Self-stabilizing spin superfluid

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Spin superfluidity is sought after as a potential route to long-range spin transport in ordered spin systems. Signatures of spin superfluidity have recently been observed in antiferromagnets, however, dipolar interactions have been predicted to destabilize the superfluid state and its realization in ferromagnets remains a challenge. Using micromagnetic simulations, we find that spin superfluidity can in fact be achieved in extended thin ferromagnetic films. We identify two unconventional superfluid states beyond the Landau instability. We uncover a surprising two-fluid state, in which spin superfluidity coexists with and is stabilized by spin waves, as well as a soliton-screened superfluid at high biases. The results of this study advance our understanding of spin superfluidity and provide guidance for its experimental realization.

The field of magnon-spintronics opens new possibilities for energy-efficient information storage, transport, and processing. Achieving low-dissipation long-range spin transport is one of the main goals of spintronics research. In magnetic insulators, magnetic damping can be low and spin currents are carried by spin waves, free of undesired electric currents 11. Spin waves, however, exhibit exponential decay over distances that can be short at high frequencies.

The bosonic nature of spin excitations in ordered magnetic materials can benefit from magnon-magnon interactions and the ensuing coherence. Bose-Einstein condensation of magnons, that was experimentally observed in various systems 2, 3, is a notable example. Another phenomenon characteristic of bosonic systems is superfluidity; resistance-free charge transport in superconductors and viscosity-free mass transport in superfluid helium are some prominent examples 7, 8. Early works by Halperin and Hohenberg proposed a hydrodynamic theory of magnons 10, which is closely related to superfluidity.

Spin superfluids can be induced in easy-plane ordered spin systems. Upon non-equilibrium spin injection with perpendicular-to-plane polarization, a global texture of magnetic order parameter in the form of a winding spiral forms (Fig. 1a). The order parameter precesses coherently in time at low frequencies and transports spin current over macroscopic distances 11. The spin current shows power-low spatial decay, thus enabling long-range spin transport well beyond the mean free path of ordinary spin waves.

Recently, signatures of spin superfluidity have been experimentally observed in antiferromagnetic spin systems 12, 13. A realization of spin superfluid in ferromagnets remains an unsolved challenge. Previous theoretical works have revealed the potential of superfluid spin transport 11, 14–23 for spintronics applications but have not systematically studied the role of dipolar interactions. Recent numerical calculations 24 for micrometer-scale thin-film ferromagnets have demonstrated that dipolar interaction can destroy the spin superfluid, sparking a discussion on the feasibility of such state 15, 19. The long-sought superfluid spin transport in ferromagnets has thus been considered in question.

Here we present a micromagnetic study of superfluid spin transport in extended ferromagnetic thin films and investigate the role of dipolar interaction. We find that, contrary to the expectation, stable spin superfluid state can be achieved. Surprisingly, we also observe that the spin superfluid is stable beyond the Landau instability.

The Landau superfluid breakdown describes a superflow-carrying state becoming energetically unstable at the critical injection bias. In ferromagnetic films, this corresponds to alignment of magnetic order parameter fully out-of-plane, which disrupts the superfluid spin transport 11. Unlike the conventional breakdown of the superfluidity or superconductivity, however, the superflow in our system is recovered. We find that the spin bias applied to the injector does not determine the spin current flowing through the magnet. The latter is rather determined self-consistently, taking into account the feedback of the magnetic dynamics near the injector. We find that this feedback regulates the spin injection through spin wave emission and coherent soliton formation.
superflow thus stays effectively below the Landau instability threshold even at large spin biases.

RESULTS

We simulate extended ferromagnetic films in the thickness range of \( t = 2-30 \text{nm} \) by applying periodic boundary conditions in the film plane to a \( 50 \mu\text{m} \times 5 \mu\text{m} \) patch. Magnetic parameters of the film are chosen (Methods) to mimic the magnetic insulator \( \text{Y}_3\text{Fe}_5\text{O}_{12} \) (YIG). Magnetization dynamics is excited by locally injecting a continuous pure spin current with out-of-plane spin polarization. It is simulated through spin-transfer torque in the middle of the film underneath a narrow spin injector. The spin injector carries electric current that translates into spin current with conversion efficiency of \( \theta_s = 0.07 \) (see Methods). At the short edges of the film patch, spin sinks are simulated by local increase of the Gilbert damping as explained in Methods. All calculations in this study are carried out at 0 K, without thermal excitations. Figure 1a shows sample geometry, spin injector, and spin sinks.

Behavior without dipolar interaction. At first, we investigate the case of omitted dipolar interaction by enforcing zero dipole fields in our simulations and introducing an artificial easy-plane anisotropy \( K_u = -10 \text{kJ m}^{-3} \) approximating the shape anisotropy of a thin film [15, 25]. For each current value, the simulations are carried out until steady state or dynamic equilibrium is reached. In Fig. 1a, a snapshot of magnetization is shown for the steady state at a current density \( j = 10^{11} \text{A m}^{-2} \). Figure 1b shows the initial velocities \( u_0 \) (calculated in the vicinity of the injector region) as a function of the current density. Three distinct regimes can be identified as indicated in the figure:

Regime I. At low current densities, the superfluid velocity linearly increases with the increasing current density, in good agreement with analytical predictions of Ref. [26]. The superfluid velocity decreases smoothly and slowly with increasing distance from the spin injector (Fig. 1c). At the spin sink, it decreases more rapidly and reaches zero value. The longitudinal spin density \( n = m_z \) (equal to the polar component of the normalized magnetization) is well below 0.5 (Supplementary Figure 1).

Regime II. At the first critical current \( j_{\text{crit}}(1) \), the superfluid starts to exhibit oscillations in real space, as shown in Fig. 1d. The superfluid velocity is calculated by averaging out these oscillations. The initial velocity shows a notable drop at the first critical current (Fig. 1b).
Underneath the injector, the magnetization is partially tilted out of the film plane by the spin current. Outside of the injector region, the longitudinal spin density remains \( n < 0.5 \).

Analysis of the temporal evolution of magnetization reveals large oscillations in the injector region. It emits incoherent spin waves into the rest of the film which superimpose with the superfluid state (Fig. 1d). We observe spin wave emission and the drop of the superfluid velocity for various injection widths \( w = 30–300 \text{ nm} \). The injector width does not affect the critical current, but modifies \([19]\) the critical current density through geometrical renormalization \( j_{\text{crit}}^{(1)} \propto I_{\text{crit}}/w \) (Supplementary Figure 2).

The temporal base frequency \( \Omega \) of the superfluid spiral is extracted for each current density by calculating the fast-Fourier transformation of the time evolution of the magnetization dynamics. As shown in Fig. 1d, both \( u_0 \) and \( \Omega \) show the distinct breakdown in the regime II.

**Regime III.** Above the second critical current density \( j_{\text{crit}}^{(2)} \), the superfluid velocity is again a smooth function of distance (Fig. 1e). No spin waves are observed. The magnetization underneath the injector is almost fully aligned out-of-plane and does not vary with time. Both initial velocity and base frequency show a reduced growth rate with increasing spin current and saturate around \( j = 8 \cdot 10^{11} \text{ A m}^{-2} \) (Fig. 1b).

**Analytical model.** To interpret these numerical results, we derive an analytical model neglecting dipolar interaction and magnetic damping. With exchange constant \( A_{\text{ex}} \), we employ the free energy:

\[
F = \int dx^3 \left[ A_{\text{ex}} (\nabla m)^2 - K_u m_z^2 \right].
\]

(1)

Taking into account that magnetization \( m \) does not vary along the \( y \) and \( z \) directions, Landau-Lifshitz equation takes the form

\[
\frac{dm}{dt} = -m \times \left( \frac{\partial^2 m}{\partial x^2} - m_z \hat{z} \right),
\]

(2)

where \( x \) and \( t \) are re-scaled in units of \( \sqrt{A_{\text{ex}}/K_u} \) and \( \mu_0 M_s/2\gamma K_u \), respectively (with the permeability of free space \( \mu_0 \) and gyromagnetic ratio \( \gamma \)). By parameterizing the magnetization with spherical coordinates, \( m = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), equation (2) becomes

\[
\dot{\theta} \sin \theta = -\partial_z (\sin^2 \theta \partial_z \phi),
\]

(3)

\[
\dot{\phi} \sin \theta = \partial_z^2 \phi + \frac{1 - (\partial_z \phi)^2}{2} \sin 2\theta.
\]

(4)

Equation (3) corresponds to a continuity equation for the longitudinal spin density. Assuming soliton solutions have the form \( \theta = \theta(x - ct) \) results (Methods) in

\[
\phi - \phi_0 = \omega t - \int_0^x \frac{c \cos \theta + a_1}{\sin^3 \theta} \, dx,
\]

(5)

\[
x - ct = x_0 \pm \frac{1}{\sqrt{2}} \int_{\theta_0}^{\theta(x,t)} \frac{d\theta'}{\sqrt{f(\theta')}}.
\]

(6)

where \( f(\theta) = a_2 - \omega \cos \theta - \frac{1}{2} \sin^2 \theta - \frac{1}{2} (c^2 - a_1^2) \csc^2(\theta) - ca_1 \cot(\theta) \csc(\theta) \). Here \( \omega, c, \phi_0, a_1, \) and \( a_2 \) are integration constants. We consider the case in which \( f(\theta) > 0 \) for some open interval \( (\theta_1, \theta_2) \subset (0, \pi/2) \), where \( \theta_1 \) and \( \theta_2 \) are zeros of \( f(\theta) \). The resulting soliton solution, \( \theta(x - ct) \), is symmetric about its minimum \( \theta_1 \) (corresponding to a spike in \( m_z \)) centered at \( x_0 = 0 \) for \( t = 0 \). Because the soliton expression (6) only describes \( \theta \) over a finite interval of \( x - ct \), to describe a full solution, the soliton and its first derivatives must be patched to suitable surrounding solutions, such as another soliton or a superfluid, or to the boundary conditions. This patching process fixes the integration constants. One such solution is an isolated soliton traveling at speed \( c \) through a surrounding spin superfluid which has constant polar angle \( \theta_2 \). The length of the soliton is determined by the characteristic length scale \( \sqrt{A_{\text{ex}}/K_u} \).

We find the relevant solutions by fixing the boundary conditions with a spin injection site and spin sink on either side. The analytically calculated transmitted spin current per spin density \( \tau = -\nabla \phi \sin^2 \theta \) is shown in Fig. 1b as the black solid line. The analytical spin current plot shows three distinct phases, similar to the three phases identified in micromagnetic simulations. The low-current regime (I) corresponds to the conventional spin superfluidity, i.e. a coherently precessing constant-\( \theta \) superflow as derived in Ref. [20].

Previous analytical study [20] has suggested that when the transmitted spin current drops to zero, the undamped spin superfluid becomes fully polarized out of plane (\( \theta = 0 \)). However, this solution is in fact unstable, even in the undamped model. It has a mode of instability which forms near the boundaries and propagates into the rest of the film. This mode of instability has superfluid-like precession and grows exponentially with time. There are no stable time-independent \( \theta \) solutions in the intermediate (II) regime between the two analytical solutions (I) and (III) plotted in Fig. 1b. The solution in the intermediate (II) regime must thus be a non-trivial dynamic state. The first critical current observed in micromagnetic simulations indicates that the dynamic instability sets in before the precipitous drop in the calculated transmitted spin current (and below the Landau criterion). The mechanisms and system parameters, that may shift the first critical current and determine the current range of the intermediate regime, are yet to be fully understood.

Above the second critical current, we find a stationary soliton solution \( c = 0 \) of particular interest. The soliton is placed at the edge of the spin superfluid with the peak at the injection region boundary. In regime III, the injector region is nearly fully polarized out-of-plane, and the local
time-dependent oscillations in $\theta$ cease. This configuration lacks the spin wave noise present in regime II. The spin current is reduced by the injector edge soliton due to the $(1-m_z^2)$ factor in the spin current. For high out-of-plane polarizations, it diminishes the transmitted spin current at the same superfluid velocity $u$. The polarization in the injector region partially blocks the spin injection, and the transmitted spin current asymptotically behaves as $\propto 1/j$ for $j \to \infty$. By virtue of this self-regulation in the injector region, the superfluid persists above biases expected for the Landau instability. We therefore suggest to name this regime "screened spin superfluid."

The exact mechanism of the superfluid stabilization in regime II remains elusive, however, the data allows us to propose two hypotheses. (i) The superfluid solution may be a hybrid periodically transitioning between the conventional superfluid and screened superfluid. The steady-state solution that matches the boundary conditions ($\theta = 0$) is unstable to variations in $\theta$, resulting in the spin texture dropping down to a $\theta$-superfluid which is unable to accommodate the large spin current at the boundaries. Therefore, the stationary soliton of a screened superfluid begins to form as spin accumulates near the edge of the injector region. However, the spin current is not strong enough to maintain a stable soliton, and the soliton decays into spin waves before reaching steady-state screening. The soliton formation and decay processes are then repeated, resulting in injector region oscillations and spin waves propagating into the film. (ii) The solution may be a large amplitude spin wave over the superfluid which would be composed of the above derived propagating solitons, fitted back to back and satisfying the boundary conditions on average. These large amplitude spin waves would have to quickly decay, possibly due to Suhl scattering [27], resulting in the observed noise of incoherent spin waves. The reduction of superfluid velocity observed in the micromagnetic simulations could be explained by the emergence of such dissipation channel for the injected spin current.

**Impact of dipolar interaction.** The dipolar interaction is expected to destroy long-range superfluid spin transport. This assertion has been made in Ref. [24] based on numerical calculations of micron-sized ferromagnetic thin films. Here, we investigate extended systems by employing periodic boundary conditions. In the following micromagnetic simulations, the dipolar interaction is enabled and the previously used uniaxial anisotropy $K_u$ is set to zero.

First, we find that the presence of the dipolar interaction suppresses spin superfluidity at low currents and imposes a threshold $j_0$ for its formation [24]. The dipolar interaction acts as an effective magnetic anisotropy that must be overcome. The effective dipole energy increases with the thickness of the film $d$ which is varied in the range of 2—30 nm in our simulations. For comparison across different film thicknesses, the current needs to be scaled by $d$. Indeed, Fig. 2a shows that such normalized threshold current $j_0/d$ increases nearly linearly with increasing film thickness.

Upon the formation of spin superfluid, its initial velocity $u_0$ presents non-monotonic dependence on the current density. Figure 2b shows a qualitatively very similar behavior as in the case of omitted dipolar interaction. Employing spatio-temporal analysis of the magnetization dynamics, we find again: (I) the low-current regime free of incoherent spin waves, (II) the intermediate regime with co-existing superfluid and incoherent spin waves, and (III) the high-current regime of screened superfluid, free of incoherent spin waves. An additional drop of the initial velocity and base frequency is observed in the middle of the intermediate regime (II). A detailed evaluation of the data reveals that $u$ and $\Omega$ show multiple non-monotonicities for both dipole case and dipole-free case. While the currents at which they occur differ, their presence seems to be universal and is likely related to the non-linear generation of spin waves in the regime II.

We further find differences of the spatial profile of su-
perfluid velocity compared to the dipole-free case. As shown in Fig. 2c, the gradient of the azimuthal angle presents two types of modulations. Due to the continuous $2\pi$-rotations of magnetization, dipolar interaction introduces a perturbation of the energy landscape with uniaxial symmetry – the magnetic charges alternate at every $\pi$-rotation. Upon these perturbations, the angle gradient shows a small magnitude modulation with periodicity being a multiple of the periodicity of the $\pi$-rotations. Another modulation with larger amplitude has a smaller periodicity (larger wavelength) that corresponds to the $\pi$-rotations of magnetization. The in-plane components of magnetization present a distorted sinusoidal profile as a function of distance (Supplementary Figure 3). The out-of-plane component of magnetization shows a small magnitude modulation with periodicity being a multiple of the periodicity of the $\pi$-rotations. The size of the soliton scales as $1/\langle \partial_x \phi \rangle_{ave}$ – the current-dependent averaged winding length of the spin superfluid.

**DISCUSSION**

In this study, spin superfluidity is found to persist over a large range of currents. The magnetization pinning by dipole fields [21] does not fully suppress the superfluidity at high biases for the case of extended films [18]. The threshold suppression of spin superfluidity at low biases has been previously discussed [11] for symmetry-breaking magnetic anisotropy. In contrast to the effect of such local anisotropy, the symmetry breaking, investigated in this study, is mediated by the non-local dipolar interaction [19]. We find the threshold current to increase linearly with increasing dipole energy.

A coupling between the superfluid order parameter (azimuthal angle $\phi$) and the longitudinal spin density $n$ is observed. The longitudinal spin density shows oscillations at twice the base frequency [19], in agreement with the symmetry order of the effective (uniaxial) magnetic anisotropy due to dipole fields. The oscillations correspond to excitations of the soliton lattice. No such behavior is observed in the absence of the dipolar interaction.

We identify three regimes of spin superfluidity, universally present with and without dipolar interaction. In the low-current regime, conventional spin superfluidity is found. Above the first critical current, the superfluid co-exists with incoherent non-thermally populated magnons. Above the second critical current, the incoherent magnons are suppressed and a soliton screened spin superfluid is found.

We discover the ability of the spin superfluid to self-stabilize beyond the Landau instability. At very high biases the superfluid is partially screened from injected spin current by soliton formation. For the intermediate-current regime, we identify non-linear magnon scattering to play a role in superfluid self-stabilization. The intermediate regime may prove of particular importance for experimental realization of spin superfluids at finite temperatures. Further theoretical efforts are called upon to elucidate the mechanism of superfluid self-stabilization.

**METHODS**

Micro magnetic simulations. The magnetostatic field was calculated by the approach presented in Ref. [29]. The material parameters were chosen to simulate YIG films [30-32]: the saturation magnetization $M_s = 130 \text{kA m}^{-1}$ and the exchange constant $A_{ex} = 3.5 \text{pJ m}^{-1}$. The magnetocrystalline anisotropy was omitted. The spin sinks were modeled by non-uniform increase of the Gilbert damping over the width (4 $\mu$m) of the spin sink regions. From the sink edge closer to the injector to the edge at the end of the film patch, the damping constant $\alpha$ was increased exponentially from 0.002 to 0.11. To ensure that the system reached a dynamic steady-state, an integration time of 500 ns was chosen. The electric current density given throughout the manuscript corresponds to the spin current via $j_s = \theta_s \frac{h}{2e} j$ with the spin conversion efficiency $\theta_s$, the Planck constant $\hbar$ and the elementary charge $e$. All micromagnetic simulations were carried out at zero temperature.

Analytical model. Numerical calculations of the analytical model resort to the same material parameters as micromagnetic simulations, but do not include magnetic damping. Here we derive equations (5) and (6). The assumption $\theta = \theta(x - ct)$ implies that the left-hand-side of Equation (3) can be written as a derivative in $x$, thus allowing Equation (3) to be integrated. The result can be solved for $\partial_x \phi$ and integrated again to express $\phi$ in terms of $\theta$. In general, the constants of integration can depend on $t$, i.e.

$$\theta = C_2(t) - \int \frac{dx'}{c} \frac{c \cos \theta + C_1(t)}{\sin \theta}. \quad (7)$$

However, the time dependence is restricted by substituting the expression for $\phi$ in terms of $\theta$ into Equation (4). Once $\theta$ has been isolated, the resulting equation should not have explicit $t$ dependence because, by assumption, $\theta$ only depends on $x - ct$. This implies that $C_1$ is independent of time and restricts $C_2$ to at most linear dependence on $t$, thus resulting in equation (5). Once Equation (4) has been expressed only in terms of $\theta$ and its derivatives, the equation can be integrated directly after multiplying by $\partial_x \theta$, resulting in equation (6).
Data availability

The data of this study is available upon request from the corresponding author.

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AUTHOR CONTRIBUTIONS

T.S. and A.K. carried out numerical simulations. D.H. and Y.T. developed the analytical model. I.B. carried out numerical simulations and supervised the project. All authors contributed to writing the manuscript.

ADDITIONAL INFORMATION

Competing financial interests

The authors declare no competing financial interests.
Supplementary Information: Self-stabilizing spin superfluid

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SUPPLEMENTARY FIGURE 1: LONGITUDINAL SPIN DENSITY

FIG. 1: Longitudinal spin density as a function of the injector current density. The values were extracted in the vicinity of the injector.
SUPPLEMENTARY FIGURE 2: INFLUENCE OF THE INJECTOR WIDTH

FIG. 2: Impact of the injector width on spin superfluid. (a) Base frequency $\Omega$ for different injector widths. (b) First critical current density decreases as $\propto 1/w$ (red line), where $w$ is the injector width.
FIG. 3: Perturbations of the magnetization spiral in the presence of dipolar interaction. (a) Re-normalized in-plane components of the magnetization \( m_x \) (blue solid line) and \( m_y \) (red solid line) deviate from sinusoidal behavior. (b) Out-of-plane component \( m_z \). The peaks in this component occur when \( m_x = -1 \) and \( m_x = 1 \). (c) The divergence of the magnetization is linked to the magnetostatic field.