A General Principle of Isomorphism: Integration of Regulator and Observer in the Control System

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Abstract
The problem is considered and the formalized method of integration of the observer and the regulator in the tracking control system of object of rather arbitrary type is offered. The solution of the problem is carried out on the basis of the General principle of isomorphism. Unlike existing methods, the structure of the control loop is not predetermined. The control loop, the regulator and the observer are determined by the structure of the control object and are formed in a uniquely defined way based on the separation theorem.

Keywords: the principle of isomorphism, the regulator, the observer, the control object, feedback, the separation theorem, integrant, the information-control system

1. Introduction
In [1, 2] the General principle of isomorphism is formulated and a new paradigm in the theory of systems is proposed, which allowed basing on this single principle to substantiate and strictly formalize all the basic concepts and properties of systems (simple, complex, large systems, emergence, causality, observability, controllability, checkability, complexability of systems, as well as postulates and principles of feedback, superposition, relativity, duality, etc.).

In work [2] the General principle of isomorphism allowed to deduce formal conditions of integration (Association) of systems in larger formations—the systems called complexes (complex or integrated systems), and to prove the known separation theorem for systems of very General type. The problems of synthesis or integration of systems into such complexes are very relevant [3–11].

In this article, the results of the General theory of integration [2] are concretized in the Appendix to the formally strict separation of the tasks of synthesis of the observer and the regulator and to their subsequent merging (integration) in a single tracking information-control system (ICS).

2. Description of the research
The problem of correct separation of synthesis tasks and subsequent integration of systems is solved. The following systems are considered: \( \Sigma_1 = (U, s_y, X) \) — control system, \( \Sigma_2 = (X_0, s_y, Z) \) — observation (evaluation) system, \( \Sigma = (X_0, s^*, X) \) — controlled object. The designations are interpreted in accordance with [2, 12–16] and will be clear from the context. As an initial premise, we note that according to [2] the map-structure \( s^*: X_0 \rightarrow X \) must be isomorphic.

It is necessary to define formal conditions of separation of tasks of synthesis of the regulator \( s_x: U \rightarrow X \) and the observer \( s_y: X_0 \rightarrow Z \), and then conditions of Association (integration) of systems of surveillance and control in single ICS of object of control. No preconditions are imposed on the structure of the observation-control loop. The formation of the contour according to formalized rules is also considered as one of the tasks of the synthesis of ICS. It is only required that the object \( \Sigma \) satisfy (defined in [2] by the corresponding theorems) the conditions of observability and controllability with accuracy up to the isomorphism \( s^* \).
For a fixed \( s' \) object structure, a commutative diagram of an integrated system combining a surveillance system and a control system is shown in Fig. 1 [2]. The General conditions for the integration of these systems are determined by the theorem proved in [2]. Its wording is given below.

![Figure 1. Commutative diagram of an integrated system.](image)

**Theorem 1 (theorem of integration).** The necessary and sufficient conditions for the integration of the surveillance system \( \Sigma = (X_0, s_x, Z) \) and the control system \( \Sigma = (U, s_y, X) \) into a single ICS that provides targeted control of the object \( \Sigma = (X_0, s', X) \) are:

1) the condition of observability \( sN = hs' \) with accuracy up to isomorphism \( s' \);

2) controllability condition \( sR = s'\beta \) up to the same isomorphism \( s' \);

3) the General condition of isomorphic determinism of the functioning of the system, determined by the existence of a composition — complex of maps

\[
\begin{align*}
s_j &= hs'\beta = h(s'\beta) = hs_x = (hs')\beta = s_y \beta, \\
s_j &= hs_x = s_y \beta. 
\end{align*}
\]

Composition (1) will be called "integrant." In accordance with theorem 1, surveillance and control systems can be combined into an integrated system-complex if and only if there is an integrant (1) that allows formally strictly and unambiguously calculate the vector \( Z \) of sensor readings from the control vector \( U \) at the input and, conversely, on the known vector \( Z \) to calculate the vector \( U \) required to obtain it, that is, to close the feedback from \( Z \) to \( U \).

Thus, the feedback here is not introduced as a postulate, as is usually done, but is calculated in the form of an integrant (1), the existence of which is a necessary and sufficient condition for the correct unification of the observer and the regulator into a single system. Integrant (1) combines both conditions—the observability condition and the controllability condition of the system with the accuracy of the same model-isomorphism \( s' \), characterizing the structure of the control object.

The indication "with the accuracy up to model-isomorphism" is very important and means that all statements made are valid strictly within the framework of a commutative diagram built on the \( s' \) isomorphism. The point is that the \( s' \) map must be isomorphic in the usual sense initially-outside of compositions with other maps, outside of the commutative diagram. In this case, it uniquely (with the accuracy up to isomorphism \( s' \)) determines the properties of all other mappings composing with it in the commutative diagram. The meaning of the statement "with the accuracy up to isomorphism" is explained in more detail in [2, 12].

The isomorphism condition of the \( s' \) map is not too strong. For example, all stationary linear dynamical systems, often used in practice, have the property of isomorphism due to the mandatory reversibility of the fundamental matrix of solutions of the corresponding systems of differential equations [5, 13, 14]. In addition, in [2] cases of integration are considered, when the property of isomorphism in the usual sense (before the conclusion in the commutative diagram) has not a map \( s \), characterizing the actual control object, but some other map, for example, \( h \) or \( \beta \). We will not dwell on these cases here.

According to the realization theorem proved in [2], relying on the General principle of isomorphism, the integrant (1) is an isomorphism with the accuracy up to the isomorphism \( s' \) and is therefore unique in the commutative diagram. This makes it possible to accurately, completely and unambiguously predict the readings of the sensors \( Z \) on the vector \( U \) and, conversely, the same accurate, complete and unambiguous determination of controls \( U \) on the readings of \( Z \) sensors. The presence of the isomorphism \( s' \) common to the observer and the regulator provides the possibility of their Union ("gluing") into a single integrated system-complex, that is, ensures the existence of a composition of maps (1), which is an integrant.

The diagram in Fig. 1 is based on the classical definitions of controllability and observability [2]. In practice, according to the indications of \( Z \) sensors, it is necessary to restore (observe) not the vector \( X_0 \), but the vector \( X \) to be regulated at the output of the object. Vector \( X \) can be observed using an isomorphic with the accuracy up to the isomorphism \( s' \) observer \( \tilde{X}_0 \), more precisely, the inverse of it observer

\[
(\tilde{X}_0) = s'(X_0)^{-1} = s'(hs')^{-1} = s'(s')^{-1} \beta = \beta^{-1},
\]

where \( \beta^{-1} \) is defined up to the isomorphism \( s' \) by the rules set out in [2, 12–16]. Then the commutative diagram is converted to the equivalent form shown in Fig. 2.

![Figure 2. Equivalent commutative diagram of an integrated system.](image)
General case of arbitrary, but satisfying the conditions of theorem 1, maps \( h, s', \beta \). Its wording is given below.

Theorem 2 (separation theorem). For a dynamical system with an isomorphic structure \( s' \), the integrant \((s_f)^{-1}\) is a composition of the regulator and the observer
\[
(s_f)^{-1} = (s_h)^{-1} (s_o)^{-1},
\]
where
\[
(s_h)^{-1} = \beta^{-1} (s')^{-1},
\]
\[
(s_o)^{-1} = h^{-1}.\]

Theorem 2 follows from theorem 1. If the ICS meets the conditions of theorems 1 and 2, then the necessary and sufficient conditions for the separation of synthesis tasks and the subsequent integration of the observer and the regulator in a single control system are fulfilled.

Separation theorem allows to decompose strictly formally, the General problem of synthesis of a control object into two "independent" tasks: the synthesis problem is of an isomorphic observer, providing the observation of the output \( X \), and the synthesis problem is of an isomorphic regulator that implements feedback from output \( X \) to the control inputs \( U \). This provides the ability to control the object in accordance with externally given purpose.

The word "independent" is quoted, because these problems must have a mandatory common element — the same isomorphic model of the object.

First, the problem of observer \((s_o)^{-1}\) synthesis is solved. Then the problem of synthesis of the regulator \((s_h)^{-1}\) is solved. The composition of the feedbacks realized by the observer and the regulator forms a General feedback from \( Z \) to \( U \) in the form of an integrant (2). The separation theorem allows us to correctly determine the structure of all feedbacks necessary to control the object, which is the practical significance of the separation theorem. In General, the separation theorem proves the necessity and sufficiency of the existence of feedback in the form of integrant (2) for unambiguous, complete and accurate control of the object.

In known works (see, for example, [6–9]) in integration problems, the structure of the observation-control loop is usually postulated in advance. In this regard, it is sometimes concluded that the classical separation theorem is incorrect [5]. In this paper, on the basis of only classical definitions of observability and controllability, the structure of the integrated circuit is determined directly as a result of solving the integration problem, taking into account the isomorphism principle proved in [1, 2]. In addition, the validity of the separation theorem is confirmed in the more General case — for nonlinear systems of General form.

Well-known extreme cases and easily verifiable examples also confirm the correctness of the results obtained. For example, for a scalar object (in which all vectors \( U, X, Z \) have only one component), the regulator in accordance with (3) is determined by the trivial inversion of the scalar "transfer function" of the object \( W_0 \) [13, 14], which is the composition \( W_0 = s' \beta \), that is
\[
(s_f)^{-1} = (W_0)^{-1} = \beta^{-1} (s')^{-1},
\]
and the observer by virtue of the fact that the only component in \( X \) in this case is measured directly (that is, \( h = 1 \)) is also trivial and according to (4) is equal to \((s_o)^{-1} = 1\).

For multivariable linear object, which in the General case, the size of the vectors \( U, X \) and \( Z \) are different (for example, \( U \) — \( r \)-vector, \( X \) an \( n \)-vector and \( Z \) an \( m \)-vector), the regulator, as before, is determined by the inverse of a "transfer function" of the object \( W_0 \) [13, 14], but the formula only in form coinciding with (5). In this case, the inverse \( \beta^{-1} \) and \((W_0)^{-1} \) in (5) must be determined up to the isomorphism \( s' \) according to the rules given in [2, 12]. According to the same rules, the \( h^{-1} \) mapping defining the structure of the observer (4) must be defined.

If we introduce the concept of "generalized control object" (GCO), which combines the composition of all three known mappings \( \beta, s', h \), shown in Fig. 2, the control loop is converted to the view shown in Fig. 3. For a linear multi-connected GCO, in whose structure all mappings are written in Laplace operator form, the contour is converted to the form shown in Fig. 4, where \((s_f)^{-1} = \beta^{-1} (s')^{-1} h^{-1} \) is a transfer function in operator form. The inverse \( \beta^{-1} \) and \( h^{-1} \) are determined up to the isomorphism \( s' \) according to the rules of works [2, 12]. Here the transfer function \( W_{GCO} = hs' \beta \) The outline in Fig. 2 and 3 have a "multiplicative" form characteristic of commutative diagrams characterizing compositions ("products") of mappings [2, 17, 18]. The outline in Fig. 4 has a traditional "additive" form, typical for operations on transfer functions in the operator form [5].
3. Conclusion

Thus, within the framework of the General principle of isomorphism, based on the classical definitions of controllability and observability, a proof of the separation theorem for the General case of nonlinear systems is given, and a fully formalized method for the synthesis of structures of integrated ICS is proposed.

As follows from the more General formulation and proof of the separation theorem, the behavior of an integrated system is completely deterministic and predictable, that is, an integrated ICS is a simple system in the sense of [2].

References

[1] Kulabukhov V S 2014 Printsip izomorfnosti v zadache realizatsii i yego prilozheniya k analizu svoystv sistem upravleniya Proceedings of the XII Vserossiysk, Soveshch po Problemam Upravleniya VSPU-2014, IPU RAN pp 438–48 http://vspu2014.ipu.ru/prcdngs
[2] Kulabukhov V S 2018 Cloud of science 3 400–72
[3] Kulabukhov V S 2007 Algebraicheskaya formalizaciya procedur analiza i sinteza slozhnyh x integrirovanhьh x sistem 6 Mezhdunarodnaya konferenciya “Aviaciya i kosmonavtika – 2007”
[4] Bukov V N, Kulabuxov V S, Kos’yanchuk V V, Ryabchenko V N, Goryunov S V and Naumov A I 2007 Osnovy’ integracji sistem aviacionnogo oborudovaniya. Uchebnoe posobie (Moscow: Izd. VVIA im. prof. N.E. Zhukovskogo)
[5] Krasovskii A A 1987 Handbook on the Theory of Automatic Control (Moscow: Nauka)
[6] Kos’yanchuk V V 2002 Avtomatika i telemekhanika 6 23–35
[7] Kos’yanchuk V V 2004 Sintez integrirovanhьh x sistem nablyudeniya i upravleniya s zadannyh x dinamicheskimi xarakteristikami Navigacija i upravlenie dvizheniem. Sbornik dokladov VI konferenciya molodyh x uchenyh x 215–21
[8] Kos’yanchuk V V 2005 Girokoptery i navigaciya 48 100
[9] Kosjanchuk V V, Bukov V N and Ryabchenko V N 2001 Automation and Remote Control 3 15
[10] Kulabukhov V S 2015 Aviakosmicheskoye priborostroyenie 12 11–31
[11] Kulabukhov V 2019 Algebraic Formalization of System Design Based on a Purposeful Approach IOP Conference Series: Materials Science and Engineering 476 012017 https://iopscience.iop.org/article/10.1088/1757-899X/476/1/012017
[12] Kulabukhov V S 2019 Symmetry 11 1301 doi:10.3390/sym11101301
[13] Kulabukhov V S 2017 Mechatronics, automation, control 18 507–15
[14] Kulabukhov V 2017 Linear Isomorphic Regulators MATEC Web of Conferences, Proceedings of the CMIA2016 9 03008 doi:10.1051/matecconf/20179903008
[15] Kulabukhov V S 2017 Radiotehnika 8 50–5
[16] Kulabukhov V S 2017 Isomorphic observers of the linear systems state IOP Conference Series, Proceedings of the Materials Science and Engineering in Aeronautics (MEA2017) 312 012016 http://iopscience.iop.org/issue/1757-899X/312/1
[17] Kostrikin A I 1994 Vvedenie v Algebra: Osnovy Algebra (Moscow: Fiziko-Matematicheskaia Literatura)
[18] Bukov V N 2006 Vlozeniye Sistem. Analiticheskiy Podkhod k Analizu i Sinteza Matrichnyh System (Kaluga: Izd-vo nauchn. Literaturny N.F. Bochkarevoy)