Mutual Visibility by Robots with Persistent Memory

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Abstract—This paper addresses the mutual visibility problem for a set of semi-synchronous, opaque robots occupying distinct positions in the Euclidean plane. Since robots are opaque, if three robots lie on a line, the middle robot obstructs the visions of the two other robots. The mutual visibility problem asks the robots to coordinate their movements to form a configuration, within finite time and without collision, in which no three robots are collinear. Robots are endowed with a constant bits of persistent memory. In this work, we consider the FSTATE computational model in which the persistent memory is used by the robots only to remember their previous internal states. Except from this persistent memory, robots are oblivious i.e., they do not carry forward any other information from their previous computational cycles. The paper presents a distributed algorithm to solve the mutual visibility problem for a set of semi-synchronous robots using only 1 bit of persistent memory. The proposed algorithm does not impose any other restriction on the capability of the robots and guarantees collision-free movements for the robots.

Keywords—Swarm robots, mutual visibility problem, semi-synchronous, persistent memory.

I. INTRODUCTION

A swarm of robots is a multi-robot system consisting of autonomous, homogeneous, small mobile robots which are capable of carrying out some task in a cooperative environment. The robots are modelled as points on the two-dimensional plane in which they can move freely. The robots do not have individual identities i.e., they are indistinguishable by their appearances. Robots are homogeneous i.e., they have same capabilities. They do not have a global coordinate system, each robot has its own local coordinate system. The robots sense the positions of the other robots w.r.t. their local coordinate systems. Each robot executes the computational cycles consisting of three phases Look-Compute-Move. In Look phase, a robot takes the snapshot of its surroundings and maps the locations of the other robots w.r.t. its local coordinate system. In Compute phase, a robot uses the information gathered in the Look stage to compute a destination point to move to. In Move phase, it moves to its computed destination point. Majority of works in the literature assume that the robots are oblivious i.e., they do not remember any data of their previous computational cycles. All the robots execute same algorithm.

Three main computational models are studied in the literature. In the asynchronous model (ASYNC or CORDA) [1], the scheduling of activities of the robots are unpredictable and independent of each other. However, the duration of each computational cycle is finite. In semi-synchronous model (SSYNC) [2], time is discretized into several rounds. In each round, a subset of robots is allowed to execute their computational cycles simultaneously. The movement of the robots are instantaneous i.e., a robot is not observed by the other robots while in motion. The fully-synchronous model (FSYNC) requires all the robots to execute their cycles in a single round. We assume a fair scheduler which activates each robot infinitely often [3].

In terms of capabilities of the robots, different assumptions are made to solve the problems. Multiplicity detection allows a robot to identify multiple occurrences of robots at a single point. Common chirality helps the robots to agree on a common orientation i.e., agreement on common clockwise direction. Rigid motion permits the robots to reach their destinations without halting in between. In persistent memory model, robots are endowed with constant amount of persistent memory (the robots are otherwise oblivious) [4]. This persistent memory can be used in three different ways: (i) the robots can set limited communications between themselves using visible lights which can assume a constant number of predefined colors to represent their different states and also to retain some constant amount of information about their previous states or (ii) only to remember information about their last states (FSTATE model) or (iii) the robots can use visible lights only to communicate with other robots in the system and they do not remember the colors of the lights of their last computational cycle (FComm model) [13]. Thus, the persistent memory can be used for communication or for internal memory or for both. In this work, robots use persistent memory only for internal memory. Unlimited visibility range allows a robot to sense other robots from any distance. Transparency of the robots provides an obstruction free vision for the robots. There can be some agreement on the direction and orientation of the local coordinate axes of the robots.

The algorithms are designed to coordinate the motion of the robots to solve a variety of problems. Fundamental geo-
metric problems like gathering, circle formation, flocking etc. have been studied extensively in the literature [17]. Recently some researchers have taken up the problem of mutual visibility [7], [22], [15], [14]. The mutual visibility problem is defined as follows: for a set of robots initially occupying distinct positions in the two dimensional plane, the mutual visibility problem asks the robots to form a configuration, within finite time and without collision, in which no three robots are collinear.

A. Earlier works

Most of the investigations on different geometric pattern formation problems assume that the robots are transparent. Obstructed visibility has been considered for fat robots (robots represented as unit discs) [5], [9], [11] as well as for the point robots [8], [23], [10]. Explicit communication among the robots using externally visible lights introduced by David Peleg [19]. Combining this limited form of communication and memory with the traditional models, different problems have been solved by many researchers [12], [13], [16], [18], [24]. Di Luna et. al. [2] presented a distributed algorithm to solve the mutual visibility problem for a set of oblivious, semi-synchronous robots. Sharma et. al. [21] analysed and modified the round complexities of the mutual visibility algorithms presented in [7] under fully synchronous model. Di Luna et al. [22] were the first to study the mutual visibility problem in the light model. They solve the problem for the semi-synchronous robots with 3 colors and for asynchronous robots with 3 colors under one axis agreement (in [6] authors claimed a solution of the mutual visibility problem for the asynchronous robots with 10 colors. However later in [22], they modified their claim and presented a solution for the asynchronous robots with 3 colors under one axis agreement). Sharma et. al. [20] proved that the problem is solvable using only 2 colors for the semi-synchronous robots and for the asynchronous robots under one axis agreement. Vaidyanathan et. al. [15] proposed a distributed algorithm for fully-synchronous robots using 12 colors. The algorithm runs in \(O(\log(n))\) rounds for \(n \geq 4\) robots. The only solution to the mutual visibility problem for asynchronous oblivious robots has been proposed in [14] under the assumption that the robots have an agreement in one coordinate axis and they have knowledge of total number of robots in the system. Thus, all the existing solutions for the mutual visible problem either assume persistent memory for both communication and internal memory purposes or one axis agreement or the knowledge of \(n\), total number of robots in the system.

B. Our Contribution

This paper studies the mutual visibility problem for a set of semi-synchronous robots on the Euclidean plane. A distributed algorithm has been proposed to solve the problem for a set of robots endowed with a constant amount of persistent memory. The proposed algorithm considers FS\textsc{State} model which does not have communication overhead of FC\textsc{omm} model. The persistent memory is used only to remember information about their previous states. The proposed algorithm does not assume any other extra assumptions like agreement on the coordinate axes or chirality, knowledge of \(n\), rigidity of movements. In spite of these weak assumptions, it is showed that the mutual visibility problem is solvable for a set of semi-synchronous robots using only 1 bit of persistent internal memory. The contribution of this paper has mainly two folds of significance. First, while all the existing solutions of the mutual visibility problem for semi-synchronous robots have considered either knowledge of \(n\) or persistent memory for both communication and internal memory purposes (combination of FS\textsc{State} and FC\textsc{omm} model), our approach assumes FS\textsc{State} model without knowledge of \(n\) (this makes system easily scalable). Secondly, in all the existing solutions for the mutual visibility problem under persistent memory model, the convex hull of the initial positions of the robots does not remain invariant and the robots move even if the configuration is completely visible to all the robots (robots do not have knowledge of \(n\)). The solution of this work maintains the convex hull of the initial robot positions if all the robots initially do not lie on a single and if the configuration is completely visible to each robot, the robots do not move. The solution also provides collision free movements for the robots. To the best of our knowledge, this paper is the first attempt to study the mutual visibility problem under FS\textsc{State} model.

II. Model and Notations

This paper considers a set of \(n\) homogeneous, autonomous robots represented by points in the two dimensional Euclidean plane. The robots are capable of moving anywhere they want. The robots neither share a global coordinate system nor a common chirality. Each robot has its own local coordinate system; the directions and the orientations of coordinate axes and the unit distance may be vary. The robots are opaque. However, the visibility range of a robot is unlimited. The robots operate in look-compute-move cycles repeatedly. The robots are semi-synchronous (SSYNC model). The robots have no knowledge about the total number of robots in the system. The movements of the robots are non-rigid i.e., a robot can be stopped by an adversary before reaching its destination. However, it is assumed that a robot, if it does not reach its destination, must travel a minimum distance \(\delta > 0\) towards its destination whenever it decides to move. The value of \(\delta\) is not known to the robots. The robots do have any explicit communication power. However, each robot has 1 bit of internal persistent memory FS\textsc{State} model. The 1 bit memory stores information about predefined specific states of the robot. This internal bit does not change automatically and it is persistent. Let \(s_i(t)\) be the binary variable which denote the value stored in the
• configurations of the robots: Let \( \mathcal{R} = \{r_1, r_2, \ldots, r_n\} \) denote the set of \( n \) robots. The position of robot \( r_i \) at time \( t \) is denoted by \( r_i(t) \). A configuration of robots, \( \mathcal{R}(t) = \{r_1(t), \ldots, r_n(t)\} \), is the set of positions occupied by the robots at time \( t \). \( \mathcal{C} \) denotes the set of all such robot configurations.

We partition \( \mathcal{C} \) into two classes: \( \mathcal{C}_L \) and \( \mathcal{C}_{NL} \), where \( \mathcal{C}_L \) is the collection of configurations in which all the robots in \( \mathcal{R} \) lie on a straight line and \( \mathcal{C}_{NL} \) consists of configurations in which there exist at least three non-collinear robot positions occupied by the robots in \( \mathcal{R} \). We say that a robot configuration \( \mathcal{R}(t) \) is in general position if no three robot positions in \( \mathcal{R}(t) \) are collinear. By \( \mathcal{C}_{GP} \), we denote the set of all configurations of \( \mathcal{R} \) which are in general position. Clearly \( \mathcal{C}_{GP} \subset \mathcal{C}_{NL} \).

• Measurement of angles: By an angle between two line segments, if not stated otherwise, we mean the angle which is less than or equal to \( \pi \).

• Vision of a robot: If three robots \( r_i, r_j \) and \( r_k \) are collinear with \( r_j \) lying in between \( r_i \) and \( r_k \), then \( r_i \) and \( r_k \) are not visible to each other. We define the vision, \( V(r_i(t)) \), of robot \( r_i \) at time \( t \) to be the set of robot positions visible to \( r_i \) (excluding \( r_i \)). The visibility polygon of \( r_i \) at time \( t \), denoted by \( STR(r_i(t)) \), is defined as follows: sort the points in \( V(r_i(t)) \) angularly in anti-clockwise direction w.r.t. \( r_i(t) \) starting from any robot position in \( V(r_i(t)) \). Then connect them in that order to generate the polygon \( STR(r_i(t)) \) (Figure 1).

![Figure 1. An example of visibility polygon](image)

- A straight line \( \mathcal{L} \) is called a line of collinearity if it contains more than two distinct robot positions. A robot occupying a position on \( \mathcal{L} \) is termed a collinear robot. For a robot \( r_i \), let \( \mathcal{B}_i \) denote the set of all lines of collinearity on which \( r_i \) is a collinear robot at time \( t \in \mathbb{N} \). Consider a line of collinearity \( \mathcal{L} \) at time \( t \). A robot \( r_i \) on \( \mathcal{L} \) is called an non-terminal robot if \( r_i(t) \) is a point in between two other robot positions on \( \mathcal{L} \). A robot which is not a non-terminal robot is called a terminal robot. Let \( r_i \) be a non-terminal robot on a line of collinearity \( \mathcal{L} \). The point \( r_i(t) \) is called a junction robot position if there is another line of collinearity \( \mathcal{L}_2 \) such that \( r_i(t) \) lies at the intersection point between \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \).

- By \( [p, q] \), we denote the closed line segment joining two points \( p \) and \( q \), including the end points \( p \) and \( q \). Let \( (p, q) \) denote the open line segment joining the points \( p \) and \( q \), excluding the two end points \( p \) and \( q \). Let \( \mathcal{L}(p, q) \) denote the length of \( [pq] \).

- \( d_{ij}^k(t) \): Let \( \mathcal{L}_{ij}(t) \) denote the straight line joining \( r_i(t) \) and \( r_j(t) \). The perpendicular distance of the line \( \mathcal{L}_{ij}(t) \) from the point \( r_k(t) \) is denoted by \( d_{ij}^k(t) \).

- \( D_i(t) \): \( D_i(t) \) is the minimum distance of any two robot positions in \( \{r_i(t), V_i(t)\} \).

### III. Algorithm

The outline of our algorithm is as follows. Consider an initial configuration \( \mathcal{R}(t_0) \) of robots. If \( \mathcal{R}(t_0) \) contains no non-terminal robot, then \( \mathcal{R}(t_0) \in \mathcal{C}_{GP} \) i.e., all the robots in the system are visible to each other. On the contrary, if \( \mathcal{R}(t_0) \) contains at least one non-terminal robot, then there are at least two robots which are not visible to each other. In this scenario, to achieve complete visibility, robots have to coordinate their movements in such a way that within finite time, they achieve complete visibility. Regarding the movements of the robots, following three things have to be decided: (i) which robots should move, terminal or non-terminal or both (ii) how much they should move and (iii) the directions of their movements. In our approach to develop a solution for the mutual visibility problem, we choose non-terminal robots for movements until there is no non-terminal robot in the system. The new destination points of the robots are computed in such way that (i) they do not create new collinearities by moving to the new positions and (ii) the total number of collinear robots in the system should decrease within finite number of movements. The algorithm terminates when system contains no non-terminal robot. A robot can easily determine whether it is a terminal robot or non-terminal robot. A terminal robot does nothing. Before describing the algorithm in details, consider the following cases:

**Case 1:** Let us consider a line of collinearity \( \mathcal{L}_1 \). Let \( r_i \) and \( r_j \) be the two end robots on \( \mathcal{L}_1 \) and both of them are terminal robots (Figure 2(a)). Suppose the robots are activated according to a semi-synchronous scheduler and new destination points of the robots are computed in such a way that no three non-collinear robots in a particular round become collinear in any of the succeeding rounds (in the following section, we describe how to compute such points). Suppose only the non-terminal robots on \( \mathcal{L}_1 \) move along directions not coincident with \( \mathcal{L}_1 \) and all of
them move together. Let $r_k$ and $r_l$ be the nearest robots of $r_i$ and $r_j$ respectively on $L_1$. After the movements of the non-terminal robots on $L_1$, at least one of the robots among $r_k$ and $r_l$ becomes terminal. For example, in figure 2(b), $r_l$ becomes terminal on line $L'_1$ (robots may move in opposite sides of $L_1$). If the non-terminal robots on $L'_1$ move again, at least one of the non-terminal robots on $L'_1$ becomes terminal. In this way, within finite number of movements, all the initially non-terminal robots on $L_1$ become terminal and visible to each other. Thus, if a line of collinearity contains two terminal robots and only the non-terminal robots move, in each round at least one non-terminal robot on this line becomes terminal.

Figure 2. An illustration of case-1 in which non-terminal robots become terminal

**Case 2:** Let $L_2$ be a line of collinearity such that at least one of the two end robot positions on this line is non-terminal position. Consider the case when $L_2$ contains exactly one terminal robot, say $r_i$. Let $r_j$ be the robot which occupies the other end robot position on $L_2$. Let $r_j$ be a non-terminal robot on a line $L_3$ (there may be multiple such lines) (Figure 3). If the line $L_3$ contains two terminal robot and exactly one junction robot position, then by case-1, the movements of the robots creates at least one terminal robot. However, if $L_3$ contains at most one terminal robot, by foregoing arguments, all the non-terminal robots on $L_3$ may remain non-terminal just like the case of $L_2$. In this way, we can get cyclic dependencies between the lines of collinearity such that the movements of the non-terminal robots may not create new terminal robots within finite number of movements (Figure 5). Let us formally define this cyclic dependency. Let

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$S = \{L_2, L_3, \ldots, L_k\}$ be a sequence of lines of collinearity. We say that this sequence of lines form a cycle if each of the lines in this sequence contains more than one junction robot positions and one junction robot position on the line.

Figure 4. An illustration of case-2 when no non-terminal robot on the line $L_2$ becomes terminal

Figure 5. An illustration when the non-terminal robots in a cycle remain non-terminal even after their movements
\( \mathcal{L}_m \) lies on the line \( \mathcal{L}_{m+1} \) and one junction robot position lies on the line \( \mathcal{L}_{m-1} \) where \( m \geq 2 \), \( \mathcal{L}_1 \) is \( \mathcal{L}_k \) and \( \mathcal{L}_{k+1} \) is \( \mathcal{L}_3 \) (Figure 6). The robot positions at the intersection point between two lines in a cycle are called **critical** points.

![Figure 6](image)

Figure 6. An example of a cycle for \( k = 4 \)

The question is how to break the cyclic dependency among the lines of collinearity? One of the ways is as follows: if the non-terminal robots move along their corresponding lines of collinearity, then this cyclic dependency can be broken within one round (Figure 7).

The strategy to break collinearity in case-2 does work for case-1 and vice versa. To break all collinearities by moving the non-terminal robots, we need to combine both the strategies applied in case-1 and case-2. Since in general robots are oblivious, we can not combine both the strategies stated in case-1 and case-2. In our model, robots are endowed with 1 bit of persistent memory and this memory can be used to get ride of the difficulties in combining the two strategies. Robots use their internal memories to remember the information about two types of movements as stated in case-1 and case-2. Robots use 0 and 1 in their persistent memory for this purpose. Initially all robots have 0 in their persistent memory.

- If a robot is terminal and its internal bit is 0, it is a terminal robot since the initial configuration.
- If a robot is terminal and its internal bit is 1, it was a non-terminal robot in the initial configuration and has become terminal during the execution of the algorithm.
- If a robot is non-terminal and its internal bit is 0, it is a non-terminal robot since the initial configuration and either it has made no move or has made a type-1 move.
- If a robot is non-terminal and its internal bit is 1, it is a non-terminal robot since the initial configuration and it has made a type-0 move.

**A. Different types of movements**

Type-0 and Type-1 moves, as defined above.

**B. States of a robot**

A robot uses its persistent 1 bit memory to remember information about its last movement. Initially all robots have 0 in their persistent memory.

- If a robot is terminal and its internal bit is 0, it is a terminal robot since the initial configuration.
- If a robot is terminal and its internal bit is 1, it was a non-terminal robot in the initial configuration and has become terminal during the execution of the algorithm.
- If a robot is non-terminal and its internal bit is 0, it is a non-terminal robot since the initial configuration and it has made a type-0 move.
- If a robot is non-terminal and its internal bit is 1, it is a non-terminal robot since the initial configuration and it has made a type-1 move.

**C. Eligible robots for movements**

The non-terminal robots are eligible for movements. The terminal robots does nothing.

**D. Computation of destination point**

Let \( r_i \) be an arbitrary non-terminal robot at time \( t \geq t_0 \). To find the new position of \( r_i \), we first decide on the direction of movement and then the amount of displacement along the this direction. While computing the new destination point of \( r_i \), two things should be taken care of. One is that the new position of \( r_i \) should not block the visibility of the other robots and the second one is that the motions of the robots should be collision free. Depending upon the current configuration \( \mathcal{R}(t) \), the destination point for \( r_i \) is computed as follows.
• **Case-1:** $\mathcal{R}(t) \in \overline{C}_{NL}$

Consider the set of angles $\Gamma(r_i(t))$ defined as follows:

$$\Gamma_i(t) = \{ \angle r_j r_i r_k : r_j, r_k \text{ are two consecutive vertices on } STR(r_i(t)) \}$$

- **The direction of movement:** Let $\alpha_i(t)$ denote the angle in $\Gamma_i(t)$ having the maximum value if the maximum value is less than $\pi$, otherwise the 2nd maximum value (tie, if any, is broken arbitrarily). The bisector of $\alpha_i(t)$ is denoted by $Bisec_i(t)$. It is a ray from $r_i(t)$. If persistent bit is 0, $r_i$ makes a type-0 move and its the direction of movement is along $Bisec_i(t)$. Before starting its movement, $r_i$ changes its persistent bit to 1. It may be noted that any other suitable direction for type-0 move would work fine for robot $r_i$. If persistent bit is 1, $r_i$ makes a type-1 move. $r_i$ randomly chooses a line of collinearity from $B_i(t)$ and moves along this line. Before starting a type-1 move, $r_i$ changes its persistent bit to 0.

- **The amount of displacement:**

Let $d_i(t) = \min \{ d_{ij}^k(t), d_{jk}^k(t), d_{ik}^k(t) : \forall r_j, r_k \in V_i(t) \}$. The amount of displacement of $r_i$ at time $t$ is denoted by $\sigma_i(t)$ and it is defined as follows,

$$\sigma_i(t) = \frac{U}{\min_{\mathcal{V}_i(t)}}$$

Where $U=\min \{ d_i(t), D_i(t) \}$ and $v_i(t) = \| \mathcal{V}_i(t) \|$. Three non-collinear robots become collinear when the triangle formed by these their positions diminishes to a line. The amount $\sigma_i(t)$ is chosen to be a small fraction of $d_{ij}^k(t)$ for all $r_j, r_k \in V_i(t)$ in order to guarantee that no new collinearity is generated during the movements of the robots. Other suitable values will also work.

- **The destination point:** Let $\bar{r}_i(t)$ be the point on $Bisec_i(t)$ at distance $\sigma_i(t)$ from $r_i(t)$ if $s_i(t) = 0$. Otherwise, $\bar{r}_i(t)$ is a point on a line $\mathcal{L} \in B_i(t)$ at distance $\sigma_i(t)$ from $r_i(t)$ (choose randomly any one of the two directions along $\mathcal{L}$). The destination point of $r_i(t)$ is $\bar{r}_i(t)$.

• **Case-2:** $\mathcal{R}(t) \in \overline{C}_L$

There is only one line of collinearity, say $\overline{L}$, in the system. Only two robots are terminal. Once one of them moves, the present configuration is converted into a configuration in $\overline{C}_{NL}$.

- **The direction of movement:** Let $\mathcal{L}^*$ be the perpendicular line to $\overline{L}$ at the point $r_i(t)$. The robot $r_i$ arbitrarily chooses a direction along $\mathcal{L}^*$ and moves along that direction. Let $\mathcal{L}^*_d$ denote the direction of movement of $r_i$. Since all robots are collinear, this movement is a type-0 move. Before starting this move, $r_i$ changes its persistent bit to 1.

- **The amount of displacement:** In this, the amount of displacement $\bar{\sigma}_i(t)$ is defined as follows:

$$\bar{\sigma}_i(t) = \frac{D_i(t)}{34}$$

- **The destination point:** Let $\bar{r}_i(t)$ be the point on $\mathcal{L}^*_d$ at the distance $\bar{\sigma}_i(t)$ from $r_i(t)$. The destination point of $r_i$ is $\bar{r}_i(t)$.

### E. Termination

A robot terminates the execution of algorithm $MutualVisibility()$ when it finds itself as a terminal robot. Thus, an initially terminal robot terminates just in one round.

Robots use the algorithm $ComputeDestination()$ to compute its destination point and use algorithm $MutualVisibility()$ to obtain complete visibility.

### F. Correctness

To prove the correctness of our algorithm, we need to prove the following: (i) three non-collinear robots in a particular round do not become collinear in any of the succeeding rounds (ii) within finite number of rounds at least one non-terminal robot becomes terminal and (iii) movements of the robots are collision free. If three non-collinear robots become collinear, then the triangle formed by their positions should collapse into either a line or a point.

Thus, for arbitrary three non-collinear robots $r_i, r_j$ and $r_k$, we prove that none of $d_{ij}^k(t), d_{ik}^k(t)$ and $d_{jk}^k(t)$ becomes zero. Without loss of generality, we prove that $d_{ij}^k(t)$ will never vanish, during the execution of our algorithm. We estimate the maximum decrement in the value of $d_{ij}^k(t)$ in a particular round, due to the movements of the robots.

**Lemma 1:** Let $r_i, r_j$ and $r_k$ be three arbitrary robots, which are not collinear at time $t \in \mathbb{N}$. During the rest of execution of algorithm $MutualVisibility()$, they do not become collinear.

**Proof.** Maximum decrement in the value of $d_{ij}^k(t)$ occurs when all the three robots move simultaneously in a round. Thus, we suppose the three robots move at time $t$. Depending upon the positions of the robots, we have the following cases.

• **Case-1:** $r_i, r_j$ and $r_k$ are mutually visible at $t_0$

  According to our approach, the displacement of a robot, in a single movement, is bounded above by $\frac{d_{ij}^k(t)}{34}$ (since $|V_i(t)| \geq 1$). Since all the three robots move simultaneously in a round, the total decrement in the value of $d_{ij}^k(t)$ is bounded above by $\frac{3}{34}d_{ij}^k(t)$. It is easy
Algorithm 1: ComputeDestination()

Input: $r_i(t), s_i(t)$ and $\mathcal{R}(t)$.  
Output: a destination point $p$  
if $|\mathcal{V}_i(t)| > 2$ then  
\[
\begin{align*}
    d_i(t) &\leftarrow \min \{d^i_{j}(t), d^i_{k}(t), d^i_{jk}(t) : \forall r_j, r_k \in \mathcal{V}_i(t)\}; \\
    D_i(t) &\leftarrow \min \{|r_j(t)r_k(t)| : \forall r_j, r_k \in \{r_i(t), \mathcal{V}_i(t)\}\}; \\
    U &\leftarrow \min \{d_i(t), D_i(t)\}; \\
    v_i(t) &\leftarrow |\mathcal{V}_i(t)|; \\
    \sigma_i(t) &\leftarrow \frac{1}{3U}U; \\
\end{align*}
\]
\[\text{if } s_i(t) = 0 \text{ then }\]
\[
\begin{align*}
    \alpha_i(t) &\leftarrow \max \{\theta_i(t) \in \Gamma_i(t) : \theta_i(t) < \pi\}; \\
    \text{Bisec}_i(t) &\leftarrow \text{Bisector of } \alpha_i(t); \\
    p &\leftarrow \text{the point on } \text{Bisec}_i(t) \text{ at a distance } \sigma_i(t) \text{ from } r_i(t); \\
\end{align*}
\]
\[\text{else }\]
\[
\begin{align*}
    \mathcal{L} &\leftarrow \text{an arbitrary line in } \mathcal{B}_i(t); \\
    \mathcal{L}^+ &\leftarrow \text{any one of the two directions along the line } \mathcal{L}; \\
    p &\leftarrow \text{the point on } \mathcal{L}^+ \text{ at a distance } \sigma_i(t) \text{ from } r_i(t); \\
\end{align*}
\]
\[\text{else }\]
\[
\begin{align*}
    D_i(t) &\leftarrow \min \{|r_j(t)r_k(t)| : \forall r_j, r_k \in \{r_i(t), \mathcal{V}_i(t)\}\}; \\
    \hat{\sigma}_i(t) &\leftarrow \frac{1}{3}D_i(t); \\
    \mathcal{L} &\leftarrow \text{the line in } \mathcal{B}_i(t); \\
    \mathcal{L}^\perp &\leftarrow \text{perpendicular line to } \mathcal{L}; \\
    \mathcal{L}_u &\leftarrow \text{any one of the two directions along the line } \mathcal{L}^\perp; \\
    p &\leftarrow \text{the point on } \mathcal{L}_u \text{ at a distance } \hat{\sigma}_i(t) \text{ from } r_i(t); \\
\end{align*}
\]
return $p$;

Algorithm 2: MutualVisibility()

Input: $\mathcal{R}(t)$, a configuration of a set robots $\mathcal{R}$ ,  
Output: $\mathcal{R}(t)$, in which no three robots are collinear.

if terminal then  
do nothing;  
else
\[\text{if } s_i(t) == 0 \text{ then }\]
\[
\begin{align*}
    p &\leftarrow \text{ComputeDestination}(r_i(t), s_i(t), \mathcal{R}(t)); \\
    s_i(t) &\leftarrow 1; \\
\end{align*}
\]
\[\text{else }\]
\[
\begin{align*}
    p &\leftarrow \text{ComputeDestination}(r_i(t), s_i(t), \mathcal{R}(t)); \\
    s_i(t) &\leftarrow 0; \\
\end{align*}
\]
Move towards $p$ along the line segment $r_i(t)p$;

to see that this bound also holds for all other scheduling of the actions of the robots. Thus, we have,

\[
d^k_{ij}(t + 1) > (1 - \frac{3}{4\beta})d^k_{ij}(t) \quad (1)
\]

Equation (1) implies that the $\triangle_{ijk}(t)$ does not collapses into a line due to the movements of the robots. Since robots are semi-synchronous and $t$ is arbitrary, these three robots never become collinear during the whole execution of the algorithm.

• Case-2: $r_i$, $r_j$ and $r_k$ are not mutually visible at $t_0$  
We show that the triangle $\triangle_{ijk}(t)$ contains another triangle whose three vertices are mutually visible to each other. By case-1, this contained triangle does not vanish during the movements of the robots and so does $\triangle_{ijk}(t)$.

– Case-2.1: Two pairs of robots are mutually visible  
Without loss of generality, suppose that $r_j(t), r_k(t) \in \mathcal{V}_i(t)$ and $r_u(t) \notin \mathcal{V}_i(t)$. Then there exist two robots $r_u$ and $r_v$ (not necessarily distinct), closest to $r_j$ and $r_k$ respectively, such that they lie on $\mathcal{L}_{jk}(t)$ (Figure 8). If $r_u \in \mathcal{V}_i(t)$, then $r_i, r_j$ and $r_u$ are mutually visible and the triangle $\triangle_{iju}(t)$ is contained within $\triangle_{ijk}(t)$. If $r_u \notin \mathcal{V}_i(t)$, there exists a robot $r_x$ such that $r_x$ lies inside the triangle $\triangle_{ijk}(t)$ and $r_x$ is visible to both of $r_i$ and $r_j$. In this case, the triangle $\triangle_{ijx}(t)$ is contained within $\triangle_{ijk}(t)$.

– Case-2.2: One pair of robots are mutually visible  
Without loss of generality, suppose that $r_k(t) \notin \mathcal{V}_i(t) \cup \mathcal{V}_j(t)$ and $r_j(t) \in \mathcal{V}_i(t)$. Then there exist (i) two robots $r_u$ and $r_v$ (not necessarily distinct), closest to $r_j$ and $r_k$ respectively, such that they lie on $\mathcal{L}_{jk}(t)$ and (ii) two robots $r_u$ and $r_v$ (not necessarily distinct), closest to $r_j$ and $r_k$ respectively, such that they lie on $\mathcal{L}_{jk}(t)$. By the same arguments as above, the triangle $\triangle_{ijx_1}(t)$ is contained within $\triangle_{ijk}(t)$, where $x_1$ is a robot (i) closest to $\mathcal{L}_{ij}(t)$ (ii) visible to both of $r_i$ and $r_j$ and (iii) lies within or on the triangle $\triangle_{ijk}(t)$ ($x_1$ may be one of $r_u$ and $r_v$).

– Case-2.3: No pair of robots is mutually visible  
In this case, $r_i(t) \notin \mathcal{V}_k(t) \cup \mathcal{V}_j(t)$ and $r_j(t) \notin \mathcal{V}_k(t) \cup \mathcal{V}_i(t)$.
Lemma 2: Let \( r_i \) be an initially non-terminal robot. During the execution of algorithm \( \text{MutualVisibility}(t) \), \( \exists \) \( t \in \mathbb{N} \) such that \( r_i \) becomes a terminal robot at time \( t \) and it remains terminal for the rest of the execution of the algorithm.

Proof.

Let \( L_1 \) be a line of collinearity in \( B_i(t) \).

- **Case-1:** \( L_1 \) does not contain a junction robot position
  In this case \( l = 1 \) i.e., \( r_i \) is a non-terminal robot on exactly one line. Since both the end robot positions on \( L_1 \) are terminal, it takes at most \( 2k - 1 \) rounds for the non-terminal robots on \( L_1 \) to become terminal, where \( k \) is number of non-terminal robots on \( L_1 \).

- **Case-2:** \( L_1 \) contains a junction robot position
  We first consider a basic scenario. Let \( L_1 \) contain exactly one junction robot position and \( r_k \) be the robot at this position. Let \( r_k \) lie exactly on two lines of collinearity and \( L_2 \neq L_1 \) be the other line of collinearity of \( r_k \). If \( L_2 \) does not contain any other junction robot position, by case-1.1, within finite round \( r_k \) at least occupies one end robot position on either on \( L_1 \) or \( L_2 \) or on both (if \( r_k \) becomes terminal, we are done). Without loss of generality, suppose \( r_k \) lies at one end of \( L_2 \) and on \( L_1 \) it is still non-terminal. If \( L_2 \) contains non-terminal robots, they may remain non-terminal due to the movement of \( r_k \), until \( r_k \) occupies one end robot position on \( L_1 \) i.e., \( r_k \) becomes terminal. Once \( r_k \) becomes terminal, by case-1.1, the collinearities among the robots initially on \( L_1 \) and \( L_2 \) are broken within finite round and \( r_i \) becomes terminal. On the other hand, suppose \( L_2 \) contains another junction robot position, say \( r_m \) and \( r_k \) and \( r_m \) are the only two robots which occupy junction position on \( L_2 \). Let \( r_m \) be \( L_3 \neq L_2 \) be a line of collinearity on which \( r_m \) lies. If \( r_k \) lies exactly on two lines of collinearity \((L_2 \text{ and } L_3) \) and \( L_3 \) does not contain a junction robot position, by the same arguments as above, within finite round \( r_i \) becomes terminal. Suppose \( L_3 \) contains another junction robot position. \( L_3 \) contains exactly two junction robot positions, we are done as above. Otherwise, continuing our arguments as above, we get a sequence \( S \) of lines of collinearity. Since there are finite number of robots, this sequence either ends with a line of collinearity \( L_k \) contain exactly one junction robot position or it contains a cycle. If former is true, as above, all the non-terminal robots in this sequence become terminal within finite time. When \( S \) contains a cycle, then a type-1 move breaks this cycle, within finite time. Thus, in this basic scenario within finite number of rounds, \( r_i \) becomes terminal.

Now consider the general scenario, in which a line of collinearity may contain more than two junction robot position. Thus, starting from \( L_1 \), we can get many such sequences of lines of collinearity. Let \( S \) denotes the set of all these sequence. Since the sequences in \( S \) may have common lines, removal of collinearities from one line may depend on the removal of collinearities from another line. If no sequence in \( S \) contains a cycle, then only type-1 movements will break all the collinearities.
in $S$. Suppose a sequence in $S$ contains a cycle $C$. Let $r_x$ be a robot at a critical robot position on a line $L_x$ in $C$. If robot $r_x$ makes a type-1 move along $L_x$, then $r_x$ does not remain as a robot at critical position and the cycle $C$ is broken. Suppose $r_x$ makes a type-1 move along another line of collinearity $L_u$. If $L_u$ does not belong to a cycle, then by above case, within finite rounds, $r_x$ does not remain non-terminal with the robots on $L_u$ and after that $r_x$ will make a type-1 move along $L_u$ to break the cycle $C$. Again, if $L_u$ belongs to a cycle, $r_x$ is a robot at a critical robot position on $L_u$ and a type-1 movement of $r_x$ along $L_u$ breaks this cycle. Thus, within finite time all the cycles in $S$ shall be broken.

Hence, within finite time, $r_i$ becomes a terminal robot. Since robots are semi-synchronous, by lemma-1, $r_i$ remains as terminal once it becomes so.

Lemma 3: The movements of the robots are collision free.

Proof. Let $r_i$ and $r_j$ be two arbitrary robots and at least one of them move. Consider a robot $r_k$ visible to at least one of $r_i$ and $r_j$. If $r_i$ and $r_j$ collide, then $r_i$, $r_j$ and $r_k$ would become collinear or remain collinear which are contradictions to lemma [1] and [2]. This implies that the movements of the robots are collision free during the whole execution of MutualVisibility().

Lemma 4: If $R(t_0) \notin C_L$, during the whole execution of algorithm MutualVisibility(), the convex hull of the robot positions in $R(t_0)$ remains invariant in size and shape.

Proof. Let $CH(t_0)$ denote the convex hull of $R(t_0)$. The robots occupying the vertices of $CH(t_0)$ are terminal robots. According to algorithm MutualVisibility(), these robots do not move. Again, the robots on the edges of $CH(t_0)$ move inside the convex hull $CH(t_0)$ and no robot, lying inside the hull, crosses any edge of the convex hull (according to the definitions of directions of movements and amount of displacement in case-1 of subsection D). Hence, $CH(t_0)$ remains invariant in size and shape.

From the above results, we can state the following theorem:

Theorem 1: Algorithm MutualVisibility() solves the mutual visibility problem without any collision for a set of semi-synchronous, communication-less robots, placed in distinct location, with 1 bit of persistent memory.

IV. Conclusion

This paper presents a distributed algorithm to solve the mutual visibility problem in finite time for a set of communication-less semi-synchronous robots endowed with constant amount of persistent memory. The proposed algorithm uses only 1 bit of persistent memory. The robots use their persistent memories only to remember information about their last movements. There is no explicit communication between the robots. The algorithm also guarantees collision free movements for the robots. The results of this paper leave many open questions. How does the internal persistent memory can help to reduce the communication overheads in the existing solutions for the mutual visibility problem, where external lights are used for communicating the internal states of the robots? How to solve the mutual visibility problem for asynchronous robots in this setting? What would be the impact of internal persistent memory in the solutions of other geometric problems?

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