Minimal Anomalous U(1)' Extension of the MSSM

Pascal Anastasopoulos; Francesco Fucito; Andrea Lionetto; Gianfranco Pradisi; Antonio Racioppi; Yassen S. Stanev

Dipartimento di Fisica dell'Università di Roma, “Tor Vergata”
and
I.N.F.N. - Sezione di Roma “Tor Vergata”
Via della Ricerca Scientifica, 1 - 00133 Roma, ITALY

Abstract: We study an extension of the MSSM by an anomalous abelian vector multiplet and a Stückelberg multiplet. The anomalies are cancelled by the Green-Schwarz mechanism. The advantage of this choice over the standard one is that it allows for arbitrary values of the quantum numbers of the extra U(1). As a first step towards the study of hadron annihilations producing four leptons in the final state (a clean signal which might be studied at LHC) we then compute the decays $Z' \to Z_0 \gamma$ and $Z' \to Z_0 Z_0$. We find that for $M_{Z'} \approx 4$ TeV the decay rate for $Z' \to Z_0 \gamma$ is $10^{-4}$.

Keywords: Anomalies, anomalous U(1), Stückelberg, Chern-Simons, $Z'$, axion, MSSM, BSM, LHC
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References
1. Introduction

The Standard Model (SM) of particle physics has been confirmed to a great accuracy in many experiments. Despite the fact that the Higgs particle remains experimentally elusive, few scientists doubt that there will be major surprises in this direction. The whole scientific community, however, knows that the SM needs to be improved. First of all, neutrino oscillation experiments have exhibited the evidence for (tiny) neutrino masses, that have to be incorporated in (an extension of) the SM. Many ideas exist on how this can be achieved and more experimental precision tests will indicate which models are viable. Second, there are also several theoretical issues that make physicists believe that the SM is only an effective manifestation of a more Fundamental Theory.

In approximately one year, the Large Hadron Collider (LHC) at CERN will start to operate at energies of order of 14 TeV in the center of mass. Apart from the search for the Higgs boson, it will probably give us some answers about the parameter space of the physics beyond the SM. Among the many issues that will be addressed, it is worth to mention: the search for supersymmetry, heavy quarks and the quark-gluon plasma, the existence of extra dimensions and the possible creation of tiny black holes.

One of the most attractive scenario for physics beyond the SM is the existence of additional massive neutral gauge bosons $[1]-[9]$. They could be one of the first discoveries at LHC if their mass is in the range of a few TeV. Many different models have been developed in the past in order to investigate this possibility. The mass could be acquired in a variety of ways: from Kaluza-Klein modes to a standard Higgs mechanism or even by adding an axionic field, $\phi$, which couples to the abelian factors (Stückelberg mechanism) $[10, 11]$. The latter is common to low energy effective field theories which appear anomalous. The anomaly cancellation is achieved by the Green-Schwarz mechanism with Stückelberg terms accompanied by axion like couplings, $\phi F \tilde{F}$, which ensure the consistency of these models $[12, 13]$.

For example, in string theory anomalous $U(1)$’s are very common. D-brane models contain several abelian factors, living on each stack of branes, and they are typically anomalous $[14]-[27]$. In the presence of these anomalous $U(1)$’s, the Stückelberg mixing with the axions cancels mixed anomalies$^1$ $[16]$, and renders the “anomalous” gauge fields massive. The masses depend non-trivially on the internal volumes and on other moduli, allowing the physical masses of the anomalous $U(1)$ gauge bosons to be much smaller than the string scale (even at a few TeV range) $[12, 28]$. However, it has been shown that axionic terms alone are not sufficient to cancel all anomalies. An important role is played by the so-called Generalized Chern-Simons terms (GCS) which are local gauge non-invariant terms. Indeed, these trilinear gauge bosons anomalous couplings are responsible for the cancellation of mixed anomalies between anomalous $U(1)$’s and non anomalous factors ensuring the consistency of the theory $[29, 30, 31]$.

In this paper, we are interested in anomaly related $Z'$ bosons. More precisely, we study an extension of the MSSM (see $[32]$ for a review) by the addition of an abelian vector multiplet $V^{(0)}$ and we assume that generically all MSSM particles are charged with

$^1$Irreducible anomalies are cancelled by the tadpole cancellation.
respect to the new $U(1)$. In order to gain in flexibility, our model is only string inspired: we do not commit to a specific brane model and this is why the charges are not fixed. The extra vector multiplet generically is anomalous and consistency of the model requires an additional Stückelberg multiplet $S$ with the proper couplings as well as GCS terms. As a consequence, the anomalous abelian boson becomes massive and behaves like a $Z'$. Moreover, in order to break supersymmetry, we add the usual soft breaking terms and the new terms coming from the fermionic sectors of $V^{(0)}$ and $S$.

Our model contains many new features: new D and F terms (which are coming from the axionic terms and not from the GCS, in accordance with [31], due to the fact that the GCS’s contain only vector multiplets in antisymmetric form), new couplings and new mass contributions in comparison with the MSSM. Explicit formulae are provide for all these terms in component fields.

Since the Higgs fields might be charged under the anomalous $U(1)$, a combination of the Stückelberg and the Higgs mechanism makes the anomalous $U(1)$ massive. An axi-Goldstone combination is eaten by the neutral gauge bosons and no physical axi-Higgs is left contrary to other studies on anomaly related $Z'$ [13] and similarly to the case of a non-anomalous related $Z'$ [11, 14].

We explicitly show how the anomaly cancellation mechanism works in our model before and after breaking the gauge symmetry. Before gauge symmetry breaking, only SM fermions contribute to the triangle diagrams. After gauge symmetry breaking, all fermions that become massive still contribute to the anomalous triangle diagrams. Their contribution is cancelled by new diagrams which involve the Nambu-Goldstone (NG) boson exchange.

In order to explore some phenomenological implications of our setting, we then analyze the decays $Z' \rightarrow Z_0\gamma$ and $Z' \rightarrow Z_0Z_0$. We numerically compute the decay rates as functions of the arbitrary $U(1)$ charges and the mass of the anomalous $U(1)$ gauge boson. We find a non-trivial dependence on all these parameters, estimating that the region that gives the largest values is for $M_{Z'} \sim 4$ TeV, where the decay rate $Z' \rightarrow Z_0\gamma$ is of the order of $10^{-4}$ GeV. These decays are part of the processes in which two colliding protons lead to a four lepton final state [13]. The final state is very clean and possibly measurable at LHC. In a future work we will push our program forward and study this signal with the aid of Monte Carlo methods [64].

The paper is organized as follows: in Section 2, we introduce the vector multiplet, $V^{(0)}$, the Stückelberg multiplet and we provide the axionic and GCS lagrangians in superfields and in components. We then discuss the anomaly cancellation both in the unbroken and in the broken phase. At the end of the Section, we add all possible soft-breaking terms. In Section 3, we describe the model set up. In particular, we discuss the kinetic mixing terms which are coming from the axionic lagrangian and the D and F terms, pointing out explicitly the new contributions. We comment on the superpotential and we compute the mass terms for all the particles, pointing out the differences from the canonical MSSM setup. Finally, in Section 4, we study some phenomenomogical implications of our model. We consider the case in which the Higgs fields are uncharged with respect to the $U(1)'$ and compute the decay rates for the two processes $Z' \rightarrow Z_0\gamma$ and $Z' \rightarrow Z_0Z_0$ which should be
relevant for the computation of hadron annihilations into four leptons. In the appendices we report the technical details and discuss the general case in which also the Higgs fields transform under the anomalous $U(1)'$.

2. Preliminaries

In this section, we discuss how to extend the Minimal Supersymmetric Standard Model (MSSM) to accommodate an additional abelian vector multiplet $V^{(0)}$ and how to cancel the anomalies with the Green-Schwarz mechanism. We assume that all the MSSM fields are charged under the additional vector multiplet $V^{(0)}$, with charges that are given in Table 1, where $Q_i, L_i$ are the left handed quarks and leptons respectively while $U^c_i, D^c_i, E^c_i$ are the right handed up and down quarks and the electrically charged leptons. The superscript $c$ stands for charge conjugation. The index $i = 1, 2, 3$ denotes the three different families. $H_u, d$ are the two Higgs scalars.

|      | SU(3)$_c$ | SU(2)$_L$ | U(1)$_Y$ | U(1)′ |
|------|-----------|-----------|----------|-------|
| $Q_i$ | 3         | 2         | 1/6      | $Q_Q$ |
| $U^c_i$ | 3       | 1         | $-2/3$   | $Q_{U^c}$ |
| $D^c_i$ | 3       | 1         | 1/3      | $Q_{D^c}$ |
| $L_i$   | 1         | 2         | $-1/2$   | $Q_L$  |
| $E^c_i$ | 1         | 1         | 1        | $Q_{E^c}$ |
| $H_u$   | 1         | 2         | 1/2      | $Q_{H_u}$ |
| $H_d$   | 1         | 2         | $-1/2$   | $Q_{H_d}$ |

Table 1: Charge assignment.

Since our model is an extension of the MSSM, the gauge invariance of the superpotential, that contains the Yukawa couplings and a $\mu$-term, put constraints on the above charges

\[
Q_{U^c} = -Q_Q - Q_{H_u} \\
Q_{D^c} = -Q_Q + Q_{H_u} \\
Q_{E^c} = -Q_L + Q_{H_u} \\
Q_{H_d} = -Q_{H_u}
\]

Thus, $Q_Q$, $Q_L$ and $Q_{H_u}$ are free parameters of the model.

2.1 Anomalies

As it is well known, the MSSM is anomaly free. All the anomalies that involve only the $SU(3)$, $SU(2)$ and $U(1)_Y$ factors vanish identically. However, triangles with $U(1)'$ in the
external legs in general are potentially anomalous. These anomalies are

\[ U(1)' - U(1)' - U(1)' \quad : \quad A^{(0)} = \sum_f Q_f^3 \quad (2.2) \]

\[ U(1)' - U(1)_Y - U(1)_Y \quad : \quad A^{(1)} = \sum_f Q_f Y_f^2 \quad (2.3) \]

\[ U(1)' - SU(2) - SU(2) \quad : \quad A^{(2)} = \sum_f Q_f \text{Tr}[T^{(2)}_{k_2} T^{(2)}_{k_2}] \quad (2.4) \]

\[ U(1)' - SU(3) - SU(3) \quad : \quad A^{(3)} = \sum_f Q_f \text{Tr}[T^{(3)}_{k_3} T^{(3)}_{k_3}] \quad (2.5) \]

\[ U(1)' - U(1)' - U(1)_Y \quad : \quad A^{(4)} = \sum_f Q_f^3 Y_f \quad (2.6) \]

where \( f \) runs over the fermions in Table 1, \( Q_f \) is the corresponding \( U(1)' \) charge, \( Y_f \) is the hypercharge and \( T^{(a)}_{k_a}, \ a = 2, 3; \ k_a = 1, \ldots, \text{dim}G^{(a)} \) are the generators of the \( G^{(2)} = SU(2) \) and \( G^{(3)} = SU(3) \) algebras respectively. In our notation \( \text{Tr}[T^{(a)}_j T^{(a)}_k] = \frac{1}{2} \delta_{jk} \). All the remaining anomalies that involve \( U(1)'s \) vanish identically due to group theoretical arguments (see Chapter 22 of [35]). Using the charge constraints (2.1) we get

\[ A^{(0)} = 3 \left\{ Q^3_{H_u} + 3Q_H Q^2_L + Q^4_L - 3Q^2_{H_u} (Q_L + 6Q_Q) \right\} \quad (2.7) \]

\[ A^{(1)} = -\frac{3}{2} (3Q_Q + Q_L) \quad (2.8) \]

\[ A^{(2)} = \frac{3}{2} (3Q_Q + Q_L) \quad (2.9) \]

\[ A^{(3)} = 0 \quad (2.10) \]

\[ A^{(4)} = -6Q_{H_u} (3Q_Q + Q_L) \quad (2.11) \]

Notice that the mixed anomaly between the anomalous \( U(1) \) and the \( SU(3) \) nonabelian factors \( A^{(3)} \) vanishes identically.

### 2.1.1 Anomalous U(1)’s and the Stückelberg mechanism

Many models have been developed in the past where all the anomalies (2.7-2.11) vanish by constraining the charges \( Q_f \) (see [1, 2] and references therein). On the contrary, in this paper we assume that the \( U(1)' \) is anomalous, i.e. (2.7)-(2.11) do not vanish. Consistency of the model is achieved by the contribution of a Stückelberg field \( S \) and its appropriate couplings to the anomalous \( U(1)' \). The Stückelberg lagrangian reads [36]

\[ \mathcal{L}_{\text{axion}} = \frac{1}{4} \left( S + S^\dagger + 4b_3 V^{(0)} \right)^2 \mid_{\theta^2 \bar{\theta}^2} \]

\[ -\frac{1}{2} \left\{ \sum_{a=0}^2 b^{(a)}_2 S \text{Tr} \left( W^{(a)} W^{(a)} \right) + b^{(4)}_2 S W^{(1)} W^{(0)} \right\} \mid_{\theta^2} + h.c. \quad (2.12) \]

where the index \( a = 0, \ldots, 3 \) runs over the \( U(1)' \), \( U(1)_Y \), \( SU(2) \) and \( SU(3) \) gauge groups respectively. The Stückelberg multiplet is a chiral superfield

\[ S = s + i\sqrt{2} \theta \psi_S + \theta^2 F_S - i\theta \sigma^\mu \bar{\theta} \partial_\mu s + \frac{\sqrt{2}}{2} \theta^2 \bar{\sigma} \partial^\mu \bar{\partial}_\mu \psi_S - \frac{1}{4} \theta^2 \bar{\theta}^2 \Box s \quad (2.13) \]
and transforms under the $U(1)'$ as
\begin{align}
V^{(0)} &\rightarrow V^{(0)} + i \left( \Lambda - \Lambda^\dagger \right) \\
S &\rightarrow S - 4i b_3 \Lambda
\end{align}
(2.14)
where $b_3$ is a constant. The lowest component of $S$ is a complex scalar field $s = \alpha + i\phi$. We assume that the real part $\alpha$ gets an expectation value by an effective potential of stringy or different origin and contributes to the coupling constants as
\begin{align}
\frac{1}{16g_a^2\tau_a} = \frac{1}{16g_a^2\tau_a} - \frac{1}{2}b_2^{(a)}\langle\alpha\rangle
\end{align}
(2.15)
where $g_a$ is the redefined coupling constant and the gauge factors $\tau_a$ take the values $1,1,1/2,1/2$.

The first line in (2.12) is gauge invariant and provides the kinetic terms and the axion-$U(1)'$ mixing. The second line is not gauge invariant and provides couplings that participate in the anomaly cancellation procedure. Notice that in (2.12) the sum over $a$ omits the $a = 3$ case since there is no mixed anomaly between the $U(1)'$ and the $SU(3)$ factors as from eq.(2.11), i.e. $b_2^{(3)} = 0$. The values of the other constants, $b_2^{(a)}$, are fixed by the anomalies.

Expanding $L_{axion}$ in component fields, using the Wess-Zumino gauge and substituting $\alpha$ by its vev we get
\begin{align}
L_{axion} &= \frac{1}{2} \left( \partial_\mu \phi + 2b_3 V^{(0)}_\mu \right)^2 + \frac{i}{4} \bar{\psi}_S \sigma^\mu \partial_\mu \bar{\psi}_S + \frac{i}{4} \bar{\psi}_S \sigma^\mu \partial_\mu \bar{\psi}_S \\
&\quad + \frac{1}{2} F_S \tilde{F}_S + 2b_3 \langle\alpha\rangle D^{(0)} - \sqrt{2}b_3 (\bar{\psi}_S \lambda^{(0)} + h.c.) \\
&\quad + \frac{1}{4} \phi \epsilon^{\mu\nu\rho\sigma} \sum_{a=0} b_2^{(a)} \text{Tr} \left( F_{\mu\nu}^{(a)} F_{\rho\sigma}^{(a)} \right) - \frac{1}{4} b_2^{(4)} \epsilon^{\mu\nu\rho\sigma} \phi F_{\mu\nu}^{(1)} F_{\rho\sigma}^{(1)} \\
&\quad + \frac{1}{2} \left( \sum_{a=0} b_2^{(a)} \right) \left[ -2\phi \text{Tr} \left( \lambda^{(a)} \sigma^\mu D_\mu \bar{\lambda}^{(a)} \right) + \frac{i}{\sqrt{2}} \text{Tr} \left( \lambda^{(a)} \sigma^\mu \sigma^\nu F_{\mu\nu}^{(a)} \right) \bar{\psi}_S \\
&\quad - F_S \text{Tr} \left( \lambda^{(a)} \lambda^{(a)} \right) \sqrt{2} \bar{\psi}_S \text{Tr} \left( \lambda^{(a)} D^{(a)} \right) \right] \\
&\quad + b_2^{(4)} \left[ \left( -\phi \lambda^{(1)} \sigma^\mu \partial_\mu \bar{\lambda}^{(0)} + i\langle\alpha\rangle \lambda^{(1)} \sigma^\mu \partial_\mu \bar{\lambda}^{(0)} \right) - \frac{1}{2} F_S \lambda^{(1)} \lambda^{(0)} \\
&\quad - \frac{1}{\sqrt{2}} \bar{\psi}_S \lambda^{(1)} D^{(0)} + \frac{i}{2\sqrt{2}} \lambda^{(1)} \sigma^\mu \sigma^\nu F_{\mu\nu}^{(0)} \bar{\psi}_S \right) + (0 \leftrightarrow 1) \right] + h.c.
\end{align}
where we omit terms which are coming from $\langle\alpha\rangle W^{(a)}W^{(a)}$, since they are absorbed in the coupling constant redefinition (2.13). This mechanism cancels some mixed anomalies and in addition provides a mass term to the anomalous $U(1)$. Therefore, the anomalous $U(1)$ behaves almost like the usual $Z'$ extensively studied in the past.
2.1.2 Generalized Chern-Simons terms

As it was pointed out in [29], the Stückelberg mechanism is not sufficient to cancel all the anomalies. Mixed anomalies between anomalous and non-anomalous factors require an additional mechanism to ensure consistency of the model: non gauge invariant Generalized Chern-Simons terms (GCS) must be added. In our case, the GCS terms have the form [37]

\[ L_{\text{GCS}} = -d_4 (V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W^{(0)}_\alpha + \text{h.c.} \]

\[ + d_5 (V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W^{(1)}_\alpha + \text{h.c.} \]

\[ + d_6 \text{Tr} \left( V^{(2)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(2)} \right) W^{(2)}_\alpha \]

\[ + \frac{1}{6} V^{(2)} D^\alpha V^{(0)} D^2 \left( [D^\alpha V^{(2)}, V^{(2)}] \right) + \text{h.c.} \] (2.17)

The constants \(d_4, d_5\) and \(d_6\) are fixed by the cancellation of the mixed anomalies. The GCS terms (2.17), expressed in component fields, are

\[ L_{\text{GCS}} = -d_4 \epsilon^{\mu\rho\sigma\tau} V^{(0)}_\mu V^{(1)}_\nu F^{(0)}_{\rho\sigma} + d_5 \epsilon^{\mu\rho\sigma\tau} V^{(0)}_\mu V^{(1)}_\nu F^{(1)}_{\rho\sigma} \]

\[ + d_6 \epsilon^{\mu\rho\sigma\tau} V^{(0)}_\mu \text{Tr} \left[ V^{(2)}_\nu F^{(2)}_{\rho\sigma} - \frac{i}{3} V^{(2)}_\nu \left[ V^{(2)}_{\rho\sigma}, V^{(2)}_\nu \right] \right] \]

\[ - d_4 \left( \lambda^{(0)} \sigma^\mu \bar{\chi}^{(0)} V^{(1)}_\mu - \lambda^{(0)} \sigma^\mu \bar{\chi}^{(1)} V^{(0)}_\mu + \text{h.c.} \right) \]

\[ + d_5 \left( \lambda^{(1)} \sigma^\mu \bar{\chi}^{(1)} V^{(0)}_\mu - \lambda^{(1)} \sigma^\mu \bar{\chi}^{(0)} V^{(1)}_\mu + \text{h.c.} \right) \]

\[ + d_6 \text{Tr} \left[ \lambda^{(2)} \sigma^\mu \bar{\chi}^{(2)} V^{(0)}_\mu - \lambda^{(2)} \sigma^\mu \bar{\chi}^{(0)} V^{(2)}_\mu + \text{h.c.} \right] \] (2.18)

These terms provide new trilinear couplings that distinguish these models from the \(Z'\) models studied in the past.

2.2 Anomaly cancellation

In the following, we illustrate the anomaly cancellation procedure both in the unbroken and broken phases by a specific example. We focus on the bosonic sector and the related diagrams, since their supersymmetric analogs are fixed by supersymmetry. The GS and GCS terms depend on unknown parameters which we fix by using the Ward identities. In theories with massive gauge bosons where the mass is acquired either by the Higgs or by the Stuckelberg mechanism, Ward identities have the following diagrammatic form [38]

\[ -ik^\mu \left( V^\mu(k) \right)_{1PI} + m_V \left( G_V(k) \right)_{1PI} = 0 \] (2.19)

where \(V_\mu\) is the massive gauge field, \(G_V\) is the corresponding Higgs or Stückelberg field (or a linear combination of them) and \(m_V\) is the coupling of the term \(V^\mu \partial_\mu G_V\). The blob denotes all the 1PI diagrams.
\[(p+q)\rho \left( V^{(0)}_\rho (p+q) + V^{(1)}_\rho (p) \right) + V^{(0)} + 2ib_3 \left( \phi \right) = 0 \]

\[p^\mu \left( V^{(0)} + V^{(1)} \right) = 0 \]

\[q^\nu \left( V^{(0)} + V^{(1)} \right) = 0 \]

Figure 1: The Ward identities for the amplitude \( V^{(0)}_\rho (p+q) \rightarrow V^{(1)}_\mu (p) V^{(1)}_\nu (q) \) in the unbroken phase include the GCS as well as the axionic couplings. The solid lines represent fermions and the wiggle lines are gauge fields. Dashed lines are scalars. Each depicted diagram also contains the exchange \((\mu, p) \leftrightarrow (\nu, q)\).

### 2.2.1 Anomaly cancellation in the symmetric phase

In our model there are two extra states in the neutral fermionic sector, namely the axino and the primeino (see Section 3.7) which do not contribute to the fermionic loop. The remaining MSSM fermionic states are a bino, a wino and the two higgsinos. Both \(U(1)_Y\) and \(SU(2)\) gauginos do not contribute to the fermionic loop due to group theoretical arguments (see Section 28.1 of [39]). The higgsino eigenstates do not participate because the \(\tilde{H}_u\) contribution is cancelled by the \(\tilde{H}_d\) one. This is due to the fact that each diagram is proportional to an odd product of charges and the two higgsinos have opposite charges (see Table [4] and the constraints (2.1)). Without loss of generality, we assume that the mixed anomaly between \(V^{(0)}\) and two \(V^{(1)}\) is non vanishing, therefore from eq. (2.3) \[A^{(1)} = \sum_f Q_f (Y_f)^2 \neq 0.\] In order to cancel the anomaly, we have to satisfy the Ward identities which are shown, in diagrammatic form, in Fig. 1. The total fermionic triangle is given by

\[\Delta^{011}_{\rho \mu \nu} (p, q; 0) = -\frac{1}{16} \sum_f Q_f (Y_f)^2 \Gamma_{\rho \mu \nu} (p, q; 0) = -\frac{A^{(1)}}{16} \Gamma_{\rho \mu \nu} (p, q; 0) \]  

The superscript indices in the l.h.s. stand for the gauge groups of the vector fields involved
in the process. $\Gamma_{\rho\mu\nu}(p, q; 0)$ can be parametrized as in \((C.7)\). For a symmetric distribution of the anomaly (see Appendix \(C.2)\), we have

\[
(p + q)^\rho \Delta^0_{\rho\mu\nu}(p, q; 0) = \frac{1}{32\pi^2} \frac{A^{(1)}_1}{2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta
\]

\[
p^\mu \Delta^0_{\rho\mu\nu}(p, q; 0) = \frac{1}{32\pi^2} \frac{A^{(1)}_1}{2} \epsilon_{\nu\rho\alpha\beta} q^\alpha p^\beta
\]

\[
q^\nu \Delta^0_{\rho\mu\nu}(p, q; 0) = \frac{1}{32\pi^2} \frac{A^{(1)}_1}{2} \epsilon_{\rho\mu\alpha\beta} q^\alpha p^\beta
\]

Denoting by

\[
(GS)^{11}_{\mu\nu} = -2ib^{(1)}_2 \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta
\]

the axion interaction vertex and by

\[
(GCS)^{011}_{\rho\mu\nu} = 2d_5 \epsilon_{\rho\nu\mu\alpha} (p - q)^\alpha
\]

the GCS coupling, the Ward identities in Fig. 1 correspond to

\[
(p + q)^\rho \left( \Delta^0_{\rho\mu\nu}(p, q; 0) + (GCS)^{011}_{\rho\mu\nu} \right) + 2ib_3 (GS)^{11}_{\mu\nu} = 0
\]

\[
p^\mu \left( \Delta^0_{\rho\mu\nu}(p, q; 0) + (GCS)^{011}_{\rho\mu\nu} \right) = 0
\]

\[
q^\nu \left( \Delta^0_{\rho\mu\nu}(p, q; 0) + (GCS)^{011}_{\rho\mu\nu} \right) = 0
\]

They fix the parameters

\[
b^{(1)}_2 b_3 = \frac{A^{(1)}_1}{128\pi^2} \quad d_5 = \frac{A^{(1)}_1}{192\pi^2}
\]

In the same way, the cancellation of the remaining mixed anomalies gives

\[
b^{(0)}_2 b_3 = \frac{A^{(0)}_1}{384\pi^2} \quad b^{(2)}_2 b_3 = \frac{A^{(2)}_1}{64\pi^2} \quad b^{(4)}_2 b_3 = \frac{A^{(4)}_1}{128\pi^2}
\]

\[
d_4 = \frac{A^{(4)}_1}{384\pi^2} \quad d_6 = \frac{A^{(2)}_1}{96\pi^2}
\]

It is worth noting that the GCS coefficients $d_{4,5,6}$ are fully determined in terms of the $A$’s by the Ward identities, while the $b^{(a)}_2$’s depend only on the free parameter $b_3$, which is related to the mass of the anomalous $U(1)$.

### 2.2.2 Anomaly cancellation in the broken phase

It is interesting to study the anomaly cancellation procedure in the broken phase. Focusing again onto the non-vanishing $A^{(1)}_1 \neq 0$, the amplitudes that contribute to the cancellation of the anomaly are given in Fig. 2, where $m_0 = Q_{\mu\nu} |v|/2$ and $m_1 = |v|/4$ with $|v| = \sqrt{v_1^2 + v_2^2}$. In the broken phase, additional contributions coming from the NG boson exchange must be added. We denote by $\Delta^0_{\rho\mu\nu}(p, q; m_f)$ the modified triangle diagram where also massive fermions circulate in the loop and by $(NG)_{\rho\mu\nu}$ the triangle diagram with a NG boson on an external leg. Note that $(GS)_{\rho\mu\nu}$ and $(GCS)_{\rho\mu\nu}$ are the same as in the unbroken phase.
The amplitude satisfies again the usual Ward identities (2.24). In order to clarify the mechanism, we will focus on a single Ward identity

\[(p + q)^\mu \left( \begin{array}{c} \Delta_{\rho\mu
u}^{001}(p, q; m_f) + \mathcal{GCS}_{\rho\mu
u}^{001} + 2ib_3(GS)_{\mu
u}^{11} + im_0(NG)_{\mu
u}^{11} = 0 \end{array} \right) \]

From now on the \((p, q; m_f)\) dependence will be explicit only when needed. Splitting \(\Delta\) and \((NG)\) terms into the sums over SM fermions and higgsinos we obtain

\[\Delta_{\rho\mu
u}^{001} = \Delta_{\rho\mu
u}^{001}|_{SM} + \Delta_{\rho\mu
u}^{001}|_{\tilde{H}_{u,d}} \]

\[(NG)_{\mu
u}^{11} = (NG)_{\mu
u}^{11}|_{SM} + (NG)_{\mu
u}^{11}|_{\tilde{H}_{u,d}} \]

Since we have

\[(p + q)^\rho \Delta_{\rho\mu
u}^{001}|_{SM} = \frac{1}{48\pi^2} \sum_{f \in SM} \left[ \frac{1}{2} t_f^{011} + t_f^{NG11} m_f^2 I_0 \right] \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \]

Figure 2: The Ward identities for the amplitude \(V^{(0)}_\rho(p+q) \rightarrow V^{(1)}_\mu(p) V^{(1)}_\nu(q)\) in the broken phase.
Table 2: Definition of $t_f^{011}$ and $t_f^{NG11}$, where $N_c = 3$ is the number of colours.

where the integral $I_0$ is

$$I_0(p, q; m_f) = - \int_0^1 dx \int_0^{1-x} dy \frac{1}{y(1-y)p^2 + x(1-x)q^2 + 2xy p \cdot q - m_f^2}$$ (2.31)

and $t_f^{011}$, $t_f^{NG11}$ are defined in Table 2, the Ward identity of the SM fermionic loop has a new contribution due to the masses of the fermions. Similarly, for the corresponding $NG$ term we get

$$im_0 (NG)_{\mu\nu}^{11}|_{SM} = - \frac{1}{48\pi^2} \sum_{f \in SM} [t_f^{NG11} m_f^2 I_0] \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta$$ (2.32)

Summing (2.30, 2.32), the massive contribution in the fermionic loop is exactly cancelled by the $NG$ ones, giving

$$[(p + q)^2 \Delta_{\mu\nu}^{011}(p, q; m_f) + im_0 (NG)_{\mu\nu}^{11}]_{SM} = \frac{1}{96\pi^2} \sum_{f \in SM} t_f^{011} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta$$

$$= (p + q)^2 \Delta_{\mu\nu}^{011}(p, q; 0)|_{SM}$$ (2.33)

The contribution of the diagrams involving the higgsinos vanishes

$$[(p + q)^2 \Delta_{\mu\nu}^{011}(p, q; m_f) + im_0 (NG)_{\mu\nu}^{11}]_{H_u,d} = \frac{1}{96\pi^2} \sum_{f \in H_u,d} Q_f Y_f^2 \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta = 0$$ (2.34)

Summing (2.33, 2.34) we get

$$[(p + q)^2 \Delta_{\mu\nu}^{011}(p, q; m_f) + im_0 (NG)_{\mu\nu}^{11}] = \frac{A^{(1)}}{96\pi^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta = (p + q)^2 \Delta_{\mu\nu}^{011}(p, q; 0)$$ (2.35)

Thus the contribution to the Ward Identities of the triangle diagrams is exactly the same as in the unbroken phase.

2.3 Soft breaking terms

The total soft breaking lagrangian can be written as

$$\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} + \mathcal{L}_{soft}^{new}$$ (2.36)
with
\[
\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \sum_{a=1}^{3} \left( M_a \lambda^{(a)} \lambda^{(a)} + h.c. \right) - \left( m_{\tilde{Q}_j} \tilde{Q}_j \tilde{Q}_j^\dagger + m_{\tilde{U}_j} \tilde{U}_j \tilde{U}_j^\dagger + m_{\tilde{D}_j} \tilde{D}_j \tilde{D}_j^\dagger \right) \\
+ m_{L_j} \tilde{L}_j \tilde{L}_j^\dagger + m_{E_j} \tilde{E}_j \tilde{E}_j^\dagger + m_{h_u} |h_u|^2 + m_{h_d} |h_d|^2 \right) \\
- \left( a_{ij} \tilde{Q}_i \tilde{U}_j^\dagger h_u - a_{ij} \tilde{Q}_i \tilde{D}_j^\dagger h_d - a_{ij} \tilde{L}_i \tilde{E}_j^\dagger h_d + b h_u h_d + h.c. \right) \quad (2.37)
\]
and
\[
\mathcal{L}_{\text{soft}}^{\text{new}} = -\frac{1}{2} \left( M_0 \lambda^{(0)} \lambda^{(0)} + h.c. \right) - \frac{1}{2} (M_S \psi_S \psi_S + h.c.) \quad (2.38)
\]
where \( \lambda^{(0)} \) is the gaugino of the added \( U(1)' \) and \( \psi_S \) is the axino. We allow a soft mass term for the axino since it couples only through GS interactions and not through Yukawa interactions [40]. Notice also that a mass term for the axion \( \phi \) is not allowed since it transforms non-trivially under the anomalous \( U(1)' \) gauge transformation (2.14).

3. Model setup

In this Section we analyze the effects of the additional terms on the rest of the lagrangian.

3.1 Kinetic diagonalization of \( U(1)' \)’s

As we mentioned before, the St"uckelberg multiplet contains a complex scalar field whose real part gets an expectation value that modifies the coupling constant (2.13). Therefore, the second line in (2.12) contributes to the kinetic terms for the gauge fields and the term \( \langle \alpha \rangle W^{(1)} W^{(0)} \) gives a kinetic mixing between the \( V^{(1)} \) and \( V^{(0)} \) gauge bosons. Redefining as usual \( V^{(0)} \to 2g_0 V^{(0)} \), \( V^{(1)} \to 2g_1 V^{(1)} \) we get
\[
\left( \frac{1}{4} W^{(0)} W^{(0)} + \frac{1}{4} W^{(1)} W^{(1)} + \frac{\delta}{2} W^{(1)} W^{(0)} \right) \bigg|_{\theta^2} \quad (3.1)
\]
with \( \delta = -4b_2^{(4)} g_0 g_1 \langle \alpha \rangle \). In order to diagonalize the kinetic terms, we use the matrix
\[
\begin{pmatrix}
V^{(0)} \\
V^{(1)}
\end{pmatrix} = \begin{pmatrix}
C_\delta & 0 \\
-S_\delta & 1
\end{pmatrix}
\begin{pmatrix}
V_C \\
V_B
\end{pmatrix} \quad (3.2)
\]
where \( C_\delta = 1/\sqrt{1-\delta^2} \) and \( S_\delta = \delta C_\delta \). Let us stress that in this case the mixing is a consequence of the anomaly cancellation procedure. Note that, since \( b_2^{(4)} \sim b_3^{-1} \sim M_{V^{(0)}}^{-1} \) (see eq. (2.26)), where \( M_{V^{(0)}} \) is the mass of the anomalous \( U(1) \) that we assume to be in the TeV range, this mixing is tiny and can be ignored for our purposes.

3.2 D and F terms

The additional fields give rise also to D and F terms. More precisely, D term contributions come from: (i) the kinetic terms of chiral multiplets and (ii) the axionic lagrangian,
Similarly, the F term contributions are

\[ \mathcal{L}_D = \frac{1}{2} \sum_{a=0}^{3} D^{(a)}_a D^{(a)}_a + \sum_{a=0}^{3} g_a D^{(a)}_a z_i (T^{(a)}_{ka})^i_j z^j + 4g_0 b_3 \langle \alpha \rangle D^{(0)} + \delta D^{(1)} D^{(0)} + \]

\[ + 2 \left[ \sum_{a=0}^{2} g_a^2 b_2^4 \sqrt{2} \psi_S \text{Tr} \left( \lambda^{(a)} D^{(a)} \right) + g_0 g_1 \frac{b_2^4}{\sqrt{2}} \psi_S \left( \lambda^{(1)} D^{(0)} + \lambda^{(0)} D^{(1)} \right) + \text{h.c.} \right] \]

where \( a = 0, 1, 2, 3 \) denotes, as usual, the gauge group factors, \( z_i \) are the lowest components of the \( i \)-th chiral multiplet (except the multiplet which contains the axion) and \( T^{(a)}_{ka} \), \( k_a = 1, \ldots, \text{dimG}^{(a)} \), are the generators of the corresponding gauge groups, \( G^{(a)} \). Solving the equations of motion for the D’s and substituting back we obtain

\[ \mathcal{L}_{DC} = -\frac{1}{2} \left\{ C_0 g_0 \sum_f Q_f |z_f|^2 - S_0 g_1 \sum_f Y_f |z_f|^2 \right\} + C_0 4g_0 b_3 \langle \alpha \rangle \]

\[ + 2 \sqrt{2} b_2^0 g_2 \left[ \psi_S (C_0^2 \lambda_C) + \text{h.c.} \right] + 2 \sqrt{2} b_2^1 g_1^2 \left[ \psi_S (S_0^2 \lambda_C - S_0 \lambda_B) + \text{h.c.} \right] \]

\[ + \sqrt{2} b_2^4 g_0 g_1 \left[ \psi_S (C_0 \lambda_B - 2C_0 \lambda_B) + \text{h.c.} \right] \]

\[ \mathcal{L}_{DB} = -\frac{1}{2} \left\{ g_1 \sum_f Y_f |z_f|^2 + 2 \sqrt{2} b_2^1 g_1^2 \left[ \psi_S (\lambda_B - S_0 \lambda_C) + \text{h.c.} \right] \right\} \]

\[ + \sqrt{2} b_2^4 g_0 g_1 \left[ \psi_S C_0 \lambda_C + \text{h.c.} \right] \]

\[ \mathcal{L}_{D(2)} = -\frac{1}{2} \sum_k \left\{ g_2 z_i^+ (T^{(2)}_k)^i_j z^j \right\} + b_2^2 g_2 \left[ \sqrt{2} \psi_S (T^{(2)}_k) + \text{h.c.} \right] \]

\[ \mathcal{L}_{D(3)} = -\frac{1}{2} \sum_k \left\{ g_3 z_i^+ (T^{(3)}_k)^i_j z^j \right\} \]

Similarly, the F term contributions are

\[ \mathcal{L}_F = \sum_{f \in \text{MSSM}} \left( F^f F^+_f - \frac{\partial W}{\partial z^f} F^f - \frac{\partial W^+_f}{\partial z^+_f} F^+_f \right) \]

\[ + \frac{1}{2} F_S F^+_S + \frac{1}{2} \left\{ F_S \left[ \sum_{a=0}^3 b_2^{(a)} \text{Tr} \left( \lambda^{(a)} \lambda^{(a)} \right) + b_2^{(4)} \lambda^{(1)} \lambda^{(0)} \right] + \text{h.c.} \right\} \]

where the first line is the standard MSSM F term contribution while the second line contains the new axionic terms. Solving the EOM, and rescaling \( V \rightarrow 2gV \) we get

\[ \mathcal{L}_{FS} = -8 \left( \sum_a b_2^{(a)} g_2^2 \text{Tr} \left( \lambda^{(a)} \lambda^{(a)} \right) + g_1 g_0 b_2^{(4)} \lambda^{(1)} \lambda^{(0)} \right) \]

\[ \times \left[ \sum_a b_2^{(a)} g_2^2 \text{Tr} \left( \lambda^{(a)} \lambda^{(a)} \right) + g_1 g_0 b_2^{(4)} \lambda^{(1)} \lambda^{(0)} \right] \]
Eq. (3.3) can also be written in the basis (3.2), but we will not need this term in the following.

We would like to mention that no D and F terms are coming from the GCS since they include only vector multiplets in an antisymmetric form. Our results are in accordance with [31].

3.3 Scalar potential

As we have seen in the previous section, the additional F terms (3.9) do not give any contribution to the scalar potential. The D, B and D terms (see eq. (3.5), (3.6) and (3.7)) provide the usual contributions to the MSSM potential. The only new contribution comes from the first line of (3.4). Thus the scalar potential can be written as

\[
V = V_{\text{MSSM}} + V_{DC} \tag{3.10}
\]

\[
V_{DC} = \frac{1}{2} \left\{ \left[ C_\delta g_0 \sum_f Q_f |z_f|^2 - S_\delta g_1 \sum_f Y_f |z_f|^2 \right] + C_\delta 4 g_0 b_3 \langle \alpha \rangle \right\}^2 \tag{3.11}
\]

Solving the equations for the minima of the potential

\[
\frac{\partial V}{\partial z_f} = 0 \tag{3.12}
\]

we get \( \langle z_f \rangle = 0 \) for all the sparticles as in the MSSM case. Inserting back these vevs into (3.10) we get the following Higgs scalar potential

\[
V_h = \left\{ |\mu|^2 + m_{h_u}^2 + 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \right\} \left( |h_u|^2 + |h_d|^2 \right) + \left\{ |\mu|^2 + m_{h_d}^2 - 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \right\} \left( |h_d|^2 + |h_d|^2 \right) + \frac{1}{2} \left( g_0 X_\delta \right)^2 + \frac{1}{8} (g_1^2 + g_2^2) \left( |h_u| + |h_u| - |h_d|^2 \right)^2 + \left\{ b (h_u^0 h_d^0 - h_u^0 h_d^0) + h.c. \right\} + \frac{1}{2} g_0^2 |h_u^0 h_d^0 + h_u^0 h_d^0|^2 \tag{3.13}
\]

which can be brought to the same form of the MSSM potential, after the following redefinitions

\[
m_{h_u}^2 + 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \to \tilde{m}_{h_u}^2
\]

\[
m_{h_d}^2 - 4g_0^2 b_3 \langle \alpha \rangle C_\delta X_\delta \to \tilde{m}_{h_d}^2
\]

\[
\left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2 \to \tilde{m}_Z^2 / 2 \tag{3.14}
\]

where

\[
g_0 X_\delta = C_\delta g_0 Q_{H_u} - \frac{1}{2} S_\delta g_1 \tag{3.15}
\]

At the minimum, we recover the MSSM result \( \langle h_u^+ \rangle = \langle h_d^- \rangle = 0 \) for the Higgs charged components. Defining \( \langle h_i^0 \rangle = v_i / \sqrt{2} \), \( v_u^2 + v_d^2 = v^2 \) and \( v_u / v_d = \tan \beta \) we can still write the tree level conditions for the electroweak symmetry breaking as

\[
b^2 > (|\mu|^2 + \tilde{m}_{h_u}^2) \left( |\mu|^2 + \tilde{m}_{h_d}^2 \right) \tag{3.16}
\]

\[
2b < 2 |\mu|^2 + \tilde{m}_{h_u}^2 + \tilde{m}_{h_d}^2 \tag{3.17}
\]
in complete analogy with the MSSM case (using \( \tilde{m} \)'s).

3.4 Higgs sector

It is worth noting that in our model there is no axi-higgs mixing. This is due to the fact that we do not consider scalar potential terms for the axion (on the contrary to [13]).

After the electroweak symmetry breaking we have four gauge generators that are broken, so we have four longitudinal degrees of freedom. One of them is the axion, while the other three are the usual NG bosons coming from the Higgs sector.

As it was mentioned above, the potential has the standard MSSM form, upon the redefinitions (3.14). The Higgs scalar fields consist of two complex \( SU(2)_L \)-doublets, or eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be NG bosons \( G^0, G^\pm \). The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars \( h^0 \) and \( H^0 \), one CP-odd neutral scalar \( A^0 \) and a charge +1 scalar \( H^+ \) as well as its charge conjugate \( H^- \) with charge \(-1\). The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as

\[
\begin{pmatrix}
  h_u^0 \\ h_d^0
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\beta_0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \tag{3.18}
\]

\[
\begin{pmatrix}
  h_u^+ \\ h_d^-
\end{pmatrix} = R_{\beta_\pm} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \tag{3.19}
\]

where the orthogonal rotation matrices \( R_\alpha, R_{\beta_0}, R_{\beta_\pm} \) are the same as in [32]. Acting with these matrices on the gauge eigenstate fields we obtain the diagonal mass terms. Expanding around the minimum (3.18) one finds that \( \beta_0 = \beta_\pm = \beta \), and replacing the tilde parameters (3.14) we obtain the masses

\[
m_{A^0}^2 = 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2 \tag{3.20}
\]

\[
m_{h^0, H^0}^2 = \frac{1}{2} \left\{ m_{A^0}^2 + \left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2 \right.
\]

\[
+ \left[ \left( m_{A^0}^2 - \left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2 \right)^2 \right]
\]

\[
+ \left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2 m_{A^0}^2 \sin^2(2\beta) \right\}^{1/2} \tag{3.21}
\]

\[
m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 = m_{A^0}^2 + g_2^2 \frac{v^2}{4} \tag{3.22}
\]

and the mixing angles

\[
\sin 2\alpha = -\frac{m_{H^0}^2 + m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2} \sin 2\beta
\]

\[
\tan 2\alpha = \frac{m_{A^0}^2 + \left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2}{m_{A^0}^2 - \left( \frac{1}{2} (g_0 X_\delta)^2 + \frac{1}{8} (g_1^2 + g_2^2) \right) v^2} \tag{3.23}
\]

\[\text{2We define } G^- = G^{++} \text{ and } H^- = H^{++}. \text{ Also, by convention, } h^0 \text{ is lighter than } H^0.\]
Notice that only the $h^0$ and $H^0$ masses get modified with respect to the MSSM, due to the additional anomalous $U(1)'$.

### 3.5 Neutral Vectors

There are two mass-sources for the gauge bosons: (i) the St"{u}ckelberg mechanism and (ii) the Higgs mechanism. In this extension of the MSSM, the mass terms for the gauge fields are given by

$$
\mathcal{L}_M = \frac{1}{2} \left( C_\mu B_\mu V_{3\mu}^{(2)} \right) M^2 \left( \begin{array}{c} C_\mu \\ B_\mu \\ V_{3\mu}^{(2)} \end{array} \right) \quad (3.24)
$$

$C_\mu, B_\mu$ are the lowest components of the vector multiplets $V_C, V_B$. The gauge boson mass matrix is

$$
M^2 = \begin{pmatrix}
M_C^2 & g_0 g_1 \frac{v^2}{\sqrt{g_1^2 + g_2^2}} X_\delta & -g_0 g_2 \frac{v^2}{\sqrt{g_1^2 + g_2^2}} X_\delta \\
& g_1 v^2 & -g_1 g_2 \frac{v^2}{\sqrt{g_1^2 + g_2^2}} \\
& & g_2 v^2
\end{pmatrix} \quad (3.25)
$$

where $M_C^2 = 16 g_0^2 b_3 C_\delta^2 + g_0^2 v^2 X_\delta^2$ and the lower dots denote the obvious terms under symmetrization. After diagonalization, we obtain the eigenstates

$$
A_\mu = \frac{g_2 B_\mu + g_1 V_{3\mu}^{(2)}}{\sqrt{g_1^2 + g_2^2}} \quad (3.26)
$$

$$
Z_0^\mu = \frac{g_2 A_3^\mu - g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}} + g_0 Q_{H_u} \frac{\sqrt{g_1^2 + g_2^2} v^2}{2M_{V(0)}^2} C^\mu + \mathcal{O}[g_0^3, M_{V(0)}^{-3}] \quad (3.27)
$$

$$
Z'^\mu = C^\mu + \frac{g_0 Q_{H_u} v^2}{2M_{V(0)}^2} (g_1 B^\mu - g_2 A_3^\mu) + \mathcal{O}[g_0^3, M_{V(0)}^{-3}] \quad (3.28)
$$

and the corresponding masses

$$
M_{Z_0}^2 = 0 \quad (3.29)
$$

$$
M_{Z_0}^2 = \frac{1}{4} (g_1^2 + g_2^2) v^2 - (Q_{H_u})^2 \left( \frac{g_1^2 + g_2^2}{4M_{V(0)}^2} \right) + \mathcal{O}[g_0^3, M_{V(0)}^{-3}] \quad (3.30)
$$

$$
M_{Z'}^2 = M_{V(0)}^2 + g_0^2 \frac{(Q_{H_u})^2}{4M_{V(0)}^2} \left( 1 + \frac{g_1^2 v^2 + g_2^2 v^2}{4M_{V(0)}^2} \right) - \frac{(\alpha) g_0^2 A_3^2}{64\pi^2 M_{V(0)}} v^2 + \mathcal{O}[g_0^3, M_{V(0)}^{-3}] \quad (3.31)
$$

where $M_{V(0)} = 4b_3g_0$ is the mass parameter for the anomalous $U(1)$ and it is assumed to be in the TeV range. Due to their complicated form, the eigenstates and eigenvalues of $M^2$ (3.25) are expressed as power expansions in $g_0$ and $1/M_{V(0)}$ keeping only the leading terms. Higher terms are denoted by $\mathcal{O}[g_0^3, M_{V(0)}^{-3}]$.

The first eigenstate (3.26) corresponds to the photon and it is exact to all orders. It slightly differs from the usual MSSM expression due to the kinetic mixing between $V^{(0)}$ and $V^{(1)}$.

For the rest of the paper, we neglect the kinetic mixing contribution since they are higher loop effects which go beyond the scope of the present paper. Then the rotation
matrix from the hypercharge to the photon basis, up to $\mathcal{O}[g_0^3, M_{V(0)}^{-3}]$ is

$$
\begin{pmatrix}
Z'_{\mu} \\
Z_{0\mu} \\
A_{\mu}
\end{pmatrix}
= O_{ij}
\begin{pmatrix}
V_{\mu}^{(0)} \\
V_{\mu}^{(1)} \\
V_{3\mu}^{(2)}
\end{pmatrix}
$$

(3.32)

$$
= \begin{pmatrix}
1 & g_1 g_0 Q_{Hu} v^2 & -g_2 g_0 Q_{Hu} v^2 \\
g_0 Q_{Hu} \sqrt{g_1^2 + g_2^2} v^2 & \frac{g_1}{\sqrt{g_1^2 + g_2^2}} & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \\
0 & \frac{g_2}{\sqrt{g_1^2 + g_2^2}} & \frac{g_1}{\sqrt{g_1^2 + g_2^2}}
\end{pmatrix}
\begin{pmatrix}
V_{\mu}^{(0)} \\
V_{\mu}^{(1)} \\
V_{3\mu}^{(2)}
\end{pmatrix}
$$

where $i, j = 0, 1, 2$.

### 3.6 Sparticles

In general, the contributions to the sparticle masses are coming from (i) the D and F terms in the superpotential and (ii) the soft-terms. However, in our case, the new contribution comes only from the $D_C$ terms

$$
V_{mass}^{DC} = \left\{ \left( C_0 g_0 Q_{Hu} + \frac{1}{2} S_0 g_1 \right) \left( \frac{v_u^2 - v_d^2}{2} \right) + 4 C_0 g_0 b_3 (\alpha) \right\} \sum_f (C_0 g_0 Q_f - S_0 g_1 Y_f) |y_f|^2
$$

(3.33)

where $y_f$ stand for all possible sparticles.

### 3.7 Neutralinos

With respect to the MSSM, now we have two new fields: $\psi_S$ and $\lambda^{(0)}$. Thus, we have

$$
\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} (\psi^0)^T M_N \psi^0 + h.c.
$$

(3.34)

where

$$
(\psi^0)^T = (\psi_S, \lambda_C, \lambda_B, \lambda^{(2)}, \tilde{e}_d^0, \tilde{e}_u^0)
$$

(3.35)

The neutralino mass matrix $M_N$ gets contributions from (i) the MSSM terms, (ii) the $h - \tilde{h} - \lambda^{(0)}$ couplings, (iii) the new soft-breaking terms $L_{soft}^{new}$, (iv) the St"uckelberg action and (v) the D terms. Finally, we obtain the symmetric matrix

$$
M_N = \begin{pmatrix}
M_S & m_{SC} & m_{SB} & \frac{2}{\sqrt{2} g_2} y^{(2)}_2 \Delta v^2 & 0 & 0 \\
\ldots & M_0 C^2_\delta + M_1 S^2_\delta & -M_1 S_\delta & 0 & -g_0 v_u X_\delta & g_0 v_u X_\delta \\
\ldots & \ldots & M_1 & 0 & -\frac{g_1 v_u}{2} & \frac{g_1 v_u}{2} \\
\ldots & \ldots & \ldots & M_2 & \frac{g_2 v_u}{2} & -\frac{g_2 v_u}{2} \\
\ldots & \ldots & \ldots & \ldots & 0 & -\mu \\
\ldots & \ldots & \ldots & \ldots & \ldots & 0
\end{pmatrix}
$$

(3.36)
Table 3: Couplings of the SM fermions with the neutral gauge bosons.

| $q_f$   | $v_f^{Z_0}$ | $a_f^{Z_0}$ | $v_f^{Z'}$ | $a_f^{Z'}$ |
|---------|-------------|-------------|-------------|-------------|
| $e, \mu, \tau$ | 0 | 1/2 | 1/2 | $Q_L$ | $Q_L$ |
| $u, c, t$ | -1 | $-1/2 + 2\sin^2 \theta_W$ | $-1/2$ | $2Q_L$ | 0 |
| $d, s, b$ | 2/3 | $1/2 - 4/3\sin^2 \theta_W$ | 1/2 | $2Q_Q$ | 0 |

where $m_{SC}$ and $m_{SB}$ are the gaugino masses coming from the soft breaking terms (2.37), and

$$m_{SC} = \sqrt{2} \left\{ 2 \left( C_0 g_0^{b(0)} + S_0 g_1^{b(1)} - C_0 S_0 g_0 g_1^{b(4)} \right) (g_0 X_0 \Delta v^2 + C_0 M_{V(0)} \alpha) \\
+ \frac{1}{2} \left( -2 S_0 g_1^{b(1)} + C_0 g_0 g_1^{b(4)} \right) g_1 \Delta v^2 + \frac{C_0}{2} M_{V(0)} \right\}$$

$$m_{SB} = \sqrt{2} \left\{ \left( C_0 g_0 g_1^{b(4)} - 2 S_0 g_1^{b(1)} \right) (g_0 X_0 \Delta v^2 + C_0 M_{V(0)} \alpha) + b_2^{(1)} g_1^3 \Delta v^2 \right\}$$

with $\Delta v^2 = v_u^2 - v_d^2$. It is worth noting that the D terms and kinetic mixing terms are only higher order corrections and they can be neglected in the computations of the eigenvalues and eigenstates.

4. Phenomenology

In this Section we compute the amplitudes for the decays $Z' \to Z_0 \gamma$ and $Z' \to Z_0 Z_0$ focusing for simplicity on the case $Q_{H_u} = 0$. In this case there is no mixing between the $V^{(0)}$ and the other SM gauge fields therefore $Z' = V^{(0)}$ (see (3.32)). Notice also that neutralino and chargino contributions to the fermionic triangles identically vanish, giving the same results, for what the decays of interest are concerned, of non-SUSY models. In Table 3 we list all the couplings of the SM fermions with the neutral gauge bosons where $q_f$ denote the electric charges, $v_f^{Z_0}$ and $a_f^{Z_0}$ are the vectorial and axial couplings with $Z_0$ and $v_f^{Z'}$ and $a_f^{Z'}$ are the vectorial and axial couplings with $Z'$, respectively (see also (D.2)).

4.1 $Z' \to Z_0 \gamma$

We compute all the relevant diagrams in the $R_\xi$ gauge, thus removing the interaction vertex $V^\mu \partial_\mu G_V$ that involves the massive gauge bosons and the St"uckelberg or NG boson. Therefore, the only diagrams that remain are the fermionic loop, the GCS vertex and a not anomalous remnant contribution (Fig. 3). It is possible to show that the last blob-diagram, that involves several diagrams, is equal to zero. For the interested reader we give further details in appendix C.3.

The decay rate for the process is given by

$$\Gamma (Z' \to Z_0 + \gamma) = \frac{p_F}{32\pi^2 M_{Z'}} \left| ATOT \right|^2 d\Omega$$

(4.1)

---

\textsuperscript{3}We would like to acknowledge T. Tomaras for discussions on this point.
Figure 3: Diagrams for $Z' \rightarrow Z_0 \gamma$.

where $A_{\text{TOT}}$ is the total scalar amplitude and $p_F$ is the momentum of the outgoing vectors in the CM frame

$$p_F = \frac{M_{Z'}}{2} \left( 1 - \frac{M^2_{Z_0}}{M^2_{Z'}} \right)$$ (4.2)

The square of the total scalar amplitude is given by

$$|A_{\text{TOT}}|^2 = \frac{1}{3} \sum_{\lambda'} \epsilon^{\rho_1}_{(\lambda')} \epsilon^{\ast \rho_2}_{(\lambda')} \sum_{\lambda_0} \epsilon^{\nu_1}_{(\lambda_0)} \epsilon^{\ast \nu_2}_{(\lambda_0)} \sum_{\lambda_\gamma} \epsilon^{\mu_1}_{(\lambda_\gamma)} \epsilon^{\ast \mu_2}_{(\lambda_\gamma)} A_{\rho_1 \mu_1 \nu_1} A_{\rho_2 \mu_2 \nu_2}$$ (4.3)

where $\epsilon$ are the polarizations of the gauge bosons, and $A_{\rho\mu\nu}$ is the Feynman amplitude of the process. The factor $1/3$ comes from the average over the $Z'$ helicity states. The polarizations obey to the following completeness relations

$$\sum_{\lambda'} \epsilon^{\rho_1}_{(\lambda')} \epsilon^{\ast \rho_2}_{(\lambda')} = -\eta^{\rho_1 \rho_2} + \frac{k^{\rho_1}_{(\lambda')} k^{\rho_2}_{(\lambda')}}{M^2_{Z'}}$$ (4.4)

$$\sum_{\lambda_0} \epsilon^{\nu_1}_{(\lambda_0)} \epsilon^{\ast \nu_2}_{(\lambda_0)} = -\eta^{\nu_1 \nu_2} + \frac{k^{\nu_1}_{(\lambda_0)} k^{\nu_2}_{(\lambda_0)}}{M^2_{Z_0}}$$ (4.5)

$$\sum_{\lambda_\gamma} \epsilon^{\mu_1}_{(\lambda_\gamma)} \epsilon^{\ast \mu_2}_{(\lambda_\gamma)} = -\eta^{\mu_1 \mu_2}$$ (4.6)

where (4.6) gives only the relevant part of the sum over helicities. Other terms are omitted since they give vanishing contributions to the decay.

The amplitude is given by the sum of the fermionic triangle $\Delta^Z_{\rho\mu\nu}$ plus the proper GCS vertex

$$A_{\rho\mu\nu}^{Z'Z_0\gamma} = \Delta^Z_{\rho\mu\nu} + (\text{GCS})_{\rho\mu\nu}^{Z'Z_0\gamma}$$

$$\Delta^Z_{\rho\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4} g_0 g_{Z_0} e \sum_f v_f^Z a_f^Z q_f \Gamma^{VAV}_{\rho\mu\nu}(p, q; m_f) + (p \leftrightarrow q, \mu \leftrightarrow \nu)$$ (4.7)
where $\Gamma_{\mu\nu}(p, q; m_f)$ is given by (C.2). It is convenient to express the triangle amplitude by using the Rosenberg parametrization [41]

$$
\Delta_{\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4\pi^2} g_0 g_{Z_0} e \left( A_1 \epsilon[p, \mu, \nu, \rho] + A_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p_\nu + A_4 \epsilon[p, q, \mu, \rho] q_\nu + A_5 \epsilon[p, q, \nu, \rho] p_\mu + A_6 \epsilon[p, q, \nu, \rho] q_\mu \right) \quad (4.8)
$$

where

$$
A_i = \sum_f v_f^Z a_f^Z g_f I_i \quad \text{for } i = 3, \ldots, 6 \quad (4.9)
$$

$I_3$, $I_4$, $I_5$ and $I_6$ are finite integrals (their explicit forms are given in (C.8)) and $\epsilon[p, q, \rho, \sigma]$ is defined after (C.7). $A_1$ and $A_2$ are naively divergent by power counting and so they must be regularized. We compute them by using the Ward identities. In this way it is possible to express $A_1$ and $A_2$ in terms of the finite integrals $I_3$, $I_4$, $I_5$ and $I_6$. The GCS term has the following tensorial structure

$$
d^{Z'Z_0\gamma} \left( \epsilon[p, \mu, \nu, \rho] - \epsilon[q, \mu, \nu, \rho] \right) \quad (4.10)
$$

so it can be absorbed by shifting the first two coefficients of the Rosenberg parametrization for the triangle. The resulting amplitude can be written as

$$
\Delta_{\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4\pi^2} g_0 g_{Z_0} e \left( \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p_\nu + A_4 \epsilon[p, q, \mu, \rho] q_\nu + A_5 \epsilon[p, q, \nu, \rho] p_\mu + A_6 \epsilon[p, q, \nu, \rho] q_\mu \right) \quad (4.11)
$$

The Ward identities (2.19) on the amplitude now read

$$
(p + q)^\rho A_{\mu\nu}^{Z'Z_0\gamma} + i M_{Z'}(GS)_{\mu\nu}^{Z_0\gamma} = 0 \quad (4.12)
$$

$$
p^\mu A_{\mu\nu}^{Z'Z_0\gamma} + i M_{Z_0}(NG)_{\mu\nu}^{Z'\gamma} = 0 \quad (4.13)
$$

$$
q^\nu A_{\mu\nu}^{Z'Z_0\gamma} = 0 \quad (4.14)
$$

where $M_{Z'} = 4b_3 g_0$ and $M_{Z_0}$ are the $Z'$ and $Z_0$ masses respectively. After some manipulations we obtain

$$
(p + q)^\rho A_{\mu\nu}^{Z'Z_0\gamma} = \frac{1}{4\pi^2} g_0 g_{Z_0} e \sum_f v_f^Z a_f^Z q_f \epsilon[p, q, \mu, \nu] \quad (4.15)
$$

$$
p^\mu A_{\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4\pi^2} g_0 g_{Z_0} e \sum_f v_f^Z a_f^Z q_f m_f^2 I_0 \epsilon[q, p, \nu, \rho] \quad (4.16)
$$

$$
q^\nu A_{\mu\nu}^{Z'Z_0\gamma} = 0 \quad (4.17)
$$

and inserting (4.11) into the above identities we get

$$
\tilde{A}_1 = (q^2 A_4 + p \cdot q A_3)
$$

$$
\tilde{A}_2 = \left( p^2 A_5 + p \cdot q A_6 + (NG)^{Z'\gamma} \right) \quad (4.18)
$$
with

\[(NG)^{Z',\gamma} = \sum_f v'_f a_f^0 q_f m_f^2 I_0 \quad (4.19)\]

where \(I_0\) is the integral given in (2.31). Substituting \(\tilde{A}_1, \tilde{A}_2\) from (4.18) into the amplitude (4.11) and performing all the contractions we finally obtain

\[|A^{Z'Z_0}\rangle^2 = |g_{Z'Z_0}|^2 \left( \frac{M_{Z'}^2 - M_{Z_0}^2}{96 M_{Z_0}^2 M_{Z'}^2 \pi^4} \right) \times \left[ \sum_f v'_f a_f^0 q_f \left( (I_3 + I_5) M_{Z_0}^2 + m_f^2 I_0 \right) \right]^2 \quad (4.20)\]

4.2 \(Z' \rightarrow Z_0 Z_0\)

The computations are similar to the previous case so we point out only the differences with the other decay. Mutatis mutandis, the decay rate for the process is given in (4.1) with

\[p_F = \frac{M_{Z'}}{2} \sqrt{1 - \frac{4M_{Z_0}^2}{M_{Z'}^2}} \quad (4.21)\]

The square of the total scalar amplitude is given by

\[|A_{\text{TOT}}|^2 = \frac{1}{8} \sum_{\lambda'} \epsilon_{\lambda'}^{\rho_1} \epsilon_{\lambda'}^{\rho_2} \sum_{\lambda_0} \epsilon_{\lambda_0}^{\nu_1} \epsilon_{\lambda_0}^{\nu_2} \sum_{\lambda_0} \epsilon_{\lambda_0}^{\mu_1} \epsilon_{\lambda_0}^{\mu_2} A_{\rho_1 \mu_1 \lambda_1 \nu_1} A_{\rho_2 \mu_2 \lambda_2 \nu_2} A^{Z'Z_0Z_0} A^{*Z'Z_0Z_0} \quad (4.22)\]

where the amplitude \(A_{\rho\mu\nu}\) is always the sum of the fermionic triangle and the (GCS) term. The contribution to the fermionic triangle is

\[A_{\rho\mu\nu}^{Z'Z_0Z_0} = \frac{1}{8g_{Z'Z_0}^2} \left[ \sum_f v'_f a_f^0 v_f^0 \Gamma_{\rho\mu\nu}^{AV} + v'_f v_f^0 a_f^0 Z_0^0 \Gamma_{\rho\mu\nu}^{AV} \right] + \sum_n \left( a_n^Z v_n^0 v_n^0 \Gamma_{\rho\mu\nu}^{AV} + a_n^Z a_n^Z v_n^0 \Gamma_{\rho\mu\nu}^{AV} \right) \quad (4.23)\]

where \(n\) runs over all the neutrinos while the \(\Gamma_{\rho\mu\nu}\)'s are given by \([C.1], [C.4], [C.2], [C.3]\). Using the fact that for the three neutrino families we have \(v_n^{Z'} = a_n^{Z'}\) and \(v_n^{Z_0} = a_n^{Z_0}\) we write the total amplitude (the sum of the triangles plus GCS terms) as

\[A_{\rho\mu\nu}^{Z'Z_0Z_0} = \frac{1}{8g_{Z'Z_0}^2} \left( \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p' \right. \]

\[+ A_4 \epsilon[p, q, \mu, \rho] q' + A_5 \epsilon[p, q, \nu, \rho] p' + A_6 \epsilon[p, q, \nu, \rho] q' \quad (4.24)\]

with

\[A_i = 2 \sum_f v'_f a_f^0 v_f^0 I_i \quad \text{for } i = 3, \ldots, 6 \quad (4.25)\]
where $\tilde{v}_n^{Z'} = 2v_n^{Z'}$ for neutrinos and $\tilde{v}_f^{Z'} = v_f^{Z'}$ for the other fermions. The Ward identities now read
\begin{align}
(p + q)\rho A_{\rho\mu
u}^{Z'Z_0Z_0} + iM_{Z'}(GS)_{\mu\nu}^{Z_0Z_0} &= 0 \quad (4.26) \\
p^\mu A_{\rho\mu
u}^{Z'Z_0Z_0} + iM_0(NG)_{\rho\mu}^{Z_0Z_0} &= 0 \quad (4.27) \\
q^\nu A_{\rho\mu
u}^{Z'Z_0Z_0} + iM_0(NG)_{\rho\mu}^{Z_0Z'} &= 0 \quad (4.28)
\end{align}
leading to
\begin{align}
(p + q)\rho A_{\rho\mu
u}^{Z'Z_0Z_0} &= \frac{1}{8\pi^2}g_0g_{Z_0}^2 \sum_f \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} \epsilon[p, q, \mu, \nu] \quad (4.29) \\
p^\mu A_{\rho\mu
u}^{Z'Z_0Z_0} &= -\frac{1}{8\pi^2}g_0g_{Z_0}^2 \sum_f \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} m_f^2 I_0 \epsilon[q, p, \nu, \rho] \quad (4.30) \\
q^\nu A_{\rho\mu
u}^{Z'Z_0Z_0} &= -\frac{1}{8\pi^2}g_0g_{Z_0}^2 \sum_f \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} m_f^2 I_0 \epsilon[q, p, \rho, \mu] \quad (4.31)
\end{align}
From these equations we find the following values for $\tilde{A}_1$ and $\tilde{A}_2$
\begin{align}
\tilde{A}_1 &= \left(q^2 A_4 + p \cdot q A_3 - (NG)^{Z'Z_0} \right) \quad (4.32) \\
\tilde{A}_2 &= \left(p^2 A_5 + p \cdot q A_6 + (NG)^{Z'Z_0} \right) \quad (4.33)
\end{align}
with
\begin{align}
(NG)^{Z'Z_0} = \sum_f \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} m_f^2 I_0 \quad (4.34)
\end{align}
where $I_0$ is the integral given in $[2.31]$. Substituting back into the amplitude and performing all the contractions we finally obtain
\begin{align}
|A^{Z'Z_0Z_0}|^2 = \frac{2}{g_0g_{Z_0}} \frac{(M_{Z'}^2 - 4M_{Z_0}^2)^2}{192M_{Z_0}^2 \pi^4} \left[ \sum_f \tilde{v}_f^{Z'} a_f^{Z_0} v_f^{Z_0} \left(2(I_3 + I_5)M_{Z_0}^2 + m_f^2 I_0 \right) \right]^2 \quad (4.35)
\end{align}

4.3 Numerical Results

In this Section we show some numerical computations for the two decay rates $\Gamma(Z' \rightarrow Z_0 + \gamma)$ and $\Gamma(Z' \rightarrow Z_0 + Z_0)$. They depend on the free parameters of the model, i.e. the charges $Q_Q$, $Q_L$ and the mass of the $Z'$. We assume that $Q_\mu = 0$ and we choose $g_0 = 0.1$. We show our results in Fig. $[\text{4.3}]$ in the form of contour plots in the plane $Q_Q, Q_L$ for $M_{Z'} = 1, 2$ and 4 TeV.

The darker shaded regions correspond to larger decay rates. The white region corresponds to the value $10^{-6}$ GeV that can be considered as a rough lower limit for the detection of the corresponding process. It is worth noting that increasing $M_{Z'}$ the mean value of the decay rate of $Z' \rightarrow Z_0\gamma$ grows while the one of $Z' \rightarrow Z_0Z_0$ decreases. We would also like to mention that increasing $M_{Z'}$, the iso-decay rate contours in the plot rotate clockwise getting more and more parallel to the $Q_L$-axis. This effect is due to the fact that the contribution of the triangle diagram with the top quark circulating inside the
loop becomes the dominant contribution for high $M_{Z'}$. In this case the decays strongly depend on the top quark charge $Q_Q$ while the lepton charges $Q_L$ become irrelevant.

Finally, we find that the region that gives the largest values (of order of $10^{-4}$ GeV) of the decay $Z' \rightarrow Z_0 \gamma$ is for $M_{Z'} \sim 4$ TeV and for $Q_Q \sim 3$, $Q_L \sim -2$.

Figure 4: $M_{Z'} = 1$ TeV.

Figure 5: $M_{Z'} = 2$ TeV.
Figure 6: $M_{Z'} = 4$ TeV.

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A. Conventions

We use the space-time metric $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$ and the spinorial conventions

\[ \epsilon_{21} = \epsilon^{12} = 1 \quad \epsilon_{12} = \epsilon^{21} = -1 \quad \epsilon_{11} = \epsilon^{11} = \epsilon_{22} = \epsilon^{22} = 0 \quad \text{(A.1)} \]

\[ \psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta \quad \psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta \quad \bar{\psi}^\dot{\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}} \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}} \quad \text{(A.2)} \]

\[ \psi \chi = \psi^\alpha \chi_\alpha \quad \bar{\psi} \bar{\chi} = \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} \quad \text{(A.3)} \]

The Dirac matrices are

\[ \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{where} \quad \left\{ \begin{array}{l} \sigma^\mu = (1, -\bar{\sigma}) \\ \bar{\sigma}^\mu = (1, \bar{\sigma}) \end{array} \right. \quad \text{(A.4)} \]

and we define

\[ \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{(A.5)} \]
B. Total lagrangian

The lagrangian of the model contains several terms

\[ \mathcal{L} = \mathcal{L}_Q + \mathcal{L}_L + \mathcal{L}_{gauge} + \mathcal{L}_H + \mathcal{L}_W + \mathcal{L}_{axion} + \mathcal{L}_{GCS} + \mathcal{L}_{Soft} \quad (B.1) \]

where

\[ \mathcal{L}_Q = \left( Q_1^\dagger e V(3) e^2 V(2) e^2 V(1) e Q L Q_i + V(0) \langle Q \rangle \right) \frac{1}{\sqrt{2}} g_2 \bar{\theta} \theta \]

\[ \mathcal{L}_L = \left( L_i^\dagger e V(3) e^{-2} V(1) e Q L \bar{V} U_i + V(0) \langle U \rangle \right) \frac{1}{\sqrt{2}} g_2 \bar{\theta} \theta \]

\[ \mathcal{L}_H = \left( H^\dagger e V(3) e^{-2} V(1) e Q H \bar{V} H + V(0) \langle H \rangle \right) \frac{1}{\sqrt{2}} g_2 \bar{\theta} \theta \]

\[ \mathcal{L}_{gauge} = \left( \frac{1}{8 g_2^2} \text{Tr} \left( W^{(3)} W^{(3)} \right) + \frac{1}{16 g_1^2} \text{Tr} \left( W^{(2)} W^{(2)} \right) + \right. \]

\[ \left. \frac{1}{16(\bar{g}_0)^2} \text{Tr} \left( W^{(1)} W^{(1)} \right) \right) \frac{1}{\sqrt{2}} g_2 \bar{\theta} \theta \]

\[ \mathcal{L}_W = \left( y_{ij}^u Q_i U_j H_a - y_{ij}^d Q_i D_j H_d - y_{ij}^e L_i E_j + \mu H_a H_d \right) \frac{1}{4} \left( S + \bar{S} + 4 b_3 V(0) \right)^2 \]

\[ \mathcal{L}_{axion} = \left. \frac{1}{4} \left( S + \bar{S} + 4 b_3 V(0) \right)^2 \right|_{\theta} \]

\[ - \frac{1}{2} \left\{ \left[ \sum_{a=0}^2 b_2(a) S \text{Tr} \left( W(a) W(a) \right) + b_2(4) S W^{(1)} W^{(1)} \right] + \text{h.c.} \right\} \]

\[ \mathcal{L}_{GCS} = -d_4 \left[ \left( V^{(1)} D^a V(0) - V^{(0)} D^a V(1) \right) W^{(0)} \langle W \rangle + \text{h.c.} \right] \frac{1}{\sqrt{2}} \bar{\theta} \theta \]

\[ + d_5 \left[ \left( V^{(1)} D^a V(0) - V^{(0)} D^a V(1) \right) W^{(1)} \langle W \rangle + \text{h.c.} \right] \frac{1}{\sqrt{2}} \bar{\theta} \theta \]

\[ + d_6 \text{Tr} \left[ \left( V^{(2)} D^a V(0) - V^{(0)} D^a V(2) \right) W^{(2)} + \right. \]

\[ \left. + \frac{1}{6} V^{(2)} D^a V(0) D^2 \left( [D^a V(2), V(2)] \right) + \text{h.c.} \right] \frac{1}{\sqrt{2}} \bar{\theta} \theta \]

\[ \mathcal{L}_{Soft} = -\frac{1}{2} \left( \sum_{a=0}^3 M_a \lambda(a) \lambda(a) + \text{h.c.} \right) - \frac{1}{2} \left( M_S \psi_S \psi_S + \text{h.c.} \right) - \left( m^2_{Q_i} \bar{Q}_i \bar{Q}_i + m^2_{U_i} \bar{U}_i \bar{U}_i + m^2_{D_i} \bar{D}_i \bar{D}_i \right) + \right. \]

\[ + m^2_{L_i} \bar{L}_i \bar{L}_i + m^2_{E_j} \bar{E}_j \bar{E}_j + m^2_{h_u} |h_u|^2 + m^2_{h_d} |h_d|^2 \]

\[ - \left( a^3_{ij} \bar{Q}_i \bar{Q}_j h_u - a^3_{ij} \bar{Q}_j \bar{Q}_i h_d - a^3_{ij} \bar{L}_i \bar{L}_j h_d + bh_u h_d + \text{h.c.} \right) \]

\[ \right) \quad (B.9) \]
where $\mathcal{L}_Q$, $\mathcal{L}_L$ and $\mathcal{L}_H$ provide the kinetic terms and the gauge interactions of the matter particles such as (s)quarks, (s)leptons, Higgs(ino)s; $\mathcal{L}_{\text{gauge}}$ contains the kinetic terms for the gauge supermultiplet; $\mathcal{L}_W$ is the usual MSSM superpotential; $\mathcal{L}_{\text{axion}}$ provides the kinetic term of the St"uckelberg multiplet and its Green-Schwarz interactions used in the anomaly cancellation procedure; $\mathcal{L}_{\text{GCS}}$ contains the Generalized Chern Simons interactions giving trilinear gauge boson couplings needed to complete the anomaly cancellation procedure; finally, $\mathcal{L}_{\text{Soft}}$ contains the usual soft breaking terms of the MSSM as well as the new terms for the primeino and the axino.

Notice that in order to include the coupling constants in the gauge interactions we need to redefine them as shown in equation (2.15) and to substitute $V \rightarrow 2gV$.

C. Amplitudes, Ward identities and Anomalies

C.1 Fermionic loop diagram

In this Subsection we give some general properties of the fermionic triangle diagram of Fig. 7. Consider a case in which only a single fermion circulates in the loop and each coupling is either axial (A) or vectorial (V) with charge equal to minus one. The fermionic triangles containing an odd number of axial couplings, denoted by AVV, VAV, VVA and AAA are

\[
\Gamma_{\rho\mu\nu}^{AVV}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} Tr \left( \gamma_5 \gamma_\rho \frac{1}{\ell - q - m_f} \gamma_\nu \frac{1}{\ell - m_f} \gamma_\mu \frac{1}{\ell + \overline{\nu} - m_f} \right) + + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]

\[
\Gamma_{\rho\mu\nu}^{VAV}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} Tr \left( \gamma_\rho \frac{1}{\ell - q - m_f} \gamma_\nu \frac{1}{\ell - m_f} \gamma_\mu \frac{1}{\ell + \overline{\nu} - m_f} \right) + + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]

\[
\Gamma_{\rho\mu\nu}^{VVA}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} Tr \left( \gamma_\rho \frac{1}{\ell - q - m_f} \gamma_5 \gamma_\nu \frac{1}{\ell - m_f} \gamma_5 \gamma_\mu \frac{1}{\ell + \overline{\nu} - m_f} \right) + + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]

\[
\Gamma_{\rho\mu\nu}^{AAA}(p, q; m_f) = \int \frac{d^4\ell}{(2\pi)^4} Tr \left( \gamma_5 \gamma_\rho \frac{1}{\ell - q - m_f} \gamma_5 \gamma_\nu \frac{1}{\ell - m_f} \gamma_5 \gamma_\mu \frac{1}{\ell + \overline{\nu} - m_f} \right) + + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]

These integrals are superficially divergent (by power counting) and thus there is an ambiguity in their definition. The internal momentum $\ell$ can, in fact, be arbitrarily shifted (see...
Section 6.2 of [42]

\[ \ell_{\sigma} \rightarrow \ell_{\sigma} + \alpha p_{\sigma} + (\alpha - \beta)q_{\sigma} \]  

(C.5)

leading to

\[ \Gamma^{AVV}_{\mu\nu}(p, q, \beta; m_f) = \Gamma^{AVV}_{\mu\nu}(p, q; m_f) - \frac{\beta}{8\pi^2} \epsilon_{\mu\nu\rho\sigma}(p - q)^{\rho} \]  

(C.6)

The amplitudes (C.1),(C.2),(C.3) and (C.4) can be written using the Rosenberg parametrization [11] as

\[ \Gamma^{AVV}_{\rho\mu\nu}(p, q; m_f) = \frac{1}{\pi^2} \left( I_1(p, q; m_f) \epsilon[p, \mu, \nu, \rho] + I_2(p, q; m_f) \epsilon[q, \mu, \nu, \rho] + I_3(p, q; m_f) \epsilon[p, q, \mu, \rho] p_{\rho} + I_4(p, q; m_f) \epsilon[p, q, \mu, \rho] q_{\rho} \right) \]  

(C.7)

with \[ \epsilon[p, q, \rho, \sigma] = \epsilon_{\mu\nu\rho\sigma} p^{\mu} q^{\nu} \] and where

\[ I_3(p, q; m_f) = - \int_0^1 dx \int_0^{1-x} dy \frac{xy}{x} \frac{y(1 - y)p^2 + x(1 - x)q^2 + 2xy p \cdot q - m_f^2}{y(1 - y)p^2 + x(1 - x)q^2 + 2xy p \cdot q - m_f^2} \]

\[ I_4(p, q; m_f) = \int_0^1 dx \int_0^{1-x} \frac{x}{1} \frac{x(x - 1)}{y(1 - y)p^2 + x(1 - x)q^2 + 2xy p \cdot q - m_f^2} \]

\[ I_5(p, q; m_f) = - I_4(q, p; m_f) \]

\[ I_6(p, q; m_f) = - I_3(p, q; m_f) \]  

(C.8)

In terms of the Rosenberg parametrization the \( \beta \) dependence of (C.6) is contained only in \( I_1 \) and \( I_2 \) (which are superficially divergent). However, using the Ward identities,

\[ (p + q)^\rho \Gamma^{AVV}_{\rho\mu\nu}(p, q, \beta; m_f) = \frac{1}{\pi^2} \left[ \frac{\beta}{4} + m_f^2 I_0(p, q; m_f) \right] \epsilon[p, q, \mu, \nu] \]

\[ p^\mu \Gamma^{AVV}_{\rho\mu\nu}(p, q, \beta; m_f) = - \frac{2 + \beta}{8\pi^2} \epsilon[q, p, \nu, \rho] \]

\[ q^\nu \Gamma^{AVV}_{\rho\mu\nu}(p, q, \beta; m_f) = - \frac{2 + \beta}{8\pi^2} \epsilon[q, p, \mu, \nu] \]  

(C.9)

where \( I_0 \) is defined in (2.31), it is possible to show that they can be expressed in terms of \( I_3 \ldots I_6 \) as

\[ I_1^{AVV}(p, q, \beta; m_f) = p \cdot q I_3(p, q) + q^2 I_4(p, q) - \frac{2 + \beta}{8} \]

\[ I_2^{AVV}(p, q, \beta; m_f) = - I_1^{AVV}(q, p, \beta; m_f) \]  

(C.10)

From now on we omit the explicit \( \beta \) dependence to get more compact formulae.

**C.2 Anomaly distribution and cancellation.**

In this Subsection we show that the sum of the triangle amplitude and of the GCS vertex are independent of \( \beta \). Since the anomaly is independent of the fermion masses we discuss
only the unbroken phase, i.e. $m_f = 0$. We consider the anomaly between $V^{(0)}$ and two $V^{(1)}$. The total fermionic triangle (the sum of AAA+AVV+VAV+VVA triangles) can be written as

$$
\Delta_{\rho\mu\nu}^{011}(p, q; 0) = -\frac{A^{(1)}}{16}\Gamma_{\rho\mu\nu}(p, q; 0) \tag{C.11}
$$

where $A^{(1)}$ is the anomaly (2.3) and $\Gamma_{\rho\mu\nu}$ is defined in (C.7). The Ward identities for the fermionic triangle are

$$
(p + q)^{\rho} \Delta_{\rho\mu\nu}^{011} = -\beta \frac{A^{(1)}}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} p^{\alpha} q^{\beta} \\
p^{\mu} \Delta_{\rho\mu\nu}^{011} = (2 + \beta) \frac{A^{(1)}}{128\pi^2} \epsilon_{\nu\rho\beta} q^{\alpha} p^{\beta} \\
q^{\nu} \Delta_{\rho\mu\nu}^{011} = (2 + \beta) \frac{A^{(1)}}{128\pi^2} \epsilon_{\rho\mu\alpha\beta} q^{\alpha} p^{\beta} \tag{C.12}
$$

For instance, $\beta = -2/3$ corresponds to a symmetric distribution of the anomaly. The gauge invariance of the theory is restored using (see Section (2.2.1))

$$
(p + q)^{\rho} \left( \Delta_{\rho\mu\nu}^{011}(p, q; 0) + (GCS)_{\rho\mu\nu}^{011} \right) + 2ib_3(GS)_{\mu\nu}^{11} = 0 \\
p^{\mu} \left( \Delta_{\rho\mu\nu}^{011}(p, q; 0) + (GCS)_{\rho\mu\nu}^{011} \right) = 0 \\
q^{\nu} \left( \Delta_{\rho\mu\nu}^{011}(p, q; 0) + (GCS)_{\rho\mu\nu}^{011} \right) = 0 \tag{C.13}
$$

The last two identities imply

$$
(2 + \beta) \frac{A^{(1)}}{128\pi^2} - 2d_5 = 0 \quad \Rightarrow \quad d_5 = \frac{2 + \beta}{2} \frac{A^{(1)}}{128\pi^2} \tag{C.14}
$$

and the first identity becomes

$$
-\beta \frac{A^{(1)}}{64\pi^2} + 4 \frac{2 + \beta}{2} \frac{A^{(1)}}{128\pi^2} + 4b_2^{(1)}b_3 = 0 \quad \Rightarrow \quad b_2^{(1)}b_3 = -\frac{A^{(1)}}{128\pi^2} \tag{C.15}
$$

It is then clear that different choices in the anomaly distribution affect only the GCS coefficient $d_5$ while the GS coefficient $b_2^{(1)}$ remains the same. This means that removing the Stückelberg coupling by gauge fixing and computing the physical amplitude $\Delta + GCS$, we get the same result and the same Ward identity. Consider the amplitude

$$
A_{\rho\mu\nu}^{011} = \Delta_{\rho\mu\nu}^{011} + (GCS)_{\rho\mu\nu}^{011} = \Delta_{\rho\mu\nu}^{011} + 2d_5 \epsilon_{\rho\mu\nu\alpha}(p - q)^{\alpha} \tag{C.16}
$$

The GCS terms can be reabsorbed by the following redefinitions

$$
\left( \frac{A^{(1)}}{16\pi^2} \right) \tilde{I}_1(p, q) = \left( \frac{A^{(1)}}{16\pi^2} \right) I_1(p, q) - 2d_5 \tag{C.17}
$$

$$
\left( \frac{A^{(1)}}{16\pi^2} \right) \tilde{I}_2(p, q) = \left( \frac{A^{(1)}}{16\pi^2} \right) I_2(p, q) + 2d_5 \tag{C.18}
$$
Imposing the $p^\mu$ and $q^\nu$ identities \([2.24]\), we get

$$I_1(p, q) = p \cdot q I_3(p, q) + q^2 I_4(p, q)$$
$$I_2(p, q) = -I_1(q, p)$$ \(\text{(C.19)}\)

that relate $I_1$ and $I_2$ to the other $I_i$’s. We can define

$$\tilde{\Gamma}_{\rho \mu \nu} = \frac{1}{\pi^2} \left( I_1 \epsilon[p, \mu, \nu, \rho] + I_2 \epsilon[q, \mu, \nu, \rho] + I_3 \epsilon[p, q, \mu, \rho]p^\nu + I_4 \epsilon[p, q, \mu, \rho]q^\nu + I_5 \epsilon[p, q, \nu, \rho]p^\mu + I_6 \epsilon[p, q, \nu, \rho]q^\mu \right)$$ \(\text{(C.20)}\)

so that the amplitude is

$$A_{\rho \mu \nu}^{011} = \Delta_{\rho \mu \nu}^{011} + (GCS)_{\rho \mu \nu}^{011} = -\frac{A^{(1)}}{16} \tilde{\Gamma}_{\rho \mu \nu}$$ \(\text{(C.21)}\)

and obeys the following Ward identities

$$(p + q)^\rho A_{\rho \mu \nu}^{011} = \frac{A^{(1)}}{32\pi^2} \epsilon_{\rho \mu \alpha \beta} p^\alpha q^\beta = -2ib_3(GS)^{11}_{\rho \mu \nu}$$
$$p^\mu A_{\rho \mu \nu}^{011} = 0$$
$$q^\nu A_{\rho \mu \nu}^{011} = 0$$ \(\text{(C.22)}\)

This result does not depend on the scheme of the anomaly distribution.

**C.3 Treatment of non anomalous diagrams**

In this section we show that the non anomalous diagrams in Fig. 3 vanish. The diagrams we consider, reported in Fig. 3, have no specific assignment for the external legs, to keep the discussion as general as possible. All the factors which are not relevant for our aim are omitted and all the possible leg exchanges are understood. Finally, we use dimensional regularization and the $R_\xi$ gauge with $\xi = 1$, in such a way that each diagram vanishes separately.

A) The Scalar triangle loop is given by

$${D}^A_{\rho \mu \nu}(p, q) = \int \frac{d^2l}{(2\pi)^2} \frac{(2l + p - q)_\rho(2l - q)_\nu(2l + p)_\mu}{[(l - q)^2 - m^2][l^2 - m^2][(l + p)^2 - m^2]} + (p \leftrightarrow q, \mu \leftrightarrow \nu)$$

$$= \int \frac{d^2l}{(2\pi)^2} \frac{(2l + p - q)_\rho(2l - q)_\nu(2l + p)_\mu}{[(l - q)^2 - m^2][l^2 - m^2][(l + p)^2 - m^2]}$$

$$+ \int \frac{d^2l}{(2\pi)^2} \frac{(2l - q + p)_\rho(2l - p)_\mu(2l + q)_\nu}{[(l - p)^2 - m^2][l^2 - m^2][(l + q)^2 - m^2]}$$ \(\text{(C.23)}\)

Performing the change of variable $l_\mu \rightarrow -l_\mu$ in the second integral, one gets

$${D}^A_{\rho \mu \nu}(p, q) = \int \frac{d^2l}{(2\pi)^2} \frac{(2l + p - q)_\rho(2l - q)_\nu(2l + p)_\mu}{[(l - q)^2 - m^2][l^2 - m^2][(l + p)^2 - m^2]}$$

$$- \int \frac{d^2l}{(2\pi)^2} \frac{(2l + p - q)_\rho(2l + p)_\mu(2l - q)_\nu}{[(l - q)^2 - m^2][l^2 - m^2][(l + p)^2 - m^2]} = 0$$ \(\text{(C.24)}\)
\[(p + q)_\rho \]

\[\rho \]

Not An. =

**Figure 8: Non Anomalous diagrams for trilinear neutral gauge boson amplitudes.**

B) The “Scalar bubble loop” is given by

\[ D^{B}\mu\nu(p, q) = -2 \int \frac{d^2\omega}{(2\pi)^2} \frac{(2l + p + q)\rho\eta_{\mu\nu}}{[l^2 - m^2]([l + p + q]^2 - m^2]} \]

\[ = -2 \int \frac{d^2\omega}{(2\pi)^2} \frac{(l + p + q)\rho\eta_{\mu\nu}}{[l^2 - m^2]([l + p + q]^2 - m^2]} \]

\[ - 2 \int \frac{d^2\omega}{(2\pi)^2} \frac{(l)\rho\eta_{\mu\nu}}{[l^2 - m^2]([l + p + q]^2 - m^2]} \quad \text{(C.25)} \]

Performing the change of variable \( l \to -l - p - q \) in the second integral one gets

\[ D^{B}\mu\nu(p, q) = -2 \int \frac{d^2\omega}{(2\pi)^2} \frac{(l + p + q)\rho\eta_{\mu\nu}}{[l^2 - m^2]([l + p + q]^2 - m^2]} \]

\[ + 2 \int \frac{d^2\omega}{(2\pi)^2} \frac{(l + p + q)\rho\eta_{\mu\nu}}{[l^2 - m^2]([l + p + q]^2 - m^2]} = 0 \quad \text{(C.26)} \]

C) Since the ghost interact with neutral vectors only through the third component of \( SU(2) \), the Ghost triangle loop is proportional to

\[ \varepsilon_{3bc}\varepsilon_{3cd}\varepsilon_{3db} = -\delta_{bd}\delta_{3db} = 0 \quad \text{(C.27)} \]
The other diagrams in Fig. 8 can also be shown to vanish after manipulations similar to the ones used in (C.24), (C.26), (C.27).

D. Decay rates. General case

In this Section we compute the amplitudes for the decays $Z' \rightarrow Z_0 \gamma$ and $Z' \rightarrow Z_0 Z_0$ in the general case $Q_{H_u} \neq 0$, still neglecting the effects coming from the kinetic mixing. We work in the limit

$$g_a v_{u,d} << \mu, M_0, M_1, M_2, M_S, M_{V(0)} \quad (D.1)$$

in which $m_{SC} \approx M_{V(0)}$, $m_{SB} \approx 0$ (see (B.37), (B.2), (3.15)). Hence, (3.36) takes the same form as in the symmetric phase in which neutralinos and charginos do not contribute to the anomaly (see Section 2.2.1). In the limit (D.1) an extension of the standard model by an extra $U(1)$ and our SUSY model give the same results for what the decays of interest are concerned.

We define the Dirac fermions $\Psi_f = \begin{pmatrix} f_L \\ f_R \end{pmatrix}$ where $f_{L(R)}$ are all the left(right) Weyl fermions in the model. The SM fermion interaction terms with the neutral gauge bosons are

$$\mathcal{L}^{int}_{Z'} = J_{Z'}^{\mu} Z_{\mu} = -\frac{1}{2} g_{Z'} \bar{\Psi}_f \gamma^\mu \left( v_{f}^{Z'} - a_{f}^{Z'} \gamma_5 \right) \Psi_f Z_{\mu} \quad (D.2)$$

$$\mathcal{L}^{int}_{Z_0} = J_{Z_0}^{\mu} Z_{0 \mu} = -\frac{1}{2} g_{Z_0} \bar{\Psi}_f \gamma^\mu \left( v_{f}^{Z_0} - a_{f}^{Z_0} \gamma_5 \right) \Psi_f Z_{0 \mu}$$

$$\mathcal{L}^{int}_{\gamma} = J_{\gamma}^{\mu} A_{\mu} = -e q_f \bar{\Psi}_f \gamma^\mu \Psi_f A_{\mu}$$

where

$$v_f^{Z'} = Q_f^{Z'} + Q_f^{Z'} \quad a_f^{Z'} = Q_f^{Z'} - Q_f^{Z'}$$

$$v_f^{Z_0} = Q_f^{Z_0} + Q_f^{Z_0} \quad a_f^{Z_0} = Q_f^{Z_0} - Q_f^{Z_0}$$

$$q_f = Q_f^{Q_0} = Q_f^{Q_0} \quad (D.3)$$

The left and right charges are defined in the following way

$$g_{Z'} Q_f^{Z_0} = g_2 T_3 O_{02} + g_1 Y_f L O_{01} + g_0 Q_f L \quad (D.4)$$

$$g_{Z'} Q_f^{Z_0} = g_1 Y_f R O_{01} + g_0 Q_f R \quad (D.5)$$

$$g_{Z_0} Q_f^{Z_0} = g_2 T_3 O_{12} + g_1 Y_f L O_{11} + g_0 Q_f L O_{10} \quad (D.6)$$

$$g_{Z_0} Q_f^{Z_0} = g_1 Y_f R O_{11} + g_0 Q_f R O_{10} \quad (D.7)$$

$$e Q_f^{L} = g_2 T_3 O_{22} + g_1 Y_f L O_{21} = g_1 Y_f R O_{21} = e Q_f R \quad (D.8)$$

where $O_{ij}$ is given in (3.32) and $T_3$ is the eigenvalue of $T_3^{(2)}$. 
D.1 \( Z' \rightarrow Z_0 \gamma \)

The amplitude is given by the sum of the fermionic triangle \( \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} \) plus the proper GCS vertex

\[
A_{\rho\mu\nu}^{Z'Z_0\gamma} = \Delta_{\rho\mu\nu}^{Z'Z_0\gamma} + (GCS)_{\rho\mu\nu}^{Z'Z_0\gamma}
\]

\[
\Delta_{\rho\mu\nu}^{Z'Z_0\gamma} = -\frac{i}{g_{Z'\gamma}} g_{\rho\mu\nu} e \sum_f \left( v_f^Z a_f^0 q_f \Gamma_{\rho\mu\nu}^{\gamma A} + a_f^Z v_f^0 q_f \Gamma_{\rho\mu\nu}^{\gamma A} \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu) \tag{D.9}
\]

The resulting amplitude can be written as

\[
A_{\rho\mu\nu}^{Z'Z_0\gamma} = -\frac{1}{4\pi^2} g_{Z'\gamma} g_{\rho\mu\nu} e \sum_f \left( v_f^Z a_f^0 q_f \right) \Gamma_{\rho\mu\nu}^{\gamma A} + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]

(D.10)

with

\[
A_i = \sum_f \left( v_f^Z a_f^0 + a_f^Z v_f^0 \right) q_f I_i \quad \text{for } i = 3, \ldots, 6
\]

(D.11)

and the integrals \( I_i \) given in (C.8). \( \tilde{A}_1 \) and \( \tilde{A}_2 \) are the new coefficients with the GCS absorbed similarly to (C.19).

The Ward identities (2.19) for the amplitude now read

\[
(p + q)^\rho A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z'} [(GS)^{Z_0\gamma} + (NG)^{Z_0\gamma}] = 0 \tag{D.12}
\]

\[
p^{\mu} A_{\rho\mu\nu}^{Z'Z_0\gamma} + iM_{Z_0} [(GS)^{Z'\gamma} + (NG)^{Z'\gamma}] = 0 \tag{D.13}
\]

\[
q^{\nu} A_{\rho\mu\nu}^{Z'Z_0\gamma} = 0 \tag{D.14}
\]

where \( M_{Z'} \) and \( M_{Z_0} \) are the \( Z' \) and \( Z_0 \) masses respectively. In both (D.12) and (D.13) we have a \( (GS) \) and a \( (NG) \) contribution due to the two Goldstone bosons which are a linear combination of the axion and \( G^0 \). We use (D.13) and (D.14) to fix \( \tilde{A}_1 \) and \( \tilde{A}_2 \) while (D.12) is automatically satisfied. Contracting with \( p^{\mu} \) we get

\[
p^{\mu} A_{\rho\mu\nu}^{Z'Z_0\gamma} = -\left\{ 8 \left[ 4g_091^2 R_{101}^{Z'Z_0\gamma} b_2^{(1)3} + 2g_092^2 R_{202}^{Z'Z_0\gamma} b_2^{(2)3} + 2g_091^2 R_{001}^{Z'Z_0\gamma} b_2^{(4)3} \right] + \right.

\]

\[
+ \frac{1}{4\pi^2} g_{Z'\gamma} g_{\rho\mu\nu} e \sum_f v_f^Z a_f^0 q_f m_f^2 I_0 \right\} \epsilon[q, p, \nu, \rho] \tag{D.15}
\]

where \( I_0 \) is the integral given in (2.31). The solution for \( \tilde{A}_1 \) and \( \tilde{A}_2 \) is

\[
\tilde{A}_1 = (q^2 A_4 + p \cdot q A_3) \tag{D.16}
\]

\[
\tilde{A}_2 = (p^2 A_5 + p \cdot q A_6) + (GS)^{Z'\gamma} + (NG)^{Z'\gamma} \tag{D.17}
\]

with

\[
(NG)^{Z'\gamma} = \sum_f v_f^Z a_f^0 q_f m_f^2 I_0 \tag{D.18}
\]

\[
(GS)^{Z'\gamma} = \frac{32\pi^2}{g_{Z'\gamma} g_{\rho\mu\nu}} \left[ 4g_091^2 R_{101}^{Z'Z_0\gamma} b_2^{(1)3} + 2g_092^2 R_{202}^{Z'Z_0\gamma} b_2^{(2)3} + 2g_091^2 R_{001}^{Z'Z_0\gamma} b_2^{(4)3} \right] \tag{D.19}
\]
The rotation factors are
\[
R_{101}^{Z'Z_0\gamma} = O_{01}O_{10}O_{21} \\
R_{202}^{Z'Z_0\gamma} = O_{02}O_{10}O_{22} \\
R_{001}^{Z'Z_0\gamma} = O_{10}O_{21}
\]
with \( O_{ij} \) given by (3.32). Substituting \( \tilde{A}_1, \tilde{A}_2 \) into the amplitude (4.11) and performing all the contractions we finally obtain
\[
|A_{\text{TOT}}|^2_{Z'Z_0\gamma} = g_{Z'Z_0}^2 g_{Z_0}^2 \frac{(M_2^{Z'} - M_2^{Z_0})^2}{96M_2^2 M_2^{Z_0} \pi^4} \times \left[ \sum_f q_f \left( v_f^Z a_f^Z + a_f^Z v_f^Z \right) (I_3 + I_5) M_2^{Z_0} + (GS)Z'\gamma + (NG)Z'\gamma \right]^2
\]
(D.21)

D.2 \( Z' \to Z_0 Z_0 \)

The contribution to the fermionic triangle is
\[
\Delta_{\mu\nu} = -\frac{1}{8} g_{Z'Z_0}^2 g_{Z_0}^2 \sum_f \left( v_f^Z a_f^Z + 2v_f^{Z'} a_f^{Z'} + a_f^{Z'} a_f^Z \right) + (p \leftrightarrow q, \mu \leftrightarrow \nu)
\]
(D.22)

where the \( \Gamma_{\mu\nu} \)'s are given by (C.1), (C.4), (C.2), (C.3). We write the total amplitude (the sum of the triangles plus GCS terms) as
\[
A_{\mu\nu}^{Z'Z_0Z_0} = -\frac{1}{8\pi^2} g_{Z'Z_0} g_{Z_0}^2 \left[ \tilde{A}_1 \epsilon[p, \mu, \nu, \rho] + \tilde{A}_2 \epsilon[q, \mu, \nu, \rho] + A_3 \epsilon[p, q, \mu, \rho] p_\nu \\
+ A_4 \epsilon[p, q, \mu, \rho] q_\nu + A_5 \epsilon[p, q, \nu, \rho] p_\mu + A_6 \epsilon[p, q, \nu, \rho] q_\mu \right]
\]
(D.23)

with
\[
A_i = \sum_f t_f^{Z'Z_0Z_0} I_i \quad \text{for } i = 3, \ldots, 6
\]
(D.24)

where
\[
t_f^{Z'Z_0Z_0} = \left( a_f^{Z'} a_f^Z v_f^Z + 2v_f^{Z'} a_f^{Z'} v_f^Z + a_f^{Z'} a_f^Z v_f^Z \right)
\]
(D.25)

and the integrals \( I_i \) are given in (C.8). The Ward identities now read
\[
(p + q)^\mu A_{\mu\nu}^{Z'Z_0Z_0} + iM_{Z'} \left[ (GS)_{\mu\nu}^{Z_0Z_0} + (NG)_{\mu\nu}^{Z_0Z_0} \right] = 0
\]
(D.26)
\[
p^\mu A_{\mu\nu}^{Z'Z_0Z_0} + iM_0 \left[ (GS)_{\mu\nu}^{Z'Z_0} + (NG)_{\mu\nu}^{Z'Z_0} \right] = 0
\]
(D.27)
\[
q^\nu A_{\mu\nu}^{Z'Z_0Z_0} + iM_0 \left[ (GS)_{\mu\nu}^{Z_0Z'} + (NG)_{\mu\nu}^{Z_0Z'} \right] = 0
\]
(D.28)

where \( M_{Z'} \) and \( M_0 \) are the \( Z' \) and \( Z_0 \) masses respectively. In (D.26)-(D.28) the (GS) and (NG) terms are present for the same reason as in the preceding Subsection. We use (D.27) and (D.28) to fix \( \tilde{A}_1 \) and \( \tilde{A}_2 \) while (D.26) is automatically satisfied.
Contracting with $p^\mu$ and $q^\nu$ we get

$$p^\mu A_{\mu\nu}^{Z'Z_0Z_0} = - \left\{ 8 \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + 
+ 2g_0g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2 g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] + 
+ \frac{1}{8\pi^2} g Z' Z_0 \sum_f \left( v_f a_f Z_0 v_f + \frac{1}{3} a_f a_f a_f a_f \right) m_f^2 I_0 \right\} \epsilon[q, p, \nu, \rho] \quad (D.29)$$

$$q^\nu A_{\mu\nu}^{Z'Z_0Z_0} = - \left\{ 8 \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + 
+ 2g_0g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2 g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] + 
+ \frac{1}{8\pi^2} g Z' Z_0 \sum_f \left( v_f a_f Z_0 v_f + \frac{1}{3} a_f a_f a_f a_f \right) m_f^2 I_0 \right\} \epsilon[q, p, \rho, \mu] \quad (D.30)$$

where $I_0$ is the integral given in (2.31). The solution for $\bar{A}_1$ and $\bar{A}_2$ is

$$\bar{A}_1 = \left( q^2 A_4 + p \cdot q A_3 \right) - \left[ (G) Z' Z_0 + (NG) Z' Z_0 \right] \quad (D.31)$$

$$\bar{A}_2 = \left( p^2 A_5 + p \cdot q A_6 \right) + (G) Z' Z_0 + (NG) Z' Z_0 \quad (D.32)$$

with

$$(NG) Z' Z_0 = \sum_f \left( v_f a_f Z_0 v_f + \frac{1}{3} a_f a_f a_f a_f \right) m_f^2 I_0 \quad (D.33)$$

$$(G) Z' Z_0 = \frac{64\pi^2}{g Z' Z_0} \left[ 4g_0^3 R_{000}^{Z'Z_0Z_0} b_2^{(0)} b_3 + 4g_0g_1^2 R_{101}^{Z'Z_0Z_0} b_2^{(1)} b_3 + 
+ 2g_0g_2^2 R_{202}^{Z'Z_0Z_0} b_2^{(2)} b_3 + 2g_0^2 g_1 R_{001}^{Z'Z_0Z_0} b_2^{(4)} b_3 \right] \quad (D.34)$$

The rotation factors are

$$R_{000}^{Z'Z_0Z_0} = O_{10} O_{10}$$

$$R_{101}^{Z'Z_0Z_0} = O_{01} O_{10} O_{11}$$

$$R_{202}^{Z'Z_0Z_0} = O_{02} O_{10} O_{12}$$

$$R_{001}^{Z'Z_0Z_0} = O_{10} O_{11} + O_{01} O_{10} O_{10} \quad (D.35)$$

with $O_{ij}$ given by (3.32). Substituting back into the amplitude and performing all the contractions we finally obtain

$$|A_{\text{TOT}}|^2_{Z'Z_0Z_0} = g Z' Z_0 \left( \frac{M_{Z_0}^2}{192 M_{Z_0}^2} \right)^2 \times$$

$$\left[ \sum_f t_f^{Z'Z_0Z_0} (I_3 + I_5) M_{Z_0}^2 + (G) Z' Z_0 + (NG) Z' Z_0 \right]^2 \quad (D.36)$$

$$\quad (D.37)$$
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