Design of Fuel Supply Curves for Aircraft Center of Gravity Balance Based on DCT Compression

Yawan Wei\textsuperscript{1}, Bo Su\textsuperscript{1}, Xuru Wang\textsuperscript{2}, Zhen Li\textsuperscript{1}, Pengfei Wang\textsuperscript{1} and Xiangchao Feng\textsuperscript{1,}\textsuperscript{*}

\textsuperscript{1}Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology, Beijing, China
\textsuperscript{2}Beijing Institute of Control Engineering, China Academy of Space Technology, Beijing, China

\textsuperscript{*}Corresponding author email: fengxiangchao@qxslab.cn

Abstract. The center of gravity (CG) has a significant influence on the controllability, stability, and fuel efficiency of the aircraft. For an aircraft with multiple fuel tanks, the CG position can be controlled by the fuel quality of different fuel tanks during flight. In this paper, an optimization method based on discrete cosine transform (DCT) compression is proposed to solve the optimization strategy of aircraft multi-tank fuel supply according to the current aircraft mission and engine working requirements. The transform coefficients of the low frequency components are used as design variables to represent the fuel supply curves. The improved KS function is proposed to deal with constraints. The numerical example is demonstrated to verify the effectiveness of the proposed method.

1. Introduction

The CG position is one of the most important parameters of the aircraft. Especially in the flight, monitoring and maintaining the CG are the main challenges of controllability and stability of the aircraft [1–3]. In addition, the CG also plays an important role in determining the fuel efficiency of aircraft [1].

The fuel consumption mode has a great influence on the CG position of an aircraft [4]. For an aircraft with multiple fuel tanks, the CG can be controlled by changing the fuel mass in a specific fuel tank or a group of fuel tanks during flight, so as to suppress the moment caused by the asymmetrical distribution of fuel. Yan et al. [5] proposed a mathematical model of aircraft’s CG varying with fuel transfer and a general scheme of the active CG control system. To obtain a better overall solution, Zheng et al. [6] extracted the aircraft centroid balance fuel supply strategy problem into several sub-problems and combined sequential quadratic programming, improved simulated annealing algorithm to solve.

Several difficulties arise in the design of fuel supply curves for the CG balance of an aircraft with multiple fuel tanks. Especially a large number of local constraints need to be considered. The Kreisselmeier-Steinhauser (KS) function [7,8] is one of the most effective approaches to transform a set of local constraints into a single global constraint, which greatly reduces the computational cost of sensitivity analysis and improves convergence. However, the original KS function is a relaxation approach that allows an individual element may slightly superior to the limit value. To address the issue, the improved KS function is proposed in section 3.
In this work, the transform coefficients of the low frequency components, as the optimization variables, are used to represent the fuel supply curve. The original KS function is improved to aggregate the constraints of mission and engine working requirements. In fact, the proposed method is a general computational framework for design of one-dimensional curves with a set of complex and strict local constraints. It will be extended in the future to treat more complex situations.

2. Problem Statement
Consider an aircraft with $m$ fuel tanks, several fuel tanks are combined to supply fuel to meet the mission requirements and engine work requirements during the flight. The fuel tanks are all cuboid and fixed inside the aircraft. The inner length, width, and height of the $k^{th}$ fuel tank are $L^{(k)}$, $W^{(k)}$, $H^{(k)}$ respectively. In the aircraft coordinate system, the CG position (without fuel) is $(0, 0, 0)$, and the CG position of the $k^{th}$ empty fuel tank is $(x_0^{(k)}, y_0^{(k)}, z_0^{(k)})$. The total mass of the aircraft (without fuel) is $M$. The fuel supply of the aircraft's fuel tank needs to meet the following constraints:

1. The fuel supply speed of the $k^{th}$ fuel tank is limited to no more than $v_{\text{max}}^{(k)}$ ($v_{\text{max}}^{(k)} > 0$). The duration of one fuel supply for each fuel tank is not less than 60 seconds.
2. The main fuel tank can directly supply fuel to the engine. The auxiliary fuel tank can only supply fuel to the corresponding main fuel tank, not directly to the engine.
3. Limited by the structure of the aircraft, the number of main fuel tanks and total fuel tanks for supplying simultaneously fuel cannot exceed the limit value.
4. The total amount of fuel supplied by fuel tanks must at least meet the fuel consumption of the engine.

As an algorithm verification, only the case of the aircraft in level flight is considered in this paper. It is convenient to extend to other situations such as pitch, roll, and yaw by replacing equation (6) with the corresponding situation.

3. Method

3.1. One-dimensional DCT
The discrete cosine transform (DCT) [9] maps a sequence $g_x = \{g_1, g_2, ..., g_{N-1}\}$ in spatial domain into the sequence $G_u = \{G_1, G_2, ..., G_{N-1}\}$ in frequency domain.

$$G_u = \sum_x \left\{ \alpha_u g_x \cos \left( \frac{\pi (2x + 1) u}{2N} \right) \right\} \quad (1)$$

While the inverse discrete cosine transform (IDCT) maps $G_u$ back into spatial domain by:

$$g_x = \sum_u \left\{ \alpha_u G_u \cos \left( \frac{\pi (2x + 1) u}{2N} \right) \right\} \quad (2)$$

where the term $\alpha_u$ is obtained as follows:

$$\alpha_u = \sqrt{\frac{2 - \delta_u}{N}} \quad (3)$$

where $\delta_u = 1$ if $u = 0$, otherwise $\delta_u = 0$.

DCT is one of the most frequently used transformations for image compression. By only a small proportion of $G_u$ corresponding to the low frequency components, the $g_x$ (where $u = 1, 2, ..., n_x$; $x = 1, 2, ..., N_X; n_x \ll N_X$) can be obtained from equation (2). And the sharp regions in spatial domain will be filtered. This property is used to filter the regions in the fuel supply curve where the duration of one fuel supply is less than 60 seconds without adding any constraints.

3.2. Formulation of fuel Supply Curve Design Method for Aircraft Fuel Tank Based on DCT
To begin, the \( n_X \) transform coefficients of the low frequency components \( x_{k}^{(i)} (i = 1, 2, ..., n_X, k = 1, 2, ..., m) \) are used as the design variables of fuel supply curve of the \( k^{th} \) fuel tank. Fuel supply speed curve of each fuel tank \( v_j^{(k)} \) (\( j = 1, 2, ..., N; N \gg n_X \)) is presented using \( N \) discrete time points (the time interval is 1 second). By using the IDCT equation (2), the speed in spatial domain \( g_j^{(k)} \) can be reconstructed by the design variables \( x_{k}^{(i)} \). Then equation (4) plotted in Figure 1(a) is used to drive \( g_j^{(k)} \) approach \( 0, v_{\text{max}}^{(k)} \) in order to obtain the fuel supply speed curve \( v_j^{(k)} \).

\[
v_j^{(k)} = v_{\text{max}}^{(k)} \frac{1 + \tanh(\beta g_j^{(k)})}{2}
\]

where \( \beta \) is the scaling parameter. Thus, the most difficult constraints mentioned in section 2 are perfectly embedded in the design variables.

With the fuel supply speed curve \( v_j^{(k)} \), the remaining fuel volume of the \( p^{th} \) main tank can be calculated by

\[
V_j^{(p)} = V_{\text{d}}^{(p)} - \sum_{i=1}^{j} \frac{v_j^{(p)} + v_{j+1}^{(p)}}{2 \rho} + \sum_{i=1}^{j-1} \frac{v_j^{(p)} + v_{j+1}^{(p)}}{2 \rho}
\]

where \( p^{th} \) is the number index of the auxiliary fuel tank corresponding to the \( p^{th} \) main fuel tank. \( V_{\text{d}}^{(p)} \) is the initial fuel volume of the \( p^{th} \) fuel tank. \( \rho \) is the density of the fuel. As for the calculation of the remaining fuel volume of auxiliary fuel tanks, only the first two terms in equation (5) need to be retained.

Considering the situation of the aircraft in level flight, the CG position of the remaining fuel in the \( j^{th} \) second \( \bar{x}_{ij} \) can be derived as follows:

\[
\bar{x}_{ij} = \begin{bmatrix} x_j^{(k)} \ y_j^{(k)} \ z_j^{(k)} - 0.5H^{(k)} + \frac{V_j^{(k)}}{2L^{(k)}W^{(k)}} \end{bmatrix}
\]

The actual position vector of the CG of the aircraft at the \( j^{th} \) second is expressed as:

\[
\bar{R}_{ij} = \frac{\sum_{k=1}^{m} \rho V_j^{(k)} \bar{x}_{ij}^{(k)}}{M + \sum_{k=1}^{m} \rho V_j^{(k)}}
\]

Hence, the functional relationship between the design variables and the aircraft’s CG is established. Then the improved KS function is used to aggregate the constraints so that only one constraint is used to deal with the other constraints mentioned in section 2.

For the constraint: the total amount of fuel supplied by fuel tanks \( v_j^* \) must at least meet the fuel consumption of the engine \( v_j^* \). By introducing a penalty parameter \( \eta \), the term \( \bar{g}_j \) of KS function [7,8]
in equation (8) is improved to equation (9), which is used to suppress that the original KS function allows an individual element to be slightly higher than the limit value.

\[ v_{ks} = \frac{1}{p} \ln \left( \frac{1}{N} \sum_{j=1}^{N} e^{p \bar{g}_j} \right) \]  

(8)

with

\[ \bar{g}_j = 1 - \frac{v_{ks} - v_{js}^*}{\eta} \]

(9)

By using the improved KS function, the constraints \( v_j \geq v_{js}^* \) for \( j = 1, 2, \ldots, N \) are aggregated to only one constraint \( v_{ks} \leq 0 \).

For the constraint: Limited by the structure of the aircraft, the number of main fuel tanks and all the fuel tanks for supplying simultaneously fuel cannot exceed the limit value. Whether the fuel tank stops supplying fuel or not is identified by equation (10) as shown in Figure 1(b).

\[ \delta_j^{(k)} = \tanh (\alpha v_j^{(k)}) \]

(10)

where \( \alpha \) is set to 10.

In this way, the integer constraint can be converted into a continuous constraint function with a sensitivity. Similarly, these integer constraints can be aggregated into \( n_{ks}^{\text{main}} \leq 0 \) and \( n_{ks}^{\text{all}} \leq 0 \) by the KS equation. And the last constraint, the remaining fuel volume of the fuel tank cannot be less than zero, is also aggregated into \( V_{ks} \leq 0 \) by the KS function.

The objective is to minimize the difference between the actual CG curve and the ideal CG curve. Thus, the optimization problem can be formulated as follows:

\[
\begin{align*}
\text{find} \quad & x^{(k)}_i \quad (k = 1, 2, \ldots, m; i = 1, 2, \ldots, n) \\
\text{min} \quad & f = (\| R - R^* \|_F)^2 \\
\text{st:} \quad & \\
& v_{ks} \leq 0 \\
& n_{ks}^{\text{main}} \leq 0 \\
& n_{ks}^{\text{all}} \leq 0 \\
& V_{ks} \leq 0 \quad (j = 1, 2, \ldots, N)
\end{align*}
\]

(11)

where \( R \) is a matrix and the \( j^{th} \) row represents \( R_{kj} \). Correspondingly, \( R^* \) contains the ideal CG curve of the aircraft.

In this paper, the method of interior-point method of the MATLAB optimization toolbox is employed to update design variables. The sensitivity information of the objective function and constraint functions can be calculated via the chain rule.

4. Numerical Example and Result Analysis

In this section, an aircraft with six fuel tanks is considered as shown in Figure 2. The fuel tanks 1 and 6 are the auxiliary fuel tanks corresponding to the main fuel tanks 2 and 3, respectively. The speed limit \( v_{\text{max}} \) of each fuel tank is \{1.1, 1.8, 1.7, 1.5, 1.6, 1.1\} kg/s, respectively. Up to two tanks can supply fuel to the engine and up to three tanks can supply fuel at the same time.

![Figure 2. Schematic of fuel tanks of the aircraft.](image-url)
A two-hour flight mission of the aircraft stipulates the fuel consumption of the engine (the red dotted line plotted in Figure 4(a)) and the ideal CG position (the red dotted line plotted in Figure 3(b)) at each moment during the flight. Since the time interval is 1 second, \( N = 7200 \). The number of transform coefficients of the low frequency components is set to \( n_x = 20 \). The initial guess of each fuel supply curve is chosen as the design variables corresponding to \( u_j^{(k)} = 0.5 u_{j \text{max}}^{(k)} \).

The entire optimization process is completed in 99 iterations and the objective function converges at 168.71. The final remaining fuel volume of each fuel tank is \{0.0016, 0.0079, 0.0440, 0.0137, 0.6097, 0.3001\} m\(^3\), which satisfies the constraint that the remaining fuel volume of the fuel tank cannot be less than zero.

As depicted in Figure 3(a), the optimized fuel supply speed curves of fuel tanks are very smooth and the duration of a single fuel supply for each fuel tank is higher than 60 seconds. According to the designed fuel supply curves, the matching of the actual and ideal CG is presented in Figure 3(b). It can be found that actual CG is very close to the ideal one.

![Figure 3.](image)

**Figure 3.** (a) The optimized fuel supply speed curves of aircraft fuel tanks (b) Comparison of components between the actual and the ideal CG position of the aircraft.

The excess fuel which is higher than the planned fuel consumption needs to be discharged from the aircraft through other devices. As illustrated in Figure 4(a), the total amount of fuel supplied by fuel tanks fully meets the fuel consumption of the engine, and only a small amount of fuel is discharged from the aircraft. It can be concluded from Figure 4(b) that the number of main fuel tanks as well as the number of all fuel tanks simultaneously supplying fuel are not higher than the limit values of 2 and 3, respectively. The results demonstrate the effectiveness of improved KS function.

Compared with the existing method for the design of the fuel supply speed curve, there are several advantages of the proposed method. First, only a small number of design variables are required. Second, the most difficult constraints (The upper limit of the fuel supply speed and the constraint that the duration of one fuel supply for each fuel tank is not less than 60 seconds) are automatically satisfied by design variables avoiding applying more complex models in constraint functions. This makes the method proposed in this paper superior to the method of dividing the total time into time slices with a length of 60 seconds reported in [6] in terms of algorithmic complexity and computational cost. The third merit of the proposed method is the smoothness of the design results. Frequent switching phenomenon (sharp change of the fuel supply speed curve) [6] is filtered.
5. Conclusion

In this paper, a curve design method based on DCT is proposed to solve the optimization strategy of aircraft multi-tank fuel supply according to the actual aircraft mission and engine working requirements. The KS function is improved to suppress that the original KS function allows an individual element to be slightly higher than the limit value. From the numerical results, the optimized fuel supply curve is very smooth and satisfies the corresponding constraints. Other constraints are also satisfied by the improved KS equation. The numerical results highlighted the advantages of the optimization method.

References

[1] Liu Y, Yang Z, Deng J and Zhu J 2018 Investigation of fuel savings for an aircraft due to optimization of the center of gravity IOP Conference Series: Materials Science and Engineering vol 322 p 072018
[2] Miao Y, Wang S and Zhao Y 2012 Study on change of aircraft center of gravity during fuel consumption IEEE 10th International Conference on Industrial Informatics pp 86–90
[3] Xiaohong Y 2018 Study on the Center Gravity Control Strategy of Tandem Aircraft Fuel Tank IET Conference Proceedings (Institution of Engineering and Technology) pp 169 (5 pp.)-169 (5 pp.) (1)
[4] Tudosie A, NEGREA P and VĂDUVESCU V 2019 METHOD OF CONTROL OF AIRCRAFT CENTER OF GRAVITY BASED ON THE FUEL CONSUMPTION ORDER Sci. Res. Educ. AIR FORCE 21 209–14
[5] Yan J, Zhang J and Li H 2016 Design of aircraft center of gravity control law based on sliding mode control 2016 IEEE International Conference on Aircraft Utility Systems (AUS) pp 438–42
[6] Zheng Z, Pei D, Zhou J, Chen Y and Pu P 2021 An Optimization Method of Aircraft Centroid Balance Fuel Supply Strategy Based on Sequential Quadratic Programming 2021 4th International Conference on Advanced Electronic Materials, Computers and Software Engineering (AEMCSE) pp 1332–6
[7] Kreisselmeier G and Steinhauser R 1979 Systematic Control Design by Optimizing a Vector Performance Index IFAC Proc. Vol. 12 113–7
[8] Verbart A, Langelaar M and Keulen F van 2017 A unified aggregation and relaxation approach for stress-constrained topology optimization Struct. Multidiscip. Optim. 55 663–79
[9] Zhou P, Du J and Lü Z 2018 A generalized DCT compression based density method for topology optimization of 2D and 3D continua Comput. Methods Appl. Mech. Eng. 334 1–21