Fuzzy-approximation-based prescribed performance control of air-breathing hypersonic vehicles with input constraints

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Abstract

In this article, aiming at the longitudinal dynamics model of air-breathing hypersonic vehicles, a fuzzy-approximation-based prescribed performance control scheme with input constraints is proposed. First, this article presents a novel prescribed performance function, which does not depend on the sign of initial tracking error. And combining prescribed performance control method with backstepping control, the control scheme can ensure that system can converge at a prescribed rate of convergence, overshoot, and steady-state error. In order to solve the problem that backstepping control method needs to be differentiated multiple times, fuzzy approximators are used to estimate the unknown functions, and norm estimation approach is used to simplify the computation of fuzzy approximator. Aiming at the problem of input saturation of actuator in subsystem of air-breathing hypersonic vehicle, the new auxiliary system is designed to ensure the stability and robustness of air-breathing hypersonic vehicle system under input constraints. Finally, the effectiveness of the proposed control strategy is verified by simulation analysis.

Keywords

Air-breathing hypersonic vehicles, fuzzy approximator, prescribed performance control, input constraints, auxiliary system

Introduction

Air-breathing hypersonic vehicle (AHV) is a new type of aerospace vehicle that can fly for a long distance at five times the speed of sound in large airspace, flying at an altitude
of 20–100 km in near space.\textsuperscript{1,2} AHV combines many advanced technologies of modern science and technology, and it has the advantages of spacecraft and aircraft. It is an important direction of future aircraft development. It has great development prospects in military and civil fields. In the military field, it is a new type of global strike weapon.\textsuperscript{3} In the civil field, if we develop reusable space vehicles, we can greatly reduce the cost of space travel, which is of great practical significance.\textsuperscript{4,5}

Compared with conventional aircraft, AHV is designed by integration of airframe and engine, and its waverider structure has special aerodynamic characteristics.\textsuperscript{6,7} During hypersonic flight, there will be strong coupling effect among propulsion system, body, and structure dynamics of aircraft.\textsuperscript{8} In addition, the aerodynamic and thermal characteristics of the aircraft also change dramatically, and the flight environment is complex, leading to the hypersonic vehicles with strong coupling, fast time variation, and nonlinear and uncertain characteristics.\textsuperscript{9–11} In practical application, in order to ensure the stable flight of hypersonic vehicles, the saturation of actuators of AHV should also be considered.

At present, many control theories have emerged in the field of AHV control system, such as robust control,\textsuperscript{12,13} sliding mode control,\textsuperscript{14,15} backstepping control,\textsuperscript{16,17} and intelligent control.\textsuperscript{18–20} Wei et al.\textsuperscript{21} proposed a $\mu$ synthesis analysis method and designed a robust control law. The control law can keep the system stable in the presence of unknown disturbances and unmodeled dynamics, but it will lose other control performance. Guo et al.\textsuperscript{22} proposed an $H_{\infty}$ control method based on online Simultaneous Policy Update Algorithm (SPUA), which converts $H_{\infty}$ state feedback control problem into solving Hamilton–Jacobi–Isaacs (HJI) equation. This method has good tracking effect, but the solution of the equation is still complex. Xu et al.\textsuperscript{23} designed an adaptive sliding mode control law to ensure that the velocity tracking error and altitude tracking error converge to the sliding mode surface and to ensure the control accuracy when the model parameters are perturbed. However, the high-frequency chattering of the control input limits the reliability of the control law. Wu et al.\textsuperscript{24} and Wang et al.\textsuperscript{25} use sliding mode observer to effectively weaken the high-frequency chattering of control input. Wang et al.\textsuperscript{26} and Tian et al.\textsuperscript{27} studied the chattering phenomenon in sliding mode control. By designing sliding mode observer or putting forward a new switching strategy of high-order sliding mode control, the accurate switching of sliding mode control can be achieved, but the stability of the control system will be slightly reduced. Sun et al.\textsuperscript{28} and Bu et al.\textsuperscript{29} introduce disturbance observer in every step of the design process of the backstepping scheme, which guarantees the robustness of the backstepping control law. However, the backstepping control requires high-precision model, and the repeated derivation of the virtual control law increases the computational complexity. Ji et al.\textsuperscript{30} design a backstepping control law based on instruction filter, which uses instruction filter to filter the virtual control law and avoids complex derivative calculation. In recent years, intelligent control has developed rapidly. Gao et al.\textsuperscript{31} use fuzzy system to approximate the unknown functions of each subsystem of AHV model online, which requires high computing power of the system.

In 2008, the Greek scholar Bechlioulis and Rovithakis\textsuperscript{32,33} proposed a new prescribed performance control strategy, which can simultaneously realize the constraint on the transient performance and steady-state performance of the system. Prescribed performance control transforms a tracking problem with performance constraints on output
errors into an unconstrained stabilization problem by introducing performance functions. By constructing error conversion functions, the performance indicators of the system are constrained. Many fields have begun to study the prescribed performance control, but the existing prescribed performance control rarely takes into account the problem of input constraints. In practical application, with the increase of AHV velocity, the efficiency of the elevator and other actuators will decrease significantly. Therefore, it is significant to develop the prescribed performance control with input constraints.

In this article, a new fuzzy-approximation-based prescribed performance control scheme with input constraints is proposed. First, the AHV control system is decomposed into velocity subsystem and altitude subsystem. Then, the control law is designed for each subsystem by using the newly designed prescribed performance function and backstepping control method. Each subsystem only needs one actual control law to be executed. Finally, the superior performance of the new control scheme proposed in this article is verified by simulation. The special contributions of this study include the following:

1. This article proposes a novel prescribed performance function. This function does not depend on the sign of initial tracking error, so it is more practical than the traditional prescribed performance function.
2. The prescribed performance function proposed in this article is combined with backstepping control method, which is more advantageous than backstepping control method.
3. In the controller, using the idea of norm estimation reduces the computational complexity of the fuzzy approximator, so that each fuzzy approximator contains only one adaptive parameter and improves the operation speed of control system.
4. In this study, a new auxiliary system to handle actuator saturation problems is introduced.

**Preliminaries**

**Prescribed performance**

*Lemma 1.* If a smoothing function \( \rho(t) : R^+ \to R^+ \) can satisfy the following two conditions, it can be called a prescribed function:

1. \( \rho(t) \) is a monotonically decreasing positive function.
2. \( \lim_{t \to \infty} \rho(t) = \rho_\infty \)

The traditional prescribed performance function takes the form of

\[
\rho(t) = (\rho_0 - \rho_\infty) e^{-lt} + \rho_\infty
\]

(1)

where \( l \), \( \rho_0 \), and \( \rho_\infty \) are design parameters, and \( l \in R^+ \), \( \rho_0 \in R^+ \), and \( \rho_\infty \in R^+ \).
According to the requirements of the prescribed performance, tracking error should meet the following relations

$$\begin{cases}
-\delta p(t) < e(t) < \rho(t), e(0) > 0 \\
-\rho(t) < e(t) < \delta p(t), e(0) < 0
\end{cases}$$

where $0 \leq \delta \leq 1$, $\delta$ is a design parameter. It can be seen from equation (2) that this function needs to predict the sign of the initial value of tracking error, but in fact the initial error is very difficult to obtain in advance. Therefore, two cases need to be analyzed in the following controller design and stability proof, which will undoubtedly greatly increase the complexity of the algorithm.

In order to eliminate the limitation that the initial error signs must be known, based on Lemma 1, we construct a new prescribed performance function

$$\rho(t) = \frac{1}{\sin h(\tau t + \lambda) + \rho_{\infty}}$$

Taking time derivative of $\rho(t)$

$$\dot{\rho}(t) = \frac{-\tau \cos h(\tau t + \lambda)}{\sin h(\tau t + \lambda)^2}$$

where $\tau$, $\lambda$, and $\rho_{\infty}$ are design parameters, and $\tau \in R^+$, $\lambda \in R^+$, and $\rho_{\infty} \in R^+$.

Obviously, $\rho(t)$ has the following properties

$$\begin{cases}
\rho(0) = \frac{1}{\sin h(\lambda) + \rho_{\infty}} > \rho_{\infty} \\
\rho(\infty) = \frac{1}{\sin h(\tau t + \lambda) + \rho_{\infty}} = \rho_{\infty}
\end{cases}$$

Therefore, the new function satisfies the definition of Lemma 1.

Let tracking error $e(t)$ satisfy the following in equation

$$-\kappa \rho(t) < e(t) < \bar{\kappa} \rho(t)$$

where $\kappa \in R^+$ and $\bar{\kappa} \in R^+$ are parameters. If we choose sufficiently small $\lambda$, $\kappa$, and $\bar{\kappa}$, there are $-\kappa \rho(0) \to -\infty$ and $\bar{\kappa} \rho(0) \to +\infty$, so whether the initial error $e(0) > 0$ or $e(0) < 0$, the following in equation (7) can be guaranteed

$$-\kappa \rho(0) < e(0) < \bar{\kappa} \rho(0)$$
Therefore, when the prescribed performance function takes the form of equation (3), the initial error sign does not need to be known, thus simplifying the subsequent controller design process.

The performance limitations for tracking errors are shown in Figure 1. As can be seen from Figure 1, $e(0)$ is the initial error, $\overline{\kappa \rho}(t)$ and $-\kappa \rho(t)$ limit the overshoot of the tracking error $e(t)$, the convergence speed of $\rho(t)$ limits the adjustment time of tracking error, and the region between $\overline{\kappa \rho}(t)$ and $-\kappa \rho(t)$ limits the steady-state range of tracking error.

By selecting appropriate parameters $\tau$, $\hat{\kappa}$, and $\rho_\infty$, constraints on the dynamic performance indexes of the control system can be realized.

Because it is difficult to construct the controller directly according to equation (6), an error conversion function is introduced to facilitate the subsequent design. The form of the function is shown as follows

$$e(t) = \rho(t)S(\varepsilon)$$  \hspace{1cm} (8)$$

where

$$S(\varepsilon) = \frac{\overline{\kappa \rho} e^\varepsilon - \kappa \rho e^{-\varepsilon}}{e^\varepsilon + e^{-\varepsilon}}$$  \hspace{1cm} (9)$$

$\varepsilon$ is the conversion error.

Since $S(\varepsilon)$ is a smooth monotone increasing function, there must be an inverse function. Its inverse function is in the form of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{prescribed_performance_graph.png}
\caption{Graphical presentation of the prescribed performance.}
\end{figure}
\[ \varepsilon = S^{-1} \begin{bmatrix} e(t) \\ \rho(t) \end{bmatrix} = \frac{1}{2} \ln \frac{e(t)/\rho(t) + \kappa}{\kappa - e(t)/\rho(t)} \]  

(10)

where

\[ \dot{\varepsilon} = \zeta \left( \frac{\dot{e} - \hat{\rho} e}{\rho} \right), \zeta = \frac{1}{2 \rho} \begin{bmatrix} 1 & -1 \\ e(t)/\rho(t) + \kappa & e(t)/\rho(t) - \kappa \end{bmatrix} \]

According to equation (9), \( S(\varepsilon) \) satisfies the properties 
\[ \lim_{\varepsilon \to +\infty} S(\varepsilon) = \bar{\kappa} \]
\[ \lim_{\varepsilon \to -\infty} S(\varepsilon) = -\kappa. \]

Thus, we can see that

\[ -\kappa < S(\varepsilon) < \bar{\kappa} \]  

(11)

Because \( \rho(t) > 0 \), multiply two sides by \( \rho(t) \)

\[ -\kappa \rho(t) < \rho(t) S(\varepsilon) < \bar{\kappa} \rho(t) \]  

(12)

According to equation (8), equation (12) is equivalent to equation (6). According to the conclusion of Bechlioulis and Rovithakis,\textsuperscript{32} if the boundedness of \( \varepsilon \) is guaranteed, the tracking error \( e(t) \) can satisfy the prescribed performance requirements. Therefore, the following controller can be designed with \( \varepsilon \) instead of tracking error \( e(t) \). As long as \( \varepsilon \) is bounded, the performance limitation of tracking error can be guaranteed.

**Fuzzy function approximator**

Wang\textsuperscript{39} proved that the fuzzy logic system with single-point fuzzification, product inference, and central average degelatinization is an universal approximator.

Assuming that the input space of a fuzzy logic system is \( U \) and the output space is \( R \), the fuzzy system performs a mapping from \( U \) to \( R \) : in the process of fuzzification, a clear point input is mapped to a fuzzy set \( A \) on \( U \). Then, the fuzzy set \( A \) on \( U \) is mapped to a fuzzy set \( B \) on \( R \) by using the fuzzy rules. Finally, the fuzzy set \( B \) on \( U \) is mapped to a clear point on \( R \) by the process of defuzzification, thus the mapping from \( U \) to \( R \) is realized.

The output of the system can be expressed as

\[ y = \theta^T \xi(X) \]  

(13)

In equation (13), the input vector is \( X = [l_1, l_2, \ldots, l_n]^T \in R^n \). The weight coefficient parameter vector of the fuzzy system is \( \theta = [\theta_1, \theta_2, \ldots, \theta_p]^T \in R^p \),
\( \xi(X) = [\xi_1(X), \xi_2(X), \ldots, \xi_p(X)]^T \in \mathbb{R}^p \) is fuzzy basis function, and the fuzzy basis function is expressed as

\[
\xi_j(X) = \frac{\prod_{i=1}^{n} \mu_{F_i}(x_i)}{\sum_{l=1}^{N} \left( \prod_{i=1}^{n} \mu_{F_i}(x_i) \right)} \tag{14}
\]

In equation (14), \( \mu_{F_i} \) is the membership function. In this article, Gauss basis function is chosen as membership function.

**Lemma 2.** For a given real continuous function \( F(X) \) on a compact set \( \Omega \), there must be a fuzzy logic system \( y \) that makes

\[
|F(X) - y| < \mu \tag{15}
\]

where \( \mu \in \mathbb{R}^+ \). In the following section, we will design a fuzzy function approximator.

**AHV model**

In the current research results, most of the work on AHV control system is carried out in the longitudinal direction. Because AHV is extremely sensitive to altitude, in order to reduce fuel consumption, lateral maneuver should be avoided during flight. The Parker et al.'s\(^{10}\) model proposed by the US Air Force Laboratory, which ignores some weak coupling factors and slow dynamic terms of AHV system, is used as the dynamic model in this article

\[
\dot{V} = \frac{T \cos \alpha - D}{m} - g \sin \gamma \tag{16}
\]

\[
\dot{h} = V \sin \gamma \tag{17}
\]

\[
\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{g}{V} \cos \gamma \tag{18}
\]

\[
\dot{\theta} = Q \tag{19}
\]

\[
\dot{Q} = \frac{M + \psi_1 \dot{\theta}_1 + \psi_2 \dot{\theta}_2}{I_{yy}} \tag{20}
\]
\[ k_1 \ddot{\eta}_1 = -2\zeta_1 \omega_1 \dot{\eta}_1 - \omega_1^2 \eta_1 + N_1 - \dot{\psi}_1 \frac{M}{I_{yy}} - \frac{\ddot{\psi}_1 \ddot{\eta}_1}{I_{yy}} \]
\[ k_2 \ddot{\eta}_2 = -2\zeta_2 \omega_2 \dot{\eta}_2 - \omega_2^2 \eta_2 + N_2 - \dot{\psi}_2 \frac{M}{I_{yy}} - \frac{\ddot{\psi}_2 \ddot{\eta}_1}{I_{yy}} \]

For rigid body states of AHV, \( V \) and \( h \) represent the flight velocity and altitude of AHV, respectively. The flight path angle is \( \gamma \), and the flight pitch angle and pitch rate are expressed as \( \theta \) and \( Q \), respectively. The distance between AHV and the earth’s center is expressed by \( r \), and \( m \) is the weight of AHV. Pitch moment of inertia is expressed by \( I_{yy} \), and define the angle of attack as \( \alpha = \theta - \gamma \). For flexible states of AHV, \( \eta_1 \) and \( \eta_2 \) represent the flexible states of AHV, \( \zeta_1 \) and \( \zeta_2 \) represent flexible damping, \( \omega_1 \) and \( \omega_2 \) are the frequencies of the flexible states, and \( N_1 \) and \( N_2 \) are generalized flexible forces.

\[
\begin{aligned}
    k_1 &= 1 + \frac{\ddot{\psi}_1}{I_{yy}} \\
    k_2 &= 1 + \frac{\ddot{\psi}_2}{I_{yy}} \\
    \ddot{\psi}_1 &= \int_{-L}^{0} \tilde{m}_f \xi \phi_f (\xi) d\xi \\
    \ddot{\psi}_2 &= \int_{0}^{L} \tilde{m}_a \xi \phi_a (\xi) s\xi
\end{aligned}
\]

\( \phi_f (\xi) \) and \( \phi_a (\xi) \) are mode functions.\(^{10}\)

The specific magnitudes of lift \( L \), thrust \( T \), drag \( D \), pitch moment \( M \), and generalized forces \( N_1 \) and \( N_2 \) of AHV can be expressed by fitting formula

\[
\begin{aligned}
    T &\approx C_T^{\alpha^3} + C_T^{\alpha^2} + C_T^{\alpha} + C_T^0 \\
    D &\approx \bar{q}S \left( C_D^{\alpha^3} \alpha^2 + C_D^{\alpha} \alpha + C_D^{\alpha^2} \delta_e^2 + C_D^0 \delta_e \right) \\
    L &\approx \bar{q}S \left( C_L^{\alpha} \alpha + C_L^{\alpha^2} \delta_e + C_L^0 \right) \\
    M &\approx z_T T + \bar{q}S \bar{c} \left[ C_{M,\alpha}^{\alpha^3} \alpha^2 + C_{M,\alpha}^{\alpha} \alpha + C_{M}^0 \delta_e \right] \\
    N_1 &\approx N_1^{\alpha^3} \alpha^2 + N_1^{\alpha} \alpha + N_1^0 \\
    N_2 &\approx N_2^{\alpha^3} \alpha^2 + N_2^{\alpha} \alpha + N_2^0 \delta_e + N_2^0 \\
    \bar{q} &= \frac{1}{2} \bar{p}V^2, \quad \bar{p} = \bar{p}_0 \exp \left( \frac{h_0 - h}{h_s} \right)
\end{aligned}
\]
The control input of AHV system is fuel-equivalent ratio $\Phi$ and elevator deflection angle $\delta_e$. The dynamic pressure of AHV is $\bar{q}$ and the average air density at altitude $h$ is $\bar{\rho}$. $h_0$ and $\bar{\rho}_0$ represent the air density at nominal altitude and nominal altitude, respectively; the reference aerodynamic area and the average aerodynamic chord length of AHV are $S$ and $\bar{c}$, respectively. Other specific parameters can be found in Parker et al.\textsuperscript{10}

**Controller design**

The control objective of AHV is to realize robust tracking of reference values $V_{\text{ref}}$ and $h_{\text{ref}}$ of $V$ and $h$ by continuously adjusting $\Phi$ and $\delta_e$ under the condition that the AHV model is unknown.

**Design of velocity controller**

Equation (16) in the velocity subsystem is transformed into the following form

$$\dot{V} = F_V + \Phi$$  \hspace{1cm} (21)

where $F_V = ((T \cos(\theta - \gamma) - D) / m) - g \sin \gamma - \Phi$, assuming that $F_V$ is an unknown nonlinear function in the model.

Tracking error of velocity is defined as

$$\tilde{V} = V - V_{\text{ref}}$$  \hspace{1cm} (22)

Combining the contents of equation (21) and taking derivative of equation (22)

$$\dot{\tilde{V}} = V - V_{\text{ref}} = F_V + \Phi - \dot{V}_{\text{ref}}$$  \hspace{1cm} (23)

Since function $F_V$ is an unknown function, we can use fuzzy approximator to approximate it. Where the estimation of $\tilde{F}_V$ of $F_V$ can be expressed as

$$\tilde{F}_V = \theta_v^* \xi_v(X_v)$$  \hspace{1cm} (24)

In equation (24), $\theta_v^* = [\theta_{v1}, \theta_{v2}, \ldots, \theta_{v_{p_v}}]^T \in \mathbb{R}^{p_v}$ is the ideal weight coefficient parameter vector, $X_v = V$ is input vector, and the fuzzy basis function is $\xi_v(X_v) = [\xi_{v1}(X_v), \xi_{v2}(X_v), \ldots, \xi_{v_{p_v}}(X_v)]^T \in \mathbb{R}^{p_v}$. $F_V$ can be expressed as

$$F_V = \theta_v^* \xi_v(X_v) + \mu_V \cdot \mu_V \leq \mu_{V_{\text{Max}}}$$  \hspace{1cm} (25)

where $\mu_V$ and $\mu_{V_{\text{Max}}}$ are fuzzy approximation error and its upper bound, respectively.
In order to reduce the computational burden, based on the idea of norm estimation approach, we define \( \phi_{VV} = \| \theta_{VV} \|_2 \), and in the next fuzzy approximation algorithm, we only need to estimate \( \phi_{VV} \), without considering the specific changes of \( \theta_{VV} \) in each calculation.

According to equation (10), the conversion error \( \varepsilon_V \) of velocity tracking error is defined as

\[
\varepsilon_V = \frac{1}{2} \ln \left( \frac{\dot{V}/\rho_V + \kappa_V}{\dot{V}/\rho_V} \right)
\]

In equation (26), \( \rho_V = 1/ \sin(h(\tau_V t + \lambda_V) + \rho_{V_{\infty}}) \) and \( \kappa_V \in R^+, \lambda_V \in R^+, \tau_V \in R^+, \lambda_V \in R^+ \), and \( \rho_{V_{\infty}} \in R^+ \) are design parameters.

Combining equation (23), the following equation can be obtained by calculating the time derivative of equation (26)

\[
\dot{\varepsilon}_V = \zeta_V \left( \dot{V} - \dot{\rho}_V \dot{V} \right) = \zeta_V \left( F_V - \dot{V}_{ref} + \Phi - \dot{\rho}_V \dot{V} \right)
\]

Based on backstepping control theory, \( \Phi \) can be designed

\[
\Phi = -k_{V,1} \varepsilon_V - k_{V,2} \int_0^t \varepsilon_V d\tau - \frac{\varepsilon_V}{2} \dot{\phi}_V \xi_V^T(X_V) \xi_V(X_V) + \dot{V}_{ref} + \dot{\rho}_V \dot{V}
\]

where \( k_{V,1} \in R^+ \) and \( k_{V,2} \in R^+ \) are design parameters, the estimation of \( \phi_V \) is \( \dot{\phi}_V \), and the adaptive law of \( \dot{\phi}_V \) is designed as

\[
\dot{\phi}_V = \frac{1}{2} c_V \varepsilon_V^2 \xi_V^T(X_V) \xi_V(X_V) - 2k_{V,1} \dot{\phi}_V
\]

where \( c_V \in R^+ \) is the design parameter.

Since the actual input of \( \Phi \) is constrained, \( \Phi \) is defined as

\[
\Phi = \begin{cases} 
\Phi_{\text{max}}, & \Phi_c > \Phi_{\text{max}} \\
\Phi_c, & \Phi_{\text{min}} \leq \Phi_c \leq \Phi_{\text{max}} \\
\Phi_{\text{min}}, & \Phi_c < \Phi_{\text{min}} 
\end{cases}
\]

where \( \Phi_{\text{min}} \) and \( \Phi_{\text{max}} \) are the lower and upper bounds of input \( \Phi \), respectively, and \( \Phi_c \) is the ideal input value to be designed

\[
\Delta \Phi = \Phi - \Phi_c
\]
where $\Delta \Phi$ is the part of constrained input. We designed a new auxiliary system to deal with the effects of input saturation

$$\dot{\chi}_V = -\sigma_1 \text{arsh} \chi_V + \Phi - \Phi_c$$

(32)

where $\chi_V$ is the state variable of the auxiliary system, $\sigma_1 \in R^+$ is the design parameter, and it should select the appropriate parameter value based on the tracking effect.

Tracking error in equation (22) is rewritten as

$$Z_V = \tilde{V} - \chi_V$$

(33)

Derivation of equation (33)

$$\dot{Z}_V = \dot{V} - \dot{\chi}_V = F_V - \dot{V}_{\text{ref}} + \sigma_1 \text{arsh} \chi_V + \Phi_c$$

(34)

Then, conversion error $\epsilon_V$ is

$$\epsilon_V = \frac{1}{2} \ln \left( \frac{Z_V / \rho_V + \kappa_V}{\kappa_V - Z_V / \rho_V} \right)$$

(35)

In equation (35)

$$\rho_V = \frac{1}{\sin h (\tau_V t + \kappa_V) + \rho_{V \infty}}$$

where $\kappa_V \in R^+$, $\kappa_V \in R^+$, $\tau_V \in R^+$, $\kappa_V \in R^+$, and $\rho_{V \infty} \in R^+$ are design parameters.

Substitute equation (34) into equation (35) and we get

$$\dot{\epsilon}_V = \xi_V \left( \dot{Z}_V - \hat{\rho}_V Z_V \right) = \xi_V \left( F_V - \dot{V}_{\text{ref}} + \sigma_1 \text{arsh} \chi_V + \Phi_c - \frac{\hat{\rho}_V Z_V}{\rho_V} \right)$$

(36)

The ideal control law is redesigned as

$$\Phi_c = -k_{V,1} \epsilon_V - k_{V,2} \int_0^t \epsilon_V d\tau - \frac{\epsilon_V}{2} \hat{\phi}_V \xi_V^T (X_V) \xi_V (X_V) + \dot{V}_{\text{ref}} - \sigma_1 \text{arsh} \chi_V + \frac{\hat{\rho}_V Z_V}{\rho_V}$$

(37)

where $k_{V,1} \in R^+$ and $k_{V,2} \in R^+$ are the design parameters.
Next, we will analyze the stability of velocity controller with input constraints theoretically.

**Theorem 1.** Consider the closed-loop system for velocity subsystem of AHV consisting of control law (37), adaptive law (29), and auxiliary compensation system (32). Then, all the signals involved are semi-globally uniformly ultimately bounded.

The proof of Theorem 1 is listed in Appendix 1.

**Remark 1.** Compared with the method in Gao et al.,31 in the process of controlling each subsystem of AHV, each weight update requires a large amount of calculation. This article adopts the design of norm estimation approach, and only one online learning parameter is required, which significantly reduces the online calculation amount.

**Design of altitude controller**

To simplify the controller design, the altitude subsystem is rewritten as

\[
\begin{align*}
\dot{h} &= V\gamma \\
\dot{\gamma} &= F_{\gamma} + \theta \\
\dot{\theta} &= Q \\
\dot{Q} &= F_Q + \delta_e
\end{align*}
\]  

(38)

In equation (38)

\[
F_{\gamma} = \frac{L + T \sin(\theta - \gamma)}{mV} - \frac{g}{V} \cos(\gamma - \theta), 
F_Q = M + \psi_1 \ddot{h} + \frac{\psi_2 \ddot{h}}{I_{yy}} - \delta_e
\]

Step 1: define the tracking error of altitude as

\[
\tilde{h} = h - h_{ref}
\]

(39)

The derivative of time for equation (39) can be obtained

\[
\dot{\tilde{h}} = V\gamma - \dot{h}_{ref}
\]

(40)

According to equation (10), the conversion error of altitude is defined as

\[
\varepsilon_h = \frac{1}{2} \ln \left( \frac{\tilde{h} / \rho_h + \kappa_h}{\tilde{h} / \rho_h} \right)
\]

(41)
In equation (41), $\rho_{h V} = 1/\sin(h(\tau + \hat{h}_{V})) + \rho_{h 0}$. Where $\kappa_{V} \in R^{+}$, $\bar{\kappa}_{V} \in R^{+}$, $\tau_{V} \in R^{+}$, $\hat{\kappa}_{V} \in R^{+}$, and $\rho_{V 0} \in R^{+}$ are design parameters.

The derivative of equation (41) can be obtained

$$\dot{\hat{h}}_V = \hat{h}_V + \frac{1}{\sin(h(\tau + \hat{h}_{V}))) + \rho_{h 0}}$$

Select virtual control $\gamma_c$ as

$$\gamma_{c} = \frac{1}{V} \left(-k_{h,1}e_h - k_{h,2} \int_0^1 e_h d\tau + \hat{h}_{ref} + \frac{\hat{\rho}_h \hat{h}}{\rho_h} \right) \tag{42}$$

where $k_{h,1} \in R^{+}$ and $k_{h,2} \in R^{+}$ are design parameters.

In order to avoid the expansion problem caused by calculation of the first derivative in the virtual control law, the tracking differentiator with high stability and rapidity (HSSTD) proposed by Yao and Cao40 is introduced to estimate the first derivative of $\gamma_c$

$$\begin{cases}
\dot{\gamma}_d = v_1 \\
\dot{v}_1 = -a_{11} [\gamma_d - \gamma_c]^m - a_{12} v_1^m - a_{13} [\gamma_d - \gamma_c]^n - a_{14} [v_1]^n \text{sgn} v_1
\end{cases} \tag{44}$$

where $a_{11} \in R^{+}$, $a_{12} \in R^{+}$, $a_{13} \in R^{+}$, $a_{14} \in R^{+}$, $0 < m_i < 1$, and $0 < n_i < 1$ are design parameters.

Step 2: define track angle tracking error as

$$\tilde{\gamma} = \gamma - \gamma_d \tag{45}$$

The derivation of equation (45) can be obtained

$$\dot{\tilde{\gamma}} = F_{\gamma} + \theta - \dot{\gamma}_d \tag{46}$$

Based on the proposed fuzzy approximator, the estimation of $F_{\gamma}$ can be expressed as

$$\hat{F}_{\gamma} = \theta_{\gamma}^T \xi_{\gamma} \left(X_{\gamma} \right) \tag{47}$$

Define the ideal weight coefficient parameter vector as $\theta_{\gamma}^*$, then the unknown function $F_{\gamma}$ can be expressed as
where $|\mu_\gamma| \leq \mu_{\gamma_{\text{Max}}}$, $\mu_\gamma$ is the fuzzy approximation error, and $\mu_{\gamma_{\text{Max}}}$ is the upper bound of approximation error.

To reduce the computational burden, define $\varphi_\gamma$ as follows

$$\varphi_\gamma = \|\theta^*_\gamma\|^2$$

Select virtual control $\theta_c$ as

$$\theta_c = -k_{\gamma,1}\ddot{\gamma} - k_{\gamma,2} \int_0^t \ddot{\gamma} d\tau - \frac{T}{2} \ddot{\gamma} \phi_{\gamma}^T(X_\gamma) \phi_{\gamma}(X_\gamma) + \dot{\gamma}_d$$

where $k_{\gamma,1} \in \mathbb{R}^+$ and $k_{\gamma,2} \in \mathbb{R}^+$ are design parameters, the estimation of $\varphi_\gamma$ is $\hat{\varphi}_\gamma$, and the adaptive law of $\hat{\varphi}_\gamma$ is designed as

$$\dot{\hat{\varphi}}_\gamma = -\frac{1}{2} c_{\gamma} \ddot{\gamma} \phi_{\gamma}^T(X_\gamma) \phi_{\gamma}(X_\gamma) - 2k_{\gamma,1} \hat{\varphi}_\gamma$$

where $c_{\gamma} \in \mathbb{R}^+$ is the design parameter.

Similarly, HSSTD is introduced to estimate the first derivative of $\theta_c$

$$\begin{cases}
\dot{\theta}_d = v_2 \\
v_2 = -a_{21} [\theta_d - \theta_c]^m - a_{22} v_2^m - a_{23} [\theta_d - \theta_c]^n - a_{24} |v_2|^n \text{ sgn} v_2
\end{cases}$$

where $a_{21} \in \mathbb{R}^+$, $a_{22} \in \mathbb{R}^+$, $a_{23} \in \mathbb{R}^+$, $a_{24} \in \mathbb{R}^+$, $0 < m_2 < 1$, and $0 < n_2 < 1$ are design parameters.

Step 3: define the pitch angle tracking error as

$$\tilde{\theta} = \theta - \theta_d$$

The derivative of time for equation (53) can be obtained

$$\dot{\tilde{\theta}} = Q - \dot{\theta}_d$$

Select virtual control $Q_c$ as
\(Q_c = -k_{\theta,1}\ddot{\theta} - k_{\theta,2}\int_0^t \dot{\theta} \, d\tau - \ddot{\gamma} + \dot{\theta}_d\)  

(55)

where \(k_{\theta,1} \in \mathbb{R}^+\) and \(k_{\theta,2} \in \mathbb{R}^+\) are the design parameters.

Get the value of virtual control \(Q_c\) by HSSTD

\[
\begin{align*}
\dot{\tilde{Q}}_d &= \nu_3 \\
\dot{\nu}_3 &= -a_{31} [Q_d - Q_c]^m_3 - a_{32} \nu_3^m_3 - a_{33} [Q_d - Q_c]^n_3 - a_{34} \nu_3^n_3 \text{sgn} \nu_3
\end{align*}
\]

(56)

where \(a_{31} \in \mathbb{R}^+\), \(a_{32} \in \mathbb{R}^+\), \(a_{33} \in \mathbb{R}^+\), \(a_{34} \in \mathbb{R}^+\), \(0 < m_3 < 1\), and \(0 < n_3 < 1\) are design parameters.

Step 4: the tracking error of pitch angular rate is defined as

\(\ddot{Q} = Q - Q_d\)

(57)

The derivative of time for equation (57) can be obtained

\(\dot{Q} = F_Q + \delta_e - \dot{Q}_d\)

(58)

Similarly, the fuzzy approximator is used to estimate the unknown nonlinear function \(F_Q\)

\(\hat{F}_Q = \theta_{Q}^{T} \xi_Q (X_Q)\)

(59)

Defining the ideal weight coefficient parameter vector as \(\theta_{Q}^{*}\), \(F_Q\) can be expressed as

\(F_Q = \theta_{Q}^{*T} \xi_Q (X_Q) + \mu_Q\)

(60)

where \(|\mu_Q| \leq \mu_{Q_{\text{Max}}}\), \(\mu_Q\) is the fuzzy approximation error, and \(\mu_{Q_{\text{Max}}}\) is the upper bound of \(\mu_Q\).

Define \(\varphi_Q\) as

\(\varphi_Q = ||\theta_{Q}^{*}||^2\)

(61)

If the estimation of \(\varphi_Q\) is \(\hat{\varphi}_Q\), then the actual control law \(\delta_{e}^{C}\) is
\[ \delta_{ec} = -k_{Q,1}Q - k_{Q,2} \int_0^t \dot{Q} d\tau - \frac{\dot{Q}}{2} \dot{\bar{\phi}} Q \xi_Q^T \left( \mathbf{X}_Q \right) \xi_Q \left( \mathbf{X}_Q \right) - \theta + \dot{Q}_d \] (62)

where \( k_{Q,1} \in R^+ \) and \( k_{Q,2} \in R^+ \) are both controller parameters.

The adaptive law of \( \bar{\phi}_Q \) is designed as

\[ \dot{\bar{\phi}}_Q = \frac{1}{2} c_Q \tilde{Q}^2 \tilde{\xi}_Q^T \left( \mathbf{X}_Q \right) \xi_Q \left( \mathbf{X}_Q \right) - 2k_{Q,1} \bar{\phi}_Q \] (63)

where \( c_Q \in R^+ \) is the design parameter.

Next, we redesign the control law in the constrained case.

The constrained elevator deflection is described as

\[ \delta = \begin{cases} \delta_{emax}, & \delta_{emax} \leq \delta_{ec} \\ \delta_{ec}, & \delta_{emin} \leq \delta_{ec} \leq \delta_{emax} \\ \delta_{emin}, & \delta_{ec} \leq \delta_{emin} \end{cases} \] (64)

where \( \delta_{ec} \) is the ideal control law and \( \delta_{emax} \) and \( \delta_{emin} \) are the upper and lower bounds of \( \delta_{ec} \), respectively.

In order to eliminate the influence of input constraints on control performance, a new auxiliary system is introduced to eliminate the deviation between the ideal control law and the control law caused by input constraints

\[ \dot{x}_h = -\sigma_h \arsh x_h + \delta_e - \delta_{ec} \] (65)

where \( \sigma_h \in R^+ \) is the design parameter.

The modified tracking error is defined as

\[ Z_Q = Q - x_h \] (66)

The derivative of equation (66) can be obtained

\[ \dot{Z}_Q = \dot{Q} - \dot{x}_h = F_Q - \dot{Q}_d + \sigma_h \arsh x_h + \delta_e \] (67)

Select the modified control law \( \delta_{ec} \)

\[ \delta_{ec} = -k_{Q,1}Z_Q - k_{Q,2} \int_0^t Z_Q d\tau - \frac{Z_h}{2} \dot{\bar{\phi}} Q \xi_Q^T \left( \mathbf{X}_Q \right) \xi_Q \left( \mathbf{X}_Q \right) - \theta + \dot{Q}_d - \sigma_h \arsh x_h \] (68)
where \( k_{Q,1} \in R^+ \) and \( k_{Q,2} \in R^+ \) are both controller parameters.

**Theorem 2.** Consider the closed-loop system for altitude subsystem of AHV consisting of control laws (43), (50), (55), and (68); adaptive laws (51) and (63); and auxiliary compensation system (65). Then, all the signals involved are semi-globally uniformly ultimately bounded.

The proof of Theorem 2 is listed in Appendix 1.

**Remark 2.** Similar to the dynamic performance presupposition of \( V \) and \( h \), the dynamic performance of tracking errors of other state variables \( \gamma, \theta \), and \( Q \) can also be limited. It is only necessary to design the prescribed performance function for these state variables and replace the tracking error with the transition error to design the virtual and actual control laws.

Therefore, the whole control system and the structure of the AHV model are shown in Figure 2.

**Simulation results**

Based on the control system and AHV model shown in Figure 2, the tracking simulation of velocity and altitude reference instructions is carried out to verify the effectiveness of the control algorithm. The fourth-order Runge–Kutta method is used in the simulation experiment, and the step size is 0.01 s.

The parameters of the controller are \( k_{V,1} = 2, k_{V,2} = 15, k_{h,1} = 12, k_{h,2} = 1, k_{\gamma,1} = 2.5, k_{\gamma,2} = 0.4, k_{\theta,1} = 8.5, k_{\theta,2} = 1.8, k_{Q,1} = 45 \), and \( k_{Q,2} = 10 \). Fuzzy adaptive parameters are \( c_V = c_\gamma = c_Q = 0.1 \). The design parameters of the differentiators are \( a_{11} = a_{12} = a_{13} = a_{14} = 30, a_{21} = a_{22} = a_{23} = a_{24} = 20, a_{31} = a_{32} = a_{33} = a_{34} = 20, m_1 = m_2 = m_3 = 0.7 \), and \( n_1 = n_2 = n_3 = 0.2 \). The values of flexible state \( \tilde{\psi}_1 = 4223.44, \tilde{\psi}_2 = 4223.55, \omega_1 = 16.02 \, \text{rad/s}, \omega_2 = 19.58 \, \text{rad/s}, \) and \( \zeta_1 = \zeta_2 = 0.02 \). Values of error conversion function \( \kappa_V = \kappa_h = 2 \) and \( \kappa_\gamma = \kappa_\theta = 2 \).

The input variables of three fuzzy approximators are selected as \( x_V = [V], x_\gamma = [V, \gamma, \theta]^T \), and \( x_Q = [V, \gamma, \theta, Q]^T \), respectively. Among them, the range of state variables is limited to \( V \in [2346.96 \, \text{m/s}, 2712.72 \, \text{m/s}], \gamma \in [-2^\circ, +2^\circ], \theta \in [-5^\circ, +5^\circ], \) and \( Q \in [-5^\circ/s, +5^\circ/s] \).

Gauss basis function is chosen as membership function

\[
\mu_{i}^{x_i}(x_i) = \exp \left\{ - \left( x_i - \mu_{ij} \right)^2 \right\}, i = V, \gamma, Q, j = 1,2,\ldots, 7
\]

The prescribed performance functions are taken into the following forms

\[
\begin{align*}
\rho_V &= 0.3048 / \sin h(0.2t + 0.9) + 0.03048 \\
\rho_h &= 0.3048 / \sin h(0.2t + 0.2) + 0.24384
\end{align*}
\]
The initial states of AHV are shown in Table 1. In the cruise state, the following two commands are mainly followed.

**Case 1. Verify the effectiveness of the prescribed performance control method**

The selected velocity reference command is a step signal, which produces a step signal of 91.44 m/s every 100 s, and the track altitude command is a square wave signal, whose amplitude $\Delta h = 91.44$ m. In order to verify that the designed prescribed performance function needs no sign of initial error, the initial error of velocity is set to $+0.5$ m/s and the initial error of altitude is set to $-1$ m. The simulation results are shown in Figures 3–9. The curves of subscript “1” in Figures 3 and 4 are the simulation results of the prescribed performance fuzzy control method proposed in this article without adding constrained auxiliary system. The curves of subscript “2” are the simulation results of the backstepping control method without prescribed performance control method and constrained auxiliary system proposed in this article.

From Figure 3, it can be seen that the velocity tracking error of “1” can converge rapidly to 0, the overshoot is only $-0.2$ m, and the adjustment time is only about 2 s. Since the velocity commands change every 100 s, the tracking error fluctuates slightly at 100 and 200 s, but the fluctuation range is within the prescribed boundary. The velocity tracking error of “2” can also reach the equilibrium state with a small overshoot and a short adjustment time within the prescribed boundary, but at 100 and 200 s, the error fluctuation amplitude of “2” is close to the prescribed boundary, which is obviously larger than the error fluctuation amplitude of “1.” Figure 4 is the altitude tracking error curve. Because the altitude command changes many times during the simulation period, the altitude tracking error fluctuates in a large range. But it can be seen that the altitude tracking error of “1” is always within the prescribed boundary, while that of “2” is
beyond the prescribed boundary many times. In addition, although the initial errors of velocity and altitude are not 0 and the sign are different in the two simulations, it does not affect the performance limitation of the controller on tracking errors. If the traditional
Figure 4. Altitude tracking and tracking error.

Figure 5. Control inputs.
Figure 6. Track angle and tracking error.

Figure 7. Pitch angle and tracking error.
Figure 8. Pitch rate and tracking error.

Figure 9. Flexible states.
prescribed performance control is adopted, two kinds of controllers need to be discussed and designed separately, which will undoubtedly greatly increase the complexity of the control algorithm.

Figure 5 shows the change curves of control input. It can be seen that the control curves are smooth as a whole, but the fuel equivalence ratio and the elevator deflection angle have many peaks. By comparing the variation curves of altitude tracking errors in Figures 4 and 5, it can be seen that the altitude tracking errors are limited by the prescribed boundary at every peak. Figures 6–9 are the curves of rigid body state and flexible state. It can be seen that both rigid body state and flexible body state are within the allowable range, and the flexible vibration suppression effect is better.

Case 2: Verify the effectiveness of the prescribed performance control method with input constraints

Select the step signal with the velocity reference command step of 329.18 m/s and the altitude track tracking amplitude of 1219.2 m. To verify the robustness of the control system, perturbation is added to all aerodynamic parameters when the simulation time is more than 100 s. The auxiliary system parameters are \( \sigma_v = 4 \) and \( \sigma_h = 5 \). In order to verify the effectiveness of the auxiliary system, the control inputs are limited to \( \Phi \in [0.2, 0.85] \) and \( \delta_e \in [11^\circ, 15^\circ] \). The method of adding auxiliary system proposed in this article is compared with the method of not adding auxiliary system. The simulation results of the methods that add auxiliary system are shown in Figures 10–17, and the simulation results of the methods that do not add auxiliary system are shown in Figures 18–23.

Figures 10 and 11 show the variation curves of control input and auxiliary system. It can be seen that the fuel equivalent ratio and elevator deflection angle are limited, especially the fuel equivalence ratio is severely limited, and the saturation state lasts for nearly 20 s. From the change in the auxiliary system, it can be seen that when the control input is constrained, the compensation system can respond quickly and make the control input out of saturation state. The continuous perturbation of aerodynamic parameters occurs after 100 s, it can be seen that the control input can follow the change of parameters and respond in time to ensure the stable tracking of the control system. Figures 12 and 13 are the variation curves of velocity and altitude tracking errors. It can be seen that due to the seriously constrained fuel equivalence ratio, the velocity tracking errors have large deviations, but still can ensure that the tracking errors are within the prescribed boundaries and meet the prescribed performance requirements. In addition, due to the persistent perturbation of aerodynamic parameters after 100 s, the altitude tracking error fluctuates to some extent, but it can always be within the prescribed boundary to meet the performance requirements. Figures 14–17 show the variation curves of rigid body state and flexible state. It can be seen that when the aerodynamic parameters are perturbed, the tracking errors of each attitude angle are kept near 0, which ensures good tracking effect. With the perturbation of aerodynamic parameters, each state variable can respond quickly and be within its allowable range.
Figure 10. Fuel equivalent ratio with addition of auxiliary system.

Figure 11. Elevator deflection angle with addition of auxiliary system.
Figure 12. Velocity tracking and tracking error with addition of auxiliary system.

Figure 13. Altitude tracking and tracking error with addition of auxiliary system.
Figure 14. Track angle and tracking error with addition of auxiliary system.

Figure 15. Pitch angle and tracking error with addition of auxiliary system.
**Figure 16.** Pitch rate and tracking error with addition of auxiliary system.

**Figure 17.** Flexible states with addition of auxiliary system.
Figure 18. Velocity tracking and tracking error without addition of auxiliary system.

Figure 19. Altitude tracking and tracking error without addition of auxiliary system.
Figure 20. Fuel equivalent ratio without addition of auxiliary system.

Figure 21. Elevator deflection angle without addition of auxiliary system.
Figure 22. Track angle, pitch angle, and pitch rate without addition of auxiliary system.

Figure 23. Flexible states without addition of auxiliary system.
Figures 18–23 are the response curves without the auxiliary system. It can be seen that the constant saturation of fuel equivalence ratio leads to the continuous increase in velocity tracking error, but the steady-state value of velocity tracking error caused by the limitation of prescribed boundary can only be changed in a very small range of $\pm 0.24384 \text{ m/s}$, which makes the control variables unable to exit the saturation state, thus causing the control mission to fail. This shows that if only the tracking error of the control system is limited and the instantaneous saturation of the input is ignored, the controller will probably fail. Therefore, in the process of practical engineering application, the performance index selection of tracking error and the input constraints of control variables should be considered comprehensively to prevent the conflicts between the two constraints on the control system.

In addition, it can be seen from Figure 21 that when there is no compensation, the elevator deflection angle can also exit from the transient saturation state, but the application of the constrained compensation strategy proposed in this article can make the control input exit from the saturation state faster and reduce the chattering phenomenon, reduce the impact of the control input constraints on trajectory tracking, and improve the performance of the control system.

**Conclusion**

A new prescribed performance controller based on fuzzy function approximator with input constraints is proposed in this article. By constructing a new prescribed performance function, the tracking error can satisfy both the prescribed transient performance and the steady-state performance. By introducing norm estimation approach into the fuzzy approximator, each fuzzy approximator only needs an adaptive parameter update law to reduce the computational load of the system. Considering input constraints, the influence of actuator constraints on the performance of the control system is analyzed, and the new type of auxiliary system is used to compensate the system. Through simulation verification, the proposed method is verified and has certain advantages. In future research, we will conduct research on fuzzy control, prescribed performance control, and non-affine systems.

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Appendix I

The proof of Theorem 1 is as follows.

The estimation error of $\phi_v$ is defined as

$$\hat{\phi}_v = \hat{\phi}_v - \phi_v$$

(69)

Substitute equation (37) into equation (36) to get

$$\dot{\epsilon}_v = \xi_v \left[ -k_{v,1} \epsilon_v - k_{v,2} \int_0^t \epsilon_v d\tau + \theta_v^T \xi_v \left( X_v \right) - \frac{\epsilon_v}{2} \dot{\phi}_v \xi_v^T \left( X_v \right) \xi_v \left( X_v \right) + \mu_v \right]$$

(70)

Select Lyapunov function as

$$W_v = \frac{1}{2\xi_v^2} \epsilon_v^2 + \frac{1}{2} k_{v,2} \left[ \int_0^t \epsilon_v d\tau \right]^2 + \frac{\hat{\phi}_v^2}{2c_v}$$

(71)

Take the derivative of equation (71) and substitute equation (36) into (72)

$$\dot{W}_v = \frac{\epsilon_v \dot{\epsilon}_v}{\xi_v} + k_{v,2} \epsilon_v \int_0^t \epsilon_v d\tau + \frac{\dot{\phi}_v \hat{\phi}_v}{c_v}$$

$$= -k_{v,1} \epsilon_v^2 + \epsilon_v \theta_v^T \xi_v \left( X_v \right) - \frac{1}{2} \epsilon_v^2 \dot{\phi}_v \xi_v^T \left( X_v \right) \xi_v \left( X_v \right) + \epsilon_v \mu_v - 2k_{v,1} \frac{\hat{\phi}_v \hat{\phi}_v}{c_v}$$

(72)

as
\[
\varepsilon \theta^T \xi (X_V) \leq \frac{1}{2} \varepsilon^T \| \theta^T \xi (X_V) \|^2 + \frac{1}{2} = \frac{1}{2} \varepsilon^2 \bar{\theta}^T \xi (X_V) \xi (X_V) + \frac{1}{2}
\]

\[-2k_{V,1} \frac{\bar{\phi}_V \phi_V}{c_V} \leq \frac{k_{V,1}}{c_V} (\phi_V^2 - \bar{\phi}_V^2) \leq \frac{k_{V,1}}{c_V} \phi_V^2, e_V \mu_V \leq \frac{1}{2} \varepsilon^2 \mu_{V Max}^2 + \frac{1}{2}
\]

Therefore, equation (72) can be rewritten as

\[
\dot{W}_V \leq -\left( k_{V,1} - \frac{1}{2} \mu_{V Max}^2 \right) e_V^2 + \frac{k_{V,1} \phi_V^2}{c_V} + 1
\]  

(73)

Define the following compact set

\[
\Omega_{e_V} = \left\{ e_V \| e_V \| \leq \begin{cases} k_{V,1} \phi_V^2 + 1 \\ k_{V,1} - \frac{1}{2} \mu_{V Max}^2 \end{cases} \right\}
\]  

(74)

Let \( k_{V,1} > (1/2) \mu_{V Max}^2 \), if \( e_V \) is outside the compact \( \Omega_{e_V} \), and then, \( \dot{W}_V \leq 0 \).

The boundedness of the new auxiliary system \( \chi_V \) proposed in this article is proved in the following. First, the Lyapunov function of the auxiliary system is defined as

\[
W^V = \frac{1}{2} \chi^2
\]  

(75)

Let \( |\Phi - \Phi_c| \leq \Phi_{Max} \), where, \( \Phi_{Max} \in R^+ \).

Combining equations (34) and (74), the derivative of equation (75) is obtained

\[
\dot{W}_\chi = -k_V \chi_V \arsh \chi_V + \chi_V (\Phi - \Phi_c) \leq -k_V |\Phi - \Phi_c| \arsh \chi_V | + \chi_V |\Phi_{Max}
\]

\[
\leq -\left( k_V |\arsh \chi_V | - \Phi_{Max} \right) |\chi_V |
\]  

(76)

Let \( k_V \geq |\arsh \chi_V | k_V \geq \Phi_{Max} \), \( \dot{W}_\chi \leq 0 \), so the auxiliary system \( \chi_V \) is bounded, and by \( Z_V = \tilde{V} - \chi_V \), we know that the tracking error \( \tilde{V} \) is bounded.

The stability of the velocity subsystem is proved.

The proof of Theorem 2 is as follows.

First, the estimation error of the differentiator is defined as

\[
\begin{align*}
e_{\gamma} &= \gamma_d - \gamma_c \\
e_{\theta} &= \theta_d - \theta_c \\
e_Q &= Q_d - Q_c
\end{align*}
\]  

(77)
It can be seen from Yao and Cao\(^{40}\) that the estimation errors \(e_\gamma, e_\theta, \) and \(e_Q\) are bounded by selecting appropriate differentiator parameters. Therefore, there must be a positive constant \(e_{i,\text{Max}}\) so that \(|e_i| \leq e_{i,\text{Max}}(i = \gamma, \theta, Q)\).

When equations (43), (45), and (77) are substituted into equation (41), there is

\[
\dot{e}_h = \zeta_h \left[ V(\tilde{y} + e_\gamma + \gamma_c) - \dot{h}_{\text{ref}} - \frac{\dot{\rho}_h}{\rho_h} \right] \\
= \zeta_h \left[ -k_{h,1}e_h - k_{h,2} \int_0^t e_h d\tau + V\tilde{y} + Ve_\gamma \right]
\]  

(78)

Combining equations (46), (48), and (50), we can get

\[
\dot{\tilde{y}} = F_\gamma + \theta_c + \tilde{\theta} - \dot{\gamma}_d + e_\theta \\
= -k_{\gamma,1}\tilde{y} - k_{\gamma,2} \int_0^t \tilde{y} d\tau + \theta_\gamma^\tau \xi_\gamma(X_\gamma) - \\
\frac{\gamma}{2} \tilde{\phi}_\gamma^\tau (X_\gamma) \tilde{\phi}_\gamma(X_\gamma) + \mu_\gamma + \tilde{\theta} + e_\theta
\]  

(79)

Substituting equations (55), (57), and (77) into equation (54), we can get

\[
\dot{\tilde{\theta}} = \tilde{Q} + Q_c - \tilde{\theta}_d + e_Q \\
= -k_{\theta,1}\tilde{\theta} - k_{\theta,2} \int_0^t \tilde{\theta} d\tau - \tilde{\gamma} + \tilde{Q} + e_Q
\]  

(80)

According to equations (67), (60), and (68), we have

\[
\dot{Z}_Q = -k_{Q,1}\tilde{Q} - k_{Q,2} \int_0^t Z_Q d\tau - \tilde{\theta} + \theta_\gamma^{\star T} \xi_\gamma(X_\gamma) - \frac{Z_{Q}}{2} \tilde{\phi}_Q^\tau (X_Q) \tilde{\phi}_Q(X_Q) + \mu_Q
\]  

(81)

The estimation error of fuzzy approximator learning variables are defined as

\[
\begin{align*}
\tilde{\phi}_V &= \phi_V - \phi_V \\
\tilde{\phi}_\gamma &= \phi_\gamma - \phi_\gamma \\
\tilde{\phi}_Q &= \phi_Q - \phi_Q
\end{align*}
\]  

(82)
The Lyapunov function is selected as

\[ W = W_h + W_\gamma + W_\theta + W_Q \]  

(83)

In the equations

\[ W_h = \frac{1}{2\xi V_{\text{Max}}^2} \xi^2 + \frac{1}{2} k_{h,2} \left( \int_0^t \xi_h d\tau \right)^2 \]  

(84)

\[ W_\gamma = \frac{1}{2} \hat{\gamma}^2 + \frac{1}{2} k_{\gamma,2} \left( \int_0^t \hat{\gamma} d\tau \right)^2 + \frac{\phi^2_\gamma}{2c_\gamma} \]  

(85)

\[ W_\theta = \frac{1}{2} \hat{\theta}^2 + \frac{1}{2} k_{\theta,2} \left( \int_0^t \hat{\theta} d\tau \right)^2 \]  

(86)

\[ W_Q = \frac{1}{2} Z_Q^2 + \frac{1}{2} k_{Q,2} \left( \int_0^t Z_Q d\tau \right)^2 + \frac{\phi^2_Q}{2c_Q} \]  

(87)

where, \( V_{\text{Max}} \geq V \).

Combining equations (51), (63), and (78)–(82), the derivative of equation (83) can be obtained

\[ \dot{W} = \dot{W}_h + \dot{W}_\gamma + \dot{W}_\theta + \dot{W}_Q = \frac{\xi_h \dot{\xi}_h}{\xi V_{\text{Max}}^2} + \frac{k_{h,2} \xi_h}{V_{\text{Max}}^2} \int_0^t \xi_h d\tau \\
+ \hat{\gamma} \ddot{\gamma} + k_{\gamma,2} \int_0^t \hat{\gamma} d\tau + \frac{\phi^2_\gamma}{c_\gamma} + \hat{\theta} \ddot{\theta} + k_{\theta,2} \int_0^t \hat{\theta} d\tau + Z_Q \dot{Z}_Q \\
+ k_{Q,2} Z_Q \int_0^t Z_Q d\tau + \frac{\phi^2_Q}{c_Q} \]  

(88)
\[
\gamma \theta_{\gamma}^T \xi_\gamma (X_\gamma) \leq \frac{1}{2} \gamma^2 \| \theta_{\gamma}^T \| \| \xi_\gamma (X_\gamma) \|^2 + \frac{1}{2} = \frac{1}{2} \gamma^2 \phi_\gamma \xi_\gamma (X_\gamma) + \frac{1}{2}
\]

\[
-2k_{\gamma,1} \frac{\phi_\gamma}{c_\gamma} \leq \frac{k_{\gamma,1}}{c_\gamma} \left( \phi_\gamma - \hat{\phi}_\gamma \right) \leq \frac{k_{\gamma,1}}{c_\gamma} \phi_\gamma^2
\]

\[
Z_\omega \theta_{\omega}^T \xi_\omega (X_\omega) \leq \frac{1}{2} Z_\omega^2 \| \theta_{\omega} \| \| \xi_\omega (X_\omega) \|^2 + \frac{1}{2} = \frac{1}{2} Z_\omega^2 \phi_\omega \xi_\omega^T (X_\omega) \xi_\omega (X_\omega) + \frac{1}{2}
\]

\[
-2k_{\omega,1} \frac{\phi_\omega}{c_\omega} \leq \frac{k_{\omega,1}}{c_\omega} \left( \phi_\omega - \hat{\phi}_\omega \right) \leq \frac{k_{\omega,1}}{c_\omega} \phi_\omega^2, \gamma_{\mu} \leq \frac{1}{2} \gamma^2 \mu_{\mu,\gamma} + \frac{1}{2}
\]

\[
Z_\omega \mu_\omega \leq \frac{1}{2} Z_\omega^2 \mu_{\omega,\gamma} + \frac{1}{2}, \chi_{\mu,\gamma} \hat{\theta} \leq \frac{1}{2} \chi_{\mu,\gamma} \hat{\theta}^2 + \frac{1}{2}
\]

\[
V \epsilon_\gamma \gamma \leq \frac{1}{2} \epsilon_\gamma^2 + \frac{1}{2} V^2 \gamma^2, V \epsilon_\mu \gamma \leq \frac{1}{2} \epsilon_\mu^2 + \frac{1}{2} V^2 \gamma^2
\]

\[
\gamma \epsilon_\gamma \leq \frac{1}{2} \gamma^2 + \frac{1}{2} \epsilon_\gamma^2, \gamma \epsilon_\omega \leq \frac{1}{2} \hat{\epsilon}^2 + \frac{1}{2} \epsilon_\omega^2
\]

So, equation (88) can be rewritten as

\[
\dot{W} \leq -\left( k_{\gamma,1} - \frac{1}{2} \mu_{\gamma,\gamma} - 1 \right) \gamma^2 - \left( k_{\omega,1} - \frac{1}{2} \mu_{\omega,\omega} - 1 \right) \hat{\theta}^2 - \left( k_{\omega,1} - \frac{1}{2} \mu_{\omega,\omega} \right) Z_\omega^2 + \Sigma
\]  

(89)

where \( \Sigma = (k_{\gamma,1} / c_\gamma) \phi_\gamma^2 + (k_{\omega,1} / c_\omega) \phi_\omega^2 + (1 / 2) \epsilon_\gamma^2 + (1 / 2) \epsilon_\omega^2 + (1 / 2) \epsilon_{\omega,\gamma}^2 + (1 / 2) \epsilon_{\omega,\omega}^2 + (5 / 2) \)

Define the following compact sets, respectively

\[
\Omega_{\epsilon} = \left\{ \epsilon_\gamma \left| \epsilon_\gamma \leq \sqrt{\frac{V_{\max}^2}{k_{\gamma,1} - 1}} \right. \right\}
\]

(90)

\[
\Omega_\gamma = \left\{ \gamma \left| \gamma \leq \sqrt{\frac{\Sigma}{k_{\gamma,1} - \mu_{\gamma,\gamma}^2 - 1}} \right. \right\}
\]

(91)
\[
\Omega_{\tilde{\theta}} = \left\{ \tilde{\theta} \mid \|\tilde{\theta}\| \leq \sqrt{\frac{\Sigma}{k_{\theta,1} - \frac{1}{2} \chi_{h,\text{Max}}^2 - \frac{1}{2}}} \right\}
\]

(92)

\[
\Omega_{\tilde{Q}} = \left\{ \tilde{Q} \mid \|\tilde{Q}\| \leq \sqrt{\frac{\Sigma}{k_{Q,1} - \frac{1}{2} \mu_{\text{Q,Max}}^2}} \right\}
\]

(93)

Let

\[
k_{h,1} > 1, k_{\gamma,1} > \frac{1}{2} \mu_{\gamma,\text{Max}}^2 + 1, k_{\theta,1} > \frac{1}{2} \chi_{h,\text{Max}}^2 + \frac{1}{2}, k_{Q,1} > \frac{1}{2} \mu_{Q,\text{Max}}^2
\]

If \(\varepsilon_{\gamma}, \varepsilon_{h}, \tilde{\gamma}, \tilde{\theta}, \text{ and } \tilde{Q}\) are all outside the compact sets \(\Omega_{\gamma}(x = \varepsilon_{\gamma}, \varepsilon_{h}, \tilde{\gamma}, \tilde{\theta}, \tilde{Q})\), then we have \(\dot{W}'' = 0\).

Define the compact set \(\Omega\)

\[
\Omega = \Omega_{\varepsilon} \cup \Omega_{\gamma} \cup \Omega_{\tilde{\theta}} \cup \Omega_{\tilde{Q}}
\]

(94)

Choosing large enough \(k_{\gamma,1}, k_{\theta,1}, k_{Q,1}\) can make the radius of compact \(\Omega\) arbitrarily small and all error signals arbitrarily small.

The above proofs are obtained on the premise that the introduced auxiliary system \(\chi_{h}\) is bounded. Then, the following proves that the introduced auxiliary system \(\chi_{h}\) is bounded. The Lyapunov function of the auxiliary system is defined as

\[
W_{\chi}^h = \frac{1}{2} \chi_{h}^2
\]

(95)

Let \(|\delta_e - \delta_{ec}| \leq \delta_{e,\text{Max}}\) and \(\delta_{e,\text{Max}}\) is a positive constant.

Combining equations (92) and (65), derivation of equation (95) is obtained

\[
\dot{W}_{\chi}^h = -\sigma_h \chi_{h} \arsh \chi_{h} + \chi_{h} (\delta_e - \delta_{ec}) \\
\leq -\sigma_h |\chi_{h}| \arsh |\chi_{h}| + |\chi_{h}| |\delta_{e,\text{Max}}| \\
\leq -\left(\sigma_h |\arsh \chi_{h}| - \delta_{e,\text{Max}}\right) |\chi_{h}|
\]

(96)
Let
\[ \sigma_h \geq \left| \arsh \chi_h \right| k_h \geq \delta_{e\text{Max}} \] (97)

Then, \( \dot{\chi}_h \leq 0 \), so the auxiliary system \( \chi_h \) is bounded, and by \( Z_Q = \tilde{Q} - \chi_h \), we know that the tracking error \( \tilde{Q} \) is bounded.

If the constrained compensation strategy proposed in Xu et al.\(^4\) is used, the form of the auxiliary system of the velocity subsystem and the altitude subsystem is as follows
\[
\begin{align*}
\begin{cases}
\dot{\chi}_V &= -\sigma_V \chi_V + (\Phi - \Phi_c) \\
\dot{\chi}_h &= -\sigma_h \chi_h + (\delta_e - \delta_{ec})
\end{cases}
\end{align*}
\] (98)

When the constraint is serious, it will lead to \( \chi_V \to \infty \) and \( \chi_h \to \infty \), which makes \( Z_V = \tilde{V} - \chi_V \to \infty \) and \( Z_Q = \tilde{Q} - \chi_h \to \infty \). Therefore, compared with the auxiliary system proposed in this article, this type of auxiliary system can only prove the boundedness of \( Z_V \) and \( Z_h \), but it is difficult to guarantee the boundedness of tracking error.