$ track for the new non-rotating models, which was omitted from the figure for clarity, and (2) for the old generation of $Z = 0.008$ models, there was an insufficient number of rotating tracks available to make a comparison to our data. Recalling that our observed sample should be complete down to $\log L/L_\odot \sim 4.8$ (which corresponds to $\sim 15 M_\odot$), we use the luminosity of the evolutionary tracks with $M > 15 M_\odot$ to define mass bins on the H-R diagram. In Table 7, we list the total number of stars we observe in each mass bin (for each set of tracks), as well as the number relative to the $15–20 M_\odot$ bin. These relative numbers are then compared to those predicted by the theoretical YSG lifetimes.

For the new $Z = 0.006$ models we present results based on a $25–40 M_\odot$ bin in addition to $25–32 M_\odot$ and $32–40 M_\odot$ bins. Both the rotating and non-rotating $Z = 0.006 32 M_\odot$ tracks actually terminate in the YSGs regime, and we wish to assess what effect this produces on the predicted number of YSGs with luminosity.

From Table 7, we see that both the $Z = 0.014$ and $Z = 0.006$ new generation S4 models relatively accurately predict our observed ratios of YSGs with luminosity. In fact, the only main variation we find is the prediction of a few high-mass ($M > 32 M_\odot$) YSGs that are not observed. The deviations are only on the order of a handful (2–5) of stars, however. To this end, we emphasize that our observations did not include every potential YSG in M33. As argued in Section 3.5, our sample should be relatively complete and representative of the relative number of stars at varying luminosities down to $\log L/L_\odot \sim 4.8$. This does not, however, preclude variations on the order of a handful of stars between our observed luminosity distribution and the true luminosity distribution for M33 YSGs. Indeed, when considering variations of only a few stars, small fluctuations in star formation rate and possible stochastic IMF sampling (see Leitherer & Ekström 2011) may become significant.
Table 6
Theoretical Yellow Supergiant Duration (yr)$^a$

| Mass $(M_\odot)$ | $Z = 0.014$ | $Z = 0.006$ | Old $Z = 0.02$ | Old $Z = 0.008$ |
|-----------------|--------------|--------------|----------------|-----------------|
|                 | $S4^b$       | $S0^b$       | $S4^b$         | $S0^b$          | $S3^b$          | $S0^b$ | $S0^b$ |
| 85              | 0            | 5200         | 0              | 3000            | 0               | 0      | 4600   |
| 60              | 3600         | 10900        | ...            | ...             | 0               | 14100  | 8300   |
| 40              | 29700        | 25500        | 29900          | 21100           | 57500           | 403500 | 104500 |
| 32              | 69200        | 64200        | 84500          | 173400          | ...             | ...    | ...    |
| 25              | 27400        | 293500       | 17900          | 48800           | 5800            | 73000  | 89900  |
| 20              | 153100       | 68000        | 68500          | 502000          | 300             | 92600  | 89200  |
| 15              | 126900       | 117700       | 71100          | 91400           | 2100            | 51100  | 200000 |

Notes.
$^a$ For the purposes of this calculation, yellow supergiants are defined as having effective temperatures between 4800 K and 7500 K. Lifetimes are determined from the Geneva group models described in the text.
$^b$ S4 has initial rotation of $0.4v_{\text{crit}}$, S3 has an initial rotation of 300 km s$^{-1}$, and S0 has an initial rotation of 0 km s$^{-1}$.

Table 7
Number of Yellow Supergiants: Observed versus Modeled

| Mass $(M_\odot)$ Range | No. of Observed$^a$ | No. of Observed$^a$ | Ratio Relative to 15–20 $M_\odot$ |
|------------------------|---------------------|---------------------|----------------------------------|
|                        | $S4/S3^b$           | $S0^b$              | $S4/S3^b$ Observed | $S4/S3^b$ Modeled | $S0^b$ Observed | $S0^b$ Modeled |
| $Z = 0.014$ Models     |                     |                     |                          |                   |                  |
| 15–20 $M_\odot$        | 33                  | 24                  | 1.0                      | 1.0               | 1.0              | 1.0              |
| 20–25 $M_\odot$        | 12                  | 28                  | 0.4                      | 0.4               | 1.2              | 1.1              |
| 25–32 $M_\odot$        | 5                   | 3                   | 0.2                      | 0.2               | 0.1              | 0.9              |
| 32–40 $M_\odot$        | 0                   | 2                   | 0.0                      | 0.1               | 0.1              | 0.1              |
| 40–60 $M_\odot$        | 0                   | 0                   | 0.0                      | 0.0               | 0.0              | 0.0              |
| 60–85 $M_\odot$        | 0                   | 0                   | 0.0                      | 0.0               | 0.0              | 0.0              |
| $Z = 0.006$ Models     |                     |                     |                          |                   |                  |
| 15–20 $M_\odot$        | 31                  | 28                  | 1.0                      | 1.0               | 1.0              | 1.0              |
| 20–25 $M_\odot$        | 7                   | 18                  | 0.2                      | 0.3               | 0.6              | 0.4              |
| 25–32 $M_\odot$        | 5                   | 4                   | 0.2                      | 0.3               | 0.1              | 0.7              |
| 32–40 $M_\odot$        | 0                   | 2                   | 0.0                      | 0.2               | 0.1              | 0.4              |
| 40–85 $M_\odot$        | 0                   | 0                   | 0.0                      | 0.1               | 0.0              | 0.1              |
| 25–40 $M_\odot$        | 5                   | 6                   | 0.2                      | 0.3               | 0.2              | 0.4              |
| Old $Z = 0.020$ Models |                     |                     |                          |                   |                  |
| 15–20 $M_\odot$        | 34                  | 29                  | 1.0                      | 1.0               | 1.0              | 1.0              |
| 20–25 $M_\odot$        | 21                  | 25                  | 0.6                      | 1.4               | 0.9              | 0.6              |
| 25–40 $M_\odot$        | 2                   | 8                   | 0.1                      | 19.3              | 0.3              | 2.4              |
| 40–60 $M_\odot$        | 0                   | 0                   | 0.0                      | 8.4               | 0.0              | 1.0              |
| 60–85 $M_\odot$        | 0                   | 0                   | 0.0                      | 0.0               | 0.0              | 0.0              |
| Old $Z = 0.008$ Models |                     |                     |                          |                   |                  |
| 15–20 $M_\odot$        | ...                 | 33                  | ...                      | ...               | 1.0              | 1.0              |
| 20–25 $M_\odot$        | ...                 | 23                  | ...                      | ...               | 0.6              | 0.3              |
| 25–40 $M_\odot$        | ...                 | 6                   | ...                      | ...               | 0.2              | 0.5              |
| 40–60 $M_\odot$        | ...                 | 0                   | ...                      | ...               | 0.0              | 0.1              |
| 60–85 $M_\odot$        | ...                 | 0                   | ...                      | ...               | 0.0              | 0.0              |

Notes.
$^a$ The S0 and S4/S3 tracks vary in luminosity for a given set of models (see Figure 20). The number of observed stars in each mass bin must therefore be measured independently.
$^b$ S4/S3 designates models that include the effects of rotation; S0 designates non-rotating models. The old generation rotating models are designated S3 because they include an initial rotation of 300 km s$^{-1}$, while the new generation rotating models are designated S4 because they include an initial rotation of $0.4v_{\text{crit}}$ (see the text).

We note that this effect is very different than that observed by Drout et al. (2009) and Neugent et al. (2010) when comparing their populations of M31 and SMC YSGs to the old generation models. In those cases, tens to hundreds of additional high-luminosity YSGs would be required to bring their observations into agreement with the evolutionary models, an effect not consistent with the completeness of their samples.

The non-rotating Geneva models fared slightly worse than the rotating models, overpredicting the number of high-luminosity $(M > 25 M_\odot)$ stars in comparison to low-luminosity stars. They would require on order of 10–15 additional stars between 25 and 40 $M_\odot$ to bring our observations in line with the predicted ratios. We do admit, however, that the mass bins become slightly confused at higher luminosities due to the number of times
the tracks seem to loop though the YSG region. This result may also be indicative of the fact that, although some massive stars are expected to be genuine slow rotators, a much larger percentage of the population likely possesses an initial rotational velocity closer to $0.4v_{\text{crit}}$. In fact, the value $0.4v_{\text{crit}}$ was chosen to correspond to the peak in the distribution of initial rotation speeds for young B-type stars in Huang et al. (2010). In a more rigorous comparison one would compute an average YSG lifetime, weighted by a velocity distribution for each initial mass.

We note that for both the S0 and S4 $Z = 0.006$ models better results are obtained by ignoring the 32 $M_{\odot}$ tracks (which, recall, terminated in the YSG regime). Including the 32 $M_{\odot}$ track leads to the prediction of $\sim 15$ and $\sim 31$ stars between $25 M_{\odot}$ and $40 M_{\odot}$ (for S0 and S4 models, respectively). On the other hand, ignoring the 32 $M_{\odot}$ tracks leads to predictions of $\sim 9$ and $\sim 11$ stars between $25 M_{\odot}$ and $40 M_{\odot}$, and we observe 5 and 6 (S0 and S4, respectively). Thus, our observed sample does not seem to contain an increase in stars at intermediate luminosities that one would expect as a result of these tracks that terminate in the YSG regime (although we emphasize that even these deviations are far less than those observed in our previous works). This will be discussed briefly in the context of YSG supernova progenitors below.

Given these considerations, we find that the relative number of YSGs at various luminosities predicted by the new generation of Geneva evolutionary tracks is relatively consistent, within our uncertainties, with our sample of observed M33 YSGs. From Table 7 we see that this agreement does reflect a real improvement over previous models. At best, the previous generation of Geneva models requires an additional 10–15 stars at high luminosity ($Z = 0.008$, S0 models) and at worst 400+ additional stars at high luminosity ($Z = 0.02$, S3 models). Similar results (i.e., a significant improvement in the performance of the new models in comparison to the old) are found by Neugent et al. (2012, a companion paper to this manuscript), who compare the YSG population of the LMC to the new Geneva $Z = 0.006$ models described here (V. Chomiene et al. 2012, in preparation).

One additional concern raised by our initial studies of YSG populations was the actual duration of the YSG phase. By comparing the (albeit uncertain) number of unevolved OB-type stars to the number of YSGs in M31 and the SMC, Drout et al. (2009) and Neugent et al. (2010) estimated that the duration of the YSG phase should be on the order of 3000 yr for $M > 20 M_{\odot}$. As can be seen from Table 6, this value is an order of magnitude below the lifetimes predicted by both the new and old generation Geneva models. At this time, we do not possess accurate enough information on the number of unevolved OB-type stars in M33 to directly assess whether this discrepancy is due to remaining deficiencies in the models or with the lifetimes predicted from observations (which should be taken as lower limits due to the effects of crowding; see Massey 2003). However, in a companion paper examining the YSG and RSG populations of the LMC Neugent et al. (2012) are able to revise this estimate of the YSG lifetime. By comparing the number of observed LMC YSGs to the number of unevolved LMC OB-type stars from Massey (2010), they estimate that the YSG lifetime is $\sim 17,000$ yr for $M > 20 M_{\odot}$. This number, although smaller than those listed in Table 6, is, in our opinion, within the uncertainties of the number of OB-type stars.

## 4.2.3. Discussion

Having demonstrated that the new generation of Geneva evolutionary models offers a dramatic improvement in predicting both the observed locations and relative numbers of YSGs as a function of luminosity, we now spend a moment discussing the natural question that arises: What is the physical cause of this improvement? As described above, the new models include several substantial modifications over the previous generation. However, despite the complex and somewhat degenerate nature of the problem, we are able to offer some modest logical arguments.

One modification that almost certainly impacts the behavior of the evolutionary tracks in this region of the H-R diagram is the adoption of a new prescription for RSG mass loss. In the new prescription, mass loss is artificially increased by a factor of three whenever any layer in the envelope of the star reaches a luminosity greater than five times the Eddington luminosity (an effect that is relevant for RSGs with masses above $\sim 15 M_{\odot}$). We emphasize that this is an ad hoc prescription, but one that is not without both theoretical and observational precedent (e.g., Heger et al. 1997; Vanbeveren et al. 2007; Yoon & Cantiello 2010; van Loon et al. 2005, 2008). As discussed in Ekström et al. (2012), this higher mass-loss rate favors blueward evolution, thus lowering the minimum mass required for blueward evolution and increasing the ratio of the blue to red lifetimes for high-mass tracks (see Section 5.5 of Ekström et al. 2012). In order to investigate whether this new mass-loss prescription can explain the improved behavior of the models in the YSG regime specifically, we again examine the YSG lifetimes listed in Table 6.

From this we see that the improved YSG ratios predicted by the new models are not simply a result of a decrease in the YSG lifetimes predicted for the highest mass tracks, nor simply an increase in the YSG lifetimes predicted for the lower mass tracks, but rather a complex modification of the predicted lifetimes at all masses. We do observe significant differences in the lifetimes predicted for some tracks that experience RSG mass loss before evolving back to the blue portion of the H-R diagram (most notably the $20 M_{\odot}$ rotating solar mass track, which only evolves back to the blue in the new generation models). However, we also observe significant differences in the predicted lifetimes for tracks that evolve straight across the H-R diagram and end their lives as RSGs (e.g., all the $15 M_{\odot}$ tracks and the $20 M_{\odot}$ non-rotating, low-metallicity tracks). These tracks do not possess blue loops, and, hence, their YSG lifetimes should be unaffected by the new prescription for RSG mass loss. The fact that we observe significant modifications to the predicted YSG lifetimes for even these tracks indicates to us that, although the modified RSG mass loss undoubtedly affects the behavior of the evolutionary tracks in this region of the H-R diagram, other effects must also be behind the modified behavior of the new models.

Thus, we emphasize a few other modifications made to the new set of models. With respect to rotation, the new models adopt the shear diffusion coefficient proposed by Maeder (1997) as opposed to that of Talon & Zahn (1997), and the initial rotation velocities included in the models have been modified. As mentioned above, the effects of rotation can also play a significant role in dictating the behavior of evolutionary tracks in this region. This is evidenced by the $20 M_{\odot}$ $Z = 0.014$ tracks, which only evolve back to the blue in the rotating models. Additionally, from Table 7 we see that, although the predictions
made by both the rotating and non-rotating tracks are improved in the new models, the rotating tracks now perform vastly better than the old, indicating that some of the improved behavior is likely due to the modified descriptions of rotation.

However, even these modifications cannot explain the contrast between the $15 M_\odot$ non-rotating $Z = 0.008$ and $Z = 0.006$ tracks, the $20 M_\odot$ non-rotating $Z = 0.008$ and $Z = 0.006$ tracks, and the $15 M_\odot$ non-rotating $Z = 0.014$ and $Z = 0.02$ tracks. All three pairs show a significant change in their predicted YSG lifetimes, and none should be affected by either RSG mass loss or rotation. To this end we note that the new models contain modified initial compositions and opacities. We are considering a transitional phase during which the star is changing shape (inflating when evolving from the blue to the red), and thus changes in the opacities may actually have important implications.

It is therefore our conclusion that no single modification in the new Geneva models can be pointed to as the main cause for their improved overall behavior in the YSG regime. Rather, when put together, the various modifications produce a set of models that are, at least heuristically, able to reproduce the observed trends. This emphasizes the role of the YSG regime as a magnifying glass for models of stellar evolution, and that much work can still be done to understand these objects from a theoretical point of view.

5. SUMMARY AND FUTURE WORK

We obtained spectra of 916 yellow stars ($\sim$68% of those initially selected) and 408 red stars ($\sim$37% of the isolated stars initially selected) toward M33. bona fide RSGs were identified through a combination of radial velocities and a two-color surface gravity discriminant. After re-characterizing the rotation curve of M33 using our newly identified RSGs, foreground Milky Way contaminants were removed from our YSG sample through a combination of radial velocities and $\Omega_1 \lambda7774$ equivalent widths. In the end we identified a sample of 189 rank 1 RSGs and 121 rank 1 YSGs.

After transforming the colors and magnitudes of our rank 1 supergiants into effective temperatures and luminosities, we placed them on the H-R diagram. A comparison of the relative numbers and locations of our supergiant populations to those predicted by the latest generation of Geneva evolutionary tracks yielded excellent results. We observed a sharp drop-off in the number of RSGs above a luminosity of $\log L/L_\odot \sim 5.2$, consistent with the upper luminosity limit predicted by the new rotating Geneva models, and an increase in the mean temperature of stars on the RSG branch with radial distance, consistent with metallicity gradient within the disk. Additionally, our observed relative number of YSGs with luminosity is consistent with the new rotating Geneva models to within a handful of stars. This represents a vast improvement over previous models.

There are several topics pertaining to YSGs, RSGs, and massive star evolution that are worthy of additional study. One such topic is the effect of binarity on massive star evolution. All the models examined here are single-star tracks. Many massive stars occur in binary systems, and, for some, this likely has an effect on their evolution through additional mass loss and interactions (e.g., Roche Lobe overflow, common envelope evolution). The level of this effect on the scale of an entire stellar population depends on many parameters that are, as yet, not well constrained (i.e., binary fraction, distribution of initial separations, distribution of mass ratios). Despite these uncertainties, it has been demonstrated that some binary models, such as those calculated with the Binary Population and Spectral Synthesis (BPASS) code as described in Eldridge et al. (2008) and Eldridge & Stanway (2009), show similar trends to rotating single-star models in their predicted ratios of supergiant and supernova types with metallicity. In a private communication J. Eldridge (2011) reveals that, at least at the high metallicities of M31, the BPASS models offer a marked improvement over non-rotating, single-star tracks with respect to the predicted number of YSGs with luminosity. Similar to the rotating Geneva models presented here, the largest discrepancy with observations appears to be an overprediction of the highest luminosity stars (on the order of those found here as opposed to our previous work).

With the addition of this paper and that of Neugent et al. (2012), we will have characterized the YSG population in four local group galaxies spanning a factor of $\sim$10 in metallicity. This will allow us to directly compare the predicted trends of the rotating single-star and non-rotating binary tracks in the YSGs region with metallicity and see what constraints, if any, our YSG populations can provide with respect to these two models.

Another topic worthy of additional consideration is the increasing number of putative supernova progenitors that fall in the YSG regime (e.g., SN 2008cn, SN 2009kr, and SN2011dh; see Elias-Rosa et al. 2009, 2010 and Maund et al. 2011, respectively), a regime inconsistent with the end points of traditional single-star tracks (with the exception of the most recent $Z = 0.006$ $2 M_\odot$ Geneva tracks described above). Although in most cases the observational data are not inconsistent with (and may even suggest) a binary system (e.g., see Elias-Rosa et al. 2009; Soderberg et al. 2011; Arcavi et al. 2011; Van Dyk et al. 2011), Georgy (2012) also demonstrates that by increasing mass-loss rates in the RSG regime the Geneva code is able to produce tracks for $12 M_\odot$ and $15 M_\odot$ single stars that terminate in the YSG region. As discussed in Georgy (2012), both the physical mechanism and frequency of such enhanced mass loss are poorly understood and warrant further investigation. Any such enhancement would likely alter the predicted ratios of YSGs with luminosity, and thus the populations of this paper, Neugent et al. (2012), Neugent et al. (2010), and Drout et al. (2009) can offer a constraint on such models.

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REFERENCES

Andrillat, Y., Jaschek, C., & Jaschek, M. 1995, A&AS, 112, 475
Arcavi, I., Abdou, Y., Abu-Zayyad, T., et al. 2011, ApJ, 742, 18
Asplund, M., Grevesse, N., & Sauval, A. J. 2005, in ASP Conf. Ser. 336, Cosmic Abundances as Records of Stellar Evolution and Nucleosynthesis, ed. T. G. Barnes & III, F. N. Bash (Austin, TX: ASP), 25
Bresolin, F. 2011, ApJ, 730, 129
Cenarro, A. J., Cardiel, N., Gorgas, J., et al. 2001, MNRAS, 326, 959
Conti, P. S., & Ebbets, D. 1977, ApJ, 213, 438
Courteau, S., & van den Bergh, S. 1999, AJ, 118, 337

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