A nearly massless graviton in Einstein-Gauss-Bonnet inflation with linear coupling implies constant-roll for the scalar field

V. K. Oikonomou\textsuperscript{1,2,3} and F. P. Fronimos\textsuperscript{1}

\textsuperscript{1} Department of Physics, Aristotle University of Thessaloniki - Thessaloniki 54124, Greece
\textsuperscript{2} Laboratory for Theoretical Cosmology, Tomsk State University of Control Systems and Radioelectronics (TUSUR) - 634050 Tomsk, Russia
\textsuperscript{3} Tomsk State Pedagogical University - 634061 Tomsk, Russia

received 3 June 2020; accepted in final form 22 July 2020
published online 25 August 2020

PACS 04.50.Kd – Modified theories of gravity
PACS 95.36.+x – Dark energy
PACS 98.80.-k – Cosmology

Abstract – The striking GW170817 event indicated that the graviton is nearly massless, since the gamma rays emitted from the two neutron stars merging arrived almost simultaneously with the gravitational waves. Thus, the graviton must also be massless during the inflationary and post-inflationary era, since there is no obvious reason to believe otherwise. In this letter we shall investigate the theoretical implications of the constraint that the graviton is massless to an Einstein-Gauss-Bonnet theory with linear coupling of the scalar field to the four-dimensional Gauss-Bonnet invariant. As we show, the constraint of having gravitational wave speed of the primordial gravitational waves equal to one severely restricts the dynamics of the scalar field, imposing a direct constant-roll evolution on it. Also, as we show, the spectral index of the primordial scalar perturbations for the GW170817-compatible Einstein-Gauss-Bonnet theory with linear coupling is different in comparison to the same theory with non-linear coupling. Thus the phenomenology of the model is expected to be different, and we briefly discuss this issue too. In addition, the constant-roll condition is always related to non-Gaussianities, thus it is interesting that the imposition of a massless graviton in an Einstein-Gauss-Bonnet theory with linear coupling may lead to non-Gaussianities, so we briefly discuss this issue too.

Copyright © 2020 EPLA

Introduction. – The precision cosmology era that we live has the advantage that there exist numerous observational data coming from low-redshift sources, the Cosmic Microwave Background and also from astrophysical sources. These data shape literally our perception of the currently accelerating Universe, and alter in some cases the way we thought that the Universe works. One of these events in the 2017 observation of gravitational waves coming from the merging of two neutron stars is known now as the GW170817 event \cite{1}. This event was astonishing because it provided useful information for theoretical cosmologists, since the gamma rays arrived almost simultaneously with the gravitational waves. This result indicated that the gravitational wave speed $c_T$ is nearly equal to that of light, that is, $c_T^2 \simeq 1$, in natural units. In effect, the graviton, which is the propagator and mediator of gravity, is nearly massless.

From a theoretical point of view there is no reason to believe that during the early Universe, the graviton should have different mass from what it has today, therefore the implication of the GW170817 is that the graviton should have nearly zero mass during the inflationary and post-inflationary era. This requirement has a dramatic effect for quite a number of modified gravities describing the early Universe which predict a non-zero mass for the graviton, since these are ruled out by the GW170817 event. For a comprehensive account on this topic see, for example, \cite{2}.

However, most of the fundamental modified gravities, like $f(R)$ gravity or Gauss-Bonnet gravity \cite{3–9}, still predict a massless graviton even at the primordial era, thus are not ruled out. Nevertheless, an interesting class of modified gravities, namely Einstein-Gauss-Bonnet gravities \cite{10–46}, became quite problematic, since the predicted primordial gravitational wave speed is non-zero, thus the tensor perturbations of the primordial Universe produce results incompatible with the GW170817 event. In view of these problems, in some previous
works [44–46] we investigated under which conditions it is possible to obtain a massless graviton in the context of Einstein-Gauss-Bonnet theories. As we demonstrated, the requirement that \( c_F^2 \approx 1 \) is obtained only when the coupling function \( \xi(\phi) \) of the scalar field to the four-dimensional Gauss-Bonnet invariant satisfies the differential equation \( \xi - H\dot{\xi} = 0 \). In refs. [45,46] we showed that the quoted differential equation can severely constrain the functional forms of the coupling function \( \xi(\phi) \) and of the scalar potential \( V(\phi) \). As we showed, the viability of the GW170817-compatible Einstein-Gauss-Bonnet models can be achieved if the slow-roll conditions hold true. Thus we can simultaneously obtain a viable inflationary era from an Einstein-Gauss-Bonnet theory that predicts massless gravitons.

However, in refs. [45,46] we did not take into account at all the case in which the coupling function \( \xi(\phi) \) is a linear function of the scalar field. The purpose of this letter is to address exactly this case, and to investigate the effects of the linear scalar coupling function on the inflationary phenomenology of the GW170817-compatible Einstein-Gauss-Bonnet theories. As we show, it has dramatic effects, since it imposes a fixed constant-roll evolution on the scalar field thus utterly affecting the scalar field evolution. We focus on the behavior of the spectral index of the scalar perturbations, and we explicitly demonstrate how the linear coupling function changes it. As we show, the phenomenology is changed in comparison to the cases in which the coupling function \( \xi(\phi) \) is non-linear, which we studied in refs. [45,46]. This result affects many theoretical frameworks which use linear functions of the scalar field coupled to the four-dimensional Gauss-Bonnet invariant, see, for example, [38,47] and also see ref. [48] for a non-local Gauss-Bonnet gravity theory which is equivalent to a particular potential-less Einstein-Gauss-Bonnet gravity with linear coupling of the scalar field to the Gauss-Bonnet invariant (for a similar study in the context of \( f(R) \) gravity see, for example, [49]).

**Linear coupling GW170817 Einstein-Gauss-Bonnet gravity: how constant-roll evolution for the scalar field is imposed.** Let us demonstrate how the inflationary dynamics of the scalar field is affected if the linear coupling function \( \xi(\phi) \) is eliminated from the differential eq. (4), thus by substituting \( \xi = \xi(\phi) \) and \( \dot{\xi} = \xi' \dot{\phi} \), we have \( \tilde{\xi} = \xi' \dot{\phi} \) and thus we can rewrite the differential eq. (4) as follows:

\[
\xi'' \tilde{\phi}^2 + \xi' \ddot{\tilde{\phi}} = H \xi' \dot{\phi}.
\]

Let us now get to the core of this work, which is based on the choice of a linear coupling function, namely,

\[
\xi(\phi) = c_1 \kappa \phi,
\]

where \( c_1 \) is a dimensionless constant. Thus, for the linear coupling choice, the term containing \( \xi'' \) is eliminated from eq. (5), thus by substituting \( \xi(\phi) \) from eq. (6) in eq. (5) we get

\[
\dot{\tilde{\phi}} = H \dot{\phi}.
\]

The above condition describes an exact constant-roll evolution for the scalar field, and in the literature it is a well-established fact that in the context of several theories, canonical scalar theories and even \( F(R) \) gravity, a viable phenomenology can be obtained, see for example refs. [50–65], and in fact with a very specific rate. Let us see how this affects the inflationary phenomenology of the GW170817 Einstein-Gauss-Bonnet theory with linear coupling. To this end, let us recall the definition of the slow-roll indices for the Einstein-Gauss-Bonnet theory [10]:

\[
\begin{align*}
c_1 &= \frac{\ddot{H}}{H^2}, & c_2 &= \frac{\ddot{\phi}}{H \dot{\phi}}, & c_3 &= \frac{\dot{F}}{2HF}, \\
c_4 &= \frac{\dot{E}}{2HE}, & c_5 &= \frac{Q_a}{2HQ_t}, & c_6 &= \frac{\dot{Q}_t}{2HQ_t}.
\end{align*}
\]
with \( Q_a = -8\epsilon_1\kappa\phi H^2 \), \( F = \frac{dF}{d\eta} = 1 \) in the case at hand and
\[ E = \frac{\omega}{3\eta + 3\epsilon_2}. \]
Therefore the third slow-roll index is actually zero. By using eq. (7) and substituting it in the slow-roll \( \epsilon_2 \) in eq. (8) we get \( \epsilon_2 = 1 \). This condition can affect significantly the inflationary phenomenology of Einstein-Gauss-Bonnet theory as we now show. Essentially, there exists no physical limitation that prohibits the rest slow-roll indices to take large values such as \( \epsilon_2 \) as well apart from the Planck 2018 data which suggests that a viable inflationary era occurs when \( \epsilon_i < \mathcal{O}(10^{-3}) \) but hereafter we shall assume that the rest slow-roll indices are lesser than unity and examine the impact of the constant-roll evolution on the observed indices. We shall mainly focus on the spectral index of the primordial scalar curvature perturbations, since this observable will be mainly affected by the constant-roll condition (7). Let us recall how the spectral index can be extracted from the scalar curvature perturbations, and following [10], the calculation of the power spectrum results in the following expression for the parameter \( z \):

\[
\frac{z''}{z} = a^2c^2(3 + \epsilon_1 + \epsilon_2 + \epsilon_4)(2 + \epsilon_2 + \epsilon_4) + a^2H(\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_4) - 2a^2Hc\left(\frac{3}{2} + \epsilon_1 + \epsilon_2 + \epsilon_4\right) \times \frac{\dot{\epsilon}_5}{1 + \epsilon_5} + \frac{a^2\epsilon_5^2}{(1 + \epsilon_5)^2},
\]

where \( z \) is defined as follows:

\[
z = \frac{a}{(1 + \epsilon_3)H}\sqrt{E},
\]

with \( a \) being the scale factor, and the parameter \( E \) was defined previously below eq. (8). The previous expression is derived from the scalar-type perturbations and has this particular form in the slow-roll regime. Also \( a_c \) and \( H_c \) denote the scale factor and the Hubble rate exactly at the first horizon crossing time instance, respectively, and furthermore all the slow-roll indices have to be evaluated at the first horizon crossing time instance. As was shown in ref. [66], by making use of the Karamata’s theorem, we can write

\[
\eta = -\frac{1}{a_cH_c}\frac{1}{1 - \epsilon_1}.
\]

where \( \eta \) is the conformal time, and recall that \( a_c \) and \( H_c \) denote the scale factor and the Hubble rate exactly at the first horizon crossing time instance. Thus, by making use of eq. (11) in conjunction with the fact that \( \epsilon_2 = 1 \), and by assuming that the rest of the slow-roll indices and their derivatives satisfy the slow-roll conditions \( \epsilon_i \ll 1 \) and \( \epsilon_i \ll 1 \) with \( i = 1,4,5,6 \), we obtain the following expression from eq. (9):

\[
\frac{z''}{z} = \frac{n_s}{\eta^2} = \frac{1}{\eta^2}(1 - \epsilon_1)^2(2 + \epsilon_1 + \epsilon_4)(3 + \epsilon_4),
\]

with the parameter \( n_s \) being defined in the following way:

\[
n_s = \frac{1}{(1 - \epsilon_1)^2}(2 + \epsilon_1 + \epsilon_4)(3 + \epsilon_4),
\]

and \( n_s \) should not be confused with the spectral index \( n_S \). The definition of the spectral index \( n_S \) in terms of the parameter \( n_s \) is [3,10]

\[
n_S = 4 - \sqrt{4n_s + 1}.
\]

Hence, by substituting eq. (13) in eq. (14) we obtain the following expression for the spectral index of the scalar primordial perturbations:

\[
n_S \simeq \frac{5\epsilon_1 + 2\epsilon_4 + 1}{\epsilon_1 - 1}.
\]

So by further expanding the above for \( \epsilon_1 \ll 1 \) and by keeping linear terms containing the slow-roll indices in the final expressions, we obtain at leading order

\[
n_S \simeq -1 - 6\epsilon_1 - 2\epsilon_4.
\]

Therefore, it is apparent from the above that the resulting phenomenology for an GW170817-compatible Einstein-Gauss-Bonnet theory with a linear coupling is quite different from the resulting phenomenology of the same theories with non-linear coupling, in which case the spectral index is equal to [10,45,46]

\[
n_S = 1 - 4\epsilon_1 - 2\epsilon_2 - \epsilon_4.
\]

This result may cast some doubt on the phenomenology of the Einstein-Gauss-Bonnet models with linear coupling function, since in order to obtain a viable phenomenology it is required that \( \epsilon_1 < 0 \) and \( \epsilon_1 \sim \mathcal{O}(10^{-1}) \). However, the Planck data [67] constrain \( \epsilon_1 \) to be of the order \( \epsilon_1 \sim \mathcal{O}(10^{-3}) \). The Planck results however are based on single scalar field cosmology, so the viability of the GW170817-compatible Einstein-Gauss-Bonnet theory with linear coupling must be checked explicitly, a task we aim to address comprehensively in the near future. The resulting phenomenology may be possibly affected by the presence of the scalar field potential and our study covers many Einstein-Gauss-Bonnet theories appearing in the literature with linear coupling, see for example [38–42,47]. We performed a full analysis of several models, and it turns out that having a slow-roll index \( \epsilon_1 \) being of order \( \mathcal{O}(10^{-1}) \) so as to obtain a scalar spectral index \( n_S \sim 0.96-0.97 \) is not a viable option either, since in most simple cases studied, such as a power-law scalar potential and/or kinetic driven inflation, a somewhat acceptable value for the scalar spectral index at least is produced; however the scalar field has a phantom kinetic term, hence it is not deemed as a suitable description either. For instance, the power-law choice for the scalar potential, meaning \( V(\phi) = V_0(\kappa\phi)^n \), is capable of producing a scalar spectral index of \( n_S = 0.967742 \) from eq. (15) and for \( V_0 = 1 \), \( c_1 = 1 \), \( \omega = -1 \), hence the phantom field mentioned before, and for \( n = -305 \) which is quite extravagant for such a simple model choice. The result is obtained for \( \epsilon_1 = -0.488 \) as stated previously and for an e-folding number \( N = 60 \). The same applies to other trivial models such
as the exponential scalar potential or kinetic driven inflation and each example can be solved only for the phantom case. Overall, the linear Gauss-Bonnet coupling is not capable of describing the inflationary era provided that the gravitational action is non-local Gauss-Bonnet gravities [48], in contrast to the previous gravitational action studied in this paper, namely the action of eq. (1). The most serious problem is that the linear Gauss-Bonnet coupling still remains a linear function of the scalar field. As a result, compatibility with the recent GW170817-compatible Einstein-Gauss-Bonnet theories with general Gaussianities predicted by the GW170817-compatible Einstein-Gauss-Bonnet theory with linear coupling. The constant-roll dynamics for the scalar field which is inherent to this theory, is known to produce primordial non-Gaussianities [50], see also ref. [56] for the canonical scalar field cases. Thus it is interesting that the imposition of having a massless graviton for the Einstein-Gauss-Bonnet theory with linear coupling, leads to a theory that may have non-Gaussianities. Thus it is worth to pursue this research issue further through the prism of primordial non-Gaussianities, which we aim to perform in a near-future study.

Another important class of theories covered by our study is that of non-local Gauss-Bonnet gravities [48], in which case the gravitational action is

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{2k^2} - \frac{\kappa^2}{2\omega} \Box^{-1} G \right),
\]

where again \( R \) is the Ricci scalar, \( \kappa = \frac{1}{M_P} \) and \( G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} \). This gravitational action corresponds to a non-local Gauss-Bonnet gravity, which, as was shown in ref. [48], corresponds to the following Einstein-Gauss-Bonnet theory with linear coupling:

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{2k^2} - \omega g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \phi G \right),
\]

which can be obtained from the non-local action by using the transformation \( \phi = -\frac{\omega}{2\kappa^2} \square^{-1} G \). In this case, the scalar potential is absent and the scalar field itself is dimensionless, in contrast to the previous gravitational action studied in this paper, namely the action of eq. (1). The same cosmological background shall be assumed, meaning that eq. (2) still applies and also, the scalar field now is, hence the constant-roll assumption is still applicable. The phenomenology of this model, which has no scalar potential, and unusual physical dimensions for the scalar field, shall also be developed in a forthcoming work. The Planck data however are consistent with the previous gravitational action studied in this paper, namely the action of eq. (1). The same cosmological background shall be assumed, meaning that eq. (2) still applies and also, the scalar field now is, hence the constant-roll assumption is still applicable. The phenomenology of this model, which has no scalar potential, and unusual physical dimensions for the scalar field, shall also be developed in a forthcoming work.

Conclusions. – In this letter we investigated the possibility of obtaining a massless graviton during the inflationary era, from an Einstein-Gauss-Bonnet theory with a linear coupling of the scalar field to the Gauss-Bonnet invariant. As we demonstrated, the implication of having gravitational wave speed \( c_s^2 = 1 \) during the inflationary era, restricts the dynamical evolution of the scalar field and it compels it to evolve dynamically in a constant-roll way. Accordingly we calculated the spectral index of scalar primordial perturbations, and as we showed, the resulting expression of it in terms of the slow-roll indices is quite different in comparison to the GW170817-compatible Einstein-Gauss-Bonnet theories with general coupling of the scalar field to the Gauss-Bonnet invariant.

In principle, the resulting phenomenology of the GW170817-compatible Einstein-Gauss-Bonnet theories with linear coupling is seriously affected, and this should be explicitly checked. The most serious problem is that the first slow-roll index, which quantifies the fact that inflation occurs since it requires \( H \ll H^2 \), must be of the order of \( \epsilon_1 \sim O(10^{-1}) \) in order to have viability with the Planck data, however, the Planck data indicate that it should be of the order \( \epsilon_1 \sim O(10^{-3}) \). The Planck data however are based on single scalar field inflation considerations, so in a forthcoming work we aim to address the phenomenology of the GW170817-compatible Einstein-Gauss-Bonnet theories with linear coupling in detail.

Another issue that is worth investigating is the non-Gaussianities predicted by the GW170817-compatible Einstein-Gauss-Bonnet theory with linear coupling. The constant-roll dynamics for the scalar field which is inherent to this theory, is known to produce primordial non-Gaussianities [50], see also ref. [56] for the canonical scalar field cases. Thus it is interesting that the imposition of having a massless graviton for the Einstein-Gauss-Bonnet theory with linear coupling, leads to a theory that may have non-Gaussianities. Thus it is worth to pursue this research issue further through the prism of primordial non-Gaussianities, which we aim to perform in a near-future study.

REFERENCES

[1] Abbott B. P. et al., Astrophys. J., 848 (2017) L12 (arXiv:1710.05833 [astro-ph.HE]).
[2] Ezquiaga J. M. and Zumalacarregui M., Phys. Rev. Lett., 119 (2017) 251304 (arXiv:1710.05901 [astro-ph.CO]).
[3] Nojiri S., Odintsov S. and Oikonomou V. K., Phys. Rep., 692 (2017) 1 (arXiv:1705.11098 [gr-qc]).
[4] Nojiri S. and Odintsov S. D., Phys. Rep., 505 (2011) 59 (arXiv:1011.0544 [gr-qc]).
[5] Nojiri S. and Odintsov S. D., eConf C, 0602061 (2006) 06; Int. J. Geom. Methods Mod. Phys., 4 (2007) 115 (hep-th/0601213).
[6] Capozziello S. and De Laurentis M., Phys. Rep., 509 (2011) 167 (arXiv:1108.6266 [gr-qc]).
[7] Faraoni V. and Capozziello S., Beyond Einstein Gravity, Fundamental Theories of Physics Series, Vol. 170 (Springer, Dordrecht) 2010, https://doi.org/10.1007/978-94-007-0165-6.
[8] De la Cruz-Dombriz A. and Saez-Gomez D., Entropy, 14 (2012) 1717 (arXiv:1207.2663 [gr-qc]).
[9] Olmo G. J., Int. J. Mod. Phys. D, 20 (2011) 413 (arXiv:1101.3864 [gr-qc]).
[10] Hwang J. C. and Noh H., Phys. Rev. D, 71 (2005) 063536 (gr-qc/0412126).
[11] Nojiri S., Odintsov S. D. and Sami M., Phys. Rev. D, 74 (2006) 046004 (hep-th/0605039).
[12] Cognola G., Elizalde E., Nojiri S., Odintsov S. and Zerbini S., Phys. Rev. D, 75 (2007) 086002 (hep-th/0611198).
[13] Nojiri S., Odintsov S. D. and Sasaki M., Phys. Rev. D, 71 (2005) 123509 (hep-th/0504052).
[14] Nojiri S. and Odintsov S. D., Phys. Lett. B, 631 (2005) 1 (hep-th/0508049).
[15] Sátoh M., Kanno S. and Soda J., Phys. Rev. D, 77 (2008) 023526 (arXiv:0706.3585 [astro-ph]).
A nearly massless graviton in Einstein-Gauss-Bonnet inflation etc.