Flow Regime, Slug Frequency and Wavelet Analysis of Air/Newtonian and Air/non-Newtonian Two-Phase Flow

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Received: 7 April 2020; Accepted: 4 May 2020; Published: 8 May 2020

Featured Application: Application of Newtonian and non-Newtonian fluids in horizontal flow loop conditions for directional drilling operations.

Abstract: This study focused on gas/Newtonian and gas/non-Newtonian two-phase horizontal fluid flow behavior by analyzing their flow regime identification and flow structural analysis on a horizontal flow loop apparatus. This involved the recognition of two-phase flow regimes for this flow loop and validation with existing flow maps in the literature. In addition, the study included flow pattern identification via wavelet analysis for gas/Newtonian and gas/non-Newtonian two-phase fluid flow in a horizontal flow loop apparatus. Furthermore, the study was extended to the detailed examination of slug frequency in the presence of air/Newtonian and air/non-Newtonian fluid flow, and the predicted slug frequency model was applied to the studied systems. The obtained results suggest that the flow regime maps and slug frequency analysis have a significant impact. The obtained pressure sensor results indicate that the experimental setup could not provide high-frequency and high-resolution data; nevertheless, wavelet decomposition and wavelet norm entropy were calculated. It offered recognizable flow characteristics for bubble, bubble-elongated bubble, and slug flow patterns. Therefore, this study can provide deep insight into intricate multiphase flow patterns, and the wavelet could potentially be applied for flow analysis in oil and gas pipelines.

Keywords: air/Newtonian and air/non-Newtonian fluid; slug frequency; two-phase flow regimes; superficial gas velocity; wavelet analysis

1. Introduction

Multiphase flows are considered as complicated flow phenomena compared to a single flow [1]. There are essential features of multiphase flows whose modeling outcome is contentious and structural explanation is still unexplored. The most common type of multiphase flow in almost all chemical, petroleum, and production industries is the two-phase gas/liquid flow [2,3]. Different forms of flow patterns may be observed when two or more phases flow simultaneously [2]. In pipe cross-sections, unpredictable turbulent flow structures generating highly asymmetric volume distribution are a challenge in experiential investigations [4,5]. This kind of unstable flow condition complicates the measurement process and may create difficulties in capturing actual flow conditions. There are also instances where the existing theoretical solution or experimental results cannot describe specific physical properties [1,6].
Flow regimes or patterns are amongst the most significant aspects of multiphase flow. The structural flow distribution of different phases in a pipe is known as the flow pattern or flow regime. The flow regime depends on the inertia force, buoyancy force, flow turbulence, and surface tension, which are altered by fluid properties, flow rates, pipe diameter, and pipe predilection [7,8]. Different forms of flow patterns may be observed when two phases of gas/Newtonian and gas/non-Newtonian flow simultaneously. Some of the conventional distributions are stratified flow, where the liquid and gas phase is separated, and the gas flows on top as it is lighter than the liquid; bubbly flow, where there is a dispersion of small-sized bubbles within the liquid; slug flow, in which each gas bubble forms a large slug shape (often a bullet shape); and annular flow where liquid flows as a film on the wall of the pipe. For gas/Newtonian and gas/non-Newtonian flow, several flow maps exist to predict the flow patterns. The Taitel and Dukler [7] and Mandhane et al. [9] flow maps for gas/Newtonian flow, and the Chhabra and Richardson [10] flow map for gas/non-Newtonian flow are the most frequently used.

Among the flow regimes, slug flow is the most frequent two-phase flow phenomena experienced in horizontal or near horizontal pipelines in practical applications [11–13]. The presence of slug flows in a pipeline can trigger a great range of design and operational problems, including the exertion of kinetic force on fittings and fittings, pressure cycling, control instability, and inadequate phase separation. If the flow rate is raised, much of the liquid is swept out, and the liquid handling capacity of the receiving facilities may be overwhelmed. In the case of a slug greater than the slow catcher capacity, the facilities can be flooded and destroyed. Therefore, multiple operational problems, such as pipeline network instability and damage to equipment due to high-pressure fluctuation or vibration of the system, can be caused by slug flow [14,15]. This can also be termed a water hammering effect. Therefore, slug flow and slug frequency analysis has been one of the primary research interests in multiphase flow.

To measure the hydrodynamic behavior of multiphase flow, it is essential to understand flow pattern under specific flow conditions [2,16]. Two-phase flow implies gas and liquid flow through a pipeline system, simultaneously. The gas and liquid interface is deformable, so it is hard to predict the region occupied by a gas or liquid phase [17]. When two phases flow through a pipeline, different types of interfacial distribution can form. The variety of flow patterns mostly depends upon the input flux of the two phases, the size and assembly of the pipe, the physical properties of the fluid, etc. Many experimental studies on gas/Newtonian or solid/Newtonian fluid flow have been conducted [18–24]. Nevertheless, a limited number of studies have observed non-Newtonian multiphase flow [25,26].

Xanthan gum is the most commonly used industrial biopolymer for thickening and stabilizing an aqueous system. Xanthan gum solution has significant thermal stability with pseudoplastic properties [27]. Due to these properties, it has a substantial application in petroleum industries. In oil industries, xanthan gum is widely used in drilling fluid [28]. Xanthan gum is also broadly used in the food industry, cosmetics, and pharmacological products [27,29].

Tutu [30] and Drahos et al. [31] characterized two-phase horizontal flow regime pressure fluctuation. Drahos et al. [31] used a probability density function (PDF), where a strain gauge pressure transducer was used in a 50 mm inner diameter (ID) Perspex pipe. Sun et al. [32] used norm entropy wavelet decomposition to analyze gas/liquid two-phase flow pressure signal data across a bluff body. Here, the inner pipe diameter was 50 mm, and, from a piezoresistive differential pressure sensor, the pressure signals were analyzed using four levels and four scales of Daubechies wavelet (db4), which provided 16 wavelet packet coefficients. This study [32] also suggested an entropy-based two-phase flow map with an identification rate of 95%. Blaney [33] used gamma rays to identify flow regimes, and continuous wavelet transforms to analyze gamma count data. Park and Kim [34] applied the wavelet packet transform to analyze pressure fluctuations in a vertical bubble column.

Furthermore, De Fang et al. [35] also used wavelet analysis to understand the gravity differential pressure fluctuation signal perpendicular to the horizontal flow of different flow patterns and the flow pattern transition of gas/liquid two-phase flow in the horizontal pipe. Here, the Haar wavelet with
six levels was used to decompose the pressure signal, and the energy value was then obtained for each scale. For identifying a two-phase flow regime, Elperin and Klochko [36] also used an eight-level db4 wavelet transformation to process time series of the measured differential pressure fluctuation. In this study, using pressure transducer signal data of different flows, the pattern was analyzed using wavelet transformation to determine the pressure signal characteristics of various flow regimes.

To identify the two-phase flow regime mapping and avoid slug flow in horizontal pipelines, a clear understanding of the Newtonian and non-Newtonian fluid flow is necessary for an accurate and inherently safer design for any flow system. The uniqueness of this study is that the experiments were performed in a setup with 73.66 mm ID and approximately 19 m flow loop in the presence of gas/Newtonian and gas/non-Newtonian two-phase horizontal flow. This flow loop has a horizontal, vertical, and inclined test section connected; however, the study is only on horizontal flow aspects. Another primary focus of this study is to understand the characteristics of slug frequency analysis for both air/Newtonian and air/non-Newtonian flow conditions in the presence of Xanthan gum as a non-Newtonian fluid. This study also recognizes the characteristics of wavelet analysis for both air/Newtonian and air/non-Newtonian flow conditions, i.e., water and Xanthan gum.

2. Materials and Methods

2.1. Materials

Water is used as a Newtonian fluid in this study. For non-Newtonian fluid experiments, a 0.1 wt% solution of biopolymer, i.e., Xanthan gum, was used (obtained from Merck, Germany). To conduct two-phase experiments, compressed air was used for gas/Newtonian fluid and gas/non-Newtonian experiments. The air was compressed via an in-house compression facility.

2.2. Experimental Methods

The experiments were performed in a flow loop system that has a horizontal, vertical, and inclined section. However, in this paper, we are only considering the 4-m horizontal section as our test section. The experimental setup is a 60-m-long closed cycle system for water and open cycle system for air. The liquid is pumped by a 5 HP pump that creates the required large volume water flow through DN80 or 2.9 ID PVC clear pipes. The airline of the flow loop comprises of DN15 and DN25 mild steel pipes, which supply air from the lab air supply at 670 kPa (100 psi) shut-in pressure. It also includes a DN 25 ball check valve just before the air and the liquid mixing zone to prevent any liquid from entering the air pipeline. There are two Omega PX603-100G5V pressure transducers with a range of 0 to 100 psi in the 2-m long horizontal test section. There were some specific experimental conditions used for this setup. The airflow range was approximately 85 L/min to 3300 L/min, and the water flow ranged from almost 250 L/min to 850 L/min. Within this range, the experimental setup mostly provides slug flow for two-phase flow, as well as providing bubble flow and wavy flow at some intervals. Figure 1 presents a schematic representation of the experimental setup. For this study, both gas/Newtonian and gas/non-Newtonian fluid flow cases were considered. A Universal Data Acquisition System from National Instruments was used to collect all types of data from the flowmeter and sensors, which consisted of four NI 9219 universal modules with four channels, each providing 100 samples per second. The experimental conditions are reported in Table 1, including brief details of the rheological model used for xanthan gum solutions.

| Table 1. The chemicals and experimental parameters used in this study. |
|---------------------------------------------------------------|
| **Newtonian Fluid** | 0.1% xanthan gum solution (Power Law Index, \(n = 0.81\)) and (Power Law Index, \(m \text{ or } k = 0.009344\)) |
| **Non-Newtonian Fluid** | Water |
| **Liquid velocity range** | 1.5 to 2.5 m/s |
| **Air velocity** | 2.9 to 6.4 m/s |
Figure 1. Schematic of the experimental setup (horizontal test section) with a data acquisition system.

Slug flow is the most frequent two-phase flow phenomenon experienced in horizontal or near horizontal pipelines in practical applications. Slug flow in pipes is encountered in different fields, such as the production and transportation of oil and gas, food industry, chemical industry, and other industrial applications. Slug frequency, in addition to water hammering, leads to various operational problems, such as pipeline network instability, equipment damage, pressure fluctuations, and vibration of the system. In the oil and gas production industries, slug flow also influences the internal corrosion rate increase of carbon steel pipelines. Slug flow creates high turbulence, which can break the pipe wall inhibitor’s protection layer [37].

2.2.1. Slug Velocity

In 1965 and 1977, Hubbard [38,39] and Otten & Fayed [40], respectively, experimentally measured the slug velocity by observing a particular slug movement in test sections. They both obtained a relationship between slug velocity and no-slip mixture velocity by plotting the experimentally measured slug velocity against the no-slip mixture velocity. The slug flow model of Hubbard [39] showed better agreement at higher slug velocity. Hubbard [39] described the relationship as:

\[ v_s = 1.25 \, v_m \]  

(1)

Hubbard & Dukler [39] also predicted the actual average gas velocity as in Equation (2), which also agreed with other experimental data [41].

\[ v_G = 1.19 \, v_m \]  

(2)
It is assumed that the Hubbard & Dukler [39] slug flow model was verified based on one dominant presupposition that the liquid slug velocity and the maximum gas phase velocity should be similar. Therefore, theoretically, the no-slip mixture velocity should be equal to the slug velocity:

\[
\frac{v_s}{v_m} = C \tag{3}
\]

where \(C\) is a constant. Theoretically, \(C\) is assumed to be 1.0 for air–water two-phase flow. Hubbard [38], Rosehart et al. [42], and Gregory & Scott [41] considered \(C\) values of 1.25, 1.26, and 1.35, respectively for air–water flow. These \(C\) values may have varied because of different experimental setups and conditions [40]. For non-Newtonian/air two-phase flow, Otten & Fayed [40] compared their results with those of Rosehart et al. [42]. The obtained results suggested that the air/Carbopol 941 concentration reported by Otten & Fayed [40] increased from 0.75% to 0.2% and \(C\) values increased from 1.36 to 1.41, whereas for the same concentration, the \(C\) values of Rosehart et al. (1975) varied from 1.54 to 1.98.

2.2.2. Slug Frequency

Different correlations can predict slug frequency. However, Gregory & Scott [41] suggested a trustworthy velocity-dependent empirical equation where slug frequency was correlated with a form of Froude number, which is described as:

\[
N_{f_s} = \frac{v_l}{g'd} \left[ \frac{(v_m^0)^2}{v_m} + v_m \right] \tag{4}
\]

Here, \(v_m^0\) was taken as 6 m/s and from the graphs of the slug frequency vs. slug Froude number by Gregory & Scott [41], the following equation was derived:

\[
f_s = 0.0157 \left( N_{f_s} \right)^{1.20} \text{sec}^{-1}. \tag{5}
\]

From Equation (5), Gregory & Scott [41] described a slug frequency correlation based on their liquid-gas two-phase flow experimental data where water and carbon dioxide are used in a 19-mm ID pipe:

\[
f_s = 0.0226 \left[ \frac{v_s^e}{g'd} \left( \frac{19.75}{v_m} + v_m \right) \right]^{1.2} \tag{6}
\]

where \(v_m\) and \(v_s^e\) represent the mixture velocity and superficial liquid velocity of liquid and gas, respectively. Therefore, this slug frequency can be combined with the Froude number established for superficial fluid velocity.

Greskovich & Shrier [43] reorganized the Gregory & Scott [41] correlation as follows:

\[
f_s = \left[ 0.0425 \frac{v_s^e}{v_m} \left( \frac{2.02}{d} + \frac{v_m^2}{g'd} \right) \right]^{0.5} \tag{7}
\]

Later, Zabaras & others [44] described another correlation based on 399 data points with the smallest average absolute error and standard deviation for both horizontal and inclined pipe flow. This correlation is a modification of the Gregory & Scott [41] correlation, using the imperial unit, which is shown in Equation (8), where \(\theta\) is the inclination angle. The experiment was conducted in a 2.54 cm and 10.16 cm ID pipe with air and water.

\[
f_s = \left[ 0.0425 \frac{v_s^e}{d} \left( \frac{1}{0.0506v_m} + v_m \right) \right]^{0.5} \left[ 0.836 + 2.7 \sin^{0.25} \theta \right] \tag{8}
\]
Heywood & Richardson [45] determined the liquid volume fraction for air–water two-phase flow utilizing the gamma-ray technique in a 4.191 cm ID horizontal pipe. To determine the liquid volume fraction, they used a power spectral density function and probability density function. These features are also helpful to decide on different slug flow characteristics, such as the value of the average film and slug volume fraction, average slug frequency, and average slug length. The slug frequency correlation was determined by curve fitting the data, resulting in Equation (9), in which $\lambda$ is the liquid volume fraction, where $\lambda = \frac{v_f}{(v_f^p + v_\delta^p)}$, and $d$ is the pipe diameter.

$$f_s = 0.0462 \lambda \left( \frac{1}{0.0126d} + \frac{v_\delta^2}{gd} \right)^{1.02} \quad (9)$$

Shea et al. [46] developed a correlation describing slug frequency as a function of pipe length. In Equation (10), $v_f^p$ is the superficial liquid velocity, $d$ is the pipe diameter, and $l_p$ is the pipe length. This correlation is based on the curve fitting of field and laboratory data, and is not based on theoretical analysis. In this equation, it is also shown that the slug frequency is inversely dependent on the pipe length $l_p$, which does not agree with the other theoretical analysis. According to Al-Safran [14], the OLGA 2000 slug tracking model had some time delay problem between two slugs; to solve this issue, the Shea et al. [46] correlation was initially used. Moreover, pipe length can be questionable for a long-distance transmission system with hilly conditions:

$$f_s = 0.47 \left( \frac{v_f^p}{l_p d^{1.5}} \right)^{0.5} \quad (10)$$

Picchi et al. [47] described a slug frequency equation that considers the rheology of the shear-thinning fluid. This equation is a modified version of the Gregory & Scott [41] correlation. In Equation (11), $Re_w = \frac{n_0 v_f^p d}{\mu_w}$ is the water Reynolds number and $Re_m = \frac{d^n v_f^p \rho_m}{m^{\alpha} \mu_m (\frac{1}{2} \rho_m v_f^p v_\delta^p)^{\frac{1}{2}}} \sqrt{g d}$ is the power-law fluid Reynolds number under superficial conditions, where $n$ and $m$ are fluid behavior indexes.

$$f_s = 0.0448 \left[ \frac{v_f^p}{gd} \left( \frac{32.2014}{v_{ms}} + v_{ms} \right) \right]^{0.88} \left( \frac{Re_m}{Re_w} \right)^{0.07} \quad (11)$$

2.2.3. Wavelet Analysis

Wavelet analysis is an effective means of signal processing. Wavelets are uneven and asymmetrical waveforms of adequately limited duration with an average value of zero. Wavelet analysis breaks up the mother wavelet signal into a shifted and scaled version. The typical wavelet transformation of a sine wave is presented in Figure 2.

![Wavelet transformation of a sine wave.](image-url)
In Fourier analysis, the signals are decomposed into different sine waves. Therefore, an irregular wavelet performs better than the steady sine wave for rapidly changing signals as it can provide better information about specific and appropriate locations. Wavelet analysis can also show any discontinuity, breakdown, trend, noise, coefficient, etc., of signals [48]. In this study, wavelet analysis was conducted using the MATLAB toolbox. There are two types of wavelet analyses, namely, discrete wavelet transform and continuous wavelet transform. There are various subgroups of these two types of wavelet transforms.

Contentious Wavelet Transform (CWT)

The continuous wavelet transforms (CWT) is a function of the shifted and scaled version of the wavelet function \( \Psi \) multiplied by the summation of the overall time of the signal. However, scaling means compressing or stretching the wavelets. A scale factor is used to represent the scaling, where the wavelet is more compressed when the scale factor is smaller. Wavelet sifting means hastening or detaining its onset.

\[
C(\text{position, scale}) = \int_{-\infty}^{\infty} f(x) \Psi(\text{position, scale, } x)dt
\]

where \( C \) is the wavelet coefficient of the CWT as a function of position and scale [48].

Discrete Wavelet Transform (DWT)

The discrete wavelet transform is a wavelet transform where the wavelets are separately sampled. In this analysis, the original signal is divided into two parts, namely, approximations and details. The estimate \( a \) is the low pass filter where the low-frequency components of the original signal are separated, and the detail \( d \) is the high pass filter where high-frequency components pass. Moreover, the original signal \( x \) is not only isolated in one level, but the approximation \( a \) is also decomposed into much lower level (\( k = 3 \)) components, known as multiple level decomposition, which is shown in Figure 3.

\[
x = a_1 + d_1 = a_2 + d_2 + d_1 = a_3 + d_3 + d_2 + d_1
\]

Figure 3. Multiple level discrete wavelet analysis.

The significant difference between CWT and DWT is that CWT operates at every scale up to the maximum value, whereas, in DWT, the range and positions can be preselected and, in that way, the size of the analysis reduces its size and is thereby faster and more precise.

Mathematically, for \( j \) scales and \( k \) levels, the approximate information \( f_a^j(x) \) can be the summation of estimated coefficients \( a_{jk} \) and the scale function \( \phi_{jk}(x) \), as shown in Equation (13). Similarly, the detail information \( f_d^j(x) \) can also be described as the summation of approximate coefficients \( d_{jk} \) and the scale function \( \Psi_{jk}(x) \) in Equation (14):

\[
f_a^j(x) = \sum_k a_{jk} \phi_{jk}(x)
\]

\[
f_d^j(x) = \sum_k d_{jk} \Psi_{jk}(x)
\]
\[ f_j^f(x) = \sum_k d_{jk} \Psi_{jk}(x) \] (14)

A standard means of applying this is via logarithmic discretization of the scale \( s \) and then connecting it to the step size. The step size is the values between the translation parameter \( \tau \). The equation is adapted from Gao & Yan (2010) [49] and shown below, where \( Z \) is an integer parameter:

\[
\begin{align*}
\{ s &= s_0^j \tau = k \tau_0 \neq 0; s_0 < 1 \ (j \in \mathbb{Z}, k \in \mathbb{Z}) \\
\Psi_{jk}(x) &= s_0^{-0.5j} \Psi\left(\frac{x}{s_0^j} - k \tau_0\right) \\
\Psi_{jk}(x) &= 2^{-0.5j} \Psi\left(\frac{x}{2^j} - k\right)
\end{align*}
\] (15)

where \( j \) is the scale and \( k \) is the level of the wavelet. Equation (18) is the base wavelet equation. Addison (2017) [50] assumed \( s_0 = 2 \) and \( \tau_0 = 1 \); therefore, Equation (19) can be derived and, finally, the discrete wavelet transform is obtained:

\[
W(j, k) = \left[ f(x), \Psi_{jk}(x) \right] = 2^{-0.5j} \int_{-\infty}^{\infty} f(x) \Psi\left(\frac{x}{2^j} - k\right) dx
\] (18)

\[
f(x) = \sum_{j,k} C_{jk} \Psi_{jk}(x)
\] (19)

In Equation (18), \( f(x) \) is the original signal and, in Equation (19), \( C_{jk} \) is the wavelet coefficient. For multilevel wavelet analysis, there are many types of orthogonal wavelet transformations, which determine the shape of the wavelet. Among them, the Daubechies wavelet is one of the most common orthogonal wavelet transformations.

**Daubechies Wavelet**

The Daubechies wavelet uses scalar products with scaling wavelets and signals to calculate moving average and difference. This method allows for obtaining a good range of signal data to compute the median and deviation. Daub4 is the most accepted and straightforward way of analyzing wavelets. If we consider a signal \( x \) constituting \( n \) number of values, then the daub4 transformation creates the mapping \( x \rightarrow (a^k|d^k) \) to its approximation \( a^k \) and details \( d^k \) sub-signals for \( k \) levels:

\[
\begin{align*}
a_m &= x . U^k_m \\
d_m &= x . \Psi^k_m
\end{align*}
\] (20)

where each value of \( a_m \) and \( d_m \) are the scaler products. \( U^k_m \) is the scaling signal and \( \Psi^k_m \) is the wavelet at level \( k \) [51].

**Wavelet Packet Analysis**

In DWT, the primary signal is decomposed into an approximation and details, and the approximation is divided into a second-level approximation and features; thus, \( n \) levels of decomposition can be undertaken. In wavelet packet analysis, both the details and the estimate can be decomposed, which means the signal can be encoded in \( 2^n \) ways. The wavelet packet decomposition tree is shown in Figure 4.
Figure 4. Wavelet packet analysis decomposition tree.

In the MATLAB toolbox, the entropy-based criterion is used to find the most desirable wavelet decomposition. Wavelet packet transformation provides many bases, and the best tree base can be found by the entropy criterion \[48\]. Wavelet packets are the general form of orthogonal wavelets. This splits detailed spaces to yield more prime decomposition. Coifman & Wickerhauser (1992) \[52\] explained the wavelet packet transformation equation as the following:

\[
\begin{align*}
    v'_{2i}(x) &= \sqrt{2} \sum_k h_k v_i(2x-k) \\
    v'_{2i+1}(x) &= \sqrt{2} \sum_k g_k v_i(2x-k) \quad ; i = 0, 1, 2, \ldots \text{and} k = 0, 1, \ldots m
\end{align*}
\]

where the two filters \(h_k\) and \(g_k\) are associated with the scaling function \(\varphi_{jk}(x)\) and the base wavelet function \(\Psi_{jk}(x)\) \[49\].

Wavelet Entropy

Wavelet entropy represents the non-uniformity of states, which is an ideal parameter for measuring the ordering of unsteady signals \[53\]. It can also provide information about the dynamic process and the signal potential. When the coefficient matrix of the wavelet transformation is represented by a probability distribution, the calculated wavelet entropy represents the randomness of the pattern \[54\]. The wavelet packet decomposition is an orthogonal function, which means the total energy entropy of the original signal should be the summation of the coefficient energy entropy \[32\]. The wavelet entropy energy can be defined as in Equation (23), where \(P_i = E_i / \sum_{i=1}^{n} E_i\) is the percentage of coefficient energy of the original signal \[55\]:

\[
EN = - \sum_{i=1}^{n} P_i \log P_i
\]

In this study, the norm entropy was used to analyze the pressure signal. In an orthonormal basis entropy, \(s\) is the signal, \(s_{ij}\) is the coefficient of \(s\), and \(E\) is the entropy function such that \(E(0) = 0\) and \(E(s) = \sum_i E(s_i)\). This entropy formula was used in MATLAB to calculate norm entropy. The concentration in \(l^p\) is a norm where \(1 \leq P < 2\). Now, \(E(s) = ||s||^p\) so \(E(s) = \sum ||s||^p = ||s||^p\) for the norm entropy method \[48\]. The wavelet entropy can find small or abnormal frequencies, and wavelet entropy can thus find different characteristics of multiphase flow. Therefore, this study aims to characterize a two-phase flow pattern using norm entropy based on wavelet packet decomposition of the pressure signal. This method followed the steps shown in Figure 5.

Wavelet Packet Analysis of the Experimental Data

In this study, a pressure transducer provided time-domain pressure fluctuations, which were analyzed using wavelet packet analysis. As mentioned previously, the existing experimental setup only provides slug flow, and the dispersed bubble flow regime and pressure signal also show specific characteristics for each kind of flow regime. The data acquisition system collected pressure transducer
signals with a sampling frequency of 100 Hz. Overall, 10,000 data points were used in 1-D wavelet analysis using MATLAB (details can be found in the above sections). The MATLAB processed data were taken into account for further analysis.

**3. Results and Discussions**

**3.1. Flow Regime Mapping for Horizontal Flow**

3.1.1. Air/Newtonian Flow Map

The experimental values were used to derive a flow regime map for horizontal pipe flow. This flow regime map was compared with that of Taitel & Dukler [7] and Mandhane et al. [9], where water and superficial air velocity was used. In the Taitel & Dukler [7] flow map for a horizontal pipe in Figure 6, most of the experimental data points fall in the respective flow regime area. However, the Taitel & Dukler [7] flow map better predicted the dispersed bubble flow for high gas/water velocity than the Mandhane et al. [9] flow map for this experimental setup.

In Figure 7, the Mandhane et al. [9] flow map is provided where the data for the slug and dispersed bubble flow data were fitted in the graph accordingly. The map can predict the slug and bubble flow regimes. Nevertheless, for high gas and water flow rates, this map cannot predict the dispersed bubble flow regime due to experimental conditions and the pipe diameter.
Researchers have also developed different flow pattern maps for horizontal, vertical, and inclined gas/non-Newtonian fluid. For horizontal gas/non-Newtonian fluid, Chhabra & Richardson [56] developed a flow pattern map by slightly modifying the Mandhane et al. [9] horizontal flow pattern map (Figure 8). This map was developed for evaluating the literature and verified using 3700 data points of gas/non-Newtonian shear-thinning liquid mixture flow with 70% certainty. However, there was not enough data to confirm the Chhabra & Richardson [56] flow map for annular and slug flows.

In Figure 8, the experimental flow regime almost matches with the Chhabra & Richardson [10] flow map, but the slug to dispersed bubble flow transition started slightly earlier for this experiment. Chhabra & Richardson [10] used the particulate suspension of china clay, aqueous polymer solutions, limestone, and coal, which is a significantly more viscous shear-thinning non-Newtonian fluid compared to the 0.1% solution of xanthan gum that was used in this experiment. This is why the dispersed bubble flow regime started earlier. It is well-known that flow patterns of gas/non-Newtonian fluid are not significantly different from gas/Newtonian fluid for horizontal flow. However, due to high viscosity, the bubbles and slug could not break easily and collided together to form more prominent and well-defined bubbles.
there was not enough data to confirm the Chhabra & Richardson [56] flow map for annular and slug flow. Nonetheless, the boundary of the flow regime varied due to the unpredictable characteristics of the transition of the flow pattern. Therefore, the results are effectively limited by the bubble flow and plug flow range, which can provide stratified, wavy, or annular flow. The Taitel & Dukler [7] and Mandhane et al. [9] flow maps for air/water two-phase horizontal flow and the Chhabra & Richardson [56] flow map for air/xanthan gum solution horizontal two-phase flow represented the flow regimes of the experimental setup relatively accurately. Nonetheless, the boundary of the flow regime varied due to the unpredictable characteristics of the transition of the flow pattern.

3.1.3. Air/Newtonian Two-phase Flow

The flow behavior of the air/Newtonian two-phase flow is shown in Figure 9. Figure 10 demonstrates that slug frequency increases with the increase of liquid superficial velocity for all studied combinations, while the superficial gas velocity was held constant for each set of data. Similar behavior was reported by Abed and Ghoben [22] for a Newtonian fluid (water). This happened due to the increase in liquid volume fraction. The liquid occupies more space in the liquid film region as the elongated bubble unit becomes smaller, which is why the slug unit increased in number.

Figure 10 shows the effect of a superficial gas ratio on slug frequency. Results from Figure 10 reveal that, for a constant liquid flow rate, slug frequency decreases with increasing gas velocity. The slug frequency initially decreases with the gas velocity up to around 5 m/s gas velocity, and then increases again, indicating additional slug formation. Moreover, to provide further insight, Figure 11 represents the slug frequency vs. the mixture velocity for air/water flow.
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Figure 9: Effect of liquid superficial velocity on slug frequency for air/water flow.

Figure 10 shows the effect of a superficial gas ratio on slug frequency. Results from Figure 10 reveal that, for a constant liquid flow rate, slug frequency decreases with increasing gas velocity. The slug frequency initially decreases with the gas velocity up to around 5 m/s gas velocity, and then increases again, indicating additional slug formation. Moreover, to provide further insight, Figure 11 represents the slug frequency vs. the mixture velocity for air/water flow.

Figure 10. Effect of gas superficial velocity on slug frequency of air/water two-phase flow.

Comparing Figures 9 and 11, it is evident that the slug frequency curves mainly depend on superficial gas velocity. Two of these graphs also show that in the range of 5 m/s to 6.5 m/s, the slug frequency reaches a minimum, and the slug frequency increases with increasing mixture velocity or gas superficial velocity. The sudden increase in slug frequency perhaps indicates the transition of the flow regime from slug to dispersed bubble flow. The higher gas flow rates induce more turbulence in the fluid flow, initiating the slug breakdown, which is attributed to the increase in the overall number of slugs in the system. It has also been observed that the number of dispersed bubbles increases in...
the slug pocket and wet film area. This indicates the beginning of the transition of the flow pattern. These findings also agree with the experimental data of Otten & Fayed [40] and Gregory & Scott [41]. Moreover, the slope of slug frequency vs. slug Froude number is reported in Figure 12.

Figure 11: Slug frequency vs. mixture velocity for air/water flow.

In Figure 12, the slope of slug frequency vs. slug Froude number yields an equation of \( f_s = 0.0673 \left( N_{fr} \right)^{0.9757} \). This equation shows a slight deviation from Gregory & Scott [41], which is shown in Equation (5), perhaps due to the experimental conditions and the assumption \( (v_m = v_g) \) for the air–water flow of this experiment. To evaluate the standard deviation and experimental accuracy of this study, Figure 13 reports the regression analysis of slug frequency and Froude number.

Figure 12. Slug frequency vs. Froude number for air/water flow.
In Figure 12, the slope of slug frequency vs. slug Froude number yields an equation of
\[ f_s = 0.0673 \cdot \left( N_{fs} \right)^{0.098}. \]
This equation shows a slight deviation from Gregory & Scott [41], which is shown in Equation (5), perhaps due to the experimental conditions and the assumption \( v_m = v_s \) for the air–water flow of this experiment. To evaluate the standard deviation and experimental accuracy of this study, Figure 13 reports the regression analysis of slug frequency and Froude number.

**Figure 13.** Relationship between slug frequency and Froude number for air–water system data (\( R^2 = 88.1\% \)).

From Figure 13, the \( R^2 \) value was found to be 88.1\%, which indicates that the slug frequency and Froude number have a good relationship. Since the experimental and modeled data are close to the regression line, the equation \( f_s = 0.0673 \cdot \left( N_{fs} \right)^{0.098} \) can explain the variability of the data around its mean.

In order to further validate and test the reliability of the obtained experimental data, the results are reported with correlations with different model results reported in various previous studies. Figure 14 shows the experimental and modeled validation with the Gregory & Scott [41] correlation model for air–water two-phase flow.

**Figure 14.** Experimental slug frequency for the air–water system compared with the prediction model of Gregory & Scott [41] correlation (\( R^2 = 73.8\% \)).

From Figure 14, it is observed that all the data points are close to the regression line, the regression has an \( R^2 \) value of 73.8\%, and all the experimental data fit well within the 95\% confidence interval. The experimental slug frequency results are also compared with the Zabaras et al. [23] predicted model for the air–water system in Figure 15. The results reveal that the data points of the Zabaras et al. [23] predicted model have comparatively deviated from the regression line, with an \( R^2 \) value of 60.3\% at a 95\% confidence interval.

**Figure 15.** Experimental slug frequency for the air–water system compared with the prediction model of Zabaras et al. [23] correlation (\( R^2 = 60\% \)).

The experimental data and the predictions of slug frequency of the Gregory & Scott [41] model have an \( R^2 \) value of 73.8\%, and the Zabaras et al. [23] model has an \( R^2 \) value of 60%. Therefore, the experimental and modeled slug frequency results can be considered accurate and reliable.
From Figure 14, it is observed that all the data points are close to the regression line, the regression has an $R^2$ value of 73.8%, and all the experimental data fit well within the 95% confidence interval. The experimental slug frequency results are also compared with the Zabaras et al. [23] predicted model for the air–water system in Figure 15. The results reveal that the data points of the Zabaras et al. [23] predicted model have comparatively deviated from the regression line, with an $R^2$ value of 60.3% at a 95% confidence interval.

The experimental data and predictions of slug frequency of the Gregory & Scott [41] model have an $R^2$ value of 73.8%, and the Zabaras et al. [23] model has an $R^2$ value of 60%. Therefore, the Gregory & Scott [41] model is found to be closer to the experimental data. In the above graphs, the differences between experimental and predicted data varied mainly due to the variation in experimental conditions and setups [22]. Since the experimental data are more similar to the Gregory & Scott [41] model, Figure 16 shows the Gregory & Scott [41] correlation with the standard deviation of three samples under the same experimental condition. The error bar indicates a 95% confidence interval of the data, and none of the confidence intervals includes zero, which means the data are statistically significant and repeatable for air/water two-phase flow.

**Figure 15.** Experimental slug frequency for the air–water system compared with the prediction model of Zabaras et al. [23] correlation ($R^2 = 60\%$).

The experimental data and predictions of slug frequency of the Gregory & Scott [41] model have an $R^2$ value of 73.8%, and the Zabaras et al. [23] model has an $R^2$ value of 60%. Therefore, the Gregory & Scott [41] model is found to be closer to the experimental data. In the above graphs, the differences between experimental and predicted data varied mainly due to the variation in experimental conditions and setups [22]. Since the experimental data are more similar to the Gregory & Scott [41] model, Figure 16 shows the Gregory & Scott [41] correlation with the standard deviation of three samples under the same experimental condition. The error bar indicates a 95% confidence interval of the data, and none of the confidence intervals includes zero, which means the data are statistically significant and repeatable for air/water two-phase flow.

**Figure 16.** Experimental slug frequency for the air–water system compared to the predictions of Gregory & Scott [41] correlation with standard deviations.
3.1.4. Air/non-Newtonian Two-Phase Flow

In this air/non-Newtonian fluid experiment, a 0.1% solution of xanthan gum was used as the non-Newtonian fluid. Non-Newtonian fluids are generally superior drilling fluids compared to Newtonian fluids due to their viscosity and the holding/uplifting capacity of the cuttings. Figure 17 illustrates the impact of liquid superficial velocity on slug frequency for the air/non-Newtonian flow system.

![Figure 17](image17.png)

**Figure 17.** Effect of liquid superficial velocity on slug frequency for air/non-Newtonian flow.

In Figure 17, the slug frequency increases with the increment of liquid non-Newtonian superficial velocity when the superficial gas velocity is held constant. Therefore, at lower superficial liquid velocity, the slug frequency increases sharply, and at higher liquid velocity, the slug frequency decreases. As the liquid velocity increases, the air requires more energy to drive the viscous fluid; nevertheless, the airflow rate is constant for each set. This explains why the number of slugs decreases as the liquid velocity increases at continuous airflow (see Figure 18). Furthermore, Figure 18 highlights the relationship between gas superficial velocity and slug frequency for air/non-Newtonian flow conditions.

![Figure 18](image18.png)

**Figure 18.** Effect of gas superficial velocity on slug frequency for air/non-Newtonian flow.

In Figure 18, slug frequency change is shown with superficial gas velocity for a constant liquid superficial velocity. Here, 0.1% xanthan gum solution was used as a non-Newtonian fluid where the power-law index, \( n \), is equal to 0.81 and \( k = 0.009344 \). From Figure 18 it is apparent that the slug frequency decreases as the gas velocity rises to 6.5 m/s. Because the gas flow rate increases in a constant...
liquid velocity, Taylor bubble formation becomes relatively more significant; therefore, the length of the slug unit increases, and slug frequency decreases. The evaluation of the slug frequency vs. mixture velocity for air/non-Newtonian fluid flow is reported in Figure 19.

![Figure 19](image_url)

Figure 19. Slug frequency vs. mixture velocity for air/non-Newtonian fluid flow.

Figure 19 represents the change of slug frequency with the mixture velocity of air–xanthan gum flow. In addition, it is seen that until 6.5 m/s, the mixture velocity slug frequency is minimal. Otten & Fayed [40] also obtained similar patterns for air/non-Newtonian flow. The same phenomena also occurred in Figure 11 for gas/Newtonian flow. However, the minimum slug frequency was around 5 m/s mixture velocity, which happened significantly earlier than the gas/non-Newtonian two-phase flow. Here, we can observe the inevitable effect of viscosity. Water viscosity at 20 °C room temperature is around 1 cP, and the experimental viscosity of 0.1% xanthan gum is 2.4 cP, which is slightly more viscous than water.

The flow mechanism of slug flow is that the gas bubble is trapped in the water and drives the water forward almost at the same velocity as the gas velocity. Nevertheless, when the liquid becomes viscous, the gas requires more energy to drive the liquid forward. At a constant airflow rate, it is hard to achieve extra energy, so the whole process becomes slow, and the slug velocity and number of slugs decrease [42]. If further experiments were conducted for gas/non-Newtonian fluids, slug frequency would increase again, with an increased gas flow rate in the slug to bubbly flow transition zone as the gas/water two-phase flow results in the transition from slug to dispersed bubble flow. The turbulence of the flow structure begins to increase, the unit slug starts to break down, and the number of slugs increases at higher gas flow rates. It has also been observed that the number of dispersed bubbles increases in the slug pocket and wet film area. This indicates the beginning of the transition of the flow pattern.

To understand this result further and demonstrate its wider applicability, the present study was extended to a non-dimensional number, i.e., the Froude number, presented in Figure 20 as the relationship between slug frequencies and Froude number for an air/xanthan gum solution.

In Figure 20, it is shown that the slope of slug frequency vs. slug Froude number for air/xanthan gum solution can be modeled using the equation $v_s = 0.0082 \left( N_f \right)^{1.5624}$, where the model strength $R^2$ is 80.62%. In order to further validate the experimental data for an air/non-Newtonian two-phase flow system, Figures 21 and 22 represent the relationship between the experimental and predicted data of the Gregory & Scott [41] proposed model with standard error bars and 95% confidence level, respectively.
In order to further validate the experimental data for an air/non-Newtonian two-phase flow system, Figure 21 represents the comparison of experimental results with the modified Gregory & Scott [41] slug frequency equation with error bars.

Figure 20. Slug frequency vs. Froude number for an air/xanthan gum solution.

Figure 21. Experimental vs. predicted data [41] with standard error bars.

Figure 22. Experimental slug frequency for the air–xanthan gum system compared to predictions by the Gregory & Scott [41] correlation, where $R^2 = 74.6\%$.
The error bar in Figure 21 shows the 95% confidence interval of the data; none of the confidence intervals includes zero, which means the data are statistically significant and repeatable for air/water two-phase flow.

Picchi et al. [47] modified the slug frequency equation of Gregory & Scott [41] for a shear-thinning non-Newtonian fluid. Figure 21 represents the comparison of experimental results with the modified Gregory & Scott [41] slug frequency equation with error bars. The $R^2$ value of 75.3% also represents the reliability and repeatability of the experimental data of this study. Figure 22 also shows the 95% confidence interval of the experimental data.

3.2. Wavelet Spectrum Analysis

The wavelet packet analysis decomposed the pressure signals into four levels. Among the wavelet decomposition methods, Daubechies’ four-scale base wavelet (db4) has been used most frequently in multiphase flow time series decompaction [57]. In this study, Daubechies’ four-scale base wavelet (db4) and norm entropy analysis method generated 16 wavelet packet coefficients. The pressure fluctuation signal achieved from the experimental data only yields a frequency of 100Hz. Thus, only four-level decomposition is sufficient because the pressure signals do not have sufficiently high-frequency and high-resolution data to obtain more detailed frequency analysis. The spectrum of the packet wavelet analysis represents the time-frequency plot, which provides a coefficient of decomposed frequencies at a different level. This spectrum represents the time and location of the fluctuation of the signal.

From Figure 23, it can be observed that the time-frequency plot divides the time-frequency plane into concentrated rectangles as a two-dimensional representation of signals. The pink color intensity of each box depends on the coefficient of the wavelet packet [34].

![Figure 23. Spectrum for slug flow under different flow conditions.](image-url)
A similar plot is shown in Figure 24, which is the time-frequency plot for the bubbly flow regime for various flow conditions. The intensity of the pink shade represents the energy level of a time-frequency cell; the lower the energy content, the lighter the shade. For the bubbly flow regime, the bubbles are smaller, so the pressure fluctuation intensity is less, which means the low-frequency response has less energy content and the pink shade is light and scattered. In the slug flow, the Taylor bubble size is more prominent. Therefore, the low-frequency cells have more energy and a darker shade [34]. In addition, the repetition of the intense pink shade after a specific time interval can be evidence of the picks of the pressure signal. Thus, with a higher-resolution and better-quality sensor, where the pressure signal picks are more precise, this map can be a helpful way to understand the flow phenomena inside the pipe. When comparing the wavelet spectrum analysis of bubbly and slug flows for the same water flow rate, it has been observed that, for bubbly flow, the color intensity is comparatively less in the low-frequency response area. However, the use of a higher-resolution pressure transducer may enhance the wavelet spectrum quality with more precise fluctuation characteristics.

Wavelet Entropy Analysis

The wavelet entropy analysis of the pressure fluctuation data represents the nonlinearity of the gas/liquid two-phase flow. After calculating the wavelet entropy of 10,000 pressure signal data points of gas/liquid two-phase horizontal flow, it was seen that the wavelet entropy increased with the increase of the pressure signal fluctuation.

The entropy values of the pressure fluctuation data were compared with the gas to liquid volume flowrate ratio (GLR) and the void fraction \( \alpha_g = \frac{v_g}{v_g + v_l} \). The void fraction \( \alpha_g \) is the ratio of the gas velocity and the liquid velocity.
In Figure 25, it can also be observed that wavelet entropy increased with the increase of gas void fraction for gas/non-Newtonian fluid. This means the fluctuation of the pressure increases with the rise in the GLR for gas/water flow. Another observation is that the wavelet entropy value for gas/non-Newtonian flow is less than that for the gas/Newtonian fluid flow. This phenomenon occurred due to the viscosity effect of the non-Newtonian fluid, which means the disturbance of the pressure signals is less than that of the Newtonian fluid.

Figure 26 shows that with the growth of the GLR, the norm wavelet entropy increased, which means the fluctuation of the pressure increases with the increase of the GLR. However, the norm entropy change at the low GLR is not consistent. Fan et al. (2013) also saw similar behavior for low GLR values, mostly in the bubble flow or bubble-slug transition flow region. As bubble flow and motion are random and complicated, it is hard for low-resolution sensors or wavelet norm entropy to detect the change. Figures 27 and 28 show the wavelet norm entropy of bubble, bubble-elongated bubble, and slug flow regimes at different GLR combinations for the experimental setup used in this study.
The flow regime analysis revealed that the slug flow regime condition was observed for most of the conditions. The slug frequency analysis illustrated that the gas/Newtonian and gas/non-Newtonian fluid flow conditions. The flow maps of both systems were established, and the type of flow regimes were determined for both air/Newtonian and air/non-Newtonian fluid flow conditions. The wavelet norm entropy value may vary with higher resolution sensors. Wavelet norm entropy depends on the pressure fluctuation signal; therefore, the more sensitively and precisely the sensor can detect the flow condition, the more accurate the wavelet norm entropy will be. However, the wavelet entropy change pattern with a different type of flow regime should remain similar. Fan et al. (2013) and Sun et al. (2013) both observed similar wavelet entropy change patterns. However, their sensors, experimental setups, and experimental conditions were different.

4. Conclusions

In this experimental work, air/Newtonian and air/non-Newtonian fluids were evaluated in horizontal flow loop conditions. The flow maps of both systems were established, and the type of flow regimes were determined for both air/Newtonian and air/non-Newtonian fluid flow conditions. The flow regime analysis revealed that the slug flow regime condition was observed for most of the studied conditions. The slug frequency analysis illustrated that the gas/Newtonian and gas/non-Newtonian fluids have a significant difference in slug properties, mainly due to the difference in viscosity of the systems. The air/water slug frequency decreased to an air velocity of approximately 5 m/s and then increased with increased air velocity. On the contrary, the air/xanthan gum solution did not demonstrate a similar effect, with the slug frequency slowly decreasing with increased air velocity within the experimental data range. Moreover, both the air/Newtonian and air/non-Newtonian

![Wavelet entropy flow map for gas/Newtonian flow.](image1)

![Wavelet entropy flow map for gas/non-Newtonian flow.](image2)
slug frequency was validated with the slug frequency correlations available in the literature, and experimental data were found to be a satisfactory analog of model predictions. Although the pressure sensor used in this study was not able to provide high-frequency and high-resolution data, the wavelet decomposition and wavelet norm entropy were nonetheless calculated, and these offered recognizable flow characteristics for bubble, bubble-elongated bubble, and slug flow patterns. In addition, 1-D wavelet packet decomposition was shown to be a useful method to identify different features of multiphase flow and for recognizing different flow patterns. Therefore, the present study can pave the way for the development of safer designs in pipe flow, especially in oil and gas pipelines, where undesired slug flow can lead to corrosion issues.

Author Contributions: M.M., M.A.R. and S.I. conceived the presented idea. M.A.R. and S.I. developed the theory, and M.M. performed the experiments. M.A.R. and S.I. supervised the findings of this work. M.S.K. and M.M. wrote the manuscript, and M.A.R. and S.I. reviewed the article. Overall, all the authors contributed to the betterment of the final manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: The APC was partially funded by Qatar National Research Fund via the grant NPRP10-0101-170091.

Acknowledgments: The authors would like to acknowledge the Faculty of Engineering and Applied Science at the Memorial University of Newfoundland for providing laboratory facilities for this research. In addition, this publication was made possible by the grant NPRP10-0101-170091 from Qatar National Research Fund (a member of the Qatar Foundation). Statements made herein are solely the responsibility of the authors.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

| Symbol | Definition |
|--------|------------|
| $v_s$ | Superficial Liquid Velocity |
| $v_l$ | Liquid Inlet Velocity |
| $\rho_w$ | Water Density = 996 Kg/M$^3$ |
| $L_p$ | Pipe Length |
| $v_m$ | No-Slip Mixture Velocity |
| $f_s$ | Slug Frequency |
| $f_{sn}$ | Slug Frequency for Non-Newtonian Fluid |
| $N_f$ | Froude Number |
| $v_{m,nn}$ | No-Slip Mixture Velocity for Non-Newtonian Fluid |
| $N_{f,nn}$ | Froude Number for Non-Newtonian Fluid |
| $Re_w$ | Water Reynolds Number |
| $\rho_n$ | Non-Newtonian Density = 1002 Kg/M$^3$ |
| $\sigma$ | Shear Stress |
| $\lambda$ | Liquid Volume Fraction |
| $v_G$ | True Average Gas Velocity in Multiphase Flow |
| $g$ | Acceleration due to gravity |
| $v_{sn,m}$ | Minimum Slug Frequency in The Graph = 6 m/s |
| $v_{ln}$ | Non-Newtonian Liquid Inlet Velocity |
| $Re_{nn}$ | Non-Newtonian Reynolds Number |
| $\gamma_{nn}$ | Shear Rate |
| $\mu_n$ | Apparent Viscosity |
| $W(j,k)$ | Wavelet Transform |
| $\psi_{j,k}(x)$ | Wavelet Base |
| $\phi_{j,k}(x)$ | Scale Function |
| $j$ | Wavelet Scales |
| $\alpha_g$ | Void fraction |
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