Characteristics of electromagnetic wave transmission and reflection from isotropic plasma coated circular nihility cylinder

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Characteristics of electromagnetic wave transmission and reflection from isotropic plasma coated circular nihility cylinder

ABSTRACT

Theoretical investigation has been carried out to study the scattering and transmission of electromagnetic energy from isotropic plasma coated circular nihility cylinder. The physical modeling of the problem is done in frame work of classical scattering theory. All the fields are expanded in terms of cylindrical vector wave functions (CVWFs). The analytical as well as numerical solution for both Transverse Electric (TE) and Transverse Magnetic (TM) polarizations is presented. The unknown scattering coefficients are computed by implementing the boundary conditions at each interface (free space/plasma and plasma/nihility). The influence of plasma parameters i.e., plasma density ($N_D$) and effective collision frequency ($v$) on the bistatic echo width is analyzed and it is concluded that the plasma coating can be used to control and tune the scattering amplitude. Further it is also concluded that for the case $\omega > \omega_p$ the scattering amplitude is much less than for the case $\omega < \omega_p$. Under special conditions, the results of present work compared with the published literature and good agreement is found.

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I. INTRODUCTION

Electromagnetic scattering has potential applications in the field of optics, photonics and optoelectronics i.e., optical sensing, image processing, remote sensing, structural defect detection and optical cloaking. In addition, different types of artificial materials, also known as metamaterials, have been realized and studied in literature, to achieve the extraordinary electromagnetic properties. Nihility medium is one of them, an electromagnetic nilpotent, in which permeability and permittivity equals to zero. This electromagnetic trinity has excited many researchers and opticians due to its novel properties. The electromagnetic scattering from nihility objects is also the concern of many researchers i.e., scattering of electromagnetic waves from nihility sphere is discussed theoretically, in the frame work of Mie theory, and it was reported that the nihility sphere has large extinction coefficient as compared to perfect electric conductor (PEC) sphere, which clearly shows that the electromagnetic response of nihility medium is quite different as compared to PEC, while in both media electromagnetic power cannot be entered.

Further, electromagnetic scattering from infinite circular nihility cylinder is also studied. To get proper control on electromagnetic scattering, the different coatings on nihility circular cylinder are theoretically studied and analyzed; metamaterial coated nihility circular cylinder of infinite length is studied and effect of different types of meta-layers i.e., double positive (DPS) and double negative (DNG), on bistatic echo width is reported. To achieve further electromagnetic transparency and control, the coating of optically active material i.e., chiral metamaterial is applied on nihility cylinder, and electromagnetic scattering from chiral coated nihility
The cylindrical nihility cylinder is studied analytically as well as numerically. The chiral metamaterials have ability to rotate the plane of polarization of the incident electromagnetic wave, i.e., a plane wave is transformed into the circular waves, where the rotation of polarization plane highly depends upon the dimension of chiral objects and the incident wave frequency. To extend the previous studies, Hamid et al., discussed the more complicated and general problem of scattering i.e., electromagnetic scattering from dielectric nihility elliptical cylinder and reported the different parameters effects on the scattering amplitude. Plasma recently realized as a metamaterial and found numerous applications in negative refractive index materials (NIMs), photonic crystals and communication. The collection of free electrons and ions, which shows the collective response to external fields, is regarded as plasma. Further to put advancements in field of metamaterials, the Plasma based composite metamaterials have been studied extensively and found numerous applications in the defense technology, photonics, functional metamaterials, microwave controlling and communication. The electromagnetic scattering from plasma coated objects (cylinders/spheres) are extensively studied in literature and it is reported that the plasma coatings can be used for the controlling and tuning the scattering amplitude, which is quite useful in the target protection and microwave controlling devices.

The main advantage of plasma coating over the other metamaterial coating is that there is no need of specific structure to control the scattering and flow of electromagnetic energy, one just have to tune the plasma to tune the electromagnetic properties. Keeping in view the above narrated applications and practical importance of the plasma coating, in this research work the electromagnetic scattering from plasma coated nihility circular cylinder is studied. To accomplish the work, the classical scattering theory is used. All the fields are expanded in terms of cylindrical vector wave functions (CVWFs). Both types of polarization i.e., transverse electric (TE) and transverse magnetic (TM) are discussed. The scattering coefficients are computed by applying the appropriate boundary conditions at each interface i.e., free space/plasma and plasma/nihility. Further, the effect of plasma parameters i.e., plasma density ($N_D$) and effective collision frequency ($\nu$) on the bistatic echo width is analyzed. In the whole study, time dependence is taken as $e^{-j\omega t}$.

**II. ANALYTICAL FORMULATIONS AND METHOD**

In this section, the analytical formulation of the above defined scattering problem is presented. The geometry of the problem is depicted in the fig. 1. The whole space is divided in three regions so that, region I ($\rho > b$) is the free space, region II ($\rho \leq b$) represents the isotropic, homogeneous plasma coating, while the region III ($\rho \leq a$) is the dielectric/nihility circular cylinder of infinite length. For region I, II and III the wave numbers are characterized as; $k_1 = \omega \sqrt{\epsilon_0}\eta_1$, $k_2 = \omega \sqrt{\epsilon_0\eta_1}$ and $k_3 = \omega \sqrt{\eta_1\eta_2\mu_2}$. The plasma relative permittivity $\epsilon_p = 1 - \frac{n_D^2}{\varepsilon_0\mu_0}$ is complex number, and fairly depends upon the incident frequency $\omega$, plasma density $N_D$ and plasma oscillation $\omega_p = \sqrt{\frac{n_D^2}{\varepsilon_0\mu_0}}$

All the incident, scattered and transmitted fields are represented and expanded in terms of CVWFs, through which the symmetry between the coordinates is secured. There are two possibilities for the normally incident fields with respect to the cylinder, either the electric field is parallel to the cylinder i.e., transverse magnetic polarization (TM) or perpendicular to the cylinder i.e., transverse electric polarization (TE). In the following, both types of polarizations states and their respective analytical formulations are presented.

**A. Transverse magnetic polarization (TM)**

The incident fields in terms of cylindrical coordinates ($\rho, \varphi$) are the following, when the electric field is perpendicular to the cylinder’s axis i.e., $x$-axis

$$E_{1z} = E_0 \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f_n(k_1\rho)e^{jn\varphi}$$

$H_{1\varphi} = -\frac{E_0}{j\eta_1} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f_n(k_1\rho)e^{jn\varphi}$

where $\eta_1 = \sqrt{\frac{\nu}{\omega}}$ is the intrinsic impedance of the free space, while $f_n(.)$, $H_n^{(1)}(.)$ and $H_n^{(2)}(.)$ are the special functions i.e., Bessel function of first kind, Hankel function of first and second kind respectively, and the primes on these special functions stands for the derivative of the function with respect to its whole argument. The scattered fields in region I are given as

$$E'_{1z} = E_0 \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f' n\rho H_n^{(1)}(k_1\rho)e^{jn\varphi}$$

$H'_{1\varphi} = -\frac{E_0}{j\eta_1} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f' n\rho H_n^{(1)}(k_1\rho)e^{jn\varphi}$

where $a_n$ is the unknown scattering coefficient of co-polarized field, and the fields in region II i.e., isotropic plasma coating with unknown coefficients $b_n$ and $c_n$, are

$$E_{2z} = E_0 \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left[b_n H_n^{(2)}(k_2\rho) + c_n H_n^{(2)}(k_2\rho)\right]e^{jn\varphi}$$

$H_{2\varphi} = -\frac{E_0}{j\eta_2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left[b_n H_n^{(2)}(k_2\rho) + c_n H_n^{(2)}(k_2\rho)\right]e^{jn\varphi}$

The $\eta_2 = \sqrt{\frac{\nu_2}{\omega}}$ is the impedance of plasma coating, while transmitted electromagnetic field in the dielectric/nihility cylinder...
along with the unknown coefficient \( d_n \) and impedance \( \eta_3 = \sqrt{\frac{\mu_3}{\varepsilon_3}} \).

are taken as

\[
E_{3z} = E_0 \sum_{n=-\infty}^{\infty} \int d_n J_n(k_3 \rho) e^{jn(\phi)}
\]

(7)

\[
H_{3\phi}^i = -\frac{E_0}{j\eta_3} \sum_{n=-\infty}^{\infty} \int d_n J_n^1(k_3 \rho) e^{jn(\phi)}
\]

(8)

B. Transverse electric polarization (TE)

The electromagnetic fields for the TE polarization state can be achieved by using the duality theorem on the above TM polarization equations as in Refs. 8 and 9, the incident fields are

\[
E_{i\phi} = -E_0 \sum_{n=-\infty}^{\infty} \int \alpha_n J'_n(k_1 \rho) e^{jn(\phi)}
\]

(9)

\[
H_{i\phi}^z = -\frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} \int \beta_n J_n(k_1 \rho) e^{jn(\phi)}
\]

(10)

while the scattered fields are

\[
E_{s\phi} = -E_0 \sum_{n=-\infty}^{\infty} \int \alpha'_n H'_n(k_1 \rho) e^{jn(\phi)}
\]

(11)

\[
H_{s\phi}^z = -\frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} \int \beta'_n H_n(k_1 \rho) e^{jn(\phi)}
\]

(12)

and transmitted fields in the plasma coating are

\[
E_{2\phi} = -E_0 \sum_{n=-\infty}^{\infty} \left[ \alpha'_n H'_n^2(k_3 \rho) + \beta'_n H_n^2(k_3 \rho) \right] e^{jn(\phi)}
\]

(13)

\[
H_{2\phi} = -\frac{E_0}{j\eta_3} \sum_{n=-\infty}^{\infty} \left[ \alpha'_n H'_n^2(k_3 \rho) + \beta'_n H_n^2(k_3 \rho) \right] e^{jn(\phi)}
\]

(14)

The fields in the dielectric/nihility cylinder are

\[
E_{3\phi} = -E_0 \sum_{n=-\infty}^{\infty} \int \alpha'_n J'_n(k_3 \rho) e^{jn(\phi)}
\]

(15)
In above equation the $a'_n$, $b'_n$, $c'_n$, and $d'_n$ are the unknown scattering coefficients, which are calculated by applying the appropriate boundary conditions at each interface i.e., free space/plasma coating and the plasma/nilility cylinder, as in Refs. 10–13. After the computation of scattering coefficients, the bistatic echo width, which is the ratio of the incident power to the scattered power, is calculated by the following formula

$$\sigma = \frac{2\pi}{W_i} \frac{W_s}{|E|^2}$$

The normalized bistatic echo width for TE case

$$\frac{\sigma}{\lambda_0} = \frac{2}{\pi} \sum_{n=-\infty}^{\infty} \left| a_n e^{jn}\phi \right|^2$$

and for TM case

$$\frac{\sigma}{\lambda_0} = \frac{2}{\pi} \sum_{n=-\infty}^{\infty} \left| a_n e^{jn}\phi \right|^2$$

III. RESULTS AND DISCUSSIONS

The numerical results of above formulated analytical solution are presented in this section. The assumption regarding constitutive parameters $(\varepsilon_r, \mu_r)$ in region III, which are used to realize the nilility, perfect electric conductor (PEC) and perfect magnetic conductor (PMC) are $\varepsilon_r \rightarrow 0$, $\mu_r \rightarrow 0$, $\mu_r \rightarrow \infty$, and $\varepsilon_r = 1$, $\mu_r = 1$, and $\varepsilon_r = 1$, $\mu_r = \infty$ respectively. Some of the results are compared with the literature under special conditions, to check the accuracy in our work as well as the numerical coding of program, and we found that our results are in accordance with the published literature which confirms the reliability and accuracy of our work, as presented in fig. 2. Under first condition i.e., isotropic plasma coating is replaced by the metamaterial coating with constitutive parameters i.e., $\varepsilon_r = 9.8$, $\mu_r = 1$, and we get the same results as in Ref. 10 for both TE and TM polarization, and our scattering problem transforms into the electromagnetic scattering from metamaterial coated nilility cylinder. For the further generalization and the accuracy, under second condition i.e., by replacing the inner nilility cylinder by PEC cylinder, our results are found to be in very good agreement with, as shown in fig. 3.
After the comparison with the published literature, further results regarding the plasma coated nihility circular cylinder are presented. The electromagnetic properties of plasma are fairly dependent on the plasma frequency ($\omega_p$), plasma density ($N_D$) and effective collision frequency ($\nu$), so the influence of these parameters on the scattering amplitude is discussed in further results. In addition to these analyses, the influence of state of polarization i.e., TE and TM, of incident electromagnetic wave is also presented separately.

The fig. 4 and 5 represents the influence of plasma density ($N_D$) on the bistatic echo width of plasma coated nihility circular cylinder for TE and TM polarization respectively. The fig. 4(a) represents the case; when the incident frequency of TM wave is greater than the plasma frequency i.e., $\omega > \omega_p$ in this case the plasma behaves as DPS material. For the case $\omega < \omega_p$ the relative permittivity of plasma is negative, it behaves as MNG material, as given in fig. 4(b). The fig. 5 deals with the TE polarization of incident wave; the fig. 5(a) and 5(b) represents the case $\omega > \omega_p$ and $\omega < \omega_p$ respectively. It is obvious from these results that with the increase of plasma density ($N_D$) the scattering width or bistatic echo width starts increasing. This similar behavior is also reported by Naito et. al., in Ref. 22, for plasma coated metallic cylinder, the scattering width increases with increase in the ratio of $\omega_p/\omega$. Similar trends on influence of plasma density, have been studied in Refs. 23 and 24 which further confirms the consistency and accuracy of present work. It can be concluded from these results that by changing the plasma density, the scattering width or amplitude can be decrease or increase, which gives us the ability to tune and control the scattering amplitude.

It is obvious from the fig. 6 & 7 that the scattering amplitude is also sensitive to the effective collision frequency ($\nu$) between neutral particles and electrons in the plasma. With the increase in the effective collision frequency, the bistatic echo width decreases with respect to the angle between the scatterer and source. The fig. 6 represents the comparison between bistatic echo widths of TM polarized wave under different values of effective collision frequency of plasma while the Fig. 7 represents the comparison of bistatic echo widths of TE polarized wave. The electromagnetic scattering of TM polarized incident wave having frequency above and below the plasma frequency is discussed in fig. 6(a) and 6(b) respectively. Similar cases i.e., $\omega > \omega_p$ and $\omega < \omega_p$ for the TE polarized incident wave...
is discussed in fig. 7(a) and (b) respectively. This is clear from these comparisons that the plasma coating has much more ability of tuning as compared to other metamaterial coating, for the case $\omega > \omega_p$ the scattering amplitude is much less than for the case $\omega < \omega_p$. So by changing the plasma parameters (plasma density and effective collision frequency), on can easily tune and control the electromagnetic (microwave) energy.

In fig. 8, a general comparison between the bistatic echo widths of the plasma coated nihility/PEC/PMC circular cylinders is presented for both TE and TM polarizations. It is clear that the scattering response of plasma coated nihility cylinder is same for both TE and TM polarization while for the case of Plasma coated PEC and PMC cylinders the electromagnetic response of TE and TM is reverse to each other.

IV. CONCLUSIONS

To further explore the importance of present work, some conclusions are drawn from the above plots, which are given below;

- The bistatic echo width is found to be sensitive to the plasma parameters and it is concluded that the scattering echo width can be controlled by the choosing the appropriate plasma parameters i.e., plasma density ($N_D$) and effective collision frequency ($\nu$).
- However, the scattering amplitude is different for TE and TM polarization but the electromagnetic response against plasma parameters is same for both polarizations.
- For case $\omega > \omega_p$ the scattering amplitude is much less as compared to the case $\omega < \omega_p$.
- Furthermore, the scattering amplitude can also be controlled by the appropriate type of the cylinder. For TM and TE polarization, the plasma coated PMC and PEC are good candidates to increase the stealth capability respectively.
- Moreover, the present scattering problem is general for the dielectric coated nihility/PEC and PMC cylinder, as well as plasmonics coatings.
- The present work can be used for the achievement of electromagnetic transparency and clocking by plasma coatings.

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