Proposals on calculating the differential properties of concrete

L R Mailyan, S A Steł'makha, E M Shcherban*, A P Korobkin, and E A Efimenko

Don State Technical University, 1, Gagarin Square, Rostov-on-Don, 344000, Russia

E-mail: au-geen@mail.ru

Abstract. Background: The paper dwells upon calculating the differential structural properties of centrifugated and vibration-centrifugated concrete as a function of the manufacturing parameters. Methods: One finding is that different layers of centrifugated and vibration-centrifugated concrete are exposed to forces of varying magnitude, resulting in these layers differing significantly in density, strength, strain, and elastic modulus. Another finding is that it is the centrifugal and centripetal forces that are key factors to be applied as arguments in calculational dependencies. Thus, the versatile calculational dependencies that make adjustments for changes in the structural properties of concrete should use these properties or their increments as functions, and the rotation inertia forces as arguments, which are in turn a function of distance from the center of rotation and the angular rotation speed. Results: This research has thus produced calculational dependencies for differential adjustment for change in all the concrete properties that have to be invoked in calculation. Conclusions: Strength- and strain-related properties of concrete that vary depthwise (i.e. across the section) are applied in the calculation procedures for more accurate and complete utilization of the available load-carrying capacity of concrete elements…

1. Introduction

Design of reinforced concrete elements based on nonlinear dependencies are listed in the standards of many countries [1-3], has set a question about their practical calculation for most of the engineers. This is because the usage of the formulas of these rules without a computer [1-3] is practically impossible. It is generally accepted that any calculation by using computers should be checked or evaluated through classical and practical techniques. Creating such a technique would allow making an assessment, verification and analysis conducted computer calculations. Simplified and approximate methods [4-7] not correspond to exact solution that is obtained by nonlinear dependencies in most of cases.

Calculation of resistibility performed by using the equation of equilibrium of external forces and internal forces, deformation diagrams of concrete and reinforcement and the function of changes of deformation by height section. As a function of diagrams of concretes deformation should be taken like function of stresses in the concrete which would correspond to the conditions of nonlinear deformation of concrete. The following dependences are proposed for calculations of nonlinear structures in the standards [1]: formula of rules Eurocode 2 (3.4), fifth degree polynomial (3.5), two- and three a linear dependence between the stresses and deformations. The dependence of stress-deformation for reinforcement is taken as two linear Prandtl diagrams. The distribution of deformation by height section at the moment of the destruction taking in a linear dependence like:
where $\varepsilon$ – relative deformation of the material at a distance $x$ from neutral line, $1/r$ – curvature of element in section.

In most cases, diagrams of concrete deformation are function of related strength and deformation characteristics: the calculated resistance of concrete to axial junction $f_c$, concrete deformation module $E_c$ and limits of concrete deformation $\varepsilon_{c1}$, $\varepsilon_{cu}$. This can be expressed in the following functional dependence

$$\sigma_c = f(f_c, E_c, \varepsilon_{c1}, \varepsilon_{cu}).$$

There are many formulas that associate deformation and strength characteristics of concrete. The following expressions can be written by its summarizing

$$E_c = f(f_c), \varepsilon_{c1} = f(f_c), \varepsilon_{cu} = f(f_c).$$

With taking into account (3) the final dependence (2) will take the following form

$$\sigma_c = f(f_c).$$

The function laid down in the current norms [1,2] will be examined for further research. It is also called Eurocode function

$$\sigma_c = \frac{E_c \varepsilon_c - f_{cm} \left( \frac{\varepsilon_c}{\varepsilon_{c1}} \right)^2}{1 + \left( \frac{E_c \varepsilon_{c1}}{f_{cm}} - 2 \right) \frac{\varepsilon_c}{\varepsilon_{c1}}} \text{ or } \frac{\varepsilon_c}{f_c} = \frac{k \eta - \eta^2}{1 + (k - 2)\eta},$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$, $k = 1.05 E_c \varepsilon_{c1} / f_c$.

Here is the expression (4) for the function (5)

$$\sigma_c = f_c \times f(k, \eta) = f_c \times f(f_c).$$

The bending reinforced concrete elements at a single reinforcement are proposed to consider. After putting equilibrium equation and conducting simple transformation with considering the hypothesis of flat sections we will get:

- for non-overreinforced

$$\left( \int_0^{\varepsilon_c} \sigma_c d\varepsilon_c - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} \sigma_c d\varepsilon_c} \right) \frac{\varepsilon_c}{f_c} \rho_f^2 f_{yd}^2 + \rho_f f_{yd} = \frac{M_{Ed}}{b d^2};$$

- for overreinforced

$$\left( \int_0^{\varepsilon_c} \sigma_c d\varepsilon_c - \frac{\varepsilon_c}{\int_0^{\varepsilon_c} \sigma_c d\varepsilon_c} \right) \frac{\varepsilon_c}{f_c} \rho_f^2 E_s \varepsilon_c^2 + \rho_f E_s \varepsilon_c = \frac{M_{Ed}}{b d^2}.$$
The following notations are introduced
\[ \omega = \frac{\rho_f f_{yd}}{E_d}, \eta_s = \frac{E_s}{f_{yd}} \] (11)

The name parameter \( \omega \) is mechanical reinforcement coefficient [4, 5]. The formulas (9) and (10) with introduced notations (11) will take the following form:

- for non-overreinforced
  \[ \int_0^{\xi_c} f(f_c)e_c d\varepsilon_c \left( \int_0^{\xi_c} f(f_c)d\varepsilon_c \right)^2 \quad \frac{\varepsilon_c}{f_c} = \frac{M_{Ed}}{f_c bd^2} + \omega \quad + \omega = \frac{M_{Ed}}{f_c bd^2}; \] (12)

- for overreinforced
  \[ \int_0^{\xi_c} f(f_c)e_c d\varepsilon_c \left( \int_0^{\xi_c} f(f_c)d\varepsilon_c \right)^2 \quad \frac{\varepsilon_c}{f_c} = \frac{M_{Ed}}{f_c bd^2} + \omega \eta_s = \frac{M_{Ed}}{f_c bd^2}. \] (13)

The notation is introduced to both equations
\[ k_z = f(f_c, \omega) = \frac{6M_{Ed}}{f_c bd^2} = \frac{M_{Ed}}{f_c W_c}. \] (14)

Thereby
- for non-overreinforced
  \[ k_z = \frac{1}{6} \left( \int_0^{\xi_c} f(f_c)e_c d\varepsilon_c \left( \int_0^{\xi_c} f(f_c)d\varepsilon_c \right)^2 \quad \frac{\varepsilon_c}{f_c} \right) \omega^2 + \omega \right); \] (15)

- for overreinforced
  \[ k_z = \frac{1}{6} \left( \int_0^{\xi_c} f(f_c)e_c d\varepsilon_c \left( \int_0^{\xi_c} f(f_c)d\varepsilon_c \right)^2 \quad \frac{\varepsilon_c}{f_c} \right) \omega^2 \eta_s^2 + \omega \eta_s \right). \] (16)

Connection between the calculated resistance of reinforced concrete and characteristics that derived above is following
\[ f_{cM} = k_z f_c \] (17)

Introduced parameter \( k_z \) generally depends on the mechanical reinforcement coefficient \( \tilde{\omega} \) and deformation characteristics of concrete. There are more details about the impact of deformation characteristics of concrete on bearing capacity of bending elements. Here is a submission of parameter dependence schedule \( k_z \) depending on the mechanical reinforcement coefficient \( \tilde{\omega} \) for different kinds of concrete. The impact of deformation characteristics of concrete (parameters \( \eta = \varepsilon_c / \varepsilon_{c1} \), \( k = 1.05 E_{c1} / f_c \) on bearing capacity of bending reinforced concrete elements for considered kinds of concrete varies with differences within 10 %. This error is completely allowed by normative coefficients of variation of strength for such elements. This makes possible to offer slightly simplified force model of reinforced concrete elements calculation. The impact of deformation characteristics of strength sections is ignored and common functional dependence is taken \( k_z = f(\tilde{\omega}) \) for all kinds of concrete. This dependence is true not only for different kinds of concrete and reinforcement and even for different duration of the load.

Formulas for non-central compression can be derived by similar arguments. Dependence of parameter \( k_z \) from the mechanical reinforcement coefficient \( \pi \) is presented in tables.

The strength conditions by a modified method of calculation resistances are formulated:

- bending elements
  \[ M_{Ed} = W_c f_{cM} \] (18)
where $f_{zM}$ – estimated resistance of reinforced concrete on bend, which depends on kinds of concrete and reinforcement, section shapes and percent of reinforcement section that defined by the expression (17). Parameter $k_z$ in expression (7) is calculated by table depending on the mechanical reinforcement coefficient $\bar{\omega}$.

- non-central compressed

$$X_{Ed} = A_c f_{z:N}$$

(19)

where $f_{zN}$ – estimated resistance of reinforced concrete on bend, which depends on kinds of concrete and reinforcement, section shapes and percent of reinforcement section, relative initial eccentricity of the force application $e_0 / d$, that defined by the expression $f_{zN} = k_z f_{c}$. Parameter $k_z$ is also calculated by table depending on the mechanical reinforcement coefficient $\bar{\omega}$; $A_c$ – working sectional area of concrete:

for rectangular elements $A_c = bd$; for round ones $A_c = \pi d^2 / 4$.

2. Materials and Methods

Concrete centrifugation is known [8-18] to utilize the ability of concrete mix to condense itself within a rotating mold due to the effects of centrifugal forces proportional to the mass of particles, the squared rotation speed and distance to the axis of rotation:

$$F_{c.f.} = \frac{4}{3} \pi \cdot r_{c.a.}^3 \cdot \rho_{c.a.} \cdot l \cdot \omega^2$$

(20)

where $l$ is the center-of-grain to center-of-rotation distance, m; $\omega$ is the angular rotation speed, rad/s:

$$\omega = \frac{\pi \cdot n}{30}$$

(21)

Centrifugal force effects on a coarse aggregate grain (Figure 1) are proportional to the radius $r_{c.a.}$ and the density $\rho_{c.a.}$ of the particle. This causes larger grains to move towards the outer layer and the smaller ones towards the inner layer.

We herein propose focusing on a different, more generic formula for calculating the centrifugal force, where $F_{c.f.} = f(\omega, l)$:

$$F_{c.f.} = |m| \cdot \omega^2 \cdot l$$

(22)

where $m$ is the mass of the rotating solid; $\omega$ is the angular rotation speed; $l$ is the center-of-grain to center-of-rotation distance.

In this regard, centrifugated concrete differs from vibrated concrete in a sense that it has less uniform depthwise distribution of coarse aggregate grains, which results in different layers having different properties, i.e. the section becomes variatropic.

This applies even more to vibration-centrifugated concrete which has even stronger variatropy. In these elements, the outer layers have a greater concentration of coarse aggregate grains, thus greater strength.

All of this shows why the strength- and strain-related difference in variatropic-section concrete must be adjusted for.
Figure 1. How centrifugal forces act on coarse aggregate grains across the wall of a centrifugated product.

To confirm this, calculate the effects of centrifugal forces, for instance, on the grains of a dense (granite) aggregate. Substitute the input data in (1) to find the centrifugal force acting on granite grains, see Table 1.

Table 1. Basic parameters of the acting centrifugal force.

| Coarse aggregate grain radius, mm | Center-of-grain to center-of-rotation distance, m | Angular speed, rad/s | Mold rotation speed when compacting the concrete, rpm | Pressure on the concrete mix, p kgf/cm² | Centrifugal force acting on the coarse aggregate grains, H |
|----------------------------------|-----------------------------------------------|----------------------|----------------------------------------------|--------------------------------------|----------------------------------|
| 10.0                             | 0.105                                         | 84                   | 800                                           | 0.7                                  | 316.5·10⁻⁶                      |
|                                   | 157                                           | 1,500                | 2.16                                          | 1105.6·10⁻⁶                         |

Using mathematical experiment design, assume a $2^k$ full factorial design FFD. Following our own recommendations [19-23], assume the concrete mix centrifugation time is 4.2 min. Table 2 presents the values of varying factors and their physical sense.

The following parameters are adopted as the response function:
- $Y1 (X1, X2)$ is the axial compressive strength, MPa;
- $Y2 (X1, X2)$ is the axial tensile strength, MPa;
- $Y3 (X1, X2)$ is the ultimate axial compressive strain, mm/m;
- $Y4 (X1, X2)$ is the ultimate axial tensile strain, mm/m;
- $Y5 (X1, X2)$ is the elastic modulus, MPa;

Table 2. Varying factor values in FFD $2^k$.

| Factor code | Physical sense of the factor | Unit | Variation range | Levels of factors |
|-------------|------------------------------|------|-----------------|------------------|

### 3. Results and Discussion

The least squares method produced basic regression equations represented by 2nd degree polynomials:

\[ Y(X_1, X_2) = B_0 + B_1 \cdot X_1 + B_2 \cdot X_2 + B_3 \cdot X_1 \cdot X_2 + B_4 \cdot X_1^2 + B_5 \cdot X_2^2 \]  

(23)

Statistical analysis of the obtained regression equations was tested for variance homogeneity, significance of the coefficient, and adequacy tested by the Fisher test.

The experiment design and the optimization parameters were based on the test results obtained in [27-31].

The significance of the equation coefficients was based on Student’s t-test. Data was processed statistically in Mathcad, which ultimately produced 30 regression equations (2nd degree polynomials) for centrifugated and vibration-centrifugated concrete.

The equations were interpreted and transformed into calculational equations for finding the differential properties of variatropic section layers in centrifugated and vibration-centrifugated concrete.

The proposed calculational equations are presented in Table 3 and Table 4.

**Table 3.** Calculational equations for finding the differential properties of variatropic section layers in centrifugated concrete.

| Concrete properties | Centrifugated concrete |
|---------------------|-------------------------|
|                     | Written equation        |
| **Inner layer**     |                         |
| Axial compressive strength, MPa | \( R_{l}=23.25+0.877 l-1.446 \omega-1.618 l \omega-1.047 l^2+0.345 \omega^2 \) |
| Axial tensile strength, MPa | \( R_{t}=3.852+0.767 l-0.204 \omega-0.892 l \omega-0.311 l^2-0.189 \omega^2 \) |
| Ultimate axial compressive strain, mm/m | \( \varepsilon_{ul}=3.247-1.245 l+0.027 \omega-0.417 l \omega+0.006 l^2-0.032 \omega^2 \) |
| Ultimate axial tensile strain, mm/m | \( \varepsilon_{ut}=2.679-0.788 l-0.215 \omega-0.509 l \omega-0.34 l^2-0.056 \omega^2 \) |
| Elastic modulus, MPa | \( E_l=E_{lt}=21.32+4.86 l-0.639 \omega+0.299 l \omega+1.168 l^2-0.38 \omega^2 \) |
| **Middle layer**    |                         |
| Axial compressive strength, MPa | \( R_{l}=27.54+1.147 l-1.226 \omega-1.394 l \omega-0.899 l^2+0.675 \omega^2 \) |
| Axial tensile strength, MPa | \( R_{t}=4.356+1.024 l-0.051 \omega-0.567 l \omega-0.181 l^2-0.137 \omega^2 \) |
| Ultimate axial compressive strain, mm/m | \( \varepsilon_{ul}=4.341-1.031 l+7.29 \cdot 10^{-3} \omega-0.203 l \omega+0.038 l^2+0.01 \omega^2 \) |
| Ultimate axial tensile strain, mm/m | \( \varepsilon_{ut}=3.021-0.532 l-0.015 \omega-0.353 l \omega-0.01 l^2+0.001 \omega^2 \) |
| Elastic modulus, MPa | \( E_l=E_{lt}=26.507+5.67 l-0.442 \omega+0.572 l \omega+1.314 l^2-0.05 \omega^2 \) |
| **Outer layer**     |                         |
| Axial compressive strength, MPa | \( R_{l}=35.893+3.166 l+1.287 \omega+0.812 l \omega-0.015 l^2+1.781 \omega^2 \) |
| Axial tensile strength, MPa | \( R_{t}=6.184+1.867 l-0.83 \omega-0.336 l \omega+0.452 l^2-0.06 \omega^2 \) |
| Ultimate axial compressive strain, mm/m | \( \varepsilon_{ul}=5.31-0.543 l+1.456 \omega+1.002 l \omega+0.569 l^2+0.59 \omega^2 \) |
| Elastic modulus, MPa | \( E_l=E_{lt}=2.886+1.592 l-0.273 \omega+0.987 l \omega+0.256 l^2+0.043 \omega^2 \) |
The findings of this study are as follows.

**Summary**

The paper experimentally and theoretically substantiates that different layers of centrifugated and vibration-centrifugated concrete are exposed to varying-in-magnitude inertia forces resulting in sectional (depthwise) variatropy and differentiation of strength- and strain-related properties.

It shows that the key affecting factors are centrifugal and centripetal forces, which are functions of the distance to the center of rotation and angular rotation speed.

To evaluate the differential properties of centrifugated and vibration-centrifugated properties, the paper proposes versatile calculational dependencies, where the concrete properties (strength, ultimate strain, and elastic modulus in compression and tension, or their absolute or relative increments) serve as the functions, while the distance to the center and the angular speed of rotation serve as the arguments.

The analytical dependencies for finding the differential strength- and strain-related properties of centrifugated and vibration-centrifugated concrete enable adjustment for the concrete properties that vary depthwise for a more complete utilization of the load-carrying capacity of structures.

| Concrete properties | Vibration-centrifugated concrete |
|---------------------|---------------------------------|
| Axial compressive strength, MPa; | $R_b = 32.31 + 8.774 l - 0.450 \omega + 0.67 l \omega + 0.308 l^2 + 0.809 \omega^2$ |
| Axial tensile strength, MPa | $R_{\sigma} = 3.882 + 0.459 l - 0.043 \omega - 0.49 l \omega + 1.282 l^2 + 1.511 \omega^2$ |
| Ultimate axial compressive strain, mm/m | $\varepsilon_{\delta b} = 3.214 - 1.412 l + 0.037 \omega - 0.82 l \omega - 1.07 l^2 + 0.086 \omega^2$ |
| Ultimate axial tensile strain, mm/m | $\varepsilon_{\delta b} = 1.95 - 0.825 l + 0.007 \omega + 1.16 l \omega + 0.778 l^2 + 0.534 \omega^2$ |
| Elastic modulus, MPa | $E_b = E_{\delta b} = 29.111 + 6.49 l + 1.65 \omega - 0.399 l \omega + 1.678 l^2 + 0.68 \omega^2$ |

| Written equation |
| Inner layer |
| Axial compressive strength, MPa; | $R_b = 32.31 + 8.774 l - 0.450 \omega + 0.67 l \omega + 0.308 l^2 + 0.809 \omega^2$ |
| Axial tensile strength, MPa | $R_{\sigma} = 3.882 + 0.459 l - 0.043 \omega - 0.49 l \omega + 1.282 l^2 + 1.511 \omega^2$ |
| Ultimate axial compressive strain, mm/m | $\varepsilon_{\delta b} = 3.214 - 1.412 l + 0.037 \omega - 0.82 l \omega - 1.07 l^2 + 0.086 \omega^2$ |
| Ultimate axial tensile strain, mm/m | $\varepsilon_{\delta b} = 1.95 - 0.825 l + 0.007 \omega + 1.16 l \omega + 0.778 l^2 + 0.534 \omega^2$ |
| Elastic modulus, MPa | $E_b = E_{\delta b} = 31.11 + 7.626 l - 0.117 l \omega - 1.12 l \omega - 5.134 l^2 - 1.223 \omega^2$ |

| Middle layer |
| Axial compressive strength, MPa; | $R_b = 38.467 + 10.506 l - 0.231 l \omega + 0.7 l \omega + 0.558 l^2 - 0.05 \omega^2$ |
| Axial tensile strength, MPa | $R_{\sigma} = 5.089 + 0.768 l - 0.02 l \omega - 0.35 l \omega - 0.04 l^2 + 0.046 l^2 + 0.005 \omega^2$ |
| Ultimate axial compressive strain, mm/m | $\varepsilon_{\delta b} = 3.983 - 1.202 l + 0.075 \omega - 0.141 l \omega + 0.04 l^2 + 0.005 \omega^2$ |
| Ultimate axial tensile strain, mm/m | $\varepsilon_{\delta b} = 2.48 - 0.637 l + 0.017 \omega - 0.052 l \omega - 0.029 l^2 + 0.083 l^2 + 0.005 \omega^2$ |
| Elastic modulus, MPa | $E_b = E_{\delta b} = 38.129 + 9.321 l - 0.51 l \omega + 0.089 l \omega - 3.713 l^2 + 0.638 \omega^2$ |

| Outer layer |
| Axial compressive strength, MPa; | $R_b = 48.45 + 13.77 l + 0.65 l \omega + 1.44 l \omega + 2.014 l^2 + 0.45 \omega^2$ |
| Axial tensile strength, MPa | $R_{\sigma} = 8.132 + 1.56 l + 1.09 l \omega + 0.321 l \omega + 0.563 l^2 + 0.12 \omega^2$ |
| Ultimate axial compressive strain, mm/m | $\varepsilon_{\delta b} = 5.451 + 0.85 l + 1.345 l \omega + 0.47 l \omega + 1.047 l^2 + 0.06 \omega^2$ |
| Ultimate axial tensile strain, mm/m | $\varepsilon_{\delta b} = 5.98 + 1.12 l - 0.003 l \omega - 0.008 l \omega + 0.844 l^2 + 0.141 \omega^2$ |
| Elastic modulus, MPa | $E_b = E_{\delta b} = 43.18 + 14.23 l + 1.231 l \omega + 1.157 l \omega - 0.09 l^2 + 1.98 \omega^2$ |
References

[1] Filatov V B, Suvorov A A 2016 Procedia Engineering 153 144–150.
[2] Panfilov D A, Pischulev A A, Romanchkov V V 2016 Procedia Engineering 153 531-536.
[3] Pol'skoi P P, Mailyan D R, Georgiev S V 2014 Engineering Herald of the Don 4.
[4] Radaikin O V 2019 Bulletin of Belgorod State Technological University named after V. G. Shukhov 10 29–39.
[5] Kholodnyak M G, Stel'makh S A, Shcherban' E M, Tret'yakov D A, Dao V N, Zaikin V I 2018 Bulletin of the Eurasian Science 10 6.
[6] Maruyama I, Lura P 2019 Cement and Concrete Research 123 105770.
[7] Kim J-J, Yoo D-Y 2019 Cement and Concrete Composites 103 213–223.
[8] Li K, Li L 2019 Cement and Concrete Research 124 105811.
[9] Kirthika S K, Singh S K 2020 Construction and Building Materials 250 118850.
[10] Hameed M, Maula B, Bahnam Q 2019 International Review of Civil Engineering 10 6 17061.
[11] Alexander M, Beushausen H 2019 Cement and Concrete Research 122 17–29.
[12] Geiker M R, Michel A, Stang H, Lepech M D 2019 Cement and Concrete Research 122 189–195.
[13] Khalaf M A, Cheah C, Ramli M 2019 Construction and Building Materials 215 73–89.
[14] Mailyan L R, Stel’makh S A, Shcherban’ E M, Kholodnyak M G 2020 Russian Journal of Building Construction and Architecture 1 (45) 6–14.
[15] Stel'makh S A, Shcherban' E M, Kholodnyak M G 2019 IOP Conf. Ser.: Mater. Sci. Eng. 687 022008.
[16] Stel’makh S A, Shcherban’ E M, Shuyskiy A I, Nazhuev M P 2018 Materials Science Forum 931 502–507.
[17] Stel'makh S A, Shcherban E M, Zholobova O A 2018 IOP Conf. Ser.: Mater. Sci. Eng. 463 022056.
[18] Shuyskiy A I, Stel'makh S A, Shcherban' E M, Kholodnyak M G 2018 Materials Science Forum 931 508–514.