Image denoising based on noise detection

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Abstract. Because of the noise points in the images, any operation of denoising would change the original information of non-noise pixel. A noise detection algorithm based on fractional calculus was proposed to denoise in this paper. Convolution of the image was made to gain direction gradient masks firstly. Then, the mean gray was calculated to obtain the gradient detection maps. Logical product was made to acquire noise position image next. Comparisons in the visual effect and evaluation parameters after processing, the results of experiment showed that the denoising algorithms based on noise were better than that of traditional methods in both subjective and objective aspects.

1. Introduction

Over the past few decades, the digital image processing has been applied more and more widely. In the process of image acquisition, conversion and transmission, an image is inevitable corrupted by noise, which will seriously attenuate the visual effect, and results in some difficulties for the subsequent segmentation, recognition, understanding and other high-level image processing [1-3]. Therefore, researches on image denoising play an important role in digital image processing. There are a lot of classic denoising algorithms, such as order-statistics, mean, median [4-6], lowpass, wiener, and other filtering algorithms, wavelet denoising method [7-9], denoising based on partial differential equations and traditional fractional integral denoising algorithm [10-13]. Though these algorithms can cut down the image noises to some extent, there is a common disadvantage that while removing noise, image edge and texture details information can usually be lost, leading to blurred vision.

In the paper, an algorithm based on the fractional calculus was proposed. To begin with, positions of the noise points were obtained by gradient images of initial image with noise in different directions. Then, from the visual effect on the subjective, traditional mean filtering, median filtering, fractional integral algorithm and these denoising algorithms based on noise points were compared. Finally, the value of evaluation parameters on the objective were calculated to indicate the practicability and feasibility of noisy detection algorithm as well as the effectiveness of denoising algorithms based noise.

2. Related theoretical guidance

Fractional Calculus came into being 300 years ago. It is generally believed that Fractional Calculus is the expansion and extension of integer order calculus [11-12]. Meanwhile, because of the genetic and memorial features of Fractional Calculus, it has become favored by many domestic scholars.
2.1. Definition of Fractional Calculus
For a function $y = f(t)$ whose argument $t$ is in the range $(a, b)$, the $v$-order fractional calculus of function $f(t)$ in $(a, b)$ is written as:

$$D^v f(t) = D^v f(t)$$

(1)

Where the order $v$ is fraction, the equation is called fractional differential while $v > 0$. When $v < 0$, the corresponding called fractional integral. There are three main definitions of fractional calculus: G-L, R-L and Caputo [13].

(1) Grünwald-Letnikov (G-L) Definition of Fractional Calculus
Deriving from the classical operation rules of integer order derivative for continuous function, G-L definition of fractional calculus comes into being after deducing its integer order form. The definition of fractional calculus is pretty suitable for signal processing. Since its operation can be converted to convolution form, G-L definition is more practical.

The fractional calculus of G-L definition can be expressed as:

$$D^v f(t) = \left[ \lim_{m \to 0} \left( \sum_{\tau = 0}^{\lfloor \frac{t}{mh} \rfloor} (-1)^{\lfloor \frac{t}{mh} \rfloor} f(t - mh) \right) \right]^{v-1}$$

(2)

(2) Riemann-Liouville (R-L) Definition of Fractional Calculus
From classical integer order derivative, for R-L definition, $n$-order differential of function or signal is calculated, followed by the $v$-order integral is operated. The complete form is expressed as follows:

$$D^v_{\tau} f(t) = \left\{ \begin{array}{ll} \frac{1}{\Gamma(n-v)} \int_{0}^{t} \frac{f(\xi)}{(t-\xi)^{v-n}} d\xi, & n-1 < v < n \\
\frac{d^n}{dt^n} f(t), & v = n \in N \end{array} \right.$$

(3)

(3) Caputo Definition of Fractional Calculus
Caputo definition is often used to solve the problems of initial boundary value of fractional differential equations. It is also an improved form based on the G-L and widely used in engineering fields.

Caputo fractional differential is defined as:

$$D^v_{\tau} f(t) = \frac{1}{\Gamma(v-n)} \int_{0}^{t} (t-\tau)^{v-n} f^{(n)}(\tau) d\tau, 0 \leq n-1 < v < n, n \in N$$

(4)

Integral form is:

$$D^v_{\tau} f(t) = \frac{1}{\Gamma(v)} \int_{0}^{t} \frac{f(t) d\tau}{(t-\tau)^{v-n}}, -1 < v < 0$$

(5)

R-L and Caputo definition are all improved from G-L definition. While the order is negative real or positive integers, they are equivalent. Otherwise, they are not. Appearance of R-L definition can simplify the calculation of the fractional derivative; Caputo definition makes the Laplace transform more concise [13].

2.2. Noise Detection based on fractional differential gradient
Because noise and its adjacent points in pixels are different, however gray value of edge in some directions is continuous and transitional. Therefore, we can effectively distinguish between noise and image information through the differential gradient in each direction.

Assuming the center point O, $c^2 = 28$ edge directions in all consist of O and its surrounding eight pixels A, B, C, D, E, F, G and H. A sample view of differential gradient in partial directions is shown as Fig.1.
The following takes the construction of fractional differential gradient mask in AOB direction for example. The masks in other directions are similar.

According to the calculation formula of fractional differential gradient, the gradient in AOB direction can be shown:

$$D'_{AOBO} = D'_{AOB} = a_1 I(x, y + 1) - a_1 I(x + 1, y + 1) + \cdots + a_1 I(x, y + n) - a_1 I(x + n, y + n) \quad (6)$$

Where $v$ is the order of differential, the coefficient

$$a_1 = \frac{v(v - 1)}{2}, \quad a_2 = \frac{v(v - 1)(v - 2)}{6}, \quad a_3 = \frac{v(v - 1)(v - 2)(v - 3)}{24}, \quad \cdots, \quad a_n = (-1)^n \frac{\Gamma(v + 1)}{n! \Gamma(v + n + 1)}$$

The mask of differential gradient in AOB direction can obtain through formula (6).

3. Experimental procedure and results analysis

3.1. Adding noise

At present, some denoising algorithms based on salt and pepper noise have great denoising effect. Gaussian noise is taken into account in the paper. The number of 500, 1000, 2000, 4000 Gaussian noise points is randomly added to the original images. Their gray values are between 100 and 200.

3.2. Detecting noise image

The application of differential gradient is the key to noise detection. The main process of detecting noise is as follows.

At first, make convolution by image with noise and the direction masks to get direction gradient maps. Then calculate the mean gray value of gradient images and round to nearest, at the same time plus 12 as a threshold value to obtain the gradient detection maps. Finally, we can get the position of noise and noise image from logical product operation of gradient detection maps.

After the comparison of experimental data, we find that direction gradient map from a set of complete direction mask under the fixed initial direction can better eliminate edge information to obtain noise of original image. Hereinafter, seven gradient detection maps based on AO-direction are employed to do logical product. From the features of convolution operation and gradient, we can find that there are displacement and overlapping between the position of noise stemming from logical product operation and the actual position, but the rule of variation is substantially the same. Therefore, appropriate inverse transform is leveraged to ascertain the position of noise. Test by randomly adding different numbers of Gaussian noise points and measure the effectiveness of detection algorithm by the number and the ratio of correct detection (1 is still corresponding to 1), error detection (0 turns to 1), and missed detection (1 turns to 0) (1 stands for noise and 0 is not noise, the value of the former is for original image and the latter is for result of detection).

Experimental data (see Table 1) show that the ratio of correct detection is all over 90%. Meanwhile, with the increase of noise points, the Cr decreases and the Mn become bigger, but the Er is gradually smaller. Thus, a moderate amount of noise points for the method has better effect on denoising. Moreover, by observing the experimental data, a strange phenomenon emerges, that is, when the number of random noise varies, En is very close. The following work can further study the principles
of detection algorithms to reduce Er and increase the accuracy of detection to better realize denoising based on noise.

Table 1 Detection parameters of algorithm based on differential gradient in Lena

| Nn  | Cn  | Cr (%) | En  | Er (%) | Mn  | Mr (%) |
|-----|-----|--------|-----|--------|-----|--------|
| 500 | 489 | 97.80  | 98  | 19.60  | 11  | 2.20   |
| 1000| 958 | 95.80  | 88  | 8.80   | 42  | 4.20   |
| 2000| 1866| 93.30  | 85  | 4.25   | 134 | 6.70   |
| 4000| 3650| 91.25  | 82  | 2.05   | 350 | 8.75   |

*Nn, Total number of noise; Cn, The number of correct detection; Cr, The ratio of Cn and Nn; En, The number of error detection; Er, The ratio of En and Nn; Mn, The number of missed detection; Mr, The ratio of Mn and Nn

3.3. Implementation and Analysis of denoising algorithms based on noise

(1) Comparison in visual aspect

The effects of traditional mean filter, median filter, fractional integral denoising and these methods based on noise are presented in Fig. 2.

![Fig. 2 Traditional denoising algorithms and these methods based on noise for Lena. (a) Original. (b) Traditional mean filtering. (c) Traditional median filtering. (d) Traditional fractional integral. (e) Noised image. (f) Mean based on noise. (g) Median based on noise. (h) Fractional integral based on noise.](image)

After contrast, in subjective or visual aspect, the images after denoising through mean filtering and fractional integral algorithms become significant blurry. Meanwhile, noise points are just reduced in gray value, and still clearly visible. The algorithms based on noise greatly improve performance on fuzzy extent. However, because there are some noise points missed to detect, nothing to be done for them so that these noise points are particularly significant, but little impact on overall recognition of the image.

For the characteristics of median filter, the algorithm based on noise does not have obvious superiority over the traditional method in visual.

(2) Comparison in evaluation parameters

In objective aspect, the PSNR (peak signal to noise ratio), MSE (mean square error) and MAE (mean absolute error) are introduced to evaluate and compare various methods.

Compared with various parameters (see Table 2), it is not difficult to find that for basic denoising methods, the algorithms based on noise have greater PSNR as well as smaller MSE and MAE than traditional methods. The various values show obvious superiority of algorithm based on noise.
Table 2 Various parameters of different methods in Lena

| Methods          | Mean filter  | Median filter | Fractional integral |
|------------------|--------------|---------------|--------------------|
| Traditional      | PSNR0        | 28.2221       | 22.7358            |
| methods          | MSE0         | 0.0015        | 0.0053             |
|                  | MAE0         | 0.0308        | 0.0709             |
| Methods based    | PSNR1        | 33.3808       | 33.3323            |
| on noise         | MSE1         | 4.6×10⁻⁴      | 5.6×10⁻⁴           |
|                  | MAE1         | 0.0181        | 0.0183             |

4. Conclusion
Because the gradient around noise is transitional and that of edge is continuous, a noise detection algorithm based on fractional differential gradient is proposed in the paper. From the visual effect and evaluation parameters, compare the traditional denoising algorithms and denoising based on noise. Simulation results show it has great denoising effect both in subjective and objective aspects. However, there is still much space for improvement. The future researches need to further reduce the ratio of missed detection. Especially when the density of noise is large, how to control the balance of the ratio of error detection and the ratio of missed detection.

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