Tidal controls on the lithospheric thickness and topography of Io from magmatic segregation and volcanism modelling

Dan C Spencer, Richard F Katz, Ian J Hewitt

Highlights

- Io is modelled with a spatially-variable tidal heating model coupled to a magma-segregation and volcanism model.
- We predict long-wavelength lithospheric thickness variations that arise from spatially variable tidal heating.
- Lithospheric thickness could either be correlated with surface heat flux (if intrusions form at a constant rate), or be weakly anti-correlated (if intrusions form at a rate proportional to magma flux).
- A Pratt-like isostasy model predicts long-wavelength topography, showing that topography is expected to be anti-correlated with lithospheric thickness.

Keywords

Io, tidal heating, heat piping, planetary volcanism, isostasy
Abstract

Tidal heating is expected to impart significant, non-spherically-symmetric structure to Jupiter’s volcanic moon Io. A signature of spatially variable tidal heating is generally sought in observations of surface heat fluxes or volcanic activity, an exploration complicated by the transient nature of volcanic events. The thickness of the lithosphere is expected to change over much longer timescales, and so may provide a robust link between surface observations and the tidal heating distribution. To predict long-wavelength lithospheric thickness variations, we couple three-dimensional tidal heating calculations to a suite of column models of magmatic segregation and volcanic eruption. We find that the lithospheric thickness could either be correlated with the radially integrated heating rate, or weakly anti-correlated. Lithospheric thickness is correlated with radially integrated heating rate if magmatic intrusions form at a constant rate in the lithosphere, but is weakly anti-correlated if intrusions form at a rate proportional to the flux through volcanic conduits. We use a simple, Pratt-like isostasy calculation to predict long-wavelength topography and find that long-wavelength topography anti-correlates with lithospheric thickness. These results will allow future observations to critically evaluate models for Io’s lithospheric structure, and enable their use in constraining the distribution of tidal heating.

1 Introduction

Io, the most volcanic body in the solar system, operates in a different tectonic regime to the terrestrial planets. The eruption and burial of lava, combined with the low surface temperature leads to the growth of a thick and cold lithosphere in spite of the high surface heat flux. This high heat flux is primarily exported from the interior by magmatic segregation in the mantle (Moore, 2001), and through volcanic ‘heat-pipes’ in the lithosphere (O’Reilly and Davies, 1981). Heat is supplied to the interior by tidal dissipation — a process of great importance in the Solar System (de Kleer et al., 2019) — and the distribution of input tidal heating is expected to control the surface heat flux distribution (Ross et al., 1990; Tackley, 2001; Kirchoff et al., 2011; Beuthe, 2013; Rathbun et al., 2018; Steinke et al., 2020). Whilst theoretical links between the tidal heating distribution and the surface heat flux are well established, the implications for interior structure, magma dynamics, tectonics and topography are not well known.

The spatial distribution of tidal heating is a longstanding and still largely unresolved problem in planetary science (Segatz et al., 1988; Roberts and Nimmo, 2008; Beuthe, 2013; Bierson and Nimmo, 2016; Renaud and Henning, 2018). The end-members generally considered for Io are those of lower-mantle heating or asthenosphere heating (Segatz et al., 1988; Ross et al., 1990; Tackley et al., 2001; Hamilton et al., 2013), though magma-ocean dissipation has also been proposed (Tyler et al., 2015; Hay et al., 2020). Lower-mantle dissipation predicts high polar heat fluxes, whereas asthenospheric dissipation predicts high equatorial heat fluxes. A number of works have sought to identify tidal dissipation patterns from surface heat fluxes (Veeder et al., 2012), volcanic activity (Rathbun et al., 2018), and volcano distributions (Ross et al., 1990; Kirchoff et al., 2011; Hamilton et al., 2013). The primary hindrance to these works is the poor polar coverage of observations, so whilst a number of these works favour an asthenosphere heating model (e.g., Ross et al., 1990; Kirchoff et al., 2011), the general consensus is that more polar observations are needed to fully address this question (Rathbun et al., 2018; de Kleer et al., 2019). Further, long-timescale, averaged heat fluxes are difficult to estimate, and it is unclear to what extent short-timescale observations of volcanic activity reflect the global dissipation structure. Tectonic features, which vary on much longer timescales, may provide a more robust link between surface observations and the distribution of tidal heating.
An important tectonic feature that is expected to relate to the surface heat flux, and thus the tidal heating distribution, is the long-wavelength lithospheric thickness (Ross et al., 1990; Steinke et al., 2020). Recent studies have proposed two hypotheses for the primary controls on the thickness of the lithosphere, which we define as the upper-most, fully solid layer of Io. Steinke et al. (2020) proposed a stagnant lid convection model where a portion of mantle heat transport occurs by convection, and this convectively-transported heat is transported though the lithosphere by conduction. This predicts that the lithosphere is thinnest where heat flux is highest. Alternatively, Spencer et al. (2020) proposed that the eruption and burial of lava results in the growth of a cold lithosphere, with a steady-state thickness that is controlled by the balance of downward advection and heat delivered by magmatic intrusions. Conduction plays a minor role in this model because the rate of burial is so large. In such a system, the lithospheric thickness is primarily controlled by the rate of melt production and the rate of intrusive heating. It should be noted that both of these previous studies referred to the surface layer as the ‘crust’; here we use ‘lithosphere’ instead because the crust is usually considered to be a petrologically distinct layer, a distinction that becomes important in the isostatic calculations below. If the thickness of the lithosphere can be related to topography and heat flow, then long-wavelength variations in lithospheric thickness could be used to infer the tidal heating distribution. Together with estimates of libration amplitude (VanHoolst et al., 2020), this can provide a powerful means of investigating Io’s interior structure.

Each of the models described above propose different controls on the lithospheric thickness and so may be expected to predict different relationships between the tidal heating distribution and thickness. Steinke et al. (2020) used radially integrated tidal heating profiles to predict the effect of spatially variable tidal heating on lithospheric thickness, finding that the thickness anti-correlates with surface heat flux. In the present work we extend the model of Spencer et al. (2020) to consider the effect of variable tidal heating on the eruption and intrusion model for lithospheric thickness such that comparisons can be made between the models of Spencer et al. (2020) and Steinke et al. (2020).

We generalise the simplified, steady-state model of Spencer et al. (2020) to allow variable tidal heating. Io is divided into a set of laterally contiguous columns that are coupled to a viscoelastic tidal heating model. The tidal heating model calculates a three-dimensional heating rate from a spherically symmetric rheological structure. This leads to a recognised limitation of these type of tidal heating models; the three-dimensional heating rate that they produce generates a non-spherically symmetric structure that cannot be used to recalculate the heating distribution without averaging over spherical shells (Roberts and Nimmo, 2008; Bierson and Nimmo, 2016). Thus, models coupling such tidal heating calculations to dynamics cannot be fully three-dimensional. We use this coupled, pseudo-three-dimensional model to investigate the links between tidal heating, lithospheric thicknesses, and long-timescale eruption rates/heat fluxes. Our results show that the relationship between lithospheric thicknesses and heat flux depends on how magmatic intrusions form within the lithosphere. If the rate of formation of permanent magmatic intrusions is independent of the (non-zero) magma flux through volcanic conduits, as may be expected if the volcanic system exploits pre-existing fractures, we predict the lithosphere to be thickest where radially integrated heating rate (and thus eruption rate and heat flux) is highest. If, however, magmatic intrusions form at a rate proportional to the magma flux in volcanic conduits, as may be expected if volcanic conduits form due to basal magma pressure that generates new pathways for magma to propagate into the lithosphere, the lithospheric thickness should be weakly anti-correlated with radially integrated heating rate. We also use a simple, Pratt-like isostasy calculation to convert dissipation-derived long-wavelength lithospheric thickness to topography, predicting
that topography anti-correlates with thickness. This relates a feature that generally has to be indirectly inferred (lithospheric thickness), to an observation that is more readily obtained (topography). Improved observations of surface heat fluxes and their relationships to lithospheric thickness and topography will test different models for the controls on Io’s lithospheric thickness (Steinke et al., 2020; Spencer et al., 2020). With a means of critically evaluating these models, the structure of tidal heating can feasibly be constrained by future estimates of lithospheric thickness.

2 Methodology

Our model consists of three parts: a theory for magmatic segregation and volcanism, another for tidal dissipation, and isostasy calculations. The one-dimensional magmatic segregation and volcanism model is a generalisation of the asymptotic approximation in (Spencer et al., 2020). In it, melting is driven by the tidal dissipation model, which most closely follows the approach of Beuthe (2013), utilising a Maxwell viscoelastic rheology. Rheological parameters required by the tidal calculation are predicted by the segregation and volcanism model, completing the coupling of the two systems. The isostasy calculations utilise the equal-pressure formulation of (Hemingway and Matsuyama, 2017).

The dynamics are described by the magmatic segregation and volcanism model (Spencer et al., 2020) derive a system where tidal heating causes the formation of magma in the mantle that rises buoyantly toward the solid lithosphere (termed crust in that work). High magma overpressure just below the base of the lithosphere facilitates a transfer of magma from the pore space into a lithospheric magmatic plumbing system, which can be thought of as a system of dikes. Magma rising in this plumbing system can freeze into the cold, surrounding lithosphere, forming permanent magmatic intrusions, delivering both mass and energy to the surroundings. The rest of the magma in the plumbing system rises to the surface and erupts, imparting a compensating downward flux of the (now cold) erupted products. (Spencer et al., 2020) found that the delivery of heat from the freezing of magmatic intrusions is required to raise the temperature of cold, downwelling lithosphere such that a lithospheric thickness within observational constraints can be maintained. This concept of the emplacement of permanent magmatic intrusions is an important one in the present work and is discussed below.

The dynamic model is coupled to tidal dissipation to yield a consistent, three-dimensional structure. We use a spherically symmetric structure to calculate a three-dimensional heating rate. This heating rate distribution (which importantly is not spherically symmetric) is applied to a suite of column models, producing a three-dimensional structure. This structure is averaged over spherical shells and used to re-calculate the heating distribution. This processes is iterated until the heating-distribution converges, yielding the three-dimensional structures presented in this work. We utilise a Maxwell viscoelastic law despite the well-documented inability of such a rheological law to produce observed dissipation rates at realistic mantle viscosities (Giersson and Nimmo, 2016; Renard and Henning, 2018). We also neglect all lateral flow, justified by the long-wavelength of the tidal forcing; the one-dimensional columns are considered isolated. This is a significant simplification that we discuss below, and we note that future work should aim to analyse the propensity for lateral flow. We also inherit some of the assumption of (Spencer et al., 2020), namely that we ignore the chemical composition and as a consequence neglect the possibility of compositional convection in the mantle. Parameter values, where not explicitly stated, are given in table 1 of (Spencer et al., 2020),
though the shear viscosity used in the tidal heating calculation is different to that used in the column models, as explained below.

### 2.1 Magmatic segregation and volcanism

The model of [Spencer et al. (2020)] is based on conservation equations for mass, momentum, and energy in a compacting two-phase medium, together with conservation of mass in a magmatic plumbing system that transports magma through the solid lithosphere. Here, we make use of the simplified model described in appendix B of that paper. In the mantle, which is at the melting temperature \( T_m \), tidal heating produces melt, and mass conservation of the melt phase reads

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 q \right) = \frac{\psi}{\rho L}
\]

where \( q = K_0 \phi^n \Delta \rho g / \eta_l \) is the Darcy segregation flux related to the porosity \( \phi \), where \( \psi \) is the local volumetric heating rate (see section 2.2) and \( L \) is the latent heat. Here \( K_0 \) is a permeability constant, \( n \) is the permeability exponent, \( \Delta \rho \) is the density difference between the solid and liquid, and \( \eta_l \) is the magma viscosity (numerical values for these and other parameters are as in table 1 of [Spencer et al. (2020)]). The magma flux is therefore

\[
q = \frac{1}{\rho L} \frac{1}{r^2} \int_{r_m}^r \psi r^2 \, dr,
\]

where \( r_m \) is the base of the mantle. At the base of the lithosphere this flux is transferred to the plumbing system, in which the flux is denoted \( q_p \). Conservation of mass and energy in the lithosphere are described by

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 (u + q_p) \right) = 0,
\]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 q_p \right) = -M,
\]

and

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 u T \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{\psi}{\rho C} + M \left( T_m + \frac{L}{C} \right),
\]

where \( u \) is the solid velocity, \( T \) is the temperature, \( M \) is the emplacement rate (the rate at which magmatic intrusions remove material from the plumbing system), and \( C \) is the specific heat capacity. The final term in equation (4) represents the heating that emplacement provides to the downwelling lithosphere. The solution of equations (3)–(4) together determines the temperature profile in the lithosphere as well as the lithospheric thickness (see [Spencer et al. (2020)] for details).

In [Spencer et al. (2020)], we assumed a temperature-dependent parametrisation of the emplacement rate \( M \), but this assumption is problematic in the present case. With a coupled calculation of the tidal heating rate \( \psi \), there is very little tidal heating of the cold lithosphere, and hence more heat is required from emplacement to limit the growth of the lithosphere. Using the temperature-dependent form for emplacement, we find that there is too little heating of the lithosphere to avoid it becoming unreasonably thick. We therefore consider two alternative parametrisations of the emplacement rate. First, that magmatic emplacement is at a constant rate, and second, that magmatic emplacement is a function of the amount of material in the
plumbing system. To allow for both possibilities we take the emplacement rate to be

\[ M = \lambda_c + \lambda_q q_p, \]  

(5)

and explore cases where only one of \( \lambda_c \) or \( \lambda_q \) is non-zero at a time. Taking a constant emplacement rate \( (\lambda_c \neq 0 \text{ and } \lambda_q = 0) \) can be interpreted as modelling a system of dikes where the number of dikes is fixed but the flux through them varies. If emplacement is a function of contact area with the host rock, such a system could result in emplacement rate being independent of the magma flux. This is similar to, but more simple than the temperature dependence taken in Spencer et al. (2020). Taking emplacement to be proportional to the amount of melt in the plumbing system \( (\lambda_c = 0 \text{ and } \lambda_q \neq 0) \) can also be interpreted as a system of dikes, but where the dikes have equal fluxes and the number of dikes varies. As the number of dikes increases, the contact area with the host rock also increases, so the total emplacement rate increases. In summary, we consider cases where emplacement is positively related to, or independent of the magma flux. We do not consider the possibility of a negative relationship between emplacement rate and magma flux as we cannot conceive of a realistic physical system that this would represent.

2.2 Tidal heating

For the calculation of tidal heating we most closely follow the methodology of Beuthe (2013). Volumetric tidal dissipation averaged over an orbit is given by (Tobie et al., 2005)

\[ \psi(r, \theta, \varphi) = \frac{\omega_f}{2} \left[ \text{Im}(\tilde{\sigma}_{ij}) \text{Re}(\tilde{\epsilon}_{ij}) - \text{Re}(\tilde{\sigma}_{ij}) \text{Im}(\tilde{\epsilon}_{ij}) \right], \]  

(6)

where \( \omega_f = 4.11 \times 10^{-5} \text{ s}^{-1} \) is the orbital frequency, \( \tilde{\sigma}_{ij} \) and \( \tilde{\epsilon}_{ij} \) are the components of the complex stress and strain tensors, and summation over components \( i \) and \( j \) is implied. We calculate the complex stress and strain tensors using the propagator matrix approach detailed in Sabadini and Vermeersen (2004), and explained in appendix A of Roberts and Nimmo (2008).

The tidal potential that forces the system arises from consideration of a synchronous eccentric orbit, to first order in eccentricity. It is given by (Kaula, 1964; Tobie et al., 2005)

\[ \Omega = r^2 \omega_f^2 e \left[ -\frac{3}{2} P_2^2(\cos \theta) \cos(\omega_f t) + \frac{1}{4} P_2^2(\cos \theta) \left[ 3\cos(\omega_f t) \cos(2\varphi) + 4\sin(\omega_f t) \sin(2\varphi) \right] \right], \]  

(7)

where \( e = 4.1 \times 10^{-3} \) is the orbital eccentricity, \( \theta \) and \( \varphi \) are the colatitude and longitude (the latter being zero at the sub-Jovian point), \( t \) is the time, and \( P_2^2 \) and \( P_2^2 \) are associated Legendre polynomials.

To couple the tidal heating model to the dynamical model we follow the approach of Bierson and Nimmo (2016). We take the shear viscosity to be a function of temperature and porosity through the relationship (Katz, 2010)

\[ \eta = \eta_0 \exp \left[ \frac{E_A}{R_g \left( \frac{1}{T} - \frac{1}{T_0} \right)} - \Lambda \phi \right], \]  

(8)

where \( E_A = 3 \times 10^5 \text{ J/mol} \) is the activation energy, \( R_g \) is the gas constant, \( \eta_0 \) is a reference viscosity at the reference temperature \( T_0 \) (taken to be the melting point), and \( \Lambda = 27 \) is a positive constant. Temperature \( T \) and porosity \( \phi \) are extracted from the model in section 2.1 and averaged over spherical shells, so \( \eta \) depends only on radius \( r \). The value of \( \eta_0 \) used is chosen so that the total global rate of tidal dissipation approximately
matches the observed dissipation rate of $\sim 1 \times 10^{14}$ W (Lainey et al., 2009). It is well documented that a Maxwell viscoelastic constitutive law requires a very low viscosity to produce the amount of tidal heating observed in Io (Segatz et al., 1988; Tackley, 2001; Bierson and Nimmo, 2016; Steinke et al., 2020). We assume that this is a failure in the present understanding of the rheology that affects dissipative processes (Bierson and Nimmo, 2016; Renaud and Henning, 2018), rather than a reasonable assessment of Io’s long-timescale mantle viscosity. Bierson and Nimmo (2016) also take a porosity dependence of the elastic shear modulus, but we neglect this small effect in line with our simplified approach. We refer to the first coupled model, using (8), as the ‘mantle heating’ model.

Numerous previous works have considered the possibility that tidal dissipation is concentrated within a lower-viscosity asthenosphere (e.g., Segatz et al. (1988); Tackley (2001); Hamilton et al. (2013); Davies et al. (2015)). Such a dissipative layer does not arise in the above formulation, even when a large decompacting boundary layer is included in the dynamic model (Spencer et al., 2020), because the porosity dependence in a Maxwell viscoelastic model is too weak. In order to investigate the lithospheric thickness and long-wavelength topography implications of such a dissipation structure, we calculate an alternative ‘asthenospheric heating’ model, where the shear viscosity in a 300 km layer beneath the lithosphere is set to be a factor of 1000 lower than the rest of the mantle. In the asthenospheric heating model, we do not include the temperature and porosity dependence of shear viscosity; in this case the heating model is decoupled from the dynamical model. We do, however, set the shear viscosity in the cold lithosphere to be effectively infinite so no dissipation occurs there, consistent with the calculated dissipation structure in the coupled mantle heating model.

The tidal heating code has been benchmarked against the radial functions in figure 2 of Tobie et al. (2005), against the TiRADE software used in Roberts and Nimmo (2008), and by reproducing figures 8 and 10 of Segatz et al. (1988).

### 2.3 Isostasy calculations

For our isostasy calculations we follow Hemingway and Matsuyama (2017) in using an equal-pressure formulation of isostasy in spherical coordinates. This assumes that compensated columns have equal pressures at their bases (the compensation depth, $r_{ced}$). Equal pressure isostasy assumes that we have

$$P = \int_{r_{ced}}^{R} \rho g \, dr,$$

where $P$ is a constant (independent of latitude and longitude), $\rho$ is the density profile, and $R$ is local planetary radius. We take gravity $g$ to be uniform for simplicity, a reasonable assumption given the likely heavy core.

We assume that density is a function of temperature only

$$\rho = \rho_0 [1 - \alpha (T - T_m)],$$

where $\alpha = 3 \times 10^{-5}$ K$^{-1}$ is the coefficient of thermal expansion, and $\rho_0$ is the reference density at the melting temperature $T_m$. It is at this point that the distinction between the crust and lithosphere becomes important for Io. In terrestrial systems, the base of the crust represents a petrological boundary between the low-density crust and the high-density mantle. Here, however, the boundary is simply an isotherm. Effective recycling of lithospheric material into the partially molten mantle is expected to remove any significant
compositional variation across this boundary (Spencer et al., In Review). As such, we expect that the cold lithosphere is more dense than the underlying hot, partially molten mantle. This inverted density structure is assumed to be stable because of the high viscosity of the rigid upper lithosphere. We also neglect density differences associated with the presence of low-density melt in the upper mantle; the density of the mantle is \( \rho_m = \rho_0 \). By considering density variations within the lithosphere, this calculation is closer to Pratt isostasy than Airy isostasy, though we are clear that this is not true Pratt isostasy as the base of the lithosphere isn’t considered to be flat (Fowler, 2004).

The integral in equation (9) can be split at the base of the lithosphere, which has a thickness \( l \) to write

\[
P = \rho_0 g (R - l - r_{ccd}) + \int_0^l \rho g \, dz,
\]

where \( z = R - l \) is the distance downward from the surface. Both \( l \) and \( \rho \) (in terms of temperature \( T \)) are known from the magmatic segregation and volcanism model, so this expression can be re-arranged to determine the variable radius \( R \) relative to its spatial average \( \bar{R} \). Since \( P \) and \( r_{ccd} \) are constant, we obtain this topography \( h = R - \bar{R} \) as

\[
h = l + \int_0^l \frac{\rho}{\rho_0} \, dz + \text{constant},
\]

where the constant is chosen to make the spatial average of \( h \) zero.

### 3 Results and discussion

Figure 1 shows model solutions for the lithospheric temperature distribution, mantle porosity, and tidal heating distribution at Io’s north pole and three points around the equator, for the (coupled) mantle heating model and the (de-coupled) asthenosphere heating model. In the mantle-heating case (figure 1a–c), heating rate is highest at the poles, and lowest at the sub- and anti-Jovian points, whereas in the asthenosphere-heating case (figure 1d–f), heating rate is highest at the sub- and anti-Jovian points, and lowest at the poles. A higher heating rate leads to increased melt production, though for the permeabilities used here melt fractions only vary by \( \sim 1\% \). Lower permeabilities lead to higher porosities and greater porosity variation between localities (Moore, 2001; Bierson and Nimmo, 2016). Throughout this work, eruption rate and surface heat flux are a proxy for radially integrated heating rate.

When the emplacement rate is a constant (\( \lambda_c \neq 0 \) and \( \lambda_q = 0 \)), integration of equation (3b) in the lithosphere yields

\[
q_p = \frac{q_e R^2}{2} + \frac{\lambda_c}{3} \left( \frac{R^3}{r^2} - r \right), \quad R - l \leq r \leq R,
\]

where \( q_e = q_p(r = R) \) is the eruption rate. Assuming negligible surface conduction, the eruption rate must be given by a column-wise energy balance as (Spencer et al., 2020)

\[
q_e = \frac{1}{R^2 (\rho L + \rho C(T_m - T_s))} \int_{r_m}^R \psi r^2 \, dr,
\]

where the integral is the total tidal heating delivered to the column. From equation (2), the plumbing flux
at the base of the lithosphere is

\[ q_p(r = R - l) = \frac{1}{(R - l)^2 \rho L} \int_{r_m}^{R-l} \psi r^2 dr. \]  

(15)

Since negligible tidal heating takes place in the lithosphere (figure 1), the integrals in (14) and (15) are essentially identical. Thus, equating (15) with (13) at the base of the lithosphere yields an analytical expression for the lithospheric thickness in terms of the local eruption rate

\[ l = R - R \left(1 - \frac{3q_e C(T_m - T_s)}{R \lambda_c L}\right)^{1/3}. \]  

(16)

A Taylor expansion of the term in brackets provides some intuition into this expression. Expanding to the first non-trivial term yields

\[ l \approx \frac{C(T_m - T_s) q_e}{\lambda_c L}. \]  

(17)

The thickness of the lithosphere is controlled by the balance between latent heat release in the lithosphere and sensible heat loss at the surface. The greater the temperature difference between erupting lava and the surface, the more heat that must be provided to downwelling material to raise it to its melting point. As the eruption rate increases, material downwells more quickly, and with no corresponding increase in emplacement rate, the thickness of the lithosphere grows. This effect can be seen in the main panels of figure 1. A higher rate of emplacement means that downwelling material is heated more rapidly, reducing the lithospheric thickness. We note that an average lithospheric thickness can be estimated using the modelled global average eruption rate of Spencer et al. (2020).

The insets in panels a and d of figure 1 show the lithospheric temperature profiles when emplacement rate is proportional to the plumbing system flux (\(\lambda_c = 0\) and \(\lambda_q \neq 0\)). In this case equation (3b) can be integrated to give

\[ q_p = \frac{R^2 q_e}{r^2} e^{-\lambda_q (R-r)}. \]  

(18)

Again assuming negligible surface conduction and equating equation (18) to the total melt production in the interior (equation (15)) gives an expression for the lithospheric thickness,

\[ l = \frac{1}{\lambda_q} \ln \left(1 + \frac{C(T_m - T_s)}{L}\right). \]  

(19)

Interestingly, this is independent of the melting rate, so lithospheric thickness is expected to be virtually constant when emplacement rate is proportional to the plumbing system flux. A Taylor expansion of (19) to first order yields equation (17) but with \(q_e/\lambda_c\) replaced by \(1/\lambda_q\), illustrating that in this case, the relationship between eruption flux and emplacement is fixed. The small variations in lithospheric thickness seen in the insets in panels a and d of figure 1 are due to conduction (which is neglected in arriving at the estimate, equation (19)), with higher heating rates producing thinner lithospheres.

Figure 2 shows lithospheric thickness, eruption rate, and topography as a function of latitude and longitude in the coupled mantle-heating model. The top row of figure 2 shows the case where emplacement rate is a constant and the bottom row shows the case where emplacement rate is proportional to the plumbing system flux. A constant emplacement rate means that lithospheric thickness correlates with the eruption rate, as specified by equation (16). Lithospheric thickness varies by about 25 km, with the most pronounced
Figure 1: Lithospheric temperature profiles, mantle porosities, and tidal heating distributions at the poles and three points around the equator, for the mantle heating (a–c) and asthenosphere heating (d–f) models, with constant emplacement rate, $\lambda = 1.66 \, \text{Myr}^{-1}$. Panels a and d show the temperature variation in the upper-most mantle and lithosphere, indicated by the green region in the other panels. Where radially integrated heating rate is highest, melt production and porosity is highest. This results in an increased eruptive flux and the growth of a thicker lithosphere. The insets in panels a and d show the case when emplacement rate is proportional to plumbing system flux, with $\lambda = 0.05 \, \text{km}^{-1}$. In this case, lithospheric thicknesses vary weakly (porosity and tidal heating profiles in the mantle are almost exactly the same as constant emplacement rate). Dots indicate estimates of lithospheric thickness using equations (17) (panels a and d) and (19) (insets). Differences between the analytical estimates and the model are caused by conduction, which is neglected in the analytical estimates.

variation being between the thick polar lithosphere and the thin equatorial lithosphere. Our isostatic model assumes that density is only a function of temperature, and so the cold lithosphere must be more dense than the underlying, partially molten mantle. This results in topographic highs where the lithosphere is thinnest. The coupled, mantle-heating model with constant emplacement rate predicts long-wavelength topography with an amplitude of about 250 m. In the case where emplacement rate is proportional to the amount of material in the plumbing system, shown in the bottom row of figure 2, the lithospheric thickness only varies by a couple of kilometres and the amplitude of long-wavelength topography is $< 40$ m. This can be understood through equation (19): increased heating and the resultant increased eruption rate is balanced by increased emplacement, resulting in an almost uniform lithospheric thickness. In this case, the long-wavelength lithospheric thickness and topography variations are a result of different conductive heat.
fluxes and so lithospheric thickness is anti-correlated with eruption rate (Ross et al., 1990; Steinke et al., 2020).

Figure 2: Solutions for lithospheric thickness, eruption rate, and topography in the case of coupled dynamics and tidal heating. Tidal heating is concentrated in the lower mantle in the coupled model, producing maximum eruption rates at the poles (see figure 1). Panels a–c show the case where emplacement rate is constant, and panels d–f show the case where emplacement rate is proportional to the plumbing system flux. Constant emplacement rate predicts a correlation of lithospheric thickness with eruption rate (or heat loss), and topographic lows where heat flux is high. An emplacement rate proportional to plumbing system flux predicts a relatively uniform lithospheric thickness and little long-wavelength topography.

Figure 3 shows the same plots as figure 2 but for the case of asthenospheric heating. All of the relationships between heating rate, eruption rate, lithospheric thickness, and topography are the same in this case, but the pattern of dissipation and so the pattern of the plotted solutions is different. Asthenospheric heating predicts higher eruption rates at the equator. If emplacement rate is constant, this predicts a thicker lithosphere at the equator (amplitude $\sim 30$ km), and topographic highs at the poles (amplitude $\sim 300$ m). If emplacement rate is proportional to the amount of material in the plumbing system, lithospheric thickness is much more uniform (amplitude $\sim 6$ km) and topography is reduced (amplitude $\sim 90$ m), with lithospheric thickness variations being controlled by variation in conductive heat fluxes.

Assuming dominantly vertical flow — a significant assumption that we discuss below — the global pattern of heat flow should be reflective of the tidal heating distribution, as has been noted elsewhere (e.g., Segatz et al. 1988; Tackley 2001; Veeder et al. 2012; Davies et al. 2015). The primary means to distinguish between lower mantle and asthenospheric heating models is on the basis of heat flux. Lower mantle heating predicts higher polar heat flux, whereas asthenosphere heating predicts higher equatorial heat flux. With the present dearth of polar observations, this is a difficult distinction to make. Rigorous observation of Io’s poles is required to understand which mode of heating is more likely to be occurring. However, if the mode
Figure 3: Solutions for lithospheric thickness, eruption rate, and topography in the case of asthenosphere heating. Panels a–c show the case where emplacement rate is constant, and panels d–f show the case where emplacement rate is proportional to the plumbing system flux. Relationships between lithospheric thickness, eruption rate, and topography are the same as in figure 2, but patterns and amplitudes are different due to the different heating mode.

of emplacement can be established, lithospheric thickness and topography can serve as a useful proxy for long-timescale heat flux.

This work predicts that the long-wavelength variations in lithospheric thickness should either correlate with the long-timescale eruption rate/heat flux, or be weakly anti-correlated, as summarised schematically in figure 4. In the constant emplacement rate model, we predict that lithospheric thickness correlates with eruption rate. An explanation for why emplacement would be independent of magma flux is that volcanic conduits are not formed by magma pressure at depth, but rather tectonic processes in the lithosphere. Io's eruption and burial tectonics are thought to form mountains by thrust faulting (McKinnon et al., 2001; Kirchoff and McKinnon, 2009). If, for example, such faults can act as conduits for magma ascent, freezing of ascending magma on their walls may be largely independent of the flux through the conduit. Alternatively, in the flux-proportional emplacement rate model, we predict that long-wavelength lithospheric thickness varies by only a few kilometers, and is weakly anti-correlated with heat flux. A rationale for why emplacement rate would be proportional to volcanic plumbing flux may be that volcanic conduits are created by overpressured magma at the base of the lithosphere. It is plausible that higher melt production in the interior would lead to a larger number of conduits. If magma in each of these conduits has a chance of stalling within the lithosphere, this would imply a positive relationship between lithospheric magma flux and emplacement rate.

The flux-proportional emplacement rate model makes predictions for variations in lithospheric thickness that are similar to the results of Steinke et al. (2020). When comparing this work to Steinke et al. (2020), it is
important to note that whilst both can predict a conductive control on lithospheric thickness variations, the controls on the absolute values of lithospheric thickness are different. In this work the lithospheric thickness is primarily controlled by the rate of magmatic emplacement, whereas in Steinke et al. (2020) the lithospheric thickness is controlled entirely by conduction through a stagnant lid. To address the relative importance of convective heat transport in the mantle likely requires a model that couples two-phase flow and convection, a significant challenge due to the different timescales on which these processes operate.

The proposed link between lithospheric thickness and topography provides a means of relating more readily-obtainable observations (topography) to the predictions of lithospheric thickness in works like this one and Steinke et al. (2020). Topography can be compared to eruption rates and volcanic heat fluxes to clarify the heat-transfer and emplacement mechanisms in the lithosphere. Alongside with recent work that demonstrates a way to constrain interior structure from libration amplitudes (VanHoolst et al., 2020), this provides a powerful means to investigate Io’s interior structure and heating distribution. We predict long-wavelength topographic highs where the lithosphere is thin. However, our isostatic calculations assume that density variations are due only to temperature differences, and that there is no compositional boundary at the base of the lithosphere. It is likely that the compositional profile in the lithosphere is complex, reflecting shallow magma fractionation, sulfur cycling, and other processes. If the vertical structure of the lithosphere is approximately uniform with latitude and longitude, and simply scaled to lithospheric thickness, the results of this work should be largely unchanged. If, however, there is significant variation in lithosphere composition with latitude and longitude, the applicability of the isostatic model presented here would be reduced. It is not clear, however, that any such variation would mirror the degree-two tidal forcing, and so may average out on the long wavelengths considered here.

White et al. (2014) created a partial stereo-topographic DEM of Io that found a system of longitudinally ar ranged alternating basins and swells near the equator, with amplitudes $\sim 1$–2 km and a wavelength $\sim 400$ km.
This amplitude is significantly greater than the largest topography predicted here (order hundreds of meters). Potential explanations for this discrepancy are that there are lateral compositional differences that result in significant additional isostatically compensated topography, or that dynamic topography caused by upwelling mantle plumes is significant (Tackley et al. 2001). It is important to note however that there are considerable discrepancies between stereo-derived and limb-profile-derived long-wavelength topography (White et al. 2014), and hence that long-wavelength topography is not well constrained. Improved observations of long-wavelength topography, particularly in the polar regions, are required to make robust comparisons between modelled topography and data.

A primary limitation of this work is the neglect of lateral flow in either the lithosphere or mantle. Differences in lithospheric thickness are expected to be counteracted by deformation of the lithosphere. Such calculations are common in studies of the ice-shells of icy satellites (Stevenson 2000; Nimmo and Stevenson 2001; Nimmo 2004), where there is a clear rheological and density transition at the base of the shell. The application of such a model to Io is not straightforward because rheological and density transitions are expected to be more gradual (Spencer et al. In Review). It is not clear whether there is an easily defined petrological ‘crust’ of Io. Nonetheless such lateral flow is possible, and would be best investigated by a two-dimensional model of upper Io. Lateral flow is also possible in the partially molten mantle. Pressure gradients would be expected to drive flow of the mobile magma phase. Pressure gradients could be produced by processes such as different melting rates or spatially variable extraction rates to the lithosphere. An investigation of lateral melt flow would likely require a two-dimensional model of the partially molten mantle. Here we simply note that the relationships proposed in this model are expected to hold if vertical motion is much greater than lateral motion, as generally expected in Io’s eruption and burial tectonics at long wavelengths.

4 Conclusions

We have demonstrated how spatially variable tidal heating leads to long-wavelength variations in lithospheric thickness in a model of magmatic segregation and volcanic eruptions. Our models predict that such variations are controlled by how magma intrudes into the lithosphere. If permanent magmatic intrusions form at a rate independent of the magma flux through volcanic conduits, the lithosphere should be thickest where tidal heating is greatest. In this case the lithosphere thickness can vary by 10s of km. If however magmatic intrusions form at a rate proportional to the magma flux through volcanic conduits, lithospheric thickness will only vary by a few km, and will be anti-correlated with eruption rates. We also predict that if density differences are predominantly derived from temperature differences, then areas of thin lithosphere will sit on topographic highs. Improved observational constraints on eruption rates, heat fluxes, and long-wavelength topography, particularly at Io’s poles, will help distinguish between different models for the controls on lithospheric thickness.

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