Chiral Properties of Quenched and Full QCD

R. D. Young¹, D. B. Leinweber¹, A. W. Thomas¹, and S. V. Wright¹,²

¹Special Research Centre for the Subatomic Structure of Matter, and Department of Physics and Mathematical Physics, University of Adelaide, Adelaide SA 5005, Australia
²Division of Theoretical Physics, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.

(Dated: March 25, 2022)

We extend a technique for the chiral extrapolation of hadron masses calculated with dynamical fermions to those generated by quenched simulations. The method ensures the correct leading and next-to-leading non-analytic behaviour for either QCD or quenched QCD in the chiral limit, as well as the correct large quark mass behaviour. We find that the primary difference between quenched and dynamical baryon masses can be described by the meson loops which give rise to the different leading and next-to-leading non-analytic behaviour.

Modern computing facilities, combined with innovations in improved actions for lattice QCD, mean that it is now possible to perform accurate quenched QCD (QQCD) simulations at quite low quark masses. For simulations with dynamical fermions (full QCD) the situation is much more difficult, but there are initial results at quark masses as low as 30 MeV. The latter development has inspired studies of chiral extrapolation aimed at using the full QCD data over a range of masses to reliably extract the physical hadron mass.

In general, effective field theories, such as chiral perturbation theory, lead to divergent or asymptotic expansions. While this raises doubts about the direct application of chiral perturbation theory to lattice data, studies of the mass dependence of hadron properties in QCD-inspired models, as well as the exactly soluble Euler-Heisenberg problem, suggest that one can develop surprisingly accurate extrapolation formulas, provided one builds in the correct behaviour in both the small and large mass limits. For the nucleon (N) and delta (Δ) masses (and by extension all other baryons), Leinweber et al. have suggested an extrapolation method which ensures both the exact low mass limit of chiral perturbation theory (technically its leading (LNA) and next-to-leading non-analytic (NLNA) behaviour) and the heavy quark limit of heavy quark effective theory (HQET). The transition between the chiral and heavy quark regimes is characterised by a mass scale Λ, related to the inverse of the size of the pion cloud source. The rapid, non-analytic variation of hadron properties, characteristic of chiral perturbation theory, is rapidly suppressed once the pion Compton wavelength is smaller than this size (i.e. \( m_π > \Lambda \)).

It is straightforward to extend the method of Ref. to QQCD. One simply includes all the Goldstone loops (including both π and η') which give rise to the LNA and NLNA behaviour of quenched chiral perturbation theory (QχPT). In principle, the parameters of the chiral Lagrangian are dependent on the number of dynamical fermion flavours, \( N_f \), but are independent of the masses of the quarks. This is a celebrated feature of partially quenched chiral perturbation theory. The extent of the \( N_f \) dependence is not well known, and a precise determination awaits dynamical fermion simulations with light dynamical quarks of varying number.

Phenomenological investigations of the role of the pion cloud in hadronic charge radii indicate that results consistent with experiment can be obtained by adding full-QCD chiral corrections to the results of quenched simulations at moderate to heavy quark masses. This suggests that the size of the pion source is not changed dramatically in going from the quenched approximation to full QCD, motivating the use of a common scale, \( \Lambda \), in QQCD and QCD. We proceed to fit quenched and dynamical lattice data assuming negligible \( N_f \) dependence in the chiral parameters and \( \Lambda \). The extent to which these assumptions hold can only be determined via further dynamical fermion calculations in the light quark regime.

By incorporating the chiral loops which give rise to the LNA and NLNA behaviour in QCD and QQCD we find a remarkable agreement between the fit parameters of each simulation. This supports our hypothesis that the behaviour of the source of the pion cloud within baryonic systems behaves much the same in both QQCD and QCD. The differences between quenched and full QCD is primarily described by the differences in the associated chiral loops. Since the chiral corrections are expected to be larger for N and Δ than for others, this suggests that a similar technique may be applicable to all baryons. This would enable quenched simulations to play a more valuable role, together with new experimental information from JLab and elsewhere, in deepening our understanding of baryon spectroscopy.

With regard to the properties of the N and Δ we find a spectacular difference in QQCD. Whereas the extrapolation of the N mass is essentially linear in the quark mass, the Δ exhibits some upward curvature in the quenched chiral limit. As a result, the Δ mass in QQCD is expected to be of the order 300–400 MeV above its mass in...
full QCD. The success of the extrapolation scheme also lends confidence to the interpretation of the $\Delta - N$ mass splitting as receiving a contribution of order 50 MeV from pion loops in full QCD and up to 250 MeV in QQCD [19]. The residual splitting in full QCD would then be naturally ascribed to some shorter range mechanism, such as the traditional one-gluon-exchange [20].

The method for extrapolating baryon masses proposed by Leinweber et al. [21] is to fit the lattice data with the form:

$$M_B = \alpha_B + \beta_B m_\pi^2 + \Sigma_B (m_\pi, \Lambda),$$

(1)

where $\Sigma_B$ is the sum of those pion loop induced self-energies which give rise to the LNA and NLNA behaviour of the mass, $M_B$. In the case of the $N$ this is the sum of the processes $N \rightarrow N\pi \rightarrow N$ and $N \rightarrow \Delta\pi \rightarrow N$, while for the $\Delta$ it involves $\Delta \rightarrow \Delta\pi \rightarrow \Delta$ and $\Delta \rightarrow N\pi \rightarrow \Delta$. In the heavy baryon limit, these four contributions ($B \rightarrow B'\pi \rightarrow B$) can be summarised as:

$$\sigma_{BB'} = -\frac{3}{16\pi^2 f_\pi^2}G_{BB'} \int_0^\infty dk \frac{k^4 u_{BB'}^2(k)}{\omega(k)(\omega_{BB'} + \omega(k))},$$

(2)

where $\omega(k) = \sqrt{k^2 + m_\pi^2}$ and $\omega_{BB'} = (M_{B'} - M_B)$, and the constants $G_{BB'}$ are standard SU(6) couplings [22].

The factor $u(k)$, which as an ultraviolet regulator, may be interpreted physically as the Fourier transform of the source of the pion field. Whatever choice is made, the form of these meson loop contributions guarantees the exact LNA and NLNA structure of chiral perturbation theory ($\chi$PT). Furthermore, such a form factor causes the self-energies to decrease as $1/m_\pi^2$ for $m_\pi >> \Lambda$. One commonly uses a dipole, $u(k) = (\Lambda^2 - \mu^2)^2/(\Lambda^2 + k^2)^2$, with $\mu$ the physical pion mass.

Quenched $\chi$PT is a low energy effective theory for quenched QCD [13,14], analogous to $\chi$PT for full QCD [21]. Sea quark loops are formally removed from QCD by a set of degenerate, bosonic quarks. These bosonic fields have the effect of cancelling the fermion determinant in the functional integration over the quark fields. This gives a Lagrangian field theory which is equivalent to the quenched approximation simulated on the lattice. The low energy effective theory is then constructed using the symmetry groups of this Lagrangian.

A study of the chiral structure of baryon masses within the quenched approximation has been carried out by Labrenz and Sharpe [14]. The essential differences from full QCD are: a) in the quenched theory the chiral coefficients differ from their standard values and b) new non-analytic structure is also introduced. The leading order form of the baryon mass expansion about $m_\pi = 0$ is

$$M_B = M_B^{(0)} + c_B^B m_\pi + c_B^B m_\pi^2 + c_B^B m_\pi^3 + c_B^B m_\pi^4 + c_{4L}^B m_\pi^4 \log m_\pi + \ldots$$

(3)

where the coefficients of terms non-analytic in the quark mass are model-independent. Throughout we use couplings as given in Ref. [14]. In addition, we have included an octet–decuplet mass splitting to explicitly give a value for $c_{4L}^B$ [19]. We also stress that the term in $m_\pi$ is absent in full QCD — such a term being unique to the quenched case.

In fitting quenched data we wish to replicate the analysis for full QCD while incorporating the known chiral structure of the quenched theory. The meson-loop, self-energy corrections to baryon masses can be described in the same form as for full QCD. The effect of quenching appears as a redefinition of the couplings in the loop diagrams in order that they yield exactly the same LNA and NLNA structure as given by $Q\chi$PT. For example, the analytic expressions for the pion-cloud corrections to the masses of the $N$ and $\Delta$ have the same form as the full QCD integrals (Eq. 1) with redefined quenched couplings. We refer to Ref. [19] for details. Assuming a weak $N_f$ dependence of the chiral parameters, we describe the quenched self-energies using the same tree level values of $D = 0.76$ and $F = 0.50$ as in full QCD.

In addition to the usual pion loop contributions, QQCD contains loop diagrams involving the flavour singlet $\eta'$ which also give rise to important non-analytic structure. Within full QCD such loops do not play a role in the chiral expansion because the $\eta'$ remains massive in the chiral limit. On the other hand, in the quenched approximation the $\eta'$ is also a Goldstone boson [13,23] and the $\eta'$ propagator is exactly the same as that of the pion.

As a consequence there are two new chiral loop contributions unique to the quenched theory. The first of these, $\delta_B^{(1)}$, corresponds to a single hairpin diagram such as that indicated in Fig. 1(a). This diagram is the source of the term proportional to $m_\pi^2$ (involving the couplings $\gamma$ and $\gamma'$ [14]) in the chiral expansion Eq. (8). The structure of this diagram is exactly the same as the pion loop contribution where the internal baryon is degenerate with the external state. The second of these new $\eta'$ loop diagrams, $\delta_B^{(2)}$, arises from the double hairpin vertex as pictured in Fig. 1(b). This contribution is particularly interesting because it involves two Goldstone boson propagators and is therefore the source of the non-analytic term linear in $m_\pi$.

The total meson loop contribution to the baryon self-energies is

$$\sigma_{BB'}^{(1)} = \frac{3}{16\pi^2 f_\pi^2} G_{BB'} \int_0^\infty dk \frac{k^4 u_{BB'}^2(k)}{\omega(k)(\omega_{BB'} + \omega(k))},$$

(2)

where $\omega(k) = \sqrt{k^2 + m_\pi^2}$ and $\omega_{BB'} = (M_{B'} - M_B)$, and the constants $G_{BB'}$ are standard SU(6) couplings [22].

![FIG. 1: Quark flow diagrams of chiral $\eta'$ loop contributions appearing in QQCD: (a) single hairpin, (b) double hairpin.](image-url)
energy within the quenched approximation is given by the sum of these four diagrams:

\[ \tilde{\Sigma}_B = \tilde{\sigma}_{\pi}^N + \tilde{\sigma}_{\eta'}(1) + \tilde{\sigma}_{\eta'}(2). \]  

(4)

As the resultant pion couplings in QQCD are quite a bit smaller than the corresponding full QCD couplings, \( \tilde{\Sigma}_B \) is smaller in magnitude than \( \Sigma_B \). We display the individual contributions to the \( \Delta \) mass for both quenched and full QCD in Fig. 2. It is notable that \( \tilde{\sigma}_{\eta'}(1) \) vanishing, so that at light quark masses the total quenched chiral loop contribution to the \( \Delta \) mass is repulsive, whereas it is attractive in full QCD.

It is now straightforward to fit the quenched lattice data with the form:

\[ \tilde{M}_B = \tilde{\alpha}_B + \tilde{\beta}_B m^2 + \tilde{\Sigma}_B(m, \Lambda). \]  

(5)

Once again the linear part describes how the mass of the pion-cloud source varies with quark mass. This form includes the expected behaviour of HQET where the \( \pi \) and \( \eta' \) loop contributions are suppressed. Since, as discussed earlier, the meson–baryon vertices are characterised by the source distribution, which is argued to be similar in quenched and full QCD, we take all vertices to have the same momentum dependence — i.e. a common dipole mass, \( \Lambda \). With this parameter fixed there are just two free parameters, \( \tilde{\alpha} \) and \( \tilde{\beta} \), to fit the quenched data for each baryon.

As described in Ref. [2], we replace the continuum integral over the intermediate pion momentum by a discrete sum over the pion momenta available on the lattice, thus encapsulating finite lattice spacing and volume artifacts.

The lattice data used for this analysis comes from a recent paper by Bernard et al. [3]. These results are obtained using an improved Kogut-Susskind quark action, which is known to give good scaling properties [24]. Unlike the standard Wilson fermion action, masses determined at finite lattice spacing are excellent estimates of the continuum limit results. The physical scale of both full and quenched data sets has been set via a variant of the Sommer scale [3]. This procedure, based on the static-quark potential where chiral corrections are negligible [19], provides a self-consistent determination of the scale for both simulations.

Fits of the the form for full QCD, Eq. 1, and quenched QCD, Eq. 5, are shown in Fig. 3. In fitting to data we choose a dipole mass of \( \Lambda = 0.8 \) GeV, which has been optimised to highlight the remarkably similar behaviour of the pion-cloud source in both quenched and dynamical simulations. Phenomenologically, this agrees with quite general expectations that it should be somewhat smaller than that for the axial form factor [25, 26, 27]. The parameters obtained from our fits are shown in Table I.

We highlight the fact that the best fit parameters, \( \alpha \) and \( \beta \), obtained from both the quenched and full simulations, agree within errors. This suggests that the quark mass behaviour of the pion-cloud source is quite similar in full and quenched QCD, lending support to the hypothe-
sis made in this analysis. This leads one to conclude that the dominant effects of quenching can be attributed to the first order meson loop corrections.

We have investigated the quark mass dependence of the $N$ and $\Delta$ masses within the quenched approximation. The leading chiral behaviour of hadron masses in quenched QCD is known to differ from the full theory. This knowledge has been used to guide the construction of a functional form which both reproduces this correct chiral structure, is consistent with current lattice simulations and encompasses the HQET properties. The success of this method in the quenched case further verifies the importance of including meson-induced self-energies when extrapolating lattice results.

We find that, although the quenched approximation gives rise to more singular behaviour in the chiral limit, these contributions are quickly suppressed with increasing quark mass. In the nucleon, the effects of quenching reduce the amount of curvature expected as lighter quark masses are simulated. In contrast, for the $\Delta$ we find some upward curvature of the mass in QQCD as the quark mass approaches zero. In addition, the $\Delta - N$ mass splitting increases to around 400 MeV at the physical point. As a consequence of this behaviour, the $\Delta$ mass in the quenched approximation is expected to differ from the physical mass by approximately 25%.

Our calculations suggest that the one-loop meson graphs which generate the leading and next-to-leading non-analytic behaviour are the primary difference between baryon masses in quenched and full QCD. Thus, rather than quenched lattice QCD being regarded as an uncontrolled approximation, we are able to make a quantitative estimate of errors over a range of quark mass. It is vital to investigate the assumptions made with further dynamical fermion simulations at low quark masses to test the extent to which the presented results hold. Nevertheless, this discovery represents a remarkable step forward in relating lattice QCD to observed hadronic properties.

We would like to thank S. Sharpe for numerous enlightening discussions concerning Q\chi PT as well as W. Detmold, M. Oettel, A. Williams and J. Zanotti for helpful conversations. This work was supported by the Australian Research Council and the University of Adelaide.

[1] UKQCD Collaboration, K. C. Bowler et al. Phys. Rev. D62 (2000) 054506.
[2] CP-PACS Collaboration, K. Kanaya et al. Nucl. Phys. Proc. Suppl. 73 (1999) 189–191.
[3] C. Bernard et al. Phys. Rev. D64 (2001) 054506.
[4] J. M. Zanotti et al. “Hadron masses from novel fat-link fermion actions,” accepted for publication in Phys. Rev. D. arXiv:hep-lat/0110216 and in preparation.
[5] CP-PACS Collaboration, S. Aoki et al. Phys. Rev. D60 (1999) 114508.
[6] F. J. Dyson Phys. Rev. 85 (1952) 631–632.
[7] J. C. Le Guillou and J. Zinn-Justin, eds., Large order behavior of perturbation theory. Amsterdam, Netherlands: North-Holland (1990) 580 p.
[8] D. B. Leinweber, D. H. Lu, and A. W. Thomas Phys. Rev. D60 (1999) 034014; E. J. Hackett-Jones, D. B. Leinweber, and A. W. Thomas Phys. Lett. B489 (2000) 143.
[9] E. J. Hackett-Jones, D. B. Leinweber, and A. W. Thomas Phys. Lett. B494 (2000) 89–99.
[10] W. Detmold et al. Phys. Rev. Lett. 87 (2001) 172001.
[11] G. V. Dunne, A. W. Thomas, and S. V. Wright. “Chiral extrapolation: An analogy with effective field theory,” accepted for publication in Phys. Lett. B, arXiv:hep-th/0110155.
[12] D. B. Leinweber, A. W. Thomas, K. Tsushima, and S. V. Wright Phys. Rev. D61 (2000) 074502.
[13] C. W. Bernard and M. F. L. Golterman Phys. Rev. D46 (1992) 853–857.
[14] J. N. Labrenz and S. R. Sharpe Phys. Rev. D54 (1996) 4595–4608.
[15] S. R. Sharpe and N. Shoresh, Phys. Rev. D 62 (2000) 094503.
[16] S. R. Sharpe and N. Shoresh, Phys. Rev. D 64 (2001) 114510.
[17] D. B. Leinweber and T. D. Cohen Phys. Rev. D47 (1993) 2147–2150.
[18] D. B. Leinweber, R. M. Woloshyn, and T. Draper Phys. Rev. D43 (1991) 1659–1678.
[19] R. D. Young, D. B. Leinweber, A. W. Thomas, and S. V. Wright, in preparation.
[20] N. Isgur Phys. Rev. D62 (2000) 054026.
[21] V. Bernard, N. Kaiser, and U.-G. Meissner Int. J. Mod. Phys. E4 (1995) 193-346.
[22] S. R. Sharpe Phys. Rev. D46 (1992) 3146–3168.
[23] D. B. Leinweber, A. W. Thomas, K. Tsushima, and S. V. Wright Phys. Rev. D64 (2001) 094502.
[24] MILC Collaboration, C. W. Bernard et al. Phys. Rev. D61 (2000) 114502.
[25] A. W. Thomas and W. Weise, The Structure of the Nucleon. Wiley-VCH, Berlin, 2001.
[26] P. A. M. Guichon, G. A. Miller, and A. W. Thomas Phys. Lett. B124 (1983) 109.
[27] A. W. Thomas and K. Holinde Phys. Rev. Lett. 63 (1989) 2025–2027.