UV (in)sensitivity of Higgs Inflation

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(in preparation).

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Motivation

- **Inflation** \( \ddot{a}(t) > 0 \)
  A.Guth, A.Linde, P. Steinhard’80

- **Who is the Inflaton?**

  ![Graph showing the relationship between two variables](image)

- **Request for simplicity,** \( \implies \) Only one scalar field in the **SM**
  LHC, PLANCK

- **Consistent simplicity**
Overview

• Higgs Inflation
  – Tree level
  – Quantum aspects (Unitarity, renormalizability ..)

• UV sensitivity
  – UV completion
  – RG flow and renormalization scale
  – Results for the CMB parameters
Higgs Inflation

\[ S = S_{SM} + \frac{M_{pl}^2}{2} \int \sqrt{-g} \left( 1 + \frac{2}{M_{pl}^2} \epsilon h^t h \right) R \]

• Einstein frame

\[ g_{E \mu \nu} = \Omega^2 g_{\mu \nu} \quad \gamma(\phi) (\partial_\mu \phi)^2 = (\partial_\mu \chi)^2 \]

\[ S = \frac{1}{2} \int \sqrt{-g_E} R(g_E) - \int \sqrt{-g_E} \left[ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + U(\chi) \right] \]

\[ U(\chi) = \frac{\lambda(\phi^2(\chi) - v_{ew}^2)}{4\Omega^4(\chi)} \]

UV (in)sensitivity of Higgs Inflation (Nikhef)
Density perturbations

\[ N_* = \int_{\chi_E}^{\chi_*} \frac{U}{dU/d\chi} \frac{d\chi}{M_{pl}} \quad \Rightarrow \quad \phi_* \simeq 9.13 M_{pl} / \sqrt{\xi} \]

\[ \Delta_R \left( \propto \frac{\lambda}{\xi^2} \right) \simeq 2.2 \cdot 10^{-9} \quad \Rightarrow \quad \xi \simeq 47000 \sqrt{\lambda} \]

\[ n_s \simeq 1 - \frac{2}{N_*} - \frac{3}{N_*^2} \simeq 0.967 \]

\[ r = 16 \epsilon_* \simeq (1 + \frac{1}{6 \xi_*}) \frac{12}{N_*^2} \simeq 0.0031 \]
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Unitarity bound

\[
S \supset \sqrt{-g} \frac{\xi}{M_{\text{pl}}^2} \phi^2 R_{\mu\nu} = \eta_{\mu\nu} + M_{\text{pl}}^{-1} h_{\mu\nu} \frac{1}{M_{\text{pl}}/\xi} \phi \Box h_{\mu\nu}
\]

Burgess '09, Barbon '09, Hertzberg '10

- Cutoff field dependent
- Considering Gauge interaction (Goldstone)

\[\mathcal{M}(\theta\theta \rightarrow \theta\theta) > 1\]

\[\Lambda_{\text{gauge}}(\phi_0) \sim \left( \frac{M_{\text{pl}}}{\xi}, \phi_0, \frac{M_{\text{pl}}}{\sqrt{\xi}} \right)\]

Bezrukov et al. '11, Burgess '14, Prokopec & Weenink '14
Renormalizability in EFT sense

- **Small regime** \( \phi \ll \frac{M_{pl}}{\xi} \)
  \( \delta_s = \xi \phi \)

- **Mid regime** \( \frac{M_{pl}}{\xi} \ll \phi \ll \frac{M_{pl}}{\sqrt{\xi}} \)
  \( \xi \rightarrow \delta_m^{-2} \xi, \quad \phi \rightarrow \delta_m^{\frac{3}{2}} \phi \)

- **Large regime** \( \phi \gg \frac{M_{pl}}{\sqrt{\xi}} \)
  \( \delta = 1/\xi \phi \)

Demand: at every order a finite number of counter terms
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UV completion

\[ \frac{\mathcal{L}_{HI}}{\sqrt{-g}} + \sum_i \frac{c_i}{\Lambda^n} \mathcal{O}^{n+4}_i \]

• Which shape for the suppression scale?
  \[ \Lambda = \Lambda_{gauge}(\phi) \]

• Which form for the higher d operators?

\[ \mathcal{L}_{new} \supset \frac{(\mathcal{H}^\dagger \mathcal{H})^3}{\Lambda^2} + \ldots \quad \mathcal{L}_{new} \supset \frac{m_h^2 \mathcal{O}^{(4)}}{\Lambda^2} \]
  – Preserving the quasi-shift symmetry
  – Effect only where really needed!

Bezrukov, Rubio and Shap.’14, Burgess, Patil, Trott ’14

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\[ \beta_i = \mu \frac{\partial \lambda_i}{\partial \ln \mu} = \beta_i^{HI} + \delta \beta_i \]

- Corrected Potential

\[ U = \frac{\lambda \phi^4}{4 \left( 1 + \frac{\xi \phi^2}{M_{pl}^2} \right)} + U^{(1)} + 2\text{-loop} + \ldots \]

- RG improvement

\[ U(\phi, g_i, \mu) = U(\phi, g_i(t), \mu(t)) \]
\[ \mu(t) = \mu e^t, \quad \frac{dg_i(t)}{dt} = \beta_i(g_j(t)) \]

Bezrukov, Grubinov, Shaposhnikov
Barvinsky, Kamenshchik, Kiefer, Starobinsky,
Simone, Hertzberg, Wilzcek
George, Mooij, Postma
Burgess, Patil, Trott

\[ \therefore (-1)^{f_i} c_i \frac{m^4(\phi)}{64 \pi^2} \ln \left( \frac{m^2(\phi)}{\mu^2} \right) \]

\[ m(\phi) = \frac{m_{\text{SM}}(\phi)}{\Omega(\phi)} \]

\[ \mu(t) \sim \frac{\phi}{\Omega} \equiv \frac{\phi}{\sqrt{1 + \xi(t) \phi^2}} \]
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Effect on the CMB observables

\[ U_{\text{eff}} = \frac{\lambda(t(\phi))\phi^4}{4 \left(1 + \frac{\xi(t(\phi))\phi^2}{M^2_{\text{Pl}}} \right)} \]

\[ t = \ln \left( \frac{\mu}{m_t} \right) = \ln \left( \frac{\phi}{m_t \sqrt{1 + \xi(t)\phi^2}} \right) \]

Inflationary regime \( \phi^2 \gg M^2_{\text{Pl}}/\xi \)

\[ \eta \approx -\frac{4}{3} \frac{1}{1 + \frac{1}{6\xi}} \delta + O(\delta^2) \]

\[ N_* \approx \frac{3}{4} Z_*^{-1} \left(1 + \frac{1}{6\xi_*} \right) \frac{1}{\delta_*} \]

\[ n_s \approx 1 - \frac{2}{N_*} + O(N_*^{-2}) \]

\[ \epsilon \approx \frac{4}{3} \frac{1}{(1 + \frac{1}{6\xi})} \delta^2 + O(\delta^3) \]

\[ \delta = \frac{1}{\xi \phi^2} \ll 1 \]

\[ 1 + \frac{\beta_\lambda}{4\lambda} \]

\[ 1 + \frac{\beta_\xi}{2\xi} \]

\[ r \approx \frac{12}{N^2_*} \left(1 + \frac{1}{6\xi_*} \right) + O(N_*^{-3}) \]

Same as tree level results
Running insensitivity

- $\mathcal{Z}_* \approx 0$
  \[ \frac{U_x}{U} \propto \mathcal{Z}\delta \]
  \[ \frac{U_{xx}}{U} \propto -\mathcal{Z}\delta \]

- Numerical results, example
  For Standard Model
  same as A.Kyle’14

- Second order, h.o. corrections $O(10^{-4})$ and suppressed
Conclusions

- Higgs inflation at tree level gives great prediction for $(n_s, r)$.
- The theory is not renormalizable, we need at least a particular UV completion.
- Threshold corrections to the slow roll parameter cancel in the expression for $r$, and independently of the form of this UV completion.

UV (in)sensitivity of Higgs Inflation
(Jacopo Fumagalli)
RG flow: \[ \Gamma[\phi_{cl}] = S[\phi_{cl}] + \Gamma^{1-loop} + \ldots \]

\[ V_{CW} = V_i(\phi_{cl}) + \frac{1}{64\pi^2} \sum_i (-1)^{f_i} s_i m_i^4(\phi_{cl}) \ln \left( \frac{m_i^2(\phi_{cl})}{\mu^2} - c_i \right) \]

Quantum corrections: two routes:
- Jordan
- Einstein

\[ V_J(\phi) + V_J^{(1)}_{CW} \xrightarrow{E} \frac{V_J(\phi)}{\Omega^4} + \frac{V_J^{(1)}}{\Omega^4} \]
\[ V_E(\phi) + V_C^{(1)}_{CW} = \frac{V_J}{\Omega^4} + V_E^{(1)} \]

- Different results in the GB sector
- Back reaction negligible only in the Einstein frame
- Approx. FLRW

Coleman-Weinberg '73

\[ g_E = \Omega^2 g_J \]

UV (in) sensitivity of Higgs Inflation

Jacopo Fumagalli (Nikhef)

S. Mooij, M. Postma '11
D. George, S. Mooij,
Masses
Higgs instability

\[ M_h = 125 \text{ GeV} \]
\[ 3\sigma \text{ bands in} \]
\[ M_t = 173.1 \pm 0.7 \text{ GeV} \]
\[ \alpha_s(M_Z) = 0.1184 \pm 0.0007 \]