Heat transfer analysis for Casson fluid flow over stretching sheet with Newtonian heating and viscous dissipation

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Abstract. A steady two-dimensional boundary layer flow and heat transfer of a Casson fluid over a stretched plate with Newtonian heating boundary condition is investigated. The governing partial differential equations are first reduced to nonlinear ordinary differential equations by using suitable similarity transformations before being solved numerically using an implicit finite difference scheme. The features of the flow and heat transfer characteristics for some embedded parameters, such as Casson fluid parameter $\beta$, Eckert number $Ec$, Prandtl number $Pr$ and conjugate Newtonian heating parameter $\gamma$, are analyzed and discussed.

1. Introduction
Due to great application in industry and manufacturing processes, flow induced by stretching sheet is important as the final desired output is very much influenced by the rate of stretching and heat transfer at the surface. Plastic and metal extrusions, production of polymer films or thin sheet and glass-fiber production are some examples where the stretching process takes place. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid [1]. Crane [2] provided the closed form solution for steady two-dimensional flow of an incompressible viscous fluid generated by stretching surface. Later, his work was extended by other researchers for various velocity and thermal boundary conditions to both Newtonian/ non-Newtonian flow, along with several physical effects (see [3]-[10]).

Most fluids are non-Newtonian in nature, eg. honey, paint, slurry, ketchup, muds, glues, lubricants and others. Hence, it should be expected that non-Newtonian fluids are highly non-linear and much more complicated compared with Newtonian fluids. Due to the complexity, there is no single constitutive equation exhibiting all properties. As such, there are few models available in the literature to cater to the characteristics which differ from one another. However, due to promising industry application, non-Newtonian fluid has been the interest of researchers looking to further explore this type of fluid, one of them being Casson fluid. Casson fluid is classified as the most popular non-Newtonian fluid, and has several applications in food processing, metallurgy, drilling operations and bio-engineering operations [11]. Casson fluid was introduced by Casson in 1959 for the prediction of the flow behavior of pigment-oil suspensions [12]. According to Dash et al. [13] and Fung [14], human blood can also be treated as Casson fluid due to the presence of several substances such as protein, fibrinogen, globulin in aqueous base plasma and red blood cells. Shawky [15] studied MHD Casson fluid flow with heat and mass transfer through a porous medium over a stretching sheet, Shateyi and Marewo [16] investigated the hydromagnetic boundary layer flow, heat and mass transfer of Casson fluid near a stagnation point over
a stretching surface in the presence of thermal radiation, viscous dissipation and chemical reaction and Chenna and Shankar [17] dealt with Casson fluid flow over a nonlinearly stretching sheet with the effects of radiation and heat source/sink. Very recently, Kumar and Gangadhar [18] considered MHD stagnation point flow of Casson fluid and heat and mass transfer over a stretching sheet with the effects of radiation, chemical reaction and thermal slip and Medikare et al. [19] analyzed MHD stagnation point flow of a Casson fluid over a nonlinearly stretching sheet with viscous dissipation. However, all the above mentioned studies have not taken into consideration the Newtonian heating effect. Das et al. [20] studied the influence of Newtonian heating on heat and mass transfer in unsteady hydromagnetic flow of Casson fluid past a vertical plate in the presence of radiation and chemical reaction, Jain [21] analyzed Casson fluid flow past an exponentially accelerated infinite vertical plate with Newtonian heating in the presence of radiation and very recently, Ullah et al. [22] examined the slip effect on MHD free convective flow of Casson fluid over a nonlinearly stretching sheet saturated in porous medium with Newtonian heating. Hence, motivated by the above works, we would like to study the effects of Newtonian heating and viscous dissipation towards Casson fluid flow moving over a stretching sheet.

To the best knowledge of the authors, such a study has never been conducted before.

2. Analysis

Consider a steady two-dimensional Casson fluid flow along a stretched plate of ambient temperature \( T_\infty \). The plate is stretched with velocity \( u_0 = ax \ (a > 0) \) and heated due to Newtonian heating. Here, the \( x \)-axis is measured along the stretched plate and the \( y \)-axis is measured in the direction normal to it. Under the Boussinesq and boundary layer approximations, the governing boundary layer equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
\frac{u}{\partial x} + \frac{v}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2}, \\
\frac{u}{\partial x} + \frac{v}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \left( \frac{\rho c_p}{\mu} \right) \left( \frac{\partial u}{\partial y} \right)^2, 
\]

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) directions, respectively. \( \nu \) is the kinematic fluid viscosity, \( \beta \) is the non-Newtonian (Casson) fluid parameter, \( T \) is the fluid temperature, \( k \) is the thermal conductivity, \( \rho \) is the density of the fluid, \( c_p \) is the specific heat at constant pressure and \( \mu \) is the dynamic viscosity. The flow is subjected to the boundary conditions:

\[
u = u_0, \quad \frac{\partial T}{\partial y} = -h_i T \ (NH) \quad \text{at} \quad y = 0, \\
u \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty, 
\]

where \( h_i \) is the heat transfer parameter for Newtonian heating. Using the following transformation:

\[
\psi = \sqrt{ax} \ f(\eta), \quad \eta = y \sqrt{\frac{a}{\nu}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}, 
\]

where \( \psi \) is defined in usual fashion, i.e \( u = \partial \psi / \partial \eta \) and \( v = -\partial \psi / \partial x \), and \( \eta \) is the similarity variable, Equation (1) is automatically satisfied and Equations (2) - (4) reduced to

\[
\left(1 + \frac{1}{\beta} \right) f'''' + f \ f''' - f' \ = 0, 
\]
\[ \theta^* + Ec \Pr f^* \Pr f \theta' = 0, \]  

(7)

along with boundary conditions

\[ f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta'(\eta) = -\gamma [1 + \theta(0)] \quad \text{at} \quad \eta = 0, \]
\[ f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \]  

(8)

where prime denotes differentiation with respect to \( \eta \), \( Ec = \frac{u_w^2}{c_p T_\infty} \) is the Eckert number, \( \Pr = \frac{\nu}{\alpha} \) is the Prandtl number and \( \gamma = h_s \frac{\nu}{\alpha} \) is the conjugate parameter for Newtonian heating.

3. Results and Discussion

Equations (6) and (7), along with boundary conditions (8), have been solved numerically for several values of Casson fluid parameter \( \beta \), Eckert number \( Ec \), Prandtl number \( \Pr \) and conjugate Newtonian heating parameter \( \gamma \). We opt to use an implicit finite-difference method, namely the Keller-box method, which involves four steps [23]:

(i) Reduce Equations (6) and (7) to a first-order system;
(ii) Write the difference equations using central differences;
(iii) Linearize the resulting algebraic equations by Newton’s method and write them in matrix-vector form; and,
(iv) Solve the linear system obtained by using a block-tridiagonal elimination technique.

\( \Delta \eta \) chosen was 0.02 and found to be satisfactory to achieve convergence of \( 10^{-5} \). The numerical method used here has been proven to be unconditionally stable and the most flexible of the common methods, being easily adaptable to solve equations of any order [23].

The effect of Eckert number along with Newtonian heating when \( Pr = 7 \) towards the heat transfer is depicted in Figure 1. It is seen that as the Eckert number increases, the heat transfer rate on the surface also increases. On top of that, Casson fluid flow is found to increase the heat transfer as compared with Newtonian flow (\( \beta = \infty \)). As the surface is being heated by Newtonian heating, it is expected that as \( \gamma \) increases, surface temperature will also increase.

![Figure 1. Variations of \( \theta(0) \) for various values of \( \beta, Ec \) and \( \gamma \) when \( Pr = 7 \).](image-url)
Figure 2 depicts the surface temperature when $Ec = 2$ and $\beta = 1$ for various values of $\gamma$. It is noted that at fixed $\gamma$, the solution for the surface temperature started at certain value of $Pr$, namely $Pr_c$, which is $Pr_c = 1.99, 6.96$ and $15.01$ for $\gamma = 1, 2$ and $3$, respectively. At this point, the surface temperature is infinitely large and decreases monotonically as $Pr > Pr_c$. No solution is found when $Pr < Pr_c$. However, the range of $Pr_c$ can be increased by decreasing $\gamma$.

![Figure 2. Variations of $\theta(0)$ for various values of $\gamma$ and $Pr$ when $\beta = 1$ and $Ec = 2$.](image)

The temperature profiles for various values of the $Ec$ numbers and $\gamma$ when $\beta = 1$ and $Pr = 50$ are plotted in Figure 3. It is seen that an increment in the Eckert number will result in an increment of the surface temperature as well. This is in line with the results obtained in Figure 1. Furthermore, the thermal boundary layer thickness is found to be thicker for a higher Eckert number. As the plate is subjected to Newtonian heating, it is expected that the surface temperature is higher when $\gamma = 2$ when compared with $\gamma = 1$.

![Figure 3. Temperature profile when $\beta = 1$ and $Pr = 50$ for several values of $Ec$ and $\gamma$.](image)
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