Production of neutron-rich heavy nuclei around $N = 162$ in multinucleon transfer reactions

Cheng Peng$^1$, Zhao-Qing Feng$^{1,a}$

$^1$ School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510640, China

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Abstract Within the framework of the dinuclear system model, the production mechanism of neutron-rich heavy nuclei around $N = 162$ has been investigated systematically. The isotopic yields in the multinucleon transfer reaction of $^{238}\text{U} + ^{248}\text{Cm}$ are analyzed and compared with the available experimental data at GSI. Systematics on the production of superheavy nuclei via the collisions of $^{238}\text{U}$ on actinide nuclei, $^{252,254}\text{Cf}$, $^{254}\text{Es}$ and $^{257}\text{Fm}$ is investigated thoroughly. It is found that the shell effect is of importance in the formation of neutron-rich nuclei around $N = 162$ owing to the enhancement of fission barrier and neutron separation energy. The fragments in the multinucleon transfer reactions manifest the broad isotopic distribution and are dependent on the beam energy. The polar angles of the fragments tend to the forward emission with increasing the beam energy. The production cross sections of new isotopes are estimated and the heavier targets are available for the neutron-rich superheavy nucleus formation. The optimal reaction system and beam energy are proposed for the future experimental measurements.

1 Introduction

The limit of mass and neutron richness of atomic nucleus is one of topical issues in nuclear physics, which is associated with the synthesis of superheavy element, rapid-neutron capture process in the big-bang nucleosynthesis, shell evolution etc. However, the nuclear fission and strongly Coulomb repulsive interaction exist in the massive nuclei [1,2]. The shell effect increases the fission barrier and enable the survival of superheavy nucleus (SHN). Exploring the limit of element or atomic nucleus is a long-term task, especially creating the SHN close to the 'island of stability' predicted by the shell model [3–5]. Up to now, there are more than 3000 nuclides existing on the nuclide chart, and most of them are located in the proton-rich region [6]. For the past several decades, the new transactinide nuclei have been synthesized by the fusion–evaporation reactions at different laboratories in the world. Before 1974, the isotopes of elements $Z = 93–106$ were created by the neutron capture reactions and heavy-ion fusion reactions. Since 1980s, the combinations with the doubly magic nuclei are usually chosen owing to the larger $Q$ values. Reactions with $^{208}\text{Pb}$ or $^{209}\text{Bi}$ based targets were firstly proposed by Oganessian et al. [7,8]. The SHEs from Bh to Cn were synthesized in the cold fusion reactions at GSI (Darmstadt, Germany) with the heavy-ion accelerator UNILAC and the SHIP separator [9,10]. Experiments on the synthesis of element Nh ($Z = 113$) in the $^{70}\text{Zn} + ^{209}\text{Bi}$ reaction have been performed successfully at RIKEN (Saitama, Japan) [11]. However, it is very difficulty to create the superheavy isotopes beyond Nh in the cold fusion reactions because of very low cross section ($\sigma < 0.1 \text{ pb}$). The superheavy elements from Fl ($Z = 114$) to Og ($Z = 118$) have been synthesized at the Flerov Laboratory of Nuclear Reactions (FLNR) in Dubna (Russia) with the double magic nuclide $^{48}\text{Ca}$ bombarding actinide nuclei [12–15], in which more neutron-rich SHN were produced and identified by the subseque $\alpha$-decay chain. With constructing the new facilities in the world such as RIBF (RIKEN, Japan) [16], SPIRAL2 (GANIL in Caen, France) [17], FRIB (MSU, USA) [18], HIAF (IMP, China) [19], the SHNs on the 'island of stability' by using the neutron-rich radioactive beams induced fusion reactions or via the multinucleon transfer (MNT) reactions might be created in experiments.

The MNT reactions and deep inelastic heavy-ion collisions were extensively investigated in experiments since 1970s, in which the new neutron-rich isotopes of light nuclei and also proton-rich actinide nuclei were observed [20–26]. The reaction mechanism and fragment formation were investigated thoroughly, i.e., the energy and angular momentum dissipation, two-body kinematics, shell effect, fission of...
actinide nuclei etc. Recently, the MNT reactions have been extensively investigated in experiments for new isotope production. The MNT reactions have the advantage of extensive isotope distributions, e.g., more than 100 nuclides with $Z = 82–100$ in the reaction of $^{48}\text{Ca} + ^{248}\text{Cm}$ and five new neutron-deficient isotopes $^{216}\text{U}, ^{219}\text{Np}, ^{223}\text{Am}, ^{229}\text{Am}$ and $^{233}\text{Bk}$ [27]. The reaction mechanism is concentrated on the neutron-rich nuclei around the neutron shell closure $N = 126$. The MNT fragments were measured in the reactions of $^{136}\text{Xe} + ^{208}\text{Pb}$ [28,29], $^{136}\text{Xe} + ^{198}\text{Pt}$ [30], $^{156,160}\text{Gd} + ^{186}\text{W}$ [31], and $^{238}\text{U} + ^{232}\text{Th}$ [32]. The neutron-rich nuclides around the neutron shell closure $N = 126$ have significant application in understanding the origin of heavy elements from iron to uranium in the r-process of nucleosynthesis. It has been confirmed that the shell closure plays an important role on the production of neutron-rich nuclei and more advantage with the multinucleon transfer (MNT) reactions in comparison to the projectile fragmentation [33].

Several models have been proposed for describing the MNT reactions, i.e., the dinuclear system (DNS) model [34–36], the GRAZING model [37,38], the dynamical model based on multidimensional Langevin equations [39–41] etc. Moreover, the microscopic approaches based on the nucleon degree of freedom, the time-dependent Hartree-Fock (TDHF) approach [42–45] and extension by incorporating fluctuation and correlation in the nuclear transfer based on the stochastic mean-field theory [46], the improved quantum molecular dynamics (ImQMD) [47–49] are also used to describe the MNT reactions. Some interesting issues have been investigated with the models, e.g., the production cross sections of new isotopes, total kinetic energy spectra and polar angle distribution of transfer fragments, structure effect on the fragment formation etc. There are still some open problems for the MNT reactions, i.e., the mechanism of pre-equilibrium cluster emission, the stiffness of nuclear surface during the nucleon transfer process, the mass limit of new isotopes with stable heavy target nuclides, interplay of nuclear fission and particle evaporation in the MNT dynamics, etc. The extremely neutron-rich beams are favorable for creating neutron-rich heavy or superheavy nuclei owing to the isospin equilibrium [50,51].

In this work, the systematics on the production of neutron-rich heavy nuclei around the neutron shell closure $N = 162$ has been investigated. The article is organized as follows. In Sect. 2 we give a brief description of the DNS model. In Sect. 3, the production cross section and angular distribution of the MNT fragments are calculated. Summary and perspective on the neutron-rich isotope production are shown in Sect. 4.

2 Brief description of the model

2.1 The MNT fragment formation

The DNS concept was assumed that the colliding system is formed at the touching configuration in nuclear collisions and proposed by Volkov at Dubna for describing the deep inelastic heavy-ion collisions [52]. The typical sticking time is several zeptoseconds. The nucleon exchange and energy dissipation take place once the DNS is formed. The nucleon transfer between the binary fragments is governed by the single-particle Hamiltonian [53,54]. Only the nucleons within the valence space are active for transfer [55,56]. The transition probability is related to the local excitation energy and nucleon transfer [53,54,57], which is microscopically derived from the interaction potential in valence space. The local excitation energy is determined by the dissipation energy from the relative motion and the potential energy surface of the DNS. The dissipation of the relative motion and angular momentum of the DNS is described by the classical trajectory with damped collisions [58]. The cross sections of the primary fragments $(Z_1, N_1)$ are calculated as follows:

$$\sigma_{pr}(Z_1, N_1, E_{c.m.}) = \sum_{J=0}^{J_{max}} \sigma_{cap}(E_{c.m.}, J) \int f(B) \times P(Z_1, N_1, E_1, J_1, B) dB \quad (1)$$

The secondary decay of the primary fragments is considered to form the final MNT fragments. The cross section is evaluated by

$$\sigma_{surf}(Z_1, N_1, E_{c.m.}) = \sum_{J=0}^{J_{max}} \sigma_{cap}(E_{c.m.}, J) \int f(B) \times \sum_{s} P(Z'_1, N'_1, E'_1, J'_1, B) \times W_{surf}(Z'_1, N'_1, E'_1, J'_1, s) dB \quad (2)$$

Here, $E_1$ and $J_1$ denote the excitation energy and the angular momentum for the fragment $(Z_1, N_1)$, respectively, which are related to the center-of-mass energy $E_{c.m.}$ and incident angular momentum $J$. The maximal angular momentum $J_{max}$ is taken to be the grazing collision of two colliding nuclei. The capture cross section is given by $\sigma_{cap} = \pi \hbar^2 (2J + 1) T(E_{c.m.}, J)/(2\mu E_{c.m.})$. The transmission probability $T(E_{c.m.}, J)$ is calculated by the classical trajectory approach with a barrier distribution by $T(E_{c.m.}, J) = 0$ and 1 for $E_{c.m.} < B + J(J+1)\hbar^2/(2\mu R_C^2)$ and $E_{c.m.} > B + J(J+1)\hbar^2/(2\mu R_C^2)$, respectively. The $\mu$ and $R_C$ denote the reduced mass and Coulomb radius by $\mu = \frac{m_1 m_2}{m_1 + m_2}$.
\[ m_n A_p A_1 / (A_p + A_1) \] with \( m_n, A_p \) and \( A_1 \) being the nucleon mass and numbers of projectile and target nuclides, respectively. The distribution function is taken as the Gaussian form

\[ f(B) = \frac{1}{\sqrt{\pi} \sigma} \exp\left[-\frac{(B - B_m)}{\Delta}^2\right], \]

with the normalization constant satisfying the unity relation \( \int f(B) dB = 1 \). The quantities \( B_m \) and \( \Delta \) are evaluated by \( B_m = (B_C + B_S)/2 \) and \( \Delta = (B_C - B_S)/2 \), respectively. The \( B_C \) and \( B_S \) are the Coulomb barrier at waist-to-waist orientation and the minimum barrier by varying the quadrupole deformation of the colliding partners.

The nucleon transfer is described by solving a set of microscopically derived master equations by distinguishing protons and neutrons [53,54,57]. The time evolution of the distribution probability \( P(Z_1, N_1, E_1, t) \) for the DNS fragment 1 with proton number \( Z_1 \) and neutron number \( N_1 \) and excitation energy \( E_1 \) is governed by the master equations as follows:

\[
\frac{dP(Z_1, N_1, E_1, t)}{dt} = \sum_{Z_1'} W_{Z_1,N_1;Z_1',N_1}(t)
\times \left[ dZ_{1',N_1} P(Z_1', N_1, E_1', t)
- dZ_{1,N_1} P(Z_1, N_1, E_1, t) \right] + \sum_{N_1'} W_{Z_1,N_1;Z_1',N_1'}(t) \left[ dZ_{1',N_1'} P(Z_1, N_1', E_1', t) \right.
- \left. dZ_{1,N_1} P(Z_1, N_1, E_1, t) \right]
\]

(3)

Here the \( W_{Z_1,N_1;Z_1',N_1}(W_{Z_1,N_1;Z_1',N_1'}) \) is the mean transition probability from the channel \((Z_1, N_1, E_1)\) to \((Z_1', N_1, E_1')\) or \((Z_1, N_1, E_1)\) to \((Z_1', N_1', E_1')\), and \( dZ_{1,N_1} \) denotes the microscopic dimension corresponding to the macroscopic state \((Z_1, N_1, E_1)\). The cascade nucleon transfer is considered in the process with the relation of \( Z_1 = Z_1 \pm 1 \) and \( N_1 = N_1 \pm 1 \). It is noticed that the quasi-fission of DNS and the fission of heavy fragments are neglected in the dissipation process. Different with the fusion-evaporation reactions [53,54], the interaction time is short for the MNT reactions at the level of several zepptoseconds. The interaction time \( \tau_{int} \) is obtained from the deflection function method [59], which depends on the relative angular momentum and colliding system. On the other hand, the interaction potential is flat at the touching distance and no potential pocket exists in the heavy systems. So the quasi-fission barrier does not appear. We assume the quasi-fission and fission do not take place before the dissipation equilibrium. The initial probabilities of projectile and target nuclei are set to be \( P(Z_{proj}, N_{proj}, E_1 = 0, t = 0) = 0.5 \) and \( P(Z_{targ}, N_{targ}, E_1 = 0, t = 0) = 0.5 \). The unitary condition is satisfied during the nucleon transfer process \( \sum_{Z_1,N_1} P(Z_1, N_1, E_1, t) = 1 \). The motion of nucleons in the interacting potential is governed by the single-particle Hamiltonian [57]. The excited DNS opens a valence space in which the valence nucleons have a symmetrical distribution around the Fermi surface. Only the particles at the states within the valence space are actively at excitation and transfer. The averages on these quantities are performed in the valence space as follows.

\[
\Delta \varepsilon_K = \frac{4\varepsilon_k^*}{g_K}, \quad \varepsilon_k^* = \frac{A K}{A}, \quad g_K = A_K/12, \quad (4)
\]

where the \( \varepsilon_k^* \) is the local excitation energy of the DNS. The microscopic dimension for the fragment \((Z_K, N_K)\) is evaluated by the valence states \( N_K = g_K \Delta \varepsilon_K \) and the valence nucleons \( m_K = N_K / 2 \) (\( K = 1, 2 \)) as

\[
d(m_1, m_2) = \binom{N_1}{m_1} \binom{N_2}{m_2}, \quad (5)
\]

The transition probability is related to the local excitation energy and nucleon transfer, which is microscopically derived from the interaction potential in valence space as

\[
W_{Z_1,N_1;Z_1',N_1} = \tau_{mem}(Z_1, N_1, E_1; Z_1', N_1, E_1') \frac{dZ_1,N_1 dZ_1',N_1' \hbar^2}{2 \pi \hbar^2} \times \sum_{ii'} |(Z_1', N_1', E_1', i') | |V[Z_1, N_1, E_1, i]|^2. \quad (6)
\]

The memory time is calculated by

\[
\tau_{mem}(Z_1, N_1, E_1; Z_1', N_1, E_1') = \left[ \frac{2 \pi \hbar^2}{\sum_{KK'} V_{KK'}^* V_{KK'}^*} \right]^{1/2} \left[ \sum_{KK'} V_{KK'}^* V_{KK'}^* > = \frac{1}{4} U_{KK'}^2 g_K g_{K'} \Delta \varepsilon_K \Delta \varepsilon_{K'} \left[ \Delta \varepsilon_{K'}^2 + \frac{1}{6} (\Delta \varepsilon_K^2)^2 \right]^{-1/2} \right]^{1/2} \quad (7)
\]

The interaction matrix element is given by

\[
\sum_{ii'} |V_{ii'}|^2 = \left[ \omega_{11}(Z_1, N_1, E_1, E_1') + \omega_{22}(Z_1, N_1, E_1, E_1') \delta_{Z_1,N_1,E_1;Z_1,N_1,E_1'} + \omega_{12}(Z_1, N_1, E_1, E_1') \delta_{Z_1,N_1,E_1;Z_1-1,N_1,E_1'} + \omega_{21}(Z_1, N_1, E_1, E_1') \delta_{Z_1,N_1,E_1;Z_1+1,N_1,E_1'} \right] \quad (8)
\]

with the relation of

\[
\omega_{KK'}(Z_1, N_1, E_1, E_1') = d_{Z_1,N_1} < V_{KK'}, V_{KK'}^* > \quad (9)
\]

The similar process for neutron transfer takes place.

In the relaxation process of the relative motion, the DNS will be excited by the dissipation of the relative kinetic energy. The local excitation energy is determined by the dissipation energy from the relative motion and the potential energy surface of the DNS as
\[ \varepsilon^* (t) = E_{diss} (t) - \left( \mu (\alpha) \right) - U (\alpha_{E+N}) . \] (11)

The entrance channel quantities \( \alpha_{E+N} \) include the proton and neutron numbers, angular momentum, quadrupole deformation parameters and orientation angles being \( Z_p, N_p, Z_T, N_T, J, R, \beta_p, \beta_T, \theta_p, \theta_T \) for the projectile-target system. The excitation energy \( E_1 \) for fragment \( (Z_i, N_i) \) is evaluated by \( E_1 = \varepsilon^* (t = \tau_{int}) A_1 / A \). The energy dissipated into the DNS is expressed as

\[ E_{diss} (t) = \frac{E_{c.m.} - B - \left( \frac{J (J + 1)}{2 \zeta_{rel}} \right) \hbar^2}{2 \zeta_{rel}} \]

(12)

Here the \( E_{c.m.} \) and \( B \) are the center-of-mass energy and Coulomb barrier, respectively. The radial energy is evaluated from

\[ \langle E_{rad} (J, t) \rangle = E_{rad} (J, 0) \exp (-t / \tau_r) \] (13)

The relaxation time of the radial motion \( \tau_r = 5 \times 10^{-22} \) s and the radial energy at the initial state \( E_{rad} (J, 0) = E_{c.m.} - B - J (J + 1) \hbar^2 / (2 \zeta_{rel}) \). The dissipation of the relative angular momentum is described by

\[ \langle J (t) \rangle = J_{st} + (J_i - J_{st}) \exp (-t / \tau_J) \] (14)

The angular momentum at the sticking limit \( J_{st} = J_{\text{rel}} / \zeta_{tot} \) and the relaxation time \( \tau_J = 15 \times 10^{-22} \) s. The \( \zeta_{rel} \) and \( \zeta_{tot} \) are the relative and total moments of inertia of the DNS, respectively. The initial angular momentum is set to be \( J_i = J \) in Eq. (1). The relaxation time of radial kinetic energy and angular momentum dissipation is associated with the friction coefficients in the binary collisions.

The potential energy surface (PES) dominates the nuclear transfer and is given by

\[ U (\alpha) = B (Z, N) + V_{CN} (J) + V (\alpha) \]

(15)

Here \( Z \) and \( N \) are the proton and neutron number of the composite system with \( Z_1 + Z_2 = Z \) and \( N_1 + N_2 = N \) [34]. The symbol \( \alpha \) denotes the quantities \( Z_1, N_1, Z_2, N_2; J, R; \beta_1, \beta_2, \theta_1, \theta_2 \). The \( B (Z, N) \) and \( V (\alpha) \) are the negative binding energies of the fragment \( (Z_i, N_i) \) and the compound nucleus \( (Z, N) \), respectively. The \( V_{CN}^{rel} \) is the rotation energy of the compound system. The \( \beta \) represent the quadrupole deformations of the two fragments at ground state. The \( \theta \) denote the angles between the collision orientations and the symmetry axes of deformed nuclei. The interaction potential between fragment \( (Z_1, N_1) \) and \( (Z_2, N_2) \) includes the nuclear, Coulomb and centrifugal parts. In the calculation, the distance \( R \) between the centers of the two fragments is chosen to be the value at the touching configuration, in which the DNS is assumed to be formed. The tip-tip orientation is chosen in the calculation, which manifests the elongation shape along the collision direction and is favorable for the nucleon transfer to produce the MNT fragments. It should be noticed that the stochastic sampling of the orientation in the PES did not improve the fusion-evaporation excitation function [60]. Actually, the collision orientation of the deformed projectile-target system is partially included by the barrier distribution approach in Eqs (1) and (2).

The survival of primary fragments formed in the nucleon transfer is described with the statistical approach based on the Weisskopf evaporation theory [61], in which the decay of hot fragments is cooled by evaporating \( \gamma \) rays and light particles including neutrons, protons, \( \alpha \) in competition with binary fission. The probability in the channel of evaporating the \( x \)th neutron, the \( y \)th proton and the \( z - \alpha \) is expressed as

\[ \Gamma_{\text{tot}} (E^*_{1}, J^*_1, x, y, z) = P (E^*_{1}, J^*_1, x, y, z) \times \frac{\prod_{i=1}^{x} \Gamma_n (E^*_{1}, J^*_1) \prod_{j=1}^{y} \Gamma_p (E^*_{1}, J^*_1) \prod_{k=1}^{z} \Gamma_{\alpha} (E^*_{1}, J^*_1)}{\prod_{k=1}^{z} \Gamma_{\text{tot}} (E^*_{1}, J^*_1)} \] (16)

Here the \( J^*_1 \) and \( J^*_1 \) are the excitation energy and the angular momentum of the primary fragment, respectively. The total width \( \Gamma_{\text{tot}} \) is the sum of partial widths of particle evaporation, \( \gamma \)-emission and fission. The excitation energy \( E^*_{s} \) before evaporating the \( s \)th particle is evaluated by

\[ E^*_{s+1} = E^*_{s} - B^p_i - B^p_j - B^p_k - 2T_s \] (17)

with the initial condition \( E^*_{1} = E_{1} \) and \( s = i + j + k \). The \( B^p_i, B^p_j \) and \( B^p_k \) are the separation energies of the \( i \)th neutron, \( j \)th proton and \( z - \alpha \), respectively. The nuclear temperature \( T_i \) is given by \( E^*_{i} = a T_i^2 - T_i \) and the level density parameter \( a \). The evaporation width and realization probability are described in detail in reference [61]. For the production of the stable and neutron-rich isotopes, the neutron evaporation is dominant. The proton and \( \alpha \) emissions have non-negligible contributions in the deexcitation of proton-rich heavy nuclei.

### 2.2 Angular distribution

The precise estimation of emission angles of fragments formed in the MNT reactions is helpful for managing the detector system in experiments. The deflection function method is used to evaluate the emission angle of fragment, which is composed of the Coulomb and nuclear deflection as [62,63]

\[ \Theta (l_i) = \Theta_C (l_i) + \Theta_N (l_i) \] (18)

The Coulomb deflection is given by the Rutherford function as

\[ \Theta (l_i)_C = 2 \arctan \left( \frac{Z_1 Z_2 e^2}{2 E_{c.m.} b} \right) \] (19)
and the nuclear deflection
\[ \Theta(l_i)_{N} = -\beta \theta^{gr}_{C} l_{gr} \left( \frac{\delta}{\beta} \right)^{l_i/l_{gr}}. \]  

Here \( \theta^{gr}_{C} \) is the Coulomb scattering angle at the grazing angular momentum \( l_{gr} \) and \( l_{gr} = 0.22 R_{int} \left[ A_{red} \left( E_{c.m.} - V(R_{int}) \right) \right]^{1/2} \). The \( l_i \) is the incident angular momentum. The \( A_{red} \) and \( V(R_{int}) \) are the reduced mass of projectile and target nuclei and interaction potential with \( R_{int} \) being the Coulomb radius, respectively. The parameters \( \delta \) and \( \beta \) are parameterized by fitting the deep inelastic scattering in massive collisions as
\[ \beta = 75 f(\eta) + 15, \quad \eta < 375 \]
\[ 36 \exp(-2.17 \times 10^{-3} \eta), \quad \eta \geq 375 \]  
and
\[ \delta = 0.07 f(\eta) + 0.11, \quad \eta < 375 \]
\[ 0.117 \exp(-1.34 \times 10^{-4} \eta), \quad \eta \geq 375 \]  
with
\[ f(\eta) = \left[ 1 + \exp \left( \frac{\eta - 235}{32} \right) \right]^{-1}. \]

The Sommerfeld parameter \( \eta = \frac{Z_1 Z_2 e^2}{\nu} \) and the relative velocity \( \nu = \sqrt{\frac{2}{A_{red}} \left( E_{c.m.} - V(R_{int}) \right)} \). For the \( i \)-th DNS fragment, the emission angle is determined by \( \Theta(i)_{l_i} = \Theta(l_i)_{\xi_i} / (\xi_1 + \xi_2) \) with the moment of inertia \( \xi_i \). Shown in Fig. 1 is the deflection angle and reaction time as a function of initial angular momentum in the reaction of \( ^{238}U + ^{248}Cm \). More relaxation time is obvious in the central collisions. The shorter interaction time in the MNT reaction is obtained in comparison with the fusion–evaporation reaction [57].

### 3 Results and discussion

The synthesis of superheavy nuclei in the fusion–evaporation reactions has entered the bottleneck stage because of the lack of neutron number of formed compound nucleus. The hunting of new reaction mechanism is expected for synthesizing the neutron-rich isotopes. There are several possible pathways to reach the ‘island of stability’, i.e., the MNT reactions, incomplete fusion reactions and complete fusion reactions induced with radioactive nuclides etc. The damped collisions of two actinide nuclei were investigated and motivated for producing superheavy nuclei in 1970s at Gesellschaft für Schwerionenforschung (GSI) [23, 24]. The experimental data were reanalyzed for investigating the MNT dynamics, in particular for the transactinide production [64]. As a test of the DNS model, the available data in collisions of \( ^{238}U + ^{248}Cm \) at the beam energy of \( E_{lab} = 7.0 \text{ MeV/nucleon} \) \( (E_{c.m.} = 850 \text{ MeV}, V_{tip-tip} = 704 \text{ MeV} \text{ and } V_{waist-waist} = 805 \text{ MeV}) \) are compared as shown in Fig. 2. It is obvious that the isotopic yields are consistent with the available data. The maximal yield of each isotopic chain is located on the line of \( \beta \)-stability. The distribution structure of the isotopic yields is related to the PES, nucleon transfer dynamics, nuclear fission, particle evaporation etc. The Gaussian-like shape appears in the isotopic spectra. The underestimation of the Californium isotopes is caused from the missing of inelastic collision in the model, which takes place before the touching configuration of two colliding nuclei. The overestimation of neutron-rich isotopes of Es, Fm and Md is resulted from the larger yields of primary fragments. The problem might be improved by implementing dynamical deformation, neck formation and inclusion of all possible orientations into the PES.

The production of neutron-rich nuclei beyond the neutron number \( N = 162 \) is a step stone to reach the isotopes on the ‘island of stability’ etc. The fission mechanism and decay modes are helpful for investigating the structure of SHN. Shown in Fig. 3 is a comparison of isotopic distribution in the \( ^{238}U \) induced reactions on \( ^{252}Cf, ^{254}Es \) and \( ^{257}Fm \) at the beam energy of 7 MeV/nucleon. The Coulomb barrier is estimated as the potential height at the touching distance in two colliding nuclei. The values of 717, 725 and 731 MeV in the tip–tip collisions and 820, 827 and 834 MeV in the waist–waist collisions are used in the barrier distribution for the reactions of \( ^{238}U + ^{252}Cf, ^{238}U + ^{254}Es \) and \( ^{238}U + ^{257}Fm \), respectively. The open symbols denote the unknown isotopes in the nuclear chart [65]. Compared to the neutron-deficient nuclei via the fusion–evaporation reactions, the MNT reactions exhibit a broad isotopic distribution. The production cross sections decrease with increasing the proton number of fragments. The heavier target \( ^{257}Fm \) is favorable for creating the neutron-rich isotopes owing to the larger isospin ratio neutron/proton of colliding system. However, the different systems almost have the same order magnitudes of isotopic yields in the proton-rich domain. The maximal yields of the isotopic spectra in the MNT reactions correspond to the line of \( \beta \)-stability, in which the shell effect and fission barrier are of significance on the spectrum structure. The experiments are expected for creating the new nuclides and verifying the shell evolution in the neutron-rich region.

The isotopic dependence of projectile-target combination in the MNT reactions is of significance for the formation of neutron-rich isotopes. Figure 4 shows the isotopic distribution of Rf, Db, Sg and Bh in the reactions of \( ^{238}U + ^{252}Cf \) (solid line) and \( ^{238}U + ^{254}Cf \) (dotted line) at \( E_{lab} = 7.0 \text{ MeV/nucleon} \). The isotopic targets are selected for producing the MNT fragments. It is obvious that the overall products move to the neutron-rich side with the neutron-rich target \( ^{254}Cf \). However, the isotopes in the proton-rich region exhibit an opposite trend. The entrance system with the larger neutron/proton ratio is available for producing the neutron-
Fig. 1 Angular momentum dependence of the deflection angle and reaction time for the system $^{238}\text{U} + ^{248}\text{Cm}$ at the beam energy of 7 MeV/nucleon

Fig. 2 Production cross sections of transcurium isotopes in the $^{238}\text{U} + ^{248}\text{Cm}$ reaction at $E_{\text{lab}} = 7.0$ MeV/nucleon and compared with the available experimental data at GSI with error bars [64]
Fig. 3 Isotopic distribution of Rf, Db, Sg and Bh in the MNT reactions at $E_{\text{lab}} = 7.0$ MeV/nucleon with $^{238}\text{U}$ bombarding $^{252}\text{Cf}$, $^{254}\text{Es}$ and $^{257}\text{Fm}$, respectively. The open symbols denote the new isotopes [65].

The neutron shell closure around $N = 126$ is favorable for enhancing the cross sections of neutron-rich isotopes and extensive investigation in the MNT reactions with the DNS model [66, 71]. The shell effect enhances the fission barrier and enlarges the separation energy of the MNT fragment. On the other hand, the strong shell effect leads to the appearance of pocket in the PES, which also enhances the fragment formation in the nucleon transfer process. A number of neutron-rich isotopes around $N = 162$ might be created with the damped collisions of two actinide nuclei. More pronounced structure effect is shown in Fig. 7 from the isotonic yields of $N = 162$ in the MNT reactions of $^{238}\text{U} + ^{252}\text{Cf}$, $^{238}\text{U} + ^{254}\text{Es}$ and $^{238}\text{U} + ^{257}\text{Fm}$ at $E_{\text{lab}} = 7.0$ MeV/nucleon.
Fig. 4 Comparison of Rf, Db, Sg and Bh production in the MNT reactions at $E_{\text{lab}} = 7.0$ MeV/nucleon with $^{238}\text{U}$ bombarding $^{252}\text{Cf}$ and $^{254}\text{Cf}$, respectively. The open symbols denote the new isotopes with the recent nuclear mass evaluation [65].

The collisions of $^{238}\text{U} + ^{257}\text{Fm}$ have the larger cross sections because of the neutron/proton ratio and primary fragment yields. The three systems have the same maximal yields around the new isotope $^{263}\text{Md}$. The isotonic yields with $Z < 102$ are the yet unknown neutron-rich nuclides in the nuclear chart indicated by the dashed line. The systematic investigation of the new isotope production around $N = 162$ is complicated both in experiments and in theories. Different with the neutron shell closure of $N = 126$, the fission barrier and decay mode is not known in the superheavy region. There are uncertainties of some physical quantities and level spectra in the domain of isotopes around $N = 162$. More experiments and dynamical models in the MNT reactions around $N = 162$ are expected.

The collisions of two actinide nuclides provide the possibility to produce the new-neutron isotopes of transfermium elements. The yields of neutron-rich and long-living SHNs in the MNT reactions might be significantly enhanced owing to the shell effect. The shell effect is of significance in the MNT fragment formation, which is manifested via the PES, fission barrier and particle evaporation in the DNS model. Figure 8 shows the production cross sections as functions of atomic number (left panel) and mass number (right panel) in the reaction of $^{238}\text{U} + ^{252}\text{Cf}$ at $E_{\text{lab}} = 7.0$ MeV/nucleon. It is obvious that the production cross sections of primary fragments are enhanced around the neutron shell closure $N = 162$, which are caused from the shell effect in the reaction dynamics. For saving the CPU time, the primary and secondary fragments with the proton number $Z \geq 90$ are taken into account in the calculation. The production of secondary fragments is reduced by several orders of magnitude owing to the deexcitation process, in particular in the superheavy region. The decay stage of the MNT fragments is associated with the local excitation energy and angular momentum at the equilibrium time, fission barrier, separation energy of evaporated particles (neutron, proton, deuteron, triton, alpha etc). The strong shell effect can enhance the fission barrier and separation energy, which is favorable the survival of the primary fragment. For the cases without the shell effect, the production cross sections of the primary and secondary fragments monotonically decrease with the mass or charge number, which reduces the formation of heavy rare isotopes in the MNT reactions. The shell effect increases the MNT yields above the two order magnitude. The results are consistent with the model of multidimensional Langevin equations [39]. Inclusion of the structure factors such as the shell effect, odd–even effect, nucleus stiffness etc is particularly significant in the dynamical models for the MNT reactions.
Fig. 5 Incident energy dependance of isotopic distribution of Rf, Db, Sg and Bh in the MNT reaction of $^{238}$U + $^{252}$Cf

Fig. 6 Excitation functions of $^{266}$Rf, $^{267}$Db and $^{268}$Sg production in the MNT reaction of $^{238}$U + $^{252}$Cf

Fig. 7 Isotopic fragments with the neutron shell closure $N = 162$ in the MNT reactions with $^{238}$U bombarding $^{252}$Cf, $^{254}$Es and $^{257}$Fm at $E_{lab} = 7.0$ MeV/nucleon
Fig. 8 Cross sections as functions of the atomic numbers and the mass numbers of primary and secondary fragments with shell effect or not in the reaction of $^{238}\text{U}+^{252}\text{Cf}$ at $E_{\text{lab}} = 7.0$ MeV/nucleon

Fig. 9 The angular distributions of superheavy nucleus of $Z = 104$–$107$ in the laboratory frame in the $^{238}\text{U}+^{252}\text{Cf}$ reaction at $E_{\text{lab}} = 7.0$ MeV/nucleon (left panels) and the angular distributions of Rf with different incident energy (right panels)

The emission of the MNT fragments is anisotropic and related to the reaction system and beam energy [29]. Accurate estimation of the angular distribution would be helpful for managing the detector system in experiments, which is associated with the dynamical characteristics in MNT reactions, i.e., the deformation of the DNS fragments, dissipation of relative energy and angular momentum, nucleon transfer etc [72]. Figure 9 shows the selected nuclides with $Z = 104$–$107$ produced in the MNT reaction of $^{238}\text{U}+^{252}\text{Cf}$ at $E_{\text{lab}} = 7.0$ MeV/nucleon (left panel) and the angular distribution of Rf at the incident energies of 7, 7.5, 8 MeV/nucleon, respectively (right panel). Slight movement of the emission polar angles with the maximal yields for the SHEs Rf, Db, Sg and Bh appears from $80^\circ$ to $95^\circ$ at the incident energy of 7 MeV/nucleon. The nuclides produced in the MNT reactions move to the forward emission with the maximal cross section from $85^\circ$ at 7 MeV/nucleon to $50^\circ$ at 8 MeV/nucleon and manifest the narrow angular distribution with increasing the beam energy.

4 Conclusions

In summary, the production of neutron-rich isotopes via the MNT reactions was investigated within the DNS model in the $^{238}\text{U}$ induced reaction on $^{252,254}\text{Cf}$, $^{254}\text{Es}$ and $^{257}\text{Fm}$ at the incident energy around Coulomb barrier. By comparing the available experimental data, the DNS model is favorable for describing the MNT reaction dynamics. The shell effect, fission barrier, neutron separation energy and nucleon transfer dynamics influence the formation of MNT fragments. A
broad isotopic spectrum is exhibited in the MNT reactions. A number of neutron-rich isotopes might be created around the Coulomb barrier energy. The energy dependence of the neutron-rich nuclide production is weak. But the proton-rich isotopes are related the incident energy. The heavier target is favorable for the SHN production. The yields of MNT fragments monotonically decrease with the mass number. The shell effect, in particular around $N = 162$, results in the appearance of the bump structure in the MNT mass spectra. The phenomena is reduced by the secondary decay process. The MNT fragments tend to be emitted in the forward direction with increasing the incident energy. The slight difference on the angular distribution exists for different isotopes. More experiments are expected for the new isotope production around $N = 162$.

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