Decohering histories and open quantum systems

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Abstract. I briefly review the “decohering histories” or “consistent histories” formulation of quantum theory, due to Griffiths, Omnès, and Gell-Mann and Hartle (and the subject of my graduate work with George Sudarshan). I also sift through the many meanings that have been attached to decohering histories, with an emphasis on the most basic one: Decoherence of appropriate histories is needed to establish that quantum mechanics has the correct classical limit. Then I will describe efforts to find physical mechanisms that do this. Since most work has focused on density matrix versions of decoherence, I’ll consider the relation between the two formulations, which historically has not been straightforward. Finally, I’ll suggest a line of research that would use recent results by Sudarshan to illuminate this aspect of the classical limit of quantum theory.

1. Introduction
Many of the most interesting developments in quantum theory since Bell’s Theorem have centered on the notion of “decoherence,” understood most generally as the absence of interference effects. Decoherence is to be avoided at all costs when constructing mesoscopic or macroscopic quantum devices, such as SQUIDs or quantum computers, while its unavoidability in certain domains is said to be an important part of why large parts of the world appear classical. One area where decoherence has played an important role is the so-called “decohering histories” or “consistent histories” formulation of quantum theory, due to Griffiths [1], Omnès [2], and Gell-Mann and Hartle [3]. While it has not been adopted universally as an interpretive framework for quantum mechanics, which is what it was originally proposed for, it is certainly a useful tool for clarifying certain aspects of the classical limit of quantum theory, which remains a challenging problem despite the enormous progress made in the last few decades. In this article I will review the basic notions underlying decohering histories, and I will also spend a little time discussing various interpretations of this formalism. Having settled for application to the classical limit, leaving questions of interpretation aside, I will then compare the decohering histories approach to decoherence as seen in the time evolution of reduced density matrices. Since I argue that demonstrating decoherence of appropriate histories is needed to properly establish the classical limit of quantum theory, connecting these two approaches is of some importance. I will run across a problem which Sudarshan and I discussed at some length near the end of my time with him, without our coming to any good solution. Finally, I will apply recent results due to Sudarshan and coworkers to suggest a potentially very useful resolution of the issue, one that could illuminate not just this area but other aspects of the classical limit of quantum mechanics.
2. Histories and decoherence
The formalism of quantum mechanics is related to empirical results by the Born rule, which states the following: If a system is in state $|\Psi\rangle$ at time 0, then the probability at time $t$ that the system will be found in state $|\alpha\rangle$ is given by

$$p(\alpha) = |\langle\alpha|U(t)|\Psi\rangle|^2$$

where $U(t)$ is the time development operator which evolves the state from time 0 to time $t$. This expression can be rewritten

$$p(\alpha) = \text{Tr}(P_\alpha(t)\rho),$$

$$= \text{Tr}(P_\alpha(t)dP_\alpha(t)),$$

where $\rho = |\Psi\rangle\langle\Psi|$ is the initial density operator of the system, $P_\alpha$ is the projection operator onto state $|\alpha\rangle$, and I have switched to the Heisenberg picture, in which observables evolve in time but states do not. The second line follows from the first because projection operators are idempotent and the trace is cyclic in its arguments. This expression is actually more general than Eq. (1) because it remains valid when $\rho$ is a general density operator, representing an incoherent statistical mixture of pure states given by projectors.

An obvious generalization of this rule gives the probability not for a single measurement result, but for a series of measurement results at a succession of times:

$$p(\alpha_1,\alpha_2,\ldots,\alpha_n) = \text{Tr}(P_{\alpha_n}(t_n)\cdots P_{\alpha_2}(t_2)P_{\alpha_1}(t_1)dP_{\alpha_1}(t_1)P_{\alpha_2}(t_2)\cdots P_{\alpha_n}(t_n)).$$

This rule was given by Wigner [4], and Aharonov, Bergmann, and Lebowitz discussed both this rule and its time-symmetric generalization [5], but it is best known now because of its use in describing decohering histories. Before I get to decoherence, though, let me introduce some simplifying notation. Let $\alpha$ denote the sequence $\alpha_1,\alpha_2,\ldots,\alpha_n$, and given a sequence of measurements as above I define the history $C_\alpha$ by

$$C_\alpha = P_{\alpha_1}(t_1)P_{\alpha_2}(t_2)\cdots P_{\alpha_n}(t_n),$$

in terms of which Eq. (3) becomes the much tidier result

$$p(\alpha) = \text{Tr}(C_\alpha^d\rho C_\alpha).$$

This expression allows for much more conceptual contact between quantum and classical theory, as a history is the quantum mechanical analog of a trajectory of a classical particle. We routinely speak of such a particle occupying a succession of positions at a series of times, perhaps as it orbits another body, and in standard expositions of quantum theory it is not obvious what the corresponding quantum object is. This immediately suggests that histories may be of use in elucidating the classical limit of quantum mechanics, and that hope will be borne out below.

Since histories are products of projection operators, they inherit some (but not all) of the properties of their factors. Projection operators come in sets that satisfy

$$P_\alpha P_\beta = P_\alpha\delta_{\alpha\beta}$$

$$\sum P_\alpha = I$$

where $I$ is the identity operator. The first condition is that the set $\{P_\alpha\}$ is exclusive; measurement results corresponding to two different $P_\alpha$ cannot occur simultaneously. [This restriction is lifted when one replaces projection operators with positive operator valued measures (POVM), but
as I’ll show in a moment exclusivity is important for defining decoherence, so how one would construct decohering histories using POVM is not clear.] The second condition guarantees that \( \{P_\alpha\} \) is exhaustive, so every possible measurement result is included. Given a set of projection operators \( \{P_\alpha\} \) at each of a predetermined set of times \( t_1, t_2, \ldots, t_n \), one can construct a set of histories \( \{C_\alpha\} \) that satisfy

\[
\sum C_\alpha = \mathbb{I},
\]

so the histories are also exhaustive. Since a given set of histories defines a complete statistical sample space for a system, it makes sense to ask whether the probabilities associated with the histories by Eq. (5) satisfy the classical Kolmogorov axioms of probability:

(1) \( p(\alpha) \geq 0 \) for all \( \alpha \).

(2) \( \sum_\alpha p(\alpha) = 1 \).

(3) \( p(\alpha \text{ or } \beta) = p(\alpha) + p(\beta) \) for all \( \alpha \neq \beta \).

Axiom 1 is clearly satisfied by Eq. (5), and Axiom 2 is satisfied thanks to Eqs. (6) and (7), but we’re in trouble with Axiom 3. This is easy to see if we recall that composite measurements in quantum theory are defined by taking the sum of the corresponding projection operators; e.g. if \( P_1 \) represents the spin of a spin-1 system being found up in the \( z \) direction, and \( P_{-1} \) represents the spin being found down, then

\[
P = P_1 + P_{-1}
\]

represents the spin being found to be nonzero. If we apply this same logic to histories then the operator representing either history \( \alpha \) or history \( \beta \) is given by

\[
C_\alpha \text{ or } C_\beta = C_\alpha + C_\beta.
\]

(Notice that this is obviously correct if the two histories have the same events at all times but one, by the remarks that led to Eq. (8).) Therefore

\[
p(\alpha \text{ or } \beta) = \text{Tr}(C^\dagger_\alpha \text{ or } C_\beta \rho C_\alpha \text{ or } C_\beta)
\]

\[
= \text{Tr}(C^\dagger_\alpha \rho C_\alpha) + \text{Tr}(C^\dagger_\beta \rho C_\beta) + \text{Tr}(C^\dagger_\alpha \rho C_\beta) + \text{Tr}(C^\dagger_\beta \rho C_\alpha)
\]

\[
= p(\alpha) + p(\beta) + 2\text{Re} \text{Tr}(C^\dagger_\alpha \rho C_\beta),
\]

and Axiom 3 holds for all histories if and only if the final term vanishes for any \( \alpha \neq \beta \). Now we recognize this term; it is nothing more than a generalization of the interference terms we see ubiquitously in quantum theory. In fact, if we were to take a two-slit interferometer as our system and consider two-event histories that consisted of a particle passing through one of the two slits and then striking the screen, this term would be exactly the usual interference term. Thus we have generalized not only the Born rule but the expression for interference, and we have seen that in general it is the presence of interference that prevents quantum mechanical probabilities from satisfying the classical axioms of probability.

All of this inspires some new definitions. Given a complete set of histories, I define the decoherence functional between any two of them to be

\[
D(\alpha, \beta) = \text{Tr}(C^\dagger_\alpha \rho C_\beta).
\]

Now \( D(\alpha, \alpha) \) is the probability that history \( \alpha \) occurs (cf. Eq. (5)), and if \( \alpha \neq \beta \) then the real part of \( D(\alpha, \beta) \) is the interference term between histories \( \alpha \) and \( \beta \). Thus a set of histories obeys the Kolmogorov rules of probability if and only if

\[
\text{Re}D(\alpha, \beta) = p(\alpha)\delta_{\alpha\beta}.
\]
This condition is called weak decoherence. In many practical calculations the stronger condition
\[ D(\alpha, \beta) = p(\alpha)\delta_{\alpha\beta}. \] (13) is actually achieved; this is called medium decoherence. (These names suggest that there ought to be some notion of strong decoherence, and several definitions have been proposed, but none have caught on, so I won’t bother describing them.) Exactly what decoherence means physically is the subject of the next section.

3. Interpretation
When decoherence of histories was introduced by Griffiths [1], who called it consistency of histories, his goal was to develop a language for speaking meaningfully about the dynamics of a closed quantum system, for which the Copenhagen interpretation has nothing to say because a closed system by definition is not being measured. His idea was to allow the mathematical formalism of the theory to tell him how to speak meaningfully about its dynamics; he derived Eq. (5) for the probability of a history, and he concluded that the formalism allows only those sets of histories which obey the Kolmogorov axioms to be assigned probabilities in a meaningful way. Thus he was led to Eq. (12), and he argued that if a set of histories of a closed quantum system exhibits weak decoherence, then it is meaningful to say that one of the histories in question actually happens in the system, with probabilities given by Eq. (5). Soon after, Omnès [2] arrived at similar expressions but with more of a focus on the logic of quantum theory, and he later integrated them into his own interpretation of quantum mechanics, which he describes as “a consistent and complete reformulation of the Copenhagen interpretation” which lacks Copenhagen’s flaws [6]. In a slightly different vein, Gell-Mann and Hartle [3] had their sights set on quantum cosmology, where it had been realized that the Copenhagen interpretation was singularly inappropriate, the universe as a whole being the very model of a system with no outside observers. They have developed a quantum theory of closed systems, in which decoherence (they first applied this term to histories) is the sieve that determines à la Griffiths which histories are candidates for reality and which are not. Various objections have been raised to the use of decohering histories in this fashion, most famously by Dowker and Kent [7], and less famously by Sudarshan, Jordan, and me at about the same time [8], which elicited thorough responses from Griffiths [9] that were further elaborated in his extensive development of his program in [10].

In contrast, in this paper I would like to focus less on interpretation and more on the relevance of decohering histories to the classical limit. Roughly speaking, the classical limit of quantum mechanics consists of two components: Classical statistics and classical dynamics. By classical statistics, I mean that the probabilities associated with histories in classical physics obey the Kolmogorov rules of probability given above; Axiom 3 in particular is an essential part of our classical intuition, from coin tossing to weather prediction. By classical dynamics, I mean that the probabilities assigned by theory are strongly peaked around histories that correspond to solutions of the classical equations of motion; it is this aspect of the classical limit that received the lion’s share of attention until about 30 years ago, when decoherence in various forms made its appearance. Both components, however, are equally necessary, which means that decoherence of an appropriately chosen set of histories is a precondition for a quantum system to have a sensible classical limit, and it is precisely those histories that classical behavior will be seen. Thus even if all we care about is establishing classical behavior for all practical purposes (precisely John Bell’s famous FAPP [11]), decoherence of appropriate histories must be demonstrated. Now I’ll describe how this has been attempted.
4. Mechanisms of decoherence

By far the most popular option in the literature for achieving decoherence is to separate the quantum system under consideration into a preferred subsystem and the remainder, often called the environment. Such a separation corresponds to an expression of the system’s Hilbert space $\mathcal{H}$ as

$$\mathcal{H} = \mathcal{H}^{S} \otimes \mathcal{H}^{E},$$

(14)

where $\mathcal{H}^{S}$ is the Hilbert space of the subsystem and $\mathcal{H}^{E}$ describes the environment. This decomposition is actually quite general, and it corresponds to the expression of the configuration space of a classical system as the Cartesian product of subspaces carrying different degrees of freedom; for example, a single particle with spin moving in space can be described as the product of two subsystems, namely the spatial and spin degrees of freedom, each with its own Hilbert space. Once this separation is defined, decoherence is achieved by considering only histories composed of projections $P^{S}$ onto the subsystem and by allowing the subsystem and environment to interact, which generally greatly accelerates the process of decoherence. This second factor leads to the use of open systems theory for decoherence, as the time evolution of the subsystem is normally far from Hamiltonian even if a Hamiltonian describes the full system.

Some of the most popular model systems for illustrating decoherence, as described by Stamp [12], can be classified as follows. The first and most famous are harmonic oscillators coupled linearly to baths of similar oscillators, typically in a thermal state with some simplified spectrum of frequencies; the preferred oscillator exhibits decoherence in its position basis due to interactions with the bath. The influence functional technique, due to Feynman and Vernon [13], was applied to this system by Caldeira and Leggett [14], and their calculation is still widely cited as an illustration of the efficiency with which environmental interaction produces decoherence. Caldeira and Leggett were at some pains to argue in their work that this model was considerably more generic than it would seem at first sight; the need for such arguments should be clear, given that their intent is to illustrate what should be one of the most widespread phenomena in nature (responsible for the entire classical world, after all). A second set of calculations has focused on finite-dimensional quantum systems coupled to spin baths [15]; the rationale was that such systems should do a good job of representing the low-energy dynamics of condensed matter systems, in which generally only a few low-lying energy levels should be excited. A third mechanism is “third-party decoherence” [12], in which correlations between subsystems which suppress interference arise not because of any interaction between them, but due to their mutual interactions with a third subsystem. (The particular example Stamp gives is of a particle passing through a two-slit interferometer which loses spatial interference because its spin interacts with the slits, even though its spin and spatial degrees of freedom are uncoupled.)

These sorts of calculations have two features in common which are worth mentioning. First, almost all of them are performed not on decoherence functionals, but on reduced density matrices describing only the preferred subsystem; for example, Caldeira and Leggett actually showed that the reduced density matrix of the preferred oscillator diagonalizes rapidly in the position representation for a variety of initial states, while its diagonal elements remain relatively unchanged. There have been exceptions, such as [16], but the majority of calculations in the literature have considered not histories but reduced density matrices or related objects. Given that I have argued for the importance of decoherence functionals in establishing classicality, it is worth asking how much light density matrix calculations shed on decohering histories. Second, a large fraction of the calculations have either assumed, or made other simplifying assumptions to guarantee, that the subsystem’s evolution is described by a master equation of some sort, often Markovian. When Sudarshan and I discussed this state of affairs, he expressed his concern that such approximations were far more restrictive than these authors perhaps realized, and that they cast some doubt on their claims to have illustrated generic features that give rise to the classical world. These issues are the subject of the next two sections.
5. Decoherence functionals vs. reduced density matrices

Particularly since the histories approach to decoherence puts so much emphasis on diagonalizing a functional, it is easy to misunderstand the importance of diagonalizing reduced density matrices in that approach (at least it was easy for me to misunderstand it). Every density matrix, reduced or not, is of course always diagonal in some basis simply because it is self-adjoint. The goal of the reduced density matrix approach is this: Given a full system with initial state $\rho$, reduced initial state $Tr_\mathcal{E}\rho$, and full time evolution operator $U(t)$, the time evolution of the reduced density matrix

$$Tr_\mathcal{E}\rho \rightarrow Tr_\mathcal{E}[U(t)\rho U^\dagger(t)] \tag{15}$$

sometimes has the property that the result diagonalizes rapidly in a particular basis, far more rapidly than the diagonal elements in that basis undergo change, and it stays diagonal over time. Further, this behavior is largely independent of the initial state $\rho$. The states making up such a basis, possibly with additional properties, are called “pointer states” [17], and the idea behind the reduced density matrix approach to decoherence is that such bases serve to define a preferred class of observables in which classical behavior emerges in appropriate systems. This is actually a very sensible idea, and in fact it deals directly with the fundamental issue raised by Dowker and Kent [7], which is that in general decoherence of histories depends far too sensitively on the exact choice of projection operators in the histories and the times at which they are placed. If anything is obvious about classicality, it is that it’s robust; one does not have to look at the positions of a planet orbiting the sun at a carefully orchestrated sequence of times in order to see different alternate trajectories failing to interfere. Thus the obvious question to ask is whether histories composed of projections onto pointer states decohere. To see whether this is so, let me consider a representative decoherence functional

$$D(\alpha, \beta) = Tr[P_{\alpha_1}^S(t_n) \cdots P_{\alpha_1}^S(t_1)\rho P_{\beta_1}^S(t_1) \cdots P_{\beta_n}^S(t_n)] . \tag{16}$$

Let me also make this expression a bit more explicit by expressing the trace as the composition of two partial traces, over the environment and subsystem, and let me also leave the Heisenberg picture and write the time evolutions explicitly, with the result

$$D(\alpha, \beta) = Tr_\mathcal{S}Tr_\mathcal{E}[P_{\alpha_1}^S U(t_n, t_{n-1}) \cdots P_{\alpha_1}^S U(t_1, t_0)\rho U(t_0, t_1) P_{\beta_1}^S \cdots U(t_{n-1}, t_n) P_{\beta_n}^S]. \tag{17}$$

Now let us suppose that the time evolution in Eq. (15) does in fact diagonalize the reduced density matrix in some basis, and further that the projections $P_{\alpha_i}^S$ project onto that very basis. Can we conclude that this functional (or at least its real part) vanishes if any $\alpha_i \neq \beta_i$? Not obviously, for the simple reason that the time evolution is repeatedly interrupted by projections before the partial trace over the environment is taken, and while the partial trace commutes with those projections, it does not commute with the time evolution. This is all obvious, of course, and this problem is discussed in some detail by Kiefer in Chapter 5 of [18]. An important reference in that book is work by Paz and Zurek [19], whose solution, with notation modified for consistency with other work shown below, is as follows. Suppose there exist operators $M(t_2, t_1)$ such that for any operator $A$ acting on the full system,

$$Tr_\mathcal{E}[U(t_2, t_1)AU(t_1, t_2)] = M(t_2, t_1)\{Tr_\mathcal{E} A\}; \tag{18}$$

in other words, suppose that the time evolution of Eq. (15) can be expressed as an operator acting on the reduced state only. In that case Eq. (17) can be written

$$D(\alpha, \beta) = Tr_\mathcal{S}[P_{\alpha_1}^S M(t_n, t_{n-1}) \cdots P_{\alpha_1}^S M(t_1, t_0)\{Tr_\mathcal{E} \rho\} P_{\beta_1}^S \cdots P_{\beta_n}^S]. \tag{19}$$

Now it’s obvious that if the times $t_i$ are sufficiently well separated for the decoherence mechanism to do its work, the result of each $M(t_i, t_{i-1})$ will be diagonal in the pointer basis and the $\alpha_i \neq \beta_i$.
terms will vanish. But is it reasonable to hypothesize such an operator $M$? For some time I struggled with this issue, because the justification offered by Paz and Zurek seemed to rely excessively on special assumptions, such as a Markovian master equation, of the type with which Sudarshan expressed such concern. However, recent work by Sudarshan and coworkers has resolved this issue rather neatly, which brings me to my final topic.

6. Decoherence functionals and the theory of open systems

Despite all my concerns with master equations and such, it was shown recently by Jordan, Shaji, and Sudarshan [20] that time evolution of operators in a subsystem of a full quantum system experiencing unitary evolution can always be expressed in a form very similar to Eq. (18) with no assumptions whatsoever. Suppressing the time indices for the moment for convenience, the expression is

$$M(\text{Tr}_E A) = L(\text{Tr}_E A) + K$$  (20)

where

$$L(\text{Tr}_E A) = \text{Tr}_E \left[ U \left( \text{Tr}_E A \otimes \frac{I_E}{\dim \mathcal{H}^E} \right) U^\dagger \right]$$  (21)

and

$$K = \text{Tr}_E \left[ U \left( A - \text{Tr}_E A \otimes \frac{I_E}{\dim \mathcal{H}^E} \right) U^\dagger \right],$$  (22)

where I have reexpressed their results using my own notation. Several comments are in order concerning this form. First, the whole thing may look like simple sleight of hand (after all, this is simply Eq. (15) with Eq. (21) added and subtracted), and it would be if this form had no advantages; but Jordan et al. prove several significant results about this form and develop useful expressions for calculating with it in [20], so its apparent simplicity is deceptive. Second, although Sudarshan has always emphasized that the most general time evolution of a density operator can always be cast in linear form, the form shown here is instead affine, and that is the source of some of its charming features. Notice that the linear term depends only on the time evolution operator $U$ and not on any correlations between the subsystem and the environment, while the contribution from the correlations has been isolated in the affine term. Third, this means that in fact $M$ is not a function of only $\text{Tr}_E A$, but $A$ in its entirety, as of course it must be; the idea here is not to neglect the correlations but to put them in a more tractable form. Finally, since this expression is available generally, it clearly follows that the $M(t_i, t_{i-1})$, while always defined, will not in general form a semigroup (as this would imply a master equation), and forming a group is almost always out of the question, as that would imply Hamiltonian evolution of the subsystem.

Casting $M(t_i, t_{i-1})$ in this form makes one point particularly clear. While it is typically assumed that the time evolution of the preferred subsystem obeys a master equation, is Markovian, or is otherwise special, all of those assumptions are for computational simplicity only; none of them are needed to express decoherence of histories. Stamp forcefully expresses a concern, hinted at previously in this paper, that the work which has been done up to this point on decoherence has not considered sufficiently general situations to convince the general population of physicists that truly generic mechanisms for producing the classical world have in fact been unearthed [12]; and while I don’t wish to endorse Stamp’s position fully, I do agree that the model calculations performed up to this point don’t quite convey the flavor of a universal phenomenon. This leads me to the fundamental point of this paper: I suggest that a study of decoherence functionals using the formalism of Jordan et al. would give a broader and more general picture of decoherence as a phenomenon than the sorts of calculations largely performed up to this point have done. Jordan et al. state that the aim of their paper is “to simply describe the Schrödinger picture before making approximations to it” (such as the introduction of master
equations), and I suggest that such a perspective would benefit the study of decoherence, which in many ways is so much more mature than it was a decade ago.

This maturity shows itself particularly in the research program of Zurek and coworkers, recently summarized in [17], in which it is recognized that decoherence is only one of many ingredients that characterize classicality. The larger picture is that the classical world is characterized by particular observables that carry several properties:

• The corresponding eigenstates are highly resistant to environmental dephasing, while their superpositions are not.
• Thus they are selected by the details of dynamics, not our arbitrary choices; it is dynamics that tells us what we can potentially observe.
• These observables imprint their values highly redundantly on the environment, so it is possible to deduce their values by sampling only a small fraction of the environment (knowing the position of an object after sampling only very few of the photons reflected from it is an obvious example).
• Finally, histories composed of projections onto eigenstates of these observables exhibit decoherence.

Many of these issues have been studied in explicit models in a master equation context, and I suggest that further study using this more general open systems perspective would be of great value.

However, the results of Jordan et al. require extension in certain directions to be useful here. The most obvious is that Eqs. (21) and (22) require the dimension of the environment’s Hilbert space to be finite, as they involve the introduction of a “state of complete ignorance” of the environment, which is defined only in the finite-dimensional case. While this is adequate for spin bath calculations, an infinite-dimensional version is clearly required for other applications. Perhaps this is easily done by replacing the state of complete ignorance with a thermal equilibrium state, but that remains to be demonstrated.

7. Conclusions

Establishing that quantum mechanics has the correct classical limit has proved to be a surprisingly challenging problem; some time passed before the full extent of the problem was even clear. Since the recognition that decoherence was an important contributor, however, various research programs have made progress on this issue. Here I have summarized results due to Griffiths, Omnès, and Gell-Mann and Hartle, and I have argued that “decoherence” or “consistency” in their sense is necessary for establishing one aspect of the classical limit, namely the validity of classical statistics. I have also pointed out that decoherence is actually only one ingredient among many for demonstrating this validity in certain regimes. After describing their formalism, I tried to bring it into contact with approaches to decoherence that consider not histories but reduced density matrices of selected subsystems, and I found that the connection seemed to depend on making certain approximations in the quantum theory of open systems, such as the existence of Markovian master equations. I also suggested that this might cast some doubts on claims that truly universal mechanisms explaining the emergence of classicality have been found. Then I described a way to analyze histories based on recent work by Jordan, Shaji, and Sudarshan that seems to justify this connection while avoiding the usual approximations, thus offering the possibility of demonstrating decoherence in a much broader range of situations. I suggest that this work offers a new path forward for performing calculations that can illuminate many different aspects of the classical limit, including but certainly not limited to decoherence of histories, with considerably greater generality.
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