Canonical interpretation of $Y(10750)$ and $\Upsilon(10860)$ in the $\Upsilon$ family

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Inspired by the new resonance $Y(10750)$, we calculate the masses and two-body OZI-allowed strong decays of the higher vector bottomonium states within both screened and linear potential models. We discuss the possibilities of $\Upsilon(10860)$ and $Y(10750)$ as mixed states via the $S - D$ mixing. Our results suggest that $Y(10750)$ and $\Upsilon(10860)$ might be explained as mixed states between $5S$- and $4D$-wave vector $b\bar{b}$ states. The $Y(10750)$ and $\Upsilon(10860)$ resonances may correspond to the mixed states dominated by the $4D$- and $5S$-wave components, respectively. The mass and the strong decay behaviors of the $\Upsilon(11020)$ resonance are consistent with the assignment of the $\Upsilon(6S)$ state in the potential models.

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I. INTRODUCTION

Very recently, the Belle Collaboration reported a new measurement of the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) cross sections at energies from 10.52 to 11.02 GeV using data collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider [1]. Besides two old vector states $\Upsilon(10860)$ and $\Upsilon(11020)$, a new resonance near 10.75 GeV, i.e. $Y(10750)$ as named in Ref. [2], was obviously found in the cross sections. The Breit-Wigner mass and width of this new structure are found to be $M = (10752.7 \pm 5.9^{+17.5}_{-11.3})$ MeV and $\Gamma = (35.5^{+17.3}_{-11.3})$ MeV. The production processes indicate that the spin-parity numbers of these three states appearing in the cross sections should be $J^{PC} = 1^{--}$. It is a great challenge for our understanding these states with the conventional $S$-, and $D$-wave bottomonium ($b\bar{b}$) states in the potential models.

The general consensus is that $\Upsilon(10860)$ and $\Upsilon(11020)$ correspond to the $S$-wave vector $b\bar{b}$ states $\Upsilon(5S)$ and $\Upsilon(6S)$, respectively [3,12]. However, if one assigns $\Upsilon(10860)$ to $\Upsilon(5S)$, we should face several problems, such as (i) the mass of $\Upsilon(5S)$ from the recent potential model calculations is about $70 - 90$ MeV lower than the observed value of $\Upsilon(10860)$ [8-12]; (ii) the mass splittings $m[\Upsilon(5S) - \Upsilon(4S)]_{\text{lab}} \approx 210$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{\text{lab}} \approx 180$ MeV predicted within various potential models [8-12] are inconsistent with the observations $m[\Upsilon(5S) - \Upsilon(4S)]_{\text{exp}} \approx 306$ MeV and $m[\Upsilon(6S) - \Upsilon(5S)]_{\text{exp}} \approx 115$ MeV.

For the newly observed $Y(10750)$, we also meet several problems if we explain it with a pure $S$-, or $D$-wave vector bottomonium state. According to the predictions in potential models, the $Y(10750)$ resonance lies between the vector states $\Upsilon(5S)$ and $\Upsilon(13D)$ [2,11]. Thus, if the $Y(10750)$ resonance correspond to a pure vector $b\bar{b}$ state, it should be assigned to either $\Upsilon(5S)$ or $\Upsilon(13D)$. However, if one assigns the $Y(10750)$ resonance to $\Upsilon(5S)$, we should meet a problem at once: how do we assign the $\Upsilon(10860)$ in the bottomonium family? On the other hand, if one assigns the $Y(10750)$ resonance to the $\Upsilon(1)(3D)$ state, we cannot understand the production of $Y(10750)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes, where the production cross sections of the $D$-wave states should be strongly suppressed for their very tiny dielectron widths predicted in theory [3,13,14].

The above analysis indicates that both $\Upsilon(10860)$ and $Y(10750)$ cannot be simply explained with a pure $S$- or $D$-wave $bb$ state with $J^{PC} = 1^{--}$. Thus, in the literature these resonances were suggested to be exotic states, such as, compact tetraquarks [2,15], or hadrobottomonium [16]. It should be emphasized that although $\Upsilon(10860)$ and $Y(10750)$ are not good candidates of a pure $S$- or $D$-wave $bb$ states, we cannot exclude them as mixed states between the $S$- and $D$-wave vector $bb$ states. In Refs. [13,14], Badalian et al. studied the dielectron widths of the vector bottomonium states, their results indicate that there might be sizeable $S - D$ mixing between the $nS$- and $(n - 1)D$-wave $(n \geq 4)$ vector states. If there is $S - D$ mixing indeed, the masses of the pure $S$- and $D$-wave states should shift to the physical states by some interactions. Then we may overcome the mass puzzles of the $\Upsilon(10860)$ as a pure $\Upsilon(5S)$ state. On the other hand, if there is $S - D$ mixing indeed, the physical states might have sizeable components of both $S$- and $D$-wave states. Considering the $Y(10750)$ as a mixed state dominated by the $D$-wave component, we may explain the large production cross sections in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) processes for its sizeable $S$-wave component. In fact, the $S - D$ mixing might also exist in the other meson spectra, such as the $\psi(3770)$ is suggested to be a $1D_1$ state with a small admixture of $2S_1$ state in the $cc$ family [5,17,19], while the $D_{s1}(2660)$ and $D_{s1}(2700)$ might be mixed states via the $2S_1 - 1D_1$ mixing in the $D$ and $D_s$ meson families, respectively [20,26].

In this work, we discuss the possibilities of $\Upsilon(10860)$ and $Y(10750)$ as mixed states via the $S - D$ mixing. By analyzing the mass spectrum of higher vector bottomonium states above the $B\bar{B}$ threshold within both screened and linear potential models, and calculating their strong decays with the $3P_0$ model, we suggest that $Y(10750)$ and $\Upsilon(10860)$ might correspond to the two mixed states between $5S$- and $4D$-wave vector $b\bar{b}$ states with a sizeable mixing angle. The $Y(10750)$ and $\Upsilon(10860)$ resonances could be the $S - D$ mixed states domi-
nated by the 4D- and 5S-wave components, respectively.

This paper is organized as follows. The mass spectrum of higher vector bottomonium is calculated in Sec. II. The \(^3P_0\) model is briefly introduced and strong decays the vector \(b \bar{b}\) states are calculated in Sec. III. Then, combining the mass and widths, we carry out a discussion about the properties of the \(J^{PC} = 1^-\) states \(Y(10750), \bar{Y}(10860)\) and \(\bar{Y}(11120)\) in Sec. IV. Finally, a short summary is presented in Sec. V.

II. MASS SPECTRUM

The mass spectrum of bottomonium has been calculated in our previous works \cite{10, 27} within the widely used linear potential model \cite{5, 7, 28, 31} and screened potential model \cite{5, 32–35}. In these potential models, the effective potential of spin-independent term \(V(r)\) is regarded as the sum of Lorentz vector \(V_\ell(r)\) and Lorentz scalar \(V_S(r)\) contributions \cite{5}, i.e.,

\[
V(r) = V_\ell(r) + V_S(r).
\]

The Lorentz vector potential \(V_\ell(r)\) can be written as the standard color Coulomb form

\[
V_\ell(r) = -\frac{4}{3} \frac{\alpha_s}{r}.
\]

The Lorentz scalar potential \(V_S(r)\) might be taken as a simple form with a linear potential \cite{5, 7, 28–31} or the screened potential as suggested in Refs. \cite{5, 32–35}, i.e.,

\[
V_S(r) = \begin{cases} 
\frac{br}{\mu} & \text{Linear potential} \\
\frac{b(1-e^{-r/\mu})}{\mu} & \text{Screened potential}.
\end{cases}
\]

Furthermore, we include three spin-dependent terms as follows. For the spin-singlet and hyperfine potential, we take \cite{31}

\[
H_{SS} = \frac{32\alpha_s^2}{9m_b^2} \tilde{\delta}_\ell(r) S_b \cdot S_b,
\]

where \(S_b\) and \(S_\bar{b}\) are spin matrices acting on the spins of the quark and antiquark. We take \(\tilde{\delta}_\ell(r) = (\sigma/\sqrt{\pi}) e^{-\sigma^2 r^2}\) as in Ref. \cite{31}. For the spin-orbit and the tensor terms, we adopt \cite{5}:

\[
H_{SL} = \frac{1}{2m_b^2 r} \left( 3 \frac{dV_\ell}{dr} - dV_S \right) S \cdot L,
\]

and

\[
H_T = \frac{1}{12m_b^2} \left( \frac{dV_\ell}{r} \frac{dr}{dr} - \frac{d^2V_S}{dr^2} \right) S \cdot T,
\]

where \(L\) is the relative orbital angular momentum of \(b\) and \(\bar{b}\) quarks, \(S = S_b + S_\bar{b}\) is the total quark spin, and the spin tensor \(S_T\) is defined by \cite{5}

\[
S_T = \frac{6}{r^2} \frac{\mathbf{S} \cdot \mathbf{r} \mathbf{S} \cdot \mathbf{r}}{r^2} - 2S^2.
\]

If the linear potential is adopted, four parameters \((\alpha_s, b, m_b, \sigma)\) in the above equations should be determined, while if the screened potential is adopted, five parameters \((\alpha_s, b, m_b, \sigma, \mu)\) should be determined.

We solve the radial Schrödinger equation by using the three-point difference central method \cite{36}. With this method, one can reasonably include the corrections from these spin-dependent potentials to both the mass and wave function of a meson state. The details of the numerical method can be found in our previous works \cite{10, 37}. With the same parameter sets determined in our previous works, we calculated the masses of the vector \(b \bar{b}\) states, \(nS\) \((n = 4, 5, 6), nD\) \((n = 3, 4, 5)\), above the \(B \bar{B}\) threshold within both the linear and screened potential models.

The calculated bottomonium masses are listed in Table I and also shown in Fig. I. It is found that both the linear and screened potential models give a similar prediction of the masses. Considering the \(\bar{Y}(10580)\) and \(\bar{Y}(11120)\) resonances as the \(\bar{Y}(4S)\) and \(\bar{Y}(6S)\) states, respectively, their observed masses are in good agreement with the potential model predictions. However, considering the \(Y(10860)\) as the \(Y(5S)\) state, the observed mass is obviously (about 70 – 90 MeV) larger than the model predictions. Fig. I shows that the newly observed state \(Y(10750)\) lies about 100 MeV above \(Y_1(3D)\), while about 40 – 50 MeV below \(Y(5S)\).

III. STRONG DECAYS

In this section, we use the \(^3P_0\) model \cite{38, 40} to evaluate the Okubo-Zweig-Iizuka (OZI) allowed two-body strong decays of the vector bottomonium. In this model, it assumes that the vacuum produces a light quark-antiquark pair with the quantum number \(0^+\) and the bottomonium decay takes place though the rearrangement of the four quarks. The transition operator \(\hat{T}\) can be written as

\[
\hat{T} = -3\gamma \sqrt{96\pi} \sum_m \sum_{m'} \langle 1m1 - m'00 \rangle \int dp_3 dp_4 \delta^3(p_3 + p_4)
\]

\[
\times \int d^{34}\left( \frac{p_3 - p_4}{2} \right) \lambda_{1m}^{34} \phi_0^{34} \alpha_0^{34} b_{3j}^i(p_3)d_{4j}^i(p_4),
\]

where \(\gamma\) is a dimensionless constant that denotes the strength of the quark-antiquark pair creation with momentum \(p_3\) and \(p_4\) from vacuum; \(b_{3j}^i(p_3)\) and \(d_{4j}^i(p_4)\) are the creation operators for the quark and antiquark, respectively; the subscripts, \(i\) and \(j\), are the SU(3)-color indices of the created quark and anti-quark; \(\phi_0^{34} = (\alpha^0 + d\bar{d} + s\bar{s})/\sqrt{3}\) and \(\alpha_0^{34} = 1/\sqrt{3}\delta_{ij}\) correspond to flavor and color singlets, respectively; \(\lambda_{1m}^{34}\) is a spin triplet state; and \(\gamma_{1m}(k) \equiv |k|^m Y_{1m}(\theta_0, \phi_0)\) is the \(l\)-th solid harmonic polynomial.

For an OZI allowed two-body strong decay process \(A \to B + C\), the helicity amplitude \(M^{M_A M_B M_C}(P)\) can be calculated as follow

\[
\langle BC|T|A\rangle = \delta(P_A - P_B - P_C) M^{M_A M_B M_C}(P).
\]

With the Jacob-Wick formula \cite{41}, the helicity amplitudes
The spectrum of the higher vector bottomonium above the $B \bar{B}$ threshold predicted within both screened potential (SP) and liner potential (LP) models. For a comparison, the experimental observations [1, 45] are plotted in the figure as well.

Table I: Predicted masses (MeV) of higher vector bottomonium states with both the linear potential (LP) and screened potential (SP) models. For a comparison, the results from recent works and experimental observations are also listed.

| State      | $n S_L$ | LP | SP | Ref. [11] | Ref. [8] | Ref. [9] | Exp. |
|------------|---------|----|----|-----------|---------|---------|------|
| $\Upsilon (4S)$ | $4^3 S_1$ | 10581 | 10597 | 10612 | 10635 | 10607 | 10579 |
| $\Upsilon (5S)$ | $5^3 S_1$ | 10794 | 10811 | 10822 | 10878 | 10818 | 10885 |
| $\Upsilon (6S)$ | $6^3 S_1$ | 10986 | 10997 | 11001 | 11102 | 10995 | 11000 |
| $\Upsilon_1 (3D)$ | $3^1 D_1$ | 10646 | 10658 | 10675 | 10698 | 10653 | ... |
| $\Upsilon_1 (4D)$ | $4^3 D_1$ | 10846 | 10858 | 10871 | 10928 | 10853 | ... |
| $\Upsilon_1 (5D)$ | $5^1 D_1$ | 11029 | 11036 | 11041 | ... | 11023 | ... |

$M^{M_A M_B M_C}(\mathbf{P})$ can be converted to the partial wave amplitudes $M^L$ via

$$M^L(A \rightarrow BC) = \frac{\sqrt{4\pi (2L+1)}}{2J_A+1} \sum_{M_{J_A} M_{J_B}} \langle L J M_{J_A} J_A M_{J_A} \rangle M^{M_A M_B M_C}(\mathbf{P}) \times \langle J_B M_{J_B} J_C M_{J_C} J M_{J_A} \rangle M^{M_A M_B M_C}(\mathbf{P}).$$

In the above equations, $(J_A, J_B$ and $J_C)$, $(L_A, L_B$ and $L_C)$ and $(S_A, S_B$ and $S_C)$ are the quantum numbers of the total angular momenta, orbital angular momenta and total spin for hadrons $A, B, C$, respectively; $M_{J_A} = M_{J_B} + M_{J_C}$, $\mathbf{J} \equiv \mathbf{J}_B + \mathbf{J}_C$ and $\mathbf{J}_A \equiv \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$. In the c.m. frame of hadron $A$, the momenta $\mathbf{P}_B$ and $\mathbf{P}_C$ of mesons $B$ and $C$ satisfy $\mathbf{P}_B = -\mathbf{P}_C \equiv \mathbf{P}$.

To partly remedy the inadequacy of the nonrelativistic wave function as the momentum $\mathbf{P}$ increases, a Lorentz boost factor...
γ_f is introduced into the decay amplitudes [42],

\[ M \rightarrow γ_f M(γ_f P) \]

where γ_f = M_B/E_B. In the decays with small phase space, the three momenta P carried by the final state mesons and corrections from the Lorentz boost are relatively small, while the relativistic effects may be essential for the decay channels with larger phase space.

Finally, the partial width A \rightarrow B + C can be given by

\[ Γ = 2π|P|^2 \frac{E_BE_C}{M_A} \sum_{JL} |M_{JL}|^2 \]

where M_A is the mass of the initial hadron A, while E_B and E_C stand for the energies of final hadrons B and C, respectively. The details of the formula of the 3P_0 model can be found in Refs [43, 44].

In the calculations, the wave functions of the initial vector bottomonium states are taken from our quark model predictions. Furthermore, we need the wave functions of the final hadrons, i.e., the B_c^+(s) and B_c^+(s) mesons and some of their excitations, which are adopted from the quark model predictions of Refs. [23, 44]. The masses of the final hadron states in the decay processes are adopted from the Particle Data Group [45] if there are data, while if there are no observations we adopt the predicted values from Refs. [23, 44]. The quark pair creation strength is determined to be \( γ = 0.232 \) by reproducing the measured width γ(10580) \rightarrow BB = 20.5 MeV [45] with the wave function calculated from the screened potential model. The γ determined in this work is also close to the values 0.217/0.234 adopted in the study of the strong decays of excited charmonium states [42]. The strong decay properties for the vector bottomonium are presented in Table II.

It is found that both the linear and screened potential models give similar predictions for the strong decay properties of the bottomonium states.

IV. DISCUSSIONS

A. \( γ(10750) \) and \( γ(10860) \)

For the \( γ(10860) \) resonance, the mass and spin-parity numbers indicate that it may be a candidate of the conventional \( γ(10860) \) state. However, with this assignment it is found that the total decay width, \( ∼ 81 \) MeV (see Table II), is about a factor of 2 larger than the measured width \( ∼ 37 \) MeV [11]. Furthermore, assigning \( γ(10860) \) as \( γ(10860) \) state we will meet a problem in the explanation of its productions in the \( e^+e^- \rightarrow γ(nS)π^+π^- \) (n = 1, 2, 3), because the production rates of D-wave states should be strongly suppressed for their tiny dielectron widths [8, 9, 11, 12, 13, 14]. The \( γ(10860) \) resonance is often explained as the \( γ(5S) \) state in the literature [7–2, 11], with this assignment, the total width is predicted to be \( ∼ 44 \) MeV (see Table IV), which is consistent with the data. However, if we assign \( γ(10860) \) to \( γ(5S) \) we should face some problems, for example, (i) the predicted mass of \( γ(5S) \) state is about 70 MeV lower than that of \( γ(10860) \); (ii) moreover, the mass splittings \( m[γ(5S) – γ(4S)]_{\text{exp}} = 210 \) MeV and \( m[γ(6S) – γ(5S)]_{\text{exp}} = 180 \) MeV predicted within various potential models (see Table II) are inconsistent with the observations \( m[γ(5S) – γ(4S)]_{\text{exp}} = 306 \) MeV and \( m[γ(6S) – γ(5S)]_{\text{exp}} = 115 \) MeV [45].

For the new structure \( γ(10750) \) observed at Belle, from Figure II one finds that it lies between the vector states \( γ(5S) \) and \( γ(1D) \). The predicted mass of \( γ(1D) \) state is about 100 MeV lower than that of \( γ(10750) \). If one assigns the \( γ(10750) \) resonance to the \( γ(1D) \) state, we cannot understand its production rates in the \( e^+e^- \rightarrow γ(nS)π^+π^- \) (n = 1, 2, 3) processes. The production cross sections of the D-wave states should be strongly suppressed for their very tiny dielectron widths predicted in theory [8, 9, 11, 12, 13, 14]. Thus, the explanation of the \( γ(10750) \) resonance as the \( γ(1D) \) state should be excluded. On the other hand, if one assigns the \( γ(10750) \) resonance to \( γ(5S) \), it is found that the decays of \( γ(10750) \) are dominated by the \( B^*B^* \) channel, and the decay width is predicted to be \( Γ ∼ 53 \) MeV, which is close to the measured value 35.5_{-11.3}^{+13.9} \) MeV at Belle [11]. However, we will meet a prob-
lem that there are no $S$-wave vector states to be assigned to $\Upsilon(10860)$, then we cannot understand largest production rates of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$ processes.

Since it is difficult to assign the $Y(10750)$ and $\Upsilon(10860)$ as the pure $S$- and $D$-wave $bb$ states simultaneously, we consider the possibilities of $Y(10750)$ and $\Upsilon(10860)$ as the $\Upsilon(5S)$-$\Upsilon_1(4D)$ mixed states with the following mixing scheme

$$
\begin{pmatrix}
Y(10750) \\
\Upsilon(10860)
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\Upsilon_1(4D) \\
\Upsilon(5S)
\end{pmatrix}.
$$

The decay widths of $Y(10750)$ and $\Upsilon(10860)$ versus the mixing angle $\theta$ are presented in Fig. 2. In Refs. [13, 14], Badalian et al. studied the dielectron widths of the vector bottomonium states, their results indicate that there might be sizeable $S - D$ mixing between the $nS$- and $(n - 1)D$-wave ($n \geq 4$) vector states with a mixing angle $\sim 27^\circ$. With this mixing angle, the decay widths of $Y(10750)$ and $\Upsilon(10860)$ can be reasonably understood (see Fig. 3).

The $S - D$ mixing mechanics can shift the masses of the pure states $\Upsilon(5S)$ and $\Upsilon_1(4D)$ to the physical states $Y(10750)$ and $\Upsilon(10860)$. The Hamiltonian of the physical states is assumed to be

$$
H = H_0 + H_1,
$$

where $H$ contributes diagonal elements to the mass matrix, while $H_1$ could contribute non-diagonal elements to the mass matrix causing the $S - D$ mixing. Then, the masses of $Y(10750)$ and $\Upsilon(10860)$ can be determined by

$$
M[\Upsilon(10860)] = \langle \Upsilon(10860) | H_0 + H_1 | \Upsilon(10860) \rangle,
$$

$$
M[Y(10750)] = \langle Y(10750) | H_0 + H_1 | Y(10750) \rangle.
$$

Combining the mixing scheme defined in Eq.(14), one finds that

$$
M[\Upsilon(10860)] = M[Y(10750)] + M[\Upsilon_1(4D)] \sin^2 \theta + M[\Upsilon(5S)] \cos^2 \theta - \Delta M_{SD} \sin(2\theta),
$$

$$
M[Y(10750)] = M[\Upsilon_1(4D)] \cos^2 \theta + M[\Upsilon(5S)] \sin^2 \theta + \Delta M_{SD} \sin(2\theta),
$$

where

$$
M[\Upsilon(5S)] = \langle \Upsilon(5S) | H | \Upsilon(5S) \rangle
$$

and

$$
M[\Upsilon_1(4D)] = \langle \Upsilon_1(4D) | H | \Upsilon_1(4D) \rangle
$$

correspond to the masses of the pure states $\Upsilon(5S)$ and $\Upsilon_1(4D)$, respectively, while

$$
\Delta M_{SD} = \langle \Upsilon_1(4D) | H | \Upsilon(5S) \rangle
$$

corresponds to the non-diagonal element. Taking a sizeable mixing angle $\theta \sim 20 - 30^\circ$ and a negative value $\Delta M_{SD} \sim -100$ MeV in Eq.(17), one finds that the physical masses of both $Y(10750)$ and $\Upsilon(10860)$ can be consistent with the experimental observations. Thus, with the $S - D$ mixing one may overcome the puzzle that the observed mass of $\Upsilon(10860)$ is obviously higher than the predicted masses of pure $\Upsilon(5S)$ in the potential models.

As a pure $\Upsilon(5S)$ state, the dielectron width of $\Upsilon(10860)$ is predicted to be $\Gamma_{ee} = 0.348$ keV in a recent work [11]. Combining it with the mixing angle $\theta \sim 27^\circ$ suggested in Ref. [13, 14], we obtain dielectron width $\Gamma_{ee} = 0.28$ keV for $\Upsilon(10860)$, which is consistent with the measured value $0.31 \pm 0.07$ keV [43]. As a mixed state containing sizeable $S$-wave component the dielectron width of $Y(10750)$ is estimated to be $\sim 0.07$ keV. Neglecting the effect of phase space, one may predict the ratio between the production rates of $Y(10750)$ and $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$ processes, i.e.,

$$
R \approx \frac{\Gamma_{ee}[Y(10750)]}{\Gamma_{ee}[\Upsilon(10860)]} \approx \frac{1}{4},
$$

where $R$ is the ratio of the production rates of $Y(10750)$ and $\Upsilon(10860)$. 

![FIG. 3: Strong decay of $\Upsilon(11020)$ versus the mixing angle $\theta$ within screened potential model.](image1)

![FIG. 4: Strong decay of $\Upsilon(10580)$ versus the mixing angle $\theta$ within screened potential model.](image2)
which can explain the observations that production cross sections of $\Upsilon(10750)$ are comparable with those of $\Upsilon(10860)$ in the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ $(n = 1, 2, 3)$ processes.

From Fig. 4 one can see that the partial widths of the strong decay modes and the ratios between them for the $\Upsilon(10750)$ and $\Upsilon(10860)$ resonances are sensitive to the mixing angle. If taking the mixing angle as $\theta \sim 30^\circ$, the decays of $\Upsilon(10860)$ are dominated by the $B^*B^*$, $BB$ and $B_s^*B_s^*$ channels, while the decays of $\Upsilon(10750)$ might be governed by both the $B^*B^*$ and $BB$ channels. The large decay rates of $\Upsilon(10860)$ into the $B^*B^*$, $B_s^*B_s^*$ decay modes can explain the observations at Belle [45] that the $B^*B^*$ cross section shows a prominent $\Upsilon(10860)$ signal, while the $B^*B_s$ and $B_sB_s$ cross sections are relatively small and do not show any significant structures. From the Review of Particle Physics [48], the branching ratios of $BB$, $B^*B^*$, $B^*B_s$, $B_sB_s$, $B^*_sB^*_s$, $B_s^*B^*_s$ decay modes are $5.5 \pm 1.0\%$, $13.7 \pm 1.6\%$, $38.1 \pm 3.4\%$, $0.5 \pm 0.5\%$, $1.35 \pm 0.32\%$, and $17.6 \pm 2.7\%$, respectively. From Table IV it can be seen that these branching ratios of $\Upsilon(10860)$ can be hardly described in the pure $\Upsilon(5S)$ interpretation, which is consistent with the analysis in Refs. [42] [43] [47]. With the mixing scheme, this problem can be partially overcome. Our results show that the $B^*B^*$ dominates in the non-strange final channels and the $B^*_sB^*_s$ is prominent in the strange decay modes, which is consistent with the experimental data. However, the predicted large $BB$ partial decay width is still in conflict with the observations. More theoretical and experimental investigations are needed to clarify this puzzle.

The intermediate $B^*B^*$ meson loop may play an important role in the $S - D$ mixing between $\Upsilon(5S)$ and $\Upsilon_1(4D)$. From Table I it is seen that both $\Upsilon(5S)$ and $\Upsilon_1(4D)$ states strongly couple to the $B^*B^*$ channel, thus, intermediate $B^*B^*$ meson loop may contribute a sizeable non-diagonal element to the mass matrix, which leads to the $S - D$ mixing. It is interesting to find that the mixing mechanism of axial-vector $D_{11}(2460)$ and $D_{33}(2536)$ has been studied via intermediate hadron loops, e.g. $DK$, to which both states have strong couplings in Ref. [47]. Also, the $S - D$ mixing scheme of $\psi(4S)$ and $\psi(2D)$ states via the meson loops are investigated in Ref. [48]. Their results indicate that the intermediate hadron loops as the mixing mechanism can lead to strong configuration mixing effects and obvious mass shifts for the physical states.

### B. $\Upsilon(10580)$ and $\Upsilon(11020)$

Taking $\Upsilon(10580)$ and $\Upsilon(11020)$ as the $\Upsilon(4S)$ and $\Upsilon(6S)$ states, respectively, their masses can be well described in the potential models (see Table I). Furthermore, their decay widths can be reasonably understood within the uncertainties (see Table II). However, their dielectron widths are overestimated as the pure $S$-wave states $\Upsilon(10580)$ and $\Upsilon(11020)$.

We also consider the $\Upsilon(11020)$ resonance as a mixed state via the $\Upsilon(6S)$-$\Upsilon_1(5D)$ mixing. The mixing scheme is adopted as follows:

$$
\begin{pmatrix}
\Upsilon(M) \\
\Upsilon(11020)
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\Upsilon_1(5D) \\
\Upsilon(6S)
\end{pmatrix}.
\tag{20}
$$

The strong decay width of $\Upsilon(11020)$ versus the mixing angle $\theta$ is plotted in Fig. 2. It is seen that the total decay width is consistent with experimental data when the mixing angle varies in large range. The current total decay width alone cannot determine the mixing angle. However, the significant leptonic decay width up to $0.130 \pm 0.030$ keV, indicates the $\Upsilon(11020)$ has a large $S$-wave component at least.

From the predicted strong decay properties of $\Upsilon(6S)$ and $\Upsilon_1(5D)$ states listed in Table II, one finds that $\Upsilon(6S)$ and $\Upsilon_1(5D)$ states mainly couple to two different channels $BB^*$ and $B^*B^*$, respectively. Furthermore, both $\Upsilon(6S)$ and $\Upsilon_1(5D)$ states are far from the thresholds of $BB^*$ and $B^*B^*$. Thus, the $\Upsilon(6S)$-$\Upsilon_1(5D)$ mixing effect via virtual meson loops may be smaller than that of $\Upsilon(5S)$-$\Upsilon_1(4D)$. If the intermediate meson loops are the main mechanism causing the $S - D$ mixing, the mixing angle for $\Upsilon(6S)$-$\Upsilon_1(5D)$ should be smaller than that for $\Upsilon(5S)$-$\Upsilon_1(4D)$. To sum up, although the $\Upsilon(6S)$-$\Upsilon_1(5D)$ mixing with a small 5$D$-wave component assignment cannot be excluded, we prefer to assign $\Upsilon(11020)$ as the pure $\Upsilon(6S)$ state. In Ref. [55] the authors also expected that there may be less $S - D$ mixing for the $\Upsilon(6S)$ state with the consideration of coupled-channel effects. To better understand the nature of $\Upsilon(11020)$, the missing $\Upsilon(11020)$ is worth looking for in future experiments. Our predictions of the mass and strong decay widths of $\Upsilon_1(5D)$ may provide helpful information for future experimental observations.

Besides the possibility of $\Upsilon(5S)$-$\Upsilon_1(4D)$ and $\Upsilon(6S)$-$\Upsilon_1(5D)$ mixing, taking the same mixing scheme the decay width of $\Upsilon(10580)$ as $\Upsilon(4S)$-$\Upsilon_1(3D)$ mixing is also shown in Fig. 3. The total decay width varies dramatically with the mixing angle, and the zero mixing angle is more favored (actually, we use this case to determine the quark pair creation strength $\gamma$). From Table II it is found that the $\Upsilon(4S)$ state mainly couples to the $BB$ channel, and $\Upsilon_1(3D)$ state has strong coupling with the $BB^*$ mode, which suggests that the $\Upsilon(4S)$-$\Upsilon_1(3D)$ mixing effects via virtual meson loops may be negligible. Thus, if the intermediate meson loops are the main mechanism causing the $S - D$ mixing, and the mixing effects can be neglected. It should be mentioned that in Ref. [55] the authors expected that there may sizeable $S - D$ mixing for the $\Upsilon(4S)$ state with the consideration of coupled-channel effects. It should mentioned that the $\Upsilon_1(3D)$ state might be a narrow state with a width of about $20 - 30$ MeV according to our calculations, its decays are governed by the $BB^*$ mode, this decay mode might be suitable to be observed in experiments. Looking for the missing $\Upsilon_1(3D)$ state is useful for better understanding the nature of $\Upsilon(10580)$.

## V. SUMMARY

In this paper, we calculate the spectrum of the higher vector bottomonium states above the $BB$ threshold within both
**TABLE II**: Strong decay properties for the higher vector bottomonium states predicted within both LP and SP models. For a comparison, other results predicted in the recent works [8, 9, 11] are also listed in the same table.

| State (LP/SP) | Decay mode | LP | SP | Ref. [11] | Ref. [8] | Ref. [9] |
|---------------|------------|----|----|-----------|----------|----------|
|               |            | $\Gamma_{th}$ (MeV) | $B_r$ (%) | $\Gamma_{th}$ (MeV) | $B_r$ (%) | $\Gamma_{th}$ (MeV) | $B_r$ (%) | $\Gamma_{th}$ (MeV) | $B_r$ (%) |
| $4^3S_1$      | $BB$       | 19.58 | 100 | 20.25 | 100 | 24.7 | 100 | 22.0 | 100 | 20.59 | 100 |
|               | Total      | 19.58 | 100 | 20.25 | 100 | 24.7 | 100 | 22.0 | 100 | 20.59 | 100 |
| $5^3S_1$      | $BB'$      | 0.96 | 2.23 | 2.25 | 7.26 | 13.7 | 30.0 | 5.35 | 19.5 | 6.22 | 22.29 |
|               | $B'B'$     | 33.73 | 78.37 | 21.77 | 70.23 | 2.58 | 5.66 | 2.42 | 8.83 | 0.09 | 0.32 |
|               | $B_sB_s$   | $\approx$ 0 | $\approx$ 0 | 0.30 | 0.97 | 0.484 | 1.06 | 0.157 | 0.573 | 0.96 | 3.45 |
| $6^3S_1$      | $BB$       | 3.22 | 26.86 | 3.80 | 18.90 | 7.81 | 20.4 | 1.32 | 3.89 | 4.18 | 5.28 |
|               | $BB'$      | 5.69 | 47.46 | 9.02 | 44.85 | 16.5 | 43.0 | 7.59 | 22.4 | 15.49 | 19.57 |
|               | $B'B'$     | 0.44 | 3.67 | 3.13 | 15.56 | 4.43 | 11.5 | 5.89 | 17.4 | 11.87 | 14.99 |
|               | $B_sB_s$   | 0.38 | 3.17 | 0.28 | 1.39 | 0.101 | 0.263 | 0.136 | 0.401 | 0.07 | 0.09 |
|               | Total      | 12.4 | 100 | 20.11 | 100 | 38.3 | 100 | 33.9 | 100 | 27.4 | 100 |
| $3^3D_1$      | $BB$       | $\approx$ 0 | $\approx$ 0 | 0.95 | 3.41 | 5.47 | 10.1 | 23.8 | 23.0 | 26.41 | 100 |
|               | $BB'$      | 26.41 | 100 | 18.76 | 67.39 | 15.2 | 28.1 | 0.245 | 0.236 | 2.02 | 2.56 |
|               | $B'B'$     | 8.13 | 29.20 | 33.4 | 61.8 | 79.5 | 76.7 | 39.18 | 100 | 71.8 | 100 |
| $4^3D_1$      | $BB$       | 11.82 | 30.17 | 12.84 | 23.87 | 27.4 | 31.4 | 3.85 | 5.36 | 3.85 | 5.36 |
|               | $BB'$      | 5.55 | 14.17 | 7.45 | 13.85 | 15.1 | 17.3 | 14.0 | 19.5 | 14.0 | 19.5 |
|               | $B'B'$     | 18.25 | 46.58 | 27.28 | 50.71 | 42.1 | 48.3 | 50.6 | 70.5 | 50.6 | 70.5 |
|               | $B_sB_s$   | 1.67 | 4.3 | 1.86 | 3.48 | 0.560 | 0.642 | 0.101 | 0.141 | 0.332 | 0.462 |
| $5^3D_1$      | $BB$       | 8.28 | 14.27 | 8.13 | 10.39 | 20.0 | 16.4 | 39.18 | 100 | 71.8 | 100 |
|               | $BB'$      | 7.89 | 13.60 | 8.49 | 10.85 | 19.3 | 15.8 | 7.89 | 13.60 | 8.49 | 10.85 |
|               | $B'B'$     | 24.08 | 41.51 | 27.41 | 35.02 | 47.1 | 38.7 | 39.18 | 100 | 71.8 | 100 |
|               | $B_sB_s$   | $\approx$ 0 | $\approx$ 0 | 0.02 | 0.03 | 0.0235 | 0.0193 | 0.44 | 0.76 | 0.46 | 0.103 | 0.0844 |
|               | $B'B'$     | 2.91 | 5.02 | 2.60 | 3.32 | 0.798 | 0.656 | 7.92 | 13.65 | 10.08 | 12.88 |
| $5^3P_1$      | $BB$       | 6.49 | 11.19 | 13.56 | 17.33 | 4.08 | 3.35 | 0.157 | 0.263 | 0.136 | 0.401 |
|               | $BB'$      | 7.61 | 9.72 | 18.1 | 14.8 | 4.08 | 3.35 | 9.23 | 7.59 | 9.23 | 7.59 |
| $5^3P_2$      | $BB$       | 58.01 | 78.26 | 100 | 121.7 | 100 | 100 | 58.01 | 78.26 | 100 | 100 |

For a comparison, other results predicted in the recent works [8, 9, 11] are also listed in the same table.
TABLE III: Strong decays for the \( \Upsilon(1(4D) \) which is assigned to be \( \Upsilon(10750) \) or \( \Upsilon(10860) \) within the screened potential model.

| State | Mode | \( \Upsilon(10750) \) \( \Gamma_{0}(\text{MeV}) \) | \( \Upsilon(10860) \) \( \Gamma_{0}(\text{MeV}) \) |
|-------|------|------------------|------------------|
| \( 4^1D_1 \) | \( BB \) | \( \simeq 0 \) | \( \simeq 0 \) |
|       | \( BB^* \) | 4.46 | 13.77 |
|       | \( B^*B \) | 25.53 | 78.84 |
|       | \( B_1B \) | 2.39 | 7.38 |
|       | \( B_sB \) | 2.16 | 2.65 |
|       | \( B^*_sB \) | 9.35 | 11.49 |
| Total |      | 32.38 | 100 |

| State | Mode | \( \Upsilon(10750) \) \( \Gamma_{0}(\text{MeV}) \) | \( \Upsilon(10860) \) \( \Gamma_{0}(\text{MeV}) \) |
|-------|------|------------------|------------------|
| \( 5^3S_1 \) | \( BB \) | 0.20 | 3.86 |
|       | \( BB^* \) | 15.63 | 29.50 |
|       | \( B^*B \) | 35.52 | 67.03 |
|       | \( B_1B \) | 1.64 | 3.09 |
|       | \( B^*_sB \) | 5.63 | 12.79 |
|       | \( B^*_sB \) | 0.81 | 1.84 |
| Total |      | 52.99 | 100 |

screened and linear potential models. Then, using the predicted masses and wave functions of these higher vector bottomonium states, their two-body \( \text{OZI} \)-allowed strong decays are investigated in the \( 3^3P_0 \) model.

Combining the productions, mass, and decay width of the higher vector bottomonium states with each other, we conclude that \( \Upsilon(10750) \) and \( \Upsilon(10860) \) might not be pure \( S \)-wave and \( D \)-wave vector bottomonium states. Then, we further discuss the possibility of \( \Upsilon(10860) \) and \( \Upsilon(10750) \) as mixed states via the \( S - D \) mixing. Our results suggest that \( \Upsilon(10750) \) and \( \Upsilon(10860) \) might be mixed states via the \( 5^3S_1 - 4^3D_1 \) mixing with a sizeable mixing angle \( \theta \simeq 20° - 30° \). The components of \( \Upsilon(10750) \) and \( \Upsilon(10860) \) are dominated by the \( 4^3D_1 \) and \( 5^3S_1 \) states, respectively.

Moreover, the strong decay behaviors of the \( \Upsilon(10580) \) and \( \Upsilon(11020) \) resonances are also discussed. If the \( \Upsilon(10580) \) and \( \Upsilon(11020) \) resonances are assigned as the \( \Upsilon(4S) \) and \( \Upsilon(6S) \) states, respectively, their observed widths together with masses are consistent with the theoretical predictions.

Finally, it should be mentioned that the mechanism for the \( S - D \) mixing is not clear. If \( \Upsilon(10750) \) and \( \Upsilon(10860) \) as mixed states, several questions should be clarified in future works:
(i) what causes the mixing between the \( 5^3S_1 \) and \( 4^3D_1 \) states;
(ii) and how the masses of the pure \( S \)- and \( D \)-wave states are shifted to the physical states by the configuration mixing.

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