Connecting Blackbody Radiation, Relativity, and Discrete Charge in Classical Electrodynamics

Timothy H. Boyer

Department of Physics, City College of the City University of New York, New York, NY 10031

Abstract

It is suggested that an understanding of blackbody radiation within classical physics requires the presence of classical electromagnetic zero-point radiation, the restriction to relativistic (Coulomb) scattering systems, and the use of discrete charge. The contrasting scaling properties of nonrelativistic classical mechanics and classical electrodynamics are noted, and it is emphasized that the solutions of classical electrodynamics found in nature involve constants which connect together the scales of length, time, and energy. Indeed, there are analogies between the electrostatic forces for groups of particles of discrete charge and the van der Waals forces in equilibrium thermal radiation. The differing Lorentz- or Galilean-transformation properties of the zero-point radiation spectrum and the Rayleigh-Jeans spectrum are noted in connection with their scaling properties. Also, the thermal effects of acceleration within classical electromagnetism are related to the existence of thermal equilibrium within a gravitational field. The unique scaling and phase-space properties of a discrete charge in the Coulomb potential suggest the possibility of an equilibrium between the zero-point radiation spectrum and matter which is universal (independent of the particle mass), and an equilibrium between a universal thermal radiation spectrum and matter where the matter phase space depends only upon the ratio $mc^2/k_BT$. The observations and qualitative suggestions made here run counter to the ideas of currently accepted quantum physics.
I. INTRODUCTION

Although blackbody radiation, relativity, and discrete charge are rarely connected in the physics literature, they are intimately connected in nature. Thus, for example, Planck’s blackbody spectrum appears as the equilibrium spectrum associated with uniform proper acceleration through a Lorentz-invariant spectrum of random classical electromagnetic radiation. Furthermore, the charge of the electron and Stefan’s blackbody radiation constant can be combined to give a universal dimensionless constant. On the other hand, the many attempts to understand blackbody radiation within classical physics using nonrelativistic statistical mechanics or nonrelativistic mechanical scattering systems with a small-charge coupling limit all lead to the Rayleigh-Jeans spectrum with its ultraviolet divergence. On this account most physicists believe that the blackbody radiation spectrum arises from a charge of arbitrary size in any (nonrelativistic) mechanical potential because of Boltzmann statistics and the quantum nature of energy exchanges. Here we reexamine the classical electromagnetic description of radiation equilibrium. We suggest that the observed Planck spectrum of blackbody radiation may have nothing to do with energy quanta and everything to do with the symmetries of relativistic classical electron theory with discrete charges.

The physicists who investigated blackbody radiation near the beginning of the twentieth century were unfamiliar with the implications of special relativity and so interpreted "classical physics" to mean "nonrelativistic classical mechanics." Furthermore, the discrete electronic charge was viewed (and still is today) as a curiosity unrelated to blackbody radiation. Thus the normal modes of oscillation of the electromagnetic field were treated as mechanical waves using nonrelativistic classical statistical mechanics, or the electric dipole oscillator in contact with the radiation field was treated by nonrelativistic classical statistical mechanics, or the scatterer of radiation was a nonrelativistic classical mechanical scatterer. Researchers around 1900 wondered how the variety of nonrelativistic mechanical systems could possibly lead to the observed universal spectrum for radiation equilibrium which was unrelated to the details of the matter producing the equilibrium. Finally around 1910 it was noted that the Rayleigh-Jeans spectrum always seemed to appear from treatments using nonrelativistic classical mechanics, and moreover nonrelativistic classical mechanics does not include any fundamental constant which could lead to a departure from
the Rayleigh-Jeans spectrum. Indeed, the principles of nonrelativistic classical mechanics (involving independent scalings of length, time, and energy) simply can not support a fundamental constant like Stefan’s constant $a_s$ connecting the energy density $u$ of thermal radiation and the absolute temperature $T$, $u = a_s T^4$.

Today relativistic physics is regarded as fundamental, not nonrelativistic mechanics. In particular, the relativistic Coulomb interaction between discrete point charges allows a separation between particle mass and the particle phase space distribution which is not possible for any other potential. Therefore it fits qualitatively with the Planck spectrum of electromagnetic radiation as nonrelativistic mechanics does not. Indeed, it is one of the ironies of the history of physics that blackbody radiation, special relativity, and discrete electronic charge all came to prominence at the beginning of the 20th century yet these were not connected. Thus at the same time that Lorentz invariance was recognized as a symmetry of electromagnetic waves, the interaction of radiation and matter was treated by nonrelativistic mechanics for particles of arbitrarily small charge. It was only half-a-century later that blackbody radiation and special relativity began to be related in connection with the Lorentz invariance of zero-point radiation and the thermal behavior associated with uniform acceleration through zero-point radiation. However, even today the textbooks of modern physics hark back to the years of disconnection of a century ago, while classical electromagnetic theory is taught as though relativistic particle motion was not important and as thought electric charge had no smallest value. It is our unproven suggestion that within classical physics, the crucial appearance of the Rayleigh-Jeans spectrum or the Planck spectrum has nothing to do with classical versus quantum physics but rather is a reflection of the differing correlations allowed by nonrelativistic or relativistic classical scattering systems with continuous or discrete charge.

A. Outline of the Discussion

This article is an attempt to exploring all the suggestive evidence for a classical explanation of blackbody radiation and to understand why the currently-accepted arguments are misleading. We start by pointing out the contrasting scaling properties of nonrelativistic classical mechanics and classical electrodynamics. It is emphasized that the solutions of classical electrodynamics found in nature involve constants which connect together the
scales of length, time, and energy. Indeed, there are analogies between the forces found in the electrostatics of discrete point charges and those found between materials in equilibrium random classical radiation. Second we consider equilibrium classical radiation and note the differing transformation properties of the zero-point spectrum and the Rayleigh-Jeans spectrum. Also, we remark on the appearance of thermal effects of acceleration within classical electromagnetism and relate them to the existence of thermal equilibrium in a gravitational field. Third, we note the unique scaling properties of the Coulomb potential and the unique separation between the phase space distribution and the particle mass. Fourth we discuss the interaction between radiation and matter. We note that the Coulomb potential with a discrete charge allows an interaction with random radiation which connects the phase space of the matter with the phase space of the radiation variables in a universal connection. Finally, we discuss our current understanding of the blackbody radiation spectrum within classical physics.

II. SCALING AND UNIVERSAL CONSTANTS

The contradictions between nonrelativistic mechanics and electromagnetism can be seen immediately from the contrasting scaling symmetries of nonrelativistic classical mechanics as compared to classical electrodynamics with relativistic particles of discrete charge. Thus attempts to explain blackbody radiation based upon nonrelativistic classical statistical mechanics or nonrelativistic classical scattering systems are doomed to failure.

A. Scaling for Nonrelativistic Mechanics

Within nonrelativistic mechanics, length, time, and energy all scale independently. The symmetries of classical mechanics allow separate dilatation factors $\sigma_l$, $\sigma_t$, $\sigma_E$ for length, time and energy, $l \to l' = \sigma_l l$, $t' \to \sigma_t t$, $E \to E' = \sigma_E E$, where the three separate dilatation factors range over all positive real numbers. Thus for any nonrelativistic mechanical system, there exists, in principle, a second system which is twice as large, has a period three times as long, and contains four times the energy. Since nonrelativistic mechanics allows independent scalings of length, time, and energy, nonrelativistic mechanics can have no fundamental constants connecting length, time, and energy. The existence in the nineteenth century of
independent standards of length, time, and energy reflects the independent scalings found in nonrelativistic classical mechanics.

B. Scaling for Classical Electromagnetism

In the electromagnetic evidence accumulated during the last half of the nineteenth century, this absence of any universality within nonrelativistic classical mechanics stood in startling contrast with the appearance of a universal wave speed \( c = 3 \times 10^{10} \text{cm/sec} \) in Maxwell’s equations, a universal energy-length-related constant \( a_s/k_B^4 = 6.25 \times 10^{64} (\text{erg cm})^{-3} \) (Stefan’s constant divided by the fourth power of Boltzmann’s constant) for blackbody radiation, and a second energy-length-related constant \( e^2 = 2.304 \times 10^{-19} \text{erg cm} \) corresponding to a smallest electric charge \( e \). Thus in contradiction to the separate scalings found in nonrelativistic mechanics, classical electromagnetism has the scales of length, time, and energy all connected. Maxwell’s equations themselves contain the speed of light \( c \) in vacuum, and this fundamental constant couples the scales of length and time. Thus if we find an electromagnetic wave in vacuum with wavelength \( \lambda \), then we know immediately that the frequency \( \nu \) of the wave is given by \( \nu = c/\lambda \). Furthermore, the solutions of Maxwell’s equations found in nature during the nineteenth century involve two other fundamental constants, one for radiation and one for matter: Stefan’s constant \( a_s \) for blackbody radiation and a smallest electric charge \( e \). Stefan’s constant \( a_s \) divided by the fourth power of Boltzmann’s constant \( k_B \) (this last connects absolute temperature \( T \) to energy) can be regarded as coupling together length and energy through the relation for the electromagnetic thermal energy \( U \) in a cubic volume of side \( l \) at temperature \( T \) given by \( U/(k_B T)^4 = (a_s/k_B^4)l^3 \).

Similarly, the smallest (nonzero) electric charge \( e \) couples the scales of length and energy for matter. Thus if two smallest charges \( e \) are separated by a distance \( r \), then the electrostatic potential energy is given by \( U = e^2/r \). Within classical electromagnetism, there is only one independent scaling in nature; the dilatation symmetry allows only one scale factor \( \sigma_\alpha \) giving

\[
l \rightarrow l' = \sigma_\alpha l, \quad t \rightarrow \sigma_\alpha t, \quad E \rightarrow E' = E/\sigma_\alpha
\]

where the dilatation factor \( \sigma_\alpha \) can assume all positive real values. Given any classical electromagnetic system, there exists, in principle, a second electromagnetic system where
all the lengths, times, and inverse energies are $\sigma_{ltE^{-1}}$ times as large.

C. Electrostatic Energy-Length Scaling Due to Discrete Charge $e$

The existence of a fundamental energy-length connection in electromagnetism limits the possible energy of an electrostatic configuration. Thus if we are told the number of elementary charges involved and the shape of the charge distribution, then we know the product of the electrostatic energy times a characteristic length of the distribution. Two elementary examples immediately come to mind. A parallel-plate capacitor with square plates of side $L$ separated by a distance $L/100$ and charged with $N$ elementary charges (spread uniformly on each plate but of opposite sign for the two plates) has an electrostatic energy $U$ where $UL = \{[1/(8\pi)][4\pi Ne/L^2L^3/100]\}L = N^2e^2/50$; a spherical conducting shell of outer radius $a$ which contains $N$ elementary electric charges (spread uniformly over the spherical shell) has an electrostatic energy $U$ where $Ua = N^2e^2/2$. In every case, the energy times the characteristic length equals a shape-dependent geometrical factor times $N^2e^2$. In every case, there is a smallest nonvanishing energy when $N = 1$.

D. Electromagnetic Energy-Length Scaling Due to Stefan’s Constant

Now it is found in nature[12] that there are van der Waals forces between macroscopic materials, and, at the absolute zero of temperature $T = 0$, these forces assume their smallest values for a given geometrical configuration. It turns out that the smallest force at $T = 0$ gives a fundamental connection between energy and length which involves Stefan’s constant and is completely analogous to that noted above in electrostatics where a minimum charge is involved.[13] Thus if we are told the shape of a distribution of conductors, then we know the product of the electromagnetic energy times a characteristic length of the distribution at absolute zero. Again two elementary examples come to mind. At temperature $T = 0$, an uncharged parallel-plate capacitor[14] with square plates of side $L$ separated by a distance $L/100$ has an energy $U$ where $UL = -(\pi^2 \times 10^6/360)[120a_s/(\pi^2 k_B^4)]^{-1/3}$; a spherical conducting shell[15] of outer radius $a$ has an energy $U$ where $Ua = 0.09[120a_s/(\pi^2 k_B^4)]^{-1/3}$. In every case, the energy times the characteristic length equals a shape-dependent geometrical factor times $(a_s/k_B^4)^{-1/3}$.  
E. Existence of a Dimensionless Universal Constant

Since nature gives us two universal electromagnetic constants connecting the scales of energy and length, it follows that their ratio must be a dimensionless constant. Thus for example, the charge of the electron is \( e = 4.80 \times 10^{-10} \text{ esu} \), so \( e^2 = 2.304 \times 10^{-19} \text{erg} - \text{cm} \) while Stefan’s constant is \( a_s = 7.56 \times 10^{-15} \text{erg} \cdot \text{cm}^{-3} \cdot \text{K}^{-4} \) and Boltzmann’s constant is \( k_B = 1.38 \times 10^{-16} \text{erg} \cdot \text{K} \). Accordingly, we note that

\[
e^2 \left( \frac{a_s}{k_B^4} \right)^{1/3} = 0.00063487 \tag{2}\]

This ratio involving nineteenth-century physical constants is not usually presented in the physics literature. It suggests the possibility that a discrete electronic charge is connected to blackbody radiation.[16]

In the analysis to follow, we will frequently use the blackbody constant \( (a_s/k_B T)^{-1/3} \). Therefore it is convenient to introduce Stefan’s second constant \( b_s \) where \[17\]

\[
b_s = \left( \frac{\pi^2 k_B^4}{120 a_s} \right)^{1/3} \tag{3}\]

The constant \( b_s \) has the units of energy \( \times \) length, just the same as \( e^2 \).

III. EQUILIBRIUM CLASSICAL RADIATION

A. Classical Zero-Point Radiation

Pure classical electromagnetic radiation is a homogeneous solution of Maxwell’s equations. Classical radiation therefore contains the length-time connection given by the wave speed \( c \) in vacuum of Maxwell’s equations, but makes no connection between energy and length or between energy and time. For example, an electromagnetic plane wave solution of Maxwell’s equations in vacuum of wavelength \( \lambda \) must have a frequency \( \nu \) given by \( \nu = \lambda/c \), but it may have any energy per unit volume associated with the electric field amplitude \( E_0 \).

It follows that any fundamental connection between energy and length involving electromagnetic radiation within classical physics must come not from Maxwell’s equations themselves but from a fundamental boundary condition on Maxwell’s equations.

Nature indeed provides a fundamental boundary condition on Maxwell’s equations. All of the experimentally observed van der Waals forces[12] between macroscopic objects can be
described in terms of classical electromagnetic forces due to random classical electromagnetic radiation. The experimentally observed van der Waals forces require that at temperature $T = 0$, there is present in space a Lorentz-invariant spectrum of random classical electromagnetic radiation with an average energy $U_\lambda$ per normal mode of wavelength $\lambda = 2\pi \lambda$ given by

$$U_\lambda(0) = \left[\frac{120}{\pi^2}(a_s/k_B^4)\right]^{-1/3}/\lambda = b_s/\lambda$$

(4)

where $b_s$ is Stefan’s second constant given in Eq. (3). The randomness of the radiation can be described in terms of random phases for the radiation and the radiation itself can be described in terms of normal modes with action-angle variables and an associated probability function $P_\lambda$ for the action variable $J_\lambda$ of the radiation mode of wavelength $\lambda$.

$$P_\lambda(J_\lambda, 0) = \frac{c}{b_s} \exp\left[ -\frac{J_\lambda c}{b_s} \right]$$

(5)

We notice in Eq. (5) that the probability function $P_\lambda(J_\lambda, 0)$ at temperature $T = 0$ is the same for every radiation mode independent of the wavelength $\lambda$ of the mode.

We expect that the entropy associated with any classical system should be related to the probability distributions of its action variables since the entropy is related to the probability distribution on phase space. The action-angle variables of multiply periodic systems give a natural division of phase space. The distribution in the angle variables is uniform, so that the only probability distribution of relevance is the action variables. By the third law of thermodynamics, we expect the thermodynamic entropy to vanish at $T = 0$. Therefore the probability distribution at $T = 0$ given in Eq. (5) corresponds to zero entropy for each radiation mode.

### B. Thermal Radiation

At finite temperature, the observed spectrum of blackbody radiation (including the zero-point radiation required by the observed van der Waals forces) can be written as an energy $U_\lambda(T)$ per normal mode

$$U_\lambda(T) = \frac{b_s}{\lambda} \coth\left( \frac{b_s}{\lambda k_B T} \right)$$

(6)

with the probability function $P_\lambda$ for the action variable $J_\lambda$ of a radiation normal mode of wavelength $\lambda$ becoming a function of temperature.

$$P_\lambda(J_\lambda, \lambda T/b_s) = \frac{c}{b_s \coth[b_s/(\lambda k_B T)]} \exp\left[ -\frac{J_\lambda c}{b_s \coth[b_s/(\lambda k_B T)]} \right]$$

(7)
Even at finite temperature, the van der Waals forces for a conducting-walled container due to random classical radiation still hold a strong analogy with the electrostatic forces due to discrete charges in their dependence upon dimensionless and scaling parameters. We saw above that an electrostatics problem was uniquely specified by giving the number of elementary charges, the shape of the charge container holding uniformly spaced charges, and one scale-determining length. In the thermal radiation problem for van der Waals forces, the pure number \( N \) (number of elementary charges) of the electrostatics problem is replaced by the pure number \( S/k_B \) corresponding to the entropy of the radiation in the container. (As usual, we have removed the inessential unit of temperature by dividing out Boltzmann’s constant.) Thus nature shows that thermal radiation in a container of given shape (specified by dimensionless parameters) is determined by exactly two parameters; viz, the volume \( V \) (which corresponds to setting the length scale \( l = V^{1/3} \) of the container of given shape) and the scale-independent entropy \( S/k_B \). All the other parameters are now fixed. For thermal radiation in a large spherical container, the radiation temperature is \( T = \left[ 3S/(4a_sV) \right]^{1/3} \), the total thermal radiation energy in the container is \( U = a_sV[3S/(4a_sV)]^{4/3} = a_sT^4 \), the energy per normal mode \( U_\lambda \) in the long wavelength (low frequency) modes is given by \( U_\lambda = k_B[3S/(4a_sV)]^{1/3} = k_BT \), and the wavelength where the thermal radiation spectrum has its maximum is given by Wien’s displacement law \( \lambda_{\text{max}} = \text{const} \times [4a_sV/(3S)]^{1/3} = \text{const}/T \).

Now classical electromagnetism is invariant under \( \sigma_{ltE^{-1}} \) scaling symmetry and even under conformal symmetry.\(^{[23]}\) Therefore an adiabatic change in the volume of the container (while maintaining its shape) is the same as a \( \sigma_{ltE^{-1}} \) change of scale. For the electrostatic situation, the energy \( U \) changes as the scaling length \( l \) changes while the number of elementary charges remains fixed. For the radiation case, the thermal energy in the container changes as the radius of the container changes while the scale-invariant entropy \( S/k_B \) is unchanged. If we imagine a spherical container, the change in the radius of the container \( r \rightarrow r' = \sigma_{ltE^{-1}}r \), leads to a consistent change of volume \( V \rightarrow V' = \sigma_{ltE^{-1}}^3V \), temperature \( T \rightarrow T' = T/\sigma_{ltE^{-1}} \), and energy \( U \rightarrow U' = U/\sigma_{ltE^{-1}} \), while the entropy is unchanged \( S \rightarrow S' = S \). All the laws of blackbody radiation hold both before and after the scale change. The probability function \( P_\lambda(J_\lambda, T) \rightarrow P_\lambda'(J_\lambda', T') \) in Eq. (7) is unchanged since the product \( \lambda k_BT = (\sigma_{ltE^{-1}} \lambda) (k_BT/\sigma_{ltE^{-1}}) = \lambda' k_BT' \) is unchanged under the scale change or adiabatic change, and this invariance is appropriate for the invariance of the radiation entropy.

The equivalence between adiabatic change in the size of a container (which retains its
shape) of electromagnetic energy and a σ<sub>HE−1</sub> scale transformation of the situation is something that can hold only for pure electromagnetic fields and not when particle masses m are involved. We do not think of particles in a container as changing mass m during an adiabatic change, whereas the the numerical value of mass is rescaled under a σ<sub>HE−1</sub> scale change, m → m′ = m/σ<sub>HE−1</sub>.

C. Electric Field Correlation Functions in the High- and Low-Temperature Limits

In the high-frequency or short-wavelength limit, the blackbody radiation spectrum (6) involves coth[x] → 1 for large x and goes over to the zero-point energy limit

\[ U_\hbar(0) = U_\omega(0) = b_s/\hbar = (b_s/c)\omega \]  

In the long-wavelength or low-frequency limit, the blackbody spectrum (6) involves coth[x] → 1/x for small x and goes over to the the Rayleigh-Jeans equipartition energy

\[ U_{RJ} = U_{RJ\omega} = k_B T \]  

This latter spectrum corresponds to the energy (with associated entropy) of traditional nonrelativistic classical statistical mechanics per normal mode. It is interesting to see the electromagnetic field correlation functions for these two limiting spectra (8) and (9), and to note the role played by the speed of light c. The electric field correlation function for the zero-point spectrum of random radiation is given by

\[ < E_i(r, t) E_j(r', t') >_{T=0} = \left( \frac{\delta_{ij}}{c^2 \partial t \partial t'} - \frac{\partial}{\partial x_i} \frac{\partial}{\partial x'_j} \right) \left( -\frac{2b_s}{\pi c^2 (t - t')^2 - (r - r')^2} \right) \]  

The other electromagnetic field correlation functions in the four-tensor expression \( \langle F_{\mu\nu}(x) F^{\mu\nu'}(x') \rangle \) can be obtained by changing the space and time indices in Eq. (10). The electric field correlation function involves space and time derivatives of the Lorentz-invariant spacetime interval \( c^2(t - t')^2 - (r - r')^2 \) between the field points \((r, t)\) and \((r', t')\). This indeed denotes the Lorentz-invariant character of the random radiation spectrum. Now we expect, but have not proved, that relativistic scattering systems which themselves maintain the Lorentz invariance of spacetime intervals will leave this spectrum invariant. We suggest that it is unreasonable to expect that nonrelativistic mechanical scattering systems,
which do not share the Lorentz symmetry of the zero-point spectrum, should leave the Lorentz-invariant zero-point spectrum unchanged.

The electric field correlation function for the Rayleigh-Jeans spectrum takes the form

\[
< E_i(r, t) E_j(r', t') >_{RJ} = \left( \frac{\delta_{ij} \frac{\partial}{\partial t} \frac{\partial}{\partial \theta} - \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j'}}{|r - r'|} \right) \frac{k_B T}{c^2 \theta(|r - r'| - c|t - t'|)} \theta(|r - r'| - c|t - t'|) \tag{11}
\]

We notice immediately that this correlation function depends not only on the Lorentz-invariant spacetime interval \(c|t - t'| - |r - r'|\) but also on the Galilean-invariant interval \(|r - r'|\). Thus the right-hand side is Galilean invariant but not Lorentz invariant. We expect that the spectrum will be preserved by electromagnetic scattering systems which maintain the invariance of the nonrelativistic invariant spatial interval. This is seen in several scattering calculations. The correlation function in Eq. (11) vanishes for time-like separations, an aspect which appears from the Lorentz-covariance of the radiation itself, not from the information carried in the radiation spectrum. The only correlations in time involve \(\delta\)-function correlations and so involve no connection between energy and time, as is true in nonrelativistic mechanics.

**D. Limiting Field Correlations Functions as \(c \to \infty\)**

In order to emphasize the distinction between zero-point radiation and the Rayleigh-Jeans spectrum, we will consider the limit \(c \to \infty\) so as to eliminate \(c\) from Eqs. (10) and (11). The \(c \to \infty\) limit is possible only when \(t = t'\). Then the zero-point radiation correlation function (10) becomes

\[
< E_i(r, t) E_j(r', t) >_{T=0} = \left( \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j'} \right) \left( \frac{-2bs/\pi}{(r - r')^2} \right) \tag{12}
\]

while that for the Rayleigh-Jeans spectrum becomes

\[
< E_i(r, t) E_j(r', t) >_{RJ} = \left( \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j'} \right) \frac{-k_B T}{|r - r'|} \tag{13}
\]

The different functional dependence upon the the spatial interval \(|r - r'|\) seen in Eqs. (12) and (13) is a reminder of the coupling between energy and length in zero-point radiation and the complete decoupling between energy and length found in the Rayleigh-Jeans spectrum. Thus the electric field (think of \(E = e/r^2\)) scales as two inverse powers of the dilatation.
factor $\sigma_{\text{uE}^{-1}}$. The left-hand sides of Eqs. (12) and (13) scale as $(1/\sigma_{\text{uE}^{-1}})^4$ and have units of energy per unit volume. For the zero-point radiation spectrum in Eq. (12), there are four powers of length on the right-hand side so that $b_s$ must have units of $\text{energy} \times \text{length}$ and can be a fundamental constant relating energy and length which is invariant under scaling by $\sigma_{\text{uE}^{-1}}$. On the other hand, the Rayleigh-Jeans spectrum in Eq. (13) has only three powers of length on the right-hand side, and therefore the parameter $k_B T$ must have the units of $\text{energy}$ and is subject to scaling by $\sigma_{\text{uE}^{-1}}$, $k_B T \rightarrow k_B T' = k_B T/\sigma_{\text{uE}^{-1}}$. Thus the energy parameter $k_B T$ can take on any non-negative value. The Rayleigh-Jeans spectrum reflects the completely independent scaling of energy and length which is typical of nonrelativistic mechanics. Indeed, when van der Waals forces between macroscopic objects are calculated using the Rayleigh-Jeans spectrum, one finds that there is no net change in the electromagnetic energy with position of the objects, all the forces are associated with changes of entropy through the Helmholtz free energy. This fits exactly with the absence of any energy-length connection within the Rayleigh-Jeans spectrum.

E. Thermal Effects of Acceleration within Classical Theory

One of the surprising realizations of the last quarter of the twentieth century was that the field correlation functions of Planck’s blackbody spectrum appear when a system undergoes uniform acceleration through zero-point radiation. Although this appearance without any apparent application of statistical mechanics has provided a profound quandary for quantum physics, it seems a natural result within classical electrodynamics with classical zero-point radiation.

The equivalence principle connects accelerations to gravitational phenomena, while acceleration through classical zero-point radiation is found to connect zero-point radiation with the Planck spectrum of thermal radiation. Thus it seems relevant to consider our sketch of relativistic classical electrodynamics with discrete charge in connection with gravitational effects. The simplest system to consider is the Rindler frame involving a time-independent coordinate system where each spatial point undergoes a uniform proper acceleration through Minkowski spacetime. We expect that thermal radiation equilibrium can exist within a gravitational field. Thus we expect thermal equilibrium to exist for the Rindler frame.
the Rindler frame involves uniform proper acceleration

\[ a = \hat{k}a = \hat{k}c^2/z \]  \hspace{1cm} (14)

for a spatial point a distance \( z \) from the event horizon at \( z = 0 \). The smallest density of random classical radiation is that given by zero-point radiation. The essential aspect is its Lorentz-invariant spectrum which can be written as

\[ U_\omega = \text{const} \times \omega \]  \hspace{1cm} (15)

for the energy \( U_\omega \) of a normal mode of frequency \( \omega \), where there is an arbitrary multiplicative constant \( \text{const.} \). It has been shown\cite{2,3,4} that the correlation function for the random classical electromagnetic fields as observed at a fixed spatial point in the Rindler frame no longer corresponds to a Lorentz-invariant spectrum but rather to a spectrum

\[ U_\omega = \text{const} \times \omega \coth(\pi\omega c/a) \]  \hspace{1cm} (16)

This result can be obtained by considering the uniform proper acceleration of a harmonic electric dipole system of fixed angular frequency \( \omega_0 \) taken in the point dipole limit as the mass of the oscillator particle goes to infinity. We notice that the expression for the radiation energy \( U_\omega \) at frequency \( \omega \) involves the hyperbolic cosine function with a dependence upon \( \omega c/a \). The proper acceleration \( a \) takes the place of the temperature in the blackbody radiation spectrum.

For frequencies \( \omega \) small compared to \( a/c \), \( \omega \ll a/c \), the spectrum is proportional to the acceleration \( a \) and independent of frequency, just like the Rayleigh-Jeans spectrum in Eq. (9). For large frequencies \( \omega \gg a/c \), the spectrum is still the Lorentz-invariant spectrum (15) which increases linearly with frequency \( \omega \). The result in Eq. (16) is entirely classical, depends crucially on the Lorentz-invariance of the original spectrum (15) in Minkowski spacetime, and has nothing to do with any fundamental constant except the speed of light in vacuum \( c \). The multiplicative scale of the of the Lorentz-invariant spectrum is given by the arbitrary constant here labeled "\( \text{const.} \)." If we choose the constant so as to fit with the experimentally observed spectrum of classical zero-point radiation, then \( \text{const} = b_s/c \) and the associated temperature is given by \( k_B T = b_s a/(\pi c^2) \). The fluctuations of the radiation can be obtained from the random phases of the radiation modes.\cite{19}
From Wien’s displacement theorem (which holds even in gravitational fields), we know that thermal equilibrium at any temperature $T$ is of the form $U_\omega = \omega f(\omega/T)$ or $U_\lambda = cf(c/\lambda T)/\lambda$ where $f$ is a universal function. In an inertial frame, we may take the limit $T \to 0$, and recover the zero-point spectrum given in Eq. (4) or (15). In the Rindler frame involving uniform acceleration, the function $f$ must follow from Eq. (16) which was found from the acceleration through zero-point radiation. Thus comparing Eq. (16) with the zero-temperature limit where the $\text{const} = b_s/c$, we find that proper acceleration $a$ through zero-point radiation corresponds to a lowest possible temperature $T_{\text{min}}$ in a gravitational field $a$ given by

$$k_B T_{\text{min}} = b_s a / (\pi c^2)$$

and any additional random radiation will increase the energy per normal mode $U_\omega$ according to the functional behavior

$$U_\omega = \frac{b_s}{c} \omega \coth \left( \frac{(b_s/c)\omega}{k_B T} \right)$$

IV. BEHAVIOR OF MATTER

Classical electromagnetic radiation within a container with perfectly reflecting walls will never come to thermal equilibrium. Rather, there must be some interaction between radiation and matter which brings the radiation to equilibrium. The state of thermal equilibrium for the radiation will reflect some fundamental aspects of the matter which causes the equilibrium. This conviction was expressed clearly by Lorentz back in the early 1900s when he writes, "...we may hope to find in what manner the value of this constant [$\lambda T = \text{const}$] is determined by some numerical quantity that is the same for all ponderable bodies." Here we are proposing that the discrete electric charge $e$ and use of relativistic interactions are the crucial elements for matter. Relativistic electromagnetic interactions begin with the Coulomb potential which involves unique properties related to scaling, phase space, and radiation emission, and these unique properties encourage the possibility of a classical explanation for blackbody radiation.
A. $\sigma_{ltE^{-1}}$ Scaling for a Central Potential

Suppose we consider a charged particle $e$ in a central potential of the form $U(r) = -k/r^n$. Now under the $\sigma_{ltE^{-1}}$ scaling of classical electromagnetism, the potential energy $U$ transforms as $U \rightarrow U' = U/\sigma_{ltE^{-1}}$ while the distance $r$ transforms as $r \rightarrow r' = \sigma_{ltE^{-1}}r$ so that $k'/r'^n = k'/(\sigma_{ltE^{-1}}r)^n = (k/r^n)/\sigma_{ltE^{-1}}$ and hence we must have $k' = \sigma_{ltE^{-1}}^{n-1}k$. This means that if in the collection of allowed systems there is a potential $U = -k/r^n$ with strength $k$, then there must also be a potential $U' = -k'/r^n$ of strength $k' = \sigma_{ltE^{-1}}^{n-1}k$. Thus only in the case of the Coulomb potential where $n = 1$ (and $k = e^2$ has the units of energy $\times$ length) do we have the possibility of a scale-independent coupling $k' = k$. For all other potentials $U(r) = -k/r^n$, $n \neq 1$, the coupling constant must allow all real values $k$. Thus only for the Coulomb potential do we preserve the $\sigma_{ltE^{-1}}$ scaling of classical electromagnetic theory rather than being forced into the traditional scaling of nonrelativistic mechanics with separate, independent scalings for energy and length. Whereas a point charge of smallest charge $e$ at a distance $r$ in a Coulomb potential of elementary strength $e$ always has a potential energy $U = e^2/r$ where $e$ is a universal value, we have no such information for a particle at radius $r$ in a general potential $U(r) = -k/r^n$ because the potential strength $k$ is freely changeable so that $U = k/r^n$ can be any real number. Thus a fundamental connection between energy and length is given by the elementary charge $e^2$ and a fundamental connection between energy and time is given by $e^2/c$. Accordingly within relativistic theory, a particle of mass $m$ and smallest charge $e$ is connected to the characteristic energy $mc^2$, the characteristic length $e^2/(mc^2)$, and the characteristic time $e^2/(mc^3)$.

B. Dependence of Orbit Speed on Action Variables

In addition to allowing a unique, $\sigma_{ltE^{-1}}$ scale-independent smallest coupling constant, the Coulomb potential also involves a unique separation between orbital speed and mass for fixed angular momentum $J$, which separation is not possible for any other central potential $U(r) = -k/r^n$. If we consider a point charge $e$ in a circular orbit of radius $r$ in a central potential $U(r) = -k/r^n$, then the equation of motion $F = dp/dt$ and the angular momentum $J$ are given by

$$m\gamma v^2/r = nk/r^{n+1} \text{ and } J = mr\gamma v$$

(19)
while the system energy is given by $H = mc^2 - k/r^n$. If we connect the equation of motion and the angular momentum $J$ in Eq. (19) so as to eliminate the orbital radius $r$, then we have $m\gamma v^2 = nk(m\gamma v/J)^n$ or $\gamma^{1-n}v^{2-n} = nkm^{n-1}/J^n$. Thus only for $n = 1$ (corresponding to the Coulomb potential $k = e^2$) does the particle mass $m$ disappear from this last equation so that we have the orbital speed $v$ of the particle determined solely by the action variable $J$, $v = e^2/J$. For every other potential $U(r) = -k/r^n$, the orbital speed $v$ is determined by both $J$ and the product $km^{n-1}$ where $k$ (as seen above) can not be a universal constant but must be a free scaling parameter.

C. Relativistic Coulomb Motion

Not only does the Coulomb potential have very special relations with $\sigma^{2}_{\mu E^{-1}}$ scaling and phase space, it is also the only mechanical potential which has been extended to a fully relativistic theory. Thus the fundamental electromagnetic system of relativistic classical electrodynamics is an elementary point charge $e$ of mass $m$ in a Coulomb potential of strength $-e$. In this case, the motion of the mechanical system can be described by action angle variables $J_1, J_2, J_3$, with an energy

$$H = mc^2 \left(1 + \frac{(e^2/c)^2}{(J_3 - J_2 + [J_2^2 - (e^2/c)^2]^{1/2})^2}\right)^{-1/2}$$  \hspace{1cm} (20)

Under the single-parameter scaling $\sigma_{\mu E^{-1}}$ which leaves classical electromagnetism invariant, only the mass $m$ and energy $H$ are rescaled, $m \rightarrow m/\sigma_{\mu E^{-1}}$, $H \rightarrow H/\sigma_{\mu E^{-1}}$. The action variables $J_i$ (with units of energy $\times$ time) as well as the constants $e$ and $c$ are all invariant under the $\sigma_{\mu E^{-1}}$ scaling of lengths, times, and energies given in Eq.(1). Thus from Eq. (20), the hamiltonian divided by the mass-energy, $H/mc^2$, is a dimensionless function which describes the shape of the particle orbit as well as the speed of the particle and is completely independent of any rescaling of the form given by a dilatation factor $\sigma_{\mu E^{-1}}$. The Coulomb potential allows the decoupling of the phase-space behavior given by the $J_i$ from the particle mass $m$, and hence allows the possibility of a universal spectrum of blackbody radiation in classical physics.

In the case of a restriction to circular orbits of angular momentum $J$, $J_1 = J_2 = J_3 = J$,  

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the energy becomes

\[ H = mc^2 \left[ 1 - \left( \frac{e^2}{Jc} \right)^2 \right]^{1/2} \]  

(21)

the orbital radius is

\[ r = \left( \frac{e^2}{mc^2} \right) \left( \frac{Jc}{e^2} \right)^2 \left[ 1 - \left( \frac{e^2}{Jc} \right)^2 \right]^{1/2} \]  

(22)

and the orbital frequency is

\[ \omega = \left( \frac{mc^3}{e^2} \right) \left( \frac{e^2}{Jc} \right)^3 \left[ 1 - \left( \frac{e^2}{Jc} \right)^2 \right]^{-1/2} \]  

(23)

while the orbital speed is simply

\[ v = r\omega = \frac{e^2}{J} \]  

(24)

We note that Eq. (20) contains a singularity at \( J_2 = e^2/c \). In the circular orbit equations (21)-(24), the lower limit for the action variable \( J \) corresponds to \( J \to e^2/c \) giving particle speed approaching the speed of light \( v \to c \), frequency diverging \( \omega \to \infty \), orbital radius approaching zero \( r \to 0 \), and total energy going to zero \( H \to 0 \). We also emphasize that here for the Coulomb potential (in contrast to all other mechanical potentials), the need for relativity has nothing to do with the magnitude of the mass \( m \), but rather is completely controlled by the ratio \( J_2/(e^2/c) \). Furthermore, for a relativistic particle \( m \) in the Coulomb potential, the total energy \( H \) in Eq. (20) for a bound particle takes on values between 0 and \( mc^2 \); thus a relativistic particle in a Coulomb potential can radiate away at most a finite amount of energy \( mc^2 \). In contrast, a nonrelativistic particle in the Coulomb potential (where for a nonrelativistic circular orbit \( E = H - mc^2 \to -(1/2)m(e^2/J)^2, r \to J^2/(me^2), \omega \to me^4/J^3, v = e^2/J \) can radiate away an infinite amount of energy as \( J \to 0 \). In the nonrelativistic limit, \( J_2 >> e^2/c \to 0 \), the constant \( e^2/c \) vanishes so that now both \( v \) and \( J \) can assume all positive real values between 0 and \( \infty \). The nonrelativistic Coulomb problem is like the rest of nonrelativistic mechanics in having no fundamental connection between energy and time.
V. MATTER-RADIATION CONNECTION

A. Equilibrium at Absolute Zero

First we consider thermal equilibrium at the absolute zero of temperature $T = 0$. We have seen above that the description of van der Waals forces between macroscopic objects within classical physics requires the presence of classical electromagnetic radiation with the Lorentz-invariant spectrum given by Eq. (4) or (15). It seems natural to expect that a Lorentz-invariant theory of classical electrodynamics would be required to accommodate a Lorentz-invariant spectrum of random radiation, and we anticipate that an elementary charge $e$ in a Coulomb potential $-e$ treated within relativistic classical electron theory will leave invariant the Lorentz-invariant spectrum of classical zero-point radiation.

Indeed, the Coulomb potential has all the qualitatively correct aspects for this invariance. If we consider an elementary charge $e$ of mass $m$ in a circular Coulomb orbit described by angular momentum $J$, then from Eqs. (22)-(24) the charge radiates a power $P_{rad}$ given by

$$P_{rad} = \frac{2e^2}{3\bar{e}^3} \omega^4 r^2 = \frac{2}{3} (mc^2) \left( \frac{mc^3}{e^2} \right) \left( \frac{e^2}{cJ} \right)^8 \left[ 1 - \left( \frac{e^2}{cJ} \right)^2 \right]^{-3}$$

or dividing by the characteristic energy per unit characteristic time

$$\frac{P_{rad}}{(mc^2)(mc^3/e^2)} = \frac{2}{3} \left( \frac{e^2}{cJ} \right)^8 \left[ 1 - \left( \frac{e^2}{cJ} \right)^2 \right]^{-3}$$

The last equation is invariant under $\sigma_1E^{-1}$ scaling and the right-hand side depends only on the action variable $J$. Furthermore the particle will emit radiation into the harmonics of the frequency $\omega$ of the mechanical motion. The radiation per unit solid angle emitted into the $n$th harmonic is given by

$$\frac{dP_{rad}}{d\Omega} = \frac{e^2 \omega^4 r^2}{2\pi c} n^2 \left\{ \left[ \frac{dJ_n(n\beta \sin \theta)}{d(n\beta \sin \theta)} \right]^2 + \frac{(\cot \theta)^2}{\beta^2} \left[ J_n(n\beta \sin \theta) \right]^2 \right\}$$

The energy radiated into the $n$th harmonic has a multiplicative factor of $n^2 \omega^4 r^2$ times a function of $n\beta$. Since in the Coulomb potential the circular orbital velocity is a function of the action variable $J$ alone and is independent of the particle mass $m$, the ratios of energy radiated into the different harmonics depend only upon the action variable $J$ (or equivalently on the particle velocity) and not on the mass $m$. Indeed, the radiation into
each of the individual modes associated with the vector spherical multipole radiation has been calculated and has the same behavior.  

Now in thermodynamic equilibrium, we expect the probability distribution for action variables $J_{\text{matter}}$ of the matter to be related to the action variables $J_{\lambda}$ of the radiation. For the harmonic dipole oscillator of frequency $\omega_0$ evaluated in the infinite-mass limit, all the radiation is exchanged with the radiation modes at the fundamental frequency $\omega_0$, and the phase space probability distribution $P_{\omega_0}(J_{\omega_0})$ is exactly the same as that given in Eq. (5) for the radiation mode of frequency $\omega_0$.

$$P_{\omega_0}(J_{\omega_0}) = \frac{c}{b_s} \exp \left[ -\frac{J_{\omega_0} c}{b_s} \right]$$  

(28)

Thus for the point harmonic oscillator, the phase space distribution $P(J)$ for matter is universal in the sense that it is independent of the scale-giving parameter $\omega_0$. Indeed, one can show that this holds generally for point mechanical systems without harmonics. However, all such systems are idealized in their interaction with radiation because they have no finite size or velocity.

In contrast with these idealized point systems with no harmonics, a charged particle in the Coulomb potential represents a realistic mechanical system of finite size and particle speed which allows this same separation of the phase space from the scale-giving parameter $m$. For a particle of mass $m$ and charge $e$ in the Coulomb potential, the probability distribution $P_C(J_i, 0)$ depends upon the exchange of energy with all the radiation modes at frequencies $\omega_n = n\omega$ which are multiples of the fundamental frequency $\omega$; however, the ratios of the power absorbed and radiated in the $n$th harmonic compared to the fundamental $\omega$ are completely independent of the mass $m$ or of the frequency of the fundamental. But then the probability distribution $P_C(J_i, 0)$ for the action variables $J_i$ of the matter will reflect information about the radiation action variables $J_{\lambda}$ and will be independent of the particle mass $m$. In addition to suggesting that the Lorentz-invariant and scale-invariant zero-point radiation spectrum is invariant under scattering by a charge $e$ in a Coulomb potential, this dependence of the $J_i$ on the $J_{\lambda}$ alone fits exactly with our idea that at temperature $T = 0$, the particle should have a probability distribution which reflects zero entropy and is independent of mass $m$. Thus we have

$$P_C(J_i, 0) = F \left( \frac{J_i}{e^2/c^2}, \frac{e^2}{b_s} \right)$$  

(29)
where $F$ is at present an unknown function. The entropy $S/k_B$ (divided by Boltzmann’s constant $k_B$) which follows from this probability distribution on phase space would also be independent of any $\sigma_{ltE^{-1}}$ scaling and independent of the mass $m$.

These ideas hold not only for circular orbits but for arbitrary orbits for an elementary charge in the Coulomb potential. In all cases, the radiation balance at zero temperature and the probability distribution for the action variables is independent of the particle mass $m$. Indeed, classical electromagnetic zero-point radiation is both scale invariant and conformal invariant. The only available scale is given by the particle mass $m$. For a relativistic Coulomb potential, considerations of scaling alone dictate that the mass $m$ can not enter the probability distribution at zero temperature. This is a first crucial step in understanding how the thermal radiation pattern can be universal within classical physics despite the interaction with matter involving various masses. It is the crucial separation of the mass parameter from the underlying conformal structure which should make possible a universal radiation equilibrium within relativistic classical physics.\[37\]

B. Equilibrium at Finite Temperature

If a charged particle in the Coulomb potential is placed in a large container with conducting walls where there is a finite amount of energy above the zero-point radiation, then the relativistic Coulomb system will scatter the radiation and presumably produce a state of thermal equilibrium. In thermal equilibrium, the available energy $U$ above the zero-point energy has been shared between the scattering system and the thermal radiation in the container. If the container is large enough, then the scattering system will absorb a negligible fraction of the available energy $U$. In equilibrium, we do not expect that the scattering system will respond to the container’s total energy $U$ (the extensive variable), but rather to the local energy per unit volume $U/V$ (the intensive variable). Under dilatation, this energy per unit volume scales with four powers of the scaling parameter $\sigma_{ltE^{-1}}$, one power of $\sigma_{ltE^{-1}}$ coming from the energy and three more powers of $\sigma_{ltE^{-1}}$ from the volume. Thus if we want to obtain an energy which scales with one power of $\sigma_{ltE^{-1}}$ and is an intensive variable, then we must choose $[U/(Va_s/k_B^4)]^{1/4}$ where $a_s$ is Stefan’s constant. Of course, this corresponds exactly to $k_B T = [U/(Va_s/k_B^4)]^{1/4}$ from Stefan’s law. Now the probability distribution $P_\lambda(J_\lambda, \lambda k_B T/b_s)$ for the action variable $J_\lambda$ of the radiation mode of wavelength $\lambda$ is given

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by Eq. (7). The probability distribution is no longer the same for all wavelengths $\lambda$, but rather involves a function of the dimensionless quantity $\lambda k_B T / b_s$ connecting the wavelength of the radiation mode and the energy $k_B T$ associated with the thermal radiation. Since the frequency of the radiation mode is directly connected to the wavelength, $\omega = c / \lambda$, the functional dependence could just as well be expressed as involving $(b_s / c) \omega / (k_B T)$.

The scattering situation at finite temperature is very similar to that at zero-temperature. For an elementary charge $e$ in a Coulomb potential, the only parameter which scales with the dilatation factor $\sigma_{l, l, E-1}$ of electromagnetism is the particle mass $m$. Thus comparing matter and thermal radiation, the ratio of characteristic energies involves $mc^2 / (k_B T)$, the ratio of characteristic lengths involves $[e^2 / (mc^2)] / [b_s / k_B T]$, and the ratio of characteristic times involves $[e^2 / (mc^3)] / [b_s / (ck_B T)]$. Every one of these ratios involves universal constants multiplying the dimensionless ratio $k_B T / (mc^2) = [U / (V a_s / k_B)]^{1/4} / (mc^2)$ or its inverse. Thus the probability distribution for the action variables $J_i$ of the mechanical system of mass $m$ is of the form

$$P_C(J_i, k_B T / mc^2) = G \left( \frac{J_i}{e^2 / c}, \frac{e^2}{b_s}, \frac{k_B T}{mc^2} \right)$$

(30)

where $G$ is at present an unknown function which goes over to the unknown function $F(J_i / (e^2 / c), e^2 / b_s)$ in Eq. (29) when $T = 0$. The crucial thing here is that the mass $m$ enters Eq. (30) only in the ratio $k_B T / (mc^2)$.

We should note the crucial parallel between radiation and matter which exists in relativistic classical electron theory with a discrete charge. Thermal radiation at absolute zero involves the same distribution (5) in the action variables $J_\lambda$ for each radiation mode $\lambda$, independent of the mode wavelength. In parallel fashion, hydrogen-like Coulomb scattering systems at absolute zero also all involve the same distribution (29) in their action variables $J_i$ independent of the mass $m$. However, the average energy $U_\lambda$ for each radiation mode is different, $U_\lambda = < J_\lambda > c / \lambda = < J_\lambda > \omega_\lambda$, because the energy $U_\lambda$ involves both the scale-giving parameter $\lambda$ and the average over the distribution of action variables $J_\lambda$, while the energy $U_m$ for each Coulomb system is different, $U_m = mc^2 < f(J_i) >$, because the energy $U_m$ involves both the scale-giving parameter $m$ times an average over the distribution of action variables. In thermal radiation, the $J_\lambda$-distribution for radiation depends on $(b_s / c) \omega_\lambda / k_B T = b_s / (\lambda k_B T)$ and the $J_i$-distribution for matter depends upon $mc^2 / (k_B T)$; in each case the single scale-giving parameter $(\omega = c / \lambda$ or $m)$ of the system is related to the
temperature $T$.

We should emphasize that, in a Coulomb potential, large and small masses act differently in thermal radiation because they couple to different radiation modes. Thus a small mass $m$ is coupled to low-frequency radiation modes since for fixed action variables $J_i$, the frequencies $\omega_i$ (for example, in Eq. (23)) vary directly as the particle mass $m$. It is these low frequencies where the thermal radiation predominates. On the other hand, a large mass $m$ is associated with high frequencies where the zero-point radiation dominates, and the thermal radiation will have little influence on the particle’s motion in its Coulomb orbit. Thus for particles in a Coulomb potential in thermal radiation, there is a transition which can be associated with the particle mass $m$, just as as there is a transition in the radiation modes which can be associated with the frequency $\omega_\lambda$ of the radiation modes. We notice that this situation is completely different from that of a nonrelativistic mass in a harmonic potential well where the natural oscillation frequency is given by $\omega_0 = (K/m)^{1/2}$, and, for fixed spring constant $K$, decreases with increasing mass.

The situation for radiation equilibrium is enormously simplified for the relativistic Coulomb potential. Thus if radiation equilibrium exists for one mass $m$ in a Coulomb potential in zero-point radiation, then it exists for all masses $m$ since the phase space distribution $P_C(J_i, 0)$ must be independent of $m$, and the interaction with radiation is determined by the phase space distribution. Furthermore, if equilibrium exists for one mass $m$ in the Planck spectrum (6) at all temperatures $T$, then the equilibrium is valid for all masses $m$ since the phase space distribution $P_C(J_i, k_B T/mc^2)$ depends only upon the ratio $k_B T/(mc^2)$. This simplification will not hold for any non-Coulomb potential function.

VI. DISCUSSION

Despite all the claims to the contrary, the blackbody radiation problem is still an unsolved problem within classical physics. Today, the textbooks and orthodox physics literature claim that classical physics can not explain the observed Planck spectrum of blackbody radiation. Indeed it was the apparent inability of classical physics to account for this spectrum which led to the introduction of quantum theory in the years after 1900. We believe that the perspective on blackbody radiation current in the physics community today arises because the physicists at the beginning of the twentieth century missed three essential aspects of
classical electrodynamics.

First, they were unaware of classical electromagnetic zero-point radiation. Classical electromagnetic zero-point radiation enters classical electromagnetic theory as the homogeneous boundary condition on Maxwell’s equations and is required to account for the experimentally observed van der Waals forces between macroscopic objects. However, in the research of the early twentieth century, there were no direct measurements of van der Waals forces and the homogeneous solution of Maxwell’s equations was taken to vanish. In his work on classical electron theory, Lorentz specifically assumes that all radiation arises at finite time; there is no classical zero-point radiation. Historically, zero-point radiation entered physics only after the advent of quantum theory and even today plays an ambiguous role. Some physicists are sure that any idea of zero-point energy must involve quantum mechanics. It is only in the second half of the twentieth century that classical electromagnetic zero-point radiation was developed in the classical description of nature.

Second, the physicists at the beginning of the twentieth century did not take seriously the requirements of special relativity. Special relativity was (and still is) regarded as a specialty subject which needs to be considered only for high-speed particles. Thus Lorentz’s classical electron theory involved point charges in nonrelativistic potentials and discussions of atomic physics were all in the context of nonrelativistic mechanics. Indeed quantum mechanics was developed as a subterfuge to fix the connection between nonrelativistic mechanics and electromagnetic radiation, and Heisenberg-Schroedinger quantum mechanics remains a nonrelativistic theory to the present day. In this same vein, even the physicists who recently developed the ideas of classical electron theory with classical electromagnetic zero-point radiation into a theory designated as stochastic electrodynamics, SED, failed to appreciate the importance of relativity for the mechanical scattering systems. Thus, for example, a review article in 1975 considers arbitrary nonrelativistic potentials and states that Newton’s second law in the nonrelativistic form $\mathbf{F} = ma$ is to be used in the analysis of particles interacting with classical electromagnetic zero-point radiation.

Third, physicists have continued the nineteenth century conception of a continuous scale of electric charge, repeatedly dealing with small electric charges in connecting radiation and matter, rather than exploring the implications of the discrete charges in nature. Thus Planck considered harmonic oscillators in the walls of the blackbody cavity, but the charge on the oscillators was merely regarded as very small and cancelled out in his final result. It is
true that in the early years of the twentieth century Planck did hope to connect the electron charge $e$ to his own constant $h$ since he noted that $e^2/c$ and $h$ have the same units. However, Lorentz, even in his account of classical electron theory in 1915, is still looking for a constant analogous to Planck’s constant without remarking that $e^2/c$ provides just such a constant. Lorentz does not note the suitability of $e^2/c$ because he is looking within nonrelativistic classical mechanics, not within classical electrodynamics. Furthermore, in the last third of the twentieth century, the developers of classical electron theory with classical electromagnetic zero-point radiation always have taken the small charge limit so as to deal with quasi-Markov stochastic processes for matter.

The blackbody radiation problem within classical physics has been explored repeatedly. In addition to the old derivations of the Rayleigh-Jeans spectrum given by the renowned physicists at the beginning of the twentieth century, there have also been repeated derivations of Planck’s spectrum. In the presence of classical zero-point radiation, the Planck spectrum within classical physics has been derived using various ideas of classical physics: energy equipartition of nonrelativistic translational degrees of freedom in the large mass limit, thermal fluctuations above zero-point radiation, comparisons between diamagnetic and paramagnetic behavior, the acceleration of point electromagnetic systems through zero-point radiation, and entropy ideas connected with Casimir forces. Most of these derivations involve harmonic oscillator-like systems which interact with radiation at a single frequency in the infinite-mass-and-zero-velocity limit. All show a natural connection between classical electromagnetic zero-point radiation and Planck’s spectrum of thermal radiation. None involves a full relativistic scattering calculation. However, all the insights of these derivations have been rejected by physicists who insist on the validity of the scattering calculations which have used nonrelativistic, nonlinear, mechanical systems to scatter classical electromagnetic zero-point radiation toward the Rayleigh-Jeans spectrum. Despite the fact that one might expect only a relativistic scattering system to maintain the invariance of the Lorentz-invariant spectrum of classical electromagnetic radiation, most physicists are so confident of the universal applicability of nonrelativistic physics that they find it hard to conceive of the possibility that relativity might be required for appropriate scatterers for thermal radiation. However, the thermal effects of acceleration through the Lorentz-invariant spectrum of classical zero-point radiation point unambiguously in the direction of a relativistic theory for blackbody radiation. And a relativistic scattering calcu-
lation using the Coulomb potential has never been done but has all the qualitative aspects appropriate to blackbody equilibrium.

It is our guess that (just as is found from uniform acceleration through the classical zero-point vacuum) the blackbody radiation spectrum has nothing to do with energy quanta and everything to do with the conformal symmetries of classical electromagnetism. In the present work, we have tried to suggest why relativity and discrete electric charge are probably crucial to understanding blackbody radiation within classical electromagnetic theory. Indeed, zero-point radiation, relativity, and discrete charge within classical physics are probably crucial to a deeper understanding of much more of atomic and statistical physics.

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See, for example, P. H. Morse, Thermal Physics, 2nd ed., (Benjamin/Cummings Publishing, Reading, Mass. 1969), p. 339. Then the dimensionless universal constant is given as 

\[
e^2 (a_S / k_B^4)^{1/3} = e^2 \left( \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} \right) / k_B^4 = \left( \frac{\pi^2}{15} \right) \left[ \frac{e^2}{(\hbar c)} \right] = 0.00063487.
\]

Thus we recognize the dimensionless constant as related to Sommerfeld’s fine-structure constant. However, for many physicists (including both the physicists of the early twentieth century and also recent referees), the use of Planck’s constant \( \hbar \) rather than Stefan’s constant \( a_s \) serves as an overwhelming distraction. The appearance of Planck’s constant seems to cause an immediate fixation on ideas of energy quanta. There are no ideas of energy quanta in the present analysis, and such ideas have no place in classical physics. Actually, Planck’s constant does not necessarily have any connection to energy quanta; it can serve as the scale factor for classical electromagnetic zero point radiation without in any way implying energy quanta. See, for example, ref. 11.

Readers will recognize \( b_s \) in terms of currently familiar constants as \( b_s = (1/2) \hbar c \). We emphasize that no energy quanta whatsoever are used in our classical analysis.

See, for example, L. L. Henry and T. W. Marshall, "A classical treatment of dispersion forces," Nuovo Cimento 41, 188-197 (1966); T. H. Boyer, Van der Waals forces and zero-point energy for dielectric and permeable materials," Phys. Rev. A 9, 2078-2084 (1974).

S. O. Rice, in Selected Papers on Noise and Stochastic Processes, edited by N. Wax (Dover, New York, 1954), p. 133.

See, for example, E. A. Power, Introductory Quantum Electrodynamics (American Elsevier, New York 1964), pp. 18-22.

T. H. Boyer, "Connection between the adiabatic hypothesis of old quantum theory and classical electrodynamics with classical electromagnetic zero-point radiation," Phys. Rev. A 18, 1238-1245 (1975).

The existence of energy fluctuations which involve no entropy requires ideas of special relativity and is foreign to nonrelativistic classical statistical mechanics.

E. Cunningham, "The Principle of Relativity in Electrodynamics and an Extension Thereof,"
Proc. London Math. Soc. 8, 77-98 (1910); H. Bateman, "The Transformation of the Electro-dynamical Equations," Proc. London Math. Soc. 8, 223-264 (1910).

[24] T. H. Boyer, "Conformal symmetry of classical electromagnetic zero-point radiation," Found. Phys. 19, 349-365 (1989), Eq. (40).

[25] The correlation function can be obtained by following the same pattern as given in ref. 24 except that we replace \( \frac{1}{2} \hbar c|k| \) by \( k_B T \) in Eq. (39). The integral then involves
\[
\int (d^3k/|k| \cos[k \cdot (r - r') - c|k|(t - t')] = \int dk \int d\theta \cos \frac{1}{2\pi} k \cos \theta \int d\phi \cos[k(R \cos \theta - c\tau)] =
\]
\[
\int_{0}^{\infty} (dk/k) \{ \sin[k(R-c\tau)] + \sin[k(R+c\tau)] \}.
\]
Finally the definite integral gives
\[
\int_{0}^{\infty} dx (\sin mx)/x = \begin{cases} 
\pi/2 & \text{if } m > 0, \\
0 & \text{if } m = 0, \\
-\pi/2 & \text{if } m < 0.
\end{cases}
\]

[26] T. H. Boyer, "Conjectured derivation of the Planck radiation spectrum from Casimir energies," J. Phys. A: Math. Gen. 36, 7425-7440 (2003), Section 6.

[27] As far as the quantum theorists are concerned, the introduction of quanta solved the blackbody problem in conjunction with statistical mechanics. Thus they are surprised when the Planck spectrum appears without the use of quantum statistics. The point of view presented here is entirely classical and is quite different. In the classical perspective present here, classical statistical mechanics plays no role but rather is a derived concept, derived from the randomness of random phases of waves, in the sense of ref. 19. The connection between conformal motions and gravity is crucial in connecting the Planck spectrum to thermal radiation. Thus if thermal radiation exists in equilibrium in a gravitational field, it must correspond to the radiation associated with uniform acceleration through zero-point radiation plus perhaps additional radiation. The finite temperature radiation must fit on top of the spectrum from acceleration through zero-point radiation. In order to satisfy Wien’s displacement theorem, the additional radiation must follow the same pattern which already appeared from the acceleration through classical zero-point radiation.

[28] W. Rindler, *Essential Relativity: Special, General, and Cosmological*, 2nd ed. (Springer-Verlag, New York1977), pp 49-51.

[29] During the early years of the twentieth century, the nonrelativistic harmonic oscillator was often connected to radiation by a very small charge \( q \). See, for example, M. Planck, *The Theory of Heat Radiation* (Dover, New York 1959). The electric dipole oscillator was said
to come to equilibrium with an energy $U$ equal to the energy $U\omega_0$ of the radiation mode at the same frequency as the natural frequency of the oscillator, $U = U\omega_0$. This equality held for fixed $\omega_0$ whether the mass $m$ and spring constant $K$ of the oscillator were both large and the velocity and amplitude were both small, or whether $m$ and $K$ were both small with the velocity and amplitude large, perhaps the velocity even exceeding $c$. However, classical electromagnetic radiation does not treat these two possible radiating systems alike because finite velocity and amplitude means that radiation will be emitted into the higher harmonics of the fundamental frequency. The traditional classical calculations are accurate only in the limit $m \to \infty$ where the velocity and amplitude vanish; in this limit, the oscillator interacts with radiation only at its fundamental frequency and so does not scatter radiation toward equilibrium.

[30] H. A. Lorentz, *The Theory of Electrons and its Application to the Phenomena of Light and Radiation Heat*, 2nd ed. (Dover, New York 1952), p. 96. This is a republication of the 2nd edition of 1915.

[31] T. H. Boyer, "Scaling Symmetry and Thermodynamic Equilibrium for Classical Electromagnetic Radiation," Found. Phys. 19, 1371-1383 (1989). D. C. Cole, "Classical Electrodynamic Systems Interacting with Classical Electromagnetic Random Radiation," Found. Phys. 20, 225-240 (1989).

[32] See, for example, H. Goldstein, *Classical Mechanics, 2nd ed.*, (Addison-Wesley Publishing, Reading, Mass. 1981), p. 498. We are using action variables which are $1/(2\pi)$ times those in Goldstein’s text.

[33] T. H. Boyer, "Unfamiliar trajectories for a relativistic particle in a Kepler or Coulomb potential," Am. J. Phys. 72, 992-997 (2004).

[34] J. D. Jackson, *Classical Electrodynamics, 2nd ed.*, (Wiley, New York 1975), p. 664.

[35] See ref. 34, p. 695.

[36] L. M. Burko, "Self-force approach to synchrotron radiation," Am. J. Phys. 68, 456-468 (2000).

[37] A complete classical electromagnetic calculation of radiation equilibrium for a relativistic particle in the Coulomb potential at zero temperature should give the value of the fine structure constant. The probability distribution for the action variables $J_i$ involves two different scales; $e^2/c$ and $b_s/c$. The scale $e^2/c$ is related to the relativistic mechanical behavior and provides a limit at small $J_2$. See ref. 33. The scale $b_s/c$ involves the energy balance with the random
zero-point radiation. Roughly, when the action variables $J_i$ are larger than $b_s/c$, then the hydrogen system is losing more energy by radiation emission than it is picking up from the zero-point radiation; when the $J_i$ are smaller than $b_s/c$, the reverse is true. See section D2 of ref. 11. We note that if $b_s/c$ were too small, $b_s/c < e^2/c$, then no hydrogen atom would exist because the energy-conserving orbits at this angular momentum would spiral into the nucleus. See ref. 33. In nature, the value is $b_s/c \approx 68e^2/c$.

[38] See ref. 30, note 6, p. 240, which gives Lorentz’s explicit assumption on the boundary condition.

[39] A review of most of the work on classical zero-point radiation is presented by L. de la Pena and A. M. Cetto, *The Quantum Dice - An Introduction to Stochastic Electrodynamics* (Kluwer Academic, Dordrecht 1996).

[40] D. C. Cole and Y. Zou, "Quantum Mechanical Ground State of Hydrogen Obtained from Classical Electrodynamics," Phys. Letters A 317, 14-20 (2003). Cole and Zou’s numerical simulation calculations suggest that classical zero-point radiation acting on the classical model for hydrogen (a point charge in a Coulomb potential) produces a steady-state probability distribution which is finite and which seems to approach the ground state obtained from solution of the Schroedinger equation for a point charge in the Coulomb potential. Cole and Zou have no adjustable parameters in their calculations. See also, T. H. Boyer, "Comments on Cole and Zou’s calculation of the hydrogen ground state in classical physics," Found. Phys. Letters 16, 613-617 (2003).

[41] See ref. (Kuhn) p. 132.

[42] On page 78 of ref. 30, Lorentz writes, "Now, if the state of radiation is produced by a ponderable body, the values of the two constants [in the blackbody spectrum] must be determined by something in the constitution of this body, and these values can only have the universal meaning of which we have spoken, if all ponderable bodies have something in common."

[43] T. H. Boyer, "Derivation of the Blackbody Radiation Spectrum without Quantum Assumptions," Phys. Rev. 182, 1374-1383 (1969); T. W. Marshall, "Brownian motion of a mirror," Phys. Rev. D 24, 1509-1515 (1981).

[44] T. H. Boyer, "Classical Statistical Thermodynamics and Electromagnetic Zero-Point Radiation," Phys. Rev. 186, 1304-1318 (1969).

[45] T. H. Boyer, "Derivation of the Planck radiation spectrum as an interpolation formula in classical electrodynamics with classical electromagnetic zero-point radiation," Phys. Rev. D
See R. Blanco, L. Pesquera, and E. Santos, "Equilibrium between radiation and matter for classical relativistic multiperiodic systems. Derivation of Maxwell-Boltzmann distribution from Rayleigh-Jeans spectrum," Phys. Rev. D 27, 1254-1287 (1983); "Equilibrium between radiation and matter for classical relativistic multiperiodic systems II. Study of radiative equilibrium with Rayleigh-Jeans radiation," Phys. Rev. D 29, 2240-2254 (1984). These articles suggest that a relativistic classical scatterer again leads to the Rayleigh-Jeans spectrum. The calculations involve a general class of potentials. The mechanical momentum of the particle is calculated relativistically but the class of potentials excludes the Coulomb potential and all purely electromagnetic scattering systems. Many physicists do not realize that such mechanical systems do not satisfy Lorentz invariance. It is the Coulomb potential and only the Coulomb potential which has been incorporated into a fully relativistic classical electron theory. A general potential (unextended to involve new acceleration-dependent forces and new radiation specific to the potential) violates the center-of-energy conservation laws which are directly related to the generator of Lorentz transformations. See T. H. Boyer, "Illustrations of the relativistic conservation law for the center of energy," Am. J. Phys. 73, 953-961 (2005).