In this talk I summarize published work on a systematic operator analysis for fermion masses in a class of effective supersymmetric SO(10) GUTs. Given a minimal set of four operators at $M_G$, we have just 6 parameters in the fermion mass matrices. We thus make 8 predictions for the 14 low energy observables (9 quark and charged lepton masses, 4 quark mixing angles and $\tan \beta$). Several models, i.e. particular sets of dominant operators, are in quantitative agreement with the low energy data. In the second half of the talk I discuss the necessary ingredients for an SO(10) GUT valid below the Planck (or string) scale which reproduces one of our models. This complete GUT should still be interpreted as an effective field theory, i.e. perhaps the low energy limit of a string theory.

$^a$This work is in collaboration with G. Anderson, S. Dimopoulos, L.J. Hall, and G. Starkman.

$^b$These are preliminary results of work in progress with Lawrence Hall.

1. Introduction

The Standard Model[SM] provides an excellent description of Nature. Myriads of experimental tests have to date found no inconsistency. Eighteen phenomenological parameters in the SM are necessary to fit all the low energy data[LED]. These parameters are not equally well known. $\alpha, \sin^2(\theta_W), m_e, m_\mu, m_\tau$ and $M_Z$ are all known to better than 1% accuracy. On the other-hand, $m_c, m_b, V_{us}$ are known to between 1% and 5% accuracy, and $\alpha_s(M_Z), m_u, m_d, m_s, m_t, V_{cb}, V_{ub}/V_{cb}, m_{Higgs}$ and the Jarlskog invariant measure of CP violation $J$ are not known to better than 10% accuracy. One of the main goals of the experimental high energy physics program in the next 5 to 10 years will be to reduce these uncertainties. In addition, theoretical advances in heavy quark physics and lattice gauge calculations will reduce the theoretical uncertainties inherent in these parameters.

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Already the theoretical uncertainties in the determination of $V_{cb}$ from inclusive B decays are thought to be as low as 5%. Moreover, lattice calculations are providing additional determinations of $\alpha_s(M_Z)$ and heavy quark masses.

Accurate knowledge of these 18 parameters is important. They are clearly not a random set of numbers. There are distinct patterns which can, if we are fortunate, guide us towards a fundamental theory which predicts some (if not all) of these parameters. Conversely these 18 parameters are the LED which will test any such theory. Note, that 13 of these parameters are in the fermion sector. So, if we are to make progress, we must necessarily attack the problem of fermion masses.

In the program discussed in this talk, we define a procedure for finding the dominant operator set reproducing the low energy data. In the minimal operator sets we have just six parameters in the fermion mass matrices. We use the six best known low energy parameters as input to fix these six unknowns and then predict the rest. These theories are supersymmetric (SUSY) SO(10) grand unified theories (GUTs). In the next two sections I want to briefly motivate these choices.

2. Why SUSY GUTs?

Looking back at the history of particle physics, it is clear that much of our understanding comes from using symmetries. This is because, even without a complete understanding of the dynamics, symmetries can be used to relate different observables. Here too we want to correlate the known low energy data, the three gauge couplings and the fermion masses and mixing angles. We want to describe these 16 parameters in terms of fewer fundamental numbers. GUTs allow us to do just that. In fact using this symmetry we can express the low energy data as follows –

\[
\text{Observable} = \text{Input parameters} \times \text{Boundary condition at } M_G \times \text{RG factor}
\]

where the observable is the particular low energy data we want to calculate, the input parameters is the set of fundamental parameters defined at the GUT scale and the last factor takes into account the renormalization group running of the experimental observable from $M_G$ to the low energy scale. The grand unified symmetry SU(5) (or SO(10), E(6) etc.) determines the boundary conditions at $M_G$. These are given in terms of Clebsch-Gordan coefficients relating different observables. Of course, these relations are only valid at the GUT scale and the RG equations are necessary to relate them to experiment. It is through the RG equations that supersymmetry enters. We will assume that only the states present in the minimal supersymmetric standard model (MSSM) are in the theory below $M_G$. We assume this because it works. Consider the GUT expression for the gauge couplings –

\[
\alpha_i(M_Z) = \alpha_G \cdot R_i(\alpha_G, \frac{M_G}{M_Z})
\]

where the boundary condition is $R_i(\alpha_G, 1) \equiv 1 + \cdots$. The input parameters are $\alpha_G$ and $M_G$ and the Clebschs in this case are all one. Thus we obtain the well known result
that given $\alpha$ and $\sin^2 \theta_W$ measured at $M_Z$ we predict the value for $\alpha_s(M_Z)$ (For recent analysis of the data, see [6]). Note that SUSY without GUTs makes no prediction, since there is no symmetry to specify the boundary conditions and GUTs without SUSY makes the wrong prediction.

I should also point out that the SO(10) operator analysis for fermion masses that I am about to describe is not new. This analysis was carried out 10 years ago with the result that the favored value of the top quark mass was about 35 GeV.

3. Why SO(10)?

There are two reasons for using SO(10).

1. It is the smallest group in which all the fermions in one family fit into one irreducible representation, i.e. the 16. Only one additional state needs to be added to complete the multiplet and that is a right-handed neutrino. In larger gauge groups, more as yet unobserved states must be introduced to obtain complete multiplets. Thus we take $\mathbf{16} \supset \{ U_i, D_i, E_i, \nu_i \}, i = 1, 2, 3$ for the 3 families with the third family taken to be the heaviest. Since SO(10) Clebschs can now relate $U, D, E$ and $\nu$ mass matrices, we can in principle reduce the number of fundamental parameters in the fermion sector of the theory. We return to this point below.

2. In any SUSY theory there are necessarily two higgs doublets – $H_u$ and $H_d$. Both these states fit into the $\mathbf{10}$ of SO(10) and thus their couplings to up and down type fermions are also given by a Clebsch. There are however six additional states in the $\mathbf{10}$ which transform as a $3 + \overline{3}$ under color. These states contribute to proton decay and must thus be heavy. The problem of giving these color triplet states large mass of order $M_G$ while keeping the doublets light is sometimes called the second gauge hierarchy problem. This problem has a natural solution in SO(10) which we discuss later.

Note that the gauge group SO(10) has to be spontaneously broken to the gauge group of the SM – $SU(3) \times SU(2) \times U(1)$. This GUT scale breaking can be accomplished by a set of states including $\{ \mathbf{45}, \mathbf{16}, \mathbf{16}, \cdots \}$. The 45 (the adjoint representation) enters into our construction of effective fermion mass operators, thus I will discuss it in more detail in the next section.

I promised to return to the possibility of reducing the number of fundamental parameters in the fermion sector of the theory. Recall that there are 13 such parameters. Using symmetry arguments we can now express the matrices $D, E$, and $\nu$ in terms of one complex 3x3 matrix, $U$. Unfortunately, this is not sufficient to solve our problem. There are 18 arbitrary parameters in this one matrix. In order to reduce the number of fundamental parameters we must have zeros in this matrix. We thus need new family symmetries to enforce these zeros.

4. The Big Picture
Let us consider the big picture (see Fig. 1). Our low energy observer measures the physics at the electroweak scale and perhaps an order of magnitude above. Once the SUSY threshold is crossed we have direct access to the effective theory at $M_G$, the scale where the 3 gauge couplings meet. Of course the GUT scale $M_G \sim 10^{16}$ GeV is still one or two orders of magnitude below some more fundamental scale such as the Planck or string scales (which we shall refer to as M). Between M and $M_G$ there may be some substructure. In fact, we may be able to infer this substructure by studying fermion masses.

In our analysis we assume that the theory below the scale M is described by a SUSY SO(10) GUT. Between $M_G$ and M, at a scale $v_{10}$, we assume that the gauge group SO(10) is broken spontaneously to SU(5). This can occur due to the vacuum expectation value of an adjoint scalar in the X direction (i.e. corresponding to the U(1) which commutes with SU(5)) and the expectation values of a 16 and a $\overline{10}$ (denoted by $\Psi$ and $\overline{\Psi}$ respectively). Then SU(5) is broken at the scale $v_5 = M_G$ to the SM gauge group. This latter breaking can be done by different adjoints (45) in the Y, B-L or $T_{3R}$ directions.

Why consider 4 particular breaking directions for the 45 and no others? The X and Y directions are orthogonal and span the two dimensional space of U(1) subgroups of SO(10) which commute with the SM. B-L and $T_{3R}$ are also orthogonal and they span the same subspace. Nevertheless we consider these four possible breaking directions and these are the only directions which will enter the effective operators for fermion masses. Why not allow the X and Y directions or any continous rotation of them in this 2d subspace of U(1) directions. The answer is that there are good dynamical arguments for assuming that these and only these directions are important. The X direction breaks SO(10) to an intermediate SU(5) subgroup and it is reasonable to assume that this occurs at a scale $v_{10} \geq v_5$. Whether $v_{10}$ is greater than $v_5$ or equal will be determined by the LED. The B-L direction is required for other reasons. Recall the color triplet higgs in the 10 which must necessarily receive large mass. As shown by Dimopoulos and Wilczek, this doublet-triplet splitting can naturally occur by introducing a 10 45 10 type coupling in the superspace potential. Note that the higgs triplets carry non-vanishing B-L charge while the doublets carry zero charge. Thus when the 45 gets a vacuum expectation value[vev] in the B-L direction it will give mass to the color triplet higgs at $v_5$ and leave the doublets massless. Thus in any SO(10) model which solves this second hierarchy problem, there must be a 45 pointing in the B-L direction. We thus allow for all 4 possible breaking vevs — X, Y, B-L and $T_{3R}$. Furthermore we believe this choice is “natural” since we know how to construct theories which have these directions as vacua without having to tune any parameters.

Our fermion mass operators have dimension $\geq 4$. From where would these higher dimension operators come? Note that by measuring the LED we directly probe the physics in some effective theory at $M_G$. This effective theory can, and likely will, include operators with dimension greater than 4. Consider, for example, our big picture looking down from above. String theories are very fundamental. They can in principle describe physics at all scales. Given a particular string vacuum, one can
obtain an effective field theory valid below the string scale $M$. The massless sector can include the gauge bosons of SO(10) with scalars in the 10, 45 or even 54 dimensional representations. In addition, we require 3 families of fermions in the 16. Of course, in a string context when one says that there are 3 families of fermions what is typically meant is that there are 3 more 16s than $\overline{16}$. The extra $16 + \overline{16}$ pairs are assumed to get mass at a scale $\geq M_G$, since there is no symmetry which prevents this. When these states are integrated out in order to define the effective field theory valid below $M_G$ they will typically generate higher dimension operators.

Consider the tree diagram in Fig. 2. The state $16_2$ contains the second generation of fermions. It has off-diagonal couplings to heavy fermions $16_i, \overline{16}_i, i = 1, 2$. If, for example, the vev $45_X > M_G$ then it will be responsible for the dominant contribution to the mass of the states labelled 1 and 2. These two states however mix by smaller off-diagonal mass terms given by the vev $45_{B-L}$ or the singlet vev (or mass term) denoted by $M_G$. When these heavy states are integrated out one generates to leading order the operator $O_{22} = 16_2 16_10^{45_{B-L}} 45_X^2 45_{45} + \text{calculable corrections of order } (v_5/v_{10})^2$.

It is operators of this type (which can be obtained by implicitly integrating out heavy 16’s) which we use to define our operator basis for fermion masses in the effective theory at $M_G$.

5. Operator Basis for Fermion Masses at $M_G$

Let us now consider the general operator basis for fermion masses. We include operators of the form

$$O_{ij} = 16_i (\cdots)_n 10 (\cdots)_m 16_j$$

where

$$(\cdots)_n = \frac{M_G^k 45_{k+1} \cdots 45_n}{M_P 45_{X}^{n-1}}$$

and the 45 vevs in the numerator can be in any of the 4 directions, $X, Y, B - L, T_{3R}$ discussed earlier.

It is trivial to evaluate the Clebsch-Gordon coefficients associated with any particular operator since the matrices $X, Y, B - L, T_{3R}$ are diagonal. Their eigenvalues on the fermion states are given in Table 1.

6. Dynamic Principles

Now consider the dynamical principles which guide us towards a theory of fermion masses.

0. At zeroth order, we work in the context of a SUSY GUT with the MSSM below $M_G$.

1. We use SO(10) as the GUT symmetry with three families of fermions $\{16_i, i = 1, 2, 3\}$ and the minimal electroweak Higgs content in one 10. SO(10) symmetry relations allow us to reduce the number of fundamental parameters.
Table 1. Quantum numbers of the four 45 vevs on fermion states. Note, if $u$ denotes a left-handed up quark, then $\bar{u}$ denotes a left-handed charge conjugate up quark.

|     | X | Y    | B - L | $T_{3R}$ |
|-----|---|------|-------|----------|
| $u$ | 1 | 1/3  | 1     | 0        |
| $\bar{u}$ | 1 | -4/3 | -1    | -1/2     |
| $d$  | 1 | 1/3  | 1     | 0        |
| $\bar{d}$ | -3 | 2/3  | -1    | 1/2      |
| $e$  | -3 | -1   | -3    | 0        |
| $\bar{e}$ | 1 | 2    | 3     | 1/2      |
| $\nu$ | -3 | -1   | -3    | 0        |
| $\bar{\nu}$ | 5 | 0    | 3     | -1/2     |

2. We assume that there are also family symmetries which enforce zeros of the mass matrix, although we will not specify these symmetries at this time. As we will make clear in section 12, these symmetries will be realized at the level of the fundamental theory defined below $M$.

3. Only the third generation obtains mass via a dimension 4 operator. The fermionic sector of the Lagrangian thus contains the term $A_{O33} \equiv A_{163} 10 163$. This term gives mass to $t$, $b$ and $\tau$. It results in the symmetry relation $- \lambda_t = \lambda_b = \lambda_\tau \equiv A$ at $M_G$. This relation has been studied before by Ananthanarayan, Lazarides and Shafi and using $m_b$ and $m_\tau$ as input it leads to reasonable results for $m_t$ and $\tan \beta$.

4. All other masses come from operators with dimension $> 4$. As a consequence, the family hierarchy is related to the ratio of scales above $M_G$.

5. [Predictivity requirement] We demand the minimal set of effective fermion mass operators at $M_G$ consistent with the LED.

7. Systematic Search

Our goal is to find the minimal set of fermion mass operators consistent with the LED. With any given operator set one can evaluate the fermion mass matrices for up and down quarks and charged leptons. One obtains relations between mixing angles and ratios of fermion masses which can be compared with the data. It is easy to show, however, without any detailed calculations that the minimal operator set consistent
with the LED is given by

\[ O_{33} + O_{23} + O_{22} + O_{12} \]  

or

\[ O_{33} + O_{23} + O_{22}' + O_{12} \]

"22" texture

\[ O_{33} + O_{23} + O_{22} + O_{12} \]  

or

\[ O_{33} + O_{23} + O_{22}' + O_{12} \]  

"23'" texture

It is clear that at least 3 operators are needed to give non-vanishing and unequal masses to all charged fermions, i.e. \( \det(m_a) \neq 0 \) for \( a = u, d, e \). That the operators must be in the \([33, 23\text{ and } 12]\) slots is not as obvious but is not difficult to show. It is then easy to show that 4 operators are required in order to have CP violation. This is because, with only 3 SO(10) invariant operators, we can redefine the phases of the three 16s of fermions to remove the three arbitrary phases. With one more operator, there is one additional phase which cannot be removed. A corollary of this observation is that this minimal operator set results in just 5 arbitrary parameters in the Yukawa matrices of all fermions, 4 magnitudes and one phase.† This is the minimal parameter set which can be obtained without solving the remaining problems of the fermion mass hierarchy, one overall real mixing angle and a CP violating phase. We should point out however that the problem of understanding the fermion mass hierarchy and mixing has been rephrased as the problem of understanding the hierarchy of scales above \( M_G \).

From now on I will just consider models with "22" texture. This is because they can reproduce the observed hierarchy of fermion masses without fine-tuning‡ Models with "22" texture give the following Yukawa matrices at \( M_G \) (with electroweak doublet fields on the right) –

\[
\lambda_a = \begin{pmatrix}
0 & z'_a C & 0 \\
z_a C & y_a E e^{i\phi} & x'_a B \\
0 & x_a B & A
\end{pmatrix}
\]

with the subscript \( a = \{u, d, e\} \). The constants \( x_a, x'_a, y_a, z_a, z'_a \) are Clebsch which can be determined once the 3 operators (\( O_{23}, O_{22}, O_{12} \)) are specified. Recall, we have taken \( O_{33} = A \, 16_3 \, 10 \, 16_3 \), which is why the Clebsch in the 33 term is independent of \( a \). Finally, combining the Yukawa matrices with the Higgs vevs to find the fermion mass matrices we have 6 arbitrary parameters given by \( A, B, C, E, \phi \) and \( \tan \beta \) describing 14 observables. We thus obtain 8 predictions. We shall use the best known parameters, \( m_e, m_\mu, m_\tau, m_c, m_b, |V_{cd}| \), as input to fix the 6 unknowns. We then predict the values of \( m_u, m_d, m_s, m_t, \tan \beta, |V_{cb}|, |V_{ub}| \) and \( J \).

Note: since the predictions are correlated, our analysis would be much improved if we minimized some \( \chi^2 \) distribution and obtained a best fit to the data. Unfortunately this has not yet been done. In the paper however we do include some tables (see for example Table 4 in this talk) which give all the predictions for a particular set of input parameters.

†This is two fewer parameters than was necessary in our previous analysis (see §1).
‡For more details on this point, see section 9 below or refer to §4.
8. Results

The results for the 3rd generation are given in Fig. 3. Note that since the parameter A is much bigger than the others we can essentially treat the 3rd generation independently. The small corrections, of order \((B/A)^2\), are however included in the complete analysis. We find the pole mass for the top quark \(M_t = 180 \pm 15\) GeV and \(\tan \beta = 56 \pm 6\) where the uncertainties result from variations of our input values of the \(\overline{\text{MS}}\) running mass \(m_b(m_b) = 4.25 \pm 0.15\) and \(\alpha_s(M_Z)\) taking values \(0.110 - 0.126\). We used two loop RG equations for the MSSM from \(M_G\) to \(M_{\text{SUSY}}\); introduced a universal SUSY threshold at \(M_{\text{SUSY}} = 180\) GeV with 3 loop QCD and 2 loop QED RG equations below \(M_{\text{SUSY}}\). The variation in the value of \(\alpha_s\) was included to indicate the sensitivity of our results to threshold corrections which are necessarily present at the weak and GUT scales. In particular, we chose to vary \(\alpha_s(M_Z)\) by letting \(\alpha_3(M_G)\) take on slightly different values than \(\alpha_1(M_G) = \alpha_2(M_G) = \alpha_G\).

The following set of operators passed a straightforward but coarse grained search discussed in detail in the paper. They include the diagonal dimension four coupling of the third generation –

\[
O_{33} = 16_3 10 16_3 \quad (1)
\]

The six possible \(O_{22}\) operators –

\[
O_{22} =
\begin{align*}
16_2 \frac{45_X}{M} & 10 \frac{45_{B-L}}{45_X} 16_2 \quad (a) \\
16_2 \frac{M_G}{45_X} & 10 \frac{45_{B-L}}{M} 16_2 \quad (b) \\
16_2 \frac{45_M}{45_X} & 10 \frac{45_{B-L}}{M} 16_2 \quad (c) \\
16_2 10 \frac{45_{B-L}}{45_X} 16_2 \quad (d) \\
16_2 10 \frac{45_X 45_{B-L}}{M^2} 16_2 \quad (e) \\
16_2 10 \frac{45_{B-L} M_G}{45_X} 16_2 \quad (f)
\end{align*}
\]

Note: in all cases the Clebschs \(y_i\) (defined by \(O_{22}\) above) satisfy

\[y_u : y_d : y_e = 0 : 1 : 3.\]

This is the form familiar from the Georgi-Jarlskog texture. Thus all six of these operators lead to identical low energy predictions.

Finally there is a unique operator \(O_{12}\) consistent with the LED –

\[
O_{12} = 16_1 \left(\frac{45}{M}\right)^3 10 \left(\frac{45}{M}\right)^3 16_2 \quad (3)
\]
The operator $O_{23}$ determines the KM element $V_{cb}$ by the relation –

$$V_{cb} = \chi \sqrt{\frac{m_c}{m_t}} \times (RG factors)$$

where the Clebsch combination $\chi$ is given by

$$\chi \equiv \frac{|x_u - x_d|}{\sqrt{|x_u x_u'|}}$$

$m_c$ is input, $m_t$ has already been determined and the renormalization group [RG] factors are calculable. Demanding the experimental constraint $V_{cb} < .054$ we find the constraint $\chi < 1$. A search of all operators of dimension 5 and 6 results in the 9 operators given below. Note that there only three different values of $\chi = 2/3, 5/6, 8/9$

$$O_{23} = \chi = 2/3$$

(1) $16_2 \frac{45y}{M} 10 \frac{M_G}{45_X} 16_3$

(2) $16_2 \frac{45y}{M} 10 \frac{45_{B-L}}{45_X} 16_3$

(3) $16_2 \frac{45y}{45_X} 10 \frac{M_G}{45_X} 16_3$

(4) $16_2 \frac{45y}{45_X} 10 \frac{45_{B-L}}{45_X} 16_3$

$$\chi = 5/6$$

(5) $16_2 \frac{45y}{M} 10 \frac{45y}{45_X} 16_3$

(6) $16_2 \frac{45y}{45_X} 10 \frac{45y}{45_X} 16_3$

$$\chi = 8/9$$

(7) $16_2 10 \frac{M_G^2}{45_X} 16_3$

(8) $16_2 10 \frac{45y y_{-L} M_G}{45_X} 16_3$

(9) $16_2 10 \frac{45y_{-L} y_{-L}}{45_X} 16_3$

We label the operators (1) - (9), and we use these numbers also to denote the corresponding models. Note, all the operators have the vev $45_X$ in the denominator. This can only occur if $v_{10} > M_G$.

At this point, there are no more simple criteria to reduce the number of models further. We have thus performed a numerical RG analysis on each of the 9 models (represented by the 9 distinct operators $O_{23}$ with their calculable Clebschs $x_a, x_a', a =
Let me make a few comments. Light quark masses (u,d,s) are \(\overline{\text{MS}}\) masses evaluated at 1 GeV while heavy quark masses (c,b) are evaluated at \((m_c, m_b)\) respectively. Finally, the top quark mass in Fig. 3 is the pole mass. Figs. 4 and 5 are self evident. In Fig. 6, we show the correlations for two of our predictions. The ellipse in the \(m_s/m_d\) vs. \(m_u/m_d\) plane is the allowed region from chiral Lagrangian analysis\(^{13}\). One sees that we favor lower values of \(\alpha_s(M_Z)\). For each fixed value of \(\alpha_s(M_Z)\), there are 5 vertical line segments in the \(V_{cb}\) vs. \(m_u/m_d\) plane. Each vertical line segment represents a range of values for \(m_c\) (with \(m_c\) increasing moving up) and the different line segments represent different values of \(m_b\) (with \(m_b\) increasing moving to the left).

In Figure 9 we test our agreement with the observed CP violation in the K system. The experimentally determined value of \(\epsilon_K = 2.26 \times 10^{-3}\). Theoretically it is given by an expression of the form \(B_K \times \{m_t, V_{ts}, \cdots\}\). \(B_K\) is the so-called Bag constant which has been determined by lattice calculations to be in the range \(B_K = .7 \pm .2\)\(^{14}\). In Fig. 9 we have used our predictions for fermion masses and mixing angles as input, along with the experimental value for \(\epsilon_K\), and fixed \(B_K\) for the 9 different models. One sees that model 4 is inconsistent with the lattice data. In Fig. 10 we present the predictions for each model, for the CP violating angles which can be measured in B decays. The interior of the “whale” is the range of parameters consistent with the SM found by Nir and Sarid\(^{15}\) and the error bars represent the accuracy expected from a B factory.

Note that model 4 appears to give too little CP violation and model 9 has uncomfortably large values of \(V_{cb}\). Thus these models are presently disfavored by the data. I will thus focus on model 6 in the second part of this talk.

9. Summary

We have performed a systematic operator analysis of fermion masses in an effective SUSY SO(10) GUT. We use the LED to lead us to the theory. Presently there are 3 models (models 4, 6 & 9) with “22” texture which agree best with the LED, although as mentioned above model 6 is favored. In all cases we used the values of \(\alpha\) and \(\sin^2 \theta_W\) (modulo threshold corrections) to fix \(\alpha_s(M_Z)\).

Table 2 shows the virtue of the “22” texture. In the first column are the four operators. In the 2nd and 3rd columns are the parameters in the mass matrix relevant for that particular operator and the input parameters which are used to fix these parameters. Finally the 4th column contains the predictions obtained at each level. One sees that each family is most sensitive to a different operator\(^{\S}\).

\(\S\)This property is not true of “23” textures.
Table 2. Virtue of “22” texture.

| Operator | Parameters | Input | Predictions |
|----------|------------|-------|--------------|
| $O_{33}$ | $\tan \beta$ | $A$ | $b \ \tau$ | $t \ \tan \beta$ |
| $O_{23}$ | $B$ | $c$ | $V_{cb}$ |
| $O_{22}$ | $E$ | $\mu$ | $s$ |
| $O_{12}$ | $C$ | $\phi$ | $V_{us}$, $u \ d$ | $\frac{V_{cb}}{V_{ub}}$, $J$ |

1. The experimentally determined values of $m_b$, $m_c$, and $\alpha_s(M_Z)$ are all subject to strong interaction uncertainties of QCD. In addition, the predicted value of $\alpha_s(M_Z)$ from GUTs is subject to threshold corrections at $M_W$ which can only be calculated once the SUSY spectrum is known and at $M_G$ which requires knowledge of the theory above $M_G$. We have included these uncertainties (albeit crudely) explicitly in our analysis.

2. In the large $\tan \beta$ regime in which we work there may be large SUSY loop corrections which will affect our results. The finite corrections to the $b$ and $\tau$ Yukawa couplings have been evaluated. They depend on ratios of soft SUSY breaking parameters and are significant in certain regions of parameter space. In particular it has been shown that the top quark mass can be reduced by as much as 30%. Note that although the prediction of Fig. 3 may no longer be valid, there is still necessarily a prediction for the top quark mass. It is now however sensitive to the details of the sparticle spectrum and to the process of radiative electroweak symmetry breaking. This means that the observed top quark mass can now be used to set limits on the sparticle spectrum. This analysis has not been done. Moreover, there are also similar corrections to the Yukawa couplings for the $s$ and $d$ quarks and for $e$ and $\mu$. These corrections are expected to affect the predictions for $V_{cb}$, $m_s$, $m_u$, $m_d$. It will be interesting to see the results of this analysis.

3. The top, bottom and $\tau$ Yukawa couplings can receive threshold corrections at $M_G$. We have not studied the sensitivity to these corrections.

4. Other operators could in principle be added to our effective theory at $M_G$. They might have a dynamical origin. We have assumed that, if there, they are subdominant. Two different origins for these operators can be imagined. The first is field theoretic. The operators we use would only be the leading terms in a power series expansion when defining an effective theory at $M_G$ by integrating out heavier states. The corrections to these operators are expected to be about 10%. We may also be sensitive to what has commonly been referred to as Planck slop, operators suppressed by some power of the Planck (or string) scale M.

$^\dagger$There is a small range of parameter space in which our results are unchanged. This requires threshold corrections at $M_G$ which distinguish the two Higgs scalars.
In fact the operator $O_{12}$ may be thought of as such. The question is why aren’t our results for the first and perhaps the second generation, hopelessly sensitive to this unknown physics? This question will be addressed in the next section.

10. Where are we going?

In the first half of Table 3 I give a brief summary of the good and bad features of the effective SUSY GUT discussed earlier. Several models were found with just four operators at $M_G$ which successfully fit the low energy data. If we add up all the necessary parameters needed in these models we find just 12. This should be compared to the SM with 18 or the MSSM with 21. Thus these theories, minimal effective SUSY GUTs[MESG], are doing quite well. Of course the bad features of the MESG is that it is not a fundamental theory. In particular there are no symmetries which prevent additional higher dimension operators to spoil our results. Neither are we able to calculate threshold corrections, even in principle, at $M_G$.

It is for these reasons that we need to be able to take the MESG which best describes the LED and use it to define an effective field theory valid at scales $\leq M$. The good and bad features of the resulting theory are listed in the second half of Table 3.

- In the effective field theory below $M$ we must incorporate the symmetries which guarantee that we reproduce the MESG with no additional operators. Moreover, the necessary combination of discrete, $U(1)$ or $R$ symmetries may be powerful enough to restrict the appearance of Planck slope.

- Finally, the GUT symmetry breaking sector must resolve the problems of natural doublet-triplet splitting (the second hierarchy problem), the $\mu$ problem, and give predictions for proton decay, neutrino masses and calculable threshold corrections at $M_G$.

- On the bad side, it is still not a fundamental theory and there may not be a unique extension of the MESG to higher energies.

11. String Threshold at $M_S$

Upon constructing the effective field theory $\leq M_S$, we will have determined the necessary SO(10) states, symmetries and couplings which reproduce our fermion mass relations. This theory can be the starting point for constructing a realistic string model. String model builders could try to obtain a string vacuum with a massless spectra which agrees with ours. Of course, once the states are found the string will

∥This statement excludes the unavoidable higher order field theoretic corrections to the MESG which are, in principle, calculable.
Table 3.

| Good | Bad |
|------|-----|
| **Eff. F.T.** | **Bad** |
| ≤ $M_G$ | Not fundamental |
| 4 op’s. at $M_G$ ⇒ LED | ⇒ Why these operators? (F.T. + Planck slop) |
| 5 para’s. ⇒ 13 observables | ⇒ Threshold corrections? |
| + 2 gauge para’s. ⇒ 3 observables | |
| + 5 soft SUSY breaking para’s. ⇒ ⋯ | |
| Total **12** parameters | |
| **Eff. F.T.** | **Symmetry** |
| ≤ $M$ | |
| $M = M_{\text{string}}$ or $M_{\text{Planck}}$ | |
| i) gives Eff. F.T. ≤ $M_G$ + corrections | |
| ii) constrains other operators | |
| GUT symmetry breaking | |
| i) d - t splitting | |
| ii) $\mu$ problem | |
| iii) proton decay | |
| iv) neutrino masses | |
| v) threshold corrections at $M_G$ | |
| **Not unique?** | |

determine the symmetries and couplings of the theory. It is hoped that in this way a fundamental theory of Nature can be found. Work in this direction by several groups is in progress. String theories with SO(10), three families plus additional $16 + \overline{16}$ pairs, 45’s, 10’s and even some 54 dimensional representations appear possible. One of the first results from this approach is the fact that only one of the three families has diagonal couplings to the 10, just as we have assumed.

12. Constructing the Effective Field Theory below $M_S$

In this section I will discuss some preliminary results obtained in collaboration with Lawrence Hall. I will describe the necessary ingredients for constructing model 6. Some very general results from this exercise are already apparent.

- **States** — We have constructed a SUSY GUT which includes all the states necessary for GUT symmetry breaking and also for generating the 45 vevs in the desired directions. A minimal representation content below $M_S$ includes $54s + 45s + 3 16s + n(\overline{16} + 16)$ pairs + 2 10s.

- **Symmetry** — In order to retain sufficient symmetry the superspace potential in the visible sector W necessarily has a number of flat directions. In particular the scales $v_5$ and $v_{10}$ can only be determined when soft SUSY breaking and quantum corrections are included. An auxiliary consequence is that the vev of $W_{\text{visible}}$ vanishes in the supersymmetric limit.
• Couplings — As an example of the new physics which results from this analysis I will show how a solution to the $\mu$ problem, the ratio $\lambda_b/\lambda_t$ and proton decay may be inter-related.

In Table 4 are presented the predictions for Model 6 for particular values of the input parameters.

Table 4: Particular Predictions for Model 6 with $\alpha_s(M_Z) = 0.115$

| Input Quantity | Input Value | Predicted Quantity | Predicted Value |
|----------------|-------------|--------------------|-----------------|
| $m_b(m_b)$     | 4.35 GeV    | $M_t$              | 176 GeV         |
| $m_t(m_t)$     | 1.777 GeV   | $\tan \beta$      | 55              |
| $m_c(m_c)$     | 1.22 GeV    | $V_{cb}$           | 0.048           |
| $m_\mu$        | 105.6 MeV   | $V_{ub}/V_{cb}$    | 0.59            |
| $m_\tau$       | 1.22 GeV    | $m_s(1GeV)$        | 172 MeV         |
| $V_{us}$       | 0.221       | $\hat{B}_K$        | 0.64            |
| $m_u/m_d$      | 0.64        | $m_s/m_d$          | 24              |

In addition to these predictions, the set of inputs in Table 4 predicts: $\sin 2\alpha = -0.46$, $\sin 2\beta = 0.49$, $\sin 2\gamma = 0.84$, and $J = 2.6 \times 10^{-5}$.

Model 6

The superspace potential for Model 6 has several pieces - $W = W_{\text{fermion}} + W_{\text{symmetry breaking}} + W_{\text{Higgs}} + W_{\text{neutrino}}$.

12.1. Fermion sector

The first term must reproduce the four fermion mass operators of Model 6. They are given by

\begin{align*}
O_{33} &= 163 101 163 \\
O_{23} &= 162 \frac{\bar{A}}{A} 101 \frac{\bar{A}}{A} 163 \\
O_{22} &= 162 \frac{\bar{A}}{M} 101 \frac{\bar{A}}{A} 162 \\
O_{12} &= 161 \left( \frac{\bar{A}}{M} \right)^3 101 \left( \frac{\bar{A}}{A} \right)^3 162
\end{align*}

There are two 10s in this model, denoted by $10_i, i = 1, 2$ but only $10_1$ couples to the ordinary fermions. The A fields are different 45s which are assumed to have vevs in the following directions - $\langle A_2 \rangle = 45_y$, $\langle A_1 \rangle = 45_{B-L}$, and $\langle \bar{A} \rangle = 45_X$. As noted earlier, there are 6 choices for the 22 operator and we have just chosen one of them, labelled a, arbitrarily here. In Figure 11, we give the tree diagrams which reproduce the effective operators for Model 6 to leading order in an expansion in the
ratio of small to large scales. The states $\Psi_a$, $\Psi_a$, $a = 1, \cdots, 9$ are massive $\mathbf{16}$, $\mathbf{16}$ states respectively with mass given by $\langle S_M \rangle \sim M$. Each vertex represents a separate Yukawa interaction in $W_{\text{fermion}}$ (see below). Field theoretic corrections to the effective GUT operators may be obtained by diagonalizing the mass matrices for the heavy states and integrating them out of the theory.

$$W_{\text{fermion}} =$$

$$16_3 16_3 10_1 + \bar{\Psi}_1 A_2 16_3 + \bar{\Psi}_1 A \Psi_1 + \Psi_1 \Psi_2 10_1$$

$$+ \bar{\Psi}_2 A \Psi_2 + \bar{\Psi}_2 A_2 16_2 + \bar{\Psi}_3 A_1 16_2$$

$$+ \bar{\Psi}_3 A \Psi_3 + \Psi_3 \Psi_4 10_1 + S_M \sum_{a=4}^9 (\Psi_a \Psi_a)$$

$$+ \bar{\Psi}_4 A 16_2 + \bar{\Psi}_5 A \Psi_4 + \bar{\Psi}_6 A \Psi_5$$

$$+ \Psi_6 \Psi_7 10_1 + \bar{\Psi}_7 A \Psi_8 + \bar{\Psi}_8 A \Psi_9 + \bar{\Psi}_9 A 16_1$$

Note that the vacuum insertions in the effective operators above cannot be rearranged, otherwise an inequivalent low energy theory would result. In order to preserve this order naturally we demand that each field carries a different value of a $U(1)$ family charge (see Fig. 11). Note also that the particular choice of a $22$ operator will affect the allowed $U(1)$ charges of the states. Some choices may be acceptable and others not.

Consider $W_{\text{fermion}}$. It has many terms, each of which can have different, in principle, complex Yukawa couplings. Nevertheless the theory is predictive because only a very special linear combination of these parameters enters into the effective theory at $M_G$. Thus the observable low energy world is simple, not because the full theory is particularly simple, but because the symmetries are such that the effective low energy theory contains only a few dominant terms.

### 12.2. Symmetry breaking sector

The symmetry breaking sector of the theory is not particularly illuminating. Two $54$ dimensional representations, $S, S'$ are needed plus several singlets denoted by $S_i, i = 1, \cdots, 7$. They appear in the first two terms and are responsible for driving the vev of $A_1$ into the B-L direction, the third term drives the vev of the $\mathbf{16}$, $\mathbf{16}$ fields $\bar{\Psi}, \Psi$ into the right-handed neutrino like direction breaking $\text{SO}(10)$ to $\text{SU}(5)$.
and forcing $\tilde{A}$ into the X direction. The fourth, fifth and sixth terms drive $A_2$ into the Y direction. Finally the last two terms are necessary in order to assure that all non singlet states under the SM gauge interactions obtain mass of order the GUT scale. All primed fields are assumed to have vanishing vevs.

Note if $\langle S_3 \rangle \approx M_S$ then two of these adjoints state may be heavy. Considerations such as this will affect how couplings run above $M_G$.

$$W_{\text{symmetry breaking}} =$$

$$A'_1(SA_1 + S_1A_1) + S'(S_2S + A^2_1)$$

$$+ \tilde{A}'(\bar{\Psi}\Psi + S_3\tilde{A})$$

$$+ A'_2(S_4A_2 + S\tilde{A} + (S_1 + S_5)\tilde{A})$$

$$+ \bar{\Psi}'A_2\Psi + \bar{\Psi}A_2\Psi'$$

$$+ A_1A_2\tilde{A}' + S_6(A'_1)^2$$

12.3. Higgs sector

The Higgs sector is introduced below. It does not at the moment appear to be unique, but it is crucial for understanding the solution to several important problems – doublet-triplet splitting, $\mu$ problem and proton decay – and these constraints may only have one solution. The $10_1A_110_2$ coupling is the term required by the Dimopoulos-Wilczek mechanism for doublet-triplet splitting. Since $A_1$ is an anti-symmetric tensor, we need at least two 10s.

The couplings of 10 to the 16s are introduced to solve the $\mu$ problem. After naturally solving the doublet-triplet splitting problem one has massless doublets. One needs however a small supersymmetric mass $\mu$ for the Higgs doublets of order the weak scale. This may be induced once SUSY is broken in several ways.

- The vev of the field $A_1$ may shift by an amount of order the weak scale due to the introduction of the soft SUSY breaking terms into the potential. In this theory the shift of $A_1$ appears to be too small.

- There may be higher dimension D terms in the theory of the form, eg.

$$\frac{1}{M_{Pl}} \int d^4\theta 10^2_1(A^*_2).$$
Then supergravity effects might induce a non-vanishing vev for the F term of $A_2$ of order the $m_W M_G$. This will induce a value of $\mu$ of order $m_W M_G / M_{Pl}$. The shift in the F-terms also appear to be negligible.

- Higher dimension D-terms with hidden sector fields may however work. Consider $\frac{1}{M_{Pl}} \int d^4 \theta 10_1^2 z^* z$ where $z$ is a hidden sector field which is connected with soft SUSY breaking. It would then be natural to have $F_z \approx \mu M_{Pl}$.

- One loop effects may induce a $\mu$ term once soft SUSY breaking terms are introduced. In this case we find $\mu \sim \frac{\lambda^4}{16 \pi^2}$ where $\lambda^4$ represents the product of Yukawa couplings entering into the graph of Figure 12.

We use the last mechanism above for generating $\mu$ in the example which follows.

$$W_{Higgs} =$$

$$+ \Psi' A_2 \Psi + \bar{\Psi} A_2 \Psi'$$

$$+ 10_1 A_1 10_2 + S_7 10^2_2$$

$$+ \bar{\Psi} \Psi' 10_1 + \Psi \bar{\Psi} 10_1$$

Note that the first two terms already appeared in the discussion of the symmetry breaking sector. They are included again here since as you will see they are important for the discussion of the Higgs sector as well. The last two terms are needed to incorporate the solution to the $\mu$ problem. As a result of these couplings to $\bar{\Psi}, \Psi$ the Higgs doublets in $10_1$ mix with other states. The mass matrix for the SU(5) $\bar{5}, 5$ states in $10_1, 10_2, \Psi, \Psi', \bar{\Psi}, \bar{\Psi}'$ is given below.

$$
\begin{bmatrix}
5_1 & 5_2 & 5_\Psi & 5_{\bar{\Psi}} \\
5_1 & 0 & A_1 & 0 & \Psi \\
5_2 & A_1 & S_7 & 0 & 0 \\
5_\Psi & 0 & 0 & 0 & A_2 \\
5_{\bar{\Psi}} & 0 & A_2 & 0 & 0 \\
\end{bmatrix}
$$

**Higgs doublets** In the doublet sector the vev $A_1$ vanishes. Since the Higgs doublets in $10_1$ now mix with other states, the boundary condition $\lambda_b / \lambda_t = 1$ is corrected at tree level. The ratio is now given in terms of a ratio of mixing angles.

**Proton decay** The rate for proton decay in this model is set by the quantity $(M^t)^{-1}_{11}$ where $M^t$ is the color triplet Higgsino mass matrix. We find $(M^t)^{-1}_{11} = \frac{S_7}{A_1}$. This may be much smaller than $\frac{1}{M_G}$ for $S_7$ sufficiently smaller than $M_G$. Note there
are no light color triplet states in this limit. Proton decay is suppressed since in this limit the color triplet Higgsinos in $10_1$ become Dirac fermions (with mass of order $M_G$), but they do not mix with each other.

12.4. Symmetries

The theory has been constructed in order to have enough symmetry to restrict the allowed operators. This is necessary in order to reproduce the mass operators in the effective theory, as well as to preserve the vacuum directions assumed for the 45s and have natural doublet-triplet splitting. Indeed the construction of the symmetry breaking sector with the primed fields allows the 45s to carry nontrivial U(1) charges. This model has several unbroken U(1) symmetries which do not seem to allow any new mass operators. It has a discrete $Z_4$ R parity in which all the primed fields, $s_{6,7}$ and $10_2$ are odd and $16_i, i = 1, 2, 3$ and $\overline{\Psi}_a, \Psi_a, a = 1, \cdots, 9$ go into $i$ times themselves. This guarantees that the odd states (and in particular, $10_2$) do not couple into the fermion mass sector. There is in addition a Family Reflection Symmetry (see Dimopoulos- Georgi) which guarantees that the lightest supersymmetric particle is stable. Finally, there is a continuous R symmetry which is useful for two reasons, (1) as a consequence, only dimension 4 operators appear in the superpotential and (2) this R symmetry is an anomalous Peccei-Quinn U(1) which naturally solves the strong CP problem.

**Neutrino sector** The neutrino sector seems to be very model dependent. It will constrain the symmetries of the theory, but I will not discuss it further here.

13. Work in Progress

The results for fermion masses and mixing angles should be considered as zeroth order results. These results receive calculable corrections from physics at the weak and GUT scales. Before we add new operators to try and improve the agreement with experiment we must evaluate the leading order corrections. In the following I will discuss preliminary results of work in progress.

1. New parameter! — We predict $\tan \beta$ to be large. It is interesting to ask whether we can lower the value of $\tan \beta$ with the addition of one new parameter. This parameter can appear naturally in any SO(10) model if the light higgs doublets are mixtures of the doublets in $10_1$ and doublets contained in other states (consider for example the higgs sector discussed in this talk). Let $H_u, H_d$ be the higgs doublets in the MSSM coupling to up, down quarks respectively and $d_{10}, \overline{d}_{10}$ be the higgs doublets in $10_1$. Then we introduce a parameter $\omega$ by the expression

$$d_{10} = H_u, \overline{d}_{10} = \omega H_d.$$ 

In this case we expect

$$\tan \beta \sim \omega \left( \frac{m_t}{m_b} \right)$$
with $\omega \leq 1$. We find\cite{2} (in a systematic operator search with 4 operators) that solutions only exist for $\omega > 0.5$ and $\tan \beta > 30$. Thus large $\tan \beta$ is unavoidable.

2. Finite SUSY corrections of order $\tan \beta$ — Large radiative corrections to the bottom quark mass have been known for some time\cite{16,17}. They can be expressed by

$$\delta m_b = (\epsilon_1 + \epsilon_2 |V_{tb}|^2) m_b$$

where the first term comes from gluino loops and the second from higgsino loops. $\epsilon_1$ and $\epsilon_2$ are proportional to $\tan \beta$. In the regime of large $\tan \beta$ these corrections can be significant. They depend on the details of the SUSY spectrum. We have considered the radiative corrections to the down quark mass matrix. These corrections can be expressed as corrections to quark masses and CKM matrix elements. We find\cite{5}

$$\delta m_a = \epsilon_1 m_a, \text{ for } a = s, d$$

and

$$\delta V_{cb} = -\epsilon_2 V_{cb} \quad \delta |V_{ub}/V_{cb}| \approx 0$$

$$\delta J = -2 \epsilon_2 J \quad \delta \alpha \approx \delta \beta \approx \delta \gamma \approx 0$$

For non-universal scalar masses $-\epsilon_1 \sim \epsilon_2$ can be $\sim 5 - 10\%$ or even larger.

3. GUT threshold corrections — The boundary conditions at $M_G$ for the two loop gauge running equations depend on $\alpha_G$, $M_G$; and $\Delta_i(GUT\ spectrum), i = 1, 2, 3$. The threshold corrections in $\Delta_i$ depend logarithmically on the spectrum of massive states at $M_G$. This affects the prediction for $\alpha_s(M_Z)$ given $\alpha$ and $\sin^2 \theta_W$. In fact, there may be an interesting constraint on the proton decay rate using the observed value of $\alpha_s(M_Z)$. This is because in order to suppress the proton decay rate we need $(M^4)^{11}_{11} = \frac{S_T}{A_1^2} \leq \frac{1}{\tan \beta M_G}$. However the two doublets in 10$_2$ get mass of order $S_T$, while the triplets all have mass of order $M_G$. Thus there may be a lower limit on the vev of $S_T$ from gauge unification. This work is now in progress. Note there may be compensating effects from allowing $S_6 \ll M_G$ and thus I can not make any definite statements at this time.

4. New $\chi^2$ analysis — Since the predictions for fermion masses are strongly correlated, the analysis done in \cite{4} using the 6 best known low energy parameters to fix the 6 input parameters is not satisfactory. Blažek is now redoing this analysis. He is using a $\chi^2$ analysis in an attempt to find the best fit to the low energy data.

14. Conclusion

In this talk, I have presented a class of supersymmetric SO(10) GUTs which are in quantitative agreement with the low energy data. With improved data these particular
models may eventually be ruled out. Nevertheless the approach of using low energy data to ascertain the dominant operator contributions at $M_G$ seems robust. Taking it seriously, with quantitative fits to the data and including the leading order corrections to the zeroth order results, may eventually lead us to the correct theory.

What is the proverbial *smoking gun* for the theories presented here? There are three observations which combined would confirm SUSY GUTs.

1. Gauge coupling unification consistent with the observed values of $\alpha, \sin^2 \theta_W, \alpha_s$.

2. Observation of SUSY particles.

3. Observation of proton decay into the modes $p \rightarrow K^+\nu$ and $p \rightarrow K^0\mu^+\nu$. Although SUSY GUTs may not predict the rate for this process, nevertheless the observation of this process would confirm SUSY GUTs.

In addition, the minimal SO(10) models presented here all demand large $\tan \beta$. Thus observation of large $\tan \beta$ would certainly strengthen these ideas. Finally, if the calculable corrections to the predictions of one of these models improve the agreement with the data, it would be difficult not to accept this theory as a true description of nature.

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