Survey of 360° domain walls in magnetic heterostructures: topology, chirality and current-driven dynamics

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Chirality and current-driven dynamics of topologically nontrivial 360° domain walls (360DWs) in magnetic heterostructures (MHs) are systematically investigated. For MHs with normal substrates, the static 360DWs are Néel-type with no chirality. While for those with heavy-metal substrates, the interfacial Dzyaloshinskii-Moriya interaction (iDMI) therein makes 360DWs prefer specific chirality. Under in-plane driving charge currents, as the direct result of “full-circle” topology a certain 360DW does not undergo the “Walker breakdown”-type process like a well-studied 180° domain wall as the current density increases. Alternatively, it keeps a fixed propagating mode (either steady-flow or precessional-flow, depending on the effective damping constant of the MH) until it collapses or changes to other types of solition when the current density becomes too high. Similarly, the field-like spin-orbit torque (SOT) has no effects on the dynamics of 360DWs, while the anti-damping SOT has. For both modes, modifications to the mobility of 360DWs by iDMI and anti-damping SOT are provided.

I. INTRODUCTION

The invention and great development of non-volatile magnetic nanodevices have led to a profound revolution in the information industry[1–3]. In these nanodevices, various magnetic solitons or magnetic domains they separate play the roles of 0 and 1 in binary world. In wide magnetic nanostrips, skyrmions/antiskyrmions[4–8], bimerons[9–13] and so on are two-dimensional (2D) isolated topologically nontrivial magnetic solitons surrounded by connected domains with uniform orientation. Under the standard definition of 2D topological charge

\[
\mathcal{W}_{2D}(\mathbf{m}) = \frac{1}{4\pi} \int_{\mathbb{R}^2} \left( \frac{\partial m_1}{\partial x} \times \frac{\partial m_2}{\partial y} \right) dx \, dy ,
\]

in which \(\mathbf{m}\) is a \(\mathbb{R}^3\) unit magnetization field locating on the \((x,y)\) plane, these solitons have an integer \(\mathcal{W}_{2D}\) and they themselves are the information carriers. While in narrow enough nanostrips which are quasi-one dimensional (Q1D) systems, the most studied magnetic solitons are the 1D (Néel or Bloch) 180° domain walls (180DWs) bearing 1/2 1D topological charge, which is defined as

\[
\mathcal{W}_{1D}(\mathbf{m}) = \frac{1}{2\pi} \int_{\mathbb{R}} \left( m_1 \frac{\partial m_2}{\partial \rho} - m_2 \frac{\partial m_1}{\partial \rho} \right) d\rho ,
\]

where \(m_{1,2}\) are magnetization components in the wall plane and the Q1D systems are supposed to extend in \(\rho\)—direction. 180DWs separate two opposite oriented domains whose orientations can be defined as 0 and 1, meantime the wall motion leads to the transformation of information. Since the famous Walker analysis[14], tremendous progress has been made on statics and dynamics of 180DWs driven by various external stimuli[15–29]. The corresponding results have laid the foundation for many mature commercial and developing magnetic nanodevices.

Interestingly, even in Q1D systems we also have some kinds of isolated magnetic solitons which have integer \(\mathcal{W}_{1D}\). Among them, the simplest ones are the so-called 360° domain walls (360DWs) in which the magnetization rotates over one full circle across the intermediate region thus bearing \(W_{1D} = \pm 1\). In the beginning of 1960s, 360DWs were first found to appear in the magnetization reversal process of thin films and their existence seem to be a nuisance since they may complicate the reversal process[30–32]. However, studies in the past three decades revealed that 360DWs themselves in 2D magnetic films have more interesting physics[33–38]. Now we know that 360DWs in lower dimensional systems, such as nanorings[39–41, 43, 44, 57] and nanostrips[45–50], can be qualified candidates to store and process information in magnetic nanodevices due to its “full-circle” topology. From the viewpoint of application, the energy barrier of nucleating a 360DW in single-domain nanorings or nanostrips is much lower than a 180DW since in the latter case one should reverse the magnetic moments in entire half. Also in many cases, a 360DW emerges from the combination of two neighboring 180DWs with opposite polarity due to the long-range magnetostatic interaction or external magnetic fields.

From the beginning of this century, a series of analytical works focus on the question whether 360DWs are genuine stable magnetization textures or just long-lived metastable states[35, 51, 52]. For 2D ferromagnetic (FM) films, the main results are as follows: (i) the magnetostatics is crucial for the existence of 360DWs; (ii) if the long-range component of magnetostatics is neglected, an in-plane external field must be applied to stabilize a 1D front of 360DW whose energy is independent of wall orientation[35]. As the films fade into narrow enough nanostrips, changes in boundary conditions further require that the external field should align with the easy axis to guarantee the existence of 360DWs. In recent device applications, narrow FM metallic nanostrips often serve as the central components of magnetic heterostructures (MHs) with heavy-metal (HM) substrates. Then the effects of interfacial Dzyaloshinskii-Moriya interaction (iDMI)[53, 54] therein to the chirality preference of 360DWs need to be clarified.
Once nucleated, 360DWs in MHs can be driven by certain external stimuli. First, external magnetic fields along easy axis can not finish this job. This can be understood by our roadmap of field-driven domain wall motion since the Zeeman energy densities in the two domains on both sides of 360DWs are the same[16]. Then, current-induced motion of 360DWs becomes the next choice. Indeed, it is the most common way to implement and manipulate in real MHs. Numerical investigations on this issue have been widely preformed in the past decade[55–61]. Alternatively, there are few analytical studies due to the complexity from the coexistence of iDMI, spin-transfer torque (STT) and spin-orbit torque (SOT) therein. In this paper, by adopting the Lagrangian-based collective coordinate models (LB-CCMs) and adequate wall ansatz, the current-driven dynamics of 360DWs is systematically explored which constitutes the second part of this work.

The paper is organized as follows. First, the magnetic Lagrangian and dissipation functional of MHs are introduced in Sec. II. Both perpendicular magnetic anisotropy (PMA) and in-plane magnetic anisotropy (IPMA) for the central FM metallic layers are considered. Then in Sec. III.A we define the proper ansatz for 1D topologically nontrivial 360DWs and then introduce three typical candidates. After integrating over the long axis of MHs, a set of unified dynamical equations is obtained in Sec. III.B and serves as the startpoint of our work. In Sec. III.C, chirality preference of 360DWs selected by iDMI is investigated. After that, the propagation mode of 360DWs under in-plane currents are systematically explored in Sec. III.D. Also, for both modes the effects of iDMI and SOT to the dynamics of 360DWs are analytically calculated. Finally discussions and concluding remarks are provided in Sec. IV and V, respectively.

II. FORMULISM

A MH under consideration is shown in Fig. 1, which is composed of three layers: a HM substrate, a central FM metallic layer and a normal caplayer. We suppose that the MH is long and narrow enough so that it can be viewed as a Q1D system extended in the long axis. For central FM layers with PMA (which will be referred to as “PMA systems”), the easy axis lies in z-axis (out-of-plane normal), the long axis of MH is along x-axis and e_x = e_y × e_z being the hard axis. While for central FM layers with IPMA (named as “IPMA systems”), the easy axis coincides with long axis and is defined as z-axis, the hard axis is along out-of-plane normal and denote as y-axis, at last e_y = e_x × e_z. In these two coordinate systems, the crystalline anisotropy energy density for PMA and IPMA systems shares the same form. However, the iDMI should be treated carefully since it is determined by the out-of-plane normal component of the magnetization vector. By setting “\( \mathbf{m} \)” as the out-of-plane normal, the iDMI energy density can be written as[62]

\[
\mathcal{E}_{\text{iDMI}} = D_i \left\{ [\mathbf{m}(\mathbf{r}) \cdot \hat{n}] \nabla \cdot \mathbf{m}(\mathbf{r}) - [\mathbf{m}(\mathbf{r}) \cdot \nabla] [\mathbf{m}(\mathbf{r}) \cdot \hat{n}] \right\},
\]  

where \( D_i \) is the iDMI strength and \( \mathbf{m}(\mathbf{r}) \) is the unit magnetization vector at position \( \mathbf{r} \). Accordingly, the total magnetic energy density \( \mathcal{E} = \mathcal{E}_0 + \mathcal{E}_{\text{iDMI}} \) includes the exchange, crystalline anisotropy, magnetostatic, Zeeman and iDMI energies. In narrow enough strips, most of the magnetostatic energy can be described by local quadratic terms of \( M_{\text{eff}} \) by means of three average demagnetization factors \( D_{x,y,z} \)[25]. In addition, for Q1D systems \( \nabla \equiv \frac{\partial}{\partial \rho} \sigma_\rho \) in which \( \rho = x(z) \) for PMA (IPMA) systems. Thus one has

\[
\mathcal{E}_0[\mathbf{m}] = \mu_0 M_t^2 \left( \frac{1}{2} k_E m_z^2 + \frac{1}{2} k_{\text{H}} m_z^2 \right)
\]

in which \( A \) is the exchange stiffness, \( \mu_0 \) is the permeability of vacuum, \( M_t \) is the saturation magnetization and \( \mathbf{H}_z = H_z \mathbf{e}_z \) is the external magnetic field along the easy axis with the strength \( H_z \). At last, \( k_{E}(k_{\text{H}}) \) denotes the total anisotropy coefficient along the easy (hard) axis of the central FM layer, namely \( k_{E} = k_1 + (D_x - D_z) \) and \( k_{\text{H}} = k_2 + (D_y - D_x) \) with \( k_{1(2)} \) being the crystalline anisotropy coefficient in easy (hard) axis.

The in-plane charge current flows along “\( \mathbf{e}_y \)” with density \( j_y \). As passing through the MH, the charge current splits into two parts. Suppose \( j_y \) is to be the component in FM (HM) layer. A simple circuit model delivers that \( j_i = j_y t_f + \theta_{t_i} / \theta_{t_f} \) and \( j_H = j_y t_H / \theta_{t_H} \), where \( t_{f(H)} \) and \( \theta_{f(H)} \) are the thickness and conductivity.
of the FM (HM) layer, respectively. For the most common FM metal (Co, Ni, Fe) and HM (Pt, Ta, Ir) materials, the conductivity varies from 10 to 20 (μΩm)^{-1}. For simplicity, we set σ_{t} ≈ σ_{0} thus j_{t} = j_{0}. The charge current component (j_{0}) in HM substrate will induce a spin current into the FM layer which is polarized in the direction of “m_{p} = n × e_{p}”, hence generate the SOT. On the other hand, in the global Cartesian coordinate system, the unit vector of magnetization in the FM layer can be fully described by its polar angle θ and azimuthal angle φ, as shown in Fig. 1. The resulting local spherical coordinate system is denoted as (e_{m}, e_{θ}, e_{φ}). Then m_{p} can be decomposed as

\[ m_{p} = ρ_{m}e_{m} + ρ_{θ}e_{θ} + ρ_{φ}e_{φ}. \]  

(5)

Based on all these preparations, the Lagrangian density \( \mathcal{L} \) and dissipation functional density \( \mathcal{F} \) of this magnetic system can be expressed as

\[ \frac{\mathcal{L}}{μ_{0}M_{s}^{2}} = \frac{- \cos θ}{γM_{s}} \frac{∂φ}{∂t} + \frac{B_{J}φ}{γM_{s}} \frac{∂(\cos θ)}{∂p} + \frac{H_{FL}p_{m}}{M_{s}} - \frac{ε_{0}}{μ_{0}M_{s}^{2}}, \]

(6)

and

\[ \frac{\mathcal{F}}{μ_{0}M_{s}^{2}} = \frac{α}{2γM_{s}} \left( \frac{∂}{∂t} \left[ \frac{βB_{J}φ}{α} \frac{∂}{∂p} \right] \right) m^{2} - \frac{H_{ADL}}{M_{s}} \left( m × m_{p} \right) \frac{∂m}{∂t}, \]

(7)

in which \( γ \), \( μ_{0}γ_{e} \), with \( γ_{e} \) being the electron gyromagnetic ratio, \( B_{J} = μ_{B}P_{J}/(eM_{s}) \) with \( e, μ_{B} \) being respectively the absolute value of electron charge and Bohr magneton, \( P \) is the spin polarization of \( j_{t} \), \( α \) is the Gilbert damping constant, \( β \) is the dimensionless coefficient describing the relative strength of the nonadiabatic STT over the adiabatic one, at last \( H_{FL} \) and \( H_{ADL} \) denotes the strength of field-like (FL) and anti-damping-like (ADL) SOT, respectively.

The dynamics of magnetization in the central FM layers of MHs is then described by the Lagrangian-Rayleigh equation

\[ \frac{d}{dt} \left( \frac{δ\mathcal{L}}{δx} \right) - \frac{δ\mathcal{L}}{δx} + \frac{δ\mathcal{F}}{δx} = 0, \]

(8)

in which \( X \) is any related local or collective coordinate. In particular, when \( X = θ \) and \( φ \) (the most common local coordinates), the resulting two equations can be combined to recover the familiar Landau-Lifshitz-Gilbert equation

\[ \frac{∂m}{∂t} = -γm × H_{eff} + αm × \frac{∂m}{∂t} + T_{STT} + T_{SOT}, \]

(9)

where \( H_{eff} = -(μ_{0}M_{s})^{-1}δε_{0}/δm, \) and

\[ T_{STT} = B_{J}\frac{∂m}{∂p} - βB_{J}m × \frac{∂m}{∂p}, \]

(10)

as well as

\[ T_{SOT} = -γH_{FL}m × m_{p} - γH_{ADL}m × (m × m_{p}). \]

(11)

However, \( θ(ρ,t) \) and \( φ(ρ,t) \) vary from point to point, hence generating a huge number of degrees of freedom. To obtain collective behaviors of magnetization system, LB-CCMs are adopted which need preset ansatz. In the beginning of next section, we will define adequate ansatz for topologically nontrivial 360DWs and introduce several typical trial profiles which contain reasonable collective coordinates. Based on them, a set of dynamical equations can be obtained, which lays the foundation of our work in this paper.

III. RESULTS

III.A Adequate ansatz for topologically nontrivial 360DWs

As we mentioned above, earlier studies confirm that in Q1D MHs if the long-range component of magnetostatics is neglected, then an external field along the easy axis is crucial for forming a 360DW. Accordingly, an analytical profile of static 360DWs has been provided based on the requirement that at equilibrium the \( e_{θ} \) component of \( H_{eff} \) disappears[35]. In this solution the azimuthal angle takes a fixed value while the polar angle changes monotonously from 0 to \( π \) as \( ρ \) runs from one end of MH to the wall center and then decreases back to 0 as \( ρ \) goes further to the other end. This nonmonotonic behavior comes from the consideration that polar angles are defined in spherical coordinate system thus can not exceed \( π \). However, we would like to point out that: a 360DW defined like this must be a topologically trivial one. In PMA (IPMA) systems, this corresponds to a “↑→↓←↑” (“↑→↓←↑”) type wall which first rotates half a circle and then returns back, thus leading to \( \mathcal{H}_{FD} = 0 \).

One possible remedy is to add a fixed value \( π \) to the azimuthal angle when the polar angle crosses the South Pole. In principle this new set of polar and azimuthal is indeed the real spherical coordinates that realizes a topologically nontrivial “↑→↓←↑” (“↑→↓←↑”) type wall for PMA (IPMA) systems, however it will artificially bring a discontinuity point in exchange energy. For the convenience of comparisons below, we denote them as \( θ_{real} \) and \( φ_{real} \). To remove the artificial discontinuity, we propose a monotonically increasing “0 to \( 2π \)” polar angle profile meanwhile keep the azimuthal angle a constant value which are defined as \( θ_{ansatz} \) and \( φ_{ansatz} \). We focus on the “\( π \) to \( 2π \)” part since this is the main region where differences occur. Obviously, we have

\[ δ_{real} = 2π - δ_{ansatz}, \]

\[ φ_{real} = φ_{ansatz} + π, \]

(12)

and they lead to the same magnetization component as follows

\[ \text{real spherical ansatz} \]

\[ m_{x} : \sin δ_{real} \cos φ_{real} = \sin δ_{ansatz} \cos φ_{ansatz}, \]

\[ m_{y} : \sin δ_{real} \sin φ_{real} = \sin δ_{ansatz} \sin φ_{ansatz}, \]

\[ m_{z} : \cos δ_{real} = \cos δ_{ansatz}. \]

(13)

In addition, the polar and azimuthal profiles in our proposal are not bothered by discontinuities in continuous Heisenberg exchange interaction, meanwhile provide \( \mathcal{H}_{FD} = +1 \). Based
on these facts, we conclude that a 360DW profile with a monotonically increasing “0 to 2π” polar angle and a constant azimuthal angle should be an adequate ansatz for 360DWs.

In this work, we use three trial profiles of 360DWs to explore their chirality preference and current-driven dynamics. The first one is inspired by the work of Muratov in 2008[35], but has been generalized to [0,2π] as we proposed above. By introducing the “traveling coordinate” $\xi = \frac{\rho-q(t)}{\Delta(t)}$, where $q(t)$ and $\Delta(t)$ are respectively the center position and width of the 360DW, it can be written as

$$\vartheta = 2 \cot^{-1} \left[ \sqrt{\frac{h}{1+h} \sinh \left( -\sqrt{1+h} \xi \right) } \right] , \quad \phi(\mathbf{r},t) = \phi(t),$$

in which $h = \frac{\mu_0 H_s}{2M_s}$ and the “cot^{-1}” function takes the range of 0 to $\pi$. Note that Eq. (14) is accurate in the absence of driving current. When in-plane currents are applied, this solution becomes an approximation since it may not hold everywhere but it does grasp the main features of dynamical 360DWs. In particular, Eq. (14) clearly ascertains the conclusion that in the absence of external magnetic fields along the easy axis (i.e. $h = 0$), $\vartheta$ keeps a constant value thus 360DWs disappear. Also, we have two other options. The second trial profile is directly generalized from the Walker ansatz, which reads

$$\vartheta = 4 \tan^{-1} e^{\xi} , \quad \phi(\mathbf{r},t) = \phi(t),$$

and the third one is

$$\vartheta = \frac{2\pi}{1+e^{-\xi}} , \quad \phi(\mathbf{r},t) = \phi(t).$$

Obviously, the latter two do not depend on $h$, thus can not be rigorous even in the absence of driving currents. However, due to their mathematical simplicity, they can be used as references. In particular when $h = 1$ ($h = \pi^2/16$), $d\vartheta/d\xi$ at $\xi = 0$ in Eq. (14) coincides with that of Eq. (15) [Eq. (16)]. The corresponding $\vartheta$ profiles are plotted in Fig. 2. Also, curves with $h = 0.1$ and $h = 5$ have been appended to illustrate the dependence of polar angle profile in Eq. (14) on $h$: as $h$ increases the effective width [not the parameter $\Delta(t)$] of 360DW is compressed.

III.B Dynamical equations

In all three trial profiles, the wall center position $q(t)$, tilting angle $\phi(t)$ and wall width $\Delta(t)$ are the three collective coordinates. In Eq. (8), by letting $X$ take $q(t)$, $\phi(t)$, $\Delta(t)$ successively, and integrating over the long axis of MHs (i.e. $\int_{-\infty}^{\infty} d\rho$), a set of dynamic equations can be obtained and expressed in a unified form for both PMA and IPMA systems:

$$\dot{\vartheta} = 2 \cot^{-1} \left[ \sqrt{\frac{h}{1+h} \sinh \left( -\sqrt{1+h} \xi \right) } \right] , \quad \phi(\mathbf{r},t) = \phi(t),$$

in which $h = \frac{\mu_0 H_s}{2M_s}$ and the “cot^{-1}” function takes the range of 0 to $\pi$. Note that Eq. (14) is accurate in the absence of driving current. When in-plane currents are applied, this solution becomes an approximation since it may not hold everywhere but it does grasp the main features of dynamical 360DWs. In particular, Eq. (14) clearly ascertains the conclusion that in the absence of external magnetic fields along the easy axis (i.e. $h = 0$), $\vartheta$ keeps a constant value thus 360DWs disappear. Also, we have two other options. The second trial profile is directly generalized from the Walker ansatz, which reads

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FIG. 2. (Color online) Trial polar angle profiles in Eq. (14) - Eq. (16). Four solid curves are those from Eq. (14) with different $h$, while the magenta (blue) dashed curve shows Eq. (15) [Eq. (16)].

$$0 = (\alpha \dot{\vartheta} + \beta \dot{B}_j) - \frac{2\pi}{\mu_0 M_s} H_{\text{ADL}} \Delta \cdot f(\varphi),$$

where an overdot means $\partial/\partial t$ and $l_0 = \sqrt{2A/(\mu_0 M_s^2)}$. For PMA systems $f(\varphi) = \cos \varphi$ while for IPMA systems $f(\varphi) = -\sin \varphi$. The five integrals ($I_1$ to $I_5$) can be defined in a general way without depending on the specific form of trial profiles (see the first column of Table I). Their values and the parameter $\lambda$ under each profile have been listed in the last three columns of Table I. We also plot them in Fig. 3 as functions of $h$ to show their evolution as $h$ increases.

Eq. (17) is the starting point for our investigations on chirality and current-driven dynamics of 360DWs in Q1D MHs. Before explicitly solving it, we would like to discuss its qualitative properties first. In the dynamical equations for 180DWs (i.e. $H_{\text{ADL}}$, FL-SOT and ADL-SOT are all present. However in Eq. (17), the FL-SOT disappears. This can be understood based on the mathematical form of SOTs in Eq. (11). The main difference lies in the fact that the FL-term is linear to the magnetization $\mathbf{m}$ while the ADL-term is quadratic (thus nonlinear). When integrating over the whole strip, the constant “$-\gamma H_{\text{FL}} \mathbf{m}_p$” factor can be brought up, leaving “$\mathbf{m}$” to be integrated over a full circle thus canceled out. However, this procedure fails for the ADL-term since the constant “$-\gamma H_{\text{ADL}} \mathbf{m}_p$” factor cannot be brought up there. This explains the presence (absence) of $H_{\text{ADL}}$ ($H_{\text{FL}}$) in Eq. (17). Similar analysis can be made to explain the presence of both $H_{\text{ADL}}$ and $H_{\text{FL}}$ in 180DW case. Furthermore, a general rule can be summarized as follows: When dealing with current-driven dy-
TABLE I. Summary of parameters in Eq. (17): definitions and values based on the three trial profiles in Eqs. (14) to (16). First to fifth rows: Integrals \( I_1 \) to \( I_5 \). Last row: Parameter \( \lambda \).

| Parameter | Definition | Value on Eq. (14) | Value on Eq. (15) | Value on Eq. (16) |
|-----------|------------|------------------|------------------|------------------|
| \( I_1 \) | \( \Delta \int_0^{2\pi} \frac{d\vartheta}{dp} \) | \( 4\sqrt{1+\hbar +2h \ln \frac{\sqrt{1+\hbar}+1}{\sqrt{1+\hbar}-1} } \) | 8 | \( \frac{2}{5} \pi^2 \) |
| \( I_2 \) | \( \int_0^{2\pi} \sin \vartheta (-\xi) \) | \( 2\ln \frac{\sqrt{1+\hbar}+1}{\sqrt{1+\hbar}-1} \) | 4 | \( \frac{2}{5} \int_0^{2\pi} \frac{1-\cos \vartheta}{\hbar} \approx \frac{4}{8753} \) |
| \( I_3 \) | \( \frac{1}{2} \int_0^{2\pi} \sin \vartheta \xi \) | \( 3\sqrt{1+\hbar} - 2h \ln \frac{\sqrt{1+\hbar}+1}{\sqrt{1+\hbar}-1} - \frac{8}{3} \) | 3 | \( \int_0^{2\pi} \frac{1-\cos \vartheta}{\hbar} \approx \frac{3.1144}{8753} \) |
| \( I_4 \) | \( \Delta \int_0^{2\pi} \frac{d\vartheta}{dp} \xi^2 d\vartheta \) | \( \frac{8h}{(1+h \hbar)} \) \( \int_0^{2\pi} \frac{\sqrt{4+4h \xi^2 \sinh \xi^2 \hbar}}{(1-\xi^2)^2} d\xi \) | \( \frac{2}{5} \pi^2 \) | \( (2\pi)^2 \int_0^{2\pi} \frac{\sin \vartheta}{\hbar} \approx \frac{8.4870}{8753} \) |
| \( I_5 \) | \( \int_0^{2\pi} \sin \vartheta \cos \vartheta (-\xi) \) | \( \frac{h}{l} \) | \( I_1 - hI_2 - I_5 \) | \( I_1 - I_2 \) |
| \( \lambda \) | \( I_1 - hI_2 - I_5 \) | \( I_1 - I_2 \) | \( \frac{h}{l} \) |

III.C iDMI-induced chirality for static 360DWs

By first choosing the easy-axis-oriented single-domain state as reference, and then integrating over the Q1D MH, the “renormalized magnetic energy” \( E_{\text{NC}} \) of the central FM layer reads

\[
\frac{E_{\text{NC}}}{\mu_0 M_s^2 S} = I_1 \frac{t_0}{\Delta} + \left[ I_3 \frac{t_0}{\Delta} \left( k_E + k_H \sin^2 \varphi \right) + k_E I_2 h \right] \Delta + 2\pi D_{\text{f}} f(\varphi) \frac{t_0}{\mu_0 M_s^2},
\]

where \( S \) is the cross section of the central FM layer. Combing with Eq. (17) for \( j_a = 0 \) (thus \( H_{\text{ADL}} = 0 \) and \( B_j = 0 \)), the chirality preference of static 360DWs can be analyzed.

iDMI is absent

First we review the simplest case where the iDMI is absent \( (D_l = 0) \). Physically this corresponds to MHS with normal substrates. Then the dynamical equations, as well as the renormalized magnetic energy for PMA and IPMA systems are the same. Since \( H_{\text{ADL}} = 0 \) and \( B_j = 0 \), Eq. (17a) provides \( \dot{q} = 0 \) meaning that the 360DW keeps static. A static wall also requires that \( \varphi = 0 \) and \( \Delta = 0 \). Putting them into Eq. (17b), one has \( \sin 2\varphi = 0 \) which means \( \varphi = \frac{\pi}{2} \). However, Eq. (18) clearly tells us that only \( \varphi = n\pi \) (i.e. \( \sin \varphi = 0 \)) minimizes \( E_{\text{NC}} \). Therefore, in the absence of iDMI, 360DWs should be Néel type, but have no chirality preference. At last, Eq. (17c) provides the static wall width \( \Delta_0 \) as

\[
\Delta_0 = \frac{t_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5}}.
\]

Note that \( \Delta_0 \) should not be obtained from the direct minimization of the first two terms in Eq. (18) since the result may not satisfy the dynamical equations. This argument also holds when iDMI appears.
For PMA systems, \( f(\varphi) = \cos \varphi \). The combination of Eq. (17b) and the static requirement (\( \varphi = 0 \) and \( \Delta = 0 \)) leads to

\[
\begin{align*}
\text{case (a)} : & \quad \sin \varphi = 0 \quad \text{or} \\
\text{case (b)} : & \quad \cos \varphi = \frac{2\pi D_i}{k_H I_3 \mu_0 M_i^2 \Delta} 
\end{align*}
\]

To determine which solution provides the real tilting angle, we must compare the corresponding “renormalized magnetic energy” in Eq. (18). For case (a), \( \sin \varphi = 0 \iff \varphi = n\pi \). However, the existence of iDMI [the last term in Eq. (18)] breaks the two-fold degeneracy of \( E_0^{re} \) upon azimuthal angle. To minimize \( E_0^{re} \), one must have

\[
\cos \varphi = -\text{sgn}(D_i),
\]

where “sgn” denotes the sign function. Correspondingly in this case the renormalized magnetic energy becomes

\[
\frac{(E_0^{re})_a}{\mu_0 M_i^2 S} = I_1 \frac{I_3^2}{\Delta} + k_E \left( \frac{T}{2} + I_2 h \right) \Delta - \frac{2\pi |D_i|}{\mu_0 M_i^2}.
\]

For case (b), direct calculation yields

\[
\frac{(E_0^{re})_b}{\mu_0 M_i^2 S} = I_1 \frac{I_3^2}{\Delta} + k_E \left( \frac{T}{2} + I_2 h \right) \Delta + \left[ \frac{2\pi D_i^2}{2k_H I_3 (\mu_0 M_i^2)^2 \Delta} + \frac{I_3}{2k_H \Delta} \right] 2\Delta.
\]

Obviously for any positive \( \Delta \), we always have \( (E_0^{re})_a < (E_0^{re})_b \). Therefore for PMA systems, Eq. (21) provides the real azimuthal angle of 360DWs, which presents definite chirality uniquely determined by iDMI. This can be understood more intuitively from the perspective of effective fields. For PMA systems, the iDMI energy density in Eq. (3) leads to the following effective field

\[
H_i = \frac{1}{\mu_0} \frac{\delta \epsilon_0^{iDMI}}{\Delta M} = -\frac{2D_i}{\mu_0 M_s} \left[ \begin{array}{c} \partial m_x \\ \partial x \\ \partial m_z \\ \partial x \end{array} \right] e_z - \left[ \begin{array}{c} \partial m_x \\ \partial x \\ \partial m_z \\ \partial x \end{array} \right] e_z.
\]

Clearly the \( x \)-component leads to the chirality of 360DWs.

At last, by putting Eq. (21) into Eq. (17c), the static wall width is found to be the same as that in Eq. (19). For the first trial profile [see Eq. (14)], \( I_2, I_3, I_5 \) and \( \lambda \) are all functions of \( h \). One can easily check that \( \sqrt{\lambda / (I_2 h + I_3)} = 1 \), which means that \( \Delta_0 \) is independent on \( h \). This is reasonable since in this profile \( \Delta \) and \( h \) appear together and are independent variables. While for the other two profiles, \( I_2, I_3, I_5 \) and \( \lambda \) are constants. Then the wall width will be compressed when an external field in the easy axis appears, which is also reasonable since \( h \) is absent in these two.

For brevity, we only list the main results here. Since \( f(\varphi) = -\sin \varphi \), the static condition then provides

\[
\begin{align*}
\text{case (a’)} : & \quad \cos \varphi = 0 \\
\text{case (b’)} : & \quad \sin \varphi = \frac{2\pi D_i}{k_H I_3 \mu_0 M_i^2 \Delta}
\end{align*}
\]

For case (a’), the iDMI-induced chirality selects \( \sin \varphi = \text{sgn}(D_i) \). However after simple calculation, it is easy to find that the renormalized magnetic energy in case (b’) is lower than that in case (a’). Therefore the correct static azimuthal angle for IPMA systems should be the one in Eq. (25b). Also, the wall acquires definite chirality determined by the iDMI.

Again, putting Eq. (25b) back into Eq. (17c), the static wall width for IPMA systems is

\[
\Delta_0 = \frac{l_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5}} \left[ 1 - \frac{k_H I_5}{\lambda} \left( \frac{2\pi D_i}{k_H I_3 \mu_0 M_i^2 l_0} \right)^2 \right].
\]

Compared the above result with that in PMA case [see Eq. (19)], a quadratic correction term of \( D_i \) appears. For the second and third trial profiles, it does not change the dependence trend of wall width on \( h \). However, for the first profile a problem emerges since now \( \Delta \) depends on \( h \). This means that in IPMA systems, Eq. (14) is not as good as it is in PMA systems. The reason lies in the fact that in IPMA systems the hard axis is along \( y \)-axis (rather than \( x \)-axis) since it is the thinnest direction of the strip thus has the largest demagnetization factor. Despite this, Eq. (14) does grasp the main features of 360DWs in IPMA systems and should be a good ansatz to explore their statics and dynamics.

**III. D Current-driven 360DW dynamics**

When in-plane currents are applied, the 360DWs will be driven to propagate along the long axis of MHSs. Generally, in Eq. (17a) the presence of “\( \tilde{H}_{ADL} \)” term will change the wall’s mobility from the pure STT-driven result by means of \( f(\varphi) \). To acquire the time-evolution of \( \varphi \), Eq. (17b) and (17b) provides

\[
\Gamma = \frac{4\pi D_i}{k_H I_3 \mu_0 M_i^2 \Delta} \frac{d}{d\varphi} - \sin(2\varphi + \kappa) = \chi dr,
\]

with

\[
\begin{align*}
\chi &= k_H \gamma M_s \left[ \alpha + \left( \frac{l_2}{\alpha l_3 I_4} \right)^2 \right]^{-1} > 0.
\end{align*}
\]

This is the fundamental equation when dealing with current-driven 360DW dynamics.
**iDMI is absent**

First we consider the simplest case where the iDMI is absent \((D_I = 0)\), which corresponds to a 360DW residing in a MH with a normal substrate. Note that at this moment the SOT is also absent. Now Eq. (27) can be directly integrated out and the result depends on the value of \(\Gamma\).

When \(|\Gamma| < 1\),

\[
\tan \left( \frac{\varphi + \kappa}{2} \right) = \frac{1}{\Gamma} - \sqrt{1 - \Gamma^2} \frac{C_1 e^{\sqrt{1 - \Gamma^2} t} + 1}{C_1 e^{\sqrt{1 - \Gamma^2} t} - 1},
\]

with

\[
C_1 = \frac{\Gamma \tan \left( \varphi_0 + \frac{\kappa}{2} \right) - 1 - \sqrt{1 - \Gamma^2}}{\Gamma \tan \left( \varphi_0 + \frac{\kappa}{2} \right) - 1 + \sqrt{1 - \Gamma^2}},
\]

and \(\varphi = \varphi_0\) at \(t = 0\). Obviously when \(t \to +\infty\) the azimuthal angle approaches the following value

\[
\varphi_\infty = \arctan \left( \frac{1}{\Gamma} - \sqrt{1 - \Gamma^2} \right) - \frac{\kappa}{2}.
\]

This means that in this case the 360DW will eventually fall into the “steady-flow” mode. By letting \(\phi = 0\) and \(\Delta = 0\), we know that the wall propagates with a constant velocity \(-\beta B_I/\alpha\) and a fixed width

\[
\Delta(\varphi_\infty) = \frac{l_0}{\sqrt{k_E}} \sqrt{\frac{\lambda}{I_2 h + I_5 + (k_H/k_E) I_5 \sin^2 \varphi_\infty}}.
\]

When \(|\Gamma| > 1\),

\[
\varphi = \arctan \left[ \frac{\sqrt{\Gamma^2 - 1}}{\Gamma} \tan \left( \frac{\sqrt{\Gamma^2 - 1}}{2} \chi t + C_2 \right) + \frac{1}{\Gamma} \right] - \frac{\kappa}{2},
\]

with

\[
C_2 = \arctan \left( \frac{\Gamma \tan \left( \varphi_0 + \frac{\kappa}{2} \right) - 1}{\sqrt{\Gamma^2 - 1}} \right).
\]

Now the azimuthal angle rotates periodically with the period

\[
T_0 = \frac{4\pi}{\chi \sqrt{\Gamma^2 - 1}},
\]

which means that the 360DW takes a “precessional-flow” mode with the same constant velocity \(-\beta B_I/\alpha\) and a periodically changing width.

It is worth noting that for a certain 360DW as the current density increases the wall always takes a specific mode (either steady-flow or precessional-flow) rather than going through a process of mode change, which is quite different from the commonly studies 180DWs. This is the direct consequence of the “full-circle” topology that 360DWs hold. Similar with the discussions in Sec. 3.2, for 180DWs [or other “\((2n + 1)\pi\)” walls] the incomplete cancellation over a half-circle rotation of \(m\) leads to the appearance of “\(\kappa H \sin 2\varphi\)” term, and then results in the famous “Walker breakdown” process. However for 360DWs (or other “\(2n\pi\)” walls), the full cancellation of anisotropic field leads to the absence of “\(\kappa H \sin 2\varphi\)” term in Eq. (17), thus results in the “fixed mode” behavior. Interestingly, both modes share the same wall mobility which is equal to that in steady-flow mode of 180DWs. This explains nearly all existing numerical observations before 360DWs change to other magnetic solitons (for example vortices) under too high currents \([55, 60, 61]\).

Furthermore, we provide the sufficient but non-necessary condition for the steady flow of 360DWs. Under the presupposition \(|\Gamma| < 1\), by putting the wall width [see Eq. (31)] back into the definition of \(\Gamma\) [see Eq. (28)], we obtain

\[
\Gamma = \frac{l_2 I_6}{a I_3 I_4} \cos 2\varphi_\infty.
\]

Thus the sufficient but non-necessary condition for \(|\Gamma| < 1\) should be \(l_2 I_6/(a I_3 I_4) < 1\), which corresponds to \(\alpha > \alpha_c \equiv l_2 I_6/(a I_3 I_4)\). For the second and third trial profiles, one has \(\alpha_c = 3/\pi^2 \approx 0.304\) and \(\alpha_c = 0.287\), respectively. While for the first profile, \(\alpha_c\) is the function of \(h\). We have plotted their dependence on \(h\) in Fig. 4. One can clear see that for all three cases \(\alpha_c\) has a upper limit \(3/\pi^2\) even when \(h\) increases to 5 which is a quite high value in real experiments. In many MHS, existing measurements show that the effective damping in FM strips is enhanced from 0.001-0.01 to 0.3-0.9 \([63, 64]\). This guarantees that experimentally 360DWs should take the steady-flow mode. As for precessional flow, since the explicit form of wall width is hard to obtain, thus it is difficult to obtain the definite range of its existence. However, from the above discussion we can reasonably infer that for sufficient small \(\alpha\), 360DWs precess. This prediction needs to be verified by future experiments and numerical simulations.

![FIG. 4. (Color online) Dependence of \(\alpha_c\) on \(h\) based on Eq. (14) - Eq. (16).](image-url)
iDMI and ADL-SOT are present

Next we study the effects of iDMI and ADL-SOT on the current-driven dynamics of 360DWs in MHs with HM substrates. In the presence of iDMI, in principle the azimuthal angle $\phi$ can not be integrated out explicitly from Eq. (27). Recently a phase diagram has been drawn to show how the types of solutions are determined by the DMI and the anisotropic parameters[65]. However in real MHs, generally the iDMI is weaker than other magnetic interactions, thus can be reasonably viewed as a small quantity. Depending on the value of $\Gamma$, different approximate treatments will be used.

When $|\Gamma| < 1$ or under the stronger condition $\alpha > \alpha_c$, at the lowest level of approximation the 360DW should eventually propagate like a rigid body with the final azimuthal angle $\phi_w$, width $\Delta(\phi_w)$ and velocity

$$\dot{q} = -\frac{\beta}{\alpha} B_j + \frac{2\pi}{\alpha l_1} \gamma H_{ADL} \Delta(\phi_w) \cdot f(\phi_w).$$

(35)

Obviously, the wall mobility is modified by the second term. However the effect of iDMI is totally submerged since it has been dropped when obtaining $\phi_w$. Note that the form of $\Delta(\phi_w)$ in Eq. (31) is not effected by this dropping.

When $|\Gamma| > 1$, the 360DW precesses. In this case for a physical quantity $O$, its time average

$$\langle O \rangle = \frac{1}{T} \int_0^T X(t) \, dt = \frac{1}{T} \int_0^T \frac{2\pi}{\phi} \, \dot{\phi} \, d\phi$$

(36)

corresponds to experimental observables, where $T$ is the precession period. Under the assumption of small iDMI, we calculate the time-averaged wall velocity $\langle \dot{q} \rangle$. First the period $T$ is replaced by $T_0$ in Eq. (33). Then the approximation $(1 - x)^{-1} \approx 1 + x + x^2$ for $|x| < 1$ is used to simplify $1/\phi$ in Eq. (27) hence the integral in Eq. (36) can be calculated. After standard algebra, we have

$$\langle \dot{q} \rangle = -\frac{\beta}{\alpha} B_j - \eta \frac{\sqrt{1 - \frac{1}{\Gamma^2}}}{\Gamma^3} \gamma H_{ADL} \frac{D_1}{\mu_0 M_s^2} \frac{4\pi^2 \cos \kappa}{\alpha k_H I_1 I_3},$$

(37)

where $\eta = +1 (-1)$ for PMA (IPMA) systems. Clearly, Eq. (37) provides the effects of both ADL-SOT and iDMI to the wall velocity in precessional flows.

IV. DISCUSSIONS

First, one should note that the premise of all our analytical results is the existence of 360DWs. The constant mobility (whether adjusted by iDMI and SOT or not) upon current increase is the direct manifestation of the wall’s “full-circle” topology. Accordingly, strong enough external stimuli would destroy the configuration of 360DWs, thereby greatly change the mobility of domain walls (not 360DWs any more). This explains the huge reduction of 360DW mobility under high currents in existing numerics[55, 60, 61].

Second, our analytics presented here is based on “0 to $2\pi$” monotonic profiles of polar angle. If $\vartheta$ is no longer monotonic but its overall change across the wall region keeps $2\pi (\vartheta_{1D} = +1$ still holds), then the results will be unchanged. In addition, for a 360DW with $\vartheta_{1D} = -1$ mathematically its profile can be transfer to that with $\vartheta_{1D} = +1$, except for an increase by $\pi$ in the azimuthal angle. The following procedure is similar to what we have presented in the main text and will not provide new physics, so we won’t repeat it.

At last, topologically the 1D 360DWs in narrow MHs under investigation here are analogous to the 1D domain wall skyrmions (DWSs) evolved from vertical Bloch lines in wide MHs with PMA[66]. Both magnetic solitons carry integer 1D topological charges ($\vartheta_{1D} = \pm 1$), hence should belong to the same topology class. The effective field of iDMI in that work plays the role of external fields along easy axis here, therefore is crucial to the formation of 1D DWSs. The current-driven results here may provide insights for exploring dynamical behaviors of 1D DWSs under external stimuli.

V. CONCLUSION

In this work, the topology, chirality and current-driven dynamics of 360DWs in Q1D MHs are systematically investigated. On one hand, the iDMI uniquely select the chirality of static 360DWs. On the other hand, the “full-circle” topology of 360DWs makes them completely different from the traditional 180DWs. For 360DWs, effective fields which are linear to the magnetization have been fully canceled out and disappear in the dynamical equations. In particular, the full cancellation of magnetic anisotropic fields directly results in the absence of “Walker breakdown”-type process under increasing currents. In a certain MH, 360DWs will take either steady-flow or precessional-flow mode, depending on the strength of effective Gilbert damping constant therein. In MHs with normal substrates, the wall mobility of both modes are the same as that in the steady-flow mode of STT-driven propagation of 180DWs. While in MHs with HM substrates, the mobility will be modified by the ADL-SOT and iDMI. These results should deepen our understanding of topological solitons in low-dimensional magnetic systems, meanwhile provide necessary theoretical basis for expanding the application of 360DWs in the field of magnetic nanodevices.

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