Apparent cosmic acceleration due to large-scale peculiar motions

Christos G Tsagas

Section of Astrophysics, Astronomy and Mechanics, Department of Physics
Aristotle University of Thessaloniki, Thessaloniki 54124, Greece
E-mail: tsagas@astro.auth.gr

Abstract. Observers drifting relative to the smooth Hubble flow have expansion rates different from that of the Universe itself. As a result, observers with small peculiar velocities in linearly perturbed Friedmann universes can experience accelerated expansion, while the universe is actually decelerating. The effect is local, but the affected scales can be large enough to give the (false) impression that the whole universe has recently entered an accelerating phase. Recent surveys reporting large-scale peculiar velocities substantially larger than previously anticipated add a certain degree of observational support to the aforementioned theoretical result.

1. Introduction
In idealised Friedmann-Robertson-Walker (FRW) cosmologies, comoving observers follow the universal expansion. In more realistic models, however, the Hubble flow is distorted and matter acquires ‘peculiar’ velocities. Thus, the dipole in the Cosmic Microwave Background (CMB) is interpreted as the result of our peculiar flow (at roughly 600 km/sec) relative to the cosmic rest-frame. This report considers the implications of such drift motions for the kinematics of the associated observers. First, we ask whether drifting observers and those following the Hubble expansion (in a dust-dominated FRW universe) can ‘measure’ substantially different deceleration parameters. Whether, in particular, it is theoretically possible for a peculiarly moving observer to ‘experience’ accelerated expansion while the universe is decelerating. We show that the answer to this question is positive. Accelerated expansion for a drifting observer does not necessarily imply the same for the universe itself. Then, we look for observational data that could turn this theoretical result into a realistic possibility. Based on the predictions of the ΛCDM model, this seems rather unlikely. However, based on recent independent surveys reporting bulk peculiar motions of up to 1000 km/sec on scales extending as far out as 1000 Mpc [1–5], the answer to the second question could be positive as well.1

2. Peculiar kinematics in perturbed FRW universes
The Microwave Background sets a preferred cosmological coordinate system, that of the smooth Hubble flow. This is the frame where the dipolar anisotropy of the CMB vanishes and with respect to which large-scale peculiar velocities can be defined and measured.

1 The reader is also referred to [6] for an alternative/complementary discussion.
2.1. The expansion rates

Suppose that \( u_a \) is the reference 4-velocity of the CMB, relative to which the universe is an exact dust-dominated FRW model. In the perturbed spacetime, a typical observer in a galaxy like our Milky Way has

\[
\tilde{u}_a = u_a + v_a,
\]

with \( v_a \) representing their drift velocity.\(^2\) Note that \( u_a v^a = 0 \) always and \( v^2 = v_a v^a \ll 1 \) in our case. The latter constraint reflects the almost-FRW nature of our model and guarantees that

\[
\gamma = (1 - v^2)^{-1/2} \approx 1.
\]

The mean kinematics of the tilded observers are determined by the volume scalar \((\tilde{\Theta} = \nabla^a \tilde{u}_a)\) of their worldline congruence \([7, 8]\). Positive values for \( \tilde{\Theta} \) imply that the mean separation between these observers increases and indicate expansion. Similarly, \( \Theta \) (with \( \Theta = \nabla^a u_a > 0 \)) monitors the expansion of the universe. To first order in \( v_a \), we have\(^3\)

\[
\tilde{\Theta} = \Theta + \vartheta,
\]

with \( \vartheta = \tilde{D}^a v_a \). This scalar measures the mean separation between neighbouring peculiar-flow lines. When \( \vartheta \) is positive, the peculiarly moving observers expand faster than the universe (i.e. \( \tilde{\Theta} > \Theta \)). Here, we will always assume positive \( \vartheta \).

In multi-component systems each group of observers has its own time-direction. So, in our case, time can be measured relative to the CMB and along the tilded frame. The expansion rate in each time-direction is determined by the associated Raychaudhuri equation \([7, 8]\). In a dust-dominated FRW model with small drift velocities, the Raychaudhuri formulae in the CMB and the tilded frames are

\[
\Theta' = -\frac{1}{3} \Theta^2 - \frac{1}{2} \rho
\]

and \[
\dot{\tilde{\Theta}} = -\frac{1}{3} \Theta^2 - \frac{1}{2} \tilde{\rho} + \tilde{D}^a \tilde{A}_a,
\]

respectively. Here, primes indicate time-differentiation along \( u_a \) and overdots are time-derivatives relative to the \( \tilde{u}_a \)-field (i.e. \( \Theta' = u^a \nabla_a \Theta \) and \( \dot{\tilde{\Theta}} = \tilde{u}^a \nabla_a \tilde{\Theta} \)). Also, \( \tilde{\rho} \) and \( \rho \) are the matter densities in the CMB and the tilded frames respectively (with \( \tilde{\rho} = \rho \) to linear order in \( v_a \) \([10]\)). Finally, \( \tilde{A}_a \) is the 4-acceleration in the tilded frame. This vector vanishes in the CMB frame (i.e. \( \tilde{A}_a = 0 \)) but is nonzero in every other relatively moving coordinate system. In particular, \( \tilde{A}_a = \tilde{v}_a + (\Theta/3)v_a \), to linear order in \( v_a \) \([10, 11]\). The 4-acceleration term in Eq. (3b) is central to our analysis. Its presence means that expressions (3a) and (3b) are different, even when matter is pressureless dust and the peculiar velocities are small. In other words, drifting observers have expansion rates different from that of the actual universe simply because of their relative motion. For our purposes, this fact represents the most significant theoretical deviation from the conventional single-fluid studies.

2.2. The deceleration parameters

We have just shown that the expansion rate of a typical observer in a dust-dominated almost FRW universe differs from that of the Hubble flow. This means that the deceleration parameters measured in the two frames are different as well. One might then ask whether it is possible to experience accelerated expansion in one frame and decelerated in the other. Whether, in particular, the peculiarly moving observer could measure a negative deceleration parameter,

\(^2\) The \( \tilde{u}_a \)-field is also timelike irrespective of the magnitude of the peculiar velocity. Each frame defines its own time direction and 3-space. The tensors \( h_{ab} = g_{ab} + u_a u_b \) and \( \tilde{h}_{ab} = g_{ab} + \tilde{u}_a \tilde{u}_b \), with \( g_{ab} \) representing the spacetime metric, project orthogonal to \( u_a \) and \( \tilde{u}_a \) respectively. They also define the orthogonally projected covariant derivative operators, by means of \( D_a = h_a^b \nabla_b \) and \( \tilde{D}_a = \tilde{h}_a^b \nabla_b \) (\( \nabla_a \) is the standard covariant derivative) \([7, 8]\).

\(^3\) Recall that we have assumed non-relativistic peculiar velocities, which implies that we can drop terms of order \( v^2 \) and higher from Eq. (2) and the rest of our formulae. We also use geometrised units with \( c = 1 = 8\pi G \).
while the universe is actually decelerating. Given that drift flows are expected to die away as we move on to successively larger scales, such an effect can only be local.

To investigate this possibility, we first need to write down the deceleration parameters as ‘measured’ in the $u_a$ and $\tilde{u}_a$ frames. Expressed in terms of the corresponding volume scalars, these read

$$q = -\left(1 + \frac{3\Theta'}{\Theta^2}\right)$$
$$\tilde{q} = -\left(1 + \frac{3\tilde{\Theta}}{\tilde{\Theta}^2}\right),$$

respectively. Substituting the above into Eqs. (3), the latter recast into

$$(1 + q)\Theta^2 = \Theta^2 + \frac{3}{2} \rho$$
$$(1 + \tilde{q})\tilde{\Theta}^2 = \tilde{\Theta}^2 + \frac{3}{2} \tilde{\rho} - 3\tilde{D}^a\tilde{A}_a.$$

Although these already show that $q$ and $\tilde{q}$ are generally different, it helps to relate the two parameters directly. First, recall that $\tilde{\rho} = \rho$ and $\tilde{A}_a = \dot{v}_a + (\Theta/3)v_a$. Then, using the definition $\vartheta = D^a v_a$, expression (2) and the (linear in $v_a$) relation $\dot{\vartheta} = D^a \dot{v}_a - \Theta \vartheta/3$ (see [11]), Eqs. (5) combine to give

$$1 + \tilde{q} = (1 + q) \left(1 + \frac{\vartheta}{\Theta}\right)^{-2} - \frac{3\dot{\vartheta}}{\Theta^2} \left(1 + \frac{\vartheta}{\Theta}\right)^{-2}. $$

This clearly shows that $q$ and $\tilde{q}$ generally differ. Moreover, as long as the right-hand side of (6) remains below unity, positive values for $q$ do not a priori guarantee the same for $\tilde{q}$. It is therefore theoretically possible for the tilded observer to experience accelerated expansion in a decelerating universe.

At this point, it is worth noting that the condition $-1 < \tilde{q} < 0$ is equivalent to $-\tilde{\Theta}^2/3 < \dot{\tilde{\Theta}} < 0$ (see Eq. (4b)), which means that $\tilde{q}$ and $\dot{\tilde{\Theta}}$ can be simultaneously negative. Analogous relations also hold between $q$, $\Theta^2$ and $\Theta'$. One should therefore distinguish between accelerated expansion with simply $-1 < \tilde{q} < 0$ and that with $\dot{\tilde{\Theta}} > 0$. We may therefore view $-1 < \tilde{q} < 0$ and $\dot{\tilde{\Theta}} > 0$ (equivalently $\tilde{q} < -1$) as the conditions for ‘weakly’ and ‘strongly’ accelerated expansion respectively. It is also important to recognise that, as long as we only require $\tilde{q}$ to lie in the (-1.0) range, the 4-acceleration term in Eqs. (3b) and (5b) does not need to dominate the right-hand side of these expressions. This implies that peculiar motions can lead to weakly accelerated expansion within the limits of the linear (the almost-FRW) approximation. Noting that the supernovae results put the deceleration parameter close to $-0.5$ [12, 13], we will focus on the $-1 < \tilde{q} < 0$ case for the rest of this report.

3. Apparent acceleration in perturbed FRW universes

The main theoretical principle following from our discussion so far, is that measuring a negative deceleration parameter in a frame drifting relative to the CMB (like that of our Local Group for example) does not necessarily imply an accelerating universe. The question is whether current observations support such a scenario.

3.1. A scenario for apparent acceleration

Consider an expanding spatial region $\mathcal{A}$ – see Fig. 1, which largely complies with the FRW symmetries, but is still endowed with a weak, bulk peculiar velocity field that ‘adds’ to the background expansion (i.e. $\vartheta > 0$, with $\vartheta \ll \Theta$). Typical observers in $\mathcal{A}$ have peculiar velocities close to the bulk flow of the patch and their deceleration parameters obey Eq. (6). The simplest case corresponds to $3\vartheta/\Theta^2 \simeq 0$, which occurs when $\vartheta$ varies very slowly with time (for example). Then, since $\vartheta/\Theta \ll 1$, a simple Taylor expansion reduces (6) to

$$1 + \tilde{q} = \left(1 + \frac{1}{2} \Omega\right) \left[1 - 2 \left(\frac{\vartheta}{\Theta}\right)\right].$$

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Figure 1. The patch \( A \) has positive \( \dot{\vartheta} = \tilde{D}^a v_a \) and so expands faster than its surroundings (see Eq. (2)). Inside region \( B \) the right-hand side of expression (6) drops below unity and there the comoving observer ‘measures’ negative deceleration parameter.

given that \( q = \Omega/2 \) in dust-dominated FRW models. Note that \( \Omega = 3\rho/\Theta^2 \) (with \( \rho \) representing the matter density in the \( u_a \)-frame) may be seen as the effective density parameter of patch \( A \), rather than that of the universe itself. Following the above, when

\[
\left( 1 + \frac{1}{2} \Omega \right) \left[ 1 - 2 \left( \frac{\vartheta}{\Theta} \right) \right] < 1. \tag{8}
\]

\( \tilde{q} \) becomes negative, though \( q \) is still positive.

Let us now turn to the last term of Eq. (6). Qualitatively speaking, a positive \( \dot{\vartheta} \) will assist the acceleration, relax the above given conditions and lead to lower values of \( \tilde{q} \). So, here, we will assume that \( \dot{\vartheta} \) is negative. We will also demand that \( \vartheta/\Theta' \approx \dot{\vartheta}/\Theta \ll 1 \), to ensure that both \( \vartheta \) and \( \dot{\vartheta} \) are small perturbations relative to their background associates. The next step is to recast Raychaudhuri’s formula (see Eq. (3a)) in the form

\[
\Theta' = -\frac{1}{3} \Theta^2 \left( 1 + \frac{1}{2} \Omega \right), \tag{9}
\]

with \( \Theta' < 0 \). Solving the above for \( \Theta^2 \), substituting into Eq. (6) and employing some straightforward algebra we arrive at

\[
1 + \tilde{q} = \left( 1 + \frac{1}{2} \Omega \right) \left( 1 - \frac{\vartheta}{\Theta} \right). \tag{10}
\]

In this case, the necessary condition for accelerated expansion, in an otherwise decelerating universe, recasts into

\[
\left( 1 + \frac{1}{2} \Omega \right) \left( 1 - \frac{\vartheta}{\Theta} \right) < 1. \tag{11}
\]

When either of conditions (8) or (11) is satisfied, around every typical observer in \( A \) there will be an essentially spherically symmetric region \( B \) (with \( B \subseteq A \)) where the expansion will be accelerated (see Fig. 1). In other words, the aforementioned observers will experience (locally) accelerated expansion in a globally decelerating universe. Whether this happens or not, as well as the scale of affected area (the size of section \( B \)), depends mainly on the speed of the drift flow. So, the next question is whether observations allow for peculiar velocities large enough to trigger apparent acceleration.
3.2. Comparison to observations

Peculiar velocities are difficult to assess, since direct measurements only provide their radial component. One also needs to subtract the Hubble expansion from the data, which requires independent knowledge of the galaxy’s distance. As a result, bulk peculiar velocities are estimated by means of statistical methods (e.g. see [14]).

The ΛCDM model predicts peculiar velocities no more than \( \sim 100 \) km/sec, on scales close to 100 Mpc, which die away as we move out to larger lengths. Recent independent surveys, however, have reported significantly larger drift velocities [1–5], putting the standard picture into doubt [15]. These surveys extend to lengths of 100 Mpc [4, 5], 300 Mpc and 500 Mpc [1–3], with \( h \) being the Hubble parameter in units of 100 km/sec Mpc. The results show bulk velocities as large as 500 km/sec [4, 5] and up to 1000 km/sec [1–3] on the corresponding scales. On smaller lengths (between 30 and 60 Mpc) the work of [16] suggests the results of [1–3] suggest that \( \tilde{\vartheta} \) lies in the (-0.4, 0) range. This condition strengthens to 0 < \( \Omega < 0.5 \) at 100 Mpc, while further out, near the 1000 Mpc mark for instance, \( \tilde{\vartheta} \) will remain positive unless 0 < \( \Omega < 0.04 \). Inserting these numbers into Eq. (7) we obtain a range of values for the deceleration parameter of the tilded observer on the corresponding scales. Thus, provided condition (8) is satisfied, \( \tilde{\vartheta} \) varies within (-0.4, 0) on scales of 50 Mpc, between (-0.2, 0) when we move to 100 Mpc and within (-0.02, 0) near the 1000 Mpc threshold. So, in this example the size of accelerated region (i.e. that of patch (B) in Fig. 1) ranges between 50 Mpc and 1000 Mpc. Within these scales \( \tilde{\vartheta} \) lies in the (-0.4, 0) range, taking its minimum value in small-scale regions of low density and approaching zero as we move on to larger lengths. These estimates are not far from those inferred by the supernovae data, which value the deceleration parameter close to -0.5 and put the transition to deceleration near \( z = 0.5 \) [12, 13]. The picture does not change much when we adopt the results of [16], the surveys of [4, 5], or those of [1–3]. Substituted into expressions (7) and (8), the former give -0.2 < \( \tilde{\vartheta} < 0 \) in regions of 50 Mpc when \( \Omega < 0.5 \) there. Similarly, close to 150 Mpc, the measurements of [4, 5] put \( \tilde{\vartheta} \) in the range (-0.1, 0), provided \( \Omega < 0.2 \) there. Finally on lengths of 450 and 800 Mpc, the results of [1–3] suggest that \( \tilde{\vartheta} \) varies the range (-0.07, 0) and (-0.04, 0) respectively, when \( \Omega < 0.15 \) and \( \Omega < 0.07 \) on the corresponding scales. Note that the same survey indicates bulk flows of 1500 km/sec on scales close to 150 Mpc. Inserted into Eqs. (7), (8) these values lead to -0.3 < \( \tilde{\vartheta} < 0 \) when \( \Omega < 0.8 \). One should keep in mind, however, that on relatively small scales the peculiar-velocity errorbars are large (see [1]).

Finally, substituting the previous values of \( \partial/\Theta \) into condition (11), we find that negative \( \tilde{\vartheta} \)'s on \( \sim 50 \) Mpc scales need \( \Omega < 0.5 \). Similarly, (11) translates into \( \Omega < 0.2 \) close to 100 Mpc and into \( \Omega < 0.02 \) near the 1000 Mpc mark, if \( \tilde{\vartheta} \) is to become negative there. In this case, the accelerated patch extends from 50 Mpc to 1000 Mpc, with \( \tilde{\vartheta} \) varying within (-0.2, 0). So, even with the last term of Eq. (6) accounted for (and in an unfavourable way), negative values for \( \tilde{\vartheta} \) are still possible. Conventional almost-FRW kinematics can accommodate accelerated expansion.

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Footnote 4: Using the approximate relation \( \vartheta = \tilde{D}_{v}\vartheta_{\alpha} \approx \partial v_{\alpha}/\partial r_{\alpha} \sim 3v/r \), where \( v \) is the magnitude of the bulk velocity and \( r \) the size of the associated region. Then, \( \vartheta/\Theta \sim v/Hr \), with \( \Theta = 3H \) and \( H \) representing the Hubble parameter.
4. Discussion

A decade ago, observations of high-redshift supernovae indicated that our universe was expanding at an accelerating pace [17, 18]. This conclusion was reached after applying the luminosity distances of the supernovae to the distance-redshift relation,

$$D_L = (1 + z) H_0^{-1} \int_0^z e^{-\int_0^x (1+q) dx} dx,$$

(12)

of an (exact) FRW model. The results have repeatedly given negative values to \(q\), indicating an accelerated expansion for our universe. In particular, the deceleration parameter was estimated close to -0.5. The same measurements also suggested that the accelerated phase was a relatively recent event, putting the transition from deceleration to acceleration around \(z = 0.5\) [12].

Explaining the supernovae results has since dominated almost every aspect of contemporary cosmology. Dark energy, an unknown and elusive form of matter with negative gravitational mass, has so far been the most popular answer. Other suggestions include modifying General Relativity, introducing extra dimensions, or abandoning the Friedmann models altogether.

The reason behind such drastic proposals was that negative values for the deceleration parameter seemed impossible in conventional FRW (as well as in perturbed, almost-FRW) cosmologies. Here, however, we have shown that this is not the case. Accelerated expansion is theoretically possible in linearly perturbed FRW universes. Peculiar motions can locally mimic the kinematic effects of dark energy. Observers moving relative to the smooth Hubble flow can have local expansion rates appreciably different than that of the actual universe. Theoretically speaking, this reflects the fact that the Raychaudhuri equations in the two coordinate systems (that of the CMB and that of a drifting observer) are not the same. The difference is due to a 4-acceleration term, which vanishes in the CMB frame but takes nonzero values in any other relatively moving reference system. As a result, weakly accelerated expansion (i.e. with \(-1 < \tilde{q} < 0\) – see § 2.2) is possible even when the drift velocities are small and matter is simple pressure-free dust, namely within the limits of the linear (almost-FRW) approximation. The effect is local, though the affected scales can be large enough to give the (false) impression that the whole universe has recently moved into an accelerated phase.

Extrapolating our drift velocity relative to the CMB frame, we found that peculiarly moving observers can measure negative deceleration parameter on scales between (roughly) 50 and 1000 Mpc, with \(\tilde{q}\) varying in the range (-0.4, 0). Based on the surveys of [4, 5] and particularly those of [1–3], the deceleration parameter was confined within (-0.3, 0). These estimates are qualitatively in the right direction. Quantitatively, also, are not far from those inferred by the supernovae data [12, 13]. Finally, we should point out that our analysis has been based on the average peculiar kinematics without incorporating anisotropies. For instance, the symmetry of region \(A\) and the observers position in it can induce anisotropy in the spatial distribution of \(\tilde{q}\). Generally, the higher the spherical symmetry of \(A\) and the closer the observer at the centre the better. These matters are less of an issue, however, when \(A\) is considerably larger than \(B\), namely as long as patch \(B\) lies well within region \(A\). The direction of the peculiar motion can also introduce an anisotropy in the \(\tilde{q}\)-distribution. This effect is maximised when the peculiar velocity maintains the same magnitude and direction throughout the integration period (i.e. from \(z \simeq 0.5\) to the present – see expression (12)). In the opposite case, when the \(v_a\)-field has been sufficiently randomised, the anisotropy will be negligible. Estimating effects like these is currently impossible, however, as it requires detailed data on the distribution of peculiar velocities within regions of several hundred Mpc. Nevertheless, if peculiar velocities are responsible for the supernovae observations, their effect should be consistent with some degree of, most likely, dipolar anisotropy in the measured distribution of the deceleration

5 To include the (linear) effects of peculiar motions into Eq. (12), we should replace \(q\) with \(\tilde{q}\) by means of (7).
parameter. Interestingly, there have been recent reports in the literature suggesting that such kind of anisotropy may actually exist in the data [19].

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References
[1] Kashlinsky A, Atrio-Barandela F, Kocevski D and Ebeling H 2008 Astrophys. J. 686, L49
[2] Kashlinsky A, Atrio-Barandela F, Kocevski D and Ebeling H 2009 Astrophys. J. 691 1479
[3] Kashlinsky A, Atrio-Barandela F, Edeling H, Edge A and Kocevski D 2010 Astrophys. J. 712 L81
[4] Watkins R, Feldman H A and Hudson M J 2009 Mon. Not. R. Astron. Soc. 392 743
[5] Feldman H A, Watkins R and Hudson M J 2010 Mon. Not. R. Astron. Soc. 407 2328
[6] Tsagas C G 2010 Mon. Not. R. Astron. Soc. 405 503
[7] Ellis G F R and van Elst H 1999 in Theoretical and Observational Cosmology Ed. M. Lachièze-Ray (Kluwer, Dordrecht) 1
[8] Tsagas C G, Challinor C and Maartens R 2008 Phys. Rep. 465 61
[9] King A R and Ellis G F R 1973 Commun. Math. Phys. 31 209
[10] Maartens R 1998 Phys. Rev. D 58, 124006
[11] Ellis G F R and Tsagas C G 2002 Phys. Rev. D 66 124015
[12] Turner M S and Riess A G 2002 Astrophys. J. 569 18
[13] Riess A G, et al 2004 Astrophys. J. 607 665
[14] Trauss M A and Willick J A 1995 Phys. Rep. 261 271
[15] Perivolaropoulos L 2008 preprint arXiv:0811.4684
[16] Li N and Schwarz D J 2008 Phys. Rev. D 78 083531
[17] Riess A G, et al 1998 Astrophys. J. 116 1009
[18] Perlmutter S, et al 1999 Astrophys. J. 517 565
[19] Cooke R and Lynden-Bell D 2010 Mon. Not. R. Astron. Soc. 401 1409