A note on closed string

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Special kind of closed strings is considered. It is shown that these closed strings behave as two (an even number of) open strings at the classical level and one open string at the quantum level. They contain massless vector field in their spectrum and can lie on D branes. Some properties of closed string field effective action are declared.

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1 Introduction

In this paper two problems in string theory are considered. The first one is the presence of open strings in the type II closed string theories. The second is the formulation of effective action for closed string fields.

With the appearance of D – Branes [1] in string theory, open strings ending on them were added to type II theories. Natural appearance of open strings in the theory of closed strings remains unclear, to my best knowledge.

More progress has been done in understanding the role of open string tachyon field both in the framework of Cubic [2] and Boundary [3]-[5] Open String Field theories (see [7]-[9] and references therein). However, we do not know much about effective action for closed string fields [10].

In the present paper, we consider a subset of closed strings and show that they behave as two (an even number of) open strings at the classical level. After quantization these closed strings contain massless vector field in their spectrum. Massive fields in their spectrum have masses twice as much as the masses of fields in the spectrum of an open string. Therefore these closed strings behave almost as one open string at the quantum level. We apply this result to the effective action for closed string fields.

In the next section we give a new look to closed strings and show that there are closed strings that behave as two open strings at the classical level. Then we quantize these strings and show that they contain massless vector field in their spectrum and behave almost as one open string at the quantum level. In section 3, we investigate properties of closed string spectrum and find some properties of closed string field effective action. Summary of results is given in section 4. In section 5 we make remarks and give solutions that represent closed strings behaving as an even number of open strings at the classical level and almost one open string at the quantum level.

2 Closed-open string duality

Action for string is

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma (-\gamma)^{1/2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

and

$$\delta S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \partial_a \{(-\gamma)^{1/2} \gamma^{ab} \partial_b X_\mu \} \delta X_\mu - \frac{1}{2\pi\alpha'} \int d\tau \partial_\sigma X_\mu \delta X_\mu |_{\sigma=\pi}$$

so we have the following boundary condition [11]-[14]

$$\partial_\sigma X_\mu \delta X_\mu (\tau, 0) - \partial_\sigma X_\mu \delta X_\mu (\tau, \pi) = 0.$$  

(2)

One of the solutions to this equation for closed string is

$$\partial_\sigma X_\mu (\tau, 0) = \partial_\sigma X_\mu (\tau, \pi), \quad X_\mu (\tau, 0) = X_\mu (\tau, \pi).$$

(3)
In this paper we investigate properties of a special solutions to the equation of motion with boundary condition (3) in conformal gauge represented by

\[ X^\mu = x^\mu + 2\alpha' p^\mu \tau + i(2\alpha')^{1/2} \sum_{n \neq 0} \frac{1}{n} \alpha^\mu_n e^{-2in\tau} \cos(2n\sigma). \] (4)

This solution has a few interesting properties as demonstrate in the sequel. They satisfy the condition

\[ \partial_\sigma X^\mu(\tau, \pi/2) = 0. \]

Note that this condition is satisfied at point \( \sigma = \pi/2 \) and (4) also satisfies

\[ X^\mu(\tau, 0) = X^\mu(\tau, \pi) \]

which means that string is closed and behaves as two open strings because Neumann boundary conditions are satisfied in two different points \( \sigma = 0 \) and \( \sigma = \pi/2 \).

Next, we quantize closed strings represented by (4). To quantize (4) we apply canonical quantization method, i.e., we impose

\[ [X^\mu(\tau, \sigma), \Pi^\nu(\tau, \sigma')] = i\eta^\mu\nu \delta(\sigma - \sigma'), \quad \Pi^\nu = \frac{1}{2\pi \alpha'} \partial_\tau X^\nu \]

which leads to the following commutation relations on the oscillators and the zero modes

\[ [x^\mu, p^\nu] = 2i\eta^\mu\nu, \quad [\alpha^\mu_n, \alpha^\nu_m] = n\delta_{n+m} \eta^\mu\nu. \]

From equation

\[ (L_0 - 1) |\psi_{phys}\rangle = 0, \quad L_0 = \frac{\alpha'}{4} p^2 + \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n + \frac{D}{2} \sum_{n=1}^{\infty} n \]

we can read the expression for the mass operator

\[ M^2 = \frac{4}{\alpha'} \left( \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n - 1 \right), \]

As it is seen closed strings (4) contain massless vector field \( A_\mu \sim \alpha^\mu_{-1} |0, k\rangle \) in their spectrum. We see that they behave as two open strings at the classical level and almost one at the quantum level, because the masses of fields are as twice as much as the masses in the spectrum of an open string.

### 3 On closed string field effective action

In this section, we assume that closed strings represented by

\[ X'^\mu = x'^\mu + 2\alpha' p'^\mu \tau + i(\alpha'/2)^{1/2} \sum_{n \neq 0} \frac{1}{n} \left( \beta^\mu_n e^{2in(\sigma - \tau)} + \beta^\mu_n e^{-2in(\sigma + \tau)} \right). \] (5)
can transform to closed strings (4). Let us show that this corresponds to process $\tilde{\beta}_n^\mu \to \beta_n^\mu$. The massless spectrum of closed strings (5) is
\[
G_{\mu\nu} \sim (\beta_{-1}^\mu \tilde{\beta}_{-1}^\nu + \beta_{-1}^\nu \tilde{\beta}_{-1}^\mu) |0, k>, \quad M^2(G_{\mu\nu}) = 0,
\]
\[
B_{\mu\nu} \sim (\beta_{-1}^\mu \tilde{\beta}_{-1}^\nu - \beta_{-1}^\nu \tilde{\beta}_{-1}^\mu) |0, k>, \quad M^2(B_{\mu\nu}) = 0,
\]
\[
\Phi \sim \beta_{-1}^\mu \tilde{\beta}_{-1}^\mu |0, k>, \quad M^2(\Phi) = 0.
\]
After the limit $\tilde{\beta}_n^\mu \to \beta_n^\mu$ the field content changes. Fields $G_{\mu\nu}$ and $\Phi$ became massive and new field $A_\mu$ appears.

\[
M^2 = \frac{2}{\alpha'} \left( \sum_{n=1}^{\infty} (\beta_{-n}\beta_n + \tilde{\beta}_{-n}\tilde{\beta}_n) - 2 \right) \to M^2 = \frac{4}{\alpha'} \left( \sum_{n=1}^{\infty} \beta_{-n}\beta_n - 1 \right),
\]
\[
G_{\mu\nu} \to G_{\mu\nu} \sim \beta_{-1}^\mu \beta_{-1}^\nu |0, k>, \quad M^2(G_{\mu\nu}) = \frac{4}{\alpha'},
\]
\[
B_{\mu\nu} \to 0,
\]
\[
\Phi \to \Phi \sim \beta_{-1}^\mu \beta_{-1}^\nu |0, k>, \quad M^2(\Phi) = \frac{4}{\alpha'},
\]
\[
A_\mu \sim \beta_{-1}^\mu |0, k>, \quad M^2(A_\mu) = 0.
\]
We see that as $\tilde{\beta}_n^\mu \to \beta_n^\mu$ spectrum of closed strings (5) coincides with the spectrum of closed strings (4). According to this result we can state that the limit $\beta_n^\mu \to \beta_n^\mu$ corresponds, in the language of fields, to
\[
S(G_{\mu\nu}, B_{\mu\nu}, \Phi, ...) \to S(G_{\mu\nu}, A_\mu, \Phi, ...), \tag{6}
\]
where $S$ is the closed string field action. This means that there is mechanism that can happen, and then the massless fields became massive and new massless vector field appears.

4 Summary of results

The main idea in this paper is that open strings attached to D–Branes in type II theories do not satisfy the condition $X^\mu(\tau, 0) = X^\mu(\tau, \pi)$. This condition should be satisfied because it is assumed from the beginning of the type II theories. We suggest the way of avoiding this contradiction.

Closed strings (4)

i) behave as two open strings at the classical level because they satisfy open string boundary conditions in two different points $\sigma = 0$ and $\sigma = \pi/2$,

ii) can lie on D-branes, because after T duality on i-th direction Neumann boundary condition becomes Dirichlet boundary condition on i-th direction. Therefore, two points of closed strings (4), $X_{closed}^i(\tau, \sigma = 0)$ and $X_{closed}^i(\tau, \sigma = \pi/2)$, can lie on D–Branes,

iii) contain massless vector field in their spectrum and behave as one open string at the quantum level.

The next result is the following: if there is transformation between closed strings (5) and closed strings (4), this corresponds to the limit $\tilde{\beta}_n^\mu \to \beta_n^\mu$ in (5), then the closed string field effective action has property (6).
5 Remarks

It is known that two or more open strings can join to form a closed string. However, there is no evidence that the resulting closed string will behave as two or more open strings. Moreover, even and odd number of open strings can join to form a closed string. In contrast, there are solutions to the equation of motion with

\[ X_{\text{closed}}^\mu = x^\mu + 2\alpha' p^\mu \tau + i(2\alpha')^{1/2} \sum_{n \neq 0} \frac{1}{n} \alpha_n e^{-2kn\tau} \cos(2kn\sigma), \quad k \in N \quad (7) \]

satisfying Neumann boundary condition in 2k points

\[ \sigma = (0, \pi/2k, \pi/k, 3\pi/2k, \ldots, \pi). \]

Hence, (7) represents closed string behaving as 2k number of open strings. Therefore, there are no closed strings behaving as an odd number of open strings. (4) is (7) with \( k = 1 \).

We note that (4) can be considered as a solution to the equation of motion with boundary conditions

\[ \partial_\sigma X^\mu(\tau, 0) = \partial_\sigma X^\mu(\tau, \pi) = 0, \quad X^\mu(\tau, 0) = X^\mu(\tau, \pi). \]

These boundary conditions are also solutions to (2).

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