We present an exact diagonalization study of the quantum Heisenberg antiferromagnet on the fractal Sierpiński gasket for spin quantum numbers $s = \frac{1}{2}$, $s = 1$ and $s = \frac{3}{2}$. Since the fractal dimension of the Sierpiński gasket is between one and two we compare the results with corresponding data of one- and two-dimensional systems. By analyzing the ground-state energy, the low-lying spectrum, the spin-spin correlation and the low-temperature thermodynamics we find arguments, that the Heisenberg antiferromagnet on the Sierpiński gasket is probably disordered not only in the extreme quantum case $s = \frac{1}{2}$ but also for $s = 1$ and $s = \frac{3}{2}$. Moreover, in contrast to the one-dimensional chain we do not find a distinct behavior between the half-integer and integer-spin Heisenberg models on the Sierpiński gasket. We conclude that magnetic disorder may appear due to the interplay of frustration and strong quantum fluctuations in this spin system with spatial dimension between one and two.

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I. INTRODUCTION

For one-dimensional (1d) and two-dimensional (2d) quantum Heisenberg antiferromagnets the question about the existence of magnetic long-range order (LRO) in the ground state (GS) is of great importance. Since only in some special cases analytical solutions exist the numerical methods like quantum Monte Carlo and exact diagonalization play an important role for this many-body problem. For the 1d linear chain with $s = \frac{1}{2}$ the famous Bethe-Ansatz \[ \text{Equation} \] provides the analytic solution: In the GS the spin-spin correlation exhibits a power-law decay to zero and consequently there is no magnetic LRO. For the 1d Heisenberg antiferromagnet (HAF) with higher spin quantum number $s > \frac{1}{2}$ no analytic solutions are available, but comprehensive studies have shown that the ground state exhibits no LRO. A special aspect of the antiferromagnetic chain is the fundamental difference between half-integer and integer spin HAF as conjectured by Haldane \[ \text{Equation} \]. While for the half-integer spin chain ($s = \frac{1}{2}, \frac{3}{2}, \ldots$) the spin-spin correlation in the GS decays to zero according to a power-law and there is no gap between the singlet GS and the first triplet excitation (i.e. the GS is critical), for the integer spin chain ($s = 1, 2, \ldots$) the spin-spin correlation in the GS exponentially decays to zero and there is a finite excitation gap, the so-called spin gap (i.e. the GS is disordered). In contrast to the lack of Néel LRO for 1d antiferromagnets, most of the 2d antiferromagnets show Néel LRO in the GS. This holds for the square lattice, the honeycomb lattice as well as for the frustrated triangular lattice. All these systems show a Néel-like magnetic order resembling the classical Néel state but with a reduced sublattice magnetization due to the quantum fluctuations. The general features of the magnetic ordering in these 2d antiferromagnets does not depend on the spin quantum number $s$, i.e. no qualitative difference between half-integer and integer spin has been observed.

In this paper we investigate the quantum HAF on the Sierpiński gasket, a fractal self-similar structure with geometrical inhomogeneities (see, e.g. \[ \text{Equation} \]). The first motivation to investigate this problem is due to the reduced fractal

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lattice dimension \( d = \ln(3) / \ln(2) \) of the Sierpiński gasket. Consequently, the question arises whether the quantum fluctuations in a system of dimension between 1 and 2 are strong enough to destroy the antiferromagnetic LRO. For the \( s = \frac{1}{2} \) HAF this problem has been considered recently [3, 6]. It has been suggested that the quantum fluctuations indeed destroy the antiferromagnetic LRO and the GS is disordered. However, from these considerations for \( s = \frac{1}{2} \) a second interesting question arises: What happens if the spin quantum number \( s \) is enlarged? Is there a basic difference between half-integer and integer spin HAF like in one dimension or is the influence of the spin quantum number on the magnetic ordering of less importance like in two dimensions? The investigation of this question is the main issue of the present paper.

The HAF on the Sierpiński gasket represents a complex quantum many-body problem with a nontranslational invariant arrangement of sites and with magnetic frustration. Hence, some of the standard methods like spin-wave theory (which suffers from the lack of translational invariance) or quantum Monte-Carlo (which suffers from the minus sign problem due to frustration) are not applicable. However, the exact diagonalization of small finite systems seems to be particularly appropriate, since it allows a direct comparison of the HAF with different spin quantum numbers. This method has been applied successfully to various problems in quantum spin magnetism, e.g. the frustrated ferrimagnet in 1d [3], the kagomé lattice in 2d [3, 11], or the HAF on the body-centered cubic lattice in 3d [12]. Though due to the limitations to small size conclusions for the thermodynamic limit have to be drawn with particular care, even the problem of magnetic LRO in the 2d HAF has been addressed successfully (see, e.g. [13–15]).

In what follows we will discuss the GS energy, the low-energy spectrum, the spin-spin correlation in the GS as well as the low-temperature specific heat for \( s = \frac{1}{2}, 1, \frac{3}{2} \) for the HAF on the Sierpiński gasket mainly for the gasket with \( N = 15 \) sites shown in Fig.1. In accordance with earlier studies on other frustrated spin systems [14, 16, 17] our investigations of the \( s = \frac{1}{2} \) case has illustrated [3, 6] that despite of the smallness of the system under consideration even the Sierpiński gasket with \( N = 15 \) seems to cover some important features of the HAF on the infinite gasket namely the fast (presumably exponential) decay of the spin-spin correlation and the peculiar low-temperature specific heat behavior.

**II. THE MODEL**

We consider the usual HAF

\[
\hat{H} = J \sum_{(i,j)} \vec{s}_i \cdot \vec{s}_j
\]

(1)

with antiferromagnetic exchange \( J = 1 \) between nearest neighbors on the Sierpiński gasket. The parameters of the system are the spin quantum number \((\vec{s}_i)^2 = s(s+1)(s = \frac{1}{2}, 1, \frac{3}{2})\) and the size of the system \(N\). Since the Heisenberg Hamiltonian commutes with the square of total spin \( \hat{S}^2 \), \( \hat{S} = \sum_i \vec{s}_i \), each eigenstate of \( \hat{H} \) belongs to a certain subspace of the Hilbert space with fixed quantum number \( S \) of the total spin \((\hat{S}^2 = S(S+1))\). To calculate the GS and low-lying excitations of \( \hat{H} \) for \( N = 15 \) sites we use the Lanczos technique.

The Hausdorff dimension of this fractal lattice is \( d_f = \frac{\log(N)}{\log(3)} \approx 1.58 \). The number of spins on the gasket is given by \( N = \frac{1}{3}(3^n + 3) \) with \( n = 1, 2, 3, \ldots \). The ground state of the classical model (spin quantum number \( s = \infty \)), where the spins \( \vec{s}_i \) effectively can be considered as classical vectors, is the same as for the HAF on the triangular lattice, namely a Néel state with 3 sublattices and an angle of 120° between neighboring spins. This classical ground state is illustrated in Fig.1.
FIG. 1. The classical ground state configuration of the HAF on the Sierpiński gasket with $N=15$: 3-sublattices A (circles), B (squares) and C (triangles) with an angle of $120^\circ$ between neighboring spins.

III. GROUND STATE ENERGY AND LOW-LYING EXCITATIONS

Since the low-lying excitations of a spin system contain signatures of the magnetic ordering \[18–20\] we discuss in this section the low-energy spectrum of finite systems. First we compare in Table I the GS energy per bond $e_b = E/N_b$ (i.e. the averaged nearest-neighbor spin-spin correlation) of the linear chain and of some 2d lattices with the GS energy of the Sierpiński gasket.

| d | Lattice            | N  | z | Frustration | $s = \frac{1}{2}$ | $s = 1$   | $s = \frac{3}{2}$ |
|---|--------------------|----|---|-------------|-------------------|-----------|-------------------|
| 1 | linear chain       | 12 | 2 | no          | -0.448949         | -1.417120 | -2.844275         |
|   | Sierpinski gasket  | 15 | 4 | yes         | -0.231181         | -0.733279 | -1.466517         |
| 2 | honeycomb lattice  | 12 | 3 | no          | -0.385048         | -1.264919 | -2.643583         |
|   | square lattice     | 10 | 4 | no          | -0.365003         | -1.227931 | -2.590695         |
|   | kagomé lattice     | 12 | 4 | yes         | -0.226869         | -0.734203 | -1.462779         |
|   | triangular lattice | 12 | 6 | yes         | -0.203443         | -0.653036 | -1.354524         |

TABLE I. Ground state energies per bond $e_b$ of different small lattices vs. spin quantum number (d - dimension, N - number of spins, z - number of nearest neighbors).
We see a clear distinction between frustrated and non-frustrated systems. For all spin quantum numbers $s$ considered the absolute value of $e_b$ for non-frustrated systems is about twice as large as for frustrated ones. We notice, that for $s \to \infty$ we have exactly a factor of two between the GS energy per bond of frustrated and non-frustrated systems, which corresponds to the $120^\circ$ angle between neighboring spins. We conclude that the averaged nearest-neighbor spin-spin correlation of the HAF on the Sierpiński gasket is rather close to that of the triangular and kagomé lattice.

Next we consider the low-lying excitations of the HAF on the Sierpiński gasket ($N = 15$) classified by the total spin $S$.

| $s = \frac{1}{2}$ | $s = 1$ | $s = \frac{3}{2}$ |
|------------------|---------|------------------|
| $S$              | $e_b$   | $D$              | $S$              | $e_b$   | $D$              | $S$              | $e_b$   | $D$              |
| $\frac{1}{2}$   | -0.231181 | 2               | $\frac{1}{2}$   | -1.466517 | 2               |
|                  | -0.229461 | 1               |                  | -1.454860 | 1               |
|                  | -0.229424 | 2               |                  | -1.452973 | 2               |
|                  | -0.222356 | 2               |                  | -1.450302 | 2               |
|                  | -0.221141 | 1               |                  |          |                 |
|                  | -0.220616 | 2               |                  |          |                 |
|                  | -0.219649 | 1               |                  |          |                 |
|                  | -0.219604 | 1               |                  |          |                 |
|                  |          |                 |                  |          |                 |
| $\frac{1}{2}$   | -0.219233 | 1               | $\frac{1}{2}$   | -1.463011 | 2               |
|                  | -0.216872 | 1               |                  | -1.455737 | 1               |
|                  | -0.214222 | 2               |                  | -1.451512 | 1               |
|                  | -0.212654 | 1               |                  | -1.448938 | 2               |
|                  | -0.212386 | 2               |                  | -1.446856 | 2               |
|                  | -0.209587 | 1               |                  | -1.44607 | 2               |
|                  |          |                 |                  |          |                 |
| $\frac{1}{2}$   | -0.186349 | 1               | $\frac{1}{2}$   | -1.446507 | 2               |
|                  | -0.185978 | 2               |                  | -1.433347 | 1               |
|                  | -0.183977 | 1               |                  | -1.429990 | 2               |
|                  | -0.183410 | 2               |                  | -1.429420 | 1               |
|                  | -0.179908 | 2               |                  | -1.423400 | 1               |
|                  | -0.179669 | 1               |                  | -1.422074 | 1               |
|                  |          |                 |                  |          |                 |
| $\frac{1}{2}$   | -0.136306 | 2               | $\frac{1}{2}$   | -1.419007 | 2               |
|                  | -0.132661 | 1               |                  | -1.402997 | 2               |
|                  | -0.127097 | 2               |                  | -1.396418 | 1               |
|                  | -0.126221 | 1               |                  | -1.39402 | 2               |
|                  | -0.118484 | 1               |                  | -1.39154 | 1               |
|                  | -0.115478 | 2               |                  | -1.39120 | 1               |

TABLE II. Lowest part of the energy spectra for the N=15 Sierpinski gasket with $s = \frac{1}{2}, 1, \frac{3}{2}$: total spin $S$, energy per bond $e_b$ and degeneracy $D$ (the trivial Kramers degeneracy due to the z-component of the total spin is not taken into account).
For $s = \frac{1}{2}$ the complete spectrum can be calculated (see [1]), but for $s = 1$ and $s = \frac{3}{2}$ the size of the matrix which has to be diagonalized is much larger and only the lowest eigen values in each subspace of total spin $S$ can be calculated. In Table [1] we present the low-lying energies and their degeneracy for the lowest four values of $S$. In general we present six eigen values in each subspace of $S$, however with some exceptions:

- For $s = \frac{1}{2}$ and $S = 1/2$ we give all eigen values below the first eigen value with $S = 3/2$.
- For $s = 1$ we need already 1.7 million basis states to calculate the singlet eigen states ($S = 0$). Using the Lanczos procedure with an extra projection onto the singlet subspace, we were able to find the lowest 16 singlet eigen states. In Table [1] we give only the lowest 6 eigen values, but in the following calculation of low-temperature thermodynamics all 16 eigen values will be used and all of them are plotted in Fig.2.
- For $s = \frac{3}{2}$ we need already 97 million states in the $S = 1/2$ subspace and large amount of computer power is necessary to calculate the lowest 4 eigen states in this subspace. In the subspaces with higher $S$ we give the lowest 6 eigen values.

Going back to a suggestion of Anderson [21], which has been recently picked-up in several papers [22,18–20] the low-energy part of the spectrum can be used to discuss the possibility of Néel ordering in the GS. The lowest levels in each subspace up to $S \sim \sqrt{N}$ should be described by an effective Hamiltonian.

$$H_{\text{eff}} = E_0 + \frac{K}{N} S^2.$$  \hfill (2)

where $K$ is a constant for a given lattice.

These so-called quasi degenerate joint states collapse to a symmetry broken Néel state in the thermodynamic limit. Furthermore, significantly above the family of quasi degenerate joint states a second family of levels describing the magnon excitations are typical for a HAF with Néel ordering.

Indeed, a linear relation between the lowest eigen values $E_{\text{min}}(S)$ and $S(S+1)$ and a similar relation for the family of magnon excitations has been observed in good approximation for the HAF on the 2d square lattice [22,23] as well as for the triangular-lattice HAF [19]. The strong deviation from this linear relation has been used as one argument that the HAF on the 2d kagome lattice has a disordered ground state [4]. A similar argumentation has been used already in [7] for the $s = \frac{1}{2}$ HAF on the Sierpiński gasket.

In Fig. 2 we compare the spectra of the HAF on the $N = 15$ Sierpiński gasket with different spin quantum number $s$.

FIG. 2. The low-lying spectra of the Sierpiński gasket with $N = 15$ vs. $S(S+1)$. The straight line connects the lowest eigen values in every subspace of $S$. Non-degenerated eigen values are denoted by $+$, degenerated eigen values by $\times$.  

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From Fig. it is obvious that for the Sierpiński gasket there is a significant deviation from the linear relation between $E_{\text{min}}(S)$ and $S(S + 1)$, well seen for $s = \frac{1}{2}$ and for $s = \frac{3}{2}$, less pronounced for $s = 1$. This deviation is much larger than in corresponding Néel ordered 2d systems with similar size (e.g. the square lattice).

Let us now discuss the excitations above the lowest states (see Fig.2 and Table I). For $s = \frac{1}{2}$ there is a number of states with $S = 1/2$ below the first state with $S = 3/2$. In a Néel ordered system usually the first excitation is a state where the quantum number $S$ of the total spin is enlarged by one (i.e. $S = S_{\text{GS}} + 1$), whereas the excitations with the same total spin as the GS ($S = S_{\text{GS}}$) are quite far above the GS since they neither are needed for the symmetry broken Néel state nor for the magnons. Hence the existence of low-lying excitations with $S = S_{\text{GS}}$ are considered as another indication for the lack of Néel ordering [24,20]. This property of the spectrum has been observed for the kagomé lattice, too [24].

For the HAF on the Sierpiński gasket with higher spin quantum number $s$ the situation seems to be different: For $s = 1$ only the ground state and one singlet with $S = 0$ exist below the first triplet excitation with $S = 1$. This second singlet is only slightly below the first triplet excitation. In this context we note the recent paper of Hida on the kagomé lattice with $s = 1$ [11]. The behavior found there seems to be very similar to ours concerning the number of the lowest singlets and the energy gap of these excitations. For $s = \frac{3}{2}$ there is no other state between the ground state with $S = 1/2$ and the first excitation with $S = 3/2$.

To be more specific concerning the deviation of the lowest excitations from the effective model (2) we have calculated the mean square deviation $\delta$ defined by

$$\delta = \sqrt{\sum_{S=S_{\text{GS}}}^{S_0} (e_i(S) - [a + b S(S + 1)])^2}$$

where $a$ and $b$ are determined to minimize $\delta$ and $e_i(S)$ are the exact eigen values divided by the corresponding GS energy. Since the linear-chain HAF has a critical GS (power-law decay of correlations to zero) for half-integer $s$ and a disordered GS (exponential decay of correlations to zero) for integer $s$ its corresponding $\delta_{\text{LC}}$ might be used as a criterion for the possibility of a disordered GS in other lattices. Since $\delta$ is influenced by several factors as system size, even or odd number of spins or possible Néel ordering with two or three sublattices we bear in mind that this criterion is rough.

We present in Table III the quotient $\delta/\delta_{\text{LC}}$ for the lattices from Table I.

| Lattice               | N  | $s = \frac{1}{2}$ | $s = 1$ | $s = \frac{3}{2}$ |
|-----------------------|----|-----------------|--------|-------------------|
| Sierpiński gasket     | 15 | 1.870           | 0.821  | 12.981            |
| honeycomb lattice     | 12 | 0.058           | 0.015  | 0.053             |
| square lattice        | 10 | 0.078           | 0.017  | 0.026             |
| kagomé lattice        | 12 | 3.408           | 2.451  | 16.038            |
| triangular lattice    | 12 | 1.543           | 0.147  | 0.239             |

TABLE III. Mean square deviation $\delta$ behavior (cf. eq.(3)) scaled by the corresponding value $\delta_{\text{LC}}$ of the linear chain with $N = 12$.

For $s = \frac{1}{2}$ the kagomé lattice has the largest $\delta$ but the next in the row is already the Sierpiński gasket. Both values are larger than $\delta_{\text{LC}}$. The $\delta$ of the triangular lattice is smaller than that of the kagomé and the Sierpiński gasket but it is still larger than $\delta_{\text{LC}}$, although the GS of the HAF on the triangular lattice is Néel ordered. For the honeycomb and the square lattice (both have Néel ordered GS) $\delta$ is more than an order of magnitude smaller than in the other lattices. With increasing spin quantum number $s$ to $s = 1$ the situation is slightly changed. The linear chain (the GS is now disordered not critical) has a slightly higher value of $\delta$ than the Sierpiński gasket. The kagomé lattice is again the lattice with the largest $\delta$, whereas all the other lattices have a $\delta$ which is significantly smaller than that of the linear chain. For $s = \frac{3}{2}$ one can see that both the kagomé lattice and the Sierpiński gasket have a $\delta$ which is one order of magnitude larger than that of the linear chain, whereas the $\delta$ for the other lattices is clearly below $\delta_{\text{LC}}$. These results would favor a non-Néel ordered GS for the Sierpiński gasket for $s = \frac{1}{2}$, $s = 1$ and $s = \frac{3}{2}$ and a close similarity to the 2d kagomé lattice.
IV. SPIN-SPIN CORRELATIONS

The spin-spin correlation \( \langle \vec{s}_i \cdot \vec{s}_j \rangle \) as function of separation \( r = |\vec{R}_i - \vec{R}_j| \) yields a direct information on the magnetic ordering. To discuss the possibility of Néel-like ordering in the GS the spin-spin correlation between the spins within one sublattice (cf. e.g. all • in Fig.1) should be considered. Because the Sierpiński gasket has no translational symmetry, the spin-spin correlation was averaged over all pairs of spins with one and the same separation \( r \). Furthermore the degeneracy of the GS for \( s = \frac{1}{2} \) and \( s = \frac{3}{2} \) was taken into account by an additional averaging over the two degenerated states.

In Fig. 3 the spin-spin correlation (scaled by the on-site spin-spin correlation \( \langle \vec{s}_i \cdot \vec{s}_i \rangle = s(s + 1) \)) is shown in a semilogarithmic plot.

![Graph showing spin-spin correlation vs. distance for the Sierpiński gasket with N=15.](image)

**FIG. 3.** The scaled spin-spin correlation \( \langle \vec{s}_i \cdot \vec{s}_j \rangle \) vs. distance \( r = |\vec{R}_i - \vec{R}_j| \) for the Sierpiński gasket with N=15.

The spin-spin correlation shows a strong decay with increasing distance. The decay is strongest for \( s = \frac{1}{2} \) but we do not see a qualitative change in the behavior for \( s = 1 \) and \( s = \frac{3}{2} \). Though, the presented correlations represent only a short length scale the linear relation in the semilogarithmic plot suggests an exponential decay to zero. We notice, that e.g. for the square lattice one has a significant deviation from this behavior [5].

As mentioned above the Sierpiński gasket is not translational invariant and therefore, in principle, the spin-spin correlation \( \langle \vec{s}_i \cdot \vec{s}_j \rangle \) may depend not only on distance \( r = |\vec{R}_i - \vec{R}_j| \) but on both site indices \( i \) and \( j \). To illustrate this in detail we show in Fig. 4 the value of the nearest-neighbor spin-spin correlation scaled by the on-site spin-spin correlation \( \langle \vec{s}_i \cdot \vec{s}_i \rangle = s(s + 1) \). The width of the line corresponds to the strength of the correlation. Indeed, we see a strong variation in the nearest-neighbor correlations from bond to bond for all three values of \( s \). This is a pure quantum effect since for the classical GS (see Fig.1) all nearest-neighbor correlations are the same. We conclude, that the interplay between quantum fluctuations and structural inhomogeneities is important even in the model with the highest spin quantum number \( s = \frac{3}{2} \).
FIG. 4. The scaled nearest-neighbor spin-spin correlation in the ground state for the Sierpiński gasket with N=15.

For $s = \frac{1}{2}$ it is interesting to note that a kind of plaquette ordering occurs (cf. the correlations between lattice sites 1-2-5-3-1). This kind of ordering does not appear in the lattices with higher spin quantum number $s$.

V. LOW-TEMPERATURE THERMODYNAMICS

As argued in [25, 10, 20] the low-temperature thermodynamics of a quantum spin system on a particular lattice, especially the specific heat and the entropy can be related to the magnetic ordering of the system in the ground state. For the $s = \frac{1}{2}$ HAF on the Sierpiński gasket such a relation between low-temperature specific heat and magnetic order had already been noticed [6]. In [11] similar considerations on the finite kagomé lattice has been reported and the authors conclude that the kagomé lattice provides an example of a new type of spin liquid with a non-magnetic excitation continuum adjacent to the ground state. Following these considerations we will examine the Sierpiński gasket with different spin quantum numbers $s$. We will start with a comparison of exactly calculated low-temperature thermodynamics of $s = \frac{1}{2}$ systems. We compare a finite square lattice (N=10) with the Sierpiński gasket with N=15.
First we observe in both systems one peak in the specific heat slightly above $k_B T = 0.5$. This peak is related to the energy scale beyond which short-range correlations start to be broken. But in the Sierpiński gasket there are two additional peaks below $k_B T = 0.2$ belonging to low-energy scales: the first one at $k_B T = 0.018$ can be attributed to the low-lying $S = 1/2$ states (see right panel of Fig. 5) and its position and even its existence may be strongly influenced by finite-size effects, but the second peak at $k_B T = 0.16$ can be related to the excitations with higher total spin $S > 1/2$ (see right panel of Fig. 5) and in particular to the magnitude of the spin gap $\Delta = E(S_{\text{min}} + 1) - E(S_{\text{min}})$.

Comparing the entropy we also observe a different behavior. In the square lattice there is about 85% of the total entropy in the high-temperature peak from $k_B T = 0.2 - \infty$. The total entropy in the Sierpiński gasket is quite different from that. First we find a non-zero entropy at $k_B T = 0$, which corresponds to the degeneracy of the GS. Second we observe that already 45% of the total entropy belong to the first and second peak in the specific heat for $k_B T = 0 - 0.2$. Only the remaining 55% of the total entropy is contained in the high-temperature peak of the specific heat. This behavior of the Sierpiński gasket is quite similar to the observations for the kagomé lattice [10].

It should also be noted that $C_v$ does not decrease exponentially below the first excitation with $S = 3/2$ (i.e. within the spin gap). This behavior can be seen in the left panel of Fig. 5. Below the peak at $k_B T = 0.16$ the specific heat is well fitted by a $T^2$ law in the range of $k_B T = 0.05 - 0.09$. This non-exponential behavior of $C_v$ is related to the presence of low-lying $S = 1/2$-states within the spin gap. However, as can be seen in the right panel of Fig. 5 to some extent higher spin channels do contribute also to $C_v$ in this range.
FIG. 6. Low-temperature specific heat $C_v$ of the $s = \frac{1}{2}$ Sierpiński gasket with $N=15$
left panel: $C_v$ is well fitted by a $T^2$ law (light line) in the range from $k_B T = 0.05 - 0.09$.
right panel: Contribution of the doublet states with $S = \frac{1}{2}$ (dotted line) to the total specific heat.

The observed $T^2$ behavior of the specific heat and the additional low-temperature maximum are very similar to
the $s = \frac{1}{2}$ kagomé lattice \[10\]. It suggests a close relation between the low-temperature physics of the HAF on the
Sierpiński gasket and on the kagomé lattice. Assuming the same scenario as for the kagomé lattice the spin gap (and
hence the second peak in $C_v$) probably survives in the thermodynamic limit, but this spin gap is filled by a continuum
of states having the same total spin $S$ as the GS (and hence the first peak in $C_v$ at $k_B T = 0.018$ disappears for
$N \to \infty$). As recently pointed out by C.Lhuillier and coworkers \[10,20\] for this kind of low-energy spectrum, we have
(in contrast to the power-law $T^2$ decay of $C_v$) a thermally activated susceptibility which decays exponentially within
the spin gap.

For the Sierpiński gasket with $N=6$ the complete thermodynamics is numerically accessible and for a comparison
we show in the left panel of Fig. 6 the specific heat data for this system for $s = \frac{1}{2}$, $s = 1$ and $s = \frac{3}{2}$. For the Sierpiński
gasket with $N=15$ only the data for $s = \frac{1}{2}$ can be calculated exactly. However, for $s = 1$ we are able to obtain the
specific heat for low temperatures $k_B T < 0.1$ by using all computed low-lying excitations. The two data sets for $N=15$
are shown in the right panel of Fig. 6.
From the figure it can be seen that for the system with \( N = 6 \) one observes one additional low-temperature peak in the specific heat. This property does not depend on the spin quantum number \( s \), only the height and the position of this first peak changes. These findings for \( N = 6 \) support our argumentation for the \( N = 15 \) system with \( s = \frac{1}{2} \). The lowest peak in \( C_v \) in this system at \( k_B T = 0.018 \) may be attributed to a finite-size effect, whereas the other additional peak at \( k_B T = 0.16 \) is a real physical effect.

The calculation of the same data for \( N = 15 \) is exact only for \( s = \frac{1}{2} \). For \( s = 1 \) we can calculate only a number of lowest energies (see section [III]) and therefore reliable results for the specific heat can be obtained for temperatures \( k_B T < 0.1 \) only. But this temperature range is sufficient to demonstrate that (i) there is no peak at very low temperature and no \( T^2 \) behavior (notice, that in difference to \( s = \frac{1}{2} \) there is no set of singlet states within the spin gap) and that (ii) the second low-temperature peak exists also in this case but it is shifted to lower temperatures (notice that the magnitude of the spin gap is smaller for \( s = 1 \) than for \( s = \frac{1}{2} \)). The high-temperature peak for \( s = 1 \) cannot be displayed, since the corresponding energy scale is beyond the temperatures we are able to consider.

For \( s = \frac{3}{2} \) we cannot give comparable results, the calculated low excitations can provide specific-heat data only for \( k_B T < 0.05 \). In this temperature range we did not observe any peculiarities in the specific heat therefore the data are not shown.

VI. CONCLUSION

The results of a numerical investigation of the ground state and low temperature thermodynamics of the Heisenberg antiferromagnet on the finite Sierpiński gasket with \( N = 15 \) are presented. These data enable us to draw conclusions with respect to the magnetic ordering of the spin system. This is done by comparing them with the corresponding data of the one-dimensional chain and the two-dimensional square, triangular, honeycomb and \textit{kagomé} lattices. For these systems the magnetic ordering is well understood. We argue that the quantum Heisenberg antiferromagnet on the Sierpiński gasket may remain disordered not only for \( s = \frac{1}{2} \), but also for \( s = 1 \) and \( s = \frac{3}{2} \). The data suggest that in contrast to the one-dimensional Heisenberg antiferromagnet there is no fundamental distinction between half-integer and integer spin quantum number \( s \). In conclusion we find a close correspondence between the physics of the quantum Heisenberg antiferromagnet on the Sierpiński gasket and on the \textit{kagomé} lattice. Assuming a short range magnetic order in both systems we relate this correspondence to the similar local geometry of both lattices with the same coordination number \( z = 4 \) and the frustration.
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[1] H. Bethe, Z. f. Phys. 71 (1931) 205.
[2] L. Hulthen, Arkiv Mat. Astr. Phys. 26a (1938) 11.
[3] F. Haldane, Phys. Lett. 93A (1983) 464.
[4] Y. Gefen, A. Aharony, B. Mandelbrot, S. Kirkpatrick, Phys. Rev. Lett. 47 (1981) 1171.
[5] P. Tomczak, A. Ferchmin, J. Richter, in: M.Suzuko, N.Kawashima (Eds.), Coherent Approaches to Fluctuations, World Scientific, 1996, p. 284.
[6] P. Tomczak, A. Ferchmin, J. Richter, Phys. Rev. B 54 (1) (1996) 395.
[7] A. Voigt, J. Richter, P. Tomczak, J. Magn. Magn. Mater. 183 (1998) 68–70.
[8] N. Ivanov, J. Richter, U. Schollwöck, Phys. Rev. B 58 (1998) 14456.
[9] P. Lecheminant, B. Bernu, C. Lhuillier, L. Pierre, P. Sindzingre, Phys. Rev. B 56 (5) (1997) 2521.
[10] P. Sindzingre, G. Misguich, C. Lhuillier, B. Bernu, L. Pierre, C. Waldtmann, H.-U. Everts, Phys. Rev. Lett. 84 (13) (2000) 2953–6.
[11] K. Hida, J. Phys. Soc. Jpn. 69 (2000) 311.
[12] D. Betts, J. Schulenburg, G. Stewart, J. Richter, J. Flynn, J. Phys. A: Math. Gen. 31 (1998) 7685.
[13] J. Oitmaa, D. Betts, Can.J.Phys. 56 (1978) 897.
[14] H. Schulz, T. Ziman, Europhys. Lett. 18 (1992) 355.
[15] R. Deutscher, H. Everts, Z. f. Phys. B 93 (1993) 77.
[16] E. Dagotto, A. Moreo, Phys. Rev. Lett. 63 (1989) 2148.
[17] J. Richter, Phys. Rev. B 47 (1993) 5794.
[18] B. Bernu, C. Lhuillier, L. Pierre, Phys. Rev. Lett. 69 (1992) 2500.
[19] B. Bernu, P. Lecheminant, C. Lhuillier, L. Pierre, Phys. Rev. B 50 (1994) 10048.
[20] C. Lhuillier, P. Sindzingre, J.-B. Fouet, preprint cond-mat/0009336.
[21] P.W. Anderson, Phys. Rev. B 86 (1952) 694.
[22] H. Neuberger, T. Ziman, Phys. Rev. B 39 (1989) 2608.
[23] J. Richter, N. Ivanov, K. Retzlaff, Proceedings of the MECO 1994 in acta physica slovaca 44 (4/5) (1994) 365.
[24] C. Waldtmann, H.-U. Evertz, B. Bernu, P. Sindzingre, C. Lhuillier, P. Lecheminant, L. Pierre, Eur. Phys. J. B 2 (1998) 501.
[25] G. Misguich, C. Lhuillier, B. Bernu, C. Waldtmann, Phys. Rev. B 60 (13) (1999) 1064.