THE PERFORMANCE EVALUATION OF MULTI USER OFDM ORTHOGONAL CHAOTIC VECTOR SHIFT KEYING SUPPORTED BY LDPC

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Abstract: Recently, LDPC code have become very important research area in wireless communication due to its ability to increase the capacity in a wireless fading environment, with low implementation complexity. In this paper, LDPC are combined with Multi User OFDM Orthogonal Chaotic Vector Shift Keying (MU-OFDM-OCVSK) communication system to improve the BER performance over multi-path Rayleigh fading channels. Two types of LDPC decoder are introduced that are Log-Domain and Min-Sum decoder. The system is simulated using MATLAB program version 2019a for different scenarios which include different number of iterations, different block lengths, different number of users and different number of spreading factor. The results show that a coding gain in a range of (4.5 – 7) dB is achieved between the coded and uncoded MU-OFDM-OCVSK system. The results also show that the Min-Sum decoder outperform the Log-Domain decoder in all scenarios.

Keywords: differential chaos shift keying, MU-OFDM, LDPC, Log-Domin, Min-Sum.

1. Introduction

Wireless communication systems, including mobile radio, are highly sensitive to the effect of multipath and signal fading. Being non periodic, wide-band, sensitive to initial conditions, random-like nature with easiness to generate, chaotic signals are more suitable for communication with the spread-spectrum and provide wireless applications [1],[2]. The non-coherent modulation technique, with a low complexity auto-correlation receiver of Differential Chaos Shift Keying (DCSK) provide an excellent performance in multipath fading environment. Among all chaotic based communication systems DCSK is preferred due to its simple e auto-correlation receiver, non-coherent ability of application and robustness against multi-path fading conditions [3-5]. A lot of DCSK schemes have been studied and proposed to assess its performance in various conditions [6-10]. In modern wireless communication systems, OFDM is the most dominating multicarrier modulation for discrete schemes [11-12]. Combining OFDM technology with DCSK system is suggested in [13-14] to reduce the complexity of the multicarrier DCSK system and provide better spectral efficiency compared to conventional DCSK systems. The authors in [15] presented the multi-user OFDM-DCSK to support multi-user communication. Though the system provides advantages like

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energy and spectral efficiency improvements, it has also some disadvantages like multiple access interference (MAI). To enhance the performance and reduce the (MAI) of MU-OFDM-DCSK system in [16] the authors increased the orthogonality of the chaotic signals and design an improved system with orthogonal chaotic vector shift keying (OCVSK) using Gram-Schmidt orthonormalization process.

Recently, Error Correction codes (ECC) are utilized in communication systems due to higher mobility and large interference that produce a large number of errors. Foreword Error Correction codes (FECC) are employed to detect and correct the error without feedback. The Low Density Parity Check (LDPC) codes are one of the most important FECC which are used for the next generation. LDPC codes have become a topic of increasing interest recently because of its remarkable performance that approaches Shannon limit which is based on a sparse matrix parallel iterative decoding algorithm with low computational complexity which makes it very suitable for high data rate applications ranging from wireless applications such as magnetic storage systems and wide-band wireless multimedia communications [17-18]. A number of research papers on the using of LDPC codes have been conducted like [19] where LDPC codes are proposed to improve the FM-DCSK system. Researchers in [20] proposed using the LDPC coding/decoding technique and the suitable spreading sequences to segregate the users in Multi-Carrier CDMA-OFDM systems. The LDPC technique results in excellent performance, and the BER values decrease rapidly as the SNR improves. Also in [21] the use of LDPC codes was proposed for MIMO-OFDM system for high data rate wireless communication.

To improve the performance of multi-user OFDM based orthogonal chaotic vector shift keying system in [16] over multi-path fading channels we propose applying LDPC codes in this paper, and investigate its performance. Two LDPC decoding algorithms; Log-Domain, and Min-Sum are studied and compared their performances in terms of BER as a function of signal to noise ratio (SNR) over multipath Rayleigh fading channel.

The remainder of this paper is organized as the following: a review of the LDPC Codes is presented in Section 2. The proposed system model is presented in Section 3. The simulation results are introduced in Section 4, followed by the conclusions at Section 5.

2. Review of the Low Density Parity Check (LDPC) Codes

An \((N, K)\) LDPC code is a linear block code identified by a very sparse binary matrix \(M \times N\) known as parity check matrix \(H\) with code rate \(R = K/N\). Where \(k\) is the information length \((K = N-M)\), \(M\) represents the number of rows, and identifies the parity check equations number of the code, and \(N\) represents the number of columns and identifies the length of the code. LDPC codes can be represented graphically by a bipartite graph or Tanner graph which consists of two sides of nodes [20].

![Tanner Graph with corresponding H matrix of 9 bit length LDPC code](image-url)
It has M check nodes on one side and N variable nodes on the other, and there is a link between the variable node $V_n$, $n \in \{1, ..., N\}$ and check node $C_m$, $m \in \{1, ..., M\}$ whenever $H_{m,n} = 1$. Figure 1 shows the Tanner graph and corresponding parity check matrix $H$ of an LDPC code with 9 bits code length.

3. LDPC-MU-OFDM-OCVSK system

3.1. System Model

Taking two users as an example, the transmitter and receiver block diagrams of LDPC-MU-OFDM-OCVSK system are shown in figures 2, and 3 respectively.

Using second order Chebyshev polynomial function (equation 1) to generate the $k^{th}$ chaotic reference sequence of the $u$th user $x_{k,u}$ [15]:

$$x_{k+1,u} = 1 - 2x_{k,u}^2$$  
(1)

For each user, the M data bits at the transmitter input are encoded by the LDPC encoder giving the binary codeword $C$. First, the parity check matrix $H_u$ is constructed achieving the condition of sparsity. Then, the generator matrix $G_u$ is obtained using

$$G_u \cdot H_u^T = 0$$  
(2)

If the information bit block for the $u^{th}$ user is $M_u$, the codeword $C_u$ will be generated as:

$$C_u = M_u \cdot G_u$$  
(3)

The codeword $C_u$ is converted to parallel data sequences then, spreading by multiplying with the chaotic reference sequence $x_u$ over a wideband to obtain a modulated signal with spreading factor $\beta$. The modulated signal is then passed through IFFT producing the OFDM symbol transmitted by each user.

The transmitted signal by user U of LDPC-MU-OFDM-OCVSK system is given by

$$S_u(t) = \sum_{k=1}^{\beta} x_{k,u} e^{2\pi j f_o(t - kT_c)} g(t - kT_c) + \sum_{i=U+1}^{M} \sum_{k=1}^{\beta} x_{k,u} C_{i,u} e^{2\pi j f_i (t - kT_c)} g(t - kT_c)$$  
(4)

After the transmission the signal of each user go through multipath fading channel with time domain channel impulse response given as:

$$h_u(t) = \sum_{u=1}^{U} \sum_{l=1}^{L} \alpha_{u,l}(t) \delta(T - \tau_{u,l})$$  
(5)

Where $\delta(t)$ is the unit impulse response and $\alpha_{u,l}(t)$ is the channel coefficient which is assumed to be zero mean and follow the Rayleigh distribution denoted by [15]

$$f(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} \quad \alpha \geq 0$$  
(6)

So, the received signal over the wireless channel will be

$$r(t) = \sum_{u=1}^{U} S_u(t) \otimes h_u(t) + n(t)$$  
(7)

where $n(t)$ is the Gaussian noise.

At the receiver side, an inverse action to that of the transmitter side with serial to parallel converter applied to the received serial signal $R(t)$ then an FFT operation is carried out for each $N$ different samples to recover the transmitted coded data $C_{u,\beta}$ and the reference signals $R_{u,\beta}$ for each user.
Figure 2. Transmitter structure of LDPC-MU-OFDM-OCVSK system with two users.

Figure 3. Receiver structure of an LDPC-MU-OFDM-OCVSK system.
Thus, after $k^{th}$ correlator process the $u^{th}$ user coded data ($C_{u,k}$) can be expressed as

$$C_{k,u} = \sum_{k=1}^{\beta} R_{k,u} R_{k-1,u}$$  \hspace{1cm} (8)

Finally, the original transmitted data stream is obtained after the decoding process of the LDPC decoder.

### 3.2. Decoding Process

The decoding process of LDPC codes is iterative and can be performed using either the hard or the soft decision decoders. Hard decision decoders are simpler and less complex but suffer from high errors in multipath conditions. The sum-product algorithm (SPA) is a soft decision message-passing algorithm that depends on passing messages between Variable Nodes (VN) and Check nodes (CN) in which the prior probabilities information from the channel represent the input of each received bit [21]. In this paper, two types of SPA soft decision LDPC decoder schemes are employed to analyze and compare the performances over multipath Rayleigh fading channel.

#### 3.2.1. Log Domain Algorithm

The Log-Domin algorithm is a soft decision message-passing algorithm where the messages to be delivered are expressed by logarithmic likelihood ratio (LLR) [23] instead of real probability. The workflow of Log Domin decoding algorithm is described as follows:

1. **Initialization.** Representing the message received by the VN from the channel as a log-likelihood ratio LLR $L(p_n)$:

   $$L(p_n) = \log \frac{1-p_n}{p_n} = \frac{4y_n}{N_o}$$  \hspace{1cm} (9)

   $$L(q_{n,m}) = \log \frac{q_{n,m}(0)}{q_{n,m}(1)}$$  \hspace{1cm} (10)

And

$$q_{n,m}(0) = 1 - p_n = P_r(C_i = 0|y_n) = \frac{1}{1 + e^{-4y_n/N_o}}$$  \hspace{1cm} (11)

$$q_{n,m}(1) = p_n = P_r(C_i = 1|y_n) = \frac{1}{1 + e^{-4y_n/N_o}}$$  \hspace{1cm} (12)

Where $y_n$ is the received data , $N_o$ is the noise variance and $q_{n,m}$ is a message sent by the VN $V_n$ to the CN $C_m$.

2. **Check node update.** The LLR estimation of the CN Messages $L(r_{m,n})$ after VN processing is:

   $$L(r_{m,n}) = 2 \tan^{-1} \left( \prod_{n'\in row_m} \tan \left( \frac{L(q_{n',m})}{2} \right) \right)$$  \hspace{1cm} (13)

   Where $r_{m,n}$ is a message to be passed from CN $C_m$ to VN $V_n$, $n' \in row_m$ indicates $n' \leq N$ of all bits in $m$ ($1 \leq m \leq M$) which have the value of one.

3. **Variable node update.** The message from CN is passed to the VN after processing. The estimated LLR of the VN $L(q_{n,m})$ is:

   $$L(q_{n,m}) = L(p_n) + \sum_{m'=col_n} L(r_{p,a})$$  \hspace{1cm} (14)

4. **Decoding decision.** The collective LLR estimation of the output bits $L(Q_o)$

   $$L(Q_o) = L(p_n) + \sum_{m\in col_n} L(r_{m,n})$$  \hspace{1cm} (15)

   If $L(Q_o) > 0$, then $\hat{C} = 0$, otherwise $\hat{C} = 1$, and the decoding process will continue to the last number of iteration unless it reach $\hat{C}, H^T = 0$ then the decoding process is over.
3.2.2. Min-Sum Algorithm

The Min-Sum algorithm is a simplified version of Log Domin decoding algorithms with reduced computational complexity by using the approximation. The steps of the two algorithms are the same, except step two (the node update) that will be modified in Min-Sum algorithm into:

\[
L(r_{m,n}) = 2 \tan^{-1}\left( \prod_{n' \in R_m} \text{sign}(L(q_{n',m})) \min(|L(q_{n',m})|) \right)
\]

(16)

4. Simulation Results

The simulation is utilized using Matlab version R2019a. The performance of the above described system is investigated using a BER as a function of SNR by applying LDPC technique. Two of LDPC decoding algorithms are used in this paper to investigate with OFDM-OCVSK system and compared over multi-path Rayleigh fading channel. The influence of the LDPC code decoding algorithm on the performance of the system is studied in different situations. The range of SNR used in the simulations is 0-15 dB. Different parameters are varied in the simulations namely number of iterations \( \in \{3, 5, 10\} \), \( M \in \{100, 200, 500\} \), code rate is 0.5, the spreading factor (\( \beta \)) \( \in \{32, 64, 128\} \), and the number of users \( \in \{2, 5, 7\} \). The channel of each user has 2 independent paths \( L_p=2 \) with average gain powers \( E[\alpha_1^2] = 2/3 \) and \( E[\alpha_2^2] = 1/3 \).

4.1. The influence of different decoding iteration on the system

The iteration in the two types of LDPC decoder is varied by 3, 5, and 10 iterations to show the effect of increasing the iteration for each type of decoder then compared with traditional MU-OFDM-OCVSK system. The simulation of the two types of decoders are applied with \( \beta = 120 \), the rate is 1/2, and \( M=100 \). The simulation results of Log-Domain are depicted in the Figure 4 and Min-Sum simulation results are shown in Figure 5.

![Figure 4](https://via.placeholder.com/150)

Figure 4. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) using Log-Domin decoding algorithm with traditional MU-OFDM-OCVSK system.

![Figure 5](https://via.placeholder.com/150)

Figure 5. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) using Min-Sum decoding algorithm with traditional MU-OFDM-OCVSK system.

As can be seen in Figures 4, and 5 the system error rate decreases, so the BER performance is enhanced by increasing the number of iterations but at the cost of increasing the delay of the system with a little improvement in the BER performance in both Log-Domain and Min-Sum.
decoders. It is also shown that Log-Domain achieves more than 5 dB improvement at $2 \times 10^{-2}$ while Min-Sum achieves more than 7 dB at $2 \times 10^{-2}$ for the same iterations compared with the MU-OFDM-OCVSK system without using LDPC, and for equal number of iterations the Min-Sum algorithm outperforms the Log-Domain algorithm.

4.2. The influence of Various Block length

The two types of LDPC decoders are applied with $M=100$ in the previous section 4-A. in this section $M$ is varied from 200 to 500 with different iterations to show the effect of increasing the block length for each type of decoder. simulation results of MU-OFDM-OCVSK system with Log-Domain, and Min-Sum decoders over multipath Rayleigh fading channel are shown in Figures. 6 to 9 respectively.

Figure 6. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) and $M=200$ using Log-Domin decoding algorithm with traditional MU-OFDM-OCVSK system

Figure 7. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) and $M=500$ using Log-Domin decoding algorithm with traditional MU-OFDM-OCVSK system

Figure 8. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) and $M=200$ using Min-Sum decoding algorithm with traditional MU-OFDM-OCVSK system

Figure 9. performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) and $M=500$ using Min-Sum decoding algorithm with traditional MU-OFDM-OCVSK system

It can be seen from the figures (6,7,8 and 9) that the increasing of the block length enhances the performance for the system in both decoders and this is an expected result, based on Shannon’s theorem which states that sending correctly coded data over a noisy communication channel,
using long lengths block, the probability of error can be reached zero at rates below the capacity of the channel. So increasing the block length of the code leads to low probability of error. As a comparison between the Log-Domain and Min-Sum Algorithms It is also shown in the Figures that Min-Sum algorithm outperforms the Log-Domain algorithm for equal block lengths.

![Figure 9](image_url)

**Figure 9.** performance comparison of LDPC-MU-OFDM-OCVSK for different iterations (3, 5, 10) and M=500 using Min-Sum decoding algorithm with traditional MU-OFDM-OCVSK system

### 4.3. The influence of Different number of users on the system

The two types of LDPC decoders are applied with two number of users in the previous sections 4-A and 4-B. In this section the number of users (U) is varied from 5 to 7 with 5 iterations, M=500, Rate=1/2, and β=120 to investigate the effect of increasing the number of users on the performance of the MU-OFDM-OCVSK system using Log-Domain, and Min-Sum decoders over multi-path Rayleigh fading channel. The results are shown in Figures 10.

![Figure 10](image_url)

**Figure 10.** BER performance of LDPC-MU-OFDM-OCVSK for various numbers of users (2,5,7) with M=500 using Min-Sum and Log-Domin decoding algorithms

It can be noticed from the Figure that increasing the number of users results in a deterioration in the BER performance of the suggested system using both Log-Domain, and Min-Sum decoders. This result is consistent with [6] where the BER performance of the system degrades by adding more users to the system because of the increasing in multiple access interference (MAI). The figure also shows a comparison with the traditional MU-OFDM-OCVSK system without using LDPC. Log-Domain decoder achieves more than 3.5 dB and 3dB improvement when the number of users is 5 and 7 respectively at $2*10^{-2}$ BER while Min-Sum achieves 4dB for 5 and 7 number of users at $2*10^{-2}$ BER as compared with the MU-OFDM-OCVSK system without using LDPC.

### 4.4. The influence of different spreading factor (β) on the system

The spreading factor (β) is varying from 32, 64, and 128 to investigate the impact of changing β on the system performance. Figures 11, and 12. illustrate the decoding performance of the
LDPC-MU-OFDM-OCVSK system using Log-Domain, and Min-Sum decoders respectively.

![Log Domain BER Performance](image1)

**Figure 11.** BER performance of LDPC-MU-OFDM-OCVSK for various numbers of $\beta$ (32, 64, 128) with $M=200, 500$ using Log-Domin decoding algorithm.

![Min Sum BER Performance](image2)

**Figure 12.** BER performance of LDPC-MU-OFDM-OCVSK for various different numbers of $\beta$ (32, 64, 128) with $M=200, 500$ using Min-Sum decoding algorithm.

It is clear from the figures that, the performance of the system at a spreading factor $\beta = 32$ is better than that at $\beta = 64, 128$. This is the result we expect, because both the reference and the data bearing signals are distorted by the channel noise in the receiver side over the channels for this kind of non-coherent modulation, a correlation between a noisy data bearing signal and a noisy reference signal, consequently will degrade the system performance.

5. **Conclusion**

In this paper, LDPC coding is proposed for MU-OFDM-OCVSK system to improve the system performance in terms of BER. The performance analysis of the suggested system is analyzed using two decoding algorithm techniques then compared with the traditional MU-OFDM-OCVSK system without coding over multi-path Rayleigh fading channel to present the effect of coding on the performance. The results show that the performance of the proposed system enhances with the increase of block length and the number of iteration at the cost of latency in both decoding techniques. The range of code gain is about $(5 - 7)$ dB is achieved by using Min-Sum decoder. While $(4.5 - 6)$ dB can be achieved when using Log-Domin decoder, for different scenarios, taking into account the complexity for each decoder. MU-OFDM-OCVSK has efficient performance when work with Min-Sum decoder with gain 1 dB and low complexity comparing with log-domain decoder. In general, we can say that using LDPC code can improve the performance of MU-OFDM-OCVSK for different numbers of $\beta$.

**Conflict of interest**

The authors confirm that the publication of this article causes no conflict of interest

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