\( \eta \) and \( \eta' \) production in nucleon-nucleon collisions near thresholds

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The production of \( \eta \) and \( \eta' \) mesons in nucleon-nucleon collisions near thresholds is considered within a one-boson exchange model. We show the feasibility of an experimental access to transition formfactors.

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1. Introduction

The pseudo-scalar mesons \( \eta \) and \( \eta' \) represent a subject of considerable interest since some time (cf. [1] for surveys). Investigations of various aspects of \( \eta \) and \( \eta' \) mesons are tightly related with several theoretical challenges and can augment the experimental information on different phenomenological model parameters. For instance, the "anomalously" large mass of the \( \eta' \) meson, as member of the \( SU_A(3) \) nonet [2], can be directly connected with the \( U(1) \) axial anomaly in QCD. A combined phenomenological analysis of \( \eta \) and \( \eta' \) production in \( N + N \) reactions together with the \( U_A(1) \) anomaly provides additional information on the gluon-nucleon coupling, which can be used to describe, e.g., the so-called "spin crisis". Also, the knowledge of the nucleon-nucleon-\( \eta' \) coupling strength allows a better understanding of the origin of the OZI rule violation in \( N + N \) reactions. A remarkable fact is that near the threshold the invariant mass of the \( N N \eta' \) system in such reactions is in the region of heavy nucleon resonances, i.e. resonances

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with isospin 1/2 can be investigated via these processes. Furthermore, the so-called "missing resonances" can be studied.

Another aspect of η and η' production in elementary hadron reactions is that both mesons have significant Dalitz decay channels into $e^+e^-\gamma$. As such, they constitute further sources of di-electrons. It is, in particular, the η which is a significant source of $e^+e^-$ pairs, competing with Δ Dalitz decays and bremsstrahlung, as the analysis of HADES data shows. One of the primary aims of the HADES experiments is to seek for signals of chiral symmetry restoration in compressed nuclear matter. For such an endeavor one needs a good control of the background processes, including the η' Dalitz decay, in particular at higher beam energies, as becoming accessible at SIS100 within the FAIR project.

The η and η' Dalitz decays depend on the pseudo-scalar transition form factor, which encodes hadronic information accessible in first-principle QCD calculations or QCD sum rules. The Dalitz decay process of a pseudo-scalar meson $ps$ can be presented as $ps \rightarrow \gamma + \gamma^* \rightarrow \gamma + e^- + e^+$. Obviously, the probability of emitting a virtual photon is governed by the dynamical electromagnetic structure of the "dressed" transition vertex $ps \rightarrow \gamma\gamma^*$ which is encoded in the transition form factors. If the decaying particle were point like, then calculations of mass distributions and decay widths would be straightforwardly given by QED. Deviations of the measured quantities from the QED predictions directly reflect the effects of the form factors and thus the internal hadron structure.

The present paper reports parameterizations of η and η' production cross sections in nucleon-nucleon collisions near the respective thresholds within a one-boson exchange model. Emphasis is put on the accessibility of transition formfactors encoding the strong-interaction η, η' structure.

2. One-boson exchange model

Cross sections of interest are

$$d^5\sigma_{NN\rightarrow NNps}^{\text{tot}} = \frac{1}{2(2\pi)^5\sqrt{\lambda(s, m^2, m^2)}} \times \frac{1}{4} \sum_{\text{spins}} |T_{NN\rightarrow NNps}|^2 ds_1' ds_2' \, dR_{N1} \, dR_{N2} \, dR_{s1s2} \, dR_{s'1's'2'}$$

with two-particle invariant phase space $P_{2ab\rightarrow cd} = \sqrt{\lambda(s_{ab}, m_{c\ell}^2, m_d^2)/(8s_{ab})}d\Omega_c$ for the production of $ps \equiv \eta, \eta'$ and

$$\frac{d\sigma}{ds_{ps}ds_{\gamma^*}} = \frac{d\Gamma_{ps\rightarrow \gamma^+e^-}}{ds_{\gamma^*}} \frac{1}{4\pi\sqrt{s_{ps}}} \frac{1}{\left(\sqrt{s_{ps}} - m_{ps}\right)^2 + \frac{1}{4}\Gamma_{ps}^2} d^5\sigma_{NN\rightarrow NNps}^{\text{tot}}$$
for the Dalitz decay. Integrating the latter one over $ds_{ps}$ or taking it at $s = m_{ps}^2$ is meant to access the electromagnetic formfactors appearing in
\[
\frac{d\Gamma_{ps\rightarrow\gamma\gamma}}{ds_{\gamma^*}} = \frac{2\alpha_{em}}{3\pi s_{\gamma^*}} \left( 1 - \frac{m_{ps}^2}{s_{\gamma^*}} \right)^3 \Gamma_{ps\rightarrow\gamma\gamma} |F_{ps\gamma\gamma^*}(s_{\gamma^*})|^2. \tag{3}
\]

### 2.1. $\eta$ channel

We employ here a one-boson exchange model, where the $\eta$ production is described by the diagrams exhibited in Fig. 1. The sum of these diagrams generate the invariant amplitude $T_{NN\rightarrow NNps}$ via interaction Lagrangians.

![Diagrams for the process $NN\rightarrow NN\gamma l_1 l_2$](image)

Fig. 1. Diagrams for the process $NN\rightarrow NN\gamma l_1 l_2$ within the one-boson exchange model. a) Dalitz decays of $\eta$ mesons from bremsstrahlung like diagrams. The intermediate baryon $N^*$ (triple line) can be either a nucleon or a nucleon resonance ($S_{11}(1535)$, $P_{11}(1440)$, $D_{13}(1520)$). Analog diagrams for the emission from Fermion line $N_2$. b) Dalitz decay of $\eta$ mesons from internal meson conversion. Exchange diagrams are not displayed. Later on we identify $l_{1,2} = e^\pm$ and denote the di-electron invariant mass by $s_{\gamma^*}$.

### 2.2. $\eta'$ channel

The calculation of $\eta'$ uses the same diagram topology as in Fig. 1 (with $\eta \rightarrow \eta'$) supplemented by $a_0$ exchange in a). The included resonances are $S_{11}(1650)$ with odd parity, and $P_{11}(1710)$ and $P_{13}(1720)$ with even parity.

### 2.3. Interaction Lagrangians

The employed interaction Lagrangians can be represented as follows.

(i) Nucleon currents:

\[
\mathcal{L}_{\sigma NN} = g_{\sigma NN} \bar{N} N \Phi_\sigma, \tag{4}
\]

\[
\mathcal{L}_{a_0 NN} = g_{a_0 NN} \bar{N} (\tau \bar{\Phi}_{a_0}) N, \tag{5}
\]

\[
\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{N} \gamma_5 \gamma^\mu \partial_\mu (\tau \bar{\Phi}_{\pi}) N, \tag{6}
\]
\[ \mathcal{L}_{\eta NN} = -\frac{f_{\eta NN}}{m_\eta} \bar{N} \gamma_\mu \gamma^\mu \partial_\mu \Phi_\eta N, \] (7)

\[ \mathcal{L}_{\rho NN} = -g_{\rho NN} \left( \bar{N} \gamma_\mu \tau N \Phi_\rho^\mu - \frac{\kappa_\rho}{2m} \bar{N} \sigma_{\mu\nu} \tau N \partial^\nu \Phi_\rho^\mu \right), \] (8)

\[ \mathcal{L}_{\omega NN} = -g_{\omega NN} \left( \bar{N} \gamma_\mu N \Phi_\omega^\mu - \frac{\kappa_\omega}{2m} \bar{N} \sigma_{\mu\nu} N \partial^\nu \Phi_\omega^\mu \right), \] (9)

(ii) Spin \( \frac{1}{2} \) resonances (\( S_{11} \) and \( P_{11} \)):

\[ \mathcal{L}_{NN^*ps}(x) \equiv \mp \frac{g_{NN^*ps}}{m_{N^*} \pm m_N} \bar{\Psi}_R(x) \left\{ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right\} \gamma_\mu \Phi_{ps}(x) \Psi_N(x) + h.c. \] (10)

\[ \mathcal{L}_{NN^*V}(x) \equiv \frac{g_{NN^*V}}{2(m_{N^*} + m_N)} \bar{\Psi}_R(x) \left\{ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right\} \sigma_{\mu\nu} V^{\mu\nu}(x) \Psi_N(x) + h.c. \] (11)

(iii) Spin \( \frac{3}{2} \) resonances (\( D_{13} \) and \( P_{13} \)):

\[ \mathcal{L}_{NN^*ps}(x) = \frac{g_{NN^*ps}}{m_{ps}} \bar{\Psi}_R(x) \left\{ \begin{array}{c} 1 \\ \gamma_5 \end{array} \right\} \partial^\alpha \Phi_{ps}(x) \Psi_N(x) + h.c. \] (12)

\[ \mathcal{L}_{NN^*V}(x) = \mp i \frac{g_{NN^*V}}{2m_N} \bar{\Psi}_R(x) \left\{ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right\} \gamma_\lambda V^{\lambda\alpha}(x) \Psi_N(x) \]

\[ - \frac{g_{NN^*V}}{4m_N^2} \partial_\lambda \Psi_R(x) \left\{ \begin{array}{c} \gamma_5 \\ 1 \end{array} \right\} V^{\lambda\alpha} \Psi_N(x) + h.c. \] (13)

with the abbreviations \( ps \equiv \pi \) or \( \eta \) or \( \eta' \), \( \Phi_{ps} \equiv (\tau \Phi_\pi(x)) \) or \( \Phi_{\eta'}(x) \), \( V \equiv V_\omega(x) \) or \( V(\tau \rho(x)) \), and \( V^{\alpha\beta} = \partial^\alpha V^\beta - \partial^\beta V^\alpha \). Furthermore needed interactions, such as \( \mathcal{L}_{ps\omega\omega} \), \( \mathcal{L}_{ps\rho\rho} \), \( \mathcal{L}_{\gamma\gamma\gamma} \), and \( \mathcal{L}_{ps\gamma\gamma} \), are listed in [7].

2.4. Formfactors

Strong formfactor are needed to dress the nucleon – nucleon (resonance) – meson vertices. These are listed in detail in [6, 7].

The electromagnetic formfactors encode non-perturbative transition matrix elements \( F_{ps\gamma\gamma} \) in [3], basically accessible within QCD. Here, however, we contrast a few parameterizations: (i) so-called QED formfactor meaning a structure-less particle with \( |F_{ps\gamma\gamma}(s_{\gamma'})|^2 = 1 \), (ii) a parametrization suggested by the vector meson dominance (VMD) model

\[ F_{ps\gamma\gamma}^{VMD}(s_{\gamma'}) \sum_{V=\omega,\phi} C_V \frac{m_V^2}{m_V^2 - s_{\gamma'}} \] (14)

with \( F_{ps\gamma\gamma}(s_{\gamma'} = 0) = 1 \), \( \sum_V C_V = 1 \) and \( m_V = m_V - i\Gamma_V/2 \). The values of \( C_V \) are quoted in [7]. For the case of \( \eta \), the kinematically accessible region
is restricted and, as a consequence, the $\rho$ contribution is sufficient. (iii) For $\eta'$, a monopole fit $F_{\eta',\gamma\gamma}(Q^2) = (1 - Q^2/\Lambda_{\eta'})^{-1}$ may be used, which does not differ too much from the VMD parametrization.

2.5. Initial state and final state interactions

Initial state interactions are accounted for by effective reduction factors for $^3P_0, ^1P_1$ waves: $\zeta = 0.277$ ($pp$), $0.243$ ($np, pp$) [8]. Final state interactions are treated by the Jost function formalism, see [9] for details.

Fig. 2. Cross sections for $\omega$ (top) and $\phi$ (bottom) production from [10, 11] (left, data for $\omega$ from [12] (open circles), [13] (triangles) and [14] (squares) and for $\phi$ from [14, 15]). The new data situation confirms these predictions (right, with data for $\omega$ from [16] and for $\phi$ from [17], both ones depicted as filled circles).

2.6. One-boson model at work

These seemingly many ingredients (coupling strengths, formfactors and their cut-offs, see [6, 7]) may cause the impression that the one-boson exchange approach to hadronic observables near threshold does not have too
much predictive power. Two counterexamples may lend more credibility to the approach. In Fig. 2 the model results of [10, 11] are exhibited (left panels). Later on the data basis has been improved confirming the model predictions (right panels). Further applications of the present approach to \( \omega \) and \( \phi \) production involving a final deuteron and including polarization observables, have been presented in [13], while [19] extends the formalism to virtual bremsstrahlung in \( NN \rightarrow NN\gamma \rightarrow NNe^+e^- \) reactions.

Fig. 3. Total cross sections for \( \eta \) (top) and \( \eta' \) (bottom) production as a function of the energy excess in \( p+p \) (left) and \( n+p \) reactions (right). For data quotation and further details cf. [6, 7].

3. Results

Numerical evaluation of the given formalism results in the total cross sections exhibited in Fig. 3. Available data are nicely reproduced in the \( p+p \) channel (a concern could be the region of excess energy \( \Delta s^{1/2} \sim 10 \text{ MeV} \) for \( \eta \)). Since now new parameters enter, the channel \( n+p \) represents a prediction, in agreement with data in case of \( \eta \); data are not (yet) available for \( \eta' \).
Fig. 4. Differential cross sections for $\eta$ (left, HADES data from [20], for $T_p = 2.2$ GeV) and $\eta'$ (right, for $T_p = 2.5$ GeV) which give access to the formfactors.

The cross sections $d\sigma/ds_{\gamma\gamma}^{1/2}$, resulting from the integration of (2) over $s_{ps}$, are exhibited in Fig. 4. There is a tiny difference when neglecting the internal strong interaction structure of $\eta$ ("QED" formfactor) or when using the "VMD" formfactor, see left panel. The situation changes drastically for $\eta'$. Here, the account of the internal structure becomes important, see right panel. Precision data would even allow for a sensible test of the VMD hypothesis. It has been shown in [6, 7] that the formfactors can be deduced from given cross sections $d\sigma/ds_{\gamma\gamma}^{1/2}$.

### 4. Summary

In summary we report on calculations of the reaction $NN \rightarrow NNps$ with $ps = \eta, \eta'$ and subsequent Dalitz decay $ps \rightarrow \gamma e^+ e^-$ within a one-boson exchange model. We point out that isolating $\eta$ and $\eta'$ contributions, e.g., in $p+p$ collisions, allows for an experimental determination of the transition formfactors $F_{ps\gamma\gamma}$. In particular, for $\eta'$ the vector meson dominance hypothesis would be testable. On the other hand, the $\eta$ Dalitz decay channel is a strong source of $e^+ e^-$ pairs in medium-energy heavy-ion collisions which need to be understood before firm conclusions on possible in-medium modifications of hadrons can be made. We emphasize that, once the model parameters are adjusted in the $p+p$ channel, the $n+p$ channel is accessible without further parameters.

For further improvements of the presented formalism we refer the interested reader to [21], where $N+N$ collisions and $\eta, \eta'$ photo-production are considered on a common footing.
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