Study of Cassegrain-type antenna for radio telescope

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Abstract. There are many critical parameters in the design of a radio telescope, such as antenna gain and antenna resolution. In telecommunications, radar, and radio telescopes, an antenna is a very important component. There are many designs of the antenna, such as dipole array and parabolic antenna. Parabolic antennas also have many sub-reflector and antenna methods that control the radio wave, such as the Cassegrain type. A Cassegrain-type antenna is a parabolic antenna in which the feed antenna is mounted at or behind the surface of the main parabolic reflector dish. For the transmitter system, the beam of radio waves from the feed antenna illuminates the secondary reflector (sub-reflector), which reflects it back to the main reflector dish and then forward to space. The Cassegrain design is widely used in parabolic antennas, for large antennas in satellite ground stations, communication satellites, and radio telescopes. In this paper discusses the design of a Cassegrain-type antenna for radio telescope, basic calculation, diameter size of the main reflector of 20 meters, the diameter size of sub-reflector of 2.5 meters, and frequency of 22 GHz and 43 GHz.

1. Introduction

A radio telescope is a device that captures radio waves emitted by celestial objects. Radio telescopes devices operate at the radio frequency of the electromagnetic spectrum where they can detect and collect information from radio sources of outer space such as stars, galaxies, black holes, quasars, and other cosmic objects, and further focus and strengthen the radio waves with an amplifier system so that they can be analyzed with a processing system. In fact, radio telescope applications are similar to optical telescope applications in that they both receive signals from the object. The difference is in the type of signal captured; radio telescopes capture radio waves, whereas optical telescopes capture light waves.

In radio telescopes, the signal received by the reflector 'dish' is then reflected to the sub-reflector and then goes to the focal point. Then there is a processing device that converts the signal to an electronic signal so that the data can be viewed on the monitor's display. In radio telescopes, the antenna is one of the most important component, that is connected to the amplifier, band-pass filter, mixer, and detector [1]; working together in one operation. Radio telescopes generally use dipole or parabolic antennas depending on the frequency to be observed, for example, at the ultra-high frequency (UHF) or higher frequencies can use a dipole antenna, at a frequency more than UHF can use a parabolic antenna. Parabolic reflectors are used in radio telescopes because the parabolic 'shaped' reflector antenna can produce the high antenna gain needed for a very low signal, long-range radio communications, and high-resolution radar applications [2]. Furthermore, reflector antennas are also used to reach UHF for a variety of reasons, one of which is caused by their high efficiency [3].

There are numerous different types of reflector antennas, and when viewed from the geometric configuration, the most popular are the plane, corner, and curved (particularly paraboloid) reflectors.
The paraboloid reflector is a common shape in radio telescopes because it collects incoming radio waves and directs them to a focal point where we (users) can efficiently place them to get the optimum efficiency. Cassegrain and Gregorian reflectors are both commonly used in radio telescopes, the difference is that the Cassegrain uses a hyperboloid subreflector while the Gregorian uses an elliptical subreflector. Because it allows for a relatively stable structure against gravitational deformation and wind pressure, the Cassegrain type of paraboloidal reflector is one of the most widely used [5]. Furthermore, in this paper, we will discuss the Cassegrain type and how it is used in radio telescopes.

2. Cassegrain-type antenna reflector

At first, the scheme of the reflector type of Cassegrain antenna was designed for optical telescopes by Laurent Cassegrain in the 17th century. He showed that incident parallel rays can connect a point by using two reflectors, namely a primary (in the form of a parabola) and a secondary (in the form of a hyperbola), the feed is placed along the axis of the parabola, usually, around the vertex [4]. This geometry can be seen in figure 1. The focal point is a point where the sub-reflector placed that can reflect radio waves to the main reflector spread evenly [2]. The design of this system was adapted for use in radio frequency systems. With this Cassegrain feed arrangement, the transmitting and/or receiving equipment can be placed behind the main reflector, making the system relatively more accessible for servicing and adjustment [4].

![Figure 1. Cassegrain feed for a paraboloidal reflector](Credit to: Department of Electrical and Electronic Engineering, Yonsei University)

The following [6] describes the measurement method and automation adjustment simulation for large radio telescopes with adjustable dual-reflectors, to keep the primary reflector and subreflector in an ideal position during operation and adjust their position in the right focus location. This system has a parabolic main reflector and a hyperbolic sub-reflector; and also two foci, the real focal point in the feed and the virtual focus point at the parabolic focus [7], which causes all parts of the wave originating from the surface to travel the same distance to the antenna [8].

The basic calculation formula for the main parameters in the antenna reflector is as follows, as cited in the literature sources [9].

\[
\text{Beamwidth} = \theta_A = \frac{1.2\lambda}{d} \tag{1}
\]

\[
\text{Aperture efficiency} = \eta_A = \frac{A}{A_g} \tag{2}
\]

\[
\text{Gain} = G = \frac{4\pi A}{\lambda^2} \tag{3}
\]
Antenna solid angle = $\Omega_A = \int_{4\pi} g(\theta, \phi) d\Omega$ \hspace{1cm} (4)

Main beam solid angle = $\Omega_m = \int g(\theta, \phi) d\Omega$ \hspace{1cm} (5)

Beam efficiency = $\eta_B = \frac{\Omega_m}{\Omega_A}$ \hspace{1cm} (6)

Scatter efficiency = $\eta_s = \exp \left[- \left(\frac{4\pi\varepsilon}{\lambda}\right)^2\right]$ \hspace{1cm} (7)

Blocking efficiency = $\eta_b = \left(1 - \frac{A_{bl}}{A_g}\right)^2$ \hspace{1cm} (8)

where $A_g$ is the geometrical aperture area, $A$ defines effective aperture area, and $A_{bl}$ is the blocked area.

In addition, the following equations and variables exist in the Cassegrain system (see figure 2) as described in [7].

$$\tan \frac{1}{2} \phi_v = \pm \frac{D_m}{4 F_m}$$ \hspace{1cm} (9)

Equation (9) above is used for the main reflector and equations (10) & (11) below are used for the sub-reflector [7].

$$\tan \frac{1}{2} \phi_v + \frac{1}{\tan \phi_r} = 2 \frac{F_c}{D_s}$$ \hspace{1cm} (10)

$$1 - \frac{\sin (\phi_v - \phi_r) / 2}{\sin (\phi_v + \phi_r) / 2} = 2 \frac{L_v}{F_c}$$ \hspace{1cm} (11)

**Figure 2.** The geometry of Cassegrain system

[Credit to: Electro Science Laboratory, Department of Electrical Engineering, The Ohio State University]
According to mathematical calculations from the following literature [7], the parabolic principal reflector is generated from

\[
X_m = \frac{Y_m^2}{4F_m}
\]  \hspace{1cm} (12)

and the hyperbolic subreflector is generated from

\[
X_s = a \left[ \sqrt{1 + \left(\frac{Y_s}{b}\right)^2} - 1 \right]
\]  \hspace{1cm} (13)

where

\[
e = \frac{\sin(\nu + \phi r)/2}{\sin(\nu - \phi r)/2}
\]  \hspace{1cm} (14)

\[
a = \frac{F}{2e}
\]  \hspace{1cm} (15)

\[
b = a\sqrt{e^2 - 1}
\]  \hspace{1cm} (16)

The equations above describe the classical Cassegrain system; many variations of the basic system can be formed using these equations; for further information, see [7].

According to reference [8], the Cassegrain system is defined by four primary variable parameters (see figure 3) namely \(D\) (diameter paraboloid), \(\theta_0\) (subtended angle of the main reflector), \(d\) (diameter hyperboloid), \(\Psi_0\) (subtended angle of the subreflector) and the equations are as follows

\[
F = \frac{D}{4} \cot\left(\frac{\theta_0}{2}\right)
\]  \hspace{1cm} (17)

\[
F_c = \frac{d}{4} \left(\cot\theta_0 + \Psi_0\right)
\]  \hspace{1cm} (18)
\[ M = \frac{F_e}{F} = \frac{\tan(\frac{\psi_0}{2})}{\tan(\frac{\theta_0}{2})} \]  

(19)

\[ \psi = 2 \arctan \left( \frac{1}{M} \tan \left( \frac{\theta}{2} \right) \right) \]  

(20)

where \( F \) is the primary reflector's focal length, \( F_c \) is the distance between the primary and secondary focus, and equation (20) shows the connection between the arbitrary feed angle \( \psi \) and the appropriate subreflector angle \( \theta \).

There are also basic calculations in the Cassegrain type sub-reflector [2], which will be briefly shown below; if you want more advanced calculations, read the reference [2].

![Figure 4. Geometry of classical hyperbolic subreflector](Credit to: Antenna Theory and Design 3rd Edition (by Stutzman, Warren L and Thiele, Gary A))

The subreflector is determined by its diameter \( D_s \) and its eccentricity \( e \) [2]. The shape of the geometry of the Cassegrain type subreflector is controlled by the eccentricity, which is defined as

\[ e = \frac{c}{a} > 1 \]  

(21)

The Cassegrain antenna is a type of symmetric dual-reflector antenna that works similarly to a paraboloid antenna (as explained before). The structure is stable, low in cost, and simple to implement. However, there are several advantages from the Cassegrain type antenna when compared to a parabolic antenna. The Cassegrain type antenna has about 10% higher efficiency than the parabolic antenna with the same primary reflector size. The Cassegrain type antenna can also be operated in dual-frequency mode, has a lower noise system temperature, better pointing accuracy, and a more flexible feed design than parabolic antennas [10], [11].

On the other hand, the Cassegrain antenna has a disadvantage that could be undesirable, namely the blocking of the aperture originating from the Cassegrain sub-reflector [10], because the position of the sub-reflector and its supporting parts will always block some of the electromagnetic waves captured and/or emitted by the main reflector. However, to overcome these shortcomings, it is feasible to utilize both of the main reflector shapes that deviate slightly from the ideal paraboloid of revolution and similarly, the sub-reflector is slightly different from the ideal hyperboloid of revolution, it is known as the "shaped" Cassegrain which can increase the aperture efficiency up to 85% or even more as shown in reference [12].
There are also solutions thanks to advances in software technology on computers that generate models and optimize the offset-reflector, but the offset-reflector model is still under development due to radiation in the antenna feed; besides that, the following literature [4] contains several solutions to increase aperture efficiency. In its mathematical form, the solution for the radio telescope’s efficiency [10],[13] is described by the aperture efficiency $\eta_A$, which is determined by equation (2) above. The following reference [8] provides a detailed description of the aperture efficiency optimization on a conventional dual-reflector Cassegrain antenna.

3. Implementation of Cassegrain-type at radio telescope

The following are some examples of Cassegrain-type reflector designs used on radio telescopes across the world [14].

![Figure 5. 45-m Radio Telescope at Nobeyama Radio Observatory](image)

[Credit to: Atsushi Nakazawa, National Astronomical Observatory of Japan]

The Nobeyama Radio Observatory (NRO) in Japan (see figure 5) has operated the Nobeyama Radio Telescope, a 45-meter diameter Cassegrain-modified-Coude reflector radio telescope that operates at frequencies between 20 - 116 GHz.

![Figure 6. From the left: the 32-m Medicina and 32-m Noto](image)

[Credit to: Salvatore Pluchino, Proceedings of the International Astronomical Union]

The Medicina and The Noto (see figure 6) are two 32-m diameter radio telescopes in Italy with identical structures using Cassegrain reflectors, that operate at frequencies in the range from 1.4 to 26 GHz and 1.4 to 43 GHz, respectively.
The Korea Astronomy and Space Science Institute (KASI) has operated The Korean VLBI Network (see figure 7) which consists of three 21-m shaped Cassegrain reflectors located in Seoul (Yonsei), Ulsan (Ulsan), and Jeju Island (Tamna) and operates in four frequency bands: 21–23 GHz, 42–44 GHz, 85–95 GHz, and 125–142 GHz (all in dual-polarization); The 22 and 43-GHz receivers will be installed first, to set up the antennas and for the initial VLBI (Very Long Baseline Interferometer) observations [15].

![Figure 7. From the left : Yonsei, Ulsan and Tamna](Credit to: Korea Astronomy and Space Science Institute (KASI))

The VLBI Exploration of Radio Astrometry (VERA) has been operated by the National Astronomical Observatory of Japan (NAOJ) and several universities in Japan. The VERA array consists of four identical 20-m radio telescopes located at Mizusawa (which also serves as the operation center), Iriki, Ogasawara, and Ishigaki-Jima (see figure 8). VERA antennas have a standard Cassegrain-type dish on an Alt-Az mount, with minimum and maximum elevation limits of 5° and 85°; with main receiving bands for VERA are 22 and 43 GHz.

VERA and KVN initially collaborated to create KaVA, a joint array, and have since expanded their collaboration to the East-Asian VLBI Network (EAVN) with the Chinese VLBI Network (CVN) and the Japanese VLBI Network (JVN) [14],[16]. KaVa has been operating since 2011 observing at 22 and 43 GHz, and has made some progress in investigations of AGN21 jets, 44 GHz methanol maser emission from major star-forming regions22, and other topics [16].

![Figure 8. Distribution of VERA stations in Japan](Credit to: National Astronomical Observatory of Japan (NAOJ))
4. Conclusion

The Cassegrain-type antenna reflector is built on the concept of using two reflectors, namely a primary reflector (in the form of a parabola) and a secondary reflector (in the form of a hyperbola), and then the feed is placed along the axis of the parabola, usually around the vertex. By using this configuration, the incident parallel rays reflected by the main reflector are subsequently reflected to the sub-reflector, causing the photon wave to be focused on the actual focal point located in the feed.

There are parameters in the antenna reflector which are not much different from the antenna in general. Beamwidth, aperture efficiency, gain, antenna solid angle, main beam solid angle, beam efficiency, scatter efficiency, blocking efficiency, and others are the main parameters of an antenna reflector. Cassegrain-type antenna reflectors are commonly used in radio telescopes, although with various designs and configurations due to the wavelength and frequency of the observations. This antenna study will be a basic feasibility for radio telescope with antenna diameter class of 20 meters and multiple-selected frequencies such as in 1.4 GHz, 22 GHz, and 43 GHz for Indonesia national observation on Timau, Kupang.

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